An Integrated Human-physical Framework for Control of Power Grids

S. Feng, M. Cucuzzella, T. Bouman, L. Steg and J. M. A. Scherpen

Abstract—In this paper, we bridge two disciplines: systems & control and environmental psychology. We develop second order Behavior and Personal norm (BP) based models (which are consistent with some studies on opinion dynamics) for describing and predicting human activities related to the final use of energy, where psychological variables, financial incentives and social interactions are considered. Based on these models, we develop a human-physical system (HPS) framework consisting of three layers: (i) human behavior, (ii) personal norms and (iii) the physical system (i.e., an AC power grid). Then, we formulate a social-physical welfare optimization problem and solve it by designing a primal-dual controller, which generates the optimal incentives to humans and the control inputs to the power grid. Finally, we assess in simulation the proposed models and approaches.

I. INTRODUCTION

Individuals’ behavior is critical for the functioning of energy systems. Accordingly, understanding the drivers behind this behavior is therefore key for the modeling and optimization of energy systems. For example, knowledge on such drivers could be employed to better understand individuals’ energy behavior that affects the energy system’s functioning, and to promote the behavior that makes the energy system function more optimally, possibly enhancing the effectiveness of technical solutions

This paper presents interdisciplinary work integrating systems & control and environmental psychology. We first develop feasible mathematical models of dynamical human behavior. Then, to model the impact of dynamical human behavior on a power grid, to further model the effects of incentives on energy use behavior, and to obtain the control inputs to the power grid as well as the incentives to humans, we develop a human-physical system (HPS) framework. Specifically, the HPS includes three layers: a behavior layer and a personal norm layer which describe social human activities, and a physical layer that describes the dynamics of an AC power grid.

A. Energy saving behavior in environmental psychology

The dynamical human behavior models proposed in this paper are inspired by and consistent with findings in psychology. Various studies have examined which factors affect energy use behavior

When focusing on energy saving behavior, which is central in this paper, two values appear particularly relevant: egoistic values that reflect a concern with self interest, status and possessions, and biospheric values that reflect a concern to protect and care about nature and the environment

B. Human-physical system (HPS) framework

In view of the last subsection, this paper develops human activity models of energy saving incorporating 1) behavior that depends on financial incentives; 2) personal norms that are influenced by egoistic and biospheric values, and social norms. The interactions between these items are shown in Figure 1. Item 1) is considered to be the behavior layer whose response is generally quick, while item 2) is considered to be the personal norm layer whose response is generally slower than the one of the behavior layer. Specifically, energy saving behavior is influenced by the underlying personal norms and financial incentives. From a control perspective, the...
incentives can be considered as “control inputs” (i.e., behavior interventions) to the behavior dynamics, while personal norms can be considered as references for the behavior in absence of incentives. From an opinion dynamics viewpoint, personal norms can be considered as the opinions of the individuals in a social network. It is worth mentioning that our models of describing human activities in energy saving behavior are partially inspired by and consistent with studies in opinion dynamics (see for instance the continuous-time Friedkin-Johnsen and high-order opinion dynamics models [13]–[18]).

For the physical layer, we consider an AC power grid where people (prosumers) share the task of current generation (current sharing) with peers in their local electricity network according to their generation capacities, regulating the voltage within permitted (safe) limits around the desired value (voltage sharing) with peers in their local electricity network. Depending on prosumers’ motives. We will clarify in the next section why people (prosumers) share the task of current generation and voltage regulation are vital control objectives for preventing energy oversteering and guaranteeing stability, respectively [19]–[22]. Our work is inspired by the recent paper [23], which studies a social-physical welfare optimization problem depending on prosumers’ motives. We will clarify in the next subsection the contributions of our work and the differences with respect to [23].

C. Contributions

We formulate a convex social-physical welfare optimization problem, whose solution corresponds to “control inputs” to both humans (i.e., behavior interventions such as financial incentives) and to the power grid. Specifically, we aim at i) maximizing the social welfare by satisfying the prosumers’ load demand and minimizing the incentives’ cost, and ii) maximizing the physical welfare by performing current sharing and voltage regulation. To achieve these goals, we design a dynamic controller, whose enforced dynamics represent the primal-dual dynamics of the considered optimization problem [23]–[26]. The contributions of this paper are four-fold:

- We bridge systems & control and environmental psychology, by developing a novel HPS framework to study and analyze energy saving dynamical behavior of humans taking into account the dynamics of the physical infrastructures of a power grid. To the best of our knowledge, such a framework is completely novel and has never been developed or studied from a control perspective. More precisely, we develop second-order models describing Behavior and Personal norm (BP) dynamics, which include intrinsic (i.e., values and personal norms) and extrinsic drivers of energy saving behavior, allowing to model their slow and fast transient processes, respectively. These are significant improvements with respect to [23], where the prosumers are supposed to have non-dynamical behavior and hence no dynamic models describing human activities are developed.

- Inspired by the findings in environmental psychology, this paper considers a scenario where values, incentives and social influence affect prosumers’ energy saving behavior instead of letting automation directly adjust prosumers’ load, as supposed in [23]. Clearly, this more realistic scenario where the prosumers’ load demand depends on dynamical human behavior makes the theoretical analysis different and more complex than the one in [23].

- We consider an AC power grid as physical layer, whose dynamics are more complex than (and include) the DC counterpart studied in [23].

- Differently from [23], where the controller generates inputs only to the power grid, the proposed HPS framework and primal-dual controller provide also optimal and socially acceptable incentives to humans, affecting their energy saving behavior.

Outline. This paper is organized as follows. In Section II, we develop the overall HPS framework including the power grid and Behavior and Personal norm models for human activities. In Section III we present the control objectives and in Section IV we design the primal-dual controller and analyze the closed-loop stability. A numerical example is presented in Section V, and finally Section VI ends the paper with conclusions and future research.

Notation. We denote by $\mathbb{R}$ the set of reals. Given $y \in \mathbb{R}$, $\mathbb{R}_{\geq y}$ denotes the set of reals no smaller than $y$. For any $w \in \mathbb{Z}$, we denote $\mathbb{Z}_{\geq w} := \{w, w+1, \cdots \}$. Let $0$ and $1$ denote column vectors of appropriate dimensions, having all 0 and 1 elements, respectively. Let $I$ denote the identity matrix with appropriate dimension. Given a vector $v$, let $\|v\|_2$ denote its $\ell_2$ norm. We let $\mathcal{N}$ denote the set of $N \in \mathbb{Z}_{\geq 2}$ prosumers and $\mathcal{E}$ denotes the set of $E \in \mathbb{Z}_{\geq 1}$ transmission lines interconnecting the prosumers. Moreover let $\mathcal{N}_i \subseteq \mathcal{N}$ denote the set of prosumers physically interconnected with prosumer $i$ in the power grid, and $\mathcal{E}_i \subseteq \mathcal{E}$ denote the set of the transmission lines connected to prosumer $i$. Let $\mathcal{S}_i \subseteq \mathcal{N}$ denote the set of the social neighbors of prosumer $i$.

II. Human-Physical System (HPS) Framework

For the readers’ convenience, before introducing the overall HPS framework, we will first introduce the AC power grid model and then the Behavior and Personal norm (BP) models.

A. AC microgrid model

In this paper, we consider a low-voltage AC power grid composed of $N$ prosumers that are connected by $E$ resistive-inductive transmission lines. From a physical point of view, we assume that every prosumer can be represented by a distributed generation unit including a Voltage Sourced Converter (VSC) and a load. Moreover, we recall that in low-voltage grids the lines are predominately resistive, leading to a strong coupling.
between the active power flows and the voltage magnitude [27], [28]. Also, we assume that the frequency is controlled in open-loop by equipping each VSC with an internal oscillator that provides the phase angle \( \delta(t) = \int_0^t \omega_0 dt \), with \( \omega_0 = 2\pi f_0 \) and \( f_0 \) denoting the nominal frequency of the grid. Then, provided that the power grid is balanced and symmetric, and all clocks of the internal oscillators are synchronized, we apply Clarke’s and Park’s transformation to obtain the system dynamics in the rotating dq-frame [29].

Given the notation in Table I the dynamics of the physical system at prosumer \( i \) can be expressed as in [30], i.e.,

\[
C_{ti} \dot{V}_{di} = \omega_0 C_{ti} V_{qi} + I_{dzi} + \sum_{k \in E_i} I_{dk} - \frac{V_{di}}{R_{Li}} - I_{Ldi} z_{li} \quad (1a)
\]

\[
C_{ti} \dot{V}_{qi} = -\omega_0 C_{ti} V_{di} + I_{tiq} + \sum_{k \in E_i} I_{qk} - \frac{V_{qi}}{R_{Li}} - I_{Lqi} z_{li} \quad (1b)
\]

\[
L_{ti} \dot{I}_{dzi} = -V_{di} - R_{ti} I_{dzi} + \omega_0 L_{ti} I_{tiq} + u_{di} \quad (1c)
\]

\[
L_{ti} \dot{I}_{tiq} = -V_{qi} - R_{ti} I_{tiq} - \omega_0 L_{ti} I_{dzi} + u_{qi} \quad (1d)
\]

where the \( d \) and \( q \) subscript represent the direct and quadrature component, respectively. For instance, \( V_d \) and \( V_q \) are the \( d \) and \( q \) components of voltage, respectively. The resistance \( R_{Li} \) can represent the base impedance load of prosumer \( i \) or simply the system damping, while \( I_{Lgi} z_{li} \) (\( g = d, q \)) represents the dynamic current load. In particular, \( I_{dzi} \) and \( I_{tiq} \) are constants that represent the load demand of prosumer \( i \), while \( z_{li} : \mathbb{R}_{\geq 0} \rightarrow [0, 1] \) is a dynamical variable depending on the behavior of prosumer \( i \). For example, if \( z_{li} = 0 \) or \( z_{li} = 1 \) for all \( t \), then the actual load of prosumer \( i \) is 0 or \( I_{Lqi}, \) respectively. The dynamics of the behavior \( z_{li} \) will be further explained in the next subsection.

In (1), \( I_{dk} \) and \( I_{qk} \) denote the current exchanged between prostomers \( i \) and \( j \) \( \in N_i \) through the line \( k \in E_i \). The ends of the transmission line connecting prostomers \( i \) and \( j \) are arbitrarily labeled by “+” and “-“. Then, the incidence matrix \( B \) for the “labeled” graph is given as \( B_{i,k} = +1 \) if prosumer \( i \) is the positive end of the labeled transmission line \( k \in E_i \), \( B_{i,k} = -1 \) if prosumer \( i \) is the negative end of the labeled transmission line \( k \in E_i \), otherwise, \( B_{i,k} = 0 \). Suppose that prosumer \( i \) is the positive end of the transmission line \( k \), then, the dynamics of \( I_{dk} \) and \( I_{qk} \) are given by

\[
L_k \dot{I}_{dk} = V_{di} - V_{dj} - R_k I_{dk} + \omega_0 L_k I_{qk} \quad (2a)
\]

\[
L_k \dot{I}_{qk} = V_{qi} - V_{qj} - R_k I_{qk} - \omega_0 L_k I_{dk}. \quad (2b)
\]

### B. Behavior and Personal norm (BP) models

In this subsection, we focus on the modeling of prostomers’ dynamics describing the consumption of energy. We will consider case i) without social influence and case ii) with social influence. The interactions between the power grid, control scheme, behavior, personal norms, social influence, values and incentives are shown in Figure 1. To facilitate the presentation of our models, we first provide the definitions of personal values, egoistic and biospheric values, personal norms and financial incentives:

- **Personal values** reflect general and desirable life goals which are used as guiding principles to evaluate actions and situations on. Research has identified a set of universal values, meaning every individual endorses these values to some extent. However, individuals differ in how strongly they endorse each value. The more an individual endorses and prioritizes a value, the more influential this value is for someone’s preferences and actions [31]–[35]. In case of energy saving behavior, two values appear of particular relevance:

  - **Egoistic values** concern goals to acquire possessions, money and status. Individuals with stronger egoistic values typically have weaker personal norms, which make them less likely to engage in pro-environmental actions such as energy saving behavior. Yet, energy saving behavior can also be associated with cost reductions, which may motivate individuals with stronger egoistic values to engage in energy saving behavior [10], [36].

  - **Biospheric values** reflect goals to care about nature and the environment. Energy savings have clear environmental benefits, which is why stronger endorsement of biospheric values is typically indicative of stronger personal norms, and thereby stronger engagement in energy saving behavior [36].

- **Personal norms** reflect a feeling of personal responsibility and feelings of moral obligation to take an action [37], e.g., the stronger an individual’s personal norm to save energy, the more likely this individual is to engage in energy saving behavior (personal norms → behavior). In the context of this paper, when someone strongly endorses biospheric and/or egoistic values, this individual is likely to experience a personal norm to take the corresponding actions (values → personal norms) [38]. Then, for instance, individuals who strongly endorse biospheric values typically feel a stronger personal norm to take actions to save energy, eventually increasing the likelihood they will engage in energy saving behavior (values → personal norms → behavior).
Financial incentives include subsidies and financial rewards (e.g., lower energy bill) that can promote energy savings by making it more attractive. The impact of financial incentives is likely more pronounced for individuals with stronger egoistic values, as such individuals care relatively much about money and possessions [10, 50].

Case i) In view of (1), it is clear that the dynamic current loads depend on the dynamics of prosumers’ behavior. Specifically, the BP dynamics of prosumer $i$ is represented by the following second order system

$$
\begin{align}
\dot{z}_{li} &= a_i(p_i - z_{li} - h_i s_i) \quad (3a) \\
\ddot{p}_i &= c_i(p_i^{ego} - p_i) + d_i(p_i^{bio} - p_i), \quad (3b)
\end{align}
$$

where $z_{li}$ and $p_i$ are the states for describing human activities (i.e., behavior and personal norms, respectively) and $s_i$ is the “control input” representing incentives. The rationale behind this model becomes clear below. For a given $s_i$, the steady-state solution of system (3) satisfies

$$
\begin{align}
\dot{z}_{li} &= p_i - h_i s_i \quad (4a) \\
\ddot{p}_i &= (c_i p_i^{ego} + d_i p_i^{bio})/(c_i + d_i). \quad (4b)
\end{align}
$$

It is clear that the compact form of (3) can be written as

$$
\begin{align}
\ddot{z}_l &= A(p - z_l - H s) \quad (5a) \\
\ddot{p} &= C(p^{ego} - p) + D(p^{bio} - p), \quad (5b)
\end{align}
$$

where $A, H, C$ and $D$ are diagonal matrices, e.g. $A = \text{diag}(a_1, ..., a_N)$. Detailed explanations about the variables and parameters of system (3) are given below.

Variables of prosumer $i$:

a) $z_{li}$: Degree of satisfaction of the load demand, with $0 \leq \dot{z}_{li} \leq 1$. It can be considered as the behavior of prosumer $i$ adjusting his/her own load $I_{L,i} z_{li}$ ($q = d, q$). The closer $z_{li}$ to 1, the more satisfied the load demand of prosumer $i$ is. This is a key variable that plays the role of interface between the grid (1) and the prosumer’s behavior (3).

b) $p_i$: Personal norms, with $0 \leq p_i \leq 1$. If we omit the “control input” $-h_i s_i$ in (3b) (see $h_i$ in item h) and $s_i$ in item c), $p_i$ can be considered as the tracking reference for $z_{li}$, i.e., if $h_i s_i = 0$, then $z_{li} \to p_i$ when time approaches infinity. This essentially describes the phenomenon for which the personal norms $p_i$ acts as a guide for the behavior $z_{li}$.

c) $s_i$: Financial incentives, with $0 \leq \bar{s}_i \leq \bar{p}_i / h_i$. $s_i$ represents the “control input” (i.e., behavior interventions) to prosumer $i$, influencing the behavior $z_{li}$ directly. Since $h_i$ is semi-positive (see item h) in the following), $s_i \geq 0$ represents financial incentives that can motivate people to save energy. Some examples of such incentives can be subsidies and lower energy bills. For a given $\bar{s}_i$ and $\bar{p}_i$, it follows from (4b) that a larger value of $\bar{s}_i$ leads to a smaller value of $\bar{z}_i$. For instance, this can describe the phenomenon for which higher subsidies generally lead to a larger energy saving of an individual, provided that his/her egoistic values are constant (see $h_i$ in item h).

From (3a) and (3b), one can see that personal norms as well as incentives together influence one’s energy saving behavior. Moreover, the inequality $\ddot{z}_i \leq \ddot{p}_i / h_i$ prevents that at the steady state the incentives are unreasonably high.

d) $p_i^{ego}$: Egoistic values, with $0 \leq p_i^{ego} \leq 1$. The influence of egoistic values on an individual is two-fold. First, egoistic values are negatively related to personal norms to save energy as pro-environmental behavior are often perceived as obstructing egoistic goals. The stronger the egoistic values are, the closer $p_i^{ego}$ to 1 is, leading to a large value of $\ddot{z}_i$ (see (4a)), i.e., more energy consumption. Second, egoistic values may also be positively related to energy saving behavior when energy savings are associated with financial benefits. Specifically, financial benefits (e.g., cost reductions) may promote energy saving behavior, and this effect will likely be larger for individuals with stronger egoistic values, see item h) for the modeling of its positive correlation to energy saving behavior.

e) $p_i^{bio}$. Biospheric values, with $0 \leq p_i^{bio} \leq 1$. The stronger the biospheric values are, the closer $p_i^{bio}$ to 0 is, leading to less energy consumption.

We assume $p_i^{ego}$ and $p_i^{bio}$ constant over the considered time windows because they change very slowly over time compared with the much faster dynamics of systems (1–3).

Parameters of prosumer $i$:

f) $a_i > 0$ indicates the time constant of the behavior dynamics $z_{li}$. One can observe that a larger value of $a_i$ implies a faster response of $z_{li}$ to $p_i$ and $s_i$ variations. It is also clear that $a_i$ does not influence the steady state of $z_{li}$.

g) $0 \leq h_i \leq 1$, with $h_i = p_i^{ego}$, indicates the degree of influence of the incentives $s_i$ on the energy saving behavior $z_{li}$. Clearly, strong egoistic values ($p_i^{ego} \to 1$) imply larger values of $h_i$ ($h_i \to 1$). Moreover, one can observe from (4b) that large values of $h_i$ indicate that incentives can have strong influence on motivating prosumer $i$ to save energy. Indeed, an individual with strong egoistic values does care about financial benefits, and hence is motivated by financial incentives to save energy.

h) $0 \leq h_i \leq 1$, with $h_i = p_i^{ego}$, indicates the degree of influence of the incentives $s_i$ on the energy saving behavior $z_{li}$. Clearly, strong egoistic values ($p_i^{ego} \to 1$) imply larger values of $h_i$ ($h_i \to 1$). Moreover, one can observe from (4b) that large values of $h_i$ indicate that incentives can have strong influence on motivating prosumer $i$ to save energy. Indeed, an individual with strong egoistic values does care about financial benefits, and hence is motivated by financial incentives to save energy.

i) $c_i \geq 0$ and $d_i \geq 0$ such that $c_i + d_i > 0$ represent the weights of egoistic and biospheric values, respectively, indicating the degree to which each of these values influences the personal norms of prosumer $i$. From (4b), one can observe that a relatively large value of $c_i$ implies $\ddot{z}_i \to p_i^{ego}$, i.e., prosumer $i$ strongly endorses the egoistic values $p_i^{ego}$. On the other hand, a relatively large value of $d_i$ implies $\ddot{z}_i \to p_i^{bio}$.

Case ii) Differently from case i), now we consider also the influence of social norms (i.e., perceptions that others save energy or expect you to save energy), which can motivate people to engage in sustainable energy-saving behavior. Then, the dynamics in (3b) become

$$
\ddot{p}_i = \sum_{j \in S_i} b_{ij} (p_j - p_i) + c_i (p_i^{ego} - p_i) + d_i (p_i^{bio} - p_i), \quad (6)
$$

where $p_j$ with $j \in S_i$ denotes the personal norms of the social neighbors of prosumer $i$, and $b_{ij} \in \mathbb{R}_{\geq 0}$ represents the weight.
of the social influence. It is clear that larger values of $b_{ij}$ imply that $\bar{p}_i$ is closer to $\bar{p}_j$.

Then, the compact model for the case with social influence is given by

$$\dot{z}_t = A(p - z_t - Hs) \quad (7a)$$
$$\dot{p} = C(p^{bio} - p) + D(p^{bio} - p) - Lp. \quad (7b)$$

where $L$ is the Laplacian matrix associated with the social network. Note that the social network topology is not necessarily identical to the physical topology of the power grid. One can observe that a relatively large $L$ with respect to $C$ and $D$ corresponds to prosumers that are likely to achieve personal norm consensus to some degree. In the following sections, we will mainly focus on case i). Then, we briefly extend the results to case ii).

According to some studies in environmental psychology [10], [36], our models include the possibility of promoting energy saving behavior in populations who more strongly endorse egoistic values by financial incentives ($s_i$, directly affects the behavior layer (3)) in the short term ($a_i$ is generally larger than $b_{ij}$, $c_i$ and $d_i$). However, in the long term, incentives may be financially unsustainable and no longer effective when removed, implying that influencing individuals’ personal norms (e.g. by strengthening biospheric values) may be more beneficial and effective.

Remark 1: The proposed models are partially inspired by and consistent with the studies on opinion dynamics. First, they are partially consistent with some opinion dynamics models in which the “topics” (state in models of one agent) are possibly more than one and logically correlated [15]. Indeed, the proposed models have two correlated topics, i.e., $z_t$ and $p$, with the topic $p$ affecting the topic $z_t$. Second, the dynamics of topic $p$ are also partially consistent with the continuous-time Friedkin-Johnsen model [14], where an individual is stubborn about his/her opinion $p_t$ (depending on $p^{geo}$ and $p^{bio}$) and is also influenced by others’ opinions.

C. Human-physical system (HPS)

In view of the dynamics of the physical AC grid in (1) and (2), and prosumers’ behavior and personal norms in (3), the overall human-physical system (HPS) considering case i) is compactly written as

$$\dot{C}l\dot{V}_d = -R_L\dot{V}_d + \omega_0C_lV_q + I_{rd} + B\dot{I}_d - L_{ld}\dot{z}_l \quad (8a)$$
$$\dot{C}l\dot{V}_q = -\omega_0C_lV_d - R_L\dot{V}_q + I_{rq} + B\dot{I}_q - L_{qz}\dot{z}_l \quad (8b)$$
$$L_{ld}\dot{I}_{td} = -V_d - R_tI_{td} + \omega_0L_tI_{tq} + u_d \quad (8c)$$
$$L_{ld}\dot{I}_{tq} = -V_q - \omega_0L_tI_{td} - R_tI_{tq} + u_q \quad (8d)$$
$$\dot{L}_d = -B\dot{V}_d - R\dot{I}_d + \omega_0\dot{L}_q \quad (8e)$$
$$\dot{L}_q = -B\dot{V}_q - \omega_0\dot{L}_d - R\dot{I}_q \quad (8f)$$
$$\dot{z}_l = A(p - z_t - Hs) \quad (8g)$$
$$\dot{p} = C(p^{geo} - p) + D(p^{bio} - p). \quad (8h)$$

where vectors and matrices have appropriate dimensions. Specifically, (8a)-(8f) and (8g), (8h) represent physical and human dynamics, respectively, where the prosumers interact with the power grid by changing their energy saving behavior ($z_l$). Moreover, $u_d$ and $u_q$ represent the inputs to control the power grid, while $s$ represents the “control input” (i.e., behavior interventions) to influence the behavior of the prosumers. Similarly, one can obtain the corresponding HPS considering case ii) by replacing (8g) and (8h) with (7a) and (7b), respectively. In the following sections, we will mainly focus on the HPS in (8), i.e., case i). Then, we briefly extend the results to case ii).

III. OPTIMIZATION PROBLEM

In this section we present the control objectives of this work and formulate a social-physical welfare optimization problem.

A. Physical welfare

It is convenient in practice to decouple the control of the active power $P_t = \frac{1}{2}(V_{ai}I_{di} + V_{qi}I_{qi})$ from the control of the reactive power $Q_t = \frac{1}{2}(V_{ai}I_{di} - V_{qi}I_{qi})$. One possible way to do this is to regulate the $q$-component of the voltage to zero. Specifically, it is desirable to achieve

$$\lim_{t \to \infty} V_{qi}(t) = 0, \quad i = 1, 2, \ldots, N. \quad (9)$$

It is then evident that the active power $P_t$ and reactive power $Q_t$ depend on the $d$-component of the voltage $V_{di}$, and the currents $I_{di}$ and $I_{qi}$, respectively.

Given any constant $\bar{u}_d, \bar{u}_q$ and $\bar{s}_t$, and considering (9), the steady-state of the power grid (8a)-(8f) satisfies

$$\dot{V}_d = -R_t\dot{I}_{td} + \omega_0L_t\dot{I}_{tq} + \bar{u}_d \quad (10a)$$
$$0 = -\omega_0L_t\dot{I}_{td} - R_t\dot{I}_{tq} + \bar{u}_q \quad (10b)$$
$$\dot{I}_{td} = I_{Ld}\dot{z}_l - \omega_0C_l\dot{I}_{td} + K\dot{V}_d \quad (10c)$$
$$\dot{I}_{tq} = I_{Lq}\dot{z}_l - \omega_0C_l\dot{I}_{tq} \quad (10d)$$

where we use $\dot{I}_d = (-R - \omega_0L^-1L)\dot{V}_d = J\dot{V}_d$ and $\dot{I}_q = -\omega_0L^-1L\dot{I}_d = K\dot{V}_d$, with $J := (-R - \omega_0L^-1L)^{-1}B$ and $K := -\omega_0R^{-1}LJ$. We also notice that the steady-state of the power grid depends on $\bar{s}_t$. Then, given any constant input $\bar{s}_t$, the steady-state of the of human behavior (8g), (8h) satisfies

$$\bar{z}_l = \bar{p} - H\bar{s} \quad (11a)$$
$$\bar{p} = (C + D)^{-1}(Cp^{geo} + Dp^{bio}). \quad (11b)$$

From (10) and (11), one can verify that given ($\bar{u}_d, \bar{u}_q, \bar{s}_t$), the forced equilibrium of the HPS (8) exists and is unique. Also, in view of $\dot{I}_{td}$ in (10c), one has

$$\sum_{i=1}^{N}(I_{Ldi}\dot{z}_l + \dot{V}_{di}/R_{Ldi}) = I^T\dot{I}_{td}, \quad (12)$$

which implies that the total (active) current load is equal to the total (active) generated current, leading to the formulation of the current sharing objective. Indeed, to avoid the over stressing of one or more energy sources, it is desirable in practice that the overall current generation is shared among all the prosumers proportionally to their generation capacities. This desire is equivalent to achieving $\pi_{ci} \dot{I}_{idi} = \pi_{cj} \dot{I}_{idi}, i, j \in N$, where $\pi_{ci} \in R_{>0}$ and $\pi_{cj} \in R_{>0}$ are constant weights depending on the generation capacity of prosumers $i$ and $j$,
respectively. For example, a relatively large $\pi_{ci}$ corresponds to a relatively small generation capacity. Therefore, to achieve this goal, in analogy with \cite{23}, we assign to every prosumer a strictly convex quadratic ‘cost’ function depending on the generated current $I_{idi}$. Then, the overall cost can be expressed as

$$C(I_{id}) = \sum_{i=1}^{N} \frac{1}{2} \pi_{ci} I_{idi}^2.$$  \hfill (13)

Note that the current sharing objective may also be interpreted as an action to pursue “fairness” in sharing tasks among all the prosumers, which can potentially enhance the energy efficiency by cooperating with each other.

Now, noticing that (3) implies that the voltage magnitude $|V_i| := (V_{di}^2 + V_{qi}^2)^{1/2}$ is determined only by $V_{di}$, we consider the regulation of $V_{di}$. Let $V_r = [V_{r1}, \ldots, V_{rN}]^T \in \mathbb{R}_{>0}^N$ be the vector of voltage references for $V_{di}$. Ideally, we would like to achieve exact voltage regulation, i.e.,

$$\lim_{t \to \infty} V_{di}(t) = V_{ri}, \quad i = 1, 2, \ldots, N.$$  \hfill (14)

However, recalling that in low-voltage grids there is a strong coupling between the active power flows and the voltage magnitude \cite{27, 23}, in order to achieve current sharing, deviations from the voltage references are unavoidable. Then, to regulate the voltages sufficiently close to the corresponding references, we aim at minimizing the voltage error, considering the strictly convex quadratic ‘cost’ function $\|V_d - V_r\|^2$.

**B. Social welfare**

In this subsection, we formulate the social (and economic) welfare problem within the considered power grid.

First, let $I_{Li} := (I_{Li,L}^2 + I_{Li,q}^2)^{1/2}$. Then, we assign to every prosumer a strictly concave quadratic ‘utility’ function $U_i(z_{li})$ depending on the energy saving behavior of prosumer $i$. Then, the overall utility can be expressed as

$$U(z_i) = \sum_{i \in N} \frac{1}{2} \pi_{ui}(I_{Li}(1 - z_{li}))^2,$$  \hfill (15)

where the parameter $\pi_{ui} \in \mathbb{R}_{>0}$ weights the satisfaction of the load demand of prosumer $i$. For example, a relatively large $\pi_{ui}$ corresponds to a relatively large request of comfort from prosumer $i$. By minimizing $-U(z_i)$, we aim at satisfying prosumers’ load demands as much as possible by making $z_{li}$ close to 1 (i.e., making $1 - z_{li}$ close to 0).

We also would like to minimize the amount of incentives $s$, which we assume to be proportional to the square of the incentives themselves. Thus, we aim at minimizing the cost function $\|s\|^2$.

**C. Social-physical welfare**

Considering the physical and social welfare introduced in Subsections \underline{IIIA} and \underline{IIIB} respectively, we now formulate the overall social-physical welfare optimization problem. Let the optimization variables be denoted by the superscript * and let $\tilde{x}_c := [z_{li}^T I_{idi}^T I_{iq}^T u_{id}^* u_{id}^T V_d^* V_d^T s^*] \in \mathbb{R}^{7N}$ be the vector of the optimization variables. Then, consider the following convex minimization problem:

$$\min_{\tilde{x}_c} \quad F(z_{li}^*, I_{idi}^*, u_{id}^*, u_{iq}^*, V_d^*, s^*)$$  \hfill (16a)

s.t. $0 = I_{Li} z_{li}^* - I_{li} - (-R_L^{-1} + BJ)V_d^*$ \hfill (16b)

$0 = I_{Li} z_{li}^* - I_{li} - C_i + BK)V_d^*$ \hfill (16c)

$0 = V_d^* + R_i I_{iq}^* - \omega_0 I_{iq}^* - \bar{u}_d^*$ \hfill (16d)

$0 = -\omega_0 I_{li} I_{iq}^* - R_i I_{iq}^* + \bar{u}_q^*$ \hfill (16e)

$0 = \bar{p} - z_{li}^* - H s^*$, \hfill (16f)

where $\bar{p}$ is given by (11b), and

$$F := \frac{1}{2} \sum_{i \in N} \pi_{ui} I_{Li}^2 (1 - z_{li}^*)^2 + \frac{1}{2} \beta \sum_{i \in N} \pi_{ci} (I_{idi}^*)^2$$

$$+ \frac{1}{2} \pi_{ui} I_{li}^2 - \|u_{id}^*\|^2 + \frac{\gamma}{2} \|u_{iq}^*\|^2 + \frac{\delta}{2} \|V_d^* - V_r\|^2 + \frac{\epsilon}{2} \|s^*\|^2,$$  \hfill (17)

where $\alpha, \beta, \gamma, \delta, \epsilon$ and $\eta$ are positive constants that can be chosen to prioritize one objective over another. We also note that the equality constraint (16d) implies $V_q^* = 0$, satisfying \cite{9}, and in (17) the terms depending on $u_{id}^*$ and $u_{iq}^*$ concern the minimization of the control efforts. Furthermore, we note that in addition to the equality constraints (16b)–(16f), inequality constraints (see e.g. [24]) may also be considered for instance to guarantee that voltages converge within a band allowing for a safe and proper functioning of the prosumers’ appliances or to avoid too high incentives that are financially unsustainable. However, for the sake of exposition and due to the page limitation, we will not include inequality constraints in the following analysis.

**IV. CONTROLLER AND STABILITY ANALYSIS**

In this section, we will present the design of a primal-dual controller to solve the optimization problem (16).

**A. Design of a primal-dual controller**

Let $\lambda := [\lambda_i^T \lambda_q^T \lambda_l^T \lambda_{ld}^T \lambda_{iq}^T]^T \in \mathbb{R}^{5N}$ denote the vector of the Lagrange multipliers corresponding to the constraints in (16b)–(16f). Moreover, let $x_c := [\tilde{x}_c^T \lambda_i^T]^T \in \mathbb{R}^{12N}$ be the state vector of the primal-dual controller we will design in this subsection. Then, the Lagrangian function corresponding to the optimization problem (16) is the following

$$l(x_c) := F(z_{li}^*, I_{idi}^*, u_{id}^*, u_{id}^* V_d^*, s^*)$$

$$+ \lambda_i^T (I_{Li} z_{li}^* - I_{li} - (-R_L^{-1} + BJ)V_d^*)$$

$$+ \lambda_q^T (I_{Li} z_{li}^* - I_{li} - C_i + BK)V_d^*$$

$$+ \lambda_l^T (V_d^* + R_i I_{iq}^* - \omega_0 I_{iq}^* - \bar{u}_d^*)$$

$$+ \lambda_{ld}^T (-\omega_0 I_{li} I_{iq}^* - R_i I_{iq}^* + \bar{u}_q^*)$$

$$+ \lambda_{iq}^T (\bar{p} - z_{li}^* - H s^*).$$  \hfill (18)
Thus, the first order optimality conditions for the optimization problem (16) are given by the Karush-Kuhn-Tucker (KKT) conditions, i.e.,

\[
\begin{align*}
0 &= -\alpha \Pi_u I_L^T (1 - z^*_t) + I_{Ld} \lambda_a + I_{Lq} \lambda_b + \lambda_c, \\
0 &= \beta \Pi_u I_{Ld} - \lambda_a + R_1 \lambda_c - \omega_0 L_d \lambda_d, \\
0 &= -\lambda_b - \omega_0 L_q \lambda_c - R_1 \lambda_d, \\
0 &= \gamma a_d - \lambda_c, \\
0 &= \delta s^* + a_d, \\
0 &= \epsilon (V_d^* - V_r) - (-R^*_L + BJ)^T \lambda_a + (\omega_0 C_t - BK)^T \lambda_b + \lambda_c, \\
0 &= \eta s^* - H \lambda_c, \\
0 &= I_{Ld} z_t^* - \bar{I}_d^* - (-R^*_L + BJ) \bar{V}_d^*, \\
0 &= I_{Lq} z_t^* - \bar{I}_q^* + (\omega_0 C_t - BK) \bar{V}_d^*, \\
0 &= \bar{V}_d^* + R_1 \bar{I}_d^* - \omega_0 L_d \bar{I}_q^* - \bar{u}_d^*, \\
0 &= -\omega_0 L_d \bar{I}_d^* - R_1 \bar{I}_q^* + \bar{u}_d^*, \\
0 &= \bar{p} - z_t^* - H \bar{s}^*,
\end{align*}
\]

with \( \Pi_u = \text{diag}(\pi_u, \ldots, \pi_u) \), \( \Pi_c = \text{diag}(\pi_c, \ldots, \pi_c) \), and \( I_{Ld}^T = \text{diag}(I_{Ld1}^T, \ldots, I_{LdN}^T) \). Since the optimization problem (16) is convex, then strong duality holds \( [39] \). Thus, \( z_t^*, \bar{I}_d^*, \bar{I}_q^*, \bar{u}_d^*, \bar{V}_d^*, s^* \) are optimal if and only if there exist \( \lambda_a, \lambda_b, \lambda_c, \lambda_d \) that satisfy (19).

Based on the KKT conditions in (19) and under the assumption that each controller can exchange information among its neighbors through a communication network with the same topology as the physical network, we design the following distributed control scheme by using the primal-dual dynamics of the optimization problem (16), i.e.,

\[
\begin{align*}
-\tau_s z_t^* &= -\alpha \Pi_u I_L^T (1 - z_t^*) + I_{Ld} \lambda_a + I_{Lq} \lambda_b + \lambda_c, \\
-\tau_d \bar{I}_d^* &= \beta \Pi_u I_{Ld} - \lambda_a + R_1 \lambda_c - \omega_0 L_d \lambda_d, \\
-\tau_q \bar{I}_q^* &= -\lambda_b - \omega_0 L_q \lambda_c - R_1 \lambda_d, \\
-\tau_u \bar{u}_d^* &= \gamma u_d + \lambda_a - pA, \\
-\tau_u \bar{u}_q^* &= \delta u_q + \lambda_c + pB, \\
-\tau_v V_d^* &= \epsilon (V_d^* - V_r) - (-R^*_L + BJ)^T \lambda_a + (\omega_0 C_t - BK)^T \lambda_b + \lambda_c, \\
-\tau_s s^* &= \eta s^* - H \lambda_c + pC, \\
\tau_a \lambda_a &= I_{Ld} z_t^* - \bar{I}_d^* - (-R^*_L + BJ) \bar{V}_d^*, \\
\tau_b \lambda_b &= I_{Lq} z_t^* - \bar{I}_q^* + (\omega_0 C_t - BK) \bar{V}_d^*, \\
\tau_c \lambda_c &= V_d^* + R_1 \bar{I}_d^* - \omega_0 L_d \bar{I}_q^* - \bar{u}_d^*, \\
\tau_e \lambda_e &= -\omega_0 L_d \bar{I}_d^* - R_1 \bar{I}_q^* + \bar{u}_d^*, \\
\tau_e \lambda_e &= \bar{p} - z_t^* - H \bar{s}^*,
\end{align*}
\]

where \( \tau_s, \tau_d, \tau_q, \tau_u, \tau_v, \tau_c, \tau_e, \tau_d, \tau_e \in \mathbb{R}^{N \times N} \) are positive diagonal matrices, which can be tuned to adjust the controller response. Moreover, the vectors \( p_A, p_B \) and \( p_C \) are the controller input ports, which will be used in the next subsection to interconnect the controller (20) with the HPS (8). We also note that since in the optimization problem (16) the objective function (17) is quadratic with respect to \( z_t^*, \bar{I}_d^*, \bar{I}_q^*, \bar{V}_d^*, s^* \) and the constraints (16b)–(16e) are linear, then, given constant \( \bar{p}_A, \bar{p}_B \) and \( \bar{p}_C \), it can be shown that the solution \( \bar{x}_e \) to (20) is unique.

### B. Stability analysis

In this subsection, we will show that the HPS (3) in closed loop with the primal-dual controller (20) is stable and converges to the solution of the optimization problem (16).

Let \( \bar{p} := \bar{p} - p = (C + D)^{-1}(C V^p + DP^p) - p \), with \( \bar{p} \) given by (11b). Then, the human dynamics (8g), (8h) can be rewritten as

\[
\begin{align*}
\dot{z}_t &= -A z_t + A(\bar{p} - \bar{p} - H z_t) \quad (21a) \\
\dot{\bar{p}} &= -(C + D) \bar{p} \quad (21b)
\end{align*}
\]

It is clear that \(-A \) and \(-(C + D) \) are Hurwitz. Thus, given any two diagonal and positive definite matrices \( Q_1 \) and \( Q_2 \), let \( P_1 \) and \( P_2 \) denote the corresponding unique solutions of the Lyapunov equations

\[
\begin{align*}
-A^T P_1 - P_1 A + Q_1 &= 0 \quad (22) \\
-(C + D)^T P_2 - P_2 (C + D) + Q_2 &= 0, \quad (23)
\end{align*}
\]

where \( P_1 \) and \( P_2 \) are positive definite matrices. Then, we interconnect the controller (20) with the HPS (8) by choosing

\[
\begin{align*}
u_d &= u_d, \quad u_q = u_q, \quad s = s^*,
\end{align*}
\]

and

\[
p_A &= I_{Ld}, \quad p_B = I_{Lq}, \quad p_C = -2H^T A^T P_1^T z_t. \quad (25)
\]

Let \( x_s := [V_d^T, V_q^T, I_{Ld}^T, I_{Lq}^T, I_{td}^T, I_{tq}^T, z_t^T, p^T]^T \in \mathbb{R}^{6N + 2E} \) denote the state of the HPS consisting of (8a)–(8f) and (21). Now we are ready to present the main result of this paper.

**Theorem 1:** The closed-loop system (3), (20), (24) and (25) converges to an equilibrium solving (16).

**Proof.** We take three steps to conduct the proof.

**Step 1.** In this step, we first propose the following storage function \( S_p \) for the HPS (8a)–(8f)

\[
S_p = \frac{1}{2} \left( V_d^T C_t V_d + V_q^T C_v V_q + \bar{I}_{td}^T L_{td} \bar{I}_{td} + \bar{I}_{tq}^T L_{tq} \bar{I}_{tq} \right)
\]

\[
+ \bar{I}_{td}^T L_{td} \bar{I}_{td} + \bar{I}_{tq}^T L_{tq} \bar{I}_{tq} \right),
\]

which satisfies

\[
\dot{S}_p = -\bar{V}_d^T R_{td}^{-1} V_d - \bar{V}_q^T R_{tq}^{-1} V_q - \bar{I}_{td}^T R_{td} \bar{I}_{td} - \bar{I}_{tq}^T R_{tq} \bar{I}_{tq}
\]

\[
- \bar{I}_{td}^T R_{td} - \bar{I}_{tq}^T R_{tq} - \bar{V}_d^T I_{Ld} z_t - \bar{V}_q^T I_{Lq} z_t
\]

\[
+ \bar{I}_{td}^T \bar{u}_d + \bar{I}_{tq}^T \bar{u}_q
\]

along the solutions to (8a)–(8f). Now, suppose without loss of generality that the loads absorb positive reactive power (i.e., the loads are predominantly inductive rather than capacitive), \( I_{Lq} \) is negative definite. Then, by virtue of the Young’s inequality \( [41] \), we have

\[
\begin{align*}
-\bar{V}_d^T I_{Ld} \bar{z}_t - \bar{V}_q^T I_{Lq} \bar{z}_t \leq \bar{V}_d^T \bar{I}_{td} \bar{V}_d + \bar{z}_t^T \bar{C}_d I_{Ld} \bar{z}_t
\]

\[
- \bar{V}_q^T \bar{I}_{tq} \bar{V}_q - \bar{z}_t^T \bar{C}_q I_{Lq} \bar{z}_t,
\]

with \( \bar{C}_d \) and \( \bar{C}_q \) being arbitrary positive reals.
Now, for the human dynamics in \((21)\) we propose the following storage function
\[
S_h = z_l^T P_1 z_l + \tilde{p}_t^T P_2 \tilde{p}_t, \tag{29}
\]
which satisfies
\[
\dot{S}_h = -z_l^T Q_1 z_l - \tilde{p}_t^T Q_2 \tilde{p}_t - 2z_l^T P_1 A(\dot{\tilde{p}} + H s^*), \tag{30}
\]
along the solutions to \((21)\). By virtue again of the Young’s inequality, one has
\[
-2z_l^T P_1 A(\dot{\tilde{p}} + H s^*) \leq z_l^T P_1 A z_l + \tilde{p}_t^T \zeta_1 P_1 A \tilde{p}_t \\
- 2z_l^T P_1 A H s^*, \tag{31}
\]
with \(\zeta_1\) being an arbitrary real. Then, for the overall HPS \((8a)-(8f)\) and \((21)\), the storage function \(S_{ph} := S_p + S_h\) satisfies
\[
\dot{S}_{ph} \leq -\dot{V}_d \left( R_L^{-1} - \frac{1}{2\zeta_2} I_{ld} \right) V_d \\
- \dot{V}_q \left( R_L^{-1} + \frac{1}{2\zeta_3} I_{q} \right) V_q \\
- z_l^T \left( Q_1 - \frac{1}{\zeta_1} P_1 A - \frac{\zeta_3}{2} I_{ld} + \frac{\zeta_3}{2} I_{q} \right) z_l \\
- \tilde{p}_t^T (Q_2 - \zeta_1 P_1 A) \tilde{p}_t \\
+ \dot{I}_{ld}^T \tilde{u}_d^* + \dot{I}_{ld}^T \tilde{u}_s^* - 2z_l^T P_1 A H s^*, \tag{32}
\]
along the solution to \((8a)-(8f)\) and \((21)\). We observe that the terms in the first and second lines can be made nonpositive by selecting sufficiently large \(\zeta_2\) and \(\zeta_3\), respectively. With the selected \(\zeta_2\) and \(\zeta_3\), also the terms in the third line can be made nonpositive by selecting sufficiently large \(Q_1\) and \(\zeta_1\). With the selected \(\zeta_1\), it is finally possible to make also the terms in the fourth line nonpositive by selecting a sufficiently large \(Q_2\). Then, we have
\[
\dot{S}_{ph} \leq -\dot{I}_{ld}^T \tilde{u}_d^* + \dot{I}_{ld}^T \tilde{u}_s^* - 2z_l^T P_1 A H s^*, \tag{33}
\]
which implies the HPS \((8a)-(8f)\) and \((21)\) is passive with respect to the supply rate \([\dot{I}_{ld}^T \dot{I}_{ld}^{-1} - 2z_l^T P_1 A H] [\tilde{u}_d^T \tilde{u}_q^T s^*]^T\) and storage function \(S_{ph}\).

Step 2. In this step, we propose for the primal-dual controller \((20)\) the following storage function
\[
S_c = \frac{1}{2} \left( \tilde{z}_l^T \tau \tilde{z}_l + \tilde{I}_{ld}^T \tau_{ld} \tilde{I}_{ld}^* + \tilde{I}_{ld}^T \tau_{ld} \tilde{I}_{ld}^* + \tilde{u}_d^T \tau_{ld} \tilde{u}_d^* \\
+ \tilde{u}_d^T \tau_{ld} \tilde{u}_q^* + \tilde{V}_d^T \tau V_d^* + s^T \tau s^* + \Lambda_c^T \tau \Lambda_c \\
+ \Lambda_0^T \tau \Lambda_0 + \Lambda_c^T \tau_{ld} \Lambda_c + \Lambda_0^T \tau_{ld} \Lambda_0 + \Lambda_c^T \tau \Lambda_c \right), \tag{34}
\]
which satisfies
\[
\dot{S}_c = -\alpha \tilde{z}_l^T \Pi_0 \tilde{z}_l^* - \beta \tilde{I}_{ld}^T \Pi_{ld} \tilde{I}_{ld}^* - \gamma \tilde{u}_d^T \tilde{u}_d^* \\
- \delta \tilde{u}_d^T \tilde{u}_q^* - \epsilon \tilde{V}_d^T \tilde{V}_d^* - \eta s^T s^* \\
- \tilde{u}_d^T \tilde{p}_A - \tilde{u}_q^T \tilde{p}_B - s^T \tilde{p}_C \\
\leq -\tilde{u}_d^T \tilde{p}_A - \tilde{u}_q^T \tilde{p}_B - s^T \tilde{p}_C \tag{35}
\]
along the solutions to \((20)\), implying that the controller \((20)\) is passive with respect to the supply rate \([-\tilde{p}_A^T \tilde{p}_B^T \tilde{p}_C^T] [\tilde{u}_d^T \tilde{u}_q^T s^*]^T\) and storage function \(S_c\), where \(\tilde{p}_A, \tilde{p}_B\) and \(\tilde{p}_C\) are given in \((25)\).

Step 3. As the last step, for the closed-loop system \((8a)-(8f)\), \((20)\) with \((24)\) and \((25)\), we propose \(S := S_{ph} + S_c\) as storage function, which satisfies \(\dot{S} \leq 0\) along the solutions to \((8a)-(8f)\), \((20)\), \((21)\) with \((24)\) and \((25)\), implying that such solutions are bounded. Therefore, there exists a forward invariant set \(\Omega\) and by LaSalle’s invariance principle the solutions that start in \(\Omega\) converge to the largest invariant set contained in
\[
\Omega \cap \{(x_s, x_c) \in \mathbb{R}^{18N+2E} | \dot{x}_s = 0, \dot{z}_l = 0, \dot{I}_{ld} = 0, \dot{u}_d = 0, \dot{u}_q = 0, \dot{V}_d = 0, \dot{s} = 0\}.
\]
Then, from \((20)\), \((20)\), \((20)\), \((20)\), \((20)\), \((20)\), \((20)\) and \((20)\) it follows that on the largest invariant set \(\lambda_c, \lambda_{ld}, \lambda_0\) and \(\lambda_c\) are also equal to zero, respectively. Moreover, we observe from \((21)\) that on the largest invariant set \(p = \tilde{p}\), with \(p\) given by \((11)\). Finally, observing from \((10)\) and \((11)\) that \(\tilde{x}_s\) is uniquely determined by \(\tilde{u}_d = \tilde{u}_d^*, \tilde{u}_q = \tilde{u}_q^*\) and \(s = s^*\), we can conclude from \((20)\) that at the steady-state the physical state variables coincide with the corresponding optimization variables.

C. HPS considering social influence

In the last two subsections, we designed the primal-dual controller and analyzed the state convergence of the closed-loop systems for case i), which does not consider social influence. In this subsection, we briefly extend our results to case ii), which includes social influence on people’s activities (in \(7\)). Thus, the resulting HPS model consists of \((8a)-(8f)\) and \((7)\).

In view of the model in \((7)\), one can verify that its steady state satisfies
\[
\tilde{p} = (C + D + L)^{-1}(Cp_{\text{exo}} + Dp_{\text{bio}}), \tag{36}
\]
\[
\tilde{z}_l = \tilde{p} - H s. \tag{37}
\]
The primal dual controller in \((20)\) is still applicable to the new HPS. For the stability analysis, some changes are needed and briefly explained in the following. Let \(\tilde{p} = \tilde{p} - p = (C + D + L)^{-1}(Cp_{\text{exo}} + Dp_{\text{bio}}) - p\), then the transformed system can be written as
\[
\dot{z}_l = -Az_l + A((C + D + L)^{-1}(Cp_{\text{exo}} + Dp_{\text{bio}}) - \tilde{p} - H s) \\
\dot{\tilde{p}} = -(C + D + L)\tilde{p}. \tag{38}
\]
One needs to replace \(C + D\) by \(C + D + L\) in the Lyapunov equation in \((23)\), where \(C + D + L\) is still a Hurwitz matrix. Then, the rest of the analysis can be conducted analogously to the proof of Theorem 1. Therefore, the following corollary holds.

Corollary 1: The closed-loop system \((7), (8a)-(8f), (20), (24)\) and \((25)\) converges to an equilibrium solving \((16)\). ■

V. Simulations

We present simulation results in this section. Specifically, we consider an AC microgrid consisting of four prosumers. Both the human models in \((5)\) (case i)) and \((7)\) (case ii)) are considered.
The parameters of the AC microgrid are listed in Table II.

The incidence matrix for the topology of the AC microgrid is given by

\[
B = \begin{bmatrix}
-1 & 0 & 0 & -1 \\
1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & 1 \\
\end{bmatrix},
\]  

(39)

The parameters for the human behavior models are given as follows, \(A = 0.5 \times I, C = \text{diag}(0.08 0.08 0.12 0.12), D = \text{diag}(0.12 0.12 0.08 0.08)\) and \(L = 0.4 \times BBT\). Moreover, we let \(p^{\text{ego}} = [0.9 0.9 0.9 0.9]^T\), \(p^{\text{bio}} = [0.6 0.6 0.6 0.6]^T\) and \(p = [0.8 0.8 0.8 0.8]^T\) at \(t = 0\).

Case i) We first consider the case of HPS in (8). The simulation results are presented in Figure 2. In the first plot of Figure 2 one can see that due to the influence of biospheric values \(p^{\text{bio}}\), \(p\) indeed decreases. However, \(p\) finally reaches the steady values 0.72 for \(p_1\) and \(p_2\) and 0.78 for \(p_3\) and \(p_4\) due to the different priorities to their egoistic and biospheric values. Prosumers 3 and 4 are more attached to their egoistic values with \(c_i = 0.6\) compared with prosumers 1 and 2 with \(c_i = 0.4\), and hence are reluctant to change \(p_i\) to a lower value depending on \(p_i^{\text{bio}} = 0.6\). In view of (11), due to the impact of incentives \(\bar{s}\), we know that \(\bar{s}_i \leq \bar{p}\). Indeed, in this example, we have \(\bar{s}_i = [0.53 0.57 0.53 0.58]^T < \bar{p}\) with \(\bar{s} = [0.21 0.17 0.28 0.22]^T\). Recalling the selected \(A, C\) and \(D\) in this example, the first plot in Figure 2 also shows the relatively fast transient process of \(z_i\) compared with \(p\), which demonstrates the fast and slow transient processes of behavior and personal norms, respectively.

For voltage regulation, from the second plot in Figure 2 one can see that \(V_d\) converges to the values around \(V_r\). We also achieve \(V_q \to 0\), and we omit the plot due to space limitation. Now we consider the status of current sharing. From the third plot in Figure 2 one can see that the prosumers approximately share the total demand due to \(\pi_{\text{cd}} = 1\) for all \(i\) (the generated currents \(I_{\text{di}}\) are similar each other). Differently, if we let for instance \(\pi_{\text{cd}} = 100\) and \(\pi_{\text{cd}} = 1\) \((i = 1, 2, 3)\), which implies that prosumer 4 has a much smaller generation capacity, then the steady current generation would be \(\bar{I}_{\text{ld}} = [35 27 35 4.2]^T\).

Case ii) The simulation results for the human behavior model in (7) including social influence are shown in Figure 3. Since the Laplacian matrix \(L\) in (7) in this example represents a connected graph, one can see that the components in \(p\) finally converge values that are closer to each other with respect to case i), despite they have different weights on egoistic and biospheric values. Furthermore, under the obtained incentives \(\bar{s} = [0.21 0.17 0.28 0.22]^T\) (which does not change in view of case i)), \(z_i\) converges to \([0.56 0.59 0.50 0.55]^T < \bar{p}\).

Finally, the total consumption reduction can be calculated by \(1^T I_{\text{ld}}(1 - \bar{z}_i)\), and the amounts are 49.87 A (44.93%) and 50.15 A (45.18%), respectively.

VI. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we formulate a framework to bridge the disciplines of systems & control and environmental psychology. Specifically, we formulate a HPS framework to describe energy saving behavior of humans in social networks and also their interactions with an AC power grid. It is clear that the developed second-order models describing human energy saving behavior and personal norms are consistent with the findings in environmental psychology and able to differentiate an individual’s extrinsic and intrinsic drivers of energy saving behavior, and also allow to model their fast and slow transient processes, respectively. With the developed HPS, we formulate a social-physical welfare optimization problem and provide the design of primal-dual controller. It is proved that the controller computing optimal incentives to humans and
control inputs to an AC power grid can stabilize the closed-loop system whose state converges to an equilibrium solving the optimization problem. The developed models of human activities not only interpret the findings in psychology from the control viewpoint, but also are consistent with the studies on opinion dynamics.

The results in this paper can be extended towards various directions in the future. Among these, one possible direction is to collect social and electrical data to identify the parameters of the proposed Behavior-Personal-norm models. Another direction is to consider inequality constraints (e.g., by considering line congestions or subsidy upper bounds) in the social-physical welfare optimization problem.

REFERENCES

[1] L. Steg, G. Perlaviciute, and E. van der Werff, “Understanding the human dimensions of a sustainable energy transition,” *Frontiers in psychology*, vol. 6, p. 805, 2015.

[2] P. C. Stern, K. B. Janda, M. A. Brown, L. Steg, E. L. Vine, and L. Lutzenhiser, “Opportunities and insights for reducing fossil fuel consumption by households and organizations,” *Nature Energy*, vol. 1, no. 5, pp. 1–6, 2016.

[3] P. C. Stern, B. K. Sovacool, and T. Dietz, “Towards a science of climate and energy choices,” *Nature Climate Change*, vol. 6, no. 6, pp. 547–555, 2016.

[4] B. K. Sovacool, “Diversity: energy studies need social science,” *Nature News*, vol. 511, no. 7511, p. 529, 2014.

[5] S. H. Schwartz and J. A. Howard, “A normative decision-making model of altruism,” *Altruism and helping behavior*, pp. 189–211, 1981.

[6] P. Sten, “Toward a coherent theory of environmentally significant behaviour,” *Journal of Social Issues*, vol. 56, no. 3, pp. 407–424, 2000.

[7] P. C. Stern, T. Dietz, T. Abel, G. A. Guagnano, and L. Kalof, “A value-belief-norm theory of support for social movements: The case of environmentalism,” *Human ecology review*, pp. 81–97, 1999.

[8] T. Dietz, A. Fitzgerald, and R. Shwom, “Environmental values,” *Ann. Rev. Environ. Resour.*, vol. 30, pp. 335–372, 2005.

[9] L. Steg, R. Shwom, and T. Dietz, “What drives energy consumers?: Engaging people in a sustainable energy transition,” *IEEE Power and Energy Magazine*, vol. 16, no. 1, pp. 20–28, 2018.

[10] D. Schwartz, W. Brune de Bruin, B. Fischhoff, and L. Lave, “Advertising energy saving programs: The potential environmental cost of emphasizing monetary savings.” *Journal of Experimental Psychology: Applied*, vol. 21, no. 2, p. 158, 2015.

[11] J. M. Nolan, P. W. Schultz, R. B. Cialdini, N. J. Goldstein, and V. Griskevicius, “Normative social influence is undetected.” *Personality and social psychology bulletin*, vol. 34, no. 7, pp. 913–923, 2008.

[12] P. W. Schultz, J. M. Nolan, R. B. Cialdini, N. J. Goldstein, and V. Griskevicius, “The constructive, destructive, and reconstructive power of social norms,” *Psychological science*, vol. 18, no. 5, pp. 429–434, 2007.

[13] M. H. DeGroot, “Reaching a consensus,” *Journal of the American Statistical Association*, vol. 69, no. 345, pp. 118–121, 1974.

[14] N. E. Friedkin and E. C. Johnsen, “Social influence and opinions,” *Journal of Mathematical Sociology*, vol. 15, no. 3-4, pp. 193–206, 1990.

[15] M. Ye, M. H. Trinh, Y.-H. Lim, B. D. Anderson, and H.-S. Ahn, “Continuous-time opinion dynamics on multiple interdependent topics,” *Automatica*, vol. 115, p. 108884, 2020.

[16] V. Nasirian, S. Mouayed, A. Davoudi, and E. L. Lewis, “Distributed cooperative control of dc microgrids,” *IEEE Transactions on Power Electronics*, vol. 30, no. 4, pp. 2288–2303, 2014.

[17] C. De Persis, E. R. Weitenberg, and F. Dörfler, “A power consensus algorithm for dc microgrids,” *Automatica*, vol. 89, pp. 364–375, 2018.

[18] M. Cucuzzella, S. Trip, C. De Persis, X. Cheng, A. Ferrara, and A. van der Schaft, “A robust consensus algorithm for current sharing and voltage regulation in dc microgrids,” *IEEE Transactions on Control Systems Technology*, vol. 27, no. 4, pp. 1583–1595, 2018.

[19] S. Trip, M. Cucuzzella, X. Cheng, and J. Scherpen, “Distributed averaging control for voltage regulation and current sharing in dc microgrids,” *IEEE Control Systems Letters*, vol. 3, no. 1, pp. 174–179, 2018.

[20] M. Cucuzzella, K. C. Kosaraju, T. Bouman, G. Schuitema, S. Johnson-Zawadzki, C. Fischione, L. Steg, and J. Scherpen, “Distributed control of dc grids: a social perspective,” *arXiv preprint arXiv:1912.07341*, 2019.

[21] T. Stegink, C. De Persis, and A. van der Schaft, “A unifying energy-based approach to stability of power grids with market dynamics,” *IEEE Transactions on Automatic Control*, vol. 62, no. 6, pp. 2612–2622, 2016.

[22] K. C. Kosaraju, M. Cucuzzella, and J. M. Scherpen, “Distributed control of dc microgrids using primal-dual dynamics,” in *IEEE Conference on Decision and Control*. IEEE, 2019, pp. 6215–6220.

[23] C. Zhao, U. Topcu, N. Li, and S. Low, “Design and stability of load-side primary frequency control in power systems,” *IEEE Transactions on Automatic Control*, vol. 59, no. 5, pp. 1177–1189, 2014.

[24] J. M. Guerrero, J. Matas, L. G. de Vicuna, M. Castilla, and J. Miret, “Decentralized control for parallel operation of distributed generation inverters using resistive output impedance,” *IEEE Transactions on industrial electronics*, vol. 54, no. 2, pp. 994–1004, 2007.

[25] M. S. Golsorkhi, M. Savaghebi, D.-C. Lu, J. M. Guerrero, and J. C. Vasquez, “A gps-based control framework for accurate current sharing and power quality improvement in microgrids,” *IEEE Transactions on Power Electronics*, vol. 32, no. 7, pp. 5675–5687, 2016.

[26] R. H. Park, “Two-reaction theory of synchronous machines generalized method of analysis-part i,” *Transactions of the American Institute of Electrical Engineers*, vol. 48, no. 3, pp. 716–727, 1929.

[27] M. Cucuzzella, S. Trip, A. Ferrara, and J. Scherpen, “Cooperative voltage control in AC microgrids,” in *IEEE Conference on Decision and Control*, 2018, pp. 6723–6728.

[28] T. Bouman and L. Steg, “Motivating society-wide pro-environmental change,” *One Earth*, vol. 1, no. 1, pp. 27–30, 2019.

[29] T. Bouman, L. Steg, and H. A. Kiers, “Measuring values in environmental research: a test of an environmental portrait value questionnaire,” *Frontiers in psychology*, vol. 9, p. 564, 2018.

[30] S. H. Schwartz, J. Cieciuch, M. Vecchione, E. Davidov, R. Fischer, C. Beierlein, A. Ramos, M. Verkasalo, J.-E. Lönnqvist, K. Demirakul et al., “Refining the theory of basic individual values.” *Journal of personality and social psychology*, vol. 103, no. 4, p. 663, 2012.

[31] L. Steg, “Values, norms, and intrinsic motivation to act proenvironmentally,” *Annual Review of Environment and Resources*, vol. 41, pp. 277–292, 2016.

[32] P. C. Stern and T. Dietz, “The value basis of environmental concern,” *Journal of social issues*, vol. 50, no. 3, pp. 65–84, 1994.

[33] E. Dogan, J. W. Bolderdijk, and L. Steg, “Making small numbers count: environmental and financial feedback in promoting eco-driving behaviours,” *Journal of Consumer Policy*, vol. 37, no. 3, pp. 413–422, 2014.

[34] T. Bouman, L. Steg, and T. Dietz, “Insights from early covid-19 responses about promoting sustainable action,” *Nature Sustainability*, pp. 1–7, 2020.

[35] P. C. Stern, “New environmental theories: toward a coherent theory of environmentally significant behavior.” *Journal of social issues*, vol. 56, no. 3, pp. 407–424, 2000.

[36] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[37] K. C. Kosaraju, M. Cucuzzella, J. M. Scherpen, and R. Pasmushary, “Differentiation and passivity for control of Brayton-Moser systems,” *IEEE Transactions on Automatic Control*, 2020.

[38] G. H. Hardy, J. E. Littlewood, G. Pólya, G. Pólya, D. Littlewood et al., *Inequalities*. Cambridge university press, 1952.