3D global simulations of RIAFs: convergence, effects of azimuthal extent and dynamo

Prasun Dhang,1∗ Prateek Sharma1
1Department of Physics and Joint Astronomy Programme, Indian Institute of Science, Bangalore, INDIA 560012

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ABSTRACT

We study the long-term evolution of non-radiative accretion flows using 3D global ideal MHD simulations and a pseudo-Newtonian gravity with three different resolutions. We find that resolving the scale height with 42 grid points is adequate to obtain convergence with the product of quality factors $⟨⟨Q_θ⟩⟩⟨⟨Q_ϕ⟩⟩ ≥ 300$ and magnetic tilt angle $θ_B ∼ 13° − 14°$. Like previous global isothermal thin disk simulations, we find stronger mean magnetic fields for the restricted azimuthal domains. Imposing periodic boundary conditions with the azimuthal extent smaller than $2\pi$ makes the turbulent field at low $m$ appear as a mean field in the runs with smaller azimuthal extent. But unlike previous works, we do not find a monotonic trend in turbulence with the azimuthal extent. We conclude that the minimum azimuthal extent should be $≥ \pi/2$ to capture the flow structure, but a full $2\pi$ extent is necessary to study the dynamo. We find an intermittent dynamo cycle, with $α$-quenching playing an important role in the nonlinear saturated state. Unlike previous local studies, we find almost similar values of kinetic and magnetic $α$-s, giving rise to an irregular distribution of dynamo-$α$. The effects of dynamical quenching are shown explicitly for the first time in global simulations of accretion flows.

Key words: accretion,accretion discs - dynamo - instabilities - magnetic fields - MHD - turbulence - methods: numerical.

1 INTRODUCTION

Accretion of matter onto compact objects is the main source power in most energetic sources in the universe; e.g. X-ray binaries (XRBs), active galactic nuclei (AGNs), Gamma ray bursts (GRBs). Shakura & Sunyaev (1973); Novikov & Thorne (1973) provided a phenomenological model for geometrically thin cold disks with $H/R ≪ 1$. Thin disks are supposed to power luminous AGN (Koratkar & Blass 1999) and XRBs in the high soft state (Remillard & McClintock 2006). The accretion rate in the thin disk ranges from few percent of the Eddington rate to the Eddington rate (for a review see Yuan & Narayan 2014). In contrast to the cold thin disk, there are geometrically thick ($H/R ∼ 0.5$ ) hot accretion flows with smaller accretion rates that power the under luminous sources; e.g. low luminous AGNs, XRBs in hard state. These hot radiatively inefficient accretion flows (RIAFs) do not radiate efficiently because of a smaller density (e.g. see Fig. 3 in Das & Sharma 2013), and hence have a larger temperature and scale-height (Ichimaru 1977; Narayan & Yi 1994, 1995; Abramowicz et al. 1995). On the other extreme, the radiation dominated slim disks (Abramowicz et al. 1988) with super-Eddington accretion rates, are also geometrically thick. In this paper we study the thick disks of the former kind, referred to as RIAFs.

Angular momentum transport in accretion flows is a long-standing problem. Microscopic viscosity alone is inadequate to give the required transport in order to explain the observed accretion rates. Shakura & Sunyaev (1973) explained the angular momentum transport in terms of an emergent turbulent viscosity. But the source of turbulence was not identified until Balbus & Hawley (1991) rediscovered (considered previously by Velikhov 1959 and Chandrasekhar 1960) and highlighted a local linear MHD instability in Keplerian flows, the magneto-rotational instability (MRI), as a solution to the angular momentum problem (for a review see Balbus & Hawley (1998)). Although the MRI ensures outward angular momentum transport in the linear regime, understanding the instability in the nonlinear regime is essential. Till date, the nonlinear regime has been studied extensively in the local shearing box simulations. While MRI is a local instability and in the geometrically thin disks ($H/R ≪ 1$) the global effects are not that important (but see Regev & Umurhan 2008; Beckwith et al. 2011) as in...
hot/non-radiative accretion flows \((H/R \sim 0.5)\), the focus of this paper.

For the numerical results to be trusted, it is critical that they are converged. For MRI simulations this means that the statistical properties of the MHD turbulent flow do not depend on the grid resolution. Several local shearing box MRI simulations with and without any explicit dissipation (viscosity or resistivity) and with and without vertical stratification have been performed to investigate convergence. Some of these studies reached divergent conclusions. While most of the zero net flux stratified simulations without explicit dissipation (Fromang & Papaloizou 2007; Pessah et al. 2007; Guan et al. 2009; Bodo et al. 2011) show that the turbulent transport diminishes as the grid resolution is increased, recent study of Shi et al. (2016) with tall boxes is able to converge. In presence of a net flux, all the unstratified (Hawley et al. 1995; Sano et al. 2004; Guan et al. 2009) and stratified (Stone et al. 1996; Miller & Stone 2000) simulations show convergence and non-zero transport in the high resolution limit. Interestingly, Davis et al. (2010) and Shi et al. (2010) claim that vertically stratified zero net flux simulations are converged, however, recent studies of Bodo et al. (2014) and Ryan et al. (2017) challenge this claim.

In contrast to local shearing boxes, global simulations are more realistic as they capture the large scale structure of the accretion flow but cannot resolve the MRI turbulence as nicely. Moreover, the shearing box setup imposes several restrictions/symmetries (e.g., no sense of the direction of mass transport, conservation of magnetic fluxes across various boundaries) on the flow that are not respected by real accretion flows. There are only a handful of works (Fromang & Nebon 2006; Flock et al. 2010; Hawley et al. 2011; Sorathia et al. 2012; Shiokawa et al. 2012; Hawley et al. 2013; Parkin & Bicknell 2013) that study convergence in global simulations. To quantify convergence in our global simulations, we rely on the statistical behaviour of the convergence metrics in the quasi-steady state (QSS) of the turbulent accretion flow, that is during the non-linear evolution of the MRI.

Self-consistent MRI simulations have to be carried out in 3D because of the impossibility of the sustenance of magnetic fields (and hence turbulence which is driven by the MRI) for long times in 2D (e.g., see Choudhuri 1998). However, 3D simulations are expensive. Therefore it is desirable to reduce the azimuthal domain size to a small fraction of \(2\pi\), provided the results (in particular, the level of transport and mean/fluctuating quantities) are similar to the full extent. Till now different studies have found different effects of azimuthal extent. By studying two different sets of simulations (namely initial vertical field and adiabatic equation of state and initial toroidal field and isothermal equation of state), Hawley (2001) concluded that azimuthal domain size of \(\Phi_0 = \pi/2\) produces a quasi-stationary turbulent state qualitatively similar to the full \(\Phi_0 = 2\pi\) extent. Unstratified global simulations of Sorathia et al. (2012) also did not show any considerable difference between the domain sizes \(\Phi_0 = \pi/4\) and \(\Phi_0 = 2\pi\). Note that both these works use a cylindrically symmetric (and not the more realistic spherically symmetric) potential. In contrast, by performing explicit comparison of global simulations with different azimuthal extents Flock et al. (2012), showed that models with smaller azimuthal domain size give rise to a higher accretion stress. They attribute the high accretion stress to the stronger mean fields produced by an \(\alpha = \Omega\) dynamo. Here, we systematically investigate the effects of azimuthal domain size for a geometrically thick RIAF \((H/R \sim 0.5)\) by looking at its different temporal and spatio-temporal quantities in the turbulent steady-state.

Large scale fields play a crucial role in producing jets (Blandford & Znajek 1977; Blandford & Payne 1982) and coronal winds in accretion flows. RIAFs, which are supposed to exist in XRBs in the hard state, are prone to outflows (Remillard & McClintock 2006). The generation of large scale fields in accretion flows is debatable. The large scale field can be the due to the advection of field (Lubow et al. 1994; Lovelace et al. 2009; Guilet & Ogilvie 2012) or due to \textit{in situ} production by a large scale dynamo (Brandenburg & Subramanian 2005). Both unstratified (with sufficient vertical extent; Lesur & Ogilvie 2008; Shi et al. 2016) and stratified (Brandenburg et al. 1995; Davis et al. 2010; Shi et al. 2010; Gressel 2010) local shearing box, and global thin disk simulations (Beckwith et al. 2011; Flock et al. 2012; Suzuki & Inutsuka 2014; Hogg & Reynolds 2016) show generation of large scale fields due to a dynamo process. While the early dynamo models were kinematic, with the mean flows and the statistical properties of the flow specified, MRI turbulence cannot be in the kinematic regime because the instability is driven by magnetic tension affecting the velocity perturbations (for a review see Balbus & Hawley 1998). We investigate the generation of large scale magnetic fields and the self-sustained dynamo process in a geometrically thick RIAF. As there is no externally imposed net field and we run our simulations for a long time, we expect the advection of field do not play a significant role in generating large scale fields.

In this work, we perform 3D global simulations of radiatively inefficient accretion flows with three aims: i) convergence, that is the minimum number of grid cells required to properly capture the nonlinear saturation of MRI turbulence and to quantify the converged solutions; ii) the effects of azimuthal extent of the computational domain on the different properties of the mean and turbulent field evolution; iii) the saturation process of magnetic energy, i.e. on the dynamo mechanism in a radiatively inefficient geometrically thick hot accretion flow.

The paper is organized as follows. In section 2 we discuss the physical set-up, solution method and different diagnostics used in the work. In section 3 we discuss the evolution of the accretion flow for our fiducial run. Convergence is discussed in section 4. We discuss the effects of the azimuthal extent on the accretion flow and the dynamo mechanism in sections 5 and 6 respectively. Finally the key points of results are discussed and summarized in sections 7 and 8.

2 METHOD

2.1 Equations solved

We solve the Newtonian MHD equations in spherical coordinates \((r, \theta, \phi)\) using the PLUTO code (Mignone et al. 2000)
2007). The equations are

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \]
\[ \frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B}) = -\rho \nabla \Phi - \nabla P^*, \]
\[ \frac{\partial E}{\partial t} + \nabla \cdot ((E + P^*) \mathbf{v} - \mathbf{B} \nabla) = -\rho \mathbf{v} \cdot \nabla \Phi, \]
\[ \frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) = 0, \]

where \( \rho \) is the gas mass density, \( \mathbf{v} \) and \( \mathbf{B} \) are the velocity and magnetic fields respectively, \( P^* \) is the total pressure given by \( P^* = P + B^2 / 2 \) (\( P \) is gas pressure), \( E \) is the total energy density which is related to the internal energy density \( e \) as \( E = e + \rho u^2 / 2 + B^2 / 2 \). To mimic the general relativistic effects close to the black hole, we use the pseudo-Newtonian potential (Paczyński & Wiita 1980) \( \Phi = GM / (r - 2r_s) \), where \( r_s = GM / c^2 \) is the gravitational radius, \( M \) and \( c \) are the mass of the accreting black hole and the speed of light in vacuum respectively. We work in dimensionless units in which \( GM = c = 1 \). Therefore, in this paper all the length scales and velocities are given in the units of \( GM / c^2 \) and \( c \) respectively. Unless stated otherwise, time scales are expressed in terms of the number of orbits a test particle would do at the inner most stable circular orbit (ISCO), and is given by

\[ N_{\text{ISCO}} = \frac{t_{\text{sim}}}{t_{\text{ISCO}}}, \]

where the simulation time is \( t_{\text{sim}} \) and the orbital period at ISCO \( t_{\text{ISCO}} = 2\pi r_s^{1/2} (r_{\text{ISCO}} - 2) \) are expressed in the units of \( GM / c^3 \). For a Schwarzschild black hole the location of ISCO is \( r_{\text{ISCO}} = 6r_s \).

PLUTO uses a conservative Godunov scheme. All quantities are stored at the cell center, except for the magnetic field components that are face centered and staggered. A staggered magnetic field is required for the constrained transport scheme to evolve the induction equation (Eq. 4; Evans & Hawley 1988). This preserved \( \nabla \cdot \mathbf{B} = 0 \) to machine precision. Out of several possible combinations of algorithms, we follow the recommendations of Flock et al. (2010) to properly capture the MRI modes. We use the HLLD solver (Miyoshi & Kusano 2005) with second-order slope limited reconstruction. For time-integration, second order Runge-Kutta (RK2) is used with the CFL number 0.3. For the calculation of the electromagnetic forces (EMFs) for the induction equation, we use the upwind CT ‘contact’ method (Gardiner & Stone 2005).

In our simulations, we find that the time step is determined by the large Alfvén speed \( v_A = B / \sqrt{\rho} \). As in Newtonian MHD there is no speed limit and \( v_A \) becomes unusually large in the low density regions near the poles. This results in impractically short time-steps due to the CFL condition. We fix this problem by imposing a density floor such that \( v_A < v_{A,\text{max}} = 5 \). For most of the runs, \( v_A \) seldom touches the limit and the total mass added by floors is insignificant. Moreover, mass is added only in the regions near the poles and in the highly magnetized ‘funnels’, which we do not incorporate in our analysis.

2.2 Initial conditions

We initialize the simulation domain with the equilibrium solution given by Papaloizou & Pringle (1984), which describes a constant angular momentum torus embedded in a non-rotating, low density hydrostatic medium. Pressure and density within the torus follow a polytropic equation of state \( P = K \rho^n \) (\( K \) is a constant). The initial density of the torus is given by

\[ \rho^{-1} = \frac{1}{(n + 1)R_0 K} \left[ \frac{R_0}{r - 2} - \frac{R_0^3}{2 (R_0 - 2)^2} \right]. \]

where \( \gamma \) is the adiabatic index, \( n = 1 / (\gamma - 1) \) is the polytropic index, \( R = rsin\theta \) is the cylindrical radius, \( R_0 \) is the cylindrical radial distance of the center of the torus from the black hole, \( d \) is the distortion parameter which determines the shape and size of the torus. The density of the torus is maximum \( (\rho = \rho_0) \) at \( r = R_0 \) and \( \theta = \pi / 2 \). This information is used to calculate

\[ K = \frac{1}{(n + 1)R_0 \rho_0} \left[ \frac{R_0}{r - 2} - \frac{R_0^3}{2 (R_0 - 2)^2} - \frac{1}{2d} \right]. \]

The constant angular momentum \( l_0 \) associated with the torus is given by its Keplerian value at \( R_0 \),

\[ l_0 = l_K(R_0) = \frac{R_0^2}{R_0 - 2}. \]

We choose \( \rho_0 = 10^6 \) in code units, \( R_0 = 20 \) and \( d = 1.15 \). The chosen value of \( d \) makes the initial torus geometrically thick \( (H / R \sim 0.1 \text{ at } R_0) \). We choose the density of ambient medium to be small enough \( (\rho_{\text{amb}} = 10^{-4} \rho_0) \), such that it does not affect our results. For all the runs the initial torus is seeded with white noise of \( |\delta \rho_0| = 10^{-7} \).

We initialize a poloidal magnetic field which threads the initial torus and is parallel to the density contours. This magnetic field is defined through a vector potential

\[ A_A = C \rho^{\gamma / 2}, \]

which guarantees the divergence free nature of the magnetic field; \( C \) is a constant that determines the field strength. The initial magnetic field strength is quantified by the ratio of volume averaged (over torus) gas to magnetic pressure

\[ \beta_{\text{ini}} = \frac{P}{B_0^2 / 2}. \]

We choose \( \beta_{\text{ini}} = 890 \) in our simulations.

2.3 Numerical set-up

We carry out six 3D MHD simulations in spherical coordinates \( (r, \theta, \phi) \). The computational domain extends from an inner radius \( r_{\text{in}} = 4 \) to an outer radius \( r_{\text{out}} = 140 \). The advantage of putting the inner boundary within the innermost stable circular orbit (ISCO) is that the accretion velocity \( v_a \) becomes supersonic (see the top left panel of Fig. 22) before reaching the inner boundary. As a result, the flow properties outside sonic radius \( (r_s) \) become independent of the details of the inner boundary conditions (McKinney & Gammie 2002). We also place the radial outer boundary far away from the torus to avoid significant mass loss through it. We use a logarithmic grid along the radial direction from
**2.4 Diagnostics**

We discuss temporal, spatial and spatio-temporal behaviour of several quantities. The averaging methods and the common diagnostics are defined below.

### 2.4.1 Averaging method

For any quantity \( q \), we consider following spatial averages,

\[
\langle q(r) \rangle = \frac{1}{V} \int q(r, \theta, \phi) dV,
\]

\[
\langle q(r, t) \rangle = \frac{1}{S_r} \int q(r, \theta, \phi) dS_r,
\]

\[
\langle q(\theta, t) \rangle = \frac{1}{S_\theta} \int q(r, \theta, \phi) dS_\theta,
\]

\[
\langle q(r, \theta, t) \rangle = \frac{1}{l_\theta} \int q(r, \theta, \phi) dl_\theta,
\]

\[
\langle q(r, \phi, t) \rangle = \frac{1}{l_\phi} \int q(r, \theta, \phi) dl_\phi,
\]

where \( V = \int dV = \int r^2 \sin \theta dr d\theta d\phi \), \( S_r = \int dS_r = \int \sin \theta d\theta d\phi \), \( S_\theta = \int dS_\theta = \int r^2 dr d\phi \), \( l_\theta = \int dl_\theta = \int \sin \theta d\phi \). The integrations are performed over the sub-domain \( S = [r_{ISCO}, 40] \times [\pi/2 - \theta_H, \pi/2 + \theta_H] \times [0, \Phi_0] \), where \( \theta_H = \cos^{-1}(H/R) \). \( S \) corresponds to the well-resolved region of the computational domain. Here, the scale-height \( H \) is given by

\[
H = \left( \langle c_s(r, \theta = \frac{\pi}{2}) \rangle \right),
\]

where \( c_s = \sqrt{\gamma P/\rho} \) and \( \Omega = v_\phi/R \) are sound speed and angular velocity respectively.

To look at the time averaged radial and meridional variation we compute,

\[
\langle q(r) \rangle = \frac{1}{\Delta T} \int_{T_1}^{T_2} (q(r, t)) dt,
\]

\[
\langle q(\theta) \rangle = \frac{1}{\Delta T} \int_{T_1}^{T_2} (q(\theta, t)) dt,
\]

respectively; \( \Delta T = T_2 - T_1 \) is the time interval over which the average is done.

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**Table 1.** In the radial direction, \( N_{r,t} \) logarithmically spaced grids are used between \( r_{in} = 4 \) to \( r_{t} = 45 \) and \( N_{r,s} \) stretched grid points are employed in the outer buffer zone extended from \( r_{t} = 45 \) to \( r_{out} = 140 \). Along the meridional direction, between \( \theta_1 = 60^\circ \) and \( \theta_2 = 120^\circ \), \( N_{\theta,u} \) number of uniform grid points are used. On both sides of this region we use stretched grids with \( N_{\theta,b} \) points. Total number of grid points in the radial \( (N_r) \) and meridional \( (N_\theta) \) are given by, \( N_r = N_{r,t} + N_{r,s} \) and \( N_\theta = N_{\theta,u} + 2N_{\theta,b} \). \( N_0 \) is the number of grid points spread uniformly over the azimuthal extent \( \Phi_0 \).

| Name | \( \Phi_0 \) | \( N_{r,t} \) | \( N_{r,s} \) | \( N_{\theta,u} \) | \( N_{\theta,b} \) | \( (r \Delta \phi/\Delta r)_{\theta = \pi/2} \) |
|------|------|------|------|------|------|------------------|
| L-P4 | \( \pi/4 \) | 144 | 32 | 64 | 16 | 32 | 1.46 |
| M-P4 | \( \pi/4 \) | 296 | 72 | 128 | 32 | 64 | 1.50 |
| M-P2 | \( \pi/2 \) | 296 | 72 | 128 | 32 | 128 | 1.50 |
| M-1P | \( \pi \) | 296 | 72 | 128 | 32 | 256 | 1.50 |
| M-2P | 2\( \pi \) | 296 | 72 | 128 | 32 | 512 | 1.50 |
| H-P4 | \( \pi/4 \) | 536 | 32 | 232 | 24 | 128 | 1.36 |

We use a pure inflow \( (v_r \leq 0) \) boundary condition at the radial inner boundary, so that the inner boundary acts as a one-way membrane. Zero gradient boundary conditions are used for \( \rho, p, v_\theta \) and \( v_\phi \). At the radial outer boundary, we fix the density and pressure to their initial values, \( v_\theta \) and \( v_\phi \) are kept free, but we set \( v_r \geq 0 \) so that matter can go out of the computation domain but can not come in. Both at the inner and outer radial boundaries, ‘force-free’ (zero-gradient) conditions are applied for the tangential components \( (B_\theta \) and \( B_\phi \) of \( B \)-field, while the normal component of \( B \)-field is determined by CT. Reflective and periodic boundary conditions are used at the meridional and azimuthal boundaries respectively.
To get a volume and time averaged quantity, we do the following,
\[ \langle q \rangle = \frac{1}{\Delta T} \int_{T_1}^{T_2} \langle q(t) \rangle dt. \] (14)

### 2.4.2 Mass and angular momentum accretion

Mass \( \dot{M}(r_{\text{ISCO}}, t) \) and angular momentum \( \dot{L}_{\text{acc}}(r_{\text{ISCO}}, t) \) accretion rates across ISCO are given by
\[ \dot{M}(r_{\text{ISCO}}, t) = r_{\text{ISCO}}^2 \int_{-\theta_H}^{\theta_H} \int_{0}^{\Phi_0} \rho v_r dS_r, \] (15)
\[ \dot{L}_{\text{acc}}(r_{\text{ISCO}}, t) = r_{\text{ISCO}}^3 \int_{-\theta_H}^{\theta_H} \int_{0}^{\Phi_0} \left( \rho v_r v_\phi + W_{\text{Max}}^{r\phi} \right) dS_r, \] (16)
respectively. Here, the \( r - \phi \) components of the total accretion stress \( W_{r\phi}^T \) is given by
\[ W_{r\phi}^T = W_{\text{Re}}^{r\phi} + W_{\text{Max}}^{r\phi}, \] (17)
where \( W_{\text{Re}}^{r\phi} \) and \( W_{\text{Max}}^{r\phi} \) are the Reynolds and Maxwell accretion stresses respectively. Following Hawley (2000), we compute the Reynolds stress \( W_{r\phi}^{\text{Re}} \) in terms of the difference between instantaneous angular momentum flux, and the average mass flux times specific angular momentum,
\[ (W_{r\phi}^{\text{Re}}(r, t)) = \langle \rho v_r v_\phi(r, t) \rangle - \langle \rho v_r(r, t) \langle v_\phi(r, t) \rangle \rangle. \] (18)
Since here MHD turbulence is subsonic, \( |v'|/c_\ast \lesssim 0.2 \) (see Table 3), density perturbations are small. Maxwell stress \( W_{r\phi}^{\text{Max}} \) is defined as,
\[ W_{r\phi}^{\text{Max}} = -B_r B_\phi. \] (19)
Therefore, the normalized stresses are given by
\[ \langle \alpha^T \rangle = \langle \alpha^{\text{Re}} \rangle + \langle \alpha^{\text{Max}} \rangle = \frac{(W_{r\phi}^{\text{Re}}) + (W_{r\phi}^{\text{Max}})}{\langle P \rangle}. \] (20)

### 2.4.3 Resolvability

To check the resolvability of the MRI we look at the two quality factors,
\[ Q_\theta = \frac{\lambda_{\text{MRI}}^{\theta}}{r \Delta \theta}, \] (21)
\[ Q_\phi = \frac{\lambda_{\text{MRI}}^{\phi}}{r \sin \theta \Delta \phi}, \] (22)
where the quality factor \( Q_\theta \) measures the number of cells in the \( \theta \)-direction across a wavelength of the fastest growing mode, \( \lambda_{\text{MRI}}^{\theta} = 2\pi |B_\theta|/\sqrt{\rho} \) and \( Q_\phi \) measures the number of cells in the \( \phi \)-direction across a wavelength of the fastest growing mode, \( \lambda_{\text{MRI}}^{\phi} = 2\pi |B_\phi|/\sqrt{\rho} \); \( \Delta \theta \) and \( \Delta \phi \) are the grid sizes in the \( \theta \) and \( \phi \)-directions respectively, \( B_\theta \) and \( B_\phi \) are the magnetic field components, \( \Omega = v_\phi/R \) is the fluid’s angular velocity. Although the quality factors were introduced to check the resolvability of the linear MRI (Sano et al. 2004, Fromang & Nelson 2006), they were found to be an important diagnostic in the fully non-linear turbulent regime too (Sorathia et al. 2012, Hawley et al. 2013). While quality factors give the information of how well the MRI is resolved,
they do not reflect the structure of turbulence in the saturated state. Magnetic tilt angle

$$\theta_B(t) = \frac{1}{2} \sin^{-1} \left( -\frac{2B_r B_\phi(t)}{(B^2(t))} \right) \tag{23}$$

above a critical value confirms the transition from linear growth of the MRI to the saturated turbulence, and the critical value of this metric is $\theta_B \approx 12^\circ$ (Pessah 2010). It is also a measure of the magnetic field anisotropy which is a key factor behind the angular momentum transport.

2.4.4 Power spectral density

We also study the azimuthal spectral structure of the turbulent flow in the non-linear regime. We compute the toroidal power spectral density (PSD)

$$P_q(r, \theta, m, t) = \frac{1}{\Delta m} \left| \int q(r, \theta, \phi, t) e^{im\phi} \, d\phi \right|^2 \tag{24}$$

of a physical quantity $q(r, \theta, \phi, t)$, where $\Delta m = 2\pi/\Phi_0$. The spatio-temporal average of $P_q(r, \theta, m, t)$ is given by

$$\langle \langle P_q(m) \rangle \rangle = \frac{\int_{T_2}^{T_2} \int_{r=40}^{r=60} \int_{\theta=\alpha}^{\beta} P_q(r, \theta, m, t) r^2 \sin \theta \, dr \, d\theta \, dt}{\Delta T \int_{r=6}^{r=40} \int_{\theta=\alpha}^{\beta} r^2 \sin \theta \, dr \, d\theta} \tag{25}$$

3 EVOLUTION OF THE FLOW

Before discussing the details of our results, we discuss the evolution of the fiducial run M-2P (the medium resolution run with the azimuthal extent $\Phi_0 = 2\pi$), which is converged (see sections 4 and 5.3).

Fig. 2 shows the time evolution of density and poloidal magnetic field (shown by streamlines) in the $\phi = 0$ plane ($xz$ plane). The first panel shows the initial condition. Shear produces toroidal field out of the initial poloidal field on a dynamical time scale ($t_{\text{dyn}} \approx 1/\Omega$, $\Omega$ is the angular speed, $\Omega \propto 1/R^2$ for the initial constant angular momentum torus). The MRI also grows on a dynamical time. As a result, both poloidal and toroidal fields grow exponentially. Closer to the central black hole, MRI grows faster as the dynamical time $t_{\text{dyn}} \propto R^{-2}$. As the MRI starts, outward angular momentum transport happens and accretion begins as seen in the second panel of the Fig. 2. Along with the breaking up of the equilibrium torus due to MRI, torus also expands under the action of magnetic pressure growing due to the background radial shear. As time passes, system enters the non-linear regime and parasitic instabilities take over (Goodman & Xu 1994, Pessah & Goodman 2009), and eventually fully developed MHD turbulence develops throughout the initial torus (see the third panel of Fig. 2). Last three panels show the snapshots of the accretion flow in the quasi-steady state (QSS). In the QSS, the accretion flow consists of two parts: the highly turbulent region on both sides of the mid-plane within one scale-height and the almost laminar region above that. This can be easily seen in last three panels of Fig. 2 and in Fig. 3.

Fig. 3 shows the three dimensional view of plasma $\beta = 2P/B^2$ in the poloidal plane. Turbulent structures can be seen in the regions around the mid-plane of the accretion flow with both high and low values of $\beta$. As expected, the structures are smaller, closer to the black hole. On the other hand, the laminar region close to the accretor and away from mid-plane is magnetically dominated ($\beta < 1$). On average (over time and spatial domain, $S$, see section 2.4.1) $\langle \langle \beta \rangle \rangle \approx 20$ (see Table 2), which shows a stronger magnetization in the QSS compared to the initial magnetization of $\beta_{\text{ini}} \approx 890$. Fig. 4 shows the radial distribution of average angular momentum ($l(r)$) at different times. Spatial average is done over the whole azimuthal domain and within one scale-height in the meridional direction. Vertical red line denotes the location of the ISCO. We begin with a constant angular momentum torus with $l_0 = l_K(r = R_0 = 20)$. Initially, for $r < R_0$, the angular momentum distribution $\langle l(r < R_0) \rangle$ is super-Keplerian, whereas for $r > R_0$, $\langle l(r > R_0) \rangle$ is sub-Keplerian. As the MRI grows, inner part of the torus loses angular momentum first and subsequently the whole torus becomes unstable against MRI. As the fully developed MHD turbulence is established, a sub-Keplerian angular momentum profile ($\langle l(r) \rangle < l_K(r) \rangle$) is developed throughout the accretion flow. The flow is more sub-Keplerian further from the accretor. Within the ISCO, as accreting matter plunges in with an almost constant specific angular momentum, $\langle l(r) \rangle/l_K$ drops rapidly. It is interesting to see that with time, $\langle l(r) \rangle$ becomes more and more sub-Keplerian. As a result, the amount of angular momentum carried by the matter accreted onto the central black hole also decreases (see the decreasing trend of the $\langle l(r) \rangle$ at ISCO with time in Fig. 4; also see the time evolution of $j_{\text{acc}} = l_l(l_{\text{ISCO}}, t)/M(r_{\text{ISCO}}, l)$ in Figs. 5 and 10). The decrease in the angular momentum reaching the black hole at late times is due to two reasons, i) the mass with initial sub-Keplerian angular momentum distribution for $r > R_0$ starts contributing to accretion onto the black hole, ii) the relative increase in total accretion stress $\alpha^2$ at late times (see Fig. 11). From Figs. 10 and 11 it looks like that the second factor is dominant over the first one because the runs (M-P2, M-1P, M-2P) that display a higher increase in $\alpha^2$, show larger decrease in $j_{\text{acc}}$ at late times.

4 CONVERGENCE

In most practical situations there are no exact analytical solutions of the fluid equations. Therefore, we discretize the continuous partial differential equations and solve them using finite difference/element/volume methods. The approximate solutions rely on discretization of the computational domain. The credibility of the resulting solution is often described in terms of its convergence. While true numerical convergence implies that the truncation errors in the solution vanish as the resolution is increased sufficiently. In case of ideal magnetohydrodynamic (MHD) simulations, or more precisely for turbulent systems in the absence of explicit dissipation, the concept of convergence is ill-defined. This is because, in ideal MHD there are no fixed viscous and resistive length scales, and the dissipation scales are proportional to the grid size. Therefore, as the resolution is increased, new structures are created. We do not include explicit viscosity and resistivity to be able to simulate as large as possible Reynolds and magnetic Reynolds numbers. Since point-wise convergence is not expected in this case, by
convergence we mean that the physically important, globally averaged observables (Maxwell stress, magnetic energy, etc.) do not change significantly with the change in numerical resolution.

To study convergence, we consider three runs L-P4, M-P4, H-P4 with three different resolutions (low, medium and high for the domain extending π/4 in the azimuthal direction). We do a convergence study for an azimuthal extent of Φ0 = π/4, because doing a higher resolution run for a larger Φ0 is computationally expensive. Following Sorathia et al. (2012), we classify the convergence metrics into three categories: physical, numerical and spectral.

4.1 Physical metrics

To check whether MRI remains well resolved, we examine the following physical metrics: mass and angular momentum accretion rate through the ISCO; normalized accretion stresses; and plasma β.

![Figure 2](image-url)

**Figure 2.** Temporal evolution of the accreting torus for our fiducial model M-2P. The color describes the density, and the instantaneous poloidal magnetic field lines are shown by the streamlines. Starting with the constant angular momentum equilibrium torus threaded by a purely poloidal weak magnetic field (first panel), MRI grows in a dynamical time τdyn = 1/Ω (second panel). Parasitic instabilities take over the MRI when the system enters into the non-linear regime and fully developed MHD turbulence is established after some time (e.g. see the third panel). The last three panels show the accretion flow in the quasi-steady state.

| Simulation results | \(\langle\alpha_{\text{Re}}\rangle\times10^{-2}\) | \(\langle\alpha_{\text{M-par}}\rangle\times10^{-2}\) | \(\langle\alpha_{\beta_{\text{sat}}}\rangle\times10^{-2}\) | \(\langle\beta_{\text{sat}}\rangle\) | \(\langle\beta_{\text{sat}}\rangle\) | \(\langle\beta_{\text{sat}}\rangle\) |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| L-P4              | 0.08 ± 0.04     | 0.24 ± 0.07     | 0.3 ± 0.1       | 73 ± 19         | 1 ± 2           | 9.7 ± 0.8       | 4.6 ± 0.6       |
| M-P4              | 0.3 ± 0.1       | 1.5 ± 0.3       | 1.9 ± 0.4       | 31 ± 5          | 10.5 ± 0.9      | 33 ± 3          | 13.4 ± 0.7      |
| M-P2              | 0.8 ± 0.3       | 3.2 ± 0.7       | 4.0 ± 0.9       | 15 ± 3          | 19 ± 4          | 54 ± 11         | 14 ± 1          |
| M-1P              | 0.8 ± 0.2       | 3.0 ± 0.5       | 3.8 ± 0.6       | 16 ± 2          | 18 ± 3          | 52 ± 7          | 14.3 ± 0.6      |
| M-2P              | 0.7 ± 0.1       | 2.4 ± 0.3       | 3.1 ± 0.3       | 20 ± 2          | 16 ± 3          | 44 ± 7          | 14.1 ± 0.4      |
| H-P4              | 0.4 ± 0.1       | 1.4 ± 0.3       | 1.8 ± 0.4       | 34 ± 6          | 20 ± 1          | 58 ± 4          | 13.8 ± 0.5      |

**Table 2.** Value of different volume and time averaged quantities for all the runs. Volume average is within the sub-domain \(S\) and time average is done in the QSS between \(N_{\text{ISCO}} = 200 – 600\). See section 2.4.1 for the details of averaging.

Top panel of Fig. 5 shows the temporal evolution of the mass accretion rate \(\dot{M}(r_{\text{ISCO}}, t)\) through the \(r = r_{\text{ISCO}}\) surface. During the linear phase of the MRI, \(\dot{M}(r_{\text{ISCO}}, t)\) attains a maximum for all three runs. After that, while for higher resolution runs (M-P4, H-P4), \(\dot{M}(r_{\text{ISCO}}, t)\) shows a similar declining trend with time, for the lowest resolution run (L-P4) it shows a non-monotonic behavior. Around \(N_{\text{ISCO}} = 300\), \(\dot{M}(r_{\text{ISCO}}, t)\) for the low resolution run shows an increase and equals that of the higher resolutions runs for some time. The reason is that the mass density in the computation domain is higher in L-P4 due to a lower accretion rate during the initial phase of evolution. To remove the effects of secular decrease of mass from the computational domain, we normalize \(\dot{M}(r_{\text{ISCO}}, t)\) by the total mass \(M(t) = \int_S \rho dV\) within the sub-domain \(S\) (\(S\) is defined in section 2.4.1). The middle panel of Fig. 5 shows the time variation of \(\dot{M}(r_{\text{ISCO}}, t)/M(t)\). For the higher resolution runs \(\dot{M}(r_{\text{ISCO}}, t)/M(t)\) is almost constant, but for the low resolution run it shows a different erratic behavior.
Figure 3. A snapshot of $\beta = 2P/B^2$ at time $N_{ISCO} = 328.51$ for our fiducial run. The turbulent region on both sides of mid-plane within one scale-height consist of patches of both low and high $\beta$. The laminar region close to the black hole above one scale-height is magnetically dominated ($\beta < 1$).

The bottom panel of Fig. 5 shows the time variation of the specific angular momentum accreted across ISCO, $j_{acc} = \bar{L}(r_{ISCO},t)/\bar{M}(r_{ISCO},t)$. Noble et al. (2010) found that $j_{acc}$ increases secularly when the MRI becomes under-resolved. In our simulations, for higher resolution runs (M-P4 and H-P4) $j_{acc}$ remains almost constant with sub-Keplerian values throughout the evolution. There is a slight decrease in $j_{acc}$ at late times for these two higher resolution runs due to the two facts discussed at the end of section 3. For the low resolution run, initially $j_{acc}$ shows an increasing trend, indicating an under-resolved MRI (Noble et al. 2010).

A direct measure of the efficiency of angular momentum transport is provided by the accretion stresses. FIG. 6 shows the variation of the normalized Reynolds stress $\alpha^{Re}(t)$ (top panel), Maxwell stress $\alpha^{Max}(t)$ (middle panel) and the total stress $\langle \alpha^T(t) \rangle = \langle \alpha^{Re}(t) \rangle + \langle \alpha^{Max}(t) \rangle$ (bottom panel) with time. During the initial linear growth of the MRI, all three stresses attain a maximum and then saturate for the non-linear evolution. We can clearly see that the normalized accretion stresses are about a factor of $\approx 6$ smaller for the low resolution run (also see Table 2).

Another physical metric we look at is the plasma $\langle \beta(t) \rangle = (P(t))/(B^2(t)/2)$, which is a measure of magnetization of the accretion flow (a smaller value of $\beta$ implies greater magnetization). FIG. 7 shows the temporal variation of $\beta$ for three different resolutions. We start with a moderately high $\beta_{ini} \approx 890$. During the linearly growing phase of MRI, $\langle \beta(t) \rangle$ decreases (i.e magnetization of the medium increases) due to the immense field amplification; after $N_{ISCO} \approx 200$, $\langle \beta(t) \rangle$ saturates. While for the low resolution run L-P4, the time-averaged (between $N_{ISCO} = 200$ and $N_{ISCO} = 600$ ) $\beta$ in the saturated state is $\langle \langle \beta_{sat} \rangle \rangle \approx 73 \pm 19$, for the higher resolution runs M-P4 and H-P4, $\langle \langle \beta_{sat} \rangle \rangle \approx 31 \pm 5$ and $\langle \langle \beta_{sat} \rangle \rangle \approx 34 \pm 6$ respectively.

So from the above discussion of physical metrics we see that the runs M-P4 and H-P4 show convergence, but the low resolution run L-P4 does not. These results quantify the minimum resolution required for converged simulations. We expect this resolution criterion (per unit scale height) to be valid for the runs with a larger azimuthal extent.

4.2 Numerical metrics and the magnetic tilt angle

To check how well the MRI is resolved, we study the poloidal ($Q_{pol}$) and the toroidal ($Q_{tor}$) quality factors (equations (21) and (22)). Along with them, we also look at the degree of
The variation of the normalized Reynolds stress $\langle \alpha^{Re}(t) \rangle$ (top panel), Maxwell stress $\langle \alpha^{Max}(t) \rangle$ (middle panel) and total stress (Reynolds+Maxwell) $\langle \alpha^T(t) \rangle$ (bottom panel) with time for three different resolutions. The match between accretion stresses of M-P4 and H-P4 indicates the convergence of medium and high resolution runs.

Figure 6.

The variation of the magnetic tilt angle $\theta_B(t)$, measured by the magnetic tilt angle $\theta_B$ (equation (23)) to study the structure of turbulence. Fig. 8 shows the variation of $\langle Q_b(t) \rangle$, $\langle Q_\phi(t) \rangle$ and $\langle \theta_B(t) \rangle$ with time for different resolutions. For all three resolution runs, $\langle Q_b(t) \rangle$ and $\langle Q_\phi(t) \rangle$ show similar trends; after initial increase in the linear phase they saturate in the non-linear regime of evolution. The time averaged (between $N_{ISCO} = 200$ and $N_{ISCO} = 600$) saturated value of $\langle Q_b(t) \rangle$ and $\langle Q_\phi(t) \rangle$ for the converged resolutions scale inversely with the grid resolution; these parameters are much smaller than this scaling for the lowest resolution run (see also Table 2). The magnetic tilt angle $\langle \theta_B(t) \rangle$ is a more physical diagnostic that is independent of resolution. It attains a quasi-steady value for the medium and high resolution runs with $\langle \theta_B(t) \rangle \approx 13^\circ$ to $14^\circ$. The lower resolution run L-P4 stands out with $\langle \theta_B(t) \rangle = (4.6 \pm 0.6)^\circ$; i.e., with much smaller magnetic field anisotropy.

Hawley et al. (2013) compared the dependence of meridional and azimuthal resolutions, and suggested a resolvability condition $\langle Q_b(t) \rangle \langle Q_\phi(t) \rangle \geq 250$. On the other hand, the criteria for convergence with respect to the magnetic tilt angle is $\delta B > 12^\circ$ (Pessah 2010). Previous global studies (Sorathia et al. 2012, Hawley et al. 2013) found $\theta_B \approx 13^\circ$ for the converged runs. Both the runs M-P4 and H-P4 satisfy all the criteria required for the quality factors and the magnetic tilt angle.
4.3 Spectral metrics

Power spectrum is an important tool to investigate the structure of turbulence. Fig. 9 shows the toroidal power spectral density (in the azimuthal direction) for thermal pressure $P$ (top left), $v_\phi$ (top right), $B_\phi$ (bottom left) and $B_{\phi r}$ (bottom right). While calculating $\langle |P_r(m)|^2 \rangle$, $\langle |P_{\phi r}(m)|^2 \rangle$ and $\langle |P_{\phi \phi}(m)|^2 \rangle$, we follow the procedure described in Sec. 2.4.4, for $B_{\phi r}$, we calculate $\langle \sqrt{P_{\phi r} P_{\phi r}(m)} \rangle$. All the spectra display the typical features of MRI turbulence; compensated spectra are flat at large scales and most of the power is concentrated at large scales. For higher resolution runs (M-P4 and H-P4), we also see the build up of an inertial range. As expected, with the increase in resolution, the inertial range extends.

The azimuthal power spectra we obtained agree with those obtained by Sorathia et al. (2012) for their zero-net-flux converged global models but not with those obtained for the zero-net-flux non-converged local simulations of Fromang & Papaloizou (2007).

5 EFFECTS OF AZIMUTHAL DOMAIN SIZE

In the previous section we see that the medium resolution is adequate to attain convergence. In this section we compare the results obtained for four medium resolution runs with different azimuthal extents $\Phi_0$: M-P4 ($\Phi_0 = \pi/4$), M-P2 ($\Phi_0 = \pi/2$), M-1P ($\Phi_0 = \pi$) and M-2P ($\Phi_0 = 2\pi$). We keep the same resolution for per unit azimuthal extent for all these runs. For details of the runs, see Table 1.

5.1 Mass and angular momentum accretion through ISCO

The three panels of Fig. 10 show the time variation of the mass accretion rate per unit azimuthal angle $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ (top left), $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ (top right), $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ (bottom right). Time average is done between $N_{\text{ISCO}} = 450$ and 600.

$\dot{M}(r_{\text{ISCO}},t)/\Phi_0$, $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ and the specific angular momentum accreted through the ISCO ($j_{\text{acc}}$) for runs with different azimuthal extents. We compute $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ instead of $\dot{M}(r_{\text{ISCO}},t)$ to compare the models with different azimuthal extents ($\Phi_0$) on equal footing. While $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ for all models are almost similar, run M-P4 shows a smaller $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ after $N_{\text{ISCO}} = 300$. For model M-P2 we see some rapid fluctuations both in $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ and $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ at certain times. A similar trend is followed by $j_{\text{acc}}$ for the run M-P4 stands out from that of the other three runs. For both M-P2 and M-1P, we observe depressions in the value of $j_{\text{acc}}$ at certain times. This is an indication of higher angular momentum transport at those times which will be discussed in the next subsections.

5.2 Accretion stresses

Fig. 11 shows the time variation of the normalized accretion stresses $\langle \alpha^R(t) \rangle$, $\langle \alpha^{\max}(t) \rangle$ and $\langle \alpha^T(t) \rangle$ respectively) for runs with different azimuthal extents. While Reynolds stresses $\langle \alpha^R(t) \rangle$ are similar for runs M-P2, M-1P and M-2P, that for M-P4 is smaller (top panel). Maxwell stress $\langle \alpha^{\max}(t) \rangle$ (middle panel) for runs M-P2 and M-1P show strong time variability and is larger than that for the runs M-P4 and M-2P. As a result, for the runs M-P2 and M-1P, the total accretion stress $\langle \alpha^T(t) \rangle$ (bottom panel) shows outbursts at certain times which is an indication of higher angular momentum transport at those instances. Sometimes this gives rise to variability in $j_{\text{acc}}(t)$ and $\dot{M}(r_{\text{ISCO}},t)/\Phi_0$ during outbursts in $\langle \alpha^T(t) \rangle$ (see Fig. 10).

It is to be mentioned that the volume averaged value of the accretion stress does not show the full picture, and...
its spatio-temporal behavior is a useful diagnostic. For instance, for run M-P2, we see three outbursts in \(\langle \alpha^T \rangle\), around \(N_{\text{ISCO}} = 250, 400, 600\). While during the outburst around \(N_{\text{ISCO}} = 250\), we see a decrease in \(M(r_{\text{ISCO}}, t)/\Phi_0\) and a dip in \(j_{\text{acc}}\); during outbursts at \(N_{\text{ISCO}} = 400, 600\), we observe a slight rise in \(M(r_{\text{ISCO}}, t)/\Phi_0\) and a steady \(j_{\text{acc}}\), indicating an efficient and steady angular momentum transport. On the other hand, we see a decrease both in \(M(r_{\text{ISCO}}, t)\) and in \(j_{\text{acc}}\) around time \(N_{\text{ISCO}} = 500\), even though \(\langle \alpha^T \rangle\) does not show any outburst. This apparent uncorrelated behaviour will be clear if we see Fig. 12.

We show the spatio-temporal distributions of dominant accretion stress \(\langle \alpha^{\text{max}}(r, t) \rangle = -(B_r B_\theta(r, t))/|P(r, t)|\) for different runs in Fig. 12. While spatio-temporal distribution of \(\alpha^{\text{max}}\) is very smooth for run M-2P, it shows strong fluctuations for runs with restricted azimuthal domains, specially for the runs M-P2 and M-1P, which show outbursts in accretion stresses at certain times. Let us correlate the fluctuations in accretion stress with that in mass accretion rate and the angular momentum flux at ISCO as described before (in Fig. 10) and to do that we consider run M-P2. For run M-P2, around \(N_{\text{ISCO}} = 400\) and \(600\), increase in \(\alpha^{\text{max}}\) happens across a large radial range, giving rise to a smooth mass accretion rate at ISCO. While around \(N_{\text{ISCO}} = 250\), the outburst in volume averaged \(\alpha^{\text{max}}\) is mostly due to the increase in its value near the central accreting black hole. The large radial gradient in \(\alpha^{\text{max}}\) gives rise to a higher angular momentum transport at small radii \((r < 20)\), and a less efficient angular momentum transport at larger radii. As a result, matter close to black hole is drained quickly due to efficient angular momentum transport, while the outer region cannot supply enough matter to the inner region due to inefficient transport. This results in a reduction in \(j_{\text{acc}}\) and in \(M(r_{\text{ISCO}}, t)/\Phi_0\). For the same reason, we see a dip both in \(M(r_{\text{ISCO}}, t)\) and \(j_{\text{acc}}\) around \(N_{\text{ISCO}} = 500\), although the volume average value of \(\alpha^{\text{max}}\) is close to the average value. Also, the long coherent structures of \(\langle \alpha^{\text{max}}(r, t) \rangle\) at certain times for models with restricted azimuthal domains hint at the sudden increase in the mean field at those instances.

### 5.3 Quality factors and magnetic tilt angle

Fig. 13 shows the temporal evolution of the poloidal \((\theta_\theta(t); \text{top panel})\) and toroidal \((\chi_B(t); \text{middle panel})\) quality factors along with the evolution of magnetic tilt angle \(\theta_B(t)\) (bottom panel). All the runs satisfy the convergence criteria \(-\langle \theta_B(t) \rangle \geq 250\) and \(\langle \theta_B(t) \rangle \geq 12^\circ\). Other than for the run M-P4 with azimuthal extent \(\pi/4\), for all other runs \(\langle \theta_B(t) \rangle\) and \(\chi_B(t)\) show an increasing trend, indicating better resolvability at late times. The most interesting part is the evolution of \(\langle \theta_B(t) \rangle\); for all runs its value is clustered around \(13^\circ - 14^\circ\) (also see Table 2). This strengthens the claim (see section 4.2 and Sorathia et al. 2012) that \(\theta_B\) is the most useful metric of convergence for MRI turbulence.

### 5.4 Evolution of the magnetic field

Fig. 14 shows the time variation of magnetic energies associated with radial (top), meridional (middle), and azimuthal (bottom) components of the magnetic field for the four runs with different azimuthal extents. All the runs show similar magnetic energy in the quasi-steady state, albeit runs with restricted azimuthal extents (M-P4, M-P2, M-1P) show larger time variability. Also, for the run M-P4, magnetic energy decreases rapidly at late times (for \(N_{\text{ISCO}} > 500\)). On
the other hand, the run M-2P with the natural azimuthal extent displays a smoother magnetic energy evolution.

Fig. 15 shows the time evolution of the net toroidal flux $F_\phi(t)$ defined as,

$$F_\phi(t) = \int_{r_{\infty}}^{r_{\infty}+\delta r_H} \int_{\theta_{\infty} - \delta \theta_H}^{\theta_{\infty} + \delta \theta_H} B_\phi(r, \theta, \phi = \Phi_c) r dr d\theta. \quad (26)$$

We calculate $F_\phi(t)$ at two locations, at $\Phi_c = 0$ and at $\Phi_c = \Phi_0/2$. Since we start with a poloidal field, $F_\phi = 0$ at the beginning. Toroidal fields build up due to shear, with different signs in the two hemispheres. Due to the underlying symmetry, net toroidal flux is zero until MRI induced MHD turbulence generates asymmetry and the disk attains a net toroidal flux. With time $F_\phi(t)$ changes sign aperiodically unlike in the previous global simulations of geometrically thin disks (Fromang & Nelson 2006; O’Neill et al. 2011; Beckwith et al. 2011; Flock et al. 2011), which show a more periodic behavior. The time reversal of net toroidal flux hints at gradual expulsion of mean $B_\phi$ from the mid-plane towards the tenuous sub-Keplerian regions and replacement with the field of opposite sign due to a dynamo process (Davis et al. 2010). The dynamo process is discussed in detail in later sections. Although the toroidal flux evolution is qualitatively similar for all the runs, runs M-P2 and M-1P show noticeably higher amplitudes of the toroidal flux. Also, for M-P2 $F_\phi(t)$ remains positive for a longer time (≈ 65 percent of the total run time) compared to the other three runs, where the ratio of time spent in positive and negative state of $F_\phi(t)$ is roughly 1 : 1. In addition, the coherence between the fluxes calculated at different $\phi = \Phi_c$ surfaces (for all runs) reflects the large scale structure of $B_\phi$.

5.5 Mean versus turbulent fields

In Fig. 12, the coherence in $\alpha^{\text{Max}}$ across a large radius for the runs with restricted azimuthal domains indicates a significant role of the mean fields in those instances. In this section we compare the relative contribution from mean and turbulent parts of the flow. We decompose some variable $q$ into a mean $\bar{q}$ and a turbulent $q'$

$$q = \bar{q} + q', \quad (27)$$

where the decomposition follows the Reynolds rules, $\bar{q} = 0$, $q' = 0$.

Fig. 16 shows the time variation of the mean (dashed line) and the turbulent (solid line) energies associated with the radial (top panel), meridional (middle panel) and azimuthal components (bottom panel) of the magnetic field for the four runs with different azimuthal extents. For all the runs and all the components of magnetic fields, the contribution from the mean fields is smaller compared to that of turbulent fields. In addition, mean field evolution shows a coherent trend – larger the azimuthal extent of the simulation domain, smaller the mean magnetic field energy (see Table
3). On the other hand, turbulent magnetic energy evolution does not show any definite trend. These results are different from Flock et al. (2012), who found a higher strength both for mean and turbulent fields for the restricted domain sizes. While the turbulent magnetic energy evolution for run M-2P shows a smooth behavior, that for the runs with restricted azimuthal extents show larger variability. On average, turbulent energies for the runs M-1P and M-2P are higher than the other two runs (see Table 3).

Fig. 17 shows the azimuthal power spectral density (PSD) \( \langle \mathcal{P}_{B_r,\phi,\phi}(m) \rangle \) (equation 25) of different magnetic field components for the runs M-P4, M-P2, M-1P and M-2P. The time average is done over \( N_{\text{ISCO}} = 200-600 \). All the runs show almost similar PSD. For all the models and magnetic field components, most of the power is concentrated at small wavenumbers (i.e., at large scales). While PSDs for \( B_r \) and \( B_{\phi} \) start saturating at \( m = 8 \), \( \langle \mathcal{P}_{B_r}(m) \rangle \) does not show any saturation. For the most dominant field component \( B_{\phi} \), run M-1P has a slightly larger power at small \( m \) compared to the other three runs. This is reflected in the time averaged value of \( B_{\phi}^2 \) in Table 3 (in accordance with the Parseval’s theorem).

Total accretion stress is comprised of the Reynolds stress and the dominant Maxwell stress. While Reynolds stress is solely due to turbulence, Maxwell stress can arise due to the statistical correlation between the turbulent fields \( (B_r',B_\phi') \) as well as between mean fields \( (B_r,B_\phi) \). Fig. 18 shows the ratio of mean Maxwell stress \( B_r B_\phi \) to turbulent Maxwell stress \( B_r' B_\phi' \) for the runs with different azimuthal extent. For all the four runs, the relative contribution from the turbulent field is higher than the mean fields. Although, as expected,
the relative contribution of the mean fields for restricted azimuthal domains is higher. Also notice the increase in the mean to turbulent stress ratio at times when the total stress and quality factors show fluctuations (compare Fig. 18 with Figs. 11 and 13). This coincidence indicates a better correlation between mean fields in those instances. It is interesting to see that $\langle \langle B_i B_0 \rangle \rangle = \langle \langle B_i B_0 \rangle \rangle + \langle \langle B'_i B'_0 \rangle \rangle$ for run M-P2 is smaller compared to that for runs M-1P and M-2P (see Table 3). Still run M-P2 shows highest value of $\langle \langle B_{\text{sat}} \rangle \rangle$ among all the runs (see Table 2). The reason is that due to a higher mass loss, run M-P2 has comparatively lower $\langle P(t) \rangle$ and hence a larger normalized stresses.

Fig. 19 shows the variation in kinetic energy per unit mass $v^2 (i \text{ stands for } r, \theta, \phi)$ associated with the mean and turbulent flows for the four runs. Unlike magnetic energy, which is an extensive variable, $v^2$ is an intensive quantity. Therefore, its evolution is independent of mass available in the simulation domain. As a result, $v^2$ does not show any secular decrease in time due to the mass loss through the inner/outer boundaries. Instead, we see a slow secular increase in $v^2$ (both for mean and turbulent flows) with time, which implies that the resolvability gets better as the time passes (also see Fig. 13). The increasing trend is less prominent for the run M-P4 compared to other three runs. One marked difference between the evolutions of $v^2$ and $B^2$ is that while the turbulent magnetic field amplitudes are always larger than the mean magnetic field amplitudes in the quasi-steady state, velocity fields do not follow any definite trend. While for the radial component of velocity, for runs M-1P and M-2P, $v_r^2 > \bar{v}_r^2$ and for run M-P2, $\bar{v}_r^2 \approx \bar{v}_r^2$ for run M-P4; for meridional component, $v_\theta^2 > \bar{v}_\theta^2$ for the runs M-P2, M-1P and M-P2, and $v_\phi^2 \approx \bar{v}_\phi^2$ for the run M-P4. Due to the dominant rotation, as expected, for all runs $v_\phi^2 \ll \bar{v}_\phi^2$. It is interesting to see that for all the velocity components, turbulent velocity amplitude is smallest for the run M-P4, compared to the other three runs which have a comparable amplitude. This is a signature of a higher level of turbulence in the runs M-P2, M-1P, M-2P.

### 5.6 Spatio-temporal evolution of mean magnetic field

In previous sub-sections, we see that accretion stress attains saturation and mean fields play a significant role in the time evolution of the flow (by causing sudden rise in $\alpha_{\text{sat}}$, for example). To get a clearer picture of the scenario, in this sub-section we look into the spatio-temporal (both radial and meridional) behavior of the mean magnetic field.

#### 5.6.1 Meridional variation

A well-demonstrated phenomenon in stratified accretion disks is an oscillating mean toroidal magnetic field due to buoyant rise of the toroidal magnetic field from the midplane to the upper coronal regions. This famous feature known as the 'butterfly diagram' is seen in local (Brandenburg et al. 1995, Hawley et al. 1996, Stone et al. 1996, Johansen et al. 2009, Davis et al. 2010, Gressel 2010, Simon et al. 2012, Bodo et al. 2012) as well as in global (O’Neill et al. 2011, Flock et al. 2012, Parkin & Bicknell 2013, Parkin 2014, Hogg & Reynolds 2016) MHD simulations with the oscillation timescale $\sim 10$ local orbits. Fig. 20 shows the variation of mean toroidal field $B_\phi(r = 20, \theta, t)$ with latitude $(\pi/2 - \theta)$ and time for our geometrically thick $(H/R \sim 0.5)$ radiatively inefficient accretion flow. Here time is expressed in local orbit at $r = R_0 = 20$, which equals $8.22 \times \text{yr}$. Unlike previous studies mentioned above, we do not see a regular cyclic behaviour of $B_\phi$, rather we observe an intermittent sign reversal in the mean toroidal field. Recently,

[Figure 18: The ratio of mean to turbulent Maxwell stress for the runs with different azimuthal domain sizes. The relative contribution of mean Maxwell stress $\bar{B}_i \bar{B}_0$ relative to the turbulent Maxwell stress $B'_i B'_0$ decreases as the azimuthal extent of the simulation domain increases.]

[Figure 19: Temporal evolution of the mean (dashed lines) and the turbulent (solid lines) velocity fields for the runs M-P4, M-P2, M-1P and M-2P with different azimuthal extents. Unlike magnetic energy density, which scales with the mass density, $v^2$ is an intensive quantity. So it does not depend on the mass available in simulation domain and hence it does not show any secular decrease in time as a result of mass accretion through the inner boundary. The details are discussed in the main text.]

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Hogg & Reynolds (2018) also observed an intermittent cycle of the mean toroidal field with $H/R \sim 0.4$. The runs with restricted azimuthal domain (M-P4, M-P2, M-1P) show stronger mean fields variations compared to the run M-2P. Also notice the ubiquity of positive $B_\phi$ at late times for the run M-P2; that is the reason why toroidal flux $F_\phi(t)$ remains positive for longer time compared to the other runs in Fig. 15.

### 5.6.2 Radial variation

In this section we study the time evolution of the mean fields in the radial direction. In the meridional direction we average from mid-plane ($\theta = \pi/2$) to $\theta_H$ in the northern hemisphere (NH), as mean fields often have opposite signs in both the hemispheres (as seen in Fig. 20).

Fig. 21 shows the time evolution of the mean radial (top panel), meridional (middle panel) and toroidal (bottom panel) magnetic fields along the radius for the four runs. For all the runs, we see an intermittent sign reversal of $\langle B_r(r, t) \rangle$ with time. Long coherent radial structures are seen, which are most prominent for runs with restricted domain sizes. Unlike the mean radial magnetic field, mean meridional magnetic field $\langle B_\phi(r, t) \rangle$ shows a patchy distribution. Beyond $r = 30$, patchy patterns are observed (specially for the run M-2P) to translate in time, which is a signature of the presence of an outflow at $r \geq 30$. Like $\langle B_r(r, t) \rangle$, the mean toroidal magnetic field $\langle B_\phi(r, t) \rangle$ shows long coherent radial structures, which are anti-correlated with $\langle B_r(r, t) \rangle$. The strong anti-correlation between the mean radial and toroidal fields at certain times give rise to enhanced accretion stresses (see Fig. 11 and Fig. 12), which results in a higher mass accretion rate through the ISCO. As expected, all the runs with restricted azimuthal domains show stronger radial filaments $\langle B_r \rangle$ and $\langle B_\phi \rangle$ / patches $\langle B_\phi \rangle$ of mean fields compared to the run M-2P.

### 5.7 Structure of the flow

In the previous sub-sections we discuss the effects of azimuthal extent of the computational domain on the temporal evolution of different quantities. In this section, we study how time averaged properties of the accretion flow differ with different azimuthal domain sizes. As we are studying the radiatively inefficient accretion flows (RIAFs), which are geometrically thick ($H/R \sim 0.5$), we concentrate only on the
radial profiles, defined by equation 13. Time average is done over $N_{ISCO} = 200 - 600$.

5.7.1 Velocities

Four panels of Fig. 22 show the radial profiles of different flow velocities for our four runs. The dashed blue vertical line shows the location of the ISCO. Radial

$$\langle v_r(r) \rangle = \frac{\langle \rho v_r(r) \rangle}{\langle \rho(r) \rangle}$$ (28)

and azimuthal

$$\langle v_\theta(r) \rangle = \frac{\langle \rho v_\theta(r) \rangle}{\langle \rho(r) \rangle}$$ (29)

velocities are compared in the top panels. The bottom panels show the radial profiles of sound

$$\langle c_s(r) \rangle = \sqrt{\frac{\langle P(r) \rangle}{\langle \rho(r) \rangle}}$$ (30)

and Alfvén

$$\langle v_A(r) \rangle = \sqrt{\frac{\langle B^2(r) \rangle}{\langle \rho(r) \rangle}}$$ (31)

speeds respectively.

At $r \sim 40$, $\langle v_r(r) \rangle$ shows a sharp gradient indicating the region of ‘inflow equilibrium’, the radius inside which the accretion flow is in a statistically stationary state. This scale is given by $r_{eq} \sim r_{visc}[v_r]$, where $r_{visc}$ is the viscous time. We see almost the same values of $r_{eq} \approx 40$ for all the runs. In the inset, the radii pointed by filled circles and $*$ denote the locations where flow becomes super-sonic ($|v_r| > c_s$) and super-Alfvénic ($|v_r| > v_A$) respectively. As subsonic and sub-Alfvénic flow moves inward, it becomes both super-sonic and super-Alfvénic within the ISCO. As a result, accretion flow outside the outermost critical point (here Alfvén point)
is not affected by any disturbances created around the inner boundary. Although, the nature of the radial profiles $\langle v_r(r) \rangle$ are approximately similar in all four runs, its absolute value for the run M-P4 is smaller by a factor of 2 compared to that for the other three runs.

While the radial velocity shows a transonic nature, azimuthal velocity $\langle v_\theta(r) \rangle$ is highly super-sonic and super-Alfvenic at all radii. Apart from the run M-P4, all other runs maintain a similar sub-Keplerian profile of $\langle v_\theta(r) \rangle$ over all radii (see also Fig. 4 for the time evolution of angular momentum for the run M-2P). For the run M-P4, the region between $r = 6 - 10$, $\langle v_\theta(r) \rangle$ has an almost Keplerian profile (for clarity see the inset), which again indicates a less efficient angular momentum transport compared to the other three runs. Alfven speed always remains sub-thermal ($\beta = 2c^2_s/\gamma v_\phi^2 > 1$) all the way up to the inner boundary for all four runs, in spite of the fact that the Alfven speed rises more steeply with decreasing radius compared to the sound speed.

Therefore, the study of the time-averaged (in QSS) radial profiles of characteristic velocities suggest a somewhat different nature of the run M-P4 compared to the other three runs.

### 5.7.2 Magnetic fields

To look at the radial structure of magnetic fields in a concise way, we define a poloidal to toroidal magnetic energy ratio as

$$f_{pt}(r) = \frac{\langle |B_r|^2(r) \rangle + \langle |B_\phi|^2(r) \rangle}{\langle |B_r|^2(r) \rangle},$$

which describes the radial variation of the poloidal to toroidal magnetic energy ratio. It is also a measure of the efficiency of production of poloidal fields (predominantly radial field) out of the toroidal fields. Fig. 23 shows the radial profiles of $f_{pt}(r)$ for four runs M-P4, M-P2, M-1P and M-2P. As we approach ISCO from the outer radii, the poloidal to toroidal field ratio increases very slowly; just outside the ISCO, we see a rapid increase in $f_{pt}$. While outside the ISCO, the total magnetic field is dominated by the toroidal component (> 80 percent of total field energy), inside it the importance of poloidal field (mainly the radial field) increases very rapidly. This result along with those shown in

![Figure 23. Radial variation of f_{pt} (see equation 32) for the four runs with different azimuthal domain sizes. Time average is done between N_{ISCO} = 200 and N_{ISCO} = 600.](image)

Fig. 22 suggest that the dynamics of the magnetic field inside ISCO is controlled by flux-freezing and almost radial free fall rather than turbulence that controls the evolution of magnetic field outside ISCO (for a more detail discussion see section 5.2 of Beckwith et al. (2011)).

Although the above mentioned trend in magnetic field configuration is followed for all runs, run M-P4 stands out with a lower efficiency of conversion of toroidal field to poloidal field. This implies, like the velocity field, the steady state nature of the magnetic field matches for the runs with azimuthal extent $\Phi_0 \geq \pi/2$.

### 6 DYNAMO

Previous local (e.g. Brandenburg et al. 1995; Gressel 2010; Bodo et al. 2012) and global (e.g. O’Neill et al. 2011; Flock et al. 2012; Parkin & Bicknell 2013) studies of MRI driven accretion flow identify the oscillating mean fields with a dynamo process. All these works simulate thin disks ($H/R \ll 1$) in a local or global frame-work, in which the scale separation is self-evident. In this work, we study the dynamo process in a geometrically thick disk in which $H/R \sim 0.5$. It is interesting to see in Fig. 20 that we don’t see any periodic oscillations of the mean toroidal fields ($B_\phi$) for any of the runs, rather we see an intermittent sign reversal of ($B_\phi$) in both the hemispheres. In this section we analyze the runs with the full 2$\pi$ azimuthal extent (M-2P).

#### 6.1 Symmetry of mean fields

The geometry of mean fields is described by its symmetry about the mid-plane ($\theta = 90^\circ$). If radial and azimuthal fields are of different signs in SH and NH, the magnetic field configuration is dipole dominated (or more precisely, dominated by the even $l$ modes). On the hand, quadrupole dominated field configuration demands the same signs of $B_r$ and $B_\phi$ in the two hemispheres. The symmetry of the mean fields within one scale-height is not very clear from Fig. 20. To examine it more carefully, we look at the time variation of $\theta$-averaged mean fields in northern ($\theta_D < \theta < 90^\circ$; NH) and southern ($90^\circ < \theta < 90^\circ + \theta_D$; SH) hemispheres calculated at $r = R_0 = 20$ for the most realistic run M-2P. Top three panels of Fig. 24 show the variations of mean radial ($\langle B_r(R_0, t) \rangle$), meridional ($\langle B_\phi(R_0, t) \rangle$) and toroidal ($\langle B_\phi(R_0, t) \rangle$) components of the magnetic field respectively. While ($B_\phi(R_0, t)$) in SH and NH does not show any correlation, ($B_r(R_0, t)$) and ($B_\phi(R_0, t)$) show oscillations between dipole and quadrupole symmetry. To describe the symmetry concisely, we look at the parity of the mean fields (following Flock et al. 2012) defined as

$$P_\theta = \frac{B_{dip} - B_{quadr}}{B_{dip} + B_{quadr}}$$

(33)

where $B_{dip} = (B_{\phi}^{AS} + (B_{\phi}^{N})^2 + (B_{\phi}^{S})^2)$ and $B_{quadr} = (B_{\phi}^{AS})^2 + (B_{\phi}^{S})^2$. Here, symmetric and anti-symmetric parts of the field are defined as $B_r^S = (B_{r}^{N})^2 + (B_{r}^{S})^2)/2$ and $B_r^{AS} = (B_{r}^{N} - B_{r}^{S})^2)/2$; $r = r, \theta, \phi$. Bottom panel of Fig. 24 shows the variation of parity with time. As we initialize with a dipolar field, in the beginning, $P_\theta = +1$. In the quasi-steady state, parity oscillates between dipole ($P_\theta > 0$) and quadrupole ($P_\theta < 0$) dominated configurations. In the
### Figure 24. Time variation of average mean fields in both the hemispheres for the most realistic run M-2P. The average is done over the $\theta$ in the northern ($90^\circ < \theta < 90^\circ + \Theta_N$; NH) and southern hemispheres ($90^\circ < \theta < 90^\circ$; SH) and calculated at $r = 20$. Top three panels show the variations of mean radial ($\langle B_r, R_0, t \rangle$), meridional ($\langle B_\phi, R_0, t \rangle$) and toroidal ($\langle B_\theta, R_0, t \rangle$) components of the magnetic field respectively. Bottom panel shows the variation of mean field parity (Eq. 33) with time. On average, dipole symmetry ($\langle Pa > 0 \rangle$ prevails over quadrupole symmetry ($\langle Pa < 0 \rangle$). Note that erratic parity at late times coincides with irregular dynamo cycles seen in the bottom right panel of Fig. 20.

### Figure 25. Variation of $\alpha_{\text{dy}n} = \mathcal{E}_s / \mathcal{E}_\phi$ with $\theta$ and time at $r = R_0$. Time is expressed in units of local orbit at $r = R_0$. $\alpha_{\text{dy}n}$ does not show any coherent pattern as seen in previous local and global simulations (see the text); instead we see a patchy distribution over $\theta$ and time. This may be a result of $\alpha$-quenching.

### Figure 26. The meridional variation of average (over time) kinetic ($\alpha_{\text{kin}}^{\text{dyn}}$) and magnetic ($\alpha_{\text{mag}}^{\text{dy}n}$) helicities at $r = 20$ (top panel) and $r = 15$ (bottom panel). The sum of the $\alpha_{\text{kin}}^{\text{dy}n}$ and the $\alpha_{\text{mag}}^{\text{dy}n}$ is shown by black line ($\alpha_{\text{dy}n} = \alpha_{\text{kin}}^{\text{dy}n} + \alpha_{\text{mag}}^{\text{dy}n}$).

### 6.2 $\alpha - \Omega$ dynamo and $\alpha$ quenching

In addition to intermittent sign flipping of mean fields, the saturation of magnetic energy in our global stratified simulation indicates a dynamo process. MRI is compatible with both direct (non-helical turbulent dynamo) and inverse (helical mean-field dynamo) dynamo processes (Blackman & Tan 2004). Direct dynamos can work both in unstratified and stratified MRI simulations, in which the field grows by random stretching and twisting (Kazantsev 1968; Parker 1979). In a stratified medium, turbulence can be helical due to two effects (Blackman & Tan 2004, Gressel 2010)- i) traditional Parker-type $\alpha - \Omega$ dynamo mechanism (Parker 1979; Choudhuri 1998), ii) due to magnetic buoyancy (Tout & Pringle 1992; Brandenburg 1998; Rüdiger & Pipin 2000). The generation of large-scale field by turbulent fields can be modeled using the non-linear mean field theory.

The mean field evolution equations are given by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ \mathbf{v} \times \mathbf{B} + \nabla \times \mathbf{B} - \lambda \nabla \times \mathbf{B} \right]$$

(34)

where $\lambda$ is the microscopic diffusivity. The first term within the bracket on the right-hand side describes the effects of mean fields and flows on its evolution. The effect of turbulence on the mean field evolution is captured through mean emf $\mathcal{E} = \nabla \times \mathbf{B}$. While shear in the mean velocity field dominates the generation of mean toroidal field $\mathbf{B}_t$, mean poloidal field (both $\mathbf{B}_r$ and $\mathbf{B}_\theta$) generation is mainly attributed to the poloidal gradient of the mean emf, $\partial \mathcal{E}_s / \partial \theta$ and $\partial \mathcal{E}_s / \partial r$. The key idea of mean field dynamo is to express mean emf in terms of mean velocity and magnetic fields and statistical properties of turbulent velocity fields. If we use a closure (e.g see Choudhuri 1998)

$$\mathcal{E}_s = \alpha_{ij} \mathbf{B}_j + \lambda_{ijk} \partial_k \mathbf{B}_k$$

(35)

we obtain the classical $\alpha - \Omega$ dynamo. Here, $\lambda$ is the turbulent diffusion and $\alpha_{ij}^{\text{dy}n}$ is the tensor which is crucial for the
mean field amplification. If we assume isotropic turbulence, $\alpha_{ij}^{\text{dyn}}$ only contains the diagonal terms. Then the term

$$\alpha_{ij}^{\text{dyn}} \approx \frac{E_i}{B_j} \tag{36}$$

(neglecting the off-diagonal terms of $\alpha_{ij}^{\text{dyn}}$ and second term in equation 35) captures toroidal to poloidal field conversion. From symmetry arguments, we expect different sign of $\alpha_{ij}^{\text{dyn}}$ in the two hemispheres. On one hand local simulations provide negative (positive) sign of $\alpha_{ij}^{\text{dyn}}$ in NH (SH) (Brandenburg et al. 1995; Brandenburg & Donner 1997; Davis et al. 2010, in Gressel 2010 for $z < H$), on the other hand global simulations show the opposite sign (Arlt & Rüdiger 2001; Flock et al. 2012). Although recently, Hogg & Reynolds (2018) claimed that they observed a very weak $\alpha_{ij}^{\text{dyn}}$ of negative sign (in NH) in global simulations.

Fig. 25 shows the variation of $\alpha_{ij}^{\text{dyn}}$ (calculated at $r = R_0$) with $\theta$ and $\phi$. Time is expressed in units of local orbit at $r = R_0$. Unlike previous studies, we do not see any coherent spatial-temporal distribution of $\alpha_{ij}^{\text{dyn}}$; instead we see a patchy distribution. This is probably due to $\alpha$-quenching (Pouquet et al. 1976; Ruediger & Kichatinov 1993) which arises due to the back reaction of the Lorentz force on the helical fluid motions. As a result, the effective $\alpha$ turns out to be the combination of kinematic and magnetic contributions: $\alpha_{ij}^{\text{dyn}} = \alpha_{ij}^{\text{kin}} + \alpha_{ij}^{\text{mag}}$ (Gressel 2010). While $\alpha_{ij}^{\text{kin}}$ gives the forcing, $\alpha_{ij}^{\text{mag}}$ quenches the kinematic forcing through non-linear response. Assuming isotropy (which need not be valid in rotating MHD turbulence), we calculate kinematic and magnetic $\alpha$ in the following way,

$$\alpha_{ij}^{\text{kin}} = \frac{1}{3} \tau \nabla \times (\nabla \times v^i), \tag{37}$$

$$\alpha_{ij}^{\text{mag}} = \frac{1}{3} \tau \frac{B^i}{\sqrt{\rho}} \left( \nabla \times \frac{B^j}{\sqrt{\rho}} \right), \tag{38}$$

where the correlation time $\tau$ is a free parameter. As we are interested in the relative importance of $\alpha_{ij}^{\text{kin}}$ and $\alpha_{ij}^{\text{mag}}$ not on their absolute values, we simply use $\tau = \Omega^{-1}$. Fig. 26 shows the meridional variation of average kinetic and magnetic $\alpha$ at $r = 20$ (top panel) and $r = 15$ (bottom panel). We also plot the sum of the $\alpha$-s to check the relative strength of the kinematic and magnetic $\alpha$-s. We look at the meridional variation of $\alpha$-s at different radii to check consistency of its trend. The vertical dashed black line describes the angle ($\theta_B$) corresponding to one scale-height. At both radii, $\alpha_{ij}^{\text{kin}}$ shows a definite trend. In the NH, close to the mid-plane, it is negative and changes sign above one scale height (see insets). In SH, $\alpha_{ij}^{\text{kin}}$ just follows the opposite trend. On the other hand, $\alpha_{ij}^{\text{mag}}$ is always negative in the NH with little bit of randomness near the mid-plane. Although at $r = 15$, due to a better behaved $\alpha_{ij}^{\text{mag}}$, the resultant $\alpha_{ij}^{\text{dyn}}$ shows a regular trend within one-scale height with a negative sign (in NH). Above one-scale height, $\alpha_{ij}^{\text{kin}}$ and $\alpha_{ij}^{\text{mag}}$ have different signs with almost equal magnitudes. Their sum have different signs at different $\theta$ depending on the relative strength of kinetic and magnetic $\alpha$. This may be the reason behind the randomness of $\alpha_{ij}^{\text{dyn}}$ seen in Fig. 25.
Our 3D global ideal adiabatic MHD simulations of RIAFs with different azimuthal extents show some similarities and dissimilarities with the previous (mostly isothermal) studies. We discuss them one by one.

7.2 Dependence on azimuthal domain size

Our 3D global ideal adiabatic MHD simulations of RIAFs with different azimuthal extents show some similarities and dissimilarities with the previous (mostly isothermal) studies. We discuss them one by one.

7.2.1 Higher accretion stress for $\Phi = \pi/2, \pi$

While runs with $\Phi = \pi/2$ and $\pi$ overestimate the accretion stress $\alpha^2$ compared to the natural azimuthal extent of $\Phi_0 = 2\pi$, $\alpha^2$ for the run M-P4 ($\Phi_0 = \pi/4$) underestimates it (see Table 1). This result does not match either with Hawley (2001), which finds independence of accretion stress on azimuthal domain size (more precisely, about 10 percent less magnetic stress on average for the converged runs) or with Flock et al. (2012), which finds a larger stress for smaller $\Phi_0$. Therefore, although we find a dependence of the accretion stress on azimuthal extent of computational domain, the trend does not follow Flock et al. (2012). The difference lies in the evolution of mean and turbulent Maxwell stresses in the two studies. Flock et al. (2012) find both higher turbulent and mean Maxwell stresses for the restricted domain sizes. On the contrary, we find higher mean Maxwell stress for the runs with smaller azimuthal domains, turbulent stress does not show any trend (see Table 3).

7.2.2 Higher mean fields for the runs with smaller azimuthal extent

Smaller azimuthal domains produce stronger mean magnetic fields (3). This is in accordance with Flock et al. (2012). The reason behind the stronger mean fields for the reduced azimuthal domain is not very clear. Hawley (2001) attributes the larger fluctuations of accretion stress seen in the model with reduced azimuthal extent to the existence of the channel solution for vertical fields. It is easier for the channel solutions to maintain a spatial coherence for longer time for the smaller domain size. We also observe larger fluctuation level for the runs with smaller azimuthal domain (see Table 2).

To understand the dominance of mean fields in the restricted domain size, we look at the effects of azimuthal boundary conditions. In Fig. 17 we see that most of turbulent energy is at the largest scales. The periodic boundary conditions used in $\phi$ enforce the fields to be the same at the two boundaries. As a result, the natural build up of low-$m$ field is prohibited. For the smaller domain size the probability of cancellation of fields is small as $B_\phi = \sum_\phi B_{\phi}(\phi)$ and the magnetic fields have tendency to appear as the mean field. This is manifested in stronger mean fields for the restricted azimuthal domains.

7.2.3 The appropriate azimuthal extent

The steady state properties of the accretion flow are very similar for the runs with the azimuthal extent $\geq \pi/2$ (M-P2, M-1P and M-2P), but the run M-P4 with $\Phi_0 = \pi/4$ stands out with a lower specific mass accretion rate (see Fig. 10) and less efficient toroidal to poloidal magnetic field conversion process (see Figs. 22 and 23). Despite many similarities, because of the boundary effects, runs with restricted azimuthal domains of $\pi/2$ and $\pi$ give rise to stronger axisymmetric mean fields compared to the run with the natural azimuthal extent $2\pi$. We conclude that the appropriate restricted azimuthal extent of the simulation domain depends on the aim of the study. If one wants to study the steady structure of the flow, the minimum requirement of the azimuthal extent is $\Phi_0 = \pi/2$. On the other hand, to study turbulence and dynamo in a geometrically thick disk ($H/R \sim 0.5$), one must use $\Phi_0 = 2\pi$ in comparison to the minimum requirement $\phi_0 = \pi$ proposed by Flock et al. (2012) for their study of geometrically thin disk ($H/R = 0.07$).
7.3 Structure of RIAFs

RIAFs are more difficult to describe as compared to the standard thin accretion disks. For the latter in steady state the mass accretion rate through the disk is constant as a function of radius, and the vertically-averaged radial structure of the accretion disk is determined by the turbulent ‘viscosity’. The conservation of angular momentum implies that the inward advection of angular momentum at each radius is almost balanced by the turbulent flux of outward angular momentum, resulting in a net inward angular momentum flux equal to $M_{\rm ISCO}$ (where $\theta$ is the specific angular momentum corresponding to the ISCO). The energy equation is also simple, with the gravitational energy release, mediated by viscous dissipation, radiated locally as an optically thick black body.

For RIAFs, $H/R \sim 0.5$ and the radial and vertical dynamics are coupled. Thermal energy produced by accretion is not radiated but can be channeled into outflows, or lead to convection that stirs accretion. Outflows can carry away angular momentum and assist accretion in the mid-plane. The standard thin disk accretion models are no longer applicable in this case.

Mean field approach provides a useful framework to describe the essentially 2-D RIAFs. Figure 27 shows the mean density and poloidal flow streamlines in steady state of our fiducial simulation. The steady state is attained only within $40r_g$. Once can clearly see a mean inflow in the equatorial plane and a mean outflow across the disk boundary at radii $\gtrsim 20r_g$. Looking at the mean flow in the poloidal plane provides a more useful way of understanding the flow structure than simply averaging over all $\theta s$ (e.g., as in Fig. 7 of Stone & Pringle 2001). Averaging over all angles cannot distinguish between outflows and circulating motions.

We compare the 1D-profiles obtained from our simulation with that from different 1D RIAF models proposed in the literature. Apart from the flows which are purely advective, we also compare the available mass to be accreted. All these idealized models allow only a small fraction of the available angular momentum and energy, scalings are provided by the RIAF study (Narayan et al. 2000) and convection (CDAF; Narayan et al. 2000; Quataert & Gruzinov 2000) allow only a small fraction of the available mass to be accreted. As indicated, the idealized models consider unmagnetized flows. Akizuki & Fukue (2006) self-similarly accretion and an ADAF with the pure toroidal magnetic fields and provide the following scalings,

$$v_r \sim -\alpha^{0.5}$, \quad v_\phi \sim r^{-0.5}$, \quad c_s \sim r^{-0.5}$, \quad v_A \sim \beta^{1.5}r^{-0.5}$; \quad (39)

where $\alpha$ is the viscosity parameter and $\beta$ is the plasma $\beta$. The classic unmagnetized ADAF scalings (Narayan & Yi 1994) are also very similar. On the other hand, the convection dominated accretion flow (CDAF) follows the scalings

$$v_r \sim -\alpha^{0.5}$, \quad v_\phi \sim r^{-0.5}$, \quad c_s \sim r^{-0.5}$, \quad v_A \sim \beta^{1.5}r^{-0.5}$; \quad (40)

If wind affects the accretion by carrying away mass, angular momentum and energy, scalings are provided by the convection-dominated inflow-outflow solution (ADIOS) with

$$v_r \sim -\alpha^{0.5}$, \quad v_\phi \sim r^{-0.5}$, \quad c_s \sim r^{-0.5}. \quad (41)

In Fig. 22 we show the radial structure of the RIAF from our M-2P run. Between $r = 10$ and $r = r_{\text{mag}} = 40$, the characteristic velocities follow the following scalings,

$$v_r \sim -r^{-2.5}$, \quad v_\phi \sim r^{-0.7}$, \quad c_s \sim r^{-0.6}$, \quad v_A \sim r^{-0.9}. \quad (42)

If we use the measured spatial variation of $\alpha \sim r^{-0.8}$, $v_r/\alpha \sim r^{-1.7}$. Therefore, the scalings we get for the characteristic velocities do not match with any of the standard RIAF models. However, from Figure 27 it is clear that outflows play a role in reducing the mass accretion rate on to the black hole. It should be noted that a much larger radial extent of the numerical simulation is needed to compare the simulation results with analytic scalings (Narayan et al. 2012). Due to a small dynamical range ($r_{\text{mag}} = 40$; $r \sim 10r_g$ is affected by non-self-similarity due to the Paczynski-Wiita potential) in our simulations, we are unable to obtain a reliable power-law scaling for all the mean physical quantities.

7.4 Dynamo

We observe the saturation of magnetic energy due to a mean field dynamo in our 3D global simulation of a geometrically thick radiatively inefficient accretion flow. The mean field dynamo give rise to an intermittent sign reversal of the mean toroidal field, unlike previous local and global simulations with Keplerian angular velocity profile ($\Omega \propto r^{-5/3}$, $q = 1.5$) which show a dynamo cycle with definite time period. Recently Nauman & Blackman (2015) found that the dynamo period becomes shorter and less well-defined (see the butterfly diagram for $q = 1.8$ in their Fig. 6) when the shear parameter $q$ is increased from its Keplerian value. Interestingly, in the quasi-steady state of our simulations the angular velocity is sub-Keplerian with $\Omega \propto v_{\phi} \propto r^{-1.7}$ (see equation (42)). Thus sub-Keplerian velocity and intermittent dynamo cycle in our work agree with the observations in previous studies.

The axisymmetric fields produced by the dynamo have admixture of dipolar and quadrupolar symmetry, with the former dominating over the latter. Unlike previous global simulations of thin disks (Arlt & Rüdiger 2001; Flock et al. 2012), which found $\alpha_{\text{dyn}} \equiv \tilde{E}_m/\tilde{B}_m$ to be positive in NH, we do not find any coherent pattern of $\alpha_{\text{dyn}}$ instead we find a random meridional distribution of it due to the occurrence of a direct dynamo (close to the mid-plane), and $\alpha$-quenching (away from the mid-plane; Blackman & Field 2002) that is due to the interplay between kinetic and current helicities.

Small and random $\alpha_{\text{dyn}}$ close to the mid-plane indicates the dominance of a direct dynamo in which the field grows due to random stretching of field lines by MRI driven turbulence. Another intriguing feature is the same sign of $\alpha_{\text{dyn}}$ and $\alpha_{\text{mag}}$ within one scale-height for both hemispheres. The same sign indicates reinforcing rather than quenching (as expected from Fig. 25) within one scale-height. This is also observed in previous studies (Brandenburg et al. 1995; Gressel 2010). Brandenburg (1998) proposed the existence of a buoyancy driven dynamo and introduced another term $\alpha_{\text{dyn}}^{\text{mag}}$ which has the opposite sign to both $\alpha_{\text{kin}}$ and $\alpha_{\text{mag}}$.

Our work probably is the first study which investigates the effects of dynamical quenching on magnetic field saturation in global accretion disk simulations. Interestingly, sign of $\alpha_{\text{mag}}^{\text{mag}}$ (negative in NH) is opposite to that found in local shearing-box simulations by Gressel (2010) (compare their
Fig. 3 with our Fig. 26), but matches with that in Oishi & Mac Low (2011). Oishi & Mac Low (2011) attributed the disagreement in the sign of $\alpha_{\text{mag}}$ to the vertical extent of the box and recovered the negative sign at larger height in the NH by extending the vertical domain size. On the other hand, although the sign of $\alpha_{\text{kin}}$ matches with Gressel (2010) within one scale-height, it differs at higher latitudes.

Another interesting difference is in the relative strength between the $\alpha$-s: both Gressel (2010) and Oishi & Mac Low (2011) found an order of magnitude stronger $\alpha_{\text{mag}}$ compared to $\alpha_{\text{kin}}$: we find $\alpha$-s of almost equal strength. This may be the reason behind the patchy pattern we obtain for $\alpha_{\text{kin}} = \tilde{E}_{\phi}/B_{\phi}$. In addition, the presence of the mean flow (see Fig. 27) in our global simulation can have significant effects on the dynamo mechanism (Choudhuri et al. 1995).

Anisotropic nature of turbulence may also be effective in our simulations so that the off-diagonal terms in $\alpha$ and $\lambda$ (see equation 35) tensors are non-negligible. In future we would like to calculate the coefficients of $\alpha$ and $\lambda$ tensors rigorously.

8 SUMMARY

In this work we perform 3D ideal MHD simulations of accretion tori with different grid resolutions and azimuthal extents. Our aim is to investigate - i) convergence, ii) effects of azimuthal extent of the simulation domain on accretion flow properties, and iii) saturation mechanism of magnetic field that is governed by the dynamo process. The key findings of the study are listed below in the order of importance.

- We see an intermittent dynamo cycle for our geometrically thick ($H/R \sim 0.5$) radiatively inefficient accretion flows (Fig. 20). By looking at the symmetry of mean fields in the northern and southern hemispheres, we find that mean field parity is an admixture of dipole and quadrupole, with the former dominating over the latter (Fig. 21 and section 6.1). The irregularity found in the spatio-temporal evolution of mean fields in our simulations (for which shear parameter $q = 1.7$) is similar to that found in the butterfly diagram of Nauman & Blackman (2015) for $q = 1.8$.

- We also find an irregular behaviour of the dynamo-$\alpha$ (Fig. 25) unlike previous global simulations in Flock et al. (2012). This is because of two reasons – i) dominance of direct dynamo close to the mid-plane, ii) suppression of kinetic $\alpha$ by the magnetic $\alpha$ of a similar magnitude away from the mid-plane (section 7.4). On the contrary, previous local studies found $\alpha_{\text{mag}} \sim 10\alpha_{\text{kin}}$ (Gressel 2010; Oishi & Mac Low 2011). Probably, the effects of $\alpha$-quenching are studied explicitly for the first time in global simulations of accretion flows.

- We get stronger mean magnetic fields for the runs with smaller azimuthal domains (similar to Flock et al. (2012)) due to the periodic boundary condition used in the azimuthal direction, and the tendency of magnetic fields to be at the largest scales (Fig. 17). However, turbulent magnetic fields do not show any trend with the azimuthal domain size unlike Flock et al. (2012). For the azimuthal extent $\Phi_0 \geq \pi/2$, the runs with restricted azimuthal domains show stronger accretion stresses compared to the run with the natural domain size $\Phi_0 = 2\pi$. For all the runs the stress due to turbulent fields dominates over that due to mean fields (section 7.2).

We conclude that the appropriate azimuthal domain size depends on the aim of the study. If one wants to study the structure of the flow, the runs with the azimuthal domain size $\Phi_0 \geq \pi/2$ are able to produce the results obtained by run with $\Phi_0 = 2\pi$. On the other hand, for the study of turbulence and dynamo in a RIAF with $H/R \sim 0.5$, $\Phi_0 = 2\pi$ is the necessary azimuthal domain size compared to the minimum requirement $\Phi_0 = \pi$ proposed by Flock et al. (2012) for the thin accretion discs with $H/R \sim 0.07$.

- Decomposing the flow into a mean and fluctuations provides a useful insight at understanding the structure of RIAFs. Although we have a small radial extent for the steady-state flow ($< 40$), Fig. 27 shows the presence of vertical outflows that can carry away substantial mass (energy and angular momentum) away from the disk. This may explain the key feature of RIAFs – the much smaller accretion rate on to the black hole compared to the available mass supply (see section 7.3).

- We attain convergence for with 42 – 64 cells per scale-height in the vertical direction. We use the number of cells in the radial and azimuthal direction such that the three dimensional structure of the cells is close to cubes. Exceeding the minimum requirements for convergence, the quality factors are $\langle \langle Q_{\phi} \rangle \rangle = 10.5 \pm 0.9$ and $\langle \langle Q_{\phi} \rangle \rangle = 33 \pm 3$. The magnetic tilt angle $\theta_B$ turns out to be an excellent indicator of convergence with $\langle \langle \theta_B \rangle \rangle = 13^\circ - 14^\circ$.

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ADDITIONAL LINKS

The movies of the evolution of torus are available in the following links

i) https://www.youtube.com/watch?v=zDTSPqxsXMl

ii) https://www.youtube.com/watch?v=k7p9yQQDDQA

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