Cosmic Ray Positrons from Cosmic Strings

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We study the spectrum of cosmic ray positrons produced by a scaling distribution of non-superconducting cosmic strings. In this scenario, the positrons are produced from the jets which form from the cosmic string cusp annihilation process. The spectral shape is a robust feature of our scenario, and is in good agreement with the results from the recent PAMELA and ATIC experiments. In particular, the model predicts a sharp upper cutoff in the spectrum, and a flux which rises as the upper cutoff is approached. The energy at which the flux peaks is determined by the initial jet energy. The amplitude of the flux can be adjusted by changing the cosmic string tension and also depends on the cusp annihilation efficiency.

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I. INTRODUCTION

Recent results from the PAMELA \cite{1} and ATIC \cite{2} experiments have indicated an excess power of the cosmic ray positron flux compared to what is predicted from astrophysical backgrounds alone. The power of the flux is observed to rise towards an upper cutoff which is in the range of 600 GeV and falls off quite sharply above this cutoff. The data is in conflict with what is expected from astrophysical backgrounds. A peak in the spectrum of cosmic ray positrons in the energy range of 80 – 300 GeV had been suggested as a signature of dark matter annihilation in the galaxy a long time ago (the positrons are either produced from jets formed by the non-leptonic decay products \cite{3,4,5,6} or by direct decay into electrons and positrons \cite{7,8,9} - for reviews see e.g. \cite{10,11}). However, the specific form of the spectrum obtained from observations is hard to reconcile with the predictions from simple dark matter annihilation models \cite{12}, although modified models have been proposed which are in better agreement with the data \cite{13}. It is also possible that the positrons are due to nearby pulsars \cite{14}. However, in the case of pulsars the decline of the flux at energies larger than the peak energy is not abrupt (see, however, \cite{15} for a different point of view on this issue).

In this Letter we would like to propose an alternative explanation for the positron excess which has nothing to do with dark matter. We investigate the possibility that the observed positron flux is due to jets from cusp annihilation of cosmic string loops. We are considering non-superconducting cosmic strings. It has been known for a long time that particle emission \cite{16} from cusps of cosmic strings leads to a spectrum which rises as a function of energy up to a cutoff set by the parameters in the cosmic string model and falls off quite sharply above this cutoff \cite{17}.

Since cosmic strings arise in a large class of particle physics beyond the Standard Model, our mechanism provides a way to test physics beyond the Standard Model independent of the existence of low-energy supersymmetry.

In the following section we review the basics of cosmic string dynamics and particle emission from cosmic strings which are important to understand our scenario. Section 3 contains the computation of the cosmic ray positron flux. In Section 4 we give a brief discussion of the change in the positron spectrum during propagation through the galaxy. We conclude with a discussion of our results.

II. PARTICLE EMISSION FROM A COSMIC STRING LOOP

In this section we review the basics of cosmic string dynamics and discuss the cusp annihilation mechanism by which cosmic string loops can emit high energy particles.

Cosmic strings are one-dimensional topological defects which form during symmetry breaking phase transitions in a wide class of gauge theory models (see \cite{18,19,20} for reviews). If the gauge symmetry group at high temperatures is $G$, and the unbroken subgroup below the transition temperature is $H$, then the criterion for the existence of cosmic strings in the theory is

$$\Pi_1(M) \neq \mathbb{Z},$$

where $M$ is the vacuum manifold of the theory below the transition temperature and $\mathbb{Z}$ is the trivial group. If the group $G$ is simply connected, then $M = G/H$. This criterion is satisfied in a large class of particle physics theories beyond the Standard Model.

In particle physics theories admitting the existence of cosmic strings, such strings inevitably arise during the symmetry breaking phase transition \cite{21}. By causality, the point in $M$ which the order parameter describing the phase transition takes at temperatures lower than the transition temperature must be uncorrelated on length scales larger than the Hubble radius $H^{-1}$, where $H$ is
the cosmological expansion rate. Hence, there is a probability of order 1 that one string will traverse any particular Hubble volume after the phase transition. Typically (in particular if matter above the transition temperature is in thermal equilibrium), the initial separation of the cosmic strings will be microscopic. The above causality argument applies at all times subsequent to the transition time. Thus, in a theory which admits cosmic strings, then at all times after the phase transition (in particular at recent cosmological times) a network of cosmic strings with separation no greater than the Hubble radius will be present.

Cosmic strings arising in gauge field theories must be closed, i.e. either string loops of “infinite” strings (defined as truly infinite strings or string loops with curvature radius larger than the Hubble radius. Thus, the system of cosmic strings at any time after the phase transition will consist of a network of “infinite” strings and a distribution of string loops. Both analytical arguments detailed in [17, 18, 19] and detailed numerical simulations [21, 22, 23] have shown that the network of infinite strings approaches a “scaling” solution characterized by a string correlation length which is a fixed fraction of the Hubble radius at all late times. Roughly speaking, we can view the long string network as a random walk with step length of the order of the Hubble radius.

The scaling solution implies that the total length in the long strings is decreasing. This decrease is realized by the inter-commutation of long strings. Such inter-commutations produce string loops. Thus, at any time \( t \), there will be distribution of loops. Neither numerical simulations nor analytical studies at this point agree on the exact nature of the loop distribution (see e.g. [24] for recent progress). We will be using a simple one-scale model for the distribution of loops which is based on the assumption that loops at time \( t \) form at a fixed fraction of the Hubble radius.

Once formed, string loops decay predominantly by gravitational radiation. The rate of gravitational radiation is governed by the string tension \( \mu \), namely

\[
\dot{R} = \gamma G \mu, \tag{2}
\]

where \( G \) is Newton’s gravitational constant and \( \gamma \) is a numerical constant whose value is of the order \( 10^2 \) [25].

Taking into account the redshift of the number density of string as well as the formation scenario and decay rate of cosmic strings discussed above, we obtain the following distribution of string loops at time \( t \) (see e.g. [26]):

\[
n(R, t) = \kappa R^{-2} t^{-2}, \quad R > t_{eq} \tag{3}
n(R, t) = \kappa R^{-5/2} t^{-1/2} \gamma \mu t_{eq}, \quad \gamma \mu t < R < t_{eq}
\]

\[
n(R, t) = \kappa (\gamma \mu)^{-5/2} t^{-2} t_{eq}^{1/2}, \quad R < \gamma \mu t, \tag{4}
\]

where \( \kappa \) is another numerical constant which depends on the details of the cosmic string scaling solution, and \( t_{eq} \) is the time of equal matter and radiation. In the above, we are assuming \( \gamma G \mu < t_{eq} \). The first line in (3) represents loops which were produced after the time of equal matter and radiation, the second line loops generated before \( t_{eq} \) which will survive gravitational radiation for more than a Hubble expansion time. The last line represents loops which are in the final stages of decay by gravitational radiation.

From the distribution (3) it can be seen that the energy density in string loops is dominated by loops of radius about \( R \sim \gamma G \mu t \). These loops will, as we show below, also dominate the positron emission from strings.

Cosmic strings as two-dimensional world sheets \( x^\mu(\sigma, \tau) \) (where \( \tau \) is a world sheet time coordinate and \( \sigma \) labels the spatial world sheet coordinate) are solutions of the Nambu-Goto equations, the same equations which describe fundamental strings. Since cosmic strings have relativistic tension, they will oscillate. It can be shown [27] that “cusps” generically occur on string loops (at least once per oscillation time). A cusp is a point on the string where

\[
x_i' = 0, \tag{4}
\]

where a prime indicates the derivative with respect to \( \sigma \).

Geometrically, a cusp corresponds to a “spike” on the string (see Figure 1). Since a cosmic string has a finite width \( w \) whose magnitude is of the order of \( \mu^{-1/2} \), at a cusp the two segments of the string at either side of the cusp overlap. By expanding the solutions of the Nambu-Goto equations about a cusp it can be shown [28] that the length of the overlap region is

\[
l_c \sim w^{1/3} R^{2/3}, \tag{5}
\]

where \( R \) is the radius of the string loop.

![FIG. 1: Sketch of a cusp on a cosmic string loop. The mean curvature radius of the string loop is \( R \), the string width is \( w \), and the length of the overlap region is \( l_c \).](image)

There is nothing that prevents this overlap region from annihilating into particle excitations of the Higgs and gauge fields which the string is made up of. Assuming that the entire overlap region decays, one gets a power of particle radiation given by [15] (see also [29])

\[
P_c \sim \mu l_c R^{-1} \sim \mu w^{1/3} R^{-1/3}. \tag{6}
\]

This particle decay rate is - for string loops of macrophysical radius \( R \) - very small compared to the power
radiated into gravitational radiation which is

$$P_g = \gamma (G\mu)\mu$$

(7)

(see (2)). Nevertheless, as was discussed in [14] (see also [33]), cusp annihilation can contribute a significant fraction to the cosmic ray flux, assuming that a substantial fraction of the string overlap region decays [46]. The expected ultra-high energy neutrino flux from cosmic strings with a tension given by the scale of Grand Unification was studied in [16, 30], and the corresponding day signatures were analyzed in [32]; these works were performed under the assumption that the distribution of cosmic string loops scales as described by (3). The analysis was extended in [33] to the case of a non-scaling loop distribution (numerical evidence for such a non-scaling loop distribution came from the analysis of [34]). However, most studies of cosmic string dynamics favor a scaling distribution of string loops (see e.g. [24]), and thus in this paper we will assume such a scaling distribution.

III. POSITRON FLUX FROM A SCALING NETWORK OF COSMIC STRINGS

As discussed above, the power of energy loss from a cosmic string loop of radius $R$ is

$$P_c = \varepsilon \mu^{5/6} R^{-1/3},$$

(8)

where $\varepsilon$ is the efficiency factor discussed above and we have made use of the fact that the string width $w$ is proportional to the inverse square root of the string tension $\mu$.

The primary decay products from cusp annihilation are quanta of the scalar and gauge fields which make up the cosmic strings. These quanta, in turn, will decay into relativistically moving standard model particles which will form jets. Following the discussion in [16], we take the primary energy of a single jet to be $m_f$. In this case, the number of jets $N$ formed per unit time is

$$N = \varepsilon \mu^{5/6} m_f^{-1} R^{-1/3} = PR^{-1/3},$$

(9)

where the last step defines the quantity $P$.

A single jet leads to the following spectrum of energies (number per energy interval) of neutrinos resulting from the jet [35]:

$$\frac{dN}{dE} = \frac{15}{16} m_f^{-1} \left( \frac{11}{3} - 6x^{1/2} - 4x^{-1/2} + \frac{2}{3} x^{-3/2} \right),$$

(10)

where

$$x = \frac{E}{m_f} < 1.$$  

(11)

We will take the same formula to give the energy spectrum of all stable leptons resulting from the decay, in particular the spectrum of positrons.

Particle physics models admitting non-superconducting cosmic string solutions will have string cusps decaying at all times. Neutrinos produced from cusp cusps will travel cosmological distances (see e.g. [16]), whereas positrons are absorbed and lose their energy on super-galactic scales. To obtain the neutrino flux at energy $E$, we have to integrate over all times $t$ the flux of particles emitted at time $t$ with energy $E(z(t) + 1)$. For electrons and positrons, we only have to integrate over times which are smaller than the current time $t_0$ by less than the “containment time” of electrons and positrons, respectively, in the galaxy. According to [6], the containment time of positrons is of the order $10^7$ yrs., i.e. much longer than the time it would take light to travel through the galaxy. The containment time corresponds to a redshift of $z_c \sim 10^{-3}$.

When computing the expected positron flux from cosmic string cusp annihilations we must therefore impose several cutoffs. First of all, only positrons emitted at redshifts smaller than $z_c$ will contribute. Secondly, only string loops located inside the galaxy may be considered.

The general expression for the differential energy flux $F(E)$ of cosmic ray positrons from cosmic string cusp annihilations is

$$F(E) = \int dt (z(t) + 1)^{-3} f((z(t) + 1)E, t),$$

(12)

where $f$ is the differential flux per unit time of positrons emitted at time $t$, the “injection spectrum”. The redshift enters in two places. Firstly, the injection number density is redshifted, and, secondly, the energy of a given positron redshifts.

The injection spectrum is obtained by integrating over all cosmic string loops present:

$$f((z(t) + 1)E, t) = \frac{dN}{dE} \bigg|_{x=(z(t)+1)E/m_f} \int dRn(R, t)PR^{-1/3},$$

(13)

where the string number density $n(R, t)$ and the constant $P$ have been defined previously. The first factor on the right-hand side of this equation is the Jacobean factor obtained by transforming between final energy $E$ and injection energy.

As follows from recalling the loop distribution [8], the integral over $R$ is dominated by loops of radius $R \sim \sqrt{G\mu}$. If we consider the string scale to be at least a couple of orders of magnitude smaller than the scale of Grand Unification, string loops of radius $R \sim \sqrt{G\mu}\sqrt{0}$ will still be present today, and their number density will be such that many string loops of such radius will be located within our galaxy.

To estimate the amplitude and shape of the spectrum, we first insert the injection flux [13] into (12) and perform the integral over loop radii. The integral is dominated by the value $R = \sqrt{G\mu}$. A good estimate of the result is obtained by integrating over the loops with radii in the
range $\gamma G\mu t < R < t_{eq}$. The result is

$$F(E) \sim P(\gamma G\mu)^{-11/6} t_{eq}^{-1/2}$$

\[ \int dt (z(t) + 1)^{-2} t^{-2} t^{-11/6} \frac{dN}{dE} \big|_{x=(z+1)E/m_f}. \]

Next is the integral over time which can be simplified by using the integration variable

$$\tilde{z} \equiv z(t) + 1.$$ (15)

To obtain an estimate of the flux, we use for $dN/dE$ the final term on the right-hand side of (10). After a couple of lines of algebra (and in particular plugging in the definition of $P$ from [36]), we obtain

$$E^3 F(E) \sim \varepsilon(\gamma G\mu)^{-11/6} t_{eq}^{-3/4}$$

\[ t_0^{-7/3} \mu^{5/6} m_f \left( \frac{E}{m_f} \right)^{3/2} \tilde{z}_c, \]

where the final factor comes from the range of integration over $t$.

Inserting numbers into (16) and expressing the result in terms of the units which experimentalists use we get

$$E^3 F(E) \sim \varepsilon(\gamma G\mu)^{-5/6} (\gamma G\mu)^{-1}$$

$m_f$|GeV $\left( \frac{E}{m_f} \right)^{3/2} \tilde{z}_c 10^{-11} m^{-2} sec^{-1} GeV^2$.

The specific signature of our predicted cosmic ray positron flux is the power law increase of $E^3 F(E) \sim E^{3/2}$ and the sharp cutoff at an energy scale set by the initial jet energy $m_f$. In contrast, the background flux of positrons (multiplied by $E^3$) from astrophysical sources is predicted to be slightly decreasing in the energy range between 10GeV and 1000GeV.

Let us first give a rough analytical treatment of the predicted positron to electron flux ratio. Both fluxes are a superposition of background and cosmic string-induced fluxes, and we will use the subscripts bg and cs, respectively, to denote these two contributions. The flux ratio $\Phi$ is

$$\Phi \equiv \frac{E^3 F(E)^+}{E^3 F(E)^- + E^3 F(E)^+}.$$ (18)

Assuming that the electron flux is dominated by the background, we obtain

$$\Phi = R_{bg} + \frac{E^3 F(E)^+_{cs}}{E^3 F(E)^-_{bg}},$$

where $R_{bg}$ is the background flux ratio. Since $E^3 F(E)^{-}_{bg}$ is roughly constant, we see that the flux ratio in the cosmic string model is predicted to be equal to the background value at low energies and gradually shift to scaling as

$$\Phi \sim E^{3/2}$$\hspace{1cm} (20)

at higher energies. At energies close to the cutoff value $m_f$, the spectrum again flattens out because terms in (16) scaling differently than the $x^{-3/2}$ term which we focused on will become important.

From the PAMELA data [1] we can read off a slope which is rising to about 0.5 at energies between 50 and 100GeV. From the ATIC data, a slope of close to 1 is inferred at energy scales between 300 and 600GeV.

To obtain a better idea of the fit of our model, we have evaluated the predicted positron flux numerically, keeping all of the terms in (10). Our results are plotted in Figures 2 - 4. The numerical results also include the processing of the spectrum during propagation as discussed in the following section.

IV. PROPAGATION OF THE POSITRONS

Positrons will lose energy not only because of red-shifting, but also because of interactions during their propagation from the source to us through the galaxy. We consider a standard diffusion model for the propagation of positrons in the galaxy.

To begin with, let us recall the physical processes which affect the propagation of charged particles in the galaxy. Firstly, when a charged particle travels through the galaxy, its movement can be affected by the galactic magnetic field. Although the magnetic gyroradius of a particle is usually very small, this particle can still possible to jump to near-by field lines due to the tangled magnetic field and so could change its orbit. We usually model this process with a diffusion equation. Secondly, during the propagation of a positron, the particle loses energy because of inverse Compton and synchrotron processes. These two factors are the most important ones. For a detailed study, we refer to Ref. [36]. In the following we focus on the above-mentioned two processes, especially the energy loss.

Neglected other effects which are present in addition to the two mentioned in the previous paragraph, and assuming a spherically symmetric diffusion process, we obtain the following propagation equation for the flux $F$ of charged cosmic ray particles:

$$\frac{\partial}{\partial t} F = D(\epsilon) \nabla^2 F + \frac{\partial}{\partial \epsilon} \left( L(\epsilon) F \right) + Q(\epsilon, \vec{x}),$$ (21)

where $\epsilon$ is defined as a dimensionless energy variable $\epsilon \equiv \frac{E_{\text{GeV}}}{m_f}$ with $a$ the scale factor of the universe, $D$ is the diffusion coefficient, $L$ is the energy loss rate and $Q$ is the source term.

In models in which dark matter annihilation is the source of the positron excess, the production of positrons is dominated by the annihilation of dark matter particles today. Therefore, usually only steady state solutions of Eq. (21) (in which he left hand side of the equation vanishes) are considered, as analyzed for example in Ref. [37].

However, in our model the source of positrons does not scale in time as the background density, and hence the
resulting flux will not be steady state-like. As discussed in the previous section, we need to integrate the flux equation from the earliest moment from which positrons will still reach us today. This time is the containment time of positrons in the galaxy which corresponds to a redshift \( z_c \approx 10^{-3} \).

In the following, we will neglect the diffusion term in Eq. (21). In models with a string tension significantly smaller than that given by the scale of Grand Unification, the separation of strings is much smaller than the radius of the galaxy. Many of these strings are a distance away from us which is smaller than the diffusion radius. Hence, we argue that we can treat the flux as quasi-homogeneous and hence neglect the diffusion term.

Thus, we consider the simplified propagation equation

\[
\frac{\partial}{\partial t} F \approx \frac{\partial}{\partial \epsilon} \left( L(\epsilon) F \right) + (z(t) + 1)^{-3} f((z(t) + 1)E, t)
\]

To solve this equation, we apply a perturbative approach and separate the flux \( F \) into infinitely many components,

\[
F = F_0 + F_1 + \ldots
\]

Each component satisfies its own propagation equation as follows,

\[
\frac{\partial}{\partial t} F_0 = (z(t) + 1)^{-3} f((z(t) + 1)E, t),
\]

\[
\frac{\partial}{\partial t} F_1 = \frac{\partial}{\partial \epsilon} \left( L(\epsilon) F_0 \right), \ldots
\]

and, more generally,

\[
\frac{\partial}{\partial t} F_i = \frac{\partial}{\partial \epsilon} \left( L(\epsilon) F_{i-1} \right), \ldots
\]

After solving these equations one by one, we then sum up all the components to obtain the result

\[
F(\epsilon) = \int_{t_0}^{t_*} dt a^3 f + \int_{t_0}^{t_*} dt \frac{\partial}{\partial \epsilon} L(\epsilon) \int_{t_0}^{t} dt' a^3 f + \ldots
\]

By changing the time integrals to all the entire integral range, we can obtain the factor \( 1/n! \) for the \( n \)-th component. Eventually, we obtain the following formal solution for the flux:

\[
F(\epsilon) = F_0(\epsilon, t_0) \times \exp \left\{ \int_{t_0}^{t_*} dt \frac{\partial}{\partial \epsilon} L(\epsilon) F_0(\epsilon, t) \right\},
\]

where \( F_0(\epsilon, t_0) \) is exactly what we have studied in Sec. III without considering the energy losing effect.

In a realistic model, we consider energy loss through synchrotron emission and inverse Compton scattering. As introduced in Ref. [38], the process can be parameterized as

\[
L(\epsilon) = \frac{\epsilon^2}{\tau_E},
\]

with the energy-loss time \( \tau_E \approx 10^{16} s \). After combining Eq. (16) and the energy-loss parametrization, we can derive the factor in the exponential term of Eq. (26). It is

\[
\sim \int_{z_0}^{z_0 + z_c} dz \frac{3t_0}{4\tau_E} (z^{-\frac{3}{2}} - z_c^{-\frac{3}{2}}) E^{-\frac{1}{2}} / z_c E^{-3/2}
\]

\[
\sim - \frac{9t_0}{32\tau_E} \frac{z_c E}{1 GeV},
\]

in the low energy regime. Correspondingly, we obtain the following approximate form of the flux

\[
F(E) \approx F_0(E) \exp \left\{ - \frac{9t_0}{32\tau_E} \frac{z_c E}{1 GeV} \right\}.
\]

The exponential factor describes the energy losing of the positrons when they are passing through the galaxy.

From the above result, we learn that because of the smallness of the containment time of positrons in the galaxy, the energy loss due to interactions is insignificant for low energy positrons. For higher energy positrons the energy loss becomes more important. This leads to a slight smoothing out of the delta function-like upper cutoff in the predicted flux. In addition, depending on the parameters, the energy corresponding to the maximum of the flux may be smaller than \( m_f \).

To obtain a better idea of the fit of our model, we have evaluated the predicted positron flux numerically, keeping all of the terms in (10) and taking into account of the energy loss effects on the propagation of positrons discussed in this section. Our results are plotted in Figures 2, 3 and 4. In the figures, we take three groups of parameters for the model as shown in the captions of these figures. We have chosen the parameters \( \nu = 13 \) (determined by numerical simulations of cosmic string evolution [23]), the containment time \( z_c = 10^{-3} \), and the energy-loss time \( \tau_E = 10^{16} s \).

Whereas the PAMELA experiment only shows a rise of the flux ratio as a function of energy, the ATIC experiment which probes the spectrum of positrons to higher energies shows a sharp upper cutoff at an energy of about 600 GeV. Thus, matching the ATIC data leads us to prefer a higher value for the initial jet energy \( m_f \).

V. CONCLUSIONS AND DISCUSSION

In this Letter we have proposed a new explanation for the observed excess of positrons over electrons in the cosmic ray flux at energies between 10 and 600 GeV. In our model, the source of the cosmic ray positrons is radiation from cosmic string cusp emission. Our model does not assume that the particle physics model manifests supersymmetry with low-scale supersymmetry breaking. In contrast, it assumes the existence of linear topological defects.

The spectral shape which we predict is insensitive to the details of the cusp annihilation process and is thus a
robust prediction of our model. The position of the peak of the flux is determined by the initial jet energy $m_f$. The amplitude of the spectrum, is not a robust prediction of our model. It depends sensitively on both the cosmic string tension and on the efficiency factor $\epsilon$ of the cusp annihilation process. In our plots, we have fixed $\epsilon = 1$. From the analysis in Section III it follows immediately that the factor which determines the amplitude of the flux is $\epsilon(\gamma^{11/6}G\mu)^{-1}$. This is the factor which can be fixed from the recent positron flux observations, assuming that our mechanism is the source of the excess.

From our analysis we can learn another important lesson: for fixed value of $m_f$, a model with non-superconducting cosmic strings predicts a positron flux with a shape given by our analysis. Even if the observed flux is not due to strings, we get an upper bound on the quantity $\epsilon(\gamma^{11/6}G\mu)^{-1}$. As our results show, for $\epsilon \sim 1$ this is a bound which rules out many models with low energy scale strings.

Let us add some more comments on the sensitivity of our predictions to the value of the efficiency factor $\epsilon$. If we were to use to value of $l_c$ given by [31], which takes into account effects which were not included in the initial analysis of [13], and is

$$l_c \sim w^{1/2}R^{1/2},$$

then the predicted amplitude of the flux decreases by a factor of $(w/t_0)^{1/6}$. Moreover, back-reaction effects on cusp formation are still not included completely in [31], and thus the actual amplitude may even be lower. The assumptions we make about the efficiency of cusp annihilation will change the value of $G\mu$ for which the amplitude of the positron flux agrees with observations.

There are large classes of particle physics models beyond the Standard Model which predict the existence of cosmic strings. Cosmic strings are also predicted in many inflationary universe models based on superstring theory [39] (for reviews see e.g. [40]). A network of cosmic strings will also remain in the string gas cosmology model [41, 42] (for a recent review see [43]).

In order for our model to be consistent with the absence of an excess in the cosmic ray anti-proton spectrum [44] we require the cosmic strings to decay predominantly leptonically.

In light of the recent positron data, our work motivates a closer look at the mechanism of cusp annihilation. Any improvement in our understanding of this process would lead to a much improved predictive power of our analysis. Another issue which merits revisiting is the determination of the initial jet mass $m_f$ resulting from cusp annihilation. Another important issue is to determine which cosmic string models can lead to predominantly leptonic jets. On the experimental side, it is interesting to explore ways to distinguish between the proposed scenarios.
to explain the positron excess (see e.g. [45]).

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