Research on the dynamic characteristics of crack damage of a seal-rotor system

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Abstract In rotor systems, the labyrinth seal system is the core component to suppress fluid leakage between the rotor and the stator. In this paper, based on the finite element method, a dynamic model of a seal-crack rotor system is established by using the Muzynska nonlinear seal force model and the cosine crack stiffness model, and the vibration characteristics of airflow excitation and single-crack and double-crack coupled faults are analyzed. This paper analyzes the vibration characteristics of the coupling of airflow-induced vibration and a crack fault. First, numerical simulation analysis and test verification were performed on the system response with no sealing force or crack failure. Subsequently, systems with a sealing force and different crack parameters were analyzed for numerical simulation analysis, and then, the influence of crack damage failure on other sealing parameters (including the sealing pressure difference, sealing gap, and sealing length) was studied. Finally, the influence of double-crack damage (damage location, damage degree, phase difference angle) on the rotor system was analyzed. The results show that when the crack depth increases to a certain value, it causes a superharmonic resonance phenomenon in the subcritical speed region of the system. When the system has a sealing force, the airflow excitation frequency of the system can be affected as the degree of crack damage increases. The coupled dynamic response of airflow excitation and crack faults shows a rich spectrum of nonlinear phenomena, which is closely related to the degree of cracks and sealing parameters. Increasing the crack angle weakens the impact of crack damage on the system. This research provides a theoretical basis for detecting and diagnosing crack faults in labyrinth seal-rotor systems.

Keywords Labyrinth seal · Crack damage · Airflow excitation · Finite element method · Crack parameters

List of symbols

- $A_x$, $A_y$: Vibration displacement of the system in the $X/Y$-direction
- $A_i$: Unit cross-sectional area
- $A_{ae}$: Remaining area of the crack section
- $C$: Damping matrix of the rotor system
1 Introduction

The labyrinth seal is an effective sealing structure widely used between the shaft end and various levels in modern aeroengines, compressors, steam turbines and other power mechanical structures. It has a simple structure, no friction, low power consumption, long service life, and no need for lubrication and maintenance. A tortuous path and other characteristics suppress fluid leakage between the stator and the rotor, thereby reducing leakage loss [1]. Muzynska pointed out that rubbing and cracking usually easily occur in seals [2]. In fact, due to installation error, fatigue loss, external force impact, etc., crack failure of the seal-rotor system easily occurs. If this failure cannot be found and handled in time, other failures will cause secondary damage to the equipment. Therefore, to improve the operating stability and safety of this type of system, it is of great significance to study the rotor system crack failure’s complex dynamic characteristics under airflow excitation force and analyze the influence of some essential factor parameters on the system.
We consider sealing force models and the study of dynamic characteristics. The models can be divided into parametric analytical models and numerical calculation models of the sealing force. The parametric analytical models include the Thomas-Alford and Muzynska models, while the numerical calculation models are mainly based on the control body models, such as Wang et al. [3], through the application of a single control body model and perturbation method to dynamic modeling and calculation of the rotor system with a labyrinth seal. Cangioli et al. [4] proposed a new labyrinth seal thermoelastic body flow model, taking the cross-tooth labyrinth seal of a steam turbine as the research object, considering different pressures, speeds, and inlet prerotation ratios. The influence of sealing stability. Kirk et al. [5, 6] used CFX-TASCflow fluid dynamics software to solve and study its influencing factors. Avza et al. [7] proposed a transient CFD method to calculate the nonlinear dynamic characteristics of the coupled seal-rotor system on this basis.

In terms of dynamic characteristics research, Subramanian et al. [8] conducted a numerical study on the rotor dynamic characteristics of the turbine seal of a gas turbine considering centrifugal force increases and found that these increases are not conducive to the stability of the system. Regarding the influencing factors of seal leakage, Sun Dan et al. [9] studied the influence of antiswirling flow on the static and dynamic characteristics of the seal from both theoretical and experimental aspects; Liu Xingwang et al. [10] used the macroenergy method and viscous flow theory to calculate the leakage in the radial labyrinth seal of the scroll compressor. Regarding coupled fault nonlinear vibration problems involving seal airflow excitation, Ma W et al. [11] studied the influence of seal length and diameter on system dynamics; Shyu SH et al. [12] applied the finite element method (FEM) to model the seal-rotor system with rolling bearing support and analyzed the influence of the geometric parameters of the seal structure on the seal-bearing-rotor system.

In terms of the research on the vibration characteristics and mechanism of crack damage in the rotor system, Rao Xiaobo et al. [13] studied the cracked rotor system with an oil film and found a chaotic “eye” phenomenon in the dynamic characteristics, which is helpful for the control and diagnosis of crack failure in the rotor system. Darpe et al. [14, 15] simulated the two-way coupled crack model with the intensity factor zero method, studied the effect of the crack on the transient vibration of the Jeffcott rotor and analyzed the unbalanced response of the system. Patel et al. [16, 17] established a finite element model of a rotor system with oblique cracks based on the theory of fracture mechanics. They studied the effect of the crack azimuth on the stiffness and the vibration response characteristics of the system. P et al. [18] applied the cosine crack breathing effect to analyze the response characteristics of the system at 1/3 critical speed under different double-crack parameters. Lu Zhenyong et al. [19] studied the effect of crack depth and crack location on the amplitude and reached important conclusions. [20, 21] established the finite element equation of an asymmetric rotor and studied the influence of transverse breathing cracks on the asymmetric rotor-bearing system.

There have been many achievements in the research on the coupling of crack faults and other faults. Palacios-Pineda Luis M. et al. [22] took the Jeffcott rotor system as the research object, considered the influence of transverse cracks on system vibration, and obtained relevant conclusions through simulation calculations and experimental verification. Luo Yuegang et al. [23] established a crack-support loose coupling fault dual-span rotor system dynamics system with sliding bearing oil film force and concluded that loose faults have a strong impact on the system. Yang Yongfeng et al. [24] studied a rotor system with loosening and crack coupling faults and used the centralized mass method to model the system, and the crack switch model used the cosine model method. The single-span single-disk rotor system considers the effect of the oil film force. Xiang Ling et al. [25] analyzed the nonlinear behavior of the rotor system coupled with multiple faults and obtained the coupling effects of cracks, rubbing and oil film instability through experiments and simulation calculations.

Judging from the existing literature, the current research results mostly use the Jeffcott rotor as the research object for modeling and analysis and focus on considering single factors such as air-induced vibration or crack failure. Research results are mostly based on the Jeffcott rotor model with rigid supports at both ends and a disk with a mass at the center of the rotating shaft. In this paper, the Timoshenko beam element is used to simulate the shaft element, considering the mass, stiffness, gyroscopic effect and shear
deformation of the shaft. The main research methods for coupled faults containing cracks are the vibration characteristics of crack-oil film instability, crack looseness and crack rubbing. However, there have been relatively few studies on crack failures that are most likely to occur in seal-rotor systems, which consider airflow excitation factors. This provides a specific theoretical basis for revealing the mechanism of the influence of crack faults on fluid excitation and the identification and diagnosis of air flow-induced vibration-crack coupling faults in seal-rotor systems.

The structure of this article is as follows: Sect. 2 establishes the dynamic equation of the labyrinth seal-rotor system, the finite element model of the rotor system, and the crack stiffness model of the damage breathing effect. Section 3 compares and analyzes the influence of crack damage on the nonlinear dynamic response of the system with or without air force in the system and analyzes the influence of different crack damage degrees and damage locations on the system. Section 4, based on the research in the third section, analyzes the vibration response characteristics of the crack-damaged rotor system under the condition of changing the important sealing parameters of the seal-rotor system and analyzes the instability speed and the change law of the excitation frequency after the occurrence of airflow excitation. Section 5 analyzes the effect of double-crack damage (damage location, damage degree, phase difference angle) on the rotor system.

2 Establishment of the seal-rotor system model

2.1 System dynamics model

The finite element model of the rotor system is shown in Fig. 1a. On the left is the disk with a sealing device with a radius of \( D = 200 \text{ mm} \) and a length of \( L = 100 \text{ mm} \). The rotating shaft adopts a stepped shaft, and there are two identical interleaved labyrinth seals on the disk. The length of the two seals is 15 mm. The air inlet is located in the center of the sealed stator. The system is divided into 27 shaft sections. The numbers in the figure indicate node numbers, totaling 28. Divide the sealing turntable into four shaft units, and the corresponding unit number is 6–9. This paper uses the Timoshenko beam element to simulate the shaft element. As shown in Fig. 1b, there are two section nodes A and B in the shaft segment, and each node has six degrees of freedom, which are translation and rotation along the three directions of \( x, y, \) and \( z \) and the corresponding translation displacement and rotation angle; they are denoted as \( x_A(x_B), y_A(y_B), z_A(z_B) \) and \( \theta_{xA}(\theta_{xB}), \theta_{yA}(\theta_{yB}), \theta_{zA}(\theta_{zB}) \), respectively. Only the radial vibration is studied here, and the axial

![Fig. 1](image-url)
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vibration is not considered, so the displacement in the z-axis direction and the rotation angle are ignored. Then, each node of the shaft element has four degrees of freedom, and the displacement vector of the nth shaft element is

$$\mathbf{u}_n = [x_A, y_A, \theta_{x_A}, \theta_{y_A}, x_B, y_B, \theta_{x_B}, \theta_{y_B}]^T, \quad n = 1, 2, \ldots, 27$$  \hspace{1cm} (1)

According to the literature [26], the mass matrix \( \mathbf{M}_n \), stiffness matrix \( \mathbf{K}_n \), and gyro matrix \( \mathbf{G}_n \) of discrete Timoshenko beam elements are assembled in the form of Fig. 2. The shaded parts in the figure indicate that the matrix elements of the common nodes contained in two adjacent shaft segments can be obtained from the 112-dimensional shaft mass matrix \( \mathbf{M}_s \). Similarly, the stiffness matrix \( \mathbf{K} \) and the gyro matrix \( \mathbf{D}_s \) can be obtained.

The unit mass matrix \( \mathbf{M}_n \) is

$$\mathbf{M}_n = \rho A l \begin{bmatrix} a & 0 & a & 0 \\ 0 & c & 0 & g \\ 0 & 0 & b & 0 \\ -d & 0 & 0 & f \\ d & 0 & 0 & c \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & c \\ 0 & 0 & 0 & g \end{bmatrix}$$  \hspace{1cm} (2)

In the matrix, \( a, b, c, d, f \) and \( g \) are

$$a = \frac{13}{35} + \frac{2}{15} f + \frac{1 + 2 f^2}{2} - \frac{6 (\frac{c}{l})^2}{(1 + f)^2},$$

$$b = \frac{9}{70} + \frac{3}{70} f + \frac{1 - 2 f^2}{2} - \frac{6 (\frac{c}{l})^2}{(1 + f)^2},$$

$$c = \frac{[\frac{11}{10} + \frac{1}{15} f + \frac{1}{2} f^2 + \left(\frac{1}{10} - \frac{1}{2} f \right) (\frac{c}{l})^2]}{(1 + f)^2}$$

$$d = \frac{[\frac{3}{140} + \frac{1}{5} f + \frac{1}{24} f^2 - \left(\frac{1}{10} - \frac{1}{2} f \right) (\frac{c}{l})^2]}{(1 + f)^2},$$

$$f = \frac{[\frac{1}{140} + \frac{1}{5} f + \frac{1}{24} f^2 + \left(\frac{1}{10} + \frac{1}{2} f - \frac{1}{6} f^2 \right) (\frac{c}{l})^2]}{(1 + f)^2},$$

$$g = \frac{[\frac{1}{105} + \frac{1}{7} f + \frac{1}{15} f^2 + \left(\frac{2}{15} + \frac{1}{6} f + \frac{1}{6} f^2 \right) (\frac{c}{l})^2]}{(1 + f)^2}$$  \hspace{1cm} (3)

The element stiffness matrix \( \mathbf{K}_n \) is

$$\mathbf{K}_n = \begin{bmatrix} h & 0 & h & 0 \\ 0 & -i & j & 0 \\ i & 0 & 0 & j \\ -h & 0 & 0 & -i & j \end{bmatrix}$$  \hspace{1cm} (4)

where \( h, i, j \) and \( k \) are

$$h = \frac{12EI}{l^3(1 + \phi)} \quad i = \frac{6EI}{l^3(1 + \phi)} \quad j = \frac{(4 + \phi)EI}{l(1 + \phi)}$$

$$k = \frac{(2 - \phi)EI}{l(1 + \phi)}$$  \hspace{1cm} (5)

The unit gyro matrix \( \mathbf{G}_n \) is

$$\mathbf{G}_n = 2 \omega \rho A l \begin{bmatrix} 0 & -p & 0 & 0 \\ -q & 0 & -s & 0 \\ 0 & -p & -q & 0 \\ 0 & -q & 0 & w & q & 0 & 0 \\ 0 & -q & -w & 0 & 0 & q & -s & 0 \end{bmatrix}$$  \hspace{1cm} (6)

where \( p, q, s \) and \( w \) are

Fig. 2 Matrix assembly
\[
p = \left(\frac{3}{2}\right) r_g^2 \frac{P}{I(1 + \phi)^2}
q = -\frac{1}{2} \left(1 - \frac{1}{2} \phi\right) \frac{r_g^2}{l^2(1 + \phi)^2}
\]

\[
s = \frac{1}{18} + \frac{1}{6} \phi + \frac{1}{3} \phi^2 \frac{r_g^2}{(1 + \phi)^2}
\]

\[
w = \frac{1}{30} + \frac{1}{6} \phi + \frac{1}{6} \phi^2 \frac{r_g^2}{(1 + \phi)^2}
\]

where \(G_s\) is the shear modulus and \(\vartheta\) is the shear coefficient, which is expressed in terms of \(v\), the Poisson ratio of the material. The shear coefficient of the solid shaft is \(\vartheta = \frac{6(1 + v)}{(7 + 6v)}\), and that of the hollow shaft is \(\vartheta = \frac{2(1 + v)}{4(1 + v)}\).

The mass of the disk of the system adopts a centralized mass model in this paper, and its mass matrix and gyro force matrix can be expressed as \(D_d\) and \(M_d\), respectively. As shown in Fig. 1c, the gyro moment matrix and mass matrix of the disk element at the m-th node position are as follows:

\[
D_d = \omega \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & J_p \\
0 & 0 & -J_p & 0
\end{bmatrix}
\]

\[
M_d = \text{diag}[m_d, m_d, J_d, J_d] \quad m = 6, 7, 8, 9, 20, 22, 24
\]

In the above formula, \(m_d\), \(J_d\) and \(J_p\) are the mass, diameter moment of inertia and polar moment of inertia of the disk, respectively. These can be calculated according to the following formula according to the disk material density and geometry (inner radius \(r\), outer radius \(R\) and thickness \(h\) of the disk):

\[
m_d = \pi \rho (R - r)^2 h 
J_d = \frac{\rho \pi (R - r)^4 h}{4} 
J_p = \frac{\rho \pi (R - r)^4 h}{2}
\]

2.2 Establishment of the finite element model of the system crack element

The crack stiffness model used in this paper is that of Mayes et al. [31], who considered the opening and closing process of the crack during the operation of the rotor system. They combined the square wave model and the cosine wave model and proposed a process for the opening and closing of the rotor crack. The crack is divided into four states in one rotation, which are open, open to closed, closed to open, and closed to open. The breathing process of the crack is described in more detail. In the research, it is assumed that the moment of inertia of the crack section to the fixed coordinate axis of the system does not change with the change in the rotation angle. When the crack occurs, the moment of inertia of the crack to the two sections does not change, but it changes the center of mass of the section.

Figure 3 shows a cross-sectional view of the crack unit. O-XY is the fixed coordinate system of the rotating shaft, where O is the center of the rotating shaft when there is no crack, \(O_1\) is the center of the rotating shaft after the crack occurs, \(e\) is the position change of the center after the crack occurs, \(z\) is the cracking angle of the crack, \(h\) is the depth of the crack, and \(R\) is the radius of the shaft.

The moment of inertia of the crack on the \(ox\) and \(oy\) coordinate axes is

\[
l_x = \frac{\pi R^4}{8} - \frac{R^4}{4}(1 - \mu) (2\mu^2 - 4\mu + 1) \gamma + \sin^{-1}(1 - \mu)
\]

\[
l_y = \frac{R^4}{12} (1 - \mu) (2\mu^2 - 4\mu - 3) \gamma + 3 \sin^{-1}(\gamma)
\]

The remaining area of the crack section \(A_{c e}\), the centroid moment \(e\), and the crack angle \(\alpha\) are

\[
A_{c e} = R^2 \left(\pi - \cos^{-1}(1 - \mu) + (1 - \mu) \gamma\right) - \pi r^2
\]

\[
e = \gamma^3 \frac{2R^3}{3A_{c e}}
\]

\[
\alpha = 2 \arccos(1 - \mu)
\]
In the above formula, the crack depth ratio \( \mu = h/R \), where \( \gamma \) is the dimensionless crack depth of the crack, which can be expressed as \( \gamma = \sqrt{\mu^2 - \mu} \).

For the crack section, the moment of inertia of the remaining section except for the crack to the new coordinate axes \( cx \) and \( cy \) is

\[
I^c_x = I - (I^0_x + A_{ce}e^2)
\]

\[
I^c_y = I - I^0_y
\]

\[
I = \pi(R^4/4)
\]

where \( I_1 = I^0_x + A_{ce}e^2, I_2 = I^0_y \). The decrease in the moment of inertia can be expressed as

\[
I^c_x = I - I_1
\]

\[
I^c_y = I - I_2
\]

According to the finite element theory of rotor dynamics, it is believed that the crack occurs in the \( lw \)-th element. The matrix \( K^w_e \) of the stiffness reduction in the crack element in the entire system is

\[
K^w_e = \frac{E}{R}
\]

\[
\begin{bmatrix}
12I_2 \frac{1}{1 + \phi_1} & 0 & 0 & 6I_1 \frac{1}{1 + \phi_1} & -12I_2 \frac{1}{1 + \phi_1} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_1} \\
0 & 12I_2 \frac{1}{1 + \phi_2} & -6I_2 \frac{1}{1 + \phi_2} & 0 & 0 & -12I_2 \frac{1}{1 + \phi_2} & 0 & 0 \\
0 & -6I_2 \frac{1}{1 + \phi_2} & \frac{4 + \phi_2}{1 + \phi_2} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_2} & 0 & 0 \\
6I_1 \frac{1}{1 + \phi_1} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_1} & -6I_1 \frac{1}{1 + \phi_1} & 0 & 0 & -6I_1 \frac{1}{1 + \phi_1} \\
-12I_1 \frac{1}{1 + \phi_1} & 0 & 0 & -12I_1 \frac{1}{1 + \phi_1} & 12I_1 \frac{1}{1 + \phi_1} & 0 & 0 & 12I_1 \frac{1}{1 + \phi_1} \\
0 & -12I_2 \frac{1}{1 + \phi_2} & 6I_2 \frac{1}{1 + \phi_2} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_2} & 0 & 0 \\
0 & -6I_2 \frac{1}{1 + \phi_2} & \frac{2 + \phi_2}{1 + \phi_2} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_2} & 0 & 0 \\
6I_1 \frac{1}{1 + \phi_1} & 0 & 0 & 6I_2 \frac{1}{1 + \phi_1} & -6I_1 \frac{1}{1 + \phi_1} & 0 & 0 & -6I_1 \frac{1}{1 + \phi_1}
\end{bmatrix}
\]
The stiffness model of the double crack mainly adopts the cosine stiffness model, and its formula is

\[ K = K_1 - f(t)K_{c1}^{lw} - f(t)K_{c2}^{lw} \]  

(17)

In the formula, \( K_1 \) is the total stiffness of the system without cracks, \( K_{c1}^{lw} \) and \( K_{c2}^{lw} \) are the stiffness reduction in the first crack and the second crack, respectively, and \( f(t) \) is the switching function. We can write

\[ f(t) = \frac{1}{2} (1 + \cos(\omega t + \beta)) \]  

(18)

In the formula, \( \beta \) is the phase difference angle of the double crack.

2.3 Labyrinth seal-Muszynska sealing force model

Compared with the Thomas-Alford and Black-Childs sealing force models, the Muszynska model [27] is obtained based on a large number of experiments and theoretical analyses, which fully reflects the nonlinear characteristics of the sealing force and is widely used in the study of the nonlinear dynamics of the seal-rotor system. The model introduces the average flow velocity ratio \( \tau \) used to represent the fluid movement in the sealing gap, which expresses the sealing force vs. The effect of inertia, damping, and stiffness produced by the rotor movement is analyzed. Figure 4 shows a radial cross-sectional view of the sealed cavity, where the circumferential angular velocity of the fluid close to the rotor, \( \omega_0 \), equals 0 at the point close to the stator. The average flow rate ratio in the sealed cavity can be expressed as \( \tau_0 \), and the sealing force can be expressed as

\[ \{ F_{\hat{p}} \} = - \begin{bmatrix} K - mf_1^2 \omega_0^2 & \tau_0 D \\ -\tau_0 D & K - mf_1^2 \omega_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \]  

\[ - \begin{bmatrix} D & 2\tau_0 m_f \\ -2\tau_0 m_f & D \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} m_f & 0 \\ 0 \quad m_f \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \]  

(19)

In the formula, \( K, D, \) and \( m_f \) are the stiffness, damping, and mass of the disturbance movement of the fluid to the rotor. \( K, D, \) and \( \tau \) are all nonlinear functions related to the disturbance displacements \( x \) and \( y \), which can be expressed in the following form:

\[ K = K_0 (1 - e_1^2)^{-n} \]  

\[ D = D_0 (1 - e_1^2)^{-n} \]  

\[ \tau = \tau_0 (1 - e_1)^b \]  

where \( e_1 \) is the relative eccentricity of the rotor; \( c_1 \) is the sealing gap (the distance between the rotor and the sealing teeth); and \( n, b \) and \( \tau_0 \) are the specific sealing structure parameters.

\[ K_0 = \mu_0 \mu_3 D_0 = \mu_1 \mu_3 T m_f = \mu_2 \mu_3 T^2 \]  

(20)

Parameters \( \mu_0, \mu_1, \mu_2, \mu_3 \) and \( T \) are expressed as

\[ \mu_0 = \frac{2\sigma^2}{1 + z + 2\sigma} E_f (1 + m_0) \]  

\[ \mu_1 = \frac{2\sigma^2}{1 + z + 2\sigma} \left( E_f + \frac{B}{2} \left( E_f + \frac{1}{6} \right) \right) \]  

\[ \mu_2 = \frac{\sigma (E_f + \frac{1}{2})}{1 + z + 2\sigma} \]  

\[ \mu_3 = \frac{\pi R_m \Delta P}{\lambda} \]  

\[ T = \frac{l}{v_a} \]  

(22)

Fig. 4  Schematic diagram of the Muszynska sealing force model
Parameters $\sigma$, $\lambda$, $E_f$ and $B$ are expressed as

$$\sigma = \frac{\lambda l}{c_1}$$

$$\lambda = n_0 R_e m_0 \left[ 1 + \left( \frac{R_e}{R_a} \right)^2 \right]^{1/n_0}$$

$$E_f = \frac{1 + z}{1 + 2\sigma}$$

$$B = 2 - \frac{m_0}{\left( \frac{R_e}{R_a} \right)^2 + 1}$$

$$R_v = \frac{R_m c_1 \gamma_0}{\gamma_0} \quad R_a = \frac{2v_a c_1}{\gamma_0}$$

In addition, $\gamma_0$ represents the hydrodynamic viscosity coefficient, $\sigma$ represents the friction loss gradient coefficient, $R_v$ represents the circumferential flow Reynolds number, $R_a$ represents the axial flow Reynolds number, $v_a$ represents the axial flow velocity of the airflow, and $\Delta P$ represents the seal pressure difference. $R_m$ represents the sealing radius. Other parameters are shown in Table 1.

### 2.4 Dynamic equations of the seal-rotor system

The mass, stiffness, and gyro matrix of each unit of the system obtained are merged into the mass, gyro matrix, and stiffness of the entire system, namely $M$, $D$, and $K_1$. The damping of the system adopts proportional damping; that is, damping $C$ is a linear combination of $M$ and $K$, and its expression is

$$C = \alpha_0 M + \beta_0 K_1$$

$$\alpha_0 = 4\pi f_1 f_2 (\xi_2 f_1 - \xi_1 f_2) / (f_1^2 - f_2^2)$$

$$\beta_0 = (\xi_2 f_1 - \xi_1 f_2) / \pi (f_2^2 - f_1^2)$$

Here, $\xi_1$ and $\xi_2$ are damping coefficients, representing the damping ratio of the first and second modes of the system, and $f_1$ and $f_2$ are the first and second frequencies of the system, respectively, so the dynamic equation of this system is

$$M \ddot{x} + (D + C) \dot{x} + (K_1 - F(t)K_m) x = Q$$

### Table 1 Parameters of the seal-bearing rotor system

| Parameter                     | Value          |
|-------------------------------|----------------|
| Rotor system model parameters |                |
| $E_i (\text{Pa})$             | $2.1 \times 10^{11}$ |
| $\rho (\text{Kg/m}^3)$        | 7850           |
| $\nu$                         | 0.3            |
| $\xi_1, \xi_2$                | 0.02, 0.04     |
| $m_e (\text{g/mm})$           | 600            |
| $f_1, f_2$ (Hz)               | 62.809, 123.55 |
| $\eta = h/R$                  | 1/6, 1/2       |
| $h$                           | 5, 15          |
| $R$                           | 30             |
| Crack parameters              |                |
| $c$ (mm)                      | 0.1            |
| $l_m$ (mm)                    | 15             |
| $R_m$ (mm)                    | 100            |
| $\Delta P$ (MPa)              | 0.3            |
| $v_a$ (m/s)                   | 5              |
| $\gamma_0$ (Pa·s)             | $1.79 \times 10^{-5}$ |
| $\beta$                       | 0.1            |
| $m_0$ [28]                    | -0.25          |
| $n_0$ [28]                    | 0.079          |
| $\tau_0$ [28]                 | 0.2            |
| $n_f$ [30]                    | 2.5            |
| $b$ [29]                      | 0.45           |
In this equation, $Q = F_p + F_f + G$ is the sum of the eccentric force, gravity, and sealing force. The expression of the eccentric force $F_p$ is

\[
F_p = meo^2
\]

\[
F_{px} = meo^2 \cos(\omega t)
\]

\[
F_{py} = meo^2 \sin(\omega t)
\]

(28)

3 Effect of a single crack on the seal-rotor system

3.1 Influence of the degree of crack damage on the rotor system

Figure 5 shows the response diagram of the system at the No. 10 shaft section at the damage position when the speed of the system increases with different degrees of crack damage. Figure 5(a1) and (a2),

Fig. 5 Bifurcation diagram and 3D waterfall chart of different crack damages: a $\mu = 1/6$ and b $\mu = 1/2$
respectively, shows the bifurcation diagram and three-dimensional waterfall diagram of the system when the crack depth ratio is 1/6. In the bifurcation diagram, the system exhibits cycle-one motion. The system can be identified in the three-dimensional waterfall diagram. The weak double frequency component is removed; at this time, the system frequency components are only $1X$ and $2X$. Figure 5(b1) and (b2), respectively, shows the bifurcation diagram and three-dimensional waterfall diagram of the system when the crack depth ratio is 1/2. In the bifurcation diagram, the system has prominent obvious fluctuations in the first-order critical speed. There is a slight fluctuation at the lower speed. The $1X$, $2X$, $3X$, and $4X$ high frequencies appear in the three-dimensional waterfall chart, and $2X$ appears at speeds equal to 2550 rev/min and 1275 rev/min. There are two peaks and a peak at 850 r/min for $3X$.

This section mainly analyzes the impact of crack damage on the system when the airflow force appears in the seal-rotor system, and the system speed increases to 10,000 rev/min. Figure 6 shows the bifurcation diagram of the system when there is no crack. When the system speed reaches 6300 r/min, the system loses stability and enters the pseudoperiodical state, and period-doubling movement occurs in the range of the 7600–7900 rev/min periodic window phenomenon.

When there is airflow, the air pressure at the sealing outlet is 0.1 MPa. Figure 7a shows a bifurcation diagram of bifurcation diagrams for different crack damage degrees when the crack depth ratio $\mu$ is 1/6 and 1/2. When cracks appear, the system’s destabilizing speed decreases significantly from 6300 r/min (no crack damage) to 6100 r/min (crack failure occurs). As the crack depth increases to a certain extent, the instability speed increases slightly, and with the continuous increase in the damage degree of the system, the period window of the system continues to advance, from 7500–7800 r/min to 7300–7500 r/min. Figure 7b shows the three-dimensional waterfall diagram under different crack damage degrees and the three-dimensional waterfall diagram of the system when $\mu$ is 1/6 and 1/2. The diagram mainly shows $1X$, $2X$, and $3X$ frequency multiplication composition, airflow excitation frequency $ff$ and some combination frequencies such as $1X + ff$, $1X + 2ff$, $1X−ff$, $2X + ff$, $2X + 2ff$, and $2X−ff$.

Figure 8 shows the amplitude curve of the seal-rotor system with crack damage failure during the speed-up process when there is no crack damage failure. Figure 7(b2) shows that when the rotational speed is greater than 6000 r/min, airflow excitation occurs. The amplitude curve of the cracks has a crack depth ratio of $\mu = 1/2$. The amplitude of a rotor system with cracks is significantly higher than that of a rotor system without cracks when there is no airflow excitation. However, the amplitude of the rotor system with cracked failure after the airflow excitation of the system is not much different from that of the noncracked rotor system. This is because the airflow force has a certain inhibitory effect on crack failure and because the system is in a high-speed area. The dynamic behavior is dominated by airflow excitation.

3.2 Effect of crack damage on the system under steady-speed conditions

Figure 9 shows a comparison diagram of the axis trajectory and time-domain waveform of the $1/n$ ($n = 1, 2$) order critical speed with or without airflow excitation force for the $\mu = 1/2$ system. When the system has no air force, the response is shown in red, and when there is an air force, the axis track is shown in blue. The amplitude when there is an air force is significantly lower than that with no air force. When the system speed is at other values, the movement range of the axis trajectory is smaller when there is airflow force. The amplitude of vibration decreases, and the waveform is more stable when there is airflow force in the domain waveform diagram. The airflow excitation force weakens the high-frequency vibration caused by cracks in the subcritical speed region and reduces the system amplitude.
3.3 Test verification

To verify the accuracy of the theoretical modeling and numerical simulation results, the built-up seal-rotor test bench is shown in Fig. 10, and a schematic diagram of the hardware structure is shown in Fig. 10a. The test bench is composed of a drive system, a supporting system, a test piece system, a lubrication system, a pneumatic system, and an electric control system. The test bench uses a motor with a power of up to 15 kW and a maximum speed of 6000 rev/min and uses a frequency converter to control the speed of the motor. Figure 10b shows a partial cross-sectional view of the test piece. The air flows in from the air inlet in the middle and flows out from the front and back ends. There is a staggered labyrinth seal structure at the front and back. Each seal structure contains three sealed teeth. The sealing length is 15 mm.

The test system composition and installation position of the seal-rotor test bench are shown in Fig. 10c.
**Fig. 9** Rotor system with or without air force response diagram: a 2550 rev/min and b 1275 rev/min

(a1) Axis trajectory diagram

(b1) Axis trajectory diagram

(a2) Time-domain waveform

(b2) Time-domain waveform

**Fig. 10** Seal-rotor test rig: a hardware schematic; b sectional view of the test specimen; c testing system
A pair of eddy current sensors, a group of photoelectric sensors and acceleration sensors are arranged at the corresponding positions of the test bench. After the amplifier conditions the signal through the signal cable, the signal is sent to the dynamic data acquisition instrument, and the steps of analog signal antimixing filtering and A/D conversion are realized in the data acquisition instrument. The signal is finally converted into a digital signal that the upper-level analysis software can process. Finally, the digital signal is uploaded to the analysis software of the computer to realize various analysis functions.

In the experimental test of the labyrinth seal-rotor system, the impact of the system’s crack damage failure in the subcritical speed region on the vibration characteristics of the system is mainly analyzed and compared with the crack failure characteristics of the rotor system in the subcritical speed region during simulation calculations. The degree and location of the crack damage of the experimental study of the sealing shaft are the same as the simulation calculation \( \mu = 0.5 \), and the damage location is the No. 10 shaft section. Figure 11 shows a comparison diagram of the system vibration response simulation and experimental test in the subcritical speed region. Figure 11(a1), (a2) and (b1), (b2) shows the first-order critical speeds, and Fig. 11(c1), (c2) and (d1), (d2)

shows the results at 1/2 times the critical speed. The fault characteristic of the crack obtained in the simulation calculation is that the system has a high-frequency doubler and the frequency doubler phenomenon appears at the 1/2 critical speed in the subcritical speed region (system superharmonic vibration). The axis trajectory is also the same as the axis trajectory during simulation.

4 Effect of cracks on different sealing parameters of the system

4.1 Effect of damage on changing seal pressure difference

The relevant calculation parameters in this section are the same as those in the previous section. The air pressure difference between the sealed chamber and the outside can be achieved by changing the inlet pressure of the sealed inlet. This experiment mainly analyzes the response characteristics of the system when the system’s sealing pressure difference is 0.2 MPa and 0.3 MPa when the operating speed is increased from 100 to 10,000 rev/min. Figure 12(a1), (a2) and (b1), (b2), respectively, shows the bifurcation diagram and three-dimensional waterfall diagram of

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Fig. 11 Simulation and experimental response graphs at different speeds: a 2550 rev/min, b 2620 rev/min (experiment), c 1275 rev/min, and d 1300 rev/min (experiment)
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the system under different sealing pressure differences. In the bifurcation diagram, the destabilizing speed of the system also delays with the increase in the sealing pressure difference. Figure 12(a1) and (a2) shows that when the sealing pressure difference is 0.2 MPa, the speed is greater than 6100 r/min, and the system enters the unstable speed state. When the sealing pressure difference is 0.3 MPa, the system enters the unstable speed at 6700 r/min. From 8700 to 9800 r/min, period-doubling motion occurs, but at this speed, the frequency-locking component of the system disappears because under certain speed and sealing conditions, the rotor system with crack fault has the same influence as the sealing system and because suppressing the excitation frequency of the airflow leads to the disappearance of the frequency-locking component in a certain rotational speed region. From Fig. 12(a1), it is not difficult to find that this trend is also observed near the rotational speed of 9000 r/min. In the three-dimensional waterfall diagrams Fig. 12(b1) and (b2) of the system, there are high-frequency $2X, 3X$ signals caused by crack faults, and there are also the airflow excitation frequency $ff$ that occurs after the system instability enters the pseudoperiod state and the combined frequency after the airflow excitation occurs. Examples include $2ff, 1X - ff, 1X + ff, 1X + 2ff, 1X - 2ff, 2X + ff, 2X + 2ff$ and other complex frequencies. However, the combined frequency components of the system instability in the three-dimensional waterfall diagram are continuously reduced with the continuous increase in the seal pressure difference.

4.2 Response characteristics of the variable seal pressure difference to the steady speed of the cracked system

The system speed is stabilized at 10,000 rev/min, and the response of the system within the range of 0.1 MPa. From Fig. 13a, b, the system performs pseudoperiodic motion in the range of 0.1 MPa. In the three-dimensional waterfall diagram, the frequencies in the range are $ff, 1X, 2X, 3X, 2ff, 1X - ff, 1X + ff, 1X + 2ff, 1X - 2ff, 2X + ff, 2X + 2ff$, etc., and as the seal pressure difference increases, the system airflow excitation frequency $ff$ also decreases from 70 to 50 Hz. When
the seal pressure difference is after 0.38 MPa, the system enters cycle-one movement and no airflow excitation occurs, and its frequency in the three-dimensional waterfall chart has only 1X, 2X, and 3X components. Figure 13c shows the amplitude curve of the system. The system amplitude is relatively stable before 0.18 MPa, and then, the system amplitude begins to drop sharply and ends in a stable state at 0.38 MPa.

4.3 Response characteristics of variable seal length to steady speed of cracked system

The operating speed of the system is stabilized at 10,000 r/min, and the change in the sealing length is controlled by increasing or decreasing the number of sealed teeth. The sealing length is used as a variable to analyze the response characteristics of the system at this time. In the bifurcation diagram in Fig. 14a, when the system is at a steady speed, the system performs a pseudoperiodical motion within the range of 10–22 mm, and the system enters a stable cyclic motion state after the seal length increases to 22 mm. In the three-dimensional waterfall chart of the method, the airflow excitation frequency $f_f$ of the system suddenly changes at a seal length of 12 mm; when the length is 10 mm, the excitation frequency is 134.2 Hz, which gradually decreases as the seal length increases past 12 mm. A sudden change to 70 Hz occurs. When the seal length is greater than 22 mm, the airflow excitation frequency of the system disappears, and the system has only power frequency and doubling frequency components and enters a stable motion state. In the amplitude curve of the system in Fig. 14b, the amplitude of the seal length increases slightly before 14.5 mm and then begins to decrease, and the amplitude starts to remain stable when the system enters the cycle at 22 mm. From the axis trajectory diagram of Fig. 14c system, as the seal length increases, the axis trajectory changes from a complex nested large circular ring shape to a single circular ring.

4.4 Effect of damage changes on the sealing gap

This section mainly analyzes the system’s response characteristics when the sealing gap is $c = 0.08$ mm and $c = 0.12$ mm. Figure 15 shows a three-

![Fig. 13 System response diagram with the change in seal pressure difference: a bifurcation diagram, b 3D waterfall chart, and c amplitude curve](image1)

![Fig. 14 System response as the seal length changes: a 3D waterfall chart, b amplitude curve, and c axis track diagram](image2)
 dimensional waterfall diagram of the system when the system speed is increased from 100 to 10,000 r/min under the two types of sealing gaps. The figure shows that before the system becomes unstable, there are only 1X, 2X, and 3X multiplier components. After the system becomes unstable, the system starts to exhibit an airflow excitation frequency $ff$ and some combined frequencies of $1X - ff$, $1X - 2ff$, $1X + ff$, $1X + 2ff$, $2ff - 1X$, $2X + ff$. When the speed increases to 8800 r/min, the airflow excitation vibration undergoes a sudden change. When the sealing gap increases to 0.12 mm (Fig. 15b), the combined frequency content continuously decreases. This result reflects that the increase in the sealing gap increases the amount of gas leakage, weakens the phenomenon of airflow excitation in the sealed cavity, and increases the sealing gap, which is conducive to the stability of the system.

4.5 Response characteristics of the variable sealing gap to the steady speed of the cracked system

We keep the system’s working speed at 8000 r/min and analyze the influence law on the sealed rotor system with crack damage by changing the sealing gap of the system. The sealing gap varies from 0.05 to 0.17 mm. Figure 16 shows the response diagram of the system with the change in the sealing gap. The bifurcation diagram in Fig. 16a shows the airflow excitation state before reaching 0.126 mm. The system is in pseudoperiodic motion, but the period window appears from 0.087 to 0.093 mm. After 0.126 mm, the system enters a stable cyclic motion state. From the three-dimensional waterfall chart in Fig. 16b, the frequency of the system before 0.126 mm contains $1X$, $2X$, $ff$, $2ff$, and other combined frequencies, and there are two sudden changes in the excitation frequency within its range. In Fig. 16c amplitude diagram of the system, the amplitude of the system also increases with the increase in the sealing gap. At a sealing gap of 0.123 mm, the amplitude starts to drop sharply and reaches a stable state when the sealing gap is 0.126 mm.

5 Effect of double-crack damage on the seal-rotor system

5.1 Vibration characteristics of double-crack damage

This section analyzes the vibration characteristics of the seal-rotor system with double-crack damage.
When the position of the double crack is at the position of the 11–12 shaft section, the damage degree is $\mu = 0.5$, and the system speed increases to 7000 r/min. The bifurcation diagram in Fig. 17a shows that the speed executes cycle-one motion before 5500 r/min, then enters the pseudoperiod state for a short period of time, and then exhibits cycle-one motion again until reaching 5700 r/min. The 6200 r/min system once again enters the pseudoperiodic regime. From Fig. 17b, the frequency components of the system mainly include $1X$, $2X$, $3X$, $4X$, $ff$ and other combined frequencies. Compared with a single-crack failure, the frequency-doubling component caused by the crack is more easily detected, and the frequency is high. If there are more components, the system enters unstable motion early, and the motion state in the high-speed zone becomes complicated.

5.2 Effect of double-crack damage location on system vibration characteristics

This section considers the positions of the double cracks between nodes 10 and 13 in Fig. 1, so this section selects three representative cases: double cracks at nodes 11–13, 12–13, and 11–12. Figure 18 shows the system’s axial trajectory and spectrogram when the system is at 1200 r/min (approximately 1/2 times the critical speed) with different crack damage positions. The damage degree of the two cracks is $\mu = 0.5$ (crack depth ratio). Figure 18(a1) and (a2) shows the response characteristics of the cracks at nodes 11–13. The $2X$ raising of the system is still based on power frequency. When the positions of the two cracks are at nodes 12–13 and 11–12, the distance between the two cracks decreases. Figure 18 (b1), (b2) and (c1), (c2) shows that the frequency of the system is mainly $2X$ and $IX$ at the auxiliary point, and the system exhibits superresonance. The axis trajectory diagram of the system shows that the axis trajectory of the system gradually moves in the direction of the arrow when the positions of the double cracks are close and that the axis trajectory changes from simple to complex. When double cracks occur, the axis track of the system exhibits a figure-eight-like complex shape, which indicates that the damage position of the double cracks has a great influence on the vibration characteristics of the system.

5.3 Effect of the phase difference angle of the double crack on the vibration characteristics of the system

A schematic diagram of the crack angle difference is shown in Fig. 19. The damage position of the double crack is selected as nodes 12–13, the crack depth ratio of the double crack is $\mu = 0.5$, and the system speed is set to 1200 r/min. The change in the different angles of the double cracks is achieved mainly by keeping the direction angle of the No. 13 node crack unchanged, and the direction angle of the No. 12 unit crack is changed in turn according to $\pi/3$, $\pi/2$, and $\pi$. From Fig. 20a, it can be concluded that when the phase difference angle is slight (less than $\pi/3$), the superharmonic vibration of the system in the subcritical speed state is excited by the effect of the double cracks. The amplitudes of systems $IX$ and $2X$ are increased. The value enhances the nonlinear characteristics of the system and shows rich high-order harmonic components in the system’s spectrogram and axis trajectory diagram. From Fig. 20(b1), (b2)

![Fig. 17](https://example.com/fig17.png)  
**Fig. 17** Response characteristics of the double-crack fault seal-rotor system under the speed-up state: a bifurcation diagram and b 3D waterfall chart

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and (c1), (c2), when the difference angle is large (greater than $\pi/3$), the double cracks inhibit each other as the difference angle increases. The power frequency component of the system gradually increases and dominates. The superharmonic vibration of the system disappears at the subcritical speed.

The influence of the different angles of the cracks on the frequency $f_f$ of the airflow excitation force generated after the instability of the system is further analyzed, and the damage degree of the double cracks $\mu = 0.8$. The different angles are $\pi/4$, $\pi/2$, and $3\pi/4$, and the position is 12–13. Figure 21 shows the frequency spectrum of the system under different crack angles when the system speed is 7000 rev/min. When there is no difference angle, the airflow excitation frequency is 63 Hz, and the amplitude is 0.13 mm. When the difference angle increases to $3\pi/4$, the $f_f$ amplitude of the system advances to 0.21 mm. In summary, the crack difference angle has a nonlinear effect on the amplitude of the airflow excitation, and an increase in the crack difference angle increases the amplitude of the airflow excitation frequency.
Fig. 20 Vibration response diagram of the 1200 r/min system with different phase difference angles: (a) \( \pi/3 \), (b) \( \pi/2 \), and (c) \( \pi \)

Fig. 21 Spectrogram of 7000 rev/min crack angle systems: (a) 0, (b) \( \pi/4 \), (c) \( \pi/2 \), (d) \( 3\pi/4 \)
5.4 Effect of double-crack damage on system vibration characteristics

Based on the original parameters, the influence of the damage degree of the double crack on the vibration characteristics of the system under the subcritical speed state is further analyzed. The main result is that the phase difference angle of the double crack is 0, and the damage degree of the double crack at the crack damage position is 12–13. Without changing the crack damage depth of unit No. 13, the depth ratio \( \mu = 0.5 \) changes the crack damage depth of unit No. 12. Figure 22 shows the axis trajectory and frequency spectrum of the system in the subcritical speed region when the crack damage degree \( \mu = 0.3, 0.4, \) and 0.5. As the crack depth increases, the amplitude of the power frequency of the system gradually decreases, and the amplitude of the high frequency of the system gradually increases. The frequency of the system is dominated by the power frequency when the crack depth ratio is less than 0.5. High multiplier frequency is supplemented, and no superharmonic vibration occurs. When the crack depth is 0.5, the 2X component of the system exceeds the power frequency and begins to dominate, and the system begins to generate superharmonic vibration. With increasing crack depth, the amplitude of the 2X and other high-frequency components increases, the amplitude of the power frequency decreases, and the axis trajectory of the system becomes more complicated.

5.5 Experimental analysis of the double-crack damage rotor system

The double-crack damage test mainly analyzes the system’s response characteristics when the depth of one of the cracks is constantly changing. The main parameters refer to the damage parameters of the double crack in Sect. 5.4. The damage location of the double cracks in shaft Sect. 12–13. The different angles of the double crack are 0 degrees, the system speed is 1270 r/min (near the 1/2-order critical speed), and the crack damage degree of shaft section No. 12 remains unchanged. As shown in Fig. 23, the crack

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**Fig. 22** System response diagram of 1200 r/min double cracks with different damage levels: a \( \mu = 0.3 \), b \( \mu = 0.4 \), and c \( \mu = 0.5 \)
damage degree of the No. 13 shaft section $\mu$ is 0.3, 0.4, and 0.5.

Figure 24 shows the response diagram of the vibration experiment of the system with different phase difference angles of 1200 r/min. The experimental result diagram corresponds exactly to the simulation result of Fig. 20. From Fig. 24(a1) and (a2), when the phase difference angle is $\pi/3$, the superharmonic vibration of the system in the subcritical speed state is stimulated by the action of the double cracks, which is shown in the system’s spectrogram and axis trajectory graph. From Fig. 24(b1), (b2) and (c1), (c2), when the phase difference angle is large, the double cracks inhibit each other, which makes the power frequency component of the system gradually increase and dominate, and the superharmonic vibration of the system disappears at the subcritical speed.

Figure 25 shows the experimental spectrum of the system with different crack angles at 7000 r/min and a crack depth of $\mu = 0.5$. The test results are consistent with the simulation result in Fig. 21. In the figure, the airflow excitation frequency is 63 Hz when there is no difference angle. As the phase difference angle increases, the amplitude increases from 0.35 to 0.80 mm. In summary, the crack difference angle has a nonlinear effect on the amplitude of the airflow excitation. The increase in the crack difference angle increases the amplitude of the airflow excitation frequency, indicating the accuracy of the simulation results.

Figure 26 shows the response diagram of the system under different levels of damage. In the spectrogram of the system, as the degree of crack damage increases, the frequency-doubling component of the system gradually becomes prominent, showing the 2X and 3X components and other frequency-doubling phenomena. The amplitude of 2X gradually increases. When the damage degree increases to a particular value, the 2X component of the system exceeds the power frequency and dominates. This phenomenon is consistent with the simulation results. The axis trajectory is also the same as the simulation results.

### 6 Conclusion

Based on a labyrinth seal-rotor system, this paper simulates the crack damage failure of the rotating shaft and considers the influence of the gyro effect and the sealing force. Based on finite element theory, the dynamic equation of the system is established, and the calculation is solved by the Newmark-\(\beta\) numerical integration method. The vibration displacement data of the system are presented in three-dimensional waterfall diagrams, bifurcation diagrams, time-domain waveform diagrams, axis trajectory diagrams, spectrogram diagrams and Poincaré cross section diagrams to analyze the crack damage in the seal-rotor system and the impact of crack damage on the system when the sealing parameters are changed. The conclusions obtained are as follows:

1. When there is no sealing force in the system, the crack damage failure causes the first-order critical speed of the system to decrease and the resonance amplitude to increase, and this outcome becomes more evident with the increase in the crack damage degree and induces superharmonic resonance in the subcritical region. After the sealing force is stored in the system, the airflow force in the system weakens the high-frequency resonance characteristics caused by cracks in the subcritical speed zone so that the amplitude of the system and the range of the axis trajectory are reduced. As the crack deepens, the crack increases the amplitude of the system and affects the instability speed of the rotor system. The dynamic characteristics of the system in the super-first-order critical speed region are mainly characterized by airflow excitation, but as the degree of crack damage increases, the $ff$ frequency and amplitude of the system gradually decrease.
2. When there is a sealing force, increasing the sealing pressure difference, sealing gap, and sealing length increases the instability speed of the system, delays the occurrence of airflow excitation, and makes the movement state of the system in the high-speed region more complicated. With increasing seal pressure difference, length, and gap, the range of pseudoperiodical motion gradually decreases. When it increases to a certain value, the system always maintains a
period-one motion state. It is found that a large sealing gap can weaken the high-frequency characteristics of the system caused by cracks, but when the sealing gap is large, the leakage of the system fluid is increased, and the sealing effect of the system is affected.

3. The influence law of sealing parameters on system airflow excitation is as follows. When keeping the system speed constant and changing the sealing pressure difference, length, and gap of the system, increasing the sealing pressure difference gradually reduces the frequency and amplitude of the excitation generated after the system is unstable. Increasing the length of the seal reduces the frequency and amplitude of the system’s airflow excitation, causing a sudden change. Increasing the sealing gap induces two sudden changes in the airflow excitation within a smaller range, which shows that the sealing parameters strongly affect the system.

4. The effect of double-crack failure on the seal-rotor system is as follows. Compared with single-crack failure, double-crack failure increases the amplitude of the frequency-doubling component of the system and also increases the high-frequency components such as $4X$. Double-crack failure advances the instability speed of the system; under the action of double cracks, the nonlinear characteristics of the system are increased. When the phase angle is large, the power frequency of the system exceeds $2X$ dominates, and the crack difference angle can enhance the influence of the crack failures to cancel each other. Increasing the damage depth of the crack gradually reduces the $1X$ amplitude of the system, and the frequency amplitude of $2X$ and other high-power components gradually increases. When the crack depth is significant, the system experiences subcritical speed superharmonic vibration.

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Data availability statement  The raw/processed data required to reproduce these findings cannot be shared at this time, as the data also form part of an ongoing study.

Declaration

Conflict of interest  The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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