A hypothetical formulation of quantum mechanics is presented so as to reconcile it with macro-realism. On the analogy drawn from thermodynamics, an objective description of wave packet reduction is postulated, in which a characteristic energy scale and a time scale are introduced to separate the quantum and classical conceptions.

I. INTRODUCTION

Despite the indisputable practical success of quantum mechanics (QM), conceptual and philosophical difficulties are still left behind. [1,2] In fact, we are so accustomed to the classical notion in real life that we cannot even imagine what a superposition of macroscopically distinct states, as stipulated by QM as a possibility, really looks like. Here we aim to present a hypothetical formulation to fill in the conceptual gap between classical and quantum mechanics. First of all, let us state our standpoint to tackle this long-standing problem by three steps, on each of which respectively one will find many proponents as well as opponents.

Firstly, we regard a wave function as elements of reality, characterizing an individual physical system, not an ensemble. We believe that the quantum formalism is something more than merely a set of observational predictive rules. We pay attention to time evolution of individual systems and assume that the wave function gives the fullest description of a quantum state. In this interpretation, wave function collapse is ultimately unavoidable, and we shall regard the collapse as a real and physical process. At this point, we deviate not only from the Copenhagen interpretation, but also from the Everett relative-state interpretation.

Secondly, as there must be no privileged observer, we assume that the wave packet reduction is a spontaneous process. In this respect, there now exists a class of notable theories stemming from the original work of Ghirardi, Rimini and Weber (GRW), [3,4] in which the time evolution equation of standard quantum theory is elaborately modified by introducing nonlinear and stochastic elements. In this approach, the classical behavior of macroscopic systems as well as the quantum properties of microscopic systems are derived altogether from a unified dynamics. Though not related directly, there are other works similar in spirit but from a different perspective, viz., quantum gravity. [5,6] These theories predict testable consequences against standard quantum theory. Most notably, one of their striking outcomes is the violation of energy conservation. [3,5,7] Somehow they introduce a characteristic scale to describe a crossover from the microscopic quantum regime to the ordinary macroscopic regime. The latter emerges from the former by state vector reduction. The nonlinear theories are soundly motivated by the crucial point of QM that the essential problem is indeed a consequence of the general and unavoidable fact that the state space as well as the evolution equation thereof are linear. In fact, it is often remarked [8] that there is no way out to render the quantum formalism ontologically interpretable but either to alter it in more or less ad hoc ways, by plugging in nonlinear terms in the Schrödinger equation, or to assume explicitly hidden variables. Nevertheless, this remark is implicitly based on an expectation that time evolution of a ‘state’, whatever it means, must be governed by a unified dynamics. It is indeed convincing, but this is the point where we break away from various types of nonlinear theories. Such unified theories could afford to make quantitative, but not qualitative, difference between two categories of natural phenomena.

As the third point, discarding the assumption of a unified dynamics or a single prescription, we propose to promote the wave function collapse to the status of the elementary process, ranked along with the unitary linear time evolution. Our approach consists in accepting from the outset that wave functions can develop temporally via two distinct ways. We do not modify the unitary evolution at all, but supplement it with a subsidiary condition for the collapse to intervene occasionally. The condition will be expressed by an inequality. We come to adopt this strategy in consideration of the status quo that any such attempts to derive, or explain the collapse process from a more fundamental level must be faced with more or less conceptual or mathematical difficulties at that level. [9,10] To get around them virtually, we shall take the less ambitious attitude in a sense, i.e., to renounce the attempts altogether at the outset. Indeed it is as old as quantum mechanics itself to postulate two fundamentally distinct laws of evolution, but in this context the second type has almost always been attributed to the act of consciousness. Divorced from such subjective approaches, we intend to formalize objectively the reduction process on the premise that the reduction is fully characterized by the input and the output states, without delving into the mechanism in between.

In our approach, quantum states are almost always governed by the linear dynamics, except when the prob-
abilistic collapses happen. The collapse mechanism operates only under a specific condition, which is to be identified. For our purpose, a remark due to von Neumann is especially noteworthy, [13] that the two kinds of processes can be distinguished unambiguously by the concept of statistical entropy. Nevertheless, it is easily conceivable that von Neumann’s entropy criterion is not sufficient. To find an answer to the problem of ‘measurement’, we need to identify not only the boundary between deterministic and stochastic, or reversible and irreversible, but also that between ‘microscopic’ and ‘macroscopic’, hopefully simultaneously. The latter boundary between system and observer is often cited as a principal cause of debate, which originates from the very fact that there is nothing in standard quantum theory to fix such a borderline.

Bearing in mind the conceptual achievements of the collapse theories, [3,5,11,12] and in the full conviction that there must be the definite boundary somewhere in the middle of the ‘micro’ and ‘macro’, we aim to remedy the above drawback by looking for the objective criterion, that, if successful, would replace some perceiving subject which von Neumann, Wigner and others have had recourse to. To fix the borderline, we will bring in some parameters as naturally as possible, without introducing any other unobservable machinery agent. In a sense, our attempt may be viewed as a step toward objectification of the wave packet reduction postulate, by which to judge if a given linear superposition is stable or unstable. We see what emerges from the synthesis to appear thereafter. It is anticipated that any unstable superposition is doomed to collapse at random, in accordance with the probability principle of QM. As a fundamental rule, the criterion must be simple and appealing. Moreover, as a stringent prerequisite, the new formalism must not spoil the statistical predictions of QM for microscopic systems, as they have been overwhelmingly confirmed without any doubt. To put it concretely, we are concerned about whether or not a given state collapses, or whether the system is ‘observed’ in a given situation. For example, an electron is ‘observed’ when injected in a cloud chamber, but not when bound in the ground state of a hydrogen atom. When ‘observed’, we need not only reproduce the predicted results as expected, but should abstract the presumed condition that is met there.

In search of a satisfactory formulation of QM, we find it instructive to cite the two guidelines according to J. S. Bell. [14] “The first is that it should be possible to formulate them for small systems.” (Otherwise, it is likely that ‘laws of large numbers’ are being invoked at a fundamental, so that the theory is fundamentally approximate.) “The second, related, point is that the concepts of ‘measurement’, or ‘observation’, or ‘experiment’, should not appear at a fundamental level.” (Because these concepts appear to be too vague to appear at the base of a potentially exact theory.)

It is the core of the paper to postulate the criterion in Section III. Before that, in order to see how the criterion serves its purpose, we provide a framework to discuss wave packet reduction in Section II. Section IV contains some applications of the theory. A brief summary is presented in Section V.

II. PREFERRED BASIS PROBLEM

States resulting from wave packet reduction are not arbitrary. Before discussing the condition for the reduction, we have to specify on what basis a given wave function is projected by reduction. This is called the preferred basis problem. [11] The GRW theory pays special attention to a position basis, onto which a wave function is reduced to realize a localized state in the real space representation. At a glance over the general structure of the transformation theory of QM, however, there seems no special reason to prefer the position basis, since we can think of many other types of linear superpositions which are equally as clumsy but not necessarily consisting of spatially far-off separated parts. In fact, a more general basis is apparently required in practice to accommodate various kinds of macroscopic quantum phenomena of current experimental interests, [15] although it may well be argued that the measurement problem should in any case boil down to the projection onto the position basis of an observer or apparatus. [16] In this section, we provide a framework within which to discuss the wave packet reduction phenomena on a general basis. In that, we aim to substantiate an insight that wave packet reduction is a process to break off weak coupling correlation developed in a wave function.

A normalized solution Ψ of the wave equation,

\[ i\hbar \frac{\partial \Psi}{\partial t} = H \Psi, \]

is expanded in terms of a complete set of orthonormalized functions Φₙ as

\[ \Psi = \sum_{n} c_n \Phi_n. \]

According to QM, the probability \( w_n \) for the initial state Ψ to be ‘observed’ in the state \( \Phi_n \) is given by

\[ w_n = |c_n|^2 = |\langle \Phi_n | \Psi \rangle|^2. \]

For instance, the state \( \Phi_n \) represents a product state of the \( n \)-th ‘reading’ of an apparatus and the corresponding

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1 The quoted sentences, published prior to the GRW theory, are meant to introduce the pilot wave interpretation of de Broglie and Bohm.
state of a system. As an outcome of reduction, $\Phi_n$ will possess some ‘classical’ properties of the apparatus. To represent the ‘classical’ basis, let us introduce the Hamiltonian $H_0$ which is diagonal in the representation $\Phi_n$,

$$H_0 \Phi_n = E_n \Phi_n.$$ 

Since $\Phi_n$ need not be an eigenstate of the true Hamiltonian $H$ of the whole system, we will generally find $H = H_0 + H_1$ and $[H_0, H_1] \neq 0$. In practice, the difference $H_1 = H - H_0$ may be formally regarded as a negligible quantum-mechanical perturbation to $H_0$, so that the ‘macro’ state $\Phi_n$ would be only quasi-stationary.

To put it differently, let us assume an eigenstate $\Phi_i$ of $H_0$ to represent a ‘macroscopic’ state. In the presence of an ever-present microscopic off-diagonal perturbation $H_1$, and in the absence of any ‘observer’, it will be developed into a grotesque linear superposition by the causal time evolution,

$$\Phi_i \rightarrow \Psi = \sum_n c_n \Phi_n.$$ 

(1)

However small $H_1$ may be, this is generally an ultimately inevitable consequence, in principle. Therefore, we have somehow recourse to the reduction process projecting $\Psi$ back again onto one of the ‘macroscopic’ states, $\Phi_m$. In terms of the wave function,

$$\Psi = \sum_n c_n \Phi_n \rightarrow \Phi_m,$$ 

(2)

or, in terms of the density matrix

$$\hat{\rho} = \sum_{m,n} c_m^* c_n |\Phi_n \rangle \langle \Phi_m| \rightarrow \sum_n w_n |\Phi_n \rangle \langle \Phi_n|.$$ 

(3)

The reduction process is characterized not only by the resulting set of states, but also by the loss of phase correlation possessed by the initial linear superposition $\Psi$. We suppose that the both aspects are built into the Hamiltonian as $H = H_0 + H_1$ in the way that $H_0$ provides a basis set on which the initial entangled configuration is projected, while $H_1$ characterizes the correlation to be lost in the reduction. In effect, all the situations encountered in ‘measurement’ seem to have the Hamiltonian of this structure. In practice, the off-diagonal matrix elements of $H_1$ may quantitatively depend on the choice of the basis defined with $H_0$. By way of illustration, we will later regard the kinetic energy of a massive particle as $H_1$ in order to realize a spatially localized wave packet. Then the matrix elements will depend on the spatial width of the resulting localized state.

The above discussion is aimed at providing a framework in which to discuss the projection postulate objectively in the following sections. In any case of our concern, the decomposition of the full Hamiltonian $H$ into an ‘independent-particle’ basis and ‘(scattering) interaction’ thereof will be considered as self-evident a priori, and it is formally regarded as a physical device to incorporate the ‘classical’ basis into our formalism in a self-contained manner. In this way we take account of a ‘macro-realistic’ assumption that there exist in nature a special set of distinct states whose linear superpositions is intrinsically prohibited. This may be regarded as a postulate to be checked experimentally. Within this framework, we shall next inquire how small the effect of $H_1$ has to be, for the off-diagonal correlations to collapse spontaneously.

### III. TRIGGER PROBLEM

The transformation (1) is causal and reversible, while the process (2) is essentially irreversible, in striking contrast. With this crucial point in mind, we examine rather phenomenologically under what condition the latter is triggered. Taking up the problem this way, we are in an objective standpoint, regarding (2) as a spontaneous elementary process inherent in Nature, which will occur independently of any observer. To characterize the process (2), we have to find relevant physical quantities. It is naturally suspected that the reduction process must have something to do with the Second Law of Thermodynamics. [6]

The first we can think of is the statistical entropy defined by

$$\Delta S = S_f - S_i = -\sum_n w_n \log w_n = -\text{Tr}(\hat{\rho}_f \log \hat{\rho}_f)$$ 

(4)

where $\hat{\rho}_f$ is the density matrix of the statistical ensemble resulting from copies of the initial state $\Psi$. Here we used the notation $\Delta S$ to signify the change of entropy due to (2), as we have $S_i = 0$ for the initial pure states. By the above expression, we still mean to represent the entropy change in each individual event. In effect, a single process (2) transforms a pure state not into a mixed state, but to another pure state in general. Still we associate $\Delta S$ not with the particular final state, but with the final mixture of states, including those which could have been but in fact not realized. In short, by the entropy we characterize the process, not the state. The entropy thus defined characterizes the probabilistic process (2) in an objective manner, and enables us to describe individual events in statistical terms. It never decreases in (2), while in (1) holds $\Delta S = 0$ identically. [13] For this very reason, and since we obviously know that not all quantum states collapse spontaneously, the appealing inequality $\Delta S > 0$ borrowed from the Second Law is disqualified as the criterion for (2). Quite the contrary, microscopic systems mostly preserve quite robust coherence for good. So we must seek another quantity.

The next to which we need pay due attention would be the change of energy $\Delta E$, defined similarly as $\Delta S$. 

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\[
\Delta E = \sum_n w_n \langle \Phi_n | H | \Phi_n \rangle - \langle \Psi | H | \Psi \rangle
= - \sum_{m,n} c_m c_n \langle \Phi_m | H_1 | \Phi_n \rangle = -\langle \Psi | H_1 | \Psi \rangle. \tag{5}
\]

This is the difference of statistical expectation values of energy; the initial and final states, \( \Psi \) and \( \Phi_n \), are generally not the eigenstates of the Hamiltonian \( H \). One may regard (5) as the coherence energy shared by the initial linear combination, but is lost in the final mixture. It is the off-diagonal contribution of interaction energy developed in (1). In particular, the reduction (2) from the ground state of \( H \) will always entail \( \Delta E > 0 \). The magnitude \( |\Delta E| \) will characterize the strength of the collapsing coupling of distinct states in superposition.

Having thus discussed, we had gone through a quintessential point intentionally tacitly. That is, by freely introducing \( \Delta E \) for the reduction process (2), we are abandoning the topmost principle of physics, that is, the law of conservation of energy. We claim that this is unpleasant but not unacceptable, since no fully accepted theoretical explanation has yet been given so far to the wave packet reduction. In effect, we find no compelling reason, but inductive inference, to conclude that energy must conserve in (2) as well. Therefore, in the following, we shall dare to allow \( \Delta E \neq 0 \) as a working hypothesis, and discuss the notable consequences.

Now, we look for the condition in terms of \( \Delta S \) and \( \Delta E \). Imagine a collection of a great number of reduction processes from a single definite state, regard them as physical and real processes, and try to make a thermodynamic description of them. By natural inference, a criterion is drawn on the analogy of the thermodynamic inequality of irreversible processes in an open system, that is,

\[
\Delta S > \frac{\Delta E}{T_0}, \tag{6}
\]

where \( T_0 \) is a constant with the dimension of energy. To sum up, we hypothesize that the wave packet reduction (2) operates when (6) is met, or that, under the condition (6), quantum states are ready to collapse spontaneously so as to provide the statistical ensembles in conformity with the probability principle of QM. Accordingly, we interpret (2) figuratively as depicting an inherent tendency of quantum systems to behave as if they were immersed in a heat bath of the temperature \( T_0 \). One may then regard \(-\Delta E/T_0\) as the entropy production in the heat bath, thereby the entropy principle recovered. We claim \( T_0 \) is a universal constant. In passing, it is of note that the ‘thermodynamic’ criterion (6) is fitted to accommodate a holistic view on non-separability of quantum states.

As mentioned in the introduction, we are not concerned about the trigger mechanism of reduction. It is presumed to be the universal process which sets in when the above entropy criterion is met. To characterize how the stochastic process operates temporally, we hypothesize that reduction is an instantaneous Poisson process with a mean frequency \( \gamma_0 \), and we do not analyze it further. Let this be contrasted with the GRW theory, in which a reduction, especially for a macroscopic body, is effected by numbers of instantaneous processes, called ‘hits’. In each of the hits, wave function is multiplied by a normalized Gaussian of width \( \sim 10^{-5}\text{cm} \), and the hitting frequency \( \lambda \) effectively depends on the number \( N \) of constituent particles comprising the wave function, \( \lambda \simeq N \times 10^{-16}\text{sec}^{-1} \). In contrast, besides \( T_0 \), we regard \( \gamma_0 \) as another universal constant, independent of the system size.\(^2\) The size dependence will be manifested through the criterion (6). By giving up inquiries about the reduction mechanism, or by the definition of ‘classical’ states, we get free from the problem of the tails of the reduced wave function as was raised against the GRW theory.\(^9\)

**IV. Discussion**

In the above formulation, the unitary evolution is interrupted by a reduction within a finite interval of time \( \tau_0 = \gamma_0^{-1} \) after the criterion is met. Hence we predict results in discord with those without reduction after the elapse of time \( t > \tau_0 \), where we can no longer expect the interference phenomena due to the off-diagonal correlation inscribed in (5), not for all practical purposes but in principle. Therefore, we may say that \( \gamma_0 \) is a characteristic time scale separating the coherent reversible quantum regime \( t < \tau_0 \) from the incoherent irreversible ‘classical’ regime. In the weak coupling limit of \( H_1 \), time evolution in the latter regime will be described by infrequent discontinuous jumps between diagonal states of \( H_0 \), or by the Pauli master equation (Appendix A). The incoherent regime appears only when one can conceive of such processes allowed by the criterion (6).

In this regard, it is remarked that the entropy criterion (6) is always met for the energy conserving collapse, namely, for \( \Delta E = 0 \). This may be the case in which the Fermi golden rule (A5) applies, where we claim that the generalised entropy principle applies to microscopic processes as well. This constitutes a part of non-trivial contents of our proposition. The entropy principle decides

\(^2\)One may argue that the frequency \( \gamma_0 \) of reduction and the energy scale \( T_0 \) that we introduced above must be related by an uncertainty relation, \( T_0 \sim \hbar \gamma_0 \). However, the argument would be presumably based on the very assumption we relinquished at the outset, that the reduction is cognate with the Schrödinger-like time evolution. Therefore, the presumed relation is no more than an unfounded hope from our standpoint to accept the two types of processes as essentially distinct and equally elementary.
In equilibrium, the instability condition (6) gives

\[ T_0 \log 2 > v_1, \quad v_1 \gg T \]
\[ T_0 > 2T, \quad T \gg v_1 \quad (8) \]

Under the condition, we can derive a dynamical equation for the density matrix \( \rho_{\sigma'} \). In terms of \( \omega_0 = v_1 / \hbar \), for \( w = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow} \) we obtain

\[ \dot{w} + \gamma_0 w + \omega_0^2 w = 0, \quad (9) \]

while for \( \delta \rho_{\pm} = \rho_{\uparrow\downarrow} \pm \rho_{\downarrow\uparrow} \),

\[ \dot{\delta \rho}_{\pm} = -i\omega_0 w - \gamma_0 \delta \rho_{\pm}, \quad (10) \]
\[ \dot{\delta \rho}_{\pm} = -\gamma_0 \delta \rho_{\pm}. \quad (11) \]

By the decay rate \( \gamma_0 \), the system tends to the diagonal equilibrium \( w = \delta \rho_{\pm} = 0 \). In general, the criterion (6) is expressed in terms of \( \Delta E = v_1 \delta \rho_{\pm} + \Delta S \), a function of \( w \) and \( \delta \rho_{\pm} \). In the equilibrium, one would rather follow the state vector as a function of time. In the ‘classical’ limit \( \omega_0 \gg 1 \), there appears an intermediate time scale \( \Delta t \) between \( \omega_0 \gg 1 \): \( \tau_0 \gg \Delta t \gg \omega_0^{-1} \). Once localized in \( | \uparrow \rangle \) or \( | \downarrow \rangle \), the gross observers who can tolerate an inaccuracy of \( \Delta t \gg \tau_0 \) will find infrequent discontinuous random switching back and forth between the classical states \( | \uparrow \rangle \) and \( | \downarrow \rangle \), fluctuating with the elongated time scale \( \omega_0^{-2} \tau_0^{-1} \).

Let us speculate further what would occur if the system under the condition (8) is open to surrounding infinite space via some interaction, i.e., when the system cannot reach statistical equilibrium. Suppose an interaction \( H_2 \) to relax the excited state \( | 1 \rangle = (| \uparrow \rangle - | \downarrow \rangle) / \sqrt{2} \) back to \( | 0 \rangle \) within a ‘microscopic’ lifetime \( \tau \), presumably by emitting a ‘photon’ \( | \omega \rangle \), e.g.,

\[ H_2 = v_2 (| \omega \rangle \langle 0 | + | 1 | \langle 0 | \omega \rangle). \quad (12) \]

In the limit \( \tau \ll \tau_0 \), reduction is relatively insignificant, and the ground state \( | 0 \rangle \), even if unstable, survives for a while. After the elapse of time \( \tau_0 \), it is spontaneously excited to populate \( | 1 \rangle \), which then immediately relax back again to \( | 0 \rangle \) within the lifetime \( \tau \). Hence, under the condition (8), we predict infinite cycles of reduction and causal evolution. The cyclic time evolution, adapted to the wave propagation in free space, has been noted in the Károlyházy model. [5] In the above example, not only an explicit demonstration of energy non-conservation, but also the proposed statistics of reduction with the mean lifetime \( \tau_0 \), are reflected in the counting of the emitted state \( | \omega \rangle \) in principle.

In practice, a macroscopic body must always have a well defined position in the objective description of reality. In our scheme, a spatially localized quasi-eigenstate of a massive particle in free space is realized similarly as above; in this case, at the cost of the last off-diagonal correlation left in the Hamiltonian, namely, the kinetic energy.
\[ H_1 = \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right), \]  
\tag{13}

for which the criterion (6) reads \( v \gtrsim \lambda_0^3 \), where \( v \) is the volume occupied by the wave packet and

\[ \lambda_0 = \frac{\hbar}{\sqrt{2\pi m T_0}} \]  
\tag{14}

is the thermal de Broglie wavelength at the temperature \( T_0 \). The reduced wave packet will unitarily develop further for the duration \( \tau_0 \) at most, or it will grow as large as \( \lambda_{0,\text{max}} \) given by

\[ \lambda_{0,\text{max}}^2 = \lambda_0^2 + \left( \frac{\hbar \tau_0}{2m \lambda_0} \right)^2. \]  
\tag{15}

The stochastic processes keep the wave packet from spreading without limit. Note \( \lambda_{0,\text{max}} \approx \lambda_0 \) if \( T_0 \tau_0 \ll 4\pi \hbar \) (cf. the second footnote).

The above consideration is directly applied to the center-of-mass motion of the many-body wave function of a macroscopic body. The body will perform a Brownian motion within the width of order \( \lambda_0 \) along the classical trajectory, as it is expected from Ehrenfest’s theorem. As a matter of fact, the results of the last paragraph remain valid as far as the potential energy \( H_0 = V(x, y, z) \) does not vary appreciably over a region of the linear dimension \( \sim \lambda_0 \). Thus, our predictions are qualitatively similar as in the GRW model and the Károlyházy model, [3,5] though quantitatively the results for the width are all different from each other. [18] For macroscopic bodies, the finite breadth \( \lambda_0 \) of the trajectory means slight departure from classical mechanics. [3,5,18] For microscopic systems, the width \( \lambda_0 \) of a wave packet must be reflected in diffraction experiments as a washout of an interference pattern, although we predict no phase shift as expected for nonlinear theories. [19] The effect of \( \lambda_0 \) will be manifested as the mass dependence on the contrast of interference fringes for material waves of a fixed de Broglie wavelength \( \lambda > \lambda_0 \).

The upper bound allowed for the numerical choice of \( T_0 \) is set by experiments on quantum interference. A double slit neutron interference experiment by Zeilinger et al., [20] in qualitatively agreement with the prediction of QM, suggests \( \lambda_0 > 243 \text{Å} \), a coherence length of the diffracted neutrons, or \( T_0 < 7 \times 10^{-26}\text{J} \). In practice, the bound should be still much lowered considerably, by reducing the bandwidth \( \Delta \lambda = 1.4 \text{Å} \) of the neutron. On the other hand, the lower bound of \( T_0 \) is set from a rough estimate on a macroscopic body, e.g., \( \lambda_0 \lesssim 1 \mu\text{m} \) for \( m = 1\text{ng} \), by which \( T_0 \gtrsim 10^{-40}\text{J} \). Therefore, as anticipated by the fundamental ‘shiftiness’ of the micro-macro boundary in the standard quantum theory, we still have a wide range left unexplored for \( T_0 \). This holds true even if a free neutron is supposed to have \( \lambda_0 > 1\mu\text{m} \).

As for the time scale \( \gamma_0 = \tau_0^{-1} \), we claim that a non-trivial aspect of our proposition \( \gamma_0 > 0 \) has been already supported experimentally by an exponential decay law of an unstable system like a radioactive nucleus. Indeed, as is well known, the decay law \( N(t) = N(0) \exp(-t/\tau) \) for the number \( N(t) \) of radioactive nuclei cannot derive strictly from the unitary time evolution of the decaying state. [21] Instead, it follows from the classical assumption of the complete independence of the nuclei state of the past history, \( N(t + t') = N(t)N(t') \). Therefore, we agree with Fonda et al. [21] and others, who point out that the exponential behavior is explained satisfactorily by random reduction processes due to ‘measurement’, although we disagree with them on the crucial point that the frequent ‘measurement’ processes be ascribed to interactions with its environment. On the contrary, one would rather expect the lifetime \( \tau \) of the unstable nucleus is an intrinsic property of the nucleus, independent of the presence or absence of any ‘observer’.

Based upon common sense, we suspect that \( \tau_0 \) would be at large of order of the human perception time, or \( \tau_0 \lesssim 10^{-2} \) second. Furthermore, we have to remark that we are severely put under the constraint that the power of anomalous energy fluctuation \( \sim T_0/\tau_0 \), that we predict rather unwillingly, must be extraordinarily small for the present theory to be viable.

V. SUMMARY

To summarize, we postulate two fundamental laws of time evolution in quantum mechanics. The first is causal and unitary, described by the Schrödinger equation. The second is unprecedented as one of the elementary processes; irreversible, stochastic, and energy non-conserving. It is proposed that quantum state \( \Psi \) of the Hamiltonian \( H = H_0 + H_1 \) is spontaneously and instantaneously projected onto one of the eigenstates \( \Phi_n \) of the reduced Hamiltonian \( H_0 \) with the probability \( |\langle \Phi_n | \Psi \rangle|^2 \) and with the mean frequency \( \gamma_0 \), when the generalized entropy criterion (6) is satisfied.

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APPENDIX A: MASTER EQUATION

In this appendix, we derive a master equation for the density matrix \( \hat{\rho}(t) \) on the basis of the formalism presented in this paper. We assume the representation in
which $H_0$ is diagonal while $H_1$ has no diagonal element, and the latter is treated by perturbation theory. The derivation is then compared with the technique due to Van Hove. [22]

To show the non-trivial effect of the constant $\gamma_0$, we consider a special case in which the generalized entropy criterion (6) is always met for the small perturbation $H_1$. Then, by prescription, the effect of reduction is described by the equation

$$\frac{\partial \hat{\rho}(t)}{\partial t} = \frac{1}{i\hbar}[H, \hat{\rho}] - \gamma_0 \delta \hat{\rho}, \quad (A1)$$

where

$$\delta \hat{\rho}(t) = \hat{\rho}(t) - \sum_n P_n \hat{\rho}(t) P_n, \quad (A2)$$

and $P_n = |n\rangle\langle n|$ is the projection operator on the $n$-th eigenstate $|n\rangle$ of $H_0$. The last term of (A1) effectively suppresses the off-diagonal elements of $\hat{\rho}$. Eq. (A1) was specifically discussed by Fonda, Ghirardi and Rimini [23] to investigate the environmental effect of random ‘measurements’ on the decay of an unstable state. On the basis of their results, we only have to investigate time evolution from general states, instead of a special initial state. Nevertheless, it is stressed that we interpret (A1) quite differently from Fonda et al. The last term of (A1) is effective only under a prescribed condition, and we regard it as an intrinsic, not extrinsic, property of the system.

Let us consider the diagonal elements of $\Delta \hat{\rho} = \hat{\rho}(\Delta t) - \hat{\rho}(0)$, for which (A1) is solved by iteration. [23]

$$\Delta \rho_{jj} = \frac{2}{\hbar^2} \sum_k \frac{|\langle j| H_1 |k \rangle|^2}{\omega_{jk}^2 + \gamma_0^2} \times \left[ \gamma_0 \Delta t + \frac{\omega_{jk}^2 - \gamma_0^2}{\omega_{jk}^2 + \gamma_0^2} \left( 1 - e^{-\gamma_0 t \cos(\omega_{jk} \Delta t)} \right) \right] - \frac{2 \gamma_0 \omega_{jk}}{\omega_{jk}^2 + \gamma_0^2} e^{-\gamma_0 t \sin(\omega_{jk} \Delta t)} (\rho_{kk} - \rho_{jj}), \quad (A3)$$

where $\hbar \omega_{jk} = E_j - E_k$ and $E_j$ is the unperturbed energy of $|j\rangle$. The diagonal elements of $\hat{\rho}$ are stationary in the zeroth order approximation. In the long time regime $\Delta t \gg \tau_0 = \gamma_0^{-1}$, from (A3), we immediately obtain the master equation

$$\frac{\Delta \rho_{jj}}{\Delta t} = \sum_k (W_{jk} \rho_{kk} - W_{kj} \rho_{jj}), \quad (A4)$$

where we define the transition probability

$$W_{jk} = \frac{2 \pi}{\hbar} |\langle j| H_1 |k \rangle|^2 \delta_{\gamma_0}(\hbar \omega_{jk}), \quad (A5)$$

in terms of

$$\delta_{\gamma_0} = \frac{\gamma_0}{\pi(\omega^2 + \gamma_0^2)}. \quad (A6)$$

It is remarked that the effect of $\gamma_0$ on the off-diagonal matrix elements is to replace the time dependent factor $e^{-i\omega_{jk} t}$ with $e^{-i\gamma_0 t} e^{-\gamma_0 t}$, so that they are suppressed effectively.

Next, let us set $\gamma_0 = 0$ in (A3) to discuss a derivation without $\gamma_0$. To derive the master equation, Van Hove assumes the peculiar weak coupling limit $\langle H_1 \rangle \rightarrow 0$ while $\Delta t \rightarrow \infty$, so as to fix $|\langle j| H_1 |k \rangle|^2 \Delta t$ as constant. [22] Indeed, in this limit too, one recovers the same equation (A4) formally, but with the genuine delta function $\delta(\omega)$, owing to the formula

$$\lim_{t \rightarrow \infty} \frac{1 - \cos \omega t}{t \omega^2} = \pi \delta(\omega),$$

instead of the above Lorentzian $\delta_{\gamma_0}(\omega)$ for $W_{jk}$. But this is only the first half of the derivation.

In order to warrant the absence of interference terms, Van Hove has to restrict the initial configuration for $\hat{\rho}(0)$ to a special class of states. In effect, such a special assumption on the absence of the initial correlations, or on the initial states of low entropy, is surely imperative to derive the irreversible equation on the basis of reversible mechanics, for it is obviously generally impossible.

Incidentally, the indeterminacy of energy $\sim \hbar \gamma_0$, as implied in (A6) for individual processes, is completely within the conventional framework of quantum mechanics. This effect should not be misidentified with (statistically significant) energy non-conservation $\Delta E \neq 0$ that we invoke for (5).

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