Magnetism via superconductivity in SF proximity structures

V. N. Krivoruchko, V. N. Varyukhin

Donetsk Physics & Technology Institute NAS of Ukraine,
Str. R. Luxemburg 72, 83114 Donetsk, Ukraine and
E-mail: krivoruc@krivoruc.fti.ac.donetsk.ua

(Dated:)

Abstract

We consider the proximity effects in hybrid superconductor (S) - ferromagnet (F) structures drawing attention to the induced ferromagnetism of the S metal. The analysis is based on a quasiclassical theory of proximity effect for metals in the dirty limit conditions. It is shown that, below the superconducting critical temperature, ferromagnetic correlations extend a distance of order of the superconducting coherence length $\xi_S$ into the superconductor, being dependent on the S/F interface parameters. We argue that the properties of mesoscopic SF hybrids may drastically depend upon the magnetic proximity effect, and recent experiments lend support to the model of SF structures where the superconducting and magnetic parameters are tightly coupled.
I. INTRODUCTION

Proximity effects are phenomena stipulated by a ‘penetration’ of an order parameter (of some state) from one material into another, which does not possess such type of the order itself, due to the materials being in contact. The leakage of superconducting correlations into a non-superconducting material is an example of the superconducting proximity effect. For a nonmagnetic normal metal (N) in contact with a superconductor (S), the proximity effect has been intensively studied and well understood many years ago [1,2]. However, in the case of SN structures we deal with a single type of order - superconductivity. When a normal nonmagnetic metal is replaced by a ferromagnet (F), the physics of proximity effect is much more interesting and rich [2-20]. There are two competing states with different order parameters: superconductivity and ferromagnetism. Due to the difference in energy between spin-up and spin-down electrons and holes under the exchange field of a ferromagnet, a singlet Cooper pair, adiabatically injected from a superconductor into a ferromagnet, acquires a finite momentum $\Delta p \sim H_e/\hbar v_F$ (here $H_e = \mu_B h_F$ is an extra energy caused by the intrinsic magnetic field $h_F$ in ferromagnet; $v_F$ is the Fermi velocity, $\hbar$ is the Planck constant, and $\mu_B$ is the Bohr magneton). As a result, proximity induced superconductivity of the F layer is spatially inhomogeneous and the order parameter contains nodes where the phase changes by $\pi$. Particularly, transport properties of tunnel SF structures in the $\pi$-phase state have turned out quite unusual. The phase shift of $\pi$ in the ground state of the junction is formally described by the negative critical current $J_C$ in the Josephson current-phase relation: $J(\varphi) = J_C \sin(\varphi)$. The $\pi$-phase state of an SFS weak link due to Cooper pair spatial oscillation was first predicted by Buzdin et al., [4,5]. Experiments that have been performed by now on SFS weak links [6-8] and SIFS tunnel junctions [9] directly prove the $\pi$-phase superconductivity (I denotes an insulator). Planar tunneling spectroscopy also reveals a $\pi$-phase shift in the order parameter, when superconducting correlations coexist with ferromagnetic order [10]. The superconducting phase was also measured directly [11] using SQUID’s made of $\pi$-junctions.

There is another interesting case of a thin F layer, $d_F << \xi_F$, being in contact with an S layer. As far as the thickness of the F layer, $d_F$, is much less than the corresponding superconducting coherence length, $\xi_F$, there is spin splitting but there is no order parameter oscillation in the F layer. Surprisingly, but it was recently predicted Refs. [12,13] that for
SFIFS tunnel structures with very thin F layers one can, if there is a parallel orientation of the F layers magnetization, turn the junction into the $\pi$-phase state with the critical current inversion; if there is an antiparallel orientation of the F layers internal fields, one can even enhance the tunnel current. It was shown in Refs. [14,15], that physics behind the inversion and the enhancement of the supercurrent in this case differs from that proposed by Buzdin et al.

While proximity induced superconductivity of the F metal in SF hybrid structures has been intensively studied, much less attention has been paid to a modification of the electron spectrum of a superconductor in a region near the S/F interface due to a leakage of magnetic correlation into the superconductor. Some feature of the induced magnetism (e.g., the spin-splitting of the density of states) were found by numerical calculations in Refs. [16-18]. To our knowledge, only recently the question of S metal magnetization has been addressed in Refs. [19,20]. (Here we do not touch S/FI systems, where FI stands for a ferromagnetic insulator (semiconductor). In such systems conduction electrons penetrate the magnetic layer on much smaller distances than in the case of metals and are totally reflected at the S/FI boundary. The S/FI boundary being magnetically active rotates spins of reflected electrons. This spin rotation occurs only as a result of a tunneling by the quasiparticle into the classically forbidden region of the boundary. Due to the spin rotation the exchange field is induced in a superconductor on a distance of order of superconducting coherence length $\xi_S$ near an S/FI surface [21-23]. However, in contrast to ferromagnetic metals, where the proximity effect is pronounced, this effect is drastically reduced in S/FI structures.)

The investigation of a ‘magnetic proximity’ effect in SF nanostructures is the purpose of this report. To tackle the physics, we consider an interesting and practicable case of an SF structure of a massive superconducting and thin ferromagnetic layers. Using a quasiclassical theory of superconductivity for proximity coupled bilayer (Sec. II), we will show that for some limits the problem can be solved analytically. Two limits will be discussed here: (i) a weak and (ii) a strong proximity effect. Section III is the key one; here we describe the examples of the magnetic proximity effect manifestation. We show that due to induced magnetism of the S metal: (i) the superconducting phase jumps at the S/F interface; as well as, there are (ii) additional suppression of the order parameter near the S/F interface; (iii) the spin splitting of the quasiparticle density of states (DOS); (iv) the appearance of the local bands inside the energy gap; and, directly, in (v) induced equilibrium electronic
magnetization of the S layer that spreads over distance of the order of the superconducting coherence length $\xi_S$. We also briefly discuss recent experiments. Summarizing the results in Conclusion we draw attention to the fact that in the general case, for proximity coupled SF hybrid structures both phenomena - induced superconductivity of the F metal and induced magnetism of the S metal - take place simultaneously and should be considered self-consistently.

II. QUASICLASSICAL THEORY OF SUPERCONDUCTIVITY OF SF BILAYER

A. Bilayer model

Let us consider proximity effects in the bilayer of a massive superconducting ($d_S >> \xi_S$) and a thin ferromagnetic ($d_F << \xi_F$) metals, with arbitrary transparency of the S/F interface. Here $\xi_S = (D_S/2\pi T_C)^{1/2}$ and $\xi_F = (D_F/2H_e)^{1/2}$ stand for the superconducting coherence lengths, $D_{S,F}$ are the diffusion coefficients, $d_{S,F}$ are the thicknesses of the S and F layers. (Henceforth, we have taken the system of units with $\hbar = k_B = 1$.) We assume the ‘dirty’ limit for both metals, i.e., $\xi_{S,F} >> l_{S,F}$ where $l_{S,F}$ are the electron mean free paths. It is also assumed that the superconducting critical temperature of the F material equals zero. All quantities are assumed to depend only on a single coordinate $x$ normal to the interface surface of the materials. We also expect that the F layer has a homogeneous (monodomain) magnetic structure with magnetization aligned parallel to the interface, so that there is no spontaneous magnetic flux penetrating into the S layer. Under these conditions, the only magnetic interaction which can affect the superconductor is the short-range exchange interaction between the superconducting quasiparticles and magnetic moments into the ferromagnet.

B. Main equations

As is well know, the superconductivity of ‘dirty’ metals is conveniently described by the quasiclassical Usadel equations [24] for the normal, $G_{\sigma\sigma'}(x, \omega)$ and $\tilde{G}_{\sigma\sigma'}(x, \omega)$, and anomalous, $F_{\sigma\sigma'}(x, \omega)$ and $\tilde{F}_{\sigma\sigma'}(x, \omega)$, Green functions, integrated over energy and averaged over the Fermi surface. (Green functions are defined in a standard way, see, e.g. Ref. [25]). It can be shown, that for singlet pairing and in the absence of spin-flip scattering, the whole
system of Usadel equations decomposes into two equivalent subgroups, which go over to each other under interchange of the spin indices ($\sigma = \uparrow, \downarrow$) $\uparrow \leftrightarrow \downarrow$ and reversal of the exchange field sign, $H_e \leftrightarrow -H_e$.

It is convenient to take into account the normalization of the Green function, $G_{F\uparrow\uparrow} \tilde{G}_{F\downarrow\downarrow} + F_{F\uparrow\downarrow}^+ F_{F\downarrow\uparrow} = 1$, explicitly and to introduce [2] modified Usadel functions $\Phi_S, \Phi_F$, defined by the relations $\Phi_S = \omega F_S / G_S, \Phi_F = \tilde{\omega} F_F / G_F$, etc. Then we can recast the equations for the S layer in terms of these functions. We specialize the discussion to a geometry when all quantities depend on a single coordinate $x$, normal to the S/F interface. For the superconducting metal we have ($x \geq 0$):

$$\Phi_S = \Delta_S + \xi^2 \frac{2\pi T_C}{\omega G_S} [G_S^2 \Phi'_S], \quad G_S = \frac{\omega}{(\omega^2 + \Phi_S \tilde{\Phi}_S)^{1/2}},$$

(1)

with the superconducting order parameter $\Delta_S(x)$ determined by the self-consistency equation:

$$\Delta_S \ln(T/T_C) + 2\pi T \sum_{\omega > 0} \left[ (\Delta_S - \Phi_S G_S) / \omega \right] = 0,$$

(2)

Here the prime denotes differentiation with respect to a coordinate $x$, and in Eq. (2) the summation over frequencies is cut off by the Debye frequency $\omega_D$. For the F metal we have ($-d_F \leq x < 0$):

$$\Phi_F = \xi^2 \frac{\pi T_C}{\tilde{\omega} G_F} [G_F^2 \Phi'_F], \quad G_F = \frac{\tilde{\omega}}{(\tilde{\omega}^2 + \Phi_F \tilde{\Phi}_F)^{1/2}},$$

(3)

Here $\tilde{\omega} = \omega + iH_e$, and $\omega \equiv \omega_n = \pi T(2n + 1), n = \pm 1, \pm 2, \pm 3, \ldots$ is Matsubara frequency. Assuming the symmetry of the system with respect to the rotation in spin space both in the F and in the S layers, we drop the spin indices, apart from the specified cases. We also put for the F metal a vanishing value of the bare superconducting order parameter $\Delta_F = 0$, while the pair amplitude $F_F \neq 0$ due to proximity with the superconductor.

The equations for the functions $\tilde{\Phi}_S$ and $\tilde{\Phi}_F$ have a form analogous to (1)-(3); note that $\tilde{\Phi}(\omega, H_e) = \Phi^*(\omega, -H_e)$. In Eq. (3) we write our formulas for the F metal using the effective coherence length of normal nonmagnetic (N) metal with the diffusion coefficient $D_F$, $\xi = (D_F / 2\pi T_C)^{1/2}$, instead of $\xi_F = (D_F / 2H_e)^{1/2}$, to have a possibility to analyze both limits $H_e \to 0$ (SN bilayer) and $H_e \gg \pi T_C$. The relation on the ferromagnetic layer thickness one may read as $d_F << \min(\xi_F, \xi)$. 

5
C. Boundary conditions

The Eqs. (1)-(3) should be supplemented with the boundary conditions in the bulk of the S metal and at the external surface of the F layer. Far from the S/F interface, \( x \gg \xi_S \), for the S layer we have the usual boundary conditions in the bulk of the S metal: \( \Phi_S(\infty) = \Delta_S(\infty) = \Delta_0(T) \), where \( \Delta_0(T) \) is the BCS value of the order parameter. At the external surface of the F metal \( \Phi'_F(-d_F) = 0 \). The relations at the S/F interface we obtain [26] by generalizing the results of Kupriyanov and Lukichev [27] for interface between two superconductors.

The first condition on the Usadel equations ensures continuity of the supercurrent flowing through the S/F boundary at any value of the interfacial transparency. Going over to the modified Usadel functions \( \Phi_S \) and \( \Phi_F \), the first boundary condition has the form:

\[
\left. \frac{1}{\omega} \gamma \xi G_{F}^{2} \Phi'_F \right|_{x=0} = \left. \frac{1}{\omega} \xi S G_{S}^{2} \Phi'_S \right|_{x=0},
\]

(4)

Here \( \gamma = \rho_S \xi_S/\rho_F \xi_F \) is the proximity effect parameter, which characterizes the intensity of superconducting correlations induced in the F layer, and vice versa, an intensity of magnetic correlation induced into the S layer; \( \rho_{S,F} \) are the resistivities of the metals in the normal state.

The boundary condition (4) takes into account the effect of quasiparticle DOS of the metals in contact. The second relation takes into consideration the effects of a finite transparency (electrical quality) of the interface. For \( \Phi(\omega, x) \) - parametrization the second boundary condition becomes

\[
\xi_{BF} G_{F} \Phi'_F |_{x=0} = \tilde{\omega} G_{S}(\Phi_S/\omega - \Phi_F/\tilde{\omega})|_{x=0},
\]

(5)

where \( \gamma_{BF} \) is the parameter that characterizes the effects of a finite transparency of the interface. For \( \gamma_{BF} = 0 \), i.e., for a fully transparent boundary, condition (5) goes over to \( \Phi_S/\omega = \Phi_F/\tilde{\omega} \). The expression for \( \gamma_{BF} \) can be written through more convenient values: \( \gamma_{BF} = R_B/\rho_F \xi_F \), where \( R_B \) is the product of the S/F boundary resistance and its area [27].

The relations (4) and (5) generalize the proximity effect problem with an arbitrary interface transparency for the case of a normal metal with ferromagnetic order. The additional physical condition we assumed is that the exchange splitting of the momentum subbands, \( p^\pm_F = \sqrt{2m \sqrt{E_F \pm H_e}} \), is substantially smaller than the Fermi energy \( E_F \) (\( m \) is the effective mass of an electron). For most magnetic materials the momentum renormalization is not so
important as the frequency renormalization, because \( H_e >> \omega_n \tilde{\pi}_T \) while \( H_e << E_F \) and due to this the difference in the DOS and transparencies of the S/F interface for electrons with opposite spin orientations can be neglected.

According to the Green functions formalism, if the functions \( G_{S,F}(x, \omega) \) and \( F_{S,F}(x, \omega) \) are known, that is all we need to be able to describe, at least in principle, any superconducting and magnetic properties of the system. We draw attention to the feature important for further conclusions: due to superconductivity these is only a single space length - the respective superconducting coherence length, \( \xi_S \) or \( \xi_F \), - that encounters into the differential equations (1) and (3). So, due to superconductivity the coordinate dependences of both superconducting and magnetic properties of each layer have the same typical space scale.

D. Analytical solutions

The proximity effect for an SF structure with a thin F metal, \( d_F << (\xi_F, \xi) \), can be reduced to consideration of the boundary value problem for the S layer [2,26,28]. Indeed, the differential equation (3) can be solved by iteration with respect to the parameter \( d_F/\xi \). To a first approximation one can neglect the nongradient term and, taking into account that \( \Phi'_F(-d_F) = 0 \), we obtain \( \Phi_F(x) = \text{const} \). In the next approximation in \( d_F/\xi \) we find, after linearizing Eq.(3),

\[
\Phi_F(\omega, x) = \frac{\tilde{\omega}_F(\omega, 0)}{\xi\pi T CG_F} (x + d_F)
\]

Here we have again taken into account the condition that \( \Phi'_F(-d_F) = 0 \). Determining \( \Phi'_F(0) \) from Eq. (6) and substituting it into boundary conditions (4) and (5), we obtain the boundary condition for the function \( \Phi_S(\omega, x) \). We have (here we restore the spin index):

\[
\xi_S G_S \Phi'_S|_{x=0} = \gamma_M \tilde{\omega}_\sigma \Phi_S \left[ \pi T_C \left( 1 + \frac{2G_S \gamma_B \tilde{\omega}_\sigma}{\pi T_C} + \frac{(\gamma_B \tilde{\omega}_\sigma)^2}{(\pi T_C)^2} \right)^{1/2} \right]^{-1} \bigg|_{x=0}
\]

where \( \tilde{\omega}_\sigma \equiv \omega + i\sigma H_e \). The unknown value of the function \( \Phi_F(\omega, x = 0) \) is defined by the relation:

\[
\Phi_F(\omega, 0) = G_S \Phi_S \left[ \omega \left( \frac{\gamma_B}{\pi T_C} + \frac{G_S}{\tilde{\omega}_\sigma} \right) \right]^{-1} \bigg|_{x=0}
\]

We introduce the effective boundary parameters, \( \gamma_M = \gamma d_F/\xi \) and \( \gamma_B = \gamma_B F d_F/\xi \), instead of \( \gamma \) and \( \gamma_B \). As a result, the problem of the proximity effect for a massive superconductor
with a thin ferromagnet layer reduces to solving the equations (1) and (3) for a semi-infinite S layer with the boundary conditions (7) on the external side and BCS type on infinity. The spatial dependence of the function $\Phi_F(\omega, x)$ in the F layer can be neglected due to the mesoscopic thickness $d_F \ll (\xi, \xi_F)$ of the latter; Eq. (8) determines the value of $\Phi_F(\omega, 0)$.

One can directly see, that via the boundary condition, Eq. (7), electronic spin ‘up’ and spin ‘down’ subbands lost its equivalence in the S metal too. Spin discrimination means magnetism of a metal. The penetration of the magnetic correlation into the superconducting layer is governed by the proximity effect parameter $\gamma_M$, i.e., by the electron density of states on contacting metals. For high quasiparticle’s density in the F metal in comparison to that in the S counterpart (a large value of $\gamma_M$) the equilibrium diffusion of these quasiparticles into the superconductor leads to an effective leakage of magnetic order into the S layer and strong suppression of superconductivity near the S/F interface. In the opposite case, $\gamma_M \ll 1$, the influence of the F layer on properties of the S metal is weak; it even vanishes if $\gamma_M \to 0$. Opposite is the behavior of the superconductivity on this parameter. So, to increase magnetic correlation near the S/F interface one should increase the parameter $\gamma_M$; in order to increase superconducting correlation one should decrease this parameter. Of course, the electric quality of the interface is also important for the penetration of magnetic and superconducting correlations from one metal into another.

There are three parameters which enter the model: $\gamma_M$ is the measure of the strength of proximity effect between the S and F metals, $\gamma_B$ describes the electrical quality of the SF boundary, and $H_e$ is the energy of the exchange correlation in the F layer. In a general case, the problem needs self-consistent numerical solution. Here, to consider the new physics we are interested in, we will not discuss the quantitative calculations, but will use analytical ones obtained earlier in Refs. [26,28] for two limits: (a) $\gamma_M \ll 1$, small strength of the proximity effect - low suppression of the order parameter in the S layer near the S/F boundary, and (b) $\gamma_M >> 1$, strong suppression of the order parameter in the S layer near the S/F boundary. Note that the results obtained are applicable to any value of the S/F boundary transparency, as we made no specific assumption about $\gamma_B$ in the derivation below.

**Weak proximity effect.** If $\gamma_M \ll 1$, one can find an explicit expression for $\Phi_S(\omega, x)$ in the form

$$\Phi_S(\omega, x) = \Delta_0 \left\{ 1 - \frac{\gamma_M \beta \tilde{\omega}}{\gamma_M \beta \tilde{\omega} + \omega A(\omega)} \exp(-\beta x/\xi_S) \right\}$$

(9)
where $\beta = [(\omega^2 + \Delta_0^2)/\pi T_C]^{1/2}$ and $A(\omega) = [1 + \gamma_B \bar{\omega}(\gamma_B \bar{\omega} + 2\omega/\beta^2)/(\pi T_C)^2]^{1/2}$. As one can expect, the magnetic correlation spreads into the S film over a distance of about $\xi_S$ and it can much exceed the distance of the superconducting correlation spreading into the F film $\xi_F$. If $H_e \rightarrow 0$ (i.e., $\bar{\omega} \rightarrow \omega$) the result (9) restores with that for the SN bilayer in the limit in question (see, e.g., Ref. [2]). For the function $\Phi_F(\omega, 0)$ we obtain

$$\Phi_F(\omega, 0) = \Delta_0 \bar{\omega}_S/(\gamma_B \bar{\omega} \beta^2 + \omega)$$

*Strong proximity effect.* When $\gamma_M >> 1$, the behavior of $\Phi_S(\omega, x)$ near the S/F boundary, $0 < x << \xi_S$, is given by

$$\Phi_S(\omega, 0) = B(T)\left\{(\pi T_C + \gamma_B \bar{\omega})/\gamma_M \bar{\omega}\right\}$$

Here $B(T) = 2T_C[1 - (T/T_C)^2][7\zeta(3)]^{-1/2}$ (see Ref. [2]) and $\zeta(3) \cong 1.2$ is the Riemann $\zeta$ function. The function $\Phi_F(\omega, 0)$ in this approximation is read

$$\Phi_F(\omega, 0) = B(T)\pi T_C/\gamma_M \omega$$

It is seen that the proximity-induced superconductivity in the F layer is independent of the boundary transparency, but decreases with increase of $\gamma_M$. To obtain the results for larger distance, $x \gtrsim \xi_S$, the equations should be solved numerically by a self-consistent procedure. We will not discuss these results here.

III. MAGNETIC PROXIMITY EFFECT MANIFESTATION

An important feature of the results obtained for the SF structure is that the modified Usadel function for the S layer $\Phi_S(\omega, x)$, Eqs. (9) and (10), directly depends on the exchange field of the F metal. That is the reason to speak about the exchange correlation that has been induced into the S layer due to superconductivity. In this section we discuss a few examples of such 'magnetic proximity effect' manifestation.

A. Phase variation at SF interface

Comparing the results for an SF structure with those for an SN bilayer, one can find a fundamental aspect, that leads to new physical consequences; namely, the $\Phi_S(\omega, x)$ is a
complex function near the S/F interface. As a result, the additional 'superconducting phase rotation' (a phase jump on the S/F interface for our approximation of a thin ferromagnetic layer $d_F << \xi_F$) occurs at the S/F interface. To illustrate this, let us take, for simplicity, a structure with favorable for magnetic effects interface parameters: $\gamma_M >> 1$ and $\gamma_B = 0$. Then, as follows from Eq. (10), the modified Usadel function at S/F interface $\Phi_S(\omega, 0)$ can be written in the form

$$\Phi_S(\omega, 0) = B(T)(\pi T_C / \gamma_M) \exp(-i\theta) \left(\frac{\omega^2 + H_e^2}{\omega^2 + H_e^2}^{1/2}\right),$$

with $\theta = \arctan(H_e/\omega)$. Taking into account that a typical value of $\omega \sim \pi T_C$, one can see that in the limit $H_e >> \pi T_C$ the correlation function acquires an additional $\pm \pi/2$ phase shift in comparison with the similar function for the SN bilayer. For an SF multilayered system with strong enough ferromagnetism of the layers the phase shift can be summarized or subtracted, depending on mutual orientation of F layers magnetizations, leading to new effects in superconductivity of SF hybrid structures. Namely, one can show, that proximity induced magnetism of the S layers makes preferable the $\pi$-phase superconductivity of the system for parallel directions of the exchange fields; for antiparallel magnetizations orientation and low temperature, the critical current can be even enhanced [12-15].

B. Suppression of the superconducting order parameter by an exchange field

Another feature of the S/F boundary is that the gap $\Delta_S(x)$ is suppressed near the interface more strongly than in the SN case. This is not surprising, since one would expect that induced ferromagnetism suppresses the superconducting order parameter at some distance into the S layer in excess of that for nonmagnetic normal layer. Suppression increases with the increase of the exchange energy $H_e$ and of electrical quality of the interface; far from the interface, $x >> \xi_S$, the bulk superconductivity is restored.

Using $\Phi_S(\omega, x)$ (9) and the self-consistency condition (2) one can find the spatial variation of the order parameter in the S layer $\Delta_S(x)$ for different values of $\gamma_B$ and $\gamma_M << 1$. The exchange interaction influence on the spatial variation of the order parameter in the S layer is shown in Fig. 1. Namely, the dependence of difference of the order parameters for the case when magnetic interaction is turned off (i.e., an SN bilayer) and with ferromagnetic correlation (a SF bilayer) as function of distance from the interface is shown; the boundary
parameters, $\gamma_M$ and $\gamma_B$, are fixed. It is seen, that influence of magnetism decreases with increasing the distance from the S/F boundary. The scale at which superconductivity reaches the value for a SN bilayer is $\xi_S$ from the interface. The curves in Fig. 1 illustrate the spatial dependencies of the induced exchange correlation in the superconductor for the case of vanishing interface resistance $\gamma_B = 0$. With an increase of the SF boundary resistance the electrical coupling of the S and F metals decreases and in the limit $\gamma_B \rightarrow \infty$ the metals become decoupled.

C. Exchange field spin-splitting of DOS and intra-gap states

Spin splitting of DOS and intergap states in the S layer are other manifestations of magnetic correlation leakage into a superconductor. Note that the magnetic layer does not influence the DOS of the normal metal. In this case the decay length is extremely small $\sim p_F^{-1} \approx 1\,\text{Å}$ and the effect can be neglected.

The Green functions for the S layer $G_{S\uparrow\uparrow}(\omega, x)$ and $G_{S\downarrow\downarrow}(\omega, x)$ for both spin subbands can be obtained using solutions for the functions $\Phi_S(\omega, x)$ with $\tilde{\omega} = \omega + iH_e$ and $\tilde{\omega} = \omega - iH_e$, respectively. Performing the analytical continuation to the complex plane by the substitution $\omega \rightarrow -i\varepsilon$ we calculate the spatial dependence of quasiparticle DOS for spin ‘up’ and ‘down’ subband: $N_{S\uparrow}(\varepsilon, x) = \text{Re} G_{S\uparrow\uparrow}(\omega, x)$ and $N_{S\downarrow}(\varepsilon, x) = \text{Re} G_{S\downarrow\downarrow}(\omega, x)$, respectively. The total density of states for quasiparticles, by definition, is given by $N_S(\varepsilon, x) = N_{S\uparrow}(\varepsilon, x) + N_{S\downarrow}(\varepsilon, x)$. Using Eqs. (9) and (10), one can obtain the explicit expressions for the total DOS, as well as for the specified spin subband. The resulting expressions, which are cumbersome to be presented here, imply that for $H_e \neq 0$, $\gamma_M \neq 0$, and $\gamma_B \neq 0$ the density of quasiparticle states is spin-split: $N_{S\uparrow}(\varepsilon, x) \neq N_{S\downarrow}(\varepsilon, x)$. This is because of the initial exchange field splitting of the Fermi surface in the F metal, which is manifested in the characteristics of the united system — the SF bilayer. The symmetry of the density of states with respect to the energy variable is also lost: $N_{S\uparrow}(\varepsilon > 0, x) \neq N_{S\uparrow}(\varepsilon < 0, x)$. However, as one can expect from the fermionic symmetry, the spin-up particles and spin-down holes have the same DOS, and likewise for spin-down particles and spin-up holes; as a result the total density $N_S(\varepsilon, x)$ is symmetric: $N_S(\varepsilon > 0, x) = N_S(\varepsilon < 0, x)$.

In Fig. 2 representative $N_{S\uparrow}(\varepsilon, x)$ dependences at different distances from the S/F interface are presented for $H_e = 5\pi T_C$ and $\gamma_M = 0.1$, and vanishing boundary resistance.
In Fig. 3 the same dependence is presented for $x/\xi_S = 1$ and different values of the exchange energy. We find that all features mentioned above are saved on a distance of a scale $\xi_S$ from the SF boundary. The spin-splitting decreases with an increase of the distance from the boundary and vanishes in the bulk of the S layer (see curve 4 in Fig. 2).

Other important features, shown in Figs. 2 and 3, are the local states that appear inside the energy gap at the distances up of a few $\xi_S$ from the S/F boundary. These intergap states are absent far from the S/F interface, and also if $H_e = 0$. For small values of $\gamma_M$ and $\gamma_B = 0$, as follows from the expression (9), $N^S(\varepsilon, x)$ has singularity for

$$
\varepsilon = \pm \Delta_0 \{1 - \frac{\gamma_M \beta \varepsilon}{\varepsilon + \gamma_M \beta \varepsilon} \exp(-\beta \varepsilon x/\xi_S)\} 
$$

where $\beta^2 = (\Delta_0^2 - \varepsilon^2)^{1/2}/\pi T_C$ and $\tilde{\varepsilon} = \varepsilon - H_e$. We found the singularity inside the superconducting gap, $-\Delta_0 < \varepsilon < \Delta_0$, by numerical calculations [19]. The local states are definitely not due to the spatial variation of the pair potential, but due to Cooper pairs breaking in the superconductor by the exchange-induced magnetic correlation. The region of their existence increases with the increasing of $H_e$, or increasing pair breaking effects. In the absence of spin-flip (e.g., spin-orbit) scattering, the subgap bands accommodate quasiparticles with a definite (‘up’ or ‘down’) spin direction. These bands bear superficial resemblance to both the bands observed at interface of superconductor and perfectly insulating ferromagnet [29] and bulk superconductor containing finite concentrations of magnetic impurities [30,31].

**D. Induced magnetization of the S layer**

As we saw above, the influence of the ferromagnet on the superconductor is reflected in a nonzero value of the difference in the DOS for spin-up and spin-down unpaired electrons, $N^S(\varepsilon, x)$ and $N^S(\varepsilon, x)$. This DOS splitting causes an effective magnetization $M^S(x)$ of the S layer, that can be found using the relation:

$$
M^S(x)/M_O = \int_0^\infty d\varepsilon \{N^S(\varepsilon, x) - N^S(\varepsilon, x)\} f(\varepsilon)
$$

where $M_O = g_s \mu_B (= \mu_B)$ is a quasiparticle magnetic moment, $s = 1/2, g = 2$ and $f(\varepsilon) = 1/\{\exp(\varepsilon/T) + 1\}$ is the Fermi distribution function. Figure 4 illustrates the mechanism of proximity induced magnetization of the S layer. For $T < T_C$ we took $f(\varepsilon) = 1$, i.e., all states below Fermi level are filled (dashed regions in Fig. 4), while all states above Fermi
energy are empty. One can directly see from the figure that the S layer acquires a nonzero magnetic moment. This suggestion is confirmed by numerical calculations of $M_s(x)$ Eq.(13) shown in Figs. 5, 6. Figure 5 shows the magnetization of the superconductor versus distance from the S/F interface for fixed boundary parameters. The same magnetic characteristics but for a SF sandwich with fixed exchange energy and boundary transparency, and different proximity effect strength are presented in Fig. 6.

E. Experiment

There are only a few experimental reports devoted to the questions discussed here. Interplay between magnetism and superconductivity in Nb/Co multilayers was recently investigated by Ogrin et al. [32]. The upper critical fields of the samples were measured for the field applied parallel to the plane, $H_{C_2||}$ and perpendicular to the plane $H_{C_2\perp}$ of the films. Effective thickness of the Co layer, $d_{eff}$, they define through the well known relation:

$$d_{eff} = \left( \frac{\Phi_0}{2\pi H_{C_2\perp}} \right)^{1/2} \frac{H_{C_2\perp}}{H_{C_2||}}$$

where $\Phi_0$ stands for flux quantum. Experiments revealed that the effective thickness of the magnetic layer in Nb/Co structures is usually much larger than its physical thickness $d_{Co}$. For example, taking the data on sample with $d_{Co} = 1.8$ nm, the authors obtained a value $d_{eff} = 12$ nm, so that $d_{eff} >> d_{Co}$. The ‘increase’ of the thicknesses was so great that in all samples, except for those with extremely thin magnetic layers, the crossover to a 3D state superconductivity is never in fact observed experimentally. This is to be contrasted with the case of nonmagnetic spacer layers, where these two length scales are comparable. Taking into account our results, we explain the rise of the effective magnetic layer thickness in the Nb/Co multilayer as an impact of proximity effect. Namely, the induced magnetic correlation into the S layer depletes Cooper pairs density at the SF boundary, which results in an excess thickness of the magnetic layer.

The modification of the DOS in mesoscopic superconducting strips of Al under the influence of magnetic proximity effect of a classical ferromagnet Ni has also been studied both theoretically and experimentally in [33]. However, since the tunnel spectroscopy experiments were carried out with a nonmagnetic probe, the authors could not measure spin-dependent local DOS in the superconducting side.
The interest in the magnetic proximity effect has been increased with the development of experimental techniques like neutron reflectometry and muon spin rotation, which allow to determine accurately the spatial distribution of magnetic moments. For example, very recently multilayered system YBa$_2$Cu$_3$O$_7$/La$_{2/3}$Ca$_{1/3}$MnO$_3$ have been studied by neutron reflectometry in [34]. Evidence for a characteristic difference between the structural and magnetic depth profiles is obtained from the occurrence of a structurally forbidden Bragg peak is a ferromagnetic state. The authors discussed findings in two possible scenarios: a sizable magnetic moment within the Slaley antiparallel to one in the F layer (inverse proximity effect), or a "dead" region in the F layer with zero net magnetic moment.

IV. CONCLUSION

In recent years, advances in materials growth and fabrication techniques have made it possible to create heterostructures with high quality interfaces. Taking into account that ferromagnet-superconductor hybrid systems have great scientific importance, and are promising for application in spin-electronics, it is not surprising that interest to these hybrid materials has been renewed. As far as the thickness of superconducting and magnetic metals in such structures may be a few atomic periods, understanding of how the proximity effects modify electronic properties of S/F interfaces is growing in importance.

We have studied in the magnetic correlations acquired by a superconductor at S/F interface due to a proximity effect. We have found that an equilibrium exchange of electrons between the F and S metals results not only in proximity induced superconductivity of the F metal, as was found earlier, but in proximity induced magnetism of the S metal, too. The magnetic correlations spread over a large distance which is of the order of the superconducting coherence length $\xi_S$ and can exceed both the ferromagnetic and the superconducting films thicknesses. That is why the existence of these magnetic properties of the S metal is quite important for SF nanoscale structures and should be taken into account while comparing theoretical results with experimental data. Summarizing the results, we should stress that for SF nanoscopic hybrid structures both phenomena, – the superconducting and the magnetic proximity effects, – take place simultaneously, and both should be paid attention to.

Acknowledgements. We wish to dedicate this paper to V.G. Bar’yakhtar, our Master who
played a significant, exceptional role in our post-student life, on the occasion of his 75th birthday, and to wish him continuing health and vigour in pursuing his scientific interest. The authors would like to thank V. V. Ryazanov, A.I. Buzdin, L. Tagirov, and M. A. Belogolovskii for valuable discussions of some questions of proximity effect phenomena. We also acknowledge E. A. Koshina for performing numerical calculations.
1. Wolf E. L. Principles of electron tunneling spectroscopy. - Oxford: University Press, 1985.
2. Golubov A. A., Kupriyanov M. Yu., Il’ichev E. //Rev. Mod. Phys. - 2004.- 76, #2.- p.411-469.
3. Izumov Yu. A., Proshin Yu. N., Khusainov M. G. //Usp. Fiz. Nauk. - 2002.- 45, #2.- p.114-154.
4. Buzdin A. I., Bulaevskii L. N., Panyukov S. V. //Sov. Phys. JETP Lett. -1982.- 35, #4.- p.147-148.
5. Buzdin A. I., Kupriyanov M. Yu. //Sov. Phys. JETP Lett. - 1991.- 53, #6.- p.308-312.
6. Ryazanov V. V., Oboznov V. A., Rusanov A. Yu., Veretennikov A. V., Golubov A. A., Aarts J. //Phys. Rev. Lett. -2001.- 86, #11-12.- p.2427-2430.
7. Ryazanov V. V., Oboznov V. A., Veretennikov A. V., Rusanov A. Yu. //Phys. Rev. B - 2002 - 65, #2. - 020501-4(R).
8. Blum Y., Tsukernik A., Karpovski M., Palevski A. //Phys. Rev. Lett. - 2002. - 89, #18.- p.187004-4.
9. Kontos T., Aprili M., Lesueur J., Genet F., Stephanidis B., Boursier R. //Phys. Rev. Lett. - 2002.- 89, #13. - 137007-4.
10. Kontos T., Aprili M., Lesueur J., Grison X. //Phys. Rev. Lett. - 2001.- 86, #2. - p.304-307.
11. Guichard W., Aprili M., Bourgeois O., Kontos T., Lesueur J., Gandit P. //Phys. Rev. Lett. - 2003.- 90, #16.- p.167001-4.
12. Bergeret F. S., Volkov A. F., Efetov K. B. //Phys. Rev. Lett. - 2001.- 86, #14.- p.3140-3143.
13. Krivoruchko V. N., Koshina E. A. //Phys. Rev. B - 2001.- 64, #17.- p.172511-4.
14. Golubov A. A., Kupriyanov M. Yu., Fominov Ya. V. //JETP Lett. - 2002.- 75, #11.- p.709-713.
15. Koshina E. A., Krivoruchko V. N. //Low. Temp. Phys. - 2003.- 29, #8.- p.642-649.
16. Halterman K., Valls O. T. //Phys. Rev. B -2002.- 65, #1.- p.014509-12.
17. Fazio R., Lucheroni C. //Europhys. Lett. - 1999.- 45, #6. - p.707-713.
18. Halterman K., Valls O. T. //Phys. Rev. B - 2004.- 69, #1.- 014517-11.
19. Krivoruchko V. N., Koshina E. A. //Phys. Rev. B - 2002- 66, #1.- 014521-6.
20. Bergeret F. S., Volkov A. F., Efetov K. B. //Phys. Rev. B -2004.- 69, #17.- p.174504-5.

21. Millis A., Rainer D., Sauls J. A. //Phys. Rev. B - 1988.- 38, #7.- p.4504-4515.

22. Tokuyasu T., Sauls J. A., Rainer D. //Phys. Rev. B - 1988.- 38, #13.- 8823-8833.

23. Fogelström M. //Phys. Rev. B - 2000.- 62, #17.- p.11812-11819.

24. Usadel K. D. //Phys. Rev. Lett. - 1970.- 25, #8.- 507-509.

25. A. M. Swidzinski. Spatially Nonuniform Problems of the Theory of Superconductivity. - Moscow: Nauka, 1982.

26. Koshina E. A., Krivoruchko V. N.//Low. Tem. Phys. - 2000. - 26, #2.- p.115-120.

27. Kupriyanov M. Yu., Lukichev V. F. //Sov. Phys. JETP - 1988.- 94, #6.- p.139-149.

28. Koshina E. A., Krivoruchko V. N.//Phys. Rev. B - 2001.- 63, #22.- 224515-8.

29. DeWeert M. J., Arnold G. B. //Phys. Rev. B - 1989.- 39, #16.- 11307-11319.

30. Shiba H. //Prog. Theor. Phys. - 1968.- 40, #3.- p.435-451.

31. Rusinov A.J. //Sov. Phys. JETP Lett. - 1969.- 9, #1.- p.85-89.

32. Ogrin F. Y., Lee S. L., Hillier A. D., Mitchell A., Shen T.-H. //Phys. Rev. B - 2000.- 62, #9.- 6021-6026.

33. Sillanpää M. A., Heikkilä T. T., Lindell R. K., Hakonen P. J. //Europhys. Lett. - 2001.- 56, #4.- p.590-595.

34. Stahn J., Chakalian J., Niedermayer Ch., Hoppler J., Gutberlet T., Voigt J., Treubel F., Habermeier H-U., Cristiani G., Keimer B., Bernhard C. //Phys. Rev. B.-2005.- 71, #14 - p.140509-4(R).
Figure Captures

Fig. 1. The difference of the superconducting order parameter in the S layer versus distance from the interface for SN and SF structures with the same boundary parameters ($\gamma_M = 0.1, \gamma_B = 0$), and different ferromagnetic field energy $H_e/\pi T_C = 8, 9, 10, 12$ and 15.

Fig. 2. Normalized density of state for spin 'up' quasiparticles in the S layer of the SF sandwich for $\gamma_M = 0.1, \gamma_B = 0$ and $H_e = 5\pi T_C$, and various distances from the S/F interface: $x/\xi_S = 0, 1, 5, \text{ and } 30$ (curves 1, 2, 3, and 4, respectively).

Fig. 3. Same as in Fig. 4 for $\gamma_M = 0.1, \gamma_B = 0.1$ and $x = \xi_S$, and various ferromagnetic field energies: $H_e/\pi T_C = 1, 2, \text{ and } 5$ (curves 1, 2, and 3, respectively).

Fig. 4. Quasiparticle density of states in the S layer near the S/F interface; $\gamma_M = 0.1, \gamma_B = 0.0, x = \xi_S$, and $H_e = 5\pi T_C$. All states above Fermi energy are empty; all states below Fermi level are filled (dashed regions in figure).

Fig. 5. Leakage of magnetization into the S material versus distance from the interface for SF sandwich for $\gamma_M = 0.1, \gamma_B = 0$, and different exchange energies $H_e/\pi T_C = 7, 5, \text{ and } 3$ (curves 1, 2, and 3, respectively).

Fig. 6. Same as in Fig. 6 for $\gamma_B = 0, H_e = 3.5\pi T_C$ and different proximity effect strength $\gamma_M = 0.1, 0.15, 0.2$ (curves 1, 2, and 3, respectively).
\[ \frac{[\Delta_s(0) - \Delta_s(H_c)]}{\Delta_s} \]

- \( H_c / \pi T_C = 15 \)
- \( T/T_C = 0.1 \)
- \( \gamma_M = 0.1 \)
- \( \gamma_B = 0 \)

\[ x/\xi_S \]
$H_e / \pi T_C = 1, 2, 3$
\[ N_q(\varepsilon) \]
$H / \pi T_C = 7 \ (1)$
$5 \ (2)$
$3 \ (3)$

$\gamma_M = 0.1$

$\gamma_B = 0$

$\gamma(x)/\xi_S$
\[ M(x)/\mu_B = \frac{3}{2} \]

\[ H_e = 3.5 \pi T_C \]

\[ \gamma_B = 0 \]

\[ \gamma_M = 0.1 \, (1) \]
\[ 0.15 \, (2) \]
\[ 0.2 \, (3) \]