Effect of microorganisms on the stability analysis in magnetic nanofluids

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Abstract. A study of onset of convection of a new type of fluid, a fluid that contains both magnetic nanoparticles and microorganisms, is presented in this paper. We consider an infinite horizontal layer of water based magnetic nanofluids (MNF) containing gyrotactic microorganisms, heated from below, in the presence of uniform vertical magnetic field. Here we utilize the Chebyshev pseudospectral method to solve the eigen value problem in gravitational environment. The effect of various important parameters which are conducive for the stability of the system is shown graphically.

Keywords: Magnetic nanofluids, Microorganisms, Bioconvection.

1. Introduction
The process when the spontaneous pattern is formed in the suspensions of swimming microorganisms is known as bioconvection[1]. Wager was the first to carry out the detailed observations of bio-convection at the beginning of 20th century. [2]. Then after a long gap of almost 50 years, the subject was taken up again by Platt [3], who apparently coined the term “bioconvection”. Following common features are observed during the pattern formation due to the upswimming of microorganisms: they are little denser than the liquid they swim in, swimming is directed upwards on an average. Moreover, the patterns disappear when the microorganisms stop swimming. Due to the upswimming of microorganisms, the upper surface of the suspensions becomes too dense to be stable. If the density gradient is sufficiently large, overturning instability sets in that causes the cells to descend along thin lines known as plumes. The underlying bioconvection process is similar to that of Rayleigh-Bénard convection but, unlike the latter, it is driven solely by the swimming of microorganisms [4].

Kuznetsov and Avramenko [5] observed that the process of bioconvection intensifies the mixing process and helps in the settling of small particles. A numerical investigation had been conducted by Kuznetsov and Geng [6] to study the effect of bioconvection on mixing of small solid particles. The authors found that the small solid particles of ideal size “decelerate” the bioconvection process. Similarly very large particles having negligible diffusivity and very heavy particles with substantial settling velocity do not affect the bioconvection process. Nield and Kuznetsov [7] performed linear stability analysis for the onset of bioconvection in a thermally conducting fluid. They found that as the gyrotactic number and Péclet number increase the critical Rayleigh number decreases. Kuznetsov [8] reported that the bottom heavy nanoparticle distribution delays the onset of convection while the top heavy arrange-
ment and upswimming microorganisms advance the onset of convection. Nield et al. [9] incorporated the gyrotactic effects in their analysis. Kuznetsov [10] considered horizontal porous layer of finite depth which consists both nanoparticles and gyrotactic microorganisms to study the onset of instability. The author demonstrated that the bioconvection Péclet number $Q$ is the key factor in determining the role of microorganisms on the convective stability. Other experimental investigations were carried out by Loeffler and Mefferd [11], Wille and Ehret [12], Nultsch and Hoff [13], Kessler [14] and, Bees and Hill [15]. Some more aspects related to bioconvection are discussed in the references [16, 17, 18, 19, 20].

MNF are very useful in microfluidic devices but the problem of proper mixing is usually a matter of concern [21]. There are active mixers available for this purpose but most of them are very costly. The process of bio-convection helps in enhancing mixing and mass transfer in micro-devices. It also contributes in enhancing the stability of the nanofluids. Thus inclusion of microorganisms in MNF seems to be a way out to this problem. In order to use this new type of fluid in microsystems and microchannels, the behavior of such fluids must be comprehended at the primary level. In this paper, therefore, an attempt has been made in this direction. The onset of convection of a fluid which contains nanoparticles and microorganisms has been examined.

2. Formulation
Incompressible infinite horizontal layer of magnetic nanofluids containing gyrotactic microorganisms is considered. Two cases are considered: (i) both the boundaries are rigid (ii) upper boundary is stress-free while lower boundary is rigid.

![Diagram of Incompressible Magnetic Nanofluid Containing Gyrotactic Microorganisms](image)

Governing equations:

\[
\nabla \cdot \mathbf{q} = 0,
\]

\[

\rho_f \left( \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{q} \right) = -\nabla p + \mu \nabla^2 \mathbf{q} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} - \rho g \mathbf{k},
\]

\[

\frac{\partial \phi}{\partial t} + \mathbf{q} \cdot \nabla \phi = \nabla \cdot (D_B \nabla \phi + D_T \frac{\nabla T}{T_c} - D_H \nabla H),
\]
\[(\rho c) \left( \frac{\partial T}{\partial t} + q \cdot \nabla T \right) = \nabla \cdot (k_1 \nabla T) + \rho_p c_p \left( D_B \nabla T \cdot \nabla \phi + D_T \frac{\nabla T \cdot \nabla T}{T_c} \right) - D_H \nabla T \cdot \nabla H, \tag{4} \]

\[\nabla \cdot B = 0, \quad \nabla \times H = 0, \quad B = \mu_0 (M + H), \tag{5} \]

\[M_{eq} = \frac{H}{H} M_s \phi L_0 (\alpha L) = \frac{H}{H} M_{eq} (H, \phi, T), \tag{6} \]

\[\frac{\partial n}{\partial t} = -\nabla \cdot j, \quad j = (n q + n w m - D_m \nabla n). \tag{7} \]

When \( w = 0 \) and \( j.k = 0 \)

then \( T = T_h, \quad \phi = \phi_0, \quad \) at \( z = 0, \)

and \( T = T_c, \quad \phi = \phi_1, \quad \) at \( z = d. \tag{8} \]

In dimensionless form, equations (1)–(7) are:

\[
\nabla \cdot q = 0, \tag{9} \\
\frac{1}{Pr} \left( \frac{\partial q}{\partial t} + q \cdot \nabla q \right) = -\nabla p + \nabla^2 q + \lambda_1 (M \cdot \nabla) H - (R_n \phi - Ra T)
+ Ra_N N' T \phi + \frac{R_b}{Lb'} n + \rho_1 - \rho_2 \phi) k, \tag{10} \\
\frac{\partial \phi}{\partial t} + q \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 T - \frac{N_A'}{Le} \nabla^2 H, \tag{11} \\
\frac{\partial T}{\partial t} + q \cdot \nabla T = \nabla^2 T + \frac{N_B}{Le} (\nabla \phi \cdot \nabla T) + \frac{N_A N_B}{Le} (\nabla T \cdot \nabla T)
- \frac{N_A'}{Le} (\nabla H \cdot \nabla T), \tag{12} \\
\chi_2 \nabla \cdot M + \nabla \cdot H = 0, \tag{13} \\
M = \frac{H}{H} \left( \frac{1 + \chi}{\chi_2} \right) \left\{ \frac{\chi}{1 + \chi} H - \frac{m_1}{m_3} T + \frac{m_1'}{m_3'} \phi + \frac{\chi_2 - \chi}{1 + \chi} \right\}, \tag{14} \\
\frac{\partial n}{\partial t} = -\nabla \cdot (n q + n \frac{Q}{Lb} p - \frac{1}{Lb} \nabla n), \tag{15} \]
where
\[ \rho_1 = \frac{d^2 p_f}{\kappa \mu} (1 + \alpha T_c) g, \quad \rho_2 = \frac{d^2 p_f}{\kappa \mu} (\phi_0 - \phi_1) \alpha T_c g, \quad \lambda_1 = \frac{\mu_0 M_0 H_0 d^2}{\kappa \mu}. \]

Here following are the non-dimensional parameters:
\[ Le = \frac{\kappa}{D_B}, \quad Ra = \frac{\rho_f g \alpha d^2 (T_h - T_c)}{\mu \kappa}, \quad N_A = \frac{D_T (T_h - T_c)}{D_B T_c (\phi_0 - \phi_1)}, \quad Pr = \frac{\mu}{\rho_f \kappa}, \]
\[ N_A' = \frac{D_H H_0}{D_B (\phi_0 - \phi_1)}, \quad N_B = \frac{(\rho_C)_{p} (\phi_0 - \phi_1)}{(\rho C)_{f}}, \quad Rn = \frac{(\rho_p - \rho_f) (\phi_0 - \phi_1) g d^3}{\mu \kappa}, \]
\[ Ra_N = (1 - \phi_0) Ra, \quad N_0 = \frac{\phi_0 - \phi_1}{1 - \phi_0}, \quad m_1 = \frac{\mu_0 \chi^2 H_0^2 (T_h - T_c)}{\rho_f g \alpha d (1 + \chi) T_h^2}, \]
\[ m_3 = \frac{\mu_0 \chi H_0^2}{\rho_f g \alpha T_h}, \quad m_3' = \frac{\mu_0 \chi H_0^2}{\rho_f g \alpha \phi_0}, \quad m_1' = \frac{\mu_0 \chi^2 H_0^2 (\phi_0 - \phi_1)}{\rho_f g \alpha (1 + \chi) \phi_0^2}, \]
\[ Rb = \frac{\Delta \rho g \nu' d^3}{\mu D_m}, \quad Lb = \frac{\kappa}{D_m}, \quad Q = \frac{w_m d}{D_m}. \]

where Ra: Thermal Rayleigh number, Rn: Nanoparticle concentration Rayleigh number, Le: Lewis number, Pr: Prandtl number, N_A, N_A’: The modified diffusivity ratios, N_B: The modified particle-density increment, m_1, m_1’, m_3, m_3’: The magnetic parameters, Rb: Bioconvection Rayleigh number, Lb: Bioconvection Lewis number and Q: Bioconvection Péclet number. Here \( \nu' \) in the definition of \( Rb \) is a dimensionless parameter defined in equation [24].

The boundary conditions now become
\[ w = 0, \quad T = \frac{T_h}{T_h - T_c}, \quad \phi = \frac{\phi_0}{\phi_0 - \phi_1}, \quad Qn = \frac{dn}{dz} \quad \text{at} \quad z = 0, \]
\[ w = 0, \quad T = \frac{T_c}{T_h - T_c}, \quad \phi = \frac{\phi_1}{\phi_0 - \phi_1}, \quad Qn = \frac{dn}{dz} \quad \text{at} \quad z = 1. \]

3. The quiescent state solution
Here
\[ q_b = 0, \quad \text{and} \quad p_b, T_b, \phi_b, M_b, H_b, n_b \]
all are functions of \( z \) only.

Equations (9)–(15) then reduce to
\[ -\frac{dp_b}{dz} + \lambda_1 M_b \frac{dH_b}{dz} - Rn \phi_b + Ra T_b - Ra_N N_b T_b \phi_b - \frac{Rb}{Lb} m_b - \rho_1 + \rho_2 \phi_b = 0, \]
\[ \frac{d^2 \phi_b}{dz^2} + \frac{N_A d^2 T_b}{dz^2} - N_A' \frac{d^2 H_b}{dz^2} = 0, \]
\[ \frac{d^2 T_b}{dz^2} + \frac{dT_b}{dz} \left\{ \frac{N_B d \phi_b}{Le} \frac{d^2 H_b}{dz^2} + \frac{N_A N_B d T_b}{Le} \frac{d^2 H_b}{dz^2} - \frac{N_A' N_B d H_b}{Le} \right\} = 0, \]
\[ \chi_2 \frac{dM_b}{dz} + \frac{dH_b}{dz} = 0, \]  
\[ M_b = \frac{1 + \chi}{\chi_2} \left\{ \frac{\chi}{1 + \chi} H_b - \frac{m_1}{m_3} T_b + \frac{m_1'}{m_3'} \phi_b + \frac{\chi_2 - \chi}{1 + \chi} \right\}, \]  
\[ \frac{d^2n_b}{dz^2} - Q \frac{dn_b}{dz} = 0. \]  

Following Kuzenetsov [22], we have

\[ n_b = \nu' e^{Qz}, \]

where

\[ \nu' = \frac{\bar{n}Q}{e^{Q} - 1}. \]  

In Equation 24

\[ \bar{n} = \int_0^1 n_b(z) dz \]

is the average dimensionless concentration of microorganisms in MNF layer.

4. Linear analysis

To study the linear stability of the quiescent state, we now take infinitesimally small perturbations as

\[ q = q_b + q', \quad p = p_b + p', \quad T = T_b + \theta', \quad n = n_b + n', \quad M = M_b + M', \quad \phi = \phi_b + \phi', \quad H = H_b + H'. \]

This gives following set of linearized perturbation equations (dropping primes)

\[ \frac{1}{Pr} \frac{\partial^{2} w}{\partial t} = \nabla^4 w - \left\{ Ra m_3 - Ra_s m_3' \right\} \frac{\partial^{2} \psi}{\partial z^2} + \left\{ Ng - Ra m_3 m_1' \right\} \frac{\partial^{2} \theta}{\partial z^2} + Ra_N (1 + N_\phi z) \nabla H \theta - \left\{ Ra m_3 m_1' - Ra_s m_1' + Rn + Ra_N N_\phi \right\} \nabla \phi \times (1 - z) \nabla \phi - \frac{R_b}{\nu' L_b} \nabla H n, \]  

\[ \text{(26)} \]
Thus we have
\[
\frac{\partial \phi}{\partial t} = w + \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 \theta - \frac{N_A'}{Le} \frac{\partial \nabla^2 \psi}{\partial z}, \tag{27}
\]
\[
\frac{\partial \theta}{\partial t} = \nabla^2 \theta + w - \frac{N_B}{Le} \frac{\partial \phi}{\partial z} + \frac{N_B}{Le} \frac{\partial \psi}{\partial z^2} - \left\{ \frac{N_B}{Le} + \frac{2N_A N_B}{Le} - \frac{N_B N_A' m_1}{Le m_3} \right\} \frac{\partial \theta}{\partial z}, \tag{28}
\]
\[
\frac{\partial^2 \psi}{\partial z^2} = -\frac{(1 + \chi_2)}{(1 + \chi)} \nabla_H^2 \psi + \frac{m_1}{m_3} \frac{\partial \theta}{\partial z} - \frac{m_1'}{m_3'} \frac{\partial \phi}{\partial z}, \tag{29}
\]
\[
\frac{\partial n}{\partial t} = \frac{1}{Lb} \nabla^2 n - \frac{Q}{Lb} \frac{\partial n}{\partial z} + \nu' e Q \frac{\partial^2 \phi}{\partial z^2} + \frac{\nu' e Q}{Lb} \left[ G(1 - \alpha_0) \nabla_H^2 w + G(1 + \alpha_0) \frac{\partial^2 w}{\partial z^2} - w \right], \tag{30}
\]
where \( \nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \), \( Ng = m_1 Ra \) and \( Ra_s = \frac{\rho_s g \alpha d (\phi_0 - \phi_1)}{\mu \kappa} \).

Here we assume
\[
[w, \phi, \theta, \psi, n] = [w(z), \phi(z), \theta(z), \psi(z), n(z)] \exp[\sigma t + i(k_x x + k_y y)]. \tag{31}
\]

Thus we have
\[
\sigma \phi = w + \frac{1}{Le} (4D^2 - k^2) \phi + \frac{N_A}{Le} (4D^2 - k^2) \theta - \frac{2N_A'}{Le} (4D^2 - k^2) D\psi, \tag{32}
\]
\[
\sigma \theta = w + (4D^2 - k^2) \theta - 2 \left\{ \frac{N_B}{Le} + \frac{2N_A N_B}{Le} - \frac{N_B N_A' m_1}{Le m_3} + \frac{N_B N_A' m_1'}{Le m_3'} \right\} D\theta \\
- \frac{2N_B}{Le} D\phi + \frac{4N_B N_A'}{Le} D^2 \psi, \tag{33}
\]
\[
4D^2 \psi = \frac{k^2 (1 + \chi_2)}{(1 + \chi)} \psi + \frac{2m_1}{m_3} D\theta - \frac{2m_1'}{m_3'} D\phi, \tag{34}
\]
\[
\sigma n = -\frac{2Q D n}{Lb} + \frac{1}{Lb} (4D^2 - k^2) n + \nu' e \frac{G(z+1)}{2} Q \frac{\partial^2 n}{\partial z^2} + \frac{4G(1 + \alpha_0) D^2 w - G(1 - \alpha_0) k^2 w - w}{Lb}, \tag{35}
\]
\[ w = 0, \quad \theta = 0, \quad \phi = 0, \quad nQ = 2Dn, \quad \text{at} \quad z = \pm 1, \]
\[ 2(1 + \chi)D\psi - k\psi = 0 \quad \text{at} \quad z = -1, \]
\[ 2(1 + \chi)D\psi + k\psi = 0 \quad \text{at} \quad z = +1, \]
\[ Dw = 0 \quad \text{at} \quad z = \pm 1 \quad \text{on rigid-rigid surface}, \]
\[ Dw = 0 \quad \text{at} \quad z = -1, \quad D^2w = 0 \quad \text{at} \quad z = +1 \quad \text{on rigid-free surface}. \]

The eigen value problem: Equations (32)-(36) with boundary conditions (37) is solved by Chebyshev pseudospectral method [23].

5. Results and discussion

The numerical results are presented here for a 1 mm thick layer of water based MNF containing the gyrotactic microorganisms. Figure 1 displays the effect \( \alpha_L \) and \( R_n \) on magnetic nanofluid containing gyrotactic microorganisms. The effect of \( R_n \) is to destabilize the system while \( \alpha_L \) stabilizes the system.

In Figure 2(a), the plot depicts decrease in \( Ra_c \) on increasing \( R_b \), thus exhibiting the destabilizing tendency of \( R_b \) on the system. Increasing \( R_b \) means increasing the average concentration of microorganisms in MNF. Due to the up swimming tendency of microorganisms they produce top-heavy unstable density stratification which causes bio convection to occur. As the value of \( R_b \) increases, more and more microorganisms accumulate near the top surface. Thus an increase in the value of \( R_b \) causes destabilizing effect on the system. Figure 2(b) displays the instability boundary in the \( (Ra_c, R_n) \)–plane. The destabilizing effect of \( R_n \) is witnessed on the system because for any fixed value of \( R_b \), the value of \( Ra_c \) keeps on decreasing as \( R_n \) increases. It is also established that variation in the values of the \( R_n \) does not affect the propensity of \( R_b \) on the qualitative grounds.

Neutral curves are displayed for different values of the bioconvection Péclet number \( Q \) in Figure 3(a). It is evident from the figure that an increase in the value of \( Q \) produces destabilizing effect on the system by reducing \( Ra_c \). Figure 3(b) displays the instability boundary in the \( (Ra_c, R_b) \)–plane for...
Figure 2. $\alpha_L = 2$, $d = 0.001$, $\Delta \phi = 0.001$, $N_A = 10$, $Le = 5000$ and $Q = 1$.

Figure 3. $\alpha_L = 2$, $d = 0.001$, $\Delta \phi = 0.001$, $N_A = 10$, $Le = 5000$ and $Rb = 10$.

various values of $Q$. The destabilizing effect of both the parameters is obvious from the figure as one can see that $Ra_c$ decreases on increasing both the parameters, viz. $Q$ and $Rb$. The figure also depict that the rate of decrease in the values of $Ra_c$ with the increase in the values of $Rb$ is higher at higher values of $Q$. It is worth mentioning here that the qualitative behavior of the bioconvection Péclet number $Q$ is unaffected by the change in the values of $Rb$.

To understand the effect of the gyrotaxis number $G$ at the onset of convection, neutral curves are displayed for its different values in Figure 4 (a). The gyrotaxis number $G$ is the deviation of the cell swimming direction from strictly vertical which causes bioconvection to occur and thus higher values of $G$ makes the system unstable [19]. In Figure 4 (a) exactly same trend is observed: $Ra_c$ decreases with an increase in the value of the gyrotaxis number $G$ depicting its destabilizing effect on the system. Figure 4 (b) shows $Ra_c$ as a function of $Rb$ for different values of $G$. It is noted that $Ra_c$ decreases with an increase in the value of $Rb$ for any particular value of $G$ but it decreases sharply with larger values of $G$. Qualitatively, no difference has been found in the behavior of $G$ with the variation in the $Rb$. 
Figure 4. $\alpha_L = 2, d = 0.001, \Delta \phi = 0.001, N_A = 10, Le = 5000, Rb = 10$ and $Q = 1$.

Figure 5. $\alpha_L = 2, d = 0.001, \Delta \phi = 0.001, N_A = 10, Le = 5000, Rb = 10$ and $Q = 1$.

It is established from Figure 5 that $L_b$ delays the convection process while $R_b$ accelerates the same. It can be interpreted that qualitative behavior of $L_b$ is not affected by the variation in the value of the $R_b$. In Table 1, three representative values of the volumetric fraction $\Delta \phi$ of nanoparticles have been chosen to display the results on both type of boundary conditions.

6. Conclusions
Linear analysis theory has been applied to study the stabilizing and destabilizing effect of various parameters in a thin layer of water based MNF layer containing the gyrotactic microorganisms. Rigid-rigid and rigid-free boundaries are considered. Combined effects of nanoparticles and microorganisms have been investigated in the gravity environment. In rigid-rigid boundary condition, destabilizing effect of microorganisms has been observed. The parameters, $\alpha_L$ and $L_b$ stabilize the system while $R_n$, $R_b$, $Q$ and $G$ advance the onset of convection.
Table 1. $\alpha_L = 2$, $d = 0.001$, $\Delta \phi = 0.001$, $N_A = 10$, $Le = 5000$, $Rb = 10$, $Q = 1$.

References

[1] Pedley T J and Kessler J O 1992 Hydrodynamic phenomena in suspensions of swimming microorganisms Annu. Rev. Fluid Mech. 24 313–58
[2] Wager H 1910 The effect of gravity upon the movements and aggregation of Euglena viridis, Ehrb. and other microorganisms Proc. of the Royal Society of London. Series B, Containing Papers of a Biological Character 83 94–6
[3] Platt J R 1961 Bioconvection Patterns in cultures of free-swimming organisms Sci. 133 1766–67
[4] Plesset M S and Winet H 1974 Bioconvection patterns in swimming microorganism cultures as an example of Rayleigh-Taylor instability Nature 248 441–43
[5] Kuznetsov A V and Avramenko A A 2004 Effect of small particles on the stability of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite depth Int. Commun. Heat Mass Transf. 31 1–10
[6] Kuznetsov A V and Geng P 2005 The interaction of bioconvection caused by gyrotactic micro-organisms and settling of small solid particles Int. J. Numer. Method H. 15 328–47
[7] Nield D A and Kuznetsov A V 2006 The onset of bio-thermal convection in a suspension of gyrotactic microorganisms in a fluid layer: Oscillatory convection Int. J. Therm. Sci. 45 990–97
[8] Kuznetsov A V 2011 Non-oscillatory and oscillatory nanofluid bio-thermal convection in a horizontal layer of finite depth Eur. J. Mech. B Fluids 30 156–65
[9] Nield D A Kuznetsov A V and Avramenko A A 2004 The onset of bioconvection in a horizontal porous-medium layer Transp. Porous Media 54 335–44
[10] Kuznetsov A V 2012 Nanofluid bioconvection in a horizontal fluid-saturated porous layer J. Porous Media 15 11–27
[11] Loefer J B and Melford R B 1952 Concerning pattern formation by free-swimming microorganisms Am. Nat. 86 325–29
[12] Wille J J and Ehret C F 1968 Circadian rhythm of pattern formation in populations of a free-swimming organism, Tetrahymena J. Protozool 15 789–92
[13] Nultsch W and Hoff E 1973 Investigation on pattern formation in Euglenae Arch. Protistenkd. 115 336–52
[14] Kessler J O 1984 Gyrotactic buoyant convection and spontaneous pattern formation in algal cell cultures In
Nonequilibrium Cooperative Phenomena in Physics and Related Fields Springer US 241–48

[15] Bees M A and Hill N A 1997 Wavelengths of bioconvection patterns J. Exp Biol. 200 1515–26

[16] Avramenko A A and Kuznetsov A V 2010 The onset of bio-thermal convection in a suspension of gyrotactic microorganisms in a fluid layer with an inclined temperature gradient Int. J. Numer. Method H. 20 111-29

[17] Bees M A and Hill N A 1998 Linear bioconvection in a suspension of randomly swimming, gyrotactic micro-organisms Phys. Fluids 10 1864–81

[18] Kuznetsov A V and Avramenko A A 2003 Stability analysis of bioconvection of gyrotactic motile microorganisms in a fluid saturated porous medium Transp. Porous Media 53 95–104

[19] Sharma Y D and Kumar V 2011 Overstability analysis of thermo-bioconvection saturating a porous medium in a suspension of gyrotactic microorganisms Transp. Porous Media 90 673–85

[20] Shaw S, Sibanda P, Sutradhar A and Murthy P V S N 2014 Magnetohydrodynamics and soret effects on bioconvection in a porous medium saturated with a nanofluid containing gyrotactic microorganisms J. Heat Transfer 136 052601(1–10)

[21] Tsai T H, Liou D S, Kuo L S and Chen P H 2009 Rapid mixing between ferro-nanofluid and water in a semi-active Y-type micromixer Sens. Actuator A Phys. vol. 153 267–73

[22] Kuznetsov A V 2010 The onset of nanofluid bioconvection in a suspension containing both nanoparticles and gyrotactic microorganisms Int. Commun. Heat Mass Transf. 37 1421–25

[23] Canuto C, Hussaini M Y, Quateroni A and Zang T 1998 Spectral Methods in Fluid Dynamics Springer, New York