Universality in Decaying Turbulence at High Reynolds Numbers

Christian Kuechler  
MPI for Dynamics and Self-Organization

Gregory Bewley  
Cornell University

Eberhard Bodenschatz  (✉️ eberhard.bodenschatz@ds.mpg.de)  
MPI for Dynamics and Self-Organization  https://orcid.org/0000-0002-2901-0144

Article

Keywords: homogeneous isotropic turbulence, velocity differences, Extended Self Similarity

Posted Date: September 30th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-587483/v1

License: ☺️ This work is licensed under a Creative Commons Attribution 4.0 International License.  
Read Full License
Universality in Decaying Turbulence at High Reynolds Numbers

Christian Küchler
Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany
Institute for Dynamics of Complex Systems, University of Göttingen, Göttingen, Germany

Gregory P. Bewley
Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY, USA

Eberhard Bodenschatz
Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany
Physics Department, Cornell University, Ithaca, NY, USA
Institute for Dynamics of Complex Systems, University of Göttingen, Göttingen, Germany

(Dated: June 3, 2021)

In the limit of very large Reynolds numbers for homogeneous isotropic turbulence of an incompressible fluid, the statistics of the velocity differences between two points in space are expected to approach universal power laws at scales smaller than those at which energy is injected. Even at the highest Reynolds numbers available in laboratory and natural flows such universal power laws have remained elusive. On the other hand, power laws have been observed empirically in derived quantities, namely in the relative scaling in statistics of different orders according to the Extended Self Similarity hypothesis. Here we present experimental results from the Max Planck Variable Density Turbulence Tunnel over an unprecedented range of Reynolds numbers. We find that the velocity difference statistics take a universal functional form that is distinct from a power law. By applying a self-similar model derived for decaying turbulence to our data, an effective scaling exponent for the second moment can be derived that agrees well with that obtained from Extended Self Similarity.

Turbulence in a three-dimensional incompressible fluid can be described by a flow of kinetic energy from large energy injection length scales \( L \) to small viscous scales \( \eta \), where internal friction dissipates this kinetic energy into heat. For intermediate scales, i.e., in the inertial range, the statistics of turbulent velocity fluctuations are well described by the moments of velocity increments. The \( n \)-th order moments of velocity increments are called structure functions, \( S_n(r) \), where \( n \) is the order. The separation between large and small scales, or the size of the inertial range, goes hand in hand with the magnitude of the main parameter capturing the intensity of a turbulent flow, which is the Reynolds number \( R_{\lambda} = u_{\text{RMS}} \lambda / \nu \). \( u_{\text{RMS}} \) is the root-mean-squared velocity fluctuation, \( \nu \) is the kinematic viscosity of the fluid, and \( \lambda \) is the length scale defined in Taylor [1], where \( L \gg \lambda \gg \eta \).

The configuration for which the scaling laws (1) have been derived is idealised. It is a practical challenge to separate viscous and non-universal large-scale effects sufficiently to form an inertial range where the scaling laws can be observed. Adequately large Reynolds numbers were available up to recently only in natural atmospheric

\[ \zeta_{n,K41} = n/3, \]
flows, which are inhomogeneous and non-stationary, or turbulence in (super)fluid helium [16–19], where measurements are extremely challenging due to the small viscous length scales [20–24]. A large body of experimental and numerical data is available at lower $R_\lambda$. At these $R_\lambda$ and at low orders $n$, $S_n \sim r^{n/3}$ approximates the existing data [e.g. 26–28]. However, viscosity influences relatively large scales compared with the dissipation scale through the so called bottleneck [2, 29] shadowing the inertial range scaling in both experiments and numerical simulations [30–34]. Additional complications are the decay of turbulent kinetic energy in many experimental flows or the energy injection typical for sheared turbulence and forced numerical simulations. These effects are known to adversely affect the buildup of power law scaling in the inertial range $[28, 35, 36]$. It is known that the effects of decay and anisotropic energy injection are typically stronger than those of the scale-local forcing in numerical simulations [30, 35, 37]. In these cases, the statistics carry a $R_\lambda$-dependence and no definite conclusions regarding universality have yet been drawn. The form of the statistics is then typically written as

$$S_n = C_n \, (\varepsilon r)^{n/3} \left( \frac{r}{L} \right)^{\mu_n} F_n(R_\lambda, r/\eta).$$  

FIG. 1. (A): $\zeta_2(r)$ for $R_\lambda = 150, 410, 660, 890, 1480, 2030, 2680, 3070, 4140$ and $5860$. The curves collapse approximately onto a universal form for $R_\lambda > 2000$ at scales extending up to $1000\eta$ ($\approx 0.1L$) as seen in the inset. This form extends from the smallest scales up to $0.1L$ and is different from a constant, which indicates that power law scaling is masked in these data. In contrast, the curves at $R_\lambda < 2000$ change shape significantly with $R_\lambda$. Inset: Zoom on the inertial range of the same curves. At the largest $R_\lambda$ a wave-like fine structure can be seen as in Sinhuber et al. [2]. Dashed lines: $r_0/\eta$ for the curves in (D). (B): Same as (A), but normalised by $L$. At the largest scales the curves follow a similar shape from the largest scales down to $0.2L$. Dashed line: $r_0/L$ for the curve in (D). (C): Structure functions $S_2$ compensated by the scale-invariant prediction, $(\varepsilon r)^{2/3}$. (D): $\zeta_2(r)$ evaluated at fixed $r_0/\eta$ given by the dashed lines in (A), and fixed $r_0/\eta$ given by the curve in (B). To the extent that the curves approach constants, these constants depend on $r_0$. Therefore, no single scaling exponent $\zeta_2$ can be isolated. Dashed lines are fits of $\alpha_1 - \alpha_2 R_\lambda^3$ to the data.
FIG. 2. Same as Fig. 1 (A) but for orders $3 < n \leq 6$, showing that the general trends observed at the second order are preserved at higher orders. $\zeta_5$ was smoothed using cubic splines, and those data do not converge as well at the highest $R_\lambda = 5890$ are in gray.

Using closure models for the statistical evolution equations [25, 37, 38], empiric parameterisations for $F_n$ [39–43], or physically motivated derivations of the large-scale terms [25, 33, 38, 44], functional forms for $F_n$ can be found to describe data at low $R_\lambda$. The results indicate that a dependence on $R_\lambda$ may not vanish before $O(R_\lambda) = 10^4$ in decaying turbulence behind a passive grid.

In this article we show how velocity increment statistics approach a fully-developed inertial range that is independent of the Reynolds number above $R_\lambda \approx 2000$ up to the experimental limit of $R_\lambda \approx 6000$. From this we conclude that $F_n(R_\lambda, r/\eta)$ is a non-trivial, $R_\lambda$-independent and universal function at high Reynolds numbers.

FIG. 3. Same as Fig. 1 (D) but for $3 < n \leq 6$, showing as in Fig. 2 that the trends observed at the second order are visible also at higher orders. The solid black lines show the result of Kolmogorov’s [3] dimensional analysis $\zeta_n = n/3$, which lies above the data for values of $r_0$ in the inertial range. Dashed lines are fits of $\alpha_1 - \alpha_2 R_\lambda^\beta$ to the data excluding the largest three $R_\lambda$.

I. MAIN

We conducted experiments in the Max Planck Variable Density Turbulence tunnel which has a volume of 88m$^3$ and is pressurized with sulphur hexafluoride (SF$_6$) at pressures between 1 and 15 bar, where an approximately homogeneous central region exists within the tunnel [45]. The turbulence was generated by an active grid with correlated forcing [29, 46]. We recorded time series of the streamwise velocity component using subminiature hot wires (Nanoscale Thermal Anemometry Probes, NSTAPs) [47] and conventional hot wires. The wire lengths were $\ll 4\eta$.

In eq. (3) we observe that the prefactors $C_n$ as well as
as expected from continuity. Around $r \approx 100\eta$, $\zeta_2(r)$ flattens as expected for the inertial range. The width of this approximate plateau increases with $R_\lambda$, with a tilt evident even at the largest $R_\lambda$. This shape appears not to change starting around $R_\lambda \approx 2000$ and above. At yet larger scales, $\zeta_2(r)$ approaches zero, its large-scale limiting value for even $n$. Panel (C) of Fig. 1 shows the corresponding structure functions $S_2(r)$ compensated by the Kolmogorov prediction eq. (2). No clear plateau can be observed even at the largest $R_\lambda$ indicating the absence of plain self-similar scaling. To better illustrate the $R_\lambda$-dependence of the local power law exponent $\zeta_2(r)$ we plot its value at specific scales $r_0$ within the inertial range as functions of $R_\lambda$ in Fig. 1 (D). Overall, $\zeta_2(r_0/\eta)$ reaches a constant for $R_\lambda > 2000$ and any fixed $r_0$ in the inertial range. Therefore, the shape of $(r/L)^\mu_\eta F_n(r/\eta)$ in the inertial range becomes independent of $R_\lambda$ for $R_\lambda > 2000$. However, the particular asymptotic values of $\zeta_2(r_0/\eta)$ found at each specific scale $r_0/\eta$ in the inertial range differ by up to 0.2 – far more than typical intermittency corrections. This implies that we cannot infer a single inertial range exponent, and cannot disentangle $\mu_\eta$ from $F$ given the data alone.

The above observations apply also at higher orders, which are shown in Figs. 2 and 3. At the largest $R_\lambda$ and smallest scales observed, the data are likely influenced by insufficient probe resolution. This is particularly important at higher orders.

In the following we investigate the extent to which our observations can be explained by finite Reynolds number models in order to facilitate the inference of $\mu_\eta$. In the present experiments turbulence is excited by the active grid and the turbulent kinetic energy decays subsequently. In freely decaying turbulence, the energy injection scale $L$ grows over time [48, 49]. In our experiments, however, the growth of $L$ is limited by the dimensions of the wind tunnel’s cross section. Decaying turbulence in a confined domain was recently modeled by Yang, Pumir and Xu [25]. The authors derive the functional forms for the viscous and large-scale cutoffs of inertial range power laws from a closure theory and self-similar decay laws (see Methods for details). In the model the effective scaling exponent of the second order structure function $(n/3 + \mu_2 F)$ is one parameter, while the other describes the decay and is related to the normalised rate of dissipation $C_\epsilon = \epsilon L / u^3$. The model can thus be used to separate the inertial-range scaling from large-scale effects in the present experiments. An alternative is the ad-hoc formula after Batchelor [39, 42], which provides smooth transitions between the different scaling regimes $(r^n, r^{\zeta_n}, r^0)$, but has no further physical justification.

In Fig. 4 we show the two models in red ([25]) and green ([42]) with parameters fitted to the experimental data. The fits indicate that the model for decaying turbulence in a confined domain [25] is a better approximation at higher Reynolds numbers than the Batchelor interpolation formula[39, 42], whereas the Batchelor formula describes the data better at lower $R_\lambda$. Both models
asymptotically approach power laws in the inertial range at very large $R_\lambda$. At second order the model in Yang et al. [25] better predicts the sustained influence of turbulence decay down to relatively small scales and is close to the data in the inertial range.

We interpret the model in Yang et al. [25] as a physical model for $(r/\eta)^{\mu_n} F_2(R_\lambda, r/\eta)$ and extract the intermittency correction $\mu_n$ from the data. In Fig. 5 we compare the intermittency correction $\mu_2F$ from this model of decaying turbulence [25] to an established method for extracting $\mu_2$ from the data alone. This latter Extended Self Similarity (ESS) method was introduced in Benzi et al. [50] and assumes that $F_\lambda \approx F_3$, such that ratios of different order structure functions show an extended scaling range with reduced effects of the finite Reynolds number and reduced uncertainty in the inertial-range scaling exponent $\zeta_{2,\text{ESS}}$. We find good agreement between this method of extended self-similarity (ESS) and the model parameter $\zeta_{SF} = \mu_2 + 2/3$.

We are finally in the position to measure the universal modulation $F(R_\lambda, r/\eta)$ at large Reynolds numbers and small scales. For this we consider the curve

$$F_2(R_\lambda, r/\eta) = \frac{S_2}{C_2(\varepsilon r)^{2/3}(r/\eta)^{\mu_2}}. \quad (5)$$

We determine $C_2$ by normalising the maximum of the resulting curves to 1 and fix $\mu_2 = 0.693$ from the ESS estimate. Fig. 5 (B) shows that $F_2$ begins to collapse around $R_\lambda \approx 1500$, i.e. assumes a universal form at asymptotically high $R_\lambda$. To show this more rigorously, we take $F(R_\lambda = 4141)$ as an approximation towards this asymptotic form and plot the relative divergences towards this reference. In the inset of Fig. 5 (A) we observe that, starting around $R_\lambda \approx 1500$ the curves are within $\pm 3\%$ of each other.

In this article we show experimental data on how the velocity increment statistics approach a fully-developed inertial range whose shape is independent of the Reynolds number. While this is in agreement with Kolmogorov’s hypothesis of universality, the scaling laws (and their intermittency corrections) anticipated for these conditions are not directly observed. That is, the inertial range is only approximately described by power laws and carries a $R_\lambda$-independent modulation, $F_2(r/\eta)$ in eq. (5). Data from entirely different flow geometries, such as a jet [28], suggest that $F_2(r/\eta)$ is sensitive to the overall flow configuration for $n = 3$, but less so for $n = 2$. We observed little variance for different active grid schemes. A careful analysis of other high $R_\lambda$-data is of great interest in the light of our results. We also show that the widely used empirical ESS scheme to obtain the intermittency correction $\mu$ [50] gives an equivalent answer at second order to a physically motivated model of the entire structure function [25].

In decaying turbulence the inertial range grows more slowly than in continuously forced turbulence, and the time-dependent term in the statistical evolution equation does not vanish [25, 28, 30, 33, 35, 51]. Indeed, a model [25] for the decay of turbulence (confined as in our experiment) predicts an influence of the decay its structure from large scales into the inertial range and allows us to quantify the intermittency correction $\mu_2F$. While the model we use is designed to approach an inertial range power law, our data suggest that above $R_\lambda \approx 2000$ the approach to a power law is halted. We show instead that above this Reynolds number the statistics are described by a universal and nontrivial shape from small scales up to $r \approx 0.1L$.

**FIG. 5.** Upper Plot: The second-order scaling exponent $\zeta_2$ measured in different ways and in different laboratory experiments. Circles show $\zeta_2$ found by fitting eq.(A4) to data from active and passive grid experiments. Squares show extended self-similarity (ESS) exponents, $S_2/[S_3]$, for the same datasets. According to the model fits, $\zeta_2$ approaches a constant $(\zeta_{SF}) = 0.698 \pm 0.011$ (dashed) larger than Kolmogorov’s prediction (dotted) [3]. We attribute the slight downward trend in the last two data points to probe effects and the anisotropic forcing that we used to reach these high $R_\lambda$. For comparison we include data from Mydlarski and Warhaft [27]. For $R_\lambda < 300$, no ESS exponent could be measured due to an insufficient inertial range. The shaded region corresponds to the range of values that the local slope $\zeta_2(r)$ takes within $100\eta < r < 0.1L$. Lower Plot: Approach of $F_2$ measured by eq. (5) towards an $R_\lambda$-universal shape. Starting around $R_\lambda \approx 1500$ the curves collapse for $r < 0.1L$. Inset: The relative difference, $(F_2 - F_2(R_\lambda = 4141))/F_2(R_\lambda = 4141)$ shows universality to within $\pm 3\%$ (indicated by the shaded area) for about three decades in $r$. The plots include measurements at a total of 29 different Reynolds numbers.
When turbulence is not isotropic, scaling laws appear only when projecting onto appropriate symmetry groups [52–54]. The instrumentation in the present experiment allows only unidirectional velocity measurements, such that anisotropy can be inferred only indirectly. The measurements might therefore represent the approach to universality in anisotropic turbulence with little consequences for the idealised Kolmogorov framework. However, the measurement volume is relatively free of mean shear and the results are remarkably robust even when the turbulence is excited using an anisotropic active grid or a classical and static grid.

We have shown that $F_n$ is a $R_\lambda$-independent and non-trivial function of the scale $r$ with indications from Figs. 2 and 3 that higher orders behave similarly. We point out that ESS means that $F_n$ is similar for all even orders. The processes that shape the asymptotic form of $F_n$ and that interfere with power-law scaling are evidently open questions. This already bears the potential for substantial advancements to applied turbulence models and the scaling seen in engineering wind tunnel studies. Future studies will need to investigate the degree to which $F_n$ changes from flow to flow. A flow-independent function of the $n$-th order statistics, if it existed, would have far-reaching implications for turbulence models and closure schemes.

We end by commenting that deviations from power-law scaling in the inertial range have in the past been dismissed as finite Reynolds number effects that were to be circumvented. Viscous effects are important when the Reynolds number is low. Our results suggest however, that deviations from power-law scaling are an important feature of naturally occurring decaying turbulence, whatever its Reynolds number.

ACKNOWLEDGEMENTS

We thank M. Hultmark and Y. Fan for providing the nanoscale hot wire probes and helping with their operation. We thank M. Sinhuber for help with using the passive grid data and helpful discussions. We thank A. Pumir, H. Xu, M. Wilczek, and D. Lohse for helpful discussions. The Max Planck Variable Density Turbulence Tunnel (VDTT) is maintained and operated by A. Kubitzek, A. Kopp, and A. Renner. The machine workshop led by U. Schminke and the electronic workshop led by O. Kurre built and installed the active grid. The Max Planck Society and Volkswagen Foundation provided financial support for building the VDTT.

[1] G. I. Taylor, Statistical theory of turbulence, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 151, 421 (1935).
[2] M. Sinhuber, G. P. Bewley, and E. Bodenschatz, Dissipative Effects on Inertial-Range Statistics at High Reynolds Numbers, Physical Review Letters 119, 134502 (2017).
[3] A. N. Kolmogorov, The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers, Proceedings: Mathematical and Physical Sciences 434, 9 (1941).
[4] J. Meyers and M. Baelmans, Determination of subfilter energy in large-eddy simulations, Journal of Turbulence 5, N26 (2004).
[5] G. K. Batchelor and A. Townsend, Decay of vorticity in isotropic turbulence, Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences 190, 534 (1947).
[6] G. K. Batchelor and Townsend, A.A., The nature of turbulent motion at large wave-numbers, Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences 199, 238 (1949).
[7] A. N. Kolmogorov, A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, Journal of Fluid Mechanics 13, 82 (1962).
[8] U. Frisch, P.-L. Sulem, and M. Nelkin, A simple dynamical model of intermittent fully developed turbulence, Journal of Fluid Mechanics 87, 719 (1978).
[9] R. Benzi, G. Paladin, G. Parisi, and A. Vulpiani, On the multifractal nature of fully developed turbulence and chaotic systems, Journal of Physics A: Mathematical and General 17, 3521 (1984).
[10] K. R. Sreenivasan and C. Meneveau, The fractal facets of turbulence, Journal of Fluid Mechanics 173, 357 (1986).
[11] C. Meneveau and K. R. Sreenivasan, Simple multifractal cascade model for fully developed turbulence, Physical Review Letters 59, 1424 (1987).
[12] L. C. Andrews, R. L. Phillips, B. K. Shivamoggi, J. K. Beck, and M. L. Joshi, A statistical theory for the distribution of energy dissipation in intermittent turbulence, Physics of Fluids A: Fluid Dynamics 1, 999 (1989).
[13] Z.-S. She and E. Leveque, Universal scaling laws in fully developed turbulence, Physical Review Letters 72, 336 (1994).
[14] B. Dubrulle, Intermittency in fully developed turbulence: Log-Poisson statistics and generalized scale covariance, Physical Review Letters 73, 959 (1994).
[15] G. I. Barenblatt and N. Goldenfeld, Does fully developed turbulence exist? Reynolds number independence versus asymptotic covariance, Physics of Fluids 7, 3078 (1995).
[16] A. Praskovskiy and S. Oncley, Measurements of the Kolmogorov constant and intermittency exponent at very high Reynolds numbers, Physics of Fluids 6, 2886 (1994).
[17] K. R. Sreenivasan, An update on the energy dissipation rate in isotropic turbulence, Physics of Fluids 10, 528 (1998).
[18] H. Kahalerras, Y. Malécot, Y. Gagne, and B. Castaing, Intermittency and Reynolds number, Physics of Fluids 10, 910 (1998).
[19] Y. Tsuji, Intermittency effect on energy spectrum in high-Reynolds number turbulence, Physics of Fluids 16, L43 (2004).
[20] C. M. White, A. N. Karpetis, and K. R. Sreenivasan, High-Reynolds-number turbulence in small apparatus:
Grid turbulence in cryogenic liquids, Journal of Fluid Mechanics 452, 189 (2002).

[21] P. Pietroplinto, C. Poullain, C. Baudet, B. Castaing, B. Chabaud, Y. Gagne, B. Hébral, Y. Ladam, P. Lebrun, O. Pirotte, and P. Roche, Superconducting instrumentation for high Reynolds turbulence experiments with low temperature gaseous helium, Physica C: Superconductivity 386, 512 (2003).

[22] G. P. Bewley and K. R. Sreenivasan, The Decay of a Quantized Vortex Ring and the Influence of Tracer Particles, Journal of Low Temperature Physics 156, 84 (2009).

[23] J. Salort, B. Chabaud, E. Lévéque, and P.-E. Roche, Energy cascade and the four-fifths law in superfluid turbulence, EPL (Europhysics Letters) 97, 34006 (2012).

[24] B. Rousset, P. Bonnay, P. Diribarne, A. Girard, J. Michel, and M. Bon Mardion, Superfluid high Reynolds von Kármán experiment, Review of Scientific Instruments 85, 103908 (2014).

[25] P.-F. Yang, A. Pumir, and H. Xu, Generalized self-similar spectrum and the effect of large-scale in decaying homogeneous isotropic turbulence, New Journal of Physics 20, 103035 (2018).

[26] S. G. Saddoughi and S. V. Veeravalli, Local isotropy in turbulent boundary layers at high Reynolds number, Journal of Fluid Mechanics 268, 333 (1994).

[27] L. Mydlarski and Z. Warhaft, On the onset of high-Reynolds-number grid-generated wind tunnel turbulence, Journal of Fluid Mechanics 320, 331 (1996).

[28] R. A. Antonia, S. L. Tang, L. Djenidi, and Y. Zhou, Finite Reynolds number effect and the 4/5 law, Physical Review Fluids 4, 084602 (2019).

[29] C. Küchler, G. P. Bewley, and E. Bodenschatz, Experimental Study of the Bottleneck in Fully Developed Turbulence, Journal of Statistical Physics 175, 617 (2019), arXiv:1812.01370.

[30] D. Fukayama, T. Oyamada, T. Nakano, T. Gotoh, and K. Yamamoto, Longitudinal Structure Functions in Decaying and Forced Turbulence, Journal of the Physical Society of Japan 69, 701 (2000), arXiv:chao-dyn/9912033.

[31] T. Gotoh, D. Fukayama, and T. Nakano, Velocity field statistics in homogeneous steady turbulence obtained using a high-resolution direct numerical simulation, Physics of Fluids 14, 1065 (2002).

[32] S. Y. Chen, B. Dhruva, S. Kurien, K. R. Sreenivasan, and M. A. Taylor, Anomalous scaling of low-order structure functions of turbulent velocity, Journal of Fluid Mechanics 533, 10.1017/S002211200500443X (2005).

[33] S. L. Tang, R. A. Antonia, L. Djenidi, L. Danaila, and Y. Zhou, Finite Reynolds number effect on the scaling range behaviour of turbulent longitudinal velocity structure functions, Journal of Fluid Mechanics 820, 341 (2017).

[34] P. K. Yeung, K. R. Sreenivasan, and S. B. Pope, Effects of finite spatial and temporal resolution in direct numerical simulations of incompressible isotropic turbulence, Physical Review Fluids 3, 064603 (2018).

[35] L. Danaila, F. Anselmet, and R. A. Antonia, An overview of the effect of large-scale inhomogeneities on small-scale turbulence, Physics of Fluids 14, 2475 (2002).

[36] R. A. Antonia, S. L. Tang, L. Djenidi, and L. Danaila, Boundedness of the velocity derivative skewness in various turbulent flows, Journal of Fluid Mechanics 781, 727 (2015).

[37] W. J. T. Bos, L. Chevillard, J. F. Scott, and R. Rubinstein, Reynolds number effect on the velocity increment skewness in isotropic turbulence, Physics of Fluids 24, 015108 (2012).

[38] F. Thiesset, R. A. Antonia, L. Danaila, and L. Djenidi, Kármán-Howarth closure equation on the basis of a universal eddy viscosity, Physical Review E 88, 011003 (2013).

[39] G. K. Batchelor, Pressure fluctuations in isotropic turbulence, Mathematical Proceedings of the Cambridge Philosophical Society 47, 359 (1951).

[40] D. Lohse and A. Müller-Groeling, Bottleneck Effects in Turbulence: Scaling Phenomena in r versus p Space, Physical Review Letters 74, 1747 (1995).

[41] S. Kurien and K. R. Sreenivasan, Anisotropic scaling contributions to high-order structure functions in high-Reynolds-number turbulence, Physical Review E 62, 2206 (2000).

[42] B. R. Dhruva, An experimental study of high Reynolds number turbulence in the atmosphere, Ph.D. Thesis, 2717 (2000).

[43] J. Meyers and C. Meneveau, A functional form for the energy spectrum parametrizing bottleneck and intermittency effects, Physics of Fluids 20, 065109 (2008).

[44] R. A. Antonia, R. J. Smalley, T. Zhou, F. Anselmet, and L. Danaila, Similarity of energy structure functions in decaying homogeneous isotropic turbulence, Journal of Fluid Mechanics 487, 245 (2003).

[45] E. Bodenschatz, G. P. Bewley, H. Nobach, M. Sinhuber, and H. Xu, Variable density turbulence tunnel facility, Review of Scientific Instruments 85, 093908 (2014).

[46] K. P. Griffin, N. J. Wei, E. Bodenschatz, and G. P. Bewley, Control of long-range correlations in turbulence, Experiments in Fluids 60, 55 (2019), arXiv:1809.05126.

[47] M. Vallikivi, M. Hultmark, S. C. C. Bailey, and A. J. Smits, Turbulence measurements in pipe flow using a nano-scale thermal anemometry probe, Experiments in Fluids 51, 1521 (2011).

[48] P. G. Saffman, The large-scale structure of homogeneous turbulence, Journal of Fluid Mechanics 27, 581 (1967).

[49] M. Sinhuber, E. Bodenschatz, and G. P. Bewley, Decay of Turbulence at High Reynolds Numbers, Physical Review Letters 114, 034501 (2015).

[50] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Masliaoli, and S. Succi, Extended self-similarity in turbulent flows, Physical Review E 48, R29 (1993).

[51] R. A. Antonia and P. Burattini, Approach to the 4/5 law and the large-scale structure functions of turbulent velocity, Journal of Fluid Mechanics 2206 (2000).

[52] M. Sinhuber, E. Bodenschatz, and G. P. Bewley, Decay of Turbulence in Fully Developed Turbulence, Physics Reports 414, 342 (2005), arXiv:nlin/0404014.

[53] P. K. Yeung, K. R. Sreenivasan, and S. B. Pope, Effects of finite spatial and temporal resolution in direct numerical simulations of incompressible isotropic turbulence, Physical Review Fluids 3, 064603 (2018).

[54] L. Danaila, F. Anselmet, and R. A. Antonia, An overview of the effect of large-scale inhomogeneities on small-scale turbulence, Physics of Fluids 14, 2475 (2002).
Appendix A: Methods

1. The Max Planck Variable Density Turbulence Tunnel

The Variable Density Turbulence Tunnel (VDTT) \cite{55} is a closed-loop wind tunnel, which can be operated with any non-corrosive gas at pressures up to 15 bar. For the experiments presented here it was operated with sulphur-hexafluoride (SF$_6$), which offers a low kinematic viscosity that decreases with density while being relatively harmless and inert. The Reynolds number of the flow in the VDTT can be finely adjusted in three largely independent ways up to levels typical for atmospheric turbulence: (i) the large-scale forcing with a novel active grid, (ii) the mean flow speed $U$ up to 5.5 m/s by adjusting the rotation frequency of its fan, and (iii) the kinematic viscosity $\nu$ by changing the static pressure.

Flow structures of variable size are introduced using a mosaic-like arrangement of individually controllable paddles ("active grid"). It allows us to obstruct the flow on finely adjustable time- and length scales \cite{55,66}. The resulting grid length scale is indicated in Fig. 7 as red vertical lines. In this way we control the energy injection scale between about 0.1m $\lesssim L \lesssim 0.6$m. $L$ is indicated as short black vertical lines in Fig. 7.

The small kinematic viscosity of pressurized SF$_6$ permits the existence of very small flow structures. The size of these structures scales with the viscous length scale $\eta = (\nu^3/\varepsilon)^{1/4}$, where $\varepsilon = 15\nu((\partial u/\partial x)^2)$. For the range of ambient pressures 1 bar $< p <$ 15 bar, this viscous length is between 250$\mu$m $\lesssim \eta \lesssim 10$um.

In our experiment, the turbulent kinetic energy $u_{RMS}^2$ decays along the length of the measurement section, but the integral length scale $L$ remains constant or also decays over time (see Fig. 6). This is in contrast to freely decaying turbulence, where $L$ grows with time \cite{49,56}. We believe that the boundaries of the measurement section with cross-section 1.2 m x 1.5 m (with 0.1 m $\lesssim L \lesssim 0.6$m) suppresses this growth. We found this to be relatively independent of the way we estimate $L$. We chose to use $L = \int_0^\infty \langle u(x)u(x+r) \rangle / u_{RMS}^2 dr$ with $\langle u(x)u(x+r) \rangle = 0$. Other definitions of $L$ impact the results at small $R_\lambda$ and the scatter of the data otherwise.

2. Measurement Technology and Data Analysis

We record time series of hot-wire signals and convert them into one-dimensional flow fields assuming that the turbulent fluctuations are passively advected across the sensor by the mean flow $U$. Thus, a time step $\Delta t$ is converted to a spatial increment $\Delta x = U\Delta t$ \cite{1}. We use a commercial constant temperature anemome-
The frequency response of the system is not perfectly flat and the frequency response of the measurement system.

To achieve converged statistics the data was acquired for $10^3 - 10^4$ eddy turnover times (up to 8 hours) between $150 < R_L < 6000$.

The frequencies (and wavenumbers) encountered in the measurements presented here are generally in a range that is not particularly demanding for this combination of sensor and anemometer circuitry [60–62]. The temporal resolution is determined by the noise filtering frequency and the frequency response of the measurement system.

The frequency response of the system is not perfectly flat anymore starting around $1$ kHz [60].

The range of scales we are interested in is therefore in the flat part of the frequency response curve. To illustrate this, the length scales corresponding to a measurement frequency of $1$ kHz are indicated in Fig. 7 as vertical lines in the color of the corresponding $\zeta_2(r)$. The noise filtering frequency is always at frequencies above $1$kHz.

The experiments presented here were taken under different ambient pressures and different active grid forcing schemes to allow for a careful check of the hot wire fidelity. We thus ensure the robustness of the results against probe- or flow geometry-induced biases. We emphasise that all conclusions presented here are independent of the frequencies where turbulent fluctuations are measured, the dissipation length scale, and the active grid forcing.

3. Fits to the Model Spectrum [25]

The evolution equation of the velocity energy spectrum $E(k, t)$ can be derived directly from the Navier-Stokes-Equation in the isotropic case and is known as the Karman-Howarth-Lin equation.

$$ \partial_t E(k, t) = -\partial_k \Pi(k, t) - 2\nu k^2 E(k, t) \tag{A1} $$

The first term on the RHS describes the nonlinear transfer of energy from small to large wavenumbers and ultimately prevents the closure of the equation, since it is a third-order term. The Pao closure [63] used in the model by Yang et al. [25] assumes that the transfer term $\Pi$ is local in wavenumber space and has a self-similar form:

$$ \Pi(k, t) = C_0 \varepsilon^{−1/3} k^{5/3} E(k, t) \tag{A2} $$

The second term on the RHS of (A1) represents the viscous dissipation at the smallest flow scales. This yields a closed form of the Karman-Howarth-Lin equation. The model by Yang et al. further assumes that the energy spectrum can be assembled by a large scale term $f_L(kL)$, a small scale term $f_\eta(k\eta)$, and a self-similar inertial range:

$$ E(k, t) = C_k \varepsilon^{2/3} k^{5/3} f_\eta(k\eta) f_L(kL) \tag{A3} $$

These assumptions are now combined with a general, self-similar decay of turbulent kinetic energy. In the case of a confined domain, where the parameter describing $dL/dt$ tends to zero, this model predicts the energy spectrum

$$ E(k) \sim \frac{-A_K}{C} (kL)^{−(\zeta_{2F}+1)} e^{3A_K/2C}(kL)^{−2/3} e^{−(1.5/C)(k\eta)^{4/3}}. \tag{A4} $$

For the purpose of measuring a scaling exponent, we replaced the term $(kL)^{−5/3}$ used in the original formulation of the spectrum with $(kL)^{−(\zeta_{2F}+1)}$, where the fitting parameter $\zeta_{2F}$ is the inertial range scaling exponent for the second order structure function[64]. The parameters $C$ and $A_K$ are related through $C = −A_K(6/\pi)^{1/3}$. In practice, $A_K$ describes the large-scale part of the energy spectrum, which is heavily influenced by the decay.

The one-dimensional versions of $S_2$ and $E(k)$ are related through the following integral transform [65]:

$$ S_2(r) = \int_0^\infty E(k) \left( \frac{1}{3} + \frac{\cos(kr)}{(kr)^2} - \frac{\sin(kr)}{(kr)^3} \right) dk. \tag{A5} $$

To obtain the fits shown in Fig. 4, we have searched for parameters $A_K$, and $\zeta_{2F}$ that yield best fits of the logarithmic derivative of eq. (A5) to the experimentally measured $\zeta_2(r)$.

It can be shown that $C = −A_K(6/\pi)^{1/3}$. This quantity is related to the dissipation constant $C_\varepsilon = \varepsilon L/u^3$ relating...
the large scale energy injection and the small scale energy transfer rate \( \varepsilon \). \( A_K \) is the non-dimensionalized time-evolution of the energy spectrum prefactor \( d(C_K \varepsilon^{2/3})/dt \), which is a free parameter.

The energy transfer spectrum \( \Pi(k) \) is related to \( S_3 \) via

\[
S_3 = 12 \int_0^\infty \frac{1}{k^2} \frac{d}{dk} \frac{d}{dr} \left( \frac{1}{3} + \frac{\cos(kr)}{(kr)^2} - \frac{\sin(kr)}{(kr)^3} \right) dk.
\]

(A6)

The second derivative has been estimated by a Taylor expansion for \( kr < 0.001 \). Therefore, the model (A4) in combination with its underlying closure hypothesis eq. (A3) implicitly predicts \( S_3 \). Note that strictly speaking the combination of the intermittency-corrected model eq. (A4) and the K41-type closure eq. (A3) yields a third order exponent \( \zeta_3 \) slightly different from 1. It is reassuring to see that instead leaving the 5/3-term in eq. (A3) as a generic scaling and fitting the resulting model to \( S_3 \) yields \( \Pi \approx \text{const} \) in the inertial range so that \( S_3 \sim r \).