Ultra-high optical nonreciprocity with a coupled triple-resonator structure

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Abstract

Optical transmission nonreciprocity as a widely investigated phenomenon is essential to various applications. Many sophisticated mechanisms have been proposed and tested for achieving the optical nonreciprocity on integrated scale, but the technical barriers still exist to their practical implementation. To have an ultra-high transmission nonreciprocity, we consider a simple physical mechanism of optical gain saturation applied to a structure of three mutually coupled cavities or fiber rings. The gain saturation processes in two of its components creates a significantly enhanced optical nonreciprocity that satisfies the requirements for the realistic applications. The structure enjoys two advantages of its wide working bandwidth and the flexibility in choosing its components. Moreover, it is possible to apply the structure to a faithful and non-reciprocal transmission of broadband pulse signals. The structure may considerably relax the constraints on the integrated photonic circuits based on the current technology.

1. Introduction

As the indispensable elements in all-optical devices, optical isolator eliminates the reflection of a light field transmitting along one direction, and optical diode allows the one way transmission of light but blocks almost all of the inverse transmission. How to efficiently realize these optical elements has attracted great attention of researches recently. A routine approach to optical isolator is the magneto-optic effect [1], which breaks the Lorentz reciprocity of an optical circuit so that the nonreciprocity can be realized (see, e.g. [2–5]). However, it is still rather challenging to place magnetic element into integrated photonic circuits [3]. As the alternatives, various non-magnetic approaches have been experimentally and theoretically investigated, including optical nonlinearity [6–8], photonic and other crystal [9–12], dynamical modulation [13, 14], parametric process [15, 16], unbalanced quantum coupling [19], Brillouin-scattering [17, 18], optomechanics [20–26], and parity-time (PT)-symmetry related phenomena [27–29].

In the current work we focus on a system developed from those of PT-symmetry optics. A typical PT-symmetric optical system consists of an active component and a passive component with their balanced gain and loss [30, 31]. This type of systems can be generalized to the structures consisting of multiple cavities (see, e.g. [32–36]), as well as carrying optical nonlinearity and other elements (see, e.g. [37–51]). Such a system working in the PT symmetry-breaking regime was found to be capable of realizing optical nonreciprocity [27]. Moreover, together with the asymmetric couplings to the different input waveguides, an optical nonreciprocity with the non-reciprocal isolation ratio ranging between −8 dB and +8 dB can be realized by the gain saturation effect in a two-cavity PT-symmetric system [28]. Such PT-symmetric systems have been developed into the other structures involving optical gain and loss for implementing non-reciprocal transmission. One example consists of three mutually coupled cavities [29], which achieves the isolation ratios between −30 dB and +30 dB. The similar structure of three mutually coupled cavities is
also proposed for simulating a topological structure (Möbius band) [52]. In performing the non-reciprocal transmission by the structure, an output field can be stronger than the input due to the optical gain.

The isolation ratio for a practical isolator should be as high as possible. In commercial fiber communication the isolation ratio higher than 40 dB is required, and it is currently achieved by the isolators based on Faraday effect [53, 54]. The development of non-magnetic optical non-reciprocity is under way currently. In 2013 a silicon optical diode with 40 dB isolation ratio, associated with the input power 230 μW and insertion loss rate 15.5 dB, was reported [55]. Later in 2017, an optomechanical circuit via synthetic magnetism achieved 35 dB isolation ratio [56], and a further higher isolation ratio 78.6 dB based on coherent light-sound interaction was reported for a waveguide-resonator system [57]. The purpose of the current work is to target toward better optical non-reciprocity based on a simpler mechanism and enjoying more flexibility in its performance. We will illustrate that the optical gain saturation in two components of a mutually coupled triple-cavity or triple-fiber-ring structure can achieve ultrahigh optical non-reciprocity. There also exist two other advantages of such structure: the working bandwidth for non-reciprocal transmission is large and the low-quality cavities with higher damping rates can be used by a proper choice of the corresponding optical gain.

The rest of the paper is organized as follows. In section 2, the system model and the associated dynamics are presented, to illustrate how non-reciprocal transmission of light fields can be implemented based on a simple mechanism of gain saturation. The operation of the system for the inputs over a wide range of frequencies and the influence of the used components are discussed in section 3. In section 4 we provide the details about how the choice of the physical parameters affect such non-reciprocal transmission and, in section 5, we have a deeper discussion on the system dynamics leading to the optical nonreciprocity. In section 6 we discuss how to faithfully implement the non-reciprocal transmission of pulse signals by the structure. The final part concludes with some remarks.

2. Structure and system dynamics

Our concerned structure is illustrated in figure 1(a), and can be made up of three coupled microcavities [58, 59] or three mutually coupled fiber rings [29]. It is different from the one in [29] by adding an additional active component (component 2) with the optical gain. Component 1 is a passive resonator with the dissipation rate \( \gamma_1 = \gamma \) (this quantity is used as a relative scale for the other parameters in the following discussions), while the other two components are the active ones with the same initial gain rate \( g_b \) but with the different dissipation rate \( \gamma_2 \) and \( \gamma_3 \), respectively. The three resonators are coupled to each other with the coupling strength \( J_{12}, J_{13}, J_{23} \), respectively, which can be adjusted by the distances between the microcavities or optical fibers. The external drives are from port \( P_1 \) or \( P_3 \), creating the clockwise (CW) mode and counter-clockwise (CCW) mode in each cavity. The field modes in the coupled components follow the equations

\[
\begin{align*}
\dot{a}_{1+}^+ &= -\gamma_1 a_{1+}^+ - iJ_{12} a_{2-} - iJ_{13} a_{3-} + \sqrt{2}\kappa_e S_{f+}, \\
\dot{a}_{1-}^- &= -\gamma_1 a_{1-}^- - iJ_{12} a_{2+}^- - iJ_{13} a_{3+}^- , \\
\dot{a}_{2+}^+ &= [g_2(t) - \gamma_2] a_{2+}^+ - iJ_{12} a_{1-}^+ - iJ_{23} a_{3-}^+ , \\
\dot{a}_{2-}^- &= [g_2(t) - \gamma_2] a_{2-}^- - iJ_{12} a_{1+}^- - iJ_{23} a_{3+}^- , \\
\dot{a}_{3+}^+ &= [g_3(t) - \gamma_3] a_{3+}^+ - iJ_{13} a_{1-}^+ - iJ_{23} a_{2-}^+ + \sqrt{2}\kappa_e S_{b+}, \\
\dot{a}_{3-}^- &= [g_3(t) - \gamma_3] a_{3-}^- - iJ_{13} a_{1+}^- - iJ_{23} a_{2+}^-.
\end{align*}
\]

in the frame rotating with the components’ resonance frequency \( \omega_c \), where the sign (+) denotes the CW (CCW) mode. Here we assume that the three components are of the same resonance frequency and the coupling \( \kappa_e \) to the opposite input waveguides is identical too. The difference in the used three components leads to the different detunings for the forward and backward input, and a variation in the waveguide couplings will be equivalent to two nonidentical inputs from the different input terminals and can be applied to enhance the transmission nonreciprocity [28]. To focus on the performance of the structure itself, we will only consider the exactly identical inputs from the front and back port, given the identical components and identical waveguide couplings.

If the gain rates are constant, the above equations will be reduced to a system of linear differential equation. Then, due to the existence of the optical gain, the system dynamics can be unstable. In reality, the gain rates for the two active microcavities actually decrease with the increasing field intensity in the
Figure 1. (a) A mutually coupled triple-cavity or triple-fiber-ring structure with one passive component ($\gamma_1 = \gamma$) and two active components (with the gain rates $g_2$ and $g_3$). (b1) The evolved forward intensities (the red solid curve) and backward intensities (the blue solid curve) with time, as compared with the results with only one active cavity in [29] (the corresponding dashed curves). The evolution of the gain rate is illustrated in the inset. The system parameters are $g_0/\gamma = 10.55$, $\gamma_2/\gamma = \gamma_3/\gamma = 9.72$, $E/\gamma = 10$, $\Delta/\gamma = 5$, $J_{12}/\gamma = J_{13}/\gamma = 1$, $\eta = J_{23}/J_{12} = 5$, and $b_0 = 10^7$. The isolation ratio is about 40 dB with present structure, while it is about 18.2 dB with the structure in [29]. (b2) The corresponding results for the drive intensity to be increased to $E/\gamma = 100$. The isolation ratio is about 23 dB, as compared to 15.6 dB with the structure in [29]. (b3) The corresponding results for the initial gain rate to be lowered to $g_0/\gamma = 9$. The isolation ratio is about 6.1 dB, as compared to 0.6 dB with the structure in [29]. (b4) The corresponding results for the cavity couplings is changed to $\eta = J_{23}/J_{12} = 1$. The average isolation ratio is about 60.5 dB, as compared to 6.1 dB with the structure in [29]. The system parameters in (b2)–(b4) are the same as those in (b1), except for the one indicated on the top of each figure.

corresponding cavities due to the gain saturation effect

$$g_j(t) = \frac{g_0}{1 + \frac{|a_j^+(t) + a_j^-(t)|^2}{I_0}}$$

for $j = 2, 3$, which is determined by the saturation intensity $I_0$ (see supplementary material of [29] for the definition of this dimensionless quantity). This saturation effect of the optical gain can lead to stably
oscillating cavity fields in the end. The nonlinearity due to the gain saturation makes the system dynamics much richer than that of a linear PT-symmetric system.

The existing transmission nonreciprocity is considered for the inputs from ports $P_1$ and $P_4$, while the non-reciprocal phenomenon for those from ports $P_2$ and $P_3$ is the same due to the symmetry. The amplitude and detuning of a single-frequency input of $\omega_1$ are $E_{f(b)}$ and $\Delta = \omega_r - \omega_1$, respectively, so that the input in equation (1) has the form $\sqrt{2\kappa}S_{gf(b)} = E_{f(b)}e^{i\Delta t}$, where $f$ indicates the input from $P_1$ and $b$ the input from $P_4$. A stronger requirement than the straightforward non-reciprocal transmission is that the reflection from the backward port should also be considered, i.e. the forward and backward inputs act together. Then, by adding up the contributions from both ports, the forward output intensity should be $|S_{gf}|^2/(2\kappa) = |a_{1f} + a_{3b}|^2$ and the backward counterpart is $|S_{gb}|^2/(2\kappa) = |a_{1f} - a_{3b}|^2$. The figure-of-merit to measure the performance under such two simultaneous input fields is the logarithmic isolation ratio defined as [29]

$$10 \log_{10} \left| \frac{a_{1f} + a_{3b}}{a_{1b} + a_{3f}} \right|^2 = 10 \log_{10} \left| \frac{a_{1f}}{a_{1b}} \right|^2. \tag{3}$$

In a special situation of the transmission resonance under the condition $\eta = J_{33}/J_{12} = \Delta/\gamma$, so that the stabilized output modes take the form $a_i^j = A_\eta e^{i\Delta t}$, where $A_\eta$ is time-independent ($i = 1, 2, 3$ and $j = +, -$), there exist the analytical solutions ($E_f = E_b = E$):

$$|a_3^+|^2 = \frac{E^2 \gamma^2}{4(\gamma_2 - \gamma_1)^2[\eta^2(\gamma_2 - \gamma_1)^2 + 4\gamma^2]},$$

$$|a_3^-|^2 = \frac{E^2[\eta^4 + 4\eta^3 + \eta^2 - 8\eta + 4]}{4(\eta^2 - 1)[\eta^2(\gamma_2 - \gamma_1)^2 + 4\gamma^2]},$$

where $\gamma_2 = \gamma_3$ and $\gamma_2 - \gamma_1 << \gamma$ (the stabilized gain rate $g_\eta$ becomes smaller than but is close to the damping rate in the same cavity, as it is seen from the exact numerical simulations). This result indicates that, to some extent, the concerned optical nonreciprocity are determined by the asymmetric coupling $J_{33} \neq J_{12}$ inside the structure. In the limit of such asymmetric coupling $J_{33}/J_{12} \to \infty$, the difference between the forward and backward output will tend to a limited value $|a_3^+ / a_1^+|^2 \to \gamma_2/(\gamma_2 - \gamma_2)^2$, being consistent with the tendency seen from the numerical simulations (the dip positions in figure 2(a) below).

In the general situation, the amplitudes $A_\eta$ of the output modes are time-dependent, so the numerical simulations based on the nonlinear differential equations, equation (1), are indispensable to illustrate the transmission nonreciprocity. Without loss of generality, we adopt the initial gain rate $g_0/\gamma = 10.55$ and the damping rate $\gamma_2/\gamma = \gamma_3/\gamma = 9.72$ converted from the real ones in an experiment [28], while the other system parameters are adjustable in the whole parameter space. As shown in all insets in figure 1(b), the time-dependent gain rates due to gain saturation reduce with the increase of the cavity field intensities to finally have the steady gain rates $g_\eta$ lower than the damping rates in the same cavities, except for one case of an almost constant gain rate in figure 1(b3). The simulation results in figure 1(b) indicate that good optical nonreciprocity can be easily realized by the structure with the configuration of double active components.

Compared to the configuration with only one active component [29], the transmission nonreciprocity can be enhanced with the current configuration with two active components, because the forward input and backward input induce more significantly different gain saturations in these two components. This fact

![Figure 2](image_url)

**Figure 2.** The relation between the isolation ratio and the drive detuning $\Delta/\gamma$, together with the comparison of the result of the current configuration (the solid curves) with the corresponding one according to the setup in [29] (the dashed curves). (a) The initial gain rate $g_0/\gamma = 10.55$ is set to be higher than the damping rates $\gamma_2/\gamma = \gamma_3/\gamma = 9.72$. (b) The initial gain rate is set to be $g_0/\gamma = 9.72$ without changing the others. The saturation intensity is fixed at $I_s = 10^7$, and the drive amplitude is $E = 10\gamma$. 

where $\gamma_2 = \gamma_3$ and $\gamma_2 - \gamma_1 << \gamma$ (the stabilized gain rate $g_\eta$ becomes smaller than but is close to the damping rate in the same cavity, as it is seen from the exact numerical simulations). This result indicates that, to some extent, the concerned optical nonreciprocity are determined by the asymmetric coupling $J_{33} \neq J_{12}$ inside the structure. In the limit of such asymmetric coupling $J_{33}/J_{12} \to \infty$, the difference between the forward and backward output will tend to a limited value $|a_3^+ / a_1^+|^2 \to \gamma_2/(\gamma_2 - \gamma_2)^2$, being consistent with the tendency seen from the numerical simulations (the dip positions in figure 2(a) below).

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Compared to the configuration with only one active component [29], the transmission nonreciprocity can be enhanced with the current configuration with two active components, because the forward input and backward input induce more significantly different gain saturations in these two components. This fact
is demonstrated by using a set of experimentally available system parameters in figure 1(b1), where the forward output intensity $|a_f|^2$ (blue solid curve) is much larger than the backward output intensity $|a_r|^2$ (red solid curve) to have an isolation ratio about 40 dB, in contrast to the isolation ratio of only 18.2 dB with the configuration of only one active cavity (the results indicated by the dashed curves). If the drive intensity is increased by one order as in figure 1(b2), the gain rates will be saturated more quickly and the isolation ratio will be reduced to 23 dB. The phenomenon implies that, for a good transmission can have a certain range of frequency, and the performance of non-reciprocal transmission for such inputs will be degraded if the input frequency is far from the transmission resonant point; for example, an average ratio 78.1 dB is realized at $\eta = 1$ as in figure 1(b4), the forward and back intensity will decrease due to a weaker coupling with one of the active cavities, but the isolation ratio will be increased to about 60.5 dB. If the field detuning is chosen further away from the point $\eta$, the backward output field intensity can be continually reduced to near zero with the increased field detuning $\Delta$. In this situation the absolute intensity of the backward output is lowered to very small value, while the forward output keeps stronger than the input, so that the structure operates like an optical diode. All these results clarify that, in conjunction with the proper choice of the system parameters, the application of a simple mechanism of gain saturation can greatly enhance the optical nonreciprocity by the structure.

3. Performance over the working bandwidth

The performance of the concerned structure are determined by various factors. First of all, the input field can have a certain range of frequency, and the performance of non-reciprocal transmission for such inputs of different frequencies is a primary concern. In building up the structure, one needs to choose the proper cavities or coupled fiber rings. In what follows we study the relevance of these essential factors.

Given a fixed resonant frequency $\omega_c$ for the coupled components, the different frequencies of inputs are equivalent to their different detunings $\Delta = \omega - \omega_c$ in evaluating their effect. By using two different initial gain rate $g_0$, we illustrate the effect for the detunings in figure 2. In figure 2(a) the initial gain rate is set to be $g_0/\gamma = 10.55$, higher than the damping rate $\gamma_f/\gamma = 9.72$ for the same cavity. An interesting observation from figure 1(b4) is that the stabilized output fields will become oscillating if the normalized detuning with respect to the damping rate $g$ for the input becomes different from the cavity coupling ratio $\eta$. Only within a narrow range ($< 0.1$) around the transmission resonant point $\eta = \Delta/\gamma$, will there exist time-independent transmission of the inputs. Farther away the detuning is from the transmission resonant point $\eta = \Delta/\gamma$, more severe oscillation of the output fields will exist. For this reason we present the averaged isolation ratios in figure 2. One can have high average isolation ratio if the detuning is far from the transmission resonant point; for example, an average ratio 78 dB is realized at $\Delta/\gamma = 12$, in contrast to the given coupling ratio $\eta = 2$. In this regime, the isolation ratio grows with the suppressed absolute intensity of the backward output due to the increased $\Delta/\gamma$; as the backward input is lowered close to zero, the system will be like an optical diode, which also has its amplified forward output (the average output intensity is higher than the input intensity). To have a time-independent isolation ratio, on the other hand, we will have to stay with a value about 30 dB around the transmission resonant point $\Delta/\gamma = \eta = 2$. This feature is different from the corresponding one in [29], as seen from the comparison in figure 2(a), which shows that the doubled optical gain in two cavities significantly enhances the optical nonreciprocity.

If the initial gain rate is reduced to, e.g. $g_0/\gamma = 9.72$ as in figure 2(b), the relation between the isolation ratio and the detuning will be totally changed. During the time evolution of the system, the two active cavities mostly behave like two passive ones, making the isolation ratios lower than those in figure 2(a). There exists a different feature that the isolation ratio at the transmission resonant point $\eta = \Delta/\gamma$ becomes maximum, and it is consistent with a feature in [29]; cf the comparison in figure 2(b).

One advantage with the involved mechanism—isotropic optical gain saturation—for such optical nonreciprocity is the feasibility to have a wide bandwidth for its performance. Theoretically the working bandwidth for the optical nonreciprocity is consistent with that of the used gain medium. For example, if the optical gain is through erbium doped fiber amplifier (EDFA) for the mutually coupled fiber ring structure, the theoretical working range of the system will be between 1300 nm and 1560 nm, corresponding to a bandwidth about 394 THz. In contrast, the asymmetric propagation of light fields based on many other mechanisms should rely on the effects depending on the propagation direction of the fields, such as the phase-matching condition of the momenta of photons (the conservation of the photon momenta) in various nonlinear processes, thus limiting the range of the workable frequencies. For example, in a non-reciprocal setup based on the mechanism of Brillouin scattering induced transparency [57], the working bandwidth is limited to
400 kHz while the setup achieves a high isolation ratio of 78.6 dB. The improvement on the figure-of-merit is a focus of the recent research. In a very recent study employing the collision effect of thermal atoms [60], a bandwidth over 1.2 GHz for the isolation ratio exceeding 20 dB is reported.

In addition to the mutually coupled fiber rings, a main approach to the non-reciprocal setup in figure 1(a) is coupling the micro-cavities as in [27, 28, 58]. High-quality microcavities were demanded for constructing such systems in the relevant researches of the past. Since the mechanism for the nonreciprocity is optical gain saturation, which bears no direct relation to the quality factor of the used cavities, the requirement for the high-quality cavities can be relaxed if the gain rate can be correspondingly improved to compensate for the cavity damping. The details about how to choose the optical gain will be discussed in section 4.2. Here we provide a specific illustration of the feature with two working ranges in figure 2(a).

Given the experimental parameters in [28], the structure works with the isolation ratio over 60 dB between point A and point B spanning the frequency range of 188.24 MHz, and operates with the isolation ratio over 50 dB in a range of 123.08 MHz between C and D. If the lower quality cavities with 10 times higher damping rate (from 36.2 MHz to 362 MHz) are used while the optical gain can be raised by one order, the corresponding operation band ranges can be 1.8 GHz and 1.2 GHz, respectively.

4. Relevance of the main system parameters

Given a specific system, one needs to know how well its transmission nonreciprocity could be for different inputs. Then it is necessary to study the dependence of the transmission on the drive amplitude $E$.

Moreover, the choice of the optical gain can significantly affect the optical nonreciprocity, so one should concern the parameters such as the initial gain rate $g_0$ and the saturation intensity $I_0$.

4.1. Drive amplitude $E$

Since the output fields are unstable when $\eta \neq \Delta / \gamma$, we fix the condition $\eta = \Delta / \gamma$ below to have the stable transmission. In most practical situations, the forward and backward input fields are switched on separately, so we will consider this pattern with the intensities of the inputs fixed to be the same, $E_f = E_b$. The nonreciprocity is then measured by the logarithmic ratio defined as

$$10 \log_{10} \left| \frac{a_{3,f}}{a_{4,b}} \right|^2.$$

The dependence of the non-reciprocal transmission on the drive amplitude will be indicated by this ratio.

Here we set the initial gain rate $g_0/\gamma = 9.8$ to be slightly higher than the damping rate $\gamma_i/\gamma = 9.72$ for $i = 2, 3$. The examples in figure 3 show the overall tendency that the increased drive intensity decreases the nonreciprocity ratio. Especially, the decreasing is rapid with the initial increase of the field intensity, because the backward output intensity will be increased under the same condition. Then the nonreciprocity ratio will tend to the fixed values with the increasing $E$, and it is due to the fast gain saturation when the inputs are very strong. Given a relatively low gain saturation intensity, e.g. $I_0 = 10^7$ in figure 3, the nonreciprocity in the transmission will be limited to the lower ratios. In analogue to that strong electrical current may cause the breakdown of a diode with low capacity, a very strong input will decrease the nonreciprocity ratio here.

One way to improve the transmission nonreciprocity is to use an optical gain medium with higher saturation intensity $I_0$. For example, if the gain saturation is $I_0 = 10^{13}$, the nonreciprocity ratio can be well preserved over 96 dB even when the drive amplitude is as high as $E/\gamma = 100$. One advantage of the present configuration is that the nonreciprocity ratio can keep high by choosing the proper saturation intensity $I_0$ for the strong input fields.

4.2. Optical gain and its saturation speed

Now we consider an important factor, the initial gain rate $g_0$ used for the structure. The physical mechanism for the non-reciprocal transmission by the structure is that the inputs from different ports induce very different optical gain saturations inside the cavities, which determine the intensities of the output fields. The use of two active components together greatly enhances such difference, to have the obtained relation between the nonreciprocity ratio and the initial gain rate as illustrated in figure 4. The nonreciprocity ratios are found negligible when $g_0 < \gamma_2 = \gamma_3$, but they increase suddenly when the gain rate $g_0$ is slightly larger than the damping rate $\gamma_i$. After crossing the maximum values, the ratios decrease gradually with the further increase of the initial gain rate, until finally converge to the fix values. The feature for the configuration in [29] is similar, except that the point for the sudden increase of the nonreciprocity ratio is around $g_0 = \gamma_2 + \gamma$. 
Figure 3. The relation between the stable nonreciprocity ratio and the varying drive amplitude, together with the comparison of the result of the current configuration (the solid curves) with the corresponding one according to the setup in [29] (the dashed curves). The equally strong forward and backward fields are switched on separately. The system parameters are set as η = J23/J12 = Δ/γ = 5, g0 = 9.8γ, γ2 = γ3 = 9.72γ, and γ1 = γ.

Figure 4. The relation between the nonreciprocity ratio and the initial gain rate g0, together with the comparison of the result of the current configuration (the solid curves) with the corresponding one according to the setup in [29] (the dashed curves). The indicated ratios increase suddenly at the point where the initial gain rate g0 is slightly higher than the corresponding damping rate γ2(3); for the configuration in [29] the jump takes place at the point where the initial gain rate g0 is slightly larger than γ2 + γ. Here, the drive intensity is E/γ = 10, and the other parameters are set as η = Δ/γ = 5, I0 = 10^7, and γ2 = γ3.

The relation illustrated in figure 4 clearly demonstrates that the requirement to achieve good non-reciprocal transmission is that the initial gain rate should be higher than the corresponding damping rate. Especially, a high nonreciprocity ratio will be available if the initial gain rate is slightly larger than the corresponding damping rate. On the other hand, if the initial gain rate is too high, the forward and backward intensities will reach their limits quickly due to the saturation effect, while the effective gain rate will decrease to be lower than the damping rate (the denominator in equation (2) increases very quickly) so that the performance of the non-reciprocal transmission becomes worse. The longer the period of time for the effective gain rate 82(3)(t) to be higher than the corresponding damping rate is, the better the non-reciprocal transmission will be. At the optimum point of the initial gain rate slightly larger than the corresponding damping rate, the time period, in which the effective gain rate is higher than the corresponding damping rate, is the longest. Under the condition a high optical nonreciprocity will be available. For example, a nonreciprocity ratio 100 dB can be obtained with g0/γ = 6.35 and γ2/γ = γ3/γ = 6.

The above mentioned feature is highly meaningful to the practical applications. No matter whether the damping rate of the cavities is high or not, it will be possible to achieve a good non-reciprocal transmission under the condition g0 > γ2(3), given an available higher gain rate g0. For this reason high-quality cavities are not necessary for the purpose, so that the constraints on making such optical isolators can be considerably relaxed. Technically, the gain rate can be heightened by increasing the power of the pump field [28, 61–63].
Figure 5. The relation between the nonreciprocity ratio and the saturation intensity $I_0$, together with the comparison of the result of the current configuration (the solid curves) with the corresponding one according to the setup in [29] (the dashed curves). The initial gain rate for the present configuration is $g_0/\gamma = 9.8$, while the gain rate for the configuration in [29] is set as $g_0/\gamma = 11.72$. The other parameters are chosen as $\gamma_2/\gamma = \gamma_3/\gamma = 9.72$ and $E = 10\gamma$.

Figure 6. The comparison between the asymmetric optical gains with the symmetric configuration and the configuration with only one active component. (a) The solid curves: $g_2(0) = g_3(0) = 10.55\gamma$ and $I_{20} = I_{30} = 10^7$; the dashed curves: $g_2(0) = g_3(0) = 10.55\gamma$, $I_{20} = I_{30} = 10^7$; the dash dotted curves: $g_2(0) = 10.55\gamma$, $I_{20} = 10^7$, but $g_3(0) = 0$. The inputs are with $E = 10\gamma$ and $\Delta/\gamma = \eta = 5$. The relative damping rates are $\gamma_2 = \gamma_3 = 9.72\gamma$.

The role of the saturation intensity can be seen in conjunction with the discussion in section 4. As shown in figure 3, when the saturation intensity is high, the isolation ratio will be stably high even if the input drive amplitude is very large. This feature is clarified further in figure 5. A large saturation intensity $I_0$ will make the gain saturation process slower, so that the fields in the cavities will be effectively amplified for longer time before they reach their limit values. Meanwhile, the condition of a higher effective gain rate than the corresponding damping rate will be preserved for longer time, to achieve a higher nonreciprocity ratio.

Another interesting configuration is that the optical gain media in two cavities are different and/or the corresponding gain saturation speeds are also different. In figure 6 we compare such asymmetric optical gains with the symmetric situation and the setup with the optical in only one cavity. It is seen that the configuration with two identical optical gains with the same saturation speed outdoes the asymmetric configuration significantly for the purpose of optical nonreciprocity. If one of the optical gains saturates too quickly (for example, under the condition $I_{20} \gg I_{30}$ as in figures 6(a) and (b)), its effective gain effect will be lost quickly, rendering the result to be similar to that with only one optical gain. Then, the relative intensities of the forward and backward outputs are similar to those due to only one optical gain. On the other hand, the comparison indicates further that the cooperated action of two optical gains is necessary for achieving the targeted optical nonreciprocity by the structure.

5. Dynamical process leading to the nonreciprocity

In this section we look at the optical nonreciprocity of the structure from a different viewpoint, and clarify why a dynamical evolution obtained with the exact numerical simulation is indispensable in studying the
phenomenon. In what follows we simply consider the individual action of the forward and backward input. First of all, we provide the following analytical solution for the concerned output modes at the transmission resonant point \( (\eta = \Delta / \gamma) \), where the stabilized output modes have the form \( A_j e^{i \Delta t} \) with the constant \( A_j \):

\[
|a_{i,j}^f| = \frac{2J_{12}^f - (g_{i,f} - \gamma_j)J_{12}^f}{4J_{12}^f + (g_{i,f} - \gamma_j)^2 \gamma_j^2 - J_{12}^f(4g_{i,f} - 4\gamma_j - \eta^2 \gamma_j)} E, \\
|a_{i,j}^b| = \frac{2J_{12}^b - (g_{i,b} - \gamma_j)J_{12}^b}{4J_{12}^b + (g_{i,b} - \gamma_j)^2 \gamma_j^2 - J_{12}^b(4g_{i,b} - 4\gamma_j - \eta^2 \gamma_j)} E. 
\]

(6)

Obviously, the optical nonreciprocity manifests with the different stabilized gains \( g_{i,f} \neq g_{i,b} \) in the end. However, these stabilized gain rates cannot fully capture the nonreciprocity of the concerned structure.

In a more general sense, the stabilized transmission of light fields by any optical circuit can be depicted by a scattering matrix like the following one

\[
\hat{S} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\
S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\
S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\
S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66}
\end{bmatrix},
\]

(7)

which can be found from the numerical simulations based on equation (1) \([29]\). Here the forward output is \( S_{16}^f E \), while the backward counterpart is given as \( S_{66}^b E \). In addition to the Lorentz reciprocity that restricts the scattering matrix to be symmetric \( S_{ij} = S_{ji} \), as in the linear optical systems (see, e.g. \([64–69]\)), there is the relation \( S_{16}^{(b)} = S_{25}^{(b)} \) when either of the forward or backward input acts individually. The nonlinearity due to the gain saturation, however, induces a different forward matrix element \( S_{16}^f \) from the backward matrix element \( S_{66}^b \). There is one interesting observation from the exact numerical calculations illustrated in figure 7—the stabilized gain rate respectively due to the forward or backward input is almost invariant with the change of the detuning of the input, but the forward scattering matrix element will become more and more different from the corresponding backward matrix element, thus having more significant optical nonreciprocity with the increased detuning \( \Delta \). The cause for such phenomenon is indicated by the inserts in figure 7(a), which show that it takes longer time for the gain rate to be stabilized at a higher detuning, so that the time period for the gain rate \( g(t) \) to be higher than the corresponding damping rate in the same cavity becomes longer. The degree of nonreciprocity is mostly determined by how much time the evolving gain rate \( g(t) \) is higher than the damping rate in the same cavity and, for this reason, one should take a dynamical approach with the complete time evolution of the cavity modes in studying the concerned nonreciprocity.

The necessity for the time evolution of gain saturation from \( g(t = 0) = g_0 \) to \( g(t \to \infty) = g_\infty \) can also be seen in another way. If the stabilized gain rate \( g_{i,f(b)0} \) is directly plugged into the dynamical equations, equation (1), these equations will be reduced to the linearized form,

\[
\frac{d}{dt} \vec{\sigma}_{f(b),0}(t) = \hat{M}_{f(b)} \vec{\sigma}_{f(b),0}(t) + \vec{\lambda}_{f(b)}(t),
\]

(8)

in which there are the definitions for the output modes

\[
\vec{\sigma}_{f(b),0} = (a^+_{f(b),0})^T a^+_{f(b),0} a^+_{f(b),0} a^+_{f(b),0} a^-_{f(b),0} a^-_{f(b),0} a^-_{f(b),0} a^-_{f(b),0})^T,
\]
and the drive terms $\tilde{\lambda}_f(t) = (E_f e^{i\Delta f}, 0, 0, 0, 0, 0)^T$ and $\tilde{\lambda}_b(t) = (0, 0, 0, 0, E_b e^{i\Delta f}, 0)^T$, while the corresponding dynamics matrix takes the form
\[
\hat{M}_{f/b} = \begin{pmatrix}
-\kappa & 0 & 0 & -iJ_{12} & 0 & -iJ_{13} \\
0 & -\kappa & -iJ_{12} & 0 & -iJ_{13} & 0 \\
0 & -iJ_{12} & g_{d(f/b)} - \gamma_2 & 0 & 0 & -iJ_{23} \\
-iJ_{12} & 0 & 0 & g_{d(f/b)} - \gamma_2 & -iJ_{23} & 0 \\
0 & -iJ_{13} & 0 & -iJ_{23} & g_{d(f/b)} - \gamma_3 & 0 \\
-iJ_{13} & 0 & -iJ_{23} & 0 & 0 & g_{d(f/b)} - \gamma_3
\end{pmatrix},
\] (9)

Then there is an analytical solution:
\[
\tilde{c}_{f/b}(t) = \int_0^t d\tau \exp\{\hat{M}_{f/b}(t - \tau)\} \tilde{\lambda}_{f/b}(\tau).
\] (10)

However, such linearized equations are insufficient to describe the non-reciprocal transmission by the structure, as illustrated by a couple of the examples in figure 8. Even in the situation of transmission resonance $\eta = \Delta/\gamma$ [figure 8(a)], there exists a huge difference between the results obtained with equation (1) and those obtained with equation (8); the contrast between forward and backward output from equation (8) is insignificant. In the non-resonant situation [figure 8(b)], the contrast predicted by the linearized equation (8) deviates even more drastically from the real one. The most important factor that determines the optical nonreciprocity is therefore the dynamical process due to the gain saturation, a mechanism we intend to clarify in the current work.

6. Faithfully non-reciprocal transmission of pulses

Most optical signals in realistic applications are in the forms of pulses, so it is necessary to study how good the currently concerned structure can work with the pulses. In addition to the non-reciprocal transmission of the field energy, we require that the input pulse spectrum or pulse shape should be preserved for the corresponding output, so the operation under this extra condition is much harder than the non-reciprocal transmission of a continuous-wave (CW) field. Without loss of generality we use Gaussian-shaped pulses with the profile $E_{f/b}(t) = E \exp\left(-\frac{(t - t_0)^2}{\tau^2}\right)$ (with $t_0$ being the peak-value moment and $\tau$ being the duration) for the example. One advantage of using Gaussian pulses is that the Fourier transform of their profiles in the time (frequency) domain are still in Gaussian profiles in the frequency (time) domain. The bandwidth $\Delta \omega$ of a Gaussian signal is simply the reciprocal of its duration $\tau$ in the time domain.

In the numerical simulation we adopt the normalized bandwidth $\Delta \omega/\gamma$ without dimension. The examples of the transmitted forward and backward output fields are depicted in figure 9 for two different values of such dimensionless bandwidth, but the central frequency of the pulses is assumed to be the same as the cavity resonant frequency. If the dimensionless bandwidth is as large as $\Delta \omega/\gamma \sim 10^{-1}$, the transmitted Gaussian profile will be totally deformed. However, the Gaussian profile can be well preserved if the dimensionless bandwidth is lowered to $\Delta \omega/\gamma \sim 10^{-2}$. In principle, the non-reciprocal transmission of the pulses with high fidelity can be better, if the pulses transmitted by the structure satisfy the condition of a low dimensionless bandwidth $\Delta \omega/\gamma$. The faithful transmission of the pulses with broad bandwidth can be nonetheless implemented by using the cavities of larger damping rate $\gamma$, since the allowed bandwidth $\Delta \omega$
Figure 9. The transmitted profiles of the Gaussian pulses with different bandwidths. The red (blue) curve is the forward (backward) output. (a) The forward and backward outputs for the input with $t_0 = 25$ and $1/\tau = 10^{-4}$. (b) The forward and backward outputs for the inputs with $t_0 = 250$ and $1/\tau = 10^{-2}$. The insets are the profiles of the input pulses. The system parameters are chosen as $g_0/\gamma = 9.7, \gamma_2/\gamma = 9.72, E/\gamma = 10$, $\Delta = 0, J_{ij}/\gamma = 0.001 \ (i \neq j)$, and $I_0 = 10$.

Figure 10. (a1) and (a2) The transmitted profiles of a Gaussian pulse under the different initial gain rates $g_0$. (b1) and (b2) The corresponding outputs for the situations of the different saturation intensities $I_0$. (c1) and (c2) The corresponding output due to the different cavity couplings $J_{ij} (i,j = 1, 2, 3 \ \text{and} \ i \neq j)$. The normalized pulse bandwidth is $\Delta \omega/\gamma = 0.01$. The other parameters are the same as those in figure 9.

can scale up with the damping rate $\gamma$. For example, given the damping rate $\gamma = 36.2 \ \text{MHz}$ [28], the allowed bandwidth for the pulses is $\Delta \omega = 0.36 \ \text{MHz}$, but this figure-of-merit can be increased to $\Delta \omega = 3.6 \ \text{MHz}$ (within the range for the commercial broadband signals) if the damping rate is raised by one order.

There exists a tricky point for the non-reciprocal transmission of pulses. By figures 10(a1) and (a2), we compare the use of two different gain rates. The use of a larger gain rate $g_0/\gamma = 10$ in figure 10(a2) totally deforms the output pulse, implying that the favorable bandwidth should be compatible with a relatively lower initial gain rate. In figure 10(a1), the faithful transmission of the pulse that maintains its shape occurs with the gain rate $g_0/\gamma = 8$ lower than the corresponding damping rate $\gamma_2/\gamma = 9.72$. However, a better nonreciprocity shown in figure 9(b) is that the gain rate $g_0/\gamma = 9.7$ should be slightly lower than the corresponding damping rate $\gamma_2/\gamma = 9.72$. Through these simulations one sees that the requirement for achieving both good nonreciprocity and faith transmission of the broadband pulses is different from the optimum condition for the non-reciprocal transmission of a CW field of single
Figure 11. The comparison between the operation at the transmission resonant point $\Delta/\gamma = \eta$ ((a) and (b)) and off the transmission resonant point with $\Delta/\gamma \neq \eta$ ((c) and (d)). The couplings between the components affect the nonreciprocity in transmission. The system parameters are the same as those in figure 9.

frequency; in the case of pulse fields the optimum gain rate should be slightly lower rather than slightly higher than the corresponding damping rate. This condition renders the outputs weaker, and the processed signals should be amplified for their restoration. The nonreciprocity for the pulse signals lies in the contrast between the forward and backward output.

The gain saturation speed is also relevant. If the saturation intensity $I_0$ is heightened from the value in figure 10(b1) to the one in figure 10(b2), the backward output will become stronger as well, spoiling the effect of the nonreciprocity. Moreover, if the mutual coupling intensity is increased from the value $J_{ij}/\gamma = 0.001$ ($i,j = 1, 2, 3$ and $i \neq j$) in figure 9(b) to those in figures 10(b1) and (b2), the backward output field will become relatively stronger. Small cavity couplings are thus more favorable to the non-reciprocal transmission of pulse signals. The extra demand on a faithful transmission of pulse signals in non-reciprocal way requires more consideration in the system design.

For the sake of keeping the high fidelity in pulse transmission, the operation at the transmission resonant point $\eta = \Delta/\gamma$ is the best because the output field of single frequency becomes completely time-independent; see the time evolutions in, for examples, figures 1(b1)–(b3). However, at this point, the contrast between the forward and backward output are less significant especially in the situation shown in figure 11(b). The contrast can be improved by deviating from this operation point (figure 11(c)) and will be enhanced to the level like an optical diode by adjusting the cavity coupling further (figure 11(d)). A slight deformation of the output pulse will exist in such situation of high nonreciprocity, and should be reduced by decreasing the normalized bandwidth $\Delta \omega/\gamma$. The simultaneous realization of both high fidelity and high nonreciprocity for broadband pulses is under such restrictions illustrated above. These constraints could be relaxed by employing another mechanism of asymmetric linear coupling in [29], but in the current work we are only concerned with the mechanism of optical gain saturation.

7. Conclusion

In recent years, various systems based on the state-of-the-art approaches and sophisticated mechanisms are under development for the realization of practical optical nonreciprocity. The purpose of the current study is to bring a fresh attention on a simple mechanism of optical gain saturation and the structures in the category of PT-symmetry optics. Through the detailed simulations with a model of three mutually coupled cavities or optical fiber rings, it is found that two active components carrying optical gain in such structure can significantly enhance the nonreciprocity of optical field transmission. In the regime of high field detuning, where not only the contrast between the forward and backward output is deep but the absolute intensity of the backward output also becomes negligible, the structure operates like an optical diode to the input fields. Unlike the reliance on the phase matching conditions of photons in many other nonreciprocal processes, the operation bandwidth for the structure is simply determined by the workable frequency range.
for the used optical gain medium, so it is feasible for the system to work in an extremely broad bandwidth. The desired figures-of-merit for such system are realized through choosing the proper optical gain, adjusting the couplings between the components, as well as selecting the suitable detuning of the inputs. If the higher optical gain rates are available, low-quality cavities can also be applied to construct the setup. We also investigate the operation of the structure on pulsed fields, the requirement of preserving a high spectrum fidelity for which imposes more restrictions on the system parameter. The approximately faithful and nonreciprocal transmission of a broadband pulse is possible by employing the suitable optical gain/loss ratios and mutual couplings between the components. The requirements in designing the system that non-reciprocally transmits the CW or pulsed fields are compatible with the current experimental technology, and the understanding of the corresponding features will be helpful to making optical diodes and optical isolators solely based on the nonlinearity of optical gain saturation.

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References

[1] Saleh B E A and Teich M C 1991 Fundamentals of Photonics (New York: Wiley)
[2] Tien M-C, Mizumoto T, Pintus P, Kromer H and Bowers J E 2011 Silicon ring isolators with bonded nonreciprocal magneto-optic garnets Opt. Express 19 11740–5
[3] Bi L, Hu J, Jiang P, Kim D H, Dione G F, Kimerling L C and Ross C A 2011 On-chip optical isolation in monolithically integrated non-reciprocal optical resonators Nat. Photon. 5 758–62
[4] Shoji Y, Mizumoto T, Yokoi H, Hsieh I-W and Osgood R M 2008 Magneto-optical isolator with silicon waveguides fabricated by direct bonding Appl. Phys. Lett. 92 071117
[5] Goto T, Onbaslò M C and Ross C A 2012 Magneto-optical properties of cerium substituted yttrium iron garnet films with reduced thermal budget for monolithic photonic integrated circuits Opt. Express 20 28507–17
[6] Fan L, Wang J, Varghese L T, Shen H, Niub, Xuan Y, Weiner A M and Qi M 2012 An all-silicon passive optical diode Science 335 447–50
[7] Sun Y, Tong Y-w, Xue C-h, Ding Y-q, Li Y-h, Jiang H and Chen H 2013 Electromagnetic diode based on nonlinear electromagnetically induced transparency in metamaterials Appl. Phys. Lett. 103 091904
[8] Xia K, Nori F and Xiao M 2018 Cavity-free optical isolators and circulators using a chiral cross-Kerr nonlinearity Phys. Rev. Lett. 121 203602
[9] Wang C, Zhong X-L and Li Z Y 2012 Linear and passive silicon optical isolator Sci. Rep. 2 674
[10] Wang D W, Zhou H T, Guo M J, Zhang J X, Evers J and Zhu S Y 2013 Optical diode made from a moving photonic crystal Phys. Rev. Lett. 110 095301
[11] Yang I, Zhang Y, Yan X B, Sheng Y, Cui C L and Wu J H 2015 Dynamically induced two-color nonreciprocity in a tripod system of a moving atomic lattice Phys. Rev. A 92 053859
[12] Shi Q Y, Dong H Y, Fung K H, Dong Z-g and Wang J 2018 Opt. Express 26 33613
[13] Yu Z and Fan S 2009 Complete optical isolation created by indirect interband photonic transitions Nat. Photon. 3 91–4
[14] Fang K J, Yu Z and Fan S 2012 Photonic Aharonov–Bohm effect based on dynamic modulation Phys. Rev. Lett. 108 153901
[15] Kang M S, Butsch A and Russell P S J 2011 Reconfigurable light-driven opto-acoustic isolators in photonic crystal fibre Nat. Photon. 5 549–53
[16] Kamal A, Clarke J and Devoret M H 2011 Noiseless non-reciprocity in a parametric active device Nat. Phys. 7 311–5
[17] Kim J, Kuzyk M C, Han K, Wang H and Bahl G 2015 Non-reciprocal Brillouin scattering induced transparency Nat. Phys. 11 275–80
[18] Dong C H, Shen Z, Zhou C L, Zhang Y L, Fu W and Guo G C 2015 Brillouin-scattering-induced transparency and non-reciprocal light storage Nat. Commun. 6 6193
[19] Xia K Y, Lu G W, Lin G W, Cheng Y Q, Niub, Y P, Gong S Q and Twamley J 2014 Reversible nonmagnetic single-photon isolation using unbalanced quantum coupling Phys. Rev. A 90 043802
[20] Hafezi M and Rabl P 2012 Optomechanically induced non-reciprocity in microring resonators Opt. Express 20 7672–84
[21] Ruesink, F, M, et al. A 2012 Nonreciprocity and magnetic-free isolation based on optomechanical interactions Nat. Commun. 7 13662
[22] Shen Z, Zhang Y-L, Chen Y, Zhou C-L, Xiao Y-F, Zou X-B, Sun F-W, Guo G-C and Dong C-H 2016 Experimental realization of optomechanically induced non-reciprocity Nat. Photon. 10 657–61
[23] M, Miri, F, Verhagen E and Alù A 2017 Optical nonreciprocity based on optomechanical coupling Phys. Rev. Appl. 7 064014
[24] Yan X B, Lu H L, Gao F and Yang L 2019 Perfect optical nonreciprocity in a double-cavity optomechanical system Front. Phys. 14 52601
[25] Shang C, Shen H Z and Yi X X 2019 Nonreciprocity in a strongly coupled three-mode optomechanical circulator system Opt. Express 27 25882–901
[26] Chen Y-T, Du L, Liu Y-M and Zhang Y 2020 Dual-gate transistor amplifier in a multimode optomechanical system Opt. Express 28 7095
[27] Peng B et al 2014 Parity-time-symmetric whispering-gallery microcavities Nat. Phys. 10 394–8
[28] Chang L, Jiang X, Hua S, Yang C, Wen J, Jiang L, Li G, Wang G and Xiao M 2014 Parity-time symmetry and variable optical isolation in active–passive-coupled microresonators Nat. Photon. 8 524
[29] He B, Yang L, Jiang X S and Xiao M 2018 Transmission nonreciprocity in a mutually coupled circulating structure Phys. Rev. Lett. 120 203904
[30] Wen J, Jiang X, Jiang L and Xiao M 2018 Parity-time symmetry in optical microcavity systems J. Phys. B: At. Mol. Opt. Phys. 51 222001
[31] El-Ganainy R, Makris K G, Khajavikhan M, Musslimani Z H, Rotter S and Christodoulides D N 2018 Non-Hermitian physics and PT symmetry Nat. Phys. 14 11
[32] He B, Yang L, Zhang Z and Xiao M 2015 Cyclic permutation-time symmetric structure with coupled gain-loss microcavities Phys. Rev. A 91 033830
[33] Hodaei H, Hassan A U, Wittek S, Garcia-Gracia H, El-Ganainy R, Christodoulides D N and Khajavikhan M 2017 Enhanced sensitivity at higher-order exceptional points Nature 548 187
[34] Wu R B, Zheng Y, Chen Q M and Liu Y X 2018 Synthesizing exceptional points with three resonators Phys. Rev. A 98 033817
[35] Jin L 2019 Flat band induced by the interplay of synthetic magnetic flux and non-Hermiticity Phys. Rev. A 99 053810
[36] Wang P, Jin L and Song Z 2019 Non-Hermitian phase transition and eigenstate localization induced by asymmetric coupling Phys. Rev. A 99 062112
[37] Jing H, Özdemir S K, Liu Y, Zhang J, Yang L and Nori F 2014 PT-symmetric phonon laser Phys. Rev. Lett. 113 053604
[38] He B, Yan S B, Wang J and Xiao M 2015 Quantum noise effects with Kerr-nonlinearity enhancement in coupled gain-loss waveguides Phys. Rev. A 91 053832
[39] He B, Yang L and Xiao M 2016 Dynamical phonon laser in coupled active-passive microresonators Phys. Rev. A 94 033802(R)
[40] Liu Y L and Liu Y X 2017 Energy-localization-enhanced ground state cooling of a mechanical resonator from room temperature in optomechanics using a gain cavity Phys. Rev. A 96 023812
[41] Gu X, Bai R, Zhang C, Jin X R, Zhang Y Q, Zhang S and Lee Y 2017 Unidirectional reflectionless propagation in a non-ideal parity-time meta-surface based on far field coupling Opt. Express 25 11778
[42] Li W, Li C and Song H 2017 Theoretical realization and application of parity-time-symmetric oscillators in a quantum regime Phys. Rev. A 95 023827
[43] Vashahri-Ghamsari S, He B and Xiao M 2017 Continuous-variable entanglement generation using a hybrid PT-symmetric system Phys. Rev. A 96 033806
[44] Antonosyan D A, Solntsev A S and Sukhorukov A A 2018 Photon-pair generation in a quadratically nonlinear parity-time symmetric coupler Photon. Res. 7 66–79
[45] Tian T, Wang Z and Song I 2019 Rotation sensing in two coupled whispering-gallery-mode resonators with loss and gain Phys. Rev. A 100 043810
[46] Vashahri-Ghamsari S, He B and Xiao M 2019 Effects of gain saturation on the quantum properties of light in a non-Hermitian gain-loss coupler Phys. Rev. A 99 023819
[47] Quesada N, Adjei E, El-Ganainy R and Braiczyk A M 2019 Non-Hermitian engineering for brighter broadband pseudothermal light Phys. Rev. A 100 043805
[48] Perina A Jr, Lukš A, Kalaga J K, Leonšík W and Miranowicz A 2019 Nonclassical light at exceptional points of a quantum PT-symmetric two-mode system Phys. Rev. A 100 053820
[49] Chakraborty S and Sarma A K 2019 Delayed sudden death of entanglement at exceptional points Phys. Rev. A 100 063846
[50] Arkhipov I L, Miranowicz A, Minganti F and Nori F 2020 Quantum and semiclassical exceptional points of a linear system of coupled cavities with losses and gain within the Scully–Lamb laser theory Phys. Rev. A 101 013812
[51] Xie Y, Cao Z, He B and Lin Q 2020 PT-symmetric phonon laser under gain saturation effect Opt. Express 28 22580–93
[52] Wu L-T, Guo R-P, Cui T-J and Chen J 2017 A photonic analog of Möbius strips using coupled optical ring resonators Opt. Express 25 123901–9
[53] Solja M and Joannopoulos J D 2014 Enhancement of nonlinear effects using photonic crystals Nat. Mater. 3 211–9
[54] Pavesi L and Lockwood D J 2004 Silicon Photonics (Berlin: Springer)
[55] Fan L, Varghese L T, Wang J, Xuan Y, Weiner A M and Qi M 2013 Silicon optical diode with 40 dB nonreciprocal transmission Opt. Exp. 21 12958
[56] Fang K, Luo J, Metelmann A, Matheny M H, Marquardt F, Clerk A A and Painter O 2017 Generalized non-reciprocity in an optomechanical circuit via synthetic magnetism and reservoir engineering Nat. Phys. 13 465–71
[57] Kim J H, Kim S and Bahl G 2017 Complete linear optical isolation at the microscale with ultralow loss Sci. Rep. 7 1647
[58] Yang C, Jiang X, Hua Q, Hua S, Chen Y, Ma J and Xiao M 2017 Realization of controllable photonic molecule based on three ultra-high-Q microtoroid cavities Laser Photon. Rev. 11 1600178
[59] Yang C, Hu Y, Jiang X S and Xiao M 2017 Analysis of a triple-cavity photonic molecule based on coupled-mode theory Phys. Rev. A 95 033847
[60] Liang C, Liu B, Xu A N, Wen X, Lu C C, Xie K Y, Tey M K, Liu Y C and You L 2020 Collision-induced broadband optical nonreciprocity Phys. Rev. Lett. 125 123901
[61] Kouki T and Tomitani M 2007 Optical microwave amplification system Opt. Express 15 7391–97
[62] Lei F, Peng B, Özdemir Ş K, Long G L and Yang L 2014 Dynamic Fano-like resonances in erbium-doped whispering-gallery-mode microresonators Appl. Phys. Lett. 105 101112
[63] Liu X-F, Lei F, Gao M, Yang X, Wang C, Özdemir Ş K, Yang L and Long G-L 2016 Gain competition induced mode evolution and mode control in erbium-doped whispering-gallery microresonators Opt. Express 24 9550
[64] Reck M, Zeilinger A, Bernstein H J and Bertani P 1994 Experimental realization of any discrete unitary operator Phys. Rev. Lett. 73 58
[65] He B and Bergou J A 2006 A general approach to physical realization of unambiguous quantum-state discrimination Phys. Lett. A 356 306
[66] He B, Bergou J and Wang Z 2008 Implementation of quantum operations on single-photon qudits Phys. Rev. A 76 042326
[67] Genovese M and Traina P 2008 Review on qudits production and their application to quantum communication and studies on local realism Adv. Sci. Lett. 1 153–60

[68] Piani M, Pitkanen D, Kaltenbaek R and Lütkenhaus N 2011 Phys. Rev. A 84 032304

[69] Tischler N, Rockstuhl C and Slowik K 2018 Quantum optical realization of arbitrary linear transformations allowing for loss and gain Phys. Rev. X 8 021017