Unsupervised Learning of Dense Optical Flow and Depth from Sparse Event Data

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Abstract—In this work we present unsupervised learning of depth and motion from sparse event data generated by a Dynamic Vision Sensor (DVS). To tackle this low level vision task, we use a novel encoder-decoder neural network architecture that aggregates multi-level features and addresses the problem at multiple resolutions. A feature decorrelation technique is introduced to improve the training of the network. A non-local sparse smoothness constraint is used to alleviate the challenge of data sparsity. Our work is the first that generates dense depth and optical flow information from sparse event data. Our results show significant improvements upon previous works that used deep learning for flow estimation from both images and events.

I. INTRODUCTION

Visual motion is evolutionary the oldest and most important cue for encoding information about the 3D motion and spatial geometry of a scene. Even the most primitive animals, such as insects and reptiles, use visual motion to interpret the space-time geometry surrounding them. Yet, even the most advanced Computer Vision algorithms are no match for the capabilities of biological systems.

Recently, there has been much progress in imaging sensor technology, offering alternative solutions to scene perception. The dynamic vision sensor (DVS) \[19\] and other event-based sensors, inspired by the transient pathway of mammalian vision, offer exciting alternatives for visual motion perception. The DVS does not record image frames, but instead - the changes of lighting occurring independently at every DVS pixel. Each of these changes is transmitted asynchronously and is called an event. By its design, this sensor accommodates a large dynamic range and provides high temporal resolution and low latency – ideal properties for real-time applications. The price for these properties is indeed heavy - they produce a lot of noise. Furthermore the data is very sparse, which can be a great advantage, but requires different treatment. The reality created by this novel type of visual sensors thus requires completely different visual processing approaches.

Most works, both in frame-based and event-based vision, fall within the reconstruction framework. A typical reconstruction pipeline computes feature-based correspondences or the optical flow field, which are universal representations for motion analysis.

With recent advancements in deep learning, the traditional feature-based scene reconstruction framework has been replaced by neural networks. Neural network based learning approaches \[34\], \[33\] have shown promising results on frame-based data in solving video reconstruction problems. However, the design of neural network architectures for event-based data is still a challenging problem because of the sparse nature of events and ambiguity in event representation for training a neural network. In this work, we propose a novel, Evenly-Cascaded neural Network (ECN) architecture and a frame-based representation to solve design challenges.

Traditionally, low level tasks such as image segmentation, depth estimation and optical flow were solved by utilizing low level features at multiple resolutions. The recently introduced deep neural networks have an encoder-decoder architecture and they address these low level tasks with high-level features universally used in deep networks \[28\], \[34\], \[9\]. They do not use low level features at multiple resolutions. In this work we introduce a novel encoding-decoding neural network architecture that utilizes both low-level and high-level features and addresses the final task from coarse-to-fine using multiple resolutions. We also utilize a sparse smoothness constraint, which is tailored for sparse data.

Our pipeline achieves good results during low-light scenes. Fig. 1 shows one example featuring night driving - the network was able to predict both depth and flow even with a low event rate and abundance of noise. One of the contributing factors for that is our event-image representation: instead of using the latest event timestamps, we use the average timestamp of the events generated at a given pixel.
The averaging helps to reduce the noise without losing the timestamp information. The main contributions of our work can be summarized as:

- The first learning-based approach to the full structure from motion using DVS input.
- A new network architecture, called ECN.
- A data representation (average time image), that improves robustness in difficult lighting conditions.
- A pre-processed MVSEC [36] dataset to allow other researchers work further on SfM with event data.

II. RELATED WORK

A. Event-based Depth Estimation

The majority of event-based depth estimation methods [27], [18], [37], [35] use two or more event cameras. As our proposed approach uses only one event camera, we focus our discussion on monocular depth estimation methods. The first works on event-based monocular depth estimation were presented in [15] and [17]. Rebocz et al. [15] used a space-sweep voting mechanism and maximization strategy to estimate semi-dense depth maps where the trajectory is known. Kim et al. [17] used probabilistic filters to jointly estimate the motion of the event camera, a 3D map of the scene, and the intensity image. More recently, Gallego et al. [11] proposed a unified framework for joint estimation of depth, motion and optical flow. So far there has been no deep learning framework to predict depths from a monocular event camera.

B. Event-based Optical Flow

Previous approaches to image motion estimation used local information in event-space. The different methods adapt in smart ways one of the three principles known from frame-based vision, namely correlation [7], [21], gradient [4] and local frequency estimation [29], [2]. The most popular approaches are gradient based and fit local planes to events [3], [25]. As discussed in [1], local event information is inherently ambiguous. To resolve the ambiguity Barranco et al. [1] proposed to collect events over a longer time intervals and compute the motion from the trace of events that contours create when moving over multiple pixels.

Recently, neural network approaches have shown promising results in various estimation problems without explicit feature engineering. Orchard and Etienne-Cummings [26] used a spiking neural network to estimate flow. Most recently, Zhu et al. [38] released the MVSEC dataset [36] and proposed self-supervised learning algorithm to estimate dense flow. Unlike [38], which uses grayscale information as a supervision signal, our proposed framework uses only events.

C. Self-supervised Structure from Motion

The unsupervised learning framework for 3D scene understanding has recently gained popularity in frame-based vision research. Zhou et. al [34] pioneered this line of work. The followed the traditional geometric modeling and built two neural networks, one for learning pose from single image frames, and one for pose from consecutive frames, which were self-supervised by aligning the frames via the flow. Follow-up works [22], [33] have used similar formulations with better loss functions and networks.

III. METHODS

A. Ego-motion Model

We assume that the camera is moving with a rigid motion with translational velocity \( v \) and rotational velocity \( \omega \), and the camera intrinsic matrix \( K \) is provided. We start with the calibrated coordinates by applying \( K^{-1} \) beforehand. Here we give a brief overview of several equations that are used in this paper [5]. Let \( X = (X, Y, Z)^T \) be the world coordinates of a point, which are related to the pixel coordinates as \( x = \frac{X}{Z}, y = \frac{Y}{Z} \). The velocity of the pixel \((x', y')\) is obtained as:

\[
X' = \frac{XZ - X'Z}{Z}, \quad Y' = \frac{YZ - Y'Z}{Z}.
\]

On the other hand, under the assumption of rigid motion, we have: \( X' = -v - \omega \times X \), where \( \times \) represents the cross product. The expanded version of the equation is:

\[
X' = -vX - \omega_0 Z + \omega_1 Y, \quad Y' = -vY - \omega_1 X + \omega_0 Y, \quad Z' = -vZ - \omega_0 Y + \omega_0 X.
\]

After basic substitutions we have:

\[
x' = -X - \omega_0 Z + \omega_1 Y, \quad y' = -Y - \omega_1 X + \omega_0 Y, \quad Z' = -Z - \omega_0 Y + \omega_0 X.
\]

Using the camera projection equations, the previous equations can be simplified into the algebraic form:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
-1 & 0 & x \\
0 & -1 & y
\end{bmatrix} \begin{bmatrix}
v_x \\
v_y
\end{bmatrix} + \begin{bmatrix}
0 & -1 & -x \\
-1 & 0 & -y
\end{bmatrix} \begin{bmatrix}
\omega_0 \\
\omega_1
\end{bmatrix} = A p (1)
\]

To put it in simpler words, for each pixel, given the inverse depth and the ego-motion velocities \( v, \omega \), we can calculate the optical flow or pixel velocity using a simple matrix multiplication (Eq. 1). Here \( p \) is the pose \( (v, \omega)^T \), \( A \) is a 2 \( \times \) 6 matrix.

B. The Pipeline

In this work we use a network with a novel encoding-decoding architecture to estimate the scaled inverse depth \( \frac{1}{\alpha} \) from a slice of event signals, rather than from normal RGB images [34]. We use another separate network to take consecutive slices of signals and predict the translational velocity \( v \) and rotational velocity \( \omega \). Under the rigid ego-motion assumption, the velocity of each pixel can be predicted from \( v, \omega \) and \( \frac{1}{\alpha} \) using simple matrix multiplication (Eq. 1). The pixel velocity, also known as optical flow is used to inversely warp the neighboring slices to the middle slice (Fig. 2). We use the \( \ell^1 \) photometric warping loss as the supervision signal.

C. Evenly Cascading Network Architecture

We use an encoder network to estimate motion pose from consecutive frames and a U-Net like encoder-decoder network [28] to estimate the scaled depth. Here, we introduces important differences to circumvent drawbacks in the standard designs.

Standard downsampling and upsampling techniques for neural networks such as pooling and transposed convolutions are limited by integer scaling factors. The networks need
A depth network (middle) with an encoder-decoder architecture is used to estimate scene depth. A pose network (right) takes consecutive frames to estimate the translational velocity and rotational velocity with respect to the middle frame. Given the poses of neighboring frames and the depth of the middle frame, we calculate the pixel velocity or optical flow. The neighboring frames are inversely warped to the middle frame and we can calculate the warping loss.

to be carefully handcrafted according to the problem size. Upscaling with transposed convolutions are also known to introduce unwanted ‘checkerboard artifacts’.

In our network, we use bilinear interpolation to resize the features, as in classic vision problems. In the encoding layers, our network evenly downscales the previous feature maps by a scaling factor \((s < 1)\) to get coarser and coarser features until the feature sizes fall below a predefined threshold. In the decoding layers, the feature maps are reversely upsampled back by scaling factor \(1/s\). Since bilinear interpolation is locally differentiable, the gradients can be easily calculated for back propagation training. The network construction is automatic and is controlled by the scaling factor.

Our transform of features is inspired by the celebrated cascade algorithm in wavelet analysis. The encoding stage [31] is analogous to the wavelet packet decomposition [6], which decomposes signals into two streams of low/high frequency coefficients. Each layer of our encoding stage contains two streams of features (Fig. 4). One stream adapts the low-level features from previous layers via residual learning [14]. The other stream generates a set of higher level features from these features. At the end of the encoding stage, the network possesses multiple levels of coarsened feature representations. Our pose prediction is made at this stage with these multi-level features. Our decoding stage is similar to the ‘merging’ operation in wavelet reconstruction. In each decoding layer, the highest level features, together with the corresponding features in the encoding layers are convolved and added back to the lower level features as modulation. At the end of the decoding stage, the network acquires a set of modulated high resolution low-level features for the final task. It is important to point out that all modulation signals are added to the lower-level features as is common in residual learning.

Our evenly-cascaded (EC) structure facilitates training by
Then we iteratively compute:

\[ f = f_1 + f_2 + \ldots + f_N \]. The leftmost pathway in Fig. 4 contains the easiest to train, lowest-level features, and is maintained throughout the whole network. Therefore this construction alleviates the vanishing gradient problem in neural network training, and allows the network to selectively enhance and utilize multiple levels of features.

\section*{D. Depth Predictions}

In the decoding stage, we make predictions from features at different resolutions and levels (Fig. 4). Initially, both high and low-level coarse features are used to predict a backbone depth map. The depth map is then upsampled with bilinear interpolation for refinement. In the middle stage, high level features as well as features in the encoding layers are merged with the low level features to serve as modulation streams. Lower level features, enhanced by the modulation streams are used to estimate the prediction residue, which are usually also low-level structures. The residue is add to the backbone estimation to refine it. The final prediction map is therefore obtained through successive up-samplings and refinements.

\section*{E. Feature Decorrelation}

Gradient descent training of neural networks can be challenging if the features are not properly conditioned. For neural networks, the feature channels are usually correlated due to the interplay of channels. The signal amplitude can also be different due to layers of transforms. Researchers have proposed normalization strategies \cite{30} to partially account for the scale inconsistency problem. We proceed one step further with a decorrelation algorithm to combat the feature collinearity problem. Straightforward decorrelation can be achieved by applying the inverse square root of the covariance matrix to the mean subtracted features. However, for neural network training, calculating inverse square roots of matrices at each iteration not only in computational expensive but introduces instability. Here we propose to apply Denman-Beavers iterations \cite{8} to decorrelate the feature channels in a simple and forward fashion. Given symmetric positive definite covariance matrix \( C \), Denman-Beavers iterations start with initial values \( Y_0 = C, Z_0 = I \). Then we iteratively compute:

\[ Y_{k+1} = \frac{1}{2} Y_k (3 I - Z_k Y_k), Z_{k+1} = \frac{1}{2} (3 I - Z_k Y_k) Z_k \]. We then have \( Z_k \rightarrow C^{-\frac{1}{2}} \) \cite{20}. In our implementation, we evenly divide the features into 16 groups as proposed in group normalization \cite{30}, and reduce the correlation between the groups by performing a few (1-10) Denman-Beavers iterations. We notice that a few iterations lead to significantly faster convergence and better results.

\section*{F. Non-local Smoothness Penalty}

To combat the sparsity in data, we utilize a sparsity constraint that promotes non-local information propagation:

\( \text{Loss}_{\text{smooth}} (I) = \sum \sum_{j \in N(i)} |I_j - I_i|^p = \sum \sum_{j \in N(i)} |I_j - I_i|^p - 2 |I_j - I_i|^2 = \sum \sum_{j \in N(i)} w_{ij} |I_j - I_i|^2. \) Here the loss is applied on the first-order derivatives of the depth estimation, and we use a sparse penalty where \( 0 < p = 1 \). The complexity of the loss is quadratic in the neighborhood size. Acceleration techniques have been applied to reduce it to \( O(1) \) \cite{32}.

\section*{G. Data Representation}

Event data consists of 3 dimensions: the pixel coordinate \( (x, y) \) and the event timestamp. In addition to that, the DAVIS camera provides event polarity - a binary value which disambiguates events which were generated on rising light intensity (positive polarity) and events generated on falling light intensity (negative polarity).

The 3D event data was projected onto a plane and converted to a 3-channel image. An example of such image can be seen on Fig. 6. The two channels of the image are the per-pixel counts of positive and negative events. The third channel is the time image used in \cite{24} - each pixel consists of the average timestamp of the events generated on this pixel. We argue that the averaging of the timestamps allows better toleration of noise, allowing our pipeline to work in low-light conditions, and handle cases of fast motion, where more recent events overwrite the previous ones.

\section*{IV. EXPERIMENTAL EVALUATION}

The main contribution of our self-supervised learning framework lies in its ability to infer both dense optical flow and depth given only the sparse event data. We evaluate our work on the MVSEC \cite{36} event camera dataset which, given a ground truth frequency of 20 Hz, contains over 40000 training images.

The MVSEC dataset is inspired by KITTI \cite{13}, \cite{23}, and it features 5 sequences of a car on the street (2 during the day and 3 during the night), as well as 4 indoor sequences shot from a flying quadrotor. MVSEC was shot in a variety of lighting conditions and features low-light and high dynamic range frames which are often challenging for an analysis with classical cameras. We find that the event-based camera allows our pipeline to perform equally well during both day and night.

For all experiments and tables, the network was trained on all frames except the \textit{outdoor day 1} sequence. During the training we did not see significant signs of overfitting, but all table entries labeled \textit{outdoor day} correspond to an unseen \textit{outdoor day} sequence. All other entries aggregate corresponding sequences.

Another important note is that the \textit{outdoor night} sequences have occasional errors in the ground truth (see for example Fig. 5 last three rows, or Fig. 8). All incorrect frames were manually removed for the evaluation.

\section*{A. Qualitative Results}

In addition to the quantitative evaluation, we present a number of samples for qualitative analysis in Fig. 5. The last three rows of the table show the night sequences, and how the pipeline performs well even when only a few events are available. The third and the fourth rows are captured indoor. The indoor sequences were relatively few and it is possible that the quality of the output would increase given a larger dataset.
Fig. 5: Qualitative results from our evaluation. The table entries from left to right: DVS input, ground truth optical flow, network output for flow, ground truth for depth, network output for depth. The event counts are overlaid in blue for better visualization. Examples were collected from sequences of the MVSEC [36] dataset: (top to bottom) outdoor day 1, outdoor day 1, indoor flying 1, indoor flying 2, outdoor night 1, outdoor night 2, outdoor night 3. It can be seen that on the ‘night’ sequences the ground truth is occasionally missing due to Lidar limitations but the pipeline performs reasonably well. Best viewed in color.

|                   | outdoor driving day | outdoor driving night | indoor flying 1 | indoor flying 2 | indoor flying 3 |
|------------------|---------------------|-----------------------|----------------|----------------|----------------|
| **AEE**          | 0.40                | 0.48                  | 0.20           | 0.25           | 0.22           |
| **% Outlier**    | 0.24                | 0.81                  | 0.01           | 0.01           | 0.01           |
| **ECN**          | 0.36                | 0.52                  | 1.1            | 0.24           | 0.21           |
| **ECNmasked**    | 0.31                | 0.46                  | 0.67           | 0.24           | 0.21           |
| **ECNrate**      | 0.49                | -                     | 1.03           | 1.72           | 1.53           |
| **SfMlearner**   | 0.53                | 0.59                  | 1.12           | 0.39           | 0.45           |
| **EV-FlowNetbest** [38] | 0.49          | -                     | 2.20           | 15.10          | 11.90          |
| **EV-FlowNet**   | 0.53                | 0.89                  | 0.59           | 0.39           | 0.39           |
| **% Outlier**    | 0.89                | 1.12                  | 0.13           | 0.39           | 0.29           |

### Table I: Evaluation of the optical flow pipeline

#### B. Quantitative Evaluation

1) **Optical Flow:** We evaluate our optical flow results in terms of Average Endpoint Error (\(AEE = \frac{1}{n} \sum \| \vec{y} - \vec{y}^* \|_2 \) with \(x\) and \(y\) the estimated and ground truth value, and \(n\) the number of events) and compare our results against the state-of-the-art optical flow method for event-based cameras: EV-FlowNet [38].

Because our network produces flow and depth values for every image pixel, our evaluation is not constrained by pixels which didn’t trigger a DVS event. Still, for consistency reasons, we report both numbers for each of our experiments (for example, **ECN** and **ECNmasked**, where the latter has errors computed only on the pixels with at least one event). In all the cases, the portion of the frame with no ground truth values was masked off. Similar to KITTI and EV-FlowNet,
we report the percentage of outliers - values with error more than 3 pixels or 5% of the flow vector magnitude.

To compare against EV-FlowNet, we account for the difference in the frame rates (EV-FlowNet uses the frame rate of the DAVIS classical frames) by downscaling our optical flow. Our results are presented in the Table I.

Our results show that the predicted flow is always very close to the ground truth. The results are typically better for the experiments with event masks except for the outdoor night. One possible explanation for that is this sequence is much noisier with events being generated not only on the edges, which leads to suboptimal masking.

We also provide the baseline results by training and evaluating the state-of-the-art SfMlearner [34] on our data, labeled SfMlearner in Table I.

2) Performance Versus Event Rate: One of our experiments was to investigate how well the neural network performs on the sparse event data in comparison to the amount of events on the scene.

Fig. 7 shows that the event rate (the number of pixels with at least one event) is inversely proportional to the error rate. That is, the more events are available to the network, the better is the quality of the predicted flow. This is an important observation since the event rate is known prior to the inference, and this property could be used for the late sensor fusion, when several systems provide optical flow.

Motivated by that, we provide an additional row to the Table I: ECN<sub>rate</sub> and report our error metrics once again only for the frames with higher than average number of event pixels across the dataset.

3) Depth Evaluation: Since there are currently no event-based methods for the depth estimation based on unsupervised learning, we provide the classical scale-invariant depth metrics, used in many works such as [10], [34], [12]:

- Accuracy: % of $y_i$ s.t. max($\frac{y_i}{\hat{y}_i}, \frac{\hat{y}_i}{y_i}$) = $\delta < rh$, SILog: $\frac{1}{n} \sum \delta_i^2 - \frac{1}{n} (\sum \delta_i)^2$, $\delta_i = \log y_i - \log \hat{y}_i^*$, Absolute Relative Difference: $\frac{1}{n} \sum |y_i - \hat{y}_i|^*, \text{Logarithmic RMSE: } \sqrt{\frac{1}{n} \sum \| \log y - \log \hat{y}^* \|^2}$.

Our results are presented in Table II for both event count-masked depth values and full, dense depth.

Applying an event mask during the evaluation increases accuracy for all scenes - this is expected, as the inference is indeed more accurate on the pixels with event data. On the contrary, the error rate increases on the outdoor scenes and decreases on the indoor scenes. This is probably due to higher variation of the outdoor scenes and also the faster motion of the cars.

4) Failure Cases: One cause of failure would be the lack of motion or relative motion, which happens often on the road. The resulting lack of events leads to the failure displayed in Fig. 8, when the static car is completely undetected. This problem can be solved by masking off the regions which have little events.

Another failure case is related to the inner workings of the pipeline itself. The internal smoothing results in blurry object boundaries, an example of which is shown in Fig. 9.

On the contrary, the pipeline seems to behave very stable in the presence of high-speed motion. On Fig. 10 the fast motion results in event counts and timestamps being overwritten by the more recent events, but the result is still close to the ground truth.

Fig. 6: A three-channel DVS data representation. The first channel represents the time image described in [24]. The second and third channels represent the per-pixel positive and negative event counts. Best viewed in color.

Fig. 7: The Average Endpoint Error (blue) and the number of pixels with at least one event (red) for the first 1500 frames of ‘outdoor day’ sequence of the MVSEC [36] dataset. Both plots are normalized so that the mean value is 0.5 for easier comparison.

Fig. 8: A common failure case: A non-moving car (visible in the middle ground truth inverse depth image) is not visible on the DAVIS camera (left image) which prevents the network to infer optical flow or depth correctly (right image is the inference inverse depth image). On the contrary, the moving car on the left side of the road is clearly visible in the event space and its depth inference is correct, but due to the Lidar limitations the depth ground truth is completely missing. This frame is taken from the ‘outdoor night 1’ MVSEC sequence.

Fig. 9: Another common failure case: the smoothing constraint causes the flow at the edges of an object to be incorrect. Left image - ground truth flow, middle image - predicted flow, right image - the per-pixel endpoint error (darker is better). The blue lines on the flow images are overlaid event counts. This frame is taken from the ‘indoor flight 2’ MVSEC sequence.

Fig. 10: A failure case where the pipeline fails to infer depth on the moving cars in a high-speed environment.
TABLE II: Evaluation of the depth estimation pipeline

|                     | Error metric | Accuracy metric |
|---------------------|--------------|-----------------|
|                     | mask | Abs Rel | RMSE log | SILog | $\delta < 1.25$ | $\delta < 1.25^2$ | $\delta < 1.25^3$ |
| outdoor driving day | -    | 0.29    | 0.34     | 0.12  | 0.80           | 0.91               | 0.96               |
| outdoor driving night | -    | 0.34    | 0.38     | 0.15  | 0.67           | 0.85               | 0.93               |
| indoor flying       | ✓    | 0.28    | 0.29     | 0.11  | 0.75           | 0.91               | 0.96               |

Fig. 10: A case of fast motion: even though the recent events overwrite older pixels, the network is still capable of predicting accurate flow. Ground truth is in the middle, the inference image is on the right. This frame is taken from the 'indoor flight 2' MVSEC sequence.

V. CONCLUSION

We have presented a novel pipeline for generating dense optical flow and depth from sparse event camera data. We also have shown experimentally that our new neural network architecture using multi-level features improves upon existing work. Future work will investigate the estimation of moving objects as part of the pipeline, using event cloud representations instead of accumulated events, and the use of space-time frequency representations in the learning.
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