Revisiting spin alignment of heavy meson in its inclusive production

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Abstract

In the heavy quark limit inclusive production rate of a heavy meson can be factorized, in which the nonperturbative effect related to the heavy meson can be characterized by matrix elements defined in the heavy quark effective theory. Using this factorization, predictions for the full spin density matrix of a spin-1 and spin-2 meson can be obtained and they are characterized only by one coefficient representing the nonperturbative effect. Predictions for spin-1 heavy meson are compared with experiment performed at $e^+e^-$ colliders in the energy range from $\sqrt{s} = 10.5\text{GeV}$ to $\sqrt{s} = 91\text{GeV}$, a complete agreement is found for $D^*$- and $B^*$- meson. For $D^{**}$ meson, our prediction suffers a large correction, as indicated by experimental data. There exists another approach by taking heavy mesons as bound systems, in which the total angular momentum of the light degrees of freedom is $\frac{1}{2}$ and $\frac{3}{2}$ for spin-1 and spin-2 meson respectively, then the diagonal parts of spin density matrices can be obtained. However, there are distinct differences in the predictions from the two approaches and they are discussed in detail.

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1. Introduction

In the heavy quark limit the momentum of a heavy meson containing one heavy quark can be approximated as the momentum of the heavy quark. This fact allows us to study properties of a heavy meson by starting from QCD directly. Activities of the study have leaded to the birth of the heavy quark effective theory (HQET) \[1,2\], with this effective theory a consistent and systematic expansion in the inverse of the heavy quark mass can be performed. HQET has been widely used in studies of decays of heavy hadrons, in this work we apply HQET for inclusive productions of a polarized heavy meson, where the heavy meson is a spin-1 or spin-2 particle.

In its inclusive production a heavy meson is formed with a heavy quark \(Q\) and with other light degrees of freedom, the light degrees can be a system of light quarks and gluons. Because its large mass \(m_Q\) the heavy quark is produced by interactions at short distance. Therefore the production can be studied with perturbative QCD. The heavy quark, once produced, will combine light degrees of freedom to form a hadron, the formation is a long-distance process, in which momentum transfers are small, hence the formed hadron will carry the most momentum of the heavy quark. The above discussion implies the production rate can be factorized, in which the perturbative part is for the production of a heavy quark, while the nonperturbative part is for the formation. For the nonperturbative part a expansion in the inverse of \(m_Q\) can systematically be performed in the framework of HQET. This type of the factorization was firstly used in parton fragmentation into a heavy hadron \[3\].

In this work we will use the factorization to predict the spin density matrix of a spin-1- or spin-2 heavy meson in its inclusive production. It turns out that the matrix is determined by perturbative coefficients and by one nonperturbative parameter, which is a ratio of matrix elements defined in HQET. The predictions are compared with experiment performed at \(e^+e^-\) colliders in the energy range from \(\sqrt{s} = 10.5\,\text{GeV}\) to \(\sqrt{s} = 91\,\text{GeV}\), a good agreement is found for \(D^*\)- and \(B^*\)-meson.

The diagonal part of spin density matrices have been studied before \[4\], in which the total angular momentum \(j\) of the light degrees of freedom in the heavy meson is taken as \(\frac{1}{2}\) and \(\frac{3}{2}\) for the spin-1- and spin-2 meson, respectively, i.e., \(j\) takes the smallest of possible values. It should be noted that this property of a heavy meson has been not derived from QCD, because solutions of eigenvalue problem of QCD hamiltonian in the heavy quark limit are not known. This property may be argued by that the system of light degrees of freedom with larger value of \(j\) is produced with a suppressed probability. For \(B^*\) this property is experimentally supported by observing the ratio of the production rates of spin-0-and spin-1 states, while for charm mesons the ratio has a 20% deviation from the prediction based on the property. In our approach we will not take the property into account. In other word our results are derived on a more general ground and corrections from higher orders in \(m_Q^{-1}\) can be systematically added. It is interesting to note that the obtained predictions of the spin density matrix of \(D^*\) also agree with experiment well, in which one expects large corrections form higher orders of \(m_c^{-1}\). For a spin-2 heavy meson a parameter is introduced for its production in the approach in \[4\], and this is the only parameter characterizing the angular distribution of its decay. In our approach the corresponding distribution is predicted as an isotropic one. Experimentally this distribution is measured with a limited data sample for \(D^{**}\). Although the distribution is not well determined in experiment, the measured distribution does not agree with our prediction, indicating that the correction to our prediction made in the
heavy quark limit is substantial, this is also theoretically expected, because the binding energy of
$D^{**}$ is quite large. We will discuss in detail the difference between the two approaches.

In this work we consider the inclusive production at $e^+e^-$ colliders, where the initial beams are
unpolarized. Our results can be generalized to the case with polarized beams, and also to inclusive
production at hadron colliders or at $ep$-colliders. It is straightforward to obtain predictions for
other inclusive predictions. Our work is organized as the following: In Sect.2 we give our result for
spin-1 meson and compare our predictions with experiment. The differences between our approach
and that in [4] are discussed in detail. In Sect.3 we present results for spin-2 meson. Sect.4 is our
summary. Throughout this work we take nonrelativistic normalization for heavy quarks and for
heavy mesons.

2. The density matrix for spin-1 heavy meson

We denote a spin-1 heavy meson as $H^*$, which contains one heavy quark $Q$. We study the
inclusive production

$$e^+(p) + e^-(p) \rightarrow H^*(k) + X,$$

where the three momenta are given in the brackets. In the process we assume that the initial
beams are unpolarized. We denote the helicity of $H^*$ as $\lambda$ and $\lambda = -1, 0, 1$. All information
about the polarization of $H^*$ is contained in a spin density matrix, which may be unnormalized or
normalized, we will call them unnormalized or normalized spin density matrix, respectively. The
unnormalized spin density matrix can be defined as

$$R(\lambda, \lambda', p, k) = \sum_X \langle H^*(\lambda)X|T|e^+e^- \rangle \cdot \langle H^*(\lambda')X|T|e^+e^- \rangle^*,$$

where the conservation of the total energy-momentum and the spin average of the initial state is
implied. $T$ is the transition operator. The cross-section with a given helicity $\lambda$ is given by:

$$\sigma(\lambda) = \frac{1}{2s} \int \frac{d^3k}{(2\pi)^3} R(\lambda, \lambda, p, k).$$

From Eq.(2) the normalized spin density matrix is defined by

$$\rho_{\lambda\lambda'}(p, k) = \frac{R(\lambda, \lambda', p, k)}{\sum_\lambda R(\lambda, \lambda, p, k)}.$$  

It should be noted that the normalized spin density matrix is measured in experiment.

The contribution to $R(\lambda, \lambda', p, k)$ can be characterized by Feynman diagrams. With the fac-
torization discussed in the introduction, each diagram can be divided into two parts. For the
process in Eq.(1) the contribution at tree-level is given in Fig.1, where we divide the diagram
with a horizontal broken line into two parts. The upper part with the black box represents the
formation, while the lower part represents the inclusive production of a heavy quark $Q$, which is
not necessarily on-shell here. At loop-level it is possible that there are gluon exchanges between
quark lines and between the lower part and the black box. The effect of the exchanges between the lower part and the black box is at higher orders in $m_Q^{-1}$, we will neglect it in this work. The exchanges of gluons between quark lines will in general bring some infrared singularities in the lower part. We can show that at the leading order of $m_Q^{-1}$ the heavy quark lines connecting the lower- and upper part represent on-shell quarks. Hence, the lower part with possible gluon exchanges is responsible for inclusive production of an on-shell quark $Q$, and it is free from infrared singularities.

**Fig.1**: The diagram for the contribution at tree-level. The line is for heavy quarks. The vertical broken line is the Cutkosky cut.

With the diagram the density matrix can be written as
\begin{equation}
R(\lambda, \lambda', p, k) = \int \frac{d^4q}{(2\pi)^4} T_{ji}(q) \Gamma_{ij}(\lambda, \lambda', q, k),
\end{equation}

where $T_{ji}(q)$ is for the lower part and can be calculated with perturbative theory, $\Gamma_{ij}(\lambda, \lambda', q, k)$ represents the formation of the heavy meson, which is defined as:

\begin{equation}
\Gamma_{ij}(\lambda, \lambda', q, k) = \int d^4xe^{-ix \cdot q} \sum_X \langle \bar{h}_v(0) | H^*(\lambda') X \rangle \langle X H^*(\lambda) | \bar{h}_v(0) \rangle.
\end{equation}

The indices $i, j$ stand for Dirac- and color indices, they connect the lower- and the upper part through the vertical quark lines. $Q(x)$ is the Dirac field for the heavy quark $Q$. In the heavy quark limit the heavy meson carries the most momentum of the heavy quark, the difference between the momentum of the heavy quark and that of the heavy meson is at order of $\Lambda_{QCD}$. This scale $\Lambda_{QCD}$ is much smaller than $m_Q$ and $\sqrt{s}$, which are scales appearing in the lower part of Fig.1. This fact indicates that we can expand $\Gamma_{ij}(\lambda, \lambda', q, k)$ in $\Lambda_{QCD}$. With this expansion for $\Gamma_{ij}(\lambda, \lambda', q, k)$ it results in that the density matrix given in Eq.(6) is expanded in $m_Q^{-1}$. For doing this we use HQET, in which the Dirac field is expanded in the field of HQET:

\begin{equation}
Q(x) = e^{-im_Qv \cdot x} \left\{ 1 + \frac{1}{2m_Q} i \gamma \cdot D_T \right\} h_v(x) + O\left( \frac{1}{m_Q^2} \right)
+ (\text{terms for anti-quark}),
\end{equation}

where $v$ is the velocity of $H^*$, $h_v(x)$ is the heavy quark field in HQET, $D_T^\mu = D^\mu - v \cdot Dv^\mu$ and $D^\mu$ is the covariant derivative. Substituting this into $\Gamma_{ij}(\lambda, \lambda', q, k)$, the integrand depends on $x$ through $\bar{h}_v(x)$ and through a exponential factor, the $x$-dependence through $h_v(x)$ is controlled by the scale at order of $\Lambda_{QCD}$, at leading order this dependence can be neglected. We then have:

\begin{equation}
\Gamma_{ij}(\lambda, \lambda', q, k) = (2\pi)^4 \delta^4(q - m_Qv) \sum_X \langle \bar{h}_v(0) | H^*(\lambda') X \rangle \langle X H^*(\lambda) | \bar{h}_v(0) \rangle |0\rangle + \cdots,
\end{equation}

where the $\cdots$ stand for neglected terms, whose contributions to the density matrix are suppressed by powers of $m_Q^{-1}$. With a detailed examination one can show these contributions are of order of $m_Q^{-2}$ or of higher orders. At the leading order the momentum of $H^*$ is approximated by $m_Qv$. To analyze the structure of $\Gamma_{ij}$ in the above equation it is convenient to work with the rest frame of $H^*$, which is obtained from the moving frame only by a Lorentz boost. We choose the $z$-direction to be the moving direction of $H^*$ and denote the polarization vector of $H^*$ in the rest-frame as $\epsilon(\lambda)$. In this frame we can define a creation operator for $H^*$:

\begin{equation}
|H^*(\lambda)\rangle = a^\dagger(\lambda) |0\rangle = \epsilon(\lambda) \cdot a^\dagger |0\rangle.
\end{equation}

In the rest frame the field $h_v$ has two non zero components and can be written as:

\begin{equation}
h_v(x) = \begin{pmatrix} \psi(x) \\ 0 \end{pmatrix}.
\end{equation}

With this notation the matrix in Eq.(8) takes the form:
The matrix element \( \langle 0|\psi a^\dagger(\lambda')a(\lambda)\psi^\dagger|0 \rangle \) is diagonal in color space, the matrix structure in spin space can be decomposed into the unit matrix and the Pauli matrices \( \sigma \). For the decomposition we define two operators:

\[
O(H^*) = \frac{1}{6}\text{Tr}\psi a_i^\dagger a_i \psi^\dagger, \quad O_s(H^*) = \frac{i}{12}\text{Tr}\sigma_i \psi a_i^\dagger a_k \psi^\dagger \varepsilon_{ijk},
\]

where \( \varepsilon_{ijk} \) is the totally antisymmetric tensor. With these operators the matrix can be written:

\[
\langle 0|\psi a^\dagger(\lambda')a(\lambda)\psi^\dagger|0 \rangle = \frac{1}{3}\langle 0|O(H^*)|0 \rangle \epsilon^*(\lambda) \cdot \epsilon(\lambda') \cdot I + \frac{i}{3} \sigma \cdot [\epsilon^*(\lambda) \times \epsilon(\lambda')] \cdot \langle 0|O_s(H^*)|0 \rangle,
\]

where \( I \) is a \( 2 \times 2 \) unit matrix in the spin space. It should be noted that the factor \( (1 + \gamma \cdot v)/2 \) in Eq.(11) is a projector and can be written as \( \sum_s u(p,s)\bar{u}(p,s) \), where \( u(p,s) \) is the spinor of the heavy quark \( Q \) in the rest frame. This means that the projector projects the quark state represented by the vertical quark lines in Fig.1 to the on-shell state. Therefore, the lower part is just for inclusive production of a on-shell heavy quark \( Q \). The two matrix elements \( \langle 0|O(H^*)|0 \rangle \) and \( \langle 0|O_s(H^*)|0 \rangle \) are universal in the sense that they do not depend on how \( Q \) is produced, they describe the long-distance process of the formation of \( H^* \). Using Eq.(8), Eq.(11) and Eq.(13) we can write the result for the unnormalized spin density matrix as:

\[
R(\lambda, \lambda', p, k) = \frac{1}{3}a(p,k)\langle 0|O(H^*)|0 \rangle \epsilon^*(\lambda) \cdot \epsilon(\lambda') + \frac{i}{3}b(p,k) \cdot [\epsilon^*(\lambda) \times \epsilon(\lambda')] \cdot \langle 0|O_s(H^*)|0 \rangle.
\]

The quantities \( a(p,k) \) and \( b(p,k) \) characterize the spin density matrix of the heavy quark \( Q \) produced in the inclusive process:

\[
e^+(p) + e^-(p) \rightarrow Q(k,s) + X
\]

where \( s \) is the spin vector of \( Q \) in its rest frame and the rest frame is related to the moving frame only by a Lorentz boost. The unnormalized spin density matrix \( R_Q(s,p,k) \) of \( Q \) can be defined by replacing \( H^*(\lambda) \) with \( Q(k,s) \) in Eq.(2). This matrix can be calculated with perturbative theory because of the heavy mass. The result in general takes the form

\[
R_Q(s,p,k) = a(p,k) + b(p,k) \cdot s
\]

where \( a(p,k) \) and \( b(p,k) \) are the same in Eq.(14). The results at tree-level for \( a(p,k) \) and \( b(p,k) \) may be found in [4]. With Eq.(14) we obtain the normalized spin density matrix:

\[
\rho(p,k) = \frac{1}{3} \begin{pmatrix} 1 + P_3, & -P_+ & 0 \\ -P_+ & 1 & -P_+ \\ 0 & -P_+ & 1 - P_3 \end{pmatrix},
\]
with

\[ P_3 = \frac{b_3(p, k)}{a(p, k)} \cdot \frac{\langle 0|O_s(H^*)|0 \rangle}{\langle 0|O(H^*)|0 \rangle}, \quad P_\pm = \frac{b_1(p, k) \pm ib_2(p, k)}{\sqrt{2}a(p, k)} \cdot \frac{\langle 0|O_s(H^*)|0 \rangle}{\langle 0|O(H^*)|0 \rangle}, \]

The indices of matrices in Eq.(17) run from -1 to 1.

Before confronting experimental results we give the following comments to our results:

1). If one uses perturbative theory to calculate the coefficients in Eq.(14) or Eq.(16), one will encounter terms with large logarithm \( \ln(m_Q/\sqrt{s}) \). One needs to resume these terms. One way to resume is to use the well known factorization formula for the process in Eq.(1), where the heavy meson \( H^* \) is produced through parton fragmentation. The definitions of the spin-dependent fragmentation functions can be found in [12]. Then one applies the approach here to the fragmentation functions as used in [3]. These functions can be calculated at the energy scale \( \mu = m_Q \) with perturbative QCD. Through renormalization group equations one obtains the functions and then the coefficients at the energy scale \( \mu = \sqrt{s} \). Before the resummation the coefficients at tree-level are singular in the energy of \( H^* \), but the ratio \( P_3 \) and \( P_\pm \) are regular. After the resummation is performed, the coefficients become regular.

2). If the parity is conserved and T-odd effects can be neglected, which are proportional to \( m_Q/\sqrt{s} \), then \( b(p, k) = 0 \), i.e., the heavy quark is unpolarized, it leads to that \( H^* \) is unpolarized too. We obtain in this case \( \rho_{00} = \rho_{-1-1} = \rho_{11} = 1/3 \), and all nondiagonal elements of the spin density matrix are zero. It should be noted that in general the nondiagonal elements can be nonzero even if the parity is conserved, e.g., a light vector meson can have tensor-polarization, which partly corresponds to the matrix element \( \rho_{-\lambda\lambda} \) for \( \lambda = \pm 1 \). Unfortunately, the spin of \( H^* \) is analyzed in experiment with its parity-conserving two-body decay, where the polarization of the decay products is not observed. This leads to that the polarization of the heavy quark, i.e., the contributions from \( b(p, k) \) to the spin matrix element, can not be determined in experiment, this is the so-called ”no-win” theorem [1].

3). Our approach can be used not only for the process in Eq.(1), but also for other processes. For other inclusive productions the same analysis can be done, where one needs to replace the lower part in Fig.1 and \( T_{ji}(q) \) in Eq.(5) with those for the corresponding production of \( Q \), while the upper part in Fig.1 and \( \Gamma_{ij} \) in Eq.(5) remain the same. With the analysis presented here it is straightforward to obtain predictions for other inclusive productions, and predictions always takes the form as given in Eq.(14) or in Eq.(17), where the coefficients \( a(p, k) \) and \( b(p, k) \) should be replaced by those characterizing the spin density matrix of \( Q \) in the corresponding processes. Without knowing these coefficients we can always conclude that in any inclusive production of \( H^* \) we have \( \rho_{00} = 1/3 \) and \( \rho_{-11} = \rho_{1-1} = 0 \) in the heavy quark limit. These predictions may be tested in other processes, e.g., in inclusive production at electron-proton colliders or at hadron-hadron colliders. It is to the author’s knowledge that the spin-measurements for \( B^*, D^* \) and \( D^{**} \) have been done only at \( e^+e^- \)-colliders by those experimental groups, whose results are listed in this work. The list may be incomplete. It would be interesting to test our predictions in experiment at an electron-proton- or a hadron-hadron collider.

The experiments to measure the polarization of \( B^* \) are performed at LEP with \( \sqrt{s} = M_Z \) by different experiments groups. To measure the polarization the dominant decay \( B^* \to \gamma B \) is used, where the polarization of the photon is not observed. Because the parity is conserved and the
distribution of the angle between the moving directions of $\gamma$ and of $B^*$ is measured, one can only determine the matrix element $\rho_{00}$. If we denote $\theta$ is the angle between the moving directions of $B^*$-rest frame and $\phi$ is the azimuthal angle of $\gamma$, then the angular distribution is given by $W_{B^* \rightarrow B \gamma}(\theta, \phi) \propto \sum_{\lambda \lambda'} \rho_{\lambda \lambda'} (\delta_{\lambda \lambda'} - Y_{1 \lambda}(\theta, \phi) Y_{1 \lambda}^*(\theta, \phi))$. Integrating over $\phi$ and using our result $\rho_{00} = 1/3$, the distribution of $\theta$ is isotropic. In experiment one indeed finds that the distribution is isotropic in $\theta$. The experimental results at $\sqrt{s} = M_Z$ are:

- $\rho_{00} = 0.32 \pm 0.04 \pm 0.03$, DELPHI [8]
- $\rho_{00} = 0.33 \pm 0.06 \pm 0.05$, ALEPH [9]
- $\rho_{00} = 0.36 \pm 0.06 \pm 0.07$, OPAL [10].

These results agree well with our prediction $\rho_{00} = 1/3$. It is interesting to note that the spin alignment is also studied with the LUND string fragmentation model [11] implemented with JETSET [12] in [13], and the result $\rho_{00} = 0.567$ is obtained, which is not in agreement with experiment. In [4] the $B^*$- and $B$ meson is taken as a bound state of a $b$-quark and other light degrees of freedom, and these light degrees have the total angular momentum $j = 1/2$. Because the parity is conserved in the formation of $B^*$, the light degrees are produced with equal probability for $j_3 = \pm 1/2$. With this argument one obtains the probability for a left-handed $b$-quark to form a $B^*$- and $B$ meson is:

$$P(\bar{B}^*(\lambda = -1)) : P(\bar{B}^*(\lambda = 0)) : P(\bar{B}^*(\lambda = 1)) : P(B) = \frac{1}{2} : \frac{1}{4} : 0 : \frac{1}{4}. \quad (20)$$

One then also obtains $\rho_{00} = 1/3$ in agreement with experiment and with our results. One can also obtains the ratio of the production rates

$$P_V = \frac{\sigma(B^*)}{\sigma(B^*) + \sigma(B)} = \frac{3}{4}, \quad (21)$$

this prediction is well tested with experiment:

- $P_V = 0.72 \pm 0.03 \pm 0.06$, DELPHI [8]
- $P_V = 0.771 \pm 0.026 \pm 0.07$, ALEPH [9].

However the prediction of the ratio for $c$-flavored mesons has a deviation at 20% level. It should be noted that in [11] only the diagonal part of the density matrix is predicted, while in our work the predictions are given for the complete matrix.

The polarization measurement for $D^*$-meson has been done with different $\sqrt{s}$, in some experiments the non-diagonal part of the spin density matrix has also been measured by measuring azimuthal angular distribution in $D^*$ decay, where the decay mode into two pseudo-scalars, i.e., $D^* \rightarrow D \pi$, is used. Denoting $\theta$ as the angle between the moving directions of $D^*$ and of $\pi$ in the $D^*$-rest frame and $\phi$ as the azimuthal angle of $\pi$, then the angular distribution of $\pi$ is given by $W_{D^* \rightarrow D \pi}(\theta, \phi) \propto \sum_{\lambda \lambda'} \rho_{\lambda \lambda'} Y_{1 \lambda}(\theta, \phi) Y_{1 \lambda}^*(\theta, \phi)$. Integrating over $\phi$ and using our result $\rho_{00} = 1/3$, the distribution of $\theta$ is again isotropic. The experimental results are summarized in Table 1 and also partly summarized in [14].
Table 1. Experimental Results for $D^*$

| Collaboration | $\sqrt{s}$ in GeV | Results               |
|--------------|-------------------|----------------------|
| CLEO [15]   | 10.5              | $\rho_{00} = 0.327 \pm 0.006$ |
| HRS [16]    | 29                | $\rho_{00} = 0.371 \pm 0.016$  
|              |                    | $\rho_{1-1} = 0.04 \pm 0.03$  
|              |                    | $\rho_{10} = 0.00 \pm 0.01$    |
| TPC [17]    | 29                | $\rho_{00} = 0.301 \pm 0.042 \pm 0.007$  
|              |                    | $\rho_{1-1} = 0.01 \pm 0.03 \pm 0.00$  
|              |                    | $\rho_{10} = 0.03 \pm 0.03 \pm 0.00$    |
| SLD [18]    | 91                | $\rho_{00} = 0.34 \pm 0.08 \pm 0.13$  
| OPAL [10]   | 91                | $\rho_{00} = 0.40 \pm 0.02 \pm 0.01$  
|              |                    | $\rho_{1-1} = -0.039 \pm 0.014$       |

From Table 1, we can see that the $\rho_{00}$ measured by all experimental groups is close to the prediction $\rho_{00} = \frac{1}{3}$, the most precise result is obtained by CLEO, its deviation from the prediction is 2%, the largest deviation of the prediction is from the result made by OPAL at $\sqrt{s} = 90$GeV, it is 20%. In general, $\rho_{00}$ depends on the energy of $H^*$. Our results in Eq.(17) give that $\rho_{00}$ is a constant in the heavy quark limit, or the energy dependence is suppressed by $m_Q^{-2}$. In experiment only a very weak energy dependence is observed, e.g., in CLEO results [15]. From our results $\rho_{1-1}$ is exactly zero in the heavy quark limit, the results from TPC and from HRS are in consistent with our result, a non zero value is obtained by OPAL, which has a 3$\sigma$ deviation from zero. These deviations may be explained with effects of higher orders in $m_c^{-1}$, these effects are expected to be substantial, because $m_c$ is not so large. It is interesting to note only results from OPAL at $\sqrt{s} = 91$GeV have the largest deviations from our predictions, while results from other groups agree well with our predictions. At $\sqrt{s} = 10.5$GeV or 29GeV, the effect of the Z-boson exchange can be neglected, hence the parity is conserved. We obtain $\rho_{10} = 0$. This prediction is also in agreement with the experimental result made by TPC and by HRS.

Since our results are derived without knowing the total angular momentum $j$ of the light degrees of freedom in the heavy meson, the agreement of our results with experiment can not be used to extract the information about $j$ from the experimental data in Eq.(19) and in Table 1., although $\rho_{00} = 1/3$ can also be obtained by taking $j = 1/2$. One way to extract $j$ may be to measure the difference between $\rho_{11} - \rho_{-1-1}$, but it seems not possible, because the polarization of $H^*$ is measured through its parity-conserved decay and the polarization of decay products is not observed in experiment. In the heavy quark limit, the nondiagonal element $\rho_{1-1}$ and $\rho_{-11}$ are zero, while the other nondiagonal matrix elements are nonzero if the parity is not conserved and the initial state is unpolarized. At higher orders in $m_Q^{-1}$ this can be changed, e.g., $H^*$ can have tensor polarization.

3. The spin density matrix for spin-2 heavy meson

In this section we consider a spin-2 heavy meson $H^\ast\ast$, which contains one heavy quark $Q$. We
We study the inclusive production
\[ e^+(p) + e^-(−p) → H^{**}(k) + X. \]

With the detailed analysis in the last section it is straightforward to obtain the spin density matrix. To avoid introducing too many notations, we will use some notations used in the last section. The used notations in this section are referred to the matrix. To avoid introducing too many notations, we will use some notations used in the last section. The unnormalized spin density matrix \( R(λ, λ', p, k) \), which is defined by replacing \( H^{*} \) with \( H^{**} \) in Eq.(2). The helicity \( λ \) takes the value from -2 to 2. We work in the \( H^{**} \)-rest frame, which is related to the moving frame only through a Lorentz boost. The \( z \)-direction is chosen as the moving direction of \( H^{**} \). In the rest frame the polarization tensor of \( H^{**} \) is \( ε_{ij}(λ) \), it has the following properties:
\[
ε_{ij}(λ) = ε_{ji}(λ), \quad ε_{ii}(λ) = 0, \quad ∑_λ ε_{ij}(λ)ε_{kl}(λ) = \frac{1}{2}(δ_{ik}δ_{jl} + δ_{il}δ_{jk}) - \frac{1}{3}δ_{ij}δ_{kl}. \]

We introduce a creation operator for \( H^{**} \) in its rest-frame:
\[
|H^{**}(λ)\rangle = ε_{ij}(λ)a_{ij}^\dagger|0\rangle, \quad \quad \quad (25)
\]
where \( a_{ij}^\dagger \) is symmetric and trace-less. In analogy to Eq.(12) for the spin-1 case two operators can be defined:
\[
O(H^{**}) = \frac{1}{6}Trψa_{ij}^\dagger a_{ij}ψ^\dagger, \quad O_s(H^{**}) = \frac{i}{12}Trσ_3ψa_{ij}^\dagger a_{ik}ψ^\daggerε_{ijk}. \quad (26)
\]
With these operators the matrix for \( H^{**} \) corresponding to that for \( H^{*} \) in Eq. (13) can be written:
\[
∑_X ⟨0|ψ|H^{**}(λ')X⟩⟨XH^{**}(λ)|ψ^\dagger|0⟩ = \frac{1}{5}⟨0|O(H^{**})|0⟩ε_{ij}^*(λ)ε_{ij}(λ') · I + \frac{i}{5}⟨0|O_s(H^{**})|0⟩ε_{ij}^*(λ)ε_{kl}(λ')ε_{ijl} · σ_i. \quad (27)
\]
the matrix is diagonal in color space. Predictions can be obtained by replacing the matrix element in r.h.s. of Eq.(11) with that given above and perform the same calculation as that in the last section. The unnormalized spin density matrix for \( H^{**} \) then reads:
\[
R(λ, λ', p, k) = \frac{1}{5}a(p, k)⟨0|O(H^{**})|0⟩ε_{ij}^*(λ)ε_{ij}(λ') + \frac{i}{5}b_i(p, k)⟨0|O_s(H^{**})|0⟩ε_{jk}^*(λ)ε_{kl}(λ')ε_{ijl}, \quad (28)
\]
where the coefficients \( a(p, k) \) and \( b_i(p, k) \) for \( i = 1, 2, 3 \) are the same as those defined in Eq.(16). With this matrix we obtain the normalized spin density matrix for \( H^{**} \):
\[
ρ(p, k) = \frac{1}{5}
\begin{pmatrix}
1 + P_3, & \frac{-1}{√2}P_+ & 0, & 0, & 0 \\
\frac{-1}{√2}P_-, & 1 + \frac{1}{2}P_3, & \frac{-√3}{2}P_+, & 0, & 0 \\
0, & \frac{-√3}{2}P_-, & 1, & \frac{-√3}{2}P_+, & 0 \\
0, & 0, & \frac{-√3}{2}P_-, & 1 - \frac{1}{2}P_3, & \frac{-1}{√2}P_+ \\
0, & 0, & 0, & \frac{-√2}{2}P_-, & 1 - P_3
\end{pmatrix}, \quad (29)
\]
with
\[ P_3 = \frac{b_3(p, k)}{a(p, k)} \cdot \frac{\langle 0|O_s(H^{**})|0 \rangle}{\langle 0|O(H^{**})|0 \rangle}, \quad P_\pm = \frac{b_1(p, k) \pm ib_2(p, k)}{\sqrt{2}a(p, k)} \cdot \frac{\langle 0|O_s(H^{**})|0 \rangle}{\langle 0|O(H^{**})|0 \rangle}, \]
(30)

The indices of matrices in Eq.(29) run from -2 to 2. The results are similar to those for the spin-1 case. The correction to the predictions in Eq.(29) is of order of \( m_Q^{-2} \) or of higher orders. The same comments made in the last section for spin-1 heavy meson also apply here.

It would be interesting to compare the predictions with experiment. Unfortunately, there is no experimental data for \( B^{**} \) meson, there is a candidate \( B^*_J(5732) \) observed in experiment, but its spin is not determined. For c-flavored meson, a spin-2 meson noted as \( D^*_2(2460) \) is observed, and the information of its spin is analyzed by ARGUS \[19\] with a limited data sample. Through the angular distribution of the decay \( D^*_2 \to D\pi \), it is found that there is no significant population of \( |\lambda| = 2 \) states. For the decay there is one form-factor in the decay amplitude, the angular distribution can then be written as:
\[ \frac{d\Gamma}{\Gamma d\phi d\cos \theta}(D^*_2 \to D\pi) \sim \sum_{\lambda\lambda'} \rho_{\lambda\lambda} Y_{2\lambda}(\theta, \phi) Y^*_{2\lambda'}(\theta, \phi), \]
(31)

where \( \theta \) is the angle between the moving directions of \( D^*_2 \) and of \( \pi \) in the \( D^*_2 \)-rest frame, \( \phi \) is the azimuthal angle of \( \pi \). With our results we obtain:
\[ \frac{d\Gamma}{\Gamma d\cos \theta}(D^*_2 \to D\pi) = \frac{1}{2}, \]
(32)
i.e., the distribution is isotropic. In the approach in \[4\] the distribution is predicted as:
\[ \frac{d\Gamma}{\Gamma d\cos \theta}(D^*_2 \to D\pi) = \frac{1}{4} \left[ 1 + 3 \cos^2 \theta - 6 \omega_{3/2} (3 \cos^2 \theta - 1) \right], \]
(33)
where an unknown parameter \( \omega_{3/2} \) is introduced, this parameter is interpreted as the probability to produce a system of light degrees of freedom with the total angular momentum \( j = 3/2 \) and \( j_3 = 3/2 \), and this system is combined with a heavy quark \( Q \) to form \( H^{**} \). ARGUS has found that the data favored a small \( \omega_{3/2} \), indicating that a small portion of \( |\lambda| = 2 \) states is produced. A re-analysis of the data is given in \[4\]. Assuming \( \omega_{3/2} > 0 \), it is found \[4\]:
\[ \omega_{3/2} < 0.24, \quad 90\% \text{ C.L.} \]
(34)

A model estimation also gives a small \( \omega_{3/2} \) \[20\]. With our prediction we obtain that the distribution is isotropic. This leads to \( \omega_{3/2} = 1/2 \). It seems that experimental results are against our predictions. However, one should note that the correction from higher orders of \( m_Q^{-1} \) can be more significant than that in the case of \( D^* \), because the excited state \( D^*_2 \) is heavier than \( D^* \). The effects of higher orders may be estimated by the mass difference:
\[ \frac{M(D^*_2) - M(D_0)}{M(D_0)} \approx 0.34, \]
(35)
this indicates that the large correction of our prediction $\omega_{3/2} = 1/2$ exists. An extension of our present work to the next-to-leading order in $m_c^{-1}$ is needed to analyze this effect, including effects from higher orders the distribution will be not isotropic. At leading order of the expansion in $m_Q^{-1}$ we obtain the relation $\rho_{\lambda\lambda} + \rho_{-\lambda-\lambda} = 2/5$, this leads to that the distribution in Eq.(32) or (33) is isotropic. If we add corrections at higher orders of $m_Q^{-1}$, the relation will be changed, the coefficient $2/5$ receives corrections which depend on $\lambda$. This will change the distribution as predicted in Eq.(32) and it becomes non-isotropic. It needs to be study further. For $B^{**}$ meson the correction is expected to be small because $m_b$ is large, hence the predictions are more accurate. It would be interesting to test our prediction in future experiment with $B^{**}$.

4. Summary

In this work we have studied the polarization of a heavy meson in its inclusive production, where the heavy meson contains one heavy quark. A QCD factorization is performed for the spin density matrix of a spin-1-and spin-2 meson. In the heavy quark limit, the spin density matrix takes a factorized form, in which the nonperturbative effect related to the heavy meson is represented by two matrix elements defined in HQET, while the perturbative part is related to the spin density matrix of the heavy quark in its inclusive production. Hence, a detailed relation between the spin density matrices of the heavy meson and of the heavy quark is established.

Without the knowledge of the two matrix elements defined in HQET, some elements of the spin density matrix can already be predicted. These elements are well measured in experiment. We find that the measured element for $B^*$-meson at LEP with $\sqrt{s} = 91$GeV agrees well with our prediction. For $D^*$-meson the polarization is measured at $e^+e^-$ colliders with $\sqrt{s} = 10.5, 29$ and 91GeV, the experimental results also agree with our predictions, except a small deviation with OPAL data, the deviation may be due to corrections from higher orders in $m_c^{-1}$. In general one expects these corrections are large because $m_c$ is not so large, the agreement for $D^*$-meson indicates that HQET can be well used to predict the polarization of $D^*$-meson. For $D^{**}$ meson not many experimental data are available. The results of a measurement with large errors does not agree with our result. The disagreement may be explained with large corrections from higher orders in $m_c^{-1}$, these large corrections for $D^{**}$ meson are more likely than those for $D^*$ meson, because the binding energy of $D^{**}$ is larger than that of $D^*$ meson.

The polarization of a heavy meson in its inclusive production has also been studied in $[4]$, where one takes the total angular momentum of light degrees of freedom in the heavy meson to be the lowest of the possible values. With this the diagonal part of the spin density matrix can be predicted. In our work we only use the QCD factorization concept by employing the expansion in the inverse of the heavy quark mass, and we obtain results not only for the diagonal part, but also for the nondiagonal part of the spin density matrix. For the spin-1 case we obtain the same matrix element $\rho_{00}$, while for the spin-2 case, our results are different than those in $[4]$, it leads to that the prediction for the angular distribution of the decay $D^{**} \rightarrow D\pi$ is different than that predicted in $[4]$. However, large corrections to our prediction are likely possible, it will be interesting to test predictions for $B^{**}$ meson in future experiments, because corrections in the case with $B^{**}$ are expected to be small.
Although we have given in this work detailed predictions for inclusive production of a polarized heavy meson at a $e^+e^-$ collider, our approach can be easily generalized to other inclusive productions, testable predictions can be made without a detailed calculation, for example, in inclusive production of $B^*$ at an electron-hadron- or a hadron-hadron collider we always have the prediction $\rho_{00} = 1/3$ and $\rho_{-11} = \rho_{1-1} = 0$ in the heavy quark limit.

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