Statistical Evidence Against Simple Forms of Wavefunction Collapse

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2011 January 4

Abstract

If the initial quantum state of the universe is a multiverse superposition over many different sets of values of the effective coupling ‘constants’ of physics, and if this quantum state collapses to an eigenstate of the set of coupling ‘constants’ with a probability purely proportional to the absolute square of the amplitude (with no additional factor for something like life or consciousness), then one should not expect that the coupling ‘constants’ would be so biophilic as they are observed to be. Therefore, the observed biophilic values (apparent fine tuning) of the coupling ‘constants’ is statistical evidence against such simple forms of wavefunction collapse.

*Alberta-Thy-01-11, arXiv:yymm.nnnn [hep-th]
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1 Introduction

Two alternative types of quantum theory are Copenhagen-type versions in which the wavefunction is said to collapse at measurements, and Everett-type versions in which the wavefunction never collapses. Often it is believed that these two types are merely matters of interpretation that have no observational consequences. I have previously demonstrated that this is not necessarily so [1, 2, 3]. Now I further show that the biophilic fine-tuning observed for the ‘constants’ of nature in our part of the universe is statistical evidence against certain simple collapse versions of quantum theory in an initial quantum state that is a multiverse superposition over many different sets of the effective coupling ‘constants’ of physics.

Consider multiverse theories (such as what one might expect from a string/M theory landscape [4, 5, 6]) in which the quantum state gives a superposition of different possible ‘constants’ of physics. In Everett versions all components of the quantum state would have real existence, and the normalized measure (‘probability’) for an observation would be given by the quantum amplitudes and some as-yet-undetermined rule for getting the measure from the quantum state (perhaps as a normalized expectation value of a positive operator, though I have shown it cannot be a projection operator as one might have expected from the simplest interpretation of Born’s rule [7, 8, 9, 10, 11]). For a suitable rule, one would expect that the measure would be weighted toward components of the quantum state in which there are more observers, or a higher density of observers, favoring components in which the ‘constants’ have biophilic values, which is indeed what we observe.

However, for simple collapse versions of quantum theory, one would expect the probability of any result from the collapse of the wavefunction to depend purely on the absolute square of the amplitude. There is no reason to expect this to favor biophilic values of the ‘constants.’ Of course, if the collapse occurred to a component of the wavefunction with absolutely no observations, this result would not be observed. Therefore, to get normalized probabilities for observations, we must condition on the occurrence of at least one observation. However, since a component of the wavefunction in which the universe is sufficiently large may have some observations even if the ‘constants’ are not particularly biophilic, one would not expect even this selection to restrict to components of the wavefunction in which the ‘constants’ have highly biophilic values, just biophilic enough to permit at least one observation. One would then expect that most of the conditional probabilities for observations (conditional on there being an observation) in collapse versions of
quantum theory to give values of the ‘constants’ that are much less biophilic than what we observe.

Therefore, the highly biophilic values that we observe for the ‘constants’ of physics give statistical evidence against the collapse of the wavefunction.

2 Consequences for toy models of multiverse states

Let us examine the difference between collapse and no-collapse versions of quantum theory with some simple toy models. By a collapse (or Copenhagen) version, I essentially mean what might more accurately be called a single-history version, in which quantum theory gives probabilities for various alternative sequences of events, but only one sequence actually occurs. Each such alternative sequence might be called a “history” or a “world.” In contrast, by a no-collapse (or Everett) version, I mean a ‘many-worlds’ version, in which all of the possible histories or worlds with nonzero quantum probabilities actually occur, with the quantum probabilities being not propensities for potential histories to be actualized (since all with nonzero quantum amplitude are actual), but instead essentially measures for the magnitudes of the existence of the various histories.

For Model I, for simplicity let there be only two alternative worlds or sequences of events, with not-so-biophilic World 1 having quantum measure $p_1 = 1 - 10^{-30}$ and $N_1 = 10^{10}$ observations, and with highly biophilic World 2 having quantum measure $p_2 = 10^{-30}$ and $N_2 = 10^{90}$ observations. Assume that the number of observations in each world is determined by the values of effective coupling ‘constants’ of physics that are observed in the respective observations, so that the observations themselves show whether the world is not so biophilic or highly biophilic. The question then arises as to which type of observation is more probable.

In collapse versions of quantum theory in which the probability of the collapse of the wavefunction to each world is the quantum measure, almost certainly (probability $1 - 10^{-30}$) the reduction of the quantum state would give the not-so-biophilic World 1 and its observed effective coupling ‘constants.’ On the other hand, in no-collapse versions in which the total measure for all observations is proportional to the number of observations as well as to the quantum measure for the corresponding world, most of the total measure for observations would occur for the highly biophilic World 2 and its observed effective coupling ‘constants.’ That is, these simple collapse versions would predict most probably the not-so-biophilic parameter val-
ues, whereas the no-collapse version would predict most probably the more biophilic parameter values.

The collapse or single-history version of quantum theory is like assigning lottery tickets to World 1 and World 2 in the ratio \( p_1 : p_2 \). Then it is as if a lottery ticket were chosen at random to select which world, and its observations, exist.

The no-collapse or many-worlds version of quantum theory is like assigning lottery tickets to each observation in World 1 and 2 with ratio \( p_1 : p_2 \), so that the ratio of the total number of lottery tickets in World 1 to that in World 2 is \( N_1 p_1 : N_2 p_2 \). All the observations exist, but with different measures for their reality, analogous to holding different numbers of lottery tickets. Choosing a measure-weighted observation at random is analogous to choosing a lottery ticket at random. The choice is not actually made (since all observations really exist in the many-worlds version), but for assigning probabilities to the observations despite the determinism of the no-collapse version of quantum theory, it is helpful to imagine such a random choice.

Let us now go to a more realistic, but still highly idealized, Model II in which we consider the multiverse variation of one single effective coupling ‘constant,’ the cosmological constant \( \Lambda \). Using here and henceforth Planck units (\( \hbar = c = G = 1 \)), the observed value of the cosmological constant in our World or part of the multiverse is extremely tiny, \( \Lambda_O \sim 3.5 \times 10^{-122} \), but string landscape considerations \([4, 5, 6]\) suggest that it could have a huge number of different values in different parts of a multiverse. Presumably it could have a magnitude at least comparable to the Planck value for either sign.

For simplicity, let us idealize the ‘discretuum’ of values suggested by the string landscape to a continuum and hypothesize a quantum measure that gives a normal distribution for the cosmological constant with standard deviation one Planck unit, so

\[
p(\Lambda)d\Lambda = \exp\left(-0.5\Lambda^2\right)d\Lambda/\sqrt{2\pi}.
\]

The details of this are not important, but only the fact that the quantum measure is normalizable and is nearly flat for values of \( \Lambda \) comparable to the tiny observed value \( \Lambda_O \).

Now Martel, Shapiro, and Weinberg \([12]\), following upon previous ideas of Weinberg \([13, 14, 15]\), have shown that the “collapsed fraction” of matter (here by gravitational collapse, not by the collapse of the wavefunction) to form potential observers is a very sensitive function of \( \Lambda \) that decreases rapidly if \( \Lambda \) is much larger than \( \Lambda_O \). Let us therefore consider an idealized model in which the number of observations is proportional to \( \exp\left[-(\Lambda/\Lambda_O)^2\right] \). This is of course only a very crude hypothesis for
what the actual dependence on the cosmological constant might be, but since it is
nearly flat for very small values of $\Lambda$ and falls off rapidly for $|\Lambda| \gg |\Lambda_0|$, as Martel,
Shapiro, and Weinberg found for the collapsed fraction, it will be sufficient for our
purposes.

For an observation to occur at all, we need to restrict to worlds in which there
is at least one observation, so we need a numerical coefficient for the hypothesized
gaussian dependence upon the cosmological constant. If the universe had infinite
size (which might be the most reasonable hypothesis), this coefficient would be
infinite, so that no matter how large $\Lambda$ were and how small the gaussian factor were,
there would still be observations somewhere in the infinite universe. Therefore,
in simple collapse versions of quantum theory, there would be no restriction on
the cosmological constant, and its observed probability distribution would be given
purely by the quantum measure. If that were the normal distribution given above,
the probability that the observed value would be as small as $\Lambda_0$ would be very nearly
$2\Lambda_0/\sqrt{2\pi} \sim 3 \times 10^{-122}$. This probability is so tiny that it is very strong statistical
evidence against the hypothesis that there is wavefunction collapse to an eigenstate
of the cosmological constant, with the probabilities given by the absolute squares
of the quantum amplitudes, from an initial quantum state that is a nearly uniform
superposition of infinite universes with the cosmological constant taking values over
a range comparable to the Planck value.

For a more conservative estimate of the probability of our observation of the
tiny value of the cosmological constant, let us suppose that the universe is finite. A
plausible (though still very highly uncertain) estimate of a finite size would be the
size to which the universe would inflate during slow-roll inflation from an inflaton
that starts near the Planck density. If the inflaton were a massive scalar field,
observations of the fluctuations of the cosmic microwave background give $m \sim 1.5 \times
10^{-6}$ [16, 17]. Then if the inflation starts with a symmetric bounce on a round
three-sphere at density $\rho_0 = 0.5m^2\phi_0^2$, the volume at the end of classical slow-roll
inflation is approximately [18] $[0.09644/(m\rho_0^2)] \exp (12\pi \rho_0/m^2) \sim \exp (17 \times 10^{12} \rho_0)$,
which would be $\sim \exp (17 \times 10^{12})$ if the initial density were the Planck density.

We have very little idea of the spatial density of observations after inflation, but
let us suppose that the absolute value of the logarithm of that density is much less
than $17 \times 10^{12}$. For example, if the density were one observation per Hubble volume
in our World today, or $\sim 10^{-184}$ in Planck units, the logarithm would be $\sim -423.7$,
which is indeed negligible in comparison with $17 \times 10^{12}$. So let us hypothesize that the
number of observations is, very crudely, \( \exp [17 \times 10^{12} - (\Lambda/\Lambda_O)^2] \). This is greater than unity for \( |\Lambda/\Lambda_O| < 4 \times 10^6 \), so this model would suggest that the collapse of the wavefunction to any value of the cosmological constant less than four million times the observed value would still permit observations. Therefore, if the wavefunction for a universe inflating from the Planck density did collapse to an eigenstate of the cosmological constant with a probability given purely by a quantum measure that is nearly uniform for values of \( \Lambda \) much less than the Planck value, the probability plausibly would be less than one in a million for observing our observed tiny value. Even this estimate of the probability is conservatively high because of the assumed very rapid drop-off in the number of observations with \( |\Lambda| \gg |\Lambda_O| \), exponentially in \( (\Lambda/\Lambda_O)^2 \). Slower drop-offs would reduce the probability significantly.

On the other hand, for a no-collapse or Everett version of quantum theory in which the measure for observations is weighted not only by the quantum measure for each world with a particular value of \( \Lambda \) but also by the number (or by the number density) of observations, most of the total measure for observations would occur for \( \Lambda \) of the same order of magnitude as \( \Lambda_O \), thus explaining the highly biophilic value observed for the cosmological constant that would be an enormously improbable fluke in a collapse version of quantum theory with the multiverse initial quantum state assumed here.

### 3 Alternative assumptions

The very strong statistical evidence deduced above against the collapse of the wavefunction did include a number of crucial assumptions. One of course is that the initial quantum state really is a multiverse state, with a wide range of values of the effective coupling ‘constants,’ and with most of the quantum measure for values that are not nearly so biophilic as what we observe.

A second assumption is that the wavefunction collapses to an eigenstate of the effective coupling ‘constants.’ If one has a very large universe in which the wavefunction collapse leads to a spatial distribution of different coupling ‘constants’ (though perhaps definite in each region, so the collapse has eliminated the local quantum uncertainty), then a random observation chosen from this spatial distribution could still tend to favor biophilic values of the local coupling ‘constants.’ One cannot really rule out a collapse version of quantum theory without knowing what it says about how the wavefunction collapses. However, it would seem simplest to assume
that the collapse would lead to an eigenstate of the effective coupling ‘constants,’ and it is that simple version that I have shown leads to huge statistical problems for plausible multiverse initial quantum states.

A third assumption is that the particular world or single history resulting from the collapse of the wavefunction has a probability given purely by the quantum measure (e.g., the absolute square of the amplitude). If the collapse itself were caused by observation (e.g., by conscious observers), then this might weight the results to favor worlds with more observations. However, it would seem simpler to assume that the collapse of the wavefunction, if it occurs at all, occurs independently of observations.

4 Conclusions

I have shown that if the initial quantum state of the universe is a multiverse state with most of the quantum measure spread over values of the effective coupling ‘constants’ that are not particularly biophilic, and if the wavefunction collapses to an eigenstate of these ‘constants’ with a probability given purely by the absolute square of the amplitude, then the probability is very small to observe the highly biophilic values that we in fact do observe. Thus our observations of highly biophilic values is strong statistical evidence against this simple form of wavefunction collapse under the multiverse hypothesis.

This research was partially done at the headquarters of the Asia Pacific Center of Theoretical Physics in Pohang, Korea, and was supported by them and by travel money from the Korea Research Foundation (KRF) Grant funded by the Korean Government (MEST) (2010-0016-422) to Sang Pyo Kim through Kunsan National University. Financial support was also provided by the Natural Sciences and Engineering Council (NSERC) of Canada.

References

[1] D. N. Page, “Observational Consequences of Many-World Quantum Theory” (University of Alberta report Alberta-Thy-04-99, April 1, 1999), quant-ph/9904004.

[2] D. N. Page, “Can Quantum Cosmology Give Observational Consequences of Many-Worlds Quantum Theory?” in General Relativity and Relativistic Astrophysics, Eighth Canadian Conference, Montreal, Quebec, 1999, eds. C. P.
Burgess and R. C. Myers (American Institute of Physics, Melville, New York, 1999), pp. 225-232 [arXiv:gr-qc/0001001 (lowest existing gr-qc number)].

[3] M. Antia, “Parallel Universes: A World Apart,” The Economist, May 22, 1999, p. 145.

[4] R. Bousso and J. Polchinski, “Quantization of Four-Form Fluxes and Dynamical Neutralization of the Cosmological Constant,” JHEP 0006, 006 (2000) [arXiv:hep-th/0004134]; “The String Theory Landscape,” Sci. Am. 291, 60-69 (2004).

[5] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, “De Sitter Vacua in String Theory,” Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240].

[6] L. Susskind, The Cosmic Landspace: String Theory and the Illusion of Intelligent Design (Little, Brown and Company, New York and Boston, 2006).

[7] D. N. Page, “Cosmological Measures without Volume Weighting,” JCAP 0810, 025 (2008) [arXiv:0808.0351 [hep-th]].

[8] D. N. Page, “Insufficiency of the Quantum State for Deducing Observational Probabilities,” Phys. Lett. B678, 41-44 (2009) [arXiv:0808.0722 [hep-th]].

[9] D. N. Page, “The Born Rule Fails in Cosmology,” JCAP 0907, 008 (2009) [arXiv:0903.4888 [hep-th]].

[10] D. N. Page, “Born Again,” arXiv:0907.4152 [hep-th].

[11] D. N. Page, “Born’s Rule is Insufficient in a Large Universe,” arXiv:1003.2419 [hep-th].

[12] H. Martel, P. R. Shapiro, and S. Weinberg, “Likely Values of the Cosmological Constant,” Astrophys. J. 492, 29-40 (1998).

[13] S. Weinberg, “Anthropic Bound on the Cosmological Constant,” Phys. Rev. Lett. 59, 2607-2610 (1987).

[14] S. Weinberg, “The Cosmological Constant Problem,” Rev. Mod. Phys. 61, 1-23 (1989).

[15] S. Weinberg, “Theories of the Cosmological Constant,” arXiv:astro-ph/9610044.

[16] A. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic Publishers, Chur, Switzerland, 1990).

[17] A. R. Liddle and D. H. Lyth, Cosmological Inflation and Large-Scale Structure (Cambridge University Press, Cambridge, 2000).

[18] D. N. Page, “Symmetric-Bounce Quantum State of the Universe,” JCAP 0909, 026 (2009) [arXiv:0907.1893].