Modeling the Decision Making of Vehicle Control in Case of Emergency Situations Based on Game Theory

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Abstract. Modern transport is a high energy-intensive system of increased danger. There are special instructions on how to comply with the rules of operation and use of such systems, ensuring safety, in accordance with the requirements. Drivers of any vehicle are trained in accordance with the regulations for this type of transport. The article deals with issues related to life safety in the driving air vehicles, in solving the problem of landing with use of manual control in the event of emergency situations. A pilot must make the right decision under the conditions of time deficiency: he should land or go to the second round. In order to gain experience, it is proposed to use decision-making modeling in case of emergency situations using game theory methods. Usually, game theory considers conflicts between the opposing parties, which include military operations, sports games, and market relations. Contingencies that are called "nature" may occur when driving. Although nature does not have a conscious antagonistic reaction, it can be considered as the opposing side. Based on the theory of games, it is possible to conduct a safety simulation while controlling transport.

1. Introduction

Ensuring transport safety is an important component of people's lives. This is especially important when driving an air vehicle, since in case of an accident a large number of people are immediately exposed to mortal danger. It goes without saying, the safety of the aircraft and people depend on experience and the level of crew training. Despite the fact that modern aircraft are equipped with automation, the pilot is indispensable when landing. The landing process is the most critical part of the flight, and emergency situations often arise when the role of the crew commander is most significant. The pilot must make the right decision as soon as possible: whether to start landing or go to the second round. When landing in emergency situations, the pilot has to evaluate a number of factors. They can be a change in speed, deviation from the glide path, a change in the angle of inclination, a strong crosswind, engine malfunction, depressurization and different situations on the land [1-4]. We propose to use the theory of games as a mathematical approach, for the decision-making rules. It is clear that this approach may not be practiced in real flight conditions, but in training.

2. A mathematical model based on game theory

Game theory formally refers to the mathematical discipline associated with decision-making in conflict situations arising from the clash of interests of the opposing sides [5,6]. Opponents in this situation may be participants in sports games, opposing parties to hostilities, participants in market
relations between seller and client. When problem setting on the basis of game theory, the relations of conflicting parties are described using a payment matrix that has a structure shown in figure 1.

![Figure 1. Payoff matrix.](image)

In the matrix in Figure 1, A and B are opposing players. They take strategies $A_i$ and $B_j$, where $i$ takes values from 1 to $m$ and $j$ takes values from 1 to $n$, respectively, with step 1. The value $a_{ij}$ indicates the amount of payment depending on the winning or losing of one of the players. To get the maximum win or minimum loss, each of the players must adhere to a certain strategy. Player A must adhere to the maximum of values which are minimal in columns in this case, let it be $\alpha$. Player B must adhere to $\beta$, which is the minimum of values which are maximal in rows of the payoff matrix.

If $\alpha$ and $\beta$ are equal, then the game is stable. This is a game with a “saddle” point, when players know all the moves. If $\alpha$ is not equal $\beta$, then a mixed strategy can be used, and the mathematical expectation of the gain of player A can be evaluated according to the expression:

$$M(P, Q) = \sum_i \sum_j a_{ij} p_i q_j$$

(1)

Elements $P = (p_1, p_2, \ldots, p_m)$ and $Q = (q_1, q_2, \ldots, q_n)$ in the formula (1) set the probabilities of the adoption of strategies by players A and B, respectively [5, 6]. In case of games of the type with dimension 2x2 or 2xn, a solution can be visually demonstrated geometrically. Let in the case payment matrix dimension 2x2 the player A has alternatives $A_1$ and $A_2$ with probabilities $p_1$ and $p_2$, accordingly. In this case, if $p_1$ is equal to $p$, then $p_2$ is equal to $1-p$. Alternatives $B_1$ will linearly depend on event probabilities according to:

$$W_1 = a_{11}p + a_{21}(1-p)$$

and

$$W_2 = a_{12}p + a_{22}(1-p)$$

(2)

Figure 2 gives an illustrative representation of decision making with a mixed strategy, where $A_1$ can take its strategies $A_i$ and $A_2$ with probabilities $p_1$ and $p_2$. The value of $V$ provides the maximum.

![Figure 2. Graphical Decision Making Method.](image)

Antagonistic conflicts in pure form take place in cases of deliberate confrontation opponent. It may be a war opposition, sports, market relations, etc. In reality, we can consider abnormal unexpected
situations as an adversary. In decision theory they are called “nature”. Although games with "nature" do not imply a conscious opposition, but due to uncertainty, they can have an even greater negative effect.

3. Decision-making modeling for aircraft control in a landing

When driving air transport an automatic flight mode is used, as a rule. But for a number of reasons pilots have to switch to manual control when landing. This may be due to the fact that not all runways can be equipped with Instrument Landing System. In addition, even if there is one, it can be turned off for repair work. Some airfields have such glide path angles that auto landing on them may be prohibited. Furthermore, equipment malfunctions are possible. In this case, the pilot has to perform a visual control in manual mode. And in case of emergency, as soon as possible he should make the right decision to sit down, go to the second round or fly to another airfield [7, 8, 9].

Let the notation is introduced for the decision made: $X_1$ means to land and $X_2$ means to go to the second round. And the following notation is introduced for possible emergency situations:

- $S_1$ – speed change;
- $S_2$ – pitch change;
- $S_3$ – glide path deviation;
- $S_4$ – engine failure;
- $S_5$ – onboard equipment malfunction;
- $S_6$ – depressurization;
- $S_7$ – fire on board.

Various abnormal situations can lead to various injuries, up to catastrophes. Let the conditional scale adopted to assess the severity of damage using numbers from 0 to 9 [5]. Numerical values characterize the catastrophic consequences that may occur on board after the adoption of certain decisions as it is shown in Table 1. Zero means successful landing and 9 means catastrophic landing. These values should be formulated by highly professional experts.

| Catastrophic level | Grades |
|--------------------|--------|
| No problem         | 0      |
| Minor damage       | 1 - 3  |
| Serious damage     | 5 - 7  |
| Serious consequences | 8 - 9 |

The decision table is presented in the form of a payment matrix in Table 2. Element $X$ there corresponds to alternatives, element $S$ corresponds to contingencies, and $R(x)$ is value for decision making.

$$R(x) = \max_{x \in X} \min_{s \in S} R(x, s)$$

Based on Wald's min-max theory of decision making under uncertainty the value $R(x)$ can be evaluated by expression (3).
Consequently, with the values given in the example the first alternative \( X_1 \) to be followed, i.e. try to land the plane. Based on game theory, the lower bound for the min-max strategy is when \( \alpha \) has value 1 and the upper one is when \( \beta \) is equal to 5. Therefore, a mixed strategy can be used.

Let introduce the system of linear equations (4) for abnormal situations based on the values of Table 2.

\[
\begin{align*}
W_1 &= 7p + 1(1-p) \\
W_2 &= 6p \\
W_3 &= 7p+2(1-p) \\
W_4 &= 2p+7(1-p) \\
W_5 &= 1p+5(1-p) \\
W_6 &= 2p+8(1-p) \\
W_7 &= 2p+9(1-p)
\end{align*}
\]

(4)

These linear equations can be represented as lines on the graph in figure 3.

Figure 3. Decision Making Chart for a Mixed Strategy.

The solution region is the space enclosed by lines 2, 5 and the lower horizontal ordinate. The maximum value is determined at the intersection point of the equation \( w_2 = w_5 \). That means:

\[
6p = 1p + 6(1-p); \text{ then } p = 6/11; \text{ or } p_1 = 6/11 \text{ and } p_2 = 5/11.
\]

The mixed strategy will be:

\[
\begin{pmatrix}
0.6 \\
0.5
\end{pmatrix}
\]

(5)

Substituting \( p_1 \) with the value 6/11 in the equation for \( w_5 \) results in the maximum of value \( v \). This value is equal to 3.27 which also corresponds to the graphical interpretation.

Thus, for given initial numerical values in Table 2, game theory recommends the adoption of the first alternative \([10, 11]\) which means landing. In this case, the damage assessment will be no more than 3.27, which is the minimum value.

4. Conclusion

Transport safety is associated with factors that include human, technical and environmental factors. Human factors can be associated with inadequate preparation for driving or psychophysiological stress, leading to errors. Technical factors are associated with failures, malfunctions or damage to individual components in the transport. Environmental factors include meteorological conditions (thunderstorm, wind, fog, etc.). All these factors affect the safety of driving. Pilots are most at risk when landing an aircraft. During the landing of an aircraft using manual control, various emergency situations can arise and in conditions of time pressure, the pilot must make the right decision: to land or go to the second round. To develop these solutions, it is proposed to model situations using the methods of game theory and uncertainty \([12]\). Decision-making modeling using game theory can also
be used in many emergency situations: in transport, during the construction of complex facilities, during fires, earthquakes, information security in networks and others.

5. References
[1] Suslov Yu 2005 *Flight safety of civil aviation aircraft (transport category)* (Ulyanovsk: UICA) p 167
[2] Shumilov I 2005 *Aviation accident prevention* (Moscow: MSTU Bauman Publishing House) p 78
[3] Zubkov B and Prozorov S 2011 *Flight safety: textbook* (Moscow: MSTUCA) p 456
[4] Sander V 2009 *A Pilot Guide to Safe Flying* (Australia: mCOVE Resources) p 204
[5] Rizaev I 2014 *Theory of decision making* (Kazan: Master Line) p 132
[6] Salim F, Reid J, Dulleck U and Dawson E 2010 Towards a game theoretic authorization model *Decision and Game Theory for Security (GameSec)* 208-19
[7] Miftakhutdinov D, Rizaev I and Shleymovich M 2017 Method of automatic detection of new motionless objects on the basis of comparison of images of the area *Russian Aeronautics* 60(4) 542-50
[8] Saaty T and Ergu D 2015 When is a decision-making method trustworthy? Criteria for evaluating multi-criteria decision-making methods *International Journal of Information Technology and Decision Making* 14(6) 1171-87
[9] Rizaev I and Takhavova E 2016 Statistical methods of making decision on repair of technical Means *2nd International Conference on Industrial Engineering, Applications and Manufacturing ICIEAM 2016 Proc.* 7911647
[10] Rizaev I and Takhavova E 2019 Solution of the Problem of Classification of Vehicles on the Basis of Statistical Estimates of Data *12th International Scientific and Technical Conference “Dynamics of Systems, Mechanisms and Machines”, Dynamics 2018* 8601417
[11] Saaty T 2016 Continuous Pairwise Comparisons *Fundemental Informaticae* 44 (3-4) 213-21
[12] Saaty T 1987 Rank generation, preservation, and reversal in the Analytic Hierarchy Decision Process *Journal of the Decision Sciences Institute* 18(2)