Exploring the evolutionary mechanisms underlying the viewing durations of learners on online courses

Zheng Xie¹,‡

¹ College of Liberal Arts and Sciences, National University of Defense Technology, Changsha, China
‡ xiezheng81@nudt.edu.cn

Abstract

We adopted survival analysis for the viewing durations of massive open online courses. The hazard function of empirical duration data is dominated by a bathtub curve and has the Lindy effect in its tail. To understand the evolutionary mechanisms underlying these features, we categorized learners into two classes, due to their different distribution patterns of viewing durations, namely power law with exponential cutoff and lognormal. Two random differential equations are provided to describe the growth patterns of viewing durations: the expected duration change rate of the learners featured by lognormal is depended on their past duration, and that of the rest learners is inversely proportional to time. Solutions of the equations predict power law with exponential cutoff and lognormal for the density function of viewing durations, as well as the Lindy effect and bathtub curve for the corresponding hazard function. The equations also reveal the feature of memory and that of memoryless for the viewing behaviors of the two learner class respectively.

Keywords: Distance education, Massive open online courses, Online learning.

Introduction

Massive open online courses (MOOCs) arise from the integration of education and information technologies, featured by unlimited participation and open access via the Internet [1,2]. These courses break the spatiotemporal boundary of traditional education, and contribute to balancing the resources of education [3,4]. Differences between these courses and traditional courses lie in several dimensions, involving admission condition, learning motivation, teaching methods, the management of learners, the interactions between teachers and learners [5,6]. Analyzing learning behaviors has become a hot topic in the MOOC
community, which includes the achievements of learners [7,8], the effect of learners’ interactions [9], the visual analysis of course data [10], the assessment of courses [11], and so on.

High dropout rate is a feature of learning MOOCs, which has been regarded as a result of the diversified expectations and motivations to learn these courses [12–15]. The learners of MOOCs are motivated not just to pass exams or to obtain certificates. They could be only interested in understanding particular concepts or some parts of course contents [16–19], and then are likely to drop after obtaining what they want. Analyzing the dropouts of MOOCs contributes to quantifying the completion and continuance of learning [20,21]. For example, how many learners who will continue to learn when they have learned for a certain time? At what rate they will drop in the future? Moreover, understanding the evolutionary mechanisms underlying the dropout behavior and modelling them mathematically contribute to profiling learners’ type [22], and to quantifying the effects of teaching methods or other explanatory variables on learning behaviors [23–26].

The log data of learning behavior collected by the platforms of MOOCs can be used to analyze the dropout rate, where viewing is a prominent learning behavior [27]. We adopted survival analysis for learners’ viewing duration on a course, where the empirical data are provided by the platform iCourse (http://www.icourse163.org). The survival function of the durations describes the fraction of the learners viewing a course past a given time, and the corresponding hazard function describes the rate of these learners dropping viewing at the given time. The hazard function of empirical data is concave but convex in the tail, namely comprising four parts: decreasing, stable, increasing, and decreasing again. The shape of the first three parts is known as bathtub curve [28], and the decreasing tail part is known as the Lindy effect [29], namely the more you have learned, the more you want to learn. The features of the hazard functions have the potential to be used to assess course attraction.

We analyzed the evolutionary mechanisms underlying these features. We categorized learners as lognormal-learners and segment-learners, due to the different features of their viewing duration distribution, namely power law with exponential cutoff and lognormal. For each category, we provided a random differential equation to describe the growth mechanism of viewing durations. For lognormal-learners, their duration change rate correlates to their past duration and to a random disturbance. For segment-learners, their duration change rate is inversely proportional to time, and their start time of viewing a course is a random variable. The equations express the factor of memory in viewing behavior for lognormal-learners, and that of memoryless for segment-learners. The solutions of the equations predict
power law with exponential cutoff and lognormal for the density function, as well as the Lindy effect and bathtub curve for the hazard function.

This paper is organized as follows. Empirical data are described in Section 2. The features of hazard function for viewing durations are described in Section 3. The evolutionary mechanisms of hazard function for viewing durations are analyzed in Sections 4-6. Discussion and conclusions are drawn in Section 7.

The data

We analyzed the log data of viewing eight MOOCs from 01/01/2017 to 10/11/2017, which are provided by the platform iCourse. The courses are selected from natural sciences, social sciences, humanities and engineering technology. Specific statistical indexes of these courses are listed in Table 1 which have been used to analyze course attraction in our previous work[30]. The data include the time length of each video. For each learner, the data include the start time of viewing each video he or she viewed, and the corresponding viewing durations.

Videos can be downloaded by iCourse app. The log data of viewing the downloaded videos are also collected, unless the app disconnects to the Internet. Accordingly, our study only involves the online viewing behavior recorded as log data. However, learners may be off-task during video playing, which cannot be measured through log data. This is a limitation of our study. In addition, some typical operations on videos are also not analyzed here, such as pausing, skipping, backward, forward, speed changing, and so on.

We concentrated on learners’ viewing duration, which is defined to be the duration of video playing, where the duration of pause is not counted. We introduced the following symbols to express the duration. Suppose that learners \( \{L_1, \ldots, L_m\} \) have viewed a course with \( n \) videos \( \{V_1, \ldots, V_n\} \). For each learner \( L_s \) \((s = 1, \ldots, m)\), denote the label set of the videos he viewed as \( S'_V \). For each video \( V_i \) \((i = 1, \ldots, n)\), denote the label set of learners who viewed it as \( S'_L \). Denote the time length of video \( V_i \) as \( l_i \), the duration of learner \( L_s \) viewing \( V_i \) as \( t^*_i \). Then the viewing duration of learner \( L_s \) on the course is \( \sum_{s=1}^{m} t^*_i \). Hereafter, the viewing duration on a course is called viewing duration for short.
Table 1. Specific statistical indexes of the data provided by iCourse.

| Course       | Course Id   | m    | n    | a    | b    | c    | d    |
|--------------|-------------|------|------|------|------|------|------|
| Calculus     | 1002301004  | 2,955| 129  | 2    | 8.081| 0.998| 0.189|
| Game theory  | 100223009   | 4,764| 38   | 66   | 7.141| 2.238| 0.427|
| Finance      | 1002301014  | 6,380| 63   | 2    | 5.368| 1.310| 0.330|
| Psychology   | 1002301008  | 3,827| 26   | 59   | 5.008| 0.913| 0.204|
| Spoken English| 1002299019 | 11,719| 46   | 7    | 3.032| 0.321| 0.106|
| Etiquette    | 1002242007  | 3,846| 41   | 22   | 7.787| 1.271| 0.205|
| C Language   | 1002303013  | 17,541| 81   | 39   | 12.47 | 1.541| 0.142|
| Python       | 1002235009  | 13,417| 53   | 28   | 10.32 | 0.896| 0.087|

Index m: the number of learners, n: the number of videos, a: the number of all-rounders who viewed all of the videos, b: the average number of viewed videos for learners, c: the average viewing duration of learners (unit: hour), and d: the average time length of videos (unit: hour).

Survival analysis for viewing durations

A learner’s viewing duration on a course can be regarded as the “lifetime” of his or her viewing behavior. The number of learners with duration \( t \) expresses the number of dropouts at “age” \( t \), where \( t \in [0, T] \), and \( T \) is the maximum viewing duration. Therefore, the density function of viewing durations, denoted as \( f(t) \), expresses the rate of dropouts at any possible \( t \). The rate of dropouts at a given \( t \) for the learners with viewing duration no less than \( t \) is calculated as \( h(t) = f(t)/S(t) \), where \( S(t) = \int_{T}^{t} f(\tau)d\tau \) is the probability of a learner’s duration no less than \( t \). In survival analysis, \( S(t) \) and \( h(t) \) are called survival function and hazard function respectively. The function \( h(t) \) is the derivative of \( \log S(t) \), and then is more informative about dropouts.

The hazard function of empirical data is dominated by a concave function and has a convex tail, namely it has four parts: decreasing, stable, increasing and decreasing again (Fig. 1). The shape of the first three parts is known as bathtub curve, the concept of which comes from product quality assessment. The curve is often used to describe the failures of products over time, which contain the decreasing rate of early failures as defective products are discarded, the random failures with constant rate during the useful life of products, and the rate of wear-out failures as the products exceed their designed lifetime. In this study, we called the dropouts of the increasing part wear-out dropouts. The last decreasing part is known as the Lindy effect, namely the future life expectancy is proportional to their current age. It means that every additional period of duration implies a longer remaining duration expectancy.

To understand why the Lindy effect and bathtub curve emerge simultaneously, we should analyze the features of the density function \( f(t) \), and mine the dynamic rules under those features, because that the hazard function \( h(t) \) is essentially based on \( f(t) \). When a learner has viewed a certain number of videos,
Figure 1. The hazard function of empirical viewing durations. Panels showed the hazard function of viewing durations (orange circles), and its trend line (red dotted lines), as well as the trend line of the hazard function for lognormal-learners (black lines) and that for segment-learners (blue lines) respectively.

his or her viewing duration follows a lognormal distribution (the results of the KS test are shown in Table 2). For the rest learners, most of their duration follow a power law with an exponential cutoff (the results of good-of-fit are listed in Table 2). To illustrate these features, we fitted the parameters of these distributions for each course, and generated synthetic durations following each distribution (Fig. 2), the number of which is the same as that of the corresponding empirical durations. The comparison between empirical duration distributions and synthetic ones are shown in Fig. 3. How do these features of the density function affect the shape of the hazard function? To answer this question, we explored the evolutionary mechanisms underlying these features in the following sections.

Wear-out dropouts, Lindy effect and lognormal

The empirical data show that the viewing durations of the learners, who have viewed no less than $\tau$ videos (Table 2), follow a lognormal distribution. We called them lognormal-learners, and had provided an algorithm to find them (Table 3 in the appendix). Following identical lognormal distribution means those learners are the samples drawn from the same population in the sense of viewing duration, thus
Figure 2. Synthetic viewing duration distributions. Panels show the density function of lognormal distribution (black lines, blue circles) and that of power law with exponential cutoff (blue lines, orange circles) respectively, as well as the mixture density function of them (red dotted lines). The parameters of these distributions are listed in Table 2.

Figure 3. Comparisons between the empirical distributions of viewing durations and synthetic ones. Panels show that the empirical distributions can be approximated by the mixture of a lognormal and a power law with an exponential cutoff.
Table 2. Fitting parameters and goodness-of-fit.

| Course        | α    | β    | µ    | σ    | τ    | p-value | ψ    |
|---------------|------|------|------|------|------|---------|------|
| Calculus      | 0.7284 | 0.0035 | 6.1694 | 0.8155 | 11 | 0.059   | 15.40% |
| C Language    | 0.5803 | 0.0028 | 6.9664 | 0.4865 | 29 | 0.067   | 12.48% |
| Etiquette     | 0.5288 | 0.0151 | 5.5873 | 0.9395 | 4  | 0.069   | 16.09% |
| Finance       | 0.7557 | 0.0024 | 6.6165 | 0.8052 | 8  | 0.092   | 14.53% |
| Game theory   | 0.6556 | 0.0023 | 6.7109 | 0.7546 | 7  | 0.105   | 16.32% |
| Psychology    | 0.5391 | 0.0062 | 6.0323 | 0.6659 | 7  | 0.355   | 23.05% |
| Python        | 0.2622 | 0.0083 | 6.0817 | 0.4989 | 19 | 0.077   | 16.08% |
| Spoken English| 0.9561 | 0.0044 | 5.9030 | 0.5965 | 10 | 0.123   | 30.36% |

The parameters of $x^\alpha e^{\beta x}$ are fitted through multiple linear regression. The parameters of Lognormal($\mu, \sigma$) are calculated based on empirical data. At significance level 5%, the KS test cannot reject that the viewing durations of the learners, who have viewed no less than $\tau$ videos, follow a lognormal ($p$-values $> 0.05$). The good-of-fit index $\psi$ is the half of the cumulative difference between the duration distribution of segment-learners and the corresponding synthetic one.

can be categorized as one class.

The density function of lognormal is $f(x) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}}$, where $x \in [1, \infty)$, $\sigma > 0$, and $\mu \in \mathbb{R}$.

The corresponding hazard function is

$$h(x) = \frac{1}{x\sigma \sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}} \left(1 - \text{erf} \left( \frac{\log x - \mu}{\sqrt{2\sigma}} \right) \right)^{-1}. \quad (1)$$

The hazard function (1) is convex, when its corresponding density function is convex [32]. Although in different level, the density function and hazard function of lognormal-learners are convex for each empirical course, namely appears the wear-out dropouts and the Lindy effect (Figs. 1, 2). To verify above analytical arguments, we showed the hazard function of synthetic durations, which gives a reasonable fit to that of empirical data (Fig. 3).

To find the evolutionary mechanism underlying the wear-out dropouts and the Lindy effect, we should go back to where lognormal comes from. Lognormal often emerges in the lifetime distribution of mechanism units [33], where the lifetime is affected by the multiplication of many small factors. Approximate these factors as a range of independent and identically distributed random variables. The central limit theorem says that the summation of these variables in log scale follows a normal distribution. Back-transforming to the original scale gives rise to that the multiplication of these factors follows a lognormal distribution. This is known as the multiplicative version of the central limit theorem, called the Gibrat’s law [34].

Lognormal-learners contain the all-rounders who viewed all of the videos. It means that the endurances of the rest lognormal-learners and those of all-rounders are homogenous, and then could be regarded
as potential all-rounders. For a potential all-rounder who have the willing to complete a course, his endurance of viewing videos (measured by his viewing duration) could be compared to a mechanical unit whose failure mode is of a fatigue-stress nature. The life of such a unit follows a lognormal distribution. The analogy enlightens us to provide

$$\frac{d}{dt} k_i(t) = k_i(t)(\mu + \sigma x(t)), \quad (2)$$

where $k_i(t)$ is the duration at time $t$ of learner $i$, $x(t)$ is a random variable of the standard normal distribution $N(0,1)$, $\mu$ and $\sigma > 0$. Supposing $k_i(t_0) = 1$ gives rise to the solution $k_i(t) = e^{\mu(t-t_0)+\sigma \int_{t_0}^{t} x(\tau) d\tau}$, which is the random variable of lognormal. Guaranteed by the central limit theorem, the result holds for any independent and identically distributed random variables $x(\tau)$ with any possible $\tau$, as long as their effects are negligible compared with that of $\int_{t_0}^{t} x(\tau) d\tau$.

The equation (1) means the viewing behavior of a lognormal-learner has memory, because the change rate of duration correlates to his or her past duration. Moreover, the expected change rate correlates to the past duration positively. It means when $t$ is large enough the duration increases exponentially, namely the more you learn, the more you want to learn. This is the status of the learners who are deeply
impressed by a course.

**Dropouts with constant rate and exponential cutoff**

The viewing durations of the learners viewing less than $\tau$ (Table 2) videos approximately follow a power law with an exponential cutoff (Fig. 3). The emergence of the cutoff is mainly due to that the durations of segment-learners, which are no less than one minute, approximately follow an exponential distribution (Fig. 5). The density function of exponential distribution is $f(x) = e^{-x/\lambda}/\lambda$, where $x \in [1, \infty)$ and $\lambda > 0$. The corresponding survivor function is $S(x) = e^{-x/\lambda}$, and then the hazard function is a constant, namely $h(x) = f(x)/S(x) = 1/\lambda$.

To find the evolutionary mechanism underlying the dropouts with a constant rate, we came back to the mechanism underlying exponential distribution. The distribution is featured by memoryless, namely it satisfies $p(T > s + t | T > s) = e^{-(s+t)/\lambda}/e^{-s/\lambda} = e^{-t/\lambda} = p(T > t)$ for any possible $T$, $s$ and $t$. In our study, the memoryless means the future viewing duration is free of the past duration. For example, the probability that a learner, who has viewed ten minutes, will view one minute is equal to the probability that a learner, who does not view videos, will view one minute.

Due to the memoryless and the fatigue of learning, it is reasonable to suppose the change rate of
viewing duration decreases with time. For simplicity, we supposed the change rate of learner \( i \)'s viewing duration \( k_i(t) \) to be

\[
\frac{d}{dt} k_i(t) = \frac{\lambda}{t}.
\]  (3)

Solving the equation in the time interval \([y, T]\) gives rise to

\[
k_i(T) = \lambda \log \frac{T}{y},
\]  (4)

where \( y \) is the start time of viewing. Supposing \( y \) is a random variable of the uniform distribution over \([T_0, T]\) gives rise to

\[
p(k_i \leq x) = p(\lambda \log \frac{T}{y} \leq x) = p(T e^{-\frac{x}{\lambda}} < y) = 1 - \frac{T}{T - T_0} e^{-\frac{x}{\lambda}}.
\]  (5)

It leads to an exponential distribution

\[
p(k_i = x) = \frac{d}{dk} p(k_i \leq x) = \frac{T}{\lambda(T - T_0)} e^{-\frac{x}{\lambda}}.
\]  (6)

When \( T_0 = 0 \), Eq. (6) is the standard exponential distribution.

To make synthetic duration distributions fit the empirical ones, we valued the parameters of the solution (4) from the information of empirical data. Let the domain of the durations of segment-learners (which are no less than one minute) be \([T_1, T_2]\), and calculate the exponent \(-1/\lambda\) of the formula (6) by fitting empirical data (Table 3). Letting the simulated duration \( \lambda \log (T/y) \) belong to \([T_1, T_2]\) gives the sampling interval \([e^{-T_2/\lambda}, e^{-T_1/\lambda}]\) for \( y/T \). Table 5 in the appendix shows the detail for the process of simulation.

Analytical arguments allow for the prediction of exponential cutoff. The simulations based on the solution (4) also provide a reasonable fit to those of empirical data (Fig. 5, the index \( \psi_1 \) in Table 4). Therefore, Eq. (3) can be regarded as an expression of the evolutionary mechanism for the exponential cutoff and for the random dropouts with a constant rate.
Table 3. Parameters of synthetic power law and exponential cutoff.

| Course        | $T$  | $\lambda$ | $N_1$ | $\psi_1$ | $a$    | $b$    | $c$    | $N_2$ | $\psi_2$ |
|---------------|------|-----------|-------|----------|--------|--------|--------|-------|----------|
| Calculus      | 460  | 116.9     | 1,217 | 26.38%   | 1.30e03| 3.682  | 7.25e-2| 1,169 | 11.89%   |
| C Language    | 1,253| 224.5     | 9,587 | 16.27%   | 8.30e03| 2.383  | 7.21e-2| 5,587 | 8.5%     |
| Etiquette     | 206  | 42.62     | 683   | 22.36%   | 1.89e01| 2.122  | 7.71e-2| 1,079 | 10.29%   |
| Finance       | 1,042| 178.1     | 2,875 | 24.59%   | 3.98e03| 4.093  | 6.98e-2| 2,448 | 7.52%    |
| Game theory   | 678  | 181.4     | 1,834 | 25.08%   | 1.47e02| 2.904  | 7.51e-2| 1,408 | 7.74%    |
| Psychology    | 495  | 93.70     | 1,718 | 31.49%   | 1.04e01| 2.170  | 5.54e-2| 1,188 | 17.51%   |
| Python        | 616  | 104.2     | 6,607 | 14.59%   | 2.70e00| 1.351  | 6.94e-2| 4,261 | 13.49%   |
| Spoken English| 916  | 93.70     | 4,062 | 25.58%   | 1.29e36| 22.78  | 5.92e-2| 7,074 | 18.52%   |

Index $T$: the maximum duration of segment-learners, $\lambda$: the parameter of Eq. (3), $N_1$ and $\psi_1$ ($N_2$ and $\psi_2$): the number of the segment-learners with duration no less than (less than) one minute and the cumulative difference between the duration distribution of those learners and the corresponding synthetic distribution, $a$ and $b$: the parameters of Eq. (8), $c$: the normalization coefficient of the formula (7).

Early decreasing dropouts and power law

Early decreasing dropout rate appears in the hazard function of empirical data, which describes a phenomenon that the dropout rate of viewing course decreases within the first minute. It describes the period of “infant mortality” where the learners, who only tour a course, drop viewing. Meanwhile, the density function of empirical data shows the viewing durations of segment-learners, which are less than one minute, can be fitted by a power law function approximately (Fig. 6). Denote the density function of these durations by $f(x) = cx^{-\alpha}$, where $c$ is the normalization coefficient, and $\alpha \in (0, 1)$. Hence the random variable $x$ is valued in a finite interval, denoted as $[R_1, R_2]$. Note that the value of the exponent $\alpha$ is different from that of degree distribution in network sciences, which is larger than one. The corresponding survivor function is $S(x) = 1 - c(x^{1-\alpha} - 1)/(1 - \alpha)$, and then the hazard function is

$$h(x) = \frac{cx^{-\alpha}}{1 - c^{-1}(x^{1-\alpha} - 1)}. \quad (7)$$

It decreases with the growth of $x$, when $x \leq R_2((\alpha + 1)/2)^{1/(1-\alpha)}$.

To find the evolutionary mechanism underlying the early decreasing dropouts, we also came back to the mechanism underlying such a power law. The durations of learners approximately following a power law are less than those approximately following an exponential distribution. Hence their learning behavior are more reasonable to think as memoryless. Therefore, we suppose their duration are also governed by Eq. (2). Meanwhile, power law reflects the heterogeneity of samples [35, 36]. It means the parameter $\lambda$ of Eq. (2) should be heterogenous over learners.
Figure 6. **The power-law part of viewing duration distribution.** Panels show the distribution of the durations of segment-learners, which are less than one minute (red circles), compared with the predictions of Eq. (9) (blue squares).

We expressed this heterogeneity by \( \lambda(\nu) = \nu^b/a \), namely

\[
\frac{d}{dt} k_i(t) = \frac{\nu^b}{at},
\]

where \( \nu \) is a random integer of the uniform distribution over \([S_1, S_2]\), \( a > 0 \), and \( b > 1 \). Solving it on interval \([y, T]\) gives rise to

\[
k_i(T) = \frac{\nu^b}{a} \log \frac{T}{y},
\]

where \( y \) is the start time of viewing, sampled from the uniform distribution over \([T_0, T]\). Hence the expected value of \( k_i(T) \) is \( \nu^b \log 2/a \), which yields

\[
p(k_i(T) \leq x) = p\left(\frac{\nu^b \log 2}{a} \leq x\right) = p\left(\nu \leq \left(\frac{ax}{\log 2}\right)^{\frac{1}{b}}\right) = \frac{1}{(S_2 - S_1 + 1)} \left(\frac{ax}{\log 2}\right)^{\frac{1}{b}}.
\]

Then the density function of viewing duration is

\[
p(k_i(T) = x) = \frac{d}{dx} p(k_i(T) \leq x) = \frac{1}{(S_2 - S_1 + 1)b} \left(\frac{a}{\log 2}\right)^{\frac{1}{b}} x^{\frac{1}{b}-1} \propto x^{\frac{1}{b}-1}.
\]
The strict deduction of the density function needs averaging over all possible $\nu$, which yields

$$p(k_i(T) = x) = \frac{1}{S_2 - S_1} \int_{S_1}^{S_2} \frac{1}{\lambda(\nu)} e^{-\frac{x}{\lambda(\nu)}} d\nu = \frac{1}{S_2 - S_1} \int_{S_1}^{S_2} \nu^{-b} e^{-\nu^{-b} x} d\nu$$

(12)

$$= \frac{a^b}{b(S_2 - S_1)} x^{b-1} \int_{a S_2^{-b} x}^{a S_1^{-b} x} \tau^{-b} e^{-\tau} d\tau \propto x^{b-1} \int_{a S_2^{-b} x}^{a S_1^{-b} x} \tau^{-b} e^{-\tau} d\tau.$$  

Differentiate the integration part to obtain

$$\frac{d}{dx} \int_{a S_2^{-b} x}^{a S_1^{-b} x} \tau^{-\frac{1}{b}} e^{-\tau} d\tau = a^{1-\frac{1}{b}} x^{-\frac{1}{b}} \left(S_1^{1-b} e^{-a S_1^{-b} x} - S_2^{1-b} e^{-a S_2^{-b} x}\right).$$

(13)

This derivative is approximately equal to 0 if $a$ is large enough, which is guaranteed by the empirical values of $a$ in Table 3. Hence the integration part is free of $x$ and $p(k_i(T) = x) \propto x^{1/b-1}$.

To make the simulated distributions fit the empirical ones, we valued the parameters of Eq. (9) based on empirical data as follows. Calculate the domain of the durations of segment-learners (which are less than one minute) $[R_1, R_2]$, and fit their distribution by power law $cx^{-\alpha}$. The fitted values of $\alpha$ and $c$ are listed in Table 2 and Table 3 respectively. Comparing the coefficients of Eq. (11) to the $\alpha$ and $c$ gives rise to $\alpha = 1 - 1/b$ and $c = (a/\log 2)^{1/b} / (S_2 - S_1 + 1)b$. Solving them obtains the value of $a$ and $b$. The expected duration $\nu b \log 2/a$ belonging to $[R_1, R_2]$ gives rise to the sampling interval for $\nu$ and then for $y/T_2$. The detail for the process of simulation is shown in Table 6 in the appendix.

Above analysis realizes a process of deriving power law from a range of exponential distributions. Moreover, it provides an explanation for the early decreasing part of the hazard function: the dropout rate $1/\lambda(\nu)$ decreases with the growth of the expected value $\lambda(\nu)$. The simulations based on the solution (9) also provide a reasonable fit to the former parts of the empirical duration distributions (Fig. 5). Therefore, Eq. (8) can be regarded as an expression of the evolutionary mechanism for power law and for the early decreasing dropout rate. In addition, the memoryless of Eq. (8) together with that of Eq. (3) can be regarded as the intrinsic meaning of the class name segment-learners.

**Discussion and conclusions**

We adopted survival analysis for the viewing behavior of learning MOOCs. We showed the features of density function and hazard function for learners’ viewing duration on a course, namely power law with
Figure 7. An illustration of the presented results. The illustrated data are from the course C language. Panel (a) shows the density function of viewing durations and its evolutionary equations. Panels (b, c) show the features of the density function, and those of the corresponding hazard function. The equation on the left describes the evolutionary mechanism for power law with exponential cutoff, for early decreasing dropouts, and for the random dropouts with constant rate. The equation on the right describes the mechanism for lognormal, for wear-out dropouts, and for the Lindy effect.

exponential cutoff, lognormal for the density function, and the Lindy effect and bathtub curve for the hazard function. To understand the emergence of those features, we categorized learners as lognormal-learners and segment-learners based on the features of their viewing duration distribution, and provided dynamic equations to describe the growth processes of their duration for each category. For lognormal-learners, the change rate of durations is depended on the past duration and on a random disturbance. For segment-learners, the change rate of durations is inversely proportional to time, and the start time of viewing is a random variable of a uniform distribution. The solutions of these equations predict the features of the density function and those of hazard function. Therefore, these equations can be regarded as proper expressions of the evolutionary mechanism underlying these features. We summarized the presented results in Fig. 7.

The presented results have the potential to illuminate specific views and implications in the broader study of learning behaviors. For example, the features of viewing duration distribution can be used to profile the type of learners, such as lognormal-learners, the learners with duration approximately following
The three types of learners are lognormal-learners (the durations of them follow a lognormal distribution), the segment-learners with duration no less than one minute (those durations follow an exponential distribution approximately), and the rest (the durations of them follow a power law approximately).

The fractions of these types vary over courses (Fig. 8). Over half of the learners studying the course *Calculus* are lognormal-learners. Almost half of the learners taking *C Language* or *Python* view the course less than one minute, where their duration approximately follows a power law. Weighting each type with a different value helps to measure the attraction of a course in a reasonable way. In addition, comparing the duration distributions before and after adopting a teaching method helps to know whether the method significantly increases or decreases learning duration. For example, if the KS test shows the duration distributions of lognormal-learners are identical, it cannot say the improvement of the adopted method is significant. This can also be used to compare the attractions of different courses. It removes the heterogeneity of the number of learners, and hence is a fair way for the courses with high quality but having few learners.

In the presented equations, the viewing durations are based on random factors, and memory or memoryless. In the beginning of studying a course, a learner does not need the knowledge of the course. When studying deeply, the learner would need the knowledge he or she has learned from the course. This process could be regarded as the transition from memoryless to memory. Meanwhile, the viewing duration distributions of empirical data have a fat tail, which is known as a feature of complexity. We showed that these tails are dominated by the tails of lognormal distributions. We also showed that viewing with memory can generate lognormal. Therefore, understanding the mechanisms underlying the transition and expressing them mathematically may contribute to understanding the role of memory in the complexity of online learning behaviors.
Availability of data and materials

The data are from the website http://www.icourse163.org. Feel free to get in contact with the corresponding author in case you need more information.

Competing interests

The author declares that he has no competing interests.

Funding

The author acknowledges the support from National Science Foundation of China (NSFC) Grant No. 61773020.

Acknowledgments

The author thanks Xiao Xiao in the Higher Education Press, Jinying Su and Jianping Li in the National University of Defense Technology for their helpful comments and feedback. The author is grateful to the MOOC platform iCourse for its empirical data.

References

1. Breslow L, Pritchard DE, DeBoer J, Stump GS, Ho AD, Seaton DT (2013) Studying learning in the worldwide classroom research into edX’s first MOOC. Res Pract Assess 8: 13-25.

2. Zhu M, Sari A, Lee MM (2018) A systematic review of research methods and topics of the empirical MOOC literature (2014-2016). Int High Educ, 37: 31-39.

3. Emanuel EJ (2013) Online education: MOOCs taken by educated few. Nature 503(7476), 342-342.

4. Reich J (2015) Rebooting MOOC research. Science 347(6217): 34-35.

5. Anderson A, Huttenlocher D, Kleinberg J, Leskovec J (2014) Engaging with massive online courses. In Proceedings of the 23rd international conference on World wide web (pp. 687-698). ACM.
6. Jona K, Naidu S (2014) MOOCs: emerging research. Dist Educ 35(2): 141-144.

7. DeBoer J, Ho AD, Stump GS, Breslow L (2014) Changing “course” reconceptualizing educational variables for massive open online courses. Educ Res 43(2): 74-84.

8. Meyer JP, Zhu S (2013) Fair and equitable measurement of student learning in MOOCs: An introduction to item response theory, scale linking, and score equating. Res Pract Assess 8: 26-39.

9. Sunar AS, Su W, Abdullah NA, Davis HC (2017) How learners’ interactions sustain engagement: a MOOC case study. IEEE T Learn Technol, 10(4), 475-487.

10. Emmons SR, Light RP, Börner K (2017) MOOC visual analytics: empowering students, teachers, researchers, and platform developers of massively open online courses. J Assoc Inf Sci Technol, 68: 2350-2363.

11. Sandeen C (2013) Assessment’s Place in the New MOOC World. Res Pract Assess 8: 5-12.

12. de Freitas SI, Morgan J, Gibson D (2015) Will MOOCs transform learning and teaching in higher education? Engagement and course retention in online learning provision. Br J Educ Technol 46(3), 455-471.

13. Greene JA, Oswald CA, Pomerantz J (2015) Predictors of retention and achievement in a massive open online course. Am Educ Res J 52(5), 925-955.

14. Hone KS, Said GRE (2016) Exploring the factors affecting MOOC retention: A survey study. Comput Educ 98, 157-168.

15. Littlejohn A, Hood N, Milligan C, Mustain P (2016) Learning in MOOCs: motivations and self-regulated learning in MOOCs. Int High Educ, 29, 40-48.

16. Barak M, Watted A, Haick H (2016) Motivation to learn in massive open online courses: Examining aspects of language and social engagement. Comput Educ 94, 49-60.

17. de Barba PG, Kennedy GE, Ainley MD (2016) The role of students’ motivation and participation in predicting performance in a MOOC. J Comput Assist Learn 32(3), 218-231.

18. Watted A, Barak M (2018) Motivating factors of MOOC completers: comparing between university-affiliated students and general participants. Int High Educ 37, 11-20.
19. Zheng S, Rosson MB, Shih PC, Carroll JM (2015) Understanding student motivation, behaviors and perceptions in MOOCs. Proceedings of the 18th ACM conference on computer supported cooperative work & social computing (pp. 1882-1895). ACM.

20. Alraimi KM, Zo H, Ciganek AP (2015) Understanding the moocs continuance: the role of openness and reputation. Comput Educ 80, 28-38.

21. Jordan K (2014) Initial trends in enrolment and completion of massive open online courses. Int Rev Res Open d Dist Learn 15, 133-160.

22. Kizilcec RF, Piech C, Schneider E (2013). Deconstructing disengagement: analyzing learner sub-populations in massive open online courses. In Proceedings of the third international conference on learning analytics and knowledge (pp. 170-179). ACM.

23. Henrie CR, Halverson LR, Graham CR (2015) Measuring student engagement in technology-mediated learning: A review. Comput Educ 90, 36-53.

24. Hew KF (2016) Promoting engagement in online courses: What strategies can we learn from three highly rated MOOCs. Br J Educ Technol 47(2), 320-341.

25. Li LY, Tsai CC (2017) Accessing online learning material: Quantitative behavior patterns and their effects on motivation and learning performance. Comput Educ 114: 286-297.

26. Cheng G, Chau J (2016) Exploring the relationships between learning styles, online participation, learning achievement and course satisfaction: An empirical study of a blended learning course. Br J Educ Technol 47(2), 257-278.

27. Li N, Kidziński L, Jermann P, Dillenbourg P(2015) MOOC video interaction patterns: What do they tell us? Design for teaching and learning in a networked world (pp. 197-210). Springer International Publishing.

28. Klutke GA, Kiessler PC, Wortman MA (2003) A critical look at the bathtub curve. IEEE T Reliab 52(1), 125-129.

29. Holman J (2013) Antifragile: things that gain from disorder. Quant Financ, 13(11), 1691-1692.
30. Xie Z (2018) Exploring the value of viewing behavior data for massive open online courses. ArXiv: 1803.01698.

31. Kleinbaum DG, Klein M (2012) Survival Analysis. Springer New York.

32. Kiefer NM (1988) Economic duration data and hazard functions. J Econ Lit 26(2), 646-679.

33. Gaddum JH (1945) Lognormal distributions. Nature 156(3964), 463.

34. Sutton J (1997) Gibrat’s legacy. J Econ Lit 35(1), 40-59.

35. Xie Z, Ouyang ZZ, Li JP, Dong EM, Yi DY (2018) Modelling transition phenomena of scientific coauthorship networks. J Assoc Inf Sci Technol 69: 305-317.

36. Xie Z, Li JP, Li M (2018) Exploring cooperative game mechanisms of scientific coauthorship networks. Complexity 9173186.

Appendix

Categorization of learners

The following algorithm comes from our previous work [30].

\textbf{Table 4. An algorithm of categorizing learners .}

\begin{center}
\begin{tabular}{l}
\hline
Input: the viewing duration $t_s$ and the number of viewed videos $n_s$ of learners $L_s$ ($s = 1, ..., m$).

\texttt{For} $k$ from 0 to $\max(n_1, ..., n_m)$ \texttt{do:}

\hspace{1cm} Do the KS test for $t_s$ of the learners $L_s$ satisfying $n_s > k$ with the null hypothesis that they follow a lognormal distribution;

\hspace{1cm} Break if the test cannot reject the null hypothesis at significance level 5%.

\texttt{Output:} the current $k$ (denoted as $\kappa$).

\hline
\end{tabular}
\end{center}

The unit of durations is millisecond. Categorize learner $L_s$ as a lognormal-learner if $n_s > \kappa$, and as a segment-learner if else.

The process of generating synthetic durations
Table 5. Modelling exponential cutoff.

| Input: the empirical durations ($\geq 1$ minute) of segment-learners. |
| Regress the coefficients of $e^{-x/\lambda}$ for the input; |
| Calculate the domain of the input: $[T_1, T_2]$; |
| For $i$ in range 1 to the number of empirical durations |
| Sample a $y/T$ from $[e^{-T_2/\lambda}, e^{-T_1/\lambda}]$; |
| Substitute it into Eq. (4) to obtain a random integer; |
| Append the integer to the list of synthetic durations. |

Output: the list of synthetic durations.

The unit of durations is 2 seconds.

Table 6. Modelling power-law part.

| Input: the empirical durations ($< 1$ minute) of segment-learners; |
| The domain $[S_1, S_2]$ of $\nu$. |
| Regress the coefficients of $cx^\alpha$ for the input; |
| Calculate the parameters of Eq. (9): $b = 1/(1 - \alpha)$, $a = (c(S_2 - S_1 + 1)b^\log 2)$; |
| Calculate the domain of the input: $[R_1, R_2]$; |
| For $i$ in range 1 to the number of empirical durations; |
| Sample a $\nu$ from $[(aR_1/\log 2)^{1/b}, (aR_2/\log 2)^{1/b}]$; |
| Sample a $y/T_2$ from $[e^{-R_2/\lambda(\nu)}, e^{-R_1/\lambda(\nu)}]$; |
| Substitute them into Eq. (9) to obtain a random integer; |
| Append the integer to the list of synthetic durations. |

Output: the list of synthetic durations.

The unit of durations is 2 seconds, $S_1 = 1$ and $S_2 = 29.$