Realizing Einstein’s Mirror: Optomechanical Damping with a Thermal Photon Gas

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Einstein described the damping and thermalization of the center-of-mass motion of a mirror placed inside a blackbody cavity [1]. Here, in analogy with Brownian motion, a dynamic equilibrium between the momentum fluctuations of the BB light and the object would bring the motional temperature of the object to that of the BB. Importantly, this was used by Einstein to understand the Planck description of BB sources [1]. This process was further explored as a potential mechanism for damping on astronomical scales [2,3] by thermal radiation from the cosmic microwave background at 3K. However, it was found that the damping time for any object was significantly longer than the lifetime of the Universe [2]. This damping process is weak, because as the temperature of a BB decreases, the number of photons also decreases since a BB has a chemical potential of zero.

Thermal sources of light with a well-defined chemical potential have only recently been realized [4,5]. These sources allow control over the chemical potential and therefore the number of photons for a fixed temperature. They are in a dynamic equilibrium such that the photons come into thermal equilibrium with an active medium via absorption and emission. Here, we show that when illuminated by these sources the motion of microscopic optomechanical objects are damped, while their center-of-mass motion thermalizes to the source temperature. This occurs on timescales of tens of seconds, making an experimental demonstration feasible.

In 1909 Einstein described how an object’s motion would be damped by the recoil of photons when placed inside a blackbody cavity [1]. Here, in analogy with Brownian motion, a dynamic equilibrium between the momentum fluctuations of the BB light and the object would bring the motional temperature of the object to that of the BB. Importantly, this was used by Einstein to understand the Planck description of BB sources [1]. This process was further explored as a potential mechanism for damping on astronomical scales [2,3] by thermal radiation from the cosmic microwave background at 3K. However, it was found that the damping time for any object was significantly longer than the lifetime of the Universe [2]. This damping process is weak, because as the temperature of a BB decreases, the number of photons also decreases since a BB has a chemical potential of zero.

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Einstein initially considered the motional damping of an object by BB radiation when placed inside the BB cavity [1]. To illustrate this process we consider a mirror placed outside the cavity and illuminated by the light emanating from the BB. The mirror is a disk of area $A$ and mass $M$ with a frequency dependent reflectivity $R(\omega)$. The spectral distribution of the BB photons is described by the Bose-Einstein (BE) distribution. We consider the disk in motion with velocity $v_z$ along the $z$ axis. Incoming photons make an angle $\theta$ with the surface normal of the disk along the $z$ axis. The number density and variance of the BB photons per unit angular frequency, solid angle, and volume are given by

$$n(\omega) = \frac{\rho_n}{\omega^2/4\pi^3c^3} \left\{ \frac{1}{\exp(\hbar\omega/k_BT) - 1} \right\}$$

and

$$\Delta N^2 = \frac{\rho_n}{\omega^2/4\pi^3c^3} \left\{ \frac{1}{\exp(\hbar\omega/k_BT) - 1} \right\} \left\{ \frac{1}{\exp(\hbar\omega/k_BT) - 1} \right\}$$

respectively [6], where $\omega$ is the angular frequency of the photons, $k_B$ is the Boltzmann constant, $\hbar$ is the reduced Planck constant, $c$ is the speed of light in vacuum, and $T$ is the bulk temperature of the blackbody source. In the moving frame of the disk, the BB source appears at an effective energy that the disk gains per second is

$$E_z = \frac{\rho_n}{\omega^2/4\pi^3c^3} \left\{ \frac{1}{\exp(\hbar\omega/k_BT) - 1} \right\}$$

and

$$\Gamma_z = \frac{\rho_n}{\omega^2/4\pi^3c^3} \left\{ \frac{1}{\exp(\hbar\omega/k_BT) - 1} \right\}$$

where $\beta_z = v_z/c$ and $v_z \ll c$. The total force [2,7] delivered by all photons incident on the moving disk from a blackbody is

$$F_z(v_z) = \int_0^{\pi/2} \int_0^{2\pi} \int_0^\infty R(\omega) cA \cos \theta \times \rho_n(v_z) 2\hbar k \cos \theta d\omega d\Omega \approx \frac{A\pi^2 k_B^4 T^4}{45c^3\hbar^4} v_z,$$

(1)

where the solid angle $d\Omega$ is $\sin \theta d\phi d\theta$ and we have set $R(\omega) = 1$ as considered by Einstein. The first term in Eq. (1) is the radiation pressure force, while the velocity dependent second term is the radiation damping due to recoil of photons at rate $\Gamma_z = (A\pi^2 k_B^4 T^4/15Mc^4\hbar^3)$. The energy that the disk loses per second [6] is $M\Gamma_z v_z^2$, where $v_z^2 = k_B T_{c.m.}/M$ and $T_{c.m.}$ is the center-of-mass (c.m.) temperature of the disk. In addition, due to the fluctuations in photon number [1,6], the energy that the disk gains per second is

$$\Delta E = \frac{1}{2M} \int_0^{\pi/2} \int_0^{2\pi} \int_0^\infty R(\omega) cA \cos \theta \times \Delta N^2 (2\hbar k \cos \theta)^2 d\omega d\Omega \approx \frac{A\pi^2 k_B^4 T^4}{15Mc^4\hbar^4}.$$

(2)
At equilibrium, the loss and the gain rate in energy are equal so that $MT_z v_z^2 = \Delta \dot{E}$, and the c.m. temperature is equal to the BB temperature, $T_{\text{c.m.}} = T$. This is the result calculated by Einstein [1] and also later calculations for objects within the 3K cosmic microwave background of the Universe [2,3].

We now consider the BB damping of a silica disk which has a radius $r = 5 \mu m$, thickness $50 \text{ nm}$, and mass density $2200 \text{ kg m}^{-3}$.

The cavity traps photons emitted by the dye molecules when optically pumped. The dye molecules are a thermal bath for the photons providing the necessary chemical potential for conserving photon number when the temperature is varied [4]. The chemical potential can be adjusted by changing the number of dye molecules or the pump power [10]. The photon statistics of these sources are still given by the Bose-Einstein distribution, with the inclusion of a chemical potential $\mu_c$ determined by the ratio of molecules in the excited state to those in the ground state [4,10,15]. The energy density inside the cavity is given by

\[
\tilde{u} = \frac{1}{V} \sum_{n_x=0}^{\infty} \sum_{n_y=0}^{\infty} 2(n_x + n_y + 1) \frac{\hbar(\omega_c + (n_x + n_y + 1)\omega)}{\exp[(\hbar(\omega_c + (n_x + n_y + 1)\omega) - \mu_c)/k_B T] - 1},
\]

where $n_x$ and $n_y$ are the transverse mode numbers of the cavity, $\omega = 2\pi c/n\sqrt{D_0 R/2}$ is the difference in frequency between two consecutive transverse modes, $\omega_c = q\pi c/nD_0$ is the angular frequency of the longitudinal cavity mode.
number $q$, and $n$ is the refractive index of the cavity filler material. The cavity mirrors have a radius of curvature $R$, with cavity volume $V_r$. In the limit where $\hbar \omega \ll k_BT$ \cite{10,16}, the average number of photons transmitted through one of the cavity mirrors, per angular frequency and solid angle is
\[ \hat{N} = \left( \frac{V_r T_r}{n q D_0} \right) \frac{\omega_r}{4 \pi c^2} \left\{ \frac{1}{\exp\left[ \hbar (\omega_e + \omega) - \mu_e \right] / k_BT - 1} \right\}, \]
where $T_r$ is the transmission coefficient of the cavity mirror (see Supplemental Material \cite{17}, Sec. iii for details). Given that $\exp\left[ \hbar (\omega_e + \omega) - \mu_e \right] / k_BT \gg 1$ \cite{4}, $\hat{N}$ can be approximated as $\left( \frac{V_r T_r}{q n D_0} \right) \frac{\omega_r}{4 \pi^2 c^2} \left\{ 1 / \left( \exp\left[ \hbar (\omega_e + \omega) - \mu_e \right] / k_BT \right) \right\}$. Output powers of tens of nanowatts have been demonstrated \cite{4,10}. Because of the relatively narrow bandwidth (±60 nm \cite{4}) of the light compared to a blackbody, it can be amplified using optical amplifiers with gain $G$. We assume that the photon statistics are not significantly modified by the amplification process by noting that an optical amplifier, in addition to replicating the photon statistics of the input, adds a small thermal field typically via amplified spontaneous emission \cite{18,19}. The addition of a thermal component to an already thermal source will not change the input statistics. This has more recently been shown to hold for broadband amplification of the thermal light produced by amplified spontaneous emission \cite{20}. Amplifiers do not, however, have a flat spectral profile. This can be ameliorated by using a spectral filter on the input to compensate for this variation \cite{11,21,22}. When illuminated with this amplified light, the force on the mirror in the moving frame is
\[ F_z = \frac{G V_r T_r \exp[\mu_e / k_BT]}{q n D_0} \left\{ \int_0^\pi \int_0^{2\pi} \int_0^\infty \mathcal{R}(\omega) 2 \hbar (\omega_e + \omega) \cos^2 \theta d\omega d\Omega \right\} \approx \frac{2G}{3c} P - \frac{G \hbar \omega_z}{2c^2 k_BT} P v_z, \]
where
\[ P = \exp\left[ \left( \mu_e - \hbar \omega_z \right) / k_BT \right] \left( V_r T_r k_B^2 T^2 \omega_e^2 / 2 q n D_0 \pi^2 \hbar^2 \right) \] is the cavity output power before amplification. The diameter of the incident light beam (see Fig. 2(a)) is equal to or smaller than that of the disk. Since the 2D thermal light source is spectrally narrow compared to a BB source, disks can have 100% reflectivities in this spectral range such that $R(\omega) = 1$. The damping rate of the disk is $\Gamma_z = (G/2Mc^2)(\hbar \omega_z / k_BT) P$. The rate of energy gain due to the fluctuation in photon number is now $\Delta \dot{E}_z \approx (G \hbar \omega_z / 2Mc^2) P$. The equilibrium center-of-mass temperature of the mirror is $T_{c.m.} = \Delta \dot{E}_z / k_BT \Gamma_z = T$ which is the same as that obtained for the BB source. In this case, however, both $\Gamma_z$ and $\Delta \dot{E}_z$ are adjustable through the chemical potential $\mu_e$, and the optical gain $G$. Figure 3 shows the damping time $2\pi/\Gamma_z$ at $T = 300$ K as a function of the normalized chemical potential $\mu_e / \hbar \omega_z$. The parameters used in our calculation are typical of 2D experimental microcavities \cite{4}. For a chemical potential $\mu_e / \hbar \omega_z = 0.92$, and an optical amplifier gain of 80 dB, we calculate a damping time of $2\pi/\Gamma_z \approx 80$ sec. This is eight orders of magnitude less than a BB source at the same temperature. If the mirror is not perfectly reflecting, the damping time will increase but importantly the equilibrium CM temperature remains the same and has been shown to hold for a linear change in the reflectivity (see Supplemental Material \cite{17}).

A levitated dielectric sphere damped by a thermal photon gas.—We consider a levitated dielectric sphere of radius $r \ll 2\pi c / \alpha$ with a scattering cross section of $\sigma = (\alpha^2 \omega_m^4 / 6\pi c^4)$ illuminated by amplified 2D thermal light from a microcavity. The frequency of the incident light is $\omega_m = \omega_e + \alpha$ and $\alpha$ is the polarizability of the particle \cite{23}. Furthermore, we assume that the amplified light is tightly focused using a lens to a spot size of area $A_m$. The particle could be levitated in a Paul trap \cite{9}, or by the thermal light itself \cite{11}. In the laboratory frame, the wave vector of an incident photon is given by $k_m = [k_x, k_y, k_z]$, where $k_m = \omega_m / c$. In the particle frame, the incident and scattered photon frequency is $\omega_m (1 + \beta)$, where $\beta = (\mathbf{v} \cdot \mathbf{k}_m / c k_m)$, and $\mathbf{v} = [v_x, v_y, v_z]$ is the velocity of the particle along the three axes. The unpolarized incident photons means that the scattered photons are isotropically distributed over $4\pi$ steradians. After a scattering event, each photon delivers momentum to the particle equivalent to $\mathbf{p} = \hbar k_m (1 + \beta)(\Theta_m - \Theta_\infty)$, where $\Theta_m = [\sin \theta_m \cos \phi_m, \sin \theta_m \sin \phi_m, \cos \theta_m]$ and $\Theta_\infty = [\sin \theta_\infty \cos \phi_\infty, \sin \theta_\infty \sin \phi_\infty, \cos \theta_\infty]$. Here, $\theta_m$ and $\phi_m$ represents the polar and the azimuthal angles that the wave vector of an incoming photon makes with the $-z$ axis and the $+x$ axis respectively. $\theta_\infty$ and $\phi_\infty$ are the corresponding angles that the wave vector of a scattered photon makes with the same reference axes. With the appropriate Lorentz transformation, the total force exerted by all photons is
where $\Lambda = [111]$. The equilibrium center-of-mass temperature of the particle along all axes is equal to the bulk temperature $T$ of the dye molecules, i.e., $T_{\text{c.m.}} = T$. This is strikingly different to the value calculated for a laser trapped neutral nanoparticle in an optical trap [12,13,29,30], in recent advances in optomechanics [11,13,14]. An experimental demonstration using a levitated optomechanical object, such as a charged nanoparticle in a Paul trap [9] or a neutral nanoparticle in an optical trap [12,13,29,30], in ultrahigh vacuum seems ideal. Although we have calculated the damping from a 2D source, a 1D source [5] could potentially be focused more tightly, leading to a higher damping. Amplified LEDs and superluminescent diodes, which produce thermal light [8,31] and have been used to trap dielectric spheres [11], could also be considered for investigating thermal radiation damping. Our results raise the possibility that by increasing or decreasing the bulk temperature of a thermalized light source one
can heat or cool the center-of-mass temperature of a levitated nanoparticle without requiring feedback cooling [11,13,27,29,30]. Lastly, although we have considered damping by a thermal light source with an adjustable chemical potential, it would also be interesting to study the interaction of an optomechanical object with a photon BEC [10,32,33], where $\mu = 1$.

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