Comments on $D$-term inflation

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Abstract

An inflationary stage dominated by a $D$-term avoids the slow-roll problem of inflation in supergravity and can naturally emerge in theories with a non-anomalous or anomalous $U(1)$ gauge symmetry. In the latter case, however, the scale of inflation as imposed by the COBE normalization is in contrast with the value fixed by the Green-Schwarz mechanism of anomaly cancellation. In this paper we discuss possible solutions to this problem and comment about the fact that the string-loop generated Fayet-Iliopoulos $D$-term may trigger the presence of global and local cosmic strings at the end of inflation.
1. It is commonly accepted that inflation looks more natural in supersymmetric theories rather in non-supersymmetric ones. This is because the necessity of introducing very small parameters to ensure the extreme flatness of the inflaton potential seems very unnatural and fine-tuned in most non-supersymmetric theories, while this naturalness is achieved in supersymmetric models. The nonrenormalization theorems in exact global supersymmetry guarantee that we can fine-tune any parameter at the tree-level and this fine-tuning will not be destabilized by radiative corrections at any order in perturbation theory. This is the advantage of invoking supersymmetry. There is, however, a severe problem one has to face when dealing with inflation model building in the context of supersymmetric theories. The generalization of supersymmetry from a global to a local symmetry automatically incorporates gravity and, therefore, inflation model building must be considered in the framework of supergravity theories.

In small-field models of inflation (values of fields smaller than the reduced Planck scale $M_{\text{Pl}} \approx 2.4 \times 10^{18}$ GeV), where the theory is under control, it is reasonable to work in the context of supergravity. This is a relatively recent activity because although small-field models were the first to be proposed they were soon abandoned in favour of models with fields first of order the Planck scale and then much bigger. Activity began again after hybrid inflation was proposed, with the realization that the model is again of the small-field type. In Ref. supersymmetric implementations of hybrid inflation were considered, in the context of both global supersymmetry and of supergravity.

The supergravity potential is rather involved, but it can still be written as a $D$-term plus an $F$-term, and it is usually supposed that the $D$-term vanishes during inflation. Now, for models where the $D$-term vanishes, the slow-roll parameter $\eta = M_{\text{Pl}}^2 V''/V$ generically receives various contributions of order $\pm 1$. This is the so-called $\eta$-problem of supergravity theories. This crucial point was first emphasized in Ref., though it is essentially a special case of the more general result, noted much earlier, that there are contributions of order $\pm H^2$ to the mass-squared of every scalar field. Indeed, in a small-field model the troublesome contributions to $\eta$ may be regarded as contributions to the coefficient $m^2$ in the expansion of the inflaton potential. Therefore, it is very difficult to naturally implement a slow-roll inflation in the context of supergravity. The problem basically arises since inflation, by definition, breaks global supersymmetry because of a nonvanishing cosmological constant (the false vacuum energy density of the inflaton). In supergravity theories, supersymmetry breaking is transmitted by gravity interactions and the squared mass of the inflaton becomes...
naturally of order of $V/M_{Pl}^2 \sim H^2$. The perturbative renormalization of the Kähler potential is therefore crucial for the inflationary dynamics due to a non-zero energy density which breaks supersymmetry spontaneously during inflation. How severe the problem is depends on the magnitude of $\eta$. If $\eta$ is not too small then its smallness could be due to accidental cancellations. Having $\eta$ not too small requires that the spectral index $n = 1 - 6\epsilon + 2\eta$ ($\epsilon = \frac{1}{2}M_{Pl}^2(V'/V)^2$ is another slow-roll parameter) be not too small, so the observational bound $|n - 1| < 0.3$ is already beginning to make an accident look unlikely.

2. Several proposals to solve the $\eta$-problem already exist in the literature \[12\]. One of the most promising solutions is certainly $D$-term inflation \[13\, 14\]. It is based on the observation that $\eta$ gets contributions of order 1 only if inflation proceeds along a $D$-flat direction or, in other words, when the vacuum energy density is dominated by an $F$-term. On the contrary, if the vacuum energy density is dominated by nonzero $D$-terms and supersymmetry breaking is of the $D$-type, scalars get supersymmetry soft breaking masses which depend only on their gauge charges. Scalars charged under the corresponding gauge symmetry obtain a mass much larger than $H$, while gauge singlet fields can only get masses from loop gauge interactions. In particular, if the inflaton field is identified with a gauge singlet, its potential may be flat up to loop corrections and supergravity corrections to $\eta$ from the $F$-terms are not present.

If the theory contains an abelian $U(1)$ gauge symmetry (anomalous or not), the Fayet-Iliopoulos $D$-term term

$$\xi \int d^4 \theta \, V = \xi D$$

is gauge invariant and therefore allowed by the symmetries. It may lead to $D$-type supersymmetry breaking. It is important to notice that this term may be present in the underlying theory from the very beginning or appears in the effective theory after some heavy degrees of freedom have been integrated out. It looks particularly intriguing, however, that an anomalous $U(1)$ symmetry is usually present in string theories \[16\]. The corresponding Fayet-Iliopoulos term is \[17\]

$$\xi = \frac{g^2}{192\pi^2} \text{Tr}Q \, M_{Pl}^2,$$

where $\text{Tr}Q \neq 0$ indicates the trace over the $U(1)$ charges of the fields present in the spectrum of the theory. The $U(1)$ group may be assumed to emerge from string theories so that and the anomaly is cancelled by the Green-Schwarz mechanism. In such a case $\sqrt{\xi}$ is expected to be of the order of the stringy scale, $(10^{17} - 10^{18})$ GeV or so.
Let us remind the reader how $D$-term inflation proceeds \[14,15\]. To exemplify the description, let us take the toy model containing three chiral superfields $S$, $\Phi_+$ and $\Phi_-$ with charges equal to 0, +1 and −1 respectively under the $U(1)$ gauge symmetry. The superpotential has the form

$$W = \lambda S \Phi_+ \Phi_-.$$  \hspace{1cm} (3)

The scalar potential in the global supersymmetry limit reads

$$V = \lambda^2 |S|^2 \left( |\phi_-|^2 + |\phi_+|^2 \right) + \lambda^2 |\phi_+ \phi_-|^2 + \frac{g^2}{2} \left( |\phi_+|^2 - |\phi_-|^2 + \xi \right)^2$$  \hspace{1cm} (4)

where $\phi_\pm$ are the scalar fields of the supermultiplets $\Phi_\pm$, $g$ is the gauge coupling and $\xi > 0$ is a Fayet-Iliopoulos $D$-term. The global minimum is supersymmetry conserving, but the gauge group $U(1)$ is spontaneously broken

$$\langle S \rangle = \langle \phi_+ \rangle = 0, \quad \langle \phi_- \rangle = \sqrt{\xi}.$$  \hspace{1cm} (5)

However, if we minimize the potential, for fixed values of $S$, with respect to other fields, we find that for $S > S_c = \frac{\xi}{\lambda} \sqrt{\xi}$, the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $S > S_c$ and $\phi_+ = \phi_- = 0$ the tree level potential has a vanishing curvature in the $S$ direction and large positive curvature in the remaining two directions $m_\pm^2 = \lambda^2 |S|^2 \pm g^2 \xi$. For arbitrarily large $S$ the tree level value of the potential remains constant $V = \frac{g^2}{2} \xi^2$ and the $S$ plays the role of the inflaton. As stated above, the charged fields get very large masses due to the $D$-term supersymmetry breaking, whereas the gauge singlet field is massless at the tree-level.

What is crucial is that, along the inflationary trajectory $\phi_\pm = 0$, $S \gg S_c$, all the $F$-terms vanish and large supergravity corrections to the $\eta$-parameter do not appear. Therefore, we do not need to make any assumption about the structure of the Kähler potential for the $S$-field: minimal $S^* S$ and non-minimal quartic terms in the Kähler potential $(S^* S)^2$ (or even higher orders) do not contribute in the curvature, since $F_S$ is vanishing during inflation.

Since the energy density is dominated by the $D$-term, supersymmetry is broken and this amounts to splitting the masses of the scalar and fermionic components of $\Phi_\pm$. Such splitting results in the one-loop effective potential for the inflaton field

$$V_{1\text{-\text{loop}}} = \frac{g^2}{2} \xi^2 \left( 1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |S|^2}{Q^2} \right).$$  \hspace{1cm} (6)

The end of inflation is determined either by the failure of the slow-roll conditions or when $S$ approaches $S_c$. COBE imposes the following normalization
\[ 5.3 \times 10^{-4} = \frac{V^{3/2}}{V' M_{Pl}^3} \]  

which can be written in the equivalent form

\[ \frac{V^{1/4}}{\epsilon^{1/4}} = 8 \times 10^{16} \text{ GeV}. \]  

More or less independently of the value of \(|S|\) at the end of inflation, this gives with the above potential

\[ \sqrt{\xi} = 6.6 \times 10^{15} \text{ GeV}. \]  

Notice that his normalization is independent from the gauge coupling constant \(g\), but depends on the numerical coefficient in the one-loop potential. This, in turn, depends upon the particle content of the specific model under consideration.

The spectral index results

\[ n = 1 - \frac{2}{N} = (0.96 - 0.98). \]  

Now, the value of \(\xi\) looks too small to be consistent with the value arising in many compactifications of the heterotic string [17] (even though some level of flexibility is allowed in M-theory [18]).

The stabilisation of the dilaton field is also an unsolved problem in heterotic string theories. One may ask whether the value of \(\sqrt{\xi}\) required by density perturbations can be motivated by a realistic string theory. At this point uncertainties come from the fact that \(\xi\) is always treated as constant (up to a coarse-graining scale dependence through the gauge-coupling). This is certainly justified in the effective field theory approach in which \(\xi\) is treated as an input parameter. In string theories the gauge and gravitational coupling constants are set through the expectation value of the dilaton field and the Fayet-Iliopoulos \(D\)-term actually is a function of the real part of the dilaton field. Since the dilaton potential most likely is strongly influenced by the inflationary dynamics, the actual value of \(\xi\) at the moment when observationally interesting scales crossed the horizon during inflation might be quite different from the one "observed" today. It seems that entire question is related to the problem of the dilaton stabilization and it is hard to make any definite statement without knowing the details of the dilaton dynamics during inflation. All the estimates made above are valid within an effective field theory description, in which the gauge and
gravitational constants can be treated as parameters whose inflaton-dependence arises from
the coarse-graining scale-dependence [19,20].

3. One might ask if the COBE normalization permits a bigger $\xi$, if the slope of the
potential is altered. Before looking at a couple of specific possibilities, we note the general
point that a dramatic increase will not be possible, unless $g$ is very small. The reason is
that slow-roll inflation requires $\epsilon \ll 1$, so that the COBE normalization requires $V_0^{1/4} \ll 8 \times 10^{16}$ GeV. In fact, barring a cancellation, the observational result $|n - 1| \lesssim 0.2$ requires $6\epsilon \lesssim 0.2$, corresponding to $V^{1/4} \lesssim 3.4 \times 10^{16}$ GeV. This translates to

$$\xi^{1/2} \lesssim \frac{4.1 \times 10^{16}}{g^{1/2}} \text{ GeV}, \quad (11)$$

but $g^2/4\pi \sim 0.1$ would correspond to $g \sim 1$, and to increase $\xi^{1/2}$ much above $4 \times 10^{16}$ GeV requires an unreasonably small $g^2$.

Still the extra order of magnitude is worth having, so let us see how it might be done.
One possibility is to increase the numerical coefficient $c$ in front of the one-loop term. If
we rewrite the one-loop effective potential as $V_{\text{1-loop}} \sim V_0 \left(1 + cg^2 \ln |S|\right)$, $c = (1/8\pi^2)$ in
the example above. This coefficient depends upon the number of degrees of freedom which
are coupled to the field playing the role of the inflaton and is expected to be quite large,
especially if the theory is embedded in some grand unified gauge group. For instance, the
superfields $\Phi_{\pm}$ might be interpreted as 126 and $\overline{126}$ Higgs superfields of $SO(10)$ and there
might be more the one vector-like pair coupled to the $S$-field.

It is easy to show that the COBE normalization gives $\xi^{1/2} \propto c^{1/4}$, so a reasonable value
$8\pi^2 c \sim 100$ would give us a modest increase to $\xi^{1/2} \simeq 2 \times 10^{16}$ GeV.

Another possibility is to suppose that the $V'$ is dominated by a tree-level term. The sim-
plest possibility is mass term, $V_S \equiv \frac{1}{2}\tilde{m}^2 \phi^2$ where $\phi \equiv \sqrt{2}|S|$ is the inflaton field. This term
may be easily generated. It might be present in the theory under the form of supersymmet-
ric mass. Otherwise, imagine that in some sector of the underlying theory supersymmetry
is broken by some $F$-term. Supersymmetry breaking may be communicated to the $S$-field
by gravitational interactions and in this case $\tilde{m}^2 \sim F^2/M_{\text{Pl}}^2$. Another possibility is that
supersymmetry breaking is transmitted by gauge interactions (with gauge group $G$). In this
case, the sector which breaks supersymmetry is assumed to first transmit supersymmetry

\[\text{[1]}\] In this case the fields $\phi_{\pm}$ get also the same mass $\tilde{m}$, but it is easy to show that, in the regime
we will be working, its presence doesn’t affect the main conclusions.
breaking to some fields (usually called the messangers) charged under $G$ by gauge interactions. In turn, these messangers are coupled to the field $S$ which receives a nonvanishing soft supersymmetry breaking squared mass term at two-loops, $\tilde{m}^2 \sim \alpha_G^2 F$, where $\alpha_G$ is the gauge coupling constant.

If $\phi \equiv \sqrt{2}|S|$ is the inflaton field, the potential during inflation is

$$V = V_0 + V_S + V_{\text{1-loop}},$$

where

$$V_0 \equiv \frac{g^2}{2} \xi^2,$$  \hspace{1cm} (13)

$$V_S \equiv \frac{1}{2} \tilde{m}^2 \phi^2,$$  \hspace{1cm} (14)

$$V_{\text{1-loop}} \equiv (8\pi^2 c) \frac{g^4 \xi^2}{16\pi^2} \ln \left( \frac{\lambda \phi}{\sqrt{2} Q} \right).$$  \hspace{1cm} (15)

The derivatives are

$$V_S' = \tilde{m}^2 \phi,$$  \hspace{1cm} (16)

$$V_{\text{1-loop}}' = (8\pi^2 c) \frac{g^4 \xi^2}{16\pi^2} \frac{1}{\phi}.$$  \hspace{1cm} (17)

We want to suppose that $V_{\text{1-loop}}'' \ll V_S'$ in the interval of interest, which is

$$2\frac{g^2}{\lambda^2} \xi < \phi^2 < 2\frac{g^2 \xi^2}{\lambda^2} e^{2x},$$  \hspace{1cm} (18)

where the lower end corresponds to $\phi_c^2$ and $x$ is defined as $x = N(n - 1)/2$, $N$ being the number of e-folds after the COBE scale leaves the horizon. If this is satisfied we shall also have $V_{\text{1-loop}}'' \ll V_S''$. Also, the loop contribution at the end of inflation will then be

$$\frac{V_{\text{1-loop}}}{V_0} \approx (8\pi c^2) \frac{g^2}{8\pi^2}$$  \hspace{1cm} (19)

which is much smaller than 1 for any reasonable choice of parameters. Thus, if Eq. (18) is satisfied, the loop is negligible in all respects.

We also want to assume that $V_0$ dominates in the interval of interest, because otherwise the COBE normalization will need $\phi \gtrsim M_{\text{Pl}}$ making it unreasonable to assume a constant $\tilde{m}$. This requires

$$\frac{\tilde{m} \phi}{g \xi} \ll 1.$$  \hspace{1cm} (20)
Then the COBE normalization can be written
\[
\frac{\xi^{1/2}}{M_{Pl}} = 3 \times 10^{-4}(n - 1) \frac{e^x}{\lambda}.
\] (21)

We used the result \( n - 1 = 2\eta \), thus ignoring \( \epsilon \).

One can verify that Eqs. (18) and (20), with the COBE condition, can be written
\[
\frac{\xi^{1/2}}{\xi_{\text{max}}^{1/2}} \leq 8\pi^2 c \left( \frac{n - 1}{2} \right)^{1/4} \frac{g^{-1/2}}{M_{Pl}}
\] (22)
where
\[
\xi_{\text{max}}^{1/2} = 2.3 \times 10^{-2} g^{-1/2}(n - 1)^{1/4} M_{Pl}
\] (23)
\[
= (3.6 \times 10^{16} \text{GeV}) \left( \frac{n - 1}{2} \right)^{1/4} g^{-1/2}.
\] (24)

The lower limit is indeed lower than the upper limit for reasonable parameter. This confirms the general conclusion, that the combination \( (g\xi)^{1/2} \) cannot be bigger than a few times \( 10^{16} \text{GeV} \). Note that as the upper limit is approached, \( \epsilon \) becomes significant which complicates matters, but barring fine tuning this will not change the general conclusion.

Another possible solution to the mismatching between the value suggested by string theories and the one imposed by the COBE normalization is to completely decouple the origin of \( \xi \) from string theories and to envisage that the \( D \)-term is generated in some low-energy effective theory after some degrees of freedom have been integrated out. However, to do so, one has presumably to break supersymmetry by some \( F \)-terms present in the sector which the heavy fields belong to and to generate the \( D \)-term by loop corrections. As a result, it turns out that \( \langle D \rangle \ll \langle F^2 \rangle \), unless some fine-tuning is called for, and large supergravity corrections to \( \eta \) appear again. Let us give an example. Consider the following superpotential where a \( U(1) \) symmetry has been imposed
\[
W = \lambda X \left( \Phi_1 \Phi_2 - m^2 \right) + M_1 \Phi_1 \Phi_2 + M_2 \Phi_2 \Phi_1.
\] (25)
For \( \lambda^2 m^2 \ll M_1^2, M_2^2 \), the vacuum of this model is such that \( \langle \phi_i \rangle = \langle \bar{\phi}_i \rangle = 0 \ (i = 1, 2) \), where \( \bar{\phi}_i \) and \( \phi_i \) are the scalar components of the superfields \( \bar{\Phi}_i \) and \( \Phi_i \), respectively. Supersymmetry is broken and \( F_X = -\lambda^2 m^2 \). This means that in the potential a term like \( V = (F_X \bar{\phi}_1 \phi_1 + \text{h.c.}) \) will appear. It is easy to show that, integrating out the \( \phi_i \) and \( \bar{\phi}_i \) scalar fields, induces a a nonvanishing Fayet-Iliopoulos \( D \)-term
\[
\xi \simeq \frac{F_X^2}{16\pi^2(M_1^2 - M_2^2)} \ln \left( \frac{M_2^2}{M_1^2} \right),
\] (26)
which is, however, smaller than $F_X$ and inflation, if any, is presumably dominated by the $F$-term.

4. Another point we would like to comment on is the following: when the field $\phi_-$ rolls down to its present day value $\langle \phi_- \rangle = \sqrt{\xi}$ to terminate inflation, cosmic strings may be form since the abelian gauge group $U(1)$ is broken to unity [21]. As it is known, stable cosmic strings arise when the manifold $\mathcal{M}$ of degenerate vacua has a non-trivial first homotopy group, $\Pi_1(\mathcal{M}) \neq 1$. The fact that at the end of hybrid inflationary models the formation of cosmic strings may occur was already noticed in Ref. [22] in the context of global supersymmetric theories and in Ref. [23] in the context of supergravity theories. In $D$-term inflation the string per-unit-length is given by $\mu = 2\pi \xi$. Cosmic strings forming at the end of $D$-term inflation are very heavy and temperature anisotropies may arise both from the inflationary dynamics and from the presence of cosmic strings. From recent numerical simulations on the cosmic microwave background anisotropies induced by cosmic strings [24,25] it is possible to infer than this mixed-perturbation scenario leads to the COBE normalized value $\sqrt{\xi} = 4.7 \times 10^{15}$ GeV [21], which is of course smaller than the value obtained in the absence of cosmic strings. Moreover, cosmic strings contribute to the angular spectrum an amount of order of 75% in $D$-term inflation [21], which might render the angular spectrum, when both cosmic strings and inflation contributions are summed up, too smooth to be in agreement with present day observations [24,25].

All these considerations and, above all, the fact that the value of $\sqrt{\xi}$ is further reduced with respect to the case in which cosmic strings are not present, would appear to exacerbate the problem of reconciling the value of $\sqrt{\xi}$ suggested by COBE with the value inspired by string theories when cosmic strings are present. However, even though cosmic strings are generally produced, this is not always true. If there is an anomalous $U(1)$ factor in the four-dimensional gauge group, since the Yukawa couplings respect the anomalous $U(1)$, this becomes global, the local symmetry being broken by the mass of the gauge boson. The global $U(1)$ can be spontaneously broken by the Fayet-Iliopoulos $D$-term. This can be understood in the following way: the Fayet-Iliopoulos $D$-term depends upon the value of the dilaton and the gauge anomalous $U(1)$ is always broken through the dilaton vacuum expectation value. Indeed in the effective field theory the $U(1)$ symmetry is realised nonlinearly from the very beginning. So whenever effective field theory makes sense the anomalous $U(1)$ gauge symmetry is already broken and gauge bosons are massive. More formally, the relevant couplings of the dilaton superfield $s$ to the $U(1)$ gauge superfield $V$ (with gauge invariant
field strengths $W^\alpha$) read in the global limit

$$\mathcal{L} = -\int d^4\theta \ln \left(s + s^\dagger - \delta_{GS}V\right)$$

$$+ \int d^2\theta \left[\frac{s}{4} k \text{Tr} W^\alpha W^\alpha + \text{h.c.}\right]$$

(27)

where $\delta_{GS}$ is the Green-Schwarz coefficient and $k$ is the Kac-Moody level of the group $U(1)$. Under a $U(1)$ gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$, $s$ is shifted as

$$s \rightarrow s + \frac{i}{2} \delta_{GS}\alpha(x).$$

(28)

The gauge boson gets a mass in string theory eating the model-independent axion and the residual anomalous global $U(1)$ may be spontaneously broken by the Fayet-Iliopoulos $D$-term, in which case global cosmic strings are formed. Moreover, in realistic four-dimensional string models, there are extra local $U(1)$ symmetries that can be also spontaneously broken by the $D$-term. This happens necessarily if there are no singlet fields charged under the anomalous $U(1)$ only. In such a case, besides the previously mentioned global cosmic strings, there may arise local cosmic strings associated to the breaking of extra $U(1)$ factors. However, the condition to produce cosmic strings is $\Pi_1(\mathcal{M}) \neq 1$ and one must consider the structure of the whole potential, i.e. all the $F$-terms and all the $D$-terms. When this is done, it turns out that, depending on the specific models, some or all of the (global and local) cosmic strings may disappear. In general there can be models with anomalous $U(1)$ that have just global strings, just local strings, both global and local strings or, more important, no strings at all \cite{24}. The latter case is, to our opinion, the most preferable case since the presence of cosmic strings renders the problem of reconciling the COBE normalized low value of $\xi$ with the one suggested by string theory even worse.

In the case in which the Fayet-Iliopoulos $D$-term is present in the theory from the very beginning because of anomaly-free $U(1)$ symmetry and not due to some underlying string theory, the value $\sqrt{\xi} \sim 10^{15}$ GeV is very natural and is not in conflict with the presence of cosmic strings. The only shortcoming seems to be a too smooth angular spectrum because cosmic strings may provide most contribution to the angular spectrum. If this problem is taken seriously and one wants to avoid the presence of cosmic strings, a natural solution to it is to assume that the $U(1)$ gauge group is broken before the onset of inflation so that no cosmic strings will be produced when $\phi_-$ rolls down to its ground state. This may be easily achieved by introducing a pair of vector-like (under $U(1)$) fields $\Psi$ and $\bar{\Psi}$ and two gauge singlets $X$ and $\sigma$ with a superpotential of the form
\[ W = X \left( \kappa \bar{\Psi} \Psi - M^2 \right) + \beta \sigma \bar{\Psi} \Phi_+ + \lambda S \Phi_+ \Phi_-, \quad (29) \]

where \( M \) is some high energy scale, presumably the grand unified scale. It is easy to show that the scalar components of the two-vector superfields acquire vacuum expectation values \( \langle \psi \rangle = \langle \bar{\psi} \rangle = M \), and \( \langle X \rangle = \langle \sigma \rangle = 0 \) which leave supersymmetry unbroken and \( D \)-term inflation unaffected. In this example, cosmic strings are produced prior to the onset of inflation and subsequently diluted.

In conclusion, an inflationary stage dominated by a \( D \)-term avoids the slow-roll problem of inflation in supergravity and can naturally emerge in theories with a non-anomalous or anomalous \( U(1) \) gauge symmetry. In the latter case, however, we have shown that the scale of inflation as imposed by the COBE normalization is in contrast with the value fixed by the Green-Schwarz mechanism of anomaly cancellation. We have discussed different possible solutions to this problem, \textit{e.g.} the inclusion of a new term in the tree-level potential. We have also commented on the fact that at the end of inflation global and local cosmic strings may be generated. If so, cosmic strings generate density perturbations which, in turn, lowers the COBE normalized value of \( \xi \) and exacerbate the problem of reconciling this value with the one suggested by string theory. However, as we pointed out, this is a very model-dependent issue and should be analyzed case by case.

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REFERENCES

[1] A. D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic, Switzerland (1990).
[2] M. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. **B159**, 429 (1979).
[3] A. D. Linde, Phys. Lett. **108B** (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] Q. Shafi and A. Vilenkin, Phys. Rev. Lett. **52**, 691 (1984).
[5] J. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, Phys. Lett. **127B**, 331 (1983).
[6] A. D. Linde, Phys. Lett. **B129**, 177 (1993).
[7] A. Linde, Phys. Lett. **B249**, 18 (1990).
[8] A. D. Linde, Phys. Lett. **B259**, 38 (1991).
[9] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D **49**, 6410 (1994).
[10] M. Dine, W. Fischler and D. Nemeschansky, Phys. Lett. **136B**, 169 (1984).
[11] G. D. Coughlan, R. Holman, P. Ramond and G. G. Ross, Phys. Lett. **140B**, 44 (1984).
[12] D. H. Lyth and A. Riotto, *Models of inflation, particle physics and the spectral index of the density perturbations*, to be submitted to Phys. Rep.
[13] E. D. Stewart, Phys. Rev. D **51**, 6847 (1995).
[14] P. Binetruy and G. Dvali, Phys. Lett. **B388**, 241 (1996).
[15] E. Halyo, Phys. Lett. **B387**, 43 (1996).
[16] M. Green and J. Schwarz, Phys. Lett. **B149**, 117 (1984).
[17] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. **B289**, 585 (1987); J. Atick, L. Dixon and A. Sen, Nucl. Phys. **B292**, 109 (1987); M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. **B293**, 253 (1987).
[18] E. Witten, Nucl. Phys. **B471**, 135 (1996).
[19] G. Dvali and A. Riotto, [hep-ph/9706408](http://arxiv.org/abs/hep-ph/9706408).
[20] T. Matsuda, [hep-ph/9705448](http://arxiv.org/abs/hep-ph/9705448).
[21] R. Jeannerot, [hep-ph/9706391](http://arxiv.org/abs/hep-ph/9706391).
[22] R. Jeannerot, Phys. Rev. **D53**, 5426 (1996).
[23] A. D. Linde and A. Riotto, [hep-ph/9703209](http://arxiv.org/abs/hep-ph/9703209), to be published in Phys. Rev. D.
[24] B. Allen, R.R. Caldwell, E.P.S. Shellard, A. Stebbins and S. Veeraraghavan, [astro-ph/9609038](http://arxiv.org/abs/astro-ph/9609038).
[25] B. Allen, R.R. Caldwell, S. Dodelson, L. Knox, E.P.S. Shellard and A. Stebbins, astro-ph/9704160; Ue-Li Pen, U. Seljak and N. Turok, astro-ph/9704165.

[26] We thank J.A. Casas for bringing the following references to our attention, J.A. Casas and C. Munoz, Phys. Lett. B216, 37 (1989) and J.A. Casas, J.M. Moreno, C. Munoz and M. Quiros, Nucl. Phys. B328, 272 (1989), where the issue of cosmic strings and anomalous $U(1)$ is discussed.

[27] P. Binetruy and E. Dudas, Phys. Lett. B389, 503 (1996).

[28] G. Dvali and A. Pomarol, Phys. Rev. Lett. 77, 3728 (1996).

[29] R. N. Mohapatra and A. Riotto, Phys. Rev. D55, 1138 (1997); Phys. Rev. D55, 4262 (1997).

[30] A.E. Nelson and D. Wright, hep-ph/9702359.

[31] G. Dvali and S. Pokorski, Phys. Rev. Lett. 78 (1997) 807. Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B396 (1997) 150, hep-ph/9612232.