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Error Analysis of Nuclear Matrix Elements

Presented by J. E. A. at 22th European Conference On Few-Body Problems In Physics: EFB22
9 - 13 Sep 2013, Krakow (Poland)

Abstract We estimate the expected errors of nuclear matrix elements coming from the uncertainty on the NN interaction. We use a coarse grained (GR) interaction fitted to NN scattering data, with several prescriptions for the long-part of the interaction, including one pion exchange and chiral two-pion exchange interactions.

Keywords NN scattering · Chiral potentials · Shell Model · Error analysis

1 Introduction

We have recently made an error analysis of nuclear two body forces based on a coarse graining of the unknown short range part of the NN interaction that allows to quantify the uncertainties in the potential parameters [1, 2, 3, 4, 5, 6, 7, 8, 9]. Many nuclear structure calculations are carried out by diagonalization of the many body nuclear Hamiltonian within the harmonic oscillator shell model basis (possibly including the needed short range correlations). However, very little is known about the expected accuracy of those calculations based on our lack of knowledge of the input NN interaction. In this talk we face the problem by deducting and propagating two-body systematic and statistical errors to provide a theoretical estimate of nuclear matrix elements and binding energy uncertainties. The impact of chiral Two Pion Exchange interactions [10, 11] in the evaluation of nuclear matrix elements based on our error analyses can also be analyzed. This may help to set up a priori the needed accuracy to solve the many body problem.

2 Statistical and systematic errors of potential parameters

Meanful error estimates require to start with a NN potential fitting the available data with $\chi^2$/d.o.f. $\sim 1$. That potential should be simple enough to allow the extraction of the errors in the fitting parameters. According to Aviles [12] one may efficiently sample the unknown part of the interaction using a coarse grained (GR)

Supported by Spanish DGI (grant FIS2011-24149) and Junta de Andalucía (grant FQM225) and the Mexican CONACYT.

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potential parameterized for each partial wave as a sum of Dirac delta functions \[5\]. Equivalently \[6; 8\], the potential can be written in operator form as a linear combination of spin-isospin-angular operators as

\[
V(r) = \sum_{\pi=1}^{21} O_n \sum_{i=1}^{N} V_n \delta(r - r_i) + V_L(r) \theta(r - r_c) \tag{1}
\]

extending the standard AV18 set \[13\]. Here \(\Delta r = 0.6\text{fm}\) is the resolution scale and \(r_c = 3\text{fm}\) is the cut-off radius. \(V_L(r)\) is the long-range part of the interaction, including one-pion exchange potential plus additional e.m. terms. Our recent partial wave analysis of NN scattering data below pion production \[6; 8\] yielding \(\chi^2/\text{d.o.f.} = 1.06\) with the GR potential, allowed to extract the statistical errors in the fitted parameters. Alternatively one can fit directly to phase-shift \(\Delta\) pseudodata for each partial wave. Systematic errors manifest when different high quality potentials show discrepancies in the predicted phaseshifts. The pseudodata are then obtained from the computed phase-shifts for several high-quality potentials \[14; 15; 13; 16; 17\]. The pseudodata are obtained as the average and standard deviation of the computed phaseshift for a given energy. A first conclusion can be extracted: systematic errors are in general more than twice larger than statistical ones in most of the observables. The estimated error in the nuclear binding energy per particle, obtained \[1\] using several approaches for different nuclei in a range of mass number, is \(\Delta B/A = 0.1 - 0.4\text{ MeV}\). In the next section we show new results obtained for separated nuclear matrix elements.

3 Errors of nuclear matrix elements

In Fig. 1 we show results for the first partial waves \(^1S_0, {}^3S_1, {}^1P_1\) and in Fig. 2 for \(^3P_0, {}^3P_1, {}^3P_2\). Statistical errors are represented by an error band labeled OPE. In the upper panels we show the error of the phaseshifts as a function of the energy. That error propagates to the expected values of the NN potential energy on harmonic oscillator wave functions, displayed in the lower panels as a function of the oscillator length. The relative error in the potential expected value is appreciably larger than in the phaseshift for S-waves.

Modern chiral perturbation theory studies of nuclear structure emphasize the universality of two-pion exchange (\(\chi\text{TPE}\)) for intermediate to long distances. Our model can easily be modified to investigate the effect that the presence of \(\chi\text{TPE}\) in the NN interaction would have on nuclear observables. Therefore in Figs 1 and 2 we show also results from a second fit with the potential modified to include TPE in the intermediate region. This involves to reduce the cut radius to \(r_c = 1.8\text{ fm}\), and upward that distance to define the potential as the
sum of one- plus two-pion exchange. \( V_L(r) = V_{\text{TPE}}(r) + V_{\text{OPE}}(r) \). Below \( r_c \), the potential is again of the GR form. This procedure reduces the number of parameters in the fit, but increases the \( \chi^2 / \text{d.o.f.} \) value to 1.10 [9, 18]. As we can see from the figures 1 and 2, the presence of \( \chi_{\text{TPE}} \) in the potential produces different results in the nuclear expected values for S-waves, taking into account the statistical error.

Recently, an optimized chiral potential has been fitted to np scattering data by setting an upper cut-off in the LAB energy to \( E_{\text{LAB}} = 125 \text{MeV} \) [19]. This corresponds to resolution scale \( \Delta r \sim 1.2 \text{fm} \), or equivalently a low-momentum interaction. Shell model calculations [20] with low-momentum effective interactions are easier to work with than G-matrix calculations, similarly to soft core NN potentials [21], and are able to provide an accurate description of nuclear structure. We explore the theoretical error arising from that approach in Figs. 3 and 4. Therein we show the results from two fits of NN data for energy below 125 MeV, using a GR

![Fig. 1](image1.png)

**Fig. 1** The same as Fig. 1 for partial waves \( ^3P_0, ^3P_1, ^3P_2 (T_{\text{LAB}} \leq 350 \text{MeV}, r_c|_{\text{OPE}} = 3 \text{fm}, r_c|_{\text{TPE}} = 1.8 \text{fm} ) \).

![Fig. 2](image2.png)

**Fig. 2** The same as Fig. 1 for partial waves \( ^3P_0, ^3P_1, ^3P_2 (T_{\text{LAB}} \leq 125 \text{MeV}, r_c|_{\text{OPE}} = 1.8 \text{fm}, r_c|_{\text{TPE}} = 1.8 \text{fm} ) \).
potential with cut radius $r_c = 1.8$ fm. In the first fit the long-range interaction includes the OPE potential only. In the second we add also $\chi$TPE. Errors are big in the expected value of the potential, the larger being those obtained with the chiral potential.

4 Conclusions

Sumarizing, we have estimated errors in nuclear matrix elements coming from the uncertainty of the NN interaction. Errors are moderate, but not negligible, setting a theoretical limit to the precision that one can reach in nuclear physics calculations. Low energy fits produce softer potentials but theoretical errors increase.

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Fig. 4 The same as Fig. 2 for $T_{LAB} \leq 125$MeV, $r_c|_{OPE} = 1.8$fm, $r_c|_{TPE} = 1.8$fm.