Approximation Method for Evaluation of Strains and Forces in LTP Reinforcement of Embankments on Columns

Beata Gajewska¹, Marcin Gajewski²

¹ Department of Geotechnical Engineering, Faculty of Civil and Environmental Engineering, Warsaw University of Life Sciences, 159 Nowoursynowska St., 02 776 Warsaw, Poland
² Faculty of Civil Engineering, Warsaw University of Technology, 16 Armii Ludowej St., 00 637 Warsaw, Poland
beata_gajewska@sggw.pl

Abstract. This article presents a method of approximate finding of forces and strains resulting from the so-called membrane effect in the reinforcement of the transmission layer of embankments located on a soft soil improved by columns (rigid inclusions). The model task needed to determine the characteristic values in geosynthetic reinforcement supported on rigid columns was formulated. The reinforcement is treated as an isotropic membrane, in which the strains are determined using the formula for the length of the curve, i.e. it deviates from the assumptions of the small strain theory. Spatial problems are reduced to equivalent one-dimensional static tasks for analysis using the isotropic membrane theory. The closed form solution of this task exists only in the case of uniform load. For other types of loads, it is necessary to approximate the formula for the length of the curve occurring in the expression defining the strains in the membrane, e.g. using the Taylor series, and then solving the strongly non-linear equation in which the membrane tension force is unknown. The article presents the analysis of the effect of the length curve approximation with usage of the Taylor series on the accuracy of the results obtained. The presented analysis is very important from the point of view of standard engineering calculations needed to design the reinforcement of the transmission layer in the case of embankments located on soft soil reinforced with rigid inclusions.

1. Introduction

In Europe, there are several guidelines for designing the reinforcement of the transmission layer in the case of embankments on soft soil reinforced with columns [1-4]. Recommendations regarding the design of reinforcement of Load Transfer Platform (LTP) may be found in ASIRI guideline [5]. In most cases, analytical determination of forces and strains in the reinforcement above the columns is recommended. Numerical solutions are not recommended because the reinforcement strains determined with them are usually underestimated [3, 6]. A comparison of the results of analytical and FEM models was presented in [7], where the inconsistencies in the currently available design methods were identified and discussed in detail.

Analytical models do not fully reflect the three-dimensional work of geosynthetic reinforcement. Research works are being continued, as a result of which new analytical models are proposed [8-12]. Fei in [13] proposed the modelling of deformed reinforcement in the form of a function that takes into account the three-dimensional nature of deformation of reinforcement and a simplified analytical model.
based on the shell theory. Van Eeekelen, based on a series of model and natural scale studies [14-15],
proposed an analytical model [16-18], which was adopted in [2].

Analytical calculations are carried out in two steps [2]. In the first step, the load is divided into the
part carried by the columns and the remaining part transferred to the reinforcement and the ground
between the columns. In the second step, the forces and strains in the reinforcement are calculated. Due
to the high complexity of the equations, the guidelines [2-3] contain nomograms that allow to determine
the strains of the reinforcement from membrane effect, with the assumed load distribution: uniform,
triangular and inverted triangular.

The aim of the article is to present the method of approximate determination of strains and normal
forces in geosynthetic reinforcement from the membrane effect, loaded in any way and verification of
its accuracy.

2. Formulation of the model task to determine the characteristic values in geosynthetic
reinforcement supported on rigid columns

As a result of arching, the load from the structure layers located above the geosynthetic reinforcement
decomposes into two main parts: the part transferred directly by the columns and the part carried by the
reinforcement and the soil between the columns. The geogrid is not loaded by the whole lump of soil
located above the reinforcement, but only by a part of it. There are many models of arching [17, 19-20].
To examine the load distribution mechanism in reinforced piled embankments, modern techniques, for
example X-ray CT, are used [21]. Depending on the adopted model, the load value assigned to the
reinforcement and the soil between the columns is determined.

In this paper, the phenomenon of arching will not be analyzed, and the load on the geosynthetic is
treated as given. At the initial stage, it was assumed that the load carried by reinforcement is constant
\( p(x, y) = p \), cf. figure 1b. The resistance from the soil placed under reinforcement is modelled as a
one-parameter Winkler type elastic foundation, characterized by a subgrade-elasticity constant \( k \).

\[
L_x = s_x - a_{eq}
\]

\[
L_y = s_y - a_{eq}
\]

\[
q_{eq} = \frac{q}{L}
\]

\[
q_{eq} = \frac{q}{L}
\]

\[
q_{eq} = \frac{q}{L}
\]

**Figure 1.** Determination of an equivalent one-dimensional static system for analysis using the
isotropic membrane theory: a) scheme of collecting the load, b) conversion of two-dimensional task
into one-dimensional

It should be noted that the adoption of a constant load for a two-dimensional membrane is debatable
from the point of view of mechanics and experimental research, however, this assumption is not
questioned in the case of design issues. Thanks to it, a relatively simple algorithm for designing
geosynthetic reinforcement placed over the column heads can be created. The next step is to reduce the
two-dimensional task with the membrane loaded in the normal direction to its surface (constant load
$p = \text{const}$ to one-dimensional problem, see figure 1b. For example, a uniform, triangular and inverse-
triangular distribution is assumed.

An extensive discussion about the advantage of the inverse triangle distribution over the uniform or
triangular distribution may be found in [17]. The transition from a two-dimensional problem to a one-
dimensional one is carried out with the following assumptions:

- For a column layout on a rectangular plan, i.e. $s_x \neq s_y$ two similar tasks (in the direction of the
  x-axis and in the y-axis direction) are analyzed and the most unfavourable design situation is
  chosen. In the case of a set of columns on a square plan, these two cases are equivalent to each
  other.
- It is assumed that the membrane deflection function $w(x, y)$ is a function of only one variable,
  i.e. $w(x)$.
- The symmetry of the function $\tilde{p}(x)$ with respect to y axis is assumed.
- A suitable load distribution is assumed in a one-dimensional task, i.e. $\tilde{p}(x)$ (for example
  $\tilde{p}(x) = q_{av}$, $\tilde{p}(x) = -q_{av} + 2q_{av}x/L$ or $\tilde{p}(x) = -2q_{av}x/L$)
- The following geometrical parameters are determined: $A_p = s_x s_y / 2$, $d = d_{eq} = (4 A_p / \pi)^{1/2}$, $a = a_{eq} = d \sqrt{\pi} / 2$, where $A_p$ stands for the area of the head of the
  column, and $d$ is a column diameter.
- Equivalence of total load i.e. $p A_x / 2 = a_{q_{av}} \int_0^{L/2} \tilde{p}(x) dx$. For example when $\tilde{p}(x) = q_{av}$ we get
  $p A_x = a_{q_{av}} A_p q_{av}$ and then a parameter $\xi$ may be defined as $\xi = q_{av} / p = A_p / a_{q_{av}} L$.
- Equivalence of reaction of elastic foundation $\int_{A_k} k w(x, y) dxdy = a_{q_{av}} \int_{-L/2}^{L/2} K w(x) dx$. If we accept
  that $w(x, y) \equiv w(x)$ then we get: $\int_{A_k} k w(x) dxdy = a_{q_{av}} \int_{-L/2}^{L/2} K w(x) dx$. It is often assumed,
  however, that $k A_x = a_{q_{av}} K L$, so as the result $\xi = K / k$ which is only true when $w(x) = \text{const}$.

3. Formulation of the boundary value problem of a one-dimensional isotropic membrane
The base for determining the force in the geosynthetic reinforcement treated as an isotropic membrane
is the solution of the membrane on the elastic subgrade. Differential equation of isotropic deflection of
a one-dimensional membrane stretched with constant force $T_y (x) = T_y$ supported on Winkler foundation
taking into account assumptions from subsection 2 has the following form:

$$T_y \frac{d^2 w(x)}{dx^2} - K w(x) = -\tilde{p}(x),$$

(1)

see figure 2a, in which all the values appearing in equation (3) were presented. In addition, internal
forces for the membrane section are shown in figure 2b. Equilibrium forces and moments were arranged
for the same section.

From the sum of projections per z axis, it follows that the vertical component of the normal force
satisfies the following differential equation ($\tilde{r}(x) = kw(x)$):

$$\frac{dT_y (x)}{dx} + \tilde{p}(x) - \tilde{r}(x) = 0.$$ (2)
From the sum of the moments relative to the point A, we get the relationship between the components of the normal and vertical force in the membrane:

\[ T_n \frac{dw(x)}{dx} = T_v(x). \]  

(3)

Then, from the Pythagorean Theorem, we can write the formula for normal force in the membrane, and then using the relation (3) transform it into a form:

\[ T(x) = \left( T_n^2 + T_v^2(x) \right)^{1/2} = T_n \left( 1 + \left( \frac{dw(x)}{dx} \right)^2 \right)^{1/2} = T_n C(x,T_n,K,\bar{\rho}). \]  

(4)

Substituting (3) into (2) and taking into account the following relationship \( r(x) = Kw(x) \) and then transferring \( p(x) = \bar{\rho}(x) \) we get the equation (1), which after substitution \( \alpha^2 = K/T_n \) may be written as:

\[ \frac{d^2 w(x)}{dx^2} - \alpha^2 w(x) = -\frac{\bar{\rho}(x)\alpha^2}{K}. \]  

(5)

After determining the deflection function on the basis of the differential equation of the membrane in the form (5), the average strain can be determined using the formula for the length of the curve:

\[ \varepsilon_{avg}^{bw} = \frac{\int_0^{L/2} \left( 1 + \left( \frac{dw(x)}{dx} \right)^2 \right)^{1/2} dx - \frac{L}{2} \int_0^{L/2} \left( C(x,T_n,K,\bar{\rho}) \right) dx}{\frac{L}{2}} = \frac{\int_0^{L/2} \left( C(x,T_n,K,\bar{\rho}) \right) dx - \frac{L}{2} \int_0^{L/2} C(x,T_n,K,\bar{\rho}) dx}{\frac{L}{2}}. \]  

(6)

Calculating the strain based on the arc length means that we deviate from the limitations of the small strain theory. In equation (6), the integrated function was noted as \( C(x,T_n,K,\bar{\rho}) \), based on which the length of the curve can be determined. When deriving the differential equation for membrane deflection under the theory of small deformations (in the form (6)), no constitutive relationship characterizing membrane material is used. Indirectly the membrane tensile force \( T_n \) takes into account the properties of the material and cross-section, because it depends on them. In general, the membrane constitutive relationship can be written as follows:

\[ T(x) = J \varepsilon(x), \]  

(7)

where \( \varepsilon(x) \) is a strain function of membrane, while \( J \) stands for membrane stiffness.
In general, in the case of linear-elastic materials, the stiffness of the membrane can be interpreted as the product of the Young’s modulus of the material and the effective cross-section area (per linear meter of the grid). On the basis of this relation (7) and knowledge of the normal force function in the membrane, it is also possible to calculate the average strain as:

$$\varepsilon_{avg}^w = \int_0^{L/2} T(x)dx \left( \frac{1}{2} \frac{LJ}{T_h} \right) = \frac{2T_h}{LJ} \tilde{C}(T_h, K, \tilde{p}).$$

(8)

Comparing (6) with (8) gives an integral equation from which, with given load, stiffness of the elastic foundation, stiffness of the reinforcing mesh and given geometrical dimensions, one can determine the membrane tensile force $$T_h$$. Solving the equation $$\varepsilon_{avg}^p = \varepsilon_{avg}^w$$ we do not find the force value with which one should stretch the grid before it is loaded, but the force that will appear after loading the initially unstretched grid. And so after the transformations we get:

$$T_h = \frac{2C - L}{2C} J.$$  

(9)

The strain function can then be written as:

$$\varepsilon(x) = \frac{T(x)}{J} = \frac{T_h}{J} C_l(x) = \frac{2C - L}{2C} C_l(x).$$  

(10)

4. The task of uniformly loaded one-dimensional membrane extension

For the task of extension a one-dimensional uniformly loaded membrane ($$\tilde{p}(x) = -q_{av}$$), differential equation (5), with the appropriate boundary conditions $$w(-L/2) = 0$$ and $$w(L/2) = 0$$ or $$w'(0) = 0$$ and $$w(L/2) = 0$$ leads to the following solution:

$$w(x) = \frac{\beta}{\eta} L \left( 1 - \text{Cosh}(\alpha x) \text{Sech}(\alpha L/2) \right),$$

(11)

where $$\text{Sech}(x) = 1/\text{Cosh}(x)$$, $$\xi = q_{av}/p$$. Parameters $$\eta = k\xi L^2/J$$ and $$\beta = q_{av} L/J$$, which have an appropriate interpretation of the ratio of the stiffness of the elastic foundation to the stiffness of the membrane and the ratio of the load collected from the length $$L$$ to membrane stiffness. It should be noted that regardless of the way the membrane is loaded, i.e. the form of the function $$\tilde{p}(x)$$ in equation (5), finding its analytical solution is not a problem. In most cases, we get a closed form solution. The problem is to evaluate the length of the curve according to the formula (6). In case of a uniform load, the integrand expression is in the form:

$$C_l(x) = \left[ 1 + \left( \frac{La\beta \text{Sin}h(\alpha x)}{\eta \text{Cosh}(\alpha L/2)} \right)^2 \right]^{1/2},$$

(12)

and the result of the integral can be written as:

$$\tilde{C} = -\frac{I}{\alpha} \text{ElipticE} \left( \frac{HL}{2}, \frac{L^2 \alpha^2 \beta^2}{\eta^2 \text{Cosh}^2(La/2)} \right).$$

(13)

Symbol $$\text{ElipticE}(\varphi, m)$$ stands for the second kind of elliptical integral, where for $$-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$$:

$$\text{ElipticE}(\varphi, m) = \int_0^\varphi \left( 1 - m \sin^2(\theta) \right)^{1/2} d\theta.$$  

(14)
According to the authors' knowledge a solution $\bar{C}$ can be found in a closed form only for uniformly distributed load. The normal force function can be determined using the equation $c_{avg}^{bp} = c_{avg}^{cr}$ and (13). The appropriate substitution gives the following equation:

$$f(T_h) \left[ Jf(T_h) + 2 I(J - T_h) \text{EllipticE} \left( \frac{1}{2} f(T_h), \frac{2J\beta^2}{T_h\eta + T_h\eta \cosh (f(T_h))} \right) \right] = 0, \quad (15)$$

where $f(T_h) = (J\eta/T_h)^{1/2}$. In order to solve a non-linear equation (15) with regard to $T_h$ it is necessary to use numerical methods.

5. Analysis of the approximation effect of the curve length result using the Taylor's series on the accuracy of the results

In the case of a task with a uniform load, there is a solution in the form of elliptical integrals. In that case, the approximation effect of the curve length result using the Taylor series may be estimated. The need to find the length of the curve results from the fact that determining the membrane tensile force requires solving equation (15). An integrand expression is developed in the Taylor's series

$$C_i(x) \approx Taylor(C_i(x), N) \quad \text{where } N = 5,10,15,30 \quad (16)$$

and then the integral is calculated as

$$\int_0^{L/2} C_i(x) \, dx. \quad (17)$$

The influence of the number of series elements on the accuracy of the received solution for an exemplary task and for various relations of $K$ to $q_{av}$ was analyzed. In the analysis, constant value of geosynthetics stiffness $J$ is adopted. The forces are analyzed for the case with columns arranged on a square plan with dimensions $s_x = s_y = s = 2.25$ m. A grid of geosynthetic reinforcement with stiffness $J = 1967$ kN/m is arranged on the columns. The columns have an equivalent diameter $a_{eq} = 0.75$ m. It has been assumed that the load from the layers deposited above the reinforcement grid can be converted into a permanent load of value $q_{av} = 15$ kPa. The reaction of soil layers below the reinforcement grid is modelled as a single-parameter a subgrade reaction $K = 100$ kN/m$^3$. Based on geometric dimensions data, i.e. $s$ and $a_{eq}$ it is possible to calculate the real span of the reinforcement band between the columns $L = 1.5$ m ($s - a_{eq} = 2.25 - 0.75 = 1.5$ m). In the considered task $A_{h} = 2.25$ m$^2$. All notation is given in figure 1.

For the above data, a strict solution was defined, which is a reference point for further considerations. Further solutions were obtained by appropriately approximating the function $C_i(x)$ with five, ten, fifteen and thirty elements of Taylor's series. Selected results obtained for the analyzed membrane are presented in table 1.

Analysing the results presented in table 1, it can be noticed that the adoption of 10 series elements allows for a solution whose absolute percentage difference is at the level of a few per miles. In the case of an approximation with the fifteen elements of the series, a very good agreement was obtained (agreement of all significant digits), and in the case of thirty elements, the conformity is full. The maximum elongation of the reinforcement determined on the basis of the nomograms from [2] is 0.007, which is consistent with the results obtained. The approximation of the result in the form of 5 element series is unacceptable. For the assumed ratio $K/q_{av} = 100/15$ the error of determined force $T_h$ and the maximum elongation is almost 100%.
Table 1. Selected results of the membrane extension task obtained by approximating the function $C_{1}(x)$ with five, ten, fifteen and thirty elements of Taylor’s series.

|               | The exact solution | 5 series el. | 10 series el. | 15 series el. | 30 series el. |
|---------------|--------------------|--------------|--------------|--------------|--------------|
| $T_{h}$ [kN/m] | 13.4188            | 0.00000000415283 | 13.3714 | 13.4248 | 13.4188 | 13.3714 \[99.999\%] | 13.4248 \[0.045\%] | 13.4188 \[0.00\%] |
| $T(L/2)$ [kN/m] | 13.9634            | 0.00000000415283 | 13.9096 | 13.9706 | 13.9634 | 13.9096 \[99.999\%] | 13.9706 \[0.052\%] | 13.9634 \[0.00\%] |
| $w(0)$ [m]    | -0.0667343         | -0.075        | -0.0667764  | -0.0666729 | -0.0667343 | -0.0667343 \[99.999\%] | -0.0667764 \[0.063\%] | -0.0666729 \[0.008\%] | -0.0667343 \[0.00\%] |
| $\varepsilon(L/2)$ | 0.00709883        | 0.000000000211125 | 0.00707145 | 0.00710251 | 0.00709883 | 0.00709883 \[99.999\%] | 0.00707145 \[0.052\%] | 0.00710251 \[0.00\%] | 0.00709883 \[0.00\%] |

* - in brackets the absolute percentage difference is given in relation to the exact solution.

6. Results and discussions

For geometric data as in the example in subsection 5, the sensitivity analysis of the method of determining forces and strains in the reinforcement was made depending on the ratio of $K$ to $q_{av}$ and the number of Taylor’s series. The approximation of function with 10, 15 and 30 Taylor’s series was applied. For the given ratio $K/q_{av}=100/15$ the values $K$ and $q_{av}$ were increased accordingly keeping their constant ratio. For the $q_{av}$ range from 15 kPa to 60 kPa, the values of membrane tensile force $T_{h}$ and maximum strains of the membrane were determined. The relative error was determined for the designated pairs $f$ in relation to the exact solution $\tilde{f}$ according to the formula:

$$\text{ref} = \left(\frac{(f - \tilde{f})^2}{\tilde{f}^2}\right)^{1/2}, \quad f = T_{h}, \varepsilon.$$

A relative error of the results obtained for the ratio $K/q_{av}$ from 25/15 to 400/15 was also determined depending on the number of elements of Taylor’s series. The obtained results are presented in figures 3-6.

Based on the obtained results it may be concluded that for the same ratio $K/q_{av}$ along with the increase of the equivalent load value $q_{av}$ and subgrade reaction $K$ the value of the membrane tensile force increases, but for a solution with 10 elements of Taylor’s series this value remains constant from $q_{av}=45$ kPa (for $K$ equal to respectively 300, 350 and 400 kN/m$^2$). Relative estimation error in relation to the exact solution for 15 and 30 series is below 1%, while for 10 series the error increases as the value $q_{av}$ and $K$ increases, and for equivalent load of 60kPa and subgrade reaction 400 kN/m$^2$ reaches a value above 15% (figure 3). Analogical solutions were obtained for the maximum elongation of the membrane, but in the case of 10 series for equivalent load of 60kPa and subgrade reaction 400 kN/m$^2$ the relative error is bigger and close to 17% (figure 4).

When increasing the ratio $K/q_{av}$ ($q_{av}=15$ kPa, and $K$ increases) relative error obtained by approximate method values $T_{h}$ and $\varepsilon$ significantly grows in both cases for 10 and 15 elements of Taylor’s series (figure 5). Interestingly the larger errors were obtained for solutions approximated by 15 elements of series. In turn, the error of the determined values of membrane tensile force and maximum elongation using a series with 30 elements is very small (order of per mille), but for $K/q_{av}=350/15$ and 400/15 the results with approximate method was not obtained because of the lack of convergence for the assumed numerical accuracy value (constant for all analysed cases). In turn,
when reducing the ratio $K / q_{av}$ ($k = 100 \text{kN/m}^2$, and $q_{av}$ increases) in the first two cases (for 10 and 15 elements of series) the errors are the order of per mille, and for 30 elements of series for all analyzed cases, the relative error in relation to the exact solution is zero.

**Figure 3.** a) The value of the membrane tensile force at a constant ratio $K / q_{av}$ for different values $q_{av}$ and $K$, b) relative error in relation to the exact solution for 10, 15 and 30 elements of Taylor's series.

**Figure 4.** a) The value of the maximum elongation of the membrane at a constant ratio $K / q_{av}$ or different values $q_{av}$ and $K$, b) relative error in relation to the exact solution for 10, 15 and 30 elements of Taylor's series.

**Figure 5.** Relative error in relation to the exact solution for 10, 15 and 30 elements of Taylor's series for constant value $q_{av}$ ($K$ increases), a) for $T_s$, b) for $\varepsilon$. 
7. Conclusions
In the paper the method of approximate determination of strains and normal forces from the membrane effect in the LTP reinforcement of piled embankments were presented. The effect on the accuracy of the results obtained with the approximation of the results on the length of the curve using the Taylor’s series were also given. On the basis of the analyses, the following conclusions may be formulated:

1. The task of extension of the membrane is a nonlinear problem, as evidenced by the obtained results of the membrane tensile force and the maximum strains at a constant ratio $K/q_{av}$ and different values of subgrade reaction and equivalent load.

2. For $K/q_{av}$ from 25/15 to 300/15 the largest relative approximation error for 30 elements in relation to the exact solution is 0.4%, and for $K/q_{av}$ from 25/15 to 100/22.5 the relative error is zero. However for values $K/q_{av}$ from 350/15 to 400/15 it was not possible to get a solution for 30 elements of series.

3. An approximation of the solution to the length of the curve with 10 elements of Taylor’s series allows getting the results of the membrane tensile force and maximum strains with the error below 1% for a range of values $K/q_{av}$ from 25/15 to 100/15, and for approximation with 15 elements for $K/q_{av}$ from 25/15 to 150/15.

4. The presented method of estimation of strains and forces in reinforcement of Load Transfer Platform of piled embankments allows determining forces and strains in geosynthetic reinforcement loaded with any distribution of equivalent load.

References
[1] BS8006-1: 2010, Code of practice for strengthened/reinforced soils and other fills, British Standards Institution, 2010.
[2] CUR 226, Design Guideline for Basal Reinforced Piled Embankment Systems, Stichting Deltarces, 2016.
[3] EBGEO Recommendations for Design and Analysis of Earth Structures using Geosynthetic Reinforcements, DGGT, Ernst & Sohn, 2011.
[4] Nordic guidelines for reinforced soils and fills (NGI), 2004.
[5] ASIRI, Recommendations for the design, construction and control of rigid inclusion ground improvements, Institut pour la recherche appliquée et l'expérimentation en génie civil (IREX), 2013.
[6] G. S. F Farag, Lateral spreading in basal reinforced embankments supported by pile-like elements,
Schriftenreihe Getechnik, Heft 20, Universität Kassel, 2008.

[7] P. Ariyarathne, and D. S. Liyanapathirana, Review of existing design methods for geosynthetic-reinforced pile-supported embankments, Soils and Foundations, 55 (1), pp. 17–34, 2015.

[8] L. Zhang, M. Zhao, Y. Hu, H. Zhao, and B. Chen, Semi-analytical solutions for geosynthetically reinforced and pile-supported embankment, Computers and Geotechnics, Vol. 44, 2012.

[9] Y. Zhuang, K. Y. Wang, and H. L. Liu, A simplified model to analyze the reinforced piled embankments, Geotextiles and Geomembranes, Vol. 42, Issue 2, pp. 154-165, 2014.

[10] Ch. Zhang, G. Jiang, X. Liu, and O. Buzzi, Arching in geogrid-reinforced pile-supported embankments over silty clay of medium compressibility: Field data and analytical solution, Computers and Geotechnics, Volume 77, Pages pp. 11-25, 2016.

[11] K. Deb, A mathematical model to study the soil arching effect in stone column-supported embankment resting on soft foundation soil, Applied Mathematical Modelling, 34, pp. 3871-3883, 2010.

[12] M. A. Nunez, L. Briançon, and D. Dias, Analyses of a pile-supported embankment over soft clay: Full-scale experiment, analytical and numerical approaches, Engineering Geology, 153 (2013), pp. 53-67, 2013.

[13] K. Fei, A Simplified Method for Analysis of Geosynthetic Reinforcement Used in Pile Supported Embankments, Hindawi Publishing Corporation, The Scientific World Journal, Vol. 2014, Article ID 273253, 9 pages, 2014.

[14] S. J. M. Van Eekelen, A. Bezuijen, H. J. Lodder, and A. F. van Tol, Model experiments on piled embankments Part I, Geotextiles and Geomembranes, 32, pp. 69-81, 2012

[15] S. J. M. Van Eekelen, A. Bezuijen, H. J. Lodder, and A. F. van Tol, Model experiments on piled embankments Part II, Geotextiles and Geomembranes, 32, pp. 82-94, 2012

[16] S. J. M. Van Eekelen and A. Bezuijen, Inversed triangular load distribution in a piled embankment, 3D model experiments, field tests, numerical analysis and consequences. EuroGeo5, Valencia, Spain, 2012.

[17] S. J. M. Van Eekelen, Basal Reinforced Piled Embankments - Experiments, field studies and the development and validation of a new analytical design model, CPI – Koninklijke Wörrmann, Zutphen, Netherlands, 2015.

[18] S. J. M. Van Eekelen, A. Bezuijen, and A. F. van Tol, An analytical model for arching in piled embankments, Geotextiles and Geomembranes, 39, pp. 78-102, 2013.

[19] D. Zaeske, Zur Wirkungsweise von unbewehrten und bewehrten mineralischen Tragschichten über pfahlartigen Gründungselementen (On the effect of unreinforced and reinforced mineral base courses over pile-like foundation elements), Schriftenreihe Geotechnik, Uni Kassel, Heft 10, 2001.

[20] W. J. Hewlett and M. F. Randolph, Analysis of piled embankments, Ground Engineering, Vol. 22, Number 3, pp. 12-18, 1988.

[21] T. Eskişar, J. Otani, and J. Hironaka, Visualization of soil arching on reinforced embankment with rigid pile foundation using X-ray CT, Geotextiles and Geomembranes, 32, pp. 44-54, 2012.