Numerical Modeling of Fracture Height Propagation in Multilayer Formations Considering the Plastic Zone and Induced Stress

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ABSTRACT: Hydraulic fracturing and acid fracturing are very effective stimulation technologies and are widely used in unconventional reservoir development. Fracture height, as an essential parameter to describe the geometric size of a fracture, is not only the input parameter of two-dimensional fracturing models but also the output parameter of three-dimensional fracturing models. Accurate prediction of fracture height growth can effectively avoid some risks. For example, petroleum reservoirs produce a large amount of formation water because wrong fracture height prediction leads to the connection between the oil or gas reservoir and the water layer. Although some fracture height prediction models were developed, few models considered the effects of the plastic zone, induced stress, and heterogeneous multilayer formation and its interaction. Therefore, considering the influence of many factors, an improved fracture-equilibrium-height model was developed in this study. The successive over-relaxation iteration method and the displacement discontinuity method were used to solve the model. We investigated the effects of the geological and engineering factors on fracture height growth by using the model, and some important conclusions were obtained. The higher the fracture height, the larger the plastic zone size, and the more obvious its influence on fracture height propagation. High overlying or underlying in situ stress and fracture toughness and low fluid density played a positive role in limiting the growth of the fracture height. Induced stress caused by fracture 1 could not only inhibit the height growth of fracture 2 but also promote its growth. The model established in this paper could be coupled to a fracturing simulator to provide a more reliable fracture height prediction.

1. INTRODUCTION

Hydraulic fracturing or acid fracturing, as one of the important technologies for reservoir stimulation, has been tested and gradually popularized in the oilfield since the 1940s.1,2 So far, hydraulic fracturing and acid fracturing have become necessary technologies for the exploitation of tight oil or gas, shale oil or gas, coalbed methane, and gas hydrate.3–10 Fracture height is one of the essential parameters to describe fracture geometry. It is the input parameter of the two-dimensional fracturing model or the output parameter of the three-dimensional fracturing model, which affects the accuracy of fracturing model calculation results and ultimately affects fracturing treatment design, safety, and economy.11–13 Therefore, precise prediction of the fracture height is a significant task for fracturing stimulation.

There are many factors affecting fracture height propagation in multilayer formations, mainly including geological factors (e.g., natural fracture, rock mechanics parameters (e.g., shear modulus, fracture toughness, shear strength, tensile strength, and Poisson’s ratio), in situ stress, and layer interface) and
engineering factors (e.g., working fluid properties (e.g., fluid density, viscosity, and filtration), induced stress caused by other artificial fractures, and construction parameters (e.g., injection displacement, pressure, and fluid volume)). Researchers have done a series of work on the influence of these factors for decades and achieved remarkable results.14−20

Liu and Valkó27 and Valkó and Economides28 stated that under the determined geological and engineering conditions, there was an upper limit of the fracture height, i.e., the equilibrium height theory. If the stress intensity factor $K_f$ is not greater than the rock fracture toughness $K_{fC}$ i.e., $K_f \leq K_{fC}$ the fracture tip will stop growing. Since the 1970s, scholars began to seek the solution of the equilibrium height problem and developed several solution models. Before the Simonson model was proposed, the fracture height was assumed as a constant (e.g., effective thickness of the reservoir) in the fracturing model.29−32 Simonson et al.33 established a fracture height prediction model in a three-layer formation, which assumed that the physical properties and mechanical parameters of the upper and lower layers were the same, and then the complex problem was simplified in a symmetrical way. However, this assumption is different from the actual situation. Later, researchers modified the Simonson model, and the upper and lower layers can be asymmetric and different rock properties.34,35 Fung et al.36 developed a fracture height prediction model for asymmetric multilayer formations with different rock properties, but it was assumed that the net pressure along the height direction was constant. Mack and Warpinski37 and Weng et al.38 modified the Fung model and considered the variation of net pressure. Liu and Valkó27 proposed a rigorous fracture-equilibrium-height prediction model suitable for six layer formations and investigated the influence of fracture toughness, in situ stress, net pressure, and their interaction on fracture height evolution.

However, the above model does not consider the effect of the plastic zone at the fracture tip during fracture height propagation. Cheng et al.39 and Li et al.40 thought that when the fracture height was large, the plastic zone at the fracture tip was also large, and its influence should be considered. In addition, many research results demonstrated that the induced stress, caused by adjacent artificial fractures, had a significant effect on fracture propagation.41−44 Most studies focused on the effect of induced stress on fracture length propagation, and there were few reports on the effect of induced stress on fracture height growth. Kresse and Weng45 viewed that when two artificial fractures had a similar height and were initiated at the same layer, the variation of induced stress along the fracture height direction can be ignored and we can assume that the induced stress was a constant value. However, when two artificial fractures were vertically offset or initiated in different layers and have different heights, the induced stress along the height direction was variable.

In this paper, an improved fracture height prediction model for asymmetric multilayer formations is established, which considers the influence of the plastic zone and induced stress. Then, the computational method used in the model is presented. Later, the analytical and numerical solutions in the literature are used to verify the model results. Finally, the effects of the plastic zone, induced stress, fracture toughness, in situ stress, fluid density, and its interaction on fracture height growth are investigated by using our model.

2. MATHEMATICAL MODEL

Several important model assumptions are made. First, the vertical in situ stress $\sigma_y$ is greater than the horizontal minimum in situ stress $\sigma_{h0}$ i.e., $\sigma_y > \sigma_{h0}$ and the fracture will be vertical. Second, the fracture is filled with fluid, and the pressure distribution of fluid along the height direction conforms to the hydrostatic pressure.

A typical physical model of the vertical fracture in six layers is presented (Figure 1). Ignoring the effect of the plastic zone, the fracture height is $2c$ (Figure 1), and the $K_{f+}$ and $K_{f-}$ can be calculated by:

$$
\begin{align}
K_{f+} &= \sqrt{\frac{1}{\pi c}} \int_{-c}^{c} p_{net}(y) \sqrt{c+y} \, dy \\
K_{f-} &= \sqrt{\frac{1}{\pi c}} \int_{-c}^{c} p_{net}(y) \sqrt{c-y} \, dy
\end{align}
$$

(1)

where $K_{f+}$ and $K_{f-}$ are the stress intensity factors at the upper and lower fracture tips, respectively (Pa·m$^{1/2}$). The fracture height $2c$ is:

$$
2c = y_u - y_l
$$

(2)

where $y_u$ and $y_l$ are the vertical depth of upper and lower fracture tips, respectively (m). The net pressure at the position in the fracture can be expressed as:

$$
p_{net}(y) = -\rho g y + p_{mid} - \sigma'_h
= -\rho g y + p_{ref} + \rho g (d_{mid} - d_{ref}) - \sigma'_h
= -\rho g y + \sigma_h
$$

(3)

where $\rho$ is the fluid density (kg/m$^3$); $p_{mid}$ is the pressure at the middle of the fracture (Pa); $d_{mid}$ and $d_{ref}$ are the vertical depth at the middle of the fracture and the perforation location, respectively (m); $p_{ref}$ is the liquid pumping pressure at the perforation location (Pa); $g$ is the gravitational acceleration (m/s$^2$); and $\sigma'_h$ is the horizontal minimum in situ stress of the rth layer (Pa).

Equation 3 is brought into eq 1, and the integration of $K_{f-}$ from location $c$ to location $y$ is:
stress intensity factor considering the plastic zone can be calculated by:

$$K_{I_2}(m, a_s, y) = \frac{1}{\pi c} \int_y^{c+y} (-m_y + a_s) \frac{x-y}{c+y} dy$$

$$= \frac{2\sqrt{c+y} (2a_s + mc) \sin^{-1} \left( \frac{c-y}{\sqrt{c+y}} \right)}{2 \sqrt{c+y}} (c+y)(2a_s-y)$$

$$K_{I_2}(m, a_s, c) = 0$$

$$K_{I_2}(m, a_s, -c) = \frac{mc}{2} - a_s \sqrt{c}$$

The total $K_{I_2}$, caused by multiple layers, can be calculated by:

$$K_{I_2} = \sum_{i=1}^{n} K_{I_2,i,r}$$

$$= \sum_{i=1}^{n} [K_{I_2,i}(m, a_s, -\gamma_{d,r}) - K_{I_2,i}(m, a_s, -\gamma_{u,r})]$$

where $\gamma_{d,r}$ and $\gamma_{u,r}$ are the bottom and top vertical depths of the $r$th layer, respectively (m). Similarly, $K_{I_2}$ can be calculated by:

$$K_{I_2} = \sum_{i=1}^{n} K_{I_2,i,r}$$

$$= \sum_{i=1}^{n} [K_{I_2,i}(m, a_s, y_{d,r}) - K_{I_2,i}(m, a_s, y_{u,r})]$$

where $y_{d,r}$ and $y_{u,r}$ are the bottom and top vertical depths of the $r$th layer, respectively (m).

Due to the obvious stress concentration in the fracture tip area, the rock in the area will change from elasticity to plasticity. The $K_{I_2}$ and $K_{I_2}$ calculated by the linear elastic fracture mechanics theory cannot characterize this phenomenon. Therefore, Dugdale and Cheng et al. thought that the actual fracture height is $2b$ considering the plastic zone (Figure 1). Referring to eq 1, the calculation formula of the stress intensity factor considering the plastic zone can be written as:

$$K_{I_2}'' = \frac{1}{\pi (c + \epsilon_0)} \int_{-(c+\epsilon_0)}^{c+\epsilon_0} \sigma_b \left( \frac{(c + \epsilon_0) + y}{c + \epsilon_0} - y \right) dy$$

$$K_{I_2}'' = \frac{1}{\pi (c + \epsilon_0)} \int_{-(c+\epsilon_0)}^{c+\epsilon_0} \sigma_b \left( \frac{(c + \epsilon_0) - y}{c + \epsilon_0} + y \right) dy$$

$$K_{I_2}'' = \frac{1}{\pi (c + \epsilon_0)} \int_{-(c+\epsilon_0)}^{c+\epsilon_0} \sigma_b \left( \frac{(c + \epsilon_0) + y}{c + \epsilon_0} + y \right) dy$$

According to the equilibrium height theory, the absolute value of the $K_{I_2}''$, and $K_{I_2}''$ caused by the in situ stress is equal to the absolute value of the $K_{I_2}'$, and $K_{I_2}'$ caused by the fluid pressure and then $\epsilon_0$ and $\epsilon_d$ can be obtained as:

$$\epsilon_0 = \frac{\pi}{2} \frac{K_{I_2}'}{\sigma_b}$$

$$\epsilon_d = \frac{\pi}{2} \frac{K_{I_2}'}{\sigma_b}$$

Let $b = (c + \epsilon_0 + \epsilon_d)/2$, then the stress intensity factors at fracture tips considering the plastic zone are:

$$K_{I_2}' = \frac{1}{\pi b} \int_{-b}^{b} p_{net}(y) \sqrt{\frac{b+y}{b-y}} dy$$

$$K_{I_2}'' = \frac{1}{\pi b} \int_{-b}^{b} p_{net}(y) \sqrt{\frac{b-y}{b+y}} dy$$

Similar to eq 5 and eq 6, considering the effect of the plastic zone, the total $K_{I_2}'$ and $K_{I_2}''$, caused by multiple layers, can be written as:

$$K_{I_2}' = \sum_{i=1}^{n} [K_{I_2,i}'(m, a_s, \gamma_{d,r}) - K_{I_2,i}'(m, a_s, \gamma_{u,r})]$$

$$K_{I_2}'' = \sum_{i=1}^{n} [K_{I_2,i}''(m, a_s, \gamma_{d,r}) - K_{I_2,i}''(m, a_s, \gamma_{u,r})]$$

If $K_{I_2}'$ is greater than $K_{I_2}''$ (i.e., $K_{I_2}' > K_{I_2}''$), the upper fracture tip will grow. If $K_{I_2}'$ is greater than $K_{I_2}''$ (i.e., $K_{I_2}' > K_{I_2}''$), the lower fracture tip will grow.

For multistage fracturing wells, it is a typical case that fracture 1 is formed first and then fracture 2 is formed. Therefore, the propagation of fracture 2 will be affected by fracture 1. Figure 2 shows that the stress distribution acting on fracture 2 is changed due to the induced stress caused by fracture 1, and the induced stress along the height direction of fracture 2 is variable. We use the displacement discontinuity method (DDM) to solve the induced stress, and the basic principle of DDM is described in detail in the literature.

Figure 2. Schematic diagram of stress acting on fracture 2.
and the normal stress $\sigma_n^j$ caused by the element $j$ at the midpoint of element $i$ can be expressed as:

$$\sigma_n^i = \sum_{j=1}^{N} C_{ij}^n D_n^j + \sum_{j=1}^{N} C_{ijn}^n D_n^j$$

where $C_{ij}^n$ and $C_{ijn}^n$ are the stress influence coefficients of element $j$ to element $i$ and are represented in Appendix A. For fracture 1, the stress balance equation is:

$$\begin{align*}
\sigma_i^j &= p_f^j - \sigma_h^j, \quad (i = 1, 2, ..., N) \\
\sigma_i^i &= 0
\end{align*}$$

where $p_f^j$ and $\sigma_h^i$ are the fluid pressure and in situ stress at element $i$, respectively (Pa). There are 2N independent linear equations to solve 2N unknowns. By solving eq 12 and eq 13 together, the shear and normal displacement discontinuities ($D_s^i$ and $D_n^i$) of each element can be obtained. By introducing $D_s^i$ and $D_n^i$ into eq 14, the induced stress caused by element $j$ at any point $p$ can be expressed as:

$$\begin{align*}
\Delta \sigma_{x}^j &= \sum_{j=1}^{N} C_{ij}^{x} D_s^j + \sum_{j=1}^{N} C_{ijn}^{x} D_n^j \\
\Delta \sigma_{y}^j &= \sum_{j=1}^{N} C_{ij}^{y} D_s^j + \sum_{j=1}^{N} C_{ijn}^{y} D_n^j \\
\Delta \tau_{xy}^j &= \sum_{j=1}^{N} C_{ij}^{xy} D_s^j + \sum_{j=1}^{N} C_{ijn}^{xy} D_n^j
\end{align*}$$

where $C_{ij}^{x}$, $C_{ij}^{y}$, $C_{ij}^{xy}$, $C_{ij}^{x}$, $C_{ij}^{y}$, and $C_{ij}^{xy}$ are the stress influence coefficients of element $j$ to point $p$ and are represented in Appendix A. In the x-direction, the total stress at point $t$ in fracture 2 is the induced stress caused by fracture 1 plus in situ stress (Figure 2), i.e.,

$$\sigma^t = \sigma_h^t + \Delta \sigma_{xx}$$

where $\sigma_h^t$ and $\Delta \sigma_{xx}$ are the in situ stress and induced stress at point $t$, respectively (Pa). By introducing eq 15 into eq 3, the calculation formula of net pressure at point t in fracture 2 considering induced stress caused by fracture 1 is:

$$p_{net}^t = -\rho g t + p_{ref} + \rho g (d_{mid} - d_{ref}) - \sigma^t$$

By introducing eq 16 into eq 11, the height of fracture 2 can be predicted considering the influence of induced stress caused by fracture 1.
3. COMPUTATIONAL METHOD

The computational method of the fracture height prediction model of multilayer formations considering the influence of the plastic zone and induced stress is shown in Figure 4. Case 1 presents that there is only one fracture in the study area, so only the effect of the plastic zone on the fracture height propagation needs to be considered. Case 2 presents that there are two fractures in the study area. Fracture 1 has been formed and will not propagate again. Therefore, the influences of the plastic zone at the fracture 2 tip and induced stress, caused by fracture 1, on fracture 2 height propagation need to be considered. If $K'_{1c} > K'_{2c}$ or $K'_{1b} < K'_{2b}$ one element needs to be added or reduced from the upper fracture tip. Similarly, if $K'_{1c} > K'_{2c}$ or $K'_{1b} < K'_{2b}$ one element needs to be added or reduced from the lower fracture tip. In our model, the linear over-relaxation iteration method (SOR).

4. MODEL VALIDATION

Since many fracturing models can solve the three-layer symmetry problem, we test the models with three-layer symmetry formations so that our model can be compared with more previous models. The input formation, perforation, and fluid parameters are shown in Table 1.

Table 1. Input Parameters of the Fracturing Simulation in Three-Layer Formations

| object               | variable | value | unit   |
|----------------------|----------|-------|--------|
| up layer             | thickness| 304.8 | m      |
|                      | stress   | 53091500.0 | Pa     |
|                      | fracture toughness | 1098882.2 | Pa·m\(^{1/2}\) |
| target layer         | thickness| 22.9 | m      |
|                      | stress   | 48954500.0 | Pa     |
|                      | fracture toughness | 1098882.2 | Pa·m\(^{1/2}\) |
| bottom layer         | thickness| 304.8 | m      |
|                      | stress   | 55849500.0 | Pa     |
|                      | fracture toughness | 1098882.2 | Pa·m\(^{1/2}\) |
| fluid                | density  | 999.6 | kg/m\(^3\) |
| perforation          | full perforation and central depth of the target layer | 3048.0 | m      |

Table 2 shows the calculation results of the fracture height of different models. The pressures at the center point of the perforated section are 50,333,500 and 53,091,500 Pa, and the corresponding net pressures are 1,379,000 and 4,137,000 Pa, respectively. When the net pressure is small, the fracture heights calculated by different models have little difference. However, when the net pressure is large, there are significant differences in the fracture height calculated by different models. If there is an aquifer above or below, the wrong height will cause unnecessary risk, which is that the oil or gas reservoir will produce a large amount of formation water or pollute drinking water. Because the influence of the plastic zone is considered in our model, our model results are slightly larger than the Liu and Valko model.

The numerical solution of induced stress in our model is compared with the analytical solution to verify our model. Warpiniski and Teufel\(^{30}\) established the analytical solution of the stress field around the vertical fracture (eq 17). Figure 5 is the physical model, and the model basic parameters are: $h = 50$ m, $\sigma_0 = 45$ MPa, $P_{net} = 8$ MPa, $G = 30$ GPa, and $v = 0.25$. The calculation results are shown in Figures 678.

$$\sigma_{xx} = -P_{net}\frac{r}{r_2}\left(h^2 + \frac{k_{12}^2}{h_{12}^2}\right)^{1.5}\sin\theta\sin\{1.5(\theta_1 + \theta_2)\}$$
$$- P_{net}\frac{r}{\sqrt{r_2^2}}\cos\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) - 1$$
$$\sigma_{yy} = P_{net}\frac{r}{h}\left(h^2 + \frac{k_{12}^2}{h_{12}^2}\right)^{1.5}\sin\theta\sin\{1.5(\theta_1 + \theta_2)\}$$
$$- P_{net}\frac{r}{\sqrt{r_2^2}}\cos\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) - 1$$
$$\sigma_{xy} = -P_{net}\frac{r}{h}\left(h^2 + \frac{k_{12}^2}{h_{12}^2}\right)^{1.5}\sin\theta\cos\{1.5(\theta_1 + \theta_2)\}$$

(17)

Figure 5. Two-dimensional fracture physical model for induced stress verification.

Figure 6. Induced stress $\Delta\sigma_{xx}/P_{net}$ distribution in the x-direction.
Figures 6 and 7 show that the numerical solution is very close to the analytical solution, and the variation law is also the same.

Figure 8 is the cloud diagram of induced stress distribution calculated by our model, which is consistent with the results obtained by other scholars using numerical methods.51

5. RESULTS AND DISCUSSION

The calculation process of case 1, as shown on the left of Figure 4, will be used to study the effects of the fluid density (ρ), plastic zone, fracture toughness (KIC), in situ stress (σh), and their interaction on fracture height propagation. In addition, the calculation process of case 2, as shown on the right of Figure 4, will be used to investigate the effect of induced stress on fracture height propagation.

5.1. Case 1. Although the multilayer fracture-equilibrium-height (MFEH) model developed by Liu and Valkó1 ignores the influence of the plastic zone, the idea of the MFEH model is similar to our model. Therefore, the basic parameters described by Liu and Valkó are used for a comparative study of the effect of the plastic zone, as shown in Table 3. Table 3 is applied to Section 5.1.1.

To analyze the influence of various factors, the symmetrical five-layer formations are set (Table 4) and applied as basic parameters in Sections 5.1.2 to 5.1.7.

Table 3. Basic Parameters Used for Simulation in Section 5.1.1

| formation       | top vertical depth (m) | thickness δ (m) | in situ stress σh (Pa) | fracture toughness KIC (Pa·m1/2) |
|-----------------|------------------------|-----------------|------------------------|-----------------------------------|
| layer 1         | 2740.2                 | 27.4            | 49299250.0             | 2197764.4                         |
| layer 2         | 2767.6                 | 27.4            | 49299250.0             |                                    |
| layer 3         | 2795.0                 | 51.8            | 39301500.0             |                                    |
| layer 4         | 2846.8                 | 12.2            | 50678250.0             |                                    |
| layer 5         | 2859.0                 | 22.9            | 39991000.0             |                                    |
| layer 6         | 2881.9                 | 59.4            | 56539000.0             |                                    |
| perforation     | middle depth of the perforated section (m) | 2820.9 |
| fluid density   | ρ (kg/m³)              | 1100.0          |                        |                                    |

The superscript r represents the rth layer.

5.1.1. Influence of the Plastic Zone. Figure 9 shows the comparison results calculated by the two models. When the fracture height is low, the difference between the results calculated by the two models is less because the plastic zone at the fracture tip is small. However, with the continuous expansion of the fracture height, the difference between the results is also gradually obvious because the size of the plastic zone is increasing by degrees. The fracture height calculated by our model breaks through layer 5 before that calculated by the

(a) Normal stress in x direction (b) Normal stress in y direction (c) Shear stress

Figure 8. Stress shadow of the artificial fracture.
Liu and Valko\textsuperscript{1} model. More accurate fracture height prediction can avoid unnecessary risks. For example, it can prevent the petroleum reservoir with bottom water from connecting with the lower water layer.

5.1.2. Influence of the Overlying In Situ Stress. We only change the overlying in situ stress ($\sigma_{\text{fl}}^h$) and set it to 40, 45, 50, or 55 MPa, respectively. Our model is used to investigate the effect of $\sigma_{\text{fl}}^h$ on fracture height propagation, and the result is shown in Figure 10. When $\sigma_{\text{fl}}^h$ is relatively low (e.g., $\sigma_{\text{fl}}^h = 40$ MPa; $\sigma_{\text{fl}}^h = 38$ MPa; $\sigma_{\text{fl}}^h$ is close to $\sigma_{\text{fl}}^b$), the upper fracture tip will quickly pass through layer 1, as shown by the blue curve in Figure 10. When $\sigma_{\text{fl}}^h$ is equal to the underlying in situ stress ($\sigma_{\text{fl}}^b$), the fracture first passes through layer 5 rather than layer 1 because the fluid pressure distribution conforms to the hydrostatic pressure distribution (pink curve in Figure 10). Generally speaking, when $\sigma_{\text{fl}}^h$ gradually increases from 40 to 55 MPa (the blue, red, green, and pink curves in Figure 10), it is more difficult for the upper fracture tip to pass through layer 1, and the growth of the fracture height in layer 1 becomes flat.

5.1.3. Influence of the Underlying In Situ Stress. Similarly, we only change the underlying in situ stress ($\sigma_{\text{fl}}^b$) and set it to 40, 45, 50, or 55 MPa to investigate the effect of $\sigma_{\text{fl}}^b$. When $\sigma_{\text{fl}}^b \leq \sigma_{\text{fl}}^b$, the fracture tip will first pass through layer 5 rather than layer 1 (Figure 11). When $(\sigma_{\text{fl}}^b - \sigma_{\text{fl}}^b) > 5$ MPa, the upper fracture tip will stay at the interface of layer 1 and layer 2 for a long time (e.g., the red and green curves in Figure 11). When $\sigma_{\text{fl}}^b$ gradually increases from 40 to 55 MPa (the blue, red, green, and pink curves in Figure 11), a greater fluid pressure is required for the lower fracture tip to pass through layer 5.

5.1.4. Influence of the Overlying Fracture Toughness. Similarly, the overlying fracture toughness ($K_{IC}^{\text{fl}}$) is set at 0.1, 5, 10, or 15 MPa-m$^{1/2}$ to investigate the effect of $K_{IC}^{\text{fl}}$ (Figure 12). When $K_{IC}^{\text{fl}}$ gradually increases, the effect of $K_{IC}^{\text{fl}}$ on the growth of the fracture height is not obvious (the blue, red, green, and pink curves in Figure 12), and the upper fracture tip extends slowly in layer 1. Comparing Figure 10 with Figure 12, we view that $K_{IC}^{\text{fl}}$ and $\sigma_{\text{fl}}^b$ have similar effects on fracture height propagation, but the effect of $K_{IC}^{\text{fl}}$ on fracture height propagation is significantly less than that of $\sigma_{\text{fl}}^b$ because $K_{IC}^{\text{fl}}$ is the integration of fluid pressure/stress.

5.1.5. Influence of the Underlying Fracture Toughness. Similarly, the underlying fracture toughness ($K_{IC}^{\text{IC}}$) is set at 0.1, 5, 10, or 15 MPa-m$^{1/2}$ to investigate the effect of $K_{IC}^{\text{IC}}$ (Figure 13). Comparing Figure 12 with Figure 13, we think that changing the fracture toughness $K_{IC}^{\text{IC}}$ of different layers (e.g., $K_{IC}^{\text{fl}}$ and $K_{IC}^{\text{IC}}$) has the same influence trend, but $K_{IC}^{\text{IC}}$ has a more significant influence on fracture height propagation under high fluid pressure. Comparing Figure 11 with Figure 13, it also proves again that the effect of fracture toughness on fracture height propagation is less than that of overlying or underlying in situ stress.

5.1.6. Interaction of the In Situ Stress and Fracture Toughness. We set $\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{IC}} = 15$ MPa-m$^{1/2}$ or $\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{fl}} = 15$ MPa-m$^{1/2}$ to investigate the interaction of low in situ stress ($\sigma_{\text{fl}}^b$) and high fracture toughness ($K_{IC}^{\text{IC}}$) on fracture height propagation (Figure 14a,b). In addition, we also set $\sigma_{\text{fl}}^h = 60$ MPa and $K_{IC}^{\text{IC}} = 0.1$ MPa-m$^{1/2}$ or $\sigma_{\text{fl}}^h = 60$ MPa and $K_{IC}^{\text{fl}} = 0.1$ MPa-m$^{1/2}$ to investigate the interaction of high in situ stress ($\sigma_{\text{fl}}^b$) and low fracture toughness ($K_{IC}^{\text{IC}}$) on fracture height propagation (Figure 14c,d).

Although the fracture toughness ($K_{IC}^{\text{IC}}$ or $K_{IC}^{\text{fl}}$) is very large, the in situ stress ($\sigma_{\text{fl}}^b$ or $\sigma_{\text{fl}}^h$) is very small. The upper or lower fracture tip will quickly pass through layer 1 or layer 5 after entering layer 1 or layer 5 and lose stability (Figure 14a,b). In addition, we found an interesting phenomenon. Comparing the blue curves in Figure 10 ($\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{IC}} = 2.19$ MPa-m$^{1/2}$) with Figure 14a ($\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{IC}} = 15$ MPa-m$^{1/2}$), $\sigma_{\text{fl}}^h$ is the same and $K_{IC}^{\text{IC}}$ in Figure 14a is greater. Therefore, the upper fracture tip in Figure 14a stays at the interface between layer 1 and layer 2 for a while. Similarly, comparing the blue curves in Figure 11 ($\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{IC}} = 2.19$ MPa-m$^{1/2}$) with Figure 14b ($\sigma_{\text{fl}}^h = 40$ MPa and $K_{IC}^{\text{IC}} = 15$ MPa-m$^{1/2}$), a similar phenomenon can be observed.
Although the fracture toughness ($K_{IC}^1$ or $K_{IC}^5$) is very small, the in situ stress ($\sigma_{1h}$ or $\sigma_{5h}$) is very large. The upper or lower fracture tip grows slowly in layer 1 or layer 5, and there is no tip jump (Figure 14c, d).

5.1.7. Influence of Fluid Density. We only change the fluid density ($\rho$) and set it to 1000, 1200, 1400, or 1600 kg/m$^3$ to investigate the effect of $\rho$ (Figure 15). The greater the fluid density, the greater the fluid pressure difference between the upper and lower fracture tip due to the action of gravity. Therefore, even in the symmetrical five-layer formation, the fracture breaks through layer 5 first, not layer 1 or at the same time. Comparing Figure 15 with Figure 12 and Figure 13, it can be seen that the effect of fluid density on fracture height propagation is slightly greater than that of fracture toughness.

5.2. Case 2. To analyze the influence of the induced stress caused by fracture 1 on height propagation of fracture 2, the physical model in Figure 16 is established. Points A, B, C, D, E, F, or G in Figure 16 are the middle position of the perforated section of fracture 2. In addition, the basic simulation parameters in Sections 5.2.1−5.2.2 are shown in Table 5.

5.2.1. Influence of the Fracture 1 Height. The middle depth of the perforated section of fracture 2 is 2850 m and is located at point A (Figure 16) and $x_1 = 25$ m. We set fracture 1 height ($h$) at 25, 50, or 100 m to investigate the influence of fracture 1 height on fracture 2 height propagation. Figure 17a shows the height profile of fracture 2 under different fracture 1 heights, Figure 17b shows the distribution of in situ stress, and Figure 17c shows the induced stress $\Delta\sigma_{xx}$ distribution caused...
by fracture 1 through point A in the y-direction. Overall, the greater the height of fracture 1, the smaller the height of fracture 2 under the same fluid pressure at the perforation position (Figure 17a). The reason is that the induced stress caused by the greater height of fracture 1 at point A is larger and positive (Figure 17c). Combined with eq 16, it can be seen that the net fluid pressure in fracture 2 is smaller. In addition, when the fluid pressure of fracture 2 at the perforation position increases from 43.6 to 43.9 MPa, the red curve in Figure 17a quickly approaches the blue curve because the induced stress \( \Delta \sigma_{xx} \) represented by the red curve in Figure 17c within a depth of 2800 to 2824 m is less than that represented by the blue curve, and the net pressure in the fracture 2 is greater than \( \sigma_3 h \) and \( \sigma_5 h \) (Figure 17b).

5.2.2. Influence of the Perforation Location. To investigate the influence of the perforation location of fracture 2 on its height propagation, we set the middle depth of the perforated section of fracture 2 at points A, B, C, D, E, F, or G (Figure 16) and \( h_1 = 60 \text{ m}, x_1 = 0.5 h, x_2 = 0.5 h, x_3 = h, x_4 = 3 h, \) and \( y_1 = 2 h \). Figure 18a1 is a height profile of fracture 2 at different perforation points, and Figure 18a3 is the induced stress \( \Delta \sigma_{xx} \) distribution caused by fracture 1 through different points in the y-direction. Figure 18b is a height profile of fracture 2 at different perforation points without considering induced stress. Under the same fluid pressure at different perforation positions, the fracture height is basically the same because \( \sigma_1 h = \sigma_3 h = \sigma_5 h, K_{1c} = K_{3c} = K_{5c}, \delta_3 = \delta_5, \) and \( \delta_1 = \delta_9, \delta_3 = \delta_{12} \) (Figure 18b). However, the phenomenon in Figure 18b cannot be observed in Figure 18a because of the influence of induced stress. When the perforation position of fracture 2 is gradually moved away from fracture 1 from point A to point D, the height of fracture 2 (solid lines in Figure 18a1) increases because the induced stress caused by fracture 1 in layer 4 and its vicinity decreases gradually (Figure 18a3). However, when the perforation position of fracture 2 is from point E to point G, the height of fracture 2 (dotted lines in Figure 18a1) decreases because the induced stress caused by
fracture 1 in layer 2 and its vicinity gradually increases from negative to zero. Although the $x$ coordinate values of point A and point E are the same (Figure 16), the $y$ coordinate values are different, and the fracture height profiles of perforation at

| formation | top vertical depth (m) | $\delta$ (m) | $\sigma_{ij}$ (MPa) | $K_{IC}$ (MPa$^{-1/2}$) | shear modulus $G'$ (GPa) | Poisson’s ratio $\nu$ (Dimensionless) |
|-----------|------------------------|--------------|---------------------|------------------------|------------------------|--------------------------------------|
| layer 1   | 2680                   | 100          | 44                  | 5                      | 50                     | 0.35                                 |
| layer 2   | 2780                   | 20           | 38                  | 2                      | 20                     | 0.17                                 |
| layer 3   | 2800                   | 40           | 44                  | 5                      | 30                     | 0.25                                 |
| layer 4   | 2840                   | 20           | 38                  | 2                      | 20                     | 0.17                                 |
| layer 5   | 2860                   | 40           | 44                  | 5                      | 30                     | 0.25                                 |
| layer 6   | 2900                   | 20           | 48                  | 7                      | 40                     | 0.3                                  |
| layer 7   | 2920                   | 100          | 53                  | 9                      | 50                     | 0.35                                 |
| perforation | middle depth of perforated section (m) | 2850 |
| fluid     | $\rho$ (kg/m$^3$)      | 1100         |                     |                        |                        |                                      |

Figure 17. Influence of fracture 1 height on fracture 2 height propagation.

Figure 18. Influence of the perforation location of fracture 2 on its height propagation.
point A and point E (solid and dotted blue lines in Figure 18a) are also different because of the different induced stresses.

6. CONCLUSIONS

A numerical model of fracture height propagation in multilayer formations, which considered the plastic zone at the fracture tip and induced stress caused by another artificial fracturing, was introduced in this study. By using the established model, the effects of some geological and engineering factors on fracture height propagation were investigated. First, the higher the fracture height is, the larger the plastic zone size is, and its influence on fracture height propagation cannot be ignored. Second, high overlying or underlying σh and KIC and low fluid density played a positive role in limiting fracture height propagation. The influences of fracture toughness and fluid density on fracture height growth were obviously less than that of overlying or underlying in situ stress, and the effect of fluid density on fracture height propagation is slightly greater than that of fracture toughness. Low overlying or underlying in situ stress made the fracture tip unstable and jump, but low fracture toughness could not lead to this phenomenon. Finally, when fracture 2 is closer to fracture 1, the height growth of fracture 2 is more affected by the induced stress generated by fracture 1. The higher the height of fracture 1, the greater the influence area of induced stress caused by fracture 1. The effect of induced stress on the height propagation of fracture 2 is different due to the different perforation positions of fracture 2 in the influence area of induced stress. Induced stress could not only inhibit the height growth of fracture 2 but also promote its growth.

7. FUTURE WORK

Considering the pressure drop of fluid in the height and length direction, the three-dimensional fracturing model in multilayer formations can be established by coupling the fracture height and length growth.

■ APPENDIX A

Four stress influence coefficients in eq 12:

\[
\begin{aligned}
C_{xx}^{\sigma} &= 2G[2f_{xx}^{\sigma} \cos 2\theta + f_{xy}^{\sigma} \sin 2\theta] \\
+ \bar{y} (f_{xyy}^{\sigma} \cos 2\beta_j - f_{yy}^{\sigma} \sin 2\beta_j) \\
C_{xy}^{\sigma} &= 2G[-f_{xx}^{\sigma} + \bar{y} (f_{xy}^{\sigma} \sin 2\beta_j + f_{yy}^{\sigma} \cos 2\beta_j)] \\
C_{yx}^{\sigma} &= 2G[-f_{xx}^{\sigma} - \bar{y} (f_{xy}^{\sigma} \sin 2\beta_j - f_{yy}^{\sigma} \cos 2\beta_j)] \\
C_{yy}^{\sigma} &= 2G[-f_{xx}^{\sigma} \sin 2\theta_j - f_{xy}^{\sigma} \cos 2\theta_j] \\
- \bar{y} (f_{xyy}^{\sigma} \cos 2\beta_j + f_{yy}^{\sigma} \sin 2\beta_j) \\
\end{aligned}
\]

(A.1)

where G is the shear modulus (Pa); \( \gamma^2 = \beta_i - \beta_f \).

Six stress influence coefficients in eq 14:

\[
\begin{aligned}
C_{xx}^{\sigma} &= 2G[2f_{xx}^{\sigma} \cos 2\beta_j + f_{xy}^{\sigma} \sin 2\beta_j] \\
+ \bar{y} (f_{xyy}^{\sigma} \cos 2\beta_j - f_{yy}^{\sigma} \sin 2\beta_j)] \\
C_{xy}^{\sigma} &= 2G[-f_{xx}^{\sigma} + \bar{y} (f_{xy}^{\sigma} \sin 2\beta_j + f_{yy}^{\sigma} \cos 2\beta_j)] \\
C_{yx}^{\sigma} &= 2G[-f_{xx}^{\sigma} - \bar{y} (f_{xy}^{\sigma} \sin 2\beta_j - f_{yy}^{\sigma} \cos 2\beta_j)] \\
C_{yy}^{\sigma} &= 2G[-f_{xx}^{\sigma} \sin 2\theta_j - f_{xy}^{\sigma} \cos 2\theta_j] \\
+ \bar{y} (f_{xyy}^{\sigma} \cos 2\beta_j + f_{yy}^{\sigma} \sin 2\beta_j) \\
\end{aligned}
\]

(A.2)

where

\[
\begin{aligned}
f'_{xx} &= \frac{1}{4\pi(1 - \nu)} \left[ \ln |(x - a)|^2 + y^2 |^{1/2} - \ln |(x + a)|^2 + y^2 |^{1/2} \right] \\
f'^{\sigma}_{xy} &= -\frac{1}{4\pi(1 - \nu)} \left[ \arctan \frac{\bar{y}}{x - a} - \arctan \frac{\bar{y}}{x + a} \right] \\
f'^{\sigma}_{yx} &= -\frac{1}{4\pi(1 - \nu)} \left[ \frac{x - a}{(x - a)^2 + y^2} - \frac{x + a}{(x + a)^2 + y^2} \right] \\
f'^{\sigma}_{yy} &= \frac{1}{4\pi(1 - \nu)} \left[ (x - a)^2 - y^2 \right] \\
- \frac{(x + a)^2}{((x + a)^2 + y^2)^2} \\
\end{aligned}
\]

\[
\begin{aligned}
f'_{yy} &= \frac{1}{4\pi(1 - \nu)} \left[ (x - a)^2 + y^2 \right] \\
- \frac{x - a}{((x + a)^2 + y^2)^2} \\
\end{aligned}
\]

(A.3)

where \( \nu \) is Poisson’s ratio. The local coordinate \((\bar{x}, \bar{y})\) of the element \(i\) relates to the element \(j\) via coordinate transformation:

\[
\begin{aligned}
\bar{x} &= (x_i - x_j) \cos \beta_j + (y_i - y_j) \sin \beta_j \\
\bar{y} &= -(x_i - x_j) \sin \beta_j + (y_i - y_j) \cos \beta_j
\end{aligned}
\]

(A.4)

where the global coordinates \((x_i, y_i)\) and \((x_j, y_j)\) are the midpoint of elements \(i\) and \(j\); \(\beta_i\) is the angle between the local coordinate \(\bar{x}\)-axis and the global coordinate \(x\)-axis.

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NOMENCLATURE
DDM displacement discontinuity method
SOR successive over-relaxation iteration method
MFEH multilayer fracture-equilibrium-height
CNPC China National Petroleum Corporation

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