Lattice QCD with Domain-Wall Fermions *

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We study the quenched lattice QCD using domain-wall fermions at $\beta = 6.0$. Behaviors of both pion mass and the explicit breaking term in the axial Ward-Takahashi identity support the existence of the chiral zero modes. We observe a good agreement between the pion decay constants $f_\pi$ from both the conserved axial current and the local current perturbatively renormalized at 1-loop. Finally the possible existence of the parity broken phase is also examined in this model.

1. Introduction

Domain-wall QCD (DWQCD) [1,2] is considered to have good properties such as exact chiral symmetry without doublers, no $O(a)$ scaling violation and the existence of conserved axial current. There exist several pilot studies [2], which seems to support these superior properties.

We study DWQCD in the quenched approximation. First we investigate the pion mass and the explicit breaking term of the axial Ward-Takahashi identity to confirm the existence of the chiral zero mode at zero current quark mass. Next we calculate the pion decay constant $f_\pi$ from both the conserved axial current and the local axial current, using the perturbative renormalization factor [3] for the latter, in order to check the reliability of the lattice perturbation theory. Finally we explore negative $m_f$ region to examine the existence of the parity broken phase predicted in [3].

2. Chiral symmetry

The fermion action is identical to the original one [2], with domain wall height $M$, bare quark mass $m_f$ and the extent of the 5th dimension $N_s$. We employ 10–30 gauge configurations, generated by the plaquette action at $\beta = 6.0$ ($a^{-1} \sim 2$ GeV) on $16^3 \times 32 \times N_s$ lattices. The unit wall source without gauge fixing is used for quark propagators. The mean-field estimate for the optimal value of $M$, $M = 1 + 4s(1 - u)$ with $u = 1/(8K_c)$, gives $M = 1.819$, from $K_c = 0.1572$ for the Wilson fermion at $\beta = 6.0$.

First we investigate the existence of the chiral zero mode in the chiral limit of the model, $m_f \rightarrow 0$ and $N_s \rightarrow \infty$. The pion mass squared is plotted as a function of $m_f a$ in Fig. 1. Since the linearity of $M_\pi^2$ in $m_f$ is well satisfied, we linearly extrapolate it to $m_f = 0$ for each $N_s$. We also evaluate a critical quark mass $m_c(N_s, M)$ at which the pion mass squared vanishes.

In Fig. 2, $M_\pi^2$ is plotted as a function of $N_s$ for $m_f a = 0.025, 0.05, 0.075$ and $\rightarrow 0$. Extrapolated values of $M_\pi^2$ at $m_f = 0$ seem to vanish exponentially in $N_s$, while $M_\pi^2$ at finite $m_f$ remains non-zero. For $N_s = 10$, $M_\pi^2(m_f = 0)$ is already as small as that for the NG pion of the KS fermion at the same $\beta$ for the same spatial lattice size. Furthermore $m_c(N_s, M = 1.819)$ and the WI-mass [4], defined by $\langle J^K_5(P)P \rangle/P \langle P \rangle$, also decrease exponentially in $N_s$, as shown in Fig. 3. All these facts indicate that the chiral symmetry is restored for $m_f \rightarrow 0$ and $N_s \rightarrow \infty$ at $\beta = 6.0$.

The lattice scale $a$ is set by the $\rho$ meson mass.

![Figure 1. Pion mass squared as a function of $m_f a$ at $M = 1.819$ and $N_s = 4, 6, 10$. Solid lines show linear fits.](image-url)
3. Pion decay constant $f_\pi$

The pion decay constant is defined as $m_\pi f_\pi / Z_A = \langle 0 | A_4 | \pi \rangle$, which is obtained from correlation functions of pseudo scalar density $P(t)$ and axial current $A_\mu(t)$ at zero momentum: $\langle X(t)Y(0) \rangle = C_{XY}G(t)$ with $G(t) = \exp(-M_\pi t)/(2M_\pi V_s)$ for $X,Y = P,A_4$. For the renormalization factor for the axial current $Z_A$, we take unity for the conserved current and use the value from the mean-field improved perturbation theory\[4\] for the local current.

In Fig. 5, $f_\pi$ is shown as a function of $m_f$. Circles are obtained from $C_{AP}$ for the conserved axial current, while squares from $C_{AP}$ and diamonds from $C_{AA}$ for the local axial current. The triangle is the experimental value. For local current, filled(open) symbols represent values with(without) perturbative corrections. Three different estimates reasonably agree for all $m_f$ within current statistics, if 1-loop corrections are included. It is also noted that the value of $f_\pi$ from the conserved current, linearly extrapolated at $m_f = 0$, is close to the experimental value.

4. Parity broken phase

For finite $N_s$, the parity broken phase may exist in negative $m_f$ regions. As $N_s$ increases, the broken phase shrinks rapidly, and the phase bound-
ary, where the pion mass vanishes, converges to $m_f = 0$. To examine this parity broken picture in DWQCD, we calculate pion masses at $m_f a = -0.120, -0.100, -0.080$ for $N_s = 4$ and $M = 1.819$. The pion propagators at these parameters form peculiar shapes similar to "W" character, which has been often observed for the Wilson fermion near or in the parity broken phase. Pion mass squared as a function of $m_f$ is shown in Fig. 6. Extrapolations of $M^2_\pi$ to zero both from positive and negative $m_f$ (two largest $m_f$ are used for negative $m_f$) indicate that a parity broken phase may exist around $m_f a \sim -0.03$ at this parameter. Needless to say, more high statistics and variation of parameters are necessary for the definite conclusion.

5. Conclusions and discussions

At $\beta = 6.0$ we have several indications that the chiral symmetry is restored at $N_s \to \infty$.

We see that all three different estimates for $f_\pi$ are consistent with each other. This shows the mean-field improved perturbation theory works well at $\beta = 6.0$ in DWQCD. Although the value of $f_\pi$ at $\beta = 6.0$ turns out to be compatible to the experimental one, detailed scaling studies at different $\beta$'s are needed before making the firm statement. The chiral symmetry in DWQCD, however, may fail to recover on the coarse lattice[5]. If this is true, the scaling studies of DWQCD become rather difficult.

We examine negative $m_f$ to see the parity broken phase in DWQCD. The result seems consistent with the parity broken picture, though further confirmations for this are definitely required.

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