Improving learning through modelling: a theoretical approach to teaching and assessment based on modelling activities

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Abstract. Modelling in physics instruction has a long history, but surprisingly, there is no general theory of modelling. However, most of the practices in constructing models are well known to physics educators: the necessity for a detailed assessment of the problem, the use of multiple representations and the need to evaluate the outcome. In this paper, I will outline the theoretical basis for these practices and show how the cognitive psychology of problem solving can lead to fresh insights into the modelling process, in particular allowing access to the very rich literature on problem solving in physics dating back to the 1980s. I show how the same steps required for the construction on a model lend themselves to the construction of interactive teaching sequences and assessments.

1. Introduction
Arguably, modelling is central to the practice of physics. Theoreticians construct mathematical models to represent the physical world or universe, experimentalists have to develop a mental model of their experiment, including sources of error, in order to construct it and computational physicists’ use of models probably needs no explanation. The importance of modelling in undergraduate curricula has long been recognised [1], but finding a clear theoretical approach to the construction of models within the literature is difficult. The essential difficulty is to define a model and the method of modelling that is specific enough to help students whilst at the same time being flexible enough to accommodate a wide range of physical phenomena and situations. In short, if the modelling method reduces to a series of well-defined steps there is a danger that it becomes essentially another algorithm that students will want to learn and apply.

The key insight behind this paper is the recognition that modelling forms an essential part of problem solving: a problem cannot be solved until it is understood, but in understanding it one has to develop at the very least a mental model of the processes and interactions at work. This connection with problem solving allows access to a very wide range of literature dating back to at least 1980, when the importance of domain-specific knowledge was just being recognised among psychologists. Both Larkin et al and Chi et al [2,3] published seminal papers making specific reference to physics and highlighted the differences between experts and novices. Since then, there has been an enormous amount of work in the cognitive sphere related to problem solving, some of which the author has summarized [4]. The
cognitive psychology of problem solving has been used by the author as the basis of a theoretical approach to modelling and the purpose of this paper is to describe that approach.

2. Towards a theory of modelling

It is too ambitious to claim to have developed a complete theory of modelling, but many of the elements that should form part of a theory have been identified. In fact, many of the practices that together form a methodology of modelling are already in widespread use within physics education, but here they are combined to create a modelling protocol. The elements of a modelling theory include:

1. **Modelling is natural.** Consider the following statement by Epstein [5]: “The first question that arises frequently—sometimes innocently and sometimes not—is simply, "Why model?" Imagining a rhetorical (non-innocent) inquisitor, my favorite retort is, "You are a modeler." Anyone who ventures a projection, or imagines how a social dynamic—an epidemic, war, or migration—would unfold is running some model.". It might seem remarkable that Epstein makes no mention of physics, but he isn’t a physicist. What strikes this author as truly remarkable is the insight, which shows just how natural it is to construct a model and run it mentally.

Giere writes that it is “probably the central notion in the cognitive sciences” that “humans (and animals) create internal representations of their environment (as well as of themselves)” [6]. There might well be different views within the cognitive sciences as to the exact nature of these representations and Giere comments that, “depending on the particular field within the cognitive sciences, one finds talk of such things as “schemata”, “cognitive maps”, “mental models” or “frames”. Despite this widespread support within the cognitive sciences for the notion that modelling is natural, the reader might well feel that the dynamic aspect of mental modelling to which Epstein so eloquently refers is not captured by the different forms referred to by Giere. However, this aspect is captured by Nersessian [7], who has developed a theory of model-based reasoning in which analogical, iconic mental models are manipulated much in the manner described by Epstein.

2. **The use of multiple representations is essential.** The value to students of being able to use different kinds of representations is now an established fact of physics education. Representations are especially helpful for understanding a problem, as Glaser [8] has pointed out: “We define a problem representation as a cognitive structure corresponding to a problem that is constructed by a solver on the basis of domain-related knowledge and its organization. At the initial stage of problem analysis the problem solver attempts to ‘understand’ the problem by constructing an initial problem representation”. Representations can be both internal, i.e. mental, or external, in the form of diagrams, graphs, sketches, etc. Nersessian [7] regards internal and external representations as a coupled pair.

Suwa and Tversky [9] were interested in the process by which designers create new forms and structures and observed that representations serve several purposes, by, for example,

- Freeing up working memory
- Cueing retrieval from long term memory
- Allowing perceptual judgements about spatial relations
- Allowing the generation of new ideas

The use of representations is thus vital to the process of thinking. In particular, the role of representations in freeing up working memory is especially important.

3. **Working memory imposes limitations.** The psychological construct of working memory is a complex tri-partite structure comprising the phonological loop, the visuo-spatial sketch pad and the central executive. Working memory plays a part in processing new information before it is transferred to and stored in long-term memory, but is not simply a buffer between the input of information and its storage. It is an active element in the processing of information. Information
being operated on is held in working memory with words and text being processed by the phonological loop and diagrams and other elements of spatial thinking being processed by the visuo-spatial sketch pad. The central executive controls the resources directed to these different elements. To a great extent, the detailed structure is not important. Of much greater relevance to the present discussion is the observation that working memory acts as a limited capacity storage system [10]. Anyone trying to solve a problem or construct a mental model without regard for these limitations is unlikely to be successful, as only a limited amount information can be retained and operated on. If this limit is exceeded some information is lost and the problem cannot be solved. Therefore, the use of external representations to take information out of working memory and onto paper or some other medium where it may be looked at afresh takes the load off working memory and allows small bits of information to be processed and worked on. This is especially the case for processing spatial information. Whenever we imagine something above or below, to the left or right or acting in a certain direction, we are using the visuo-spatial sketch pad and there is a clear limit to the complexity of spatial information that can be held in the head and operated on.

4. **Qualitative reasoning precedes quantitative.** The ability to reason qualitatively is perhaps the most under-recognised aspect of modelling, but perhaps also the most crucial. It is probably also related to the limitations of working memory in as much as it is entirely consistent with the inability to retain a lot of detailed information. In constructing mental models the tendency is to use qualitative relationships rather than detailed mathematical arguments. Qualitative relationships might be causal in nature, for example, a net force causes an acceleration, spatial, such as the direction of a force, or probabilistic, as occurs for example in statistical phenomena. Qualitative reasoning is thus a crucial aspect of mental modelling, but it is not an ability possessed by many students, especially early in their academic studies. Writing in 1987 in the context of AI and the use of computers in education, Andrea di Sessa [11] commented that, “novice physics students have been documented to display the kind of inefficient blind search through equations that … programmes avoid by using qualitative knowledge.” Moreover, “such flow of control from qualitative dependencies into calculation seems quite general”.

5. There are four components to mathematical models in physics. David Hestenes [12] first put forward his modelling methodology over thirty years ago and made explicit what is often implicitly understood by professional physicists, but is alien to students: namely, that a model is more than a set of equations, which in themselves are just a representation of a physical situation in which objects of one sort or another interact with others. Hestenes therefore defined the components of a model as:

- A set of names for the objects and agents that interact with them
- A set of descriptive variables representing properties of the objects
- The equations of the model, which comprise its structure and determine its evolution
- The interpretation of the model, which includes an evaluation and possibly application to other situations.

These five elements together define a model and a methodology. Before representing a physical situation mathematically, the modeller has to identify all the objects and interactions and, using whatever representations are appropriate, whether internal or external, construct a qualitative mental model as illustrated in figure 1. This model is then translated into a mathematical representation, which might then require further mathematical manipulation and representation back to the physical domain in order to evaluate the model and fully appreciate the implications. A key aspect of this methodology, therefore, is the ability to translate between different representations and this is illustrated in the following example of a teaching activity.
Teaching activities can generally take one of two forms: either participatory activities in which students discuss a problem and construct a model in groups or pairs, or teaching sequences in which a model is developed interactively, with guidance from the instructor, using the preceding ideas. The following example illustrates a teaching sequence based on Newton’s third law, a topic that is notoriously misunderstood by students. These difficulties even extend to situations that correspond directly to the statement of the law with which the students are most familiar. The author has analysed students’ thinking behind their answers to a question concerning which exerts the greater force on the other when a car is pushing a truck and both are accelerating [13]. This particular question is one that few students get right and those who answer incorrectly overwhelmingly apply Newton’s second law on the basis that an acceleration requires a nett force and therefore identify the force of the car on the truck as being greater than the force of the truck on the car. Even students who stated the third law in terms of the force exerted by an object A on an object B being equal and opposite to the force by object B on A, which directly applies to this situation, applied Newton’s second law in the same incorrect manner.

The teaching sequence is illustrated in figure 2. The physical situation is depicted schematically with reference to masses $m$ and $M$ representing the car and the truck respectively. Newton’s second law is applied directly from this diagram to both masses combined. Further mathematical manipulation corresponds in this case to simple re-arrangement of the terms to allow the nett force on each mass to be identified via Newton’s second law as $ma$ and $Ma$ respectively. For other, more complex models the mathematics might be significantly more involved. The nett force on the small mass is seen to comprise two terms: the external force and an opposing term, as indicated by the negative sign, consisting of the nett force on the large mass. We see from these equations that the small mass, $m$, acts on the large mass,
$M$, with a force $Ma$, and the large mass acts on the small mass with an equal and opposite force, -$Ma$. The net force on the small mass is $F-Ma$, which is always greater than zero. The acceleration is constant if $F$ is constant, as $a$ cannot increase, but neither can it decrease because there is no opposing force.

Figure 2. A teaching sequence showing the translation between representations in a model based on Newton’s 3rd law. The sequence starts with a depiction of the physical system, the translation to mathematical form by applying Newton’s second law, mathematical manipulation to separate out the individual forces acting on each mass and, finally, the representation of this mathematics in free-body diagrams.

Splitting the composite body into its two components and constructing a free-body diagram showing the force acting on each shows that both the second and the third laws apply, but crucially the third law pairs apply to different objects and therefore do not cancel out. If there is a net external force, there will be a net acceleration, but Newton’s third law still holds. In reality, we know that if $m$ corresponds to a car and $M$ to a truck, as in the FCI, at some point the acceleration will drop to zero as the car reaches cruising speed, but this is impossible in the context of this model. As long as there is only one external force acting there will be a net acceleration. It is clear, therefore, that for the acceleration to go to zero there must be another force opposing the motion and that this force must increase with speed so that at some point the net force on the system is zero. Students can play with the free body diagrams and explore the action of an opposing force, which can act on either body. If it acts on $m$ then at cruising speed there will be no force on $M$, which seems unphysical but is perfectly consistent with the laws of motion; if it acts on $M$ students are left with a seeming paradox. According to the model illustrated in figure 2, the 3rd law forces will go to zero, but they must exist and be equal to the external forces so that the net force on each body is zero. In fact, it is necessary to construct a new model taking into account the opposing external force on $M$. One then finds that the magnitude of the third law force is the sum of $Ma$ and the external opposing force so when $a=0$ all that is left is the opposing force.

The preceding highlights the explanatory power of a model expressed through different representations. It also highlights the importance of starting from first principles and constructing a
mathematical representation from the physical situation rather than taking a representation and adding elements to it. In the specific example used here, constructing free-body diagrams for each of the masses separately shows that the forces comprising the action-reaction pair are acting on different bodies. The representations thus provides the element missing in most students’ comprehension of the law and why they assume the forces must be unequal if a nett force exists. A similar model can be constructed for a number of equal masses being pushed or pulled, such as might occur in a train comprising a number of carriages.

4. Assessing modelling activities

In deciding how to assess modelling activities, there are two main considerations: first, the situation to be modelled and, secondly, the assessment criteria. There is a very wide range of possible models, but the assessment criteria should be simple enough to be consistent between models. As such, assessment should focus on the process of building models rather than the outcome of the model, which could be a numerical value in some cases or a representation in others. For example, if students were asked to model the motion of a composite system subjected to a single force, as in figure 2, to show how Newton’s 3rd law applies, a sufficient outcome might be considered to be the free body diagrams showing the nett force on each mass. However, this is only useful as an outcome if the students have actually derived the mathematics and interpreted the equations that lead to that outcome. In other words, if the model has been constructed correctly the result will follow naturally.

Despite the enormous variety of problems that can be modelled, including conventional end-of-chapter problems, this approach to assessment implies that assessing representations and the translation between them provides perhaps the simplest and most consistent assessment criteria. To recap, for the modelling protocol presented here, the model must allow for representations at the three stages of the modelling process: during the construction of the qualitative mental model, during the construction of the mathematical model, and during the evaluation stage. Not all models lend themselves to a clear distinction between stages, as the construction of a mental model very often implies the mathematical structure to follow. However, the distinction made here is that the assessment stage is concluded when the mental model has been constructed and the problem is understood. That is, the interactions have been identified and represented effectively. These representations then provide the basis for the construction of the mathematical model comprising an equation or set of equations that represent the mental model. The forward projection in time or the use of limiting cases to check for the validity of the model is reserved for the evaluation stage. Experience shows that the evaluation stage is hardest for students to grasp.

A model as defined here should therefore require at least three representations, as shown for example in figure 2: the initial representation, the mathematical equations and the interpretation as represented by the free-body diagram. The model in figure 2 is very simple and more complicated examples might require additional representations in order to model fully the problem or situation. The temptation to assess the effectiveness of each representation employed is probably best resisted as this would imply that the number of representations should be specified at the outset. This might be appropriate in some circumstances, but not in all as it places a constraint on the model to be constructed. There is value, for example, in having students construct free-body diagrams, but it should also be possible for a student to construct a model without having to construct a particular representation and ideally the assessment criteria should allow for that.

The method of assessment recommended here is to decide upon the minimum number of representations, which will usually be three, and to assign marks for the effectiveness or otherwise of those representations. My own preference is for a simple four level scheme:

- 0 for absent,
- 1 for present but needs substantial work,
- 2 for nearly there and
- 3 for effective use of representations.
A similar scheme can be used to award additional marks for the explanation behind the translation from one representation to another as the modeller moves between the stages. This is particularly important for models constructed within groups in which individual students have access to the representations constructed by the group but will not necessarily understand the reasoning behind them. Assessing the translations between representations guards against this difficulty.

5. Conclusion

The elements of a theory of modelling have been described. There are five elements and three stages to the modelling process. The five elements comprise the structure of a model in physics, which has been adapted from Hestenes [12] and augmented with findings from the cognitive psychology of problem solving [4]. The three stages comprise assessment of the problem or situation to be modelled, which is equivalent to constructing a mental model, construction of the mathematical model, which requires the elements of the mental model to be translated into mathematical equations, and the running, or evaluation, of the model. Exactly how the last stage is conducted depends on the nature of the model. It could be a simple forward projection in time, the running of a computer code, or possibly varying other parameters to examine limiting cases.

An example of a teaching activity has been described in order to illustrate how translation between representations works in practice. An assessment methodology has also been described based on two keys considerations. First, the emphasis should be on the process of modelling rather than the end result, such as a number or calculation: if a situation is assessed correctly and the representations are translated effectively into other representations, the model should be correct in as much as a model can be said to be so. It should be possible, therefore, to trace errors in thinking to particular steps in the modelling process and thereby provide effective feedback to students. The second consideration is that the process should be flexible enough to take into account individual differences in thinking, such as what assumptions are made and why and what representations are used, as well as allow for the very wide variety of different situations that can be modelled. The method adopted here is to assess how effectively representations are used at each stage and how effectively students translate between representations.

The question then arises as to how effective this methodology has proven to be. The lack of space has prevented discussion of this aspect, but this methodology has been used in the teaching of mechanics for five years and is now being extended to electricity and magnetism. Student learning in mechanics has been assessed using the Force Concept Inventory [14] both before and after instruction and improvements in the average FCI score of between 4 and 5 questions have been recorded each year. The modelling methodology adopted is interactive and interactive teaching in itself is known to produce learning gains, so these gains cannot be ascribed unambiguously to modelling, but there is another aspect of this methodology that is certainly beneficial: students appreciate that this affords a richer way of looking at problems and feedback suggests that many appreciate the approach. In conclusion, the modelling methodology presented here is based on sound experimental findings and has proven to be effective as a method of teaching concepts.

References

[1] McDermott L 2001 Oersted Medal Lecture 2001: “Physics Education Research—The Key to Student Learning”. Am. J. Phys. 69(11) 1127–37. DOI: https://doi.org/10.1119/1.1389280 1–48

[2] Larkin J, McDermott J, Simon D and Simon H 1980 Expert and novice performance in solving physics problems Science 208(4450) 1335–42. DOI: https://doi.org/10.1126/science.208.4450.1335

[3] Chi M T, Feltovich P J and Glaser R 1981 Categorization and representation of physics problems by experts and novices Cogn. Sci. 5(2) 121–152. DOI: https://doi.org/10.1207/s15516709cog0502_2

[4] Sands D and Overton T 2018 Cognitive psychology and problem solving in the physical sciences
Available at: https://journals.le.ac.uk/ojs1/index.php/new-directions/article/view/374 [Accessed 6 Sep 2018]

[5] Epstein J 2008 Why model? J. Artif. Soc. Soc. Simul. 11(4) 12
[6] Giere R 1988 Explaining science Chicago: University of Chicago Press
[7] Nersessian N 2010 Creating scientific concepts Cambridge, Mass: MIT Press
[8] Glaser R 1984 Education and thinking: The role of knowledge. Am. Psych. 39(2) 93–104
[9] Suwa B and Tversky B 2002 External representations contribute to the dynamic construction of ideas. In: Hegarty M, Meyer B and Hari Narayanan N (Eds) Diagrammatic Representation and Inference, Proc. Inference 2002, Lecture Notes in Artificial Intelligence series, Springer 341–3
[10] Reid N 2009 Working memory and science education: conclusions and implications. Res. Sci. Techn. Educ. 27(2) 245–50. DOI: https://doi.org/10.1080/02635140902853681
[11] DiSessa A 1987 The third revolution in computers and education J. Res. Sci. Teach. 24(4) 343–67. DOI: https://doi.org/10.1002/tea.3660240407
[12] Hestenes D 1987 Toward a modeling theory of physics instruction Am. J. Phys. 55(5) 440–54. DOI: https://doi.org/10.1119/1.15129
[13] Sands D 2018 Evidence for the applicability of dual processing theory in physics problem solving icpe2013.org. Available at: http://www.icpe2013.org/uploads/ICPE-EPEC_2013_ConferenceProceedings.pdf [Accessed 6 Sep. 2018].
[14] Sands D and Marchant A 2018 Enhanced conceptual understanding in first year mechanics through modelling. Available at: https://journals.le.ac.uk/ojs1/index.php/new-directions/article/view/490 [Accessed 6 Sep 2018]