Black Crunch

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ABSTRACT: We study the growth of fluctuations in collapsing cosmologies, extending old work of Lifschitz and Khalatnikov. As examples of systems where the fluctuations have a different composition than the background we study scalar fields with general improvement terms. Fluctuations always grow, and often dominate the homogeneous background. We argue that even for very dilute fluctuations, scattering processes inevitably lead to a dense gas of black holes. This leads us to hypothesize that the generic final state of a Big Crunch is described by a collapsing $p = \rho$ FRW cosmology. We conjecture that the black hole fluid is invariant under the conformal Killing symmetry of this metric, so that the final state is in fact stationary.

KEYWORDS: Cosmological singularities.
1. Introduction

A wide variety of cosmological initial conditions for Einstein’s equations lead to a future spacelike singularity colloquially known as a Big Crunch. The energy density and curvature invariants all become singular on an entire spacelike hypersurface. For many years, physicists have speculated about the meaning of this singularity. A popular scenario, which has recently been revived in the context of string theory [2] is that quantum effects would lead to a ”bounce” followed by reexpansion of the universe.

In the present paper, we will present a different view. We will argue that the generic final state in a Big Crunch is a maximally stiff \( p = \rho \) fluid, and further, that even the collapsing cosmological solution in the presence of such a fluid, is in some sense, a stationary state. In earlier papers [1][3] we have argued that such a fluid is the appropriate semiclassical initial state for the Big Bang. In particular, we showed there that a mechanistic model for such a fluid is a ”dense gas of black holes”. This picture will form the basis for our intuitive discussion of the Big Crunch\(^1\).

\(^1\)We emphasize that although the equation of state is the same, the quantum states of the Big Bang and Big Crunch are very different. The Big Bang state consists of a collection of systems with limited correlation between them because of the existence of a finite particle horizon. In the Big Crunch, we imagine that all the states of the universe are correlated.
One of the motivations for this picture of the Big Crunch is a series of early papers by Lifshitz and Khalatnikov[7](LK). These authors developed a general formalism for the study of fluctuations, and behavior near a singularity, and applied it primarily to the study of cosmologies with the conventional equations of state of nonrelativistic and ultrarelativistic gases. They concluded that in these cosmologies, fluctuations come to dominate the energy density as one approaches the singularity. We will review this calculation in Section 2.

The LK analysis applies to general equations of state of the form \( p = \kappa \rho \). They assume that the inhomogeneous fluctuations satisfy the same equation of state. Clearly, this is a rather special assumption. If we want to study a more general class of fluctuations we must present a real field theoretical model for them, rather than characterize them by an equation of state. We will do this in Section 3 by studying scalar fields with a general coefficient of the improvement term in the stress tensor. Again we find growing fluctuations near a Big Crunch. In section 4, we discuss the fate of these fluctuations. For the case of fluctuations that can be thought of as a dilute gas of relativistic particles (which we argue is a sort of worst case scenario for our conjecture) we argue that particle collisions will lead to a dense fluid of black holes with equation of state \( p = \rho \). We then outline our reasons for believing that this system is conformally invariant under the conformal Killing transformation of the \( p = \rho \, F(riedmann)-R(oberston)-W(alker) \) cosmology. This implies that the \( p = \rho \) fluid is in some sense a quiescent, stationary state. All flat FRW universes whose scale factor is a pure power of the time, have conformal Killing vectors. The metric rescales by a constant factor when the cosmic time, and spatial coordinates are rescaled in a correlated fashion. This is in no sense a symmetry of the physics in a generic FRW universe. The Hamiltonian for particles and waves propagating in such a universe are not invariant under this conformal isometry. However, if we believe that the \( p = \rho \) fluid has no such localized excitations, then there are no apparent observables that detect the change of scale.

The rigorous definition of black hole regions of spacetime is, “The complement of the causal past of null infinity”. However, there is clearly a more local, approximate notion of black hole associated with the creation of trapped surfaces. We think that a quantum version of the Cosmic Censorship conjecture would read something like the following: Any time a trapped surface forms, and classical GR predicts a singularity, a region containing the trapped surface is excited to a state of maximal entropy, that is a superposition of a large number of almost degenerate states. This region is then called

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2We will avoid talking to any person who asks us the unpleasant question of what the observables in a \( p = \rho \) gas are!

3The number is the exponential of one quarter the area of the region, and the typical energy splitting is of order the inverse of this number.
a black hole. A dense fluid of thus defined black holes, with typical separation of order their size, satisfies $\sigma \propto \rho^{1/2}$, where $\sigma$ and $\rho$ are the entropy and energy densities. If, as time goes on, the black holes merge in order to preserve this relation, then the fluid has equation of state $p = \rho$. This is the dense black hole fluid that we discussed in [3].

2. Review of the Lifshitz-Khalatnikov analysis

In most of this paper we will study a flat infinite Big Crunch solution. Some remarks about the effect of the global topology of the Big Crunch universe will be presented in the Conclusions. Following [7], we work in conformal coordinates for the FRW universe:

$$ds^2 = a^2(\tau)(-d\tau^2 + (dx^i)^2). \quad (2.1)$$

The linearized stress tensor has the form:

$$\delta T^k_i = (p + \rho)(u_i \delta u^k + u^k \delta u_i) + (\delta p + \delta \rho)u_i u^k + \delta^k_i \delta p \quad (2.2)$$

LK show that

$$\frac{\delta \rho}{\rho} = \frac{1}{3\rho a^2} [n^2(\lambda + \mu) + 3a' a\mu']e^{i n_j x^j}. \quad (2.3)$$

Primes denote derivatives with respect to conformal time $\tau$, and $n_i$ is the dimensionless wave number of the fluctuation. $\lambda$ and $\mu$ appear in the parametrization of the spatial metric perturbations $h_\alpha^\beta$

$$h_\alpha^\beta = [\lambda(\tau)\left(\frac{1}{3}\delta_\alpha^\beta - \frac{n_\alpha n^\beta}{n^2}\right) + \frac{\mu(\tau)}{3}\delta_\alpha^\beta]e^{i n_j x^j}. \quad (2.4)$$

We are in a synchronous gauge, where $h_{00} = 0 = h_{0\alpha}$.

The linearized Einstein equations determine $\lambda$ and $\mu$ in terms of four auxiliary functions $\lambda_0, \mu_0, \psi, \zeta$. These are defined by the following four equations

$$\lambda_0 = -n^2 \int \frac{d\tau}{a(\tau)} \quad (2.5)$$

$$\mu_0 = -\lambda_0 - \frac{3a'}{a^2} \quad (2.6)$$

$$\psi' + \psi\left[\frac{2a''}{a^2} + \frac{a'}{a}(-2 + 3\kappa/2)\right] + \kappa\zeta/2 = 0 \quad (2.7)$$

$$\zeta' + \zeta\frac{a'}{a}(1 + 3\kappa/2) + \psi[-2n^2 + 3\frac{a''}{a} + \left(\frac{a'}{a}\right)^2(9\kappa/2 - 6)] = 0 \quad (2.8)$$
In these equations, \( \kappa \) parametrizes the equation of state of both the background and the fluctuations: \( \delta p = \kappa \delta \rho \). Then

\[
\lambda + \mu = (\lambda_0 + \mu_0) \int \psi d\tau \tag{2.9}
\]

\[
\lambda' - \mu' = (\lambda'_0 - \mu'_0) \int \psi d\tau + \frac{\zeta}{a} \tag{2.10}
\]

For generic values of \( \kappa \), we can solve equation (2.7) for \( \zeta \) and obtain an equation depending only on \( \psi \). This strategy does not work for \( \kappa = 0 \), and we will discuss this special case first. The relevant formulae for the coefficients in our equations, when the background equation of state is \( p = \kappa \rho \) are:

\[
a = \tau^{\frac{2}{3 + \kappa}} \tag{2.11}
\]

\[
\frac{a''}{a'} = \frac{1 - 3\kappa}{(1 + 3\kappa)\tau} \tag{2.12}
\]

\[
\frac{a'}{a} = \frac{2}{(1 + 3\kappa)\tau} \tag{2.13}
\]

\[
\frac{a''}{a} = \frac{2(1 - 3\kappa)}{(1 + 3\kappa)^2\tau^2} \tag{2.14}
\]

\[
\rho a^2 \sim \tau^{-2} \tag{2.15}
\]

The equation for \( \psi \) when \( \kappa = 0 \) has the simple solution

\[
\psi = \psi_0 \tau^2 \tag{2.16}
\]

It is then easy to see that the term proportional to \( \psi \) in the equation for \( \zeta \) is negligible near the singularity. Note that this includes all the dependence on the comoving wave number, as long as that quantity is fixed. The equation for \( \zeta \) now becomes

\[
\zeta' + \frac{2}{\tau} \zeta = 0. \tag{2.17}
\]

So

\[
\zeta = \zeta_0 \tau^{-2}. \tag{2.18}
\]

These equations can now be plugged into (2.9) and (2.10) to obtain \( \lambda \) and \( \mu \) and eventually \( \frac{\delta \rho}{\rho} \). The lower limit of integration can be changed at will by gauge transformations that preserve the synchronous gauge [7]. We take it to be zero, to cancel off singular, pure gauge terms. Note that the powers of \( \tau \) appearing in these terms are different than the powers in the gauge invariant terms that we retain. The first, momentum dependent, term in Equation (2.3) is subleading, and we find
\[ \frac{\delta \rho}{\rho} \sim \zeta_0 \tau^{-3} \]  

(2.19)

Inhomogeneous nonrelativistic fluctuations dominate the background near the singularity.

We now turn to the general case, eliminating \( \zeta \) in terms of \( \psi \). We find a second order equation

\[ \psi'' - 2\psi \frac{(1 - 3\kappa)}{(1 + 3\kappa)^2 \tau^2} = 0. \]  

(2.20)

This has solutions of the form \( \tau^A \) where

\[ A^2 - A - 2 \frac{(1 - 3\kappa)}{(1 + 3\kappa)^2} = 0. \]  

(2.21)

As before, we choose the most singular power. The power law behavior of \( \zeta \) is \( \tau^{(A-1)} \). We again find that the momentum dependence drops in the most singular terms (i.e. all comoving momentum modes grow in the same way) and we find:

\[ \frac{\delta \rho}{\rho} \sim \left[ \frac{18(1 + \kappa)}{(A + 1)(1 + 3\kappa)^2} - 3 \frac{1}{1 + 3\kappa} + \frac{1}{\kappa} (A - \frac{3\kappa + 2}{1 + 3\kappa}) \right] \tau^{x + 3\kappa}, \]  

(2.22)

where \( x = A(1 + 3\kappa) - 2 \). \( x \) satisfies

\[ x^2 + 3x(1 - \kappa) = 0. \]  

(2.23)

One root of this equation vanishes, reflecting a perturbation which is simply a rescaling of the background, while the other is negative, reflecting an unstable growth of inhomogeneous fluctuations on all length scales.

3. Improved fluctuations

The LK analysis relies on equal coefficients in the background and fluctuation equations of state, and is inconsistent if this assumption is not made. Nonetheless it is clear that fluctuations which are not the same kind of matter as the homogeneous background are possible. In any such case, one must specify the dynamics of the matter in a way that goes beyond the use of an equation of state. One could for example consider a general scalar field Lagrangian or multiple scalars. It turns out that potential terms in the Lagrangian are irrelevant near the singularity. Scalars with non-canonical kinetic terms (a metric on field space) all behave like a minimally coupled scalar. Couplings of the form \( f(\phi)R \), do affect the behavior near the singularity. At the linearized level
in which we will be working, it is sufficient to keep the improvement term\[10]. These are the simplest models in which we can analyze fluctuations that do not satisfy the equation of state of the background.

We will set the classical value of the scalar field to zero. This has the advantage that the gravitational backreaction to the fluctuations is a higher order correction. Thus, we can work in FRW coordinates and many of the gauge fixing questions in the LK analysis do not arise. We are describing inhomogeneous scalar fields on a fixed homogeneous background manifold, which is undergoing a Big Crunch.

The equations for scalar field perturbations in FRW coordinates are

\[
\ddot{\phi}_k + 3 \frac{\dot{a}}{a} \dot{\phi}_k + \left( \frac{k^2}{a^2} \right) \phi_k + \xi R \phi_k = 0.
\]

The value \( \xi = 1/6 \) corresponds to conformal coupling. In this case, there is a cancellation of the most singular term in the fluctuations. We will discuss it separately below.

In the mostly plus metric convention, the scalar curvature of an FRW background satisfies

\[
R = \rho - 3p = 3(1 - 3\kappa)\left( \frac{\dot{a}}{a} \right)^2.
\]

The scale factor \( a \) satisfies \( a \sim t^\beta \), where \( \beta = \frac{2}{3(1+\kappa)} \). The scalar equation then has solutions of the form \( \phi_k \sim t^\alpha \) near \( t = 0 \). Note that for \( \kappa > -1/3 \), \( \alpha \) is independent of \( k \), because the momentum term in equation 3.1 is subleading. We will not study these very negative values of \( \kappa \) because we find it implausible that such negative pressure matter will dominate the universe near the Crunch. Other forms of energy density grow much more rapidly near the singularity. The exponent \( \alpha \) satisfies

\[
\alpha^2 + (3\beta - 1)\alpha + 3\xi(1 - 3\kappa)\beta^2 = 0
\]

whose solutions are

\[
\alpha = \frac{1}{2 + 2\kappa} \left[ (\kappa - 1) \pm \sqrt{(\kappa - 1)^2 + \frac{16\xi}{3}(3\kappa - 1)} \right].
\]

For \( \kappa > -\frac{1}{3} \) the argument of the square root is positive for \( 0 < \xi < \frac{1}{6} \). It vanishes in the case \( \kappa = 1 \) and \( \xi = 0 \). In this case we have a logarithmic solution as well. For general \( \xi \) the leading singularity in the stress tensor does not cancel and its order can be estimated just by calculating \( \dot{\phi}^2 \). In any FRW metric with power law scale factor, the Friedmann equation implies that the background energy density scales as \( t^{-2} \) near the singularity. Thus the power law in \( \frac{2\dot{\rho}}{\rho} \) is just \( t^{2\alpha} \). For all values of \( \kappa > -\frac{1}{3} \) and \( 0 < \xi \leq 1/6 \), there is always one negative root of \( \alpha \) and we find that the fluctuations
are growing relative to the background. For the special case $\xi = 0$, $\kappa = 1$, both roots vanish. This is the situation where we also have a logarithmic solution, but this also gives a constant $\delta \rho / \rho$. Note however that this case is also covered by the LK analysis, which takes into account the back reaction of the fluctuating energy density on the metric. That analysis seems to give a logarithmic singularity in $\delta \rho / \rho$. There has been some controversy about this in the literature\cite{9}.

Finally, we turn to the conformally coupled case. In this case the trace of the fluctuating stress tensor vanishes identically as a consequence of the scalar field equation of motion. Covariant conservation then implies that $\delta \rho \sim a^{-4} \sim t^{-\frac{8}{9(1+\kappa)}}$. This grows more rapidly than the background only when $\kappa < \frac{1}{3}$. Thus the conformally coupled case has tamer fluctuations than more general values of $\xi$. One can check that this is due to a cancellation of the most singular term in the stress tensor, for this value of $\xi$. Again, for $\kappa = \frac{1}{3}$ the LK analysis applies and implies that gravitational backreaction makes the fluctuations more singular. Intuitively this behavior is expected because gravity is attractive and leads to the growth of inhomogeneous fluctuations.

In an expanding universe, finding that $\delta \rho$ grows more rapidly than $\rho$ is a signal that one is about to enter an era of local gravitational collapse and black hole formation. This would seem even more likely in the contracting case. We can see that the situation of a very dilute gas of conformally invariant fluctuations is in some sense the worst case scenario for black hole formation. In a generic background, in the approximation we have used in this section, the conformally coupled scalar fluctuations grow less rapidly than any other form of scalar energy density. Similar behavior is found for photons or any other form of conformal matter. In the next section we will examine this case and argue that even here, scattering processes, which are not taken into account either in the analysis of this section, or that of LK, will lead to black hole formation.

4. Scattering of inhomogeneities

Let us imagine then that we begin with an FRW universe which is heading towards a Big Crunch. We superpose on this background a spectrum of fluctuations, consisting of relativistic particles. The energy density in the $k$th mode at some initial time is equal to $\epsilon_k$. Here $k$ is the dimensionless comoving momentum. We imagine that the particles have initial $k$ values which are all of the same order of magnitude. At later times, the fluctuating energy density is

$$\delta \rho_k \sim \frac{\epsilon_k}{a^4} \quad (4.1)$$

The typical center of mass energy in collisions of these particles will be $\sim k/a$. Their
number density is \( n_k \sim \epsilon_k / k a^3 \). Thus their typical impact parameter is

\[
I \sim a \frac{k^{1/3}}{\epsilon_k^{1/3}}
\]  

(4.2)

There are strong arguments[8] that when the impact parameter is of order the Schwarzschild radius of the center of mass energy, a finite fraction of the collisions will lead to black hole formation. The criterion for this to occur is

\[
a \frac{k^{1/3}}{\epsilon_k^{1/3}} \sim \frac{k}{a M_P^2}.
\]  

(4.3)

This criterion is reached when

\[
a^2 M_P^2 \sim \epsilon_k^{-2/3} k^{4/3}
\]  

(4.4)

If \( \epsilon_k / k^4 \ll 1 \) then this occurs when the energy density of the fluctuations is much smaller than the Planck scale. This distribution cannot be thermal, but by assumption we are talking about small inhomogeneous fluctuations on a homogeneous background, so we do not expect them to be thermal. For a given initial distribution \( \epsilon_k \), black hole formation will occur first at some particular value of \( k \). Other particles in the distribution will be swallowed by the gas of black holes that has been created.

At the time of black hole formation, the separation between black holes is of the order of their Schwarzschild radii. Thus, the black hole gas satisfies the entropy density/energy density relation \( \sigma \propto \sqrt{\rho} \). The black holes are being pushed together by the contraction of the universe. Thus they will begin to undergo a process of continual merger, creating the dense black hole fluid that we have identified as a mechanistic model of the \( \rho = p \) equation of state. Eventually the entropy density will hit the holographic bound[5]. We have argued[3] that in this situation there can be no inhomogeneous energy fluctuations of the black hole fluid. The entropy of the system resides in internal black hole states, which are (with exponential accuracy as the black hole mass gets large) degenerate in energy. Note that this conclusion depends on quantum mechanics through the Bekenstein-FSB bound. We cannot expect to see it coming out of the classical GR equations for fluctuations in the \( p = \rho \) background. In particular, although these equations can encode the equation of state, they have no information about whether the holographic bound is saturated. In a relatively dilute system of black holes (e.g., one in which the typical separation between black holes is 100 times their Schwarzschild radius), the holographic bound is not saturated. Inhomogeneous fluctuations will obviously grow in such a gas, resulting in the production of larger black holes. Our point is that the holographic bound tells us that this process eventually stops.
Although the $p = \rho$ FRW Big Crunch proceeds to a singularity in finite time, we believe that this is an illusion. The maximally saturated black hole fluid seems like a quiescent stationary state. It is self similar. We conjecture that the mathematical representation of this intuition is the invariance of the quantum mechanics of the $p = \rho$ fluid under the conformal Killing vector of the FRW geometry. Indeed, a system invariant under this transformation feels the Big Crunch as a geometry completely equivalent to flat Minkowski space. The singularity has disappeared because the physical system which experiences it is invariant under the time evolution in a collapsing FRW universe.

5. Conclusions

In this paper, we have reviewed and extended earlier work, which shows that fluctuations grow in Big Crunch spacetimes. We argued that even mild growth of fluctuations (i.e. even those for which $\frac{\delta \rho}{\rho}$ remains bounded) would, via scattering, lead to production of a dense black hole fluid filling spacetime. We conjectured that this is the proper description of the final state of a Big Crunch. Its global geometry is that of a collapsing, homogeneous $p = \rho$ FRW cosmology. We conjectured that the black hole fluid was in fact invariant under the conformal Killing symmetry of the geometry, so that the Black Crunch is in fact a stationary state and the singularity is an illusion. There are no observables of the physical system filling the universe, which can feel the singularity.

This paper dealt only with infinite flat universes. The neglect of spatial curvature is likely to be unimportant, but the neglect of spatial topology might not be. In previous work with Motl[11] we have argued that certain Big Crunch singularities in cosmological solutions of the low energy field equations of M-theory might be reinterpreted by duality transformations and the singularity resolved. These all have the following character: after the duality transformation the spacetime is approaching a large radius compact geometry in either 11 dimensions, or weakly coupled type II string theory. The singularity consists of the fact that one approaches infinity in moduli space in finite time. We conjectured that production of the light Kaluza-Klein modes of the expanding background would slow the expansion rate and eliminate the singularity.

For such singularities, there are two competing processes going on. Imagine approaching the singularity in a duality frame where it looks like some of the dimensions are contracting, and let the universe be filled primarily with unwound matter states in this frame. The matter density is homogeneous, with small inhomogeneous fluctuations. As we approach the singularity, scattering processes among these states can produce light winding states, which are the light KK modes of the nonsingular duality frame. The inhomogeneities can also scatter to produce black holes. The question of
which of these processes dominate may depend on the precise initial state, and the fate of the singularity may depend on which process dominates. The issue is also complicated by the possibility of black hole to black brane transitions. So it seems possible that the mechanism of [11] provides an alternate endpoint for this kind of Big Crunch. However, much work remains to be done to sort out what really happens.

In [11] we also pointed out another class of singularities that could not be removed by duality transformation. The system approaches asymptotic regions of moduli space that are not dual to any semiclassical description. In this case, we expect that the Black Crunch described in this paper will be the appropriate end point for the evolution of the system.

There are several directions of research which are affected if the Black Crunch is indeed the fate of cosmological singularities. The first is research into models where the universe passes through a Big Crunch and bounces back to a large expanding universe. In our opinion, models of that type only make sense if the singularity is in fact dual to an expanding universe. The analysis of what happens as one passes through the classical singularity is very complicated and involves the competition of black hole and KK mode production. It seems unlikely that any straightforward translation of information about fluctuations before the crunch into predictions about fluctuations after the crunch could be performed. One would have to solve a complicated system of space time dependent rate equations. The geometry after the crunch is not related to that prior to the crunch by any local transformation, and the nature of the light states changes completely. None of the models in the literature exhibit any of these features.

The second direction which we find problematic is the attempt to find a resolution of singularities within the domain of weakly coupled string theory. In a Big Crunch singularity, particle energies are blueshifting and impact parameters are getting smaller. It is well known that string perturbation theory breaks down at high energy and a range of impact parameters that grows with the energy. The kinematic regime that particles experience in a Big Crunch is one in which black hole production is important. Note that even if our fantasies about the resolution of certain singularities by duality turn out to be correct, the system always passes through regions where the string coupling is of order one and string perturbation theory is an inadequate guide to the physics. It is Planck scale, rather than string scale physics which is involved when it comes to the crunch.

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