Investigation on the Cavity Backwater of the Jet Flow from the Chute Aerators

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Abstract

In order to protect discharge works against cavitation erosion, the air concentration in the flow needs to reach some extent, which demands the jet flow from an aerator to form a good flow pattern with an enough cavity space for aeration. Based on the hydrodynamic analysis and the momentum equation of the jet flow from an aerator, the equation for the backwater depth in the cavity of jet flow from the chute aerators has been theoretically established. Together with the trajectory equation of jet flow, the backwater depth in the flow cavity of aerators can be calculated. Also the reason for the appearance of the backwater is analyzed. Comparison shows that the calculation results agree well with the observed experimental data.

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1. Introduction

The chute bottom aerator, to prevent cavitation in high velocity flows, has become more and more popular as it has proved to be an economical, effective and successful measure. It has been widely used in the discharge works of hydraulic engineering. In order to protect discharge works against cavitation erosion, the air concentration in the flow needs to reach some extent, which demands the jet flow from an aerator to form a good flow pattern with an enough cavity space for aeration. In some hydraulic projects, however, the phenomena that some backwater, much or less, exists in the flow cavity can be observed.

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The existence of the backwater in the cavity will affect flow and aeration characteristics. The backwater makes flow jet shorter and cavity room smaller so as to weaken flow aeration. Severely, the backwater can make the cavity disappear or the aerator submerged. In this case, the aeration of flow cannot occur and the aerator will lose its function. The investigation on the backwater in the cavity of jet flow from chute aerator is very important both in theory and in practical use.

There are a lot of factors to affect an aerator’s backwater in the cavity, such as the boundary conditions: the chute slope, the aerator style and size (step height, ramp’s angle and height etc.) and the flow conditions: the approaching flow velocity and depth, the incidence angle of jet flow on the chute, the sub-pressure in the cavity, the cavity length and the air resistance, etc. The cavity length is an important factor to affect aeration. There are some methods to calculate the cavity length of the jet flow from chute aerators: dimensionless empirical formulas [1], projectile formulas [2~4] and numerical simulations [5~9]. But the backwater which may exist in the cavity of jet flow is not taken into account in all these methods. Up till now, there is no really satisfied method for calculating the backwater in the cavity of jet flow from chute aerators, with both relative high precision and simple in calculation. The research on backwater in the cavity of jet flow from an aerator is seldom found. To weaken the backwater in the cavity on the chute with mild slope, the concept of “equivalent height of ramp” is brought forward and the layout of a concave aeration ramp is suggested for aerator [10]. The hydraulics and aeration characteristics of low Froude number flow over a step aerator were investigated and the calculation of the water volume contained in cavity was also discussed by Yang [11]. Yang’s method found a good way to solve the backwater question, however, a few assumptions and calculations, such as the choice of the control water body for momentum equation and the calculation of water pressure, are not quite rational.

Mainly based on the hydrodynamic analysis of the jet flow from an aerator, an equation for backwater depth in the cavity is tried to be theoretically established in this paper. Also the reason for the appearance of the backwater is analyzed.

2. Hydrodynamic analysis for the backwater in the cavity

2.1. Theoretic analysis for backwater depth

Fig.1 Scheme of jet flow over an aerator

![Diagram of jet flow over an aerator](image)

Fig.1 shows a flow over a chute aerator, in which, \( AB \) and \( ED \) are the cross-sections of jet flow that are vertical to the flow trajectory. If the air resistance of jet flow is neglected, the horizontal velocity of the jet, \( v \cos \beta \), can be considered unchangeable. So the average flow velocity and the depth in cross-section \( AB \) are as follows:

\[
v_i = \frac{v \cos \beta}{\cos (\alpha + \theta_i)}
\]

\[
h_i = \frac{v}{v_i} = \frac{h \cos (\alpha + \theta_i)}{\cos \beta}
\]
Where, \( v \) and \( h \) are the approaching average flow velocity and the flow depth respectively, \( v_1 \) and \( h_1 \) are the average flow velocity and the depth in cross-section \( AB \) respectively, \( \theta_1 \) is the included angle between the jet trajectory and \( x \) axis at point \( B \), \( \beta \) and \( \alpha \) is the chute slope upstream and downstream the aerator respectively.

The average flow velocity \( v_2 \) in cross-section \( ED \) and its depth \( h_2 \) are:

\[
v_2 = \frac{v \cos \beta}{\cos(\alpha + \theta_2)}
\]

\[
h_2 = \frac{h \cos(\alpha + \theta_2)}{\cos \beta}
\]  

(3)  

(4)

Where, \( \theta_2 \) is the included angle between the trajectory of jet flow and \( x \) axis at point \( D \).

Take the water volume \( ABCDEA \) as the control water body to establish the momentum equation of the unit-wide flow in the \( x \) direction. In this way, if the movement of the backwater is not considered, all the friction of flow on the soleplate of the chute in the control body does not exist, which is just the difficult factor in Yang’s method [11]. The forces exerted on the control body are: hydrodynamic pressures \( F_1 \) on the cross-section \( AB \) and \( F_2 \) on the cross-section \( ED \); \( F_c \) on water surface \( BC \); gravity \( G_1 \) of water volume \( BCD \) and \( G_2 \) of water volume \( ABDE \); the force \( F \), vertical to the \( x \) direction, which does not affect the momentum equation in the \( x \) direction. These forces can be expressed as the follows:

\[
F_1 = \frac{p_A + p_B}{2} h_1 = -\frac{\Delta p h \cos(\alpha + \theta_2)}{2 \cos \beta}
\]  

(5)

Where, \( p_A = p_a \) and \( p_B = p_c \), \( \Delta p = p_a - p_c \), \( p_c \) is the air pressure in the cavity. It must be emphasized that the pressure in cross section \( ED \) does not meet the static pressure distribution law because the flow pattern in this section is not a gradual change flow. However, if the pressures in points \( E \) and \( D \) are known, the pressure force in all the section can be achieved approximately. Because the backwater in the cavity can be regarded as static water, the water pressure at point \( D \) is

\[
P_D = p_c + \rho g \frac{\sin(\alpha + \theta_2)}{\sin \theta_2}
\]  

(6)

Where, \( y \) is the distance from point \( B \) to \( x \) axis, i.e. the depth of the backwater \( H \), \( \rho \) and \( g \) are the water density and the gravity acceleration.

Therefore,

\[
F_2 = \frac{1}{2} (p_c - p_a) h_2 + \frac{1}{2} \rho g \frac{\sin(\alpha + \theta_2)}{\sin \theta_2} h_2 = -\frac{\Delta p h \cos(\alpha + \theta_2)}{2 \cos \beta} + \frac{1}{4} \rho g y \sin(2(\alpha + \theta_2)) \sin \theta_2 \cos \beta
\]  

(7)

\[
G_1 = \frac{1}{2} y CD \rho g + \frac{1}{2} \rho g y^2 (\cot \alpha + \cot \theta_2) = \frac{1}{2} \rho g y^2 \frac{\sin(\alpha + \theta_2)}{\sin \theta_2}
\]  

(8)

\[
G_2 \approx \frac{\rho g h_1 + h_2}{2} BD = \frac{\rho g y}{2 \cos \beta \sin \theta_2} (\cos(\alpha + \theta_1) + \cos(\alpha + \theta_2))
\]  

(9)

\[
F_3 = (p_c - p_a) \frac{y}{\sin \alpha} = -\Delta p \frac{y}{\sin \alpha}
\]  

(10)

The momentum equation for the control body in \( x \) direction is

\[
F_1 \cos \theta_1 + (F_c + G_1 + G_2) \sin \alpha - F_2 \cos \theta_2 = \rho g (v_2 \cos \theta_2 - v_1 \cos \theta_1)
\]

It can be shortly written as:
\( k_1 y^2 + k_2 y + k_3 = 0 \) \hspace{1cm} (11)

where, \( q \) is flow unit-discharge

\[
k_1 = \frac{1}{2} \rho g \frac{\sin(\alpha + \theta_2)}{\sin \theta_2}
\]

\[
k_2 = \frac{1}{2} \rho g h \frac{\sin \alpha}{\cos \beta \sin \theta_2} (\cos(\alpha + \theta_1) + \cos(\alpha + \theta_2)) - \Delta p - \frac{\rho g h}{4} \frac{\sin 2(\alpha + \theta_2)}{\tan \theta_2 \cos \beta}
\]

\[
k_3 = \frac{\Delta p h}{2 \cos \beta} (\cos \theta_2 \cos(\alpha + \theta_2) - \cos \theta_1 \cos(\alpha + \theta_1)) - \rho q v \cos \beta \left( \frac{\cos \theta_2}{\cos(\alpha + \theta_2)} - \frac{\cos \theta_1}{\cos(\alpha + \theta_2)} \right)
\]

2.2 Equation of the jet flow trajectory

Xu established a differential equation for jet trajectory from aerators [12]. This equation is based on the hydrodynamic analysis of a micro water unit of the jet flow, in which the negative pressure in the bottom cavity of the jet as well as the centrifugal force of flow are considered. In the coordinate system shown in Fig. 1, the jet trajectory equation is

\( y_g = y'_{gh} \frac{\cos \alpha - y'(1 + y'^2) \sin \alpha + \frac{\Delta p}{\rho g h} (1 + y'^2)^{\frac{3}{2}}}{g} y'^2 \)

(15)

Its fixed-solution conditions are

\[
\begin{cases}
y'(0) = \tan \theta \\
y(0) = \Delta + y_0
\end{cases}
\]

(16)

Where, \( \Delta \) is the height of the ramp and \( \theta \) is its takeoff angle; \( y_0 \) is the height of the vertical step of chute.

Together with the backwater Eq. (11) and the jet trajectory Eq. (15), the backwater depth in the cavity and the net jet length can be calculated.

3. Solution for equations

If the style and size of an aerator and the flow conditions are known, the downside jet trajectory can be obtained from Eq. (15) written as:

\( y = f_1(x) \)

(17)

Meanwhile from the curve of Eq. (16), \( \theta_2 \) can be calculated out. When \( \theta_2 \) is known, parameter \( k_1 \) in Eq. (11) is a known number, and parameters \( k_2 \) and \( k_3 \) in Eq. (13) and Eq. (14) are the functions of \( \theta_1 \) alone. From Fig. 1, it is known that \( y'(x) = \tan(\pi - \theta_1) = -\tan \theta_1 \), so, \( y \) in Eq. (11) is also a function of \( x \):

\( y = f(\theta_1) = f_2(x) \)

(18)

The curves of Eq. (17) and Eq. (18) are shown in Fig. 2. \( y \), corresponding to the intersection point \( B \) of the two curves, is the backwater depth in cavity, and \( L_{\text{net}} \) is the net length of the jet. If the intersection point \( E \) of the two curves is superposed with point \( D \), it is meant that the backwater depth is zero, i.e. no backwater is contained in the cavity. If the point of intersection \( B \) of the two curves is located at point \( K \), it is theoretically meant that the cavity is full of water, i.e. no cavity exists. In this case, the aerator loses its function completely.
From the deduction of equation (11) and Fig.1, it can be seen that when the backwater depth \( y \) is zero, the cross-sections \( AB \) and \( ED \) are superposition. In this case, the momentum equation loses its meaning (the volume of flow control body is zero.). Under certain boundary and hydraulic conditions, if there is backwater in the cavity, to keep the flow momentum to be equilibrium, the backwater depth must be a unique one and meets the momentum equation. The reason why the backwater is contained in the cavity is that in these conditions the flow needs backwater in the cavity to keep its momentum equilibrium.

5. Calculation Results

In the reference [11], the model chute is 0.50m wide, 0.40m high and 18m long. The height of the vertical step of the chute \( y_0 \) is 0.104m, the slope of the chute \( \alpha = \beta = 5^\circ \), and the ramp angle of the aerator \( \theta = 0^\circ \). In the experiments, the flow Froude number \( F_r \) varies from 2.81 to 5.95. Fig.3 and Fig.4 show the comparison of the net length \( L_{net} \) and the backwater depth \( y \) between the calculated values and the experimental data with the flow \( F_r \), respectively. Fig.5 is the Relation of \( L_{net} \) with discharge \( q \) and Fig.6 is the relation of \( y \) with \( q \). Calculated and experimental results show that \( y \) decreases with the increasing of \( F_r \). With the increasing of \( F_r \), the \( L_{net} \) increases first and then decreases. It can be found that the calculation results agree well with the observed experimental data.
6. Conclusions

(1) Based on the strict hydrodynamic deduction and clear mechanics conceptions, an equation for the backwater depth in a jet cavity from chute aerators is set up in this paper, by which the backwater depth in the cavity and the net length of the cavity can be achieved.

(2) The reason of backwater contained in the cavity is that in certain boundary and flow conditions the flow needs backwater in the cavity to keep its momentum equilibrium.

(3) With the increasing of flow Froude number, the backwater depth in the cavity decreases monotonously, but the net length of cavity increases first and then decreases.

(4) The factors to affect the backwater contained in the jet cavity include the boundary conditions such as the type of aerator, the slope of the chute etc and the flow conditions such as the flow Froude number, the sub-pressure etc. How these factors affect the backwater needs to be further investigated.

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