A Novel Possibility to Determine the $CP$-violating Phase $\gamma$ and the $B_s^0$-$\bar{B}_s^0$ Mixing Parameter $y_s$ at the $\Upsilon(5S)$ Resonance

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Abstract

We show that a $CP$-violating phase can be model-independently determined from the time-independent measurement of coherent $B_s^0\bar{B}_s^0$ decays into $(D_s^{(*)}\pm K^{(*)}\mp})(D_s^{(*)}\pm K^{(*)}\mp)$ states at the $\Upsilon(5S)$ resonance. This phase amounts to $(-\gamma)$ within the standard model, where $\gamma$ is the well-known angle of the quark mixing unitarity triangle. It is also possible to determine or constrain the $B_s^0$-$\bar{B}_s^0$ mixing parameter $y_s$ from the same measurement.

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Today the weak decays of $B_d$ and $B_s$ mesons are playing crucial roles in the study of flavor mixing and $CP$ violation beyond the neutral kaon system. The best place to produce $B_d^0$ and $B_d^{0*}$ events with high statistics and low backgrounds is the $\Upsilon(4S)$ resonance, on which both the symmetric $B$ factory at Cornell and the asymmetric $B$ factories at KEK and SLAC are based. Similarly a wealth of coherent $B_s^0$ and $\bar{B}_s^0$ mesons can be achieved at the $\Upsilon(5S)$ resonance. Recently some interest has been paid to the possibilities to investigate $CP$ violation and probe new physics in weak $B_s$ decays at the $\Upsilon(5S)$ resonance [1, 2], although it remains an open question whether the existing $B$ factories running at the $\Upsilon(4S)$ energy threshold will finally be updated to run at the $\Upsilon(5S)$ energy threshold.

In this paper we point out a novel idea, which works for the coherent decays of $B_s^0\bar{B}_s^0$ pairs into $(D_s^\pm K^\mp)(D_s^\pm K^\mp)$, $(D_s^{*\pm} K^\mp)(D_s^{*\pm} K^\mp)$, $(D_s^\pm K^{*\mp})(D_s^\pm K^{*\mp})$ and $(D_s^{*\pm} K^{*\mp})(D_s^{*\pm} K^{*\mp})$ states at the $\Upsilon(5S)$ resonance, to extract the $CP$-violating phase

$$\phi \equiv \arg \left[ \frac{q}{p} \cdot \frac{V_{ub}V^*_{ub}}{V_{cb}V_{ub}} \right],$$

(1)

where $q/p$ describes the weak phase of $B_s^0\bar{B}_s^0$ mixing, and $V_{ub}, V_{cs}, V_{cb}$ and $V_{us}$ are four elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Within the standard model $q/p = (V_{tb}V_{cs})/(V_{tb}V_{ts}^*)$ is an excellent approximation, therefore $\phi$ amounts to $(-\gamma)$ to a good degree of accuracy, where

$$\gamma \equiv \arg \left[ -\frac{V_{ub}V_{ud}}{V_{cb}V_{cd}} \right]$$

(2)

denotes one inner angle of the well-known CKM unitarity triangle [3]. So far numerous methods have been proposed towards a clean determination of $\gamma$ in the weak decays of $B$ mesons [4].

The advantages of our present approach are remarkable: (1) it is completely independent of specific models or approximate symmetries in treating the relevant hadronic matrix elements of $B$ transitions; (2) its feasibility does not require any time-dependent measurement of $B_s$ decays, which is quite difficult due to the expected rapid rate of $B_s^0\bar{B}_s^0$ oscillation; and (3) it remains valid to determine $\phi$ and able to shed light on $\gamma$, even if there exists a kind of yet unknown new physics in $B_s^0\bar{B}_s^0$ mixing.

The transitions $B_s^0 \rightarrow D_s^{(*)-}K^{(*)+}$ and $\bar{B}_s^0 \rightarrow D_s^{(*)-}K^{(*)+}$ occur only through the tree-level quark diagrams with the CKM factors $(V_{cb}V_{us})$ and $(V_{ub}V_{cs}^*)$, respectively (see Fig. 1 for illustration). Their $CP$-conjugate processes $B_s^0 \rightarrow D_s^{(*)+}K^{(*)-}$ and $\bar{B}_s^0 \rightarrow D_s^{(*)+}K^{(*)-}$ have the corresponding CKM factors $(V_{cb}V_{us}^*)$ and $(V_{ub}V_{cs})$. Hence each of the four decay amplitudes involves only a single weak phase and a single strong phase [3]. For the study of $CP$ violation it is convenient to define two rephasing-invariant measurables:

$$\lambda_f \equiv \frac{q}{p} \cdot \frac{\langle f | \bar{B}_s^0 \rangle}{\langle f | B_s^0 \rangle},$$

$$\lambda_{f'} \equiv \frac{q}{p} \cdot \frac{\langle f | \bar{B}_s^0 \rangle}{\langle f | B_s^0 \rangle},$$

(3)
where $f$ and $\bar{f}$ are charge-conjugate states:

$$f = D_s^- K^+ , D_s^* K^+ , D_s^- K^{*+} , D_s^* K^{*+} ;$$

$$\bar{f} = D_s^+ K^- , D_s^{*-} K^- , D_s^+ K^{-*} , D_s^{*-} K^{-*} .$$  (4)

Note that $|q/p| = 1$ holds up to an accuracy of $\mathcal{O}(10^{-4})$ in the standard model [6], and it is expected to remain valid up to an accuracy of $\mathcal{O}(10^{-2})$ even if there exists a kind of yet unknown new physics with large CP violation in $B_s^0 - \bar{B}_s^0$ mixing [6, 7]. We therefore take $|q/p| \approx 1$ as a good approximation in the subsequent discussions. Then $\lambda_f$ and $\lambda_{\bar{f}}$ can be explicitly parametrized as [6]

$$\lambda_f = \rho e^{i(\phi+\delta)} ,$$

$$\lambda_{\bar{f}} = \frac{1}{\rho} e^{i(\phi-\delta)} ,$$  (5)

where $\phi$ is the overall weak phase defined already in Eq. (1), $\delta$ is the relevant strong phase difference, and

$$\rho \equiv \frac{\langle f|\bar{B}_s^0 \rangle}{\langle f|B_s^0 \rangle} = \frac{\langle \bar{f}|B_s^0 \rangle}{\langle \bar{f}|\bar{B}_s^0 \rangle}$$  (6)

measures the ratio of two decay amplitudes. Of course both $\rho$ and $\delta$ depend upon the specific final state $f$.

The coherent $B_s^0$ and $\bar{B}_s^0$ pairs with odd or even charge-conjugation (C) parity can be produced at the $\Upsilon(5S)$ resonance [8]. The joint decay rates of $B_s^0 - \bar{B}_s^0$ mesons into $f_1$ and $f_2$ states, in both $C = -1$ and $C = +1$ cases, have been derived in Refs. [8, 10]. Here we are only interested in the time-independent measurements. The generic formula for the time-integrated rate of a joint $B_s^0 - \bar{B}_s^0$ decay mode reads [9]

$$\mathcal{R}(f_1, f_2)_C \propto |\langle f_1|B_s^0 \rangle|^2 |\langle f_2|\bar{B}_s^0 \rangle|^2 \left[ \frac{1 + Cy_s^2}{1 - y_s^2} \left( |\xi_C|^2 + |\zeta_C|^2 \right) - \frac{2(1 + C)y_s}{(1 - y_s^2)^2} \text{Re}(\xi_C^* \zeta_C) 
- \frac{1 - Cy_s^2}{1 + x_s^2} \left( |\zeta_C|^2 - |\xi_C|^2 \right) + \frac{2(1 + C)x_s}{(1 + x_s^2)^2} \text{Im}(\zeta_C^* \xi_C) \right] .$$  (7)
where \( x_s \equiv \Delta M / \Gamma \) and \( y_s \equiv \Delta \Gamma / (2 \Gamma) \) are two dimensionless parameters of \( B^0_s - \bar{B}^0_s \) mixing, and
\[
\xi_C = \frac{p}{q} (1 + C \lambda_{f_1} \lambda_{f_2}) ,
\]
\[
\zeta_C = \frac{p}{q} (\lambda_{f_2} + C \lambda_{f_1}) .
\]

The definition for \( \lambda_{f_1} \) and \( \lambda_{f_2} \) is similar to that for \( \lambda_f \) in Eq. (3). The present experimental bound for \( x_s \) is \( x_s \geq 14 \) at the 95\% confidence level \([3]\). In addition, a detailed calculation based on the standard model yields \( y_s \sim O(10^{-2}) \) up to 0.1 \([11]\). This value will always be reduced, if \( B^0_s - \bar{B}^0_s \) mixing receives \( CP \)-violating contributions from new physics \([4, 10]\). Therefore the formula in Eq. (7) can be simplified by neglecting the \( O(y_s^2) \) and \( O(x_s^{-2}) \) terms. Up to the corrections of \( O(10^{-2}) \), we arrive for \( C = -1 \) at
\[
\mathcal{R}(f_1, f_2)_- \propto |\langle f_1 | B^{0}_{s} \rangle|^2 |\langle f_2 | B^{0}_{s} \rangle|^2 \left[ 1 + |\lambda_{f_1}|^2 + |\lambda_{f_2}|^2 + |\lambda_{f_1}|^2 |\lambda_{f_2}|^2 - 4 \text{Re} \lambda_{f_1} \text{Re} \lambda_{f_2} \right] ;
\]
and for \( C = +1 \) at
\[
\mathcal{R}(f_1, f_2)_+ \propto |\langle f_1 | B^{0}_{s} \rangle|^2 |\langle f_2 | B^{0}_{s} \rangle|^2 \left[ 1 + |\lambda_{f_1}|^2 + |\lambda_{f_2}|^2 + |\lambda_{f_1}|^2 |\lambda_{f_2}|^2 + 4 \text{Re} \lambda_{f_1} \text{Re} \lambda_{f_2} \right.
\]
\[
- 4 y_s \left( 1 + |\lambda_{f_1}|^2 \right) \text{Re} \lambda_{f_2} - 4 y_s \left( 1 + |\lambda_{f_2}|^2 \right) \text{Re} \lambda_{f_1} \right] .
\]

If \( y_s \sim O(10^{-3}) \) held, the relevant \( O(y_s) \) terms in \( \mathcal{R}(f_1, f_2)_+ \) could also be neglected. Here and hereafter we treat \( y_s \) as a free parameter of \( O(10^{-2}) \). It can be seen later on that the formulas in Eqs. (9) and (10) allow one to determine the weak phase \( \phi \) and the mixing parameter \( y_s \), model-independently, from the joint decays of \( B^0_s - \bar{B}^0_s \) pairs into \( (f, f) \), \( (\bar{f}, \bar{f}) \) and \( (f, \bar{f}) \) states.

Now taking \( (f_1, f_2) = (f, f), (\bar{f}, \bar{f}) \) and \( (f, \bar{f}) \) respectively, we obtain two ratios of the three joint decay rates for the \( C = -1 \) case:
\[
R^\prime f_- = \frac{\mathcal{R}(f, f)_-}{\mathcal{R}(f, f)_+} = \frac{(1 + \rho^2)^2 - 4 \rho^2 \cos(\phi + \delta)}{(1 + \rho^2)^2 - 4 \rho^2 \cos(\phi + \delta) \cos(\phi - \delta)} ,
\]
\[
R^\prime f_+ = \frac{\mathcal{R}(\bar{f}, \bar{f})_-}{\mathcal{R}(f, f)_-} = \frac{(1 + \rho^2)^2 - 4 \rho^2 \cos(\phi - \delta)}{(1 + \rho^2)^2 - 4 \rho^2 \cos(\phi + \delta) \cos(\phi - \delta)} ;
\]
and another two ratios for the \( C = +1 \) case:
\[
R^\prime f_+ = \frac{\mathcal{R}(f, f)_+}{\mathcal{R}(f, f)_-} = \frac{(1 + \rho^2)^2 + 4 \rho^2 \cos(\phi + \delta) - 8 y_s \rho (1 + \rho^2) \cos(\phi + \delta)}{(1 + \rho^2)^2 + 4 \rho^2 \cos(\phi + \delta) \cos(\phi - \delta) - 8 y_s \rho (1 + \rho^2) \cos \phi \cos \delta} ,
\]
\[
R^\prime f_+ = \frac{\mathcal{R}(\bar{f}, \bar{f})_+}{\mathcal{R}(f, f)_+} = \frac{(1 + \rho^2)^2 + 4 \rho^2 \cos(\phi - \delta) - 8 y_s \rho (1 + \rho^2) \cos(\phi - \delta)}{(1 + \rho^2)^2 + 4 \rho^2 \cos(\phi + \delta) \cos(\phi - \delta) - 8 y_s \rho (1 + \rho^2) \cos \phi \cos \delta} ;
\]
The four observables $R_f^{(\pm)}$ and $R_f^{(-)}$ depend upon four unknown quantities $\rho$, $\phi$, $\delta$ and $y_s$, therefore the latter can be determined from the former with some discrete ambiguities (only for $\phi$ and $\delta$). Such ambiguities can be resolved if the coherent $B_s^0\bar{B}_s^0$ decays into four different final states listed in Eq. (4), which involve the universal $\phi$ and $y_s$ but the different $\rho$ and $\delta$, are taken into account.

Note that $\delta = 0$ or $\pi$ would leads to

$$R_f^{(-)} = R_f^{(-)} = R_f^{(+)} = R_f^{(+)} = 1.$$  \hspace{1cm} (13)

In this case it is impossible to extract the weak phase $\phi$ from these observables. However, the four strong phases $\delta(D_sK)$, $\delta(D_s^*K)$, $\delta(D_sK^*)$ and $\delta(D_s^*K^*)$ are in general expected to take values different from one another. Thus one can always pin down the magnitude of $\phi$ from the measurements of $R_f^{(\pm)}$ and $R_f^{(\pm)}$ for different final states $f$. A special value of $\phi$ ($=0$ or $\pi$) could also result in the relationship in Eq. (13), but this possibility should not happen within the standard model, where $\phi \approx \gamma$ with $0 < \gamma < \pi$.

An estimate of $\rho$ in the naive factorization approximation yields $\rho \sim \mathcal{O}(1)$. Therefore the $B_s^0\bar{B}_s^0$ mixing parameter $y_s$ of $\mathcal{O}(10^{-2})$ may affect the magnitudes of $R_f^{(+)}$ and $R_f^{(+)}$ significantly, in particular if the $\cos(\phi \pm \delta)$ terms are of $\mathcal{O}(1)$. This provides an interesting opportunity to determine or constrain $y_s$ model-independently and time-independently.

As the branching ratios of $B_s^0$ decays into $D_s^{\mp}K^{\pm}$, $D_s^{*\mp}K^{\pm}$, $D_s^{\mp}K^{*\pm}$ and $D_s^{*\mp}K^{*\pm}$ states are all at the $\mathcal{O}(10^{-4})$ level \cite{2}, to observe signals of the joint $B_s^0\bar{B}_s^0$ decays into these final states needs about $10^8 B_s^0\bar{B}_s^0$ events at the $\Upsilon(5S)$ resonance. The similar number of $B_s^0\bar{B}_s^0$ pairs is required to apply the idea proposed in Ref. \cite{2} for a determination of $\gamma$ using the partial rates for $CP$-tagged $B_s$ decays into $D_s^{(*)\mp}K^{(*)\pm}$ states at the $\Upsilon(5S)$ resonance.

Finally let us give a brief comparison between our new approach to determining the $CP$-violating phase $\gamma$ and those presented in Ref. \cite{4} and Ref. \cite{3}, which all make use of the $B_s$ decays into $D_s^{(*)\pm}K^{(*)\mp}$ states. The method of Ref. \cite{3} requires the time-dependent measurements, therefore its feasibility relies crucially upon the knowledge of $B_s^0\bar{B}_s^0$ mixing (i.e., the known values of $x_s$ and $y_s$). To apply the time-independent method of Ref. \cite{4}, one needs the quantitative information on the relevant decay amplitudes of pure $B_s^0$ and $\bar{B}_s^0$ mesons, which can be experimentally achieved only after the values of $x_s$ and $y_s$ have been fixed. For our approach, the precise knowledge of $B_s^0\bar{B}_s^0$ mixing is not a prerequisite. Indeed it is even possible to measure the magnitude of $y_s$ using our method at the $\Upsilon(5S)$ resonance. Within the standard model the three approaches can be complementary to one another for the determination of $\gamma$.

In summary, we have proposed a new method to extract the weak phase $\gamma$ and to determine the $B_s^0\bar{B}_s^0$ mixing parameter $y_s$ from the coherent $B_s^0\bar{B}_s^0$ decays into $(D_s^{(*)\pm}K^{(*)\mp})(D_s^{(*)\pm}K^{(*)\mp})$ states at the $\Upsilon(5S)$ resonance. It will become realistic and useful, perhaps in the second (or final) round of $B$-factory experiments, once the present $e^+e^-$ colliders operating at the $\Upsilon(4S)$ resonance are updated to run at the $\Upsilon(5S)$ energy threshold.
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