Clusters and condensates in Fermi systems

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Abstract. Superconductivity, superfluidity, condensation, cluster formation, etc. are phenomena that might occur in many-particle systems. These are due to residual interactions between the particles. To explain these phenomena consistently in a microscopic approach, at some point, one needs to solve few-body equations that are modified because of the Pauli principle and the interactions of the many particles around.

1 Introduction

Correlations in many-particle systems are responsible for a number of interesting and exciting properties of the system. In an infinite, equilibrated system the presentation of these properties (or phases of matter) are usually organized in a phase diagram. This is a plot of the dominant state of matter separated by “critical lines” depending on the two basic variational parameters of quantum statistics relevant for a grand canonical ensemble, i.e. the temperature $T$ and the chemical potential $\mu$. Instead of the chemical potential $\mu$ that is difficult to access experimentally one might by use of proper thermodynamic relations chose the particle density $n(\mu, T)$. Note, however, that the relation between $\mu$ and $n(\mu, T)$ is not simple and depends on the specific system considered.

The phase diagram becomes particularly rich when the chemical potential $\mu$ is in the order of MeV to GeV. This is at rather large particle densities, where nuclei are close together. This is the domain where Quantum Chromodynamics (QCD) dominates the dynamics of the system. Since nucleons are made of quarks it is obvious that the relevant degrees of freedom to describe the system might change from nucleons (nuclear matter) to quarks (quark matter) at some point. In addition gluons might appear as additional degrees of freedom that are otherwise absent at lower densities. Because of the free (color) charge this phase is called a (quark gluon) plasma. Indeed, lattice QCD and model calculations suggest a variety of phenomena in the QCD dominated phase of matter. These are superfluidity, critical points, first and second order phase transitions, color...

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Figure 1. Schematic view of the phase diagram of nuclear matter. The phase diagram is empirical accessible by heavy ion collisions, excited nuclei, observation of neutron stars and the early universe as indicated in the diagram. New plans at GSI aim at exploring the color superconducting phase as well.

super conductivity, plasma phase, among others. A sketch of the phase diagram is given in Fig. 1. To explain all these different states of matter one needs a treatment that goes beyond the simple quasiparticle picture.

To tackle the complicated many-particle system the Green function approach is a good starting point. Equations for Green functions are hierarchically coupled and hence an approximate solution of the problem is unavoidable. If we are interested in two-body correlations only, the three-body Green functions are approximated. Quite a few phenomena are already accessible at this level. However, there are good reasons to go beyond two-particle correlations, e.g.:

- Particle production even in a dense environment such as deuteron formation in a heavy ion reaction, need a third particle to conserve energy-momentum.
- To study the properties of α-particles or determine the critical temperature of a possible α-particle condensate needs an in-medium four-body equation.
- Recent results in the Hubbard model indicate, that three-particle contributions may lead to a different (lower) critical temperature compared to the simple Thouless criterion. Question of this type have not been
addressed for nuclear matter.

- The chiral phase transition is often discussed along with a confinement-deconfinement transition based on investigating mesons (quark-antiquark states). Does this transition happen for nucleons (three-quark states) at the same density/temperature?

Some of these issues related to the nuclear matter phase are addressed in the present paper. The ones related to quark matter are given by Mattiellio et al \[8\].

2 Theory

We use Dyson equations to tackle the many-particle problem, see e.g. Ref. \[3\]. This enables us to decouple the hierarchy of Green functions. The Dyson equation approach used here is based on two ingredients: i) all particles of a cluster are taken at equal time ii) the ensemble averaging for a cluster is done for an uncorrelated medium. The resulting decoupled Green functions may be economically written as resolvents in the $n$-body space, where $n = 2, 3, 4, \ldots$ is the number of particles in the considered cluster.

The solution of the one-particle problem in Hartree-Fock approximation leads to the following quasi-particle energy

$$\varepsilon_1 = \frac{k_1^2}{2m_1} + \sum_V V_2(12, \tilde{12}) f_2 \simeq \frac{k_1^2}{2m_1^{\text{eff}}} + \Sigma^{\text{HF}}(0).$$

The last equation introduces the effective mass that is a valid concept for the rather low densities considered here and $\mu^{\text{eff}} \equiv \mu - \Sigma^{\text{HF}}(0)$. The Fermi function $f_i \equiv f(\varepsilon_i)$ for the $i$-th particle is given by

$$f(\varepsilon_i) = \frac{1}{e^{\beta(\varepsilon_i - \mu)} + 1}. \quad (2)$$

The resolvent $G_0$ for $n$ noninteracting quasiparticles is

$$G_0(z) = (z - H_0)^{-1} N \equiv R_0(z) N, \quad H_0 = \sum_{i=1}^n \varepsilon_i \quad (3)$$

where $G_0$, $H_0$, and $N$ are formally matrices in $n$ particle space. The Matsubara frequency $z_\lambda$ has been analytically continued into the complex plane, $z_\lambda \to z \mp \pi i$. The Pauli-blocking for $n$-particles is

$$N = \bar{f}_1 \bar{f}_2 \cdots \bar{f}_n \pm f_1 f_2 \cdots f_n, \quad \bar{f} = 1 - f \quad (4)$$

where the upper sign is for Fermi-type and the lower for Bose type clusters. The full resolvent $G(z)$ is given by

$$G(z) = (z - H_0 - V)^{-1} N, \quad V \equiv \sum_{\text{pairs } \alpha} N_2^\alpha V_2^\alpha. \quad (5)$$
Note that $V^\dagger \neq V$. For the two-body case as well as for a two-body subsystem embedded in the $n$-body cluster the standard definition of the $t$ matrix leads to the Feynman-Galitskii equation for finite temperature and densities \[1\],

$$T_2^\alpha(z) = V_2^\alpha + V_2^\alpha N_2^\alpha R_0(z) T_2^\alpha(z).$$  \(6\)

Introducing the Alt Grassberger Sandhas (AGS) transition operator $U_{\alpha\beta}(z)$ the effective inhomogeneous in-medium AGS equation reads

$$U_{\alpha\beta}(z) = (1 - \delta_{\alpha\beta}) R_0^{-1}(z) + \sum_{\gamma \neq \alpha} N_2^\gamma T_2^\gamma(z) R_0(z) U_{\gamma\beta}(z).$$  \(7\)

The homogeneous in-medium AGS equation uses the form factors defined by

$$|F_\beta\rangle = \sum_{\gamma} \delta_{\beta\gamma} N_2^\gamma V_2^\gamma |\psi_{B_3}\rangle$$  \(8\)

to calculate the bound state $\psi_{B_3}$

$$|F_\alpha\rangle = \sum_{\beta} \delta_{\alpha\beta} N_2^\beta T_2^\beta(B_3) R_0(B_3) |F_\beta\rangle.$$  \(9\)

Finally, the four-body bound state is described by

$$|F_{\sigma\beta}\rangle = \sum_{\tau\gamma} \delta_{\sigma\tau} U_{\beta\gamma}(B_4) R_0(B_4) N_2^\gamma T_2^\gamma(B_4) R_0(B_4) |F_{\beta\gamma}\rangle,$$  \(10\)

where $\alpha \subset \sigma, \gamma \subset \tau$ and $\sigma, \tau$ denote the four-body partitions. The two-body input is given in \(6\) and the three-body input by \(7\). Note that, although we have managed to rewrite the above equations in a way close to the ones for the isolated case, they contain all the relevant in-medium corrections in a systematic way, i.e. correct Pauli-blocking and self energy corrections. The numerical solution requires some mild approximations that are however well understood in the context of the isolated few-body problem.

3 Results

An experiment to explore the equation of state of nuclear matter is heavy ion collisions at various energies. Here we focus on intermediate to low scattering energies and compare results to an experiment $^{129}$Xe+$^{119}$Sn at 50 MeV/A by the INDRA collaboration \[10\]. A microscopic approach to tackle the heavy ion collision is given by the Boltzmann equation for different particle distributions and solved via a Boltzmann Uehling Uhlenbeck (BUU) simulation \[2\]. The reaction rates appearing in the collision integrals are \textit{a priori} medium dependent. We use the in-medium AGS equations \(7\) that reproduce the experimental data in the limit of an isolated three-body system. For details on the specific interaction model see Ref. \[11\]. We investigate the influence of medium dependent rates in the BUU simulation of the heavy ion collision as compared
to use of isolated rates. Figure 2 shows that the net effect (gain-minus-loss) of deuteron production becomes larger for the use of in-medium rates (solid) compared to using the isolated rates (dashed). The change is significant, however, a comparison with experimental data is difficult since deuterons may also be evaporating from larger clusters that has not been taken into account in the present calculation so far. The ratio of protons to deuterons may be better suited for a comparison to experiments that is shown in Figure 3. The use of in-medium rates (solid) lead to a shape closer to the experimental data (dots) than the use of isolated rates (dashed). Within linear response theory for infinite nuclear matter the use of in-medium rates leads to faster time scales for the deuteron life time and the chemical relaxation time as has been shown in detail in Refs. [12, 13]. This faster time scales should have consequences for the freeze out of fragments.

![Figure 2. BUU simulation of deuteron formation in the collision of $^{129}\text{Xe}+^{119}\text{Sn}$ at 50 MeV/A. Use of in-medium rates lead to a 20% enhancement.](image)

![Figure 3. Ratio of proton to deuteron numbers as a function of c.m. energy. The experimental data are from the INDRA collaboration.](image)

![Figure 4. Due to the Mott effect clusters exist only above the respective curves. Below no bound states exist because of Pauli blocking.](image)

![Figure 5. Part of the phase diagram with lines of critical temperatures. Solid line four-body AGS type calculation. Others earlier calculation [4].](image)
As does the nucleon, see eq. \([1]\), the cluster changes its mass. The binding energy changes as well until the Pauli blocking is too strong for bound states to exist (Mott effect). This effect depends on the momentum of the cluster. This important effect for modeling of heavy ion collisions is shown in Figure 4.

In Figure 5 part of the phase diagram of nuclear matter is shown. The condition for the onset of superfluidity for \(\alpha\)-particles is
\[
B(T_c, \mu_c, P = 0) = 4\mu_c
\]
where \(B\) is the binding energy. The critical temperature found by solving the homogeneous AGS equation for \(\mu < 0\) confirms the onset of \(\alpha\) condensation even at higher values (solid line) as given earlier (dashed, from \([1]\)) which was based on a variational calculation using the 2+2 component of the \(\alpha\) particle. For \(\mu > 0\) the condition \(E = 4\mu\) for the phase transition can also be fulfilled. However, the significance for a possible quartetting needs further investigation.

To conclude, strongly correlated Fermi systems such as nuclear/quark matter provide an excellent field for few-body techniques.

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