Multiple Scales in the Fine Structure of the Isoscalar Giant Quadrupole Resonance in $^{208}$Pb

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Abstract

The fine structure of the isoscalar giant quadrupole resonance in $^{208}$Pb, observed in high-resolution (p,p') and (e,e') experiments, is studied using the entropy index method. In a novel way, it enables to determine the number of scales present in the spectra and their magnitude. We find intermediate scales of fluctuations around 1.1 MeV, 460 keV and 125 keV for an excitation energy region 0–12 MeV. A comparison with scales extracted from second RPA calculations, which are in good agreement with experiment, shows that they arise from the internal mixing of collective motion with two particle-two hole components of the nuclear wavefunction.

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1 Introduction

The decay of giant resonances in nuclei provides important information on how a well-ordered collective excitation dissolves into a disordered motion of internal degrees of freedom in fermionic quantum many-body systems (see e.g. [2]). This can be understood to result from the decay of the collective
modes towards compound nuclear states leading to internal mixing. Besides this internal mixing, the nuclear states may also decay into a continuum of escaping states giving rise to the deexcitation of the system through particle emission.

It is generally agreed that internal mixing occurs through a hierarchy of couplings towards more and more complex degrees of freedom in the nucleus. Collective states are constructed in mean-field theory as a coherent superposition of one particle-one hole (1p-1h) excitations and are generally treated in the random-phase approximation (RPA) \[2, 3\]. In order to understand their decay towards the compound nucleus, one has to go beyond the mean field and introduce two-body effects as embodied in the second RPA (SRPA) \[4\]. Such approaches are valid under the assumption that collective motion is preferentially damped by 2p-2h components of the many-body wavefunction, reflecting the two-body nature of the nuclear interaction. The description can be extended by introducing more and more complex components such as 3p-3h...np-nh. Indeed, all transport theories assume a classification in increasing degrees of complexity \[5, 6, 7\].

Since this picture is based on a particular hierarchy, one should be able to extract experimental information on it by studying scales present in the decay of collective motion. One expects a hierarchy of lifetimes linked to a hierarchy of energy scales starting from the typical scale associated with collective states, the full width at half maximum (FWHM) which is of the order of a few MeV, going down to scales characterized by the width of long-lived compound nuclear states which is of the order of a few eV. In order to test this framework, an experimental identification of scales involved in the decay of giant resonances appears as an important issue.

Experimental evidence for scales associated with the coupling between collective states and internal and external degrees of freedom is a long-standing problem. On the one hand, the spectral analysis requires high-resolution experiments. Proton and electron scattering experiments, which may reach resolutions better than 50 keV, present promising candidates for this type of analysis. The appearance of fine structure in the isoscalar giant quadrupole resonance (ISGQR) of \(^{208}\)Pb has been reported already a long time ago in high-resolution electron scattering \[9, 10\]. This finding, which has led to considerable debate, was finally confirmed \[11\] when proton scattering data of comparable resolution became available \[12\]. On the other hand, one has to develop tools to extract the information from a complex signal where several scales of different nature are mixed. A variety of methods has been
proposed to study fluctuation properties of the experimental spectra either using a doorway model and microscopic calculations [13] or taking advantage of autocorrelation techniques [14] assuming a statistical distribution of the decay channels [15]. However, such analyses remain dependent on underlying model hypotheses and become difficult to handle when more than one scale of the fine structure exists.

In the present paper, we reanalyze the $^{208}\text{Pb}(p,p')$ and $^{208}\text{Pb}(e,e')$ experiments using a model-independent method, the entropy index, which is especially suited for the study of multiscale fluctuations [16]. In particular, this method does not make any a priori assumptions on the decay mechanism. In the next section the entropy index method is briefly summarized. Its application is then illustrated for the $^{208}\text{Pb}(p,p')$ experiment where fine structure at different scales is found. Results of electron and proton scattering are then compared showing a perfect agreement between these two independent measurements. We finally analyze the results of SRPA calculations of the ISGQR in $^{208}\text{Pb}$ in order to understand the origin of the fluctuations. A good agreement of scales extracted from the calculated and experimental results is found. This supports the interpretation that the two-body nature of the nuclear interaction governs the dominant decay channels.

## 2 The entropy index method

The principle of the entropy index method is to have a measure of the fluctuations at a given resolution in energy $\delta E$. Suppose that we have an experimental spectrum in the excitation energy region $\Delta E = [E_{\text{min}}, E_{\text{max}}]$. In order to study fluctuations, we can divide this interval into $n$ bins with $n = \Delta E/\delta E$. If we call $\sigma(E)$ the fluctuating function, i.e., the cross section or the strength, we define a coefficient $D_j$ in each bin $j$ as

$$D_j(\delta E) = \int_{E_{j-1}}^{E_j} dE \sigma(E) \Omega_j(E)$$

where $E_j = E_{\text{min}} + j\delta E$. In this expression, $\Omega_j(E)$ denotes a function which takes a non-zero value in the interval $[E_{j-1}, E_j]$. In the following, we suppose that $\Omega_j(E) = \text{sign}(E - (j - 1/2)\delta E)$. With this definition, $D_j$ represents

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\footnote{It should however be noted that the choice of $\Omega_j(E)$ is not unique and other odd functions with respect to the center of the interval could be used.}
a coarse-grained derivative of the function $\sigma$, and the fluctuations of these coefficients are directly related to the fluctuations of $\sigma$ at the considered scale. In order to infer global properties of these fluctuations, we can define an entropy $K$

$$K(\delta E) = -\frac{1}{n} \sum_{j=1,n} W_j(\delta E) \log W_j(\delta E)$$

(2)

where $W_j(\delta E) = |D_j| / \langle |D_j| \rangle$ stands for the absolute value of the coefficients $D_j$ normalized to their averaged value ($\langle |D_j| \rangle = 1/n \sum_{j=1,n} |D_j|$).

This technique has been initially proposed in order to study self-similar fluctuations in heartbeats which could be identified through a linear dependence of $K(\delta E)$ on the logarithm of the resolution $\delta E$. It has recently been shown that the entropy index method is also suitable in situations where well-separated scales of fluctuations exist. Then, the linear increase is replaced by a change in curvature of the entropy which corresponds to transitions of $\delta E$ from one scale to another. Note finally that, in order to avoid problems due to the limited number of bins for large $\delta E$, in the results presented below we have considered a function defined as 31 repetitions of the analyzed spectra as described in Ref. [16].

3 Results

In the following, we apply the presented method to the experimental data. A sample spectrum of the $^{208}$Pb(p,p') reaction [12] is displayed in Fig. 1 at $\theta = 8^\circ$, where $\Delta L = 2$ transitions - and thus the population of the ISGQR - are enhanced. With an experimental resolution of about 50 keV fine structure has been observed at excitation energies below 12 MeV. Indeed, in a doorway picture one expects a large increase of the density of states to which collective modes are coupled with increasing excitation energy, $E_x$. As a result, scales present at lower excitation energies might differ from those at higher energies.

In Fig. 2 we show the entropy variation obtained by selecting the excitation energy interval $E_x > 6.5$ MeV of the (p,p') spectrum displayed in Fig. 1. We select this interval in order to exclude strongly excited low-lying levels which are outside the scope of this method. Figure 2 exhibits sudden variations in the evolution of the entropy as a function of $\delta E$ which can be associated with the appearance of different dominant scales in the fine structure. Although the number of dominant scales appears directly from
the analysis, their precise determination is not straightforward from the localization of the curvature changes.

In order to gain a deeper insight, we have extended the work performed in \[16\]. We have shown that for the models considered in \[16\] the entropy index can be properly fitted by a function \( F(\delta E) \) defined as \( F(\delta E) = \sum_n K_n(\delta E) \) where \( n \) is an index running on the different scales and where \( K_n(\delta E) \) is defined as a Fermi-Dirac like distribution function

\[
K_n(\delta E) = \frac{k_n}{1 + \exp \left( \frac{\ln(\delta E) - d_n}{\Delta_n} \right)}
\]  

with parameters \( k_n \), \( d_n \) and \( \Delta_n \). An example of the fit obtained with \( F \) is shown as solid line in Fig. 2. In the model investigations \[16\], where the scales \( \Gamma_n \) are known, an empirical relation could be established between the fitting functions and the scales, viz. \( K_n(\Gamma_n)/k_n = 0.92 \pm 0.01 \). Application of the same relation to the experimental data allows the identification of fine-structure scales at 1.1 MeV, 460 keV and 125 keV. It should be noted that the value of the smallest scale might already be affected by the experimental resolution. Furthermore, the procedure described above can only serve as an indication of the range of fluctuations since it is based on an empirical observation in model cases only.

Considering the larger scales, it is interesting to note that assuming a two-doorway picture for the ISGQR in \(^{208}\)Pb, the authors of Ref. \[13\] predicted spreading widths \( \Gamma_\downarrow \) of 490 keV and 740 keV, respectively. The first scale in particular seems to corroborate our result while the second one is somewhat smaller than what is deduced here. We would like to emphasize, however, that our present method does not suppose any number of doorways.

Besides the analysis of the proton inelastic scattering data, we can also apply the entropy index method to the \(^{208}\)Pb(e,e’') data \[10\]. An experimental spectrum obtained for \( E_e = 50 \text{ MeV} \) and \( \Theta = 93^\circ \) is presented in the upper part of Fig. 3. In the (e,e’') case, the high-resolution part of the experimental spectrum is limited to \( E_x = 7.6 - 11.7 \text{ MeV} \), the background is removed and the experimental resolution is again around 50 keV. The part of the (p,p’’) spectrum corresponding to the same energy interval is plotted in the lower part of Fig. 3.

A detailed correspondence exists in the fine structure, up to about 10 MeV even on a level-by-level basis \[11\]. Similarities in the fine structure observed between the two experiments is indeed confirmed by the entropy
index analysis. Figure 3 presents the variation of $K(\delta E)$ as a function of $\delta E$ in the electron scattering case (circles) as well as for proton scattering (crosses). The similarity in the fine structures observed in Fig. 3 is reflected in a perfect superposition of the curves obtained from these two independent experiments. This agreement provides further confidence in the estimated localization of curvature changes in $K(\delta E)$.

4 Comparison with second RPA results

The interpretation of the fine-structure scales is far from being straightforward. Indeed, as we pointed out in the introduction, having precise information on the scales present in the damping of giant resonances is of particular interest since it may help to understand which mechanisms are involved in the internal mixing. On the theory side, many physical effects might contribute to the damping \cite{18}. Already at the mean-field level, the Landau fragmentation may introduce a typical scale. However, qualitative agreement with experiment can only be achieved when models include two-body effects as in extended RPA approaches \cite{4, 11, 19}. In that case, we expect different effects due to the coupling of 1p-1h with 2p-2h states. The coherent coupling to low-lying collective surface vibrations may introduce fragmentation \cite{20}. In addition, strong coupling may lead to a reduction of widths, either from interferences due to coupling through common decay channels \cite{21, 22, 23} or due to motional narrowing \cite{24}. Furthermore, different assumptions are used in models in order to treat the decay channels of the collective states and/or the interaction matrix elements. For example, statistical assumptions might be needed when the excitation energy increases \cite{25, 26}, while in a microscopic picture like the SRPA, in particular at low excitation energy, only a few 2p-2h states are coupled to the collective states and statistical assumptions may break down.

In this section, we apply the entropy index method to SRPA results for the isoscalar giant quadrupole response in $^{208}$Pb. The calculation is based on the M3Y interaction \cite{27} with some adjustment of the short-range part which allows to reproduce the experimental centroids of the low-multipolarity electric giant resonances in $^{208}$Pb. A truncation of the 2p-2h configuration space is necessary, e.g., at the upper limit of the calculation $E_x = 20$ MeV one would have to include about $1.5 \times 10^4$ 2p-2h states. The method used here focuses on diagonal matrix elements in the 2p-2h subspace. Their dis-
tribution can be approximated by a Gaussian assuming random fluctuations. All configurations associated with matrix elements exceeding this Gaussian fit are included in the further analysis (about 3000 in the present example). The complex SRPA selfenergy is chosen to attain a finite resolution similar to the experimental data. The calculated strength function of the ISGQR is presented in Fig. 5. At the RPA level (not shown) the strength function consists essentially of a single collective state around 12 MeV. By introducing 2p-2h components, the FWHM strongly increases and fine structure appears on top of the global shape.

The result obtained from the entropy index method analysis for this SRPA spectrum and $0 \text{MeV} < E_x < 12 \text{ MeV}$ is shown in Fig. 6. The upper energy limit is set to exclude any modification of the results due to the truncation of the 2p-2h model space. Using the same fitting procedure as in the experimental case permits to identify scales of fluctuations at 2.1 MeV, 400 keV, and 120 keV. The two smaller values show very good agreement with the experimental observation while the first one is somewhat larger. This difference might result from the presence of multipolarities other than $L = 2$ in the $(p, p')$ spectrum. For example, the isoscalar giant monopole resonance centered around 14 MeV is clearly visible in the data. The generally good agreement suggests that the origin of the fine-structure scales is indeed due to the coupling of the 2p-2h excitations to the collective states and that their magnitude characterizes the two-body components of the nuclear Hamiltonian, i.e. the coupling matrix elements and density of states.

5 Conclusion

In this paper, using the model-independent entropy index method in high-resolution proton and electron elastic scattering, we show that multiple scales of the fine structure appear in the damping of the isoscalar giant quadrupole resonance of $^{208}$Pb. In addition to the entropy index method a fitting procedure has been developed that enables a more precise estimate of the magnitude of these scales. Fine-structure scales appear at 1.1 MeV, 460 keV and 125 keV, in perfect agreement between electron and proton scattering data. The application of the entropy index method to a SRPA strength function gives results in very satisfactory agreement with experiment while the RPA alone is not able to exhibit any of these scales. This provides a strong argument that the observed scales result from the decay of collective modes.
into 2p-2h states. In particular, they should be connected to the density of 2p-2h states and the coupling matrix elements of the in-medium residual interaction associated with this damping mechanism which is a rather unique way to infer properties of two-body components in the nuclear system. The quantitative knowledge of the scales may help to improve effective interactions which are generally fitted to mean-field properties only. In addition, the results argue in favor of a hierarchy of complexity in the internal components of the nucleus: one-body, two-body, three-body... One may hope in the near future, with improved experiments, to uncover even smaller fine structure scales that are connected to more complex internal degrees of freedom and go a step further in our understanding of the transition from order to chaos in nuclear systems.

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Figure 1: The $^{208}$Pb(p,p$'$) experimental spectrum at a beam energy $E_p = 200$ MeV and $\theta = 8^\circ$.
Figure 2: The variation of the entropy index as a function of the resolution $\delta E$ obtained with the $^{208}$Pb(p,p') data when excitation energies $E_x > 6.5$ MeV are selected (crosses). The solid line corresponds to a fit of Eq. (3) using a sum of four Fermi-Dirac functions.

Figure 3: Top: the $^{208}$Pb(e,e') spectrum (where the background is removed) at $E_e = 50$ MeV and $\theta = 93^\circ$ in the excitation energy range $E_x = 7.6 - 11.7$ MeV. Bottom: the corresponding part of the $^{208}$Pb(p,p') spectrum shown in Fig. [1].
Figure 4: Comparison between the entropy index evolutions obtained from the $^{208}\text{Pb}(p,p')$ reaction (crosses) and the $^{208}\text{Pb}(e,e')$ reaction (circles). In both cases, the energy interval is $E_x = 7.6 - 11.7 \text{ MeV}$.

Figure 5: The SRPA isoscalar giant quadrupole strength function in $^{208}\text{Pb}$. The presented result was calculated with a width of the complex selfenergy to give a resolution similar to the experimental spectra in Figs. 1 and 3.
Figure 6: The variation of the entropy index $K(\delta E)$ as a function of the scale $\delta E$ obtained from the $^{208}$Pb SRPA strength function for $0 \text{ MeV} < E_x < 12 \text{ MeV}$. The solid line is a fit of expression (B) using a sum of four Fermi-Dirac functions.