Ground State Mass Predictions of Heavy-Light Hybrids from QCD Sum-Rule Analysis ($J^P = \{0^\pm, 1^\pm\}$) *

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Abstract

We present QCD Laplace sum-rule predictions of ground state masses of heavy-light open-flavour hybrid mesons. Having computed leading-order diagonal correlation functions, including up to dimension six gluon condensate contributions, we extract hybrid mass predictions for all $J^P \in \{0^\pm, 1^\pm\}$, and explore possible mixing effects with conventional meson states. Similarities are found in the mass hierarchy in both charm and bottom systems with some exceptions that are discussed.

Keywords: Hadron Spectroscopy, Laplace Sum Rules, Exotic Hadrons, Hybrids, Charm Quarks, Bottom Quarks, Mixing

1. Correlation Functions of Heavy-Light Open-Flavour Hybrid Mesons

The first investigations of heavy-light hybrids using QCD sum-rules were performed by Govaerts, Reinders, and Weyers [1] (abbreviated GRW). In that work, they considered four currents covering $J \in \{0, 1\}$ in an effort to compute a comprehensive collection of ground state hybrid masses. For each heavy-light hybrid ground state, the square of the predicted mass was close to the continuum threshold (with separations of roughly 10–15 MeV); GRW noted that even a modest hadronic resonance width would result in concern of contamination from the continuum [1].

We review work done in [2] where we extended the work of GRW [1] by including higher dimensional condensate contributions (dimension-five mixed and dimension-six gluon) in the calculation of our correlator. In the case of heavy-light hybrids, condensates involving light quarks are enhanced by a heavy quark mass allowing for the possibility of significant contributions to the correlator and to the sum-rules. This was noted previously by GRW [1]. Thus, the dimension-five mixed condensate could become a significant component of a QCD sum-rules application to the heavy-light hybrid systems. Further, sum-rules analyses of closed-flavor, heavy hybrid mesons [3] have demonstrated that the dimension-six gluon condensate can have a significant stabilizing effect on what were in previous studies [1,4,5] unstable analyses.

We define our open-flavour hybrid interpolating currents in the same fashion as GRW [1] by including higher dimensional condensate contributions (dimension-five mixed and dimension-six gluon) in the calculation of our correlator. In the case of heavy-light hybrids, condensates involving light quarks are enhanced by a heavy quark mass allowing for the possibility of significant contributions to the correlator and the sum-rules. This was noted previously by GRW [1]. Thus, the dimension-five mixed condensate could become a significant component of a QCD sum-rules application to the heavy-light hybrid systems. Further, sum-rules analyses of closed-flavor, heavy hybrid mesons [3] have demonstrated that the dimension-six gluon condensate can have a significant stabilizing effect on what were in previous studies [1,4,5] unstable analyses.

We define our open-flavour hybrid interpolating currents in the same fashion as GRW,

$$ j_\mu = \frac{g_s}{2} \bar{Q} \gamma^\rho \lambda^q G^{\rho \mu}_{\lambda q}, $$

(1)

where $g_s$ is the strong coupling and $\lambda^q$ are the Gell-Mann matrices. The field $Q$ represents a heavy charm or bottom quark with mass $M_Q$ while $q$ represents a light up, down, or strange quark with mass $m_q$. The Dirac matrix $\gamma^\rho$ satisfies $\gamma^\rho \in \{\gamma^\rho, \gamma^\rho \gamma_5\}$ and the tensor $G^{\rho \mu}_{\lambda q}$, the portion of $j_\mu$ containing the gluonic degrees of free-

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dom, satisfies
\[ G_{\mu
u}^a \in \{ G_{\mu
u}^a, \tilde{G}_{\mu
u}^a = \frac{1}{2} \epsilon_{\mu
u\alpha\beta} G_{\alpha\beta}^a \}, \]
where \( G_{\mu
u}^a \) is the gluon field strength and \( \tilde{G}_{\mu
u}^a \) is its dual defined using the Levi-Civita symbol \( \epsilon_{\mu\nu\alpha\beta} \).

For each of the currents defined through \( j_\mu \) above, we consider a diagonal, two-point correlation function
\[
\Pi_{\mu\nu}(q) = i \int d^4x e^{i q \cdot x} \langle \Omega | T [ j_\mu(x) j^\dagger_\nu(0)] | \Omega \rangle
\]
\[
= \frac{q_\mu q_\nu}{q^2} \Pi^{(0)}(q^2) + \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi^{(1)}(q^2),
\]
where \( \Pi^{(0)} \) probes spin-0 states and \( \Pi^{(1)} \) probes spin-1 states. We will reference each of the \( \Pi^{(0)} \) and \( \Pi^{(1)} \) according to the \( J^{PC} \) combination, respectively.

We briefly review the details of the Laplace sum-rule (LSR) methodology \([9]\). The formation of the sum-rule involves subtraction constants (a polynomial in \( Q^2 \)), and \( t_0 \) represents the appropriate physical threshold. The dispersion relation \([8]\), satisfied by each of the components \( \Pi^{(0)}, \Pi^{(1)} \), encapsulates

\[ \Pi(Q^2) = \frac{Q^4}{\pi} \int_0^\infty \frac{\text{Im} \Pi(t)}{t^2 (t + Q^2)} \, dt + \cdots, \quad Q^2 > 0 \]

We utilize the operator product expansion (OPE) to calculate the correlators \([5]\). Within the OPE, perturbation theory is supplemented by non-perturbative terms, each of which is the product of a perturbatively computed Wilson coefficient and a non-zero vacuum expectation value (VEV), referred to as a condensate. We include
\[
\langle \bar{q}q \rangle = \langle \bar{q}_i q_i^a \rangle \quad (4)
\]
\[
\langle q^a G^2 \rangle = \langle q^a_i G^a_{\mu\nu} G_{\mu\nu}^a \rangle \quad (5)
\]
\[
\langle g G q G \rangle = \langle g s_i q_i^a G_{\mu\nu}^a G_{\mu\nu}^a \rangle \quad (6)
\]
\[
\langle g G^3 \rangle = \langle g s_i v a b G_{\mu\nu}^a G_{\mu\nu}^b G_{\mu\nu}^c \rangle, \quad (7)
\]
where the VEVs \((4)-(7)\) are respectively referred to as the 3d quark condensate (i.e. the dimension-three quark condensate), the 4d gluon condensate, the 5d mixed condensate, and the 6d gluon condensate.

The Wilson coefficients are computed to leading-order (LO) in \( g_s \) (see \([6]\) for a review of calculational methods). Light quark masses are included in perturbation theory through a light quark mass expansion, but have been set to zero in all other OPE terms. The contributing Feynman diagrams are depicted in Figure \([1]\).
quark-hadron duality: a connection between hadrons and QCD. The
left side of (8) is calculated from the OPE discussed earlier in this section, while Im\(\Pi(t)\) on
the right side represents the hadronic spectral function. In a traditional sum-rules analysis, this spectral function
is parameterized in terms of hadronic quantities within
a model that are then fit to the OPE result. To form our sum-rule, we apply to (8) the Borel transform,
\[
\hat{B} = \lim_{t \to \infty} \frac{(-Q^2)^N}{\Gamma(N)} \left( \frac{d^N}{dQ^2} \right)^N.
\]
(9)

This enhances the lower-lying ground state resonances and suppresses excited states described by our correlator, and conveniently eliminates the subtraction constants in (8). This gives the \(k^N\)-order Laplace sum-rule
\[
\mathcal{R}_k(\tau) = \int_{s_0}^{\infty} t^e^{-\tau t} \frac{1}{\pi} \text{Im}\Pi(t) \, dt
\]
(10)
\[
= \frac{1}{\tau} \hat{B} \left\{ (-Q^2)^N \Pi(Q^2) \right\}.
\]
(11)

Employing a “single narrow resonance plus continuum” model for our spectral function (9)
\[
\frac{1}{\pi} \text{Im}\Pi(t) = f_H^2 m_H^8 \theta(t - m_H^2) + \theta(t - s_0) \frac{1}{\pi} \text{Im}\Pi/MPL(t)
\]
(12)
gives us a final form for our (continuum-subtracted) sum-rule,
\[
\mathcal{R}_k(\tau, s_0) = \mathcal{R}_k(\tau) - \int_{s_0}^{\infty} t^e^{-\tau t} \frac{1}{\pi} \text{Im}\Pi/MPL(t) \, dt
\]
(13)
\[
= \frac{f_H^2 m_H^8 \theta}{\tau} e^{-m_H^2 \tau}
\]
(14)
where \(m_H\) is the ground state resonance mass, \(f_H\) is its coupling strength, \(\theta\) is a Heaviside step function, \(s_0\) is the continuum threshold and \(\text{Im}\Pi/MPL\) is the imaginary part of the QCD expression for \(\Pi\). This form allows us to extract the hadronic mass as a ratio of these continuum-subtracted sum-rules,
\[
\frac{\mathcal{R}_1(\tau, s_0)}{\mathcal{R}_0(\tau, s_0)} = m_H^2.
\]
(15)

To compute stable mass predictions using (15), we require a suitable range of values for our Borel scale \(\tau\) within which reliable results can be extracted (known as a Borel window). Within this Borel window, we perform a fitting procedure in order to find an optimized value of the continuum onset \(s_0\) associated with our resulting mass prediction. We determine our Borel window by requiring convergence of the OPE, and that the pole mass contributes a certain percentage to the overall mass prediction. We follow our previous work done
in closed-flavour heavy hybrid systems [13] to enforce OPE convergence and obtain an upper-bound on the window, we require that contributions to the 4d condensate be less than one-third that of the perturbative contribution, and the 6d gluon condensate contribute less than one-third of the 4d condensate contributions. For a lower-bound on the Borel window, we require a pole contribution of at least 10%.

2. Mixing Effects

A consequence of the open-flavour systems in question is the preclusion of exotic quantum numbers; as no definite C-parity number exists for the \(J^P, J^{PC}\) systems in question, we cannot access clean, “smoking gun” signals that exotic quantum numbers might allow. As such, we must consider possible effects of mixing with conventional meson systems. For a preliminary examination of these effects, we extend our single narrow resonance model to a double narrow resonance model coupling to a conventional meson state,
\[
\frac{1}{\pi} \text{Im}\Pi(t) = f_H^2 m_H^8 \delta(t - m_H^2) + f_{\text{com}} m_{\text{com}}^8 \delta(t - m_{\text{com}}^2)
\]
\[
+ \theta(t - s_0) \frac{1}{\pi} \text{Im}\Pi/MPL(t).
\]
(16)

With this change in resonance model, our resulting sum-rule in (15) becomes
\[
\frac{\mathcal{R}_1(\tau, s_0)}{\mathcal{R}_0(\tau, s_0)} = f_{\text{com}} m_{\text{com}}^{10} e^{-m_{\text{com}}^2 \tau} = m_H^2.
\]
(17)

While this is no replacement for a full mixing analysis involving off-diagonal contributions, it serves as a simple model to examine possible mixing effects. To examine the effects of conventional state mixing, we have anchored our double narrow resonance model to the corresponding lowest-lying resonance for a given set of quantum numbers using a conventional meson mass \(m_{\text{com}}\) obtained from the PDG [19]. Along with our mass predictions from the single narrow resonance model, we also report the maximal mixing effect from considering (17) in Tables 5 [8]. \(\Delta m_{\text{mix}}\) is the increased mass range with mixing up to \(\left| \frac{f_{\text{com}}}{f_H} \right| = \frac{1}{2} \) due to coupling to the lowest-lying conventional state with appropriate quantum numbers according to PDG values summarized in Table 2. In all cases, the results of the mixing tends to increase the resulting mass prediction, though the charm flavoured channels appear particularly insensitive to this mixing model.
The quark masses and $\alpha_s$ reference values used in our analysis are

$$M_c = (1.275 \pm 0.025) \text{ GeV} \quad (18)$$
$$M_b = (4.18 \pm 0.03) \text{ GeV} \quad (19)$$
$$m_n(2 \text{ GeV}) = (3.40 \pm 0.25) \text{ MeV} \quad (20)$$
$$m_s(2 \text{ GeV}) = (93.5 \pm 2.5) \text{ MeV} \quad (21)$$
$$\alpha_s(M_c) = 0.330 \pm 0.014 \quad (22)$$
$$\alpha_s(M_b) = 0.1185 \pm 0.0006. \quad (23)$$

as well as the mass ratios

$$\frac{M_c}{m_n} = 322.6 \pm 13.6, \quad \frac{M_c}{m_s} = 11.73 \pm 0.25, \quad (24)$$
$$\frac{M_b}{m_n} = 1460.7 \pm 64.0, \quad \frac{M_b}{m_s} = 52.55 \pm 1.30. \quad (25)$$

The condensate values used are

$$\langle \alpha_G^2 \rangle = (0.075 \pm 0.020) \text{ GeV}^4$$

$$\langle g^3 G_1 \rangle = (8.2 \pm 1.0) \text{ GeV}^2 \langle \alpha_G^2 \rangle$$

$$\frac{\langle g G q \rangle}{\langle q \bar{q} \rangle} \equiv M^2_c = (0.8 \pm 0.1) \text{ GeV}^2,$$

where $\langle q \bar{q} \rangle$ is given by PCAC using the coupling values

$$f_s = 92.2 \pm 3.5 \text{ MeV}, \quad f_c = 110.0 \pm 4.2 \text{ MeV}. \quad (29)$$

See Ref. [2] for references to the sources for these parameters and uncertainties.

3. Results of Laplace Sum-Rules Analysis

Performing a Laplace sum-rules analysis of all eight distinct $J^P(C)$ combinations defined according to Table 1, we present the mass predictions and estimated uncertainties in Figure 2 and in Tables 5-8. For the eight channels described in Table 1, only four stabilized for each combination of flavours ($\bar{c}Gq$, $\bar{c}Gs$, $\bar{b}Gq$, $\bar{b}Gs$). See Ref. [2] for an in-depth discussion and all results (including Borel windows and continuum parameters). A full uncertainty analysis was performed accounting for variations in QCD parameters (condensate values, heavy quark masses, mass ratios, $\alpha_s$ reference values, truncation of the OPE, and variations in the Borel window and renormalization scale) with contributions added in quadrature. Within the computed uncertainty, mass degeneracy between strange and non-strange channels cannot be precluded. Comparisons between our results and those found previously by GRW are shown in Tables 5 and 6 where we report $J^P$ numbers because a change in channel stability was observed in our analysis; in GRW, the stable channels were $J^P(C) \in \{0^+, 0^-, 1^+, 1^-\}$ for all heavy-light flavour hybrids. We see in Tables 5 and 6 that the central values of our predictions differ significantly from GRW, except in the case of $1^+$ charm-nonstrange. Additionally, there emerge similar mass hierarchies between the charm and bottom channels; excluding the $0^+$ states, the $1^+$, $1^-$ and $0^-$ states form a pattern where the $1^+$ state is lighter than essentially degenerate $1^-$ and $0^-$ states.

| Flavour | $J^P$ | PDG State | $m_{conv}\text{(GeV)}$ |
|---------|-------|-----------|------------------|
| $\bar{c}Gq$ | 0$^+$ | $D_0^0\ (2400)^0$ | 2.318 |
| | 0$^-$ | $D^0$ | 1.865 |
| | 1$^-$ | $D^*$\ (2007)$^0$ | 2.007 |
| | 1$^+$ | $D_1^+(2420)^0$ | 2.420 |
| $\bar{c}Gs$ | 0$^+$ | $D_0^*(2317)^+$ | 2.318 |
| | 0$^-$ | $D_s^+$ | 1.969 |
| | 1$^-$ | $D_{s\pi}^+$ | 2.112 |
| | 1$^+$ | $D_{s1}(2460)^+$ | 2.460 |
| $\bar{b}Gq$ | 0$^+$ | - | - |
| | 0$^-$ | $B_0$ | 5.279 |
| | 1$^-$ | $B_s^+$ | 5.324 |
| | 1$^+$ | $B_1(5721)^0$ | 5.726 |
| $\bar{b}Gs$ | 0$^+$ | - | - |
| | 0$^-$ | $B_{s0}^0$ | 5.367 |
| | 1$^-$ | $B_{s1}^+$ | 5.416 |
| | 1$^+$ | $B_{s1}(5830)^0$ | 5.828 |

Table 3: Comparison of central values against GRW mass predictions for $\bar{q}qG$ hybrids ($q = \{u, d\}$).

| $J^P$ | $m_{GRW}\text{(GeV)}$ | $m_{11}\text{(GeV)}$ |
|-------|------------------|------------------|
| 0$^+$ | 4.0 | 4.54 |
| 0$^-$ | 4.5 | 5.07 |
| 1$^-$ | 3.6 | 4.40 |
| 1$^+$ | 3.4 | 3.39 |

Table 4: Comparison of central values against GRW mass predictions for $\bar{q}qG$ hybrids ($q = \{u, d\}$).
While discrepancies in a shared hierarchy seem to appear with the inclusion of the $0^-$ states, we note that the charm and bottom $0^-$ mass predictions arise from interpolating currents with different C-parity. Although open-flavour systems have no well-defined C quantum number, Ref. [11] attributes physical meaning to the charm and bottom $0^-$ states, we note that the $0^-\bar{0}^+$ structure is heavier than the $0^+\bar{0}^-$, consistent with the pattern we observe in Fig. 2.

Table 5: QCD sum-rules analysis results for ground state charm-nonstrange hybrids, including effect on hybrid mass prediction from mixing with conventional meson states; $\delta m_{mix}$ is the increased mass range with mixing up to $\frac{\Delta m_{mix}}{m_{H}} = \frac{1}{2}$ due to coupling to the lowest-lying conventional state with appropriate quantum numbers according to PDG values summarized in Table 2.

| $J^{PC}$ | $m_H \pm \delta m_{H}$ (GeV) | $f_{HJ}^2 \times 10^6$ | $+\delta m_{mix}$ (GeV) |
|----------|-------------------------------|----------------------|--------------------------|
| $0^{(+)}$ | 5.45 ± 0.43 | 7.47 | 0.02 |
| $0^{(-)}$ | 5.07 ± 0.31 | 7.28 | 0.00 |
| $1^{(-)}$ | 4.40 ± 0.19 | 12.4 | 0.01 |
| $1^{(+)}$ | 3.39 ± 0.18 | 9.87 | 0.05 |

Table 6: QCD sum-rules analysis results for ground state charm-strange hybrids, including effect on hybrid mass prediction from mixing with conventional meson states; $\delta m_{mix}$ is the increased mass range with mixing up to $\frac{\Delta m_{mix}}{m_{H}} = \frac{1}{2}$ due to coupling to the lowest-lying conventional state with appropriate quantum numbers according to PDG values summarized in Table 2.

| $J^{PC}$ | $m_H \pm \delta m_{H}$ (GeV) | $f_{HJ}^2 \times 10^6$ | $+\delta m_{mix}$ (GeV) |
|----------|-------------------------------|----------------------|--------------------------|
| $0^{(+)}$ | 4.49 ± 0.40 | 7.36 | 0.02 |
| $0^{(-)}$ | 4.98 ± 0.39 | 2.03 | 0.00 |
| $1^{(-)}$ | 4.28 ± 0.19 | 11.0 | 0.02 |
| $1^{(+)}$ | 3.15 ± 0.14 | 8.45 | 0.06 |

Table 7: QCD sum-rules analysis results for ground state bottom-nonstrange hybrids, including effect on hybrid mass prediction from mixing with conventional meson states; $\delta m_{mix}$ is the increased mass range with mixing up to $\frac{\Delta m_{mix}}{m_{H}} = \frac{1}{2}$ due to coupling to the lowest-lying conventional state with appropriate quantum numbers according to PDG values summarized in Table 2.

| $J^{PC}$ | $m_H \pm \delta m_{H}$ (GeV) | $f_{HJ}^2 \times 10^6$ | $+\delta m_{mix}$ (GeV) |
|----------|-------------------------------|----------------------|--------------------------|
| $0^{(+)}$ | 8.57 ± 0.51 | 1.28 | - |
| $0^{(-)}$ | 7.01 ± 0.21 | 0.516 | 0.19 |
| $1^{(-)}$ | 8.74 ± 0.25 | 1.76 | 0.32 |
| $1^{(+)}$ | 8.26 ± 0.41 | 1.66 | 0.74 |

4. Conclusions

Ground state mass predictions of heavy-light (strange and nonstrange) hybrid mesons for $J^P \in \{0^+, 1^+\}$ have been briefly presented, utilizing QCD sum-rules and improving upon previous calculations [1] by updating the non-perturbative parameters in the calculation, and including higher dimensional condensates in the OPE shown important to sum-rule stability in other contexts. A complete discussion of the analysis and results may be found in [2]. A degeneracy is observed in the heavy-light and heavy-strange states, and stabilization in the previously unstable $0^{(-)}$ and $1^{(-)}$ channels [11] driven by the addition of the higher dimensional 5d mixed and 6d gluon condensate contributions. As a consequence of these higher dimensional contributions, the $1^{(+)}$ channel is destabilized from the original analysis of [1]. Possible mixing effects are examined, and in our simplest mixing model we find our predictions serve as lower bounds on ground state mass predictions.

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