Buckling analyses of carbon nanotube reinforced functionally graded composite cylindrical panels

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Abstract. This paper investigates the buckling analyses of carbon nanotube (CNT) reinforced composite cylindrical panels via a geometrically nonlinear finite element model with large rotations based on the first-order shear deformation (FOSD) hypothesis. Fully geometrically nonlinear strain-displacement relations and large rotation of shells are considered in the model. First, the proposed model is validated by a frequency analysis of a simply supported CNT reinforced composite cylindrical panel from an existing reference. Then, the model is applied to simulate the behaviors of carbon nanotube reinforced functionally graded (CNT-FG) composite cylindrical panels. The effects of curvature ratio, different buckling behaviors and four representative forms of CNT distributions are studied for their material performance comparatively.

1. Introduction
Recently, an advanced material known as carbon nanotube (CNT) has been drowned much attention for its excellent thermal, electrical and mechanical properties. Due to its high strength, stiffness and aspect ratio, and low density, the so-called “material for the 21st century” [1] poured into the new vigor for the investigation of composite structures. Hence, numerous researchers have reported a large number of analyses on physical and mechanical properties of composite structures with CNT. 

In order to investigate the structural response of CNT composite cylindrical panels, some papers took the classical shell theory (CST) based on the Kirchhoff-Love assumption which neglects the transverse shear deformations[1], then the first-order shear deformation (FOSD) hypothesis was widely used. Fruitful dynamic behaviors of different CNT structures were obtained in the articles, see Dai et al.[7-10]. Zhang & Lei[2,6] investigated flexural strength and free vibration responses of CNT reinforced composites cylindrical panels based on the FOST using the mesh-free kp-Ritz method. They have also analyzed the free vibration of carbon nanotube (CNT) reinforced functionally graded rotating to reveal the influences of volume fraction of carbon nanotubes, edge-to-radius ratio and rotation speed on the frequency characteristics of cylindrical panels. But few references have mentioned geometrically nonlinear modeling. In thin-walled structures, it is easily undergoing larger displacement or rotations, which requires fully geometrically nonlinear with large rotation theory, but not simplified nonlinear theories.

Therefore, this paper developed a fully geometrically nonlinear model with large rotations for the nonlinear behavior analysis of the CNT-FG composite cylindrical panels. First the nonlinear model is
validated by a cylindrical panel. Then several simulation examples are presented to show the effects of nanotube volume fraction, the thickness of the panel and different representative forms of CNT distributions.

2. CNT reinforced composite panels

In the present article, the geometry of the CNT-FG composite cylindrical panel are presented in Figure 1, where “h” is the thickness of the panel, “R” is the radius , “β” is the angle, and “L” is the half length of the side along Θ1 direction, in the cylindrical coordinate.

![Figure 1. Geometry of the CNT-FG composite cylindrical panel](image-url)

In this paper, the CNTs are assumed to be aligned in axial direction and four representative forms of CNT distributions, namely uniform, V-shaped, O-shaped, and X-shaped distributions, denoted by U, V, O, X, respectively. The mathematical expression of the effective volume fractions of each type is as follow[3]:

\[
V_{\text{CNT}}^* = \frac{\omega_{\text{CNT}}}{\omega_{\text{CNT}} + (\frac{\rho_{\text{CNT}}}{\rho_m} - (\frac{\rho_{\text{CNT}}}{\rho_m})\omega_{\text{CNT}})}
\]

(2)

where

\[
V_{\text{CNT}}^* = \frac{\omega_{\text{CNT}}}{\omega_{\text{CNT}} + (\frac{\rho_{\text{CNT}}}{\rho_m} - (\frac{\rho_{\text{CNT}}}{\rho_m})\omega_{\text{CNT}})}
\]

(2)

in which \( V_{\text{CNT}}^* \) is the total volume fraction of CNT, \( \rho_{\text{CNT}} \) and \( \rho_m \) are the densities of the CNTs and the matrix, respectively, and \( \omega_{\text{CNT}} \) is the mass fraction of the CNT.

The effective Young’s modulus, shear modulus and other effective material properties can be expressed as[3]:

\[
E_{11} = \eta_1 V_{\text{CNT}}^* E_{11}^\text{CNT} + V_m E_m
\]

(3)

\[
\frac{\eta_2}{E_{22}} = \frac{V_{\text{CNT}}^*}{E_{22}^\text{CNT}} + \frac{V_m}{E_m}
\]

(4)

\[
\frac{\eta_3}{G_{12}} = \frac{V_{\text{CNT}}^*}{G_{12}^\text{CNT}} + \frac{V_m}{G_m}
\]

(5)
where $E_{11}$, $E_{22}$ and $G_{12}$ are the effective Young’s moduli of composite along and transverse direction and the effective shear modulus, respectively. $E_{11}^{CNT}$, $E_{22}^{CNT}$ and $G_{12}^{CNT}$ are the Young’s moduli and shear modulus of the CNT, $E^m$ and $G^m$ are the corresponding properties of the matrix, $\eta_i$ (i=1,2,3) are the CNT efficiency parameters, $V_m$ and $V_{CNT}$ are the matrix and CNT volume fractions.

2.1 Geometrically nonlinear model

The Green-Lagrange strain tensors of the in-plane terms, the transverse shear and transverse normal terms, which include full geometric nonlinearities based on the FOSD hypothesis can be obtained as

$$
\varepsilon_{\alpha\beta} = \epsilon_{\alpha\beta} + \Theta^3 \varepsilon_{\alpha\beta} + \left( \Theta^3 \right)^2 \varepsilon_{\alpha\beta}
$$

(6)

$$
\varepsilon_{\alpha3} = \varepsilon_{\alpha3} + \Theta^3 \varepsilon_{\alpha3}
$$

(7)

$$
\varepsilon_{33} = \varepsilon_{33}
$$

(8)

Here, the strain terms have their own physical meanings, the in-plane longitudinal strains ($\varepsilon_{11}$, $\varepsilon_{22}$), the in-plane shear strains ($\varepsilon_{12}$, $\varepsilon_{21}$), the bending strains ($\varepsilon_{11}$, $\varepsilon_{22}$), the torsional strains ($\varepsilon_{12}$, $\varepsilon_{21}$), the transverse shear strains ($\varepsilon_{13}$, $\varepsilon_{23}$), and the transverse normal strain ($\varepsilon_{33}$)[11].

With the assumption of an inextensible shell director, the transverse normal strain will be $\varepsilon_{33} = 0$, and that will lead to $\varepsilon_{\alpha3} = 0$, the strain components with six parameters can be expressed as[5]

$$
2 \varepsilon_{\alpha\beta} = \nu_{\alpha,\beta} + \nu_{\beta,\alpha} + 3 \nu_{3,\alpha} \nu_{3,\beta} + \nu_\delta, \alpha \nu_\delta, \beta,
$$

(9)

$$
2 \varepsilon_{\alpha\beta} = \nu_{\alpha,\beta} + \nu_{\beta,\alpha} + 3 \nu_{3,\alpha} \nu_{3,\beta} + \nu_\delta, \alpha \nu_\delta, \beta + \nu_\delta, \alpha \nu_\delta, \beta,
$$

(10)

$$
2 \varepsilon_{\alpha3} = \nu_{\alpha} + \nu_{3,\alpha} + \nu_\delta, \alpha \nu_\delta + \nu_{3,\alpha} \nu_{3}.
$$

(12)

with

$$
\nu_{\alpha} = \nu_{\alpha} + \Theta^3 \nu_{\alpha},
$$

(13)

$$
\nu_{3} = \nu_{3} + \Theta^3 \nu_{3}.
$$

(14)

where $0 \nu_{\alpha}$ and $0 \nu_{3}$ denote the translational displacements at the mid-surface, $1 \nu_{\alpha}$ and $1 \nu_{3}$ represent the generalized rotational parameters, $\Theta^3$ is the distance from the mid surface. The lower Greek symbols $\alpha$ can be 1 or 2, $1 \nu_{3}$ is usually neglected under the condition of the small and moderate rotations.
3. Numerical results and discussions

In the present analysis, the properties of the CNT at 300K and the matrix materials are considered as Table 1. It is assumed that $G_{12}^{CNT} = G_{13}^{CNT} = G_{23}^{CNT}$, and in the non-dimensional central deflection, $\bar{w}_{\text{MAX}} = W_{\text{MAX}} / h$, in the formula, $W_{\text{MAX}}$ is the maximum central deflection.

**Table 1.** Material properties of the CNT at 300K

|        | CNT          | Matrix       |
|--------|--------------|--------------|
| $Y_{11}^{CNT}$ | 5646.6 GPa   | $Y^m = 3.52$ GPa |
| $Y_{22}^{CNT}$ | 7080.0 GPa   | $\rho^m = 1150$ kg / m$^3$ |
| $G_{12}^{CNT}$ | 1945.5 GPa   | $\nu^m = 0.34$ |
| $\rho^{CNT}$   | 1400 kg / m$^3$|              |
| $\nu_{12}^{CNT}$ | 0.175        |              |

3.1 Linear simulation

In this part, the non-dimensional central deflection of the four type CNT-FG composite cylindrical panel are presented in the Table 2 for different curvature ratio with all edges simply supported (SSSS) supported condition under 1MPa surface force, and the parameters are $R=1$m, $L=0.5$m, $\beta=0.05$rad, $h=0.002$m.

**Table 2.** Non-dimensional central deflection of simply supported cylindrical panel for different curvature ratio

| Curvature ratio | Type | Present | Ref[3] | Deviation |
|-----------------|------|---------|--------|-----------|
| 5               | U    | 11.30   | 11.47  | -1.48%    |
|                 | V    | 16.00   | 16.154 | -0.95%    |
|                 | O    | 19.80   | 21.048 | -5.93%    |
|                 | X    | 7.95    | 7.906  | 0.56%     |
| 10              | U    | 11.30   | 11.47  | -1.48%    |
|                 | V    | 16.00   | 16.154 | -0.95%    |
|                 | O    | 19.80   | 21.048 | -5.93%    |
|                 | X    | 7.95    | 7.906  | 0.56%     |
| 20              | U    | 11.50   | 11.583 | -0.72%    |
|                 | V    | 16.30   | 16.481 | -2.28%    |
|                 | O    | 20.30   | 21.527 | -5.70%    |
|                 | X    | 8.05    | 7.947  | 0.63%     |
| 50              | U    | 11.55   | 11.615 | -0.56%    |
|                 | V    | 16.35   | 16.59  | -3.02%    |
The example shows that the non-dimensional central deflection with different forms of CNT distribution of the simply supported cylindrical panel does not change obviously as the curvature ratio increases. The O-shaped distribution type has the largest displacement, and the X-shaped distribution type is the most stable one. As shown in the table, perfect agreement is achieved between the results presented and those of Ref [3]. Hence, the present model is accurate enough to be employed in the following simulation.

### 3.2 Buckling analysis of CNT-FG shells

In this part, the non-dimensional central deflections of the four types of CNT-FG composite cylindrical panels are presented in Figure 2, using different theories abbreviated as RVK5, MRT5, LRT5, and LRT56[4] with two straight edges hinged and the two curved ones free (HFHF) support condition under 2500N concentrated force, and the parameters are as follow: $R=1m$, $L=0.16m$, $\beta=0.1rad$, $h=0.002m$.

![Figure 2](image_url)

**Figure 2.** Non-dimensional central deflection of simply supported cylindrical panel for (a) different theories and (b) different types of CNT distributions.

Fig 2 (a) shows that using different theories lead to similar outputs, that means the model does not have a large rotation. And Fig 2 (b) shows different buckling behaviors of four forms of CNT distributions based on the same theory, the trend of deformation is the same as in previous examples that the X-shaped distribution type has the largest stiffness, and the O-shaped distribution type is on the contrary.

### Conclusions

In this paper, the behavior of carbon nanotube reinforced functionally graded (CNT-FG) composite cylindrical panels of four representative forms of CNT distributions has been analyzed using the geometrically nonlinear model. The model is verified through the comparison to the reference and the effects of curvature ratio and CNT distribution are shown in the linear simulation, then by computing the responses for the different theories and distributions, the nonlinear behaviors of model are presented. From the results it is obvious that curvature ratio does not affect much on the non-dimensional central deflection and the theories has little effect on the final displacement, but the CNT distributions make a difference, the X-shaped distribution type has larger stiffness compared to other three types.
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