Impact Localization of Thin Plate Structures Using Pzt-Array Based Passive Wave Method

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Abstract. Impact of foreign objects is one of the main reasons for the damage of thin plate structures, resulting in property losses or unexpected accidents. Therefore, a rapid and effective impact localization of thin plate structures is of great value. In this paper, a localization method is proposed based on the passive wave method, aiming at locating the impact quickly and accurately. First, a square PZT array composed of 4 PZT sensors was designed, and a hyperbolic localization algorithm was derived according to time-different of arrival (TDoA) for different sensors in the array. Then, Matlab calculation software was used to perform DB wavelet decomposition on impact signals, the time delay was calculated by Lamb wave of A0 mode in D5 detail signals, and the velocity of A0 mode wave was determined by the dispersion curve for 1 mm thick aluminum plate. Finally, the impact point was located to experimentally verify the effectiveness of the proposed localization method. Experimental results show that the hyperbolic positioning algorithm based on TDoA can accurately identify impact locations, and wavelet decomposition can effectively identify single mode Lamb waves. The proposed method has the characteristics of fastness and accuracy, which is suitable for single object impact localization on thin plate structures.

1. Introductions

1.1. Impact on Thin Plate Structures
Thin plate structure is a common structural form in which the geometry in one dimension is much smaller than the other two dimensions. Thin plate structure is widely used in the fields of industrial manufacturing, mechanical processing, civil engineering, aerospace, automobile and shipbuilding, such as pressure vessel enclosures, aircraft skins, building curtain walls, etc. Foreign object impacts may cause damages to thin plate structures, such as bird impacts on airplane, cosmic garbage impacts on aircraft, rock impacts on ship wall and even the impact of flying stones on vehicle, which sometime may cause serious casualties and catastrophic consequences. Therefore, quickly and accurately identifying the impact on thin plate structures, assessing the location and extent of the damage, and taking corresponding effective measures to reduce the damage caused by impacts are of great significance. The paper aims at proposing an effective method to identify the impact location on thin plate structures by passive wave based technology.

1.2. PZT-Array Based Impact Localization
Various researches have been carried out and fruitful results have been achieved on impact localization and damage identification of thin plate structures. At present, damage identification (DI) and structural health monitoring (SHM) based on piezoelectric ceramics (such as lead zirconate titanate,
PZT) are mainly divided into active and passive wave methods. The active wave method requires the PZT sensor array attached on the surface of structure to actively generate specific stress waves for DIs or SHMs, such as Lamb waves in thin plate structures, combining with specific DI algorithm, damage assessment can be carried out. The active wave method is characterized by not paying attention to the impact process, which requires a certain degree of permanent damage to the monitored thin plate structure, so that the received signal can carry the damage information. The passive wave method needs to receive the impact signals directly by using the surface-attached PZT sensor array, then use the impact signals themselves to locate the impact point (sound source), which requires sensitive signal capturing capabilities of the PZT sensors and effectiveness of localization algorithms, also the passive wave method needs to pre-determine the range of impact area and pre-arrange the PZT array with a specified form. When an impact occurs, the generated stress wave propagates in a certain form and will be received by different sensors in the PZT array, and the position of impact points can be calculated by the time-difference of arrival (TDoA) of this stress wave.

The passive wave method does not require PZT to apply excitation on thin plate structures as a driver which is suitable for certain cases where the DI stress wave cannot be excited due to actual conditions. Because of the flexibility and convenience of passive wave method, it has been widely studied by scholars all over the world (Salvermoser et al. 2015, Chehami et al. 2015 & Caizzone et al. 2015). Based on the received waveform of impact, a passive wave method based on time reversal was proposed to achieve the impact localization on anisotropic materials (Ciampa & Meo 2012). Impact localization was performed on composite plates, and the effects of signal filtering on impact localization were compared, the experimental results showed that the error caused by the filtering process on the impact location is insignificant (Seydel & Chang 2001). A passive wave method was used to detect the impact and impact force of simple structures based on transfer function of structural and physical properties (Park & Chang 2005). The strain on the surface of a sandwich composite was measured by sensors, and its properties under low-speed impact were analyzed (Anderson & Madenci 2000). Wavelet decomposition and Newton-based optimization techniques were used to measure the velocity of A0 mode wave and locate the sound source (Ciampa & Meo 2010).

It can be seen that although the current researches on the impact localization of thin plate structures using passive wave method have achieved certain results, there are still some aspects that need further improvement, such as accurately determining the time delay of the sensing signals, quickly evaluating the Lamb wave velocity, and besides efficient algorithms are needed to reduce error and improve localization accuracy. Based on the existing research results, a square PZT array was designed and a hyperbolic localization algorithm for impact was proposed in this paper. The algorithm is computed by time delay and wave velocity of sensing signals from different channels, in order to accurately estimate the arrival time of the sensing signals and determine the wave velocity at the specified frequency and wave mode with the theoretical dispersion curves, DB wavelet decomposition was utilized to obtain the fifth level of detail signals (D5, detail signals), and finally the position of the impact point was calculated.

2. Theoretical Basis

2.1. Hyperbolic Localization Algorithm

For the monitored thin plate structure, a set of square PZT array (consisting of 4 PZTs) was designed, the intersection point of two diagonal lines of the square PZT array is taken as the origin, a local coordinate system with the PZT positions are shown in Figure 1.
Figure 1. The schematic of thin plate structure and local coordinate system as well as PZT locations.

As shown in Figure 1, the solid rectangles represent PZT sensors which are pasted at positions of (-c1, 0), (c1, 0), (0, c2), and (0, -c2) and respectively marked as PZT1, PZT2, PZT3, and PZT4. The solid circle represents the actual impact position with the coordinates of (x, y), and the hollow circles represent the suspected impact positions with the coordinates of (x’, y’). The time delay between the impact signals received by the PZT1 and PZT2 is recorded as Δt12, and the time delay between the impact signals received by PZT3 and PZT4 is recorded as Δt34.

According to the occurrence sequence of impact signals received by four PZT sensors, the quadrant position of impact point can be determined as follow:

1) If PZT2 and PZT3 receive signals earlier than PZT1 and PZT4, then the impact point locates in the first quadrant; if PZT1 and PZT3 receive signals earlier than PZT2 and PZT4, then the impact point locates in the second quadrant; and so is the case of impact point locates in the third and fourth quadrant.

2) If PZT1 and PZT2 receive signals at the same time, then the impact point locates on the Y axis; similarly, according to the signal arriving time sequence of PZT3 and PZT4, the impact point can be determined on the positive or negative half of Y axis; for the impact point on the X axis, the same method is used.

3) If PZT1, PZT2, PZT3 and PZT4 all receive the signal at the same time, then the impact point locates at the origin.

After the preliminary determination of the impact point, the exact coordinates of the impact point are located by the hyperbolic algorithm as follow:

According to the definition of hyperbolic function, the absolute value of its distance difference between two fixed points F1 and F2 in a plane is equal to a constant (the constant is 2a, 2a=|F1F2|), which is a hyperbola, where the two fixed points F1 and F2 are the two focal points of hyperbola, a is the semi-real axis of hyperbola. Based on the above definition, the distance difference from the impact point to PZT1 and PZT2 can determine a hyperbola f1, similarly the distance difference from the impact point to PZT3 and PZT4 can determine another hyperbola f2, meanwhile the distance difference from the impact point to two sensors equals to the wave velocity multiplied by the time difference of the signals received by the corresponding sensors, thus the intersection coordinates (x’, y’) of these two hyperbolas f1 and f2 are the suspected coordinates of the impact point. The hyperbola f1 and f2 can be derived as Equation 1 and Equation 2 by above principles and positions of PZT sensors, suspected im-
impact points coordinates can be obtained by the consociation of f1 and f2, shown as Equation 3 and Equation 4.

\[
\frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 1, \quad \text{where} \quad 2a_1 = v \times \Delta t_{12}, \quad b_1^2 = c_1^2 - a_1^2 \tag{1}
\]

\[
\frac{y^2}{a_2^2} - \frac{x^2}{b_2^2} = 1, \quad \text{where} \quad 2a_2 = v \times \Delta t_{34}, \quad b_2^2 = c_2^2 - a_2^2 \tag{2}
\]

\[
x' = \pm \frac{v \Delta t_{12}}{2} \sqrt{\frac{(4c_1^2 - v^2 \Delta t_{34}^2)(4c_1^2 - v^2 \Delta t_{12}^2 + v^2 \Delta t_{34}^2)}{(4c_1^2 - v^2 \Delta t_{12}^2)(4c_2^2 - v^2 \Delta t_{34}^2) - v^4 \Delta t_{12}^2 \Delta t_{34}^2}} \tag{3}
\]

\[
y' = \pm \frac{v \Delta t_{34}}{2} \sqrt{\frac{(4c_1^2 - v^2 \Delta t_{12}^2)(4c_2^2 - v^2 \Delta t_{34}^2 + v^2 \Delta t_{12}^2)}{(4c_1^2 - v^2 \Delta t_{12}^2)(4c_2^2 - v^2 \Delta t_{34}^2) - v^4 \Delta t_{12}^2 \Delta t_{34}^2}} \tag{4}
\]

Where \( v \) is the velocity of the stress wave generated by the impact, which can be determined according to its mode and frequency combining with the dispersion curve. \( c_1 \) is the distance from PZT1 or PZT2 to the origin, and \( c_2 \) is the distance from PZT3 or PZT4 to the origin. Other symbols have the same meaning as mentioned before.

After these two steps, the coordinates of the actual impact point \((x, y)\) can be uniquely determined.

2.2. Lamb Wave Generated by Impact in Tested Aluminum Plate
As an elastic wave propagates in thin plate structures, Lamb wave will be excited when the plate thickness and the wavelength are in a same order of magnitude, which is also known as "plate wave". When Lamb wave propagates through a structure, both surfaces and various mass points inside the plate will vibrate, this vibration spreads throughout the thickness of the plate, meanwhile the originally transmitted transverse waves and longitudinal waves will be coupled when reflected by the upper and lower surfaces, which makes the ultrasonic wave propagates throughout the plate structure. The stress wave generated by impact in the aluminum plate belongs to Lamb wave, its multiple modes and dispersion phenomenon can be described by the dispersion curve, as shown in Figure 2. It is highly difficult to directly use Lamb waves for the localization of the impact point due to the complexity of multimodal and dispersive characteristics; therefore, it is necessary to select a specified mode at a specified frequency to locate impact points. It can be seen from the dispersion curve that Lamb wave of the 1 mm thick aluminum plate exists only in \( A_0 \) and \( S_0 \) modes in rather low frequency range of 0-500 kHz, where the wave velocity changes slowly with frequencies, utilizing Lamb wave in this frequency range can largely avoid multiple modes and frequency dispersion. Since the amplitude of \( A_0 \) mode is larger than \( S_0 \) mode in the low frequency range, \( A_0 \) mode is used for localization to facilitate the extraction of time delay.
2.3. Wavelet Decomposition

The impact signal received by PZT sensor is a typical non-stationary signal, so the traditional short-time Fourier transform cannot meet the accuracy requirements of the signal in time. Wavelet transform is a time-frequency joint analysis method with low time resolution and high frequency resolution in the low frequency range and high time resolution and low frequency resolution in the high frequency range. According to the theory of multi-resolution analysis proposed by Mallat in 1987 (Mallat 1987), the algorithm of wavelet decomposition with discrete signal at $2^j$ resolution can decompose the signal into different frequency bands at different scales, which can be written as:

$$S = A_n + D_n + \cdots + D_j + \cdots + D_1$$ (5)

Where $A_n$ is the approximate signal of layer $n$; $D_n$, ..., $D_j$, ..., $D_1$ are the detail signals of layer $n$, ..., $j$, ..., 1, respectively.

An advantage of wavelet decomposition is that all approximation signals and detail signals have the same time resolution as the original signal, the original signal will be formed when all signals are reconstructed together, which lays a foundation for using detail signals to extract the time delay. Another advantage of wavelet decomposition is that the algorithm itself can be regarded as a bandwidth filter, according to Nyquist sampling law, the frequency of approximation signal $A_j$ is distributed between 0 Hz and $F_s/2^{j+1}$ Hz, correspondingly the frequency of detail signal $A_j$ is distributed between $F_s/2^j$ Hz and $F_s/2^{j+1}$ Hz, where $F_s$ is the sampling frequency of signal. Thus, detail signal $D_j$ can be regarded as the original signal passing through a set of bandwidth filters from $F_s/2^j$ Hz to $F_s/2^{j+1}$ Hz, which allows wavelet decomposition to convert the wide-band original signal into a set of narrow-band signals and maintains precision well in time.

2.4. Selection of Wave Velocity and Time Delay

At present, methods for obtaining the time delay of difference signals mainly consist of threshold method, cross-correlation function method, wavelet analysis method, and Hilbert Huang transform method, etc. Based on Matlab calculation software, the original impact signals were processed by DB wavelet decomposition, and $D_5$ detail signal with the highest time resolution and the needed signal recognition level was selected for impact localization.

In this paper, the time delay between different channels was calculated by the first wave (head wave) packet peaks of $A_0$ mode in $D_5$ detail signals, determination of signal arriving time in one $D_5$ detail signal was shown in Figure 3. According to Nyquist sampling law, the frequency of $D_5$ detail signal is distributed between 39.06 kHz and 78.13 kHz, in order to accurately identify the center frequen-
cy of the D₅ detail signal, Fourier transform was performed, and it was found that the center frequency was mostly concentrated around 40 kHz in the experiment, as shown in Figure 4, thus the group velocity of A₀ mode in the D₅ detail signal was determined to be 1233 m/s in combination with the dispersion curve, which was selected for the impact localization.

![D₅ detail signal](image)

**Figure 3.** Determination of signal arriving time in a D₅ detail signal.

![Fourier transform of D₅ detail signal](image)

**Figure 4.** Fourier transformation figure of a D₅ detail signal.

### 3. Impact Localization Experiment and Error Analysis

#### 3.1. Impact Localization

The experiment aims at validating the efficiency of the proposed method for impact localization. The experimental setup and object (a thin aluminum plate) are shown in Figure 5 and Figure 6, respectively. The tested aluminum plate with a size of 800mm×800mm×1mm is fixed on a steel bracket, with plasticine glued around to reduce the influence of reflected waves. A Cartesian coordinate system is established with the center of the aluminum plate as the origin, PZT1, PZT2, PZT3 and PZT4 sensors are correspondingly arranged at (–0.25m, 0m), (0.25m, 0m), (0m, 0.25m) and (0m, –0.25m) forming a square array. A steel ball with a diameter of 10 mm was freely dropped from a height of 1 m above the tested plate, impacting at different positions of the plate, in order to precisely induce impact at the des-
ignated point, a PVC tube with the length of 1m and the inner diameter of 15mm was used to trajec
the steel ball. A four-channel RIGOL DS1104B oscilloscope was used to collect impact signals, and
four channels simultaneously receive impact signals from PZT1, PZT2, PZT3 and PZT4 with a sam-
pling frequency of 2.5 MHz. According to the localization algorithm, the signals of different channels
were decomposed by wavelet decomposition so as to obtain narrow-banded signals for impact locali-
zation, then D5 detail signal was adopted for the impact localization. Time delay was determined by
the arrival time of the head wave packets from different D5 signals, which is also defined as the head
wave method, combining with the theoretical Lamb wave velocity and the hyperbolic algorithm, im-
 pact location can be pinpointed. The implementation process of the impact localization is shown in
Figure 7.

A series of impact localization experiments were carried out on different positions of the aluminum
plate, experimental error was calculated by Formulas (6) and (7):

\[
\Delta = \sqrt{(x - x_0)^2 + (y - y_0)^2}
\]

\[
\delta = \Delta/l \times 100\%
\]

Where \(\Delta\) is absolute error, indicating the absolute distance between the evaluated position and actual
one; \(\delta\) is the relative error, indicating the relative distance between the calculated value and the actual
one; The point of \((x, y)\) is the actual impact point coordinates, \((x_0, y_0)\) is the calculated coordinates of
the impact point; \(l\) is the gauge distance between a group of PZT plates. Impact point localization re-
sults and errors are shown in Table 1.

![Figure 5. Experimental setup.](image-url)
Figure 6. Tested aluminum thin plate.

Figure 7. Implementation process of impact localization.
3.2. Error Analysis

The experimental results show that the localization method proposed in this paper is of great accuracy and effectiveness, the maximum absolute error of experiment is 0.04m, which meets the requirements of engineering precision. The main possible reasons for experimental errors were analyzed and given as follow:

(1) Error caused by PZT size: the size of PZT plate itself is 0.01 m × 0.005 m, which is not an ideal point and may cause a certain localization error. Therefore, under the condition of meeting the precision requirement, the dimensions of the PZT sensors are suggested to be as small as possible to reduce the PZT sensor dimension influence.

(2) Error caused by time delay calculation: when the PZT plate is too close to impact point, it may cause the first wave peaks of A0 mode to be deviated due to the superposition of the A0 and S0 modes. Therefore, when the impact point is close to a PZT sensor, the sensor is not suggested to be used, the other sensor in the PZT array can be applied by using the same algorithm but different sensor group.

(3) Error caused by theoretical speed selection: although Fourier transform was performed to determine the center frequency of D3 detail signal, the actual center frequency ranges between 39-70 kHz. To simplify the localization process, the velocity of 1233 m/s for A0 mode corresponding to 40 kHz was adopted, which will produce a certain error.

(4) Error caused by impact operating: in the experiment, a PVC pipe with an inner diameter of 15 mm was used to guide the steel ball to impact, which may cause the actual impact position to deviate from the expected position.

4. Conclusions

In this paper, a thin plate structure impact localization method using piezoelectric ceramic (PZT) sensor array and passive wave method was proposed. An explicit hyperbolic algorithm for the evaluation of the impact point location was derived by designing a square PZT array. We can conclude the research results as following:

(1) The PZT- array based impact location monitoring system is highly efficient practical for thin plate structures. By placing PZT sensors in a specific array, almost all positions in the monitoring field in the plate structure can be passively monitored.

(2) The proposed hyperbolic localization algorithm is also effective for the impact localization evaluation based on the passive Lamb wave technology. The multiple modes and frequency dispersion characteristics of Lamb wave makes the monitoring procedure difficult, but it can be effectively solved by the in-depth knowledge of dispersion curves of the Lamb waves.
(3) Original impact signals can be decomposed by DB wavelet decomposition to obtain Ds detail signal, Lamb wave of A₀ mode in Ds detail signal can be used for the localization of the impact points, which can efficiently avoid the influence of dispersion and multiple modes caused by Lamb wave.

(4) The time delay of the signals received by different sensors can be determined by wave propagation time difference of arrival (TDOA), the wave velocity of A₀ mode in Lamb wave was defined by dispersion curve of the tested aluminum plate.

(5) The proposed impact localization method has the characteristics of fastness and accuracy, the maximum relative error in the test can be controlled within 8%, which meets the actual engineering requirements.

The PZT-array based impact localization system and the hyperbolic localization algorithm can be effectively used in the impact location online monitoring for thin plate structures. Combined with propagation characteristics of Lamb waves and data processing technology, the proposed impact point monitoring technology for the thin plate structures may have a wide application in the future.

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