Channel Estimation and Signal Detection for MIMO-AFDM under Doubly Selective Channels

Haoran Yin, Yanqun Tang*
School of Electronics and Communication Engineering, Sun Yat-sen University, China
email: {yinhr6@mail2, tangyq8@mail}.sysu.edu.cn

Abstract—On the heels of orthogonal time frequency space (OTFS) modulation, the recently discovered affine frequency division multiplexing (AFDM) is a promising waveform for the sixth-generation wireless network due to its strong delay-doppler resilience against the double dispersive channels. With the superiorities of high multiplexing and diversity gain of multiple-input multiple-output (MIMO), we derive a vectorized input-output formulation of the MIMO-AFDM system. Correspondingly, we also propose an efficient single pilot aided with minimum guard (SPA-MG) scheme to perform channel estimation in the discrete affine Fourier transform (DAFT) domain. Furthermore, the message passing based iterative detector is explored for signal detection. Finally, the bit error ratio (BER) performances are simulated under doubly selective channels. It is worth emphasizing that the MIMO-AFDM system can achieve outstanding performance similar to MIMO-OTFS. Additionally, compared to ideal channel state information, our proposed SPA-MG scheme is verified to provide marginal difference with the least overhead.

Index Terms—MIMO-AFDM, DAFT domain, doubly selective channels, channel estimation, message passing detection.

I. INTRODUCTION

The sixth-generation wireless network is envisioned to provide ultra-reliable, high data rate and low latency communications in high mobility scenarios, including vehicle-to-vehicle (V2V), unmanned aerial vehicles (UAV) and high-speed trains, etc. These dynamic channels therein are characterized by heavy delay-doppler spread, which cast a huge challenge to the current widely adopted waveforms, such as orthogonal frequency division multiplexing (OFDM). The non-negligible doppler shift devastates greatly the orthogonality between the subcarriers in OFDM, which is a serious problem especially in the case of higher frequency bands used in the future communication systems.

Many efforts have been made to design a new modulation waveform to accommodate time- and frequency-selective channels. An alternative two-dimensional modulation waveform named orthogonal time frequency space (OTFS) outperforms OFDM significantly [1]–[3]. However, OTFS suffers from heavy guard overhead when conducting the pilot aided channel estimation [4]. Affine frequency division multiplexing (AFDM), a newly discovered waveform, always attains full diversity in doubly selective channels due to its strong propagation paths separability, which guarantees a thorough delay-doppler representation [5], [6]. Information symbols in AFDM are multiplexed on a set of orthogonal chirps via inverse discrete affine Fourier transform (DAFT) [7], [8]. Numerical results in [5], [6] show that AFDM has the identical remarkable bit error ratio (BER) performance in doubly selective channels just as OTFS, but possesses the advantage on fewer channel estimation overhead due to its one-dimensional structure.

Multiple-input multiple-output (MIMO) techniques have been exploited to enhance the spectral and energy efficiency compared to single-input single-output (SISO) system in terms of space diversity gain. MIMO-OFDM has come to great success in the past and present wireless networks, while several works on MIMO-OTFS, including channel estimation and signal detection, have emerged recently [9]. A single pilot aided scheme is introduced for channel estimation of MIMO-AFDM system and AFDM based multiuser system in [10] from the perspectives of feasibility and simplicity. However, to the best of our knowledge, a comprehensive introduction to MIMO-AFDM system has not been established in the literature.

Our contributions can be summarized as follows. We first derive the vectorized input-output formulation of the MIMO-AFDM system and analyze the sparsity of its effective channel matrix. Based on which, a novel channel estimation scheme called single pilot aided with minimum guard (SPA-MG) is proposed to reduce the pilot overhead to the greatest extent. In addition, a message passing (MP) based iterative detector is exploited for signal detection of MIMO-AFDM system. We compare the BER performance of SISO-AFDM, MIMO-AFDM and MIMO-OTFS. Numerical results validate that the MIMO-AFDM possesses the superiority over SISO-AFDM and the similarity with MIMO-OTFS.

The rest of this paper is organized as follows. Section II reviews the basic concepts of AFDM and introduces MIMO-AFDM system, which lay down the foundations for its development of channel estimation scheme and signal detection in Section III and Section IV respectively. Simulation results are presented in Section V, followed by the conclusions in Section VI.

Notations: \( \delta(\cdot) \) denotes the Dirac delta function; \( (\cdot)^H \) and \( (\cdot)^N \) denote the Hermitian transpose and \( N \) modulus operation respectively; \( \mathbb{Z}^+ \), \( \mathbb{Z} \) and \( \mathbb{C} \) denote the set of all non-negative integers, integers and complex numbers respectively; terms “Rx” and “Tx” are the abbreviations for the receive antenna and the transmit antenna correspondingly.

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II. SYSTEM MODEL

In this section, the basic concepts of AFDM are reviewed, based on which we derive the vectorized input-output relationship of MIMO-AFDM system and analyzes the sparsity of its effective channel matrix.

A. SISO-AFDM system

Let $x$ denote a vector of $N$ quadrature amplitude modulation (QAM) symbols that reside on the DAFT domain. The $N$ points inverse DAFT (IDAFT) is performed to map $x$ to the time domain as

$$s_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x_m e^{j2\pi (c_2m^2 + \frac{1}{N}mn + c_1n^2)}$$

with $n = 0, \ldots, N - 1$ denoting the index of subcarrier and $\Lambda_c = \text{diag} \left( e^{-j2\pi cn^2}, n = 0, 1, \ldots, N - 1 \right)$. Before transmitting $s$ into the channel, a chirp–periodic prefix (CPP) should be added with a length which is any integer greater than or equal to the value in samples of the maximum delay spread of the wireless channel with the following impulse response

$$g_n(l) = \sum_{i=1}^{P} h_i e^{-j2\pi \frac{2c_1}{N} \delta (l - l_i)}$$

where $P$ is the number of propagation paths, $h_i$ is the channel coefficient, $l_i$ and $\alpha_i$ are the normalized delay and doppler. While in this paper, we assume $l_i \in \mathbb{Z}^+$ and $\alpha_i \in \mathbb{Z}_1$

After transmitting through the channel, serial to parallel, discarding CPP and $N$ points DAFT are performed. Thus, the received time domain samples $d$ are transformed to DAFT domain samples $y$ with

$$y_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_n e^{-j2\pi (c_2m^2 + \frac{1}{N}mn + c_2n^2)}$$

where $m = 0, \ldots, N - 1$.

B. MIMO-AFDM system

We present the MIMO-AFDM modulation/demodulation block diagrams as Figure 1. Let $N_t$ and $N_r$ denote the number of Tx and Rx respectively. Then the linear system model based input-output relationship between the $r$-th Rx and $N_t$ Tx from a $N_t \times N_r$ MIMO-AFDM system can be described as

$$y_{r,m} = \sum_{t=1}^{N_r} \sum_{i=1}^{P_r} h_{r,t,i} e^{j2\pi \left(N_c l_{t,r,i}^2 - q_{r,t,i}^n + N_c (q^2 - m^2)\right)} x_{t,q} + w_r$$

where $m = 0, \ldots, N - 1$, integer $r \in [1, N_r]$ and $t \in [1, N_t]$ denote the index of the Rx and Tx respectively, $P_{r,t} \geq 1$ is the number of paths between the $r$-th Rx and $t$-th Tx, $\alpha_{r,t,i} \in [-\tilde{\alpha}_{\text{max}}, \tilde{\alpha}_{\text{max}}]$ is the corresponding doppler shift and $\tilde{\alpha}_{\text{max}}$ is the maximum doppler shift among all the paths, $l_{t,r,i}$ is the delay spread and $h_{r,t,i}$ is the channel coefficient, $q = (m + \text{loc}_{r,t,i})_N$ and $\text{loc}_{r,t,i} = \alpha_{r,t,i} + 2Nc_1 l_{t,r,i}$, $c_1 = \frac{2q_{\text{max}} + 1}{2N}$ and $c_2$ is either an arbitrary irrational number or a rational number sufficiently smaller than $\frac{1}{\sqrt{N}}$.

The received impulse response can be denoted as

$$y_1 = H_{1,1}x_1 + H_{1,2}x_2 + \cdots + H_{1,N_t}x_{N_t} + w_1$$

$$y_2 = H_{2,1}x_1 + H_{2,2}x_2 + \cdots + H_{2,N_t}x_{N_t} + w_2$$

$$\vdots$$

$$y_{N_r} = H_{N_r,1}x_1 + H_{N_r,2}x_2 + \cdots + H_{N_r,N_t}x_{N_t} + w_{N_r}$$

with $H_{r,t} = \Lambda_{\alpha_{r,t}} F \Lambda_{c_1} H_{r,t} \Lambda_{c_2}^H \Lambda_{\alpha_{r,t}}^H$ representing the effective channel matrix between the $r$-Rx and $t$-Tx, $H_{r,t}$ being the associated delay-time channel matrix, $F$ denoting the discrete Fourier transform (DFT) matrix, noise vector $w \sim \mathcal{CN}(0, N_0 \mathbf{I})$.

For the sake of compactness, we define the effective MIMO channel matrix for the above MIMO-AFDM system as

$$H_{\text{MIMO}} = \begin{bmatrix}
H_{1,1} & H_{1,2} & \cdots & H_{1,N_t} \\
H_{2,1} & H_{2,2} & \cdots & H_{2,N_t} \\
\vdots & \vdots & \ddots & \vdots \\
H_{N_r,1} & H_{N_r,2} & \cdots & H_{N_r,N_t}
\end{bmatrix}_{N_N \times N_N}$$
where $H_{\text{MIMO}} \in \mathbb{C}^{N_{r} \times N_{t} \times N_{t}}$, the transmitted vector $x_{\text{MIMO}} = [x_{1}^{T}, x_{2}^{T}, \ldots, x_{N_{t}}^{T}]^{T} \in \mathbb{C}^{N_{r} \times 1}$, received vector $y_{\text{MIMO}} = [y_{1}^{T}, y_{2}^{T}, \ldots, y_{N_{r}}^{T}]^{T} \in \mathbb{C}^{N_{t} \times 1}$, and noise vector $w_{\text{MIMO}} = [w_{1}^{T}, w_{2}^{T}, \ldots, w_{N_{t}}^{T}]^{T} \in \mathbb{C}^{N_{t} \times 1}$. Then the equation (3) can be rewritten as

$$y_{\text{MIMO}} = H_{\text{MIMO}} x_{\text{MIMO}} + w_{\text{MIMO}}. \quad (7)$$

### C. Sparsity analysis

Let $P_{\text{max}} = \max\{P_{r,t}| r \in [1, N_{r}], t \in [1, N_{t}]\}$. For each submatrix $H_{r,t}$ in the equation (6), the number of non-zero elements of each row and column equals to the number of paths. $P_{r,t} \leq P_{\text{max}} \ll N_{t}$, since the number of non-negligible propagation paths between arbitrary transmit and receive antenna is small in practice. Therefore in $H_{\text{MIMO}}$, the number of non-zero elements in the $b$-th row is $R_{b} = \sum_{t=1}^{N_{t}} P_{r,t+1}^{b}$, and the number of non-zero elements in the $a$-th column is $C_{a} = \sum_{r=1}^{N_{r}} P_{r,a+1}^{s}$ (start from the zero). Then the maximum number of non-zero elements for all the rows and columns can be expressed as $R_{\text{max}} = \max\{R_{b}| b \in [0, N_{N_{t}} - 1]\}$ and $C_{\text{max}} = \max\{C_{a}| a \in [0, N_{N_{t}} - 1]\}$ respectively. Subsequently, we have

$$R_{\text{max}} \leq N_{t} P_{\text{max}} \ll N_{N_{t}}, \quad C_{\text{max}} \leq N_{r} P_{\text{max}} \ll N_{N_{r}}. \quad (8)$$

This means $H_{\text{MIMO}}$ inherits the sparsity characteristic from $H_{r,t}$.

### III. CHANNEL ESTIMATION

In our prior work [10], we consider that the maximum delay $\Delta_{\text{max}}$ is identical between all pairs of Rx and Tx for simplicity, and a conventional single pilot aided (SPA) channel estimation scheme is proposed for MIMO-AFDM system. However, this approximation results in spectral efficiency degradation in many scenarios, such as distributed MIMO and multiuser systems. Therefore, we relax that assumption in the following and propose a novel channel estimation scheme, named single pilot aided with minimum guard (SPA-MG), to estimate all the channel state information (CSI) simultaneously at the receiver. The SPA-MG scheme is performed in the DAFT domain and consumes the minimum guard symbols (GS) among all the pilot based channel estimation schemes by exploiting the sparsity and circulation characteristics of each submatrix $H_{r,t}$ flexibly. A comparison between SPA-MG and SPA for a $2 \times 2$ MIMO-AFDM system are given in Figure 2.

Let’s define $Q_{r,t} = (l_{r,t,\text{max}} + 1)(2\alpha_{\text{max}} + 1) - 1$, where $l_{r,t,\text{max}}$ is the maximum delay between the $r$-th Rx and $t$-th Tx, $Q_{t} = \max\{Q_{r,t}| r \in [1, N_{r}]\}$, $Q_{\text{max}} = \max\{Q_{t}| t \in [1, N_{t}]\}$ and $S_{t} = \sum_{t=1}^{N_{t}} (1 + Q_{t}), t \in [1, N_{t}]$. For each submatrix $H_{r,t}$, as an example $H_{1,1}$ shown in Figure 2, there are $Q_{r,t} + 1$ possible non-zero entries in all the columns corresponding to $Q_{r,t} + 1$ possible propagation paths [5], [10]. According to the equation (4), the row and column coordinates of a non-zero entry are determined by the delay and doppler of the associated path jointly, also as the explicit value which depends on the channel coefficient. Therefore, all the columns in $H_{r,t}$ contain the same and intact CSI. Moreover, when $N$ is sufficiently large, different CSI of different channels can be estimated at the same time by extracting the distinguish columns of the corresponding effective channel matrix with pilots and sufficient guard symbols.

#### A. Conventional SPA scheme

In the conventional SPA scheme [10], $Q_{r,t}$ are all replaced by $Q_{\text{max}}$ roughly and the symbol arrangement of the $t$-th Tx are

$$x_{t}[m] = \begin{cases} \text{pilot} & m = (Q_{\text{max}} + 1)t - 1 \\ \text{guard} & m \in [0, (Q_{\text{max}} + 1)N_{t} + Q_{\text{max}} - 1]N_{r} \\ \text{blank} & m \neq (Q_{\text{max}} + 1)t - 1 \end{cases} \quad (9)$$

with an overhead of $(Q_{\text{max}} + 1)N_{t} + Q_{\text{max}}$ for each Tx and a total overhead $O_{\text{SPA}} = (Q_{\text{max}} + 1)N_{r}^{2} + Q_{\text{max}}N_{t}$.

#### B. SPA-MG scheme

In the sequel, we first present the implementation of SPA-MG scheme and then demonstrate its superiority over the conventional SPA. The pilot and guard symbols of the $t$-th Tx are arranged in SPA-MG as

$$x_{t}[m] = \begin{cases} \text{pilot} & m = S_{t} - 1 \\ \text{guard} & m \in [0, S_{N_{r}} + Q_{t} - 1]N_{r} \\ \text{blank} & m \neq S_{t} - 1 \end{cases} \quad (10)$$

where the overhead for channel estimation at the $t$-th Tx is $O_{t} = S_{N_{r}} + Q_{t}$, and the total estimation overhead for a $N_{r} \times N_{r}$ MIMO-AFDM system applying the SPA-MG scheme can be expressed as

$$O_{\text{SPA-MG}} = \sum_{i=1}^{N_{t}} Q_{i} = S_{N_{r}}N_{t} + \sum_{i=1}^{N_{t}} Q_{i}. \quad (11)$$

By comparing the first and second part of $O_{\text{SPA-MG}}$ and $O_{\text{SPA}}$ respectively, we can conclude that without losing estimation accuracy, SPA-MG has the advantage on higher spectral efficiency over SPA. This improvement will be more prominent in ultra-high speed scenarios where the maximum doppler $\alpha_{\text{max}}$ is especially high, resulting in the difference enlargement between $O_{\text{SPA-MG}}$ and $O_{\text{SPA}}$. Based on the symbol arrangement in (10), $N$ in SPA-MG should satisfy $S_{N_{r}} \leq N$, whereas $(Q_{\text{max}} + 1)N_{t} \leq N$ are required in conventional SPA according to the symbol arrangement in (9).

We next demonstrate that the number of guard symbols consumed by SPA-MG is minimum among all the pilot aided channel estimation schemes. The ultimate goal of SPA-MG is to separate the $N_{t}$ groups of received pilot symbols at all the Rx with the minimum GS, which can also be divided into $N_{t}$ groups at each Tx (with overlaps) according to the received pilot symbols they protect. To reduce the number of GS, we need to enlarge the overlapped parts among them to the greatest extent, which indicates that the pilots of all the Tx should be placed as close as possible at their own transmitted frame. Since the smallest space between the pilots of $(t-1)$-th
The left blank slots can be further explored to enhance the spectral efficiency by transmitting data symbols (also known as embedded pilot), or the channel estimation accuracy by transmitting extra pilots since the noise contained in different received symbols is independent identically distributed.

IV. SIGNAL DETECTION

Inspired by the message passing detector proposed for OTFS
in [11] and the sparsity analysis of $H_{\text{MIMO}}$ mentioned in section II, we present a MP based iterative algorithm for the detection of MIMO-AFDM system.

Let $\mathcal{I}(b)$ and $\mathcal{J}(a)$ denote the sets of non-zero indexes in the $b$-th row and $a$-th column of $H_{\text{MIMO}}$. According to $[3]$, $|\mathcal{I}(b)| \leq R_{\text{max}} \ll NN_t$ and $|\mathcal{J}(a)| \leq C_{\text{max}} \ll NN_t$. Then the equation (7) can be modeled as a sparsely connected factor graph with $NN_t$ observation nodes corresponding to the received vector $y_{\text{MIMO}}$ and $NN_t$ variable nodes corresponding to the transmitted vector $x_{\text{MIMO}}$. Each observation node $y_b$ is connected to a set of variable nodes $\{x_m, m \in \mathcal{I}(b)\}$. Analogously, each variable node $x_a$ is connected to a set of observation nodes $\{y_m, m \in \mathcal{J}(a)\}$.

The joint maximum a posterior (MAP) probability detection rule for the equation (7) is given by

$$\hat{x}_{\text{MIMO}} = \arg \max_{x_{\text{MIMO}}} \Pr(x_{\text{MIMO}} | y_{\text{MIMO}}, H_{\text{MIMO}})$$

with $\Lambda$ denoting the modulation alphabet applied. Since the complexity of joint MAP detection is too high for implementation in practice, MAP with symbol by symbol is used instead for the detection of $x_a$, $0 \leq a \leq NN_t - 1$. Thus, we can get

$$\hat{x}_a = \arg \max_{\lambda_j \in \Lambda} \frac{1}{|\Lambda|} \Pr(y_{\text{MIMO}} | x_a = \lambda_j, H_{\text{MIMO}})$$

which can be solved by applying the message passing based iterative algorithm under the assumption of equal probability among all the transmitted symbols $\lambda_j \in \Lambda$. And $P_{a,\lambda} = \{p_{a,\lambda}(\lambda_j) | \lambda_j \in \Lambda\}$ represents the probability mass function (pmf) vector passing
from the variable node \( x_a \) to the observation node \( y_b \), while the effective mean \( \mu_{b,a} \) and the variance \( \sigma^2_{b,a} \) are passed from \( y_b \) to \( x_a \). Let \( H[b,a] \) denote the element in the \( b \)-th row and \( a \)-th column of \( \mathbf{H}_{\text{MIMO}} \). Here, we elaborate the implementation of MP detection:

1) **Inputs:** \( y_{\text{MIMO}} , \mathbf{H}_{\text{MIMO}} \)
2) **Initialization:** iteration count \( i = 0 \); pmf \( P^{(0)}_{a,b}(\lambda_j) = \frac{1}{|\mathcal{A}|} \), for \( \forall a \in [0,NN_t-1] \), \( b \in \mathcal{J}(a) \) and \( \lambda_j \in \mathcal{A} \);
3) **Messages passed from** \( y_b \) **to** \( x_a \): for each pair of nodes \( (y_b, x_a) \), \( \forall a \in \mathcal{J}(b) \), \( y_b \) can be decomposed into two parts as follow:

\[
y[b] = x[a]H[b,a] + \sum_{m \in \mathcal{J}(b), m \neq a} x[m]H[b,m] + z[b]
\]

where the interference term \( \zeta^{(i)}_{b,a} \) can be approximately modeled as Gaussian random variable with the effective mean \( \mu^{(i)}_{b,a} \) and the effective variance \( \sigma^2_{b,a} \), which can be calculated as:

\[
\mu^{(i)}_{b,a} = \mathbb{E} \left[ \zeta^{(i)}_{b,a} \right] = \sum_{m \in \mathcal{J}(b), m \neq a} \sum_{j=1}^{[\mathcal{A}]} p_{m,b}^{(i-1)}(\lambda_j) \lambda_j H[b,m]
\]

and

\[
\left( \sigma^2_{b,a} \right)^2 = \sum_{m \in \mathcal{J}(b), m \neq a} \left( \sum_{j=1}^{[\mathcal{A}]} p_{m,b}^{(i-1)}(\lambda_j) |\lambda_j|^2 |H[b,m]|^2 \right) - \sum_{j=1}^{[\mathcal{A}]} p_{m,b}^{(i-1)}(\lambda_j) \lambda_j H[b,m] \bigg|_{\lambda_j = \lambda} \bigg) \bigg) + \sigma^2.
\]

4) **Messages passed from** \( x_a \) **to** \( y_b \): the pmf scalar \( p^{(i)}_{a,b}(\lambda_j) \) can be updated as:

\[
p^{(i)}_{a,b}(\lambda_j) = \Delta \tilde{p}^{(i)}_{a,b}(\lambda_j) + (1-\Delta) p^{(i-1)}_{a,b}(\lambda_j), \quad \lambda_j \in \mathcal{A}
\]

where \( \Delta \in (0,1] \) is the damping factor which affects the convergence speed, and

\[
\tilde{p}^{(i)}_{a,b}(\lambda_j) \propto \prod_{m \in \mathcal{J}(a), m \neq b} \text{Pr} \left( y_m | x_a = \lambda_j, \mathbf{H}_{\text{MIMO}} \right) = \prod_{m \in \mathcal{J}(a), m \neq b} \frac{\xi^{(i)}(m,a,j)}{\xi^{(i)}(m,a,k)}
\]

where \( \xi^{(i)}(m,a,k) = \exp \left( \frac{-|y[m]-\mu^{(i)}_{m,a} - H[m,a]k|^2}{\sigma^2_{m,a}} \right) \).

5) **Stopping criterion:** repeat steps 3 and 4 until the maximum number of iterations \( N_{\text{iter}} \) is reached or the following condition is satisfied:

\[
\max_{a,b,\lambda_j} \left| p^{(i)}_{a,b}(\lambda_j) - p^{(i-1)}_{a,b}(\lambda_j) \right| < \epsilon
\]

where \( \epsilon \) is a small value indicating the convergence quality.

6) **Output:** the final detected symbol is

\[
\hat{x}_a = \arg\max_{\lambda_j \in \mathcal{A}} p_a(\lambda_j), \quad a \in [0,NN_t-1]
\]

where

\[
p_a(\lambda_j) = \prod_{m \in \mathcal{J}(a)} \text{Pr} \left( y_m | x_a = \lambda_j, \mathbf{H}_{\text{MIMO}} \right).
\]

V. SIMULATION RESULTS

We illustrate the superiority of MIMO-AFDM over SISO-AFDM and compare MIMO-AFDM with MIMO-OTFS system in terms of BER performance. The SPA-MG scheme is applied to perform channel estimation and the MP detector is used for signal detection with \( \Delta, N_{\text{iter}} \) and \( \epsilon \) being set to 0.5, 50 and 0.01 respectively. The delay (\( \mu s \)) and doppler (Hz) profile applied in the simulation are \( [0, 2.2, 4.4, 6.6, 8.8] \) and \( [0, 444, 888, 1332, 1776] \) respectively, with the variance of channel coefficient is 1/5. The carrier frequency, subcarrier spacing and AFDM frame size \( N \) are set to 4 GHz, 444 Hz and 1024 respectively, corresponding to a maximum UE speed of 480 Kmph. The OTFS frame size \( N_{\text{OTFS}} = 32 \), \( M_{\text{OTFS}} = 32 \) is adopted to ensure the same resources are occupied \( N_{\text{OTFS}} \times M_{\text{OTFS}} = N \). BPSK signaling is adopted and the pilot signal for signal-to-noise ratio (SNR) is denoted as SNRp.

Figure 3 shows the BER performance comparisons among SISO-AFDM, SISO-OTFS, MIMO-AFDM and MIMO-OTFS systems with ideal CSI. We can observe that the \( 2 \times 2 \) MIMO-AFDM system outperforms SISO-AFDM system significantly due to the space diversity gain, and brings in the advantages of linear increase in spectral efficiency with the number of Tx. Moreover, the \( 2 \times 2 \) MIMO-AFDM system establishes the identical BER with the \( 2 \times 2 \) MIMO-OTFS system. However, by considering the lower overhead of MIMO-AFDM system when conducting the pilot aided channel.
estimation due to its one-dimensional structure, MIMO-AFDM will enjoy higher spectral efficiency.

![Fig. 4. BER versus SNR of 2 x 2 MIMO-AFDM system applying SPA-MG scheme with different SNRp.](image)

We next investigate the effect of different SNRp on the BER performance of MIMO-AFDM system applying the SPA-MG scheme. We can observe from Figure 4 that the BER performance enhances as SNRp increases, since the larger energy of the pilots used, the more accurate CSI can be obtained by SPA-MG. We can also notice that when SNRp is at a low level, increasing the SNRp by 5 dB will enhance the BER performance greatly. Moreover, when SNRp reaches 30 dB, the BER of MIMO-AFDM system using the estimated CSI shows only marginal degradation compared to that with ideal CSI, which validates the effectiveness of the SPA-MG scheme.

Figure 5 presents the effect of different SNRp on the number of iteration when performing the MP detection. The number of iteration of all the cases decreases as the SNR increases, because the interference from the noise become smaller during the iteration in the equation \( \mu_{b,a}^{(i)} \). Furthermore, as the SNRp increases, the number of iteration decreases since the effective mean \( \mu_{b,a}^{(i)} \), the effective variance \( \sigma_{b,a}^{(i)} \) and the pmf vector \( \mathbf{P}_{a,b} \) can be calculated more precisely with a more accurate \( \mathbf{H}_{MIMO} \) in each iteration, which results in a higher speed of convergence.

VI. CONCLUSION

In this paper, a comprehensive introduction to the MIMO-AFDM system is presented. We derive the vectorized formulation of the input-output relationship and propose a novel channel estimation scheme in the DAFT domain to reduce the overhead to the greatest extent. We also exploit the message passing algorithm for signal detection by taking advantage of the sparse characteristic of its effective channel matrix. Simulation results verify that the proposed SPA-MG scheme can provide precise CSI for signal detection, and MIMO-AFDM has the BER performance similar to MIMO-OTFS with MP detector in doubly selective channels.

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