Abstract. Through the last decades, edge detection algorithms have obtained a great degree of sophistication, not being the same with the tools that evaluate their performance. The selection of the best possible edge map output for a given image in an unsupervised way, without prior knowledge of the real edge structure, is still an image processing open problem.

In this work we define a method to evaluate the performance of Edge detection (ED) techniques without any knowledge of the true edge structure, besides the ED output. The method studies the quality of an edge map through a new statistical complexity measure that searches for the balance between the edge equilibrium and the edge information estimated from the ED image. In the ED context, edge equilibrium refers to perfect shape made with very few edge points, while edge information refers to image structure made with many edge points. In order to measure edge equilibrium, a cosine based similarity index is made projecting the image into a family of edge patterns that score the continuity and width of edges in fixed size windows of the ED image. The information is measured by an edge map entropy function based on the Kolmogorov Smirnov test statistic. The statistical complexity measure is thus defined as the product of the similarity index that measures equilibrium and the entropy that measure information.

Our experiments made over selected images of the South Florida and Berkeley databases show that the new statistical complexity measure is able to score meaningfully different man made Ground Truths, and to select the best edge map from a large set of outputs of six different edge detectors, Canny, Sobel, Prewitt, Roberts, Laplacian of Gaussian, and Zero cross, compared with supervised selections made with Pratt’s Figure of merit (FOM) measure.

1. Introduction

In most image processing techniques, the detection and handling of the edge structure of the input image is very important. From object detection to image transmission, the quality of the edge manipulation takes a big part in the success of the operation. Nevertheless, there is no universal definition of the notion of edge. In [AP79], an edge is defined as a local change in luminance or discontinuity in the luminance intensity of the image, while in [KR81] it is pointed out that the edge concept depends of the type of processing and analysis in which it is involved.

Key words and phrases. image processing, discriminating measures, edge detection.

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Therefore, many researchers have designed Edge Detection algorithms which are nearly optimum related to some property or other of the edge structure, but only a few have studied how to measure the edge strength and quality of general edge maps,\textsuperscript{PP11}. Such measures can be classified by the need of a reference map called Ground Truth (GT) (supervised or unsupervised measures) and the type of score that they output, quantitative or qualitative.

Some well known examples of quantitative supervised measures, called also discrepancy measures, are Pratt Figure of Merit (FOM) \textsuperscript{Pra78}, Kappa index \textsuperscript{Coh60}, and Baddeley error measure \textsuperscript{Bad92}. Without the guide of a Ground Truth, the unsupervised methods that are found in the literature look for characteristics of the input edge map, like coherence, continuity, smoothness and good continuation, \textsuperscript{BB79} \textsuperscript{KR81} \textsuperscript{Zhu96} \textsuperscript{HSSB97}, the empirical bootstrap likelihood of detecting a real edge, \textsuperscript{CMC97}, or an specific pattern identification, \textsuperscript{Ber91}, between others. Bowyer et. al (1999) \textsuperscript{BKD99} already pointed out that these metrics based on qualitative properties of detected edges should be considered as additional secondary metrics, but they should not be the primary measures of edge quality. A more detailed discussion on various edge detectors and edge-detection evaluation methods can be found in \textsuperscript{BKD99} y \textsuperscript{PP11}.

In this paper, we propose a quantitative unsupervised measure that searches for a compromise between two extreme values in the space of edge maps: a map with few edge points in a perfect shape (equilibrium) and many edge points located randomly (Information). To our knowledge, there is no previous work that attempts to define notions of Equilibrium and Information (or Entropy) in the space of possible edge maps. We propose as Equilibrium function a discrepancy measure that combines edge map projections into a family of edge patterns. Also, in order to measure the Information present in the edge map we define a new concept of entropy as a function of the Kolmogorov-Smirnov test statistic. The combination of both measures produces a new statistical complexity measure that is capable to score an edge map output in a way completely different from all other measures in the literature.

Our work is organized in the following manner. In section 2 we introduce a discrepancy measure $q_{GT}$ that will be later the basis of the Equilibrium measure. The measure $q_{GT}$ can be compare with the well known Pratt measure to assess the fairness of the scoring. In the section 3 we introduce the concept of Equilibrium and Entropy, and define the final statistical complexity measure. We show some experiments in section 4 and leave the conclusion and comments for section 5.
2. A COSINE DISCREPANCY MEASURE $q_{GT}$.

Our main goal in this paper is to define a new quality measure to score edge maps without knowledge of the Ground Truth. We will introduce now a discrepancy measure $q_{GT}$ only as an intermediate step in the definition of the final measure $C$.

Let $I$ be an image with size $N \times M$, $b$ a edge map associated with $I$, this is, $b$ is a binary image with the same size than $I$, and $GT$ is its Ground Truth. Figure 1 is an example of such images. A simple measure of discrepancy between $b$ and $GT$ is the cosine of the angle $\beta$ between them, when they are seen as 1-d vectors (concatenating all columns one under another).

\begin{equation}
q_{GT}(b) = \cos(\beta) = \frac{GT^T b}{\|GT\| \|b\|}
\end{equation}

being $\beta$ the angle between $GT$ and $b$, and $\|x\| = \sqrt{x^T x}$.

If there is more than one Ground Truth available, $F = \{GT_1, GT_2, \ldots, GT_n\}$, for a given image, the index is defined as the maximum of all scores

\begin{equation}
q_F(b) = \max_{1 \leq i \leq n} \frac{GT_i^T b}{\|GT_i\| \|b\|}
\end{equation}

The Cauchy Schwartz inequality implies that the index is upper bounded by one, being one only when the map is optimums, ($b \in F$), Since the edge maps and GT’s are binary images, the index is lower bounded by 0, being 0 only in the absence of any similarity ($b$ is orthogonal to $F$).

![Figure 1. (a) Original image , (b) Ground Truth , (c) Edge map made with Sobel edge detector, $q_{GT} = 0.3976$.](image)

3. A STATISTICAL COMPLEXITY MEASURE

In this section we define a statistical complexity measure that evaluates the performance of Edge detection (ED) technique without any knowledge besides the ED output. The measure needs two
complementary indexes, an equilibrium index $q(b)$ and an entropy index $H(b)$ that score over the space of all edge maps. We will follow the general structure of complexity measures describes in [LRNC95], so we define our statistical complexity measure $C$ as

$$C(b) = q(b)H(b)$$

We say that an edge map is well balanced (reached equilibrium) if it is structurally simple. In this sense, the map in panel (d) of Figure 2 is better balanced than the edge map in panel (c) and in turn, the one in (c) is better balanced than the one in (b). In the other hand, we say that a map has more information than another if it characterizes better the discontinuities, textures and shapes of the analyzed image. The overabundance of information produce chaotic (cluttered) edge maps like in (b), and the absence of information produce poor edge maps like in (d). Thus, Equilibrium and Information are two complementary concepts, and the product of them, the complexity, searches for a balance point between them. In order to measure the equilibrium of an edge map, we modified the cosine measure of section (2), replacing the family of GT by a family of edge patterns, that make sure the correct identification and value of the usual local characteristics of edges. In the other hand, the entropy is a concept that measures the amount of information of a system, which is maximized when the system reaches a random state. Thus we quantify the randomness of an edge map with a function based on Kolmogorov-Smirnov (KS) goodness of fit statistics, that measures the statistical distribution of the edge patterns against the uniform distribution.

3.1. Equilibrium measure. Abdou and Pratt [?] introduced in its seminal paper the notion of figure of merit, in order to score edge patterns that are fragmented, offset and smeared related to the ideal edges present the Ground Truth. We want our equilibrium index to do a similar task in the unsupervised case, so we will replace the Ground truth with a family $\mathcal{B}$ of carefully chosen binary edge patterns.
Abusing notation, let \( \mathcal{B} = \{b_1/\|b_1\|, b_2/\|b_2\|, \ldots, b_n/\|b_n\|\} \) be the collection of all the edge patterns transformed into column vectors. Sliding a window over the edge map \( b \), centered in each pixel position \( k \), we can extract the edge sub-maps \( b_{(k)} \), and measure the local correlation of the edge map \( b \) respect to the family \( \mathcal{B} \), with:

\[
q_{\mathcal{B}}(b_{(k)}) = \max_{1 \leq j \leq n} \frac{b_j^T b_{(k)}}{\|b_j\| \|b_{(k)}\|}
\]

The equilibrium of \( b \), respecto to the family of edges \( \mathcal{B} \) is the average of the local measures computed only on edge pixels \( k \):

\[
q_{\mathcal{B}}(b) = \frac{1}{\#E} \sum_{k=1}^{\#E} q_{\mathcal{B}}(b_{(k)}) = \frac{1}{\#E} \sum_{k=1}^{\#E} \max_{1 \leq j \leq n} \frac{b_j^T b_{(k)}}{\|b_j\| \|b_{(k)}\|}
\]

where \( E \) is the set of all edge pixels in the binary edge map \( b \).

3.1.1. A family of edge patterns. The family \( \mathcal{B} \) of edge patterns could be very general, but in this paper, following the ideas of Kitchen et al. [KR81], we work only with line-like edge patterns, see Figure 3.

![Figure 3. Patrones de 7 × 7 considerados](image)

The line segment is an essential graphic primitive, so it can be used to construct many other objects. Our line patterns are made with an accurate and efficient raster line-generating algorithm made by Bresenham [Bre65]. Bresenham showed that his line algorithms provide the best-fit approximations to the true lines by minimizing the error (distance) to the true primitive. Beginning with ray traces that go though the origin we constructed 140 edge patterns of size 7x7.

In Figure 4 we can observe the value of the measure Equilibrium index (5) on different patterns that appear in an edge map computed from the Block image. Noisy patterns (b)-(e), reach an index value lower than 0.54. The edge pattern (c), (f), (g) and (h) show the performance of the index when the edges are closer to line segments. The maps (h)-(k) plot the behavior of (5) in presence of thick edges. The maximum value is reached in (h), a pattern of a line of width one pixel.
3.2. Information and the Kolmogorov-Smirnov statistic. Whenever we make statistical observations, or design and conduct statistical experiments, we seek information. How much can we infer from a particular set of statistical observations or experiments about the sampled population? Shannon [Sha48] quantified the information provided by an observation proportionally to how improbable it is. Relating this notion with our edge detection problem, if we have three points aligned in an edge map, the probability of having a fourth point next to them is higher than the probability of having a point further away. So, observing a point in place with low probability gives more information than observing a point in an expected place.

We define our notion of information through a new entropy function, that assess the randomness of an edge map through the Kolmogorov-Smirnov (KS) test of goodness of fit. For a given edge map $b$, we select all edge pixels $E$ and map their positions $(i, j)$ to unit square $[0, 1] \times [0, 1]$ with a suitable injective function $\phi$, and test the goodness of fit of such a sample with the uniform $U$ distribution on the unit square.

Let $D$ be Kolmogorov-Smirnov bidimensional statistic defined as by [JPZ97]

$$D(b) = \sup_{(x, y) \in \mathbb{R}^2} |F_b(x, y) - F(x, y)|$$

where $F$ is the cumulative distribution function of an uniform distributed bidimensional vector, and $F_b$ the empirical distribution function of the sample $\phi(E)$ given by

$$F_b(x, y) = \frac{\#\{(i, j) \in E/ i - \frac{1}{2} \leq x, j - \frac{1}{2} \leq y\}}{\#E}$$

We use the efficient algorithm developed in [JPZ97] to compute $D(b)$. The KS statistic takes values between 0 and 1, rejecting the uniform hypothesis for values closer to 1, so we define our entropy
measure $H$ as

$$H(b) = 1 - D(b)$$

3.3. **Statistical Complexity Measure for edge scoring.** We are now ready to define our notion of statistical complexity measure $C$, that will score edge maps with a value between zero and one, balancing the information with the equilibrium scores estimated from the edge map $b$

$$C(b) = q_F(b)H(b).$$

4. **Results and Analysis.**

We will explore the behavior of our measure in two main situations:

1. Selecting the best map $b_{BD}$ from a set of edge maps $S_{BD}$ made with different detectors over the same image

$$b_{BD} = \arg \max_{b \in S_{BD}} C(b)$$

2. Selecting the best map $b_{BP}$ over a set of edge maps $S_{BP}$ made with the same detector, moving the parameters in a wide range.

$$b_{BP} = \arg \max_{b \in S_{BP}} C(b)$$

We will use selected images of the public dataset of the University of South Florida [BKD99]. This image dataset includes two subsets, one of 50 natural images and other with 10 aerial images, all of them with their ground truth edge images. Each of the fifty images of the first subset contains a single object approximately centered in the image, set against a natural background. The second set has images of man made constructions. We will work also with selected images of the Berkeley database [MFTM01], which have several different ground truth edge images available for each image. We will analyze the performance of our measure studying 6 gradient based Edge Detectors, Canny, Prewitt, Sobel, Roberts Laplacian of Gaussian and Zerocross, using the Matlab implementation on the edge function.

We will also compute the well known Pratt’s FOM discrepancy measure to correlate the values with our measure.

$$Pratt = \frac{1}{\max\{\#E_{ED}, \#E_{GT}\}} \cdot \sum_{k \in E_{DE}} \frac{1}{1 + \alpha d^2}$$
4.1. **Canny edge detector.** In our first example, we work with the well known detector Canny. We have computed a set $S_{BP}$ with 100 edge maps moving Canny’s parameters in the following fashion: we have fixed the smoothness parameter $\sigma = \sqrt{2}$, sampled the hysteresis parameter HT (high threshold) 100 times from zero to one, and defined the LT parameter (low threshold) as $LT=0.4*HT$. Over such database of edge maps we have computed the equilibrium measure, the entropy measure and the complexity measure $q_F$, $H$ y $CE$, respectively. As the test images have ground truth available, we have computed our cosine discrepancy measure and Pratt’s FOM measure.

![Figure 5](image)

**Figure 5.** (a) Original image i109, (b) Ground Truth, (c)-(d) Canny’s extreme edge maps with high threshold HT=0.01, 0.99; low threshold LT=0.4*HT; and $\sigma = \sqrt{2}$.

In Figure 5 we have shown in panel (a) an example image from the South Florida Dataset, named 109; in panel (b) the Ground Truth available from such database; in panel (c) Canny’s edge map with HT=0.01, and in panel (d) Canny’s edge map with HT=0.99. The last two edge maps are extreme possibilities in the set $S_{BP}$, compared with the GT, since the first has too high sensitivity, and many texture details transformed in short edges, and the second has very low sensitivity, with almost no edges selected. The other 98 edge maps are comprised in between these two extreme edge maps.

In panel (a) of Figure 6 we show a plot of the equilibrium measure $q_F$, the entropy measure $H$ and the complexity measure $C$, as a function of the HT values. This is a very interesting plot, because the Statistical Complexity measures are a compromise between Equilibrium and Information, and the maximum value of $C$ over the range of parameters sinteties such a compromise. In Panel (b) of Figure 6 we show a plot of Pratt’s FOM measure and our cosine discrepancy measure, over the same parameter’s range.

The edge map selected by

$$b_{BP} = \text{argmax}_{b \in S_{BP}} C(b)$$

(shown in panel (a) of Figure 7), can be compared visually with the edge map selected maximizing the supervised measure Pratt (shown in panel (b) of Figure 7). Also the value of the thresholds can
Figure 6. (a) $q_F$, $H$ and $C$ vs high threshold (HT), (b) Pratt and $q_{GT}$ vs high threshold (HT).

Figure 7. (a)-(b) edge maps obtained with Canny with high threshold $HT=0.11, 0.14$; low threshold $LT=0.4*HT$; and $\sigma = \sqrt{2}$. (c) best edge map selected with $C$, (d) best edge map selected with Pratt’s and $q_{GT}$.

be compared; in this case, the $C$ measure selected a map with threshold $HT = 0.11$ and Pratt a map with threshold $HT = 0.14$.

4.2. Roberts, Prewitt, Laplacian of Gaussian, Canny, Zerocross and Sobel edge detectors. In this example, we consider other classical gradient based detectors, Roberts, Prewitt, Sobel, Laplacian of Gaussian and Zerocross, which are implemented in the function edge in the Image Processing Toolbox of Matlab. We selected the image ”woods” from the South Florida Database and sampled 100 equispaced instances of the parameter space of each detector in order to produce a clutter and white map as extremes. In Figure 8 we show a plot of the $C$ measures related to the set of each detector. The maximum value of each of the $C$ curves have the same order of magnitude.
Figure 8. Complexity measures for all detectors computed over woods image.

| Edge Detector          | Matlab optimal parameters     | $q$  | $H = 1 - D$ | $C$  | Pratt | $q_{GT}$ |
|------------------------|-------------------------------|------|-------------|------|-------|----------|
| Canny                  | $T=[0.076;0.19]; \sigma = \sqrt{2}$ | 0.7472 | 0.9646 | 0.7208 | 0.3061 | 0.2876 |
| Laplacian of Gaussian  | $T=0.0076; \sigma = 2$      | 0.7760 | 0.9201 | 0.7139 | 0.3951 | 0.2654 |
| Prewitt                | $T=0.064$                    | 0.7384 | 0.9262 | 0.6839 | 0.4197 | 0.3787 |
| Roberts                | $T=0.0052$                   | 0.6515 | 0.9368 | 0.6104 | 0.3649 | 0.3348 |
| Sobel                  | $T=0.064$                    | 0.7326 | 0.9293 | 0.6808 | 0.4137 | 0.3775 |
| Zerocross              | $T=0.0076$                   | 0.7760 | 0.9201 | 0.7139 | 0.3951 | 0.2654 |
| GT                     |                               | 0.8288 | 0.7739 | 0.6456 |        |          |

Table 1. Table of $C$ scores over all best ED outputs and the Ground Truth of image Woods.

In Table 1 we show the scores given to the best of each detector’s maps, and the score assigned to the GT, and in Figure 9 we show the maps selected by the measure $C$. We also score the selected ED with $q_{GT}$ and Pratt’s measure using the provided GT. All the measures, the two supervised and our unsupervised one, considered the maps alike, a result that can be visually corroborated.

4.3. Several Ground Truth. Our last example was made with an image from the Berkeley Segmentation Database [MFTM01], a benchmark database for boundary detection algorithms that has 300 images with several hand made segmentations offered as Ground Truth. The level of detail of the different GT segmentations is diverse, and it represents the human opinion about what are the structural edges of the objects in the images.

In Figure 11 we show six different Ground Truth available for the image 86000, Building. The first GT is very detailed, so any supervised measure computed with this GT will give high marks to a cluttered edge map, but a measure computed using GT in panel (GT5) will certainly score as better a map with very few edge points. But which map will receive high marks from a non supervised measure as our complexity measure?
To answer that question, we made different experiments using the set of Ground Truth as ED outputs, also with a 100 Canny’s outputs computed just like our first experiment.

We show the scores each GT with our unsupervised measure, and show such values in Table 2. The best of all GT was the second most detailed one, which describes all the building characteristics without being too cluttered, see Figure 11.

In Figure 22 we can see the $C$ measure computed over 100 realization of Canny’s algorithm with the same parameter range that our first example. We also show plots of our discrepancy measure and Pratt’s measure computed using the most detailed ground truth.

Having several Ground Truth to choose from, we can select the best edge map from the set of 100 Canny’s outputs with Pratt’s measure using different Ground Truth maps. In Figure 11 first row, we can see all GT available. In the second row, in panel (a) we show the original image, and next to it, in panel (b), the best map selected by our measure $C$. In panel (c) we show the optimum
Figure 10. (a) $q_F$, $H$ and $C$ vs high threshold (HT), (b) Pratt and $q_{GT}$ vs high threshold (HT), computed with the most detailed GT.

The edge map selected using Pratt’s measure with the most detailed GT called (GT). That map was also selected with the supervised measure $q_{GT}$ using the set of all ground truth available. In panel (d) we show the edge map selected by $q_{GT}$ using all GT but the first one. In panel (e) we show the edge map using Pratt’s measure with (GT3). Visual inspection tell us that the maps selected by Pratts’s and $C$ are almost identical, when the GT is the most detailed one. But the differences are very striking when Pratt is using GT3 as ground truth; the map selected has lost the structure of the building.

Figure 11. Panel (GT) most detailed Ground Truth, (GT1)-(GT5) Ground Truth from the Berkeley database. Second row, panel (a) Image 86000,(b) best map selected with measure $C$, (c) best map selected by Pratt’s with GT, (d) best map selected by $q_{GT}$ with all GT but the first, (e)best map selected by Pratt’s with GT3.
In this example we should point out three conclusions

- using supervised measures, the degree of details of the GT impacts in the quality of the edge map selected. Pratt’s measure selects a better map using a detailed ground truth than using a less detailed ground truth.
- when using our supervised measure, using a set of ground truth compensates for the lack of details of each GT in the set.
- measure $q_{GT}$ selects edge maps that are as good as the ones selected by Pratt’s measure, and can accommodate for the use of a whole set of ground truth to select the best edge maps moving parameters in a fixed range.
- measure $C$ selects edge maps that are as good as the ones selected by Pratt’s measure when the GT is detailed enough, and selects better maps than Pratt’s when Pratt’s reference it is too sketchy.

5. Conclusions

In this paper we introduce new ideas of edge equilibrium and edge information that lead into the definition of a new statistical complexity measure for scoring binary maps. To measure edge equilibrium, we defined a similarity index projecting the ED image into a family of edge patterns that score the continuity and width of edges in fixed size windows of the ED image. We measured the information with a entropy function based on the Kolmogorov Smirnov test statistic. The statistical complexity measure was thus defined as the product of the similarity index that measures equilibrium and the entropy that measure information.

Our experiments made over selected images of the South Florida and Berkeley databases showed that the new statistical complexity measure is able to score meaningfully different man made Ground Truths, and to select the best edge map from a large set of outputs of six different edge detectors, Canny, Sobel, Prewitt, Roberts, Laplacian of Gaussian, and Zerocross, compared with supervised selections made with Pratt’s Figure of merit (FOM) measure.

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