STUDY ON THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS BASED ON CURVED BEAM ON ELASTIC FOUNDATION THEORY

Haobo Fan¹,², Siyue He*,², Dongping Zhao¹, Xiuling Wang², Hao Wang³, Hao Ding²

1. China Railway Eryuan Engineering Group Co.Ltd, Chengdu 610031, China, hesiyue@chd.edu.cn
2. School of Highway, Chang'an University, Xi'an 710064, China
3. School of Civil & Construction Engineering, Oregon State University, 101 Kearney Hall, Corvallis, OR 97331 USA

ABSTRACT
Tunnel invert is the weak section of curved beam on tunnel foundation, and it is easy to break down. Based on the curved beam theory on elastic foundation, the curved beam model of tunnel invert was established, the displacement equations of tunnel invert under external load were deduced, and the formulas of settlement of tunnel bottom and the pressure of rock mass were presented. By means of the calculating formulas, the distribution laws of settlement of tunnel bottom and pressure of rock mass were obtained when tunnel bottom was strengthened and not strengthened by high pressure jet grouting piles. The final formulas in the paper are precise to predict the settlement of tunnel bottom and pressure of rock mass, so they are of great value for tunnel design and construction.

KEYWORDS
Tunnel invert, Curved beam, Elastic foundation, Displacement equation, Pressure of rock mass

INTRODUCTION
There are many problems in the construction of curved beams, such as embankment culvert, underground pipeline and tunnel lining, and these problems can be used the curved beam model to solve [1-3]. At present, the researches of curved beams are mostly concentrated on the study of the basic theory [4-5]. Pagano et al. [6-7], Chien et al. [8], Shen et al. [9], and Tornabene et al. [10] used the linear elastic theory to study laminated composite plates under the condition of column bending and gave the exact solutions. Lekhnitskii et al. [11] gave the general solutions of anisotropic bending beam under moment load. This method was also adopted to calculate the interlaminar tensile stress for four point bending experiment of curved beam (ASTMD6415/D6415M-06a) [12]. Shenoi et al. [13], Arici et al. [14] established the model for bending behavior of elastically curved beam and obtained the elastic solutions of the anisotropic beam. Higher-order shear deformation theory can accurately calculate in-plane deformation and stress for shell structure with span to thickness ratio than four, but cannot calculate the interlaminar stresses for composite plates and shells [15, 16]. Aköz et al. [17] used the finite element method to analyze circular beam with variable cross section on elastic foundation under arbitrary load. Banan et al. [18] found the general finite element formula for spatial curved beams and arches on elastic foundation. Çalim et al. [19, 20] presented an effective method for analyzing the dynamic performance of straight beams and curved beams on elastic foundation. Wang et al. [21], Öz et al. [22], Chen et al. [23], Malekzadeh et al. [24], and Wei et al. [25] studied the vibration problem of curved beams on elastic foundation. Based on Euler beam theory, Zhong et al. [26], Adineh et al. [27], Cong et al. [28], Duc et al. [29, 30], and Dung et al. [31] studied the nonlinear transient thermal response for FGM beam with unstable heat conduction and revealed the mechanical characteristics of FGM beams on the basis of tension free. From the point of view, the academic researches on the curved beam on elastic foundation have made a wealth of
research results, but there is little research in practical engineering applications.

In tunneling engineering, the tunnel lining is closely contacted with rock mass, the tunnel lining can be regarded as curved beam on elastic foundation. Dai et al. [32] put forward the calculation model of tunnel lining deformation and internal force on the basis of the Qingdao overlapping tunnel on the beam theory on elastic foundation. Based on principle of initial parameter method, Sun et al. [33] established the initial parameter matrix equations for solving the internal force and deformation of concrete lining on curved beam on elastic foundation. Through on-site monitoring of steel arch stress and pressure of rock mass, Wen et al. [34] deduced an analytical formula for internal force of the primary support of tunnel on curved beam on elastic foundation. Based on the beam model on double elastic foundation and influence of uneven settlement of soft soil, Li et al. [35] established a plane numerical model and analyzed the arrange of settlement joint and the longitudinal stress and deformation of the tunnel. In summary, more emphasis focuses on the analysis of the internal force and the uneven settlement in the longitudinal range of the superstructure of the tunnel, while the analysis of the tunnel invert is relatively rare.

In this paper, the curved beam model of the tunnel invert was established and the displacement equations of curved beam on elastic foundation under external load were deduced based on the differential equations of beam on elastic foundation. The displacement equations were used to calculate the settlement of tunnel bottom and the pressure of rock mass when the tunnel foundation of soft loess was strengthened and not strengthened by high pressure jet grouting piles.

DIFFERENTIAL EQUATIONS OF CURVED BEAM ON ELASTIC FOUNDATION

The Winkler Foundation assumes that the settlement at any point on the surface of the foundation is proportional to the pressure on the unit area of the point, this assumption is the foundation for a series of independent spring simulation on rigid base. When a point on the foundation surface is subjected to pressure \( p \), because the springs are independent of each other, only the local settlement \( y \) is generated at this point, and no settlement occurs in other places. The foundation model is also called the local elastic foundation model. The formula is expressed in equation (1).

\[
p = Ky
\]

Where the \( K \) is the coefficient of foundation reaction which represents the pressure intensity required to produce the unit deformation at a point on the foundation (kN/m); \( p \) is the pressure intensity at any point on the foundation (kPa) and \( y \) is the deformation of the foundation at the point of pressure (m).

The local elastic foundation model is still applicable in the derivation of curved beam on elastic foundation. There is a constant cross-section beam on the Winkler Foundation (Figure 1(a)). The inner and outer radius of curved beam is \( r_0 \) and \( r_1 \) respectively, and the thickness of curved beam is \( h = r_1 - r_0 \), the sectional area is \( A \), the sectional inertia moment is \( I \), and the center radius is \( r_i = \frac{(r_1 + r_0)}{2} \). Assuming that there is no friction effect, the resistance of the foundation is proportional to the radial displacement of the curved beam.
We selected an arbitrary element \( r_d \theta \) from the beam as the object of analysis, as shown in Figure 1(b). The \( y(\theta) \) is radial displacement; \( K \) is the coefficient of foundation reaction; the \( M(\theta) \) is the bending moment of initial section; the \( Q(\theta) \) is shear force and the \( N(\theta) \) is axial force. All the forces are positive in the direction of the Figure 1.

According to the force state of the element, as shown in Figure 1(b), the radial, tangential and center moment balance equations of the infinitesimal element are established, as shown in equation (2) to Equation (4).

\[
\begin{align*}
N(\theta)d\theta + Ky(\theta)r_d\theta + dQ(\theta) &= 0 \quad (2) \\
Q(\theta)d\theta - dN(\theta) &= 0 \quad (3) \\
dM(\theta) - Q_d\theta &= 0 \quad (4)
\end{align*}
\]

The radial displacement \( y(\theta) \) of curved beam has the following relationship with internal force [34]:

\[
d^2y(\theta) + y(\theta) = \frac{M(\theta)}{EI} + \frac{N(\theta)r_d}{EA} \quad (5)
\]

Where \( E \) is the elastic modulus of curved beam.

To derive \( \theta \) in the Equation (5) once and three times, we can get the following equations:

\[
\begin{align*}
\frac{d^3y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta} &= \frac{r_d^2M(\theta)}{EAd\theta} + \frac{r_dN(\theta)}{EA} \quad (6) \\
\frac{d^4y(\theta)}{d\theta^4} + \frac{d^3y(\theta)}{d\theta^3} &= \frac{r_d^2M(\theta)}{EAd\theta^3} + \frac{r_d^2N(\theta)}{EA} \quad (7)
\end{align*}
\]

Then we can obtain the Equation (8) by add Equation (6) and Equation (7):

\[
\frac{d^5y(\theta)}{d\theta^5} + 2\frac{d^4y(\theta)}{d\theta^4} + \frac{dy(\theta)}{d\theta} = \frac{r_d^2M(\theta)}{EAd\theta^5} + \frac{r_d^2N(\theta)}{EA} + \frac{r_d^2M(\theta)}{EAd\theta^3} + \frac{r_d^2N(\theta)}{EA} \quad (8)
\]

The Equation (9) can be obtained by simultaneous Equation (3), Equation (4) and Equation (6):

\[
Q(\theta) = \left(\frac{r_d^3}{EI} + \frac{r_d}{EA}\right)\left[\frac{d^3y(\theta)}{d\theta^3} + \frac{dy(\theta)}{d\theta}\right] \quad (9)
\]

The Equation (10) can be got by simultaneous Equation (2) and Equation (9):

\[
N(\theta) = -\left(\frac{r_d^3}{EI} + \frac{r_d}{EA}\right)\left[\frac{d^4y(\theta)}{d\theta^4} + \frac{d^2y(\theta)}{d\theta^2}\right] - Kr_dy(\theta) \quad (10)
\]

And the Equation (11) can be obtained by simultaneous Equation (5) and Equation (10):
The differential equations of curved beam on elastic foundation under symmetric load

Section forms of the highway tunnel are generally symmetrical about the vertical axis, and no asymmetric loading tunnel, so it can be assumed that the load of tunnel rock mass is also symmetrical about vertical axis. The section where the tunnel center is located is taken as the initial section, the symmetry of structure and load can be used.

\[ M(\theta) = \frac{EI}{r_i^3} \left\{ \frac{d^2 y(\theta)}{d\theta^2} + y(\theta) + \left( \frac{r_i^2 A}{I} + 1 \right) \left( \frac{d^3 y(\theta)}{d\theta^4} + \frac{d^2 y(\theta)}{d\theta^2} \right) + \frac{r_i}{EA} K_r y(\theta) \right\} \]  \hspace{1cm} (11)

We can get a five order radial displacement \( y(\theta) \) differential equation with constant coefficients based on simultaneous Equation (2), Equation (4) and Equation (8):

\[ \frac{d^5 y(\theta)}{d\theta^5} + 2 \frac{d^4 y(\theta)}{d\theta^4} + n^2 \frac{d^3 y(\theta)}{d\theta^3} = 0 \]  \hspace{1cm} (12)

where

\[ n^2 = 1 + K_r \left( \frac{r_i^2}{EI} + \frac{r_i}{EA} \right) \]

So the general solution of the Equation (12) is:

\[ y(\theta) = C_0 + C_1 \sin \theta + C_2 \cos \theta + C_3 \sin \theta \cos \theta + C_4 \sin \theta \cos \theta \sin \theta \]  \hspace{1cm} (13)

Finally, we can get analytical Equation of internal force of curved beam by simultaneous Equation (2) to Equation (6) and Equation (13):

\begin{align*}
Q(\theta) &= T \left[ C_0 (A \sin \theta \cos \theta + A_1 \sin \theta \cos \theta) + C_1 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) + C_2 (A \sin \theta \cos \theta - A_1 \sin \theta \cos \theta) + C_3 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) \right] \\
N(\theta) &= -K_r C_0 - C_1 (2 \alpha \beta T \sin \theta \cos \theta) - C_2 (2 \alpha \beta T \sin \theta \cos \theta) + C_3 (2 \alpha \beta T \sin \theta \cos \theta) + C_4 (2 \alpha \beta T \sin \theta \cos \theta) \\
M(\theta) &= [(K_r B_1 + 1)C_0 + C_1 (2 \alpha \beta T \sin \theta \cos \theta) \sin \theta \cos \theta + C_2 (2 \alpha \beta T \sin \theta \cos \theta) \sin \theta \cos \theta - C_3 (2 \alpha \beta T \sin \theta \cos \theta) \sin \theta \cos \theta - C_4 (2 \alpha \beta T \sin \theta \cos \theta) \sin \theta \cos \theta] / B_2
\end{align*}

Where \( A_1 = -2 \alpha \beta \); \( A_2 = -2 \alpha^2 \beta \); \( B_1 = r_i / (EA) \); \( B_2 = r_i^2 / (EI) \); \( T = \left( \frac{r_i^2}{EI} + \frac{r_i}{EA} \right) \)

\[ (14) \]

\[ (15) \]

\[ (16) \]

The differential equations of curved beam on elastic foundation under symmetric load

Section forms of the highway tunnel are generally symmetrical about the vertical axis, and no asymmetric loading tunnel, so it can be assumed that the load of tunnel rock mass is also symmetrical about vertical axis. The section where the tunnel center is located is taken as the initial section, the symmetry of structure and load can be used.

\[ \left. \left[ \frac{dy(\theta)}{d\theta} \right] \right|_{\theta=0} = 0 \]

\[ \left. \left[ Q(\theta) \right] \right|_{\theta=0} = 0 \]

That is:

\[ \alpha C_2 + \beta C_3 = 0 \]

\[ \beta C_2 - \alpha C_3 = 0 \]

So we can get \( C_2 = C_3 = 0 \). Then, Equation (13) to Equation (16) can be simplified as follows:

\[ y(\theta) = C_0 + C_1 \sin \theta + C_2 \cos \theta + C_3 \sin \theta \cos \theta + C_4 \sin \theta \cos \theta \sin \theta \]  \hspace{1cm} (17)

\[ -Q_0 = T \left[ C_1 (A \sin \theta \cos \theta + A_1 \sin \theta \cos \theta) + C_2 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) + C_3 (A \sin \theta \cos \theta - A_1 \sin \theta \cos \theta) + C_4 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) \right] \\
-\dot{Q}_0 = T \left[ C_1 (A \sin \theta \cos \theta + A_1 \sin \theta \cos \theta) + C_2 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) + C_3 (A \sin \theta \cos \theta - A_1 \sin \theta \cos \theta) + C_4 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) \right] \\
-\ddot{Q}_0 = T \left[ C_1 (A \sin \theta \cos \theta + A_1 \sin \theta \cos \theta) + C_2 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) + C_3 (A \sin \theta \cos \theta - A_1 \sin \theta \cos \theta) + C_4 (A_1 \sin \theta \cos \theta + A_2 \sin \theta \cos \theta) \right] \\
\]  \hspace{1cm} (18) \hspace{1cm} (19) \hspace{1cm} (20)
In Equation (17) ~ (20), the internal force analytical equation of curved beam on elastic foundation can be obtained after the integral constants $C_0$, $C_1$ and $C_4$ are calculated. And the integral constants $C_0$, $C_1$ and $C_4$ can be obtained by the boundary conditions at both ends of the curved beam.

**ESTABLISHMENT OF CURVED BEAM MODEL ON ELASTIC FOUNDATION OF TUNNEL INVERT**

When the tunnel invert was completed, it would be affected by the weight of the lining and the load of rock mass. The settlement of tunnel bottom would produce pressure on the foundation; similarly, the tunnel foundation would also produce the counterforce to the tunnel invert. Therefore, the tunnel invert could be regarded as a curved beam on the elastic foundation. The self-weight of tunnel lining and the pressure of rock mass could be equivalent to the load acting on both ends of tunnel invert, and the boundary conditions at the ends of the tunnel invert could be obtained by the equivalent load. Assuming that the tunnel invert was rigidly connected with the secondary lining of the tunnel, the equivalent load could be simplified as shear $Q_0$, axial force $N_0$ and bending moment $M_0$, as shown in Figure 2.

![Fig. 2 - Simplified calculation model of tunnel invert](image)

Then the boundary condition of the beam end was:

\[
\begin{align*}
Q(\theta_0) &= -Q_0, \\
N(\theta_0) &= -N_0, \\
M(\theta_0) &= -M_0
\end{align*}
\]  

(21)

So we could establish a three-variable linear equation by simultaneous Equation (18) ~ (20), (21), as shown in Equation (22) ~ (24):

\[
\begin{align*}
Q_0 &= T[C_1(A\sin\theta_0\cos\beta\theta_0 + A_2\sin\theta_0\cos\beta\theta_0) + C_4(A\sin\theta_0\cos\beta\theta_0 - A_2\sin\theta_0\cos\beta\theta_0)] \\
N_0 &= -K_T C_0 - C_1(2\alpha\beta\sin\theta_0\sin\beta\theta_0) + C_4(2\alpha\beta\sin\theta_0\sin\beta\theta_0) \\
M_0 &= [(K_T - 1)C_0 + C_1(2\alpha\beta)(B_T - 1)\sin\theta_0\sin\beta\theta_0 - C_4(2\alpha\beta)(B_T - 1)\sin\theta_0\sin\beta\theta_0]/B_z
\end{align*}
\]  

(22)  

(23)  

(24)

The pressure of rock mass of tunnel invert could be calculated according to literature (JTGD70-2004) [36]. The internal force of the corner of tunnel invert could be obtained according to the elastic center method in Structural Mechanics. Then the shear force $Q_0$, axial force $N_0$ and bending moment $M_0$ acting on both ends of tunnel invert could be obtained.

**CALCULATION EXAMPLES OF THE SETTLEMENT OF TUNNEL BOTTOM AND PRESSURE OF ROCK MASS IN THE DAYOUSHAN TUNNEL**

The foundation reinforcement section of the Dayoushan tunnel was selected as the typical section, and the above theoretical formulas were tested by the engineering case. Firstly, the settlement of the tunnel bottom and pressure of rock mass were calculated when the foundation was not strengthened, and the law of the force and settlement of the tunnel bottom was explored. Then
the settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting piles were calculated.

The composite lining was adopted in the Dayoushan tunnel, the excavation radius was 6.41m, and the excavation was constructed by bench cut method. The primary support of shallow section was sprayed with C25 concrete, and its thickness was 26cm; the secondary lining used C25 reinforced concrete, and its thickness was 50cm. The analysis of unit length of tunnel invert in shallow section was carried out. The geometric parameters and calculation parameters of tunnel invert were shown in Table 1 and Table 2 respectively.

| Thickness $h$ (m) | External radius $r_1$ (m) | Inner radius $r_0$ (m) | Central angle $\theta_0$ (°) | Cross-sectional area $A$ (m$^2$) |
|-------------------|--------------------------|------------------------|-----------------------------|-------------------------------|
| 50                | 12.561                   | 12.061                 | 26.9                        | 5.87                          |

**Tab. 1 - Geometric parameters of tunnel invert**

**Tab. 2 - Calculation parameters of tunnel invert**

| Parameters                                      | Value            | Material                                |
|------------------------------------------------|------------------|-----------------------------------------|
| Elastic modulus of concrete $E_c$ (GPa)         | 29.5             | C25 concrete                            |
| Elastic modulus of reinforcement $E_s$ (GPa)    | 200              | Φ8 double layer steel mesh              |
| Sectional area of tunnel invert steel bar (m$^2$) | 5.03×10$^{-5}$   | Φ8 double layer steel mesh              |
| Coefficient of foundation reaction $K_1$ (kN/m$^3$) | 8000             | Soft foundation                         |
| Coefficient of foundation reaction $K_2$ (kN/m$^3$) | 60000            | Foundation reinforcement by high pressure jet grouting pile |

The composite elastic modulus of reinforced concrete could be calculated according to formula (25):

$$EA = E_c A_c + E_s A_s$$  \hspace{1cm} (25)

Where $E_c$ and $E_s$ is the elastic modulus of concrete and steel bars of curved beam on foundation reinforcement, respectively; $A_c$ and $A_s$ is the sectional area of concrete and steel bars.

$$E = (E_c A_c + E_s A_s) / A = 29.54 \text{ (GPa)}$$

The section ZK3+160 of Dayoushan tunnel was selected for the study. The depth of this section was 30m, and the rock mass was 13m collapsible loess and 17m non collapsible loess. The lateral pressure coefficient was calculated according to the lateral pressure and vertical pressure obtained from the previous monitoring, and $\lambda = 0.5$. The pressure of rock mass of shallow tunnel was calculated according to the literature (JTGD70-2004) [36], the vertical pressure $q = 348 kN/m$ and horizontal lateral...
pressure $e = 174 \text{kN/m}$ were calculated, after the load was reduced by 60% (The pressure of rock mass acting on secondary lining accounted for about 60%), so $q_1 = 208 \text{kN/m}$ and $e_1 = 104 \text{kN/m}$. The internal force at the corner of tunnel invert could be obtained according to the elastic center method in Structural Mechanics. The shear force $Q_1 = 342\text{kN}$, the axial force $N_1 = 1233\text{kN}$ and the bending moment $M_0 = 591\text{kN}\cdot\text{m}$ were obtained.

Considering the influence of the weight of secondary lining, the cross section area of secondary lining was $11.52\text{m}^2$, and the bulk density was $25\text{kN/m}^3$. So the weight of unit length of the secondary lining was $281.25\text{kN}$. The force produced by the weight of secondary lining acting on the tunnel invert could be calculated by the following formulas:

- Shear force: $Q_2 = (281.25/2) \times \cos \theta_0 = 125 \text{(kN)}$
- Axial force: $N_2 = (281.25/2) \times \sin \theta_0 = 63 \text{(kN)}$

Therefore, the total force acting on both ends of the tunnel invert were:

- Shear force: $Q_0 = Q_1 + Q_2 = 467 \text{(kN)}$
- Axial force: $N_0 = N_1 + N_2 = 1296 \text{(kN)}$
- Bending moment: $M_0 = 591 \text{(kN}\cdot\text{m)}$

**Settlement of tunnel bottom and pressure of rock mass when tunnel foundation was not strengthened**

When the tunnel foundation was not reinforced, the tunnel foundation was soft soil, and the foundation stiffness was small, and the coefficient of foundation reaction was selected according to the empirical value [1], $K_{1} = 8000 \text{kN/m}^3$. The MATLAB program was compiled to solve the Equations (22) ~ (24), and the integral constant $C_0, C_1$ and $C_4$ in differential equations could be obtained.

- $C_0 = 12.39 \times 10^{-3}$
- $C_1 = 4.16 \times 10^{-3}$
- $C_4 = 7.95 \times 10^{-3}$

We could get the Equation (26) ~ (29) when we substitute $C_0, C_1$ and $C_4$ into equation (17) ~ (20):

\[
y(\theta) = [12.39 + 4.16\sigma\alpha\theta\cos\beta\theta + 7.95\sigma\alpha\theta\sin\beta\theta] \times 10^{-3}
\]

\[
Q(\theta) = T[4.16(A_{c}\sigma\alpha\theta\cos\beta\theta + A_{c}\sigma\alpha\theta\sin\beta\theta) + 7.95(A_{c}\sigma\alpha\theta\sin\beta\theta - A_{c}\sigma\alpha\theta\cos\beta\theta)] \times 10^{-3}
\]

\[
N(\theta) = [-12.39K_{1} - 4.16(2\alpha\beta T\sigma\alpha\theta\sin\beta\theta) + 7.95(2\alpha\beta T\sigma\alpha\theta\cos\beta\theta)] \times 10^{-3}
\]

\[
M(\theta) = [12.39(K_{1}B_{1} + 1) + 4.16(2\alpha\beta)(B_{1}T - 1)\sigma\alpha\theta\sin\beta\theta - 7.95(2\alpha\beta)(B_{1}T - 1)\sigma\alpha\theta\cos\beta\theta)] \times 10^{-3} / B_{1}
\]

From Equations (26) to (29), the deflection, shear force, axial force and bending moment of any section of the tunnel invert can be obtained.

**Internal force of tunnel invert**

We calculated the internal force of any section of tunnel invert by Equation (27) ~ (29). Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagrams were shown in Figure 3~ Figure 5.
As shown in Figure 3 to 5, the center and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress. However, the outer sides bore tension stress and the inner sides bore compressive stress at the corner of tunnel invert. The internal force at the corner of tunnel invert was greater, and there was stress concentration phenomenon.

The axial force of the tunnel invert was relatively large, and the maximum axial force was located at the corner of tunnel invert, which was $-1282kN$, and the direction was pointing to the inside of tunnel. The shear force of the tunnel invert was relatively small, and the maximum shear force was located at the corner of tunnel invert, which was $-410kN$. The bending moment was relatively large, and the positive moment $425kN\cdot m$ was located at the center of the tunnel invert. The force form was not conducive to the stability of the tunnel invert, and the inside of the tunnel invert was easy to be pulled to crack.
Settlement of tunnel bottom

The radial displacement of any section of tunnel invert could be calculated by equation (26), the vertical displacement of different sections of tunnel invert could be obtained by formula $S_i = y_i \cos \theta_i$, the calculation results were shown in Table 3.

| The angle between the calculation section and central line $\theta_i (°)$ | Horizontal distance between calculation section and tunnel invert center $L \ (m)$ | Radial displacement $y \ (mm)$ | Settlement of tunnel bottom $S \ (mm)$ |
|---|---|---|---|
| 0  | 0  | 16.6 | 16.6 |
| 5  | 1.1 | 17.3 | 17.2 |
| 10 | 2.1 | 19.4 | 19.1 |
| 15 | 3.2 | 22.5 | 21.8 |
| 20 | 4.2 | 26.2 | 24.6 |
| 25 | 5.2 | 29.4 | 26.7 |

As shown in Table 3, the settlement of tunnel bottom was generally large when the tunnel foundation of soft loess was not reinforced. The settlement reached 26.7mm at the corner of tunnel bottom, and the settlement at the center of the tunnel bottom reached 16.6mm. The maximum differential settlement within the cross-section of tunnel bottom was as follows:

$$\Delta S_{\max} = 26.7 - 16.6 = 10.1 \ (mm)$$

The horizontal distance between the center and corner point of tunnel bottom was 5.2m, therefore, the differential settlement of unit length was as follows:

$$\frac{\Delta S_{\max}}{L} = 1.94 \ (mm/m)$$

There is a big difference between the center and corner point of tunnel bottom, so the tunnel invert was easy to crack. According to the distribution of the bending moment of the tunnel invert, the risk of cracking at the center of tunnel invert was the biggest.

Pressure of rock mass of the tunnel bottom

After calculating the radial displacement $y$ of the tunnel invert, the elastic resistance of the tunnel foundation to the tunnel invert could be obtained by the formula (1). The normal elastic resistance was approximately equal to the pressure of rock mass of the tunnel bottom, assuming that the frictional force was not taken into account. The calculation results of pressure of rock mass were shown in Figure 6.
As shown in Figure 6, rock mass at the tunnel bottom was mainly subjected to compressive stress. The compressive stress at the center of tunnel invert center was the minimum, 132kPa. The pressure of rock mass increased uniformly from the center to the corner of tunnel invert, and the maximum compressive stress was reached 235kPa at the corner of tunnel invert. The stress concentration was easy to occur due to the structural reasons at the corner of tunnel invert. The weight of secondary lining and the pressure of rock mass acted at the corner of tunnel invert, and the corner of tunnel invert bore the large load. There should be smooth transition between side wall and the corner of tunnel invert, and the section size at the corner of tunnel invert could be increased appropriately to improve the force situation in the process of tunnel construction.

Settlement and pressure of rock mass of foundation reinforcement by high pressure jet grouting piles

When the tunnel foundation was reinforced by high pressure jet grouting piles, the bearing capacity and the stiffness of the tunnel foundation were greatly improved. The coefficient of foundation reaction was determined by field loading test, \( K_2 = 60000 \text{ kN/m}^3 \). The MATLAB program was compiled to solve the equations (22) ~ (24), and the integral constant \( C_0 \), \( C_1 \) and \( C_4 \) in differential equations could be obtained.

\[
C_0 = 16.6 \times 10^{-4}, \quad C_1 = -1.4 \times 10^{-4}, \quad C_4 = 6.8 \times 10^{-4}
\]

We can get the equation (30) ~ (33) when substitute \( C_0 \), \( C_1 \) and \( C_4 \) into equation (17) ~ (20):

\[
y(\theta) = [16.6 - 1.4 \theta \cos \beta \theta + 6.8 \theta \sin \beta \theta] \times 10^{-4} \quad (30)
\]

\[
Q(\theta) = [16.6K \theta + 1.4(2\alpha \theta \cos \beta \theta + 6.8(2\alpha \theta \sin \beta \theta - 6.8(2\alpha \theta \theta \cos \beta \theta))] \times 10^{-4} \quad (31)
\]

\[
N(\theta) = [-16.6K \theta + 1.4(2\alpha \theta \cos \beta \theta + 6.8(2\alpha \theta \theta \cos \beta \theta))] \times 10^{-4} \quad (32)
\]

\[
M(\theta) = [16.6K \theta + 1.4(2\alpha \theta \cos \beta \theta - 6.8(2\alpha \theta \theta \cos \beta \theta))] \times 10^{-4} / B_2 \quad (33)
\]

From equations (30) ~ (33), the deflection, shear force, axial force and bending moment of any section of the tunnel invert could be obtained.

Internal force of tunnel invert

We calculated the internal force of any section of tunnel invert by equation (31) ~ (33). Considering the symmetry of tunnel invert, half of the tunnel structure was calculated. The internal force diagram was shown in Figure 7 ~ Figure 9.

![Fig.7 - The shear force of tunnel invert (kN)](image-url)
As shown in Figure 7~ Figure 9, after the reinforcement of the tunnel foundation, the internal force of the tunnel invert was similar to that without reinforcement. The inside at the center and the waist of tunnel bottom bore the tension stress, and the lateral bore compressive stress. The outer side at the corner of tunnel invert bore tension stress, and the inner side bore compressive stress. However, the stress condition of tunnel invert had been improved. The axial force at the center the tunnel invert increased slightly and the shear force and bending moment decreased. The maximum positive bending moment at the center of the tunnel invert decreased to 98kN·m, which greatly improved the stress state of the tunnel invert.

**Settlement of tunnel bottom**
The radial displacement of any section of tunnel invert could be calculated by equation (30), the vertical displacement of different sections of tunnel invert could be obtained by formula $S_y = y \cdot \cos \theta$, the calculation results were shown in Table 4.

| The angle between the calculation section and central line $\theta$ (°) | Horizontal distance between calculation section and tunnel invert center $L$ (m) | Radial displacement $y$ (mm) | Settlement of tunnel invert $S$ (mm) |
|---|---|---|---|
| 0 | 0 | 1.5 | 1.5 |
| 5 | 1.1 | 1.7 | 1.7 |
| 10 | 2.1 | 2.2 | 2.2 |
| 15 | 3.2 | 3.1 | 3.0 |
| 20 | 4.2 | 4.1 | 3.9 |
| 25 | 5.2 | 4.7 | 4.3 |

Fig. 8 - The axial force of tunnel invert (kN)

Fig. 9 - The bending moment of tunnel invert (kN.m)
As shown in Table 3 and Table 4, the settlement of the tunnel bottom was greatly reduced when the tunnel foundation was reinforced by jet grouting piles. The settlement at the corner of tunnel invert decreased to 4.3mm, and the differential settlement of unit length was as follows:

$$\Delta S_{\text{max}} = \frac{\Delta S_{\text{max}}}{L} = 0.54\,(\text{mm/m})$$

The settlement of the tunnel invert was smaller and the overall settlement was more harmonious, which met the requirements of the standard. Thus, the bearing capacity and stiffness of tunnel foundation were greatly improved after the jet grouting piles was used to reinforce the tunnel foundation, and the reinforcement effect was good.

**Pressure of rock mass of the tunnel bottom**

The calculation results of pressure of rock mass were shown in Figure 10.

As shown in Figure 10, the pressure of rock mass at tunnel bottom was similar to that without reinforcement, the compressive stress at the center of tunnel invert was the smallest, and the maximum compressive stress at the corner of tunnel invert, after foundation reinforcement, the compressive stress at the corner of tunnel invert increased more obviously, and increased from 235kPa to 282kPa. The stress concentration phenomenon at the corner of tunnel invert was more obvious, which was related to the substantial increase of the bearing capacity of the tunnel foundation after the foundation reinforcement.

**CONCLUSION**

(1) When the tunnel foundation was not strengthened, the inside at the center and the waist of tunnel invert bore the tension stress, and the lateral bore compressive stress; the outside at the foot of tunnel invert bore tension stress and the inside bore compressive stress; the internal force at the foot of tunnel invert was greater, the axial force and bending moment of the tunnel invert were relatively large, and the greater positive moment was the inside at the center of the tunnel invert. The maximum positive bending moment at the center of the tunnel invert decreased greatly after the reinforcement of the tunnel foundation.

(2) When the tunnel foundation was not strengthened, the settlement of tunnel bottom was generally large; the settlement at the corner of tunnel invert reached 26.7mm, and the maximum differential settlement within the range of tunnel invert reached 10.1mm; the risk of tension crack at the center of tunnel invert was larger. After tunnel foundation was strengthened, the settlement of tunnel bottom decreased greatly, and the overall settlement was more coordinated, and the reinforcement effect was good.

(3) When the tunnel foundation was not strengthened, theoretical calculation results showed that the rock mass at tunnel bottom was mainly subjected to compressive stress; the compressive stress at the center of the tunnel invert was minimum, and it was large at the corner of tunnel invert. After the tunnel foundation was strengthened, the pressure of rock mass at tunnel bottom was similar to
that without reinforcement; the compressive stress at the corner of tunnel invert increased obviously and the stress concentration phenomenon was more obvious.

COMPETING INTERESTS
The authors declare that there is no conflict of interests regarding the publication of this paper.

ACKNOWLEDGEMENTS
This work is financially supported by the Brainstorm Project on Social Development of Shaanxi Provincial Science and Technology Department (No. 2016SF-412) and the Special Fund for Basic Scientific Research of Central Colleges of Chang'an University (No. 310821172004, No. 310821153312, No. 310821165011) and the National Key R&D problem of China (No. 2017YFC0805306). We also acknowledge the editor for the valuable suggestion.

REFERENCES
[1] S. Chakraborty, S. K. Sarkar, “Analysis of a curved beam on uncertain elastic foundation”, *Finite Elements in Analysis and Design*, vol. 36, no. 1, pp. 73-82, 2000.
[2] A. M. Talmon, A. Bezuijen, “Analytical model for the beam action of a tunnel lining during construction”, *International Journal for Numerical & Analytical Methods in Geomechanics*, vol. 37, no. 2, pp. 181-200, 2013.
[3] L. Zhang, M. H. Zhao, C. J. Shi, H. Zhao, “Settlement Calculation of Composite Foundation Reinforced with Stone Columns”, *International Journal of Geomechanics*, vol. 13, no. 3, pp. 248-256, 2013.
[4] W. Wang, R. A. Shenoi, “Delamination modelling of a curved composite beam subjected to an opening bending moment”, *Journal of Strain Analysis for Engineering Design*, vol. 38, no. 5, pp. 453-457, 2003.
[5] E. L. Tan, B. Uy, “Experimental study on straight composite beams subjected to combined flexure and torsion”, *Journal of Constructional Steel Research*, vol. 65, no. 8, pp. 1855-1863, 2009.
[6] N. J. Pagano, “Analysis of The Flexure Test of Bidirectional Composites”, *Journal of Composite Materials*, vol. 1, no. 4, pp. 336-342, 1967.
[7] N. J. Pagano NJ, “Exact solution for composite laminates in cylindrical bending”, *Solid Mechanics & Its Applications*, vol. 3, no. 3, pp. 398-411, 1969.
[8] R. D. Chien, C. S. Chen, “Nonlinear vibration of laminated plates on a nonlinear elastic foundation”, *Composite Structures*, vol. 70, no. 1, pp. 606-615, 2005.
[9] H. S. Shen, F. Lin, Y. Xiang, “Nonlinear vibration of functionally graded graphene-reinforced composite laminated beams resting on elastic foundations in thermal environments”, *Nonlinear Dynamics*, vol. 90, no. 2, pp. 899-914, 2017.
[10] F. Tornabene, N. Fantuzzi, M. Bacciocchi, J. N. Reddy, “A posteriori stress and strain recovery procedure for the static analysis of laminated shells resting on nonlinear elastic foundation”, *Composites Part B-Engineering*, vol. 126, pp. 162-191, 2017.
[11] S. G. Lekhnitskii, “Bending of a curved rod by moments and a force applied at the ends, theory of elasticity of an anisotropic body”, *Moscow: Mir. Pub., 1981.*
[12] ASTM D6415/D6415M-06a, “Standard test method for measuring the curved beam strength of a fiber-reinforced polymer matrix composite”, *West Conshohocken: American Society for Testing and Materials*, 2013.
[13] R. A. Shenoi, W. Wang, “Flexural behaviour of a curved orthotropic beam on an elastic foundation”, *Journal of Strain Analysis for Engineering Design*, vol. 36, no. 1, pp. 1-15, 2001.
[14] M. Arici, M. F. Granata, “Generalized curved beam on elastic foundation solved by transfer matrix method”, *Structural Engineering and Mechanics*, vol. 40, no. 2, pp. 279-295, 2011.
[15] K. H. Lo, R. M. Christensen, E. M. Wu, “Stress solution determination for high order plate theory”, *International Journal of Solids & Structures*, vol. 14, no. 8, pp. 655-662, 1978.
[16] J. N. Reddy, “A Simple Higher-Order Theory for Laminated Composite Plates”, *Journal of Applied Mechanics*, vol. 51, no. 4, pp. 745-752, 1984.
[17] A. Y. Aköz, F. Kadioğlu, “The mixed finite element solution of circular beam on elastic foundation”, *Computers & Structures*, vol. 60, no. 4, pp. 643-651, 1996.
[18] M. R. Banan, G. Karami, M. Farshad, “Finite element analysis of curved beams on elastic foundations”, *Computers & Structures*, vol. 32, no. 1, pp. 45-53, 1989.
[19] F. F. Çalış, F. G. Akkurt, “Static and free vibration analysis of straight and circular beams on elastic foundation”, Mechanics Research Communications, vol. 38, no. 2, pp. 89-94, 2011.
[20] F. F. Çalış, “Forced vibration of curved beams on two-parameter elastic foundation”, Applied Mathematical Modelling, vol. 36, no. 3, pp. 964-973, 2012.
[21] T. M. Wang, W. F. Brannen, “Natural frequencies for out-of-plane vibrations of curved beams on elastic foundations”, Journal of Sound & Vibration, vol. 84, no. 2, pp. 241-246, 1982.
[22] H.R.Öz, M. Pakdemirli, E. Özkaya, “Non-linear vibrations of a slightly curved beam resting on a non-linear elastic foundation”, Journal of Sound & Vibration, vol. 212, no. 2, pp. 295-309, 1998.
[23] C. S. Chen, A. H. Tan, R. D. Chien, “Non-linear oscillations of orthotropic plates on a non-linear elastic foundation”, Journal of Reinforced Plastics and Composites, vol. 28, no. 7, pp. 851-867, 2009.
[24] P. Malekzadeh, M. R. G. Haghighi, M. M. Atashi, “Out-of-plane free vibration of functionally graded circular curved beams in thermal environment”, International Journal of Applied Mechanics, vol. 2, no. 3, pp. 635-652, 2010.
[25] C. L. Wei, C. S. Chen, C. S. Shih, Y. C. Chang, Nonlinear vibration of initially stressed hybrid composite plates on elastic foundations, Mechanics of Composite Materials, vol. 48, no. 4, pp. 467-482, 2012.
[26] J. Zhong, Y. Fu, Y. Chen, Y. Li, “Analysis of nonlinear dynamic responses for functionally graded beams resting on tensionless elastic foundation under thermal shock”, Composite Structures, vol. 142, pp. 272-277, 2016.
[27] M. Adineh, M. Kadkhodayan, “Three-dimensional thermo-elastic analysis and dynamic response of a multi-directional functionally graded skew plate on elastic foundation”, Composites Part B-Engineering, vol. 125, pp. 227-240, 2017.
[28] P. H. Cong, P. T. N. An, N. D. Duc, “Nonlinear stability of shear deformable eccentrically stiffened functionally graded plates on elastic foundations with temperature-dependent properties”, Science and Engineering of Composite Materials, vol. 24, no. 3, pp. 455-469, 2017.
[29] N. D. Duc, P. D. Nguyen, N. D. Khoa, “Nonlinear dynamic analysis and vibration of eccentrically stiffened S-FGM elliptical cylindrical shells surrounded on elastic foundations in thermal environments”, Thin-walled structures, vol. 117, pp. 178-189, 2017.
[30] N. D. Duc, V. D. Quang, V. T. T. Anh, “The nonlinear dynamic and vibration of the S-FGM shallow spherical shells resting on an elastic foundations including temperature effects”, International Journal of Mechanical Sciences, vol. 123, pp. 54-63, 2017.
[31] D. V. Dung, P. M. Vuong, “Analytical investigation on buckling and postbuckling of FGM toroidal shell segment surrounded by elastic foundation in thermal environment and under external pressure using TSDT”, Acta Mechanica, vol. 228, no. 10, pp. 3511-3531, 2017.
[32] C. Q. Dai, L. Wang, Y. J. Wang, “Deformation study of Lining on Overlap Tunnels Based on Elastic Foundation Beam”, Applied Mechanics & Materials, vol. 94, no. 10, pp. 1875-1878, 2011.
[33] F. X. Sun, X. H. Cai, Y. H. Zhu, “Analytical solution of internal force and displacement in multi-center circular arc tunnel lining based on initial parameter method”, Rock & Soil Mechanics, vol. 30, no. 4, pp. 1127-1130, 2009. (in Chinese)
[34] J. Z. Wen, Y. X. Zhang, C. Wang, “Back analysis of internal force of initial support in tunnel based on touch stress”, Rock & Soil Mechanics, vol. 32, no. 8, pp. 2467-2472, 2011. (in Chinese)
[35] Z. Li, S. Y. Chen, “Calculation and analysis of longitudinal settlement joint of shield tunnel based on dual elastic foundation beam”, World Automation Congress. IEEE, pp. 1-6, 2012.
[36] Code for Design for Highway Tunnel. JTGD70-2004. (in Chinese)