Helicity asymmetries in double pion photoproduction on the proton

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Abstract

Based on a prior model on double pion photoproduction on the proton, successfully tested in total cross sections and invariant mass distributions, we make a theoretical study of the angular dependence of helicity asymmetries from the interaction of circularly polarized photons with unpolarized protons. We show that this observable is sensitive to details of the internal mechanisms and, thus, represents a complementary test of the theoretical model.

1 Introduction

The photoproduction of two pions on nucleons at low and intermediate energies (up to $E_\gamma \sim 1 \text{ GeV}$) has been the subject of intense experimental [1–6] and theoretical [7–19] study. The works have been mainly motivated to understand the role of the many baryonic resonances involved in the process. The fact that there are three particles in the final state, $N\pi\pi$, gives way to different mechanisms in which baryonic resonances play an important role, and this has led to obtain useful information on some resonances not attainable with other reactions. Much of the work has been done in unpolarized observables, mostly total cross sections and invariant mass distributions. In particular, the work of [15], which is the one we will use along this work, is based on around 25 Feynman diagrams considering the coupling of photons to several baryonic resonances able to influence the energy region up to $E_\gamma \sim 800 \text{ MeV}$. The main advantage is the use of no free parameters. This model succeeded in reproducing total cross sections and invariant mass distributions for all the charge channels with a good accuracy, not only in nucleons but also in nuclei. Specially remarkable was the application of the model of [15] to the study of the photoproduction of two pions in nuclei [20]. It succeeded in describing the shift of strength in the double
pion invariant mass distribution towards the $2m_\pi$ masses, due to the modification of the $\sigma$-meson mass in nuclear matter, where the $\sigma$-meson is dynamically generated as a $\pi\pi$ rescattering in scalar-isoscalar channel. This prediction was confirmed by the experiment of [21]. Therefore, the model of [15] has widely proven his efficiency in reproducing and predicting unpolarized observables in the energy range from threshold up to $E_\gamma \sim 800$ MeV. Nonetheless, a more demanding test to the model can be done by evaluating polarization observables, since it can be sensible to details of the model not visible when integrating over polarization degrees of freedom in the unpolarized observables. In this line, a test of the model of [15] was done in [22] when evaluating the spin $1/2$ and $3/2$ amplitudes and the contribution to the GDH sum rule of the double pion channel, in fair agreement with Mainz results [23, 24], and the evaluation of beam asymmetries under experimental study at GRAAL [5]. These observables are based on differences of total differential cross sections dependent on polarization, which provide a valuable information on the internal dynamics of the reaction. However, these observables still rely on integrated cross sections and no angular distributions are provided from where more information can be obtained.

The aim of the present work is to evaluate angular dependences of the cross section asymmetry $\sigma^+ - \sigma^-$ for the absorption of circularly polarized photons by unpolarized protons. This observable is very sensitive to the internal mechanisms of the reaction and, therefore, can be a very useful test to impose constraints on the theoretical models. The work has been partly motivated by preliminary experimental results, for the $\gamma p \rightarrow \pi^+\pi^- p$ channel, with the CLAS detector at Jefferson Lab [25] which shows strong and not trivial angular dependences of this observable, and prospects of measurements at Mainz [26] for the $\gamma p \rightarrow \pi^0\pi^0 p$ channel.

2 Summary of the $\gamma p \rightarrow \pi\pi p$ model

In this section we briefly summarize the model of [9, 10, 15] for the double pion photoproduction on nucleons. This model is intended to reproduce the total cross sections and invariant mass distributions up to photon energies of $E_\gamma \sim 800$ MeV. The model is based on a set of tree level mechanisms, depicted in Fig. 1 for the $\pi^+\pi^-$ channel. For the $\pi^0\pi^0$ channel only the mechanisms $e$, $f$, $g$, $h$, $k$, $l$, $m$, $o$, $p$, $q$, $r$ and $u$ contribute. These Feynman diagrams involve pions, $\rho$-mesons, nucleons and nucleonic and $\Delta$ resonances. The baryon resonances included in the model are: $\Delta(1232)$ or $P_{33}$ ($J^\pi = 3/2^+$, $I=3/2$), $N^*(1440)$ or $P_{11}$ ($J^\pi = 1/2^+$, $I=1/2$), $N^*(1520)$ or $D_{13}$ ($J^\pi = 3/2^-$, $I=1/2$) and $\Delta(1700)$ or $D_{33}$ ($J^\pi = 3/2^-$, $I=3/2$). The contribution of the $N^*(1440)$ is small but it was included due to the important role played by that resonance in the $\pi N \rightarrow \pi\pi N$ reaction and the fact that the excitation of the $N^*(1440)$ peaks around 600 MeV photon energy in the $\gamma N$ scattering. The $N^*(1520)$ has a large coupling to the photons and is an important ingredient due to its interference with the dominant term of the process, the $\gamma N \rightarrow \Delta N$ transition called the $\Delta$-Kroll-Ruderman ($\Delta$KR) contact term. (The $\Delta$KR term is not present in the $\gamma p \rightarrow \pi^0\pi^0 p$ channel). Several $\rho$ and $\Delta(1700)$ terms were included in the last version of the model [15] because of important interference effects. The consideration of the $\rho$ terms was of crucial
Figure 1: Mechanisms used in the model for $\gamma p \rightarrow \pi^+\pi^- p$. Solid lines without labels are nucleons. $\Delta$ means $\Delta(1232)$. For the $\gamma p \rightarrow \pi^0\pi^0 p$ channel only the $e, f, g, h, k, l, m, o, p, q, r$ and $u$ mechanisms contribute.
importance in the analysis of Ref. [27], when studying \( \rho \) meson photoproduction in nuclei. The \( y \) and \( z \) diagrams considering a \( \rho \) exchange were not considered in [15] since they give negligible contribution to the cross section in the energy region of concern. However they were considered in the work of [27] by completeness when considering \( \rho \) meson photoproduction and we also include them here since they can produce a non-negligible influence in the polarization asymmetry.

No other resonances were considered in the model since they cannot appreciably change the results in the energy range up to \( E_\gamma \sim 800 \) MeV, because their widths are small and lie at too high energies, because the decay width rates into \( \Delta \pi \) or \( \rho N \) are small or because a combination of various of these effects [15].

The diagrams u), v) and x) of Fig. 1 are the main modifications of [15] with respect to [10]. In the first work of [9] they included more than 50 diagrams for the \( \gamma p \rightarrow \pi^+ \pi^- p \) channel, but many of them were shown to be negligible at energies up to \( E_\gamma = 800 \) MeV. The non-negligible contributions come from the diagrams of Fig. 1.

The amplitudes are evaluated from effective interaction Lagrangians which are shown in the Appendices of Ref. [15], using a non relativistic approximation exact up to order \( p/M_p \), that is, removing terms of order \( (p/M_p)^2 \) and higher.

It is important to stress that this model has no free parameters, in the sense that there is no parameter to be fitted to the experimental double pion photoproduction observable. All input needed is obtained uniquely from properties of resonances and their decays. Where there are doubts about relative signs of couplings, one resorts to quark models or chiral perturbation theory to fix them [28].

3 Photon helicity asymmetry

We will consider the absorption of circularly polarized photons by non-polarized protons. For real photons the polarization vectors of a circularly polarized photon can be expressed as:

\[
\vec{\epsilon}^\pm = \frac{1}{\sqrt{2}}(\mp 1, -i, 0)
\]

where \( \vec{\epsilon}^+ \) or \( \vec{\epsilon}^- \) represent a right-handed or left-handed circularly polarized photon respectively.

The helicity asymmetry that we are going to consider in the present work can be defined as

\[
A \equiv \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}
\]

where \( d\sigma^{+(-)} \) is the differential cross section for the interaction of a right-handed (left-handed) circularly polarized photon with an unpolarized proton.

Three different frames have been commonly used in the literature to describe angular distributions in three body final state processes [29–31]. These frames are called Gottfried-Jackson, helicity and Adair systems, differing in the choice of the \( z' \) axis from which the azimuthal and polar angles are defined. The choice of a particular frame is of relevance.
Figure 2: Angular and kinematics definition in the helicity frame. The angle $\phi$ represents the angle between the scattering plane ($\vec{k}\vec{p}_2$) and the two pions plane ($\vec{p}_{\pi^+}\vec{p}_{\pi^-}$). (See the text for the exact definition for the origin and sign of $\phi$).

when studying particular production processes, like vector-meson photoproduction, due to particular angular distributions for a certain spin of the intermediate meson. In the present work, in order to test all the possible mechanisms contributing to the $\gamma p \rightarrow \pi^+\pi^- p$ process, any of these frames is useful for our purposes. Therefore, in order to allow comparison with preliminary experimental results with the CLAS detector at Jefferson Lab [25], we will use the helicity frame, which is defined as having the $z'$ axis in the direction of the sum of the momentum of both pions in the overall ($\gamma p$) c.m. frame. The use of the other frames would produce similar qualitative results for the discussion done in the present work. In Fig. 2 the kinematics for the $\gamma(k)p(p_1) \rightarrow \pi^+(p_{\pi^+})\pi^- (p_{\pi^-})p(p_2)$ reaction in this helicity frame is shown.

The $\phi$ angle, which we will use along the present work, accounts for the angle between the scattering plane (containing the photon, $\vec{k}$, and the final proton, $\vec{p}_2$) and the plane containing the two pions, and is defined by:

$$\cos \phi = \frac{(\vec{k} \times \vec{q}) \cdot (\vec{q} \times \vec{p}_{\pi^+})}{|\vec{k} \times \vec{q}| |\vec{q} \times \vec{p}_{\pi^+}|}$$

$$\sin \phi = -\frac{|(\vec{k} \times \vec{q}) \times \vec{q} \cdot (\vec{q} \times \vec{p}_{\pi^+})|}{|(\vec{k} \times \vec{q}) \times \vec{q}| |\vec{q} \times \vec{p}_{\pi^+}|}$$

with $\vec{q} \equiv \vec{p}_{\pi^+} + \vec{p}_{\pi^-}$, and with all the momenta in the overall center of mass frame. The expressions in Eq. (3) establish the definition of $\phi$ without ambiguities in the origin or sign, and uses the same convention as in Ref. [30].

In the present work we will consider the $\phi$ dependence of the $A$ observable since it is very sensitive to the particular mechanisms involved in the reaction by different reasons.
First, it is sensible to differences of cross sections, \((\sigma^+ - \sigma^-)\), and thus mechanisms which give small contribution to the total cross section can produce a sizeable contribution to the helicity asymmetry if the mechanisms are strongly helicity dependent. Second, it is very sensitive to interferences between the different diagrams of the model, as we explain in detail below.

Let us write the amplitude for the process as

\[ T = \epsilon_\mu T^\mu. \]  

(4)

In order to evaluate the cross sections one has to consider the squared \(T\)-matrix averaged over the initial spin of the proton, since we are considering non-polarized target, and summed over the spins of the final proton:

\[
\sum_{s_i, s_f} \langle m_{s_i} | \epsilon_\mu T^\mu | m_{s_f} \rangle \langle m_{s_f} | \epsilon^*_\nu T^\dagger_{\nu} | m_{s_i} \rangle = \text{Tr} \{ \epsilon_\mu \epsilon^*_\nu T^\mu T^\dagger_{\nu} \}. 
\]  

(5)

By using Coulomb gauge, where \(\epsilon_0 = 0\) and transversality, \(\epsilon_z = 0\), with \(\hat{z}\) the direction of the photon momentum \((\vec{k})\), Eq. (5) reads

\[
\text{Tr} \left\{ |\epsilon_x|^2 T^\dagger_x T_x + |\epsilon_y|^2 T^\dagger_y T_y + \epsilon_x \epsilon_y T^\dagger_y T^\dagger_x + \epsilon^*_x \epsilon^*_y T_x T^\dagger_y \right\}. 
\]  

(6)

Should we use linearly polarized photons \((\vec{\epsilon} = (1, 0, 0)\) or \((0, 1, 0)\)), we would obtain for the numerator of Eq.(2) up to phase space integrals, \(\text{Tr} \{ T^\dagger_x T_x - T^\dagger_y T_y \}\) \(^1\) (which is roughly the beam asymmetry \(\Sigma\) already studied in [22]). If we use circularly polarized photons, the numerator of Eq.(2) goes as (up to phase space integrals):

\[
d\sigma^+ - d\sigma^- \sim -2i \text{Tr} \{i(T^\dagger_x T^\dagger_x - T^\dagger_y T^\dagger_y)\} = 4\mathcal{I}m \{ \text{Tr} \{ T^\dagger_x T^\dagger_y \} \} = -2i \text{Tr} \{ (\vec{T} \times \vec{T}^\dagger)_z \} \]  

(7)

where \((\vec{T} \times \vec{T}^\dagger)_z\) means the component in the photon direction of the cross product of \(\vec{T}\) and \(\vec{T}^\dagger\).

There is an interesting necessary condition for the helicity asymmetry \(A(\phi)\), that is

\[ A(\phi) = -A(2\pi - \phi). \]  

(8)

This is true since the change \(\phi \rightarrow (2\pi - \phi)\) can be interpreted as a reflection of the \(\vec{p}_\pi + \vec{p}_\pi^-\) plane with respect to the \(\vec{k}\vec{p}_2\) plane (see Fig. 2). This is equivalent to changing the sign of the \(y\) coordinate and, therefore, by looking at Eq. (1), to the exchange of the role right-handed \(\leftrightarrow\) left-handed, what means \(A \rightarrow -A\). The condition \(A(\phi) = -A(2\pi - \phi)\) implies that \(A\) can be expanded as

\[ A(\phi) = \sum_{n=1}^{\infty} a_n \sin(n\phi), \text{ with } n = 1, 2, 3, 4, \ldots \]  

(9)

\(^1\)In the nomenclature of "hadronic tensor" \(W^{\mu\nu}\), and "structure functions" it would be \(\sim (W^{xx} - W^{yy}) = W_{TT}\). (See, for instance, ref. [32]).
The proportionality on \( \sin(\phi) \) implicit in Eq. (9), implies the asymmetry to be proportional to \((\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})_z\), since \( \sin(\phi) \) is proportional to \((\vec{p}_{\pi^-} \times \vec{p}_{\pi^+}) \cdot \vec{k} \) as can be easily obtained from Eq. (3).

This \((\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})_z\) proportionality relation can also be obtained from the general structure that the double pion photoproduction amplitude can take, that was shown in Ref. [11]. This general expression of the amplitude can be obtained [11] by using Lorentz covariance and gauge invariance and can be written as:

\[
\vec{T} = F_1 \vec{p}_{\pi^-} + F_2 \vec{p}_{\pi^+} + F_3 \vec{G} \cdot (\vec{p}_{\pi^-} + \vec{p}_{\pi^+}) \vec{\sigma} \cdot \vec{p}_{\pi^-} - \vec{p}_{\pi^+} + F_4 \vec{G} \cdot (\vec{p}_{\pi^-} + \vec{p}_{\pi^+}) \vec{\sigma} \cdot \vec{p}_{\pi^-} - \vec{p}_{\pi^+} + F_5 \vec{G} \cdot \vec{p}_{\pi^-} + \vec{p}_{\pi^-} + F_6 \vec{G} \cdot \vec{p}_{\pi^-} + \vec{p}_{\pi^-} + F_7 \vec{G} \cdot \vec{p}_{\pi^-} + \vec{p}_{\pi^-} + F_8 \vec{G} \cdot \vec{p}_{\pi^-} - \vec{p}_{\pi^-} + F_9 \vec{G} \cdot \vec{p}_{\pi^-} - \vec{p}_{\pi^-} - F_{10} \vec{G} \cdot \vec{p}_{\pi^-} - \vec{p}_{\pi^-} + F_{11} \vec{k} \cdot \vec{G} + F_{12} \vec{G} \cdot (\vec{p}_{\pi^-} + \vec{p}_{\pi^+}) \vec{\sigma} \cdot \vec{p}_{\pi^-} - \vec{k} \times \vec{\sigma}).
\] (10)

The \( F_i \) coefficients are, in general, complex functions of the momenta accounting for the dynamics of the different mechanisms (propagators, momentum dependences of the vertices, etc). \( \vec{T} \) has to change sign under parity (since \( \vec{T} \cdot \vec{e} \) has to be invariant and \( \vec{e} \) is a vector). This implies that all the \( F_i \) coefficients are scalars under parity. Time reversal symmetry implies also that \( F_i \) for \( i = 1 - 10 \), are invariant under time reversal but \( F_{11} \) and \( F_{12} \) has to change.

After applying Eq. (7) to the amplitude in Eq. (10) we have checked that the resulting non-vanishing terms can always be rearranged in terms of the form

\[
\frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \sum G \imath m(\eta F_m F_n^*)(\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})_z;
\] (11)

Where the \( G \)'s are scalar functions of \( \vec{p}_{\pi^-}, \vec{p}_{\pi^+} \) and \( \vec{k} \). In Eq. (11), \( \eta \) is 1 if \( F_m F_n^* \) is invariant under time reversal or \( i \) if it is not.\(^2\)

Let us discuss some important consequences that can be concluded from Eq. (11) on the sensitivity of the helicity asymmetry to the internal structure of the mechanisms and to the interferences between different diagrams. In some mechanisms\(^3\) it is allowed to have either a \( \pi^+ \) or a \( \pi^- \) in both the two external pion lines (we will call it type-I mechanisms), while in other mechanisms only one charge configuration is possible (type-II). (Type-II mechanisms are \( a, b, c, d, e, f, g, h, k, p, s, t, v, x, y \) and \( z \) of Fig. 11 and type-I are the rest). For instance, in the \( \Delta KR \) term (\( i \) mechanism of Fig. 11) it is possible to have a \( \pi^+ \) in the \( \gamma N \Delta \pi \) vertex and a \( \pi^- \) in the \( \Delta N \pi \) vertex or vice-versa, while in the \( a \) mechanism only the diagram where the \( \pi^+ \) is in the \( \gamma NN \pi \) vertex and the \( \pi^- \) in the \( NN \pi \) vertex is possible. If we take any individual diagram the asymmetry will necessarily vanish. This is so since the general complex structure of propagators is the same in all the pieces of the amplitude of Eq. (10) if one considers only one diagram, and therefore the \( F_i \) coefficients can be factorized as \( F_i = aa_i \), where \( \alpha \) contains all the structure of propagators and \( a_i \) are

\(^2\)Recall that time reversal changes \( i \) by \( -i \).

\(^3\)In the following discussion we will call ”mechanism” to the different graphs of Fig. 11 without specifying the charge configuration, and we will call ”diagram” to the different charge configurations that a mechanism can have.
real for \( i = 1 - 10 \) and purely imaginary for \( i = 11, 12 \). Therefore, in Eq. (11) we have \( |\alpha|^2 \Im(\eta a_n a_n^*) \) with \( \eta a_n a_n^* \) being real. On the other hand, if we have two diagrams then the \( \alpha \) coefficient can be different for some terms of the amplitude and the \( |\alpha|^2 \) factorization does not hold. In conclusion, associating mechanisms to diagrams, only the mechanisms that have two possible diagrams associated, i.e., those of type-I, could by themselves be nonzero (although some of them can also be zero). On the other hand, the fact that one needs \( \Im(\eta F_n F_n^*) \neq 0 \) implies the coefficients \( F_i \) to be generally complex, and this is provided by the propagator structure of the diagrams. This is the reason why this observable is so sensitive to the internal mechanisms of the reaction. In addition, the fact that the \( (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})_z \) factor of Eq. (11) makes the numerator of \( A \) to be proportional to \( \sin \phi \) implies that the difference from a simple \( \sin \phi \) dependence comes from the momentum dependence of the \( F_i \) coefficients of the amplitude. Thus, the angular dependence of the helicity asymmetry is strongly reflecting the internal structure of the various mechanisms.

On the other hand, when allowing the different mechanisms to interfere between them, even if some individual mechanisms do not produce an asymmetry by themselves, they can produce a non-vanishing asymmetry when adding them coherently. This is why the interferences in the \( \phi \) dependence of the helicity asymmetry are so important. The angular dependence of the denominator of the helicity asymmetry, \( d\sigma^+ + d\sigma^- \) (which is proportional to the total cross section), does not modify qualitatively the previous discussion.

Let us illustrate the previous discussion with an example: let us consider the \( \Delta KR \) term (I mechanism of Fig. 11). The amplitude for the process where the \( \pi^+ \) is emitted before the \( \pi^- \) is given by [15]

\[
\tilde{T}_{\Delta KR} = \frac{1}{9} e \left( \frac{f^*}{m_\pi} \right)^2 G_\Delta(p_2 + p_{\pi^-}) F_\pi((p_{\pi^+} - k)^2)[2\vec{p}_{\pi^-} - i(\vec{\sigma} \times \vec{p}_{\pi^-})],
\]

where \( f^* = 2.13 \), \( G_\Delta \) is the \( \Delta(1232) \) propagator and \( F_\pi \) is a form factor. For the process where the \( \pi^- \) is emitted before the \( \pi^+ \) the amplitude is obtained by exchanging \( p_{\pi^+} \leftrightarrow p_{\pi^-} \) and writing the appropriate isospin coefficients

\[
\tilde{T}_{\Delta KR} = -\frac{1}{3} e \left( \frac{f^*}{m_\pi} \right)^2 G_\Delta(p_2 + p_{\pi^-}) F_\pi((p_{\pi^+} - k)^2)[2\vec{p}_{\pi^+} - i(\vec{\sigma} \times \vec{p}_{\pi^+})].
\]

With these expressions for the amplitude, we have for the last term of Eq. (11)

\[
Tr\{(\tilde{T} \times \tilde{T}^\dagger)_z\} = 20i \Im(\alpha \beta^*)(\vec{p}_{\pi^-} \times \vec{p}_{\pi^+})_z,
\]

with \( \alpha = \frac{1}{9} e \left( \frac{f^*}{m_\pi} \right)^2 G_\Delta(p_2 + p_{\pi^-}) F_\pi((p_{\pi^+} - k)^2) \) and \( \beta = -\frac{1}{3} e \left( \frac{f^*}{m_\pi} \right)^2 G_\Delta(p_2 + p_{\pi^-}) F_\pi((p_{\pi^+} - k)^2) \). This means that the \( \Delta KR \) mechanism (accounting for two Feynman diagrams) gives by itself a non-vanishing angular dependence of the helicity asymmetry thanks to the imaginary part in the \( \Delta \) propagator, and deviation from a simple \( \sin \phi \) dependence (as will be shown in the Results section) is due to the momentum dependence of the propagator and the form factor, (although this latter one is a smooth function).

The kind of reasoning presented in this section stresses the importance of small details of the theoretical models, making the \( \phi \) dependence of the helicity asymmetry a very useful
and powerful tool to check the heart of the theoretical models. Therefore, even if the model succeeds to reproduce unpolarized observables (like total cross sections, invariant mass distributions, etc), it could fail to reproduce the kind of polarization observables studied in this work, simply because of small details.

4 Results

In Fig. 3 we show, for a given $\gamma p$ energy of $\sqrt{s} = 1500$ MeV ($E_{Lab} \approx 730$ MeV), the $\phi$ distribution of the helicity asymmetry, $A$, for different mechanisms contributing to the $\gamma p \rightarrow \pi^+\pi^-p$ process. From left to right and up to down the plots represent: $\Delta$KR term ($i$ diagram of Fig. 1), pion pole term ($j$ diagram), nucleon intermediate mechanisms (diagrams from $a$ to $q$), $\Delta(1232)$ (diagrams $h$ to $k$ and $m$ to $p$), $\rho$-meson intermediate contribution (diagrams $v$ to $z$), $N^*(1440)$ resonance (diagrams $q$ to $t$), $N^*(1520)$ (diagrams $l$ and $v$), $\Delta(1700)$ (diagrams $u$ and $x$), all the mechanisms except the nucleon intermediate diagrams, all the mechanisms except the $\Delta$KR term and, finally, the full model (all the diagrams). (The plots of the $\Delta$KR term alone and the $N^*(1520)$ have been multiplied by 100 and $1/2$ respectively to make the curves visible inside the represented scale). In the plots the condition $A(\phi) = -A(2\pi-\phi)$, Eq. (8), is clearly visible.
Figure 4: Angular distribution of the helicity asymmetry for different energies with the full model, for the $\gamma p \rightarrow \pi^+\pi^-p$ channel. Preliminary experimental results from [25]. (The data is integrated over the full CLAS acceptance while the theoretical calculations cover the full phase space).

One can see in Fig. 3 the very strong dependence on the mechanisms considered and the crucial role of the interferences. For instance, even if the nucleon intermediate mechanisms give a vanishing contribution by themselves, the interference with the $\Delta(1232)$ mechanisms produces strong changes in the distribution with respect to considering the $\Delta(1232)$ terms alone. From the discussion of the previous section it can be understood why the nucleon mechanisms give a zero asymmetry: the nucleon propagators do not have width and therefore do not have a complex structure needed to produce a non-zero imaginary part in Eq. (11). Another quantitative example of the important role of the interferences can be seen, for instance, by looking at the figures evaluated with all the mechanisms except the nucleons or except the $\Delta KR$ term. For this latter case, despite the asymmetry for the $\Delta KR$ being very small, it has an important influence in the full result. On the other hand, by comparing the ”all except nucleon” with the ”all” plot, one can see the dramatic influence of the nucleon intermediate mechanism in the angular distribution of the helicity asymmetry, despite these mechanisms contributing only around 10% to the total cross section.

In order to show the sensitivity to the energy of the angular distribution of the helicity
asymmetry, we show in Fig. 4 the results with the full model for different energies. The experimental data, still preliminary, are obtained from Ref. [25], measured with the CLAS detector at Jefferson Lab. It is important to stress that these data are integrated over the full CLAS acceptance, while the theoretical model covers the full phase space. Thus, given the sensitivity of the observable to these details, one has to be cautious when making conclusions from this naive comparison. With this caveat, and after the remarks on the sensitivity of this observable to small details of the model, the comparison of the theoretical predictions of the present work and the data of [25] shown in Fig. 4 would be seen as an indication that the model contains the basic mechanisms. The strength of the theoretical results and experiment is similar, and this is not a trivial theoretical result given the large range of values found in Fig. 3 for different options of partial results of the model. The discrepancies found in the shape for the two lower energies are more worrisome, but in view of the preliminary character of the experimental data, and the fact that they are not $4\pi$ integrated, it is probably too early to draw conclusions from there. We would like to note that the theoretical results reported here are similar in strength and shape as those reported in [25] as private communication, calculated with the model of [33]. In view of this, it is important that definitive data are provided and that direct calculations adapted to the acceptance of the experimental setup are carried out. This comparison should help in the future to improve the present models of double pion photoproduction. We are aware, that given the important role of the sources of complex part in the amplitudes in this polarized observable, other sources of complex part not considered in our model could be relevant for the polarization observables in spite that they could be not so important in the unpolarized observables. For instance, in Ref. [34] it was pointed out, as private communication from Mokeev, that complex relative phases in the amplitudes could produce sizeable effects in these polarization observables, whereas in unpolarized observable they do not. On the other hand, final state interaction of the produced particles could also influence these observables. Therefore, we are well aware of the limitations of our model, but the present work can serve to establish, from the comparison with experiments, how much room for improvement one can expect on the theoretical models.

Next we show the results for the $\gamma p \rightarrow \pi^0\pi^0 p$ channel, for which there are prospects to be measured at Mainz [26]. In Figs. 5 and 6 we show the contribution of different mechanisms, or combinations between them, for two different energies. In the $\pi^0\pi^0$ channel there are less mechanisms allowed than in the $\pi^+\pi^-$ case, like, for instance, the $\Delta KR$ term (which gives the most important contribution in the $\pi^+\pi^-$ channel). From left to right and up to down the plots in Figs. 5 and 6 represent: $k$ diagram (which is important in the total cross section [15]), $\Delta(1232)$ (diagrams $h$, $k$, $m$, $o$ and $p$), $\Delta(1232)$ plus nucleon terms, all the mechanisms except $k$, all except $\Delta(1232)$, all except nucleon, all except $N^*(1440)$, all except $N^*(1520)$ and , finally, the full model.

In this channel, apart from the condition $A(\phi) = -A(2\pi - \phi)$, there is another extra condition which is $A(\phi) = A(\phi + \pi)$. This happens since the two $\pi^0$ are identical particles and the observables cannot depend on permuting the two $\pi^0$, but the exchange of the two pions means the change $\phi \rightarrow \phi + \pi$ (see Fig. 2). The conditions $A(\phi) = -A(2\pi - \phi)$ and
Figure 5: Angular ($\phi$) distribution of the helicity asymmetry, $A$, for different contributions in the $\gamma p \rightarrow \pi^0 \pi^0 p$ channel for a $\gamma p$ energy of $\sqrt{s} = 1400$ MeV.
Figure 6: Same as Fig. 5 for $\sqrt{s} = 1500$ MeV.
\( A(\phi) = A(\phi + \pi) \) imply that, in the series of Eq. (10), only the \( n = \text{even} \) terms are possible. That is why the angular dependence of the helicity asymmetry for the \( \pi^0\pi^0 \) channel manifests, essentially, a \( \sin(2\phi) \) shape. For this reason, the shape for this channel is less rich in variety of structures than in the \( \pi^+\pi^- \) case, since the \( \sin(\phi) , \sin(3\phi) , \ldots \) terms are forbidden. Nonetheless, despite the plots in Figs. 5 and 6 manifesting mainly a \( \sin(2\phi) \) dependence, one can see there the important role of interferences in determining the strength and phase (sign) of the distributions, and the energy dependence of the effect. For instance, for \( \sqrt{s} = 1400 \text{ MeV} \), despite the nucleon mechanisms giving a vanishing contribution by themselves (not shown in the figure), the interference with the \( \Delta(1232) \) mechanisms produces a very small asymmetry. Another example, quite spectacular, is the role of the \( k \) diagram since, in spite that by itself gives a similar distribution for both energies, the distribution removing it from the full model looks dramatically different in the two energies: in fact, at 1400 MeV, the angular distribution of the asymmetry is negligible if one removes the \( k \) mechanism from the full model, while at 1500 MeV the effect is very large. The \( N^*(1520) \) reduces the strength at 1500 MeV if it is removed from the full model, while no significant effect is visible at 1400 MeV. Therefore, in spite the \( \pi^0\pi^0 \) channel having less richness in different shapes for the distribution, the large variation in the strength and sign stresses the importance of this observable in elucidating the mechanisms involved in the reaction.

The calculations done in the present work are just an example of the type of studies that one can make, in the sense that other energies, angles or kinematical cuts can be implemented, but it serves as an example of the strong dependence on the internal mechanisms and interferences that is obtained from this kind of polarization experiments.

5 Conclusions

We have made calculations of angular distributions of helicity asymmetries in \( \gamma p \rightarrow \pi^+\pi^- p \) and \( \gamma p \rightarrow \pi^0\pi^0 p \) for the interaction of circularly polarized photons with unpolarized protons. We have used a well tested theoretical model successfully applied in the evaluation of several unpolarized observables. The study of polarization observables of the kind of those discussed in the present work, can serve to challenge the theoretical models when more demanding refinement of the details can be crucial. We have shown the strong dependence of the shape and strength of the calculations on the internal mechanisms and interferences among different contributions to the process. We have shown that, in spite that some mechanisms do not give structure by themselves, they can be crucial to produce the final result due to subtle interferences with other mechanisms. Furthermore, mechanisms which give a small contribution in unpolarized observables, like total cross sections, can be of strong relevance in the contribution to the difference between the polarization cross sections. Therefore, these polarization observables have different sensitivities to the internal details of the model than other observables.

Further experimental results would be of importance to discriminate between models
but being aware that sizeable discrepancies between theoretical and experimental results can be due to small details which are irrelevant when applying the model to the evaluation of unpolarized observables.

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