Development of Particle-fluid Drag Model for Individual Wheat Straw Particle

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Abstract. To deal with the sedimentation of wheat straw particles in the process of conveying and mixing, the drag model of the non-spherical particle is to be developed. In this work the settling velocity of sheared and the defibered particles are acquired by image analysis. The simulating settling velocity of particle is acquired by ANSYS Fluent. As a result, the KG-based drag models with shape factors of Π, AR, BR, and Φ are obtained respectively by fitting the relation between drag coefficient and particle Reynold for 24 sheared particles. The settling velocity of 24 particles are acquired by simulation. In addition, the six drag models are used to simulate settling velocity of 134 defibered particles. At last, the means value of relative error of velocity between the shear and the defibered particles are acquired, they are 20%, 23%, 19%, 59%, respectively, corresponding to the shape factors of Π, AR, BR and Ψ. Considering the p-value of curve fitting, the drag model with shape factor Π is best for simulating the wheat straw particles in particle-liquid flow.

1. Introduction
Approximately 529 million tons’ wheat [1] is produced every year over the world, and a nearly equal amount of wheat straw is produced. As the global energy structure changes to clean and low-carbon, the use of crop straw as a traditional fuel has been abandoned and as a new clean energy to convert it to biogas have been gradually adopted. Unfortunately, the sedimentation of wheat straw during conveying and mixing process is severely to reduce its energy conversion efficiency.

Computation fluid dynamics (CFD) is commonly applied to simulate the particles’ motion. Commonly, single phase flow is use to simulate the particle-fluid’s motion in anaerobic digesters [2-5]. But the we can’t see the sedimentation of the particle from single phase model. Two-phase flow models can reflect the particles’ motion in with the drag model included in the CFD code. For instance, Patrí ci Metolina et al. [6] studied liquid–solid flow in tapered and cylindrical fluidised beds using the Eulerian–Eulerian two-phase model in which the spherical drag correlation is used. Infecho, wheat straw particle has irregular shape, the spherical drag used in CFD can result in bad accuracy. Pierfrancesco Dellino et al. [7] studied the settlement process of pumice particles using simulated by the spherical and non-spherical drag models and obtained the prediction errors of settling velocity of 65% and 22%.

In this work, the terminal settling velocity of sheared particles and defibered particles are measured through the experiment. The particle shape is acquired and calculated by different shape factors. Then, a drag model of KG-based model was developed with a new shape factor Π. Comparing with relative
error of particle settling velocity between measurement and simulation, the best drag model with the appropriate shape factor is acquired for simulating the wheat straw particles in solid-liquid phase flow.

2. Model setup

2.1. Derivation of fluid-particle drag model

For a single particle drag $F_D$ comes from fluid can be expressed by equation (1). $F$ is given by equation (2) [8]. Substitute equation (2) to equation (1), drag coefficient $C_D$ is expressed by equation (3).

Because of the drag $F_D$ has relation of particle shape and particle Reynolds number [9], then non-spherical drag model expression can be expressed as equation (4).

$$F_D = m_p F(\bar{u}_f - \bar{u}_p) C_D \frac{Re_p}{24} = S^B(1 + b \cdot Re_p^A) S^C$$  

(1)  

$$F = \frac{18 \mu}{\rho_p d_p^2} C_D \frac{Re_p}{24} = \frac{1}{u} \frac{V_p(\rho_p - \rho_f) g d_v}{12 \rho_p \mu}$$  

(2)  

$$C_D = 24 \frac{\rho_f F_D}{\rho_p d_p^2 \mu}$$  

(3)  

$$C_D = 24 \frac{f(S, Re_p)}{Re_p}$$  

(4)  

Where, $\bar{u}_f$ is the velocity of flow fluid, $\bar{u}_p$ is the particle velocity, $\mu$ is the viscosity of fluid, $\rho_f$ is the density of fluid, $\rho_p$ is density of particle, $d_p$ is diameter of particle, $Re_p$ is particle Reynolds.

2.2. Form of drag model

There are several non-spherical drag models for wheat straw particles, Kishore and Gu [10] studied ellipsoidal particles moving at a moderate Reynolds number and proposed a drag model with the shape factor of particle aspect ratio. Haider and Levenspiel [11] developed a non-spherical drag correlation and an explicit formula of the settling velocity by using the shape factor of sphericity $\Psi$. Richter and Nikrityuk [12] studied the drag coefficient of particles with shaped of spheres, squares and ellipses at low Reynolds numbers and considered the ratio of the particle length parallel to the flow direction to the diameter of an equivalent-volume sphere as the shape factor. Fabio Dioguardi et al.[13] studied the drag correlation of pumice particles settling in water using the shape factor $\Phi$, which refers to the ratio of the $\Psi$ of a particle to its circularity $X$. The four drag models are listed in Table 1.

| Model   | Conditions                                                                 | Literature                        |
|---------|----------------------------------------------------------------------------|-----------------------------------|
| $C_{DKG}$ | $\frac{24 \cdot AR^{0.49}}{Re_p} = (1.05 + 0.152Re_p^{0.687}AR^{0.671})$ | $1 \leq Re_p \leq 200$, $0.25 \leq AR \leq 2.5$ | Kishore and Gu(KG) [10] |
| $C_{D,Hl}$ | $\frac{24}{Re_p} (1 + A \cdot Re^B)$                                     | $Re_p < 2.6 \cdot 10^5, \Psi > 0.67$ | Haider and Levenspiel(HL) [11] |
| $C_{DRN}$  | $0.21 + \frac{20}{Re_p} SR^{0.58} + \frac{6.9}{\sqrt{Re_p}} SR^{-1.4}$  | $10 \leq Re_p \leq 250$          | Richter and Nikrityuk(RN) [12] |
\[ C_D = \frac{0.9627}{Re^{0.079} \Phi^{1.6}} \quad 1 \leq Re \leq 10^5 \]

2.3. Shape factors
Five shape factors used in this paper, they are Π(Π1), aspect ratio AR, BR, sphericity Ψ, and Φ. The calculation of the shape factors is listed in table 2.

| Shape factor | Definition | Explanation |
|--------------|------------|-------------|
| Π            | Π = ZX     | Heywood shape factor [14] |
| AR           | AR = \frac{2D_{f,\text{max}}}{D_{f,\min} + d_{f,\text{max}}} | \( D_{f,\text{max}} \) is circumscribed circle diameter and \( D_{f,\min} \) is intangential circle diameter of maximum projection area, \( d_{f,\text{max}} \) is maximum of lateral view |
| BR           | BR = \frac{L}{B} | L and B is length and width of the particle |
| Ψ            | Ψ = \frac{A_{\text{sph}}}{A_p} | \( A_{\text{sph}} \) is area of the equal-spherical volume, \( A_p \) is the real superficial area of the particle |
| Φ            | Φ = Ψ/X    | Ψ is sphericity, X is circularity |

2.4. Calculation method
According to equation (4) and model KG, a KG-based drag model is acquired by equation (5). \( C_D \frac{Re_p}{24} \) is calculated by the balance of the drag force \( F_{\text{drag}} \) and effective gravity \( F_g \) [15]. Particle settling relative velocity \( u \), volume \( V_p \), density \( \rho_p \), particle diameter \( d_v \) and projection area \( A_p \), density \( \rho_f \) and viscosity \( \mu \) are also calculated by experiments.

\[ C_D \frac{Re_p}{24} = S^B \left(1 + b \cdot Re_p^A \right) \]  
(5)

\[ C_D \frac{Re_p}{24} = \frac{1}{u} \frac{V_p(\rho_p-\rho_f)g}{12A_p \mu} \]  
(6)

In order to fit for equation (6), a transformation of equation (5) should be conducted. First of all, let \( y_1 = \frac{C_D \frac{Re_p}{24}}{S^B} - 1 \), then \( y_1 = AR e^B S^E \), next, Logarithm is used, equation (7) is obtained. Let \( y = \ln y_1 \), \( x_1 = \ln Re \), \( x_2 = \ln S \), \( \beta_0 = \ln A \), \( \beta_1 = B \), \( \beta_2 = E \), the finale linear regression function is obtained by equation (8). At last, matlab is used to get the coefficients and determination coefficient \( R^2 \).

\[ \ln y_1 = \ln A + \ln Re + \ln SC_D \frac{Re_p}{24} = S^B \left(1 + b \cdot Re_p^A \right) \]  
(7)

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \frac{Re_p}{24} = \frac{1}{u} \frac{V_p(\rho_p-\rho_f)g}{12A_p \mu} \]  
(8)

3. Experiment

3.1. Particle geometry and density
The wheat straw was obtained from Funan City, Anhui Province, China. 24 sheered wheat straw particles were picked randomly with length of 10mm and width of 1-6mm. Before measuring the particle geometry and density, the particles must be entirely wet. Outline of a single wheat straw particle is presented in Fig. 1. The maximum projection length \( L \), the maximum projection width \( B \) and the stretched width \( B' \) are measured as shown in Fig. 1. The
The maximum projection area $A_{p\text{max}}$ is calculated by the equation $LB$. The particle mass $m_p$ is measured by a precision balance with a measurement range of 0–120 g and an accuracy of ±0.1 mg. The density $\rho_p$ of a single particle and the density of water $\rho_f$ are measured by a standard Gay–Lussac pycnometer with the volume of 100 mL. The volume of a particle $V_p$ is calculated using the formula $m_p/\rho_p$. The particle equivalent thickness $T$ is calculated by $V_p/A_{p\text{max}}$.

![Figure 1. Outline of a single wheat straw particle.](image)

3.2. Free-settling velocity

The free-settling experiment is conducted in a square tank made of acrylic glass (Figure 2). The length, width, height and thickness of the tank are 100 mm, 100 mm, 1 m and 5 mm, respectively. A white paper is established on one side of the tank, and a V20-3D camera is installed on the opposite side of the paper. The frame rate of the camera is 30 fps and its resolution is 1080*1920 pixel. A 120 W LED light source is used to illuminate the region of interest in the tank. A millimetre-scale ruler is attached to the left side of the water tank and used to assess the falling position of the particles. The distance between the camera and the water tank is denoted as $H_c$, and the camera focus is adjusted to ensure that the ruler can clearly be seen. The height of the camera’s visual scope is 500 mm.

![Figure 2. Diagram of the experimental setup.](image)

At the beginning of the experiment, the recorder of the camera is switched on. The particles are added to the tank one by one and allowed to fall to the bottom of the vessel. The video recorded by the
camera are converted into photographs frame by frame using Photoshop software, and 30 pictures in PNG format are produced in 1s. MATLAB software is used to process the pictures. Firstly, the colour images are converted into grey images. Secondly, the grey images are binarised and denoised. Finally, the centroid y coordinate and the orientation angle θ of the particle in every image is obtained, as shown in Fig. 3. The terminal settling velocity $u_{\text{meas}}$ will be obtained by the relationship of y vs t. The maximum projection area $A_{p\text{max}}$ is calculated using the formula $A_{p\text{max}}\cos\theta; d_p$, X and Z are also calculated.

### 4. Numerical setup

#### 4.1. Theory of ANSYS Fluent DPM model

\[
\frac{\partial (\rho f \tilde{u}_f)}{\partial t} + \nabla \cdot (\rho f \tilde{u}_f \tilde{u}_f) = 0 \quad \text{CD}_{24} \quad \text{Re}_f = \frac{V_f}{\mu_f} = S_f \left( 1 + b \cdot \text{Re}_p f \right)
\]

\[
\frac{\partial (\rho f \tilde{u}_f \tilde{u}_f)}{\partial t} + \nabla \cdot (\rho f \tilde{u}_f \tilde{u}_f) = -\rho f \nabla p + \nabla \cdot (\rho f \tilde{g} \tilde{f}) + \rho f g f + R_{\text{f}} f \text{CD}_{24} = \frac{1}{u} \left( |\rho_f| - |\rho_f| \right) \text{gt} \nu_i \mu_f
\]

For dispersed phase, the single particle’s motion equation is as follows:

\[
\frac{d\tilde{u}_f}{dt} = \tilde{F} (\tilde{u}_f - \tilde{u}_p) + \frac{\tilde{m}}{\tilde{p}_p} + F
\]

Take consideration of the fluid drag force on the particle, number of particles M in the calculation cell, the volume of $V_{\text{cell}}$ and the force on the $m_{th}$ particle of $\tilde{F}_{\text{D},m}$. Drag $R_{f,p}$ [16] of unit volume of calculation cell is:

\[
R_{f,p} = \frac{\nu_f^m \tilde{m}_f \tilde{D}_m}{V_{\text{cell}}}
\]

#### 4.2. Theory of ANSYS Fluent DPM model

Drag in ANSYS Fluent UDF function is defined by equation (13).

\[
\text{DEFINE}_{\text{DPM,DRAG}} = \frac{18 \text{C}_d \text{Re}_f}{24}
\]

### 5. Results and discussion

#### 5.1. Geometry parameters and shape factors

Basic parameters for water and two type of handled particles are listed in Table 3. The mass, volume, size and shape parameters of the 24 particles are listed in Table 4.

| Table 3. Basic parameters for water and two kinds of treated particles. |
|---------------------------|---------------------------|---------------------------|
|                           | Water                     | Sheared particles         | Defibering particles    |
| $\rho_f$(g/m$^3$)         | 996.62                    | 1138                      | 1050                    |
| $\mu_f$(Pa·s)             | 0.00089                   | -                         | -                       |

| Table 4. Mass, volume, size and shape parameters of the 24 sheared particles. |
|---------------------------|---------------------------|---------------------------|
|                           | $m_p$(g) | $V_p$(mm$^3$) | L(mm) | B(mm) | T(mm) | $A_{\text{max}}$(mm$^2$) | Dp(mm) | Dv(mm) | $\Psi$ | $\Phi$ | $\Pi$ | AR |
|---------------------------|----------|----------------|------|-------|-------|------------------------|--------|--------|-------|-------|-------|----|
| 1                         | 0.0234   | 20.56          | 10.26| 4.4   | 0.46  | 45.14                  | 7.58   | 3.40   | 0.37  | 0.30  | 0.038 | 4.23|
| 2                         | 0.021    | 18.45          | 10.42| 4.31  | 0.41  | 44.91                  | 7.56   | 3.28   | 0.39  | 0.31  | 0.034 | 4.41|
| 3                         | 0.0147   | 12.92          | 9.72 | 3.62  | 0.37  | 35.19                  | 6.69   | 2.91   | 0.38  | 0.30  | 0.034 | 4.88|
| 4                         | 0.0159   | 13.97          | 10.16| 3.72  | 0.37  | 37.80                  | 6.94   | 2.99   | 0.43  | 0.34  | 0.033 | 4.97|
| 5                         | 0.0145   | 12.74          | 10.1 | 3.68  | 0.34  | 37.17                  | 6.88   | 2.90   | 0.42  | 0.33  | 0.031 | 5.02|
| 6                         | 0.0159   | 13.97          | 10.2 | 3.42  | 0.40  | 34.88                  | 6.66   | 2.99   | 0.40  | 0.31  | 0.036 | 5.34|
5.2. Particle settling velocity, drop angle and projection area

Fig. 4 gives the vertical position and velocity of a single particle falling in still water. The trajectory of a particle is a polynomial equation of time $t$ when $t < 2$ s and a linear equation when $t > 2$ s. The drop velocity of a wheat straw particle is calculated by the slope of the trajectory.

Fig. 5 illustrates the horizontal projection position and the falling angle $\theta$ of the particle changes with time in every 0.1s. On the left side of Fig. 5, describes the particle falls in the still water, the particles orient themselves with their largest projection area normal to the flow [17]. The maximum area $A_{\text{pmax}}$ of a particle is always orient downwards. The vertical projection area is equal to $A_{\text{pmax}} \cos \theta_p$, where $\theta_p$ is the terminal settling angle when the velocity is in the condition of terminal settling. Due to the falling angle $\theta$ changes with times, the mean value in a period of time is calculated as the terminal settling angle $\theta_p$. Settling velocity and related parameters of the 24 particles are listed in Table 5.

| 7  | 0.0146 | 12.83 | 9.76 | 3.44 | 0.38 | 33.57 | 6.54 | 2.91 | 0.44 | 0.34 | 0.036 | 5.11 |
|---|--------|-------|------|------|------|-------|------|------|------|------|-------|------|
| 8  | 0.0133 | 11.69 | 10.96 | 3.26 | 0.33 | 35.73 | 6.74 | 2.82 | 0.39 | 0.29 | 0.028 | 6.11 |
| 9  | 0.0103 | 9.05  | 10.52 | 2.7  | 0.32 | 28.40 | 6.01 | 2.59 | 0.46 | 0.33 | 0.030 | 6.97 |
| 10 | 0.0118 | 10.37 | 10.62 | 3.3  | 0.30 | 35.05 | 6.68 | 2.71 | 0.41 | 0.31 | 0.026 | 5.91 |
| 11 | 0.0113 | 9.93  | 10.32 | 2.92 | 0.33 | 30.13 | 6.19 | 2.67 | 0.46 | 0.34 | 0.031 | 6.35 |
| 12 | 0.0083 | 7.29  | 10.36 | 2.2  | 0.32 | 22.79 | 5.39 | 2.41 | 0.49 | 0.33 | 0.031 | 8.22 |
| 13 | 0.0057 | 5.01  | 10.6  | 1.98 | 0.24 | 20.99 | 5.17 | 2.12 | 0.42 | 0.27 | 0.023 | 9.56 |
| 14 | 0.0044 | 3.87  | 11.1  | 1.08 | 0.32 | 11.99 | 3.91 | 1.95 | 0.58 | 0.29 | 0.033 | 15.83 |
| 15 | 0.0064 | 5.62  | 10.48 | 2.5  | 0.22 | 26.20 | 5.78 | 2.21 | 0.36 | 0.25 | 0.020 | 7.72 |
| 16 | 0.0101 | 8.88  | 10.76 | 2.2  | 0.38 | 23.67 | 5.49 | 2.57 | 0.54 | 0.36 | 0.036 | 8.36 |
| 17 | 0.0125 | 10.98 | 10.62 | 3.56 | 0.29 | 37.81 | 6.94 | 2.76 | 0.39 | 0.30 | 0.025 | 5.52 |
| 18 | 0.0089 | 7.82  | 10.3  | 2.72 | 0.28 | 28.02 | 5.97 | 2.46 | 0.42 | 0.30 | 0.026 | 6.87 |
| 19 | 0.0085 | 7.47  | 10.46 | 3.42 | 0.21 | 35.77 | 6.75 | 2.43 | 0.32 | 0.25 | 0.019 | 5.77 |
| 20 | 0.0136 | 11.95 | 11    | 4.1  | 0.27 | 45.10 | 7.58 | 2.84 | 0.35 | 0.28 | 0.022 | 5.04 |
| 21 | 0.0104 | 9.14  | 9.8   | 3.28 | 0.28 | 32.14 | 6.40 | 2.59 | 0.35 | 0.27 | 0.027 | 5.50 |
| 22 | 0.007  | 6.15  | 10.28 | 2.4  | 0.25 | 24.67 | 5.60 | 2.27 | 0.41 | 0.28 | 0.024 | 7.76 |
| 23 | 0.0038 | 3.34  | 10.1  | 1.12 | 0.30 | 11.31 | 3.80 | 1.85 | 0.57 | 0.30 | 0.032 | 14.27 |
| 24 | 0.0067 | 5.89  | 10.64 | 2.12 | 0.26 | 22.56 | 5.36 | 2.24 | 0.43 | 0.29 | 0.025 | 8.94 |

**Figure. 4.** Falling position and drop velocity of wheat straw particle changing with time.

**Figure. 5.** The horizontal projection and settling angle of wheat straw particle during falling down.
Table 5. Settling velocity and related parameters of the 24 particles.

|   | $u_{\text{meas}}$(m/s) | $\theta_p$($^\circ$) | $A_p$(mm$^2$) | $f(Re_p,S)$ | $Re_p$ | $Re_s$ |
|---|------------------------|----------------------|---------------|-------------|---------|---------|
| 1 | 0.0227                 | 0                    | 45.144        | 19.688      | 191.333 | 85.77   |
| 2 | 0.0206                 | 1.98                 | 44.883        | 19.489      | 173.468 | 75.25   |
| 3 | 0.0212                 | 3.51                 | 35.120        | 15.020      | 157.552 | 68.60   |
| 4 | 0.0197                 | 0.41                 | 37.794        | 16.824      | 152.139 | 65.52   |
| 5 | 0.0209                 | 2.26                 | 37.139        | 14.617      | 159.698 | 67.28   |
| 6 | 0.0196                 | -0.37                | 34.883        | 17.582      | 145.586 | 65.29   |
| 7 | 0.0204                 | 0.98                 | 33.569        | 15.816      | 148.606 | 66.01   |
| 8 | 0.0187                 | 1.3                  | 35.720        | 15.233      | 140.560 | 58.70   |
| 9 | 0.0171                 | 1.442                | 28.395        | 14.490      | 114.429 | 49.21   |
|10 | 0.0166                 | 1.79                 | 35.029        | 15.342      | 123.816 | 50.16   |
|11 | 0.0166                 | 0                    | 30.134        | 15.916      | 114.297 | 60.37   |
|12 | 0.0166                 | -0.13                | 22.792        | 13.442      | 99.4019 | 44.40   |
|13 | 0.0176                 | 3.03                 | 20.959        | 9.0780      | 101.081 | 41.53   |
|14 | 0.0148                 | -0.99                | 11.986        | 11.002      | 64.3783 | 32.09   |
|15 | 0.0192                 | 1.8                  | 26.187        | 8.3556      | 123.306 | 47.13   |
|16 | 0.0203                 | -0.81                | 23.670        | 13.126      | 123.869 | 57.96   |
|17 | 0.0154                 | 0.076                | 37.807        | 16.905      | 119.032 | 47.33   |
|18 | 0.0169                 | -1.06                | 28.011        | 12.724      | 112.604 | 46.43   |
|19 | 0.0225                 | 0.95                 | 35.768        | 8.0761      | 169.434 | 60.90   |
|20 | 0.0196                 | 3.13                 | 45.033        | 13.214      | 165.69  | 62.07   |
|21 | 0.020                 | -0.16                | 32.144        | 11.764      | 142.33  | 57.71   |
|22 | 0.0149                 | -3.09                | 24.636        | 12.114      | 93.026  | 37.77   |
|23 | 0.0192                 | -1.01                | 11.310        | 7.5534      | 80.989  | 39.57   |
|24 | 0.0195                 | -0.34                | 22.556        | 9.2641      | 116.43  | 48.67   |

5.3. Form selection of drag model

Non-spherical drag model of KG HL RN and FD are used to simulated 24 sheared particles. relative error between simulation velocity and measurement velocity for the 24 particles using four drag models described in Fig. 6. KG model is best for the wheat straw particle with the smallest error of 95%. The KG-based model is used for the drag model development.

![Figure 6](image_url)

Figure 6. Relative error between simulated velocity and measured velocity for the 24 particles using four KG HL RN and FD models.

5.4. Fitting results of KG-based drag models using sheared particles

Shape factors of $\Pi$, $AR$, $BR$ and $\Phi$ are used to fit for the KG-based drag model. Assigns a series of discontinuous values for $B$, and fit the relationship between $f(Re_p,S)$ and $Re_p$, corresponding coefficients of $\beta_0, \beta_1, \beta_2$ are acquired.
According to $R^2$ and $R^2_{(adj)}$ (Adjusted R-Squared) and $p$-Value, the best $B$ and coefficients are determined. See Fig. 7 and Table 6 and Table 7.

![Figure 7](image)

**Figure 7.** $R^2$ and $R^2_{(adj)}$ changes with $B$ by curve fitting with the shape factor $\Pi$, $AR$, $BR$ and $\Phi$ used in the drag model.

| Coefficient | $\Pi$ | $AR$     | $BR$     | $\Phi$ |
|-------------|-------|----------|----------|--------|
| $\beta_0$   | 0.0082911 | 0.066814 | 0.16903  | 0.0535 |
| $\beta_1$   | 0.0040152 | 0.99445  | 0.61977  | 0.011961 |
| $\beta_2$   | 1.7789e-17 | 5.3474e-14 | 1.4163e-15 | 2.92E-17 |
| overall      | 3.05E-20  | 2.06E-19  | 3.05E-20  | 1.45E-16 |

**Table 6.** $p$Value of coefficient from curve fitting of different shape factors.

5.5. *Prediction effects of KG-based drag models*

Drag models with shape factors of $\Pi$, $AR$, $BR$, and $\Psi$ are used to simulate the 24 sheared particles moving in the still water and Drag models with shape factors of $\Pi_1$, $AR$, $BR$, $\Phi$ and $\Psi$ are used to simulate the 134 defibered particles moving in the still water. The relative error of particle velocity between simulation and measurement are listed in Fig. 8 and Fig. 9.

According to Fig. 8 and Fig. 9, drag model with shape factor $\Pi$, $AR$, $BR$ have the lower prediction relative error for sheared wheat straw particles, and drag model with shape factor $\Pi_1$, $AR$, $BR$, and $\Psi$ have the lower prediction relative error for sheared wheat straw particles. A comprehensive evaluation is applied by the mean relative error obtained by drag models with shape factors of $\Pi$, $AR$, $BR$ and $\Psi$, see Table 8. Base on the mean error in Table 8 and $p$Value in Table 7, KG-based model with shape factor is selected for simulating the settling velocity of wheat straw particle moving in the still water.
Figure 8. Relative error between simulation velocity and measurement velocity for the 24 particles using shape factors Π, AR, BR, and Ψ.

Figure 9. Relative error between simulation velocity and measurement velocity for the 134 particles using drag models with shape factors of Π, Π1, AR, BR, Φ and Ψ.

Table 7. Relative error prediction by drag models with shape factors of Π, AR, BR and Ψ.

|                | Π    | AR   | BR   | Ψ     |
|----------------|------|------|------|-------|
| Sheered wheat straw particles | 9.20%| 13.89%| 11.94%| 78.78%|
| Defibered wheat straw particles | 31%  | 32%  | 26%  | 39%   |
| Mean error     | 20%  | 23%  | 19%  | 59%   |

5.6. Error analysis
The error of the drag model to predict the falling velocity mainly come from the experimental measurement. The falling angel and the optical reflection is non-negligible reasons.

6. Conclusions
A KG-based drag model is developed with the form of $C_D = \frac{24H^B}{Re_p^2} \left( 1 + b \cdot Re_p^A \Pi^C \right)$, coefficients of A, B, C and b are 0.48452, -5.1988, 6 and $e^{3.0613}$, respectively.
The drag model with shape factor Π has higher prediction accuracy than other shape factors with the mean relative error of 20% for wheat straw particles.

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Acknowledgements
This research was funded by the key technology and demonstration project of the efficient recycling handles of human manure on rural toilets (2018YFC1903204) of the ministry of Science and Technology of China.