Higgs Scalar-Pseudoscalar Mixing in the Minimal Supersymmetric Standard Model

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Abstract

In the minimal supersymmetric Standard Model, the heaviest CP-even Higgs boson $H$ and the CP-odd Higgs scalar $A$ are predicted to be almost degenerate in mass at the tree level, for the wide kinematic range $M_A > 2M_Z$ and $\tan \beta \geq 2$. However, if large soft-CP-violating Yukawa interactions involving scalar quarks of the third generation are present in the theory, then the CP invariance of the Higgs potential can be maximally broken beyond the Born approximation, and the high degree of mass degeneracy between $H$ and $A$ may be lifted through a sizeable $HA$ mixing. After taking the CP-odd tadpole renormalization of the $A$ boson into account, we find that the small mass difference $M_H - M_A$, which is about 1% of the $A$-boson mass at the tree level, can be substantially enhanced to the 25% level at the one-loop order. We also find that the loop-induced mixing between the lightest CP-even Higgs boson $h$ and the $A$ boson may be of comparable size to $M_h$. We briefly discuss the main phenomenological implications of the predicted $hA$ and $HA$ mixings for the general Higgs-boson mass spectrum and for CP-violating observables at collider and lower energies.

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It is known \[1\] that the Higgs potential of the minimal supersymmetric Standard Model (MSSM) is invariant under the transformations of charge conjugation and parity (CP) at the tree level. The reason is that supersymmetry (SUSY) imposes an additional (holomorphic) symmetry on the Higgs sector of a general two-Higgs doublet model, which entails flavour conservation in tree-level neutral currents and absence of CP-violating scalar-pseudoscalar mixings in the Born approximation. Beyond the Born approximation, recent studies have shown that CP invariance of the Higgs potential may in principle be broken spontaneously through radiative corrections \[2\] if the CP-odd Higgs scalar \( A \) is sufficiently light \[3\] \[4\]. However, this possibility has now been ruled out by experiment \[3\] \[4\].

Here, we shall study in more detail another interesting possibility of CP non-conservation within the context of the MSSM. In this case, CP violation in the Higgs sector is induced by loop effects due to the presence of additional interactions in the theory, which violate CP explicitly. For example, such CP-violating interactions may occur in the trilinear Yukawa couplings of the Higgs fields to scalar quarks. However, the phenomenological viability of such a scenario of explicit CP violation is often questioned in the literature. The standard reasoning for the latter goes as follows. Unless the scalar quarks have TeV masses, all new CP-violating phases in the MSSM not present in the SM \[3\] \[4\] \[5\] \[6\] \[7\] must be suppressed at least by a factor of order \( 10^{-3} \), otherwise one-loop effects may exceed the current experimental limit on the neutron electric dipole moment (EDM). According to these general arguments, CP violation in the Higgs sector of the MSSM, which arises at one loop, was estimated to be dismally small, so as to bear any phenomenological relevance \[3\]. In this paper, we shall show that without any further assumptions on the model, this rough estimate based on the neutron EDM limit and the naive counting of loop suppression factors is fairly inaccurate and may be misleading in general. In particular, we find that scalar-pseudoscalar mixings can still be large in the MSSM giving rise to observable CP-violating phenomena, if the Yukawa sector involving the scalar top and bottom quarks contains relatively large CP-violating couplings compatible with universal boundary conditions imposed by minimal supergravity models at the unification scale \( M_X \).

The MSSM introduces several new phases in the theory which are absent in the SM \[3\]. The mixing mass parameter \( \mu \) involving the two Higgs chiral superfields in the superpotential, the soft-SUSY-breaking gaugino masses \( m_\lambda \) (with \( \lambda = \tilde{g}, \tilde{W} \) and \( \tilde{B} \) representing the gauginos of the SU(3)_c, SU(2)_L and U(1)_Y gauge groups, respectively), the soft bilinear Higgs mixing mass \( m_{12}^2 \) (sometimes denoted as \( B\mu \) in the literature) and the soft trilinear Yukawa couplings \( A_f \) are all complex numbers. If the universality condition at \( M_X \) is assumed, the gaugino masses \( m_\lambda \) are then related to each other and have the same phase, while the different trilinear couplings \( A_f \) are all equal, \( i.e., A_f = A \). Not all phases of the
four complex parameters \( \{ \mu, \, m_{12}^2, \, m_\lambda, \, A \} \) are independent of the fields [3]. For instance, one can make \( m_\lambda \) real by redefining the gaugino field \( \lambda \). Also, as we will see below, minimization conditions on the Higgs potential lead to the constraint that the phase of \( m_{12}^2 \) should be equal to the phase difference of the two Higgs doublets in the MSSM. As a result, there are only two independent CP-violating phases in this constrained version of the MSSM. Usually, these are taken to be \( \arg(\mu) \) and \( \arg(A) \). Limits coming from the electron EDM may be avoided to a great extent by requiring that the phases of \( m_\lambda \) and \( \mu \) be aligned to \( m_{12}^2 \), i.e., \( \Im(m_\lambda^2 \mu) = 0 \) [8]. In this case, chargino and neutralino mass matrices are real and conserve CP. Furthermore, it has been argued [8] that bounds obtained from the neutron EDM leave \( \arg(A) \) essentially unconstrained, even for slightly smaller than TeV soft scalar masses [8]. Notice that constraints on the scalar- top and bottom sector do not come directly from the neutron EDM but rather indirectly via the universality conditions at \( \hat{M}_X \). From the above discussion, it is clear that for soft scalar masses in the range \( 0.5 \leq M_0 \leq 1 \) TeV, \( \arg(A) \) can safely be considered to be the only large CP-violating phase in the theory of order unity. As a consequence, significant scalar-pseudoscalar mixings in the MSSM are only expected to come from loop effects of scalar top and bottom quarks, whereas chargino and neutralino contributions being proportional to the vanishingly small CP-violating phase \( \arg(\mu) \) may be neglected.

We start our discussion by considering the Higgs potential of the MSSM. Because of the holomorphic property of SUSY, one needs two Higgs doublets at least, denoted as \( \tilde{\Phi}_1 \) and \( \Phi_2 \), with opposite hypercharges, \( Y(\Phi_2) = -Y(\tilde{\Phi}_1) = 1 \), in order to give masses to both up- and down-quark families and, at the same time, cancel the triangle anomalies induced by the fermionic SUSY partners of the Higgs field. After integrating over the Grassmann-valued coordinates in the SUSY action and including the soft-SUSY-breaking masses for the Higgs fields, one arrives at the Lagrangian describing the Higgs potential

\[
L_V = \mu_1^2(\tilde{\Phi}_1^\dagger \Phi_1) + \mu_2^2(\tilde{\Phi}_2^\dagger \Phi_2) + m_{12}^2(\tilde{\Phi}_1^\dagger \Phi_2) + m_{12}^2(\tilde{\Phi}_2^\dagger \Phi_1) + \lambda_1(\tilde{\Phi}_1^\dagger \Phi_1)^2 + \lambda_2(\tilde{\Phi}_2^\dagger \Phi_2)^2 + \lambda_3(\tilde{\Phi}_1^\dagger \Phi_1)(\tilde{\Phi}_2^\dagger \Phi_2) + \lambda_4(\tilde{\Phi}_1^\dagger \Phi_2)(\tilde{\Phi}_2^\dagger \Phi_1),
\]

(1)

where \( \Phi_1 = -i\tau_2 \tilde{\Phi}_1^* \) (\( \tau_2 \) is the Pauli matrix) and

\[
\mu_1^2 = -m_1^2 - |\mu|^2, \quad \mu_2^2 = -m_2^2 - |\mu|^2, \quad \lambda_1 = \lambda_2 = -\frac{1}{8}(g^2 + g'^2), \quad \lambda_3 = -\frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = \frac{1}{2}g^2.
\]

(2)

The complex parameter \( m_{12}^2 \) in Eq. (1) as well as the real parameters \( m_1^2 \) and \( m_2^2 \) in Eq. (4) are soft Higgs-scalar masses. Furthermore, \( g \) and \( g' \) are the usual SU(2)_L and U(1)_Y gauge couplings, respectively. It is interesting to remark that the MSSM Higgs potential \( L_V \) contains fewer quartic couplings than that of the general two-Higgs doublet model; all
quartic couplings $\lambda_i$ in $\mathcal{L}_V$ are uniquely specified by SUSY. This makes the Higgs sector of the MSSM highly predictive.

In order to determine the ground state of the MSSM Higgs potential, we consider the linear decompositions of the Higgs fields

$$
\Phi_1 = \left( \frac{1}{\sqrt{2}} (v_1 + H_1 + iA_1) \right), \quad \Phi_2 = e^{i\xi} \left( \frac{1}{\sqrt{2}} (v_2 + H_2 + iA_2) \right),
$$

where $v_1$ and $v_2$ are the moduli of the vacuum expectation values (VEV’s) of the Higgs doublets and $\xi$ is their relative phase. The positive parameters $v_1$ and $v_2$ and the phase $\xi$ are entirely fixed by the minimization conditions on $\mathcal{L}_V$. This can be accomplished by requiring the vanishing of the following tadpole parameters:

$$
T_{H_1} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial H_1} \right\rangle = v_1 \left[ \mu_1^2 + \Re(e^{i\xi}m_{12}) \tan \beta - \frac{1}{2} M_Z^2 \cos 2\beta \right],
$$

$$
T_{H_2} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial H_2} \right\rangle = v_2 \left[ \mu_2^2 + \Re(e^{i\xi}m_{12}) \cot \beta + \frac{1}{2} M_Z^2 \cos 2\beta \right],
$$

$$
T_{A_1} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial A_1} \right\rangle = v_2 \Im(m_{12}^2 e^{i\xi}),
$$

$$
T_{A_2} \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial A_2} \right\rangle = -v_1 \Im(m_{12}^2 e^{i\xi}),
$$

where $\tan \beta = v_2/v_1$ and $M_Z^2 = (g^2 + g'^2)v^2/4$ is the Z-boson mass squared with $v^2 = v_1^2 + v_2^2$. Variations of $\mathcal{L}_V$ with respect to $\phi_1^+$ and $\phi_2^+$ vanish identically, reflecting the fact that a physical ground state should conserve charge [9]. If we now perform the orthogonal rotation

$$
\begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} =
\begin{pmatrix}
\cos \beta & -\sin \beta \\
\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
G^0 \\
A
\end{pmatrix},
$$

the Higgs potential shows up a flat direction with respect to the $G^0$ field, i.e., $\langle \partial \mathcal{L}_V / \partial G^0 \rangle = 0$. Then, the $G^0$ field becomes the true would-be Goldstone boson eaten by the longitudinal component of the Z boson. In this weak basis, the tree-level mass matrix of the CP-odd scalars becomes $\text{diag}(0, M_A^2)$, where $M_A^2 = \Re(e^{i\xi}m_{12}^2)/(s_\beta c_\beta)$ is the tree-level A-boson mass squared. Furthermore, the orthogonal rotation (8) leads to a non-trivial CP-odd tadpole parameter given by

$$
T_A \equiv \left\langle \frac{\partial \mathcal{L}_V}{\partial A} \right\rangle = -v \Im(m_{12}^2 e^{i\xi}).
$$

As has been shown explicitly in [10], the tadpole renormalization of the A boson is very crucial to render all $H_1G^0$, $H_2G^0$, $H_1A$ and $H_2A$ mixings ultra-violet (UV) finite (see also Fig. 1). It is now important to notice that the phase difference $\xi$ between the two Higgs
VEV’s is no more arbitrary but completely specified by virtue of Eq. (9). At the tree level, one has \( T_A = 0 \) and \( m_{12}^2 e^{i \xi} \) is a real number. Beyond the Born approximation however, \( m_{12}^2 e^{i \xi} \) acquires a small imaginary part. In fact, the non-vanishing tadpole graph of the \( A \)-boson should be compensated by the tadpole counter-term (CT) \( T_A \), such that the true ground state of the effective Higgs potential does not get shifted. Without any loss of generality, we can therefore redefine \( \Phi_2 \) as \( e^{-i \arg(m_{12}^2)} \Phi_2 \), thereby resulting in a weak basis in which \( \xi = 0 \) at zeroth order. Then, \( m_{12}^2 \) is real at the tree level but effectively receives an imaginary part at higher orders which is determined by the tadpole renormalization condition on \( T_A \). With this simplification, one can avoid unnecessary \( \xi \)-dependent phases in the MSSM.

After including all tadpole contributions, the Lagrangian relevant for the Higgs-boson mass matrix can be cast into the general form

\[
L^H_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} H_1, & H_2, & G^0, & A \end{pmatrix} \begin{pmatrix} \mathcal{M}^2_S & \mathcal{M}^2_{SP} \\ \mathcal{M}^2_{PS} & \mathcal{M}^2_P \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ G^0 \\ A \end{pmatrix}.
\]

(10)

Employing the usual short-hand notations \( s_x = \sin x \) and \( \cos x = c_x \), the 2 \( \times \) 2 sub-matrices in Eq. (10) are given by

\[
\mathcal{M}^2_S = \begin{pmatrix} c^2 \! M_Z^2 + s^2 \beta M_A^2 - T_{H_1} / v_1 & -s \beta c \beta (M_Z^2 + M_A^2) \\ -s \beta c \beta (M_Z^2 + M_A^2) & s^2 \beta M_Z^2 + c^2 \beta M_A^2 - T_{H_2} / v_2 \end{pmatrix},
\]

(11)

\[
\mathcal{M}^2_P = \begin{pmatrix} -c \beta T_{H_1} + s \beta T_{H_2} & s \beta T_{H_1} - c \beta T_{H_2} \\ s \beta T_{H_1} - c \beta T_{H_2} & M_A^2 - s \beta \tan \beta T_{H_1} + c \beta \cot \beta T_{H_2} \end{pmatrix},
\]

(12)

\[
\mathcal{M}^2_{SP} = -\frac{T_A}{v} \begin{pmatrix} s \beta & c \beta \\ -c \beta & s \beta \end{pmatrix},
\]

(13)

and \( \mathcal{M}^2_{PS} = (\mathcal{M}^2_{SP})^T \). In the Born approximation, the CP-even mass eigenstates \( h \) and \( H \) are obtained by diagonalizing \( \mathcal{M}^2_S \) through the orthogonal transformation

\[
\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix},
\]

(14)

with

\[
\tan(2\alpha) = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \tan(2\beta).
\]

(15)

The tree-level mass eigenvalues of \( \mathcal{M}^2_S \) are then given by

\[
M^2_{h(H)} = \frac{1}{2} \left[ M_Z^2 + M_A^2 - (+) \sqrt{(M_Z^2 + M_A^2)^2 - 4 M_A^2 M_Z^2 \cos^2 2\beta} \right].
\]

(16)
Notice that for $\tan \beta \geq 2$, one has $\cos^2 2\beta \approx 1$, $M_h \approx M_Z$ and $M_H \approx M_A$, that is, $H$ and $A$ are nearly degenerate in the MSSM. However, radiative corrections due to the large top Yukawa coupling affect the $h$-boson mass in a significant manner. In fact, $h$ can be as heavy as 130 GeV and is heavier than the $Z$ boson for the largest bulk of the parameter space [11]. On the other hand, the high degree of mass degeneracy persists even beyond the tree level in the CP-invariant limit of the theory, especially when $M_A > 2M_Z$ and $\tan \beta \geq 2$. In this kinematic range, $\tan \beta \approx \tan \alpha$ and quantum effects seem to affect equally $M_H$ and $M_A$, such that the small mass splitting of $H$ and $A$ remains still valid [12]. As we will see however, large CP-violating Yukawa couplings of scalar top and bottom quarks can give rise to sizable $HA$ mixings at the one loop level and to a substantial enhancement of the mass difference $M_H - M_A$. In the calculation of scalar-pseudoscalar self-energy transitions, one has to include the relevant CP-violating mass CT's given by $\mathcal{M}_{SP}^2$ in order to arrive at UV-safe analytic results.

We shall now discuss the interactions of the neutral Higgs fields with the scalar top and bottom quarks in the presence of CP-violating terms [13]. The scalar quarks of the third generation are represented by

$$
\hat{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}, \quad \hat{U}^* = \tilde{t}_R, \quad \hat{D}^* = \tilde{b}_R,
$$

(17)

with $Y(\hat{Q}) = 1/3$, $Y(\hat{U}) = -4/3$, and $Y(\hat{D}) = 2/3$. If we denote the neutral component of the Higgs doublets $\Phi_1$ and $\Phi_2$ by $\phi_1^0$ and $\phi_2^0$, there are then three contributions of $\phi_1^0$ and $\phi_2^0$ to scalar-quark masses and their respective couplings. The relevant Lagrangian, $\mathcal{L}_V^0$, receives contributions from the soft-SUSY-breaking sector, and from the so-called $F$ and $D$ terms (or components) of the chiral and vector superfields, respectively. Specifically, we have

$$
\mathcal{L}_V^0 = \mathcal{L}_{soft}^0 + \mathcal{L}_F^0 + \mathcal{L}_D^0,
$$

(18)

where

$$
- \mathcal{L}_{soft}^0 = M_0^2 (\tilde{t}_L^* \tilde{t}_L + \tilde{b}_L^* \tilde{b}_L) + \tilde{M}_t^2 \tilde{t}_R^* \tilde{t}_R + \tilde{M}_b^2 \tilde{b}_R^* \tilde{b}_R + \left( f_1 \Lambda_t \phi_1^0 s_\beta^* \tilde{t}_L + f_2 \Lambda_t \phi_2^0 s_\beta^* \tilde{t}_R + H.c. \right),
$$

$$
- \mathcal{L}_F^0 = f_1^2 \phi_1^0 \phi_1^0 (\tilde{b}_L^* \tilde{b}_L + \tilde{b}_R^* \tilde{b}_R) + f_2^2 \phi_2^0 \phi_2^0 (\tilde{t}_L^* \tilde{t}_L + \tilde{t}_R^* \tilde{t}_R) + \left( \mu f_2 \phi_1^0 \phi_2^0 \tilde{t}_L^* \tilde{t}_R + \mu f_1 \phi_2^0 \phi_2^0 \tilde{b}_L^* \tilde{b}_R + H.c. \right),
$$

$$
- \mathcal{L}_D^0 = \frac{M_Z^2}{v^2} \left[ (\phi_1^0 \phi_1^0 - \phi_2^0 \phi_2^0) (1 - 2e_t s_w^2) \tilde{t}_L^* \tilde{t}_L + 2e_t s_w^2 \tilde{t}_R^* \tilde{t}_R - (1 + 2e_b s_w^2) \tilde{b}_L^* \tilde{b}_L + 2e_b s_w^2 \tilde{b}_R^* \tilde{b}_R \right],
$$

(19)
with \( s_w = \sin \theta_w \) being the weak mixing angle, \( e_t = 2/3 \), \( e_b = -1/3 \), \( f_1 = \sqrt{2} m_b / v_1 \) and \( f_2 = \sqrt{2} m_t / v_2 \). Furthermore, \( \tilde{M}_Q, \tilde{M}_t \) and \( \tilde{M}_b \) are soft-scalar quark masses and are usually considered to be all equal to \( M_0 \) at the unification scale \( M_X \). Notice that all operators of dimension four in \( \mathcal{L}_V \) and \( \mathcal{L}_V^0 \) satisfy an additional global U(1)\(_Q\) symmetry with \( Q \) charges: \( Q(\Phi_1) = 2, Q(\Phi_2) = 1, Q(\bar{Q}) = 0, Q(\bar{U}) = -2 \) and \( Q(\bar{D}) = -1 \); the bilinear operator \( \Phi_1^1 \Phi_2 \) and the \( A \)-dependent trilinear terms break it only softly. In fact, if the Higgs-mixing term \( \mu \) in the superpotential is absent, the U(1)\(_Q\) symmetry can appropriately be extended to the whole MSSM Lagrangian. As has extensively been discussed in [10] and is also valid for the case of the MSSM, the simultaneous soft-breaking of the symmetries U(1)\(_Q\) and CP is sufficient to assure the renormalizability of Higgs scalar-pseudoscalar transitions to all orders in perturbation theory.

It is straightforward to obtain the scalar top and bottom mass matrices from \( \mathcal{L}_V^0 \) which may conveniently be expressed as

\[
\tilde{M}_q^2 = \begin{pmatrix} \tilde{M}_Q^2 + m_q^2 + \cos 2\beta M_Z^2 (T_2^q - e_q s_w^2) & m_q (\mu R_q + A_q) e^{i\delta_q} \\ m_q (\mu R_q + A_q) e^{-i\delta_q} & \tilde{M}_q^2 + m_q^2 + \cos 2\beta M_Z^2 e_q s_w^2 \end{pmatrix},
\]

(20)

where \( q = t, b \), \( T_2^t = 1/2, T_2^b = -1/2 \), \( R_t = \cot \beta, R_b = \tan \beta \). The phase \( \delta_q \) is determined from the requirement that the scalar-quark mass matrix becomes positive definite by a judicious re-definition of the right-handed scalar quark fields, \( i.e., \tilde{q}_R \rightarrow e^{i\delta_q} \tilde{q}_R \) with \( \delta_q = \arg(\mu R_q + A_q) \). In this weak basis, we can diagonalize \( \tilde{M}_q^2 \) through the orthogonal rotation

\[
\begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} = \begin{pmatrix} \cos \theta_q & \sin \theta_q \\ -\sin \theta_q & \cos \theta_q \end{pmatrix} \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix},
\]

(21)

where the rotation angle \( \theta_q \) may be obtained by

\[
\tan(2\theta_q) = -\frac{2 m_q |\mu R_q + A_q^*|}{M_Q^2 - M_q^2 + \cos 2\beta M_Z^2 (T_2^q - 2 e_q s_w^2)}. \]

(22)

The masses for two scalar-quark mass eigenstates \( \tilde{q}_1 \) and \( \tilde{q}_2 \) are then given by

\[
M_{\tilde{q}_1(\tilde{q}_2)}^2 = \frac{1}{2} \left\{ \tilde{M}_Q^2 + \tilde{M}_q^2 + 2 m_q^2 + T_2^q \cos 2\beta M_Z^2 \right. \\
\left. - (+) \sqrt{[ \tilde{M}_Q^2 - \tilde{M}_q^2 + \cos 2\beta M_Z^2 (T_2^q - 2 e_q s_w^2)]^2 + 4 m_q^2 |\mu R_q + A_q^*|^2} \right\}.
\]

(23)

It is easy to see that the scalar quarks of the first two families are almost degenerate for large universal soft-scalar quark masses, \( e.g., \) bigger than 0.5 TeV. However, this mass pattern is not in general true for the third family owing to the non-negligible top and bottom masses.
Defining as \( \varphi_k^0 = \sqrt{2} (\phi_k^0 - \langle \phi_k^0 \rangle) = H_k + i A_k \) with \( k = 1, 2 \), we are then able to write the trilinear couplings of the Higgs fields to the scalar quarks in the generic form

\[
-L_{\text{int}} = \sum_{q=t,b} \left[ \left( g_1^{Lq} \varphi_1^0 + g_2^{Lq} \varphi_2^0 \right) \tilde{q}_L \tilde{q}_L + \left( g_1^{Rq} \varphi_1^0 + g_2^{Rq} \varphi_2^0 \right) \tilde{q}_R \tilde{q}_R \right]
+ \left[ \left( h_1^0 \varphi_1^0 + h_2^0 \varphi_2^0 \right) \tilde{b}_L \tilde{b}_L + \left( h_1^0 \varphi_1^0 + h_2^0 \varphi_2^0 \right) \tilde{b}_R \tilde{b}_R + \text{H.c.} \right],
\]

where the different coupling parameters are given by

\[
\begin{align*}
g_1^{Lq} &= \frac{c_3 m_Z^2}{v} \left( \frac{1}{2} - e_t s^2_w \right), & g_2^{Lq} &= \frac{m_t^2}{s_\beta v} - \frac{s_\beta M_Z^2}{v} \left( \frac{1}{2} - e_t s^2_w \right), \\
g_1^{Rq} &= \frac{c_3 m_Z^2}{v} e_t s^2_w, & g_2^{Rq} &= \frac{m_t^2}{s_\beta v} - \frac{s_\beta M_Z^2}{v} e_t s^2_w, \\
g_1^{Lb} &= \frac{m_b^2}{c_\beta v} - \frac{c_\beta m_Z^2}{v} \left( \frac{1}{2} + e_b s^2_w \right), & g_2^{Lb} &= \frac{s_\beta M_Z^2}{v} \left( \frac{1}{2} + e_b s^2_w \right), \\
g_1^{Rb} &= \frac{m_b^2}{c_\beta v} e_b s^2_w, & g_2^{Rb} &= -\frac{s_\beta M_Z^2}{v} e_b s^2_w, \\
h_1^t &= \frac{m_t}{s_\beta v} \mu e^{-i \delta}, & h_2^t &= \frac{m_t}{s_\beta v} A_t e^{-i \delta}, & h_1^b &= \frac{m_b}{c_\beta v} A_b e^{-i \delta}, & h_2^b &= \frac{m_b}{c_\beta v} \mu e^{-i \delta}.
\end{align*}
\]

From Eqs. (24) and (25), it is then not difficult to compute the tree-level couplings \( H_1 \tilde{q}_i \tilde{q}_j \), \( H_2 \tilde{q}_i \tilde{q}_j \), \( A_q \tilde{q}_i \tilde{q}_j \) and \( G_q \tilde{q}_i \tilde{q}_j \) \((i, j = 1, 2)\). These couplings may respectively be given by the following matrices:

\[
\begin{align*}
\Gamma_{H_1 \tilde{q} \tilde{q}} &= \begin{pmatrix}
s_2 q \text{Re} h_k^q - c_2 q g_k^q - s_2 g_k^q + 2 i T \text{Im} m_h^q \\
-c_2 q \text{Re} h_k^q - s_2 c_q (g_k^q - g_q^R) - 2 i T \text{Im} m_h^q
\end{pmatrix}, \\
\Gamma_{A \tilde{q} \tilde{q}} &= i \frac{s_\beta}{T} \begin{pmatrix}
-i s_2 q \text{Im} m_h^q \\
-T \left( 2 \text{Re} h_1^q - \frac{c_\beta s_2 q \Delta M_q^2}{v} \right) + i c_2 q \text{Im} m_h^q
\end{pmatrix}, \\
\Gamma_{G \tilde{q} \tilde{q}} &= T \begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix},
\end{align*}
\]

where \( k = 1, 2 \) and \( \Delta M_q^2 = M_q^2 - M_{q_1}^2 \). Note that \( \text{Im} m_h^q = -\tan \beta \text{Im} h_2^q \) and \( \text{Re} h_1^q = -\tan \beta \text{Re} h_2^q + (s_q c_\beta \Delta M_q^2 / v) \) in the weak basis we are working.

As has been discussed above, the tadpole parameter \( T_A \) is determined by the A-boson tadpole graph, \( \Gamma^A(0) \), shown in Fig. 1(c). Using the renormalization condition \( T_A + \Gamma^A(0) = 0 \), we easily find that

\[
T_A = -\sum_{q=t,b} N_c^q \int \frac{d^4 k}{(2\pi)^n} \text{Tr} \left[ i \Delta \tilde{q} (k) i \Gamma_{H_1 \tilde{q} \tilde{q}} \right]
= -\frac{1}{16\pi^2} \sum_{q=t,b} N_c^q \frac{s_2 q}{s_\beta} \text{Im} m_h^q \Delta M_q^2 B_0(0, M_q^2, M_{q_1}^2).
\]

(27)
In Eq. (27), $N_c^q = 3$ is the colour factor for quarks, $\Delta \tilde{q}(k) = \text{diag}[(k^2 - M_{q_1}^2)^{-1}, (k^2 - M_{q_2}^2)^{-1}]$ is the scalar-quark propagator matrix and $B_0(s, m_{1}^2, m_{2}^2)$ is the known Veltman-Passarino function defined as

$$B_0(s, m_{1}^2, m_{2}^2) = C_{\text{UV}} - \ln(m_1 m_2) + 2 + \frac{1}{s} \left[ (m_2^2 - m_1^2) \ln \left( \frac{m_1}{m_2} \right) + \lambda^{1/2}(s, m_1^2, m_2^2) \cosh^{-1} \left( \frac{m_1^2 + m_2^2 - s}{2m_1 m_2} \right) \right], \quad (28)$$

where $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and $C_{\text{UV}} = 1/\varepsilon - \gamma_E + \ln(4\pi \mu^2)$ is an UV infinite constant. For $s = 0$, the one-loop function in Eq. (28) takes on the simple form

$$B_0(0, m_{1}^2, m_{2}^2) = C_{\text{UV}} - \ln(m_1 m_2) + 1 + \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_2}{m_1} \right). \quad (29)$$

A straightforward calculation of the individual contributions to $H_i A$ self-energies (with $i = 1, 2$), shown in Figs. 1(a) and (b), yields

$$\Pi_{(a)}^{H_i A}(s) = \sum_{q=t,b} N_c^q \int \frac{d^4 k}{(2\pi)^4 i} \text{Tr} \left[ i \Delta \tilde{q}(k) i \Gamma_0^{H_i \tilde{q}^* \tilde{q}} i \Delta \tilde{q}(k+p) i \Gamma_0^{A \tilde{q}^* \tilde{q}} \right]$$

$$= \frac{1}{16\pi^2} \sum_{q=t,b} N_c^q \Im h_1^q \left\{ \frac{r_{i} s_{2q}}{s_{\beta} v} \Delta M_q^2 B_0(s, M_{q_1}^2, M_{q_2}^2) \right.$$ \hspace{1cm} (30)

$$+ \Re h_1^q \frac{s_{2q}}{s_{\beta}} \left[ B_0(s, M_{q_1}^2, M_{q_2}^2) + B_0(s, M_{q_2}^2, M_{q_1}^2) - 2B_0(s, M_{q_1}^2, M_{q_2}^2) \right]$$

$$+ \left( s_{2q} \frac{L_q^R}{s_{\beta}} + s_{2q} \frac{L_q^L}{s_{\beta}} \right) \left[ B_0(s, M_{q_2}^2, M_{q_1}^2) - B_0(s, M_{q_1}^2, M_{q_2}^2) \right] \right\}, \quad (30)$$

$$\Pi_{(b)}^{H_i A}(s) = - (M_{ZP})_{ij} = \frac{r_i T_A}{v}$$

$$= - \frac{1}{16\pi^2} \sum_{q=t,b} N_c^q \Im h_1^q \frac{r_{i} s_{2q}}{s_{\beta} v} \Delta M_q^2 B_0(0, M_{q_1}^2, M_{q_2}^2), \quad (31)$$

where $s = p^2$, $r_1 = \cos \beta$ and $r_2 = \sin \beta$. It is easy to verify that the $H_i A$ self-energies are UV finite, only after the CP-odd tadpole contribution given in Eq. (31) is taken into account. The CP-violating $h A$ and $H A$ self-energy transitions are then obtained by

$$\left( \begin{array}{c} \Pi_{t}^{h A}(s) \\ \Pi_{t}^{H A}(s) \end{array} \right) = \left( \begin{array}{cc} \cos \alpha \Pi_{t}^{h A}(s) + \sin \alpha \Pi_{t}^{H A}(s) \\ -\sin \alpha \Pi_{t}^{h A}(s) + \cos \alpha \Pi_{t}^{H A}(s) \end{array} \right). \quad (32)$$

For the kinematic range $M_A > 2M_Z$ and $\tan \beta > 2$ ($\tan \alpha \approx \tan \beta$), we have $\Pi_{t}^{h A}(s) \approx \Pi_{t}^{H A}(s)$ and $\Pi_{t}^{H A}(s) \approx -\Pi_{t}^{H A}(s)$. Furthermore, the rotation angles $\theta_t$ and $\theta_b$ are almost equal to $\pi/4$, i.e., $s_{2q} \approx 1$, since the off-diagonal elements of the scalar-quark mass matrix
$$\tilde{M}_q^2$$ are much bigger than the difference of its diagonal entries. For relatively low \(\tan \beta\) values, the biggest contribution occurs when the mass difference between \(\tilde{t}_1\) and \(\tilde{t}_2\) is rather large. Specifically, for \(2 \leq \tan \beta < 5\), we have

$$\Pi^{HA}(0) \approx \frac{3}{8\pi^2} \Im h_1^t \left[ \Re h_2^t \left( \frac{x_t^2 + 1}{x_t^2 - 1} \ln x_t - 1 \right) - \frac{m_t^2}{v} \ln x_t \right],$$

(33)

$$\Pi^{hA}(0) \approx -\frac{3}{8\pi^2} \Im h_1^t \Re h_1^t \left( \frac{x_t^2 + 1}{x_t^2 - 1} \ln x_t - 1 \right),$$

(34)

with \(x_t = M_{\tilde{t}_2}/M_{\tilde{t}_1} \geq 1\). In Eqs. (33) and (34), the CP-violating quantities \(\Im h_1^t \Re h_1^t\) and \(\Im h_1^t \Re h_2^t\) satisfy the inequalities

$$|\Im h_1^t \Re h_1^t| \leq \frac{1}{2} \frac{m_t^2}{v^2 s_\beta^2} |\mu|^2, \quad |\Im h_1^t \Re h_2^t| \leq \frac{1}{2} \frac{m_t^2}{v^2 s_\beta^2} |\mu A_t| .$$

(35)

For larger \(\tan \beta\) values, scalar-pseudoscalar mixings receive significant contributions from scalar-bottom quarks as well, leading to a more involved kinematic dependence.

We shall now examine numerically the dependence of \(HA\) and \(hA\) mixings on the various kinematic parameters. Motivated by minimal supergravity models, we consider the soft scalar-quark masses and trilinear couplings to be universal: \(\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = M_0 = 0.5\) TeV and \(A_t = A_b = A\) with \(\arg(A) = 90^\circ\) \([14]\). The soft parameters \(m_1^2\) and \(m_2^2\) in the Higgs potential (c.f. Eq. (2)) are also equal to \(M_0^2\) at \(M_X\). However, their renormalization-group-equation (RGE) runnings from \(M_X\) down to \(M_Z\) are very sensitive to further model-dependent details of the physics near the unification scale \([14]\). Therefore, we assume that \(m_1^2\) and \(m_2^2\) are free parameters not larger though than few TeV, being subject into the tadpole constraints in Eqs. (4) and (5). For our numerical analysis, we consider the parameters \(\{\tan \beta, \mu, M_0, A\}\) as independent and take \(M_A = M_0\). We expect that the above considerations will be in good agreement with a more elaborate treatment. Refinements due to the RGE running of the MSSM parameters are beyond the scope of the present paper and may be studied elsewhere.

In Fig. 2, we display the dependence of the \(HA\) and \(hA\) self-energies on the parameter \(\mu\) at vanishing momentum transfer, \(s = 0\), for three selected values of \(|A|\). With \(\mu\) increasing, both the CP-violating quantities in Eq. (35) and the scalar-top mass ratio \(x_t\) increase and so the \(HA\) and \(hA\) mixings. Indeed, we observe that the mass difference \(M_H - M_A \approx \Pi^{HA}(0)/M_A\) can be as large as \(25\% \times M_A\), for \(\mu \gtrsim 2\) TeV \((M_A = 0.5\) TeV\) and \(M_{\tilde{t}_1}, M_{\tilde{b}_1} > M_Z\). This should be contrasted with the tree-level prediction of \(1.07\% \times M_A\) for the same value of \(\tan \beta = 2\). As has been discussed in \([10],[13]\), an \(HA\) mass splitting of order \(10\%\) of the \(A\)-boson mass is sufficient to give rise to significant contributions to the neutron EDM through the two-loop Barr-Zee mechanism \([16]\) close to the observable.
Furthermore, we find that \( \Pi^{hA}(0) \) may reach the (100 GeV)\(^2\) level in the same kinematic region and so be comparable in size with \( M_h \) at one loop. Nevertheless, the net effect of \( hA \) mixing on the \( h \)-boson mass will be a modest reduction of the \( M_h \) one-loop value by \( -\frac{(\Pi^{hA})^2}{2M_h^2M_A^2} \), ranging from \(-2\%\) to \(-10\%\) \( (M_h \approx 100 \text{ GeV}) \) in the \( A \)-boson mass interval 500 – 200 GeV.

In Fig. 3, we plot the dependence of the scalar-pseudoscalar mixings as a function of energy \( (E_{cms} = \sqrt{s}) \). We find that \(-\Re \Pi^{hA}(s)\) (solid line) can increase by one order of magnitude for \( s \approx M_A^2 \) in comparison to the \( s = 0 \) value. The cause for this strong energy dependence appears to be the subtle cancellations occurring at low energies between terms proportional to \( \Re h_2^2 \) and \( m_t^2/v \) in Eq. (32) which get less important at higher energies. For similar reasons, \( \Re \Pi^{HA}(s) \) (solid line) shows up a milder energy dependence because of the absence of such destructive terms (see also Eq. (34)). As can be seen from Fig. 3 too, the absorptive parts \( \Im m^{HA}(s) \) and \( \Im m^{hA}(s) \) (dashed lines) become significant above the \( \tilde{t}_1 \) threshold. In general, we have that both \( |\Re \Pi^{HA}(M_A^2)| \) and \( |\Im m^{HA}(M_A^2)| \) can formally be of comparable order. Thus, the necessary conditions for resonant CP violation through \( HA \) mixing [18] at future high-energy machines, such as the LHC, \( \mu^+\mu^- \) collider, etc., may comfortably be satisfied [15]. In particular, at a \( \mu^+\mu^- \) collider with an integrated luminosity of 50 fb\(^{-1}\), one may be sensitive to \( HA \) mixings up to order \( 10^{-3} \) TeV\(^2\), corresponding to mass differences \( M_H - M_A \gtrsim 1\% \times M_A \). This is in agreement with an earlier observation made in [18] that CP asymmetries of order unity due to resonant Higgs scalar-pseudoscalar transitions can naturally occur even within the MSSM [19]. Finally, the dependence of \( HA \) and \( hA \) mixings on tan \( \beta \) is presented in Fig. 4. We see that \( \Pi^{HA}(0) \) decreases in general for large values of \( \tan \beta \), while \(-\Pi^{hA}(0)\) exhibits a significant enhancement for \( \tan \beta \gtrsim 50 \) close to the perturbative bound of the theory. The reason for this enhancement is due to the sizable left-right mixing of the scalar bottom quarks in the large \( \tan \beta \) domain.

The fact that scalar-pseudoscalar mixings may be large within the MSSM should not be very surprising and may be understood in simple terms as follows. All quartic couplings of the MSSM Higgs potential as well as \( M_h \) are suppressed by gauge coupling constants as a result of SUSY. On the other hand, despite the typical loop suppression factor \((4\pi)^{-2}\), Higgs scalar-pseudoscalar mixings are enhanced by the large top Yukawa couplings proportional to \( m_\mu/v^2 \) and \( m_A/v^2 \). In this case it is therefore evident that naive dimensional analysis of counting loop suppression factors can lead to a dramatic underestimation of the actual size of the quantum effects. As has been shown by the present paper, a large Higgs scalar-pseudoscalar mixing is indeed possible within the MSSM and can lead to observable CP-violating phenomena in near future experiments.
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Figure captions

Fig. 1: Feynman graphs contributing to the $H_1A$ and $H_2A$ mixings: (a) One-loop self-energy graph, (b) CP-odd tadpole renormalization, (c) Tadpole graph of the $A$ boson.

Fig. 2: $HA$ and $hA$ self-energies at $s = 0$ as a function of the parameter $\mu$.

Fig. 3: $HA$ and $hA$ self-energies versus energy $E_{cins} = \sqrt{s}$. Solid lines indicate dispersive parts of the scalar-pseudoscalar self-energies, while dashed lines correspond to their absorptive parts.

Fig. 4: $HA$ and $hA$ self-energies at $s = 0$ as a function of $\tan \beta$.

\begin{figure}[h]
\centering
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{fig1a}
\caption{(a) $H_1, H_2 \times \tilde{q}_1, \tilde{q}_2, \tilde{q}_1^*, \tilde{q}_2^*$}
\end{subfigure}
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{fig1b}
\caption{(b) $A \leftrightarrow T_A \leftrightarrow H_1, H_2$}
\end{subfigure}
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=0.8\textwidth]{fig1c}
\caption{(c) $\tilde{q}_1, \tilde{q}_2$}
\end{subfigure}
\caption{Fig. 1}
\end{figure}
$\tan \beta = 2 \quad M_0 = 0.5 \text{ TeV}$

$\arg(A) = 90^0$

- $|A| = 0.6 \text{ TeV}$
- $|A| = 0.8 \text{ TeV}$
- $|A| = 1.0 \text{ TeV}$

Fig. 2
Fig. 3

$\Gamma_{hH}^{\text{et}}(s) \ [\text{TeV}^2]$  

$E_{\text{cms}} \ [\text{GeV}]$

- $\mu = 1 \text{ TeV}$
- $\tan \beta = 2$
- $M_0 = 0.5 \text{ TeV}$
- $|A| = 1 \text{ TeV}$
- $\arg(A) = 90^0$

$\Gamma_{hH}^{\text{et}}(s) \ [\text{TeV}^2]$  

$E_{\text{cms}} \ [\text{GeV}]$

- $\mu = 1 \text{ TeV}$
- $\tan \beta = 2$
- $M_0 = 0.5 \text{ TeV}$
- $|A| = 1 \text{ TeV}$
- $\arg(A) = 90^0$
\[ M_0 = 0.5 \text{ TeV} \]
\[ |A| = 1 \text{ TeV} \]
\[ \text{arg}(A) = 90^\circ \]

\[ m = 1.0 \text{ TeV} \]
\[ m = 0.3 \text{ TeV} \]

Fig. 4