THE GLOBAL NORMAL DISK OSCILLATIONS AND THE PERSISTENT LOW-FREQUENCY QUASI-PERIODIC OSCILLATIONS IN X-RAY BINARIES

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Received 2000 June 22; accepted 2000 August 30; published 2000 October 3

ABSTRACT

We suggest that persistent low-frequency quasi-periodic oscillations (QPOs) detected in the black hole (BH) sources XTE J1118+480, GRO J1655−40, and LMC X-1 at ~0.1 Hz, in HZ Her/Her X-1 at ~0.05 Hz, and in neutron star (NS) binaries 4U 1323−62, 4U 1746−31, and EXO 0748−76 at ~1 Hz are caused by the global disk oscillations in the direction normal to the disk (normal mode). We argue that these disk oscillations are a result of the gravitational interaction between the central compact object and the disk. A small displacement of the disk from the equatorial plane results in a linear gravitational restoring force opposite to this displacement. Our analysis shows that the frequency of this mode is a function of the mass of the central object and that it also depends on the inner and outer radii of the disk, which in turn are related to the orbital period of the binary system. We derive an analytical formula for the frequency of the normal disk mode and show that these frequencies can be related to the persistent lower QPO frequencies observed in the NS and BH sources. We offer a new independent approach to the BH mass determination by interpreting this low QPO frequency as the global disk oscillation frequency. The implementation of this method combined with the independent method recently developed by Shrader & Titarchuk that uses the X-ray energy spectra results in stringent constraints for the BH masses.

Subject headings: accretion, accretion disks — black hole physics — diffusion — stars: individual (EXO 0748−676, GRO J1655−40, Hercules X-1, LMC X-1, XTE J1118+480, 4U 1323−62, 4U 1746−31) — stars: neutron — X-rays: stars

1. INTRODUCTION

In previous papers (Osherovich & Titarchuk 1999; Titarchuk & Osherovich 1999, hereafter TO99; Titarchuk, Osherovich, & Kuznetsov 1999), we have suggested a new two-oscillator (TO) model, which has led to the classification of quasi-periodic oscillations (QPOs) in frequency range from ~1 Hz to around a kilohertz. All oscillations in the TO model are related to the local conditions in the disk.

The classification offered by the TO model does not include the persistent low-frequency oscillations (~10−2−1 Hz) that have been observed in many neutron star (NS) and black hole (BH) candidate systems (see reviews by van der Klis 1995, 2000). We suggest in this Letter that these persistent low-frequency oscillations are related to the global disk oscillations under the influence of the gravitational force of the central object—in contrast to the variable frequencies that are characteristic of the local properties of the disk and the magnetosphere. Below we show that these persistent frequencies carry information about the system as a whole, namely, the size of the disk and the mass of the central object.

Persistent low-frequency QPOs with ~1 Hz frequencies were discovered in NS binaries 4U 1323−62 by Jonker, van der Klis, & Wijnands (1999), EXO 0748−676 by Homan et al. (1999, hereafter H99), and 4U 1746−31 by Jonker et al. (2000). Recently, Boroson et al. (2000) have found a significant QPO feature with ~0.05 Hz in the power density spectrum (PDS) of UV flux from the HZ Her/Her X-1 system. Similar persistent QPO features with ~0.1 Hz frequencies have been found in the transient source XTE J1118+480 in the far-ultraviolet and optical (Haswell et al. 2000) wavelengths. The QPO feature with ~0.08 Hz in XTE J1118+480 was first detected in X-rays in data from the Proportional Counter Array on the Rossi X-Ray Timing Explorer (RXTE; Revnivtsev, Sunyaev, & Borozdin 2000). Ebisawa, Mitsuda, & Inoue (1989) have also found a narrow 0.08 Hz QPO peak in the LMC X-1 PDS in the 1−16 keV range.

It has been emphasized by H99 that several properties of the low-frequency QPO, particularly its frequency and its relatively unchanged persistence during bursts and dips, are remarkably similar among different stars. We suggest these QPOs are caused by vertical oscillations of the disk, which we call the global disk mode (GDM) oscillations.

In § 2 we derive the GDM frequency. We compare the calculations of the GDM with the QPO observations in BH sources and present a new method of the BH mass determination using the derived GDM oscillation frequency in § 3. Comparisons with the observations in NS sources are made in § 4. Conclusions follow in § 5.

2. GLOBAL DISK MODE OSCILLATIONS: FORMULATION OF THE PROBLEM AND SOLUTION

We consider the oscillations of the disk as the whole body under the influence of gravity of the central compact object shown as a filled circle in Figure 1. We approximate the surface density distribution in the geometrically thin Shakura-Sunyaev
disk (Shakura & Sunyaev 1973) by the formula
\[
\Sigma = \Sigma_0 = \text{constant for } R_{\text{in}} \leq R \leq R_{\text{adj}},
\]
\[
\Sigma = \Sigma_0 \left( \frac{R}{R_{\text{adj}}} \right)^{-\gamma} \text{ for } R_{\text{adj}} \leq R \leq R_{\text{out}},
\]
where \( R_{\text{in}} \) is the innermost radius of the disk, \( R_{\text{adj}} \) is an adjustment radius in the disk, and \( R_{\text{out}} \) is the outer radius of the disk. Titarchuk, Lapidus, & Muslimov (1998; see also Titarchuk & Osherovich 2000) calculated the size of the transition disk region between the Keplerian disk and the inner disk edge, and they found that the typical radius of the adjustment radius depending on the effective Reynolds’s number is within \((2-3)R_{\text{in}}\).

We assume that the disk as a whole is perturbed and that it deviates from equatorial plane by a small distance \( h \). In Figure 1 we have enlarged that distance for the purpose of illustration. The restoring force \( F(h) \) caused by gravitational attraction of the central object is
\[
F(h) = GM_X \pi h \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{\Sigma 2R \, dR}{(h^2 + R^2)^{3/2}},
\]
where \( M_X \) is the mass of the central object and \( G \) is the gravitational constant. After integration using the density distribution (eqs. [1] and [2]) with assumptions that \( R_{\text{out}} \ll R_{\text{adj}} \) and \( \dot{h} \ll R_{\text{in}} \), we get
\[
F(h) \approx \frac{2\pi GM_X \Sigma_0 \pi h}{R_{\text{in}}} \left[ 1 - \frac{\gamma}{(\gamma + 1)} \frac{R_{\text{in}}}{R_{\text{adj}}} \right].
\]
The mass of the disk \( M_d \) from \( R_{\text{in}} \) to \( R_{\text{out}} \) is
\[
M_d = 2\pi \int_{R_{\text{in}}}^{R_{\text{out}}} \Sigma R \, dR \approx \frac{2\pi \Sigma_0 \pi R_{\text{out}}^{2/\gamma}}{2 - \gamma} \left( \frac{R_{\text{out}}^{1-\gamma}}{R_{\text{adj}}^{1-\gamma}} \right) \text{ for } \gamma \neq 2.
\]
The vertical oscillations of the disk as a whole can be described by the following equation of motion:
\[
M_d \ddot{h} + F = 0,
\]
which can be written in the form
\[
\ddot{h} + \omega_0^2 \dot{h} = 0, \quad \omega_0^2 = (2\pi \nu_0)^2 = \frac{F}{h M_d}.
\]

Using equations (4), (5), and (7), we present the GDM frequency as
\[
\nu_0 = \frac{2.2 \times 10^3}{m} \left( \frac{2 - \gamma [1 - \gamma/(\gamma + 1)]}{R_{\text{in}}^{1-\gamma} R_{\text{adj}}^{1-\gamma}} \right)^{1/2},
\]
where \( x_{\text{adj}} = R_{\text{in}}/3R_{\text{out}} \), \( r_{\text{out}} = R_{\text{adj}}/R_{\text{out}} \), \( x_{\text{adj}} = R_{\text{adj}}/R_{\text{out}} \), \( m = M_{\text{X}}/M_{\odot} \), and \( R_{\text{X}} = 2GM_{\text{X}}/c^2 \) is the Schwarzschild radius.

According to the Shakura-Sunyaev disk model (Shakura & Sunyaev 1973), the index \( \gamma \) of the surface density can be either \( \frac{3}{2} \) or \( \frac{2}{3} \). For \( \gamma = \frac{3}{2} \) and for typical parameter values of the disk model around NSs—\( m = M_{\text{X}}/M_{\odot} = 1.4 \), \( x_{\text{adj}} = 1 \), \( r_{\text{adj}} = 3 \), and \( r_{\text{out}} = 10 \)—equation (8) reads \( \nu_0 \approx 2 \text{ Hz.} \) For \( \gamma = \frac{2}{3} \) the value of \( \nu_0 \) is also close to 2 Hz.

The size of the disk \( R_{\text{out}} \) can be estimated using the size of the Roche lobe \( R_L \). Following de Jong, van Paradijs, & Augusteijn (1996), we assume that the relative size of the accretion disk \( f = R_{\text{out}}/R_L \) is approximately 0.5. De Jong et al. adopted this empirical size from eclipse-mapping observations on cataclysmic variables (Rutten, van Paradijs, & Timbergen 1992). Having in mind that according to Paczyński (1967)
\[
R_L = 0.46a \left( \frac{M_X}{M_{\odot} + M_{\text{opt}}} \right)^{1/3}
\]
and using the third Kepler law relating the distance \( a \) between the optical and X-ray counterparts of the binary, the masses \( (M_X \text{ and } M_{\text{opt}}) \) and the orbital period of the binary \( P \),
\[
a = \left( \frac{P}{2\pi} \right)^{2/3} \left[ G(M_X + M_{\text{opt}}) \right]^{1/3},
\]
we find
\[
R_{\text{out}} \approx 0.5R_L = 0.23(GM_X)^{1/3} \left( \frac{P}{2\pi} \right)^{2/3}.
\]
For periods of the binary \( P \) measured in units of 3 hr and masses of the X-ray source \( M_X \) measured in solar units, we have the following size of the disk:
\[
R_{\text{out}} = 1.7 \times 10^{10} m^{1/3} P^{2/3} \text{ cm},
\]
\[
r_{\text{out}} = 1.92 \times 10^4 m^{-1/3} P^{2/3} x_{\text{adj}}^{-1} \text{ cm}.
\]

But in fact, fitting the UV data obtained from IUE, Howarth & Wilson (1983, hereafter HW83) found that the size of the disk in HZ Her/Her X-1 is approximately 1.5 times larger than that given by equation (12). They gave \( R_{\text{out}} = (1.69 \pm 0.25) \times 10^{11} \text{ cm} \) instead of \( R_{\text{out}} = 1.08 \times 10^{11} \text{ cm} \) obtained from equation (12) for \( P = 1.7 \text{ days} \) and \( m = 1.4 \). Thus, an
uncertainty of a factor of 1.5 exists in the estimate of the disk size by equation (12).

Using equation (13) for the value of $r_{\text{out}}$ and $\gamma = \frac{3}{2}$, we get the GDM frequency $\nu_0$ from equation (8) as

$$\nu_0 \approx 2(\text{Hz})^{x_{\infty}^{8/15} m_{\text{BH}}^{8/15} P_1^{7/15}} r_{\text{adj}}^{-0.3}. \quad (14)$$

3. BLACK HOLE MASS DETERMINATION AND THE GLOBAL MODE OSCILLATION FREQUENCY

Given the success of the bulk motion Comptonization model (Titarchuk, Mastichiadis, & Kylafis 1997; Titarchuk & Zannias 1998; Shrader & Titarchuk 1999, hereafter ShT99) in reproducing the high soft state continuum, ShT99 have investigated the potential predictive power of this model. From the spectral shape and normalization, one can calculate an effective disk size and the mass-to-distance ratio $m/d$. Other quantities, such as the BH mass and mass accretion rate, can be determined if the distance or mass are known independently. This is dependent on an additional factor $T_h$, the so-called hardening factor, which represents the ratio of color to effective temperature. If one can identify several “calibrators,” i.e., sources for which distance and mass are well determined, the model can be applied to derive $T_h$ over the available dynamic range in luminosity. Three such sources are (1) GRO J1655–40, for which extensive analysis was presented in previous papers (BOR99 and ShT99), (2) LMC X-1, for which the distance is well constrained, and (3) Nova Muscae 1991, for which the binary parameters are reasonably constrained. In the latter two cases, the hardening factor $T_h \approx 2.6$, was used, which was previously derived from the analysis of GRO J1655–40 (BOR99). The temperature-flux curve presented in BOR99 and ShT99 provide a self-consistency test of this assumption.

Thus, using the energy spectra of the high soft state only, one can get strong constraints on the BH masses. Here we have verified this energy spectrum method using the global oscillation frequency model for LMC X-1 and GRO J1655–40, and then we determine the BH mass for XTE J1118+480 using the persistent QPO frequency ~0.1 Hz discovered by independent groups in the different energy bands.

3. LMC X-1

ShT99 have estimated the BH mass in LMC X-1 as $m = (16 \pm 1) (0.5/\cos i)^{1/2}$, and they inferred that $R_{\infty} \approx 3R_{\text{eq}}$. Using the mass $m = 16$, the period $P = 4.22$ days found by Hutchings et al. (1983), and the estimates of $x_{\infty} = 1$ and $r_{\text{adj}} = 2$ (for high mass accretion rate), we find from equation (14) that $\nu_0 = 0.072$ Hz. This value of $\nu_0$ is very close to the narrow 0.08 ± 0.009 Hz QPO peak found by Ebisawa et al. (1989) with a harmonic seen in the 1–16 keV range. The agreement of $\nu_0$ with the observed frequency may be viewed as a confirmation of the correct mass estimate given by ShT99 for this source.

3.2 GRO J1655–40

Remillard et al. (1999) have found the relatively stable frequency near 0.1 Hz that appears in the high soft state. The mass $m \approx 7$ and the period $P = 2.62$ days for this source is known from the observations by Orosz & Bailyn (1997). Using the values in equation (14), $x_{\infty} = 1$ (BOR99), and $r_{\text{adj}} = 2$, we find that $\nu_0 = 0.14$ Hz, which is comparable with the observable value of $\nu_0 = 0.1$ Hz.

3.3 XTE J1118+480

The source XTE J1118+480 is presumably a BH candidate (Revnivtsev et al. 2000). The source is observed in the low hard state during the rising phase of the 2000 April outburst (Hynes et al. 2000). In this source, almost three identical QPO frequencies were detected in optical (0.1 Hz), in ultraviolet (0.08 Hz) by the Hubble Space Telescope (HST; Haswell et al. 2000), and in X-ray by RXTE (0.08 Hz; Revnivtsev et al. 2000) and by ASCA (0.11 Hz; Yamaoka et al. 2000). The stability and independence of this frequency on the photon energy is a necessary feature of the GDM when the disk oscillates as a whole body. We do not exclude that these oscillations of the outer part of the disk may be amplified by the resonance effect due to reprocessing of X-ray flux oscillating with the same frequency. In fact, a preliminary analysis by Haswell et al. (2000) indicates that ultraviolet variability lags are behind the X-ray by 1–2 s. This is likely due to the X-ray reprocessing in the disk. The energy spectrum of XTE J1118+480 is a typical thermal Comptonization spectrum (Sunyaev & Titarchuk 1980) with the best-fit parameters optical depth $\tau = 3.2$ and $kT_\text{in} = 33$ keV.

Recently, Wood et al. (2000) have demonstrated from extensive observations of XTE J1118+480 using the Unconventional Stellar Aspect (USA) experiment and RXTE that the QPO shows an upward drift from 0.07 to 0.15 Hz, while the source intensity slowly rises and then decreases. We can interpret this behavior of the QPO frequency by a significant change of the disk size $r_{\text{out}}$ during the spectral evolution of the source. The spectral evolution is related to the change of the mass accretion rate in the disk (Chakrabarti & Titarchuk 1995), which in turn causes the change of the disk size. R. Soria (2000, private communication) argues that such a change is seen in GRO J1655–40. In other words, when the disk mass accretion rate drops, the spectrum gets harder and the QPO frequency increases (see eq. [8]) because the size of the disk ($r_{\text{out}}$) decreases. If our interpretation is correct, we expect that the QPO frequency correlates with the spectral hardness but not with the source flux.

Following ShT99, we suggest that in the low hard state the inner disk edge retreats approximately at 17 Shwarschild radii as a result of evaporation of the innermost part. From equation (14) we infer the BH mass in XTE J1118+480 to be $20 M_\odot$ with an accuracy of 10%, taking the observed 0.1 Hz as the GDM frequency, $x_{\infty} = 17/3 = 5.7$ (ShT99), $P = 4.1$ hr (Uemura et al. 2000), and $r_{\text{adj}} = 3$. This mass is comparable to that ShT99 have found in LMC X-1 and Nova Muscae.

4. COMPARISON OF GLOBAL MODE OSCILLATION FREQUENCY WITH PERSISTENT LOW FREQUENCIES OBSERVED IN A NUMBER OF NS SOURCES

4.1 Her X-1

Averintsev, Titarchuk, & Sheffer (1992) made a numerical simulation of the process of occultation of the X-ray emission region on the NS surface and compared their results with the observed X-ray emission of Her X-1. They demonstrated constraints for a number of geometrical parameters of the Hz Her/Her X-1 system; in particular, they found the position of the inner edge of the accretion disk at $r_{\text{in}} = (10 \pm 2)R_{\text{NS}}$. Taking $R_{\text{in}} = 10R_{\text{NS}}$, $R_{\text{out}} = 1.69 \times 10^{11}$ cm (HW83), $P = 1.7$ days...
(Deeter et al. 1991), $m = 1.4$, and $r_{\text{adj}} = 3$ we find from equation (14) that $\nu_0 \approx 0.067$ Hz, which is close to the QPO frequency 0.05 Hz obtained by Boroson et al. (2000) by analyzing the PDS of HST UV data.

4.2 4U 1323–62, EXO 0748–676, and 4U 1746–37

The low-mass X-ray binary sources 4U 1323–62, EXO 0748–676, and 4U 1746–37 exhibit QPO frequencies of about 1 Hz (see references in § 1). It was noted by Jonker et al. (1999) that a medium modulating the radiation from a central source is a promising explanation. Here we put forward the idea that the oscillations of X-radiation with the low frequency can be a result of the GDM modulation of the X-radiation from a central source. The frequency of GDM is expected to be very stable because it depends mostly on the intrinsic characteristics of the system such as mass and orbital period (or the disk size). These characteristics are comparable for the three aforementioned NS sources. The periods are 2.93, 3.82, and 5.7 hr for 4U 1323–62, EXO 0748–676, and 4U 1746–37, respectively (van Paradijs 1995). To estimate $\nu_0$, we take $x_{\text{in}} = 1$ (TO99) and $m = 1.4$, which is a typical value of the NS mass. The dependence of $\nu_0$ on the period according to equation (14) is not strong, $\nu_0 \propto R^{-7/15}$, which means almost the same value of $\nu_0$ for each of the sources within uncertainties caused by assumptions regarding the disk size and the adjustment radius.

The stability of $\nu_0$ can exist until the inner part of the disk is disrupted or just moved to the larger radii. In principle, magnetic force and radiation pressure can modify the restoring force of the GDM. The related effects are amenable, and they will be taken into account at later time.

5. CONCLUSIONS

In this Letter we have presented a model of the global disk oscillation mode. Our analysis shows that the frequencies of this mode of 0.01–1 Hz are closely related to the large ratio of sizes (10$^3$) of the disk and its inner portion. This ratio reduces the characteristic kHz Keplerian frequency of the inner part of the disk by 3–4 orders of magnitude. The GDM is a vertical mode, and thus it should be often seen from the systems with the high inclination angle. However, because of X-ray processing in the outer part of the disk and deformation of the disk, there is also a chance of detecting these QPOs from systems with small inclination angles. The GDM frequency is expected to be also independent of the photon energy and should be seen in all energy bands that the disk emits. The particular part of the viscous disk is presumably seen in the specific energy range because the effective temperature of the disk $T_{\text{eff}}$ is a function of radius $R$, namely, $T_{\text{eff}} \propto R^{-3/4}$.

This Letter offers a new method of BH mass determination based on the persistent low QPO frequency measurement; it supplements the X-ray energy spectrum method and the dynamical optical method of mass function determination. This new approach, in our view, is very promising, and it allows an independent verification of BH mass obtained by other methods.

The authors acknowledge discussions with Chris Shrader and Bram Boroson and thank Joe Fainberg for valuable comments on an earlier version of this Letter.

REFERENCES

Averintsev, M., Titarchuk, L. G., & Sheffer, E. K. 1992, AZh, 69, 71
Boroson, B., et al. 2000, ApJ, in press
Borozdin, K., Revnivtsev, M., Trudolyubov, S., Shrader, C. R., & Titarchuk, L. G. 1999, ApJ, 517, 367 (BOR99)
Chakrabarti, S., & Titarchuk, L. G. 1995, ApJ, 455, 623
Deefer, J. E., et al. 1991, ApJ, 383, 324
de Jong, J., van Paradijs, J., & Augusteijn, T. 1996, A&A, 314, 484
Ebisawa, K., Mitsuda, K., & Inoue, H. 1989, PASJ, 41, 519
Haswell, C. A., Skillman, D., Patterson, J., Hynes, R. I., & Cui, W. 2000, IAU Circ. 7427
Homan, J., Jonker, P. G., Wijnands, R., van der Klis, M., & van Paradijs, J. 1999, ApJ, 516, L91 (H99)
Howarth, I. D., & Wilson, B. 1983, MNRAS, 202, 347 (HW83)
Hutchings, J. B., Crampton, D., & Cowley, A. P. 1983, ApJ, 275, L43
Hynes, R. I., Mauche, C. W., Haswell, C. A., Shrader, C. R., Cui, W., & Chaty, S. 2000, ApJ, 539, L37
Jonker, P. G., van der Klis, M., Homan, J., Wijnands, R., van Paradijs, J., Méndez, M., Kuikkers, E., & Ford, E. 2000, ApJ, 531, 453
Jonker, P. G., van der Klis, M., & Wijnands, R. 1999, ApJ, 511, L41
Laurent, Ph., & Titarchuk, L. G. 1999, ApJ, 511, 289 (LT98)
Orosz, J. A., & Bailyn, C. D. 1997, ApJ, 477, 876
Osherovich, V. A., & Titarchuk, L. G. 1999, ApJ, 522, L113
Paczyński, B. 1967, Acta Astron., 17, 287
Remillard, R. A., Morgan, E. H., McClintock, J. E., Bailyn, C. D., & Orosz, J. A. 1999, ApJ, 522, 397
Revnivtsev, M., Sunyaev, R. A., & Borozdin, K. I. 2000, A&A, in press (astro-ph/0005212)
Rutten, R. G. M., van Paradijs, J., & Tinbergen, J. 1992, A&A, 260, 213
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337 (SS73)
Shrader, C. R., & Titarchuk, L. G. 1999, ApJ, 521, L121 (ShT99)
Sunyaev, R. A., & Titarchuk, L. G. 1980, A&A, 86, 121
Titarchuk, L., Lapidus, I. I., & Muslimov, A. 1998, ApJ, 499, 315
Titarchuk, L., Mastichiadis, A., & Kylafis, N. D. 1997, ApJ, 487, 834
Titarchuk, L., & Osherovich, V. 1999, ApJ, 518, L15 (TO99)
———. 2000, ApJ, 537, L39
Titarchuk, L., Osherovich, V., & Kuznetsov, S. 1999, ApJ, 525, L129 (TOK)
Titarchuk, L., & Zannias, T. 1998, ApJ, 493, 863 (TZ98)
Uemura, M., et al. 2000, PASJ, in press (astro-ph/00042450)
van der Klis, M. 1995, in X-Ray Binaries, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 252
———. 2000, ARA&A, in press (astro-ph/0001167)
van Paradijs, J. 1995, in X-Ray Binaries, ed. W. H. G. Lewin, J. van Paradijs, & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 536
Wood, K. S., et al. 2000, ApJ, in press (astro-ph/0006234)
Yamaoka, K., Ueda, Y., Dotani, T., Drououchoux, P., & Rodríguez, J. 2000, IAU Circ. 7427