Comprehending finger flexor tendon pulley system using a computational analysis

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Abstract

Existing prosthetic/orthotic designs are rarely based on kinetostatics of a biological finger, especially its tendon-pulley system (TPS) which helps render a set of extraordinary functionalities. Studies on computational models or cadaver experiments do exist. However, they provide little information on TPS configurations that lead to lower tendon tension, bowstringing, and pulley stresses, all of which a biological finger may be employing after all. A priori knowledge of such configurations and associated trade-offs is helpful not only from the design viewpoint of, say, an exoskeleton but also for surgical reconstruction procedures. We present a parametric study to determine optimal TPS configurations for the flexor mechanism. A compliant, flexure-based computational model is developed and simulated using the pseudo rigid body method, with various combinations of pulley/tendon attachment point locations, pulley heights, and widths. Deductions are drawn from the data collected to recommend the most suitable configuration. Many aspects of the biological TPS configuration are explained through the presented analysis. We reckon that the analytical approach herein will be useful in arriving at customized (optimized) hand exoskeletal designs.

Keywords: Tendon-pulley system, finger biomechanical model, finger flexion, bionic hand devices

1. Introduction

Nature has gifted human fingers with the ability to perform extraordinarily diverse movements that are combinations of the four basic types - flexion, extension, and adduction. Anatomically, flexor tendon pulley system (TPS) and extensor mechanism are responsible for transferring power from respective muscles to phalangeal bones, to perform these movements. In case of injury or post-stroke cognitive impairment, the patient may need TPS reconstruction surgery or artificial devices depending on cases and severity. Therefore, it is crucial to understand the biomechanics of the TPS involved.

Flexion-extension mechanics of a finger is studied mainly in two ways: (i) performing surgical procedures on cadaver hands, and (ii) using computational models. Computational models provide a non-destructive alternative to studies using cadaver hands thereby reducing the number of cadaver surgeries required. Given the complex anatomy of the hand, developing an accurate and generic model is still being actively researched and remains a challenging task.

Several studies exist on the role of each pulley and tendon in the flexor TPS (Fig. 1). Among annular pulleys, A2 is the widest, followed by A4. Stress in pulley fibers depend on pulley location and width. High pulley stress can cause discomfort in finger movement (Schweizer (2008)). Some cadaver studies suggest that smaller width of A2 and A4 pulleys can be used without compromising much on the flexion range (Mitsionis et al. (1999); Chow et al. (2014); Lee and Coert (2014)). Those by Solonen and Hoyer (1967) and Hume et al. (1991) that employ computational models contradict on the effect of change in pulley positions. Loosen-
ing/removal of pulleys can cause bowstringing (i.e., tendon moving away from bones during flexion-extension), which reduces the range of flexion (ROF), creating difficulty in forming a fist, or grasping (Dy and Daluiski (2013); Brand et al. (1975)). While designing artificial systems or performing TPS reconstruction surgery, proper knowledge of pulley locations, widths, and heights (i.e., loosening), and, exclusion of pulleys/tendon if required and the trade-offs involved, will help make the best possible decisions.

Existing literature does not focus much on the tendon tension requirement for finger flexion. It is desirable to have the highest range of flexion (ROF) for a given tendon tension for the corresponding muscle load to be reduced. This is even more critical when designing bionic artificial devices, as, selection of actuators, and battery power requirements would pose a limit on this tension. This paper presents a parametric study to arrive at optimal flexor TPS configurations, which maximizes ROF while keeping bowstringing and pulley stress as small as possible. We also investigate whether having both FDS and FDP tendons improves the flexion range, or one can be ignored to simplify the design of an artificial device without significant loss in functionality.

The paper is organized as follows. The computational biomechanical model developed is presented in section 2. Parametric study and results are described in section 3. TPS configurations are recommended and comparisons with the biological finger made in section 4, followed by conclusion in section 5.

2. Methods

We modeled the flexor mechanism of an index finger\(^1\) as a beam-string arrangement described in Fig. 2. Human finger joints have non-fixed axes of rotation and inherent stiffnesses (van Nierop et al. (2008)). Hence, we modeled them as flexure hinges which possess similar deformation characteristics (Guo and Lee (2013)). Biological finger tendons experience minimal strain (Pring et al. (1985)), and thus, were modeled as inextensible strings. C–pulleys are cross-shaped (cruciate), flexible–inextensible, and remain loose unless pulled by a tendon. We implemented these characteristics through flexible–inextensible string loops (Fig. 3).

2.1. Mathematical Formulation

For full range flexion, the flexure hinges must undergo large bending deflection with small strain. A 2R or 3R pseudo-rigid-body model (PRBM) of such a flexure is computationally simpler than the nonlinear finite element method (FEM), and gives much smaller approximation error compared to a 1R PRBM (Su (2009) and Yu et al. (2012)). Nevertheless, we chose 1R PRBM because (i) in our analysis, it gave results sufficiently close to those from FEM and the experiments performed (Appendix A and Appendix B), and (ii) this study concerns only with relative responses of different TPS configurations. The proposed model of the overall TPS is thus, 3R, as shown in Fig. 4a. We assumed that the finger moves slowly while changing its posture. Hence, joint velocities and accelerations were considered zero. All external (contact and non-contact) forces were assumed absent. Friction was neglected in tendon-pulley contacts. With these assumptions, governing equations of the quasistatic system can be written from Fig. 4b as:

\[ K_j(\theta_j - \theta_{j0}) = \sum_{i=1}^{2} T_i d_{ij}, \quad j = 1, 2, 3, \quad d_{32} = 0 \tag{1} \]

where, \( K_j \) is the stiffness and \( \theta_{j0} \) is the neutral position of the torsional spring at \( j^{th} \) joint. Here, \( j = 1, 2, 3 \) identify the three joints MCP,PIP, and DIP (and the three phalanges PP, IP, and DP), respectively. \( \theta \) is the corresponding joint angle. Moment arm \( d_{ij} \) at the \( j^{th} \) joint is \( ||Z_jN_{j1}|| \) (Solonen and Hoyer (1967)) for \( r^{th} \) tendon tension \( T_r \), as shown in Fig. 4c–d. Indices \( r = 1, 2 \) correspond to FDP and FDS tendons, respectively. Since FDS tendon does not exert moment on the DIP joint, the moment arm \( d_{32} \) was set to zero. Moment

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\(^1\)Index finger is the most dexterous among all four fingers, and is, therefore, a good choice for framing a generic model.
Figure 4: Equivalent 3R Pseudo-Rigid-Body Model (PRBM) shown in (a): Each flexure in its undeformed state is replaced by a revolute joint at its center (Zj) and a torsional spring. Moment balance to solve the joint kinetics: Free body diagram (FBD) shown in (b) of the distal portion of finger attached to torsional spring at the PIP joint. Assumed external force absent. Κ j is the stiffness of θωj, the normal position of torsional spring equivalent to the PIP joint. Computation of moment arm at jth joint with inactive pth pulley shown in (c) and active pth pulley in (d): Pulley proximal to the joint, shown as an example. All vertices of ΔZjSjQj+1 (xi = p or p − 1), and therefore moment arm dp of the tension Tj can be expressed in terms of joint angles θj, pulley locations xj, heights hj, and widths wj. 2bij is the bone width of the jth phalanx. t = 1, and 2 denote FDP and FDS tendons, respectively. The pulley angle βp is −90° for a stiff pulley, and needs to be computed for a flexible-inextensible C−pulley.

The moment arm dp can be computed in terms of the TPS parameters, as follows.

Let p = 0, 1, 2, ⋯ denote the sequence of pulleys/TAPs in which they are connected with a given tendon. Ground pulley is always indexed as 0. As an example, p = 1 and 2 for pulleys C1 and A2, respectively, if pulley C1 is proximal to pulley A2. Let xj, wj, and hj be relative position, width, and height of pth pulley (Fig. 4). The pulley end is marked with points Qp, Rj, and Sj (Fig. 4d) occupying positions qj, rj, and sj, respectively. Let jth joint (Zj) occupy position zj. Using complex algebra, we may write:

\[
\begin{align*}
  z_1 &= 0, \quad z_2 = l_1 e^{i\theta_1}, \quad z_3 = z_2 + l_2 e^{i(\varphi_2 + \theta_2)} \\
  r_p &= z_j + \left[ x_p - ih_j + \frac{h_p}{2} e^{i(\varphi_j + \frac{\pi}{2} + \sum_{k=1}^{p-1} \varphi_k)} \right] e^{i(\sum_{k=p}^{j} \varphi_k)} \\
  q_p &= r_p - \frac{w_p}{2} e^{i(\varphi_j + \frac{\pi}{2} + \sum_{k=1}^{p-1} \varphi_k)} \\
  s_p &= r_p + \frac{w_p}{2} e^{i(\varphi_j + \frac{\pi}{2} + \sum_{k=1}^{p-1} \varphi_k)} \\
  \text{if } p \text{th pulley is on } j \text{th phalange}
\end{align*}
\]

Here, lj and 2bj are length and nominal bone-width of jth phalange, respectively. We assumed βp = −π/2 for stiff pulleys. If the flexible-inextensible C−pulley is active, as in Figs. 3 and 4d, it orients itself along the angle bisector of the two segments of tendon in contact. This ensures zero bending moment in that C−pulley. Hence, the corresponding angle βp is solved by minimizing the following objective:

\[
\Delta \phi = |\phi_1 - \phi_2| \quad (3)
\]

where, φ1 and φ2 are angles with the pulley direction (UpRj) made by the left and right tendon segments (QpSp−1) and (SpQp+1), respectively, as shown in Fig. 4d (enlarged view). For a flexible pulley indexed p, φ1 and φ2 can be expressed in terms of pulley parameters and joint angles as follows:

\[
\phi_1 = \arg \left( \frac{r_p - u_p}{\overline{s_p} - q_p} \right), \quad \phi_2 = \arg \left( \frac{q_{p+1} - s_p}{r_p - u_p} \right) \quad (4)
\]

pth pulley becomes active when the shortest distance of the tendon segment Sp−1Qp+1 from the base Up becomes equal to or smaller than the pulley height. In that case, the moment arm base Nj lies on SpQp+1, otherwise on Sp−1Qp+1. In case flexible pulley is located distally relative to the joint, Nj lies on Sp−1Qp when the pulley is active, and Sp−1Qp+1 otherwise.

Point Nj (nj) on the nearest segment SjQj (xi = p or p − 1, η = p or p + 1) can be found using the following two conditions:

(i) ZjNj ⊥ SjQj implying:

\[
(n_j - z_j)(s_j - q_j)^\ast + (n_j - z_j)^\ast(s_j - q_j) = 0 \quad (5)
\]

(ii) Nj lies on the line SjQj. Therefore,\n
\[
n_j = (1 - \alpha)s_j + \alpha q_j \quad (6)
\]
Figure 5: Procedure for solving Eq. (1) involving joint-locking and two tendons. $T_s$ is the sum $T_1 + T_2$ of tensions in FDP and FDS tendons. $\gamma$ is the ratio $T_j/T_s$ which we choose a priori when both tendons are actuated. Subscript $m$ indicates maximum permissible values. Iterations continue until maximum tension is reached or all joints lock. Owing to discrete nature of numerical simulations, $\partial_j$ can exceed the limit $\theta_{\text{im}}$ in an incremental step, for one or more joints.

4d) due to FDP and FDS tendons as follows:

$$\begin{align*}
\sigma_{\text{axial}} &= \frac{1}{wD}(T_1 + T_2)(\cos \phi_1 + \cos \phi_2) \\
\sigma_{\text{bending}} &= h(T_1 + T_2)(\sin \phi_1 - \sin \phi_2) \frac{w/2}{I} \\
\sigma_{\text{net}} &= |\sigma_{\text{axial}}| + |\sigma_{\text{bending}}|
\end{align*}$$

where, $h$, $w$, $D$ and $I = Dw^3/12$ are the height, width, depth and area moment of inertia of the pulley. The maximum resultant stress exists at either $V_p$ or $W_p$ on the pulley base (Fig. 4d), where bending stress is maximum. When computing stresses in flexible-inelastic pulleys, the pulley-width is assumed to remain $w$ even when pulley bends, and the pulley tip-line QS normal to the pulley length RU. This works well for pulleys of small widths. Biologically, FDS tendon terminates just proximally to the $A_4$ pulley. Likewise, it may not interact with pulleys $A_4$ and $C_3$ if they are located distally to FDS-TAP in a candidate TPS configuration. In that case, we substitute $T_2 = 0$ for them in Eqs. (10). We defined the highest $\sigma_{\text{net}}$ among all pulleys as the critical value $PS$.

3. Results

Using the 3R PRBM, we determined the effect of TPS parameters on the range of flexion (ROF) and critical values of bowstringing $B_w$ and pulley stresses $PS$. Various finger model data used in the PRBM are listed in Tab. 1 and results are summarized in Tab. 2.

To explain the observations, we qualified pulleys and TAPs as proximal (P), central (C), or distal (D) based on their locations on the respective phalanges (Fig. 6). We also named TPS configurations using the characters P, C, and D in groups separated by hyphens (−) or double hyphens (=). The leftmost group describes pulleys on the proximal phalanx. The rightmost character describes TAP. For example, C–D–P indicates one central pulley on the proximal phalanx, one distal pulley on the intermediate phalanx, and proximal FDP-TAP on the distal phalanx. A hyphen (−) indicates that only one tendon is active, whereas double-hyphen (=) indicates that both FDP and FDS tendons are active. In the above example, only FDP tendon is active. Another example C–C–
Table 1: Finger Model Data: Joint stiffnesses $K_1$, $K_2$, and $K_3$ in Nmm/deg, as per Kim et al. (2019); Dionysian et al. (2005); Kamper et al. (2002). Phalange lengths $l_1$, $l_2$, and $l_3$ in mm, measured joint-to-joint, inclusive of flexure length $l_f$ (Fig. 4). Default values of pulley widths $w_0$, heights $h_0$, out-of-plane thicknesses $t_0$, bone-widths $b_0$, and offsets $e_0$ (Fig. 6) in mm. Bone-width assumed equal for all phalanges. Ground pulleys $G_1$ and $G_2$ assumed coincident at $(X_g, Y_g)$. Tendon tension incremented in $n$ steps upto $T_0$ N. Neutral positions $\phi_{1m}, \phi_{2m}$, and $\phi_{3m}$ of finger joints, assumed to coincide with fully extended state, to simulate full range of motion as flexion. Finger joint limits $\phi_{1m}, \phi_{2m}$, and $\phi_{3m}$, as per Zheng and Li (2010).

| Constants | Values | Constants | Values | Constants | Values | Constants | Values |
|-----------|--------|-----------|--------|-----------|--------|-----------|--------|
| $K_1$     | 0.95   | $l_1$     | 42.0   | $\phi_{10}$ | 0      | $\phi_{1m}$ | 90°    |
| $K_2$     | 0.60   | $l_2$     | 27.0   | $\phi_{20}$ | 0      | $\phi_{2m}$ | 100°   |
| $K_3$     | 0.60   | $l_3$     | 19.5   | $\phi_{30}$ | 0      | $\phi_{3m}$ | 80°    |

Table 2: Range of Flexion (ROF) and critical values of bowstringing ($B_w$), and pulley stress (PS) for FDP-TPS configurations yielding ROF $> 240°$ and FDS-TPS configurations yielding ROF $> 150°$. Subscripts to $B_w$ and PS values indicate the joints and pulleys, respectively, where those critical values occur. Table arranged in descending order of ROF at 8 N tension in FDP or FDS tendon individually. Superscript † implies ROF, $B_w$, and PS values at 6.4 N tension in the FDP and FDS tendons when both tendons are actuated simultaneously with equal tensions. Offsets $e_j$ of locations P and D, of 10% distance from joints (marked with superscript 10) calculated as percentage of bone lengths as: $e_j = 0.1e_{10} = (l_j - l_{10})/10 + l_{10}/2$, where $j = 1, 2, 3$ indicates PF, IP, and DP, respectively. Heights $h_j$ and widths $w_j$ of pulleys $A_3$ and $A_4$. Similarly, $h_j$ and $w_j$ are heights and widths of pulleys $C_1$ and $C_2$. Heights of FDP-TAP and FDS-TAP $h_j$ in all cases. In some cases, MCP joint was ignored while computing $B_w$ (Figs. 10a, 10b, 10d).

| ROF at 5 N | Configuration | $B_w$ (mm) | PS (MPa) | $e_j$ | $h_a$ | $h_c$ | $w_a$ | $w_c$ | Refer to marked curves in figures |
|-----------|---------------|------------|--------|------|-------|-------|-------|-------|-------------------------------|
| 251° † 270° | C-D=D=C      | 9.2 MCP    | 1.2 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 252° † 270° | C-D=D=D      | 9.2 MCP    | 1.1 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 176° 256° | C-D=C-P      | 8.9 PIP    | 1.1 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 176° 256° | C-D=C-D      | 9.0 MCP    | 2.2 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 200° 256° | C-D=C-D      | 9.8 PIP    | 1.1 C1 | 0.5 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 200° 256° | C-D=C-D      | 10.3 PIP   | 0.2 A4 | — | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 223° 256° | C-D=C-C      | 13.1 PIP   | 0.9 A4 | — | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 223° 256° | C-D=C-D      | 13.6 PIP   | 2.2 A4 | — | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 223° 256° | C-D=C-D      | 13.8 PIP   | 7.5 A4 | — | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10c |
| 224° 256° | C-D=C-D      | 14.1 PIP   | 7.3 A4 | — | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 226° 254° | C-D=C-C      | 14.3 PIP   | 13.7 A4 | — | 3.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 177° 252° | C-D=C-D      | 8.9 PIP    | 2.1 C1 | 10 $e_j$ | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 200° 250° | C-D=C-D      | 13.6 PIP   | 2.4 A4 | 10 $e_j$ | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 148° 247° | C-D=C-D      | 9.0 MCP    | 1.4 C1 | 10 $e_j$ | 0.5 h0 | 8.0 | 8.0 | w0 | — | Fig. 10c |
| 166° 246° | C-D=C-D      | 7.9 PIP    | 8.1 C1 | 10 $e_j$ | 0.5 h0 | 8.0 | 8.0 | w0 | — | Fig. 10b |
| 176° 246° | C-D=C-D      | 8.9 PIP    | 2.2 C1 | 10 $e_j$ | 3.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 184° 246° | C-D=C-D      | 18.9 PIP   | 1.8 A2 | — | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 151° 245° | C-D=C-D      | 9.0 MCP    | 1.5 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 151° 245° | C-D=C-D      | 9.0 MCP    | 1.6 C1 | 10 $e_j$ | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 187° 243° | C-D=C-D      | 18.9 PIP   | 1.8 A2 | — | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 187° 242° | C-D=C-D      | 13.6 PIP   | 2.4 A4 | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 187° 242° | C-D=C-D      | 18.9 PIP   | 1.8 A4 | — | 0.5 h0 | 2.0 | 2.0 | w0 | — | Fig. 10d |
| 114° 176° | C-D=CD      | 9.0 MCP    | 2.2 C1 | 10 $e_j$ | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 143° 176° | C-D=C-D      | 13.6 PIP   | 0.8 A2 | — | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 143° 176° | C-D=C-D      | 18.9 PIP   | 1.8 A2 | — | 2.0 | 2.0 | w0 | — | Fig. 12 |
| 114° 174° | C-D=CD      | 9.0 MCP    | 2.8 C1 | 10 $e_j$ | 2.0 h0 | 2.0 | 2.0 | w0 | — | Fig. 12 |
implying nothing on the distal phalange, indicates FDS tendon, with central pulley on the proximal and central FDS-TAP on the intermediate phalange. CP=CD= C indicates one central and one proximal pulley on the proximal phalange, and one central and one distal pulley on the intermediate phalange. Both tendons are active. Further, tilde over P indicates that the corresponding pulley is flexible–inelastic ones increases flexion range to 256° (Fig. 10b). Bowstringing becomes 8.9 mm. With proximal C–pulleys, as in TPS configuration CP–CD–C, flexion range is smaller, and bowstringing and pulley stresses are higher (Tab. 2). Without C–pulleys, increasing widths of pulleys A₂ and A₄ from 0.5 mm (very thin) to 2 mm reduces pulley stress from 7.5 MPa to 0.9 MPa, without affecting flexion range much (Fig. 10c, Tab. 2). Increasing heights of pulleys A₂ and A₄ with width w₀, from 0.5 mm to 3.5 mm, increases the pulley stress from 2.4 MPa to 13.7 MPa (Fig. 10d). Flexion range does not change much, while bowstringing increases from 13.6 mm to 14.8 mm. In presence of flexible C–pulleys, the pulley stress increases from 1.8 MPa to 2.2 MPa, while bowstringing remains unaffected (Fig. 10d). The flexion range decreases from 256° to 246°.

To study the FDS-TPS, we analyzed case of the FDS tendon with one pulley per phalange. Figure 11a describes nine TPS configurations, formed from three candidate positions of A₂ pulse and FDS—TAP. As observed in Fig. 11b–d, TPS configuration C–C gives the highest flexion range of 176°, bowstringing B₀, of 13.6 mm, and pulley stress PS of 0.8 MPa. TPS configuration C–D results in higher bowstringing of 18.9 mm, and higher pulley stress of 1.8 MPa. To reduce bowstringing, we added C₁ pulley with offset e₁ = 0.1 w₁, width w₀, and height 2 mm. With the resulting TPS configurations CD–C– and CD–D–, B₀ reduces to 9.0 mm, and pulley stress increases to 2.2 and 2.8 MPa respectively, without affecting the flexion range (Tab. 2).

To study the combined actuation of FDP and FDS tendons, we arrived at the common configuration CD=CD=C, as follows. For highest flexion range, FDP–TPS configurations are CD–CD–C and CD–CD–D, and FDS–TPS configurations are CD–C– and CD–D– (Tab. 2). Both bowstringing and pulley stress are much lower for FDS–TPS configuration CD–C–. Hence, we merged it with FDP-TPS configuration CD–CD–C. When both tendons were actuated in this configuration CD=CD=C, sharing equal loads, full finger flexion (270°) was achieved at much lower individual tendon tension of 6.4 N (Fig. 12, Tab. 2). Pulley-stress increased to 2.8 MPa from 2.2 MPa for full flexion with only FDP-tendon achieved at 10.1 N.

Overall, critical bowstringing was observed mostly at MCP or PIP joint, while critical stress at pulley A₄ or C₁ (Tab. 2).

4. Discussion

Results for FDP-TPS with one pulley per phalange show that pulley locations for high ROF (> 240°, Tab. 2) suffer from high bowstringing and high pulley stress. This problem can be addressed without affecting ROF adversely, (i) by adding flexible–inelastible C–pulleys slightly away from joints, and (ii) by either increasing width or decreasing height of annular pulleys or both. Including FDS tendon increases ROF further. We quantified bowstringing ≈ 9 mm as small, a limit obtained as sum of bone semi-width 3.5 mm, pulley height 2 mm, and the clearance 3 mm beyond pulley height.
4.1. The Biological Tendon Pulley System

We observed that the centrally located annular pulleys $A_2$ and $A_4$ result in very high flexion ranges (Tab. 2), thus agreeing with Dy and Daluiski (2013) and Chow et al. (2014). Role of flexible-inelastic biological C–pulleys is evident from sig-
(a) Role of flexible-inelastic C–pulleys and their heights: \( h_a = h_0, \; w_a = w_c = w_0, \; e = e_0. \) Higher the height \( h_c \) of C–pulleys, higher the ROF, \( B_w \), and PS. Flexible-inelastic C–pulleys yield higher flexion range and lower pulley stress.

(b) C-Pulley locations: \( h_c = 2.0 \text{ mm}, \quad \text{and} \quad h_a = h_0. \) The next three values are for the case with stiff C–pulleys. The last three values correspond to the case with flexible-inelastic C–pulleys. For each kind of C–pulley, values are listed in the order of legends corresponding to C–pulley heights \( h_c \). Subscripts show locations where these critical values occur. Similar order is followed in (b)–(d) too. In (b), for each kind of C–pulley, values are listed in the order of legends corresponding to C–pulley offsets \( e_j \), whereas in (c) to A–pulley heights \( h_a \), and in (d) to A–pulley widths \( w_a \). Bowstringing near MCP joint was observed higher (9 mm) in some cases. We excluded it in (a), (b) and (d) to demonstrate the effect of changes in parameters. Star and diamond (hollow) markers indicate PIP and DIP joint locking respectively.

A slight increase in heights of the annular pulleys increases pulley stress significantly in the absence of C–pulleys (Fig. 10d). This result explains why loosening of the main pulleys \( A_2 \) and \( A_4 \) during injury is painful when C–pulleys get torn. In this case, high bowstringing is also observed. However, it remains unexplained why ROF reduces in the case of biological TPS. Biologically, FDS–TAP is immediately proximal to pulley \( A_4 \), and therefore nearly central on IP. Further, both FDP and FDS tendons share the same set of pulleys. Both...
Figure 11: a–d, top–bottom, FDS–TPS flexion response: (a) shows the conventions followed in (b), (c), and (d): Three locations — proximal (P), central (C), and distal (D) considered for A2 pulley (differentiated by colors) and FDS–TAP (differentiated by markers). Star-marker indicates PIP joint locking. In (c), critical value $w$ of bowstringing is the maximum of MCP and PIP moment arms. The subplot (d) shows critical values of resultant stress at the base of A2 pulley. Pulley/FDS–TAP heights $h = h_0$, and widths $w_1 = w_0$; $i = 1, 2$. In (b), higher the curve, better is the FDP–TPS configuration. In (c) and (d), lower the curve, better is the TPS configuration.

Figure 12: Combined actuation of FDP and FDS tendons in TPS configurations CD–CD–X, CD–X–, and CD=CD=X, where FDP/FDS-TAP is central (X = C) or distal (X = D); A2 pulley and FDS–TAP are assumed at the same location in a given TPS configuration. Delta, square, and disk marked curves correspond to cases when load is exerted by (i) FDP tendon only, (ii) FDS tendon only, and (iii) both tendons together. Pentagon, star, and diamond markers indicate MCP, PIP, and DIP joint locking, respectively. The first three values correspond to the case with central FDP/FDS-TAP, while the next three to distal FDP/FDS-TAP. For each case, bowstringing ($B_w$) and pulley stress ($PS$) values are listed in the order of the legends for markers. Pulley stress is lower for $X = C$.

4.2. Bionic devices based on tendon-pulley system

Recently, flexure hinges and TPS based robotic hand devices have shown to be useful in rehabilitation and daily assistance of hand-impaired patients (Hofmann et al. (2018); Mutlu et al. (2015)). This study may help improve existing designs by offering the right TPS configuration as per the requirements. To exemplify, consider developing a TPS based hand orthosis with one pulley per phalange and one (FDP) tendon. Results for FDP–TPS at 8 N tendon tension (Figs. 7–9) show that for small bowstringing ($B_w \leq 9$ mm), the highest ROF attainable is $210 \times 10^{-6}$ which can be increased to $225 \times 10^{-6}$, if the $B_w$ limit is increased to 10 mm. Thus, only a sub-optimal design can be obtained. The same ROF can be obtained if desired, at a lower tendon tension with two pulley per phalange designs. In that case, we recommend the TPS configuration CD–CD–C with flexible-inelastic C–pulleys of 2 mm height, offset from joints by 10% of the respective bone lengths (Fig. 10b). This configuration offers the ROF of $210^\circ$ at 5.5 N tension and the full ROF ($270^\circ$) at 10.1 N tension without compromising on both the bowstringing and pulley stresses. Adding FDS tendon helps achieve the full ROF at a
much lower individual tendon tension of 6.4 N (Fig. 12). In this case, two smaller actuators may be used with a control system to distribute the load in both tendons, and thereby generate several finger flexion postures (Fig. 12). However, the design may become bulky and also require a complex control system. A single FDP tendon may be sufficient when designing orthosis for hand open-close exercises, in case forming different hand postures is not essential. In this case, the TPS configuration suggested by Hume et al. (1991) (explained in section 4.3) is also optimal. These examples highlight the importance of the parametric study in choosing an optimal flexor TPS configuration given the design requirements and making aware of the trade-offs needed.

4.3. Comparison with TPS Reconstruction Literature

The average FDP tendon tension in biological fingers for full flexion is 8.15 N (Yang et al. (2016)). When only the FDP tendon is active, nearly full flexion at 8 N FDP tension is observed with FDP-TPS configuration CD–CD–C (Fig. 12). This reinforces the argument of reconstructing only the FDP tendon, if just one tendon can be repaired (Kotwal and Gupta (2005)). The TPS configuration with two pulleys around each joint at the flare of the metaphysis of the phalangeal bones, of small height as suggested by Hume et al. (1991) for reconstruction surgery, is also observed to be good in our simulations (Fig. 13). High flexion range, in this case, can be attributed to the fact that the two pulleys on each of the proximal and intermediate phalanges behave like a single, very wide pulley located centrally, resulting in the TPS configuration C–C–C. The effectively large width also helps in lowering both pulley stress and bowstringing. Recommendations of Solonen and Hoyer (1967) to use one pulley per phalange near or on joints result in low ROF, as observed in Fig. 7.

4.4. Limitations, Advantages, and Future Scope

Results herein are based on a specific set of finger joint stiffness values (section 3), assuming the fully extended finger as its neutral state. With different sets of stiffnesses and neutral states, the relative positioning of curves corresponding to different TPS configurations is expected to remain similar. Therefore, it may not affect the choice of TPS configuration much. This can be explained geometrically from Fig. 4a based on equilibrium moment-arm variations. Nevertheless, one may need to verify by regenerating all graphs as per his/her finger joint stiffnesses. Some patients having spasticity or otherwise, have either joint neutral positions or joint stiffnesses or both altered. An advantage with the 3R PRBM computational model used herein is that it is readily adaptable in such situations.

This study does not address much on the coordination between FDS and FDP tendons. That requires simulating grasping using contact mechanics, which can provide insight into how the two tendons share the load in forming different finger postures. An extensor mechanism can also be included, as it is known to contribute during grasping.

5. Conclusions

The presented study facilitates choosing an optimal flexor tendon pulley system (TPS) configuration based on one’s design requirements, while also making aware of the trade-offs needed. It also explains several aspects of the biological TPS. This fact validates our study, as well as indicates that the objective of high flexion range with low tendon tension, bowstringing, and pulley stress is in accordance with nature. We also demonstrated that TPS configurations superior to those used in existing hand prosthetic devices could be employed.

2Spasticity is a condition in which fingers are always in the flexed state and resist extension.

Figure 13: FDP–TPS configurations in literature: Case (i) C–C–C, with  \(w_o = 4 \text{ mm} \) (Delph et al. (2013); Nycz et al. (2015)), Case (ii) D–D–C, with  \(w_o = 2 \text{ mm} \) (Bajaj et al. (2020)), Case (iii) PD–PD–C, with  \(w_o = 2 \text{ mm} \), \(h_i = h_o \), offsets \(e_i = e_o \) (Jung et al. (2009); Xu et al. (2012); Xu and Todorov (2016)), Case (iv) PD–PD–P by Hume et al. (1991):  \(w_o = w_i = 2 \text{ mm} \), \(e_i = x_i(l_i - l_i) + l_i/2, k = 1, 2, \cdots, 5 \), and \(J = [k/2, \text{ where } x = (0.21, 0.31, 0.25, 0.21, 0.27)^T \) (metaphysis dimensions from Schulte-Ellis and Lazar (1984)), Case (v) CD–CD–C (current study) with flexible-inelastic C–pulleys. \(e_i = 10e_i \),  \(w_o = 2 \text{ mm} \), \(h_i = 2 \text{ mm} \). Default values used for heights of A–pulleys (\(h_o \)), and widths of C–pulleys (\(w_o \)) in all cases unless mentioned above. Star, and diamond (hollow) markers represent PIP and DIP joint locking respectively. Numerical values are arranged in the order of legends. The parameter values in cases (i)-(iv) are only estimates, and may not be accurate.

We also analyzed various TPS configurations employed in the existing robotic hands (Fig. 13) and found them mostly suboptimal. These configurations involve only a single (FDP) tendon. FDP–TPS configuration C–C–C used by Delph et al. (2013) and Nycz et al. (2015) suffers from high bowstringing (3.2 mm above the limit). Configuration D–D–C with 2 mm pulley width by Bajaj et al. (2020) results in small bowstringing, but also a much smaller flexion range (225° at 10 N). Configuration PD–PD–C by Jung et al. (2009), Xu et al. (2012), and Xu and Todorov (2016) results in even lower flexion range (215°) and higher pulley stress.
without introducing much additional complexity. Results herein may alter, but only quantitatively, in presence of hand abnormalities through variations in joint stiffnesses and neutral positions.

This study may find applications in – (i) understanding flexion biomechanics of the human finger, (ii) designing cost-effective robotic devices for hand, and (iii) surgery related to tendon pulley reconstruction.

Appendix A. Nonlinear Finite Element Model

In the nonlinear finite element method (FEM) formulation, 1D co-rotation frame elements were used which can undergo large bending deflection but permit small strain. The small strain was ensured via sufficiently large number of elements per flexure. Discretization of the model geometry and boundary conditions are shown in Fig. A.14. Detailed formulation for 1D co-rotational frame elements based on the works of Crisfield (1993); Belytschko and Hsieh (1973); Belytschko and Glaum (1979) is given in Mankame (2004). String tensions $T_1$ and $T_1 + T_2$ ($T_1$ in FDP and $T_2$ in FDS tendon) at each boundary node $B_1$, $B_2$, and $B_3$ (Fig. A.14) are always directed towards the neighbouring nodes lying on the string. This makes the nodal external force vector $f_{\text{ext}} = f_{\text{int}}(T, u)$, i.e., dependent on $u$ where $u$ is the nodal displacement vector. To solve nonlinear equilibrium equations using the Newton-Raphson technique, with $g = f_{\text{int}} - f_{\text{ext}}$ as the force residual where $f_{\text{int}}$ is the nodal internal force vector, the tangent stiffness matrix $K_i$ was computed as

$$K_i = \frac{\partial g}{\partial u} = \frac{\partial f_{\text{int}}}{\partial u} - \frac{\partial f_{\text{ext}}}{\partial u} \quad (A.1)$$

One notes that $\frac{\partial f_{\text{int}}}{\partial u} \neq 0$.]

Figure A.14: Finite Element Method: Analysed the model with one-pulley per phalange and single (FDP or FDS) tendon during validation, without loss of generality. TPS configuration with pulleys $A_2$ and $A_4$ and FDP/FDS-TAP placed centrally on respective phalanges. Geometry (in gray) discretized using using 1D co-rotation frame elements. Black disks represent nodes, and thick black line segments depict elements. Two elements per flexure shown as an example. Simulations involve 20 elements per flexure. Frictionless point contact is assumed between tendons and pulleys.

Appendix B. Validation of the computational model

To validate the two computational models, we developed a prototype of the biomechanical model (Fig. B.15). C-Pulleys were not included. All three phalanges were 3D-printed (FDM) using ABS plastic (2000 MPa, Young’s modulus), and had the same cross-section (20 mm × 6 mm). A neoprene rubber (9 MPa, Young’s modulus) strip of cross-section 11.6 mm × 2.1 mm was used as flexure for finger joints. 7.5 mm height (includes bone-width in the model, Fig. 4) was chosen for all pulleys including the guiding pulley, and the TAPs. The guiding pulley $G_1$ location was chosen to be (−10, −7.5) mm (refer Fig. A.14).

To reduce friction between the platform and the prototype, we embedded a 4 mm carbon-steel ball on each phalange. A pull-type force dynamometer with 2 gf resolution was employed to measure the string tension. To account for measurement errors, we conducted five trials of finger flexion for each of the FDP and FDS tendons. Finally, we compared the mean and standard deviation of the tendon tension and flexion range with the simulation results from both FEM and 3R PRBM.

To account for manufacturing tolerances (available rubber strip thickness = 2.1 ± 0.1 mm, width = 11.6 ± 0.1 mm), we simulated the computational model for flexure dimensions in this tolerance band. The strip width was chosen equal to the finger width. The strip thickness was fixed to ensure that flexure stiffness matches that of the respective finger joint. The moment exerted on the right portion of the finger (FBD in Fig. 4b) can be obtained from FEM as the moment at its leftmost node. This moment should be equal to that obtained from PRBM. With this understanding, we performed some trial and error with the strip thickness for each flexure to arrive at its appropriate thickness. Equivalent stiffness of all three joints with the nominal dimensions is ≈ 0.27 N mm. The upper tolerance value corresponds to
Figure B.16: Comparison of experimental and simulation results for (a) FDP and (b) FDS tendon: Tendons not actuated simultaneously. To account for measurement errors, five trials conducted for each tendon. Standard deviation for the experimental curve shown with the red patch. Rubber strip manufacturing tolerances simulated with the 3R PRBM and FEM, and its results shown with blue and yellow patches respectively. Curves are quite close up to 110° of flexion defined by $\sum \theta_i$. The experimental curve flattens thereafter, possibly due to material nonlinearity in the neoprene rubber.

$\approx 0.31 \text{ N mm}$, whereas the lower one to $\approx 0.23 \text{ N mm}$. PRBM is used to simulate the TPS configuration C–C–C or C–C–C for these three stiffness sets. FEM directly uses the highest and lowest dimensions in addition to the nominal dimensions. As a result, we obtained the bands of tension-flexion response as shown in Fig. B.16.

All three methods yield very similar results in the limit of standard deviation, up to $\sum \theta_i = 120^\circ$, at tendon tension of 1.2 N for FDP tendon. After that, the experimental curve diverges, which may be explained by the material nonlinearity associated with the neoprene rubber corresponding to large deflections. Both FEM and PRBM as implemented herein, disregard material nonlinearity. Results for the FDS tendon are similar (Fig. B.16b).

References

Bajaj, A., Jain, V., Kumar, P., Unal, A., Saxena, A., 2020. Soft hand exoskeleton for adaptive grasping using a compact differential mechanism, in: Mechanism and Machine Science. Springer, pp. 733–746.
Belytschko, T., Glaum, L.W., 1979. Applications of higher order corotational stretch theories to nonlinear finite element analysis. Computers & Structures 10, 175–182.
Belytschko, T., Hsieh, B., 1973. Non-linear transient finite element analysis with convected co-ordinates. International Journal for Numerical Methods in Engineering 7, 255–271.
Brand, P.W., Cranor, K., Ellis, J., 1975. Tendon and pulleys at the metacarpophalangeal joint of a finger. The Journal of bone and joint surgery. American volume 57, 779–784.
Chow, J.C., Sensinger, J., McNeal, D., Chow, B., Amiouch, E., Gonzalez, M., 2014. Importance of proximal a2 and a4 pulleys to maintaining kinematics in the hand: a biomechanical study. Hand 9, 105–111.
Crisfield, M.A., 1993. Non-linear finite element analysis of solids and structures. volume 1. Wiley New York.
Delph, M.A., Fischer, S.A., Gauthier, P.W., Luna, C.H.M., Clancy, E.A., Fischer, G.S., 2013. A soft robotic exoskeleton glove with integrated semg sensing for hand rehabilitation, in: 2013 IEEE 13th International Conference on Rehabilitation Robotics (ICORR), IEEE, pp. 1–7.
Dionysian, E., Kabo, J.M., Dorey, F.J., Meals, R.A., 2005. Proximal interphalangeal joint stiffness: measurement and analysis. The Journal of hand surgery 30, 573–579.
Dy, C.J., Daluiski, A., 2013. Flexor pulley reconstruction. Hand clinics 29, 235–242.
Guo, J., Lee, K.M., 2013. Compliant joint design and flexure finger dynamic analysis using an equivalent pin model. Mechanism and Machine Theory 70, 338–353.
Dy, C.J., Daluiski, A., 2013. Flexor pulley reconstruction. Hand clinics 29, 235–242.
Guo, J., Lee, K.M., 2013. Compliant joint design and flexure finger dynamic analysis using an equivalent pin model. Mechanism and Machine Theory 70, 338–353.
Dy, C.J., Daluiski, A., 2013. Flexor pulley reconstruction. Hand clinics 29, 235–242.
Guo, J., Lee, K.M., 2013. Compliant joint design and flexure finger dynamic analysis using an equivalent pin model. Mechanism and Machine Theory 70, 338–353.
Dy, C.J., Daluiski, A., 2013. Flexor pulley reconstruction. Hand clinics 29, 235–242.
van Nierop, O.A., van der Helm, A., Overbeeke, K.J., Djajadiningrat, T.J., 2008. A natural human hand model. The Visual Computer 24, 31–44.

Nycz, C.J., Delph, M.A., Fischer, G.S., 2015. Modeling and design of a tendon actuated soft robotic exoskeleton for hemiparetic upper limb rehabilitation, in: 2015 37th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), IEEE. pp. 3889–3892.

Pring, D., Amis, A., Coombs, R., 1985. The mechanical properties of human flexor tendons in relation to artificial tendons. The Journal of Hand Surgery: British & European Volume 10, 331–336.

Schulter-Ellis, F.P., Lazar, G.T., 1984. Internal morphology of human phalanges. The Journal of Hand Surgery 9, 490–495.

Schweizer, A., 2008. Biomechanics of the interaction of finger flexor tendons and pulleys in rock climbing. Sports Technology 1, 249–256.

Solonen, K.A., Hoyer, P., 1967. Positioning of the pulley mechanism when reconstructing deep flexor tendons of fingers. Acta Orthopaedica Scandinavica 38, 321–328.

Su, H.J., 2009. A Pseudorigid-Body 3R Model for Determining Large Deflection of Cantilever Beams Subject to Tip Loads. Journal of Mechanisms and Robotics 1. 021008.

Xu, Z., Kumar, V., Matsuoka, Y., Todorov, E., 2012. Design of an anthropomorphic robotic finger system with biomimetic artificial joints, in: 2012 4th IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics (BioRob), IEEE. pp. 568–574.

Xu, Z., Todorov, E., 2016. Design of a highly biomimetic anthropomorphic robotic hand towards artificial limb regeneration, in: 2016 IEEE International Conference on Robotics and Automation (ICRA), IEEE. pp. 3485–3492.

Yang, T.H., Lu, S.C., Lin, W.J., Zhao, K., Zhao, C., An, K.N., Jou, I.M., Lee, F.Y., Kuo, L.C., Su, F.C., 2016. Assessing finger joint biomechanics by applying equal force to flexor tendons in vitro using a novel simultaneous approach. PloS one 11, e0160301.

Yu, Y.Q., Feng, Z.L., Xu, Q.P., 2012. A pseudo-rigid-body 2r model of flexural beam in compliant mechanisms. Mechanism and Machine Theory 55, 18 –33.

Zheng, R., Li, J., 2010. Kinematics and workspace analysis of an exoskeleton for thumb and index finger rehabilitation, in: 2010 IEEE International Conference on Robotics and Biomimetics, IEEE. pp. 80–84.