Berry’s phase at quantum vacuum level

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The quantum vacuum contribution to Berry’s geometric phase of photon fields inside a noncoplanarly curved (coiled) fiber is considered by means of the second-quantization formulation. It is shown that the quantum vacuum Berry’s phases of left- and right-handed circularly polarized light have the equal magnitudes but opposite signs, and are therefore eliminated entirely by each other. In order to realize such a novel vacuum effect, a scheme to separate the quantum vacuum Berry’s phase of one polarized light from another by using the chiral medium fiber is suggested. We think the present study might be the first treatment for the time evolution of vacuum zero-point energy.

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During the past two decades, the topics on geometric phases and time-dependent quantum systems have attracted extensive attention of a large number of investigators in various fields, including quantum optics, condensed matter physics, nuclear physics, gravity theory as well as molecular physics (molecular chemical reaction). Recently, geometric phase has practical applications in the subjects of quantum decoherence and geometric (topological) quantum computation. As for the purely theoretical aspect of geometrical phases, historically, Berry established a semiclassical connection between the quantum geometric phase γ and the classical Hannay’s angle ∆θ(I) as follows: ∆θ(I) = −∂γ/∂nI. Here, the geometric phase is associated with the eigenstates with quantum numbers n = {nI}, and the Hannay’s angle ∆θ(I) is a shift in the Ith angle variable for motion round a phase-space torus with actions I = {Ii} 2. According to Berry’s relation, one can obtain γ = −nI ∆θ(I) + γ0 (here the summation over the repeated indices is implied), where γ0 is an integral constant. Thus, a new question that might have never been considered before is left to us: what is the physical meanings of the nonvanishing integral constant γ0 (should such exist)? We believe that such an integral constant may have close relation to the quantum vacuum contribution to the time-dependent quantum system, namely, γ0 may be viewed as Berry’s phase at quantum vacuum level, which arises from the quantum fluctuation of vacuum.

Quantum vacuum effects have so far captured intensive attention of many researchers in quantum optics, quantum field theory and atomic (molecular) physics. These effects are as follows: the Casimir effect, anomalous magnetic moment of electron, vacuum polarization, Lamb’s shift as well as Casimir-Polder potentials. More recently, some new vacuum effects caused by quantum fluctuation in the inhomogeneous and anisotropic electromagnetic materials have been predicted theoretically or observed experimentally. These include the dramatic modification of spontaneous emission in photonic crystals and EIT media (multilevel atomic ensemble), magnetoelectric birefringences of quantum vacuum and vacuum contribution to the momentum of anisotropic media (e.g., magnetoelectric materials). Even though some researchers investigated the problem of light propagation in a noncoplanarily curved (coiled) fiber by means of various methods, including the classical electromagnetic, differential geometry (parallel transport) as well as the first-quantization formulation, to the best of our knowledge, the treatment based on the second-quantization formulation has so far never been considered in references. In this Letter, we will study the wave propagation of the coiled light (and hence the quantum vacuum contribution to Berry’s phase) inside the framework of second quantization, namely, a physical realization will be proposed for photon Berry’s phase at vacuum level, which originates from the quantum vacuum fluctuation. It should be noted that since the left- and right-handed (LRH) circularly polarized photons acquire the quantum-vacuum Berry’s phases with equal magnitudes but opposite signs, such vacuum contributions to the LRH polarized light are always eliminated completely inside the isotropic media and may therefore have no observable effects experimentally. However, in some certain anisotropic media such as chiral media, gyrotrropic (gyroelectric or gyromagnetic) media and magnetoelectric materials, the quantum vacuum contribution may no longer be exactly cancelled for the LRH circularly polarized light since the noncompensation effect of a pair of LRH vacuum modes will arises in these anisotropic media. We suggest an experimentally feasible scheme to test this new vacuum effect by using the noncoplanar (coiled) chiral-medium fiber in Tomita-Chiao experiment.

First, we consider the wave propagation inside the noncoplanar fiber. According to the Maxwellian equations, the field vectors \( G_\pm = E \pm i\nu B \) agree with the following...
with the phase velocity \( v = c/n \), where \( n \) denotes the optical refractive index of the linear electromagnetic medium. Consider a planar electromagnetic wave whose field vector \( G_+ \propto \exp(i \mathbf{k} \cdot \mathbf{r}) \). Thus, Eq. (1) can be rewritten as \( \mathbf{k} \times \mathbf{G}_+ = (1/v) \frac{\partial}{\partial t} \mathbf{G}_+ \). By using the relations \( \mathbf{S} \times \mathbf{S} = i\hbar \mathbf{S} \) and \( [\mathbf{a} \cdot \mathbf{S}, \mathbf{b} \cdot \mathbf{S}] = i\hbar (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{S} \), where \( \mathbf{S} \) denotes the spin operator of the photon field, one can further obtain \( [\mathbf{k} \cdot \mathbf{S}, \mathbf{G}_+ \cdot \mathbf{S}] = i\hbar (\partial/\partial t) \mathbf{G}_+ \cdot \mathbf{S} \). Further analysis shows that the field vectors \( \mathbf{G}_\pm \) (or the operator \( \mathbf{G}_\pm \)) satisfy (for simplicity, the subscript \( \pm \) will be omitted)

\[
\frac{\partial}{\partial t} \mathbf{G} \cdot \mathbf{S} + \frac{1}{i\hbar} [\mathbf{G} \cdot \mathbf{S}, \mathbf{v} \mathbf{k} \cdot \mathbf{S}] = 0, \tag{2}
\]

which is just a form of Liouville-von Neumann equation \( \frac{d}{dt} \mathbf{I} + \{i[H, \mathbf{I}]/\hbar\} = 0 \). Here, \( \mathbf{I} \) and \( H \) denote the Lewis-Riesenfeld invariant [19] and the Hamiltonian of the quantum system, respectively. It follows from Eq. (2) and the Liouville-von Neumann equation that the operator \( \mathbf{v} \mathbf{k} \cdot \mathbf{S} \) can be regarded as the effective interaction Hamiltonian that characterizes the wave propagation of circularly polarized light inside the noncoplanar fiber. Here, \( \mathbf{k} \) denotes the wave vector of one-dimensional longitudinal propagation along the guiding direction. As the relation between the frequency \( \omega \) and the modulus of wave vector of the electromagnetic wave in a linear medium is \( \omega = kv \), the effective Hamiltonian \( \mathbf{v} \mathbf{k} \cdot \mathbf{S} \) can be rewritten as \( \omega \mathbf{n} \cdot \mathbf{S} \), where the unit vector \( \mathbf{n} = \mathbf{k}/k \), which can be expressed as \( (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \). In the noncoplanar fiber, the angle displacements, \( \theta(t) \) and \( \varphi(t) \), in the three-dimensional spherical polar coordinate system are the time-dependent functions. Note that here \( \theta(t) \) and \( \varphi(t) \) can characterize the geometric shape of the noncoplanarly curved fiber, since it was assumed that in waveguides (fibers) for photons (or their atomic counterparts) [20], the wave packet describing the propagation follows the channel smoothly and without too much distortion. Such assumptions derive from some underlying smoothness of the guiding structures and thus the adiabaticity will hold in the propagation process [20].

In view of the above discussion, the eigenvalue equation of instantaneous Hamiltonian \( H(t) = \omega \mathbf{n}(t) \cdot \mathbf{S} \) governing the wave propagation of the light in the noncoplanarly curved fiber reads \( \omega \mathbf{n}(t) \cdot \mathbf{S} |\mathbf{k}(t), \pm\rangle = E_{\pm}(\mathbf{k}(t), \pm) \). If the initial condition for the photon wave vector is \( |\mathbf{k}(t = 0)\rangle = (0, 0, k) \) (and hence \( \theta(t = 0) = 0 \)), then the solutions of this equation are of the form \( |\mathbf{k}(t), \pm\rangle = V(t)|\pm\rangle \), where \( |\pm\rangle = |\mathbf{k}(t = 0), \pm\rangle \), and the time-dependent unitary operator in the solutions is

\[
V(t) = \exp \left\{ \left[ -\frac{\theta(t)}{2\hbar} e^{-i\varphi(t)} \right] S_+ - \left[ -\frac{\theta(t)}{2\hbar} e^{i\varphi(t)} \right] S_- \right\}. \tag{3}
\]

Here the operators \( S_\pm = S_1 \pm iS_2 \). Further calculation shows that the explicit expressions for the above solutions take the form

\[
|\mathbf{k}(t), +\rangle = \left[ \cos \frac{\theta(t)}{2} |+\rangle + e^{i\varphi(t)} \sin \frac{\theta(t)}{2} |-\rangle \right],
\]

\[
|\mathbf{k}(t), -\rangle = \left[ \cos \frac{\theta(t)}{2} |+\rangle - e^{-i\varphi(t)} \sin \frac{\theta(t)}{2} |+\rangle \right]. \tag{4}
\]

Note that for the propagating electromagnetic wave in the noncoplanar fiber, the cyclic evolution of photon wavefunction will yield the original state plus a phase shift \( [\mathbf{I}] \), which is a sum of a dynamical phase \( -\gamma^d(\mathbf{T}) \) and a geometric phase shift \( \gamma^g(\mathbf{T}) \), namely, if the initial state is \( | \pm \rangle \), then the state at \( T \) is

\[
|\mathbf{k}(T), \pm\rangle = \exp \left\{ i\gamma^d(\mathbf{T}) \right\} \exp \left\{ -i\gamma^g(\mathbf{T}) \right\} V(T)|\pm\rangle, \tag{5}
\]

where \( \gamma^d(\mathbf{T}) = h^{-1} \int_0^T \langle \pm | \mathbf{V}(t') H(t') V(t')|\pm\rangle dt' \), and \( \gamma^g(\mathbf{T}) = \int_0^T \langle \pm | \mathbf{V}(t') \mathbf{i}\partial \mathbf{V}(t')/\partial t'| |\pm\rangle dt' \). Here the explicit expressions for the geometric phase shift \( \gamma^g(\mathbf{T}) = -h^{-1}(\pm S_3|\pm\rangle \int_0^T \dot{\varphi}(t') [1 - \cos \theta(t')] dt', \) where dot denotes the time derivative.

In the following, we assume that the noncoplanarly curved fiber has the shape of coil [21]. Consider a typical case where the precessional frequency \( \dot{\varphi} \) of photon moving on the fiber helicoid is constant (denoted by \( \Omega \)) and the nutational frequency \( \dot{\theta} \) vanishing. After one period \( (T = 2\pi/\Omega) \) that it takes to complete a precessional cycle in the coiled fiber, the cyclic geometric phase (Berry’s phase shift) is \( \gamma^g(\mathbf{T}) = -2\pi(1 - \cos \theta)|\pm S_3|\pm\rangle/h, \) where \( 2\pi(1 - \cos \theta) \) is an expression for the solid angle subtended at the center by a curve traced by the photon wave vector in the propagation process inside the coiled fiber.

Now we consider the expectation value, \( |\pm \rangle \langle \pm | S_3 |\pm \rangle \), of the third component of photon spin operator in \( \gamma^g(\mathbf{T}) \). Substitution of the Fourier expansion series of three-dimensional magnetic vector potentials \( \mathbf{A}(\mathbf{x}, t) \) into the expression \( \langle S_{3|\pm\rangle \langle \pm | S_3 |\pm \rangle} = -\int (\dot{A}_x A_z - \dot{A}_z A_x) d^3x \) for the spin operator of the photon field yields the monomode spin operator \( S_3 = (i\hbar/2)[a(k, 1)a^\dagger(k, 2) - a(k, 1)a^\dagger(k, 2) - a(k, 2)a^\dagger(k, 1) + a^\dagger(k, 2)a(k, 1)] \) with \( a(k, \lambda) \) and \( a^\dagger(k, \lambda) \) \((\lambda = 1, 2)\) being the creation and annihilation operators of polarized photons corresponding to the two mutually perpendicular polarization vectors. Here, \( \lambda \) denotes the label of the polarization vectors [22]. In what follows, we will define the creation and annihilation operators, \( a^\dagger_R(k), a^\dagger_L(k), a_R(k), a_L(k) \), of right- and left-handed circularly polarized light [22], which are expressed in terms of \( a^\dagger(k, \lambda) \) and \( a(k, \lambda) \), i.e., \( a_R(k) = [a^\dagger(k, 1) + ia(k, 2)]/\sqrt{2}, a_L(k) = [a(k, 1) - ia(k, 2)]/\sqrt{2}, \)

\[
a_R(k) = [a^\dagger(k, 1) - ia(k, 2)]/\sqrt{2} and a_L(k) = [a(k, 1) + ia(k, 2)]/\sqrt{2}. \] Thus, the spin operator of monomode photon.
ton can be rewritten in terms of the creation and annihilation operators of left- and right-handed polarized photons, i.e., $S_3 = \left[ a_R(k)a_R^\dagger(k) + a_L^\dagger(k)a_R(k) \right] \hbar/2 - \left[ a_L(k)a_R^\dagger(k) + a_R^\dagger(k)a_L(k) \right] \hbar/2$, which can also be rewritten as the form $S_3 = \left[ a_R^\dagger(k)a_R(k) + 1/2 \right] \hbar - \left[ a_L^\dagger(k)a_L(k) + 1/2 \right] \hbar$. Hence, Berry’s phases at second-quantization level of left- and right-handed circularly polarized light accumulated in one precessional period ($T = 2\pi/\Omega$) are $\gamma_{L/R}^{(g)}(T) = +2\pi(1 - \cos \theta)(n_L + 1/2)$ and $\gamma_{L/R}^{(d)}(T) = -2\pi(1 - \cos \theta)(n_R + 1/2)$, where $n_L$ and $n_R$ denote the occupation numbers of left- and right-handed polarized photons, respectively.

It is physically interesting to consider the connection between the second- and first-quantized Berry’s phases. Such a connection can be demonstrated by taking account of the expectation value of the operator of electric field strength in the coherent state: specifically, the coherent state of the polarized light (say, the right-handed polarized light) at $t = T$ can be constructed in terms of the photon states $|\alpha, T\rangle = \exp \left(-\frac{\alpha^*\alpha}{2}\right) \sum_{n_R=0}^{\infty} \frac{\alpha^{n_R}}{\sqrt{n_R!}} \exp [i\gamma_{n_R}(T)] |n_R, T\rangle$, i.e.,

$$|\alpha, T\rangle = \exp \left(-\frac{\alpha^*\alpha}{2}\right) \sum_{n_R=0}^{\infty} \frac{\alpha^{n_R}}{\sqrt{n_R!}} \exp [i\gamma_{n_R}(T)] |n_R, T\rangle,$$

where $\gamma_{n_R}(T) = \gamma_{n_R+1}(T) - \gamma_{n_R}(T)$ and $|n_R, T\rangle = \hat{V}(T)|n_R\rangle$. The expectation value of the operator $\hat{q} = \left(\alpha_R + \alpha_R^\dagger\right)/\sqrt{2}$ of the electric field strength in the coherent state is

$$\langle \hat{q} \rangle(T) = \frac{1}{\sqrt{2}} \left\{ \alpha^* \exp [\pm i\Delta_R(T)] + \alpha \exp [\mp i\Delta_R(T)] \right\},$$

where $\Delta_R(T) = \gamma_{n_R+1}(T) - \gamma_{n_R}(T)$, which is a phase shift independent of the photon occupation number $n_R$. Thus, the geometric phase contribution in the phase shift $\Delta_R(T)$ is

$$\gamma_R^{(g)} = -2\pi(1 - \cos \theta),$$

which is a first-quantized Berry’s phase acquired by the electromagnetic wave in the cyclic process. It should be noted that the result obtained here is consistent with the previous studies based on the differential geometry method, Maxwellian equations and quantum mechanics (first quantization) [17, 18]. So, we think the treatment presented in this Letter is self-consistent. Apparently, there exists no Berry’s phases at quantum vacuum level in the expression since $\Delta_R(T)$ in $\gamma^{(g)}_R$ is merely a phase shift at first-quantization level.

It follows from the expressions for $\gamma_{n_R}^{(g)}(T)$ and $\gamma_{n_R}^{(d)}(T)$ that Berry’s phases of the left- and right-handed circularly polarized vacuum modes are always having the equal magnitudes but opposite signs, i.e., $\gamma_{0L}^{(g)}(T) = +\pi(1 - \cos \theta)$ and $\gamma_{0R}^{(d)}(T) = -\pi(1 - \cos \theta)$. Unfortunately, such a novel vacuum effect may not manifest the observable phenomena experimentally. Indeed, historically, Berry’s phases at quantum vacuum level inside the curved fiber have not been detected in the previous experiments [17] just due to such a cancellation.

However, it is reasonably believed that since the vacuum mode structures can be influenced dramatically by the anisotropic and inhomogeneous environment, some quantum vacuum effects will unavoidably arise in anisotropic (and inhomogeneous) media. Here, we will take into account the wave propagation in the chiral medium fiber, in which the quantum vacuum contribution to the left- and right-handed polarized fields will no longer be eliminated by each other. A homogeneous chiral medium can be characterized by the following constitutive relations [24]

$$D = \epsilon_0 \epsilon \mathbf{E} + i\zeta \mathbf{B}, \quad H = i\zeta \mathbf{E} + \frac{\mathbf{B}}{\mu_0},$$

where $\epsilon$ and $\zeta$ denote the relative permittivity and the chirality parameter, respectively. As a preliminary consideration, let us first analyze the problem of field quantization in chiral media. By expanding the three-dimensional electromagnetic vector potentials $\mathbf{A}(\mathbf{x}, t)$ as a Fourier series $\mathbf{A}(\mathbf{x}, t) = \sum_i A_i(\mathbf{x}) f_i(t)$, where $A_i(\mathbf{x}) = \mathbf{u}_i \exp (i\mathbf{k}_i \cdot \mathbf{x})$ ($\mathbf{u}_i$ is a unit vector) and $f_i(t) = |f_i| \exp (-i\omega_i t)$, one can obtain the expression $W = 2(\epsilon_0 + \mu_0 \zeta^2) V \sum_1 \omega_i^2 f_i^* f_i$ for the energy of electromagnetic field in the chiral medium, where $V$ denotes the volume of the medium. It follows that for the electromagnetic energy density in the chiral medium, the role of the chirality parameter $\zeta$ is just to make a correction to the scalar factor $\epsilon_0$. Thus, we believe that the conventional mathematical formalism of the field quantization in isotropic homogeneous electromagnetic media can apply in chiral media: specifically, by introducing $q_i = [(\epsilon_0 + \mu_0 \zeta^2) V]^{1/2} (f_i + f_i^*)$ and $p_i = -i [(\epsilon_0 + \mu_0 \zeta^2) V]^{1/2} \omega_i (f_i - f_i^*)$, the above expression for the field energy in the chiral medium can be rewritten as $W = \sum_i (p_i^2 + \omega_i^2 q_i^2)/2$. By using the quantization technique, in which the annihilation and creation operators of photon fields are defined as follows: $b_i = (2\hbar \omega_i)^{-1/2}(\omega_i q_i + i\hbar p_i)$, $b_i^\dagger = (2\hbar \omega_i)^{-1/2}(\omega_i q_i - i\hbar p_i)$, the formula of the field energy in the chiral medium can be written in the form $W = (1/2) \sum \hbar \omega_i \left(b_i^\dagger b_i + b_i b_i^\dagger\right)$. This, therefore, means that the treatment and results of the quantum vacuum contribution to Berry’s phase presented above is still valid for the case of chiral media.

According to the constitutive relations, a time harmonic electromagnetic wave agrees with the wave equation

$$\nabla^2 - \epsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} (E_1 \pm iE_2) \equiv 2\zeta \mu_0 \hbar \omega (E_1 \pm iE_2) = 0.$$ 

(10)
Therefore, the wave vectors $k_{L,R}$ corresponding to the wave amplitudes $E_1 \pm i E_2$ fulfill the two quadratic equations $k_{R}^2 + 2\zeta \mu_0 \omega k_{R} - \epsilon \epsilon_0 \mu_0 \omega^2 = 0$ and $k_{L}^2 - 2\zeta \mu_0 \omega k_{L} - \epsilon \epsilon_0 \mu_0 \omega^2 = 0$, respectively. The solutions of the quadratic equations are given by $k_R = \mu_0 \omega (-\zeta + \sqrt{\zeta^2 + \epsilon \epsilon_0 / \mu_0})$ and $k_L = \mu_0 \omega \left( +\zeta + \sqrt{\zeta^2 + \epsilon \epsilon_0 / \mu_0} \right)$. Note that here both the negative roots of $k_{L,R}$ have been ruled out since it is assumed that the wave vectors of the left- and right-handed polarized light have the same direction in the propagation process inside the coiled fiber. It is thus clear that the wave vector $k_R$ of the right-handed polarized light is different from $k_L$ of the left-handed polarized light because of the chirality parameter $\zeta$, and that the difference between them is $\Delta k = -2\zeta \mu_0 \omega$. This may lead to the possibility to differentiate between quantum-vacuum Berry's phases of left- and right-handed polarized fields, which can be interpreted as follows: the precessional frequency of electromagnetic wave propagating on the helicoid reads $\Omega = 2\pi v / \sqrt{d^2 + (4\pi a)^2}$, where $v$ denotes the phase velocity of light inside the coiled fiber; $a$ is the radius of the helix and $d$ the helical pitch length. If the parameter $\zeta$ in the constitutive relations [9] is a small number (compared with $\sqrt{\epsilon \epsilon_0 / \mu_0}$), then the difference between the precessional frequencies $\Omega_R$ and $\Omega_L$ is $\Delta \Omega = 4\zeta \pi a^2 / [\epsilon \sqrt{d^2 + (4\pi a)^2}]$. Meanwhile, the difference between the precessional periods of right- and left-handed circularly polarized light propagating on the helicoid of the coiled fiber is $\Delta T = -2\zeta \mu_0 \sqrt{d^2 + (4\pi a)^2}$. It follows that because of the existence of the chirality parameter $\zeta$, the precessional frequencies (and hence the precessional periods) are different between the left- and right-handed polarized light inside the coiled fiber. This may enable us to separate one of the circularly polarized light from another, and thus no longer allows the quantum vacuum contribution to Berry's phases to be cancelled totally by each other in one precessional cycle. Hence, such a novel vacuum effect is possible to be observed in the coiled fiber experiments [16], if one utilizes the fibers fabricated by the chiral media instead of those made of regular (isotropic) media.

To summarize, we considered a new vacuum effect caused by the time evolution of the zero-point energy of quantum vacuum fluctuation. As the noncompensation effect of a pair of counter-propagating (or -spinning) vacuum modes will arise in anisotropic media [10], the physical effects resulting from quantum vacuum fluctuation of left- and right-handed polarized modes will no longer be exactly cancelled by each other. This may lead to an observable effect of the quantum vacuum contribution to Berry's phase in time-dependent quantum systems. Since Berry's phase at quantum vacuum level may have close relation to some fundamental problems, e.g., the time evolution of vacuum background, field quantization, time-dependent field theory as well as cavity QED, we hope such an interesting vacuum effect would be realized experimentally in the near future.

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