Synthetic topological Kondo insulator in a pumped optical cavity

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Abstract

Motivated by experimental advances on ultracold atoms coupled to a pumped optical cavity, we propose a scheme for synthesizing and observing the Kondo insulator in Fermi gases trapped in optical lattices. The synthetic Kondo phase arises from the screening of localized atoms coupled to mobile ones, which in our proposal is generated via the pumping laser as well as the cavity. By designing the atom–cavity coupling, it can engineer a nearest-neighbor-site Kondo coupling that plays an essential role for supporting topological Kondo phase. Therefore, the cavity-induced Kondo transition is associated with a nontrivial topological features, resulting in the coexistence of the superradiant and topological Kondo state. Our proposal can be realized with current technique, and thus has potential applications in quantum simulation of the topological Kondo insulator in ultracold atoms.

1. Introduction

The experimental realization and manipulation of ultracold atoms provide a versatile platform with feasible controllability to simulate the many-body physics in strongly correlated systems [1]. Taking advantage of current techniques, theoretic and experimental investigations on ultracold atoms advance in creating artificial gauge fields [2–6] and a variety of optical lattices [7–10], which opens the way to explore and predict unconventional properties of strongly correlated systems, such as the topological the Kondo insulator [11, 12].

The Kondo phase arises from the screening of localized electrons hybridized with mobile ones, forming a so-called Kondo insulator [13]. It captures many unusual properties of strongly correlated systems such as the heavy fermion materials. With coexistence of the spin–orbital coupling, it brings rich physics with the discovery of the topological Kondo insulator [14–17]. Recent studies of ultracold atoms shed light on the realization of the Kondo insulator by proposing a variety of schemes that are widely based on the optical coupling [18], the Bloch-orbital hybridizations [19], and the orbital Feshbach resonance [20–25]. Especially in the earlier work [19], artificial gauge fields generated by the laser-induced Raman coupling make it possible to detect the topological Kondo insulator. On the other hand, the study of Fermi gases in an optical cavity [26–31] offers another framework for engineering artificial gauge fields [32–37]. It predicts the existence of the topological superradiant states [38–40]. This motivates us to find an alternative scheme to synthesize a topological Kondo insulator by implementing the cavity field.

Here in this paper, we present a proposal for realizing the synthetic topological Kondo insulator in a pumped optical cavity. The paper is organized as follows. In section 2, we start with the Hamiltonian describing atoms coupled to a pumped optical cavity, and design it to obtain the Kondo lattice Hamiltonian. In section 3, we show the phase transition by changing experimental controllable parameters, and show the topological features of the superradiant Kondo phase. The extension of our proposal to a higher-dimensional case are discussed in section 4. In section 5, we summarize the work. The details of the effective Hamiltonian and the slave boson approach, which we use to study the Kondo phase, are formulated in appendices A and B.
2. The model

2.1. Effective Hamiltonian

We consider ultracold fermionic atoms trapped in a one-dimensional (1D) optical lattice oriented in x direction. The atomic level structure is illustrated in figure 1(a). In practice, for $^6$Li alkali atoms as an example, we can choose two nuclear spin states with $|F, m_F\rangle = |1/2, 1/2\rangle$ and $|1/2, = -1/2\rangle$ as $|g, \uparrow\rangle$, and $|3/2, 3/2\rangle$ and $|3/2, 1/2\rangle$ as $|e, \uparrow\rangle$, respectively. The spinful atoms are initially prepared in $|g, \uparrow\rangle$ with double filling in each site.

The experimental setup for our proposal is sketched in figure 1(b). When placed into a high-finesse cavity, the Raman transition between $|g, \uparrow\rangle$ and $|e, \uparrow\rangle$ ($s = \uparrow$, $\sigma = \downarrow$) is driven by a plane-wave pumping laser $h(r)$ with frequency $\omega_p$ and linear polarization in accompany with a single-mode standing-wave cavity field $\eta(x)$ with frequency $\omega_c$ and $\sigma$ polarization. The selection rule in the Raman transition can suppress the unwanted atomic transitions. The optical cavity is oriented in the x axis, while the pumping laser is placed in the $x-z$ plane. In order to realize the Raman transition by using the same laser and cavity fields, we introduce an AC-Stark shift individually to $|g, \downarrow\rangle$, which can be generated by a far-detuned laser. This system is described by a Hamiltonian composed of three terms,

$$H = H_A + H_C + H_I.$$  

(1)

The first term $H_A$ describes the atom subsystem,

$$H_A = \int dx \left\{ H_L(x) - \sum_{\sigma} \Delta_{a\sigma} \psi_{g\sigma}^\dagger(x) \psi_{e\sigma}(x) \right\}.$$  

(2)

Here

$$H_L(x) = \sum_{\lambda, \sigma} \psi_{g\sigma}^\dagger(x) \left[ -\frac{\nabla^2}{2m} + V_L(x) \right] \psi_{g\sigma}(x)$$  

(3)

describes free atoms confined in the lattice. $\{\psi_{g\sigma}, \psi_{e\sigma}^\dagger\}$ are the annihilation and creation field operators for the level $\lambda = g/e$, respectively. $V_L(x) = V_0 \sin^2(\lambda x)$ is the optical lattice trap potential where $k_L = \pi/d$ with $d$ as the lattice constant. $\Delta_{a\sigma}$ is the detuning between $|g, \sigma\rangle$ and $|e, \sigma\rangle$ ($\sigma = \uparrow$, $\downarrow$). For simplicity without loss of generality, we assume $\Delta_{a\uparrow} = \Delta_{a\downarrow} = \Delta_a$. In the whole paper we set $\hbar = 1$. The second term $H_C$ describes the cavity subsystem,

$$H_C = -\Delta_c a^\dagger a.$$  

(4)

Here $\{a, a^\dagger\}$ are the annihilation and creation operators for cavity fields, respectively. $\Delta_c$ is the cavity–pump detuning. The last term $H_I$ describes the interaction between the atom and cavity subsystems,

$$H_I = \int dx \ g(x) \sum_{\sigma} [a \psi_{g\sigma}^\dagger(x) \psi_{e\sigma}(x) + \text{H.c.}].$$  

(5)

Here $g(x)$ denotes the atom–cavity coupling mode, which is originated from $h(r)$ and $\eta(x)$. The term H.c. stands for Hermitian conjugation. The details of the Hamiltonian (1) are given in appendix A.

From the Hamiltonian (1), taken the cavity decay into considerations, the Heisenberg equation for the cavity fields $a$ is written as

$$i\partial_t a = [a, H] = -\left(\Delta_c + i\kappa\right)a + g(x) \sum_{\sigma} \psi_{g\sigma}^\dagger(x) \psi_{e\sigma}(x),$$  

(6)
where $\kappa$ is the decay rate. Hereafter we make the mean-field approximation $\langle \alpha \rangle = \alpha$. As known in previous works [31], the system undergoes a superradiant transition characterized by $\alpha$, which is driven by tuning experimental parameters, e.g. the pumping laser amplitude $\Omega_p$. When $\alpha$ is nonzero, the system is noted as the superradiant state. Due to the cavity decay, the system is indeed a non-equilibrium one. For steady cavity field, $\alpha$ satisfies $\partial_\alpha = 0$, and thus we obtain

$$\alpha = \frac{\eta}{\Delta_0 + i\kappa}, \quad \eta = \frac{1}{L} \int dx \sum_\sigma \varphi(x) \langle \psi_{g\sigma}^\dagger(x) \psi_{e\sigma}(x) \rangle.$$

Here $L$ is the length of the 1D lattice. Inserting $\alpha$ back into equation (1), the Hamiltonian is recast as

$$\hat{R} = \int dx \sum_\sigma \left\{ \hat{H}_c(x) - \Delta_0 \psi_{g\sigma}^\dagger(x) \psi_{g\sigma}(x) + [\alpha g(x) \psi_{e\sigma}^\dagger(x) \psi_{e\sigma}(x) + \text{H.c.}] \right\}.$$

In order to derive a Hubbard Hamiltonian to quantum simulate the Kondo physics, we use the tight-binding approximation and expand $\varphi(x)$ in terms of Wannier wave functions $W_i(x)$. Besides, the following two manipulations are required: (i) atoms in $|g\rangle$ are deeply trapped in the lattice to make the tunneling amplitude $t_g \ll t_e$. This can be achieved by means of two groups of counter-propagate lasers to create optical lattices separately trapping $|g\rangle$ and $|e\rangle$ with different depth. (ii) Via Feshbach resonance, we introduce strong repulsive interaction between $|g\rangle$ with opposite spins in the same site. For example of $^{6}$Li [41], we can make broad Feshbach resonance between $|g\rangle$, $|\uparrow\rangle$ and $|g\rangle$, $|\downarrow\rangle$. By contrast in the same parameter region, the scattering between $|e\rangle$, $|\uparrow\rangle$ and $|e\rangle$, $|\downarrow\rangle$ is off resonance, hence their interaction is ignorable. After accomplishing the above manipulations, $|g\rangle$ can simulate the localized fermions in Kondo problems, while $|e\rangle$ simulates the conduction fermions. The effective Hamiltonian is then expressed as

$$H = -\sum_{\langle ij \rangle} t_{ij} \lambda_{ij}^g \lambda_{ij}^e - \Delta_0 \sum_j g_{j\sigma}^g \bar{g}_{j\sigma} + U \sum_j g_{j\uparrow}^g g_{j\downarrow}^g + \sum_{\langle ij \rangle, \sigma} \left( V_{ij} e_{i\sigma}^g \bar{g}_{j\sigma} + \text{H.c.} \right),$$

where $\sum_{\langle ij \rangle}$ denotes the summation between nearest-neighbor sites, $\{ \lambda_{ij}^g, \lambda_{ij}^e \}$ are the annihilation and creation operators for atoms on $j$th site, and $U$ is the repulsive interaction strength. The synthetic Kondo coupling $V_{ij} \equiv \alpha I_{ij}$ with $I_{ij} = \int dx g(x) W_{i\sigma}(x - x_i) W_{j\sigma}(x - x_j)$.

From [12], we know that, aiming to synthesize the gapped Kondo state, i.e. the Kondo insulator, it is required that the tunneling of the localized fermions hosts an opposite sign compared with the conduction fermions. Unfortunately, this requirement is not easily satisfied, because the signs of the tunneling for the lowest Bloch bands, i.e. $t_g$ and $t_e$, are usually the same. On the other hand, if we invoke the following transformation

$$\bar{g}_{j\sigma} \rightarrow g_{j\sigma} e^{i\varphi},$$

into the Hamiltonian (9), the atom fields $g$ and $e$ will host tunneling with opposite signs. However, the new operator representation may lead to a staggered Kondo coupling $(V_{ij} e_{i\sigma}^g e_{j\sigma}^e \bar{g}_{j\sigma} + \text{H.c.})$. Next, we design the pumping laser and cavity mode to eliminate the staggered phase.

### 2.2. Synthetic Kondo coupling

The synthetic Kondo coupling $V_{ij} = \alpha K_{ij}$ is generated by the pumping laser as well as the atom–cavity coupling. It is obviously proportional to the superradiance order $\alpha$, hence works only in the superradiant state. As we focus on the physics of the 1D lattice system along the $x$ axis, the plane-wave mode of the pumping laser can be approximately treat as a constant strength $h(r) = \Omega_p$. From [12], we know the on-site Kondo coupling does not introduce topological nontrivial properties to the system. On the other hand, the on-site components with odd parity will exhibit features of spin–orbital couplings, and host the possibility to harbor a topological Kondo state. For this sake, we design the standing-wave mode of the cavity as $\Omega(x) = \Omega \sin(k_c x)$, where $\Omega$ is the atom–cavity coupling strength and $k_c$ is the cavity mode momentum. In this way, the on-site component in $I_{ij}$ vanishes while the nearest-neighbor-site component dominates. When we tune $k_c$ to match $k_c/2$, it will impose a phase $e^{-i\varphi}$ into $I_{ij}$, eliminating the staggered phase $e^{i\varphi}$ generated by the new operator representation (10). Thus we can obtain $I_{ij} \approx \pm K \delta_{ij} e^{-i\Omega x}$. Here we denote $K = \tilde{\Omega} \int dx \sin(k_c x/2) W_{i\sigma}(x) W_{j\sigma}(x - a)$. $\tilde{\Omega}$ is the coupling strength between $|g\rangle$ and $|e\rangle$, and its detailed form is given in appendix A.

Based on the above designs, the final expression of the effective Hamiltonian is written as

$$H = \sum_{\langle ij \rangle, \sigma} \left( t_{ij} \bar{g}_{j\sigma}^g e_{i\sigma}^g - t_e e_{i\sigma}^e \bar{g}_{j\sigma} + \Delta_0 \sum_{j\sigma} g_{j\sigma}^g \bar{g}_{j\sigma} + U \sum_j g_{j\uparrow}^g g_{j\downarrow}^g + \sum_{\langle ij \rangle, \sigma} \left( V_{ij} e_{i\sigma}^g \bar{g}_{j\sigma} + \text{H.c.} \right) \right).$$


where $V_{ij} = \pm \alpha K_b_{j\pm 1}$. At the limit $t_g \ll t_c$ and $U \to \infty$, the Hamiltonian (11) describes the Kondo lattice model associated with the off-site Kondo coupling [12].

2.3. Slave boson method

We employ the slave boson method [13] to explore the possible Kondo phase arising from our model Hamiltonian (11). In the slave boson method, the localized fermion operator $\hat{g}$ are decomposed into a fermion operator $\hat{b}$ associated with an auxiliary boson operator $\hat{b}$,

$$\hat{g}_{ij} = \hat{b}_{ij}^{\dagger}.$$ (12)

In this paper, we make the mean-field approximation to the boson operator $\hat{b} = (\hat{b}_l) \equiv b \equiv b$ and $\hat{b}_l^\dagger = (\hat{b}_l^\dagger) \equiv b^\dagger \equiv b^\dagger$, and recognize $b$ as the order parameter of the emergent Kondo phase. We focus on the results at zero-temperature limit, because the mean-field method can capture the qualitative and the topological features of the lattice system. After processing the standard approach (see appendix B), we can get the effective action

$$S = \int_0^\beta d\tau \left[ \sum_{j\rho} (\hat{g}_{ij}^{\dagger} \partial_\tau \hat{g}_{ij} + e_{ij}^\rho \partial_\tau e_{ij}) - \mathcal{H}_{\text{eff}} \right].$$ (13)

Here $\tau \equiv i\tau$ is the imaginary time. $\beta \equiv 1/(k_B T)$ with temperature $T$ and we set the Boltzmann constant $k_B = 1$ in the whole paper. $\mathcal{H}_{\text{eff}}$ is expressed as [42]

$$\mathcal{H}_{\text{eff}} = E_0 + \sum_{(j),\sigma} \hat{g}_{ij}^{\dagger} \hat{g}_{ij} - t_e \sum_{j,\sigma} \hat{g}_{ij}^{\dagger} e_{ij}^\sigma e_{ij} + \sum_{j,\sigma} \left( \lambda_\sigma - \Delta_0 \right) \hat{g}_{ij}^{\dagger} \hat{g}_{ij}$$

$$+ \sum_{(j),\sigma} \left( V_{ij} e_{ij}^\sigma b_{ij}^{\dagger} \right) + \text{H.c.}$$ (14)

with $E_0 \equiv \sum_\lambda \lambda_\lambda (|b|^2 - 1) + \Delta_0 |\alpha|^2$. $\lambda_\lambda$ is the Lagrange multiplier introduced by the number conservation on each site, and we can see $\Delta_0 - \lambda_\lambda$ characterizes the effective chemistry potential of the localized $\hat{g}$ fermions. The ground state at zero-temperature limit can be determined by self-consistently minimizing the free energy $\mathcal{F}$ with respect to the order parameter $b$ and $\lambda$ (see appendix B): (i) when $b$ is nonzero, the system is a Kondo state, and moreover is a superradiant Kondo state if $\alpha \neq 0$. (ii) When $b$ vanishes, the system is a normal gas state.

3. Phase transition and topological features

In figure 2, we plot the Kondo order $b$ and the superradiance order $\alpha$ with respect to $K$ at zero-temperature limit. Experimentally, $K$ can be tuned by the pumping laser strength $\Omega_p$ and the configuration of the optical cavity. The gap of the Kondo phase is the manifest signature distinct from the normal gas state, resulting in a Kondo insulator [13]. It originates from the screening of localized $\hat{g}$ fermions coupled to the mobile $e$ fermions, and is evaluated by the nonzero order parameter $b$. In figure 2 we can see there exist two states as the ground state of the lattice system. When $K$ exceeds a threshold, the system simultaneously processes two types of phase transitions: the Kondo transition from the normal gas state to the gapped Kondo phase, and the superradiant transition from the normal gas state to the superradiant state. This is because in section 2.2 we have know the Kondo coupling works only in a superradiant state. In another words, the atom–cavity coupling is responsible to the Kondo
screening. As both the two order parameters exhibit discontinuous evolutions across the phase transition, it yields that both the Kondo and superradiant transitions are of first order.

What interests us more here is whether the cavity-induced superradiant Kondo phase is a topological nontrivial state. To analyze its possible topological properties, we make a Fourier transformation to the effective Hamiltonian (14). In the base \( \Psi_k = (\hat{g}_k, \hat{e}_k, \hat{e}_f)^T \), it is written as

\[
\mathcal{H}_{\text{eff}}(k) = \begin{pmatrix}
\epsilon_g(k) & bV^*_k \\
b^*V_k & \epsilon_e(k)
\end{pmatrix} \otimes I_{2 \times 2},
\]

where \( \epsilon_g(k) = 2t_c \cos(kd) + \lambda - \Delta_o, \epsilon_e(k) = -2t_c \cos(kd), V_g = -i2\alpha K \sin(kd), \) and \( I_{2 \times 2} \) is the 2 \( \times \) 2 identity matrix. We treat \( b \) as a real number here, since its phase can be rotated off in \( \mathcal{H}_{\text{eff}} \) and does not change the physics. The Hamiltonian (15) can be decompose into two parts: \( \mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_t \) with the notations \( \mathcal{H}_0(k) \equiv \epsilon_g(k) + \epsilon_e(k)I_{2 \times 4} \) and \( \mathcal{H}_t(k) \equiv (d_k \cdot \sigma) \otimes I_{2 \times 2} \). Here \( d_k = (d_x^0, d_y^0, d_z^0) \) with

\[
d_x^0 = 0, d_y^0 = -2\alpha a K \sin(kd), \text{ and } d_z^0 = \frac{\epsilon_g(k) - \epsilon_e(k)}{2}, \sigma_i(i = x, y, z) \text{ are Pauli matrices. By making a unitary transformation } U = \exp(i\mathcal{H}_0t), \text{ we can eliminates } \mathcal{H}_0 \text{ in } \mathcal{H}_{\text{eff}}, \text{ and obtain}
\]

\[
\widehat{\mathcal{H}}_{\text{eff}} = U\mathcal{H}_{\text{eff}}U^\dagger = \mathcal{H}_t + [\mathcal{H}_0, \mathcal{H}_t] + \frac{1}{2} [\mathcal{H}_0, [\mathcal{H}_0, \mathcal{H}_t]] + \cdots,
\]

where we have used the Campbell–Baker–Hausdorff expansion. It is easy to demonstrate \([\mathcal{H}_0, \mathcal{H}_t] = 0 \) since \( \mathcal{H}_0 \) behaves like an identity matrix, and then we obtain \( \widehat{\mathcal{H}}_{\text{eff}} = \mathcal{H}_t \). Therefore, the lattice system described by \( \mathcal{H}_{\text{eff}} \) shares the same topological properties from the eigenstates of \( \mathcal{H}_t \) \([43, 44]\). We can see that the Hamiltonian \( \mathcal{H}_t \) respects the particle–hole symmetry: \( \Xi \mathcal{H}_t(k) \Xi^{-1} = -\mathcal{H}_t(-k) \), where \( \Xi = \sigma_y \mathcal{K} \otimes I_{2 \times 2} \) and \( \mathcal{K} \) is the complex conjugate operator. It indicates the superradiant Kondo phase belongs to a \( D \) topological class \([45]\), which is characterized by a \( \mathbb{Z}_2 \) topological invariant \([46]\).

In figure 3(a), we display the Bogoliubov–de Gennes (BdG) quasiparticle spectrum (see appendix B) for the superradiant Kondo phase. Distinct from the trivial Kondo phase, inside the band gap, the superradiant Kondo phase hosts edge states whose wave functions are localized on the chain ends, as shown in figure 3(b). This is the key signature as a topological insulator for the superradiant Kondo phase. The lattice model has no spin.
hybridization and is spin degenerate. Therefore, the edge states are four-fold degenerate. They are protected by the particle–hole symmetry discussed above.

4. Discussions

Our proposal is readily extended to a two-dimensional (2D) case. It can be realized by engineering a 2D optical lattice and two optical cavities in the $x$–$y$ plane. We assume the cavity modes $\eta_x(x) = \Omega_x \sin(k_x x/2)$ and $\eta_y(y) = \Omega_y \sin(k_y y/2)$. Then the Kondo coupling originated from the Raman transition can give rise to a Chern Kondo insulator state, which has also been proposed via Bloch-orbital hybridizations in ultracold Fermi gases [19]. It is noted that, in the 2D extension of our proposal, the emergence of the Chern Kondo insulator state is still ascribed to the cavity field.

5. Conclusions

In summary, we propose a scheme for synthesizing the Kondo insulator in ultracold Fermi gases placed in a pumped optical cavity. The synthetic Kondo coupling originates from the Raman transition, which in our proposal is driven by the pumping laser as well as the cavity field. Distinguished from the trivial Kondo phase, the atom–cavity coupling gives rise to a synthetic nearest-neighbor-site Kondo coupling, and is the key ingredient for supporting the topological superradiant Kondo phase with edge states gapped from the bulk. Our proposal is simple and reliable based on current experimental technique, and can provide a versatile platform for quantum simulating and studying the many-body physics and topological phases of the Kondo insulator.

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Appendix A. Effective Hamiltonian

For the model illustrated in figure 1, we start with Jaynes–Cummings Hamiltonian [32]:

$$H(x) = H_L(x) + H_I(x) + H_2(x).$$  \hfill (A.1)

The first term describes the free atoms confined in the optical lattice,

$$H_L(x) = \sum_{\lambda=g,e,\sigma} \psi_{\lambda,\sigma}^\dagger(x) \left[ -\frac{\nabla^2}{2m} + V_L(x) \right] \psi_{\lambda,\sigma}(x).$$  \hfill (A.2)

The second term describes the transition between $|g\rangle$ and $|s\rangle$ (|s\rangle denote levels with the highest energy in the Raman transition, i.e. the upper gray levels in figure 1(a)),

$$H_I(x) = \sum_\sigma \Gamma_{s,\sigma} \psi_{s,\sigma}^\dagger(x) \psi_{g,\sigma}(x) + \sum_\sigma [h(r) e^{-i\omega t} \psi_{s,\sigma}^\dagger(x) \psi_{g,\sigma}(x) + \text{H.c.}].$$  \hfill (A.3)

Here $\Gamma_{s}$ is the atomic frequency difference between $|g\rangle$ and $|s\rangle$. The last term describes the transition between $|e\rangle$ and $|s\rangle$,

$$H_2(x) = \sum_\sigma \Gamma_{e,\sigma} \psi_{e,\sigma}^\dagger(x) \psi_{s,\sigma}(x) + \omega_e a^\dagger a + \eta(x) \sum_\sigma [a \psi_{s,\sigma}^\dagger(x) \psi_{e,\sigma}(x) + \text{H.c.}].$$  \hfill (A.4)

Here $\Gamma_{e}$ is the atomic frequency difference between $|g\rangle$ and $|e\rangle$. As we choose $|g\rangle$ and $|e\rangle$ by hyperfine states with the same nuclear spin, $\Gamma_{g}$ and $\Gamma_{e}$ can be tuned to be approximately independent of the pseudo-spins $\sigma$. We make a unitary transformation

$$U = \exp \left\{ i \sum_\sigma (\Gamma_s \psi_{s,\sigma}^\dagger \psi_{g,\sigma} + \Gamma_e \psi_{e,\sigma}^\dagger \psi_{s,\sigma}) + \omega_e a^\dagger a \right\}.$$

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In the rotating frame, the Hamiltonian $H(x)$ is rewritten as

$$H' = U^\dagger H(x) U - U^\dagger \partial_x U = H_L + \sum_{\sigma} \left[ h(r) e^{i \hat{W}_x \psi_{\sigma \sigma}} + \eta(x) e^{i \hat{W}_x \psi_{\sigma \sigma}} + \text{H.c.} \right],$$

(A.6)

where $\Gamma_p = \Gamma_p - \omega_p$ and $\Gamma_c = \Gamma_c - \omega_c$. Here we make the notations $\Delta \equiv (\Gamma_p + \Gamma_c)/2$ and $\delta \equiv (\Gamma_p - \Gamma_c)/2$, and make the unitary transformation

$$U' = \exp \left\{ i \left( \hat{a}' \hat{a}' - \sum_{\sigma} \psi_{\sigma \sigma}' \psi_{\sigma \sigma}' \right) t \right\}.$$

(A.7)

The Hamiltonian $H'(x)$ is rewritten as

$$H'' = U^\dagger H'(x) U' - U^\dagger \partial_x U' = H_L + \delta \hat{a}' \hat{a}' - \delta \sum_{\sigma} \psi_{\sigma \sigma}' \psi_{\sigma \sigma}' + \sum_{\sigma} \left[ h(r) e^{i \Delta \tau_{\sigma \sigma}^+ \psi_{\sigma \sigma}} + \eta(x) e^{i \Delta \tau_{\sigma \sigma}^+ \psi_{\sigma \sigma}} + \text{H.c.} \right].$$

(A.8)

Adiabatically eliminating $|\psi\rangle$, we obtain the final form of the effective Hamiltonian in section 2.1:

$$\mathcal{H}(x) = H_L + \delta \hat{a}' \hat{a}' - \delta \sum_{\sigma} \psi_{\sigma \sigma}' \psi_{\sigma \sigma}' + g(x) \sum_{\sigma} [a \psi_{\sigma \sigma}'(x) \psi_{\sigma \sigma}(x) + \text{H.c.}],$$

(A.9)

where $g(x) = h(r) \eta(x)/\Delta = \tilde{\Omega} \sin(k_{\perp} L/2x)$ with $\tilde{\Omega} \equiv \Omega_{\perp} \Omega_{\parallel}/\Delta$. In section 2.1, we set $\Delta_{\perp} = -\delta$. $\Delta_{\parallel}$ can be changed by $\delta$ associated with an AC-Stark shift generated by a far-detuned auxiliary laser.

The parameters we choose in the phase diagram are accessible in practice. They can be estimated as follows. The numeric calculation via the maximally localized Wannier functions gives the tunneling $t \approx t_{\perp} \approx 0.143E_R$ with $V_c = 2E_R$, and $t_{\parallel} \approx 0.308E_R \approx 0.215t$ with the trap depth $V_{\perp} = 8E_R$. Here $E_R \equiv h^2/2md^2$ is the lattice recoil energy. When tuning $\Delta \approx 300E_R$, $\Omega_{\perp} \approx 100E_R$ and $\Omega_{\parallel} \approx 600$, it gives $K \approx 12.03t$. Therefore, the parameter regions displayed in figure 2 are available in real ultracold Fermi gases.

### Appendix B. Slave boson approach

In the slave boson method, the localized fermion operator $\hat{g}$ is written as a decomposition of a fermion operator $\hat{g}$ as well as an auxiliary boson operator $\hat{b}$,

$$\hat{g}_{\sigma \sigma} = \hat{g}_{\sigma \sigma} \hat{b}^\dagger_j.$$

(B.1)

Here $\hat{g}$ and $\hat{b}$ satisfy the single occupancy constraint on each site,

$$\hat{b}^\dagger_j \hat{b}_j + \sum_{\sigma} \hat{g}_{\sigma \sigma}^\dagger \hat{g}_{\sigma \sigma} = 1.$$

(B.2)

Then by excluding the $U$-term, the Kondo lattice Hamiltonian (9) can now be recast as

$$\mathcal{H} = t_{\perp} \sum_{\langle j, j \rangle, \sigma} \hat{g}_{\sigma \sigma}^\dagger \hat{g}_{\sigma \sigma} - t_{\parallel} \sum_{\langle j, j \rangle, \sigma} e_{\sigma \sigma}^j e_{\sigma \sigma} - \Delta_{\parallel} \sum_{j, \sigma} \hat{g}_{\sigma \sigma}^\dagger \hat{g}_{\sigma \sigma} + \alpha K \sum_{j, \sigma} \left( \hat{b}^\dagger_j e_{\sigma \sigma}^j \hat{g}_{\sigma \sigma} - \hat{b}^\dagger_j e_{\sigma \sigma}^j \hat{g}_{\sigma \sigma} + \text{H.c.} \right).$$

(B.3)

We make the mean-field approximation to the boson operator

$$\hat{b}_j \approx \langle \hat{b}_j \rangle \approx b_j, \quad \hat{b}^\dagger_j \approx \langle \hat{b}^\dagger_j \rangle \approx b^*.$$

(B.4)

The effective action is written as $S = \int dt \mathcal{L} \equiv \int dt (\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2)$ with

$$\mathcal{L}_0 = \sum_{\sigma \sigma} \left( \hat{g}_{\sigma \sigma}^\dagger \partial_t \hat{g}_{\sigma \sigma} + e_{\sigma \sigma}^j \partial_j e_{\sigma \sigma} \right) - \mathcal{H}_t,$$

$$\mathcal{L}_1 = -\sum_j \lambda \left( \| b_j \|^2 + \sum_{\sigma} \hat{g}_{\sigma \sigma}^\dagger \hat{g}_{\sigma \sigma} - 1 \right), \quad \mathcal{L}_2 = -\Delta_{\parallel} |\alpha|^2.$$

(B.5)
Here $L_1$ is introduced by the constraint from the particle number conservation (B.2) with the Lagrange multiplier $\lambda_j$, and $L_2$ is induced from photons. The effective Hamiltonian is represented by combining $\mathcal{H}$ and $L_1 + L_2$

$$
\mathcal{H}_{\text{eff}} = E_0 + t_x \sum_j \left[ \hat{g}_{0j} \hat{g}_{0j} - t_x \sum_{\langle ij \rangle, \sigma} \hat{e}_{ij}^\dagger \hat{e}_{ij} + \sum_j (\lambda_j - \Delta_c) \hat{g}_{0j}^\dagger \hat{g}_{0j} \right]
\quad + \alpha K \sum_{j, \sigma} \left( b_j^* \hat{e}_{j+1, \sigma}^\dagger \hat{g}_{0j} - b_j^\dagger \hat{e}_{j-1, \sigma} \hat{g}_{0j} + \text{H.c.} \right).
\quad (B.7)
$$

where $E_0 = \sum_j \lambda_j | \langle b_j |^2 - 1 | + \Delta_c | \alpha |^2$. The Lagrange action is then written as

$$
\mathcal{L} = \sum_{\langle ij \rangle, \sigma} \left[ \hat{g}_{0j} \hat{g}_{0j} + \hat{e}_{ij}^\dagger \hat{e}_{ij} \right] - \mathcal{H}_{\text{eff}}. \mathcal{H}_{\text{eff}}$ can be diagonalized into a quadratic form

$$
\mathcal{H}_{\text{eff}} = \sum_{\eta = 1}^{4L} E_\eta \hat{c}_\eta^\dagger \hat{c}_\eta + E_0
\quad (B.8)
$$

by employing the BdG transformation

$$
\hat{g}_j = \sum_{\eta = 1}^{2L} u_\eta^j \alpha_\eta, \quad e_j = \sum_{\eta = 1}^{2L} v_\eta^j \alpha_\eta.
\quad (B.9)
$$

Here $E_\eta$ is the BdG quasiparticle spectrum. The coefficients $u^\eta \equiv (u_1^\eta, \cdots, u_{2L}^\eta)^T$ and $v^\eta \equiv (v_1^\eta, \cdots, v_{2L}^\eta)^T$ obey the BdG equation expressed as

$$
\mathcal{H}_{\text{BdG}} (v^\eta) = E_\eta (v^\eta), \quad \mathcal{H}_{\text{BdG}} = \begin{pmatrix}
\hat{h}_{g\uparrow} & 0 & \hat{\nu}^\dagger \\
0 & \hat{h}_{g\downarrow} & 0 \\
\hat{\nu} & 0 & \hat{h}_{e\uparrow} \\
0 & \hat{\nu} & \hat{h}_{e\downarrow}
\end{pmatrix},
\quad (B.10)
$$

where $[\hat{h}_{g\sigma}]_{ij} = t_x \delta_{i,j+1} + (\lambda_j - \Delta_c) \delta_{ij}, [\hat{h}_{e\sigma}]_{ij} = -t_x \delta_{i,j+1}$, and $\hat{\nu}_{ij} = \pm \alpha K b_j^\dagger \delta_{i,j+1}$. The mean-field variables $b_j \approx b$ and $\lambda_j \approx \lambda$ are determined by minimizing the free energy [42]

$$
F = E_0 - \frac{1}{\beta} \sum_{k, \sigma} \ln \left[ 1 + e^{-\beta E_k} \right],
\quad (B.11)
$$

where $E_k$ is eigenvalues of $\mathcal{H}_{\text{BdG}}$ in the momentum space, and $\beta \equiv 1/T$. This is achieved by self-consistently solving the following equations

$$
\frac{\partial F}{\partial \theta} = 0 \quad (\theta = b, \lambda)
\quad (B.12)
$$

as well as the steady-state condition of the cavity (see equation (6)) which in the tight-binding model is formulated as

$$
\alpha = \frac{\eta}{\Delta_c + \Delta_e}, \quad \eta = \frac{K}{L} \sum_{j, \sigma} b_j (\langle \hat{g}_{j+1, \sigma}^\dagger \hat{g}_{0j} \rangle - \langle \hat{g}_{j-1, \sigma}^\dagger \hat{g}_{0j} \rangle). \quad (B.13)
$$

References

[1] Bloch I, Dalibard J and Nascimbène S 2012 Nat. Phys. 8 267–76
[2] Dalibard J, Gerbier F, Juzeliūnas G and Öhberg P 2011 Rev. Mod. Phys. 83 1523
[3] Goldman N, Juzeliūnas G, Öhberg P and Spielman I B 2014 Rep. Prog. Phys. 77 126401
[4] Jakšč D and Zoller P 2003 New J. Phys. 5 56.1–56.11
[5] Aidelsburger M, Atala M, Lohse M, Barreiro J T, Paredes B and Bloch I 2013 Phys. Rev. Lett. 111 185301
[6] Miyake H, Sivilevog G A, Kennedy C J, Burton W C and Ketterle W 2013 Phys. Rev. Lett. 111 185302
[7] Soltan-Panahi P et al 2011 Nat. Phys. 7 434
[8] Tarruell L, Greif D, Uehlinger T, Itozú G and Esslinger T 2012 Nature 483 302
[9] Jo G-B, Guzman J, Thomas C K, Housar P, Vishwanath A and Stamper-Kurn D M 2012 Phys. Rev. Lett. 108 045305
[10] Becker C, Soltan-Panahi P, Kronjäger J, Dörscher S, Bongs K and Sengstock K 2010 New J. Phys. 12 065025
[11] Dzero M, Sun K, Coleman P and Galitski V 2012 Phys. Rev. B 85 045130
[12] Dzero M, Xia J, Galitski V and Coleman P 2016 Annu. Rev. Condens. Matter Phys. 7 249–80
[13] Tsunetsugu H, Sigrist M and Ueda K 1997 Rev. Mod. Phys. 69 809
[14] Dzero M, Sun K, Galitski V and Coleman P 2010 Phys. Rev. Lett. 104 106408
[15] Miyazaki H, Hajiri T, Ito T, Kunii S and Kimura S 2012 Phys. Rev. B 86 075105
[16] Lu F, Zhao J Z, Weng H, Fang Z and Dai X 2013 Phys. Rev. Lett. 110 096401
[17] Feng X Y, Dai J, Zhang C H and Si Q 2013 Phys. Rev. Lett. 111 016402
[18] Nakagawa M and Kawakami N 2015 Phys. Rev. Lett. 115 165303
[19] Chen H, Liu X-J and Xie C 2016 Phys. Rev. Lett. 116 046401
[20] Gorshkov A V et al 2010 Nat. Phys. 6 289–95
[21] Scazza F, Hofrichter C, Hofer M, De Groot P C, Bloch I and Fölling S 2014 Nat. Phys. 10 779–84
[22] Isaev L, Schachenmayer J and Rey A M 2016 Phys. Rev. Lett. 117 135302
[23] Foss-Feig M, Hermele M and Rey A M 2010 Phys. Rev. A 81 051603(R)
[24] Sundar B and Mueller E J 2016 Phys. Rev. A 93 023635
[25] Zhang R, Zhang D, Chen Y, Chen W, Zhang P and Zhai H 2016 Phys. Rev. A 93 043601
[26] Ritsch H, Domokos P, Brennecke F and Esslinger T 2013 Rev. Mod. Phys. 85 555
[27] Maschler C and Ritsch H 2005 Phys. Rev. Lett. 95 260401
[28] Baumann K, Guerlin C, Brennecke F and Esslinger T 2010 Nature 464 1301–6
[29] Baumann K, Mottl R, Brennecke F and Esslinger T 2011 Phys. Rev. Lett. 107 140402
[30] Keeling J, Bhaseen M J and Simons B D 2014 Phys. Rev. Lett. 112 143002
[31] Chen Y, Yu Z and Zhai H 2014 Phys. Rev. Lett. 112 143004
[32] Deng Y, Cheng J, Jing H and Yi S 2014 Phys. Rev. Lett. 112 143007
[33] Dong L, Zhou L, Wu B, Ramachandhran B and Pu H 2014 Phys. Rev. A 89 011602(R)
[34] Kollath C, Sheikhan A, Wolf S and Brennecke F 2016 Phys. Rev. Lett. 116 060401
[35] Sheikhan A, Brennecke F and Kollath C 2016 Phys. Rev. A 93 043609
[36] Zheng W and Copper N R 2016 Phys. Rev. Lett. 117 175302
[37] Ballantine K E, Lev B L and Keeling J 2017 Phys. Rev. Lett. 118 045302
[38] Pan J-S, Liu X-J, Zhang W, Yi W and Guo G-C 2015 Phys. Rev. Lett. 115 045303
[39] Sheikhan A, Brennecke F and Kollath C 2016 Phys. Rev. A 94 061603(R)
[40] Mivehvar F, Ritsch H and Piazza F 2017 Phys. Rev. Lett. 118 073602
[41] Chin C, Grimm R, Julienne P and Tiesinga E 2010 Rev. Mod. Phys. 82 1225
[42] Coleman P 2015 Introduction to Many-Body Physics (Cambridge: Cambridge University Press)
[43] König M et al 2008 J. Phys. Soc. Jpn. 77 031007
[44] Zheng Z, Pu H, Zou X and Guo G 2017 Phys. Rev. A 95 013616
[45] Schnyder A P, Ryu S, Furusaki A and Ludwig A W W 2008 Phys. Rev. B 78 195125
[46] Alexandrov V and Coleman P 2014 Phys. Rev. B 90 115147