Recreation of Small Strains Phenomenon under Pavement Structure and Consequences of Failure to Address It

Lidia Fedorowicz¹, Marta Kadela²

¹ Katowice School of Technology, ul. Rolna 43, 40-555 Katowice, Poland
² Building Research Institute (ITB), ul. Filtrowa 1, 00-611 Warsaw, Poland

m.kadela@itb.pl

Abstract. This paper describes the small strains phenomenon which occurs in the subgrade under a pavement, a phenomenon documented through in-situ tests and recreated in numerical analyses, which lends a practical engineering aspect to the subject matter. The analyses were preceded by: 1) presentation of the role of constitutive models in structure-subgrade system analysis, 2) reference to methods of modelling in mechanistic procedures and possibility of reliable assessment of criterial values in road structures. These studies were coupled with a description of field tests, which recorded strains in subgrade under a loaded pavement: in zone I directly under the pavement – variable, depending on stiffness of the pavement and the load (about 200÷1000.10⁻⁶) and below that, in zone II – ‘stabilised’ (about 1÷5.10⁻⁶). In summary, it has been found that the accuracy of numerical analyses of structure-subgrade systems is dependent on the adopted constitutive model of the soil and the numerical calculation area representing the subgrade. Recreation and analysis of the pavement-subgrade system behaviour employed the MCC(OC) critical state model. It was determined that a reliable response of the computational model to the load path used can be obtained with a model that has been previously properly calibrated. The paper justifies the need to carry out further, directed field tests, coupled with numerical analyses employing relevant constitutive models for description of the soil’s performance.

1. Introduction – problem description

The characteristic (structurally complex) composition of soil means that its properties are not constant; they easily change due to changes in moisture content or porosity. The considerable heterogeneous nature of soil behaviour has caused some of its properties to still not be identified in detail despite years of research. The needs of engineering – tied to new, ever larger projects – have shaped the contemporary development of strictly research and numerical methods. One of the topics drawing researchers’ attention is the complex behaviour of soils in the scope of small and very small strains. The interest in small and very small strains (10⁻³ and less) goes as far back as the 1970s and is tied to specific research centres. Resonant columns played a key part in this research. Resonant frequency has provided a basis for calculation of the so-called dynamic shear modulus \( G \). Their values are at least an order or magnitude higher than the value of the Kirchhoff modulus used in designing.

The last two decades of research, which is still being carried out, has finally confirmed that the stress-strain relation is highly non-linear. Sources credited here should include precursors as well as scientists involved later (e.g. [1-7]), Polish ones among them – e.g. Świdziński, Jastrzębska [8-15].
Figure 1. Approximate ranges of reliable application of various measuring techniques for soil stiffness characteristics [13]

It would be appropriate here to reference the representative graph of soil stiffness characteristic, a result of research of multiple scholars (figure 1). The graph compiles ranges of strain values in soil, which correspond to performance of different types of structures. It can be said that cognitively the phenomenon of small strains has been fully acknowledged in geotechnics. However, the application side of the problem still seems to be underdeveloped. This applies mainly to structures which interact with the subgrade with strains estimated within the \((10^{-5}÷10^{-3})\) range. In this scope, tests record a sudden drop of soil stiffness – coupled with an increase in strain.

In reality, when soil is loaded with the weight of the structure we are dealing with (due to the stiffness or the span of the structure) with small strains in the soil, which are accompanied by a high or very high soil stiffness. It was only recently that discussions started on the subject of overestimating structure settlement due to failure to account for the small strains phenomenon; even though, as figure 2 below illustrates, respective in-situ tests were carried out as early as in the 1980s and 1990s. The drawing presents, respectively: figures 2 a-b (quoting [16]) – function of strain and settlement of the till under a residential high rise and figure 2c (quoting [17]) – results of field tests documenting a highly non-linear response of the soil in the scope of small strains.

Figure 2c additionally suggests how significantly settlement calculated with linear elasticity may be overestimated.

Development of laboratory testing and attempts at describing soil behaviour in the scope of small strains contribute to creation of complex constitutive models (in Poland, e.g. by Gryczmański [18,19], Sternik [20], Cudny [21]). However, applicability of this phenomenon in engineering is dependent on identification and proper interpretation of in-situ tests.

This paper deals about:

1)  experimental and practical aspect of the small strains phenomenon present in the subgrade under a pavement structure, and
2)  the effect of ignoring this phenomenon on evaluation of criterial values in mechanistic procedures and on the pavement structure durability estimation.

These analyses were preceded by a presentation of the role of adequate numerical modelling in general-purpose analyses of structure-subgrade systems – chapter 1.

A wider discussion of the methods of reliable estimation of criterial values for pavements and the methods of modelling in mechanistic procedures is included in chapter 2.

The above deliberations were coupled with a description of field tests (chapter 3), which recorded strains in subgrade under a loaded pavement: in zone I directly under the pavement – variable, depending on the stiffness of the pavement and the load (about 200÷1000 \(10^{-6}\)) and below that, in zone II – ‘stabilised’ (about 1÷5 \(10^{-6}\)).
2. Numerical analyses of pavement-subgrade systems – linearly elastic constitutive soil model

Specific displacement mechanisms, and therefore structure settlement [22], are present in the soil. These mechanisms are tied to phenomena such as compaction, consolidation or changes in water conditions (as well as disturbance of the stability of the rock mass during excavation or fossil fuel mining).

We should therefore consider a procedure that is to result in a numerical estimation of structure settlement (in this case, a foundation with a specified stiffness). In reality, commonly used terms such
as ‘analysis on elastic base’ or ‘on elastic halfspace’ are poorly defined in numerical calculations. Figure 3 provides a simple illustration of the problem connected with reflectionless application of a linearly elastic soil model for structure-subgrade contact problems.

The $s$ functions shown in figure 3a) indicate settlements of circular foundations on elastic halfspace (recreated numerically – with an assumed $E$ modulus error equal to $\delta E$). Functions $s_w$ – indicate numerically determined settlement of circular foundations on an elastic layer. Every solution refers to a numerical $s(i)$ area – the response of the subgrade model ($e$) to loads. The $s(i)$ areas are defined by their height $h$ – being the thickness of the subgrade.

It should be noted that the modelled elastic layer is not a very reliable method to record the behaviour of a loaded subgrade and may be the source of incorrect estimation of internal dimensions in a structure (those dependent on the adopted thickness of the layers).

Additional consequences of assuming elastic performance of the soil are presented in figure 3b, which shows the impossibility to recreate the following in numerical models:

- the real effect of the limited deformation response of the loaded soil (in this case, under test load), and thus
- unambiguous determination of stiffness of the soil in an elastic numerical model; which is evident from in-situ tests.

The error of representing the stiffness of a given medium (figure 3b) will depend on the thickness and stiffness of the layers, mainly on the height of the numerical subgrade area $h_p$. The response of the subgrade model to load transferred from the structure is measured with the equivalent modulus $E_{\text{response}}$ (which depends on angle $\alpha_{\text{num}}$). Determining the ‘expected’ stiffness of the soil under a structure (example horizontal line from point A in figure 3c) indicates possible various combinations of layer stiffness in the analysed model). Additionally, this combination is ambiguous without formulating rules for determining $h_p$, that is adequate for a given system under analysis.

When choosing a more complex (not linearly elastic) model, typically constitutive models are used which are easy to interpret in engineering analyses in terms of parameters. This applies to models with Coulomb-Mohr, or Drucker-Prager yield surface, which in their fundamentals (outside the elastic zone) are based on strength-based description of material performance. We need to remember that the problems indicated above concerning reliable estimation of the area of subgrade model’s response to loads transferred from the structure apply to all constitutive models of soil which originate from the linearly elastic description.

3. Role of proper numerical modelling in the analysis of structure-subgrade systems

Below, we present a suggestion of overcoming some of the above problems (related to creating a reliable computational model for structure-subgrade systems) by using the Modified Cam-Clay (MCC) critical state model to characterise the performance of soil.

Conditions significant from the point of view of proper modelling of contact problems may be phrased as shown below [23], where the two first conditions are detailed in figure 4:

1. A minimum, not disturbed by boundary conditions, area of numerical (displacement) response of the model to load may only be correctly defined for an adequate constitutive soil model; in-situ tests are preferable. We will refer to this problem as base problem.
2. Every numerical area larger than the minimum, with $h > h_{\text{min}}$ used as computational model of the subgrade allows (for the right constitutive model) to repeat the base problem, recreated with a model with $h_{\text{min}}$.
3. Recreating the base problem (load-settlement relation obtained through in-situ testing) is equivalent to recreating the phenomena recorded on the surface.

At the same time, the condition of reproducibility of phenomena occurring within the loaded soil in the computational model should also be met. This way, we obtain the ability to properly predict the behaviour of a real structure-subgrade system.

The nature of relation between deformations and soil strength (used in critical state models) is shown in figure 5 [24], presenting, in sequence:
- strength characteristics of soils obtained through shear tests,
- soil strength at shear strength peak and in critical state, and
- normal consolidation line and rebound line as a stress-volume change relation (expressed by the change of void ratio \( e \)).

![Figure 4. Base problem recreated using the Modified Cam-Clay (MCC) model](image)

In the critical state model descriptions (such as the Modified Cam-Clay (MCC) model used here), the SBS surface presented in figure 6. constitutes the image of ‘coupling’ the strains and stress in the soil with the void ratio.

![Figure 6. SBS surface with lines indicating states of soil strengthening and weakening](image)

Description of soil behaviour in the MCC model may apply both to cohesive and non-cohesive soils (as shown in figure 6), and soils ‘modified on the surface’ [25]. It should be emphasised that the
reliability of representing reality through numerical simulation is strongly related to the reliability of determining geostatic stresses. This is shown in figure 7, which presents a simulation of settlement for foundations according to different evaluation criteria. For the MCC(OC) model, medium-consolidated soil was assumed (preconsolidation pressure \( p_{oc} = 157 \) kPa at the depth of 0.75 m, \( K_o \) distribution profile \( K_o(OC)=1.5 \) to a depth of 1.5 m, below that linearly varying to a value of \( K_o(OC)=0.47 \) at depth of 40 m).

![Figure 7. Foundation settlement values \( D_i \) according to different evaluation criteria](image)

4. Response to load of the pavement structure-subgrade system in in-situ tests and numerical analyses

In engineering, for the description of the soil under pavement it is assumed that the stresses caused by traffic load is concentrated mostly in the pavement itself, and when transferring into the soil, they do not penetrate it very deeply and disperse with depth. This leads to the practice of assuming subgrade of very low thickness in the contact zone. However, as the analyses in chapter 1 show, proper evaluation of subgrade stiffness is a priority problem in analyses of structure-subgrade systems.

The problem connected with arbitrary estimation of the thickness of subgrade interacting with the structure is made apparent by the results of numerical mechanistic analyses [26-28]. Incorrect characterisation of the behaviour of soil causes the mechanistic procedure to yield an incorrect estimation of criterial values and pavement performance [29,30]. This can be observed if the subgrade calculation area is excessively increased in an elastic model; it is also confirmed by research in [31]. Ambiguity in mechanistic evaluations using elastic models is illustrated in figure 8.

Using advanced models (which account for load history) seems necessary if there is a need to precisely determine strains in the subgrade, and the settlement function shows clear dependence on in-situ stress profile. Changes in geostatic stresses may in general be a result of loads applied (changes affect values \( \sigma_v \) and \( \sigma_h \)) or forced displacement, e.g. during soil compaction, where changes likely only affect \( \sigma_h \) values (e.g. [25]).

Based on experience from field tests [32] and numerical analyses [31,33,34], it is possible to introduce into mechanistic procedures analyses based on non-elastic soil performance models, which realistically couple the stress state under a structure with soil volume changes. Figure 9a shows example results from in-situ tests – deformation response of the soil under a KR4 type pavement, registered with (vibrating wire strain sensors) installed in the vertical soil profile.

Figures 9 b-c show a general image of the area of recorded soil response to loads from the rear axle of a vehicle transferred onto the tested vertical profile (see figure 9a), through:

- substructure (summer 2011); early stage of performance, with no asphaltic concrete
- full structure (summer 2012).
Figure 8. Fatigue strength as a function of subgrade thickness calculated based on a) asphalt cracking criterion, b) subgrade deformation criterion for different type of traffic category and pavement type.

Figure 9. a) Influence of HGV type on subgrade response and division of subgrade response area into zones I and II, b) for system with no asphaltic concrete, c) for complete system

In all records of the response, two zones may be distinguished:
- zone I of behaviour – strains measured in the ‘active’ soil performance, with a depth range of c. 1.0 m
- zone II of behaviour – area of strains which can be classified as very small strains (registered depth range of c. 2.0 m).

In zone I with ‘active’ behaviour, compression of subgrade was recorded, a result of vehicle traffic. Figures 9 b-c are the image of stabilised subgrade response, obtained after the process of soil compaction (medium and fine sands with minor intrusions of silk and clay).

For MCC model parameters (in figure 10b), a distribution of strains is obtained (figure 10a) corresponding to zone I and II of soil behaviour (consistent with in-situ observations) and correlated to the displacement under the structure in both computational models (figure 10b).
It should be emphasised that the matrix of distribution for parameter $\kappa$ (and $G$) in the MCC model was created as a ‘mirror’ of the identified strains in the soil bearing the load from the pavement. This allowed to recreate the real behaviour of the subgrade – in the form of ‘response’ consisting in high stiffness in the zone where very small strains caused by soil load were found (i.e. in zone II).

5. Summary and conclusions
Summing up the problems described above, it may be stated that result accuracy of numerical analyses for structure-subgrade systems is related both to the constitutive model of the soil employed, and to the numerical calculation area representing the subgrade. The reliability of the computational model response to the used load path may be evaluated with the help of model calibration. Figure 10a (loaded substructure-subgrade system) may be said to represent a sufficiently calibrated numerical model which uses a combination of two (e)-(MCC) constitutive models. Matrix of MCC model parameters (here parameters $\kappa$ and $G$) was created as a ‘mirror’ of the strains in the soil bearing the load from the pavement, identified in tests.

Considering the experiments presented and subsequent analyses in [34], the need to carry out further field tests becomes apparent. Their aim would be to confirm the presented interpretation of in-situ tests, and further to obtain the ability to unambiguously take into account the real layer of improved subgrade for modelling pavement-subgrade systems. Such approach would, firstly, enable evaluation of the system’s performance as a whole, and secondly, would allow to optimise the thickness of particular layers of the pavement and the improved subgrade, and to use new materials in those layers [35-39].

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