Description of giant resonances with Skyrme forces

J Kvasil\textsuperscript{1}, V O Nesterenko\textsuperscript{2}, P Vesely\textsuperscript{1,3}, W Kleinig\textsuperscript{2,4}, and P-G Reinhard\textsuperscript{5}

\textsuperscript{1}Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Praha 8, Czech Republic
E-mail: kvasil@ipnp.troja.mff.cuni.cz, vesely@ipnp.troja.mff.cuni.cz
\textsuperscript{2}Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region, 141980, Russia
E-mail: nester@theor.jinr.ru, kleinig@theor.jinr.ru
\textsuperscript{3}Department of Physics, Univ. of Jyvaskyla, P.O. Box 35 (YFL), 40014, Jyvaskyla, Finland
\textsuperscript{4}Technische Universität Dresden, Institut für Analysis, D-01062, Dresden, Germany
\textsuperscript{5}Institut für Theoretische Physik, Universität Erlangen, D-91058, Erlangen, Germany

Abstract. Some representative cases of the description and analysis of giant resonances within the self-consistent separable random-phase-approximation model (SRPA) based on the Skyrme functional are discussed. It is shown that SRPA with SLy6 force well describes the dipole giant resonance in rare-earth and actinide regions. The sensitivity of the E1 strength near the particle thresholds to nuclear deformation is scrutinized. Finally, the open problems of description of spin-flip M1 resonance are discussed.

1. Introduction

Nowadays, the Skyrme Hartree-Fock (SHF) mean-field models represent one of the most widely used theoretical tools for investigation of nuclear structure and dynamics, see e.g. reviews [1, 2]. These models provide high performance in description of ground state properties but description of dynamics is still a developing field, in particular concerning the exploration of giant resonances (GR). In the latter case, the SHF implementations with high accuracy and yet modest computational expense are desirable, especially for investigation of nuclear excitations in heavy spherical and deformed nuclei.

In this connection, the separable random-phase-approximation (SRPA) model with Skyrme forces was recently developed [3, 4, 5] and widely applied to description of electric [3, 4, 7, 8, 9, 10, 11] and magnetic [5, 12, 13] GR. SRPA belongs to self-consistent theories of excitations where both the static mean field and the residual interaction are derived from the same Skyrme energy-density functional [14, 15, 16]. The residual interaction includes all contributions from the functional (including those from the time-odd current and spin densities) as well as the Coulomb (direct and exchange) and pairing (at BCS level) terms. SRPA employs a self-consistent factorization of the residual interaction which considerably reduces the computational expense while maintaining high accuracy. The model is particularly suitable for systematic studies and transparent for the analysis.

In this paper, we present some of our recent results for isovector E1(T=1) and spin-flip M1(T=1) GR [5, 8, 9, 12]. First of all, we demonstrate the high accuracy of SRPA for E1(T=1)
GR in the rare-earth and actinide regions [8]. As a second step, we analyze the impact of nuclear deformation on E1 strength near the neutron and proton thresholds [9]. Being rather weak, this strength is nonetheless important for understanding astrophysical processes [17]. Its shown that the impact of deformation may be negligible or be of both sign, depending on the energy interval between the thresholds and E1(T=1) GR. Finally we inspect the problem of simultaneous description of gross-structure of spin-flip M1(T=1) GR in spherical and deformed nuclei [5, 12, 13]. We show that this problem is still a challenge for Skyrme forces and discuss possible solutions including the implementation of tensor forces.

2. Skyrme-RPA model

Starting point is the Skyrme functional [1, 2]

\[
E = \int d\mathbf{r} (\mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{Sk}}(\rho_q, \tau_q, s_q, J_q, T_q) + \mathcal{H}_{\text{pair}}(\chi_q) + \mathcal{H}_{C}(\rho_p))
\]

which consists of kinetic, Skyrme, pairing, and Coulomb terms, respectively. It involves time-even (nucleon $\rho_q$, kinetic-energy $\tau_q$, spin-orbit $J_q$) and time-odd (current $J_q$, spin $s_q$, vector kinetic-energy $T_q$) densities, as well as the pairing density $\chi_q$. The label $q$ denotes protons and neutrons. The time-odd densities do not contribute to the ground states of even-even nuclei but may be essential for the ground states of odd nuclei [18, 19] and nuclear dynamics including rotation [20] and giant resonances [3, 4, 7, 8, 10, 11]. These densities restore the Galilean invariance of the functional [7, 16, 20] and are important for the spin-flip M1(T=1) GR [3, 4, 5]. The functional (1) includes the tensor contributions $J^2 = (J_n + J_p)^2$ and $\sum_q J_q^2$, expressed through the squared spin-orbital densities [21], which can be also important for the spin-flip M1(T=1) GR [5, 12, 22].

The SRPA residual interaction is fully determined by the given Skyrme functional and does not add any additional parameters. It is expressed through the second functional derivatives

\[
\frac{\delta^2 E}{\delta \rho_q \delta \rho_q}, \frac{\delta^2 E}{\delta \tau_q \delta \rho_q}, \frac{\delta^2 E}{\delta \rho_q \delta J_q}, \frac{\delta^2 E}{\delta \rho_q \delta T_q}
\]

for electric modes and

\[
\frac{\delta^2 E}{\delta J_q \delta s_q}, \frac{\delta^2 E}{\delta s_q \delta J_q}, \frac{\delta^2 E}{\delta J_q \delta T_q}, \frac{\delta^2 E}{\delta T_q \delta s_q}
\]

for magnetic modes. The isovector E1 and M1 GR are analyzed in terms of strength functions

\[
S(E1(M1); E) = \sum_{\nu \neq 0} E^{E1}_{\nu} |\langle \Psi_\nu | \hat{M} | \Psi_0 \rangle|^2 \zeta(E - E_\nu)
\]

where $\Psi_0$ is the ground state, $\nu$ runs over the RPA states ($K^\pi = 0^-, 1^-$ for E1 and $K^\pi = 1^+$ for M1) with energies $E_\nu$ and wave functions $\Psi_\nu$. Furthermore, $\zeta(E - E_\nu) = \Delta/[2\pi((E - E_\nu)^2 + \Delta^2/4)]$ is a Lorentz weight with the averaging parameter $\Delta$ intended to simulate broadening effects beyond SRPA (escape widths, coupling with complex configurations). The energy weight is used for E1 (L=1) but not for M1 (L=0). The strength function (4) is computed directly, i.e. without calculation of RPA states $\nu$, which additionally reduces the computation expense.

The E1(T=1) GR is computed with the proton and neutron effective charges $e_p^{eff} = N/A$ and $e_n^{eff} = -Z/A$. The spin-flip M1 transition operator is calculated with the proton and neutron spin g-factors $g_p^0 = 5.58g_p$ and $g_n^0 = -3.82g_n$ quenched by $g_p^{\text{eff}}=0.68$ and $g_n^{\text{eff}}=0.64$. The corresponding isoscalar (T=0) and isovector (T=1) g-factors are $g_s^0 = g_p^0 + g_n^0 = 1.35$ and $g_s^1 = g_p^1 - g_n^1 = 6.24$. Note that $g_s^1 > g_s^0$, i.e. spin-flip M1 GR is mainly isovector. In the present study the orbital part in M1 operator is omitted.
3. Results and discussion

3.1. E1(T=1) giant resonance

Results of SRPA calculations [8] with the Skyrme force SLy6 [23] for E1(T=1) GR in axially-deformed nuclei of rare-earth and actinide regions are given in Figs. 1 and 2. The Lorentz averaging Δ=2 MeV is used. The figures demonstrate the deformation splitting between the μ = 0 and μ = 1 resonance branches. An excellent agreement with the experimental data is seen. This proves high SRPA accuracy and encourages application of the model to other kinds of nuclear dynamics discussed in the next subsections.

![Figure 1](image1.png)

**Figure 1.** E1(T=1) GR in $^{156,160}$Gd and $^{166,168}$Er [8]. The calculated strength (solid curve) is compared to the experimental data [24] for total photo-absorption (triangles) and neutron product (rhombus). The GR branches with μ = 0 (left) and μ = 1 (right) are depicted by dotted curves.

![Figure 2](image2.png)

**Figure 2.** The same as in Fig. 1 for $^{232}$Th and $^{234,236,238}$U [8].

3.2. Threshold E1 strength

The E1 strength near neutron and proton emission thresholds is a subject of intense research in connection to astrophysical needs, see e.g. [17]. The strength was explored within inconsistent models with Woods-Saxon potential [25, 26] and fully self-consistent approaches, relativistic covariant RPA [27] and Skyrme SRPA [9]. The phenomenological studies claim that nuclear deformation always increases the threshold E1 strength [25, 26]. On the other hand, SRPA calculations with the Skyrme force SLy6 for Mo isotopes [9] show that this is not the case and the deformation impact can vanish (due to the compensation of μ = 0 and μ = 1 resonance contributions) or be of both sign, depending on the proximity of the thresholds to E1(T=1) GR. The similar effect, weakening the pygmy resonance strength due to the deformation, was found in Sn isotopes near the drip line [27]. In the previous SRPA study [9], likely triaxial Mo isotopes were treated as axial. Here we consider a more refine case of the axial prolate nucleus $^{152}$Nd.

In Fig. 3 the fraction $\Sigma(E) = \int_{E_{min}}^{E} dE \Sigma(T; E) = 60NZ/A$ mb MeV is depicted for the energy interval $[E_{min}, E]$ with $E_{min} = 4$ MeV. The cases with and without the equilibrium deformation $\beta_2 = 0.347$
are compared. A large and a small Lorentz smoothing are used to embrace the E1(T=1) GR tail ($\Delta=1$ MeV) or local dipole excitations alone ($\Delta=0.1$ MeV).

Fig. 3 shows that near thresholds the influence of deformation on the total E1 strength is negligible for $\Delta=1$ MeV or may be both positive and negative for $\Delta=0.1$ MeV. The effect of deformation is always positive only at $E > 11$ MeV i.e. in close vicinity to the lower resonance branch ($\mu = 0$) at $E_{\mu=0}=12$ MeV. The middle and bottom panels of the figure show that deformation effects for $\mu = 0$ and $\mu = 1$ are of the opposite sign and, to a large extent, compensate each other in the total E1 strength. The splitting due to the prolate deformation down-shifts the $\mu = 0$ and up-shifts the $\mu = 1$ GR branches. As a result, the $\mu = 0$ branch adds strength to the lower threshold region while the $\mu = 1$ one reduces it. Being of the opposite sign, the effects have a similar magnitude since the weaker $\mu = 0$ branch is placed closer to the thresholds while the stronger $\mu = 1$ branch is more remote. Near the resonance the $\mu = 0$ contribution strictly dominates and we always have the increase of the strength as was predicted in [25, 26]. However, the thresholds often lie much below the resonance and then the compensation effect becomes decisive. In this case, the impact of the deformation can be negligible or, depending on the compensation, slightly positive or negative. This result is important for a realistic estimate of the threshold E1 strength in deformed nuclei and subsequent astrophysical conclusions.

3.3. Spin-flip M1 GR
The spin-flip M1 GR is known to be a major source of our knowledge on spin correlations and robust test for the spin-orbit interaction [28]. During the last decades the resonance was widely investigated within various (mainly empirical) models and its basic properties were properly clarified. The availability of self-consistent SHF calculations motivated the revision of the M1 GR studies on new theoretical grounds, see e.g. [29, 30]. The first systematic SHF study of this GR was performed within the SRPA [5, 12]. It was shown that none of numerous Skyrme parameterizations is able to describe simultaneously the one-bump gross structure of M1 strength in doubly magic nuclei and two-bump structure in heavy deformed nuclei. That was an alarming message for both spin-flip M1 GR and its Gamow-Teller counterpart.

The problem is illustrated in Fig. 4 where the resonance strength (4) for the forces SkO [31],
SG2 [32], and SV-bas [33] is compared to the experimental data in spherical 208Pb and deformed 158Gd. It is seen that SkO gives one bump in both nuclei, which is good for 208Pb but not for 158Gd. Instead, SG2 and SV-bas always give two-bumps, which is correct for 158Gd but not for 208Pb. The gross structure of spin-flip M1 GR is known to be determined by a fragile balance between the spin correlations (= residual interaction) and spin-orbital splitting (forming the unperturbed proton and neutron peaks) [5, 12, 28]. If the correlations are strong enough then the neutron and proton peaks are mixed and we obtain the one-bump structure. Otherwise, the two-bump proton-neutron structure is preserved. Fig. 4 shows that the given Skyrme parameterizations do not keep the proper balance but distort it in favor of the correlations (SkO) or spin-orbital splitting (SG2 and SV-bas).

As seen from Fig. 5, the description can be improved by replacing the M1 response by the M1(T=1) one. This leads to redistribution of the strength and formation of mainly one-bump structure in 208Pb even for forces like SV-bas [5]. An additional significant improvement can be obtained by including the tensor contribution, see also [22]. Both these two factors have to be in focus of further upgrade of SHF description of spin-flip M1 GR.

**Figure 4.** The spin-flip M1 strength in 158Gd and 208Pb for the forces SkO, SG2, and SV-bas. The experimental data are given by boxes with bars for 158Gd [34] and vertical arrows for 208Pb [35]. The Lorentz width $\Delta=1$ MeV is used.

**Figure 5.** The spin-flip M1 (left) and M1(T=1) (right) strengths in 208Pb and 158Gd for the force SV-bas with (bold solid curve) and without (solid curve) tensor contribution. The experimental data are like in Fig. 4. The Lorentz width is $\Delta=1$ MeV.
4. Conclusions
It was demonstrated that the Skyrme SRPA model performs very well in the description and analysis of various kinds of nuclear giant resonance (GR) excitations. For the threshold E1 strength, a simple compensation scheme was proposed to explain a small deformation effect. The open problems in description of spin-flip M1 GR were discussed. These examples, as well as other SRPA studies of multipole giant resonances [3, 4, 7, 8, 9, 10, 11], show that the model is indeed very effective, accurate, economical and universal theoretical tool.

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