Redshift-Space Density versus Real-Space Velocity Comparison

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Abstract. I propose to compare the redshift-space density field directly to the real-space velocity field. Such a comparison possesses all of the advantages of the conventional redshift-space analyses, while at the same time it is free of their disadvantages. In particular, the model-dependent reconstruction of the density field in real space is unnecessary, and so is the reconstruction of the velocity field in redshift space. The redshift-space velocity field can be reconstructed only at the linear order, because only at this order it is irrotational. Unlike the conventional redshift-space density–velocity comparisons, the comparison proposed here does not have to be restricted to the linear regime. Nonlinear effects can then be used to break the $\Omega$–bias degeneracy plaguing the analyses based on the linear theory. I present a degeneracy-breaking method for the case of nonlinear but local bias.

1. Introduction

Comparisons between density fields of galaxies and the fields of their peculiar velocities, a powerful tool to measure the cosmological parameter $\Omega$, are commonly performed in real space. A necessary ingredient of real-space analyses is the reconstruction of the galaxy density field in real space from the observed redshift-space galaxy field. This reconstruction is model-dependent: performing it, one has to correct the redshifts for peculiar velocities of galaxies and the amplitude of these velocities depends on $\Omega$. (Or, in the case of inclusion of the bias between the galaxy and the mass distributions, on $\beta$.)

To avoid this problem, the comparisons in redshift space have been proposed (Nusser & Davis 1994). However, the velocity field in redshift space is irrotational only at the linear order, so that the redshift-space analyses must be restricted to the linear regime. This is unsatisfactory, since the derived amplitude of the density fluctuations from current redshift surveys (e.g., Fisher et al. 1995) and from the POTENT reconstruction of density fields (e.g., Dekel et al. 1990) slightly exceeds the linear regime. Moreover, nonlinear effects may lead to breaking the degeneracy between $\Omega$ and bias (Dekel et al. 1993, Bernardeau et al. 1999).

Chodorowski & Nusser (1999) have shown that the nonlinear redshift-space velocity field is irrotational in the distant observer limit. However, the catalogs of peculiar velocities are not yet deep enough to satisfy this assumption.
Here I propose to compare the redshift-space density field directly to the real-space velocity field. Such a comparison enables one to avoid the model-dependent reconstruction of the density field in real space. Also, the vorticity of the velocity field is no longer a problem, because (before shell crossings) the real-space velocity field is irrotational without any restrictions. Therefore, the comparison proposed here combines all advantages of real-space and redshift-space analyses, while at the same time it is free from their disadvantages.

2. Exact relation

The transformation from the real space coordinate, \( r \), to the redshift space coordinate, \( s \), is (Kaiser 1987)

\[
\begin{align*}
\mathbf{s} &= \mathbf{r} + v_r \hat{\mathbf{r}}, \\
\hat{\mathbf{r}} &= \mathbf{r}/r, \quad v_r(r) &= \mathbf{v} \cdot \hat{\mathbf{r}}, \quad \mathbf{v}(r) \text{ is the real-space velocity field, and velocities} \\
& \text{are measured relative to the Local Group. The relation between the redshift-space galaxy density field,} \\
& \delta_s^{(g)}(s), \text{and the real-space velocity field is (Chodorowski 1999b)} \\
& \delta_s^{(g)}(s) = \sum_{n=0}^{\infty} \left[ \frac{(-1)^n}{n!} \left\{ \frac{1}{r^2} \frac{\partial^n}{\partial r^n} \left[ r^2 v_r^n \left( \frac{\phi(r)}{\phi(s)} \right) \left[ B \circ \mathcal{F}(\partial v_i/\partial r_j) + 1 \right] \right] \right\} \right] \bigg|_{\hat{\mathbf{r}}=s}.
\end{align*}
\]

Here, \( \phi \) is the selection function and \( J \) is the Jacobian of the transformation (1), 
\( J(r) = (1 + v_r/r)^2(1 + v'_r) \), where \( v' \equiv \partial/\partial r \). The function \( \mathcal{F} \) (e.g., Bernardeau 1999 and references therein) relates the real-space velocity field to the real-space mass density field,

\[
\delta(r) = \mathcal{F} \left[ \partial v_i/\partial r_j(r), \Omega \right].
\]

The function \( \mathcal{B} \) relates locally the real-space mass density field to the real-space galaxy density field, \( \delta^{(g)} \), (the, so-called, local bias model),

\[
\delta^{(g)}(r) = \mathcal{B} \left[ \delta(r) \right].
\]

If the selection function is given by a power law, \( \phi \propto s^{-p} \), then \( \phi(r)/\phi(s) = (1 + v_r/r)^p \). If not, the expression \( \phi(r)/\phi(s) \) should be expanded explicitly, and in any case it will be a function of \( v_r/r \). Therefore, both \( \phi(r)/\phi(s) \) and \( J \) are functions of the real-space velocity field and its derivatives, and equation (2) is indeed a relation between the redshift-space galaxy density and the real-space velocity.

The key point of equation (2) is that it relates the redshift-space density at a point \( s \) to an expression involving the real-space velocity field evaluated at \( \hat{\mathbf{r}} = s \). (\( \hat{\mathbf{r}} \) is a real-space point, in general different from \( r \), which is related to \( s \) by eq. (1)) This is very convenient, since now we can treat the two fields as if they were given in the same coordinate system.
3. Approximate relation

Relation (2) involves an infinite series in velocity. If the density–velocity comparison is performed on scales large enough so that they are only weakly nonlinear, then this series can be truncated.

**Linear relation.** The linear density–velocity relation is

\[
\delta_s^{(g)}(s) = \left[ -\beta^{-1} \nabla_r \cdot v - v'_r - \left( 2 + \frac{d \ln \phi}{d \ln r} \right) \frac{v_r}{r} \right] \bigg|_{\tilde{r} = s},
\]

where \( \beta \equiv \Omega_0^{0.6}/b \) and \( b \) is the linear bias parameter. This equation coincides with equation (6) of Nusser & Davis (1994).

**Second-order relation.** To second order, the function \( F \) is (e.g., Chodorowski 1997)

\[
F = -\Omega^{-0.6} \theta(r) + \frac{4}{\pi} \Omega^{-1.2} \left[ \theta^2(r) - \frac{3}{2} \Sigma^2(r) \right].
\]

Here, \( \Sigma^2 \equiv \Sigma_{ij} \Sigma_{ij}, \Sigma_{ij} \equiv \frac{1}{2} (\partial v_i/\partial r_j + \partial v_j/\partial r_i) - \frac{1}{3} \delta_{ij} \theta, \) and \( \theta \equiv \nabla_r \cdot v. \) The symbol \( \delta^K_{ij} \) denotes the Kronecker delta. The function \( B \) is

\[
B = b \delta(r) + \frac{1}{2} b_2 \left[ \delta^2(r) - \langle \delta^2 \rangle \right].
\]

Here, \( b_2 \) is the nonlinear (second-order) bias parameter. This yields

\[
\delta_s^{(g)}(s) = \left\{ -\beta^{-1} \theta - v'_r - (2 + D_1)v_r/r + \left[ v_r (\beta^{-1} \theta + v'_r) \right] \right\} \bigg|_{\tilde{r} = s},
\]

where \( D_1 = d \ln \phi/d \ln r, \) and \( D_2 = (\phi'' r^2)/(2 \phi). \)

Equation (8) can be used to reconstruct the real-space velocity field from the associated redshift-space galaxy density field. Since the real-space velocity field is irrotational, it can be described as a gradient of the velocity potential,

\[
v(r) = -\nabla_r \Phi_v.
\]

Equation (8) reduces then to a nonlinear differential equation for the velocity potential, which can be solved iteratively. First, we solve its linear part. Next, we find the second-order solution by solving again the linear equation, with the source term resulting from the density modified by nonlinear contributions approximated by first-order solutions. Specifically,

\[
\beta^{-1} \Delta_r \Phi_v^{(2)} + \frac{\partial^2 \Phi_v^{(2)}}{\partial r^2} + \frac{2 + D_1}{r} \frac{\partial \Phi_v^{(2)}}{\partial r} = \delta_s^{(g)}(\tilde{s} = r) - \mathcal{N}_2 \left[ \Phi_v^{(1)}(r) \right],
\]

where \( \mathcal{N}_2 \) is a sum of all terms quadratic in velocity in equation (8), expressed as functions of the potential.
4. Breaking the Ω–bias degeneracy

Based on (10), from the associated redshift galaxy field we can reconstruct the nonlinear real-space velocity field. The latter can then be compared to measured radial velocities of galaxies. This comparison will yield the best-fit values of two parameters: $\beta$, and

$$\beta_2 \equiv \left( \frac{4}{21b} + \frac{b_2}{2b^2} \right)^{-1} \beta^2$$

(11)

(see eq. $8$). They are a combination of three physical parameters: $\Omega$, $b$, and $b_2$. Therefore, we need an additional constraint on these parameters. As this constraint one can adopt the large-scale galaxy density skewness (Bernardeau et al. 1999).

We can measure the redshift galaxy density skewness, $S_{3s}^{(g)}$, (e.g., Kim & Strauss 1998), while gravitational instability theory can predict the value of the redshift mass density skewness, $S_{3s}$, (Hivon et al. 1995). The relation between the two is approximately (for details see Chodorowski 1999a)

$$S_{3s}^{(g)} = S_{3s} + S \frac{b_2}{b^2}$$

(12)

Now we have three independent constraints for the parameters $\Omega$, $b$ and $b_2$, so that in principle we can measure $\Omega$ and bias separately.

The additional constraint on $\Omega$ and bias (the skewness) is to be inferred from the density field alone, making any additional observations unnecessary. In conclusion, there is enough information in the density field and the associated velocity field to break the $\Omega$–bias degeneracy.

For a more detailed discussion of the problem, see Chodorowski (1999b).

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