Designing Roller Coaster Loop’s By Using Extended Uniform Cubic B-Spline

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Abstract. Roller coaster is one of the attractions to the public. The design of roller coaster plays an important role for the smoothness and safety of each roller coaster rail. This research is about designing a smooth roller coaster rail loop using extended cubic uniform B-spline method as one of the alternative in designing a smooth roller coaster rail. Two-dimensional design of roller coaster rail loop is form by using extended cubic uniform B-spline with degree 3, 4 and 5 and shape parameter, λ= 0.5, and 1. Then, three-dimensional cubic B-spline is formed by using sweep surface which is translation method. The loop is analysed by the distance between curve and control polygon. Moreover, the G-felt with the different radius of the loop has been calculated. The value of G-felt for each method used will be compared to the G-felt value of the original roller coaster. The result from this research showed that by using extended cubic B-spline method degree 4, λ=0.5 is the best way to design roller coaster rail because it has the best G-force value compared to others designs.

1. Introduction
Roller coaster is one of the thrill rides in every theme park. It is an elevated railway with steep inclines and descents that carries a train of passengers through sharp curves and sudden changes of speed and direction for a brief thrill ride [1] - [3]. Most of the actual design and layout of a roller coaster is done on a computer. In the roller coasters with all those drops, twists, turns and changes of the loop have gravitational force and acceleration force that acting on it. G-force is a force of gravity on an object and it was measured in g’s where 1 g is equal to the gravitational force at the Earth’s surface which is 9.8\(\text{m/s}^2\) [1]. The engineers and the designers of the roller coasters will make sure those forces interplays for the passengers really feel it acting on their body. Furthermore, inertia also happens in the roller coaster ride. Inertia is the resistance of an object to a change in its state of motion. The force that the passengers feel is the opposite of the force that acting on the passengers itself [2], [4]. Therefore, the safety of the roller coaster is determined by the design of the loops.

Extended cubic B-spline is one of the techniques that can be applied in system of design or in creating new objects. Extended cubic B-spline is an extension of B-spline [5]. One free parameter, \(\lambda\), is introduced within the basis function where this parameter can be used to alter the shape of the generated curve. The value of \(\lambda\) can be varied to obtain different numerical results [6]. Extended cubic B-spline curves have the same properties of cubic uniform B-spline which is symmetry, geometric invariability, and convex hull [5].
The main objective of this paper is to design two and three dimensional roller coaster loop by using extended cubic uniform B-spline curves degree 4, 5 and 6 with $\lambda=0.5$ and 1. Then, G-force for each two-dimensional roller coaster loop design is analyzed to determine the best method to be used in roller coaster loop design.

2. Methodology

2.1. Extended Cubic B-Spline Curve

Extended Cubic Uniform B-Spline is used to design curve and generate the object. The basis functions are as follows [5].

Basis function for degree 4

\[
\begin{align*}
b_0^4(t) &= \frac{1}{24} \left(4 - \lambda - 3\lambda t\right) (1 - t)^3 \\
b_1^4(t) &= \frac{1}{24} \left[6 + 2\lambda - 12(2 + \lambda)t^2 + 12(1 + \lambda)t^3 - 3\lambda t^4\right] \\
b_2^4(t) &= \frac{1}{24} \left[4 - \lambda + 12t + 6(2 + \lambda)t^2 - 12t^3 - 3\lambda t^4\right] \\
b_3^4(t) &= \frac{1}{24} \left[4(1 - \lambda) + 3\lambda t\right] t^3
\end{align*}
\]

(1)

The basis functions of extended cubic uniform B-spline degree 5 for $t \in [0,1]$ as follows:

\[
\begin{align*}
b_0^5(t) &= \frac{1}{40} \left[5 - \lambda - 4\lambda t\right] (1 - t)^4 \\
b_1^5(t) &= \frac{1}{40} \left[30 + 2\lambda - 20(3 + \lambda)t^2 + 40(1 + \lambda)t^3 - 5(1 + 7\lambda)t^4 + 12\lambda t^5\right] \\
b_2^5(t) &= \frac{1}{40} \left[5 - \lambda + 20t + 10(3 + \lambda)t^2 - 20(1 + \lambda)t^3 - 5(1 - 5\lambda)t^4 - 12\lambda t^5\right] \\
b_3^5(t) &= \frac{1}{40} \left[5(1 - \lambda) + 4\lambda t\right] t^4
\end{align*}
\]

(2)

The basis functions of extended cubic uniform B-spline degree 6 for $t \in [0,1]$ as follows:

\[
\begin{align*}
b_0^6(t) &= \frac{1}{60} \left[6 - \lambda - 5\lambda t\right] (1 - t)^5 \\
b_1^6(t) &= \frac{1}{60} \left[48 + 2\lambda - 30(4 + \lambda)t^2 + 40(3 + 2\lambda)t^3 - 30(2 + 3\lambda)t^4 + 6(3 + 7\lambda)t^5 + 5\lambda t^6\right] \\
b_2^6(t) &= \frac{1}{60} \left[6 - \lambda + 30t + 15(4 + \lambda)t^2 - 20(3 + 2\lambda)t^3 + 15(2 + 3\lambda)t^4 - 6(3 + 2\lambda)t^5 - 5\lambda t^6\right] \\
b_3^6(t) &= \frac{1}{60} \left[6(1 - \lambda) + 5\lambda t\right] t^5
\end{align*}
\]

(3)
Theorem 1: The basis functions \( b_k^i(t) \), where \( k=4,5,6 \), and \( i=0,1,2,3 \) satisfy

i. \( \sum_{i=0}^{3} b_k^i(t) = 1 \)

ii. \( b_k^i(t) = b_{k-1}^{i+1}(1-t) \)

iii. When \( -k(k-2) \leq \lambda \leq 1 \), \( b_k^i(t) \geq 0 \), \( t \in [0,1] \).

The polynomial curve segments for \( u \in [u_i,u_{i+1}], i=3,4,...,n \) are defined as follows [5].

\[
C_{j,k}(\lambda : t) = \sum_{i=0}^{3} b_k^i(t)P_{j+i,3}, \quad k = 4,5,6
\]

Where, \( P_j \in \mathbb{R}^d \) (\( d = 2,3, i = 0,1,2,...,n \)) is control point and knots is \( u_1 < u_2 < ... < u_{n+1} \).

The polynomial curve is defined as follows [5].

\[
C_k(\lambda ; u) = C_{i,k}\left( \lambda ; \frac{u - u_i}{h_i} \right), \quad u \in [u_i, u_{i+1}]
\]

Where,

\[
t = \frac{u - u_i}{h_i}, \quad h_i = u_{i+1} - u_i.
\]

**Figure 1.** Extended cubic B-spline curves degree 4, 5 and 6 with the different shape of parameter, \( \lambda \)

Figure 1 above show that the extended cubic B-spline curves degree 4, 5 and 6 with the different shape of parameter \( \lambda = 0, 0.5 \) and 1 in the same control polygon. The result shows that all the curves obtained does not touch the start and end point. It is called as close curve. The distant between close curve and control polygon is changed based on the values of shape parameter, \( \lambda \) and the degree of Extended Cubic Uniform B-spline curve. The curves degree 4 with \( \lambda = 0, 0.5 \) and 1 is far from the control polygon while extended cubic B-spline curves degree 6 with \( \lambda = 0, 0.5 \) and 1 is the closest curve from control polygon for all values of shape parameter. Curve with \( \lambda = 0 \) is closest to the control polygon compared to curve with \( \lambda = 0.5 \) and 1.

2.2 **Translation Technique**

Translation of sweep surface method is known as transition of sweeping where the contour moves from one end of the line segment to another line segment perpendicularly. The transformation in three dimensional spaces which is moves in any direction is also called as translation [7] - [11]. Figure 2 shows the transformation of two dimensional curve to three dimensional object by using the equations of translation technique below [12], [13];

\[
g(u) = [g_1(u), g_2(u)]^T, \quad r(v) = [r_x(v), r_y(v), r_z(v)]^T
\]

(6)
\[ T(v) = \frac{r'(v)}{\|r'(v)\|}, \quad N(v) = \frac{T'(v)}{\|T'(v)\|}, \quad \text{and} \quad B(v) = T(v) + N(v) \]  

(7)

\[ S(u, v) = r(v) + g_1(u)N(v) + g_2(u)B(v) \]  

(8)

where, \( g(u) \) = contour blended, \( r(v) \) = trajectory curve, \( N(v) \) = normal vector, \( T(v) \) = the unit tangent vector of the trajectory, and \( B(v) \) = binormal vector.

2.3 \( G \)'s felt

A roller coaster is considered for amusement parks if it falls into the Goldilocks zone. The \( G \)-forces in the range 0.5g to 2g are suitable for children rides and for family rides is in range to 0g to 3g. Thrill rides often involve 4 g or more, as well as negative \( g \)'s, where the body lifts from the seat and must be held in place by the restraint system. The roller coaster will be too dangerous if it has many \( g \)'s and too boring if it has few \( g \)'s [3], [14]. Therefore, this is important to the designer keeping these within safe values.

For gravity force, the number represents how many times the force of gravity is felt at a particular point. In everyday life, everybody experiences 1G regular force of gravity when in still position. If a theme park says a roller coaster rider will experience 3G then the rider will briefly experience three times the force of gravity [1].

Positive \( G \)-Force occurs at the bottom of a drop when the roller coaster wants to continue moving in the same direction, but the track forces it in a different direction. The sharpness of this change of direction determines the positive \( G \). Positive \( G \) forces typically occur when the train pulls up a hill after going down a drop. Passengers will experience \( G \)'s at the loops. Formula from circular motion will be used calculate the \( G \)'s felt. The centripetal acceleration must be calculated inside the loop [1].

\[ a_{\text{centripetal}} = \frac{V^2}{R} \]  

(9)

After the centripetal acceleration is obtained, the \( G \)'s felt can be calculated

\[ \text{G's force} = \frac{a_{\text{centripetal}}}{9.8 \frac{m}{s^2}} \]  

(10)

Lastly, the value of \( G \)-force will be used to obtain the g-felt.

\[ \text{G's felt} = \text{G's force} - 1 \]  

(11)
3. Results and Discussion

3.1. Roller coaster’s loop design
Extended cubic B-spline curve degree 4, 5 and 6 can be used to design a roller coaster’s loop in two and three dimensional by using shape parameter $\lambda = 0.5$ and 1. The degree of curve and value shape parameter used will determine the smoothness of roller coaster loop. Figure 3, figure 4, figure 5 and figure 6 show the design obtained.

![Figure 3](image1.png)  
(a) Degree 4  
(b) Degree 5  
(c) Degree 6  
**Figure 3.** Two dimensional curves by using shape parameter $\lambda = 0.5$

![Figure 4](image2.png)  
(a) Degree 4  
(b) Degree 5  
(c) Degree 6  
**Figure 4.** Two dimensional curves by using shape parameter $\lambda = 1$

![Figure 5](image3.png)  
(a) Degree 4  
(b) Degree 5  
(c) Degree 6  
**Figure 5.** Three dimensional curves by using shape parameter $\lambda = 0.5$
3.2. G-force value

Table 1 represents the result for G-felt in the loop of extended cubic B-spline degree 4, 5 and 6 with shape parameter \( \lambda = 0.5 \) and 1.

| Position          | On the top | On the right | On the left |
|-------------------|------------|--------------|-------------|
| Real roller coaster | -1         | 0.58         | 0.53        |
| Degree 4, \( \lambda = 0.5 \) | -1         | 0.00         | 0.04        |
| Degree 4, \( \lambda = 1 \) | -1         | 0.00         | -0.12       |
| Degree 5, \( \lambda = 0.5 \) | -1         | 0.16         | 0.14        |
| Degree 5, \( \lambda = 1 \) | -1         | -0.12        | -0.11       |
| Degree 6, \( \lambda = 0.5 \) | -1         | 0.19         | 0.00        |
| Degree 6, \( \lambda = 1 \) | -1         | 0.04         | -0.08       |

The result obtained is compared to the G-force value of the real roller coaster’s loop. It is clearly shows that extended cubic B-spline method degree 5 with \( \lambda = 0.5 \) is the best way to design roller coaster loop because it has the best G-force value and most accurate to the G-force value of the real roller coaster loop.

4. Conclusion

In conclusion, extended uniform B-spline can be used to form two-dimensional roller-coaster loop’s design. Then, it can be transformed into three dimensional loop by using translation method. The smoothness of loop’s design is depending on the values of shape parameter used. G-felt values for each loops are calculated in order to determine which degree and value shape parameter is the best use to design the roller coaster loops. After that, the G-force for each of the roller coaster loop are calculated. G-force value is really important for the safety and comfort of the roller coaster’s passengers. As the result, the best method in designing the loop of roller coaster is by using extended cubic B-spline method degree 5 with \( \lambda = 0.5 \). This is because the G-force value is almost the same with the standard G-force value of the real roller coaster’s loop. Nevertheless, using extended cubic B-spline method degree 5 with \( \lambda = 0.5 \) produce the smooth roller coaster loop.

The extended cubic B-spline loops can be used as one of the alternative in design the roller coaster. However, this research can be more interesting and improved if further studies is applied in getting a better curve of the roller coaster rail. Furthermore, the next research can apply B-spline method in designing the whole roller coaster rail from the start point until its end point.
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