Meson-baryon coupling constants of the SU(3) baryons with flavor SU(3) symmetry breaking

Ghil-Seok Yang$^1$, and Hyun-Chul Kim$^{2,3}$

$^1$Department of Physics, Soongsil University, Seoul 06978, Republic of Korea
$^2$Department of Physics, Inha University, Incheon 22212, Republic of Korea
$^3$School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea

(Dated: July 25, 2018)

We investigate the strong coupling constants for the baryon octet-octet, decuplet-octet, and decuplet-decuplet vertices with pseudoscalar mesons within a general framework of the chiral quark-soliton model, taking into account the effects of flavor SU(3) symmetry breaking to linear order in the expansion of the strange current quark mass. All relevant dynamical parameters are fixed by using the experimental data on hyperon semileptonic decays and the singlet axial-vector constant of the nucleon. The results of the strong coupling constants for the baryon octet and the pseudoscalar meson octet are compared with those determined from the Jülich-Bonn potential and the Nijmegen extended soft-core potential for hyperon-nucleon scattering. The results of the strong decay widths of the baryon decuplet are in good agreement with the experimental data. The effects of SU$_f$(3) symmetry breaking are sizable on the $\eta'$ coupling constants. We predict also the strong coupling constants for the $\Omega$ baryons.

Keywords: meson-baryon coupling constants, strong decay widths, the chiral quark-soliton model

$^*$ E-mail: ghsyang@ssu.ac.kr
$^†$ E-mail: hchkim@inha.ac.kr
I. INTRODUCTION

The meson-baryon coupling constants are the essential quantities in understanding the structure of SU(3) baryons and in describing various productions such as meson-baryon scattering, baryon-baryon scattering, photoproduction and electroproduction of hadrons. The strong coupling constants are often determined with flavor SU(3) symmetry assumed. Knowing the $\pi N N$ coupling constants and the ratio $\alpha = F/(F + D)$, where $F$ and $D$ are the two couplings arising from the SU(3) Wigner-Eckart theorem for computing the matrix elements of the axial-vector current [1], one can determine the pseudoscalar meson octet ($P_s$) and baryon octet ($B_s$) coupling constants:

$$g_{P_s^B B_s^B} = g \left[ i \alpha f_{ijk} + (1 - \alpha)d_{ijk} \right],$$

with $g = g_{\pi NN}$. The ratio $\alpha$ can be found from the five known experimental data on hyperon semileptonic decay (HSD) constants $(g_1/f_1)B_s^s \rightarrow B_s^s$ [2, 3]. Almost all theoretical works on the hyperon-nucleon interaction use it obtained in this way [4-9]. However, the empirical values of $F$ and $D$ determined from the HSD constants contain tacitly the effects of flavor SU(3) symmetry breaking, though $F$ and $D$ are defined with SU(3) symmetry assumed.

The strong coupling constants for the baryon decuplet ($B_{10}$)-octet and pseudoscalar meson octet vertices are less known. Even the $\pi N \Delta$ coupling constant, which is the essential quantity in describing the $NN$ and $\pi N$ interactions, is not at all given in consensus. The $\pi N \Delta$ coupling constant is usually determined by the decay width of $\Delta \rightarrow \pi N$, which yields $f_{\pi N \Delta} \approx 2.24$. In describing $\pi N$ scattering, $f_{\pi N \Delta} \approx 2.0 - 2.5$ was used [10-12]. On the other hand, the full Bonn potential for the $NN$ interaction [13], $f_{\pi N \Delta} = 1.678$ was employed, which was taken from the relation in an SU(6) quark model $f_{\pi N \Delta}^\text{SU(6)} = 72f_{\pi NN}^\text{SU(3)}/25$ [14]. A recent work determined $f_{\pi N \Delta} = 1.256$, which is much smaller that that from the decay width, based on the global fit to the $\pi N$ and $\gamma N$ data [15]. When it comes to the coupling constants for the other members of the baryon decuplet, information is much less known.

In the mean time, new experimental programs with strangeness of $S = -3$ are now under way at the J-PARC [16] and a new excited $\Omega$ resonance was reported by the Belle Collaboration [17]. The HAL Collaboration in lattice QCD predicted the dibaryon ($\Omega \Omega$) with strangeness $S = -6$ [18]. The $N \Omega$ interaction was studied in a meson-exchange picture [19] very recently. The baryon decuplet and octet interactions were investigated [20]. In this regard, it is highly required to provide information on the baryon and pseudoscalar meson coupling constants in a quantitative manner.

In the present work, we want to study the coupling constants for the vertices of the baryon decuplet-octet (also decuplet) and pseudoscalar mesons in a pion mean-field approach that is often called the the chiral quark-soliton model (χSM). In Refs. [21, 22], we reexamined the mass splittings of the SU(3) baryon octet and the decuplet, fixing all the parameters unequivocally to the experimental data. The effects of SU(3) symmetry breaking and isospin symmetry breaking due to both the electromagnetic interaction and current quark mass difference [21, 22] were systematically included, which made it possible to exploit the experimental data to fix the parameters. Since we have fixed all unknown parameters in the baryon wavefunctions, we can proceed to the study of the axial-vector transitions, again fixing relevant parameters by utilizing the experimental data on the HSD constants and the flavor-singlet axial-vector charge $g_A^{(0)}$. Though similar works were done already [24, 27], it was then not possible to fix all the parameters unambiguously because of the absence of isospin symmetry breaking which is inevitable in incorporating the experimental data for the baryon octet. Recently, we have shown that all the relevant parameters for the HSD constants can be fixed without any ambiguity [28]. Once they are known, we can compute all possible axial-vector transitions between the baryon multiplets. As a result, we are able to determine the coupling constants for the vertices of the baryon decuplet-octet (decuplet) and pseudoscalar mesons without any additional parameters introduced, taking into account the effects of explicit SU(3) symmetry breaking.

This paper is outlined as follows: Section 2, we briefly review the general formalism of the χSM to compute the axial-vector transitions between the baryon multiplets and show how to fix the parameters for the axial-vector transitions. In Sec. 3, we present the results of the coupling constants for the baryon multiplets and pseudoscalar meson vertices. We show also the decay widths of the baryon decuplet to the octet. In Sect. 4, we discuss the results for the $\eta$ ($\eta'$), and baryon coupling constants, applying a usual mixing between the octet $\eta_8$ and the singlet $\eta_0$. In the final Section we summarize the present work and draw conclusions.

II. BARYON MATRIX ELEMENTS OF THE AXIAL-VECTOR CURRENTS

The baryon matrix elements of the axial-vector currents are expressed in terms of three form factors

$$\langle B_s | A_{\mu}^s | B_s \rangle = \bar{u}_{B_s}(p_2, s_2) \left[ g^{B_s^s \rightarrow B_s^s}(q^2)\gamma_\mu + ig_2^{B_s^s \rightarrow B_s^s}(q^2)\sigma_{\mu\nu}q^{\nu} + g_3^{B_s^s \rightarrow B_s^s}(q^2)q_\mu \right] \gamma_5 u_{B_s}(p_1, s_1).$$

\[ (2) \]
where the axial-vector currents are defined as

$$A^a_\mu(x) = \bar{\psi}(x) \gamma_\mu \gamma_5 \frac{1}{2} \lambda^a \psi(x).$$

The $\lambda^i$ stand for flavor Gell-Mann matrices for strangeness conserving $\Delta S = 0$ transitions ($i = 3, 8, (1 \pm i 2)$) and for $\Delta S = 1$ ones ($i = 4, 5 \pm i 5$), respectively. The $q^2 = -Q^2$ denotes the square of the momentum transfer $q = p_2 - p_1$. The form factors $g_i$ are real quantities due to CP-invariance, depending only on the square of the momentum transfer. We can neglect $g^B_s \to B_s$, because its contribution to the decay rate is proportional to the ratio $m^2_l/M^2_B \ll 1$, where $m_l$ represents a mass of the lepton ($e$ or $\mu$) in the final state and that of the baryon in the initial state, $M_B$, respectively.

The $g^B_s \to B_s$ is finite only with the effects of SU$_f(3)$ symmetry and isospin symmetry breakings because of its opposite $G$ parity to the axial-vector current, so it is very small for the baryon octet.

In the $\chi$QSM, the collective operator for the axial-vector constants can be defined in terms of the SU$_f(3)$ Wigner $D$ functions [24, 26]:

$$\hat{g}_1 = a_1 D^{(8)}_{X^3} + a_2 d_{pq3} D^{(8)}_{X p q} j^3_q + \frac{a_3}{\sqrt{3}} D^{(8)}_{X^8} j^8_3 + \frac{a_4}{\sqrt{3}} d_{pq3} D^{(8)}_{X p} D^{(8)}_{s q}$$

$$+ a_5 \left( D^{(8)}_{X^3} D^{(8)}_{s 8} + D^{(8)}_{X^8} D^{(8)}_{s 3} \right) + a_6 \left( D^{(8)}_{X^3} D^{(8)}_{s 8} - D^{(8)}_{X^8} D^{(8)}_{s 3} \right),$$

where $a_i$ denote dynamical parameters encoding the specific dynamics of a $\chi$QSM [29, 31]. Note that $a_1$ parametrizes the leading-order contribution, $a_2$ and $a_3$ come from the rotational $1/N_c$ corrections, and $a_4$, $a_5$, and $a_6$ are originated from SU$_f(3)$ symmetry breaking, in which the strange current quark mass $m_s$ is contained. $j_q$ and $j_3$ stand for the $q$-th and third components of the collective spin operator of the baryons, respectively. The $D^{(8)}_{ab}$ are the SU(3) Wigner $D$ functions in the octet representation.

The baryon wavefunctions for the baryon octet and decuplet are written in terms of the SU$_f(3)$ Wigner $D$ functions in the $\chi$SM [21, 32]:

$$\langle A|R, B(Y T T_3, Y' J J_3) \rangle = \Psi^{(R; Y T T_3)}_{(R^*; Y' J J_3)}(A)$$

$$= \sqrt{\text{dim}(R)} (-)^{J_3 + Y'/2} D^{(R)_*}_{(Y, T, T_3)(-Y', J, J_3)}(A),$$

where $R$ designates the allowed irreducible representations of the SU$_f(3)$ group, i.e. $R = 8, 10, \cdots$. $Y, T, T_3$ denote the corresponding hypercharge, isospin and its third component, respectively. The right hypercharge is constrained to be $Y' = 1$ in such a way that it selects a tower of allowed SU$_f(3)$ representations. The baryon octet and decuplet, which are the lowest representations, coincide with those of the quark model. This has been considered as a success of the collective quantization and gives a hint about certain duality between the chiral soliton picture and the constituent quark model.

When the effects of SU$_f(3)$ symmetry breaking are taken into account, a baryon state is no more pure state but the state mixed with those in higher representations. Thus, the wavefunctions for the baryon octet and the decuplet are given by

$$|B_8\rangle = |8_{1/2}, B\rangle + c^B_{10} |10_{1/2}, B\rangle + c^B_{27} |27_{1/2}, B\rangle,$$

$$|B_{10}\rangle = |10_{3/2}, B\rangle + a^B_{27} |27_{3/2}, B\rangle + a^B_{35} |35_{3/2}, B\rangle,$$

where the spin indices $J_3$ have been dropped from the states. The mixing coefficients in Eq. (6) contain the strange current quark mass $m_s$ and are expressed as

$$c^B_{10} = c^B_{27} = \frac{\sqrt{5}}{6}, c^B_{27} = c_{27} = \begin{bmatrix} \sqrt{3} \\ 2/\sqrt{6} \end{bmatrix}, a^B_{27} = a_{27} = \begin{bmatrix} \sqrt{5}/2 \\ 2/\sqrt{3} \end{bmatrix}, a^B_{35} = a_{35} = \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5}/7 \end{bmatrix}.$$
As for the explicit definitions of $I_i$ where $I_i$ is a moment of inertia for the soliton. $\alpha$ and $\gamma$ are the parameters appearing in the collective Hamiltonian. For the explicit definitions of $I_i$, $\alpha$ and $\gamma$, we refer to Ref. [21], where one can find also a detailed discussion as to how they are fixed unambiguously, and relevant references.

Since the baryon wavefunctions contain the corrections of linear SUf(3) symmetry breaking as shown in Eq. (6), the axial-vector transition constants $g^A_{1 \to \Lambda B}$ acquire yet another linear $m_s$ corrections, when the collective operator $g_1$ is sandwiched between the baryon states. Thus, we have the two different linear $m_s$ corrections to the axial-vector transition constants, i.e., one from $a_4$, $a_5$ and $a_6$, and the other from the baryon wavefunctions. Recently, we have shown how the parameters $a_i$ are unequivocally fixed in detail [28]. The experimental data on the HSD constants $(g_1/f_1)^{B_s_1 \to B_s}$ and the flavor-singlet axial-vector charge $g^{(0)}_A$, listed in Table 1, will be the input for fixing $a_i$. The parameters $a_i$ are related to the experimentally known axial-vector HSD constants and $g^{(0)}_A$ in a form of the matrix equation:

$$g = B \cdot a,$$

where

$$g = \left( (g_1/f_1)^{n \to p} , (g_1/f_1)^{\Lambda \to p} , (g_1/f_1)^{\Sigma^- \to n} , (g_1/f_1)^{\Xi^- \to \Lambda} , (g_1/f_1)^{\Xi^0 \to \Sigma^+} , g^{(0)}_A \right),$$

$$B = \begin{bmatrix}
-\frac{7}{10} & -\frac{3}{40} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\
-\frac{1}{15} & \frac{1}{6} & \frac{1}{30} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} \\
\frac{1}{15} & \frac{1}{6} & \frac{1}{30} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} \\
\frac{1}{15} & \frac{1}{6} & \frac{1}{30} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} \\
\frac{1}{15} & \frac{1}{6} & \frac{1}{30} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} \\
0 & 0 & 1 & 0 & 1 & 0 
\end{bmatrix},$$

$$a = (a_1, a_2, a_3, a_4, a_5, a_6).$$

Inverting $B$, we can easily derive the parameters $a_i$, of which the numerical values are listed in Table 1. All other unmeasured HSD constants for the baryon octet and decuplet were predicted in Ref. [28].

| Experimental data | References |
|-------------------|------------|
| $g_1/f_1 (n \to p)$ | 1.2723 ± 0.0023 | PDG [2] |
| $g_1/f_1 (\Lambda \to p)$ | 0.718 ± 0.015 | PDG [2] |
| $g_1/f_1 (\Sigma^- \to n)$ | −0.340 ± 0.017 | PDG [2] |
| $g_1/f_1 (\Xi^- \to \Lambda)$ | 0.25 ± 0.05 | PDG [2] |
| $g_1/f_1 (\Xi^0 \to \Sigma^+)$ | 1.22 ± 0.05 | PDG [2] |
| $g^{(0)}_A$ | 0.36 ± 0.03 | Bass et al. [33] |
TABLE II. Numerical values of the dynamical parameters $a_i$

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|-------|-------|-------|-------|-------|-------|
| $-3.509 \pm 0.011$ | $3.437 \pm 0.028$ | $0.604 \pm 0.030$ | $-1.213 \pm 0.068$ | $0.479 \pm 0.025$ | $-0.735 \pm 0.040$ |

III. COUPLING CONSTANTS FOR THE $B_8$-B$_{10}$ AND $P_8$-B$_{10}$-B$_{10}$ VERTICES

The matrix elements of the $B_{10} \rightarrow B_8$ and $B_{10} \rightarrow B_{10}$ transitions with the axial-vector current are parametrized in terms of the Adler form factors $C^A_{4,5}, B_{10} \rightarrow B_8$ [34, 36]

\[
\langle B_8(p', s') | A^A_\mu | B_{10}(p, s) \rangle = \pi(p', s') \left\{ \frac{C^A_{4,5} B_{10} \rightarrow B_8(q^2)}{M_8} \gamma^\alpha + \frac{C^A_{4} B_{10} \rightarrow B_8(q^2)}{M_8^2} p^\alpha \right\} (q_\alpha g_{\mu \nu} - q_\nu g_{\alpha \mu})
\]

\[
+ C^A_5 B_{10} \rightarrow B_8(q^2) g_{\mu \nu} + \frac{C^A_6 B_{10} \rightarrow B_8(q^2)}{M_8} q_\mu q_\nu \right\} u^\nu(p, s),
\]

(13)

\[
\langle B_{10}(p', s') | A^A_\mu | B_{10}(p, s) \rangle = \pi(p', s') \left\{ g_{\alpha \beta} \left( h_1(q^2) \gamma^\mu \gamma_5 + h_3(q^2) \frac{q_\mu}{2M_{10}} \gamma_5 \right)
\]

\[
+ \frac{g_{\alpha \beta}}{4M_{10}} \left( h_1'(q^2) \gamma^\mu \gamma_5 + h_3'(q^2) \frac{q_\mu}{2M_{10}} \gamma_5 \right) \right\} u^\beta(p, s),
\]

(14)

where the $u^\nu$ represents the Rarita-Schwinger spinor for the baryon decuplet. $q_\mu$ denotes the momentum transfer $q_\mu = (p' - p)_\mu$. The axial-vector constant $C^A_{4,5} B_{10} \rightarrow B_8$ can be related to the strong coupling constants for $P_8$-B$_{10}$-B$_{10}$ vertices by the partially conserved axial-vector current (PCAC) hypothesis. The pseudoscalar meson decay constant $f_8$ is defined as the transition matrix element of the axial-vector current from the physical pion state to the vacuum

\[
\langle 0 | A^a_{\mu}(x) | \pi^b(p) \rangle = ip_\mu f_8 e^{-ipx} \delta^{ab},
\]

(15)

which will be used for the relations of the pseudovector coupling constants $f_{P_8 B_8 B_{10}}$ and $f_{P_8 B_{10} B_{10}}$ to the Adler form factors. In the present work, we will determine only $C^4_{5}$ and $h_1$.

The effective Lagrangians for the $P_8 B_8 B_{10}$ and $P_8 B_{10} B_{10}$ vertices are expressed as

\[
\mathcal{L}_{P_8 B_8 B_{10}} = \frac{f_{P_8 B_8 B_{10}}}{m_8} \bar{B}_{10}^{\mu} Z_{\mu \nu} I \left( \begin{array}{c} 3 \\ 2 \\ 2 \\ 2 \end{array} \right) B_{8} \partial^\nu M_8 + \text{h.c.},
\]

\[
\mathcal{L}_{P_8 B_{10} B_{10}} = \frac{f_{P_8 B_{10} B_{10}}}{m_8} \bar{B}_{10}^{\alpha} Z_{\alpha \beta} I \left( \begin{array}{c} 3 \\ 2 \\ 3 \\ 2 \end{array} \right) B_{10}^\beta \partial_\beta M_8 + \text{h.c.},
\]

(16)

where the pseudovector coupling constants are defined as

\[
f_{P_8 B_8 B_{10}} = \frac{m_8}{f_8} C^4_5(0),
\]

(17)

\[
f_{P_8 B_{10} B_{10}} = \frac{m_8}{f_8} h_1(0),
\]

(18)

$m_8$ denotes the mass of the pseudoscalar meson. The field operators $B_{10}^{\mu}$, $B_8$, and $P_8$ correspond respectively to a decuplet baryon, an octet baryon, and a pseudoscalar octet meson. The $Z_{\mu \nu}$ and $Z_{\alpha \beta}$ stand for the tensors including the off-shell effects arising from the Rarita-Schwinger field quantization, defined as $Z_{\mu \nu} = g_{\mu \nu} - x_{\Delta} \gamma_\mu \gamma_\nu$ with the off-shell parameter $x_{\Delta}$. $I(3/2, 1/2)$ and $I(3/2, 3/2)$ are isospin transition matrices.

For completeness, we also want to mention that the pseudoscalar strong coupling constants can be derived from the generalized Goldberger-Treiman (GT) relation [37, 38], which is defined as

\[
g_{P_8 B_8 B_{10}} \approx \frac{M_8 + M_{10}}{f_8} C^4_5(0).
\]

(19)
However, there is a caveat in Eq. (19). Keeping in mind that certain effects on the GT relation will arise from the flavor SU(3) symmetry breaking. In Ref. [39], it was shown that loop corrections to the GT relation, which come from the pion mass, are indeed very small (~2%). So, we expect that the strange current quark mass will not yield much effects on the relation. Thus, as often assumed in the hyperon-nucleon potentials, one still can use Eq.(19), if one wants to derive the strong coupling constants $g_{PBB_{10}}$.

In effect, the numerical values of the $C^A_{f}$ (0) were already presented in the previous work [28]. Thus, we will show the results for the pseudovector coupling constants and decay widths of the baryon decuplet in this work, using the experimental data on the meson decay constants, $f_{π} = 92.4$ MeV and $f_{K} = 113.0$ MeV. In Table III we list the results of the pseudoscalar coupling constants for the various $Q_{BB_{10}}$ vertices, i.e. $g_{PBB_{10}}$. The second column represents those in the SU(3) symmetric case, whereas the third one denotes those with explicit SU(3) symmetry breaking taken into account. The results are compared with those determined from the extended soft-core Nijmegen hyperon-nucleon $(YN)$ potential (ESC08a) [8] and Jülich-Bonn $YN$ potential, employing the generalized GT relation for kaon vertices. Except for the coupling constants of the vertices $πΣΣ$ and $KΩ$, the present results are in good agreement with the those from both the Nijmegen and Jülich-Bonn potentials. When the effects of the SU(3) symmetry breaking are taken into account, the present results are more deviated from those taken from the Nijmegen potential. Note that both the Nijmegen and Jülich-Bonn potentials have assumed SU(3) symmetry and the following relations for the $Q_{BB_{10}}$ vertices are obtained in exact SU(3):

$$f_{πNΔ} = \sqrt{2}f_{πΛΣ^*} = -\sqrt{2}f_{πΣΣ^*} = \sqrt{2}f_{πΞΞ^*},$$

$$f_{KΣΔ} = \sqrt{2}f_{KNN^*} = -\sqrt{2}f_{KΣΣ^*} = -\sqrt{2}f_{KΞΞ^*} = \sqrt{2}/3f_{KΞΩ},$$

which can be found in various works already.

| $Q_{BB_{10}}$ | $g_{PBB_{10}}$ | $g_{PBB_{10}}^{(total)}$ | ESC08a [8] | Jülich-Bonn [4, 7] |
|---------------|---------------|----------------|----------|----------------|
| $πNN$         | 3.524 ± 0.012 | 3.638 ± 0.018 | 3.639    | 3.795         |
| $πΛΣ$         | 3.129 ± 0.011 | 3.229 ± 0.016 | 3.328    | 2.629         |
| $πΣΣ$         | 3.356 ± 0.014 | 3.197 ± 0.019 | 3.290    | 3.036         |
| $πΞΞ$         | -1.240 ± 0.009 | -0.985± 0.015 | -1.475   | ...           |
| $KNΛ$         | -3.185 ± 0.030 | -3.180 ± 0.032 | -3.217   | -3.944        |
| $KΝΣ$         | 0.820 ± 0.009 | 0.905 ± 0.011 | 0.975    | 0.759         |
| $ΚΛΞ$         | 1.076 ± 0.013 | 1.316 ± 0.017 | 0.942    | ...           |
| $ΚΞΞ$         | -3.855 ± 0.037 | -3.793 ± 0.037 | -3.980   | ...           |

TABLE IV. Pseudovector coupling constants for the $P_{BB_{10}}$ vertices. The second column lists the results for the SU(3) symmetric case, whereas the third one does those with explicit SU(3) symmetry breaking taken into account. The last column lists the values of the coupling constants taken from the Jülich-Bonn hyperon-nucleon potential.

| $M_{BB_{10}}$ | $g_{PBB_{10}}^{(total)}$ | $g_{PBB_{10}}^{(total)}$ | Jülich-Bonn [4] |
|---------------|----------------|----------------|----------------|
| $πNΔ$         | 1.646 ± 0.006 | 1.777 ± 0.008 | 1.68           |
| $πΛΣ^*$       | 1.164 ± 0.004 | 1.178 ± 0.006 | 1.18           |
| $πΣΣ^*$       | -1.164 ± 0.004 | -1.059 ± 0.007 | -0.68         |
| $πΞΞ^*$       | 1.164 ± 0.004 | 1.111 ± 0.007 | ...           |
| $KΝΣ^*$       | -4.815 ± 0.046 | -4.551 ± 0.045 | -4.90         |
| $KΣΣ^*$       | -3.404 ± 0.032 | -3.667 ± 0.038 | -2.00         |
| $KΞΞ^*$       | 3.404 ± 0.032 | 3.450 ± 0.033 | ...           |
| $KΞΩ$         | 8.339 ± 0.079 | 8.130 ± 0.080 | ...           |

In Table IV we list the results of the pseudovector coupling constants for the $P_{BB_{10}}$ vertices. We find that the present value of $f_{πΣΣ^*}$ is different from that taken from the Jülich-Bonn potential by almost 50%. The value of $f_{KΣΣ^*}$ differs by approximately 45%. However, we want to emphasize that the present results of the coupling constants reproduce the experimental data on the decay widths of the decuplet hyperons very well, which will be discussed now.
The partial width for the decay from the baryon decuplet to the octet and pseudoscalar meson $P_b$ is expressed in terms of the pseudovector coupling constant as follows

$$\Gamma_{B_{10} \to \varphi B_b} = \frac{|k|^3}{8\pi m_{b}} \frac{M_{b}}{M_{10}} f_{\varphi B_{10}}^2,$$

(21)

where $|k|$ denotes the three momentum of the pseudoscalar meson in the rest frame of the baryon decuplet. $m_b$ represents the mass of the pseudoscalar meson involved in the decay process. Summing all possible transitions with averaging over the initial states, we can write the decay width for each member of the baryon decuplet as

$$\Gamma \left[ \Delta \to \pi N \right] = \frac{3}{2} \Gamma \left[ \Delta^+ \to \pi^0 p \right],$$

$$\Gamma \left[ \Sigma^* \to \pi A \right] = \Gamma \left[ \Sigma^0 \to \pi^0 A \right],$$

$$\Gamma \left[ \Sigma^- \to \pi \Sigma^+ \right] = 2 \Gamma \left[ \Sigma^+ \to \pi^0 \Sigma^0 \right],$$

$$\Gamma \left[ \Xi^* \to \pi \Xi \right] = 3 \Gamma \left[ \Xi^0 \to \pi^0 \Xi^0 \right].$$

(22)

Except for the $\Delta$ decay, the present results are in good agreement with the experimental data as shown in Table V. There exist also experimental data on the ratio of the decay widths for $\Sigma^* \to \Sigma$ and $\Sigma^* \to \Lambda$. The present result is comparable with the data as shown in the following

$$\frac{\Gamma \left[ \Sigma^* \to \Sigma \right]}{\Gamma \left[ \Sigma^* \to \Lambda \right]} = 0.180 \pm 0.002 \quad \text{ (experimental data} \ [2]: 0.135 \pm 0.011 \).$$

(23)

### Table V. Partial ($\Gamma_i$) and full decay widths ($\Gamma$) for the decays $B_{10} \to B_8 + \pi$ in units of MeV.

| Decay modes          | $\Gamma_i^{(8)}$ | $\Gamma_i^{(total)}$ | $\Gamma$ | $\Gamma^{(Exp.)}[2]$ |
|----------------------|------------------|---------------------|--------|----------------------|
| $\Delta \to N\pi$    | 75.98 ± 1.01     | 88.58 ± 1.31        | 116 - 120 |
| $\Sigma^+ \to \Sigma^- \pi^0$ | 2.59 ± 0.03    | 3.22 ± 0.06         |        |
| $\Sigma^+ \to \Sigma^0 \pi^-$ | 3.17 ± 0.05    | 2.62 ± 0.05         | 36.25 ± 0.42  |
| $\Sigma^+ \to \Lambda \pi^+$ | 29.68 ± 0.26   | 30.41 ± 0.33        |        |
| $\Sigma^0 \to \Sigma^- \pi^+$ | 0               | 0                   |        |
| $\Sigma^0 \to \Sigma^\pi^- \pi^+$ | 3.61 ± 0.11    | 2.98 ± 0.1           | 37.21 ± 0.69  |
| $\Sigma^0 \to \Lambda \pi^+$ | 31.15 ± 0.47   | 31.92 ± 0.52        |        |
| $\Sigma^- \to \Sigma^- \pi^+$ | 3.50 ± 0.06    | 2.89 ± 0.06         |        |
| $\Sigma^- \to \Sigma^\pi^- \pi^+$ | 3.64 ± 0.06    | 3.01 ± 0.06         | 38.18 ± 0.48  |
| $\Sigma^- \to \Lambda \pi^+$ | 31.50 ± 0.30   | 32.28 ± 0.37        |        |
| $\Xi^\pi \to \Xi^- \pi^+$ | 4.76 ± 0.05    | 4.33 ± 0.06         | 11.26 ± 0.17  |
| $\Xi^\pi \to \Xi^- \pi^+$ | 7.61 ± 0.08    | 6.93 ± 0.10         |        |
| $\Xi^\pi \to \Xi^- \pi^+$ | 8.20 ± 0.13    | 8.68 ± 0.16         | 13.01 ± 0.21  |

(24)

### Table VI. Pseudovector coupling constants for the $P_b B_{10} B_{10}$ vertices. The second column lists the results for the SU$_j(3)$ symmetric case, whereas the third one does those with explicit SU$_j(3)$ symmetry breaking taken into account.

| $P_b B_{10} B_{10}$ | $f_{\varphi B_{10}}^{(8)}$ | $f_{\varphi B_{10} B_{10}}^{(total)}$ |
|---------------------|-----------------------------|------------------------------------|
| $\pi \Delta \Delta$ | 0.769 ± 0.003               | 0.780 ± 0.004                      |
| $\pi \Sigma^+ \Sigma^*$ | 0.688 ± 0.003               | 0.703 ± 0.004                      |
| $\pi \Xi^- \Xi^*$ | 0.421 ± 0.002               | 0.469 ± 0.002                      |
| $\pi \Omega$ | 0                           | 0                                  |
| $K \Delta \Sigma^* | -1.423 ± 0.014               | -1.375 ± 0.014                      |
| $K \Sigma^\pi \Xi^- | -2.013 ± 0.020               | -2.014 ± 0.020                      |
| $K \Xi^- \Omega | -2.466 ± 0.024               | -2.507 ± 0.025                      |

(25)

In Table VI, we list the results on the pseudovector coupling constants for the $P_b B_{10} B_{10}$ vertices. The $\pi \Omega$ coupling constant vanishes, since the isoscalar $\Omega$ baryon cannot be coupled to the pion. Note that as the absolute value of strangeness increases, the magnitude of the $P_b B_{10} B_{10}$ coupling constant tends to increase. For example, the magnitude of $|f_{K \Xi^- \Omega}|$ is approximately three times larger than that of $f_{\pi \Delta \Delta}$. 

(26)
IV. COUPLING CONSTANTS FOR THE $\eta$-B, $\eta'$-B VETICES

In this Section, we provide the numerical values of the coupling constants when $\eta$ and $\eta'$ are involved. In order to compute them, we have to consider the mixing between the octet $\eta_B$ and the singlet $\eta_B$ coupling constants. Following the mixing scheme suggested in Ref. \cite{8} given as

\begin{align}
g_{\eta B_B B_S} &= \cos\theta_p \, g_{\eta_B B_B B_S} - \sin\theta_p \, g_{\eta_0 B_B B_S}, \\
g_{\eta' B_B B_S} &= \sin\theta_p \, g_{\eta_B B_B B_S} + \cos\theta_p \, g_{\eta_0 B_B B_S},
\end{align}

one can easily determine the coupling constants for the $\eta$ and $\eta'$ coupling constants. Using the values of $f_0 = 94.0$ MeV, $f_\pi = 94.1$ MeV taken from Refs. \cite{40, 42} and mixing angle $\theta_p = -23.00^\circ$ from Ref. \cite{8}, we obtain the pseudoscalar coupling constants for the $\eta_B B_S$ and $\eta' B_B B_S$ coupling constants.

| \eta\eta' | g^{(0)}_{\eta\eta' B_B B_S} | g^{(0)}_{\eta\eta' B_B B_S} | g^{(0)}_{\eta\eta' B_B B_S} | g^{(0)}_{\eta\eta' B_B B_S} |
|------------|----------------|----------------|----------------|----------------|
| $\eta N\eta'$ | 1.583 ± 0.126 | -0.328 ± 0.027 | -0.015 ± 0.002 | 1.241 ± 0.103 |
| $\eta' N\eta$ | 1.241 ± 0.103 | -0.637 ± 0.044 | 0.007 ± 0.001 | 0.611 ± 0.088 |
| $\eta\eta' \Lambda$ | -1.947 ± 0.153 | 1.169 ± 0.097 | -0.053 ± 0.005 | -0.831 ± 0.086 |
| $\eta\eta' \Sigma$ | 3.189 ± 0.199 | 2.272 ± 0.155 | 0.024 ± 0.002 | 5.486 ± 0.329 |
| $\eta\eta' \Xi$ | 3.772 ± 0.288 | -1.026 ± 0.086 | -0.006 ± 0.003 | 2.740 ± 0.214 |
| $\eta'\eta\Xi$ | -3.590 ± 0.274 | 1.470 ± 0.124 | -0.042 ± 0.004 | -2.161 ± 0.177 |
| $\eta'\eta' \Xi$ | 4.346 ± 0.266 | 3.747 ± 0.253 | 0.019 ± 0.001 | 8.111 ± 0.484 |

The corresponding numerical results are listed in Table \textbf{VII} and are compared with those from the Nijmegen potentials. Since the effects of SU(3) symmetry breaking seem rather important, we examine the contributions from the SU(3) symmetry breaking more closely. In the case of exact SU(3) symmetry, the results are very similar to those from the Nijmegen potentials. However, when the effects of explicit SU(3) symmetry breaking are taken into account, the values of the $\eta$ and $\eta'$ coupling constants are in general much changed. As shown in Table \textbf{VII}, there are two different contributions of the SU(3) symmetry breaking: The one arises directly from the collective operator for the axial-vector constant given in Eq. \textbf{1} and the other comes from the wavefunctions mixed with the states from higher representations as in Eq. \textbf{3}. As clearly shown in the fourth column of Table \textbf{VII}, the wavefunction corrections are negligibly small. However, the linear $m_s$ corrections from the collective operator, in particular, when it comes to the $\eta' B_S B_S$ coupling constants, are sizable, even compared with the contributions of the SU(3) symmetric terms.

In order to understand this, we need to examine carefully the expression for the singlet axial-vector constant $g^{(0)}_{\eta' A}$. As discussed in detail in Ref. \cite{26}, the singlet axial-vector operator $g^{(0)}_{\eta' A}$ is written as

$$g^{(0)}_{\eta A} = a_3 \hat{J}_3 + \sqrt{3}(a_5 - a_6)D^{(8)}_{83},$$

where the leading-order contribution with $a_3$ vanish. It means that $a_3$, which is subleading in the $1/N_c$ expansion, plays a leading role \cite{43, 44}. The parameter $a_3$ in Eq. \textbf{26} comes from the anomalous part of the effective chiral action in the $\chi$QSM while in the Skyrme model it arises from the Wess-Zumino term and vanishes in the version of the pseudoscalar mesons. Thus, the effects of SU(3) symmetry breaking are crucial in determining the value of $g^{(0)}_{\eta A}$ quantitatively. Since $a_5$ and $a_6$ have different signs as shown in Table \textbf{II}, the $m_s$ correction given in the second term of Eq. \textbf{26} becomes large. As a result, the effects of SU(3) symmetry breaking turn out to be sizable, in particular, in the case of the $\eta' B_S B_S$ coupling constants for which the singlet contributions are large. Thus, the present results imply physically that the effects of SU(3) symmetry breaking are crucial in determining the $\eta$ and $\eta'$ coupling constants quantitatively.

In Table \textbf{VIII} we list the results of the $\eta B_S B_{10}$ and $\eta' B_S B_{10}$ coupling constants. In general, the effects of explicit SU(3) symmetry breaking reduce the magnitudes of these coupling constants noticeably. Table \textbf{IX} lists the results of the $\eta$ and $\eta'$ coupling constants for the baryon decuplet. Interestingly, the effects of explicit SU(3) symmetry breaking are marginal except for the $\eta\Omega\Omega$ coupling constant, since the matrix elements of $D^{(8)}_{83}$ are small for the baryon decuplet. Note that the $\eta' \Sigma^+ \Sigma^+$ does not acquire any contribution from explicit SU(3) symmetry breaking. In general, the values of the $\eta'$ coupling constants are much larger than those of the $\eta$ ones.
In the present work, we have investigated the strong coupling constants for the meson-baryon-baryon vertices within the general framework of the chiral soliton model, taking into account the effects of flavor SU(3) symmetry breaking to linear order. All the relevant dynamical parameters were fixed by using the experimental data on the hyperon semileptonic decays and the singlet axial-vector constant. We were able to determine the strong coupling constants for the baryon octet and pseudoscalar meson octet vertices, those for the transition from the baryon decuplet to the baryon and pseudoscalar meson octets, and those for the baryon decuplet and pseudoscalar meson octet vertices. Except for the $\pi \Xi^+$, $\pi \Sigma^+$, and $K \Sigma^+$ vertices, the present results were in good agreement with those determined from the Nijmegen and Jülich-Bonn potentials. We also computed the decay widths of the baryon decuplet to the baryon octet and the pion. Apart from the $\Delta$ decays, the results are in good agreement with the experimental data. We also presented the strong coupling constants for the $\eta$ and $\eta'$ mesons. The effects of SU(3) symmetry breaking are in general quite sizable on the $\eta'$ coupling constants. This can be understood that the leading contribution to the singlet axial-vector constant vanishes in the $1/N_c$ expansion within the present framework and the subleading-order terms play a leading role. Thus, the corrections of the strange current quark mass become relatively more important in the case of the $\eta'$ coupling constants.

The strong coupling constants for the vector mesons and the baryon octet and decuplet can be examined within the same framework. Since the vector mesons have spin 1, the structure of the coupling constants is more involved. The related work is under investigation.

**ACKNOWLEDGMENTS**

H.-Ch.K is grateful to M. V. Polyakov for the discussion and hospitality during his visit to the Institute für Theoretical Physics II, Ruhr-Universität Bochum, where part of the work was done. The present work was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (Grant No. NRF-2016R1C1B1012429 (Gh.-S. Y.) and 2018R1A2B2001752(H.-

---

**TABLE VIII.** Pseudovector coupling constants $f_{\eta B_0 B_{10}}$ and $f_{\eta' B_0 B_{10}}$ divided by $\sqrt{4\pi}$. The second column lists the results for the SU$_f$(3) symmetric case, whereas the third one corresponds to those from $a_4$, $a_5$, and $a_6$ of the collective operator for the axial-vector constants given in Eq. (4). The fourth one represents the corrections from the symmetry-breaking parts of the collective wavefunctions in Eq. (4). The fifth column presents the total results of the coupling constants.

| $f_{\eta B_0 B_{10}}$ | $f_{\eta B_0 B_{10}}^{(wf)}$ | $f_{\eta B_0 B_{10}}^{(op)}$ | $f_{\eta B_0 B_{10}}^{(total)}$ |
|-----------------------|-------------------------------|-------------------------------|-------------------------------|
| $f_{\eta B_0 B_{10}}^{(total)}$ | 3.21 ± 0.25  | -0.74 ± 0.06  | 0.02 ± 0.01  | 2.48 ± 0.20  |
| $\eta \Sigma^*$       | 3.46 ± 0.31  | -0.01 ± 0.01  | 0.45 ± 0.28  |
| $\eta \Xi^*$          | 3.21 ± 0.25  | -0.68 ± 0.06  | -0.10 ± 0.01 | 2.42 ± 0.19  |
| $\eta' \Xi^*$         | 3.46 ± 0.31  | -3.04 ± 0.21  | 0.08 ± 0.01  | 0.49 ± 0.28  |

**TABLE IX.** Pseudovector strong coupling constants of the baryon decuplet with $\eta$ and $\eta'$, divided by $\sqrt{4\pi}$. The second column lists the results for the SU$_f$(3) symmetric case, whereas the third one corresponds to those from $a_4$, $a_5$, and $a_6$ of the collective operator for the axial-vector constants given in Eq. (4). The fourth one represents the corrections from the symmetry-breaking parts of the collective wavefunctions in Eq. (4). The fifth column presents the total results of the coupling constants.

| $f_{\eta B_0 B_{10}}$ | $f_{\eta B_0 B_{10}}^{(wf)}$ | $f_{\eta B_0 B_{10}}^{(op)}$ | $f_{\eta B_0 B_{10}}^{(total)}$ |
|-----------------------|-------------------------------|-------------------------------|-------------------------------|
| $f_{\eta B_0 B_{10}}^{(total)}$ | 1.77 ± 0.15  | -0.21 ± 0.02  | -0.04 ± 0.01  | 1.51 ± 0.13  |
| $\eta \Delta$         | 4.58 ± 0.35  | -0.02 ± 0.01  | -0.01 ± 0.01  | 3.79 ± 0.32  |
| $\eta' \Sigma^*$      | 5.06 ± 0.37  | -0.01 ± 0.01  | -0.01 ± 0.01  | 5.05 ± 0.37  |
| $\eta' \Xi^*$         | 0.56 ± 0.07  | -0.21 ± 0.02  | -0.03 ± 0.01  | 0.75 ± 0.08  |
| $\eta' \Omega$        | 5.53 ± 0.39  | 0.83 ± 0.06   | 0.02 ± 0.01   | 6.39 ± 0.43  |
| $\eta' \Omega$        | 6.00 ± 0.41  | -0.83 ± 0.06  | 0.01 ± 0.01   | 5.18 ± 0.38  |
[1] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963) [Erratum-ibid. 37, 326 (1965)].
[2] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[3] Y. Goto et al. [Asymmetry Analysis Collaboration], Phys. Rev. D 62, 034017 (2000).
[4] B. Holzenkamp, K. Holinde and J. Speth, Nucl. Phys. A 500, 485 (1989).
[5] A. Reuber, K. Holinde, H.-Ch. Kim and J. Speth, Nucl. Phys. A 608, 243 (1996).
[6] T. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Phys. Rev. C 59, 21 (1999).
[7] J. Haidenbauer and U.-G. Meißner, Phys. Rev. C 72, 044005 (2005).
[8] T. A. Rijken, M. M. Nagels and Y. Yamamoto, Prog. Theor. Phys. Suppl. 185, 14 (2010).
[9] J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga and W. Weise, Nucl. Phys. A 915, 24 (2013).
[10] C. Schütz, K. Holinde, J. Speth, B. C. Pearce and J. W. Durso, Phys. Rev. C 51, 1374 (1995).
[11] T. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Prog. Theor. Phys. Suppl. 185, 14 (2010).
[12] H. Polinder and T. A. Rijken, Phys. Rev. C 72, 065211 (2005).
[13] R. Machleidt, K. Holinde and C. Elster, Phys. Rept. 149, 1 (1987).
[14] G. E. Brown and W. Weise, Phys. Rept. 22, 279 (1975).
[15] H. Kamano, S. X. Nakamura, T.-S. H. Lee and T. Sato, Phys. Rev. C 88, no. 3, 035209 (2013).
[16] H. Takahashi, Nucl. Phys. A 914, 553 (2013).
[17] J. Yelton et al. [Belle Collaboration], [arXiv:1805.09384 [hep-ex]].
[18] S. Gongyo et al., Phys. Rev. Lett. 120, 212001 (2018) [arXiv:1709.00654 [hep-lat]].
[19] T. Sekihara, Y. Kamiya and T. Hyodo, arXiv:1805.04024 [hep-ph].
[20] J. Haidenbauer, S. Petschauer, N. Kaiser, U. G. Meißner and W. Weise, Eur. Phys. J. C 77, no. 11, 760 (2017) [arXiv:1708.08071 [nucl-th]].
[21] G.-S. Yang and H.-Ch. Kim, Prog. Theor. Phys. 128, 397 (2012).
[22] G.-S. Yang and H.-Ch. Kim, J. Korean Phys. Soc. 61, 1956 (2012).
[23] G.-S. Yang, H.-Ch. Kim and M. V. Polyakov, Phys. Lett. B 695, 214 (2011).
[24] H.-Ch. Kim, M. Praszalowicz and K. Goeke, Phys. Rev. D 61, 114006 (2000).
[25] H.-Ch. Kim, M. Praszalowicz and K. Goeke, Acta Phys. Polon. B 31, 1767 (2000).
[26] H.-Ch. Kim, M. Praszalowicz and K. Goeke, Acta Phys. Polon. B 32, 1343 (2001).
[27] G.-S. Yang, H.-Ch. Kim and K. Goeke, Phys. Rev. D 75, 094004 (2007).
[28] G. S. Yang and H.-Ch. Kim, Phys. Rev. C 92, 035206 (2015).
[29] C. V. Christov, A. Blotz, H.-Ch. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola and K. Goeke, Prog. Part. Nucl. Phys. 37, 91 (1996) [hep-ph/9604441].
[30] H.-Ch. Kim, M. V. Polyakov, M. Praszalowicz and K. Goeke, Phys. Rev. D 57, 299 (1998).
[31] T. Ledwig, A. Silva, H.-Ch. Kim and K. Goeke, JHEP 0807, 132 (2008).
[32] A. Blotz, D. Diakonov, K. Goeke, N. W. Park, V. Petrov and P. V. Pobylitsa, Nucl. Phys. A 555, 765 (1993).
[33] S. D. Bass, Rev. Mod. Phys. 77, 1257 (2005) [hep-ph/0411005].
[34] S. L. Adler, Annals Phys. 50, 189 (1968).
[35] C. H. Llewellyn Smith, Phys. Rept. 3, 261 (1972).
[36] C. Alexandrou, E. B. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato and A. Tsapalis, Phys. Rev. D 87, no. 11, 114513 (2013) [arXiv:1304.4614 [hep-lat]].
[37] L. J. General and S. R. Cotanch, Phys. Rev. C 69, 035202 (2004).
[38] C. Alexandrou, G. Koutsou, T. Leontiou, J. W. Negele and A. Tsapalis, Phys. Rev. D 76, 094511 (2007) Erratum: [Phys. Rev. D 80, 099901 (2009)].
[39] L. Zhu and M. J. Ramsey-Musolf, Phys. Rev. D 66, 076008 (2002).
[40] R. M. Barnett et al. [Particle Data Group], Phys. Rev. D 54, 1 (1996).
[41] H. J. Behrend et al. [CELLO Collaboration], Z. Phys. C 49, 401 (1991).
[42] T. Aihara et al. [TPC/Two Gamma Collaboration], Phys. Rev. Lett. 64, 172 (1990).
[43] M. Wakamatsu and T. Watabe, Phys. Lett. B 312 (1993) 184.
[44] C. V. Christov, A. Blotz, K. Goeke, P. Pobylitsa, V. Petrov, M. Wakamatsu and T. Watabe, Phys. Lett. B 325 (1994) 467 [hep-ph/9312279].