The Pion Cloud In Quenched QCD

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Diagrammatic methods and Large $N_c$ QCD are used to argue that the nucleon as calculated in quenched QCD contains physics which can be ascribed to the pion cloud of the nucleon. In particular, it is argued that the physics corresponding to one-pion loops in chiral perturbation theory are included in the quenched approximation.
I. INTRODUCTION

At the present time, the most promising technique for eventually deriving the low energy properties of hadrons directly from quantum chromodynamics (QCD) is via numerical Monte Carlo simulations of the functional integral in a lattice regularized version of the theory. Unfortunately, given the computational power currently available, it is not possible to calculate hadron properties in a completely realistic manner in which quarks of physical mass move in a large lattice volume with lattice spacings fine enough to guarantee asymptotic scaling. A common approximation that significantly reduces the numerical demands of the simulations is the so-called quenched approximation. The computational gains associated with this approximation allow one to greatly improve on systematic and statistical uncertainties, and to probe the extremes of the parameter space.

In the Euclidean space formulation of the QCD functional integral, the weighting function has the following continuum form

\[
W = \det \left[ \mathcal{D} + m_q \right] \exp(-S_{YM}),
\]

where \( S_{YM} \) is the Euclidean action for a pure Yang-Mills theory (i.e. only gluons). Integration over the quark Grassmann fields gives rise to the functional determinant. The quenched approximation consists of setting the functional determinant to a constant, independent of the gluon field configuration. Thus, in the quenched approximation, the weighting function is simplified to

\[
W_q = \exp(-S_{YM}).
\]

This form is obviously amiable to Monte Carlo integration techniques.

From a diagrammatic point of view, the omission of the fermion determinant in the weighting function corresponds to the neglect of all diagrams containing closed quark loops which are not connected to external sources.

Given the fact that quenched calculations will continue to be used for some time to come, it is an important practical problem to determine what, if any, essential physics is lost when one makes the quenched approximation. This issue is particularly important if one wishes to use the results of quenched lattice QCD calculations to gain insight into what physics should be included in models of hadrons.

The physics of the pion cloud is believed to play an essential role in hadronic structure. Several models of the nucleon, including the Skyrmion [1], the cloudy bag model [2], the hybrid or chiral bag model [3] and chiral quark-meson models [4] stress the role of the pion cloud in the nucleon structure. Thus, the question of whether the quenched approximation to QCD contains the physics of the pion cloud becomes important. In this comment, we will argue that pion cloud effects are, in fact, included in quenched calculations of nucleon properties. This result may seem to be counterintuitive, but as shall be shown, this conclusion is reached both in a large \( N_c \) analysis and in an explicit study of diagrams. The arguments are somewhat heuristic but we believe rather compelling.

Apart from the role of the pion cloud in various models, there is also a systematic method to estimate the role of pions in hadronic observables, namely chiral perturbation theory (\( \chiPT \)). This approach is an expansion which is based on a separation of scales between
the pion mass and other scales in hadronic physics \cite{3,8}. This approach should rigorously reproduce the low energy properties of QCD if the quark mass (and hence the pion mass squared) is sufficiently light. Chiral perturbation theory predicts that certain quantities are \textit{nonanalytic} in the quark mass as one approaches the chiral limit due to the effects of the infrared behavior of pion loops. For our present purpose, we note that one can use the existence of this nonanalytic behavior as a signature of pion cloud effects.

Before proceeding it is worth noting that the need for quenched calculations is particularly strong if one wishes to study the role of the pion cloud. At present, completely realistic lattice simulations of pion cloud effects are not possible. The basic difficulty stems from the use of unphysically large quark masses in the lattice simulations which leads to pions which are heavy and consequently of short range.

A number of challenges are presented as one attempts to decrease the quark mass. The long range nature of the pion demands larger physical lattice volumes which ultimately leads to larger lattices if one wishes to maintain a reasonable lattice spacing. Furthermore, calculations of fermion propagators and Monte-Carlo estimates of the functional determinant become increasingly difficult as the quark mass drops and convergence of the algorithms slows.

For finite computer resources one can always investigate larger lattice volumes and lighter quark masses in the quenched approximation than in full QCD. It is likely that there will be a significant period of time when quenched calculations are able to probe the parameter space in areas where pion cloud effects are expected to be significant whereas full QCD simulations will remain restricted to a more limited parameter space. Thus, it is important to establish whether or not quenched calculations are capable of describing the physics of the pion cloud.

In this spirit, it is important to make a more restrictive definition of the quenched approximation. Within the usual definition there remain diagrams which are extremely cumbersome to calculate and as a result are generally omitted in quenched analyses. Figure 1 displays two such diagrams with quark loops connected to external sources. These quark line disconnected diagrams require knowledge of the spatial diagonal elements of the inverse fermion matrix whereas a standard quark propagator requires knowledge of a single column. Hence, they are computationally quite difficult and for the purposes of this investigation we will broaden our definition of the quenched approximation to exclude diagrams of the type illustrated in figure 1.

To make contact with physical observables, it is necessary to extrapolate from the large values of quark mass currently used in lattice calculations to their physical masses. As has been stressed elsewhere \cite{9}, nonanalytic terms in \(\chi PT\) can lead to important corrections to this extrapolation in calculations of charge radii and other observables. It is clearly of importance to determine whether these chiral corrections to the extrapolations are present in the quenched approximation or only in full QCD.

**II. PION PROPERTIES**

The question of whether the physics of pion loops is present in the quenched approximation has been discussed previously \cite{10,11}. This discussion has been centered on the
properties of pions and specifically whether the pion mass squared contains a term proportional to $m_q^2 \ln(m_q)$ which is predicted in $\chi$PT. The argument is that a new form of $\chi$PT must be developed to deal with quenched QCD. In this quenched version of $\chi$PT there are no nonanalytic effects arising from the pion cloud surrounding a pion. This does not mean that nonanalytic behavior is absent from mesonic observables. Nonanalytic chiral behavior can have its origin in the cloud associated with the isoscalar $\eta'$ meson. While in nature the $\eta'$ is heavy due to anomalies and topological effects [12], in the quenched approximation the $\eta'$ is degenerate with the pion and $\eta'$ loops can yield nonanalytic chiral behavior.

This discussion suggests that quenched QCD calculations of pion properties will not correctly reproduce the meson cloud effects of full QCD. For example, in full QCD the pion charge radius diverges logarithmically as the quark mass goes to zero [13]; quenched $\chi$PT predicts it will remain finite in quenched QCD. The key point is that the $\eta'$ cloud cannot correctly simulate the role of the pion cloud for electromagnetic properties since the $\eta'$ is neutral and does not couple directly to photons.

### III. NUCLEON PROPERTIES

#### A. Diagrammatic considerations

We will focus our attention on the properties of the nucleon and will argue that pion loop effects do contribute to nucleon properties even in the quenched approximation. At first thought this may seem absurd since pion loops require an intermediate state with a minimum of one $\bar{q}q$ pair and the formation of quark-antiquark pairs is apparently forbidden by the restrictions imposed by the quenched approximations. However, as noted in Ref. [14–16] this restriction is only apparent. Although the quenched approximation limits quark lines to those which are connected to external currents, the quark propagators used are fully relativistic Dirac propagators. Such propagators contain “Z-gra phs” in which the quark is scattered into a negative energy state and back. With the conventional hole interpretation of the Dirac propagator such processes are the creation and annihilation of $\bar{q}q$ pairs. The restriction imposed by the quenched approximation is merely that once a virtual pair is created along one quark line it must be annihilated on the same line. Thus, it is at least possible that quenched QCD may contain $\bar{q}q$ pairs and such pairs might be pionic in nature.

The question of whether pion-loop physics is included in quenched calculations of correlation functions ultimately comes down to the question of whether intermediate states are reached which have overlap with physical states containing pions plus other hadrons. This suggests that one should study old-fashioned energy denominator type time-ordered diagrams and ask whether one can reach states which can be expressed as the product of more than one color singlet operator operating on the vacuum with at least one of these operators having the quantum numbers of the pion. It is clear that this is a necessary but not sufficient condition to establish the presence of pion loop physics. In addition, it must be shown that the singularity structure corresponds to pion plus hadron states.

At the hadronic level, some of the pion cloud physics which accounts for nonanalytic behavior in $\chi$PT can be represented as a single pion loop. The one-pion-loop physics is represented by the hadronic time-ordered diagram in figure 2a. The corresponding diagram
at the quark level commonly thought to give rise to pionic dressings but not surviving in the
quenched approximation is illustrated in figure 2b.

Compare this with the quark level diagram for a nucleon correlation function in figure 2c which survives in the quenched approximation [14]. The intermediate state contains a $\overline{q}q$ structure (at the top of figure 2c) which has both color-singlet and color-octet pieces. The quenched nature of the calculation requires that the antiquark must be the same flavor as one of the original quarks in the nucleon interpolating field. Since there are always two distinct flavors of quark in a nucleon, the $\overline{q}q$ structure can be either isospin zero or one. The three quark structure below the $\overline{q}q$ structure has a piece which is color singlet and isospin one half. It is highly plausible that the part of this graph which consists of a color singlet isospin one $\overline{q}q$ piece along with a color singlet isospin one half $qqq$ piece has some overlap with the physical pion-nucleon scattering state. Hence it appears that figure 2c contains, among other things, the essential physics of figure 2a. Of course, the diagram in figure 2c is only one of an infinite class of diagrams which appear to have nonzero overlap with pion-nucleon scattering states. One can add to the diagram an arbitrary number of gluons.

It is worth noting at this point that a similar diagrammatic analysis would not give pionic intermediate states for meson correlation functions. For example in figure 3a we show a one loop hadronic diagram for the $\rho$ meson channel. The imaginary part of this graph gives the $\rho$ to two pion decay. In analogy to figure 2c we construct figure 3b. This diagram certainly contains two isovector structures. However, unlike in the nucleon case in figure 2c, these structures do not have pion quantum numbers. They are color 8 or $\overline{3}$ diquarks with baryon number $\pm 2/3$. Alternatively, one could arrange the diagram as in figure 3c. There is a component in which both structures are color singlets but one sees that at least one of the structures is isoscalar and cannot represent a pion. The isoscalar piece has overlap with the $\eta'$ meson discussed in Ref. [10]. Thus, this simple diagrammatic analysis shows that the physics of figure 3a cannot be reproduced in the quenched approximation. This result is consistent with Ref. [15,11] and Ref. [10].
B. Large $N_c$ Analysis

The preceding diagrammatic analysis is suggestive but not conclusive since it gives no information about the analytic properties of the diagram and we do not know whether there is any spectral strength corresponding to nucleon plus pion states. The presence of such spectral strength will lead to nonanalytic behavior in $m_\pi^2$ around zero. How can one learn whether there is any nonanalytic behavior with respect to $m_\pi^2$ in quenched QCD given the fact that explicit simulations with light quark masses are currently impractical? Large $N_c$ QCD \[17,18\] provides considerable insight. The key point is that QCD to leading order in a $1/N_c$ expansion is quenched. As shown by ’t Hooft \[17\], in any diagram there is a $1/N_c$ suppression factor associated with each closed fermion loop in a large $N_c$ expansion. Thus the diagrams which contribute to the leading order expression for any correlation function have the minimum number of fermion loops. This is precisely the condition imposed by the quenched approximation. It is amusing to note that the nonquenched diagram of figure 2b, commonly thought to give rise to pionic dressings of the nucleon are $1/N_c$ suppressed relative to that of figure 2c.

Of course, the leading order large $N_c$ approximation is a more drastic approximation than the quenched approximation since some graphs which do not contain internal fermion loops (e.g. some non-planar graphs) are also $1/N_c$ suppressed. Since the large $N_c$ approximation is more severe than the quenched approximation and contains the quenched approximation it is clear that if there is pion loop physics (as evidenced by nonanalytic behavior in $m_\pi^2$) in large $N_c$ QCD, then the same physics should be present in the less severe quenched approximation.

There are two distinct arguments which suggest that pion loop physics is present in the leading order large $N_c$ approximation. One way is to study large $N_c$ hadrodynamics \[19–21\] (i.e. a dynamical model based on hadron degrees of freedom). The other is via the study of models such as the Skyrme model which capture the correct leading order $N_c$ physics from QCD.

The basic idea of large $N_c$ hadrodynamics is that if one produces an effective hadronic model which reproduces the underlying physics of QCD then all of the parameters of this hadrodynamical model must scale with $N_c$ in the manner prescribed by large $N_c$ QCD \[17,18\]:

\begin{align}
\Gamma_n^m & \sim N_c^{(1-n/2)}; \quad M_m \sim 1; \quad M_B \sim N_c; \quad g_{mBB} \sim N_c^{1/2}; \quad \Lambda_{Bff} \sim 1, \quad (3)
\end{align}

where $\Gamma_m^n$ is a meson $n$-point vertex, $M_m$ a meson mass, $M_B$ the baryon mass, $g_{mBB}$ a meson baryon coupling and $\Lambda_{Bff}$ is a baryon form factor mass. The study of loops in large $N_c$ hadrodynamics goes back to Witten \[18\] who showed that meson loops always give corrections to the tree level meson properties which are suppressed in $1/N_c$.

Consider the one meson loop contribution to the meson propagator. The propagators in the loop are all of order unity but from (3) the three-meson vertices are order $N_c^{-1/2}$. There are two such vertices so the net effect is order $1/N_c$ which is indeed suppressed compared to the leading order propagator which is of order unity. Note however, that this argument does not exclude the possibility of mesonic loop dressings to mesons in the quenched approximation since the large $N_c$ approximation is more severe than the quenched approximation.
As studied elsewhere [19–21], the situation is more interesting when one studies baryon properties in large $N_c$ hadrodynamics. For present purposes, the important issue is the meson loop contribution to the nucleon mass. Consider the nucleon self-energy at the hadronic level. This receives a contribution from a one-meson-loop diagram as in figure 2a. It is easy to see that this self-energy is order $N_c$. There are two $N_c^{1/2}$ factors at the meson-baryon couplings. The meson propagator is order unity since the mass is order one and the loop integral is cutoff by the form factor whose falloff is also order unity. In the large $N_c$ limit where nucleon recoil can be neglected, the nucleon propagator (working in the rest frame of the nucleon) goes like the inverse of the meson three momentum, i.e., order $1$. Thus, one sees that in large $N_c$ hadrodynamics (which is assumed to correctly reflect large $N_c$ QCD), one meson loop contributions to the nucleon mass are of order $N_c$ which is leading order in $N_c$ counting according to (3). As discussed above, the leading order terms in the $1/N_c$ expansion are quenched and hence the one-meson-loop physics, including one-pion-loop physics, should be present in the quenched approximation.

The preceding argument is consistent with the leading nonanalytic behavior for pion loops obtained in $\chi$PT. Perhaps the easiest place to look for this nonanalyticity is to study $d^2 M_N/d(m_\pi^2)^2$. The leading order nonanalyticity from $\chi$PT comes from a one-pion-loop diagram as in figure 2a. One obtains

$$\frac{d^2 M_N}{d(m_\pi^2)^2} = -\frac{9}{128\pi} \frac{g_A^2}{m_\pi f_\pi^2} + O(1)$$  \hspace{1cm} (4)

It is well known that $g_A \sim N_c$ and $f_\pi \sim N_c^{1/2}$ so that

$$\frac{d^2 M_N}{d(m_\pi^2)^2} \sim N_c$$  \hspace{1cm} (5)

Integrating twice with respect $m_\pi^2$ gives this pion loop contribution to $M_N$ scaling like $N_c$ which is the leading order.

The preceding argument has a drawback in that it is based on large $N_c$ hadrodynamics and not directly on QCD. While it is highly plausible that the $N_c$ behavior of hadrodynamics (including baryons) reproduces that of QCD, it has not been proven rigorously. Ideally, one should work directly in large $N_c$ QCD which unfortunately is not possible. Instead, one can look at models which are believed to correctly reproduce QCD’s large $N_c$ behavior. The analysis here will be based on the Skyrme model although all chiral large $N_c$ soliton models of the nucleon (e.g. the chiral bag model, the chiral quark meson soliton model) behave the same way. The issue of whether the pion cloud physics associated with loops is present in large $N_c$ QCD or models of large $N_c$ QCD (such as the Skyrme model) is complicated. For the present purposes it is reasonable to assert that pion cloud physics is present in large $N_c$ QCD if various quantities calculated to leading order in $N_c$ depend on the pion mass in the same way as pion loop calculations of the same quantities in a hadronic picture. The lore of $\chi$PT is that the leading nonanalytic behavior in $m_\pi^2$ is given by one-pion-loop graphs in a hadronic model. Thus, ultimately the issue is whether the large $N_c$ model calculations produce the same nonanalytic behavior near the chiral limit as one-pion-loop hadronic calculations.

Does the Skyrme model correctly reproduce the leading nonanalytic behavior in $m_\pi^2$ predicted in $\chi$PT? The answer is a qualified yes [22]. As discussed in detail in Ref. [22],
the Skyrme model reproduces the leading nonanalytic properties of $\chi$PT in the following sense. If one confines attention to vector-isovector and scalar-isoscalar operators (i.e. whose expectation values which do not vanish in the hedgehog intrinsic state), and chooses the Skyrme model parameters to give the correct value of $g_A$, then the Skyrme model prediction for nucleon matrix elements will precisely reproduce the leading nonanalytic behavior of $\chi$PT up to an overall factor which depends only on the quantum numbers of the operator.

For scalar-isoscalar operators, the Skyrme model will always give a coefficient for the leading nonanalytic term which is a factor of three larger than $\chi$PT. The origin of this factor of three lies in the fact that the Skyrme model result depends on the order in which the chiral and large $N_c$ limits are taken. In $\chi$PT, it is explicitly assumed that the pion mass is small compared to all relevant hadronic scales in the problem. Thus, the only physical states energetically near the nucleon are states with nucleon plus one pion. On the other hand, in the large $N_c$ limit for a hedgehog model the $\Delta$ is degenerate with the nucleon. The $N - \Delta$ splitting goes as $1/N_c$. Therefore, in the large $N_c$ limit, loops with $\pi - \Delta$ intermediate states should be included along with $\pi - N$ intermediate states when doing $\chi$PT. The inclusion of $\pi - \Delta$ intermediate states in $\chi$PT, assuming degenerate $N$ and $\Delta$ masses and using $g_{\pi N \Delta}$ as calculated in the Skyrme model, precisely accounts for the factor of three. Therefore, at least for this class of observables, the nonanalytic behavior associated with pion loops is in fact present.

An explicit example may help clarify this point. Once again, consider $d^2 M_N/d(m_{\pi}^2)^2$. The one pion loop contribution in $\chi$PT (including only nucleon intermediate states is given in (4). If we also include a diagram analogous to figure 2a with a $\Delta - \pi$ intermediate state and assume $M_\Delta = M_N$ with $g_{\pi N \Delta}$ given by the Skyrme model value we obtain

$$
\frac{d^2 M_N}{d(m_{\pi}^2)^2} = -\frac{27}{128\pi} \frac{g_A^2}{m_{\pi} f_{\pi}^2} + \mathcal{O}(1) \quad (6)
$$

Here the coefficient is precisely three times the usual $\chi$PT result.

One can also calculate $d^2 M_N/d(m_{\pi}^2)^2$ directly from the Skyrme model. With the identity

$$
\frac{d M_N}{d(m_{\pi}^2)} = \langle N | \frac{1}{4} f_{\pi}^2 \text{tr}(U - 1) | N \rangle,
$$

along with the asymptotic properties of $U = \exp(i\vec{\tau} \cdot \vec{\phi}/f_{\pi})$ where $\vec{\phi}$ is the nonlinear realization of the pion field, and the standard Skyrme model expression for $g_A$, one can compute $d^2 M_N/d(m_{\pi}^2)^2$ rather easily. The result is found to be the expression of (6). Thus, we see explicitly that a $1/N_c$ model such as the Skyrme model does have nonanalytic behavior in $1/m_{\pi}^2$ precisely as one anticipates from a calculation of pion dressings of the nucleon with a $\Delta$ degenerate in mass.

IV. SUMMARY

In summary, we have given heuristic—but compelling—arguments as to why simulations of nucleons in quenched QCD contain pion cloud physics which chiral perturbation theory ascribes to one pion loop contributions at the hadronic level. We have shown that diagrams surviving in the quenched approximation contain pieces which “look like” nucleon plus pion
states. We have also argued that one can use the $1/N_c$ approximation as a surrogate for the quenched approximation since the $1/N_c$ approximation is quenched. Two distinct arguments, one based on large $N_c$ hadrodynamics and one based on $1/N_c$ hedgehog models such the Skyrme model, both suggest that pion loop physics is present in large $N_c$ QCD and hence in quenched QCD. These results contrast the superficial perception that the quenched approximation is incapable of including the physics of pionic dressings of baryons.

We thank Wojciech Broniowski and Manoj Banerjee for helpful conversations. D.B.L. thanks Richard Woloshyn and Terry Draper for early discussions which stimulated his interest in these issues. This work is supported in part by the U.S. Department of Energy under grant DE-FG05-87ER-40322. T.D.C. acknowledges additional financial support from the National Science Foundation though grant PHY-9058487.

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FIGURES

FIG. 1. Skeleton diagrams of disconnected quark loops connected to external sources which are generally not included in quenched QCD analyses. Diagram (a) contributes to the three-point correlation function of a baryon current matrix element. Diagram (b) contributes to the two-point correlation function of an iso-scalar meson. The diagrams may be dressed with an arbitrary number of gluons.

FIG. 2. Time ordered diagrams for one-pion-loop dressings of the nucleon. Time flows from left to right. For illustrative purposes we have selected the proton with a $\pi^+ - n$ intermediate state for all three diagrams. Figure (a) illustrates the hadronic level dressing and figure (b) describes a quark level diagram naively thought to exclusively account for pionic dressings of the nucleon. This diagram is excluded in the quenched approximation. Figure (c) illustrates a quark level diagram whose quantum numbers overlap with that of figure (a) and which survives in the quenched approximation.

FIG. 3. Time ordered diagrams for two pion intermediate states of the $\rho$-meson. Figure (a) illustrates the hadronic level intermediate state $\pi^0 \pi^+$. Figure (b) describes a quark level diagram analogous to that in figure 1c but which has no overlap with that of figure 2a. Figure (c) illustrates a quark level diagram which has overlap with a two meson intermediate state. However, one of the mesons is an isoscalar and cannot be identified with a pion.
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At the present time, the most promising technique for eventually deriving the low energy properties of hadrons directly from quantum chromodynamics (QCD) is via numerical Monte Carlo simulations of the functional integral in a lattice regularized version of the theory. Unfortunately, given the computational power currently available, it is not possible to calculate hadron properties in a completely realistic manner in which quarks of physical mass move in a large lattice volume with lattice spacings fine enough to guarantee asymptotic scaling. A common approximation that significantly reduces the numerical demands of the simulations is the so-called quenched approximation. The computational gains associated with this approximation allow one to greatly improve on systematic and statistical uncertainties, and to probe the extremes of the parameter space.

In the Euclidean space formulation of the QCD functional integral, the weighting function has the following continuum form

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where \( S_{YM} \) is the Euclidean action for a pure Yang-Mills theory (\( i.e. \) only gluons). Integration over the quark Grassmann fields gives rise to the functional determinant. The quenched approximation consists of setting the functional determinant to a constant, independent of the gluon field configuration. Thus, in the quenched approximation, the weighting function is simplified to

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Given the fact that quenched calculations will continue to be used for some time to come, it is an important practical problem to determine what, if any, essential physics is lost when one makes the quenched approximation. This issue is particularly important if one wishes to use the results of quenched lattice QCD calculations to gain insight into what physics should be included in models of hadrons.

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A number of challenges are presented as one attempts to decrease the quark mass. The long range nature of the pion demands larger physical lattice volumes which ultimately leads to larger lattices if one wishes to maintain a reasonable lattice spacing. Furthermore, calculations of fermion propagators and Monte-Carlo estimates of the functional determinant become increasingly difficult as the quark mass drops and convergence of the algorithms slows.

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To make contact with physical observables, it is necessary to extrapolate from the large values of quark mass currently used in lattice calculations to their physical masses. As has been stressed elsewhere [9], nonanalytic terms in $\chi$PT can lead to important corrections to this extrapolation in calculations of charge radii and other observables. It is clearly of importance to determine whether these chiral corrections
FIG. 1  Skeleton diagrams of disconnected quark loops connected to external sources which are generally not included in quenched QCD analyses. Diagram (a) contributes to the three-point correlation function of a baryon current matrix element. Diagram (b) contributes to the two-point correlation function of an iso-scalar meson. The diagrams may be dressed with an arbitrary number of gluons.
to the extrapolations are present in the quenched approximation or only in full QCD.

II. PION PROPERTIES

The question of whether the physics of pion loops is present in the quenched approximation has been discussed previously [10, 11]. This discussion has been centered on the properties of pions and specifically whether the pion mass squared contains a term proportional to $m_q^2 \ln(m_q)$ which is predicted in $\chi$PT. The argument is that a new form of $\chi$PT must be developed to deal with quenched QCD. In this quenched version of $\chi$PT there are no nonanalytic effects arising from the pion cloud surrounding a pion. This does not mean that nonanalytic behavior is absent from mesonic observables. Nonanalytic chiral behavior can have its origin in the cloud associated with the isoscalar $\eta'$ meson. While in nature the $\eta'$ is heavy due to anomalies and topological effects [12], in the quenched approximation the $\eta'$ is degenerate with the pion and $\eta'$ loops can yield nonanalytic chiral behavior.

This discussion suggests that quenched QCD calculations of pion properties will not correctly reproduce the meson cloud effects of full QCD. For example, in full QCD the pion charge radius diverges logarithmically as the quark mass goes to zero [13]; quenched $\chi$PT predicts it will remain finite in quenched QCD. The key point is that the $\eta'$ cloud cannot correctly simulate the role of the pion cloud for electromagnetic properties since the $\eta'$ is neutral and does not couple directly to photons.

III. NUCLEON PROPERTIES

A. Diagrammatic considerations

We will focus our attention on the properties of the nucleon and will argue that pion loop effects do contribute to nucleon properties even in the quenched approximation. At first thought this may seem absurd since pion loops require an intermediate state with a minimum of one $\bar{q}q$ pair and the formation of quark-antiquark pairs is apparently forbidden by the restrictions imposed by the quenched approximations. However, as noted in Ref. [14–16] this restriction is only apparent. Although the quenched approximation limits quark lines to those which are connected to external currents, the quark propagators used are fully relativistic Dirac propagators. Such propagators contain “Z-graphs” in which the quark is scattered into a negative energy state and back. With the conventional hole interpretation of the Dirac propagator such processes are the creation and annihilation of $\bar{q}q$ pairs. The restriction imposed by the quenched approximation is merely that once a virtual pair is created along one quark line it must be annihilated on the same line. Thus, it is at least possible that quenched QCD may contain $\bar{q}q$ pairs and such pairs might be pionic in nature.

The question of whether pion-loop physics is included in quenched calculations of correlation functions ultimately comes down to the question of whether intermediate states are reached which have overlap with physical states containing pions plus other
hadrons. This suggests that one should study old-fashioned energy denominator type

time-ordered diagrams and ask whether one can reach states which can be expressed

as the product of more than one color singlet operator operating on the vacuum with

at least one of these operators having the quantum numbers of the pion. It is clear

that this is a necessary but not sufficient condition to establish the presence of pion

loop physics. In addition, it must be shown that the singularity structure corresponds
to pion plus hadron states.

At the hadronic level, some of the pion cloud physics which accounts for nonanalytic

behavior in $\chi$PT can be represented as a single pion loop. The one-pion-loop

physics is represented by the hadronic time-ordered diagram in figure 2a. The corre-

sponding diagram at the quark level commonly thought to give rise to pionic dressings

but not surviving in the quenched approximation is illustrated in figure 2b.

Compare this with the quark level diagram for a nucleon correlation function

in figure 2c which survives in the quenched approximation [14]. The intermediate

state contains a $\bar{q}q$ structure (at the top of figure 2c) which has both color-singlet and

color-octet pieces. The quenched nature of the calculation requires that the antiquark

must be the same flavor as one of the original quarks in the nucleon interpolating field.

Since there are always two distinct flavors of quark in a nucleon, the $\bar{q}q$ structure can

be either isospin zero or one. The three quark structure below the $\bar{q}q$ structure has a

piece which is color singlet and isospin one half. It is highly plausible that the part

of this graph which consists of a color singlet isospin one $\bar{q}q$ piece along with a color

singlet isospin one half $qqq$ piece has some overlap with the physical pion-nucleon

scattering state. Hence it appears that figure 2c contains, among other things, the

essential physics of figure 2a. Of course, the diagram in figure 2c is only one of an

infinite class of diagrams which appear to have nonzero overlap with pion-nucleon

scattering states. One can add to the diagram an arbitrary number of gluons.

It is worth noting at this point that a similar diagrammatic analysis would not
give pionic intermediate states for meson correlation functions. For example in figure

3a we show a one loop hadronic diagram for the $\rho$ meson channel. The imaginary

part of this graph gives the $\rho$ to two pion decay. In analogy to figure 2c we construct

figure 3b. This diagram certainly contains two isovector structures. However, unlike

in the nucleon case in figure 2c, these structures do not have pion quantum numbers.

They are color 8 or $\frac{3}{2}$ diquarks with baryon number $\pm 2/3$. Alternatively, one could

arrange the diagram as in figure 3c. There is a component in which both structures

are color singlets but one sees that at least one of the structures is isoscalar and
cannot represent a pion. The isoscalar piece has overlap with the $\eta'$ meson discussed

in Ref. [10]. Thus, this simple diagrammatic analysis shows that the physics of figure

3a cannot be reproduced in the quenched approximation. This result is consistent

with Ref. [15, 11] and Ref. [10].
FIG. 2 Time ordered diagrams for one-pion-loop dressings of the nucleon. Time flows from left to right. For illustrative purposes we have selected the proton with a $\pi^+ - n$ intermediate state for all three diagrams. Figure (a) illustrates the hadronic level dressing and figure (b) describes a quark level diagram naively thought to exclusively account for pionic dressings of the nucleon. This diagram is excluded in the quenched approximation. Figure (c) illustrates a quark level diagram whose quantum numbers overlap with that of figure (a) and which survives in the quenched approximation.
FIG. 3  Time ordered diagrams for two pion intermediate states of the $\rho$-meson. Figure (a) illustrates the hadronic level intermediate state $\pi^0 \pi^+$. Figure (b) describes a quark level diagram analogous to that in figure 1c but which has no overlap with that of figure 2a. Figure (c) illustrates a quark level diagram which has overlap with a two meson intermediate state. However, one of the mesons is an isoscalar and cannot be identified with a pion.
B. Large $N_c$ Analysis

The preceding diagrammatic analysis is suggestive but not conclusive since it gives no information about the analytic properties of the diagram and we do not know whether there is any spectral strength corresponding to nucleon plus pion states. The presence of such spectral strength will lead to nonanalytic behavior in $m_\pi^2$ around zero. How can one learn whether there is any nonanalytic behavior with respect to $m_\pi^2$ in quenched QCD given the fact that explicit simulations with light quark masses are currently impractical? Large $N_c$ QCD [17, 18] provides considerable insight. The key point is that QCD to leading order in a $1/N_c$ expansion is quenched. As shown by 't Hooft [17], in any diagram there is a $1/N_c$ suppression factor associated with each closed fermion loop in a large $N_c$ expansion. Thus the diagrams which contribute to the leading order expression for any correlation function have the minimum number of fermion loops. This is precisely the condition imposed by the quenched approximation. It is amusing to note that the nonquenched diagram of figure 2b, commonly thought to give rise to pionic dressings of the nucleon are $1/N_c$ suppressed relative to that of figure 2c.

Of course, the leading order large $N_c$ approximation is a more drastic approximation than the quenched approximation since some graphs which do not contain internal fermion loops (e.g. some non-planar graphs) are also $1/N_c$ suppressed. Since the large $N_c$ approximation is more severe than the quenched approximation and contains the quenched approximation it is clear that if there is pion loop physics (as evidenced by nonanalytic behavior in $m_\pi^2$) in large $N_c$ QCD, then the same physics should be present in the less severe quenched approximation.

There are two distinct arguments which suggest that pion loop physics is present in the leading order large $N_c$ approximation. One way is to study large $N_c$ hadrodynamics [19–21] (i.e. a dynamical model based on hadron degrees of freedom). The other is via the study of models such as the Skyrme model which capture the correct leading order $N_c$ physics from QCD.

The basic idea of large $N_c$ hadrodynamics is that if one produces an effective hadronic model which reproduces the underlying physics of QCD then all of the parameters of this hadrodynamical model must scale with $N_c$ in the manner prescribed by large $N_c$ QCD [17, 18]:

$$\Gamma_m^m \sim N_c^{1-n/2}; \quad M_m \sim 1; \quad M_B \sim N_c; \quad g_{mBB} \sim N_c^{1/2}; \quad \Lambda_{BfJ} \sim 1,$$

where $\Gamma_m^m$ is a meson $n$-point vertex, $M_m$ a meson mass, $M_B$ the baryon mass, $g_{mBB}$ a meson baryon coupling and $\Lambda_{BfJ}$ is a baryon form factor mass. The study of loops in large $N_c$ hadrodynamics goes back to Witten [18] who showed that meson loops always give corrections to the tree level meson properties which are suppressed in $1/N_c$.

Consider the one meson loop contribution to the meson propagator. The propagators in the loop are all of order unity but from (3) the three-meson vertices are order $N_c^{-1/2}$. There are two such vertices so the net effect is order $1/N_c$ which is
indeed suppressed compared to the leading order propagator which is of order unity. Note however, that this argument does not exclude the possibility of mesonic loop dressings to mesons in the quenched approximation since the large $N_c$ approximation is more severe than the quenched approximation.

As studied elsewhere [19–21], the situation is more interesting when one studies baryon properties in large $N_c$ hadrodynamics. For present purposes, the important issue is the meson loop contribution to the nucleon mass. Consider the nucleon self-energy at the hadronic level. This receives a contribution from a one-meson-loop diagram as in figure 2a. It is easy to see that this self-energy is order $N_c$. There are two $N_c^{1/2}$ factors at the meson-baryon couplings. The meson propagator is order unity since the mass is order one and the loop integral is cutoff by the form factor whose falloff is also order unity. In the large $N_c$ limit where nucleon recoil can be neglected, the nucleon propagator (working in the rest frame of the nucleon) goes like the inverse of the meson three momentum, i.e. order 1. Thus, one sees that in large $N_c$ hadrodynamics (which is assumed to correctly reflect large $N_c$ QCD), one meson loop contributions to the nucleon mass are of order $N_c$ which is leading order in $N_c$ counting according to (3). As discussed above, the leading order terms in the $1/N_c$ expansion are quenched and hence the one-meson-loop physics, including one-pion-loop physics, should be present in the quenched approximation.

The preceding argument is consistent with the leading nonanalytic behavior for pion loops obtained in $\chi$PT. Perhaps the easiest place to look for this nonanalyticity is to study $d^2M_N/d(m_\pi^2)^2$. The leading order nonanalyticity from $\chi$PT comes from a one-pion-loop diagram as in figure 2a. One obtains

$$\frac{d^2 M_N}{d(m_\pi^2)^2} = -\frac{9}{128\pi} \frac{g_A^2}{m_\pi f_\pi^2} + \mathcal{O}(1) \quad (4)$$

It is well known that $g_A \sim N_c$ and $f_\pi \sim N_c^{1/2}$ so that

$$\frac{d^2 M_N}{d(m_\pi^2)^2} \sim N_c \quad (5)$$

Integrating twice with respect $m_\pi^2$ gives this pion loop contribution to $M_N$ scaling like $N_c$ which is the leading order.

The preceding argument has a drawback in that it is based on large $N_c$ hadrodynamics and not directly on QCD. While it is highly plausible that the $N_c$ behavior of hadrodynamics (including baryons) reproduces that of QCD, it has not been proven rigorously. Ideally, one should work directly in large $N_c$ QCD which unfortunately is not possible. Instead, one can look at models which are believed to correctly reproduce QCD’s large $N_c$ behavior. The analysis here will be based on the Skyrme model although all chiral large $N_c$ soliton models of the nucleon (e.g. the chiral bag model, the chiral quark meson soliton model) behave the same way. The issue of whether the pion cloud physics associated with loops is present in large $N_c$ QCD or models of large $N_c$ QCD (such as the Skyrme model) is complicated. For the present purposes
it is reasonable to assert that pion cloud physics is present in large $N_c$ QCD if various quantities calculated to leading order in $N_c$ depend on the pion mass in the same way as pion loop calculations of the same quantities in a hadronic picture. The lore of $\chi$PT is that the leading nonanalytic behavior in $m_\pi^2$ is given by one-pion-loop graphs in a hadronic model. Thus, ultimately the issue is whether the large $N_c$ model calculations produce the same nonanalytic behavior near the chiral limit as one-pion-loop hadronic calculations.

Does the Skyrme model correctly reproduce the leading nonanalytic behavior in $m_\pi^2$ predicted in $\chi$PT? The answer is a qualified yes [22]. As discussed in detail in Ref. [22], the Skyrme model reproduces the leading nonanalytic properties of $\chi$PT in the following sense. If one confines attention to vector-Isoscalar and Scalar-Isoscalar operators (i.e. whose expectation values which do not vanish in the hedgehog intrinsic state), and chooses the Skyrme model parameters to give the correct value of $g_A$, then the Skyrme model prediction for nucleon matrix elements will precisely reproduce the leading nonanalytic behavior of $\chi$PT up to an overall factor which depends only on the quantum numbers of the operator.

For scalar-Isoscalar operators, the Skyrme model will always give a coefficient for the leading nonanalytic term which is a factor of three larger than $\chi$PT. The origin of this factor of three lies in the fact that the Skyrme model result depends on the order in which the chiral and large $N_c$ limits are taken. In $\chi$PT, it is explicitly assumed that the pion mass is small compared to all relevant hadronic scales in the problem. Thus, the only physical states energetically near the nucleon are states with nucleon plus one pion. On the other hand, in the large $N_c$ limit for a hedgehog model the $\Delta$ is degenerate with the nucleon. The $N-\Delta$ splitting goes as $1/N_c$. Therefore, in the large $N_c$ limit, loops with $\pi-\Delta$ intermediate states should be included along with $\pi-N$ intermediate states when doing $\chi$PT. The inclusion of $\pi-\Delta$ intermediate states in $\chi$PT, assuming degenerate $N$ and $\Delta$ masses and using $g_{\pi N\Delta}$ as calculated in the Skyrme model, precisely accounts for the factor of three [22]. Therefore, at least for this class of observables, the nonanalytic behavior associated with pion loops is in fact present.

An explicit example may help clarify this point. Once again, consider $d^2 M_N/d(m_\pi^2)^2$. The one pion loop contribution in $\chi$PT (including only nucleon intermediate states is given in (4). If we also include a diagram analogous to figure 2a with a $\Delta-\pi$ intermediate state and assume $M_\Delta = M_N$ with $g_{\pi N\Delta}$ given by the Skyrme model value we obtain

$$\frac{d^2 M_N}{d(m_\pi^2)^2} = -\frac{27}{128\pi} \frac{g_A^2}{m_\pi f_\pi^2} + O(1) \tag{6}$$

Here the coefficient is precisely three times the usual $\chi$PT result.

One can also calculate $d^2 M_N/d(m_\pi^2)^2$ directly from the Skyrme model. With the identity

$$\frac{d M_N}{d(m_\pi^2)} = \langle N | \frac{1}{4} f_\pi^2\ tr(U-1) | N \rangle \tag{7}$$
along with the asymptotic properties of \( U = \exp(i \vec{r} \cdot \vec{\phi}/f) \) where \( \vec{\phi} \) is the nonlinear realization of the pion field, and the standard Skyrme model expression for \( g_A \), one can compute \( d^2 M_N/d(m_\pi^2)^2 \) rather easily. The result is found to be the expression of (6). Thus, we see explicitly that a \( 1/N_c \) model such as the Skyrme model does have nonanalytic behavior in \( 1/m_\pi^2 \) precisely as one anticipates from a calculation of pion dressings of the nucleon with a \( \Delta \) degenerate in mass.

IV. SUMMARY

In summary, we have given heuristic—but compelling—arguments as to why simulations of nucleons in quenched QCD contain pion cloud physics which chiral perturbation theory ascribes to one pion loop contributions at the hadronic level. We have shown that diagrams surviving in the quenched approximation contain pieces which “look like” nucleon plus pion states. We have also argued that one can use the \( 1/N_c \) approximation as a surrogate for the quenched approximation since the \( 1/N_c \) approximation is quenched. Two distinct arguments, one based on large \( N_c \) hadrodynamics and one based on \( 1/N_c \) hedgehog models such the Skyrme model, both suggest that pion loop physics is present in large \( N_c \) QCD and hence in quenched QCD. These results contrast the superficial perception that the quenched approximation is incapable of including the physics of pionic dressings of baryons.

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