Baryon flow at SIS energies *

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Abstract

We calculate the baryon flow $\langle P_x/A(y) \rangle$ in the energy range from .25 to $\leq 2.5$A$GeV$ in a relativistic transport model for $Ni + Ni$ and $Au + Au$ collisions employing various models for the baryon self energies. We find that to describe the flow data of the FOPI Collaboration the strength of the vector potential has to be reduced at high relative momentum or at high density such that the Schrödinger-equivalent potential at normal nuclear density decreases above 1$GeV$ relative kinetic energy and approaches zero above 2$GeV$.

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1 Introduction

The nuclear equation of state (EOS) at high density ($\rho \geq 3\rho_0$) is still an unsettled issue though many experimental efforts have been made in the last couple of years to address this question in a more systematic way [1, 2, 3, 4]. Experimentally, the baryon sidewards flow and subthreshold particle production are the most promising observables. Recently [3, 5], the flow has been measured for heavy-ion collisions at SIS energies ($\leq 2AGeV$) for $Ni + Ni$ and $Au + Au$ systems, which provides further constraints on the hadronic models and the EOS employed.

From the theoretical point of view relativistic transport models [6, 7, 8] have been used to describe heavy-ion collisions at energies from the SIS at GSI to the AGS at BNL and the SPS regime at CERN ($\leq 200AGeV$). There are two main ingredients in the transport model: the mean fields (i.e. scalar and vector self energies) and in-medium baryon-baryon and meson-baryon cross sections. The scalar and vector mean fields are usually derived from some hadronic Lagrangian (which gives a well defined EOS) while the baryon-baryon and meson-baryon cross sections are taken from experimental data for the related processes in free space. In-medium modifications of the inelastic channels, furthermore, are constrained by the experimental data on the particle rapidity distributions which control the amount of ‘stopping’.

In principle, the baryon selfenergies should be determined by a Dirac-Brueckner approach including all relevant hadronic couplings. However, calculations for configuration dependent phase-space distributions are not very reliable yet and limited in density as well as in momentum [9, 10, 11, 12].

As mentioned before, the scalar and vector self energies for nucleons with their explicit momentum and density dependence are the key quantities that determine the nuclear EOS. In this work we will perform a systematic study of $Ni + Ni$ and $Au + Au$ collisions in order to extract further information on these quantities in comparison to the recent experimental data on the collective flow of baryons.

Our work is organized as follows: In Section 2 we will briefly describe the relativistic transport approach employed as well as the known constraints on the momentum dependence of the scalar and vector self energies. Section 3 is devoted to a systematic comparison of the calculated flow - employing various Lagrangian models - to the experimental data while Section 4 concludes with a summary and discussion of open problems.
2 The transport model

In this work we perform the theoretical analysis along the line of a relativistic transport approach which is based on a coupled set of covariant transport equations for the phase-space distributions \( f_h(x, p) \) of a hadron \( h \) \([7, 13]\), i.e.

\[
\left\{ \left( \Pi_\mu - \Pi_\nu \partial_\mu U^\nu_h - M_h^* \partial_\mu U^S_h \right) \partial_\mu + \left( \Pi_\nu \partial_\mu U^\nu_h + M_h^* \partial_\mu U^S_h \right) \partial_\mu \right\} f_h(x, p) \\
= \sum_{h_2h_3h_4} \int \, d^2d^3d^4 \left[ G^+G \right]_{12 \rightarrow 34} \delta^4_\Gamma(\Pi + \Pi_2 - \Pi_3 - \Pi_4) \\
\times \left\{ f_{h_3}(x, p_3)f_{h_4}(x, p_4) f_h(x, p) f_{h_2}(x, p_2) \\
- f_h(x, p)f_{h_2}(x, p_2)f_{h_3}(x, p_3) f_{h_4}(x, p_4) \right\}.
\] (1)

In Eq. (1) \( U^S_h(x, p) \) and \( U^\mu_h(x, p) \) denote the real part of the scalar and vector hadron self energies, respectively, while \( [G^+G]_{12 \rightarrow 34} \delta^4_\Gamma(\Pi + \Pi_2 - \Pi_3 - \Pi_4) \) is the ‘transition rate’ for the process \( 1+2 \rightarrow 3+4 \). Though in quantum many-body systems the transition rate is partly off-shell - as indicated by the index \( \Gamma \) of the \( \delta \)-function - we use the semi-classical on-shell limit \( \Gamma \rightarrow 0 \) since this approximation is found to describe reasonably well hadronic spectra in a wide dynamical regime. The hadron quasi-particle properties in (1) are defined via the mass-shell constraint \( \delta(\Pi_\mu \Pi^\mu - M_h^2) \) \([13]\) with effective masses and momenta given by

\[
M_h^*(x, p) = M_h + U^S_h(x, p) \\
\Pi^\mu(x, p) = p^\mu - U^\mu_h(x, p),
\] (2)

while the phase-space factors

\[
\bar{f}_h(x, p) = 1 - f_h(x, p)
\] (3)
account for fermion Pauli-blocking. The transport approach (1) is fully specified by \( U^S_h(x, p) \) and \( U^\mu_h(x, p) (\mu = 0, 1, 2, 3) \), which determine the mean-field propagation of the hadrons, and by the transition rates \( G^+G \delta^4_\Gamma(\ldots) \) in the collision term that describe the scattering and hadron production/absorption rates. For the latter we employ free cross sections as in Ref. \([14]\) that are parameterized in line with corresponding experimental data. In the relativistic transport approach we explicitly propagate nucleons and \( \Delta \)'s with their isospin degrees of freedom. For more details we refer the reader to Ref. \([15]\).
Before going over to a discussion of the scalar and vector self ener gies we start with the cascade limit \( U^S = U^\nu = 0 \) and compare the flow \( F \) defined by

\[
F = \frac{d}{dy} (P_x(y))_{\bar{y}=0},
\]

where \( \bar{y} \) is the nucleon rapidity in the center-of-mass system normalized by the projectile rapidity, i.e. \( \bar{y} = y_{cm}/y_{proj} \). Fig. 1 shows the calculated flow \( F \) (divided by \( A^{1/3}_1 + A^{1/3}_2 \)) as a function of beam energy for \( Ni + Ni \) and \( Au + Au \) systems. We first observe that in the cascade mode we obtain a scaling of the flow \( F \) with the system size expressed by \( A^{1/3}_1 + A^{1/3}_2 \) for both the BUU [16] and RBUU model, which will be solely used later on. It can be observed that the two models agree very well below 1 AGeV, whereas they differ in the energy range above. This difference comes mainly from the fact that the BUU includes all higher resonances up to a mass of 2 GeV whereas the RBUU treats only the \( \Delta(1232) \) explicitly. These high mass resonances, though only present in a small percentage of all particles in the high density phase, contribute significantly to the flow at high energies. In order to correct the RBUU results for this effect when including potentials we add the difference between BUU and RBUU in the cascade mode to the RBUU results. This correction of course neglects higher order effects caused by the smaller velocity of a heavier resonance in the surrounding nuclear matter; nevertheless, we expect these corrections to be small.

The flow generated without any mean fields is due to baryon-baryon col- lisions and underestimates the experimental data for \( Ni + Ni \) at all energies considerably as shown in Fig. 1. The difference between the cascade results and the data thus is caused by the baryon self energies which we try to simulate in the following.

In order to compare our flow values with other theoretical groups, we display the quantity

\[
\langle P_x/N \rangle^{dir} = \frac{1}{N} \int_{-y_{cm}}^{y_{cm}} dy \langle p_x/N \rangle(y) \frac{dN}{dy} \text{sgn}(y)
\]

versus beam energy in Fig. 2 for \( Au + Au \) systems. The UrQMD calculations of the Frankfurt group predict the flow for \( Au + Au \) collisions to increase with bombarding energy at least up to 4 AGeV [17] while hydrodynamical calculations including only hadronic matter predict a decrease of the baryon flow above 4 – 5 AGeV [18] for the heavy system \( Au + Au \). Furthermore,
hydrodynamical calculations including a transition to a quark-gluon-plasma (QGP) phase predict a vanishing baryon flow at $4-5 A GeV$ \cite{18}. In Fig. 2 we also show the results of RBUU (hard), BUU (hard) and UrQMD (hard) EOS calculations as well as our BUU cascade calculation in comparison with a UrQMD cascade calculation. The assignment 'hard' here corresponds to a nuclear incompressibility $K \approx 380$ MeV. We see, first, that all transport models agree quite well when using a 'normal' hard equation of state as well as in the pure cascade mode. All models predict an increasing baryon flow with bombarding energy. Second, the calculation with a hydrodynamical model shows an even stronger flow at the energies considered here. Third, the RBUU results with a special momentum-dependence of the mean fields (NL3*, see below) show a decreasing flow above $1 A GeV$. This specific model will be discussed in more detail later.

In Fig. 3 we plot the flow $F$ (divided by $A_1^{1/3} + A_2^{1/3}$) as a function of the beam energy for both hard and soft EOS without explicit momentum dependence of the potentials. The soft EOS is taken from the work of Furnstahl et al. \cite{19} with a nuclear incompressibility $K \sim 194$ MeV while the hard EOS is the NL3 parameter set from \cite{14} with a nuclear incompressibility $K = 380$ MeV. We notice from this figure that the baryon flow in the RBUU approach does not sensitively depend on the nuclear incompressibility. Therefore, in the following we focus on the momentum dependence of these potentials only.

In another RBUU approach \cite{8, 20} calculations have been done recently for heavy-ion collisions by employing density dependent scalar and vector potentials self-consistently as well as the momentum dependence of these potentials while taking care of chiral symmetry constraints \cite{19}. In these approaches the flow has been calculated also for $Ni+Ni$ and $Au+Au$ systems and been found to overestimate the experimental data at 1.5 and 2$AGeV$ for $Ni + Ni$ considerably. Thus also the latter transport calculations yield an increasing (or at least constant) flow with bombarding energy, whereas the experimental data indicate a decrease for $Ni + Ni$ above 1$AGeV$. We will argue that this decrease of the flow $F$ puts stringent constraints on the momentum- and density dependence of the mean fields.

The model inputs for the mean fields are related to the nuclear incompressibility $K$ at density $\rho_0$ as well as to the momentum dependence of the mean fields as first pointed out by Gale et al. \cite{21} and incorporated later on also in relativistic transport models by several authors \cite{3, 4, 8}. In the RBUU approach - due to covariance - the scalar and vector mean fields have to be explicitly momentum dependent \cite{13} in order to describe properly the
Schrödinger-equivalent optical potential \[22\] defined by

\[ U_{\text{sep}}(E_{\text{kin}}) = U_s + U_0 + \frac{1}{2M}(U_s^2 - U_0^2) + \frac{U_0}{M}E_{\text{kin}} \]  

as a function of the nucleon kinetic energy \( E_{\text{kin}} \) with respect to the nuclear matter rest frame. However, above \( E_{\text{kin}} = 1 \text{GeV} \) the Schrödinger-equivalent optical potential is not well known experimentally, such that the flow data from the FOPI Collaboration could provide further constraints also on this quantity.

In this work we use a similar Lagrangian density as proposed by Walecka \[9\] for the description of nuclear matter, which has been used in the relativistic BUU model before \[14\]. This Lagrangian contains nonlinear self-interactions of the scalar field \( U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}B\sigma^3 + \frac{1}{4}C\sigma^4 \) where the parameters \( m_\sigma, B, C \) are calculated by fitting the saturation density, binding energy, effective nucleon mass as well as the compression modulus at nuclear matter density (cf. NL3 parameters set from Table 1 in Ref. \[14\]). An additional coupling between the vector and scalar fields in the Lagrangian leads to a relatively soft EOS \[13, 23\]. In the present calculation we do not consider this effect, first, because our main object is to concentrate on the momentum dependence on the vector and scalar fields in comparison to the flow data, second, because the influence of the momentum dependence is much stronger than that of the density or incompressibility as pointed out before.

The energy density in mean-field theory (for nuclear matter) in these models can be written as \[14\]

\[ \varepsilon(m^*, n_b) = g_vV_0n_b - \frac{1}{2}m_v^2V_0^2 + \frac{m_\sigma^2}{2g_s^2}(m - m^*)^2 + \frac{B}{3g_s^3}(m - m^*)^3 \]

\[ + \frac{C}{4g_s^4}(m - m^*)^4 + \frac{\gamma}{(2\pi)^3} \int_0^{k_f} \frac{d^3p}{\sqrt{(p^2 + m^*)}} \]  

(7)

where \( m^* = m - g_s\sigma \) is the effective nucleon mass, \( n_b \) is the baryon density and the spin and isospin degeneracy is \( \gamma = 4 \). \( \sigma \) and \( V_0 \) are the scalar and vector fields with mass \( m_\sigma \) and \( m_v \), which couple to nucleons with coupling constants \( g_s \) and \( g_v \), respectively. The quantities \( B \) and \( C \) are constant parameters and \( p \) is the nucleon momentum which has to be integrated up to the fermi momentum \( k_f \). In this model the vector and scalar potentials are density dependent, however, the vector potential increases only linearly with density. Recently \[20\] this Lagrangian has been extended - maintaining
chiral symmetry constraints - to include a nonlinear dependence of the vector potential with density, too. We found, however, that this type of Lagrangian density with momentum dependent fields underestimates the flow data at all beam energies for Ni + Ni as well as Au + Au systems. The calculated flow values follow the cascade results as the scalar and vector mean fields cancel each other approximately for such type of EOS (cf. Fig. 1).

In our present calculation, we use the energy density Eq. (7) for calculating the scalar and vector potentials as a function of density. Momentum dependent potentials are obtained by fitting the Schrödinger equivalent potential according to Dirac phenomenology for intermediate energy proton-nucleus scattering [22]. We use the approach from Ref. [7] to take into account this momentum dependence by introducing additional scalar and vector cutoffs $\Lambda_s$, $\Lambda_v$. The scalar and vector form factors at the vertices are

$$\frac{\Lambda_s^2 - (p - \langle p \rangle)^2}{\Lambda_s^2 + (p - \langle p \rangle)^2} \quad \text{and} \quad \frac{\Lambda_v^2 - (p - \langle p \rangle)^2}{\Lambda_v^2 + (p - \langle p \rangle)^2},$$

respectively, where $(p - \langle p \rangle)$ accounts for the difference of the one-particle momentum to the average momentum of the surrounding nuclear matter. The values of $\Lambda_s$ and $\Lambda_v$ vary from 0.95 to 1.05 GeV and 0.9 to 1.0 GeV to get a good fit to the data. This momentum dependence is not computed self-consistently on the mean field level since it leads only to a small change in the original parameters of the model as well as in the fitting of the Schrödinger equivalent potential. So this approximation does not effect much our flow results, because for nuclear matter the energy scale involved is much smaller ($k_F \ll \Lambda_s, \Lambda_v$) than in the initial stage of high-energy heavy-ion collisions.

The Schrödinger equivalent potential (1) is shown in Fig. 4 as a function of the nucleon kinetic energy with respect to the nuclear matter at rest; also plotted are the data from Hama et al. [22]. The solid line in Fig. 4 is for the special momentum dependence including form factors $\frac{\Lambda_s^2 - (p - \langle p \rangle)^2}{\Lambda_s^2 + (p - \langle p \rangle)^2}$ and $\frac{\Lambda_v^2 - (p - \langle p \rangle)^2}{\Lambda_v^2 + (p - \langle p \rangle)^2}$. The dashed line is obtained using the scalar and vector form factors $\frac{\Lambda_s^2}{\Lambda_s^2 + p^2}$ and $\frac{\Lambda_v^2}{\Lambda_v^2 + p^2}$ with $\Lambda_s = 0.95$ GeV and $\Lambda_v = 0.9$ GeV, respectively.

3 Comparison to experimental data

We use the same parameter sets as for the Schrödinger equivalent potential in our flow calculations. The calculations are performed for the impact parameter $b = 4$ fm for Ni + Ni and $b = 6$ fm for Au + Au systems, since for
these impact parameters we get the maximum flow, which corresponds to
the multiplicity bins \( M_3 \) and \( M_4 \) as defined by the Plastic Ball collaboration [23]. We have calculated the flow, i.e. the slope parameter (4) by fitting a
straight line from \(-0.3 < \bar{y} < 0.3\) for \( Ni + Ni \) and \( Au + Au \) systems at all energies. Higher order terms (e.g. fitting a polynomial of 3rd grade) didn’t
change the results systematically.

In Fig. 5 and 6 the flow \( F \) (divided by \( A_1^{1/3} + A_2^{1/3} \)) as in Fig. 1) is
displayed in comparison with the data from Refs. [3, 4] for different sys-
tems. Fig. 5 shows the flow for \( Ni + Ni \) in the energy range up to 2.5\( AGeV \)
and Fig. 6 for \( Au + Au \), respectively. In both Figs. 5 and 6 the solid line
(NL3) is obtained without explicit momentum dependence of the self en-
ergies, whereas the dashed line corresponds to the momentum dependent scalar
and vector potentials and the dashed-dotted line (NL3*) corresponds to the
special momentum dependence shown in Fig. 4.

From Fig. 5 we observe that the dashed-dotted line (NL3*) is in good
agreement with the flow data for \( Ni + Ni \), whereas for \( Au + Au \) (Fig. 6)
the dashed-dotted line (NL3*) is slightly below the flow data from [3, 4] but
closer to the plastic ball data (open triangles).

The important point to be noted here is that the flow rises up to 1\( AGeV \)
and decreases above 1\( AGeV \) for \( Ni + Ni \), whereas it saturates above 1\( AGeV \)
for \( Au + Au \). The physics behind is that the repulsive force due to the vector
mean fields must decrease considerably at high beam energy such that the
Lorentz force on the particles vanishes in the initial phase of the collision. In
subsequent collisions, which are more important in the \( Au+Au \) case due to its
size, the kinetic energy of the particles moving relative to the local rest frame
is then in a range where the Schrödinger equivalent potential is determined
by [22]. We thus conclude that to explain the flow data up to 2\( AGeV \) one
has to reduce the vector mean field considerably. In other words, there is
only a weak repulsive force at high relative momenta and high densities.

Another interesting point is that the flow in our calculation for \( Ni + Ni \)
decreases earlier with beam energy than for the \( Au + Au \) system. This is due
to the fact that the flow is governed by the average transverse pressure generated
due to the number of nucleons. For these reasons the flow for \( Au + Au \) first
saturates and then decreases at a higher beam energy (\( \geq 2.5 AGeV \)). This
implies that the observed \( A^{1/3} \) scaling of the flow up to 1\( AGeV \) does no
longer hold for higher energies. Here it would be very interesting to have -
besides the \( Ni + Ni \) system recently measured at FOPI - systematic data
for higher mass systems and beam energies above 1\( AGeV \). For \( Au + Au \)
first preliminary results have been presented by the EOS Collaboration indicating a gradual decrease of the flow $F$ from 2-8AGeV.

Finally, we show in Fig. 7 the EOS as well as the scalar and vector potential energy associated with the special momentum dependence (NL3*). The upper part shows the energy per nucleon in comparison to the standard NL3 parametrization \footnote{14} (dashed line). For NL3* (solid line) the approximate equation of state has been derived by fitting the vector potential with an appropriate polynomial in the baryon density. The lower part shows the corresponding vector and scalar potential energy as a function of the baryon density. The vector part for NL3* is substantially lower at high baryon density as compared to the NL3 parameter set as well as to the original $\sigma-\omega$-model \footnote{9}. The vector potential at $3\rho_0$ is about 460, 653 and 1020 MeV for NL3*, NL3 and the $\sigma-\omega$-model, respectively.

\section{Summary}

In this work we have calculated the baryon flow in the energy range up to 2.5AGeV in a relativistic transport model for Ni+Ni and Au+Au. We found that in order to properly describe the flow data of the FOPI Collaboration at high beam energies, the strength of the vector potential has to be reduced considerably in the RBUU model at high relative momenta and/or densities. Otherwise, too much flow is generated in the early stages of the reaction and cannot be reduced at later phases where the Schrödinger equivalent potential is experimentally known and constrained.

This assumed decrease of the vector and scalar potentials destroys, however, the $A^{1/3}$ scaling of the flow - observed in heavy ion collisions below 1AGeV - at higher energies. Therefore it would be interesting to have systematic studies for different mass systems for higher bombarding energies. From these studies it might even be possible to extract the dependence of the potentials on density and momentum separately.

One shortcoming of the transport model used here is the restriction to binary nucleon-nucleon or meson-nucleon scattering. Especially in meson-nucleon reactions at 2AGeV bombarding energy the baryon excitations become very high ($\sqrt{s} \approx 2.5GeV$) such that multi-particle final states can occur which may lead to a decrease of directed flow. Despite of these uncertainties above $\approx 2AGeV$, the decrease of the Schrödinger equivalent potential above 1AGeV should have clearly been demonstrated.
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**Fig. 1** The flow $F(y)$ versus beam energy per nucleon for $Ni + Ni$ and impact parameter $b = 4$ fm and $Au + Au$ systems with impact parameter $b = 6$ fm in the cascade mode. The solid line corresponds to the RBUU results for $Au + Au$, the dashed line to the RBUU results for $Ni + Ni$, whereas the dotted and dashed-dotted lines correspond to the BUU results for $Au + Au$ and $Ni + Ni$ systems, respectively. The experimental data for $Ni + Ni$ are from Ref. [3].

**Fig. 2** $\langle P_x/N \rangle_{dir}(y)$ values versus beam energy per nucleon for $Au + Au$ at $b = 4$ fm. A hydrodynamical calculation (for $b = 3$ fm) is shown versus UrQMD (hard EOS), BUU (hard momentum dependent EOS) and RBUU with momentum dependence (NL3) according to [7]. Also shown are the UrQMD cascade calculations from Ref. [18] and the BUU cascade calculations [16], which are found to agree very well. The dashed line results from a RBUU calculation with special momentum dependence (NL3$^*$).

**Fig. 3** The flow $F(y)$ versus beam energy per nucleon for $Ni + Ni$ at $b = 4$ fm. The dashed line results from the soft EOS from Ref. [19] while the solid line is obtained for a hard EOS [14]. The data points are from the FOPI Collaboration [3].

**Fig. 4** The Schrödinger equivalent potential (6) as a function of the nucleon kinetic energy $E_{kin}$. The solid curve is for the special momentum dependence NL3$^*$ discussed in the text and the dashed curve for the momentum dependent parameter set NL3 [7] (see text). The data points are from Hama et al. [22].

**Fig. 5** The flow $F(y)$ versus the beam energy per nucleon for $Ni + Ni$ at $b = 4$ fm. The solid line results for the parameter set NL3, the dashed line includes an explicit momentum dependence of NL3 and the dashed-dotted line is for the special momentum dependence NL3$^*$, where the vector potential decreases at high energies (c.f. Fig. 4). The data points are from the FOPI Collaboration [3].

**Fig. 6** Same as Fig. 5 for $Au + Au$ at $b = 6$ fm. The data points are from the Plastic Ball, FOPI and EOS Collaborations, Ref. [3].

**Fig. 7** Upper part: The solid line shows the equation of state for the parameter set NL3$^*$ in comparison to NL3 (dashed line). Lower part: Vector (upper solid line) and scalar (lower solid line) potential energy per nucleon for NL3$^*$ in comparison to NL3 (dashed line).
Flow / \( \left( A_{1}^{1/3} + A_{2}^{1/3} \right) \) [MeV]

- FOPI Ni+Ni
- Furnstahl-EOS (soft)
- NL3 (hard)

\( E_{\text{kin}} \) [MeV]
Flow / (A_{1}^{1/3} + A_{2}^{1/3}) [MeV]

FOPI Ni+Ni

NL3

NL3 (mom-dep)

NL3*

E_{kin} [MeV]
