Parallel Algorithm for Calculating the Dynamics of the Electron Beam Current Distribution during Pulsed Heating of a Metal Target

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Abstract. Speedup and efficiency of two parallel algorithms for calculating the dynamics of the current distribution when the surface of a tungsten sample is heated by an electron beam pulse are presented. The algorithms are implemented using OpenMP technology. A special case of axial symmetry without taking into account electric driving forces is considered. The temperature in the sample, calculated on the basis of the two-phase Stefan problem, is necessary for solving electrodynamic equations. To implement them, the methods of Jacobi and the successive over-relaxation methods were used. Pipelining is used to parallelize the successive over-relaxation algorithm. The current is considered as a possible source of rotation of the substance, which is observed in the experiment.

1. Introduction
The problem of melting a tungsten plate heated by an electron beam pulse is considered. To determine the dynamics of the rotational speedup of the tungsten melt, it is necessary to develop a parallel algorithm. This problem can be solved using existing solvers. Integration of the solver into the developed software package presents an additional difficulty. Therefore, it was decided to parallelize the algorithm using OpenMP technology. The calculated area is a cross-section of the sample, an area measuring 12 mm × 3 mm. The calculation time is determined by the moment of the last measurement of the surface temperature. The parameters of the model are taken from the experiments on the Beam of Electrons for materials Test Applications (BETA) stand [1]. This stand was designed in the INP SB RAS.

2. The Statement of the Problem
To determine the temperature on the heated surface, the temperature distribution in the sample is calculated based on the solution of the Stefan problem [2]. The position and speed of the phase...
boundary depends on nonlinear coefficients. Stefan problem is used to calculate the temperature in the tungsten plate surface:

\[
\begin{align*}
\left\{ \begin{array}{l}
(c(T)\rho^*(T))\frac{\partial T}{\partial t} & = \frac{1}{r} \frac{\partial}{\partial r} r \lambda(T) \frac{\partial T}{\partial r} + \frac{\partial}{\partial z} \lambda(T) \frac{\partial T}{\partial z}, \\
\lambda(T) \frac{\partial T}{\partial r} |_{S} & = L_m v_n, \\
(n, \nabla T) |_{\gamma} & = \frac{W(t,r)-N(t,r)}{\lambda(T)}, \\
(n, \nabla T) & = 0, \\
T & = T_0 \quad \text{at} \quad t = 0,
\end{array} \right.
\]

(1)

where \(T(r,z,t)\) is the temperature, \(c(t)\) is the specific heat, \(\rho^*(T)\) is the density, \(\lambda(T)\) is the thermal conductivity, \(W(t,r)\) is the power of the heat flux on the surface \(\gamma\), \(N(t,r)\) is the power loss, \(n\) is the normal to the surface, and \(T_0\) is the initial temperature, \(S\) is the free boundary, \(L_m\) is the enthalpy of the phase transition, \(v_n\) is the speed of the phase transition boundary. The melting point is \(T_m = 3695^\circ\text{K}\). At each time step in the numerical simulation, the variable \(W_{max}(t)\) is taken from the experimental data file, individual for each experiment. The power density \(W(r)\) has a radius distribution close to normal.

The resulting temperature distribution (1) allows us to calculate the specific conductivity in the sample [3]. The process of propagation of current in the sample can be considered stationary, as the characteristic time of change is larger than the time of balancing the electrodynamics equations on the scale of the problem [4]. Taking into account the evaporation process ensures that the temperature increasing in the sample is limited, which corresponds to the experimental data. Maxwell’s equation system for calculating current in the sample is modified for a stationary case in a cylindrical coordinate system without taking into account electrical driving forces. Enter the vector potential \(F = (F_r, F_\varphi, F_z)\) for current \(j = (j_r, j_\varphi, j_z)\): \(\vec{j} = \nabla \times \vec{F}\). The equation for potential has the form [5]:

\[
\frac{\partial^2 F_\varphi}{\partial r^2} + \frac{\partial^2 F_\varphi}{\partial z^2} + \frac{\partial F_\varphi}{\partial r} \left( \frac{1}{\rho_e} + \frac{1}{r} \right) + \frac{\partial F_\varphi}{\partial z} \frac{1}{\rho_e} + F_\varphi \left( \frac{1}{\rho_e} + \frac{1}{r^2} \right) = 0.
\]

Introduce a variable \(G = r F_\varphi\) that determines the current within the ring of the specified radius of \(r\) and the height of \(z\). Denote \(\Phi = \ln(\rho_e/r)\) and write the problem as follows:

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial^2 G}{\partial r^2} + \frac{\partial^2 G}{\partial z^2} + \frac{\partial G}{\partial r} \frac{1}{\rho_e} + \frac{\partial G}{\partial z} \frac{1}{\rho_e} = 0, \\
\Phi = \ln(\rho_e/r), \quad G |_{(r,z) = (r_{max},z)} = 0, \\
G |_{(r,z) = (r,0)} = \frac{I(t)}{2\pi} \left( 1 - \exp \left( -\frac{r^2}{a^2} \right) \right).
\end{array} \right.
\]

(2)

Then the current density can be restored by the formula:

\[
\vec{j} = (j_r, 0, j_z) = \left( -\frac{\partial F_\varphi}{\partial z}, 0, \frac{1}{r} \frac{\partial(r F_\varphi)}{\partial r} \right) = \left( -\frac{1}{r} \frac{\partial G}{\partial z}, 0, \frac{1}{r} \frac{\partial G}{\partial r} \right).
\]

Speedup of melt is determined by the expression:

\[
g_\varphi = j_r B_z / \rho_t,
\]

where \(\rho_t = 1.6 \cdot 10^4 \text{ kg/m}^3\) is melt density, \(a\) is a beam radius, \(B_z = 0.3\) Tesla is a magnetic field, and \(I(t)\) is the current density in the center of the beam that are known from the experimental data. Note that the beam radius has one value for each series of experiments. The beam radius is used to calculate the distribution of power across the surface of the heat flow, which is a part of the Stefan problem boundary condition on the heated surface (1).

Boundary conditions are a defining characteristic for the problem (2), since the area does not contain current sources. This is true at the stage of modeling the current beam without taking
into account the thermal emission. The solution of the problem depends only on the boundary conditions for the current and temperature in the sample. It is important to correctly determine the current on the boundary, heated by impulse impact. A beam of electrons falls on the surface, not only charging it, but also raising the surface temperature, giving energy to electrons.

### 3. Parallel Algorithm

An analysis of the operation of a parallel version of the program, which is based on OpenMP technologies [6], is presented. The results were obtained at the NKS-1P cluster (Computing Centre of SB RAS). To analyze the parallel algorithm, speedup and efficiency are calculated using the formulas [7]:

\[ S_p = \frac{T_1}{T_p} \quad E_p = \frac{T_1}{pT_p} = \frac{S_p}{p} \]

Here \( T_n \) is the counting time for \( n \) cores, \( p \) is the number of cores. The speedup is equal to the counting time on \( p \) cores compared to counting on one core.

#### 3.1. Current Distribution obtained by the Jacobi method

We write the derivatives in equation (2) in the form of central differences. Then we will get a finite-difference method on a 5-point stencil "cross". The matrix of this system of linear equations has a strictly diagonally dominant. The simplest method for solving equation (2) is the Jacobi method [8, 9, 10]. The counting time was reduced with an increase in the number of processors used (fig. 1 a).

![Graphs](image)

Figure 1. Graphs of the dependence of time speedup on the number of cores for the current distribution obtained by the Jacobi method (a) and by the successive over-relaxation method (b).

There is a shared memory for all the cores of the NKS-1P cluster. Each core has its own individual cache memory of 40 MB. Access to the cache is the fastest and data is swapped through it. The calculation uses several large (2400 × 600) two-dimensional arrays at each time step. For the number of cores 2, 4, 8, all parts of the arrays distributed among the cores are not placed in the core cache. Since the access to the array elements is not the same, there is no double speedup when the number of cores is doubled (Fig. 2a). When increasing the cores from 8 to 16 or from 16 to 32, the speedup also doubles, since a significant part of the array bands is placed in the cache. The efficiency (Fig. 2b) for the number of cores more than 8 is almost the same due to the fact that access to all elements of arrays is the same for all arrays.
Figure 2. Graphs of the dependence of speedup (a) and efficiency (b) on the number of cores for the current distribution obtained by the Jacobi method.

Since the typical calculation time on 32 cores took more than 8 hours, the search for the optimal algorithm was continued. To improve the efficiency of the entire algorithm, it is necessary to use the successive over-relaxation method for the system (2).

3.2. Current Distribution obtained by the successive over-relaxation method
Solution of the equation (2) by the successive over-relaxation (SOR) method at each time step allows constructing an economical algorithm with the relaxation parameter $\omega = 2 - O(h)$ [11]. The choice of an instruction pipelining for parallelizing the SOR algorithm has no alternative. The pipelining gives good speedup and efficiency with good filling of the pipelining, for example, in model problems. Application to the practical problem turns out that a barrier arises in the algorithm. This barrier gives waiting for all threads at each step, which slows down the count. The barrier is due to the computation of the global residual at each step. Calculation of the global residual requires global synchronization and prevents the pipelining from accelerating. The counting time is not uniform across the streams. The residual value is different in different threads. The implementation required the correct choice of the OpenMP reduction directive. The counting time decreases nonlinearly with an increase in the number of processors. If on one core the calculation takes almost 3 thousand seconds, then on 16 cores 1300 seconds (Fig. 1a). The decrease is not significant, therefore the speedup does not grow much. For example, on 8 cores, the counting is accelerated 2 times compared to one core (Fig. 3a). The efficiency is not high (Fig. 3a).

4. Conclusion
The speedup and the efficiency of two parallel algorithms for calculating the dynamics of the current distribution when the surface of a tungsten sample is heated by an electron beam pulse are considered. An axially symmetric formulation is considered. It is known from experiments that the melt rotates. The problem is based on the solution of the Stefan problem and the equation for potential current. To implement them, the methods of Jacobi and the successive over-relaxation methods were used. Pipelining is selected for parallelization of the SOR algorithm. The parallel algorithm for the SOR method showed the best counting time, good speedup and efficiency for the problem of determining the acceleration of the melt rotation.
Figure 3. Graphs of the dependence of speedup (a) and efficiency (b) on the number of cores for the current distribution obtained by SOR method.

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References
[1] Vyacheslavov L., Arakcheev A., Burdakov A., Kandaurov I., Kasatov A., Kurkuchekov V., Mekler K., Popov V., Shoshin A., Skovorodin D., Trunev Y., Vasilyev A. Novel electron beam based test facility for observation of dynamics of tungsten erosion under intense ELM-like heat loads // AIP Conference Proceedings. 2016. 1771, 060004.
[2] Arakcheev A. S., Apushkinskaya D. E., Kandaurov I. V., Kasatov A. A., Kurkuchekov V. V., Lazareva G. G., Maksimova A. G., Popov V. A., Snytnikov A. V., Trunev Yu. A., Vasilyev A. A., Vyacheslavov L. N. Two-dimensional numerical simulation of tungsten melting under pulsed electron beam // Fusion Engineering and Design. 2018. V. 132. P. 13-17.
[3] Tolias P. Analytical expressions for thermophysical properties of solid and liquid tungsten relevant for fusion applications // Nuclear Materials and Energy. 2017. V. 13. P. 42-57.
[4] Jackson J.D. Classic electrodynamics. 3rd edn. John Wiley & Sons, Inc., New York. 1998.
[5] Lazareva G. G., Popov V. A., Arakcheev A. S., Burdakov A. V., Schwab I. V., Vaskevich V. L., Maksimova A. G., Ivashin N. E., Oksogoeva I. P. Mathematical modeling of the electron beam current distribution during pulsed heating of a metal target, Journal of Applied and Industrial Mathematics. 2021. V. 15, No. 2.
[6] OpenMP Homepage, http://www.openmp.org, last accessed 2021/06/12.
[7] Korneev V. D. Parallel programming of clusters. NSTU Publishing House, Novosibirsk. 2008. (in russian)
[8] Gene H. Golub, James M. Ortega. Scientific Computing and Differential Equations: An Introduction to Numerical Methods. Marshfield, Mass.: Pitman Pub. 1981.
[9] James W. Demmel. Applied numerical linear algebra. SIAM. 1997.
[10] Nemnyugin S. A., Stesik O. L. Parallel programming for multiprocessor computing systems. St. Petersburg: BHV-Petersburg. 2002. (in russian)
[11] Strongin R. G., Gergel V. P., Grishagin V. A., Barkalov K. A. Parallel computing in global optimization problems-Moscow: Moscow State University Publishing House. 2013. 280 P. (in russian)