Models for MAGDM with dual hesitant $q$-rung orthopair fuzzy 2-tuple linguistic MSM operators and their application to COVID-19 pandemic

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Abstract

In this article, we introduce dual hesitant $q$-rung orthopair fuzzy 2-tuple linguistic set (DH$q$-ROFTLS), a new strategy for dealing with uncertainty that incorporates a 2-tuple linguistic term into dual hesitant $q$-rung orthopair fuzzy set (DH$q$-ROFS). DH$q$-ROFTLS is a better way to deal with uncertain and imprecise information in the decision-making environment. We elaborate the operational rules, based on which, the DH$q$-ROFTL weighted averaging (DH$q$-ROFTLWA) operator and the DH$q$-ROFTL weighted geometric (DH$q$-ROFTLWG) operator are presented to fuse the DH$q$-ROFTL numbers (DH$q$-ROFTLNs). As Maclaurin symmetric mean (MSM) aggregation operator is a useful tool to model the interrelationship between multi-input arguments, we generalize the traditional MSM to aggregate DH$q$-ROFTL information. Firstly, the DH$q$-ROFTL Maclaurin symmetric mean (DH$q$-ROFTLMSM) and the DH$q$-ROFTL weighted Maclaurin symmetric mean (DH$q$-ROFTLWMSM) operators are proposed along with some of their desirable properties and some special cases. Further, the DH$q$-ROFTL dual Maclaurin symmetric mean (DH$q$-ROFTLDMSM) and weighted dual Maclaurin symmetric mean (DH$q$-ROFTLDWMSM) operators with some properties and cases are presented. Moreover, the assessment and prioritizing of the most important aspects in multiple attribute group decision-making (MAGDM) problems is analysed by an extended novel approach based on the proposed aggregation operators under DH$q$-ROFTL framework. At long last, a numerical model is provided for the selection of adequate medication to control COVID-19 outbreaks to demonstrate the use of the generated technique and exhibit its adequacy. Finally, to analyse the advantages of the proposed method, a comparison analysis is conducted and the superiorities are illustrated.

KEYWORDS

COVID-19 outbreaks, dual hesitant $q$-rung orthopair fuzzy 2-tuple linguistic set, dual Maclaurin symmetric mean operator, Maclaurin symmetric mean operator, MAGDM

1 | INTRODUCTION

Coronavirus (COVID-19) is an extremely infectious virus. On 31 December 2019, the initial case was discovered in Wuhan, China's capital of Hubei province. The term ‘Coronavirus’ is derived from the Latin word ‘Corona’, which refers to a crown of light. This virus exhibits symptoms similar to pneumonia and influenza. The World Health Organization (WHO) has classified the COVID-19 as a pandemic. It is extremely hard to
detect the appearance of this fatal virus because the symptoms resemble those of influenza, coughing and fever. From the time of entering the living organism, this virus begins to show symptoms between 7 and 14 days. In the absence of vaccination, social distance is the most extensively used approach for minimizing and controlling its effects. COVID-19 can infect a healthy individual's body if he/she comes into close touch with an infected person or his/her things. Infected individuals who are unaware of their disease may leave their homes infected. Recently, scholars and professionals have focused their concern on the investigation of COVID-19 transmission (Ahmadi et al., 2020; Boldog et al., 2020). Medication is also critical for an effective pandemic strategy. Therefore, selecting the best medication to treat the COVID-19 outbreak is a decision-making problem. A MADM method explains how attribute data will be analysed in order to make a decision. MAGDM is a decision-making method in which a group of decision-makers (DMs) chooses the most suitable alternative from a collection of alternatives based on multiple attributes. We face many problems in the decision-making environment, such as defining the complications and vagueness of attributes. To overcome these limitations, Zadeh (Zadeh, 1965) developed the definition of a fuzzy set (FS). As FS just has the membership degree (MD), describing certain complex data may be difficult. Intuitionistic fuzzy set (IFS) devised through Atanassov (Atanassov, 1986), as an extension of the FS, facilitated DMs with pair form of MD and non-membership degree (NMD) to demonstrate their fuzziness. IFS assumes that the sum of each ordered pair of the MD and NMD is less than or equal to one \((0 \leq \mu + \nu \leq 1)\). Yager (Yager, 2014) introduced the Pythagorean fuzzy set (PFS) to give more space to DMs for dealing with uncertainties. The square sum of MD and NMD is less than or equal to one \((0 \leq \mu^2 + \nu^2 \leq 1)\) in the PFS, which is the generalization of the IFS. Yager (Yager, 2016) proposed the \(q\)-ROFS, which provides the sum of the \(q\)th power of the MD and the NMD is less than or equal to one \((0 \leq \mu^q + \nu^q \leq 1)\). The \(q\)-ROFS are more efficient and useful than IFSs and PFSs because they have a wide membership and non-membership representation space. Although the methodologies discussed above do not take into account human hesitance. To cope with human hesitance decision-making concerns adequately, hesitant fuzzy sets (HFSs) (Torra, 2010) and dual hesitant fuzzy sets (DHFSs) (Zhu et al., 2012) were formulated. The MD features in the HFSs are extremely important as they simplify things to interpret the view of a panel of experts, particularly whenever the experts are quite distinct. But, the DHFS enables a more robust and varied approach for assigning quantities to every domain component, and that can manage two types of hesitation. To address the MADM issues, Naz and Akram (Naz & Akram, 2019) designed a model for decision-making based on the graph theory within HF factors to express judgement feedback. By incorporating the generalized DHFS to PFS, the notion of dual hesitant PFS is introduced by Wei and Lu (Wei & Lu, 2017). Xu et al. (Xu et al., 2018) proposed some Heronian mean operators for the MADM environment and formulated the notion of the dual hesitant \(q\)-ROF set (DHq-ROFS) by incorporating the advantages of the two FSs, such as \(q\)-ROFS and DHFS. As a consequence, in real MADM implementations, DHq-ROF numbers could indeed describe assessment details with greater ease than other types of fuzzy numbers. Sarkar and Biswas (Sarkar & Biswas, 2021) combined the Bonferroni mean (BM) operator and Dombi \(t\)-norms and \(t\)-conorms in DHq-ROF environment and proposed the Dombi BM, weighted Dombi BM, Dombi geometric BM and Dombi weighted geometric BM aggregation operators with DHq-ROFNs.

Processing manner of 2-tuple linguistic (2TL) knowledge can efficiently avoid the distortion and loss of information. Herrera and Martinez (Herrera & Martinez, 2000) introduced the 2TL interpretation framework which is among the most important ways for dealing with decision-making problems related to human language. Later on, several decision-making approaches and 2TL aggregation operators are developed. Deng et al. (Deng et al., 2019) proposed the generalized 2TL Pythagorean fuzzy Heronian mean aggregation operators by expanding the generalized Heronian mean aggregation operators and its weighted form with 2TL Pythagorean fuzzy numbers. Wei and Gao (Wei & Gao, 2020) utilized power geometric and power average operations with Pythagorean fuzzy 2TL information to propose some Pythagorean fuzzy 2TL power aggregation operators to solve the MADM problems. Wei et al. (Wei et al., 2018) described the MD and the NMD of an element to a 2TL variable with some of its operational laws by introducing the idea of \(q\)-rung orthopair fuzzy 2TL (\(q\)-ROFTL) sets. Further, they proposed some \(q\)-ROFTL Heronian mean aggregation operators. Ju et al. (Ju et al., 2020) developed an approach to resolve MAGDM issues with \(q\)-ROFTL information based on the \(q\)-ROFTL weighted averaging and the \(q\)-ROFTL weighted geometric operators. The \(q\)-ROFTL Muirhead mean and the dual Muirhead mean operators are also presented by them. Many researchers have developed several aggregation operators and decision-making methods in generalized fuzzy environment (Akram, Adeel, Al-Kenani, & Alcantud, 2021; Akram et al., 2019; Akram, Ali, Butt, & Alcantud, 2021; Akram, Kahraman, & Zahid, 2021; Akram, Luqman, & Alcantud, 2021; Akram, Luqman, & Kahraman, 2021; Akram, Naz, Edalatpanah, & Mehreen, 2021; Akram, Naz, & Ziaa, 2021; Ali & Mahmood, 2020; Garg et al., 2021; Liu et al., 2021; Ma et al., 2021; Mahmood et al., 2019; Mahmood et al., 2021; Naz et al., 2021).

The critical factor in the decision-making framework is performed by the information aggregation operator, which is especially crucial in MAGDM. Information is combined using aggregation operators, which are mathematical functions. That is, they are used to combine \(N\) data (e.g. \(N\) numerical values) in a single data. In practical MAGDM problems, the preferences of DM's change dynamically, and there always exist various interactions among different considered multi-attributes. The most popular aggregation operators are the arithmetic mean and the weighted mean. The major difference between the arithmetic and weighted means is that the latter allows us to weight the different data according to their significance. A traditional aggregation operator, first introduced by Maclaurin (Maclaurin, 1729), the MSM operator is utilized to integrate quantitative valuation in global communication fusion methodology. It is directly aimed at capturing the interconnections between the given logic. Qin and Liu (Qin & Liu, 2015) devised the dual Maclaurin symmetric mean (DMSM) operator utilizing the MSM operator. Wei et al. (Wei et al., 2019) incorporated MSM and DMSM operators to \(q\)-ROFSs and introduced the \(q\)-ROFMSM, \(q\)-ROFDMSM, \(q\)-ROF weighted MSM and \(q\)-ROF weighted DMSM operators.
1.1 | Motivation and objectives

The motivational factors for writing this article are as follows: (1) Traditional DH$q$-ROFS approaches fail to perceive vagueness utilizing the 2TL terms, which has a greater potential to modify linguistic forms and may also prevent data error reduction during decision-making concerns. To reduce this issue, in this article, we develop the new concept of DH$q$-ROFTLSs utilizing DH$q$-ROFSs and 2TL terms with corresponding basic notions, which would expand DH$q$-ROFS frameworks for theoretical considerations and enables the decision experts to convey outcome measures; (2) When collecting the experts’ preferences, data processing is critical. Furthermore, the association of the indicated attributes must be considered in a variety of realistic problems. Because of the MSM operator’s amazing capability, various DH$q$-ROFTLMSM operators have been developed to deal with imprecise data; (3) Assessment but also the selection of best medication has regarded as a crucial and active field within decision-making environment. Due to the extreme uncertainty as well as the variability, many assessment approaches must be investigated in order to properly examine the selection of best medication to treat COVID-19.

1. The innovation of this research article according to the above motivation is given in the following points:
2. We develop a powerful approach to cope with uncertain phenomena, called DH$q$-ROFTLSs, by incorporating the theory of DH$q$-ROFSs into 2-tuple linguistic terms and provide its operational laws to compute the DH$q$-ROFTLN.
3. We develop a family of DH$q$-ROFTL Maclaurin symmetric mean operators, known as DH$q$-ROFTLMSM operator, the DH$q$-ROFTLDMSM operator, and their weighted variants to fuse the DH$q$-ROFTLN. Some special cases and basic properties of the proposed aggregation operators are also provided.
4. The newly developed DH$q$-ROFTLWMSM and DH$q$-ROFTLWDMSM operators are used to propose a novel MAGDM model that can tackle the problems where the attributes have interrelationships.
5. An illustrative example concerning the selection of the adequate medication to control COVID-19 outbreaks is presented to demonstrate the usefulness and effectiveness of our proposed MAGDM model. Further, the influence of parameters on concerned MAGDM problem is interpreted.

1.2 | Organization of the proposed study

This article’s structure is organized in the following manner in order to accomplish the objectives. Section 2 introduces some initial concepts related to 2TL terms, DH$q$-ROFSs, and MSM aggregation operator. In Section 3, a new information representation form, that is DH$q$-ROFTLS is defined, along with its basic theories, such as some basic operational rules, score value and accuracy function of DH$q$-ROFTLN. Further, the DH$q$-ROFTL weighted averaging and weighted geometric operators are developed. In Section 4, we propose the DH$q$-ROFTLMSM and DH$q$-ROFTLWMSM operators utilizing MSM and further, the DH$q$-ROFTLDMSM and DH$q$-ROFTLWDMSM operators utilizing DMSM with some special cases. Section 5 presents the MAGDM model utilizing DH$q$-ROFTLWMSM and DH$q$-ROFTLWDMSM operators. In Section 6, a numerical illustration for the selection of the adequate medication to control COVID-19 outbreaks with DH$q$-ROFTLN, parameter analysis, comparison analysis, and advantages are conducted to exemplify the methodology outlined in this article. Finally, in Section 7, we provide a summary of this article.

The graphical representation of the proposed work in this manuscript is shown in Figure 1.

2 | PRELIMINARIES

In this section, the basic notions of 2TL representation model, DH$q$-ROFS and MSM operator along with its weighted form is provided to facilitate the next sections.

2.1 | 2-Tuple linguistic representation model

Tai and Chen (Tai & Chen, 2009) proposed the concept of 2TL as follows:

**Definition 1.** (Tai & Chen, 2009) Let $S = \{s_0, s_1, s_2, \ldots, s_n\}$ be a LTS, $\xi \in [0, 1]$ be a number value representing the aggregation result of linguistic symbolic. Then the function $\Delta$ used to obtain the 2TL information equivalent to $\xi$ is defined as follows:
\[ \Delta : [0,1] \rightarrow S \times \left[ \begin{array}{c} 1 \\ 1 \\ 2 \tau \\ 2 \tau \end{array} \right], \]

where \( \Delta(\xi) = (s_i, \gamma) \) with

\[ s_i = \text{round}(\xi / \tau), \quad \gamma = \xi / \tau, \gamma \in \left[ \frac{1}{2\tau}, \frac{1}{2\tau} \right], \]

where \( \text{round}(\cdot) \) is the usual round operation, \( s_i \) has the closest index label to \( \xi \), and \( \gamma \) is the value of the symbolic translation.

**Definition 2.** (Tai & Chen, 2009) Let \( S = \{s_0, s_1, s_2, \ldots, s_\tau\} \) be a LTS, \( (s_i, \gamma) \) is 2TL information, then there exists an inverse function \( \Delta^{-1} \), which can transform 2TL information into its equivalent numerical value \( \xi \in [0,1] \). The inverse function \( \Delta^{-1} \) is defined as follows:

\[ \Delta^{-1} : S \times \left[ \frac{1}{2\tau}, \frac{1}{2\tau} \right] \rightarrow [0,1], \]

\[ \Delta^{-1}[s_i, \gamma] = s_i / \tau + \gamma = \xi. \]

### 2.2 Dual hesitant \( q \)-rung orthopair fuzzy set

**Definition 3.** (Xu et al., 2018) Let \( R \) be an ordinary fixed set. A DH\( q \)-ROFS \( \mathfrak{J} \) defined on \( R \) is given by

\[ \mathfrak{J} = \{ (\mathcal{E}, h: \xi, g: \xi) | \mathcal{E} \in P(R) \}, \]

in which \( h: \xi \) and \( g: \xi \) are two sets of values in \([0,1]\) indicating the possible MD and NMD of the element \( \mathcal{E} \in R \) to the set \( \mathfrak{J} \), respectively, with the restrictions

\[ \zeta^q + \delta^q \leq 1, (q \geq 1), \]

where \( \zeta \in h: \xi, \delta \in g: \xi \) for all \( \mathcal{E} \in R \). For convenience, the pair \( \mathfrak{H}(\mathcal{E}) = (h: \xi, g: \xi) \) is called a DH\( q \)-ROFN denoted by \( \mathfrak{H} = (h, g) \) with the conditions \( \zeta, \delta \in [0,1] \). Evidently, DH\( q \)-ROFS is reduced to Zhu et al.'s (Zhu et al., 2012) DHFS and Wei and Lu's (Wei & Lu, 2017) DHPFS, when \( q = 1 \) and \( q = 2 \), respectively.
Definition 4. (Wang et al., 2019) Let \( N = (h, g) \) be a DHq-ROFN. The score value \( S^{DH} \) and the accuracy value \( H^{DH} \) of a DHq-ROFN \( N \) are defined as:

\[
S^{DH}(N) = \frac{1}{2} \left( 1 + \left( \frac{1}{|h|} \sum_{\xi \in h} \xi^q - \left( \frac{1}{|g|} \sum_{\delta \in g} \delta^q \right) \right) \right)
\]

\[
H^{DH}(N) = \frac{1}{2} \left( 1 + \left( \frac{1}{|h|} \sum_{\xi \in h} \xi^q + \frac{1}{|g|} \sum_{\delta \in g} \delta^q \right) \right)
\]

where \(|h|\) and \(|g|\) represent the number of elements in sets \( h \) and \( g \), respectively.

Let \( N_i = (h_i, g_i) (i = 1, 2) \) be any two DHq-ROFNs, then.
1. if \( S^{DH}(N_1) > S^{DH}(N_2) \), then \( N_1 > N_2 \);
2. if \( S^{DH}(N_1) = S^{DH}(N_2) \), then: (a) if \( H^{DH}(N_1) > H^{DH}(N_2) \), then \( N_1 > N_2 \); (b) if \( H^{DH}(N_1) > H^{DH}(N_2) \), then \( N_1 < N_2 \).

Definition 5. (Xu et al., 2018) Let \( N = (h, g), N_1 = (h_1, g_1), \) and \( N_2 = (h_2, g_2) \) be any three DHq-ROFNs and \( \lambda \) be a positive real number, then

\[
N_1 \oplus N_2 = \bigcup \{ C_{1,2} \in N_1 \cup N_2 : \{ (\xi_1 + \xi_2, \delta_1 + \delta_2) \} \},
\]

\[
N_1 \otimes N_2 = \bigcup \{ C_{1,2} \in N_1 \cup N_2 : \{ (\xi_1 \xi_2, \delta_1 \delta_2) \} \},
\]

\[
\lambda N = \bigcup \{ C_{1,2} \in N : \{ (1 - (1 - \lambda^q) \xi^q, \delta^q) \} \}, \lambda > 0;
\]

\[
N^\lambda = \bigcup \{ C_{1,2} \in N : \{ (\lambda^q, (1 - (1 - \lambda^q) \xi^q) \} \}, \lambda > 0.
\]

Definition 6. (Liu et al., 2019) Let \( L(\delta_j(\xi_i)) \) be the least common multiple (LCM) of \( l(h_{\delta_1}(\xi_i)), l(h_{\delta_2}(\xi_i)), l(h_{\delta_3}(\xi_i)), \ldots, l(h_{\delta_n}(\xi_i)) \), then the DHq-ROFSs \( \delta = \{ \delta_1, \delta_2, \ldots, \delta_m \} \) can be extended to multiple DHq-ROFSs \( \delta^* = \{ \delta_1^*, \delta_2^*, \ldots, \delta^n^* \} \) as follows:

\[
\delta_1^*(\xi) = \left\{ \xi_{\delta_1^*(\xi)}(\delta_1^*(\xi)) \right\}_q;
\]

\[
\delta_2^*(\xi) = \left\{ \xi_{\delta_2^*(\xi)}(\delta_2^*(\xi)) \right\}_q;
\]

\[
\ldots;
\]

\[
\delta_m^*(\xi) = \left\{ \xi_{\delta_m^*(\xi)}(\delta_m^*(\xi)) \right\}_q;
\]

where the numbers of \( \xi_{\delta_j^*(\xi)}(\xi) \) and \( \delta_{\delta_j^*(\xi)}(\xi) \) are determined by \( N(\xi_{\delta_j^*(\xi)}(\xi)) = \frac{L(\delta_j(\xi))}{l(h_{\delta_j}(\xi))} \) and \( N(\delta_{\delta_j^*(\xi)}(\xi)) = \frac{L(\delta_j(\xi))}{l(h_{\delta_j}(\xi))} \) respectively.

2.3 MSM operator and its weighted form

Let \( t_j (j = 1, 2, \ldots, n) \) be any collection of non-negative real numbers, then the following operators are defined as follows:
1. Maclaurin symmetric mean (Maclaurin, 1729): 
   \[ \text{MSM}^{(t)}(t_1,t_2,...,t_n) = \left( \frac{\sum_{i=1}^{n} t_i \ln t_i}{C_2} \right)^{\frac{1}{t}}; \]

2. Weighted Maclaurin symmetric mean (Maclaurin, 1729): 
   \[ \text{WMSM}^{(t)}(t_1,t_2,...,t_n) = \left( \frac{\sum_{i=1}^{n} t_i \ln t_i^{\gamma}}{C_2^{\gamma}} \right)^{\frac{1}{t}}; \]

3. Dual Maclaurin symmetric mean (Qin & Liu, 2015): 
   \[ \text{DMSM}^{(t)}(t_1,t_2,...,t_n) = \left( \frac{\sum_{i=1}^{n} t_i \ln t_i^{\gamma}}{C_2^{\gamma}} \right)^{\frac{1}{t}}; \]

4. Weighted dual Maclaurin symmetric mean (Qin & Liu, 2015): 
   \[ \text{WDMSM}^{(t)}(t_1,t_2,...,t_n) = \left( \frac{\sum_{i=1}^{n} t_i \ln t_i^{\gamma}}{C_2^{\gamma}} \right)^{\frac{1}{t}}; \]

where \( z \) is a parameter and \( z = 1,2,...,n \), \( t_1,t_2,...,t_n \) are \( z \) integer values taken from the set \( \{1,2,...,n\} \) of \( j \) integer values, \( C_2^n \) denotes the binomial coefficient, and \( C_2^n = n! / z!(n-z)! \).

### 3 Dual Hesitant q-Rung Orthopair Fuzzy 2-Tuple Linguistic Set

In this section, we introduce the new generalization of q-ROFTLSs in hesitant scenario, called DHq-ROFTLSs. Further, we develop the DHq-ROFTLWA and DHq-ROFTLWG operators and discuss their desirable properties.

#### 3.1 Definition of DHq-ROFTLS

Inspired by the ideas of DHq-ROFSs and TLSs, we develop the new concept of DHq-ROFTLSs. The mathematical representation of DHq-ROFTLSs is described as follows:

**Definition 7.** Let \( R \) be a non-empty set of the universe, \( S \) be a LTS with odd cardinality, then the DHq-ROFTLSs is defined as:

\[
D = \{ (s_{\varphi(\ell)}, (h_0(\ell),g_0(\ell))) | \ell \in R \},
\]

where \( s_{\varphi(\ell)} \in S, \varphi \in [-1,1] \), \( h_0(\ell) \) and \( g_0(\ell) \) are two sets of values in \([0,1]\) denoting the possible MD and NMD of \( \ell \) to the 2-tuple linguistics variable \( (s_{\varphi(\ell)},\varphi) \), respectively, which satisfy the following conditions:

\[
0 \leq (\zeta_0(\ell))^q + (\delta_0(\ell))^q \leq 1, \forall \ell \in R \text{ and } q \geq 1,
\]

where \( \zeta \in h_0(\ell), \delta \in g_0(\ell) \) for all \( \ell \in R \). For convenience, the pair \( \Xi(\ell) = (s_{\varphi(\ell)},(h_0(\ell),g_0(\ell))) \) is called a DHq-ROFTLN denoted by \( \Xi = (s_{\varphi(\ell)},(h_0,\delta_0)) \).

**Definition 8.** Let \( \Xi = (s_{\varphi(\ell)},(h_0,\delta_0)) \) be a DHq-ROFTLN, then the score value \( S^{\text{DHTL}}(\Xi) \) and the accuracy function \( H^{\text{DHTL}}(\Xi) \) are defined as follows:

\[
S^{\text{DHTL}}(\Xi) = \Delta^{-1}(s_{\varphi(\ell)}) \times \left( 1 + \frac{1}{P_{\zeta} \sum_{\zeta \in h} C_{q}^{\zeta} - \left( \frac{1}{P_{\zeta} \sum_{\zeta \in h} C_{q}^{\zeta}} \right) \right), \quad (1)
\]

\[
H^{\text{DHTL}}(\Xi) = \Delta^{-1}(s_{\varphi(\ell)}) \times \left( \frac{1}{P_{\zeta} \sum_{\zeta \in h} C_{q}^{\zeta} + \left( \frac{1}{P_{\zeta} \sum_{\zeta \in h} C_{q}^{\zeta}} \right) \right). \quad (2)
\]

**Definition 9.** Let \( \Xi_1 = ((s_1,\varphi_1),(h_1,\delta_1)) \) and \( \Xi_2 = ((s_2,\varphi_2),(h_2,\delta_2)) \) be two DHq-ROFTLNs, then \( \Xi_1 \) and \( \Xi_2 \) can be compared according to the following rules:

1. if \( S^{\text{DHTL}}(\Xi_1) > S^{\text{DHTL}}(\Xi_2) \), then \( \Xi_1 > \Xi_2 \).
2. If \( a_{\text{DH}}(\Xi_1) = a_{\text{DH}}(\Xi_2) \), then

- if \( a_{\text{DH}}(\Xi_1) > a_{\text{DH}}(\Xi_2) \), then \( \Xi_1 > \Xi_2 \);
- if \( a_{\text{DH}}(\Xi_1) = a_{\text{DH}}(\Xi_2) \), then \( \Xi_1 = \Xi_2 \).

**Definition 10.** Let \( \Xi = (s, \gamma, (h, g)) \), \( \Xi_1 = ((s_1, \gamma_1), (h_1, g_1)) \) and \( \Xi_2 = ((s_2, \gamma_2), (h_2, g_2)) \) be three DHq-ROFTLN sets, then

1. \( \Xi_1 \oplus \Xi_2 = \left\{ \left( \Delta \left( \Delta^{-1}(s_1, \gamma_1) + \Delta^{-1}(s_2, \gamma_2) \right), \cup_{\zeta_1 \in H_1, \zeta_2 \in H_2} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\};

2. \( \Xi_1 \otimes \Xi_2 = \left\{ \left( \Delta \left( \Delta^{-1}(s_1, \gamma_1) \times \Delta^{-1}(s_2, \gamma_2) \right), \cup_{s_1 \in H_1, \zeta_2 \in H_2} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\};

3. \( \lambda \Xi = \left\{ \left( \Delta \left( \Delta^{-1}(s, \gamma) \right), \cup_{\zeta \in H} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\}, \lambda > 0;

4. \( \Xi^q = \left\{ \left( \Delta \left( \Delta^{-1}(s, \gamma)^q \right), \cup_{\zeta \in H} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\}, \lambda > 0.

### 3.2 DHq-ROFTLWA and DHq-ROFTLGW operators

In this subsection, we will propose two kinds of weighted aggregation operators: DHq-ROFTLWA and DHq-ROFTLGW operators.

**Definition 11.** Let \( \Xi_j = ((s_j, \gamma_j), (h_j, g_j)) (j = 1, 2, ..., n) \) be a collection of DHq-ROFTLN sets. The DHq-ROFTLWA operator is a mapping \( \mathcal{H}^a \rightarrow \mathcal{H} \) such that

\[
\text{DHq-ROFTLWA}(\Xi_1, \Xi_2, ..., \Xi_n) = \bigoplus_{j=1}^{n} \omega_j \Xi_j.
\]

\( \mathcal{H} \) is the set of DHq-ROFTLN sets, \( \omega = (\omega_1, \omega_2, ..., \omega_n)^T \) is the weight vector of \( \Xi_j (j = 1, 2, ..., n) \), such that \( \omega_j \in [0, 1] \) and \( \sum_{j=1}^{n} \omega_j = 1 \).

**Theorem 1.** Let \( \Xi_j = ((s_j, \gamma_j), (h_j, g_j)) (j = 1, 2, ..., n) \) be a collection of DHq-ROFTLN sets with weight vector \( \omega = (\omega_1, \omega_2, ..., \omega_n)^T \), which satisfies \( \omega_j \in [0, 1] \) and \( \sum_{j=1}^{n} \omega_j = 1 \), then

\[
\text{DHq-ROFTLWA}(\Xi_1, \Xi_2, ..., \Xi_n) = \left\{ \left( \Delta \left( \sum_{j=1}^{n} \omega_j \Delta^{-1}(s_j, \gamma_j) \right), \cup_{\zeta \in H} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\}.
\]

**Proof.** We prove that the Equation (3) satisfies using mathematical induction method for positive integer \( n \).

For \( n = 1 \)

\[
\omega_1 \Xi_1 = \left\{ \left( \Delta \left( \omega_1 \times \Delta^{-1}(s_1, \gamma_1) \right), \cup_{\zeta \in H} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\}.
\]

Thus, Equation (3) holds for \( n = 1 \).

a. Suppose that Equation (3) holds for \( n = m \),

\[
\text{DHq-ROFTLWA}(\Xi_1, \Xi_2, ..., \Xi_n) = \left\{ \left( \Delta \left( \sum_{j=1}^{m} \omega_j \Delta^{-1}(s_j, \gamma_j) \right), \cup_{\zeta \in H} \left\{ \left\{ \alpha^{q_1} \Delta_1^{q_1}, \alpha^{q_2} \Delta_2^{q_2} \right\} \right\} \right) \right\}.
\]

For \( n = m + 1 \), we have
Therefore, we can deduce that Equation (3) satisfies for positive integer \( n = m + 1 \). Thus, by mathematical induction method, we know that Equation (3) satisfies for any \( n \geq 1 \).

**Theorem 2.** Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) and \( \Xi'_j = \langle (s'_j, \gamma'_j), (h'_j, g'_j) \rangle \) \((j = 1, 2, \ldots, n)\) be two sets of DHq-ROFTLN s, then the DHq-ROFTLWA operator satisfies the following properties:

1. (Idempotency) If all \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) are equal for all \( j \), then

\[
\text{DHq-ROFTLWA}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \Xi
\]

2. (Monotonicity) If \( \Xi_j \preceq \Xi'_j \) \((j = 1, 2, \ldots, n)\), then

\[
\text{DHq-ROFTLWA}(\Xi_1, \Xi_2, \ldots, \Xi_n) \preceq \text{DHq-ROFTLWA}(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).
\]

3. (Boundedness) Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be a collection of DHq-ROFTLN s, and let

\[
\Xi^- = \left\langle \min_j (s_j, \gamma_j), \cup_s \left\{ \left\{ \min_j \{s_j\} \right\}, \left\{ \max_j \{\delta_j\} \right\} \right\} \right\rangle,
\]

\[
\Xi^+ = \left\langle \max_j (s_j, \gamma_j), \cup_s \left\{ \left\{ \max_j \{s_j\} \right\}, \left\{ \min_j \{\delta_j\} \right\} \right\} \right\rangle.
\]

Then,

\[
\Xi^- \preceq \text{DHq-ROFTLWA}(\Xi_1, \Xi_2, \ldots, \Xi_n) \preceq \Xi^+.
\]

**Definition 12.** Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be a collection of DHq-ROFTLN s. The DHq-ROFTLWG operator is a mapping \( \mathcal{H}^n \to \mathcal{H} \), such that

\[
\text{DHq-ROFTLWG}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \bigotimes_{j=1}^n \Xi_j^n.
\]

\( \mathcal{H} \) is the set of DHq-ROFTLN s, \( \mathbf{w} = (w_1, w_2, \ldots, w_n)^T \) is the weight vector of \( \Xi_j (j = 1, 2, \ldots, n) \), such that \( w_j \in [0, 1] \) and \( \sum_{j=1}^n w_j = 1 \).

**Theorem 3.** Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be a collection of DHq-ROFTLN s with weight vector \( \mathbf{w} = (w_1, w_2, \ldots, w_n)^T \), which satisfies \( w_j \in [0, 1] \) and \( \sum_{j=1}^n w_j = 1 \). Then

\[
\text{DHq-ROFTLWG}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\langle \prod_{j=1}^n (\Delta^{-1}(s_j, \gamma_j))^m, \cup_s \left\{ \prod_{j=1}^n (1 - (\delta_j)^m) \right\} \right\rangle.
\]

**Proof.** This theorem’s proof is analogous to that of Theorem 1.

DHq-ROFTLWG operator has the same properties as DHq-ROFTLWA operator.
4 | SOME DHq-ROFTLMSM AGGREGATION OPERATORS

As the DHq-ROFTLS is a powerful tool for expressing values in the decision-making process, in this section, we generalize the MSM and DMSM operators to DHq-ROFTLSs, and develop the DHq-ROFTLMSM and DHq-ROFTLDMSM operators and its weighted forms to aggregate DHq-ROFTLNs, and study its desirable properties and special cases.

4.1 | The DHq-ROFTLMSM operator

Definition 13. Let \( \Xi = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be any collection of DHq-ROFTLNs, then we define the DHq-ROFTLMSM as follows:

\[
\text{DHq-ROFTLMSM}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \frac{\bigoplus_{1 \leq j \leq n} \bigotimes_{j=1}^n (s_j, \gamma_j)}{C_0} \right)^{1/2},
\]

(5)

Theorem 4. Let \( \Xi = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be any collection of DHq-ROFTLNs, then their aggregated value utilizing DHq-ROFTLMSM operator is also a DHq-ROFTLN, and

\[
\text{DHq-ROFTLMSM}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) = \delta \left( \frac{\sum_{1 \leq j \leq n} \left( \prod_{j=1}^n s_j \gamma_j \right)}{C_0} \right)^{1/2},
\]

(6)

\[
\bigcup_{\xi \in \mathcal{A}_0} \left\{ \frac{1}{\sqrt{C_0}} \left( \prod_{j=1}^n s_j \gamma_j \right) \right\}^{1/2}
\]

Proof. Utilizing Definition 10, we have

\[
\bigotimes_{j=1}^n \Xi_j = \delta \left( \frac{\sum_{1 \leq j \leq n} \left( \prod_{j=1}^n s_j \gamma_j \right)}{C_0} \right)^{1/2},
\]

and

\[
\bigoplus_{1 \leq j \leq n} \left( \bigotimes_{j=1}^n \Xi_j \right) = \delta \left( \frac{\sum_{1 \leq j \leq n} \left( \prod_{j=1}^n s_j \gamma_j \right)}{C_0} \right)^{1/2},
\]

Thus, we obtain

\[
\frac{1}{C_0} \bigoplus_{1 \leq j \leq n} \left( \bigotimes_{j=1}^n \Xi_j \right) = \delta \left( \frac{\sum_{1 \leq j \leq n} \left( \prod_{j=1}^n s_j \gamma_j \right)}{C_0} \right)^{1/2},
\]

\[
\bigcup_{\xi \in \mathcal{A}_0} \left\{ \frac{1}{\sqrt{C_0}} \left( \prod_{j=1}^n s_j \gamma_j \right) \right\}^{1/2}
\]


Accordingly,

\[
\text{DH}_q - \text{ROFTLMSM}^{(q)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\langle \Delta \left( \sum_{1 \leq i < j \leq n} \frac{\prod_{j=1}^{n-1} \Delta^{-1}(s_j, \gamma_j)}{C_n} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - \phi_j \gamma_j \right) \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - \phi_j \gamma_j \right) \right)^{q_b}} \right)^{1/2} \right\}.
\]

The \text{DH}_q-\text{ROFTLMSM} operator can be easily demonstrated to get the following properties. (Idempotency) If all \(\Xi_j = \langle (s_j, \gamma_j), (\eta_j, g_j) \rangle \) \(j = 1, 2, \ldots, n\) are equal, for all \(j\), then

\[
\text{DH}_q - \text{ROFTLMSM}^{(q)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \Xi.
\]

Proof. Since \(\Xi_j = \Xi = \langle (s_j, \gamma_j), (\eta_j, g_j) \rangle \) \(j = 1, 2, \ldots, n\), on the basis of Theorem 4, we have

\[
\text{DH}_q - \text{ROFTLMSM}^{(q)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\langle \Delta \left( \sum_{1 \leq i < j \leq n} \frac{\prod_{j=1}^{n-1} \Delta^{-1}(s_j, \gamma_j)}{C_n} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - \phi_j \gamma_j \right) \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - \phi_j \gamma_j \right) \right)^{q_b}} \right)^{1/2} \right\}.
\]

\[
= \left\langle \Delta \left( \frac{1}{C_n} \sum_{1 \leq i < j \leq n} \left( \Delta^{-1}(s_j, \gamma_j) \right)^{q_b} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - (\xi_j) \phi_j \right) \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( \prod_{j=1}^{n-1} \left( 1 - (\xi_j) \phi_j \right) \right)^{q_b}} \right)^{1/2} \right\}.
\]

\[
= \left\langle \Delta \left( \frac{1}{C_n} (C_n) \left( \Delta^{-1}(s_j, \gamma_j) \right)^{q_b} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( 1 - (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( 1 - (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} \right\}.
\]

\[
= \left\langle \Delta \left( (\Delta^{-1}(s_j, \gamma_j))^{q_b} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( 1 - (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( 1 - (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} \right\}.
\]

\[
= \left\langle \Delta \left( (\Delta^{-1}(s_j, \gamma_j))^{q_b} \right) \right\rangle \\
\cup_{\xi_i \in [\xi_0, \xi_n], \xi_i \in \mathbb{G}} \left\{ \left( \sqrt{1 - \left( (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} - \left( \sqrt{1 - \left( (\xi_j) \phi_j \right)^{q_b}} \right)^{1/2} \right\}.
\]

\[
= \langle (s_j, \gamma_j), (\eta_j, g_j) \rangle.
\]

**Property 2.** (Commutativity) If \(\Xi_j = \langle (s_j, \gamma_j), (\eta_j, g_j) \rangle\) be any collection of \(\text{DH}_q-\text{ROFTLNs}\), and \(\Xi_j = \langle (s_j, \gamma_j), (\eta_j, g_j) \rangle\) \(j = 1, 2, \ldots, n\) is any permutation of \(\Xi_j \) \(j = 1, 2, \ldots, n\), then

\[
\text{DH}_q - \text{ROFTLMSM}^{(q)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \text{DH}_q - \text{ROFTLMSM}^{(q)}(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).
\]

Proof. Because \(\Xi'_j \) \(j = 1, 2, \ldots, n\) is any permutation of \(\Xi_j \) \(j = 1, 2, \ldots, n\), utilizing the definition of \(\text{DH}_q-\text{ROFTLMSM}\) in Equation (5), we have
Because $z \geq 1$, we have $\zeta_i \geq \zeta'_i$. Utilizing assumption condition, for all $i,j (i=1,2,...,n; j=1,2,...,z)$, we get

$$\Delta^{-1}(s_i, y_i) \geq \Delta^{-1}(s'_i, y'_i)$$

Then, we have

$$\prod_{j=1}^{z} \Delta^{-1}(s_i, y_i) \geq \prod_{j=1}^{z} \Delta^{-1}(s'_i, y'_i)$$

$$\Delta \left( \sum_{1 \leq i_1 < i_2 < ... < i_L \leq n} \prod_{j=1}^{z} \Delta^{-1}(s_i, y_i) \right) \geq \Delta \left( \sum_{1 \leq i_1 < i_2 < ... < i_L \leq n} \prod_{j=1}^{z} \Delta^{-1}(s'_i, y'_i) \right)$$

Because $z \geq 1$, we have $\zeta_i \geq \zeta'_i$. Utilizing assumption condition, for all $i,j (i=1,2,...,n; j=1,2,...,z)$, we get

$$\prod_{j=1}^{z} \zeta_i \geq \prod_{j=1}^{z} \zeta'_i \Rightarrow 1 - \left( \prod_{j=1}^{z} \zeta_i \right) \leq 1 - \left( \prod_{j=1}^{z} \zeta'_i \right)$$

$$\Rightarrow \prod_{1 \leq i_1 < i_2 < ... < i_L \leq n} \left( 1 - \left( \prod_{j=1}^{z} \zeta_i \right) \right) \leq \prod_{1 \leq i_1 < i_2 < ... < i_L \leq n} \left( 1 - \left( \prod_{j=1}^{z} \zeta'_i \right) \right)$$

$$\Rightarrow \left\{ \sqrt[1/2]{1 - \left( \prod_{1 \leq i_1 < i_2 < ... < i_L \leq n} \left( 1 - \left( \prod_{j=1}^{z} \zeta_i \right) \right) \right)^2} \right\} \geq \left\{ \sqrt[1/2]{1 - \left( \prod_{1 \leq i_1 < i_2 < ... < i_L \leq n} \left( 1 - \left( \prod_{j=1}^{z} \zeta'_i \right) \right) \right)^2} \right\}$$

Similarly, we have

$$\Delta \left( \sum_{1 \leq i_1 < i_2 < ... < i_L \leq n} \prod_{j=1}^{z} \Delta^{-1}(s_i, y_i) \right) \geq \Delta \left( \sum_{1 \leq i_1 < i_2 < ... < i_L \leq n} \prod_{j=1}^{z} \Delta^{-1}(s'_i, y'_i) \right)$$
\[ \delta_i \leq \delta_i' \Rightarrow 1 - (\delta_i')^q \geq 1 - (\delta_i)^q \Rightarrow 1 - \prod_{j=1}^{q} ((1 - (\delta_i)^q)) \leq 1 - \prod_{j=1}^{q} ((1 - (\delta_i')^q)) \]

\[ \prod_{1 \leq i \leq n} ((1 - (\delta_i')^q)) \leq \prod_{1 \leq i \leq n} ((1 - (\delta_i)^q)) \]

\[ 1 - \prod_{1 \leq i \leq n} ((1 - (\delta_i')^q)) \leq 1 - \prod_{1 \leq i \leq n} ((1 - (\delta_i')^q)) \]

\[ 1 - \prod_{1 \leq i \leq n} ((1 - (\delta_i')^q)) \leq 1 - \prod_{1 \leq i \leq n} ((1 - (\delta_i')^q)) \]

\[ \left\{ \begin{array}{l}
\min_{\mathcal{A} \in \mathcal{A}_i} \left( \text{min}_{\mathcal{A}_i} \right) \\
\max_{\mathcal{A} \in \mathcal{A}_i} \left( \text{max}_{\mathcal{A}_i} \right)
\end{array} \right\} 
\]

Therefore, \( DH_q - \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, ..., \Xi_n) \geq DH_q - \text{ROFTLMSM}^{(2)}(\Xi'_1, \Xi'_2, ..., \Xi'_n) \).

**Property 4.** (Boundedness) Let \( \Xi = (\langle s_j, y_j \rangle, (s_j, y_j)) \) \((j = 1, 2, ..., n)\) be a collection of DHq-ROFTLNs, and let

\[ \Xi^- = \min_{\mathcal{A}_i} = \left\{ \text{min}_{\mathcal{A}_i} \right\}, \quad \Xi^+ = \max_{\mathcal{A}_i} = \left\{ \text{max}_{\mathcal{A}_i} \right\} \]

Then

\[ \Xi^- \leq \Xi \leq \Xi^+ \]

**Proof.** Utilizing the Properties 1 and 3, we get

\[ DH_q - \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, ..., \Xi_n) \geq DH_q - \text{ROFTLMSM}^{(2)}(\Xi^-, \Xi^-, ..., \Xi^-) = \Xi^- \]

\[ DH_q - \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, ..., \Xi_n) \leq DH_q - \text{ROFTLMSM}^{(2)}(\Xi^+, \Xi^+, ..., \Xi^+) = \Xi^+ \]

**Theorem 5.** Let \( \Xi_i(j = 1, 2, ..., n) \) be any collection of DHq-ROFTLNs, then

\[ \max \left( DH_q - \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, ..., \Xi_n) \right) = \left( \delta_i \left( \sum_{1 \leq i \leq n} \left( \prod_{j=1}^{q} \text{max}^{q-1} (s_j, y_j) \right) \right) \right)^{\frac{1}{q}} \]

\[ \min \left( DH_q - \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, ..., \Xi_n) \right) = \left( \delta_i \left( \sum_{1 \leq i \leq n} \left( \prod_{j=1}^{q} \text{min}^{q-1} (s_j, y_j) \right) \right) \right)^{\frac{1}{q}} \]

**Theorem 6.** For given DHq-ROFTLNs \( \Xi_i(j = 1, 2, ..., n) \) and \( z = 1, 2, ..., n \), according to the parameter \( z \), the DHq-ROFTLMSM operator decreases monotonically.
We obtain some exceptional cases of the DH$\text{-}q$ROFTLMSM operator w.r.t the parameter z by using Theorem 4.

Case 1. When $z = 1$, the DH$\text{-}q$ROFTLMSM operator reduces to the DH$\text{-}q$ROFT average operator as follows:

$$
\text{DH} \text{-} \text{ROFTLMSM}^{(1)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\{ \frac{1}{n} \sum_{1 \leq i < j \leq n} \Delta^{-1}(s_i, y_j) \right\}^{\frac{1}{2}}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - \left( \prod_{j=1}^{i} \left( 1 - (\delta_j q)^{\frac{1}{n}} \right) \right) \right) \right) \right) \right)} \right\}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - ((\delta_i) q)^{\frac{1}{n}} \right) \right) \right) \right)} \right\}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - ((\delta_i) q)^{\frac{1}{n}} \right) \right) \right) \right)} \right\} \right\}
$$

Case 2. When $z = 2$, the DH$\text{-}q$ROFTLMSM operator reduces to the DH$\text{-}q$ROFT Bonferroni mean operator as follows:

$$
\text{DH} \text{-} \text{ROFTLMSM}^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\{ \frac{1}{n^2} \sum_{1 \leq i < j \leq n} \Delta^{-1}(s_i, y_j) \right\}^{\frac{1}{2}}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - \left( \prod_{j=1}^{i} \left( 1 - (\delta_j q)^{\frac{1}{n}} \right) \right) \right) \right) \right) \right)} \right\} \right\}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - ((\delta_i) q)^{\frac{1}{n}} \right) \right) \right) \right)} \right\} \right\}
$$

$$
\bigcup_{j \in A \in \mathbb{B}} \left\{ \left\{ \sqrt{1 - \left( \prod_{1 \leq i < j \leq n} \left( 1 - \left( \prod_{j=1}^{i-1} \left( 1 - \left( \prod_{j=1}^{i} \left( 1 - (\delta_j q)^{\frac{1}{n}} \right) \right) \right) \right) \right) \right)} \right\} \right\}
$$

$$
\text{DH} \text{-} \text{ROFTLB}^{(1,1)}(\Xi_1, \Xi_2, \ldots, \Xi_n).
$$

Case 3. When $z = n$, the DH$\text{-}q$ROFTLMSM operator reduces to the DH$\text{-}q$ROFT geometric mean operator as follows:
The DH operator constraint, we shall propose a DH (Commutativity). If

\[ \Delta = \left( \sum_{j=1}^{n} \prod_{i=1}^{j-1} \left( 1 - \prod_{k=j+1}^{n} \left( 1 - \left( \frac{1}{\prod_{k=1}^{j} \left( 1 - (\delta_i)^q \right)} \right) \right) \right) \right)^{\frac{1}{q}} \]

\[ \Delta = \left( \prod_{j=1}^{n} \Delta^{-1}(s_j, \gamma_j) \right)^{\frac{1}{q}} \]

\[ \Delta = \left( \prod_{j=1}^{n} \Delta^{-1}(s_j, \gamma_j) \right)^{\frac{1}{q}} \]

4.2 | The DHq-ROFTLWMSM operator

In Section 4.1, the DHq-ROFTLWMSM operator does not acknowledge the aggregated arguments as being significant. In many real economic situations, however, particularly in MAGDM, the weights of attributes play a crucial role in the aggregation process. To resolve the DHq-ROFTLWMSM operator constraint, we shall propose a DHq-ROFTLWMSM operator.

**Definition 14.** Let \( \Xi_j = (s_j, \gamma_j, (h_j, g_j)) \) \( (j = 1, 2, \ldots, n) \) be any collection of DHq-ROFTLNs, \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) be the weight vector of \( \Xi_j \) \( (j = 1, 2, \ldots, n) \), and \( m_j > 0, \sum_{j=1}^{n} m_j = 1 \). DHq-ROFTLWMSM operator is defined as:

\[ \text{DHq} - \text{ROFTLWMSM}^{(\omega)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \frac{\Theta_{1 \leq i \leq n, s_i < g_i, s_i \neq g_i} \left( \sum_{j=1}^{n} \prod_{i=1}^{j-1} \left( 1 - \prod_{k=j+1}^{n} \left( 1 - (\delta_i)^q \right) \right) \right)^{\frac{1}{q}}}{C_n} \right)^{\frac{1}{q}}. \] (7)

**Theorem 7.** Let \( \Xi_j = (s_j, \gamma_j, (h_j, g_j)) \) \( (j = 1, 2, \ldots, n) \) be any collection of DHq-ROFTLNs with weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \), which satisfies \( m_j \in [0, 1] \) and \( \sum_{j=1}^{n} m_j = 1 \). Then the aggregated value using by DHq-ROFTLWMSM operator is also a DHq-ROFTLN, and

\[ \text{DHq} - \text{ROFTLWMSM}^{(\omega)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \Delta \left( \sum_{j=1}^{n} \prod_{i=1}^{j-1} \left( 1 - \prod_{k=j+1}^{n} \left( 1 - (\delta_i)^q \right) \right) \right)^{\frac{1}{q}} \right)^{\frac{1}{q}} \]

\[ \Delta = \left( \prod_{i=1}^{n} \Delta^{-1}(s_i, \gamma_i) \right)^{\frac{1}{q}} \]

The DHq-ROFTLWMSM operator can be easily demonstrated to get the following properties.

**Property 5.** (Commutativity). If \( \Xi_j = (s_j, \gamma_j, (h_j, g_j)) \) be any collection of DHq-ROFTLNs, and \( \Xi_j' = (s_j', \gamma_j', (h_j', g_j')) \) \( (j = 1, 2, \ldots, n) \) is any permutation of \( \Xi_j \) \( (j = 1, 2, \ldots, n) \), then

\[ \text{DHq} - \text{ROFTLWMSM}^{(\omega)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \text{DHq} - \text{ROFTLWMSM}^{(\omega)}(\Xi_1', \Xi_2', \ldots, \Xi_n'). \]
Property 6. (Monotonicity). Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) and \( \Xi_j^* = \langle (s_j', \gamma_j'), (h_j', g_j') \rangle \) \((j = 1, 2, \ldots, n)\) be two collections of DHq-ROFTLNs, if \( \Xi_j \succeq \Xi_j^* \) for all \( j \), then

\[
\text{DHq – ROFTLWMSM}_w^{[2]}(\Xi_1, \Xi_2, \ldots, \Xi_n) \succeq \text{DHq – ROFTLWMSM}_w^{[2]}(\Xi_1^*, \Xi_2^*, \ldots, \Xi_n^*).
\]

Property 7. (Boundedness). Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be a collection of DHq-ROFTLNs, and let

\[
\Xi^- = \min_j \Xi_j = \left( \min_j (s_j, \gamma_j), \bigcup_{z \in A_j \in B} \min_j (z, \min_j) \right), \Xi^+ = \max_j \Xi_j = \left( \max_j (s_j, \gamma_j), \bigcup_{z \in A_j \in B} \max_j (z, \min_j) \right).
\]

Then

\[
\Xi^- \leq \text{DHq – ROFTLWMSM}_w^{[2]}(\Xi_1, \Xi_2, \ldots, \Xi_n) \leq \Xi^+.
\]

The idempotency property does not hold in DHq-ROFTLWMSM. We obtain some exceptional cases of the DHq-ROFTLWMSM operator w.r.t the parameter \( z \) by using Theorem 7.

Case 1. When \( z = 1 \), the DHq-ROFTLWMSM operator reduces to the DHq-ROFTLW averaging operator as follows:

\[
\text{DHq – ROFTLWMSM}_w^{[1]}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \Delta \left( \sum_{j=1}^n \frac{1}{\varsigma_j} \right) \right)^\frac{1}{2}, \bigcup_{z \in A_j \in B} \left\{ \left( 1 - \prod_{j=2}^n \left( 1 - \left( \frac{\varsigma_j}{\varsigma_j} \right)^{\varsigma_j} \right) \right) \right\}.
\]

Case 2. When \( z = 2 \), the DHq-ROFTLWMSM operator reduces to the DHq-ROFTLW Bonferroni mean operator as follows:

\[
\text{DHq – ROFTLWMSM}_w^{[2]}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \Delta \left( \sum_{j=1}^n \frac{1}{\varsigma_j} \right) \right)^\frac{1}{2}, \bigcup_{z \in A_j \in B} \left\{ \left( 1 - \prod_{j=2}^n \left( 1 - \left( \frac{\varsigma_j}{\varsigma_j} \right)^{\varsigma_j} \right) \right) \right\}.
\]

Case 3. When \( z = n \), the DHq-ROFTLWMSM operator reduces to the DHq-ROFW geometric operator:

\[
\text{DHq – ROFTLWMSM}_w^{[n]}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left( \Delta \left( \prod_{j=1}^n \left( \frac{1}{\varsigma_j} \right)^{\varsigma_j} \right) \right)^\frac{1}{2}, \bigcup_{z \in A_j \in B} \left\{ \prod_{j=2}^n \left( 1 - \left( \frac{\varsigma_j}{\varsigma_j} \right)^{\varsigma_j} \right) \right\}.
\]

4.3 The DHq-ROFTLDMSM operator

Definition 15. Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \((j = 1, 2, \ldots, n)\) be any collection of DHq-ROFTLNs, then we define the DHq-ROFTLDMSM operator as follows:

\[
\text{DHq – ROFTLDMSM}_w^{[2]}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \frac{1}{2} \left( \times_{1 \leq i < j \leq n} \left( \text{DHq – ROFTLDMSM}_w^{[2]}(\Xi_i, \Xi_j) \right) \right).
\]
Theorem 8. Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \( (j = 1, 2, \ldots, n) \) be any collection of DHq-ROFTLNs, then their aggregated value utilizing DHq-ROFTLDMSM operator is also a DHq-ROFTLN, and

\[
DHq - ROFTLDMSM^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\{ \bigcup_{\zeta_j \in h_j g_j} \left\{ \sqrt[n]{1 - \prod_{1 \leq j < k \leq n} \left( 1 - \frac{1}{2} \left( \frac{1 - (\zeta_j - \zeta_k)^q}{(1 - \zeta_j)^q} \right)^{1/q} \right)} \right\} \right\}.
\]

The DHq-ROFTLDMSM operator can be easily demonstrated to get the following properties.

Property 8. (Idempotency) If all \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \( (j = 1, 2, \ldots, n) \) are equal, for all \( j \), then

\[
DHq - ROFTLDMSM^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = \Xi.
\]

Property 9. (Commutativity). If \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) be any collection of DHq-ROFTLNs, and \( \Xi'_j = \langle (s'_j, \gamma'_j), (h'_j, g'_j) \rangle \) \( (j = 1, 2, \ldots, n) \) is any permutation of \( \Xi_j \) \( (j = 1, 2, \ldots, n) \), then

\[
DHq - ROFTLDMSM^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) = DHq - ROFTLDMSM^{(2)}(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).
\]

Property 10. (Monotonicity). Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) and \( \Xi'_j = \langle (s'_j, \gamma'_j), (h'_j, g'_j) \rangle \) \( (j = 1, 2, \ldots, n) \) be two collections of DHq-ROFTLNs, if \( \Xi_j \succeq \Xi'_j \) for all \( j \), then

\[
DHq - ROFTLDMSM^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) \succeq DHq - ROFTLDMSM^{(2)}(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).
\]

Property 11. (Boundedness). Let \( \Xi_j = \langle (s_j, \gamma_j), (h_j, g_j) \rangle \) \( (j = 1, 2, \ldots, n) \) be a collection of DHq-ROFTLNs, and let

\[
\Xi^- = \min_{j} \Xi_j = \left\{ \min_j (s_j, \gamma_j), \bigcup_{\zeta_j \in h_j g_j} \left\{ \min_j \zeta_j, \max_j \delta_j \right\} \right\}, \quad \Xi^+ = \max_{j} \Xi_j = \left\{ \max_j (s_j, \gamma_j), \bigcup_{\zeta_j \in h_j g_j} \left\{ \max_j \zeta_j, \min_j \delta_j \right\} \right\}.
\]

Then

\[
\Xi^- \leq DHq - ROFTLDMSM^{(2)}(\Xi_1, \Xi_2, \ldots, \Xi_n) \leq \Xi^+.
\]

We obtain some exceptional cases of the DHq-ROFTLDMSM operator w.r.t the parameter \( z \) by using Theorem 8.

Case 1. The DHq-ROFTLDMSM operator becomes the DHq-ROFTL geometric mean operator when \( z = 1 \).

Case 2. The DHq-ROFTLDMSM operator becomes the DHq-ROFTL geometric Bonferroni mean operator when \( z = 2 \).

Case 3. The DHq-ROFTLDMSM operator becomes the DHq-ROFTL average operator when \( z = n \).

4.4 | The DHq-ROFTLWDMMSM operator

In Section 4.3, the DHq-ROFTLDMSM operator does not acknowledge the aggregated arguments as being significant. In many real economic situations, however, particularly in MAGDM, the weights of attributes play a crucial role in the aggregation process. To resolve the DHq-ROFTLDMSM operator constraint, we shall propose a DHq-ROFTLWDMMSM operator.
Definition 16. Assume that $\Xi = \{\langle s_i, y_i \rangle, h_j, \langle g_j, y_j \rangle \}$ \((j = 1,2,\ldots,n)\) be a collection of DHq-ROFTLNs, and then we define the DHq-ROFTLWDMSM operator as follows:

$$\text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) = \frac{1}{z} \left( \bigotimes_{1 \leq i \leq n} \left( \bigoplus_{j=1}^{z} (m_i, \Xi_j) \right) \right)^{\frac{1}{z}}. \quad (11)$$

Theorem 9. Let $\Xi = \{\langle s_i, y_i \rangle, h_j, \langle g_j, y_j \rangle \}$ \((j = 1,2,\ldots,n)\) be a collection of DHq-ROFTLNs with weight vector $w = (m_1, m_2, \ldots, m_n)^T$, which satisfies $m_j \in [0,1]$ and $\sum_{j=1}^{n} m_j = 1$. Then the aggregated value by using DHq-ROFTLWDMSM operator is also a DHq-ROFTLN, and

$$\text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) = \left\{ \begin{array}{l}
\left\{ 1 - \prod_{i=1}^{n} \sum_{j=1}^{q} \left( 1 - \left( s_i \right)_{m_j} \right) \right\}^{\frac{1}{q}},
\left\{ 1 - \prod_{i=1}^{n} \sum_{j=1}^{q} \left( 1 - \left( g_i \right)_{m_j} \right) \right\}^{\frac{1}{q}} \end{array} \right\} \quad (12)
$$

The DHq-ROFTLWDMSM operator can be easily demonstrated to get the following properties. (Commutativity). If $\Xi = \{\langle s_i, y_i \rangle, h_j, \langle g_j, y_j \rangle \}$ be any collection of DHq-ROFTLNs, and $\Xi = \{\langle s'_i, y'_i \rangle, h'_j, \langle g'_j, y'_j \rangle \}$ \((j = 1,2,\ldots,n)\) is any permutation of $\Xi$, \((j = 1,2,\ldots,n)\), then

$$\text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) = \text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).$$

Property 13. (Monotonicity). Let $\Xi = \{\langle s_i, y_i \rangle, h_j, \langle g_j, y_j \rangle \}$, $\Xi' = \{\langle s'_i, y'_i \rangle, h'_j, \langle g'_j, y'_j \rangle \}$ \((j = 1,2,\ldots,n)\) be two collections of DHq-ROFTLNs, if $\Xi \preceq \Xi'$ for all $j$, then

$$\text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) \succeq \text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi'_1, \Xi'_2, \ldots, \Xi'_n).$$

Property 14. (Boundedness). Let $\Xi = \{\langle s_i, y_i \rangle, h_j, \langle g_j, y_j \rangle \}$ \((j = 1,2,\ldots,n)\) be a collection of DHq-ROFTLNs, and let

$$\Xi^- = \min_{\Xi} = \left\{ \min_{i} \langle s_i, y_i \rangle, \cup_{x \in \mathbb{N}} \left\{ \begin{array}{l}
\min_{j} s_i, \max_{j} y_j \end{array} \right\} \right\}, \Xi^+ = \max_{\Xi} = \left\{ \max_{i} \langle s_i, y_i \rangle, \cup_{x \in \mathbb{N}} \left\{ \begin{array}{l}
\max_{j} s_i, \min_{j} y_j \end{array} \right\} \right\}.$$

Then

$$\Xi^- \leq \text{DHq-ROFTLWDMSM}_{\text{DH}}^q(\Xi_1, \Xi_2, \ldots, \Xi_n) \leq \Xi^+.$$
Zhang (Zhang, 2016), group decision-making problems can be solved from two angles: (a) aggregation stage; (b) exploitation stage. In the aggregation stage, collective evaluation values are obtained from the individual evaluation values of the alternatives. In the exploitation stage, the optimal alternative(s) is selected according to the priorities of the cumulative evaluation values. We will employ the DHq-ROFTLWMSM and DHq-ROFTLWDMSM operators to combine the individual decision matrices into a group decision matrix.

To solve the MAGDM problem under DHq-ROFTL environment, we choose a set of m alternatives $\mathcal{I}_i (i = 1, 2, ..., m)$ and set of n attributes $h_j (j = 1, 2, ..., n)$. Let $w = (w_1, w_2, ..., w_n)^T$ be the weighting vector of attributes satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $\mathcal{D} = \{D_1, D_2, ..., D_p\}$ be the collection of experts having weights $\alpha_j = (\alpha_{1j}, \alpha_{2j}, ..., \alpha_{pj})^T$ satisfy $\sum_{j=1}^{p} \alpha_j = 1$. Suppose $F' = (\Xi_{ij})_{m \times n}$ is the individual DHq-ROFTL decision matrix, where $\Xi_{ij} = \left( (s_i^1, r_i^j), \langle h_i, g_j \rangle \right)$ is a DHq-ROFTL provided by the decision expert $D_q$ for the alternative $\mathcal{I}_i$ according to the attribute $h_j$. To choose the most suitable alternative(s), the proposed operators are used to construct a method for MAGDM in a DHq-ROFTL environment that includes the following steps:

Step 1. Calculate the least common multiple of $l(h_j), l(g_j)$ ($i = 1, 2, ..., m$), denoted as $L(l_{ij})_{m \times n}$. Then construct the multiple DHq-ROFTL matrices $F' = (\Xi_{ij})_{m \times n}$ where $\Xi_{ij} = \left( (s_i^1, r_i^j), \left\{ \langle c_{1i}^1, c_{2i}^1, ..., c_{8i}^1 \rangle \right\}, \left\{ \langle s_i^2, r_i^2, ..., s_i^8 \rangle \right\} \right)$ and $l(h_j), l(g_j)$ represent the number of elements in $h_j$, $g_j$ respectively.

Step 2. Utilize the DHq-ROFTLWMSM in Equation (8) and the DHq-ROFTLWDMSM in Equation (12) to aggregate all individual DHq-ROFTL decision matrices $(F')^{\ell} = (\Xi_{ij}^{\ell})_{m \times n}$ into a collective DHq-ROFTL decision matrix $F' = (\Xi_{ij})_{m \times n}$ as:

$$F' = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \cdots & \Xi_{1n} \\
\Xi_{21} & \Xi_{22} & \cdots & \Xi_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
\Xi_{m1} & \Xi_{m2} & \cdots & \Xi_{mn}
\end{bmatrix} = \begin{bmatrix}
\langle (s_{11}^1, r_{11}), \langle h_1, g_1 \rangle \rangle & \langle (s_{12}^1, r_{12}), \langle h_1, g_2 \rangle \rangle & \cdots & \langle (s_{1n}^1, r_{1n}), \langle h_1, g_n \rangle \rangle \\
\langle (s_{21}^2, r_{21}), \langle h_2, g_1 \rangle \rangle & \langle (s_{22}^2, r_{22}), \langle h_2, g_2 \rangle \rangle & \cdots & \langle (s_{2n}^2, r_{2n}), \langle h_2, g_n \rangle \rangle \\
\cdots & \cdots & \cdots & \cdots \\
\langle (s_{m1}^m, r_{m1}), \langle h_m, g_1 \rangle \rangle & \langle (s_{m2}^m, r_{m2}), \langle h_m, g_2 \rangle \rangle & \cdots & \langle (s_{mn}^m, r_{mn}), \langle h_m, g_n \rangle \rangle
\end{bmatrix}$$

Step 3. Aggregate the DHq-ROFTL evaluation values of alternative $\mathcal{I}_i$ on all attributes $h_j (j = 1, 2, ..., n)$ into the overall evaluation value of the alternatives $\mathcal{I}_i (i = 1, 2, ..., m)$ by using the DHq-ROFTLWMSM operator in Equation (8) and the DHq-ROFTLWDMSM operator in Equation (12) to derive the overall preference values of the alternatives $\mathcal{I}_i (i = 1, 2, ..., m)$.

Step 4. Determine the score values $S^\text{DHFTL}$ of overall assessment values $\mathcal{I}_i (i = 1, 2, ..., m)$ utilizing Equation (1).

Step 5. Arrange all the alternatives $\mathcal{I}_i (i = 1, 2, ..., m)$ according to $S^\text{DHFTL}$ and choose the best one(s).

Step 6. End.

The graphical representation of the established approach is shown in Figure 2.

6 | ILLUSTRATIVE EXAMPLE

6.1 | Assessment process of the developed method

The use of the novel proposed strategy is demonstrated in this part by solving a real-world MAGDM problem. Recently, Coronavirus (COVID-19) is the top cause of mortality other than older diseases, which is quickly becoming a universal threat. In 2019, the COVID-19 was firstly discovered in Wuhan, China’s capital (Hubei Province in China). It is considered a medical emergency and characterized by fever, nausea, chest pain, throat pain, migraine, as well as other symptoms. Yet, there is no effective and adequate medication for it. In medical research, medications to control COVID-19 outbreaks are inherently generic drugs to treat influenza, sore throat, a weak body’s immune, and so on. In medical care, medical professionals incorporate a variety of medications for treating COVID-19 outbreaks. Following an initial assessment, let $\{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_5\}$ be a set of five medications and let $\{h_1, h_2, h_3, h_4\}$ be a set of four symptoms. Suppose, five medications are evaluated by three experts $\{D_1, D_2, D_3\}$ (Doctors and physicians with experience to control the impact of spreading COVID-19), with weight vector $w = (0.30, 0.25, 0.45)^T$ for choosing the best medication to treat COVID-19 outbreak. Three experts $D_q (q = 1, 2, 3)$ give their evaluations on the basis of linguistic term set $S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{medium}, s_4 = \text{good}, s_5 = \text{very good}, s_6 = \text{extremely good}\}$. Based on their experience, each decision expert has opinion for the selection of best medication according to four symptoms, including:

1. Fever $(h_1)$;
2. Nausea $(h_2)$;
3. Chest pain $(h_3)$;
4. Throat pain $(h_4)$.
The decision experts subjectively assign the essential weights to the symptoms as \( \omega = (0.18, 0.25, 0.23, 0.34)^T \). In order to avoid the risk of mistreatment and over-diagnosis in the medication of COVID-19 disease, experts should evaluate the effective qualities of medications in relation to all symptoms and identify the most suitable medication to cure the symptoms, according to the guidelines. Each decision expert uses DH\(_q\)-ROFTLNs to assess each medication’s ability to control the effect of COVID-19 for each symptom. The decision matrix \( \mathbf{F} = \mathbf{\Xi}^{\mathbf{C}_{3}} \) summarizes the assessment values provided by the three experts for each attribute of each alternative, presented in Tables 1 to 3, respectively.

To solve this MAGDM problem, we use the proposed approach to select the optimal medication to control COVID-19 outbreaks.

### Decision making process based on the DH\(_q\)-ROFTLWMSM operator

Since all attributes have an impact on one another and attribute values take the form of DH\(_q\)-ROFTLNs, the proposed DH\(_q\)-ROFTLWMSM operator is used to construct an approach to MAGDM in a DH\(_q\)-ROFTL environment for the selection of adequate medication to control COVID-19 outbreaks, which consists of the following steps:

**Step 1.** Utilize the DH\(_q\)-ROFTLWMSM in Equation (8) to aggregate all individual DH\(_q\)-ROFTL decision matrices \( (\mathbf{F}^{(1)})^\chi = (\mathbf{\Xi}^{\mathbf{C}_{3}})^{\chi}_{i,j} \) into a collective DH\(_q\)-ROFTL decision matrix \( \mathbf{F}^* = (\mathbf{\Xi}^{\mathbf{C}_{3}})^{\chi}_{5,4} \), shown in Table 7 (suppose \( z = 3 \) and \( q = 4 \)).
### Table 1: Decision matrix with DHq-ROFTLNs by the first expert \(D_1\)

| \(T_i\) | \(h_1\) | \(h_2\) |
|---|---|---|
| \(T_1\) | \((s_{0.2}, \{(0.4,0.5), (0.3,0.6)\})\) | \((s_{0.3}, \{(0.6,0.7), (0.3,0.4)\})\) |
| \(T_2\) | \((s_{0.1}, \{(0.2,0.5), (0.3,0.4,0.5)\})\) | \((s_{0.3}, \{(0.4,0.5,0.6), (0.2,0.4)\})\) |
| \(T_3\) | \((s_{0.4}, \{(0.4,0.6), (0.4,0.5,0.7)\})\) | \((s_{0.2}, \{(0.5,0.6,0.7), (0.1,0.2)\})\) |
| \(T_4\) | \((s_{0.5}, \{(0.5,0.7), (0.3,0.5)\})\) | \((s_{0.4}, \{(0.4,0.7), (0.4,0.5,0.6)\})\) |
| \(T_5\) | \((s_{0.1}, \{(0.1,0.4), (0.5,0.7)\})\) | \((s_{0.2}, \{(0.1,0.2,0.3), (0.4,0.6,0.7)\})\) |

### Table 2: Decision matrix with DHq-ROFTLNs by the second expert \(D_2\)

| \(T_i\) | \(h_1\) | \(h_2\) |
|---|---|---|
| \(T_1\) | \((s_{0.2}, \{(0.2,0.4,0.5), (0.3,0.7)\})\) | \((s_{0.1}, \{(0.3,0.5), (0.1,0.2,0.3)\})\) |
| \(T_2\) | \((s_{0.1}, \{(0.2,0.5), (0.5,0.7)\})\) | \((s_{0.3}, \{(0.5,0.6,0.7), (0.3,0.4)\})\) |
| \(T_3\) | \((s_{0.3}, \{(0.3,0.7), (0.5,0.6)\})\) | \((s_{0.2}, \{(0.5,0.6,0.7), (0.4,0.6)\})\) |
| \(T_4\) | \((s_{0.1}, \{(0.2,0.6), (0.3,0.4)\})\) | \((s_{0.3}, \{(0.5,0.6,0.7), (0.3,0.4)\})\) |
| \(T_5\) | \((s_{0.6}, \{(0.6,0.7), (0.2,0.6)\})\) | \((s_{0.2}, \{(0.3,0.6), (0.4,0.5)\})\) |

Step 2. Aggregate the DHq-ROFTL evaluation values of alternative \(T_i\) \((i = 1, 2, 3, 4, 5)\) on all attributes \(h_j\) \((j = 1, 2, 3, 4)\) into the overall evaluation values of the alternatives \(T_i\) using DHq-ROFTLWMSM operator in Equation (8) (suppose \(q = 4\) and \(z = 3\)). Then below are the aggregated evaluation values of the alternatives \(T_i\) \((i = 1, 2, 3, 4, 5)\):

\[
\begin{align*}
T_1 &= \langle (s_{0.6}, -0.0713), \{(0.8929, 0.8929, 0.9091, 0.9443, 0.9534, 0.9534, 0.1551, 0.1551, 0.1762, 0.2718, 0.2718, 0.2718, 0.2741, 0.2941)\} \rangle_q, \\
T_2 &= \langle (s_{0.6}, -0.0651), \{(0.8976, 0.8976, 0.9003, 0.9456, 0.9522, 0.9522, 0.1444, 0.1444, 0.1742, 0.2362, 0.2538, 0.2538)\} \rangle_q, \\
T_3 &= \langle (s_{0.5}, 0.0785), \{(0.9137, 0.9137, 0.9187, 0.9576, 0.9661, 0.9661, 0.1705, 0.1705, 0.1952, 0.2648, 0.2648, 0.2998, 0.2998)\} \rangle_q, \\
T_4 &= \langle (s_{0.6}, -0.0563), \{0.9147, 0.9147, 0.9287, 0.1693, 0.1693, 0.2008\} \rangle_q, \\
T_5 &= \langle (s_{0.6}, -0.0374), \{0.9591, 0.9632, 0.9632, 0.2634, 0.2948, 0.2948\} \rangle_q. \\
\end{align*}
\]

Step 3. Determine the score values \(S^DHTL_i\) of overall assessment values \(T_i\) \((i = 1, 2, 3, 4, 5)\) utilizing Equation (1).

\[
\begin{align*}
S^DHTL_1 &= 0.8032, S^DHTL_2 = 0.8091, S^DHTL_3 = 0.8105, S^DHTL_4 = 0.8405, S^DHTL_5 = 0.8087.
\end{align*}
\]
### Table 3: Decision matrix with DHq-ROFTLNs by the third expert $D_3$

| $\mathbb{A}_i$ | $h_1$                     | $h_2$                     |
|--------------|--------------------------|--------------------------|
| $\mathbb{A}_1$ | $\langle (s_2,0),(0.3,0.5),(0.2,0.3) \rangle_q$ | $\langle (s_5,0),(0.4,0.7),(0.1,0.4) \rangle_q$ |
| $\mathbb{A}_2$ | $\langle (s_1,0),(0.3,0.4),(0.1,0.3,0.5) \rangle_q$ | $\langle (s_5,0),(0.2,0.4,0.6),(0.2,0.3) \rangle_q$ |
| $\mathbb{A}_3$ | $\langle (s_4,0),(0.1,0.5),(0.3,0.5,0.7) \rangle_q$ | $\langle (s_5,0),(0.3,0.4,0.7),(0.2,0.4) \rangle_q$ |
| $\mathbb{A}_4$ | $\langle (s_3,0),(0.5,0.6),(0.2,0.3) \rangle_q$ | $\langle (s_4,0),(0.3,0.4),(0.3,0.5,0.7) \rangle_q$ |
| $\mathbb{A}_5$ | $\langle (s_1,0),(0.2,0.5),(0.3,0.6) \rangle_q$ | $\langle (s_5,0),(0.2,0.3,0.4),(0.2,0.5,0.6) \rangle_q$ |

### Table 4: Decision matrix with multiple DHq-ROFTLNs by the first expert $D_1$

| Alternatives | Attributes |
|-------------|------------|
| $\mathbb{A}_1$ | $\mathbb{E}_{11} = \langle (s_2,0),(0.4,0.4,0.4,0.5,0.5,0.5),(0.3,0.3,0.3,0.6,0.6,0.6) \rangle_q$; $\mathbb{E}_{12} = \langle (s_6,0),(0.6,0.6,0.6,0.7,0.7,0.7),(0.3,0.3,0.3,0.4,0.4,0.4) \rangle_q$; $\mathbb{E}_{13} = \langle (s_4,0),(0.2,0.2,0.2,0.5,0.5,0.5),(0.3,0.3,0.3,0.7,0.7,0.7) \rangle_q$; $\mathbb{E}_{14} = \langle (s_1,0),(0.3,0.3,0.3,0.5,0.5,0.5),(0.1,0.1,0.2,0.2,0.3,0.3) \rangle_q$ |
| $\mathbb{A}_2$ | $\mathbb{E}_{21} = \langle (s_1,0),(0.2,0.2,0.2,0.5,0.5,0.5),(0.3,0.3,0.4,0.4,0.5,0.5) \rangle_q$; $\mathbb{E}_{22} = \langle (s_5,0),(0.4,0.4,0.5,0.5,0.6,0.6),(0.2,0.2,0.2,0.4,0.4,0.4) \rangle_q$; $\mathbb{E}_{23} = \langle (s_2,0),(0.1,0.1,0.1,0.5,0.5,0.5),(0.2,0.2,0.2,0.5,0.5,0.5) \rangle_q$; $\mathbb{E}_{24} = \langle (s_5,0),(0.5,0.5,0.5,0.7,0.7,0.7),(0.4,0.4,0.4,0.5,0.5,0.5) \rangle_q$ |
| $\mathbb{A}_3$ | $\mathbb{E}_{31} = \langle (s_3,0),(0.4,0.4,0.4,0.6,0.6,0.6),(0.4,0.4,0.5,0.5,0.7,0.7) \rangle_q$; $\mathbb{E}_{32} = \langle (s_2,0),(0.5,0.5,0.6,0.6,0.7,0.7),(0.1,0.1,0.1,0.2,0.2,0.2) \rangle_q$; $\mathbb{E}_{33} = \langle (s_5,0),(0.3,0.3,0.3,0.7,0.7,0.7),(0.5,0.5,0.5,0.6,0.6,0.6) \rangle_q$; $\mathbb{E}_{34} = \langle (s_1,0),(0.5,0.5,0.5,0.6,0.6,0.6),(0.4,0.4,0.4,0.6,0.6,0.6) \rangle_q$ |
| $\mathbb{A}_4$ | $\mathbb{E}_{41} = \langle (s_3,0),(0.5,0.5,0.5,0.7,0.7,0.7),(0.3,0.3,0.3,0.5,0.5,0.5) \rangle_q$; $\mathbb{E}_{42} = \langle (s_4,0),(0.4,0.4,0.4,0.7,0.7,0.7),(0.4,0.4,0.5,0.5,0.6,0.6) \rangle_q$; $\mathbb{E}_{43} = \langle (s_1,0),(0.2,0.2,0.2,0.6,0.6,0.6),(0.3,0.3,0.3,0.4,0.4,0.4) \rangle_q$; $\mathbb{E}_{44} = \langle (s_5,0),(0.5,0.5,0.6,0.6,0.7,0.7),(0.3,0.3,0.3,0.4,0.4,0.4) \rangle_q$ |
| $\mathbb{A}_5$ | $\mathbb{E}_{51} = \langle (s_1,0),(0.1,0.1,0.1,0.4,0.4,0.4),(0.5,0.5,0.5,0.5,0.7,0.7) \rangle_q$; $\mathbb{E}_{52} = \langle (s_6,0),(0.1,0.1,0.1,0.2,0.2,0.3,0.3),(0.4,0.4,0.6,0.6,0.7,0.7) \rangle_q$; $\mathbb{E}_{53} = \langle (s_4,0),(0.6,0.6,0.6,0.7,0.7,0.7),(0.2,0.2,0.2,0.6,0.6,0.6) \rangle_q$; $\mathbb{E}_{54} = \langle (s_6,0),(0.3,0.3,0.3,0.6,0.6,0.6),(0.4,0.4,0.4,0.5,0.5,0.5) \rangle_q$ |

Step 4. Rank all alternatives $\mathbb{A}_i$ ($i = 1,2,3,4,5$) based on the score index as follows:

$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_1$.

Therefore, the fourth medication $\mathbb{A}_4$ to control COVID-19 outbreaks is the optimal one.

Step 5. End.
### Table 5: Decision matrix with multiple DH₄₉-ROFTLNs by the second expert D₂

| Alternatives | Attributes |
|--------------|------------|
| T₁           | \([t_{12}, 0.5, 0.1, 0.2, 0.3, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{13}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{14}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₂           | \([t_{21}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{22}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{23}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₃           | \([t_{31}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{32}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{33}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₄           | \([t_{41}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{42}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{43}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₅           | \([t_{51}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{52}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{53}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₆           | \([t_{61}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{62}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{63}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |

### Table 6: Decision matrix with multiple DH₄₉-ROFTLNs by the third expert D₃

| Alternatives | Attributes |
|--------------|------------|
| T₁           | \([t_{11}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{12}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{13}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₂           | \([t_{21}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{22}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{23}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₃           | \([t_{31}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{32}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{33}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₄           | \([t_{41}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{42}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{43}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₅           | \([t_{51}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{52}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{53}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
| T₆           | \([t_{61}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{62}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
|              | \([t_{63}, 0.2, 0.3, 0.4, 0.5, 0.4, 0.7, 0.9, 0.5, 0.6, 0.7, 0.8, 0.9])_q^1;\) |
6.1.2 | Decision making process based on the DH$q$-ROFTLWDMSM operator

Since all attributes have an impact on one another and attribute values take the form of DH$q$-ROFTLNs, the proposed DH$q$-ROFTLWDMSM operator is used to construct an approach to MAGDM in a DH$q$-ROFTL environment regarding the selection of adequate medication to control COVID-19 outbreaks, which consists of the following steps:

Step 1. Utilize the DH$q$-ROFTLWDMSM in Equation (8) to aggregate all individual DH$q$-ROFTL decision matrices $(F^c)^T = (\Xi^c)_5^{s,4} (c = 3, 4, 6)$ into a collective DH$q$-ROFTL decision matrix $F^c = (\Xi^c)_5^{s,4}$, shown in Table 8 (suppose $z = 3$ and $q = 4$).

Step 2. Aggregate the DH$q$-ROFTL evaluation values of alternative $T_i$ on all attributes $h_i (i = 1, 2, 3, 4)$ into the overall evaluation value of the alternatives $T_i (i = 1, 2, 3, 4, 5)$ using DH$q$-ROFTLWDMSM operator in Equation (12) (suppose $q = 4$ and $z = 3$). Then below are the aggregate evaluation values of the alternatives $T_i (i = 1, 2, 3, 4, 5)$:

\[
\begin{align*}
T_1 &= ((s_1, -0.0701), \{(0.1889, 0.1889, 0.2048, 0.3023, 0.3283, 0.3283), (0.8767, 0.8767, 0.8910, 0.9298, 0.9400, 0.9400)\})_{q^{-}} \\
T_2 &= ((s_1, -0.0724), \{(0.1837, 0.1837, 0.1970, 0.2969, 0.3178, 0.3178), (0.8690, 0.8690, 0.8788, 0.9269, 0.9318, 0.9318)\})_{q^{-}} \\
T_3 &= ((s_0, 0.0710), \{(0.2150, 0.2150, 0.2315, 0.3456, 0.3705, 0.3705), (0.8787, 0.8787, 0.8856, 0.9286, 0.9346, 0.9346)\})_{q^{-}} \\
T_4 &= ((s_1, -0.0595), \{(0.2175, 0.2175, 0.2658, 0.3455, 0.3655, 0.3655), (0.8931, 0.8931, 0.9000, 0.9378, 0.9428, 0.9428)\})_{q^{-}} \\
T_5 &= ((s_1, -0.0306), \{(0.1690, 0.1690, 0.1804, 0.2965, 0.3088, 0.3088), (0.8928, 0.8928, 0.9110, 0.9504, 0.9547, 0.9547)\})_{q^{-}}
\end{align*}
\]

Step 3. Determine the score values $S^DHTL_i$ of overall assessment values $T_i (i = 1, 2, 3, 4, 5)$ utilizing Equation (1).

\[
S^DHTL_1 = 0.0154, S^DHTL_2 = 0.0161, S^DHTL_3 = 0.0117, S^DHTL_4 = 0.0159, S^DHTL_5 = 0.0180.
\]

Step 4. Rank all alternatives $T_i (i = 1, 2, 3, 4, 5)$ based on the score index as follows:

\[
T_3 > T_2 > T_4 > T_1 > T_5.
\]

Thus, the optimal medication to control COVID-19 outbreaks is $T_2$.

Step 5. End.

6.2 | Parameter sensitivity analysis

The parameters play a critical role in the decision procedure, and the strategy has an impact on the final decision outcome in the computations mentioned above. As a result, in this subsection, we will perform a parameter analysis for the parameters $z$ and $q$. To complete the analysis in this subsection, we will use the DH$q$-ROFTLWMSM and DH$q$-ROFTLWDMSM operators to solve the previously described example. To explain the influence of parameters $z$ and $q$ on the score values and ranking results, we discuss the effects from the following two aspects:

1. For a fixed value of parameter $q (q = 4)$, the influence of parameter $z$ on ranking results is evaluated.
2. For a fixed value of parameter $z (z = 3)$, the influence of parameter $q$ on ranking results is evaluated.

Details are in Tables 9–12, and Figures 3–6. Tables 9 and 10 show that the ranking results differ slightly when the parameter $z$ is changed according to the expert’s subjective preferences, indicating that the DH$q$-ROFWMSM and DH$q$-ROFWDM operators can represent the expert’s risk preferences. Furthermore, as the parameter $z$ increases for the same alternative, the score values returned by the DH$q$-ROFWMSM operator become smaller, while the score values returned by the DH$q$-ROFWDM operator become larger. Experts can choose the appropriate value depending on their risk preferences in real-world decision-making situations. Generally in practical problems, we choose $z = \lfloor n/2 \rfloor$, where $\lfloor n \rfloor$ represents a round function and $n$ specifies the number of attributes. This is not only simple and precise, but it also guarantees that the expert’s risk preferences are relatively stable and the interrelationships between the various arguments can be considered.

We take a fixed value of $q = 4$ and the influence of the parameter $z$ is observed. If $z$ increases, the scores according to DH$q$-ROFTLWMSM operator will decrease and decrease, while the scores according to DH$q$-ROFTLWDMSM operator will increase and increase (see Figures 3, 4).
### TABLE 7 The aggregated DHq-ROFTL decision matrix by DHq-ROFTLWMSM operator

| Alternatives | Aggregated results |
|--------------|--------------------|
| $\Downarrow_1$ | $\langle s_1, 0.0267\rangle, \{0.6661, 0.6661, 0.6661, 0.7937, 0.7937, 0.7937\}, \{0.2039, 0.2039, 0.2039, 0.3617, 0.3617, 0.3617\} \rangle_q$. |
| $\Downarrow_2$ | $\langle s_2, 0.0\rangle, \{0.7242, 0.7242, 0.7242, 0.8879, 0.8879, 0.8879\}, \{0.2805, 0.2805, 0.2805, 0.3657, 0.3657, 0.3657\} \rangle_q$. |
| $\Downarrow_3$ | $\langle s_3, 0.0402\rangle, \{0.5271, 0.5271, 0.7194, 0.7194, 0.8476, 0.8476\}, \{0.2125, 0.2125, 0.2125, 0.4913, 0.4913, 0.4913\} \rangle_q$. |
| $\Downarrow_4$ | $\langle s_4, 0.0503\rangle, \{0.6299, 0.6299, 0.7793, 0.7793, 0.7793\}, \{0.1399, 0.1399, 0.2621, 0.2621, 0.4040, 0.4040\} \rangle_q$. |
| $\Downarrow_5$ | $\langle s_5, 0.0503\rangle, \{0.6584, 0.6584, 0.6584, 0.7676, 0.7676, 0.7676\}, \{0.2337, 0.2337, 0.3515, 0.3515, 0.4423, 0.4423\} \rangle_q$. |
| $\Downarrow_6$ | $\langle s_6, 0.0\rangle, \{0.6483, 0.6483, 0.6790, 0.7534, 0.8434, 0.8434\}, \{0.1422, 0.1422, 0.1422, 0.2778, 0.2778, 0.2778\} \rangle_q$. |
| $\Downarrow_7$ | $\langle s_7, 0.0267\rangle, \{0.5456, 0.5456, 0.5456, 0.7791, 0.7791, 0.7791\}, \{0.1709, 0.1709, 0.1709, 0.3375, 0.3375, 0.3375\} \rangle_q$. |
| $\Downarrow_8$ | $\langle s_8, 0.0\rangle, \{0.7216, 0.7216, 0.7216, 0.8676, 0.8676, 0.8676\}, \{0.2308, 0.2308, 0.2308, 0.3534, 0.3534, 0.3534\} \rangle_q$. |

### TABLE 8 The aggregated DHq-ROFTL decision matrix by DHq-ROFTLWMSM operator

| Alternatives | Aggregated results |
|--------------|--------------------|
| $\Downarrow_1$ | $\langle s_1, -0.0556\rangle, \{0.2050, 0.2050, 0.2050, 0.3820, 0.3820, 0.3820\}, \{0.6299, 0.6299, 0.6299, 0.7349, 0.7349, 0.7349\} \rangle_q$. |
| $\Downarrow_2$ | $\langle s_2, 0.0\rangle, \{0.3650, 0.3650, 0.3650, 0.5438, 0.5438, 0.5438\}, \{0.5924, 0.5924, 0.5924, 0.7621, 0.7621, 0.7621\} \rangle_q$. |
| $\Downarrow_3$ | $\langle s_3, 0.0556\rangle, \{0.1325, 0.1325, 0.2906, 0.2906, 0.4897, 0.4897\}, \{0.6109, 0.6109, 0.6109, 0.8442, 0.8442, 0.8442\} \rangle_q$. |
| $\Downarrow_4$ | $\langle s_4, 0.0556\rangle, \{0.2039, 0.2039, 0.2039, 0.3837, 0.3837, 0.3837\}, \{0.5456, 0.5456, 0.6712, 0.6712, 0.7751, 0.7751\} \rangle_q$. |
| $\Downarrow_5$ | $\langle s_5, 0.0503\rangle, \{0.5150, 0.5150, 0.5150, 0.7983, 0.7983, 0.7983\}, \{0.3145, 0.3145, 0.3145, 0.5119, 0.5119, 0.5119\} \rangle_q$. |
| $\Downarrow_6$ | $\langle s_6, -0.0590\rangle, \{0.5644, 0.5644, 0.6584, 0.6584, 0.7293, 0.7293\}, \{0.2308, 0.2308, 0.4110, 0.4110, 0.5190, 0.5190\} \rangle_q$. |
| $\Downarrow_7$ | $\langle s_7, 0.0402\rangle, \{0.6936, 0.6936, 0.6936, 0.8634, 0.8634, 0.8634\}, \{0.2198, 0.2198, 0.2198, 0.4123, 0.4123, 0.4123\} \rangle_q$. |
| $\Downarrow_8$ | $\langle s_8, 0.0\rangle, \{0.5489, 0.5489, 0.5489, 0.7817, 0.7817, 0.7817\}, \{0.3007, 0.3007, 0.3007, 0.4066, 0.4066, 0.4066\} \rangle_q$. |
## Table 9: Score values and ranking results by DHq-ROFTLWMSM operator (q = 4 and z = 1, 2, 3, 4)

| Parameter | Score values | Ranking |
|-----------|--------------|---------|
| z = 1     | 0.8117, 0.8115, 0.8133, 0.8172 | T₄ > T₃ > T₂ > T₁ |
| z = 2     | 0.8062, 0.8099, 0.8115, 0.8113 | T₄ > T₃ > T₂ > T₁ |
| z = 3     | 0.8032, 0.8091, 0.8105, 0.8087 | T₄ > T₁ > T₂ > T₃ |
| z = 4     | 0.8010, 0.8086, 0.8096, 0.8068 | T₄ > T₃ > T₂ > T₁ |

## Table 10: Score values and ranking results by DHq-ROFTLWDM operator (q = 4 and z = 1, 2, 3, 4)

| Parameter | Score values | Ranking |
|-----------|--------------|---------|
| z = 1     | 0.0054, 0.0052, 0.0045, 0.0053 | T₅ > T₁ > T₄ > T₂ > T₃ |
| z = 2     | 0.0113, 0.0116, 0.0089, 0.0116 | T₅ > T₄ > T₂ > T₁ |
| z = 3     | 0.0154, 0.0161, 0.0117, 0.0159 | T₅ > T₂ > T₄ > T₁ > T₃ |
| z = 4     | 0.0070, 0.0074, 0.0052, 0.0072 | T₅ > T₂ > T₄ > T₁ |

## Table 11: Score values and ranking results according to the parameter q by DHq-ROFTLWMSM operator

| Parameter | Score values | Ranking |
|-----------|--------------|---------|
| q = 1     | 0.8523, 0.8633, 0.8434, 0.8728 | T₄ > T₃ > T₂ > T₁ |
| q = 2     | 0.8486, 0.8571, 0.8457, 0.8761 | T₄ > T₃ > T₂ > T₁ |
| q = 3     | 0.8273, 0.8343, 0.8299, 0.8603 | T₄ > T₃ > T₂ > T₁ |
| q = 4     | 0.8032, 0.8091, 0.8105, 0.8405 | T₄ > T₃ > T₂ > T₁ |
| q = 5     | 0.7797, 0.7849, 0.7908, 0.8204 | T₄ > T₃ > T₂ > T₁ |
| q = 7     | 0.7372, 0.7413, 0.7541, 0.7828 | T₄ > T₃ > T₂ > T₁ |
| q = 11    | 0.6704, 0.6730, 0.6935, 0.7204 | T₄ > T₃ > T₂ > T₁ |
| q = 13    | 0.6444, 0.6463, 0.6688, 0.6947 | T₄ > T₃ > T₂ > T₁ |
| q = 17    | 0.6033, 0.6040, 0.6281, 0.6523 | T₄ > T₃ > T₂ > T₁ |
| q = 19    | 0.5870, 0.5873, 0.6113, 0.6346 | T₄ > T₃ > T₂ > T₁ |
| q = 23    | 0.5609, 0.5604, 0.5833, 0.6051 | T₄ > T₃ > T₂ > T₁ |
| q = 29    | 0.5331, 0.5319, 0.5519, 0.5718 | T₄ > T₃ > T₂ > T₁ |

## Table 12: Score values and ranking results according to the parameter q by DHq-ROFTLWDM operator

| Parameter | Score values | Ranking |
|-----------|--------------|---------|
| q = 1     | 0.0093, 0.0093, 0.0077, 0.0110 | T₅ > T₄ > T₃ > T₁ |
| q = 2     | 0.0100, 0.0104, 0.0080, 0.0110 | T₅ > T₄ > T₃ > T₁ |
| q = 3     | 0.0126, 0.0131, 0.0097, 0.0132 | T₅ > T₄ > T₃ > T₁ |
| q = 4     | 0.0154, 0.0161, 0.0117, 0.0159 | T₅ > T₄ > T₃ > T₁ |
| q = 5     | 0.0181, 0.0189, 0.0137, 0.0186 | T₅ > T₄ > T₃ > T₁ |
| q = 7     | 0.0230, 0.0239, 0.0173, 0.0237 | T₅ > T₄ > T₃ > T₁ |
| q = 11    | 0.0303, 0.0312, 0.0227, 0.0317 | T₅ > T₄ > T₃ > T₁ |
| q = 13    | 0.0330, 0.0338, 0.0247, 0.0348 | T₅ > T₄ > T₃ > T₁ |
| q = 17    | 0.0371, 0.0378, 0.0277, 0.0396 | T₅ > T₄ > T₃ > T₁ |
| q = 19    | 0.0387, 0.0393, 0.0288, 0.0415 | T₅ > T₄ > T₃ > T₁ |
| q = 23    | 0.0412, 0.0415, 0.0306, 0.0445 | T₅ > T₄ > T₃ > T₁ |
| q = 25    | 0.0436, 0.0436, 0.0324, 0.0475 | T₅ > T₄ > T₃ > T₁ |
The ranking outcomes for the different values of parameter \( q \) by the DH\(_q\)-ROFTLWMSM and DH\(_q\)-ROFTLWDMSM operators may differ, as shown in Tables 11 and 12. However, \( T_4 \) or \( T_5 \) is the best option. Depending on their preferences, DMs can select the appropriate parameter value \( q \). Furthermore, the score values based on the DH\(_q\)-ROFTLWMSM operator become smaller and smaller as the value of the parameter \( q \) increases, while the score values based on the DH\(_q\)-ROFTLWDMSM operator become larger and larger as the value of the parameter \( q \) increases.

It can be seen easily from Figure 5, scores according to DH\(_q\)-ROFTLWMSM operator will decrease and decrease when \( q \) increases and taking the fixed value of parameter \( z \) and there is a contradiction with DH\(_q\)-ROFTLWDMSM operator (see Figure 6).
To verify the validity of the developed approach, we utilize different approaches to solve the above mentioned MAGDM problem in Section 6.1. These methods are based on the DHₜ-ROFTLWA operator, DHₜ-ROFTLWG operator, DHₜ-ROFTL weighted Bonferroni mean (DHₜ-ROFTLWBM) operator, DHₜ-ROFTL weighted geometric Bonferroni mean (DHₜ-ROFTLWGBM) operator, DHIFTL weighted MSM (DHIFTLWMSM) operator, DHIFTL weighted DMSM (DHIFTLWDMSM) operator, DHPFTL weighted MSM (DHPFTLWMSM) operator, and DHPFTL weighted DMSM (DHPFTLWDMSM) operator.

Detailed evaluation results obtained utilizing different MAGDM approaches are given in Tables 13 and 14.

Detailed evaluation results using different MAGDM approaches are given in Figures 7, 8.

From the comparison of developed technique, in this manuscript, with some other different techniques by using the same example it is observed that the ranking results are slightly different, however the best alternative is T₄ or T₅. This shows that the proposed DHₜ-ROFTLWMSM and DHₜ-ROFTLWDMSM operators are best choices for MAGDM problems with DHₜ-ROFTLNs. In the MAGDM process, DHₜ-ROFTLs are becoming more comprehensive and contain more information. As a result, in MAGDM, our created approach delivers more general and powerful information. Further DHₜ-ROFTLWA and DHₜ-ROFTLWG operators are specific cases of the DHₜ-ROFTLWMSM and DHₜ-ROFTLWDMSM operators, respectively. The DHₜ-ROFTLWA and DHₜ-ROFTLWG operators have two drawbacks: (a) The model

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**TABLE 13** Comparison of alternatives for different MAGDM methods

| Alternatives | Scores and ranking results by DHₜ-ROFTLWA operator | Scores and ranking results by DHₜ-ROFTLWBM operator | Scores and ranking results by DHₜ-ROFTLWMSM operator |
|--------------|---------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| T₁           | S¹DHₜ = 0.8117                                    | S¹DHₜ = 0.8062                                    | S¹DHₜ = 0.8032                                    |
| T₂           | S²DHₜ = 0.8115                                    | S²DHₜ = 0.8099                                    | S²DHₜ = 0.8091                                    |
| T₃           | S³DHₜ = 0.8133                                    | S³DHₜ = 0.8115                                    | S³DHₜ = 0.8105                                    |
| T₄           | S⁴DHₜ = 0.8473                                    | S⁴DHₜ = 0.8423                                    | S⁴DHₜ = 0.8405                                    |
| T₅           | S⁵DHₜ = 0.8172                                    | S⁵DHₜ = 0.8113                                    | S⁵DHₜ = 0.8087                                    |
| Ranking      | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         |

| Alternatives | Scores and ranking results by DHIFTLWMSM operator | Scores and ranking results by DHPFTLWMSM operator | Scores and ranking results by DHₜ-ROFTLWMSM operator |
|--------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| T₁           | S¹DHFTL = 0.8523                                  | S¹DHFTL = 0.8486                                  | S¹DHₜ = 0.8032                                    |
| T₂           | S²DHFTL = 0.8633                                  | S²DHFTL = 0.8571                                  | S²DHₜ = 0.8091                                    |
| T₃           | S³DHFTL = 0.8434                                  | S³DHFTL = 0.8457                                  | S³DHₜ = 0.8105                                    |
| T₄           | S⁴DHFTL = 0.8728                                  | S⁴DHFTL = 0.8761                                  | S⁴DHₜ = 0.8405                                    |
| T₅           | S⁵DHFTL = 0.8661                                  | S⁵DHFTL = 0.8605                                  | S⁵DHₜ = 0.8087                                    |
| Ranking      | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         |

| Alternatives | Scores and ranking results by DHPFTLWDMSM operator |
|--------------|---------------------------------------------------|
| T₁           | S¹DHₜ = 0.8523                                    |
| T₂           | S²DHₜ = 0.8633                                    |
| T₃           | S³DHₜ = 0.8434                                    |
| T₄           | S⁴DHₜ = 0.8728                                    |
| T₅           | S⁵DHₜ = 0.8661                                    |
| Ranking      | T₄ ≻ T₅ ≻ T₂ ≻ T₁ ≻ T₃                         |
TABLE 14  Comparison of alternatives for different MAGDM methods

| Alternatives | Scores and ranking results by DHq-ROFTLWGM operator | Scores and ranking results by DHq-ROFTLWGBM operator | Scores and ranking results by DHq-ROFTLWDMSM operator |
|--------------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| T1           | S_{DHTL}^{DHq} = 0.0054                             | S_{DHTL}^{DHq} = 0.0116                             | S_{DHTL}^{DHq} = 0.0154                             |
| T2           | S_{DHTL}^{DHq} = 0.0052                             | S_{DHTL}^{DHq} = 0.0116                             | S_{DHTL}^{DHq} = 0.0161                             |
| T3           | S_{DHTL}^{DHq} = 0.0045                             | S_{DHTL}^{DHq} = 0.0089                             | S_{DHTL}^{DHq} = 0.0117                             |
| T4           | S_{DHTL}^{DHq} = 0.0053                             | S_{DHTL}^{DHq} = 0.0116                             | S_{DHTL}^{DHq} = 0.0159                             |
| T5           | S_{DHTL}^{DHq} = 0.0056                             | S_{DHTL}^{DHq} = 0.0132                             | S_{DHTL}^{DHq} = 0.0180                             |
| Ranking      | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        |

| Alternatives | Scores and ranking results by DHIFTLWDMSM operator | Scores and ranking results by DHPFTLWDMSM operator | Scores and ranking results by DHq-ROFTLWDMSM operator |
|--------------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| T1           | S_{DHTL}^{DHq} = 0.0093                             | S_{DHTL}^{DHq} = 0.0100                             | S_{DHTL}^{DHq} = 0.0154                             |
| T2           | S_{DHTL}^{DHq} = 0.0093                             | S_{DHTL}^{DHq} = 0.0104                             | S_{DHTL}^{DHq} = 0.0161                             |
| T3           | S_{DHTL}^{DHq} = 0.0077                             | S_{DHTL}^{DHq} = 0.0080                             | S_{DHTL}^{DHq} = 0.0117                             |
| T4           | S_{DHTL}^{DHq} = 0.0110                             | S_{DHTL}^{DHq} = 0.0110                             | S_{DHTL}^{DHq} = 0.0159                             |
| T5           | S_{DHTL}^{DHq} = 0.0112                             | S_{DHTL}^{DHq} = 0.0116                             | S_{DHTL}^{DHq} = 0.0180                             |
| Ranking      | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        | T_5 ≻ T_4 ≻ T_2 ≻ T_1 ≻ T_3                        |

FIGURE 7  Comparison with some different approaches
depending on these operators assumes that the input factors are self-contained; (b) The model depending on these operators does not take into account the interrelationships between the input factors. But, the novel developed operators in this article may also take into account the interrelationships between multiple factors and are thus an extension of the majority of conventional aggregation operators. As a consequence, the presented methodologies are more broad and robust unlike the D\textsubscript{HQ}ROFTLWA and D\textsubscript{HQ}ROFTLWG operators for solving MAGDM problems.

The ranking results of our proposed method and the D\textsubscript{HQ}ROFTLWBM and D\textsubscript{HQ}ROFTLWGBM operators are slightly different because both operators reflect the relationship between attributes, which demonstrates the developed framework's objectivity and reliability. In addition, the D\textsubscript{HQ}ROFTLWBM and D\textsubscript{HQ}ROFTLWGBM operators require two input parameters, and our approach requires only one parameter. This shows that the proposed method is simpler and better than D\textsubscript{HQ}ROFTLWBM and also it is a specific case of the proposed operator when \( z = 2 \). Further, when \( q = 1 \), DHIFTLS is a specific case of D\textsubscript{HQ}ROFTLWS. We derived distinctive rankings in order to compare the developed model to the DHIFTLWMSM and DHIFTLWDMMSM operators, however, the best alternative is \( \Sigma_4 \) or \( \Sigma_5 \). These operators are incapable of adequately handling the evaluation quantity because they only integrate the DHIFTLNS. The majority of the assessment throughout such an illustration does not really satisfy the given criteria. Consequently, the functionality scope of these operators is restricted, and they are unsuitable for such illustration of decision-making process. In comparison to the developed model, the DHPFTLWMSM and DHPFTLWDMMSM operators produce identical rankings, however all such operators can only integrate the DHPFTLNS. DHPFTLS has a broader variety than DHIFTLS but a smaller scope unlike D\textsubscript{HQ}ROFTLWS because, when \( q = 2 \) it is a specific case of D\textsubscript{HQ}ROFTLWS. This makes the approach under concern quite feasible.

When compared to other approaches, our technique has significant advantages and superiorities. The following is a summary of the benefits of our created approach:

1. A wide information space provided to experts: The D\textsubscript{HQ}ROFTLWS can express more comprehensive evaluation information since it combines the outstanding features of the linguistic term set with the D\textsubscript{HQ}ROFS. Moreover, the D\textsubscript{HQ}ROFTLWS is capable of dealing with practical problems from both a quantitative and qualitative viewpoint. Further the DHIFTLNS and DHPFTLNS are special cases of D\textsubscript{HQ}ROFTLNS. When \( q \)
1. DHq-ROFTLNs reduce to DHIFTLNs, and when q = 2, DHq-ROFTLNs reduce to DHPFTLNs. The proposed DHq-ROFTLNs have fewer constraints than DHIFTLNs and DHPFTLNs, giving DMs greater freedom to describe their assessment values comprehensively.

2. The ability to capture the interrelationships among multi-input attributes: The capacity of the MSM operator to reflect the interrelationship between any number of attributes is well known. The MSM operator is powerful and sufficient for dealing with MAGDM problems in reality because of this feature. As a result, our proposed strategy for dealing with MAGDM is quite effective.

3. The ability to minimize the negative consequences of extreme evaluation values: In real-world decision-making situations, DMs may use inappropriate or extreme assessment values for a variety of reasons, which usually has a negative impact on decision results. There are numerous causes for this, including a lack of expertise, insufficient time, and inherent prejudice. Such negative impacts should be minimized during the decision-making process in order to make a sensible choice. As our developed approach is based on the DHq-ROFTLMMSM and DHq-ROFTLWDMSM operators, it can effectively deal with DMs’ unreasonable or irrational bias, resulting in more reasonable final decisions.

7 | CONCLUSIONS

In this study, a group decision-making approach based on linguistic information for diagnosing a suspected COVID-19 infection with the use of the best medication is presented. As a new generalization of the DHq-ROFS theory, the DHq-ROFTLS is defined by integrating the DHq-ROFS and 2-tuple linguistic term to deal with imprecise and ambiguous knowledge in a complex MAGDM environment. Some new MSM and DMSM operators for aggregating DHq-ROFTL cognitive information have been developed, such as the DHq-ROFTLMMSM operator, DHq-ROFTLMMSM operator, and DHq-ROFTLWDMSM operator, which takes into consideration the interaction between DHq-ROFTLNs and can generate better results than those of the existing operators. This will lead to the development of MSM and DMSM operators. Some basic properties and special cases of these aggregation operators are stated. The fact is that our designed operators can take into account human hesitance as well as the interconnection among arguments that are fused. The DHq-ROFTLWMMSM and DHq-ROFTLWDMSM operators are differentiated from several operators not just do they incorporate the DHq-ROFTLNs, but also they take into account the interrelated arguments. Subsequently, a model to MAGDM based on DHq-ROFTLWMMSM operator and DHq-ROFTLWDMSM operator is developed in DHq-ROFTL circumstances. Further, a numerical instance related to the choice of adequate medication to control COVID-19 outbreaks is described to validate the effectiveness of the proposed method. Furthermore, the developed approach can vary the parameter dynamically dependent on the behavioural intentions of the DMs. The parameters z and q of the proposed aggregation operators have an impact on the alternative ranking. A comparative analysis is performed as well as the superiorities are demonstrated. We will expand the proposed approach to the different environments in future research and then apply it to fields of pattern recognition, decision theory, data measures, and clustering.

CONFLICT OF INTEREST

Authors declare that there is no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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