Higher-order non-symmetric counterterms in pure Yang-Mills theory

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Abstract

We analyze the restoration of the Slavnov-Taylor (ST) identities for pure massless Yang-Mills theory in the Landau gauge within the BPHZL renormalization scheme. The Zimmermann-Lowenstein IR regulator $M(s-1)$ is introduced via a suitable BRST doublet, thus preserving the nilpotency of the BRST differential. We explicitly obtain the most general form of the action-like part of the symmetric regularized action $\Gamma_s$, $s < 1$ obeying the ST identities and all other relevant symmetries of the model, to all orders in the loop expansion, and show that non-symmetric counterterms arise in $\Gamma_s$ starting from the second order in the loop expansion, unless a special choice of normalization conditions is done. We give a cohomological characterization of the fulfillment of BPHZL IR power-counting criterion, guaranteeing the existence of the physical limit $s \to 1$.

The technique analyzed in this paper is needed in the study of the restoration of the ST identities for those models, like the MSSM, where massless particles are present and no invariant regularization scheme is known to preserve all the relevant ST identities of the theory.

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1 Introduction

In a preceding paper \[1\] the quantum restoration of the Slavnov-Taylor (ST) identities for anomaly-free gauge theories has been shown to be equivalent, in the absence of IR problems, to the recursive parameterization of the action-like part of the symmetric quantum effective action

\[ \Gamma = \sum_{j=0}^{\infty} \Gamma^{(j)} \]

in terms of suitable ST functionals, associated to the cohomology classes of the classical linearized ST operator \( S_0 \). In the above equation \( \Gamma^{(j)} \) is the coefficient of order \( j \) in the \( \hbar \)-expansion of \( \Gamma \). It has also been shown \[1\] that even for models where a regularization-invariant scheme\(^2\) exists, at orders higher than one non-symmetric terms can enter in \( t^j \Gamma^{(j)}, j \geq 2 \), as a consequence of the bilinear form of the ST identities. In the case discussed in \[1\] these non-invariant terms can be put equal to zero only by a special choice of normalization conditions.

The method developed in \[1\], as well as the techniques proposed in \[2\]-\[6\], allows to recursively construct the symmetric 1-PI Green functions, fulfilling the relevant ST identities, without explicitly computing the ST breaking terms which can appear at the regularized level.

In comparison with the methods based on the explicit recursive evaluation of the ST breaking terms and of the corresponding finite counterterms designed to recover them \[8\]-\[11\], the techniques aiming at the direct restoration of the ST identities \[1\]-\[6\] have the advantage to be regularization-scheme independent and to reduce the amount of computations needed in order to obtain the correct symmetric 1-PI Green functions of the theory.

If the expansion of the action-like part of the symmetric vertex functional is performed on a basis of Lorentz-scalar monomials in the fields, the antifields and their derivatives, a set of consistency conditions \[2, 7\] among the superficially convergent \( n \)-th order 1-PI Feynman amplitudes and known lower orders 1-PI vertex functions appears, reflecting the nilpotency of the classical linearized ST operator \( S_0 \) \[2\]. These consistency conditions must be considered while recursively solving the linear system whose solutions are the coefficients of the action-like monomials entering into the expansion of the correct symmetric 1-PI quantum effective action \[2\]. They become extremely involved for supersymmetric models. On the contrary, when the symmetric 1-PI Green functions are parameterized in terms of the ST functionals, according to the prescription given in \[1\], these consistency conditions are automatically taken into account.

\[^2\] For instance dimensional regularization \[12\] for non-chiral gauge models or the modified subtraction prescription given in \[13\] for some chiral non-supersymmetric models.
We remark that whenever no invariant regularization scheme is known, the regularized Green functions do break the ST identities, and therefore under these circumstances the explicit computation of the finite restoring counterterms, required to fulfill the relevant ST identities, cannot be avoided.

This is the case for instance of supersymmetric models like the Minimal Supersymmetric Standard Model (MSSM), for which no invariant regularization scheme is known to fulfill all the relevant symmetries of the theory, due to the presence of the $\gamma^5$ matrix and of the completely antisymmetric tensor.

The complexity of the MSSM makes it very difficult to carry out first the recursive explicit computation of the ST breaking terms at the regularized level and then to obtain the finite counterterms designed to recover them. Therefore it would be very useful to be able to directly reconstruct the symmetric 1-PI Green functions. Unfortunately the procedure of [1] can only be applied in those cases where the relevant 1-PI Green functions can be Taylor-expanded around zero momentum. As a consequence this method, as well as the one analyzed in [2], cannot be used to deal with theories where massless particles are present.

The purpose of this paper is to extend the method discussed in [1] to those theories where massless particles appear. The technique analyzed in the present paper will allow to deal with the Standard Model (SM) and the MSSM. This in turn provides the last building block required in order to achieve the complete characterization, within the framework of the direct restoration of the ST identities for anomaly-free gauge theories, of the symmetric 1-PI Green functions for the SM and especially of the MSSM, to all orders in the loop expansion.

We will illustrate this extension in the case of pure massless Yang-Mills theory in the Landau gauge. This example is simple enough not to obscure the essential features of the method, while retaining all the main properties which make it useful in the study of the SM and of the MSSM. We will obtain the explicit form, to all orders in the loop expansion and for arbitrary normalization conditions, of the counterterms in the BPHZL scheme, required to fulfill the ST identities of the model.

The choice of the Landau gauge simplifies the computations, but similar results can be derived for any Lorentz-covariant gauge. We choose to work within the BPHZL regularization scheme [14]-[16],[19], following the Lowenstein-Zimmermann prescription [18]-[20] to handle massless propagators. Therefore all massless fields are assigned a mass

$$m^2 = M^2(s - 1)^2,$$

where $s$ is an auxiliary parameter ranging between 0 and 1.

The relevant zero-momentum subtractions on 1-PI Green functions, required by the BPHZL scheme, take place both at $s = 0$ and at $s = 1$ ac-
According to the prescriptions given in [18]-[20]. Both subtractions are needed in order to guarantee the existence of the massless limit $s \to 1$.

We will perform the renormalization of the relevant ST identities for the intermediate regularized symmetric quantum effective action $\Gamma_s$, $0 \leq s < 1$, by combining a variant of the approach first pioneered in [17] with the use of a BRST-invariant IR regulator introduced via a BRST-doublet [21]. This allows to maintain nilpotency of the full BRST differential and moreover provides the tool to discuss the interplay between the cohomological properties of $S_0$ and the subtraction prescriptions required to guarantee the existence of the massless limit $s \to 1$.

The use of the IR regulator in eq.(1) allows to perform a Taylor expansion of the 1-PI Green functions generated by $\Gamma_s$, $s < 1$, around zero momentum [18, 20]. $\Gamma_s$, $s < 1$ is required to fulfill a suitable extended ST identity where the IR regulator $m$ in eq.(1) enters via a BRST doublet. Then we can explicitly derive the most general solution of the action-like part $t^4 \Gamma_s$ of $\Gamma_s$, $s < 1$ by making use of the results given in [1]. Since $s < 1$, $t^4 \Gamma_s$ is well-defined and is obtained by expanding $\Gamma_s$ into a sum of linearly independent Lorentz-scalar monomials in the fields, the antifields and their derivatives and by keeping those terms of degree at most 4 in the mass dimension.

We find that non-symmetric counterterms enter into $\Gamma_s^{(j)}$, $j \geq 2$, unless a special choice of normalization conditions has been done for $\Gamma_s^{(k)}$, $k < j$. The coefficients of the non-symmetric counterterms in $\Gamma_s^{(j)}$ can be parameterized in terms of the coefficients $\lambda_1^{(k)}$, $\rho_1^{(k)}$ of suitable invariant ST functionals appearing in $t^4 \Gamma_s^{(k)}$, $k < j$.

It turns out that the limit of $\Gamma_s$ for $s \to 1$ is well-defined and free of IR singularities, as a consequence of the fulfillment of the IR and UV power-counting criteria stated in [19, 20]. The fulfillment of the IR power-counting criterion, guaranteeing the absence of zero-mass singularities in the limit $s \to 1$, can be understood in terms of purely cohomological properties of the classical linearized ST operator $S_0$.

This provides a novel cohomological interpretation of the IR power-counting criterion first introduced in [19, 20].

The quantum effective action $\Gamma$ for pure massless Yang-Mills model is finally obtained by

$$\Gamma = \lim_{s \to 1} \Gamma_s.$$

Physical unitarity stems from the ST identities obeyed by $\Gamma$.

We find that the non-symmetric counterterms entering into $\Gamma_s^{(j)}$ at order $j \geq 2$ do not vanish in the limit $s \to 1$. Hence they also affect $\Gamma_s^{(j)}$, unless a special choice of normalization conditions has been done for $\Gamma_s^{(k)}$, $k \to j$.

We point out that no explicit computation of the ST breaking terms at the regularized level is needed in this construction.
Finally we comment on the dependence of physical observables on the coefficients $\lambda_1^{(k)}, \rho_1^{(k)}$ parameterizing the non-invariant counterterms. This shows some of the advantages provided by the ST parameterization introduced in [1] in discussing the physical consequences of the non-invariant higher order counterterms.

The plan of the paper is the following. In Sect. 2 we discuss the ST identities for the model at hand and provide the most general solution to the symmetric regularized quantum effective action $\Pi_s$, $s < 1$ to all orders in the loop expansion. We show how the BPHZL IR regulator originally proposed in [18] can be introduced under the form of a suitable BRST doublet. This in turn allows to derive the proper ST identities to be fulfilled by $\Pi_s$, $s < 1$. We analyze the appearance of non-invariant counterterms in $\Pi_s$, $j \geq 2$. We show that they do not disappear unless a special choice of normalization conditions is done for $\Pi_s$, $k < j$. We give a cohomological interpretation of the IR power-counting criterion stated in [19, 20] and show that it is related to the structure of the extended BRST differential, thus establishing a connection between the BPHZL treatment of IR divergences and cohomology. In view of the fulfillment of the IR power-counting criterion, which we prove to hold true on the basis of purely cohomological arguments, the limit of $\Pi_s$ for $s \to 1$ exists. Therefore we can identify the fully renormalized quantum effective action $\Pi$ as the limit $s \to 1$ of $\Pi_s$. We discuss this identification in Sect. 3 where we also comment on the dependence of physical observables on the parameters controlling the non-invariant counterterms. Finally conclusions are presented in Sect. 4.

2 Higher-order non-symmetric counterterms

The ST identities for the classical action $\Gamma^{(0)}$ (see Appendix A) of Yang-Mills theory in the Landau gauge with an IR regulator $m$ introduced via a BRST doublet $(\bar{\rho}, m)$ are

$$S(\Gamma^{(0)}) = (\Gamma^{(0)}, \Gamma^{(0)}) + m \frac{\partial \Gamma^{(0)}}{\partial \bar{\rho}} = 0,$$  

where the bracket $(\Gamma^{(0)}, \Gamma^{(0)})$ is defined according to

$$(X, Y) = \int d^4x \left( \frac{\delta X}{\delta A_{\mu}^{* \alpha}} \frac{\delta Y}{\delta A_{\mu}^{\alpha}} + \frac{\delta X}{\delta \omega^{\alpha}} \frac{\delta Y}{\delta \omega^{\alpha}} + \frac{\delta X}{\delta \bar{\omega}^{\alpha}} \frac{\delta Y}{\delta \bar{\omega}^{\alpha}} \right).$$

The linearized classical ST operator $S_0$, given by

$$S_0 = \int d^4x \left( \frac{\delta \Gamma^{(0)}}{\delta A_{\mu}^{* \alpha}} \frac{\delta \Gamma^{(0)}}{\delta A_{\mu}^{\alpha}} + \frac{\delta \Gamma^{(0)}}{\delta A_{\mu}^{\alpha}} \frac{\delta \Gamma^{(0)}}{\delta A_{\mu}^{* \alpha}} + \frac{\delta \Gamma^{(0)}}{\delta \omega^{\alpha}} \frac{\delta \Gamma^{(0)}}{\delta \omega^{\alpha}} + \frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}^{\alpha}} \frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}^{\alpha}} \right) + m \frac{\partial \Gamma^{(0)}}{\partial \bar{\rho}},$$

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is nilpotent: $S_0^2 = 0$.

The BRST partner $\bar{\rho}$ of the mass regulator $m$ can be reabsorbed by the following antifield redefinition:

$$A^{\alpha s'}_\mu = A^{\alpha s}_\mu - \bar{\rho} m A^\alpha_\mu, \quad \omega^{\alpha s'} = \omega^{\alpha s} + \bar{\rho} m \bar{\omega}^\alpha, \quad \bar{\omega}^{\alpha s'} = \bar{\omega}^{\alpha s} - \bar{\rho} m \omega^\alpha. \quad (6)$$

This is a consequence of the fact that $(\bar{\rho}, m)$ are cohomologically trivial, pairing into a BRST doublet. In the new variables in eq.(6) $\Gamma^{(0)}$ becomes

$$\Gamma^{(0)} = \int d^4x \left\{ -\frac{1}{4g^2} G^a_\mu G^{\mu\nu a} - \bar{\omega}^a \partial_\mu (D^\mu \omega)^a + B^a \partial A^a \\
+ A^{\alpha s'}_\mu (D^\mu \omega)^a - \omega^{\alpha s'} \frac{1}{2} f^{abc} \omega^b \omega^c + \bar{\omega}^{\alpha s'} B^a + \frac{1}{2} m^2 (A^a_\mu)^2 + m^2 \bar{\omega}^a \omega^a \right\}. \quad (7)$$

The classical action $\Gamma^{(0)}$ in eq.(7) obeys a set of additional symmetries:

$$\frac{\partial \Gamma^{(0)}}{\partial \bar{\rho}} = 0, \quad (8)$$

the $B$-equation

$$\frac{\delta \Gamma^{(0)}}{\delta B^a} = \partial A^a + \bar{\omega}^{\alpha s'}, \quad (9)$$

the ghost equation

$$\frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}^a} + \partial_\mu \frac{\delta \Gamma^{(0)}}{\delta A^{\alpha s'}_\mu} = m^2 \omega^a, \quad (10)$$

and the anti-ghost equation

$$\int d^4x \left( \frac{\delta \Gamma^{(0)}}{\delta \omega^a} - f^{abc} \omega^b \frac{\delta \Gamma^{(0)}}{\delta B^c} \right) = \int d^4x \left( m^2 \bar{\omega}^a - f^{abc} A^{\beta s'}_\mu A^{\mu c} + f^{abc} \omega^{s'} \omega^c \right). \quad (11)$$

Notice that the R.H.S. of eqs.(9)-(11) are linear in the quantum fields.

The ST identities in eq.(3) can be regarded from the point of view of the Batalin-Vilkovisky formalism [28] as the master equation [28, 29] for Yang-Mills theory in the presence of an IR regulator giving mass to the gauge and ghost fields. In eq.(3) $A^{\alpha s}_\mu$, $\omega^{\alpha s}$ and $\bar{\omega}^{\alpha s}$ denote the antifields [27] coupled in $\Gamma^{(0)}$ in eq.(12) with the BRST variation respectively of the quantum fields $A^a_\mu$, $\omega^a$ and $\bar{\omega}^a$. The constant anticommuting parameter $\bar{\rho}$ pairs with the IR regulator $m$ into a $S_0$-doublet.

Since the relevant classical linearized ST operator $S_0$ in eq.(3) is nilpotent, the cohomological analysis of the IR-regularized Yang-Mills theory,
whose classical action is given by $\Gamma^{(0)}$ in eq.(42), can be performed by making use of the methods of Algebraic Renormalization [25, 29, 32].

In particular, since the IR regulator $m$ forms a $S_0$-doublet together with $\bar{\rho}$, the cohomology of $S_0$ is isomorphic to that of the classical linearized ST operator of pure Yang-Mills theory [25, 32, 30, 31].

By exploiting this result it can be shown on purely algebraic grounds [25, 32] that the extended ST identities for the IR-regularized Yang-Mills theory can be restored at the quantum level. Therefore it is possible to construct a symmetric quantum effective action $\Gamma_m$ fulfilling the extended ST identities

$$S(\Gamma_m) \equiv (\Gamma_m, \Gamma_m) + m \frac{\partial \Gamma_m}{\partial \bar{\rho}} = 0$$

(12)
to all orders in the loop expansion. The bracket in eq.(12) is defined by eq.(4). Nevertheless, this is not enough to guarantee the existence of the massless limit $m \to 0$. Indeed it may very well happen that the limit

$$\Gamma = \lim_{m \to 0} \Gamma_m$$

(13)
is ill-defined, although $\Gamma_m$ exists and fulfills eq.(12).

In order to discuss this point and to carry out properly the renormalization of the model we choose to work within the BPHZL regularization scheme [14]-[16], by following the Lowenstein-Zimmermann prescription [18]-[20] to handle massless propagators.

For that purpose we identify

$$m = M(s - 1),$$

(14)
where $0 \leq s \leq 1$ and $M$ is a constant with the dimension of a mass. The subtraction operator $t_\gamma$ for a given divergent 1-PI graph or subgraph $\gamma$ involves both a subtraction around $p = 0, s = 0$ and around $p = 0, s = 1$ [17, 18, 19, 20]:

$$(1 - t_\gamma) = (1 - t^{\rho(\gamma)}_{p,s-1})(1 - t^{\delta(\gamma)}_{p,s}),$$

(15)
where $\rho(\gamma)$ is the IR subtraction degree and $\delta(\gamma)$ the UV subtraction degree for $\gamma$ [17, 18, 19, 20]. We point out that both subtractions around $s = 0$ and $s = 1$ are needed in order to guarantee the absence of IR singularities of the 1-PI Green functions in the physical limit $s \to 1$ ($m \to 0$).

The assignments of UV dimension for the fields and external sources, required to compute $\rho(\gamma)$ and $\delta(\gamma)$ for a given graph $\gamma$ involving the fields and the antifields of the model, are as follows: $A_\mu^a, \omega^a, \bar{\omega}^a$ have UV dimension 1, $B_a, A^{a*}_\mu, \omega^{a*}, \bar{\omega}^{a*}$ have UV dimension 2. The IR dimension coincides with the UV dimension.
Let us denote by $\Gamma_s$, $s < 1$ the symmetric quantum effective action, constructed according to the BPHZL subtraction prescription, fulfilling the ST identities
\[ S(I \Gamma_s) = \int d^4x \left( \frac{\delta \Gamma_s}{\delta A_{\mu}^a} \frac{\delta \Gamma_s}{\delta A^{\mu a}} + \frac{\delta \Gamma_s}{\delta \omega^a} \frac{\delta \Gamma_s}{\delta \omega^a} + \frac{\delta \Gamma_s}{\delta \bar{\omega}^a} \frac{\delta \Gamma_s}{\delta \bar{\omega}^a} \right) + m \frac{\partial \Gamma_s}{\partial \bar{\rho}} = 0. \tag{16} \]

During the renormalization procedure we will always keep $s < 1$. Only in the very end we will take the physical limit $s \to 1$. We remark that for $s \neq 1$ the ST identities in eq. (16) give rise to a violation of physical unitarity, due to the soft breaking term
\[ M(s - 1) \frac{\partial \Gamma_s}{\partial \bar{\rho}}. \tag{17} \]

This can be explicitly verified by using methods close to the one discussed in \[26\]. We recover physical unitarity in the limit $s \to 1$. It can be proven by using the standard methods discussed e.g. in \[25\] that the functional identities in eqs. (8)-(11) can be restored at the quantum level. So we assume that they are also fulfilled by the symmetric quantum effective action $\Gamma_s$:
\[ \frac{\partial \Pi^{(j)}_s}{\partial \bar{\rho}} = 0, \quad \frac{\delta \Pi^{(j)}_s}{\delta B^a} = 0, \quad \frac{\delta \Pi^{(j)}_s}{\delta \omega^a} + \partial^\mu \frac{\delta \Pi^{(j)}_s}{\delta A^a_{\mu}} = 0, \]
\[ \int d^4x \left( \frac{\delta \Pi^{(j)}_s}{\delta \omega^a} - f^{abc} \bar{\omega}^b \frac{\delta \Pi^{(j)}_s}{\delta B^c} \right) = 0, \quad j \geq 1. \tag{18} \]

From the third equation in the first line of eq. (18) we conclude that $\Pi^{(j)}_s$, $j \geq 1$ depends on $\bar{\omega}^a$ only through the combination
\[ \hat{A}^{a}_{\mu} = A^{a}_{\mu} + \partial_\mu \bar{\omega}^a. \tag{19} \]

### 2.1 One-loop order

At one-loop order the ST identities read
\[ S_0(\Pi^{(1)}_s) = 0. \tag{20} \]

The action-like part of the most general solution $\Pi^{(1)}_s$ to eq. (20), compatible with the additional symmetries in eq. (18), is constrained to have the form
\[ t^4 \Pi^{(1)}_s = \lambda_1^{(1)} \int d^4x G_{\mu}^a G^{\mu a} + \rho_1^{(1)} S_0(\int d^4x \hat{A}^{a}_{\mu} A^a_{\mu}), \tag{21} \]

where $t^4$ is the projection operator on the sector of dimension $\leq 4$ in the fields, the antifields and their derivatives. $t^4 \Pi^{(1)}_s$ exists since $s < 1$. 

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Let us comment on the R.H.S. of eq. (21). At one-loop level only $S_0$-invariant terms appear in $t^4 \Pi_s^{(1)}$. Moreover, we notice that they are all IR-safe (all monomials entering into the R.H.S. of eq. (21) have IR degree equal to 4). This follows since

$$S_0(\Phi^{s'}) = \frac{\delta \Gamma^{(0)}_{m=0}}{\delta \Phi}$$  \hspace{1cm} (22)$$

for $\Phi^{s'} = A_\mu^{a_s'}, \omega^{a_s'}, \bar{\omega}^{a_s'}$. We will discuss this point further in the next subsection.

$\lambda_1^{(1)}, \rho_1^{(1)}$ are free parameters entering into the solution, unconstrained by the ST identities and the additional symmetries in eq. (18). They can be fixed by providing a choice of normalization conditions. As an example, one might choose

$$\xi^{(1)}_{G_{\mu \nu} G_\alpha_a} = 0, \quad \xi^{(1)}_{A_\mu^{a'}} \partial_\mu \omega^a = 0,$$  \hspace{1cm} (23)$$
yielding

$$\lambda_1^{(1)} = 0, \quad \rho_1^{(1)} = 0. \hspace{1cm} (24)$$

In the following we will not restrict ourselves to a special choice of normalization conditions, so we will keep $\lambda_1^{(1)}, \rho_1^{(1)}$ free.

**2.2 Higher orders**

At orders higher than one the ST identities read

$$S_0(\Pi_s^{(n)}) = - \sum_{j=1}^{(n-1)} (\Pi_s^{(n-j)}, \Pi_s^{(j)}). \hspace{1cm} (25)$$

The brackets are given in eq. (11). Eq. (25) is an inhomogeneous linear equation whose unknown is the action-like part of $\Pi_s^{(n)}$. The fact that the non-local terms from the R.H.S. of the above equation cancel against the non-local contributions from the L.H.S. is a consequence of the Quantum Action Principle [33] and of the assumption that the ST identities have been restored up to order $n - 1$. Therefore we can restrict ourselves to the local approximation of $\Pi_s^{(n)}$, the formal power series in the fields, the antifields

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3The notation is as follows. We expand $t^4 \Pi_s^{(1)}$ into a sum of linearly independent, Lorentz-scalar action-like functionals $\mathcal{M}_i(x)$ in the fields, the antifields and their derivatives, providing a basis for the space to which $t^4 \Pi_s^{(1)}$ belongs, and write accordingly $t^4 \Pi_s^{(1)} = \sum \int d^4 x \xi^{(1)}_i \mathcal{M}_i(x)$. $\xi^{(1)}_i$ is the coefficient of $\mathcal{M}_i(x)$ in this expansion. As an example, $\xi^{(1)}_{G_{\mu \nu} G_\alpha_a}$ is the coefficient of $G_{\mu \nu} G_\alpha_a$, $\xi^{(1)}_{A_\mu^{a'} \partial_\mu \omega^a}$ the coefficient of $A_\mu^{a'} \partial_\mu \omega^a$ in the expansion of $t^4 \Pi_s^{(1)}$. 

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and their derivatives corresponding to the Taylor expansion of all relevant 1-PI Green functions around zero momentum. This is possible since \( s < 1 \).

The solution \( t^4 \Pi_s^{(n)} \) to eq. \( \text{(25)} \) is \( [1] \)

\[
\lambda_1^{(n)} \int d^4 x A^a_\mu \delta A^a_\mu + \rho_1^{(n)} \mathcal{S}_0 \left( \int d^4 y \hat{A}^{\alpha \nu} A^a_\mu \right)
\]

\[
= \int d^4 x A^a_\mu \delta A^a_\mu \left[ \sum_{j=1}^{n-1} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4 y G^{\sigma \rho} G^{\sigma \rho} \right]
\]

\[
+ \sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \int d^4 x \frac{1}{g^2} \left( \square A^d_\rho - \partial_\rho (\partial A)^d \right) A^d_\rho
\]

\[
+ \sum_{j=1}^{n-1} \rho_1^{(n-j)} \rho_1^{(j)} \left( \int d^4 x \frac{2}{g^2} f^{d m} A^m_\sigma \left( \partial_\sigma A^m_\rho - \partial_\rho A^m_\sigma \right) A^d_\rho
\]

\[
+ \int d^4 x \frac{1}{4g^2} f^{q k} f^{k r d} A^q_\sigma A^r_\sigma A^d_\rho
\].

(26)

In contrast with one-loop level, non-symmetric counterterms enter in \( t^4 \Pi_s^{(n)} \). They appear due to the inhomogeneous term in the R.H.S. of eq. \( \text{(25)} \), which depends on \( \Pi^{(k)}_s \), \( k < j \).

In an arbitrary Lorentz-covariant gauge an additional invariant appears in the R.H.S. of eq. \( \text{(21)} \), given by

\[
\rho_2^{(1)} \mathcal{S}_0 \left( \int d^4 x \omega^{a \nu} \omega^a \right).
\]

(27)

This in turn implies that the R.H.S. of eq. \( \text{(25)} \) gets more involved, forbidding the application of the elegant homotopy techniques \( [1] \) which in the case of the Landau gauge allow to easily solve the inhomogeneous problem in eq. \( \text{(25)} \). As we will show in a moment, the IR properties of the theory and their relationship with the cohomology of the operator \( \mathcal{S}_0 \) do not depend on the choice of the gauge. Therefore we choose to restrict ourselves to the Landau gauge, for which there exists the simple and compact form for the general solution \( t^4 \Pi_s^{(n)} \) given by eq. \( \text{(26)} \).

The non-symmetric counter-terms, depending on the lower order contributions, disappear if one chooses to impose the following normalization condition for \( t^4 \Pi_s^{(j)} \):

\[
\rho_1^{(j)} = 0, \quad j = 1, 2, \ldots, n - 1
\]

(28)
equivalent to
\begin{equation}
\xi^{(j)}_{\hat{A} a^{\mu} \partial^\mu \omega_a} = 0, \quad j = 1, 2, \ldots, n - 1.
\end{equation}
(29)

We might supplement it by the choice
\begin{equation}
\lambda_1^{(j)} = 0, \quad j = 1, 2, \ldots, n - 1,
\end{equation}
equivalent to
\begin{equation}
\xi_{\hat{G}^a_{\mu \nu}, G_{\mu \nu} a}^{(j)} = 0, \quad j = 1, 2, \ldots, n - 1.
\end{equation}
(31)

Eqs. (29) and (31) extend the one-loop normalization conditions in eq. (23).

One can verify that all monomials in the R.H.S. of eq. (26) are IR safe by eq. (22). We wish to comment on the IR-safeness of the monomials entering into $t^4 I_\Gamma^{(n)}$. In the present approach this property stems from the fact that the classical linearized ST operator $S_0$ in eq. (5) has in the primed variables in eq. (6) definite degree +1 with respect to the counting operator of the fields, the antifields and their derivatives. The existence of the antifield redefinition in eq. (6) is in turn a consequence of the fact that $(\tilde{\rho}, m = M(s - 1))$ form a BRST doublet. Since this is true also in an arbitrary Lorentz-covariant gauge, the above result extends to that case too.

In [19, 20] the IR power-counting criteria were obtained by means of convergence arguments only and they did not display any relationship with the cohomological properties of $S_0$. Their cohomological interpretation in terms of the degree of $S_0$, made possible by the identification in eq. (14), now shows that they can be actually understood on purely cohomological grounds.

3 The limit $s \to 1$

In the previous section we have derived the most general form of the action-like part of $I_\Gamma$, $s < 1$ to all order in the loop expansion. We have checked that all action-like terms in $I_\Gamma$, $s < 1$ are IR-safe. UV convergence criteria are also satisfied. This is a sufficient condition [17, 18, 19, 20] to guarantee the existence of the limit $s \to 1$:

\begin{equation}
I_\Gamma = \lim_{s \to 1} I_\Gamma^s.
\end{equation}
(32)

In the limit $s \to 1$ ($m \to 0$) the primed antifields in eq. (6) reduce to their unprimed counterparts. The ST identities obeyed by $I_\Gamma$ read
\begin{equation}
S(I_\Gamma) = \int d^4x \left( \frac{\delta I_\Gamma}{\delta A^a_{\mu}}, \frac{\delta I_\Gamma}{\delta A^a_{\mu}} + \frac{\delta I_\Gamma}{\delta \omega^a}, \frac{\delta I_\Gamma}{\delta \omega^a} + \frac{\delta I_\Gamma}{\delta \bar{\omega}^a}, \frac{\delta I_\Gamma}{\delta \bar{\omega}^a} \right) = 0.
\end{equation}
(33)
The soft-breaking term in eq. (17) has disappeared. This ensures the physical unitarity of the model [34]-[37]. The non-symmetric counterterms entering into $\Pi^{(j)}$ at order $j \geq 2$ do not vanish in the limit $s \to 1$. Hence they also appear in $\Pi^{(j)}$, unless the special choice of normalization conditions in eq. (29) has been done for $\Pi^{(k)}$, $k < j$.

We wish to comment on the dependence of physical observables on the parameters $\rho^{(j)}_{1}$. $\rho^{(j)}_{1}$ enters in $\Pi^{(j)}$ as the coefficient of the $S_{0}$-exact functional $S_{0}(\int d^{4}x A^{a}_{\mu} A^{a}_{\mu})$. Hence physical observables should not depend on $\rho^{(j)}_{1}$. The study of the dependence of the Green functions of local BRST invariant operators on these parameters can be carried out according to the standard procedure [25, 38], relying on the extension of the BRST differential $s$ in such a way to incorporate $\rho^{(j)}_{1}$ into a BRST doublet together with its partner $\theta^{(j)}_{1}$:

$$s\rho^{(j)}_{1} = \theta^{(j)}_{1}, \quad s\theta^{(j)}_{1} = 0.$$  \hfill (34)

The corresponding ST identities yield for $\Pi$

$$S' (\Pi) = S (\Pi) + \sum_{j} \theta^{(j)}_{1} \frac{\partial \Pi}{\partial \rho^{(j)}_{1}} = 0,$$  \hfill (35)

where $S (\Pi)$ is given in eq. (33). Upon taking the Legendre transform $W$ of $\Pi$ with respect to the quantized fields of the model the ST identities read for the connected generating functional $W$:

$$S' (W) = - \int d^{4}x \left( J^{a}_{\mu} \delta W + J^{a}_{\omega} \frac{\delta W}{\delta \omega^{a}} + J^{a}_{\bar{\omega}} \frac{\delta W}{\delta \bar{\omega}^{a}} \right) + \sum_{j} \theta^{(j)}_{1} \frac{\partial W}{\partial \rho^{(j)}_{1}} = 0.$$  \hfill (36)

In the above equation $J^{a}_{\mu}$ is the external source coupled to $A^{a}_{\mu}$, $J^{a}_{\omega}$ the source coupled to $\omega^{a}$ and $J^{a}_{\bar{\omega}}$ the source coupled to $\bar{\omega}^{a}$. Now we differentiate eq. (36) with respect to $\theta^{(j)}_{1}$ and with respect to the sources $\beta_{1}(x_{1}), \ldots, \beta_{n}(x_{n})$, coupled to local BRST-invariant operators $\mathcal{O}_{1}(x_{1}), \ldots, \mathcal{O}_{n}(x_{n})$ and go on-shell ($J = \beta = \theta = 0$). We obtain

$$\frac{\delta^{(n+1)}W}{\delta \rho^{(j)}_{1} \delta \beta_{n}(x_{n}) \ldots \delta \beta_{1}(x_{1})} \bigg|_{\text{o.s.}} = 0.$$  \hfill (37)

Therefore the Green functions of local BRST-invariant operators are $\rho^{(j)}_{1}$-independent, as a consequence of the ST identities in eq. (35). At the level of the 1-PI generating functional this property is encoded into the Nielsen-like identity [41, 38, 39, 40] obtained by differentiating eq. (35) w.r.t. $\theta^{(j)}_{1}$:

$$\frac{\partial \Pi}{\partial \rho^{(j)}_{1}} = \hat{S}_{\Gamma} \left( \frac{\partial \Pi}{\partial \theta^{(j)}_{1}} \right).$$  \hfill (38)
where

\[
\hat{S}_{\Pi} = \int d^4x \left( \frac{\delta \Pi}{\delta A^a_\mu} \frac{\delta}{\delta A^a_{\mu}} + \frac{\delta \Pi}{\delta A^a_\mu} \frac{\delta}{\delta A^a_{\mu}} + \frac{\delta \Pi}{\delta \omega^{a*}} \frac{\delta}{\delta \omega^a} + \frac{\delta \Pi}{\delta \omega^{a*}} \frac{\delta}{\delta \omega^a} + \delta \frac{\Pi}{\delta A^a_\mu} \delta \frac{\Pi}{\delta A^a_{\mu}} + \delta \frac{\Pi}{\delta \omega^{a*}} \delta \frac{\Pi}{\delta \omega^a} \right) + \sum_k \theta^{(k)}_1 \frac{\partial}{\partial \theta^{(k)}_1}.
\]  

(39)

The possible deformations of eq.(38), compatible with the Quantum Action Principle and nilpotency of \(\hat{S}_{\Pi}\), can be studied by using the methods developed in [38, 42]. Under the assumption that the ST identities in eq.(33) hold true (so that \(\hat{S}_{\Pi}^2 = 0\)) the most general structure of the renormalized Nielsen-like identity turns out to be

\[
\frac{\partial \Pi}{\partial \rho^{(j)}_1} = (1 + \sigma^\rho) \hat{S}_{\Pi} \left( \frac{\partial \Pi}{\partial \theta^{(j)}_1} \right) + \beta^\rho \frac{\partial \Pi}{\partial g} + \sum_\varphi \gamma_\varphi^\rho N_\varphi \Pi.
\]  

(40)

In the above equation \(\beta^\rho\) parameterizes the explicit dependence of the coupling constant \(g\) on \(\rho^{(j)}_1\). Such a dependence might be induced by the choice of normalization conditions for the physical parameter \(g\) that explicitly depend on \(\rho^{(j)}_1\). \(\varphi\) stands for any of the fields of the model and \(N_\varphi\) denotes the corresponding counting operator. \(\sigma^\rho\), \(\beta^\rho\) and \(\gamma_\varphi^\rho\) control the deformations of the Nielsen identity in eq.(38).

It is possible to recursively choose the counterterms of the model, order by order in the loop expansion, in such a way that

\[
\sigma^\rho = 0, \quad \beta^\rho = 0, \quad \gamma_\varphi^\rho = 0.
\]

In this case we recover eq.(38).

However, one can relax the conditions on \(\sigma^\rho\) and \(\gamma_\varphi^\rho\) while preserving the property of the independence of physical observables of \(\rho^{(j)}_1\). Indeed, since the terms \(N_\varphi \Pi\) are equivalent to the insertion of \(\hat{S}_\Gamma\)-exact local operators [25, 42], they do not affect the dependence of physical Green functions on \(\rho^{(j)}_1\) [42]. The same is true for the first term in the R.H.S. of eq.(40).

Therefore the only condition needed in order to guarantee the independence of physical observables of \(\rho^{(j)}_1\) is

\[
\beta^\rho = 0.
\]  

(41)

Otherwise said, no spurious dependence on \(\rho^{(j)}_1\) must be generated via \(\rho^{(j)}_1\)-dependent normalization conditions for the coupling constant \(g\) (equivalently, for the \(\lambda_1^{(k)}\)-counterterms).

We remark that the above analysis relies on the fulfillment of the ST identities in eq.(33). The non-symmetric counterterms in eq.(26) are required in order to ensure that eq.(33) holds true at orders \(n \geq 2\). They must be included in order to guarantee the validity of eq.(37).
4 Conclusions

In this paper we have analyzed the restoration of the ST identities for pure massless YM theory in the Landau gauge within the BPHZL renormalization scheme and the Zimmermann-Lowenstein prescription for handling massless propagators. We have explicitly obtained the most general form of the action-like part of the symmetric regularized action $\Gamma_s$, $s < 1$, to all orders in the loop expansion, and we have shown that non-symmetric counterterms arise in $\Gamma^{(j)}_s$, $j \geq 2$, unless a special choice of normalization conditions for $\Gamma^{(k)}_s$, $k < j$ is done.

We have verified that both UV and IR power-counting criteria are fulfilled for $t^4 \Gamma_s$, thus guaranteeing the existence of the physical limit $\Gamma = \lim_{s \to 1} \Gamma_s$ (absence of zero-mass singularities).

We have provided a cohomological interpretation of the IR power-counting criterion, by noticing that it follows from the fact that the IR regulator $m = M(s - 1)$ enters into the classical action, together with its BRST partner $\bar{\rho}$, only via a cohomologically trivial term.

We have shown that the non-symmetric counterterms appearing in $\Gamma^{(j)}_s$ at orders $j \geq 2$ do not vanish in the limit $s \to 1$. We have analyzed the dependence of the Green functions of physical observables on the coefficients $\rho^{(k)}_1$ entering into the parameterization of the non-symmetric counterterms and we have discussed the associated Nielsen-like identities.

The proper inclusion of the non-symmetric counterterms is strictly necessary in order to guarantee the fulfillment of the ST identities at orders higher than one in the loop expansion, for general lower-orders normalization conditions.

Among the models with massless particles the most phenomenologically important ones are undoubtedly the SM and the MSSM. The proof that the relevant ST identities can be restored to all orders in perturbation theory has been given for the SM in [22, 23] and for the MSSM in [24]. Still the explicit construction of the symmetric 1-PI generating functional for the SM and the MSSM poses some problems, due to the lack of an invariant regularization scheme. For the SM and especially for the MSSM the explicit recursive evaluation of the ST breakings at the regularized level and of the finite counterterms required to recover them is significantly involved already at one loop and becomes unfeasible at higher orders.

The direct restoration of the ST identities, together with the method for handling massless particles analyzed in the present paper, seems to be a promising tool in order to explicitly construct the symmetric 1-PI Green functions for the SM and the MSSM, to all orders in the loop expansion.
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A The classical action

The classical action of pure massless Yang-Mills theory in the Landau gauge with an IR regulator $m$ introduced via the BRST doublet $(\bar{\rho}, m)$ is

$$
\Gamma^{(0)} = \int \! d^4 \! x \left\{ -\frac{1}{4g^2} G^a_{\mu \nu} G^{\mu \nu \cdot a} - \bar{\omega} a \partial_\mu (D_\mu \omega)^a + B^a \partial A^a + A^a_{\mu} (D^\mu \omega)^a \\
- \omega^a \frac{1}{2} f^{abc} \omega^b \omega^c + \bar{\omega}^a B^a \right\} + \int \! d^4 \! x s \left( \frac{1}{2} \bar{\rho} m (A^a_\mu)^2 + \bar{\rho} m \bar{\omega}^a \omega^a \right) \\
= \int \! d^4 \! x \left\{ -\frac{1}{4g^2} G^a_{\mu \nu} G^{\mu \nu \cdot a} - \bar{\omega} a \partial_\mu (D_\mu \omega)^a + B^a \partial A^a + A^a_{\mu} (D^\mu \omega)^a \\
- \omega^a \frac{1}{2} f^{abc} \omega^b \omega^c + \bar{\omega}^a B^a + \frac{1}{2} m^2 (A^a_\mu)^2 + m^2 \bar{\omega}^a \omega^a \\
- \bar{\rho} m A^a_\mu \partial^\mu \omega^a - \bar{\rho} m B^a \omega^a - \frac{1}{2} \bar{\rho} m \bar{\omega}^a f^{abc} \omega^b \omega^c \right\}. \quad (42)
$$

$G^a_{\mu \nu}$ is the field strength

$$
G^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu. \quad (43)
$$

$\omega^a$ is the ghost field, $\bar{\omega}^a$ the antighost field, $B^a$ the associated Nakanishi-Lautrup multiplier field.

BRST transformations

$$
sA^a_\mu = (D_\mu \omega)^a = \partial_\mu \omega^a + f^{abc} A^b_\mu \omega^c, \quad s\omega^a = -\frac{1}{2} f^{abc} \omega^b \omega^c, \\
s\bar{\omega}^a = B^a, \quad sB^a = 0, \\
s\bar{\rho} = m, \quad sm = 0. \quad (44)
$$

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