The Supersymmetric Axion and Cosmology

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Abstract

In this lecture\textsuperscript{1} we review several cosmological issues associated with the axion. Axion solves the strong CP problem and is a good candidate for the dark matter. Limits, which are imposed by the value of isocurvature fluctuations fraction in the observed CMBR from WMAP data and the domain wall problem are discussed in a supersymmetric flat direction axion model.

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1 Introduction

At present astrophysical data suggest that the observable part of the Universe contributes only a few percents to the total energy density. Most of the content of the Universe is in the form of the dark matter (25%) and dark energy (70%). In this lecture we will review one of the most viable candidate for the dark matter, which is axion particle,\(^2\) that originally arose as a solution to the strong CP problem. Axion decay constant must be very large compared to the electroweak scale, in order to not confront astrophysical data, which yields the so-called ”invisible axion” models. Since axion interacts with other particles with the strength inversely proportional to its decay constant it turns to be a good dark matter candidate.

It is worth to mention that string theory provides many periodic fields, associated with global symmetries spontaneously broken down to some discrete subgroups. Although, naively the decay constants of string axions are of order of \(\frac{M_{pl}}{32\pi^2} \sim 10^{16}\)GeV, which is too large for a QCD axion decay constant, one might hope to construct a QCD axion model out of string theory axions. The rest of these periodic fields may yield a solution to the dark energy if they get stuck away from the minimum of their potentials.

Below we discuss some axion models stressing some important issues which arise if the QCD axion does exist and contribute to the dark matter. In section 2 we briefly remind the usual QCD story and the originally proposed ”invisible” axion models. In the section 3 we discuss supersymmetric axion which corresponds to some flat direction. Finally we discuss axion isocurvature fluctuations and the domain wall problem in the context of the supersymmetric axion model. The last two issues impose a significant constraints on any axion model. We review how one can avoid corresponding difficulties.

2 The Strong CP Problem and The ”Invisible” Axion.

The Lagrangian of QCD, besides ordinary terms, contains the total derivative term or \(\theta\)-term:

\[
\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^{a}_{\mu\nu} \tilde{G}^{a\mu\nu},
\]

where \(\tilde{G}^{a\mu\nu}\) is the dual to the field strength tensor. This term is gauge and Lorentz invariant, therefore it should be incorporated in the full QCD Lagrangian. However, one

\(^2\)There are two other candidates for the dark matter, which are neutralino or LSP and mirror world matter, which we do not discuss here.
can check easily that this $\theta$-term is a total derivative since it can be rewritten as

$$\mathcal{L}_\theta = \theta \partial_\mu K^\mu$$  \hspace{1cm} (2)

with

$$K^\mu = \frac{1}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} (A^\nu_\alpha A^\alpha_\beta + \frac{1}{3} f_{abc} A^a_\nu A^b_\alpha A^c_\beta).$$  \hspace{1cm} (3)

This term being a total derivative does not affect the equation of motion and if one would have trivial vacuum structure like that in QED this term would not affect any physics at all. However, because of the rather complicated vacuum structure of QCD, i.e. the presence of instantons, the integral over this total derivative does not vanish. Therefore, this term contributes to the action of QCD and must be retained.

First, let us note that this term violates CP symmetry, thus it can give rise to neutron electric dipole moment. Current experiments set a strong bound on the value of $\theta^3$, which is

$$\theta < 10^{-9}. \hspace{1cm} (4)$$

It has to be explained why this number is so unnaturally small. Peccei and Quinn [2] proposed a very elegant solution to that problem (although not unique). Namely, consider the case when the Lagrangian has an additional $U(1)_{PQ}$ symmetry. If this symmetry is broken at some energy scale, which we denote by $f_a$, there will be NG-boson generated. This NG-boson is called axion$^4$. It is massless and it couples to the gauge bosons and to the matter fields with the strength which is inversely proportional to the scale of the PQ-symmetry breaking $f_a$. Nonperturbative instanton QCD effects, however, give an axion a potential of the form, which is well approximated by

$$V_{QCD}(T) = f_a^2 m_a^2(T) [1 - \cos \left( \frac{aN}{f_a} \right)], \hspace{1cm} (5)$$

breaking $U(1)$ down to discrete subgroup $Z_N$, with the mass of the axion depending on the quark masses and on the temperature of the quark–gluon plasma. This naturally sets the coefficient in front of axion-gluon-gluon term in the QCD Lagrangian to zero, since minimum of that potential is at $a = 0$. Thus, PQ solution solves the strong CP problem.

At zero temperature the axion mass is given by

$$m_a^2 = \frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u} \frac{\Lambda_{QCD}^3}{f_a^2} \sim \frac{f_a^2 m_a^2}{f_a^2}, \hspace{1cm} (6)$$

where $\Lambda_{QCD}$ is the QCD scale and can be related to the pion mass and decay constant by $m_a^2 = 2(m_u + m_d)\Lambda_{QCD}^3/f_a^2$. If $\Lambda_{QCD} \ll f_a$ then the axion turns out to be very light.$^3$

$^3$Neutron electric dipole moment can be computed using current algebra methods [1].

$^4$One of the other two known solutions to the strong CP-problem is what is called Nelson-Barr mechanism[3]. Second possibility is to set the mass of $u$-quark to be zero.
comparing to the QCD scale. At high temperature axion is even lighter and its mass is temperature dependent [4]

\[ m_a(T) = 0.1 m_a(T = 0) \left( \frac{\Lambda_{QCD}}{T} \right)^{3.7}. \] (7)

There are three historically consequential models of the axion:

- Weinberg-Wilczek axion
- KSVZ axion (Kim; Shifman, Vainshtein, Zakharov) [5]
- ZDFS axion (Zhitnitskii; Dine, Fishler, Srednicki) [6]

In the first model $U(1)_{PQ}$ symmetry is broken at the EW scale. There are two Higgs doublets and the axion is

\[ a = \frac{1}{v} (v_\phi I m\phi_0 - v_\chi I m\chi_0), \] (8)

where $\phi_0$ and $\chi_0$ are the neutral components of the Higgs doublets. The PQ-symmetry is broken at $v = \sqrt{v_\phi^2 + v_\chi^2}$, where $v \approx 250 \text{GeV}$. Such an axion is already ruled out by the experiment because it would lead to disastrous loss of energy by various cosmological objects via axion emission. The other constraint which is even stronger comes from accelerator experiments.

The second model and the third are so-called ”invisible” axion models. In the KSVZ model one introduces a complex scalar field $\Phi$ which couples to the hypothetical quark field in the fundamental representation of $SU(3)_c$:

\[ \delta \mathcal{L} = \Phi \bar{Q}_R Q_L + \text{h.c.} \] (9)

$\Phi$ is supposed to develop large expectation value $f_a/\sqrt{2}$ ($f_a \gg \Lambda_{QCD}$) and axion is defined to be

\[ a = f_a \text{Arg}\Phi. \] (10)

The low-energy coupling of the axion to gluons is then given by

\[ \delta \mathcal{L} = \frac{1}{32\pi^2} \frac{a}{f_a} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}, \] (11)

such that $\theta \rightarrow \theta + \frac{\phi}{f_a}$ in the lagrangian and one may apply Peccei and Quinn solution.

In the ZDFS model one starts with the additional scalar SM singlet field $\Sigma$. The scale of PQ-symmetry breaking is separated from the EW scale and the axion field is like in the Weinberg-Wilczek model but with $\Sigma$ field added:

\[ a = \frac{1}{V} (v_\phi I m\phi_0 - v_\chi I m\chi_0 + v_\Sigma I m\Sigma_0), \] (12)
where \( V = \sqrt{v_\phi^2 + v_\chi^2 + v_\Sigma^2} \approx v_\Sigma \) if \( v_\Sigma \) is large compared to the Higgs expectation value. It can be as large as GUT or even Planck scale.

Last two models are minimal extensions of the standard model, in which one can generate effectively the required coupling of the axion to gluons. At the same time in both cases one has to introduce a new scale, which is not defined by the model. This scale is essentially time independent and constrained by astrophysical and cosmological data to be in the range \( 10^9 \text{GeV} < f_a < 10^{12} \text{GeV} \). In the next section we discuss a model where one can go a little further then just introducing a new scale, but rather relating \( f_a \) to other parameters, such as SUSY breaking scale and the Hubble constant. This would have an important consequences, which are discussed in the next two sections.

### 3 Supersymmetric Case

All previous models require some extension of the electroweak model. However, one may want to consider some supersymmetric version of the original picture. Such version would be, for example, an NMSSM model, which also yields a solution to \( \mu \) problem in the MSSM. One adds additionally at least one singlet chiral field with the most general superpotential. For a particular choice of couplings this looks like supersymmetrised version of previously mentioned ZDFS axion model. One can generate axion-gluon-gluon term and apply PQ solution. However, supersymmetric case is different. Besides light axion one has additionally another PQ charged scalar, which is the radial part of a complex PQ field. This partner of axion is called saxion. Saxion acquires large mass due to supersymmetry breaking and may influence evolution of the axion field. There are flat directions associated with different fields and one can imagine the following potential\(^5\)

\[
V(S) = \frac{\lambda |S|^6}{M_{pl}^2} - H^2 |S|^2 - m_{3/2}^2 |S|^2 + V_{QCD}(T),
\]

where \( H \) is the Hubble constant, \( S = |S|\exp(a/|S|) \) is the PQ superfield, \( |S| \) is saxion, and \( a \) is axion. Such potential naturally appears as a result of lifting of the flat direction \( S \) by

- supersymmetry breaking effects in the early Universe, which leads to the masses of order of \( H^2 \) [8],

- nonrenormalizable operators, suppressed by the Planck scale, the lowest of which we take as an example in the potential above,

- zero curvature term of order of the SUSY breaking scale.

\(^5\)This is similar to [7].
This potential has a form of the "mexican hat", and corresponding $U(1)$ symmetry is broken at the energy scale

$$f_a = |S| \sim (HM_{pl})^{1/2}$$

at early time. Note, that this effective scale of $U(1)$ breaking is time–dependent until $H \sim m_3/2$, when it gets frozen at $f_a \sim (m_3/2M_{pl})^{1/2} \sim 10^{11}$GeV. We will discuss simple model with one inflaton which decays in the end of inflation quickly reheating the Universe up to some temperature $T_R$. In that case Universe becomes first radiation dominated. While it expands, it cools and at temperature $T \lesssim \Lambda_{QCD}$ the NG boson, which travels along the bottom of "mexican hat" gets the mass $m_a(T \lesssim \Lambda_{QCD}) \approx f_\pi m_\pi/f_a$ and starts to oscillate. This happens at the value of the Hubble constant given by

$$H_{QCD} = \left( \frac{\Lambda_{QCD}}{T_R} \right)^2 H_I,$$

where $H_I$ is the value of the Hubble constant during inflation. Taking $T_R \sim 10^9$GeV, and $H_I \sim 10^{13}$GeV one obtains

$$H_{QCD} \sim 10^{-5}$$

The initial amplitude of the axion field is $a_0 \sim f_a$, which corresponds to $\frac{\theta_a}{f_a} = \theta_a \sim \mathcal{O}(1)$.

Therefore from the kinetic term one can get a rough estimate of the axion energy density at this moment

$$\rho \sim m_a^2 f_a^2 \approx f_\pi^2 m_\pi^2 \approx (0.1\text{GeV})^4,$$

which constitutes a tiny fraction of the total energy density

$$\frac{\rho_a}{\rho_{tot}} \sim \frac{f_a^2}{M_{pl}^2} \approx 10^{-16}.$$

More accurate estimates, including decays of the axionic strings and domain walls lead to a larger value by an order of magnitude (for review, see, for example [9, 10]). Axion is so light and weakly interacting that it remains being unthermalized, coherently oscillating field until present times contributing to the CDM density.

Since axion exists in the form of the cold matter its energy density decreases slower by a factor of $a$, then that of radiation, with $a$ being the scale factor in Robertson-Walker metric. Because of that, this small fraction grows over time and eventually starts to dominate the energy density. In the simple model, which we consider, that restricts $f_a$ to be somewhere lower than $10^{11} - 10^{12}$GeV, which is consistent with the value we have obtained above.

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6There is no reason why initially the phase of the axion should be much different from unity, unless one employs anthropic principle.
4 Isocurvature Fluctuations and Domain Walls

First issue which shall be discussed is the limit coming from the observed CMB anisotropy [11] on the value of isocurvature fluctuations produced by axion. Isocurvature fluctuations appear when there are two or more fields, which generate quantum fluctuations during inflation. Fluctuations in the density of one of them may be compensated by some other field, so that curvature remains unperturbed. These kind of fluctuations thus are called *isocurvature* fluctuations. In the case of massless (or nearly massless) axion and massive saxion isocurvature fluctuations are fluctuations in PQ charge or, in other words, fluctuations in the phase $\theta$. The size of primordial spectrum of such fluctuations is set by

$$\frac{\delta \theta}{\theta} = \Omega_a \frac{H_I}{2\pi f_a},$$

(19)

where $\Omega_a$ is axion contribution to the total density. This can be understood as follows. First, one finds that the equation of motion

$$\ddot{\tilde{\theta}}_k + 3H\dot{\tilde{\theta}}_k + \frac{k^2}{a^2}\theta = 0$$

(20)

after rescaling $\theta_k \rightarrow \frac{\tilde{\theta}_k}{a}$ and going to conformal time reads as

$$\tilde{\theta}''_k + (k^2 - \frac{a''}{a})\tilde{\theta}_k = 0$$

(21)

and has a solution

$$\tilde{\theta}_k = \frac{e^{-ik\tau}}{\sqrt{2k}}(1 + \frac{i}{k\tau}),$$

(22)

which gives at superhorizon scales

$$|\theta_k|^2 = \frac{1}{2a^3k} = \frac{H^2}{2k^3}.$$  

(23)

Since power spectrum is defined as

$$P(k) = \frac{k^3}{2f_a^2\pi^2}|\theta_k|^2,$$

(24)

one can see that the size of the phase fluctuations is $H_I/2\pi f_a$.

The observed size of CMB anisotropy is of order of $10^{-5}$, which means that

$$\Omega_a \frac{H_I}{2\pi f_a} < 10^{-5}$$

(25)
at least\textsuperscript{7}. Moreover, the observation tell us that fluctuations are almost pure adiabatic which restricts (25) even stronger. Although, it is well known that isocurvature fluctuations tends to decay into adiabatic ones [13], in the case of the axion the limit above still is a considerable underestimate.

However, even (25) impose stringent constraints. Having Hubble constant of order of $10^{12}\text{GeV}$ requires $f_a > 10^{16}\text{GeV}$, which is far above the upper limit on $f_a$ imposed in order to not overclose the Universe. In the scenario described in the previous section this problem is naturally avoided, since $f_a$ is time dependent. During inflation its value is given by $(H_I M_{pl})^{1/2} \sim 10^{16}\text{GeV}$, which has just about right size.\textsuperscript{8} After inflation the value of $f_a$ decreases with time as $t^{-1/2}$ until it freezes at $(m_3/2 M_{pl})^{1/2} \sim 10^{11}\text{GeV}$. Such scenario seems quite plausible because it solves isocurvature fluctuations problem, and one does not overclose the Universe at the same time.

The other question which shall be discussed is the domain walls problem. The problem arises as following. When $U(1)$ is broken down to $Z_n$, the vacuum of the theory falls into $N$ degenerate, but disconnected and, thus, distinct regions. As it is well known, this leads to topological defects which appears as kink solutions between each of the disconnected piece of vacuum manifold. Simple example is the Higgs–like potential of the real scalar field which possess $Z_2$ discrete symmetry. Namely, this theory has two vacua $\phi = \pm v$. If there are two regions in space lying in different vacua, one can easily obtain kink solution of the classical equation of motion, with boundary conditions

$$\phi(x = -\infty) = -v, \quad \phi(x = \infty) = v. \quad (26)$$

It easy to see that there is energy density between boundaries, which has maximum at $x = 0$, and its characteristic thickness would be inversely proportional to the mass of the scalar. This object is a simple example of a domain wall.

Topologically formed domain walls in case of the exact discrete symmetry are absolutely stable. Since domain walls are two-dimensional objects their energy density behaves as $1/a$, while the Universe is expanding. This is much slower than that for matter $(1/a^3)$ and radiation $(1/a^4)$, thus, leading to cosmological disaster very fast. Since we do not observe our Universe to be domain walls dominated, means that if they were ever formed their density either was diluted up to cosmologically safe densities, i.e. less then one per horizon, or there are no exact discrete symmetries, so that domain walls collapse before they dominate Universe.\textsuperscript{9}

Below we will discuss both possibilities to avoid domain walls problem. A simple

\textsuperscript{7}For more detailed analysis see [12]

\textsuperscript{8}If one includes higher order nonrenormalizable terms of the form $\frac{\lambda |S|^{2n+4}}{M_{pl}^{2n}}$ the PQ symmetry is broken at higher values $f_a > 10^{16}\text{GeV}$, which relaxes the constraint from isocurvature fluctuations.

\textsuperscript{9}One other issue is that domain walls, living long enough, lead to a nongaussianity of CMB, which is in direct contradiction with current observation [14].
resolution comes in the case when PQ phase transition occurs before or during inflation. Parts of the Universe sitting in different vacua grow into very large regions, such that our Universe turns out to be living in one of those. We need to make sure, that reheating of the Universe does not restore $U(1)$ at any time after inflation, so that the Universe stays in the same vacuum until QCD effects turn on. Provided both conditions are satisfied, domain walls which might be formed at $T \sim \Lambda_{QCD}$ are not stable topologically. They do not separate different vacua and, thus, will decay.

For that to work we have to require that

$$f_a > H_I, T_R,$$  \hspace{1cm} (27)

which seems plausible in the picture described in the previous section, since at early times the value of $f_a$ is

$$f_a \sim (H_I M_{pl})^{\frac{1}{2}} \sim 10^{16} \text{GeV},$$  \hspace{1cm} (28)

when one takes $H_I \sim 10^{12} \text{GeV}$ and $T_R \sim 10^9 \text{GeV}$ as in the simplest chaotic inflation scenario. Therefore, the simple construction described in the Section 2 works well to solve the domain wall problems too.

There is one more case one may want to look at. Similar to (5) we can imagine the potential of the form

$$V(S) = \frac{\lambda |S|^6}{M_{pl}^2} + H^2 |S|^2 - m_{3/2}^2 |S|^2 + V_{QCD}(T).$$  \hspace{1cm} (29)

In that case phase transition occurs late, with

$$f_a \sim (m_{3/2} M_{pl})^{\frac{1}{2}} \sim 10^{11} \text{GeV}.$$  \hspace{1cm} (30)

This automatically solves isocurvature fluctuations problem, since there are none generated during inflation. But one is still confronted with the domain walls problem in that case. There is still a possibility to made these walls to collapse. For that one needs to break $Z_n$ symmetry, adding, for example, to the potential

$$\delta V = \mu^4 \cos(\theta),$$  \hspace{1cm} (31)

where $\mu$ is some scale. However, if one uses the restriction from neutron electric dipole moment from one hand, and require domain walls to collapse fast enough to not became cosmologically dangerous from another, one arrives to the following criterium

$$\left(\frac{m_a}{M_{pl}}\right) < \left(\frac{\mu}{f_a}\right)^2 < 10^{-9/2} \left(\frac{m_a}{f_a}\right),$$  \hspace{1cm} (32)

which numerically gives

$$10^{-5} \text{GeV} < \mu < 10^{-4} \text{GeV}$$  \hspace{1cm} (33)
up to factors of order one. This seems rather narrow window, which is hard to satisfy. Although, there might be constructed a model, where these limits are much broader [15].

In the end, we should note that the domain walls problem is safely avoided, if the discrete group is simply $Z_1$. This case can be constructed, and domain walls will not be topologically stable. Therefore, the issue with domain walls is significantly relaxed.

Finally, we should mention that there are other cosmological scenarios, i.e. those where the Universe is matter dominated soon after inflation [16], due to the moduli oscillations, such that Universe reheats several times. First is due to inflaton decay, second and further ones are due to moduli decay. There might also be an issue of parametric resonance effects [17], if one imagine that axion potential is very flat. Such scenario solves isocurvature fluctuations problem. However, it poses a domain wall problem, because after inflation axion acquires large fluctuations due to the instability, which is similar to that in Mathieu equations. This case is safe from the domain walls problem only in $N = 1$ models$^{10}$.

5 Conclusion

In this work we briefly went over axion story and two important issues which are

- the limits on axion models coming from the bound on the value of isocurvature fluctuations and
- the domain walls problem

Both issues seem to be very restrictive and provide a good test on the viability of any axion model. One can construct reasonable supersymmetric models which may safely avoid these problems. The domain walls may be either diluted if PQ transition occurs prior to the inflation, or one can break $Z_N$ symmetry by a tiny bit, making one of the vacua more preferable then the others. One can construct $Z_1$ models, which are domain walls problem free. Isocurvature fluctuations problem is solved in a model with time dependent PQ scale. More extended discussion of supersymmetric axion models is given in [18].

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$^{10}$ $N = 1$ domain walls are discussed in details in [19]
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