Low-lying Dipole Modes in $^{26,28}$Ne in the Quasiparticle Relativistic Random Phase Approximation

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The low-lying isovector dipole strengths in neutron rich nuclei $^{26}$Ne and $^{28}$Ne are investigated in the quasiparticle relativistic random phase approximation. Nuclear ground state properties are calculated in an extended relativistic mean-field theory plus BCS method where the contribution of the resonant continuum to pairing correlations is properly treated. Numerical calculations are tested in the case of isovector dipole and isoscalar quadrupole modes in the neutron rich nucleus $^{22}$O. It is found that in present calculation low-lying isovector dipole strengths at $E_x < 10$ MeV in nuclei $^{26}$Ne and $^{28}$Ne exhaust about 4.9% and 5.8% of the Thomas-Reiche-Kuhn dipole sum rule, respectively. The centroid energy of the low-lying dipole excitation is located at 8.3 MeV in $^{26}$Ne and 7.9 MeV in $^{28}$Ne.

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I. INTRODUCTION

Nuclear giant resonances have been known since 50 years ago for the dipole mode and more than 30 years for the other modes. But the research field was limited to the excitations of nuclei along the $\beta$-stability line\textsuperscript{1,2,3}. Recently, the radioactive ion beam physics has become one of the frontiers in nuclear physics. It offers the possibility to broaden the study of the giant resonance to weakly bound nuclei. Nuclei close to the drip line present some unique properties: a small separation energy of the valence nucleon, the smearing density distribution and a strong coupling between the bound

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state and the particle continuum. Those exotic properties attract more attentions both experimentally and theoretically. Low-lying electric dipole modes may appear in these weakly bound nuclei, which are so-called Pygmy Dipole Resonances. Although carrying only a small fraction of the full dipole strength these states are of particular interest because they are expected to reflect the motion of the neutron skin against the core formed with an equal number of protons and neutrons. Recent experiments have shown that the increase of the dipole strength at low energies in neutron-rich nuclei could affect the corresponding radiative neutron capture cross section considerably [4], which has a significance in astrophysics. Over the last decade, much experimental and theoretical efforts have been dedicated to investigate properties of the low-lying dipole mode in light neutron-rich nuclei, in particular, to answer whether or not these dipole excitations can be attributed to the collectivity [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

Recently, Beaumel et al. [17] have measured the inelastic scattering of $^{26}$Ne $^{208}$Pb using a 60 MeV/u $^{26}$Ne secondary beam at RIKEN. This reaction is dominated by Coulomb excitations and selective for $E1$ transitions. The experimental data are now under analysis [18]. As a subsequent work, they will continue the experiment using a more neutron rich projectile $^{28}$Ne. Therefore the theoretical investigation of low-lying dipole modes in $^{26}$Ne and $^{28}$Ne has a practical significance. $^{26}$Ne and $^{28}$Ne are neutron rich nuclei, whose Fermi surfaces are close to the particle continuum. Therefore the description of those nuclei has to explicitly include the coupling between bound states and the particle continuum. The contribution of the particle continuum to the nuclear properties at low-energies can mainly be attributed to a few resonant states [19, 20, 21, 22, 23]. On the other hand, it is well known that pairing correlations play an important role in describing properties of open shell nuclei. In order to depict the collective excitations of those nuclei pairing correlations have to be taken into account. Recently, a number of theoretical works has devoted to study the properties of low-lying dipole modes in the framework of the quasiparticle random phase approximation (QRPA) [4, 24, 25, 26, 27]. Paar and his co-workers [28] have studied the evolution of the low-lying dipole strength in Sn-isotopes in the quasiparticle relativistic random phase approximation (QRRPA) in the configuration space formalism.

In this paper, we aim at the investigation of the properties of low-lying dipole modes in neutron rich nuclei $^{26}$Ne and $^{28}$Ne in the QRRPA which is formulated in the response function method. The QRRPA is an extension of the fully consistent RRPA [29, 30, 31]
by taking into account the effect of pairing correlations. A consistent treatment of RRPA within the RMF approximation requires the configurations including not only the pairs formed from the occupied Fermi states and unoccupied states but also the pairs formed from the Dirac states and occupied Fermi states. It has been emphasized in Refs. [29, 30] that the inclusion of configurations built from the positive energy states in the Fermi sea and negative energy states in the Dirac sea is essential to give correctly a quantitative description in the excitation energies of isoscalar giant multipole resonances as well as to ensure the current conservation and decouple the spurious states. In present calculations, we pay more attentions to the energy weighted moment $m_1$ and the centroid energy of the low-lying dipole strength as well as the contribution of states around the Fermi surface to the low-lying dipole strength. Although some theoretical investigations show that $^{26}$Ne and $^{28}$Ne are deformed and strongly anharmonic [32, 33, 34], a spherical symmetry is assumed in the present investigation. In order to show the applicability of the method with a spherical assumption we also study the quadrupole excitations in these nuclei and compare the calculated position and transition strength of the lowest $2^+$ states with the experimental data.

In this work, the ground state properties of neutron rich nuclei $^{26}$Ne and $^{28}$Ne are calculated in the extended relativistic mean field and Bardeen-Cooper-Schrieffer (RMF+BCS) approximation [22], where the resonant continuum is properly treated. The empirical pairing gaps deduced from odd-even mass differences are adopted in the BCS calculation in this work. All calculations are performed with the parameter set NL3 [35], which gives a good description of not only the ground state properties [36] but also the collective giant resonance [29, 30, 31, 37, 38, 39].

The paper is arranged as follows. In Sec.II we present the formalism of the QRRPA in the extended RMF+BCS ground state in the response function approach. A test of the numerical calculation of the QRRPA in the neutron rich nucleus $^{22}$O is performed and compared with the results in Ref. [28], which is given in Sec.III. In Sec.IV the ground state properties of $^{26}$Ne and $^{28}$Ne are studied in the extended RMF+BCS approach. Then the QRRPA in the response function formalism is applied to study the properties of isovector giant dipole resonances in nuclei $^{26}$Ne and $^{28}$Ne. Finally we give a brief summary in Sec.V.
II. THE QUASIPARTICLE RELATIVISTIC RANDOM PHASE APPROXIMATION

There are usually two methods to obtain the RPA strength in the study of nuclear collective excitations. One is working in a particle-hole configuration space and solving the RPA equation by a matrix diagonalization method [30], and another is based on the linear response theory [40]. In the response function formalism one solves a Bethe-Salpeter equation by inversion. In both methods the starting point is a self-consistent solution of the nuclear ground state. In this paper we shall work in the response function formalism and study nuclear dipole excitations in neutron rich nuclei.

In the RRPA calculation we first solve the Dirac equation and equations of meson fields self-consistently in the coordinate space. The continuum is discretized by expanding nucleon spinors on a complete set basis, such as eigenfunctions in a spherical harmonic oscillator potential. Those single particle states are used to build the RRPA configurations: a set of particle-hole pairs ($ph$) and pairs formed from the negative energy state in the Dirac sea and the hole state in the Fermi sea ($\alpha h$).

The response function of a quantum system to an external field is given by the imaginary part of the polarization operator:

\[ R(Q, Q; k, k'; E) = \frac{1}{\pi} Im \Pi^R(Q, Q; k, k'; E) , \]

where $Q$ is an external field operator. The RRPA polarization operator is obtained by solving the Bethe-Salpeter equation:

\[ \Pi(Q, Q; k, k', E) = \Pi_0(Q, Q; k, k', E) - \sum_i g_i^2 \int d^3k_1 d^3k_2 \Pi_0(Q, \Gamma_i; k, k_1, E) \]

\[ D_i(k_1, k_2, E) \Pi(\Gamma_i, Q; k_2, k', E) , \]

In the RRPA, the residual particle-hole interactions are obtained from the same Lagrangian as in the description of the nuclear ground state. They are generated by exchanging various mesons: the isoscalar scalar meson $\sigma$, the isoscalar vector meson $\omega$ and the isovector vector meson $\rho$. Therefore, in Eq.(2) the sum $i$ runs over $\sigma$, $\omega$ and $\rho$ mesons, and $g_i$ and $D_i$ are the corresponding coupling constants and meson propagators. They are $\Gamma_i = 1$ for $\sigma$ meson and $\Gamma_i = \gamma^\mu, \gamma^\mu\tau_3$ for $\omega$ and $\rho$ mesons, respectively. The meson propagators in the non-linear model are non-local in the momentum space, and therefore have to be calculated numerically. The detailed expressions of non-linear meson
propagators $D_i(k_1, k_2, E)$ can be found in Ref. [40]. $\Pi_0$ is the unperturbed polarization operator, which in a spectral representation has the following retarded form:

$$
\Pi_0^R(P, Q; k, k'; E) = \frac{(4\pi)^2}{2L+1} \left\{ \sum_{h,p} (-1)^{j_h + j_p} \left[ \frac{\langle \phi_h \| P_L \| \phi_p \rangle \langle \phi_h \| Q_L \| \phi_p \rangle}{E - (\varepsilon_p - \varepsilon_h) + i\eta} - \frac{\langle \phi_p \| P_L \| \phi_h \rangle \langle \phi_h \| Q_L \| \phi_p \rangle}{E + (\varepsilon_p - \varepsilon_h) + i\eta} \right] 
+ \sum_{h,\alpha} (-1)^{j_h + j_{\alpha}} \left[ \frac{\langle \phi_h \| P_L \| \phi_{\alpha} \rangle \langle \phi_h \| Q_L \| \phi_{\alpha} \rangle}{E - (\varepsilon_{\alpha} - \varepsilon_h) + i\eta} - \frac{\langle \phi_{\alpha} \| P_L \| \phi_h \rangle \langle \phi_h \| Q_L \| \phi_{\alpha} \rangle}{E + (\varepsilon_{\alpha} - \varepsilon_h) + i\eta} \right] \right\} \tag{3}
$$

The unperturbed polarization operator includes not only the positive energy particle-hole pairs but also pairs formed from the Dirac sea states and Fermi sea states. In Refs. [29, 30] the authors show that the inclusion of configurations built from the positive energy states in the Fermi sea and negative energy states in the Dirac sea is essential to give correctly a quantitative description in the excitation energies of isoscalar giant multipole resonances as well as to ensure the current conservation and decouple the spurious states.

The pairing correlation and coupling to the continuum are important for exotic nuclei. A proper treatment of the resonant continuum to pairing correlations has been recently investigated in the Hartree-Fock (HF) Bogoliubov or the HF+BCS approximation [19, 20, 21] and the extended RMF+BCS approximation [22, 23]. It shows that the simple BCS approximation in the resonant continuum with a proper boundary condition works well in the description of ground state properties even for neutron rich nuclei. We shall treat the pairing correlation in the BCS approximation in this work and the resonant continuum is calculated by imposing an asymptotic scattering boundary condition.

When pairing correlations are taken into account, the elementary excitation is a two-quasiparticle excitation, rather than a particle-hole excitation. The unperturbed polarization operator in the QRRPA in the response function formalism can be constructed in a similar way:

$$
\Pi_0^R(P, Q; k, k'; E) = \frac{(4\pi)^2}{2L+1} \left\{ \sum_{\alpha,\beta} (-1)^{j_{\alpha} + j_{\beta}} A_{\alpha\beta} \left[ \frac{\langle \phi_\alpha \| P_L \| \phi_\beta \rangle \langle \phi_\beta \| Q_L \| \phi_\alpha \rangle}{E - (E_\alpha + E_\beta) + i\eta} - \frac{\langle \phi_\beta \| P_L \| \phi_\alpha \rangle \langle \phi_\alpha \| Q_L \| \phi_\beta \rangle}{E + (E_\alpha + E_\beta) + i\eta} \right] 
+ \sum_{\alpha,\beta} (-1)^{j_{\alpha} + j_{\beta}} \gamma^2_{\alpha} \left[ \frac{\langle \phi_\alpha \| P_L \| \phi_{\beta} \rangle \langle \phi_{\beta} \| Q_L \| \phi_\alpha \rangle}{E - (E_\alpha + \lambda - \varepsilon_{\beta}) + i\eta} - \frac{\langle \phi_{\alpha} \| P_L \| \phi_\beta \rangle \langle \phi_\beta \| Q_L \| \phi_{\alpha} \rangle}{E + (E_\alpha + \lambda - \varepsilon_{\beta}) + i\eta} \right] \right\} \tag{4}
$$
with
\[ A_{\alpha\beta} = (u_\alpha v_\beta + (-1)^L v_\alpha u_\beta)^2 (1 + \delta_{\alpha\beta})^{-1} , \] (5)

where \( v_\alpha^2 \) is the occupation probability and \( u_\alpha^2 = 1 - v_\alpha^2 \). \( E_\alpha = \sqrt{\varepsilon_\alpha^2 - \lambda^2 + \Delta^2} \) is the quasiparticle energy, where \( \lambda \) and \( \Delta \) are the Fermi energy and pairing correlation gap, respectively. In the BCS approximation, the \( \phi_\alpha \) is the eigenfunction of the single particle Hamiltonian with an eigenvalue \( \varepsilon_\alpha \). In Eq.(4), terms in the first square bracket represent those excitations with one quasiparticle in fully or partial occupied states and one quasiparticle in partial occupied or unoccupied states. Terms in the second square bracket describe all excitations between positive energy fully or partial occupied states and negative energy states in the Dirac sea. For unoccupied positive energy states outside the pairing active space, their energies are \( E_\beta = \varepsilon_\beta - \lambda \), occupation probabilities \( v_\beta^2 = 0 \) and \( u_\beta^2 = 1 \). For fully occupied positive energy states, the quasiparticle energy and the occupation probability are \( E_\alpha = \lambda - \varepsilon_\alpha \) and \( v_\alpha^2 = 1 \) in Eq.(4). States in the Dirac sea are not involved in pairing correlations. Therefore those quantities \( v_\beta^2 \) and \( u_\beta^2 \) are set to be 0 and 1, respectively. Once the unperturbed polarization operator in the quasiparticle scheme is built, the QRRPA response function can be obtained by solving the Bethe-Salpeter equation (2) as usually done in the RRPA.

### III. NUMERICAL CALCULATION AND TEST OF THE QRRPA

In this section, we first check the validity of the present QRRPA calculations. We apply the QRRPA to calculate the response function of the isovector giant dipole resonance (IVGDR) and the isoscalar giant quadrupole resonance (ISGQR) in the neutron rich nucleus \(^{22}\text{O}\). Similar calculations for the nucleus \(^{22}\text{O}\) were recently performed by Paar et al \[28\] in the framework of the Relativistic Hartree-Bogoliubov (RHB) + QRRPA in the configuration space formalism.

The ground state properties of the nucleus \(^{22}\text{O}\) are calculated in the extended RMF+BCS approach \[22\] with the parameter set NL3. The neutron pairing gap is obtained from the experimental binding energies of neighboring nuclei, \( \Delta_n = 1.532 \text{ MeV} \). In the QRRPA calculation particle-hole residual interactions are taken from the same effective interaction NL3, which is used in the description of the ground state of \(^{22}\text{O}\). Fully occupied states and states in the pairing active space are calculated self-consistently in the extended RMF+BCS approach in the coordinate space. The BCS active space is
FIG. 1: IVGDR strengths in $^{22}$O. The solid curve represents the result calculated in the QRRPA approach. The result performed in the RRPA approach is shown by a dashed curve. All results are calculated with the effective Lagrangian parameter set NL3.

taken as all states in the $sd$ shell as well as $1f_{7/2}$ state, which is a resonant state in $^{22}$O. A scattering boundary condition is imposed in the resonant continuum. Unoccupied states outside of the pairing active space are obtained by solving the Dirac equation in the expansion on a set of the harmonic oscillator basis. The response functions of the nuclear system to the external operator are calculated at the limit of zero momentum transfer. It is also necessary to include the space-like parts of vector mesons in the QR-RPA calculations, although they do not play role in the ground state[41]. The consistent treatment guarantees the conservation of the vector current.

In Fig.1, we show the response function of the IVGDR mode in $^{22}$O calculated in the RRPA and QRRPA approaches. The isovector dipole operator used in the calculations is[42]:

$$Q = e \frac{N}{A} \sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - e \frac{Z}{A} \sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i),$$

which excites an $L=1$ type electric (spin-non-flip) $\Delta T = 1$ and $\Delta S = 0$ giant resonance with $J^\pi = 1^-$. The spurious state for exotic nuclei may appear at the energy around 1 MeV in the IVGDR strength in numerical calculations due to the mixture of the isoscalar
In our present calculations, the spurious state is removed by slightly adjusting the coupling constant of the $\sigma$ meson in the residual interaction by less than 1%, that does not affect the general results.

In general, the IVGDR strengths in light stable nuclei are expected to be fragmented substantially. This also occurs in the response function of the IVGDR in neutron rich nuclei. More fragmented distributions around the GDR region in $^{22}$O are observed in Fig.1. In addition to the characteristic peak of the IVGDR at the energy around 20 MeV, the low-lying dipole strength appears at the excitation energy below 10 MeV. It can be seen that the inclusion of pairing correlations enhances the low-lying dipole strength and has a slight effect on the strength at the normal dipole resonance. This illustrates the importance of including pairing correlations in the study of the low-lying isovector dipole strength in neutron rich nuclei. The effect of pairing correlations on the isovector dipole strength in $^{22}$O observed in our calculation is consistent with that obtained in the RHB+QRRPA in the configuration formalism (Fig.2 of Ref. [28]).

![Figure 2: ISGQR strengths in $^{22}$O. Notations are the same as in Fig.1.](image)

The response functions of the ISGQR mode in $^{22}$O calculated in the RRPA and QRRPA approaches are shown in Fig.2. In present calculation the isoscalar quadrupole
TABLE I: The energy weighted moment $m_1$ at $E_x < 60$ MeV for the electric isovector dipole and isoscalar quadrupole excitations in $^{22}$O. DC represents the result from double commutators in the non-relativistic approach.

|                    | DC ($e^2 f m^2$ MeV) | RRPA ($E_x < 60$ MeV) | QRRPA ($E_x < 60$ MeV) |
|--------------------|-----------------------|------------------------|------------------------|
| IVGDR ($e^2 f m^2$ MeV) | 75.9                  | 82.08                  | 81.71                  |
| ISGQR ($e^2 f m^4$ MeV) | 2018                  | 1971                  | 2159                  |

The operator is taken from Ref. [42]:

$$Q = e \frac{Z}{A} \sum_{i=1}^{A} r_i^2 Y_{2M} (\hat{r}_i),$$ (7)

The inclusion of pairing correlations shifts the low-lying quadrupole strength to higher energy region and enhances the low-lying quadrupole strength, while this only slightly affect the strength at the normal giant resonance region. A similar result on the isoscalar quadrupole strength in $^{22}$O has been observed in Fig. 3 of Ref. [28] in the framework of the RHB+QRRPA.

In Table I we show the energy weighted moment $m_1$ at $E_x < 60$ MeV for electric isovector dipole and isoscalar quadrupole excitations in $^{22}$O. DC represents the result from double commutators in the non-relativistic approach. In the isovector dipole mode the value corresponds to the Thomas-Reiche-Kuhn (TRK) dipole sum rule. It is shown that the $m_1$ obtained by the integration of the RPA strength till to 60 MeV is slightly larger than that obtained by the double commutator (DC) in the dipole mode. While in the quadrupole mode both RRPA and QRRPA results are close to the DC value.

IV. ISOVECTOR DIPOLE EXCITATION IN NEUTRON RICH NUCLEI

$^{26}$Ne AND $^{28}$Ne

A. Ground state properties of nuclei $^{26}$Ne and $^{28}$Ne

Ground state properties of nuclei $^{26}$Ne and $^{28}$Ne are studied in the extended RMF+BCS with the parameter set NL3, where a spherical symmetry is assumed. The continuum is calculated by imposing a scattering boundary condition and the width of the resonant state is not considered in this work. Constant pairing gaps are adopted in
TABLE II: Neutron and proton pairing gaps in $^{26}$Ne and $^{28}$Ne, and calculated ground state properties: neutron and proton Fermi energies, binding energies as well as neutron and proton rms radii. Values in the parenthesis are the corresponding experimental data of the binding energy.[44].

|          | $^{26}$Ne | $^{28}$Ne |
|----------|-----------|-----------|
| $\Delta_n (MeV)$ | 1.436     | 1.400     |
| $\Delta_p (MeV)$ | 2.025     | 2.101     |
| $\lambda_n (MeV)$ | -5.325    | -4.290    |
| $\lambda_p (MeV)$ | -14.168   | -16.247   |
| $E_B (MeV)$ | 201.8(201.6) | 210.5(206.9) |
| $r_n (fm)$ | 3.179     | 3.348     |
| $r_p (fm)$ | 2.784     | 2.833     |

the calculation of pairing correlations, which are obtained from the experimental binding energies of neighboring nuclei by the formula:

$$\Delta_p = \frac{1}{8} (B(Z-2,N) - 4B(Z-1,N) + 6B(Z,N) - 4B(Z+1,N) + B(Z+2,N)) ,$$

(8)

$$\Delta_n = \frac{1}{8} (B(Z,N-2) - 4B(Z,N-1) + 6B(Z,N) - 4B(Z,N+1) + B(Z,N+2)) .$$

(9)

In our calculations, the neutron pairing active space in nuclei $^{26}$Ne and $^{28}$Ne includes states up to the $N = 28$ major shell and 2p$_{3/2}$ state, which are 1d$_{5/2}$, 2s$_{1/2}$, 1d$_{3/2}$, 2p$_{3/2}$, and 1f$_{7/2}$. The BCS active space for proton is taken as all states in the sd shell. In Table II we list the neutron and proton pairing gaps in nuclei $^{26}$Ne and $^{28}$Ne derived from Eqs.(8,9) and the calculated ground state properties, including neutron and proton Fermi energies, total binding energies as well as the neutron and proton rms radii. The values in the parenthesis are corresponding experimental binding energies taken from Ref.[44]. Neutron single particle energies and BCS occupation probabilities for those states near the neutron Fermi energy are shown in Table III, where levels (2p$_{3/2}$ and 1f$_{7/2}$) with positive energies are the single particle resonant states.
TABLE III: Neutron single-particle energies $\varepsilon_\alpha$ and occupation probabilities $v_\alpha^2$ of levels near the neutron Fermi energy in nuclei $^{26}\text{Ne}$ and $^{28}\text{Ne}$.

|       | $^{26}\text{Ne}$ |       | $^{28}\text{Ne}$ |
|-------|------------------|-------|------------------|
|       | $\varepsilon_\alpha$(MeV) | $v_\alpha^2$ | $\varepsilon_\alpha$(MeV) | $v_\alpha^2$ |
| $1d_{5/2}$ | -10.548 | 0.982 | -10.836 | 0.989 |
| $2s_{1/2}$ | -6.549 | 0.824 | -7.054 | 0.946 |
| $1d_{3/2}$ | -3.408 | 0.099 | -4.299 | 0.503 |
| $2p_{3/2}$ |  | 0.786 | 0.018 |
| $1f_{7/2}$ | 2.946 | 0.007 | 2.223 | 0.011 |

FIG. 3: Neutron and proton density distributions in nuclei $^{26}\text{Ne}$ and $^{28}\text{Ne}$. All results are calculated in the RMF+BCS.

Thus the nucleon density with pairing correlations can be written as:

$$
\rho(r) = \sum_\alpha \frac{(2j_\alpha + 1)}{4\pi} v_\alpha^2 \phi_\alpha^\dagger(r) \phi_\alpha(r),
$$

where the summation runs over all states weighted by the factor $v_\alpha^2$. In Fig.3 we show the calculated nucleon densities in $^{26}\text{Ne}$ and $^{28}\text{Ne}$. It is shown in Table II that proton rms
TABLE IV: The calculated energies and B(E2) values for the lowest $2^+$ states in $^{26}$Ne and $^{28}$Ne. The values in the parenthesis are the corresponding experimental data taken from Ref.[45].

|         | $E_{2^+}$ (MeV) | $B(E2)$ ($e^2 fm^4$) |
|---------|-----------------|-----------------------|
| $^{26}$Ne | 1.46(1.99±0.012) | 223(228±41)           |
| $^{28}$Ne | 1.29(1.32±0.020) | 156(269±136)          |

radii are much smaller than those of neutron. The neutron densities have far extending tails seen in Fig.3, which clearly shows neutron skins formed in those nuclei.

B. Low-lying isovector dipole modes in $^{26}$Ne and $^{28}$Ne

In this work a spherical assumption is adopted in the RMF and RRPA calculation, although $^{26}$Ne and $^{28}$Ne are deformed and anharmonic. In order to see how far the present model can be effective one first studies the quadrupole excitations in these nuclei and compare with the experimental data. The calculated energies and B(E2) values for the lowest $2^+$ states in $^{26}$Ne and $^{28}$Ne are listed in Table IV. The values in the parenthesis are the corresponding experimental data taken from Ref.[45]. Although the B(E2) value in $^{28}$Ne is very close to the lower limit of experimental data, the present calculations reasonably reproduce the lowest $2^+$ states and its B(E2) values.

We now apply the QRRPA approach to investigate the isovector dipole response in $^{26}$Ne and $^{28}$Ne. We focus our attention on properties of the isovector low-lying dipole strength. In Fig.4 we present the Hartree and perturbed strengths for the isovector dipole mode in nuclei $^{26}$Ne (upper panel) and $^{28}$Ne (lower panel) calculated in the RRPA and QRRPA approaches. Short-dashed curves represent the RRPA strengths, the QRRPA response functions are denoted by solid ones. In Fig.4, it can be seen that the perturbed strengths at the energy above 10 MeV for those neutron rich nuclei are very fragmented. Compared to the Hartree strengths, the RPA strengths are shifted to higher energy region due to the repulsive particle-hole residual interaction generated mainly by exchanging $\rho$ meson.

In addition to the characteristic peak of the IVGDR around energy of 20 MeV, low-lying dipole strengths appear at the excitation energy below 10 MeV. It is shown in Fig.4 that the effect of pairing correlations on the isovector dipole strength in $^{26}$Ne and $^{28}$Ne shifts the low-lying dipole strength to higher energy region and decreases the
low-lying dipole strength, especially in $^{26}\text{Ne}$. The situation slightly differs from that in $^{22}\text{O}$, where the low-lying strength is increased. The decrease of the low-lying strength is mainly due to the fact that two-quasiparticle excitations in the dipole mode are weakened by a factor of $\nu_0^2$ when the pairing correlation is switched on. In addition, the two-quasiparticle excitation energy is larger than the corresponding particle-hole excitation energy in the dipole mode [27]. On the contrary, when the pairing correlation is taken into account the configuration space becomes larger, which allows for the particle-particle and hole-hole transitions. This enlarged configuration space may increase the low-lying strength. As analyzed below, the state $2s_{1/2}$ produces a large contribution to the low-lying strength of the dipole mode in $^{26,28}\text{Ne}$ and $^{22}\text{O}$. It is known that the $2s_{1/2}$ state is located below and above the Fermi surface in $^{26,28}\text{Ne}$ and $^{22}\text{O}$, respectively. Therefore particle-particle excitations, especially due to the particle state $2s_{1/2}$, largely enhance the low-lying strength in $^{22}\text{O}$, which is not true in the case of $^{26}\text{Ne}$ and $^{28}\text{Ne}$.

In comparison with the Hartree strength it is found that the RPA strength of the isovector dipole mode at the low energy region in $^{26}\text{Ne}$ and $^{28}\text{Ne}$ shown in Fig.4 remains at its position and is contributed mainly from a few particle-hole configurations, which shows a single particle like property. The low-lying strength is slightly attracted back to the lower energy, which is due to the correlations of the isoscalar operator in the isovector mode [43]. Differing from the normal IVGDR response, the low-lying resonance can be interpreted as the excitation of the excess neutrons out of phase with the core formed with an equal number of protons and neutrons [14]. Analyzing the Hartree strength of the isovector dipole mode in $^{26}\text{Ne}$ at the energy below 10 MeV calculated with pairing correlations, one finds a pronounced peak around 8.5 MeV, which is formed mainly from the neutron configurations of $\nu(2s_{1/2}^{-1}2p_{3/2})(8.482 \text{ MeV})$ and $\nu(2s_{1/2}^{-1}2p_{1/2})(9.232 \text{ MeV})$, where the value in the parenthesis is its Hartree energy. Since 16 neutrons in $^{26}\text{Ne}$ fill neutron orbits up to $2s_{1/2}$ and form a sub-closed shell, the occupation probabilities at $1d_{3/2}$ and $1f_{7/2}$ states in $^{26}\text{Ne}$ are relatively small, see Table III. Therefore the contribution from neutron states $1d_{3/2}$ and $1f_{7/2}$ to the low-lying Hartree strength is insignificant. In contrast the Hartree strength at the low energy region in $^{28}\text{Ne}$, in addition to the peaks formed from the neutron configurations of $\nu(2s_{1/2}^{-1}2p_{3/2})(8.523 \text{ MeV})$ and $\nu(2s_{1/2}^{-1}2p_{1/2})(9.080 \text{ MeV})$, a few more peaks appear, which are formed from the neutron excitation between the bound level $1d_{3/2}$ and levels in the continuum. It is found that the Hartree strengths at the low-lying dipole in $^{26}\text{Ne}$ and $^{28}\text{Ne}$ are mainly due to neutron excitations
FIG. 4: Isovector dipole strength functions in neutron rich nuclei $^{26}$Ne (upper panel) and $^{28}$Ne (lower panel). The QRRPA responses with the pairing (solid curves) are compared with the RRPA calculation without the pairing (short-dashed curves). The thick and thin curves represent perturbed and Hartree strengths, respectively.
FIG. 5: The contribution of proton to the isovector dipole strength in $^{26}$Ne (left panel) and $^{28}$Ne (right panel) in the QRRPA approach. The solid curves are the full QRRPA strengths. The short-dashed curves represent the strengths from proton.

TABLE V: The non-energy weighted moment $m_0$ and energy weighted moment $m_1$ of isovector dipole strengths in $^{26}$Ne and $^{28}$Ne in the QRRPA calculations. We separate the energy region into the low energy ($0 \text{ MeV} \leq E_x \leq 10 \text{ MeV}$) and the high energy ($10 \text{ MeV} \leq E_x \leq 30 \text{ MeV}$). The values in the last two column are obtained from the classical TRK dipole sum rule and TRK cluster sum rule ($e^2 fm^2 MeV$). The units are $e^2 fm^2$ and $e^2 fm^2 MeV$ for $m_0$ and $m_1$, respectively.

|        | $m_0$  | $m_1$  | $m_0$  | $m_1$  | $S_{\text{TRK}}$ | $S_{\text{Clus}}$ |
|--------|--------|--------|--------|--------|-----------------|-----------------|
| $^{26}$Ne | 0.542  | 4.525  | 5.032  | 103.9  | 91.7            | 17.2            |
| $^{28}$Ne | 0.705  | 5.606  | 5.648  | 111.2  | 95.8            | 21.3            |

near the Fermi surface.

Although the Hartree low-lying strength is mainly formed from the neutron excitations, the RPA strengths are fully correlated and contributed from both neutron and
TABLE VI: Centroid energies of the isovector dipole response functions in $^{26}$Ne and $^{28}$Ne. The centroid energies are calculated within $0 \sim 10$ MeV and $10 \sim 60$ MeV, respectively. All energy values are in unit of MeV.

|            | $^{26}$Ne | $^{28}$Ne |
|------------|-----------|-----------|
| $E(0 \sim 10)$ | 8.34      | 7.94      |
| $E(10 \sim 60)$ | 22.32     | 21.13     |

proton. To illustrate the contribution of the proton to the full strength we set a very small value of the neutron effective charge in the dipole operator instead of $eZ/A$. Results are plotted in Fig.5. The short-dashed curves represent the strengths from proton, which are compared with the full QRRPA strengths denoted by solid curves. It clearly shows that the proton also plays an important role in the perturbed strength even at low-lying dipole states.

In Ref.[14], the authors have studied the evolution of collectivity in the isovector dipole response in the low-lying region for neutron-rich isotopes of O, Ca, Ni, Zr, and Sn using RRPA method. They conclude that in light neutron-rich nuclei, such as neutron-rich isotopes of O and Ca, the onset of dipole strength in the low-lying region is due to single-particle excitations of the loosely bound neutrons. By analyzing the structure of RRPA strengths in low-lying region in nuclei $^{26}$Ne and $^{28}$Ne, we find that there are only several configurations contributing to the low-lying RRPA strengths and the RRPA strengths remain their positions compared to the Hartree strengths in the low-energy region, which mean that the RRPA strengths in low-lying region in nuclei $^{26}$Ne and $^{28}$Ne are dominated by single-particle transitions.

In order to give a more clear description of those isovector low-lying dipole states obtained in the QRRPA approach, we calculate various moments of isovector dipole strengths at a given energy interval:

$$m_k = \int_{E_1}^{E_2} R^L(E) E^k dE,$$

where $E_1$ and $E_2$ are the lower and upper energies of the integral, respectively. The RPA equation is solved till $E = 60$ MeV in the present calculations. The non-energy weighted moment $m_0$ and the energy weighted moment $m_1$ of isovector dipole strengths in $^{26}$Ne and $^{28}$Ne are calculated at two energy intervals: the low energy region ($0$ MeV $\leq E_x \leq 10$ MeV) and the high energy region ($10$ MeV $\leq E_x \leq 30$ MeV), which are listed in Table
IV. The values obtained from the Thomas-Reiche-Kuhn (TRK) dipole sum rule are listed in the sixth column. In our present QRRPA calculations the low-lying isovector dipole strengths in $^{26}\text{Ne}$ and $^{28}\text{Ne}$ exhaust about 4.93% and 5.85% of the TRK dipole sum rule, respectively. This is consistent with the recent experimental observations in $^{18}\text{O}$, $^{20}\text{O}$ and $^{22}\text{O}$, where the low-lying isovector dipole strengths exhaust about 5% of the TRK dipole sum rule. In general the percentage of the low-lying isovector dipole strength becomes large as the increase of the neutron excess. The energy weighted moment $m_1$ at $E_x < 10$ MeV in $^{28}\text{Ne}$ is about 1.0% larger than that in $^{26}\text{Ne}$.

On the other hand, the cluster sum rule is usually used to understand the properties of low-lying dipole strength in exotic nucleus excitations. Here we choose $^{20}\text{Ne}$ as the core. In Table V we also list the values obtained from TRK cluster dipole sum rule, say 17.2 $e^2 fm^2$MeV and 21.3 $e^2 fm^2$MeV for $^{26}\text{Ne}$ and $^{28}\text{Ne}$, respectively. It shows that the low-lying dipole excitations in these two neutron rich nuclei exhaust about 26.3% of the TRK cluster dipole sum rule. Similar results are obtained in Ref. for neutron rich Oxygen isotopes.

The centroid energy of the response function is defined as:

$$E = \frac{m_1}{m_0}. \quad (12)$$

We separate the energy interval into two regions: 0 MeV $< E_x < 10$ MeV and 10 MeV $< E_x < 60$ MeV. The centroid energies in these two energy regions are listed in Table V. In the QRRPA calculations the centroid energies of low-lying isovector dipole strengths in $^{26}\text{Ne}$ and $^{28}\text{Ne}$ are 8.34 MeV and 7.94 MeV, respectively. Whereas the centroid energies of the normal IVGDR strengths are located at 22.3 MeV for $^{26}\text{Ne}$ and 21.1 MeV for $^{28}\text{Ne}$.

V. SUMMARY

In this paper we have studied the properties of low-lying isovector dipole resonances in neutron rich nuclei $^{26}\text{Ne}$ and $^{28}\text{Ne}$ in the framework of the QRRPA with the effective Lagrangian parameter set NL3. The ground state properties are calculated in the extended RMF+BCS approach, where the resonant continuum is properly treated. Constant pairing gaps extracted from the experimental binding energies of neighboring nuclei are adopted in the BCS calculation. In the QRRPA calculation the negative energy states in the Dirac sea are included due to the completeness. It is shown that the inclusion of pairing correlations has a relatively strong effect on the low-lying isovector dipole
strength in neutron rich nuclei. In the QRRPA calculation the low-lying isovector dipole strengths in $^{26}$Ne and $^{28}$Ne exhaust about 5% and 26.3% of the TRK dipole sum rule and TRK dipole cluster sum rule, respectively. The centroid energies of the low-lying dipole excitation in nuclei $^{26}$Ne and $^{28}$Ne are located at the energy around 8.0 MeV.

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