A SOCP-Based ACOPF for Operational Scheduling of Three-Phase Unbalanced Distribution Systems and Coordination of PV Smart Inverters

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Abstract—The proliferation of distributed energy resources (DERs) imposes new challenges to distribution system operation, e.g., power quality issues. To overcome these challenges and enhance system operation, it is critical to effectively utilize all available resources and accurately characterize unbalanced distribution networks in operational tools. This paper proposes a convex second-order-cone programming (SOCP)-based AC optimal power flow (ACOPF) model for three-phase unbalanced distribution networks, including smart inverters and Volt-Var controller (VVC) devices. Reactive power-voltage (Q-V) characteristics of smart inverters of solar photovoltaic (PV) units are also modeled. Moreover, the settings of Q-V characteristics of VVC are co-optimized within the proposed ACOPF, considering the allowable range of the IEEE 1547-2018 standard. Furthermore, dynamic analyses are conducted to verify the stability of optimal settings of VVC. The proposed models are tested on an actual 1747-node primary distribution feeder in Arizona. The results illustrate the effectiveness of the proposed ACOPF for unbalanced systems in providing an optimal solution while capturing the non-linearity and non-convexity of ACOPF. By co-optimizing settings, system operation is improved due to the flexibility of adjusting reactive power output from PV units with VVC. The time-domain simulations show that the optimal settings cause no stability issue for the distribution system.

Index Terms—AC optimal power flow, distributed energy resources, second-order cone programming, three-phase unbalanced distribution system, Volt-Var controller.

NOMENCLATURE

Indices and Sets

| Index Set | Description |
|-----------|-------------|
| $d$       | $d \in D$   | Index and set of demands. |
| $g$       | $g \in G$   | Index and set of nodes connected to substations. |

Indices and Sets

| Index Set | Description |
|-----------|-------------|
| $h$       | $h \in H$   | Index and set of all solar photovoltaic (PV) units. |
| $i$       | $i \in I$   | Index and set of nodes (bus $i$ and phase $\phi$). |
| $k$       | $k \in K$   | Index and set of all PV units having Volt-Var controller (VVC). |
| $l$       | $l \in L$   | Index and set of lines. |
| $n$       | $n \in N$   | Index and set of voltage settings of VVC. |
| $\phi$    | $\phi \in \Phi$ | Index and set of phases. |
| $\rho$    | $\rho \in \rho^\phi$ | Energy prices from bulk system and PV units. |

Decision Variables

| Variable | Description |
|----------|-------------|
| $P^d_n$  | Active power rating of PV unit $n$. |
| $Q^d_n$  | Reactive power rating of PV unit $n$. |
| $P^\phi_{pv,h}$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,h}$ | Apparent power rating of PV unit.
| $P^\phi_{pv,k}$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,k}$ | Apparent power rating of PV unit.
| $P^\phi_{pv,h}d$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,h}d$ | Apparent power rating of PV unit.
| $P^\phi_{pv,k}$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,k}$ | Apparent power rating of PV unit.
| $P^\phi_{pv,h}d$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,h}d$ | Apparent power rating of PV unit.
| $P^\phi_{pv,k}$ | Apparent power rating of PV unit.
| $Q^\phi_{pv,k}$ | Apparent power rating of PV unit.

Appendix A

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\[ P^G, Q^G \] Active and reactive powers from upstream system in node \( g \).
\[ P^l, Q^l \] Active and reactive power flows of branch \( l \).
\[ P^h, Q^h \] Active and reactive powers of PV unit \( h \).
\[ Q_{max}^{pv, k} \] Maximum reactive power setting for PV unit \( k \) with VVC.

I. INTRODUCTION

A. Motivation and Background

The increasing penetration level of distributed energy resources (DERs) has made them an indispensable part of modern distribution network operation. Different IEEE Standards have been established to enhance power system operation and overcome challenges associated with integrating DERs in grid operation. For example, Standards Coordinating Committee 21 (SCC21) has developed IEEE Standard 1547-2018 for the interconnection and interoperability of DERs in distribution systems [1]. This standard provides a uniform criterion and sets the requirements relevant to the performance, operation testing, safety considerations, and maintenance of the interconnection of DERs such that they can be universally adopted. With the issued IEEE Standard 1547-2018, there is a need to update system operational tools to improve the representation of DERs in distribution operation.

Rooftop solar photovoltaic (PV) unit is one type of DER, which has been widely adopted in distribution systems. When operated in the Reactive power-voltage (Q-V) controller mode, Vol-VAr controllers (VVC) of smart inverters can adjust the power outputs of PV units and maintain the voltage within an acceptable range in [1]. For example, excessive PV generation in distribution systems can lead to over-voltage issues, which can be mitigated via two options: (i) reactive power support under Q-V mode and (ii) active power curtailment of PV units. Thus, PV units with smart inverters constitute a set of dispatchable resources whose optimal operation can enhance the performance of distribution networks.

As a result of the high R/X ratio of distribution feeders, the assumption of DC optimal power flow typically made in transmission systems is not valid. Moreover, unlike balanced transmission grids, the distribution networks are commonly unbalanced, and the mutual coupling between the three phases cannot be ignored. Therefore, a three-phase unbalanced AC optimal power flow (ACOPF) model is essential for distribution system operational scheduling. However, the three-phase unbalanced ACOPF is highly nonlinear and non-convex. Directly solving a nonlinear and non-convex model is not preferred because of the quality of the solution and the computational requirements.

Therefore, with the increasing penetration of DERs and the recommendations of IEEE standard 1547-2018, there is an urgent need to formulate a convex three-phase unbalanced ACOPF that incorporates the dispatching and performance of DERs in distribution grids.

B. Literature Review

Numerous studies have been conducted for the convexification of nonlinear and non-convex ACOPF. For the balanced system, two main convex relaxation techniques, which are semidefinite programming (SDP) [2], [3], [4], [5], [6] and second-order-cone programming (SOCP) [7], [8], [9], [10], [11], [12], [13], are explored to obtain the optimal solution of ACOPF. Reference [2] is among the first papers that proposed SDP-based ACOPF and solved it using the interior point method (IPM) algorithm. Reference [3] implements three decomposition techniques to decrease the computational time of the SDP-based ACOPF approach. Stronger and tighter SDP relaxation is discussed in [4], [5], [6] to mitigate the non-exact issue of the SDP relaxation. The ACOPF is first convexified and reformed as a SOCP problem for radial networks in [7]. Reference [8] uses hierarchies of linear programming with SOCP to alleviate the computational burden. Reference [9] utilizes a SOCP-based ACOPF to obtain optimal online control of devices. A mixed-integer SOCP problem is presented for the reactive optimal power flow (OPF) to determine the status of shunt elements and tap ratio of transformers in [10]. Another mixed-integer SOCP model is proposed by [11] to alleviate the unbalance issue from the demand-side with DERs. In [12], SOCP-based ACOPF is used in a security-constrained OPF problem for the worst contingencies of the system. The authors in [13] use a SOCP-based model to improve the grid operation with DERs for balanced systems. The exactness of SOCP and SDP relaxation and sufficient conditions of exactness are studied and proved in references [14], [15], [16], [17], which also show that SOCP can be more computationally efficient in comparison with SDP. In addition, references [18], [19] present the tractability of mixed-integer SOCP (MISOCP) compared to mixed-integer nonlinear programming (MINLP) and mixed-integer SDP (MISDP) with existing solvers. The main shortcomings of works [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] are that those works consider the balanced system which may not be sufficient because of the primary and low-voltage level distribution grids that are generally unbalanced due to unbalanced loads, DERs, and line segments. The assumptions of ACOPF for balanced system in [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19] may not be accurate enough to address the condition of the unbalanced system and ensure the secure and economic operation in the three-phase unbalanced distribution grid.

Recently, some research efforts have been conducted to formulate the convexified three-phase unbalanced ACOPF model. The SDP-based three-phase unbalanced ACOPF model in [20] is proposed and solved by alternating direction method of multipliers (ADMM). Reference [21] proposes a chordal relaxation-based SDP model for ACOPF in unbalanced systems considering DERs and voltage regulation transformers (VRTs) and provides a tighter convex model for VRTs to mitigate solution inexactness. Reference [22] convexifies nonlinear three-phase unbalanced ACOPF through a moment relaxation-based SDP model with a two-stage hierarchical algorithm to obtain the exact feasible solution. The SDP-based ACOPF for an unbalanced system in [23] accounts for the mixture of wye and delta-connection loads, DERs, and step voltage regulators. All references [20], [21], [22], [23] utilize the SDP technique to convexify the ACOPF. The SOCP-based model is another potential option for convexifying ACOPF for three-phase unbalanced systems.
but rarely discussed in literature. To fill this gap, this paper proposes a convex SOCP-based ACOPF model for unbalanced distribution grids, and the performance of the proposed method is evaluated on an actual distribution feeder. In addition, the proposed model is computationally tractable in obtaining an exact solution for a large actual three-phase unbalanced distribution system. The proposed model is extended to MISOC-based ACOPF and shown to be computationally tractable in obtaining the exact solutions in the actual unbalanced distribution network.

The modeling of PV units in the distribution grid operation has been addressed in the literature. In [24], [25], the active and reactive power and power factor limits of PV units are considered in OPF. Reference [26] proposes three different control strategies with active and reactive power limits for PV units to improve the voltage profile in distribution networks. However, the PV models in [24], [25], [26] may not satisfy IEEE standard 1547-2018 recommendations. References [27], [28], [29], [30], [31], [32], [33] have addressed IEEE standard 1547-2018. Reference [27] incorporates Q-V and active power-voltage (P-V) characteristics together in their model. Reference [28] utilizes the Q-V characteristic to mitigate the voltage fluctuations caused by PV generation. Reference [29] identifies the optimal settings of VVC in Q-V mode. The simulations in reference [27], [28], [29] are conducted on a balanced distribution system. However, the distribution networks are commonly three-phase unbalanced. References [30], [31] identify the optimal settings of VVC in the three-phase unbalanced distribution system using a nonlinear and nonconvex ACOPF, which may not be tractable in a large-scale system. The author in [32] formulates a decentralized approach to account for the standard characteristic in IEEE standard 1547-2018 for balanced systems. Reference [33] proposed a two-level Volt-VAr control scheme of PV units in which a 15-min dispatch and real-time adjustment are considered for PV units to enhance system operation. However, the unbalanced three-phase distribution grid characteristics are not well captured in [32], [33]. To improve the modeling of DERs in system operational tools in unbalanced distribution grids, this paper incorporates the IEEE standard 1547-2018 characteristic of PV units with a MISOC-based ACOPF. Two different models of settings of VVC are considered: fixed default settings and optimal settings.

C. Contributions and Organization of the Paper

According to the gaps mentioned above, the main contributions of this paper can be summarized as follows:

- A convex SOCP-based ACOPF which accounts for the unique characteristics of distribution networks, i.e., single-phase and three-phase lines, unbalanced network, and mutual impedances between phases, is proposed for three-phase unbalanced distribution systems. A two-stage algorithm is developed to solve the proposed model. Stage 1 solves the convex SOCP-based ACOPF while updating Taylor series base points, i.e., \( u_{ij}^{(t,1)} \) and \( \theta_{ij}^{(t,1)} \). Stage 2 checks the absolute SOC relaxation errors and ends the algorithm if the absolute relaxation errors are small enough. Otherwise, the SOC constraint is added, and the algorithm goes back to Stage 1. The convex SOCP-based ACOPF is tested on a real 1747-node primary distribution feeder in Arizona with two different loading conditions, i.e., over-voltage and under-voltage. The simulation results indicate that the proposed model can capture the three-phase unbalanced characteristics of distribution grids and obtain an optimal solution that is exact for the distribution system in different loading conditions.

- The proposed SOCP-based ACOPF is extended and converted into a MISOC-based ACOPF model to account for the Q-V characteristics of PV units equipped with VVCs based on the IEEE standard 1547-2018. Two different types of models for VVC are studied: (i) default settings and (ii) optimal settings. The simulation results show that the proposed MISOC-based ACOPF model can capture the Q-V characteristic of VVC and the characteristics of the unbalanced network.

- This work also shows the advantages of optimal settings of VVC in comparison with default settings. Case studies show that co-optimizing the settings of VVC within the allowable range of IEEE standard 1547-2018 in the ACOPF model enables more flexibility to adjust the reactive power output of PV units with VVC, which can mitigate voltage issues, improve system operation, and reduce operational cost. Moreover, due to the Big-M method being introduced to formulate the Q-V characteristics of PV units with VVC, the impact of Big-M values on the performance of the proposed models is evaluated in this paper. The scalability of MISOC-based ACOPF models is also discussed and exhibited by testing the proposed models on a 1747-node network with a large number of PV units with VVC. In addition, dynamic analyses are conducted to verify the control stability of PV units under optimal settings of VVCs and load changes for different operational conditions. The time-domain simulations show that the decision of optimal settings is valid and does not result in any stability issues in the system.

Section II explains the formulations and the algorithm of the proposed convex SOCP-based ACOPF. The Q-V characteristic of PV units and corresponding formulations in MISOC-based ACOPF are introduced in Section III. Numerical results of the proposed models for different loading conditions and dynamic simulation are presented and discussed in Section IV. Finally, Section V concludes the paper.

II. CONVEX ACOPF FOR THREE-PHASE UNBALANCED DISTRIBUTION SYSTEMS

A. Convex SOCP-Based ACOPF for Unbalanced Systems

The complex power flow on the line \( S_{ij}^{L} \) from bus \( i \) to bus \( j \) at phase \( \phi \) can be calculated through the voltage \( V_{ij}^{\phi} \) multiplied by the conjugate of the line current (phasors are in bold font). The formulation of power flow of a three-phase line is shown in (1).

\[
S_{ij}^{L} = V_{ij}^{\phi} \left( \sum_{\phi' \in \Phi} (V_{i\phi'} - V_{j\phi'}) Y_{ij}^{\phi\phi'} \right) \quad \text{(1)}
\]
where $V_{i\phi}$ is the voltage at bus $i$ on phase $\phi'$; $Y_{ij \phi'}$ is the admittance of the path from bus $i$ phase $\phi$ to bus $j$ phase $\phi'$; note that $Y_{ij \phi'}$ indicates self-impedance of phase $\phi$ and $Y_{ij \phi'}$ indicates mutual impedances between phases $\phi$ and $\phi'$ with phasor $V_{i\phi} = V_{i\phi} \angle \theta_{i\phi}$ and $V_{ij \phi'} = g_{ij \phi'} + j b_{ij \phi'}$. $V_{i\phi}$ and $\theta_{i\phi}$ are the voltage magnitude and the voltage angle at bus $i$ phase $\phi$ respectively. By substituting $V_{i\phi}$ and $Y_{ij \phi'}$ in (1), $S_{ij \phi'}$ can be written as (2).

$$S_{ij \phi'} = V_{i\phi} V_{j\phi'} \left( g_{ij \phi'} - j b_{ij \phi'} \right) + \sum_{\phi \not\in \Phi} V_{i\phi} V_{j\phi'} \left( \cos \theta_{ij \phi} + j \sin \theta_{ij \phi} \right) \left( g_{ij \phi'} - j b_{ij \phi'} \right) - \sum_{\phi \in \Phi} V_{i\phi} V_{j\phi'} \left( \cos \theta_{ij \phi} + j \sin \theta_{ij \phi} \right) \left( g_{ij \phi'} - j b_{ij \phi'} \right)$$

Three auxiliary variables (3)–(5) are introduced to reformulate the nonlinear three-phase unbalanced formulation (2) and eventually convexify it in (6)–(8) and (10).

$$e_{ij \phi'} = V_{i\phi} V_{j\phi'} \cos \theta_{ij \phi'}$$

(3)

$$e_{ij \phi'} = V_{i\phi} V_{j\phi'} \sin \theta_{ij \phi'}$$

(4)

$$u_{ij} = (V_{i\phi})^2$$

(5)

By substituting these auxiliary variables into (2), the active and reactive power flows on the line from bus $i$ to bus $j$ on phase $\phi$ can be reformulated as a convex form (6)–(7). The relationship of the auxiliary variables $e_{ij \phi'}$ and $s_{ij \phi'}$ are presented via (8)–(9).

$$P_{ij \phi'} = \frac{g_{ij \phi'} e_{ij \phi'} + b_{ij \phi'} e_{ij \phi'}}{e_{ij \phi'}} = \sum_{\phi \in \Phi} S_{ij \phi'}$$

(6)

$$Q_{ij \phi'} = \frac{b_{ij \phi'} u_{ij} + \sum_{\phi \not\in \Phi} \left( g_{ij \phi'} e_{ij \phi'} - b_{ij \phi'} e_{ij \phi'} \right)}{u_{ij}} = \sum_{\phi \not\in \Phi} \left( g_{ij \phi'} e_{ij \phi'} - b_{ij \phi'} e_{ij \phi'} \right)$$

(7)

The constraint (9) shows the exact relationship between auxiliary variables; however, it introduces non-convexity in the model. Therefore, the non-convex constraint (9) is relaxed as a convex SOC constraint in (10).

$$\left( \frac{e_{ij \phi'}}{e_{ij \phi'}} \right)^2 + \left( \frac{e_{ij \phi'}}{e_{ij \phi'}} \right)^2 = u_{ij} u_{ij}$$

(10)

A general three-phase unbalanced ACOPF can be formulated as a convex SOCP-based optimization problem as shown in (11)–(18) with the core constraints (6)–(8), (10). In this general model, the PV units without VVC are non-controllable. As a result, these PV units inject maximum available active power with zero reactive power into the distribution grid.

$$\min \sum_{\phi \in G} P_{j\phi}' \left( P_{j\phi}' - P_{j\phi} \right) + \rho_{PV} \left( \sum_{i \phi \in D} P_{i\phi}' \right)$$

(11)

$$\sum_{\phi \in G} P_{j\phi}' \left( P_{j\phi}' - P_{j\phi} \right) + \rho_{PV} \left( \sum_{i \phi \in D} P_{i\phi}' \right) + \sum_{\phi \in B} P_{j\phi}' \left( P_{j\phi}' - P_{j\phi} \right)$$

(12)

$$\sum_{\phi \in G} Q_{j\phi}' \left( Q_{j\phi}' - Q_{j\phi} \right) + \sum_{\phi \in B} Q_{j\phi}' \left( Q_{j\phi}' - Q_{j\phi} \right)$$

(13)

The objective function (11) minimizes the total system operating cost including the cost of energy purchased from the upstream wholesale market and the cost of solar generation surplus paid to PV owners. The active and reactive power balances at each node are constrained by (12)–(13). The VVC enables changing the active and reactive power output of PV units and provide voltage regulation service. For the PV units without VVC, the PV units’ active and reactive outputs are modeled as (14)–(15). Voltage limits of nodes are given by (16). Constraints (17)–(18) fix voltage magnitude and angle for nodes connected to the substation.

However, the reformed formulations (6)–(8), (10) provide no guarantee of a correct and exact solution. One reason is that the mutual impedance of the three-phase line creates the virtual loops shown in Fig. 1, which makes the network non-radial and complicates the determination of an exact solution using the SOCP-based formulations (6)–(8) and (10).
Another reason is that the active and reactive power flow constraints are not completely bounded by the auxiliary variable constraint (8). For example, unlike SOCP-based ACOPF constraints in the balanced system, the reformulated formulations (6)–(8) and (10) cannot ensure that the active power line loss is nonnegative in the unbalanced distribution system. In the nonlinear and non-convex line flow constraint (2), based on trigonometry, \( \cos \theta_{ij}^{\phi\phi'} \) and \( \sin \theta_{ij}^{\phi\phi'} \) between different phase pairs have a strong correlation. However, the constraint (8) only ensures the correlation of auxiliary variables that are related to the phase pairs with the same phase in line flow formulation (6). For example, the line power flow \( P_{ij, \phi}^{L} \) and \( P_{i, \phi}^{L} \) are affected by both actual and virtual paths due to the mutual impedance shown in Fig. 1. Only the red path for the similar phase is bounded to ensure the correlation in the line flow constraints (e.g., \( c_{ij} = c_{ij}^{\phi\phi} \), \( e_{ij} = -c_{ij}^{\phi\phi} \)). In contrast, the terms with auxiliary variables related to phase pairs with different phases (e.g., phase \( \phi \) and phase \( \phi' \) for \( c_{ij}^{\phi\phi'} \) in \( P_{ij, \phi}^{L} \) and \( c_{ij}^{\phi\phi'} \) in \( P_{i, \phi}^{L} \)) have no limit and mathematical correlation for line power flow constraints. Those unbounded auxiliary variable pairs make it rather hard to obtain an exact solution. To overcome the challenges mentioned above, new bounding constraints are proposed for the three-phase unbalanced SOCP-based ACOPF and solved by a two-stage algorithm.

### B. New Bounding Constraints and Two-Stage Algorithm for Solving Three-Phase Unbalanced SOCP-based ACOPF

This paper proposes new bounding constraints for auxiliary variables \( c_{ij}^{\phi\phi'} \) and \( e_{ij}^{\phi\phi'} \) to narrow down the feasible region, address the problems of virtual paths created by mutual impedances, and bound the terms associated with different phases via Taylor series approximation. The formulations (19)–(20) are linearized expressions of auxiliary variables \( c_{ij}^{\phi\phi'} \) and \( e_{ij}^{\phi\phi'} \) in (3)–(4) by applying the first-order Taylor series approximation.

\[
\begin{align*}
  c_{ij}^{\phi\phi'} &= \sqrt{u_{i\phi}^{(t)} u_{j\phi'}^{(t)}} \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) \sin \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) \\
  &+ \frac{\sqrt{u_{i\phi}^{(t)}}}{2u_{j\phi'}^{(t)}} \cos \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) u_{i\phi} \\
  &+ \frac{\sqrt{u_{i\phi}^{(t)}}}{2u_{j\phi'}^{(t)}} \cos \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) u_{j\phi'} - \sqrt{u_{i\phi}^{(t)} u_{j\phi'}^{(t)}} \sin \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) \\
  &\times \left( \theta_{i\phi} - \theta_{j\phi'} \right) \\
  e_{ij}^{\phi\phi'} &= -\sqrt{u_{i\phi}^{(t)} u_{j\phi'}^{(t)}} \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) \cos \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) \\
  &+ \frac{\sqrt{u_{i\phi}^{(t)}}}{2u_{j\phi'}^{(t)}} \sin \left( \theta_{i\phi}^{(t)} - \theta_{j\phi'}^{(t)} \right) u_{i\phi} \\
\end{align*}
\]

Since Taylor series first-order expansion base points (e.g., \( u_{i\phi}^{(t)} \) and \( \theta_{i\phi}^{(t)} \)) are introduced in (19)–(20), those base points need to be iteratively updated in the SOCP-based ACOPF model to obtain an exact and feasible solution. In this regard, a two-stage algorithm is developed to solve the proposed three-phase unbalanced SOCP-based ACOPF model with new bounding constraints (19)–(20).

The overall two-stage algorithm is presented in Fig. 2. The algorithm updates the Taylor series base points and adds SOC constraints if needed. Stage 1 starts from an initial point, which can be a flat start for all nodes with voltage magnitudes of the substation and voltage angles \( 0^\circ \), \(-120^\circ\), \(+120^\circ\) for phases \( a, b \) and \( c \), respectively. Moreover, a power flow solution of voltage magnitudes and angles from a similar system condition can be used as an initial point. With the selected initial point, the initial Taylor series base points (i.e., \( u_{i\phi}^{(t)} \) and \( \theta_{i\phi}^{(t)} \)) can be calculated for the linearized constraints (19)–(20). Then, Stage 1 goes into the loop to make the voltage magnitudes and angles converged. In the first iteration of the loop, the initial Taylor series base points are used to update linearized constraints (19)–(20) and the convex SOCP-based ACOPF (6)–(8), (11)–(20) is solved. After obtaining the solution of the convex ACOPF model (6)–(8),

![Fig. 2. The proposed two-stage algorithm to solve the proposed SOCP-based ACOPF for three-phase unbalanced systems.](image-url)
III. Q-V CHARACTERISTIC OF PV UNITS MODELED IN ACOPF

A. Q-V Characteristic of Photovoltaic Units With VVC

In the previous section, the convex SOCP-based ACOPF is proposed with the non-dispatchable PV units that are equipped with VVC. This section extends the proposed ACOPF model to account for Q-V characteristics of PV units with VVC to enable effective operational scheduling of these resources. To this end, the SOCP-based ACOPF model is modified to a MISOCP-based ACOPF. Moreover, two different models are developed for the default and optimal settings of VVC shown in Section III.B and Section III.C, respectively.

IEEE standard 1547-2018 indicates that the DERs with VVC should be capable of injecting and absorbing reactive power and participating in voltage regulation [1]. Furthermore, this standard provides four general modes for the reactive power control functions of DERs [1]. This paper considers the Q-V mode presented in Fig. 3, which is a mandatory requirement for both Category A and Category B 1547-compliant inverters.

$V_L$ and $V_H$ are the lower and upper limits for DER continuous operation in Fig. 3. $Q_{\text{PV}}^{\text{max}}$ is the setting for the maximum reactive power output of DER. As the auxiliary variable $u_{ij}$ is defined to represent squared voltage magnitude in the proposed SOCP-based ACOPF, the voltage settings for the separation of different operating zones $V_1, V_2, V_3, V_4, V_L$ and $V_H$ are squared as $v_1, v_2, v_3, v_4, v_L$ and $v_H$, respectively. The mathematical expression of the Q-V characteristic can be expressed as (21).

$$Q_{\text{PV}}^{\nu} = \begin{cases} Q_{\text{PV}}^{\text{max}}_{p,k} & \text{if } v_L \leq u_k < v_1 \\ \frac{Q_{\text{PV}}^{\text{max}}_{p,k}}{v_2 - v_1} (v_2 - u_k) & \text{if } v_1 \leq u_k < v_2 \\ 0 & \text{if } v_2 \leq u_k < v_3, \forall k \in K \\ \frac{Q_{\text{PV}}^{\text{max}}_{p,k}}{v_3 - v_4} (u_k - v_3) & \text{if } v_3 \leq u_k < v_4 \\ Q_{\text{PV}}^{\text{max}}_{p,k} & \text{if } v_4 \leq u_k < v_H \end{cases}$$

The output of PV units, which are equipped with VVC, can be controlled and adjusted. The active and reactive power output limits of PV units with VVC are modeled as (22)–(23).

$$(P_{\text{PV}}^{\nu})^2 + (Q_{\text{PV}}^{\nu})^2 \leq (Q_{\text{PV}}^{\text{max}}_{p,k})^2, \forall k \in K$$

$$P_{\text{PV}}^{\nu} \leq P_{\text{PV}}^{\text{max}}_{p,k}, \forall k \in K$$

B. Formulations of Q-V Characteristic of Photovoltaic Units in ACOPF With Default Settings

The expression of the Q-V characteristic in (21) is piecewise linear. In this section, first, the expression (21) is converted into five different operating zones using a linear representation. Binary variables, i.e., $z_1,k, z_2,k, z_3,k, z_4,k,$ and $z_5,k$ are introduced for Zones 1-5, respectively. The Big-M method is used to formulate if-then conditional statements for the five operating zones. The square voltage settings $v_1, v_2, v_3,$ and $v_4$ of VVC are fixed default values based on IEEE standard 1547-2018, which are 0.94², 0.98², 1.02², and 1.06², respectively. The maximum reactive power setting $Q_{\text{PV}}^{\text{max}}_{p,k}$ is 60% of the apparent power rating of the PV unit $k$. The formulations of the Q-V characteristic (Fig. 3) are presented in (24)–(35). Note $z_{n,k} = 0$ indicates that the operating Zone $n$ is activated for PV unit $k$.

The if-then conditional statement for Zone 1 is given by:

$$u_k \leq v_1 + M z_1,k, \forall k \in K$$

$$- M z_1,k \leq Q_{\text{PV}}^{\nu} - Q_{\text{PV}}^{\text{max}}_{p,k} \leq M z_1,k, \forall k \in K$$

The if-then conditional statement for Zone 2 is given by:

$$- M z_2,k + v_1 \leq u_k \leq v_2 + M z_2,k, \forall k \in K$$

$$- M z_2,k \leq Q_{\text{PV}}^{\nu} - Q_{\text{PV}}^{\text{max}}_{p,k} (v_2 - u_k) \leq M z_2,k, \forall k \in K$$

The if-then conditional statement for Zone 3 is given by:

$$- M z_3,k + v_2 \leq u_k \leq v_3 + M z_3,k, \forall k \in K$$

$$- M z_3,k \leq Q_{\text{PV}}^{\nu} - Q_{\text{PV}}^{\text{max}}_{p,k} (v_3 - u_k) \leq M z_3,k, \forall k \in K$$

The if-then conditional statement for Zone 4 is given by:

$$- M z_4,k + v_3 \leq u_k \leq v_4 + M z_4,k, \forall k \in K$$

$$- M z_4,k \leq Q_{\text{PV}}^{\nu} - Q_{\text{PV}}^{\text{max}}_{p,k} (u_k - v_3) \leq M z_4,k, \forall k \in K$$

The if-then conditional statement for Zone 5 is given by:

$$- M z_5,k + v_4 \leq u_k, \forall k \in K$$

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\[-M z_{5,k} \leq Q_k^{pv} + Q_{pv,k}^{max} \leq M z_{5,k}, \forall k \in K \tag{33}\]

The constraints (34)–(35) ensure that at least one operating zone is activated for each PV unit.

\[
z_{1,k} + z_{2,k} + z_{3,k} + z_{4,k} + z_{5,k} \leq 4, \forall k \in K \tag{34}
\]

\[
z_{1,k}, z_{2,k}, z_{3,k}, z_{4,k}, z_{5,k} \in \{0, 1\}, \forall k \in K \tag{35}
\]

C. Formulations of Q-V Characteristic of PV Units in ACOPF With Dispatch-VVC-Settings Co-Optimization

The settings of the Q-V curve for PV units with VVC are considered as parameters based on default values of IEEE standard 1547-2018 in Section III.B. However, these settings can be adjusted for the VVC of each PV unit to meet the different needs of the system and improve the distribution grid operation. In this paper, the proposed ACOPF is extended to optimally identify the settings of VVC curves of smart inverters within the allowable range of IEEE standard 1547-2018. Formulations (36)–(45) show the additional constraint for the ACOPF with co-optimization of VVC curve settings, i.e., \(\bar{v}_{1,k}, \bar{v}_{2,k}, \bar{v}_{3,k}, \bar{v}_{4,k}\), and \(Q_{max}^{pv,k}\).

\[
u_k \leq \bar{v}_{1,k} + M z_{1,k}, \forall k \in K \tag{36}
\]

\[-M z_{1,k} \leq Q_k^{pv} - Q_{pv,k}^{max} \leq M z_{1,k}, \forall k \in K \tag{37}\]

\[-M z_{2,k} + \bar{v}_{1,k} - u_k \leq \bar{v}_{2,k} + M z_{2,k}, \forall k \in K \tag{38}\]

\[-M z_{2,k} \leq Q_k^{pv} - \frac{Q_{pv,k}^{max}}{v_{2,k} - v_{1,k}} (\bar{v}_{2,k} - u_k) \leq M z_{2,k}, \forall k \in K \tag{39}\]

\[-M z_{3,k} + \bar{v}_{2,k} - u_k \leq \bar{v}_{3,k} + M z_{3,k}, \forall k \in K \tag{40}\]

\[-M z_{3,k} \leq Q_k^{pv} - M z_{3,k}, \forall k \in K \tag{41}\]

\[-M z_{4,k} + \bar{v}_{3,k} - u_k \leq \bar{v}_{4,k} + M z_{4,k}, \forall k \in K \tag{42}\]

\[-M z_{4,k} \leq Q_k^{pv} - \frac{Q_{pv,k}^{max}}{v_{3,k} - v_{4,k}} (u_k - \bar{v}_{3,k}) \leq M z_{4,k}, \forall k \in K \tag{43}\]

\[-M z_{5,k} + \bar{v}_{4,k} - u_k \leq \bar{v}_{5,k}, \forall k \in K \tag{44}\]

\[-M z_{5,k} \leq Q_k^{pv} + Q_{pv,k}^{max} \leq M z_{5,k}, \forall k \in K \tag{45}\]

The formulations (36)–(37), (38)–(39), (40)–(41), (42)–(43), and (44)–(45) are if-then conditional statements of co-optimization of the VVC curve settings for Zone 1, Zone 2, Zone 3, Zone 4, and Zone 5, respectively. However, making the settings of VVC as variables in the proposed model introduces non-convexity in the model due to two nonlinear terms \(Q_{pv,k}^{max} / (v_{2,k} - v_{1,k})\) and \(Q_{pv,k}^{max} / (v_{3,k} - v_{4,k})\) from formulations (39) and (43). To handle the non-convex issue in the model, the first-order Taylor series approximation is employed to convexify the nonlinear terms \(Q_{pv,k}^{max} / (v_{2,k} - v_{1,k}) (v_{2,k} - u_k)\) and \(Q_{pv,k}^{max} / (v_{3,k} - v_{4,k}) (u_k - \bar{v}_{3,k})\) as two linearized formulations denoted as functions \(f_2\) and \(f_4\) shown in (46)–(47), respectively.

\[
f_2\left(\tilde{Q}_{pv,k}^{max}, u_k, \bar{v}_{1,k}, \bar{v}_{2,k}\right) = \frac{v_{2,k} - u_k}{v_{2,k} - v_{1,k}} \tilde{Q}_{pv,k}^{max} - \frac{v_{2,k} - u_k}{v_{2,k} - v_{1,k}} \bar{v}_{2,k} \tag{46}\]

\[
f_4\left(\tilde{Q}_{pv,k}^{max}, u_k, \bar{v}_{3,k}, \bar{v}_{4,k}\right) = u_k - \frac{v_{3,k} - u_k}{v_{3,k} - v_{4,k}} \tilde{Q}_{pv,k}^{max} - \frac{v_{3,k} - u_k}{v_{3,k} - v_{4,k}} \bar{v}_{3,k} \tag{47}\]

By substituting functions \(f_2\) and \(f_4\) into constraints (39) and (43), the linearized formulations of Zone 2 and Zone 4 for the ACOPF with co-optimization of VVC curve settings become formulations (48)–(49). In this paper, Taylor series base points in formulations (48)–(49) and (19)–(20) are updated at the same time. Moreover, the linearization errors of \(f_2\) and \(f_4\) and SOC relaxation errors are checked simultaneously in the proposed approach (Fig. 2).

\[-M z_{2,k} \leq Q_k^{pv} - f_2\left(\tilde{Q}_{pv,k}^{max}, u_k, \bar{v}_{2,k}, \bar{v}_{3,k}\right) \leq M z_{2,k}, \forall k \in K \tag{48}\]

\[-M z_{4,k} \leq Q_k^{pv} - f_4\left(\tilde{Q}_{pv,k}^{max}, u_k, \bar{v}_{3,k}, \bar{v}_{4,k}\right) \leq M z_{4,k}, \forall k \in K \tag{49}\]

Moreover, IEEE standard 1547-2018 provides allowable ranges for settings of VVC. The settings limits are considered and modeled in constraints (50)–(54).

\[0.82^2 \leq \bar{v}_{1,k} \leq \bar{v}_{2,k} - 0.04, \forall k \in K \tag{50}\]

\[0.97^2 \leq \bar{v}_{2,k} \leq 1, \forall k \in K \tag{51}\]

\[1 \leq \bar{v}_{3,k} \leq 1.03^2, \forall k \in K \tag{52}\]

\[\bar{v}_{3,k} + 0.04 \leq \bar{v}_{4,k} \leq 1.18^2, \forall k \in K \tag{53}\]

\[0 \leq \tilde{Q}_{pv,k}^{max} \leq \tilde{Q}_{pv,k}^{max}, \forall k \in K \tag{54}\]

With the proposed default setting model of VVC (22)–(35) and optimal setting model of VVC (22)–(23), (34)–(38), (40)–(42), (44)–(54), the proposed SOCP-based ACOPF model is extended into two MISOPC-based ACOPF models, which are tested on the real 1747-node unbalanced distribution system.
In the original 1747-node distribution primary network shown in Fig. 4, there is no PV unit with VVC installed. As a result, all PV units inject their maximum active power with unity power factor, i.e., zero reactive power, into the grid. For the over-voltage loading condition on March 15th, 2019, the high penetration of PV units without VVC results in over-voltage issues, i.e., voltage magnitude exceeds 1.05 p.u., for some nodes at the studied over-voltage snapshot. For the under-voltage loading condition on July 15th, 2019, the large residential demand and lower PV generation at 7:00 pm leads to an under-voltage issue, i.e., voltage magnitude less than 0.95 p.u. for some nodes. The testing system is modified by including 8 to 14 VVCs for the PV units in the case study. Two MISOPC-based ACOPF models, i.e., default and optimal setting models of VVC, are tested on the modified distribution network under two different loading conditions.

The convex SOCP-based and MISOPC-based ACOPF models for the three-phase unbalanced system are implemented in Python and solved utilizing the Gurobi solver. To validate the system voltage stability with the optimal settings of PV units with VVC, a detailed dynamic model of the VVC is developed as a dynamic link library (DLL) in OpenDSS, which is a software tool developed by the Electric Power Research Institute (EPRI) for distribution system studies. Simulations are conducted via a laptop with an Intel Core i7-10750H CPU, 16GB DDR4, and 1TB PCIe SSD. The PV generation price is modeled based on the net surplus compensation rates of Pacific Gas and Electric (PG&E) on March 15th and July 15th, 2019 [37]. The wholesale electricity price is obtained from the locational marginal price (LMP) map of the California Independent System Operator (CAISO) for the PG&E area for March 15th and July 15th, 2019 [38]. In this paper, the PV generation and wholesale electricity prices are 21.79 $/MWh and 55.71 $/MWh, respectively, for the studied over-voltage snapshot. The PV generation and wholesale electricity prices for the under-voltage snapshot are 27.81 $/MWh and 101.84 $/MWh, respectively.

D. Data and Communication Requirements

The proposed model is designed for a three-phase unbalanced distribution system operation with VVC modeling based on the IEEE standard 1547-2018 to improve the system operation and help the utility to identify optimal settings of VVC. The topology of the network is needed, which can be provided by the topology processor [34]. PV and load forecast are the other inputs to the proposed model. As mentioned in IEEE standard 1547-2018, the settings of smart inverter controllers can be updated remotely, which relies on communication infrastructure. This framework is used for this proposed work.

IV. CASE STUDIES AND NUMERICAL RESULTS

A. Test System Data and Assumptions

The proposed models are tested on an actual 1747-node three-phase unbalanced distribution network of a local electric utility in Arizona [35] with different feeders and laterals configurations, i.e., single-phase and three-phase. The topology of the testing system is directly provided by the local utility in this work. The total number of branches is 1744. An overview of this distribution feeder is shown in Fig. 4. The modeled load and PV generation data represent two snapshots at 2:00 pm on March 15th, 2019, and 7:00 pm on July 15th, 2019, which refers to over-voltage and under-voltage loading conditions, respectively. For the 2:00 pm on March 15th snapshot, the instantaneous penetration of PV is 232%. The total active and reactive power demands are 1563.3 kW and 258.9 kVAR, respectively. The total active power injection from PV units is 3625.2 kW. For the 7:00 pm on July 15th snapshot, the instantaneous penetration of PV is 1.54%. Due to the extremely hot weather in Arizona in July, the energy computation for air-conditioning is large. The total active and reactive power demands are 13908.7 kW and 3837.1 kVAR, respectively. The total active power injection from PV units is 214.4 kW, which is low due to the little sun radiance at 7:00 pm.

In the original 1747-node distribution primary network shown in Fig. 4, there is no PV unit with VVC installed. As a result, all PV units inject their maximum active power with unity power factor, i.e., zero reactive power, into the grid. For the over-voltage loading condition on March 15th, 2019, the high penetration of PV units without VVC results in over-voltage issues, i.e., voltage magnitude exceeds 1.05 p.u., for some nodes at the studied over-voltage snapshot. For the under-voltage loading condition on July 15th, 2019, the large residential demand and lower PV generation at 7:00 pm leads to an under-voltage issue, i.e., voltage magnitude less than 0.95 p.u. for some nodes. The testing system is modified by including 8 to 14 VVCs for the PV units in the case study. Two MISOPC-based ACOPF models, i.e., default and optimal setting models of VVC, are tested on the modified distribution network under two different loading conditions.

The convex SOCP-based and MISOPC-based ACOPF models for the three-phase unbalanced system are implemented in Python and solved utilizing the Gurobi solver. To validate the system voltage stability with the optimal settings of PV units with VVC, a detailed dynamic model of the VVC is developed as a dynamic link library (DLL) in OpenDSS, which is a software tool developed by the Electric Power Research Institute (EPRI) for distribution system studies. Simulations are conducted via a laptop with an Intel Core i7-10750H CPU, 16GB DDR4, and 1TB PCIe SSD. The PV generation price is modeled based on the net surplus compensation rates of Pacific Gas and Electric (PG&E) on March 15th and July 15th, 2019 [37]. The wholesale electricity price is obtained from the locational marginal price (LMP) map of the California Independent System Operator (CAISO) for the PG&E area for March 15th and July 15th, 2019 [38]. In this paper, the PV generation and wholesale electricity prices are 21.79 $/MWh and 55.71 $/MWh, respectively, for the studied over-voltage snapshot. The PV generation and wholesale electricity prices for the under-voltage snapshot are 27.81 $/MWh and 101.84 $/MWh, respectively.

B. Three-Phase Unbalanced Convex SOCP-Based ACOPF

The proposed SOCP-based ACOPF is tested on the over-voltage and under-voltage loading conditions of the three-phase unbalanced distribution network in Fig. 4. The accuracy of the proposed model is evaluated. It is worth noting that the upper and lower voltage limits of nodes are relaxed only in this section due to the over-voltage and the under-voltage issues in the utility primary distribution feeder at the studied snapshots.

Table I presents the maximum relaxation and linearization errors by applying the proposed convex SOCP-based ACOPF under different loading conditions. In addition, the total simulation times for the tests of over-voltage and under-voltage loading conditions are 9.2 and 10.7 seconds, respectively. Since the ACOPF convexifies formulation (9) into a SOC form in constraint (10), the SOC relaxation error $u_{i,j}t_{i,j} - (e_{i,j}^D)^2 - (e_{i,j}^Q)^2$ becomes a critical factor in checking the exactness of the obtained solution.
TABLE I
MAXIMUM RELAXATION AND LINEARIZATION ERROR OF CONVEX SOCP-BASED ACOPF FOR THREE-PHASE UNBALANCED SYSTEM

| Loading condition | Max SOC error (p.u.) | Max absolute linearization error for $c^φ\phi_{ij}$ (p.u.) | Max absolute linearization error for $e^φ\phi_{ij}$ (p.u.) | Compute time (s) |
|-------------------|---------------------|------------------------------------------------------------|------------------------------------------------------------|-----------------|
| Over-voltage      | $1.1 \times 10^{-10}$ | $4.9 \times 10^{-11}$ | $8.1 \times 10^{-11}$ | 9.2             |
| Under-voltage     | $2.1 \times 10^{-10}$ | $2.8 \times 10^{-10}$ | $1.4 \times 10^{-10}$ | 10.7            |

For the 1747-node three-phase unbalanced network, the maximum SOC relaxation errors are $1.1 \times 10^{-10}$ and $2.1 \times 10^{-10}$ p.u. for over-voltage and under-voltage loading conditions of the system, respectively. The SOC errors are sufficiently small to ensure that the obtained solutions are exact. The linearization errors of $c^φ\phi_{ij}$ and $e^φ\phi_{ij}$ need to be checked as well because Taylor series approximation is used to linearize the non-convex auxiliary variable formulations (19)–(20). Maximum linearization errors of $c^φ\phi_{ij}$ and $e^φ\phi_{ij}$ shown in Table I are considerably low, which implies that the approximations of two auxiliary variables are accurate enough in the proposed model to ensure an acceptable solution for both two loading conditions.

Figs. 5 and 6 present the SOC relaxation errors, $c^φ\phi_{ij}$ and $e^φ\phi_{ij}$ linearization errors for all paths from node $iϕ$ (bus $i$ phase $ϕ$) to node $jϕ$ (bus $j$ phase $ϕ$) for the 1747-node three-phase unbalanced distribution feeder for over-voltage and under-voltage loading conditions. In Figs. 5 and 6, most of the convex SOCP-based ACOPF model can accurately handle relaxation and linearization errors and capture the three-phase unbalanced characteristics of a distribution system and obtain an exact optimal solution.

Two voltage profiles are shown in Figs. 7 and 8 for the over-voltage and under-voltage loading conditions of the system. Some voltage magnitudes exceed the system voltage limit 1.05 p.u. in Fig. 7 due to the excessive PV penetration. The maximal system voltage magnitude is 1.054 p.u. for the over-voltage loading condition. In Fig. 8, there are node voltage magnitudes less than 0.95 p.u. due to the large residential load and less PV generation at 7:00 pm. The minimal system voltage magnitude is 0.948 p.u. for the under-voltage loading condition. In addition, the voltage unbalanced level between different phases can also be revealed from the two voltage profile plots.

C. Results of MISOCP-Based ACOPF With Q-V Characteristic of PV Units: Default and Optimal Settings

1) Over-Voltage Loading Condition: In this subsection, two MISOCP-based ACOPF models are tested using the modified 1747-node system with 8 VVCs for the nodes of PV units having the worst over-voltage issues. The location of 8 PV units with VVC for the modified 1747-node distribution feeder is shown in Fig. 9. The over-voltage issue is eliminated after adding VVCs because of the capability of providing reactive power support and curtailing active power from PV units. To further compare the performance of the default and optimal setting models of VVC, two MISOCP-based ACOPF models are tested on the modified system with different number of VVCs.

Since the proposed MISOCP-based ACOPF model also involves SOC relaxation and Taylor series approximation, the relaxation errors and linearization differences need to be checked to ensure the exactness of the solution. The following results show the solutions of MISOCP-based ACOPF model for default and optimal setting models of VVC.
Table II shows the maximum SOCP relaxation errors of the proposed MISOCP-based ACOPF model for default and optimal settings of 8 PV units with VVCs. As shown in Table II, relaxation errors are small enough, which implies that the obtained solutions are exact for two MISOCP-based ACOPF models. The relaxation errors of default and optimal setting models are shown in Tables III and IV, respectively. The linearization errors of default and optimal setting models are shown in Figs. 10–11, respectively. The linearization errors of SOC relaxation error, $c_{ij}^{\phi
u'}$ and $e_{ij}^{\phi
u'}$, linearization error for default setting model for over-voltage loading condition.

Fig. 9. Topology of 1747-node utility feeder with 8 VVC locations.

TABLE II
MAXIMUM RELAXATION ERROR OF SOCP-BASED ACOPF WITH 8 PV UNITS WITH VVC FOR OVER-VOLTAGE LOADING CONDITION

|                      | Default settings | Optimal settings |
|----------------------|------------------|------------------|
| Max SOC error (p.u.) | $1.14 \times 10^{-7}$ | $1.09 \times 10^{-7}$ |

TABLE III
MAXIMUM LINEARIZATION ERROR OF SOCP-BASED ACOPF WITH DEFAULT SETTINGS OF 8 VVCs OVER-VOLTAGE LOADING CONDITION

| Model                  | Max absolute linearization error for $c_{ij}^{\phi
u'}$ (p.u.) | Max absolute linearization error for $e_{ij}^{\phi
u'}$ (p.u.) |
|------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| Default settings       | $5.2 \times 10^{4}$                                           | $9.8 \times 10^{4}$                                           |

TABLE IV
MAXIMUM LINEARIZATION ERROR OF SOCP-BASED ACOPF WITH OPTIMAL SETTINGS OF 8 VVCs OVER-VOLTAGE LOADING CONDITION

| Model                  | Max absolute linearization error for $c_{ij}^{\phi
u'}$ (p.u.) | Max absolute linearization error for $e_{ij}^{\phi
u'}$ (p.u.) | Max linearization error for Q-V characteristic (p.u.) |
|------------------------|---------------------------------------------------------------|---------------------------------------------------------------|--------------------------------------------------------|
| Optimal settings       | $5.0 \times 10^{4}$                                           | $9.5 \times 10^{4}$                                           | $1.2 \times 10^{2}$                                     |

Fig. 10. SOC relaxation error, $c_{ij}^{\phi
u'}$ and $e_{ij}^{\phi
u'}$, linearization error for default setting model for over-voltage loading condition.

Fig. 11. SOC relaxation error, $c_{ij}^{\phi
u'}$ and $e_{ij}^{\phi
u'}$, linearization error for optimal setting model for over-voltage loading condition.

Fig. 12. Linearization error of Q-V characteristic for optimal setting model for over-voltage loading condition.

setting model is shown in Table IV. It can be seen in Table IV that the maximum linearization error is $1.2 \times 10^{-2}$ p.u., which is small enough for a PV unit. Tables III and IV imply that the proposed constraints based on Taylor approximations are accurate and sufficient to ensure acceptable solutions for both MISOCP-based ACOPF models.

Figs. 10–11 present relaxation and linearization errors for all paths of the distribution network by applying the default and optimal setting models. The majority of SOC relaxation errors and $c_{ij}^{\phi
u'}$ linearization errors are zero for both models. The $e_{ij}^{\phi
u'}$ linearization errors are scattered from the maximum (around $10^{-7}$) to zero. Fig. 12 shows the linearization errors of $f_4$ for 8 PV units with VVCs in the ACOPF model with controller setting co-optimization. As it can be seen, only one PV unit has non-zero linearization error, which is relatively small. Figs. 10–12 confirm that the proposed MISOCP-based ACOPF models for default and optimal settings of VVC can accurately represent the Q-V characteristic of VVC in the model and capture the characteristics of the unbalanced system.
Figs. 13–14 illustrate the active and reactive power comparison of 8 PV units with VVCs for default and optimal setting models. Note that only 8 nodes of PV units having the worst over-voltage issues are selected to have VVC to mitigate the problems in the system. The system-level scheduling ensures that the PV units with VVC manage voltage at their local nodes and alleviate voltage violations across the feeder. Both models need to select a solution with active power curtailment and reactive power absorption from PV units with VVC to avoid over-voltage in the system due to the excessive PV active power injection. For the default setting model in Fig. 13, active power outputs of PV units 5, 6, and 8 are almost completely curtailed. However, there can be much less curtailment if settings are co-optimized in the model as shown in Fig. 13. Under default settings, there are active power curtailments for PV units with VVC in Figs. 13–14, while reactive power absorptions do not reach their maximum, i.e., $Q_{\text{max}}$. This issue is due to the fixed characteristic of VVC for the default setting case. Zone 5 is activated if the node voltage reaches VVC voltage setting $V_4$, however, due to the voltage limit (i.e., 1.05 p.u.) being smaller than the fixed VVC voltage setting $V_4$ (i.e., 1.06 p.u.), Zone 5 will not be activated under default settings. Thus, these VVCs operate in Zone 4 and the reactive power absorption is smaller than maximum reactive power absorption due to the fixed Q-V characteristic and settings (Fig. 3). In contrast, for the optimal setting model, the settings of VVC are adjusted to provide needed support to the distribution feeder, which results in less need for PV active power curtailment. The optimal setting model of VVC enables the ACOPF model to attain more flexibility to adjust the reactive power output by changing the settings of VVC and facilitates reactive power support where it is most needed. Figs. 13–14 confirms that co-optimizing the settings of VVC can result in more effective utilization of reactive power support by these units. The PV units 1, 2, 3, and 7 have less reactive power absorption, however, the rest of the PV units absorb more reactive power, which leads to less active power curtailments for the optimal setting model.

The voltage profiles for the default and optimal setting models are shown in Figs. 15 and 16. As observed from these figures, the over-voltage issue is mitigated (no voltage magnitude greater than 1.05 p.u.) after integration of 8 PV units with VVC in the system for both default and optimal setting cases.

![Fig. 13. Active power comparison of 8 PV units with VVC for default and optimal settings for over-voltage loading condition.](image)

![Fig. 14. Reactive power comparison of 8 PV units with VVC for default and optimal settings for over-voltage loading condition.](image)

![Fig. 15. The voltage profile of 1747-node utility feeder for the default setting model of VVC for over-voltage loading condition.](image)

![Fig. 16. The voltage profile of 1747-node utility feeder for the optimal setting model of VVC for over-voltage loading condition.](image)

![Fig. 17. Default settings v.s. optimal settings for one PV unit with VVC for over-voltage loading condition.](image)
Fig. 17 shows the default and optimal settings obtained by the proposed MISOCP-based ACOPF models for one PV unit with VVC for the over-voltage loading condition. As seen in Fig. 17, the optimal setting model enables much higher reactive power absorption from the PV units with VVC to mitigate over-voltage issues compared to the default setting model. The increased reactive power absorption can finally reduce the PV active power curtailment and improve system operation with the optimal VVC settings model. In addition, the voltage magnitudes of the nodes connected to PV unit with VVC is reduced due to the increased reactive power absorption.

The enhanced flexibility from the optimal settings of VVCs results in an improvement of the system operation as well as a reduction in operational cost and PV active power curtailment as shown in Table V. The cost savings and percentage of the active power curtailments in this paper are calculated via (55) and (56), respectively.

| No. of VVCs |
|---|
| Default settings | Optimal settings | Cost saving (%) |
|---|---|---|
| 8 | 113.8 | 158.1 | 17 | 6.8 |
| 9 | 153.9 | 196.5 | 8 | 7.4 |
| 10 | 182.8 | 229.1 | 3 | 6.3 |
| 11 | 217.9 | 259.0 | 0 | 6.1 |
| 12 | 238.5 | 274.1 | 0 | 5.7 |
| 13 | 268.4 | 294.2 | 0 | 5.4 |
| 14 | 306.4 | 326.9 | 0 | 4.0 |

Cost difference between cases w/ default and optimal settings

\[
\frac{\text{default settings cost}}{\text{Optimal settings cost}} \times 100\% \tag{55}
\]

Total active power curtailed from PV units with VVC

\[
\frac{\text{Total active power of PV units with VVC}}{\text{Total active power curtailed from PV units with VVC}} \times 100\% \tag{56}
\]

Case studies with different number of VVCs are conducted to compare the performance of the default and optimal setting models of VVC. To this end, more VVCs are considered for PV units of the nodes with the worst over-voltage issues. Table V shows the comparison and benefit of co-optimizing the settings of VVC within the ACOPF model. Note that the computational times for the default and optimal setting models are about 15 and 40 seconds, respectively, for all simulations. Moreover, all obtained solutions are checked to ensure to have sufficiently small relaxation and linearization errors. As the number of VVCs increases, the PV curtailment decreases for both default and optimal setting models because more PV units can have higher ability to provide reactive power support. As shown in Table V, the default setting model always receives higher PV curtailment than the model with the optimal settings of VVC. For instance, the optimal setting model reaches 0\% PV curtailment, while the default setting model still has 16\% PV curtailment in the case with 11 PV units with VVC. Also, the amounts of cost saving between the default and optimal setting models of VVC are shown in Table V. Due to the lower PV generation price compared to wholesale electricity price, less PV curtailment leads to less energy purchase from the more expensive wholesale electricity market. This implies that the system operating cost can be reduced by co-optimizing the settings of VCC as shown in Table V. Furthermore, the 9 VVCs case gains the highest cost-saving 7.4\% by co-optimizing the settings in the ACOPF model.

2) Under-Voltage Loading Condition: In this subsection, the proposed MISOCP-based ACOPF models are tested for an under-voltage loading condition at 7:00 pm on July 15th. The modified 1747-node utility feeder with 8 VVCs is used to evaluate the performance of modeling VVC for default and optimal settings in the ACOPF. The under-voltage issue is mitigated after the addition of the VVCs because of their capability to provide reactive power support.

The relaxation and linearization errors of the proposed MISOCP-based ACOPF model are shown in Tables VI–VIII for the default and optimal setting models of 8 PV units with VVCs. As shown in Table VI, the maximum SOC relaxation errors are very small, which ensures the exactness of the obtained solutions by using the proposed MISOCP-based ACOPF models. The linearization errors of the default and optimal setting models are shown in Tables VII and VIII. It is worth noting that all 8 PV units with VVC operate in either Zone 1 or Zone 2 to inject reactive power and mitigate the under-voltage issue in the system due to the large residential demand and less PV generation at 7:00 pm. The small linearization errors of the default and optimal setting models in Tables VII and VIII imply that the proposed
The voltage profiles for the default and optimal setting models are shown in Figs. 18 and 19. As it can be seen, the under-voltage issue can be eliminated (no voltage magnitude lower than 0.95 p.u.) after having 8 PV units with VVC in the system for both default and optimal setting models. In addition, the optimal setting model results in a higher system minimal voltage magnitude as shown in Figs. 18 and 19.

Fig. 20 shows the default and optimal settings obtained by the proposed MISOPC-based ACOPF models for one PV unit with VVC for the under-voltage loading condition. As can be seen from Fig. 20, the optimal setting model enables much higher reactive power injection from the PV unit with VVC compared to the default setting model. In addition, the voltage magnitude of the nodes connected to PV units with VVC is slightly increased due to the increased reactive power injection.

Fig. 21 presents the voltage magnitude of PV units with VVC comparison for the under-voltage loading condition. The x-axis of Fig. 21 shows the testing cases for the default and optimal setting models. The numbers on the x-axis denote the number of PV units with VVC in the network. The ‘d’ and ‘o’ denote the default and optimal setting models, respectively. For example, ‘8d’ on the x-axis refers to the testing case of the default setting model for the system with 8 PV units with VVC. It is worth noting that there is no active power curtailment for the under-voltage loading condition within all testing cases. The computational times of all simulations are about 15 and 60 seconds for default and optimal setting models, respectively. In addition, sufficiently small relaxation and linearization errors are achieved to ensure the exactness of obtained solutions. As can be seen in Fig. 21, the optimal setting model leads to a higher nodal voltage compared to the default setting model. It is because the optimal setting model enables the PV units with VVC to have more flexibility to provide more reactive power support to improve the grid operation.

3) Big-M Values: The proposed MISOPC-based ACOPF model introduces the Big-M method to formulate if-then conditional statements for the five operating zones of Q-V characteristic. The Big-M value is related to the voltage and reactive power for the Q-V characteristic Constraints (24)–(33) and (36)–(45). The selected Big-M value should be sufficiently large to make the constraints unbounded when the operating zone is not activated. For the voltage related constraints in the default setting model (24), (26), (28), (30), and (32), the Big-M values should be sufficiently larger than the maximal absolute difference between $u_k$ and voltage settings, i.e., $v_1$, $v_2$, $v_3$, and $v_4$. The range of $u_k$ is $0.95^2\sim1.05^2$, and $v_1$, $v_2$, $v_3$, and $v_4$ are default fixed values. Then, the maximal absolute difference is $0.22\left(1.06^2-0.95^2\right)$. The maximal $Q_{\text{max}}^\text{pv,k}$ for the default settings is $1.82$ p.u. among all PV units with VVC in the test system. Since the maximal absolute difference between $Q_k$ and reactive power for the Q-V characteristic (24)–(33) and (36)–(45). The selected Big-M value should be sufficiently larger than 3.64 p.u. for the reactive power related constraints, i.e., (25), (27), (29), (31), and (33). In the optimal setting model, due to the range of the variable voltage settings $\tilde{v}_{1,k}$, $\tilde{v}_{2,k}$, $\tilde{v}_{3,k}$, and $\tilde{v}_{4,k}$, the select Big-M value should be sufficiently
TABLE IX
COMPUTATION TIME FOR THE DEFAULT SETTINGS WITH DIFFERENT BIG-M VALUES

| Big-M values for Constraints (24), (26), (28), (30), and (32) | Big-M values for Constraints (25), (27), (29), (31), and (33) | Computational time (s) |
|---------------------------------------------------------------|---------------------------------------------------------------|-----------------------|
| 0.7                                                           | 10.9                                                          | 16.5                  |
| 1.1                                                           | 18.2                                                          | 16.7                  |
| 1×10²                                                         | 1×10²                                                         | 20.5                  |
| 1×10³                                                         | 1×10³                                                         | 17.8                  |
| 1×10⁴                                                         | 1×10⁴                                                         | 17.7                  |

TABLE X
COMPUTATION TIME FOR THE OPTIMAL SETTINGS WITH DIFFERENT BIG-M VALUES

| Big-M values for Constraints (36), (38), (40), and (44) | Big-M values for Constraints (37), (39), (41), (43), and (45) | Computational time (s) |
|----------------------------------------------------------|---------------------------------------------------------------|-----------------------|
| 1.5                                                      | 14.6                                                          | 28.2                  |
| 2.4                                                      | 24.3                                                          | 31.8                  |
| 1×10²                                                    | 1×10²                                                         | 54.9                  |
| 1×10³                                                    | 1×10³                                                         | 55.2                  |
| 1×10⁴                                                    | 1×10⁴                                                         | 59.0                  |

larger than maximal absolute difference 0.49 (1.18²-0.95²) for the voltage related Constraints (36), (38), (40), (42), and (44). Since $Q_{pv,k}^\text{max}$ can reach the maximum apparent power rating $P_{pv,k}^\text{max}$, the select Big-M should be sufficiently larger than the maximal absolute difference $2 \times P_{pv,k}^\text{max}$ (i.e., 4.85 p.u.) for the reactive power related constraints in the optimal setting model (37), (39), (41), (43), and (45).

To evaluate the impact of different Big-M values on the performance of the proposed MISOCP-based ACOPF model, the sensitivity analyses are conducted for the default and optimal setting models in this subsection. Three and five times maximal absolute differences and extremely large Big-M values, i.e., $10^2$, $10^3$, and $10^4$, are tested to show the impact of Big-M values on the proposed model. The modified 1747-node system with 8 VVCs for the over-voltage loading condition is used for this sensitivity analysis. The simulation results show that different Big-M values within the aforementioned ranges only affect the computational time of the proposed model while the solutions are the same among all these cases. Tables IX and X exhibit the computational time for 8 PV units with VVC under different Big-M values for the default and optimal setting models. It is worth mentioning that the SOC errors are sufficiently small (around $1.1 \times 10^{-7}$) for all obtained solutions. As it can be seen in Tables IX and X, the computational time increases as the Big-M values increase for the optimal setting model. One reason can be that the larger Big-M value results in a larger feasible region, which can make the proposed model take more time to converge. Both Tables IX and X confirm that the optimal solution and tractable computational time can be ensured if the select Big-M value is sufficiently large for the constraints (24)–(33), (36)–(45) for the default and optimal setting models.

4) Scalability of MISOCP-Based ACOPF: 77 PV Units With VVC: In the 1747-node utility feeder, there are a total of 249 PV units. To demonstrate the scalability of the proposed model as the number of PV units with VVC increases, this work considers 77 out of 249 PV units with VVC since 77 is the minimum number of VVC needed to mitigate the over-voltage issue in the system without any active power curtailment for the default setting model. The proposed MISOCP-based ACOPF model is tested on a 1747-node network with 77 PV units with VVC for both over-voltage and under-voltage loading conditions. The simulation results are shown in Tables XI and XII. It can be seen in Table XI that the relaxation and linearization errors are very small, which ensures the exactness of the solutions obtained by the proposed models. In addition, the proposed MISOCP-based ACOPF models are still tractable for the default and optimal setting models for the system with a large number of PV units with VVC in Table XII.

5) Dynamic Simulation for Stability Analysis of PV Unit: Optimal Settings: Since the improper selection of the VVCs’ settings may cause system instability [1], dynamic analysis needs to be conducted to verify the control stability of PV units with VVC for optimal settings. In this paper, the stability of the optimal settings and the impact of load change disturbance are evaluated through dynamic analysis. The dynamic analyses are conducted via a DLL in OpenDSS, which models the dynamic characteristics of the LCL filter, current control loop, and phase-lock loop. The Q-V characteristic of PV units with VVC is programmed and developed in a DLL file and called by OpenDSS. The details of these dynamic models for PV units with VVC are shown in [39]. The result of the 14 VVCs case is
results in a large voltage drop at time 0.5 seconds, while there is an obvious increase due to the activation of VVCs. Then, it can be observed in Fig. 22–23 that the reactive power output of VVC can be maintained at its original value within a short period after the activation of VVC and load change. As can be seen from the voltage magnitude, active power, and reactive power curves in Figs. 22 and 23, the active power output of VVC can be maintained at its original value within a short period after the activation of VVC and load change. Dynamic simulation results imply that the obtained optimal settings solutions are acceptable and cause no stability issues for the system for both operational conditions.

V. CONCLUSION

In this paper, a convex SOCP-based ACOPF model is proposed for three-phase unbalanced distribution systems. A two-stage iterative-based algorithm is developed to solve the proposed SOCP-based ACOPF. The Taylor series approximation is employed to create a linear relationship among the auxiliary variables to make the SOCP-based approach suitable for unbalanced distribution systems. The Q-V characteristics of PV units with VVC is considered based on the guidance of IEEE Standard 1547-2018. The proposed SOCP-based ACOPF is converted and extended into two MISOCP-based ACOPF models to account for the Q-V characteristic of PV units with VVC with default and optimal settings. All proposed models are tested on an actual 1747-node unbalanced distribution feeder for different operational conditions.

To the best of the authors’ knowledge, a convex SOCP-based ACOPF for the unbalanced distribution network is rarely discussed in the literature. This paper fills this research gap by proposing a convex SOCP-based ACOPF model for the three-phase unbalanced distribution grid operation while accounting for IEEE Standard 1547-2018 requirements for smart inverters. The proposed models can capture the characteristics of three-phase unbalanced network. The simulation results show that the proposed models can obtain the optimal solution with very small relaxation and linearization errors, which means that the obtained solution is exact for the system. Moreover, the Q-V characteristic of PV units with VVC is well represented by the proposed MISOCP-based ACOPF. Besides, the results of the MISOCP-based ACOPF show that the system operation can be improved if settings of VVC are co-optimized in the model due to the flexibility of the optimal setting model to adjust the reactive power output of PV units. In addition, the scalability of the proposed models and the impact of Big-M values are discussed in this work. Furthermore, dynamic analysis is conducted to ensure the stability of the system. The dynamic simulation results confirm that the optimal settings are valid and cause no stability concern for the distribution system.

It should be mentioned that one of the limitations of the proposed models would be the requirement of the communication infrastructure that can remotely update the settings of smart inverter controllers. In addition, IEEE standard 1547-2018 includes four types of reactive power control modes and one voltage-active power control mode that can simultaneously be enabled with one of the reactive power control modes if

![Fig. 22. Voltage magnitude, active power, and reactive power of a PV unit with VVC for over-voltage case.](image1)

![Fig. 23. Voltage magnitude, active power, and reactive power of a PV unit with VVC for under-voltage case.](image2)
required. The proposed three-phase unbalanced SOCP-based ACOPF model only incorporates the Q-V mode characteristics of the PV unit with VVC for the default and optimal settings models. The modeling of the other types of control modes and the voltage-active power control mode in the proposed ACOPF model will be explored in the future work.

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