Study on parameter identification of vibration absorption system based on acceleration signal

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Abstract. Based on the analysis and processing of acceleration signals, the time domain least square method and frequency domain analytic method are proposed to identify the parameters of vibration absorption system. The mathematical model of vibration reduction and Simulink simulation system are established, two parameter identification methods are validated by known system parameters and vibration signals; and then, based on the acceleration signal, the influence of signal processing on identification accuracy is analyzed. The two identification methods proposed in this paper can accurately identify the parameters of the vibration absorption system, Finally, the experimental results show that the two identification methods have good engineering applicability and scientific research applicability.

1. Introduction
Rubber shock absorber is widely used in military and civil fields. Its dynamic stiffness characteristics and other parameters are important basis for shock absorber design; the dynamic stiffness characteristic is the regular pattern that the dynamic stiffness of the system changes with the excitation frequency; through the dynamic stiffness characteristics and the working state of the designed shock absorber, the designed shock absorber can avoid the resonance zone and keep it in a good state of vibration reduction\cite{1}.

Wan \textit{et al.} \cite{2} used polynomial fitting method to derive the identification process of dynamic stiffness and other parameters. He \textit{et al.} \cite{3} studied the influence of vibration frequency, amplitude and preload on dynamic stiffness. Li \textit{et al.} \cite{1} obtained the dynamic stiffness varying with frequency according to the static stiffness value and the formula of single degree of freedom. Overall, compared with the performance of shock absorber, the research on dynamic stiffness of shock absorber is less, and the research on parameter identification methods such as dynamic stiffness is less.

In this paper, two methods of identifying the parameters of vibration absorption system are proposed, which are the time domain least square method and frequency domain analytic method based on acceleration signal. The core of identification is to analyze the input signal and the output of the system to obtain the parameters of the vibration absorption system; the input and output include signals of acceleration, velocity and displacement. In vibration reduction experiments, the acceleration signal is easier to measure than the velocity signal, especially the displacement signal, acceleration sensor is selected in this experiment, the required signals of velocity and displacement are obtained by integrating the acceleration signals. Integration of acceleration signal is an important problem in identification process.

The integration of acceleration signal can be divided into time domain integration and frequency
domain integration. Gu [4] of Tsinghua University verified the two integration methods through simulation and comparison; the research of Zhou [5] of the aeronautical industry helicopter Institute shows that the effect of frequency domain integration is better than that of time domain integration.

In this paper, the mathematic model of vibration absorption system and Simulink simulation system are established, and then two identification methods are validated by known system parameters, input signals and output signals of system, respectively; the acceleration signal is integrated in frequency domain to obtain the velocity and displacement signals which are substituted in the identification process to obtain the factors affecting the identification accuracy.

2. Least squares identification

2.1. Basic principles

Identification of parameters of vibration system by the time domain least square method is a process of data fitting [6]. After determining the mathematical model of the vibration system, the sum of squares of the difference between the left and right ends of the equation of the mathematical model is minimized according to the measured data, and the optimal system parameters are identified. Figure 1 is a mass-spring single-degree-of-freedom vibration system with damping.

![Figure 1. Excited vibration system.](image)

In the picture m is the system mass, c and k are the system damping and stiffness parameters. There is the equation (1) for the harmonic excitation force f acting on the system.

\[ f = m\ddot{x} + c\dot{x} + kx \]  \hspace{1cm} (1)

In the formula, \( \ddot{x} \), \( \dot{x} \) and \( x \) are the signals of system vibration acceleration, velocity and displacement, respectively. Transform the equation into

\[ f - m\ddot{x} = c_1\dot{x} + k_1x \]  \hspace{1cm} (2)

According to N sets of data of input and output collected in the simulation system, N covers at least one cycle of data, formula (2) makes N sets of difference on both sides of equation, and obtains formula (3).

\[
\begin{align*}
\Delta_1 &= [f_1 - m\ddot{x}_1] - [c_1\dot{x}_1 + k_1x_1] \\
\Delta_2 &= [f_2 - m\ddot{x}_2] - [c_1\dot{x}_2 + k_1x_2] \\
& \vdots \\
\Delta_n &= [f_n - m\ddot{x}_n] - [c_1\dot{x}_n + k_1x_n]
\end{align*}
\]  \hspace{1cm} (3)

The best value of the parameter minimizes the sum of squares, so there is a formula (4).

\[ \Delta = \Delta_1^2 + \Delta_2^2 + \ldots + \Delta_n^2 = \min \]  \hspace{1cm} (4)

When \( \Delta \) takes the minimum value, the partial derivatives of \( k_1 \) and \( c_1 \) should be zero.
\[ \begin{aligned}
\frac{\partial \Delta}{\partial c_k} &= 2\Delta_1(-\dot{x}_1) + 2\Delta_2(-\dot{x}_2) + \ldots + 2\Delta_n(-\dot{x}_n) = 0 \\
\frac{\partial \Delta}{\partial k_1} &= 2\Delta_1(-x_1) + 2\Delta_2(-x_2) + \ldots + 2\Delta_n(-x_n) = 0
\end{aligned} \]  

(5)

Formula (6) is converted into a matrix:

\[ \left[ \begin{array}{c}
\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n \\
x_1, x_2, \ldots, x_n \\
\Delta_1, \Delta_2, \ldots, \Delta_n 
\end{array} \right] = 0 \]  

(6)

Set \( A = \left[ \begin{array}{c}
\dot{x}_1, x_1 \\
\dot{x}_2, x_2 \\
\vdots \\
\dot{x}_n, x_n 
\end{array} \right], \quad F = \left[ \begin{array}{c}
f_1 - m \dot{x}_1 \\
f_2 - m \dot{x}_2 \\
\vdots \\
f_n - m \dot{x}_n 
\end{array} \right], \quad \Delta = \left[ \begin{array}{c}
\Delta_1 \\
\Delta_2 \\
\vdots \\
\Delta_n 
\end{array} \right], \quad X = \left[ \begin{array}{c}
c_1 \\
k_1 
\end{array} \right], \]

then formulas (3) and (6) can be expressed as formula (7).

\[ \begin{aligned}
\Delta &= F - AX \\
A^T \Delta &= 0
\end{aligned} \]  

(7)

Solution (7) obtains the parameter matrix of vibration absorption system.

\[ X = (A^T A)^{-1} A^T F \]  

(8)

2.2. Simulation verification of least square identification

A mass-spring system with damping is built in Simulink, Setting the mass \( m \), stiffness \( K \) and damping parameter \( C \) in vibration absorption system. The excitation force \( f(t) \) is input to the vibration absorption system to generate response signals of acceleration, velocity, displacement, and then the parameters \( k_1 \) and \( c_1 \) are identified. The accuracy of the identification method is obtained by comparing the errors between the identification parameters and the real parameters of the system. The verification process is shown in figure 2.

![Figure 2. Verification process.](image)

Set the system parameter mass \( m = 4 \), system stiffness \( k = 1000 \), damping ratio 0.1, that is, damping parameter \( C = 12.6491 \). After the excitation force is input, the generated signal is substituted into the identification process, and the parameters of the system are obtained as shown in table 1.

From table 1, it can be seen that the time domain least square method can accurately identify the parameters according to the known input signals and output signals of the system.
Table 1. Identification of parameters of vibration system.

| Excitation frequency | Sampling frequency | System stiffness k | Identification stiffness k1 | System damping c | Identification of Damping c1 |
|----------------------|--------------------|-------------------|-----------------------------|-----------------|-----------------------------|
| 10hz                 | 512hz              | 1000.000          | 1000.0000                   | 12.6491         | 12.6491                     |
| 50hz                 | 1024hz             | 1000.000          | 1000.0000                   | 12.6491         | 12.6491                     |
| 100hz                | 2048hz             | 1000.000          | 1000.0000                   | 12.6491         | 12.6491                     |

2.3. Parameter identification based on the time domain least square method of acceleration signal

In this section, the signals of velocity and displacement are obtained by frequency domain integrating the acceleration signals, and then the parameters are identified.

Firstly, the simulated output acceleration \( \ddot{x} \) is used as the basic data. Output velocity \( \dot{x} \) and displacement \( x \) are obtained by frequency domain integration. The simulation acceleration \( \ddot{x} \), integral velocity \( \dot{x} \) and integral displacement \( x \) are substituted into the identification process, and the influence of the processing of acceleration signal on the system identification is obtained by comparing \( k_1 \) and \( c_1 \) with the real \( k \) and \( c \) of the system.

Frequency domain integration firstly transforms the time domain signal into Fourier transform, and then calculates the frequency domain signal of velocity and displacement. Finally, the inverse Fourier transform of the frequency domain signal of velocity and displacement is used to obtain the time domain signal of velocity and displacement.

Fourier component of acceleration time domain signal at any frequency can be expressed as formula (9).

\[
\ddot{x}(t) = \ddot{X}e^{i\omega t}
\]  

(9)

The component of velocity signal can be obtained by integrating the component of acceleration signal, and the component of displacement signal can be obtained by integrating the component of velocity signal, as shown in formulas (10) and (11).

\[
\dot{x}(t) = \int_{0}^{t} \ddot{X}e^{i\omega t} dt = \frac{\ddot{X}e^{i\omega t}}{i\omega} = \dot{X}e^{i\omega t}
\]  

(10)

\[
x(t) = \int_{0}^{t} \frac{\ddot{X}e^{i\omega t}}{i\omega} dt = -\frac{\ddot{X}e^{i\omega t}}{\omega^2} = Xe^{i\omega t}
\]  

(11)

In the formula, \( \ddot{x}(t) \), \( \dot{x}(t) \) and \( x(t) \) are the Fourier components of acceleration signal, velocity signal and displacement signal under frequency \( \omega \), respectively. Frequency \( \omega \) is a variable, \( \ddot{X} \), \( \dot{X} \) and \( X \) are corresponding coefficients, \( i \) is imaginary.

Setting \( \ddot{X}(\omega) \), \( \dot{X}(\omega) \) and \( X(\omega) \) as frequency domain signals of acceleration, velocity and displacement respectively, the conversion relations among signals of acceleration, velocity and displacement in frequency domain can be obtained from formulas (9), (10) and (11), as shown in formulas (12) and (13).

\[
\ddot{X}(\omega) = \frac{\dot{X}(\omega)}{i\omega}
\]  

(12)

\[
X(\omega) = -\frac{\ddot{X}(\omega)}{\omega^2}
\]  

(13)

The obtained time domain signals of velocity and displacement are substituted into the time domain
least square method for system parameter identification. The results are shown in Table 2.

| Excitation frequency | Sampling frequency | System stiffness k | Identification stiffness k1 | System damping c | Identification of Damping c1 |
|----------------------|--------------------|-------------------|----------------------------|-----------------|-----------------------------|
| 10hz                 | 512hz              | 100.000           | 1.0086e3                   | 11.6854         |                             |
| 50hz                 | 1024hz             | 1000.000          | 1.5813e3                   | 12.6491         | 12.0458                     |
| 100hz                | 2048hz             | 2.1596e3          | 12.2947                    |                 |                             |

From Table 2, it can be seen that the accuracy of identification results of system parameters decreases with the increase of excitation frequency domain. The larger the sampling frequency is, the richer the information in the frequency domain integration is, the closer the obtained signal is to the real signal, and the more accurate the identification results will be. The identification results of the simulation experiments under 100 Hz input excitation at different sampling frequencies are shown in Table 3.

| Excitation frequency | Sampling frequency | System stiffness k | Identification stiffness k1 | System damping c | Identification of Damping c1 |
|----------------------|--------------------|-------------------|----------------------------|-----------------|-----------------------------|
| 100hz                | 256hz              | 1000.000          | 5.5469e3                   | 12.6491         | 10.1388                     |
| 100hz                | 2048hz             | 2.1596e3          | 12.2947                    |                 | 12.5767                     |
| 100hz                | 8192hz             | 1.2908e3          | 12.5767                    |                 | 12.6301                     |
| 100hz                | 32768hz            | 1.0657e3          | 12.6301                    |                 |                             |

From the data in Table 3, it can be seen that the larger the sampling frequency, the more accurate the identification results are. This is because the large sampling frequency substitutes more abundant signal information into the integration process, which makes the signal of frequency domain integration closer to the real output value of the system.

3. Identification of frequency domain analytic method

3.1. Basic principles

Frequency domain analytic method is the process of transforming time domain equation into frequency domain equation and finally obtaining analytic solution. Formula (1) is transformed by Laplace transform to obtain the frequency domain equation. Transform according to Laplace transform integral theorem:

\[ F(s) = m\ddot{X}(s) + c_i \frac{\dot{X}(s)}{s} + k_i \frac{X(s)}{s^2} \]  \hspace{1cm} (14)

\[ \frac{F(s)}{\ddot{X}(s)} = m + \frac{c_i}{s} + \frac{k_i}{s^2} \]  \hspace{1cm} (15)

\[ \frac{F(s)}{\ddot{X}(s)} \] is the acceleration impedance of the system when replace \( s \) with \( i\omega \), the ratio of the input signal of the system to the output signal of the system in complex form. It characterizes the amplitude-frequency and phase-frequency characteristics of the input signal and the output signal of the system.

Set \( P = \frac{F(s)}{\ddot{X}(s)} = P_R + P_i \) \, \( P_R \) is its real part and \( P_i \) is its imaginary part. The modulus \( P \) is the
amplitude ratio of input excitation force signal to output acceleration signal. $P$ is converted into angle, which is the phase difference between the input excitation force signal and the output acceleration signal [9].

The amplitude ratio and phase difference of signals of the input and output are calculated to obtain $P$ [7], and the P-incorporation formula (15) and the formula (16) are obtained.

$$P_R + P_i i = \left( m - \frac{k_1}{\omega^2} \right) - \frac{c_1 i}{\omega}$$ (16)

The numbers of formula (16) is complex on both sides of the equal sign, so the real parts on both sides are equal and the imaginary parts on both sides are equal. Solution formula (16) gets (17).

$$\begin{cases} k_1 = \omega^2 (m - P_R) \\ c_1 = -\omega P_i \end{cases}$$ (17)

3.2. Simulation and verification of frequency domain analytic method

The parameters $k_1$ and $c_1$ are identified from the output data of the system with known parameters $k$ and $c$, and compared with the real values of $k$ and $c$ of the system. The results are shown in table 4.

| Excitation frequency | sampling frequency | System stiffness k | Identification stiffness $k_1$ | System damping c | Identification of Damping $c_1$ |
|----------------------|--------------------|-------------------|-------------------------------|------------------|-------------------------------|
| 10 hz                | 512 hz             | 1.0105e3          | 12.4102                       |                  |                                |
| 50 hz                | 1024 hz            | 1000.000          | 1.5901e3                      | 12.0491          | 10.1428                       |
| 100 hz               | 2048 hz            | 2.2011e3          | 12.2985                       |                  | 12.0491                       |

As can be seen from table 4, there are errors in the identification results. The main factors affecting the identification results are $P_R$ and $P_I$, which are determined by the amplitude ratio and phase difference of the input signal and the output signal of the system. In the process of calculating the amplitude ratio and phase difference of input and output signals, the accuracy is still affected by sampling frequency [7]. The results of the frequency domain analytic method under different sampling frequencies with simulation experiment of 100 Hz input excitation are shown in table 5.

| Excitation frequency | sampling frequency | System stiffness k | Identification stiffness $k_1$ | System damping c | Identification of Damping $c_1$ |
|----------------------|--------------------|-------------------|-------------------------------|------------------|-------------------------------|
| 100 hz               | 512 hz             | 5.6170e3          | 10.1428                       |                  |                                |
| 100 hz               | 2048 hz            | 1000.000          | 2.2011e3                      | 12.2985          |                                |
| 100 hz               | 8192 hz            | 1.3034e3          | 12.5795                       |                  |                                |
| 100 hz               | 32768 hz           | 1.0760e3          | 12.6329                       |                  |                                |

From the data in table 5, it can be seen that the frequency domain analytic method can accurately identify the system parameters, and the larger the sampling frequency, the more accurate the identification results are.

4. Examples verification

4.1. Experiments and identification processes

4.1.1. Introduction to experiments. In this section, a spacecraft electronic instrument box model is
taken as an example for vibration absorption experiments. Vibration absorption system consisting of
damper material and electronic instrument box model fixed on vibration frame. Figure 3 is a vibration
absorption model and figure 4 is a vibration absorption experiment.

\[ m\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) = 0 \]  (18)

\( \ddot{x}_1, \dot{x}_1 \) and \( x_1 \) are the signals of acceleration, velocity and displacement of the electronic instrument
box model respectively; \( \ddot{x}_2, \dot{x}_2 \) and \( x_2 \) are signals of the acceleration, velocity and displacement of the
shaking table respectively.

The shaking table is vibrated step by step in the range of 5-20 Hz with the step length of 1 HZ. Acceleration signals are collected in the experiment. Velocity and displacement signals are obtained by frequency domain integration.

4.1.2. Identification of experimental vibration system parameters by the time domain least square method. Formula (18) is converted to formula (19) when the parameters of the experimental system are identified by the time domain least square method.

\[ m\ddot{x}_i = c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) \]  (19)

4.1.3. Identification of experimental vibration system parameters by the frequency domain analytic method. When the parameters of the experimental system are identified by the frequency domain analytic method, formula (18) is transformed by Laplace transform integral theorem, and formula (20) is obtained.

\[ [m + \frac{c}{s} + \frac{k}{s^2}]\ddot{X}_1 = [\frac{c}{s} + \frac{k}{s^2}]\ddot{X}_2 \]  (20)

Substituting \( s = i\omega \) and \( \ddot{X}_1(s) = P_R + P_I i \) into formulas (20), (21) and (22) are obtained.

\[ k = \frac{m\omega^2 (P_R^2 + P_I^2 + P_R)}{P_R^2 - 2P_R + P_I^2 + 1} \]  (21)

\[ c = -m\omega^2 P_R + P_R k - k \]  (22)

4.2. Results and analysis
The dynamic stiffness and damping parameters of the experimental system identified by the two identification methods are shown in figures 5 and 6.

![Figure 5. Dynamic stiffness of system.](image1)

![Figure 6. Dynamic damping of system.](image2)

From figures 5 and 6, it can be seen that the parameters of the experimental system identified by two different methods are in good agreement. The two methods are mutually corroborative and can confirm that the identification results are correct.

5. Conclusion
- The time domain least square method and the frequency domain analytic method proposed in this paper have good application of scientific research and engineering.
- Sampling frequency of signal is an important factor in parameter identification. The larger the sampling frequency, the more accurate the identification result is, but the calculation amount will increase accordingly.

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