Design and evaluation of traffic delays in toll plaza using combination of queueing and simulation

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Abstract. In this paper, the Simulation approach is used to evaluate the toll plaza performance measures for four different types of vehicle categories F1, F2, F3 and F4 Simultaneously. Here we propose an alternate approach for studying toll plaza queueing system by employing Monte Carlo Simulation. It is a quite effective way for analyzing the performance measures for the queueing system and also we compare these simulation results with analytic method.

Key words: Arrival distribution, service distribution, arrival rate, service rate, Monte Carlo simulation, queueing model, queue length, queue time, system length, system time.

1. Introduction
Queueing Theory is the study of waiting line theory and the origin of Queueing theory started in the beginning of the 20th century, when Erlang (1909) [2] published his fundamental paper on congestion in telephone traffic. Queue analysis presents an adequate service for analyzing a system and it is not an optimization technique, which helps to find out the performance procedures for analyzing queues. Queueing system having a service facility to provide service for arrivals (vehicles) that enters into the system in order to get service with always a first-come first-served policy [3].

Simulation is a numerical technique for conducting experiment for random events to describe the behavior of a complex real world system (Ravindran A. Philips (1987)) [6] and also its mimic the behavior of a real system. The term simulation is defined as, to imitating an existing system without disturbing its actual process for predicting their future behavior. So that, the administration can takes a necessary action that will help to improve the system performance in an effective manner. But, due to the uncertainty in the real world system for future prediction is based on the probability distribution. Monte Carlo simulation is a common technique for analyzing the uncertainty based on the various aspects of a system for predicting their future behavior. Monte Carlo means using random numbers in scientific computing that is, using random numbers as a tool to compare something that is not random. The important feature of Monte Carlo method is, the management makes most happy is that simple, clever idea can lead to enormous practical improvement in system performance. This method is based on the generation of multiple trials to determine the expected value of a random variable and it is helpful for obtaining numerical solutions to the real world problems.
which are too complicated to solve analytically. It was named by S. Ulam, who in 1946 [5] became the first mathematician to distinguish this approach and Nicolas Metropolis also made important contributions for the development of this method.

This research focused on a single server analysis of a toll plaza as a case study [4] and exhibits the performance measures for four types of vehicles queue simultaneously. In addition to this, this research shows simulation techniques gives the opportunity for calculating performance measures and addresses the issue of how queues formed in the toll plaza system. In this paper, we continued our ongoing examination of a single server queue with four types of vehicles category and it is relatively a next step of the previous work with the comparison of analytic performance measures for a toll plaza system. In toll plaza analysis, both queueing and discrete event simulation models are used extensively, but compared with queueing models, discrete event simulation is more flexible for modeling the vehicle flows in a system [1].

In stochastic queueing process is to analyze steady-state conditions are

1. $L_S$ = Average number of vehicles in toll plaza queueing system or System length
2. $L_q$ = Queue length
3. $W_S$ = System time
4. $W_q$ = Queue time

2. Data collection and analysis

We investigate the collected data for four types of vehicles category at the toll plaza for example, F1 (car/jeep), F2 (Light commercial vehicles-LCV), F3 (Truck/Bus) and F4 (Multi Axel vehicles-MAV) also time punch for each vehicle were also part of the data. At this toll plaza, the toll way is a dual carriage way with four lanes in each direction and we analyze the entering lanes only for computing the performance measures. A vehicle may enter the toll plaza system and to pay the toll fee through the toll gate in charge, if the server is busy, the customer (vehicle) should wait in the system; this is the service request time in our simulation model. The study purpose of this paper is to test different types of vehicle flow at each tollgate and observing the effect on queue length, arrival times and service times, which are followed on Markovian distributions, these are calculated numerically from a collected data. In this simulation, we have used 100 trials for the analysis of a toll plaza system, because more trials will lead to a higher accuracy.

We presented a single server based simulation model for F1 (car/jeep), F2 (Light commercial vehicles-LCV), F3 (Truck/Bus) and F4 (Multi Axel vehicles-MAV) individually. Next, we proposed the comparative study for the analysis of toll plaza performance in both single and multiserver model for F1 (car/jeep) using simulation methods. Then we discussed the comparison study for the analysis of toll plaza performance in both single and multiserver models for F2 (Light commercial vehicles-LCV) using simulation methods. Also, we visualized [8] F1 (car/jeep) & F2 (Light commercial vehicles-LCV) simultaneously with the performance measures on both crisp and Monte Carlo simulation methods. Next we presented the simulation study of the single server queueing model in toll plaza by considering vehicle type [7]. Now in this paper, we analyze the simulation as well as with analytical method for each lane according to the vehicles arrive.
Consider the arrival distribution for F1, F2, F3 & F4 in following tables 1, 2, 3, & 4.

**Table 2.1. Arrival distribution for F1**

| S. No. | Inter Arrival Time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------------|-----------------|-------|------------|---------|
| 1      | 0-1                      | 85              | 0.425 | 0.425      | 000 – 424 |
| 2      | 1-2                      | 46              | 0.23  | 0.655      | 425 – 654 |
| 3      | 2-3                      | 27              | 0.135 | 0.79       | 655 – 789 |
| 4      | 3-4                      | 14              | 0.07  | 0.86       | 790 – 859 |
| 5      | 4-5                      | 10              | 0.05  | 0.91       | 860 – 909 |
| 6      | 5-6                      | 4               | 0.02  | 0.93       | 910 -929  |
| 7      | 6-7                      | 5               | 0.025 | 0.955      | 930 – 954 |
| 8      | 7-8                      | 1               | 0.005 | 0.96       | 955 – 959 |
| 9      | 8-9                      | 1               | 0.005 | 0.965      | 960-964   |
| 10     | 9-10                     | 2               | 0.01  | 0.975      | 965 – 974 |
| 11     | 10-11                    | 0               | -     | -          | -        |
| 12     | 11-12                    | 0               | -     | -          | -        |
| 13     | 12-13                    | 1               | 0.005 | 0.98       | 975 – 979 |
| 14     | 13-14                    | 0               | -     | -          | -        |
| 15     | 14-15                    | 1               | 0.005 | 0.985      | 980 – 984 |
| 16     | 15-16                    | 0               | -     | -          | -        |
| 17     | 16-17                    | 0               | -     | -          | -        |
| 18     | 17-18                    | 1               | 0.005 | 0.99       | 985 -989 |
| 19     | 18-19                    | 1               | 0.005 | 0.995      | 990 -994 |
| 20     | 19-20                    | 0               | -     | -          | -        |
| 21     | 20-21                    | 0               | -     | -          | -        |
| 22     | 21-22                    | 0               | -     | -          | -        |
| 23     | 22-23                    | 0               | -     | -          | -        |
| 24     | 23-24                    | 1               | 0.005 | 1          | 995 -999 |
| 25     | 24-25                    | 0               | -     | -          | -        |
| 26     | 25-26                    | 0               | -     | -          | -        |
| 27     | 26-27                    | 0               | -     | -          | -        |
| 28     | 27-28                    | 0               | -     | -          | -        |
| 29     | 28-29                    | 0               | -     | -          | -        |
| 30     | 29-30                    | 0               | -     | -          | -        |

**TOTAL** 200
| Sl. No | Inter Arrival Time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------------|-----------------|-------|------------|----------|
| 1      | 0-1                      | 27              | 0.216 | 0.216      | 000 - 215|
| 2      | 1-2                      | 27              | 0.216 | 0.432      | 216 - 431|
| 3      | 2-3                      | 25              | 0.2   | 0.632      | 432 - 631|
| 4      | 3-4                      | 8               | 0.064 | 0.696      | 632 - 695|
| 5      | 4-5                      | 12              | 0.096 | 0.792      | 696 - 791|
| 6      | 5-6                      | 6               | 0.048 | 0.84       | 792 - 839|
| 7      | 6-7                      | 9               | 0.072 | 0.912      | 840 - 911|
| 8      | 7-8                      | 2               | 0.016 | 0.928      | 912 - 927|
| 9      | 8-9                      | 3               | 0.024 | 0.952      | 928 - 951|
| 10     | 9-10                     | 1               | 0.008 | 0.96       | 952 - 959|
| 11     | 10-11                    | 0               | -     | -          | -        |
| 12     | 11-12                    | 0               | -     | -          | -        |
| 13     | 12-13                    | 1               | 0.008 | 0.968      | 960 - 967|
| 14     | 13-14                    | 0               | -     | -          | -        |
| 15     | 14-15                    | 0               | -     | -          | -        |
| 16     | 15-16                    | 0               | -     | -          | -        |
| 17     | 16-17                    | 2               | 0.016 | 0.984      | 968-983  |
| 18     | 17-18                    | 0               | -     | -          | -        |
| 19     | 18-19                    | 0               | -     | -          | -        |
| 20     | 19-20                    | 1               | 0.008 | 0.992      | 984 - 991|
| 21     | 20-21                    | 0               | -     | -          | -        |
| 22     | 21-22                    | 0               | -     | -          | -        |
| 23     | 22-23                    | 0               | -     | -          | -        |
| 24     | 23-24                    | 0               | -     | -          | -        |
| 25     | 24-25                    | 0               | -     | -          | -        |
| 26     | 25-26                    | 0               | -     | -          | -        |
| 27     | 26-27                    | 0               | -     | -          | -        |
| 28     | 27-28                    | 0               | -     | -          | -        |
| 29     | 28-29                    | 0               | -     | -          | -        |
| 30     | 29-30                    | 1               | 0.008 | 1          | 992 - 999|

|       | **Total**               | 125             |       |            |          |

**Table 2.2. Arrival distribution for F2**
Table 2.3. Arrival distribution for F3

| Sl. No | Inter Arrival Time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------------|-----------------|-------|------------|----------|
| 1      | 0-1                      | 23              | 0.211 | 0.211      | 000 - 210 |
| 2      | 1-2                      | 25              | 0.229 | 0.44       | 211 - 439 |
| 3      | 2-3                      | 12              | 0.11  | 0.55       | 440 - 549 |
| 4      | 3-4                      | 12              | 0.11  | 0.66       | 550 - 659 |
| 5      | 4-5                      | 11              | 0.101 | 0.761      | 660 - 760 |
| 6      | 5-6                      | 8               | 0.073 | 0.834      | 761 - 833 |
| 7      | 6-7                      | 6               | 0.056 | 0.89       | 834 - 889 |
| 8      | 7-8                      | 2               | 0.018 | 0.908      | 890 - 907 |
| 9      | 8-9                      | 1               | 0.009 | 0.917      | 908 - 916 |
| 10     | 9-10                     | 2               | 0.018 | 0.935      | 917 - 934 |
| 11     | 10-11                    | 0               | -     | -          | -        |
| 12     | 11-12                    | 2               | 0.018 | 0.953      | 935 - 952 |
| 13     | 12-13                    | 0               | -     | -          | -        |
| 14     | 13-14                    | 0               | -     | -          | -        |
| 15     | 14-15                    | 1               | 0.01  | 0.963      | 953 - 962 |
| 16     | 15-16                    | 0               | -     | -          | -        |
| 17     | 16-17                    | 0               | -     | -          | -        |
| 18     | 17-18                    | 0               | -     | -          | -        |
| 19     | 18-19                    | 1               | 0.009 | 0.972      | 963 - 971 |
| 20     | 19-20                    | 1               | 0.009 | 0.981      | 972 - 980 |
| 21     | 20-21                    | 0               | -     | -          | -        |
| 22     | 21-22                    | 0               | -     | -          | -        |
| 23     | 22-23                    | 0               | -     | -          | -        |
| 24     | 23-24                    | 0               | -     | -          | -        |
| 25     | 24-25                    | 0               | -     | -          | -        |
| 26     | 25-26                    | 0               | -     | -          | -        |
| 27     | 26-27                    | 1               | 0.01  | 0.991      | 981 - 990 |
| 28     | 27-28                    | 1               | 0.009 | 1          | 991 - 999 |
| 29     | 28-29                    | 0               | -     | -          | -        |
| 30     | 29-30                    | 0               | -     | -          | -        |
|        | Total                    | 109             |       |            |          |
Table 2.4. Arrival distribution for F4

| Sl. No | Inter Arrival Time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------------|-----------------|-------|------------|----------|
| 1      | 0-1                      | 12              | 0.15  | 0.15       | 000 – 149|
| 2      | 1-2                      | 11              | 0.138 | 0.288      | 150 – 287|
| 3      | 2-3                      | 9               | 0.112 | 0.4        | 288 – 399|
| 4      | 3-4                      | 12              | 0.15  | 0.55       | 400 – 549|
| 5      | 4-5                      | 7               | 0.088 | 0.638      | 550 – 637|
| 6      | 5-6                      | 7               | 0.088 | 0.726      | 638 – 725|
| 7      | 6-7                      | 7               | 0.088 | 0.814      | 726 – 813|
| 8      | 7-8                      | 3               | 0.037 | 0.851      | 814 – 850|
| 9      | 8-9                      | 1               | 0.013 | 0.864      | 851 – 863|
| 10     | 9-10                     | 0               | -     | -          | -        |
| 11     | 10-11                    | 0               | -     | -          | -        |
| 12     | 11-12                    | 2               | 0.025 | 0.889      | 864 – 888|
| 13     | 12-13                    | 2               | 0.024 | 0.913      | 889 – 912|
| 14     | 13-14                    | 0               | -     | -          | -        |
| 15     | 14-15                    | 0               | -     | -          | -        |
| 16     | 15-16                    | 0               | -     | -          | -        |
| 17     | 16-17                    | 0               | -     | -          | -        |
| 18     | 17-18                    | 3               | 0.037 | 0.95       | 913 – 949|
| 19     | 18-19                    | 0               | -     | -          | -        |
| 20     | 19-20                    | 1               | 0.013 | 0.963      | 950 – 962|
| 21     | 20-21                    | 0               | -     | -          | -        |
| 22     | 21-22                    | 1               | 0.012 | 0.975      | 963 – 974|
| 23     | 22-23                    | 0               | -     | -          | -        |
| 24     | 23-24                    | 1               | 0.013 | 0.988      | 975 – 987|
| 25     | 24-25                    | 0               | -     | -          | -        |
| 26     | 25-26                    | 0               | -     | -          | -        |
| 27     | 26-27                    | 1               | 0.012 | 1          | 988 – 999|
| 28     | 27-28                    | 0               | -     | -          | -        |
| 29     | 28-29                    | 0               | -     | -          | -        |
| 30     | 29-30                    | 0               | -     | -          | -        |
| **Total** |                      | **80**           |       |            |          |
The following tables are shows the service distribution for F1, F2, F3 & F4.

Table 2.5. Service distribution for F1

| S. No. | Service time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------|-----------------|-------|------------|----------|
| 1      | 1.00               | 92              | 0.46  | 0.46       | 000 – 459 |
| 2      | 1.50               | 46              | 0.23  | 0.69       | 460 – 689 |
| 3      | 2.00               | 26              | 0.13  | 0.82       | 690 – 819 |
| 4      | 2.50               | 20              | 0.1   | 0.92       | 820 – 919 |
| 5      | 3.00               | 16              | 0.08  | 1.00       | 920 – 999 |
| **Total** |                    | **200**         |       |            |          |

Table 2.6. Service distribution for F2

| S. No. | Service time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------|-----------------|-------|------------|----------|
| 1      | 1.00               | 41              | 0.328 | 0.328      | 000 – 327 |
| 2      | 1.50               | 32              | 0.256 | 0.584      | 328 – 583 |
| 3      | 2.00               | 22              | 0.176 | 0.76       | 584 – 759 |
| 4      | 2.50               | 14              | 0.112 | 0.872      | 760 – 871 |
| 5      | 3.00               | 16              | 0.128 | 1.00       | 872 – 999 |
| **Total** |                    | **125**         |       |            |          |

Table 2.7. Service distribution for F3

| S. No. | Service time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------|-----------------|-------|------------|----------|
| 1      | 1.00               | 34              | 0.312 | 0.312      | 000 – 311 |
| 2      | 1.50               | 26              | 0.239 | 0.551      | 312 – 550 |
| 3      | 2.00               | 20              | 0.183 | 0.734      | 551 – 733 |
| 4      | 2.50               | 11              | 0.101 | 0.835      | 734 – 834 |
| 5      | 3.00               | 18              | 0.165 | 1.00       | 835 – 999 |
| **Total** |                    | **109**         |       |            |          |

Table 2.8. Service distribution for F4

| S. No. | Service time (min) | No. of Vehicles | Prob. | Cum. Prob. | Tag Nos. |
|--------|--------------------|-----------------|-------|------------|----------|
| 1      | 1.00               | 26              | 0.325 | 0.325      | 000 – 324 |
| 2      | 1.50               | 14              | 0.175 | 0.5        | 325 – 449 |
| 3      | 2.00               | 10              | 0.125 | 0.625      | 500 – 624 |
| 4      | 2.50               | 22              | 0.275 | 0.9        | 625 – 899 |
| 5      | 3.00               | 8               | 0.1   | 1.00       | 900 – 999 |
| **Total** |                    | **80**          |       |            |          |
3. Results and discussions
From this study, we conclude that toll plaza can be described as the queueing system with time and state dependent arrival rates, and general service time distributions because discrete-time modelling approach can deal successfully with time-dependent arrival rates and general service time distribution in the most promising method.

3.1 Analytic Results:
The main performance measures in the analytic method are in terms of the following:

| Performance measures                      | F1  | F2  | F3  | F4  |
|------------------------------------------|-----|-----|-----|-----|
| Average arrival time in minutes          | 2.245| 3.476| 3.918| 4.584|
| Average service time in minutes          | 1.555| 1.728| 1.784| 1.825|
| Arrival rate per minute                  | 0.445| 0.288| 0.255| 0.218|
| Service rate per minute                  | 0.643| 0.579| 0.561| 0.548|
| Queue Length                             | 1.555| 0.491| 0.380| 0.262|
| System Length                            | 2.247| 0.988| 0.835| 0.661|
| Queue time in minutes                    | 3.494| 1.704| 1.492| 1.207|
| System time in minutes                   | 5.049| 3.431| 3.275| 3.032|

3.2 Simulation Results:
In simulation analysis, we used the random numbers for arrival times and service times and shown the calculated performance measures in the following table.

| Performance measures                      | F1  | F2  | F3  | F4  |
|------------------------------------------|-----|-----|-----|-----|
| Average arrival time in minutes          | 2.400| 3.550| 4.070| 5.330|
| Average service time in minutes          | 1.540| 1.775| 1.820| 1.895|
| Queue Length                             | 0.480| 0.320| 0.310| 0.220|
| System Length                            | 1.123| 0.821| 0.758| 0.576|
| Queue time in minutes                    | 0.865| 0.360| 0.365| 0.235|
| System time in minutes                   | 2.405| 2.135| 2.185| 2.130|

4. Conclusion
This paper presents a simple and efficient traffic monitoring behaviour in the tollgate system analysis. The computation performance measures are based on the knowledge of simulation generated by the Monte Carlo method and compared with analytic method. These comparison study shows that simulation results are coincide with analytic results. The advantage of this work is most favorable in monitoring traffic intensity at tollgates and these results can help to design the toll gate system, although additional practical constraints must be considered in any real-life implementation.
3.3 Comparative study for Analytic and Simulation results:

Table 3.3.1. Comparative study for Analytic and Simulation results

| Performance Measures       | F1 Analytic | F1 Simulation | F2 Analytic | F2 Simulation | F3 Analytic | F3 Simulation | F4 Analytic | F4 Simulation |
|----------------------------|-------------|---------------|-------------|---------------|-------------|---------------|-------------|---------------|
| Average arrival time in minutes | 2.245 | 2.400 | 3.476 | 3.550 | 3.918 | 4.070 | 4.584 | 5.330 |
| Average service time in minutes | 1.555 | 1.540 | 1.728 | 1.775 | 1.784 | 1.820 | 1.825 | 1.895 |
| Queue Length | 1.555 | 0.480 | 0.491 | 0.320 | 0.380 | 0.310 | 0.262 | 0.220 |
| System Length | 2.247 | 1.123 | 0.988 | 0.821 | 0.835 | 0.758 | 0.661 | 0.576 |
| Queue time in minutes | 3.494 | 0.865 | 1.704 | 0.360 | 1.492 | 0.365 | 1.207 | 0.235 |
| System time in minutes | 5.049 | 2.405 | 3.431 | 2.135 | 3.275 | 2.185 | 3.032 | 2.130 |

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