Precise Ratios for Neutrino-Nucleon and Neutrino-Nucleus Interactions

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Abstract

It is shown that several ratios of neutral to charged current cross sections are determined accurately, because the largest contribution is determined by isospin symmetry and the smaller terms are estimated from data or theoretical calculations. This way the theoretical uncertainty is very small. It is further discussed that the ratios can be measured at various distances from the origin of the neutrino beams, because an increase with distance will be indication for neutrino oscillations and will allow a precise determination of the oscillation parameters. Finally, coherent scattering is discussed as a useful reaction for oscillations and as a means of searching for new light particles.
1 Introduction

A new generation of neutrino experiments is under construction and they will be running soon. The analysis and interpretation of their results requires accurate calculations of the neutrino–nucleon and neutrino–nucleus cross sections. Several ratios of cross sections are also accessible in these experiments: among them are ratios of neutral to charged current reactions. It is shown in this article that several ratios can be calculated precisely, with the largest contribution obtained from symmetry considerations and smaller terms are determined from data and/or theoretical estimates of reactions. Having these estimates at our disposal, we can use them to interpret measurements at various distances from the origin of the neutrino beams. Changes of the ratios with the distance from the origin will point to a change of the neutrino or antineutrino fluxes and consequently oscillations to other species of neutrinos.

The article follows the standard model of the electroweak theory with the charged current given by

\[ J_\mu^+ (x) = (V_{\mu}^1 + iV_{\mu}^2) - (A_{\mu}^1 + iA_{\mu}^2) \]  

and neutral current given by

\[ J_\mu^{NC} = (1 - 2 \sin^2 \theta_W)V_{\mu}^3 - A_{\mu}^3 - 2 \sin^2 \theta_W V_{\mu}^0. \]

\[ V_{\mu}^3 \] and the isoscalar part of the current have the quark content

\[ V_{\mu}^3 = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \quad V_{\mu}^0 = \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s), \]

whose contribution to various reactions is determined by comparing electron induced reactions on protons and neutrons.

Ratios of the cross sections were useful [1]–[4] in establishing lower bounds for neutral current processes and determined the Weinberg angle [5, 6]. In the meanwhile many
matrix elements of the currents are known. The experience accumulated so far will be used to express ratios of reactions as equalities.

2 Cross Sections

Pion–inclusive Production

Most of the new experiments will be using medium and heavy nuclei where the interactions of the final pion are known to be important. We shall consider the scattering of neutrinos on isospin–zero nuclei and we shall not make the assumption of single nucleon interactions. Instead we consider the scattering of the current from the complete nucleus leading to a final pion and a recoiling system of hadrons with isospin $I = 0, 1, 2$. For the isospin analysis we adopt the standard notation [3] with $U^{(I)}$, $U_0^{(I)}$ being the contributions of the isovector current to the cross sections $\sigma_-$ and $\sigma_0$ for charged and neutral current, respectively, and $I$ denotes the isospin of the recoiling hadronic state, i.e. the states $x_1, \ldots, x_4$ below. Similarly, $S_0$ is the isoscalar contribution to $\sigma_0$ by itself, which occurs only for the $I = 1$ final state. The quantum mechanical structure of the cross section gives the relation

$$\frac{\sigma_0}{\sigma_{ch} - \sigma_0} = \frac{U_{0}^{(0)} + \frac{2}{5}U_{0}^{(2)} + \frac{1}{3}S_{0}}{U_{-}^{(0)} + \frac{2}{5}U_{-}^{(2)}}$$

(4)

with

$$\sigma_{ch} = \sigma (\nu_\mu + (I = 0) \rightarrow \mu^- + \pi^+ + x_1) + \sigma (\nu_\mu + (I = 0) \rightarrow \mu^- + \pi^- + x_2)$$

$$\sigma_0 = \sigma (\nu_\mu + (I = 0) \rightarrow \mu^- + \pi^0 + x_3)$$

$$\sigma_0 = \sigma (\nu_\mu + (I = 0) \rightarrow \nu_\mu + \pi^0 + x_4)$$

(5)

with $x_1, \ldots, x_4$ undetected hadronic states.

The corresponding cross sections for antineutrinos are obtained by charge symmetry and by changing the sign of the vector $\otimes$ axial–vector term. Thus, adding neutrino and
antineutrino reactions eliminates the vector⊗axial–vector interference term. It follows now

\[
R_0 = \frac{\sigma^0_0 + \bar{\sigma}^0_0}{(\sigma^ch - \sigma^0_0) + (\bar{\sigma}^ch - \bar{\sigma}^0_0)} = \frac{1}{2} \left\{ 1 - \frac{\sin^2 2\theta_W \left[V_{em}^{(0)} + V_{em}^{(2)}\right]}{\frac{1}{2} \left[(\sigma^ch - \sigma^0_0) + (\bar{\sigma}^ch - \bar{\sigma}^0_0)\right]} \right\}
\]

(6)

with \(\theta_W\) the Weinberg angle and

\[
\frac{dV_{\text{em}}^{(I)}}{dQ^2 d\nu} = \frac{G^2}{\pi} \frac{Q^4}{4\pi\alpha^2} \frac{d\sigma_{\text{em}}^{(I)}}{dQ^2 d\nu}
\]

(7)

being the isovector contribution of the electromagnetic current to a final state \(x_4\) with isospin \(I = 0, 1, 2\). Similarly, \(|S_0|^2\) is the corresponding expression for the isoscalar contribution of the electromagnetic current with only the \(I = 1\) state contributing to \(x_4\). Since the ratio within the parenthesis is positive, as will be shown in eq. (15), there is the upper bound

\[
R_0 \leq \frac{1}{2}.
\]

(8)

The ratio in eq. (6) is an exact equality. We have now sufficient information for neutrino, antineutrino and electron interactions with nucleons and nuclei to arrive at an accurate estimate of the right-hand side, without invoking the inequality. It will be shown below that the quotient in the bracket of eq. (6) is relatively small so that even general allowances for the uncertainties still give an accurate value for \(R_0\).

We shall use next information on the electroproduction and neutrino induced reactions in order to estimate eq. (6). We can complete the electromagnetic cross section by adding and subtracting the appropriate isoscalar term

\[
R_0 = \frac{1}{2} \left[ 1 - \sin^2 2\theta_W \frac{V_{em}}{\frac{2}{3}(\Sigma + \bar{\Sigma})} + (4 \sin^4 \theta_W + \sin^2 2\theta_W) \frac{1}{3} \frac{|S_0|^2}{\frac{2}{3}(\Sigma + \bar{\Sigma})} \right].
\]

(9)

with \(\Sigma = \sigma^ch - \sigma^0_0\) and a corresponding expression \(\bar{\Sigma}\) for antineutrinos. The results so far are general. If we had data for the cross sections in nuclei we could substitute them
in eq. (9). This data is not yet available and we shall use cross sections on protons and neutrons.

For the mixing angle we adopt the value \( \sin^2 \theta_W = 0.2227 \pm 0.0004 \) \([5, 6]\). For \( V_{em} \) we use the data of Galster et al. \([7]\) and W. Bartel et al. \([8]\) on hydrogen to estimate

\[
V_{em}(e + p \rightarrow e + \Delta^+) = 0.16 \times 10^{-38} \text{ cm}^2
\]

at 2.0 GeV. The data show \([7]\) that \( V_{em}(\pi^0)/V_{em}(ch) = 1.5 \) and there is at most a 20% incoherent background. The neutrino cross sections on protons and neutrons were measured in the early experiments:

| Reaction               | Cross section in \(10^{-38} \text{ cm}^2\) | Reference  |
|------------------------|--------------------------------------------|------------|
| \( \nu p \rightarrow \mu^- p \pi^+ \) | \(0.70 \pm 0.10\) | \([9, 10]\) |
| \( \nu n \rightarrow \mu^- n \pi^0 \)  | \(0.20 \pm 0.05\) | \([9, 10]\) |
| \( \nu n \rightarrow \mu^- n \pi^+ \)  | \(0.20 \pm 0.07\) | \([9, 10]\) |
| \( \bar{\nu} n \rightarrow \mu^+ n \pi^- \) | \(0.31\) | theory |
| \( \bar{\nu} p \rightarrow \mu^+ p \pi^- \) | \(0.14\) | theory |
| \( \bar{\nu} p \rightarrow \mu^+ n \pi^0 \) | \(0.12\) | theory |

Table 1: Neutrino and antineutrino cross sections at \( E_\nu = 2 \text{ GeV} \).

The neutrino cross sections were measured on Hydrogen and Deuterium where the nucleons are practically free. The experimental data were collected together in ref. \([11]\) where they are also compared with theoretical calculations. For the antineutrino reactions there is no data at low energies and we use theoretical values from ref. \([12]\). In going from protons and neutrons to nuclei, it is necessary to include nuclear effects, which brings in a model dependence.

The new experiments will be using medium and heavy nuclei as targets where nuclear effects are important. Among the corrections are Pauli blocking at the weak interaction vertex and at the rescatterings, absorption of the pions and charge exchange from the
subsequent scatterings. For nuclear corrections we shall use a model with multiple scatterings [13] which has been compared in a few cases with the experimental data [14] and was found to be consistent. The charge exchange matrix has been calculated for several nuclei [15, 11]. We shall consider the scattering on $^8O^{16}$ where the charge exchange matrix is [11]

$$M(^8O^{16}) = A \begin{pmatrix} 0.78 & 0.16 & 0.06 \\ 0.16 & 0.68 & 0.16 \\ 0.06 & 0.16 & 0.78 \end{pmatrix}.$$  

The factor $A$ includes the Pauli factor and part of the absorption. It is the same for all three semileptonic reactions and drops out in the ratios. Using the experimental values from Table 1 together with the mixing matrix we obtain

$$(\sigma_-^{ch} - \sigma_0^0)_{\text{Nucl. corr.}} = A(0.27 \pm 0.05) \times 10^{-38} \text{ cm}^2$$ (11)

$$(\sigma_+^{ch} - \sigma_0^0)_{\text{Nucl. corr.}} = A \times 12 \times 10^{-38} \text{ cm}^2$$ (12)

and

$$V_{em} = A \times 0.09 \times 10^{-38} \text{ cm}^2$$ (13)

for $E_\nu = 2.0$ GeV. The values for the antineutrino cross sections are theoretical and for this reason we do not quote any errors. In the electroproduction data the errors are very small. Using the above values we obtain the ratio

$$\sin^2 2\theta_W \frac{V_{em}(\pi^0)}{\frac{1}{2}(\Sigma + \bar{\Sigma})} = 0.70 \cdot \frac{0.09}{\frac{1}{2}(0.39)} = 0.32.$$ (14)

The corresponding isoscalar term is small and is included as a 20% background to the electroproduction [7]. The final ratio is

$$R_0 = \frac{1}{2}(1 - 0.32 + 0.05) = 0.37$$ (15)

with the second and third terms coming from the isovector and isoscalar part of $V_{em}$, respectively.

Experimental errors and/or other uncertainties stem from the second and third terms in the parenthesis of eq. (9), which are much smaller than one. Thus the overall error
originating from these terms is reduced; for instance a 20% error from eq. (14) translates to the value $R_0 = 0.37 \pm 0.03$. The numerical calculation illustrates the method to be followed when better data becomes available.

There are several other ratios which one can discuss, but we shall postpone this topic for a more extensive article and concentrate on different kinds of reactions which bring out new aspects of the calculations. An alternative is to use only ratios of neutrino cross sections. In this case the vector–axial interference term must be calculated theoretically, as was done explicitly in various articles [11, 16].

**Total cross sections**

We consider again isoscalar targets and sum over final states. In this case there is a popular ratio [1] where the uncertainties cancel out

$$D = \frac{\sigma_0 - \bar{\sigma}_0}{\sigma_+ - \bar{\sigma}_-} = \frac{1}{2} (1 - 2 \sin^2 \theta_W) = 0.274 \pm 0.002.$$  

(16)

This ratio is known precisely and the new experiments can measure the cross sections at various distances from the accelerator, where the neutrino and antineutrino beams are produced. An increase of $D$ with the distance is an indication for oscillation to active neutrinos, which provides a model independent method for extracting oscillation parameters.

An alternative ratio is obtained by adding neutrino and antineutrino cross sections [1]

$$R_1 = \frac{\sigma_0 + \bar{\sigma}_0}{\sigma_+ + \bar{\sigma}_-} = \frac{1}{2} \left[ 1 - \frac{\sin^2 2\theta_W V_{em}^3 - 4 \sin^4 \theta_W |J_0^0|^2}{\frac{1}{2}(\sigma_- + \sigma_+)} \right]$$  

(17)

where $V_{em}^3$ is the isovector contribution (in the sense of eq. (7)) to deep inelastic scattering and $|J_0^0|^2$ the contribution from the isoscalar part of the current. We can proceed as in eq. (9) to replace $V_{em}^3$ with the complete electromagnetic contribution. The ratio

$$\frac{2V_{em}}{\sigma_- + \sigma_+} = 0.58 \pm 0.08$$  

(18)
with the neutrino and antineutrino cross sections taken from the particle data group and $V_{em}$ from electroproduction data. The value in eq. (18) is close to the prediction of the parton model \[17\]. Deviations from this value will come from the strange and antistrange quark contributions, which are small. For the errors we assign a 14 % error to the electroproduction cross section and a 2 % to the neutrino cross sections to arrive at

$$R_1 = 0.33 \pm 0.03$$ \hspace{1cm} (19)

For the calculation we completed the electromagnetic term by adding the isoscalar contribution, which was also subtracted in the last term of eq. (17), in direct analogy to the method used in eq. (9). We note that the error in eq. (19) is small.

**Coherent Scattering**

In neutrino experiments, when the final lepton is almost parallel to the initial neutrino then the leptonic cross section is related to the pionic one \[18\]

$$\frac{d\sigma}{dQ^2 d\nu d\Gamma} = \frac{G^2}{2\pi^2} \frac{1}{\nu \bar{E}} E' F_\pi^2 \left( \frac{m_\pi^2}{m_\pi^2 + Q^2} \right)^2 \frac{d\sigma_\pi}{d\Gamma}.$$ \hspace{1cm} (20)

We adopted the standard notation for the variables with the phase space element $d\Gamma$ referring to the final hadronic states and $F_\pi$ is the pion decay coupling constant, $F_\pi \approx 0.9m_\pi$. The equation holds for neutral and charged current reactions provided that we restrict the kinematics to the region where coherent scattering is dominant. Coherent scattering occurs when the following conditions are met:

(i) the magnitude of the momentum transfer is comparable to the square of the mass of the particle exchanged, so that a forward peak is produced,

(ii) the momentum transfer is small, so that the exchanged particle propagates over a region comparable to its wave length. If in addition the momentum of the incident neutrino is small, its wave function overlaps with several scattering centers.
In coherent scattering the quantum numbers of the region under investigation add up in the amplitude. For example, an interaction proportional to the baryon number counts the baryons contained in the region of investigation. Similarly, the electromagnetic interactions count the charge enclosed in the region of investigation.

A forward peak has been observed in single pion production by charged currents and the results were reported to be consistent with the PCAC hypothesis. It is possible that other very light particles couple directly to the weak currents in a way similar to that of the pions. Low energy neutrino experiments at the MeV or lower energies can search for them with the help of coherent scattering.

3 Neutral to Charged current ratios in Oscillations

In the long base line experiments, a beam of muon–type neutrinos is produced near an accelerator. As the neutrinos travel a distance $L$, they can oscillate to other species of active and/or sterile neutrinos. At a distance $L$ the beam is a mixture of muon–type neutrinos with flux $F_\mu(L)$, of another species of active neutrinos $F_\nu(L)$ and, perhaps, of sterile neutrinos $F_S(L)$. The initial condition is $F_\mu(0) = 1$ and $F_\nu(0) = F_S(0) = 0$. Let us also assume that the energy of the experiments is low enough so that, for threshold reasons, the new type of active neutrinos do not contribute to the charged current reactions. This is the case in the oscillation of muon– to tau–neutrinos. Taking the ratio of neutral to charge current reactions, one obtains for the number of events the ratio

$$\frac{N_{NC}}{N_{CC}} = \frac{F_\mu(L) + F_\nu(L)}{F_\mu(L)} \cdot \frac{\sigma_{NC}}{\sigma_{CC}},$$

which was suggested as a means for analyzing oscillation phenomena. The ratio carries two uncertainties: one coming from the values of the cross sections and the other from the fluxes. For this reason, the analyses rely on double ratios. The results of this article indicate that for several channels the ratio of cross sections is known. This
will be useful in the analysis of the data and may eliminate the need to use double ratios.

4 Summary

We have shown that, in the electroweak theory, ratios of neutral to charged current reactions are related to electromagnetic reactions and can be estimated in terms of available data. The relations (9), (16) and (17) are general and hold at low and high energies. They are particularly useful at specific energy regions. Eq. (9) is more useful in the low energy region $E_\nu < 2.0 \text{ GeV}$, where there are few particles in the final state. Eq. (16) has been used extensively at various energy regions. Eq. (17) is more suitable at higher energies where the incoherent scattering from the quarks gives the accurate prediction of eq. (19).

The relations were derived for isoscalar targets and hold for $I = 0$ nuclei as a whole, which eliminates the assumption of single nucleon interactions. Since electroproduction and antineutrino data is not available in nuclei, we used a model for estimating nuclear effects. We found out that the final values are rather insensitive to experimental uncertainties, because they cancel out in ratios and the dominant term follows from isospin symmetry. In all cases, the contribution from the isoscalar part of the weak neutral current is small. We hope that these results will be useful in analyzing oscillation experiments.

The last topic is coherent scattering, which relates neutrino and antineutrino induced reactions to the pion induced reactions. In this kinematic region the pion induced reactions determine the absolute neutrino cross sections. When the square of the momentum transfer and the energy transfer are small, the exchange current probes a region comparable to its wavelength. These reactions make possible the searches for light particles that couple to neutrinos and hadrons. An interesting case is the coupling of neutrinos to the baryon current, which will bring a large enhancement. The searches using coherent scattering are analogous to the Primakoff effect, which has been very useful in the past.
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When the cross sections are symmetrized over neutrinos and antineutrinos they are given by expressions like

$$\sigma_{\text{tot}} = \frac{G^2 M E}{\pi} \frac{4}{3} \int F_2(x) dx$$

with $F_2(x)$ the scaling function. Relations among the structure functions of electroproduction and neutrino-induced reactions appear in books; see, for instance, D.H. Perkins, Introduction to High-Energy Physics, 4th edition (Cambridge University Press), eq. (5.48) and fig. (5.14).