The Mass of the $b$ Quark from Lattice NRQCD

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We present results for the mass of the $b$ quark in the $\overline{MS}$ scheme obtained by calculating the binding energy of the $B$ meson in the static limit. The self energy of a static quark, $E_0^\infty$, needed for this purpose, is now known to $O(\alpha^3)$ in the quenched approximation. We find a preliminary value of $m_b(\overline{m}_b) = 4.34(7)$ GeV at $n_f = 0$. The error is dominated by the remaining uncertainty in $E_0^\infty$. In addition, using $E_0^\infty$ at $O(\alpha^2)$, we estimate that the quark mass is reduced by approximately 70 MeV when two flavours of dynamical quarks are introduced.

1 Introduction

The mass of the $b$ quark in the $\overline{MS}$ scheme ($m_b$) can be extracted on the lattice using NRQCD, via the pole mass, $M_{pole}$, by calculating the binding energy of the $B$ meson, $\Lambda_{bind}$:

$$M_{pole} = M_{expt} - \Lambda_{bind}$$

where $M_{expt}$ is the spin-average of the experimental $B$ and $B^*$ masses and $\Lambda_{bind} = E_{sim} - E_0$. $E_{sim}$ is the energy of the $B$ meson (at rest) in NRQCD and $E_0$ is the $b$ quark self energy. The pole mass is then converted to $m_b$ at some scale $\mu$ using the continuum perturbative factor $Z_{cont}$:

$$m_b(\mu) = Z_{cont}(\mu)M_{pole}.$$ 

While $M_{pole}$ has an $O(\Lambda_{QCD})$ renormalon ambiguity, this is cancelled by similar effects in $Z_{cont}$: $m_b$ is well defined.

At present, $E_0$ is only known to $O(\alpha)$ for the $b$ quark. However, in the limit of the $b$ quark mass becoming infinite, $E_0$ is known to $O(\alpha^3)$ if internal quark loops are neglected (the quenched approximation). The tadpole-improved formula can be expressed as

$$E_0^\infty = 1.070\alpha_p + 0.118\alpha^2_p - 0.3(1.4)\alpha^3_p : \alpha_p = \alpha_p^{(n_f=0)}(0.84/a). \quad (1)$$

The $\alpha_p^3$ coefficient has been determined by Lepage et. al. The error on the coefficient is numerical and quite large. However, it provides a realistic estimate of the uncertainty in $E_0$, compared to using $2-loop$ perturbation theory and assuming the contribution of higher order terms is $1\alpha_p^3$.

In Eq. $1$, $E_0$ is expressed in terms of a coupling constant defined on the lattice from the plaquette, $\alpha_p$, evaluated at a characteristic gluon momenta, $q^* = 0.84/a$, calculated using the BLM procedure. In addition, the lattice calculation of $E_{sim}$ has been tadpole improved, whereby all gauge fields on the lattice are divided by a ‘mean-field’ approximation to the gluon field, $u_0$, 

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to obtain more continuum-like operators. The corresponding tadpole improvement of $E_0$ leads to the addition of the perturbative series for $\ln u_0$. These ingredients result in a well behaved perturbative series for $E_0$. This is certainly not the case if the bare lattice coupling, $\alpha_L = g_0^2/(4\pi)$ is used:

$$E_0^\infty = 2.1173\alpha_L + 11.152\alpha_L^2 + 82.7(1.4)\alpha_L^3$$

(2)

Di Renzo et. al. have also determined the $O(\alpha^3)$ coefficient. They obtain $86.2(6)$, when $\alpha_L$ is used. Encouragingly, the two determinations, which have very different systematic errors, agree within $3\sigma$.

$Z_{cont}$ has been calculated to $\alpha^3$ by Melnikov and van Ritbergen:

$$Z_{cont} = 1 - 0.4244\alpha_p - 0.4771\alpha_p^2 - 1.814\alpha_p^3 : \alpha_p = \alpha_p^{(0)}(0.62m_b).$$

(3)

The series is well-behaved and we estimate the uncertainty in $Z_{cont}$ to be $3\alpha_p^4$.

The error introduced by working in the static limit, i.e. ignoring $O(\Lambda_{QCD}/M)$ contributions to $\Lambda_{bind}$, leads to $\approx 1\%$ uncertainty in $m_b$. The error arising from working in the quenched approximation is also likely to be around $1\%$ (assuming a $10-20\%$ shift in $E_{sim}$ when sea quarks are included). These effects are the same size as the error arising from the numerical error in $E_0$.

We obtained $E_{sim}^\infty$ by extrapolating the simulation energy calculated at finite heavy quark mass. The latter was obtained as part of a high statistics study of the $B$ meson spectrum in the quenched approximation at three lattice spacings ($a$), with $a^{-1} = 1.0-2.5$ GeV. For details of the simulations see reference 7. Note that we use the spin-average of the experimental masses for the $B$ and $B^*$ mesons in the expression for $M_{pole}$ in order to reduce the error in using $E_{sim}^\infty$. In addition, we performed a study of sea quark effects using results obtained from a simulation including two flavours of sea quarks ($n_f = 2$) with $a^{-1} \sim 2$ GeV. Only the $O(\alpha_p^2)$ coefficient for $n_f = 2$ has been computed and hence the comparison with $m_b$ at $n_f = 0$ is performed using $E_0$ and $Z_{cont}$ to this order.

2 Results

Tables 1 and 2 summarize our results. Within the combined statistical and systematic errors we see that this is the case and we take the result at $\beta = 6.0$ as our best determination of $m_b(0) = 4.34(7)$ GeV at $n_f = 0$. Note that the numerical error in $E_0$ dominates the uncertainty in $m_b$.

In addition, using the results at $\beta = 6.0$ at $n_f = 0$ from the $B_s$ meson and those obtained at $n_f = 2$ we see that the $b$ quark mass decreases by 70 MeV at $O(\alpha_p^2)$ when sea quarks are introduced. Assuming the systematic (perturbative) errors for the two simulations are correlated this is $\sim 2\sigma$ in
Table 1: $\Lambda_{\text{bind}}^\infty$ and $m_b(m_b)$ in GeV from the $B$ meson at $n_f = 0$. The statistical and main systematic errors are estimated, including those due to determining the inverse lattice spacing ($a^{-1}$) and residual discretisation effects in $E_{\text{sim}} \sim O((\Lambda_{\text{QCD}}a)^2)$.

| $\beta$ | stat. | $E_0^\infty$ | $a^{-1}$ | disc. | $E_{\text{cont}}^\infty$ | $a^{-1}$ | disc. |
|---------|-------|---------------|----------|-------|----------------|----------|-------|
| 5.7     | .24   | (1)           | (11)     | (1)   | (9)            | 4.43     | (3)   | (1)   | (10)   | (1)   | (8)   |
| 6.0     | .35   | (2)           | (6)      | (1)   | (4)            | 4.34     | (3)   | (2)   | (5)    | (1)   | (3)   |
| 6.2     | .36   | (8)           | (5)      | (2)   | (2)            | 4.32     | (3)   | (7)   | (4)    | (2)   | (2)   |

Table 2: The change in $m_b(m_b)$ from $n_f = 0$ to 2, where 2-loop perturbation theory is used.

| $n_f$ | $m_b(m_b)$ | stat. error |
|-------|------------|-------------|
| 0     | 4.45       | .01         |
| 2     | 4.38       | .02         |

the (remaining) statistical errors and the same size as the error in $m_b(m_b)$ at $O(\alpha_s^3)$. Further work is necessary to reduce the error in $E_0$, in order for sea quark and $O(\Lambda_{\text{QCD}}/M)$ effects to be significant.

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