Charmless $B \to VV$ Decays in QCD Factorization: Implications of Recent $B \to \phi K^*$ Measurement

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Abstract

In the heavy quark limit, both vector mesons in the charmless $B \to VV$ decay should have zero helicity and the corresponding amplitude is proportional to the form factor difference $(A_1 - A_2)$. The first observed charmless $B \to VV$ mode, $B \to \phi K^*$, indicates that the form factors $A_1(q^2)$ and $A_2(q^2)$ cannot be very similar at low $q^2$ as shown in some form-factor models. The approach of QCD-improved factorization implies that the nonfactorizable correction to each partial-wave or helicity amplitude is not the same; the effective parameters $a_i$ vary for different helicity amplitudes. The leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit and hence it is necessary to take into account twist-3 distribution amplitudes of the vector meson in order to have renormalization scale and scheme independent predictions. Branching ratios of $B \to VV$ decays are calculated in two different models for form factors, and the predicted decay rates are different by a factor of $1.5 \sim 2$. Owing to the absence of $(S - P)(S + P)$ penguin contributions to the $W$-emission amplitudes, tree-dominated decays tend to have larger branching ratios than the penguin-dominated ones.
1. It is known that the decay amplitude of a $B$ meson into two vector mesons is governed by three unknown form factors $A_1(q^2)$, $A_2(q^2)$ and $V(q^2)$ in the factorization approach. It has been pointed out in [1] that the charmless $B \to VV$ rates are very sensitive to the form-factor ratio $A_2/A_1$. This form-factor ratio is almost equal to unity in the Bauer-Stech-Wirbel (BSW) model [2], but it is less than unity in the light-cone sum rule (LCSR) analysis for form factors [3]. In general, the branching ratios of $B \to VV$ predicted by the LCSR are always larger than that by the BSW model by a factor of $1.6 \sim 2$ [1]. This is understandable because in the heavy quark limit, both vector mesons in the charmless $B \to VV$ decay should have zero helicity and the corresponding amplitude is proportional to the form factor difference $(A_1 - A_2)$. These two form factors are identical at $q^2 = 0$ in the BSW model. We shall see that the first observed charmless $B \to VV$ mode, $B \to \phi K^*$, recently measured by CLEO [4], BELLE [5] and BABAR [6], clearly favors the LCSR over the BSW model for $B - V$ transition form factors.

In the present paper we will embark on a study of $B \to VV$ decays in the approach of QCD-improved factorization which enables us to compute nonfactorizable corrections in the heavy quark limit. In the so-called generalized factorization approach, it is assumed that nonfactorizable effects contribute to all partial-wave or helicity amplitudes in the same weight. We shall see that it is not the case in QCD factorization. Moreover, we will show that the leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit. Hence it is necessary to go beyond the leading-twist approximation for transversely polarized states.

2. In general the $B \to VV$ amplitude consists of three independent Lorentz scalars:

$$ A[B(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)] \propto \varepsilon_1^\mu \varepsilon_2^\nu (ag_{\mu\nu} + bp_{\mu}p_{\nu} + ic\epsilon_{\mu\alpha\beta}p_1^\alpha p_2^\beta), \quad (1) $$

where the coefficient $c$ corresponds to the $p$-wave amplitude, and $a, b$ to the mixture of $s$- and $d$-wave amplitudes. Three helicity amplitudes can be constructed as

$$ H_{00} = \frac{1}{2m_1m_2} [(m_B^2 - m_1^2 - m_2^2)a + 2m_B^2p_2^2b], $$

$$ H_{\pm\mp} = a \mp m_Bp_c, \quad (2) $$

where $p_c$ is the c.m. momentum of the vector meson in the $B$ rest frame and $m_1$ ($m_2$) is the mass of the vector meson $V_1$ ($V_2$). For $H_{\pm\mp}$ to occur quark spin in the emitted vector meson $V_2$ has to be flipped. Therefore, the amplitude $H_{\pm\mp}$ is suppressed by a factor of $m_2/m_B$ [7]. The $H_{++}$ amplitude is subject to a further chirality suppression of order $m_1/m_B$. In general, it is thus expected that $|H_{00}|^2 > |H_{\pm\mp}|^2 > |H_{++}|^2$. The total decay rate is given by

$$ \Gamma(B \to V_1V_2) = \frac{p_c}{8\pi m_B^2} (|H_{00}|^2 + |H_{++}|^2 + |H_{\pm\mp}|^2). \quad (3) $$

In terms of the decay constant and form factors defined by [2]:

2
\begin{align}
\langle V(p, \epsilon)|V_\mu|0\rangle &= f_V m_V \epsilon^*_\mu, \\
\langle V(p', \epsilon)|V_\mu|P(p)\rangle &= \frac{2}{m_P + m_V} \epsilon_{\mu \nu \alpha \beta} \epsilon^{*\nu} p^\alpha p^\beta V(q^2), \\
\langle V(p', \epsilon)|A_\mu|P(p)\rangle &= i \left\{ (m_P + m_V) \epsilon^*_\mu A_1(q^2) - \frac{\epsilon^*_\mu \cdot p}{m_P + m_V} (p + p')_\mu A_2(q^2) - 2m_V \frac{\epsilon^*_\mu \cdot p}{q^2} q_\mu [A_3(q^2) - A_0(q^2)] \right\},
\end{align}

where \( q = p - p' \), \( A_3(0) = A_0(0) \), and

\[ A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2), \]

one has the factorizable \( B \to V_1 V_2 \) amplitude:

\begin{align}
X^{(BV_1, V_2)} &= \langle V_2|(\bar{q}_2 q_3)_{\nu-A}|0\rangle \langle V_1|(\bar{q}_1 b)_{\nu-A}|B\rangle \\
&= -if_{V_1} m_2 \left[ (\epsilon_1^* \cdot \epsilon_2^*) (m_B + m_1) A_1^{BV_1}(m_2^2) - (\epsilon_1^* \cdot p_B) (\epsilon_2^* \cdot p_B) \frac{2A_2^{BV_1}(m_2^2)}{(m_B + m_1)} + i\epsilon_{\mu \nu \alpha \beta} \epsilon^{*\nu} \epsilon^{*\mu} p^\alpha p^\beta \frac{2V^{BV_1}(m_2^2)}{(m_B + m_1)} \right].
\end{align}

Take the decay \( B \to \phi K^* \) as an example. In the naive factorization approach for hadronic weak decays, the decay amplitude of \( B^- \to \phi K^{*-} \) reads (in units of \( G_F/\sqrt{2} \))

\[ A(B_u^- \to K^{*-} \phi) = V_{ub} V_{u1}^* a_1 X^{(B_u^-, \phi K^{*-})} - V_{tb} V_{t1}^* \left\{ a_3 + a_4 + a_5 \\
- \frac{1}{2} (a_7 + a_9 + a_{10}) \right\} X_s^{(B^- K^{*-}, \phi)} + (a_4 + a_{10}) X^{(B^-, K^{*-}, \phi)} - 2(a_6 + a_8) \langle K^{*-} \phi|\bar{s}(1 + \gamma_5) u|0\rangle \langle 0|\bar{u}(1 - \gamma_5) b|B^- \rangle, \]

where \( a_{2i} = c_{2i} + \frac{1}{N_c} c_{2i-1}, a_{2i-1} = c_{2i-1} + \frac{1}{N_c} c_{2i} \). Neglecting the annihilation contributions from the last two terms in Eq. (7), we obtain

\begin{align}
H_{00} &= \frac{\tilde{a}(\phi K^*) f_\phi}{2m_{K^*}} \left[ (m_B^2 - m_{K^*}^2, m_{\phi}^2) (m_B + m_{K^*}) A_1^{BK^*}(m_{\phi}^2) - \frac{4m_B^2 p_{\phi}^2}{m_B + m_{K^*}} A_2^{BK^*}(m_{\phi}^2) \right], \\
H_{\pm \pm} &= \tilde{a}(\phi K^*) m_\phi f_\phi \left[ (m_B + m_{K^*}) A_1^{BK^*}(m_{\phi}^2) + \frac{2m_B p_{\phi}}{m_B + m_{K^*}} V^{BK^*}(m_{\phi}^2) \right],
\end{align}

where \( \tilde{a}(\phi K^*) = a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \). In the heavy quark limit, it is clear that

\begin{align}
H_{00} &= \frac{\tilde{a}(\phi K^*) f_\phi m_B^3}{2m_{K^*}} [A_1^{BK^*}(0) - A_2^{BK^*}(0)], \\
H_{\pm \pm} &= \tilde{a}(\phi K^*) f_\phi m_\phi m_B [A_1^{BK^*}(0) \mp V^{BK^*}(0)].
\end{align}
In the so-called generalized factorization, nonfactorizable effects are parametrized in terms of $N_c^{\text{eff}}$, the effective number of colors. This amounts to assuming that nonfactorizable corrections weight in the same way to all partial-wave or helicity amplitudes. For example, the coefficient $\tilde{a}(\phi K^*)$ appearing in Eq. (9) is postulated to be the same for $S$, $P$ and $D$ (or $H_{00}$ and $H_{0\pm}$) amplitudes after including nonfactorizable contributions. Clearly there is no any known physical argument for justifying this assumption.

Fortunately, the QCD-improved factorization approach advocated recently in [9] allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the $(BV_1)$ system and $V_2$ survive in the $m_b \to \infty$ limit. Naive factorization is recovered in the heavy quark limit and to the zeroth order of QCD corrections. In this approach, the light-cone distribution amplitudes (LCDAs) play an essential role. The LCDAs of the light vector meson of interest are given by [10,9]

$$\langle V(p, \varepsilon)|\bar{q}_\alpha(y)q'_\beta(x)|0\rangle = \frac{f_Vm_V}{4} \int_0^1 du \, e^{i(up \cdot y + \bar{u}p \cdot x)} \left[ f^\parallel_\Phi \Phi^V_\parallel(u) + f^\perp_\Phi \Phi_\perp(u) \right]$$

where $z = y - x$ with $z^2 = 0$, $\varepsilon^\mu_\parallel$ ($\varepsilon^\mu_\perp$) is the polarization vector of a longitudinally (transversely) polarized vector meson, $u$ is the light-cone momentum fraction of the quark $q$, $\bar{u} = 1 - u$, $f_V$ and $f^T_V$ are vector and tensor decay constants, respectively, but the latter is scale dependent. To a good approximation one has $\varepsilon^\mu_\parallel = p^\mu_V/m_V$ for a light vector meson. It follows that $p \cdot \varepsilon_\perp = 0$. It is easily seen from Eq. (11) that $\Phi_\parallel$ contributes to $S$ and $D$ amplitudes, while $\Phi_\perp$ to $P$ and $S$ waves. In Eq. (11), $\Phi_\parallel(u)$ and $\Phi_\perp(u)$ are twist-2 DAs, while $g^{(v)}_\perp$ and $g^{(a)}_\perp$ are twist-3 ones. Note that twist-3 longitudinally polarized distribution amplitudes $h^{(v)}_\parallel$ and $h^{(a)}_\parallel$ [11] are not shown in Eq. (11). The reason for keeping the twist-3 transversely polarized distribution amplitudes $g^{(v,a)}_\perp$ rather than the longitudinal ones $h^{(v,a)}_\parallel$ will become clear shortly.

3. We will now study charmless $B \to VV$ decays within the framework of QCD-improved factorization. The power corrections such as the annihilation diagrams can be neglected in the heavy quark limit. It turns out that the twist-2 DA $\Phi_\perp(u)$ contributes to the vertex corrections and hard spectator interactions vanish in the chiral limit. Hence, we will work

*For the color-suppressed mode $B \to J/\psi K^*$, the twist-2 transverse polarized distribution amplitude is on the same footing as the longitudinal one since $J/\psi$ is not massless in heavy quark limit.
entering into the helicity amplitudes $H_{00}$ and $H_{\pm\pm}$ are not the same; they are given by

\begin{align}
a^h_1 &= c_1 + \frac{c_2}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_2 F^h, \\
a^h_2 &= c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_1 F^h, \\
a^h_3 &= c_3 + \frac{c_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_4 F^h, \\
a^h_4 &= c_4 + \frac{c_3}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \left\{ c_3 [F^h + G^h(s_q) + G^h(s_b)] - c_1 \left( \frac{\lambda_u}{\lambda_t} G^h(s_u) + \frac{\lambda_c}{\lambda_t} G^h(s_c) \right) \right. \\
&\left. + (c_4 + c_6) \sum_{i=u}^{b} G^h(s_i) + \frac{3}{2} (c_8 + c_{10}) \sum_{i=u}^{b} c_i G^h(s_i) + \frac{3}{2} c_9 [c_q G^h(s_q) - \frac{1}{3} G^h(s_b)] + c_9 G^h \right\}, \\
a^h_5 &= c_5 + \frac{c_6}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_6 (-F^h - 12), \\
a^h_6 &= c_6 + \frac{c_5}{N_c}, \\
a^h_7 &= c_7 + \frac{c_8}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_8 (-F^h - 12) - \frac{\alpha}{9\pi} N_c C^h_e, \\
a^h_8 &= c_8 + \frac{c_7}{N_c}, \\
a^h_9 &= c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_{10} F^h - \frac{\alpha}{9\pi} N_c C^h_e, \\
a^h_{10} &= c_{10} + \frac{c_9}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_9 F^h - \frac{\alpha}{9\pi} C^h_e, 
\end{align}

where $C_F = (N_c^2 - 1)/(2N_c)$, $s_i = m_i^2/m_b^2$, $\lambda_q = V_{qb} V_{q'd}^*$, $q' = d,s$ and the superscript $h$ denotes the polarization of the vector mesons: $h = 0$ for helicity 00 states, and $h = \pm$ for helicity $\pm\pm$ states.

There are QCD penguin-type diagrams induced by the 4-quark operators $O_i$ for $i = 1, 3, 4, 6, 8, 9, 10$. The corrections are described by the penguin-loop function $G^h(s)$ given by

\begin{align}
G^0(s) &= \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 dx \Phi^V(x) \int_0^1 du \, u(1-u) \ln[s - xu(1-u)], \\
G^{\pm}(s) &= \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 dx \sum_{i=0}^{2} \left\{ \ln[xu(1-u)] + \frac{xu(1-u)}{s - xu(1-u)} \right\}
\end{align}

In Eq. (12) we have also included the leading electroweak penguin-type diagrams induced by the operators $O_1$ and $O_2$:

\[
C^h_e = \left( \frac{\lambda_u}{\lambda_t} G^h(s_u) + \frac{\lambda_c}{\lambda_t} G^h(s_c) \right) \left( c_2 + \frac{c_1}{N_c} \right).
\]
The dipole operator $O_g$ will give a tree-level contribution proportional to

$$G_g^0 = -2 \int_0^1 dx \frac{\Phi_V^V(x)}{x}, \quad G_g^{\pm} = -2 \int_0^1 dx \frac{g^{(v)}_{\pm}(x)}{x} + \frac{1}{2}(1 \mp 1/2) \int_0^1 dx \frac{g^{(a)}_{\pm}(x)}{x}. \quad (14)$$

In Eq. (11), the vertex correction in the naive dimensional regularization (NDR) scheme for $\gamma_5$ is given by

$$F^h = -12 \ln \frac{\mu}{m_b} - 18 + f_I^h + f_{II}^h, \quad (15)$$

where the hard scattering function $f_I$ arises from vertex corrections and $f_{II}$ from the hard spectator interactions with a hard gluon exchange between the emitted vector meson and the spectator quark of the $B$ meson. Note that the twist-2 transversely polarized distribution amplitude $\Phi_\perp$ does not contribute to $F$: it does not give rise to the scale and scheme dependent terms $-12 \ln(\mu/m_b) - 18$ and the hard scattering kernels $f_I^{\pm}$ and $f_{II}^{\pm}$ are proportional to the light quark mass and hence can be neglected. Therefore, the parameters $a_i^{\pm}$ at the twist-2 level are not renormalization scale and scheme independent. Consequently, it is necessary to take into account the twist-3 effects for transversely polarized vector meson states. This is why we keep twist-3 DAs $g_{\perp(a,v)}^{(s,t)}$ in Eq. (14). As for the helicity zero case, it is dominated by the leading-twist one and hence the twist-3 DAs $h_{\parallel(a,s)}^{(a,s)}$, which are power suppressed by order of $m_V/m_B$, are not considered there. It should be stressed that $a_i^{h,8}$ do depend on the choice of the renormalization scale and scheme. Their scale and scheme dependence is compensated by the corresponding $(S - P)(S + P)$ hadronic matrix elements.

An explicit calculation for $f_I^h$ yields

$$f_I^0 = \int_0^1 dx \Phi_V^V(x) \left( 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi \right),$$

$$f_I^{\pm} = \int_0^1 dx g^{(v)}_{\pm}(x) \left( 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi \right), \quad (16)$$

where $f_I^0$ has the same expression as the hard scattering kernel $f_I$ in $B \rightarrow \pi \pi \pi$. The hard kernel $f_{II}^h$ for hard spectator interactions have the expressions ($V_1$: recoiled meson, $V_2$: emitted meson):

$$f_{II}^0 = \frac{4\pi^2}{N_c} 2 f_B f_{V_1} m_1 \int_0^1 d\bar{\rho} \frac{\Phi_B^B(\bar{\rho})}{\bar{\rho}} \int_0^1 d\bar{\eta} \frac{\Phi_V^{V_1}(\bar{\eta})}{\bar{\eta}} \int_0^1 d\xi \frac{\Phi_V^{V_2}(\xi)}{\xi},$$

$$f_{II}^{\pm} = -\frac{4\pi^2}{N_c} \frac{2 f_B f_{V_2}}{m_B h_{\pm}}(1 \mp 1) \int_0^1 d\bar{\rho} \frac{\Phi_B^B(\bar{\rho})}{\bar{\rho}} \int_0^1 d\bar{\eta} \frac{\Phi_V^{V_1}(\bar{\eta})}{\bar{\eta}} \int_0^1 d\xi \frac{g_{\perp}^{V_2}(\xi)}{\xi},$$

$$+ \frac{4\pi^2}{N_c} \frac{2 f_B f_{V_1} m_1}{m_B^2 h_{\pm}} \int_0^1 d\bar{\rho} \frac{\Phi_B^B(\bar{\rho})}{\bar{\rho}} \int_0^1 d\bar{\eta} d\xi \left\{ g_{\perp}^{V_1}(\bar{\eta}) g_{\perp}^{V_2}(\xi) \frac{\xi + \bar{\eta}}{\xi \bar{\eta}^2} \right\}, \quad (17)$$

$$\pm \frac{1}{4} g_{\perp}^{V_1}(\bar{\eta}) g_{\perp}^{V_2}(\xi) \frac{\xi + \bar{\eta}}{\xi^2 \bar{\eta}^2} \pm \frac{1}{4} g_{\perp}^{V_1}(\bar{\eta}) g_{\perp}^{V_2}(\xi) \frac{2\xi + \bar{\eta}}{\xi \bar{\eta}^3}. $$

where
\[ h_0 = (m_B^2 - m_1^2 - m_2^2)(m_B + m_1)A_1^{BV}(m_2^2) - \frac{4m_Bp_c^2}{m_B + m_1}A_2^{BV}(m_2^2), \]

\[ h_\pm = (m_B + m_1)A_1^{BV_0}(m_2^2) + \frac{2m_Bp_c}{m_B + m_1}V^{BV_0}(m_2^2), \]  

(18)

and we have neglected the light quark masses and applied the approximation \( \bar{\rho} \approx 0 \), and the \( B \) meson wave function \[ \Phi \]:

\[ \langle 0 | \bar{q}_a(x) b_\beta(0) | \bar{B}(p) \rangle |_{x_+ = x_- = 0} = -\frac{iF_B}{4}([\bar{\rho} + m_B] \gamma_5)_{\beta \gamma} \int_0^1 d\bar{\rho} e^{-i\bar{\rho}p \cdot x} [\Phi_0^B(\bar{\rho}) + \Phi_2^B(\bar{\rho})]_{\gamma \alpha}, \]

(19)

with \( n_- = (1, 0, 0, -1) \). Note that the presence of logarithmic and linear infrared divergences in \( f_\perp^2 \) implies that the spectator interaction is dominated by soft gluon exchanges in the final states. Hence, factorization breaks down at the twist-3 order for transversely polarized vector meson states. We will introduce a cutoff of order \( \Lambda_{\text{QCD}}/m_b \) to regulate the linear and logarithmic divergences. The choice of the cutoff is not important here since the transversely polarized amplitudes are suppressed anyway.

Two remarks are in order. (i) Since \( \langle V | \bar{q}_1 q_2 | 0 \rangle = 0 \), \( B \rightarrow VV \) decays do not receive factorizable contributions from \( a_6 \) and \( a_8 \) penguin terms except for spacelike penguin diagrams \[ \Pi \]. (ii) The first two terms \(-12 \ln(\mu/m_b) - 18\) in Eq. (15) for helicity \( \pm \) states arise from the twist-3 DA \( g^{(\gamma)}_\perp(u) \) and will render the parameters \( a_\pm^\gamma \) (except for \( a_\pm^6 \) and \( a_\pm^8 \) ) scale and scheme independent.

4. To proceed we compute the branching ratios using LCSR and BSW models for heavy-light form factors (see Table I). The factorized amplitudes of \( B \rightarrow VV \) modes are given in \[ \Pi \]. Note that the original BSW model assumes a monopole behavior for all the form factors. This is not consistent with heavy quark symmetry for heavy-to-heavy transition. Therefore, we will employ the BSW model for the heavy-to-light form factors at zero momentum transfer but take a different ansatz for their \( q^2 \) dependence, namely a dipole dependence for \( A_0, A_2 \) and \( V \). In the light-cone sum rule analysis, the form-factor \( q^2 \) dependence is given in \[ \Pi \].

To proceed we use the next-to-leading Wilson coefficients in the NDR scheme \[ \Pi \]

\[ c_1 = 1.082, \quad c_2 = -0.185, \quad c_3 = 0.014, \quad c_4 = -0.035, \quad c_5 = 0.009, \quad c_6 = -0.041, \]

\[ c_7/\alpha = -0.002, \quad c_8/\alpha = 0.054, \quad c_9/\alpha = -1.292, \quad c_{10}/\alpha = 0.263, \quad c_9 = -0.143, \]

(20)

with \( \alpha \) being an electromagnetic fine-structure coupling constant. For the LCDAs, we use the asymptotic form for the vector meson \[ \Pi \]

\[ \Phi^V_\parallel(x) = \Phi^V_\perp(x) = g^{(\gamma)}_\perp(x) = 6x(1 - x), \]

\[ g^{(\gamma)}_\perp(x) = \frac{3}{4} \left[ 1 + (2x - 1)^2 \right], \]

(21)

and the \( B \) meson wave function

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\[
\Phi_1^B(\bar{\rho}) = N_B \bar{\rho}^2 (1 - \bar{\rho})^2 \exp \left[ -\frac{1}{2} \left( \frac{\bar{\rho} m_B}{\omega_B} \right)^2 \right],
\]
(22)

with \( \omega_B = 0.25 \text{ GeV} \) and \( N_B \) being a normalization constant. For the decay constants, we use

\[
f_\rho = 216 \text{ MeV}, \quad f_{K^*} = 221 \text{ MeV}, \quad f_\omega = 195 \text{ MeV}, \quad f_\phi = 237 \text{ MeV},
\]
(23)

and we will assume \( f^T_V = f_V \) for the tensor decay constant.

TABLE I. Form factors at zero momentum transfer for \( B \to P \) and \( B \to V \) transitions evaluated in the light-cone sum rule (LCSR) analysis [3]. The values given in the square brackets are obtained in the BSW model [2]. We have assumed SU(3) symmetry for the \( B \to \omega \) form factors in the LCSR approach.

| Decay          | \( V \)   | \( A_1 \)     | \( A_2 \)     | \( A_3 = A_0 \) |
|---------------|-----------|---------------|---------------|-----------------|
| \( B \to \rho^\pm \) | 0.338 [0.329] | 0.261 [0.283] | 0.223 [0.283] | 0.372 [0.281]   |
| \( B \to \omega \)      | 0.239 [0.232] | 0.185 [0.199] | 0.158 [0.199] | 0.263 [0.198]   |
| \( B \to K^* \)        | 0.458 [0.369] | 0.337 [0.328] | 0.283 [0.331] | 0.470 [0.321]   |

To illustrate the non-universality of nonfactorizable effects for helicity amplitudes, we give a few numerical results for the parameters \( a^h_4 \):

\[
\begin{align*}
 a^0_1 &= 1.04 + 0.01i, & a^+_1 &= 1.02 + 0.01i, & a^-_1 &= 1.11 + 0.04i, \\
 a^0_2 &= 0.09 - 0.08i, & a^+_2 &= 0.17 - 0.08i, & a^-_2 &= -0.38 - 0.25i, \\
 a^0_4 &= -0.033 - 0.004i, & a^+_4 &= -0.026 - 0.004i, & a^-_4 &= -0.040 - 0.007i,
\end{align*}
\]
(24)

in the LCSR model for form factors. Therefore, nonfactorizable corrections to helicity amplitudes are not universal. From Table II we see that the branching ratios predicted by LCSR is larger than that by the BSW model by a factor of \( 1.5 \sim 2 \). Evidently, the experimental results for \( B \to \phi K^* \) favor the LCSR form factors for \( B \to V \) transition. It should be stressed that thus far we have not taken into account power corrections such as annihilation diagrams and higher-twist wave functions for the longitudinally polarized vector meson. In particular, weak annihilations induced by the \((S - P)(S + P)\) penguin operators are no longer subject to helicity suppression and hence can be sizable (see the last term in Eq. (7) and [13]). However, contrary to the \( PP \) and \( PV \) modes, the annihilation amplitude in the \( VV \) case does not gain a chiral enhancement of order \( m_B^2/(m_q m_b) \). Therefore, it is truly power suppressed in the heavy quark limit.
It is also clear from Table II that the tree-dominated modes $\rho^+\rho^-$, $\rho^-\rho^0$, $\rho^-\omega$ have larger branching ratios of order $(2 \sim 3) \times 10^{-5}$ than the penguin-dominated ones. This is ascribed to the fact that the $(S - P)(S + P)$ penguin operators do not contribute to factorizable $W$-emission amplitudes. By contrast, the $\rho^0\rho^0$ and $\omega\omega$ modes have rather small branching ratios because the parameter $a_2$ is small in QCD factorization. We have also computed $|H_{00}|^2$ and $|H_{\pm\pm}|^2$ for each channel and found that $|H_{-\pm}/H_{00}|^2 = (5 \sim 20)\%$ and $|H_{++}/H_{00}|^2 = (10^{-5} \sim 10^{-3})$.

5. We have analyzed $B \to VV$ decays within the framework of QCD factorization. Contrary to phenomenological generalized factorization, nonfactorizable corrections to each partial-wave or helicity amplitude are not the same; the effective parameters $a_i$ vary for different helicity amplitudes. The leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit and hence it is necessary to take into account twist-3 distribution amplitudes of the vector meson in order to have renormalization scale and scheme independent predictions. Branching ratios of $B \to VV$ decays are calculated in two different models for form factors, and the predicted decay rates are different by a factor of $1.5 \sim 2$. In the heavy quark limit, both vector mesons in the charmless $B \to VV$ decay should have zero helicity and the corresponding amplitude is proportional to the form factor difference $(A_1 - A_2)$. The recent observation of $B \to \phi K^*$ indicates that the form factors $A_1(q^2)$ and $A_2(q^2)$ cannot be very similar at low $q^2$ as implied by the BSW model. Owing to the absence of $(S - P)(S + P)$ penguin operator contributions to $W$-emission amplitudes, tree-dominated $B \to VV$ decays tend to have larger branching ratios than the penguin-dominated ones.

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TABLE II. Branching ratios (in units of $10^{-6}$) averaged over CP-conjugate modes for charmless $B \to VV$ decays. Two different form-factor models, the LCSR and the BSW models, are adopted and the unitarity angle $\gamma = 60^\circ$ is employed. Experimental limits and results are taken from [14-16,4-6 and the limits indicated by * are quoted from [14] only for the helicity zero states.

| Decay                  | LCSR | BSW | Expt. |
|------------------------|------|-----|-------|
| $\bar{B}^0 \to \rho^- \rho^+$ | 35.0 | 21.2 | < 2200 |
| $\bar{B}^0 \to \rho^0 \rho^0$     | 0.26 | 0.20 | < 5.9*  |
| $\bar{B}^0 \to \omega \omega$     | 0.30 | 0.22 | < 19   |
| $B^- \to \rho^- \rho^0$           | 21.8 | 12.8 | < 120  |
| $B^- \to \rho^- \omega$           | 21.0 | 13.8 | < 47   |
| $\bar{B}^0 \to K^{*-} \rho^+$    | 4.84 | 3.12 | -      |
| $\bar{B}^0 \to K^{*-0} \rho^0$   | 0.99 | 0.71 | < 19*  |
| $\bar{B}^0 \to K^{0*} K^{*0}$    | 0.32 | 0.16 | < 10*  |
| $B^- \to K^{*-} \rho^0$           | 5.59 | 2.94 | < 54*  |
| $B^- \to K^{0*} \rho^-$          | 6.70 | 4.01 | -      |
| $B^- \to K^{*-} K^{*0}$           | 0.34 | 0.15 | < 50*  |
| $\bar{B}^0 \to \rho^0 \phi$      | 0.003| 0.002| < 13   |
| $\bar{B}^0 \to \omega \phi$      | 0.003| 0.002| < 21   |
| $B^- \to \rho^- \phi$             | 0.007| 0.004| < 16   |
| $\bar{B}^0 \to \rho^0 \omega$    | 0.12 | 0.07 | < 11   |
| $\bar{B}^0 \to K^{*-0} \omega$   | 3.66 | 2.15 | < 19   |
| $B^- \to K^{*-} \omega$          | 3.12 | 1.88 | < 52   |
| $B^- \to K^{*-} \phi$             | 9.30 | 4.32 | 9.7^{+4.2}_{-3.4} ± 1.7 [3] |
|                                      |     |     | 10.6^{+6.4+1.8}_{-4.9-1.6} [3] |
|                                      |     |     | < 36 [3] |
| $\bar{B}^0 \to K^{*-0} \phi$     | 8.71 | 4.62 | 8.6^{+2.8}_{-2.4} ± 1.1 [3] |
|                                      |     |     | 11.5^{+4.5+1.8}_{-3.7-1.7} [3] |
|                                      |     |     | 15^{+8}_{-6} ± 3 [3] |