Structured light pulses and their Lorentz-invariant mass

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Abstract
The Lorentz-invariant mass and mean propagation speed have been found for structured light pulses in a vacuum considered as relativistic objects. We have solved the boundary problem for such widely known field configurations as Gauss, Laguerre–Gauss, Bessel–Gauss, Hermite–Gauss and Airy–Gauss. The pulses were taken as having a temporal envelope of finite duration. We discovered that Lorentz-invariant mass and mean propagation speed significantly depend on the spatial–temporal structure of pulses. We found that mean propagation speed is independent of the full energy of the pulse and is less than the speed of light.

Keywords: Lorentz-invariant mass, structured light pulses, groups of photons, light propagation

1. Introduction

Infinitely extended in space and time, electromagnetic plane waves propagate in a vacuum at the speed of light. But in reality electromagnetic fields have the form of light pulses with finite energy and duration and occupy a finite spatial volume. Moreover, light pulses can have a specific space–time structure, which can impart unique properties to these pulses. Theoretical and experimental research on structured light pulses has been performed for decades. This field of optics has proven to be a fruitful area for practical applications in the fields of quantum information, spectroscopy, particle state control, etc. (for examples see the reviews [1–3] and references therein). Recently it was discussed that some specific types of structured light propagate in a vacuum with a mean propagation speed of less than the speed of light [4, 5]. This conclusion was confirmed in various experiments involving quantum states of light [6]. In [7] the authors emphasized that a nontrivial configuration of the spatial structure of structured light leads to deviation of its mean propagation speed from the speed of light in a vacuum, c. However, the problem of the mean propagation speed of light pulses in a vacuum can also be attacked from a different point of view. In accordance with Fourier analysis, a light pulse consists of a very large number of plane waves. One may associate these plane waves with photons. These plane waves propagate at the speed of light, c, but in different directions. Thus, the whole pulse is a result of interference of plane waves in superposition. Consequently, the mean propagation speed of a pulse can be defined as a weighted superposition of velocities of individual plane waves. It was proved [8, 9] that the mean propagation speed defined in this way is less than the speed of light in a vacuum. This approach is analogous to the problem of propagation of a group of particles in the special theory of relativity, including groups of photons. In relativistic physics the mass of an arbitrary group of particles [10–12] is defined by the total energy of this group, \( \varepsilon \), and its momentum, \( \vec{p} \):

\[
m^2 c^4 = \varepsilon^2 - c^2 \vec{p}^2. \tag{1}
\]

Note that for a single photon equation (1) gives zero mass, as must be the case. On the other hand, let us consider as an example a pair of photons with equal frequencies but non-collinear wave vectors. Their scalar product is given by
$(\vec{k}_1 \cdot \vec{k}_2) = \omega^2/c^2 \cos(\theta), \theta \in (0, \pi]$. The total momentum of a photon pair is a vector with an absolute value smaller than total energy of the photon pair divided by $c$. In accordance with (1) the considered system of photons can be characterized by its Lorentz-invariant mass equal to

$$m = \frac{2\hbar \omega}{c^2} \sin \left( \frac{\theta}{2} \right). \tag{2}$$

The physical meaning of Lorentz-invariant mass is related to the mean velocity of a group of particles as a whole, the absolute value of which is given by

$$v = \frac{c^2 |\vec{p}|}{\epsilon} = c \sqrt{1 - \frac{m^2 c^4}{\epsilon^2}}. \tag{3}$$

One can treat an arbitrary light pulse as a system consisting of photons, where the number of photons in the classical limit is very large. Thus, one can apply the concepts of Lorentz-invariant mass and mean propagation speed to arbitrary light pulses [8, 9].

In the present work, we calculate, using equations (3) and (1), the mean propagation speed of localized light pulses specified by their spatio-temporal structure. In section 2.1 we start our analysis from the most general perspective by defining the relation between Lorentz-invariant mass and the Fourier image of the light pulse’s electric field. In section 2.2, we define the electric field component of the pulse by setting the boundary problem. The boundary field is taken on spatial and temporal electric field component of the pulse by setting the boundary conditions for the transversal coordinates (e.g. see book [14]). The spectra of such beams directly depend on the absolute value of the transverse component of a wave vector $\vec{E}(\vec{k}) = \vec{E}(k_x, k_y)$. It is evident that the mentioned symmetry of a spectrum $\vec{E}(\vec{k})$ determines which component of the wave vector makes the most significant contribution to the momentum calculation and, consequently, to the mass. Thus, owing to the axial symmetry of the spectra, the calculation of mass is simplified due to vanishing transverse components of the momentum in (6):

$$m^2 = \frac{1}{(2\pi)^2} \left[ \left( \int |\vec{E}(\vec{k})|^2 d\vec{k} \right)^2 - \left( \int |\vec{E}(\vec{k})|^2 \frac{k}{|\vec{k}|} d\vec{k} \right)^2 \right]. \tag{6}$$

There is important family of beam models frequently used in the optics of structured light. For this family of beams the spatial spectrum possesses an axial symmetry with respect to the propagation axis. Models having axial symmetry stem from the Helmholtz equation in the paraxial limit in cylindrical coordinates (e.g. see book [14]). The spectra of such beams directly depend on the absolute value of the transverse component of a wave vector $\vec{E}(\vec{k}) = \vec{E}(k_x, k_y)$. It is evident that the mentioned symmetry of a spectrum $\vec{E}(\vec{k})$ determines which component of the wave vector makes the most significant contribution to the momentum calculation and, consequently, to the mass. Thus, owing to the axial symmetry of the spectra, the calculation of mass is simplified due to vanishing transverse components of the momentum in (6):

$$m^2 = \frac{1}{(2\pi)^2} \left[ \int |\vec{E}(\vec{k})|^2 \left( 1 - \frac{k_x}{|\vec{k}|} \right) d\vec{k} \right. \times \left. \int |\vec{E}(\vec{k})|^2 \left( 1 + \frac{k_x}{|\vec{k}|} \right) d\vec{k} \right]. \tag{7}$$

2. Lorentz-invariant mass of an arbitrary light pulse

2.1. General description

Two problems are considered in this section. First, we derive a general expression of Lorentz-invariant mass of light pulses in a vacuum. Second, we discuss propagation of an arbitrary pulse in the space–time domain, defined by its boundary problem for the electric field of the pulse. We will demonstrate how the Fourier image of the boundary conditions can be used for calculation of the Lorentz-invariant mass.

In [8, 13] the authors derived a method of calculation of Lorentz-invariant mass for the case of a Gaussian light pulse propagating in a vacuum. It was shown that Lorentz-invariant mass is defined by a pulse spectrum $|\vec{E}(\vec{k})|$, where $\vec{E}(\vec{k})$ is a Fourier image of a pulse electric field in the space–time domain:

$$E(\vec{r}, t) = \frac{1}{(2\pi)^2} \int \vec{E}(\vec{k}) e^{-i\vec{k}\cdot\vec{r}} e^{\omega t} d\vec{k}, \tag{4}$$

where, in accordance with the Maxwell equations, $\omega^2 = c \sqrt{k_x^2 + k_y^2}$. In terms of Fourier images of $\vec{E}(\vec{k})$ the energy and momentum of a pulse are given by

$$\epsilon = \frac{1}{2\pi} \int |\vec{E}(\vec{k})|^2 d\vec{k},$$

$$\vec{p} = \frac{1}{2\pi c} \int |\vec{E}(\vec{k})|^2 \frac{\vec{k}}{|\vec{k}|} d\vec{k}. \tag{5}$$

These equations and (1) yield the following general expression for Lorentz-invariant mass:

$$m^2 = \frac{1}{(2\pi)^2} \left[ \left( \int |\vec{E}(\vec{k})|^2 d\vec{k} \right)^2 - \left( \int |\vec{E}(\vec{k})|^2 \frac{\vec{k}}{|\vec{k}|} d\vec{k} \right)^2 \right]. \tag{6}$$

2.2. Boundary problem

In the previous section we derived the most general form of Lorentz-invariant mass which is completely defined by the pulse Fourier image $\vec{E}(\vec{k})$ of an electric field. In turn, $\vec{E}(\vec{k})$ can be specified by the boundary conditions for the transverse components of the electrical field. To complete the pulse description one needs to derive the component $E$, and the magnetic field from the Maxwell equations. In experiments one can construct an optical system with lenses, phase plates, nonlinear elements, etc to give one or the other spatio-temporal structure.
For instance, the action of linear optical elements is especially straightforward in Fourier space and is reduced to algebraic transformation of a field’s Fourier image at some plane. We apply Fourier optics methods which are commonly used and proved to be one of the most effective tools for describing the light pulse in space by its boundary problem (14).

Now let us turn to the boundary problem and specify its form. Let the pulse’s electric field component at the plane \( z = 0 \) be defined as follows:

\[
\vec{E}_\perp (\vec{r}_\perp, t, z = 0) = A(x, y) T(t, \omega_0)
\] (8)

where \( A(x, y) \) is the spatial part and \( T(t, \omega_0) \) represents a temporal function depending on the carrier frequency \( \omega_0 \). We choose \( T(t, \omega_0) = T(t) \sin (\omega_0 t) \), where \( T(t) \) is a real finite function (a temporal envelope) which commonly varies slowly with respect to highly oscillating \( \sin (\omega_0 t) \) function of carrier frequency. Let us perform a Fourier transform of (8) with Fourier images considered in a space–time domain \( \zeta = 0 \). In this case our calculations can only be performed numerically. Therefore, the resulting Fourier image of the boundary conditions \( \vec{E}_\perp (\vec{k}_\perp, \omega) \) on the transfer function \( h (\vec{k}_\perp, \omega, \zeta) \). Consequently, this Fourier expansion yields:

\[
h (\vec{k}_\perp, \omega, \zeta) = e^{i\vec{k}_\perp \cdot \vec{r}_\perp}.
\] (10)

It worth emphasizing that it is especially useful to operate with Fourier images of field in the \( \vec{k}_\perp, \omega \) domain describing localized pulses, particularly in the context of the calculation of Lorentz-invariant mass. Thus, we imply that \( k_\perp = \sqrt{\omega^2/c^2 - k^2_\perp} \) in accordance with the dispersion equation and fix the direction of the pulse propagation, \( k_\perp > 0 \). Frequently, structured light pulses and beams are considered within the limits of paraxial approximation. In this case, one assumes that \( |k| \ll k_\perp \). In the present work we are not bounded by the limits of paraxial approximation for the obtained field in \( z > 0 \). We will show that effects related to Lorentz-invariant mass and mean propagation speed appear more clearly when pulse divergence is significant. That is equal to the fulfillment of \( w_0 \sim \lambda_0 \), so small \( w_0 \) should be understood in this context as \( w_0 < 50 \lambda_0 \). In this case our calculations can only be performed numerically. Thereby, the resulting Fourier image of the pulse is

\[
\vec{E}_\perp (\vec{k}_\perp, \omega) = h (\vec{k}_\perp, \omega, \zeta) \vec{E} (\vec{k}_\perp, \omega) |_{z=0}.
\] (11)

Now we can use the Maxwell equation \( \text{div} (\vec{E}_\perp) = 0 \) to find the \( E_z (\vec{k}_\perp, \omega) \) component of the field:

\[
E_z (\vec{k}_\perp, \omega) = -\frac{k_x E_x (\vec{k}_\perp, \omega) + k_y E_y (\vec{k}_\perp, \omega)}{\sqrt{\omega^2/c^2 - k^2_\perp}}.
\] (12)

This consequently gives \( \vec{E} (\vec{k}_\perp, \omega) \).

In the third step we restore the electromagnetic field in a space–time domain \( \zeta = 0 \) by performing an inverse Fourier transform in equation (11). This yields

\[
\vec{E} (\vec{r}, t) = \frac{1}{(2\pi)^2} \int \vec{E} (\vec{k}_\perp, \omega) e^{-i\vec{k}_\perp \cdot \vec{r} - i\omega t} d\vec{k}_\perp d\omega.
\] (13)

In its turn, the magnetic field must be defined as follows:

\[
\vec{H} (\vec{r}, t) = \int [\vec{c} \times \vec{E} (\vec{k}_\perp, \omega)] e^{-i\vec{k}_\perp \cdot \vec{r} - i\omega t} d\vec{k}_\perp d\omega,
\] (14)

where \( k_\perp = \sqrt{\omega^2/c^2 - k^2_\perp} \). In the next step we obtain a Fourier image \( \vec{E} (\vec{k}_\perp, k_\perp) \). Owing to \( k_\perp = \sqrt{\omega^2/c^2 - k^2_\perp} \), we make a variable change in the integral (4):

\[
\vec{E} (\vec{r}, t) = \frac{1}{(2\pi)^2} \int \frac{\omega \vec{E} (\vec{k}_\perp, k_\perp)}{c^2 \sqrt{\omega^2/c^2 - k^2_\perp}} e^{-i\vec{k}_\perp \cdot \vec{r} - i\omega t} d\vec{k}_\perp d\omega.
\] (15)

Finally, one can find the connection between spectra by direct comparison of the fields in (15) and (13). As a result, we have:

\[
\vec{E} (\vec{k}_\perp, k_\perp) = \vec{E} (\vec{k}_\perp, \omega) |_{z=0} \frac{c^2}{\sqrt{\omega^2/c^2 - k^2_\perp}}.
\] (16)

This permits one to obtain the Lorentz-invariant mass from the Fourier image of the boundary conditions for various types of light pulses. One needs to calculate the Fourier image of the pulse’s boundary condition and substitute it into the integral (6) after the variable change similar to (15). Calculation of the mass has an especially elegant form in the case of pulses with an axially symmetric spectrum. It can be noticed in (7) that, in fact, this calculation can be reduced to the calculation of the following integral:

\[
p_{z=\text{sym}} = c \int \vec{E} (\vec{k}_\perp, \omega) |_{z=0}^2 \left( 1 - \frac{k^2_\perp c^2}{\omega^2} \right) d\vec{k}_\perp d\omega.
\] (17)

Consequently, we can write:

\[
m = \frac{1}{c^2} \sqrt{\epsilon^2 - \frac{c^2 p^2_{z=\text{sym}}}{4\epsilon^2}}.
\] (18)

In the next section we examine Lorentz-invariant mass and the mean propagation speed of light pulses with well-known spatial profiles coincident with the Laguerre–Gauss (LG), Bessel–Gauss (BG), Hermite–Gauss (HG) and Airy-Gauss (AG) beams at the boundary plane \( z = 0 \).
3. Lorentz-invariant mass and mean propagation speed for the Gauss-family pulses

3.1. Temporal part of the boundary conditions

In [8, 9, 13] a model of a pulse with spatial and temporal envelopes of Gaussian form was considered. It is a good approximation of real pulses, especially in paraxial approximation, but formally such a pulse has infinite duration. In contrast, here we adhere to a model of a pulse which has a finite duration in the time domain. In accordance with (9) the Fourier image of temporal part reads

\[ S(\omega) = \int_{-\infty}^{\infty} T(t) \sin (\omega_0 t) e^{-i \omega t} \, dt, \quad (19) \]

Note, that \( T(t) \sin (\omega_0 t) \) is the real function. It is imposed that \( S(\omega) = S(-\omega) \). Let us specify the model of the temporal envelope \( T(t) \) with finite duration. For simplicity, we choose

\[ T(t) = \begin{cases} 1 & \text{if } t \in [0, t_p] \\ 0, & \text{otherwise}, \end{cases} \quad (20) \]

where \( t_p \) is pulse duration. In accordance with (9), straightforward calculation of the Fourier image of the temporal part reads

\[ S(\omega) = \frac{(-i)^{2} t_p}{2} e^{-\frac{i \pi}{2}} \sin \left( \frac{\omega_0 - \omega}{2} \right) \]

\[ - e^{-i \omega_0} \sin \left( \frac{\omega_0 + \omega}{2} \right), \quad (21) \]

One needs to change the Fourier image of the temporal function of a field (21) to operate correctly with the complex representation (or analytical signal) as follows:

\[ S(\omega) \rightarrow \begin{cases} 2S(\omega) & \text{if } \omega \geq 0 \\ 0, & \text{otherwise}. \end{cases} \quad (22) \]

The width of \( S(\omega) \) is determined by the pulse duration. We assume that the parameter \( \omega_0 t_p \gg 1 \), i.e. the duration of the pulses, is quite big. Thus, one may assume that \( S(\omega) \) has non-zero values at the interval between the center at \( \omega_0 \) and the width \( \sim 1/ t_p \).

The model of the pulse’s temporal part differs significantly from a Gaussian envelope modulated by the highly oscillating function \( \exp\left( -\frac{\pi}{8 t_p^2} \right) \sin (\omega_0 t) \), which was utilized previously in [8, 13]. The main difference is that the Fourier image of the latter has only an imaginary part which is non-zero. In contrast, if a temporal envelope has a finite time duration then its Fourier image consists of an imaginary and real part. This fact does not significantly affect calculation of the Lorentz-invariant mass of a single coherent light pulse. However, as we believe, it can be important in the analysis of superposition of pulses, including partially coherent and incoherent cases, as well as analysis of chirped pulses.

3.2. Spatial part of the boundary conditions

Once the temporal part is determined, we consider the spatial profile of the pulses. As a first example let us consider pulses with axial symmetry. We specify the pulse type by the spatial profile of the boundary conditions which are being chosen as the complex amplitude of LG and BG beams at the plane \( z = 0 \):

\[ \tilde{A}_{\text{LG}}(\rho) |_{z=0} = C_{\text{LG}} \tilde{e}_\phi \left( \frac{\rho}{w_0} \right)^{l - \frac{q^2}{2}} I_q \left( \frac{2 \rho}{w_0} \right) e^{-i \phi}, \]

\[ \tilde{A}_{\text{BG}}(\rho) |_{z=0} = C_{\text{BG}} \tilde{e}_\phi I_1(\beta \rho) e^{-\frac{\rho^2}{2}}, \quad (23) \]

where \( C_{\text{LG}} \) and \( C_{\text{BG}} \) are normalization constants, \( \rho = \sqrt{x^2 + y^2} \) and \( w_0 \) is a pulse waist at the plane \( z = 0 \), which is assumed to be the same for both types of pulse. \( I_q (\cdot) \) are generalized Laguerre polynomials and \( l, m \) are positive integer numbers. We choose azimuthal polarization \( \tilde{e}_\phi \) for both pulse boundary conditions. This is a common choice for beams that satisfy the paraxial Helmholtz equation in cylindrical coordinates [15–18].

Another important type of boundary condition stems from HG and AG beams [19]:

\[ A_{\text{HG}} = \tilde{e}_{4s^s} C_{\text{HG}} H_l \left( \frac{\sqrt{2} y}{w_0} \right) e^{-\frac{x^2}{w_0^2}} H_q \left( \frac{\sqrt{2} y}{w_0} \right) e^{-\frac{y^2}{w_0^2}}, \]

where \( l, q \) are, again, positive integers. \( \tilde{e}_{4s^s} = 1/\sqrt{2} (\tilde{e}_x + \tilde{e}_y) \) denotes linear polarization. The case of an AG beam is given by

\[ A_{\text{AG}} = \tilde{e}_{4s^s} C_{\text{AG}} A_l \left( \frac{x}{w_0} \right) e^{-\frac{x^2}{w_0^2}} A_l \left( \frac{y}{w_0} \right) e^{-\frac{y^2}{w_0^2}}, \]

where \( A_l (\cdot) \) is the Airy function.

Now let us calculate Fourier images of the spatial part of the boundary problem, performing Fourier transform with respect to the transverse spatial coordinates. After algebraic transformation and integration, in the case of LG-type beams we have

\[ F_{\text{LG}}(k_x) = 2\pi C_{\text{LG}} w_0^2 e^{i \phi} e^{i \pi q} \left( k_x w_0 \right) \frac{1}{2} \times e^{-\frac{x^2}{w_0^2}} I_q \left( k_x w_0 \right) \tilde{e}_\phi. \]

\[ \quad (26) \]

Fourier transform in the case of a BG pulse yields

\[ F_{\text{BG}}(k_x) = 2\pi C_{\text{BG}} w_0^2 e^{-i \frac{\beta^2 x^2}{2}} I_1(\beta k_x w_0) \tilde{e}_\phi. \]

Finally, performing Fourier transform in case of a HG pulse for arbitrary \( l, q \) we have

\[ F_{\text{HG}}(k_x, k_y) = \pi w_0^2 e^{i \phi} e^{-i \frac{x^2}{w_0^2}} \times H_l \left( k_x w_0 \frac{1}{\sqrt{2}} \right) \frac{1}{\sqrt{2}} H_q \left( k_y w_0 \frac{1}{\sqrt{2}} \right). \]

\[ \quad (28) \]

For an AG pulse we perform only numerical calculations.

In our calculations of the spatial spectrum we adopted several solutions from [20] which contains integrals involving special functions of mathematical physics. Different structures of pulses significantly influence the Lorentz-invariant mass and mean propagation speed of each type of a pulse. To demonstrate this, we have performed numerical calculations...
of integrals (6) and (7) assuming that the total energy and

time duration $t_p$ of the pulses are fixed. Therefore the param-

eters of the calculations are: wavelength $\lambda_0 = 404 \text{ nm}$, waist

$w_0$, a pulse waist at the plane $z = 0$, measured in numbers of carrier

wavelengths $\lambda_0 = 404 \text{ nm}$. Numerical calculations were performed

with respect to the fixed full energy $\varepsilon = 10 \text{ mJ}$ and pulse duration

time $t_p = 0.539 \text{ ps}$ or 400 periods of the carrier wave.

Figure 1. Lorentz-invariant mass of the Gauss, LG ($l = 1, q = 0$),

BG, HG (symmetrical $l = 1, q = 1$) and AG light pulses. $w_0$ is a pulse waist at the plane $z = 0$, measured in numbers of carrier wavelengths $\lambda_0 = 404 \text{ nm}$. Numerical calculations were performed with respect to the fixed full energy $\varepsilon = 10 \text{ mJ}$ and pulse duration time $t_p = 0.539 \text{ ps}$ or 400 periods of the carrier wave.

Figure 2. Lorentz-invariant mass of a LG pulse as a function of the quantum number $l$, which corresponds to the orbital angular momentum. Different lines correspond to different parameters $q$.

Figure 3. Lorentz-invariant mass of a HG pulse as a function of number $l$ with different parameters $q$. Energy, duration and wavelength are fixed and are the same as in figures 1 and 2. The pulse waist is chosen to be $w_0 = 10\lambda_0$.

Figure 4. Lorentz-invariant mass as a function of energy for different types of pulses. In this case, duration, carrier wavelength and pulse waist ($w_0 = 10\lambda_0$) are fixed. The functions are linear for all types of pulse, which indicates that velocity does not depend on energy and is defined by the structure of a pulse.

Figure 5. Relative deviation of mean propagation speed of Gauss, LG, BG, HG and AG pulses, calculated by means of Lorentz-invariant mass (3) as a function of pulse waist. Other parameters are fixed.

Lorentz-invariant mass, but these effects are non-negligible only in the case of a wide spatial spectrum.

4. Conclusion

In the present research, Lorentz-invariant mass is calculated for structured light pulses in accordance with special relativity theory. Different pulses were described by different boundary conditions induced by beams, commonly used in practice, and a special temporal envelope that provides strict localization of the pulses in the temporal domain. Based on general analytical derivations, all specific calculations were done numerically. The open-access Python source code for the calculations is available online [21]. In all calculations and derivations the vector structure of the field was completely taken into account. The obtained Lorentz-invariant mass appeared to be independent of the pulse duration at the given full energy of the pulse. On the other hand, Lorentz-invariant mass significantly depends on the waist of the pulse in all cases. Increasing the waist leads to a decrease of the Lorentz-invariant mass (figure 1). This is not surprising since the infinite waist corresponds to a plane wave, the mass of which equals zero. Thus, significant effects associated with non-zero Lorentz-invariant mass only
occur at small $w_0$. For such $w_0$, the paraxial approximation is not valid, hence we perform the calculations numerically. By means of Lorentz-invariant mass we found the mean propagation speed of the pulse. This behaves in some way similar to the mass. Mean propagation speed is also independent of the pulse duration and phase of the temporal boundary conditions. It significantly depends on the waist of the pulse, but in contrast to the mass increases and tends to $c$ with increase of the waist. Linear dependences in figure 4 together with equation (3) indicate independence of the mean propagation speed of the pulse on its energy. Moreover, mean propagation speed does not depend on the average power. Consequently, mean propagation speed is fully defined by the space–time structure of the pulse. In accordance with physical meaning, mean propagation speed determines the rest frame of pulses. We believe that the structure of the field in its rest frame is an interesting subject. Finally, in the present work we consider coherent light pulses. The case of incoherent superposition of different light pulses within the context of Lorentz-invariant mass will be discussed elsewhere.

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