Statefinder diagnosis for the interacting model of holographic dark energy

Jingfei Zhang,1 Xin Zhang,2 and Hongya Liu1

1School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, People’s Republic of China

2Kavli Institute for Theoretical Physics China, Institute of Theoretical Physics, Chinese Academy of Sciences (KITPC/ITP-CAS), P.O.Box 2735, Beijing 100080, People’s Republic of China

Abstract

In this paper, we investigate the holographic dark energy model with interaction between dark energy and dark matter, from the statefinder viewpoint. We plot the trajectories of the interacting holographic dark energy model for different interaction cases as well as for different values of the parameter $c$ in the statefinder-plane. The statefinder diagrams characterize the properties of the holographic dark energy and show the discrimination between the two cases with and without interaction. As a result, we show the influence of the interaction on the evolution of the universe in the statefinder diagrams. Moreover, as a complement to the statefinder diagnosis, we study the interacting holographic dark energy model in the $w−w’$ plane, which can provide us with a dynamical diagnosis.
Today it has been confirmed that our universe is undergoing an accelerating expansion through numerous cosmological observations, such as type Ia supernovae (SNIa) \(^1\), large scale structure (LSS) \(^2\), and cosmic microwave background (CMB) \(^3\). This cosmic acceleration is attributed to a mysterious dominant component, dark energy, with negative pressure. The combined analysis of cosmological observations suggests that the universe is spatially flat, and consists of about 70% dark energy, 30% dust matter, and negligible radiation. Many candidates have been proposed to interpret or describe the properties of dark energy, though its nature still remains enigmatic. The most obvious theoretical candidate of dark energy is the cosmological constant \(\lambda\) \(^4, 5\) which has the equation of state \(w = -1\). However, as is well known, there are two difficulties arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the “fine-tuning” problem and the “cosmic coincidence” problem \(^6\). Theorists have made lots of efforts to try to resolve the cosmological constant problem but all these efforts were turned out to be unsuccessful.

Also, there is an alternative proposal to dark energy — the dynamical dark energy scenario. The dynamical dark energy scenario is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. A lot of scalar-field dark energy models have been studied, including quintessence \(^7\), K-essence \(^8\), tachyon \(^9\), phantom \(^10\), ghost condensate \(^11\) and quintom \(^12\) etc.. In addition, other proposals on dark energy include scenarios of interacting dark energy \(^13\), braneworld \(^14\), Chaplygin gas \(^15\), and so forth. By far, obviously, it is not yet clear if dark energy is a cosmological constant or a dynamical field. Generally, theorists believe that we can not entirely understand the nature of dark energy before a complete theory of quantum gravity is established \(^16\).

However, in this circumstance, we still can make some efforts to probe the properties of dark energy according to some principle of quantum gravity. The holographic dark energy model is an example of such effort, which stems from the holographic principle and can provide us with an intriguing way to interpret the dynamics of dark energy. The holographic principle is an important result of the recent researches of exploring the quantum gravity and is enlightened by investigations of the quantum property of black holes \(^17\). According to the holographic principle, the number of degrees of freedom for a system within a finite region should be finite and should be bounded roughly by the area of its boundary. In the cosmological context, the holographic principle will set an upper bound on the entropy of the universe. Motivated by the Bekenstein entropy bound, it seems plausible that one may require that for an effective quantum field theory in a box of size \(L\) with UV cutoff \(\Lambda\), the total entropy should satisfy \(S = L^3 \Lambda^3 \leq S_{BH} \equiv \pi M_P^2 L^2\), where \(S_{BH}\) is the entropy of a black hole with the same size \(L\). However, Cohen et al. \(^18\) pointed out that to saturate this inequality some states with Schwartzchild radius much larger than the box size have to be counted in. As a result, a more restrictive bound, the energy bound, has been proposed to constrain the degrees of freedom of the system, requiring the total energy of a system with size \(L\) not to exceed the mass of a black hole with the same size, namely, \(L^3 \Lambda^4 = L^3 \rho_{de} \leq L M_P^2\). This means that the maximum entropy is in the order of \(S_{BH}^{3/4}\). When we take the whole universe into account, the vacuum energy related to this holographic principle is viewed as dark energy, usually dubbed holographic dark energy. The largest IR cut-off \(L\) is chosen by saturating the inequality, so that we get the holographic dark energy density

\[
\rho_{de} = 3 c^2 M_P^2 L^{-2},
\]

where \(c\) is a numerical constant \(^\star\) (note that \(c > 0\) is assumed), and as usual \(M_P\) is the reduced Planck
mass. If we take $L$ as the size of the current universe, for instance the Hubble scale $H^{-1}$, then the dark energy density will be close to the observed value. However, Hsu [19] pointed out that this yields a wrong equation of state for dark energy. Li [20] subsequently proposed that the IR cutoff $L$ should be given by the future event horizon of the universe,

$$R_{eh}(a) = a \int_{\frac{t}{a}}^{\infty} \frac{dt'}{a(t')} = a \int_{a}^{\infty} \frac{da'}{Ha'^2}. \quad (2)$$

Such a holographic dark energy looks reasonable, since it may provide simultaneously natural solutions to both dark energy problems, as demonstrated in Ref. [20]. Meanwhile, other applications of the holographic principle in cosmology [21] show that holography is an effective way to investigate cosmology. For other extensive studies, see e.g. [22]–[25].

Besides, some interacting models are discussed in many works because these models can help to understand or alleviate the coincidence problem by considering the possible interaction between dark energy and cold dark matter due to the unknown nature of dark energy and dark matter. In addition, the proposal of interacting dark energy is compatible with the current observations such as the SNIa and CMB data [20]. For the interacting model of holographic dark energy see [23].

On the other hand, since more and more dark energy models have been constructed for interpreting or describing the cosmic acceleration, the problem of discriminating between the various contenders is becoming emergent. In order to be capable of differentiating between those competing cosmological scenarios involving dark energy, a sensitive and robust diagnosis for dark energy models is a must. In addition, for some geometrical models arising from modifications to the gravitational sector of the theory, the equation of state no longer plays the role of a fundamental physical quantity, so it would be very useful if we could supplement it with a diagnosis which could unambiguously probe the properties of all classes of dark energy models. For this purpose a diagnostic proposal that makes use of parameter pair \{r, s\}, the so-called “statefinder”, was introduced by Sahni et al. [27]. The statefinder probes the expansion dynamics of the universe through higher derivatives of the scale factor $a$ and is a “geometrical” diagnosis in the sense that it depends on the scale factor and hence on the metric describing space-time. Since different cosmological models involving dark energy exhibit different evolution trajectories in the $s - r$ plane, the statefinder can be used to diagnose different dark energy models [23].

In this paper, we focus on a model of holographic dark energy with interaction between dark energy and dark matter and study the influence of the interaction to the cosmic evolution. Moreover, we use the statefinder to diagnose various cases with different interaction strength and different parameter $c$ in the holographic model.

Let us start with a spatially flat Friedmann-Robertson-Walker (FRW) universe with dust matter and holographic dark energy. The Friedmann equation reads

$$3M^2_hH^2 = \rho_{de} + \rho_m, \quad (3)$$

where $\rho_m$ is the energy density of matter and $\rho_{de} = 3c^2M^2_h R^{-2}$ is the dark energy density. The total energy density satisfies a conservation law,

$$\dot{\rho}_{de} + \dot{\rho}_m = -3H(\rho + P), \quad (4)$$

where $\rho = \rho_m + \rho_{de}$ is the total energy density of the universe, and $P = P_{de} = w\rho_{de}$ is the total pressure ($w$ denotes the equation of state of dark energy). Note that since the matter component is
mainly contributed by the cold dark matter, we ignore the contribution of the baryon matter here for simplicity. By introducing $\Omega_{\text{de}} = \rho_{\text{de}}/(3M_{\text{Pl}}^2H^2)$ and $\Omega_{\text{m}} = \rho_{\text{m}}/(3M_{\text{Pl}}^2H^2)$, the Friedmann equation can also be written as $\Omega_{\text{de}} + \Omega_{\text{m}} = 1$. Furthermore, if we proceed to consider a scenario of interacting dark energy, $\rho_{\text{m}}$ and $\rho_{\text{de}}$ do not satisfy independent conservation laws, they instead satisfy\[\dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = Q,\] and\[\dot{\rho}_{\text{de}} + 3H(1 + w)\rho_{\text{de}} = -Q,\] where $Q$ describes the interaction between dark energy and dark matter. It is obvious that the interaction term $Q$ could not be introduced by considering some micro-process currently, so a phenomenological way is the must. One possible choice for the interaction term is setting\[Q = 3b^2H\rho,\] where $b$ is a constant describing the coupling strength. This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a pressureless cold dark matter component, in order to get a scaling solution to the coincidence problem [23].

Taking the ratio of energy densities as $\mu = \rho_{\text{m}}/\rho_{\text{de}}$ and using the Friedmann equation $\Omega_{\text{de}} + \Omega_{\text{m}} = 1$, we have $\mu = (1 - \Omega_{\text{de}})/\Omega_{\text{de}}$ and $\dot{\mu} = -\Omega_{\text{de}}/\Omega_{\text{de}}$. Furthermore, from (5), (6) and (7), we obtain\[\dot{\mu} = 3b^2H(1 + \mu)^2 + 3H\mu w.\] Combining these results, we easily get the equation of state of dark energy\[w = -\frac{\Omega_{\text{de}}/\Omega_{\text{de}}^2 - 3b^2H(1 + \mu)^2}{3H\mu},\] where prime denotes the derivative with respect to $x = \ln a$.

Using the definition of the holographic dark energy (11) and the Friedmann equation, the future event horizon (2) can be expressed as $R_h = c\sqrt{1 + \mu}/H$. Then, for this expression, taking the derivative with respect to $t$ and reducing the result, we get\[\frac{\Omega_{\text{de}}'}{\Omega_{\text{de}}^2} = (1 - \Omega_{\text{de}}) \left[ \frac{1}{\Omega_{\text{de}}} + \frac{2}{c\sqrt{\Omega_{\text{de}}}} - \frac{3b^2}{\Omega_{\text{de}}(1 - \Omega_{\text{de}})} \right].\] It is notable that this differential equation governs the whole dynamics of the interacting model of holographic dark energy. Substituting (10) to (9) yields\[w = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\text{de}}}}{3c} - \frac{b^2}{\Omega_{\text{de}}}.\] Then we can compute the deceleration parameter \[q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2}w\Omega_{\text{de}} = \frac{1}{2}\left(1 - 3b^2 - \Omega_{\text{de}} - \frac{2}{c}\Omega_{\text{de}}^2\right).\] In order to show the influence of interaction to the cosmic evolution, the cases with dependence of the parameter $b^2$ for the deceleration parameter $q$ are shown in Fig. 1. In Fig. 1, we fix $c = 1$ and...
Figure 1: Evolution of the deceleration parameter $q$ with a fixed parameter $c$. In this plot, we take $c = 1$, $\Omega_{\text{de}0} = 0.73$, and vary $b^2$ as 0, 0.02, 0.06, and 0.10, respectively.

Figure 2: Evolution of the deceleration parameter $q$ with a fixed coupling $b^2$. In this plot, we take $b^2 = 0.10$, $\Omega_{\text{de}0} = 0.73$, and vary $c$ as 0.9, 1.0 and 1.1, respectively.
take the coupling constant $b^2$ as 0, 0.02, 0.06, and 0.10, respectively. Besides, the cases with a fixed $b^2$ and various values of $c$ are also interesting. In Fig. 2, fixing the coupling constant $b^2 = 0.10$, we plot the evolution diagram of the deceleration parameter $q$ with different values of parameter $c$ (here we take the values of $c$ as 0.9, 1.0, and 1.1, respectively). From Figs. 1 and 2 we learn that the universe experienced an early deceleration and a late time acceleration. Fig. 1 shows that, for a fixed parameter $c$, the cosmic acceleration starts earlier for the cases with interaction than the ones without coupling (for this point see also, e.g., [29]). Moreover, the stronger the coupling between dark energy and dark matter is, the earlier the acceleration of universe began. However, the cases with smaller coupling will get bigger acceleration finally in the far future. In addition, Fig. 2 shows that the acceleration starts earlier when $c$ is larger for the same coupling $b^2$, but finally a smaller $c$ will lead to a bigger acceleration. It should be pointed out that, in the interacting holographic dark energy model, the interaction strength has an upper limit because of the evolutionary behavior of the holographic dark energy. For detailed discussions about correlation of the coupling $b^2$ and the parameter $c$, see [26]. It is remarkable that, with the interaction between dark energy and dark matter, the case of $c = 1$ could not enter a de Sitter phase in the infinite future. In short, the influence of the interaction between dark energy and dark matter to the cosmic evolution is obvious, as manifested by Figs. 1 and 2. On the other hand, nevertheless, as Eq. (12) shows, the deceleration parameter $q$ carries the information of the equation of state of dark energy $w$, the property of dynamical evolution for $w$ can not be read out from $q$. For diagnosing properties and evolutionary behaviors of dark energy models exquisitely, more powerful diagnostic tool is a must.

Now we turn to the statefinder diagnosis. For characterizing the expansion history of the universe, one defines the geometric parameters $H = \dot{a}/a$ and $q = -\ddot{a}/aH^2$, namely the Hubble parameter and the deceleration parameter. It is clear that $\dot{a} > 0$ means the universe is undergoing an expansion and $\ddot{a} > 0$ means the universe is experiencing an accelerated expansion. From the cosmic acceleration, $q < 0$, one infers that there may exist dark energy with negative equation of state, $w < -1/3$ and likely $w \sim -1$, but it is hard to deduce the information of the dynamical property of $w$ (namely the time evolution of $w$) from the value of $q$. In order to extract the information on the dynamical evolution of $w$, it seems that we need the higher time derivative of the scale factor, $\dddot{a}$. Another motivation for proposing the statefinder parameters stems from the merit that they can provide us with a diagnosis which could unambiguously probe the properties of all classes of dark energy models including the cosmological models without dark energy describing the cosmic acceleration. Though at present we can not extract sufficiently accurate information of $\dot{a}$ and $\ddot{a}$ from the observational data, we can expect, however, the high-precision observations of next decade may be capable of doing this. Since different cosmological models exhibit different evolution trajectories in the $s-r$ plane, the statefinder parameters can thus be used to diagnose the evolutionary behaviors of various dark energy models and discriminate them from each other. In this paper, we apply the statefinder diagnosis to the interacting holographic dark energy model.

The expansion rate of the universe is described by the Hubble parameter $H$, and the rate of acceleration/deceleration of the expanding universe is characterized by the deceleration parameter $q$. Furthermore, in order to find a more sensitive discriminator of the expansion rate, let us consider the general expansion form for the scale factor of the universe

$$a(t) = a(t_0) + \dot{a}|_{t_0}(t-t_0) + \frac{\ddot{a}|_{t_0}}{2}(t-t_0)^2 + \frac{\dddot{a}|_{t_0}}{6}(t-t_0)^3 + \ldots.$$  \hspace{1cm} (13)

Generically, various dark energy models give rise to families of curves $a(t)$ having vastly different properties. In principle, we can confine our attention to small value of $|t-t_0|$ in (13) because the acceleration of the universe is a fairly recent phenomenon. Then, we see, following [27], that a new diagnostic of dark
energy dubbed statefinder can be constructed using both second and third derivatives of the scale factor. The second derivative is encoded in the deceleration parameter $q$, and the third derivative is contained in the statefinder parameters $\{r, s\}$. The statefinder parameters $\{r, s\}$ are defined as

$$r \equiv \frac{\dddot{a}}{aH^2}, \quad s \equiv \frac{r - 1}{3(q - \frac{1}{2})}. \quad (14)$$

Note that the parameter $r$ is also called cosmic jerk. Thus the set of quantities describing the geometry is extended to include $\{H, q, r, s\}$. Trajectories in the $s - r$ plane corresponding to different cosmological models exhibit qualitatively different behaviors, so the statefinder can be used to discriminate different cosmological models. The spatially flat LCDM (cosmological constant $\lambda$ with cold dark matter) scenario corresponds to a fixed point in the diagram

$$\{s, r\}_{\text{LCDM}} = \{0, 1\}. \quad (15)$$

Departure of a given dark energy model from this fixed point provides a good way of establishing the “distance” of this model from spatially flat LCDM [27]. As demonstrated in Refs. [28]–[45], the statefinder can successfully differentiate between a wide variety of dark energy models including the cosmological constant, quintessence, phantom, quintom, the Chaplygin gas, braneworld models and interacting dark energy models, etc.. We can clearly identify the “distance” from a given dark energy model to the LCDM scenario by using the $r(s)$ evolution diagram. The current location of the parameters $s$ and $r$ in these diagrams can be calculated in models. The current values of $s$ and $r$ are evidently valuable since we expect that they can be extracted from data coming from SNAP (Supernova Acceleration Probe) type experiments. Therefore, the statefinder diagnosis combined with future SNAP observations may possibly be used to discriminate between different dark energy models. The statefinder parameter-pair also can be expressed as

$$r = 1 + \frac{9(\rho + P)\dot{P}}{2\rho}, \quad s = \frac{(\rho + P)\dot{P}}{P\dot{\rho}}, \quad (16)$$

where $\rho$ is the total density and $P$ is the total pressure. Then, by using the Friedmann equation, we can obtain the following concrete expressions

$$r = 1 - \frac{3}{2} \Omega_{de} w' + 3 \Omega_{de} w \left(1 - \frac{1}{c} \sqrt{\Omega_{de}}\right), \quad (17)$$

$$s = 1 + w - \frac{w'}{3w} + \frac{b^2}{\Omega_{de}}. \quad (18)$$

Directly, from Eq. (11), we have

$$w' = \frac{\Omega_{de}'}{\Omega_{de}^2} \left(b^2 - \frac{1}{3c} \Omega_{de}^{3/2}\right)$$

$$= (1 - \Omega_{de}) \left(b^2 - \frac{\Omega_{de}^{3/2}}{3c}\right) \frac{1}{\Omega_{de}} - \frac{3b^2}{\Omega_{de}(1 - \Omega_{de})} + \frac{2}{c\sqrt{\Omega_{de}}}, \quad (19)$$

where the prime denotes the derivative with respect to $x = \ln a$. Note that the whole dynamics of the universe in the interacting holographic dark energy model is governed by the differential equation [10]. So by solving Eq. (10) we can get the evolution solution of $\Omega_{de}$ and then hold all the cosmological quantities of interest and the whole dynamics of the universe.

\[1\]It should be noted that the opinion of other authors may not be so optimistic, see, e.g. [10].
Figure 3: The statefinder diagrams $r(s)$ for the interacting holographic dark energy with a fixed parameter $c$ and different coupling $b^2$. Selected curves of $r(s)$ are plotted by fixing $c = 1$, $\Omega_{de0} = 0.73$ and varying $b^2$ as 0, 0.02, 0.06 and 0.10, respectively. A star denotes the LCDM fixed point $(0, 1)$. The dots show today’s values for the statefinder parameters $(s_0, r_0)$.

Figure 4: The statefinder diagrams $r(s)$ for the holographic dark energy model with different values of parameter $c$. In the plot, we take $\Omega_{de0} = 0.73$ and vary $c$ as 0.9, 1.0, 1.1, 1.2 and 1.5, respectively. The left panel is for the holographic model without interaction between the dark energy and dark matter ($b^2 = 0$), while the right one is for the case including the interaction ($b^2 = 0.10$). A star denotes the LCDM fixed point $(0, 1)$. The dots show today’s values for the statefinder parameters $(s_0, r_0)$. 
In what follows we shall diagnose the interacting holographic dark energy model employing the statefinder method. We shall analyze the cases with fixed coupling constant $b^2$ and with fixed parameter $c$, respectively. As demonstrated above, the information of this model can be acquired by solving the differential equation (10). Making the redshift $z$ vary in a large enough range involving far future and far past, one can solve the differential equation (10) numerically and then get the evolution trajectories in the statefinder $s-r$ planes for this model. For instance, we plot the statefinder diagram in Fig. 3 for the cases of $c = 1$ with various values of coupling such as $b^2 = 0, 0.02, 0.06$ and $0.10$, meanwhile the present density parameter of dark energy is taken to be $\Omega_{\text{de}0} = 0.73$. The case $b^2 = 0$ corresponds to the holographic dark energy model without interaction between dark energy and dark matter. The arrows in the diagram denote the evolution directions of the statefinder trajectories and the star corresponds to \{r = 1, s = 0\} representing the LCDM model. This diagram shows that the evolution trajectories with different interaction strengths exhibit different features in the statefinder plane. When the interaction is absent, the $r(s)$ curve for holographic dark energy ends at the LCDM fixed point, i.e., the universe of this case will evolve to the de Sitter phase in the far future. However, taking the interaction into account, the endpoints of the $r(s)$ curves could not arrive at the LCDM fixed point $(0, 1)$, though all of the evolution trajectories tend to approach this point. It should be mentioned that the statefinder diagnosis for holographic dark energy model without interaction has been investigated in detail in [37], where the focus is put on the diagnosis of the different values of parameter $c$. The statefinder analysis on the holographic dark energy in a non-flat universe see [42]. In [37], it has been demonstrated that from the statefinder viewpoint $c$ plays a significant role in this model and it leads to the values of \{r, s\} in today and future tremendously different. In this paper, by far, we have clearly seen that the interaction between holographic dark energy and dark matter makes the statefinder evolutionary trajectories with the same value of $c$ tremendously different also. If the accurate information of \{r_0, s_0\} can be extracted from the future high-precision observational data in a model-independent manner, these different features in this model can be discriminated explicitly by experiments, one thus can use this method to test the holographic dark energy model as well as other dark energy models. Hence, today’s values of \{r, s\} play a significant role in the statefinder diagnosis. We thus calculate the present values of the statefinder parameters for different cases in the interacting holographic dark energy model and mark them on evolution curves with dots. It can be seen that stronger interaction results in longer distance to the LCDM fixed point. The interaction between holographic dark energy and dark matter prevents the holographic dark energy from behaving as a cosmological constant $\lambda$ ultimately in the far future.

We also plotted the statefinder diagram in the $s-r$ plane for different values of parameter $c$ with $b^2 = 0$ and $0.10$ in Fig. 4. The left panel is for the holographic dark energy without interaction while the right one is for the case involving the interaction. The star in the figure also corresponds to the LCDM fixed point and the dots marked on the curves represent the present values of the statefinder parameters. Note that the true values of $(s_0, r_0)$ of the universe should be determined in a model-independent way, we can only pin our hope on the future experiments to achieve this. We strongly expect that the future high-precision experiments (e.g. SNAP) may provide sufficiently large amount of precise data to release the information of statefinders \{H, q, r, s\} in a model-independent manner so as to supply a way of discriminating different cosmological models with or without dark energy. From Fig. 4, we can learn that the $r(s)$ evolutions have the similar behavior, i.e. the curves almost start from a fixed point for both cases in the $s-r$ plane. Evidently, the interaction between dark components makes the value of $r$ smaller and the value of $s$ bigger. Also, obviously, the parameter $c$ plays a crucial role in the holographic model.

As a complement to statefinder diagnosis, we investigate the dynamical property of the interacting
holographic dark energy in the $w - w'$ phase plane, where $w'$ represents the derivative of $w$ with respect to $\ln a$. Recently, this method became somewhat popular for analyzing dark energy models. Caldwell and Linder [47] proposed to explore the evolving behavior of quintessence dark energy models and test the limits of quintessence in the $w - w'$ plane, and they showed that the area occupied by quintessence models in the phase plane can be divided into thawing and freezing regions. Then, the method was used to analyze the dynamical property of other dark energy models including more general quintessence models [48], phantom models [49] and quintom models [50], etc.. The $w - w'$ analysis undoubtedly provides us with an alternative way of classifying dark energy models using the quantities describing the dynamical property of dark energy. But, it is obviously that the $(w, w')$ pair is related to statefinder pair $(s, r)$ in a definite way, see Eqs. (17) and (18). The merit of the statefinder diagnosis method is that the statefinder parameters are constructed from the scale factor $a$ and its derivatives, and they are expected to be extracted in a model-independent way from observational data, although it seems hard to achieve this at present. While the advantage of the $w - w'$ analysis is that it is a direct dynamical diagnosis for dark energy. Hence, the statefinder $s - r$ geometrical diagnosis and the $w - w'$ dynamical diagnosis can be viewed as complementarity in some sense.

Now let us investigate the interacting model of holographic dark energy in the $w - w'$ plane. In Fig. 5, we plot the evolutionary trajectories of the holographic dark energy in the $w - w'$ plane where the selected curves correspond to $c = 0.8, 1.0$ and $1.2$, respectively. The left graph is an illustrative example without interaction to which we can compare the evolution of the interacting holographic dark energy in the right diagram. Fig. 5 shows clearly that the parameter $c$ and the interaction $b^2$ both play important roles in the evolution history of the universe. The left graph tells us: $c \geq 1$ makes the holographic dark energy behave as quintessence-type dark energy with $w \geq -1$ and $c < 1$ makes the holographic dark energy behave as quintom-type dark energy with $w$ crossing $-1$ during the evolution history. However, when the interaction between dark components is present, the situation becomes somewhat ambiguous because that the equation of state $w$ loses the ability of classifying dark energies definitely, due to the fact that the interaction makes dark energy and dark matter be entangled in each other. In this circumstance,
the conceptions such as quintessence, phantom and quintom are not so clear as usual. But, anyway, we can still use these conceptions in an undemanding sense. It should be noted that when we refer to these conceptions the only thing of interest is the equation of state $w$. The right panel of Fig. 5 tells us: with the interaction (a case of strong coupling, $b^2 = 0.10$), $c \leq 1$ makes the holographic dark energy behave as phantom-type dark energy with $w \leq -1$ and $c > 1$ makes the holographic dark energy behave as quintom-type dark energy with $w$ crossing $-1$ during the evolution history. In this diagram, the effect of the interaction is shown again. When the coupling between the two components is absent, the value of $w'$ first decreases from zero to a minimum then increases again to zero meanwhile the value of $w$ decreases monotonically. Nevertheless, for the case involving the interaction, $w$ increases first to a maximum and then decreases meanwhile $w'$ decreases from a maximum to a negative minimum first and then increases to zero again. Therefore, we see that the $w - w'$ dynamical diagnosis can provide us with a useful complement to the statefinder geometrical diagnosis.

In summary, we have studied the interacting holographic dark energy model from the statefinder viewpoint in this paper. Since the accelerated expansion of the universe was found by astronomical observations, many cosmological models involving dark energy component or modifying gravity have been proposed to interpret this cosmic acceleration. This leads to a problem of how to discriminate between these various contenders. The statefinder diagnosis provides a useful tool to break the possible degeneracy of different cosmological models by constructing the parameters $\{r, s\}$ using the higher derivative of the scale factor. Thus the method of plotting the evolutionary trajectories of dark energy models in the statefinder plane can be used to as a diagnostic tool to discriminate between different models. Furthermore, the values of $\{r, s\}$ of today, if can be extracted from precise observational data in a model-independent way, can be viewed as a discriminator for testing various cosmological models. On the other hand, though we are lacking an underlying theory of the dark energy, this theory is presumed to possess some features of a quantum gravity theory, which can be explored speculatively by taking the holographic principle of quantum gravity theory into account. So the holographic dark energy model provides us with an attempt to explore the essence of dark energy within a framework of fundamental theory. In addition, some physicists believe that the involving of interaction between dark energy and dark matter leads to some alleviation and more understanding to the coincidence problem. It is thus worthwhile to investigate the interacting model of holographic dark energy. We analyzed the interacting holographic dark energy model employing the statefinder parameters as a diagnostic tool. The statefinder diagrams show that the interaction between dark sectors can significantly affect the evolution of the universe and the contributions of the interaction can be diagnosed out explicitly in this method. At last, as the complement to the statefinder geometrical diagnosis, a dynamical diagnosis was also studied, which diagnoses the dynamical property of the interacting holographic dark energy in the $w - w'$ phase plane. We hope that the future high-precision observations can offer more and more accurate data to determine these parameters precisely and consequently shed light on the essence of dark energy.

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\footnote{It should be noted that the old version of holographic cosmology is not compatible with the phantom energy, see e.g., \textsuperscript{61} \textsuperscript{62}}
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