Modifications to Accelerate the Iterative Algorithm for the Single Diode Model of PV Model

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Abstract

This paper discussed the solution of an equivalent circuit of solar cell, where a single diode model is presented. The nonlinear equation of this model has suggested and analyzed an iterative algorithm, which work well for this equation with a suitable initial value for the iterative. The convergence of the proposed method is discussed. It is established that the algorithm has convergence of order six. The proposed algorithm is achieved with various values of load resistance. Equation by means of equivalent circuit of a solar cell so all the determinations is achieved using MATLAB in ambient temperature. The obtained results of this new method are given and the absolute errors are demonstrated.

Key words

Three step method, Newton Raphson method, convergence, iterations, iterations, absolute error.

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Introduction

Advanced mathematical applications are related to numerical analysis of partial differential equations and control problems. This activity was developed using MATLAB to study problems coming from the real world, from industrial and medical applications, and from complex systems [1-8]. The manufacture of these cells began in the fifties and the first solar cell was made of silicon and since that time and so far have made many modifications in how to manufacture these cells as well as expanding the base of materials that fit these cells. Numerical analysis is a branch of mathematics used in many fields, for example, authors in [1] studied the convergence of shifted Chebyshev wavelets while generalized Laguerre polynomials utilized in [2] for solving special optimal control problems. In [3-4], the solution of integral equation was considered. Various problem were solved with the aid of different orthogonal functions and wavelets [5-9]. One can solve the IV curve of solar cell numerically depends on how the equation of IV curve model looks like the one-diode or two-diode.
model, with or without series and/or shunt resistance. Usually, to obtain a single point of the IV curve one has to solve an implicit equation, which can be reformulated as a zero-finding problem. This means that you have to perform the numerical solution repeatedly, i.e., in a loop (as many times as the number of desired amount of data points). Other researchers used the numerical methods for solving nonlinear Kepler's and Barker's equations in celestial mechanics [10-13]. The parameters of the solar cells using different methods and numerical algorithms such as visual studio program, fuzzy logic method, fuzzy set technique [14-17]; mathematical modeling have been implemented and studied [18-19].

Researches are underway in order to reduce the cost of these cells, which are still high, and are currently looking for cheap and high-efficiency models such as silicon solar cells and inorganic and organic cells [20-24].

In this study, a new three-step method is presented in order to solve nonlinear equation of a solar cell. The purpose of the present work is to reduce the root of iteration in the calculations of the nonlinear equation root. It is systematic as the following steps: section 2 characterizing the analytical model of a single-diode design of the solar cell; section 3 establishing the root finding of Newton Raphson method; while in section 4 three step method has been described; section 5 results and discussion; section 6 conclusions of the acquired results.

Characteristics of single-diode solar cells equation

In order to understand the behavior of the solar cell, an equivalent electrical model had to be made. This model is based on known electrical components that are easy to study and analyze.

In electrical engineering and science, an equivalent circuit refers to a theoretical circuit that maintains all the electrical properties of a given circuit. The equivalent circuit is believed to simplify calculations, and on a larger scale, it is the simpler form of a more complex circuit to aid in its analysis. In its common form, the equivalent circuit is made up of linear and passive components, but more complex parabolic circuits approximate the nonlinear behavior of the original circuit as well. In order to understand the behavior of the solar cell, an equivalent electrical model had to be made. This model is based on known electrical components that are easy to study and analyze.

The simple equivalent electric circuit of a solar cell is illustrated in Fig. 1. The cell is appear as a source of electric current with a uniform diode while a resistance is put in parallel, the shunt resistance, and another resistance respectively, as a simulation of reality.

The ideal model of solar cells, the cell is represented by a source of electric current with a uniform diode, but practically there is no ideal solar cell, so a resistance is placed in parallel, the shunt resistance, and another resistance respectively, as a simulation of reality. The simple equivalent electric circuit of a solar cell shown in Fig. 1.

Fig. 1. Single-diode electrical equivalent circuit model of a solar cell.
By applying Kiehoff’s current law for the circuit, the equation of this equivalent circuit is given by the current from the solar cell = the current from the source \( I_L \) - the current passing through the diode combiner \( I_D \) - the current passing through the resistor in the parallel \( R_{SH} \). Voltage on both ends of the combiner diode = Voltage on terminals \( V \) + Current × Resistance (IR). Thus;

\[
I = I_{ph} - I_D \tag{1}
\]

\[
I_D = I_0 \left( \frac{V_{pv}}{nVT} - 1 \right) \tag{2}
\]

\[
I = I_{ph} - I_0 \left( \frac{V_{pv}}{nVT} - 1 \right) \tag{3}
\]

where \( I_{ph} \) is the photocurrent \( (A) \); \( I_0 \) is reverse saturation current of the diode \( (A) \); \( I \) and \( V_{pv} \) are the delivered current and voltage, respectively \( (V) \); \( V_T = \frac{kT}{q} = 0.0259 \text{ V} \) is thermic voltage = 27.5 \( \pm \) 26 mV at \( (T = 25 \text{ } ^\circ \text{C} \text{ Air-Mass} = 1.5) \); \( m \) is the recombination factor closeness to an ideal diode \( (1 < m < 2) \); \( k \) is Boltzmann constant \( = 1.38 \times 10^{-23}/K \); \( T \) is \( p - n \) junction temperature \( (K) \); \( q \) is the electron charge \( = 1.6 \times 10^{-19} \text{ C} \).

\[
I_{ph} = I_{source} \tag{4}
\]

\[
I_D = I_s \times \left( \frac{V_D}{nVT} - 1 \right) \tag{5}
\]

Merge Eq. (4) in Eq. (5) we get:

\[
(I_{source}) - 10^{-12} \left( e^{\frac{-V}{1.2 \times 0.026}} - 1 \right) = \frac{V}{R} \tag{6}
\]

where \( I_s \) reverse saturation current \( = 10^{-12} A \). In parallel, \( V_D = V_{pv} = V \).

According to Eq. (6) one can calculate \( V \) of the cell numerically based on the first derivative of this equation.

**Newton Raphson method**

The following NRM algorithm suggestion for solving Eq.(6).

**INPUT** initial approximate solution \( x_0 = 1 \), tolerance \( \varepsilon \), maximum number of iterations \( N \).

**OUTPUT** approximate solution \( x_{n+1} \)

**Step 1:** Set \( x = 0 \)

**Step 2:** while \( i \leq x_0 \)

**Step 3:** Calculate

\[
 x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0, 1, 2, \ldots
\]

**Step 4:** if \( |x_i - x_{i-1}| < \varepsilon \); then OUTPUT \( x_{n+1} \) and stop.

**Step 5:** Set \( n = n + 1; i = i + 1 \) and go to Step 2.

**Step 6:** OUTPUT

**Three Steps Method (TSM)**

To compare the different methods of iterations algorithms 1 and 2 against the proposed algorithm (algorithm 3); in addition; to demonstrate the performance of the new method, we solve Eq.(6). We shall determine the consistency and stability of results by examining the convergence of the proposed method. The results are tested using three iterative methods.

**Step 1:** Applying Newton Raphson Method (NRM): 

\[
D_{n+1} = D_n - \frac{f(D_n)}{f'(D_n)}
\]
Step 2: Using two-step method (TM): \( y_n = D_n - \frac{f(D_n)}{f'(D_n)} \)

\[ D_{n+1} = y_n - \frac{f(D_n) = 2 \times f(y_n)}{f(D_n) = 4 \times f(y_n)} \]

Step 3: Utilizing three-step method (TSM): given by the following formulas

\[ y_n = D_n - \frac{f(D_n)}{f'(D_n)} \]
\[ z_n = y_n - \frac{f(D_n) - 2 \times f(y_n)}{f(D_n) - 4 \times f(y_n)} \]
\[ D_{n+1} = z_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0, 1, 2, \ldots \]

We take \( \varepsilon = 10^{-9} \) as a tolerance.

The following criteria is used for estimating the zero

\[ \sigma = |D_{n+1} - D_n| < \varepsilon, |f(D_n)| < \varepsilon \]

For convergence criteria, it was required that \( \sigma \) the distance between two consecutive iterates was less than \( 10^{-9} \), \( n \) represents the number of iterations and \( f(D_n) \), the absolute value of the function. In addition, the computational order of convergence (COC) can be approximated using the expression.

\[ \text{COC} = \frac{\ln \left( \frac{D_{n+1} - D_n}{D_n - D_{n-1}} \right)}{\ln \left( \frac{D_n - D_{n-1}}{D_{n-1} - D_{n-2}} \right)} \]

Results and discussion

Consider the Eq. (6) is modeled in the form single-diode solar cell has obtained the following approximate solutions and both the NRM and TSM are applied with the first initial value \( x_0 \). In Table 1 the NRM and TSM of the solution results voltage \( V_{pv} \) of the solar cell. The absolute error and the order of the convergence are presented and listed in this table when the load resistance \( R = 1 \). The goal in the present work is to reach the optimal value of parameters of solar cell in minimum iterations. According to Table 1, the NRM gives the solution after nine iterations while TSM needed seven iterations to converge the optimal solution depending on the specific minimum stopping criteria. The computational order of convergence means the order of the corresponding sequence in each case are examined and listed in Table 1.

### Table 1: The obtained values using NRM and TSM.

| Iterations | \( V_{pv} \)-IM | \( V_{pv} \)-TSM | \( \sigma \)-NRM | \( \sigma \)-TSM | COC-NRM | COC-TSM |
|-----------|----------------|----------------|---------------|---------------|----------|----------|
| 1         | 0.942812862    | 0.979429776    | 0.028583139   | 0.020570224   | 2.596719229 | -141.1918796 |
| 2         | 0.922004414    | 0.958997978    | 0.024684255   | 0.020431798   | 2.456821567 | -1.231624109 |
| 3         | 0.912933605    | 0.905989832    | 0.0168669     | 0.05308146    | 2.239971168 | 4.935792787 |
| 4         | 0.91658633     | 0.922373301    | 0.006617812   | 0.016383469   | 2.059662439 | 1.528064426 |
| 5         | 0.921613448    | 0.922423127    | 0.000813893   | 4.98269E-05   | 2.003977562 | 2.029615837 |
| 6         | 0.922412182    | 0.922423135    | 1.08636E-05   | 7.10254E-09   | 1.92563053 | 1.92125053 |
| 7         | 0.922423133    | 0.922423135    | 0.011022E-16  | 1.9025E-09    | 1.92563053 | -1.029615837 |
| 8         | 0.922423135    | 0.0000000000   | 0.011022E-16  | 1.9025E-09    | 1.92563053 | -1.029615837 |

Fig. 2 presents the obtained solutions of the study result. The plots illustrate the difference between two adjacent values \( x_n - x_{n-1} \) for problem, Eq. (6), for number of iterations \( N=9 \) and 7 using NRM and TSM respectively. The best error obtained by
the proposed two methods NRM and TSM was improved and this figure shows that TRM is more rapid as compared with the results obtained in NRM.

Fig.2: Obtained solutions of the study result.

The curves of the best solution in the $\sigma$-plane confirm the suggested TSM method needs few iterations compared with the other method. It is also noted that the behavior of the solution using TSM technique together with the starting value $x_0 = 1$ produces a smallest error tolerance compared with NRM.

In Table 2 the NRM and TSM of the solution results voltage $V_{pv}$ of the solar cell. The absolute error and the order of the convergence are presented and listed in this table when the load resistance $R = 2$. From this table; the result of the iterative process, and the minimum stopping criterion $\varepsilon$, in which the objective function is the difference between two adjacent values which starts with Newton-Raphson method (NRM) and after the $9^{th}$ iteration the algorithm changes to three step method (TSM) that converges to the solution in $7^{th}$ iterations.

Table 2: The obtained values using NRM and TSM.

| Iterations | $V_{pv}$-IM | $V_{pv}$-TSM | $\sigma$-NRM | $\sigma$-TSM | COC-NRM | COC-TSM |
|------------|-------------|--------------|--------------|--------------|---------|---------|
| 1          | 1           | 0.978756325  | 0.028969528  | 0.021243675  | 2.598408663 | 44.05017045 |
| 2          | 0.971030472 | 0.956222512  | 0.025608505  | 0.022533813  | 2.480264742 | 44.05017045 |
| 3          | 0.945421967 | 0.653711503  | 0.01858749   | 0.302511009  | 2.279358069 | 53.16317033 |
| 4          | 0.926834474 | 0.9168757    | 0.008395731  | 0.02533813   | 2.083498269 | 1.062490583 |
| 5          | 0.918438746 | 0.917035321  | 0.001371861  | 0.000159621  | 2.007591262 | 1.929826514 |
| 6          | 0.917066885 | 0.917035382  | 3.14863E-05  | 6.09416E-08  | 2.00014089 |
| 7          | 0.917035399 | 0.917035382  | 1.61176E-08  | 4.21885E-15  |
| 8          | 0.917035328 | 4.21885E-15  |               |              |
| 9          | 0.917035382 |              |               |              |
Fig. 3 presents the obtained solutions of the study result. The plots illustrate the difference between two adjacent values $x_n - x_{n-1}$ for problem, Eq. (6), for number of iterations $N=9$ and 7 using NRM and TSM respectively. The best error obtained by the proposed two methods NRM and TSM was improved and this figure shows that TRM is more rapid as compared with the results obtained in NRM.

The curves of the best solution in the $\sigma$-plane confirm the suggested TSM method needs few iterations compared with the other method. It is also noted that the behavior of the solution using TSM technique together with the starting value $x_0 = 1$ produces a smallest error tolerance compared with NRM.

In Table 3 the NRM and TSM of the solution results voltage $V_{pp}$ of the solar cell. The absolute error and the order of the convergence are presented and listed in this table when the load resistance $R = 3$.

**Table 3: The obtained values using NRM and TSM.**

| Iterations | $V_{pp}$-IM | $V_{pp}$-TSM | $\sigma$-NRM | $\sigma$-TSM | COC-NRM       | COC-TSM       |
|------------|--------------|--------------|--------------|--------------|---------------|---------------|
| 1          | 1            | 0.978098497  | 0.029356208  | 0.021901503  | 2.591572295   | -18.88739431  |
| 2          | 0.970643792  | 0.953715087  | 0.02655956   | 0.02438341   | 2.49817873    | -1.324885788  |
| 3          | 0.944084232  | 0.95692545   | 0.020489989  | 0.003210363  | 2.323673159   | -1.628471945  |
| 4          | 0.923594243  | 0.909809334  | 0.010716403  | 0.047116116  | 2.118829629   | 1.572474582   |
| 5          | 0.91287784   | 0.910402763  | 0.002376578  | 0.000593429  | 2.015254521   | 1.878816564   |
| 6          | 0.910501262  | 0.910403374  | 9.77309E-05  | 6.10916E-07  | 2.000319404   |               |
| 7          | 0.910403531  | 0.910403374  | 1.57416E-07  |               |               |               |
| 8          | 0.910403374  | 4.07563E-13  |               |               |               |               |
| 9          | 0.910403374  | 0.910403374  |               |               |               |               |
From this Table; the result of the iterative process, and the minimum stopping criterion ε, in which the objective function is the difference between two adjacent values which starts with Newton-Raphson method (NRM) and after the 9th iteration the algorithm changes to three step method (TSM) that converges to the solution in 7th iterations.

Fig.4 presents the obtained solutions of the study result. The plots illustrate the difference between two adjacent values $x_n - x_{n-1}$ for problem, Eq.(6), for number of iterations $N=9$ and 7 using NRM and TSM respectively. The best error obtained by the proposed two methods NRM and TSM was improved and this figure shows that TRM is more rapid as compared with the results obtained in NRM.

Fig.4: Obtained solutions of the study result.

The curves of the best solution in the $\sigma$-plane confirm the suggested TSM method needs few iterations compared with the other method. It is also noted that the behavior of the solution using TSM technique together with the starting value $x_0 = 1$ produces a smallest error tolerance compared with NRM.

In Table 4 the NRM and TSM of the solution results voltage $V_{pv}$ of the solar cell. The absolute error and the order of the convergence are presented and listed in this table when the load resistance $R = 4$. From this table; the result of the iterative process, and the minimum stopping criterion $\varepsilon$, in which the objective function is the difference between two adjacent values which starts with Newton-Raphson method (NRM) and after the 9th iteration the algorithm changes to three step method (TSM) that converges to the solution in 7th iterations.
Table 4: The obtained values using NRM and TSM.

| Iterations | $V_{pp}$-IM | $V_{pp}$-TSM | $\sigma$-NRM | $\sigma$-TSM | COC-NRM | COC-TSM |
|------------|-------------|-------------|--------------|--------------|---------|---------|
| 1          | 1           | 0.97745446  | 0.029743178  | 0.02254554  | 2.567991249 | -3.871764615 |
| 2          | 0.970256822 | 0.951381278 | 0.027538101  | 0.026073182 | 2.501358034 | -1.662284557 |
| 3          | 0.94271872  | 0.936530256 | 0.022595711  | 0.014851022 | 2.367921894 | -2.690538547 |
| 4          | 0.920123009 | 0.898678908 | 0.013776515  | 0.037851349 | 2.171620277 | 2.369587074 |
| 5          | 0.906346494 | 0.901732761 | 0.004268788  | 0.03053854  | 2.032918698 | 1.755437978 |
| 6          | 0.902077706 | 0.901740602 | 0.000335204  | 7.84038E-06 | 2.001372388 |
| 7          | 0.901742503 | 0.901740602 | 1.90082E-06  | -          |         |
| 8          | 0.901740602 | 6.06911E-11 |             | -          |         |
| 9          | 0.901740602 | 0.901740602 |             | -          |         |

Fig.5 presents the obtained solutions of the study result. The plots illustrate the difference between two adjacent values $x_n - x_{n-1}$ for problem, Eq.(6), for number of iterations $N=9$ and 7 using NRM and TSM respectively. The best error obtained by the proposed two methods NRM and TSM was improved and this figure shows that TRM is more rapid as compared with the results obtained in NRM.

The curves of the best solution in the $\sigma$-plane confirm the suggested TSM method needs few iterations compared with the other method. It is also noted that the behavior of the solution using TSM technique together with the starting value $x_0 = 1$ produces a smallest error tolerance compared with NRM.

In Table 5 the NRM and TSM of the solution results voltage $V_{pp}$ of the solar cell. The absolute error and the order of the convergence are presented and listed in this
table when the load resistance \( R = 5 \). From this table, the result of the iterative process, and the minimum stopping criterion \( \varepsilon \), in which the objective function is the difference between two adjacent values which starts with Newton-Raphson method (NRM) and after the 9\( ^{th} \) iteration the algorithm changes to three step method (TSM) that converges to the solution in 7\( ^{th} \) iterations.

### Table 5: The obtained values using NRM and TSM.

| Iterations | \( V_{pp-IM} \) | \( V_{pp-TSM} \) | \( \sigma \)-NRM | \( \sigma \)-TSM | COC-NRM | COC-TSM |
|------------|-----------------|-----------------|-----------------|----------------|---------|---------|
| 1          | 0.976822639     | 0.949161044     | 0.023177361     | 2.505513836   | -1.343899756 |
| 2          | 0.96986956      | 0.927351432     | 0.024928888     | 2.386636648   | -0.074404984 |
| 3          | 0.916395843     | 0.690900718     | 0.017860198     | 2.243841533   | 39.98697013  |
| 4          | 0.898535645     | 0.888927794     | 0.008058636     | 2.078376466   | 1.091896281  |
| 5          | 0.890477009     | 0.889092643     | 0.001351246     | 2.007526042   |
| 6          | 0.889125763     | 0.889092715     | 3.30291E-05     | 1.91907E-08   |
| 7          | 0.89092734      | 0.889092715     | 0.000164849     | 0.889092715   |
| 8          | 0.889092715     | 0.889092715     | 1.91907E-08     | 0.889092715   |

Fig.6 presents the obtained solutions of the study result. The plots illustrate the difference between two adjacent values \( x_n - x_{n-1} \) for problem, Eq.(6), for number of iterations \( N=9 \) and 7 using NRM and TSM respectively.
The best error obtained by the proposed two methods NRM and TSM was improved and this figure shows that TRM is more rapid as compared with the results obtained in NRM.

The curves of the best solution in the $\sigma$-plane confirm the suggested TSM method needs few iterations compared with the other method. It is also noted that the behavior of the solution using TSM technique together with the starting value $x_0 = 1$ produces a smallest error tolerance compared with NRM.

The presented method produces minimum error after few iterations and this in turn is illustrating its efficiency as shown in Tables 1 to 5.

Conclusions

In this paper, we observed that the efficiency of the new three step iterative method considerably improve that of Newton method and the given two step method. Remark that only 8 iterations are needed to reach the exact solution with small tolerance, while Newton's method requires 9 iterations. Data acquired from the proposed method TSM were found to be sufficient and values for single diode solar cell were determined with fast convergence, more capable to determine these parameters and establishing towards the final values. The obtained results proved that the TSM technique is more suitable than NRM because the mathematical method with the least number of iterations is better and faster when TSM in utilized.

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