Vibration control of nonlinear vibration of suspended cables based on quadratic delayed resonator

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\textbf{Abstract.} Based on Galerkin method and multiple scales method, the first-order symmetric mode responses the suspension cable under longitudinal time-delayed velocity feedback control are obtained. The influence of system parameters, such as time delay, control gain and excitation amplitude on the controlled system, is investigated by the numerical examples. According to the feedback gain increasing, the amplitude of the system response is significantly suppressed. And the efficiency of vibration control can be improved by selecting control gain and the time-delay value.

1. Introduction

As an engineering component, suspended cables are widely used in bridge engineering, structural engineering, and electric power engineering. Due to the characteristics of the suspension cable, such as low damping, light mass, and large flexibility, it is easy to cause degradation due to large vibration under the action of external load\textsuperscript{[1-3]}, and its vibration control has attracted extensive attention from scholars. With the diversity of the nonlinear phenomena of the suspension cable, the mathematical model is more complicated. It making the control problem more challenging. Many scholars have studied the linear\textsuperscript{[4]} and nonlinear\textsuperscript{[5]} dynamic behavior of the cable and obtained some results. Susumpow et al.\textsuperscript{[6]} studied the active control of small sag cables based on bilinear control theory and axial support motion. Fujino et al.\textsuperscript{[7]} used active stiffness control to control cable vibration and utilize an active boundary control system. The method of suppressing the vibration of the cable is proposed\textsuperscript{[8, 9]}. In addition, in order to reduce lateral vibration, the passive damper is a commonly used device\textsuperscript{[10]}. Gattulli et al.\textsuperscript{[11]} analyzed the nonlinear vibration of a controlled suspension under in-plane excitation using the longitudinal displacement of movable support.

In the process of vibration control, the existence of time delay is inevitable, and the influence on the control effect is also cannot be negligible\textsuperscript{[12,13]}. With the increasing attention of scholars on time-delay dynamics, time delay vibration absorption technology has new development\textsuperscript{[14-16]}. The development of the time delay absorption technique provides an idea of the vibration control of suspension cables. This paper studies the dynamic behavior of suspension cables under time-delay feedback control.
2. Mathematical model and perturbation solution

As shown in figure 1, nonlinear in-plane equation of motion of the horizontal suspension cable can be obtained by using Hamilton variational principle as follows\textsuperscript{[1,11]}.

\[
m\ddot{v} + 2c\dot{v} - Hv^\prime - \frac{EA}{L} (y^\prime + v^\prime) \times \left[ u_c + \int_0^L (y'' + 1/2v^2) dx \right] = F(x,t). \tag{1}\]

where \(v\) is the in-plane displacement, \(u_c\) is the longitudinal displacement of one support (\(x=L\)), \(m\) is the mass per unit length, \(c\) is the damping coefficient of the cable, \(E\) is the elastic modulus, \(A\) is the cross-sectional area, \(H\) is the horizontal tension, \((H = mgL^2/8b, H \leq EA)\). \(L\) is the span of the cable, \(b\) is the sag, \(F(x,t) = P(x)\cos(\Omega t)\) is the external excitation, \(\Omega\) is the frequency of external excitation, and \(P(x)\) is the distribution function of external excitation. This paper assumes that the ratio of suspension span is small \((f = b/L \leq 1/8)\), therefore its shape can be described as a parabola as \(y(x) = 4b[x/L - (x/L)^2]\).

Introducing the dimensionless parameter as following

\[
x^* = x/L, y^* = y/L, v^* = v/L, \alpha = EA/H, P^* = PL/H,
\]

\[
t^* = t\sqrt{g/8b}, \Omega^* = \Omega\sqrt{8b/g}, c^* = (c/m)\sqrt{8b/g}, u_c^* = u_c/L. \tag{2}\]

The asterisk are dropped for convenience, then the non-dimensional equations can be obtained

\[
\ddot{v} + 2c\dot{v}^\prime - \alpha (y^\prime + v^\prime) \left[ u_c + \int_0^1 (y'' + 1/2v^2) dx \right] = P(x)\cos(\Omega t) \tag{3}\]

The control strategy \(u_c\) is a function of the in-plane transverse velocity and displacement of a monitored point (\(x_c = L/2\)). Moreover, the quadratic feedback control strategy with time-delayed is adopted

\[
u_c = g_q v(x_c, \tau) \ddot{v}(x_c, t-\tau) \tag{4}\]

We express the dimensionless displacement \(v(x,t)\) as

\[
v(x,t) = \sum_{n=1}^{\infty} q_n(t)\phi_n(x) \tag{5}\]

where \(q_n(t)\) is the generalized coordinate function, \(\phi_n(x)\) is the \(n\)th order mode shape function of the suspension cable. Substituting equation (5) into equation (3) to obtain

\[
\ddot{q}_n(t) + \omega_n^2 q_n(t) + 2\mu_n \dot{q}_n(t) + \sum_{i,j=1}^{\infty} \Gamma_{ij} q_i(t)q_j(t) + \sum_{i,j=1}^{\infty} \Lambda_{ij} q_i(t)\dot{q}_j(t) + \sum_{i=1}^{\infty} g_{i,j} \dot{q}_i(t)q_j(t) = f_c \cos(\Omega t) \tag{6}\]

where
\[ \Gamma_{m} = -\alpha \left[ \phi''(x) \int_{\phi'(x) y}^{\phi'(x) y'} \phi(x) dx - \frac{1}{2} \alpha \int_{\phi'(x) y}^{\phi'(x) y'} \phi(x) \phi'(x) dx \right] \phi(x) dx , \]

\[ \Lambda_{m} = -\alpha \left[ \phi''(x) \int_{\phi'(x) y}^{\phi'(x) y'} \phi(x) \phi'(x) dx \right] \phi(x) dx , \]

\[ f_n = \int_{0}^{1} P(x) \phi'(x) dx, \quad \mu_n = \int_{0}^{1} \phi(x) dx. \]

\[ c_i = -\alpha \phi''(x) \int_{0}^{1} \phi(x) \phi'(x) dx, \]

We consider the symmetric mode

\[ \phi_n(x) = \xi_n \left[ 1 - \cos(\omega_n x) - \tan(\frac{\omega_n}{2}) \sin(\omega_n x) \right] \]

where \( \xi_n \) can be obtained according to the modal normalization, frequency \( \omega_n \) can be obtained from the following equation:

\[ \tan(\frac{\omega_n}{2}) = \frac{\omega_n}{2 - \lambda_i^2} \quad (n = 1,3,5...) \]

According to the multi-scale method, import a small amount \( \varepsilon \) (\( \varepsilon \ll 1 \)), adjust the coefficients of equation (6):

\[ \mu_n = \varepsilon^2 \mu_n, \quad \Gamma_{mn} = \varepsilon^2 \Gamma_{mn}, \quad \Lambda_{mn} = \varepsilon^2 \Lambda_{mn}, \quad f_n = \varepsilon^2 f_n, \quad \Omega \approx \omega_n + \varepsilon^3 \sigma. \]

\( \sigma \) is the tuning parameter. Let the solution of equation (6) can be represented by an expansion in the form

\[ q_n(x,\varepsilon) = q_{0n}(T_0,T_1,T_2) + \varepsilon q_{1n}(T_0,T_1,T_2) + \varepsilon^2 q_{2n}(T_0,T_1,T_2) + O(\varepsilon^3) \]

\( T_i = \varepsilon^i \tau (m = 0,1,2) \). Substituting equation (9) into equation (6), and equating the coefficients of \( \varepsilon^0 \), \( \varepsilon^1 \) and \( \varepsilon^2 \) to zero, we obtain

\[ D_0^2 q_{0n} + \omega_n^2 q_{0n} = 0 \]

\[ D_1^2 q_{1n} + \omega_n^2 q_{1n} = -2D_0 D_2 q_{0n} - \Gamma_{mn} q_{1n} \]

\[ D_0^2 q_{2n} + \omega_n^2 q_{2n} = -2D_0 D_2 q_{0n} - D_0^2 q_{1n} - 2 \mu_n D_0 q_{1n} - 2 \Gamma_{mn} q_{1n} q_{0n} \]

To eliminate duration terms, we find that

\[ -(2i \omega_n A_n + 2i \mu_n \omega_n A_n) + \frac{10 \Gamma_{mn}}{3 \omega_n^3} A_n^3 \overline{A_n} = -3 \Lambda_{mn} A_n^2 \overline{A_n} - i \omega_n g_n c_n A_n^2 \overline{A_n} \exp(i \omega_n \tau) \]

\[ \frac{-2i \omega_n g_n c_n A_n^3 \overline{A_n} \exp(-i \omega_n \tau) + f_n}{2} \exp(i \sigma T_2) = 0 \]

We write \( A_n \) in the polar form \( A_n = A_n \exp(i \beta_n) \), where \( A_n \) and \( \beta_n \) are real. Then the amplitude-frequency response equation is obtained as follows

\[ \frac{F_n^2}{4 \omega_n^3} = \left[ \mu_n + \frac{3 \varepsilon c_n A_n^2 \cos(\omega_n \tau)}{8} \right] a_n^2 + \left[ \sigma - \frac{g_n c_n A_n^2 \sin(\omega_n \tau)}{8} + \frac{10 \Gamma_{mn}^2 - 9 \Lambda_{mn}^2 A_n^2}{24 \omega_n^3} \right] a_n^2 \]

3. Numerical analysis and discussion

In this section, a numerical example is provided to illustrate the primary resonance response of the first-order mode of the controlled cable. The geometrical dimensions and material properties of the cables are as follows: cross-sectional area \( A = 0.1257 \text{mm}^2 \), span \( L = 600.5 \text{mm} \), line weight \( m = 4.8655 \times 10^{-6} \text{kg/m} \), elastic modulus \( E = 1340.83 \text{MPa} \), the excitation amplitude \( f_t = 0.003 \), and the damping coefficient \( \mu_t = 0.005 \).
Figure 2. The amplitude-frequency curves of the first mode primary resonance response of the suspend cable with control feedback gain

Figure 2 shows the amplitude-frequency curves of the first mode primary resonance response of the suspended cable with control feedback gain, and the $\tau = 0.45\pi$. When $q_g = 0$, namely in the uncontrolled state, the amplitude is large, when the delay feedback control is adopted ($g_q \neq 0$), as the control gain value $q_g$ increases, the response amplitude decreases rapidly. And the amplitude-frequency curve gradually changes from hardening to softening.

Figure 3. The amplitude-frequency curves of the first mode primary resonance response of the suspend cable with time delay

Figure 3 is the amplitude-frequency curves of the first mode primary resonance response of the suspend cable with a time delay when the control gain $g_q = 0.01$. When $\tau \in (0, \pi/(2\omega_1))$, with the increase of time delay $\tau$, the primary resonance amplitude gradually increases.
Figure 4. The response-excitation amplitude curves of the primary resonance with time delay and a detuning parameter

Figure 4 shows the response amplitude-excitation amplitude curves of the suspended cable under different time-delay values. These curves are directly obtained by equation (14). When the tuning parameters change, the curve has a single value and multi-value, and it is not difficult to find that the unstable value appears on the multi-value curve. As the value of the tuning parameter increases, the excitation frequency increases, and a single to multi-value conversion occurs. At the same time, it can be found that as the time delay increases, the system response amplitude increases correspondingly, and the curve curvature increases.

Figure 5. The time history curves of the primary resonance with control feedback gain

Figure 5 shows the response of the time history curve with different control gains when $\tau = 0.45 \pi, \sigma = 0.05$. As the control gain increases, the response amplitude is significantly suppressed. The amplitude of the control gain $g_q = 0.5$ is reduced by 50% compared to the uncontrolled time ($g_q = 0$), and the curves exhibit periodic motion.

4. Conclusion

In this paper, the mechanism of quadratic delay feedback control is used to study the mechanical analysis of the suspended cable system under the parameter excitation. The nonlinear control equation of the control system is established, and the multi-scale method is used to solve the vibration control equation. The amplitude-frequency curve and the time-history curve under different delay and different gain are obtained. The results obtained by analysis and comparison are as follows: When the control gain value is taken, within a certain period of time, the increase of the time delay value will cause the response of the system to become significantly larger, and may even be unstable. When
designing an axial time-delay feedback control system, the control gain, and the time-delay value are adjusted to achieve the best control effect.

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