Energy Management Strategies for Hybrid Electric Vehicles: Review, Classification, Comparison, and Outlook

Fengqi Zhang 1,*, Lihua Wang 1, Serdar Coskun 2, Hui Pang 1, Yahui Cui 1 and Junqiang Xi 3

1 School of Mechanical and Precision Instrument Engineering, Xi’an University of Technology, Xi’an 710048, China; cyhxut@xaut.edu.cn (Y.C.)
2 Department of Mechanical Engineering, Tarsus University, Tarsus, Mersin 33400, Turkey; serdarcoskun@tarsus.edu.tr
3 School of Mechanical Engineering, Beijing Institute of Technology, Beijing 100081, China; xijunqiang@bit.edu.cn
* Correspondence: zfqdy@126.com

Received: 29 April 2020; Accepted: 28 June 2020; Published: 30 June 2020

Abstract: Hybrid Electric Vehicles (HEVs) have been proven to be a promising solution to environmental pollution and fuel savings. The benefit of the solution is generally realized as the amount of fuel consumption saved, which by itself represents a challenge to develop the right energy management strategies (EMSs) for HEVs. Moreover, meeting the design requirements are essential for optimal power distribution at the price of conflicting objectives. To this end, a significant number of EMSs have been proposed in the literature, which require a categorization method to better classify the design and control contributions, with an emphasis on fuel economy, providing power demand, and real-time applicability. The presented review targets two main headlines: (a) offline EMSs wherein global optimization-based EMSs and rule-based EMSs are presented; and (b) online EMSs, under which instantaneous optimization-based EMSs, predictive EMSs, and learning-based EMSs are put forward. Numerous methods are introduced, given the main focus on the presented scheme, and the basic principle of each approach is elaborated and compared along with its advantages and disadvantages in all aspects. In this sequel, a comprehensive literature review is provided. Finally, research gaps requiring more attention are identified and future important trends are discussed from different perspectives. The main contributions of this work are twofold. Firstly, state-of-the-art methods are introduced under a unified framework for the first time, with an extensive overview of existing EMSs for HEVs. Secondly, this paper aims to guide researchers and scholars to better choose the right EMS method to fill in the gaps for the development of future-generation HEVs.

Keywords: Hybrid Electric Vehicles (HEVs); energy management strategies (EMSs); driving cycle prediction; optimization

1. Introduction

Hybrid Electric Vehicles (HEVs) are composed of different types of energy sources and power converters, which generally refer to vehicles consisting of an internal combustion engine (ICE) with an electric motor. HEVs seem to be the most economically viable solution so far and probably for the upcoming decades. The general goal to develop HEVs is to reduce fuel consumption and emissions while ensuring drivers’ power demands by investigating the appropriate energy management strategies (EMSs). Energy management aims to obtain an optimal power split in view of complex
driving conditions, as well as to minimize fuel consumption and emissions. It is commonly acknowledged that improvements in the fuel economy of HEVs, and thus the consequent reduction in emissions, depend crucially on their energy management strategies (EMSs) [1]. The complex configuration and behavior of multi-source hybrid energy systems introduce challenges to the performance of EMSs. Regardless of the topology of the powertrain, the EMS aim is to instantaneously manage the power flows from the energy converters to achieve the control objectives [2]. The optimal control algorithms employed under a given driving cycle are therefore the representative research outline in the field of energy management strategies.

Various EMSs for HEVs have been conducted in recent years. In the existing literature reviews, a number of classifications for the energy management strategy are reported [3–7]. Generally, EMSs can be divided into three categories: rule-based EMSs, local optimization-based EMSs, and global optimization-based EMSs [3]. An overview of EMSs for plug-in Hybrid Electric Vehicles is presented in [4]. The classification of energy management, such as rule-based control strategies and optimization-based control strategies, are introduced according to their mathematical models and the approach commonly used. In [5], EMSs are divided into two categories as rule-based and optimization-based methods for parallel Hybrid Electric Vehicles, and the pros and cons of each approach are compared. Finally, some real-time implementation issues are discussed from different aspects (e.g., computational burden and optimality). The different classifications for hybrid vehicles focusing on hydraulic drives is introduced and discussed in [6]. Different kinds of approaches like offline and online strategies are classified and compared. As intelligent transportation system (ITS) technology has emerged and machine learning methods have been widely used, some new EMSs have been developed to improve the performance requirements (e.g., adaptability and real-time implementation). However, there is still a need for a comprehensive review of the EMSs to better elucidate the state-of-the-art approaches and potential future research directions. To this end, the present review, different from the aforementioned review papers in EMSs, proposes a comprehensive hierarchical classification scheme for the first time. In the first category, offline EMSs are presented based on the level of driving information under global optimization-based EMSs and rule-based EMSs. In the second category, online EMSs are layered as instantaneous optimization-based EMSs, predictive EMSs, and learning-based EMSs. Since the presented scheme covers various approaches in terms of targeted solution objectives, optimality, and real-time implementation, an important number of literature studies are extensively overviewed. The principle of each approach along with its pros and cons are illustrated and compared within the design and operational characterization of the proposed scheme. Finally, a good number of emerging innovative EMSs and recent literature that have not been covered in previous review papers are summarized and important future trends for HEVs are highlighted. This study is intended to serve as a comprehensive reference for researchers in the field of development and optimization of EMSs.

The remainder of the paper is organized as follows. Different powertrain topologies of Hybrid Electric Vehicles are briefly discussed and compared in Section II. In Section III, a hierarchical classification scheme of EMSs is presented. In the following Sections IV and V, offline and online EMSs categories are stated in more detail. Each approach is elaborated and compared according to its principles, as well as pros and cons. Some important future trends of EMSs are discussed in Section VI.

2. The Powertrain Topologies of Hybrid Electric Vehicles

It is well known that there are mainly three kinds of topologies for Hybrid Electric Vehicles: series, parallel, and power-split. A series hybrid powertrain is regarded as a simple extension of a battery-powered electric vehicle that is propelled only by motor. The engine drives a generator, producing electrical power, which can be summed to the electrical power coming from the energy storage system and then transmitted, via an electric bus, to the electric motor(s) driving the wheels [8]. In principle, the advantage of the series hybrid powertrain is that only electrical connections between the main power conversion devices are required. Thus, vehicle packaging and design are simplified. Meanwhile, the engine that is completely off the wheels offers great freedom in selecting
speed and load, thus allowing the engine to operate at a high-efficiency region. On the other hand, the series hybrid powertrain requires two energy conversions (i.e., from mechanical to electrical in the generator, and from electrical to mechanical in the motor), which result in a loss of efficiency, even when there is a direct mechanical connection between the engine and the wheels in the existing configuration. As a result, in some cases, a series hybrid electric vehicle consumes more fuel than a traditional vehicle, especially in highway driving. Furthermore, one of the two electromechanical energy converters must be sized to meet the maximum power demand of the vehicle, as it is the primary source of propulsion [2,8]. The series topology is shown in Figure 1.

![Figure 1. Series hybrid electric vehicle.](image)

As for the parallel topology, the engine is connected to the powertrain by a mechanical coupling device while the motor propels the vehicle. Either engine or motor could propel the vehicle according to different load conditions, which makes it possible to greatly increase the fuel economy. The motor provides the power when the vehicle operates at lower speed to reduce fuel consumption. Thus, this configuration is capable of maintaining a higher efficiency and better fuel economy. The power summation is mechanical rather than electrical, and the engine and the electric machines (one or more) are connected with a gear set, a chain, or a belt; thus, their torques are summed and transmitted to the wheels [8]. In this configuration, there is no need to size one of the two electromechanical energy converters to meet the maximum power demand for parallel hybrid powertrain; however, unless it is significant oversize, the electric motors have less power than those used in a series hybrid powertrain (since not all the mechanical power goes through them), thus reducing the possibility of regenerative braking. Meanwhile, the engine operating conditions cannot be regulated as freely as in a series hybrid powertrain, since the engine speed is mechanically related (via the transmission system) to the vehicular velocity [2,8]. The parallel topology is illustrated in Figure 2.

![Figure 2. Parallel hybrid electric vehicle.](image)

As for the power-split topology, the most important improvement is the ability to operate as either a series or parallel topology, which provides more operation modes to substantially improve the overall efficiency under complex driving conditions. Although the series path is generally avoided because it is less efficient, the main feature of this design is that the engine, generator, and motor speed are decoupled, allowing additional freedom in control. The engine and two electric machines are connected to a power split device (usually a planetary gear set), so that the power from
the engine and the electric machines can be merged through both a mechanical and an electrical path, allowing series and parallel operations [8]. Compared to the parallel hybrid powertrain, the power-split architecture is the most flexible and represents a higher control ability on the engine operating conditions while adopting the double energy conversion, which is typical of a series operation only in a small portion of the total power demand, thus decreasing overall losses [2,8]. The power-split topology is presented in Figure 3.

3. The Classification of EMSs

In this paper, we propose a new hierarchical classification scheme of EMSs for all kinds of Hybrid Electric Vehicles via two main headlines: (1) offline EMSs are categorized according to the information level of the driving conditions utilized, including global optimization based-EMSs and rule-based EMSs; and (2) online EMSs are represented as instantaneous optimization-based EMSs, predictive EMSs, and learning-based EMSs. The classification of the EMSs is illustrated in Figure 4. It is noted that a flexible EMS can include a mixture of various techniques (offline and online) to form an integrated EMS for improving the fuel economy and performance. Thus, in this paper, these combinations with other techniques may be included while providing a particular EMS classification. For offline EMSs, two categorizations are illustrated: the global optimization-based and rule-based EMSs. The main goal of global optimization-based EMSs is to achieve a global optimal power split under a given driving cycle and provide modified online EMSs. They are not directly applicable in real-time control due to their computational complexity and the requirement of a priori knowledge of the entire driving cycle. However, it can be used as a benchmark to adjust the control parameters. Typical methods, such as dynamic programming, can implement global optimization over given driving cycles, but it cannot be directly employed in a real vehicle. Therefore, this method can be used to evaluate the performance of other optimization methods to extract the control rules. Rule-based EMSs are considered as an offline method since the rules are derived from pre-production tests. Rule-based EMSs are based on pre-defining a series of control rules to determine the power split while it cannot achieve optimal allocation of power as compared to offline globally optimized energy management. Online EMSs, however, are based on local optimization and causal with the potential of being applied in real-time control. Among these strategies, instantaneous optimization EMSs can minimize the instantaneous fuel consumption at each instant without a priori knowledge of the entire driving cycle and only obtain local optimal results. The instantaneous optimization-based EMSs are 1) the equivalent consumption minimization strategy (ECMS); 2) adaptive-ECMS (A-ECMS); and 3) robust control (RC). As a fundamental method, ECMS can be used for real-time implementation due to its adjustability, which is related to the equivalent factor (EF). It is realized that the performance of ECMS is closely tied to the equivalent factor. The next question on how to select an appropriate equivalent factor remains a key issue for ECMS. Therefore, different methods are proposed to adjust the equivalent factor online and split the power on the basis of ECMS, for example A-ECMS. Next, the discussion continues for the predictive EMSs, whereby the main idea is to optimize the power split based on the predicted velocity over a certain horizon. The future power
demand over the horizon is calculated via the traffic information received through ITS and GPS. As the intelligent transportation system technologies are increasingly utilized in traffic management systems, useful information of the preceding vehicle through communication channels among the vehicles lead to an implementation of predictive control that distributes the power by maximizing the fuel economy over a certain time window. Thus, the driving cycle prediction is significant for predictive EMSs. As a common solution method, model predictive control (MPC), which depends on the accuracy of a vehicle model for prediction, can be implementable in predictive control for HEVs. Learning-based EMSs mainly update the control parameters by training data to improve the adaptability to the changing driving conditions.

![Diagram of EMS categories]

**Figure 4.** The proposed categorization of the energy management strategies.

4. Offline EMSs

*Global Optimization-Based EMSs*
These types of methods are non-causal and seek global optimal solutions since they need a prior knowledge of the typical driving cycle. Because of the non-causal solution, they cannot directly be employed in real-time problems; however, non-causal optimal solutions can be obtained offline under a given driving cycle, which can provide a benchmark for other algorithms or modified online EMSs. Thus, as a benchmark, these methods can be adopted to obtain globally optimal results under a specific driving cycle. The commonly methods, such as dynamic programming (DP), stochastic dynamic programming (SDP), genetic algorithm (GA), game theory (GT), robust control (RC), pseudospectral method, and convex optimization, are illustrated and compared in this section. To clearly illustrate the pros and cons of each approach, a comparison of different approaches is shown in Table 1. The “computational complexity” requires low computational burden to score well since this is desirable for fast operation and efficiency. The “adaptability” refers to the flexibility of the EMSs adapted in different driving cycles. It scores well when the control parameters are easy to adjust to different driving cycles for fuel economy. The SDP can provide the best adaptability in comparison with other methods. The “prior knowledge of driving cycle” denotes the amount of driving future information required for calibration and formulation. For these methods, the DP requires the most a priori knowledge of the future information of the driving cycle and obtains the best fuel economy.

Table 1. Comparison of different approaches.

| Approaches    | Main advantages                                      | Main disadvantages                                      | Literature |
|---------------|------------------------------------------------------|--------------------------------------------------------|------------|
| DP            | ● achieves global optimal results                    | ● less adaptability to changeable driving cycles        | [9–22]     |
|               | ● benchmark for other EMSs                           | ● highest computational complexity (3-level)           |            |
| SDP           | ● more adaptability                                  | ● highest computational complexity (3-level)           | [23–28]    |
|               | ● achieves near-optimal fuel economy                 | ● requires driving cycle database                      |            |
| GA            | ● global optimality                                  | ● higher computational complexity (2-level)           | [29–33]    |
|               | ● good global search performance                     | ● less adaptability                                    |            |
| GT            | ● trade off among conflicting objectives             | ● highest computational complexity (3-level)           | [34–45]    |
|               | ● consider driver behaviors in EMSs                  | ● poor adaptability                                    |            |
| Pseudospectral | ● global optimality                                  | ● higher computational complexity (2-level)           | [46–49]    |
| method        | ● more accurate numerical computation                | ● requires analytic expressions for vehicle models     |            |
| Convex        | ● fast computation                                  | ● requires convex models                               | [50–54]    |
| optimization  | ● easy to implement                                 | ● limited applications                                 |            |
| PMP           | ● achieve near-optimal results                       | ● complex mathematical models                          | [55–68]    |
|               | ● lower computational burden                         | ● require co-state estimation                          |            |

Note: The computational complexity of other algorithms refers to the computation time compared to dynamic programming (DP). The smaller of the level represents less computation burden compared to DP.

4.1.1. Dynamic Programming (DP)

Dynamic programming, as an offline optimization approach, can realize a global optimal solution for a given driving cycle; however, it cannot directly be used in a real vehicle EMS because it is impossible to know the future driving conditions (speed, road slope as well as traffic dynamics). DP also suffers from considerable computing time for solving the optimal problem of the backward duration of the trip from the future state to find the initial control input in a feasible region. Especially, the computation burden increases as the dimension of the system states raise. However, as a benchmark, it can be used to determine the operating conditions that yield a globally optimal fuel...
consumption, which is then further used to evaluate the performance of other energy management algorithms and extract some heuristic rules. Moreover, it can be employed to obtain an optimal solution over a prediction horizon for model predictive control, such as in [9].

The basic principle of DP is illustrated as shown in Figure 5. The optimal process is formulated as to find the best cost function from A to F. Firstly, the feasible region is discretized and cast into the grid to calculate all possible paths from A to F. Then, starting from F and proceeding backwards, the best path is computed from F to E at time $t$. Similarly, the global optimal solution is calculated step-by-step starting from E and to an ending at the initial state. The shortest path is A-B-J-H-E-F and the minimum cost is $1.2 + 0.6 + 0.7 + 0.6 + 0.8 = 3.9$. The general optimal objective function is defined as follows

$$J = \sum_{t=0}^{N-1} [L(x(k),u(k))] + G(x(N)) = \sum_{t=0}^{N-1} [m_{fuel}(k) + \mu \cdot NO_x(k) + \alpha \cdot PM(k)] + \beta(SOC(N) - SOC)$$

(1)

where $N$ is the driving cycle time; $L$ is the cost function, including fuel consumption; $NO_x$ is emissions, etc.; and $G$ is the constraint of the state-of-charge (SOC) and gear shifting.

Guzzella et al. [10] put forward an energy management strategy with DP for parallel Hybrid Electric Vehicles. Dynamic programming is used to design an optimal gear shift strategy in [11]. A cost function representing a combination of fuel consumption and emissions over a driving cycle is defined to sustain battery SOC. The optimal gear shifting schedule that can be implemented to a real vehicle is extracted from DP by splitting the power between the engine and the motor. To reduce the computational complexity and implement easily, Patil et al. [12] proposed a novel dynamic programming that is calculated by a backward simulation model for a series hybrid electric vehicle. This approach evaluates state constraints before choosing the optimal paths rather than using penalty functions, which can avoid the requirement interpolation by considering transitions to only the finely discretized nodes of the state space.

A novel dynamic programming, on the basis of machine learning, is proposed in [13]. An EMS for a power-split HEV with an on-line trained neural network is developed to predict traffic congestion and road types. DP is adopted to split the power between engine and motor over a specific driving cycle. The neural network is utilized to predict the traffic conditions and road type with vehicle historical data. This is called an on-line intelligent energy management strategy by combining a machine self-learning algorithm and dynamic programming. In [14], a gear shifting strategy and a power allocation strategy for a hydraulic hybrid vehicle were obtained by dynamic programming, which is utilized in a real-time controller by extracting the control rules. Simulation results show that fuel savings can be improved by 47% over a conventional vehicle. Kutter et al. [15] combined dynamic programming with an equivalent consumption minimization strategy to solve the conflict between global optimality and real-time capability, which is performed by an independent calculation of the main control parameters using dynamic programming, and the power split is optimized online by
the ECMS. In [16], a weighted, improved dynamic programming technique is proposed to allocate the power for a hybrid fuel cell vehicle, proving that it converges faster than the traditional dynamic programming methods that suffer from a dimensionality problem. Simulation results reveal that, when compared to the rule-based EMSs, lower costs and a lower hydrogen consumption are achieved using the weighted, improved dynamic programming. To improve the computation efficiency, Zhuang et al. [17] extracted a mode shift map for a multi-mode hybrid powertrain with the DP optimal results using the support vector machine. This can be combined with ECMS to implement real-time control. More works can be viewed in related studies [18–21].

It is well known that DP is a numerical method to solve a dynamic optimal control problem. However, it may lead to optimization inaccuracy when the continuous states are implemented in a discrete framework. To address this issue, Berkel et al. [22] proposed a new implementation method by extending the discrete method by storing the quantization residual after the nearest neighbor, rounding of the continuous state at each node. This can avoid the implementation difficulty of the interpolation method and the inaccuracy of the discrete method.

4.1.2. Stochastic Dynamic Programming (SDP)

Although DP is regarded as a useful tool to obtain a global optimal solution, it is impossible to know exactly the whole driving cycle conditions (speed, road slope, etc.) in advance. To address this issue, stochastic dynamic programming is proposed by researchers. The basic principle of stochastic dynamic programming is that assuming that the sequence of values can be modeled using Markov chain power, the state transition matrix map of the future driver’s power demand is generated to estimate the driver’s power demand. The power sequence demand is calculated by discretizing the historical driving data at a certain step, and the determination of the current power demand is made in terms of the vehicular speed. The maximum likelihood estimation method is utilized to obtain the state transition probability from the current state to the next one by distributing the total power using discrete dynamic programming. It has been successfully applied as a promising approach for obtaining a quasi-optimal policy that is implementable on-line and in real time, since only historical driving data is needed without a priori knowledge of the driving cycle. However, there are differences between the power demand using the Markov chain model and actual driver power demand, leading to poor adaptability to different driving cycles because of the complexity and randomness of the actual driving cycles. Moreover, the computation process for solving the SDP is still time consuming due to the policy iterations. The future discounted costs are chosen based on the mathematical expediency, leading to difficulties in validation on engineering applications.

The state transition probability is described as in Formula (2).

\[
p_{i,j,a} = P[ P_{\text{dem}}(k+1) = i | P_{\text{dem}}(k) = j, v(k) = a ]
\]

(2)

where \( l \) and \( j \) is the power demand at state \( k+1 \) and \( k \), respectively, and \( a \) is the velocity at state \( k \).

In [23], a stochastic dynamic programming algorithm for a power-split hybrid vehicle is proposed, which is performed by establishing drive power sequence demand over different driving cycles based on the Markov process to obtain a state transfer matrix of the driver’s power demand. The optimal problem is formulated to maximize the fuel and electricity economy as the objective function in a constraint domain, on the condition of the torque of the engine and motor, as well as the battery charging and discharging power. The energy price was introduced into the objective function. The simulation results are compared with that of the charge-depleting and charge sustainability (CD-CS) strategy in terms of fuel consumption, engine control principle, engine start-stop control, and energy price.

Researchers mainly focus on minimizing the fuel consumption by using SDP. To incorporate the drivability, Opila et al. [24] formulated a stochastic dynamic programming to gain a trade-off between fuel economy and drivability, including engine start–stop and gear shifting time. The driving cycle is modeled by the Markov process considering driver power demand as a stochastic process. The simulation results in FTP and NEDC demonstrate that fuel savings of the proposed EMSs improve by 11%. The influence of engine start–stop and gear shifting time on fuel economy is
also investigated and compared with baseline EMSs. In [25], an optimal energy management for a series hybrid electric vehicle is presented on the basis of SDP and considering the fuel consumption and emissions. However, the computational burden is intractable for SDP due to the large state space in this problem. Thus, a new neurodynamic programming (NDP) is proposed to solve the issue. Finally, an SDP controller and NDP controller are compared with a baseline one, indicating that both SDP and NDP can achieve significant fuel economy compared to rule-based EMSs. References [26–28] can be referred to for more information on the subject.

4.1.3. Genetic Algorithm (GA)

The genetic algorithm (GA) in evolutionary computing has become one of the most popular algorithms among modern optimization algorithms due to its good global search performance and low algorithm complexity [29]. As a random search method, the genetic algorithm is performed by global searching to converge to an optimal solution based on the law of biological evolution. These advantages are well suited to optimizing the rules, parameters, or evaluation criteria in EMS for better performance [29]. The optimization problem is solved by simulating biological phenomena, such as genetic variation. GA can be applied in EMSs to obtain global optimal solutions; however, the computational load is heavy, especially for more variables due to the repeated searches, and can be regarded as an offline optimization method, which guides researchers to select the optimal parameters (e.g., engine size and battery size) for an HEV. Zhou et al. [30] obtained the optimal parameters by GA and analyzed the energy management for fuel cell Hybrid Electric Vehicles. Figure 6 shows the basic flow of the genetic algorithm. The main steps to implement the GA are as follows:

1. Initial population: Select an initial population in a feasible solution domain.
2. Genetic operation: A new population is generated by the selection, crossover and variation of the initial population to converge to the global optimal solution.
3. Decide if the population meets the ending criteria, referring to the iterations of the intelligent optimal algorithm.

![Figure 6. The flow of the genetic algorithm.](image)

Piccolo et al. [31] put forward an energy management strategy using the genetic algorithm to implement global optimization, which can be performed by adjusting the control parameters to minimize fuel consumption and emissions. This method can obtain the global optimal solution and
yield better robustness; however, the computational complexity is higher than the other EMSs. To improve the optimal performance of a genetic algorithm, Liu et al. [32] proposed a hybrid genetic algorithm for a series hybrid electric vehicle, with faster convergence and better adaptability compared to the traditional GA that performs the global search randomly. The proposed algorithm can acquire fast convergence to a global solution using the quadratic programming algorithm. In addition, the GA is combined with other algorithms to address the energy management optimization problem. In [33], an energy management strategy is proposed based on fuzzy logic and genetic algorithm optimization. The membership function of a fuzzy logic controller is optimized using the genetic algorithm. The simulation results show that the presented EMSs are clearly capable of improving the fuel economy and reducing the gas emissions as compared to the deterministic rules without adjustment by the GA.

4.1.4. Game Theory (GT)

As a branch of operational research, game theory is commonly used in multi-subject optimization problems by taking into account the forecast and actual behavior of individuals in a game. In the 1950s, cooperative game theory enjoyed its peak and non-cooperative game theory began to develop [34]. During this time, a legendary figure, John F. Nash, deserves special mention for his two essays in 1950 [35] and 1951 [36], firstly using rigorous mathematical language and then simple words to accurately define the Nash Equilibrium, which was a significant milestone in game theory history. The basic idea of game theory is to determine, through formal reasoning alone, what strategies the players ought to choose in order to pursue their own interests rationally, and what outcomes will result if they do so [37]. In recent years, game theory-based EMSs, which are sensitive to the variations in vehicle parameters, have been developed.

Gielniak et al. [38] proposed an integrated system approach based on game theory for automotive electrical power and energy management systems. The objective of the players is to maximize their payoff that is a function of vehicle performance and powertrain efficiency. Yin et al. [39] formulated the energy management problem as a non-cooperative current control game. The Nash equilibrium is analytically derived as a balanced solution that compromises the different preferences of the independent devices. Drexireit et al. [40,41] designed a controller for a parallel hybrid electric vehicle using game theory with the objectives of fuel economy and emission. First, the vehicle operating conditions and the powertrain are viewed as two players in a finite-horizon non-cooperative game. A cost function of this game is formed by weighting the fuel consumption, NOx emissions, and the deviation of the battery SOC from the setpoint, as well as the deviation from the vehicle operating conditions. The policy is established as a function of wheel speed, torque, and battery SOC to decide the control mode of the engine, motors, and battery. Compared to traditional EMSs, this control policy is independent of the time and driving cycle. Therefore, it can achieve better performance under different driving cycles. Test results validate that the game theory controller substantially outperforms the baseline controller under NEDC. Xu et al. [42] proposed a game-theoretic energy management strategy with velocity prediction for a hybrid electric vehicle. A recurrent neural network structure was realized to predict the future velocities and Nash equilibrium of game-theoretic energy management, and was implemented through the best response functions. Chen et al. [43] developed a game-theoretic approach for solving the complete vehicle energy management problem of a hybrid heavy-duty truck with a high-voltage battery and an electric refrigerated semi-trailer. The solution concept is based on a two-level single-leader multi-follower game model. The game-theoretic approach presented the optimal performance in the simulation. Chen et al. [44] introduced an adaptive game-theoretic approach for solving the complete vehicle energy management problem of a hybrid heavy-duty truck with a high-voltage battery and an electric refrigerated semi-trailer. The proposed method enhances the game-theoretic approach, such that the strategy is able to adapt to real driving behavior. The fuel reduction results are compared and the adaptive game-theoretic approach shows improved and more robust performance over different drive-cycles compared to the non-adaptive one. A game-theoretic solution concept for solving the complete vehicle energy management (CVEM) of a hybrid heavy-duty truck can be found in [45].
4.1.5. Pseudospectral Method

Pseudospectral method, also known as the discrete variable representation method [46], is a direct numerical algorithm for optimal control problems. In the energy management problem, the optimal control theory is utilized to optimize the energy distribution. The pseudo spectral method can be used as a direct numerical method to obtain the optimal energy distribution. The continuous energy management optimization problem can be solved by discretizing and transforming it into a nonlinear programming problem. Hu et al. [47] proposed a double objective charging optimization strategy for two kinds of lithium-ion batteries, by considering the influence of battery charging time and charging energy loss on HEV energy management. A multi-objective optimal charging control problem was constructed, and then solved by using the Radua pseudospectral method. Zhou et al. [48] utilized the pseudospectral method to solve an HEV energy management problem and optimized the energy management and co-state trajectory simultaneously. The results showed that the computation efficiency of the pseudospectral method is higher than that of DP, while the optimization performance is close to DP. Wu et al. [49] developed a hierarchical EMS with the pseudospectral method for Hybrid Electric Vehicles, which incorporates velocity planning, with a tradeoff between fuel consumption and path tracking accuracy.

4.1.6. Convex Optimization

As an optimization algorithm, convex optimization is utilized for solving convex problems [3], whose objective function and constraints are convex. In convex optimization problems, the results of local optimization and global optimization are consistent, which greatly simplifies the solution process [50]. As compared to other global optimization algorithms, it is easy to obtain optimal solutions with a higher computation efficiency. The optimization of HEV energy management can be regarded as a nonlinear programming problem, which can be transformed into a semi convex problem by using a convex optimization method that offers a simplified calculation process and better optimization effect. Murgovski et al. [51] presented an EMS with convex optimization for a plug-in hybrid electric bus. The influence of battery size, gearshift, and engine on/off on energy management was investigated by transforming these problems into semi convex problems with a convex optimization method. In addition, the optimal results obtained from the convex optimization were compared with dynamic programming. Nafisi et al. [52] considered the influence of the power grid on the energy management of plug-in HEVs, and proposed a two-level optimization method based on convex optimization to reduce the energy loss. However, the disadvantage of convex optimization is that the objective function and inequality constraint must be convex [53], and it yields limited applications. Especially for a parallel HEV, the gearshift strategy should be devised separately, instead of optimizing the gearshift and power split simultaneously, such as in [54].

4.1.7. Pontryagin’s Minimum Principle (PMP)

PMP is an analytical optimization method to solve optimal control problems to provide a necessary condition. PMP transforms a global optimization problem into an instantaneous Hamiltonian optimization problem, derived from DP through a variational approach. Thus, an optimal solution can be obtained by minimizing the instantaneous Hamiltonian that includes fuel consumption and battery SOC. Similar to ECMS, an optimal co-state is a key factor that needs to be determined appropriately. A shooting method is commonly adopted to calculate the optimal co-state \( \lambda \), for example in [55]. More works can be found in [56–59]. The form of instantaneous optimization shown in PMP makes it possible to implement real-time control. The basic principle is generally formulated as Equations (4)–(8). It is obvious that a differentiable objective function is required for deriving the optimal solution; however, it is difficult to obtain a continuous Hamiltonian for Hybrid Electric Vehicles, especially for a parallel HEV. To this end, a simplified PMP is proposed in [60] to avoid the adaptation mechanism of the co-state for real-time applications. The main drawback of the control concept is that the PMP-based EMS will not guarantee optimality if no information regarding the future driving condition is provided [61].
The augmented cost function for a general problem can be given as Equation (3):

\[ Q = \varphi(x_f, t_f, u) + \int_{t_0}^{t_f} L(x, u, t)dt \]  

(3)

where \( L(x, u, t) \) is the cost function and \( \varphi(x_f, t_f, u) \) is presented as Equation (4):

\[ \varphi(x_f, t_f, u) = \varphi(x_f, t_f) + u^T \psi(x_f, t_f) \]  

(4)

The state dynamic is described as Equation (5) and \( x(t_0) = x_0 \) is also satisfied:

\[ \dot{x} = g(x, u, t) \]  

(5)

Thus, the Hamiltonian function can be formulated as Equation (6):

\[ H(x, u, t) = L(x, u, t) + \lambda^T g(x, u, t) \]  

(6)

where \( \lambda^T \) is the co-state. Given the problem settings in Equations (3)–(6) and assuming the problem is convex, the necessary condition that minimize Equation (3) are given as Equation (7).

\[ \begin{cases} 
\dot{\lambda} = -H_c \text{ and } \varphi_{x_i} = \lambda^T (t_f) \\
\dot{x} = g(x, u, t) \text{ and } x(t_0) = x_0 \\
H_x = 0 \\
(\varphi + H)_ux = 0 \\
\psi(x, t_f) = 0
\end{cases} \]  

(7)

In [62], three kinds of EMSs, namely DP, PMP, and ECMS, are conducted and compared. By comparing ECMS and PMP, it is found that they are similar in terms of equivalent factor and co-state. The author suggested that the ECMS becomes the implementation of the optimal solution of PMP, which also obtains results close to the DP optimal solution, with an improvement in comparison to the traditional ECMS. To adjust the control parameters, adaptive PMP is proposed using the total trip length and the average cycle speed in [63]. The results demonstrate that improvement in fuel consumption can reach 20% compared to an on-board controller. Kim et al. [64] proposed an EMS-based on PMP considering the battery efficiency of the plug-in Hybrid Electric Vehicles (PHEVs) and derived an additional condition for the inequality state constraints. The results prove that the PMP can achieve similar performance to the global optimal results obtained by DP. In [65], PMP is introduced by solving the Hamiltonian function to find the battery current command, and the simulated annealing algorithm is used to calculate the engine-on power and the maximum current coefficient. The simulation results demonstrate that the proposed algorithm can reduce the fuel consumption as compared with charge-depleting and charge-sustaining EMS. Although PMP is utilized to solve the optimal control problems for the energy management by simplifying the engine fuel map, engine on/off control is not considered. To address this issue, the approximate PMP is proposed in [66]. A piecewise linear approximation to fit the fuel rate map for a plug-in HEV has been developed based on PMP to avoid distortion in the fuel map. The results show that the engine state switching frequency is reduced by 43.40% with engine on/off optimal EMSs.

Previous works mainly focus on the determination of an optimal co-state with future driving cycles or a prior knowledge of the driving cycles, such as [67]. Kim et al. [61] presented an adaptive energy management strategy with PMP by analyzing the past driving patterns and updating the control parameters with an assumption that vehicles operate under repeated driving conditions (e.g., commuting buses). In real conditions, the driving cycle is affected by numerous factors, for example, driver behaviors and traffic conditions. To this end, Park et al. [68] investigated a PMP-based energy management strategy for plug-in HEVs incorporating the driver’s characteristics to improve the adaptability of PMP.

4.2. Rule-Based EMSs
Generally, rule-based EMSs can be performed by predefining the logical rules according to the HEV system characteristics and operation mode. The rules are determined based on the battery SOC, driver power demand, and vehicle velocity through an “if-then” structure. Given these rules, the power split can be performed to meet the driver power demand and maintain the SOC at a certain range. Instead of a prior knowledge of the driving cycle, this method mainly depends on logical rules and local constraints. The control parameters cannot be tuned due to a lack of future information on the driving cycle, making it less adaptable to varying driving conditions. The typical methods, like deterministic rule-based control and fuzzy rule-based methods, are introduced in the following sequel.

4.2.1. Deterministic Rule-Based EMSs

In this method, based on the engine map and motor efficiency map, a series of logical rules are predefining to split the power between the engine and motor, considering the efficiency of the motor and engine and battery SOC simultaneously. The control rules are easy to implement on-line by a look-up table due to its simplicity. Thus, it is widely utilized in the commercial application of vehicle controllers. The rules are commonly devised based on specific driving cycles (e.g., ECE). However, the varying traffic conditions make it less adaptable to different driving cycles. Peng et al. [69] present a rule-based EMS for a parallel hybrid electric vehicle. Thus, conventional rule-based power management is not optimal for real driving cycles since a unique approach to design the logical rules does not exist. In most cases, this depends on the engineer’s experiences and driving cycles. In the following subsections, rule-based strategies, including on/off and power follower EMSs, are discussed in more detail.

1) on/off EMSs

As for this strategy, a battery SOC is always maintained between its preset minimum and maximum thresholds by turning the engine on/off. The basic control rules are as follows:

- The engine starts to work at the highest efficiency region or sub-optimal emissions area and supplies constant power when the battery SOC is lower than the preset minimum threshold. A portion of the engine power is provided to the motor to satisfy the power requirement while the rest is used on charging the battery.

- The engine is shut off when the battery SOC increases to the pre-set maximum threshold and only the battery provides the driving power.

In some cases, a surge of instantaneous power may be supplied from the battery with this EMS, which makes battery charge and discharge period shorter and the engine start–stop frequently. The main advantage is that the average efficiency of the engine is higher and the battery charging and discharging period became shorter, but leads to negative effects, such as more power loss due to frequent engine start–stop, less total energy efficiency, and shorter battery life [70]. Although this method is simple relative to the optimal EMSs, it cannot satisfy the vehicle power demand at all operating conditions.

2) The power follower EMSs

Based on the battery SOC and vehicle load, the output engine power as well as the moment to start or shut off the engine are determined to satisfy the driver power demand. The control rules are as follows:

- If the power demand is less than the maximum engine power at its operating speed, the operation point is adjusted to work at the minimum output power line.

- If the battery SOC is higher than the preset minimum value and lower than maximum value while driver power demand is less than the battery capacity and greater than the maximum engine power at the operating speed, the engine operates at the maximum output power line and the rest of the power demand is supplied by the battery.

- If only the battery SOC is higher than the preset maximum value and able to satisfy the power demand, the engine should be shut off.

The main advantage of this strategy is that it can reduce the frequency of battery charging and discharging and lower the system energy loss to extend the battery life. This method yields better
adjustability for engine output power to the power demand, but the engine operation region becomes wider to lower the overall efficiency.

The rule-based EMSs is easy to implement on-line; however, it is not optimal and cannot guarantee the optimality for different driving cycles. It is also not capable of adjusting the control parameters to achieve the best fuel economy due to the complexity of the driving conditions.

4.2.2. Fuzzy Logic-Based EMSs

Fuzzy logic control theory is composed of fuzzy set theory and fuzzy logic. The former is an extension of TRUE and FALSE (1 and 0) set theory and the latter is an extension of conventional logic in how the system determines the output [71]. Fuzzy relations depend largely on the similarity or the degree of similarity between data sets, and fuzzy reasoning is represented by the IF–THEN format, giving birth to some popular reasoning approaches, for example, the Mamdani method [72] and Takagi–Sugeno method [73]. Fuzzy logic-based EMSs have been conducted throughout the years in the literature [74–77]. Fuzzy logic-based EMSs aim to split the power with fuzzy rules. In this method, the fuzzy logic rules are usually developed according to the driver power demand and SOC. A fuzzy logic controller consists of a set of linguistic rules and each of them includes one antecedent and two consequents. Looking into a hybrid system as a nonlinear and time-varying plant, fuzzy logic controllers are adjustable to implement in real-time with sub-optimal control by a set of fuzzy logical rules. Moreover, it is important to devise a membership function in optimizing the power split. Thus, GA is adopted to optimize the membership function in the reference [78]. Some other forms of modified fuzzy logic-based EMSs can be referred to in [79–81].

In [82], a fuzzy logic controller (FLC) for parallel Hybrid Electric Vehicles is designed. In [83], a multi-input fuzzy logic controller for a power-split hybrid vehicle is presented and compared to rule-based EMSs in terms of fuel economy and emissions. Given the desired driver torque, vehicle speed, and battery SOC, the power is distributed using the FLC method. This method achieves a better fuel economy with good adjustability compared to the conventional rule-based EMSs. Lee et al. [84] presented a fuzzy logic-based energy management strategy to minimize the NO_x emissions while meeting the driver power demand. The proposed fuzzy logic controller uses an electrical motor speed as well as an acceleration pedal stroke as the control inputs. It is claimed that the proposed fuzzy logic controller could reduce about 20% of the NO_x emissions compared with the conventional vehicle. However, the main challenge of this method is that it cannot guarantee the SOC charge-sustainability of the battery. To address this problem, Lee et al. [85] proposed a more sophisticated fuzzy logic controller that includes a power balance controller and a driver’s intention predictor for the energy management. Baumann et al. [86] developed an inclusive fuzzy logic controller based on road load estimation to compensate for the difference between the actual engine torque and the required torque. To enhance the adaptability of the fuzzy-based EMS, Tian et al. [87] presented an EMS for a plug-in hybrid electric bus using adaptive fuzzy logic-based with an optimal SOC reference generated by a neural network and followed by a fuzzy logic controller.

In principle, the fuzzy rule-based EMSs can be utilized to adjust the control parameters to a limited extend by predefining a set of fuzzy rules. However, this approach yields less adaptability due to the difficulty in selecting a proper membership function based on different inputs.

5. Online EMSs

Online EMSs are causal and local optimization-based since they generally do not require a priori knowledge of the whole driving cycle. They can be implemented in real-time with a limited computational burden by converting the global optimization problem of off-line EMSs into an instantaneous optimization problem. Due to less computational effort, on-line EMSs yields the potential of being implemented in real-time control problems. Three categories are included, namely instantaneous optimization-based EMSs, predictive EMSs, and learning-based EMSs. The instantaneous optimization-based EMSs determines the power split with optimal algorithm utilizing the current driving cycle information while the predictive EMSs mainly employ future information to optimize the power split. Furthermore, the instantaneous optimization EMSs mainly focus on
determining the optimal power split by minimizing the performance indexes (e.g., fuel economy, emissions, and drivability) at each instant. In the following subsection, these EMSs are extensively reviewed and important headlines are highlighted.

5.1. Instantaneous Optimization-Based EMSs

This kind of approach is to optimize the power split by minimizing the instantaneous fuel consumption and other performances (e.g., emissions and drivability) at each instant. These EMSs can achieve the best performance at each instant without a priori knowledge of the driving cycle and it is easy to implement in real-time. Instead of predefining the logical rules, instantaneous optimization EMSs mainly focus on optimization and implementation on-line, resulting in better fuel economy and adjustability compared to simple rule-based EMSs. However, only local optimal results can be obtained instead of global optimization as is possible in offline EMSs.

Due to its reasonable computation burden and no requirements of previewed knowledge, these are capable of being applied to a real-time controller and achieving approximate optimal results in comparison with DP. In recent years, many researchers focus on instantaneous optimization EMSs, including equivalent consumption minimization strategies (ECMS), adaptive-ECMS, Pontryagin’s minimum principle (PMP), and robust control. In the following section, these are introduced and discussed in more detail.

5.1.1. Equivalent Consumption Minimization Strategy (ECMS)

The main idea of ECMS is that the power is distributed by minimizing the instantaneous equivalent fuel consumption at each instant by converting the electricity consumption into the equivalent fuel consumption. In contrast to other EMSs, the control variable in ECMS is the equivalent factor (EF), which is defined as the relation between the energy consumption of the secondary power source and power requirement. The equivalent factor plays a significant role in improving the fuel economy. Thus, selecting a suitable equivalent factor according to different driving cycles is a key issue. For this method, it is easy to implement for real-time control, achieving sub-optimal results without prior knowledge of the driving cycle. The standard ECMS generally adopts a constant optimal EF obtained from an iterative method; however, it cannot adapt to the varying driving conditions. Thus, other forms of ECMSs are proposed, such as adaptive ECMS [88,89], telemetry ECMS [90], predictive ECMS [91], ANFIS-based ECMS [92], artificial neural network-enhanced ECMS [93], and a driving-style based ECMS [94]. Since fuel consumption is the main design objective, two key issues need to be considered for ECMS implementation. One is the drivability, in that the optimal torque usually jumps frequently at each instant without incorporating engine or motor response time, which may lead to oscillation of the powertrain. Another is the computation efficiency, in that it cannot directly be utilized in a real vehicle controller although yielding a lower computational burden compared to DP. Instead, it can be implemented online in a look-up table. Additionally, it is more challenging to adjust the EF in real driving cycles.

The basic principle of an ECMS is illustrated in Figure 7, which is depicted for a parallel HEV. The energy flow when the battery is discharging is shown in Figure 7a. In this state, the electric motor supplies mechanical power. The route of the red dots is concerned with the return of the used instantaneous electrical energy in the future, which means that the used electricity is converted into equivalent consumption. The energy flow when the battery charging is shown in Figure 7b. In this state, the engine supplies the mechanical power. The mechanical energy is received and converted into electrical energy by the motor, and then is stored in the battery. The red dotted route is related to the use of this electrical energy for generating mechanical power in the future. This part of the mechanical energy will not have to be generated by the engine, which is considered as fuel-saving. The power split is then determined by minimizing the equivalent fuel consumption.
The equivalent fuel consumption rate is given as Equation (8).

\[ m_{eq} = \dot{m}_f + \dot{m}_e = \dot{m}_f + \frac{s}{Q_{lhv}} P_e \]  

where \( \dot{m}_f \) is the engine instantaneous fuel consumption; \( s \) is the equivalent factor; \( P_e \) is the motor power, where the value is negative when braking and the value is positive when driving; and \( Q_{lhv} \) is the fuel lower heating value.

As an instantaneous optimization method, an equivalent consumption minimization strategy (ECMS) is firstly introduced by [95] and an instantaneous optimization algorithm is supplemented (see [96–98]). Nüesch et al. [99] proposed an approach that minimizes the fuel consumption using ECMS for a diesel hybrid electric vehicle while tracking a given reference trajectory for both battery SOC and \( NO_x \) emissions adjusted by a PI controller. By hardware-in-the-loop (HIL) experiments, the proposed method not only improves the fuel economy but also implements feedback regulation of the SOC and \( NO_x \) emissions. Gao et al. [100] introduced an ECMS for series Hybrid Electric Vehicles in comparison with on/off EMSs and power follower strategy. The on/off EMS mainly optimizes the operation region of the engine while the power follower EMS optimizes the operation region of the battery charging and discharging. The main objective of the ECMS is to implement system optimization in terms of battery and engine efficiency, which can achieve better fuel economy.

To ensure battery SOC charge-sustainability and keep the EMSs simple to implement, Skugor et al. [101] proposed an energy management strategy for a power-split hybrid electric vehicle, integrating rule-based EMS and ECMS to optimize the fuel economy. One-dimensional directional search-based and two-dimensional directional search-based instantaneous ECMSs were analyzed, in which the former was performed in two variants, corresponding to the engine maximum torque target line and constant-power target line, while the latter gave special attention to the offline optimization of the target region size. The simulation results indicate that the optimization solution of the rule-based + ECMS is close to that of dynamic programming under an HWFET (Highway Fuel Economy Test) cycle.

In [102], ECMS is deployed to solve the optimization problem for a hybrid system of fuel cells and batteries, obtaining suitable energy management of the hybrid system by minimizing the hydrogen consumption. In [103], Park et al. applied ECMS for the power distribution between the engine and the motor of Hybrid Electric Vehicles. To find the optimal equivalent factor for a certain
driving cycle, a parameter optimization method based on a model applying a genetic algorithm was studied. The results represent a promising improvement in fuel economy and the optimal equivalent factor is considered as a good initial value for vehicle calibration.

5.1.2. Adaptive Equivalent Consumption Minimization Strategy (A-ECMS)

As explained previously, the performance of an ECMS for real-time control is closely related to the equivalent factor. Therefore, how to tune the equivalent factor is essential to improve the performance of energy management strategies. The equivalent factor is generally decided by the future power requirement and the current SOC as well. To achieve this goal, A-ECMS is proposed by refreshing the control parameters according to the future power demand and current one. The basic principle of an A-ECMS is that the equivalent factor is regulated accordingly by the current SOC, predicting the velocity and driver’s power demand in real-time, keeping the SOC in a certain range and minimizing the fuel consumption. The PI adaptor is commonly adopted in [104]; however, the PI parameters needs to be adjusted appropriately. Thus, a fuzzy logic-based PI adaptor is proposed in [105] to adapt to the changing driving conditions. Furthermore, incorporating the uncertainty of the driving cycles and future information from ITS are utilized in adjusting the EF. The typical structure of an A-ECMS with ITS is illustrated in Figure 8. With the GPS/ITS and feedback information, the future power demand is estimated over a certain horizon. The equivalent factor is estimated and tuned online to maintain the prescribed SOC by the adaptor. The A-ECMS can be implemented in real-time control without a priori knowledge of the driving cycle.

![Figure 8. Structure of the adaptive equivalent consumption minimization strategy (A-ECMS). Note that $s(t)$ is the equivalent factor, and $\alpha(t)$ and $\beta(t)$ is the acceleration and deceleration pedal, respectively.](image)

In [106], Musardo et al. put forward an A-ECMS by estimating the equivalent factor based upon different road loads to update the control parameters, which minimize fuel consumption and maintain the battery SOC at a certain range. Sezer et al. [107] introduced a novel ECMS for series Hybrid Electric Vehicles considering the efficiency of the engine, battery, and generator to gain the combined fuel consumption and emissions cost map, which optimizes the engine-generator set and ensures battery charge sustainability. Sciarretta et al. [97] proposed a new approach for redefining an equivalent factor according to the coefficient of charging and discharging of the battery, which presents great robustness and reduces the fuel consumption by 30% in comparison to the traditional approaches.

Other approaches to estimate the equivalent factor by combining the ECMS with other optimization algorithms are proposed in [91–93]. In [108], Zhang et al. proposed two kinds of methods, such as DP and backward ECMS to estimate the equivalence factor and adopted a backward ECMS sweeping over the estimated future velocity and exacting the future 3-D terrain information to adjust the parameters of the ECMS. In [109], Kim et al. developed a method based on Pontryagin’s minimum principle (PMP) to calculate the optimal equivalence factor. He et al. [110] presented an
energy management strategy that combines rule-based strategy and ECMS for fuel cell vehicles to reduce the hydrogen consumption.

The development of Intelligent Transportation Systems offers a promising way to predict the velocity and estimate the equivalent factor for an ECMS. The velocity and position of each vehicle as well as the traffic information in front of a target vehicle can be provided through vehicle-to-vehicle communication (V2V) and vehicle-to-infrastructure communication (V2I) with a DSRC protocol to make it possible to adjust the equivalent factor according to the updated predicted velocity. Serrao et al. [111] indicate that PMP can be shown as the underlying optimization principle for ECMS, but online implementation is unfeasible due to its iterations in finding the initial value of the dynamic equivalent factor for charge-sustaining (CS) operation. Mohd et al. [112] proposed a velocity prediction method combining a car-following model and cell-transmission model (CTM) based on Inter-Vehicle Communication (IVC) and Vehicle-Infrastructure Integration (VII). A computationally efficient CS HEV powertrain optimization strategy was then analytically derived based on the PMP and CS condition to adjust the co-state according to the predicted velocity to evaluate the performance of the proposed strategy. Zhang et al. [113] proposed an adaptive ECMS on the basis of velocity prediction through V2V and V2I communications to improve the robustness of the ECMS and maintain a good SOC charge sustainability. To incorporate the future information into the EF adaptor, Sun et al. [114] developed an adaptive-ECMS to improve the fuel economy by updating the EF periodically, with the predicted velocity obtained from neural networks.

On the basis of recognizing the driving pattern, the equivalent factor can also be assessed. The driving cycle pattern can be identified by the previous driving pattern. Thus, the equivalent factor is adjusted adaptively based on driving pattern recognition. In [115,116], Gurkaynak et al. put forward an energy management strategy using ECMS for a parallel hybrid electric vehicle to obtain sub-optimal results. The optimal performance is related to the vehicle model and equivalent factor, which is updated by driving cycle identification using a neural network algorithm. Simulation results demonstrate that ECMS can obtain approximate optimization results in comparison to DP. In [98], a novel method is developed to calculate the equivalent factor determined by the change rate of the SOC in ECMS without a priori knowledge of the entire driving cycle. The robustness and adjustability are demonstrated through different driving cycles compared with that of estimating the equivalent factor under specific driving cycles. To catch energy-saving opportunities, Rezaei et al. [117] proposed a novel energy management based on an adaptive equivalent consumption minimization strategy for series Hybrid Electric Vehicles by determining a range for the optimal EF of ECMS. Most of the literature ignore the vehicle lateral dynamic in devising the EMS; to this end, Li et al. [118] developed an energy management strategy considering the vehicle lateral dynamic with an adaptive ECMS.

5.1.3. Robust Control

Robust control is a branch of control theory whose approach to controller design explicitly deals with uncertainty. The robust control method is utilized in designs for them to function properly, provided that there are uncertain parameters and that disturbances exist within some forms (parametric or structural) [119]. As for this method, the energy management is formulated as an optimal problem, represented by the state-space equation. The HEV model generally needs to be simplified to devise a closed-loop system, which can be stable and of strong anti-jamming ability by designing state-feedback gain matrices, as well as achieving sub-optimal results with higher computational complexity.

In [120], to overcome the presence of parameter uncertainty in the optimal problem, an optimal-heuristic EMS is presented. The solution is real-time implementable since it is based on a discrete-time description of the system and the optimal solution can be analytically found. In [121], a robust energy management strategy for a fuel cell hybrid vehicle is proposed to solve the sensitivity issue regarding driving cycle uncertainty. This approach improves the robustness of the energy management strategy against driving cycle variations while minimizing the H₂ consumption. Pisu et al. [122] discussed three kinds of energy management approaches for a parallel hybrid electric
vehicle, namely rule-based EMSs, an adaptive equivalent consumption minimization strategy (A-ECMS), and H<sub>oo</sub> control, compared with DP that presents the disadvantages of computational complexity and requiring a priori knowledge of the driving cycle. The rule-based EMSs that is of lower computational burden is easy to implement by pre-defining a series of control rules, dependent on the brake and accelerator pedal angle, battery SOC, and the torque demand. The A-ECMS is implemented by establishing an optimization cost function, which takes into account electricity consumption, fuel consumption, and NO<sub>x</sub> emissions, and adds a penalty function on an equivalent factor. The state–feedback H<sub>oo</sub> control method aims at minimizing fuel consumption by computing a control gain matrix. Simulation results show that an A-ECMS achieves a similar performance in comparison with DP. For the A-ECMS, the optimal control can be calculated offline and stored in the controller as a look-up table to reduce the computational load, whereas the dynamic characteristics of the components is neglected.

In principle, although the robust control method can provide dynamic optimization to adjust the control parameters, it only can achieve sub-optimal solutions because of its simplification of the models.

To clearly show the pros and cons of each method, a comparison of different approaches is summarized in Table 2.

Table 2. Comparison of different approaches.

| Approaches | Main Advantages | Main Disadvantages | Literature |
|------------|-----------------|--------------------|------------|
| ECMS       | • easy to implement | • less adaptability | [88–103] |
|            | • on-line implementation |                   |            |
| A-ECMS     | • on-line implementation | • complex EF adaptor | [104–118] |
|            | • more adaptability | • obtain local optimal results |            |
| RC         | • robustness with uncertainty parameters | • high computational complexity | [119–122] |
|            | • more adaptability | • higher vehicle model complexity |            |

Note: The adaptability refers to the flexibility of the energy management strategies (EMSs) adapted in different driving cycles.

5.2. Predictive EMSs

The main purpose of predictive EMSs is to optimize the power split utilizing predictive information related to the uncertainty and disturbance of a driving cycle. This strategy requires future driving cycle information (e.g., future velocity) that can be predicted with available information (e.g., road conditions and traffic conditions). Thus, to a large extent, the performance of this strategy depends on the power reference provided at each prediction horizon. In other words, it is mainly based on the predicted velocity on a flat road without considering road slope. Therefore, it is significant to predict the vehicular velocity accurately in implementing such approach. Generally, it is impossible to predict the whole cycle accurately. Alternatively, it should be partially predicted if only a small part of the upcoming trip is considered [123]. In addition, the factors affecting the prediction accuracy include driver behavior, road condition, dynamic traffic conditions, preceding vehicles, etc. Inaccurate prediction may worsen an EMS’s performance. Therefore, in order to improve the prediction accuracy, more surrounding information needs to be effectively considered. The optimal control input is obtainable by minimizing the performance indexes (e.g., fuel consumption and emissions) over a certain horizon, and this approach is in real-time implementation to adapt to the changing driving conditions. In view of this, researchers increasingly adopt predictive EMSs to improve the fuel economy. Model predictive control (MPC) is commonly employed to implement predictive energy management. Apart from this approach, predictive ECMS [124] can also be performed.
The general cost function of the predictive EMSs is commonly formulated as Equation (9) and the constraint is given as Equation (10). The optimal problem can be solved by minimizing Equation (9) under the constraint Equation (10).

\[
J = \int_{t_k}^{t_{k+1}} \left[ (\dot{m}_f(u(t))^2 + \lambda F(t)) \right] dt
\]

\[
\begin{align*}
SOC_{\text{min}} & \leq SOC \leq SOC_{\text{max}} \\
W_{e_{\text{min}}} & \leq W_e \leq W_{e_{\text{max}}} \\
W_{m_{\text{min}}} & \leq W_m \leq W_{m_{\text{max}}} \\
P_{m_{\text{min}}} & \leq P_m \leq P_{m_{\text{max}}} \\
P_{e_{\text{min}}} & \leq P_e \leq P_{e_{\text{max}}} \\
\end{align*}
\]

where \( J \) is the cost function; \( H_0 \) is the prediction horizon; \( \dot{m}_f(u(t)) \) is the fuel consumption; \( u(t) \) is the control input (e.g., engine torque, motor torque, and gearshift); \( F(t) \) is the other performance factors, such as emissions and drivability, etc.; \( \lambda \) is the penalty coefficient; \( SOC_{\text{min}}(t) \) and \( SOC_{\text{max}}(t) \) are the minimum \( SOC \) and maximum \( SOC \), respectively, with \( SOC \) being the state of charge; \( W_{e_{\text{min}}}(t) \) and \( W_{e_{\text{max}}}(t) \) are the minimum and maximum speed of the engine; \( W_{m_{\text{min}}}(t) \) and \( W_{m_{\text{max}}}(t) \) are the minimum and maximum speed of the motor, respectively. \( P_{m_{\text{min}}}(t) \) and \( P_{m_{\text{max}}}(t) \) are the minimum and maximum power of the motor; \( P_{e_{\text{min}}}(t) \) and \( P_{e_{\text{max}}}(t) \) are the minimum and maximum power of the engine; \( P_m(t) \) and \( P_e(t) \) are the power of the motor and engine, respectively; and \( W_m(t) \) and \( W_e(t) \) are the speed of the motor and engine, respectively.

In the following subsection, typical prediction techniques as well as predictive EMSs are elaborated.

5.2.1. The Driving Cycle Prediction Approach

It is important to predict the driving cycle for EMSs, especially for predictive EMSs. The main challenge of EMSs is that the power split is conducted under a given standard driving cycle, which cannot achieve the best fuel economy due to the uncertainty of the driving cycles. Especially for the city condition, many uncertain factors exist, such as traffic congestion and driving habits. Thus, it is very important to predict the driving cycle for the energy management of HEVs. In this section, typical prediction methods are introduced. More predictive techniques can be advised in [125].

A. Driving pattern recognition

The driving database can be obtained by dividing the standard driving cycles into several segments to extract the feature parameters, including velocity, acceleration, and deceleration. The whole driving cycle can then be constructed by comparing the current driving pattern with all the past databases to find a match. At present, this approach is widely used in recognizing driving patterns. However, the identified driving cycle may be different from the actual one due to the complexity and uncertainty of the real driving cycle. For this approach, the fuzzy recognition method as well as artificial neural network are commonly adopted.

Langari et al. [126] proposed an intelligent energy management strategy for a parallel hybrid electric vehicle on the basis of the driving pattern identification. They utilize vehicle static information (e.g., velocity and acceleration) to improve fuel economy in different driving conditions. The cycle characteristic parameters, such as maximum speed, minimum speed, acceleration, and deceleration, are used to recognize the driving cycle. Wu et al. [127] proposed a learning vector quantization (LVQ) algorithm by extracting the driving condition parameters to recognize the driving pattern, which can be integrated into a fuzzy torque distribution controller for improved adaptability. Simulation results demonstrate that this method enhances the fuel economy more effectively than that of without driving cycle recognition. Murphey et al. [128] also extracted the driving characteristic parameters from standard driving cycles by dividing them into several segments and classifying historical data for different roadway types. The collected data can be trained with a neural network (NN) to identify the type of driving cycle. Finally, the current driving cycle...
can be identified according to the input parameters from the NN. Simulation results show that the EMS with driving identification can significantly improve the fuel economy.

Driving pattern recognition is usually adopted in optimization for a city bus due to its relatively fixed route. Zhu et al. [129] proposed a dynamic optimization method based on driving cycle self-learning in view of a relatively fixed route for a series of hybrid city buses. The velocity and mileage for certain routes are accumulated by an on-board information unit. The database server receives the data through GPRS and extracts the kinematics segment, and then the clustering approach is used to construct the entire driving cycle. Finally, dynamic programming is utilized to optimize the control parameters and load them into a hybrid controller unit. Bender et al. [130] presented an energy management strategy for hybrid hydraulic vehicles based on driving cycle prediction in terms of the repetitive operation characteristics for a city bus. The velocity and acceleration are captured by a GPS and on-board unit. After data processing and filtering, current driving data, including the beginning and ending position of the interval, is extracted to obtain the speed–position profiles as a history database. In the following, the current velocity profile can be predicted by comparing the start and stop position of each new interval with that of the acceleration process included in the history database if the set threshold value is satisfied. Finally, DP was implemented according to the predicted velocity profile to evaluate the effect of prediction error on fuel economy. The results show that fuel savings increased by 5% with the recognition of the driving cycle.

B. Traffic flow modeling

The velocity can also be obtained by modeling a driving cycle approximately with the help of a traffic flow model in the field of transportation. Due to the relationship between vehicle speed and traffic flow, the velocity is estimated using a mathematical model as well as a probability method with historical traffic data (e.g., traffic volume, speed, and occupancy). The traffic flow models (e.g., macroscopic or microscopic models) are utilized to predict the velocity, only reflecting the regular characteristics approximately and neglecting other factors. Furthermore, it is difficult to accurately represent the actual cycle conditions because of the uncertainty of the actual driving conditions.

In [131], a piecewise modeling approach is proposed to obtain an entire driving cycle assuming that velocity and acceleration are kept constant at different intervals. The velocity is also given by analyzing the historical data. Meanwhile, considering the influence of road slope on fuel economy, an energy management strategy based on Multi-Information Integrated Trip modeling is developed. The influence of interval length on fuel economy is analyzed, indicating that a long interval length leads to less computational time and a worse fuel economy using dynamic programming. In [132], an optimal EMS is introduced based on a traffic flow model called the gas-kinetic model for highways. Simulation results show that the gas-kinetic model can reflect the dynamic characteristic of the actual driving conditions and improve the fuel economy under different driving cycles.

C. Driving cycle prediction based on an Intelligent Transportation System (ITS)

An Intelligent Transportation System (ITS) aims to provide innovative services related to traffic management and enables various users to be better informed about traffic conditions and having a safe trip. The ITS does not only offer traffic information for an energy management strategy but also provides a promising way to enhance the road traffic safety via intelligent vehicle technology. One way to do so, the vehicular velocity can be predicted over a certain horizon by accumulating real-time traffic data (e.g., traffic condition, signal phase and timing, and road grade) with Global Position System (GPS), Geographic Information System (GIS), vehicle-to-vehicle communication, an on-board units. The predictive EMSs can be then be implemented considering this future information. The corresponding performances of the EMSs can be highly improved, since multi-source information from ITS, GPS, and GIS could be combined for reducing the uncertainty of future driving conditions to further improve the prediction accuracy [125].

To improve the prediction accuracy, He et al. [133] presented a driving cycle prediction method with real-time traffic data from the communication between a vehicle and infrastructure (V2I) to predict the velocity using neural networks. The predicted velocity is sent to a vehicle’s on-board unit
to calculate the driver’s power demand. The influence of prediction error, penetration rate, and window size on fuel economy are also analyzed. Simulation results show that fuel savings can increase by 14% under the UDDS (Urban Dynamometer Driving Schedule) and the average velocity prediction error with V2I communication is 13.2%. Considering the influence of road terrain on an energy management strategy, Zhang et al. [134] proposed a new strategy for solving the problem of not achieving the best fuel economy with traditional energy management due to a lack of information about the upcoming driving cycle. With future road terrain determined by Geographic Information System (GIS), the optimal results using DP and ECMS in comparison to rule-based EMSs are analyzed for the case of having a terrain preview and no preview. The results show that fuel savings with a terrain preview increase from 1% to 4% and enhance the longevity of the battery on the uphill. Fu et al. [135] proposed a real-time optimal energy management strategy based on driving cycle prediction, utilizing information attainable from Intelligent Transportation Systems (ITS). The effect of prediction error on the optimal results is also analyzed. The results of using model predictive control (MPC) and A-ECMS are compared, respectively, which is based on different prediction errors using standard and actual driving cycles as a prediction benchmark. The results indicate that a small deviation in the final SOC and fuel economy are introduced when the prediction error is small. Thus, it is important to investigate the influence of prediction error on fuel economy due to sensor precision and delay of ITS. Gong et al. [136–138] modeled a driving cycle using real-time traffic data from ITS, GIS, and GPS. Two kinds of models are introduced, utilizing historical driving data and only real-time driving data. The accumulated historical data is classified into urban, highway, and countryside conditions. The characteristic parameters (e.g., maximum acceleration, maximum speed limit, and average waiting time) of each segment are extracted to generate an approximate driving cycle. Simulation results under the two kinds of models were analyzed, indicating that the EMSs with historical data modeling is better than those without them. The fuel economy of the proposed energy management algorithm is better than the rule-based EMS.

D. Driving cycle prediction using artificial intelligence

Machine learning is the science of having computers to act without being explicitly programmed, which can be used to build smart robots (for perception and control), text understanding (web search and anti-spam), computer vision, medical informatics, audio, database mining, and other areas. As a machine learning algorithm, Artificial Neural Network (ANN) is deployed in classification, prediction, pattern recognition, and clustering. The application of ANN to predict driving and handling behaviors [139], city power load [140], and traffic flows [141] have demonstrated its strong capability in predicting nonlinear dynamic behaviors. In [142], three kinds of prediction methods, including exponentially varying, the Markov process, and ANN, are compared. The prediction is performed over each receding horizon and the predicted velocity is utilized for energy optimization of a power-split HEV. The results show that the ANN-based velocity predictor yields the best performance for predictive energy management. In [143], considering the vehicle-to-vehicle communication (V2V) and vehicle-to-infrastructure communication (V2I) information, a Bayesian Network approach is presented to predict the velocity by assuming a stochastic model of the velocity of the preceding vehicle. The results demonstrate that the prediction yields a higher accuracy within a certain horizon.

5.2.2. Model predictive control (MPC)

Model predictive control (MPC) describes the development of tractable algorithms for uncertain, stochastic, and constrained systems. As a mathematical method, model predictive control aims to optimize a future system output by calculating the system input trajectory [144]. The main idea of MPC is that the future control output is predicted by an online optimization according to historical information, as well as by future input and output. The principle diagram of an MPC is shown in Figure 9. Upon the error between the reference and the predictive output, the control sequence can be obtained by combining the historical input, historical output, and predictive input. However, it
requires a higher computational burden if the vehicle model is complex. It can be used as a real-time energy management strategy when the computational load is decreased.

In recent years, MPC has been widely adopted in EMSs. The purpose of MPC-based EMSs is to optimize the power split over a prediction horizon and update the control input, by transforming the global optimization problem into a local optimization for the whole driving condition. Compared with other EMSs, MPC is a rolling horizon optimization method based on system prediction information. The main advantages of an MPC are that it can deal with constraints explicitly, i.e., state variables, input, and output constraints. In addition, the constraints can be formulated as quadratic or nonlinear programming problems, by predicting the system dynamic behavior [145]. The traditional optimization algorithm cannot effectively deal with the impact of the uncertainty of the future working conditions on vehicle performance. In view of this, MPC adopts local optimization, rolling optimization and feedback correction to solve this problem efficiently. To this end, some forms of MPC has been developed in optimizing the power split, such as hybrid MPC [146], distributed MPC [147,148], variable horizon MPC [149], adaptive MPC [150], and tube-based MPC [151]. Generally, linear MPC and nonlinear MPC are commonly formulated in optimizing the power split. Borhan et al. [115] presented an MPC-based energy management strategy for the power split of a hybrid electric vehicle. Energy management is a constrained nonlinear optimal problem. The MPC is utilized to split the power between the engine and motor to regulate the engine operating point at each sample time. Simulation results of the nonlinear MPC show a noticeable improvement in the fuel economy with respect to linear time-varying MPC. In [152], the power management based on nonlinear MPC with an adaptive prediction time horizon is proposed. An MPC-based control algorithm based on load profile prediction is proposed. In this approach, results show that the MPC-based solution yields better performance for total energy consumption in comparison to the conventional approach and strongly depends on the performance of the prediction algorithm. If the predicted velocity could match well with the measured velocity, then the time horizon increases, and vice versa. Thus, it is significant to decrease the computational load to improve the performance of energy management.

In [153], an integrated predictive power management controller is studied. A model-based control approach for a plug-in HEV is proposed to minimize the overall CO₂ emissions. The energy management is formulated as a global optimization problem and cast into a local problem by applying Pontryagin’s Minimum Principle. Simulation is conducted to calibrate the control parameters (e.g., environmental factors, vehicle usage condition, and geographic scenarios) and investigate their influence on the fuel economy. Results show that the sensitivity of proposed EMSs on the driving cycle is not significant. To design a torque controller for a parallel hybrid electric vehicle, He et al. [154] developed a torque demand control approach based on MPC. The torque distribution controller has the function of the torque split, torque demand, torque compensation, and torque limit. The engine torque controller is designed based on a nonlinear mean-value model with MPC. The difference between the demand torque and the actual engine torque is compensated for by motor torque because of the nonlinear and lag of the engine torque response. The motor torque
controller is developed based on a linear MPC and the transient torque load of a hybrid powertrain is estimated with a PI observer.

5.2.3. Stochastic model predictive control (SMPC)

The common MPC generally utilizes the predicted velocity provided by an exponential estimation or neural network, which have been well studied in [155,156]. This method is based on the standard driving cycles without considering the uncertainty of the driving cycles. Thus, it yields less adaptability to the changing driving cycles. To this end, the stochastic model predictive control (SMPC) is proposed in [157–159], which mainly employs the predicted velocity by a Markov chain and an MPC in optimizing the power split. In this method, the distribution of the driver’s future power demand can be obtained by a Markov chain and the MPC is then adopted to obtain the optimal power split. A linear optimization method is utilized for solving the SMPC with a lower computational burden, which can be regarded as an online EMS. Stochastic model predictive control (SMPC) accounts for model uncertainties and disturbances based on their probabilistic description [160].

In [161], a new model predictive control for a series hybrid electric vehicle is proposed based upon the Markov chain process. In this method, the driver power demand is modeled as a Markov chain to represent the driver future power request. All possible distribution of power demand in the next step with all possible Markov states at each time step are generated iteratively. In the following, SMPC is used to split the power between the engine and the motor over a distribution of future power demand given the current one at each sample time. Simulation is conducted under standard driving cycles to obtain a better fuel economy, compared to other deterministic approaches. The advantage of SMPC is that its optimization is feasible in real-time control with respect to SDP. Xie et al. [162] proposed a model predictive energy management for plug-in HEVs based on Pontryagin’s Minimum Principle. The design utilizes a Markov chain model to predict the velocity and achieves a higher computation efficiency. Most of the literature works do not consider the battery aging in devising EMSs, especially for a plug-in HEV. For this purpose, Chen et al. [163] developed a nonlinear model predictive control for a power-splitted HEV considering battery aging. Better battery aging performance is achieved compared with that without considering battery aging while obtaining a similar fuel economy performance.

5.2.4. Learning-Based SMPC

Learning-based SMPC aims to integrate an MPC with machine learning algorithms to improve the performance of the MPC controller in a data-driven way. In contrast to SMPC, which assumes that the driver’s power demand can be modeled offline based on the Markov chain, learning-based SMPC can update the Markov chain by online learning, which allows adjusting to variations in the driver behavior with minimal computational effort in real-time control. Therefore, it can dynamically adapt to the changing driving behaviors, such as environmental changes and varying traffic conditions. This approach is more realistic than SMPC in terms of capturing driver actions as well as driving styles. In addition, a traditional MPC generally assumes that the vehicle model parameters are time-invariant; however, actual vehicle models are usually time-varying, such as the vehicle load and battery life that change with time, for construction vehicles or hybrid electric buses. Thus, to devise a robust MPC-based energy management, a new learning-based model predictive control should be developed by adjusting the model parameters adaptively and updating the system model dynamically with online learning, which can capture the dynamic characteristics of the control objectives.

In [164], the driver’s power demand Markov chain model is updated by online learning, which can be reconfigured in real-time for accommodating the changes in driver behavior. To capture the driver behaviors, online learning of the Markov chain is introduced to tackle the uncertainty that arises from the environment around the vehicle. By updating the Markov model, the controller can adapt to the changes in driver behavior with less computational effort. Learning-based SMPC is adopted to determine the power split of a series hybrid electric vehicle, where the driver model
predicts the future power request that relates to the driving style and driving cycle. Simulation results for standard and real-world driving cycles show that learning-based SMPC improves the performance of classical MPC with the learned pattern of driver behavior.

5.3. Learning-Based EMSs

The learning-based EMSs aim to update the control parameters of EMSs online by interacting with the environment to adapt to the various traffic conditions. They generally employ massive historical and real-time driving-related data to obtain the optimal solution. For this method, the precise model data is not required. Reinforcement learning (RL) and machine learning are commonly used to devise such EMSs. This method can capture the dynamic traffic conditions and yield to potential real-time applications. In [165], the concept of EMSs based on learning is introduced to combine the optimal EMS with the learning method to enhance the robustness of the EMSs. A predictive energy management strategy for a parallel HEV is designed by means of a reinforcement learning approach. Similar work can also be found in [166–169]. Moreover, an overview of reinforcement learning-based EMSs can be found in [170]. A reinforcement learning system is composed of two items: a learning agent and an environment where the learning agent interacts continuously with the environment. The state of the environment can be observed at each instant for the learning agent. The learning agent then selects an action, which is subsequently inputted to the environment. The reward associated with the transition is calculated and fed back to the learning agent while the environment shifts to a new state because of the action. Together with each state transition, the agent can receive an immediate reward to produce a control policy that represents the current state to the best control action for that state. At each instant, the agent makes a decision based on its control policy. Finally, the optimal policy can lead the learning agent executing the best series of actions to maximize the cumulated reward over time, which can be learned after satisfactory training. The advantage is that the design is a model-free control and provides more adaptability compared to other EMSs. However, these require more driving-related data for training.

To obtain a trade-off between optimal fuel savings and real-time performance, Qi et al [171] proposed a reinforcement learning-based real-time EMS for PHEVs by learning the optimal decisions from historical driving cycles. In [172], a deep reinforcement learning-based PHEV EMS is devised to autonomously learn the optimal behaviors from its own historical driving cycles to adapt to the changes in driving conditions. Most of the works in the literature ignore battery health in devising learning-based EMSs; to this end, in [173], a reinforcement learning-based real-time energy management is developed for PHEV by considering the battery health. To further achieve higher computation efficiency, Sun et al. [174] developed a reinforcement-learning-based EMS by combining the ECMS for fuel cell Hybrid Electric Vehicles.

Apart from the learning-based EMSs, the distributed optimization (DO) approach was recently proposed to solve the complete vehicle energy management problem. Romijn et al. [175] proposed a distributed optimization (DO) approach for a hybrid truck with a refrigerated semi-trailer, an air supply system, an alternator, a dc–dc converter, a low-voltage battery, and a climate control system. A dual decomposition is firstly applied to the optimal control problem such that the problem related to each subsystem can be solved separately. Then, an Alternating Direction Method of Multipliers method is used to efficiently solve the optimal control problem for every subsystem in the vehicle. Simulation results show that the fuel consumption can be reduced up to 0.52% by including auxiliaries in the energy management problem, assuming that the auxiliaries are continuously controlled. The computation time is reduced by a factor of 64 up to 1825, compared with solving a centralized convex optimization problem.

6. Conclusion and Future Trends

The EMSs of Hybrid Electric Vehicles have been extensively studied and compared. The offline EMSs aim to minimize fuel consumption globally. Although they cannot be directly implemented in a real vehicle, they provide a benchmark for other energy management strategies and obtaining modified online EMSs. The online EMSs are relatively easy to implement in a real vehicle due to a
lower computational burden and no prior knowledge of the whole driving cycle, while achieving similar performance (e.g., fuel economy) as compared to the offline EMSs. The instantaneous optimization-based EMSs are a promising way to compromise real-time implementation and fuel consumption minimization. Driving cycle prediction is important in predictive EMSs. As the ITS technology is increasingly developed, the predictive EMSs are capable of better adjustability and represent a better performance compared to other EMSs. Although different EMSs have been conducted in recent years, offering remarkable solutions, some important future trends need to be further considered.

6.1. The Predictive EMSs Considering Dynamic Traffic Conditions with ITS

The main challenge of current EMSs is that solutions are generally devised under specific driving cycles, which bring about the impossibility to attain optimal results in a real cycle. Despite the global optimization that obtains optimal results in theory, it is hard to implement in real-time vehicle controllers because of computational complexity. Predicting the driving cycle is an effective way for real-time optimization-based energy management strategies considering dynamic traffic conditions and future information. In this design, the real-time traffic data can be dynamically obtained with intelligent transportation system (e.g., vehicle-to-vehicle communication technology).

If the driving cycle can be predicted as accurate as possible by taking into consideration traffic congestion and road slope, the EMSs can be effectively performed. Therefore, incorporating dynamic traffic conditions into EMSs and investigating predictive EMSs based on driving cycle prediction are possible future trends.

6.2. Real-Time EMSs Incorporating Components Response and Accurate Vehicle Models

Currently, most of the EMSs aim to minimize the fuel consumption and emissions to obtain global results. Although these EMSs can provide a benchmark for researchers, it is more challenging to implement in real-time. The vehicle controller not only determines the power split but also is responsible for acquiring data, monitoring the operation state, and diagnosing the fault required for a high real-time performance. This is essential for vehicle prompt response once the control command has been received. The computational complexity of global optimization is acceptable for simulation, whereas it is impossible to update control parameters for real-time application. The adopted numerical optimization methods for simplified vehicle models can reduce the computational complexity, which, on the other hand, become less attractive if the nonlinear characteristic of the vehicle model is considered. Considering the nonlinear characteristics of the vehicle model results in obtaining a higher accuracy for the optimization results.

Rule-based EMSs are commonly performed in real vehicles. Because the other optimization algorithms are hard to implement due to their computational complexity, they utilize simplified vehicle models, which lead to unexpected energy management results in practice. Consequently, how to simplify the vehicle model and reduce the computational complexity to ensure the real-time performance of the optimization algorithm will be an urgent problem that needs to be solved in the near future.

6.3. Multi-Objectives EMSs Incorporating Battery Aging and Drivability

It is well known that a hybrid electric vehicle is a complex and nonlinear system composed of many components: engine, motor, and transmission, which are highly coupled. Multiple performance indexes (e.g., drivability and fuel economy) are influenced by each other. Thus, it is important to trade-off different performance objectives since these are conflicted with each other in a variety of operating mode switches. In addition, battery aging will also affect energy efficiency and fuel economy.

Currently, the energy management strategy mainly aims at minimizing fuel economy and emissions while neglecting other performance factors involving battery life as well as drivability.
Thus, how to incorporate these performance indexes to implement them in an integrated optimization is a key issue.

6.4. Adaptive EMSs Considering Driver Characteristics and More Influential Factors

To the best of our knowledge, most of the EMSs are demonstrated by simulation over a specific driving cycle. However, the actual driving condition is complex and diverse; for instance, traffic congestion in cities, highways, urban, and suburban areas. In addition, driving behaviors (e.g., driving styles) is another important factor in the driving cycle. Different drivers may take different actions toward the same situation, leading to uncertainty in driving cycles. The optimal results of the EMSs are strongly dependent on the driving cycle while it is hard to adapt to various driving conditions using existing EMSs. Therefore, to develop adaptive EMSs may be a promising solution for the HEVs.

6.5. Multi-Dimension EMSs Including Route Planning and Velocity Planning

It is well known that the performance of EMSs is related to the vehicular velocity and traffic conditions. The changing traffic conditions make it challenging to implement a high energy-efficiency-oriented energy management strategy. This is due to the uncertainty of the vehicle route and velocity affected by traffic conditions. Moreover, different routes present a distinct traffic condition, even if the vehicle operates on the same route since the traffic condition may be diverse. All these factors bring uncertainty and disturbance for optimizing EMSs. Traditional EMSs mainly consider fuel-to-powertrain optimization instead of combining economic route planning and optimal velocity planning. Thus, how to integrate powertrain optimization, route, and velocity planning to further improve the energy efficiency is a key challenge.

Author Contributions: Wrote the original manuscript, F.Z.; edited and proofread the manuscript, L.W and S.C.; reviewed and provided good suggestions, H.P, Y.C and J.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by National Natural Science Foundation of China (Grant No.51905419) and Natural Science Basic Research Program of Shaanxi (Grant No.2019JQ-503), and the Fundamental Research Fund for the Central Universities of China (grant No. 300102229514 and No.300102229502).

Conflicts of Interest: The authors declare no conflicts of interest.

References
1. Zhang, F.Q.; Hu, X.S.; Langari, R.; Cao, D.P. Energy management strategies of connected hevs and phevs: Recent progress and outlook. Prog. Energy Combust. Sci. 2019, 73, 235–256.
2. Onori, S.; Serrao, L.; Rizzoni, G. Hybrid Electric Vehicles: Energy Management Strategies; Springer: Berlin/Heidelberg, Germany, 2016.
3. Wang, Q.; You, S.; Li, L.; Yang, C. Survey on energy management strategy for plug-in hybrid electric vehicles. J. Mech. Eng. 2017, 53, 1–19.
4. Wirasingha, S.G.; Emadi, A. Classification and review of control strategies for plug-in hybrid electric vehicles. IEEE Trans. Veh. Technol. 2011, 60, 111–122.
5. Salmasi, F.R. Control strategies for hybrid electric vehicles: Evolution, classification, comparison, and future trends. IEEE Trans. Veh. Technol. 2007, 56, 2393–2404.
6. Karbaschian, M.; Söffker, D. Review and comparison of power management approaches for hybrid vehicles with focus on hydraulic drives. Energies 2014, 7, 3512–3536.
7. Tran, D.-D.; Vafaipour, M.; Baghdadi, M.E.; Barrero, R.; Hegazy, O. Thorough state-of-the-art analysis of electric and hybrid vehicle powertrains: Topologies and integrated energy management strategies. Renew. Sustain. Energy Rev. 2019, 119, 109596.
8. Miller, J.M. Propulsion Systems for Hybrid Vehicles; The Institution of Electrical Engineers: London, UK, 2004; Volume 45.
9. He, H.; Zhang, J.; Li, G. Model predictive control for energy management of a plug-in hybrid electric bus. Energy Procedia 2016, 88, 901–907.
10. Donitz, C.; Vasile, I.; Onder, C.; Guzzella, L. Dynamic programming for hybrid pneumatic vehicles. In Proceedings of the 2009 American Control Conference, St. Louis, MO, USA, 10–12 June 2009; IEEE: New York, NY, USA, 2009; pp. 3956–3963.

11. Lin, C.C.; Peng, H.; Grizzle, J.W.; Kang, J.M. Power management strategy for a parallel hybrid electric truck. IEEE Trans. Control Syst. Technol. 2003, 11, 839–849.

12. Patil, R.M.; Filipi, Z.; Fathy, H.K. Comparison of supervisory control strategies for series plug-in hybrid electric vehicle powertrains through dynamic programming. IEEE Trans. Control Syst. Technol. 2014, 22, 502–509.

13. Murphey, Y.L.; Park, J.; Kiliaris, L.; Kuang, M.L.; Abul Masrur, M.; Phillips, A.M.; Wang, Q. Intelligent hybrid vehicle power control-part ii: Online intelligent energy management. IEEE Trans. Veh. Technol. 2013, 62, 69–79.

14. Wu, B.; Lin, C.C.; Filipi, Z.; Peng, H.; Assanis, D. Optimal power management for a hydraulic hybrid delivery truck. Veh. Syst. Dyn. 2004, 42, 23–40.

15. Kutter, S.; Bäker, B. Predictive online control for hybrids: Resolving the conflict between global optimality, robustness and real-time capability. In Proceedings of the 2010 IEEE Vehicle Power and Propulsion Conference, Lille, France, 1–3 September 2010; pp. 1–7.

16. Fares, D.; Chedid, R.; Panik, F.; Karaki, S.; Jabr, R. Dynamic programming technique for optimizing fuel cell hybrid vehicles. Int. J. Hyd rog. Energy 2015, 40, 7777–7790.

17. Zhuang, W.; Zhang, X.; Li, D.; Wang, L.; Yin, G.J.A.E. Mode shift map design and integrated energy management control of a multi-mode hybrid electric vehicle. Appl. Energy 2017, 204, 476–488.

18. Peng, J.; He, H.; Xiong, R. Rule based energy management strategy for a series–parallel plug-in hybrid electric bus optimized by dynamic programming. Appl. Energy 2017, 185, 1633–1643.

19. Yang, Y.; Hu, X.; Pei, H.; Peng, Z. Comparison of power-split and parallel hybrid powertrain architectures with a single electric machine: Dynamic programming approach. Appl. Energy 2016, 168, 683–690.

20. Liu, B.; Li, L.; Wang, X.; Cheng, S. Hybrid electric vehicle downshifting strategy based on stochastic dynamic programming during regenerative braking process. IEEE Trans. Veh. Technol. 2018, 67, 4716–4727.

21. Liu, J.; Chen, Y.; Zhan, J.; Shang, F. Heuristic dynamic programming based online energy management strategy for plug-in hybrid electric vehicles. IEEE Trans. Veh. Technol. 2019, 68, 4479–4493.

22. van Berkel, K.; de Jager, B.; Hofman, T.; Steinbuch, M. Implementation of dynamic programming for optimal control problems with continuous states. IEEE Trans. Control Syst. Technol. 2015, 23, 1172–1179.

23. Moura, S.J.; Fathy, H.K.; Callaway, D.S.; Stein, J.L. A stochastic optimal control approach for power management in plug-in hybrid electric vehicles. IEEE Trans. Control Syst. Technol. 2011, 19, 545–555.

24. Opila, D.F.; Wang, X.Y.; McGee, R.; Gillespie, R.B.; Cook, J.A.; Grizzle, J.W. An energy management controller to optimally trade off fuel economy and drivability for hybrid vehicles. IEEE Trans. Control Syst. Technol. 2012, 20, 1490–1505.

25. Johri, R.; Filipi, Z. Optimal energy management of a series hybrid vehicle with combined fuel economy and low-emission objectives. Proc. Inst. Mech. Eng. Part D J. Automob. Eng. 2014, 228, 1424–1439.

26. Zou, Y.; Kong, Z.; Liu, T.; Liu, D. A real-time markov chain driver model for tracked vehicles and its validation: Its adaptability via stochastic dynamic programming. IEEE Trans. Veh. Technol. 2016, 66, 3571–3582.

27. Xu, F.; Jiao, X.; Sasaki, M.; Wang, Y. Energy management optimization in consideration of battery deterioration for commuter plug-in hybrid electric vehicle. In Proceedings of the 2016 55th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE), Tsukuba, Japan, 20–23 September 2016; pp. 218–222.

28. Du, Y.; Zhao, Y.; Wang, Q.; Zhang, Y.; Xia, H. Trip-oriented stochastic optimal energy management strategy for plug-in hybrid electric bus. Energy 2016, 115, 1259–1271.

29. Lü, X.; Wu, Y.; Lian, J.; Zhang, Y.; Chen, C.; Wang, P.; Meng, L. Energy management of hybrid electric vehicles: A review of energy optimization of fuel cell hybrid power system based on genetic algorithm. Energy Convers. Manag. 2020, 205, 112474.

30. Zhou, S.; Wen, Z.; Zhi, X.; Jin, J.; Zhou, S. Genetic Algorithm-Based Parameter Optimization of Energy Management Strategy and Its Analysis for Fuel Cell Hybrid Electric Vehicles; 0148-7191; SAE Technical Paper: New York, NY, USA, 2019.

31. Piccolo, A.; Ippolito, L.; Galdi, V.Z.; Vaccaro, A. Optimisation of energy flow management in hybrid electric vehicles via genetic algorithms. In Proceedings of the 2001 IEEE/ASME International Conference on
Advanced Intelligent Mechatronics. Proceedings (Cat. No.01TH8556), Como, Italy, 8–12 July 2001; IEEE: New York, NY, USA, 2001; pp. 434–439.

32. Xudong, L.; Yanping, W.; Jianmin, D. Optimal sizing of a series hybrid electric vehicle using a hybrid genetic algorithm. In Proceedings of the 2007 IEEE International Conference on Automation and Logistics, Jinan, China, 18–21 August 2007; pp. 1125–1129.

33. Zhang, Y.; Meng, D.; Zhou, M.; Lu, D. Management strategy based on genetic algorithm optimization for phev. Int. J. Control Autom. 2014, 7, 399–408.

34. Zhang, H.; Su, Y.; Peng, L.; Yao, D. A review of game theory applications in transportation analysis. In Proceedings of the 2010 International Conference on Computer and Information Application, Tianjin, China, 3–5 December 2010; pp. 152–157.

35. Nash, J.F. Equilibrium points in n-person games. Proc. Natl. Acad. Sci. USA 1950, 36, 48–49.

36. Nash, J. Non-cooperative games. Ann. Math. 1951, 54, 286–295.

37. Colman, A.M. Game Theory and Its Applications: In the Social and Biological Sciences; Psychology Press: London, UK, 2013.

38. Gielen, M.J.; Shen, Z.J. Power management strategy based on game theory for fuel cell hybrid electric vehicles. In Proceedings of the IEEE 60th Vehicular Technology Conference, 2004. VTC2004-Fall. 2004, Los Angeles, CA, USA, 26–29 September 2004; pp. 4422–4426.

39. Yin, H.; Zhao, C.; Li, M.; Ma, C.; Chow, M.-Y. A game theory approach to energy management of an engine-generator/battery/ultracapacitor hybrid energy system. IEEE Trans. Ind. Electron. 2016, 63, 4266–4277.

40. Dextreit, C.; Kolmanovsky, I.V. Game theory controller for hybrid electric vehicles. IEEE Trans. Control Syst. Technol. 2014, 22, 652–663.

41. Dextreit, C.; Assadian, F.; Kolmanovsky, I.; Mahtani, J.; Burnham, K. Hybrid Electric Vehicle Energy Management Using Game Theory; 0148-7191; SAE Technical Paper: New York, NY, USA, 2008.

42. Xu, J.; Alsabbagh, A.; Yan, D.; Ma, C. Game-theoretic energy management with velocity prediction in hybrid electric vehicle. In Proceedings of the 2019 IEEE 28th International Symposium on Industrial Electronics (ISIE), Vancouver, BC, Canada, 12–14 June 2019; pp. 1084–1089.

43. Chen, H.; Kessels, J.; Donkers, M.; Weiland, S. Game-theoretic approach for complete vehicle energy management. In Proceedings of the 2014 IEEE Vehicle Power and Propulsion Conference (VPPC), Coimbra, Portugal, 27–30 October 2014; pp. 1–6.

44. Chen, H.; Kessels, J.T.; Weiland, S. Online adaptive approach for a game-theoretic strategy for complete vehicle energy management. In Proceedings of the 2015 European Control Conference (ECC), Linz, Austria, 15–17 July 2015; pp. 135–141.

45. Chen, H. Game-Theoretic Solution Concept for Complete Vehicle Energy Management. Ph. D. Thesis, Technische Universiteit Eindhoven, Eindhoven, The Netherlands, 2016.

46. Orszag, S.A. Comparison of pseudospectral and spectral approximation. Stud. Appl. Math. 1972, 51, 253–259.

47. Hu, X.; Li, S.; Peng, H.; Sun, F. Charging time and loss optimization for linmc and lifepo 4 batteries based on equivalent circuit models. J. Power Sources 2013, 239, 449–457.

48. Zhou, W.; Zhang, C.; Li, J.; Fathy, H.K. A pseudospectral strategy for optimal power management in series hybrid electric powertrains. IEEE Trans. Veh. Technol. 2016, 65, 4813–4825.

49. Wu, J.; Zou, Y.; Zhang, X.; Du, G.; Du, G.; Zou, R. A hierarchical energy management for hybrid electric tracked vehicle considering velocity planning with pseudospectral method. IEEE Trans. Transp. Electrific. 2020, 6, 703–716.

50. Martinez, C.M.; Hu, X.; Cao, D.; Velenis, E.; Gao, B.; Wellers, M. Energy management in plug-in hybrid electric vehicles: Recent progress and a connected vehicles perspective. IEEE Trans. Veh. Technol. 2016, 66, 4534–4549.

51. Murgovski, N.; Johannesson, L.; Sjöberg, J.; Egardt, B. Component sizing of a plug-in hybrid electric powertrain via convex optimization. Mechatronics 2012, 22, 106–120.

52. Nafisi, H.; Agah, S.M.M.; Abyaneh, H.A.; Abedi, M. Two-stage optimization method for energy loss minimization in microgrid based on smart power management scheme of phevs. IEEE Trans. Smart Grid 2015, 7, 1268–1276.

53. Boyd, S.; Boyd, S.P.; Vandenberghe, L. Convex Optimization; Cambridge university press: Cambridge, UK, 2004.
54. Nüssch, T.; Elbert, P.; Flankl, M.; Onder, C.; Guzzella, L. Convex optimization for the energy management of hybrid electric vehicles considering engine start and gearshift costs. Energies 2014, 7, 834–856.
55. Xie, S.; Li, H.; Xin, Z.; Liu, T.; Wei, L. A pontryagin minimum principle-based adaptive equivalent consumption minimum strategy for a plug-in hybrid electric bus on a fixed route. Energies 2017, 10, 1379.
56. Kang, C.; Song, C.; Cha, S. A costate estimation for pontryagin’s minimum principle by machine learning. In Proceedings of the 2018 IEEE Vehicle Power and Propulsion Conference (VPPC), Chicago, IL, USA, 27–30 August 2018; pp. 1–5.
57. Zhang, J.; Zheng, C.; Cha, S.W.; Duan, S. Co-state variable determination in pontryagin’s minimum principle for energy management of hybrid vehicles. Int. J. Precis. Eng. Manuf. 2016, 17, 1215–1222.
58. Li, X.; Wang, Y.; Yang, D.; Chen, Z. Adaptive energy management strategy for fuel cell/battery hybrid vehicles using pontryagin’s minimal principle. J. Power Sources 2019, 440, 227105.
59. Ghasemi, M.; Song, X. A computationally efficient optimal power management for power split hybrid vehicle based on pontryagin’s minimum principle. In Proceedings of the ASME 2017 Dynamic Systems and Control Conference, Tysons, VA, USA, 11–13 October 2017.
60. Nguyen, B.-H.; German, R.; Trovão, J.P.F.; Bouscayrol, A. Real-time energy management of battery/supercapacitor electric vehicles based on an adaptation of pontryagin’s minimum principle. IEEE Trans. Veh. Technol. 2018, 68, 203–212.
61. Kim, N.; Jeong, J.; Zheng, C. Adaptive energy management strategy for plug-in hybrid electric vehicles with pontryagin’s minimum principle based on daily driving patterns. Int. J. Precis. Eng. Manuf. Green Technol. 2019, 6, 539–548.
62. Serrao, L.; Onori, S.; Rizzoni, G. A comparative analysis of energy management strategies for hybrid electric vehicles. J. Dyn. Syst. Meas. Control 2011, 133, 031012–031012.
63. Onori, S.; Tribioli, L. Adaptive pontryagin’s minimum principle supervisory controller design for the plug-in hybrid gm chevrolet volt. Appl. Energy 2015, 147, 224–234.
64. Kim, N.; Rousseau, A.; Lee, D. A jump condition of pmp-based control for phev. J. Power Sources 2011, 196, 10380–10386.
65. Chen, Z.; Mi, C.C.; Xia, B.; You, C.W. Energy management of power-split plug-in hybrid electric vehicles based on simulated annealing and pontryagin’s minimum principle. J. Power Sources 2014, 272, 160–168.
66. Hou, C.; Ouyang, M.G.; Xu, L.F.; Wang, H.W. Approximate pontryagin’s minimum principle applied to the energy management of plug-in hybrid electric vehicles. Appl. Energy 2014, 115, 174–189.
67. Zhu, M.; Wu, X.; Xu, M. Adaptive Energy Management Strategy for Hybrid Vehicles Based on Pontryagin’s Minimum Principle; 0148-7191; SAE Technical Paper. New York, NY, USA, 2020.
68. Park, K.; Son, H.; Bae, K.; Kim, Y.; Kim, H.; Yun, J.; Kim, H. Optimal control of plug-in hybrid electric vehicle based on pontryagin’s minimum principle considering driver’s characteristic. In Proceedings of the International Conference on Vehicle Technology and Intelligent Transport Systems, Porto, Portugal, 22–24 April 2017; pp. 151–156.
69. Jimming, L.; Huei, P. Modeling and control of a power-split hybrid vehicle. IEEE Trans. Control Syst. Technol. 2008, 16, 1242–1251.
70. Ehsani, M.; Gao, Y.; Longo, S.; Ebrahimi, K. Modern Electric, Hybrid Electric, and Fuel Cell Vehicles: Fundamentals, Theory, and Design; CRC press: Boca Raton, FL, USA, 2004.
71. Liu, W. Introduction to Hybrid Vehicle System Modeling and Control; John Wiley & Sons: Hoboken, NJ, USA, 2013.
72. Mamdani, E.H. Application of fuzzy algorithms for control of simple dynamic plant. Inst. Electr. Eng. 1974, 121, 1585–1588.
73. Takagi, T.; Sugeno, M. Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. Syst. Man Cybern. 1985, SMC-15, 116-132.
74. Syed, F.U.; Kuang, M.L.; Smith, M.; Okubo, S.; Ying, H. Fuzzy gain-scheduling proportional–integral control for improving engine power and speed behavior in a hybrid electric vehicle. IEEE Trans. Veh. Technol. 2008, 58, 69–84.
75. Denis, N.; Dubois, M.R.; Desrochers, A. Fuzzy-based blended control for the energy management of a parallel plug-in hybrid electric vehicle. IET Intell. Transp. Syst. 2014, 9, 30–37.
76. Dawei, M.; Yu, Z.; Meilan, Z.; Risha, N. Intelligent fuzzy energy management research for a uniaxial parallel hybrid electric vehicle. Comput. Electr. Eng. 2017, 58, 447–464.
77. Li, S.G.; Sharkh, S.; Walsh, F.C.; Zhang, C.-N. Energy and battery management of a plug-in series hybrid electric vehicle using fuzzy logic. IEEE Trans. Veh. Technol. 2011, 60, 3871–3885.
78. Yu, H.; Tarsitano, D.; Hu, X.; Cheli, F. Real time energy management strategy for a fast charging electric urban bus powered by hybrid energy storage system. Energy 2016, 112, 322–331.
79. Li, J.; Zhou, Q.; Williams, H.; Xu, H. Back-to-back competitive learning mechanism for fuzzy logic based supervisory control system of hybrid electric vehicles. IEEE Trans. Ind. Electron. 2019, 67, 8900–8909.
80. Ma, K.; Wang, Z.; Liu, H.; Yu, H.; Wei, C. Numerical investigation on fuzzy logic control energy management strategy of parallel hybrid electric vehicle. Energy Procedia 2019, 158, 2643–2648.
81. Li, J.; Zhou, Q.; He, Y.; Williams, H.; Xu, H. Driver-identified supervisory control system of hybrid electric vehicles based on spectrum-guided fuzzy feature extraction. IEEE Trans. Fuzzy Syst. 2020, doi:10.1109/TFUZZ.2020.2972843
82. Salman, M.; Schouten, N.J.; Kheir, N.A. Control strategies for parallel hybrid vehicles. In Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No.00CH36334), Chicago, IL, USA, 28–30 June 2000; Volume 521, pp. 524–528.
83. Montazeri-Gh, M.; Mahmoodi-k, M. Development a new power management strategy for power split hybrid electric vehicles. Transp. Res. Part D Transp. Environ. 2015, 37, 79–96.
84. Hyeoun-Dong, L.; Seung-Ki, S. Fuzzy-logic-based torque control strategy for parallel-type hybrid electric vehicle. Ieee Trans. Ind. Electron. 1998, 45, 625–632.
85. Hyeoun-Dong, L.; Euh-Suh, K.; Seung-Ki, S.; Joohn-Sheok, K.; Kamiya, M.; Ikeda, H.; Shinohara, S.; Yoshida, H. Torque control strategy for a parallel-hybrid vehicle using fuzzy logic. Ind. Appl. Mag. 2000, 6, 33–38.
86. Baumann, B.M.; Washington, G.; Glenn, B.C.; Rizzoni, G. Mechatronic design and control of hybrid electric vehicles. IEEE/ASME Trans. Mechatron. 2000, 5, 58–72.
87. Tian, H.; Wang, X.; Lu, Z.; Huang, Y.; Tian, G. Adaptive fuzzy logic energy management strategy based on reasonable soc reference curve for online control of plug-in hybrid electric city bus. IEEE Trans. Intell. Transp. Syst. 2017, 19, 1607–1617.
88. Onori, S.; Serrao, L. On adaptive-ecms strategies for hybrid electric vehicles. In Proceedings of the International Scientific Conference on Hybrid and Electric Vehicles, Malmaison, France, 6–7 December 2011.
89. Zeng, Y.; Cai, Y.; Kou, G.; Gao, W.; Qiu, D. Energy management for plug-in hybrid electric vehicle based on adaptive simplified-ecms. Sustainability 2018, 10, 2060.
90. Geng, B.; Mills, J.K.; Sun, D. Energy management control of microturbine-powered plug-in hybrid electric vehicles using the telemetry equivalent consumption minimization strategy. IEEE Trans. Veh. Technol. 2011, 60, 4238–4248.
91. Han, J.; Kum, D.; Park, Y. Synthesis of predictive equivalent consumption minimization strategy for hybrid electric vehicles based on closed-form solution of optimal equivalence factor. IEEE Trans. Veh. Technol. 2017, 66, 5604–5616.
92. Tian, X.; He, R.; Sun, X.; Cai, Y.; Xu, Y. An anfis-based ecms for energy optimization of parallel hybrid electric bus. IEEE Trans. Veh. Technol. 2019, 69, 1473–1483.
93. Xie, S.; Hu, X.; Qi, S.; Lang, K. An artificial neural network-enhanced energy management strategy for plug-in hybrid electric vehicles. Energy 2018, 163, 837–848.
94. Yang, S.; Wang, W.; Zhang, F.; Hu, Y.; Xi, J. Driving-style-oriented adaptive equivalent consumption minimization strategies for hevs. IEEE Trans. Veh. Technol. 2018, 67, 9249–9261.
95. Paganelli, G.; Delprat, S.; Guerra, T.M.; Rimaux, J.; Santin, J.J. Equivalent consumption minimization strategy for parallel hybrid powertrains. In Proceedings of the Vehicular Technology Conference, 2002. VTC Spring 2002, Birmingham, AL, USA, 6–9 May 2002.
96. Kleimaier, A.; Schroder, D. An approach for the online optimized control of a hybrid powertrain. In Proceedings of the 7th International Workshop on Advanced Motion Control. Proceedings (Cat. No.02TH8623), Maribor, Slovenia, 3–5 July 2002; pp. 215–220.
97. Sciarretta, A.; Back, M.; Guzzella, L. Optimal control of parallel hybrid electric vehicles. IEEE Trans. Control Syst. Technol. 2004, 12, 352–363.
98. Khodabakhshian, M.; Feng, L.; Wikander, J. Improving fuel economy and robustness of an improved ecms method. In Proceedings of the 2013 10th IEEE International Conference on Control and Automation (ICCA), Hangzhou, China, 12–14 June 2013; pp. 598–603.
99. Nüesch, T.; Cerofolini, A.; Mancini, G.; Cavina, N.; Onder, C.; Guzzella, L. Equivalent consumption minimization strategy for the control of real driving nox emissions of a diesel hybrid electric vehicle. *Energies* **2014**, *7*, 3148–3178.

100. Gao, J.P.; Zhu, G.M.G.; Strangas, E.G.; Sun, F.C. Equivalent fuel consumption optimal control of a series hybrid electric vehicle. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* **2009**, *223*, 1003–1018.

101. Skugor, B.; Deur, J.; Cipek, M.; Pavkovic, D. Design of a power-split hybrid electric vehicle control system utilizing a rule-based controller and an equivalent consumption minimization strategy. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* **2014**, *228*, 631–648.

102. Torreglosa, J.P.; Jurado, F.; García, P.; Fernández, L.M. Hybrid fuel cell and battery tramway control based on an equivalent consumption minimization strategy. *Control Eng. Pract.* **2011**, *19*, 1182–1194.

103. Park, J.; Park, J.H. Development of equivalent fuel consumption minimization strategy for hybrid electric vehicles. *Int. J. Automot. Technol.* **2012**, *13*, 835–843.

104. Sun, C.; He, H.; Sun, F. The role of velocity forecasting in adaptive-ecms for hybrid electric vehicles. *Energy Procedia* **2015**, *75*, 1907–1912.

105. Zhang, F.; Liu, H.; Hu, Y.; Xi, J. A supervisory control algorithm of hybrid electric vehicle based on adaptive equivalent consumption minimization strategy with fuzzy pi. *Energies* **2016**, *9*, 919.

106. Musardo, C.; Rizzoni, G.; Staccia, B. A-ecms: An adaptive algorithm for hybrid electric vehicle energy management. In Proceedings of the 2005 and 2005 European Control Conference. CDC-ECC ‘05. 44th IEEE Conference on Decision and Control, Seville, Spain, 12–15 December 2005; pp. 1816–1823.

107. Sezer, V.; Gokasan, M.; Bogosyam, S. A novel ecms and combined cost map approach for high-efficiency series hybrid electric vehicles. *IEEE Trans. Veh. Technol.* **2011**, *60*, 3557–3570.

108. Chen, Z.; Vahidi, A. Route preview in energy management of plug-in hybrid vehicles. *Control Syst. Technol. IEEE Trans.* **2012**, *20*, 546–553.

109. Kim, N.W.; Lee, D.H.; Zheng, C.; Shin, C.; Seo, H.; Cha, S.W. Realization of pmp-based control for hybrid electric vehicles in a backwards-looking simulation. *Int. J. Automot. Technol.* **2014**, *15*, 625–635.

110. Hemi, H.; Ghouili, J.; Cheriti, A. A real time energy management for electrical vehicle using combination of rule-based and ecms. In Proceedings of the 2013 IEEE Electrical Power & Energy Conference, Halifax, NS, Canada, 21–23 August 2013; pp. 1–6.

111. Serrao, L.; Onori, S.; Rizzoni, G. Ecms as a realization of pontryagin’s minimum principle forhev control. In Proceedings of the 2009 American Control Conference, St. Louis, MO, USA, 10–12 June 2009; pp. 3964–3969.

112. Mohd Zulkifli, M.A.; Zheng, J.; Sun, Z.; Liu, H.X. Hybrid powertrain optimization with trajectory prediction based on inter-vehicle-communication and vehicle-infrastructure-integration. *Transp. Res. Part C* **2014**, *45*, 41–63.

113. Zhang, F.; Xi, J.; Langari, R. Real-time energy management strategy based on velocity forecasts using v2v and v2i communications. *IEEE Trans. Intell. Transp. Syst.* **2017**, *18*, 416–430.

114. Sun, C.; Sun, F.; He, H. Investigating adaptive-ecms with velocity forecast ability for hybrid electric vehicles. *Appl. Energy* **2017**, *185*, 1644–1653.

115. Borhan, H.; Vahidi, A.; Phillips, A.M.; Kuang, M.L.; Kolmanovsky, I.V.; Di Cairano, S. Mpc-based energy management of a power-split hybrid electric vehicle. *IEEE Trans. Control Syst. Technol.* **2012**, *20*, 593–603.

116. Gurkaynak, Y. *Neural Adaptive Control Stategy for Hybrid Electric Vehicles with Parallel Powertrain*; Illinois Institute of Technology: Chicago, IL, USA, 2011.

117. Rezaei, A.; Burl, J.; Solouk, A.; Zhou, B.; Rezaei, M.; Shahbakhty, M. Catch energy saving opportunity (ceso), an instantaneous optimal energy management strategy for series hybrid electric vehicles. *Appl. Energy* **2017**, *208*, 655–665.

118. Li, L.; Coskun, S.; Zhang, F.; Langari, R.; Xi, J. Energy management of hybrid electric vehicle using vehicle lateral dynamic in velocity prediction. *IEEE Trans. Veh. Technol.* **2019**, *68*, 3297–3293.

119. Robust control. Available online: https://en.wikipedia.org/wiki/Robust_control (accessed on 30 June 2020).

120. Morales-Morales, J.; Cervantes, I.; Cano-Castillo, U. On the design of robust energy management strategies for hecv. *IEEE Trans. Veh. Technol.* **2015**, *64*, 1716–1728.

121. Motapon, S.N.; Dessaint, L.A.; Al-Haddad, K. A robust h2-consumption-minimization-based energy management strategy for a fuel cell hybrid emergency power system of more electric aircraft. *IEEE Trans. Ind. Electron.* **2014**, *61*, 6148–6156.
122. Pisu, P.; Rizzoni, G. A comparative study of supervisory control strategies for hybrid electric vehicles. *IEEE Trans. Control Syst. Technol.* 2007, 15, 506–518.

123. Karbowski, D.; Kim, N.; Rousseau, A. Route-based online energy management of a phev and sensitivity to trip prediction. In Proceedings of the 2014 IEEE Vehicle Power and Propulsion Conference (VPPC), Coimbra, Portugal, 27–30 October 2014; pp. 1–6.

124. Vadamaluri, R.; Beidl, C.; Barth, S.; Rass, F. Multi-objective predictive energy management framework for hybrid electric powertrains: An online optimization approach 27th aachen colloquium. In Proceedings of the 27th Aachen Colloquium Automobile and Engine Technology, Aachen, Germany, 8–10 October 2018.

125. Zhou, Y.; Ravey, A.; Péra, M.-C. A survey on driving prediction techniques for predictive energy management of plug-in hybrid electric vehicles. *J. Power Sources* 2019, 412, 480–495.

126. Langari, R.; Jong-Seob, W. Intelligent energy management agent for a parallel hybrid vehicle-part i: System architecture and design of the driving situation identification process. *IEEE Trans. Veh. Technol.* 2005, 54, 925–934.

127. Wu, J.; Zhang, C.H.; Cui, N.X. Fuzzy energy management strategy for a hybrid electric vehicle based on driving cycle recognition. *Int. J. Automot. Technol.* 2012, 13, 1159–1167.

128. Murphy, Y.L.; Zhichang, C.; Kiliaris, L.; Jungme, P.; Ming, K.; Masrur, A.; Phillips, A. Neural learning of driving environment prediction for vehicle power management. In Proceedings of the IEEE International Joint Conference on Neural Networks, Hong Kong, China, 1–8 June 2008; pp. 3755–3761.

129. Zhu, D.W.; Hui, X.; Yin, Y.; Song, Z.B. The dynamic optimization of control strategy for hybrid city bus based on driving condition self-learning. *J. Mech. Eng.* 2010, 46, 33–38.

130. Bender, F.A.; Kaszynski, M.; Sawodny, O. Drive cycle prediction and energy management optimization for hybrid hydraulic vehicles. *IEEE Trans. Veh. Technol.* 2013, 62, 3581–3592.

131. Yang, B.; Yaoyu, L.; Quiming, G.; Zhong-Ren, P. Multi-information integrated trip specific optimal power management for plug-in hybrid electric vehicles. In Proceedings of the 2009 American Control Conference, St. Louis, MO, USA, 10–12 June 2009; pp. 4607–4612.

132. Quiming, G.; Yaoyu, L.; Zhong-Ren, P. Trip based optimal power management of plug-in hybrid electric vehicles using gas-kinetic traffic flow model. In Proceedings of the American Control Conference, 2008, Seattle, WA, USA, 11–13 June 2008; pp. 3225–3230.

133. He, Y. Vehicle-Infrastructure Integration Enabled Plug-in Hybrid Electric Vehicles for Energy Management; Clemson University: Clemson, SC, USA, 2013.

134. Zhang, C.; Vahidi, A.; Pisu, P.; Li, X.; Tennant, K. Role of terrain preview in energy management of hybrid electric vehicles. *IEEE Trans. Veh. Technol.* 2010, 59, 1139.

135. Fu, L.; Ozguner, U.; Tulpule, P.; Marano, V. Real-time energy management and sensitivity study for hybrid electric vehicles. In Proceedings of the American Control Conference (ACC), 2011, San Francisco, CA, USA, 29 June–1 July 2011; pp. 2113–2118.

136. Gong, Q.; Li, Y.; Peng, Z.R. Optimal power management of plug-in hev with intelligent transportation system. In Proceedings of the 2007 IEEE/ASME international conference on advanced intelligent mechatronics, Zurich, Switzerland, 4–7 September 2007.

137. Gong, Q.; Li, Y.; Peng, Z.R. Optimal power management of plug-in hybrid electric vehicles with trip modeling. In Proceedings of the ASME International Mechanical Engineering Congress and Exposition, IMECE 2007, Seattle, WA, USA, 11–15 November 2007; American Society of Mechanical Engineers: New York, NY, USA, 2008; Volume 16, pp. 53–62.

138. Quiming, G.; Yaoyu, L.; Zhong-Ren, P. Trip-based optimal power management of plug-in hybrid electric vehicles. *IEEE Trans. Veh. Technol.* 2008, 57, 3393–3401.

139. Lin, Y.; Tang, P.; Zhang, W.J.; Yu, Q. Artificial neural network modelling of driver handling behaviour in a driver-vehicle-environment system. *Int. J. Veh. Des.* 2005, 37, 24–45.

140. Santos, P.J.; Martins, A.G.; Pires, A.J. Designing the input vector to ann-based models for short-term load forecast in electricity distribution systems. *Int. J. Electr. Power Energy Syst.* 2007, 29, 338–347.

141. Vlahogianni, E.; Golas, J.C.; Karlaftis, M.G. Short-term traffic forecasting: Overview of objectives and methods. *Transp. Rev.* 2004, 24, 533–557.

142. Sun, C.; Hu, X.S.; Moura, S.J.; Sun, F.C. Velocity predictors for predictive energy management in hybrid electric vehicles. *IEEE Trans. Control Syst. Technol.* 2014, 23, 1197–1204.

143. Moser, D.; Waschl, H.; Schmied, R.; Efendic, H.; del Re, L. Short term prediction of a vehicle’s velocity trajectory using its. *SAE Int. J. Passeng. Cars Electron. Electr. Syst.* 2015, 8, 364–370.
144. Kouvaritakis, B.; Cannon, M. Model Predictive Control; Springer International Publishing: Cham, Switzerland, 2016.
145. Huang, Y.; Wang, H.; Khajepour, A.; He, H.; Ji, J. Model predictive control power management strategies for hevs: A review. J. Power Sources 2017, 341, 91–106.
146. Li, G.; Goerges, D. Hybrid modeling and predictive control of the power split and gear shift in hybrid electric vehicles. In Proceedings of the 2017 IEEE Vehicle Power and Propulsion Conference (VPPC), Belfort, France, 11–14 December 2017; pp. 1–6.
147. Joševski, M.; Abel, D. Distributed predictive control approach for fuel efficient gear shifting in hybrid electric vehicles. In Proceedings of the 2016 European Control Conference (ECC), Aalborg, Denmark, 29 June–1 July 2016; pp. 2366–2373.
148. Joševski, M.; Abel, D. Gear shifting and engine on/off optimal control in hybrid electric vehicles using partial outer convexification. In Proceedings of the 2016 IEEE Conference on Control Applications (CCA), Buenos Aires, Argentina, 19–22 September 2016; pp. 562–568.
149. Cao, J.; Peng, J.; He, H. Research on model prediction energy management strategy with variable horizon. Energy Procedia 2017, 105, 3565–3570.
150. Zhou, F.; Xiao, F.; Chang, C.; Shao, Y.; Song, C. Adaptive model predictive control-based energy management for semi-active hybrid energy storage systems on electric vehicles. Energies 2017, 10, 1063.
151. Joševski, M.; Abel, D. Tube-based mpc for the energy management of hybrid electric vehicles with non-parametric driving profile prediction. In Proceedings of the 2016 American Control Conference (ACC), Boston, MA, USA, 6–8 July 2016; pp. 623–630.
152. Marx, M.; Soffker, D. Optimization of the powerflow control of a hybrid electric powertrain including load profile prediction. In Proceedings of the 2012 IEEE Vehicle Power and Propulsion Conference (VPPC), Seoul, Korea, 9–12 October 2012; pp. 395–400.
153. Stockar, S.; Marano, V.; Canova, M.; Rizzoni, G.; Guzzella, L. Energy-optimal control of plug-in hybrid electric vehicles for real-world driving cycles. IEEE Trans. Veh. Technol. 2011, 60, 2949–2962.
154. He, L.; Shen, T.; Yu, L.; Feng, N.; Song, J. A model-predictive-control-based torque demand control approach for parallel hybrid powertrains. IEEE Trans. Veh. Technol. 2013, 62, 1041–1052.
155. Zhang, S.; Xiong, R.; Sun, F. Model predictive control for power management in a plug-in hybrid electric vehicle with a hybrid energy storage system. Appl. Energy 2017, 185, 1654–1662.
156. Xiang, C.; Ding, F.; Wang, W.; He, W. Energy management of a dual-mode power-split hybrid electric vehicle based on velocity prediction and nonlinear model predictive control. Appl. Energy 2017, 189, 640–653.
157. Li, L.; You, S.; Yang, C.; Yan, B.; Song, J.; Chen, Z. Driving-behavior-aware stochastic model predictive control for plug-in hybrid electric buses. Appl. Energy 2016, 162, 868–879.
158. Xie, S.; He, H.; Peng, J. An energy management strategy based on stochastic model predictive control for plug-in hybrid electric buses. Appl. Energy 2017, 196, 279–288.
159. Xie, S.; Hu, X.; Xin, Z.; Li, L. Time-efficient stochastic model predictive energy management for a plug-in hybrid electric bus with an adaptive reference state-of-charge advisory. IEEE Trans. Veh. Technol. 2018, 67, 5671–5682.
160. Mesbah, A.; Kolmanovsky, I.V.; Di Cairano, S. Stochastic model predictive control. In Handbook of Model Predictive Control; Springer: Berlin/Heidelberg, Germany, 2019; pp. 75–97.
161. Ripaccioli, G.; Bernardini, D.; Di Cairano, S.; Bemporad, A.; Kolmanovsky, I.V. A stochastic model predictive control approach for series hybrid electric vehicle power management. In Proceedings of the 2010 American Control Conference (ACC), Baltimore, MD, USA, 30 June–2 July 2010; pp. 5844–5849.
162. Xie, S.; Hu, X.; Xin, Z.; Brighton, J. Pontryagin’s minimum principle based model predictive control of energy management for a plug-in hybrid electric bus. Appl. Energy 2019, 236, 893–905.
163. Cheng, M.; Chen, B. Nonlinear model predictive control of a power-split hybrid electric vehicle with consideration of battery aging. J. Dyn. Syst. Meas. Control 2019, 141, 81108.
164. Di Cairano, S.; Bernardini, D.; Bemporad, A.; Kolmanovsky, I.V. Stochastic mpc with learning for driver-predictive vehicle control and its application to hev energy management. IEEE Trans. Control Syst. Technol. 2014, 22, 1018–1031.
165. Liu, T.; Hu, X.; Li, S.E.; Cao, D. Reinforcement learning optimized look-ahead energy management of a parallel hybrid electric vehicle. IEEE/ASME Trans. Mechatron. 2017, 22, 1497–1507.
166. Xu, B.; Malimir, F.; Rathod, D.; Filipi, Z. *Real-Time Reinforcement Learning Optimized Energy Management for a 48v Mild Hybrid Electric Vehicle*; SAE Technical Paper: New York, NY, USA, 2019; doi:10.4271/2019-01-1208.
167. Lian, R.; Peng, J.; Wu, Y.; Tan, H.; Zhang, H. Rule-interposing deep reinforcement learning based energy management strategy for power-split hybrid electric vehicle. *Energy* 2020, 197, 117297.
168. Wang, P.; Northrop, W. *Reinforcement Learning Based Energy Management of Plug-in Hybrid Electric Vehicles for Commuter Route*; 0148-7191; SAE Technical Paper: New York, NY, USA, 2020.
169. Han, X.; He, H.; Wu, J.; Peng, J.; Li, Y. Energy management based on reinforcement learning with double deep q-learning for a hybrid electric tracked vehicle. *Appl. Energy* 2019, 254, 113708.
170. Hu, X.; Liu, T.; Qi, X.; Barth, M. Reinforcement learning for hybrid and plug-in hybrid electric vehicle energy management: Recent advances and prospects. *IEEE Ind. Electron. Mag.* 2019, 13, 16–25.
171. Qi, X.; Wu, G.; Boriboonsomsin, K.; Barth, M.J.; Gonder, J. Data-driven reinforcement learning–based real-time energy management system for plug-in hybrid electric vehicles. *Transp. Res. Rec.* 2016, 2572, 1–8.
172. Qi, X.; Luo, Y.; Wu, G.; Boriboonsomsin, K.; Barth, M.J. Deep reinforcement learning-based vehicle energy efficiency autonomous learning system. In Proceedings of the 2017 IEEE Intelligent Vehicles Symposium (IV), Los Angeles, CA, USA, 11–14 June 2017; pp. 1228–1233.
173. Xiong, R.; Cao, J.; Yu, Q. Reinforcement learning-based real-time power management for hybrid energy storage system in the plug-in hybrid electric vehicle. *Appl. Energy* 2018, 211, 538–548.
174. Sun, H.; Fu, Z.; Tao, F.; Zhu, L.; Si, P. Data-driven reinforcement-learning-based hierarchical energy management strategy for fuel cell/battery/ultracapacitor hybrid electric vehicles. *J. Power Sources* 2020, 455, 227964.
175. Romijn, T.C.J.; Donkers, M.; Kessels, J.T.; Weiland, S. A distributed optimization approach for complete vehicle energy management. *IEEE Trans. Control Syst. Technol.* 2018, 27, 964–980.

© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).