CLUSTER CORRELATION IN MIXED MODELS

A. Gardini and S. A. Bonometto
Dipartimento di Fisica G. Occhialini, Università di Milano Bicocca, INFN sezione di Milano, Via Celoria 16, I-20133 Milano, Italy; gardini@mi.infn.it, bonometto@mi.infn.it

G. Murante
Osservatorio Astronomico di Torino, Strada Osservatorio, 20 Pino Torinese, I-10025, Italy; giuseppe@to.astro.it

AND

G. Yepes
Departamento de Fisica Teorica C–XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain; gustavo.yepes@uam.es

Received 1999 June 17; accepted 2000 May 24

ABSTRACT

We evaluate the dependence of the cluster correlation length, \( r_c \), on the mean intercluster separation, \( D_c \), for three models with critical matter density, vanishing vacuum energy (\( \Lambda = 0 \)), and COBE normalization: a tilted cold dark matter (tCDM) model (\( n = 0.8 \)) and two blue mixed models with two light massive neutrinos, yielding \( \Omega_b = 0.26 \) and 0.14 (MDM1 and MDM2, respectively). All models approach the observational value of \( \sigma_8 \) (and hence the observed cluster abundance) and are consistent with the observed abundance of damped Ly\( \alpha \) systems. Mixed models have a motivation in recent results of neutrino physics; they also agree with the observed value of the ratio \( \sigma_8/\sigma_{25} \), yielding the spectral slope parameter \( \Gamma \), and nicely fit Las Campanas Redshift Survey (LCRS) reconstructed spectra. We use parallel AP3M simulations, performed in a wide box (of side 360 \( h^{-1} \) Mpc) and with high mass and distance resolution, enabling us to build artificial samples of clusters, whose total number and mass range allow us to cover the same \( D_c \) interval inspected through Automatic Plate Measuring Facility (APM) and Abell cluster clustering data. We find that the tCDM model performs substantially better than \( n = 1 \) critical density CDM models. Our main finding, however, is that mixed models provide a surprisingly good fit to cluster clustering data.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general —
  large-scale structure of universe — methods: n-body simulations

On-line material: color figures

1. INTRODUCTION

The study of the clustering of galaxy clusters in the early 1980s allowed a basic advancement in our understanding of large-scale structure (LSS). The discrepancy between the galaxy correlation length, \( r_p \), and the cluster correlation length, \( r_c \) (Bahcall & Soneira 1983; Klypin & Kopylov 1983; but see also Hauser & Peebles 1973), led to the introduction of the concept of bias (Kaiser 1984). Data on \( r_c \) were then worked out in further detail for Abell clusters by Peacock & West (1992) and Postman, Huchra, & Geller (1992), as well as for clusters in the Automatic Plate Measuring Facility (APM) and Edinburgh-Durham Southern Galaxy catalogs by Dalton et al. (1992), Nichol et al. (1992), and Croft et al. (1997).

These analyses show that the value of \( r_c \) depends on the mass threshold (\( M_{th} \)) of the cluster sample, through its mean intercluster separation, \( D_r = n^{-1/3}(M_{th}) \), and that \( r_c \) increases with \( D_r \). However, \( r_c \) values obtained from Abell and APM data seem only partially consistent. This can in part be ascribed to different cluster definitions; Bahcall & Burgett (1986), Bahcall & Cen (1992), and Bahcall & West (1992) suggested that observational ambiguities are wide enough to allow one to conjecture that the scaling relation \( r_c \propto 0.4D_r \) holds for \( 20 < D_r < 100 \) \( h^{-1} \) Mpc. Here, we refer to this relation as the BW conjecture. It should be borne in mind that above \( \sim 50 \) \( h^{-1} \) Mpc, such a conjecture hinges on the estimates of \( r_c \) for 55 and 94 \( h^{-1} \) Mpc mean separations, for richness \( R \geq 1 \) and \( R \geq 2 \) Abell clusters, while APM data, for the same \( D_r \) range, give smaller \( r_c \).

Dekel et al. (1989) and Sutherland & Efstathiou (1991) suggested that the projection effects and peculiar inhomogeneities in the Abell sample might have biased \( r_c \) upward at large \( D_r \). Peacock & West (1992) instead confirmed such points (see also Jing, Plionis, & Valdarnini 1992). Altogether, it may be fair to say that the controversy surrounding the observational behavior of \( r_c \) for high \( D_r \) values has not yet been solved, although, as we will see, there may be good reasons to think that different cluster definitions play a key role.

This paper is devoted to a comparison of cluster clustering, as it emerges from such data, with simulations of three cosmological models: a tilted cold dark matter (tCDM) model and two mixed models (MDM1 and MDM2) with cold + hot dark matter. All models have critical matter density, vanishing vacuum energy, and are COBE normalized. During the last few years, much attention has been devoted to models with a positive cosmological constant \( \Lambda \), because of the remarkable data sets on Type Ia supernovae (SNe Ia; see, e.g., Riess et al. 1998; Perlmutter et al. 1998, and references therein). In this work, we will not debate whether mixed models can still offer a fair fit to all cosmological data; they certainly do not fit SNe Ia data, unless their current interpretation has been misled by some systematic bias. In a number of cases, however, mixed models were just not tested, and the success of \( \Lambda \) models in fitting some data set was directly taken as further evidence in their favor. In the case of cluster clustering, we
show that mixed models perform quite well and are surely better than any other model with a matter density parameter $\Omega_m = 1$ considered until now.

In order to fit cluster data with a model, a large simulation volume is required; in fact, we need a fair sample of galaxy clusters for large $D_c$, as well as adequate mass and force resolutions, to identify clusters in a reliable way for small $D_c$. Simulation parameters are therefore set so as to allow a sample of 90 clusters, at least, for large and $D_c$ galaxy clusters for large as well as adequate mass and observation volume is required; in fact, we need a fair sample of the parameter considered until now.

We shall see, 60 particles corresponds to $10^9 M_\odot$.

Cluster clustering has been studied by various authors in simulations. In particular, the behavior of $r_c$ versus $D_c$ for standard CDM and open CDM was studied by Bahcall & Cen (1992), Watanabe, Matsubara, & Suto (1994), Croft & Efstathiou (1994), Eke, Cole, & Frenk (1996), Croft et al. (1997), and Governato et al. (1999). Their results allow us to conclude that CDM models with $n = 1$ may approach the observed behavior of $r_c$ versus $D_c$ only for $\Omega_m < 1$. The behavior of $r_c$ versus $D_c$ in a mixed model was also studied, using particle-mesh (PM) simulations, by Klypin & Rhee (1994) and Walter & Klypin (1996). Their work treated a different mix from those considered here, using a smaller box and resolution. Accordingly, they could inspect only the $D_c$ interval running from ~20 to 45 $h^{-1}$ Mpc. The behavior they found is only marginally consistent with a constant $r_c/D_c$ ratio, but their model does not exhibit much improvement in respect to pure CDM.

The mixed models we consider here were selected on the basis of recent tests on $v$-flavor mixing, which seem to support a nonvanishing $v$ mass. Mixing data come from the solar $v$ deficit (see, e.g., Hampel et al. 1996 for GALEX, and Abdurashitov et al. 1996 for SAGE), the atmospheric $v$ anomaly (Fukuda et al. 1994), and the Liquid Scintillator Neutrino Detector (LSND) experiment (Anthanassopoulos et al. 1995) on $v$'s arising from $\mu^-$ and $\pi^-$ decay. Barger, Weiler, & Whisnant (1998) and Sarkar (1999) show that all the results listed above can agree if a fourth sterile $v$ exists, which can be added without harming big bang nucleosynthesis or Large Electron-Positron (LEP) standard results. Diagonalizing the mass matrix, they eventually obtain the four $v$ mass eigenvalues, which split into two nearly degenerate pairs, corresponding to $m_1 \approx 0$ and $m_2 \approx 1.4 - 1.5$ eV. It must be pointed out that within this picture there remains no contradiction among different experimental results, at variance with earlier analyses, which seemed to find contradictions between LSND and other $v$-mixing results.

In a cosmological context, however, mixed models have been considered for a long time. The transfer function for several mixed models was first computed by Bonometto & Valdarnini (1984). Results for mixed models were then found by a number of authors (see, e.g., Achilli, Occhionero, & Scaramella 1985; Bonometto & Valdarnini 1985; Holtzmann 1989 for results obtainable from the linear theory; and Davis, Summers, & Schlegel 1992; Klypin et al. 1993; Ghigna et al. 1994 for early simulations). After the release of LSND data, Primack et al. (1995) performed simulations of models with two massive $v$'s, yielding $\Omega_b = 0.20$, and found that such a mixture eased some problems encountered by greater $\Omega_b$ models. The possibility of considering mixed models together with blue spectra (primeval spectral index $n > 1$) was first considered by Liddle et al. (1996) and Lucchin et al. (1996). In the former paper, blue mixed models capable of fitting all linear and analytical constraints were shown to exist. In the latter paper, inflationary models leading to blue spectra were discussed, and results of an $N$-body simulation of blue mixed models were reported. Unfortunately, the model considered violated some observational constraints. A systematic study of blue mixed models was recently performed by Bonometto & Pierpaoli (1998) and Pierpaoli & Bonometto (1999), selecting those consistent with cosmic microwave background (CMB) data and data predictable from the linear theory.

In the next section we show that the models considered here, on the basis of $v$-physics, are also suited to fulfill the main observational constraints. In § 3 we review the technique used to simulate their nonlinear evolution. In § 4 we describe how clusters are selected in simulations. Then in § 5 we describe how the two-point correlation function and its error estimates were worked out. In § 6, we show the main results of the $r_c$ versus $D_c$ behavior derived from fits to the two-point functions. Section 7 is devoted to a discussion of the results and the main conclusions we derive from this work.

2. MODEL PARAMETERS

The mixed models discussed in this paper have already been considered in a previous work by Gardini, Bonometto, & Murante (1999; hereafter Paper I). They are models with two equal-mass massive $v$'s, selected by requiring agreement with data that can be fitted using the linear theory. In more detail, we first required agreement with observations at the top and bottom scales, i.e., with COBE data and with the observed damped Ly$\alpha$ system (DLAS) abundance (Storrie-Lombardi et al. 1995). Assuming two massive $v$'s with $m_1 \geq 1.5$ eV, we adjusted the spectral index $n$ so as to agree with the above top and bottom data, choosing the minimum allowed value for the spectral amplitude ($A_{\psi}$). Over intermediate scales, the main constraints to be tested are at 8 and 25 $h^{-1}$ Mpc, where we evaluated the mass variances, $\sigma_8$ and $\sigma_4$, (see below). From such values we can work out the expected spectral slope and cluster abundance (again, see below); comparing their values with observations, we see that $m_1$, values up to ~3 eV can be considered without violating such constraints. Accordingly, we considered two values for the hot dark matter (HDM) density parameter $\Omega_h$, yielding $m_1$, at the top and bottom of the allowed interval.

A CDM model, selected so as to fit the same data in a similar way, was also studied, for the sake of comparison. While mixed models require $n > 1$, even with low $A_{\psi}$, CDM may fit COBE data only if $n < 1$.

Model parameters are shown in detail in Table 1, while Figure 1 shows the spectra obtained from the linear transfer function, $T(k)$, against APM reconstructed spectral points (Baugh & Gatzanaga 1996). The wavenumber $k$ is related to the comoving length scale, $L = 2\pi/k$, and to the mass scale, $M = (4\pi/3)\rho_0 L^3$, where $\rho_0$ is the present density of the universe. Figure 1 also shows other data and results, which are discussed below.

The $\Lambda$CDM model approximates the APM galaxy spectrum slightly better than the standard CDM (SCDM), thanks to its increased slope. However, it still lies quite below the spectral points around the peak at $k \approx 5 \times 10^{-2}$ $h$ Mpc$^{-1}$ (comoving length $\lambda \approx 100 - 120$ $h^{-1}$ Mpc). Blue
mixed models, on the other hand, show a stronger spectral peak, occurring where $\nu$ free-streaming bends a primordial spectrum steeper than the Zeldovich. This causes a steeper downward spectrum for $5 \leq \lambda \lesssim 50$ $h^{-1}$ Mpc, which might be related to the reasons why blue mixed models approach the $\nu_c$ versus $D_\epsilon$ behavior up to large $D_\epsilon$. Mass variances are defined according to the relation

$$\sigma^2(L) = \frac{\pi}{9} \left( \frac{x_0}{L} \right)^{n+3} A_p \int_0^{\infty} du u^{n+2} T^2 \left( \frac{u}{L} \right) W^2(u),$$

with a top-hat window function $W(u) = 3(\sin u - u \cos u)/u^2$ and

$$P(k) = \frac{2\pi^3}{3} \frac{A_p}{x_0^3} (x_0 k)^n,$$

where $x_0$ is the comoving horizon distance. Using equation (2.1), we estimate the parameter

$$\Gamma = 7.13 \times 10^{-3} \left( \frac{\sigma_8}{\sigma_{25}} \right)^{10/3},$$

which (only for pure CDM models) is often approximated as $\Gamma \approx \Omega_h^2$ (see Efstathiou, Bond, & White 1992). Peacock & Dodds (1994), using APM data, and Borgani et al. (1994), using Abell/ACO samples, constrained $\Gamma$ within the $(2 \sigma)$ intervals 0.19–0.27 and 0.18–0.25, respectively. This parameter therefore tests the spectral slope above cluster mass scales.

Values of $\sigma_8$ are directly related to the expected cluster number densities. A direct fit of data with simulations for the cluster mass function is given in Paper I, where, however, a different definition of cluster mass was used. This point will be discussed again below and will be deepened in a forthcoming work.

3. THE SIMULATIONS

The three simulations considered here were used in Paper I, to which we refer the reader for details. They were performed using a parallel N-body code, based on the serial public AP3M code of Couchman (1991), extended in order to treat variable-mass particle sets and used varying the time steps, when needed. We considered a box of side $L = 360$ $h^{-1}$ Mpc (where $h$ is the Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$); here CDM + baryons were represented by $180^3$ particles, whose individual mass is $m_{180} = 2.22 \times 10^{12} h^{-1}$ $M_\odot$. Mixed models also involve two massive $\nu$'s with $m_\nu \approx 0.02$ and 1.63 eV, to yield $\Omega_\nu = 0.26$ and 0.14 (for MDM1 and MDM2, respectively). Hence, slow particles, representing CDM + baryons, have masses of $\Omega_\nu m_{180}$, where $\Omega_\nu = \Omega_{\text{CDM}} + \Omega_{\text{bar}} = 0.74$ and 0.86, respectively, while fast particles, representing hot dark matter (HDM), have masses of $3\Omega_\nu m_{180}$ (the ratio 2:1 between fast and slow particle numbers is required to set initial conditions with locally vanishing linear momentum). Our force resolution can be reported to a Plummer-equivalent smoothing parameter $\epsilon_p \approx 40.6$ $h^{-1}$ kpc. The comoving force and mass resolutions approach the limits of

- **Table 1:** Parameters of the Models

| Parameter | MDM1 | MDM2 | tCDM |
|-----------|------|------|------|
| $\Omega_b$ | 0.26 | 0.14 |      |
| $m_\nu$ (eV) | 3.022 | 1.627 |      |
| $\Omega_c \times 10^3$ | 6.8 | 9 | 6 |
| $n$ | 1.2 | 1.05 | 0.8 |
| $Q_{\text{rms,}\mu}$ (h K) | 12.1 | 13 | 17.4 |
| $\sigma_8$ | 0.75 | 0.62 | 0.61 |
| $\Gamma$ | 0.18 | 0.23 | 0.32 |
| $N_d$ (PS, $\delta_c = 1.69$) | 14.0 | 5.2 | 5.7 |
| $N_d$ (sim) | 10.0 | 4.7 | 6.0 |
| $L_{\text{cl}}$ | 1.3 | 1.2 | 1.3 |

Note: This table lists input parameters or quantities derived from the linear theory, and also the value of $N_d$ (number of clusters of mass greater than $4.2 \times 10^{14}$ $h^{-1}$ $M_\odot$ in a box side of 100 $h^{-1}$ Mpc) as obtained in Paper I. The normalization to COBE quadrupole was deliberately kept at the $\sim 3 \sigma$ lower limit, in order leave some room for the contribution of tensor modes, but keeping consistent with the data. The expected interval for $N_d$ is 4–6, but models with $N_d$ up to 8–10 cannot be safely rejected. $L_{\text{cl}} = 2.25 \Omega_b \Omega_m (\delta_c = 4.25, M = 5 \times 10^9 h^{-1} M_\odot) \times 10^7$ accounts for the amount of gas expected in damped Ly$\alpha$ systems. More details can be found in Paper I. Data provided by Storrie-Lombardi et al. (1995) give $L_{\text{cl}} > 2.2 \pm 0.6$. This is one of the most stringent indicators of the model capacity to produce high-$\sigma$ objects.

- **Figure 1:** Spectra of the three models at $z = 0$. Solid curves give the linear power spectrum and the spectrum corrected for nonlinearity, according to Peacock & Dodds (1996). Open squares show the simulation spectra corrected for CIC (cloud-in-cell; see Paper I for more detail). Open circles with 2 $\sigma$ error bars show the power spectrum measured from the APM survey.
the computational resources of the machine we used (an HP Exemplar SPP2000 X-class processor of the CILEA consortium at Segrate-Milan). The numerical resolution of our simulations were similar to other simulations of pure CDM, with different initial conditions performed by Colberg et al. (1997), Thomas et al. (1998), Cole et al. (1997), and Governato et al. (1999). Mixed-model simulations with a comparable dynamical range have only been performed by Gross et al. (1998), but in a smaller volume.

4. CLUSTER SELECTION

Different criteria can be used to select clusters in simulations. In Paper I, we identified clusters as virialized halos. Here we give results obtained with a cluster definition aimed at more closely approaching their observational definition. However, we widely test and compare the results ensuing from different definitions, and a comprehensive discussion of the fit between particle sets obtained with various criteria will be published elsewhere. Let us state, however, that two-point function outputs are fairly robust; although specific values of the clustering length, \( r_s \), at various \( D_e \) have even significant variations, when the cluster definition is changed, the general trend is always preserved. The mixed models discussed here, however, fit observational data. We illustrate this point with a few examples, without giving outputs for whole sets of different cluster definitions.

In order to approach the observational pattern, here clusters were found with a spherical overdensity (SO) algorithm, based on a fixed sphere radius \( R_e = 1.5 \ h^{-1} \) Mpc. The details of the procedure are close to those suggested by Croft & Efstathiou (1994) and Klypin & Rhee (1994). Hence, effects arising from the limiting magnitude of (observational) samples, border effects, and projection effects are not included. (In addition, the sphere radius is fixed to mimic the Abell cluster definition; clusters found in the APM survey were also selected with smaller \( R_e \). Our simulations were also used to test whether systematic effects arise from a different choice of \( R_e \); the differences we found have no significant impact on the results shown here.)

In more detail, we start the procedure with a friends-of-friends (FOF) algorithm, which finds sets of \( N \) CDM-baryon particles closer than / times the average interparticle separation. Results reported here are obtained using \( f = 0.2 \) and \( N = 25 \). Centers of mass (CM) of FOF groups are then inspected as possible centers for SO. Starting from them, we follow an iterative procedure: CDM baryon particles within a distance \( R_e \) from CM and their CM are found; this is repeated until we reach a stable particle set and fix their CM. Only particle sets containing at least 25 particles are kept, however. When two spheres intersect during the iterative procedure, only the most massive particle set is kept. Our procedure aims to find all clusters above a suitable mass scale. Loose requirements were therefore set on \( f \), in order to explore any possible matter condensation; the dependence of our results on \( N \) was also tested. Reducing \( N \) obviously leads to more FOF groups, and a number of them survive the iteration procedure defined above. Most of such “extraclusters,” however, do not contain many particles. The result of such tests can be summarized by stating that extraclusters of more than ~60 particles, found by lowering \( N \) down to 12, are less than ~0.3%, in all cases; this percentage has no further increase when still lower \( N \) are taken. Hence, for \( N > 60 \), i.e., for \( M > 1.3 \Omega \_b \times 10^{14} \ h^{-1} \ M_\odot \equiv M_{\text{min}} \), our cluster samples can be considered complete. In Figure 2, we show the relation between cluster masses and \( D_e \) values at \( z = 0 \) and 0.8.

Among other tests, we also verified the size of virialized halos contained in clusters as a function of their mass, \( M \). Let \( R_e \) be the radius encompassing a sphere inside which the density contrast is 180, found by starting from the CM of each cluster, but whose actual center is attained through
to their we compute the two-point correlation function, bootstrap error bars for particles within from it. Then let be the mass of all suitable number of iterations, so it is the CM of all particles within \( R_v \) from it. Then let \( M_v \) be the mass of all CDM + baryon particles within \( R_v \). For large clusters, \( R_v \) may exceed \( R_c \); when \( M_v \) is approximately less than \( 7\Omega_m \times 10^{14} h^{-1} M_\odot \), in general it is \( M_v < M_c \). In Figure 3 we show the values of \( M_v \), as a function of \( M_c \), for tCDM. The trend is quite similar for mixed models.

Although \( M_v \) tends to increase with \( M_c \), the trend is clearly not monotonic; this is due to the spread of the values we find for \( R_v \) at any \( M_v \). In spite of that, for \( M > M_{\text{min}} \), all clusters contain a virialized halo. However, if we order the cluster set by mass, using \( M_v \) instead of \( M_c \), we find a different result. Hence, cluster sets, whose mean distance is \( D_c \), are different if we use \( M_v \) instead of \( M_c \). It is then significant to compare the dependence of \( r_c \) on \( D_c \) for the two different orderings. In Paper I, clusters were given \( M_v \) as mass; such a definition is farther from observational criteria, but is likely to be closer to physical requirements, e.g., if we aim to compare simulation outputs with the expectations of a Press-Schechter approach. Below, in a few cases, we test how the clustering length depends on \( D_c \) when \( M_v \) replaces \( M_c \). As we shall see, outputs depend significantly on the model, but are substantially independent of the cluster definition.

5. CLUSTER TWO-POINT CORRELATION FUNCTION

Using clusters in our simulation box, ordered according to their \( M_c \), we compute the two-point correlation function, \( \xi(r) \), for a set of \( D_c \) values, by applying the estimator

\[
\xi(r) = \frac{D^2 v \text{pairs}(r)}{L^2 \delta V(r)} - 1. \tag{5.1}
\]

Here \( \text{pairs}(r) \) is the number of cluster pairs in the radial bin of volume \( \delta V(r) \), centered on \( r \), and \( L^2 \) is the box volume. Error bars for \( \xi(r) \) were estimated using the standard bootstrapping procedure. (We checked the convergence of the estimator of the standard deviation evaluated from bootstrap realizations by inspecting the third moment of the bootstrap distribution. In all cases, convergence was attained when the number of bootstrap realizations matched the number of points in the catalogs; see, e.g., Bradley 1982.) We compared such errors with the usual Poisson errors, which were found to be systematically smaller by a factor of \( \sim 2 \).

We then performed two different fits to a power law,

\[
\xi(r) = \left( \frac{r}{r_c} \right)^{-\gamma}, \tag{5.2}
\]

over the distance range \( 5 < r < 25 h^{-1} \) Mpc: (1) a constrained fit, assuming a constant \( \gamma = 1.8 \) (as an example, in Fig. 4 we show such fit for \( D_c = 30 h^{-1} \) Mpc), and (2) an unconstrained fit, allowing both \( r_c \) and \( \gamma \) to vary. Points were weighted by the corresponding bootstrap errors, and \( r_c \) best-fit values are also given with bootstrap errors. Such errors are obviously smaller for the constrained fit, where our ignorance of \( \gamma \) is hidden. In general, large \( D_c \) clusters yield best-fit \( \gamma \) values approaching 2, although 1.8 always lies within 1 \( \sigma \). Our \( r_c \) estimates are performed at \( z = 0 \) and 0.8, to inspect cluster clustering evolution.

6. RESULTS

In Figure 5 we report the \( r_c \) versus \( D_c \) behavior for tCDM, MDM1, and MDM2, for fixed \( \gamma = 1.8 \). In Figure 6 we give results for the same cases, obtained with two-parameter fits on \( r_c \) and \( \gamma \). Errors bars represent 1 \( \sigma \) bootstrap errors (see § 3). Of course, error bars are smaller in the single-parameter fits, where our ignorance of \( \gamma \) is hidden.

Together with the \( r_c \) values obtained from our simulations, we also plot APM and Abell cluster data, the BW conjecture, and the results from simulations performed by Bahcall & Cen (1992) and Croft & Efstathiou (1994). Recent results obtained by Governato et al. (1999) for a critical CDM model lie between the last two curves. Observational points and error bars given in our figures were obtained from original work. We draw the reader’s attention on the fact that in some recent work studying cluster clustering in simulations, observational points and error bars are not accurately reported.

A comparison of tCDM with the data shows that simulated and APM data points are in fair agreement. In view of the better fits obtainable with mixed models, shown in the same figures, one might tend to overlook the improvement of tCDM with respect to CDM models with \( n = 1 \), which in fact is significant. Our tCDM model, however, seems to systematically miss the Abell catalog points, and thus is far from the BW conjecture, which tries for a compromise between the Abell and APM results.

From this point of view, the performance of mixed models is better. For low \( D_c \), MDM1 tends to give \( r_c \) values above the BW conjecture (see, however, Lee & Park 1999).
A similar, but less pronounced, effect exists also for MDM2. On intermediate scales, MDM1 sticks on the BW conjecture curve and meets two of the APM points at the 2 σ level only. MDM2 instead seems to try to compromise between the APM and Abell points. On the top scales, the behaviors shown here by the two models are opposite. However, the MDM1 behavior at such scales seems somehow anomalous; these are the scales that are most likely to be affected by cosmic variance, and the MDM1 behavior at $D_c > 65–70 \, h^{-1} \text{Mpc}$ should certainly be tested with different model realizations. Furthermore, unconstrained fits tend to indicate that such a discrepancy arises from different correlation-function slopes.

It may also be significant to consider the unconstrained fit obtained by ordering clusters according to $M_v$ masses, which is shown in Figure 7. Let us note that (1) in most cases, error bars are smaller, and (2) the peculiar feature for MDM1 at large $D_c$ has disappeared. It is likely that such an improvement is related to a more direct physical significance of the mass $M_v$ and the (variable) radius $R_v$. Taking instead a fixed radius, $R_a$, risks accentuating a dependence on local peculiar features. In principle, this is more likely to occur for small-mass clusters, for which a significant volume still unaffected by virialization processes lies within $R_a$. In our simulation volume, however, we have a large number of low-mass clusters, and this allows an efficient averaging over local realizations. At the top mass end, the sample is more restricted, and we must mostly rely on the virialization process rather than sample averaging to smear off local peculiarities. Our results seem to indicate that significant memory of initial conditions is also kept below $R_v$.

In Figures 8 and 9, we report a comparison between the two-point function results at $z = 0$ and 0.8, obtained using $M_c$. Unconstrained fits at $z = 0.8$ are rather noisy at large $D_c$, in particular for the top scales. Constrained fits, on the other hand, might be taken as an indication of clustering evolution on the top scales. Here, perhaps, there is further evidence of an anomaly in MDM1, the only case in which clustering seems weaker at $z = 0$ than at $z = 0.8$. Apparently, all models seem to indicate a greater clustering length at scales between 50 and 65–70 $h^{-1} \text{Mpc}$ for $z = 0.8$. In MDM1, however, this is inverted above 70 $h^{-1} \text{Mpc}$.

Figure 10 shows the results of cluster evolution based on $M_v$ ordering and using constrained fits. The kind of evolution found above seems to be confirmed, while the MDM1 anomaly is reduced.
7. DISCUSSION

Previous numerical results on cluster clustering, based on models with $\Omega_m = 1$, gave a behavior of $r_c$ versus $D_c$ a few $\sigma$ below observational results. The only exception is in Bahcall & Cen (1992), whose numerical study involves peculiar extrapolations, which, however, succeeds in meeting two APM points at large $D_c$ only. When considering subcritical CDM models (OCDM), the same authors obtain a behavior close to the BW conjecture. However, this is not fully confirmed by later numerical studies; although they clearly indicate that in OCDM models the $D_c$ dependence on $r_c$ is consistent with APM points, Abell cluster points seem to require a still steeper dependence than in OCDM. Such findings, however, were interpreted as an indication that observational data on cluster clustering could be approached only by models with $\Omega_m < 1$, and led to arguments that the observed dependence of $r_c$ on $M_{\text{tot}}$ is somehow related to early cluster formation.

The behavior we find for tCDM does not support such an inference. Taking a spectral index of $n \neq 1$ has no substantial effect on the time of cluster formation, which is quite similar to standard CDM. In view of the more striking outputs for mixed models, we must not disregard the result we find for tCDM. The $r_c$ versus $D_c$ behavior of clusters in such models is analogous to previous outputs for OCDM and lies well above the behavior obtained by Bahcall & Cen (1992) for standard CDM. Our findings are that, taking $n < 1$, the clustering of clusters in an $\Omega_m = 1$ model approaches the behavior obtained from APM cluster data.

Even more striking is the cluster clustering behavior for the mixed models considered in this work, which are based on low-mass $\nu$'s. In this case, the slope of the $r_c$ versus $D_c$ behavior approaches the BW conjecture and, more significantly, within 1 or 2 $\sigma$ bootstrap error bars, we mostly find consistency both with APM and Abell results. The only exception could be the point of Abell clusters of richness $R > 2$, which is approached only by MDM2. Our results, however, support previous claims that, at the top mass end, wider samples may be required to suppress the cosmic variance.

In general, it may be reasonable to consider cluster clustering as a measure of the spectral power on scales exceeding $\sim 25$ h$^{-1}$ Mpc. All models with $\Omega_m = 1$ ought to have similar values of $\sigma_8$, in order to be consistent with the observed cluster abundance. Accordingly, the power at scales $\sim 25$ h$^{-1}$ Mpc is basically gauged by the value of the $\Gamma$ parameter. It may not be the case that models perform in a better way as their $\Gamma$'s approach the observational interval.

Surely, in the case of mixed models, the situation is complicated by the presence of a hot component, which may slow down the gravitational growth in the nonlinear regime, in a scale-dependent fashion. Previous results for
mixed models concerned a mix including 30% HDM, due to a single $l$ with mass $eV$. Here we deal with $l$'s $3-4$ times lighter; accordingly, when cluster formation begins, their speeds are $\sim 3-4$ times greater. Peculiar effects of MDM are therefore significantly reinforced. Hence, in addition to having a suitable spectral slope, the mixed models treated here differ still more from standard CDM, because of the late $l$ derelativization.

In this paper we show that critical CDM models, with a blue spectrum suitably "compensated for" by a light-$l$ component, in addition to fitting most LSS and CMB data, are able to follow APM and Abell cluster clustering data. This result adds to those discussed in Paper I, in which critical blue mixed models were shown to provide a good fit to the cluster mass function and to be in agreement with Donahue at al. (1998) findings concerning high-$z$ cluster abundance.

REFERENCES

Abdurashitov, J. N., et al. 1996, Phys. Rev. Lett., 77, 4708
Achilli, S., Occhionero, F., & Scaramella, R. 1985, ApJ, 299, 577
Athanassopoulos, C., et al. 1995, Phys. Rev. Lett., 75, 2650
Bahcall, N., & Cen, R. 1992, ApJ, 398, L81
Bahcall, N. A., & Soneira, R. M. 1983, ApJ, 270, 20
Bahcall, N. A., & West, M. J. 1992, ApJ, 392, 419
Barger, V., Weiler, T. J., & Whisnant, K. 1988, Phys. Lett. B, 242, 97
Baugh, C. M., & Batista, E. 1994, MNRAS, 280, L37
Bonometto, S. A., & Pierpaoli, E. 1998, NewA, 3, 391
Bonometto, S. A., & Valdarnini, R. 1984, Phys. Lett. A, 103, 1399
Boylan, E. 1985, ApJ, 398, L71
Bradley, E. 2002, The Jackknife, the Bootstrap, and Other Resampling Plans (Philadelphia: Society for Industrial and Applied Mathematics)
Colberg, J. M., et al. 1997, in Large Scale Structure: Proc. Ringberg Workshop September 1996, ed. D. Hamilton (preprint astro-ph/9702086)
Cole, S., Weinberg, D. H., Frenk, C. S., & Ratra, B. 1997, MNRAS, 289, 37
Couchman, H. M. P. 1991, ApJ, 368, L23
Croft, R. A. C., Dalton, G. B., Efstathiou, G., & Sutherland, W. J. 1997, MNRAS, 291, 305
Croft, R. A. C., & Efstathiou, G. 1994, MNRAS, 267, 390
Dalton, G. B., Efstathiou, G., Maddox, S. J., & Sutherland, W. J. 1992, ApJ, 390, L1
Davis, M., Summers, F. J., & Schlegel, M. 1992, Nature, 359, 392
Dekel, A., Blumenthal, G. R., Primack, J. R., & Olivier, S. 1989, ApJ, 338, L5
Donahue, M., Voit, G. M., Gioia, I., Luppino, G., Hughes, J. P., & Stocke, J. T. 1998, ApJ, 502, 550
Efstathiou, G. Bond, J. R., & White, S. D. M. 1992, MNRAS, 258, 1
Fukuda, Y., et al. 1994, Phys. Lett. B, 335, 237
Gardini, A., Borgani, S., Bonometto, S. A., & Murante, G. 1999, ApJ, 524, 510 (Paper I)
Ghigna, S., Borgani, S., Bonometto, S. A., Guzzo, L., Klypin, A., Primack, J. R., Giovanelli, R., & Haynes, M. 1999, ApJ, 437, L71
Governato, F., Babul, A., Quinn, T., Tozzi, P., Baugh, C. M., Katz, N., & Lake, G. 1999, MNRAS, 307, 949
Gross, M. A. K., Somerville, R. S., Primack, J. R., Holtzman, J., & Klypin, A. 1998, MNRAS, 301, 81

Fig. 9.—Same as Fig. 8, but from simultaneous fits of $r_c$ and $\gamma$. MDM1 shows some anomaly. See the electronic edition of the Journal for a color version of this figure.

Fig. 10.—Same as Fig. 8, but $D_c$ are obtained by ordering clusters according to $M_c$. Plots are less noisy when such mass determination is used. See the electronic edition of the Journal for a color version of this figure.
Hampel, W., et al. 1996, Phys. Lett. B, 388, 384
Hauser, M. G., & Peebles, P. J. E. 1973, ApJ, 185, 757
Holtzman, J. A. 1989, ApJS, 71, 1
Jing, Y. P., Plionis, M., & Valdarnini, R. 1992, ApJ, 389, 499
Kaiser, N. 1984, ApJ, 284, L9
Klypin, A., Holtzman, J., Primack, J., & Reggós, E. 1993, ApJ, 416, 1
Klypin, A., & Kopylov, A. I. 1983, Soviet Astron. Lett., 9, 41
Klypin, A., & Rhee, G. 1994, ApJ, 428, 399
Lee, S., & Park, C. 1999, ApJ, submitted (preprint astro-ph/9909008)
Liddle, A. R., Lyth, D. H, Viana, P. T. P., & White, M. 1996, MNRAS, 282, 281
Lucchin, F., Colafrancesco, S., De Gasperis, G., Matarrese, S., Mei, S., Mollerach, S., Moscardini, L., & Vittorio, N. 1996, ApJ, 459, 455
Nichol, R. C., Collins, C. A., Guzzo, L., & Lumsden, S. L. 1992, MNRAS, 255, 21P
Peacock, J. A., & Dodds, S. J. 1994, MNRAS, 267, 1020
Peacock, J. A., & Dodds, S. J. 1996, MNRAS, 280, L19
Peacock, J. A., & West, M. J. 1992, MNRAS, 259, 494
Perlmutter, S., et al. 1998, Nature, 391, 51
Pierpaoli, E., & Bonometto, S. A. 1999, MNRAS, 305, 425
Postman, M., Huchra, J. P., & Geller, M. J. 1992, ApJ, 384, 404
Primack, J. R., Holtzman, J., Klypin, A., & Caldwell, D. O. 1995, Phys. Rev. Lett., 74, 2160
Riess, A. G., Nugent, P., Filippenko, A. V., Kirshner, R. P., & Perlmutter, S. 1998, ApJ, 504, 935
Sarkar, U. 1999, Phys. Rev. D, 59, 031301
Storrie-Lombardi, L. J., McMahon, R. G., Irwin, M. J., & Hazard, C. 1995, preprint (astro-ph/9503089)
Sutherland, W. J., & Efstathiou, G. 1991, MNRAS, 248, 159
Thomas, P. A., et al. 1998, MNRAS, 296, 1061
Walter, C., & Klypin, A. 1996, ApJ, 462, 13
Watanabe, T., Matsubara, T., & Suto, Y. 1994, ApJ, 432, 17