The process of student analytical thinking in understanding and applying lattice method to solve mathematical problem

A M Annizar¹, Sofiah¹, A C Lestari¹, S Dalimarta¹, Y N Wulandari¹

¹State Islamic Institute of Jember

E-mail: anasmularfannizar@iain-jember.ac.id

Abstract. One of the educational objectives that is to improve analytical thinking skills. An improvement in analytical thinking skills can be initiated by analyzing the student's thinking process when faced with a new problem. Meanwhile, the Lattice Method is a method of multiplication which is very rarely taught in the school so that few students know about it. Therefore, this research aims to describe the process of analytical thinking students in understanding and applying the Lattice Method to solve mathematical problems. The method used in this research is descriptive with a qualitative approach. The data has collect with tests and interviews. The subject’s determination uses a purposive sampling technique. The results showed that four subjects had interesting thought processes in understanding and applying the Lattice Method that they had never previously studied. S1 is able to understand the Lattice Method in multiplication by explaining in detail the correct systematic and correlating the problem using the Lattice Method. S2 is able to understand the problem but still not able to explain properly. S3 is able to understand the problem in its own way with the concept taught earlier. While, S4 is able to understand the problem in its own way without the concept of Lattice Method.

1. Introduction

Education is one of the most important aspects that need to be considered in the process of human self-development. Because in general, education is a process of human change marked by not understanding being understood, not being able to be able, and so forth that is obtained from families, educational institutions or schools, as well as the community environment with the aim of instilling positive things in life. Haryanto states that education is an effort in fostering humans, in order to develop quality human characters physically and mentally [1]. The opinion is in line with the opinion of Nurkholis [2] which defines education as an effort that aims to produce quality individuals who include knowledge and moral values through the development of self-potential possessed by each individual so that they can flow like cycles in fostering, lead, support and guide students. While the Ministry of National Education (Depdiknas) revealed that education aims to make students into human beings who believe in and fear God Almighty, have noble character, be healthy, knowledgeable, capable, creative, independent and become democratic and responsible citizens. Based on these definitions it can be concluded that education is a learning process for students to be able to understand, comprehend, and improve moral values and also develop knowledge and skills in thinking. Thinking skills that need to be developed because it is closely related to decision making in everyday life one of which is analytical thinking skills.
Analytical thinking reveals that analytical thinking is the ability to think that describes one by one sub problem in detail as well as analyzing various information to understand and solve a problem [3]. Meanwhile, according to Qolfathiriyus [4], analytical thinking is the ability to group and describe a problem into interrelated parts. Based on this opinion it can be concluded that analytical thinking is a cognitive process that occurs in humans when trying to find solutions to the problems faced by dividing the problem into several partitions, exploring and analyzing these parts to find the linkages so that they can be put back together in a way the new one.

On the other side, analytical thinking skills in daily life are closely related to students' ability to solve mathematical problems. This is in line with the opinion of Annizar [5–7] who states mathematics as a science that has an important role in creating technological progress in everyday life. According to Fahradina [8] mathematics is one of the subjects that must be given to students in order to improve critical, logical, creative, and analytical thinking skills. In line with Annizar [9] which states that mathematics is a branch of knowledge in the form of information that can be found through numerical experience and knowledge with creative, innovative, critical, logical and analytical mindsets.

For most people the opinion that mathematics is procedural. This is not wrong but not completely correct. Because for some creative people, it can make mathematical problems become unique problems so they are able to develop higher order thinking skills (far from procedural impressions). Whereas examples of mathematical topics that seem procedural are multiplication calculations. Most students at various levels are generally able to perform multiplication operations (the most famous is the multiplication method). Almost all students in Indonesia are taught multiplication multiplication methods in completing multiplication operations. Actually this method is not difficult, but if it involves too many digits, sometimes students still find it difficult and inaccurate, so that the results obtained are not right. The multiplication method that is still foreign in Indonesia is the lattice method. Therefore, researchers use the lattice method to determine students’ analytical thinking processes in solving mathematical problems. Students with knowledge of the multiplication procedure are felt to have enough stock to carry out analytical thinking in order to uncover the lattice method independently.

The lattice method is an operation method that multiplies two multi-digit numbers by using a grid or table [10]. While Irawati [11] state that the lattice method is a multiplication operation method that separates dozens with units and is presented in tabular form. This method is different from conventional methods or tiered methods, because in the lattice method the process is neater and the results of calculations have been placed according to the grid so as to reduce the level of student error in completing multiplication operations [7]. In conclusion the lattice method is a multiplication method of two numbers with the help of tables that separate units, tens, hundreds, thousands and so on so that it further increases accuracy in calculating.

Based on the description above, the researcher feels it is important to analyze the students’ analytical thinking processes in understanding and applying lattice methods to solve mathematical problems. This research is expected to contribute in improving the mathematical abilities of Indonesian students in the future, so that they can compete globally.

2. Literature Review

2.1. Mathematical Problems

According to Woolfolk [12], a problem is a situation to achieve several goals by finding a way to solve it. Meanwhile, according to Suherman [13] the problem is a problem that contains the situation so that it can encourage someone to solve it, but do not know directly what to do to solve it.

Mathematical problems according to Fahradina [14] are problems that students can solve without using routine methods or algorithms. Thus, a mathematical problem is a problem that is solved, not directly known by students, so they need a certain way or algorithm to solve it.
2.2. Problem Solving

Problem solving is an ability where students must find a way out to achieve goals by requiring readiness, creativity, knowledge and application in daily life [15, 16]. Meanwhile, according to Ihsan [17], the ability to solve problems in mathematics is an intellectual activity to find a mathematical solution faced with the provision of knowledge as measured by indicators of mathematical problem solving. So problem solving is a form of thinking that is directly directed to find a solution or a solution to a problem.

The ability to solve mathematical problems according to Nova [14] is the ability to find a way to solve mathematical problems faced by using logical relationships. Meanwhile, according to Harahap [18] the ability to solve mathematical problems is an attempt to translate mathematics, which includes the ability to apply mathematical ideas to the context of the problems and the cooperative ability to arrange and solve the problems. So the ability to solve mathematical problems is the ability of students to find solutions to mathematical problems in accordance with the ability to think logically by applying mathematical ideas in solving problems.

2.3. Analytical Thinking

According to Ilma, et al [19] the ability to think analytically is the ability of students to describe the concepts of becoming more detailed parts and explain the relationships between these parts. This is in line with the opinion of Sanjaya [19] which states that analytical thinking is the ability to describe or break a problem into several parts. Meanwhile, according to Widiyastuti [3] analytical thinking ability is the ability to break problems in mathematics into basic elements, distinguish relevant elements or not with existing problems in mathematical problems, find ways about how elements of problems in mathematics can be interrelated. So it can be concluded that analytical thinking skills are students' skills in presenting answers to a problem being solved and being able to associate those answers.

2.4. Lattice Method

The lattice multiplication method is an alternative method that can be given to students to complete the calculation of longer multiplication operations, for example multiplication operations of tens and hundreds of natural numbers. According to Mujib [11] the lattice method is a multiplication method presented in tabular form, between tens and units separated. The lattice multiplication method is a multiplication method that uses a lattice to multiply multi-digit numbers [10]. The lattice multiplication method was introduced in Europe by Italian mathematician Leonardo Fibonacci in the 13th century which could be used as an alternative to solving long multiplication. In this method, the multiplication calculation uses a grid with each cell divided diagonally in half. The number of grids is adjusted according to the number of numbers multiplied. The results of multiplying the two numbers are placed in a table arranged by units and tens. The upper diagonal line is filled with tens and the lower diagonal is filled with unit numbers [20].

3. Method

This research was conducted at one of the Madrasah Aliyah located in Jember. The subject of this research used 10 students of XI Science class. The research instrument used was a test and interview guide. The test consists of 2 parts namely the first part contains examples of questions and discussion (in it using the lattice method), and the second part contains 1 question to do. The following are the tests used in this research:

1. Understand and analyse the question and solution below to answer question number 2!
   Jojo is very fond of reading books. Every day he read as many as 87 pages of the book. How many pages does Jojo read, if Jojo reads books every day in October?
   Solution:
   The number of pages of the book Jojo read in October was 2697 pages, from multiplications 87 to 31.
The lattice method was chosen because students still do not know this method, so that researchers can examine in detail the students’ analytical thinking processes in understanding the concept of the lattice method and applying it to the given problems. Based on the subjects’ completion result, then 4 subjects were taken as respondents for in depth interviews to uncover the analytical thinking process. The choice of subject or respondent uses a purposive sampling method that is choosing a subject by considering certain factors, namely the diversity and attractiveness of the resulting answers. Besides that the interview method used is semi-structured. This means that the researcher uses interview guidelines where the questions given are in accordance with the analytical thought process indicators. The analytical thinking indicators used in this research are as follows:

### Table 1. Indicators of analytical thinking.

| Analytical Thinking | Indicator                                                                 |
|---------------------|---------------------------------------------------------------------------|
| Differentiating     | Understand every word in the given story matter                           |
|                     | Predict and find information that is known to the given story matter      |
|                     | Plan a concept to solve questions on a given story problem                |
| Organizing          | Arranging the concept that has been planned                                 |
|                     | Detailed concepts that have been arranged                                 |
|                     | Examine and rationalize the detailed concepts                             |
| Attributing         | Correlating and connecting concepts that have been applied                |
|                     | Apply and interpret concepts that have been linked                       |
|                     | Summing up the concepts that have been applied                            |

87 × 31 =

![Lattice Method Diagram](image)

The result = 2697
So, the number of pages of books that Jojo read was 2697 pages

2. Do this question like solving the problem above!

One day, Jeje’s birthday. He prepared 75 packages of snacks to be distributed. How much snacks should Jeje prepare if each packet contains 1 kodi of various snacks?
4. Results
Based on the work and interviews conducted on 4 research subjects obtained data as follows:

Table 2. Suitability of Subject Analytical Thinking Process based on Indicators.

| Analytical Thinking | Indicator   | S1 | S2 | S3 | S4 |
|---------------------|-------------|----|----|----|----|
| Differentiating     | Understand  | ✓  | ✓  | ✓  | ✓  |
|                     | Predict     | ✓  | ✓  | ✓  | ✓  |
|                     | Plan        | ✓  |    |    |    |
| Organizing          | Organize    | ✓  | ✓  |    |    |
|                     | Detailing   | ✓  | ✓  |    |    |
|                     | Analyze     | ✓  | ✓  |    |    |
| Attributing         | Correlate   | ✓  | ✓  | ✓  | ✓  |
|                     | Apply       | ✓  | ✓  | ✓  | ✓  |
|                     | Conclude    | ✓  |    |    |    |

While the summary of the analytical thought processes of the 4 subjects is as follows:

Table 3. Analytical thinking processes of all four subjects.

| S1 | S2 | S3 | S4 |
|----|----|----|----|
| Able to understand the lattice method in multiplication by explaining in detail systematically correct and correlating problems using the lattice method. In the next question, student 1 is able to work independently without the help of a teacher or assistant. | Understanding of student problems is still not able to explain correctly. The concept that is embedded in him is still not true. The student experiences difficulties in understanding the next example or problem. By teaching the lattice method, the student still does not fully understand without guidance. | Able to understand questions in their own way with concepts taught before. However, do not understand the lattice method in the matter. Thus to go to the next problem it will be difficult to do it with the lattice method. | Able to understand questions in their own way without the concept of the lattice method. After going to the next problem, students can understand the course of the problem without knowing the way it is understood. |

5. Discussion

5.1. Many quotas are needed before the pandemic occurs
When S1 is given a test instrument in the form of sample questions along with discussion and one question that must be solved, first S1 tries to understand every word in the sample questions given, then S1 finds information in the form of numbers that are known in the sample questions, namely many pages of books that are Jojo read 87 pages and found the numbers implied like the number of days in October, which is 31 days. After S1 finds information on explicit and implicit numbers, S1 arranges concepts by encircling numbers, numbers 87 and 31 in the lattice method table contained in the discussion of sample problems as a sign that S1, understands the laying and order of numbers as the initial stage in doing multiplication operations using the lattice method. After that, based on the interview S1 has been able to detail the concept of the lattice method by explaining that the results of the multiplication of numbers 3 with number 8 are written 2 on the top left as tens and 4 below number
2, but still in the same box. In analyzing and rationalizing the concept of the lattice method, S1 can explain that when unit multiplication is the number 8 multiplied by 1, then what is written in the lattice method table is 08, it must add 0 if the result of the multiplication is a unit. After that, S1 can correlate or connect the concept of the lattice method by extending the slash in the lattice method table so that S1 produces 4 parts of the extended slash and then sums them up based on the slope of the line that S1 has extended, and results in the addition of unit numbers, tens, then hundreds of thousands which is the answer of 87 multiplied by 31 using the lattice method. So that from the thought process carried out, S1 can deduce the concept of the lattice method correctly. Whereas in solving the second problem given, S1 can interpret and apply the concept of the lattice method that S1 has understood by doing the same steps. First S1 read to understand the contents of the problem, then find information in the form of numbers that are known to the problem that is 75 snacks, and S1 can find implied number information, which is 1 score totaling 20 pieces. Then S1 makes a table and arranges and puts the numbers that S1 has found in the problem, which is 75 and 20. After that S1 breaks down and rationalizes the numbers in the lattice method table, so that it gets some multiplication results, 14, 0, 10, and 0 then put S1 in the lattice method table that S1 has created. In the last step, S1 correlates the multiplication numbers in the lattice method table by extending the slash in the lattice method table, the same as that of the previous S1, so that S1 can complete the 75 x 20 product with the correct answer, which is 1500.

In S2, the test instrument is also given in the form of sample questions and discussion and one question that must be resolved. The first step taken by the S2 is to read carefully the example of the questions given by the researcher so that the S2 can find information that has been known to the problem, namely 87 pages of the book. By reading the sample questions repeatedly, S2 can also find information in the form of numbers implied in the example problems, namely the number of days in October which amounts to 31 days. Then S2 tries to predict and plan concepts by paying attention to the lattice method table in discussing example problems with fairly long periods of time. Based on interviews, S2 can understand the origins of laying numbers 87 and 31 of the information in the form of sample numbers of questions that S2 has found before. S2 can also detail and rationalize the concept of the lattice method in the discussion of example problems by showing that the numbers 2 and 4 in the lattice method table in the discussion of sample problems are obtained from the product of 8 by 3, the number 21 is obtained by multiplying between the numbers 7 by 3, arranging the numbers the number S2 has found. In addition, S2 is also able to examine the number 08 in the lattice method table in the discussion of example problems obtained from 8x1, and S2 explains that the number 0 is added so that the 1digit unit, can be changed to 2 digits. Then S2 correlates the numbers contained in
the lattice method table in the discussion of example problems by giving arrows to show the sum of results in parallel lines such as the number 7 given an arrow indicating the number 7 in the final result of the latticemoede multiplication, numbers 8, 0, and 1 is given a down arrow showing number 9, the results of the sum of the three numbers, numbers 0, 4, and 2 are given an arrow pointing to number 6 below the table as a result of adding up hundreds, and number 2 which is located in the upper left corner given a down arrow pointing to number 2, the result of the number thousands at the completion of 87 x 31 using the lattice method. From this process S2 can deduce the concept of the lattice method correctly in the example problem.

Whereas when S2 is confronted with the questions, the stages carried out remain the same as the previous ones. But S2 is wrong in understanding and finding information in the form of numbers implied in the questions given. S2 states that 1 score is 12, not 20. So that because of an error in understanding and finding information implicit in the problem, S2 can solve the problem with the concept of the correct lattice method, which S2 has understood before but the answer error occurs because of differences in numbers that are misinterpreted. S2 writes 75 x 12 on the latos method table that S2 made before, with 900 results instead of 1500.

When S3 is given the same test instrument, S3 first reads and understands the sample questions given, then S3 finds some information in the form of numbers, 87 and 31 by underlining the sentence "87 pages" and the sentence "October". After that, S3 looks at the lattice method table repeatedly in the discussion of sample problems while predicting and planning the concept of the lattice method that has never been taught. Based on interviews, S3 has been able to arrange the numbers that will be multiplied in the example problems, namely the placement of numbers 87 and 31 in the lattice method table that has been listed. Then S3 detailed and rationalized the numbers contained in the lattice method table in the discussion of problems with the results of their own predictions and understanding, such as numbers 2 and 4 in the lattice method table in the discussion of example problems, S3 stated that the numbers came from factoring 8, numbers 0 and 7 came from factoring number 7, numbers 0 and 8 from factoring number 8, but the numbers 2 and 1 in the lattice method table, S3 did not know and could not explain where the numbers came from. In addition S3 did not know and could not explain where the numbers 2697 (the results of the multiplication of 87 to 31) were under the lattice method table in the discussion of sample problems so that S3 could not deduce the concept of the lattice method correctly.

Whereas when S3 starts working on the questions given by the researcher, S3 applies and interprets the results of understanding the lattice concept that S3 has. First, S3 can understand the contents of the problem then find information on the numbers implicit and explicit in the problem, namely the
When S4 was given a test instrument in the form of sample questions along with discussions and questions that had to be solved, S4 initially read and understood each word in the example questions repeatedly so that S4 could find information in the form of numbers, 75 and 31 which were many days in October. After that, S4 starts planning and predicting the concept of the lattice method by paying close attention and for a considerable amount of time. Based on interviews, S4 has been able to find out the origin of the laying of numbers to be multiplied using the lattice method by showing researchers that numbers 8, 7, 3, and 1 which are located above and to the right of the lattice method table (in the discussion of example problems) are obtained of 87 which will be multiplied by 31. In addition, S4 can also detail and rationalize the numbers contained in the lattice method table in the discussion of sample problems by showing the researcher that numbers 2 and 4 are separate but still in one box obtained from multiplication between 8 with 3 that produce numbers 24, numbers 2 and 1 are obtained by multiplying 7 by 3, numbers 0 and 8 get from 8x1, and 0 and 7 are obtained by multiplying 7 by 1. Then S4 looks back at the numbers under the table of lattice method with a long enough time, until finally S4 can connect numbers below the table with the numbers contained in the table by stating that the numbers the ka below the table is the sum of the numbers delimited by slashes. The concept of the lattice method understood by S4 is correct, although S4 requires a long thought process. Therefore, S4 is not able to conclude well with the concept of the lattice method. This can be proven when S4 is faced with the second problem that must be solved. In reading the given questions, S4 can understand the contents of the questions and find information on the numbers on the questions, which are 75 and 1 score which is 20 pieces. However, to solve the problem, S4 cannot interpret and re-apply the concept of the lattice method that S4 has understood before.

Figure 3. S3 completion result that meet differentiating and attributing indicators.
Figure 4. S4 completion result that meet differentiating and attributing indicators.

6. Conclusion
Based on the results and discussion of the research, it can be concluded that the students’ analytical thinking processes in understanding and applying the concepts of the lattice method have never been taught to be very diverse and have their own uniqueness. S1 is able to understand the lattice method in multiplication by explaining in detail systematic correct and correlating problems using the lattice method. In the next problem, S1 can work independently without the help of a teacher or assistant. S2 understands the problem but still cannot explain it correctly. The concept that is embedded in him is still not true. The student experiences difficulties in understanding the next example or problem. By teaching the lattice method, the student still does not fully understand without guidance. S3 is able to understand the questions in his own way with the concepts taught before. However, do not understand the lattice method in the matter. Thus to go to the next problem it will be difficult to do it with the lattice method. S4 is able to understand questions in its own way without the concept of the lattice method. After going to the next problem, students can understand the course of the problem without knowing the way it is understood.

7. Suggestion
Based on the results of research that have been presented researchers have suggestions including (1) for students are advised to learn more about the use of lattice method as an alternative to multiplication with a high level of accuracy, while (2) for teachers need to practice making problems more than procedural problems, and it is also necessary to inform students about indicators that can be used as stages of analytical thinking.

Acknowledgment
In this section the authors would like to thank the partners who have jointly supported and helped so that this research can go well. In addition, researchers also thankful to all students who have taken their time to fill out the test and become the subject of this research. The cost of the research conducted is credited to the research team's personal funds.

References
[1] Islam S 2017 Karakteristik Pendidikan Karakter; Menjawab Tantangan Multidimensional Melalui Implementasi Kurikulum 2013 Edureligia 1 89–101
[2] Nurkholis 2013 Pendidikan Dalam Upaya Memajukan Teknologi J. Kependidikan 1 24–44
[3] Widiyastuti E and Jazuli A 2018 Description of Mathematical Analytical Thinking Ability of Mathematica Education Students 8th Univ. Res. Colloq 401–409
[4] Qolfathiriyyus A, Sujadi I and Indriati D 2019 Characteristic profile of analytical thinking in mathematics problem solving *IOP Conf. Ser. J. Phys.* **1157**

[5] Annizar A M, Sisworo and Sudirman 2018 Pemecahan Masalah menggunakan Model IDEAL pada Siswa Kelas X Berkategori Fast-Accurate *J. Pendidik. Teor. Penelitian, dan Pengemb.* **3** 634–640

[6] Annizar A M, Masrurrotullaily, Jakaria M H D, Mukhlis M and Apriyono F 2020 Problem solving analysis of rational inequality based on IDEAL model *J. Phys. Conf. Ser.* **1465** 1–14

[7] Aini A N, Mukhir M, Annizar A M, Jakaria M H D and Septiadi D D 2020 Creative thinking level of visual-spatial students on geometry HOTS problems Creative thinking level of visual-spatial students on geometry HOTS problems *J. Phys. Conf. Ser.* **1465** 1–7

[8] Ramadania F, Wulandari N I and Nahlini 2017 Peranan Komunikasi Bahasa Dalam Pembelajaran Matemayika Pada Siswa Kelas V SDN Keraton Martapura *Math Didact. J.* **3** 23–32

[9] Annizar A M, Maulyda M A, Khairunnisa G F and Hijriani L 2020 Kemampuan Pemecahan Masalah Matematis Siswa dalam Menyelesaikan Soal PISA pada Topik Geometri *J. Elemen* **6** 39–55

[10] Zubaidah, Margiati and Kresnadi H 2015 Peningkatan Hasil Belajar Matematika Dengan Metode Lattice Di Kelas III Sekolah Dasar *2* 1–17

[11] Irawati 2020 Metode Perkalian Silang VS Metode Perkalian Latis *J. Pendidik. dan Pembelajaran Terpadu* **2** 37–45

[12] Nezhad A S and Vahedi M 2011 The role of educational psychology in teacher education programs *Procedia-Social Behav. Sci.* **30** 327–330

[13] Rahmi 2013 Menciptakan Pembelajaran Matematika yang Kreatif dan Menyenangkan *J. Pelangi* **6** 1–8

[14] Fahradina N, Ansari B I and Saiman 2014 Peningkatan Kemampuan Komunikasi Matematis dan Kemandirian Belajar Siswa SMP dengan Menggunakan Model Investigasi Kelompok *J. Didakt. Mat.* **1** 54–64

[15] Fitriani S and Yarmayani A 2018 Development of Rubric Creativity Thinking Skill Senior High School In Mathematical Problem Solving *J. Mosharafa.* **7** 33–38

[16] Maulyda M A, Annizar A M, Hidayati V R and Mukhlis M 2020 Analysis of students’ verbal and written mathematical communication error in solving word problem *J. Phys. Conf. Ser.* **1538** 1–11

[17] Ihsan M 2016 Pengaruh Metakognisi dan Motivasi terhadap Kemampuan Pemecahan Masalah Matematika Melalui Kreativitas Siswa Kelas VIII SMP Negeri di Kecamatan Kindang Kabupaten Bulukumba *Al-Khwarizmi J. Pendidik. Mat. dan Ilmu Pengetah. Alam* **4** 129–140

[18] Harahap E R and Surya E 2017 Kemampuan Pemecahan Masalah Matematis Siswa Kelas VII Dalam Menyelesaikan Persamaan Liniar Satu Variabel *Edumatica* **7** 44–54

[19] Ilma R, Hamdani A S and Lailiyah S 2017 Profil Berpikir Analitis Masalah Aljabar Siswa Ditinjau dari Gaya Kognitif Visualizer dan Verbalizer *J. Rev. Pembelajaran Mat.* **2** 1–14

[20] Fatmala F W, Muzaki A and Pujilestari 2018 Pengaruh Penerapan Lattice Multiplication Method Untuk Mengatasi Kesulitan Menyelesaikan Operasi Perkalian *Media Pendidik. Mat.* **6** 68–78