Getting $\beta - \alpha$ without penguins

David Atwood

Dept. of Physics and Astronomy, Iowa State University, Ames, IA 50011

Amarjit Soni

Theory Group, Brookhaven National Laboratory, Upton, NY 11973

(Dated: November 2, 2018)

Oscillation effects in $B^0 \rightarrow K_S D^0$ and related processes are considered to determine $\delta \equiv \beta - \alpha + \pi = 2\beta + \gamma$. We suggest that $D^0$ decays to CP eigenstates used in concert with inclusive $D^0$ decays provide a powerful method for determining $\delta$ cleanly i.e. without any complication from penguin processes. The CP asymmetry is expected to be $\lesssim 40\%$ for $D^0$ decays to non-CP eigenstates and $\lesssim 80\%$ for decays to CP eigenstates. This method can lead to a fairly accurate determination of $\delta$ with $O(10^5 - 10^6)$ $B$-mesons.

PACS numbers: 12.15.Hh; 11.30.Er; 13.25.Hw

The two asymmetric B-factories have made remarkable progress in determining one of the angles ($\beta$) of the unitarity triangle; the world average now stands at $\sin 2\beta^{W,A} = 0.78 \pm 0.08$ \cite{1,2}. This is in very good agreement with the expectations from the Standard Model (SM), $\sin 2\beta^{SM} = 0.70 \pm 0.10$ \cite{11}. However, considerable amount of theoretical input has to be used to deduce $\sin 2\beta^{SM}$ and progress in reducing the theory error is likely to be rather slow. Thus, methods that determine the angles without the uncertainties of hadronic matrix elements are crucial in testing the CKM paradigm\cite{4} to an increasing degree of accuracy in an effort to search for CP-odd phase(s) due to physics beyond the SM.

In the SM, CP violation is controlled by only one CP-odd phase. Therefore, different decays which measure the same angle of the unitarity triangle (UT) may give inconsistent results if physics beyond the SM is present. Likewise other apparent failure of unitarity of the CKM matrix, such as the failure of the UT to close, would also indicate new physics. Beyond the phase $\beta$, the determinations of $\alpha$ and $\gamma$, therefore, provide key SM tests.

Two extensively studied methods for determining $\alpha$ already exist, via $B \rightarrow \pi \pi$ \cite{3} and $B \rightarrow \rho \pi$ \cite{3}. In these approaches, in addition to some experimental difficulties, considerable theoretical input is essential as these modes receive large QCD-penguin as well as some electro-weak penguin (EWP) contributons. While efforts at these methods should certainly continue, it is also very important that, in our drive towards precision, we develop methods that require no theoretical assumption and therefore have zero theory error. The key point is that effect of beyond the SM CP-odd phase(s) on B-physics may be quite small so any residual theory error on the determined unitarity angles may mask the effect of new physics and thwart experimental searches.

In this work we wish to report on our study of a method to extract $\delta \equiv \beta - \alpha + \pi = 2\beta + \gamma$ that uses interference between $b \rightarrow u$ and $b \rightarrow c$ tree graph exchanges only; no penguin contribution, strong or EW, or any theoretical assumption is involved. Given that $\beta$ is already well measured, this method is very effective in determining $\alpha$ “cleanly”, i.e. without QCD complications. In addition, this method can also be used to simultaneously extract $\beta$, allowing a crucial check against the value of $\beta$ determined with the $B \rightarrow J/\psi K_S^0$ approach \cite{3,3}. As mentioned before, a difference in the two determinations of $\beta$ may then be an indication of new physics. The basic idea behind the method has already received some attention \cite{8,9,10,11,12,3}. We extend and complement these earlier studies in several ways so that it becomes now a powerful approach to determine $\alpha$, and possibly $\beta$, without any complication from penguins.

In principle, a comparison of time dependent CP asymmetry measurement in $B^0(B^0) \rightarrow K_S D^0$ with that in $K_S D^0$ suffices to give $\delta$ \cite{3}. In practice, though, as has already been noted previously, flavor tagging of $D^0(D^0)$ appears extremely difficult \cite{10,12}. Semi-leptonic tags suffer from very serious background from prompt B-decays, $B \rightarrow l \nu X_C$; therefore, here we will not consider the possibility of semi-leptonic tags further. Hadronic tags of $D^0$ (say via $D^0 \rightarrow K^- \pi^+$) receive appreciable corrections from doubly-Cabibbo-suppressed decays of $D^0$. As in the case of $\gamma$ extraction with $B^\pm$ \cite{3}, this interference can be used to our advantage in determining $\delta$ as Kayser and London (KL) have discussed \cite{10}.

In this letter, we would like to highlight at least two
additional methods which will be shown to have great practical importance in extracting $\delta$. First of all, $\delta$ may be determined if $D^0$ decays to CP eigenstates (CPES) are observed, provided both CP$=+1$ and CP$=-1$ states are used. Although we find that while neither this CPES method, nor the CP non-eigenstate (CPNES) method of KL[1], can separately provide an especially strong determination of $\delta$, a great improvement is achieved if both approaches are used together because both data sets depend on a common set of parameters in the amplitude. Secondly, we will generalize these methods from single final states to inclusive sets of final states. In this way we can use the entire observable hadronic branching ratio of the $D^0$ greatly enhancing the statistical power. Finally we will briefly discuss methods whereby ancillary information constraining $\delta$ may also be obtained. The methods which we describe share with KL[1] the feature that the amplitude parameters are overdetermined and therefore a value of $\beta$, in addition to $\delta$, may also be extracted from the same data providing a valuable comparison to $\beta$ obtained from $B \to J/\psi K_S$.

Consider now the case where $B^0(t)/\bar{B}^0(\bar{t}) \to K_S D^0$, $K_S \bar{D}^0$ followed by the decay $D^0/\bar{D}^0 \to F$; $F$ denotes an inclusive set of states $F = \{f_i\}$ and in general $F \neq F$. For example, the set $\{f_i\}$ may range over states of different particle content (e.g. $K^- + \pi^+$) or different points in phase space [14] (e.g. each $f_i$ is a point on the $K^- \pi^+ \pi^0$ Dalitz plot) or a combination of both. For each $f_i$ the four relevant amplitudes are:

$$A_1(f_i) \equiv A(\bar{B}^0 \to K_S D^0 \to f_i) = A$$
$$A_2(f_i) \equiv A(B^0 \to K_S \bar{D}^0 \to f_i) = A r_D e^{i \eta_D}$$
$$A_3(f_i) \equiv A(\bar{B}^0 \to K_S \bar{D}^0 \to f_i) = A r_D e^{i \eta_D}$$
$$A_4(f_i) \equiv A(B^0 \to K_S \bar{D}^0 \to f_i) = A r_D e^{i \eta_D}$$

where, we have adopted the Wolfenstein[13] representation of the CKM matrix, and without loss of generality, we can choose the strong phase convention so that $A_1 = A$ is real. The quantity $r_D$ is the ratio $|A(D^0 \to f_i)|/|A(D^0 \to f_i)|$ which we will assume is known from the study of $D^0$ decay. The strong phase $\eta_D(f_i) = arg(A(\bar{B}^0 \to f_i)/A(D^0 \to f_i))$ we will assume to be not known apriori. Likewise the parameter $r_B$ and the strong phase $\eta_B$ given by $r_B e^{i \eta_B} = e^{-i \tau} A(B^0 \to K_S D^0)/A(B^0 \to K_S \bar{D}^0)$ are also assumed to be not known apriori. Note that $\{r_D, \eta_D, A\}$ depend on the state $f_i$ while $\{r_B, \eta_B\}$ are independent.

The time dependent decay rates for this decay is:

$$2\Gamma(B^0/\bar{B}^0(t) \to K_S F) = e^{-\tau} \left| \langle X(F) + b Y(F) \cos(x_B \tau) - b Z(F) \sin(x_B \tau) \rangle \right|^2$$

where $\tau = \Gamma_{Bt}$ and $x_B = \Delta m_B/\Gamma_B$ while $b = +1$ for $B(t)$ and $b = -1$ for $\bar{B}(t)$. Defining $A(f_i) = A_2(f_i) + A_4(f_i)$ and $\bar{A}(f_i) = A_1(f_i) + A_3(f_i)$, the coefficients $X, Y$ and $Z$ in Eqn. (2) are given by

$$X(F) = \sum_i (|A(f_i)|^2 + |\bar{A}(f_i)|^2)$$
$$Y(F) = \sum_i (|A(f_i)|^2 - |\bar{A}(f_i)|^2)$$
$$Z(F) = \sum_i Im(e^{-2i\eta_B} A(f_i)^* A(f_i))$$

we can expand these quantities in terms of eqn. (3) and obtain

$$X(F) = \left( (1 + r_D^2) + (1 - r_D^2) / 2 \right)$$
$$+ 2 R_F r_D \cos(\eta_D - \gamma) \cos(\eta_B) \bar{A}^2$$
$$Y(F) = - \left( (1 - r_D^2) + (1 - r_D^2) / 2 \right)$$
$$- 2 R_F r_D \sin(\eta_D - \gamma) \sin(\eta_B) \bar{A}^2$$

where $\bar{A}^2 = \sum_i (A_2(f_i) + \bar{A}_4(f_i)) \bar{A}_2(f_i) / \bar{A}_2^2$ and $R_F e^{i \eta_B} = \sum_i (A_4(f_i) r_D (f_i) e^{-i \eta_B (f_i)})/(\bar{A}_2 r_D)$. The corresponding quantities for $F$ are given by $X(\bar{F})(\eta_B, \eta_D, \gamma) = X(F)(-\eta_B, \eta_D, \gamma)$; $Y(\bar{F})(\eta_B, \eta_D, \gamma) = -Y(F)(-\eta_B, \eta_D, \gamma)$ and $Z(F)(\eta_B, \eta_D, \gamma) = Z(F)(-\eta_B, \eta_D, \gamma)$ assuming that there is no additional CP violation in $D^0$ decay [13].

Initial we will assume that $\beta$ is well determined. Let us now consider the special case where $F$ consists of CPES with eigenvalue $\sigma = \pm 1$. In this case, the modes add coherently and so $R_F = 1$, $\bar{r}_D = 1$ and $\bar{\eta}_D = 0$ or $\pi$ for $\sigma = +1$, $-1$ respectively. The three observables $X(F), Y(F)$ and $Z(F)$ thus depend on the four parameters $\{A, r_B, \eta_B, \delta\}$. If we have the two data sets, for $\sigma = +1$ and for $\sigma = -1$, then there are five independent observables (note that $Y(\sigma = +1) + Y(\sigma = -1) = 0$) determining the same four parameters and so the system is overdetermined and one may solve for $\delta$.

Some examples of CP$=\pm 1$ final states [17] include $K_S \pi^0$ (BR$=1\%$), $K_S \eta$ ($0.35\%$), $K_S \rho^0$ ($0.6\%$), $K_S \omega$ ($1.1\%$), $K_S \phi'$ ($0.9\%$) and $K_S \phi$ ($0.4\%$) giving a total of about $4.4\%$. CP$=-1$ final states include $K_S \rho^0$ ($0.3\%$, $\pi^+ \pi^- (0.07\%)$) and $K^+ K^- (0.21\%)$ for a total of $0.6\%$. For each of the modes with a $K_S$ one can construct a mode of the opposite CP by changing the $K_S$ to a $K_L$. We can also change the $K_S$ which arises from the $B^0$ decay i.e. in $B^0 \to K_S D^0$ (which we refer to as the fast kaon) to a $K_L$. Switching the fast kaon to $K_L$ changes $\eta_B \to \eta_B + \pi$ and thus gives the same information as switching the slow kaon (i.e. the kaon arising from $D^0$ decay). It should be emphasized that, in this instance, including the final states both with $K_S$ and with $K_L$, increases the number of observables and, as mentioned above, enables the system of equations to become soluble. This is in contrast, for example, with the case of $B^0 \to J/\psi K_S$ versus $B^0 \to J/\psi K_L$ where switching the kaon merely improves statistics but does not provide additional independent observables.

We can extend this CPES method to consider inclusive final states. If $F$ is defined in a CP invariant manner (e.g.
$F = K_S + X, \text{ BR}=21\%$) the resultant observables will be 
similar to the pure eigenstate case. Here again $r_D = 1$
and $\eta_D=0$ or $\pi$ but $R_F$, which measures the purity of
$F$, will not be $1$. The 3 observables are thus dependent
on 5 parameters $\{\hat{A}, r_B, \eta_B, \delta, R_F\}$. As before, we can
obtain a solution by changing the fast $K^0$ to a $K_L$ and/or
changing the slow $K^0$, in the case where $K_S = F$. Again,
these $K^0$ changes will lead us to 5 observables.

In [10], KL studied the special case where $F$ consists
of a single quantum state which is a CPNES (e.g. $f =$
$K^−\pi^+$). Then $R_F = 1$ but $\{\hat{A}, r_B, \eta_B, \delta, \eta_D\}$ are 
not known. If we take the point of view that $\beta$ and all the
relevant $D^0$ branching ratios are well determined then
as discussed in [10] there are 6 observables $\{X(f), Y(f),
Z(f), X(\bar{f}), Y(\bar{f}), Z(\bar{f})\}$ determining these 5 parameters
and so the system is overdetermined; therefore, one can
extract $\delta$. Furthermore, as in [10] one can also take the
point of view that $\beta$ is a free parameter and solve for
both $\beta$ and $\delta$ from the same six observables. In this
context, as in the CPES case, taking the fast kaon to be $K_L$
(rather than $K_S$) provides 6 more independent observables
dependent on the same parameters rendering the system even more overdetermined.

There is a great advantage to combining the CPES and
CPNES methods above since the parameters involved in
the CPES case are a subset of those for a CPNES. Thus,
combining information from CPES and CPNES methods
can increase the number of observables to nine or
eleven depending on whether one or both CP eigenvalues
are included, respectively. Indeed, if also the fast $K_L$
is taken with the CPNES then the number of observables
increases to seventeen. The number of parameters, of
course, stays the same, i.e. five (or six if we also
include $\beta$ as an unknown). Thus, not only there is
enough information but in fact there is considerable degree
of redundancy to solve for the unknown parameters.

Likewise considering several CPNES can enhance the
degree of over determination. For each CPNES we add,
we have six new observables but introduce only one new parameter $(\eta_D(F))$ giving a net gain of 5. Indeed there
are several candidate modes: $K^−\pi^+$ (branching ratio
3.8%), $K^−\rho^+$ (10.8%), $K^+\pi^−$ (5.0%), $K^{*0}\pi^0$ (3.1%),
$K^−\rho^+$ (6.1%) and $K^−a_1^+$ (7.3%) giving a total 36%.
In this method one would have to separate the quasi two
body modes from the broad resonances (eg $K^−\rho^+$) mak-
ing it somewhat difficult.

Generalizing the CPES case to inclusive states should
provide the most statistically powerful data to determine
$\delta$. For instance, the inclusive $D^0 \to K^−+X$ has a branching
ratio of 53% [17]. In this case, we have the general
case of eqn. (3) and so six observables $\{X(F), Y(F),
Z(F), X(\bar{f}), Y(\bar{f}), Z(\bar{f})\}$ are determined by the six para-
eters $\{\hat{A}, r_B, \eta_B, r_D, \eta_D, \delta\}$ and so the system can
be solved with some discrete (8-fold) ambiguities.

This may be improved in two ways. Firstly, one can
segaret the set $F$ into several subsets. Thus each
additional set $F$ provides six new observables but in-
roduces only two new parameters $(\eta_D$ and $R_F)$ giv-
ing a net gain of 4. For instance, a substantial fraction
of $K^−+X$ is made up of the exclusive state $K^−\pi^+$
(4%) together with the inclusive (in the sense that these
modes depend on phase space variables) states $K^−\pi^+\pi^0$
(13.9%), $K^−\pi^+\pi^−\pi^−$ (7.5%), $K^−\pi^+\pi^−\pi^0$ (10.0%) and
$K^−\pi^+\pi^−\pi^0$ (4.0%) giving a total branching fraction of
about 40%. Another approach would be to divide the
$K^−+X$ into separate bins according to the energy of the
$K^−$ in the $D^0$ frame. This would approximate the above
since a higher energy $K^−$ would tend to be associated
with fewer pions; then one would not have to identify
the content of the $X$ state. Secondly, one could combine
the $F$ (inclusive) method with the CPES method.

The magnitude of the time dependent CP asymmetry
for various final states can be seen in the expression for $Z$ in Eqn. (3). If $D^0 \to F$ is Cabibbo allowed then $r_D \approx
\sin^2 \theta_c \approx 0.05$ while $r_B \approx |V_{ub}||V_{cb}|/(|V_{ub}|V_{us}|) \approx 0.36$. The
dominant term in $Z$ is thus the fourth term which will
lead to CP violation of $\lesssim 36\%$. Note that this term
does not become small in the limit $R_F \to 0$. In the case
where $F$ is a CP eigenstate, so $r_D = 1$, the second term
$\propto \sin 2\beta \approx 0.8$ becomes dominant. If $R_F$ is small then
the third and fourth terms are dominant, giving again a
contribution $\propto r_B \approx 0.36$.

Now, in order to illustrate the relative power of
the different methods, let us consider the following
toy model. First, we estimate $BR(D^0 \to D^0K_S) \approx
\sin^2 \theta_c (1/N^2_{K_S}) BR(B^0 \to D^* \pi^+)/2 \approx 10^{-5}$. For
this example, we will arbitrarily take $\eta_B = 50^\circ$ and $\eta_D = 70^\circ$
with $\gamma = 60^\circ$ and $\beta = 25^\circ$, consistent with the B factory
values and so $\delta = 110^\circ$.

In Fig. 1 we plot the $\chi^2$ which would be obtained as-
suming $N_B = (number \ of \ B \ mesons)/(acceptance) =
10^9$ for various combinations of the data. The thin solid
line gives the minimum value of $\chi^2$ as a function of $\delta$
obtained for the data from the single final state $K^−\pi^+$. Clearly
discrete ambiguities in the solution tend to con-
spire to keep the value of $\chi^2$ relatively low. The dashed
line gives the minimum $\chi^2$ using CP eigenstates contain-
ing a $K_S$ and a $K_L$. The dotted line shows the case
when $K^−\pi^+$ data is combined with CPES containing $K_S$.
The dashed-dotted line gives the result for the inclusive
$K^−+X$ alone where we have taken for the purposes of
illustration $\eta_D = 70^\circ$, $r_D$ the same as for the $K^−\pi^+$ and
$R_F = 0.1$ and the thick solid line combines this with the
CP$=−1$ eigenstates. In all cases, we have assumed that
the overall tagging efficiency for the $B^0$ flavor is 25%.

Table 1 shows the one sigma error on $\delta$ for various in-
puts. Clearly, the best results are obtained when the ob-
servables over determine the parameters and a large frac-
tion of the BR is included in the sample. Thus when
$K^−+X$ is used together with $K_S$ CPES, error on $\delta$ is
$\pm 2.5^\circ$ with $N_B = 10^9$; with $N_B = 10^8$ this error
increases to $\pm 11.4^\circ$, so is still quite useful.
through $B^0 \to D^0 K_S$ can be greatly enhanced by considering $D^0$ decays to CP eigenstates and by using inclusive sets of $D^0$ decays. In particular, using inclusive $D^0$ decays such as $K^- + X$ together with CP eigenstates, our illustrative calculation suggests that as the number of available $B$ mesons increases from $10^8$ to $10^9$ a determination of $\delta$ with a one sigma error of $\lesssim \pm 11.4^\circ$ becomes feasible even with a modest acceptance of $O(10\%)$. This error can, of course, be reduced to the level of a few percent as the acceptance is improved. The method described to make use of inclusive states is likely to have wider application to the extraction of CP violating phases.

This research was supported by Contract Nos. DE-FG02-94ER40817 and DE-AC02-98CH10886.

Since overdetermining the system of equations is key to improving accuracy on $\delta$ it may be useful to introduce additional constraints. First of all, one can replace the $D^0$ and the $K_S$ with higher resonances which will tend to increase the total statistics. In addition, as suggested in [10], if the $D^0$ is replaced with a $D^{0*}$ then we can tag the flavor of the $D^{**}$ through the decay $D^{0*} \to D^+ \pi^-$. The analysis of decays with this tag thus reduces to that of $[3, 11]$. There is the practical problem implementing this method that one must separate the $D_1$ and $D_2$ states which are 40 MeV apart. Secondly, as suggested in [12], using the methods of $[13, 14]$ one can directly determine $R_S$ and $\hat{\eta}_D$ from studies at a $\psi(3770)$ charm factory.

Finally, the technique discussed here for replacing a single state with an inclusive one in the interference of two amplitudes has an immediate application to getting $\gamma$. In $[15]$ we consider this for the method of $[16]$ for extracting $\gamma$ from $B^- \to K^- D^0$ with various $D^0$ final states. In particular, this gives a model independent way of analyzing three body $B$ and $D$ final states.

In conclusion, we show that the ability to determine $\delta$

![Graph](image-url)