Numerical Calculation of Two-Dimensional Subsea Cable Tension Problem Using Minimization Approach

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Abstract Subsea cable laying is a risky and challenging operation faced by engineers, due to many uncertainties arise during the operation. In order to ensure that subsea cables are laid out diligently, the analysis of subsea cable tension during the laying operation is crucial. This study focuses on the fatigue failure of cables that will cause large hang-off loads based on catenary configuration after laying operation. The presented problem was addressed using mathematical modelling with consideration for a number of defining parameters, which include external forces such as current velocity and design parameters such as cable diameter. There were two types of subsea cable tension analyses studied: tensional analysis of catenary configurations and tensional analysis of lazy wave configurations. The latter involved a buoyancy module that was incorporated in the current catenary configuration that reduced subsea cable tension and enhanced subsea cable lifespan. Both analyses were solved using minimization through the gradient-based approach concerning on the tensional analysis of the subsea cable in different configurations. Lazy wave configurations were shown to successfully reduce cable tension, especially at the hang-off section.

Keywords Two-dimensional, Steady state problem, Catenary configuration, Lazy wave configuration, Minimization with gradient-based approach.

Mathematics Subject Classification 65D99, 65K05

1 Introduction

Subsea cable systems have been extensively used worldwide since the mid-19th century. Ever since subsea cable has played an important role in providing communications due to
their massive capacity, great reliability, and excellent communication quality. The rapid development of telecommunication systems has led to the exploitation of ocean environment resources and the intensive deployment of subsea cables. During the subsea cable deployment process, any incorrect manipulation can cause unreparable subsea cable damage. Therefore, the tension analysis of subsea cables has been explored by researchers over the past few decades.

Attention has been paid for the last few decades to theoretical studies about the static and dynamic responses of subsea cables during the deployment process. Zajac [1] was the first to develop a steady state theory for subsea cables in a typical deployment process in which the model assumed as a straight line and excluded the effect of transient motions. Followed from the theory developed by Zajac [1], many studies have considered different analysis approaches for both static and dynamic subsea cable response [2-3]. After Zajac [1], Patel and Vaz [4] extended the steady state model into a two-dimensional model of transient subsea cable behavior in which the developed model was used to analyze cable top tension, required payout rate as a function of time, and cable configuration whenever a ship accelerates or decelerates. The two-dimensional model was then extended into a three-dimensional model that considered the sheared currents effect, subsea cable elastic deformation, and vessel speed during the laying process using the conventional finite element method and Runge-Kutta technique [5-10].

The numerical and experimental dynamic behavior results of subsea cables have been a great interest to the subsea cable design process. A comparison of the numerical and experimental results of this problem have been discussed in past studies in [11-14] in which subsea cable bending stiffness was calculated using the cable shape to determine where the cable experienced the greatest stress and thrust from the laying ship. A recent study by Han et al. [15] established a mathematical model in which the initial-boundary value problem was addressed using a Partial Differential Equation (PDE). These mathematical models have been used to describe the three-dimensional motion of subsea cables being laid onto the seabed of varying depths.

Subsea cable installation is an extremely important lifetime process that requires very precise control. One of the challenges faced during subsea cable deployment is slack formation as stated by Abidin et al. [16]. Slack formed during the subsea cable laying process increases the possibility of the cable being laid too long for planned route, causing a buckling issue at the touchdown point (TDP). According to Wang et al. [17], the configuration and performance of subsea cables are essential to subsea cable installation. A free hanging subsea cable hangs from a floating production vessel to the seabed, forming a catenary shape has a lower manufacturing cost and good ability to resist high temperature. However, this kind of configuration may face buckling issues at the touchdown zone. Variations in catenary configurations were developed considering the existence of buoyancy modules, which are a logical extension of the catenary shape. These new variations in catenary configurations were modeled by installing buoyancy modules into subsea cables at a certain part to eliminate partial tension. According to Wang and Duan [18], buoyancy modules equipped onto subsea cables tend to decouple catenary configurations from surface dynamics in the touchdown region, which ensures that the strength, fatigue performance, and platform payload of the cable are within acceptable limits.

In summary, most studies have mainly focused on subsea cables without buoyancy modules to allow for simple and easy subsea cable installation. Hence, this study extends the formulated mathematical model by including buoyancy modules to provide insight into subsea cables catenary configurations, which are very sensitive to fatigue and have a high top tension. The implementation of buoyancy modules onto subsea cables in past studies only focused on the
pipeline itself to show the buoyancy modules ability to reduce tension along the pipeline. Thus, this study was conducted to provide insight into the work ability of buoyancy modules on subsea cables as both subsea cables and pipelines are slender structures subject to the similar accidental impact loads with different local damage mechanisms due to the discrepancies in cable diameter, structural configuration, and flexural rigidity.

The problem considered in this study considered subsea cables in the (1) catenary configuration, and (2) lazy wave configuration. The study theory was formulated and used to solve problems related to subsea cable tension analysis in engineering applications. In this study, subsea cable tension analysis was based on the following assumptions:

- Continuity: Subsea cables were considered continuous in terms of tension and force analysis
- Flexibility: Bending stiffness was neglected, and bending was only considered except momentarily at the sea surface node of the top boundary condition
- The subsea cable was assumed to be fully immersed in sea water
- The drag coefficient \((C_D)\), lift coefficient \((C_L)\), water density \((\rho_w)\), and cable density \((\rho_c)\) were considered constant.

2 Theoretical Model

The first model formulated in this paper was based on subsea cables in catenary configurations. The aim of this model was to analyze the position of maximum tension on the subsea cable during its practical installation in a steady state condition. This model was further enhanced by the installation of a buoyancy module on certain parts of the subsea cable to create a lazy wave configuration.

2.1 Equilibrium Equations for the Subsea Cable in Water

The model formulated in this study was based on the Cartesian coordinate system. This paper established a two-dimensional subsea cable mathematical model that considered ocean environmental effects such as lift force, drag force, and ocean current for steady state cases. When a subsea cable is laid from the floating production vessel to the TDP, the components that contribute to the tensional forces \((T)\) along the subsea cable can be described as:

- Gravity force \((W\Delta s)\), which is due to subsea cable self-weight
- Buoyancy force \((B)\), which is equal to the weight of its displaced fluid
- Drag force \((D)\), which is the force a flowing fluid exerts on a body in the flow direction
- Lift force \((L)\), which is the force a flowing fluid exerts on a body in the normal flow direction

Considering a cable length as an infinitesimal element, with a leftward \(\Delta s\) and water flow direction, the possible forces acting on a subsea cable in water is illustrated in Figure 1.
2.2 Mathematical Model of Subsea Cables in Catenary Configurations

Catenary shape of subsea cables can be described as a free hanging subsea cable without a buoyancy module hanging from a floating production vessel to the seabed. It has become a popular configuration for the deployment of subsea cables near platforms as it is a low cost alternative to subsea cable design.

In steady state conditions, the subsea cable is considered to be laid onto the seabed and the last point of the cable touching the seabed is known as the touchdown point (TDP), which is fixed in place. The TDP is the initial point from which the length of the subsea cable is measured to the floating production vessel. The parameter, \( s \) (arc length) was introduced to denote the length of the subsea cable. The study model does not consider the sea surface effect (wave) on the subsea cable.

Based on the Cartesian coordinate system, the cable laying ship starts at origin, \( o \). The ship sails and lays the subsea cable from point \( P \) along the prescribed lines. The arc length, \( s \) is introduced to denote the length of the cable from point \( P \) to any point until the desired cable configuration is achieved. Figure 2 illustrate a subsea cable in a catenary configuration. Based on Figure 2, \( V_x \) is the velocity of water (current velocity) and \( H \) is the water depth.
The two-dimensional subsea cable steady state model was derived based on some physical and mechanical subsea cable laws. This study’s notations are described as follows:

- \( r \) - The radius of subsea cable, where \( d = 2r \) is the diameter of subsea cable.
- \( g \) - Gravitational acceleration.
- \( W \) - Self-weight of a cable per unit length in water.
- \( T_x(i) \) - Tension force at \( i^{th} \) point in the \( x \)-direction.
- \( T_y(i) \) - Tension force at \( i^{th} \) point in the \( y \)-direction.
- \( \rho_c \) - Density of subsea cable.
- \( \rho_w \) - Density of seawater.
- \( \rho g \Delta s \) - Gravity of cable segment \( \Delta s \), \( \rho = \pi r^2 \rho_c \).
- \( \rho_o g \Delta s \) - Buoyancy of cable segment \( \Delta s \), \( \rho_o = \pi r^2 \rho_w \).
- \( D_x \) - Drag force of seawater in the \( x \)-direction.
- \( D_y \) - Drag force of seawater in the \( y \)-direction.
- \( L_x \) - Lift force of seawater in the \( x \)-direction.
- \( L_y \) - Drag force of seawater in the \( y \)-direction.

Generally, the drag force, \( D \) and lift force, \( L \) were obtained when the upstream velocity (relative to body) of fluid, \( V \) and fluid density, \( \rho_f \) were measured using flow over body using Equation (1) and (2), respectively.

\[
D = C_D \frac{\rho_f}{2} V^2 A_p \tag{1}
\]
\[
L = C_L \frac{\rho_f}{2} V^2 A \tag{2}
\]

\( C_D \) in Equation (1) denote the total drag coefficient while \( C_L \) in Equation (2) is the total lift force. \( A_p \) is the frontal area which means the projected area seen by a person looking toward the object from a direction parallel to the upstream velocity \( V \) and variable \( A \) is the planform area seen by a person looking toward the object from a direction normal to the upstream velocity \( V \).

In this study, the water velocity along the \( y \)-direction was neglected as it is very small in the ocean where \( v_y(x, y) = 0 \). Water velocity along the \( x \)-direction was written as \( v_x = V_x(x, y)\vec{m} = (U_x, U_y) \) was the velocity of the cable segment relative to the water while \( \vec{\ell} \) was a vector of \( \vec{\ell} = (\Delta x, \Delta y) \). The drag and lift force in the \( x \)-direction and \( y \)-direction were defined using Equation (3) and (4) respectively considering the vector of \( \vec{m} \) and \( \vec{\ell} \).

\[
D_{xi} = -K_D U_{xi} \left| \vec{\ell}_i \times \vec{m}_i \right| \tag{3}
\]
\[
D_{yi} = -K_D U_{yi} \left| \vec{\ell}_i \times \vec{m}_i \right| \tag{3}
\]
\[
L_{xi} = K_L U \left| \vec{\ell}_i \cdot \vec{m}_i \right| \frac{E(1,0)}{2} \text{sgn} \left( \vec{\ell}_i \cdot \vec{m}_i \right) \tag{4}
\]
\[
L_{yi} = K_L U \left| \vec{\ell}_i \cdot \vec{m}_i \right| \frac{E(0,1)}{2} \text{sgn} \left( \vec{\ell}_i \cdot \vec{m}_i \right) \tag{4}
\]
where the $K_D$ and $K_L$ used in Equations (3) and (4) were drag and lift coefficient defined using Equations (5) and (6) respectively, while parameter $E(a,b)$ was defined using Equation (7)

$$K_D = C_D \frac{\rho w}{2} d$$

(5)

$$K_L = C_L \frac{\rho w}{2} d$$

(6)

$$E(a,b) \left(U^2 - aU_{xi}^2 - bU_{yi}^2\right) \left(\frac{\partial x}{\partial s} + b \frac{\partial y}{\partial s}\right) - \left[(1-a)U_{xi} \frac{\partial x}{\partial s} + (1-b)U_{yi} \frac{\partial y}{\partial s}\right] \left(aU_{xi} + bU_{yi}\right)$$

(7)

The direction of the drag and lift forces with respect to the direction of the upstream velocity, $V$ is illustrated in Figure 3.

![Diagram](image1)

(a) Direction of Force if $\ell_i \cdot \vec{m}_i \geq 0$

![Diagram](image2)

(b) Direction of Force if $\ell_i \cdot \vec{m}_i < 0$

Figure 3: Direction of Force with Respect to Upstream Velocities.

The equilibrium equation of a subsea cable in catenary configuration for steady state cases in $x$-direction and $y$-direction can be given by Equations (8) and (9) respectively.

$$T_{xi+1} \cos \alpha_{i+1} + T_{xi} \cos \alpha_i + D_{xi} + L_{xi} = 0$$

(8)

$$T_{yi} \cos \beta_i + T_{yi+1} \cos \beta_{i+1} + D_{yi} + L_{yi} + W\Delta s_i = 0$$

(9)

where the angle calculation for the subsea cable elements $\alpha(s)$ and $\beta(s)$ were obtained by considering the dot product between vector $\vec{T}$, $\vec{i}$, and/or $\vec{j}$ as given in Equation (10).
\[ T_i \cdot i = |T_i| |i| \cos \alpha_i, \]
\[ T_i \cdot j = |T_i| |j| \cos \beta_i, \]
\[ T_{i+1} \cdot i = |T_{i+1}| |i| \cos \alpha_{i+1}, \]
\[ T_{i+1} \cdot j = |T_{i+1}| |j| \cos \beta_{i+1}. \]  

(10)

2.2.1 Boundary Conditions for Subsea Cable Problem in Catenary Configuration

Boundary conditions for subsea cable are important to the analysis process. In this paper, subsea cable were modelled by discretizing them into smaller elements as shown in Figure 4.

Based on Figure 4, \((x_1, y_1)\) denotes TDP and the boundary condition at this point is based on a single moment of balance at the hanging point in the floating production vessel. This condition was considered as the subsea cable was assumed to have the properties of a rigid body element. The equilibrium force was considered in the clockwise and anti-clockwise direction using Equation (11) and (12), respectively.

**Force in anti-clockwise rotation**

\[ W \cdot \Delta s_i \cdot r_{x_i} \]  

(11)

**Force in clockwise rotation**

\[ Force \cdot R_y \]  

(12)

By combining both Equations (11) and (12), while considering drag and lift force, the equilibrium force at TDP from a single moment at sea surface node can be given as Equation (13).
\[ \sum_{i=1}^{N_s} (W \cdot \Delta s_i \cdot r_{x_i} + D_{x_i} \cdot r_{y_i} + L_{y_i} \cdot r_{x_i}) = Force \cdot R_y, \]  

(13)

\[ Force = \frac{\sum_{i=1}^{N_s} (W \cdot \Delta s_i \cdot r_{x_i} + D_{x_i} \cdot r_{y_i} + L_{y_i} \cdot r_{x_i})}{R_y}. \]

Boundary condition at TDP can be said as equal to the Force obtained from a single moment at sea surface node \((T_1 = Force)\). The boundary condition at the top node was obtained by projecting the force into the \(x\)-direction and \(y\)-direction. Total force at the top subsea cable node was described by Equations (14) and (15). \(T_1\) in equation (14) denotes the tension at TDP, which was calculated using the total force based on a single moment at sea surface point.

\[ T_{xN_s} \cos \alpha_{N_s} = \bar{T}_1 \]  

(14)

\[ T_{yN_s} \cos \beta_{N_s} = \sum_{2}^{N_s} (W \Delta s_i + L_{y_i}) \]  

(15)

The input data in this study are tabulated as in Table 1. The parameter values were only for mathematical simulation and were not used as a benchmark for any actual problem.

Table 1: Input Data

| Parameter                          | Value                  |
|------------------------------------|------------------------|
| Diameter of cable, \(d\)           | 0.023 m                |
| Water depth, \(H\)                 | 80 m                   |
| Density of cable, \(\rho_c\)       | 2888 kg/m\(^3\)       |
| Density of water, \(\rho_w\)       | 1024.8103 kg/m\(^3\)  |
| Cable weight in water, \(W\)       | \(\pi \left(\frac{d}{2}\right)^2 (\rho_c - \rho_w) g\) kg/ms\(^2\) |
| Drag coefficient, \(C_D\)          | 1.2                   |
| Lift coefficient, \(C_L\)          | 0.024                  |
| Gravity, \(g\)                     | 9.8 m/s\(^2\)         |
| Relative velocity, \(V\)           | 1.2 m/s               |

2.3 Mathematical Model for Subsea Cables in a Lazy Wave Configuration

The second model was based on subsea cable in a lazy wave configuration. This model was developed to reduce slack formation and improve fatigue performance in subsea cable designs in catenary configuration. The model considered in this case illustrated in Figure 5.
This mathematical model for subsea cables was obtained by extending the formulation obtained for catenary problem. The equilibrium equation in $y$-direction was modified to consider the buoyancy force to the left and right region of the buoyancy module as illustrated in Figure 5. The equilibrium equation related to the $x$-direction was obtained directly from Equation (8). The equilibrium equation for subsea cables with buoyancy module in a two-dimensional steady state can be written in the $x$-direction and $y$-direction using Equation (16) and (17) respectively.

$x-$direction

\[ T_{x_{i+1}} \cos \alpha_{i+1} + T_{x_i} \cos \alpha_i + D_{x_i} + L_{y_i} = 0 \]  

(16)

$y-$direction

\[ T_{y_i} \cos \beta_i + T_{y_{i+1}} \cos \beta_{i+1} + D_{y_i} + L_{y_i} + W \Delta s_i = 0 \]

- Left element of the buoyancy module

\[ T_i \cos \beta_i + \sum_{i=1}^{Left element of buoyancy module} (W \Delta s_i + L_{y_i}) = 0 \]  

(17)

- Right element of the buoyancy module

\[ T_{i+1} \cos \beta_{i+1} + \sum_{i=1}^{Right element of buoyancy module} (W \Delta s_i + L_{y_i}) = 0 \]

- At the buoyancy module point

\[ T_i \cos \beta_i + T_{i+1} \cos \beta_{i+1} - F_{bm} = 0 \]
Note that the equilibrium equation at the buoyancy module point in Equation (17) was formulated to determine the buoyancy force of the buoyancy module. The buoyancy force of the buoyancy module was calculated considering the subsea cable element in which we proposed to implement the buoyancy module. The equation for calculating the buoyancy force of buoyancy module was written using Equation (18), where \( p \) denotes the desired buoyancy module weight percentage that will balance the buoyancy module in certain conditions while \( L \) denotes the length of the buoyancy module region.

\[
\text{Buoyancy Force of the buoyancy module, } F_{\text{bm}} = L \cdot p \quad (18)
\]

Boundary conditions for this problem were also based on the theory developed for the catenary configuration with slight modifications.

### 2.3.1 Boundary Conditions for Subsea Cable in Lazy Wave Configurations

The subsea cable configuration with a buoyancy module was created using an interpolation technique that discretized the subsea cable into \( N \) segments. For the boundary condition at TDP, Equation (13) was considered but the calculation was made based on the buoyancy module region illustrated in Figure 5 to produce Equation (19).

\[
\begin{align*}
\sum_{i=1}^{Buoyancy \ Node} & (W \cdot \Delta s_i \cdot r_{x_i} + D_{x_i} \cdot r_{y_i} + L_{y_i} \cdot r_{x_i}) = \text{Force} \cdot R_y, \\
\text{Force} = & \sum_{i=1}^{Buoyancy \ Node} (W \cdot \Delta s_i \cdot r_{x_i} + D_{x_i} \cdot r_{y_i} + L_{y_i} \cdot r_{x_i}) \quad (19)
\end{align*}
\]

Next, the boundary condition at the top node was calculated using Equations (14) and (15) but the calculations were only made on the floating production vessel region as shown in Figure 5. The equation for the boundary condition at the top node was calculated using Equations (20) and (21) in the \( x \)-direction and \( y \)-direction respectively.

\[
\begin{align*}
T_{xN_s} \cos \alpha_{N_s} = & T_1 \\
T_{yN_s} \cos \beta_{N_s} = & \sum_{Floating \ Production \ Vessel}^{N_s} (W \Delta s_i + L_{y_i}) \quad (21)
\end{align*}
\]

The data considered for this case shown in Table 1. The effectiveness of the new formulated mathematical model in steady state condition was validated and compared with previous studies. A comparison between present and previous model was made using subsea cable tension analysis and cable configurations based on different parameters such as cable weight and cable diameter. The cable configuration of this study was similar to Yang et al. [13]. A comparison of subsea cable tension analysis between this study and previous study showed good agreement.
for performance measurements such as cable diameter, water depth and current velocity. The formulated mathematical subsea cable model in a catenary configurations was then extended through the addition of buoyancy module. The results of both models are discussed in the next section.

3 Results and Discussion

The mathematical subsea cable model in this study was developed based on Yang et al. [13] and Han et al. [15]. The formulated model was updated with appropriate boundary conditions. To show the validity of the model, comparisons between the developed model and Yang et al. [13] were conducted as shown in the next section.

3.1 Comparison of Subsea Cable Tension with Past Study

The comparison of subsea cable tension analysis in this study and previous study was performed. The comparison showed good agreement for some performance measurements based on the input data given in Table 1. The comparison results are shown in Figure 6.

As shown in Figure 6, the cable tension calculated in this study was smaller than Yang et al. [13] except for a very small portion around the sea surface. The top tension obtained in this study is larger than Yang et al. [13]. At the TDP the cable tension of this study was smaller than Yang et al. [13].

3.2 Tension Analysis of Subsea Cables in Catenary Configurations

Throughout the analyses, cable configurations is taken to be differ in their distance from the floating production vessel as illustrated in Figure 7. The objective function \( f(T_1, T_2, \ldots, T_{Ns}) \) was created to minimize subsea cable tension for analysis purposes based on the equilibrium

Figure 6: Tension Distribution Comparison for Past and Present Studies
equation obtained in Equations (8) and (9) as well as boundary conditions of Equations (13) to (15) Table 2 shows the corresponding tensions in each cable element. The objective function was written as follows:

Let

\[ netFy = \sum_{i=2}^{Ns} (W \Delta s_i + L_{yi}) \]

\[ x\text{-direction} \]

\[ f = f + \sum_{i=2}^{Ns-1} (T_{xi+1} \cos \alpha_{x_{i+1}} + T_{xi} \cos \alpha_{xi} + D_{xi} + L_{xi})^2 \]

\[ y\text{-direction} \]

\[ f = f + \sum_{i=2}^{Ns-1} (T_{yi} \cos \beta_{yi} + T_{yi+1} \cos \beta_{yi+1} + D_{yi} + L_{yi} + W \Delta s_i)^2 \]

Boundary condition at TDP

\[ f = f + (\bar{T}_1 - \text{Force})^2 \]

Boundary condition at top node

\[ f = f + \left( T_i \cos \alpha_i - \bar{T}_1 \right)^2 + \left( T_i \cos \beta_i - netFy \right)^2 \]

Figure 7: Cable Configuration of Different Distances from The Floating Production Vessel
Table 2: Tension Distribution Along Subsea Cables in Catenary Configurations (Starts from TDP)

| Water depth (m) | 10 | 20 | 30 | 40 | 50 | 60 |
|----------------|----|----|----|----|----|----|
| Cable length (m) | 83.224 | 88.0197 | 93.6104 | 99.7614 | 106.3452 | 113.2799 |
| Cable weight (N) | 651.3675 | 665.7419 | 710.1545 | 756.8173 | 806.7638 | 859.3726 |
| Tension distribution along the subsea cable (N) | 16.7657 | 10.2317 | 44.6708 | 60.1627 | 77.4239 | 95.9095 |
| | 24.1715 | 45.6149 | 66.9973 | 89.452 | 111.7473 | 135.5564 |
| | 31.4381 | 60.8502 | 89.1685 | 117.3657 | 145.9080 | 175.0417 |
| | 38.4285 | 75.6349 | 110.8247 | 145.2455 | 179.5066 | 213.9573 |
| | 45.3028 | 90.7289 | 131.5416 | 172.0855 | 211.9972 | 251.7168 |
| | 52.6001 | 103.3051 | 151.1901 | 197.4880 | 242.8479 | 287.7005 |
| | 61.1201 | 117.0094 | 170.0420 | 221.4451 | 271.8017 | 321.4738 |
| | 70.6049 | 131.9018 | 188.9586 | 244.5071 | 299.1070 | 353.0263 |
| | 81.1180 | 148.0300 | 208.7753 | 267.5942 | 325.5233 | 382.8843 |
| | 91.0049 | 155.3509 | 229.0024 | 291.4442 | 351.9539 | 411.9217 |
| | 107.4282 | 183.9342 | 251.9354 | 316.2588 | 378.9480 | 440.9078 |
| | 125.3017 | 204.3881 | 275.2025 | 341.9342 | 406.5674 | 470.1558 |
| | 148.0801 | 227.7183 | 300.1260 | 368.5679 | 434.7391 | 499.6382 |
| | 177.2562 | 255.3804 | 327.7465 | 396.7601 | 463.6999 | 529.5681 |
| | 214.4303 | 288.9881 | 359.5013 | 427.6336 | 494.2127 | 559.7873 |
| | 261.2033 | 330.2468 | 397.1006 | 462.7225 | 527.5530 | 591.8752 |
| | 319.6746 | 381.1018 | 442.4432 | 503.8771 | 565.3982 | 627.1010 |
| | 391.4430 | 443.5232 | 497.5678 | 553.0681 | 609.7171 | 667.3144 |
| | 478.0078 | 519.6002 | 564.6317 | 612.5426 | 662.8870 | 714.6419 |

For each configuration, tension increases from TDP to its maximum value at the hanging point. This is due to a gravitational effect in which the cable element at the TDP is resting horizontally on the seabed while the cable elements near the hanging point does not have support and has to withstand the weight of the cables below. For the cable length in the x-direction of 60m, it can be seen clearly in Table 2 that the tension distribution for the subsea cable reached more than 500N. Apart from this, the longer the cable, the higher the tension at hanging point for each cable. In order to reduce cable tension, a support is needed such as a buoyancy module which will create a lazy wave configuration that will reduce the extreme stresses and fatigue of the subsea cable to within an acceptable limit.

3.3 Tension Analysis of Subsea Cables for Lazy Wave Configurations

The tension analysis of subsea cables with a lazy wave configuration problem were based on Equations (16) and (17). The cable configuration for this case was created based on several different buoyancy module distances from the seabed (30m, 40m and 50m) and from the floating production vessel (20m, 30m and 40m). Note that the x-distance of the cable length was fixed at 60m, so it could be compared to the last column of Table 2 for subsea cable configurations without a buoyancy module. The objective function $f(T_1, T_2, \ldots, T_{Ns})$ was created based on the equilibrium equation obtained in Equations (16), (17), (19), (20) and (21).
3.3.1 Tension Analysis of Subsea Cables with Buoyancy Module 30m from Seabed

The buoyancy module was installed 30m from the seabed illustrated in Figure 8, with different distances from the floating production vessel of 20m, 30m and 40m respectively.

![Figure 8: Cable Configuration with Buoyancy Module 30m from Seabed (a)20m, (b)30m and (c)40m distance from floating Production Vessel](image)

Table 3: Tension Distribution Along Subsea Cables With Buoyancy Module 30m from Seabed With Varying Distances from the Floating Production Vessel.

| Distance from Production floating vessel (m) | 20   | 30   | 40   |
|---------------------------------------------|------|------|------|
| Cable Length, L, m (m)                      | 128.3809 | 124.6065 | 122.7833 |
| Cable Weight, w (N)                         | 973.9330  | 945.2994  | 931.4683  |
| Length of buoyancy module region, Lbm (m)   | 57.4130   | 50.8960   | 52.1399   |
| Percentage of buoyancy force of buoyancy module from cable weight (%) | 44.7208 | 40.8454 | 42.465 |
| Buoyancy force of buoyancy module, Fbm (N)   | 435.5506  | 386.1113  | 395.5480  |
| Tension distribution along subsea cable      |       |       |       |
| 116.7582                                    | 76.6283  | 77.9930  |
| 10.3627                                     | 11.2398  | 31.6236  |
| 66.9689                                     | 90.5690  | 111.7599 |
| 141.7811                                    | 180.4623 | 207.7665 |
| 226.0713                                    | 263.4413 | 289.6859 |
| 320.7865                                    | 347.6782 | 368.6213 |
| 425.5860                                    | 435.5577 | 448.1036 |

Table 3 shows the corresponding tensions in each cable element. The buoyancy module point was outlined by a black color in Table 3. All the results had an appropriate tension distribution along the subsea cable that did not exceed the critical value of 500N.
### 3.3.2 Tension Analysis of Subsea Cables with Buoyancy Module 40m from Seabed

This section discusses on the results for the buoyancy module 40m from the seabed which are shown in Figure 9 with a distance from the floating production vessel of 20m, 30m and 40m respectively.

![Figure 9: Cable Configuration with Buoyancy Module 40m from Seabed (a)20m, (b)30m and (c)40m distance from floating Production Vessel.](image)

| Distance from Production floating vessel (m) | 20  | 30  | 40  |
|--------------------------------------------|-----|-----|-----|
| Cable Length, L (m)                        | 115.7250 | 123.9171 | 129.4244 |
| Cable Weight, w (N)                        | 877.9295 | 940.0698 | 981.8493 |
| Length of buoyancy module region, Lhm (m)  | 57.9431 | 59.5587 | 65.3475 |
| Percentage of buoyancy force of buoyancy module from cable weight (%) | 50.0693 | 48.0633 | 50.4909 |
| Buoyancy force of buoyancy module, Fhm (N) | 439.5732 | 451.8286 | 495.7445 |

![Table 4: Tension Distribution Along Subsea Cable With Buoyancy Module 40m from Seabed With Varying Distances from Floating Production Vessel](table)

The buoyancy module point was outlined using a black color in Table 4. The cable configuration with buoyancy module position of 30m or 40m from the floating production vessel also had an appropriate tension distribution along the subsea cable less than 500N.
3.3.3 Tension Analysis of Subsea Cable with Buoyancy Module 50m from Seabed

This section discusses the results for the buoyancy module 50m from the seabed as shown in Figure 10 with different distances from the floating production vessel of 20m, 30m and 40m respectively.

![Graphs showing tension distribution](image)

Figure 10: Cable Configuration with Buoyancy Module 50m from Seabed (a) 20m, (b) 30m and (c) 40m distance from floating Production Vessel.

| Distance from Production floating vessel (m) | 20    | 30    | 40    |
|---------------------------------------------|-------|-------|-------|
| Cable Length, L (m)                        | 114.7503 | 123.1169 | 129.8866 |
| Cable Weight, m (N)                        | 870.5274 | 933.9989 | 983.3554 |
| Length of buoyancy module region, Lbm (m)  | 66.2816 | 72.2137 | 74.7230 |
| Percentage of buoyancy force of buoyancy module from cable weight (%) | 57.7616 | 58.6546 | 57.5294 |
| Buoyancy force of buoyancy module, Fbm (N) | 502.8306 | 547.8333 | 566.8691 |
| Tension distribution along subsea cable     | 133.5274 | 118.2690 | 123.9489 |
|                                            | 186.9246 | 146.9233 | 198.3902 |
|                                            | 299.7340 | 244.8775 | 295.0623 |
|                                            | 407.5620 | 350.3030 | 389.8611 |
|                                            | 501.6933 | 448.0700 | 486.1186 |
|                                            | 588.2340 | 542.8555 | 492.2768 |
|                                            | 529.1723 | 526.7026 | 684.9125 |
|                                            | 12.2269  | 34.6975  | 34.5218  |
|                                            | 4.1796   | 3.9363   | 10.6727  |
|                                            | 64.8008  | 64.4133  | 88.7526  |
|                                            | 125.6519 | 135.3596 | 173.1213 |
|                                            | 177.6867 | 196.0328 | 238.3586 |
|                                            | 227.6672 | 254.4308 | 296.3969 |
|                                            | 277.7345 | 313.0019 | 351.1620 |

The buoyancy module point was outlined by using a black color in Table 5. Based on the three models, only cable configurations with a buoyancy module position 40m from the floating production vessel had an appropriate tension distribution along the subsea cable that was less than 500N.
4 Conclusion and Recommendations

Mathematical models of subsea cable for both catenary and lazy wave configurations were generated with assumption that velocity was constant, the seabed was flat, and the effects of wind and waves were insignificant. Two solutions were presented in this paper: tension analysis for subsea cables without buoyancy modules (catenary configuration), and tension analysis for subsea cables with buoyancy modules (lazy wave configuration). Both solutions used the same unconstrained minimization approach. It can be seen clearly in the report that the tension distribution for subsea cables in the catenary configuration was greater than 500N, which will cause a large hang-off load at the floating production vessel.

Buoyancy modules were installed in a suitable section of the catenary cable shape, forming a lazy wave configuration that reduced the tension of the subsea cable at the floating production vessel. From the nine different buoyancy module positions, the best buoyancy module position was 30m from the seabed and 30m from the vessel because its tension distribution was the lowest and was below the acceptable limit of 500N. Moreover, the buoyancy force used for this configuration was also smaller than other configurations. The proposed buoyancy module should have a buoyancy force percentage within 40% of the total subsea cable weight.

In future research, this study suggests researchers to investigate more reliable lazy wave configurations since different shapes will produce different amounts of tension along the subsea cable. Subsea cable configurations should be developed based on different criteria such as different buoyancy force, buoyancy module distance from the seabed, and buoyancy module distances from the floating production vessel.

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