Higher Twist Effects in Hadronic B Decays

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Received 30 June 2003

Within the framework of QCD factorization, we discuss various important corrections arising from higher twist distribution amplitudes of mesons in the hadronic B decays.

PACS: 13.20.Hw, 12.39.St, 12.38.Bx

Key words: QCD factorization, light-cone distribution amplitude

1 Factorization with QCD improvement

Consider the charmless \(B \to M_1 M_2\) decays with \(M_2\) being emitted, as shown in Fig. 1. The decays can be studied by means of the QCD factorization (QCDF) approach [1]. Since the energies of final state mesons \(M_1, M_2\) are order of \(m_B/2\), the soft corrections between the two mesons will therefore decouple in order of \(\Lambda_{QCD}/m_b\). Only hard interactions between \((BM_1)\) and \(M_2\) survive in the heavy \(b\) quark mass limit, \(m_b \to \infty\), and soft effects are confined to \((BM_1)\) system. In the QCDF approach, the transition matrix element of the 4-quark operator \(O_i\), depicted in Fig. 1, is given by

\[
\langle M_1 M_2|O_i|B \rangle = \mathcal{F}^{BM_1}(m_B^2) \int_0^1 du T^{I}(u)\Phi_{M_2}(u) + \int_0^1 d\xi du dv T^{II}(\xi, u, v)\Phi_\mathcal{B}(\xi)\Phi_{M_1}(v)\Phi_{M_2}(u),
\]

where \(T^{I}, T^{II}\) are hard scattering functions, \(\mathcal{F}^{BM_1}\) the \(B \to M_1\) transition form factor, \(\Phi_{M_1,2}\) the light-cone distribution amplitudes (LCDAs) of the final state mesons.

2 Light-cone distribution amplitudes of light mesons

The nonlocal quarks (and gluon(s)) sandwiched between the vacuum and the final state meson can be expressed in terms of a set of LCDAs. For the processes of \(B \to VP\), where \(V\) and \(P\) denote the vector and pseudoscalar mesons, respectively, we find that the weak annihilation diagrams induced by \((S - P)(S + P)\) penguin operators, which are formally power-suppressed by order \((\Lambda_{QCD}/m_b)^2\), are chirally and logarithmically enhanced owing to the non-vanishing end-point behavior of the twist-3 LCDA of the pseudoscalar meson \(P\) [2]. The two-parton LCDAs of the light

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The pseudoscalar meson of interest are given by

\[ \langle P(p)|q_\alpha(x)q_\beta^\dagger(0)|0\rangle = \frac{iF_P}{4} \int_0^1 du e^{iup.x} \times \left[ f_5 \Phi(u) - \mu_\chi \gamma_5 \left( \Phi_p(u) - \frac{1}{6} \sigma_{\mu\nu} p^\nu x^\mu \Phi_\sigma(u) \right) \right]_{\beta\alpha}, \]

where \( \Phi \) is the leading twist (twist-2) LCDA and \( \Phi_P \) and \( \Phi_\sigma \) are of twist-3. The above LCDAs are defined by

\begin{align*}
\langle P(p)|\bar{q}_1(0)\gamma_\mu \gamma_5 q_2(x)|0\rangle &= -iF_P p_\mu \int_0^1 d\bar{\eta} e^{i\bar{\eta}p.x} \phi^K(\bar{\eta}), \\
\langle P(p)|\bar{q}_1(0)i\gamma_5 q_2(x)|0\rangle &= F_P \mu_\chi \int_0^1 d\bar{\eta} e^{i\bar{\eta}p.x} \Phi_p^P(\bar{\eta}), \\
\langle P(p)|\bar{q}_1(0)\sigma_{\mu\nu} \gamma_5 q_2(x)|0\rangle &= -\frac{i}{6} F_P \mu_\chi \left[ 1 - \left( \frac{m_1 + m_2}{m_P} \right)^2 \right] \\
&\times \left( p_\mu x_\nu - p_\nu x_\mu \right) \int_0^1 d\bar{\eta} e^{i\bar{\eta}p.x} \Phi_\sigma^P(\bar{\eta}),
\end{align*}

where \( \mu_\chi^P = m_P^2/(m_1 + m_2) \), with \( m_{1,2} \) being the current quark masses of \( q_{1,2} \), and the asymptotic forms of LCDAs are:

\[ \Phi_P^P(x) = 6x(1-x), \]
\[ \Phi_\sigma^P(x) = 1, \quad \Phi_\sigma^P(x) = 6x(1-x). \]
Higher Twist Effects in Hadronic B Decays

CLEO, BaBar and Belle recently reported the results

\[
B(B^\pm \to \phi K^\pm) = \begin{cases} 
(5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6} & \text{CLEO,} \\
(10.0 \pm 0.9 \pm 0.5) \times 10^{-6} & \text{BaBar,} \\
(14.6 \pm 3.0^{+2.8}_{-2.0}) \times 10^{-6} & \text{Belle,}
\end{cases}
\]

\[
B(B^0 \to \phi K^0) = \begin{cases} 
(5.4^{+2.7}_{-2.7} \pm 0.7) & \text{CLEO,} \\
(7.6 \pm 1.3 \pm 0.5) & \text{BaBar,} \\
(13.0^{+6.1}_{-5.2} \pm 2.6) & \text{Belle.}
\end{cases}
\]

In absence of annihilation contributions, the resulting branching ratios are $Br(B^- \to \phi K^-) = (3.8 \pm 0.6) \times 10^{-6}$ and $Br(B^0 \to \phi K^0) = (3.6 \pm 0.6) \times 10^{-6}$, which are small compared with the data. The relevant weak annihilation diagrams for $\phi K$ modes, which are penguin-dominated processes, are depicted in Fig. 2. The annihilation contributions induced by $(S-P)(S+P)$ penguin operators are not subject to helicity suppression and could be sizable. As shown in [2], these annihilation contributions, which are formally power-suppressed by order of $(\Lambda_{QCD}/m_b)^2$, are chirally and logarithmically enhanced. The logarithmatic divergence (or enhancement) is owing to the non-vanishing end-point behavior of the twist-3 LCDA of the kaon. The annihilation amplitude for $\phi K$ modes is approximately given by

\[
A_{\text{ann}} \simeq -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* f_B f_\phi b_3(\phi, K),
\]

where

\[
b_3(\phi, K) \approx \frac{C_F}{N_c} \left(c_6(\mu_h) + c_5(\mu_h)/N_c\right) \times 6\pi\alpha_s \frac{2\mu_\chi^2}{m_B} (2X_A^2 - X_A),
\]

with $X_A$ being usually parametrized as $X_A = (1 + \rho_A) \ln(m_B/\Lambda_\chi)$. Note that the gluon propagator in the annihilation diagrams, as shown in Fig. 2, is not as hard as in the vertex diagrams. Since the virtual gluon’s momentum squared there is $(\bar{\eta}p_{\eta} + \bar{\xi}\eta p_{\xi})^2 \approx \bar{\eta}k m_B^2 \sim 1 \text{ GeV}^2$, where $\bar{\eta}m_B \sim \Lambda_\chi$ and $\xi \sim 0.5$, the relevant scale for the annihilation topology should be $\mu_h \sim 1 \text{ GeV}$. 
By comparing calculation results with the data, we therefore know that annihilation contributions are not negligible and could give 50% corrections to the decay amplitudes of $\phi K$. Moreover, because the annihilation amplitudes give constructive contributions to $\phi K$ modes, they need to have the large real part, $\text{Re}(\rho_A) \geq 0.7$ for adopting $f_B = 180$ MeV. A totally different conclusion has been made in [7]. Their calculation gives almost imaginary annihilation contributions to the decay amplitudes. Nevertheless, their resulting branching ratios are instead enhanced by matching the full theory to effective theory at the confinement scale, where the Wilson coefficients become much larger. Using the QCDF approach, we obtain $\phi K^*/\phi K \sim 1\ [2, 9]$, while in the pQCD study [7, 8] the ratio is $\sim 1.5$. The result should be testable in the near future measurements.

4 Higher twist effects in $B \to VV$

The $B \to VV$ amplitude consists of three independent Lorentz scalars:

$$A(B(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)) \propto \varepsilon_1^{\mu} \varepsilon_2^{\nu}(ag_{\mu\nu} + bp_{\mu}p_{\nu} + ic\varepsilon_{\mu\nu\alpha\beta}p^\alpha p^\beta),$$

where $c$ corresponds to the $p$-wave amplitude, and $a$, $d$ are related to the mixture of $s$- and $d$-wave amplitudes. The three helicity amplitudes are given by

$$H_{00} = \frac{1}{2m_1m_2}\left[\left(m_B^2 - m_1^2 - m_2^2\right)a + 2m_2^2 p_c^2 b\right],$$

$$H_{\pm\pm} = a \mp m_B p_c c,$$

where $p_c$ is the c.m. momentum of the vector meson in the $B$ rest frame and $m_{1,2}$ is the mass of the vector meson $V_{1,2}$. Take $B \to \phi K^*$ as an example, which is shown in Fig. 3. When compared with $H_{00}$, to occur in the $H_{--}$ helicity amplitude, the spin of the $\bar{s}$ in the emitted vector meson $\phi$ has to be flipped. Therefore, the amplitude $H_{--}$ is suppressed by a factor of $m_\phi/m_B$. The $H_{++}$ amplitude is subject to a further spin flip and therefore is suppressed by $(m_\phi/m_B) \times (m_{K^*}/m_B)$. It is thus expected that $|H_{00}|^2 \gg |H_{--}|^2 \gg |H_{++}|^2$. The QCDF results indicate that the nonfactorizable correction to each helicity amplitude is not the same; the effective Wilson coefficients $a_i$ vary for different helicity amplitudes. The leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit and hence it is necessary to take into account twist-3 DAs $\tilde{g}_1^{(c,a)}$ of the vector meson in order to have renormalization scale and scheme independent predictions. Because the $(S-P)(S+P)$ penguin contributions to the $W$-emission amplitudes are absent and annihilation is always suppressed by helicity mismatch, tree-dominated decays tend to have larger branching ratios than the penguin-dominated ones [9].

5 Higher twist effects in $B \to J/\psi K$

The $B \to J/\psi K^{(*)}$ modes [3, 10, 11] are of great interest because they are only few of color suppressed modes in hadronic $B$ decays that have been measured.
Higher Twist Effects in Hadronic B Decays

Fig. 3. The directions of quark spins in $B \rightarrow \phi K^*$ helicity amplitudes. Here the decays are assumed to happen via a $(V - A)(V - A)$ 4-quark operator.

Fig. 4. Spectator corrections to $B \rightarrow J/\psi K$.

They receive large nonfactorizable corrections. Under factorization, the $B \rightarrow J/\psi K$ decay amplitude reads

$$A(B \rightarrow J/\psi K) \equiv \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} a_2 f_{J/\psi} m_{J/\psi} F_{BK}^B (m_{J/\psi}^2) (2 \epsilon^* \cdot p_B). \quad (8)$$

$|a_2|$ can be extracted from the data and its value is $|a_2| \simeq 0.27 \pm 0.04$, where the error depends on the form factor model of $F_{BK}$. The QCDF result for $a_2$ is

$$a_2 = c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_1 \left[ -18 - 12 \ln \frac{\mu}{m_b} + f_I + \frac{F_{BK}^B (m_{J/\psi}^2)}{F_{BK}^B (m_{J/\psi}^2)} g_I + f_{II} + f_{III} \right], \quad (9)$$

where $f_I, g_I$ arise from the vertex corrections, $f_{II}^2$ from the twist-2 hard spectator interaction, and $f_{III}^2$ from the twist-3 hard spectator interaction. In Fig. 4 we plot the spectator corrections to the decay amplitude of $B \rightarrow J/\psi K$. To leading-twist order, i.e. neglecting $f_{III}^2$, we have $a_2 (J/\psi K) \sim 0.15$ which is too small when compared with the data. The contribution of two-parton kaon LCDAs of twist-3 to the
spectator diagrams is

\[ f_{11}^3 = \left( \frac{2\mu^K}{m_B} \right)^2 \int \frac{dxdydz}{x} \frac{\Phi^B_B(x) \Phi^{J/\psi}(y) \Phi^K_{\sigma}(z)}{6z^2}, \]

where \( \Phi^K_{\sigma} \) is the two-parton kaon LCDA of twist-3 which has been shown in Eqs. (3) and (4). In the above equation, the integral of \( \Phi^K_{\sigma} \) is divergent. However, it is known that the collinear expansion cannot be correct in the end point region owing to the non-zero transverse momentum \( \langle k_\perp \rangle \) of the quark. Thus we parametrize the integral as

\[ \int_0^1 \frac{dz}{z} \Phi^K_{\sigma}(z) \sim \int_0^1 \frac{dz}{z + \langle k_\perp \rangle / m_b} - 1 \simeq \ln(m_B/\Lambda_s)(1 + \rho_H) - 1 \]  

To account for the experimental value of \(|a_2|\), the parameter \(\rho_H\) should be chosen as large as \(\sim 1.5\) [3]. It implies that \(a_2(J/\psi K)\) may be largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA \(\Phi^K_{\sigma}\), which are formally power-suppressed but chirally and logarithmically enhanced [3].

Nevertheless, since the contribution of the twist-3 kaon LCDA \(\Phi^K_{\sigma}\) to the spectator diagram is end-point divergent in the collinear expansion, the vertex of the gluon and spectator quark should be considered to be inside the kaon wave function. I.e., the kaon itself is at a three-parton Fock state. Instead of considering the contribution of the two-parton kaon LCDA of twist-3 to spectator diagrams, we calculate the subleading corrections from the three-parton LCDA of the kaon and get \(a_2 = 0.27 + 0.05i\) [12] which is well consistent with the data. The result also resolves the long-standing sign ambiguity of \(a_2\) which turns out to be positive for its real part.

6 Can we understand why \(K\omega/\pi\omega \sim 1\)?

The history of searching for the \(B^- \to \omega K^-\) rate is very interesting. The \(\omega K^-\) mode was first reported by CLEO in 1998 [13] with a large branching ratio \(\sim 15 \times 10^{-6}\), but disappeared soon after analyzing a larger data set [14] which was confirmed by the later BABAR measurement [15]. However, recently Belle observed a large \(\omega K^-\) rate, \((9.2^{+2.6}_{-2.3} \pm 1.0) \times 10^{-6}\), and \(\omega K^-/\omega\pi^- \sim 2\) [16]. The non-small \(\omega K^-\) rate is also shown in the newly BABAR data [3] with \(\omega K^- \sim \omega K^0 \sim \omega\pi^- \sim 5 \times 10^{-6}\). From the theoretical point of view, large \(\omega K\) rates are hard to understand. The ratio \(\mathcal{K}^0/\omega/\pi^-\omega\) reads

\[ \frac{\mathcal{K}^0}{\omega/\pi^-\omega} \sim \left| \frac{V_{cb}}{V_{ub}} \right|^2 \left( \frac{f_K}{f_{\pi}} \right)^2 \frac{a_4 - a_6r^{K}_{\chi} + (F^{B\pi}_{1} f_{\pi}/(F^{B\pi}_{1} f_{K}))r_{1}a_{9}/2 + f_{B} b_{3}(K,\omega)}{a_{1} + r_{1}a_{2}} \right|^2, \]
Higher Twist Effects in Hadronic B Decays

Fig. 5. The contributions of the $d\bar{u}g$ Fock state of the pion to the $B^- \to \pi^-\omega$ amplitude.

where $r_1 = f_\omega F_1^{B\pi}/f_\pi A_0^{B\omega}$, the chirally enhanced factor $r^K_\chi = \frac{2m_K^2}{m_b(m_u + m_s)}$ with $m_{s,\omega}$ being the current quark masses, and $b_3(K, \omega)$ is the annihilation contribution of $K\omega$ modes [17]. The $\pi^-\omega$ rate weakly depends on the annihilation effects. Without annihilation, since the $a_4$ and $a_6 r^K_\chi$ terms are opposite in sign in the $K_0\omega$ amplitude, the $K^0\omega/\pi^-\omega$ ratio should be very small. Choosing smaller $m_s$ could enhance the ratio, but does not help much in understanding data. We consider the contributions of the three-parton Fock state of the final state $\pi$ (or $K$) meson to the decay amplitudes, as shown in Fig. 5. We find that they can give significant corrections to decays with $\omega$ in the final states. The decay amplitudes with corrections from the three-parton Fock states are given by

$$A(B^- \to \pi^-\omega) \simeq \cdots + G_F m_\omega (\epsilon^*_\omega \cdot p_\pi) f_\omega F_1^{B\to\pi}(m_\omega^2)(V_{ub}V_{ud}^* c_1 - V_{tb}V_{td}^* (2c_4 - 2c_6 + c_3)) f_3,$$

$$A(\bar{B}^0 \to \bar{K}^0\omega) \simeq \cdots + G_F m_\omega (\epsilon^*_\omega \cdot p_K) f_\omega F_1^{B\to K}(m_\omega^2)(V_{ub}V_{us}^* c_1 - V_{tb}V_{ts}^* (2c_4 - 2c_6)) f_3,$$

(13)

where \ldots denote the contributions from two-parton Fock states of the final state mesons, the normalization scale of $c_1$ is $\sim 1$ GeV, and

$$f_3 = \frac{\sqrt{2}}{m_B f_\omega F_1^{B\to\pi}(m_\omega^2)} \langle \omega\pi^- | O_1 | B^- \rangle_{qgs}$$

$$= -4 \frac{\alpha_\pi g_4 m_B F_1^{B\to\pi}(m_\omega^2)}{\overline{\alpha}_g m_B F_1^{B\to\pi}(m_\omega^2)} p_\omega^0 \langle \pi^- | \bar{d} \gamma_5 \gamma_\alpha g_A b | B^- \rangle \simeq 0.12,$$

(14)

with $O_1 = \bar{d}_\alpha \gamma^\mu (1 - \gamma_5) u_\alpha \cdot \bar{u}_\beta \gamma_\lambda (1 - \gamma_5) b_\beta$, and $\overline{\alpha}_g \approx 0.23$ being the averaged fraction of the pion momentum carried by the gluon. More details of the present work would be given elsewhere [12]. We plot in Fig. 6 the branching ratios for $K\omega, \pi\omega$ vs the weak angle $\gamma$. Taking $\gamma = 90^\circ$, we obtain the branching ratios $\pi^-\omega : K^-\omega : K^0\omega \approx 5.5 : 4.5 : 4.3$ in units of $10^{-6}$, which are in good agreement with the present data.
7 Summary

Using the QCD factorization approach, we study various important corrections from higher twist distribution amplitudes of mesons in the hadronic $B$ decays. (i) For the case of $B \to \phi K$, the weak annihilation diagrams induced by $(S-P)(S+P)$ penguin operators, which are power-suppressed by order of $(\Lambda_{QCD}/m_b)^2$, are chirally and in particular logarithmically enhanced owing to the non-vanishing end-point behavior of the twist-3 light-cone distribution amplitudes of the kaon. (ii) For the case of $B \to VV$, it is necessary to take into account twist-3 distribution amplitudes of the vector meson in order to have renormalization scale and scheme independent predictions. (iii) $a_2(J/\psi K)$ may be largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA $\Phi^K_3$, which are formally power-suppressed but chirally and logarithmically enhanced. However, since the contribution of the twist-3 kaon LCDA $\Phi^K_3$ to the spectator diagram is end-point divergent in the collinear expansion, the vertex of the gluon and spectator quark should be considered to be inside the kaon wave function. Instead of considering the contribution of the two-parton kaon LCDA of twist-3 to spectator diagrams, we calculate the subleading corrections originating from the three-parton LCDAs of the kaon and obtain $a_2 = 0.27 + 0.05i$ which is well consistent with the data and can solve the long-standing sign ambiguity of $a_2$. (iv) We study the subleading corrections arising from the three-parton Fock states of mesons in $B$ decays. Our results can account for the observation of $\omega K \sim \omega \pi^-$.  

Acknowledgement: I wish to thank Hai-Yang Cheng for collaboration. This work is supported in part by the National Science Council of R.O.C. under Grant No. 91-2112-M-033-013.
Higher Twist Effects in Hadronic B Decays

References

[1] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B591, 313 (2000).

[2] H. Y. Cheng and K. C. Yang, Phys. Rev. D 64, 074004 (2001) arXiv:hep-ph/0012152.

[3] H. Y. Cheng and K. C. Yang, Phys. Rev. D 63, 074011 (2001) arXiv:hep-ph/0011179.

[4] CLEO Collaboration, R.A. Briere et al., hep-ex/011032.

[5] BaBar Collaboration, M. Pivk, talk presented at the XXXVIII Rencontres de Moriond, Les Arcs, Savoie, France, March 15-29, 2003.

[6] Belle Collaboration, T. Tomura, talk presented at the XXXVIII Rencontres de Moriond, Les Arcs, Savoie, France, March 15-29, 2003.

[7] C. H. Chen, Y. Y. Keum and H. n. Li, Phys. Rev. D 64, 112002 (2001) arXiv:hep-ph/0101655.

[8] C. H. Chen, Y. Y. Keum and H. n. Li, Phys. Rev. D 66, 054013 (2002) arXiv:hep-ph/0204166.

[9] H. Y. Cheng and K. C. Yang, Phys. Lett. B 511, 40 (2001) arXiv:hep-ph/0104000.

[10] H. Y. Cheng, Y. Y. Keum and K. C. Yang, Phys. Rev. D 65, 094023 (2002) arXiv:hep-ph/0111094.

[11] H. Y. Cheng, Y. Y. Keum and K. C. Yang, Int. J. Mod. Phys. A 18, 1437 (2003) arXiv:hep-ph/0112257.

[12] K. C. Yang, hep-ph/0308005.

[13] CLEO Collaboration, T. Bergfeld et al., Phys. Rev. Lett. 81, 272 (1998).

[14] CLEO Collaboration, C.P. Jessop et al., Phys. Rev. Lett. 85, 2881 (2000).

[15] BABAR collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 221802 (2001).

[16] Belle collaboration, R.S. Lu, et al. Phys. Rev. Lett. 89, (2002).

[17] D. s. Du, H. j. Gong, J. f. Sun, D. s. Yang and G. h. Zhu, Phys. Rev. D 65, 094025 (2002) [Erratum-ibid. D 66, 079904 (2002)]