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The evidence for unusual gravity from the large-scale structure of the Universe

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Under the assumption that General Relativity (GR) correctly describes the phenomenology of our Universe, astronomical observations provide compelling evidence that (1) the dynamics of cosmic structure is dominated by dark matter (DM), an exotic matter mostly made of hypothetical elementary particles, and (2) the expansion of the Universe is currently accelerating because of the presence of a positive cosmological constant $\Lambda$. The DM particles have not yet been detected and there is no theoretical justification for the tiny positive $\Lambda$ implied by observations. Therefore, over the last decade, the search for extended or alternative theories of gravity has flourished.

1.1 The evidence for dark matter and $\Lambda$

1.1.1 Systems of galaxies

The first evidence of the existence of DM was found by Zwicky [68, 69]. He measured the redshift of the eight brightest galaxies in the Coma cluster, attributed their redshifts to the Doppler effect, and assumed the cluster to be in virial equilibrium. The galaxy velocity dispersion thus promptly gives the total mass of the cluster, that turns out to be $\sim 100$ times larger than the sum of the masses of the individual galaxies. The obvious way out, that the cluster has a positive total energy, namely that it is expan-
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ding, is not convincing, because the cluster should have just formed and if this were the case for the many clusters we see on the sky, it would be unlikely to observe them. The cluster could also be a chance fluctuation in the galaxy distribution, but this hypothesis is called into question by the morphological and photometric homogeneity of the galaxy population of the cluster. In the following decades, this very same problem also appeared in nearby groups of galaxies [2].

In the early 1970s, the measure of the 21-cm line emission of neutral hydrogen showed that the rotation curves of spirals do not fall off at large radii, as expected if most of the galaxy mass were concentrated in the optically luminous component [56] (see [57] for a recent review). These findings appeared to be essential to explain the stability of the disk of spiral galaxies: in fact, self-gravitating disks, which are prone to bar instabilities, become dynamically stable when they are embedded within massive halos [51, 25].

At the same epoch, clusters of galaxies were discovered to be strong X-ray emitters [30]. If the emission mechanism is thermal, due to line emission or bremsstrahlung which originate in the ion-ion and ion-electron collisions in the intracluster plasma, the plasma temperature is an indicator of the depth of the gravitational potential well. The derived mass of the cluster agrees, within a factor of two in most cases, with the traditional estimate based on the virial theorem or the Jeans equations which only use kinematic data [9]. Moreover, the very existence of the intracluster plasma is a strong indication that there must be a massive halo that keeps the plasma confined. In 1990s, hot plasma was also observed in many groups (Figure 1.1), showing that they roughly are a rescaled version of clusters [48].

Both the galaxy kinematics and the X-ray analysis assume that the cluster is in virial equilibrium. In 1990s, photometry reached enough accuracy that one could start using clusters as gravitational lenses: the images of background galaxies are distorted by the gravitational potential well of the cluster and the cluster mass can be inferred from the amount of this distortion [38]. This technique does not need any assumption about the dynamical state of the cluster. Another mass-estimation technique which does not assume virial equilibrium, known as the caustic method, was developed by Diaferio and Geller [20]; this technique is only based on the distribu-
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Figure 1.1. Diffuse X-ray emission in the NGC 2300 group of galaxies, also known as Hickson compact group HG92. From [48]. The presence of a confined X-ray emitting intergalactic plasma indicates that the compact group is gravitationally bound [21].

In 1965, Abell made the argument that the mean mass density of the Universe must be smaller than the mean density within clusters and larger than the mean density computed including only the richest clusters [1]. He thus estimated a density of the Universe $\Omega_m \approx 0.2$, which is remarkably close to the currently accepted value $\Omega_m = 0.26$ [62]. This value implies a cosmic mass-to-light ratio $\langle M/L_B \rangle = \Omega_m \rho_c/\rho_L \approx 400 h M_\odot L_\odot^{-1}$, where $\rho_c = 3H_0^2/8\pi G$ is the critical density of the Universe and $\rho_L \sim 1.7 \times 10^8 h L_\odot$ Mpc$^{-3}$ is the luminosity density of the Universe derived from the galaxy luminosity function. Including the contribution of their DM halo, galaxies have a typical mass-to-light ratio $\langle M/L_B \rangle \sim 10 h M_\odot L_\odot^{-1}$. Therefore, galaxies globally contribute less than 3% to the mass in the Universe and we must conclude that DM is mostly distributed in structures of size much larger than the size of galaxies.
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Figure 1.2. Each dot is a galaxy in the wedge with declination in the range $26.5 - 32.5^\circ$ and right ascension and redshift $cz$ as shown. The upper panel shows the 1061 galaxies with $m_B < 15.5$ and $cz < 15,000$ km s$^{-1}$ and the lower panel the 182 galaxies with $m_B < 14.5$ and $cz < 10,000$ km s$^{-1}$. From [19].

1.1.2 The large-scale structure

In 1980s, advances in the optical detector technology made possible the realization of large redshift surveys. The most striking result appeared in 1986 when de Lapparent, Geller and Huchra [19] published the first slice of the extension of the Center for Astrophysics redshift survey [18]: with $\sim 1100$ galaxies in a $6^\circ \times 117^\circ$ strip on the sky and a depth of 15,000 km s$^{-1}$ in redshift, this survey imposed the view, contrary to what was commonly believed at that time, that the distribution of galaxies is largely dishomogeneous on scales of tens of megaparsecs (Figure 1.2). More recent surveys [15, 67] confirmed this result. Currently, the largest survey is the SDSS with $\sim 700,000$ galaxy redshifts.

How does this large cosmic structure form? Gravitational instability is the simplest driving process we can think of. In 1981, the inflationary scenario was proposed to solve the three classical problems of the standard hot Big Bang cosmology: the horizon, the flatness and the magnetic mono-
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The evidence for dark matter and $\Lambda$ pole density [31]. The inflationary scenario also gives, as a bonus, the initial conditions of the matter density field: small perturbations due to quantum fluctuations are inflated to cosmic scales by the tremendous expansion of the Universe [47].

Superclusters of galaxies typically have matter overdensities $\delta$ of the order of $\sim 1 - 10$. Gravitational instability yields a growth rate $\propto (1 + z)^{-1}$, with redshift $z$, or slower. Thus, the temperature anisotropies $\delta T / T = \delta / 3$ in the Cosmic Microwave Background (CMB), which formed at $z \sim 10^3$, should be larger than one to yield the cosmic structure we see today on large scales. On the contrary, observations yield $\delta T / T \sim 10^{-5}$ on $\theta \sim 7^\circ$ angular scales, corresponding to superclusters and larger structures [61]. To reconcile these tiny CMB anisotropies with the gravitational instability paradigm, we need to assume that the DM density perturbations start evolving at the time of equivalence, when the density perturbations in the baryonic matter\(^1\) are still coupled to the radiation field. The baryonic perturbations fall later into the DM gravitational potential wells when, at recombination time, they find a dishomogeneous distribution of the dynamically dominant DM. Therefore, we are forced to assume the existence of a non-baryonic DM, that can decouple much earlier than the baryons responsible for the CMB anisotropies.

Further evidence of the non-baryonic nature of DM is the abundance of light elements which are synthetized in the early Universe. Measures of the primordial abundance of deuterium, for example, which is particularly sensitive to the photon-to-baryon ratio, implies a baryon density $\Omega_b h^2 = 0.0214 \pm 0.0020$ [41], which is sensibly smaller than the total matter density $\Omega_m = 0.26$, and agrees with the baryon density $\Omega_b h^2 = 0.02229 \pm 0.00073$ implied by the CMB anisotropies [62].

It was soon clear that DM cannot be mostly made of neutrinos, because the large-scale distribution would be much fuzzier than observed [66], and cosmic structure would have formed in a top-down rather than in a bottom-up fashion, as indicated, for example, by the existence of quasars and galaxies at high redshift [27]. Therefore, DM must be mostly made of cold collisionless particles (CDM), namely particles that were not relativi-

\(^1\) In the astronomical jargon, baryonic or ordinary matter is all the matter made of quarks and electrons, although, rigorously, only strongly interacting fermions are baryons. All the other matter is generically called non-baryonic matter, including neutrinos and the hypothetical Weakly Interacting Massive Particles (WIMPs).
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castic at the time of decoupling [17].

These conclusions can be reached by exploiting the fact that DM dominates the dynamics of the large-scale structure, and we can, to first approximation, neglect the complications due to the dissipative nature of baryons on scales larger than galaxy clusters. However, a more detailed comparison with the real Universe requires the modeling of galaxy formation. Various methods at different level of sophistication show that a CDM scenario reproduces the evolution of kinematic, photometric and clustering properties of galaxies [40, 63]. Discrepancies remain [58, 13], but they are likely to be due to the approximated galaxy formation recipes rather than to an incorrect DM modelling.

To partially avoid the difficulty of baryon physics, we can resort to weak gravitational lensing: the light emitted by galaxies at high redshift reaches our telescopes after being deflected by the dishomogeneities of the DM distribution which increase with time [37]. Thus, the weak lensing analysis probes both the amount of DM and the history of structure formation.

The validity of the CDM scenario obtained a relevant success with quasar spectra: the short-wavelength side of these spectra contains the Lyman-α forest, a large number of absorption lines. These lines were commonly attributed to clouds of neutral hydrogen between the quasar and the observer. N-body/hydrodynamical simulations of a CDM universe, initially not conceived to reproduce the statistical properties of these absorption lines, indeed produced synthetic spectra of quasars in amazingly good agreement with observations, indicating that the Lyman-α “cloud” interpretation was unnecessarily elaborate [32].

When all this information on the matter distribution on large scales, CMB, galaxy distribution, cluster abundance and dynamics, weak lensing and Lyman-α forest measures, is combined into a power spectrum of the large-scale structure density fluctuations, we find that the best fit is obtained with a non-null cosmological constant Λ (Figure 1.3) [64]. Therefore, the cosmological community was not too surprised when, at the end of last century, some high-z supernovae were observed to be fainter than expected in a decelerating universe, namely a universe with a null cosmological constant [55, 52]. More recent supernova samples confirm that Λ must be positive, but both the statistical and systematic uncertainties remain large
In conclusion, cosmological observations lead us to a suspicious cosmic energy budget: baryon mass density $\Omega_b \sim 0.04$, non-baryonic dark matter density $\Omega_{DM} \sim 0.22$, and cosmological constant $\Omega_{\Lambda} \sim 0.74$ [62]; therefore, $\sim 96\%$ of the matter and energy in the Universe is elusive. We describe below how we suppose that these numbers fit in our picture of the physical world.

1.2 Standard solutions

Extensions of the Standard Model of particle physics have many DM particle candidates [7]. In supersymmetric models, possible DM particles are neutralinos, sneutrinos, gravitinos, axinos. Among non-supersymmetric candidates, there are sterile neutrinos, which are neutrinos that do not have Standard Model weak interactions; axions, that were introduced to sol-
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ve the CP problem in strong interactions; Kaluza-Klein excitations of the Standard Model fields, like the first excitation of the hypercharge gauge boson, in theories with extra space dimensions.

DM particles are supposed to have masses in the range between $\sim 1$ GeV and $\sim 100$ TeV, typically, depending on the various assumptions of the individual model, but scalar particles with masses below 1 GeV can also have the required properties of DM particles, namely the relic abundance achieved when they freeze out of thermal equilibrium (this happens when their self-annihilation rate becomes smaller than the expansion rate of the Universe), the measured $\gamma$-ray fluxes expected by the self-annihilation, and the limits from particle physics experiments [8]. Very massive DM particles, the so-called *wimpzillas*, with mass $> 10^{10}$ GeV, are also possible candidates but they must have been out of thermal equilibrium during freeze-out and thus their relic abundance is independent of their annihilation cross section. These supermassive particles can be produced gravitationally at the end of inflation.

Search for the DM particle candidates can be separated into direct and indirect detections [28]. If the DM particle cross section for scattering off matter is large enough, one can hope to measure the nuclear recoils in sensitive detectors located underground, to minimize the background noise due to cosmic rays and radioactive decays from earth rocks. Indirect detections rely on the possibilities of detecting the $\gamma$ photons or the particles (neutrinos, positrons, antiprotons, antideuterons [23]) produced during the annihilations of DM particles; these annihilations occur in high density regions of our cosmic surroundings, mainly the Sun, which is relevant as a source of neutrinos, the Galactic center, and the DM halo of the Milky Way itself in which the earth is embedded. Despite the large effort poured into the many past and present experiments, none has yet provided a detection which can convincingly be interpreted as due to a DM particle (see, e.g., [6]). Much hope is of course put into the data coming from the LHC experiments.

The cosmological constant problem appears to be more serious than the DM problem. Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + \frac{\Lambda}{c^2}g_{\mu\nu},$$

(1.1)
with the usual meaning of the symbols, admit the presence of an arbitrary constant \( \Lambda \). The Friedmann solutions with a positive constant \( \Lambda \) fit very satisfactorily the observational evidence of an accelerating universe.

The problem arises when one wishes to attach a physical interpretation to \( \Lambda \). A classical physics approach is to consider the \( \Lambda \) term a contribution to the energy-momentum tensor \( T_{\mu\nu} \) and to introduce a dark energy fluid with equation of state \( \rho_{\Lambda} = -p_{\Lambda}/c^2 = \Lambda c^2/8\pi G \). Since observations indicate \( \Lambda > 0 \), the dark energy fluid has negative pressure. Extensions of the equation of state \( p_{\Lambda} = w\rho_{\Lambda} \) consider an evolving \( w \) (see, e.g., [16] for a review), but current observations suggest \( w = -1 \) at all probed epochs [4], so models more sophisticated than a simple constant \( \Lambda \) seem unnecessary. It remains to be seen what sets the value of \( \Lambda \).

In the context of quantum field theory, one interprets the energy density \( \rho_v(t_0) = \Lambda/8\pi G \) at the present time \( t_0 \), as the ground state of the vacuum. In the standard hot Big Bang model, the phase transitions occurring in the very early Universe decrease the vacuum energy density by the quantity \( \Delta \rho_v \sim m^4 \), where \( m \) is the mass characteristic of the symmetry break. Specifically, one finds \( \Delta \rho_v^{\text{GUT}} \sim 10^{60} \text{ GeV}^4 \), \( \Delta \rho_v^{\text{SUSY}} \sim 10^{12} \text{ GeV}^4 \), \( \Delta \rho_v^{\text{EW}} \sim 10^8 \text{ GeV}^4 \), \( \Delta \rho_v^{\text{QCD}} \sim 10^{-4} \text{ GeV}^4 \) for the phase transitions which are supposed to occur. Now, we must have \( \rho_v(t_P) = \rho_v(t_0) + \sum \Delta \rho_v \), where \( t_P \) is the Planck time. It follows that \( \rho_v(t_P) = (1 + 10^{-108}) \sum \Delta \rho_v \), because \( \rho_v(t_0) = 10^{-48} \text{ GeV}^4 \). The \( \Lambda \) problem thus translates into an extreme fine-tuning problem, because \( \rho_v(t_P)/\sum \Delta \rho_v \) is extremely close to 1 but not exactly 1. The problems of course would disappear if \( \Lambda \) were exactly zero [65].

The \( \Lambda \) problem complicates when we consider the time evolution of the matter and \( \Lambda \) contributions to the energy budget of the Universe. Friedmann equation yields \( \Omega_m(t) = \Omega_m(t_0)/E(a) \) and \( \Omega_\Lambda(t) = \Omega_\Lambda(t_0)a^3/E(a) \), where \( E(a) = \Omega_m(t_0) + [1 - \Omega_m(t_0) - \Omega_\Lambda(t_0)]a + \Omega_\Lambda(t_0)a^3 \) and \( a(t) \) is the scale factor. Figure 1.4 shows that we live exactly at the transition time between a universe dominated by \( \Omega_m \) and a universe dominated by \( \Lambda \). In other words, we are living at a very special epoch for our Universe and this situation is somewhat embarrassing unless we resort to the anthropic principle.

\[ ^2 \text{I switch to natural units for this brief discussion.} \]
Figure 1.4. The relative contribution of the cosmological constant and matter and radiation to the energy density budget of the Universe as a function of the scale factor $a$. From [42].

1.3 Ways out?

When applied to cosmic structure on galactic and larger scales, GR and its newtonian weak-field limit fail at describing the observed phenomenology. To reconcile the theory with observations, we need to assume that $\sim 85\%$ of the mass is seen only through its gravitational effect and that $\sim 74\%$ of the energy content of the Universe is due either to an arbitrary cosmological constant or to a not yet well defined dark energy (DE) fluid. The alternative conclusion we can draw from this failure is that GR is not correct on these large cosmic scales.

An extended/modified theory of gravity is required if we want to unify quantum field theory with GR. The final theory either will provide us with a natural explanation for the existence of DM and DE or will explain the observed phenomenology without one or either of them. Thus, the scientific community has considered simplified models hoping that they can be derived as effective theories from the yet unknown ultimate unification theory. Over the last 25 years, the number of proposed theories of gravity is enormous and I will not attempt to list them all here. I will rather mention a few (random) examples, to provide the reader with a taste of the wealth
Figure 1.5. In the left panel, the solid lines show the time evolution of the DE density in quintessence models with $w = -0.999$ today and different initial values of the scalar field; the dashed curve is the attractor solution. The right panel shows the DE density evolution for non-minimally coupled models with the same initial values of the scalar field as in the quintessence models; however, the range of initial DE densities they span is much wider than in the quintessence models because these latter models are forced to have a flat potential $V$ to yield $w = -1$ today. The dashed and dot-dashed curves in the right panel show the matter and radiation energy density evolution. From [46].

of suggestions appeared in the literature.

Consider the gravitational field action $S \propto \int \mathcal{L} \sqrt{-g} d^4x$, with the lagrangian density $\mathcal{L} = R + \Lambda$ in the case of the Einstein-Hilbert action. The simplest modification to the lagrangian is to assume $\mathcal{L} = f(R)$, where $f$ is a generic function (see [12] for a review). A simple power law $f(R) \propto R^n$ can describe both the rotation curves of galaxies and the supernovae Hubble diagram, but not with a power $n$ that admits a viable matter dominated epoch followed by an acceleration epoch [3].

Assuming the existence of an additional scalar field $\phi$, we can write a lagrangian $\mathcal{L} = f(\phi, R) - \partial^\mu \phi \partial_\mu \phi/2 - V(\phi)$, where $V(\phi)$ is the scalar field potential. Models with $f(\phi, R) = R$ are known as quintessence models and models with $f(\phi, R) = F(\phi) R$ are known as non-minimally coupled scalar-tensor theories of gravity (see [29] for a review). Both theories have a dynamic DE fluid that evolves towards the current value of $\Omega_\Lambda$. Unlike the quintessence models, the non-minimally coupled theories can easily solve the fine-tuning problem (Figure 1.5) [46]; neither model however solves the coincidence problem.
More drastically, conformal gravity chooses the contraction of the Weyl tensor as the lagrangian density \[ \mathcal{L} = C_{\mu\nu\lambda\kappa}C^{\mu\nu\lambda\kappa} \]. This fourth-order theory of gravity [59] is invariant under the conformal transformations \[ g_{\mu\nu} \rightarrow \Omega(x)g_{\mu\nu} \], where \( \Omega(x) \) is a function of the 4 space-time coordinates. Conformal gravity is claimed to describe both the rotation curves of galaxies and the acceleration of the universe without DM and DE [45], but it is unable to produce enough deuterium during a high-density/high-temperature phase of the early universe [26], and it does not describe correctly the phenomenology of gravitational lensing [53, 54] and of clusters of galaxies [33].

Inspired by string theories, braneworld models assume that our 3+1-dimensional universe is a mem-brane embedded in a 3+D+1-dimensional space-time. In string theories, the \( D \) extra space dimensions are compactified on scales much smaller than the elementary particle scales. Gravity can leak into these extra space dimensions, thus explaining the hierarchy problem, i.e. the weakness of gravity compared to the other fundamental forces. In braneworld cosmologies one dimension is not compactified and can be an infinite bulk. Gravity also leaks into this dimension and the attenuation of gravity on very large scales can be responsible for the late-time acceleration of the universe (see [44] for a review). The gravity attenuation has consequences on the formation of cosmic structures. For example, the
model suggested by Dvali, Gabadadze and Porrati (DGP) \([24, 43]\) yields a weak lensing converging power spectrum sensibly smaller than the standard \(\Lambda\)CDM model (Figure 1.6) \([36]\).

The braneworld models only attempt to solve the DE problem and still assume the existence of DM. The Unified Dark Matter models (see, e.g., \([5]\)) assume the existence of a single exotic fluid responsible for both DM, at high density, and DE, at low density. A celebrated example is the Chaplygin gas \([39]\) that is assumed to have the equation of state \(p \propto -\rho^{-\alpha}\).

All these extended/modified theories of gravity have been conceived to describe either the Universe expansion history or the dynamics of cosmic structure generally in virial equilibrium (mostly galaxies) or both (see e.g. \([11]\)). It remains to be seen if the formation and evolution of cosmic structure can be successfully reproduced in these theories; to this task, assuming that gravitational instability is the driving process, we need to calculate the initial field of the density perturbations. Some attempts towards this direction have been accomplished, for example, for the TeVeS model \([60]\).

Among the models proposing new physics, the Quasi-Steady State Cosmology (QSSC) has the longest history and is the most revolutionary (see \([49]\) for a review). It originates from the classical steady-state model of Bondi, Gold and Hoyle \([10, 34]\), the first model to require (rather than explain) a negative deceleration parameter \(q_0\) \([35]\). The QSSC introduces a scalar field \(C\) which is responsible for the continuous creation of matter in active galactic nuclei, rather in a single big bang event, and for the long-term expansion of the Universe. The scalar field \(C\) enters the action with a negative kinetic energy term to compensate for the creation of matter. In this model, large-scale structure does not form by gravitational instability but is determined by the process of mass ejection from randomly distributed creation centers \([50]\). The model explains many observables without DM and DE, but it is unclear if it is able to reproduce in detail the observed evolution of galaxy clustering.

1.4 Conclusion

Astronomy has posed relevant problems in physics over the centuries. We are living at exciting times, where the phenomenology that we see on the
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sky fits unsatisfactorily in the laws that, in our mind, rule the natural world. To constrain the ideal picture that we wish to formulate to describe nature, we need to keep extracting information from the sky above us. Formulating this picture is a paramount task that does not seem to be close to success yet. But the goal is unpredictable, and the path is fascinating. It’s worth a try.

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