Atom interferometry and the gravitational redshift

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Abstract
From the principle of equivalence, Einstein predicted that clocks slow down in a gravitational field. Since the general theory of relativity is based on the principle of equivalence, it is essential to test this prediction accurately. Müller, Peters and Chu claim that a reinterpretation of decade old experiments with atom interferometers leads to a sensitive test of this gravitational redshift effect at the Compton frequency. Wolf et al dispute this claim and adduce arguments against it. In this paper, we distil these arguments to a single fundamental objection: an atom is not a clock ticking at the Compton frequency. We conclude that atom interferometry experiments conducted to date do not yield such sensitive tests of the gravitational redshift. Finally, we suggest a new interferometric experiment to measure the gravitational redshift, which realizes a quantum version of the classical clock 'paradox'.

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1. Introduction
The gravitational redshift (GRS) is the first prediction [1] made by Einstein from the principle of equivalence: clocks slow down in a gravitational field. When identical clocks are compared at different locations in a gravitational field, the lower clock ticks slightly slower, its frequency ν being reduced by

\[ \frac{\delta \nu}{\nu} = \frac{\Delta U}{c^2}, \]

where \( \Delta U \) is the gravitational potential difference between the locations of the clocks and \( c \), the speed of light. The effect has been experimentally measured using clocks on a tower [2], an aircraft [3] and a rocket [4]. More recently, the experiment of Chou et al [5] has measured the GRS effect, by comparing two Al+ ion clocks separated in height by just 33 cm. The GRS is at the foundation of Einstein’s general relativity (GR) and supports the idea that gravity is encoded in the curvature of spacetime. There is every reason to measure the GRS with as
much accuracy as possible. Indeed, there is a proposal [6] to further improve the accuracy by putting an atomic clock ensemble in space.

In a recent paper, Müller, Peters and Chu [7] (MPC) have suggested that existing experiments on atom interferometry [8, 9] can be reinterpreted as a sensitive test of the GRS effect. If this claim is correct, one could achieve high accuracy without the trouble and expense of a space mission. The claim was based on the Compton frequency of an atomic mass \( m \): one writes \( E = mc^2 = h\nu \) and arrives at \( \nu_{\text{Compton}} = \frac{mc^2}{h} \). The advantage of a Compton frequency clock is that it ticks at the frequency \( \sim 10^{25} \) Hz which is considerably (about \( 10^{10} \) times) higher than the optical frequencies. As a result, \( \delta \nu \) in equation (1) is larger for higher \( \nu \) and easier to detect as a fringe shift in interferometry.

However, the claim of MPC is disputed by Wolf et al (WBBRSC) [10, 11], who note that the atom interferometer experiments only constitute a test of the universality of free fall (UFF) and not of the GRS. Since the GRS applies to all clocks, it is also referred to as the universality of clock rates (UCR). WBBRSC object to MPC’s claim on the grounds that a detailed analysis [12] of atom interferometer experiments shows that the Compton frequency does not appear in the final answer for the calculated fringe shift. The analysis presented in [12] is performed for quadratic Lagrangians describing the atoms and the propagator is explicitly calculated. WBBRSC also suggest [11] that the GRS requires a continuous exchange of signals between the participating clocks and such exchange would be equivalent to welcherweg information which destroys the interference pattern. They argue that the atom does not deliver a physical signal at the Compton frequency. MPC, however, stand by [13] their claim, which is approvingly quoted by the authors of [14] and repeated in [15, 16], which include some of MPC as co-authors. The matter evidently is not settled.

Our purpose here is to incisively confront the claim of MPC by sharpening the objections raised by WBBRSC. Of the objections raised by WBBRSC, one of them stands out as being fundamental: there is nothing physical about the Compton frequency in this experiment. We will rest our case entirely on this objection. We will theoretically examine some conceptual questions raised by this controversy. We start by critically examining in section 2 the notion of a ‘clock’ in GR. In section 3, we theoretically analyze an atom interferometry experiment and show that it does not test the redshift at the Compton frequency. In section 4, we propose a ‘clock interferometry’ experiment which does test the redshift, though not at the Compton frequency. Finally, in the discussion in section 5, we make a number of comments and note that our proposed experiment is a quantum version of the classical clock ‘paradox’.

2. What is a clock?

Einstein’s principle of equivalence implies both the UFF and the UCR. To test the principle of equivalence, it is important to test both these effects independently. The UFF can be tested by dropping masses, as in Galileo’s famous experiment or by constructing sensitive torsion balances with suspended masses, as in the Eötvös experiments. For testing the GRS (or the UCR), it is evident that one needs to have clocks not just masses. What, then, is a clock? A clock is anything which ticks–delivers a periodic signal. It is usually a dynamical system which executes a periodic motion like a pendulum, a planetary orbit, the moons of Jupiter or a crystal oscillator. The period defines the ‘ticks’ of the clock, which gives the least count in time measurement. Precise clocks have high tick rates. The most precise clocks in use today are atomic clocks operating at optical frequencies, ticking at the rate of \( 10^{15} \) Hz. These clocks operate in a quantum superposition of nondegenerate energy eigenstates

\[
|\psi(t)\rangle = e^{-i\frac{\Delta E}{\hbar}}|\psi_1\rangle + e^{-i\frac{\Delta E}{\hbar}}|\psi_2\rangle
\]
where $|\psi_1\rangle$ and $|\psi_2\rangle$ are the stationary states (eigenstates of the energy) and $E_1$ and $E_2$ are the corresponding distinct energies. The oscillation frequency of the atomic clock is given by the difference of the two energies

$$v_{\text{clock}} = \frac{E_2 - E_1}{\hbar}.$$  

(3)

For atoms, typical differences in energy (called spectral terms in spectroscopy) are in the eV range. If this frequency is in the optical range, the ticks of the clock are at $10^{15}$ Hz. This is an improvement over microwave clocks, which operate at a lower frequency. What makes a quantum clock tick is the superposition of at least two stationary states. The ticking rate is given by the beat frequency between these states. Classical clocks such as crystal oscillators are in fact superpositions of many highly excited stationary states and can be described in quantum mechanics as coherent states. In contrast, an atom in a stationary state

$$|\tilde{\psi}(t)\rangle = e^{-\frac{i}{\hbar}E_1 t} |\psi_1\rangle$$  

(4)

is not a clock because it does not execute any periodic motion. While the wavefunction in equation (4) solving Schrödinger’s equation does appear to have periodic time dependence, it is important to realize that the wavefunction is not directly observable: only bilinears constructed from the wavefunction are. Thus, bilinears constructed from the wavefunction in equation (4) would be time independent while bilinears made from equation (2) would have the interference term

$$2\text{Re}[e^{-ivt}\psi_1^*(t,r)\psi_2(t,r)],$$  

(5)

which is a measurable quantity reflecting the oscillating charge density of the electronic motion within the atom. In fact, this oscillation leads to an oscillating dipole moment for the atom, which couples to radiation during atomic transitions.

MPC suggest that an atom is a clock at the Compton frequency. We contradict this suggestion and show by general arguments that the Compton frequency is not a gauge-invariant observable in the atom interferometry experiment. Atoms in stationary states are masses and do not tick. They cannot be viewed as clocks ticking at the Compton rate.

3. Quantum interference of atoms

Consider the interference between atoms emitted at 1 and received at 2 (figure 1). If an atom is initially in an atom trap at 1 (red) and is moved with a probability amplitude $1/\sqrt{2}$ to a higher trap (blue), one can observe interference between the two possible histories $\Gamma_a$ and $\Gamma_b$. Our analysis below shows that the effects of the Compton frequency can be eliminated in all observables. Thus, an atom in a stationary state is not a clock ticking at the Compton frequency. It takes two energy states to beat, just as it takes two hands to clap.

The experiment may be described either by the Schrödinger equation or the equivalent Feynman path integral. The phase picked up by the atom in following a path $\Gamma$ is $S_\Gamma/\hbar$, where

$$S_\Gamma = \int_\Gamma [(mc) \, ds - V(t,r) \, dt]$$  

(6)

where $ds$ is the proper interval measured along $\Gamma$ and $V(t,r)$ is the effect of non-gravitational potentials which may be used to manipulate the atom. The gravitational field of the earth can be described by the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dr^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\Omega^2,$$  

(7)
and to the required accuracy, the Lagrangian describing the motion of the atom is
\[ L = -mc^2 + \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{GMm}{r} - V(t, r), \] (8)
which leads to the Hamiltonian
\[ H = \frac{p^2}{2m} + V(t, r) - \frac{GMm}{r} + mc^2. \] (9)

(Strictly, since we are testing the theory, we ought to go beyond GR and allow for the possibility that the masses appearing in the potential and kinetic terms differ. We gloss over this point since our interest is only in the effect of the constant term.)

The energy corresponding to the ‘Compton frequency’ is present in the Hamiltonian (9) as an additive constant $mc^2$. This term cancels out in all energy differences (or spectral terms) and is therefore unobservable. The unobservability of the constant is driven home by noting that the problem admits a ‘gauge symmetry’. If $\psi(t, r)$ is a solution, then
\[ \psi'(t, r) = U(t, r)\psi(t, r), \] (10)
where $U(t, r) = \exp i\chi(t, r)$ is a solution of the Schrödinger equation with Hamiltonian
\[ H' = UHU^{-1} + i\hbar UU^{-1}. \] (11)

The choice $\chi(t, r) = \frac{mc^2}{\hbar}$ results in a new Hamiltonian
\[ H' = \frac{p^2}{2m} - \frac{GMm}{r} + V(t, r), \] (12)
in which the Compton frequency disappears. Since one only measures bilinears in the wavefunction, the gauge argument shows quite generally that the effects of the Compton frequency can be eliminated in all observables.
The argument can easily be translated to the Feynman path integral formalism. Under a gauge transformation the Lagrangian changes by a total time derivative

$$L' = L + \frac{d\chi}{dt}$$

and as a result we find that the propagator

$$K(t_1, r_1; t_2, r_2) = \sum \exp iS$$

(14)

(expressed as a Feynman path integral over all spacetime paths $\Gamma$ which go from $(t_1, r_1)$ to $(t_2, r_2)$) transforms as

$$K'(t_1, r_1; t_2, r_2) = \exp i\chi(t_1, r_1)K(t_1, r_1; t_2, r_2) \exp -i\chi(t_2, r_2).$$

(15)

All physical results are unchanged. The freedom to add a constant to the Hamiltonian is present for particles in external fields, gravitational or otherwise. This freedom is lost only when one considers the gravitational field of the particle itself, in this case the atom. This gravitational field is clearly negligible in the present context. We conclude that the rest mass of an atom in this experiment does not deliver a physical signal at the Compton frequency and therefore an atom is not a clock ticking at the Compton frequency.

In the gedanken experiment shown in figure 1, the phase difference of the atoms arriving at the detector is easily calculated. One observes interference between two histories $\Gamma_a$ and $\Gamma_b$, each of which has equal amplitude $\sqrt{2}$. In $\Gamma_a$, the atom is moved to a higher trap and spends a coordinate time $T$ in the higher trap, while in $\Gamma_b$, the atom stays in the lower trap for a time $T$ and is then lifted to the higher trap by an external force (supplied by lasers). We can suppose that the non-gravitational potential has the same value 0 in both traps and that the phases picked up in the rising (green) sectors cancel exactly. Since the atoms are stationary in the traps, there is no effect of motion and for a residence time $T$, the phase difference is $mT/\hbar$ times the difference of the gravitational potential $U$ between the two traps separated in height by $z$. The final answer is

$$\delta \phi = \frac{m \Delta U T}{\hbar} \approx \frac{mgzT}{\hbar}$$

(16)

which is in complete agreement with MPC. It is only the interpretation of this result as a detection of the redshift effect at the Compton frequency that we dispute. We interpret equation (16) as the phase shift due to fall under the gravity $g$, not as a redshift. This is because the atoms are in stationary states and therefore are not clocks.

Our result (16) and that of MPC are apparently at variance with that of [10, 11]. The difference is easily understood. References [10, 11] compute the quantum propagator explicitly, which is only possible in the quadratic approximation. In order to better compare our results with those of [10–12] we assume that $V$ is time independent and expand $V(r) + GMm/r$ in a Taylor series around its minimum at $r_0$ to quadratic order in $(r - r_0)$. We find the Lagrangian

$$L = -mc^2 + \frac{1}{2}m \left( \frac{dr}{dt} \right)^2 - \frac{k}{2} (r - r_0)^2 - V(r_0),$$

(17)

where $k$ is an effective spring constant. This Lagrangian describes the simple harmonic oscillator.

In order to cause interference we must consider two classical histories that start and end at the same spacetime point. For example, $r_1(t) = r_0 + A_1 \sin \omega t$ and $r_2(t) = r_0 + A_2 \sin \omega t$ which intersect at $t = 0$ and $t = \frac{2\pi}{\omega}$, respectively. Such pairs are conjugate points and as is well known, the total phase acquired by an oscillator is independent of the amplitude. The phase difference is therefore zero, in agreement with [12]. There is no conflict, however, between the
phase-shift calculations of MPC and [10–12]. These computations apply to different situations and they are both correct. The phase shift (16) is calculated semiclassically for a non-harmonic potential.

Our objection to MPC is not that their computed phase shift is incorrect but that their interfering atoms are in stationary states and therefore not clocks.

4. Quantum interference of clocks

Can one use atom interferometry to test UCR? The answer is yes: we need to observe the interference of atomic clocks rather than atomic masses. To do this, we must have a source of atoms in a coherent superposition of different energy states. The phase picked up by an atom is $\exp \left(-\frac{i \epsilon \tau}{\hbar} \right)$, where $\tau$ is the proper time measured along its worldline and $\epsilon$ is its proper energy.

Consider an atom in an initial state $\frac{1}{\sqrt{2}} (|1\rangle + |2\rangle)$, a superposition of two states with proper energies $\epsilon_1$ and $\epsilon_2$. A beam splitter at the source causes the atom to follow paths $\Gamma_a$ and $\Gamma_b$ with equal amplitude. The proper times for the rising (green), upper horizontal (blue) and lower horizontal (red) sections are respectively $\tau$, $\tau_a$ and $\tau_b$ (see figure 1). On arrival at the detector, the state of the atom is given by a sum of two amplitudes

$$|\psi_a\rangle = \frac{1}{\sqrt{2}} \left( \exp \left(-\frac{i \epsilon_1 (\tau + \tau_a)}{2\hbar} \right) |1\rangle + \exp \left(-\frac{i \epsilon_2 (\tau + \tau_a)}{2\hbar} \right) |2\rangle \right)$$

and

$$|\psi_b\rangle = \frac{1}{\sqrt{2}} \left( \exp \left(-\frac{i \epsilon_1 (\tau + \tau_b)}{2\hbar} \right) |1\rangle + \exp \left(-\frac{i \epsilon_2 (\tau + \tau_b)}{2\hbar} \right) |2\rangle \right).$$

The interference between the two alternatives $\Gamma_a$ and $\Gamma_b$ gives a term $2 \Re \langle \psi_a | \psi_b \rangle$ which is easily computed to be given by

$$\left[ \cos \frac{m \Delta \omega T}{\hbar} \right] \cos \frac{\Delta \epsilon \Omega T}{2 c^2 \hbar},$$

where $m = \frac{\epsilon_1 + \epsilon_2}{2c^2}$ and $\Delta \epsilon = \epsilon_1 - \epsilon_2$. The first term in square brackets is the old term in equation (16). However, the phase of the second term is a measurement of the redshift effect.

For an atom in a superposition of states with proper energies $\epsilon_1$ and $\epsilon_2$, the expected interference term at the detector in figure 1 is given by equation (20), where $m = \frac{\epsilon_1 + \epsilon_2}{2c^2}$ and $\Delta \epsilon = \epsilon_1 - \epsilon_2$. The first factor measures UFF and couples to the mass of the atoms, while the second measures UCR and couples to the energy difference between the superposed states.

Experiments of this kind can be described as clock interferometry and have not been done, to the best of our knowledge. They can be thought of as a quantum version of the clock ‘paradox’, in which one uses a single clock in a quantum superposition, instead of the two clocks compared in the classical clock ‘paradox’, which is experimentally realized in [5]. In the quantum version, a single clock traverses both alternative world lines and interferes with itself. Such experiments constitute a good example of the use of the internal degrees of freedom of an atom in interferometry [12]. They would lead to new tests of UCR at optical frequencies.

5. Discussion

It is known [17, 18] that if one assumes UFF and energy conservation, it follows that UCR must also hold. This conclusion can be arrived at by considering systems which can exist in various energy states and make transitions between them emitting quanta [17, 18]. Indeed, one can construct perpetual motion machines of the first kind if UFF holds but UCR does not
hold. One striking realization of such a machine is Bondi’s system of buckets and pulleys [18], using excited atoms to perpetually outweigh identical atoms in their ground state. However, more quantitatively, a test of UFF to a certain level of accuracy leads to an accuracy of UCR which is reduced by a factor $\frac{\Delta E}{E}$ [17–19], where $E$ is the absolute energy and $\Delta E$ is the energy difference between the states of interest. In atomic systems, the energy separations are much smaller than the rest mass, $\frac{\Delta E}{E} \sim 10^{-10}$, which is why the present accuracy of UCR tests is considerably lower than that of UFF.

Apart from atomic clocks, which are held together by electromagnetic interactions, one can also consider clocks held together by other forces. Gravitational clocks (such as the moons of Jupiter) are held together by gravity and their binding energies are extremely small compared to the mass of the clock. The clock consists of the entire system, Jupiter plus its moons, and the clock rate depends on the solar gravitational potential. In contrast, nuclear binding energies are often relatively large, around 1% of the rest mass and such systems could be explored to improve the accuracy of UCR tests. This point has been noted in [16] in their recent preprint.

We have defined a clock as something which ‘ticks’ periodically. One can also use clocks based on decay rates [17] or transition rates between two energy states. Our arguments also apply to such decay clocks, since all we need is that the system must be in a superposition of at least two states.

We have been using the word ‘atom’ to mean an atom in a stationary state and ‘atomic clock’ to describe atoms which are in a superposition of at least two states. Needless to say, the discussion applies also to ions, which may be easier to manipulate experimentally. Our discussion here is at the level of ‘gedanken experiments’ and the translation to a laboratory experiment may involve some changes.

Our final result for the expected phase shift is in agreement with [7]. Our disagreement is more subtle: we do not agree with [7] that this experiment is a measurement of the redshift. It only constitutes a measurement of the phase shift due to the gravitational acceleration $g$. Thus, the atom interferometry experiment only measures the UFF and not UCR. For the latter one needs to have genuine clocks, not just masses interpreted as clocks. If one could use atomic masses to generate a periodic signal, this would lead to unprecedented accuracy in time keeping. Present day atomic clocks work at optical frequencies and lose no more than a second in the age of the Universe. A Compton frequency clock would lose no more than a nanosecond in the age of the Universe. It seems clear that present technology is far from achieving such precision in time keeping.

Can one construct clocks at the Compton frequency? The answer, in principle, is yes. What one has to do is to superpose states which differ in the number of atoms, i.e generate a quantum state which is not in a number eigenstate. Such states exist in a quantum field theory. An example of such a state is a single-mode coherent state (with $Z$ a complex number) $|Z\rangle = \exp\left\{-|Z|^2 \sum_{n=0}^{\infty} \frac{Z^n}{\sqrt{n!}} |n\rangle\right\}$ which superposes different numbers of particles. A laser beam is an example of a coherent photon field which can be regarded as a clock at optical frequencies. Needless to say, experiments using superpositions of atom numbers in interferometry have not so far been performed.

To summarize, Einstein’s equivalence principle implies UFF and UCR. It is important to test both of these. Tests of UFF entail the use of masses, whereas one needs clocks to check UCR. In this paper, we explicitly show that an atom is not a clock ticking at the Compton frequency and therefore one cannot achieve an advantage of ten orders of magnitude in precision compared to existing tests of UCR.

In the popular relativity literature, considerable attention has been devoted to the classical twin ‘paradox’. Two twins are separated at birth and follow different world lines. (In popular
accounts, one stays home.) When they meet after some years, one has aged relative to the other. The two twins evidently carry biological clocks, which are synchronized at birth and compared when they meet. (There is no need for the twins to exchange signals between these events.) In an actual experiment, Chou et al [5] have realized this effect using two Al⁺ ions to play the role of the two twins. Like the twins, the ions are clocks and clocks ‘age’ differently on different world lines. (It is of course necessary that the ions are in a superposition of at least two stationary states so that they tick!) It is possible to come up with a quantum version of this effect: we do not need two ‘twins’. Starting with a single ion source of ion clocks (these have to be in a superposition of internal states to qualify as clocks) we perform a split beam experiment so that the single ion has equal amplitude to traverse the two arms of an interferometer. On recombining the beams, the interference between the arms will reveal the presence of differential aging.

Our proposed experiment in figure 1 to measure GRS is exactly of this kind. Atomic clocks are separated by a beam splitter into two beams, one of which lies deeper in a gravitational field than the other. When the beams are recombined their interference will reveal the presence (20) of a different number of ticks in the two arms. Of course, it is necessary that the atoms are genuine clocks, i.e they must be in a superposition of internal energy states. GRS measurements can thus be performed in atom interferometry by causing quantum interference between atomic clocks. We hope to interest the atom interferometric community in developing a realization of this gedanken experiment.

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Erratum and Addendum: Atom interferometry and the gravitational redshift (2011 *Class. Quantum Grav.* 28 145018)

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Abstract

We correct a typographical error in our earlier paper and remark that an analogue version of the experiment suggested there has been realized.

1. Correction

In [1], equation (20) contains a typographical error of a misplaced ‘/’ symbol. The equation should read

$$\cos\left(\frac{m\Delta UT}{\hbar}\right) \times \cos\left(\frac{\Delta\epsilon\Delta UT}{2c^2\hbar}\right).$$

(1)

2. Addendum

As a matter of interest, we add that a version of the experiment proposed in [1] has been recently performed and is reported in [2]. Because of the experimental difficulty of detecting the gravitational slowing down of clocks within the small size of their atom chip, the authors of [2] simulate gravity by introducing a relative slowing down of clocks in one interferometric arm using a magnetic field gradient.

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