Interlayer heat transfer in bilayer carrier systems

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Abstract. We study theoretically how energy and heat are transferred between the two-dimensional layers of bilayer carrier systems due to the near-field interlayer carrier interaction. We derive the general expressions for interlayer heat transfer and thermal conductance. Approximation formulae and detailed calculations for semiconductor- and graphene-based bilayers are presented. Our calculations for GaAs, Si and graphene bilayers show that the interlayer heat transfer can exceed the electron–phonon heat transfer below the (system-dependent) finite crossover temperature. We show that disorder strongly enhances the interlayer heat transport and pushes the threshold toward higher temperatures.

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1. Introduction

Interlayer momentum transfer (the drag effect) has been extensively investigated in bilayer carrier systems, where two two-dimensional (2D) carrier gases are separated by a thin barrier. The drag effect is a manifestation of near-field interlayer interaction, and bilayer carrier systems provide a unique laboratory for probing charge carrier interactions and interaction-driven phases (for a review see \[1, 2\]). After the pioneering experiment on electron–electron drag between two coupled 2D electron gas (2DEG) layers in GaAs–AlGaAs heterostructure \[3\], 2D carrier bilayers have been demonstrated in a variety of semiconductor structures. Recently, the drag effect was experimentally investigated also in graphene bilayer, where two single-layer graphene flakes are separated by a dielectric \[4\].

Investigations of bilayer carrier systems have focused on the drag phenomenon, but the interlayer interaction also mediates an energy and heat transfer between the layers (see figure 1) and such a near-field energy/heat transfer is the topic of this paper. Considerable efforts have been devoted to understanding near-field heat transfer via different channels between bodies that are separated by a small vacuum gap \[5–7\]. One of the most significant heat exchange channels is built from inter-body photon coupling. Surface excitations involving optical phonons and plasmons can play an important role and these so-called polariton effects can strongly enhance the near-field energy transfer \[6\]. Recently, a near-field heat transfer channel arising directly from lattice vibrations was also proposed \[8, 9\]. Near-field heat transfer is naturally always present between closely spaced systems, even in the case of a solid contact, but then the effect is expected to be strongly masked by competing heat dissipation channels formed by solid heat conduction and/or electron–phonon coupling. One of the motivations for this work is to challenge this line of thought and, indeed, by detailed calculations we will show that in bilayer carrier systems the near-field heat transfer can become the dominant interlayer heat transfer mechanism.

In this work, we derive a general expression for the charge fluctuation-induced interlayer energy transfer rate, which is applicable to semiconductor and graphene bilayers. In the derivation, we use perturbation theory and fluctuation–dissipation relations. Our formula for the interlayer thermal conductance, \(G_{12}\), has a strong connection to the drag resistance formulae \[10\]. The interlayer thermal conductance is intimately connected to fluctuations and
Figure 1. (a) Illustration of near-field scattering processes (momentum and energy transfer) between 2D carrier layers 1 and 2 separated by a distance $d$ and embedded in a solid with dielectric constant $\varepsilon_b$. The layers are at local temperatures $T_1$ and $T_2$. A carrier in layer 1 (layer 2) experiences momentum scattering $k_1 \rightarrow k_1 + q$ ($k_2 \rightarrow k_2 - q$) due to interlayer interaction. During the process, energy $\Delta E$ is transferred between the layers. (b) The equivalent thermal circuit. $P_L$ is the input heating/cooling power ($L = 1, 2$) and $G_{12}$ is the interlayer thermal conductance. Due to the interlayer scattering processes, $G_{12} \neq 0$ and power (or heat) $P_{12}$ flows between the layers. Layers couple to the phonon bath, which is at temperature $T_p$, via electron–phonon thermal conductance $G_{LP}$ and power $P_{LP}$ flows to the bath. Illustration of the conduction band diagram and electron wavefunctions of (c) GaAs and (d) Si bilayer. (e) Graphene bilayer with a dielectric barrier.

dissipative properties of the individual layers. This is explicitly seen as the presence of the imaginary parts of the layer susceptibilities in the $G_{12}$ formula and it is a manifestation of fluctuation–dissipation theorem. Approximation formulae and detailed calculations of $G_{12}$ in the case of screened Coulomb interlayer interaction are presented and we show that interlayer thermal transport is strongly enhanced due to disorder. As the layers are in the same solid there exist competing energy relaxation channels. At the temperatures of interest, electron–phonon coupling to the bulk thermal phonons is the relevant competing heat dissipation mechanism (see figure 1(b)). It is shown that remarkably $G_{12}$ can dominate over the electron–phonon coupling. Therefore, near-field heat transfer can become a dominant heat transfer mechanism even in the case of a solid contact.

2. Theory

In this section, we derive the general expression for the interlayer thermal conductance $G_{12}$. Then we introduce approximation formulae for $G_{12}$ and, on the basis of the existing literature, discuss electron–phonon coupling, which is a competing dissipation channel.

2.1. Interlayer thermal conductance

The scattering events depicted in figure 1(a) are mediated by interlayer interaction, which is described by matrix element $M_q$ (to be defined later). The interlayer Hamiltonian $H$
is given by

$$H = \frac{1}{2A} \sum_{q} M_q \rho_{1q} \rho_{2q}, \quad (1a)$$

$$\rho_{Lq} = \sum_k \sum_{\sigma, \sigma'} \sum_{s, s'} c_{k-s, \sigma}^\dagger F_{k-q, \sigma}^\dagger F_{k, \sigma} c_{k, \sigma}, \quad (1b)$$

where $A$ is the area, $\rho_{Lq}$ is the electron density operator for layer $L = 1, 2$ and $c_{k, \sigma}$ is the electron annihilation (creation) operator. Variables $k, s$ and $\sigma$ are the wavevector, band index and spin index, respectively (here we will assume spin degeneracy). All electron variables depend on the layer index $L$, but this is typically not written explicitly (e.g. $k = k_L$). The factor $F_{k,s}$ is defined by the wavefunction of the single-particle states and product $F_{k-q,s}^\dagger F_{k,s}$ defines a band form factor. For an ideal 2DEG, we have $F_{k,s} = 1$ and summation over band indices $s, s'$ can be ignored. For graphene, we have $F_{k,s} = \frac{1}{\sqrt{2}} (1 - \text{se}^{\bar{\theta}_k})^\dagger$, where $s = +1$ and $-1$ denote conduction and valence bands, respectively, and $\bar{\theta}_k = \arctan(k_y/k_x)$.

Next, $H$ will be considered as a perturbation Hamiltonian that will cause transitions from the initial state $|i1, i2⟩ = |i1⟩|i2⟩$ with energy $E_i = E_{i1} + E_{i2}$ to the final state $|f1, f2⟩ = |f1⟩|f2⟩$ with energy $E_f = E_{f1} + E_{f2}$. Here $|iL⟩ (|fL⟩)$ is the initial (final) state of layer $L$. The transition rate $\Gamma_{fi}$ from initial state $i$ to final state $f$ is given by the golden rule formula

$$\Gamma_{fi} = \frac{2\pi}{\hbar} |\langle f2, f1 | H | i1, i2 ⟩|^2 \delta(E_i - E_f). \quad (2)$$

By multiplying $\Gamma_{fi}$ by the energy change $\Delta E_i = E_{i1} - E_{f1}$ and performing an ensemble average over the initial electronic states, and summing over the final electronic states, we obtain the energy transfer rate (heat transfer)

$$P_{12} = \frac{2\pi}{\hbar} \frac{1}{2A} \sum_q \sum_{i1, f1/f2} \sum_{i2, f2} \Delta E_i \hat{w}_{i1} \hat{w}_{i2} |M_q|^2 |\langle f1 | \rho_{1q}^\dagger | i1 ⟩|^2 |\langle f2 | \rho_{2q} | i2 ⟩|^2 \times \delta(E_{i1} + E_{i2} - E_{f1} - E_{f2}), \quad (3)$$

where $\hat{w}_{Li}$ is the weighting factor of the carrier layer $L$ in state $i$. We assume that each layer $L$ can be described by a local temperature $T_L$ and, therefore, $\hat{w}_{Li} = \hat{w}_{Li} \exp[(E_i - E_f)/k_B T_L]$. By using the identity $\delta(E_A + E_B) = \hbar \int_{-\infty}^{+\infty} d\omega \delta(E_A - \hbar \omega)\delta(E_B + \hbar \omega)$ and the definition of the correlator

$$C_L(q, \omega) = 2\pi \hbar \sum_{n, m} \hat{w}_{Ln} \left|\langle nL | \rho_{Lq} | mL⟩\right|^2 \delta(E_{Ln} - E_{Ln} + \hbar \omega), \quad (4)$$

we find that

$$P_{12} = \frac{1}{2\pi \hbar^2} \left(\frac{1}{2A}\right)^2 \int_{-\infty}^{+\infty} d\omega \sum_q \hbar \omega |M_q|^2 C_1(q, -\omega) e^{\hbar \omega/k_B T_L} C_2(q, -\omega). \quad (5)$$

As we assume internal equilibrium for the different layers, we can adopt the fluctuation–dissipation relation [11] \((1 - e^{-\hbar \omega/k_B T_L})C_L(q, \omega) = -2\hbar A \text{Im}\{\chi_L(q, \omega)\}\), where $\chi_L(q, \omega)$ is the susceptibility, which can depend on $T_L$. Using the fluctuation–dissipation relation and the property $C_L(q, -\omega) = e^{-\hbar \omega/k_B T_L} C_L(q, \omega)$, we found the general expression for the interlayer heat transfer

$$P_{12} = \int_{0}^{+\infty} \frac{d\omega}{2\pi} \sum_q \hbar \omega |M_q|^2 \text{Im}\{\chi_1(q, \omega)\}\text{Im}\{\chi_2(q, \omega)\} \left[n_1(\hbar \omega) - n_2(\hbar \omega)\right], \quad (6)$$

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where \( n_L(h\omega) = (\exp(h\omega/k_BT_l) - 1)^{-1} \). At the limit \( T_1, T_2 \to T \) it is useful to define the interlayer thermal conductance \( G_{12}(T) = P_{12}/(T_1 - T_2) \). From equation (6), we found

\[
G_{12}(T) = \frac{1}{4k_BT^2} \int_{0}^{\infty} \frac{d\omega}{2\pi} \sum_{q} (h\omega)^2 |M_q|^2 \frac{\text{Im} \{\chi_1(q, \omega)\} \text{Im} \{\chi_2(q, \omega)\}}{\sinh^2 (h\omega/2k_BT)},
\]

which has a striking similarity to the bilayer drag resistance formula [10]. Equation (7) has only a single temperature and, therefore, is more suitable for the case studies than equation (6).

In the following, we will assume that the interlayer interaction is mediated by screened Coulomb interaction, when the matrix element is given by \( M_q = \epsilon_{12}^{-1}(q, \omega)U(q)F_{12}(d) \), where \( U(q) = \epsilon^2/2\epsilon_b q \) is the 2D Fourier transform of Coulomb potential (\( \epsilon_b \) is the background dielectric constant) and \( F_{12}(d) \) is the spatial form factor, which depends on the spatial extent of the electron wavefunctions and layer distance \( d \). For graphene the extent is practically zero and for the sake of simplicity here we assume vanishing extent for the semiconductor systems as well. Thus, we use \( F_{12}(d) = \exp(-qd) \). The inter-layer dielectric function \( \epsilon_{12}(q, \omega) \) is given by \( \epsilon_{12}(q, \omega) = [1 - U(q)\chi_1(q, \omega)][1 - U(q)\chi_2(q, \omega)] - F_{12}^2 U(q)^2 \chi_1(q, \omega)\chi_2(q, \omega) \) [1, 10].

In the ballistic limit the carrier mean free path \( l_c \) exceeds the layer distance \( l_c \gg d \) and we use the ideal 2D susceptibilities. For 2DEG, we have [12]

\[
\chi_L(q, \omega) = v(2\zeta)^{-1}[2\zeta - \Omega_-(z, u) - \Omega_+(z, u) + \mathcal{F}_-(z, u) + \mathcal{F}_+(z, u)],
\]

where \( \Omega_{\pm}(z, u) = C_{\pm}\sqrt{(z \pm u)^2 - 1}, F_{\pm}(z, u) = iD_{\pm}\sqrt{1 - (z \pm u)^2}, z = q/2k_F, \ u = \omega/qv_F, C_{\pm} = (z \pm u)/|z \pm u| \) and \( D_{\pm} = 0 \) for \( |z \pm u| > 1 \), and \( C_{\pm} = 0 \) and \( D_{\pm} = 1 \) for \( |z \pm u| < 1 \). Here, \( v_F \) (\( k_F \)) is the Fermi velocity (wave vector) and \( v = v(\varepsilon_F) \) is the density of states at Fermi level \( \varepsilon_F \gg k_B T \). For ballistic graphene the expression for \( \chi_L(q, \omega) \) is quite lengthy and will not be presented here. It can be found in, for example, [13].

2.2. Approximation formulae

Even though there are some fundamental differences between graphene and 2DEGs, the response of these systems is similar at low frequencies and small \( q \). Indeed, for the Taylor series expansion of 2DEG (equation (8)) and graphene susceptibilities [13], we find the same result

\[
\chi_L(q, \omega) \approx -v \left( 1 + i \frac{\omega}{v_F q} \right).
\]

Respectively, in the diffusive limit \( (\omega\tau, l_c/d \ll 1) \) the susceptibility can be approximated as

\[
\chi_L(q, \omega) \approx -v \frac{iDq^2}{\omega + iDq^2},
\]

where \( D = v_F^2 \tau/2 \) is the diffusion coefficient and \( \tau = l_c/v_F \) is the momentum relaxation time.

By using equation (9) in (7) for two similar ballistic 2DEG and graphene layers, we find the asymptotic low-temperature result

\[
G_{12}(T) \approx \frac{f(kd)\hbar}{2d^{2-\alpha}v_F^\alpha} \left( \frac{k_B}{\hbar} \right)^{2+\alpha} T^{1+\alpha},
\]
where $\alpha = 1.9$, $\kappa = \frac{\nu^2}{2k_b}$ is the screening wave vector and $f(a) \simeq (a^{-2} + 2.21a + 1.24)^{-1}$. The above equation provides a good approximation when $k_B T < 2E_F$. Note that parameter $\kappa d$ characterizes the screening of the interlayer interaction: large (small) $\kappa d$ means strong (weak) screening.

In the diffusive limit, we use equation (10) in (7) and we find the low-temperature approximation formula

$$G_{12}(T) \simeq \frac{3A_3 e_0}{16\pi d^2} \left( \frac{k_B}{\hbar} \right)^3 T^2. \quad (12)$$

Here $\sigma = e^2 v_D$ is the dc conductivity of a single layer and $A_n = \Gamma(n) \zeta(n) = \int dx x^{n-1}/[\exp(x) - 1]$. Equation (12) is applicable when $(l_v/v_F)k_B T/\hbar < (l_v/\ell)^2$. Note that the diffusive $G_{12}$ (equation (12)) greatly exceeds that in the ballistic case (equation (11)), which is a manifestation of the enhanced fluctuations and dissipation due to disorder.

2.3. Electron–phonon coupling

As depicted in the thermal circuit of figure 1(b), $G_{12}$ competes with the electron–phonon thermal conductance $G_{LP}$, which at the limit $T_L, T_p \to T$ is given by $G_{LP}(T) = P_{LP}/(T_L - T_p)$. In 2DEGs at low temperatures the electron–phonon energy transfer is dominated by screened deformation potential ($G_{LP}^{DP}$) and piezoelectric ($G_{LP}^{PE}$) interaction with total thermal conductance $G_{LP}(T) = G_{LP}^{DP}(T) + G_{LP}^{PE}(T)$. For the deformation potential contribution, we have [14, 15]

$$G_{LP}^{DP}(T) = \sum_\lambda \frac{F_{\lambda n} 2^n}{l_p^2 K^2} \langle f_n(\theta)K^2 \rangle v_{\lambda n}^2 v_{\lambda}^{-6} T^{6-n}, \quad (13)$$

where $n = 0 \ (1)$ represents ballistic (diffusive) limit of electron–phonon coupling, for which we have $q_{\lambda T} l_v > 1 \ (-1)$. Here $q_{\lambda T} = k_B T/\hbar v_{\lambda T}$ is the thermal phonon wave vector, the factor $F_1 = \frac{\nu v_g}{2\pi^2 \rho v_{\perp}}$, $v_{\lambda T}(T)$ is the longitudinal (transversal) phonon velocity and $\rho$ is the mass density of the crystal. The brackets $\langle \cdots \rangle$ stand for solid angle average and $\theta$ is the angle with respect to the $z$-axis, which is perpendicular to the layer plane. $\langle f_n(\theta)K^2 \rangle$ is an effective deformation potential coupling and $f_0(\theta) = \sin \theta$ and $f_1(\theta) = \frac{\sin^2 \theta}{\alpha + \sin^2 \theta}$. Parameter $\alpha = (\ell_v/v_F/v_j)^{-2}$ and as a result $f_1(\theta) \approx 1$. The piezoelectric coupling gives rise to the contribution [14–16]

$$G_{LP}^{PE}(T) = \sum_\lambda \frac{F_{\lambda n} 2^n}{l_p^2 K^2} \langle f_n(\theta)K^2 \rangle v_{\lambda n}^2 v_{\lambda}^{-4} T^{4-n}, \quad (14)$$

where $\langle f_n(\theta)K^2 \rangle$ is the effective piezo coupling. For graphene, the electron–phonon coupling is dominated by deformation potential coupling [17] and vector potential coupling [18] ($G_{LP}^{VP}(T)$), giving $G_{LP}(T) = G_{LP}^{DP}(T) + G_{LP}^{VP}(T)$. For both these contributions, we will directly use the results of [18].

3. Results and discussion

In this section, we calculate the interlayer thermal conductance $G_{12}$ of selected semiconductor and graphene bilayer systems at the ballistic and diffusive limit and discuss possible experimental configurations to investigate $G_{12}$. Interlayer thermal conductance will be compared with electron–phonon thermal conductance $G_{LP}$. Diffusive $G_{12}$ is considered at a
Figure 2. The interlayer thermal conductance ($G_{12}$) and deformation potential ($G_{DP}^L$) and piezoelectric ($G_{PL}^{PE}$) electron–phonon thermal conductance at the ballistic limit for a GaAs bilayer system with electron density $n = 10^{15}$ m$^{-2}$. The solid curve is the result of numerical integration using the ballistic response function. The dashed curve was obtained using equation (11).

larger interlayer separation than the ballistic one in order to ensure that the diffusive response formula (equation (10)) is valid and the condition $k_F l_e > 1$ is fulfilled.

3.1. Calculations for different bilayers

Figure 2 shows $G_{12}(T)$ obtained numerically from equations (7) and (8) in the case of symmetric high- mobility (ballistic) GaAs bilayer [3] (depicted in figure 1(c)) with single-layer electron density $n = 10^{15}$ m$^{-2}$ and $d = 20$ nm. The asymptotic limit formula of equation (11) is also plotted. In the phonon contribution, we have $\sum_\lambda \langle \sin \theta \Xi_2^2 \rangle v_{\lambda}^{-6} = \frac{1}{4} \pi \Xi_d^2 v_L^{-6}$, where $\Xi_d = 10$ eV is the dilatational deformation potential constant, and $\sum_\lambda \langle \sin \theta K^2 \rangle v_{\lambda}^{-4} = (e_{14}^2) \psi L (\frac{89}{1023} v_L^{-4} + \frac{107}{1023} v_F^{-4})$, where $e_{14} = -0.16$ C/N is the only non-zero element of the piezotensor. Other parameters can be found in [19]. Equations (13) and (14) are plotted as symbols in figure 2 in the ballistic limit of electron–phonon coupling. Below a few kelvin, piezoelectric coupling fully dominates and as a result the temperature regime where $G_{LP} < G_{12}$ is pushed toward relatively low, but experimentally achievable, temperatures. The crossover occurs at $T \sim 140$ mK.

For silicon-based bilayer (see [20, 21] and figure 1(d)) we consider both ballistic and diffusive limits at electron density $n = 5 \times 10^{15}$ m$^{-2}$. Parameters for Si can be found in [19]. The curves in figure 3 are calculated for a symmetric high (low) mobility Si bilayer system with mobility $\mu = 2.5$ (0.2) m$^2$ V$^{-1}$ s$^{-2}$, mean free path $l_e = 200$ (16) nm and layer distance $d = 20$ (200) nm. For the high-mobility device, we have used the ballistic limit response
Figure 3. The interlayer thermal conductance \((G_{12})\) and deformation potential \((G_{DP}^{L_p})\) electron–phonon thermal conductance for two Si bilayer devices with \(n = 5 \times 10^{15} \text{m}^{-2}\). For device A (B), solid curves are the results of numerical integration using the ballistic (diffusive) susceptibility and dashed curves are obtained from the approximation formula of equation (11) (equation (12)). The circle marks the crossover where electron–phonon coupling changes from ballistic to diffusive.

Si
A: \(d = 20 \text{ nm}, l_e = 200 \text{ nm}\)
B: \(d = 200 \text{ nm}, l_e = 16 \text{ nm}\)

(equation (8)) and for the low-mobility one the diffusive response (equation (10)). Silicon is not piezoelectric, so for \(G_{L_p}\) we need to consider only \(G_{DP}^{L_p}\). Due to finite electron mean free path (even for the high-mobility device), we will include the ballistic and diffusive limits of \(G_{DP}^{L_p}\). For simplicity we plot \(G_{DP}^{L_p}\) so that it changes abruptly from a diffusive to a ballistic formula (note that equation (13) is not valid close to \(q \lambda_{fi} = 1\)). For Si 2DEG, we have

\[
\sum_{\lambda} \langle \sin \theta \Sigma^2 \rangle v_\lambda^{-6} = \frac{1}{15} \pi \left(4 \Sigma_a \Sigma_u + 8 \Sigma_0^2 + \Sigma_u^2 \right) v_L^{-6} + \frac{1}{3} \pi \Sigma_0^2 v_F^{-6} \quad \text{and} \quad \sum_{\lambda} \langle \Sigma^2 \rangle v_\lambda^{-6} = \left(\frac{2}{3} \Sigma_a \Sigma_u + \Sigma_0 + \frac{1}{3} \Sigma_u^2 \right) v_L^{-6} + \frac{2}{3} \Sigma_0^2 v_L^{-6},
\]

where \(\Sigma_u\) is the uniaxial deformation potential constant. We use the typical values \(\Sigma_{du} = -11.7(9.0) \text{ eV}\). For the high- and low-mobility Si systems the crossover temperature where \(G_{12} = G_{L_p}\) is 660 mK and 1.4 K, respectively. Even though we have set \(d\) an order of magnitude larger for the diffusive device, still the crossover occurs at higher temperature, which is a signature of the enhancement of fluctuations/dissipation and, thereby, interlayer coupling due to disorder. Note that deformation potential electron–phonon coupling is also enhanced due to disorder.

Next we consider a graphene bilayer [4] that is depicted in figure 1(e). The curves in figure 4 are calculated for a symmetric high (low) mobility device with a mobility \(\mu = 1.59 (0.17) \text{ m}^2 \text{V}^{-1} \text{s}^{-2}\), mean free path \(l_e = 10 \mu\text{m} (20 \text{ nm})\), layer distance \(d = 20 (200) \text{ nm}\) and electron density \(n = 10 \times 10^{15} \text{m}^{-2}\). We used \(v_F = 10^6 \text{ m s}^{-1}\) and assumed AlO dielectric between the layers. As above, for the high-mobility and the low-mobility device we used ballistic and diffusive response functions, respectively. The screened deformation potential
Figure 4. The interlayer thermal conductance \((G_{12})\), deformation potential \((G_{\text{DP}}^{\text{LP}})\) and vector potential \((G_{\text{VP}}^{\text{LP}})\) electron–phonon thermal conductance for two graphene bilayer devices with \(n = 1 \times 10^{16} \text{ m}^{-2}\). \(G_{\text{DP}}^{\text{LP}}\) and \(G_{\text{VP}}^{\text{LP}}\) are taken from [18]. For device A (B) solid curves are the results of numerical integration using the ballistic (diffusive) susceptibility and dashed curves are obtained from the approximation formula of equation (11) (equation (12)).

and vector potential electron–phonon contributions are plotted in figure 4 as symbols. For high-mobility graphene, \(G_{\text{VP}}^{\text{LP}}\) dominates at the lowest temperatures and the crossover where \(G_{12} = G_{\text{LP}}\) occurs at a relatively low temperature of \(T = 40 \text{ mK}\). The disorder enhancement of the interlayer heat transfer pushes the threshold for low-mobility graphene to \(T = 3.0 \text{ K}\). Note that the vector potential electron–phonon coupling is decreased with disorder in contrast to the deformation potential coupling.

Another widely explored semiconductor bilayer carrier system which can be realized using compound semiconductors [22] or Si [23, 24] is the electron–hole bilayer. The complexity of the valence band makes this system more difficult to analyze theoretically. We will not present \(G_{12}\) for such a system here explicitly, but it should behave in a similar fashion as its electron–electron counterpart. However, one thing that may differ drastically from the electron–electron bilayer system is the carrier–phonon coupling. Due to asymmetry in the deformation potential coupling between the different layers the carrier–phonon coupling can be unscreened and, as a result, \(G_{\text{LP}}\) can be strongly enhanced at low temperatures [15]. The enhancement factor depends on the details of the system, but in many cases it is of the order of \((\kappa/q_{\lambda}T)^2\), which suggests that for semiconductor electron–hole bilayers, \(G_{\text{LP}}\) can dominate over \(G_{12}\) even down to very low temperatures. Note also that in symmetric electron bilayers, \(G_{\text{LP}}\) can be affected by the presence of another carrier system in a non-trivial way, but a significant enhancement is not expected [15].
3.2. Possible experimental realizations

The interlayer heat transfer can be investigated experimentally by varying the input powers $P_L$ while measuring the electron temperatures $T_L$ (see figure 1(b)). A uniform input power follows, for example, from Ohmic heating. This technique has been broadly utilized in electron–phonon coupling measurements. Indeed, the electron–phonon contributions $G_{LP}$ can be investigated independently from $G_{12}$ at a balanced input power that gives $T_1 = T_2$. In the case of semiconductor bilayer, the other layer can also be depleted to get a handle on $G_{LP}$. Note that the Ohmic heating technique has also been utilized in the investigation on coupling of Johnson–Nyquist noise heating between two resistors at different temperatures [25], which is conceptually very close to the case presented here.

It is not necessarily trivial to measure the electron temperature of individual layers. For example, the quantum corrections of resistivity and Shubnikov–de Haas oscillations, which have been used as an electron thermometer, can be affected by the other layer in a complicated way. More local temperature probes based on, e.g., quantum point contacts and quantum dots have also been investigated [26–28]. Noise thermometry provides an attractive way of probing the electron temperature. It has recently been used for single-layer graphene [29] and could be adopted for investigations of $G_{12}$.

Interlayer heat transfer can be investigated also in a more indirect way, by coupling the individual layers to metallic electrodes, to which the input power is fed and where the temperature is sensed. As metals have quite a large electron–phonon coupling, the volume of metallic islands should be sufficiently small in order not to hide $G_{12}$. Especially in the case of Si, doped contact regions can serve as metallic islands. This is an attractive approach as the electron–phonon coupling in doped semiconductors can be relatively weak so that $G_{12}$ still dominates. The sign of $P_L$ can be also reversed, which is equivalent to cooling. This can be achieved by using quantum dots [27] or semiconductor–superconductor contacts [30].

As $G_{12}$ (and $G_{LP}$) depends on the electron densities and on the interlayer density balance, it is desirable to adjust the layer densities by external gates. In general, $G_{12}$ could be used as a gate voltage-controlled thermalization path (thermal switch). However, it is important to note (as already pointed out in [15]) that a similar near-field thermal coupling to $G_{12}$ can exist between the 2D carriers and the external gate electrodes.

4. Summary and conclusions

In summary, a near-field heat transfer effect due to interlayer interaction in bilayer carrier systems was investigated. By using perturbation theory and fluctuation–dissipation relations, we derived a general expression for the near-field interlayer energy transfer rate (equation (6)) and thermal conductance (equation (7)). Our formulation can be applied to, e.g., semiconductor- and graphene-based bilayers. We presented analytical approximation formulae and detailed calculations of the interlayer heat transfer due to screened Coulomb interaction for GaAs-, Si- and graphene-based bilayers. It was shown that remarkably the interlayer heat transfer can dominate over the electron–phonon coupling to the thermal bath below a crossover temperature that depends on the system parameters. We found a crossover temperature of 140 mK (660 mK) for ballistic GaAs (Si) bilayer with $d = 20$ nm layer distance and carrier density $n = 10^{15}$ m$^{-2}$ ($5 \times 10^{15}$ m$^{-2}$). A strong vector potential electron–phonon coupling in ballistic graphene results in low crossover temperature of 40 mK ($n = 10 \times 10^{15}$ m$^{-2}$). Interlayer heat transfer is enhanced

New Journal of Physics 15 (2013) 033043 (http://www.njp.org/)
by disorder and for low-mobility Si (graphene) bilayer with \( d = 200 \text{ nm} \) the crossover occurs already at \( \sim 1.4 \text{ K} \) (3.0 K). The crossover temperatures reported here can be accessed by standard experimental equipment, and we introduced possible experimental configurations to investigate the interlayer heat transfer.

Finally, we note that by lowering the electron densities and/or by increasing the temperature, plasmons and virtual phonons may start to play a role in interlayer interaction. These excitations are known to enhance the bilayer drag effect [31, 32], and the enhancement should also be observable in the interlayer heat transfer. In a very dilute and strongly interacting system, an enhancement of drag, which cannot be explained with plasmons or virtual phonons, has also been observed [33]. Therefore, depending on the system parameters, the crossover temperature below which the interlayer heat transfer starts to dominate over the electron–phonon coupling to the thermal bath can be significantly higher than those given in this work. The study of plasmonic effects, virtual phonon excitations, dilute carrier regime and elevated temperatures is left for future investigations. At elevated temperatures the effect described in this paper can also be of relevance for inter-flake heat transfer in thermal interface materials fabricated from graphene composites [34]. The concepts presented in this work can be extended to coupled one-dimensional carrier systems.

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