The method of intermediate-energy Coulomb excitation has been widely used to determine absolute $B(E2; 0_1^+ \to 2_1^+)$ quadrupole excitation strengths in exotic nuclei with even numbers of protons and neutrons. Transition rates measured with intermediate-energy Coulomb excitation are compared to their respective adopted values and for the example of $^{26}$Mg to the $B(E2; 0_1^+ \to 2_1^+)$ values obtained with a variety of standard methods. Intermediate-energy Coulomb excitation is found to have an accuracy comparable to those of long-established experimental techniques.

PACS numbers: 25.70.De, 23.20.-g
stopping power of the nuclei in the material is known, the Doppler shifts of the $\gamma$ rays emitted by the recoiling reaction residues determine the points in time at which emission occurred and hence the lifetime of the excited state (suitable for $\tau < 1$ ps) [17]. The RDSS method similarly uses the Doppler shift of an excited, recoiling nucleus to determine the lifetime of the state. A stopper is placed downstream of the target and the intensity ratio of $\gamma$ rays emitted in-flight and stopped for different target-stopper distances provides a measure of the lifetime in the range $10^{-9} \text{ s} < \tau < 10^{-12} \text{ s}$ [18].

Nuclear resonance fluorescence (NRF), electron scattering, and Coulomb excitation determine transition rates through the measurement of cross sections. In a typical NRF experiment, a continuous photon spectrum (bremsstrahlung) irradiates a target of stable nuclei. The target nuclei are excited by the radiation and de-excitation $\gamma$ rays are subsequently emitted with an angular distribution depending on the transition. The energy-integrated cross section of the scattered $\gamma$ rays is inversely proportional to the lifetime of the excited state [19]. Electron scattering utilizes a simplified form of low-energy Coulomb excitation where the form factor in the Born approximation is related to the multipolarity of the transition. The transition rate can be extracted from the value of the form factor [20]. Due to the well-understood nature of the interaction and the ease of producing a large projectile flux, electron scattering is one of the most accurate methods of determining transition probabilities.

In Coulomb excitation, the interaction of the electromagnetic fields of the target nuclei and projectile nuclei leads to excitations with subsequent $\gamma$-ray emissions. The number of photons $N_{\gamma,f \rightarrow i}$ observed in an inverse-kinematics Coulomb excitation experiment with $\gamma$-ray tagging is related to the excitation cross section by

$$\sigma_{i \rightarrow f} = \frac{N_{\gamma,f \rightarrow i}}{N_T N_B \epsilon}$$

where $N_T$ is the number of target nuclei, $N_B$ is the number of beam nuclei, and $\epsilon$ is the efficiency of the experimental setup. $N_B$ can be determined prior to interaction with the target, and $N_T$ is given by the target thickness. The efficiency accounts for the intrinsic and geometric efficiencies of all detector systems involved. Equation 2 assumes only one excited state; if more states than one are excited, possible feeding from higher excited states must be considered (see Figure 1). The excitation cross section can be related to the reduced transition probability through various approaches. At beam energies below the Coulomb barrier of the projectile-target system, a Rutherford trajectory is assumed. For intermediate-energy Coulomb excitation, we use the relativistic theory developed by Winther and Alder, which involves a semi-classical approach with first-order perturbation theory. Distorted-wave Born approximation calculations have also been used to determine transition rates from cross sections [8] and are in agreement with the excitation theory developed by Winther and Alder.

**INTERMEDIATE ENERGY COULOMB EXCITATION**

The most important difference between low- and intermediate-energy Coulomb excitation is that nuclear interactions can occur above the Coulomb barrier. However, the inclusion of nuclear contributions to the measurement of electromagnetic transition rate can be prevented in heavy-ion reactions by considering only those events scattered within a maximum scattering angle representing a “safe” minimum impact parameter $b_{\text{min}}$ (see Figure 2). The radius $R_{\text{int}}$ beyond which the Coulomb interaction dominates defines the minimum impact parameter to be allowed in the experiment. Wilcke, et al. use elastic scattering data to predict $R_{\text{int}}$ for interactions between various nuclei [21]. For $^{46}$Ar it has been shown that varying $b_{\text{min}}$ where $b_{\text{min}} \geq R_{\text{int}}$ has little effect on the measured transition rate value [22]. In contrast, for light nuclei (approximately $Z < 10$) nuclear interactions may occur even for particles scattered at small angles and care must be taken to disentangle the nuclear and Coulomb contributions to the cross section [23].

The adiabatic cutoff of the Coulomb excitation process occurs at a maximum excitation energy

$$E_{x}^{\text{max}} \approx \frac{\gamma hc \beta}{b}$$

where $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ are the velocity and Lorentz factor of the beam and $b$ is the impact parameter. Intermediate-energy beams can excite states at higher excitation energies compared to low-energy beams. For example, $^{26}$Mg impinging on a $^{209}$Bi target with a beam velocity of $\beta = 0.36$ has an adiabatic cutoff of $E_{x}^{\text{max}} \approx 6$ MeV [24]. However, the possibility

---

**FIG. 1:** Schematic of Coulomb excitation of a nucleus from an initial state $|i\rangle$ to a final bound state $|f\rangle$ and the ensuing $\gamma$ decay with a possible feeding transition from a higher state shown.
of feeding from excitations to states above the first $2^+$ state must be considered when calculating the excitation cross section $\sigma$. Photons are used to identify the inelastic scattering process to bound excited states and hence target thickness is not constrained by the need to preserve momentum resolution to differentiate elastic and inelastic scattering. Higher energy beams allow for the use of thicker targets, and the number of scattering centers can be increased by as much as a factor of 1000 over low-energy experiments, permitting an equivalent decrease in the number of required projectile nuclei. In typical intermediate-energy Coulomb excitation experiments, 1 beam particle in $10^3$–$10^4$ interacts with the target nuclei and multiple excitations are significant only to this small factor $\beta$. The wide range of scattering angles inherent in low-energy Coulomb scattering require large solid-angle detectors; a few degrees of acceptance suffices for intermediate-energy Coulomb excitation.

At the NSCL, SeGA, an array of eighteen 32-fold segmented high-purity Ge $\gamma$-ray detectors, and APEX, twenty-four position-sensitive NaI(Tl) crystals, are used for Coulomb-excitation measurements in conjunction with a phoswitch detector or the S800 spectrograph for event-by-event particle identification. Similar setups are employed at GANIL, GSI, and RIKEN. For a more detailed description of intermediate-energy Coulomb excitation, see Refs. 5, 8, 30.

**ACCURACY OF INTERMEDIATE-ENERGY COULOMB EXCITATION**

The advantages of intermediate-energy Coulomb excitation are most pronounced when the method is applied to exotic nuclei with low production rates. Under these circumstances, the statistical uncertainty dominates. This difficulty is present irrespective of the method applied, and, therefore, only high-statistics intermediate-energy Coulomb excitation measurements will be considered in determining the method’s accuracy. A summary of intermediate-energy Coulomb excitation measurements of previously-published transition rates along with their respective adopted values can be found in Figure 4. For these Coulomb excitation test cases, no feeding was observed. The adopted values are those compiled by Raman, where four or more independent transition rate measurements using any of the above techniques have been made for each nucleus. In the calculation of the adopted transition rate for $^{40}$Ar, one of eight experimental values was measured using intermediate-energy Coulomb excitation, and for $^{36}$Ar, two of eight. The error bars on the adopted values represent the relative uncertainties. The average difference from the adopted value is 6% and only one data point exceeds 10%. Note that all measurements are in agreement with their respective adopted values.

Figure 4 shows the relative differences between measured $B(E2; 0^+ \rightarrow 2^+_1)$ transition rates and the adopted value $\sigma$ for $^{26}$Mg. The shaded area represents the uncertainty of the adopted value. The measurements were made using low-energy ($x,x'\gamma$) Coulomb excitation, NRF, DSAM, RDDS, and electron scattering. These traditional transition rate measurements have an average difference of 23% from the adopted value for $^{26}$Mg. The right-most data point, which deviates from the adopted value by 3%, was measured by Church et al. using intermediate-energy Coulomb excitation at a beam energy of 66.8 MeV/nucleon. This specific measurement illustrates the more general point made in Figure 3a that intermediate-energy Coulomb excitation measurements that are not limited by statistics can readily measure transition rates with an accuracy of about 5% to a precision of about 10%.

**CONCLUSION**

The extent of the Coulomb excitation method to projectiles at intermediate beam energies allows for the measurement of transition rates in nuclei far from stability. Thick targets allow for experiments on isotopes with low projectile fragmentation production rates. Intermediate-energy Coulomb excitation has been shown to produce results within error of the adopted values for transitions measured with long-established experimental techniques. Additionally, the accuracy of the $^{20}$Mg transition rate measurement from intermediate-energy Coulomb excitation exceeds the average accuracy of the other measurements.

This work is supported by the National Science Foundation through grants PHY-0110253.
FIG. 3: a) The percent differences between adopted and measured $B(E2; 0^+ \rightarrow 2^+)$ transition rates for published test cases in intermediate-energy Coulomb excitation measurements. The average difference is 6% compared to an intermediate-energy Coulomb excitation measurement (right-most) of the same transition. The 3% difference of the intermediate-energy Coulomb excitation measurement compares favorably with the average absolute value of the difference of 23% for the other measurements.

[1] B. A. Brown, Prog. Part. Nucl. Phys. 47, 517 (2001).
[2] T. Otsuka, M. Homma, T. Mizusaki, N. Shimizu, and Y. Usuno, Prog. Part. Nucl. Phys. 47, 319 (2001).
[3] M. Bender, P. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. 75 (2003).
[4] G. A. Lalazissis, S. Raman, and P. Ring, Atom. Data Nucl. Data Tables 78, 1 (2001).
[5] K. Alder and A. Winther, *Electromagnetic excitation: theory of Coulomb excitation with heavy ions* (North-Holland Pub. Co., Amsterdam, 1975).
[6] A. Winther and K. Alder, Nucl. Phys. A 319, 518 (1979).
[7] T. Motobayashi et al., Phys. Lett. B 346, 9 (1995).
[8] T. Glaesmer, Annu. Rev. Part. Sci. 48, 1 (1997).
[9] H. Scheit et al., Eur. Phys. J. A 25, s01, 397 (2005).
[10] O. Niedermaier et al., Nucl. Phys. A 752, 273c (2005).
[11] O. Niedermaier et al., Phys. Rev. Lett. 94, 172501 (2005).
[12] J. Åystö, Eur. Phys. J. A 25, s01, 767 (2005).
[13] B. V. Pritychenko et al., Phys. Lett. B 461, 322 (1999).
[14] V. Chisté et al., Phys. Lett. B 514, 233 (2001).
[15] A. Bohr and B. R. Mottelson, *Nuclear Structure*, vol. I (World Scientific Publishing Co. Pte. Ltd., Singapore, 1998).
[16] D. B. Fossan and E. K. Warburton, *Nuclear Spectroscopy and Reactions* (Academic Press, New York and London, 1974), p. 307.
[17] H. Morinaga and T. Yamazaki, *In-beam Gamma-ray Spectroscopy* (North-Holland Pub. Co., Amsterdam, 1976), p. 404.
[18] U. Kneissl, H. H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. 37, 349 (1996).
[19] D. G. Ravenhall, Rev. Mod. Phys. 30, 430 (1958).
[20] W. W. Wilce, J. R. Birkelund, H. J. Wollersheim, A. D. Hoover, J. R. Huizenga, W. U. Schröder, and L. E. Tubbs, Atom. Data Nucl. Data Tabl. 25, 391 (1980).
[21] A. Gade et al., Phys. Rev. C 68, 014302 (2003).
[22] T. Glaesmer, Nucl. Phys. A 693, 1 (2001).
[23] J. A. Church et al., Phys. Rev. C 72, 054320 (2005).
[24] W. Mueller, J. A. Church, T. Glaesmer, D. Gutknecht, G. Hackman, P. G. Hansen, Z. Hu, K. L. Miller, and P. Quirin, Nucl. Instrum. Methods Phys. Res. A 466, 492 (2001).
[25] B. C. Perry, C. M. Campbell, J. A. Church, D.-C. Dinca, J. Enders, T. Glaesmer, Z. Hu, K. L. Miller, W. F. Mueller, and H. Olliver, Nucl. Instrum. Methods Phys. Res. A 505, 85 (2003).
[26] D. Bazin, J. A. Cagiano, B. M. Sherrill, J. Yurkon, and A. Zeller, Nucl. Instrum. Methods Phys. Res. B 204, 629 (2003).
[27] O. Sorlin et al., Phys. Rev. Lett. 88, 092501 (2002).
[28] A. Buergel et al., Acta Phys. Pol. B 36, 1249 (2005).
[29] J. F. Bertulani and G. Baur, Prog. Rep. 163, 299 (1988).
[30] R. B. Ibbotson et al., Phys. Rev. Lett. 80, 2081 (1998).
[31] P. D. Cottle, M. Fauerbach, T. Glaesmer, R. W. Ibbotson, K. W. Kemper, B. Pritychenko, H. Scheit, and M. Steiner, Phys. Rev. C 60, 031301(R) (1999).
[32] P. D. Cottle, B. V. Pritychenko, J. A. Church, M. Fauerbach, T. Glaesmer, R. W. Ibbotson, K. W. Kemper, H. Scheit, and M. Steiner, Phys. Rev. C 64, 057304 (2001).
[33] P. D. Cottle, B. V. Pritychenko, J. A. Church, M. Fauerbach, T. Glaesmer, R. W. Ibbotson, K. W. Kemper, H. Scheit, and M. Steiner, Phys. Rev. C 71, 014302(R) (2005).
[34] D. S. Andreev, V. D. Vasiliev, G. B. Gusinskii, K. I. Erolkina, and K. L. Kember, Columbia Tech. Transl. 25, 842 (1962).
[35] V. K. Rasmussen, F. R. Metzger, and C. P. Swann, Phys. Rev. 123, 1386 (1961).
57, 403 (1964).
[41] O. Hausser, T. K. Alexander, and C. Broude, Can. J. Phys. 46, 1035 (1968).
[42] S. W. Robinson and R. D. Bent, Phys. Rev. 168, 1266 (1968).
[43] P. R. de Kock, J. W. Koen, and W. L. Mouton, Nucl. Phys. A140, 190 (1970).
[44] A. B. McDonald, T. K. Alexander, O. Hausser, and G. T. Ewan, Can. J. Phys. 49, 2886 (1971).
[45] J. L. Durell, P. R. Alderson, D. C. Bailey, L. L. Green, M. W. Greene, A. N. James, and J. F. Sharpey-Schafer, J. Phys. A: Math. Nucl. Gen. 5, 392 (1972).
[46] D. Schwalm, Ph.D. thesis, Universitaet Heidelberg (1973).
[47] J. L. Eberhardt, R. E. Horstman, H. A. Doubt, and G. Van Middlekoop, Nucl. Phys. A244, 1 (1975).
[48] P. Wagner, M. A. Ali, J. P. Coffin, and A. Gallmann, Phys. Rev. C 11, 1622 (1975).
[49] D. Schwalm, E. K. Warburton, and J. W. Olness, Nucl. Phys. A293, 425 (1977).
[50] K. Dybdal, J. S. Forster, P. Hornshoj, N. Rud, and C. A. Straede, Nucl. Phys. A359, 431 (1981).
[51] R. H. Spear, T. H. Zabel, M. T. Esat, A. M. Baxter, and S. Hinds, Nucl. Phys. A378, 559 (1982).
[52] V. M. Khvastunov, N. G. Afanasev, V. D. Afanasev, E. V. Bondarenko, I. S. Gulkarov, G. A. Savitskii, and N. G. Shevchenko, Sov. J. Nucl. Phys. 12, 5 (1971).
[53] E. W. Lees, A. Johnston, S. W. Brain, C. S. Curran, W. A. Gillespie, and R. P. Singhal, J. Phys. A: Math. Nucl. Gen. 6, L116 (1973).
[54] E. W. Lees, A. Johnston, S. W. Brain, C. S. Curran, W. A. Gillespie, and R. P. Singhal, J. Phys. A: Math. Nucl. Gen. 7, 936 (1974).