Neutrinos in Strong Magnetic Fields

A. Pérez Martínez\textsuperscript{1,2}, A. Amézaga Hechavarria\textsuperscript{1}, D. Oliva Agüero\textsuperscript{1}, H. Pérez Rojas\textsuperscript{1,3} and S. Rodríguez Romo\textsuperscript{2}

\textsuperscript{1} ICIMAF, Calle E No. 309, 10400 La Habana, Cuba
\textsuperscript{2} Centro de Investigaciones Teóricas, FES-Cuautitlán, UNAM, México.
\textsuperscript{3} CINVESTAV-IPN, A.P. 14-740, 07000 México, D.F.

Abstract

We compute the dispersion curves for neutrinos propagating in an extremely dense electroweak plasma, in the presence of very strong magnetic fields of order $B \leq M_W^2/e$. The neutrino self-energy is calculated in the one-loop approximation. We consider only contributions of the first Landau level to the propagator of the $W$-bosons, and distinguish between motion parallel or perpendicular to the external magnetic field. We find that the neutrino dispersion curve for parallel propagation to the field suggests a superfluid behavior. An interesting analogy with fractional QHE is pointed out. We obtain a neutrino effective mass which increases with the magnetic field.

I. INTRODUCTION

The present paper is addressed to investigate the dispersion equation of massless neutral fermions, interacting with charged fermions and massive vector bosons, propagating in a medium at finite temperature and density, and in presence of an extremely strong magnetic field.

The propagation of neutrinos in an electroweak plasma has been studied and the dispersion equation for the quasiparticles was obtained\cite{1–3}. The spectrum found exhibits, in some extreme conditions, a superfluid behavior.

In the present paper we consider the role of extremely strong magnetic fields as a possible mechanism for generating an effective neutrino mass in a very dense medium. We find again a superfluid behavior for the neutrinos moving parallel to the external magnetic field, provided it is strong enough.

The propagation of neutrinos in magnetized media, assuming no dependence of the $W$-propagator on the magnetic field, has been done by computing two types of diagrams (bubble and tadpole)\cite{12}. We work with the effective action, the generating functional of irreducible one-particle inverse Green’s functions. Thus, in the evaluation of the inverse Green’s function, whose zeros give the dispersion equation, the tadpole diagrams do not appear. The tadpole diagram is reducible and does not contribute to it.
For strong enough magnetic fields, a gas of charged bosons undergoes Bose-Einstein condensation. This suggests that it is the ground state of the bosons which play the main role. In our calculation we will take into account only the part of the $W$ propagator which contains the ground state energy. When the momentum is small and the magnetic field is high enough $eB \leq M_W^2$, so that the term $1/\sqrt{M_W^2 - eB}$ dominates, the main contribution to the propagator comes from the low momentum gauge bosons ($W$-condensate).

The expressions we derive for the neutrino dispersion equation are valid for strong fields, close to the limiting value $M_W^2/e$. Such fields are at present conjectured to exist in the cores of neutron stars, and they may have also existed in the early universe, in which case the observed galactic and intergalactic magnetic fields are viewed as relics of huge primordial fields. However, on a more general basis, our results apply to any massless fermions interacting with vector bosons of mass $M$ in strong magnetic fields $eB \leq M^2$.

The first part of this paper (section 2) presents some general expressions of Green functions in a magnetized medium, which will be used subsequently. In section 3, we find the mass operator in the limit of high magnetic field, after performing the sum over $p_4$. These results are very general and apply equally well to the limiting cases of high temperature and high density. In section 4, we analyze the limit $T \to 0$, corresponding to a degenerate fermion gas (large fermion density). We also consider the role of $W$-boson condensation. In section 5, the dispersion equations are discussed. We analyze two cases of neutrino propagation, parallel and perpendicular to the magnetic field. For parallel motion, an effective mass results only in the sense of $\lim_{k_3 \to 0} \omega \neq 0$. When the motion is perpendicular to the magnetic field, a minimum appears in the dispersion equation at non-zero momentum and it is possible to define an effective mass in the more traditional sense of being a local minimum of the dispersion equation.

Our results depend linearly on the magnetic field and, in the degenerate case, also linearly on the chemical potential $\mu$. Other studies of the mass operator in QED in presence of a magnetic field analyze different limits: high temperature, high density, and high magnetic field. The first two show similar dispersion equations if one substitutes $M = e^2 T^2/8$ with $M = e^2 \mu^2/(8\pi^2)$.

II. NEUTRINO SELF-ENERGY: GENERAL EXPRESSIONS

The neutrino two-point inverse Green function in presence of a magnetic field reads as

$$S^{-1}_\nu(k^*) = P_R(-i\gamma_\mu k^*_\mu + \Sigma^W(k^*))P_L,$$

where $k^*_\lambda = k_\lambda - i\mu_\nu \delta_{4,\lambda}$, and $P_R = (1 + \gamma_5)/2$ and $P_L = (1 - \gamma_5)/2$ are the left- and right-handed projection operators. We recall that the following relation holds among the chemical potentials $\mu_\nu = \mu_e - \mu_W$. Note that, in equation (1), the self-energy of the $Z$-boson is not included. It can be neglected since it does not interact with the magnetic field (at the one-loop level); it is of order $g^2/M_Z^2$, whereas the $W$ term is of order $g^2 eB/M_W^2$.

The expression for the self-energy $\Sigma^W$ is the following, in configuration space:

$$\Sigma^W(x, x') = -i \frac{g^2}{2\pi^2} \gamma_\mu G^\mu(x, x')D^W_{\mu\nu}(x - x')\gamma_\nu,$$
It represents the self-energy due to electron-W polarization.

In Euclidean space and in the gauge $A_\mu = (0, Bx, 0, 0)$, the propagator of the electron is

$$G^\text{e}(x, x') = -\frac{1}{(2\pi)^2} \sum_{n=0}^{\infty} \sum_{p_4} \frac{dp_3 dp_2}{\beta(2\pi)^3(p_4^2 + p_3^2 + m_e^2 + 2eBn)} \cdot \{(i p_4 - \mu)\gamma_4 + i p_3 \gamma_3 - m_\nu (\sigma_+ \psi_n \psi_n + \sigma_- \psi_{n-1} \psi_{n-1}) + 1/2 \sqrt{2eBn}[\gamma_+ \psi_{n-1} - \gamma_- \psi_{n-1} \psi_n]\}$$

(3)

$$\cdot \exp[ip_4^4(x_4 - x_4') + ip_3(x_3 - x_3') + ip_2(x_2 - x_2')]$$

where $\xi = \sqrt{eB}(x_1 + x_o)$, $\xi' = \sqrt{eB}(x_1 + x_o)$, $x_o = p_{\perp}/eB$, $\sigma^\pm = 1/2[1 \pm \sigma_2]$, $\gamma^\pm = 1/2[\gamma_1 \pm i \gamma_2]$, $\sigma_3 = i/2[\gamma_1 \gamma_2]$, and $p_{\perp} = p_\perp - i eB \delta_{4, \lambda}$.

The W-propagator in a magnetic field has the form

$$D^W_{\mu\nu}(x, x') = \frac{1}{(2\pi)^2} \int dp_3 dp_2 \left[ \frac{R^- + R^+}{2} \Psi^1_{\mu\nu} + R^0 \Psi^2_{\mu\nu} + i \left( \frac{R^- - R^+}{2} \Psi^3_{\mu\nu} \right) \right]$$

(4)

$$\cdot \psi_n(\xi) \psi_n(\xi') \exp[ip_4^4(x_4 - x_4') + ip_3(x_3 - x_3') + ip_2(x_2 - x_2')]$$

where $R^\pm = [p_4^2 + E_n^0 \pm 2eB]^{-1}$, $R^0 = [p_4^2 + E_n^0]^{-1}$, with $E_n^0 = M^2_W + p_3^2 + 2eB(n + 1/2)$; $\Psi^1_{\mu\nu} = \frac{1}{B^2} G^0_{\mu\nu}$, $\Psi^2_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{B^2} G^0_{\mu\nu}$, and $\Psi^3_{\mu\nu} = \frac{1}{B} G^0_{\mu\nu}$ ($G^0_{\mu\nu}$ is the field tensor of the SU(2)xU(1) electromagnetic external field). Concerning the gauge fixing term, we are taking $D^W_{\mu\nu}$ in a transverse gauge which is expected to guarantee the gauge independence of the neutrino spectrum.

The poles of $D^W_{\mu\nu}$ are located at

$$E^W_n = E^W_{n,1} = \sqrt{p_3^2 + m_N^2 - eB},$$

(5)

which is the ground state energy, and at

$$E^W_n = \sqrt{p_3^2 + m_N^2 + 2eB(n + 1/2)},$$

(6)

where $n = 0, 1, 2, \ldots$ with degeneracy $\beta_n = 3 - \delta_{0n}$. The ground state energy (5) is unstable for $p_3^2 < eB - M^2_W$. The analog of the Euler-Heisenberg vacuum energy due to vector boson polarization is

$$U_W = -\frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t^2} e^{-M^2_W t} [eB \text{csch}(eBt)(1 + 2 \cosh 2eBt) - \frac{3}{4} - \frac{7}{2} e^2 B^2 t].$$

(7)

Convergence of this expression is only possible for $eB < M^2_W$, i.e. the vacuum becomes unstable for $eB \geq M^2_W$. This problem has been the subject of investigation mainly by Nielsen, Olesen and Ambjorn [10], [11]. In the last reference, a static magnetic solution of
classic electroweak equations, corresponding to a vacuum condensate of $W$ and $Z$ bosons, is found. It is valid above the critical value $B_c = M_W^2/e$. The vacuum bears the properties of a ferromagnet or an anti-screening superconductor.

We are interested in the Fourier transform of $(2)$. It requires rather long calculations involving the Fourier transform for two Hermite functions, which lead to functions of generalized Laguerre polynomials. Eventually we find

\[
\Sigma^W(k) = \frac{g^2}{2\pi^2} \sum_{p_4} \frac{dp_3}{(2\pi)^2} G_c(p_3 + k_3, p_4 + k_4, n') \Sigma_{\alpha\beta} P_L,
\]

where

\[
G_c(p_3 + k_3, p_4 + k_4, n') = \left((p_3 + k_3)^2 + (p_4 + k_4)^2 + m_e^2 + 2eBn'\right)^{-1}
\]

and $\Sigma_{\alpha\beta}$ is a $4 \times 4$ matrix whose elements are the following ($\Sigma_{12} = \Sigma_{21} = \Sigma_{34} = \Sigma_{43} = 0$):

\[
\Sigma_{11} = B_1(i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + B_1(-i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + 2A_1(i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{13} = -2A_1(p_3 + k_3)T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{14} = -2iB_1\sqrt{2eBn'T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} + 2iC\sqrt{2eBn'T_{n'-1,n}^{*} T_{n'-1,n}^{*} T_{n'-1,n}}}
\]

\[
\Sigma_{22} = B_1(i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + B_1(-i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + 2A_1(i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{23} = -2iB_1\sqrt{2eBn'T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} + 2iB_1\sqrt{2eBn'T_{n'-1,n}^{*} T_{n'-1,n}^{*} T_{n'-1,n}} - 2iC\sqrt{2eBn'T_{n'-1,n}^{*} T_{n'-1,n}^{*} T_{n'-1,n}}}
\]

\[
\Sigma_{24} = 2A_1(p_3 + k_3)T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{31} = 2A_1(p_3 + k_3)T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{32} = 2iB_1\sqrt{2eBn'T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} + 2iB_1\sqrt{2eBn'T_{n'-1,n}^{*} T_{n'-1,n}^{*} T_{n'-1,n}} + 2iC\sqrt{2eBn'T_{n'-1,n}^{*} T_{n'-1,n}^{*} T_{n'-1,n}}}
\]

\[
\Sigma_{33} = B_1(i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + B_1(-i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n} + 2A_1(-i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{34} = 2A_1(p_3 + k_3)T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{42} = 2A_1(p_3 + k_3)T_{n'-1,n}^{*} T_{n'-1,n},
\]

\[
\Sigma_{44} = B_1(i(p_4^* + k_4^*))T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} + B_1(-i(p_4^* + k_4^*))T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} T_{n'}^{*} - 2A_1(-i(p_4^* + k_4^*))T_{n'-1,n}^{*} T_{n'-1,n},
\]

where $A_1 = -(R^+ + R^-)/2 + 2R^0$, $B_1 = R^0$, $C = -i/2(R^+ - R^-)$,

\[
T_{n,m} = \left(\frac{n!}{m!}\right)^{1/2} \left(\frac{k_1 + ik_2}{2}\right)^{m-n} e^{-\frac{k_1 k_2}{4}} L_{m-n}^{(m-n)}((k_1^2 + k_2^2)/2),
\]

and $L_{m-n}^{(m-n)}$ are the Laguerre polynomials.
III. NEUTRINO SELF-ENERGY IN THE HIGH MAGNETIC FIELD LIMIT

Let us now consider the limit of an extremely strong magnetic field (\(eB\) close to, but smaller than, \(M^2_W\)). The \(W\) propagator is dominated by the Landau ground state term (\(n = 0\)), which means we keep in (4) only terms proportional to \(R^-\).

Furthermore, the condition \(eB > \mu_e^2 - m^2\) implies that the only electron state which contributes to the mass operator is \(n' = 0\). The sum \(\sum_{n'=0}^{\infty}\) can be approximated to \(\sum_{n'=0}^{n_n}\), where \(n_n\) is the integer part of \((\mu_e^2 - m^2)/2eB\), which is zero whenever \(\mu_e^2 < eB\), i.e., for most cases of interest. Hence, we concentrate on the case when both electron and \(W\)-boson are in the Landau ground state, and their quantum numbers are \(n = n' = 0\).

Keeping in mind the above approximations, equation (8) becomes

\[
\Sigma^W(k) = \frac{g^2eB}{(2\pi)^2} \int dp_3 G^\alpha_\nu(p_3 + k_3, p_4 + k_4) \Sigma_{\alpha\beta} P_L, \tag{9}
\]

with \(\Sigma_{\alpha\beta} = R^-(p_3, p_4)e^{-k^2_4/eB}S_{\alpha\beta}^\prime\), and \(R^- = \left[M^2_W + p^2_4 + p^2_3 - 2eB\right]^{-1}, G^\nu(p_3 + k_3)^{-1} = \left[(p_3 + k_3)^2 + (p + k_3)^2 + m^2_e\right]^{-1}\). The matrix \(\Sigma_{\alpha\beta}^\prime\) simplifies and takes the form

\[
\Sigma_{\alpha\beta}^\prime = \begin{pmatrix}
(i(p_4^4 + k_4) & 0 & (p_3 + k_3) & 0 \\
0 & 0 & 0 & 0 \\
-(p_3 + k_3) & 0 & -(i(p_4^4 + k_4) & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \tag{10}
\]

After performing in (9) the sum over \(p_4\) and taking the analytic continuation \((k_4^4 \rightarrow ik_o)\) we get a function of the new variable \(k_o = \mu_\nu\). The singularities in this variable lead to the gauge-invariant, physically relevant spectrum. We have

\[-i(k_4 - i\mu_\nu) = (k_o - \mu_\nu) = \omega + i\Gamma\]

where \(\omega\) is the energy and \(\Gamma\) the inverse lifetime of the neutrino quasiparticles.

We obtain the following expression for \(\Sigma^W\):

\[
\Sigma_{11}^W = \frac{g^2eB}{(2\pi)^2} [\omega I_2 - I_1 + I_3], \quad \Sigma_{33}^W = -\Sigma_{11}^W
\]

\[
\Sigma_{13}^W = -\Sigma_{31}^W = \frac{g^2eB}{(2\pi)^2} [-k_3 I_2 + I_4 - I_5], \tag{11}
\]

where the integrals \(I_i\) can be written as (neglecting the vacuum contributions),

\[
I_1 = \int \frac{dp_3}{2Q^2} \left( (J_{oo} - 2p_3k_3)(n_e - n_\nu) - 2\omega E_e(n_e + n_\nu) \right) e^{-k_4^2/2eB},
\]

\[
I_2 = \int \frac{dp_3}{2E_WQ} \left( (J_{oo} - 2p_3k_3)(n_{W^-} + n_{W^+}) + 2\omega E_W(n_{W^-} - n_{W^+}) \right) e^{-k_4^2/2eB},
\]

\[
I_3 = \int \frac{dp_3}{2Q^2} \left( (J_{oo} + 2p_3k_3)(n_{W^-} - n_{W^+}) + 2E_W\omega(n_{W^-} + n_{W^+}) \right) e^{-k_4^2/2eB},
\]

\[
I_4 = \int \frac{p_3dp_3}{2E_WQ^2} \left( (J_{oo} - 2p_3k_3)(n_e - n_\nu) - 2\omega E_e(n_e - n_\nu) \right) e^{-k_4^2/2eB},
\]

\[
I_5 = \int \frac{p_3dp_3}{2E_WQ^2} \left( (J_{oo} + 2p_3k_3)(n_{W^-} + n_{W^+}) + 2E_W\omega(n_{W^-} - n_{W^+}) \right) e^{-k_4^2/2eB}. \tag{12}
\]
with

\[ Q = (J_{oo} - 2p_3k_3)^2 - 4\omega^2 E_e^2 \]
\[ Q' = (J'_{oo} + 2p_3k_3)^2 - 4\omega^2 E_W^2 \]

and

\[ J_{oo} = z_1 - eB - m_e^2 + M_W^2, \]
\[ J'_{oo} = z_1 + eB + m_e^2 - M_W^2. \]

In the above formulae \( E_e = \sqrt{((p_3 + k_3)^2 + m_e^2)} \) and \( E_W = \sqrt{(p_3^2 + M_W^2 - eB)} \) are the Landau ground state energies of the electron and the W-boson, respectively, whereas

\[ n_{e,p} = [e^{(E_{e,p} + \mu_e \beta)} + 1]^{-1}, \quad n_{W-,W^+} = [e^{(E_{W^+} + \mu_W - \mu_e \beta)} - 1]^{-1} \]

are respectively the distribution functions of the electrons, positrons, \( W^- \) and \( W^+ \) in our plasma.

The mass operator given by the expressions (8) is general and holds also in the high temperature and high density limits. The branch points in the denominators \( Q \) and \( Q' \) can be identified as thresholds for neutrino absorption in the plasma. We postpone a careful study of the analytic properties of the mass operator and their implications for the dispersion equation to future work.

**IV. DEGENERATE CASE**

In this section we consider the case of degenerate electrons (formally equivalent to the limit \( T \rightarrow 0 \)). Our results are of interest in the theory of neutron stars as well as in the early universe, when there might have been magnetic fields \( eB \approx M_W^2 (10^{22} \text{ gauss}) \). In the degenerate case, the distribution of the electrons is just a step function, \( n_e = \theta(\mu_e - E_e) \), and there are no positrons left, \( n_p = 0 \). Charge neutrality is ensured thanks to some \( W^+ \) background. From the behavior of the distributions of \( W^\pm \) it has been argued that \( W \)-condensation in the presence of a magnetic field may indeed take place. Thus, the distribution of \( W^+ \) can be approximated by \( (2\pi)^2 \delta(k_3) \frac{N_W}{eB} \) (\( N_W \) is the total density of \( W \)-particles in the medium), and for the excited states \( n_{W^+} = 0 \).

In this limit, equations (12) become

\[ I_1 = \frac{g^2 e Be^{-k_1^2/2eB}}{2(2\pi)^2} \int \frac{dp_3}{2k_3^2 - 2p_3k_3 - m_e^2 - \omega^2 + 2\omega E_e - d^2} \theta(\mu_e - E_e), \]
\[ I_2 = \frac{g^2 N_W e^{-k_1^2/2eB}}{2(2\pi)^2} \int \frac{dp_3}{2(2E_W)^2} \frac{\delta(p_3)}{k_3^2 + 2p_3k_3 + m_e^2 - \omega^2 - 2\omega E_W - d^2}, \]
\[ I_3 = \frac{g^2 N_W e^{-k_1^2/2eB}}{2(2\pi)^2} \int \frac{dp_3}{2k_3^2 + 2p_3k_3 + m_e^2 - \omega^2 - 2\omega E_W - d^2} \delta(p_3), \]
\[ I_4 = \frac{g^2 e Be^{-k_1^2/2eB}}{2(2\pi)^2} \int \frac{dp_3}{2(2E_e)^2} \frac{\delta(p_3)}{k_3^2 - 2p_3k_3 - m_e^2 - \omega^2 + 2\omega E_e + d^2} \theta(\mu_e - E_e), \]
\[ I_5 = \frac{g^2 N_W e^{-k_1^2/2eB}}{2(2\pi)^2} \int \frac{dp_3}{2(2E_W)^2} \frac{\delta(p_3)}{k_3^2 + 2p_3k_3 + m_e^2 - \omega^2 - 2\omega E_W - d^2} \theta(\mu_e - E_e), \]

where \( d = \sqrt{M_W^2 - eB} \).
V. DISPERSION EQUATION

Before solving the dispersion equation, note that we work far from the thresholds for neutrino absorption. In order to get the dispersion equation we must solve

$$\det(-i\gamma_\mu k_\mu + \Sigma^W) = 0,$$

which yields

$$\left(k_3^2 - \omega^2\right)\left[-\omega^2 - (\Sigma_{11} - \Sigma_{33})\omega + \Sigma_{11}\Sigma_{33} + (k_3 - \Sigma_{13})^2\right] + k_\perp^2 \left[2k_3(k_3 - \Sigma_{13}) + k_\perp^2 - 2\omega^2 - \omega(\Sigma_{11} - \Sigma_{33})\right] = 0,$$

Let us remark that the limit $eB \to 0$ does not mean that $\Sigma^W = 0$. The dispersion equation (16), when $k_3 \to 0$ $k_\perp \to 0$, leads to a value for $\omega$ different from zero; this value corresponds to a sort of “effective mass” proportional to $eB\mu_e/d^2$. This means that, for fixed electron density, if the magnetic field grows up to near $M^2_W/e$ the “effective mass” grows too. As pointed out before, the vacuum is unstable for $eB \geq M^2_W$. We shall show below that when the motion is perpendicular to the magnetic field, the “effective mass” becomes a mass in strict sense, since it is also a minimum of the dispersion curve: in formulas, $\partial\omega/\partial k_\perp|_{k_\perp=0} = 0$ and $\partial^2\omega/\partial k_\perp^2|_{k_\perp=0} > 0$.

Since we are considering the degenerate case, the contribution of terms containing the $W$-boson distribution function can be safely neglected: for huge magnetic fields, $N_W \approx C << eB$, $(C$ is the $W$-condensate), whence only $I_1$ and $I_4$ contribute in (13).

In order to solve numerically equation (16), we distinguish two cases: motion parallel to the field ($k_\perp \to 0$) and motion perpendicular to it ($k_3 \to 0$). This equation involves only $I_1$ and $I_4$:

$$\left(k_3^2 - \omega^2 + k_\perp^2\right)^2 - \left(k_3^2 - \omega^2 + k_\perp^2\right)(2k_3I_1) + (k_3^2 - \omega^2)I_4^2 = 0,$$

A. Motion parallel to magnetic field

Equation (17) for neutrino propagation along the magnetic field becomes

$$\left(k_3^2 - \omega^2\right)^2 + \left(k_3^2 - \omega^2\right)\left[2I_4^2 - 2k_3I_1\right] = 0.$$

Figure (1) shows the neutrino dispersion curves in this case, having fixed $\mu = 100 \text{ m}_e$ and $eB = 0.9M^2_W$. It has two branches. One of them corresponds to the light cone. The second one arises due to the magnetic field and the finite density. The non-zero intercept at $k_3 = 0$ is a sort of “effective mass” (for our typical values it is approximately $0.92 \text{ m}_e$). Besides, the curve has a gap ($0.68 \text{ m}_e$).

Interestingly, the curve shows a close analogy to the collective excitations arising in the fractional quantized Hall effect. In paper [18], a theory of the excitation spectrum in the fractional Hall effect analogous to Feynman’s theory for the excitation spectrum of superfluid helium was proposed. A magneto-roton minimum for the collective excitation spectrum was
found, which has a remarkable analogy with the minimum obtained in the present case, when the neutrino propagates parallel to the magnetic field.

It is possible to interpret the gap of the quasiparticle spectrum as the symptom of a superfluid behavior. A similar interpretation has been done in the case of the dispersion of neutrinos in a hot medium without magnetic field. But here we must observe that the neutrinos interacting with the $W$-s and electrons must align their spins also along the magnetic field, leading to weak coupling in pairs, and to condense.

B. Motion perpendicular to the magnetic field

When the neutrino moves perpendicularly to the field, we get the following expression for the dispersion equation

\[- \omega^2(-\omega^2 + 2I_1^2) + k_{\perp}^2(k_{\perp}^2 - 2\omega^2) = 0. \tag{19}\]

The numerical solution also yields two branches (figure 2); one of them gives the same “effective mass” as in the parallel case. This is a mass in the proper sense since it is the minimum of the dispersion equation.

In spite of the same value for the effective mass, the dispersion curves at zero momentum have different slopes for motion parallel and normal to the field. The behavior of both curves is quite different in both cases. This conclusion is to be expected since the magnetic field produces an anisotropy in the system and the motion in these two directions have different physical properties. However, the most notable result here is the behavior of the effective mass $m_{\nu_{\text{eff}}} \sim \mu_e eB/(M_W^2 - eB)$, which increases without bound as $eB \to M_W^2$. For fields $eB \geq M_W^2$, the neutrino magnetic mass problem, requires further research along with the Higgs mechanism in external fields, taking into account the results of refs. [10], [11].

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VII. APPENDIX

We present here the result of the calculation of $I_i$ far from the thresholds. Taking into account the condition

\[ k_3^2 - \omega^2 + d^2 > \mu_e(k_3 - \omega), \]

we get
\[ I_1 = \frac{g^2 r}{2(2\pi)^2} \frac{1}{k_3^2 - \omega^2 + d^2}, \]
\[ I_2 = \frac{g^2 N_w}{2(2\pi)^2} \frac{1}{d(k_3^2 - (\omega + d)^2)}, \]
\[ I_3 = \frac{g^2 N_w}{2(2\pi)^2} \frac{1}{k_3^2 - (\omega + d)^2}, \]
\[ I_4 = I_1, \]
\[ I_5 = 0. \]

(20)

where \( r = eB\mu_e \).
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Fig 1. Curve of propagation of neutrino parallel to the magnetic field

Fig 2. Curve of propagation of neutrino normal to the magnetic field