Cryptographic Enforcement of Information Flow Policies without Public Information

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Abstract

Cryptographic access control has been studied for over 30 years and is now a mature research topic. When symmetric cryptographic primitives are used, each protected resource is encrypted and only authorized users should have access to the encryption key. By treating the keys themselves as protected resources, it is possible to develop schemes in which authorized keys are derived from the keys explicitly assigned to the user’s possession and publicly available information. It has been generally assumed that each user would be assigned a single key from which all other authorized keys would be derived. Recent work has challenged this assumption by developing schemes that do not require public information, the trade-off being that a user may require more than one key. However, these new schemes, which require a chain partition of the partially ordered set on which the access control policy is based, have some disadvantages. In this paper we define the notion of a tree-based cryptographic enforcement scheme, which, like chain-based schemes, requires no public information. We establish that the strong security properties of chain-based schemes are preserved by tree-based schemes, and provide an efficient construction for deriving a tree-based enforcement scheme from a given policy that minimizes the number of keys required.

1 Introduction

Access control is a fundamental security service in modern computing systems. Informally, an access control system filters attempts by users to interact with protected resources, only allowing those interactions that are authorized by a policy, which is configured by the resource owner(s). Implementations of access control in software are vulnerable to compromise of the machine hosting the software. Moreover, such enforcement mechanisms do not work when protected resources are stored by an untrusted or semi-trusted third party, as is increasingly common.

In some situations, therefore, we may wish to use cryptographic techniques to enforce some form of access control. Such an approach is useful when data objects have the following characteristics: read often, by many users; written once, or rarely, by the owner of the data; and transmitted over unprotected networks. In such circumstances, protected data (objects) are encrypted and authorized users are given the appropriate cryptographic keys. When cryptographic enforcement is used, the problem we must address is the efficient and accurate distribution of encryption keys to authorized users.

In recent years, there has been a considerable amount of interest in key encrypting or key assignment schemes. In such schemes, a user is given a secret value – typically a single key – which enables the user to derive some collection of encryption keys which decrypt the objects for which she is authorized. Key derivation is performed using the secret value and some information made publicly available by the scheme administrator. These schemes are particularly suitable for policies that can be represented in terms of information flow.

Ideally, such a scheme should minimize the amount of public information and the time required to derive a key. Unsurprisingly, it is not possible to realize both objectives simultaneously, so trade-offs
have been sought. Crampton et al. [10] provide a survey of, and taxonomy for, key assignment schemes, and the various factors that affect the parameters described above.

In 2010, Crampton et al. [9] introduced a new type of scheme in which users may receive several keys. The significant advantage of this scheme is that no public information is required. Moreover, the simplicity of the underlying structure of the scheme makes it possible to prove the scheme possesses very strong security properties [12].

An information flow policy is defined by a partially ordered set \( X \) and a function mapping users and resources to elements in \( X \). Most key assignment schemes are derived directly from \( X \). The innovation introduced by Crampton et al. was to consider a partition of \( X \) into chains (or total orders). It is particularly easy to work with chains, but the partition breaks some of the “connectivity” of the partial ordering. These breaks are “repaired” by issuing more than one key to some users. However, one question that remains open is how best to choose the chain partition of a partially ordered set: there may be many such partitions and different choices may lead to chain partition schemes with different characteristics.

In this paper, we show that it is possible to work with trees, rather than chains, without reintroducing the need for public information. We define a tree-based, cryptographic enforcement scheme and provide a rigorous construction for such schemes from a given partially ordered set. We identify a number of different parameters that may be important in the context of a tree-based enforcement scheme. In particular, we consider the total number of keys that may be required in such a scheme and prove that a tree-based enforcement scheme with a minimal number of keys can be constructed in time \( O(|X|^2) \).

We show that a tree-based enforcement scheme for a given \( X \) will typically require fewer keys than a chain-based scheme. Moreover, we have an efficient algorithm for computing the best choice of tree from the information flow policy, in contrast to chain-based methods (which assume that a chain partition is given).

Our approach is based on constructing a weighted directed acyclic graph from \( X \) and then constructing a minimum weight spanning out-tree from the graph. In this paper, we establish a number of results about this out-tree that are likely to provide the foundation for further study of tree-based enforcement schemes.

In the next section, we introduce notation, relevant background material and related work. Then, in Sec. 3 we define a tree-based enforcement scheme, provide a method for constructing such schemes for a given information flow policy, and prove that all the resulting schemes have the property of strong key indistinguishability. In Sec. 4 we address the problem of finding a tree-based enforcement scheme that minimizes the total number of keys required to enforce a given policy, culminating in a polynomial-time algorithm for computing such a scheme. We conclude the paper with a summary of our contributions and some suggestions for future work.

2 Background and Related Work

In this paper, we consider the cryptographic enforcement of access control policies. In particular, we focus on the enforcement of information flow policies using symmetric cryptographic primitives.

2.1 Definitions and Notation

A directed graph (or digraph) \( G = (V(G), E(G)) \) is defined by a vertex set \( V(G) \) and an arc set \( E(G) \subseteq V(G) \times V(G) \). An arc in \( E(G) \) is written in the form \( xy \), where \( x, y \in V(G) \). A directed path is a sequence of arcs \( v_1v_2, v_2v_3, \ldots, v_{p-1}v_p \), which we may also write as the sequence of vertices \( v_1v_2\ldots v_p \) through which the path passes. We write \( x \leadsto_G y \) if there exists a directed path from \( x \) to \( y \) in \( G \). For all \( x \in V(G) \), we define \( x \leadsto_G x \).

The in-degree of a vertex \( v \in V(G) \) is defined to be the number of arcs of the form \( uv \) in \( E(G) \). Given an undirected rooted tree, we may orient each edge in such a way that the root has in-degree 0 and all other vertices have in-degree 1; the resulting (acyclic) digraph is called an out-tree. Thus if a directed path exists between a pair of two vertices in an out-tree then it is unique. \( H \) is a spanning subgraph of a graph \( G \) if \( V(H) = V(G) \). A spanning out-tree is a spanning subgraph that is an out-tree.

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\[ \text{There exists a large body of work on the enforcement of attribute-based policies using asymmetric cryptographic primitives, notably attribute-based encryption [6,14].} \]
A partially ordered set or poset is a pair \((X, \leq)\), where \(\leq\) is a binary, reflexive, anti-symmetric, transitive relation. Given a poset \((X, \leq)\), we write \(x < y\) if \(x \leq y\) and \(x \neq y\); and we may write \(x \geq y\) if \(y \leq x\). We write \(x \leq y\) and say \(y\) covers \(x\) if \(x < y\) and there does not exist \(z \in X\) such that \(x < z < y\). We say \(x\) is incomparable to \(y\), denoted \(x \nleq y\) if \(x \notin y\) and \(y \notin x\). We say \(Y \subseteq X\) is an antichain if for all \(x, y \in Y\), either \(x = y\) or \(x \nleq y\): \(Y\) is a maximum antichain if \(|Y| = |Z|\) for every other antichain \(Z \subseteq X\); the width of \(X\) is the cardinality of a maximum antichain.

Given a poset \((X, \leq)\), we define the graph \(H = (X, E_0)\), where \(xy \in E_0\) if and only if \(x > y\). \(H\) is called the Hasse diagram of \((X, \leq)\) and is a directed acyclic graph. A Hasse diagram of a simple poset is shown in Fig. 1 (on page 4). We may also define the graph \(H^* = (X, E_0^\ast)\), where \(xy \in E_0^\ast\) if and only if \(x > y\). The graph \(H^\ast\) is obtained by taking the transitive closure of \(H\).

An information flow policy is defined by a partially ordered set of security labels \((X, \leq)\), a set of users \(U\), a set of (protected) objects \(O\), and a security function \(\lambda : U \cup O \to X\). We say \(u \in U\) is authorized to read \(o \in O\) if \(\lambda(u) \succeq \lambda(o)\) \(^5\).

### 2.2 Basic Methods of Cryptographic Enforcement

A natural way to enforce an information flow policy is to define a cryptographic key \(\kappa(x)\) for each \(x \in X\), encrypt object \(o\) with \(\kappa(\lambda(o))\) and give \(u\) (or enable \(u\) to derive) all keys \(\kappa(x)\) such that \(x \leq \lambda(u)\). More specifically, let \(G = (X, E(G))\) be an acyclic directed graph such that \(E_0 \subseteq E(G) \subseteq E_0^\ast\). Thus the transitive closure of \(G\) is equal to \(H^\ast\) and \(x \rightharpoonup H\ y\) if and only if \(x \rightharpoonup G\ y\). By publishing key derivation information for each arc in \(E(G)\), it is possible to derive \(\kappa(y)\) from \(\kappa(x)\) if \(x \rightharpoonup G\ y\). Thus, the total amount of key derivation information required is proportional to \(|E(G)|\), while the number of key derivations will depend on the lengths of the directed paths in \(G\). We provide a more formal account of the functionality required of a cryptographic enforcement scheme in Sec. 2.4.

Typically, key derivation information is generated using an appropriate symmetric cryptographic algorithm \(^4\): for arc \(xy \in E(G)\), the inputs to the cryptographic algorithm will include \(\kappa(x)\) and \(\kappa(y)\). We write \(Enc(m, \kappa)\) to denote the encryption of message \(m\) with key \(\kappa\). There are three very well known ways to implement cryptographic enforcement of information flow policies \(^10\):

- **Basic** – give \(u\) the set of keys \(\{\kappa(x) : x \leq \lambda(u)\}\);
- **Iterative** – give \(u\) a single key \(\kappa(\lambda(u))\) and publish \(\{Enc(\kappa(x), \kappa(y)) : x \leq y\}\);
- **Direct** – give \(u\) a single key \(\kappa(\lambda(u))\) and publish \(\{Enc(\kappa(x), \kappa(y)) : x \leq y\}\).

We may evaluate different implementations by considering a number of parameters. Let \(k(x)\) be the number of keys required by a user associated with \(x\). Then we write \(k\) to denote the maximum value of \(k(x)\) taken over all \(x\) and \(K\) to denote \(\sum_{x \in X} k(x)\). We write \(p\) to denote the number of items of public information, \(^4\) and \(d\) to denote the number of key derivation operations a user may be required to perform to derive a key. Let \(n\) denote the cardinality of \(X\). Then the characteristics of the three schemes described above are summarized in Table 1.

| Scheme   | Keys for \(u\) | Public Information | \(K\) | \(k\) | \(p\) | \(d\) |
|----------|----------------|---------------------|-------|-------|------|------|
| Basic    | \{\kappa(x) : x \leq \lambda(u)\} | –                   | \(n + |E_0^\ast|\) | \(O(n)\) | 0    | 0    |
| Iterative| \(\kappa(\lambda(u))\) | \{Enc(\kappa(x), \kappa(y)) : x \leq y\} | \(n\) | \(1\) | \(|E_0|\) | \(O(n)\) |
| Direct   | \(\kappa(\lambda(u))\) | \{Enc(\kappa(x), \kappa(y)) : x \leq y\} | \(n\) | \(1\) | \(|E_0^\ast|\) | 1    |

Table 1: How the parameters of various key assignment schemes vary

Naturally, there is a trade-off between the amount of public information we need to compute and store centrally, and the number of key derivation operations that are required. The direct scheme, for example, minimizes the cost of key derivation at the expense of an increase in public information. Consider the example in Fig. 1; the Hasse diagram of the poset has 8 vertices and 10 arcs, and the width of the poset is 2; the graph of the transitive closure has 23 arcs.

\(^2\)It is assumed that the structure of the poset \((X, \leq)\) is known to all participants of a cryptographic enforcement scheme.
More complex schemes have been devised to reduce the number of derivation operations by increasing $|E(G)|$ \cite{2,8,11}. In particular, Atallah \textit{et al.} introduced a scheme for policies where $X$ is a total order, in which the number of derivation operations was no greater than 2 and $|E(G)| = O(|X| \log |X|)$ \cite{2}. Crampton extended these ideas to arbitrary interval-based access control policies \cite{8}.

### 2.3 Chain Partition Techniques

We may consider other ways of enforcing an information flow policy. Crampton \textit{et al.} observed that one possibility is to decompose a partially ordered set $(X, \leq)$ into disjoint chains and then use one-way functions to derive keys \cite{9}. In this case, the arc set $E(G) \subseteq E_0$ and the transitive closure of $G$ (the graph representing the chain partition) is not necessarily equal to $H^*$ (as illustrated in Fig. 1b, in which deleted arcs are shown as gray dashed lines).

The advantage of such a scheme is that no public information is required. We simply select a key for the top element in each chain and then use a (public) one-way function $F$ to iteratively compute the keys for the remaining elements in each chain. (In particular, if $x \prec y$ in a chain, then $\kappa(x) = F(\kappa(y))$.) Thus a user can simply derive keys by repeated applications of the one-way function. The trade-off in this case is that the user may need as many as $w$ keys, one for each of $w$ chains. In Fig. 1b, for example, a user assigned to vertex $d$ will require $\kappa(d)$ and $\kappa(c)$. In short, it may be advantageous to eliminate public information, in which case each user may require multiple keys to support key derivation.

### 2.4 Formalization and Constructions

Recent work has formalized the security properties required of a cryptographic enforcement scheme (CES) for information flow policies \cite{1,3,12}. Atallah \textit{et al.} introduced the concepts of key recovery and key indistinguishability \cite{1}. The former, informally, is the requirement that a coalition of users $V \subseteq U$ (the “adversary”) can derive $\kappa(x)$ only if there exists $v \in V$ such that $\lambda(v) \geq x$. In other words, compromising users cannot lead to non-derivable keys being compromised. This is, essentially, the weakest security requirement that one might require of a CES. The schemes described in Sec. 2.2 have this property (provided the encryption scheme has reasonable properties).

However, in the interests of integrating a CES with other cryptographic tools, the stronger notion of indistinguishability was introduced. This property requires that the adversary cannot distinguish between $\kappa(x)$ and a random string (of the same length). The schemes discussed in Sec. 2.2 do not have this property (see \cite{1}, for example).

Informally, treating encryption keys as “just another encrypted data object” cannot be the basis for a robust cryptographic enforcement scheme. Specifically, the derivation of keys has to be separated from the decryption of data objects. We achieve this by introducing a secret value $\sigma(x)$ for each $x \in X$ from which $\kappa(x)$ may be derived. More formally, a CES for $(X, \preceq)$ comprises the $\text{SetUp}$ and $\text{Derive}$ algorithms, the first being used to generate keys and the data used to derive keys, and the second to derive keys. Let $K$ denote an arbitrary key space (typically $K = \{0, 1\}^l$ for some $l \in \mathbb{N}$).
• **SetUp** takes as input a security parameter $\rho$ and a poset $(X, \preceq)$ associated with an information flow policy. It outputs, for each element $x \in X$, a pair $(\sigma(x), \kappa(x))$: $\sigma(x)$ is used to derive keys $\kappa(y) \in \mathcal{K}$, where $y \preceq x$; and $\kappa(x)$ is used to encrypt data objects associated with security label $x$. We write $\sigma$ and $\kappa$ to denote, respectively, vectors that list the values $\sigma(x)$ and $\kappa(x)$ for all $x \in X$. The SetUp algorithm also outputs a set of public information Pub, which is used to support key derivation.\(^3\)

• **Derive** takes as input $(X, \preceq)$, Pub, start and end points $x, y \in X$ and $\sigma(x)$.

  It outputs $\kappa(y) \in \mathcal{K}$ if and only if $y \preceq x$. (In particular, $\kappa(x)$ can be derived from $\sigma(x)$.)

Atallah et al. described a CES in which two keys $\tau(x)$ and $\kappa(x)$ are derived from $\sigma(x)$ using a pseudorandom function and $(\tau(y), \kappa(y))$ is directly derivable from $\tau(x)$ only if $y \prec x$. (Thus, $\kappa(y)$ is iteratively derivable from $\sigma(x)$ if $x \sim y$.) The main innovation here is to separate the derivation and encryption functions of $\kappa(x)$, meaning that knowledge of the object decryption key $\kappa(x)$ does not help in deriving $\kappa(y)$. (Of course, exposure of $\tau(y)$ will allow for the derivation of $\tau(y)$ and hence $\kappa(y)$.)

Freire et al. introduce a security property called strong key indistinguishability \(^2\). The adversary selects a vertex $x$ to attack and may then learn $\{\sigma(y) : y \not\preceq x\}$ (as in the security model for key indistinguishability) and $\{\kappa(y) : y \neq x\}$; the adversary’s task is to distinguish $\kappa(x)$ from random. They then define a CES for total orders that has the property of strong key indistinguishability, in which a key $\kappa(x)$ is derived from $\sigma(x)$ using a pseudorandom function and $\sigma(y)$ is directly derivable from $\sigma(x)$ only if $y \preceq x$. Finally, they demonstrate how this CES can be extended to arbitrary posets using the chain partition construction described in Sec. 2.3.

3 Tree-Based Enforcement Schemes

In this work, we are interested in enforcing an information flow policy, defined in terms of the Hasse diagram of a partially ordered set $(X, \preceq)$, using cryptographic primitives. We may enforce the policy in any way we see fit. We may, for example, increase the number of arcs (by including some subset of the transitive arcs), thereby decreasing the lengths of the directed paths in the graph and the number of key derivations that are required. Thus there is a trade-off between (increasing) the number of arcs and (decreasing) the amount of storage required for public information. In particular, we could include all transitive arcs, so that all paths are of length 1 (as in the direct scheme). Alternatively, we may increase the number of keys given to each user and reduce the derivation time (keeping the number of arcs constant). This corresponds to allowing the user to start from multiple points in the graph.

In practice, there may be constraints that will dictate what kind of cryptographic enforcement schemes will be appropriate. There may be constraints, for example, on the computational power and/or storage of the end-user devices; or it may not be possible to provide an on-line server to store public information. As noted in Table 1\(^\!*\) there are four parameters that are likely to be of interest: $k$, $K$, $p$, and $d$. We may wish to minimize or impose an upper bound on one or more of these parameters. Certain choices have been well studied, particularly those for which $k = 1$ (when each user is given exactly one key and $E_0 \subseteq E(G) \subseteq E_0^*\)$. In the upper part of Table 2\(^4\) we summarize some of the requirements that have been studied in the literature.

The lower part of the table considers the possibility of eliminating public information, at the expense of an increase in the number of keys assigned to each vertex. It is these types of schemes that we consider in the remainder of this paper. In particular, we consider the problem of minimizing $K$.

In the special case that the Hasse diagram $H = (X, E_0)$ is a spanning out-tree, we may use simpler cryptographic primitives to enforce an information flow policy. Specifically, we know there is a unique directed path from $x$ to $y$ whenever $y < x$. Hence, for all $x, y \in X$ such that $y < x$, we define $\kappa(y)$ to be $F(\kappa(x) \parallel y)$, where $F$ is an appropriate one-way function \(^5\) and $\parallel$ denotes string concatenation.\(^6\) In other words, keys are determined by the vertices, rather than the arcs, through which a directed path passes. In this case, we require no public information (apart from a description of the poset), because keys are derived only from a (secret) key and a (public) vertex label.

\(^3\)In some schemes, it may be the case that $\kappa(y) = \sigma(y)$ for all $y \in X$; and in some schemes, it may be that the set of public information is empty.

\(^4\)This method is not appropriate for arbitrary posets because we may have $y < x$ and $y \preceq z$ \(^7\).

\(^5\)This method is not appropriate for arbitrary posets because we may have $y < x$ and $y \preceq z$ \(^7\).
### Table 2: Requirements and Enforcement Schemes

| Requirements Scheme Parameter Values | Parameter Values |
|-------------------------------------|------------------|
| Requirement | Scheme | \( K \) | \( k \) | \( p \) | \( d \) | \( \min \) | \( n \) | \( |E_0^*| \) | \( O(n) \) |
| Iterative | 1 | min | 1 | 1 | \( |E_0^*| \) | 1 | \( O(n) \) |
| Direct | 1 | 1 | 0 | Basic | \( n + |E_0^*| \) | 0 | 0 | 0 | 0 | Tree-based | 0 | 0 | 0 | 0 | 0 |

In general, of course, \( H \) is not an out-tree. We may assume without loss of generality, however, that our poset has a maximum element. If \((X, \preccurlyeq)\) has more than one maximal element then we add a new element to \( X \) which is defined to be greater than all elements in \( X \). (In this case, no user or object would be assigned to such an element.) Thus, we may assume that \( H^* \) has only one vertex of in-degree zero and so has a spanning out-tree \[4\] Prop. 1.7.1.

#### 3.1 Constructing an Enforcement Scheme

In this paper, then, we investigate ways of constructing a spanning out-tree from \( H^* = (G, E_0^*) \) (in order to eliminate the need for public information) by selecting an arc set that is a subset of \( E_0^* \). However, we have to “repair” the Hasse diagram by allocating some users more than one key (because some of the paths will have been broken by the deletion of arcs). Thus it is interesting to consider how to select the arcs for deletion in such a way that the increase in the number of keys is minimized (either on a per-vertex basis or in total).

Figure 2 illustrates three out-trees derived from the poset in Fig. 1. Removing arcs to create an out-tree inevitably means that certain paths are broken. The out-tree in Fig. 2a, for example, means that a user associated with vertex \( h \) only requires a single key and derivation requires no more than one hop. However, every other vertex (except \( a \)) requires additional keys in order to bridge the gaps. The above observations motivate the following definition.

**Definition 1.** Given an information flow policy \((X, \preccurlyeq)\), \( E(T) \subseteq X \times X \) defines a derivation out-tree \( T = (X, E(T)) \) if

- \( T \) is a spanning out-tree;
- \( xy \in E(T) \) implies \( y < x \).

Thus if \( x \not\preccurlyeq y \), then \( x \not\Rightarrow_T y \). Equivalently, \( E(T) \subseteq E_0^* \) and so \( E(T) \) cannot be used to derive a key that is unauthorized.
Lemma 1. Let $D = (V, E)$ be an acyclic digraph with only one vertex $r$ of in-degree zero. Then by selecting one in-bound arc for each vertex $x \neq r$ we obtain a spanning out-tree of $D$. Furthermore, any spanning out-tree of $D$ can be constructed in this way.

Proof. First, let us prove that $T$ is a spanning out-tree. Clearly, $T$ has no directed cycle and every vertex of $x \neq r$ has in-degree $1$. It remains to show that $T$ is connected and contains $r$. Consider a vertex $y_1 \neq r$ and a longest directed path of $T$ terminating at $y_1$: $P = y_1 y_2 \cdots y_t$. Since $T$ has no directed cycle all vertices of $P$ are distinct and since $P$ is longest, $y_t = r$. Thus, every vertex of $T$ is reachable from $r$ showing that $T$ is connected and contains $r$.

Now let $T$ be a spanning out-tree. Note that for every vertex $x \neq r$ there is exactly one arc to $x$. Thus, $T$ can be constructed by the procedure of the lemma.

Definition 2. Given an information flow policy $(X, \preceq)$, a tree-based enforcement scheme is a pair $(T, \phi)$, where $T$ is a derivation out-tree and $\phi : X \to 2^X$ is a key allocation function such that:

- $x \in \phi(x)$;
- if $y \preceq x$, then there exists a unique $z \in \phi(x)$ such that $z \rightsquigarrow_T y$;
- if $y \nless x$, then, for all $z \in \phi(x)$, $z \not\rightsquigarrow_T y$.

In a tree-based enforcement scheme $(T, \phi)$, directed paths in $T$ are used to derive keys: $E(T)$ determines the paths and $\phi$ determines the starting points of those paths (and hence the set of keys that should be given to each user). In particular, $\phi(x) \setminus \{x\}$ is a set of vertices that were reachable from $x$ in $H^*$ that are no longer reachable in $T$. Given an enforcement scheme $(T, \phi)$, for each user assigned to vertex $x$, we give the user $\kappa(z)$ for each $z \in \phi(x)$. Hence

$$|\phi| \overset{\text{def}}{=} \sum_{x \in X} |\phi(x)|$$

determines how many keys in total are required for $T$ to preserve the reachability defined by the directed paths in $H^*$. Note the requirement that there should be a unique $z \rightsquigarrow_T y$ (for some $z \in \phi(x)$) in the derivation out-tree whenever $y \prec x$. Note also that there exists an efficient algorithm to compute a spanning out-tree for $H^* = (X, E_0)$.

We now explain how to generate a tree-based enforcement scheme, given an arbitrary spanning out-tree $T$ of $H^*$. We first introduce some notation and prove a useful result. Given a poset $(X, \preceq)$ and arc $yz \in E_0^*$, define

$$\gamma(yz) \overset{\text{def}}{=} \{ x \in X : x \geq z, x \nless y \}.$$

Lemma 2. For any tree-based enforcement scheme $(T = (X, E), \phi)$, every non-root vertex $x \in X$ and all $yz \in E$,

$$z \in \phi(x) \text{ if and only if } x \in \gamma(yz).$$

Proof. Suppose $z \in \phi(x)$. If $z \nless x$ then, for each $t \in \phi(x)$, there is no path from $t$ to $z$ in $T$, implying that $z \nsubseteq \phi(x)$. Thus, by contradiction, $z \subseteq x$. Moreover, $x \nless y$ (otherwise there would exist $t \in \phi(x)$ such that $t \rightsquigarrow_T y$ and hence $t \rightsquigarrow_T z$ with $t \neq z$, violating the uniqueness condition). That is, $x \in \gamma(yz)$.

Now suppose $x \in \gamma(yz)$. Then $x \nless y$, by definition, and there does not exist $t \in \phi(x)$ such that $t \rightsquigarrow_T y$. However, $x \geq z$; hence, there exists $t \in \phi(x)$ such that $t \rightsquigarrow_T z$. Since $t \not\rightsquigarrow_T y$ for any $t \in \phi(x)$ and $yz$ is the only arc into $z$, it must be the case that $z \in \phi(x)$. \hfill \Box

Corollary 1. Given $E \subseteq E_0^*$ such that $T = (X, E)$ is a derivation out-tree, we may compute the tree allocation function $\phi$ associated with $E$ in time $O(|X|^2)$.

Proof. By Lemma 2, we can compute $\phi(x)$ for each $x \in X$ by computing $\gamma(yz)$ for each $yz \in E$. Thus, an algorithm to compute $\phi$ comprises an outer loop which iterates through the elements of $X$ and an inner loop that iterates through the elements of $E$. We can compute $\gamma(yz)$ by testing whether $x \geq z$ and $x \nless y$; we can compute the adjacency matrix of $H^*$ in time $O(|X|^2)$ and can test whether $x \geq z$ (and $x \nless y$) in constant time. Moreover, $|E| = |X| - 1$ (since every vertex except the root has in-degree 1). Thus our algorithm runs in time $O(|X|^2)$.
3.2 Generating Keys

We now describe how to instantiate a tree-based enforcement scheme using a pseudorandom function. The scheme is a natural extension of the one used by Freire et al. for total orders \[^{12}\] Let \( \rho \) be a security parameter and \( F : \{0,1\}^\rho \times \{0,1\}^* \rightarrow \{0,1\}^\rho \) be a pseudorandom function.

**SetUp:** The inputs to the algorithm are \( \rho \) and a poset \( (X, \leq) \). Consider an arbitrary tree-based enforcement scheme \( (T = (X, E), \phi) \) for \( (X, \leq) \).

Let \( r \) be the root vertex of \( T \). Select secret value \( s(r) \) uniformly at random from \( \{0,1\}^\rho \). Set

\[
\kappa(r) \overset{\text{def}}{=} F(s(r), 0 \parallel r)
\]

and, recursively, if \( v_i \prec v \) is the \( i \th \) child of vertex \( v \),

\[
s(v_i) \overset{\text{def}}{=} F(s(v), 1 \parallel v_i)
\]

\[
\kappa(v_i) \overset{\text{def}}{=} F(s(v_i), 0 \parallel v_i)
\]

Thus, for \( y \prec x \), \( s(y) \) is derived from \( s(x) \) and the label of \( y \), while \( \kappa(y) \) is derived from \( s(y) \) and the label of \( y \).

Finally, define \( \sigma(x) = \{s(y) : y \in \phi(x)\} \).

**Derive:** Given \( y, x \) and \( \sigma(x) \), with \( y \leq x \), there (uniquely) exists \( z \in \phi(x) \) such that \( z \sim_T y \).

If \( z = y \), then (since \( s(z) \in \sigma(x) \)), compute \( \kappa(z) = F(s(z), 0 \parallel z) \). If \( z \neq y \), then for each intermediate vertex \( t_i \) on the path \( t_1 \ldots t_m \) between \( t_1 = z \) and \( t_m = y \), compute \( s(t_i) = F(s(t_{i-1}), 1 \parallel t_i) \).

Finally, compute \( \kappa(y) = F(s(y), 0 \parallel y) \).

The principle behind our scheme is also illustrated in Fig. 3.

3.3 Security Analysis

We start by specifying what we understand by a PRF. Our definition is not the most general possible and is tailored to the requirements of our construction (as described in Sec. 3.2); specifically, we assume that the keyspace and range of the PRF are the same set.

**Definition 3.** A pseudorandom function \( (F_{\rho})_{\rho \in \mathbb{N}} \) is a family of efficient functions \( F_{\rho} : \mathcal{K} \times \{0,1\}^* \rightarrow \mathcal{K} \), where we understand \( \rho \) as a security parameter and \( \mathcal{K} = \{0,1\}^\rho \) as the keyspace.

We will usually write \( F_{\rho,K}(x) \) to denote \( F_{\rho}(K, x) \) for any \( K \in \mathcal{K} \). To further simplify the notation, we will omit \( \rho \) when no confusion can arise. We write \( \mathcal{D} \Rightarrow 1 \) to denote a configuration where \( \mathcal{D} \) is a probabilistic poly-time Turing machine that has oracle access to a function \( \mathcal{O} \) and outputs a bit with value 1.

**Definition 4.** Given a pseudorandom function \( F \), we define the advantage of a distinguisher \( \mathcal{D} \) to be

\[
\text{Adv}_{\mathcal{D}}(F) = \left| \Pr[K \leftarrow_R \mathcal{K} ; \mathcal{D}^{F^\mathcal{K}}() \Rightarrow 1] - \Pr[\phi \leftarrow_R \langle \{0,1\}^* \rightarrow \mathcal{K} \rangle ; \mathcal{D}^{\phi}() \Rightarrow 1] \right|
\]

where \( \langle \{0,1\}^* \rightarrow \mathcal{K} \rangle \) denotes the universe of all functions mapping \( \{0,1\}^* \) to \( \mathcal{K} \). We say \( F \) is indistinguishable from a random function if the advantage of any efficient distinguisher \( \mathcal{D} \) is negligible.

We next make precise the level of security that we target. We refer to \[^{11,12}\] for recent discussions and comparisons of security models that are specific enough to allow the analysis of CESs using the formalisms of provable security. We reproduce here the strongest model from \[^{12}\], that is, the one formalising the highest level of security, which is based on the security experiment \( \text{Exp}^{\text{kat,6}}_{X,x,A}(1^\rho) \) defined in Fig. 4.

[^1]: In the special case of a total order, we obtain the scheme of Freire et al., modulo some differences in the choice of the second input to the pseudorandom function.
Definition 5. Let $(X, \leq)$ be an arbitrary poset. A CES for $(X, \leq)$ is strongly key indistinguishable with respect to static adversaries if, for all $x \in X$, the advantage of all efficient adversaries $A$ that interact in experiment $\text{Expt}^{\text{kist}}_{X,x,A}$ is negligible, where we define

$$\text{Adv}^{\text{kist}}_{X,x,A}(\rho) = \left| \text{Pr} \left[ \text{Expt}^{\text{kist},1}_{X,x,A}(1^{|x|}) \Rightarrow 1 \right] - \text{Pr} \left[ \text{Expt}^{\text{kist},0}_{X,x,A}(1^{|x|}) \Rightarrow 1 \right] \right|$$

and set $\text{Corrupt}_{X,x} = \{\sigma(v) : v \in X, x \not\leq v\}$ and $\text{Keys}_{X,x} = \{\kappa(v) : v \in X \setminus \{x\}\}$.

Observe that in this definition, and in contrast to other models discussed in [1, 12], the adversary obtains, in principle, all secrets embedded in the system (that is, all $\sigma(x)$ and $\kappa(x)$ values), excluding only those that would allow distinguishing the target key by trivial means (e.g., by invoking the Derive algorithm)\(^6\).

The final step of our analysis is to prove that our tree-based enforcement scheme from Sec. 3.2 is strongly key indistinguishable. Observe that this implies that our scheme is secure in all the models considered in [12]. More formally, we have the following result.

Theorem 1. Our tree-based enforcement scheme is strongly key indistinguishable in the sense of Definition 5. More precisely, for any poset $(X, \leq), x \in X$, and efficient adversary $A$, there exists a constant $0 \leq c \leq |X|$ and efficient distinguishers $D_0, \ldots, D_c, D_1, \ldots, D_1$ against the underlying PRF such that

$$\text{Adv}^{\text{kist}}_{X,x,A} \leq \text{Adv}^{F}_{D_0} + \cdots + \text{Adv}^{F}_{D_c} + \text{Adv}^{F}_{D_1} + \cdots + \text{Adv}^{F}_{D_1}.$$
Figure 4: Security experiment for strong key indistinguishability

Proof. The argument proceeds using sequences of \(|X| = n\) hybrid games that interpolate between experiments \(\Exp^{\text{kat,0}}_{X,x,a}(1^p)\) and \(\Exp^{\text{kat,1}}_{X,x,a}(1^p)\). In each hybrid step, if specific conditions are met, we replace one PRF instance by a random function; from the point of view of the adversary, the distance between each two consecutive hybrids is not greater than \(\Adv^F\), for an appropriate PRF distinguisher.

Fix a poset \((X, \leq)\), a derivation out-tree \(T = (X, E(T))\) for \(X\), a label \(x \in X\), and an efficient adversary \(A\). Without loss of generality, let \(x_0 \prec x_{n-1} \prec \cdots \prec x_2 \prec x_1 = r\) be a linear extension of \(X\). We consider the elements of \(X\) in reverse order (determined by \(\prec\)), and, for \(b \in \{0, 1\}\), we set \(G^b_0 = \Exp^{\text{kat,b}}_{X,x,a}\) and define games \(G^b_1, \ldots, G^b_n\) such that, if \(x \not\subset x_b\) then \(G^b_k\) and \(G^{b-1}_k\) are identical, and if \(x \leq x_b\) then the difference between games \(G^b_k\) and \(G^{b-1}_k\) is precisely that all PRF invocations with key \(\sigma(x_b)\) are replaced by assignments with values in \(K\) drawn uniformly at random. Let \(S^b_k\) denote \(\Pr[G^b_k \Rightarrow 1]\) for all \(b, k\).

Observe that we replace PRF invocations by random assignments for precisely those labels \(x\) that do not have a corresponding entry in \(\Corrupt_{X,x}\). Observe also that, as we consider labels \(x \in X\) in reverse order (determined by \(\prec\)), for all switchings from a PRF to a random function we have that the corresponding PRF key \(\sigma(x)\) was replaced with a uniform random value before. Hence, by a standard reductionist argument, in the cases \(x \leq x_k\) we have

\[
|S^b_k - S^b_{k-1}| = |\Pr[G^b_k \Rightarrow 1] - \Pr[G^b_{k-1} \Rightarrow 1]| \leq \Adv^F_D,
\]

for a specific (efficient) distinguisher \(D\); in addition, whenever \(x \not\subset x_k\) we have \(G^b_k = G^{b-1}_k\) and hence \(|S^b_k - S^b_{k-1}| = 0\). Now, by repeated application of the triangle inequality and (4), we have

\[
|S^b_0 - S^b_n| \leq \sum_{i=1}^{n} |S^b_{i-1} - S^b_i| \leq \sum_{i=1}^{c} \Adv^F_{D^i},
\]

where \(c = \{|x' \in X : x \leq x'|\}\) and distinguishers \(D^i\) are constructed as specified. We now consider games \(G^0_n\) and \(G^n_0\). In both cases \(\kappa(x)\) is picked uniformly at random, thus lines 2 and 3 in the experiment implement the same operation. Hence \(G^0_n\) is identical to \(G^n_0\) and \(|S^0_n - S^n_0| = 0\). Thus, we obtain

\[
\Adv^{\text{kat}}_{X,x,a} = |S^0_0 - S^0_n| \leq |S^0_0 - S^1_n| + |S^1_n - S^0_n| + |S^0_n - S^0_1| \leq \Adv^F_{D^1} + \ldots + \Adv^F_{D^c} + 0 + \Adv^F_{D^c} + \ldots + \Adv^F_{D^1},
\]

as required.

4 Minimizing \(K\) in a Tree-based Enforcement Scheme

So far, we have shown that it is possible to construct a tree-based enforcement scheme for an information flow policy \((X, \leq)\) that is strongly key indistinguishable. As we observed before, we will usually require our tree-based enforcement scheme to have some particular properties, such as minimizing the total number of keys or ensuring that all derivation paths are no longer than some threshold value. Hence, we

\footnote{That is, if \(x \not\subset y\) (in \(X\)) then \(x \not\subset y\) (in the linear extension). Every (finite) partial order has at least one linear extension, which may be computed, in linear time, by representing the partial order as a directed acyclic graph and using a topological sort [2] [22.3].}
require an algorithm to compute a derivation out-tree that satisfies the desired requirements, since, by Corollary 1 we can then compute the associated key allocation function in polynomial time.

In this section, we consider two questions: how to minimize $K$, the total number of keys allocated to vertices (by the key allocation function $\phi$); and how to minimize $\hat{K}$, the total number of keys distributed to users. The second question is interesting because, in practice, we might want to reduce the exposure of keys by ensuring that very few keys are associated with vertices to which many users are assigned. We solve both questions, demonstrating that it is surprisingly efficient to compute the required tree-based enforcement schemes in polynomial time. This is possible because of a particular connection between $\phi$ and $\gamma$, which we state and prove in Theorem 2. We then state and prove Theorem 3, the main result of this section.

Our basic approach is to define a weight for each arc in $E_0^*$ and construct a minimum weight spanning out-tree. Accordingly, given an information flow policy $((X, \preceq), \lambda, U, O)$, where $\lambda : U \cup O \to X$, let $U(x) = \{ u \in U : \lambda(u) = x \}$, and let $H = (X, E_0)$ be the Hasse diagram of $X$. Then we define the weight function $\omega : E_0^* \to \mathbb{N}$, where

$$\omega(yz) \overset{\text{def}}{=} \sum_{x \in \gamma(yz)} |U(x)|.$$

**Theorem 2.** Let $(T = (X, E), \phi)$ be any tree-based enforcement scheme for $(X, \preceq)$. Then

$$\sum_{x \in X \atop x \neq r} |U(x)| \cdot |\phi(x)| = \sum_{e \in E} \omega(e).$$

**Proof.** By Lemma 2 we have, for every $x \neq r$,

$$|\phi(x)| = |\{ yz \in E : x \in \gamma(yz) \}|$$

and so

$$|U(x)| \cdot |\phi(x)| = |U(x)| \cdot |\{ yz \in E : x \in \gamma(yz) \}|$$

Hence

$$\sum_{x \in X \atop x \neq r} |U(x)| \cdot |\phi(x)| = \sum_{x \in X \atop x \neq r} |U(x)| \cdot |yz \in E : x \in \gamma(yz)|$$

and, since $r \notin \gamma(yz)$ for any $yz \in E$, we have

$$\sum_{x \in X \atop x \neq r} |U(x)| \cdot |\phi(x)| = \sum_{yz \in E} \sum_{x \in \gamma(yz)} |U(x)| = \sum_{yz \in E} \omega(yz).$$

**Theorem 3.** Given an information flow policy $((X, \preceq), U, O, \lambda)$, we can compute a tree-based enforcement scheme $(T, \phi)$ such that $\hat{K}$ is minimized in time $O(|E_0^*| + |X|^2)$.

**Proof.** For brevity, we write $E$ for $E(T)$. By Theorem 2

$$\hat{K} = |U(r)| + \sum_{e \in E} \omega(e).$$

Since $|U(r)|$ is fixed, we minimize $\hat{K}$ by computing a derivation out-tree that minimizes $\sum_{e \in E} \omega(e)$. By Lemma 1 we can achieve this by selecting, for each non-root vertex $x \in X$, the minimum weight arc to $x$, where the weights are given by $\omega$. We need only consider each arc (in $E_0^*$) once, which takes time $O(|E_0^*|)$. The resulting set of arcs forms a spanning out-tree of minimum weight and the number of additional keys required is $\sum_{e \in E} \omega(e)$. We can derive the associated key allocation function in time $O(|X|^2)$, by Corollary 1 the result follows.
Corollary 2. Given an information flow policy \(((X, \preceq), U, O, \lambda)\), we can compute a tree-based enforcement scheme such that \(K\) is minimized in time \(O(|E_0^*| + |X|^2)\).

Proof. We simply set \(|U(x)| = 1\) and apply Theorems 2 and 3.

Corollary 3. We can find, in time \(O(|E_0^*| + |X|^{3/2} |E_0^*|^{1/2})\), a minimum weight spanning out-tree that has the minimum number of leaves among such trees.

Proof. Replace \(H^*\) by its subgraph \(D = (X, E)\) obtained as follows: for each vertex \(x \neq r\) delete all arcs to \(x\) apart from those of minimum weight (among arcs to \(x\)). Observe that \(D\) can be constructed in time \(O(|E_0^*|)\). Find an out-tree with minimum number of leaves using algorithm \textsc{MinLeaf} [14]. It remains to observe that \textsc{MinLeaf}'s runtime is \(O(|E| + |X|^{3/2} |E|^{1/2})\).

It is useful to find a minimum weight spanning out-tree with a minimum number of leaves because the number of leaves will impose an upper bound on \(|\phi(x)|\). Note, however, that \(|\phi(x)|\) may be greater than the width of \(X\) (and it is not difficult to construct such an example). This is because the set of arcs in the graph that is input to \textsc{MinLeaf} will, in general, be a strict subset of \(E_0^*\). Thus, the size of the maximal independent set in the graph that is input to \textsc{MinLeaf} can exceed the width of the poset (which is the equal to the size of the maximal independent set in \(G = (X, E_0^*)\)).

We now prove some further properties of \(\gamma\). This enables us to reduce the running time of our algorithm because we show it is sufficient to consider only arcs in \(E_0\) (rather than \(E_0^*\)) when constructing the minimum weight spanning out-tree.

Lemma 3. Let \((X, \preceq)\) be a partially ordered set. Then for all \(x, y, z \in X\) such that \(z < y < x\),

\[
\gamma(xy) \cap \gamma(yz) = \emptyset \quad \text{and} \quad \gamma(xz) \supseteq \gamma(yz) \cup \gamma(xy)
\]

Proof. Suppose \(t \in \gamma(xy) \cap \gamma(yz)\). Since \(t \in \gamma(yz)\), we have \(t \geq z\) and \(t \neq y\); since \(t \in \gamma(xy)\), we have \(t \geq y\), immediately leading to the desired contradiction.

Now suppose \(t \in \gamma(xy)\). Then \(t \geq y\) and \(t \neq x\). Hence, we have \(t > z\), by transitivity: thus \(t \in \gamma(xz)\) and \(\gamma(xy) \subseteq \gamma(xz)\). Finally, suppose \(t \in \gamma(yz)\). Then \(t \geq z\) and \(t \neq y\). Now \(t \neq x\) (otherwise, we would have \(t > y\) by transitivity) and hence \(t \in \gamma(xz)\); thus \(\gamma(yz) \subseteq \gamma(xz)\).

Corollary 4. Let \((X, \preceq)\) be a partially ordered set with Hasse diagram \(H = (X, E_0)\). Then, for any path \(x_1x_2 \ldots x_p\) in \(H^*\), \(p > 2\), we have

\[
\omega(x_1x_p) \geq \sum_{i=1}^{p-1} \omega(x_ix_{i+1}).
\]

Proof. Consider the case \(p = 3\), with \(x > y > z\). Using Lemma 2 and the fact that \(|U(t)| \geq 0\) for all \(t\), we have

\[
\omega(xz) = \sum_{t \in \gamma(xz)} |U(t)|
\geq \sum_{t \in \gamma(xy)} |U(t)| + \sum_{t \in \gamma(yz)} |U(t)|
= \omega(xy) + \omega(yz).
\]

Now suppose the result holds for all \(p < P\) and consider a path \(x_1 \ldots x_P\) containing \(P\) vertices. Then \(x_1x_{P-1} \in E_0^*\) and, by Lemma 2 and the inductive hypothesis, respectively, we have

\[
\omega(x_1x_P) \geq \omega(x_1x_{P-1}) + \omega(x_{P-1}x_P)
\geq \omega(x_1x_2) + \cdots + \omega(x_{P-2}x_{P-1}) + \omega(x_{P-1}x_P)
= \sum_{i=1}^{P-1} \omega(x_i x_{i+1})
\]

Thus the result holds by induction.
Corollary 5. Let \((X, \preceq)\) be a partially ordered set with Hasse diagram \(H = (X, E_0)\). Then there exists a minimum weight spanning out-tree \(T = (X, E)\) with \(E \subseteq E_0\).

Proof. Let \(T' = (X, E')\) be a minimum weight spanning out-tree for \((X, \preceq)\), and suppose arc \(xy\) is in \(E'\) but not in \(E_0\). Then \(x \rightarrow y\) and let \(zy\) be the last arc in this path. Since \(\omega(uv) \geq 0\) for each arc \(uv\) and by Corollary 4, \(\omega(zy) \leq \omega(xy)\). Therefore by removing \(xy\) from \(E'\) and adding \(zy\), we have a spanning out-tree with weight at most that of \(T'\). By replacing every arc in \(E'\setminus E_0\) in this way, we have a spanning out-tree \(T = (X, E)\) of weight at most that of \(T'\), and therefore of minimum weight. \(\square\)

Corollary 6. We can compute a tree-based enforcement scheme for information flow policy \((X, \preceq)\) in time \(O(|E_0| + |X|^2)\).

Proof. By Corollary 5 we may restrict our attention to arcs in the Hasse diagram. Thus we can compute the minimum weight derivation tree in time \(O(|E_0|)\) and we can compute \(\phi\) in time \(O(|E_0| + |X|^2)\). \(\square\)

Remark 1. In practice, we expect that \(|U(x)| > 0\), although our proofs do not make this assumption. If we do make this assumption, it is possible to strengthen the statement in Corollary 5 and assert that a minimum weight spanning out-tree can only contain arcs from the Hasse diagram.

Fig. 5 illustrates the construction of the minimum weight spanning out-tree for the poset in Fig. 1 (assuming there is a single user for each vertex). The weight on arc \(ec\) is 3, for example, because \(\gamma(ec) = \{c, d, f\}\). (The effect of retaining arc \(ec\) would be that \(\kappa(c)\) would be required for each of \(c, d\) and \(f\). Equivalently, \(c \in \phi(d)\) and \(c \in \phi(f)\) if we were to choose \(ec\) to belong to our derivation out-tree.) To construct a minimum weight spanning out-tree, we must select arcs \(ca\) and \(dc\) (and we select one or other of \(fd\) and \(gd\)). One possible scheme, when \(gd\) is retained rather than \(fd\) is illustrated in Fig. 5b; the scheme requires a total of 11 keys, being the sum of the weights on the retained arcs plus an extra one for the root vertex.

![Weights on arcs](a) Weights on arcs

![Derivation out-tree](b) Derivation out-tree

Figure 5: The minimum weight derivation tree for Fig. 1

Remark 2. Our construction will almost always require fewer keys than a scheme based on chain partitions. This follows by noting that any vertex \(x\), such that \(x > y\), \(x > z\) and \(\{y, z\}\) is an antichain, necessarily requires (at least) two keys in a chain partition scheme, but this is not necessarily true of our construction (since the derivation tree may include many antichains). Consider the chain partition in Fig. 1b and the derivation tree in Fig. 5b. The former would require 13 keys, while the latter only 11.

5 Conclusion

In this paper, we have introduced a new form of cryptographic scheme for the enforcement of information flow policies. Our scheme has the advantage that no public information is required for the derivation of decryption keys. Moreover, our tree-based scheme requires fewer keys (when \(X\) is not a total order), compared to existing chain-based approaches, to enforce a given policy. Nevertheless, our scheme retains the strong security properties that have recently been established for chain-based schemes [12]. From a
practical perspective, we provide an efficient algorithm for computing an optimal derivation tree, in the sense that it requires the smallest number of keys. This is in sharp contrast to chain-based approaches, which provide no guidance on how best to select a chain partition of the poset (of which there may be many) nor provide a way of computing the number of keys required for a given partition. Thus, there are particular practical advantages to using a tree-based approach.

There are several interesting opportunities for future work. From a mathematical perspective, it would be interesting to establish the minimum total number of keys required by a chain-based scheme and, if possible, to quantify the benefits offered by a tree-based scheme. This is, however, likely to be non-trivial, as it is not clear that there exists a weight function for chain-based schemes that can be used to formulate a result analogous to Theorem 2. From a more practical perspective, it would be interesting to find an algorithm that can compute a derivation tree such that (i) no user requires more than \( w \) keys, where \( w \) is the width of the poset (ii) the total number of keys is as small as possible. In particular, such a construction may be useful in scenarios where the user devices have limited secure storage for keys. Our preliminary work on this problem suggests that no efficient algorithm exists, but whether it is an NP-hard problem remains open. We also intend to investigate whether a forest-based enforcement scheme, which would share some of the characteristics of tree- and chain-based schemes, would offer advantages in terms of reducing (i) the maximum number of steps required for key derivation (ii) the administrative effort required following key revocation (since we can limit key updates to those vertices within a tree in the forest). In Fig. 5b, for example, we could delete arc \( gd \) to yield a forest of two trees: each user assigned to vertex \( h \) or \( g \) would require an additional key (\( \kappa(d) \)) but worst-case key derivation would require two, rather than four, hops.

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