Impurity Scattering in Carbon Nanotubes with Superconducting Pair Correlations

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Abstract. Effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes are studied theoretically. The backward scattering of electrons vanishes in the normal state. In the presence of the superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such the scatterings between electrons and holes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur, and the carbon nanotube is a good conductor for Cooper pairs. Relations with experiments are discussed.

INTRODUCTION

Recent investigations (1,2) show that the superconducting proximity effect occurs when the carbon nanotubes contact with conventional superconducting metals and wires. The superconducting energy gap appears in the tunneling density of states below the critical temperature $T_c$. On the other hand, the recent theories discuss the nature of the exceptionally ballistic conduction (3) and the absence of backward scattering (4) in metallic carbon nanotubes with impurity potentials at the normal states.

In this paper, we study the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes, using the continuum $k \cdot p$ model for the electronic states. We find the absence of backward scatterings of electron- and hole-like quasiparticles in the presence of superconducting proximity effects, and the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. Therefore, the carbon nanotube is a good conductor for Cooper pairs as well as in the normal state. This finding is interesting in view of the recent experimental progress of the superconducting proximity effects of carbon nanotubes (1,2).
IMPURITY SCATTERING IN NORMAL NANOTUBES

We will study the metallic carbon nanotubes with the superconducting pair potential. The model is as follows:

\[ H = H_{\text{tube}} + H_{\text{pair}}, \]

where \( H_{\text{tube}} \) is the electronic states of the carbon nanotubes, and the model based on the \( \mathbf{k} \cdot \mathbf{p} \) approximation (4,5) represents electronic systems on the continuum medium. The second term \( H_{\text{pair}} \) is the pair potential term owing to the proximity effect.

The hamiltonian of a graphite plane by the \( \mathbf{k} \cdot \mathbf{p} \) approximation (4,5) in the secondly quantized representation has the following form:

\[ H_{\text{tube}} = \sum_{\mathbf{k},\sigma} \Psi_{\mathbf{k},\sigma}^\dagger E_{\mathbf{k}} \Psi_{\mathbf{k},\sigma}, \]

where \( E_{\mathbf{k}} \) is an energy matrix:

\[
E_{\mathbf{k}} = \begin{pmatrix}
0 & \gamma(k_x - ik_y) & 0 & 0 \\
\gamma(k_x + ik_y) & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma(k_x + ik_y) \\
0 & 0 & \gamma(k_x - ik_y) & 0
\end{pmatrix},
\]

\[ \mathbf{k} = (k_x, k_y), \] and \( \Psi_{\mathbf{k},\sigma} \) is an annihilation operator with four components: \( \Psi_{\mathbf{k},\sigma}^\dagger = (\psi_{\mathbf{k},\sigma}^{(1)}, \psi_{\mathbf{k},\sigma}^{(2)}, \psi_{\mathbf{k},\sigma}^{(3)}, \psi_{\mathbf{k},\sigma}^{(4)}) \). Here, the first and second elements indicate an electron at the A and B sublattice points around the Fermi point \( K \) of the graphite, respectively. The third and fourth elements are an electron at the A and B sublattices around the Fermi point \( K' \). The quantity \( \gamma \) is defined as \( \gamma \equiv (\sqrt{3}/2)a\gamma_0 \), where \( a \) is the bond length of the graphite plane and \( \gamma_0 (\approx 2.7 \text{ eV}) \) is the resonance integral between neighboring carbon atoms. When the above matrix is diagonalized, we obtain the dispersion relation \( E_{\pm} = \pm \gamma \sqrt{k_x^2 + \kappa_{\nu\phi}^2(n)} \),

where \( k_x \) is parallel with the axis of the nanotube, \( \kappa_{\nu\phi} (n) = (2\pi/L)(n + \phi - \nu/3) \), \( L \) is the circumference length of the nanotube, \( n = 0, \pm 1, \pm 2, ... \) is the index of bands, \( \phi \) is the magnetic flux in units of the flux quantum, and \( \nu (= 0, 1, \text{or} \ 2) \) specifies the boundary condition in the \( y \)-direction. The metallic and semiconducting nanotubes are characterized by \( \nu = 0 \) and \( \nu = 1 \) (or 2), respectively. Hereafter, we consider the case \( \phi = 0 \) and the metallic nanotubes \( \nu = 0 \).

The second term in Eq. (1) is the pair potential:

\[
H_{\text{pair}} = \Delta \sum_{\mathbf{k}} (\psi_{\mathbf{k},\uparrow}^\dagger \psi_{-\mathbf{k},\downarrow}^\dagger + \psi_{\mathbf{k},\downarrow}^\dagger \psi_{-\mathbf{k},\uparrow} + \psi_{\mathbf{k},\uparrow}^\dagger \psi_{-\mathbf{k},\downarrow}^\dagger + \psi_{\mathbf{k},\downarrow}^\dagger \psi_{-\mathbf{k},\uparrow} + h.c.)
\]

where \( \Delta \) is the strength of the superconducting pair correlation of an \( s \)-wave pairing. We assume that the spatial extent of the regions where the proximity effect occurs is as long as the superconducting coherence length.
Now, we consider the impurity scattering in the normal metallic nanotubes. We take into account of the single impurity potential located at the point \( r_0 \):

\[
H_{\text{imp}} = I \sum_{\mathbf{k}, \mathbf{p}, \sigma} e^{i(\mathbf{k} - \mathbf{p}) \cdot r_0} \psi_{\mathbf{k}, \sigma}^\dagger \psi_{\mathbf{p}, \sigma},
\]

where \( I \) is the impurity strength.

The scattering \( t \)-matrix at the \( K \) point is

\[
t_K = I [1 - \frac{2}{N_s} \sum_\mathbf{k} G_K(\mathbf{k}, \omega)]^{-1},
\]

where \( G_K \) is a propagator of a \( \pi \)-electron around the Fermi point \( K \). The discussion about the \( t \)-matrix at the \( K' \) point is qualitatively the same, so we only look at the \( t \)-matrix at the \( K \) point. The sum for \( \mathbf{k} = (k, 0) \), which takes account of the band index \( n = 0 \) only, is replaced with an integral:

\[
\frac{2}{N_s} \sum_\mathbf{k} G_K(\mathbf{k}, \omega) = \rho \int d\varepsilon \frac{1}{\omega^2 - \varepsilon^2} \left( \begin{array}{cc} \omega & \varepsilon \\
\varepsilon & \omega \end{array} \right) \simeq -\rho \pi i \text{sgn} \omega \left( \begin{array}{cc} 1 & 0 \\
0 & 1 \end{array} \right),
\]

where \( \rho = a/2\pi L\gamma_0 \) is the density of states at the Fermi energy. Therefore, we obtain

\[
t_K = \frac{I}{1 + I \rho \pi i \text{sgn} \omega \left( \begin{array}{cc} 1 & 0 \\
0 & 1 \end{array} \right)}. \tag{8}
\]

The transformation into the energy-diagonal representation where the branches with \( E = \pm \gamma |k| \) are diagonal has the same form of \( t_K \).

The scattering matrix \( t_K \) in the energy-diagonal representation is diagonal, and the off-diagonal matrix elements vanish. This means that only the scattering processes from \( k \) to \( k \) and from \( -k \) to \( -k \) are effective. The scatterings from \( k \) to \( -k \) and from \( -k \) to \( k \) are cancelled. Such the absence of the backward scattering has been discussed recently (4).

**IMPURITY SCATTERING WITH SUPERCONDUCTING PAIR POTENTIAL**

We consider the single impurity scattering when the superconducting pair potential is present. In the Nambu representation, the scattering \( t \)-matrix at the \( K \) point is

\[
\tilde{t}_K = \tilde{I} [1 - \frac{2}{N_s} \sum_\mathbf{k} \tilde{G}_K(\mathbf{k}, \omega) \tilde{I}]^{-1},
\]

where \( \tilde{G}_K \) is the Nambu representation of \( G_K \) and

\[
\tilde{I} = I \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 \end{array} \right). \tag{10}
\]
The sign of the scattering potential for holes is reversed from that for electrons, so the minus sign appears at the third and fourth diagonal matrix elements.

The sum over $k$ is performed as in the previous section, and we obtain the scattering $t$-matrix (with the same form in the energy-diagonal representation):

$$
\tilde{t}_K = \frac{I}{1 + (I\rho\pi)^2} \begin{pmatrix}
1 + \alpha\omega & 0 & -\alpha\Delta & 0 \\
0 & 1 + \alpha\omega & 0 & -\alpha\Delta \\
-\alpha\Delta & 0 & -1 + \alpha\omega & 0 \\
0 & -\alpha\Delta & 0 & -1 + \alpha\omega
\end{pmatrix}
$$

(11)

where $\alpha = I\rho\pi i/\sqrt{\omega^2 - \Delta^2}$.

Hence, we find that the off-diagonal matrix elements become zero in the diagonal $2 \times 2$ submatrix. This implies that the backward scatterings of electron-line and hole-like quasiparticles vanish in the presence of the proximity effects, too. Off-diagonal $2 \times 2$ submatrix has the diagonal matrix elements whose magnitudes are proportional to $\Delta$. The finite correlation gives rise to backward scatterings of the hole of the wavenumber $-k$ when the electron with $k$ is incident. The back scatterings of the electrons with the wavenumber $-k$ occur for the incident holes with $k$, too. Negative and positive currents induced by such the two scattering processes cancel each other. Therefore, the nonmagnetic impurity does not hinder the supercurrent in the regions where the superconducting proximity effects occur. This effect is interesting in view of the recent experimental progress of the superconducting proximity effects (1,2).

**SUMMARY**

We have investigated the effects of the superconducting pair potential on the impurity scattering processes in metallic carbon nanotubes. The backward scattering of electrons vanishes in the normal state. In the presence of the superconducting pair correlations, the backward scatterings of electron- and hole-like quasiparticles vanish, too. The impurity gives rise to backward scatterings of holes for incident electrons, and it also induces backward scatterings of electrons for incident holes. Negative and positive currents induced by such the scatterings between electrons and holes cancel each other. Therefore, the carbon nanotube is a good conductor for the Cooper pairs coming from the proximity effects.

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