Predictions of the Generalized Glauber Model for the coherent 
$\rho$-production at RHIC and the STAR data

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ABSTRACT

We calculate the rapidity distribution and the total cross section of coherent and incoherent $\rho$-production in the heavy ion ultraperipheral collisions (UPC) at $\sqrt{s_{NN}} = 130$ GeV using the generalized vector dominance model (GVDM) and the Gribov-Glauber approach. We find the coherent cross section of $\rho$-production $\sigma_{coh} = 490$ mb compared to the $\sigma_{coh} = 370 \pm 170 \pm 80$ mb recently reported by the STAR collaboration at RHIC. The predicted cross section inside the acceptance of the experiment, $|y| \leq 1$, agrees with the data within half a standard deviation. It is emphasized that measurements of the rapidity distribution will provide a much more stringent test of the model.

1 Introduction

Ultraperipheral collisions (UPC) of relativistic heavy ions at RHIC and LHC open a promising new avenue for experimental studies of the photon induced coherent and incoherent interactions with nuclei at high energies (see [1, 2, 3] for the reviews and extensive lists of references). Really, the LHC heavy ion program will allow studies of photon-proton and photon - nucleus collisions at the energies exceeding by far those available now at HERA for $\gamma - p$ scattering.

Hence, it is very important to check our basic understanding of the UPC processes using the reactions which have smaller theoretical uncertainties on the level of the $\gamma A$ interactions. Recently the STAR collaboration released the first data on the cross section of the coherent $\rho$-meson production in gold-gold UPC at $\sqrt{s_{NN}} = 130$ GeV [4]. This provides a first opportunity to check the basic features of the theoretical models and main approximations which include the Weizsäcker-Williams (WW) approximation for the spectrum of the equivalent photons, an approximate procedure for removing collisions at small impact parameters where nuclei interact strongly, and the model for the vector meson production in the $\gamma A$ interactions. In the case of the $\rho$-meson production the basic process is understood much better than for other photoproduction processes. Hence, checking the theory for this case is especially important for proving that UPC could be used for learning new information about photon - nucleus interactions.

Earlier we published [6] predictions for the cross section of this process at higher energy $\sqrt{s_{NN}} = 200$ GeV. Hence, a direct comparison of the STAR result with [6] is difficult. In this paper we perform
the analysis of the $\rho$-production at $\sqrt{s_{NN}} = 130$ GeV including the effects due to the cuts of the STAR experiment.

2 Outline of the model

Production of $\rho$-mesons in ultraperipheral heavy ion collisions can be expressed in the Weizsäcker-Williams approximation\[8\] through the $\gamma A \rightarrow \rho A$ cross section

$$\frac{d\sigma_{A_1A_2\rightarrow A_1A_2V}}{dy} = n_{A_1}(y)\sigma_{\gamma A_2\rightarrow \rho A_2}(y) + n_{A_2}(-y)\sigma_{\gamma A_1\rightarrow \rho A_1}(-y).$$

(1)

The quantity $y = \ln \frac{2\omega}{M_{\rho}}$ is the rapidity of the produced $\rho$ meson and $n_{\gamma}(y)$ is the flux of photons with the energy $\omega_{\gamma} = \gamma_c q_0$ emitted by one of nuclei ($\gamma_c$ is the Lorentz factor for colliding nuclei, and $q_0$ is the photon momentum in the coordinate system of moving nucleus). The photoproduction cross section $\sigma_{\gamma A_1\rightarrow \rho A_2}(y)$ can be calculated in the Glauber model\[9\]

$$\sigma_{\gamma A_1\rightarrow \rho A_2}(y) = \int_{-\infty}^{t_{min}} dt \frac{\pi}{k_{\rho}^2} |F_{\gamma A_1\rightarrow \rho A_2}(t)|^2 = \frac{\pi}{k_{\rho}^2} \int_{0}^\infty dt_\perp \frac{|i k_{\rho}}{2\pi} \int d\vec{b} e^{i q_\perp \cdot \vec{b}} \Gamma(\vec{b})|^2.$$

(2)

Here $q_\perp^2 = t_\perp = t_{min} - t$, $-t_{min} = \frac{M_{\rho}^4}{4q_0^2}$ is longitudinal momentum transfer in $\gamma - \rho$ transition, and $\Gamma(\vec{b})$ is the diffractive nuclear profile function

$$\Gamma(\vec{b}) = \lim_{z\rightarrow\infty} \Phi(\vec{b}, z).$$

(3)

To calculate the eikonal function $\Phi(\vec{b}, z)$ the Glauber approach\[10\] was combined with the generalized vector dominance (GVD) model\[11\]. More properly such an approximation should be called the Gribov-Glauber model\[12\] because the space-time evolution of high energy processes is different in quantum mechanical models and in quantum field theory and therefore theoretical foundations for the high-energy model are different. In particular, in QCD in difference from quantum mechanics a high-energy projectile interacts with all nucleons at the same impact parameter almost at the same time\[12\]. Such formulae allow to extend the domain of applicability of the Glauber model to the description of high energy phenomena where inelastic (high multiplicity) particle production gives dominant contribution to the total cross section. As we are mostly interested in the accurate estimate of the coherent diffractive production of the vector meson $\rho$ with $M_{\rho} = 0.77$ GeV, the spectrum of the higher corresponding hadronic states of $M \leq 2$ GeV can be approximated by one effective $\rho'$-meson with some reasonable fixed mass, say $M_{\rho'} = \sqrt{3} M_{\rho}\[13\]$. We want to draw attention that the value and sign of the $\rho'$ contribution as taken from quenched GVDM is fitted to describe approximate Bjorken scaling in the case of highly virtual photon. Thus the model used in the paper correctly accounts for the fluctuations of cross section including color transparency phenomenon\[14\].

Then the GVD model comprises elementary amplitudes

$$f_{\gamma N\rightarrow \rho N} = \frac{e}{f_{\rho}} f_{\rho N\rightarrow \rho N} + \frac{e}{f_{\rho'}} f_{\rho' N\rightarrow \rho' N},$$

$$f_{\gamma N\rightarrow \rho' N} = \frac{e}{f_{\rho'}} f_{\rho' N\rightarrow \rho' N} + \frac{e}{f_{\rho}} f_{\rho N\rightarrow \rho' N}.$$  

(4)
In the optical limit ($A \gg 1$), with accuracy $O(\sqrt{\alpha_{em}})$ the eikonal functions $\Phi_{\rho, \rho'} (\vec{b}, z)$ are determined by the solutions of the coupled two-channel equations

$$2ik_\rho \frac{d}{dz} \Phi_\rho (\vec{b}, z) = U_{\gamma A \rightarrow \rho A} (\vec{b}, z) e^{iq_\parallel A z} +$$

$$+ U_{\rho A \rightarrow \rho A} (\vec{b}, z) \Phi_\rho (\vec{b}, z) + U_{\rho A \rightarrow \rho' A} (\vec{b}, z) e^{iq_\parallel \rho' z} \Phi_{\rho'} (\vec{b}, z)$$

(5)

$$2ik_{\rho'} \frac{d}{dz} \Phi_{\rho'} (\vec{b}, z) = U_{\gamma A \rightarrow \rho' A} (\vec{b}, z) e^{iq_\parallel A z} +$$

$$+ U_{\rho A \rightarrow \rho' A} (\vec{b}, z) \Phi_{\rho'} (\vec{b}, z) + U_{\rho A \rightarrow \rho A} (\vec{b}, z) e^{iq_\parallel \rho z} \Phi_\rho (\vec{b}, z)$$

(6)

with the initial condition $\Phi_{\rho, \rho'} (\vec{b}, -\infty) = 0$. The exponential factors $e^{iq_\parallel j z}$ are responsible for the coherent length effect, $i, j = \gamma, \rho, \rho'$, $q_\parallel j = \frac{M_\rho^2 - M_{\rho'}^2}{2\gamma c}$. The generalized Glauber-based optical potentials in the short-range approximation are given by the expression

$$U_{iA \rightarrow jA} (\vec{b}, z) = -4\pi f_{iN \rightarrow jN} (0) g (\vec{b}, z).$$

(7)

Here $f_{iN \rightarrow jN} (0)$ are the forward elementary amplitudes, and $g (\vec{b}, z)$ is the nuclear density normalized by the condition $\int d\vec{b} dz \ g (\vec{b}, z) = A$. We calculated $g (\vec{b}, z)$ in the Hartree-Fock-Skyrme (HFS) model which provided a very good (with an accuracy $\approx 2\%$) description of the global nuclear properties of spherical nuclei along the periodical table from carbon to uranium and the shell momentum distributions in the high energy $(p,2p)$ and $(e,e'p)$ reactions.

Following the simple suggestion of Ref. [13] which is quite reasonable in the case of light vector mesons, we fixed the elementary scattering amplitudes and coupling constants by relations

$$f_{\rho'N \rightarrow \rho'N} = f_{\rho N \rightarrow \rho N}, \quad f_{\rho N \rightarrow \rho'N} = f_{\rho'N \rightarrow \rho N} = -\varepsilon f_{\rho N \rightarrow \rho N}, \quad f_{\rho'} = \frac{M_{\rho'}}{M_{\rho}} \cdot f_{\rho},$$

(8)

with the value of the $\rho$-meson coupling constant $f_{\rho}/4\pi = 2.01$. The diagonal amplitude $f_{\rho N \rightarrow \rho N}$ was taken in parameterization of the Landshoff-Donnachie model while the value of parameter $\varepsilon = 0.18$ was found from a best fit to the differential cross section of the $\rho$-meson photoproduction off lead at $\omega_\gamma = 6.2$ GeV and $t_\perp = 0.001 GeV^2$. With all parameters fixed, we compared our calculations with all available data on $\rho$-meson photoproduction off nuclei at low and intermediate energies and found a very good description of the absolute cross section and of the momentum transfer distributions (see [3]). Hence, it was natural to expect that this model should provide a reliable parameter-free predictions for production of $\rho$-mesons in high energy heavy ion UPC. Note here that the inelastic shadowing effects which start to contribute at high energies still remain a few percent correction at energies $\lesssim 100$ GeV relevant for the STAR kinematics. For LHC energy range one should account for the blackening of interaction with nuclei. In this case cross section of inelastic diffraction in hadron-nucleus collisions should tend to 0. Also it leads to a suppression of the $\rho'$ contribution to the cross section of the diffractive $\rho$ meson photoproduction [3].

3 Results and discussion

The calculated momentum transfer distributions at the rapidity $y = 0$ and the momentum transfer integrated rapidity distribution for gold-gold UPC at $\sqrt{s_{NN}} = 130$ GeV are presented in Figs. 3a,b.
Figure 1: (a) Momentum transfer dependence of the coherent and incoherent $\rho$-meson production in AuAu UPC at $\sqrt{s_{NN}} = 130$ GeV calculated in Generalized Glauber model (GGM). (b) Rapidity distributions for coherent $\rho$-meson production in the gold-gold UPC at $\sqrt{s_{NN}} = 130$ GeV calculated in GGM.
Figure 2: Energy dependence of the cross sections for coherent $\rho$-meson production in the gold-gold UPC calculated in the GGM and the STAR results: a) The dashed line is the total cross section in GGM, the solid line is the cross section calculated accounting for the STAR cut on the momentum transfer, star is the STAR cross section based on the Monte-Carlo extrapolation of the measured value to the full detector acceptance; b) Comparison of the GGM cross section in the interval of rapidities $|y| < 1$ with the value measured by the STAR.
Let us briefly comment on our estimate of the incoherent $\rho$-meson production cross section. The momentum transfer distribution (dashed line in Fig. 2a) is practically flat in the discussed $t_\perp$ range. The total incoherent cross section obtained by integration over the wide range of $t_\perp$ is $\sigma_{\text{inc}} = 120$ mb. To select the coherent production the cut $t_\perp \leq 0.02$ GeV$^2$ was used in the data analysis [4]. Correspondingly, the calculated incoherent cross section for this region of $t_\perp$ is $\sigma_{\text{inc}} = 14$ mb. Our calculations of incoherent production which are based on accounting for only the single elementary diffractive collision obviously present the lower limit. The residual nucleus will be weakly excited and can evaporate only one-two neutrons. The events $A + A \to \rho + xn + A_1 + A_2$ were detected by the STAR and identified as a two-stage process - coherent $\rho$-production with the subsequent electromagnetic excitation and neutron decay of the colliding nuclei [20]. In particular, the cross section estimated by the STAR for the case when only one of the nuclei is excited and emits several neutrons is $\sigma_{\rho,xn,0n} = 95 \pm 60 \pm 25$ mb. The momentum transfer distribution for these events is determined by the coherent production. Hence, it differs from that for incoherent events but in the region of very low $t_\perp$ it is hardly possible to separate them experimentally and obviously the measured cross section $\sigma_{\rho,xn,0n}$ includes contribution of incoherent events on the level of 15%.

The total rapidity-integrated cross section of coherent $\rho$-meson production calculated in the GVDM for the range of energies available at RHIC is shown in Fig. 2a (dashed line). We find $\sigma_{\text{th coh}} = 540$ mb at $\sqrt{s_{NN}} = 130$ GeV. The value $\sigma_{\text{exp coh}} = 370 \pm 170 \pm 80$ mb was obtained at this energy by the STAR from the data analysis at the low momentum transfer $t_\perp \leq 0.02$ GeV$^2$. Thus, before making a comparison we should take into account this cut. It leads to a reduction of the cross section by $\approx 10\%$ (the solid line in Fig. 2a). In our calculations we did not account for the $t_\perp$-dependence of the elementary amplitudes which are rather flat in the considered range of energies and momentum transfers as compared to that for the nucleus form factor. So, in the region of integration important for our analysis neglect by this slope is a reasonable approximation but, nevertheless, an account of this effect would slightly reduce our estimate of the total cross section. Also we neglected a smearing due to the transverse momentum of photons and the interference of the production amplitudes from both nuclei [23]. This latter phenomenon results only in the narrow dip in the coherent $t_\perp$-distribution at $t_\perp \leq 5 \cdot 10^{-4}$ GeV$^2$. All these effects does not influence noticeably the value of the $t_\perp$-integrated cross section but can be easy treated and taken into account in a more refined analysis. Thus we find $\sigma_{\text{coh}} = 490$ mb to be compared to the STAR value $\sigma_{\text{exp coh}} = 370 \pm 170 \pm 80$ mb. Since our calculation does not have any free parameters, this can be considered as a reasonable agreement.

At this point we would like to comment on the statement of [4] that our prediction for coherent $\rho$-production in gold-gold UPC at $\sqrt{s_{NN}}=200$ GeV [6] is 50% higher than the value given by model in [21]. We already briefly explained in [6] that this discrepancy originates from a number of approximations made in the model of Ref. [21] which differs from the conventional Glauber model. The coherent photoproduction cross section was defined in [21] by the expression:

$$\sigma_{\gamma A \to \rho A} = \frac{d\sigma_{\gamma A \to \rho A}(t = 0)}{dt} \cdot \int_{-\infty}^{t_{\text{min}}} |F_A(t)|^2 dt. \quad (9)$$

where $F_A(t)$ is the nuclear form factor (two-dimensional Fourier transformation of the parameterized nuclear density) and the forward photoproduction cross section was estimated using the vector dominance model and optical theorem

$$\frac{d\sigma_{\gamma A \to \rho A}(t = 0)}{dt} = \frac{\alpha_{\text{em}}}{4f_\rho^2} \sigma_{\text{tot}}(\rho A). \quad (10)$$
The total cross section of the $\rho A$ interaction was found in [21] using the formula:

$$\sigma_{\text{tot}}(\rho A) = \int \! d\vec{b} \left[ 1 - \exp \left[ -\sigma_{\rho N} \left( \int_{-\infty}^{\infty} \varrho(b, z) \, dz \right) \right] \right].$$  \hspace{1cm} (11)

Eq.\hspace{1cm} (11) presents the classical mechanics model with standard for this approach expression for the total cross section $\sigma_{\text{tot}}(\rho A)$. The quantum mechanics expression is given by the Gribov-Glauber model (here for simplicity we give the expression in the limit of Re/Im=0):

$$\sigma_{\text{tot}}^\text{qm}(\rho A) = 2 \int \! d\vec{b} \left[ 1 - \exp \left( -\frac{1}{2} \sigma_{\rho N} T_{\rho A}(b) \right) \right].$$  \hspace{1cm} (12)

In the Black Body Limit($\sigma_{V N} \to \infty$) the total $\gamma A \to VA$ cross section estimated with the use of the classical mechanics ($\sigma_{\text{tot}}^\text{cm} = \pi R_A^2$) and the quantum mechanics ($\sigma_{\text{tot}}^\text{qm} = 2\pi R_A^2$) expressions in Eq.(10) differ by a factor of four. The difference for the case of the gold nucleus and reasonable value of the $\rho N$ elementary cross section $\sigma_{\rho N} \approx 25\text{ mb}$ can be found using the simplified model of the nucleus of constant density $\varrho_0 \approx 0.16\text{ fm}^{-3}$ and radius $R_A$. With the value of radius $R_{Au} \approx 6.5\text{ fm}$ one can obtain the reasonable estimate of the ratio

$$\frac{\sigma_{\text{tot}}^\text{qm}(\rho A)}{\sigma_{\text{tot}}^\text{cm}(\rho A)} \approx 2 \left[ 1 - \frac{3}{2\sigma_{\rho N}^2 \varrho_0^2 R_A^2} \right] \approx 1.55.$$

Hence, we find that due to use of the classical mechanics expression (11) instead of the Gribov-Glauber model expression (12) the total $\gamma A \to \rho A$ cross section was underestimated in Ref.\hspace{1cm} [21] by a factor $\approx 2.5$. Note, that in the range of the photon energies essential in photoproduction of $\rho$-mesons in UPC at RHIC the elementary $\rho N$ cross section still weakly depends on the energy of $\rho$-mesons. Hence, this factor weakly depends on $\sqrt{s_{NN}}$.

Next important approximation made in Ref.\hspace{1cm} [21] in deriving formulae Eqs.\hspace{1cm} (9) and (10) was the neglect by the coherence length effects. This requires $q_{z}^{\gamma \to \rho z} = \frac{M_{\rho}^2 z}{2\varrho_0 q_0} \ll 1$. This neglect is not justified because large longitudinal distances are essential in the diffractive $\rho$-photoproduction. The $\rho$-meson can be formed far from the nucleus. Besides, the photon flux is large at small $q_0 \ll R_A^{-1}$, i.e. in the region where the coherence length effect is important. We estimated that the cross section for $AuAu \to \rho AuAu$ at the energy $\sqrt{s_{NN}} = 200\text{ GeV}$ is overestimated by a factor $\approx 1.5$ if one neglects the coherence length effect. The coherence length effect becomes more essential with decrease of the energy. As a result at $\sqrt{s_{NN}} = 130\text{ GeV}$ this effect suppresses the cross section by a factor $\approx 2$. On the contrary, at much higher energies where the coherence length is very large (for example, at LHC) this effect will be small.

Note in passing that the calculations in Ref.\hspace{1cm} [21] were performed neglecting the real part of the elementary $\rho N$ amplitude which we accounted for using the Landshoff-Donnachie parameterization. In the high energy domain, for example, in the region of the central rapidities at RHIC the real part of the $\rho N$ amplitude is negligible but one should account for $Re f_{\rho N \to \rho N}$ at the edges of rapidity distribution which correspond to the photoproduction at intermediate energies of photons. Since the contribution of this region to the total cross section is enhanced by the high photon flux, the

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7 In the Appendix we demonstrate how the model used in Ref.\hspace{1cm} [21] but with the correct high energy expression for the total cross section can be obtained from the Gribov-Glauber based Generalized Vector Dominance Model and what essential approximations have to be done on the way.
total cross section of the coherent $\rho$ production at $\sqrt{s_{NN}} = 130$ GeV would be underestimated by $\approx 10\%$.

Thus we emphasize that a number of approximations were made in the model of the Ref.\,[21] which are inconsistent with the conventional Glauber model and their interplay depends on the photon energy. Thus the estimates obtained within this model contain systematic uncertainties. We want to draw attention that the model we have used is well theoretically justified. It correctly calculates the nuclear form factor in the coherent photoproduction. We checked \[6\] that this model provided a very good description of the coherent $\rho$-photoproduction off nuclei at low and intermediate energies along the periodic table without any free parameters.

At present the comparison of our predictions with STAR data is still preliminary because the experimental errors are too large and there are few points in the procedure of the data analysis which should be discussed. Really the acceptance of STAR is very strongly $y$ dependent being maximal at $y = 0$ and going to zero at $|y| = 1$ (Fig.3a in \[4\]) while the theoretical distribution is expected to have a double bump shape, see Fig.1b, which is simply due to the symmetry of collision and the interplay of the energy dependence of the photon flux and $\gamma A \rightarrow \rho A$ cross section.

Due to the acceptance conditions $Au\rightarrow \rho Au Au$ events were detected in the range of rapidities $|y| \leq 1$, while the cross section reported in \[4\] is corrected for the $|y| \geq 1$ using the Monte-Carlo extrapolation based on the model \[21\] in which

$$R = \sigma_{4\pi}/\sigma_{|y| \leq 1} = 2.7.$$  \hspace{1cm} (13)

Thus the cross section in the region $|y| \leq 1$ is

$$\sigma_{|y| \leq 1} = 140 \pm 60 \pm 30 \text{ mb}.$$ \hspace{1cm} (14)

The errors for this cut are scaled accordingly. However the model \[21\] differs from ours in many aspects, described above. In particular, the neglect by the coherence length effect leads to a significant modification of the value of factor $R$. The rapidity interval $|y| \leq 1$ corresponds high energies of the photon where this effect is negligible. On the other hand due to the neglect by the coherent length effects at the edges of the rapidity distribution the relative contribution of photoproduction cross section at $|y| \geq 1$ was overestimated in Ref.\,[21]. Hence their value of $R$ is lower than the one we obtain from our rapidity distribution which gives $R = 3.2$, and the cross section $\sigma_{|y| \leq 1} = 170$ mb. It is this value of the cross section which should be compared (Fig.2b) to the experimentally measured cross section \[14\]. Obviously a more detailed comparison would require a detailed study of the sensitivity of the analysis to the assumed $y$-distribution.

## 4 Conclusions

We demonstrate that the cross section of coherent $\rho$-meson production in high energy heavy ion UPC calculated within the GVDM is in a good agreement with experimental results of the STAR collaboration. More stringent test will involve comparison of the model predictions with the cross section and the rapidity distributions of the $\rho$-meson production measured with higher precision.

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\[2\] We thank S.Klein for the discussion of this issue.
5 Appendix

For the completeness let us discuss how classical mechanical formulae used in Ref.\[21\] arise from the quantum mechanical ones. Since the Vector Dominance model was used in \[21\] we should neglect by the nondiagonal $\rho \rightarrow \rho'$ transitions in Eq.\[4\]. Thus we need to put $\varepsilon = 0$ in Eq.\[8\] instead of $\varepsilon = 0.18$ which is used in our calculations. If we keep all other parameters of the model fixed, we pay for such a reduction by the increase of the $\rho$-photoproduction cross section by a factor $\approx (1 + 2\varepsilon/\sqrt{3}) \approx 1.2$. With $\varepsilon = 0$ equations \[8\] and \[8\] become decoupled and the solution of Eq.\[8\] gives the eikonal function of the $\rho$-meson

$$\Phi_{\rho}(\vec{b}, z) = \frac{1}{2ik_\rho} e^{\frac{ik_\rho}{\sqrt{m_\rho \mu}}} \int_{-\infty}^{\infty} U_{\rho N \rightarrow \rho N}(\vec{b}, z') dz' \int_{-\infty}^{\infty} dz' U_{\rho N \rightarrow \rho N}(\vec{b}, z') e^{i\frac{\rho N}{\sqrt{m_\rho \mu}} z'} e^{-\frac{ik_\rho}{\sqrt{m_\rho \mu}}} \int_{-\infty}^{\infty} U_{\rho N \rightarrow \rho N}(\vec{b}, z'') dz''. \quad (15)$$

Using Eq.\[2\] and Eq.\[4\] and standard expression for the elementary amplitude

$$f_{\rho N}(0) = \frac{ik_\rho \sigma_{\rho N}}{4\pi} [1 - i\beta_{\rho N}] \quad \beta_{\rho N} = \frac{Re f_{\rho N}(0)}{Im f_{\rho N}(0)},$$

we obtain the amplitude of the $\rho$-meson photoproduction off the nucleus in the optical limit of the standard Glauber plus the Vector Dominance model\[22\]

$$F_{\gamma A \rightarrow \rho A} = f_{\gamma N \rightarrow \rho N}(0) \int_{0}^{\infty} d\vec{b} e^{i\vec{q} \cdot \vec{b}} \int_{-\infty}^{\infty} dz' g(\vec{b}, z') e^{i\frac{\rho N}{\sqrt{m_\rho \mu}} z'} e^{-\frac{\sigma_{\rho N}}{2} \int_{-\infty}^{\infty} g(\vec{b}, z'') dz''}. \quad (16)$$

Note that accounting for the real part of the $\rho N$ amplitude leads to appearance of the phase factor $exp[i\beta_{\rho N} \int_{-\infty}^{\infty} g(\vec{b}, z') dz']$ which is similar to that describing the coherence length effect and which is important in the same energy domain. Following the assumptions of Ref.\[21\] where both the coherence length effect and real part of the $\rho N$ amplitude were neglected we remove these exponential factor from Eq.\[16\]. Then in the limit of the purely imaginary elementary $\rho N$ amplitude we obtain

$$F_{\gamma A \rightarrow \rho A} = f_{\gamma N \rightarrow \rho N}(0) \int_{0}^{\infty} d\vec{b} e^{i\vec{q} \cdot \vec{b}} \int_{-\infty}^{\infty} dz' g(\vec{b}, z') e^{-\frac{\sigma_{\rho N}}{2} \int_{-\infty}^{\infty} g(\vec{b}, z'') dz''} = \quad (17)$$

$$= \frac{f_{\gamma N \rightarrow \rho N}(0)}{\sigma_{\rho N}} \cdot 2 \cdot \int_{0}^{\infty} d\vec{b} e^{i\vec{q} \cdot \vec{b}} \int_{-\infty}^{\infty} dz \frac{d}{dz} \left[ -\frac{\sigma_{\rho N}}{2} \int_{-\infty}^{\infty} g(\vec{b}, z') dz' \right] = \quad (18)$$

Now using the vector dominance relation $f_{\gamma N \rightarrow \rho N} = \frac{f_{\rho N \rightarrow \rho N}}{t_0} f_{\rho N \rightarrow \rho N}$ we can write the formula for the forward $\gamma A \rightarrow \rho A$ cross section in the optical limit of the Glauber +VD model

$$\frac{d\sigma_{\gamma A \rightarrow \rho A}(t = 0)}{dt} = \frac{\pi}{k_\rho^2} |F_{\gamma A \rightarrow \rho A}(t = 0)|^2 = \frac{\alpha_{\text{em}}}{4f_{\rho}^2} \sigma_{\rho A}^2 \quad (19)$$
where
\[ \sigma_{\rho A} = 2 \cdot \int d\vec{b} \left[ 1 - \exp \left( -\frac{\sigma_{\rho N}}{2} \int_{-\infty}^{\infty} g(\vec{b}, z)dz \right) \right] \]  
(20)

Thus in the Glauber+VD model we have got the expression for the forward cross section of the photoproduction coinciding with Eq.(10) used in calculations performed in Ref. [21] but with high energy quantum mechanics formula for the total cross section.

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