A particle probing thermodynamics in a three-dimensional black hole

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Abstract
We have shown the thermodynamic changes in a (2+1)-dimensional rotating black hole when it absorbs a particle. The microscopic changes that the black hole undergoes are interpreted using the AdS/CFT correspondence. Using the particle-absorption phenomenon, we formulate an irreducible mass and black hole entropy directly related to the particle momenta. We describe the black hole evolution that preserves the second law of thermodynamics. A number of microstates that evaluate the entropy are analyzed at each of the black hole evolution stages.

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1. Introduction

The black hole information can be obtained from the Bekenstein–Hawking entropy [1], which is described by the surface area regardless of the internal structure in the black hole.

In asymptotically flat spacetimes, the Bekenstein–Hawking entropy can be derived from considering the particle absorption independent of the area law. This may provide the black hole to change the mass and the rotation angular momentum. The particle-absorption process will increase the entropy by increasing the irreducible mass of black hole. The irreducible mass squared is shown to be proportional to the Bekenstein–Hawking entropy for the four-dimensional Kerr black hole [2] and for the higher dimensional extension [3]. The physical meaning of the irreducible mass is the energy distributed on the black hole’s surface. Interestingly, the irreducible mass always increases when a particle is absorbed to the black hole, consistent with the second law of thermodynamics. This issue can be related to the following gedanken experiment for the black hole’s horizon [4]. If the particle absorption could provide the increase of the spin parameter in the four-dimensional extremal Kerr black hole more than that of the mass, the horizon might disappear. However, this never occurs for the four-dimensional and higher dimensional extremal rotating black holes [3, 4]. In other words, the particle absorption does not cause the horizon destruction problem.
These pictures have not yet been elucidated in a Banados–Teitelboim–Zanelli (BTZ) black hole, which is a rotating black hole solution with a cosmological constant in (2+1) dimensions [5, 6]. The BTZ black hole has asymptotically AdS geometry, in which AdS/CFT correspondence principle can be applied. We can study the thermodynamic behaviors in terms of the AdS/CFT dual dictionary, which has been well investigated. In the duality, the microscopic origin of the AdS black hole entropy is explained from a quantum field theory [7]. According to the AdS/CFT dictionary, the AdS black hole entropy is interpreted as the asymptotic growth of states described by the conformal field theory (CFT) [8, 9]. In the AdS/CFT duality, the properties of BTZ black holes are interpreted in terms of two-dimensional CFT. In this respect, the earlier works regarding a rotating black hole entropy are mainly concerned with the BTZ black hole. On the other hand, BTZ black holes can occur from the string theory. Hence, obtaining the entropy and the gray-body factor for the BTZ black hole can give direct information about these black holes [10]. There are other approaches to a black hole entropy. Using the D-brane description, the result for the Bekenstein–Hawking entropy is reproduced as the logarithm of the microstate counting of the D-brane configurations in the specific black hole [11]. Recently, for asymptotically flat spacetime, the Bekenstein–Hawking entropy for the Kerr black hole has been interpreted as a microscopic entropy for the specific CFT, which leads to the conjecture of Kerr/CFT correspondence [12].

In this paper, we will compute and describe the second law of thermodynamics and evolution of the BTZ black hole arising in the (2+1)-dimensional spacetime with a cosmological constant. The thermodynamic behaviors will be microscopically interpreted in terms of the AdS/CFT correspondence. The BTZ black hole entropy duality has been derived by Strominger [8]. In his derivation, the AdS black hole entropy is obtained by Cardy’s formula [13] using diffeomorphism operators. We give a brief review about this work in section 2 of the paper. Then, we obtain an exact expression of the energy dispersion relation for particle absorption by using the Hamilton–Jacobi method. Integrating the dispersion relation, we will formulate an irreducible mass and show that the change in the irreducible mass is directly related to the particle radial momentum. Since the change in the irreducible mass is proportional to the particle radial momentum, the change in the Bekenstein–Hawking entropy is exactly proportional to the particle radial momentum. These relations also lead to the exact expression for the change in the outer horizon. Using this expression, the microstate changes which the black hole undergoes during particle absorption are obtained as the particle momenta. In addition, the energy equation for the particle confines the changes in black hole momenta. It means imposing the particle energy and momentum on the changes in the black hole’s properties [3]. From this point of view, the changes in the black hole mass and angular momentum is limited to the particle momenta, and the irreducible mass always increases as we previously explained. This procedure prevents an unphysical evolution that breaks the second law of thermodynamics. It is the same process that the cosmological black hole undergoes when it absorbs particles. From the CFT perspective, the particle energy and momentum change the black hole’s energy states, which are composed as the sum of the quantum numbers for the left- and right-moving operators. In the process, the particle effects work differently for each ratio of the black hole mass and angular momentum. Herein, we have shown what a (2+1)-dimensional rotating black hole undergoes and how its microstates change during particle absorption. In addition, we have demonstrated that the increase of the BTZ black hole mass is greater than that of the angular momentum in particle absorption. Therefore, we conclude that the outer horizon does not disappear during the absorption of any particle.

This paper is organized as follows. First, we briefly review the derivation of the Bekenstein–Hawking entropy formula as the asymptotic growth of states described by the
CFT. Second, we describe how thermodynamic changes in a black hole are obtained in terms of the infalling particle momenta. Third, the microstate excitations on the CFT side are interpreted as a particle carrying momenta. Fourth, thermodynamic and microscopic changes are analyzed at each of the evolution stages of a black hole. Finally, we summarize and discuss our results and conclusions.

2. Three-dimensional black hole and CFT

In this section, we briefly review about the three-dimensional rotating black hole entropy from the asymptotic growth of states described by the CFT [8]. The gravitational action in the three-dimensional spacetime is given by

$$S = \frac{1}{16\pi G} \int d^3x \sqrt{-g}(R + 2\ell^{-2}),$$  \hspace{1cm} (1)

where the cosmological constant $\Lambda$ is $-\ell^{-2}$. Note that our discussion is valid in the semiclassical regime $\ell \gg G$. The AdS$_3$ vacuum solution can be obtained from the action. The AdS$_3$ is included in the SL(2, $R$) group manifold, and the isometry group of that is $SL(2, R)$.$_1 \otimes$SL(2, $R$)$_2$. The boundary condition should be determined to define the full quantum theory on AdS$_3$. Thus, the condition required is written as

$$g_{tt} = -\ell^{-2}r^2 + O(1), \hspace{1cm} g_{\phi\phi} = O(1), \hspace{1cm} g_{\phi t} = O(1/r^2), \hspace{1cm} g_{rr} = \ell^2 r^{-2} + O(1/r^4), \hspace{1cm} g_{d\phi} = O(1/r^4), \hspace{1cm} g_{\phi\phi} = r^2 + O(1).$$  \hspace{1cm} (2)

The non-trivial diffeomorphism is obtained from the vector fields preserving the boundary condition. The generators of the diffeomorphism are defined as $L_m$ and $\bar{L}_m$ in $-\infty < n < \infty$. The generators obey the Virasoro algebra of which the central charge is $c = 3\ell/2G$. Therefore, the CFT lies on AdS$_3$. The BTZ black hole [5, 6] which preserves the required boundary condition in equation (2) can be obtained by a discrete identification of the AdS$_3$ vacuum solution and written [8]

$$\text{d}s^2 = -N^2 \text{d}t^2 + \rho^2(N^\phi \text{d}t + \text{d}\phi)^2 + \frac{r^2}{N^2 \rho^2} \text{d}r^2,$$

$$N^2 = \frac{\ell^2 (r^2 - r_h^2)}{\ell^2 \rho^2}, \hspace{1cm} N^\phi = -\frac{4GJ}{\rho^2}, \hspace{1cm} \rho^2 = r^2 + 4GM\ell^2 - \frac{1}{2}r_h^2, \hspace{1cm} r_h^2 = 8G\ell \sqrt{M^2 \ell^2 - J^2},$$  \hspace{1cm} (3)

where $M$, $J$ and $r_h$ are the black hole mass, angular momentum and horizon distance. Note that the black hole is in the Hilbert space of the CFT and goes to the AdS$_3$ vacuum when $M = -1/\ell$. The black hole mass and angular momentum are redefined in terms of additional constants $L_0$ and $\bar{L}_0$, which will be matched with the generators of the previous CFT. The redefined mass and angular momentum are

$$M = \frac{1}{\ell}(L_0 + \bar{L}_0) \hspace{1cm} \text{and} \hspace{1cm} J = L_0 - \bar{L}_0.$$  \hspace{1cm} (4)

The black hole solution is assumed to be the excitations of the vacuum obtained as $M = J = 0$. The vacuum is a different spacetime, globally, but there exists only one constant curvature metric locally in three dimensions. Therefore, it is equivalent to AdS$_3$, locally. In super CFT (sCFT), the Ramond ground state of the periodic boundary condition has zero mass, which corresponds to the vacuum solution. For an anti-periodic condition, the Neveu–Schwarz ground state gets a mass shift by $L_0 = \bar{L}_0 = -c/24$. The central charge of the CFT lying on AdS$_3$ and
equation (4) gives $M = -1/8G$ [15], and further evidences are in [8]. The asymptotic growth of states for given the CFT can be shown to obey Cardy’s formula [13]

$$S_{CFT} = 2\pi \sqrt{\frac{c n_R}{6} + 2\pi \sqrt{\frac{c n_L}{6}}}$$

where $n_R$ and $n_L$ are the eigenvalues of $L_0$ and $\bar{E}_0$, respectively, and $n_R + n_L \gg c$ in the semiclassical regime. Using equation (4), the asymptotic growth of states is rewritten in terms of the BTZ black hole mass and angular momentum:

$$S_{BH} = \pi \sqrt{\frac{\ell(\ell M + J)}{2G}} + \pi \sqrt{\frac{\ell(\ell M - J)}{2G}}$$

which is exactly the same as the Bekenstein–Hawking entropy [8].

3. Thermodynamics in the particle absorption process

The black hole’s properties are controlled by the particle momenta. To obtain the first-order geodesic equations, the separation of variable method is applied similar to the case of Kerr black hole [16]. The Hamiltonian and Hamilton–Jacobi action are, respectively,

$$\mathcal{H} = \frac{1}{2} g^{\mu \nu} p_\mu p_\nu$$

and

$$S = \frac{1}{2} m^2 \lambda - Et + L\phi + S_t(r),$$

where the conserved quantities are obtained from translation symmetries for $t$ and $\phi$ of the metric. The first-order geodesic equations are obtained as follows:

$$\dot{r} = p^r = \frac{N^2 \rho^2}{r^2} \sqrt{R(r)}, \quad \dot{t} = \frac{E - N^2 \lambda}{N^2}, \quad \phi = \frac{-N^2 L + EN^2}{N^2} = \frac{L}{\rho^2},$$

$$R(r) = \frac{r^2}{N^2 \rho^2} \left[ -m^2 + \frac{1}{N^2} [E^2 + 2N^2 EL + N^2 J^2] - \frac{L^2}{\rho^2} \right].$$

Using the radial geodesic equation, the dispersion relation described by the particle location and momenta is constructed as

$$E^2 + 2N^2 EL + N^2 J^2 = \frac{r^2 p^2}{N^2 \rho^2} - m^2 - \frac{L^2}{\rho^2} = 0.$$  \hspace{1cm} (9)

The particle energy is a positive-valued solution, which lies in the future-forwarding geodesics. The particle energy is absorbed to the black hole when it reaches to the outer horizon $r_h$. The particle energy constructed in terms of momenta at the horizon is obtained as

$$E = \frac{4GJ}{\rho_h^2} + \frac{r_h |p^r|}{\rho_h} \geq E_{\text{min}} = \frac{4GJ}{\rho_h^2} L, \quad \rho_h = \sqrt{4GM^2 + \frac{1}{2} r_h^2},$$

where the energy has a minimum from the positive sign of the coefficient $E$ square. It is achieved when $p^r = 0$ at the horizon. The black hole mass and angular momentum are changed as much as those of the particle, so it can be written as $\delta M = E$ and $\delta J = L$. The rewritten equation (10) is given by

$$\delta M = \frac{4GJ}{\rho_h^2} \delta J + \frac{r_h |p^r|}{\rho_h} \geq E_{\text{min}}.$$  \hspace{1cm} (11)

The black hole mass is a sum of the irreducible mass and rotational energy [2, 17]. After removing the rotational energy, the irreducible mass and its change are obtained:

$$M_{\mu} = \left( \frac{1}{2} \rho_h^2 + 4GM^2 \right)^{1/4} = \rho_h, \quad \delta M_{\mu} = \delta \left( \frac{1}{2} \rho_h^2 + 4GM^2 \right)^{1/4} = \frac{|p^r| r_h \ell}{4 \sqrt{\rho_h (M^2 \ell^2 - J^2)}}.$$  \hspace{1cm} (12)
where the change in the irreducible mass depends only on the particle radial momentum. The change in the irreducible mass has no dependence on the particle angular momentum. The black hole rotational energy can decrease or increase according to the sign of the particle angular momentum, so the irreducible mass is obtained from the black hole mass change and extracted from the change in the black hole rotational energy. Therefore, the change in the irreducible mass is only dependent on the particle radial momentum and always increased. The black hole mass can be described in terms of irreducible mass and angular momentum:

\[
M = \frac{M_i^4}{8G\ell^2} + \frac{2GJ^2}{M_i^2},
\]

(13)

Thus, the black hole mass can be divided into the irreducible mass and rotational energy. The black hole horizon area is proportional to the square of irreducible mass in equation (12), so the Bekenstein–Hawking entropy

\[
S_{BH} = \frac{A_H}{4G} = \frac{2\pi \rho_h}{4G} = \frac{\pi \sqrt{16GM\ell^2 + 2r_h^2}}{4G}, \quad \rho_h = \frac{1}{2}r_h^2 + 4GM\ell^2,
\]

(14)

where the black hole horizon area is denoted as \(A_H\). The change of the black hole entropy after the particle absorption is

\[
\delta S_{BH} = \frac{2\pi \delta \rho_h}{4G} = \frac{\pi M_i \delta M_i}{G} = \frac{\pi |p'| r_h \ell}{4G\sqrt{M^4 \ell^2 - J^2}} \geq 0,
\]

(15)

which depends only on the particle radial momentum. The entropy change is proportional to the absolute value of particle radial momentum, so the entropy always increases in this process. It saves the second law of thermodynamics. The changes in the thermodynamic properties of the black hole can be classified into four cases according to the particle rotating direction and radial momentum at the horizon. If the particle rotates in the same direction as that of the black hole, the black hole mass is increased. For a particle with a non-zero radial momentum, the entropy and irreducible mass are increased from equations (12) and (14). It is interpreted as an irreversible process, since \(\delta S_{BH} > 0\). The horizon location \(r_h\) is changed due to the absorption, because the black hole mass and angular momentum also change. From the increase in \(M_i\) from equation (12), the distance of the horizon is shown to be

\[
\delta r_h = \frac{32G^2\ell^4M}{r_h^2 \rho_h} |p'| - \frac{16G^2\ell^2J}{r_h \rho_h \ell}.
\]

(16)

The location of the horizon \(r_h\) can change by particle momenta. It is distinguished from the increase in the horizon area \(A_H\) in equation (14).

4. Microstate excitations in the dual CFT

The change of the black hole entropy in equation (15) is equivalent to that in equation (12). Thus, it can be written as the changes of \(n_R\) and \(n_L\):

\[
\delta S_{BH} = \delta \left[ \pi \sqrt{\ell(M + J)/2G} + \pi \sqrt{\ell(M - J)/2G} \right] = \pi \sqrt{\frac{c}{6n_R}} \delta n_R + \pi \sqrt{\frac{c}{6n_L}} \delta n_L = \delta S_{CFT},
\]

(17)

where the central charge \(c\) is invariant since the boundary is still AdS3 even after the absorption. The entropy change due to the microstate change should be the same as that of the Bekenstein–Hawking entropy. The state numbers can be rewritten in terms of the black hole mass and angular momentum through equation (4),

\[
n_R = \frac{1}{2}(\ell M + J), \quad n_L = \frac{1}{2}(\ell M - J), \quad c = \frac{3\ell}{2G}.
\]

(18)
The exact relations in equation (11) and (12) lead the changes in total and each state numbers possible to the exact expressions. The changes in the state numbers are
\[
\delta n_R = \frac{\ell r_h}{2\rho_h} |p'| + \frac{4G\ell J + \rho_h^2}{2\rho_h^2} L, \quad \delta n_L = \frac{\ell r_h}{2\rho_h} |p'| + \frac{4G\ell J - \rho_h^2}{2\rho_h^2} L.
\]
\[
\delta n_R + \delta n_L = \frac{\ell r_h}{\rho_h} |p'| + \frac{4G\ell J}{\rho_h^2} L. \quad (19)
\]
Note that \(4G\ell J - \rho_h^2 \leq 0\) for \(\ell M \geq J\). Each state numbers behave differently for given particle momenta. The particle radial momentum contributes similarly to \(n_R\) and \(n_L\), but the angular momentum contributes differently. The change in the total number of states is not matched with the entropy change, because the entropy is increased to maximize the state degeneracy. Therefore, the microstate structure and the degeneracy can be elucidated from the state number changes due to a particle absorption. Under the description,
\[
\delta S_{\text{CFT}} = \pi \sqrt{c \left[ \frac{\ell r_h}{6n_R} |p'| + \frac{4G\ell J + \rho_h^2}{2\rho_h^2} L \right] + \pi \left[ \frac{\ell r_h}{6n_L} |p'| + \frac{4G\ell J - \rho_h^2}{2\rho_h^2} L \right]} \quad (20)
\]
\[
= \pi \sqrt{\frac{\ell}{2G(J + \ell M)} \left( \frac{1}{2} (\delta M \ell^2 + \delta J) \right) + \pi \sqrt{\frac{\ell}{2G(-J + \ell M)} \left( \frac{1}{2} (\delta M \ell^2 - \delta J) \right) + \pi \sqrt{\frac{\ell}{2G(-J + \ell M)} \left( \frac{1}{2} (\delta M \ell^2 - \delta J) \right)}}
\]
\[
= \delta S_{\text{BH}}.
\]
Therefore, the two entropies are the same at the variation level. Oppositely, the black hole properties are described in terms of the CFT state numbers:
\[
M = \frac{1}{\ell} (n_R + n_L), \quad \delta M = \frac{1}{\ell} (\delta n_R + \delta n_L), \quad J = n_R - n_L, \quad \delta J = \delta n_R - \delta n_L.
\]
\[
M_{\mu} = [4G\ell (n_R + n_L + \sqrt{n_R n_L})]^{1/4} = [4G\ell (\sqrt{n_R} + \sqrt{n_L})]^{1/2}, \quad (21)
\]
\[
\rho_h = M_{\mu}^2 = \sqrt{4G\ell (\sqrt{n_R} + \sqrt{n_L})}.
\]
The dependence on the particle angular momentum in the change of the field theory entropy cancels. The entropy variation for \(S_{\text{CFT}}\) is
\[
\delta S_{\text{CFT}} = \pi \sqrt{\frac{c}{6} \left[ \frac{\delta n_R}{\sqrt{n_R}} + \frac{\delta n_L}{\sqrt{n_L}} \right]} = 2\pi \sqrt{\frac{c}{6} \delta [\sqrt{n_R} + \sqrt{n_L}]}
\]
\[
= 2\pi \sqrt{\frac{c}{6} \delta [\sqrt{n_R} + \sqrt{n_L} + 2\sqrt{n_R n_L}]}
\]
\[
= 2\pi \sqrt{\frac{c}{6} \left[ \frac{\ell r_h}{\sqrt{n_R} + \sqrt{n_L}} \right] \left[ \frac{\delta n_R + \delta n_L + n_R \delta n_L + n_L \delta n_R}{\sqrt{n_R n_L}} \right]}
\]
\[
= 2\pi \sqrt{\frac{c}{6} \left[ \frac{\ell r_h}{\rho_h} |p'| + \frac{4G\ell J L}{\rho_h^2} \right] + \frac{4G\ell J L}{\rho_h^2} \left( \frac{\ell^2 M_{\mu} |p'|}{\rho_h} + \frac{(4G\ell^2 - \rho_h^2)J L}{\rho_h^2} \right) \}
\]
\[
= \frac{4\pi}{\sqrt{n_R} + \sqrt{n_L}} \left[ \frac{c}{6} \ell r_h \right] \geq 0. \quad (22)
\]
Thus, the change in the field theory entropy is only dependent on the particle radial momentum and always increases.

5. Phase and microstate changes in particle absorption

We consider the black hole evolution by adding a particle. Starting with a black hole of given mass and angular momentum, the evolution shows how drastically black hole properties get
changed. Also, the changes are interpreted as microstate changes. In the process, the second law of thermodynamics is conserved when we change the black hole properties, so it is more similar to what an observer can watch in three-dimensional spacetime. In particular, the black hole horizon disappears for the specific case \( J = \ell M \). The possibility of this to happen will be shown from the particle-absorption point of view. We consider that a black hole has the angular momentum \( 0 < J < \ell M \) and mass \( M \). The behaviors depend on the particle radial momentum. The phenomenon is divided into three different cases. If the particle radial momentum is larger than \(-\frac{4GJ}{\ell r_{\text{ph}}}L\), the changes are

\[
|p'| > -\frac{4GJ\ell - \rho_h^2}{\ell r_{\text{ph}}}L \rightarrow \delta r_h \geq \frac{16G^2\ell^2}{r_h^2}\left(\frac{L}{\ell M} - J\right) + 8\ell^3GM, \\
\delta n_L \geq 0, \quad \delta n_R \geq L, \quad \delta n_{\text{tot}} \geq L,
\]

so the large value of the radial momentum increases \( n_L, n_R \) and \( n_{\text{tot}} \). Despite of the negative contribution of angular momentum in \( n_L \), the radial momentum increases \( n_{\text{tot}} \). If the particle radial momentum is in-between \( \frac{Jr_h}{2\ell^2M\rho_h}L \leq |p'| < \frac{4GJ - \rho_h^2}{\ell r_{\text{ph}}}L \rightarrow 0 \leq \delta r_h < \frac{16G^2\ell^2}{r_h^2}\left(\frac{L}{\ell M} - J\right) + 8\ell^3GM, \)

\[
-\frac{M\ell - J}{2\ell M}L \leq \delta n_L < 0, \quad \frac{M\ell + J}{2\ell M}L \leq \delta n_R < L, \quad \frac{J}{\ell M}L \leq \delta n_{\text{tot}} < L.
\]

In this case, the left-moving state number \( n_L \) is decreased due to the increase in the angular momentum, but the sum of states increase. When the particle radial momentum is smaller than \( \frac{Jr_h}{2\ell^2M\rho_h}L \), the changes are

\[
0 \leq |p'| < \frac{Jr_h}{2\ell^2M\rho_h}L \rightarrow -\frac{16G^2\ell^2J}{r_h^2L}L \leq \delta r_h < 0, \\
\frac{4G\ell J - \rho_h^2}{2\rho_h^2}L \leq \delta n_L < 0, \quad \frac{4G\ell J + \rho_h^2}{2\rho_h^2}L \leq \delta n_R < \frac{M\ell + J}{2\ell M}L, \quad \frac{4G\ell J}{\rho_h^2}L \leq \delta n_{\text{tot}} < \frac{J}{\ell M}L,
\]

where the horizon distance and \( n_L \) is decreased, but the entropy increases. Note that the black hole mass and the angular momentum are increased due to the particle absorption in the above three cases. If the black hole absorbs a particle satisfying equation (25), it can slowly attain an angular momentum.

Now, in the case of \( J \sim \ell M \), the particle momenta change black hole properties as

\[
0 \leq |p| \leq \frac{r_hL}{2\rho_h\ell}, \quad -\frac{4G\ell}{r_h}L \leq \delta r \leq 0, \quad \delta n_R \sim L, \quad \delta n_L \sim 0, \quad \delta n_{\text{tot}} \sim L,
\]

where \( r_h \) is approximately zero from equation (3), such that the decrease in the horizon distance is almost impossible. When a particle falls into the black hole, its radial momentum will be bigger than zero. The non-zero radial momentum changes the black hole properties such that

\[
|p| > \frac{r_hL}{2\rho_h\ell}, \quad \delta r > 0, \quad \delta n_R > L, \quad \delta n_L > 0, \quad \delta n_{\text{tot}} > L.
\]

Thus, adding mass and rotational energy makes a horizon distance bigger in size than before in almost all the practical falling. In these cases, the particle energy increases the black hole energy states. The black hole horizon distance becomes smaller in the specific range of a particle radial momentum, but the range becomes smaller when the particle angular momentum come closer to \( M\ell \). Finally, a non-zero radial momentum particle causes to increase the horizon distance and the black hole mass, so the horizon does not disappear.
6. Summary and discussions

In this paper, we studied the thermodynamic properties of the black hole absorbing a particle. The effects of particle absorption on a black hole evolution are analyzed in specific black hole mass and angular momentum ratios. In a (2+1)-dimensional black hole, the black hole thermodynamic properties are investigated by the momenta carried by a particle. The black hole irreducible mass and rotational energy are controlled by the particle total energy and momenta. In this process, the particle radial momentum always increases the irreducible mass of the BTZ black hole. On the other hand, the rotational energy can be increased or decreased depending on whether the particle rotating direction is parallel or anti-parallel. The particle-absorption process satisfies the second law of thermodynamics because it increases an irreducible mass, which is proportional to the entropy. The constraint on the changes in the black hole irreducible mass and rotational energy coming from the dynamics of the particle-absorption process prevents such unphysical cases as the violation of second law of thermodynamics.

The particle total energy and momenta transfer to the black hole mass and momenta in the particle-absorption process. To be described in terms of the particle momenta, the particle energy is obtained from first-order particle geodesic equations by the Hamilton–Jacobi method. Using geodesic equations, the particle energy is derived and determined by particle momenta. An inequality in equation (11) gives a relation between black hole mass and particle momenta in particle absorption. Since the black hole rotational energy can increase or decrease, removing rotational energy from the black hole mass change gives a continually increasing property. It is defined as an irreducible mass. The Bekenstein–Hawking entropy is proportional to the square of the irreducible mass. The changes in the irreducible mass and black hole entropy are only proportional to the particle radial momentum. Therefore, the entropy always increases in the particle absorption and saves the second law of thermodynamics.

If a particle has a non-zero radial momentum, the black hole irreducible mass increases, so the entropy also increases. It is a thermodynamically irreversible process. Differently, the behavior of the black hole horizon distance and mass are dependent on the relative rotating direction of the black hole and the particle. As a specific case, if a zero-radial momentum particle falls into the black hole, the change in the irreducible mass and entropy is zero, so it is a reversible process. In this case, the black hole rotational energy only changes.

Since the particle absorption changes the black hole properties, black hole microstates are also changed. These changes are obtained in terms of particle momenta, using the AdS/CFT duality. The entropy change is rewritten as a left and right quantum number in equation (6). The particle momenta can change these quantum numbers. The derived quantum number changes are in equation (19). For a given black hole mass and angular momentum, a particle radial momentum increases $n_L$ as much as $n_R$, but a particle angular momentum provides a positive contribution to $n_R$ and a negative contribution to $n_L$. For increasing a black hole angular momentum, the change of $n_R$ is larger than that of $n_L$, and the total numbers of states $\delta n_R + \delta n_L$ increases. For decreasing a black hole angular momentum, the change of $n_R$ is smaller than that of $n_L$, and the total numbers of states $\delta n_R + \delta n_L$ decreases.

The black hole configuration and microstates are investigated for given $M$ and $J$ ratios and for the general case of a black hole in $0 < J < M\ell$. Next, we investigate near extremal case $J \sim M\ell$. This (2+1)-dimensional black hole angular momentum has an upper bound at $J = M\ell$. In the case, the black hole has a very small horizon distance. In this case, the horizon distance, $n_R$, $n_L$ and $n_{\text{tot}}$ are increased for $|p_r| > \frac{\rho_{\text{hL}}}{2\rho_{\text{h}}}$. However, if a particle has a radial momentum in $0 \leq |p_r| \leq \frac{\rho_{\text{hL}}}{2\rho_{\text{h}}}$, the horizon distance can be smaller than before, in an ideal case. In $J \sim M\ell$ case, a horizon distance approaches to zero size, but $\rho_{\text{h}}$ is similar
to $4GM^2$, so making a smaller horizon distance is only possible in zero-radial momentum cases. When a particle falls into the horizon distance, the radial momentum is increased by the gravitational force. Thus, a zero-radial momentum particle is practically impossible. The black hole horizon distance is increased in the (2+1)-dimensional spacetime.

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