The large-scale three-point correlation function of the SDSS BOSS DR12 CMASS galaxies

Zachary Slepian,1* Daniel J. Eisenstein,1* Florian Beutler,2 Chia-Hsun Chuang,3,4 Antonio J. Cuesta,5 Jian Ge,6 Héctor Gil-Marín,7,8,9 Shirley Ho,10 Francisco-Shu Kitaura,4 Cameron K. McBride,1 Robert C. Nichol,9 Will J. Percival,9 Sergio Rodríguez-Torres,11,12,13 Ashley J. Ross,14 Román Scoccimarro,15 Hee-Jong Seo,16 Jeremy Tinker,15 Rita Tojeiro17 and Mariana Vargas-Magaña18

1Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA
2Lawrence Berkeley National Lab, 1 Cyclotron Rd, Berkeley CA 94720, USA
3Instituto de Física Teórica, (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain
4Leibniz-Institut für Astrophysik Potsdam (AIP), An der Sternwarte 16, D-14482 Potsdam, Germany
5Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona (IEEC-UB), Martí i Franquès 1, E-08028 Barcelona, Spain
6Astronomy Department, University of Florida, 211 Bryant Space Science Center, Gainesville, FL 32611, USA
7Sorbonne Universités, Institut Lagrange de Paris (ILP), 98 bis Boulevard Arago, F-75014 Paris, France
8Institute of Cosmology and Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth PO1 3FX, UK
9Bruce and Astrid McWilliams Center for Cosmology, Department of Physics, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA
10Instituto de Física Teórica, (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain
11Instituto de Física Teórica, (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain
12Campus de International UAM+CSIC, Cantoblanco, E-28049 Madrid, Spain
13Departamento de Física Teórica M8, Universidad Autonoma de Madrid (UAM), Cantoblanco, E-28049 Madrid, Spain
14Center for Cosmology and Astroparticles Physics, Department of Physics, The Ohio State University, OH 43210, USA
15Center for Cosmology and Particle Physics, New York University, 4 Washington Place, NY 10003, New York, USA
16Department of Physics and Astronomy, Ohio University, Clippinger Labs, Athens, OH 45701, USA
17University of St Andrews, North Haugh, St Andrews Fife KY16 9SS, UK
18Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, México

1 INTRODUCTION

The spatial distribution of galaxies on large scales contains a wealth of information on both the Universe’s contents and the laws under which these contents evolved. The three-point correlation function (3PCF) is an important tool for quantifying this distribution (Peebles & Groth 1975; Groth & Peebles 1977; Peebles 1980; Bernardeau et al. 2002). It measures the excess probability over random of finding three galaxies whose separations form a given triangle. In the consensus picture of structure formation, density perturbations were set up at the end of inflation as a nearly Gaussian random field (Starobinsky 1982; Bardeen, Steinhartd & Turner 1983). These primordial fluctuations are thus expected to be nearly fully characterized by their two-point correlation function (2PCF), with a negligible 3PCF (Planck Collaboration XVII 2016). However, subsequent evolution generates a 3PCF as well as imprinting additional

ABSTRACT

We report a measurement of the large-scale three-point correlation function of galaxies using the largest data set for this purpose to date, 777,202 luminous red galaxies in the Sloan Digital Sky Survey Baryon Acoustic Oscillation Spectroscopic Survey (SDSS BOSS) DR12 CMASS sample. This work exploits the novel algorithm of Slepian & Eisenstein to compute the multipole moments of the 3PCF in O(N^2) time, with N the number of galaxies. Leading-order perturbation theory models the data well in a compressed basis where one triangle side is integrated out. We also present an accurate and computationally efficient means of estimating the covariance matrix. With these techniques, the redshift-space linear and non-linear bias are measured, with 2.6 per cent precision on the former if σ_8 is fixed. The data also indicate a 2.8σ preference for the BAO, confirming the presence of BAO in the three-point function.

Key words: cosmology: observations – distance scale – large-scale structure of Universe.
information in the 2PCF. Further, the small level of primordial non-Gaussianity (PNG) generically predicted at the end of inflation produces its own unique signature in the 3PCF (Desjacques & Seljak 2010, for a review; Sefusati & Komatsu 2007 for forecasts; for recent observational work using large-scale structure, see Xia et al. 2010, 2011; Ross et al. 2013; Giamman Toni et al. 2014; for cosmic microwave background constraints, see Planck Collaboration XVII 2016).

This picture applies within the context of perturbation theory (PT) for the cosmic density field, i.e. for small perturbations around a homogeneous background. The galaxies we observe are tracers of this density field, but do so with some bias because galaxy formation is a complicated process (Kaiser 1984; Dekel & Rees 1987). In addition to information on the initial conditions and on high-redshift physics, the 2PCF and 3PCF therefore illuminate galaxy formation, revealing how faithfully galaxies trace the underlying matter density.

Since the 1970s, both 3PCF and 2PCF have been measured in galaxy redshift surveys of increasingly large volume and number of objects. In the last decade, a particular focus of 2PCF measurements has been to use the Baryon Acoustic Oscillation (BAO) method that exploits a sharp bump in the 2PCF, imprinted by sound waves in the early Universe, to constrain the Universe’s expansion history and thus the cosmological parameters (Eisenstein, Hu & Tegmark 1998; Blake & Glazebrook 2003; Hu & Haiman 2003; Linder 2003; Seo & Eisenstein 2003). The 3PCF is intrinsically weaker than the 2PCF, and consequently has not been a powerful additional lever on the BAO scale. Moreover, BAO analyses now routinely use reconstruction, a technique to move galaxies backwards along their inferred velocity vectors to try to undo some of the low-redshift non-linear evolution. Reconstruction reduces the amplitude of the 3PCF further and puts some of the distance information in the 3PCF back into the 2PCF (Eisenstein et al. 2007b; Padmanabhan, White & Cohn 2009; Burden et al. 2014). Schmittfull, Baldauf & Seljak (2015a) explicitly shows how three- and four-point statistics of the unreconstructed density field enter the reconstructed 2PCF.

Measurements of the 3PCF have further been complicated by its scaling with the number of objects. In contrast to the 2PCF, which requires pair counting and hence scales as \( N(nV_{\text{max}}) \), the 3PCF involves triplets of galaxies and so scales as \( N(nV_{\text{max}}^3) \), where \( N \) and \( n \) are the number and number density of galaxies and \( V_{\text{max}} \) is the volume within a sphere of radius \( R_{\text{max}} \), the maximal scale at which the correlations are measured. While \( nV_{\text{max}} \) is much smaller than \( N \) for large-volume surveys, this scaling has none the less mostly inhibited exploitation of the 3PCF on the acoustic scale of \( r_s \sim 100 \text{ Mpc} h^{-1} \).

It is not only measuring the 3PCF of the data that is time-consuming; in most methods, the number of points must also include a set of random points, much larger than the actual galaxy set, in order to model the survey geometry. Further, computing covariance matrices to fit parameters requires using potentially thousands of mock catalogues generated by \( N \)-body simulations combined with some prescription for matching galaxies to dark matter haloes. There is also a problem of visualization: the 3PCF depends on three independent variables (three triangle sides, or two triangle sides and an angle), so it has not been clear how to show the results of an analysis. In practice, what is often done is to show the angle dependence of the amplitude for certain fixed triangle side lengths or ratios.

For these reasons, 3PCF measurements have been limited in ability to exploit the full data sets used by the 2PCF. They often have focused on only certain triangle configurations (two-to-one, isosceles) and worked on scales smaller than the BAO scale. Even so, these measurements have been computationally challenging.

Historically, the 3PCF traces back to the pioneering work of Groth & Peebles (1977); our list of works here focuses on the recent and is not intended to be exhaustive.\(^1\) To our knowledge, the largest sample used for the 3PCF to date is 220 000 galaxies from SDSS DR6 in McBride et al. (2011a,b), with two-to-one triangles up to \( 27 \text{ Mpc} h^{-1} \). There have been a number of other works also using the SDSS data. Nichol et al. (2006) use 36 738 LRGs for two-to-one, three-to-one and four-to-one triangles on scales up to \( \sim 40 \text{ Mpc} h^{-1} \); Kulkarni et al. (2007) use 50 967 SDSS DR3 LRGs for isosceles, two-to-one, and three-to-one triangles on scales up to \( 30 \text{ Mpc} h^{-1} \); Marin (2011) uses 105 831 luminous red galaxies (LRGs) from SDSS DR3 for two-to-one triangles on scales up to \( 90 \text{ Mpc} h^{-1} \); Guo et al. (2014) use DR7 LRGs to measure isosceles, two-to-one, and three-to-one configurations up to \( 40 \text{ Mpc} h^{-1} \). Guo et al. (2015) use LRGs in the DR11 CMASS sample to measure two-to-one configurations up to \( 40 \text{ Mpc} h^{-1} \).

Data from the Two-degree-Field Galaxy Redshift Survey (2dFGRS) have also been used for 3PCF measurements: Jing & Börner (2004) and Wang et al. (2004) use, respectively, \( \sim 60000 \) and \( \sim 250000 \) galaxies from this survey on scales up to, respectively, \( 40 \text{ Mpc} h^{-1} \) and \( \sim 20 \text{ Mpc} h^{-1} \).\(^2\) The 3PCF has also been measured using the WiggleZ Dark Energy Survey: Marin et al. (2013) use isosceles, two-to-one and three-to-one triangles for scales up to \( 90 \text{ Mpc} h^{-1} \).

By concentrating only on particular triangle configurations, these measurements leave information in the 3PCF unexploited. Importantly, they also do not probe the BAO scale, which in principle offers cosmological distance information.

The only work to probe the BAO scale in the 3PCF thus far has been Gaztañaga et al. (2009) using a sample of \( \sim 40000 \) LRGs from SDSS DR7. They find a 2\( \sigma \)–3\( \sigma \) detection of the BAO using all opening angles of a single triangle configuration with side lengths \( r_1 = 33 \text{ Mpc} h^{-1} \) and \( r_2 = 88 \text{ Mpc} h^{-1} \). They did not fit for the BAO scale to extract distance information.

Similar data sets to those discussed above have been used for measuring the bispectrum, the Fourier space analogue of the 3PCF. In principle, the signal to noise is the same for the bispectrum and the 3PCF. In practice, whether Fourier space or configuration space is preferable for a given analysis will depend on the details of the sky survey and the details of the application. There are some disadvantages of the bispectrum relative to the 3PCF. First, edge correction is not straightforward in the bispectrum, as even simple survey boundaries in real space lead to complicated, infinite-support ringing in Fourier space. Secondly, the BAO features in the 3PCF are localized but in the bispectrum are dispersed, much as the sharp bump in the 2PCF corresponds to extended wiggles in the power spectrum. This localization is likely to make it easier in practice to fit for BAO information in the 3PCF than in the bispectrum.

None the less, a major advantage of the bispectrum is that it is obvious how to estimate it quickly. Even the most naive approach only requires comparison of pairs of wavevectors rather than galaxy triplets. Additional speed can come from using templates to directly exploit a sharp bump in the 2PCF, imprinted by sound waves in the early Universe, to constrain the Universe’s expansion history and consequently has not been a powerful additional lever on the BAO scale. Moreover, BAO analyses now routinely use reconstruction, a technique to move galaxies backwards along their inferred velocity vectors to try to undo some of the low-redshift non-linear evolution. Reconstruction reduces the amplitude of the 3PCF further and puts some of the distance information in the 3PCF back into the 2PCF (Eisenstein et al. 2007b; Padmanabhan, White & Cohn 2009; Burden et al. 2014). Schmittfull, Baldauf & Seljak (2015a) explicitly shows how three- and four-point statistics of the unreconstructed density field enter the reconstructed 2PCF.

Measurements of the 3PCF have further been complicated by its scaling with the number of objects. In contrast to the 2PCF, which requires pair counting and hence scales as \( N(nV_{\text{max}}) \), the 3PCF involves triplets of galaxies and so scales as \( N(nV_{\text{max}}^3) \), where \( N \) and \( n \) are the number and number density of galaxies and \( V_{\text{max}} \) is the volume within a sphere of radius \( R_{\text{max}} \), the maximal scale at which the correlations are measured. While \( nV_{\text{max}} \) is much smaller than \( N \) for large-volume surveys, this scaling has none the less mostly inhibited exploitation of the 3PCF on the acoustic scale of \( r_s \sim 100 \text{ Mpc} h^{-1} \).

It is not only measuring the 3PCF of the data that is time-consuming; in most methods, the number of points must also include a set of random points, much larger than the actual galaxy set, in order to model the survey geometry. Further, computing covariance matrices to fit parameters requires using potentially thousands of mock catalogues generated by \( N \)-body simulations combined with some prescription for matching galaxies to dark matter haloes. There is also a problem of visualization: the 3PCF depends on three independent variables (three triangle sides, or two triangle sides and an angle), so it has not been clear how to show the results of an analysis. In practice, what is often done is to show the angle dependence of the amplitude for certain fixed triangle side lengths or ratios.

For these reasons, 3PCF measurements have been limited in ability to exploit the full data sets used by the 2PCF. They often have focused on only certain triangle configurations (two-to-one, isosceles) and worked on scales smaller than the BAO scale. Even so, these measurements have been computationally challenging.

\(^1\) Different authors use different parametrizations; to standardize, we quote the largest triangle side possible in each work.

\(^2\) Wang et al. (2004) do not quote an exact number of galaxies; they use a sub-sample from 2dFGRS that in total has \( \sim 250 000 \) galaxies.

\[ \text{Downloaded from https://academic.oup.com/mnras/article-abstract/468/1/1070/2972997 by guest on 16 March 2020} \]
a finite number of separable basis functions (Schmittfull, Regan & Shellard 2013). Further, it has long been known that the opening-angle-averaged bispectrum can be computed using FTs (Scoccimarro 2000), a result recently extended to the full, unaveraged bispectrum (Scoccimarro 2015). Previous work on the bispectrum has been Feldman et al. (2001), Scoccimarro et al. (2001) and Verde et al. (2002), with the most recent measurement, using 690 827 LRGs from the SDSS DR11 CMASS sample, by Gil-Marín et al. (2015). This measurement uses isocases and two-to-one triangles in Fourier space up to scales of \( k = 0.03 \ h/\text{Mpc} \sim 30 \ \text{Mpc}^{-1} \). Gil-Marín et al. (2015) use only diagonal elements of the covariance matrix for parameter fitting since the large number of matrix elements relative to the number of mock catalogues available means the full covariance cannot be accurately computed empirically.

In this paper, we present the first measurements of the large-scale 3PCF for the CMASS sample of 777 202 LRGs within SDSS DR12, working on scales from 750 to 180 \( \text{Mpc}^{-1} \). We use the novel algorithm of Slepian & Eisenstein (2015b, hereafter SE15b; see also Slepian & Eisenstein 2015c) that computes the 3PCF’s multipole moments in \( \mathcal{O}(N(n V_{\text{max}})) \) time using spherical harmonic decompositions; the covariance matrix also turns out to be tractable in the multipole basis (SE15b). The main outcomes of this work are as following:

1. A highly accurate covariance matrix derived analytically but matching mock catalogues well.
2. Demonstrating that PT is in agreement with the data on these scales.
3. Measurement of the redshift-space linear and non-linear bias with 2.6 per cent precision on the former.
4. A 2.8\( \sigma \) detection of the BAO feature in the 3PCF.

The paper is laid out as follows. In Section 2, we briefly introduce the data set, the random catalogues used for edge correction and the mock catalogues used to verify our analysis pipeline and covariance matrix. Section 3 presents our multipole basis for the 3PCF and reviews the algorithm used to do the measurement. Section 4 recapitulates a compression scheme developed in previous work that we apply to the 3PCF here. Section 5 details our PT model, while Section 6 describes our approach to obtaining the covariance matrix. Section 7 shows the results of our analysis on both mock catalogues and the CMASS data. Section 8 concludes.

2 DATA, RANDOMS AND MOCKS

Here, we summarize the data set, random catalogues and mock catalogues used. The data are 777 202 LRGs within the CMASS sample of DR12 (Alam et al. 2015) of the Baryon Oscillation Spectroscopic Survey (BOSS; Eisenstein et al. 2011; Dawson et al. 2013) in the redshift range 0.43–0.7, with 568 776 in the North Galactic Cap over 6934 deg\(^2\) and 208 426 in the South Galactic Cap over 2560 deg\(^2\), for a total of 9493 deg\(^2\) (Reid et al. 2016, table 2). The sample is colour-selected to have roughly constant stellar mass and observations were completed in 2014. Target selection and the algorithms used for creating the galaxy catalogues and the associated random catalogues are presented in Reid et al. (2016), with the treatment for observational systematic biases described in Ross et al. (2012) and Ross et al. (2017). The 2PCF of this data set is used to constrain the cosmic distance scale via the BAO method in Cuesta et al. (2016). For our analysis, both data and randoms were FKP weighted (Feldman, Kaiser & Peacock 1994).

In total, the SDSS (York et al. 2000), comprising SDSS I, II (Abazajian et al. 2009) and SDSS III (Eisenstein et al. 2011), imaged more than one-third of the sky (14 555 deg\(^2\)) with a drift-scanning mosaic CCD camera (Gunn et al. 1998) in five photometric bandpasses (Fukugita et al. 1996; Smith et al. 2002; Doi et al. 2010) on the 2.5-m Sloan Telescope (Gunn et al. 2006) at Apache Point Observatory in New Mexico. The data underwent astrometric calibration (Pier et al. 2003), photometric reduction (Lupton et al. 2001), photometric calibration (Padmanabhan et al. 2008) and reprocessing for Data Release 8 (Aihara et al. 2011). Within BOSS, targets were assigned to tiles with an adaptive algorithm (Blanton et al. 2003), spectra obtained using double-armed spectrographs (Smeel et al. 2013) and redshifts derived as described in Bolton et al. (2012).

We also ran our pipeline on 299 mock catalogues developed for DR12 known as the MULTIDARK-PATCHY BOSS DR12 mocks (Kitaura, Yepes & Prada 2014; Kitaura et al. 2015, 2016). These catalogues used second-order Lagrangian perturbation theory (2LPT) combined with a spherical collapse model on small scales (Kitaura & Heß 2013). They were calibrated on accurate \( N \)-body-based reference catalogues using halo abundance matching to reproduce the number density, clustering bias, selection function and survey geometry of the BOSS data (Rodríguez-Torres et al. 2016).

3 BASIS AND ALGORITHM

We parametrize the 3PCF using two triangle sides \( r_1 \) and \( r_2 \) and the cosine of the angle between them, \( \cos \theta_{12} = r_1 \cdot r_2 \). We then project the angular dependence on to the basis of Legendre polynomials \( P_l \) of \( \cos \theta_{12} \), so that the 3PCF is written as

\[
\zeta(r_1, r_2; \hat{r}_1 \cdot \hat{r}_2) = \sum_l \zeta_l(r_1, r_2) P_l(\hat{r}_1 \cdot \hat{r}_2).
\]  

The multipole coefficients \( \zeta_l \) capture the 3PCF’s dependence on side lengths at each multipole. We bin in the two side lengths but our measurement uses the exact angle between the triangle sides when projecting on to the multipoles. The Legendre polynomials of even or odd \( l \) involve a falling sequence of even or odd powers of \( \cos \theta_{12} \), with \( l \) giving the maximal power. This expansion was first proposed by Szapudi (2004) but has not been used in any previous works save for the monopole term. Since \( P_0 = 1 \), the monopole corresponds to measuring the angle-averaged 3PCF (Pan & Szapudi 2005). The motivation for this expansion is that at leading order in PT, prior to cyclically summing over all triangle vertices, the 3PCF has only \( l = 0, 1, 2 \) moments (the PT is reviewed in Bernstein et al. 2002). Cyclically summing around the triangle vertices then introduces higher moments, but they do not contain substantial new information, tending to be dominated by the same population of squeezed triangles. These points are further discussed in Slepian & Eisenstein (2015a,b, hereafter SE15a and SE15b).

The Legendre polynomials and radial coefficients of equation (1) form a complete, orthogonal basis for any functions depending only on the relative angle \( \hat{r}_1 \cdot \hat{r}_2 \). This basis builds in isotropy, which is in reality not true due to redshift-space distortions (RSDs; see Hamilton 1998 for a review). By using this basis, we spherically average over all orientations to the line of sight, so the effects of RSD get integrated over much as is the case in the spherically averaged, isotropic 2PCF. We discuss RSD further in Section 5.2.

We directly measure the 3PCF’s multipole coefficients \( \zeta_l \) using an algorithm presented in SE15b that substantially accelerates the computation. We construct the counts over triplets of galaxies or randoms as follows. For each galaxy in the sample successively, we bin the surrounding galaxies into concentric spherical shells set by our radial binning around that galaxy and store the unit vectors for
each of the separations. On each spherical shell, we compute the multipole moments of the unit vectors and then convert these to the spherical harmonic decomposition $a_{lm}$ of the galaxy field. We then form all of the cross-powers between the shells as a function of the order $l$, summing over the azimuthal parameter $m$. We finally sum the resulting cross-powers over all central galaxies. The spherical harmonic computation around each galaxy is $O(nV_{\text{max}})$ and we must do this around each of the $N$ galaxies in the survey, leading to a 3PCF measurement that scales as $N(nV_{\text{max}})$. Further details such as implementation, tests and runtimes are given in SE15b.

We now briefly recapitulate the edge-correction framework of SE15b. We measure the projection on to multipoles of the Szapudi & Szalay (1998) estimator for the 3PCF,

$$\zeta = \frac{NNN}{RRR} = \sum \zeta(r_1, r_2) P_l(\hat{r}_1 \cdot \hat{r}_2), \quad (2)$$

where $N = D - R$, $D$ denotes a data point and $R$ denotes a random point. It can be shown that if the $NNN$ and $RRR$ counts are given by the multipole series

$$NNN = \sum_{l} N_l(r_1, r_2) P_l(\hat{r}_1 \cdot \hat{r}_2) \quad (3)$$

and

$$RRR = \sum_{l} R_l(r_1, r_2) P_l(\hat{r}_1 \cdot \hat{r}_2) \quad (4)$$

then

$$\frac{N_l}{R_0} = \zeta_l + \sum_{l} \zeta(2k+1) \sum_{l' > 0} \left( \begin{array}{ccc} l & l' & k \\ 0 & 0 & 0 \end{array} \right)^2 f_{l'}. \quad (5)$$

The $\zeta_l$ are the desired multipole coefficients of the 3PCF and the $f_{l'} = R_{l'}/R_0$ are the ratios of higher multipoles of the randoms’ triple count to its monopole. The Wigner 3j-symbol represents angular momentum coupling; it satisfies the triangle inequality $|l - l'| \leq k \leq |l + l'|$ and $l + l' + k$ must be even by parity.

We see that a vector of measured $N_l/R_0$ is simply a linear transformation of the vector of $\zeta_l$ we desire, and, given the matrix $I + M$ where $I$ is the identity matrix and $M$ has elements

$$M_{ij} = (2k + 1) \sum_{l' > 0} \left( \begin{array}{ccc} l & l' & k \\ 0 & 0 & 0 \end{array} \right)^2 f_{l'}, \quad (6)$$

the edge-correction equation becomes

$$N/R_0 = (1 + M) \zeta = A \zeta, \quad (7)$$

where $N = (N_0, N_1, \ldots, N_{l_{\text{max}}})$, and analogously for $\zeta$, with $l_{\text{max}}$ the maximum multipole to which we measure. This edge-correction equation can then be solved for $\zeta$ by matrix inversion. In practice, the $f_{l'}$ fall rapidly with $l'$, meaning we can truncate them to $l' = 10$ and compute edge-corrected $\zeta_l$ up to $l = 9$, as discussed in SE15b.

It is necessary to use a large number of random points so that the Poisson error from them is much smaller than the Poisson error from the galaxy data. We do this by computing $NNN$ with 1.5 times more randoms than data, weighting the randoms so that the sum of their weights is equal to that of the data (e.g. each random is given 1/1.5 the weight of a galaxy). The calculation of $NNN$ is repeated with 32 different sets of randoms, and the results then averaged to get our final $NNN$; this is equivalent to using 1.5 times more randoms than data when computing the data–random pairs. As described in SE15b, this process optimizes the minimization of the Poisson error for a fixed amount of computational time, by avoiding an expensive excess of random–random pairs. When computing the $NNN$ of mock catalogues, we use only 10 sets of randoms.

This incurs slightly more Poisson error in the resulting covariance matrix, but is threefold faster computationally. We estimate $RRR$ and the resulting $R_0$ and $f_{l'}$ factors using simply one set of randoms, 1.5 times more numerous than the data. This is sufficient for accurate answers, as these terms do not involve the difference of two nearly canceling ingredients as $N$ does.

4 COMPRESSION

Using an idea first advanced in SE15a, we compress the radial coefficients $\zeta(r_1, r_2)$ by integrating over $r_2$ in a region fixed by $r_1$. This has two motivations.

First, the integration region for $r_2$ can be set so as to avoid the squeezed limit where $r_2 = r_1$ and the third triangle side, $r_3$, can become zero. In this squeezed limit, as $\theta_{12} \rightarrow 0$, the two galaxies separated by $r_1$ begin to interact with each other strongly via gravity and so PT will likely be invalid. Further, the 3PCF becomes large when one triangle side is small, so these squeezed, third-side-zero triangles actually dominate the isosceles $r_1 = r_2$ signal. By choosing an integration region in $r_2$ that constrains $r_2$ to be greater than some minimum and also well different from $r_1$, we can avoid this squeezed limit. The compression scheme is shown in Fig. 1.

Secondly, compression lowers the dimension of the problem, reducing the dimension of the covariance matrix, easing parameter fitting and permitting simpler visualization of the data, as a 1D line plot versus $r_1$ rather than a 2D colour map of $\zeta(r_1, r_2)$ amplitudes versus $r_1$ and $r_2$. This framework simplifies analysis of the 3PCF.

However, we believe that the compression is not the optimal choice for extraction of BAO information. As SE15a shows that the full 3PCF in the multipole basis has structure where $r_3 = r_1 + r_2$ equals the BAO scale of 100 Mpc $h^{-1}$. This structure gets averaged out in this compression. In this work, we present the compressed approach as a first step, in that it can validate the covariance matrix and the basic broad-band agreement with PT. Even despite the non-optimality of the compression for this purpose, we find a BAO signal, and we will extend our fitting to the full two triangle side and multipole space soon.

We now present the explicit form of our compression scheme. The radial binning means the original compression scheme of SE15a, which took $r_1$ and $r_2$ as continuous variables, required modifications, described in SE15b. The compression used in that work and here is

$$\zeta'(r_1) = \frac{\sum_{r_2 \in S(r_1)} \zeta(r_1, r_2) \Delta V(r_2)}{\sum_{r_2 \in S(r_1)} \Delta V(r_2)}, \quad (8)$$

Figure 1. A visualization of the compression scheme; dots represent galaxies, and $r_1$ and $r_2$ are the triangle sides. The grey annulus contains allowed $r_2$ values given a particular $r_1$. In this work, the inner radius of the annulus is fixed to be $30$ Mpc $h^{-1}$ and the outer edge $r_1 = 30$ Mpc $h^{-1}$.
where superscript \(c\) denotes compressed, \(\Delta V(r_2)\) is the volume of bin \(r_2\) and \(S(r_2)\) is the set of all bins in \(r_2\) where \(r_2\) is greater than \(3\Delta r\) and less than \(r_2 - 3\Delta r\), with \(\Delta r\) the bin width. For our bin width of \(\Delta r = 10\) Mpc \(h^{-1}\), this prescription guarantees that \(r_2 > 30\) Mpc \(h^{-1}\) and \(r_2 - r_1 > 30\) Mpc \(h^{-1}\), meaning by the triangle inequality that \(r_2 > 30\) Mpc \(h^{-1}\). As an example, for \(r_1 = 100\) Mpc \(h^{-1}\), the \(r_2\) range integrated over is \(30\) Mpc \(h^{-1} - 70\) Mpc \(h^{-1}\), or four \(r_2\) bins given our \(\Delta r\). This compression scheme means that for \(r_1 < 70\) Mpc \(h^{-1}\), there are no allowed \(r_2\) bins, so it places a minimum on \(r_1\) as well.

Overall, the compression guarantees that all the galaxies in the triangle are well separated, so PT should describe them well. The framework of compression and its sensitivity to linear, non-linear and baryon-dark matter relative velocity bias are further discussed in SE15a. SE15b shows how this compression scheme translates to the covariance matrix that we will also briefly summarize when we turn to the covariance matrix in Section 6.

5 Model

5.1 Standard Perturbation Theory

We use standard perturbation theory (SPT) to compute the 3PCF at leading (fourth) order in the linear density field \(\delta\). Summarizing the calculation performed in SE15a, we begin with the real-space local (Fry & Gaztaña 1993) Eulerian galaxy bias model

\[
\begin{align*}
\delta(r) &= b_1 \delta_m(r) + b_2 \left[ \delta_p^2(r) - \langle \delta_m^2(r) \rangle \right],
\end{align*}
\]

where \(\delta_p\) is the galaxy overdensity at \(r\), \(b_1\) is the linear bias and \(b_2\) is the non-linear bias. We do not include a bias term coupling to the tidal tensor here. \(\delta_m\) is the matter density, given in terms of the linear density field as

\[
\delta_m(r) = \delta(r) + \delta^{(2)}(r)
\]

with \(\delta^{(2)}\) the usual second-order density field, most often given in Fourier space as

\[
\begin{align*}
\delta^{(2)}(k) &= \int d^3k_1 d^3k_2 \delta_D^3(k_1 + k_2 - k)F^{(2)}(k_1, k_2)\delta(k_1)\delta(k_2),
\end{align*}
\]

with \(\delta_D^3\) a 3D Dirac delta function and \(F^{(2)}\) the second-order kernel

\[
\begin{align*}
F^{(2)}(k_1, k_2) &= \frac{17}{21} P_0(k_1, k_2) + \frac{1}{21} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) P_1(k_1, k_2) + \frac{4}{21} P_2(k_1, k_2),
\end{align*}
\]

(11) (Bernardeau et al. 2002, equation 45).

We calculate the 3PCF to fourth order using the bias model (9) and focus on the pre-cyclic terms. This means that we choose one vertex of the triangle of galaxies at which to define \(\theta_{12}\) and the two sides enclosing it, \(r_1\) and \(r_2\). Note that in the product of three copies of \(\delta_p\) needed to form the 3PCF, each of the three galaxies can contribute a \(\delta^2\) and a \(\delta^{(2)}\). The pre-cyclic term is written by choosing one galaxy to contribute each of these (it may be the same one). We have chosen the third galaxy to contribute these more complicated terms. Since we can then take this galaxy to be at the origin, this approach simplifies the calculation. However, to connect with observations, where there is no ‘preferred’ vertex (galaxy), we eventually must sum cyclically, giving each galaxy in the survey the chance to contribute \(\delta^2\) and \(\delta^{(2)}\). Denoting ‘pre-cyclic’ with a subscript \(pc\), we find

\[
\begin{align*}
\xi_{pc}(r_1, r_2; \hat{r}_1 \cdot \hat{r}_2) &= \sum_{l=0}^{2} \xi_{pc}(r_1, r_2) P_l(\hat{r}_1 \cdot \hat{r}_2),
\end{align*}
\]

(13) with the coefficients as

\[
\begin{align*}
\xi_{pc0}(r_1, r_2) &= \left[ 2b_1^2 b_2 + \frac{34}{21} b_1^3 \right] \xi_1 \xi_2, \\
\xi_{pc1}(r_1, r_2) &= -b_1^2 \left[ \xi_1^{[1]} - \xi_2^{[1]} + \xi_1^{[1]} \xi_2^{[1]} + \xi_1^{[1]} \xi_2^{[1]} \right] \\
\xi_{pc2}(r_1, r_2) &= \frac{8}{21} \xi_1^{[2]} \xi_2^{[2]}.
\end{align*}
\]

(14) (15) (16) Subscripts indicate sides \(r_1\) or \(r_2\) as a function’s argument and \(\xi\) is the linear theory matter correlation function; \(\xi^{[1]}\) and \(\xi^{[1]+}\) are defined as

\[
\begin{align*}
\xi^{[1]}(r) &= \int_0^\infty \frac{dk}{2\pi^2} k^2 j_1(kr) P(k) k^{[1]},
\end{align*}
\]

(17) and

\[
\begin{align*}
\xi^{[2]}(r) &= \int_0^\infty \frac{dk}{2\pi^2} k^2 j_2(kr) P(k),
\end{align*}
\]

(18) where \(P(k)\) is the matter power spectrum.

We will consider two different matter power spectra: the physical power spectrum, denoted \(P_{phys}\), and a ‘no-wiggle’ power spectrum without BAO, denoted \(P_{nw}\). This no-wiggle power spectrum is computed using Eisenstein & Hu (1998)’s fitting formula for the no-wiggle transfer function, while the physical power spectrum is computed, following Eisenstein, Seo & White (2007a) and Anderson et al. (2012), as

\[
\begin{align*}
P_{phys}(k) &= [P(k) - P_{nw}(k)] \exp[-k^2 \Sigma_{BAO}^2/2] + P_{nw}(k),
\end{align*}
\]

(19) where \(\Sigma_{BAO} = 8\) Mpc \(h^{-1}\) is a non-linear smoothing scale to represent non-linear structure formation and RSD.

We pause to note an important feature of equation (17) regarding the BAO. The additional factor of \(k\) in \(\xi^{[1]+}\) amplifies the sharp, localized real-space BAO bump encoded in the power spectrum (SE15a). This factor of \(k\) stems from the \(\ell = 1\) term in the \(F^{(2)}\) kernel (12) that in turn is generated primarily (\(\sim 70\) per cent) by density gradients parallel to the velocity (SE15b; Bernardeau et al. 2002). The density Green’s function (response to an initial Dirac-delta function overdensity at high redshift) has a sharp BAO bump (Slepian & Eisenstein 2016), and so its divergence goes from positive to negative with a zero crossing at the BAO scale. Multiplying by the velocity amplifies structure at and beyond the BAO scale relative to that on much smaller scales because the velocity is larger at and beyond the BAO scale (SE15a). The remaining 30 per cent of the \(\ell = 1\) term in \(F^{(2)}\), which also enters \(\xi^{[1]+}\), stems from gradients of the velocity divergence parallel to the velocity that also contain BAO information. Mathematically, the weighting by \(k\) in equations (12) and (17) occurs because the gradient operator’s Fourier Transform scales as \(k\). As Section 7 will show, these points lead to an order-unity BAO feature in the weighted, compressed 3PCF’s dipole (\(\ell = 1\)).

After cyclically summing, we re-project on to the basis of Legendre polynomials to find the full radial coefficients in our multipole expansion (1) of the 3PCF as

\[
\begin{align*}
\xi_{\ell}(r_1, r_2) &= \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu_{12} \xi_{pc}(r_1, r_2, \mu_{12}) \\
&+ \xi_{pc}(r_2, r_3, \mu_{23}) + \xi_{pc}(r_3, r_1, \mu_{31}) \right] P_{\ell}(\mu_{12}),
\end{align*}
\]

(20)
with $\mu_{12} \equiv \cos \theta_{12}$ and analogously for $\mu_{21}$ and $\mu_{31}$. Note that $r_{12}$, $r_{23}$ and $r_{13}$ are all functions of $r_{1}$, $r_{2}$ and $r_{12}$, easily found using the law of cosines. The factor of $(2l+1)/2$ is necessary because $\int_{-1}^{1} P_\ell (\mu) P_\ell (\mu) d\mu = 2/(2l+1) \delta_{\mu_{\text{gu}}, 0}$; that is, the Legendre polynomials are an orthogonal but not orthonormal basis. Where in the pre-cyclic terms, we only found terms up to $l = 2$, cyclically summing introduces higher orders. These coefficients are shown versus $r_1$ and $r_2$ in SE15a fig. 9 split out by bias coefficient; since we set $b_i = 0$, only the first two columns of this three-column figure are relevant to this analysis. Also note that this figure is in Mpc, whereas this work uses Mpc $h^{-1}$.

The power spectrum used in these predictions is computed from the matter transfer function $T_m$ given by CAMB (Lewis 2000). We have $P = A k^\gamma T_m^2$, with $A$ a normalization set by matching $\sigma_8$ and $n_s$ the scalar spectral tilt. The transfer function is on a grid equally spaced in log $k$ with 5000 divisions per decade from $k = 3.46 \times 10^{-6}$ to 1.87 hMpc$^{-1}$. To avoid ringing in the transforms of equations (17) and (18), we multiply by a Gaussian smoothing in Fourier space $\exp \left[-k^4\right]$, which suppresses structure below 1 Mpc $h^{-1}$. The predictions are not sensitive to the exact value of the smoothing scale.

We use a geometrically flat $\Lambda$ cold dark matter cosmology with parameters matching those used for the MULTIDARK-PATCHY mock catalogues (Kitaura et al. 2016): $\Omega_m = 0.048, \Omega_{\Lambda} = 0.307 115, h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.6777, n_s = 0.9611$, $\sigma_8(\zeta = 0) = 0.8288, T_{\text{CMB}} = 2.7255$ K. These parameters do not differ substantially from the Planck values (Planck Collaboration XIII 2016). We rescale $\sigma_8$ by the ratio of the linear growth factor at the survey redshift to the linear growth factor at redshift zero. We take the survey redshift to be $z_{\text{survey}} = 0.565$ by simply averaging 0.43 and 0.7; since the redshift distribution of objects is roughly symmetric about the middle of the redshift interval, this is a good approximation.

Broadly, it is worth emphasizing that the PT model we use is fourth order in the linear density field: this is the lowest order at which the 3PCF is non-zero. It does not incorporate any modelling of spherical collapse and virialization. However, on the large scales ($> 30$ Mpc $h^{-1}$) we present here, it is likely that such effects are not important, especially because our compression scheme (Section 4) prevents any triangle side from becoming smaller than 30 Mpc $h^{-1}$.

5.2 Redshift-space distortions

Our model does not include any treatment of the effects of RSDs. In principle, RSD can generate additional angular and side-length-dependent structure in the 3PCF. In practice, however, we believe that in our compressed basis, the dominant effect of RSD is simply to rescale the multipole moments by some overall factor that depends only mildly on side length and multipole (SE15b). Consequently, the no-RSD PT model presented above can fit the data well, but the bias parameters appearing in it are actually redshift-space quantities rather than the usual real-space biases.

In SE15b, we measured the real-space and redshift-space compressed 3PCF multipole moments on scales from 45 – 90 Mpc $h^{-1}$ for the LasDamas SDSS DR7 mock catalogues. SE15b fig. 12 shows that RSD rescale the 3PCF by a factor of $\sim 1.5$–1.8 roughly independent of side length and multipole. Recent analytic work on the bispectrum by Scoccimarro, Couchman & Frieman (1999; hereafter SCF99) has shown that the rescaling is mostly set by $\beta = f/b_1$, where $f = d \ln D / d \ln a \approx 1 - 1.52 \ln a$ is the logarithmic derivative of the linear growth rate $D$ with respect to scale factor $a$ and $b_1$ is the real-space linear bias. For the LasDamas mocks, we found $\beta = 0.33$; using the real-space linear bias $b_1 = 2.086$ from the CMASS power spectrum (Gil-Marín et al. 2015) and the cosmology given in Section 5.1, $\beta = 0.37$ for this work. Given the similarity of the CMASS and LasDamas values of $\beta$, we expect that the impact of RSD on the CMASS compressed 3PCF is comparable to that shown in SE15b fig. 12.

Importantly, since at fixed $\gamma = 2b_2/b_1$ the 3PCF is proportional to $b_1^3$, any overall rescaling of the 3PCF due to RSD will only enter the measured redshift-space $b_1$ as a cube root.$^3$ Thus, rescaling factors in the range 1.5–2 will only change the implied real-space $b_1$ by 14–26 per cent.

In detail, the rescaling due to RSD does also depend explicitly on $b_1$. The real-space linear bias $b_1 = 1.90$ for LasDamas, comparable to the value $b_1 = 2.086$ Gil-Marín et al. (2015) find for CMASS, so the $b_1$-dependence is unlikely to make the effect of RSD different between the two. The rescaling also depends on $\gamma$; for LasDamas, we found $\gamma = 0.99$, while using the Gil-Marín et al. (2015) value for $b_2$ from the power spectrum, we have $\gamma = 0.43$ for CMASS. While these values differ, the dependence on $\gamma$ is weak, $\sim 3$ per cent in $\ell = 0$ and $\ll 1$ per cent for $\ell = 1$ and 2 for a factor of 2 change in $b_1$. Thus, the empirical result that RSD rescale the LasDamas compressed 3PCF roughly independently of side length and multipole is likely to hold for the CMASS compressed 3PCF as well.

We now offer physical intuition as to why this rescaling is roughly a constant. One can show analytically that the rescaling is independent of side length at $O(\beta)$ and nearly so at $O(\beta^2)$. Over a realistic range of $b_1$ and $b_2$, the variation with multipole is also modest. Given that $\beta^3 \sim 3$ per cent, higher order corrections might be expected to be unimportant.

In more detail, at $O(\beta)$, simplifying SCF99, we find the following rescaling factors:

$$\frac{\xi_{0,0}}{\xi_0} = 1 + \frac{94 + 42(b_1 + \gamma)}{102 + 63\gamma} \beta,$$
$$\frac{\xi_{1,1}}{\xi_1} = 1 + \left(1 + \frac{b_1}{3}\right) \beta,$$
$$\frac{\xi_{2,2}}{\xi_2} = 1 + \frac{4}{3} \beta. \quad (21)$$

Subscript $s$ denotes redshift-space. At $O(\beta^2)$, simple analytic formulae are not available but numerically taking the multipole moments of the SCF99 expression for the bispectrum (averaged over all orientations with respect to the line of sight) shows that dependence on the magnitudes of $k_1$ and $k_2$ is negligible. We have used these $O(\beta^2)$ results to estimate the rescaling expected for the LasDamas compressed 3PCF. For $\ell = 0$, 1 and 2, the empirical rescalings from SE15b are 1.50, 1.71 and 1.72. Our $O(\beta^2)$ results using SCF99 are 1.52, 1.70 and 1.80, which differ from the SE15b values by respectively 1.2 per cent, $-0.59$ per cent and 4.7 per cent. This agreement is encouraging and suggests that our approach could be used to relate redshift-space and real-space measurements to reasonable accuracy. For the CMASS sample, we estimate the rescalings are 1.74, 1.87 and 1.99 for $\ell = 0$, 1 and 2. While we expect these factors are accurate at the $\sim 5$ per cent level, we defer detailed modelling of RSD to future work. Here, we do not use these estimated rescalings to recover real-space biases from our redshift-space fits, but

---

$^3$ The term in $b_2^2b_3$ of equation (14) can be rewritten as $b_1^3\gamma$. $\gamma$ is defined following SCF99, but their real-space $b_3$ is defined such that it is twice the real-space $b_2$ of this work.
simply report the redshift-space best-fitting biases for the model equation (9).

We now summarize the approaches to RSD of other recent 3PCF and bispectrum works. For the bispectrum, we focus on Gil-Marín et al. (2015), the most recent work on the bispectrum and the one using the largest sample to date. This work uses the full bispectrum model of SCPP99 with an additional overall damping factor to describe fingers-of-God (Jackson 1972) from velocity dispersions inside virialized structures. The SPT kernels of SCPP99 are also modified to ‘effective’ kernels, though this is to better describe the smaller scale, weakly non-linear regime rather than to address RSD. Gil-Marín et al. (2015) also add a non-local bias term, which effectively rescales the $P_2$ and $P_3$ terms in the $\tilde{P}^{(2)}$ kernel.

For the 3PCF, we focus on Gaztañaga et al. (2009), which measures the 3PCF on BAO scales, and on McBride et al. (2011a,b), the largest sample used for the 3PCF previous to this work. Gaztañaga et al. (2009) measure the reduced 3PCF, which is the 3PCF divided by the hierarchical ansatz that $\xi(r_1, r_2, r_3) \sim \xi(r_1)\xi(r_2) + \xi(r_3)$, with $\xi$ the linear correlation function (Groth & Peebles 1977). They note that on large scales, RSD cancel out of the reduced 3PCF, as shown in their fig. 2.

McBride et al. (2011a,b) compare their observed reduced 3PCF with that computed from the dark-matter clustering of the Hubble Volume simulations, including RSD in the dark matter particles’ positions by using their peculiar velocities. They find that the reduced 3PCF of their sample is roughly a constant rescaling of the simulated dark-matter-only reduced 3PCF (their fig. 8) save on the smallest scales they measure ($\lesssim 6$ Mpc $h^{-1}$). If a simple linear bias model is assumed, this enables a measurement of the linear bias.

Finally, the MULTIdark PATCHY mocks used for parts of this analysis have been tuned to match small-scale quasi-virialized motions and the concomitant fingers-of-God, but for triangle opening angles $\theta = 0$ and $\pi$ do not reproduce the observed 3PCF well (Kitaura et al. 2016; fig. 6). However, this mismatch is for a triangle with $r_1, r_2 = 10$, 20 Mpc $h^{-1}$, and on the larger scales in this analysis, we expect that fingers-of-God will not be an important effect. In particular, the compression scheme guarantees that all triangle sides are greater than 30 Mpc $h^{-1}$, and further averages two triangle sides ($r_2$ and $r_3$) over rather large bins, additionally suppressing any finger-of-God effect.

5.3 Integral constraint model

We consider whether the randoms used for edge correction may have been incorrectly normalized and hence fail to satisfy the integral constraint that the mean number density of randoms matches the true number density of galaxies. Such a misnormalization can occur because we only know the measured number density of galaxies, which may be different from the true number density were one to perform the survey over an infinite volume.

We incorporate a possible failure of the integral constraint into the PT model fit to the data by adding a small fractional shift $c$ to the randoms, taking our 3PCF estimator from

$$\zeta = \frac{(D - R)}{RRR} \Rightarrow \frac{D - (1 + c)(1 + c)}{RRR(1 + c)^2}. \quad (22)$$

Expanding this out and simplifying, we find

$$\zeta^M = \zeta^T + c^2 \zeta_{cyc} - c^3. \quad (23)$$

where $\zeta^M$ is the measured 3PCF and $\zeta^T$ is the true 3PCF. $\zeta_{cyc}$ is the cyclic sum of the linear theory 2PCF around the three triangle sides,

$$\zeta_{cyc}(r_1, r_2, r_3) \equiv \zeta(r_1) + \zeta(r_2) + \zeta(r_3). \quad (24)$$

The 2PCF fits have often included similar nuisance polynomials, including a constant in $\xi$ (e.g. Anderson et al. 2012). Including this constant in the BAO fits helps them robust to observational systematics (Osomi et al. 2015; Ross et al. 2012, 2017). This term is also a good approximation of anomalous systematic power at small wavenumber.

6 COVARIANCE MATRIX

6.1 A hybrid approach

The covariance matrix $C$ measures the independence of different multipole and radial bin combinations from each other. In the limit of fully independent measurements, $C$ is diagonal. The $\chi^2$ for a parameter fit is calculated via $\mathbf{v}^T C^{-1} \mathbf{v}$, where $\mathbf{v}$ is a vector of the data points minus the model being fit. Minimizing the $\chi^2$ according to this prescription is guaranteed to give the optimal parameter fit if the likelihood is Gaussian.

However, this approach requires that the covariance matrix be invertible. Consequently, if the covariance matrix is determined empirically by measuring the 3PCF of mock catalogues, a large number of mocks for each matrix element is desirable (Percival et al. 2014). Given the large dimension of the covariance matrix, this is computationally costly, and so we have adopted an alternative approach. We compute the covariance matrix analytically and treat the effective survey volume and number density as free parameters to be fit by matching the empirical covariance derived from mock catalogues. This empirical covariance matrix, while likely too noisy to invert, is sufficient to constrain the volume and shot noise and reveal whether our analytic calculation describes the survey well.

Assuming that the density field is purely Gaussian and that the survey is boundary-free, the covariance of our multipole decomposition of the 3PCF can be analytically computed as (SE15b)

$$C_{GRF,l'l''l'}(r_1, r_2; r'_1, r'_2) = \frac{4\pi}{V} \int r^2 dr \sum_{l_1 l_2 l_3} \left[ \begin{array}{ccc} 1 & l'' & l_1 \\ 0 & 0 & 0 \end{array} \right] \left( \begin{array}{c} l'' \\ 0 \\ 0 \end{array} \right)^2 \times \left\{ \begin{array}{c} (1 - l)^3 \xi(r) \left[ f_{l''l'}(r; r_1, r'_1) f_{l''l''}(r; r_2, r'_2) \\
+ f_{l''l''}(r; r_2, r'_1) f_{l''l'}(r; r_1, r'_2) \right] + (-1)^{(l'' + l_1 + l_2)/2} \times \left[ f_{l''}(r; r_1) f_{l''}(r; r_2) f_{l'}(r; r_1, r'_1) f_{l'}(r; r_2, r'_2) \\
+ f_{0}(r; r_1) f_{l''}(r; r'_1) f_{l'}(r; r_1, r'_2) \right] + f_{0}(r; r_2) f_{l''}(r; r'_2) f_{l'}(r; r_2, r'_1) \right\}, \quad (25)$$

with

$$f_{l'}(r; r_1) = \int \frac{k^2 dk}{2\pi^2} \left[ P(k) + \frac{1}{n} \right] j_l(k r_1) j_l(k r), \quad (26)$$

and

$$f_{l''l'}(r; r_1, r'_1) = \int \frac{k^2 dk}{2\pi^2} \left[ P(k) + \frac{1}{n} \right] j_l(k r_1) j_{l'}(k r_1) j_{l'}(k r), \quad (27)$$

where $V$ is the effective survey volume and $n$ is the effective number density of the survey. The $1/n$ term added to the power spectrum above characterizes the shot noise contribution to the covariance.
matrix. The covariance as above can be easily binned radially and compressed by integrating out \( r_1 \) and \( r_2 \), as detailed in SE15b.

Note that linear PT is insufficient to obtain the 3PCF for a purely Gaussian Random Field, because as an odd moment the 3PCF vanishes. However, linear theory does give the covariance because it is a six-point function and hence an even moment. An analogy here is to a distribution with zero mean (the expected signal, the 3PCF), but non-zero width (the covariance).

We now wish to find the volume \( V \) and number density \( n \) (or shot noise \( 1/n \)) to use in our analytic covariance that will most closely mimic the true covariance of the data as estimated from the mocks. This can be done using a likelihood proposed in Xu et al. (2012) (their equation 22). This prescription computes the likelihood of a given analytic covariance matrix using vectors \( x_i \) of the differences between the mock multipoles \( \zeta(r_1, r_2) \) for the \( i \) th mock and their mean taken over all mocks. The dimension of each \( x_i \) is the number of multipoles times the number of radial bins squared. For the \( i \) th mock, the covariance matrix of the 3PCF can be formed from \( x_i \) and this covariance can then be averaged over all mocks for a more accurate estimate of the empirical covariance.

We explicitly evaluate the Xu et al. (2012) likelihood for a list of number densities using bisection to converge on the best value. Obtaining the best number density and volume turns out to require only a 1D rather than a 2D search: the likelihood-maximizing volume \( V_{\text{best}} \) is uniquely determined by the number density. In particular, at a fixed number density, we have

\[
V_{\text{best}} = \frac{d}{\langle x_i^2 \rangle},
\]

where \( d \) is the dimension of the covariance matrix and \( \langle x_i^2 \rangle \) is the mean \( x_i^2 \) of all the \( x_i \) (essentially the mock covariances) with respect to the analytic covariance matrix with the given number density and with the volume set to unity. Explicitly,

\[
\langle x_i^2 \rangle = \frac{1}{N_{\text{mock}}} \sum_{i=0}^{N_{\text{mock}}} x_i \mathbf{C}_{i,\text{GRF}}^{-1} x_i^T,
\]

where \( \mathbf{C}_{\text{GRF}} \) is the analytic covariance matrix with \( V = 1 \) and a given shot noise. The \( x_i \) are defined in the previous paragraph above.

We find a best-fitting number density of \( n = 1.80 \times 10^{-4} \) \( h^3 \text{Mpc}^{-3} \)

\[
= 5.60 \times 10^{-4} \text{Mpc}^{-3}
\]

and a best-fitting volume of \( 1.92 \text{Gpc}^3 \) \( = 6.17 \text{Gpc}^3 \). The average number density of the CMASS sample is \( 7.20 \times 10^{-5} \text{Mpc}^{-3} \) and the true volume \( 10.8 \text{Gpc}^3 \) (Cuesta et al. 2016). It is encouraging that the number density and volume found from our covariance matrix fitting are comparable to the true values for the survey. However, we caution that any differences should not be overinterpreted. Our best-fitting volume and shot noise are simply parameters in our analytic covariance matrix. They are likely driven by differences between the analytic and empirical covariance, such as non-linear structure formation, as well as by the raw survey number density and volume.

Fig. 2 shows the reduced covariance matrix used in our parameter fitting, i.e. that given by equation (25) after compression and with the values of \( V \) and \( n \) quoted above. Away from the diagonal, the covariance is strongest for low multipoles, indicated by the higher amplitude in the lower left corner.
4.3 Details of analytic covariance matrix numerical implementation

Here, we briefly detail our numerical implementation of the compressed version of equations (25)–(27). As discussed in SE15b, the covariance can be efficiently computed following the prescription of Section 4 by compressing the $f$ tensors equations (26) and (27) first when their arguments include either $r_2$ or $r_1'$ or both. However, for some terms in the covariance, the tensors only have $r_1$ and $r_1'$ as arguments, meaning we still need the full uncompressed tensors. We form these tensors on a linearly spaced grid in $k$ with 3000 points from $10^{-4}$ hMpc$^{-1}$ to 2 hMpc$^{-1}$. These tensors also involve a dummy variable $r$ that is integrated over at the end of the covariance calculation (see equation 25); for this, we use a linearly spaced grid with 4100 points from $10^{-5}$ Mpc$^{-1}$ to 100 Mpc$^{-1}$. A number of different endpoints for each grid and number of points for each grid were tested and the results found to be well converged for the choices above.

Binning adds an additional complication to the covariance calculation; as discussed in SE15b, one can analytically bin-average the spherical Bessel functions within the $f$ tensors (equations 26 and 27) to incorporate this. In practice, we found that simply numerically averaging them over the required bins was more accurate. To obtain

the $f$ tensors, we use Python’s internal implementation of the spherical Bessel functions to produce lookup tables; these need only be computed once even if one wishes to test a variety of cosmological parameter values in the 3PCF predictions.

Finally, for the covariance matrix, we wish to match as closely as possible the observed redshift-space galaxy power spectrum. We do so by rescaling $kP(k)/(2\pi^2)$ to be 90 h$^{-2}$ Mpc$^2$ at its maximum to match the observed CMASS power spectrum (Anderson et al. 2014). This normalization incorporates RSDs and linear bias as they enter the power spectrum; empirically, we find that the required rescaling factor is 4.43.

7 RESULTS

7.1 Mock and data results

For both the CMASS data and the 299 Multidark-Patchy mocks, we used the algorithm of SE15b to measure the 3PCF in 18 radial bins of 10 Mpc h$^{-1}$ width each, covering triangle sides from 0 to 180 Mpc h$^{-1}$. Our compression scheme means that there are no allowed $r_2$ bins for $r_1 < 70$ Mpc h$^{-1}$ (Section 4), so after compression, we have 11 bins for $r_1$ going from 70 Mpc h$^{-1}$ to 180 Mpc h$^{-1}$. The run time was 2 h per mock on a 36-core node, about 20k core-hours for the full computation. Computing the 3PCF of data and mocks was, by far, the dominant time cost of the calculation, with all other calculations taking negligible time. These include computing the covariance matrix and fitting for the best volume and shot noise, as well as fitting redshift-space linear and non-linear bias and the integral constraint parameter $c$ to the data and mock results.

For all fits, we use 10 multipoles, from $l = 0$ to 9 and 11 radial bins at each multipole. These fits for the mean of the 299 Multidark-Patchy mocks are displayed in Figs 4 and 5. The error bars plotted are from the diagonal of the covariance matrix divided by $\sqrt{299}$ as is appropriate for a measurement of the mean of 299 mocks. We have
multiplied the compressed 3PCF multipoles by \((r_1/10)^4\) to remove the large-scale fall-off of the 3PCF, which from the hierarchical ansatz, we expect to scale like the square of the 2PCF, \(\propto \ell^2 \propto r^{-4}\) (Groth & Peebles 1977). Notice the good match between the data and the PT. In particular, note the features at the BAO scale of \(r_1 = 100\, \text{Mpc}\, h^{-1}\) in both \(\ell = 5\) and 6. Note the similarity of these higher \(\ell\) panels to each other, and indeed to \(\ell = 4\) in Fig. 4. This similarity indicates there is less information in the higher multipoles, for reasons further discussed in Sections 3 and 5.1. Essentially, prior to cyclic summing, the leading-order 3PCF has only \(\ell = 0, 1,\) and 2 terms, and so in our model higher multipoles stem from the purely geometric effect of cyclic summing. RSD can in principle introduce higher-multipole angular structure even at the pre-cyclic level, but it seems this effect is not substantial.

This fit shows that lowest-order PT in our compressed multipole basis appears to describe the 3PCF of these mocks fairly well. In particular, it supports our earlier claim that the primary effect of RSD in this compressed basis is a roughly multipole and side-length-independent rescaling of the 3PCF.

None of the less, the fit's \(\chi^2/\text{d.o.f.} = 405.53/107 = 3.79 > 1\), meaning that the PT is not a perfect match to these 299 mocks. We suspect that this imperfect fit stems both from higher-order corrections to our leading-order PT model as well as possible scale or multipole-dependent effects of RSD. None of the less, this \(\chi^2\) for the mocks suggests that the model should be more than sufficient to fit the data well. Given that \(\chi^2 \propto C \propto V\), we expect the model only begins to be insufficient for surveys with \(V \sim 80\) that of SDSS DR12. Indeed, the PT model is an adequate fit to the data, as we will soon show.

Table 1 gives our best-fitting values of \(b_1, b_2\) and \(c\) for the mean of the 299 mocks. Again note these biases are redshift-space quantities. The error bars quoted in Table 1 are from integrating over the probability distribution given by \(P \propto \exp[ -\chi^2/2]\) to compute \(\sigma_{b_1} = \sqrt{\langle b_1^2 \rangle - \langle b_1 \rangle^2}\) from \(\langle b_1 \rangle\) and \(\langle b_1^2 \rangle\) and the same for \(b_2\) and \(c\). They are the error bars we might expect if we did the measurement on a volume 299 times that of the SDSS DR12 CMASS sample. To verify our analysis procedure, we also examined the probability contours in \(b_1 - b_2\) space having marginalized over \(c\), checking that the contours were consistent with our quoted error bars on \(b_1\) and \(b_2\). We do not show this plot here though we will display it for the CMASS data.

Finally, Rodríguez-Torres et al. (2016; see fig. 14) measure the linear bias for a set of 100 MULTIDARK-PATCHY mocks using the 2PCF on scales up to 60 Mpc\(h^{-1}\). That work’s model involves only linear bias and uses the Kaiser (1987) formula to remove the effects of RSD. They find some scale-dependence to the bias; the largest scale they measure is that most directly comparable to this work. At this scale, the real-space \(b_1 \simeq 1.8\), comparable to our inferred real-space bias of 1.90–2.00 if the rescaling factors of Section 5.2 are applied to the redshift-space mock bias \(b_1 = 2.390\). The difference between our inferred real-space \(b_1\) and Rodríguez-Torres et al.’s measured value should not be overinterpreted for several reasons. First, the RSD factors used to convert our redshift-space bias are approximate; secondly, the bias model of Rodríguez-Torres et al. (2016) includes only linear bias (ours includes non-linear bias as well) and thirdly, they compute the bias by comparing the measured 2PCF to that from the dark matter, while we compare the measured 3PCF to the PT model of Section 5.1.

We fit the CMASS data in the same way as the mocks, with the results displayed in Figs 6 and 7. The error bars plotted are again from the diagonal of the covariance matrix. Again, notice the match between data and PT. Though the match looks visually worse for the CMASS data than for the mocks, in fact the \(\chi^2/\text{d.o.f.} = 107.64/107 = 1.01\), meaning the PT model is adequate for the data set, whereas for the mocks it was not.

In the case of the mocks, one might have worried that, since they are based on 2LPT, by construction PT would fit them well. However, the fair match between data and model shows that on these large scales and in our compressed basis, leading-order PT describes the 3PCF with reasonable fidelity.

The best-fitting parameters for the data are in Table 1. We measure the redshift-space linear bias \(b_1\) with 2.60 per cent precision at fixed \(\sigma_8\). Further, in concert with the power spectrum or 2PCF, our work can be used to place a constraint on \(\sigma_8\). If the power spectrum, \(P \propto b_1^4\sigma_8^2\), and the redshift-space to real-space rescaling are known to high accuracy, then dividing \(b_1^4\sigma_8^2\) by \(P^{1/2}\) isolates \(\sigma_8\) with 7.8 per cent precision (this approach was originally proposed in Fry 1994).

### Table 1

| b_1          | b_2          | c         | \(\Delta \chi^2\) no BAO vs. BAO |
|--------------|--------------|-----------|----------------------------------|
| MULTIDARK-PATCHY mocks | 2.390 ± 0.003 | 0.32 ± 0.04 | 0.0000 ± 0.0006 | 3234.34 |
| CMASS SDSS DR12 sample | 2.23 ± 0.06 | 0.3 ± 0.7 | -0.023 ± 0.007 | 7.58 |

---

**Figure 5.** Same as Fig. 4 but now for multipoles \(\ell = 5 - 9\). Notice the clear bump at the BAO scale of \(r_1 = 100\, \text{Mpc}\, h^{-1}\) in both \(\ell = 5\) and 6. Note the similarity of these higher \(\ell\) panels to each other, and indeed to \(\ell = 4\) in Fig. 4. This similarity indicates there is less information in the higher multipoles, for reasons further discussed in Sections 3 and 5.1. Essentially, prior to cyclic summing, the leading-order 3PCF has only \(\ell = 0, 1,\) and 2 terms, and so in our model higher multipoles stem from the purely geometric effect of cyclic summing. RSD can in principle introduce higher-multipole angular structure even at the pre-cyclic level, but it seems this effect is not substantial.
Figure 6. A fit of PT predictions, computed using the physical power spectrum, to the compressed 3PCF’s multipoles $\ell = 0 - 4$ for the CMASS sample. Notice the peaks in $\ell = 0, 1, 4$ around the BAO scale of $r_1 = 100 \text{ Mpc}$ $h^{-1}$. In particular, compare the $\ell = 1$ panel here with that of Fig. 4 to aid in identifying the peak and trough theting induce in the 3PCF’s dipole moment. The points in the peak are anti-correlated with those in the trough, as shown in Fig. 2 (second tile on the diagonal). These points are therefore more constraining than the error bars shown would suggest. The error bars plotted are the diagonal of the covariance matrix, and the $\chi^2$/d.o.f. = 107.64/107.

Figure 7. Same as Fig. 6 but now for multipoles $\ell = 5 - 9$. These higher multipoles appear noisier than their lower-$\ell$ counterparts, as indicated by the larger number of points more than 1$\sigma$ distant from the model. While the error bars are similar in magnitude to those in Fig. 6, the signal is reduced relative to the largest in Fig. 6 (i.e. $\ell = 2$ and 3).

We obtain essentially no constraint on $b_2$. On these large scales, the 3PCF in our compressed basis seems to be insensitive to the redshift-space non-linear bias aside from indicating its existence.

Figure 8. The probability contours for the redshift-space biases $b_1$ and $b_2$ having marginalized over the integral constraint. The red ellipse contains 68 per cent of the probability and the light blue 95 per cent. One can see that our measurement obtains a good constraint on $b_1$ but has very little constraining power on $b_2$, a conclusion borne out quantitatively by the large error bar on $b_2$ relative to that on $b_1$ quoted in Table 1.

7.2 Searching for the BAO

To determine the significance of a BAO signal in our compressed multipole measurements of the 3PCF, we fit PT predictions for the 3PCF computed using the no-wiggle model to both mocks and data, and compute the $\Delta \chi^2$ relative to our fits of Section 7.1, which used the physical power spectrum equation (19). We emphasize that the BAO significance always stems from comparing the no-wiggle model to the physical power spectrum model. In this work, we do not fit for the BAO scale itself to extract distance information, but this will be a direction of future work.

For the mean of the MultiDark-Patchy mocks, we find a clear preference for the BAO model. The comparison is between Figs 4 (with BAO) and 9 (without BAO). The $\Delta \chi^2$ is 3234.34, meaning if we had a survey volume 299 times as large as CMASS, we would expect a 56.9$\sigma$ BAO detection even in our compressed 3PCF. The reason for the large $\chi^2$ penalty of the no-BAO model over the model with BAO can be seen by the visual comparison of Figs 4 and 9 around the BAO scale of $r_1 = 100 \text{ Mpc}$ $h^{-1}$, most prominently in $\ell = 1$ but also in $\ell = 0$ and 4.

For the CMASS data, we again find a preference for the BAO, with $\Delta \chi^2 = 7.58$, meaning a 2.8$\sigma$ preference for the BAO. One can see, comparing Figs 6 and 10, that both physical and no-wiggle models fit the data reasonably well, but that around the BAO scale of $r_1 = 100 \text{ Mpc}$ $h^{-1}$, the no-wiggle model fits less well. Scaling the 56.9$\sigma$ detection from the 299 mocks’ mean down by $\sqrt{299}$ to mirror the volume of CMASS, we expect on average a 3.29$\sigma$
no-wiggle power spectrum differs slightly from the physical power spectrum on small scales and $b_2$ changes to absorb this difference.

8 CONCLUSIONS

We have measured the large-scale 3PCF of the SDSS DR12 CMASS sample of 777,202 LRGs. The novel multipole algorithm of SE15b permits, us for the first time, to take advantage of all triangle configurations. We have used a compression scheme first developed in SE15a to reduce the dimension of the full 3PCF multipole coefficients and to avoid the triangles for which PT is likely invalid. We have shown that, in this basis, the analytic covariance matrix of SE15b, which assumes a Gaussian Random Field density and a boundary-free survey, is a good match to the empirical covariance matrix derived from 299 MULTIDARK-PATCHY mock catalogues. Using our analytic covariance matrix with volume and shot noise derived from the mocks, we have fit for the redshift-space linear and non-linear bias as well as a constant to marginalize over possible failure to satisfy the integral constraint. We measure the redshift-space linear bias with 2.6 per cent precision. We also find a 2.8σ preference for the BAO in the data by comparing a physical model with BAO to the Eisenstein & Hu (1998) 'no-wiggle' model.

This work uses the largest number of galaxies to date for the 3PCF by a factor of roughly 4, and is unique in exploiting all triangle configurations. We also measure the 3PCF on significantly larger scales (roughly a factor of 2) than previous works. Our error bar on the redshift-space linear bias is competitive with that found on the real-space linear bias from recent bispectrum studies such as Gil-Marín et al. (2015), which measures the combination $b_1^{1060}\sigma_8(z_{	ext{eff}})$ with 3.5 per cent precision (their table 2). If $\sigma_8$ is fixed, as we do in this work, their measurement translates to 2.6 per cent precision on $b_1$.

Gil-Marín et al. (2015) use 690,827 LRGs in the CMASS sample of DR11 for their bispectrum, finding $b_1^{1060}\sigma_8(z_{	ext{eff}}) = 1.672 \pm 0.060$. This work uses a detailed, complex model for removing RSD to obtain the real-space bias. Using our value of $\sigma_8$ to convert $b_1^{1060}$ to $b_1$ translates this measurement to a real-space linear bias $b_1 = 2.03$. Gil-Marín et al. (2015) also measure $b_1$ and $b_2$ from the power spectrum, finding $b_1 = 2.086$ and $b_2 = 0.902$; they do not quote error bars on the parameters derived from the power spectrum.

Our measured redshift-space $b_1$ is likely enhanced by $\sim 20$–25 per cent relative to its real-space value based on the rescaling factors computed in Section 5.2. Reducing our value of 2.23 by 25 per cent relative to its real-space value based on the rescaling factors computed in Section 5.2. Reducing our value of 2.23 by 25 per cent relative to its rescaled value leads to an inferred real-space linear bias of $\sim 2.0$. From the projected 2PCF, Guo et al. (2013) infer a real-space linear bias of $b_1 = 2.16 \pm 0.01$ using subsamples from the CMASS sample of DR9 with a maximum of 260,000 galaxies. Guo et al. (2013) uses projected separations of 3–25 Mpc $h^{-1}$ and a detailed model of non-linear structure formation. That work’s cosmology is WMAP7, with $\sigma_8(z = 0) = 0.8$, whereas in this work, $\sigma_8(z = 0) = 0.8288$. Rescaling the Guo et al. (2013) measurement to our $\sigma_8$ yields a real-space linear bias of $b_1 = 2.1$. Note that the projected measurement is insensitive to RSD.
From the full 2PCF, Ross et al. (2014) uses 540,505 CMASS galaxies in DR10 to measure the real-space linear bias, finding \( b_1 = 1.96 \pm 0.05 \). \( \sigma_z(z = 0) = 0.8 \) for this work, and rescaling as discussed above reduces this value to \( b_1 = 1.89 \), comparable to our expected real-space range of \( b_1 \sim 1.77-1.85 \). We caution that the galaxy samples used in Guo et al. (2013) and Ross et al. (2014) are again proper subsets of our own (roughly 30 percent and 70 percent, respectively), so one would not necessarily expect the bias values to be equal.

The 2PCF and 3PCF depend on different combinations of \( b_1 \) and \( \sigma_z \) (Fry 1994), so measuring the two in concert can permit isolating a value for each. In this work, we have not pursued a combined fit to the 2PCF and 3PCF; future work might explore this approach. Such work would require the covariance between the 2PCF and the 3PCF, a topic explored for the Fourier-space analogues in Sefusatti et al. (2006), who found that squeezed triangles dominate this covariance. As our compression scheme avoids such triangles, it may be beneficial in the context of joint 2PCF–3PCF fitting.

Regarding squeezed triangles, the position-dependent two-point correlation function, developed in Chiang et al. (2015), likely offers interesting complementary information not captured by our compression. This technique evaluates the 2PCF within sub-volumes conditioned on the mean overdensity of those subvolumes, and is therefore sensitive to an integral of the 3PCF dominated by squeezed triangles. Chiang et al. (2015) have already applied the technique to the SDSS DR10 CMASS sample to measure the non-linear bias \( b_1 \). In future, it would be worthwhile to explore joint parameter constraints using the position-dependent 2PCF and our compressed 3PCF.

We will also explore more complicated bias models, such as Lagrangian schemes (shown to be more accurate in Chan, Scoccimarro & Sheth 2012) that include tidal tensor bias (McDonald & Roy 2009; Baldauf et al. 2012). Adding a tidal tensor bias might modestly raise our error bars on the measured biases. However, it is unlikely to alter the BAO significance because the tidal tensor enters at \( \ell = 0 \) and 2 pre-cyclically, whereas the BAO’s dominant contribution is at \( \ell = 1 \) (see Section 5). Additional modelling of RSD to permit precise extraction of the real-space bias values will also be important moving forward. It may also be worthwhile to further explore the impact of observational systematics on the 3PCF, for instance, by reanalyzing the sample with systematic weights such as the stellar density weight turned off.

Finally, in future work, we will extend our BAO search from the compressed statistic to the full multipole moments of the 3PCF. We developed the compressed approach to ease the handling of the covariance matrix and fitting as well as to avoid regimes where PT is invalid, but it may be overly conservative. In practice, we have found that the covariance matrix and fitting are computationally by far not the limiting steps in our analysis, and the good agreement between the compressed PT and our measurements argues that there is some room to incorporate more strongly squeezed triangles before PT breaks down.

Regarding the BAO, Gaztañaga et al. (2009)’s finding of a 2\( \sigma \)−3\( \sigma \) preference using a single triangle configuration measured from 40,000 LRGs suggests that our measurement of the 3PCF might contain an even more significant BAO feature than we have found. Gaztañaga et al. (2009) did find a high value of the baryon fraction \( f_b = 0.28 \), significantly larger than the Planck-compatible value \( f_b = 0.16 \) used in this work; this is likely related to the strength of their BAO feature. Since our work uses many triangle configurations but also compresses, the significance of their BAO evidence cannot be simply scaled up as the square root of survey volume or galaxy number. None the less, this is an additional motivation for future work examining analysis variations in the context of the BAO.

**ACKNOWLEDGEMENTS**

ZS thanks Neta Bahcall, Patricia Burchat, Blakesley Burkhardt, Aaron Bray, Douglas Finkbeiner, Lehman Garrison, Margaret Geller, JR Gott III, James Guillochon, Elizabeth Krause, Abraham Leob, Ramesh Narayan, Stephen Portillo, Katie Rubin, Jeff Scargle, Marcel Schmittfull, Uroš Seljak, Joshua Suresh, David Spiegel, Alexander Wiegand, Risa Wechsler and Matias Zaldarriaga for valuable discussions. We also thank the anonymous referee for comments that improved the presentation and content of this work. This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1144152; DJE is supported by grant DE-SC0013718 from the U.S. Department of Energy. AIC is supported by supported by the European Research Council under the European Community’s Seventh Framework Programme FP7-IDEAS-Phys.LSS 240117. Funding for this work was partially provided by the Spanish MINECO under projects FPA2011-29678-C02-02 and MDM-2014-0369 of ICCUB (Unidad de Excelencia ‘María de Maeztu’). HGM acknowledges support from the Labex ILP (reference ANR-10-LABX-63), part of the Idex SUPER, receiving state financial aid managed by the Agence Nationale de la Recherche, as part of the programme Investissements d’avenir under the reference ANR-11-IDEX-0004-02. FSK acknowledges support from the Leibniz Society for the Karl-Schwarzschild fellowship.

Funding for SDSS-III has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation and the U.S. Department of Energy Office of Science. The SDSS-III website is http://www.sdss3.org/. SDSS-III is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS-III Collaboration including the University of Arizona, the Brazilian Participation Group, Brookhaven National Laboratory, University of Cambridge, Carnegie Mellon University, University of Florida, the French Participation Group, the German Participation Group, Harvard University, the Instituto de Astrofísica de Canarias, the Michigan State/Notre Dame/JINA Participation Group, Johns Hopkins University, Lawrence Berkeley National Laboratory, Max Planck Institute for Astrophysics, Max Planck Institute for Extraterrestrial Physics, New Mexico State University, New York University, Ohio State University, Pennsylvania State University, University of Portsmouth, Princeton University, the Spanish Participation Group, University of Tokyo, University of Utah, Vanderbilt University, University of Virginia, University of Washington and Yale University.

**REFERENCES**

Abazajian K. N. et al., 2009, ApJS, 182, 543
Aihara H. et al., 2011, ApJS, 193, 29
Alam S. et al., 2015, ApJS, 219, 12
Anderson et al., 2012, MNRAS, 427, 3435
Anderson et al., 2014, MNRAS, 441, 24
Baldauf T., Seljak U., Desjacques V., McDonald P., 2012, Phys. Rev. D, 86, 8
Bardeen J. M., Steinhardt P. J., Turner M. S., 1983, Phys. Rev. D, 28, 679
Bernardeau F., Colombi S., Gaztañaga E., Scoccimarro R., 2002, Phys. Rep., 367, 1
Blake C., Glazebrook K., 2003, ApJ, 594, 665
Blanton M. et al., 2003, AJ, 125, 2276

Downloaded from https://academic.oup.com/mnras/article-abstract/468/1/1070/2972997 by guest on 16 March 2020
