Localization by Fusing a Group of Fingerprints via Multiple Antennas in Indoor Environment

Xiansheng Guo, Member, IEEE, and Nirwan Ansari, Fellow, IEEE

Abstract—Most existing fingerprints-based indoor localization approaches are based on some single fingerprints, such as received signal strength (RSS), channel impulse response (CIR), and signal subspace. However, the localization accuracy obtained by the single fingerprint approach is rather susceptible to the changing environment, multi-path, and non-line-of-sight (NLOS) propagation. Furthermore, building the fingerprints is a very time consuming process. In this paper, we propose a novel localization framework by Fusing A Group Of Fingerprints (FAGOT) via multiple antennas for the indoor environment. We first build a Group Of Fingerprints (GOOF), which includes five different fingerprints, namely, RSS, covariance matrix, signal subspace, fractional low order moment, and fourth-order cumulant, which are obtained by different transformations of the received signals from multiple antennas in the offline stage. Then, we design a parallel GOOF multiple classifiers based on AdaBoost (GOOF-AdaBoost) to train each of these fingerprints in parallel as five strong multiple classifiers. In the online stage, we input the corresponding transformations of the real measurements into these strong classifiers to obtain independent decisions. Finally, we propose an efficient combination fusion algorithm, namely, MUltiple Classifiers mUltiple Samples (MUCUS) fusion algorithm to improve the accuracy of localization by combining the predictions of multiple classifiers with different samples. As compared with the single fingerprint approaches, the prediction probability of our proposed approach is improved significantly. The process for building fingerprints can also be reduced drastically. We demonstrate the feasibility and performance of the proposed algorithm through extensive simulations as well as via real experimental data using a Universal Software Radio Peripheral (USRP) platform with four antennas.

Index Terms—GrOup Of Fingerprints (GOOF), MUltiple Classifiers mUltiple Samples (MUCUS) fusion localization, multiple antennas, USRP, AdaBoost

I. INTRODUCTION

INDOOR localization using radio signals has attracted increasing attention in the field of target positioning and tracking [1]. Global Positioning System (GPS) is a very accurate system, which is used broadly in many outdoor localization fields, such as commercial, personal, and military applications. However, the performance of GPS degrades drastically in indoor environment, which is full of multi-path and non-line-of-sight (NLOS) propagation. Hence, indoor localization has attracted more attention in asset management, public safety, and military domains.

As compared with the outdoor localization environment, the indoor localization channel exhibits severe multi-path and low probability of line-of-sight (LOS) signal propagation between the transmitter and receiver. Furthermore, the changing environment, as a result of moving people, and closing/opening of doors and windows, also presents a big challenge for indoor localization. These factors make it difficult to design an accurate and robust indoor localization approach in a real indoor scenario.

The existing approaches of indoor localization available in literature can be categorized into two groups: the range-based approach and fingerprint-based approach. The former is to estimate the position of a target by gathering distance estimates from some parametric information such as received signal strengths (RSS) [2], angles of arrival (AOA) [3], times of arrival (TOA) [4], and time differences of arrival (TDOA) [5] about the position of the target. It is well known that the range-based approach by measuring parameters (RSS, AOA, TOA, TDOA) or their combinations (TDOA with AOA or RSS) fails to provide high accuracy in a complex indoor environment [6].

Comparatively speaking, the fingerprint-based approach does not need to estimate distance between the transmitter and receiver. It achieves better performance than the range-based approach in a complex indoor environment. However, most of the existing fingerprint-based approaches are based on some single fingerprints. The RSS is the most popular one to be used as a fingerprint for indoor localization in that it is easy to obtain for some mobile equipments. However, the major challenge of the RSS fingerprint is its fluctuation with time and changing environment. So the accuracy of RSS fingerprint-based approaches shows low accuracy and poor robustness in practice. To overcome this drawback, several different kinds of fingerprints, namely, channel impulse response (CIR) [7]–[10], signal strength difference (SSD) [11], [12], signal subspace [13], [14], power delay doppler profile (PDDP) [15], and others [16], have been proposed to improve the accuracy and robustness of the RSS fingerprint. These fingerprints can, to some extent, improve the accuracy of indoor localization. All in all, they all belong to the single fingerprint localization framework.

It is well known that another drawback of the fingerprint-based localization is the big burden of building fingerprints. To reduce this burden, some fingerprint building strategies have been proposed, including crowdsourcing techniques [17].
Our proposed indoor localization framework by FAGOT using multiple antennas consists of two phases: an offline GOOF building and training phase, and an online localization phase, as summarized below.

• **The offline phase: GOOF building**
  
  Assume that we have \( Q \) grids in an indoor environment, the received array is deployed in the origin, the received array with \( M \) antennas is deployed at the origin, and \( L \) snapshots are collected at each grid; then we can obtain multiple measurements of received signals \( \mathbf{Y} = \{ y(1), y(2), \ldots, y(L) \} \) of size \( M \times L \). We can build GOOF by using \( \mathbf{Y} \) through different transformations. The \( Q \) different labels are also added into the GOOF for classification.

• **The offline phase: Training**
  
  After obtaining the GOOF, we divide each fingerprint in the GOOF into two parts. One is used to train the parallel GOOF multiple classifiers based on AdaBoost (GOOF-AdaBoost). The rest is used for online localization. Assume that we can obtain \( \mathcal{H} \) different kinds of fingerprints in the GOOF, from which we can train \( \mathcal{H} \) strong classifiers.

• **The online phase: Localization**
  
  We input \( J (J \ll L) \) samples of each fingerprint into our trained \( \mathcal{H} \) strong classifiers, and each of them will output an \( J \times Q \) prediction matrix \( \mathbf{P} \), in which each row has only one nonzero entry and the other entries are zeros. The aggregated prediction matrix is \( \{ \mathbf{P} \}_{i=1}^{\mathcal{H}} \), upon which we propose an efficient fusion localization algorithm, namely, MultiClassifiers MultiSample (MUCellus) fusion algorithm, to determine final localization results. Our proposed FAGOT localization algorithm can synthesize not only the predictions of \( \mathcal{H} \) strong classifiers but also the \( J \) predictions of each strong classifier. Thus, FAGOT is more robust and accurate as compared with most existing single fingerprint based indoor localization methods.

The main contributions of this work are summarized below:

• We adopt the GOOF instead of the conventional single fingerprint to localize a target in the indoor environment. As compared with some conventional single fingerprint-based methods, GOOF can capture the characteristics of the complex indoor environment, such as severe multi-path propagation, unknown noise, and is robust to the changing environment.

• We design a parallel GOOF multiple classifiers based on AdaBoost (GOOF-AdaBoost), which is simple to implement, fast, and less susceptible to overfitting. AdaBoost can generate \( \mathcal{H} \) strong classifiers with \( \mathcal{H} \) fingerprints in the GOOF, and thus offers more decisions than those of the single fingerprint based localization approach.

• We propose an efficient fusion algorithm, namely, MultiClassifiers MultiSample (MUCUS) fusion algorithm to improve the accuracy of localization by combining the predictions of multiple classifiers with different samples. MUCUS can improve the estimation accuracy drastically, as compared with any kind of single fingerprint based localization methods.

• The burden of building fingerprints is reduced because our framework can obtain multiple fingerprints based on the measurements of the same received antennas.

The rest of the paper is organized as follows. Related works are presented in Section III. The array model used in this study is described in Section III-A. The GOOF building...
approach is introduced in Section III-B. The GOOF-AdaBoost approach is addressed in Section III-C. Our proposed MUCUS fusion localization algorithm is delineated in Section III-D. The experimental results, including simulation and real data, are presented in Section IV to evaluate and demonstrate the localization accuracy and efficiency of our proposals. Finally, conclusions are drawn in Section V.

II. RELATED WORKS

The main bottlenecks of indoor localization come from its accuracy and robustness. In the past few decades, many indoor localization frameworks based on different networks were proposed, including wireless sensor networks (WSNs) [11], wireless local area network (WLAN) [22], radio frequency identification (RFID) technology [23], RADAR [24], and other techniques [25]. Array signal processing has advanced tremendously and many high resolution algorithms have been studied in the past three decades, including MUSIC [26], ESPRIT [27], and their deviations [28]. Most of these algorithms work well in the environment without multi-path and obstacles. Their performance will degenerate in a complex indoor environment. Advances of antenna technologies and high speed baseband processing integrated circuit (IC) have facilitated the development of small array processing platforms, such as USRP, which has been used to build a small base station [29], mobile anti-terrorism devices [30], etc., and hence, indoor localization using a small platform with multiple antennas becomes feasible and has been a hot research subject.

Machine learning has also been a well research topic in many fields for many years, such as image processing, computer vision, and signal processing [31]. Many machine learning frameworks such as neural network [22], support vector machine [32], AdaBoost [33], and random forest [34] have been proposed to solve various pattern recognition and regression problems. Applying machine learning to address indoor localization is being unfolded. Recently, some machine learning based indoor localization algorithms have been proposed to improve the accuracy and robustness of indoor localization. Bozkurt et al. [35] compared the performance of several machine learning algorithms in the WLAN environment. The compared machine learning algorithms include nearest neighbor (NN), decision tree, Naïve Bayes, AdaBoost, and Bagging. It was found that AdaBoost and Bagging are the two best classifiers for indoor localization. Taniuchi et al. [36] proposed an ensemble learning algorithm to improve the performance of RSS based indoor localization in the WLAN environment. After training several multiple weak learners, a weighted average strategy is adopted to yield the final position estimate. Pan et al. [37] considered users tracking in WSN via semi-supervised colocalization. Neural network based indoor localization approaches were studied for both the WSN and WLAN environment [6], [22], [38], which are also in RSS based localization framework. All of these approaches can improve the performance of the conventional RSS fingerprint based indoor localization to some extent.

In this work, we propose a novel localization framework by Fusing A Group Of fingerprints (FAGOT) via multiple antennas for a complex indoor environment. The advantage of our proposed system is that it can extract several different kinds of fingerprints. This system can offer more data to fuse in reaching the final localization result. We first build multiple strong learners by using GOOF-AdaBoost from some simple weak learners. Then, we propose an efficient fusion algorithm, namely, MUltiple Classifiers mUltiple Samples (MUCUS) fusion algorithm to improve the accuracy of localization by combining the predictions of multiple classifiers with different samples. As compared with the existing machine learning algorithms, our proposed system can improve the robustness and accuracy of indoor localization and is robust to changing environment, multi-path, and unknown noise distributions.

III. METHODOLOGY

A. Signal Model

Consider an indoor environment deployed with a uniform linear array (ULA) in which M antenna elements are equally spaced apart, with an inter-distance of d, as shown in Fig. 1. Denote by \( y_m(t) \) as the received signal at the \( m \)th antenna element with channel gain \( \alpha_i \), delay \( \tau_i \), and angle of arrival (AoA) \( \theta_i \). Note that the received signal of each path consists of an enormous number of unresolvable signals received around the mean of AoA in each element in a complex indoor scenario. A vector of the received signals \( \mathbf{y}(t) = [y_1(t), y_2(t), \cdots, y_M(t)]^T \) in the ULA can be expressed as

\[
\mathbf{y}(t) = \sum_{i=1}^{I} \alpha_i \mathbf{a} (\theta_i) s (t - \tau_i) + \mathbf{n}(t),
\]

where \( I \) denotes the number of paths received by each antenna element and \( \mathbf{a} \) is an array steering vector. The location \( \mathbf{x} = [x, y]^T \) of the transmitted signal \( s(t) \) is to be estimated. The unknown noise vector \( \mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T \) with \( n_m(t) \) being the noise of the \( m \)th antenna element. The array steering vector is defined as \( \mathbf{a} (\theta) = [a_1(\theta), a_2(\theta), \cdots, a_M(\theta)]^T \), where its \( m \)th element is

\[
a_m(\theta) = f_m(\theta) e^{-j2\pi(m-1)(d/\lambda) \sin \theta},
\]

where \( f_m(\theta) \) denotes a complex field pattern of the \( m \)th array element and \( \lambda \) is the carrier wavelength. The received signal in Eq. 1 can be expressed in the following integral form:

\[
\mathbf{y}(t) = \iint \mathbf{a} (\theta) h(\theta, \tau) s (t - \tau) \, d\tau \, d\theta + \mathbf{n}(t),
\]

where \( h(\theta, \tau) \) represents the channel as a function of azimuth-delay spread (ADS). The average power azimuth-delay spectrum (PADS) is given as

\[
P(\theta, \tau) = E \left\{ \sum_{i=1}^{I} |\alpha_i|^2 \delta(\theta - \theta_i, \tau - \tau_i) \right\},
\]

where \( E \{ \} \) is the expectation operator. The central angular of arrival (CAoA) \( \theta_0 \) and angular spread (AS) \( \sigma_A \) are defined as

\[
\begin{align*}
\theta_0 &= \int \theta P_A(\theta) \, d\theta, \\
\sigma_A &= \sqrt{\int (\theta - \theta_0)^2 P_A(\theta) \, d\theta},
\end{align*}
\]
where \( P_A(\theta) = \int P(\theta, \tau) \, d\tau \) is the power angular spectrum (PAS).

Similarly, the average delay spread (ADS) and delay spread (DS) are given by
\[
\begin{align*}
\tau_0 &= \int \theta P_D(\tau) \, d\tau, \\
\sigma_D &= \sqrt{\int (\tau - \tau_0)^2 P_D(\tau) \, d\tau},
\end{align*}
\]
where \( P_D(\tau) = \int P(\theta, \tau) \, d\theta \) is the power delay spectrum (PDS). The indoor localization problem using ULA is to estimate the location of \( s(t) \) from the \( T \) measurements \( y(t) \) of Eq. \( \text{(1)} \).

### B. GOOF Building

Here, we address how to build our proposed GOOF from the received signals \( y(t) \) by using \( T \) snapshots. Assume that we divide the indoor environment into \( Q \) grids with equal spacing. The signal \( s(t) \) is transmitted from one antenna located at the \( q \)-th grid, and the received signals vector of \( M \) antenna elements at time \( t \) is denoted by \( y^q(t) \).

- **Covariance matrix fingerprints (CMFs)**

  We can estimate the covariance matrix by using \( T \) snapshots at the \( q \)-th grid without any knowledge of noise distributions as follows:

  \[
  \hat{R}^q = \frac{1}{L} \sum_{t=1}^{L} y^q(t) y^q(t)^H.
  \]

  Note that the estimated covariance matrix \( \hat{R}^q \) can be expressed as

  \[
  \hat{R}^q = \begin{bmatrix}
  r(0) & r(-1) & \cdots & r(-M) \\
  r^*(0) & r(-1) & \cdots & r(-M+1) \\
  \vdots    & \vdots    & \ddots & \vdots    \\
  r^*(-M) & r^*(-M+1) & \cdots & r(0)
  \end{bmatrix}
  \]

  The \( (i,j) \)-th entry of \( \hat{R}^q \) is the correlation between the outputs of the \( i \)-th and \( j \)-th antennas. We can estimate the RSS from \( \hat{R}^q \) as follows.

- **RSS fingerprints (RSSFs)**

  It is well known that the \( i \)-th diagonal element of the estimated covariance matrix \( r(0) \) in Eq. \( \text{(8)} \) denotes the autocorrelation of the received signals \( y_i(t) \) of the \( i \)-th antenna element, i.e.,

  \[
  r_i(0) = \frac{1}{L} \sum_{t=1}^{L} y_i(t) y_i(t) = \frac{1}{L} \sum_{t=1}^{L} |y_i(t)|^2.
  \]

  In other words, we can build the RSS fingerprints by taking the diagonal elements of the estimated covariance matrix \( \hat{R}^q \), i.e.,

  \[
  \text{RSS}^q = [r_1^q(0), r_2^q(0), \cdots, r_M^q(0)]^T = \text{diag} \{ \hat{R}^q \},
  \]

  where \( \text{diag} \{ \cdot \} \) is an operator of extracting the diagonal elements of a matrix.

  In comparing with Eqs. \( \text{(8)} \) and \( \text{(10)} \), it is remarkable that the covariance matrix fingerprint can offer more information about the indoor channel than that of the RSS fingerprint because the covariance matrix fingerprint has much correlation information between each antenna element. So, we have enough reasons to believe that the covariance matrix fingerprint can yield a more accurate location estimate than that of the RSS fingerprint.

- **Signal subspace fingerprints (SSSFs)**

  By taking eigen-decomposition (ED) of the estimated covariance matrix, we have

  \[
  \hat{R}^q = [U_s^q \Sigma_s^q U_n^q]^H,
  \]

  where \( \Sigma_s^q \) is the signal subspace corresponding to the \( k \) largest eigenvalues whose elements are the diagonal elements of the diagonal matrix \( \Sigma_s^q \); \( U_n^q \) is the noise subspace, which corresponds to the \( M - k \) smallest eigenvalues. Signal subspace methods are empirical linear methods for dimensionality reduction and noise reduction. They have also been demonstrated to be robust robustness to multi-path propagation in indoor localization \[13\]. Note that we just build the signal subspace fingerprints by taking the first column of \( U_s^q \) instead of finding the \( k \) columns of \( U_s^q \) for simplicity.

- **Fourth-order cumulant fingerprints (FOCFs)**

  The fourth-order cumulants of the received signals \( y(t) \) can be given by

  \[
  C_{4,y} = \text{cum} \{ y_{k_1}, y_{k_2}, y_{k_3}, y_{k_4} \} = E \{ y_{k_1} y_{k_2} y_{k_3} y_{k_4} \} - E \{ y_{k_1} y_{k_3} \} E \{ y_{k_2} y_{k_4} \} - E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} y_{k_4} \} - E \{ y_{k_1} y_{k_2} y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} - E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} y_{k_4} \} - E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} - E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} - E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \} + E \{ y_{k_1} y_{k_2} \} E \{ y_{k_3} \} E \{ y_{k_4} \}.
  \]

  It is well known that the fourth-order cumulants are generally robust to color noise \[39\].

- **Fractional low order moments fingerprints (FLOMFs)**

  Impulsive noise distorts the source signal and causes the degeneration of localization accuracy for some direction-finding methods. Studies in \[40\] have shown that the symmetric alpha-stable (S\&S) processes are able to model the impulsive noise better. We can calculate the fractional low order moments fingerprint by using the following formulas \[41\].

  \[
  C_{f,y}^q = E \{ y_{k_1}(t) |y_k(t)|^{p-2} y_k(t) \}, 1 < p < \alpha \leq 2,
  \]

  where \( 0 < \alpha \leq 2 \) is the characteristic exponent of an S\&S processes. Note that when \( p = 2 \), Eq. \( \text{(15)} \) is the special case of Eq. \( \text{(7)} \). However, for impulse noises, the fractional low order moments are unbounded. The fractional low order moments are a good statistic to be used to estimate DOA of sources in array signal processing.

So far, we have addressed how to build the GOOF based on the received signals in an indoor environment. Note that
the dimensions of the five proposed fingerprints in the GOOF are not the same. Except for the RSSFs, the rest of them are complex values. For the complex fingerprints, we just take absolute values of them and drop the phase information, which is sensitive to the noise level. We adjust the dimensions and data types of the constructed GOOF, as shown in Table I.

For comparisons, we summarize the procedures of building the above five fingerprints as the GOOF building algorithm in Algorithm 1. In order to obtain as many fingerprints as the GOOF building algorithm, we partition the L snapshots into M groups. Each group has L/M snapshots, and we just use the L/M snapshots to estimate each fingerprint.

It is worth to note that our proposed GOOF building strategy can reduce the fingerprints building burden as compared with the traditional fingerprints building approaches [18]–[21]. The GOOF building strategy can obtain multiple types of fingerprints with different dimensions from the same measurements, while the traditional fingerprints building strategies can only obtain one fingerprint from the same measurements. Hence, from this viewpoint, the efficiency of our GOOF building strategy is much higher than the traditional fingerprints building strategies. In other words, GOOF can obtain the same localization results with less fingerprints building time as compared with the traditional fingerprints building strategies. Furthermore, some fingerprints building approaches adopted in the single fingerprints can only reduce the fingerprints building burden; our GOOF strategy can not only reduce the fingerprints building burden, but also can offer more different fingerprints information about the indoor environment for further localization, which is very attractive for complex indoor environment localization.

C. GOOF-AdaBoost

Adaptive Boosting (AdaBoost) is a popular method to increase the accuracy of any supervised learning technique through resampling and arcing [42], [43]. AdaBoost itself is not a learning algorithm, but rather a meta-learning technique that “boosts” the performance of other learning algorithms, known as weak learners, by weighting and combining them. The basic premise is that multiple weak learners can be combined to generate a more accurate ensemble, known as a strong learner, even if the weak learners perform little better than random. AdaBoost and its variants have been applied to diverse domains with great success, owing to their solid theoretical foundation, accurate prediction, and great simplicity [44].

Unlike other ordinary single classifier based on AdaBoost, which just classifies the training data by many weak learners and combine them as one final classifier. Our proposed Parallel GOOF Multiple Classifiers based on AdaBoost (GOOF-AdaBoost) needs to build multiple strong classifiers from the constructed GOOF. So, we design the GOOF-AdaBoost by constructing the five strong classifiers, one for each fingerprint in GOOF, in parallel. Each strong classifier yields its final location estimation of the target. The procedure of our proposed GOOF-AdaBoost is illustrated in Fig. 2.

![Fig. 2. The framework of our proposed GOOF-AdaBoost approach.](image-url)
given fingerprint, where \( r_i \) is the \( i \)th fingerprint vector and \( q_i \in \{1, \ldots, Q\} \) is the corresponding location label. \( M \) is the total number of training data. A weak learner \( w \) \((w = 1, 2, \ldots, N)\) is the total number of weak learners in the \( \gamma \)th fingerprint, \( \gamma = 1, \ldots, H \) with \( H \) being the total number of different kinds of fingerprints in GOOF), which is based on a learning algorithm \( \mathcal{L} \) and the initialized distribution \( D_1 \), will output a hypothesis \( h_\gamma(r) \); \( H_\gamma(r) \), the combination of outputs of all the \( N \) weak learners, can be expressed as

\[
H_\gamma(r) = \sum_{w=1}^{N} \alpha_w \gamma h_w(r) = \alpha h_\gamma(r),
\]

where \( \alpha = [\alpha_1^\gamma, \ldots, \alpha_N^\gamma]^T \) with \( \alpha_w^\gamma \) being the weight of the \( w \)th weak learner in the \( \gamma \)th fingerprint, as shown in Fig. 2 and \( h_\gamma(r) = [h_1^\gamma(r), \ldots, h_N^\gamma(r)]^T \) with \( h_w(r) \) being the output hypothesis of the \( w \)th weak learner in the \( \gamma \)th fingerprints. In this respect, AdaBoost actually solves two key problems, i.e., how to generate the hypothesis \( h_\gamma(r) \) and how to determine the proper weights \( \alpha_w^\gamma \).

In order to facilitate a highly-efficient error reduction process, AdaBoost tries to find the best weight \( \alpha_w^\gamma \) by minimizing the following exponential loss

\[
\text{loss}_{\text{exp}}(h_\gamma) = \mathbb{E}_{r \sim D_q} \left[ e^{-q h_\gamma(r)} \right],
\]

where \( q h_\gamma(r) \) is the classification margin of the hypothesis \( h_\gamma(r) \). The weight \( \alpha_w^\gamma \) can be found by taking derivative of the following combination loss and setting it to zero, that is

\[
\frac{\partial \text{loss}_{\text{exp}}(H + \alpha h_\gamma(r))}{\partial \alpha_w^\gamma} = 0.
\]

By solving (18), we can get the weight \( \alpha_w^\gamma \) as follows

\[
\alpha_w^\gamma = \frac{1}{2} \ln \frac{1 - \epsilon_w^\gamma}{\epsilon_w^\gamma},
\]

in which the error \( \epsilon_w^\gamma \) is written as

\[
\epsilon_w^\gamma = \text{Pr} (h_w^\gamma(r_i) \neq q_i).
\]

The hypothesis is adopted for a given weak learner \( \mathcal{L} \) by AdaBoost as \( h_\gamma(r) = \mathcal{L}(D_\gamma) \), and so the weight is updated as follows:

\[
D_{\gamma+1}^i = \frac{D_\gamma^i h_w^\gamma}{Z_\gamma^i},
\]

where \( Z_\gamma \) is a normalization factor which enables \( D_\gamma+1 \) to be a distribution. The final output of the \( \gamma \)th classification is

\[
H_\gamma(r) = \text{sign} \left( \sum_{w=1}^{N} \alpha_w^\gamma h_w^\gamma(r) \right),
\]

where \( \text{sign}(\cdot) \) is a sign function. It should be pointed out that the outputs of each classifier are either +1 or -1, but we can realize multiple classification by comparing every two samples one by one in each fingerprint to obtain the final predictions.

Our proposed GOOF-AdaBoost is summarized in Algorithm 2. Note that the GOOF-AdaBoost algorithm consists of two stages: the training stage and the prediction stage. The training stage is done offline, while the prediction stage is the on-line localization phase. After training the GOOF-AdaBoost as the multiple classifiers, we can input some online measurements to the GOOF-AdaBoost as the multiple predictors to predict the output of the localization. How to fuse these predictions is, however, the key to improve the accuracy of indoor localization, which will be addressed next.

**Algorithm 2 GOOF-AdaBoost**

Input: 1) The training data \( \mathcal{F} = \{ (r_1, q_1), (r_2, q_2), \ldots, (r_M, q_M) \} \in \mathbb{F} \). 2) The number of weak learners in each fingerprints \( N \). 3) The initialize distribution \( D_1 \). 4) The base learning algorithm \( \mathcal{L} \). 5) The number of training data in each fingerprints \( M \).

Output: The \( H \) strong classifiers \( H(r) \).

1: for \( \gamma = \{1, \ldots, H\} \) do
2: \hspace{1cm} for \( i = \{1, \ldots, M\} \) do
3: \hspace{2cm} Initialize the weight \( \gamma h_w^i \) \( i = 1/M \)
4: \hspace{1cm} end for
5: for \( w = \{1, \ldots, N\} \) do
6: \hspace{1cm} Set \( D_w^\gamma = \frac{\gamma h_w^\gamma}{\frac{1}{\mathbb{D}}} \)
7: \hspace{1cm} Call the weak learner \( \mathcal{L} \); \( h_w^\gamma = \mathcal{L}(\mathcal{F}, D_w^\gamma) \)
8: \hspace{1cm} Calculate the error of \( h_w^\gamma ; \epsilon_w^\gamma = \text{Pr} (h_w^\gamma(r_i) \neq q_i) \)
9: \hspace{1cm} if \( \epsilon_w^\gamma > 0.5 \) then
10: \hspace{2cm} break
11: \hspace{1cm} end if
12: \hspace{1cm} Set \( \epsilon_w^\gamma = \text{Pr} (h_w^\gamma(r_i) \neq q_i) \) by using Eq. (20)
13: \hspace{1cm} Set \( \gamma h_w^\gamma = \frac{1}{\mathbb{D}} \) by using Eq. (19)
14: \hspace{1cm} Update \( D_{w+1}^\gamma \) by using Eq. (21)
15: \hspace{1cm} end for
16: \hspace{1cm} end for
17: \hspace{1cm} \( H(r) = \frac{H(r)}{H(r) H(r)} \)
18: \hspace{1cm} end for
19: \hspace{1cm} return \( \hat{H}(r) \)

**D. Multiple Classifiers mUltiple Samples (MUCUS) Fusion Localization Algorithm**

In the online phase, we assume that we can obtain \( J \) samples for each kind of fingerprints at the grid \( q \) (each sample can be calculated from a given number of snapshots). Let \( \mathcal{G} = \{g_1, q, g_2, q, \ldots, g_J, q\} \) be the \( j \)th sample fingerprint obtained from the received signal \( y \), where \( g_j \) is the \( j \)th fingerprint vector and \( q \) is the corresponding location label. We can input the \( J \) samples of the \( \gamma \)th group test fingerprint one by one into the \( \gamma \)th strong classifier, which has been trained by GOOF-AdaBoost. Then, the \( \gamma \)th strong classifier will work as a predictor to yield the prediction \( P_\gamma \) with dimension \( J \times Q \), in which each row of \( P_\gamma \) has one nonzero entry and the others are zeros. The total prediction matrix of the output of the \( H \) strong classifiers can be written as \( \hat{P} = \{ P_1, \ldots, P_H \} \) with size of \( J \times H \). In order to compare the predictions of the \( H \) different strong classifiers with multiple samples, we transform the total prediction matrix \( \hat{P} \) into a final prediction matrix \( \hat{P} \) with dimension of \( J \times H \). The \( \hat{(.)} \)th entry of \( \hat{P} \) denotes the output of the \( \gamma \)th classifier.
when inputting the $j$th test sample, as shown in Fig. 3. This figure shows the prediction results of all the $J$ samples at the $q$th grid. For the $j$th test sample, the $H$ strong classifiers produce different prediction results $\{q, q_1, q_3\}$. While for the $\gamma$th strong classifier, different test samples may yield different prediction results $\{q, q_1, q_2, q_3, q_4\}$. From these prediction results, we find that the outputs of all the strong classifiers with different samples can be combined to give a more accurate fusion result. We demonstrate our proposed MUCUS fusion localization algorithm as follows.

Fig. 3. The diagram of the prediction results of MUCUS framework.

The total number of different prediction results is denoted as $A$. Let $\{\hat{q}_1, \ldots, \hat{q}_A\}$ be the predictions of all the $H$ strong classifiers and $\hat{q}_i, (i \in \{1, \ldots, A\} \in Q)$ be the $i$th prediction. The frequency of $\hat{q}_i$ is denoted as $N_i$. Hence, for the multiple strong classifiers, we can build an exponential weight as follows:

$$\hat{q} = \text{round}\left\{\frac{A}{\sum_{i=1}^{A} \hat{q}_i \exp(N_i)} \right\},$$

where $\text{round}\{X\}$ rounds $X$ to the nearest integer.

Note that Eq. (23) does not take the predictions of multiple samples into account. The predictions of multiple samples for the same classifier represent the robustness of this classifier, and so we can combine the predictions of different classifiers of multiple samples to give a more accurate prediction. Assume that $B$ is the total number of different prediction results of different classifiers of multiple samples. $\hat{q}_j, (j \in \{1, \ldots, B\} \in Q)$ is the $j$th prediction. The frequency of $\hat{q}_j$ is denoted as $M_j$. So, we can build a jointly exponential weight based on multiple classifiers and multiple samples as follows

$$\hat{q} = \text{round}\left\{\frac{1}{2} \left(\frac{A}{\sum_{i=1}^{A} \hat{q}_i \exp(N_i)} + \frac{B}{\sum_{j=1}^{B} \hat{q}_j \exp(M_j)}\right)\right\},$$

Equation (24) denotes our final prediction result based on our proposed MUCUS. We can summarize our proposed MUCUS algorithm in Algorithm 3. The occurrence frequency in Algorithm 3 is defined as the frequency of the appearance of an entry appears.

### Algorithm 3 MUCUS

**Input:** 1) The final prediction matrix $\hat{P}$.

**Output:** The final prediction $\hat{q}$.

1: for $j = \{1, \ldots, J\}$ do
2: for $i = \{1, \ldots, H\}$ do
3: Find the different predictions $\hat{q}_i$
4: Find the occurrence frequency $N_i$ of $\hat{q}_i$
5: Find the total number of different predictions $A$
6: end for
7: Find the different predictions $\hat{q}_j$
8: Find the occurrence frequency $M_j$ of $\hat{q}_j$
9: Find the total number of different predictions $B$
10: end for
11: Calculate the final predict by using Eq. (24)
12: return $\hat{q}$

### E. Performance Analysis

The MUCUS algorithm takes the effects of multi-path, environment changing, unknown environment noise, and robustness into account. The predictions of the different strong classifiers can cope with multi-path, changing environment, and unknown environment noise adaptively because each of the above effects is robustly addressed by one of the classifiers. Furthermore, the predictions of different samples of the same classifier demonstrate the robustness of this classifier. Therefore, combining predictions of multiple classifiers and multiple samples can improve the final prediction drastically.

The exponential weighting strategy we adopted can strength the prediction of higher occurrence frequency, which is defined as the frequency of the appearance of an entry appears. The higher occurrence frequency of the prediction in $\hat{P}$, the more probability it will be selected. Meanwhile, the exponential weights can also address the following two special cases.

- All the predictions have the same frequency. The final prediction may be simplified as

$$\hat{q} = \text{round}\left\{\frac{1}{2} \left(\frac{A}{\sum_{i=1}^{A} \hat{q}_i \exp(N_i)} + \frac{B}{\sum_{j=1}^{B} \hat{q}_j \exp(M_j)}\right)\right\},$$

which takes the average of all the possible predictions.

- Each prediction has a different frequency. Our algorithm has the tendency to choose the prediction with the highest frequency.

Assume that the true grid location is $q$; to evaluate the performance of our proposed algorithm, we define a metric of the prediction probability $\varrho$ as

$$\varrho = \frac{\sum_{j=1}^{J} \hat{q}_j = q}{J},$$

which will be used to evaluate the performance of our proposed FAGOT localization framework.
IV. EXPERIMENTAL RESULTS

In this section, we will employ simulation data and real data to test the performance of our proposed FAGOT localization framework. In the simulation part, we use our proposed signal model in Section III-A and in the real experimental setup, we employ a USRP receiver platform with four antennas, and a USRP transmitter platform with one antenna.

A. Simulation Data

Assume we have a ULA with 5 antennas with carrier frequency at 1 GHz. The interspace between adjacent antennas is half wavelength. The uniform PAS model is adopted, i.e., \( P_A(\theta) = 1/(2\sqrt{\pi} \sigma_A) \), where the AS is defined by (5).

Assume that a 6m x 6m indoor environment is divided into \( Q = 36 \) grids with equal interspace of 1m. The location of the \( q \)th grid is denoted as \([x_q, y_q]\), the ULA received array with 5 antennas is deployed at the corner of this room with the location of the central element being \([0, 0]\), and its normal direction points to the diagonal of the indoor area. The central AoA (CAoA) \( \theta_0 \) and average time delay \( \tau_0 \) of the transmitted signal are calculated from the locations of the received array and the \( q \) location \([x_q, y_q]\). The time delay spread (DS) and angular spread (AS) are \( \tau_0/10 \) and 30°, respectively. We add 10 multi-paths to each LOS at each grid.

![Fig. 4](image)

**Fig. 4.** The prediction performance of different algorithms versus different SNRs: Gaussian noise.

First, Gaussian white noise with different SNRs is added to the generated signals. The signal-noise-ratio (SNR) is defined as \( \text{SNR} = 10 \log \frac{\sigma_s^2}{\sigma_n^2} \), where \( \sigma_s^2 \) and \( \sigma_n^2 \) are signal and noise variance, respectively. The total number of snapshots is 51200 at each grid, and we get \( J = 100 \) samples with each sample having 512 snapshots. The SNRs are set from 0 to 30 dB with 6 dB interspace. \( J = 5 \) fingerprints are considered in this case. We build the five fingerprints by using Algorithm 1 and then we divide each of these five fingerprints into two groups: one with 50 samples is used to train the five AdaBoost strong classifiers, and the other with 50 samples is used to obtain the prediction results from the five AdaBoost strong classifiers. The number of weak learners is 30. Finally, the final prediction results are generated by using our proposed MUCUS algorithm.

![Fig. 5](image)

**Fig. 5.** The prediction performance of different algorithms versus different SNRs: impulse noise.

Now, we add some impulse noise to the generated signals. The symmetric alpha-stable (S\( \alpha \)S) processes are considered here whose SNR is defined as \( \text{SNR} = 10 \log \left( \frac{E[|x(t)|^2]}{\gamma} \right) \), where \( \gamma \) is the dispersion parameter of an isotropic complex S\( \alpha \)S variable. The characteristic exponent \( \alpha = 1.4 \), the skewness parameter \( \beta = 0 \), and the location parameter \( \delta = 0 \). The SNRs are also set from 0 to 30 dB with 6 dB increment. Fig. 5 shows the prediction results of our proposed MUCUS algorithm versus different SNRs for the impulse noise. Note that the FLOMFs fingerprint yields better prediction results than the other fingerprints, thus demonstrating that FLOM is robust to impulse noise. The other fingerprints perform poorly especially when SNRs are low. As compared with the five fingerprints, our proposed MUCUS algorithm obtains the best prediction results for different levels of impulse noise. We can conclude from Figs. 4 and 5 that our proposed FAGOT localization framework performs the best in the unknown complex indoor environment.
B. Real Data

The experimental receiver platform is based on two Universal Software Radio Peripheral (USRP) units, each equipped with two antennas (i.e., a total of four antenna elements), and the transmitter platform is one USRP with one antenna, as shown in Fig. 6(a) and Fig. 6(b) respectively.

![Fig. 6. The experimental testbed.](image)

The experimental indoor is the KB508 laboratory at University of Electronic Science and Technology of China (UESTC), which has many desks, partitions, and about 30 graduate students. The topological layout of this laboratory is illustrated in Fig. 7. The length and width of our laboratory are 9.8m and 6.3m, respectively. We select 18 transmitting grids, which are depicted as red circles in Fig. 7. The receiver array with 4 antennas is deployed at the corner of the laboratory at the height of 1.5m, as depicted as yellow circles in Fig. 7.

We transmit a cosine signal with carrier frequency of 700MHz at each grid and build the GOOF by using the signals received at the four antennas. 400 snapshots are taken, and are divided into 40 groups (samples), each group having 10 snapshots. We just use 10 \((T = 10)\) snapshots to estimate each fingerprint at each grid. Hence, there are 40 fingerprints in our final GOOF. We use 30 \((M = 30)\) of them to train the GOOF-AdaBoost strong classifiers, and 10 \((J = 10)\) of them to obtain the prediction results. Fig. 8 shows the average prediction probability of 18 grids. From this figure, we find that our proposed MUCUS algorithm can obtain the best prediction probability as compared with other five fingerprints based AdaBoost methods. The prediction probabilities of these methods at each grid are plotted in Fig. 9. It is seen that our proposed MUCUS algorithm achieves better prediction performance than the other methods at each grid. Note that our results are given without any knowledge of the noise types, multi-path, and environment changing. So, our algorithm is very robust to real complex indoor environment.

![Fig. 7. The topological layout of our laboratory.](image)

![Fig. 8. The average prediction probabilities of 18 grids.](image)

V. Conclusion

We have proposed a novel localization framework by Fusing A Group Of fingerprintTs (FAGOT) via multiple antennas in indoor environment. This indoor localization framework can overcome drawbacks of single fingerprint based indoor localization methods. Our proposed indoor localization framework mainly consists of three steps: 1) GOOF construction; 2) the training of parallel GOOF multiple classifiers based on AdaBoost (GOOF-AdaBoost); and 3) multiple classifiers multiple samples (MUCUS) fusion. The first and the second steps are done offline, and the third step is executed online. Five typical fingerprints, including RSSFs, CMFs, SSFs, FoCFs, and FLOMFs, are constructed from the same received signals of a ULA. Obviously, one can build as reasonably many
fingertips as possible into the GOOF to improve the accuracy and robustness of the final localization results. Generally speaking, more fingerprints will offer more information to be fused.

Note that we just test two types of noise, namely, Gaussian and impulse noise in our simulation results. We believe that our proposed localization framework is also robust to the other types of noise, such as color noise and the mixed noise. In the real experiment, we are not aware of the noise type, multipath, and environmental changes in constructing GOOF; the localization performance of our proposed framework is still robust to the unknown localization environment.

We consider only five fingerprints in constructing our GOOF, even though other fingerprints, such as CIR, PDDP, general array manifold, can be incorporated into the GOOF to obtain more information of the indoor environment. Our proposed framework can be applied to as many kinds of fingerprints as possible. Generally, the more fingerprints the better the performance.

It is worth to note that our proposed GOOF strategy can reduce the burden of building fingerprints as compared with the traditional fingerprints building approaches. Our GOOF strategy can not only reduce the burden of building fingerprints, but can also offer more information about the indoor environment for further localization, and is thus very attractive for localization in complex indoor environment.

**REFERENCES**

[1] X. Guo, L. Chu, and X. Sun, “Accurate localization of multiple sources using semidefinite programming based on incomplete range matrix,” *IEEE Sensors Journal*, vol. 16, no. 13, pp. 5319–5324, 2016.

[2] X. Guo, L. Chu, Y. Pi, and Y. Zou, “Two stages signal strength difference localization algorithm using SDP relaxation,” in *Digital Signal Processing (DSP), 2015 IEEE International Conference on*. IEEE, 2015, pp. 957–961.

[3] X. Guo, Y. Huang, B. Li, and L. Chu, “DOA estimation of mixed circular and non-circular signals using uniform circular array,” in *Image and Signal Processing (CISP), 2014 7th International Congress on*. IEEE, 2014, pp. 1043–1047.

[4] G. Wang, H. Chen, Y. Li, and N. Ansari, “NLOS error mitigation for TOA-based localization via convex relaxation,” *Wireless Communications, IEEE Transactions on*, vol. 13, no. 8, pp. 4119–4131, 2014.

[5] G. Wang, Y. Li, and N. Ansari, “A semidefinite relaxation method for source localization using TDOA and FDOA measurements,” *Vehicular Technology, IEEE Transactions on*, vol. 62, no. 2, pp. 853 – 862, 2013.

[6] C. Nerguizian, C. Despins, and S. Affes, “Indoor geolocation with received signal strength fingerprinting technique and neural networks,” in *Telecommunications and Networking-ICT 2004*. Springer, 2004, pp. 866–875.

[7] C. Nerguizian, C. Despins, and S. Affes, “Geolocation in mines with an impulse response fingerprinting technique and neural networks,” *Wireless Communications, IEEE Transactions on*, vol. 5, no. 3, pp. 605–611, 2006.

[8] Y. Jin, W.-S. Soh, and W.-C. Wong, “Indoor localization with channel impulse response based fingerprint and nonparametric regression,” *Wireless Communications, IEEE Transactions on*, vol. 9, no. 3, pp. 1120–1127, 2010.

[9] C. Nerguizian and V. Nerguizian, “Indoor fingerprinting geolocation using wavelet-based features extracted from the channel impulse response in conjunction with an artificial neural network,” in *Industrial Electronics, 2007. ISIE 2007. IEEE International Symposium on*. IEEE, 2007, pp. 2028–2032.

[10] Z. Yang, Z. Zhou, and Y. Liu, “From RSSI to CSI: Indoor localization via channel response,” *ACM Computing Surveys (CSUR)*, vol. 46, no. 2, p. 25, 2013.

[11] A. Mahtab Hossain, Y. Jin, W.-S. Soh, and H. N. Van, “SSD: A robust RF location fingerprint addressing mobile devices’ heterogeneity,” *Mobile Computing, IEEE Transactions on*, vol. 12, no. 1, pp. 65–77, 2013.

[12] X. Guo, L. Chu, B. Li, B. Xu, Q. Wan, and Y. Shen, “A robust vector matching localization approach based on multiple channels SSD fingerprinting of ZigBee networks,” *Progress In Electromagnetics Research*, vol. 144, pp. 133–140, 2014.

[13] H. Tsuji, S. Kikuchi, and M. Kaveh, “Indoor localization using subspace matching: An experimental evaluation,” in *Sensor Array and Multichannel Processing. 2006. Fourth IEEE Workshop on*. IEEE, 2006, pp. 541–545.

[14] S. Ikeda, H. Tsuji, and T. Ohtsuki, “Effects of spatial correlation between signal subspaces on indoor localization using subspace matching,” in *TENCON 2007-2007 IEEE Region 10 Conference*. IEEE, 2007, pp. 1–4.

[15] T. Öktiem and D. Stokl, “Power delay doppler profile fingerprinting for mobile localization in NLOS,” in *Personal Indoor and Mobile Radio Communications (PIMRC), 2010 IEEE 21st International Symposium on*. IEEE, 2010, pp. 876–881.

[16] Y. Shi, Y. Huang, J. Zhang, P. Coué, P. Cheng, J. Chen, and K. G. Shin, “Gradient-based fingerprinting for indoor localization and tracking,” 2015.

[17] C. Wu, Z. Yang, and Y. Liu, “Smartphones based crowdsourcing for indoor localization,” *Mobile Computing, IEEE Transactions on*, vol. 14, no. 2, pp. 444–457, 2015.

[18] S. Nikitaki, G. Tsagkatakis, and P. Tsakalides, “Efficient training for fingerprint based positioning using matrix completion,” in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*. IEEE, 2012, pp. 195–199.

[19] Z. Gu, Z. Chen, Y. Zhang, Y. Zhu, M. Lu, and A. Chen, “Reducing fingerprint collection for indoor localization,” *Computer Communications, 2015.

[20] J. Talvitie, M. Renfors, and E. S. Lohan, “Distance-based interpolation and extrapolation methods for RSS-based localization with indoor wireless signals,” *Vehicular Technology, IEEE Transactions on*, vol. 64, no. 4, pp. 1340–1353, 2015.

[21] K. Arai and H. Tolle, “Color radiomap interpolation for efficient fingerprint WiFi-based indoor location estimation,” *International Journal of Advanced Research in Artificial Intelligence*, vol. 2, no. 3, pp. 10–15, 2013.

[22] S.-H. Fang and T.-N. Lin, “Indoor location system based on discriminant-adaptive neural network in IEEE 802.11 environments,” *Neural Networks, IEEE Transactions on*, vol. 19, no. 11, pp. 1973–1978, 2008.

[23] S. S. Saab and Z. S. Nakad, “A standalone RFID indoor positioning system using passive tags,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 5, pp. 1961–1970, 2011.

[24] P. Bahl and V. N. Padmanabhan, “RADAR: An in-building RF-based user location and tracking system,” in *INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 2. IEEE, 2000, pp. 775–784.

[25] N. B. Priyantha, A. Chakraborty, and H. Balakrishnan, “The cricket location-support system,” in *Proceedings of the 6th annual international
conference on Mobile computing and networking. ACM, 2000, pp. 32–43.

[26] R. O. Schmidt, “Multiple emitter location and signal parameter estimation,” Antennas and Propagation, IEEE Transactions on, vol. 34, no. 3, pp. 276–280, 1986.

[27] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” Acoustics, Speech and Signal Processing, IEEE Transactions on, vol. 37, no. 7, pp. 984–995, 1989.

[28] T. Schmid, O. Sekkat, and M. B. Srivastava, “An experimental study of network performance impact of increased latency in software defined radios,” in Proceedings of the second ACM international workshop on Wireless network testbeds, experimental evaluation and characterization. ACM, 2007, pp. 59–66.

[29] J. Friedman, A. Davitian, D. Torres, D. Cabric, and M. Srivastava, “Angle-of-arrival-assisted relative interferometric localization using software defined radios,” in Military Communications Conference, 2009. MILCOM 2009. IEEE, 2009, pp. 1–8.

[30] L. Wang, Support vector machines: theory and applications. Springer Science & Business Media, 2005, vol. 177.

[31] M. H. Changrampadi, “A fusion-based multiclass AdaBoost for classifying object poses using visual and ir images,” 2011.

[32] E. Jedari, Z. Wu, R. Rashidzadeh, and M. Saif, “Wi-Fi based indoor location positioning employing random forest classifier,” in Indoor Positioning and Indoor Navigation (IPIN), 2015 International Conference on. IEEE, 2015, pp. 1–5.

[33] S. Bozkurt, G. Elibol, S. Gunal, and U. Yayan, “A comparative study on machine learning algorithms for indoor positioning,” in Innovations in Intelligent SysTems and Applications (INISTA), 2015 International Symposium on. IEEE, 2015, pp. 1–8.

[34] D. Taniuchi and T. Maekawa, “Robust Wi-Fi based indoor positioning with ensemble learning,” in Wireless and Mobile Computing, Networking and Communications (WMC), 2014 IEEE 10th International Conference on. IEEE, 2014, pp. 592–597.

[35] J. J. Pan, S. J. Pan, J. Yin, L. M. Ni, and Q. Yang, “Tracking mobile users in wireless networks via semi-supervised colocalization,” Pattern Analysis and Machine Intelligence, IEEE Transactions on, vol. 34, no. 3, pp. 587–600, 2012.

[36] H. Dai, W.-h. Ying, and J. Xu, “Multi-layer neural network for received signal strength-based indoor localisation,” IET Communications, vol. 10, no. 6, pp. 717–723, 2016.

[37] C. L. Nikias, “Higher-order spectral analysis,” in Engineering in Medicine and Biology Society, 1993. Proceedings of the 15th Annual International Conference of the IEEE. IEEE, 1993, pp. 319–319.

[38] X. Zhong, A. Premkumar, and A. Madhukumar, “Particle filtering for acoustic source tracking in impulsive noise with alpha-stable process,” Sensors Journal, IEEE, vol. 13, no. 2, pp. 589–600, 2013.

[39] T.-H. Liu and J. M. Mendel, “A subspace-based direction finding algorithm using fractional lower order statistics,” Signal Processing, IEEE Transactions on, vol. 49, no. 8, pp. 1605–1613, 2001.

[40] Y. Freund, R. E. Schapire, “A decision-theoretic generalization of on-line learning and an application to boosting,” Journal of computer and system sciences, vol. 55, no. 1, pp. 119–139, 1999.

[41] Y. Freund, R. Schapire, and N. Abe, “A short introduction to boosting,” Journal-Japanese Society For Artificial Intelligence, vol. 14, no. 771, 780, p. 1612, 1999.

[42] P. Viola and M. J. Jones, “Robust real-time face detection,” International journal of computer vision, vol. 57, no. 2, pp. 137–154, 2004.