Preparation of Low Entropy Correlated Many-body States via Conformal Cooling Quenches

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We analyze a method for preparing low-entropy many-body states in isolated quantum optical systems of atoms, ions and molecules. Our approach is based upon shifting entropy between different regions of a system by spatially modulating the magnitude of the effective Hamiltonian. We conduct two case studies, on a topological spin chain and the spinful fermionic Hubbard model, focusing on the key question: can a “conformal cooling quench” remove sufficient entropy within experimentally accessible timescales? Finite temperature, time-dependent matrix product state calculations reveal that even moderately sized “bath” regions can remove enough energy and entropy density to expose coherent low temperature physics. The protocol is particularly natural in systems with long-range interactions such lattice-trapped polar molecules and Rydberg dressed atoms where the magnitude of the Hamiltonian scales directly with the density. To this end, we propose a simple implementation of conformal cooling quenches in a dilutely-filled optical lattice, where signatures of quantum magnetism can be observed.

Ultracold quantum gases have reached the extraordinary realm of sub-nanokelvin temperatures [1][2], revealing, along the way, phenomena ranging from Bose-Einstein condensation and Cooper-paired superfluidity to Mott insulators and localization [3][7]. This scientific impact owes, in part, to a flexible array of cooling techniques that can effectively quench the kinetic energy of atomic systems; indeed, the laser cooling of atomic registers in optical tweezers has enabled the observation of few-particle quantum interference and entanglement [8][9], while the evaporative cooling of Bose gases has realized temperatures nearly two orders of magnitude smaller than that required for condensation [10].

Nevertheless, these temperatures are still too high to emulate a number of more exotic- and delicate- quantum phases including antiferromagnetic spin liquids, fractional Chern insulators and high-temperature superconductivity [11][13]. The figure of merit for observing such physics is not the absolute temperature, but rather the dimensionless entropy density in units of $k_B$ [14]. Reaching ultra-low entropy densities remains a major challenge for many-body quantum simulations despite the multitude of kinetic cooling techniques. This challenge is particularly acute for gases in deep optical lattice potentials, for which transport, and thus evaporative cooling, is slowed [15]. Moreover, in lattice systems representing models of quantum magnetism, the entropy resides primarily in spin, rather than motional, degrees of freedom [16]. Expelling such entropy through evaporative cooling requires the conversion of spin excitations to kinetic excitations, a process that is typically inefficient [17][19].

Two broad approaches have been proposed toward overcoming this challenge. The first is adiabatic preparation, where one initializes a low entropy state and changes the Hamiltonian gradually until the desired many-body state is reached [20][21]. However, the final entropy density is bounded from below by the initial entropy density, and experimental constraints or phase transitions may preclude a suitable adiabat.

The second approach is to ‘shift entropy elsewhere’ [11][22][23], using the system’s own degrees of freedom as a bath.

Indeed, this approach helps to stabilize the Mott-insulating phase of the Bose-Hubbard model, where the low-density wings of the system serve as an entropy sink allowing for in situ evaporative cooling.

In this work, we analyze a class of methods—termed ‘conformal cooling quenches’—for shifting entropy by spatially modulating the magnitude of the Hamiltonian [25]. The intuition behind our approach is best illustrated as follows: Suppose that we take a system’s Hamiltonian $H$ and either suddenly or adiabatically reduce it by a factor $\lambda < 1$, taking $H \rightarrow \lambda H$. Since $k_B T$ has units of energy, the temperature $T$ is accordingly reduced by $T \rightarrow T/\lambda$. When applied to the entire system, this “cooling” is trivial, since it amounts to a change of units without reducing the entropy density. How-
ever, if the reduction by λ instead occurs for only a portion of the system, which we call the ‘bath,’ the change in temperature is physical, and establishes a temperature gradient; during equilibration, entropy will then flow out of the system and into the bath.

One virtue of the conformal cooling approach is that it can “cool” a system within a metastable state-space. For example, conforming cooling can be applied to a gas equilibrating at negative kinetic or spin temperature [26], bringing the system toward zero temperature from below. It can also be applied to gases equilibrating in high-energy manifolds of states, i.e. in excited bands of an optical lattice [27, 28]. Systems equilibrating at negative temperatures or in higher bands can exhibit strong frustration without complicated band engineering. We will begin by introducing the thermodynamics of our approach, focusing on two questions: 1) how much entropy can a cooling quench remove and 2) how long does it take? Next, we perform a numerical study of both a 1D topological spin-chain and the fermionic Hubbard model, demonstrating that realistic cooling quenches can remove enough entropy to reveal their low-temperature physics. Finally, we propose experimental implementations of our approach in systems with power-law interactions and the half-filled Fermi-Hubbard model.

**General Strategy**—We envision spatially demarcating the degrees of freedom into a “bath” (B) and “system” (S) which are placed “end-to-end,” so that the coupling between their boundaries scales sub-extensively with their volume (Fig. 1) [29]. We assume that the Hamiltonian $H_B$ (bath) is identical to $H_S$ (system), except that its magnitude is scaled by a factor $\lambda < 1$. The entropy (s) versus energy density (E) curves in the two regions are then related by $s_B(E) = s_S(E/\lambda)$ and their temperatures by $T_B(E) = \lambda T_S(E/\lambda)$ (Fig. 1). In the following, we will consider two protocols, “quenched” and “adiabatic.”

**Quench Protocol**—In the quench approach, the Hamiltonians are time-independent with $H_B = \lambda H_S$. At $t = 0$, we prepare a uniform initial state (e.g. a product state) and simply let it evolve. Equivalently, one can begin in thermal equilibrium with $H_B = H_S$, and then suddenly reduce $H_B$ to $H_B = \lambda H_S$. The overall system is now in local equilibrium, with the local density matrices in $B$ and $S$ identical, and thus, $s_B = s_S$ and $T_B = \lambda T_S$. As the system evolves toward global equilibrium, entropy will follow the thermal gradient and flow from $S$ to $B$. Thus, by probing region $S$, one can explore physics at an entropy density far lower than that of the initial state.

The final equilibrium temperature $T_f^{(q)}$ is determined by energy conservation,

$$ (V_S + \lambda V_B) E_S(T_f) = V_S E_S(T_f^{(q)}) + V_B \lambda E_S(T_f^{(q)}/\lambda) \quad (1) $$

where $V_S, V_B$ are the volumes of the system and bath and $T_f$ is the initial temperature of the system. When $\lambda V_B \gg V_S$, we have $T_f^{(q)} = \lambda T_f$, but more generally one should choose $\lambda$ so as to minimize $T_f^{(q)}$ based on the precise form of $E_S(T_f)$. While we have assumed a sharp distinction between $S$ and $B$ for simplicity, one can let the spatial modulation $\lambda(x)$ vary smoothly, in which case Eq. (1) is replaced by an integral over the energy density.

**Adiabatic Protocol**—The cooling is more effective if the magnitude of $H_B = \Lambda(t) H_S$ is instead slowly reduced in time, with $\Lambda(t = 0) = 1$ and $\Lambda(t \to \infty) = \lambda$. In the isentropic limit, the final system temperature $T_f^{(a)}$ is determined by

$$ (V_B + V_S) s_S(T_f) = V_B s_S(T_f^{(a)}) + V_S s_S(T_f^{(a)}) \quad (2) $$

with $T_f^{(a)} \leq T_f^{(q)}$. When the bath and system are end-to-end, diffusive dynamics imply that the equilibration time, $t_{eq}$, scales as $L_S^2/\Lambda(t)$ (q.v. Eq. (6)) where $L_S$ is the linear extent of the system and adiabaticity requires $\partial_t \Lambda \ll 1/t_{eq}$. For small $\Lambda$, the bath and system will eventually fall out of equilibrium and additional entropy will be produced, though the temperature will always be upper-bounded by $T_f^{(q)}$.

To demonstrate that conformal cooling can shift significant entropy out of the system even for moderate bath sizes and short time-scales, we numerically investigate two distinct settings: the $S = 1$ Haldane topological anti-ferromagnet and the fermionic Hubbard model.

**Conformal cooling in an $S = 1$ Haldane chain**—Consider a one dimensional chain of $S = 1$ spins with Hamiltonian

$$ H[\lambda_x] = \sum_x \lambda_x h_x = J \sum_x \lambda_x \left[ S_x \cdot S_{x+1} + \frac{\gamma}{3} (S_x \cdot S_{x+1})^2 \right]. \quad (3) $$

At both the Heisenberg point $\gamma = 0$ and the AKLT point $\gamma = 1$, the spin-chain is a gapped topological paramagnet in the Haldane phase [30, 31]. The topology of the phase has
a striking signature in a finite-length chain, which has pair of localized spin-1/2 edge states. At temperatures below the bulk gap, \( T < \Delta \sim J \), these localized edge states can coherently store quantum information for long times, providing a sharp experimental signature of the topological phase \([32]\).

Calculating the thermodynamics using exact diagonalization reveals that a modest bath size of \( V_B/V_S \approx 2-3 \) is sufficient to cool from the Neél product state \( |↑↓↑↓⋯\rangle \), which corresponds to an initial temperature \( T_i = 1.45\Delta \), to well below the gap, \( T_f^{(q)} \approx 0.7\Delta \) \([33]\). Since the spin chain is diffusive \([44]\), the time-scale required for cooling is determined by Fourier’s law. When \( \lambda(x) \) varies smoothly compared to the lattice scale, the local thermal conductivity \( \kappa \) and specific heat \( c \) are determined by rescaling, \( \kappa(T,x) = \lambda(x)\kappa_S(T(x)/\lambda(x)) \) and \( c(T,x) = c_S(T(x)/\lambda(x)) \), where \( \kappa_S(T) \) and \( c_S(T) \) are defined with \( \lambda = 1 \). Applying this within a simple lumped element model predicts that temperature will decrease as \([33]\).

\[
T_S(t) \sim T_f^{(q)} + \frac{\kappa L_S}{\sqrt{D_B}}(T_i - T_f^{(q)})
\]  

where \( D_B \) is the thermal diffusivity of the bath, \( L_S \) is the linear extent of the system and \( K \) is an \( O(1) \) geometrical factor. For bath temperatures above \( \lambda J/h \), the diffusivity will generically saturate to a temperature-independent value, \( D_B \propto \lambda J/h \) \([35]\), implying that \( t_{eq} \sim L_S^2(\lambda J/h)^{-1} \).

To verify these dynamics, we simulate the evolution of a finite-energy density pure state using the TEBD-algorithm \([36]\). It is exponentially difficult to simulate finite temperature dynamics and our simulations require an MPS bond-dimension of \( m = 20,000 \), limiting our system to \( L = 30 \) sites (Fig. 2) \([37]\). We initialize a uniform state \( |\Psi(0)\rangle = e^{-\frac{iH}{\hbar}|\lambda_z=1\rangle|↑↓↑↓⋯\rangle \), where \( \tau = 0.35/J \), resulting in an energy density that corresponds to temperature \( T_S = 0.51J \) after local equilibration \([38]\). The system is then quenched into a spatially non-uniform \( H[\lambda_z] \) (Fig. 2). Using the optimal \( \lambda_0 = 0.17 \) in the ‘bath’ leads to a final predicted temperature: \( T_f^{(q)} = 0.29J \).

The evolution of the local energy density \( \langle \lambda_z \hat{h}_z(t) \rangle \) during the cooling quench is depicted in Fig. 2a. The energy density in region \( S \) at time \( t = 100/J \) corresponds to \( T_S = 0.34J \), within 14% of the expected \( T_f^{(q)} \) \([33]\). Moreover, the relaxation dynamics are roughly consistent with \( T_S \sim T_f^{(q)} + (T_i - T_f^{(q)})/t_{eq}/t \), where \( t_{eq} \approx 0.22(K LS)^2(\lambda J/h)^{-1} \), consistent with the expectation \( 1/D_B \sim 0.19/\lambda \) \([33,35]\).

Even for a relatively small bath size, the cooling quench has a dramatic effect on the dynamical correlation function of the topological edge mode. Since the edge state in region \( S \) will generically have overlap with the right-most spin \( S^\mu_{end} \), its coherence can be probed via the correlation function

\[
C_{zz}(t) = \langle \Psi| S^\mu_{end}(t + t_f) S^\mu_{end}(t_f) |\Psi \rangle,
\]

where the measurement only begins after the cooling quench is complete \( (t_f = 100/J) \). At \( T = 0 \), these correlations should asymptote to a finite constant [Fig. 3a], while at large \( T \) [Fig. 3d], they will decay exponentially. We compare \( C_{zz}(t) \) under four preparation scenarios described in Fig. 3. The conformal cooling quench improves the coherence time by more than an order of magnitude.

Adiabatic conformal cooling in the fermionic Hubbard model—We next consider the adiabatic protocol applied to the fermionic Hubbard model, \( H = -\sum_{i<j,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_{i\sigma} \). Here, we focus on the Mott insulating phase at half-filling with \( t/U \ll 1 \) and \( T < U \). While the fermions’ motion is quenched, their spins interact via an effective anti-ferromagnetic Heisenberg interaction, \( H_{\text{eff}} = \sum_{(i,j)} J_{ij} [S_i \cdot S_j - \frac{1}{4}] \), where \( J = 4\frac{\lambda J}{\mu} \) is the super-exchange coupling.
In the Mott regime, adiabatic cooling is naturally realized if one can weakly modulate the depth of the optical potential, \( V(\vec{x}) = -V_0(\vec{x}) \sum_{i=1}^d \cos(k_i x_i)^2 \), where \( V_0(\vec{x}) \) is slowly varying. Changing \( V_0 \) has three effects on the effective Hamiltonian: \( U \) will increase, as the orbitals are further localized, \( \mu \) will increase, as the trap is deeper and \( t \) will decrease due to the barrier height. Since \( t \) is exponentially more sensitive than \( U \) to the trap-depth, \( \xi = \sqrt{\frac{\mu}{E_r}} \) (\( E_r \) is the recoil energy), the dominant effect is to modulate the hoppings \([39]\). Assuming \( \mu \) is compensated to maintain half-filling, the super-exchange energy becomes \( J_{ij} \propto \xi^2(x) e^{-4t(x)} \), precisely the desired modulation (Fig. 3b). Fortunately, a small modulation in \( V_0 \) is already capable of dramatically reducing the system’s temperature: for example, in the the 3D cubic Heisenberg model, a \( 6\% \) change in the trap depth can cool the system from \( 1.4T_N \) \([40]\) down to the Néel temperature, \( T_N \) \([33]\). While the above discussion assumed that \( t/U \ll 0 \), \( T \ll U \), using determinantal Quantum Monte Carlo, we find that cooling occurs even outside the Mott regime, so long as the initial entropy is less than \( k_B \log(2) \) \([33]\).

To confirm the effectiveness of the adiabatic protocol, we simulate the dynamics of the spinful 1D fermionic Hubbard model. We use a purified TEBD method to time evolve a finite-temperature ensemble \([41]\). At time \( t = 0 \) the Hamiltonian is uniform, \( U = 1, t_{ij} = 0.1 \), with an initial thermal state \( \rho = e^{-H/T}/Z \) at \( T = 1.4J \). We then time evolve the ensemble with a Hamiltonian in which \( t_{ij} \) decreases adiabatically in the bath. Since energy is not conserved, we divide it into heat and work, \( \dot{E} = \dot{Q} - \dot{W} \) \([33]\), and plot the evolution of the heat-density \( \dot{Q} \) in Fig. 5(a). Total heat is conserved, but with clear transport from \( S \) to \( B \). As a more qualitative thermometer, we note that at \( T = 0 \), the system should display algebraic anti-ferromagnetic correlations. To reveal them, we place a small Zeeman field \( H = 0.05S_z \) on the right edge spin, both in the initial thermal state and the subsequent dynamics. As depicted in Fig. 5 the finite temperature of the initial thermal state disorders the magnetization \( \langle S_z \rangle \), but as the dynamics proceed and cooling occurs, the antiferromagnetic correlations become clearly manifest.

Experimental implementation—Our conformal cooling protocols are well suited to systems with long-ranged interactions, such as polar molecules, Rydberg atoms, and trapped ions \([24, 44–47]\). To implement the quench protocol, we envision a setup where the average spacing between particles is larger in the bath than in the system, \( r_B > r_S \). Assuming power-law interactions \((1/R^\alpha)\), the Hamiltonian in \( B \) will be reduced by a constant factor \( \lambda = (r_S/r_B)^\alpha \) relative to that in \( S \) (Fig. 1b). This approach is particularly applicable to current polar molecule experiments where the optical lattice filling, \( v < 1 \), leads to random dilution \([24]\). Fortunately, the cooling quench is perhaps simplest to implement in this randomly diluted case, since one can make \( r_B \gg r_S \) merely by modulating the average density, without having to ensure the particles in \( B \) lie on a particular sub-lattice. In this case, simply time-evolving an initial product state in the presence of this density modulation will cool the high-density region, and could provide a simple route towards studying, for example, algebraically correlated random-singlet phases \([48–50]\).

We now propose an experimental route toward realizing the adiabatic cooling protocol in a cold-atomic fermi-Hubbard simulation \([17, 19]\). The key challenge is the generation of a modulated optical potential, \( V_0(\vec{x}) \), while keeping the chemical potential nearly constant. While the required change in trap depth is modest, the modulated potential nonetheless requires the ability to create two differing lattice potentials in the system and bath regions. The intermediate transitional region does not need to be spatially sharp and a “ramp” region (Fig. 5) will help to ensure that thermal diffusion remains efficient across this boundary.

In principle, a quantum gas microscope combined with digital mirror devices or spatial light modulators allows one to paint a near-arbitrary optical potential. However, since we require only the special case of a “binary” optical potential, we propose a simpler strategy based upon overlaying two distinct optical intensity patterns that are projected onto a single object plane. Rather than using a mirror for retro-reflection, each of the lattices \( B, S \) can be generated by two incoming counter-propagating beams, for a total of four beams: \( B_1, B_2, S_1, S_2 \) (Fig. 4a). By modulating the intensity of both \( B_1 \) and \( B_2 \), one can independently control the periodic component \( V_0 \) and the slow component \( \mu \) (likewise for \( S \)). Before reaching the object plane, we envision a binary spatial filter that transmits pattern \( S_1, S_2 \) to the system’s spatial region and pattern \( B_1, B_2 \) to the bath’s spatial region. Such spatial filtering can be easily achieved in two ways: 1) by utilizing a spatially patterned wave-plate followed by a polarizing filter or 2) by using a dichroic interference filter (Fig. 4a).

In summary, we have proposed a general method for prepar-
ing low-entropy many-body states in isolated quantum systems. Our approach can be naturally implemented in systems with power law interactions by simply diluting the particle density of the bath region; moreover, we also provide a simple experimental blueprint for implementing conformal cooling in the spinful fermionic Hubbard model. Looking forward, our proposal raises a number of intriguing questions: is it possible to implement a refrigeration cycle by repeated preparation of the bath state? Can one optimize a side-by-side geometry which could reduce the equilibration time? By performing conformal cooling during a quantum phase transition, can one reduce the rate of defect formation?

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Thermodynamics of the quench protocol for the $S = 1$ chain

Here we report the final system energy density, entropy density, and temperature as a function of both the initial energy density in units of the gap, $E_i/\Delta$, and the ratio of bath to system length $L_B/L_S$. The thermodynamic relation $T(E)$ was computed using exact diagonalization Eq. (3) for 10-site chain, which we then use to numerically solve Eq. (1). A table of results is reported in Fig. 6.

![Fig. 6](https://via.placeholder.com/150)

**FIG. 6.** Thermodynamics of the cooling quench for a spin-1 Heisenberg chain at $\gamma = 3/4$ (results for other $\gamma$ are similar). The final state of the system depends on the ratio of bath to system size, $L_B/L_S$, and the initial energy density above the ground state in units of the gap, $E_i/\Delta$. For comparison, the state $|\uparrow\downarrow\cdots\rangle$ is at $E_i/\Delta = 0.54$. (a) The optimal choice of bath scaling $\lambda_{opt}$. Larger baths allow for smaller $\lambda_{opt}$, and hence greater cooling. (b) The final system entropy $S_f$ in units of the infinite-temperature value $\log(3)k_B$. (c) The final system energy density above the ground state, $E_f$, in units of the gap. (d) The final system temperature, $T_f$, in units of the gap.

Diffusion times for the cooling quench

Solving the heat diffusion equation exactly is not possible, since the temperature dependence of the heat capacity and thermal conductivity leads to a non-linear PDE that depends on the details of the model. As an illustrative approximation, note that temperature gradients in $S$ decay exponentially over the diffusion time $\tau_S \sim L_S^2/D_S$, while when $V_B$ is large the average...
temperature $T_S$ of the system decays as a much slower power law, $T_S \sim 1/\sqrt{t}$. Since the dynamics of $T_S$ are so much slower than the dynamics of the temperature gradients in $S$, we can consider a simplified model which lumps the system together into a single heat capacity $C_S(T_S) = V_SC_S(T_S)$. As for the bath, we assume it is at a relatively high temperature in terms of the natural units $\lambda J$. In a high-temperature expansion for a spin model, both the heat capacity and thermal conductance scale as $1/T^2$, so the diffusivity $D_B = \kappa/c$ is nearly constant ($D_B = 5.3\lambda J/\hbar$ for the AKLT chain [35]). Thus we consider a continuous medium of diffusivity $D_B$ coupled to a single heat capacity $C_S$, where at time $t = 0$ the system temperature is $T_S = T_i$ and the bath temperature is $T_B = \lambda T_i$. Ignoring the temperature dependence of $c_S/c_B$, Fourier’s law can be solved to obtain

$$T_S(t) \sim T_i + \kappa \frac{c_S}{c_B} \frac{L_S}{\sqrt{T_D}} (T_i - T_B).$$

(6)

where $\kappa$ is an order-one geometrical factor and $T_i$ is the final equilibrium temperature. After the system reaches $T_S < \Delta$, with $\Delta$ the gap, $c_S/c_B$ will decrease exponentially with $T_S$ (since the system’s heat capacity is activated), so ignoring the temperature dependence of $c_S/c_B$ is no longer justified. This does not accelerate the cooling, however; the $t^{-1/2}$ behavior will remain since the temperature gradient in the bath must still relax. In Fig. 7 we show $T_S(t)$ for the cooling quench of the Haldane model.

**Definition of $Q$ for adiabatic quench.**

In the adiabatic protocol the Hamiltonian $H(t) = \sum_i h_i(t)$ changes in time, so energy is not conserved. Nevertheless, we would like a convenient way to show heat is being transported between the bath and system. For a large system we could use the local temperature as inferred from the local energy density, but for the small chains studied here we find artifacts near the edge and ramp which make it less useful to visualize.

Instead, we note that the energy changes according to $\partial_t E = \sum_i \langle t| (\partial_t h_i)| t \rangle$, and that the change in energy can be divided into the heat $Q$ and work $W$, $dE = \delta Q - \delta W$. In the adiabatic limit $\delta Q = 0$ globally, but heat can be transported between regions (of course transport of energy is driven by entropy production, so this transport itself must be arbitrarily slow in the adiabatic limit). This motivates the definition $Q = \sum_i Q_i$ where $\partial_t Q_i = \partial_t (\langle t| h_i | t \rangle) - \langle t| (\partial_t h_i) | t \rangle$. By construction $\partial_t Q = 0$, though each $Q_i$ may change. It is these $Q_i$ which are shown in the Fig. 5 of the main text. The practical implementation of this subtraction is illustrated in Fig. 8.

While the $Q_i$ can be defined in this manner away from the adiabatic limit, and $\sum_i Q_i$ is always constant, it does not have the precise definition of ‘heat,’ since the diffusive transport itself generates entropy, and hence heat $TdS$. Note, however, that in the system $\partial_t h_i = 0$, so there $Q_i$ reduces to the energy density. Since our experiment shows $\partial_t Q_i < 0$ in the system, it is unambiguously cooling.
FIG. 8. (a) Total energy $E$ as a function of time for the 1D Hubbard simulations. For numerical convenience, the coupling constants are changed in a series of small, discrete steps, leading to step-discontinuities in the energy interlaced with periods of $E$-conserving evolution. Likewise, the local energy density $E_i$ has step-discontinuities interlaced with slow dynamics. $Q_i$ is conveniently obtained from the $E_i$ by dropping all the step-discontinuities. (b) The total $Q$ is conserved to better than $10^{-4}J$; the error arises from numerical errors like the finite Trotter step used in the finite-temperature TEBD dynamics.

Adiabatic cooling of Hubbard model outside the Mott regime

The analysis of the Hubbard model in the main text assumed the Mott limit $t/U \to 0$, so that $t,U$ only appear through the super-exchange energy $J = \frac{4t^2}{U}$. To confirm the approach is effective outside the Mott regime, we note that for the adiabatic cooling to work we only require that the temperature satisfies $\partial_{s}T(s; t, U) > 0$, where $s$ is the entropy density. Using determinantal quantum Monte Carlo to determine $T$ for the 2D Hubbard model, we see this condition is satisfied whenever $s < \log(2)k_B$.[42]

FIG. 9. Thermodynamics of the 2D fermionic Hubbard model: dependence of the temperature $T$ on the entropy density $s$ and the model parameters $t, U$, at half-filling. We see that at fixed entropy, $\partial_{s}T|_{s} > 0$ whenever $s < \log(2)k_B$, implying the adiabatic scheme works outside the Mott limit. Calculations were done using the QUEST DQMC package on an $8 \times 8$ torus with extrapolation of the Trotter step to zero.