CRITICAL SUPERSTRING VACUA FROM NONCRITICAL MANIFOLDS
A Novel Framework for String Compactification

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ABSTRACT

A new framework is found for the compactification of supersymmetric string theory. It is shown that the massless spectra of Calabi–Yau manifolds of complex dimension $D_{\text{crit}}$ can be derived from noncritical manifolds of complex dimension $2k + D_{\text{crit}}$, $k \geq 0$. These higher dimensional manifolds are spaces whose nonzero Ricci curvature is quantized in a particular way. This class is more general than that of Calabi–Yau manifolds because it contains spaces that correspond to critical string vacua with no Kähler deformations, i.e. no antigenerations, thus providing mirrors of rigid Calabi–Yau manifolds. The constructions introduced here lead to new insights into the relation between exactly solvable models and their mean field theories on the one hand and Calabi–Yau manifolds on the other. They also raise fundamental questions about the Kaluza–Klein concept of string compactification, in particular regarding the rôle played by the dimension of the internal theories.

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1. Introduction

It is believed that the heterotic string without torsion can propagate consistently in a manifold only if this manifold is complex, Kähler and admits a covariantly constant spinor, i.e. has vanishing first Chern class. Manifolds of this type, so-called Calabi–Yau manifolds, are examples of left–right symmetric string vacua with $N = 2$ supersymmetry on the world sheet. It is further believed that the configuration space of such ground states features an important symmetry, not at all manifest in the construction of the superstring: mirror symmetry. The predictions of this symmetry, which has been discovered in the context of Landau–Ginzburg vacua in [1] and proven to exist in this framework in ref. [2], have been shown to be correct in all computations performed so far [3,4]. Independent evidence for this symmetry has been found in the context of orbifolds of exactly solvable tensor models by Greene and Plesser [5].

Mirror symmetry creates a puzzle. There are well–known Calabi–Yau vacua which are rigid, i.e. they do not have string modes corresponding to complex deformations of the manifold, fields that describe generations in the low energy theory. Since mirror symmetry exchanges complex deformations and Kähler deformations of a manifold, the latter describing the antigenerations seen by a four–dimensional observer, it would seem that the mirror of a rigid Calabi–Yau manifold cannot be Kähler and hence does not describe a consistent string vacuum. It follows that the class of Calabi–Yau manifolds is not the appropriate setting in which to discuss mirror symmetry and the question arises what the proper framework might be.

It is the purpose of this article to introduce a new class of manifolds which generalizes the class of Calabi–Yau spaces of complex dimension $D_{crit}$ in a natural way. The manifolds involved are of complex dimension $(2k + D_{crit})$ and have a positive first Chern class which is quantized in multiples of the degree of the manifold. Thus they do not describe, a priori, consistent string groundstates. Surprisingly, however, it is possible to derive from these higher dimensional manifolds the spectrum of critical string vacua. This can be done not only for the generations but also for the antigenerations. For particular types of these new manifolds it is in fact possible to construct $D_{crit}$–dimensional Calabi–Yau manifolds directly from the $(2k + D_{crit})$–dimensional spaces.

This new class of manifolds is, however, not in one to one correspondence with the class of Calabi–Yau manifolds as it contains manifolds which describe string vacua that do not contain massless modes corresponding to antigenerations. It is precisely this new type of manifold that is needed in order to construct mirrors of rigid Calabi–Yau manifolds without generations. The results presented in this article suggest that the noncritical manifolds described here are no less physical than critical manifolds and indeed define the appropriate generalization of the Calabi–Yau framework of string compactification. They also lead to important ramifications regarding the relation between Landau–Ginzburg theories and critical manifolds.
2. Higher Dimensional Manifolds with Quantized Positive First Chern Class

Consider the class of manifolds of complex dimension $N$ embedded in a weighted projective space $\mathbb{P}_{(k_1,...,k_{N+2})}$ as hypersurfaces

$$M_{N,d} = \mathbb{P}_{(k_1,k_2,...,k_{N+2})}[d] = \{ p(z_1, \ldots, z_{N+2}) = 0 \} \cap \mathbb{P}_{(k_1,...,k_{N+2})}$$

(1)

defined as the zero set of some transverse polynomial $p$ of degree $d$. Here the integers $k_i$ describe the weights of the ambient weighted projective space. $\mathbb{P}_{(k_1,k_2,...,k_{N+2})}[d]$ is called a configuration. Assume that for the hypersurfaces (1) the weights $k_i$ and the degree $d$ are related via the constraint

$$\sum_{i=1}^{N+2} k_i = Qd,$$

(2)

where $Q$ is a positive integer. Relation (2) is the defining property of the class of manifolds I will consider in this article. It is a rather restrictive condition in that it excludes many types of varieties which are transverse and even smooth but are not of physical relevance.

Alternatively, manifolds of the type above may be characterized via a curvature constraint. Because of (2) the first Chern class is given by

$$c_1(M_{N,d}) = (Q - 1) \ d \ h,$$

(3)

where $h$ is the pullback of the Kähler form of the ambient space. Hence the first Chern class is quantized in multiples of the degree of the hypersurface. For $Q = 1$ the first Chern class vanishes and the manifolds for which condition (2) holds are Calabi–Yau manifolds, defining consistent ground states of the supersymmetric closed string. For $Q > 1$ the first Chern class is nonvanishing and therefore these manifolds cannot possibly describe vacua of the critical string, or so it seems.

It will be shown below that these spaces are closely related to string vacua of critical dimension

$$D_{crit} = N - 2(Q - 1)$$

(4)

i.e. the critical dimension is offset by twice the coefficient of the first Chern class of the normal bundle. The evidence for this is twofold. First it is possible to derive from these higher dimensional manifolds the massless spectrum of critical vacua. Furthermore it is shown that it is possible to construct Calabi–Yau manifolds $M_{CY}$ of dimension $D_{crit}$ and complex codimension $\text{codim}_Q(M_{CY}) = Q$ directly from certain subclasses of hypersurfaces of type (2).

1The erudite reader will recognize that this definition is rather natural in the context of Landau–Ginzburg compactification with an arbitrary number of scaling fields as will become clear below. A particularly simple manifold in this class, the cubic sevenfold $\mathbb{P}^7[3]$, has been the subject of recent investigations [6–8].
the critical dimension and the codimension the class of manifolds to be investigated below can be described as the projective configurations

\[ \mathbb{P}(k_1, \ldots, k_{D_{\text{crit}} + 2Q}) \left[ \frac{1}{Q} \sum_{i=1}^{D_{\text{crit}}+2Q} k_i \right]. \tag{5} \]

The class defined by (5) contains manifolds with no antigenerations. Hence it is necessary to have some way other than Calabi–Yau manifolds to represent string ground states in order to establish a relation between such higher dimensional manifolds and string vacua. One possible way to achieve this is via Landau–Ginzburg theories: manifolds of type (5) can be viewed as a projectivization via a weighted equivalence of an affine noncompact hypersurface defined by the same polynomial

\[ C_{(k_1, \ldots, k_{N+2})}[d] \ni \{ p(z_1, \ldots, z_{N+2}) = 0 \}. \tag{6} \]

Because the polynomial \( p \) is assumed to be transverse in the projective ambient space the affine variety has a very mild singularity: it has an isolated singularity at the origin, defining what is called a catastrophe in the mathematics literature.

The complex variables \( z_i \) parametrizing the ambient space are to be viewed as the field theoretic limit, \( \varphi_i(z, \bar{z}) = z_i \), of the lowest components of the order parameters \( \Phi_i(z_i, \bar{z}_i, \theta^\pm_i, \bar{\theta}^\pm_i) \) described by chiral \( N = 2 \) superfields of a 2–dimensional Landau–Ginzburg theory. It was the important insight of Martinec [9] and Vafa and Warner [10] that such Landau–Ginzburg theories are useful for the understanding of string vacua and also that much information about such ground states is already encoded in the associated catastrophe (6). A crucial piece of information about a vacuum, e.g., is its central charge. Using a result from singularity theory, it is easy to derive that the central charge of the conformal fixed point of the LG theory is

\[ c = 3 \sum_{i=1}^{N+2} (1 - 2q_i), \]

where \( q_i = k_i/d \) are the U(1) charges of the superfields. It is furthermore possible to derive the massless spectrum of the GSO projected fixed point of the Landau–Ginzburg theory defining the string vacuum directly from the catastrophe (6) via a procedure described by Vafa [11]. The manifolds (5) therefore correspond to (in general somewhat unconventional) Landau–Ginzburg theories with central charge

\[ c = 3(N - 2(Q - 1)) = 3D_{\text{crit}} \tag{7} \]

where the relation (4) has been used.

In certain benign situations the subring of monomials of charge 1 in the chiral ring describes the generations of the vacuum [12]. For this to hold at all it is important that the GSO projection is the canonical one with respect to the cyclic group \( \mathbb{Z}_d \), the order of which is the degree \( d \) of
the superpotential \[ W = \sum \Phi_i^3 \]. Thus the generations are easily derived for this subclass of theories in (3) because the polynomial ring is identical to the chiral ring of the corresponding Landau–Ginzburg theory. In general a more sophisticated analysis, involving the resolution of higher dimensional singularities, will have to be done [14].

It remains to extract the second cohomology. In a Calabi–Yau manifold there are no holomorphic 1–forms and hence all of the second cohomology is in \( H^{(1,1)} \). Because of Kodaira’s vanishing theorem the same is true for manifolds with positive first Chern class and therefore for the manifolds under discussion. At first sight it might appear hopeless to find a construction which would allow one to relate the antigenerations of the critical vacuum to the \((1,1)\)–cohomology of the higher dimensional manifold because of the following example. Consider the orbifold \( T_3^3/\mathbb{Z}_3^2 \) where the two actions are defined as \( (z_1, z_4) \rightarrow (\alpha z_1, \alpha^2 z_4) \), all other coordinates invariant and \( (z_1, z_7) \rightarrow (\alpha z_1, \alpha^2 z_7) \), all other invariant. Here \( \alpha \) is the third root of unity. The resolution of the singular orbifold leads to a Calabi–Yau manifold with 84 antigenerations and no generations [15]. This is precisely the mirror flipped spectrum of the exactly solvable tensor model \( 1^9 \) of \( 9 \) copies of \( N = 2 \) superconformal minimal models at level \( k = 1 \) [16] which can be described in terms of the Landau–Ginzburg potential \( W = \sum \Phi_i^3 \) which belongs to the configuration \( C_{(1,1,1,1,1,1,1,1,1)} [3] \).

This Landau–Ginzburg theory clearly is a mirror candidate for the resolved torus orbifold just mentioned [6-8] \( 3 \) and the question arises whether a manifold corresponding to this LG potential can be found. Since the theory does not contain modes corresponding to \((1,1)\)–forms it seems that the manifold cannot be Kähler and hence not projective. Thus it appears that the 7–dimensional manifold \( \mathbb{P}_8[3] \) whose polynomial ring is identical to the chiral ring of the LG theory is merely useful as an auxiliary device in order to describe one sector of the critical LG string vacuum: Even though there exists a precise identity between the Hodge numbers in the middle cohomology group of the higher dimensional manifold and the middle dimensional cohomology of the Calabi–Yau manifold this is not the case for the second cohomology group.

3. Relation between Critical and Noncritical Manifolds

It turns out that by looking at the manifolds of the type described by (3) in a particular way it is indeed possible to extract the second cohomology in a canonical manner (even if there is

\[ \text{It does not hold for projections that involve orbifolds with respect to different groups such as those discussed in [13]. This is to be expected as these modified projections can be understood as orbifolds of canonically constructed vacua. The additional moddings generate singularities the resolution of which introduces, in general, additional modes in both sectors, generations and antigenerations.} \]

\[ \text{A detailed comparison of the Yukawa couplings of the Landau–Ginzburg theory with those of the ‘instanton corrected’ resolved orbifold has been performed in [7].} \]
The way this works is as follows: the manifolds will, in general, not be described by smooth spaces but will have singularities which arise from the projective identification. The basic idea now is to associate the existence of antigenerations in a critical string vacuum with the existence of singularities in these higher dimensional noncritical spaces.

Since the structure of these geometrical singularities depends on the precise form of the polynomial constraint it is difficult to prove the correctness of this idea in full generality. Instead I will, in the following, make the ideas involved more precise and illustrate how they work with a few particularly simple classes of theories, leaving a more detailed investigation of other types of manifolds to a more extensive discussion [14]. As an unexpected bonus this derivation will provide new insight into the Landau–Ginzburg/Calabi–Yau connection.

It is useful to first consider an example in some detail. The GSO projected LG theory based on the superpotential

$$W = \sum_{i=1}^{3} \left( \Phi_{i}^{3} \Psi_{i} + \Psi_{i}^{3} \right) + \Psi_{4}^{3}$$

(8)

describes a vacuum with 35 generations and 8 antigenerations. Associated to this groundstate is the affine configuration $C_{(2,3,2,3,3,3)}[9]$ which induces, via projectivization, a 5–dimensional weighted hypersurface $\mathbb{P}_{(2,2,3,3,3,3)}[9]$. This compact manifold has two types of orbifold singularities:

$$\mathbb{Z}_{3} : \mathbb{P}_{3}[3] \ni \{ p_{1} = \sum_{i=1}^{4} x_{i}^{3} = 0 \}$$

$$\mathbb{Z}_{2} : \mathbb{P}_{2}.$$  

(9)

The $\mathbb{Z}_{3}$–singular set is a smooth cubic surface which supports seven (1,1)–forms whereas the $\mathbb{Z}_{2}$–singular set is just the projective plane and therefore adds one further (1,1)–form. Hence the singularities induced on the hypersurface by the singularities of the ambient weighted projective space give rise to a total of eight (1,1)–forms. A simple count leads to the result that the subring of monomials of charge 1 is of dimension 35. Thus we have derived the spectrum of the critical theory from the noncritical manifold $\mathbb{P}_{(2,2,3,3,3,3)}[9]$.

It is presumably possible to derive this result via a surgery process on the singular space, but more important is, at this point, that the idea introduced above of relating the spectrum of the string vacuum to the singularity structure of the noncritical manifold also makes it possible to derive from these higher dimensional manifolds the Calabi–Yau manifold of critical dimension! This leads to a canonical prescription which allows to pass from the Landau–Ginzburg theory to its geometrical counterpart when the model has antigenerations.

This works as follows: Recall that the structure of the singularities of the weighted hypersurface just involves part of the superpotential, namely the cubic polynomial $p_{1}$ which determined
the $\mathbb{Z}_3$ singular set described by a surface. The superpotential thus splits naturally into the two parts $p = p_1 + p_2$, where $p_2$ is the remaining part of the polynomial. The idea is to consider the product $\mathbb{P}_3[3] \times \mathbb{P}_2$, where the factors are determined by the singular sets of the higher dimensional space and to impose on this 4–dimensional space a constraint described by the remaining part of the polynomial which did not take part in constraining the singularities of the ambient space. In the case at hand this leaves a polynomial of bidegree $(3, 1)$ and hence we are lead to a manifold embedded in

$$\mathbb{P}_2 \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ni \begin{cases} p_1 = y_1^3 x_1 + y_2^3 x_2 + y_3^3 x_3 = 0 \\ p_2 = \sum_{i=1}^4 x_i^3 = 0 \end{cases}. \quad (10)$$

But this is a well known Calabi–Yau manifold of complex dimension 3, first constructed in [17]!

The ideas just described are general. A subclass of manifolds of a different type which can be discussed in this framework rather naturally is defined by the projective configurations

$$\mathbb{P}_{(2k,k,k,k+3,k+4,k_5)}[2K] \quad (11)$$

where $K = k + k_3 + k_4 + k_5$ and it is assumed, for simplicity, that $K/k$ and $K/k_i$ are integers. The potentials are

$$W = \sum_{i=1}^{2} (x_i^{K/k} + x_i y_i^2) + x_3^{K/k_3} + x_4^{K/k_4} + x_5^{K/k_5} \quad (12)$$

and the singularities in these manifolds are of two types,

$$\mathbb{Z}_2 : \mathbb{P}_{(k,k,k_3,k_4,k_5)}[K] \ni \begin{cases} p_1 = \sum_{i=1}^5 x_i^{K/k_i} = 0 \end{cases}$$

$$\mathbb{Z}_{K-k} : \mathbb{P}_1. \quad (13)$$

The $\mathbb{Z}_2$–singular set is 3–fold with positive first Chern class embedded in weighted $\mathbb{P}_4$ whereas the $\mathbb{Z}_{K-k}$–singular set is just the sphere $S^2 \sim \mathbb{P}_1$.

In complete analogy with the previous discussion the manifolds in this class lead to critical manifolds embedded in

$$\mathbb{P}_1 \begin{bmatrix} 2 & 0 \\ k & K \end{bmatrix} \ni \begin{cases} p_1 = y_1^2 x_1 + y_2^2 x_2 = 0 \\ p_2 = x_1^{K/k} + x_2^{K/k} + x_3^{K/k_3} + x_4^{K/k_4} + x_5^{K/k_5} = 0 \end{cases}. \quad (14)$$

That this correspondence is in fact correct can be inferred from the work of [18] where it was shown that these codimension–2 weighted CICYs correspond to $N = 2$ minimal exactly solvable tensor models of the type

$$\left[2 \left( \frac{K}{k} - 1 \right) \right]_D^2 \cdot \prod_{i=3}^5 \left( \frac{K}{k_i} - 2 \right)_A. \quad (15)$$

where the subscripts indicate the affine invariants chosen for the individual levels.
The general picture that emerges from these constructions then is the following: Embedded in the higher dimensional manifold is a submanifold which is fibered, the base and the fibers being determined by the singular sets of the ambient manifold. The Calabi–Yau manifold itself is a hypersurface embedded in this fibered submanifold. A heuristic sketch of the geometry is shown in the Figure 1.

The examples above illustrate the simplest situation that can appear. In more complicated manifolds the singularity structure will consist of hypersurfaces whose fibers and/or base themselves are fibered, leading to an iterative procedure. The submanifold to be considered will, in those cases, be of codimension larger than one and the Calabi–Yau manifold will be described by a submanifold with codimension larger than one as well. In the most general situation the fiber bundle will presumably not be simply a product bundle as in the previous examples but will presumably involve nontrivial twists.

The relation between the noncritical manifolds of type (5) and critical string vacua is not 1–1. Indeed, by using the construction of ‘splitting’ and ‘contracting’ Calabi–Yau manifolds introduced in [19] it follows that noncritical manifolds of different dimensions can lead to one and the same critical vacuum. Thus there exist nontrivial relations between the spaces of type (5). A more detailed discussion of these aspects will appear in [14].

In the framework described above it becomes clear what is special about string vacua that do not have modes corresponding to antigenerations. Consider again the example related to the tensor model $\mathbb{C}^9$. Its LG theory describes an affine cubic surface in $\mathbb{C}^9$ the naive compactification of which leads to

$$\mathbb{P}^8[3] \ni \{ p(z_1, \ldots, z_9) = \sum_{i=1}^9 z_i^3 = 0 \}. \quad (16)$$

Counting monomials leads to the spectrum of 84 generations found previously for the corresponding string vacuum and because this manifold is smooth no antigenerations are expected in this model! Hence there does not exist a Calabi–Yau manifold that describes this ground state. A second theory in the space of all LG vacua with no antigenerations is

$$\left(2^6\right)_{A^6}^{(0,90)} \sim \mathcal{C}_{(1,1,1,1,1,1,2)}^9[4] \ni \left\{ \sum_{i=1}^6 z_i^4 + z_7^2 = 0 \right\} \quad (17)$$

with an obviously smooth manifold $\mathbb{P}^\mathcal{C}_{(1,1,1,1,1,1,2)}[4]$.

4. Generalization to Arbitrary Critical Dimensions

Even though the examples discussed in the previous section are all concerned with critical vacua of central charge $c = 9$ and the way they are related to the new class of noncritical spaces of
dimension $3 + 2k$, it should be clear that these considerations are not specific to this particular set of string groundstates. Instead of considering ‘compactifications’ of the heterotic string down to the physical dimension, 4, we can contemplate compactifying down to 2, 6 or 8 dimensions, or else, discuss the class of manifolds type (5) independently from string considerations.

To illustrate this point consider the infinite class of $(n + 1)$–dimensional manifolds

$$\mathbb{P}_{(2, n-1, 2, n-1, 2, \ldots, 2)}[2n] \ni \{ p = \sum_{i=1}^{2} (x_i^n + x_i y_i^2) + x_3^n + \cdots + x_{n+1}^n = 0 \}. \quad (18)$$

According to the ideas of the previous sections these spaces are related to Calabi–Yau manifolds embedded in products of projective spaces

$$\mathbb{P}_1 \left[ \begin{array}{cc} 2 & 0 \\ 1 & n \end{array} \right] \ni \left\{ \begin{array}{l} p_1 = y_1^2 x_1 + y_2^2 x_2 = 0 \\ p_2 = \sum_{i=1}^{n+1} x_i^n = 0 \end{array} \right\}, \quad (19)$$

corresponding to critical vacua with central charge $c = 3(n-1)$. Particularly interesting is the case of K3 because it shows that the procedure also works for smooth configurations involving non–Fermat type potentials. It should be emphasized that the constructions of section 3 are not restricted to the classes of spaces to which I have confined the present analysis for the sake of brevity. A more complete discussion is involved and will appear elsewhere [14].

5. Conclusion

Mirror symmetry cannot be understood in the framework of Calabi–Yau manifolds. Assuming that mirror symmetry is indeed a symmetry of the space of left–right symmetric vacua and that the geometrical framework is general enough would lead one to suspect the existence of a space of a new type of noncritical manifolds which contain information about critical vacua, such as the mirrors of rigid Calabi–Yau manifolds. Mirrors of spaces with both sectors, antigenerations and generations, are, however, again of Calabi–Yau type and hence the noncritical manifolds which correspond to such groundstates should make contact with Calabi–Yau manifolds in some manner.

It has been shown that the class of higher dimensional Kähler manifolds of type (5) with positive first Chern class, quantized in a particular way, generalizes the framework of Calabi–Yau vacua in the desired way: For particular types of such noncritical manifolds Calabi–Yau manifolds of critical dimension are embedded algebraically in a fibered submanifold. For string vacua which cannot be described by Kähler manifolds and which are mirror candidates of rigid Calabi–Yau manifolds the higher dimensional manifolds still lead to the spectrum of the critical vacuum and a rationale emerges that explains why a Calabi–Yau representation is not possible in such theories. Thus these manifolds of dimension $c/3 + 2k$ define an appropriate framework in which to discuss mirror symmetry.
There are a number of important consequences that follow from the results of the previous sections. First it should be realized that the relevance of noncritical manifolds suggests the generalization of a conjecture regarding the relation between superconformal field theories with \(N=1\) spacetime supersymmetry and central charge \(c = 3D\), where \(D \in \mathbb{N}\), on the one hand and Kähler manifolds of complex dimension \(D\) with vanishing first Chern class on the other. It was suggested by Gepner [20] that this relation is 1–1. It follows from the results above that instead superconformal theories of the above type are in correspondence with Kähler manifolds of dimension \(c/3 + 2k\) with a first Chern class quantized in multiples of the degree.

A second consequence is that the ideas of section 3 lead, for a large class of Landau-Ginzburg theories, to a new canonical prescription for the construction of the critical manifold, if it exists, directly from the 2D field theory.

Recently Batyrev [21] introduced a new construction of mirrors of Calabi–Yau manifolds based on dual polyhedra. His method appears to apply only to manifolds defined by one polynomial in a weighted projective space or products thereof. The method of toric geometry that is used in [21] is however not restricted to Calabi–Yau manifolds and therefore the constructions described in sections 3 and 4 lead to the exciting possibility of extending Batyrev’s results to Calabi–Yau manifolds of codimension larger than one by proceeding via noncritical manifolds.

A final remark is that in this framework the role played by the dimension of the manifolds becomes of secondary importance. This is as it should be, at least for an effective theory, which tests only matter content and couplings. It is, however, somewhat mysterious that via ineffective splittings [19], manifolds of different dimension describe one and the same critical vacuum.

It is clear that the emergence in string theory of manifolds with quantized first Chern class should be understood better. The results presented here are a first step in this direction. They indicate that these manifolds are not just auxiliary devices but may be as physical as Calabi–Yau manifolds of critical dimension. In order to probe the structure of these models in more depth it is important to get further insight into the complete spectrum of these theories and to compute the Yukawa couplings of the fields. The spectra of the higher dimensional manifolds contain additional modes beyond those that are related to the generations and antigenerations of the critical vacuum and the question arises what physical interpretation these fields afford.

A better grasp on the complete spectrum of these spaces should also give insight into a different, if not completely independent, approach toward a deeper understanding of these higher dimensional manifold, which is to attempt the construction of consistent \(\sigma\)-models defined via these spaces. Control of the spectrum will shed light on the precise relation between the \(\sigma\)-models based on Calabi–Yau manifolds and noncritical \(\sigma\)-models.
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