Highly Excited Hadrons in QCD and Beyond\textsuperscript{1}

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Abstract

I discuss two issues related to high “radial” excitations which attracted much attention recently: (i) chiral symmetry restoration in excited mesons and baryons, and (ii) universality of the $\rho$-meson coupling in QCD and AdS/QCD. New results are reported and a curious relation between an AdS/QCD formula and 1977 Migdal’s proposal is noted.

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1 Introduction

A renewed interest to highly excited mesons in QCD is explained by at least two reasons.

First, the gravity/gauge correspondence, originally established for conformal field theories on the gauge side, is being aggressively expanded to include closer relatives of QCD, with the intention to get a long-awaited theoretical control over QCD proper. The present-day “bottom-up” approach is as follows: one starts from QCD and attempts to guess its five-dimensional holographic dual. In this way, various holographic descriptions of QCD-like theories emerge and their consequences are being scrutinized with the purpose of finding the fittest model.

This approach goes under the name “AdS/QCD.” Since the limit $N = \infty$ is inherent to AdS/QCD, the meson widths vanish, and one can unambiguously define masses and other static characteristics of excited states. Explorations of this type were reported in Refs. [1–17]. In refined holographic descriptions expected to emerge in the future one hopes to get asymptotically linear primary and daughter Regge trajectories, and obtain residues and other parameters and regularities pertinent to hadronic physics, and demonstrate their compatibility with experiment — a goal which has not yet been achieved.

The recently suggested orientifold large-$N$ expansion [18] which is complementary to the ’t Hooft one [19], provides another framework which can be used for studying excited states.

The second reason is a clear-cut demonstration that the chiral symmetry of the QCD Lagrangian is empirically restored in excited mesons and baryons, due to Glozman and collaborators [20–23]. As is well-known from the early days of QCD, highly excited hadrons can be described quasiclassically (see e.g. [24]; recapitulated recently in Ref. [25] at the qualitative level). The quasiclassical description implies, in particular, asymptotically linear Regge trajectories. Needless to say, it also implies linear realization of all symmetries of the QCD Lagrangian at high energies, in particular, in high radial excitations. Indeed, in such states the valent quarks on average have high energies — high compared to $\Lambda_{QCD}$ — and, thus, can be treated quasiclassically so that effects due to the quark condensate are inessential and can be neglected.

Nevertheless, the fact that excited states do exhibit the full linearly-realized chiral symmetry of QCD seemingly caught some theorists by surprise, probably, due to an over-concentration on the low-lying states for which the chiral symmetry is broken. (It would be more exact to say that the axial SU($N_f$) where $N_f$ is the number of massless flavors is realized non-linearly, in the Goldstone mode.) According to [20–23], the symmetry is largely restored already for the first radial excitations; for
instance, three excited pions and one excited scalar-isoscalar meson form an almost degenerate dimension-4 representation of SU(2)×SU(2)∼O(4).\(^2\)

The question is what can be said quantitatively on the rate of the symmetry restoration. In other words, what is the \(n\)-dependence (\(n\) is the excitation number) of the splittings \(\delta M_n\) of the radially excited states from one and the same representation of SU(2)×SU(2)?

In the first part of my talk I suggest some natural “pedestrian” estimates of the rate of the chiral symmetry restoration. In the second part I will focus on an aspect of AdS/QCD which was recently discussed in [13, 14]: implementation of the vector meson dominance (VMD) and universality of the \(\rho HH\) coupling. AdS/QCD-based results will be confronted with QCD expectations.

2 Chiral symmetry restoration: generalities

If both vector and axial SU(2)'s are linearly realized, hadronic states must fall into degenerate multiplets of the full chiral symmetry. The degeneracy is lifted by an SU(2)×SU(2)→SU(2) breaking that dies off with energy (or the excitation number, which is the same). I will first briefly review an appropriate representation theory [22]. My next task will be an estimate of \(\delta M_n\) versus \(n\) at \(n \gg 1\) where \(\delta M_n\) is the mass difference in particular chiral representations, for instance, the mass difference of the scalar–pseudoscalar mesons or the vector–axial-vector ones.

It is clear that to define highly excited states \textit{per se} we need large \(N\). In tricolor QCD, as we will shortly see, resonance widths rapidly grow with the excitation number; as a result, for large \(n\) the very separation of mesons from continuum becomes impossible, and the question of mass splittings and other similar questions cannot be addressed. \textit{I will systematically exploit the large-\(N\) limit.} I will consider the massless quark sector consisting of two flavors. Extension to three flavors is rather straightforward.

The large-\(N\) limit sets an appropriate theoretical framework for consideration of excited mesons. As for excited baryons, there is no reason for their widths to be suppressed at \(N \to \infty\). Therefore, theoretical analysis of highly excited baryonic

\(^2\)I assume for simplicity that there are two massless flavors, ignoring the strange quark. The chiral U(1) also gets restored as a valid flavor symmetry: the U(1) chiral anomaly dies off in the limit of large number of colors. Even if the number of colors \(N\) is kept fixed, at large energies (i.e. high \(n\)), where perturbation theory becomes, in a sense, applicable, the axial U(1) gets restored since one can always redefine the quark U(1) current by adding a Chern-Simons current in such a way that the U(1) charge is conserved in perturbation theory. Restoration of the axial U(1) in radially excited mesons is clearly seen in experiment, cf. e.g. [22].
states becomes problematic, as at $n \gg 1$ they should form a continuum. I will limit my discussion to mesonic states. (Empirically, isolation of excited baryon resonances from existing hadronic data seems possible, and data seem to indicate restoration of the full chiral symmetry for excited baryons [20–23]. Representations of unbroken $\text{SU}(2) \times \text{SU}(2)$ for baryons had been studied long ago, even before the advent of QCD [26].)

Analysis of the chiral symmetry is more conveniently performed in terms of the Weyl rather than Dirac spinors. Each Dirac spinor is a pair of two Weyl ones. We can take them both to be left-handed, $\chi$ and $\eta$, the first being triplet with respect to $\text{SU}(3)_{\text{color}}$, the second antitriplet.$^3$ To form color singlets we convolute $\chi \eta$ or, alternatively, $\bar{\chi} \chi$, $\bar{\eta} \eta$. The Dirac spinor $\Psi$ combines one left-handed and one right-handed spinor, $\Psi \sim \{\chi_\alpha, \bar{\eta}_\dot{\beta}\}$ where $\alpha$ and $\dot{\beta}$ are spinorial indices. Each chiral spinor carries a flavor $\text{SU}(2)$ index. Since there are two linearly realized $\text{SU}(2)$’s, there are two flavor $\text{SU}(2)$ indices, $\chi^k$ and $\eta^{\dot{a}}$ ($k, \dot{a} = 1, 2$) which are independent. We can call them “left” and “right” isospin. Conventional isospin entangles “left” and “right” isospins.

The exact conserved quantum numbers of QCD, namely, conventional isospin, total angular momentum and parity, do not always completely specify the full structure of a quark-antiquark meson, as we will see shortly. Distinct patterns of “left” and “right” isospin additions can lead to distinct mesons having the same conventional isospin, total angular momentum and parity.

Let us now discuss the simplest representations. The scalar (pseudoscalar) mesons are of the type

$$\chi \eta \pm \bar{\chi} \bar{\eta}. \quad (1)$$

Its Lorentz structure is $(1/2, 0) \times (1/2, 0) \rightarrow (0, 0)$ and $(0, 1/2) \times (0, 1/2) \rightarrow (0, 0)$. The isospin structure is $(1/2, 1/2)$. In terms of conventional isospin this is a triplet plus a singlet. In terms of $\text{SU}(2) \times \text{SU}(2) \sim \text{O}(4)$ we have two four-dimensional chiral representations: The first includes scalar isoscalar plus pseudoscalar isovector, the second pseudoscalar isoscalar plus scalar isovector.

In fact, the symmetry that gets restored is higher than just $\text{SU}(2) \times \text{SU}(2) \sim \text{O}(4)$. Indeed, at $N \rightarrow \infty$ the two-point functions of the currents $\bar{\Psi} \Psi$ and $\bar{\Psi} \tau^a \Psi$ are degenerate (here $\tau^a$ are the conventional isospin generators, $a = 1, 2, 3$), since the quark-gluon mixing that can occur in the isoscalar — but not isovector — channel is suppressed by $1/N$ and, thus, can be neglected. The above degeneracy is in one-to-one correspondence with the fact that the full flavor symmetry of the QCD

$^3$If the gauge group is extended to $\text{SU}(N)$, with respect to $\text{SU}(N)_{\text{color}}$ the field $\chi$ transform as $N$, while $\eta$ as $\overline{N}$.
Lagrangian is $U(1) \times U(1)_A \times SU(2) \times SU(2)$. The vector $U(1)$, the baryon charge, plays no role in the meson sector. The linear realization of $U(1)_A \times SU(2) \times SU(2)$ implies that the two four-dimensional representations of $SU(2) \times SU(2)$ are combined in one irreducible eight-dimensional representation of $U(1)_A \times SU(2) \times SU(2)$.

If we pass to nonvanishing angular momenta, we observe that the vector and axial-vector mesons can be of two types,

$$\bar{\chi}_\alpha \chi_\alpha \pm \bar{\eta}_\alpha \eta_\alpha, \tag{2}$$

$$\chi_{\{\alpha \beta\}} \pm \text{h.c.}, \tag{3}$$

where the braces denote symmetrization. The first one is Lorentz $(1/2, 1/2)$, the second $(1, 0) + (0, 1)$. In terms of the “left” and “right” isospins, it is the other way around. The state $\bar{\chi}_\alpha \chi_\alpha$ has isospin $(1/2, 0) \times (1/2, 0) \rightarrow (1, 0) + (0, 0)$. It is a triplet plus a singlet with respect to the conventional isospin. Taking into account the second term in (2), we have a vector isovector plus an axial-vector isovector plus two isosinglets. At large $N$ they all, taken together, form an eight-dimensional representation. The state (3) forms $SU(2) \times SU(2)$ quadruplets $(1/2, 0) \times (0, 1/2)$: a vector isoscalar plus an axial-vector isovector, and vice versa, a vector isovector plus an axial-vector isoscalar. Again, both quadruplets are combined into an eight-dimensional representation at $N \rightarrow \infty$.

A physical ground-state $\rho$ meson which is roughly an equal mixture of (2) and (3) is “polygamous” and has two distinct chiral partners [22]: an axial-vector isovector and an axial-vector isoscalar.

3 The rate of the chiral symmetry restoration

For simplicity I will discuss the mass splittings of scalar versus pseudoscalar excited mesons, produced from the vacuum by the operators $\bar{\Psi} \Psi$ and $i\bar{\Psi} \gamma_5 \Psi$, respectively. As was mentioned, at $N \rightarrow \infty$ their isotopic structure is inessential. The mass splittings in other chiral multiplets must have the same $n$ dependence.

The chiral symmetry is broken by the quark mass term, which I will put to zero. Then the local order parameter representing the chiral symmetry breaking is $\langle \bar{\Psi} \Psi \rangle$. Unfortunately, it is rather hard to express the mass splittings in the individual multiplets in terms of this local parameter. Generally speaking, the fact that its mass dimension is 3 tells us that various chiral symmetry violating effects will die off as

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4This fact was, of course, known to QCD practitioners from the very beginning, and was repeatedly exploited in QCD and in models in various contexts, e.g. [27].
inverse powers of $M_n$ (i.e. as positive powers of $n^{-1/2}$). Today’s level of command of QCD does not allow us to unambiguously predict the laws of fall off of the chiral symmetry violating effects in terms of $\langle \bar{\Psi} \Psi \rangle$. In some instances a minimal rate can be estimated, however. In some instances there are reasons to believe that the actual rate may be close to the minimal one.

3.1 On quasiclassical arguments

As a warm-up exercise I will derive textbook results: equidistant spacing of radially excited mesons and $n$ independence of $\Gamma_n/M_n$ where $\Gamma_n$ is the total width of the $n$-th excitation. In the following simple estimates I will try to be as straightforward as possible, omitting inessential numerical constants and assuming that the only mass dimension is provided by $\Lambda_{QCD} \equiv \Lambda$. In this “reference frame” the string tension is $\Lambda^2$, while the $\rho$-meson mass is $\Lambda$.

When a highly excited meson state (say, $\rho_n$) is created by a local source (vector current), it can be considered, quasiclassically, as a pair of free ultrarelativistic quarks; each of them with $E = p = M_n/2$. These quarks are produced at the origin, and then fly back-to-back, eventually creating a flux tube of the chromoelectric field. Since the tension of the flux tube is $\sigma \sim \Lambda^2$, the length of the tube is

$$L \sim \frac{M_n}{\Lambda^2}.$$ (4)

Using the quasiclassical quantization condition

$$\int p \, dx \sim p \, L \sim \frac{M_n^2}{\Lambda^2} \sim n$$ (5)

we immediately arrive at

$$M_n^2 \sim \Lambda^2 \, n.$$ (6)

(Let me parenthetically note that asymptotically linear $n$ dependence of $M_n^2$ can be analytically obtained in the two-dimensional ‘t Hooft model [28, 29] where linear confinement is built in.)

Let us now discuss total decay widths of high radial excitations. The decay probability (per unit time) is determined, to order $1/N$, by the probability of producing an extra quark-antiquark pair. Since the pair creation can happen anywhere inside the flux tube, the probability must be proportional to $L$. As a result [30],

$$\Gamma_n \sim \frac{1}{N} L \Lambda^2 = \frac{B}{N} M_n,$$ (7)
where $B$ is a dimensionless coefficient independent of $N$ and $n$, see Eq. (4).

Thus, the width of the $n$-th excited state is proportional to its mass which, in turn, is proportional to $\sqrt{n}$. The square root formula for $\Gamma_n$ was numerically confirmed [31] in the 't Hooft model. It is curious that both, in actual QCD and in two dimensions, $B \sim 0.5$.

Equation (7) demonstrates that the limits $N \to \infty$ and $n \to \infty$ are not commutative. We must first send $N$ to infinity, and only then can we consider high radial excitations.

Asymptotic linearity of the Regge trajectories (Eq. (6) at $n \gg 1$) must emerge in any sensible string-theory-based description of QCD. As discussed in detail by Schreiber [32], this is indeed the case in the picture of mesons as open strings ending on a probe D brane in an appropriate background. In the same work, using an open string analog of the well-known Witten’s argument, Shreiber shows [32] that treating radial excitation of low-spin mesons as fluctuations of the probe D branes one obtains, generally speaking, a wrong behavior, $M_n^2 \sim \Lambda^2 n^2$. (This remark which pre-emps the beginning of Sect. 4 will be explained there in more detail.)

### 3.2 Analyzing chiral pairs: the slowest fall off of $\delta M_n$

For definiteness I will focus on scalar–pseudoscalar mesons. Semi-quantitative results to be derived below are straightforward and general. The only “serious” formula I will use is that for the Euler function,

$$
\psi(z) = -\gamma - \frac{1}{z} + \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{z+n} \right). \tag{8}
$$

At large positive $z$

$$
\psi(z) \to \ln z - \frac{1}{2z} + O\left(z^{-2}\right). \tag{9}
$$

The two-point correlators we will deal with are defined as

$$
\Pi(q) = i \int d^4x e^{ix} \langle T\{J(x), J(0)\}\rangle, \tag{10}
$$

where

$$
J = \bar{\Psi}\Psi \text{ and } J_5 = i \bar{\Psi}\gamma^5\Psi \tag{11}
$$

for scalars and pseudoscalars, respectively (to be denoted as as $\Pi$ and $\Pi_5$). I will consider flavor-nonsinglet channels. Then

$$
\Pi(Q) - \Pi_5(Q) = \sum_n \left( \frac{f_n}{Q^2 + M_n^2} - \frac{\bar{f}_n}{Q^2 + \bar{M}_n^2} \right), \tag{12}
$$
where the untilded quantities refer to the scalar channel while those with tildes to the pseudoscalar one.

The operator product expansion (OPE) for \( \Pi(Q) - \Pi_5(Q) \) at large Euclidean \( Q^2 \) was built long ago \[33\]. In conjunction with the large-\( N \) limit which justifies factorization of the four-quark operators it implies

\[
\Pi(Q) - \Pi_5(Q) \sim \frac{1}{N} \frac{\langle \bar{\Psi}\Psi \rangle^2}{Q^4}, \quad Q^2 \to \infty ,
\]

(modulo possible logarithms). Now, the residues \( f_n \) are positive numbers of dimension \( \Lambda^4 \). They can be normalized from the relation

\[
\Pi(Q) \sim N Q^2 \ln Q^2 , \quad Q^2 \to \infty .
\]

In fact, it is easy to show that the equidistant spectrum \(6\) combined with Eq. \(8\) lead to

\[
f_n \sim N \Lambda^2 M^2_n . 
\]

Now we are in position to estimate the splittings. As was explained above, one expects that asymptotically, at large \( n \),

\[
|\delta f_n| \ll f_n , \quad |\delta M^2_n| \ll M^2_n , 
\]

where

\[
\delta f_n = f_n - \bar{f}_n , \quad \delta M^2_n = M^2_n - \bar{M}_n^2 .
\]

The scalar-pseudoscalar difference in Eq. \(12\) depends on \( \delta f_n \) and \( \delta M^2_n \). We first set \( \delta f_n = 0 \), i.e. assume perfectly degenerate residues. (Shortly this degeneracy will be lifted, of course.) Then, taking account of \(15\), we get the following sum-over-resonances representation:

\[
\Pi(Q) - \Pi_5(Q) = N \Lambda^2 Q^2 \sum_n \frac{\delta M^2_n}{(Q^2 + M^2_n)^2} 
\]

\[
\rightarrow N \Lambda^2 Q^2 \frac{\partial}{\partial Q^2} \sum_n \frac{\delta M^2_n}{Q^2 + M^2_n} .
\]

To evaluate the convergence of \( \delta M^2_n \) to zero, the last expression must be matched with the asymptotic formula \(13\) at \(Q^2 \to \infty\). Needless to say, matching an infinite sum to a single OPE term one cannot expect to get a unique solution for \( \delta M^2_n \). In
fact, what we are after, is the slowest pattern of the chiral symmetry restoration still compatible with the OPE.  

Even if $\delta M^2_n$ falls very fast at large $n$, but the sign of $\delta M^2_n$ is $n$-independent, there is no matching. Indeed, Eq. (18) implies then $1/Q^2$ rather than $1/Q^4$ behavior. The only way to enforce the $1/Q^4$ asymptotics is to assume that $\delta M^2_n$ is a sign alternating function of $n$, say,

$$\delta M^2_{2k} > 0, \quad \delta M^2_{2k+1} < 0, \quad |\delta M^2_{2k}| = |\delta M^2_{2k+1}|. \quad (19)$$

Let us show that the slowest possible decrease of $|\delta M^2_n|$ versus $n$ is

$$M^2_n \delta M^2_n = \text{sign alternating const}. \quad (20)$$

Equation (13) implies that $\delta \Pi(Q^2)$ is analytic in the complex plane, with a $Q^{-4}$ singularity at the origin. This implies, in turn, that the sum

$$\sum_n \delta M^2_n \quad (21)$$

is convergent and vanishes. The slowest-rate solution is $\delta M^2_n \sim (-1)^n n^{-1}$, which is the same as (20). Under the condition (20) the right-hand side of (18) becomes

$$NA^6 Q^2 \frac{\partial}{\partial Q^2} \sum_k \left\{ \frac{1}{M^2_{2k} (Q^2 + M^2_{2k})} - \frac{1}{M^2_{2k+1} (Q^2 + M^2_{2k+1})} \right\}$$

$$\rightarrow NA^4 Q^2 \frac{\partial}{\partial Q^2} \frac{1}{Q^2} \ln \frac{Q^2 + A^2}{Q^2} \rightarrow NA^6 \frac{1}{Q^4}, \quad (22)$$

*quod erat demonstrandum.* Here $A^6$ must be identified with $N^{-2} \langle \bar{\Psi} \Psi \rangle^2$.

Thus, $\delta M_n$ is sign alternating and falls off as

$$|\delta M_n| \sim \frac{A^6}{n^{3/2}} \quad (23)$$

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5As we will see in Sect. 4, the advent of holographic ideas in QCD definitely revived interest in combining OPE with Regge-trajectory-based constructions. For instance, attempts to reconcile OPE in the chirality-breaking two-point function $\langle V+A, V-A \rangle$ with strictly linear Regge trajectories were reported in [34]. Of course, no reconciliation can be achieved under the assumption of exact linearity. Similar arguments were later used in [35] to address the issue of asymptotic deviations of the Regge trajectories from linearity.
or faster. Whether or not it actually falls off faster than $n^{-3/2}$ depends on the behavior of higher order terms in OPE for $\Pi(Q) - \Pi_5(Q)$. Next-to-nothing can be said on this at the moment. Even with the slowest possible regime (23) the rate of the chiral symmetry restoration is pretty fast, see Fig. 1.

![Figure 1: $\delta M_n$ versus $n$, in arbitrary units.](image)

### 3.3 The impact of $\delta f_n \neq 0$

Now, let us relax the (unrealistic) assumption $\delta f_n = 0$. Again, our task is to find the slowest fall off compatible with both the OPE and resonance representations. If both $\delta f_n \neq 0$ and $\delta M_n \neq 0$, the difference $\Pi(Q) - \Pi_5(Q)$ acquires an additional term

$$\Pi(Q) - \Pi_5(Q) = \sum_n \frac{\delta f_n}{Q^2 + M_n^2}.$$  \hspace{1cm} (24)

It is not difficult to see that, barring an unlikely and subtle conspiracy between $\delta f_n$ and $\delta M_n$, the previous estimate for $\delta M_n$ stays intact, and, in addition, one gets an estimate for $\delta f_n$. Copying the consideration above one finds that $\delta f_n$ must be sign-alternating, and the fall-off regime of $|\delta f_n|$ must be $N\Lambda^4/n$ or faster, so that

$$|\delta f_n|/f_n \sim 1/n^2.$$  \hspace{1cm} (25)

Here I used the fact that $f_n$ grows linearly with $n$, see Eq. (15).

### 3.4 Excited $\rho$ mesons of two kinds

In Ref. [22] it was noted that, if the chiral symmetry is restored at high $n$, two distinct varieties of excited $\rho$ mesons must exist. Let us define $\rho$ meson as a $J^{PC} = 1^{--}$ quark-antiquark state with isospin 1. Then, such mesons are produced from the vacuum by
the following two currents\(^6\) which belong to two distinct chiral multiplets (see Eqs. (2) and (3)):

\[ \bar{\Psi} \gamma_\mu \Psi, \quad (26) \]

\[ \bar{\Psi} \sigma_{\alpha\beta} \Psi. \quad (27) \]

Were the chiral symmetry exact, the above two currents would not mix. The ground state \(\rho\) meson would be coupled to the first current and would have vanishing residue in the second. There will be two distinct types of the \(J^{PC} = 1^{--}\) mesons. In actuality, they do mix, however; the \(\rho\) mesons show up in both currents, see Fig. 2. The \(J^{PC} = 1^{--}\) excited states of the first kind couple predominantly to the first current while those of the second kind to the second current. I want to evaluate the coupling of \(\rho_n\) of a given kind to the "wrong" current (as a function of \(n\)).

![Figure 2: Spectral densities in the channels corresponding to the currents (26) and (27). Contributions due to \(\rho_n\) of the "wrong kind" die off as \(1/n^3\) at large \(n\).](image)

To this end we will consider a "mixed" correlation function

\[ \langle \bar{\Psi} \gamma_\mu \Psi, \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle_{q} \sim \left( g_{\alpha\mu} q_{\beta} - g_{\beta\mu} q_{\alpha} \right) \langle \bar{\Psi} \Psi \rangle \frac{1}{q^2} \quad (28) \]

modulo logs of \(Q^2\). Saturating this expansion by resonances and defining

\[ \langle \text{vac} | \bar{\Psi} \sigma_{\alpha\beta} \Psi | \rho_n \rangle = \left( g_{\alpha\mu} p_{\beta} - g_{\beta\mu} p_{\alpha} \right) f_n \varepsilon^\alpha_{\mu} \quad (29) \]

\(^6\)The current (27) also produces \(1^+\) mesons; this is irrelevant for what follows.
we get, say, for the resonances “strongly” coupled to $\Psi \bar{\tau} \gamma_\mu \Psi$

$$
\sum_n \frac{M_n^2}{g_n} f_n \sim \langle \bar{\Psi} \Psi \rangle ,
$$

(30)

implying, in turn, that

$$
f_n \sim \frac{\Lambda}{n^{3/2}},
$$

(31)

or faster. Thus, barring conspiracies, the contribution, of an excited $\rho$ of the “wrong” chiral structure in the diagonal two-point function of two vector currents $\Psi \bar{\tau} \gamma_\mu \Psi$ scales as $f_n^2 \sim 1/n^3$, to be compared with that of the “right” chiral structure, $M_n^2/g_n^2 \sim n^0$. The relative suppression is $1/n^3$ or faster.

One can arrive at the very same statement using a slightly different argument. The wrong-chirality $\rho_n$’s in the diagonal two-point function of two vector currents $\Psi \bar{\tau} \gamma_\mu \Psi$ must be associated with the operator $\langle \bar{\Psi} \Psi \rangle^2$. Therefore, the sum $\sum_n f_n^2 M_n^4$ must converge, again leading to the estimate $f_n^2 \sim 1/n^3$ or faster.

4 AdS/QCD versus QCD

As I have already mentioned, AdS/CFT correspondence [36] inspired a general search for five-dimensional holographic duals of QCD. This trend flourished in the last few years, e.g. [1–17]. Generic holographic models of QCD describe gauge theories that typically (but not always!) have color confinement and chiral symmetry breaking as built-in features, and are dual to a string theory on a weakly-curved background. One should remember that they describe strong coupling theory — i.e. asymptotic freedom is not properly incorporated. In the majority of the holographic duals discussed so far $M_n \sim n$ rather than $\sqrt{n}$ required by linearity of the Regge trajectories. In some more contrived holographic descriptions (e.g. [16]) asymptotically linear Regge trajectories do emerge, however.

In discussing flavor physics in AdS/QCD one should keep in mind that at present there are two methods of introducing dynamical quarks in the fundamental representation into the gravity/string duals of QCD: (i) the so-called flavor probes (for $N_f \ll N$); (ii) fully back-reacted backgrounds that incorporate flavor — in this case, obviously, there are no restrictions on $N_f$. The latter direction is not yet sufficiently developed. Burrington et al. [9] suggested a background based on D3/D7 system, which seems singular, however. This construction has some undesirable features, serious drawbacks, which cannot be discussed here. Klebanov and Maldacena incorporated flavor [10] in a conformal theory claimed to be dual to the infrared fixed
point of $\mathcal{N} = 1$ SQCD, a theory which is in no way close to QCD. This seems to exhaust the list of developments in this direction.

The probe approach pioneered by Karch and Katz [1] is way more advanced. In the original work [1] Karch and Katz added D7 branes to the $\text{AdS}_5 \times S^5$ background and obtained a model which had no confinement. A remedy was found shortly. Flavor in a confining background was introduced in Ref. [4] in the context of the Klebanov-Strassler model. A simpler and more illuminating paper [5] followed almost immediately. It was based on adding D6 flavor branes to the model of Witten of near-extremal D4 branes. Although Ref. [5] was very inspiring, the authors themselves were aware of the fact this model did not incorporate chiral symmetry in the ultraviolet and, hence, dynamical chiral symmetry breaking of QCD was not addressed.

Dedicated designs allowing one to include chiral symmetry breaking and related low-energy phenomena in AdS/QCD were worked out in [6,8,11,15]. In fact, it will be very interesting to check whether fluctuations of the probe branes that correspond to meson excitations will explain chiral symmetry restoration in high excitations, and if yes, whether the rate of restoration in highly excited mesons will be properly reproduced.

This last remark presents a nice bridge between the contents of Sect. 3 and the remainder of this talk devoted to implementations of universality. First AdS/QCD-based analyses of the issue were reported in Refs. [13, 14]. After a brief review I confront them with QCD expectations which I derived for this conference. One should note that, notwithstanding their stimulating character, the models [13, 14] are not based on backgrounds and probes proven to be duals of QCD. In fact, in these models the Regge trajectories are not asymptotically linear. Moreover, they share a general feature inherent to all probe-based constructions. Ignoring back reaction means that the flavor-carrying quarks in these models are, in essence, nondynamical. A conceptual parallel that immediately comes to one’s mind is quenching in lattice QCD. This does not seem to be a serious drawback, though, given that $N = \infty$.

One could say that unless the above stumbling blocks are eliminated there is no point in analyzing consequences of present-day AdS/QCD and confronting them with QCD proper. Such a standpoint, although legitimate, is not constructive. The more aspects we study the more chances we have for eliminating drawbacks and finding a “perfectly good” holographic dual.
4.1 Implementation of universality in AdS/QCD and QCD

To begin with, I would like to dwell on the work of Hong et al. [13] devoted to the issue of VMD and universality of the $\rho$-meson coupling.

The notion of VMD is known from the 1960’s [37]. Let us consider, for definiteness, the vector isovector current

$$J^\mu = \frac{1}{2} (\bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d) .$$

If it is completely saturated by the $\rho$ meson,

$$J^\mu \equiv (\frac{M^2_\rho}{g_\rho}) \rho^\mu ,$$

with no higher excitations, then the $\rho HH$ coupling is obviously universal and depends only on the isospin of the hadron $H$. Indeed, consider the formfactor of $H$ for the $J^\mu$-induced transition. At zero momentum it equals to the isospin of $H$. On the other hand, saturating the formfactor by the $\rho$ meson, we get

$$(g_{\rho HH}) \ g^{-1}_\rho = H \ isospin .$$

This is the famous VMD formula. Say, for the pion, $g_{\rho\pi\pi} = g_{\rho}$. In what follows I will keep in mind $H = \pi$ as a typical example.

Equation (34) is approximate since the absolute saturation (33) is certainly unrealistic. Higher radial excitations are coupled to the isovector current too,

$$\langle \text{vac} | J^\mu | \rho_n \rangle = \left( \frac{M^2_n}{g_n} \right) \epsilon_n^\mu \neq 0, \quad n = 1, 2, 3, ....$$

The exact formula replacing VMD is

$$(g_{\rho HH}) \ g^{-1}_\rho + \sum_{n=1}^{\infty} \frac{g_{\rho_n HH}}{g_n} = H \ isospin .$$

If the sum over excitations on the left-hand side is numerically small, for whatever reason, one still recovers the universality relation (34) which will be valid approximately rather than exactly.

\[7\] In this case the (dimensionless) coupling constant $g_{\rho\pi\pi}$ is defined as

$$\langle \rho | \pi^+ \pi^- \rangle = g_{\rho\pi\pi} \epsilon_\mu (p_+^\mu - p_-^\mu) .$$

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There are two distinct regimes ensuring suppression of the sum: (a) each term \( n = 1, 2, 3, \ldots \) is individually small; (b) each term is of the same order as \((g_{\rho HH}) g_{\rho}^{-1}\), but successive terms are sign-alternating and compensate each other. The first option is an approximate VMD and, hence, leads to a natural universality, while the second one is in fact an “accidental” or “fortuitous” universality.

It is the latter regime which takes place in the holographic model considered in Ref. [13], see Fig. 3 illustrating numerical results reported in this paper.\(^8\) For the ground state the authors get [13] \((g_{\rho HH}) \approx 1.49 g_{\rho}\). The factor 1.49 is to be compared with unity in VMD.

\[
\text{Figure 3: The residue } g_{\rho n HH} (g_n)^{-1} \text{ versus } n \text{ for } n = 1, 2, \ldots \text{ Borrowed from [13].}
\]

I would like to show that QCD proper gives the former regime rather than the latter — suppression of individual contributions of high excitations. The divergence between QCD and AdS/QCD must cause no surprise since AdS/QCD does not accurately describe short-distance QCD dynamics which governs high excitations.

Now, let us discuss QCD-based expectations in more detail.

\(^8\)Whether or not poor convergence of \((g_{\rho,HH})^n g_{\rho}^{-1}\) is a general feature of AdS/QCD, or, perhaps, of a certain class of holographic models, remains to be seen. According to M. Stephanov’s private communication, the AdS/QCD model of Ref. [14], which builds on previous results [6, 8], yields \(n^{-5/2}\), the rate of fall-off that is even steeper than that expected in QCD, see Sect. 4.2. An intermediate regime, with the fall-off rate \(\sim n^{-1/2}\), is reported in [38]. It would be instructive to discuss in detail particular reasons explaining the above distinctions.
4.2 Sign alternating residues

As well-known (see e.g. [39] for extensive reviews), at large (Euclidean) momentum transfer the pion formfactor is determined by the graph of Fig. 4 and scales as

$$F_\pi(Q^2) \sim \frac{1}{Q^2 \ln Q^2}. \quad (37)$$

It is important that the fall off is faster than $1/Q^2$. Comparing Eq. (37) with the sum-over-resonances representation,

$$F_\pi(Q^2) = \sum_{n=0}^{\infty} g_{\rho_n \pi \pi} \frac{g_n}{g_n} \frac{M_{\rho_n}^2}{Q^2 + M_{\rho_n}^2}, \quad (38)$$

we immediately conclude that successive terms must be sign-alternating — otherwise the asymptotic fall off would be $1/Q^2$. Thus, QCD supports the sign-alternation feature of AdS/QCD.

4.3 Large $n$ suppression of the residues

Now, our task is to estimate the large $n$ behavior of $g_{\rho_n \pi \pi}/g_n$ using, as previously, the quasiclassical approximation. A high radial excitation of the $\rho$ meson can be viewed as an ultrarelativistic quark-antiquark system, each quark having energy $m_n/2$. Conversion to the pion pair proceeds via the diagram of Fig. 5. A straightforward examination of this graph allows us to conclude that

$$g_{\rho_n \pi \pi} \sim \frac{\hat{f}_\pi^2}{g_n} \frac{1}{M_n^2} \frac{1}{[\ln(M_n^2/\Lambda^2)]^2},$$

\[ (39) \]
where both factors, $M_n^{-2}$ and the square of the logarithm, are the consequences of the two-gluon exchange. The logarithm is due to $\alpha_s$ (see below).

Deviations from VMD in Eq. (36) are determined by the residues

$$\frac{g_{\rho_n\pi\pi}}{g_n} \sim \frac{f_\pi^2}{g_n^2} \frac{1}{M_n^2} \frac{1}{\ln(M_n^2/\Lambda^2)}.$$  \hspace{1cm} (40)

Taking into account the fact that $M_n^2/g_n^2$ is asymptotically $n$-independent, we arrive at the following suppression factor:

$$\left| \frac{g_{\rho_n\pi\pi}}{g_n} \right| \sim \frac{1}{n^2} \frac{1}{(\ln n)^2}.$$ \hspace{1cm} (41)

the fall off of the residues of high excitations is quite steep. Formally, Eq. (41) is valid at large $n$. It is reasonable to ask whether a suppression persists for the first or second excitation. The answer seems to be positive. Even if $n \sim 1$, there is a numeric suppression coming from $\alpha_s^2$ of the type

$$(\lambda)^2 = \left( \frac{N\alpha_s}{2\pi} \right)^2 \sim (4 \ln M_n/\Lambda)^{-2},$$ \hspace{1cm} (42)

where $\lambda$ is the 't Hooft coupling, small in QCD and large in AdS/QCD. According to the above estimate, the first excitation contributes in Eq. (36) at the 10% level. This is in accord with the experience I gained from multiple analyses of the QCD sum rules [33] in which I had been involved in the past.
5 In conclusion...

I would like to conclude this talk on a curious note showing that, perhaps, indeed, new is a well-forgotten old. Descending down from conceptual summits to down-to-earth technicalities let us ask ourselves what we learn from AdS/QCD with regards, say, to the $\rho$-meson channel at operational level. To this end let us have a closer look at results reported in [14]. Operationally, the bare-quark-loop logarithm is represented in this work as an infinite sum over excited $\rho$ mesons whose masses and residues are adjusted in such a way that the above infinite sum reproduces pure logarithm of $Q^2$ up to corrections exponentially small at large $Q^2$ (there are no power corrections).

A similar question was raised long ago, decades before AdS/QCD, by A. Migdal [40], who asked himself what is the best possible accuracy to which $\log Q^2$ can be approximated by an infinite sum of infinitely narrow discrete resonances, and what are the corresponding values of the resonance masses and residues. He answered this question as follows: “the accuracy is exponential at large $Q^2$ and the resonances must be situated at the zeros of a Bessel function.” This is exactly the position of the excited $\rho$ mesons found in Ref. [14]!

6 Conclusions

◊ Chiral symmetry restoration in high radial excitations occurs at the rate

$$|\delta M_n| \sim \Lambda n^{-3/2}$$

or faster (this is related to the quark condensate $\langle \bar{\Psi}\Psi \rangle$);

◊ ◊ Relatively simple versions of AdS/QCD (the majority of the holographic duals analyzed so far!), along with the wrong $n$ dependence of $M_n$ (the well-known fact) also lead to a wrong pattern for the $n$ dependence of the residue $g_{\rho_n \pi \pi}/g_n$. Thus, although the $g_{\rho HH}$ universality is implemented in AdS/QCD, it may be implemented in a wrong way!

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