Localized tadpoles and anomalies in 6D orbifolds

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In this talk, we review the instability due to radiatively induced FI tadpoles in \( \mathcal{N} = 2 \) supersymmetric gauge theories on orbifolds in six dimensions. Even with the localized FI tadpoles, we have the unbroken supersymmetry at the expense of having the spontaneous localization of bulk zero mode. We find the non-decoupling of massive modes unlike the 5D case. We also comment on the local anomaly cancellation.

I. INTRODUCTION AND SUMMARY

Recently models with extra dimensions have drawn a great attention from particle physicists due to the interesting possibility that all or part of the Standard Model particles are confined to the hypersurfaces in higher dimensions, the so called branes. We call the orbifold fixed points of extra dimensions also the branes. When we have in mind the embedding of field theoretic orbifolds into the string or M-theory, they often contain the remnant supersymmetry from higher dimensional supersymmetry which could describe the supersymmetric standard model in four dimensions at low energies.

In the \( d = 4 \) supersymmetric theory with a \( U(1) \) factor, it is known that the Fayet-Iliopoulos term (FI) can be radiatively generated for the nonvanishing sum of \( U(1) \) charges\(^1\). The FI term in \( d = 4 \) could introduce the quadratic divergence even in the supersymmetric theory for not breaking the anomalous \( U(1) \). However, the situation is somewhat different in orbifold models. In this case, the FI term can be radiatively generated at the orbifold fixed points and pose the instability problem in a different way. With globally vanishing but locally nonzero FI term on orbifolds in \( d = 5 \), the supersymmetry condition gives rise to the dynamical localization of the bulk zero mode and the heavy massive modes\(^\!*\!). This could open a new possibility for explaining the fermion mass hierarchy and other scale problems in particle physics with some overlap of wave functions. The presence of localized FI terms is consistent up to the introduction of a bulk Chern-Simons term to cancel the locally nonvanishing \( U(1) \) mixed gravitational anomalies\(^2, 3, 4, 5, 6, 7, 8\).

In this talk, we examined the more complicated case of co-dimension 2 in the framework of an \( \mathcal{N} = 2 \) supersymmetric orbifold theory in \( d = 6 \)\(^9\). The co-dimension 2 case should be more relevant for the discussion of compactified superstring theories in \( d = 10 \) where we have 3 complex extra dimensions. We find a localization phenomenon of the bulk zero mode but the situation differs from the co-dimension 1 case in the sense that the bulk field retains its six-dimensional nature. The spectrum of massive modes turns out to be equivalent to a spectrum in the presence of a constant Wilson line. The potential problem of localized gauge or gravitational anomalies is cured with the help of a generalized Green-Schwarz mechanism.

II. SETUP OF \( d = 6 \) SUPERSYMMETRIC ORBITFOLD

We consider a \( d = 6 \) \( \mathcal{N} = 2 \) supersymmetric \( U(1) \) gauge theory compactified on an orbifold \( T^2/Z_2 \). Due to the absence of central charge in the \( d = 6 \) supersymmetry algebra, there is no off-shell formulation possible for hypermultiplets. The \( d = 6 \) supersymmetry has \( SU(2)_R \) as the automorphism group.

First let us construct the bulk Lagrangian for the vector multiplet and the hypermultiplets. The \textit{off-shell} vector multiplet consists of the gauge boson \( A_M(M = 0, 1, 2, 3, 5, 6) \), the \( SU(2)_R \) doublet gaugino \( \Omega^i (i = 1, 2) \) and the \( SU(2)_R \) triplet auxiliary field \( \tilde{D} = (D_1, D_2, D_3) \). The gaugino is subject to the right-handed symplectic Majorana-Weyl conditions\(^1\) such as \( \Omega_i = \varepsilon_{ij} (\Omega^j)^TC \) and \( \Gamma^I \tilde{\Omega} = \tilde{\Omega}^i \). The Lagrangian for the abelian vector multiplet is given

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\(^1\) The metric convention is \( \eta_{MN} = \text{diag}(+\ldots) \). The gamma matrices in \( d = 6 \) are \( 8 \times 8 \) matrices, \( \Gamma^M = \begin{pmatrix} 0 & \gamma^M \\ \gamma^M & 0 \end{pmatrix} \) with \( \gamma^M = (\gamma^a, \gamma^5, -\mathbf{1}_4) \) and \( \gamma^M = (\gamma^a, \gamma^5, \mathbf{1}_4) \) where \( a = 0, 1, 2, 3 \) are four-dimensional indices. The \( d = 6 \) chirality operator is given by...
is tells that the hyperinos have opposite chirality to the one of the gaugino. Then the Lagrangian for the hypermultiplets and the supersymmetry transformation laws give constraints to hyperinos:

$$\bar{\zeta} = \bar{\zeta}^\dagger \epsilon \bar{\Omega} = \bar{\Omega} \epsilon \bar{\zeta}^\dagger$$

where $tr$ is the trace over $SU(3)$ and for the hypermultiplets, $x$ for the torus are $Z_2$ orbifold action on the field space can be read from the Lagrangian as the following: for the vector multiplet,

$$Z_2: (x^5, x^6) \rightarrow (-x^5, -x^6).$$

So there are four fixed points on $T^2/Z_2$ orbifold: $(x^5, x^6) = (0, 0), (\pi R_5, 0), (0, \pi R_6)$, and $(\pi R_5, \pi R_6)$. Then the orbifold action on the field space can be read from the Lagrangian as the following: for the vector multiplet,

$$A_m(x^5, x^6), \quad A_{5,6}(x^5, x^6) = -A_{5,6}(x^5, x^6),$$

and for the hypermultiplets,

$$h(-x^5, -x^6) = -\eta(1 \otimes \tau_3)h(x^5, x^6) \tau_3, \quad \zeta(-x^5, -x^6) = i\eta(1 \otimes \tau_3)\gamma_5 \zeta(x^5, x^6)$$

where $\eta = \pm 1$. Defining a $d = 6$ spinor such as $\psi = (\psi_L, \psi_R)$ in the four-dimensional Weyl representation, the orbifold action for the gaugino and the hyperinos becomes respectively

$$\Omega_R(-x^5, -x^6) = i(1 \otimes \tau_3)\gamma_5 \Omega_R(x^5, x^6),$$

$$\zeta_L(-x^5, -x^6) = i\eta(1 \otimes \tau_3)\gamma_5 \zeta_L(x^5, x^6)$$

where the subscripts $R, L$ denote the $d = 6$ chiralities.

$$\Gamma^7 = -\tau^3 \otimes 1_4$$ and the charge conjugation is $C = -i\tau^2 \otimes C$ where $C$ is the $d = 5$ charge conjugation.
The coefficient of FI term can be computed by considering eqs. (18) and (19) with the standard procedure as in the fixed points. We note that the bulk contribution has both quadratically divergent and logarithmically divergent pieces which are written in terms of two-component Weyl spinors of \( \chi_\pm \) as \( \chi = \chi_+ - \chi_- \) and \( C\chi^T = \chi_+ + \chi_- \). As a result, a four-dimensional \( N = 1 \) vector multiplet is composed of four-dimensional fields at the fixed points is composed of 

\[
(A_m, \chi_{+L}, -D_3 + F_{56}),
\]

i.e. the four-dimensional auxiliary field is not \( D_3 \) as one might have naively expected but \( -D_3 + F_{56} \) which is the gauge covariant generalization of \( -D_3 + \partial_5 \Phi \) in \( d = 5 \). Therefore, we can couple chiral multiplets which live at the fixed points to the \( N = 1 \) vector multiplet. Then, the total Lagrangian contains

\[
\mathcal{L}_{bulk} = \sum_{\pm} ([D_M \phi_{\pm}]^2 + g\phi_{\pm}^4 D_3) + i\bar{\psi} i^M D_M \psi + \cdots,
\]

\[
\mathcal{L}_{brane} = \sum_{I=1}^4 \delta(x^5 - x_I^5) \delta(x^6 - x_I^6) |[D_m \phi_I|^2 + g\phi_I^4 q_I \phi_I (-D_3 + F_{56}) + \cdots|
\]

where we omitted \( \alpha \) indices in the bulk Lagrangian, \((x_I^5, x_I^6)\) label the fixed points and \( q_I \) are the charge matrices at the fixed points.

### III. FAYET-ILIOPOULOS TADPOLES

As discussed in the previous section, the \( D \) field belonging to the four-dimensional vector multiplet is given by \( D = -D_3 + F_{56} \). So the form of our Fayet-Iliopoulos(FI) term is

\[
\mathcal{L}_{FI} = \xi (-D_3 + F_{56}).
\]

The coefficient of FI term can be computed by considering eqs. 18 and 19 with the standard procedure as in \( d = 5 \). The sum of bulk and brane contributions to the FI term is

\[
\xi = \sum_{I} (\xi_I + \xi''(\partial_5^2 + \partial_6^2)) \delta(x^5 - x_I^5) \delta(x^6 - x_I^6)
\]

with

\[
\xi_I = \frac{1}{16\pi^2} g\Lambda^2 \left( \frac{1}{4} tr(q) + tr(q_I) \right),
\]

\[
\xi'' = \frac{1}{16\pi^2} \frac{1}{4} \ln \Lambda^2 tr(q).
\]

We note that the bulk contribution has both quadratically divergent and logarithmically divergent pieces which are equally distributed at the fixed points whereas the brane contribution has only the quadratic divergence.

Then, from the effective potential with the FI tadpoles, we find the conditions for unbroken supersymmetry

\[
\langle D_3 \rangle = \langle F_{56} \rangle = g(\langle \phi_+ \rangle^q \langle \phi_+ \rangle - \langle \phi_- \rangle^q \langle \phi_- \rangle + \xi + g \sum_I \delta(x^5 - x_I^5) \delta(x^6 - x_I^6) \delta \langle \phi_I \rangle^q q_I \langle \phi_I \rangle
\]
together with
\[ \langle \phi_+ \rangle T q \langle \phi_- \rangle = 0 \quad \text{and} \quad \langle (D_5 + iD_6)\phi_\pm \rangle = 0. \] (25)

Provided that the ground state does not break the \( U(1) \), i.e. \( \langle \phi_\pm \rangle = \langle \phi_I \rangle = 0 \), the supersymmetry condition (24) becomes
\[ \langle F_{56} \rangle = \xi. \] (26)

After integrating both sides over the extra dimensions, the Stokes theorem with no boundary tells
\[ \sum_I \xi_I = 0 \quad \text{or} \quad \text{tr}(q) + (q_1) + (q_2) + (q_4) + (q) = 0. \] (27)

This consistency condition ensures the absence of overall mixed gauge-gravitational anomalies. If (27) is violated, we would expect the \( U(1) \) to be broken at a high scale either spontaneously or through a variant of Green-Schwarz mechanism\[11\]. Even if the consistency condition is satisfied, we need the local version of Green-Schwarz mechanism for the local anomaly cancellation\[12, 13, 14, 15\]. As a result, the \( U(1) \) is broken or unbroken depending on whether the dual axion lives on the brane or in the bulk\[9, 12, 13, 14, 15\].

Then, taking the following ansatz\[2\]
\[ \langle A_5 \rangle = -\partial_6 W \quad \text{and} \quad \langle A_6 \rangle = \partial_5 W, \] (28)
eq (26) becomes a sort of Poisson equation\[^3\]
\[ \partial \bar{\partial} W' = \sum_I \xi_I \delta^2(z - z_I), \] (29)
with
\[ W' = 2 \left( W - \frac{2}{R_5^2} \sum_I \xi'' \delta^2(z - z_I) \right). \] (30)

Consequently, the solution to eq. (29) is given by the propagator of a bosonic string for a toroidal world sheet as
\[ W' = \frac{1}{2\pi} \sum_I \xi_I \left[ \ln \left| \partial_1 \left( \frac{z - z_I}{2\pi} \right) \right|^2 - \frac{1}{2\pi\tau_2} [\text{Im}(z - z_I)]^2 \right], \] (31)
where \( \tau_2 = \text{Im} \tau = R_6/R_5 \). Note that in order for \( W' \) in the above to be a solution to (29), eq. (27) must hold.

IV. LOCALIZATION OF THE BULK ZERO MODE AND MASS SPECTRUM

As shown before, in the presence of localized FI tadpoles, the unbroken gauge symmetry and supersymmetry requires the nontrivial profile of the extra dimensional components of gauge field. This nontrivial background modifies the wave functions and the mass spectrum of bulk modes.

First let us consider the solution for the zero mode in the presence of the background solution. The equation for the zero mode is
\[ (\bar{\partial} + gq \bar{\partial} W)\phi_+ = 0 \] (32)

\[^2\] Here we have fixed the gauge implicitly
\[^3\] We are using complex coordinates \( z = \frac{1}{R_5} x^5 + \frac{i}{R_6} \tau x^6 \), with the torus modulus \( \tau = iR_6/R_5 \). The periodicities on the torus then are \( z \approx z + 2\pi \approx z + 2\pi \tau \).
where \( A = A_5 - i A_6 = -(2i/R_5) \partial W \) in the complex coordinates. Thus, we find the exact solution for the zero mode as

\[
\phi_+ = f_+ e^{gqW} = f_+ \prod_I \left| \vartheta_1 \left( \frac{z-z_I}{2\pi} \right) \right|^{1/2} e^{gq\xi_I} \times \exp \left[ -\frac{1}{8\pi^2 \tau_2} gq\xi_I [\text{Im}(z-z_I)]^2 + \frac{gq\xi''}{R_5^2} \delta^2(z-z_I) \right]
\]

(33)

where \( f_+ \) is a complex integration constant which is determined by the normalization condition

\[
1 = \int_0^{\pi R_5} dx_5 \int_0^{\pi R_6} dx^6 |\phi_+|^2.
\]

(34)

Therefore, from the asymptotic limit of the theta function

\[
\vartheta_1 \left( \frac{z-z'}{2\pi} \right) \rightarrow (\eta(\tau))^3(z-z') \quad \text{for} \quad z \rightarrow z'
\]

(35)

where \( \eta(\tau) \) is the Dedekind eta function, the \( \vartheta_1 \) term shows the similar tendency for localization as the \( e^{(\text{Im})^2} \) term but it would mean a strong localization of the zero mode due to the divergence at the fixed point(s) with \( q \xi_I < 0 \). Moreover, the \( e^{\delta^2} \) term also seems to give a strong (de)localization for \( q \xi'' > 0, q \xi'' < 0 \) as in the five-dimensional case \( i \).

To understand the localization of the zero mode explicitly we have to maintain two regularization scales: the momentum cutoff \( \Lambda \) and the brane thickness \( \rho \); both \( \rho \) and \( 1/\Lambda \) are small compared to \( R_5, R_6 \). The localization induced by \( \xi \) is typically exponential in \( \Lambda \) while the one induced by \( \xi'' \) is power like. Thus as long as \( \rho \) is not very small compared to \( 1/\Lambda \) the effect of the logarithmic FI-term will be subleading (naturally one could expect \( \rho \) and \( 1/\Lambda \) to be of the same order of magnitude).

Next let us consider the equation for the massive modes with nonzero gauge field background

\[
(\partial \pm gq \partial W)(\partial \mp gq \partial W)\phi_\pm = -\frac{1}{4} m^2 R_5^2 \phi_\pm.
\]

(36)

By substituting in eq. \( 33 \)

\[
\phi_\pm = e^{gqW} \tilde{\phi}_\pm,
\]

(37)

we get a simpler form

\[
\partial \tilde{\partial} \tilde{\phi}_\pm \pm 2 gq \partial W \tilde{\phi}_\pm = -\frac{m^2}{4} R_5^2 \tilde{\phi}_\pm.
\]

(38)

Since there appear derivatives of delta functions in this equation, we need to regularize the delta function. Let us take the regularizing function \( 4 \) as \( \Delta^2(z-z_I) \) which satisfies \( \lim_{\delta \rightarrow 0} \int d^2 z' \Delta^2(z-z') h(z', \bar{z'}) = h(z, \bar{z}) \) for an arbitrary complex function \( h \) and is zero only for \( |z-z_I| > \rho/\Lambda \) with \( \rho \) being the brane thickness. Then, we get the holomorphic derivative of \( W \) as

\[
\partial W = \frac{1}{2} \sum_I \xi_I \int d^2 z' \partial G(z-z') \Delta^2(z'-z_I) + \frac{2}{R_5^2} \sum_I \xi'' \partial \Delta^2(z-z_I)
\]

(39)

where \( G(z-z') \) is the string propagator on the torus satisfying \( \partial \bar{\partial} G(z-z') = \delta^2(z-z') - 1/(8\pi^2 \tau_2) \). Then, since \( \partial W \) is holomorphic for \( |z-z_I| > \rho_I/R_5 \), eq. \( 33 \) becomes solvable with a separation of variables as \( \tilde{\phi}_\pm = \chi_\pm(z) \varphi_\pm(\bar{z}) \).

After all, imposing the periodicity on \( \tilde{\phi}_\pm \), we find the mass spectrum

\[
m^2 = \frac{4}{R_5^2} \left| c_{\pm} \pm \frac{gq}{8\pi^2 \tau_2} \sum_I \xi_I \bar{z}_I \right|^2,
\]

(40)

\[4 \text{ See, for instance, Ref. } 3.\]
with
\[ c_\pm = \frac{1}{2} \left( \frac{n'}{\tau_2} \pm in \right), \] (41)
where \( n \) and \( n' \) are integers. The structure of the resulting mass spectrum is so different from the \( d = 5 \) case in the sense that there also appear linear terms in \( \xi_I' \)'s. In particular, for \( \sum_I \xi_I z_I = 0 \), which is the case with no net dipole moments coming from FI terms, even the nonzero localized FI terms do not modify the mass spectrum at all. Even for \( \sum_I \xi_I z_I \neq 0 \) with large FI terms, there generically appears a normal KK tower of massive modes starting with large integers \( n \) and \( n' \) which cancel the shift due to local FI terms.

Comparing this mass spectrum with the one obtained in the five-dimensional case, we observe a qualitative difference. There, the Kaluza-Klein excitations of the bulk mode became very heavy with the cut-off \( \Lambda \) and in the limit \( \Lambda \to \infty \) we just retained a massless zero mode localized at a fixed point. Effectively the bulk field underwent a dimensional transmutation and became a brane field. In the present six-dimensional case such a radical effect does not happen. The zero mode bulk field shows a localization behaviour as illustrated in Ref. [2] but the Kaluza-Klein excitations are not removed and the bulk field retains its six-dimensional nature.

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