MAGNETO-ACOUSTIC WAVE OSCILLATIONS IN
SOLAR SPICULES

A. Ajabshirizadeh$^{1,2,3}$, E. Tavabi$^{1,4}$, S. Koutchmy$^4$

1 Department of Theoretical Physics and Astrophysics, Tabriz University, 51664 Tabriz,

2 Research Institute for Astronomy and Astrophysics of Maragha, (RIAAM),

3 Research Institute for Applied Physics and Astronomy of Khajeh Nassiraldin, Iran,

4 Institut d’Astrophysique de Paris and UPMC, 98 Bis Boulevard Arago, F-75014 Paris, France.

(E-mail: a-adjab@tabrizu.ac.ir, tavabi@tabrizu.ac.ir , koutchmy@iap.fr).

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Abstract

Some observations suggest that solar spicules show small amplitude and high frequency oscillations of magneto-acoustic waves, which arise from photospheric granular forcing. We apply the method of MHD seismology to determine the period of kink waves. For this purposes, the oscillations of a magnetic cylinder embedded in a field-free environment is investigated. Finally, diagnostic diagrams displaying the oscillatory period in terms of some equilibrium parameters are provided to allow a comparison between theoretical results and those coming from observations.

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1 Introduction

Spicules are jet-like chromospheric structures and are usually seen all around the limb of the Sun arising in different directions. The mechanism of spicule formation and evolution is not well understood (for the propulsive mechanisms, see reviews of Sterling 2000; Lorrain and Koutchmy 1996; Filippov et al. 2006). Spicules are relatively homogeneous in height along their life time of approximately 5-15 min., i.e. they are short-lived and comparable to the photospheric granules lifetime. They have typical up flow speeds of 20-50km/s, spicules diameter at chromospheric layers are of the order of 200-500km. The mean number of spicules per supergranule cells at height of km is approximately 40 (Pataraya et al. 1990), which is covered about 1 percent of the Sun’s surface and they are usually concentrated between supergranule cells. Their temperatures and density are higher than those of the surrounding environment, Parenti et al. (1999) estimated $n_e = 10^{10} cm^{-3}$ and $T_e \approx 2 \times 10^5 ^\circ K$ in giant spicules see Koutchmy and Loucif, also named macrospicules, are observed over 20,000 km off-limb and live 40 min. (Xia et al. 2005) in case of macro-spicules. This means that a magnetic field of 10G or more is needed for the low-$\beta$ (ratio of the thermal pressure to the magnetic pressure) conditions (Wilhelm 2000) in case of macro-spicules. Spicule usually have oscillation behavior, the existence of 5 minutes oscillations in spicules have been firstly reported by Kulizhanishvili and Nikolsky (1978) and others including spectroscopically resolved observations. Recently image sequences were studied by De Pontieu et al. 2003, 2004; Xia et al. (2005); Ajabshirizadeh et al. (2007). These oscillations seem to be related to p-modes, but it is evident that if spicules are driven by p-modes, crucial details about their formation are still missing. Clearly, not all spicular flows are periodic, whereas most photospheric oscillations are. In addition, the horizontal scale for amplitude coherence of p-modes ($\approx 8000 km$) is well beyond the width of fibrils (De Pontieu et al. 2003). On the other-hand, oscillations in spicule with shorter period have been reported by Nikolsky and Platova (1971). They found that
spicules oscillate along the limb with a characteristic period of about 1 min. If spicules are formed in thin magnetic flux tubes, then the periodic displacement of the axis observed by them was probably due to the propagation of kink waves. More recently, Kukhianidze et al. 2006 have reported the observational signature of propagating kink waves in spicules. The period of waves was estimated to be 35-70s for a spicule with height 3500 km which may carry photospheric mechanical energy into the corona. The cutoff period of kink waves due to stratification in the hydrostatic photosphere is $\approx 660\text{s}$ so the expected period of kink waves is well below the cutoff value (Singh and Dwivedi, 2007). The wavelength was found to be $\approx 1000\text{km}$ at the photosphere level which indicated a granular origin of the waves. Magnetic flux tubes support transverse kink waves that can be generated in photospheric magnetic flux tubes through buffeting action of granular motions (Roberts, 1979; Hasan and Kalkofen, 1999) although the extrapolation of the network magnetic field toward the corona is still a matter of discussion. In this study, the effect of gravitational stratification has been ignored, Singh and Dwivedi 2007 have considered kink mode periods with this effect and confirms with our results.

2 Basic MHD Equations

2.1 The Dispersion Relations

Magneto-hydrodynamics (MHD) is one of the key tools to understand the hydrodynamics of magnetized plasmas. It concerns virtually all phenomena observed in the solar atmosphere: coronal loops, filaments, spicules, etc. Thanks to high spatial resolution, image processing, and time cadence capabilities of the SoHO and TRACE spacecraft, oscillating loops (and spicules) and propagating waves have been identified and localized in the transition region (TR) and chromosphere, and studied in detail since 1996. Using seeing free observations, they evidently complement what has been studied for a long time at ground-based, including spectroscopic
analysis. These discoveries established a new discipline has become know as solar atmosphere seismology. Many astrophysical plasmas are characterized by a set of equations that is called ideal MHD equations and includes the continuity, the momentum, Maxwell’s equations and Ohm’s law. To understand the various oscillations and waves we observe in the spicule plasma we have to find wave solutions of the MHD equations (Roberts, 1981), the existence of wave solutions is generally derived by introducing a small perturbation of physical parameters (such as: density, velocity and magnetic field) of the plasma, and to derive dispersion relations \( \omega(k) \), which tell us either the group velocity or phase speed of wave. The cylindrical flux tube appearance of many magnetic structures in the low-\( \beta \) plasmas of the magnetosphere and more specifically the solar chromosphere and corona encourages an investigation of propagation in cylindrical geometries. Edwin and Roberts (1983) consider a uniform cylinder of magnetic field \( B_0 \hat{z} \) confined to a region of radius \( 2b \), surrounded by a uniform magnetic field \( B_e \hat{z} \), the gas pressure and density within the cylinder are \( P_0, \rho_0 \), outside \( P_e, \rho_e \) respectively (see figure 1). The Fourier form of the velocity disturbance \( v_1 \) in cylindrical coordinate is:

\[
v_1 = v_1(r) \exp[i(\omega t + n\theta - kz)], \tag{2.1}
\]

where \( n \) is integer (\( n=0, 1, 2 \)) which describes the azimuthally behavior of the oscillating tube i.e. the cylindrically symmetric (sausage or pulsation mode given by \( n=0 \), the asymmetric (kink or taut-wire) mode given by \( n=1 \) and higher mode with \( n=2, 3 \) are called the fluting or interchange modes. The general governing equation is (Edwin and Roberts 1983):

\[
\frac{d}{dr}\left[ \rho_\alpha(r) \left( \frac{k^2 V_{2\alpha}^2 - \omega^2}{m_\alpha^2 + \frac{k^2}{V_{2\alpha}^2}} \right) \frac{d}{dr}(rv_1) \right] - \rho_\alpha(r) \left( k^2 V_{2\alpha}^2 - \omega^2 \right) v_1 = 0, \tag{2.2}
\]

where \( m_0 \) and \( m_e \) are defined by \( m_\alpha \) (with \( \alpha = 0 \) or \( \alpha = e \) inside or outside of the tube respectively):

\[
m_\alpha = \frac{(k^2 C_{2\alpha}^2 - \omega^2)(k^2 V_{2\alpha}^2 - \omega^2)}{(C_{2\alpha}^2 + V_{2\alpha}^2)(k^2 C_{2\alpha}^2 - \omega^2)} \tag{2.3}
\]
where $C_\alpha = \left(\frac{\gamma P_\alpha}{\rho_\alpha}\right)^{1/2}$ and $V_{A\alpha} = \frac{B_\alpha}{(\mu_\rho_\alpha)^{1/2}}$ are the sound and Alfvén speed inside (or outside) the cylinder, and $C_{T\alpha}$ is defined as:

$$C_{T\alpha} = \frac{C_\alpha V_{A\alpha}}{\sqrt{(C^2_\alpha + v^2_{A\alpha})}} \quad (2.4)$$

($\gamma$ is the ratio of specific heats.) The external and internal solutions of MHD equations need to be matched at the boundary by the continuity of pressure and the perpendicular component of velocity. After some algebra one gets the dispersion relation for magneto-acoustic waves in a cylindrical magnetic flux tube is found to be (Edwin and Roberts 1983; Daz et al. 2002):

$$\rho_e(\omega^2 - k^2 v^2_{Ae})m_0 I'_n(m_0b) + \rho_0(\omega^2 - k^2 v^2_{A0})m_e K'_n(m_e b) = 0, \quad (2.5)$$

where $I_n$ and $K_n$ are modified Bessel functions of order $n$, with $I'_n$ and $K'_n$ being the derivatives with respect to the argument $x$. This dispersion relation describes both surface (for $m_0^2 > 0$) and body waves (for $m_0^2 < 0$).

### 2.2 Kink-mode period

Magneto-acoustic oscillations of kink mode have now been directly observed in $H\alpha$ line using 53-cm coronagraph the Abastumani Astrophysical Observatory at different heights above the photosphere (Kukhianidze et al. 2006). The ratio of the spicule width $2b$ to the spicule full length $2L$ is $\frac{b}{L} = 0.01 - 0.4$, which is correspond to the dimensionless wave number ($kL$). Therefore, the observed kink-mode oscillations are in the long-wavelength regime of $kL << 1$, where the phase speed of kink-mode is practically equal to the kink speed (Spruit, 1981; Roberts et al. 1984) given by:

$$C_k = \left(\frac{\rho_0 V^2_{A0} + \rho_e V^2_{Ae}}{\rho_0 + \rho_e}\right)^{1/2}, \quad (2.6)$$

In the low-$\beta$ plasma limit and for the field free environment, the expression for the kink speed $C_k$ reduces to:

$$C_k \approx \left(\frac{2}{1 + \frac{\beta}{\rho_0}}\right)^{1/2}V_{A0}, \quad (2.7)$$
If we denote the full spicule length $l=2L$, the wavelength of the fundamental standing wave is the double spicule length (due to the forward and backward propagation) i.e., $\lambda = 2l$ and thus the wave number of the fundamental mode ($N=1$) is $k_z = \frac{2\pi}{\lambda} = \frac{\pi}{l}$, while higher harmonics ($N=2, \ldots$) would have wave numbers $k_z = N\left(\frac{\pi}{l}\right)$, then the time period $P$ of a kink-mode oscillation at the fundamental harmonic is:

$$P \approx \frac{2l}{C_k} = \frac{2l}{V_{A_0}} \left(1+\frac{\rho_e}{\rho_0}\right)^{\frac{1}{2}},$$

(2.8)

and for higher harmonics,

$$P \approx \frac{2l\sqrt{\mu N B_0}}{V_{A_0}} \left(\frac{\rho_0+\rho_e}{2}\right)^{\frac{1}{2}}.$$

(2.9)

### 3 Results

Such as has been pointed out before, the determinant coming from dispersion equation (2-5) must be truncated by taking into account a finite number of basis functions and we will use surface wave ($m_0^2 > 0$ in Eq. 2-5). As we know, the kink wave is essentially non-dispersive and has a phase speed equal to the kink speed, $v_{ph} = \frac{\omega}{k_z} \approx C_k \approx V_{A_0}$ so we introduce dimensionless frequency which is given by $\frac{\omega L}{V_{A_0}}$ for odd modes and the fundamental modes and their harmonics have a cutoff frequency for odd modes equal $\pi$ (see Diaz et al. 2002).

In figure 2 the eigenfrequencies of the kink (n=1) odd modes ($\omega_{cutff} = \pi$) have been plotted in term of the spicule half-thickness. In this plot we can see that for small value of $\frac{b}{L}$ only the fundamental and lower harmonics could appear and for large ratio of $\frac{b}{L}$ the frequency of the fundamental mode is insensitive to the spicule thickness. One of the essential results of these plots is that the oscillatory frequency is quite insensitive to the exact value of the ratio $\frac{b}{L}$ (the ratio of the spicule thickness to the half-length of magnetic field lines inside the spicule), i.e., for a given length $2L$, spicules with different thickness oscillate with the same frequency, this is perhaps an interesting result, and this subject have been firstly reported by Daz et al. 2002, for kink waves in the prominence fibrils. We continue by choosing two typical values of the
spicule full length, namely, \(l=3500\) and \(8500\)km, and then plot the frequency of odd modes below the cutoff as a function of \(\left(\frac{\rho_e}{\rho_0}\right)\) (see figure 3-a, b). In these plots, we keep \(\frac{b}{L} = 0.05, 0.03\) constant for two lengths of spicule respectively. For all harmonic modes the frequency is seen to slowly decrease with increasing \(\left(\frac{\rho_e}{\rho_0}\right)\) and more oscillatory modes are present for large values of this quantity. To compute the corresponding periods for each harmonic modes, we used \(l=3500\) and \(8500\)km, \(B_0 = 30G\) and \(\rho_0 = 3 \times 10^{-10}\)kgm\(^{-3}\). The periods obtained from these quantities and using Eq. (2-9) is labeled \(P\) and correspond to the right vertical axes in figure 3 (see also Table 1).

The period, \(P\), (figure 3- a, b right vertical axes and table 1) for different values of \(\frac{b}{L}\), \(L\) calculated from the expression of Eq. (2-5) and (2-9), show that the fundamental and first harmonic have typical periods of \(\approx 30 - 80\)s. These periods which are obtained from a simple MHD approach are found in the more recently reported observational results (Kukhianidze et al. 2006), where oscillation periods in the range of kink wave of \(\approx 35 - 70\)s are found for spicules with height 3500-8700 km. From the theoretical point of view, we therefore expect kink wave periods in the range of \(\approx 80 - 120\)s for length of \(\approx 10,000 - 14,000\)km which could be found from future observations of kink wave inside spicules (Hinode observations).

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Figure Captions

Figure-1: Fig.1- Sketch of the equilibrium configuration used in this study. The density and magnetic field inside the spicule are $B_e, \rho_e$, and in chromosphere environment are $B_0, \rho_0$. The magnetic field is uniform and along z-axis.

Figure-2: Fig. 2. Variation of the frequency with the spicule half-thickness for Kink-Modes for the set of value $2L=8500km$, and for the density ratio $\rho_e/\rho_0=0.03$.

Figure-3: Fig. 3- a, b. Frequency of the kink odd modes vs. $\rho_e/\rho_0$ for two full length of spicules and $\frac{h}{L} = 0.05, 0.03$ for the parameters. The right-hand axis provides the period $P$ after estimation that magnetic field strength, the spicule density, and full-length of spicules are $\rho_0 = 3 \times 10^{-10} kgm^{-3}$ (corresponding to a $2 \times 10^{11}$ number density), $B_0 = 30G$, and $2L=3500, 8500km$, respectively.

Figure-4: TABLE 1