TeV-Scale Black Holes in Warped Higher-Curvature Gravity * †

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Abstract

We examine the properties of TeV-scale extra dimensional black holes (BH) in Randall-Sundrum-like models with Gauss-Bonnet higher-curvature terms present in the action. These theories naturally lead to a mass threshold for BH production in TeV particle collisions which could be observable at LHC/ILC. The lifetimes of such BH are examined and, in particular, we focus on the predicted lifetime differences between the canonical and microcanonical thermodynamical descriptions of BH decaying to Standard Model brane fields and the possibility of long-lived relics. The sensitivity of these results to the particular mix of fermions and bosons present in the Standard Model spectrum is also briefly examined.

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1 Introduction

The Randall-Sundrum (RS) model\textsuperscript{[1]} provides a geometric solution to the hierarchy problem through an exponential warp factor whose size is controlled by the separation, $\pi r_c$, of two 3-branes with non-zero tension, situated at the $S^1/Z_2$ orbifold fixed points, which are embedded in 5-d Anti-deSitter space, $AdS_5$. It has been shown that this interbrane distance can be naturally stabilized at a value necessary to produced the experimentally observed ratio of the weak and Planck scales\textsuperscript{[2]}. In the original RS model, Standard Model (SM) matter is confined to one of the 3-branes while gravity is allowed to propagate in the bulk. A generic signature of this kind of scenario is the existence of TeV-scale Kaluza-Klein (KK) excitations of the graviton. These states would then appear as a series of spin-2 resonances\textsuperscript{[3]} that might be observable in a number of processes at both hadron and $e^+e^-$ colliders which probe the TeV-scale. Another possible RS signature is the copious production of TeV scale black holes, though this is not a qualitatively unique feature of the RS model as they also appear in non-warped scenarios\textsuperscript{[4]}. Here we will consider BH production in RS employing the most general self-consistent bulk action.

One can easily imagine that this simple RS scenario is incomplete from either a top-down or bottom-up perspective. Apart from the placement of the SM matter fields, we would generally expect some other ‘soft’ modifications to the details of the scenario presented above, hopefully without disturbing the nice qualitative features of the model in the gravity sector. It is reasonable to expect that at least some aspects of the full UV theory may leak down into these collider measurements and may lead to potentially significant quantitative and/or qualitative modifications of simple RS expectations that can be experimentally probed. These modifications may make their presence known via higher dimensional, possibly loop-induced, operators involving invariant products of the curvature tensor. Of course not all possible operators are allowed if we require that the generic setup of the RS model remain valid or be stable with respect to quantum corrections as we will discuss below.
One such extension of the basic RS model is the existence of higher curvature invariants in the action which might be expected on general grounds from string theory, quantum gravity or other possible high-scale completions. A certain string-motivated class of such terms with interesting properties was first generally described by Lovelock and, hence, are termed Lovelock invariants.

These invariants are particularly important for a number of reasons. Consider extending the EH action by an arbitrary set of various higher curvature/derivative terms. The resulting equations of motion will generally be higher than second order. If the usual 5-d metric and set-up of the RS model are assumed to remain valid, this implies that derivatives of $\delta-$functions will be generated in the equations of motion that cannot be canceled by matter source terms. Thus, for consistency we must demand that the equations of motion in standard RS be no higher than second order. This is a rather strong restriction on what terms are allowed when generalizing the RS action. Other constraints may be of comparable importance but are more wide ranging beyond the RS scenario. For example, if one examines the 5-d propagating fields in the general case one usually obtains a set of massive tensor ghosts which need to be avoided. Lovelock long ago showed that these and other problems have a unique solution: construct the full action as a sum of Lovelock invariants.

The Lovelock invariants come in fixed order, $m$, which we denote as $L_m$, that describes the number of powers of the curvature tensor, contracted in various ways, out of which they are constructed, i.e.,

$$L_m \sim \delta_{C_1 D_1 \cdots C_m D_m}^{A_1 B_1 \cdots A_m B_m} R_A^{A_1 B_1} R_{C_1 D_1} \cdots R_{A_m B_m}^{C_1 D_1 \cdots C_m D_m},$$ (1)

where $\delta_{C_1 D_1 \cdots C_m D_m}^{A_1 B_1 \cdots A_m B_m}$ is the totally antisymmetric product of Kronecker deltas and $R^{CD}_{AB}$ is the $D$-dimensional curvature tensor. When added to the EH action in any combination the resulting generalized Einstein equations of motion remain second order, are tachyon and ghost free, lead to a unitary perturbation theory due to the absence of derivatives of the metric higher than first in the action and to a naturally conserved and symmetric stress-energy tensor for the matter source fields. Because of this the basic RS setup is stable when these new terms are added to the action. For a
fixed number of dimensions the number of the non-zero Lovelock invariants is quite restricted, \( e.g. \), for \( D = 4 \), only \( \mathcal{L}_0 = 1 \) and \( \mathcal{L}_1 = R \), the ordinary Ricci scalar, can be present in the action; all higher order invariants can be shown to vanish by the properties of the curvature tensor. Adjusting the numerical coefficients of these terms we see that the resulting action is just the ordinary Einstein-Hilbert (EH) action of General Relativity with a cosmological constant. When \( D = 5 \), as in the RS model, \( \mathcal{L}_2 = R^2 - 4R_{MN}R^{MN} + R_{MNA}R^{MNA} \), the Gauss-Bonnet (GB) invariant, can also be present in the action with an arbitrary coefficient, \( \alpha \), with all other \( \mathcal{L}_{m>2} = 0 \). Thus the GB term is the unique addition to the bulk EH action in the Lovelock framework in \( D = 5 \); no higher order terms are allowed. The bulk action in the most general self-consistent RS model is now specified. Furthermore, it is important to recall that the GB term is in fact the leading (and here only allowed) stringy correction to the EH action\[5, 6\].

In principle, one might consider also adding brane terms of the GB form. However, since the branes live in \( D = 4 \) these potential GB brane terms are at most surface terms as can be shown using the various curvature identities; they will not contribute to the equations of motion. The addition of the GB term to the EH action is thus a rather unique possibility from (i) the string point of view, (ii) our desire to maintain the setup and stability of the RS model and (iii) the nice behavior of the resulting equations of motion and usual structure graviton propagators. For these reasons we will concentrate on its influence on RS BH properties below.

Some of the modifications of the RS model due the presence of GB terms have been discussed by other authors (\( e.g. \), Refs. \[8, 9, 10, 11, 12, 13, 14, 15, 16\]). \[8\] were the first to notice that the addition of the GB term to the EH action did not alter the basic structure of the RS model. In fact they were the first to argue that it is the only allowed addition of its kind; any other invariant in the \( D = 5 \) action would lead to a destabilization of the model.

Recently, we have begun a phenomenological examination of the effect of the presence of GB invariants in the action on the predictions of the RS model\[17\]. In that work we concentrated the modifications of graviton KK properties due to the GB term and how the value of the parameter \( \alpha \)
can be extracted from this collider data. In other work\cite{18} we have shown that the presence in the action of Lovelock invariants can lead to TeV-scale BH in ADD-like models with properties that can differ significantly from the usual EH expectations including the possibility that BH may be stable in $n$-odd dimensions. The purpose of the present paper is to examine the effects of GB terms in the action on TeV scale Schwartzschild-AdS BH in the RS model. The possible lifetimes of these BH will be examined and, in particular, we focus on the predicted differences between canonical and microcanonical thermodynamical descriptions of BH decays and the possibility of long-lived relics.

In the next section we provide a brief overview of the essential aspects of the GB-extended RS model necessary for our analysis. In Section 3, we discuss the basic properties of the BH in this model and calculate their corresponding production cross sections for TeV colliders. We also discuss the differences between the use of the canonical ensemble and microcanonical ensemble descriptions for BH Hawking radiation when the BH mass is comparable to the 5D effective Planck scale as might be expected at future colliders. In Section 4, we present the results of a numerical study of BH decay rates in the GB-extended RS model. We perform a detailed comparison of the two possible statistical descriptions for BH decay and show the sensitivity of these results to variations in the model parameters. The sensitivity of BH mass loss to the statistics of the final state particles is also discussed. The last section of the paper contains a summary and our conclusions.

2 RS Background

The basic ansatz of the RS scenario is the existence of a slice of warped, Anti-deSitter space bounded by two ‘branes’ which we assume are fixed at the $S^1/Z_2$ orbifold fixed points, $y = 0, \pi r_c$, termed the Planck and TeV branes, respectively\cite{1}. The 5-d metric describing this setup is given by the conventional expression

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2.$$  (2)
As usual, due to the $S^1/Z_2$ orbifold symmetry one requires $\sigma = \sigma(|y|)$ and, in keeping with the RS solution, we have $\sigma = k|y|$ with $k$ a dimensionful constant of order the fundamental Planck scale. As first shown in Ref.[8] the inclusion of GB terms is the only one that does not alter this basic setup. Based on the above discussion, the action for the model we consider takes the form

$$S = S_{bulk} + S_{branes},$$

where

$$S_{bulk} = \int d^5x \sqrt{-g} \left[ \frac{M^3}{2} R - \Lambda_b + \frac{\alpha M}{2} \left( R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \right) \right],$$

describes the bulk with $M$ being the $D = 5$ fundamental Planck scale, $\Lambda_b$ the bulk cosmological constant and $\alpha$ the above mentioned dimensionless constant which measures the relative strength of the GB interaction. If we anticipate that the GB terms be (mainly) subdominant to the EH ones as might arise in some sort of loop perturbative expansion then one can argue[18, 19] that the natural size of this parameter is $|\alpha| \sim 1/D^2 \sim 1/16\pi^2 \sim O(0.01)$ which we will assume in what follows. In order to avoid both ghosts and tachyons[20] in both the perturbative graviton KK and radion[21] sectors, $\alpha$ must be chosen negative. Similarly

$$S_{branes} = \sum_{i=1}^{2} \int d^4x \sqrt{-g_i} \left( \mathcal{L}_i - \Lambda_i \right),$$

describes the two branes with $g_i$ being the determinant of the induced metric and $\Lambda_i$ the associated brane tensions; the $\mathcal{L}_i$ describe possible SM fields on the branes. In what follows we will assume as usual for simplicity that the SM fields are all localized on the TeV brane at $y = \pi r_c$. The (Einstein) equations of motion then lead to the following parametric relations:

$$\Lambda_{Planck} = -\Lambda_{TeV} = 6kM^3 \left( 1 - \frac{4\alpha k^2}{3M^2} \right)$$

and

$$6k^2 \left( 1 - \frac{2\alpha}{M^2 k^2} \right) = -\frac{\Lambda_b}{M^3}.$$
where we explicitly see the $\alpha$-dependent modifications to the conventional RS results. In the classic RS case, one ordinarily assumes that the ratio $k^2/M^2$ is small to avoid higher curvature effects. Here such terms are put in from the beginning and we need no longer make this *a priori* assumption; we will allow a wide range of $k^2/M^2$ values in the analysis that follows. For later convenience we will define the parameter $M_* = M_\epsilon = M e^{-\pi kr_c}$, which is the warped-down fundamental scale, $\sim$ TeV. This parameter in many ways acts in a manner similar to the fundamental $\sim$ TeV scale in ADD models when discussing BH production on the RS TeV brane as will be seen in the following analysis.

Since collider experiments with center of mass energies $\sqrt{s} \sim$ TeV are directly probing the mass scale $M$, we might imagine that higher order terms in the gravity action may be of some importance when considering RS signatures. Here we examine how the GB term will modify RS BH properties.

3 Black Hole Properties

How do we describe a Schwartzschild-like BH in $AdS_5$ that is created on the TeV brane through SM particle collisions when GB terms are present in the action? In the usual BH analysis within the ADD framework\[22\], one employs a modified form of the conventional $D$-dimensional Schwartzschild solution\[23, 24\] assuming all dimensions are infinitely large. Under the assumption that the size of the BH, given by its Schwartzschild radius, $R_s$, is far smaller than the size of the compactified dimension, $\pi R_c$, this is a valid approximate description. This approximation is seen to hold to a very high degree in the usual ADD models with low values of $D$. The analogous requirement in the RS case for a TeV brane BH is that $(\epsilon R_s/\pi r_c)^2 << 1$ which is also well satisfied as we will see below. A further requirement in the RS case is that the bulk is now $AdS_5$ and not Minkowski-like as in ADD so that asymptotically flat $D = 5$ Schwartzschild-like solutions are not directly applicable here. This becomes immediately obvious when we think of the curvature of the space-time, $\sim k\epsilon$
as measured on the TeV brane, becoming comparable to $R_s^{-1}$. Fortunately, the solutions for this asymptotic $AdS$-Schwartzschild BH were found long ago\cite{24} and such BH have had their properties discussed in some detail in the literature; we will make use of these results in the analysis that follows. Clearly, RS BH in models with large values of $k^2/M^2$ may differ significantly from their ADD cousins and this is particularly true when higher order curvature terms are present in the action as they are here.

The first step in our analysis is to determine the relationship between $R_s$, $M_*$ and the BH mass $M_{BH}$. Note that it is $M_*$ that appears in these relations and not $M$ since all the SM matter is assumed to be on the TeV brane. Following, e.g., Cai\cite{24}, and employing the definitions above as well as the relationship between $k$ and $\Lambda_b$ implied by the Einstein equations of motion in Eq.(6), we find that

$$m = c \left[ x^2 + 2\alpha + \gamma x^4 \right],$$

(7)

where we employ the dimensionless quantities $x = M_* R_s$, $m = M_{BH}/M_*$,

$$\gamma = \frac{k^2}{M^2} \left(1 - 2\alpha \frac{k^2}{M^2}\right),$$

(8)

and the numerical factor $c = 3\pi^2$. Note that $\gamma$ is a measure of the curvature of the $AdS_5$ space; when $\gamma \to 0$ the bulk becomes ‘flat’, i.e., Minkowski-like and we recover the ADD result for $D = 5$ with GB terms in the action. To cover all eventualities we will consider the range $10^{-4} \leq \gamma \leq 1$ in our subsequent numerical analysis. Since we generally want $x(m)$ and not $m(x)$ as above we simply invert the expression in Eq.(7) to give

$$x^2 = (2\gamma)^{-1} \left[ -1 + \left[1 + 4\gamma(m/c - 2\alpha)\right]^{1/2} \right],$$

(9)

where the sign has been chosen to insure that $x^2 \geq 0$. In $D = 5$ ADD-like models, to insure that $x^2 \geq 0$ one requires that $m \geq 2\alpha c$ (since there $\alpha$ is positive) which would indicate the existence of a mass threshold for BH production. Here there is no apparent threshold of this type arising
from Eq.(9) since $\alpha$ is negative. Note that when $m \to 0$, $x$ remains finite; conversely, note that $x = 0$ corresponds to a negative value of $m$ since $\alpha < 0$. This behavior is quite distinct from that in ADD-like models even with Lovelock terms present. However, the typical BH radii obtained from this expression are numerically quite comparable to those obtained in the usual ADD model so that large BH cross sections should be expected at the LHC.

Given the mass-radius relationship we can now ask if the requirement $(\epsilon R_s/\pi r_c)^2 << 1$ discussed above is satisfied. Recall that we do not want our BH to ‘see’ the fact that it is living in a bounded space which would invalidate our solution. Using the notation above we can re-write this condition as $(k^2/M^2)(\pi kr_c)^{-2}x^2 << 1$; if we take typical values of these quantities: $k^2/M^2 \lesssim O(1)$, $x \sim O(1)$ and $kr_c \sim 11$, we see that this constraint is very easily satisfied for our range of parameters.

The BH Hawking temperature can be obtained from the derivative of the metric tensor evaluated on the horizon in the usual manner; following, e.g., Cai\cite{24} we obtain

$$T = \frac{1}{2\pi} \frac{x + 2\gamma x^3}{x^2 + 4\alpha}. \quad (10)$$

Since $\alpha$ is negative while $\gamma$ is positive it is clear that $x^2$ must be bounded from below if we demand that the BH Hawking temperature is to remain positive, i.e., $x^2 \geq -4\alpha$. Through the mass-radius relationship this implies that there is a corresponding critical lower bound on the BH mass, $m_{\text{crit}}$:

$$m_{\text{crit}} = -6\pi^2 \alpha(1 - 8\gamma\alpha), \quad (11)$$

which is of $O(1)$ when $|\alpha|$ is $O(0.01)$; smaller BH masses lead to negative temperatures. Note that the BH temperature is infinite precisely at $m = m_{\text{crit}}$ instead of at $m = 0$ in the usual flat-space EH case. This is also unlike the ADD-like models where a minimum BH mass also arises from the presence of Lovelock terms in the action in odd numbers of extra dimensions. In that case both $x, T \to 0$ at a fixed value of $m$\cite{18} producing the mass threshold. Here, as a function of the BH radius, $x$, the temperature starts off infinite at $x^2 = -4\alpha$, goes through a minimum at some fixed
radius and then grows rapidly again as \( x \rightarrow \infty \) due to the finite value of the curvature factor \( \gamma \). The threshold temperature behavior in GB augmented RS is thus more like that obtained from the traditional \( D \)-dimensional EH action where the temperature diverges as \( x \rightarrow 0 \).

Given the BH mass-radius relationship we can calculate the cross section for BH at colliders. The leading approximation for the subprocess cross-section for the production of a BH of mass \( M_{BH} \) is just its geometric size [22]:

\[
\hat{\sigma} = f \pi R_s^2 (\sqrt{s} = M_{BH}),
\]

(12)

where \( R_s \) is the 5-dimensional Schwarzschild radius corresponding to the mass \( M_{BH} \) and \( f \) is some factor of order unity. The specific range of values for this factor has been discussed extensively in the literature [25]. For our numerical purposes we will assume \( f = 1 \) in the analysis that follows.

Fig. 1 shows the numerical results we obtain for the BH cross section as a function of \( m/m_{crit} \) for different choices of the \( \alpha, \gamma \) parameters. Several features are immediately apparent: (i) BH do not form for masses below \( m_{crit} \) as this leads to negative temperatures. However, the Schwarzschild radius for a BH of mass \( m_{crit} \) is non-zero which implies a step-like threshold behavior for the cross section. This is unlike the case of the ADD-like models where Lovelock terms produce a very smooth threshold behavior for both odd and even numbers of dimensions [18]. In a more realistic UV completed theory it is likely that this threshold is smoothed out to some degree by quantum corrections. (ii) The cross section above threshold has an approximately linear \( m \) dependence; the deviations from linearity are related to the amount of curvature, \( i.e., \) the size of \( \gamma \). For the range of parameters we consider the qualitative sensitivity to \( \gamma \) is generally rather weak; the numerical scale of the cross section is set by \( M_s^{-2} \). (iii) The overall magnitude of the cross section is approximately linear in \( -\alpha \). Properties (i)-(iii) imply that experimental measurements of the BH cross section at colliders can be used to determine the basic GB-extended RS model parameters.

We now turn to the issue of BH decays through Hawking radiation; for simplicity we will ignore the influence of possible grey-body [22, 26] factors that may be present. It has been shown in the flat space case that the inclusion of such factors for brane fields when the GB term is present
Figure 1: The scaled BH production cross section as a function of $m/m_{\text{crit}}$ for $\gamma = 10^{-4}$ (solid) and 1 (dashed) for different values of $\alpha$. From bottom to top the curves correspond to $\alpha = -0.005, -0.010...$ increasing in magnitude by 0.005 for each subsequent curve.

does not modify the standard expectation that BH decays to brane fields dominates over bulk modes. Cardoso, Cavaglia and Gualtieri [24] have, however, shown that bulk grey-body factors could upset these expectations when the number of extra dimensions. Assuming these results are applicable to the present case we are relatively safe from these effects as $n = 1$.

The usual thermodynamical description of the Hawking radiation produced by TeV-scale BH decays is via the canonical ensemble (CE) [22] which has been employed in most analyses in the literature (in particular, our previous analysis of ADD-like BH). However, as pointed out by several groups [31], though certainly applicable to very massive BH, this approach does not strictly apply when $M_{BH}/M_*$ is not much greater than $O(1)$ or when the emitted particles carry an energy comparable to the BH mass itself due to the back-reaction of the emitted particles on the properties of the BH. This certainly happens when the resulting overall BH Hawking radiation multiplicity is low. In the decay of TeV-scale BH that can be made at a collider, the energy of the emitted particles is generally comparable to both $M_*$ as well as the mass of the BH itself thus requiring
following the MCE treatment. In the CE approach the BH is treated as a large heat bath whose temperature is not significantly influenced by the emission of an individual particle. While this is a very good approximation for reasonably heavy BH it becomes worse as the BH mass approaches the $M_*$ scale as it does for the case we consider below. Furthermore, the BH in an asymptotically flat space (which we can assume is relatively good approximation here for small $\gamma$ since the BH Schwarzschild radii, $R_s$, are far smaller than the compactification radius as noted above) cannot be in equilibrium with its Hawking radiation.

It has been suggested\cite{31} that all these issues can be dealt with simultaneously if we instead employ the correct, \textit{i.e.}, microcanonical ensemble (MCE) approach in the statistical mechanics treatment for BH decay. As $M_{BH}/M_*$ grows larger, $\gtrsim 10^{-20}$, the predictions of these two treatments will be found to agree, but they differ in the region which is of most interest to us since at colliders we are close to the BH production threshold where $M_{BH}/M_*$ is not far above unity. Within the framework of the EH action it has been emphasized\cite{31} that TeV-scale BH lifetimes will be increased by many orders of magnitude when the MCE approach is employed in comparison to the conventional CE expectations. This is not due to modifications in the thermodynamical quantities, such as the temperature, themselves but how they enter the expressions for the rate of mass loss in the decay of the BH. Here we will address the issue of how these two statistical descriptions may differ in the BH mass range of interest to us when the additional higher-curvature GB term is present in the RS action. Here we address these CE vs. MCE differences in the case of the RS model with GB terms present. It is important to remember that the values of the various thermodynamic quantities themselves, \textit{e.g.}, the BH temperature, $T$, are the same in both approaches.

4 Numerical Results: Mass Loss Rates and Lifetimes

In order to be definitive in our calculation of the BH Hawking decay rates we follow the formalism of Hossenfelder\cite{31}; to simplify our presentation and to focus on the GB modifications to RS, as
well as potential MCE and CE differences, we remind the reader that we will ignore the effects due
to grey-body factors[22] in the present analysis. Since bulk decays are expected to be generally
sub-dominant and since the only bulk modes are gravitons which have KK excitations that are
quite heavy in the RS model, ∼ TeV and the BH masses themselves, we will here assume that
the only (numerically) important BH decays are into SM brane fields. In this approximation the
rate for BH mass loss (time here being measured in units of $M_\ast^{-1}$) due to decay into brane fields
employing the MCE approach is given by

$$\left[ \frac{dm}{dt} \right]_{\text{brane}} = \Omega_{d+3} \frac{2}{(2\pi)^3} \zeta(4)x^2 \sum_i \int_{m_{\text{crit}}}^{m} dy \ (m - y)^3 N_i \left[ e^{S(m)-S(y)} + s_i \right]^{-1},$$

(13)

where, $x = M_\ast R_s$ as above, $S$ in the corresponding entropy of the BH, $i$ labels various particle
species which live on the brane and obey Fermi-Dirac(FD), Boltzmann(B), or Bose-Einstein(BE)
statistics, corresponding to the choices of $s_i = 1, 0, -1$, respectively, with the corresponding number
of degrees of freedom $N_i$, $\zeta$ is the Riemann zeta function with $\zeta(4) = \pi^4/90$, and as usual

$$\Omega_{d+3} = \frac{2\pi^{(d+3)/2}}{\Gamma((d + 3)/2)},$$

(14)

so that $\Omega_3 = 4\pi$. To proceed further we need to know the BH entropy $S$; this entropy can be
calculated using the familiar thermodynamical relation

$$S = \int dx \ T^{-1} \frac{\partial m}{\partial x},$$

(15)

which yields

$$S = \frac{4\pi}{3} c \left( x^3 + 12\alpha x + K \right),$$

(16)

where $K$ is an integration constant. Since only the entropy difference $S(m) - S(y)$ enters into
the expression above this constant can be chosen arbitrarily within our present analysis. However,
perhaps the most convenient choice is to choose $K = 16(-\alpha)^{3/2}$ such that $S(m_{\text{crit}}) = 0$. In this
special case \( S \) starts at zero when \( x^2 = -4\alpha \) and monotonically increases as \( x \) increases since \( \partial S/\partial x \) is always positive. The free energy of these BH, \( F = m - TS \), is also always positive in this case.

How sensitive is the BH mass loss rate and lifetime to the statistics of the particles in the final state? In the CE analysis, to be discussed below, this sensitivity is only at the level of \( 5 - 10\% \) and is due to a simple overall multiplicative factor. To address this issue for the MCE approach we display in Fig. 2 the results obtained by assuming BH decays to the pure BE, B and FD statistical final states taking \( N = 60, \alpha = -0.01 \) and \( \gamma = 10^{-4} \) for purposes of demonstration. While the B and FD statistics cases are rather close numerically over the entire mass range, the BE choice leads to a more rapid decay and a substantially shorter lifetime as can be seen in Fig. 2. Here we see that BH decaying only to fermions may have a lifetime which differs from one decaying only into bosons by a few orders of magnitude, \( \sim 10^3 - 10^4 \). Note that at larger values of \( m \), the MCE \( \rightarrow \) CE limit, all three curves become rather close, at the level of \( \sim 10\% \) as expected, while still differing at smaller \( m \) values. Though the detailed meaning of these these curves will not be discussed until later in this section it is clear that these differing statistics can in general be quite important when performing the BH mass loss analyses using the MCE approach.

In the CE case, the expression above simplifies significantly as there are no non-trivial integrals remaining. The reason for this is that in the CE treatment, the factors \( S(m) \) and \( S(m - y) \) appearing in the exponential factor above are considered nearly the same since backreaction is neglected, \( i.e., \) one replaces this difference in the MCE expression above by the leading term in the Taylor series expansion \( S(m) - S(m - y) \approx y\partial_m S = y/T \). Taking the limit \( m \rightarrow \infty \), \( i.e., \) no recoil, and integrating over \( y \) then produces the familiar CE result:

\[
\left[ \frac{dm}{dt} \right]_{brane} = Q \frac{\Omega_3^2}{(2\pi)^3} \zeta(4) x^2 \Gamma(4) T^4,
\]

where \( Q \) takes the value \( \pi^4/90(1, 7\pi^4/720) \) for BE(B, FD) statistics. Note that the difference in statistics here in the CE case is essentially trivial: just a simple multiplicative factor which is close to unity unlike in the MCE approach as seen above where there is a functional difference at low
Figure 2: The BH mass loss rate(top) and lifetime(bottom) in the MCE approach as a function of $m/m_{\text{crit}}$ assuming an initial value of $m = 5m_{\text{crit}}$, with $\alpha = -0.01$ and $\gamma = 10^{-4}$. In the top panel, from top to bottom, the curves correspond to the choice of BE, B or FD statistics, respectively. In the bottom panel, the order for the statistics choice is reversed.
m values. In practical calculations, especially since we are concerned with decays to SM brane fields where the numbers of fermionic degrees of freedom (48, assuming only light Dirac neutrinos) is somewhat larger than the number for bosons (14), the value of $s_i$ does not play much of an important role. In our numerical analysis that follows we will for simplicity take $s_i = 0$ and assume that the number of SM fields is 60. The reason why this is a good approximation is that (i) the SM is mostly fermionic and the results for Boltzmann and FD statistics lie rather close to one another and (ii) the B distribution lies between the BE and FD ones. It would be interesting to know how this approximation fares for BH decays in the ADD-like case when several Lovelock terms are present in the action simultaneously.

Given the results above we can now address a number of issues, in particular, (i) how do BH decay rates and lifetimes depend on the values of the parameters $\alpha$ and $\gamma$ and (ii) how sensitive are these results to the choice of the MCE or CE analysis approach. To be specific we first perform our analysis by following the usual CE approach; Figs. 3 and 4 show the results of these specific calculations which were performed for two widely different values of $\gamma = 10^{-4}$ and $\gamma = 1$, respectively. Any realistic value for this parameter must lie within this range. Assuming a BH with an initial mass of $m = 5m_{\text{crit}}$, as might be expected at TeV colliders, the top panel in both figures shows the rate of mass loss, $dm/dt$, for this BH. It is important to note that the mass loss rate is predicted to increase dramatically using the CE approach as $m \rightarrow m_{\text{crit}}$ making the BH radiate faster and faster. This is to be expected as in the CE approach the mass loss rate is proportional to $T^4$ and $T$ increases dramatically as the BH looses mass and gets smaller. This implies that in the CE analysis the final state classically stable remnant is reached in a reasonably short amount of time. The bottom panel for both $\gamma$ cases shows the time taken for the initial BH to radiate down to a smaller mass; when $m = m_{\text{crit}}$ this is the BH radiative ‘lifetime’, i.e., the time taken to decay down to $m_{\text{crit}}$. In this CE case this time is short and finite, $\sim 0.1M_{\text{s}}^{-1}$, as might be naively expected; it is not quite clear what happens to the remnant without a more complete theory which includes quantum gravity contributions but within the present framework we are left with a stable
Figure 3: The BH mass loss rate (top) and lifetime (bottom) as a function of $m/m_{\text{crit}}$ assuming $\gamma = 10^{-4}$ employing the CE scheme and assuming an initial mass $m = 5m_{\text{crit}}$. In the top panel, from top to bottom, the curves correspond to the choices $\alpha = -0.005, -0.010, \ldots$. In the bottom panel, the order is reversed.
remnant. This differs from the case of the ADD-like model with Lovelock terms in odd dimensions where the decay down to the finite mass remnant takes essentially an infinite amount of time. We further note that the BH specific heat, \( C = \partial m / \partial T \), here remains negative, as is typical for EH BH, over the relevant RS parameter space. This again differs from the ADD-like model with Lovelock terms where \( C \) can have either sign depending on the values of the Lovelock parameters and the BH mass.

One might wonder if this classical stable BH remnant is unusual and something that happens only in extra dimensions (as we already know that it can also occur for flat ADD extra dimensions for \( n \) odd). Surprisingly, the existence of a classically stable BH remnant is a common feature in many models which go beyond the conventional EH description including what may happen for a 4-d BH when a renormalization group running of Newton’s constant is employed\([27]\) in order to approximate leading quantum corrections; such a threshold scenario can also be seen to occur in theories with a minimum length\([28]\), in loop quantum gravity\([29]\) and in non-commutative theories\([30]\). Of course, the details of this phenomena differ in all these approaches. In, e.g., the case of a minimum length, stable remnants occur for all numbers of extra dimensions. It is interesting to note that this phenomena occurs in all these models where one tries to incorporate quantum corrections in some way; though the quantitative nature of such remnants differ in detail in each of these models, it would be interesting to learn whether or not this is a general qualitative feature of all such approaches.

For fixed \( \gamma \), Figs. 3 and 4 show that both the BH mass loss rate and lifetime have only modest dependence on the value of \( \alpha \) (for the parameter range considered here). The exact \( \alpha \) sensitivity is itself, however, dependent on the specific value of \( \gamma \) as a comparison of these two figures show. Generally the BH decay rate decreases (and the corresponding lifetime increases) as the magnitude of \( \alpha \) is raised.

Do these conclusions continue to be valid when we follow the MCE prescription? This question can be answered by examining the results presented in Figs. 5 and 6. In the MCE case,
Figure 4: Same as the previous figure but now assuming $\gamma = 1$. 
Figure 5: Same as Fig. 3 with the opposite ordering of the $\alpha$ dependence of the curves, but now using the MCE approach.
the BH mass loss rate rapidly decreases as \( m \) approaches \( m_{\text{crit}} \); this is the complete opposite behavior from what happens for the CE approach as discussed above. Generally we see that there is not a very large \( \gamma \) dependence, as we saw before in the CE analysis, but now the BH with the smaller (in magnitude) value of \( \alpha \) leads to the smallest mass loss rates, \( \frac{dm}{dt} \). This is again the completely opposite behavior to that seen in the CE analysis. We note that for \( m = 5m_{\text{crit}} \), the largest value shown, \( \frac{dm}{dt} \) is not so different in the CE and MCE cases. For much large values of \( m \gtrsim 10 - 20 \) one can check that the two sets of calculations yield essentially identical numerical results. The fact that \( \frac{dm}{dt} \) here decreases as \( m \to 0 \) implies that following the MCE approach a BH lives far longer than if one employs the CE analysis. The bottom panels of Figs. 5 and 6 clearly demonstrate this result where we see the vastly longer BH lifetimes obtained for the MCE approach. In comparison to CE analysis these BH lifetimes are observed to be greater by factors of order \( \sim 10^{10} \) or so. This result is certainly in line with what might have been expected based on the previously performed MCE versus CE analyses performed in the ADD model assuming the EH action[31].

The differences in BH mass loss and lifetimes we have obtained here in the RS model due to the choice of the MCE vs.CE thermodynamical description demonstrates how much we can learn from BH if they are produced at future colliders.

5 Summary and Conclusions

In this paper we have analyzed the properties of TeV-scale black holes in the Randall-Sundrum model with an extended action containing Gauss-Bonnet terms. As discussed above there are many reasons to consider such terms as augmentations to the EH action within the RS context: (i) model stability, (ii) string theory origin and (iii) good behavior of the resulting equations of motion and graviton propagators. In particular we have performed a detailed comparison of BH mass loss rates and lifetimes obtained by analyses based on the canonical and microcanonical ensemble descriptions. In addition, we have obtained expressions for BH production cross sections
Figure 6: Same as the previous figure but now for $\gamma = 1$. 

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in these models for future colliders. As in the ADD case, these cross sections are rather large yielding millions of events for typical parameter choices and luminosities.

Overall, the detailed quantitative behavior of BH in the RS model with GB terms in the action were found to be quite different than those in ADD-like models with Lovelock terms present. Our specific results are as follows:

(i) The restriction that the Hawking temperature of a BH be positive leads to a lower bound on its radius, \( R_s^2 \geq -4\alpha/M_s^2 \). This implies a corresponding mass threshold, \( M_{BH} \geq -6\pi^2\alpha(1 - 8\gamma\alpha)M_* \), which is \( \sim M_* \) for canonical parameter values, below which BH will not be produced at colliders. The resulting production cross section is found to have a step-like behavior, since \( R_s \) is finite at threshold, and to grow approximately linearly with the BH mass and value of \( |\alpha| \).

(ii) We performed a comparison of the two possible approaches to BH thermodynamics based on the canonical and microcanonical ensemble descriptions for the RS model. While yielding the same results for large BH masses, as was explicitly verified, the two differ in a number of ways when the BH mass is TeV-scale as was considered here. First, the BH mass loss rate and lifetime displayed in the CE analysis is well-known to display a very trivial dependence on the statistics of the particles into which it decays. For light BH we showed that this is not generally true in the MCE approach. For example, employing the MCE a BH decaying only to fermions may have a lifetime which differs from one decaying only into bosons by several orders of magnitude, \( \sim 10^3 - 10^4 \) in the specific case examined. For practical calculations involving BH decays only to SM fields the statistically weighted mass loss rate was found to be similar to that obtained for decays to particles with classical Boltzmann statistics. For light BH the CE and MCE treatments were shown to lead to drastically different lifetimes over the entire model parameter space. This can be traced to the fact that in the CE analysis the BH mass loss rate, which goes as \( \sim T^4 \), grows rapidly as \( m \) decreases since \( T \) is then also increasing. Given our parameter ranges, this then led to BH lifetimes which were typically \( \sim 0.1/M_*^2 \). On the other hand, the mass loss rate as \( m \to m_{crit} \) was found to behave in just the opposite manner when the MCE approach was followed. Since the
mass loss rate decreases so rapidly the corresponding BH lifetimes were found to be enhanced by factors of order $\sim 10^{10}$ relative to those of the CE case. This greatly increases the chance of long lived relics remaining after the usual BH decay process.

The differences between ADD-like and RS model BH is rather striking as are those between the CE and MCE thermodynamical descriptions. It is clear from this discussion that the observation of BH at future TeV colliders will provide an important probe of new high scale physics.

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