A locking-free DPG scheme for Timoshenko beams

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ABSTRACT

We develop a discontinuous Petrov–Galerkin scheme with optimal test functions (DPG method) for the Timoshenko beam bending model with various boundary conditions, combining clamped, supported, and free ends. Our scheme approximates the transverse deflection and bending moment. It converges quasi-optimally in $L^2$ and is locking free. In particular, it behaves well (converges quasi-optimally) in the limit case of the Euler–Bernoulli model. Several numerical results illustrate the performance of our method.

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