Derivative Portal Dark Matter

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We propose a new kind of Dark Matter: Derivative Portal Dark Matter. This kind of Dark Matter connects to the Standard Model through a massive mediator, which links to the Standard Model in derivative form. The derivative of a mediator in momentum space corresponds to the mediated momentum, which vanishes in the zero momentum transfer limit. As a result, this kind of Dark Matter can evade stringent constraint from the Dark Matter direct detection while fitting the Dark Matter relic density observation naturally.

I. INTRODUCTION

The ever-improving sensitivities of Dark Matter (DM) direct detection experiments have put the famous Weakly Interacting Massive Particles (WIMPs) DM models under pressure. Cold massive DM can explain the observed DM relic density through thermal production, with the requirement of weak interaction, which can be naturally interpreted as electroweak interaction possessed by the Standard Model (SM). This scenario also predict DM with hundreds of GeV mass which can be explored by DM direct detection experiment. However, with the ever-improving sensitivities of DM direct detection experiments, no DM has been found. A smaller interaction coupling can be adopted to explain the null result of the DM direct detection search, while a smaller interaction coupling will also result in insufficient DM thermal production. This raises a question: how can WIMPs explain the DM direct detection null result without diminishing of DM thermal production? One way out of this is to introduce an cancellation mechanism which works in DM direct detection only. In recent years, there have been studies exploring models where direct detection interaction is cancelled by two scalar mediators in the zero momentum transfer limit \cite{1-5}. In our previous work \cite{6} we have constructed a cancellation model by adding one U(1) gauge symmetry to the SM, where the extra gauge boson will mix with the Z boson from the mass matrix. In that model the direct detection mediated by the Z boson will be cancelled by the extra gauge boson. However, the kinetic mixing between the photon and the extra gauge boson will ruin the cancellation. In this work, we propose a new kind of DM model where the DM and the SM fermions are linked by the kinetic mixing between the Z boson and a count for the observed DM relic density.

The cancellation occurs when the scattering between DM and the SM fermions, mediated by two mediators, cancels each other out, resulting in an amplitude proportional to the momentum transfer. Therefore, the amplitude vanishes in DM direct detection since we usually adopt the zero momentum transfer limit in direct detection, while the DM relic density is not diminished since the momentum transfer in the annihilation process surpasses two times the DM mass and thus can not be disregarded. The usual way to prove the cancellation mechanism is to directly calculate scattering amplitude, which will be proportional to the momentum transfer. Alternatively, we can also prove it by noting that the momentum transfer is equal to the momentum of the mediators, which is equal to the derivative of the mediators in momentum space. Therefore, we can denote the interactions in models with the cancellation mechanism in the form of the derivative of the mediators (i.e., in the form of kinetic mixing between mediators). This allows us to see the cancellation property immediately. Therefore, in this work we will construct a new kind of model from the perspective of the derivative of the mediators.

In our previous work \cite{6}, we have constructed a cancellation model by adding one U(1) gauge symmetry to the SM, where the extra gauge boson will mix with the Z boson from the mass matrix. In that model the direct detection mediated by the Z boson will be cancelled by the extra gauge boson. However, the kinetic mixing between the photon and the extra gauge boson will ruin the cancellation. In this work, we propose a new kind of DM model where the DM and the SM fermions are linked by the kinetic mixing between the Z boson and a

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\textsuperscript{1} In ref. \cite{7}’s Appendix B, there are also discussions of cancellation between vector mediators.
massive dark vector boson \(^2\), and we will show that this kind of DM model possesses the cancellation property. There are lots of works have studied models where interaction between DM and the SM fermions comes from kinetic mixing term \([8–17]\). While the distinctive point in our construction is that the dominant kinetic mixing is between massive vector bosons and the kinetic mixing between the photon and the dark vector boson should be naturally negligible (e.g., the kinetic mixing between the photon and the dark vector boson comes from two-loop corrections). The reason why the photon should be out of the picture is that the propagator of the photon contains a momentum transfer \(t\) in its denominator, which will cancel the momentum transfer in the numerator and thus ruin the cancellation.

II. DERIVATIVE PORTAL DARK MATTER

The key Lagrangian of the Derivative Portal Dark Matter (DPDM) model is

\[
\mathcal{L} \supset J_\mu^f Z_\mu - \frac{\epsilon}{2} Z^{\mu\nu} Z'^{\mu\nu} + J^{\mu}_{DM} Z'_\mu, \tag{1}
\]

where \(Z_\mu\) and \(Z'_\mu\) are massive vector mediators, while \(J_\mu^f\) and \(J^\mu_{DM}\) are the current of the SM fermions and DM respectively. \(\frac{\epsilon}{2} Z^{\mu\nu} Z'^{\mu\nu}\) is the derivative portal which connects the SM and the dark sector. Then the dark matter SM fermion scattering is depicted by Fig. 1, where we use \(\chi\) and \(f\) to denote DM and the SM fermion respectively. Since the derivative portal contains derivative of mediators, which in momentum space is equal to the mediators’ momentum, the scattering amplitude will be proportional to the mediators’ momentum and thus the momentum transfer \(t\):

\[
i\mathcal{M} \propto \frac{p_1 - p_3}{t - m_Z^2} \frac{p_4 - p_2}{t - m_{Z'}^2} = \frac{t}{(t - m_Z^2)(t - m_{Z'}^2)}, \tag{2}
\]

where \(m_Z\) and \(m_{Z'}\) represent the mass of \(Z\) and \(Z'\) boson. Therefore the amplitude goes to zero in the zero momentum transfer limit\(^3\). From Eq. (2) we see that when the mass of one mediator goes to zero, there will be a \(t\) in the denominator which will cancel the \(t\) in the numerator and thus ruin the \(t\)-proportional property. Hence the massless photon is not suitable for building the derivative portal. Now let us look back at the derivative portal at Eq. (1), it is actually a kinetic mixing term between the \(Z\) boson and the \(Z'\) boson. For Abelian gauge bosons, one can write down the kinetic mixing term directly. For non-Abelian gauge bosons the kinetic mixing term can originate from loop corrections as shown in Fig. 2, where \(\Phi\) and \(\Psi\) represent the scalar and fermion which contribute to the kinetic mixing respectively. The kinetic mixing can thus be estimated as \([6, 8, 17]\)

\[
\epsilon \sim \sum_i \frac{g_i g'_i}{48\pi^2} \ln \frac{\mu^2}{m_i^2} - \sum_i \frac{g_i g'_i}{12\pi^2} \ln \frac{\mu^2}{m_i^2}, \tag{3}
\]

where the first term and the second term represent contribution from scalars and fermions respectively, and \(g_i, g'_i\) and \(m_i\) are the couplings and mass of the \(i\)th particle which contribute to the kinetic mixing. One thing should be kept in mind is that the kinetic mixing between the photon and the massive gauge boson which couples to DM should be naturally small, which means the leading loop corrections to their kinetic mixing should be at least two-loop corrections.

\(^2\) The \(Z\) boson can be replaced by another massive neutral-charged gauge boson which couples to the SM fermions.

\(^3\) The usual way of proving the cancellation mechanism can be seen in Appendix A.
III. BUILDING THE DERIVATIVE PORTAL

Building the derivative portal is simple, while making a naturally small kinetic mixing between the photon and the dark gauge boson which couples to DM is non-trivial (especially for the case where the SM $Z$ boson is in the derivative portal). Because generally one can write down a kinetic mixing term between the photon and another U(1) gauge boson directly. However, this can be avoided by assuming the kinetic mixing is in the same order of magnitude as its leading loop corrections or by embedding the dark gauge boson into a non-Abelian gauge group. In the following we will present three DPDM models and show the origination of their derivative portal and the kinetic mixing between the photon and the dark gauge boson. In the first two models, we assume the kinetic mixing between two U(1) gauge bosons is in the same order of magnitude as its leading loop corrections, and we show that in these two models the kinetic mixing between the photon and the dark gauge boson is truly in two-loop corrections level. In the third model, we embed the dark gauge boson into a non-Abelian gauge group, and show that the kinetic mixing between the photon and the dark gauge boson originates from two-loop corrections level.

A. The $U(1)_{B-L} \times U(1)_X$ model

A simple and direct construction of the DPDM model is extending an $U(1)_{B-L}$ model with an extra $U(1)_X$ gauge symmetry. The relevant Lagrangian can be given by:

$$
L = -\frac{1}{4} C^{\mu\nu} C_{\mu\nu} - \frac{1}{4} X^{\mu\nu} X_{\mu\nu} - \frac{\epsilon}{2} C^{\mu
u} X_{\mu\nu} + \sum_f g_{B\mu} C_{\mu} \bar{f} \gamma^\mu f + g_X X_{\mu} \bar{\chi} \gamma^\mu \chi + \frac{1}{2} m_C^2 C_{\mu} C^{\mu} + \frac{1}{2} m_X^2 X_{\mu} X^{\mu} - m_\chi \bar{\chi} \chi,
$$

where $C$ and $X$ are gauge bosons of $U(1)_{B-L}$ and $U(1)_X$ symmetry\textsuperscript{4}. The kinetic mixing term can be written directly or generated from loop corrections. To make it consistent we will consider all kinetic mixing comes from loop corrections. Therefore the derivative portal can be generated from the following Lagrangian:

$$
\mathcal{L} = g_{B\mu} C_{\mu} \bar{\Psi} \gamma^\mu \Psi + g_X X_{\mu} \bar{\Psi} \gamma^\mu \Psi.
$$

With the above interactions the derivative portal can be generated through the second diagram in Fig. 2. While kinetic mixing between the SM $B_{\mu}$ and $X_{\mu}$ is generated from two-loop corrections as displayed in Fig. 3. Because there is no particle coupling directly to both $B_{\mu}$ and $X_{\mu}$. Note that there will also be kinetic mixing between $C_{\mu}$ and $B_{\mu}$, however this will not affect the cancellation mechanism since DM $\chi$ only couples to $X_{\mu}$.

\textsuperscript{4} We also constructed a model which extends an $U(1)_{B-L}$ model with an $U(1)_X$ gauge symmetry in [6]. While in that construction the two extra gauge bosons are linked by mass mixing rather than kinetic mixing.
B. The SU(2)_L × U(1)_Z' model

Another construction of the DPDM model is taking the SM Z boson as one of the mediators in the derivative portal. The difficulty in this setting is that: since both the Z boson and the photon are combinations of B_µ of W_µ, it is not easy to make the extra gauge boson couples to the Z boson at one-loop level while couples to the photon at two-loop level. Fortunately there are particles in the SM which couple to the Z boson but the photon, which are the Higgs boson and neutrinos. Therefore we can use the neutrinos to generate a derivative portal between the SM Z boson and the extra gauge boson Z', while leaving the photon out of the picture. One possible UV complete model of the DPDM model can thus be written as:

\[
L = L_{SM} + (D^\mu \Phi)^\dagger D_\mu \Phi + \mu_\Phi^2 |\Phi|^2 - \lambda_\Phi |\Phi|^4 + \lambda_H |H|^2 |\Phi|^2
- \frac{1}{2} Z^{0\mu} Z^{0\nu} + i \chi \gamma^\mu D_\mu \chi - m_\chi \chi + i \overline{\psi}_L \gamma^\mu D_\mu \psi_L
+ i N_R \phi \overline{N}_R - \frac{1}{2} M_N \overline{N}_R N_R - \nu_\nu H \overline{L}_L N_R - \nu_\nu \overline{\psi}_L N_R + h.c.,
\]

where \( \chi \) and \( \Phi \) are DM and dark scalar respectively, \( L \) is the SM lepton doublet (it can also be an extra fermion doublet), and \( N_R \) is a right-handed “neutrino” that will give mass to either the extra fermion \( \psi_L \) or the L’s neutral component \( \nu_L \). The covariant derivatives are given by:

\[
\begin{align*}
D_\mu \Phi &= (\partial_\mu - ig_\chi Z'_\mu) \Phi \\
D_\mu \chi &= (\partial_\mu - ig_\chi_n Z'_\mu) \chi \\
D_\mu \psi_L &= (\partial_\mu - ig_\chi Z'_\mu) \psi_L,
\end{align*}
\]

where \( g_\chi \) and \( n_\chi \) are the gauge coupling and the quantum number of DM \( \chi \). After \( H \) and \( \Phi \) get their vacuum expectation value \( v_H \) and \( v_\Phi \), we can write the mass matrix of \( \nu_L \), \( N_R \) and \( \psi_L \) as:

\[
\frac{1}{2} \begin{pmatrix}
0 & Y_\nu v_H & 0 \\
Y_\nu v_H & M_N & Y_\psi v_\Phi \\
0 & Y_\psi v_\Phi & 0
\end{pmatrix}.
\]

In these three particles the Z boson couples to \( \nu_L \) and the Z' boson couples to \( \psi_L \), therefore after diagonalizing these three particles to their mass eigenstates, they all couple to the Z and Z' bosons simultaneously, without coupling to the photon. Thus these particles can generate the kinetic mixing between the Z and Z' bosons through one-loop corrections. While the kinetic mixing between the photon and the Z' boson are generated through two-loop corrections, as illustrated in Fig. 3 (with \( X_\mu \), \( C_\mu \), and \( B_\mu \) replaced by \( Z'_{\mu} \), \( Z_{\mu} \), and \( A_\mu \), and \( \Psi \) replaced by neutrinos). Note that with a large \( v_\Phi \), the Z' \( \bar{f} f \) coupling originated from mixing between H and \( \Phi \) can be neglected.

C. The SU(2)_L × SU(2)_Z' model

To avoid the assumption that the kinetic mixing between the photon and the DM is in the same order of magnitude as its leading loop corrections, alternatively, we can embed the Z' boson in the SU(2)_L × U(1)_Z' model to be a member of a multiplet, then there will be no tree-level kinetic mixing between the photon and the Z' boson. For example, to embed Z' in a non-Abelian gauge group SU(2)_Z', Eq. (6) can be modified to:

\[
(6') L = L_{SM} + (D^\mu \Phi)^\dagger D_\mu \Phi + \mu_\Phi^2 |\Phi|^2 - \lambda_\Phi |\Phi|^4 + \lambda_H |H|^2 |\Phi|^2
- \frac{1}{2} W_\mu^{a\nu} W_\mu^{a\nu} + i \chi \gamma^\mu D_\mu \chi - m_\chi \chi + i \overline{\psi}_L \gamma^\mu D_\mu \psi_L
+ i N_R \phi \overline{N}_R - \frac{1}{2} M_N \overline{N}_R N_R - \nu_\nu H \overline{L}_L N_R - \nu_\nu \overline{\psi}_L N_R + h.c.,
\]

where \( \Phi \), \( \chi \) and \( \psi_L \) are SU(2)_Z' doublet now, and the Z' boson becomes a neutral component of the SU(2)_Z' gauge filed \( W_\mu^{a} \). Therefore the covariant derivatives are given by:

\[
\begin{align*}
D_\mu \Phi &= (\partial_\mu - ig_\chi \nu_{\mu}^a x^a) \Phi \\
D_\mu \chi &= (\partial_\mu - ig_\chi n_{\nu} W_{\nu_{\mu}^a} x^a) \chi \\
D_\mu \psi_L &= (\partial_\mu - ig_\chi \nu_{\mu}^a x^a) \psi_L
\end{align*}
\]

Compared to Eq. (6), there are more particles in Eq. (9). For example, there are two DMs in Eq. (9) (i.e., both components of SU(2)_Z' doublet \( \chi \) are DM). These two DMs couple to the Z' boson in the same way, and the direct detection cancellation works for both DMs since the building of the derivative portal is the same as that in the SU(2)_L × U(1)_Z' model. In this model there is no tree-level kinetic mixing between the photon and the Z' boson. Their kinetic mixing originates from two-loop corrections, as illustrated in Fig. 3 (with \( X_\mu \), \( C_\mu \), and \( B_\mu \) replaced by \( Z'_{\mu} \), \( Z_{\mu} \), and \( A_\mu \), and \( \Psi \) replaced by neutrinos). Just like that in the SM, the imaginary part of the neutral component of \( \Phi \) and the charged component of \( \Phi \) will be eaten by the Z' boson and the W^{+/-} bosons respectively, and thus giving mass to these gauge bosons. Note that the mass of the W^{±} bosons will be exactly the same as the mass of the Z' boson, which is safe since
the charge the $W^{±}$ bosons possess is not the same as the electric charge in the SM. To keep the $W^{±}$ bosons not involved in the derivative portal, we have imposed a global U(1) symmetry to Eq. (9), under which $\Phi$ and $\psi_L$ charged oppositely. This symmetry will prohibit the $\Phi \psi_L N_R$ term, and thus prohibiting the charged component of $\psi_L$ get into mass matrix like Eq. (8). Therefore the $W^{±}$ bosons will not couple to the fermions which couple to the SM Z boson. Also one might note that the Higgs portal can also generate DM-SM fermions interactions, however the Higgs portal can always be neglected naturally by setting the dark scalar mass much heavier than the Higgs boson mass. Large dark scalar mass will lead to negligible mixing between the dark scalar and the Higgs boson, thus keeping the Higgs portal influence out of the picture.

In these models we adopt fermions in the loop to build the derivative portal, and the construction of derivative portal through scalars in the loop is left for future works.

IV. DM RELIC DENSITY

In this section we will study the constraints from observed DM relic density. We adopt a effective DPDM model where the SM Z boson and a dark boson $Z'$ are chosen as mediators, then the relevant Lagrangian can be written as:

$$\mathcal{L} = -\frac{1}{4} Z^{\mu \nu} Z_{\mu \nu} - \frac{1}{4} Z'^{\mu \nu} Z'^{\mu \nu} - \frac{\epsilon}{2} Z^{\mu \nu} Z'^{\mu \nu} + \sum_f Z_{\mu} f \gamma^\mu (g_V - g_A \gamma^5) f + g_\chi Z_{\mu} \chi \gamma^\mu \chi + \frac{1}{2} m_Z^2 Z_{\mu} Z^{\mu} + \frac{1}{2} m_Z'^2 Z'^{\mu} Z'^{\mu} - m_\chi \bar{\chi} \chi,$$

(11)

where the first, the second, and the third lines represent the kinetic terms, the coupling terms, and the mass terms respectively. Since the kinetic mixing term comes from loop corrections, this Lagrangian is not UV complete. The possible UV complete models can be seen in Sec. III.

To normalize the kinetic terms, we can apply the following transformation:

$$\begin{pmatrix} Z_{\mu} \\ Z'^{\mu} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{1 - \epsilon^2} & \sqrt{1 + \epsilon^2} \\ \sqrt{1 + \epsilon^2} & \sqrt{1 - \epsilon^2} \end{pmatrix} \begin{pmatrix} \tilde{Z}_{\mu} \\ \tilde{Z}'^\mu \end{pmatrix}.$$

(12)

After this operation there will be a mixing term in the mass matrix of $\tilde{Z}$ boson and $\tilde{Z}'$ boson. After diagonalizing these bosons to their mass eigenstates, one can prove the cancellation in the usual way. See Appendix A for more details.

We implemented this model in FeynRules 2 [18], and utilized the MadGraph [19] plugin MadDM [20] to calculate the DM relic density. The results are shown in Fig. 4, where the lines are contours that saturate the Planck experiment [21] observation of DM relic density. There are four free parameters in the DPDM model, which are $m_\chi$, $\epsilon$, $g_\chi$ and $m_{\tilde{Z}}$ (the mass of mass eigenstates of $Z'$), with the measured SM Z boson mass being an input parameter. We adopt the DM mass $m_\chi$ vs the kinetic mixing coupling $\epsilon$ and the gauge coupling $g_\chi$ in the left and right panel of Fig. 4 respectively. In the left panel of Fig. 4 we fix $g_\chi = 0.1$ and use green line and blue line to denote cases where $m_{\tilde{Z}} = 1000$ GeV and $m_{\tilde{Z}} = 2000$ GeV respectively. In the right panel of Fig. 4 we fix $m_{\tilde{Z}} = 1000$ GeV and use green line and blue line to denote cases where $\epsilon = 0.01$ and $\epsilon = 0.1$ respectively. Area below the lines is parameter space where DM relic density is larger than the Planck experiment observation, and thus these area is excluded by the Planck experiment. There are dips which correspond to the resonant annihilation that happens when DM is around half of $m_{\tilde{Z}}$. There are also dips around $m_{\tilde{Z}}$ which are caused by DM coannihilating with the dark mediator $Z'$.

From Fig. 4 we see that the kinetic mixing coupling should be in the order similar or larger than $O(0.01)$ to not being constrained severely by DM relic density. Larger $\epsilon$ or $g_\chi$ will result in larger relic density since larger kinetic mixing or coupling represents larger interaction and hence leads to larger annihilation cross section of DM. When the DM mass is around half of the mass of the dark vector boson or is about the same as the mass of the dark vector boson, there will be resonant annihilation or coannihilation which will strongly enhance the annihilation cross section of DM. Since the mass of both DM and the dark vector boson are free parameters, this leaves large parameters space for future phenomenology studies.

V. CONCLUSION AND DISCUSSION

In this work we have proposed a new kind of DM model where DM interacts with the SM fermions through the derivative portal. We have proved that in this kind of model the scattering amplitude between DM and the SM fermions is proportional to the momentum transfer, therefore the DM direct detection goes to zero in the zero momentum transfer limit. We have also studied the DM
relic density predicted by the DPDM model. Possible UV completions of this kind of model are also discussed.

In this work we focused on the framework of the DPDM model, while the derivative portal can link to a vast variety of DM models. For example, one can easily implement UV complete DPDM models with two extra massive U(1) gauge bosons by coupling these two vector bosons to a same heavy particle. Though it is not easy to do the same for the SM Z boson and an extra vector boson, in Sec. III we adopt neutrino to build the derivative portal and presented two possible UV complete models. In these constructions, the dark sector is not deeply involved, and a mirror world might be preferred in the dark sector of these UV complete DPDM models [22, 23].

Also there are lots of works can be done in the future: the possible UV completion model in Sec. III might be able to give mass to neutrinos; the derivative portal might come from scalars in UV complete theory. Phenomenology studies like electroweak oblique parameters constraints, and collider search can be explored in the future.

Appendix A: Proof of cancellation mechanism in mass eigenstates

In the proof of cancellation mechanism in the DPDM model, the mass term of the DM is irrelevant. Therefore the relevant Lagrangian reads:

\[
\mathcal{L} = -\frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} - \frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} - \frac{\alpha}{2} Z^{\mu\nu} Z'^{\mu\nu}
\]

(A1)

After the following transformation

\[
\begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \sqrt{1-\epsilon} \\ \epsilon & \sqrt{1+\epsilon} \end{pmatrix} \begin{pmatrix} \tilde{Z}_\mu \\ \tilde{Z}'_\mu \end{pmatrix}
\]

(A2)

the kinetic terms are normalized and the Lagrangian becomes

\[
\mathcal{L} = -\frac{1}{4} \tilde{Z}^{\mu\nu} \tilde{Z}_{\mu\nu} - \frac{1}{4} \tilde{Z}'^{\mu\nu} \tilde{Z}'_{\mu\nu} + g_\chi (k_1 \tilde{Z}_\mu + k_2 \tilde{Z}'_\mu) \tilde{\chi}^\gamma \chi
\]

(A3)

\[
+ \sum_f (k_1 \tilde{Z}_\mu + k_2 \tilde{Z}'_\mu) \tilde{f} \gamma^\mu (g_V - g_A \gamma^5) f
\]

\[
+ \frac{1}{2} m_\chi^2 (k_1 \tilde{Z}_\mu + k_2 \tilde{Z}'_\mu)^2 + \frac{1}{2} m_\chi^2 (k_1 \tilde{Z}_\mu + k_2 \tilde{Z}'_\mu)^2,
\]

where \( k_1 = 1/\sqrt{2-2\epsilon} \) and \( k_2 = 1/\sqrt{2+2\epsilon} \). Then the mass matrix of the vector mediators can be written as:

\[
\frac{1}{2} \begin{pmatrix} \tilde{Z}_\mu & \tilde{Z}'_\mu \end{pmatrix} \begin{pmatrix} m_2^2 & 0 \\ 0 & m_2'^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}_\mu \\ \tilde{Z}'_\mu \end{pmatrix} = \frac{1}{2} \begin{pmatrix} m_2^2 Z_\mu \\ 0 m_2'^2 Z'_\mu \end{pmatrix}
\]

(A4)

where we have defined \( M_1 = m_2^2 + m_2'^2 \), \( M_2 = m_2'^2 - m_2^2 \), and \( O \) is an orthogonal matrix that diagonalizes the mass matrix. \( O \) can be defined as:

\[
O = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},
\]

(A5)
with \( \tan 2\theta \) being formulated as:

\[
\tan 2\theta = \frac{2k_1k_2M_2}{(k_2^2 - k_1^2)M_1}. \tag{A6}
\]

After diagonalization, the Lagrangian becomes

\[
\mathcal{L} = -\frac{1}{4} \hat{Z}^{\mu\nu} \hat{Z}_{\mu\nu} - \frac{1}{4} \hat{Z}^{\mu\nu} \hat{Z}_{\mu\nu} + \frac{1}{2} m_2 Z_\mu^2 + \frac{1}{2} m_2 \hat{Z}_\mu^2 + \sum_l ((-k_2 \sin \theta - k_1 \cos \theta) \hat{Z}_\mu + (-k_1 \sin \theta + k_2 \cos \theta) \hat{Z}_\mu) \\
+ \hat{f} \gamma^\mu (g_V - g_A \gamma^5) f \\
+ g_\chi ((k_1 \cos \theta - k_2 \sin \theta) \hat{Z}_\mu + (k_2 \cos \theta + k_1 \sin \theta) \hat{Z}_\mu) \hat{\chi} \gamma^\mu \chi.
\]

With this Lagrangian we can write the scattering amplitude between the SM fermions and DM as:

\[
i\mathcal{M} = (-i)^2 \tilde{u}(p_3)(-\gamma^\mu (g_V - g_A \gamma^5))u(p_1) \\
\left( -ig_{\mu\nu}(-k_2 \sin \theta - k_1 \cos \theta)(k_1 \cos \theta - k_2 \sin \theta) \frac{1}{t - m_2^2} \\
+ ig_{\mu\nu}(-k_1 \sin \theta + k_2 \cos \theta)(k_2 \cos \theta + k_1 \sin \theta) \frac{1}{t - m_2^2} \right) \\
\tilde{u}(p_4)(-g_\chi \gamma^\nu)u(p_2) \\
\propto \left( \frac{(-k_2 \sin \theta - k_1 \cos \theta)(k_1 \cos \theta - k_2 \sin \theta)}{t - m_2^2} \\
+ \frac{(-k_1 \sin \theta + k_2 \cos \theta)(k_2 \cos \theta + k_1 \sin \theta)}{t - m_2^2} \right) \frac{1}{t(...)} \\
\frac{1}{(t - m_2^2)(t - m_2^2)} \\
\frac{m_2^2 (k_2^2 \sin^2 \theta - k_1^2 \cos^2 \theta) + m_2^2 (k_2^2 \cos^2 \theta - k_1^2 \sin^2 \theta)}{(t - m_2^2)(t - m_2^2)}. \tag{A7}
\]

In the result of the above equation we have extracted the key structure of the cancellation mechanism. If the amplitude is proportional to the momentum transfer \( t \), then it goes to zero in the zero momentum transfer limit. Thus the cancellation is valid when the last line of the above equation equals to zero. Which means:

\[
\frac{m_2^2 (k_2^2 \sin^2 \theta - k_1^2 \cos^2 \theta) + m_2^2 (k_2^2 \cos^2 \theta - k_1^2 \sin^2 \theta)}{(t - m_2^2)(t - m_2^2)} = 0.
\]

This equation is equivalent to

\[
\frac{k_2^2 \sin^2 \theta - k_1^2 \cos^2 \theta}{k_2^2 \cos^2 \theta - k_1^2 \sin^2 \theta} = -\frac{m_2^2}{m_2'}, \tag{A9}
\]

and we will prove that this equation is true. From Eq. (A4) we can write:

\[
\frac{-m_2^2}{m_2'} = \frac{k_1 k_2 M_2 \cos^2 \theta - k_1^2 M_1 \sin \theta \cos \theta}{k_1 k_2 M_2 \sin^2 \theta + k_2^2 M_1 \sin \theta \cos \theta}. \tag{A10}
\]

After replacing \( k_1 k_2 M_2 \) with \((k_2^2 - k_1^2) \tan 2\theta / 2\) and simplification, we have:

\[
\frac{-m_2^2}{m_2'} = \frac{k_2^2 \tan^2 \theta - k_1^2}{k_2^2 - k_1^2 \tan^2 \theta} = \frac{k_2^2 \sin^2 \theta - k_1^2 \cos^2 \theta}{k_2^2 \cos^2 \theta - k_1^2 \sin^2 \theta}. \tag{A11}
\]

This means the amplitude is truly proportional to the momentum transfer, and we see that the DPDM model do possess the cancellation mechanism.

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NOTE ADDED

Recently the CDF Collaboration has measured the \( W \) boson mass being \( 7\sigma \) level deviation from the SM prediction [24]. Interestingly, the DPDM model proposed in this paper can explain both the \( W \) boson mass anomaly and DM nicely [25].

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