Research on infrared passive ranging algorithm based on unscented Kalman filter and modified spherical coordinates

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Abstract. Target range detection is one of the key technologies for intelligent unmanned vehicle to work reliably and efficiently. Infrared passive ranging can estimate the relative distance information of the target in real time by measuring the angle information with noise. It has many advantages such as low cost, good real-time performance and so on, which has a good application prospect in the range detection. In this paper, the mathematical model of infrared passive ranging of intelligent unmanned vehicle based on modified spherical coordinates (MSC) is established, and the algorithm flow based on unscented Kalman filter (UKF) is given. Finally, the simulation experiment and analysis are carried out. The results show that: in the problem of target range detection of intelligent unmanned vehicle, the method adopted has high ranging accuracy and good stability.

1. Introduction

With the rapid development of artificial intelligence, all kinds of intelligent unmanned vehicles, such as driverless cars, intelligent robots, unmanned aerial vehicles, are used more and more widely in human society [1]. The range detection of the target object is one of the key technologies to ensure that the intelligent unmanned vehicle has a safe track and works reliably and efficiently. Range detection technology can be divided into active ranging and passive ranging. The former is to detect the target by transmitting high-power signals, and determine the target range by analyzing the information. The detection accuracy is high, but the detection range is limited and the cost is high [2]. The latter is to determine the target range by detecting the radiation information, angle information or other information of the target itself and analyzing it. This technology does not need to transmit detection signals, and can omit high-cost transmitting units. It has a high cost performance ratio, and compared with active ranging technology, passive ranging technology has a farther detection capability [3].

As an important passive ranging technology, infrared detection estimates the target range by measuring the angle information with noise. Because of its low cost and good real-time performance, it has a good application prospect in the target range detection of intelligent unmanned vehicle [4]. But for infrared passive ranging, the relative motion equation of the target is nonlinear. In dealing with this kind of nonlinear filtering problem, extended Kalman filter (EKF) is a classical and widely used method. In EKF, the nonlinear function is approximated by linearization, while higher-order terms are
ignored or approximated to solve the nonlinear problem. Although the application of EKF to the state estimation of nonlinear systems has been recognized by the academic and engineering circles, it has the following shortcomings [5]:

1) When the higher-order terms of Taylor expansion of nonlinear function cannot be ignored, linearization will cause large errors and even divergence of the system;

2) In many practical problems, it is difficult to obtain the derivation of Jacobian matrix for nonlinear function;

3) EKF needs to take derivatives, so it is necessary to clearly understand the specific form of nonlinear functions, which cannot be packaged in black boxes, so it is difficult for modular application.

In order to improve the filtering effect of nonlinear problems, Julier et al. proposed UKF method based on unscented transformation (U-transform) [6]. In order to reduce the estimation error, the U-transform is used to process the state equation, and then the U-transform state quantity is used to estimate the filter. Its computational complexity is of the same order as EKF, but it is easier to realize, with higher accuracy and faster convergence [7].

In this paper, UKF is introduced to study the problem of infrared passive ranging, and MSC is used to overcome the problem of poor filtering stability in rectangular coordinate system. The following arrangement of the paper is as follows: firstly, the mathematical model of infrared passive ranging of intelligent unmanned vehicle is established based on MSC, then the basic algorithm flow of UKF is introduced briefly, and finally the simulation experiment of infrared passive ranging based on UKF/MSC is carried out.

2. Mathematical model of infrared passive ranging
The research shows that the filtering stability in rectangular coordinate system is poor, sometimes even divergent [8]. In order to overcome this problem, MSC is used in this paper. As shown in Figure 1, the infrared detector is located at the coordinate origin O, the target is located at \((x, y, z)\). The distance between the target and the infrared detector is \(r\), and the projection of \(r\) on the \(X-Y\) plane is \(r_h\), the angle between \(r_h\) and \(x\) axis is the target azimuth \(\theta\), and the angle between \(r_h\) and \(r\) is the target pitch \(\varphi\). The target acceleration components \(a_{R_x}\), \(a_{H_y}\) and \(a_{V_z}\) are along the three axis directions of the antenna coordinate system respectively, specifically: \(a_{R_x}\) is along the \(r\) direction, \(a_{H_y}\) is in the horizontal plane and parallel to \(\varphi\). The acceleration components of the infrared detector on the three axes of the antenna coordinate system are \(a_{R_0}\), \(a_{H_0}\) and \(a_{V_0}\), respectively.

Define the state vector as:

\[
X = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [\varphi, \dot{\varphi}, \theta, \dot{\theta}, \frac{r}{r}, \frac{1}{r}]^T
\]

where, \(\omega = \dot{\theta} \cdot \cos \varphi\).
According to Figure 1, it can get:
\[ x = r \cos \phi \cos \theta \]
\[ y = r \cos \phi \sin \theta \]
\[ z = r \sin \phi \]  
(2)

The second derivative of equation (2) can be obtained:
\[ \dot{x} = \dot{r} \cos \phi \cos \theta - 2 r \dot{\phi} \sin \phi \cos \theta - 2 r \dot{\theta} \cos \phi \sin \theta \]
\[ - r \ddot{\phi} \cos \phi \cos \theta - r \dot{\phi}^2 \cos \phi \cos \theta + 2 r \ddot{\theta} \sin \phi \sin \theta \]
\[ - r \ddot{\theta} \cos \phi \sin \theta - r \dot{\theta}^2 \cos \phi \cos \theta \]
\[ \ddot{y} = \dot{r} \cos \phi \sin \theta - 2 r \dot{\phi} \sin \phi \sin \theta + 2 r \ddot{\theta} \cos \phi \cos \theta \]
\[ - r \ddot{\phi} \sin \phi \sin \theta - r \dot{\phi}^2 \sin \phi \cos \theta - 2 r \ddot{\theta} \cos \phi \sin \theta \]
\[ + r \ddot{\theta} \cos \phi \cos \theta - r \dot{\theta}^2 \cos \phi \cos \theta \]
\[ \ddot{z} = \dot{r} \sin \phi + 2 r \ddot{\phi} \cos \phi + r \dot{\phi} \cos \phi - r \dot{\phi}^2 \sin \phi \]  
(3)

The coordinate rotation matrix from rectangular coordinate system to antenna coordinate system can be expressed as:
\[
\begin{bmatrix}
 a_{R_t} - a_{R_o} \\
 a_{H_t} - a_{H_o} \\
 a_{V_t} - a_{V_o}
\end{bmatrix} = \begin{bmatrix}
 \cos \phi \cos \theta & \cos \phi \sin \theta & \sin \phi \\
 -\sin \theta & \cos \theta & 0 \\
 -\sin \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi
\end{bmatrix} \begin{bmatrix}
 \dot{x} \\
 \dot{y} \\
 \dot{z}
\end{bmatrix}
\]  
(4)

Substituting the expression of \( x, y, z \) in equation (3) into equation (4), we can get:
\[ a_{R_t} - a_{R_o} = \ddot{r} - r \dot{\phi}^2 - r \omega^2 \]
\[ a_{H_t} - a_{H_o} = 2r \omega - r \dot{\phi} \omega \tan \phi + r \dot{\omega} \]
\[ a_{V_t} - a_{V_o} = 2r \ddot{\phi} + r \dot{\phi} \omega \tan \phi \]  
(5)

therefore:
\[ \ddot{r} = \dot{\phi}^2 + \omega^2 + \frac{a_{R_t} - a_{R_o}}{r} \]
\[ \ddot{\phi} = \dot{\phi} \omega \tan \phi - \ddot{r} \omega + \frac{a_{H_t} - a_{H_o}}{r} \]
\[ \dot{\omega} = -2 \ddot{r} \tan \phi + \frac{a_{V_t} - a_{V_o}}{r} \]  
(6)

According to equation (1) and equation (6), the equation of motion state is as follows:
\[
\dot{X} = \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4 \\
 x_5 \\
 x_6
\end{bmatrix} = \begin{bmatrix}
 x_2 \\
 -2x_2 x_5 - x_4^2 \tan x_1 + x_6 (a_{V_t} - a_{V_o}) \\
 x_4 \sec x_1 \\
 -2x_4 x_5 + x_2 x_4 \tan x_1 + x_6 (a_{H_t} - a_{H_o}) \\
 x_2^2 + x_4^2 - x_5^2 + x_6 (a_{R_t} - a_{R_o}) \\
 -x_5 x_6
\end{bmatrix}
\]  
(7)

In practical application, because the acceleration of the target is unknown, \( a_{R_t}, a_{H_t}, a_{V_t} \) are generally treated as process noise.

3. The Unscented Kalman Filter
For discrete-time nonlinear system:
\[ x_{k+1} = F(x_k, u_k, w_k) \]
\[ z_k = H(x_k, v_k) \]
where, \( k \in N \) is the time indicator, \( x_i \in R^n \) is the state quantity at time \( k \), \( w_k \) is the \( n \) -dimension process noise, \( z_i \in R^m \) is the measurement value of system state at time \( k \), and \( v_k \) is the \( m \) -dimension measurement noise.

Initially, process noise and measurement noise are independent of each other and are satisfied:
\[
E(x_i) = \bar{x}_i, \quad \text{cov}(x_i) = P_i
\]
\[
\text{cov}(w) = Q, \quad \text{cov}(v) = R
\]

Then the general steps of UKF algorithm can be described as follows:

1. Expand the dimension of the state vector, and let
\[
\begin{bmatrix}
    x^T \\
    v^T
\end{bmatrix} = \begin{bmatrix}
    x^T \\
    0
\end{bmatrix}
\]
\[
P_k^x = E((x_i - \bar{x}_i)(x_i - \bar{x}_i)^T) =
\begin{bmatrix}
    P_i & 0 & 0 \\
    0 & Q & 0 \\
    0 & 0 & R
\end{bmatrix}
\]

2. Initial conditions
\[
\begin{bmatrix}
    x^T_0 \\
    v^T_0
\end{bmatrix} = \begin{bmatrix}
    x^T_0 \\
    0
\end{bmatrix}
\]

3. For \( k \in \{1, \cdots, N\} \)
\[
x^T_k = [\bar{x}_{i,k-1} \bar{z}_{i,k-1} (\sqrt{(L+\lambda)P_{z,i,k-1}}) \bar{z}_{i,k-1} - (\sqrt{(L+\lambda)P_{z,i,k-1}})]
\]

where, \((\sqrt{(L+\lambda)P_{z,i,k-1}})\) represents the \( j \) column of the square root of matrix.

4. Time update
\[
\bar{x}_i = \sum_{i=0}^{2\lambda} \phi^{(\alpha)} \xi_{i,k} - \bar{x}_k
\]
\[
\bar{z}_i = \sum_{i=0}^{2\lambda} \phi^{(\beta)} \xi_{i,k} - \bar{z}_k
\]
\[
P = \sum_{i=0}^{2\lambda} \phi^{(\gamma)} \xi_{i,k} - \bar{z}_i
\]
\[
\bar{z}_i = \sum_{i=0}^{2\lambda} \phi^{(\zeta)} \xi_{i,k} - \bar{z}_k
\]

5. Measurement update
\[
P_{z,i} = \sum_{i=0}^{2\lambda} \phi^{(\alpha)} \xi_{i,k} - \bar{z}_i
\]
\[
P_{z,i} = \sum_{i=0}^{2\lambda} \phi^{(\beta)} \xi_{i,k} - \bar{z}_i
\]
\[
K_i = P_{z,i}^{-1} P_{\alpha}^{-1}
\]
\[
\bar{x}_k = \bar{x}_k + K(z_i - \bar{z}_i)
\]
\[
P_k = P_k - K \bar{x}_k P_{z,k} K^T
\]

where, \( \lambda = \alpha^2(n+\kappa) - n \) is the scaling factor, \( \phi^{(\alpha)} \) is the weight used for mean weighting, and \( \phi^{(\gamma)} \) is the weight used for covariance weighting.

4. Results and discussion

Generally speaking, for the target with low mobility, the 3-dimensional motion can be divided into two 2-dimensional motions, namely \( r-\theta \) 2-dimensional motion and \( r-\theta \) 2-dimensional motion, so as to reduce the computation. Because of the similarity of two 2-dimensional motions, it is enough to select a 2-dimensional motion for simulation experiment. Next, take \( r-\theta \) 2-dimensional motion as an example for simulation experiment.

The state quantity of the target is:
\[
X = [x_1, x_2, x_3, u_4]^T = [\theta, \phi, \frac{\dot{r}}{r}, \frac{\dot{\theta}}{r}, 1]^T
\]

Then, the equation of nonlinear continuous motion can be obtained from equation (7):
\[
\dot{X} = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix} = \begin{bmatrix}
x_2 \\
-2x_2x_3 \\
x_3^2 - x_3^2 \\
-x_3x_4 \\
\end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_{R_T} - \alpha_{H_T} \\ \alpha_{R_T} - \alpha_{R_O} \\ 0 \\
\end{bmatrix}
\]

If the infrared detector moves at a constant speed, i.e. \( \alpha_{H_O} = \alpha_{R_O} = 0 \), the discrete motion equation is:

\[
X(k+1) = X(k) + T \begin{bmatrix}
x_2(k) \\
-2x_2(k)x_3(k) \\
x_3^2(k) - x_3^2(k) \\
-x_3(k)x_4(k) \\
\end{bmatrix} + T \begin{bmatrix} 0 & 0 & x_4(k) & 0 \\ 0 & 0 & 0 & \alpha_{R_T} \\
\end{bmatrix}
\]

Where, \( w(k) = [a_{R_T}, a_{H_T}]^T \) is the process noise.

The system measurement equation is:

\[
Z(k) = HX(k) + \nu(k)
\]

Where, \( \nu(k) \) is the measurement noise, and \( \nu(k) \) is a Gaussian white noise sequence independent of \( w(k) \). \( H = [1,0,0,0] \) is the measurement matrix.

The conditions of simulation as follows: target initial position is \([x_0, y_0] = [9000m, 0m] \), its speed is \( v = 500m/s \), and speed direction is \( \theta \). Then the target's trajectory is shown in Figure 2:

**Figure 2.** The target's trajectory.

The simulation parameters are set as follows: the standard deviation of process noise are \( \sigma_{R_T} = \sigma_{H_T} = 0.5m/s^2 \), the standard deviation of measurement noise is \( \sigma_{\theta} = 0.001rad \), the simulation cycle is \( T = 1s \), the simulation time is 200s, the covariance matrix for the initial state is \( P_0 = \text{diag}(1^2, 0.01^2, 0.01^2, 0.0001^2) \). The angle estimation error curve, distance estimation error curve, distance estimation relative error curve and distance tracking curve are shown in Figure 3~6.

**Figure 3.** The angle estimation error curve.  **Figure 4.** The distance estimation error curve.
According to the simulation results, the relative distance error after filtering convergence is better than 1%, and the ranging accuracy is high. Moreover, in many simulation experiments, the filter has good convergence and stability.

5. Conclusions
In this paper, the infrared passive ranging algorithm based on UKF/MSC is discussed in detail and verified by simulation. Simulation results show that the algorithm has high ranging accuracy and good stability for low maneuvering moving targets. However, because the motion equation of the target in MSC is generally a complex nonlinear equation, the passive ranging effect of the high maneuvering target is not very ideal. In addition, for the passive detection using only angle information, there are inevitable nonlinear problems, and the amount of information available is small, so the convergence speed of the filter is slow. These problems need further study.

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