Weyl symmetric Abelian Decomposition and Monopole Condensation in SU(3) QCD

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We generalize the previous result of SU(2) QCD and demonstrate the monopole condensation in SU(3) QCD. We present the gauge independent and Weyl symmetric Abelian (Cho-Duan-Ge) decomposition of the SU(3) QCD, and obtain an infra-red finite and gauge invariant integral expression of the one-loop effective action. Integrating it gauge invariantly imposing the color reflection invariance (“the C-projection”) we show that the effective potential generates the stable monopole condensation which generates the mass gap.

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I. INTRODUCTION

One of the most challenging problems in theoretical physics is the confinement problem in quantum chromodynamics (QCD). The outstanding conjecture of the confinement mechanism is the monopole condensation [1–3]. It has long been argued that the confinement in QCD can be triggered by the monopole condensation. Indeed, if one assumes the monopole condensation, one can easily argue that the ensuing dual Meissner effect could guarantee the color confinement. Proving the monopole condensation, however, has been extremely difficult.

A natural way to establish the monopole condensation in QCD is to show that the quantum fluctuation triggers a phase transition similar to the dimensional transmutation observed in massless scalar QED [6]. There have been many attempts to demonstrate this. Savvidy has first calculated the effective action of SU(2) QCD integrating out the colored gluons in the presence of an ad hoc color magnetic background, and has almost “proved” the magnetic condensation which is known as the Savvidy vacuum [7].

Unfortunately, the subsequent calculation repeated by Nielsen and Olesen showed that the effective action has an extra imaginary part which destabilizes the Savvidy vacuum. This is known as the “Savvidy-Nielsen-Olesen (SNO) instability” [8–10]. The origin of this instability can be traced to the tachyonic modes in the gluon functional determinant, and how to justify that.

We emphasize, however, that the most serious defect of the SNO vacuum is not that it is unstable but that it is not gauge invariant. So even if the Savvidy vacuum were made stable, it can not be the QCD vacuum. Because of this Nielsen and Olesen has proposed the so-called “Copenhagen vacuum”, the randomly oriented piecewise Savvidy vacuum [8]. But one can not obtain a gauge invariant vacuum simply by randomly orienting something which is not gauge invariant.

The gauge independent Abelian decomposition known as the Cho-Duan-Ge (CDG) decomposition can cure these defects. First, it tells that the Abelian potential is made of two parts, the non-topological (Maxwellian) Abelian part and the topological (Diracian) monopole part [2, 3]. Moreover it tells that only the Diracian background is gauge invariant [13, 14]. This means that there are actually two possible magnetic backgrounds, the Maxwellian background and Diracian background. More importantly this means that we must choose the Diracian background to calculate the QCD effective action.

The gauge independent Abelian decomposition also plays the crucial role to cure the SNO instability. It tells that the decomposition has the color reflection invariance as a discrete symmetry, so that we have to integrate the colored gluons imposing this color reflection when we calculate the QCD effective action [13, 14]. And this assures the stable monopole condensation.

The fact that the monopole should play the crucial role in the color confinement has been well established string theory we have the tachyonic vacuum when we do not make the theory supersymmetric and modular invariant with the Gliozzi-Scherk-Olive (GSO) projection [11, 12]. The question here is how to remove the tachyonic modes in the gluon functional determinant, and how to justify that.

We emphasize, however, that the most serious defect of the SNO vacuum is not that it is unstable but that it is not gauge invariant. So even if the Savvidy vacuum were made stable, it can not be the QCD vacuum. Because of this Nielsen and Olesen has proposed the so-called “Copenhagen vacuum”, the randomly oriented piecewise Savvidy vacuum [8]. But one can not obtain a gauge invariant vacuum simply by randomly orienting something which is not gauge invariant.

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The fact that the monopole should play the crucial role in the color confinement has been well established
by now. First, using the CDG decomposition one can prove that the Abelian part of the potential is responsible for the confining force in Wilson loop [15]. Of course, this establishes only the Abelian dominance, this strongly implies the monopole dominance because the Maxwellian Abelian part is known to play no role in Wilson loop.

In fact, implementing the gauge independent CDG decomposition on the lattice, the KEK-Chiba and the SNU-Konkuk Lattice Collaborations independently have confirmed that the confining force comes from the monopole part of the Abelian projection numerically [16] [17]. The SNU-Konkuk result of SU(3) QCD is shown in Fig 1, where the slope of the lines represents the string tension of the Wilson loop. Clearly all three potentials, the full potential, the Abelian potential, and the monopole potential generate the same confining force. This confirms that the monopole plays the crucial role in the confinement. The importance of these lattice results is that they are the first lattice calculations which demonstrate that the essential features of SU(2) QCD, the dimensional transmutation by the monopole condensation which generates the mass gap remains the same. Finally in section VII we discuss the physical implications of our result.

The paper is organized as follows. In section II we review the restricted QCD (RCD), the extended QCD (ECD), and the Abelianized QCD (ACD) in SU(2) for later purpose. In section III we discuss the color reflection invariance which plays the crucial role in QCD. In section IV we repeat the calculation of the one-loop effective action of SU(2) QCD which plays the essential role for the SU(3) QCD. In section V we show how to generalize the Abelian decomposition to SU(3), and obtain the Weyl symmetric RCD, ECD, and ACD. In section VI we calculate SU(3) QCD effective action and demonstrate that the monopole condensation becomes the Weyl symmetric vacuum in SU(3) QCD. In particular we show that the essential features of SU(2) QCD, the dimensional transmutation by the monopole condensation which generates the mass gap remains the same. Finally in section VII we discuss the physical implications of our result.

II. RCD, ECD, AND ACD: A REVIEW

To prove the monopole condensation in SU(3) QCD it is crucial to understand the SU(2) QCD first. So we start from the SU(2) QCD, and review the gauge independent Abelian decomposition first. Let $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$ be a righthanded orthonormal basis and choose $\hat{n} = \hat{n}_3$ to be the Abelian direction. Impose the magnetic isometry to the gauge potential $\hat{A}_\mu$ to make the Abelian projection

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g \hat{A}_\mu \times \hat{n} = 0,$$

$$\hat{A}_\mu \rightarrow \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} = A_\mu + C_\mu,$$

$$A_\mu = A_\mu \hat{n}, \quad C_\mu = -\frac{1}{g} \hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \hat{A}_\mu. \tag{1}$$

Notice that $C_\mu$ describes the Wu-Yang monopole when $\hat{n} = \hat{r}$ [19] [20]. This tells that the potential $\hat{A}_\mu$ which leaves $\hat{n}$ invariant under the parallel transport is made of the “naive” (non-topological) Abelian part $A_\mu$ and the topological monopole part $C_\mu$.

To understand the meaning of this dual structure of $\hat{A}_\mu$, notice that

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g \hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu}) \hat{n},$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$H_{\mu\nu} = \frac{1}{g} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) = \partial_\mu C_\nu - \partial_\nu C_\mu,$$

$$C_\mu = -\frac{1}{g} \hat{n}_1 \cdot \partial_\mu \hat{n}_2. \tag{2}$$

where $C_\mu$ becomes exactly the Dirac’s monopole potential [2] [3]. This tells that $A_\mu$ and $C_\mu$ (or equivalently $A_\mu$ and $C_\mu$) represent the non-topological Maxwellian “electric” potential and the topological Diracian “magnetic” potential.

With the Abelian projection we have the Abelian decomposition known as the Cho-Duan-Ge (CDG) decomposition or the Cho-Faddeev-Niemi (CFN) decomposi-
ticted QCD (RCD) which describes the Abelian sub-
composition uniquely defines \( \hat{X} \) which doubles the gauge symmetry \([24, 25]\). This is because the decomposition (3) contains the non-Abelian topology of QCD, and contains the full non-Abelian gauge symmetry because \( \hat{n} \) has the full gauge freedom. In fact, ECD and ACD have not only the classical gauge symmetry but also the quantum gauge symmetry \([18, 26]\).

\[ \hat{D}_\mu X_\nu = (\partial_\mu + igB_\mu)X_\nu, \]
\[ B_\mu = A_\mu + C_\mu, \quad X_\mu = \frac{1}{\sqrt{2}}(X_\mu^1 + iX_\mu^2). \]

Formally this is what we can obtain from the Yang-Mills Lagrangian with \( \hat{A}_\mu = (X_\mu^1, X_\mu^2, B_\mu) \). But this is a gauge dependent Abelianization. In comparison, \( \hat{A}_\mu \) is gauge independent, because here we have never fixed the gauge to obtain this Lagrangian.

Under the infinitesimal gauge transformation we have
\[ \delta A_\mu = \frac{1}{g} \hat{n} \cdot \partial_\mu \hat{\alpha}, \quad \delta \hat{X}_\mu = \frac{1}{g} \hat{D}_\mu \hat{\alpha}, \]
\[ \delta \hat{X}_\mu = -\hat{\alpha} \times \hat{X}_\mu. \]

So \( \hat{A}_\mu \) by itself describes an SU(2) connection which enjoys the full SU(2) gauge degrees of freedom. Moreover, \( \hat{X}_\mu \) becomes gauge covariant. Most importantly, the decomposition is gauge independent. Once \( \hat{n} \) is given, the decomposition uniquely defines \( \hat{A}_\mu \) and \( \hat{X}_\mu \), independent of the choice of gauge \([2, 3]\).

With the Abelian decomposition we obtain the restricted QCD (RCD) which describes the Abelian sub-dynamics of QCD \([2, 3]\).

\[ \mathcal{L}_{RCD} = -\frac{1}{4} \hat{F}^2_\mu = -\frac{1}{4} \hat{F}^2_\mu + \frac{1}{2g} F_{\mu\nu} \hat{n} \times (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) - \frac{1}{4g^2} (\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2. \]

It has the full SU(2) gauge freedom, in spite of the fact that it is simpler than QCD. Because of this it retains the non-Abelian topology of QCD, and contains the monopole degrees explicitly. This makes RCD an ideal platform for us to discuss the monopole dynamics gauge independently.

Furthermore, with \([3]\) we have the extended QCD (ECD),

\[ \mathcal{L}_{ECD} = -\frac{1}{4} F^2_\mu = -\frac{1}{4} \hat{F}^2_\mu - \frac{1}{4} (\hat{D}_\mu \hat{X}_\nu - \hat{D}_\nu \hat{X}_\mu)^2 \]
\[ -\frac{g}{2} F_{\mu\nu} \hat{n} \times (\hat{X}_\mu \times \hat{X}_\nu) - \frac{g^2}{4} (\hat{X}_\mu \times \hat{X}_\nu)^2. \]

This shows that QCD can be viewed as RCD made of the binding gluons, which has the colored valence gluons as its source \([2, 3]\). Notice, however, that ECD has more gauge symmetry: In addition to the classical (slow) gauge symmetry of QCD, it has the extra quantum (fast) gauge symmetry. This is because the decomposition \([3]\) automatically put \([6]\) to the background field formalism which doubles the gauge symmetry \([24, 25]\).

With this we can actually Abelianize ECD and have the Abelianized QCD (ACD) \([2, 3]\).

\[ \mathcal{L}_{ACD} = -\frac{1}{4} G^2_\mu = -\frac{1}{2} |\hat{D}_\mu X_\nu - \hat{D}_\nu X_\mu|^2 \]
\[ + igG_{\mu\nu} X_\mu^* X_\nu - \frac{1}{2} g^2 [(X_\mu^* X_\mu)^2 - (X_\mu^2)^2 (X_\mu^2)^2], \]
\[ G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu} + H_{\mu\nu}. \]

Obviously the physics should not change under this change of basis. On the other hand, the isometry condition \([1]\) is insensitive to this change. So we have two different Abelian decompositions imposing the same isometry using two different bases, without changing the physics. This tells that the color reflection (8) which originally was introduced as a gauge transformation now becomes a discrete symmetry of RCD, ECD, and ACD, after the Abelian decomposition \([2, 3]\).

To amplify this notice that, under the color reflection we have
\[ \hat{A}_\mu \rightarrow \hat{A}_\mu^{(c)} = -A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} = -A_\mu + C_\mu, \]
\[ A_\mu \rightarrow A_\mu^{(c)} = -A_\mu, \quad C_\mu \rightarrow C_\mu^{(c)} = C_\mu, \]
\[ X_\mu \rightarrow X_\mu^{(c)} = X_\mu^* = \frac{X_\mu^1 - iX_\mu^2}{\sqrt{2}}, \]
\[ F_{\mu\nu} \rightarrow F_{\mu\nu}^{(c)} = (-F_{\mu\nu} + H_{\mu\nu})\hat{n}. \]

Clearly the valence gluon change the color. Moreover, \( A_\mu \) and \( C_\mu \) (as well as \( A_\mu \) and \( C_\mu \)) transform oppositely under \([5]\). In particular the Diracian magnetic (topological) part \( C_\mu \) (and \( H_{\mu\nu} \)) remains invariant under the color
reflection while the Maxwellian electric (non-topological) part $A_\mu$ (and $F_{\mu\nu}$) changes the signature.

This assures that the colored objects must become unphysical, because the color reflection which changes the color of the valence gluons is a symmetry which should not change the physics. This, of course, amounts to the color confinement. So, after the Abelian decomposition the color reflection invariance plays the role of the non-Abelian gauge invariance.

As importantly this tells that in QCD the monopole is equivalent to the anti-monopole. This is because the monopole quantum number $\pi_2(S^2)$ defined by $\tilde n$ changes the signature under (8), but the magnetic potential $C_\mu$ remains unchanged. So the monopole and anti-monopole are physically undistinguishable in QCD [18 27].

This should be compared with the Maxwellian electric potential $A_\mu$ (equivalently $A_\mu$). Unlike $C_\mu$ it changes the signature. So $A_\mu$ and $C_\mu$ (equivalently $A_\mu$ and $C_\mu$) have the negative and positive color charge conjugation quantum number, respectively. Moreover, this tells that $F_{\mu\nu}$ is not color reflection invariant and thus can not be observable, while $H_{\mu\nu}$ is color reflection invariant and is qualified to be an observable.

The above analysis confirms that $\vec X_\mu$ and $A_\mu$ are not color reflection invariant (and thus can not be observables), which is not surprising. What is surprising is that the gauge potential contains a color reflection invariant (and thus observable) part $C_\mu$, and that we can separate this part gauge independently by the Abelian projection.

In the fundamental representation the color reflection group of SU(2) is the 4 element subgroup generated by

$$\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},$$

which contains the center group $Z_2$ [2 3]. But notice that it is defined up to the Abelian U(1) rotation.

The two potentials $A_\mu$ and $C_\mu$ have another important difference. Consider the space inversion $P$ (the parity)

$$\vec x \rightarrow -\vec x.$$ (10)

Under this the Maxwellian $A_\mu$ (just like the Abelian gauge potential in QED) behaves as an ordinary vector, so that it must have negative parity [28]. But under the space inversion we have

$$C_\mu = -\frac{1}{g}(1 - \cos \theta)\partial_\mu \phi \rightarrow C^{(\mu)}_\mu$$

$$= \frac{1}{g}(1 + \cos \theta)\partial_\mu \phi.$$ (11)

So simply moves the Dirac string in $C_\mu$ from the negative $z$-axis to the positive axis, which does not change the monopole physically. This means that $C_\mu$ should be interpreted as an axial vector which has positive parity [3].

From this we conclude that $J^{PC}$ of the electric potential $A_\mu$ (or $A_\mu$) becomes $1^{--}$ while $J^{PC}$ of the magnetic potential $C_\mu$ (or $C_\mu$) becomes $1^{++}$, where $C$ represents the color (not ordinary) charge conjugation number. This tells that the electric background made of $F_{\mu\nu}$ is not gauge invariant, while the monopole background made of $H_{\mu\nu}$ is so. So we must use the $CP$-invariant topological background in the calculation of the QCD effective action [18 26].

Moreover, the monopole background should really be understood as the monopole-antimonopole background, because they are gauge equivalent. These are the lessons from the above analysis which we have to keep in mind in the followings.

IV. EFFECTIVE ACTION OF SU(2) QCD

To obtain the one-loop effective action we must divide the potential to the classical and quantum parts and integrate out the quantum part in the presence of the classical background. Let us start from ACD and let the background be $B_\mu$. In this case the effective action is expressed by the gluon and ghost loop determinants given by $K$ and $M$,

$$\Delta S = i \frac{1}{2} \ln \text{Det} K - i \ln \text{Det} M,$$

$$\text{Det}^{-1/2} K_{\mu\nu} = \text{Det} \left( - g_{\mu\nu} \bar D^2 + 2i g \bar G_{\mu\nu} \right),$$

$$\text{Det} M^{1/2} = \text{Det} \left( - \bar D^2 \right).$$ (12)

where $\bar D_\mu$ is the covariant derivative defined by the classical background.

Savvidy and others chose the Savvidy background $\bar A_\mu$ [10]

$$\bar G_{\mu\nu} = \bar F_{\mu\nu}, \quad \bar F_{\mu\nu} = H \delta^{(1)}_{[\mu} \delta^2_{\nu]},$$ (13)

where $H$ is a constant chromomagnetic field of $\bar F_{\mu\nu}$ in $z$-direction. In this case the calculation of the functional determinant of the gluon loop integral amounts to finding the energy spectrum of a charged vector field moving around a constant magnetic field, which is given by [29]

$$E^2 = 2gH(n + \frac{1}{2} - q S_3) + k^2,$$ (14)

where $S_3$ and $k$ are the spin and momentum of the vector fields in the direction of the magnetic field, and $q = \pm 1$ is the charge (positive and negative) of the vector fields. This is schematically shown in Fig. 2 (A). Notice that
for both charges the energy spectrum contains negative (tachyonic) eigenvalues which violate the causality.

From (14) one has the integral expression of the effective action [7-10]

\[
\Delta \mathcal{L} = \lim_{\epsilon \to 0} \frac{\mu^2}{16\pi^2} \int_0^\infty \frac{dt}{t^{2+\epsilon}} \frac{gH}{\sinh(gHt/\mu^2)} \times \left[ \exp(-2gHt/\mu^2) + \exp(+2gHt/\mu^2) - 2 \right],
\]

where \(\mu^2\) is a dimensional parameter. Clearly the second term has a severe infra-red divergence, but this can be regularized with the standard \(\zeta\)-function regularization. With this regularization one obtains the SNO effective action [7,9]

\[
\mathcal{L}_{eff} = -\frac{H^2}{2} \left[ \frac{11g^2H^2}{48\pi^2} \ln \frac{gH}{\mu^2} - c \right] + g^2H^2 \frac{2}{8\pi},
\]

where \(c\) is an integration constant. This contains the well-known imaginary part which destabilizes the Savvidy vacuum [8]. Obviously the imaginary part originates from the tachyonic eigenstates.

In general for an arbitrary chromo-electromagnetic background \(A_\mu\) the functional determinants are given by [7,10]

\[
\ln \det K = 2 \ln \det (-\bar{D}^2 + 2a)(-\bar{D}^2 - 2a) + 2 \ln \det (-\bar{D}^2 - 2ib)(-\bar{D}^2 + 2ib),
\]

\[
\ln \det M = 2 \ln \det (-\bar{D}^2),
\]

\[
a = \frac{g}{2} \sqrt{\bar{H}^4 + (\bar{H}F)^2 + F^2},
\]

\[
b = \frac{g}{2} \sqrt{\bar{H}^4 + (\bar{H}F)^2 - F^2}.
\]

From this we have the well known expression of QCD effective action [9,10]

\[
\Delta S = i \ln \det (-\bar{D}^2 + 2a)(-\bar{D}^2 - 2a) + i \ln \det (-\bar{D}^2 - 2ib)(-\bar{D}^2 + 2ib)
\]

\[
- 2i \ln \det (-\bar{D}^2),
\]

and

\[
\Delta \mathcal{L} = \lim_{\epsilon \to 0} \frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t^{2+\epsilon}} \frac{abt^2}{\sinh(at/\mu^2)} \times \left[ \exp(-2at/\mu^2) + \exp(+2at/\mu^2) + \exp(+2ibt/\mu^2) + \exp(-2ibt/\mu^2) - 2 \right].
\]

Here the first four terms are the gluon loop contribution, but the last term comes from the ghost loop. When \(a = gH\) and \(b = 0\), this becomes identical to [15]. Notice that the second and fourth terms have a severe infra-red divergence.

There are two critical defects in the old calculations. First, the Savvidy background [13] is neither gauge invariant nor parity conserving, as we have emphasized. Second, the gauge invariance is completely overlooked in the calculation of the functional determinants. In particular, the color reflection invariance (the C-parity) is not correctly implemented in the old calculations.

To calculate the effective action correctly, we choose the monopole background \(C_\mu\) obtained from the gauge independent Abelian decomposition [3].

\[
\bar{G}_{\mu\nu} = \bar{H}_{\mu\nu}, \quad \bar{H}_{\mu\nu} = H^{\delta}_{[\mu} \delta_{\nu]}^{\delta^2},
\]

where now \(\bar{H}\) is the chromomagnetic field of \(\bar{H}_{\mu\nu}\). This should be compared with the Savvidy background [13].

To be general, however, we will let \(\bar{H}_{\mu\nu}\) arbitrary but constant and define \(a\) and \(b\) by \(\bar{H}_{\mu\nu}\)

\[
a = \frac{g}{2} \sqrt{\bar{H}^4 + (\bar{H}F)^2 + F^2},
\]

\[
b = \frac{g}{2} \sqrt{\bar{H}^4 + (\bar{H}F)^2 - F^2}.
\]

With this we can integrate out the colored gluons gauge invariantly, imposing the color reflection invariance. Consider the case \(a = gH\) and \(b = 0\) shown in Fig. 2 again. Clearly the color reflection (the C-parity) changes (A) to (B), so that they are gauge equivalent. But since this reflection does not change the spin of the valence gluons, the physical eigenvalues must be invariant under the reflection for each spin polarization separately.

Obviously the lowest two eigenvalues for both \(S_3 = +1\) in (A) and \(S_3 = -1\) in (B) do not satisfy this requirement, so that they must be discarded. This, of course,
removes the tachyonic states. This is the C-projection which restores the gauge invariance in the gluon loop integral. This neglect of gauge invariance is the critical mistake of the conventional calculations \[7\] \[10\].

Notice that the C-parity here plays exactly the same role as the G-parity in string theory. It is well known that the GSO projection (the G-projection) restores the supersymmetry and modular invariance in NSR string by projecting out the tachyonic vacuum \[11\] \[12\]. Just like the G-projection in string, the C-projection in QCD removes the tachyonic modes and restores the gauge invariance of the effective action.

Exactly the same argument applies to \( \text{Det}(-\hat{D}^2 + 2ib) \) in \[18\]. Here again they are the C-parity counterpart of each other, so that they must have exactly the same contribution. This tells that the correct effective action is given by \[18\]

\[
\Delta S = 2i \ln \text{Det} \left[ (-\hat{D}^2 + 2a)(-\hat{D}^2 - 2ib) \right] - 2i \ln \text{Det}(-\hat{D}^2),
\]

\[
\Delta \mathcal{L} = \lim_{\epsilon \to 0} \frac{1}{8\pi^2} \int_0^{\infty} dt \frac{ab}{t^{1/2}} \sinh(\epsilon a t/\mu^2) \sinh(\epsilon b t/\mu^2)
\times \left[ \exp(-2\epsilon a t/\mu^2) + \exp(+2\epsilon b t/\mu^2) - 1 \right]. \tag{22}
\]

This is the new integral expression of QCD effective action which should be compared with \[19\]. Obviously the C-projection makes \[22\] gauge invariant. As importantly, it removes the infra-red divergence of \[19\]. This tells that, we do not need the \( \zeta \)-function (or any) regularization if we calculate the effective action correctly.

At first thought this might be surprising, but actually is not so. The gauge invariance implies the confinement. This implies the generation of a mass gap, which should make the theory infra-red finite. So it is natural that the gauge invariance makes \[22\] infra-red finite.

Integrating \[22\] we have

\[
\mathcal{L}_{\text{eff}} = \begin{cases} 
\frac{a^2}{2g^2} - \frac{11a^2}{48\pi^2} \left( \ln \frac{a}{\mu^2} - c' \right), & b = 0 \\
\frac{b^2}{2g^2} + \frac{11b^2}{48\pi^2} \left( \ln \frac{b}{\mu^2} - c' \right) & \text{if } a = 0 \\
-\frac{1}{96\pi}, & \text{if } b = 0
\end{cases} \tag{23}
\]

Notice that when \( b = 0 \) the effective action has no imaginary part which destabilized the Savvidy vacuum. But when \( a = 0 \) it has a negative imaginary part, which implies the pair annihilation of gluons \[14\] \[30\]. This must be contrasted with the QED effective action where the electron loop generates a positive imaginary part \[31\] \[32\]. This difference is a direct consequence of the Bose-statistics of the gluon loop. Of course the quark loop, due to the Fermi-statistics, will generate a positive imaginary part \[13\] \[14\].

This has a very important meaning. The positive imaginary part in QED means the pair creation which generates the screening. On the other hand in QCD we must have the anti-screening to explain the asymptotic freedom, and the negative imaginary part is what we need for the asymptotic freedom \[13\] \[14\] \[30\].

The effective action has an important symmetry, the electric-magnetic duality \[13\]. Clearly the two effective actions for \( a = 0 \) and \( b = 0 \) are related. We can obtain one from the other simply by replacing \( a \) with \(-ib\) and \( b \) with \( ia\). This duality, which states that the effective action should be invariant under the replacement

\[
a \to -ib, \quad b \to ia, \quad \tag{24}
\]

was first discovered in the QED effective action \[32\]. But subsequently this duality has been shown to exist in the QCD effective action \[13\] \[14\]. This tells that the duality should be regarded as a fundamental symmetry of the effective action of gauge theory, Abelian and non-Abelian. The importance of this duality is that it provides a very useful tool to check the self-consistency of the effective action. The fact that the two effective actions are related by the duality assures that they are self-consistent.

The effective action \[23\] generates the much desired dimensional transmutation in QCD. From this we have the following effective potential when \( b = 0 \)

\[
V = \frac{H^2}{2} \left[ 1 + \frac{11g^2}{24\pi^2} \left( \ln \frac{gH}{\mu^2} - c \right) \right], \tag{25}
\]

where \( H \equiv a/g \) now represents the magnetic field of the Diracian background. From this we define the running coupling \( \bar{g} \) by \[13\] \[14\]

\[
\frac{\partial^2 V}{\partial H^2} \bigg|_{H=\mu^2/g} = \frac{g^2}{\bar{g}^2}, \tag{26}
\]

and obtain the well known \( \beta \)-function \[33\]

\[
\beta(\bar{g}) = \bar{g}^3 \frac{\partial \bar{g}}{\partial \bar{g}} = -\frac{11\bar{g}^3}{24\pi^2}, \tag{27}
\]
In terms of the running coupling the renormalized potential is given by

\[ V_{\text{ren}} = \frac{H^2}{2} \left[ 1 + \frac{11\bar{g}^2}{24\pi^2} \left( \ln \frac{\bar{g}H}{\bar{\mu}^2} - \frac{3}{2} \right) \right], \tag{28} \]

which generates a non-trivial local minimum at

\[ \langle H \rangle = \frac{\bar{g}^2}{\bar{g}} \exp \left( - \frac{24\pi^2}{11g^2} + 1 \right). \tag{29} \]

This is nothing but the desired dimensional transmutation (the generation of a mass gap) by the monopole condensation. The corresponding effective potential is plotted in Fig. 3, where we have assumed $\bar{\alpha} = 1$ and $\bar{\mu} = 1$.

It has been suggested that the existence of the tachyonic modes is closely related to the asymptotic freedom \[8\]. Our analysis tells that this is not true. Obviously \[27\] is consistent with the stable monopole condensation.

V. ABELIAN DECOMPOSITION OF SU(3) QCD

Now, we generalize the above result to the real SU(3) QCD. To do that we have to select the Abelian direction first. But in SU(3) there are two Abelian directions because we have two Abelian subgroups. Let $\hat{n}_i$ ($i = 1, 2, ..., 8$) be a local orthonormal octet basis of SU(3). Choose $\hat{n} = \hat{n}_3$ to be the $\lambda_3$-like unit vector which selects one Abelian direction at each space-time point, and impose the magnetic isometry

\[ D_\mu \hat{n} = 0. \tag{30} \]

This automatically selects the other ($\lambda_8$-like) Abelian direction $\hat{n}' = \hat{n}_8$, because \[30\] guarantees \[3\] \[20\].

\[ D_\mu \hat{n}' = 0, \quad \hat{n}' = \sqrt{3} (\hat{n} * \hat{n}), \tag{31} \]

where $*$ denotes the d-product ($\hat{n}' = \sqrt{3} d_{\mu}^a \hat{n}^a \hat{n}^b$). This is because SU(3) has the d-product as well as the f-product (the symmetric as well as the anti-symmetric product). Of course $\hat{n}'$ becomes identical to $\hat{n}$ when $\hat{n}$ is $\lambda_3$-like. But notice that when $\hat{n}$ is $\lambda_3$-like, $\hat{n}'$ becomes $\lambda_8$-like. This tells us that we must choose the Abelian direction to be $\lambda_3$-like $\hat{n}$, which automatically gives us the $\lambda_8$-like Abelian direction $\hat{n}' \ [3\] \ [20]$.

The Abelian projection \[30\] uniquely determine the restricted potential $\tilde{A}_\mu$, the most general Abelian gauge potential in SU(3) QCD,

\[ \tilde{A}_\mu = A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g} \hat{n}' \times \partial_\mu \hat{n}', \tag{32} \]

where $A_\mu = \hat{n} \cdot \tilde{A}_\mu$ and $A'_\mu = \hat{n}' \cdot \tilde{A}_\mu$ are the Maxwellian chromoelectric potentials. Introducing three unit octets $\hat{n}^p$ ($p = 1, 2, 3$) in red, blue, and yellow (i-spin, u-spin, and u-spin) directions in color space and the corresponding Abelian potentials $A^p_\mu$, we can express this in a manifestly Weyl symmetric form,

\[ \tilde{A}_\mu = \sum_p \frac{2}{3} A^p_\mu, \quad \tilde{A}'_\mu = A^p_\mu \hat{n}^p - \frac{1}{g} \hat{n}^p \times \partial_\mu \hat{n}^p, \]

\[ A^1_\mu = A_\mu, \quad A^2_\mu = -\frac{1}{2} A_\mu + \frac{\sqrt{3}}{2} A'_\mu, \]

\[ A^3_\mu = -\frac{1}{2} A_\mu - \frac{\sqrt{3}}{2} A'_\mu, \quad \hat{n}^1 = \hat{n}, \]

\[ \hat{n}^2 = \frac{1}{2} \hat{n} + \frac{\sqrt{3}}{2} \hat{n}', \quad \hat{n}^3 = -\frac{1}{2} \hat{n} - \frac{\sqrt{3}}{2} \hat{n}'. \tag{33} \]

The advantage of this expression, of course, is that $\tilde{A}_\mu$ is explicitly invariant under the Weyl symmetry, the six-element permutation subgroup of three colors of $SU(3)$ which contains the cyclic $Z_3$.

With this the most general SU(3) QCD potential is written as

\[ \tilde{A}_\mu = \tilde{A}_\mu + \tilde{X}_\mu = \sum_p \left( \frac{2}{3} \tilde{A}^p_\mu + \tilde{W}^p_\mu \right), \]

\[ \tilde{W}^1_\mu = \tilde{W}^1_\mu + X^1_\mu \hat{n}_1 + X^2_\mu \hat{n}_2, \quad \tilde{W}^2_\mu = X^6_\mu \hat{n}_6 + X^2_\mu \hat{n}_7, \]

\[ \tilde{W}^3_\mu = X^4_\mu \hat{n}_1 - X^4_\mu \hat{n}_5, \tag{34} \]

where $\tilde{X}_\mu = \tilde{W}^1_\mu + \tilde{W}^2_\mu + \tilde{W}^3_\mu$ is the valence potential. This is the Weyl symmetric CDG decomposition of SU(3) QCD.

The decomposition \[34\] allows two types of gauge transformation, the background gauge transformation described by

\[ \delta \tilde{A}_\mu = \frac{1}{g} D_\mu \tilde{\alpha}, \quad \delta \tilde{X}_\mu = -\tilde{\alpha} \times \tilde{X}_\mu, \tag{35} \]

and the quantum gauge transformation described by

\[ \delta \tilde{A}_\mu = 0, \quad \delta \tilde{X}_\mu = \frac{1}{g} D_\mu \tilde{\alpha}. \tag{36} \]

Notice that, just as in SU(2), $\tilde{A}_\mu$ by itself enjoys the full SU(3) gauge degrees of freedom, even though it describes the Abelian part of the potential. Moreover, the valence potential $\tilde{X}_\mu$ transforms covariantly. Most importantly the decomposition \[34\] is gauge independent. Once the color direction $\hat{n}$ is selected, the decomposition follows automatically, independent of the choice of gauge $\tilde{\alpha}$.

From the restricted potential \[32\] we have

\[ F_{\mu \nu} = G_{\mu \nu} \hat{n} + G'_{\mu \nu} \hat{n}' = \sum_p \frac{2}{3} F^p_{\mu \nu}, \]

\[ F^p_{\mu \nu} = \frac{2}{3} F^\mu_{\mu \nu}, \quad \hat{n}' \text{ and } \hat{n} \text{ are } \lambda_8 \text{-like and } \lambda_3 \text{-like, respectively.} \]
\[
G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu} + H_{\mu\nu}, \quad B_\mu = A_\mu + C_\mu, \\
C'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu = F'_{\mu\nu} + H'_{\mu\nu}, \quad B'_\mu = A'_\mu + C'_\mu, \\
H_{\mu\nu} = -\frac{1}{g} \hat{\rho} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) = \partial_\mu C_\nu - \partial_\nu C_\mu, \\
H'_{\mu\nu} = -\frac{1}{g} \hat{n}' \cdot (\partial_\mu \hat{n}' \times \partial_\nu \hat{n}') = \partial_\mu C'_\nu - \partial_\nu C'_\mu,
\]
\[
\hat{F}'_{\mu\nu} = F'_{\mu\nu} \hat{n}' - \frac{1}{g} \partial_\mu \hat{n}' \times \partial_\nu \hat{n}' = G'_{\mu\nu} \hat{n}', \\
G'_{\mu\nu} = \partial_\mu B'_\nu - \partial_\nu B'_\mu = F'_{\mu\nu} + H'_{\mu\nu}, \\
H'_{\mu\nu} = -\frac{1}{g} \hat{n}' \cdot (\partial_\mu \hat{n}' \times \partial_\nu \hat{n}').
\]

where \( C_\mu \) and \( C'_\mu \) are the Diracian monopole potentials and \( B'_\mu \) are the dual potentials in \( t \)-spin, \( u \)-spin, and \( v \)-spin direction. This confirms that the restricted potential has the dual structure.

With (32) we have SU(3) RCD which has the full SU(3) gauge symmetry
\[
\mathcal{L}_{RCD} = -\frac{1}{4} \hat{F}^2_{\mu\nu} = -\frac{1}{6} \sum_p (F^p_{\mu\nu})^2 = -\frac{1}{4} (G^2_{\mu\nu} + G'^2_{\mu\nu}) = -\frac{1}{6} \sum_p (G^p_{\mu\nu})^2.
\]

Notice the change of the coefficient 1/4 to 1/6. It is really remarkable that the SU(3) RCD can be written in a manifestly Weyl symmetric form.

With the Abelian decomposition (34) we have
\[
\hat{F}_{\mu\nu} = \hat{F}_\mu \hat{X}_\nu + D_\mu \hat{X}_\nu - D_\nu \hat{X}_\mu + g \hat{X}_\mu \times \hat{X}_\nu, \\
= \sum_p \left( \frac{2}{3} F^p_{\mu\nu} + \hat{D}^p_{\mu} \hat{W}^p_{\nu} - \hat{D}^p_{\nu} \hat{W}^p_{\mu} + g \hat{W}^p_{\mu} \times \hat{W}^p_{\nu} \right), \\
\hat{W}^p_{\mu} = (\partial_\mu + ig B^p_\mu) \hat{W}_\mu.
\]

From this we have the Weyl symmetric SU(3) ECD
\[
\mathcal{L}_{ECD} = -\frac{1}{4} \hat{F}^2_{\mu\nu} - \frac{1}{4} \hat{F}'^2_{\mu\nu} - \frac{1}{4} (\hat{D}^p_{\mu} \hat{X}^p_{\nu} - \hat{D}^p_{\nu} \hat{X}^p_{\mu})^2 \\
= \frac{g}{2} \hat{F}^p_{\mu\nu} \cdot (\hat{X}^p_{\mu} \times \hat{X}^p_{\nu}) + \frac{g^2}{2} (\hat{X}^p_{\mu} \times \hat{X}^p_{\nu})^2 \\
= \sum_p \left\{ -\frac{1}{6} (F^p_{\mu\nu})^2 - \frac{1}{4} (\hat{D}^p_{\mu} \hat{W}^p_{\nu} - \hat{D}^p_{\nu} \hat{W}^p_{\mu})^2 \\
- \frac{g}{2} F^p_{\mu\nu} \cdot (\hat{W}^p_{\mu} \times \hat{W}^p_{\nu}) - \frac{g^2}{4} (\hat{W}^p_{\mu} \times \hat{W}^p_{\nu})^2 \right\},
\]

which has the extended (classical and quantum) gauge symmetry. Again this confirms that QCD can be viewed as the restricted QCD which has the three gauge covariant valence gluons \( \hat{W}^p_\mu \) as the colored source.

With this we can obtain the SU(3) ACD. Introducing three complex valence gluon fields \( W^p_\mu \)
\[
W^1_\mu = \frac{1}{\sqrt{2}} (X^1_\mu + iX^2_\mu), \quad W^2_\mu = \frac{1}{\sqrt{2}} (X^6_\mu + iX^1_\mu), \\
W^3_\mu = \frac{1}{\sqrt{2}} (X^3_\mu - iX^5_\mu), \quad (41)
\]
we can express (40) as
\[
\mathcal{L}_{ACD} = \sum_p \left\{ -\frac{1}{6} (G^p_{\mu\nu})^2 - \frac{1}{2} (\hat{D}^p_{\mu} W^p_{\nu} - \hat{D}^p_{\nu} W^p_{\mu})^2 \\
+ ig G^p_{\mu\nu} W^p_{\nu} W^p_{\mu} - \frac{g^2}{2} [(W^p_{\nu} W^p_{\mu})^2 - (W^p_{\mu})^2 (W^p_{\nu})^2], \\
- (W^p_{\mu})^2 (W^p_{\nu})^2 \right\},
\]

\[
D^p_{\mu} W^p_{\nu} = (\partial_\mu + ig B^p_\mu) W^p_{\nu}. \quad (42)
\]

This is the Weyl symmetric ACD. Notice that, although the potentials \( B^p_\mu \) which couple to three valence gluons are not independent, \( W^p_\mu \) are independent.

Clearly the RCD, ECD, and ACD have the residual Abelian gauge symmetry even after the Abelian directions are fixed. Moreover, they retain the full SU(3) gauge symmetry because \( \hat{n} \) and \( \hat{n}' \) have the full non-Abelian freedom. In fact ECD and ACD have the extended (quantum as well as classical) non-Abelian gauge symmetry, because \( \hat{A}_\mu \) and \( \hat{X}_\mu \) can be treated as the classical and quantum fields.

Of course, it goes without saying that the above RCD, ECD, and ACD, must have the discrete color reflection invariance. Just as in SU(2), the physics should not change under the color reflection. The only difference is that in SU(3) the color reflection symmetry has more freedom, because here the Abelian direction can be chosen by any of the three \( \hat{n}^p \), so that the physics should not under the color reflection \( \hat{n}^p \rightarrow -\hat{n}^p \).

For S(3) the color reflection group (in the fundamental representation) can be identified as the 27 element subgroup generated by
\[
\left( \begin{array}{ccc} 0 & 1 & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{array} \right), \quad \left( \begin{array}{ccc} 0 & 1 & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{array} \right).
\]

Of course the color reflection group could be enlarged to include the trivial center group \( Z_3 \), but here we have devided out the center group for simplicity.

The advantage of the above Abelian decomposition of SU(3) QCD, in particular the Weyl symmetric RCD, ECD, and ACD, is unmistakable. Clearly this allows us to generalize the SU(2) result directly in the calculation of the effective action of SU(3) QCD.
VI. EFFECTIVE ACTION AND MONPOLE CONDENSATION IN SU(3) QCD

To obtain the SU(3) effective action we have to integrate out the colored gluons $X_\mu$ gauge invariantly (treating them as the quantum field) in the presence of the gauge invariant and parity conserving Diracian background, imposing the color reflection invariance with the gauge fixing $\tilde{D}_\mu X_\mu = 0$. But this is straightforward, because the above Weyl symmetric Abelian decomposition of SU(3) QCD tells us that this simply amounts to integrating three colored gluons $\tilde{W}_\mu^a$ gauge invariantly in the presence of the background $H_{\mu\nu}^a$ with the gauge fixing $\tilde{D}_\mu \tilde{W}_\mu^a = 0$.

This effectively reduces the calculation of the SU(3) QCD effective action to that of the SU(2) QCD which we already know how to do. This, of course, is made possible with the Weyl symmetric Abelian decomposition. Without this Weyl symmetric decomposition the calculation of the SU(3) effective action would have been very difficult.

Now, all we have to do is to add the SU(2) result in a Weyl symmetric way. With the Diracian background $H_{\mu\nu}^a$ which comes from the monopole potentials $C_\mu^a$

$$a_p = \frac{g^2}{2} \sqrt{H_{\mu\nu}^a + (H_{\mu} H_{\nu})^2 + H_{\mu}^2},$$
$$b_p = \frac{g^2}{2} \sqrt{H_{\mu\nu}^a + (H_{\mu} H_{\nu})^2 - H_{\mu}^2},$$

we have

$$\Delta S = 2i \sum_p \ln \text{Det} \left[ (-\tilde{D}_\mu^2 + 2a_p)(-\tilde{D}_\mu^2 - 2ib_p) \right] - 2i \sum_p \ln \text{Det} (-\tilde{D}_\mu^2).$$

From this we obtain

$$\Delta \mathcal{L} = \lim_{\epsilon \to 0} \sum_p \int_0^\infty \frac{a_p b_p}{8\pi^2} \frac{dt}{t^{1-\epsilon}} \left[ \exp(-2a_p t/\mu^2) + \exp(+2ib_p t/\mu^2) - 1 \right] \sinh(a_p t/\mu^2) \sin(b_p t/\mu^2).$$

Notice that for the magnetic background we have $b_p = 0$, but for the electric background we have $a_p = 0$.

So, for the magnetic background we have

$$\mathcal{L}_{eff} = -\sum_p \left( \frac{a_p^2}{3g^2} + \frac{11a_p^2}{48\pi^2} (\ln \frac{a_p}{\mu^2} - c) \right).$$

For the electric background we have

$$\mathcal{L}_{eff} = \sum_p \left( \frac{b_p^2}{3g^2} + \frac{11b_p^2}{48\pi^2} (\ln \frac{b_p}{\mu^2} - c) - \frac{11b_p^2}{96\pi} \right).$$

Just as in SU(2), here the imaginary part has a negative signature. Moreover, the effective action has the dual symmetry. It is invariant under the dual transformation $a_p \to -ib_p$ and $b_p \to ia_p$.

We can express the effective action in terms of three Casimir invariants, $(\tilde{F}_{\mu\nu})^2$, $(\tilde{F}_{\mu\nu} \cdot \tilde{F}_{\rho\sigma})^2$, and $|\tilde{F}_{\alpha\beta} \cdot (\tilde{F}_{\alpha\beta} \cdot \tilde{F}_{\gamma\delta})|^2$, replacing $a_p$ and $b_p$ by the Casimir invariants. But notice that the imaginary part of the effective action is quadratic in $g$ and depends only on one Casimir invariant, $(\tilde{F}_{\mu\nu})^2$.

To obtain the effective potential from the effective action, notice that the constant monopole background in SU(3) is given by two magnetic fields $\tilde{H}$ and $\tilde{H}'$ in $\tilde{n}$ and $\tilde{n}'$ directions, which in principle can have different space orientation. So the effective potential is given by

$$V_{eff} = \frac{1}{2} (H^2 + H'^2) + \frac{11g^2}{48\pi^2} \left\{ H^2 \ln \left( \frac{gH_\mu^2}{\mu^2} - c \right) + H'^2 \ln \left( \frac{gH_-^2}{\mu^2} - c \right) \right\},$$
$$H = |\tilde{H}|, \quad H' = |\tilde{H}'|,$$
$$H^2 = \frac{1}{4} (H^2 + \frac{3}{4} H'^2 + \frac{3\sqrt{3}}{2} H H' \cos \theta),$$
$$\cos \theta = (\tilde{H} \cdot \tilde{H}')/HH'.$$

Notice that the classical potential depends only on $H^2 + H'^2$, but the effective potential depends on three independent variables $H$, $H'$, and $\theta$. As we have remarked this is because the effective action depends on three Casimir invariants.

One can renormalize the potential by defining a running coupling $\tilde{g}^2(\tilde{\mu}^2)$

$$\frac{\partial^2 V_{eff}}{\partial H^2} \big|_{H=H'=\tilde{\mu}^2/g,\theta=\pi/2} = \frac{\partial^2 V_{eff}}{\partial H'^2} \big|_{H=H'=\tilde{\mu}^2/g,\theta=\pi/2} = \frac{g^2}{\tilde{g}^2} = 1 + \frac{11g^2}{16\pi^2} (\ln \frac{\tilde{\mu}^2}{\mu^2} - c + \frac{5}{4}),$$

from which we retrieve the QCD $\beta$-function obtained perturbatively.

$$\beta(\tilde{\mu}) = \tilde{\mu} \frac{d\tilde{g}}{d\tilde{\mu}} = -\frac{11\tilde{g}^3}{16\pi^2}. \quad (51)$$

This confirms that QCD has the asymptotic freedom.

With this we have the renormalized potential

$$V_{ren} = \frac{1}{2} (H^2 + H'^2 + H''^2) + \frac{11g^2}{48\pi^2} \left\{ H^2 \ln \left( \frac{gH_\mu^2}{\mu^2} - \frac{5}{4} \right) + H'^2 \ln \left( \frac{gH_-^2}{\mu^2} - \frac{5}{4} \right) \right\},$$

We emphasize that the renormalized potential depends on three variables because $\cos \theta$ can be arbitrary. The
FIG. 4. The QCD effective potential with $\cos \theta = 0$, which has a unique minimum at $H = H' = H_0$.

potential has the absolute minimum at $H = H' = H_0$ and $\cos \theta = 0$,

$$V_{\text{min}} = -\frac{11\bar{\mu}^4}{32\pi^2} \exp \left( -\frac{32\pi^2}{11g^2} + \frac{3}{2} \right),$$

$$\langle H \rangle = \langle H' \rangle = \frac{\bar{\mu}^2}{g} \exp \left( -\frac{16\pi^2}{11g^2} + \frac{3}{4} \right) = H_0.$$

(53)

Notice that when $\hat{H}$ and $\hat{H}'$ are parallel (i.e., when $\cos \theta = 1$) it has two degenerate minima at $H = 2^{1/3}H_0$, $H' = 0$ and at $H = 2^{-2/3}H_0$, $H' = \sqrt{3} \times 2^{-2/3}H_0$.

We plot the effective potential for $\cos \theta = 0$ in Fig. 4 and for $\cos \theta = 1$ in Fig. 5 for comparison, where we have put $\bar{\mu} = 1$ and $\bar{\alpha}_s = 1$. Notice that the effective potential breaks the original $SO(2)$ invariance of $H^2 + H'^2$ of the classical potential.

This demonstrates the followings. First, of course, SU(3) QCD has the stable monopole condensation which could be identified as the vacuum. Second, the monopole condensation naturally reproduces (and thus consistent with) the asymptotic freedom. Third, the chromoelectric flux makes the pair annihilation of colored gluons. This confirms that essentially all qualitative features of the SU(2) QCD translate to the SU(3) QCD.

We emphasize that, just as in SU(2) QCD, the color reflection invariance implemented by the C-projection plays the crucial role in SU(3) QCD. It is this symmetry which assures the the gauge invariance of the effective action and the stability of the monopole condensation. Clearly this symmetry forbids colored objects from the physical spectrum in ECD and ACD. This necessitates the confinement of color.

The monopole condensation induces two scales, the correlation length of the monopoles and the penetration length of the color flux [2][3]. So it is natural to expect the existence of two magnetic glueballs, the $0^{++}$ and $1^{++}$ modes of the vacuum fluctuation of the monopole condensation, whose masses are fixed by the two scales. It would be very interesting to confirm these modes experimentally. The experimental confirmation of these modes could be viewed as an indirect evidence of the monopole condensation.

VII. DISCUSSION

In this paper we have shown how to generalize the calculation of the SU(2) QCD effective action of the proceeding paper to that of the SU(3) QCD. The Weyl symmetric Abelian decomposition plays the crucial role. This decomposition separates the monopole potential (as well as the colored potential) gauge independently and allows us to choose the gauge invariant monopole background. Moreover this naturally introduces the color reflection symmetry which plays the crucial role for us to implement the gauge invariance in the calculation of the effective action.

But what is really remarkable about the Abelian decomposition is that it allows a straightforward generalization of the SU(2) result to any SU(N) in a manifestly Weyl symmetric form. From the practical point of view this is a most important advantage of the decomposition. Without this Weyl symmetric Abelian decomposition we could not have succeeded to calculate the effective action of the SU(3) QCD so easily.

Our result confirms the monopole condensation. But we emphasize that this monopole condensation should really be understood as the monopole-antimonopole condensation. This is because in QCD the monopole and anti-monopole are gauge equivalent, because they are the C-parity partners [18, 27].

This has a deep meaning. It has often been claimed that the color confinement in QCD comes from “the dual Meissner effect” generated by the monopole condensation [1]. We emphasize, however, that in QCD the confinement mechanism is not exactly dual to the Meissner effect which confines the magnetic flux in ordinary super-
conductor [2, 3].

In superconductor the magnetic flux is confined by the supercurrent generated by the electron pairs only, and there is no positron pairs. But in QCD the chromoelectric flux is confined by the current made of the gauge invariant monopole and antimonopole pairs. So the two confining mechanisms are not exactly dual to each other. This is a very important point of our analysis. As a consequence in QCD the colored flux which bounds $q\bar{q}$ pairs has no sense of helicity, which plays important role in the hadron spectrum in connection with the parity doubling problem [11].

Moreover our result confirms that the chromoelectric flux annihilates the colored gluons. This has an important implication. Since the Abelian decomposition separates the gauge covariant valence gluons, we could have “the gluon model” which predicts many glueballs. In other words, just like the quarks the valence gluons can be treated as the constituents of the hadrons, and we can construct the glueballs with two or three of them [2, 3]. But experimentally we have few candidates of glueballs, which has been a big mystery.

The pair annihilation could explain this because the annihilation makes the glueballs unstable. So the gluons can not form stable glueballs. Notice, however, they could form the hybrid hadrons with quarks. For example we can have $q\bar{q}g$ states with only one valence gluon. It would be very interesting to confirm the existence of such hybrid states experimentally.

To prove that the monopole condensation becomes the true vacuum of QCD, of course, we have to integrate the effective action \[ \mathcal{A}(a_p, b_p) \] for arbitrary $a_p$ and $b_p$. This can be done, and the monopole condensation remains the true vacuum. Basically this is because the chromoelectric background creates an imaginary part which makes the condensation unstable [35]. So only when there is no chromoelectric background the effective potential becomes real.

In this paper we have neglected the quarks. We simply remark that the quarks, just as in the asymptotic freedom, tend to destabilize the monopole condensation. But if the number of quarks are small enough, the condensation remains stable. In fact we can show that the stability puts exactly the same constraint on the number of quarks as the asymptotic freedom [35].

The details of the QCD effective action including the quark loop will be published elsewhere [35].

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