Primordial black holes' relevancy with dark matter

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Abstract. Primordial black holes (PBHs) are probable candidates of dark matter (DM). In this review, we gathered data, evidence, and opinions from previous literatures to support the theory that PBHs are a form of DM. Besides, the formation of PBHs is introduced as well to give a more comprehensive description of PBHs. According to the results, the PBHs could assemble DM, or be DM itself, or generates DM, or influence DM. On the other hand, DM could have the similar influence on PBHs. Different relationships are discussed between PBHs and DM, including entropy, microlensing, distribution, growth rate, mass, and different detections (e.g., gamma ray). These results offer a guideline for future studies of PBHs.

1. Introduction
Primordial back holes (PBHs) are the black holes formed at the time of inflation due to quantum fluctuation or other effects. Generally, the mass of PBHs can be smaller than the Chandrasekhar limit and has been considered the possible DM component of the recent universe [1]. The origin of the theory of PBHs can be traced back to the 1960s, proposed by Zel'dovich, Ya., Novikov, I. D. [2], and Hawking [3]. However, the theory of PBHs had been desolated for decades since they are unable to be sustained based on Hawking evaporation [4].

Contemporarily, the theory becomes popular again after the first detection of a gravitational wave by LIGO [5]. The main reason is that PBHs are possible to exist until today based on Planck mass stable PBHs theory [6]. Another hot topic of PBHs is the generation of dark matter (DM) via PBHs, which pave a feasible path to explain the origination of dark matter in our universe [7].

In this paper, we will first introduce the formation of the PBHs in Sec. 2. Subsequently, considering the gravitational wave discovery of LIGO as the milestone, the PBHs DM theories are separated into two different parts. Specifically, Sec. 3 will summarize the progress of the theories before 2016 (the first discovery), while Sec. 4 discusses the analytical developments after 2016. Finally, we give a summary in Sec. 5.
2. Formation of PBH

2.1 Historical Background

As they discover more in-depth in astrophysics since the 20th century, they captured more detailed features of black holes and re-categorized the subclass of black holes. PBHs are one of the interesting subclasses defined in the 1960s [8]. This black hole was considered a hypothesized black hole, which is not formed by the gravitational collapse at the last phase of large-massed stars. Instead, it is made up of the extremely high density during the beginning of the universe’s early inflation. As the basic principle of the formation black hole is when the matter condenses into an unimaginably small-scale ratio, it tends to form the black hole. Coincidentally, the early phase of the universe somehow possesses the condition to generate the black hole, such as the dramatically high pressure and temperature. After further investigation, the areas originally occupied by the PBH would be dispersed as the expansion of the universe, yet the PBH keeps its shape till literally now.

PBH is a term first proposed by Yakov Borisovich Zel'dovich and Igor Dmitriyevich Novikov in 1966. They claimed the existence of such black holes [9]. Then, Stephen Hawking made a more in-depth case study on it [3] and put forward the idea black holes are not black. Depending on the scale of PBHs, they could have entirely different masses. Normally, their masses are in the interval of \(10^{14}\) kilograms and \(10^{23}\) kilograms.

According to the Hawking radiation, there would have some PBHs whose original mass is lower than \(10^{11}\) kilograms. Nevertheless, they could not sustain their existence until the present, meaning that PBHs with mass greater than \(10^{11}\) kilograms. Besides, a relatively recent event, the gravitational wave event GW150914, was detected by the Laser Interferometer Gravitational Wave Observatory (LIGO) on September 14, 2015. Based on the database, scholars approximate two black holes that have 24 times mass greater than the sun [10,11]. Moreover, it was found that the normal mass of the black hole is outweighed attributed by the death stars, i.e., LIGO might detect the PBH.

2.2 PBH Formation and Equations:

Combined with previous references, the formation of a PBH is able to start after fewer than one second after the big bang [12]. This specific time period is the so-called radiation-dominated era. As mentioned earlier, the high density of the universe is equipped with the conditions to form a PBH. In that case, the fluctuating density and gravitational collapse would be a crucial component or the prerequisite of it. More specifically, the PBH is enabled to form after meeting a unique density satisfy \(\delta \rho / \rho \sim 0.1\). Thus, the connection between the mass of PBH and the horizontal mass could be expressed as [13]:

\[
M \sim (c^3 t)/G \sim 10^{15} \left( t/10^{-23} s \right) g
\]  

(1)

To calculate the mass of a black hole, we derive the following equation:

\[
M_{BH} = \left( 4\pi \sqrt{3} M_P^3 \right) / \sqrt{\rho_f} \cong 0.05 M_\odot (g_*/100)^{-2/3} (T_f/GeV)^{-2},
\]

\[
\cong 1.4 \times 10^{13} M_\odot (g_*/100)^{-1/6} (k_f/\text{Mpc}^{-1})^{-2}
\]

(2)

where \(M_{BH}\) is the mass of the black hole; \(M_P \cong 2.4 \times 10^{18} GeV\) is the reduced Planck mass, \(M_\odot \cong 2 \times 10^{33}\) is the solar mass, \(g_*\) stands for the light degrees of freedom in thermal equilibrium, \(\rho_f, T_f\) and \(k_f\) are the density of energies[14].

Apart from this, under the Friedman model, there were more fluctuations as the universe collapsed in its early days. Before further analysis, we should first know something about the formation of a black hole. We define its mass \(M_{bh}\), as the gravitational mass, \(m\), surrounded by the innermost shell, consistent with \(e^{-\phi} \geq 10^{-10}\), the time evolution of all shells with little \(e^{-\phi}\) being essentially frozen (with respect to the proper time of a distant observer). Due to the abrupt rise of \(e^{-\phi}\) at the event horizon, the
selection of the precise cutoff value will not affect $M_{bh}$ within the accuracy range reported in this paper. Unless otherwise stated, we will refer to $M_{bh}$ in units of initial event horizon mass, $M_h$, and proper time in units of initial time, $\tilde{t}_0$.[15]

An example of a PBH's mass varied in different periods is shown in figure 1. According to the results, PBH is tended to from the interior and slow pace of fluctuation, but later on, it goes strikingly at the very first tenths of a second.

Figure 1. Time evolution of a near-critical Mexican-Hat density perturbation with (a) initial $\delta = 0.6780$ forming a mass $M_{bh} = 0.37$ and (b) initial $\delta = 0.7175$, forming a black hole with mass $M_{bh} = 0.36$.[15]

3. The relationship between PBHs and DM before 2016

In 1971, Stephan Hawking studied primordial black holes (PBH) in depth, a hypothetical type of black holes that formed soon after the Big Bang. As research shows more and more interest in primordial black holes, the idea that primordial black holes are a plausible candidate for dark matter has been raised. As a result, many papers discussing the relationships between PBHs and dark matters are published.

3.1 Accretion of matter

A study in 2006 [16] discussed the possibility that the accretion of dark matter made primordial black holes grow. To calculate the accretion, they set up accretion models in both the radiation era and the matter era. As a result, they found that a PBH could grow up by two orders of magnitude by accumulating dark matter halo from early in the rational era to $z\sim30$. In this case, one concludes that the accumulation of dark matter halos will affect the growth of PBHs, i.e., means that there are some connections between dark matter and PBHs. Furthermore, they also calculated the density parameter with the matter fraction proportion to the clothed PBH mass:

$$\frac{\omega_{BH,f}}{\omega_{BH,i}} = \frac{m_{BH,f}}{m_{BH,i}}$$

They found that the proportional mass augment depends on the proportion of dark matter made up of PBHs, instead of on the proportion of the mass of PBHs. When the dark matter begins to be dominated by PBHs, dark matter density will decrease, which will make them grow less. Therefore, PBHs will grow quickly after formation because of the accretion of dark matter halos. In addition, it means that clothed black holes will make up an arrestive fraction of dark matter.

3.2 Entropy

In 2009, Paul H. Frampton listed three possible candidates for dark matter in his paper [17]: axions, WIMPs, and MACHOs. He believed that black holes, as a category of MACHOs, are the most possible candidates. As revisiting wide binaries in Ref. [18], the range of mass of MACHOs increase from 30–43 solar mass to 30–500 solar mass, which made it more likely that dark matters are in the forms of MACHOs. Besides, it is also stated that according to theoretical cosmology, black holes are the most
likely solution to dark matter. Calculating the entropy of the black holes, with the energy of the universe divided as 0.04 baryons, 0.24 dark matter, and 0.72 dark energy, the entropy is almost entirely come from black holes, with only $10^{-15}$ from other places. Since dark matter also contributes to the universe's entropy, dark matter may be actually black holes.

### 3.3 As all dark matter

Generally, dark matter is not included in the standard model. Whereas there are actually candidates in the standard model framework [14]. They point out that PBHs with a greater mass than $10^{15}$g will be able to survive the Hawking evaporation, which contributes to dark matter density. They also point out that PBHs are attractive dark matter candidates because of their long lifetime due to their evaporation rate. Although this could just be a coincidence, it offers a powerful evidence.

Moreover, by means of a smooth-hybrid new double inflation model, they proved that it is possible to produce PBHs with a mass of $10^{-8}$ solar mass to $10^{5}$ solar mass. Hence, they mentioned an interesting idea that if PBHs of $10^{5}$-solar mass can explain dark matter, then it might account for the size evolution of elliptic galaxies by a dynamical fraction.

### 3.4 All or none

Brian C. Lacki et al. concluded that PBHs either make up almost all the dark matter or almost none of it in 2010[18]. They included in their paper that a halo is expected to form around PBHs by acquiring dark matter from the surrounding if they are not made up of dark matter. Those dark matter halos are called Ultracompact Minihalos. Assuming dark matter is a self-annihilating thermal relic which is not originated from PBHs, a combination of results about the abundance is proposed of PBHs in the cosmic gamma-ray background and Milky Way gamma-ray background constraints. Nevertheless, their analysis only works when all the dark matter is not made of PBHs. In this case, one concludes that PBHs either make up almost all the dark matter or almost none of the dark matter.

### 3.5 Microlensing

In 2011, M. R. S. Hawkins reviewed the arguments of PBHs made up dark matters [19]. He pointed out that observations showed that dark matter density is far greater than the density of baryonic materials, indicating that dark matter must be in the form of non-baryonic form. Since this fact rules out many elementary particles, PBHs seemed to be a great candidate. If PBHs are made up of dark matter, then there should be gravitational lensing in lines of sight. This demonstrated that microlensing should be observable in compact light sources' curves. To collect data, they used the Fourier power spectrum:

$$P(S_i) = \frac{1}{N} \left[ \sum_{j=1,N} m(t_j) \cos \frac{2\pi j i}{N} \right]^2 + \frac{1}{N} \left[ \sum_{j=1,N} m(t_j) \sin \frac{2\pi j i}{N} \right]^2$$

where $i$ runs over $N$ equally spaced epochs of observation separated by time $t$, $m(t_j)$ represents the magnitude at epoch $t_j$. They used Fourier power spectrum to compare with the result of numerical simulations and the prediction for time dilation, color change, and statistical symmetry from microlensing near-critical density PBHs. Their result is that dark matter bodies best explain the microlensing that is observed in many quasar systems. Unfortunately, it cannot be certain since there is no intrinsic variation models at present for them to compare.

More importantly, the search for MACHOs in the Galactic halo supported that dark matters are in the form of compact bodies. The mass of a population of compact bodies is the optical depth to microlensing, defined as

$$\tau = \frac{4\pi G}{c^2} \int_0^L \rho(l) {l(l-1) \over l} dl$$

where $L$ is the distance between the observer and the source. This could present the probability that the object is being micro-lensed by a compact object. The halo models of MACHO collaboration are restricted to three, which are S, B, F in Table 1, according to Hawkins [19]. Figure 2 demonstrated that the amplitude of intrinsic variations would decrease, for more luminous quasars [19]. Moreover, light curves become more dominated at higher redshifts.
4. Theories of PBHs as DM after the detection of gravitational wave

In recent years, there has been an increasing interest in the existence of PBHs due to the detection of the gravitational wave. As a result, a large volume of published studies describes the relationship between PBHs and DM. In this section, we will focus on a few of those theories. There are two different approaches in terms of relationship: PBHs itself as DM and DM generated by PBHs, which will be discussed separately.

4.1 PBHs itself as DM

PBHs themselves are perfectly substance while considering the extinction of dark matter. Theories suggest two different approaches to prove the hypothesis, which are correlated exist observation and problems for PBHs or suggesting theories for PBHs to withstand the Hawking evaporation.

In 2018 S’ebastien Clesse and Juan Garc’ia-Bellido related existing observation data and possible detection on PBHs [20]. The first point they propose is the rate and mass of black hole binary mergers detected by AdvLIGO-Virgo. Two clustering models for PBHs (Dominant clustering scale and Extended halo mass function) reduce PBHs merger rates, which correlate with the ones interpret by LIGO. Besides, by using the Markov-Chain Monte-Carlo method, they show that the Adv-LIGO is able to detect a subdominant fraction (between 0.1% and 1% for the preferred scenarios) of PBH mergers involving a PBH of mass $M_A \approx 5 M_\odot$ and a PBH with a mass smaller than the Chandrasekhar mass, $M_B \lesssim M_{ch} \approx 1.4 M_\odot$ [20]. The second evidence is the spin distribution. The spin of the BH from a pre-existing star binary will tend to become the same with its orbital angular momentum [21]. In the simplest cases, the effective spin ($x_{eff}$) we could expect will be $x_{eff} \approx 0.8$ whereas LIGO mergers have effective spins centered on $x_{eff} \approx 0$. The result of the calculation agreed with the observation detected by LIGO BHB. The third evidence is the detection of microlensing of distant quasars and stars in M31. The results

| Model | S | B | F | H1 | H2 | H3 |
|-------|---|---|---|----|----|----|
| $\beta$ | - | -0.2 | 0 | 0 | 0 | 0 |
| $Q$ | - | 1 | 1 | 1 | 1 | 1 |
| $R_c$(kpc) | 5 | 5 | 25 | 5 | 5 | 5 |
| $R_0$(kpc) | 8.5 | 8.5 | 7.9 | 8.5 | 8.5 | 8.0 |
| $\Sigma_0(M_\odot pc^{-2})$ | 50 | 50 | 80 | 67 | 50 | 50 |
| $R_d$(kpc) | 2.5 | 2.5 | 3.0 | 2.7 | 2.3 | 2.0 |
| $\Theta_0$(km s$^{-1}$) | 192 | 223 | 218 | 220 | 220 | 226 |
| $\tau_{LMC}$($10^{-7}$) | 4.7 | 8.1 | 1.9 | 1.58 | 1.64 | 1.45 |

Figure 2. The V-band light curve from quasar 3C273 (left panel) and the B$_v$-band light curve from Field 287 survey in a higher redshift (right panel) [19].
of 24 gravitationally lensed quasars in optical and X-ray [22] showed 20 M ± 5% of the total matter in the galaxy lens is made of any type of compact objects with masses in the range 0.05M⊙ ≤ M ≤ 0.45. The observation results strongly support the tension with the expected stellar component. Hence, an important MACHO component. In figure 3 the regions in the (μ, σ) plane leading to 20 − 35 % of PBH in the range [0.5 − 1]M⊙ and to 15 − 25 % in the range [0.05 − 0.45]M⊙, as suggested by M31 and quasar microlensing. These two regions overlap for μ ∼ 3M⊙ and σ ∼ 0.6M⊙, which exactly falls within the ellipse obtained by the mass spectrum reconstruction with LIGO events.

![Figure 3. Reconstruction of the PBH mass spectrum [20].](image)

While S'ebastien Clesse and Juan García-Bellido's paper suggests possible evidence for with the present data and PBHs, paper proposes by Sunghoon Jung and TaeHun Kim in 2020 has suggested use Gamma-ray burst (GRBs) lensing parallax to probe PBHs [23].

\[
0.1 \lambda \lesssim \frac{2GM}{c^2} (1 + z_L) \approx 10 - 5 \text{sec}(1 + z_L)M, \quad (6)
\]

\[
\delta \Lambda \equiv \frac{|A_1 - A_2|}{A_1 + A_2} \gtrsim \epsilon, \quad (7)
\]

\[
\left( \frac{M}{10^{-12}M} \right) \gtrsim \epsilon \left( \frac{D_{\text{Gpc}}}{}\right)^{-1} \left( \frac{r_e}{r_s} \right)^2. \quad (8)
\]

By solving equation (6)~(8) and consider the restriction mention in [23], we obtain the range of the PBH DM mass that can be probe: 10^{-16}M⊙ ≤ M⊙ ≤ 10^{-11}M⊙. By adding the possible probing range for the Near-by star [23], we can use GRBs to probe full range of PBH DM mass. Not only to that, SC and JG also show that we are likely to done this by present day technologies.

Aside From relating observation data, there is also the theory that suggests the existing PBHs. In 2020, Yang Bai and Nicholas Orlofsky pointed that some of the PBHs can escape from Hawking evaporation and Schwinger discharge effects, remain stable, and last till today [24-29]. They suggest that after PBHs formed during radiation domination following reheating and PBHs kept absorbed surrounding dark electrons and positrons, some of the light PBHs absorbs both charges in equal numbers though no charge occurs. As a result, they evade Schwinger discharge effects and light enough to escape Hawking evaporation. Additionally, high-energy neutrinos, gamma rays, and multimessage approach can detect the merge event of two equal mass primordial extremal black holes for observation proving.

4.2 PBHs generate DM
In 2020, Paolo Gondolo, Pearl Sandick, and Barmak Shams Es Haghi demonstrated that PBHs can produce DM under certain circumstances [7]. For weakly interacting massive particles (WIMP) and strongly interacting massive particles (SIMP), they are able to generate by freeze-out (reaching a
constant comoving dark matter density by dropping out of chemical equilibrium either between dark matter particles and the thermal bath (WIMP case) or among dark matter particles themselves (SIMP case)). For WIMP, they can produce by PBHs if PBHs evaporate after the freeze-out. At that time, the particles produce by Hawking evaporation can neither thermalize with the bath nor annihilate efficiently with each other. As a result, they will be directly contributed to the final dark matter density. For SIMP, it is a similar case. If the PBHs were evaporated after freeze-out, then the dark matter particles produced by Hawking evaporation can thermalize neither among themselves nor with the thermally produced dark matter particles. Additionally, PBHs can also generate nonthermal super-massive particles by Hawking evaporation.

5. Conclusion
In summary, the formation of PBHs as well as the progress of PBHs DM is reviewed systematically. In detail, we first introduced the historical background of PBHs' formation as well as its general equations and analyzed a PBH's mass with different initial δs. Additionally, the theory supported for the PBHs and DM and their correlation with existing observations are discussed. Lastly, the progress of the PBHs DM before and after the gravitational detection in 2016 are demonstrated separately. This paper contributes to making an overarching perspective of discoveries on PBHs DM so far along the historical timeline. The summary of the analytical developments of PBHs will help understand the universe.

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