E$_7$(7) on the Light-Cone

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Motivation

Explore the structure of maximally supersymmetric theories in the light-cone formalism

Maximally supersymmetric theories in $d=4$

- **N=4 Super Yang-Mills (spin 1)**
  - superconformal PSU(2,2|4) symmetry
  - UV finite theory, shown in the light-cone formalism
    
    [Mandelstam '83] [Brink, Lindgren, Nilsson '83]
  - In the light-cone frame, the Hamiltonian is a quadratic form
    
    [Ananth, Brink, SK, Ramond '05]

- **N=8 Supergravity (spin 2)**
  - $\kappa$ coupling has dimension of the length, and thus not a conformal theory
  - but has on-shell $E_{7(7)}$ duality symmetry
    
    [Cremmer, Julia '78]
  - In light-cone superspace, the Hamiltonian is also a quadratic form to order $\kappa^2$
    
    [Brink et al '83] [Ananth at al '06]
• **Similarities**

  • Tree level amplitude of Supergravity $\sim$ square of that of super Yang-Mills
    
    [Kawai et al '86]

  • $N=8$ theory might be UV finite
    
    [Bern et al '07]

  • Dimensionally reduced theories form higher dimensional theories
    e.g. $N=4$ Super Yang-Mills from $N=1$ Super Yang-Mills in $d=10$,
    $N=8$ Supergravity from $N=1$ Supergravity in $d=11$
N=8 Supergravity

- Maximally supersymmetry theory with maximum spin of 2

- 256 massless states: \(128\) Bosonic + \(128\) Fermionic

  Helicity: \(2\) \(3/2\) \(1\) \(1/2\) \(0\) \(-1/2\) \(-1\) \(-3/2\) \(-2\)

  States: \(1\) \(8\) \(28\) \(56\) \(70\) \(56\) \(28\) \(8\) \(1\)

\(h\) \(\psi^i\) \(B_{ij}\) \(\chi^{ijk}\) \(\bar{D}_{ijkl}\) \(\bar{\chi}^{ijk}\) \(\bar{B}_{ij}\) \(\bar{\psi}_i\) \(\bar{h}\)

- \(E_7(7)\) duality symmetry

  [Cremmer & Julia '79]

- Non-compact \(E_7\) with the maximal compact subgroup \(SU(8)\)

- On-shell electro-magnetic duality symmetry

- Non-linearly realized on the scalars and the fields strengths.
Duality symmetry

E-M duality: Maxwell’s equations are invariant under a rotation of E-fields and B-fields into each other. In relativistic notation, it means that rotations between the fields strength $F^{\mu\nu}$ and its dual $\tilde{F}^{\mu\nu}$ ($\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$)

\[(I) \quad \partial_\mu F^{\mu\nu} = 0 \quad \text{Equations motions}\]
\[(II) \quad \partial_\mu \tilde{F}^{\mu\nu} = 0 \quad \text{Bianchi identities}\]

Duality symmetry: Equations of motion (I) $\leftrightarrow$ Bianchi identities (II)

- Not a symmetry of Lagrangian!
  - e.g.
    \[L = \frac{1}{2} (E^2 - B^2) \quad H = \frac{1}{2} (E^2 + B^2)\]
- For interacting theories, eq. of motion
  \[\partial_\mu G^{\mu\nu} = 0 \quad \text{where} \quad G^{\mu\nu} = \frac{\partial L (F_{\mu\nu})}{\partial F_{\mu\nu}}\]
Duality symmetry in covariant theory for N=8 Supergravity

N=8 Lagrangian (to order $\kappa^2$)  
[de Wit and Friedman] [Cremmer and Julia] [de Wit and Nicolai]

$$\mathcal{L}_{dWF} = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{\text{others}}$$

$$\mathcal{L}_S = -\frac{1}{48} \left\{ \partial_\mu C^{ijkl} \partial^\mu C^{ijkl} + \frac{\kappa^2}{2} C^{ijkl} C^{klmn} \partial_\mu C^{mnpq} \partial^\mu C^{pqij} + \mathcal{O}(\kappa^3) \right\},$$

where the scalar fields satisfy

$$C^{ijkl} = \frac{1}{4!} \epsilon^{ijklmn} C^{mnpq}.$$
\begin{align*}
\mathcal{L}_V &= -\frac{1}{8} \mathcal{F}^{ij}_{\mu\nu} \mathcal{G}^{\mu\nu \ ij} + c.c. , \\
\mathcal{F}^{ij}_{\mu\nu} &= \frac{1}{2} F^{\mu\nu \ ij} + \frac{i}{2} \tilde{F}^{\mu\nu \ ij} ,
\end{align*}

and

\begin{align*}
\mathcal{G}^{\mu\nu \ ij} &= \mathcal{F}^{\mu\nu \ ij} + \kappa \bar{C}^{ijkl} \mathcal{F}^{\mu\nu \ kl} + \frac{\kappa^2}{2} \bar{C}^{ijkl} \mathcal{C}^{klnm} \mathcal{F}^{\mu\nu \ mn} + \mathcal{O}(\kappa^3) 
\end{align*}

Equations of motion:
\begin{align*}
\partial_\mu \left( \mathcal{G}^{\mu\nu \ ij} + \bar{G}^{\mu\nu \ ij} \right) &= 0 , \\
\partial_\mu \left( \mathcal{F}^{\mu\nu \ ij} - \bar{F}^{\mu\nu \ ij} \right) &= 0 .
\end{align*}

Bianchi identities:

SU(8) Duality symmetry

\begin{align*}
\mathcal{G}^{\mu\nu \ ij} + \mathcal{F}^{\mu\nu \ ij} &\sim 28 \quad \mathcal{G}^{\mu\nu \ ij} - \mathcal{F}^{\mu\nu \ ij} \sim \overline{28} 
\end{align*}
E₇(7) symmetry

E₇ : 133 parameter group
fundamental representation : 56

Things to know about E₇(7) (a non-compact E₇)
1. contains SU(8) as maximal compact subgroup.

\[ E₇ \supset SU(8) : \ 56 = 28 + \overline{28}, \quad Z = \begin{pmatrix} x^{ab} \\ y_{ab} \end{pmatrix} \]

2. Coset E₇(7)/SU(8) transformations: non-compact

\[ \delta x^{ab} = \Xi^{abcd} y_{cd}, \quad \delta y_{ab} = \Xi_{abcd} x^{cd}, \]

\[ \Xi^{abcd} = \frac{1}{4!} \epsilon^{abcdefgh} \Xi_{efgh}, \]

3. The subscript (7) denotes
\[ #(\text{non-compact}) - #(\text{compact}) = 70 - 63 = 7 \]
We choose the 56 of $E_7$ \[ Z_{\mu \nu} = \begin{pmatrix} G_{\mu \nu}^{ij} + F_{\mu \nu}^{ij} \\ G_{\mu \nu}^{ij} - F_{\mu \nu}^{ij} \end{pmatrix} \equiv \begin{pmatrix} X_{\mu \nu}^{ab} \\ Y_{\mu \nu}^{ab} \end{pmatrix} \]

The two components are

\[
\begin{align*}
X_{\mu \nu}^{ab} &= 2F_{\mu \nu}^{ij} + \kappa C^{ijkl} F_{\mu \nu}^{kl} + \frac{k^2}{2} C^{ijkl} C^{klmn} F_{\mu \nu}^{mn} + O(\kappa^3), \\
Y_{\mu \nu}^{ab} &= \kappa C^{ijkl} F_{\mu \nu}^{kl} + \frac{k^2}{2} C^{ijkl} C^{klmn} F_{\mu \nu}^{mn} + O(\kappa^3),
\end{align*}
\]

related by constraint

\[ Y_{\mu \nu}^{ab} - \frac{k}{2} C_{abcd} X_{\mu \nu}^{cd} + O(\kappa^2) = 0. \]

The equations of motion + the Bianchi identities

\[ \partial_\mu \left( Z_{\mu \nu} + \tilde{Z}_{\mu \nu} \right) = 0, \]

where

\[ \tilde{Z}_{\mu \nu} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tilde{Z}_{\mu \nu} = \begin{pmatrix} \tilde{G}_{\mu \nu}^{ij} - \tilde{F}_{\mu \nu}^{ij} \\ \tilde{G}_{\mu \nu}^{ij} + \tilde{F}_{\mu \nu}^{ij} \end{pmatrix} = \begin{pmatrix} \tilde{Y}_{\mu \nu}^{ab} \\ \tilde{X}_{\mu \nu}^{ab} \end{pmatrix} \]
Duality transformations:

\[ \delta X^{\mu\nu \ ab} = \Xi^{abcd} Y^{\mu\nu \ cd}, \]
\[ \delta Y^{\mu\nu \ ab} = \overline{\Xi}_{abcd} X^{\mu\nu \ cd}, \]

Under the coset $E_7(7)/SU(8)$, both the scalars and the field strengths transform non-linearly (required by the constraint)

\[ \delta \overline{C}_{abcd} = \frac{2}{\kappa} \Xi_{abcd} - \frac{\kappa}{2} C_{ef[ab} \overline{C}_{cd]} mn \Xi^{efmn} + \mathcal{O}(\kappa^2), \]

\[ \delta \mathcal{F}^{\mu\nu \ ij} = -\Xi^{ijkl} \mathcal{F}^{\mu\nu \ kl} + \frac{\kappa}{2} \left( \Xi^{ijkl} - \Xi^{ij kl} \right) \overline{C}^{klmn} \mathcal{F}^{\mu\nu \ mn} \]
\[ + \frac{\kappa^2}{4} \left( \Xi^{ijkl} - \Xi^{ij kl} \right) \overline{C}^{klmn} C^{mnpq} \mathcal{F}^{\mu\nu \ pq} + \mathcal{O}(\kappa^3). \]
Light-Cone formalism

Light-Cone Coordinate: \( \eta^{\mu\nu} = (-, +, +, +) \)

\[
x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^3) ; \quad \partial^\pm = \frac{1}{\sqrt{2}} (- \partial_0 \pm \partial_3) ,
\]
\[
x = \frac{1}{\sqrt{2}} (x_1 + i x_2) ; \quad \bar{\partial} = \frac{1}{\sqrt{2}} (\partial_1 - i \partial_2) ,
\]
\[
\bar{x} = \frac{1}{\sqrt{2}} (x_1 - i x_2) ; \quad \partial = \frac{1}{\sqrt{2}} (\partial_1 + i \partial_2) ,
\]

Light-Cone Gauge: \( A^+ = \frac{1}{\sqrt{2}} (A^0 + A^3) = 0 \)

physical d.o.f. : \( \bar{A} = \frac{1}{\sqrt{2}} (A^1 + i A^2) \quad A = \frac{1}{\sqrt{2}} (A^1 - i A^2) \)

\[
A^- = \frac{1}{\sqrt{2}} (A^0 - A^3) \quad \text{is replaced by eq. of motions:}
\]
\text{LC2 formalism}
We then make non-linear field redefinitions to get rid of “time derivatives” in the interaction terms.

\[ A^{ij} \rightarrow B^{ij} - \frac{\kappa}{2} \frac{1}{\partial^+} \left( D_{ijkl} \partial^+ B^{kl} \right) + \cdots \]

\[ C^{ijkl} \rightarrow D^{ijkl} - \frac{\kappa^2}{4} \frac{1}{\partial^+} \left( D^{pq[ij} \partial^+ D^{kl]mn} \bar{D}_{pqmn} \right) + \cdots \]

In \( E_{7(7)}/SU(8) \) transformation, the scalars contain terms quadratic in the vector fields.

\[
\delta B^{ij} = - \frac{\kappa}{4} \Xi_{mnkl} D^{ijkl} B^{mn} + \frac{\kappa}{4} \Xi^{ijkl} \frac{1}{\partial^+} \left( D_{mnkl} \partial^+ B^{mn} \right) + O(\kappa^2),
\]

\[
\delta D^{ijkl} = \frac{2}{\kappa} \Xi^{ijkl} - \frac{\kappa}{2} \Xi_{mnpq} \frac{1}{\partial^+} \left( D^{mn[kl} \partial^+ D^{ij]pq} \right) \]
\[ + \frac{\kappa}{2} \Xi^{pq[ij} \frac{1}{\partial^+} \left( \partial^+ D^{kl]mn} \bar{D}_{pqmn} \right) \]
\[ - 3 \kappa \left( \frac{\Xi^{mn[kl}}{\partial^+} \left( \partial^+ B^{ij]} \bar{B}_{mn} \right) + \epsilon_{ijklrstu} \Xi^{tumn} \frac{1}{4! \partial^+} \left( B^{mn} \partial^+ \bar{B}_{rs} \right) \right) \]
\[ + O(\kappa^2).\]
Light-Cone Superspace

• SUSY is manifest.
• A single superfield captures all physical degrees of freedom.

\[
\varphi(y) = \frac{1}{\partial_{+2}} h(y) + i \theta^m \frac{1}{\partial_{+2}} \overline{\psi}_m(y) + \frac{i}{2} \theta^m \theta^n \frac{1}{\partial_{+}} \overline{B}_{mn}(y)
\]

- \frac{1}{3!} \theta^m \theta^n \theta^p \frac{1}{\partial_{+}} \overline{\chi}_{mnp}(y) - \frac{1}{4!} \theta^m \theta^n \theta^p \theta^q \overline{D}_{mnpq}(y)

+ \frac{i}{5!} \theta^m \theta^n \theta^p \theta^q \theta^r \epsilon_{mnprstu} \chi_{stu}(y)

+ \frac{i}{6!} \theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \epsilon_{mnprstu} \partial^+ B_{tu}(y)

+ \frac{1}{7!} \theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \epsilon_{mnprstu} \partial^+ \psi^u(y)

+ \frac{4}{8!} \theta^m \theta^n \theta^p \theta^q \theta^r \theta^s \theta^t \theta^u \epsilon_{mnprstu} \partial^+ \hat{h}(y) .

Chiral coordinate:

\[
y = (x, \bar{x}, x^+, y^- \equiv x^- - \frac{i}{\sqrt{2}} \theta^m \bar{\theta}_m)
\]

• Chirality constraints

\[
d^m \varphi = 0, \quad \overline{d}_m \varphi = 0 ,
\]

where

\[
d^m \equiv - \frac{\partial}{\partial \theta^m} - \frac{i}{\sqrt{2}} \theta^m \partial^+ , \quad \overline{d}_m \equiv \frac{\partial}{\partial \theta^m} + \frac{i}{\sqrt{2}} \bar{\theta}_m \partial^+
\]

satisfying

\[
\{d^m, \overline{d}_n\} = -i \sqrt{2} \delta^m_n \partial^+
\]

Inside out constraints

\[
\varphi = \frac{1}{4 \partial_{+4}} d^1 d^2 \cdots d^8 \varphi ,
\]
Kinematical Supersymmetry

- Kinematical supersymmetry generators: the spectrum generating part

\[ q^m = - \frac{\partial}{\partial \theta_m} + \frac{i}{\sqrt{2}} \theta^m \partial^+ , \quad \bar{q}_m = \frac{\partial}{\partial \bar{\theta}^m} - \frac{i}{\sqrt{2}} \bar{\theta}^m \partial^+ \]

which satisfy anticommutation relation

\[ \{ q^m , \bar{q}_n \} = i \sqrt{2} \delta^m_n \partial^+ , \]

- SUSY transformations:

\[ \delta_s \varphi(y) = \bar{\epsilon}_m q^m \varphi(y) , \]

where \( \epsilon_m \) eight susy transformation parameters.

In component form,

\[ \delta_s \bar{B}_{mn} = - 2i \sqrt{2} \epsilon_{[m} \bar{\psi}_{n]} , \]

\[ \delta_s \bar{\psi}_m = - \sqrt{2} \epsilon_m \partial^+ h , \]

\[ \ldots \]
What about the other fields?

- All fields are related by supersymmetry.
- A coset $E_{7(7)}/SU(8)$ should commute with SUSY.
  (There is no $E_{7(7)}$ supergroup)
- fermions and graviton, do not transform ??

Following the algebra, we found that other fields including the graviton transform by requiring $[E_{7(7)}/SU(8), \text{susy}] = 0$.

The claim is then “gauge dependent” statement: in the light-cone gauge, all physical fields transform under $E_{7(7)}$!
Demanding $[\text{SUSY, } E_7(7)/\text{SU}(8) ] = 0$

Let's choose a vector field $\overline{B}_{12}$ to demonstrate how to find the $E_7(7)/\text{SU}(8)$ transformation satisfying $[\text{SUSY, } E_7(7)/\text{SU}(8) ] = 0$

$$[\delta_s, \delta] \overline{B}_{ij} = 0 \quad \rightarrow \quad \delta_s \delta \overline{B}_{12} = \delta \delta_s \overline{B}_{12}$$

This equation must hold for all susy parameters $\epsilon_m$.

Since $$\delta_s \overline{B}_{12} = -2i\sqrt{2} \bar{\epsilon}_{1[\bar{\psi}_2]}$$, susy transformations on $\overline{B}_{12}$ along $\bar{\epsilon}_1$ (or $\bar{\epsilon}_2$) yields the gravitino $\bar{\psi}_2$ (or $\bar{\psi}_1$)

However, susy transformations along $\bar{\epsilon}_m (m = 3, \ldots, 8)$ vanish!!
Demanding $[\text{SUSY, } E_7(7)/\text{SU}(8)] = 0$

Let's choose a vector field $B_{12}$ to demonstrate how to find the $E_7(7)/\text{SU}(8)$ transformation satisfying $[\text{SUSY, } E_7(7)/\text{SU}(8)] = 0$

$$[\delta_s, \delta] B_{ij} = 0 \quad \rightarrow \quad \delta_s \delta B_{12} = \delta \delta_s B_{12}$$

This equation must hold for all susy parameters $\epsilon_m$.

Since $\delta_s B_{12} = -2i\sqrt{2} \bar{\epsilon}_1 \bar{\psi}_2$, susy transformations on $B_{12}$ along $\bar{\epsilon}_1$ (or $\bar{\epsilon}_2$) yields the gravitino $\bar{\psi}_2$ (or $\bar{\psi}_1$)

However, susy transformations along $\bar{\epsilon}_m (m = 3, \ldots, 8)$ vanish!!

Therefore, if one uses, for instance, $\bar{\epsilon}_3$, then RHS vanishes!
Now, check the LHS
Recall the coset transformation with $\Xi^{1234}$

$$\delta \overline{B}_{12} = -\kappa \overline{D}_{1234} \Xi^{3412} \overline{B}_{12}$$

Then take the susy transformation along $\overline{\epsilon}_3$

$$\delta_s \delta \overline{B}_{12} = -\kappa \Xi^{3412} (\delta_s \overline{D}_{1234} \overline{B}_{12} + \overline{D}_{1234} \delta_s \overline{B}_{12})$$

$$= -\kappa \sqrt{2} \Xi^{3412} \overline{\epsilon}_3 \overline{\chi}_{124} \overline{B}_{12} \neq 0$$

LHS is not zero, but RHS is zero!

Thus the transformation $\delta \overline{B}_{12}$ should be modified!!

We keep adding terms to make LHS zero.
In this way, one can find the coset $E_7(7)/SU(8)$ transformation on vectors

$$
\delta B_{ij} = -\kappa \Xi^{klmn} \left( \frac{1}{4} \bar{D}_{ijkl} B_{mn} + \frac{1}{4!} \frac{1}{\partial^+} \bar{D}_{klmn} \partial^+ \bar{B}_{ij} - \frac{1}{4!} \epsilon_{ijklmnrs} \frac{1}{\partial^+} B^{rs} \partial^+ h \\
+ \frac{i}{3!} \frac{1}{\partial^+} \bar{\chi}_{klm} \bar{\chi}_{ijn} - \frac{i}{3!} \epsilon_{ijklrst} \frac{1}{\partial^+} \chi^{rst} \bar{\psi}_n \right) \\
+ \kappa \Xi_{ijkl} \frac{1}{\partial^+} \left( \frac{1}{4} D^{klmn} \partial^+ \bar{B}_{mn} - \frac{1}{\partial^+} B^{kl} \partial^+ h \\
+ \frac{i}{4(3!)^2} \bar{\chi}_{mnp} \bar{\chi}_{rst} \epsilon^{klmnprst} - 3i \frac{1}{\partial^+} \chi^{kln} \partial^+ \bar{\psi}_n \right)
$$

Similarly,

$$
\delta_s \delta B_{ij} = -2i\sqrt{2} \bar{\epsilon}_{[i} \delta \bar{\psi}_{j]}
$$

leads to the $E_7(7)/SU(8)$ transformation on the gravitinos

$$
\delta \bar{\psi}_i = -\kappa \Xi^{mnpq} \left( \frac{1}{4!} \frac{1}{\partial^+} \bar{D}_{mnpq} \partial^+ \bar{\psi}_i + \frac{1}{3!} \bar{D}_{mnpq} \bar{\psi}_q \\
- \frac{1}{4!} \epsilon_{mnpqirst} \frac{1}{\partial^+} \chi^{rst} \partial^+ h + \frac{1}{4} \bar{\chi}_{imn} \bar{B}_{pq} + \frac{1}{3!} \frac{1}{\partial^+} \bar{\chi}_{mnp} \partial^+ \bar{B}_{iq} \right)
$$
It follows that \( \delta_s \delta \overline{\psi}_i = -\sqrt{2} \bar{\epsilon}_i \partial^+ \delta h \)
leads to how the graviton transforms under \( E_{7(7)}/SU(8) \)

\[
\delta h = - \kappa \Xi^{ijkl} \left( \frac{1}{8} B_{ij} B_{kl} + \frac{1}{4!} \frac{1}{\partial^+} D_{ijkl} \partial^+ h + \frac{i}{6} \frac{1}{\partial^+} \bar{\chi}_{ijk} \overline{\psi}_l \right) + \mathcal{O}(\kappa^3)
\]

In superfield language,

\[
\delta \varphi = \delta^{(-1)} \varphi + \delta^{(1)} \varphi + \mathcal{O}(\kappa^3)
\]

\[
= - \frac{2}{\kappa} \theta^{klmn} \Xi_{klmn} + \frac{\kappa}{4!} \Xi^{mnqp} \frac{1}{\partial^+} \left( \bar{d}_{mnpq} \frac{1}{\partial^+} \varphi \partial^+ \varphi + 4 \bar{d}_{mpl} \varphi \bar{d}_q \partial^2 \varphi + 3 \bar{d}_{mn} \partial^+ \varphi \bar{d}_{pq} \partial^+ \varphi \right)
\]
Dynamical Supersymmetry

involves the interactions, and generates the Hamiltonian.

What is known:

To order $\kappa$, the Hamiltonian is simple. [Brink, et al '83]

$$ \mathcal{H} = 2\bar{\varphi} \frac{\partial\bar{\varphi}}{\partial+4} \varphi + 2\kappa \left( \frac{1}{\partial + 2} \bar{\varphi} \partial\varphi \bar{\varphi} + c.c. \right) + \mathcal{O}(\kappa^2) $$

and is of a quadratic form to order $\kappa^2$

$$ \mathcal{H} = \left( \delta^{\text{dyn}}_{\bar{q}^-} \varphi, \delta^{\text{dyn}}_{\bar{q}^-} \varphi \right) = \int \delta^{\text{dyn}}_{\bar{q}^-} \varphi \frac{1}{\partial + 3} \delta^{\text{dyn}}_{\bar{q}^-} \varphi $$
Dynamical susy generator:

\[
\delta_{s}^{\text{dyn}} \varphi = \delta_{s}^{\text{dyn} (0)} \varphi + \delta_{s}^{\text{dyn} (1)} \varphi + \delta_{s}^{\text{dyn} (2)} \varphi + \mathcal{O}(\kappa^3),
\]

\[
= \epsilon^m \left\{ \frac{\partial}{\partial^+} \bar{q}_m \varphi + \kappa \frac{1}{\partial^+} \left( \bar{\partial} \bar{d}_m \varphi \partial^{2} \varphi - \partial^+ \bar{d}_m \varphi \partial^+ \bar{\partial} \varphi \right) + \mathcal{O}(\kappa^2) \right\}
\]

We require that

\[
\left[ \delta, \delta_{s}^{\text{dyn}} \right] \varphi = 0
\]

At order \( \kappa \),

\[
\left[ \delta^{(-1)}, \delta_{s}^{\text{dyn} (2)} \right] \varphi + \left[ \delta^{(1)}, \delta_{s}^{\text{dyn} (0)} \right] \varphi = 0
\]

which determines \( \delta_{s}^{\text{dyn} (2)} \varphi \)
Conclusion and future work

- How \( E_7(7) \) symmetry is realized on the LC

- \([\text{susy}, E_7(7)] = 0\)
  - All physical fields transform under \( E_7(7) \)
  - yields the LC Hamiltonian.

- Closed form to all orders. [Kallosh and Sorouch ’08]

- Might lead to a better understanding of UV divergent structure of the N=8 theory.