Gravitational Rainbows: LIGO and Dark Energy at its Cutoff

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The recent direct detection of a neutron star merger with optical counterpart has been used to severely constrain models of dark energy that typically predict a modification of the speed of gravitational waves. We point out that the energy scales observed at LIGO, and the particular frequency of the neutron star event, lie very close to the strong coupling scale or cutoff associated with many dark energy models. While it is true that at very low energies one expects gravitational waves to travel at a speed different than light in these models, the same is no longer necessarily true as one reaches energy scales close to the cutoff. We show explicitly how this occurs in a simple model with a known partial UV completion. Within the context of Horndeski, we show how the operators that naturally lie at the cutoff scale can affect the speed of propagation of gravitational waves and bring it back to unity at those scales. We discuss how further missions including LISA and PTAs could play an essential role in testing such models.

Dark Energy after GW170817 and GRB170817A: The recent direct detections of gravitational waves have had an unprecedented impact on our understanding of gravity at a fundamental level. The first event alone (GW150914 [1]) was already sufficient to put bounds on the graviton with better precision than what we know of the photon. Last year, the first detection of gravitational waves from a neutron star merger (GW170817), some $10^{15}$ light seconds away, which arrived simultaneously with an optical counterpart (GRB170817A) within one second, allowed us to constrain the speed of gravitational waves with remarkable precision [2–4]

$$-3 \times 10^{-15} \leq \frac{c_T}{c_\gamma} - 1 \leq 7 \times 10^{-16},$$

(1)

with $c_T$ being the speed of gravitational waves (more precisely the phase velocity) and $c_\gamma$ the speed of light.

Such a constraint has had far-reaching consequences for models of dark energy. Within the context of the effective field theory (EFT) for dark energy [5], it was very rapidly pointed out that the constraint (1) was sufficient to suppress the EFT operator(s) that typically predict non-luminal gravitational propagation when considered as a candidate for dark energy [6–9] (and more recently also [10–12]). In particular, within the framework of scalar-tensor theories of gravity, Horndeski [13] has played a major part in the past decade as providing a consistent ghost-free EFT in which the scalar degree of freedom could play the role of dark energy. Yet the interplay between the scalar and gravity typically implies that gravitational waves would not travel luminally in these models. The LIGO constraint on the speed of propagation of gravitational wave was hence able to remove two out of the four free functions present in Horndeski, leaving only the generalization of the cubic Galileon [14]. In parallel, other observations severely constrain that operator, and as a result the Horndeski EFT by itself is almost entirely ruled out as a dark energy candidate. In addition, the impact of (1) is not limited to scalar-tensor theories of gravity—other models of dark energy, such as vector-tensor gravity or scalar-vector-tensor gravity, have also seen their parameter space remarkably affected (see however [10, 12] for models of dark energy that survive the bound).

In what follows we shall not discuss any further which particular models may or may not be consistent with the recent bound (1), but rather emphasize the point that the recent LIGO bound is a statement on the speed of gravitational waves at a frequency of $10 – 100$Hz, while the EFT for dark energy is “constructed” as an effective field theory for describing cosmology on scales 20 orders of magnitude smaller. When it comes to constraining a parameter such as the sound speed of gravitational waves, it is therefore important to recall that such a quantity could in principle depend on scale: generically, the speed of gravitational waves may depend on the frequency at which it is measured, $c_T = c_T(k)$. The LIGO bound (1) should therefore be read as a constraint on $c_T(k)$ at frequencies on the order of $k \sim 10 – 100$Hz, and from their very construction we expect EFTs such as Horndeski to break down at a cutoff $\sim 100$Hz if not much lower. If the theory is to ever admit a Lorentz-invariant Ultra-Violet (UV) completion, then the front velocity [15] must be luminal which directly implies that the sound speed $c_T(k)$ will necessarily asymptote to exactly luminal at high frequencies. While the EFT of dark energy may predict a sound speed for GWs that departs from unity at low energy, it is nonetheless very natural to expect a speed very close to luminal at higher frequencies. Precisely what we mean by “higher frequencies” depends on the details of the UV completion, but in the case of Horndeski having a cosmological background generally requires that new physics ought to enter at (or even parametrically before) the energy scales observed
at LIGO, where it may therefore be natural to observe a luminal velocity. We shall present how this would naturally occur in a simple scalar field model (see Fig. 1) before turning to the full-fledged scalar-tensor theory and discussing the implications of Lorentz-invariant UV completions to Horndeski.

![Graph showing sound speed for the light field fluctuations](image)

**FIG. 1.** Sound speed (phase velocity) for the light field fluctuations in the example partial UV completion (6). At low frequencies $k \ll M$, the field fluctuations are subluminal. Luminality is recovered at frequencies above the cutoff, chosen here at $M = 10^{-2}\Lambda \approx 2.6\text{GHz}$. Also shown are the Hubble rates today and at recombination, as well as the frequency sensitivities of Pulsar Timing Arrays ($\sim 10^{-9} - 10^{-7}\text{Hz}$), LISA ($\sim 10^{-4} - 10^0\text{Hz}$) and LIGO ($\sim 10^1 - 10^3\text{Hz}$). The leading order EFT can safely describe everything between today and beyond recombination on cosmological scales, but may break down (receive order one corrections) in the LIGO band. In this realization, the cutoff of the partial UV completion is as high as $100\Lambda \sim 10^4\text{Hz}$, and can be trusted at LIGO frequencies.

**Scalar EFT:** Before jumping into the subtleties related with gravitational and dark energy theories, it is useful to get a sense of what happens in a simpler yet representative example. Take for instance the following shift-invariant scalar field low-energy effective field theory [16]

$$\mathcal{L}_M = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{2\Lambda^4}(\partial\phi)^4 + \mathcal{O}\left(\frac{(\partial^2\phi)^2}{M^2}\right),$$

where $M$ is the scale of new physics. In a standard EFT picture (that would not invoke a Vainshtein mechanism) we expect new physics to enter at $M \lesssim \Lambda$ to “save” the EFT from perturbative unitarity breaking that would otherwise occur at the scale $\Lambda$. Here $\phi$ could be a placeholder for the dark energy field and acquires a non-trivial profile $\phi(t)$. For instance, let us set $\langle \phi \rangle = \alpha\Lambda^2 t$ as the background and consider fluctuations of the field $\phi = \langle \phi \rangle + \delta\phi$, where for now we do not put any a priori restriction on the dimensionless coefficient $\alpha$. As is well known, on this spontaneously Lorentz-breaking background the speed of sound for the $\delta\phi$ fluctuations is

$$c_s^2 = 1 - \Delta_0 = 1 - \frac{4\alpha^2}{1 + 6\alpha^2}.$$  

We clearly see an order one deviation from luminal propagation if the parameter $\alpha$ is of order 1 and $c_s^2 \rightarrow 1/3$ if $|\alpha| \gg O(1)$. At this stage, we may wonder if we can trust a background configuration which occurs close to the strong coupling scale $\Lambda$ of the low-energy EFT. This question has been the subject of extensive work and we refer the reader to [17] for careful considerations. Here, we take the approach that the EFT can be re-organized as a derivative expansion in which, while the field gradient may be “large”, higher derivatives of the field are suppressed. This means that a profile with $\phi \sim \Lambda$ may be considered without going beyond the regime of validity of the EFT so long as higher derivatives are suppressed: $\partial^n\phi \ll \Lambda^{n+1} \lesssim \Lambda^{n+1}$ for any $n \geq 2$. We follow this approach here as it is the one used in the context of Horndeski models of dark energy. Concretely, this implies that we may consider background configurations with $\alpha \sim O(1)$ without necessarily needing to worry about the contributions of other irrelevant operators.

**Physics at the Cutoff:** Having established the (potential) consistency of the background, we now turn to that of perturbations. The model provided in (2) predicts a speed of sound (3) which appears to be the same irrespective of the frequency associated with the $\delta\phi$ fluctuations. Yet if we consider $\delta\phi$-waves at sufficiently high frequencies, they should be insensitive to the spontaneously Lorentz-breaking background ($\phi$). At sufficiently high energy we would expect Lorentz invariance to be restored [18] and hence the speed of propagation of the high-frequency $\delta\phi$ waves to be exactly luminal. The reason this behaviour is not manifest as yet is because we are working within the low-energy EFT (2) which is of course only consistent at frequencies much smaller than the EFT cutoff, $M$. Interestingly, in the context of the GW170817 detection, the frequency of the GWs were starting at about $24\text{Hz}$ and spanning towards a few hundred Hz, which is perilously close to the strong coupling scale associated with Horndeski dark energy models,

$$M \lesssim \Lambda_{\text{Horndeski}} \sim (M_{\text{Pl}}H_0^3)^{1/3} \sim 260\text{Hz}$$

where $H_0$ is related to the Hubble parameter today. At those scales, the EFT (2) can no longer be the appropriate description for the $\delta\phi$-waves, as we have neglected operators of the form $(\partial^2\phi)^2/\Lambda^2$, where $M$ is the cutoff [19]. The existence of such higher derivative operators cannot be ignored—they are mandated by positivity bounds if this theory is to admit a sensible Wilsonian UV completion [20, 21].

**Speed of Sound when Approaching the Cutoff:** The low-energy EFT (2) is appropriate when considering $\delta\phi$-waves at frequencies $k/M \ll 1$, however at higher frequencies one should include the irrelevant operators that naturally enter the EFT at the scale $M$ and hence
modify the dispersion relation,
\[ c_s^2(k) = 1 - \Delta_0 + \Delta_2 \frac{k^2}{\Lambda^2} + O \left( \frac{k^4}{\Lambda^4} \right), \tag{5} \]
where the running \( \Delta_2 \) is controlled by the higher order operators. This scale-dependence of the sound speed is unavoidable: not only are the next-to-leading order operators required in order to properly renormalization divergences within the EFT, they also naturally arise from a generic UV completion. Of course when reaching the scale \( M \), we lose control of the EFT and the precise details of the UV completion are essential in determining the sound speed of \( \delta \phi \)-waves (even if—as we have argued—the background configuration itself may not be sensitive to the UV completion).

To give a precise example of how the UV physics [22] may affect the sound speed at frequencies close to \( M \), consider the following specific situation where the massless scalar \( \phi \) couples to a heavy scalar \( \chi \) via,
\[ \mathcal{L}_{\Lambda_*} = -\frac{1}{2} (\partial \phi)^2 - \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} M^2 \chi^2 + \frac{\chi}{\Lambda_*} (\partial \phi)^2 \tag{6} \]
where \( \chi \) becomes dynamical around \( M \) and strongly coupled at a scale \( \Lambda_* \). For this two-field model to represent a (partial) completion of the low-energy EFT (2) with an extended region of validity, we require the scale hierarchy \( \Lambda_* \gg M \) which implies
\[ M \ll \Lambda = (M \Lambda_*)^{1/2} \ll \Lambda_* , \tag{7} \]
and we hence see that even though the low-energy EFT (2) only becomes strongly coupled at the scale \( \Lambda \), its cutoff (within this realization) is in fact even smaller \( M \ll \Lambda \) (this hierarchy also appears in the case of Galileons [21] and massive gravity [23]). Integrating out \( \chi \) at tree level gives the EFT (2) with additional higher order operators
\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + \frac{1}{2\Lambda^2} (\partial \phi)^4 \tag{8} \]
\[ -\frac{1}{\Lambda^4} (\partial \phi)^2 \left( \frac{\Box / M^2}{1 - \Box / M^2} \right)^2 (\partial \phi)^2 , \]
and so this two-field theory (6) represents a partial UV completion of the single field theory, lifting the cutoff from \( M \) to \( \Lambda_* \). Including the irrelevant operators that are generated from the high energy theory, we directly see a modification to the dispersion relation
\[ \omega^2 = k^2 - \frac{8\alpha^2}{1 + 2\alpha^2} \frac{\omega^2 M^2(\omega^2 - k^2 + M^2/2)}{(\omega^2 - k^2 + M^2/2)^2} \tag{9} \]
\[ = \frac{1 + 2\alpha^2}{1 + 6\alpha^2} k^2 + \mathcal{O}(k^4/M^2) \tag{10} \]
and clearly we recover the same leading order sound speed as from the EFT (2) at sufficiently small frequencies \( k \ll M \), but at frequencies \( M \ll k(\ll \Lambda_*) \) the sound speed is luminal. The exact behavior of the sound speed as a function of frequency for various values of \( \alpha \) is depicted in Fig. 1. Since the consistency of the two-field model requires the hierarchy between the scales (7), for concreteness we can imagine an example where \( M = 10^{-2} \Lambda \), so that the partial UV completion (6) remains a valid description up to the scale \( \Lambda_* = 100 \Lambda \).

In that case if we were to draw an analogy with the frequencies observed at LIGO (i.e. starting at about 24Hz), and considering the scale \( \Lambda \) to be given by about 260Hz as in eqn. (4), then \( k_{\text{LIGO}} > 10^{-1} \Lambda \sim 10 M \), and we clearly see from Fig. 1 that at those frequencies we expect the sound speed to be luminal, despite the low-frequency sound speed being potentially significantly subluminal. It is worth noting that these scales should be taken with a grain of salt—they are merely provided to illustrate the point in this simple scalar field model.

**Horndeski EFT:** We now turn to Horndeski as a dark energy EFT. As is well-known, the scalar field present in Horndeski can in principle play the role of a dark energy fluid driving the late-time acceleration of the Universe. In doing so, the Universe is filled with a medium (the dark energy condensate) which in turn affects the speed of propagation of gravitational waves (without affecting those of the other massless particles such as photons). For illustration purposes, consider the following Horndeski dark-energy model,
\[ \mathcal{L}_H = \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} G^{ab} \partial_a \phi \partial_b \phi , \tag{12} \]
with
\[ G^{ab} = g^{ab} + c_2 \frac{M_{\text{Pl}}}{\Lambda^3} G^{ab} + c_3 \frac{M_{\text{Pl}}}{\Lambda^6} L^{ab} \nabla_\mu \nabla_\nu \phi , \tag{13} \]
\( G_{ab} \) being the Einstein tensor and \( L_{\mu\nu} \), the dual Riemann tensor, and we have defined the scale \( \Lambda \) as \( (H_0^2 M_{\text{Pl}})^{1/3} \) as given in (4). The accelerated solution is given by a non-trivial configuration for \( \phi, \langle \phi \rangle = \alpha M_{\text{Pl}} H_0 t \), leading to an accelerated expansion with Hubble parameter \( H = \beta H_0 \), where the coefficients \( \alpha \) and \( \beta \) are determined in terms of \( c_2 \) and \( c_3 \) and are of order one when \( c_{2,3} \) are order unity. There is a region in parameter space for \( c_{2,3} \) where the accelerated solutions are stable (no ghost nor gradient instabilities). In order to understand the scales involved, it is useful to normalize the metric fluctuations \( g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}/M_{\text{Pl}} \), so that the term proportional to \( c_2 \) and \( c_3 \) enters at the scale \( \Lambda \),
\[ \mathcal{L}_H \supset (\partial h)^2 + (\partial \phi)^2 + (\partial \delta \phi)(\partial \delta h) + \frac{1}{\Lambda^2} \partial^2 h(\partial \delta \phi)^2 , \tag{14} \]
At first sight we may have suspected that the terms proportional to \( c_2 \) and \( c_3 \) in (12) also lead to operators that enter at a much lower scale, for instance \( \langle \phi \rangle \partial^2 h \partial \delta \phi / \Lambda^3 \sim \partial^2 h \partial \delta \phi / H_0 \), however such an operator is a total derivative thanks to the features of the Horndeski EFT.
In this model, the tensor modes have a well-known subluminal sound speed at low frequency,

\[ c_T^2(k) = 1 - \frac{2c_2\alpha^2\beta^2 + 6c_3\alpha^3\beta^3}{2 + c_2\alpha^2\beta^2 + 6c_3\alpha^3\beta^3} + \mathcal{O}\left(\frac{k^2}{M^2}\right), \quad (15) \]

which is only valid for \( k \ll M \), where \( M \) is the cutoff of the theory. In a typical EFT, we expect the cutoff to be at most the strong coupling scale, but it may be lower [24], \( M \lesssim \Lambda \). In what follows, we explore the possibility that Horndeski be a “standard” effective field theory with a cutoff at or below \( \Lambda \) [25].

As was already the case for the scalar field theory presented previously, the existence of a UV completion mandates the existence of other irrelevant operators in addition to the Horndeski ones. Precisely which operators we would expect from the UV completion, it would if those were at all representative of the types of operators that would typically enter depends on the UV completion and within an EFT approach one should allow for all operators to be present. However, for concreteness, we present here a class of operators that would return to luminal. Indeed the operators presented previously, the existence of a UV completion mandates that Horndeski be a “standard” effective field theory at a scale \( M \lesssim \Lambda \),

\[ \mathcal{L}_{\text{higher-der}}^{(n)} = (M^2_0 G_{\mu\nu}) \frac{\Box^n}{M^{2n+4}_n} \partial^\mu \phi \partial^\nu \phi, \quad (16) \]

with \( n \geq 2 \) and appropriate scales \( M_n \), which we now study. First, notice that such operators affect the background solutions by an amount proportional to

\[ \frac{\mathcal{E}^{(n)}}{\mathcal{E}_H} \sim \frac{H_0^{2(n-1)} \Lambda^6}{M^{2n+4}_n}, \quad (17) \]

where symbolically \( \mathcal{E}^{(n)} \) is the contribution from \( \mathcal{L}^{(n)} \) to the background equations of motion and \( \mathcal{E}_H \) that from the Horndeski Lagrangian (12). Trusting the background provided by the Horndeski EFT (12) requires this ratio to be small. So in principle the scale of the higher derivative operator \( \mathcal{L}^{(n)} \) could be as small as say \( M_n^{2n+4} \sim H_0^{2n-4} \Lambda^8 \ll \Lambda^{2n+4} \) and these operators would still not significantly affect the background. Furthermore, on this background the higher derivative terms (16) lead to operators that scale as (for \( n \geq 2 \))

\[ \mathcal{L}^{(n)} \sim \frac{\Lambda^6 (\Box^n h)^2}{M^{2n+4}_n} \left( 1 + \frac{\partial^2 h}{\Lambda^2} \left[ 1 + \mathcal{O}\left(\frac{h}{M_1}, \frac{H_0}{\partial} \right) \right] \right) \frac{\Lambda^3 (\partial^{n+1} h)^2}{H_0 M^{2n+4}_n} \left[ 1 + \mathcal{O}\left(\frac{h}{M_1}, \frac{H_0}{\partial}, \frac{H_0 h \partial \delta \phi}{\Lambda^3} \right) \right], \quad (18) \]

so if those were at all representative of the types of operators we would expect from the UV completion, it would mean that the Horndeski EFT (12) can be trusted until the strong coupling scale \( \Lambda_* \),

\[ \Lambda_* = \min_n \left( M_n^{2n+4} H_0 \Lambda^{-3} \right)^{1/(2n+2)}. \quad (19) \]

It will depend on the precise UV completion whether all the scales \( M_n \) are the same order (maybe all set to \( \Lambda \) or a lower scale \( M \)) or whether they scale so that \( \Lambda_* \gtrsim \Lambda \).

For now we simply point out that we have a large deal of flexibility in the scales \( M_n \), they could even potentially be much lower and as we have seen not affect the background evolution. Yet they will affect the speed of gravitational waves at scales close to \( \Lambda \). Indeed, if one considers the following,

\[ \mathcal{L} = \mathcal{L}_H + \sum_{n \geq 2} c_n \mathcal{L}_{\text{higher-der}}^{(n)} \quad (20) \]

the higher derivative operators lead to modifications of the gravitational wave dispersion relation which would (symbolically) scale as follows

\[ \omega^2 \sim (\omega_k^2(0) k^2 + O(H_0^2)) \]

\[ + \sum_{n \geq 2} \frac{c_n \Lambda^6}{3 M_n^{2n+4}} \left( -\omega^2 + k^2 \right)^{n-1} \left( \omega^2 + O(k^2, H_0^2) \right), \]

where \( \omega_k^2(0) \) is given by (15), and at frequencies close to \( M_n \) the term proportional to \( -\omega^2 + k^2 \) pushes the speed of gravitational waves arbitrarily close to unity.

**Conspiracy vs Lorentz-Invariant UV completion**: The fact that the cutoff of Horndeski is relatively close to the frequencies observed by LIGO (and particularly for the GW170817 event) was already noticed in [6], who pointed out that from a bottom-up approach it would seem extremely unlikely that order one effects that enter at the cutoff conspire to precisely cancel \( c_T^2 - 1 \) within an accuracy of one part to the \( 10^{15} \). From the point of view of the low energy EFT this would indeed appear surprising. However from a top-down approach, it is very unlikely that the UV completion knows anything about the special structure of the Lorentz-breaking background. Quite the opposite, we expect that at sufficiently large energies modes should be insensitive to the spontaneous Lorentz-breaking cosmological solution and we would naturally expect a speed of propagation that returns to luminal. Indeed the operators presented in (16) (or the partial completion (6) in the scalar case) have in no way been tuned so as to precisely cancel \( c_T^2 - 1 \). Rather the operators simply satisfy Lorentz invariance and at sufficiently high energy that symmetry should be restored for the gravitational modes as well. What we mean by “sufficiently” high energy depends on the context, but for the GW170817 event, it is not inconceivable (and one may even argue natural) that the pure Horndeski theory (eg. (12)) has broken down by that scale and the speed of gravitational waves for those observed frequencies has already returned to unity. It is important to notice that for the speed of GWs to be unity at LIGO’s frequencies, the break down of the low-energy EFT must have occurred at scales parametrically lower that \( \Lambda \).

**Horndeski and Modified Gravity**: Part of the motivation in studying Horndeski, and other related
EFTs, is that (within some regime of validity) these scalar-tensor theories can mimic the behaviour of some models of modified gravity. This was first pointed out within the context of the decoupling limit of DGP which was covariantized as a special Horndeski theory in [26]. The same can be achieved for instance for cascading gravity [27] and for massive gravity [28]. The initial study of Galileon scalar fields in [29] was indeed motivated as a way to capture some of the relevant physics of infrared models of modified gravity. Since some Horndeski EFTs arise from the decoupling limit of various theories of modified gravity, it is clear that Horndeski can be seen as an EFT with an infrared cutoff (of the order of the Hubble parameter today), as well as a UV cutoff and we could take the perspective that these models of modified gravity are in fact what (partially) “completes” those Horndeski theories. Interestingly in all these models of modified gravity, while the dispersion relation is modified at very low frequencies (of the order of the effective graviton mass), the sound speed remains luminal independently of the background configuration. This suggests that Horndeski EFTs could very easily be implemented within some completion for which the speed of gravitational waves as observed at LIGO frequencies are luminal within an impeccable precision. All such EFTs may still have a non-vanishing region of validity in the wake of GW170817. 

Gravitational Rainbows: Throughout this work, we have raised the possibility that the frequencies observed at LIGO are at the edge of (or even beyond) the regime of validity of the Horndeski EFT and shown how the speed of gravitational waves could be close to unity at those scales even though the low-energy EFT may predict a subluminal propagation. By no means do we suggest that every time an observation is performed, one should simply shield the EFT from any constraints by simply invoking a lower cutoff. The only reason we contemplate this possibility within the context of Horndeski and current observations from LIGO is that the frequencies observed are observed close to the expected cutoff if the EFT is to describe dark energy.

Turning towards future surveys, the upcoming LISA mission will have peak sensitivity near $10^{-3}$ Hz, at which scale $k/\Lambda \sim 10^{-5}$. If LISA were to bound the speed of gravitational waves with a similar precision as LIGO but at such low frequencies, it would be very hard for a Horndeski EFT to remain alive as a model of dark energy and still have an interesting regime of predictability.

Interestingly, modifications to the dispersion relation that would be induced by the higher derivative operators may also be sufficient to rule out these models altogether. Indeed, referring for simplicity to the scalar field example in Fig. 1, in the case where $M$ is not much smaller than $\Lambda$, the modification to the dispersion relations within the observable window of LIGO would be so dramatic that they would be sufficient to rule out such a possibility without the need of an optical counterpart, unless the transition between the low-energy and high-energy values of $c_T^2(k)$ happens extremely fast. If not, then for the example provided for Horndeski, it would require the higher derivative operators to enter at a scale at least $9$ orders of magnitude away from the observed scale so that the transition between the low energy modes and the high energy modes is well passed and settled once we get to the frequencies within reach of LIGO and the modification to the dispersion relation is minimal.

**Outlook for the EFT of Dark Energy:** In one of its simplest formulations, the EFT of dark energy has only four free functions of time, (in addition to those that determine the background cosmological history), [30]. One of those free functions (which we may call $m_4$) can be seen to be directly related to the speed of gravitational waves. While recent observations have been extremely successful at reducing the large parameter space and in particular forcing $m_4$ to vanish, through this work we simply point out that those quantities are (to some extent) scale-dependent (in addition to their time dependence) and the current constraints set $m_4(k_{\text{LIGO}}) = 0$ which may not necessarily imply that $m_4(k = 0) = 0$ or $m_4(k \sim H_0 \lesssim 10^{-20}k_{\text{LIGO}}) = 0$.

Throughout this work we have focused on a picture where the low-energy EFT has a Lorentz invariant UV completion and where physics may enter even before the strong coupling scale so as to restore perturbative unitarity. We stress however that even if the UV completion were to be manifestly Lorentz breaking, one would not expect the scale of Lorentz breaking at high energy to be linked to the scale of spontaneous Lorentz breaking at low energy and thus we would still expect a running of the speed of gravitational waves (if its value in the low-energy EFT departed from one).

We emphasize that the aim of this work is not to revive Horndeski or any specific EFT as a particular model for dark energy. Rather the aim is to bring across the subtleties related with measurements such as the sound speed when dealing with EFTs, especially when the effective cutoff may be relatively low and comparable to the scale associated with the measurement. In the coming age of precision cosmology, correctly interpreting what EFT corrections mean for these measurements will be more important than ever before and go a long way in discriminating between different classes of models.

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[15] Strictly speaking, the front velocity is defined as the speed of the front of a disturbance [31, 32]. In practice, the front velocity is defined as the high frequency limit of the phase velocity [31–33],

\[
\frac{c_{\text{front}}} = \lim_{k \to \infty} c_{\text{phase}}(k) \quad (22)
\]

[16] The argument goes through essentially unaffected if instead of (2) we had chosen an arbitrary function \( \Lambda^4 P(\phi) \phi^2 / \Lambda^4 \). We chose the particular form of (2) for concreteness.

[17] C. de Rham and R. H. Ribeiro, JCAP 1411, 016 (2014), arXiv:1405.5213 [hep-th].
[18] Even if the UV completion was not Lorentz invariant, it would be surprising that high energy physics knows about the scale of the spontaneously Lorentz breaking background.
[19] Note that the existence of higher derivative operators in this EFT should not be confused with the existence of an Ostrogradsky ghost. Indeed, higher derivative operators naturally enter from integrating out heavy degrees of freedom, and just manifest the fact that the EFT breaks down at the cutoff scale.
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[21] Lavinia Heisenberg, Atsushi Naruko, Andrew Tolley, and S.-Y. Zhou, JHEP 09, 072 (2017), arXiv:1702.08577 [hep-th].
[22] P(X) theories such as (2) can be seen as the low energy EFT of a high energy U(1) theory, broken to nothing as one integrates over the (massive) radial component. In that case the completion could be renormalizable and could then be a complete (rather than partial) UV completion of (2). For the case of Horndeski, it will certainly not be our aim to find a renormalizable completion and for simplicity we shall consider (2) as a potential completion here. We thank Paolo Creminelli and Filippo Vernizzi for pointing this out.
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[24] In theories that admit a Vainshtein mechanism [34, 35], we may hope to be able to trust the theory at scales of order \( \Lambda \) and to invoke a Vainshtein redressing to push the regime of validity of the theory to higher scales, however the Vainshtein redressing is negligible for the physical setup considered here.
[25] A skeptic reader may worry about an EFT with such a low cutoff of the order of \( 10^{-13} \text{eV} \) when GR is clearly valid and predictive over a much broader set of scales. Yet we should bear in mind that such a theory is typically introduced to tackle dark energy and would be valid from a scale of the order \( 10^{-33} \text{eV} \) that is at 20 orders of magnitude lower than that cutoff.
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