Controlling 3-d model of two exoskeleton links with variable length

A Borisov¹,², V Borisova², L Konchina¹, and K Maslova¹

¹Branch of the "National Research University "MPEI" in Smolensk, Energeticcheskiy proezd, house 1, Smolensk, 214013, Russia
²Smolensk State Agricultural Academy, st. Bolshaya Sovetskaya, 10, Smolensk, 214000, Russia

E-mail: ³borisowandrej@yandex.ru

Abstract. 3-D models of one-link and two-link exoskeletons with variable-length links are considered. The construction of a multi-link model is done from a simple one to a complex one. The 3-D analysis of the one-link model with one fixed point is done at first. After that, the two-link model with variable-length links is considered. It has been found that increasing the number of links leads to difficulties in composing the system of differential equations of motion relating to a longer time required for building it, even if the modern systems of computer mathematics are used. The proposed link model can be used for designing the working exoskeleton or anthropomorphic robot. The exoskeleton link consists of two absolutely rigid weighty sections at the link ends and of a weightless section between them. The analytically synthesised trajectories of the exoskeleton motion are given. The problem of finding controlling torques and forces is solved numerically and the solution results are presented graphically. The application of the proposed variable-length link model to an exoskeleton will reduce the load on the user’s joints, increase the user’s comfort and the force applied by the exoskeleton operator, as well as the period of its uninterrupted operation.

1. Introduction

The controlled motion simulation of anthropomorphic systems is currently important and actual considering the ample scope of their possible use in studying the characteristics of human musculoskeletal system, developing exoskeletons and anthropomorphic robots. Nevertheless, the problems of effective controlling at present are not fully solved. The kinematic method of controlling the motion of anthropomorphic system consisting of two variable-length links is developed in this article. The simulation results can be applied in developing the telescopic robotic arm. Developing exoskeletons is pressing and demanded domain in scientific research and engineering work [1-16]. The practical tasks solved by the proposed model are as follows: increasing the comfort for exoskeleton users, partial unloading the muscles of human musculoskeletal system, increasing the operating period for exoskeleton users, and rehabilitation of the human motor activity. In this article, the 2-D model of the variable-length link proposed in the paper [1-2] is generalized to 3-D case.

2. The model of one exoskeleton link of variable length

Consider the model of one fixed exoskeleton link of variable length in space. Let’s introduce the fixed right-hand Cartesian system of coordinates Oxyz, in which the mechanism motion takes place (Figure 1.).
Consider the link model consisting of the two weighty absolutely rigid parts performing the relative motion along the $A_1A_2$ line passing through the beginning and the end of the link (Figure 1). The combination of two cylindrical hinges enabling the link rotation in the two mutually orthogonal directions (Figure 1) is situated in the point $A_1$ firmly fixed with the supporting surface. The movement of the $C_1A_2$ section relative to the $A_1B_1$ section along the direction $A_1A_2$ takes place under the gravity and reactions from the neighbouring rods (not shown on the Figure 1). Thus, the length change of the link on the section $B_1C_1$ is implemented.

The system has two weighty absolutely rigid rods: $A_1B_1$ and $C_1A_2$. The Figure 1 schematically shows the link $A_1A_2$ and the corresponding notation. The lengths of the links are as follows $A_1B_1 = l_{11}$, $C_1A_2 = l_{12}$. The double indexing relates to the building of the multi-link exoskeleton model: the first index $i$ corresponds to the link number, the second index $\alpha$ corresponds to the number of the weighty section on the link. The change of the link length is implemented due to the relative motion of the $C_1A_2$ section along the $A_1A_2$ link. The variable-length section $B_1C_1$ is considered weightless. It has a spring applying the force $F_1$ which ensures the change of the link length in accordance with the load applied to the link from the other links.

The position of the weighty link section depends on three parameters and can be unambiguously defined by the angles $\phi_1(t)$, $\psi_1(t)$ and by the variable length of the link $\xi_1(t)$. The considered system has three degrees of freedom. The controlling torques applied in the hinge $A_1$ are denoted as $M_{1\phi}$ and $M_{1\psi}$. The link rotation can be controlled, for example, by electric motors with reduction gears or by artificial muscles. The lengthwise force $F_1$ controls the change of the link length. The pneumatic or hydraulic cylinder can be used for implementing the controlled change of the link length. The stepper electric motor with screw-type or rack and pinion gears can be also used. The controlling devices allowing changing the mechanism configuration are not specified in this model. They are simulated with the torques $M_{1\phi}$ and $M_{1\psi}$ concentrated in the hinges and the lengthwise force $F_1$, applied along the link.

The kinetic energy of the link is the sum of the kinetic energies of the sections $A_1B_1$ and $C_1A_2$

$$ T = T_{A1B1} + T_{C1A2}.$$ 

The differential equations of motion composed based on the Lagrange equation of the second kind are as follows:

$$ 2T = (\xi_1^2)m_{12}l_{11}l_{12} + (l_{11}^2 + l_{12}^2 + m_{12}(l_{11}^2 + 2\xi_1l_{11} + \xi_1^2 + \xi_1^2 + \xi_1^2))(\phi_1^2 \cos^2 \psi_1 + \psi_1^2) \quad (1) $$

Figure 1. The model of the exoskeleton link of variable length in space.
Thus, the system of differential equations of motion, describing the 3-D variable-length link model of exoskeleton has been composed.

3. The 3-D model of two exoskeleton links of variable lengths

Let’s introduce the fixed right Cartesian coordinate system \( Oxyz \), in which the mechanism motion takes place (Figure 2).

![Figure 2. The 3-D model of two mobile exoskeleton links of variable lengths.](image)

Consider the mechanism model consisting of two variable-length links. Each link consists of two absolutely rigid parts performing relative motion along the line passing through the beginning and the end of the link (Figure 2). Similarly to the one-link model, the combination of the cylindrical hinges is available in the firmly fixed point \( A_1 \). The following link sections are considered absolutely rigid: \( A_1B_1 = l_{11} \) with mass \( m_{11} \) and with the moment of inertia relative to the axis perpendicular to the link and...
passing through it beginning \( I_{11} \), and \( C_1A_2 = l_{12} \) with inertial characteristics \( m_{12}, I_{12} \). Under the action of the internal controlling force \( F_1 \), the required link length change is ensured on the weightless section \( B_1C_1 = \xi_1(t) \) of variable length. This change is implemented owing to the movement of the section \( C_1A_2 \) relative to the section \( A_1B_1 \) along the \( A_1A_2 \). The point \( A_2 \) is selected as the pole for the second link \( A_2A_3 \). The design of the second links as well as its work are similar with those of the first link.

The mechanism position depends on six parameters and can be unambiguously defined by the angles \( \phi_1(t), \psi_1(t), \phi_2(t), \psi_2(t) \) and by the variable lengths of the link sections \( \xi_1(t) \) and \( \xi_2(t) \). Therefore, the considered mechanical system has six degrees of freedom.

The kinetic energy of the mechanism can be obtained by integrating the weighty sections of the inertial parts \( A_1B_1, C_1A_2, A_2B_2, \) and \( C_2A_3 \)

\[
2T = \sum_{i=1}^{2} \sum_{\alpha=1}^{2} \int_0^{l_{1\alpha}} V_{1\alpha}^2 \rho_{1\alpha} \, dx_{i\alpha}.
\]  

(5)

In this formula, \( x_{i\alpha} \) is the coordinate of the infinitesimally small particle of the \( \alpha \)-th inertial section of the \( i \)-th link, \( \rho_{1\alpha} \) – the density of the \( \alpha \)-th section of the \( i \)-th link, foregoing \( m_{1\alpha} = \rho_{1\alpha} l_{1\alpha} \), \( I_{1\alpha} = \rho_{1\alpha} l_{1\alpha}^3/3 \), where \( l_{1\alpha} \), \( m_{1\alpha} \), \( I_{1\alpha} \) – the length, the mass, the moment of inertia respectively of the \( \alpha \)-th section of the \( i \)-th link, \( V_{1\alpha}^2 \) – the square of the velocity of the infinitesimally small particle of the \( \alpha \)-th section in the \( i \)-th link. For example, for the section \( C_1A_2 \) this value is as follows:

\[
V_{12}^2 = \dot{x}_1^2 + (l_{11} + \xi_1 + x_{12})^2 (C_1^2 \dot{x}_1^2 + \dot{x}_2^2).
\]

(6)

The system of differential equations of motion in the form of Lagrange equations of the second kind is listed below.

\[
\begin{align*}
\zeta_{11}[(C_1^\psi)^2 \ddot{\phi}_1 - 2C_1^\psi S_1^\psi \dot{\phi}_1 \dot{\psi}_1] + \zeta_{12}(C_1^\psi)^2 \ddot{\xi}_1 \dot{\phi}_1 - \\
- \eta_{11}[C_1^\psi C_2^\psi \ddot{\phi}_2 + C_1^\psi S_2^\psi \ddot{\phi}_2 + C_1^\psi C_2^\psi S_1^\psi \dot{\phi}_2 + C_1^\psi S_2^\psi S_1^\psi \dot{\phi}_2] + \\
+ C_1^\psi S_1^\psi S_2^\psi \ddot{\psi}_2 - 2C_1^\psi S_2^\psi C_2^\psi \dot{\phi}_2 \dot{\psi}_2] - \\
- \eta_{12}[C_1^\psi C_2^\psi S_1^\psi \ddot{\xi}_2 \dot{\phi}_2 + 2C_1^\psi C_2^\psi \ddot{\xi}_2 \dot{\phi}_2 + 2C_1^\psi S_2^\psi S_1^\psi \ddot{\phi}_2 \dot{\psi}_2] = M_1\phi,
\end{align*}
\]

(7)

\[
\begin{align*}
\zeta_{21}[C_1^\psi C_2^\psi \ddot{\phi}_1 - C_1^\psi S_1^\psi \ddot{\phi}_1] - S_1^\psi C_2^\psi S_1^\psi \ddot{\psi}_1 - \\
- \eta_{21}[C_1^\psi C_2^\psi \ddot{\phi}_1 - C_1^\psi S_1^\psi \ddot{\phi}_2 - 2C_1^\psi S_1^\psi C_2^\psi \dot{\phi}_1 \dot{\psi}_1] + \\
+ 2\eta_{22}[C_1^\psi C_2^\psi \ddot{\xi}_2 \dot{\phi}_2 - C_2^\psi S_1^\psi \ddot{\phi}_2 + 2C_2^\psi S_1^\psi \ddot{\phi}_2 \dot{\psi}_2] + \\
+ \eta_{22}[C_1^\psi C_2^\psi S_1^\psi \ddot{\xi}_2 \dot{\phi}_2 + 2(C_2^\psi)^2 \ddot{\xi}_2 \dot{\phi}_2] = M_2\phi,
\end{align*}
\]

(8)

\[
\begin{align*}
\zeta_{11}[\ddot{\psi}_1 + C_1^\psi S_2^\psi \ddot{\phi}_2] + 2\zeta_{11} \ddot{\xi}_1 \dot{\phi}_1 - \eta_{21}[S_1^\psi C_2^\psi S_1^\psi \ddot{\phi}_2 + (C_1^\psi C_2^\psi + \\
S_1^\psi S_2^\psi C_1^\psi \ddot{\phi}_2] + \\
+ S_1^\psi C_2^\psi C_2^\psi \ddot{\phi}_2 + (S_1^\psi C_2^\psi C_2^\psi - C_1^\psi S_2^\psi \ddot{\phi}_2 + 2S_1^\psi S_2^\psi S_1^\psi \ddot{\phi}_2)%
\end{align*}
\]

(9)
4. Controlling the motion of the two-link exoskeleton model

The initial lengths of non-deformed links are as follows: \( l_1^* = 0.385\) m, \( l_2^* = 0.477\) m. The lengths are distributed over the link as follows: \( l_1 = 0.15\) m, \( \xi_1 = 0.085\) m, \( l_2 = 0.2\) m, \( \xi_2 = 0.077\) m, \( l_1 = l_2 = (i = 1, 2)\). The moments of inertia of the weighty parts of the links relative to the axes passing through the bottom points of the weighty link parts are as follows: \( I_{11} = 0.011\) kg m², \( I_{21} = 0.060\) kg m², \( I_{12} = I_{22} = (i = 1, 2)\). The acceleration due to gravity is \( g = 9.81\) m/s². The period during which the single-supporting step phase
takes place, i.e. half of the walking period is $t_k = 0.36$ s. The coefficient of the link length change is $l = 0.25$. The walk parameters are $j_1 = j_2 = 0.25$, $f_1 = \pi/2$, $f_2 = 0.687$.

The method of controlling the exoskeleton motion based on the analytically specified kinematic parameters of its motion, which is described in the papers [5-7], is used in this research. This method has been modified for the considered model.

$$\psi_1(t) = \pi/2 + j_1 \sin(f_1 - (1 - \cos[2\pi t / T]) \pi/2), \quad \psi_2(t) = \pi/2 + j_2 \cos[f_2 - (1 - \cos[2\pi t / T]) \pi/2],$$

$$\phi_1(t) = j_1 \sin[2\pi t / T], \quad \phi_2(t) = j_2 \cos[2\pi t / T], \quad l_1(t) = l_1 + l_1 \sin[2\pi t / T], \quad l_2(t) = l_2 + l_2 \sin[2\pi t / T].$$

In these formulas $T$ is the walking period, $j_i$ and $f_i$ are the walk parameters, $l_i$ is the initial length of the non-deformed link, $l$ is the coefficient of the link length change.

The analytical expressions (13) and the values of the walk parameters for them are specified based on the condition of synthesizing the anthropomorphic periodic walk.

The specified kinematic parameters of the mechanism motion with their first and second derivatives are shown on the Figure 3-5.

**Figure 3.** The curves representing the rotation angles and the link length as functions of time.
Figure 4. The curves representing the angular velocities and linear velocity of the link as functions of time.

Figure 5. The curves representing angular accelerations and linear acceleration of the link as functions of time.

The listed below graphs of controlling torques (Figure 6) and lengthwise forces (Figure 7) have
been obtained as solution results of the inverse problem of dynamics if the kinematic of motion is specified by (13).

![Graph showing controlling torques in exoskeleton hinges-joints as functions of time.](image1)

**Figure 6.** The curves representing the controlling torques in the exoskeleton hinges-joints as functions of time.

![Graph showing lengthwise forces applied along exoskeleton links as functions of time.](image2)

**Figure 7.** The curves representing lengthwise forces applied along the exoskeleton links as functions of time.

The kinogram frames of the pictographic animated visualization of the 3-D model motion of the anthropomorphic mechanism with the kinematics specified by (13) are shown on the Figure 8.

![Kinogram frames of 3-D exoskeleton model motion.](image3)

**Figure 8.** The kinogram frames of the 3-D exoskeleton model motion.
5. Conclusion. 
Thus, the problem of synthesizing the controlling actions for the given exoskeleton motion has been solved. The controlling method for 3-D model of two exoskeleton links has been proposed. The 3-D model of new generation of exoskeletons with variable-length links has been developed. These links are more comfortable in operation. The developed model can be directly used for designing the working exoskeleton prototype. The obtained results can be applied in designing an exoskeleton for enhancing the capabilities of physical labour workers, for rehabilitating patients’ motor activity in healthcare, for training purpose in sports and military. Furthermore, since the motions of exoskeleton, anthropomorphic robot, and human musculoskeletal system are described by the same equations, the proposed models can be applied to all of them. This considerably extends the variety of mechanisms that can be designed based on the developed models.

Acknowledgement
The reported study was funded by RFBR and Smolensk region, project number 19-48-670002

References
[1] Borisov A V, Rozenblat G M 2017 Matrix method of constructing the differential equations of motion of an exoskeleton and its control Journal of Applied Mathematics and Mechanics 81, pp 351-359.
[2] Borisov A V, Rozenblat G M 2018 Modeling the dynamics of an exoskeleton with control torques in the joints and a variable length of the links using the recurrent method for constructing differential equations of motion ISSN 1064-2307, Journal of Computer and Systems Sciences International, vol 57, No 2, pp 319-347.
[3] Borisov A V 2019 Two-dimensional and three-dimensional models of anthropomorphic robot and exoskeleton with links of variable length Proc. of 24th Int. Conf. ‘MECHANIKA 2019’. 17 May 2019 Kaunas University of Technology, Lithuania. pp 26-39.
[4] Borisov A V, Chigarev A V 2020 The Causes of a Change in the Length of a Person’s Link and Their Consideration When Creating an Exoskeleton. Biomedical Journal of Scientific and Technical Research. ISSN: 2574 -1241. Vol 25, Iss 1 P 18769-18771 DOI: 10.26717/BJSSTR.2020.25.004137
[5] Golubev Yu, Melkumova E 2018 Two-legged Walking Robot Prescribed Motion on a Rough Cylinder AIP Conference Proceedings 1959, 030009. Published by the American Institute of Physics; doi: 10.1063/1.5034589
[6] Mukharlyamov R G, Tleubergenov M I 2017 Control of System Dynamics and Constraints Stabilization. In: Vishnevskiy V., Samouylov K., Kozyrev D. (eds) Distributed Computer and Communication Networks. DCCN 2017. Communications in Computer and Information Science, vol 700. Springer, Cham. – p.p. 431-442.
[7] Jaume Llibre, Rafael Ramirez. (2016) Inverse Problems of Ordinary Differential Equations and Applications. Springer International Publishing Switzerland, 266 p.
[8] Kaspriovich I E 2016 Application of Constraint Stabilization to Nonholonomic mechanics 2016 2nd International Conference on Industrial Engineering, Applications and Manufacturing (ICIEAM), DOI 10.1109/ICIEAM.2016.7910921. IEEE Conference Publications.
[9] Piña-Martinez E, Rodriguez-Leal E 2015 Inverse Modeling of Human Knee Joint Based on Geometry and Vision Systems for Exoskeleton Applications Mathematical Problems in Engineering. Vol 2015, Article ID 145734, 14 pages http://dx.doi.org/10.1155/2015/145734
[10] Kumani D S Chaudhary H 2015 Hexahedron Point Mass Model and Teaching Learning Based Optimization for Balancing of Industrial Manipulators 2nd International and 17th National Conference on Machines and Mechanisms (iNaCoMM2015) by IIT Kanpur at IIT Kanpur. P. 28-36.
[11] Gupta V, Chaudhary H, Saha S K 2015 Dynamics and actuating torque optimization of planar robots // Journal of Mechanical Science and Technology. Vol 29 P 2699-2704
[12] Chaudhary K, Chaudhary H 2014 Optimum Balancing of Slider-Crank Mechanism Using Equimomental System of Point-Masses 2nd International Conference on Innovations in Automation and Mechatronics Engineering, ICIAME 2014 by Springer at G H Patel Coll. of Eng. & Tech., Vallabh Vidyanar, 12.

[13] Bortole M, Ama A, Rocon E 2013 A robotic exoskeleton for overground gait rehabilitation Proceedings of the IEEE International Conference on Robotics and Automation (ICRA ’13), May 2013. P. 3356-3361.

[14] Hassan M, Kadone H, Suzuki K, Sankai Y 2012 Exoskeleton robot control based on cane and body joint synergies Proceedings of the 25th IEEE/RSJ International Conference on Robotics and Intelligent Systems (IROS ’12), October 2012. P 1609-1614.

[15] Tsukahara A, Hasegawa Y, Eguchi K, Sankai Y 2015 Restoration of gait for spinal cord injury patients using HAL with intention estimator for preferable swing speed IEEE Transactions on Neural Systems and Rehabilitation Engineering. Vol 23, № 2. P 308-318

[16] Tsukahara A, Hasegawa Y, Sankai Y 2011 Gait support for complete spinal cord injury patient by synchronized leg-swing with HAL Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS ‘11), September 2011 P 1737-1742