Gravitational Radiation – Observing the Dark and Dense Universe

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Abstract

Astronomical observations in the electromagnetic window – microwave, radio and optical – have revealed that most of the Universe is dark. The only reason we know that dark matter exists is because of its gravitational influence on luminous matter. It is plausible that a small fraction of that dark matter is clumped, and strongly gravitating. Such systems are potential sources of gravitational radiation that can be observed with a world-wide network of gravitational wave antennas. Electromagnetic astronomy has also revealed objects and phenomena – supernovae, neutron stars, black holes and the big bang – that are without doubt extremely strong emitters of the radiation targeted by the gravitational wave interferometric and resonant bar detectors. In this talk I will highlight why gravitational waves arise in Einstein’s theory, how they interact with matter, what the chief astronomical sources of the radiation are, and in which way by observing them we can gain a better understanding of the dark and dense Universe.

1. Introduction

Einstein’s theory of gravity admits wave-like solutions that are in many ways similar to electromagnetic radiation but with important differences, two among them being most crucial: Universal, but weak, interaction and non-linearity. The former property has profound consequences: It implies that one cannot infer the influence of gravitational waves by watching an isolated particle in space, one would need at least two well-separated particles, just as one would in Einstein’s gedanken lift-experiment to infer the presence of the Earth’s gravitational field. The weakness of the gravitational interaction means that on the one hand it will be very difficult to observe them, but on the other the radiation carries the true signature of the emitting source, be it the core of a neutron star or a supernova, the quasi-normal mode oscillations of a black hole, or the birth of the Universe, thereby making it possible to observe phenomena and objects that are not directly accessible to the electromagnetic, neutrino or the cosmic-ray window. The latter property, namely the non-linearity of the waves, means that the waves interact with the source resulting in a rich structure in the shape of the emitted signals.
Therefore, gravitational radiation should also facilitate both quantitatively and qualitatively new tests of Einstein’s theory including the measurement of the speed of gravitational waves, and hence (an upper limit on) the mass of the graviton, polarisation states of the radiation, non-linear effects of general relativity untested in solar system or Hulse-Taylor binary pulsar observations, uniqueness of axisymmetric spacetimes, and so on.

The influence of gravitational radiation on an antenna can be characterized by a dimensionless amplitude which is a measure of the deformation caused by the wave as it passes through the detector. For instance, in an interferometric antenna of length $\ell$ a wave of amplitude $h$ causes a change in length $\delta \ell = h\ell/2$. Typical astronomical events, say a binary black hole merger at 100 Mpc, would have an amplitude $h \sim 10^{-23}$ at a frequency $\sim 100$ Hz and such events can be expected to occur at a once every few years. Nearer and/or stronger events could produce amplitudes that are several orders of magnitudes larger, but their event rate would be too low. The technology needed to observe such tiny amplitudes has become available only in the past decade or so. Many resonant bars and interferometers are currently taking data near their design sensitivity in the range $10^{-21} - 10^{-23}$ and should soon be in a position to observe some of the most violent phenomena in the Universe.

In this talk I will begin with a brief overview of gravitational wave (GW) theory and the interaction of the waves with matter and how that is used in the construction of the detectors. The main focus of this talk will be the astronomical sources, tests of general relativity, and astrophysical and cosmological measurements afforded by GW observations. Sec. 5 lists our choice of units and the conventions used in making estimates of source strengths.

2. Gravitational wave theory - A brief overview

Newtonian gravity is described by a scalar potential $\varphi(t, \mathbf{x})$, which obeys the Poisson equation, $\nabla^2 \varphi(t, \mathbf{x}) = 4\pi \rho(t, \mathbf{x})$, where $\rho(t, \mathbf{x})$ is the density distribution, whose formal solution is given by

$$\varphi(t, \mathbf{x}) = \int \frac{\rho(t, \mathbf{x}') \, d^3x'}{|\mathbf{x} - \mathbf{x}'|}. \quad (1)$$

The key point is that because the potential satisfies a Poisson equation there are no retardation effects. Since the same time $t$ appears both on the LHS and the RHS in the above equation, any change in the distribution of density at the source point $\mathbf{x}'$ would instantaneously change the potential at the remote field point $\mathbf{x}$.

2.1. Wave equation

In Einstein’s theory the metric components of the background spacetime are the gravitational potentials and they satisfy a “wave” equation and hence
there will be retardation effects. This is explicitly seen in the linearized version of Einstein’s equations. Under the assumption of weak gravitational fields one can assume that the background metric $g_{\alpha\beta}$ of spacetime to be only slightly different from the Minkowski metric $\eta_{\alpha\beta} = \text{Diag}(-1, 1, 1, 1)$: $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$. Here $h_{\alpha\beta}$ is a part of the metric that describes the departure of the spacetime from flatness. For weak gravitational fields, there exists a coordinate system in which each component of $h_{\alpha\beta}$ is numerically small compared to unity, i.e. $|h_{\alpha\beta}| \ll 1$. For this reason $h_{\alpha\beta}$ is termed as the metric perturbation. Furthermore, even when the source that produces the weak field is relativistic, Einstein’s equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is the energy-momentum tensor, reduce, on keeping only terms linear in $h_{\alpha\beta}$, to a set of wave equations for the metric perturbation: $\Box h_{\alpha\beta} = 16\pi T_{\alpha\beta}$, where $2\Box h_{\alpha\beta} \equiv 2h_{\alpha\beta} - \eta_{\alpha\beta}h_\mu^\mu$, and $\Box$ is the wave operator: $\Box \equiv \eta^{\alpha\beta}\partial_\alpha\partial_\beta$. These equations have the formal solution

$$h_{\alpha\beta}(t, x) = 4 \int \frac{T_{\alpha\beta}(t - |x - x'|/c, x') d^3x'}{|x - x'|}.$$  

(2)

In this solution, the metric perturbation at the field point $x$ at the time $t$ is determined by the configuration of the source $T_{\alpha\beta}$ at a retarded time $t - |x - x'|/c$ ($c$ being the speed of light), and hence disturbances in the source travel only at a finite speed. Indeed, any non-stationary source $T_{\alpha\beta}$ will give rise to wave-like solutions for the potentials $h_{\alpha\beta}$, which extract energy, momentum and angular-momentum from the source, propagating at the speed of light and have other attributes similar to electromagnetic waves.

2.2. Polarisation states and principle of detection

Although to begin with there are 10 independent components of the metric, because the theory is covariant under general coordinate transformations and invariant under gauge transformations, one can make a choice of coordinate system and gauge such that only two independent components of the metric are non-zero. Therefore, just as in electromagnetic theory, there are only two independent polarisations of the field, denoted as $h_+$ (h-plus) and $h_\times$ (h-cross).

When a wave of plus- or cross-polarisation is incident perpendicular to a plane containing a circular ring of beads, the ring is deformed in the manner shown in Fig. 11. Monitoring the distance from the centre of the ring to the beads at the ends of two orthogonal radial directions can best measure the deformation of the ring. This is the principle behind a laser interferometer antenna wherein highly reflective mirrors (losses $\sim 10^{-5}$) are freely suspended (quality factors $\sim 10^6$) at the ends of two orthogonal arms (length $\ell \sim$ km) inside vacuum tanks (pressure $\sim 10^{-8}$ mbar) and high power lasers (effective power of 10 kW) are used to measure extremely tiny strains ($\delta\ell/\ell \sim 10^{-21}$–$10^{-23}$ for transient bursts and $10^{-25}$–$10^{-27}$ for continuous wave sources observed over several months), in
Fig. 1. The response of a circular ring of free beads to waves of plus- (left) and cross-polarisation (right) as a monochromatic signal of period $P$. The ring (radius $C$) is continuously deformed into an ellipse (semi-major axis $(1 + h/2)C$, semi-minor axis $(1 - h/2)C$) after one quarter of the period. The configuration of the beads after a time $T = P/4, P/2, 3P/4$ is shown. Also depicted are the locations of the beam-splitter and mirrors that are freely suspended inside a vacuum tube in an interferometric detector.

A wide range (10–1000 Hz) of frequency. Gravitational wave interferometers are quadrupole detectors with a good sky coverage. Indeed, an interferometer will have 40% of the peak sensitivity over 40% of the sky area! A single antenna, except when it is a spherical resonant detector, cannot determine the polarisation state of a transient wave or the direction to the source that emits the radiation. Interferometers and resonant bars don’t measure the two polarisations separately but rather a linear combination of the two given by:

$$h(t) = F_+ (\theta, \varphi, \psi) h_+ (t) + F_\times (\theta, \varphi, \psi) h_\times (t),$$

where $F_+$ and $F_\times$ are the antenna patterns. To infer the direction $(\theta, \varphi)$ to the source, the polarisation amplitudes $(h_+, h_\times)$, and the polarisation angle $\psi$, it is necessary to make five measurements which is possible with three interferometers: Each interferometer gives a response, say $h_1(t)$, $h_2(t)$, and $h_3(t)$, and one can infer two independent delays, say $t_1 - t_2$, and $t_2 - t_3$, in the arrival times of the transient at the antennas. Therefore, a network of antennas, geographically widely separated so as to maximise the time delays and hence improve directionality, is needed for GW observations. Moreover, detecting the same event in two or more instruments helps to remove the non-Gaussian and non-stationary backgrounds, while adding a greater degree of confidence to the detection of an event. In the case of continuous waves the motion of the detector relative to the source causes a Doppler modulation of the response which can be de-convolved from the data.
to fully reconstruct the wave.

2.3. Amplitude, luminosity and frequency

The amplitude $h$ and luminosity $\mathcal{L}$ of a source of GW is given in terms of the famous quadrupole formula:

$$h_{mn}(t, r) = \frac{2}{r} \ddot{I}_{mn}(t - r), \quad \mathcal{L} = \frac{1}{5} \langle \dot{I}_{mn} \dot{I}^{mn} \rangle,$$

where an overdot denotes derivative with respect to time; angular brackets denote a suitably defined averaging process (say, over a period of the GW); $I_{mn}$ is the reduced (or trace-free) quadrupole moment tensor which is related to the usual quadrupole tensor $I^{mn} \equiv \int T^{00} x^m x^n \text{d}^3 x$, via $I_{mn} \equiv I_{mn} - \delta_{mn} I^k_k / 3$. In simple terms, for a source of size $R$, mass $M$ and angular frequency $\omega$, located at a distance $r$ from Earth,

$$h \sim \epsilon_h \frac{M}{r} R^2 \omega^2, \quad \mathcal{L} \sim \epsilon_\mathcal{L} M^2 R^4 \omega^6.$$

where $\epsilon_h, \mathcal{L}$ are dimensionless efficiency factors that depend on the orientation of the system relative to the observer (in the case of $h$ only) and how deformed from spherical symmetry the system is. $\epsilon_h, \mathcal{L} \sim 1$ for ideally oriented and highly deformed sources. The amplitude of the waves, just as in the case of electromagnetic radiation, decreases as inverse of the distance to the source. However, there is a crucial difference between EM and GW observations that is worth pointing out: Let $r_l$ be the largest distance from which an EM or a GW detector can observe standard candles. In the case of electromagnetic telescopes $r_l$ is limited by the smallest flux observable which falls off as the inverse-square of the distance. This is because astronomical EM radiation is the superposition of waves emitted by a large number of microscopic sources, each photon with its own phasing; we cannot follow each wave separately but only a superposition of many of them. This, of course, is the reason why in conventional astronomy the number counts of standard candles increase as $r_l^{3/2}$. In the case of GW, signals we expect to observe are emitted by the coherent bulk motion of large masses and hence it is possible to observe each cycle of the wave as it passes through the antenna. Indeed, one can fold many wave cycles together to enhance the visibility of the signal buried in noise, provided the shape of the signal is known beforehand. Because we can follow the amplitude of a wave the number of sources which an antenna can detect increases as $r_l^3$.

For a self-gravitating system, say a binary system of two stars of masses $m_1$ and $m_2$ (total mass $M = m_1 + m_2$ and symmetric mass ratio $\eta = m_1 m_2 / M^2$), the linear velocity $v$ and angular velocity $\omega$ are related to the size $R$ of the system via Kepler’s laws: $\omega^2 = M / R^3$, $v^2 = M / R$. It turns out that the efficiency factors
for such a system are $\epsilon_h = 4\eta C$, $\epsilon_L = 32\eta^2/5$, so that

$$h \simeq 4\eta C \frac{M M}{r R^3}, \quad \mathcal{L} \simeq \frac{32}{5} \eta^2 v^{10}, \quad f_{GW} = 2f_{orb}.$$  

(6)

where $C \sim 1$ is a constant that depends on the orientation of the source relative to the detector, $f_{GW}$ is the GW frequency which is equal to twice the orbital frequency $f_{orb}^*$. The above relations imply that the amplitude of a source is greater the more compact it is and the luminosity is higher from a source that is more relativistic. The factor to covert the luminosity from $G = c = 1$ units to conventional units is $L_0 \equiv c^5/G \simeq 3.6 \times 10^{59}$ erg s$^{-1}$. Since $v < 1$, $L_0$ denotes the best luminosity a source could ever have and generally $\mathcal{L} \ll L_0$.

2.4. *The Hulse-Taylor and other binaries*

As an illustration of these order-of-magnitude estimates let us consider the Hulse-Taylor binary pulsar PSR 1913+16 \[57\]. Discovered in 1974 the system consists of two neutron stars each of mass $1.4 M_\odot$, in a tight orbit with a period $P_b \sim 7.75$ Hrs at a distance of 5 kpc from Earth. The expected amplitude, luminosity and frequency of GW are $h \sim 6 \times 10^{-23}$, $\mathcal{L} \sim 1.4 \times 10^{29}$ erg s$^{-1}$ and $f_{GW} \sim 7.17 \times 10^{-5}$ Hz, respectively. Although the frequency of GW is beyond the reach of the current ground-based, and future space-based detectors, the emission of GW causes the orbital period $P_b = 2\pi M/v^3$ to decrease in course of time. Demanding that the energy dissipated into GW should be balanced by a loss in the binding energy $E = -\eta Mv^2/2$, i.e. $\mathcal{L} = -dE/dt$, we can deduce that

$$\dot{P}_b = -\frac{192\pi\eta}{5} \left(\frac{2\pi M}{P_b}\right)^{2/3}.$$  

(7)

In reality, since the binary pulsar is in an eccentricity orbit with ellipticity $e = 0.617$, the rate of change of the period is larger by a factor of 10 giving $\dot{P}^\text{GR}_b = (-2.4047 \pm 0.00002) \times 10^{-12}$ s s$^{-1}$ \[61\]. Thus, general relativity predicts that the period of the binary should change each year by $\Delta P_b \simeq -76$ micro seconds. By monitoring the binary pulsar for over 25 years it has been possible to measure $\dot{P}_b$ very accurately \[61\] $\dot{P}^\text{Obs}_b = (-2.4086 \pm 0.0052) \times 10^{-12}$ s s$^{-1}$ and it agrees with the theoretical estimate to better than 0.25%. The neutron stars in this system would spiral-in towards each other and eventually coalesce in about 300 million years. The radiation emitted during the last few minutes before coalescence of such systems would be the target of ground-based detectors.

Since 1974 two more compact binaries that would coalesce within the Hubble time have been discovered (cf. Table \[1\], B1534+12 \[63\] and J0737-3039 \[14\].

\*For a binary consisting of two equal masses the configuration of the system is identical on rotation by $\pi$, rather than $2\pi$, radians. This is the reason why the frequency of GW is twice the orbital frequency. In general, the wave would contain the orbital frequency and its harmonics with twice the orbital frequency being the dominant.
Table 1. The orbital period $P_b$, eccentricity $e$, derived masses of the pulsar $m_p$ and its companion $m_c$, the measured/expected rate of decay of the period $\dot{P}_b$ and time to coalescence $\tau$ of 3 binary pulsars that would coalesce within the Hubble time.

| Binary       | $P_b$/s | $e$   | $(m_p, m_c)/M_\odot$ | $\dot{P}_b/10^{-12}$ | $\tau$/Myr |
|--------------|---------|-------|----------------------|-----------------------|-------------|
| B1913+16     | 27907   | 0.617 | (1.44, 1.39)         | -2.40                 | 302         |
| B1534+12     | 36352   | 0.274 | (1.33, 1.35)         | -0.14                 | 2730        |
| J0737-3039   | 8835    | 0.0877| (1.24, 1.35)         | -1.24                 | 86          |

The binary pulsar J0737-3039 has particularly improved the prospect of detecting GW with the upcoming detectors. \[39\].

3. Gravitational Wave Detector Projects

There are chiefly two types of GW detectors that are currently in operation taking sensitive data: (i) resonant bars and (ii) laser interferometers. The sensitivity of a detector is defined in terms of the smallest discernible dimensionless strain caused by an astronomical source against background noise of the instrument. Because a GW antenna can follow the phase of GW, the sensitivity of an antenna is given in terms of the amplitude noise power spectral density as a function of frequency and is measured in Hz$^{-1/2}$. Fig. 2 shows in solid lines the design sensitivity goals of three generations of ground-based interferometers (shown here for the American initial and advanced LIGO, and a possible third generation European detector EURO). The inset shows the same for the space-based LISA. Also plotted in Fig. 2 are source strengths to be discussed in Sec. 4.

3.1. Bar detectors

Resonant bars operate in a narrow band of 10–50 Hz at a frequency of about 950 Hz (see Schutz \[53\] for a fuller description). In a bar detector the vibrations induced in a seismically isolated, cryogenic Aluminium or Niobium cylindrical bar is amplified using a transducer. There are currently five such detectors operating around the world, one in Australia (NIOBE), three in Italy (NAUTILUS \[19\], AURIGA \[20\], Explorer$^*$) and one in the US (ALLEGRO). Bar detectors are limited by background noises caused by internal thermal noise and the quantum uncertainty principle. Current detectors have a strain sensitivity of about $10^{-21}$ Hz$^{-1/2}$ and are mainly sensitive to supernovae in the neighbourhood of the Milkyway and in-band continuous wave sources.

$^*$The Explorer detector is operated by an Italian group but located in CERN
3.2. Ground-based Interferometers

Interferometers operate in a broad band (1 kHz) at a central frequency of 150 Hz (see Schutz [53] for a fuller description). In a laser interferometric antenna the tidal deformation caused in the two arms of a Michelson interferometer is sensed as a shift in the fringe pattern at the output port of the interferometer. The sensitivity of such a detector is limited at low frequencies (10–40 Hz) by anthropogenic sources and seismic disturbances, at intermediate frequencies (40–300 Hz) by thermal noise of optical and suspended components, and at high frequencies (> 300 Hz) by photon shot noise. Three key technologies have made it possible to achieve the current level of sensitivities: (1) An optical layout that makes it possible to recycle the laser light exiting the interferometer and build effective powers that are 1000’s of times larger than the input thereby mitigating...
the photon shot noise. This technique allows us to operate the interferometer either in the wide band mode (as in Fig. 2.), or with a higher sensitivity in a narrow band of about 10–50 Hz centered at a desired frequency, say 300 Hz, but at the cost of worsened sensitivity over the rest of the band. This latter mode of operation is called signal recycling and is particularly useful for observing long-lived continuous wave sources. (2) Multiple suspension systems that filter the ground motion and keep the mirrors essentially free from seismic disturbances. (3) Monolithic suspensions that help isolate the thermal noise to a narrow frequency band.

There are currently six long baseline detectors in operation: The American Laser Interferometer Gravitational-Wave Observatory (LIGO) [2], which is a network of three detectors, two with 4 km arms and one with 2 km arms, at two sites (Hanford, Washington and Livingston, Louisiana), the French-Italian VIRGO detector with 3 km arms at Pisa [17], the British-German GEO 600 [44] with 600 m arms at Hannover and the Japanese TAMA with 100 m arms in Tokyo [58]. Australia has built a 100 m test facility with a plan to build a km-size detector sometime in the future.

Plans are well underway both in Europe and the USA to build, by 2008, the next generation of interferometers that are 10–15 times more sensitive than the initial interferometers. With a peak sensitivity of $h \sim 10^{-24} \text{Hz}^{-1/2}$ these advanced detectors will be able to detect NS ellipticities as small as $10^{-6}$ in our Galaxy, BH-BH binaries at a redshift of $z \sim 0.5$, stochastic background at the level of $\Omega_{\text{GW}} \sim 10^{-8}$. In the longer term, over the next 6 to 10 years, we might see the development of 3rd generation GW antennas. The sensitivity of the current and next generation instruments is still far from the fundamental limitations of a ground-based detector: The gravity gradient noise at low frequencies due to natural (winds, clouds, earthquakes) and anthropogenic causes, and the quantum uncertainty principle of mirror position at high frequencies. A detector subject to only these limitations requires the development of new optical and cryogenic techniques that form the foundation of a third generation GW detector in Europe called EURO [28], whose expected noise performance is also shown in Fig. 2.

### 3.3. Space-based Interferometers

ESA and NASA have resolved to place three spacecraft in heliocentric orbit, 60 degrees behind the Earth, in an equilateral triangular formation of size 5 million km [7]. These craft constitute the Laser Interferometer Space Antenna (LISA) scheduled to fly in 2011. LISA’s sensitivity is limited by difficulties with long time-scale ($\lesssim 10^{-4}$ Hz) stability, photon shot-noise ($\sim 10^{-3}$ Hz) and large size ($> 10^{-1}$ Hz). LISA will be able to observe Galactic, extra-Galactic and cosmological point sources as well as stochastic backgrounds from different astrophysical populations and perhaps from certain primordial processes. In addition to LISA there have been proposals to build an antenna in the frequency gap of LISA.
and ground-based detectors. The Deci-Hertz Interferometer Gravitational-Wave Observatory (DECIGO) \cite{56} by the Japanese team and the Big-Bang Observer (BBO), a possible follow-up of LISA \cite{8}, are aimed as instruments to observe the primordial GW background and to answer cosmological questions on the expansion rate of the Universe and dark energy.

4. Sources of gravitational waves

Gravitational wave detectors will unveil dark secrets of the Universe by helping us to study sources in extreme physical environs: strong non-linear gravity, relativistic motion, extremely high density, temperature and magnetic fields, to list a few. We shall focus our attention on compact objects (in isolation or in binaries) and stochastic backgrounds.

4.1. Compact Binaries

Compact binaries, consisting of a pair of compact objects (i.e., NS and/or BH), are an astronomer’s standard candles \cite{52}: A parameter called the chirp mass $M \equiv \eta^{2/3} M$, completely fixes the absolute luminosity of the system. Hence, by observing GW from a binary we can measure the luminosity distance to the source provided the source chirps, that is the orbital frequency changes, by as much as $1/T$ during an observational period $T$. This feature helps to accurately measure cosmological parameters and their variation as a function of red-shift. The dynamics of a compact binary consists of three phases: (i) inspiral, (ii) plunge and (iii) merger. In the following we will discuss each in turn.

(i) The early inspiral phase: This is the phase in which the system spends 100’s of millions of years and the power emitted in GW is low. This phase can be treated using linearized Einstein’s equations and post-Newtonian theory with the rough energy balance between the binding energy and the emitted radiation (cf. Sec. 2.4.). The emitted GW signal has a characteristic shape with its amplitude and frequency slowly increasing with time and is called a chirp waveform. Formally, the inspiral phase ends at the last stable orbit (LSO) when the effective potential of the system undergoes a transition from having a well-defined minimum to the one without a minimum, after which stable orbits can no longer be supported. This happens roughly when the two objects are separated by $R \simeq 6 GM/c^2$, or when the frequency of GW is $f_{\text{LSO}} \simeq 4400 (M_\odot/M)$ Hz. The signal power drops as $f^{-7/3}$ and the photon shot-noise in an interferometer increases as $f^2$ beyond about 200 Hz so that it will only be possible to detect a signal in the range from about 10 Hz to 500 Hz (and a narrower bandwidth of 40–300 Hz in initial interferometers) during which the source brightens up half-a-million fold (recall that the luminosity $\propto v^{10} \propto f^{10/3}$). For $M \lesssim 10M_\odot$, inspiral phase is the only phase sensed by the interferometers and lasts for a duration of
\[ \tau = 5576 \eta^{-1} (M/M_\odot)^{-5/3} \text{ s}, \] starting at 10 Hz. The phase development of the signal is very well modelled during this epoch and one can employ matched filtering technique to enhance the visibility of the signal by roughly the square root of the number of signal cycles \( N_{\text{cyc}} \sim 16\tau \), starting at 10 Hz. Since a large number of cycles are available it is possible to discriminate different signals and accurately measure the parameters of the source such as its location (a few degrees each in co-latitude and azimuth) \[ 35 \], mass (fractional accuracy of 0.05–0.3% in total mass and a factor 10 worse for reduced mass, with greater accuracy for NS than BH), and spin (to within a few percents) \[ 22 \].

(ii) The late inspiral, plunge and merger phase: This is the phase when the two stars are orbiting each other at a third of the speed of light and experiencing strong gravitational fields with the gravitational potential being \( \varphi = GM/Rc^2 \sim 0.1 \). This phase warrants the full non-linear structure of Einstein’s equations as the problem involves strong relativistic gravity, tidal deformation (in the case of BH-BH or BH-NS) and disruption (in the case of BH-NS and NS-NS) and has been the focus of numerical relativists \[ 13 \] for more than two decades. However, some insights have been gained by the application of advanced mathematical techniques aimed at accelerating the convergence properties of post-Newtonian expansions of the energy and flux required in constructing the phasing of GW \[ 13 \[ 16 \[ 24 \]. This is also the most interesting phase from the point of view of testing non-linear gravity as we do not yet fully understand the nature of the two-body problem in general relativity. Indeed, even the total amount of energy radiated during this phase is highly uncertain, with estimates in the range 10% \[ 31 \] to 0.07% \[ 16 \]. Since the phase is not well-modelled, it is necessary to employ sub-optimal techniques, such as time-frequency analysis, to detect this phase and then use numerical simulations to gain further insights into the nature of the signal.

(iii) The late merger phase: This is the phase when the two systems have merged to form either a single NS or a BH, settling down to a quiescent state by radiating the deformations inherited during the merger. The emitted radiation can be computed using perturbation theory and gives the quasi-normal modes (QNM) of BH and NS. The QNM carry a unique signature that depends only on the mass and spin angular momentum in the case of BH, but depends also on the equation-of-state (EOS) of the material in the case of NS. Consequently, it is possible to test conclusively whether or not the newly born object is a BH or NS: From the inspiral phase it is possible to estimate the mass and spin of the object quite precisely and use that to infer the spectrum of normal modes of the BH. The fundamental QNM of GW from a spinning BH, computed numerically and then fitted, is \[ 27 \] \( f_{\text{QNM}} = 750[1 - 0.63(1 - a)^{0.3}](100M_\odot/M) \text{ Hz} \), with a decay time of \( \tau = 5.3/[f_{\text{QNM}}(1 - a)^{0.45}] \text{ ms} \), where \( a \) is the dimensionless spin parameter of the hole taking values in the range \([0, 1]\). The signal will be of the
form $h(t; \tau, \omega) = h_0 e^{-t/\tau} \cos(\omega t)$, $t \geq 0$, $h_0$ being the amplitude of the signal that depends on how deformed the hole is.

It has been argued that during the late stages of merger the energy emitted in the form of QNM might be as large as 3% of the system’s total mass \[31\]. By matched filtering, it should be possible to detect QNM resulting from binary mergers of mass in the range $60-10^3 M_\odot$ at a distance of 200 Mpc in initial LIGO and from $z \sim 2$ in advanced LIGO. In Fig. 2 filled circles (connected by a dotted line) show the amplitude and frequency of QNM radiation from a source at $z = 2$, and total mass 1000, 100 or 10 $M_\odot$. Such signals should serve as a probe to test if massive black holes found at galactic cores initially formed as small BHs of $10^3 M_\odot$ and then grow by rapid accretion. Moreover, there is a growing evidence \[33\] that globular clusters might host BH of mass $M \sim 10^3 M_\odot$ at their cores. If this is indeed true then the QNM from activities associated with such BHs would be observable in the local Universe, depending on how much energy is released into GW when other objects fall in. EURO could also observe QNM in stellar mass black holes of mass $M \sim 10-20 M_\odot$.

The span of an interferometer to binaries varies with the masses as $\eta^{1/2} M_5^{5/6}$, greater asymmetry in the masses reduces the span but larger total mass increases the span. However, for $M > 100 M_\odot$ the sensitivity worsens as the seismic and thermal noise begin to dominate the noise spectrum at lower frequencies. In Fig. 3 we have plotted the distance up to which binaries can be seen as a function of the binary’s total mass for an equal mass system when including both the inspiral and merger part of the signal. This estimate is based on the effective-one-body approach \[16\], which predicts 0.07% of the total mass in the merger waves. The plunge phase increases the signal-to-noise ratio (SNR) by about a factor 1.5 over the inspiral phase which leads to an increase in the event rate, over that based purely on the inspiral stage, by a factor of $1.5^3 \simeq 3.4$.

**NS-NS binaries**

Double NS can be seen in advanced LIGO to a distance of 300 Mpc as shown in Fig. 2. Based on the observed small population of double NS binaries which merge within the Hubble time (cf. Table 1), Kalogera et al. \[39\] conclude that the Galactic coalescence rate is $\sim 1.8 \times 10^{-4} \text{ yr}^{-1}$ which would imply an event rate of NS-NS coalescences of 0.25 and 1500 yr$^{-1}$, in initial and advanced LIGO, respectively. As the spins of NS are very small ($a \ll 1$) and because the two stars would merge well outside the LIGO’s sensitivity band, the current state-of-the-art theoretical waveforms \[10\] will serve as good templates for matched filtering. However, detailed relativistic hydrodynamical simulations (see, e.g. Ref. \[40\]) would be needed to interpret the emitted radiation during the coalescence phase, wherein the two stars collide to form a bar-like structure prior to merger. The bar hangs up over a couple of dynamical time-scales to get rid of its deformity by
Fig. 3. The span of initial and advanced LIGO and EURO for compact binary sources when including both the inspiral and merger waveforms in our search algorithms (left panel). BH mergers can be seen out to a red-shift of $z = 0.55$ in advanced LIGO and $z = 2$ in EURO. On the right we plot the SNR achieved by LISA for mergers at $z = 1$ of BH of mass as on the $x$-axis with a MBH of mass is indicated on the plot.

emitting strong bursts of GW. Observing the radiation from this phase should help to deduce the EOS of NS bulk matter. Also, an event rate as large as in advanced LIGO and EURO will be a valuable catalogue to test astronomical predictions, for example if $\gamma$-ray bursts are associated with NS-NS or NS-BH mergers [29].

**NS-BH binaries**

These are binaries consisting of one NS and one BH and are very interesting from an astrophysical point of view: The initial evolution of such systems can be treated analytically fairly well, however, the presence of a BH with large spin can cause the NS to be whirled around in precessing orbits due to the strong spin-orbit coupling. The evolution of such systems is really not very well understood. However, it should be possible to use the “point-mass” approximation in which the NS is treated as a point-particle orbiting a BH, in getting some insight into the dynamics of the system. The evolution will also be complicated by the tidal disruption of the NS immediately after reaching the last stable orbit. It should be possible to accurately measure the onset of the merger phase and deduce the radius of the NS to $\sim 15\%$ and thereby infer the EOS of NS [60].

Advanced interferometers will be sensitive to NS-BH binaries out to a distance of about 650 Mpc. The rate of coalescence of such systems is not known empirically as there have been no astrophysical NS-BH binary identifications. However, the population synthesis models give [34] a Galactic coalescence rate in the range $3 \times 10^{-7}$–$5 \times 10^{-6}$ yr$^{-1}$. The event rate of NS-BH binaries will be worse than BH-BH of the same total mass by a factor of $(4\eta)^{3/2}$ since the SNR goes
down as \( \sqrt{4\eta} \). Taking these factors into account we get an optimistic detection rate of NS-BH of 1 to 1500 in initial and advanced LIGO, respectively.

**BH-BH binaries**

Black hole mergers are the most promising candidate sources for a first direct detection of GW. These sources are the most interesting from the viewpoint of general relativity as they constitute a pair of interacting Kerr spacetimes experiencing the strongest possible gravitational fields before they merge with each other to form a single BH, and serve as a platform to test general relativity in the non-linear regime. For instance, one can detect the scattering of GW by the curved geometry of the binary \[11\], \[12\], and measure, or place upper limits on, the mass of the graviton to \(2.5 \times 10^{-22} \) eV and \(2.5 \times 10^{-26} \) eV in ground- and space-based detectors, respectively \[62\]. High SNR events (which could occur once every month in advanced LIGO) can be used to test the full non-linear gravity by comparing numerical simulations with observations and thereby gain a better understanding of the two-body problem in general relativity. As BH binaries can be seen to cosmological distances, a catalogue of such events compiled by LIGO can be used to measure Cosmological parameters (Hubble constant, expansion of the Universe, dark energy) and test models of Cosmology \[29\].

The span of interferometers to BH-BH binaries varies from 100 Mpc (with the inspiral signal only) to 150 Mpc (inspiral plus merger signal) in initial LIGO and to a red-shift of \(z = 0.4–0.55\) in advanced LIGO, and \(z = 2\) in EURO (cf. Fig. 2. and 3.). As in the case of NS-BH binaries, here too there is no empirical estimate of the event rate. Population synthesis models are highly uncertain about the Galactic rate of BH-BH coalescences and predict \[34\] a range of \(3 \times 10^{-8}–10^{-5}\) yr\(^{-1}\), which is smaller than the predicted rate of NS-NS coalescences. However, owing to their greater masses, BH-BH event rate in our detectors is larger than NS-NS by a factor \(M^5/2\) for \(M \lesssim 100 M_\odot\). The predicted event rate is a maximum of 1 yr\(^{-1}\) in initial LIGO and 500 yr\(^{-1}\) to 20 day\(^{-1}\) in advanced LIGO.

**Massive black hole binaries**

It is now believed that the centre of every galaxy hosts a BH whose mass is in the range \(10^6–10^9 M_\odot\) \[48\]. These are termed as massive black holes (MBH). There is now observational evidence that when galaxies collide the MBH at their nuclei might get close enough to be driven by gravitational radiation reaction and merge within the Hubble time \[41\]. For a binary with \(M = 10^6 M_\odot\) the frequency of GW at the last stable orbit is \(f_{\text{LSO}} = 4\) mHz, followed by merger from 4 mHz to the QNM at 40 mHz (if the spin of the black holes is not close to 1). This is in the frequency range of LISA which has been designed to observe the MBH: their formation, merger and activity.
The SNR for MBH-MBH mergers in LISA is shown in Fig. 3. These mergers will appear as the most spectacular events requiring no templates for signal identification, although good models would be needed to extract source parameters. Mergers can be seen to $z \sim 30$ and, therefore, one could study the merger-history of galaxies throughout the Universe and address astrophysical questions about the origin, growth and population of MBH. The recent discovery of a MBH binary \cite{41} and the association of X-shaped radio lobes with the merger of MBH \cite{45} has raised the optimism concerning MBH mergers and the predicted rate for MBH mergers is the same as the rate at which galaxies merge, about $1 \text{ yr}^{-1}$ out to a red-shift of $z = 5$ \cite{35}.

**Smirches**

The MBH environment of our own galaxy is known to constitute a large number of compact objects and white dwarfs. Three body interaction will occasionally drive these compact objects, white dwarfs and other stars into a capture orbit of the central MBH. The compact object will be captured in an highly eccentric trajectory ($e > 0.99$) with the periastron close to the last stable orbit of the MBH. Due to relativistic frame dragging, for each passage of the apastron the compact object will experience several turns around the MBH in a near circular orbit. Therefore, long periods of low-frequency, small-amplitude radiation will be followed by several cycles of high-frequency, large-amplitude radiation. Waveforms from two such orbits is shown in Fig. 4. The apastron slowly shrinks, while the periastron remains more or less at the same location, until the final plunge of the compact object before merger. There is a lot of structure in the waveforms (cf. Fig. 4) which arises as a result of a number of different physical effects: Contribution from higher order multipoles, precession of the orbital plane that changes the polarisation of the waves observed by LISA, etc. This complicated structure smears the power in the signal in the time-frequency plane \cite{50} as compared to a sharp chirp from a non-spinning BH binary and for this reason this *spin modulated chirp* is called a *smirch* \cite{51}.

As the compact object tumbles down the MBH it will sample the spacetime geometry in which it is moving and the structure of that geometry will be imprint in the GW emitted in the process. By observing smirches, LISA offers a unique opportunity to directly map the spacetime geometry around the central object and test whether or not this structure is in accordance with the expectations of general relativity \cite{49}. Indeed, according to Einstein’s theory the geometry of a rotating black hole is uniquely determined to be the Kerr metric involving just two parameters, the mass of the MBH and its spin. Thus, the various multipole-moments of the source are uniquely fixed once we have measured the mass and spin of the BH. With the observed smirch one can basically test (i) whether general relativity correctly describes the spacetime region around a generic BH
and (ii) if the central object is indeed a BH or some other exotic matter.

The SNR from smirches will be between 10–50 depending on the mass of the central object (cf. Fig. 4.), but it might be very difficult to match filter them due to their complicated shapes. The events rate is expected to be rather high. Indeed, a background population of these smirches will cause confusion noise and only sources in the foreground will be visible in LISA. The event rate is as yet highly uncertain ranging from 1–10 yr$^{-1}$ within 1 Gpc [47].

4.2. Neutron stars

Neutron stars (NS) are the most compact stars in the Universe. With a density of $2 \times 10^{14}$ g cm$^{-3}$, and surface gravity $\varphi \equiv M/R \sim 0.1$, they are among the most exotic objects whose composition, equation-of-state and structure, are still largely unknown. Being highly compact they are potential sources of GW. The waves could be generated either from the various normal modes of the star, or because the star has a tiny deformation from spherical symmetry and is rotating about a non-axisymmetric axis, or because there are density inhomogeneities caused by an environment, or else due to certain relativistic instabilities. We will consider these in turn.

*Supernovae and Birth of NS*

The birth of a NS is preceded by the gravitational collapse of a highly evolved star or the core collapse of an accreting white dwarf. Type II supernovae (SN) are believed to result in a compact remnant. In any case, if the collapse is non-spherical then GW could carry away some of the binding energy and angular momentum depending on the geometry of the collapse. It is estimated that in a typical SN, GW might extract about $10^{-7}$ of the total energy [40]. The waves could come off in a burst whose frequency might lie in the range $\sim 200–1000$ Hz.
Advanced LIGO will be able to see such events up to the Virgo supercluster with an event rate of about 30 per year.

**Equation of State and Normal Modes of NS**

In order to determine the equation of state (EOS) of a neutron star, and hence its internal structure, it is necessary to independently determine its mass and radius. Astronomical observations cannot measure the radius of a neutron star although radio and X-ray observations do place a bound on its mass. Therefore, it is not been possible to infer the EOS. Neutron stars will have their own distinct normal modes and GW observations of these modes should resolve the matter here since by measuring the frequency and damping times of the modes it would be possible to infer both the radius and mass of NS. The technique is not unlike helioseismology where observation of normal modes of the Sun has facilitated insights into its internal structure. In other words, GW observations of the normal modes of the NS will allow *gravitational asteroseismology* \(^1\).

Irrespective of the nature of the collapse a number of normal modes will be excited in a newly formed NS. The star will dissipate the energy in these modes in the form of GWs as a superposition of the various normal modes and soon the star settles down to a quiescence state. Normal modes could also be excited in old NS because of the release of energy from star quakes. The strongest of these modes, the ones that are important for GW observations, are the \(p\) - and \(w\) -modes for which the restoring forces are the fluid pressure and space-time curvature, respectively. Both of these modes will emit transient radiation which has a generic form of a damped sinusoid: 

\[
h(t; \nu, \tau) = h_0 \exp(-t/\tau) \sin(2\pi \nu t),
\]

where \(h_0\) is the amplitude of the wave that depends on the external perturbation that excites the mode and \(\nu\) and \(\tau\) are the frequency and damping time of the mode, respectively, and are determined by the mass and radius of the NS for a given EOS.

To make an order-of-magnitude estimate let us assume that the mass of the NS is \(M_\star = 1.4 M_\odot\) and that its radius is \(R_\star = 10\) km. For the \(p\)-modes, which are basically fluid modes, the frequency of the fundamental mode, also called the \(f\)-mode, is simply the dynamical frequency of the fluid, namely \(\nu_f \sim \sqrt{\rho}\), where \(\rho\) is the density of the fluid. For a NS of radius \(R_\star\) and mass \(M_\star\) this corresponds to a frequency of \(\sqrt{3M_\star/(4\pi R_\star^3)} \sim 3\) kHz. If the star radiates all the energy deposited in the mode at a luminosity \(\mathcal{L}\), the damping time of the mode would be \(\tau \sim E/\mathcal{L}\). Since \(E \propto M_\star^2/R_\star\) and \(\mathcal{L} \propto M_\star^2 R_\star^4 \omega^6 = M_\star^5/R_\star^5\), we get \(\tau \sim R_\star^4/M_\star^3\). Indeed, detailed mode calculations for various EOS have been fitted to yield the following relations for \(f\)-modes \(^1\):

\[
\nu_f = \left[0.78 + 1.635 \left(\frac{M_\star}{R_\star^3}\right)^{1/2}\right] \text{kHz}, \quad \tau_f^{-1} = \frac{M_\star^3}{R_\star^4} \left[22.85 - 14.65 \frac{M_\star}{R_\star}\right] \text{s}, \quad (8)
\]

and similarly for \(w\)-modes. The \(f\)- and \(w\)-mode frequencies nicely separate into
two distinct groups even when considering more than a dozen different EOS: The \( f \)-modes are in the frequency range 1–4 kHz, \( w \)-modes are in the range 8–14 kHz, and therefore, detecting a signal at these frequencies places it in one or the other category. The frequency and damping time, together with the relations above, can then be used to fix the radius and mass of the star. Observing several systems should then yield a mass-radius curve which is distinct for each EOS and thereby helps to address the question of NS structure.

The amplitude of \( f \)- and \( w \)-modes corresponding to 12 different EOS from NS at 10 kpc to 15 Mpc is shown in Fig. 2 as two shaded regions. In a typical gravitational collapse the amount of energy expected to be deposited in \( f \)- or \( w \)-modes, \( \sim 10^{-8} M_\odot \), makes it impossible to detect them in initial LIGO and barely in advanced LIGO instruments, even for a Galactic source. However, EURO should be able to detect these systems with a high SNR. The event rates for these systems would be at least as large as the supernova rate, i.e. about 0.1–0.01 yr\(^{-1}\) in our galaxy, increasing to 10–100 yr\(^{-1}\) within the Virgo supercluster.

**Relativistic Instabilities in NS**

NS suffer dynamical and secular instabilities caused by hydrodynamical and dissipative forces, respectively. What is of interest to us is the secular instability driven by gravitational radiation. GW emission from a normal mode in a non-spinning NS would always lead to the decay of the mode. However, the situation might reverse under certain conditions: Imagine a NS spinning so fast that a normal mode whose angular momentum (AM) in the star’s rest frame is opposite to its spin, appears to an inertial observer to be co-rotating with the spin. In the inertial frame, GW extracts positive AM from the mode; therefore the mode’s own AM should become more negative. In other words, the amplitude of the mode should grow as a result of GW emission, and hence the instability. The energy for the growth of the mode comes from the rotational energy of the star, which acts like a pump field. Consequently, the star would spin down and eventually halt the instability. It was expected that this instability, called the CFS instability [18, 32], might drive the \( f \)-modes in a NS unstable, but the star should spin at more than 2 kHz (the smallest \( f \)-mode frequency) for this to happen. Moreover, it has been shown that due to viscous damping in the NS fluid the instability would not grow sufficiently large, or sustain for long, to be observable (see e.g. Ref. [4]).

It was recently realized [3] that modes originating in current-multipoles, as opposed to mass-multipoles which lead to the \( f \)-mode, could be unstable even in a non-spinning NS. These modes, called the \( r \)-modes, have received a lot of interest because they could potentially explain why spin frequencies of NS in low-mass X-ray binaries are all clustered in a narrow range of 300–600 Hz or why no NS with spin periods smaller than 1.24 ms have been found. The role of \( r \)-modes in these circumstances is as yet inconclusive because the problem involves
very complicated astrophysical processes (magnetic fields, differential rotation, superfluidity and superconductivity), microphysics (the precise composition of NS – hyperons, quarks) and full non-linear physics of general relativity. It is strongly expected that $r$-modes will be emitted by newly formed NS during the first few months of their birth [43], [4]. The frequency of these modes will be $4/3$ of the spin frequency of the star and might be particularly important if the central object in a low-mass X-ray binary is a strange star [3]. The radiation might last for about 300 years and the signal would be detectable in initial LIGO with a few weeks of integration.

**NS Environment**

A NS with an accretion disc would be spun up due to transfer of AM from the disc. Further, accretion could lead to density inhomogeneities on the NS that could lead to the emission of GW. The resulting radiation reaction torque could balance the accretion torque and halt the NS from spinning up. It has been argued [9] that GW emission could be the cause for spin frequencies of NS in low-mass X-ray binaries to be locked up in a narrow frequency range of 300–600 Hz. It is also possible that $r$-modes are responsible for the locking up of frequencies instead, in which case the waves would come off at a different frequency [5]. These predictions can be tested with advanced LIGO or EURO as Sco-X1, a nearby low-mass X-ray binary, would produce quite a high SNR (marked as $\star$ in Fig. 2).

**Spinning NS with Asymmetries**

Our galaxy is expected to have a population of $10^8$ NS and they normally spin at high rates (several to 500 Hz). Such a large spin must induce some equatorial bulge and flattening of the poles. The presence of a magnetic field may cause the star to spin about an axis that is different from the symmetry axis leading to a time-varying quadrupole moment [21]. Gravitational waves emitted by a typical NS a distance of $r = 10$ kpc from the Earth will have an amplitude [59] $h \sim 8 \times 10^{-26} f_{\text{kHz}}^2 \epsilon^{-6}$, where $f_{\text{kHz}}$ is the frequency of GW in kHz and $\epsilon^{-6}$ is the ellipticity of the star in units of $10^{-6}$. Fig. 2 plots the signal strength expected from a NS with $\epsilon = 10^{-6}$ at 10 kpc integrated over 4 months.

The ellipticity of neutron stars is not known but one can obtain an upper limit on it by attributing the observed spin-down rate of pulsars as entirely due to gravitational radiation back reaction, namely that the change in the rotational energy is equal to GW luminosity. The ellipticity of the Crab pulsar inferred in this way is $\epsilon \leq 7 \times 10^{-4}$. The GW amplitude corresponding to this ellipticity is $h \leq 10^{-24}$. Noting that Crab has a spin frequency of 25 Hz (GW frequency of 50 Hz), on integrating the signal for $10^7$ s one obtains $h = 3.3 \times 10^{-21}$ Hz$^{-1/2}$, which is easily reachable by LIGO. It is unlikely that the ellipticity is so large and hence the GW amplitude is probably much less. However, seeing Crab at a hundredth
of this ellipticity is quite good with advanced LIGO as indicated by a diamond in Fig. 21 (Note that Crab is at 2 kpc, so with an ellipticity of $\epsilon = 7 \times 10^{-6}$ the signal strength would be 35 times higher than the NS line.)

4.3. Stochastic Background

A population of background sources and quantum processes in the early Universe produce stochastic signals that fills the whole space. By detecting such a stochastic signal we can gain knowledge about the underlying populations and physical processes. A network of antennas can be used to discover stochastic signals buried under the instrumental noise backgrounds. It is expected that the instrumental backgrounds will not be common between two geographically well-separated antennas. Thus, by cross-correlating the data from two detectors we can eliminate the background and filter the interesting stochastic signal. However, when detectors are not co-located the SNR builds only over GW wavelengths longer than twice the distance between antennas which in the case of the two LIGO antennas means over frequencies $\lesssim 40$ Hz [1]. The visibility of a stochastic signal integrated over a period $T$ and bandwidth $f$ only increases as $(fT)^{1/4}$ since cross-correlation uses a ‘noisy’ filter. But the noise in a bandwidth equal to frequency $f$ is $\sqrt{fS_h(f)}$. Thus, the signal effectively builds up as $(T/f)^{1/4}$.

Astronomical Backgrounds

There are thousands of white dwarf binaries in our galaxy with their period in the range from a few hours to 100 seconds. Each binary will emit radiation at a single frequency, but over an observation period $T$ each frequency bin of width $\Delta f = 1/T$ will be populated by many sources. Thus, unless the source is nearby it will not be possible to detect it amongst the confusion background created by the underlying population. However, a small fraction of this background population will be detectable as strong foreground sources. The parameters of many white dwarfs are known so well that we can precisely predict their SNRs in LISA and thereby use them as to calibrate the antenna. In the inset panel of Fig 21 the curve labelled WDB is the expected confusion noise from Galactic white dwarfs [36], [34]. NS and BH populations do not produce a large enough background to be observable. Note that the white dwarf background imposes a limitation on the sources we can observe in the frequency region from 0.3 mHz to about 1 mHz – the region where we expect smirches to occur.

Primordial Background

A cosmological background should have been created in the very early Universe and later amplified, as a result of parametric amplification, by its coupling to the background gravitational field [34]. Imprint on such a background are the physical conditions that existed in the early Universe as also the nature of the
physical processes that produced the background. Observing such a background is therefore of fundamental importance as this is the only way we can ever hope to directly witness the birth of the Universe. The cosmic microwave background, which is our firm proof of the hot early phase of the Universe, was strongly coupled to baryons for 300,000 years after the big bang and therefore the signature of the early Universe is erased from it. The GW background, on the other hand, is expected to de-couple from the rest of matter $10^{-24}$ s after the big bang, and would therefore carry uncorrupted information about the origin of the Universe.

The strength of stochastic GW background is measured in terms of the fraction $\Omega_{GW}$ of the energy density in GW as compared to the critical density needed to close the Universe and the amplitude of GW is given by [59]:

$$h = 8 \times 10^{-19} \frac{\Omega_{GW}^{1/2}}{f},$$

for $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}$. By integrating for $10^7$ s, over a bandwidth $f$, we can measure a background density at $\Omega_{GW} \approx 4 \times 10^{-5}$ in initial LIGO, $5 \times 10^{-9}$ in advanced LIGO and $10^{-10}$ in LISA (cf. Fig. 2. dot-dashed curves marked $\Omega_{GW}$). In the standard inflationary model of the early Universe, the energy density expected in GW is $\Omega_{GW} \lesssim 10^{-15}$, and this will not be detected by future ground-based detectors or LISA. However, space missions currently being planned (DECIGO/BBO) to exploit the astrophysically quiet band of $10^{-2} \text{–} 1 \text{ Hz}$ might detect the primordial GW and unveil the origin of the Universe.

5. Conventions on Source Strengths and Units

This article chiefly deals with compact objects, namely neutron stars (NS) and black holes (BH). Unless specified otherwise we shall assume that a NS has a mass of $M = 1.4M_\odot$ and radius $R = 10 \text{ km}$, and by a stellar mass BH we shall mean a black hole of mass $M = 10M_\odot$. While dealing with the detectability of a source we shall assume that a broadband source of known phase evolution is integrated over a bandwidth equal to its frequency, for continuous waves an integration time of $10^7$ s, for stochastic signals an integration over $10^7$ s over a bandwidth $f$, and for quasi-normal modes an integration over one $e$-folding time. These operations will convert the raw dimensionless amplitude of GW into units $\text{Hz}^{-1/2}$, thereby allowing us to compare source strengths with the antenna’s amplitude noise spectral density $\sqrt{S_h(f)}$, which is also measured in units of $\text{Hz}^{-1/2}$. For a 1% false alarm probability during the course of observation it is typically necessary to set signal-to-noise ratio (SNR) threshold of about 8 in a single detector. In a network consisting of three equally sensitive detectors, in order that the network-SNR is 8, each instrument must register an SNR of $\sim 5$. We shall therefore deem that a source is observable if the SNR it produces is at least 5. The SNR achievable for point sources depends on the orientation of the source relative to the detector. We shall assume that sources occur with random orientations and consider our typical source to have an “RMS” orientation. The amplitude of a source with an “RMS” orientation is smaller than an optimally
oriented source by a factor of $5/2$. We shall assume a flat Universe with a cold dark matter density of $\Omega_M = 0.3$, dark energy of $\Omega_\Lambda = 0.7$, and a Hubble constant of $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We shall use a system of units in which $c = G = 1$, which means $1 M_\odot \approx 5 \times 10^{-6} \text{ s} \approx 1.5 \text{ km}$, $1 \text{ Mpc} = 10^{14} \text{ s}$.

There have been many reviews and books on this topic to which we refer the reader for further reading. Our depiction of the noise spectral density and source strengths in Fig. 2 is motivated by Ref. [23].

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