CROSSOVER AND MASS GAP OF THE 
SU(2)-HIGGS MODEL AT HIGH TEMPERATURE\textsuperscript{a}

W. Buchmüller

\textit{Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany}

Analytic estimates of screening masses in the symmetric, high-temperature phase of the SU(2)-Higgs model are reviewed. The size of the mass gap in the symmetric phase is closely related to the critical Higgs mass where the first-order electroweak phase transition changes to a smooth crossover. We also discuss some conjectures concerning the connection between the screening masses of propagators in a fixed gauge and screening masses of gauge-invariant operators.

The electroweak phase transition\textsuperscript{1} is of great cosmological significance because baryon number and lepton number violating processes are in thermal equilibrium at temperatures above the critical temperature of the transition\textsuperscript{2}. In recent years the thermodynamics of the phase transition has been studied in detail by means of perturbation theory and numerical lattice simulations. The essential non-perturbative aspects of the transition can be investigated in the pure SU(2)-Higgs model neglecting the effects of fermions and the mixing between photon and neutral vector boson, which can be included perturbatively. For the dimensionally reduced theory lattice simulations for the SU(2)\times U(1)-Higgs model have been carried out. We now know that the transition is first-order for Higgs boson masses below 70 GeV, and that around $\sim 80$ GeV the first-order transition changes to a smooth crossover\textsuperscript{3}.

The electroweak transition is influenced by non-perturbative effects whose size is characterized by a ‘magnetic screening length’, the inverse of a ‘magnetic mass’. In perturbation theory a magnetic mass appears as a cutoff which regularizes infrared divergencies\textsuperscript{4}. The size of this cutoff is closely related to the confinement scale of the effective three-dimensional theory which describes the high-temperature limit of the SU(2) Higgs model\textsuperscript{5}. In the following we shall review estimates of the magnetic mass by means of gap equations. In particular, we shall discuss the connection between the size of the magnetic mass and the critical Higgs mass for the onset of a crossover behaviour. We

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\textsuperscript{b}For recent reviews, see\textsuperscript{3}.\textsuperscript{1}}
shall also consider some conjectures concerning the connection between the magnetic screening mass and screening masses of gauge-invariant operators.

1 Gap equations for the magnetic mass

Consider the SU(2) Higgs model in three dimensions which is defined by the action

\[
S = \int d^3x \text{Tr} \left( \frac{1}{2} W_{\mu\nu} W^{\mu\nu} + (D_\mu \Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi + 2 \lambda (\Phi^\dagger \Phi)^2 \right),
\]

with \( \Phi = \frac{1}{2}(\sigma + i \vec{\pi} \cdot \vec{\tau}) \), \( D_\mu \Phi = (\partial_\mu - ig W_\mu) \Phi \), \( W_\mu = \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \).

Here \( \vec{W}_\mu \) is the vector field, \( \sigma \) is the Higgs field, \( \vec{\pi} \) is the Goldstone field and \( \vec{\tau} \) is the triplet of Pauli matrices. The gauge coupling \( g \) and the scalar coupling \( \lambda \) have mass dimension 1/2 and 1, respectively. For perturbative calculations gauge fixing and ghost terms have to be added. The parameters of the 3d Higgs model are related to the parameters of the 4d Higgs model at finite temperature by means of dimensional reduction:

\[
g^2 = \bar{g}^2(T) T, \quad \lambda = \bar{\lambda}(T) T + \ldots, \quad \mu^2 = \left( \frac{3}{16} \bar{g}^2(T) + \frac{1}{2} \bar{\lambda}(T) \right) (T^2 - T_b^2) + \ldots,
\]

where \( T \) is the temperature and \( \bar{g} \), \( \bar{\lambda} \) and \( T_b^2 \) are the parameters of the zero temperature 4d theory. Note, that a variation of temperature in the 4d theory corresponds to a variation of the parameter \( \mu^2/g^4 \) in the effective 3d theory whereas the ratio \( g^2/\lambda \) stays constant.

We are interested in the propagators \( G_\sigma \) and \( G_W \) of Higgs field and vector field, respectively, which at large separation \( |x - y| \) fall off exponentially,

\[
G_\sigma(x - y) = \langle \sigma(x)\sigma(y) \rangle \sim e^{-M|x - y|}, \quad G_W(x - y)_{\mu\nu} = \langle W_\mu(x) W_\nu(y) \rangle \sim e^{-m|x - y|}.
\]

For \( \mu \gg g^2 \) one has \( M \simeq \mu \), whereas \( m \) cannot be computed in perturbation theory. An estimate for the vector boson mass can be obtained from a coupled set of gap equations for Higgs boson and vector boson masses as follows. One shifts the Higgs field \( \sigma \) around its vacuum expectation value \( v \), \( \sigma = v + \sigma' \), which yields the tree level masses

\[
m_0^2 = \frac{g^2}{4} v^2, \quad M_0^2 = \mu^2 + 3\lambda v^2.
\]
The masses $m_0^2$ and $M_0^2$ are now expressed as

$$m_0^2 = m^2 - \delta m^2, \quad M_0^2 = M^2 - \delta M^2,$$

where $m$ and $M$ enter the propagators in the loop expansion, and $\delta m^2$ and $\delta M^2$ are treated perturbatively as counter terms. Together with the mass resummation a vertex resummation is performed. One then obtains a coupled set of gap equations for Higgs boson and vector boson masses (cf. Fig.1),

$$\delta m^2 + \Pi_T(p^2 = -m^2, m, M, \xi) = 0,$$
$$\delta M^2 + \Sigma(p^2 = -M^2, m, M, \xi) = 0,$$

where $\Pi_T(p^2)$ is the transverse part of the vacuum polarization tensor. The calculation has been carried out in $R_\xi$-gauge. In order to obtain masses $M$ and $m$ which are independent of the gauge parameter $\xi$, it is crucial to perform a vertex resummation in addition to the mass resummation and to evaluate the self-energy terms on the mass shell. This yields the screening lengths defined
in Eq. (3). ‘Magnetic masses’ defined at zero momentum are gauge-dependent

Together with a third equation for the vacuum expectation value $v$, determined by the condition $\langle \sigma' \rangle = 0$, the gap equations determine Higgs boson and vector boson masses for each set of values $\mu^2/g^4$ and $\lambda/g^2$. For negative $\mu^2$ one finds a unique solution, corresponding to the familiar Higgs phase, with masses close to the results of perturbation theory (cf. Fig. 2). In the case of small positive $\mu^2/g^4$ and sufficiently small $\lambda/g^2$ there exist two solutions, corresponding to the Higgs phase and the symmetric phase with a small, but finite vector boson mass, respectively. This is the metastability range characteristic for a first-order phase transition. For large positive $\mu^2/g^4$ only the solution corresponding to the symmetric phase remains. Here the Higgs boson mass is $M \approx \mu$, and the vector boson mass, which is rather independent of $\mu$ and $\lambda$, is given by

$$m_{SM} = Cg^2,$$

$$C = \frac{3}{16\pi}\left(\frac{21}{4}\ln 3 - 1\right) \approx 0.28 . \quad (9)$$
In the symmetric phase, where ordinary perturbation theory breaks down, the vector boson mass is dominated by the graphs a-d in Fig. 1, which correspond to the one-loop contributions of the non-linear SU(2)-Higgs model. Note, that the mass gap is a direct consequence of the non-abelian gauge interactions. In the abelian Higgs model no vector boson mass is generated in the symmetric phase.

The general strategy and the status of attempts to calculate the mass gap in the 3d SU(N)-gauge theory by means of gap equations has recently been discussed by Jackiw and Pi. The starting point is always a resummation which is generated by adding and subtracting an auxiliary action \( S_m \) from the Yang-Mills action \( S_G \),

\[
S_{\text{eff}} = \frac{1}{l} \left( S_G(\sqrt{l}W) + S_m(\sqrt{l}W) \right) - S_m(\sqrt{l}W). \quad (10)
\]

In a perturbative expansion the coefficient of \( l^n \) gives the contribution of all \( n \)-loop graphs. \( S_m \) is some conveniently chosen, gauge-invariant mass term. The one-loop gap equation reads

\[
\Pi_{\text{1-loop}}(p^2 = -m^2) = -m^2. \quad (11)
\]

In the case of the non-linear \( \sigma \)-model, \( S_m = S_\sigma \), the calculation can be considerably simplified. The functional integral for the partition function is

\[
Z = \int DW \, D\pi \Delta \, \exp \left[ -\frac{1}{l} \left( S_G(\sqrt{l}W) + S_\sigma + S_{GF} - lS_\sigma \right) \right]. \quad (12)
\]

where \( S_{GF} \) is some gauge fixing term and \( \Delta \) is the corresponding Faddeev-Popov determinant. The integral over Goldstone fields and ghost fields can be carried out exactly yielding a massive Yang-Mills theory without any additional gauge fixing terms.

\[
Z \propto \int DW \exp \left[ -\frac{1}{l} \left( S_G(\sqrt{l}W) - m^2 \text{Tr} \int d^3 x \sqrt{l}W_\mu \sqrt{l}W^\mu \right) \right]. \quad (13)
\]

The exact gap equation now reads

\[
\Pi_T(p^2 = -m^2) = -m^2. \quad (14)
\]

Massive Yang-Mills theory corresponds to the non-linear \( \sigma \)-model in unitary gauge. One may therefore expect that the solution of Eq. (14) agrees with
Eq. (9), whereas the off-shell self energies should only agree in the limit $\xi \rightarrow \infty$. This is indeed the case.

For the 3d SU(N) gauge theory also other gap equation have been considered which are based on the Chern-Simons eikonal and on the non-local action:

$$S_{m}^{J} = m^{2} \text{Tr} \int d^{3}x F_{\mu} \frac{1}{D^{2}} F_{\mu},$$

(15)

where $F_{\mu} = \frac{1}{2} \varepsilon_{\mu\nu\rho} F_{\nu\sigma}$. Amazingly, the one-loop gap equation of Alexanian and Nair yields a magnetic mass closely related to the one obtained for the non-linear $\sigma$-model,

$$m_{A,N} = \frac{Ng^{2}}{8\pi} \left( \frac{21}{4} \ln 3 - 1 \right) = \frac{4}{3} m_{SM}.$$

(16)

The choice $S_{m}^{J}$ yields a complex ‘magnetic mass’,

$$m_{J,p} = \frac{Ng^{2}}{8\pi} \left[ \left( \frac{116}{16} \ln 3 - \frac{67}{12} \right) \pm i \frac{13}{16} \right].$$

(17)

Note, that Eq. (15) can be modified such that the generated mass gap is real.

The evaluation of a magnetic mass by means of gap equations is not a systematic approach to determine the mass gap in the 3d SU(N) gauge theory since the loop expansion does not correspond to an expansion in a small parameter. This is an obvious shortcoming. On the other hand, it may very well be that the the one-loop results for $m_{SM}$ and $m_{A,N}$ represent reasonable approximations of the true mass gap. In this respect an extension of the gap equations to two-loop order would be very valuable. If the whole approach makes sense the two-loop correction should be of order $m_{SM} - m_{A,N}$. It is also encouraging that the magnetic masses $m_{SM}$ and $m_{A,N}$ are consistent with the propagator mass obtained in a numerical lattice simulation in Landau gauge.

$$m_{SM}^{(L)} = 0.35(1) g^{2}.$$

(18)

The lattice simulation also yields a dependence of the vector boson mass on the parameter $\mu^{2} / g^{4}$ which is very similar to the solution of the gap equation shown in Fig. 3.

2 Mass gap and crossover

A direct consequence of the mass gap in the symmetric phase is the change of the first-order phase transition at small Higgs masses to a smooth crossover at
some critical Higgs mass $\bar{m}_H$. Fig. 3 shows the solution of the gap equations for a physical Higgs mass equal to the W-boson mass of about 80 GeV. The solution is very different from the one shown in Fig. 2 which corresponds to a Higgs mass of about 20 GeV. In Fig. 3 the variation of vector and scalar screening masses is continuous as $\mu^2/g^4$ is increased from the region of the Higgs phase to the domain of the symmetric phase. Based on the gap equations it was predicted that the critical Higgs mass $\bar{m}_H$ should be below 100 GeV. Recently, convincing evidence for a crossover has indeed been obtained in numerical lattice simulations, and the critical Higgs mass has been determined to be about 80 GeV.

In the SU(2)-Higgs model a crossover behaviour was first observed for the finite-temperature 4d theory in numerical lattice simulations for large Higgs masses by Evertz, Jersáš and Kanaya, who also discussed in detail the phase diagram of the average action approach to the electroweak phase transition a change from a first-order transition to a crossover is expected. In this framework, information about non-perturbative properties of the theory from elsewhere is at present needed to obtain an estimate for the critical Higgs mass which could be as large as 200 GeV or larger. The general

Figure 3: Vector boson and Higgs boson masses for $\lambda/g^2 = 1/8$ which corresponds to a physical Higgs mass equal to the W-boson mass. Gap equations: $m$ (full line), $M$ (dashed line); perturbation theory: $m$ (dash-dotted line), $M$ (dotted line). From Ref. [4].
argument, that three-dimensional confinement forbids massless particles and therefore a second-order transition, is at variance with the observed change from a first-order transition to a crossover and the general expectation that a first-order critical line ends in a second-order transition. It is important to clarify the nature of the endpoint for the electroweak transition.

The essence of the connection between mass gap and critical Higgs boson mass in the gap equation approach can be easily understood as follows. Consider the one-loop effective potential in unitary gauge ($\vec{\pi} = 0$),

$$V_{1l} = \frac{1}{2} \mu^2 \sigma^2 + \frac{1}{4} \lambda \sigma^4 - \frac{1}{16\pi} g^3 \sigma^3,$$  \hspace{1cm} (19)

where we have neglected the scalar contributions for simplicity. At the beginning of the metastability range, $\mu^2 = 0$, the Higgs vacuum expectation value is $\sigma_0 = \frac{3g^4}{16\pi\lambda}$, which corresponds to the vector boson mass $m_W(\mu^2 = 0) = \frac{3g^4}{32\pi\lambda}$. \hspace{1cm} (20)

It is reasonable to expect that the first-order phase transition disappears at a critical scalar coupling where the vector boson mass in the Higgs phase reaches the magnetic mass of the symmetric phase. The condition $m_W(\mu^2 = 0) = m_{SM}$ determines a critical coupling $\lambda_c$. The corresponding zero-temperature critical Higgs boson mass is given by

$$\tilde{m}_c^H = \left( \frac{3}{4\pi C} \right)^{1/2} \tilde{m}_W \simeq 74 \text{ GeV},$$ \hspace{1cm} (21)

where $\tilde{m}_W$ is the zero-temperature vector boson mass. Eq. (21) clearly shows that the crossover point is determined by the constant $C$, i.e., the size of the magnetic mass. The obtained value of the critical Higgs mass agrees rather well with the result of recent numerical simulations. In contrast, for vanishing magnetic mass the first-order transition never changes to a crossover, while taking $C > 1.0$ corresponding to the measured bound state mass $m_V$ (cf. Eq. (28), table 1) grossly underestimates $\tilde{m}_c^H$.

From the effective potential (19) one can easily determine the critical line of the first-order phase transition. For $\hat{\lambda}(\mu^2) < \lambda_c$, the conditions

$$0 = V_{1l}(\sigma_0) = \frac{\partial}{\partial \sigma} V_{1l} \bigg|_{\sigma = \sigma_0}$$ \hspace{1cm} (22)

yield for the critical line $\hat{\lambda}(\mu^2)$,

$$\hat{\lambda}(\mu^2) = \frac{g^4}{128\pi^2 \mu^2} \cdot$$ \hspace{1cm} (23)
The corresponding phase diagram is shown in Fig. 4. The critical value of the mass parameter at the crossover point is given by

$$\frac{\mu^2}{g^4} \simeq \frac{C}{8\pi}.$$  \hfill (24)

Using the matching relations to the finite-temperature Higgs model one can evaluate the critical temperature as function of the zero-temperature Higgs boson mass.

3 Gauge-invariant correlation functions

It is known that the SU(2) Higgs model has only a single phase, and that the Higgs and the confinement regime are analytically connected. All physical properties of the model can be obtained by studying correlation functions of gauge-invariant operators. This is of particular importance for numerical lattice simulations where in general the gauge is not fixed.

In the literature the following operators for scalar states with $J^{PC} = 0^{++}$ have been studied,

\begin{align}
R(x) &= \text{Tr} \left( \Phi^\dagger(x) \Phi(x) \right), \\
L(x) &= \text{Tr} \left( (D_\mu \Phi)^\dagger D_\mu \Phi(x) \right),
\end{align}

\hfill (25) \hfill (26)
\[ P(x) = \frac{1}{2} \text{Tr} (W_{\mu\nu} W^{\mu\nu}) = -\frac{1}{8g^2} \text{Tr} ([D_\mu, D_\nu][D_\mu, D_\nu]) \, . \] (27)

The standard operator for vector states with \( J^{PC} = 1^{--} \) is
\[ V^a_\mu (x) = \frac{1}{2} \text{Tr} \left( \Phi^\dagger (x) \tilde{D}_\mu \Phi (x) \tau^a \right) \, . \] (28)

In the numerical simulations, screening masses have been determined from the 2-point functions of the operators \( R \) and \( V^a_\mu \),
\[ G_R (x - y) = \langle R(x)R(y) \rangle \sim e^{-m_R|x-y|} \, , \] (29)
\[ G_V (x - y)_{\mu\nu} = \langle V^\mu_\mu (x)V^\nu_\nu (y) \rangle \sim e^{-m_V|x-y|} \, . \] (30)

In screening masses have also been measured for the operators \( L(x) \) and \( P(x) \),
\[ G_L (x - y) = \langle L(x)L(y) \rangle \sim e^{-m_L|x-y|} \, , \] (31)
\[ G_P (x - y) = \langle P(x)P(y) \rangle \sim e^{-m_P|x-y|} \, . \] (32)

The screening masses \( m_R, m_V, m_L \) and \( m_P \) have been determined for positive and negative values of \( \mu^2 \), i.e., in the symmetric phase and in the Higgs phase. In the latter case, as expected, the results agree with perturbation theory and gap equations. The value for \( m_P \) is consistent with an intermediate state of two massive vector bosons \( V \) contributing to \( G_P \). This is the leading contribution if one expands \( P(x) \) in powers of \( g^2 \).

In the symmetric phase, however, the numerical results for \( m_R \) and \( m_V \) do not agree with the predictions of the gap equations. Since also in the symmetric phase the vacuum expectation value of the Higgs field is different from zero, it was suggested that the magnetic mass \( m_{SM} \) should determine the asymptotic behaviour of \( G_V \) (cf. (33)). The numerical simulations show no sign of this.

What is the connection between the gauge-dependent 2-point function \( G_W \) and the gauge-invariant 2-point function \( G_V \)? This question has been addressed by Fröhlich, Morchio and Strocchi in their detailed study of the Higgs phenomenon in terms of gauge-invariant operators. As they have pointed out, gauge-invariant correlation functions are approximately proportional to gauge-dependent correlation functions as calculated in standard perturbation theory, if for the chosen gauge and renormalization scheme the fluctuations of the Higgs field are small compared to the vacuum expectation value. For instance, for the scalar correlation functions one has \( \langle \sigma = v + \sigma', \langle \sigma' \rangle = 0 \),
\[ \langle R(x)R(y) \rangle \sim v^2 \left( \langle \sigma'(x)\sigma'(y) \rangle + O \left( \frac{\sigma'}{\langle \sigma \rangle}, \frac{\vec{\pi}}{\langle \sigma \rangle} \right) \right) \, . \] (33)
As a measure for the relative size of the fluctuation terms one may consider the ratio
\[ \zeta = \frac{\langle \Phi^\dagger \Phi \rangle}{\langle \sigma \rangle^2}. \] (34)

At one-loop order one obtains in $R_\xi$-gauge,
\[ \langle \Phi^\dagger \Phi \rangle = \langle \sigma^2 + \vec{\pi}^2 \rangle = v^2 + \langle \sigma'^2 + \vec{\pi}'^2 \rangle \] (35)
\[ = v^2 - \frac{1}{4\pi} \left( M + 3\sqrt{\xi}m \right). \] (36)

Here linear divergencies have been subtracted by means of dimensional regularization. Deep in the Higgs phase, where $\mu^2 < 0$, $v_0^2 = -\mu^2/\lambda$, $M_0^2 = 2\lambda v_0^2$ and $m_0^2 = g^2 v_0^2/4$, one finds
\[ \zeta_H = 1 - \frac{3}{8\pi} \left( \sqrt{\xi} + \frac{2\sqrt{2}\lambda}{3g} \right) \frac{g}{v} + \ldots. \] (37)

In the relevant range of parameters one has $g/v < 1$. Hence, the deviation of $\zeta_H$ from 1 is small and ordinary perturbation theory is reliable.

On the contrary, in the symmetric phase the situation is very different. Here the gap equations yield for the vacuum expectation value $g/v \approx 10$. Inserting in the definition of the ratio (34) solutions of the gap equations for $M$ and $m$ one finds in the symmetric phase that $\zeta_{SM}$ deviates from 1 by more than 100%. Hence, we cannot expect that the gauge-dependent 2-point functions give a good approximation to the gauge-invariant 2-point functions.

## 4 Gauge-invariant screening masses

What is the physical meaning of the propagator masses obtained from gap equations as well as numerical simulations in the symmetric phase? Several years ago the notion of a ‘screening energy’ has been introduced in connection with an analysis of the SU(2) Higgs model at zero temperature\(^{28}\). The authors considered the gauge-invariant correlation function
\[ G_\Phi(T, R) = \langle \text{Tr} \left( \Phi(y)U(\Gamma)\Phi(x) \right) \rangle, \] (38)
where
\[ U(\Gamma) = P \exp \left( ig \int_\Gamma ds \cdot W \right) \equiv U^\dagger(R, y)U(T)U(R, x), \] (39)
and the path $\Gamma$
\[ \Gamma \equiv \Gamma(y, R) \circ \Gamma(T) \circ \Gamma(R, x) \] (40)
is shown in Fig. 5. For large $T$, with $R$ fixed, an exponential fall-off was found,

$$G_\Phi(T, R) \sim e^{-m_\Phi T},$$

with $m_\Phi$ being independent of $R$. In temporal gauge the 2-point function takes the form,

$$G_\Phi(T, R) = \langle \text{Tr} \left( \Phi^\dagger(x)U^\dagger(R, x)e^{-HT}U(R, x)\Phi(x) \right) \rangle,$$

where $H$ is the hamiltonian. Comparison of Eqs. (41) and (42) suggests that $m_\Phi$ is the energy of a dynamical charge bound by an external charge. If the energy of the infinitely heavy external charge is properly subtracted, $m_\Phi$ corresponds to the ‘constituent’, or screening mass of the bound scalar $\Phi$. In the case $R = 0$ the 2-point function $G_\Phi$ reduces to the gauge-invariant propagator

$$\tilde{G}_\Phi(x - y) = \langle \text{Tr} \left( \Phi^\dagger(y)U_{yx}\Phi(x) \right) \rangle \sim e^{-m_\Phi |x - y|},$$

where the superscript $\mathcal{A}$ denotes SU(2) matrices in the adjoint representation.

The contribution from the phase factor to the masses $m_\Phi$ and $m_W$, which depend on the mass parameter $\mu^2$, are linearly divergent. Renormalized screening masses can be defined by matching $m_\Phi$ and $m_W$ to the masses $m_R$ and $m_V$ of the gauge-invariant correlation function at some value $\mu_0^2$ in the Higgs phase. The corresponding screening masses $m_\Phi(\mu^2; \mu_0^2)$ and $m_W(\mu^2; \mu_0^2)$ satisfy the boundary conditions

$$m_\Phi(\mu_0^2; \mu_0^2) = m_R(\mu_0^2), \quad m_W(\mu_0^2; \mu_0^2) = m_V(\mu_0^2).$$
These screening masses, as functions of $\mu^2$, should behave similarly as the solutions $M(\mu^2)$ and $m(\mu^2)$ of the gap equations, respectively.

What is the role of the screening masses in the correlation functions of gauge-invariant operators? As discussed above, the fluctuations dominate in the symmetric phase. Hence, one may expect that multi-particle states of ‘constituent’ scalar and vector bosons dominate the exponential fall-off of the 2-point functions. This is similar in spirit to the bound state model of Dosch et al.\cite{Dosch:1987}. According to Fig. 6, for $G_R$ this should be a $(\Phi\Phi^\dagger)$ state (a), for $G_L$ a $(\Phi^\dagger WW\Phi)$ state (b), for $G_P$ a $(WWW)$ state (c) and for $G_V$ a $(\Phi\Phi^\dagger)$ state (d). Here we have identified a covariant derivative $D_\mu$ with a constituent vector boson $W$, since for bound states in the symmetric phase an expansion in powers of $g^2$ is not justified. Neglecting binding effects, this yields the mass formulae

$$m_R \simeq 2m_\Phi, \quad m_L \simeq 2m_\Phi + 2m_W, \quad m_P \simeq 4m_W, \quad m_V \simeq 2m_\Phi + m_W.$$ \hfill (46)

These relations can be compared with results from lattice simulations. A screening mass $m_W$ for the vector boson was determined in\cite{Hatsuda:1991}, $m_W = 0.35(1)g^2$. No scalar screening mass has been measured so far, hence we choose $m_\Phi = m_R/2$. This yields three predictions for $m_L$, $m_P$ and $m_V$ which are compared with the results of Ref.\cite{Hatsuda:1991} in table 1. The qualitative agreement supports the constituent picture. Note, that in the bound state model\cite{Dosch:1987} no prediction has so far been made for the W-ball mass $m_P$.

The proposed picture can be tested by measuring the gauge-invariant propagators $\hat{G}_\Phi$ and $\hat{G}_W$ as functions of $\mu^2$. The masses $m_\Phi(\mu^2; \mu_0^2)$ and $m_W(\mu^2; \mu_0^2)$ should behave like the solutions $M(\mu^2)$ and $m(\mu^2)$ of the gap equations. In particular, at a first-order transition from the Higgs phase to the
symmetric phase, both screening masses should jump to smaller values. With increasing $\lambda/g^2$ the jump should decrease and eventually vanish at the critical coupling where the crossover behaviour sets in.

We hope that further numerical and analytical investigations will unequivocally clarify the relation between the various screening masses and provide a clear physical picture of the symmetric phase. This will be crucial in order to understand better real-time correlation functions at high temperature, in particular the sphaleron rate.\textsuperscript{30}

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| $J^{PC} = 0^{++}$ | $J^{PC} = 1^{--}$ |
|-------------------|-------------------|
| $R$ | $L$ | $P$ | $V$ |
| lattice simulations | 0.839(15) | 1.47(4) | 1.60(4) | 1.27(6) |
| constituent model | - | 1.54 | 1.40 | 1.18 |

Table 1: Comparison of screening masses from lattice simulations and a constituent model. $m_R$ is used to fix the constituent scalar mass. From Ref.\textsuperscript{22}. 
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