Multifractality of Cloud Base Height Profiles

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Abstract

Cloud base height profiles measured with laser ceilometer are studied using multifractal approach. The irregular structure of the signals is a benchmark for nonlinear dynamical processes. A hierarchy of generalized dimensions determines the intermittency of the signal. The multi-affine properties are described by the $h(\gamma)$ function.

Key words: time series analysis, fractals, cloud physics

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1 Introduction

Cloud base height profiles (CBHP) are known to have highly fluctuating and irregular structure. The dynamics of CBHP evolution is determined by a variety of processes in the atmosphere \cite{1}. This irregular structure is a benchmark for nonlinear dynamical processes. We apply the multifractal approach to study the scaling properties of the CBHP. In this report we present an analysis of the multi-affine properties of a CBHP signal $y(t)$ (Fig.1) measured with a ground-based laser ceilometer\textsuperscript{1} having a temporal resolution of 30 seconds, taken on September 23-25, 1997 at the Southern Great Plains (SGP) (Oklahoma, USA) site of the Atmospheric Radiation Measurement Program of the Department of Energy.

\textsuperscript{1} http://www.arm.gov
Fig. 1. Cloud base height profile data measured at SGP site on Sept. 23-25, 1997. Time resolution : 30 sec; 7251 data points.

2 Multi-affinity and Intermittency

First, we tested the scaling properties of the power spectral density $S(f)$ of the CBHP signal and obtained (figure not shown) that $S(f) f^{-\beta}$ with $\beta = 1.46 \pm 0.06$ for frequencies lower than $1/15$ min$^{-1}$ and $\beta \approx 0$ for higher frequencies.

The multi-affine properties of $y(t)$ can be described by the so-called “q-th” order structure functions $c_q = \langle |y(t+r) - y(t)|^q \rangle$ ($i = 1, 2, \ldots, N-r$), where the averages are taken over all possible pairs of points that are $\tau = t_i + r - t_i$ apart from each other with $r > 0$. Assuming a power law dependence of the structure function, the $H(q)$ spectrum is defined through $c_q(\tau) \sim \tau^{qH(q)}$ with $q \geq 0$.

The intermittency of the signal can be studied through the so-called singular measure analysis. The first step that this technique requires is defining a basic measure $\varepsilon(1;l)$ as $\varepsilon(1;l) = |\Delta y(1;l)|/ < \Delta y(1;l) > (l = 0, 1, \ldots, N-1)$, where $\Delta y(1;l) = y(t_{i+1}) - y(t_i)$ is the small-scale gradient field and $< \Delta y(1;l) > = \frac{1}{N} \sum_{l=0}^{N-1} |\Delta y(1;l)|$. It should be noted that we use spatial/temporal averages rather than ensemble averages, thus making an ergodicity assumption [5] as our only recourse in such an empirical data analysis. Next we define a series of ever more coarse-grained and ever shorter fields $\varepsilon(r;l)$ where $0 < l < N - r$ and $r = 1, 2, 4, \ldots, N = 2^m$. The average measure in the interval $[l; l + r]$ is $\varepsilon(r;l) = \frac{1}{r} \sum_{l'=l}^{l+r-1} \varepsilon(1;l')$ ($l = 0, \ldots, N - r$) The scaling properties of the generating function are then searched for through $\chi_q(\tau) = < \varepsilon(r;l)^q >$
Thus the multi-fractal properties of the CBHP signal are expressed by two sets of scaling functions, $H(q)$ describing the roughness of the signal and $K(q)$ describing its intermittency. The intermittency of the signal can be also expressed through the generalized dimensions $D(q)$ as introduced by Grassberger [6] and Hentschel and Procaccia, [7] $D(q) = 1 - K(q)/(q - 1)$.

The multi-affinity of $y(t)$ means that one should use different scaling factors $H(q) = H_q$ in order to rescale such a signal. This also implies that local roughness exponents $\gamma$ exist [8] at different scales. The density of the points $N_\gamma(\tau)$ that have the same roughness exponent is assumed [9] to scale over the time span $\tau$ as $N_\gamma(\tau) \sim \tau^{-h(\gamma)}$. From Ref. [10] the following relations are found: $\gamma(q) = d(qH(q))/dq$ and $h(\gamma_q) = 1 + q\gamma(q) - qH(q)$. In Fig. 3 the CBHP multi-affine properties are presented via the $h(\gamma)$-curve.

3 Conclusions

We have demonstrated the multi-affine structure of cloud base height profiles. Further work will be directed toward relating these statistical parameters to the dynamical properties of the clouds, an important step toward understanding, modeling and predicting their dynamical behaviour.
Fig. 3. The $h(\gamma)$-curve for the CBHP data in Fig. 1

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