Optical forces in photonic Weyl system

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Abstract

Topological photonics has attracted extensive attention, since it allows for a platform to explore and exploit versatile nano-optics systems. In particular, the ideal Weyl metamaterials have recently been demonstrated with fascinating phenomena such as chiral zero mode and negative refraction. In this work, we apply the photonic Weyl metamaterials into the optical tweezers. Based on the effective medium approach, the optical force generated by the body state of the Weyl metamaterial is systematically investigated. Interestingly, theoretical results show that for oblique incidence, the optical force spectra present a valley around Weyl frequency with zero magnitude exactly at the Weyl frequency, and the forces show strong optical circular dichroism. In addition, due to the bi-anisotropic properties, transmissions through the Weyl metamaterial exhibit a significant linear-to-circular polarization conversion and the transmitted wavefront acquires spin momenta of photons, which induces abnormal force on chiral particles. Our study may provide potential applications in the optical manipulations, polarization conversions, and wavefront engineering optics.

Since the pioneering work of Ashkin [1], optical tweezers have become an indispensable tool in manipulating micro-particles. This technique allows for trapping and manipulation of various nanoparticles [2–6], nanostructures [7], atoms [8] and even cells and virus [9, 10] without mechanical contact. So far, optical tweezers have been widely used in multiple disciplines such as nanophotonics [11, 12], near field optics [13], plasmonics [14–16], optical cooling [17] and biological science [18, 19]. Optical force, which transfers the momentum from electromagnetic (EM) radiation to the particle, plays a vital role in these important optical manipulating applications [20–34].

Topology, which describes invariant properties under continuous deformation, have come into focus recently [35–39]. Topologically nontrivial photonic systems, including photon topological insulators [40, 41], photonic topological semimetals [42], have been proposed and realized both in theory and experiment. As a highly robust gapless topological state, Weyl points are the point degeneracies of linearly dispersing energy bands in three-dimensional momentum space [43, 44]. Photonic Weyl metamaterial exhibits a lot of exceptional properties, including the existence of Fermi arcs at the interfaces, diminishing density of states at the Weyl frequency [45, 46], all angle negative refraction [47, 48], chiral zero modes and chiral magnetic effects [49, 50]. However, photonic Weyl systems have not yet been studied in the field of optical manipulations. Here we carry out a comprehensive theoretical studies on the optical force exerted on a particle by the transmitted wave passing through the ideal Weyl metamaterial slab. The general effective Hamiltonian of Weyl point is

\[ H_{\text{eff}} = N_x k_x \sigma_x + N_y k_y \sigma_y + N_z k_z \sigma_z + T_c k_I, \]
Figure 1. (a) Unit cell of photonic Weyl metacrystal: a saddle-shaped solid metallic coil with D2d point group symmetry in the tetragonal lattice, lattice constants of the unit structure are $p_x = p_z = 3 \mu m$ and $p_y = 4.5 \mu m$. (b) Band structures of the Weyl metacrystal, the type-I Weyl point reside along the $k_x = k_z$ directions with respect to $k_y = 0$. EFCs of the Weyl metamaterial (blue line) in $k_x - k_z$ plane at the frequency: (c) 0.936$\omega_{\text{Weyl}}$ and (d) 1.063$\omega_{\text{Weyl}}$. The red circle in the center shows the EFS of air at the same frequency. The incident and refracted group velocities pointing to gradient directions of the EFCs are denoted by the blue and red arrows, respectively.

where $\sigma_{x,y,z}$ are Pauli matrices, $I$ is a $2 \times 2$ identity matrix, $N_{x,y,z}$ are the Fermi velocities and $T_x$ is the tilted velocity on the $x$ direction. The parameter that describing the tilting of Weyl dispersion cone can be defined as $\alpha_{\text{Weyl}} = T_x / N_x$, for $\alpha_{\text{Weyl}} < 1$ corresponds to type-I Weyl system, and $\alpha_{\text{Weyl}} > 1$ corresponds to type-II Weyl system [39].

Configuration of unit cell of the ideal Weyl metamaterial is shown in figure 1(a), a saddle-shaped solid metallic structure with D2d point group symmetry are embedded in an isotropic dielectric medium with dielectric constant 2.2. Band structures of the Weyl metamaterial in figure 1(b), it shows the type-I Weyl points formed by the linear crossing between the longitudinal mode (LM) and the transverse modes (TM) are located at the frequency of 14.8 THz and are well separated from any other bands.

The calculated equifrequency contours (EFCs) of the Weyl metacrystal and air are shown in the figure 1(c) 0.936$\omega_{\text{Weyl}}$ and (d) 1.063$\omega_{\text{Weyl}}$, since the unit cell is rotated 45° along the $y$ axis, subsequently the four elliptical EFCs are rotated accordingly. It indicates that the four elliptical EFCs of type-I Weyl points reside in the first Brillouin zone symmetrically, and the red circle in the center represents the air cone on the same frequency. When the frequency is lower than the Weyl frequency, as the arrows indicated in figure 1(c), the EFC of Weyl metamaterial shrinks with negative group velocity, meaning that negative refraction would occur at the interface between air and Weyl metamaterial. The black vertical line indicates the tangential wavevector $k_z$ is conserved. When the frequency is higher than the Weyl frequency in
Weyl frequency can be obtained as the frequency of longitude mode by the TE polarized Gaussian beam passing through the Weyl metacrystal slab. Under a steady state or trap a particle, this is the well known optical manipulations. Next, we investigate the optical force exerted when the frequency is above the Weyl frequency in figure 2(d), the Gaussian beam experiences a positive refraction.

When the frequency is below the Weyl frequency in figure 2(c), the EFC of the Weyl metamaterial shrinks to a point where the group velocity is zero, thus the frequency, the Gaussian beam undergoes a negative refraction. When approaching the Weyl frequency in figure 1(d), positive refraction emerges naturally between the two media, EFCs of Weyl metamaterial expands with positive group velocity. Since the robustness and linear dispersion of the Weyl medium, it provides major advantages over traditional negative index media.

The EM response of the saddle metallic coil can be modeled as two splitting resonance rings (SRR), then the effective material tensors of the bi-anisotropic Weyl metamaterial can be obtained as

\[
\begin{align*}
\varepsilon & = \begin{bmatrix}
\varepsilon_\perp & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \varepsilon_\perp
\end{bmatrix}, & \varepsilon_\perp &= 1 + \frac{F}{L} \frac{1}{\omega_0^2 - \omega^2 - i\rho\omega}, \\
\mu & = \begin{bmatrix}
\mu_\perp & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \mu_\perp
\end{bmatrix}, & \mu_\perp &= 1 + \frac{A^2}{L} \frac{1}{\omega_0^2 - \omega^2 - i\rho\omega}, \\
\xi & = \begin{bmatrix}
\gamma & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -\gamma
\end{bmatrix}, & \gamma &= \frac{IA}{L} \frac{1}{\omega_0^2 - \omega^2 - i\rho\omega},
\end{align*}
\]

where \( L, A, \) and \( l \) are the inductance, effective area and length of SRR, \( \rho \) is the collision frequency, then the Weyl frequency can be obtained as the frequency of longitude mode \( \omega_{\text{Weyl}} = \sqrt{\omega_0^2 L + F} / (L - A^2) \) [44], where we neglect the non-local effect in the Weyl metamaterial. Here, the Weyl frequency is far below the plasmonic resonance of the saddle metallic coil (\( \omega_0 \approx 0.2\omega_{\text{Weyl}}, \) see appendix A). Using the constitutive relations,

\[
\begin{align*}
D &= \varepsilon_0 \varepsilon_\perp \vec{E} + i\sqrt{\varepsilon_0} \mu_\perp \vec{H} \\
B &= \mu_0 \mu_\perp \vec{H} - i\sqrt{\mu_0} \varepsilon_\perp \vec{E}
\end{align*}
\]

and the Maxwell’s equations in the frequency domain,

\[
\begin{align*}
\nabla \times \vec{E} &= i\omega \vec{B} \\
\nabla \times \vec{H} &= -i\omega \vec{D}
\end{align*}
\]

we derive the dispersion relation for the ideal Weyl metamaterial:

\[
(\varepsilon_\perp \mu_\perp - \gamma^2) \bar{\omega}^4 - (\varepsilon_\perp + \mu_\perp) (\varepsilon_\perp \mu_\perp - \gamma^2) |k|^2 \bar{\omega}^2 + \varepsilon_\perp \mu_\perp |k|^4 - \gamma^2 (k_x^2 - k_y^2)^2 = 0,
\]

where \( \bar{\omega} = \omega / c, \) \( c \) is the speed of light in vacuum, the propagating wavevector in the Weyl metamaterial is obtained as

\[
k_{\text{Weyl}}^{1,2,3,4} = \pm \frac{1}{\sqrt{2}} \left[ \frac{\bar{\omega}^2 (\varepsilon_\perp + \mu_\perp) - 2k_x^2 \pm \sqrt{\left( (\varepsilon_\perp - \mu_\perp)\right)^2 + 4\gamma^2 \left[ (\varepsilon_\perp - \mu_\perp)^2 + 4\gamma^2 \right]}}{8\bar{\omega}^2 k_x^2 \gamma^2 (\varepsilon_\perp + \mu_\perp) \left( \varepsilon_\perp \mu_\perp - \gamma^2 \right) + 16k_x^2 \varepsilon_\perp \mu_\perp \gamma^2 \left( \varepsilon_\perp \mu_\perp - \gamma^2 \right)^2} \right].
\]

Subsequently, we use the commercial software Comsolf Multiphysics [51] to perform full wave simulations of a focused Gaussian beam with waist radius 60 \( \mu m \) propagating through the Weyl metamaterial slab. Gaussian beam is incident from the air with an angle of 20°. Electric field amplitude distribution is shown in figure 2(a), as indicated by the black arrows, all angle negative refraction of the Gaussian beam is demonstrated at the interfaces between the air and the Weyl metamaterial slab.

Figures 2(d)–(f) illustrate the electric field \( E_y \) (perpendicular to the \( Nz \) plane) with the same incident angle 20° at different frequencies. As shows in figure 2(b), when the frequency is lower than the Weyl frequency, the Gaussian beam undergoes a negative refraction. When approaching the Weyl frequency in figure 2(c), the EFC of the Weyl metamaterial shrinks to a point where the group velocity is zero, thus the Gaussian beam undergoes near zero refraction and the intensity of transmitted wave declines dramatically. When the frequency is above the Weyl frequency in figure 2(d), the Gaussian beam experiences a positive refraction.

When light is scattered or absorbed by a dielectric particle, the transfer of optical momentum can push or trap a particle, this is the well known optical manipulations. Next, we investigate the optical force exerted by the a TE polarized Gaussian beam passing through the Weyl metacrystal slab. Under a steady state
condition, the radiation force $\mathbf{F}$ exerted on the particle is derived by integrating the dot product of the outwardly directed normal unit vector $\mathbf{n}$ and the Maxwell’s stress tensor $\mathbf{T}$ over a surface enclosing the particle:

$$\langle \mathbf{F} \rangle = \left\langle \oint \mathbf{n} \cdot \mathbf{T} dS \right\rangle,$$

where $\left\langle \right\rangle$ represents a time average, and the Maxwell’s stress tensor $\mathbf{T}$ is

$$\mathbf{T} = \frac{1}{4\pi} \left( \mathbf{E}\mathbf{E} - \mathbf{H}\mathbf{H} - \frac{1}{2} \left( \mathbf{E}\mathbf{E}^\ast + \mathbf{H}\mathbf{H}^\ast \right) \mathbf{I} \right).$$

Since the particle radius is much smaller than the wavelength, we simplify the configuration by considering the particle as a dipole, in which case the time averaged optical force acting on a dipolar particle can be analytically addressed as $[22, 23, 27]$:

$$\langle \mathbf{F} \rangle = -\frac{1}{2} Re \left[ (\nabla \mathbf{E}^\ast) \cdot \mathbf{p} + (\nabla \mathbf{H}^\ast) \cdot \mathbf{m} - c k_0^3/6\pi (\mathbf{p} \times \mathbf{m}^\ast) \right],$$

where the dipolar polarizabilities ($\alpha, \beta, \chi$) for a spherical nanoparticle in the quasi-static limit can be expressed

$$\alpha = 4\pi R^3 \left( \varepsilon_r - 1 \right) / \left( \varepsilon_r + 2 \right) \left( \mu_r + 2 \right) - \kappa^2,$$

$$\beta = 4\pi R^3 \left( \varepsilon_r + 2 \right) / \left( \varepsilon_r + 2 \right) \left( \mu_r + 2 \right) - \kappa^2,$$

$$\chi = 12\pi R^3 \kappa / \left( \varepsilon_r + 2 \right) \left( \mu_r + 2 \right) - \kappa^2,$$

where $\varepsilon_r, \mu_r$ and $\kappa$ are the electric permittivity, magnetic permeability, and chirality of the particle, respectively $[27]$, the real part (imaginary part) is associated with optical rotation (circular dichroism), $\varepsilon_d, \mu_d$ are the electric permittivity and magnetic permeability of the surrounding medium, and $\varepsilon_s = \varepsilon_r/\varepsilon_d, \mu_s = \mu_r/\mu_d$. The optical force in equation (9) can be divide into two parts: the dipole force term $\mathbf{F}_{\text{dipole}} = 1/2 Re \left[ (\nabla \mathbf{E}^\ast) \cdot \mathbf{p} + (\nabla \mathbf{H}^\ast) \cdot \mathbf{m} \right]$, and the recoil force term

$$\mathbf{F}_{\text{recoil}} = -1/2 Re \left[ c k_0^3/6\pi (\mathbf{p} \times \mathbf{m}^\ast) \right].$$

We first consider the polystyrene particle as an electric dipolar particle without chirality ($\varepsilon_R = 2.25, u_1 = 1, \kappa = 0$).

Optical forces as a function of particle displacement on the lateral direction with normal incidence are shown in figure 3(a); the solid lines represent analytical results obtained by our effective medium approach and the hollow symbols indicate the results obtained by full wave simulations (see appendix B for tuning bi-anisotropic weak form) with incident power 10 mW. As it shows, the scattering force $F_s$ increases to the maximum at the center of transmitted Gaussian beam waist and the bidirectional oriented gradient force $F_s$ would pull the particle to the center of the beam spot laterally. The analytical predictions are in good agreement with the simulation results. Correspondingly, the optical force obtained by simulation on different conditions are shown in figures 3(b)–(d): (b) negative refraction (0.915$\omega_{\text{Weyl}}$), (c) non zero
refraction (around Weyl frequency), (d) positive refraction ($1.085\omega_{\text{Weyl}}$). It shows the optical forces on the different refraction conditions exhibit the similar behavior, while its magnitude near the Weyl frequency is much less than those above or below the Weyl frequency.

Furthermore, due to the strong dispersion of the Weyl metamaterial, in a specific location ($x = -50\ \mu m$), as indicated by the green triangles in figures 3(b) and (d), the lateral optical force changes from negative to positive on different frequency that the particle would be pulled to the opposite horizontal direction. That means we can effectively manipulate the gradient force direction by just tuning the frequency in a small range around the Weyl frequency, demonstrating a new degree of freedom of Weyl metamaterials for optical force modulation.

As indicated in figure 2, the Gaussian wave experiences a transition from negative refraction to positive refraction through the Weyl frequency, thus it is expected that the corresponding optical force vary significantly around the Weyl frequency. The optical force spectra around the Weyl frequency for different polarized waves are shown in figure 4. For an incident wave of arbitrary polarization, when increasing to the Weyl frequency, both the gradient force $F_x$ and scattering force $F_z$ decrease to zero (close to zero refraction), and the magnitude of optical forces increase gradually after the frequency exceeds the Weyl frequency. The optical force spectra exhibit a valley shape around the Weyl frequency with the dip slightly lower than the Weyl frequency, rather than exactly at the Weyl frequency. This is due to our effective medium model does not consider the non-local effect in the Weyl metamaterial. Furthermore, this zero optical forces might be used for particle suspensions or manipulations. Since the Weyl frequency can be easily tuned by just changing size of unit structure of the Weyl metamaterial, it provides more advantages in optical tweezers.

More importantly, as shown in figures 4(a) and (b), the two orthogonal linearly polarized wave (TE/TM) show the same order of magnitude of optical forces, while for circularly polarized waves (RCP/LCP), the optical force shows a substantial difference between opposite polarizations in figures 4(c) and (d), where the difference in magnitude exceeds one order of magnitude. The physical origin of the

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**Figure 3.** Optical forces versus particle displacement along the lateral direction ($x$ axis): gradient force $F_x$ (black line) and scattering force $F_z$ (red line). (a) Normal incidence; (b)–(d) full wave simulation results with incident angle $20^\circ$ on different refraction conditions: (b) negative refraction, (c) near zero refraction, (d) positive refraction. Particle radius $R = 5\ \mu m$. 

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significant difference in RCP/LCP waves can be understood from the band structures and the EFC of the ideal Weyl metamaterial. Since the two Weyl cones involved in the refraction process have the same topological charge, transmissions through the Weyl metamaterial would exhibit a strong circular dichroism (CD), resulting in a considerable difference in the optical force spectra \([\text{figures 4(c) and (d)}]\) and also in the transmission for RCP and LCP waves \([\text{figure 5(a)}]\).

Gradient force \(F_x\) and scattering force \(F_z\) generated by RCP wave for different incident angles are shown in \([\text{figures 4(e) and (f)}]\). It shows that for normal incidence, the optical force spectra exhibit a flat band with high amplitude near the Weyl frequency. In contrast, both the gradient force and scattering force exhibit a dip at the Weyl frequency for oblique incidence. When the incident angle increases, the dip is broadened gradually and the corresponding magnitude declines.

Since the fundamental building blocks of life are built of chiral amino acids and chiral sugar, the optical forces exerted on chiral objects has recently initiated a lot of interests. Due to the bi-anisotropic characteristic of ideal Weyl metamaterial, it is expected that the transmitted wave passing through the Weyl
metamaterial slab would generate strong optical couplings with chiral particles. The corresponding optical forces versus the particle’s chirality $\kappa$ exerted by TE wave is illustrated figure 6(a), the solid lines represent analytical results obtained by the effective medium approach with dipolar approximation and the hollow symbols are the results obtained by full wave simulations. It shows that the magnitude of both gradient force and scattering force increases gradually with $|\kappa|$, and they are not symmetric with respect to $\kappa = 0$. In contrast, figure 6(b) shows the optical forces exerted by the TE wave transmitted through an isotropic slab with the same size, it presents a symmetrical form with respect to chirality. To further clarify the physical mechanism on the different chiral responses, we show the electric field magnitude around the particle in figure 6 with different chirality $\kappa = \pm 1$ respectively for different transmitted incident waves through the Weyl metamaterial slab (c) and (d) and the isotropic slab (e) and (f). It shows the scattering patterns of the particles are the same for opposite chirality with incident transmitted wave through an isotropic slab, but
are different for incident transmitted wave through the Weyl metamaterial slab. Specifically, for particle chirality $\kappa = 1$ in figure 6(c), most of the energy is coupled into the particle, which is completely different from chirality $\kappa = 1$ in figure 6(d). The scattering field patterns reflect different mutual interactions between the particle and incident field, which would induce different optical forces for opposite chirality.

The difference on the chiral response originates from the mutual interaction of chiral particle with the transmitted beam wavefront that carries optical spin density. When the Gaussian beam passes through the Weyl metamaterial slab, the wavefront of transmitted wave acquires spin angular momentum (SAM) of photons. As the light is scattered by a chiral particle, the transfer of SAM induces an abnormal force on the particles with opposite chirality. The unsymmetric behavior is mainly contributed to the recoil force term stemming from dipolar self-interactions.

The chirality flow that corresponds to the time-averaged SAM densities is given by the summation of electric and magnetic field ellipticities as $\Phi = \omega (\epsilon \Phi_E + \mu \Phi_H) / 2$, where $\Phi_E = -1/2 \text{Im} (E \times E^*)$ and $\Phi_H = -1/2 \text{Im} (H \times H^*)$ [22, 27]. The chirality flow vector can be analytically obtained as $\Phi_{\text{chiral}} = - \omega \epsilon \Pi \cdot \text{Im} (E_t E_s^* \sin \theta, 0, E_t E_s^* \cos \theta)$, where $E_t$ and $E_s$ is the transmission amplitude for TE and TM waves, respectively (see appendix A). This can be confirmed by the transmission for linearly polarized wave figure 5(b), a TE wave impinges on the Weyl metamaterial slab, the two orthogonally polarized transmitted waves have comparable magnitudes, indicating a significant linear-to-circular polarization conversion and thus acquiring an SAM on the wavefront of transmitted wave. For the linearly polarized wave passes through an isotropic medium, one of $E_t$ or $E_s$ will be zero, such that polarization conversion would not occur and there is no optical SAM carried on the wavefront, therefore the corresponding optical response is the same for the positive and negative chirality of the particle.

Conclusions

We have carried out a systematic investigation on the optical force exerted on a dipolar particle by the transmitted wave passing through the Weyl metamaterial slab theoretically. It shows the optical force spectra exhibit a dip that the magnitude approaches zero at the Weyl frequency. When the incident angle increases, the dip is broadened gradually. Meanwhile, the magnitude and direction of gradient force could be modified effectively by simply turning the frequency. Furthermore, the ideal Weyl metamaterials demonstrate a significant optical CD and linear-to-circular polarization conversion effect, which could be used in versatile optical manipulations and wavefront engineering. The analytical approach on dipolar approximation indicates a reasonable description on the optical forces. Our findings may inspire searches for other photonic topological devices’ applications such as optical tweezers and wavefront engineering optics.

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Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: 10.1088/1367-2630/ac5e88.

Appendix A

In the appendix, we derive the analytical formula of an EM Gaussian beam transmitted through the Weyl metamaterial slab. A focused Gaussian EM wave impinges on the Weyl metamaterial with incident angle $\theta_i$ is given by

$$
E_i = \int_{-\infty}^{\infty} \mathbf{e}_r \Phi (s_x) \exp \left[ i f^i (x, z) \right] \, dk_x,
$$

$$
H_i = \sqrt{\frac{\varepsilon_0}{\mu_0}} \int_{-\infty}^{\infty} \mathbf{h}_r \Phi (s_x) \exp \left[ i f^i (x, z) \right] \, dk_x,
$$

$$
f^i = k_x (x - x_0) + k_z (z - z_0),
$$

(11)
where $\Phi (z_c)$ is the Fourier spectrum for Gaussian beam with the center located at $(x_0, z_0)$:

$$\Phi (z_c) = \exp \left(-\frac{z_c^2}{2\sigma_0^2}\right),$$

$$s_x = k_x \cos \theta_i - k_z \sin \theta_i, \quad s_z = k_x \sin \theta_i + k_z \cos \theta_i,$$

and

$$e_i = E_i^2 y + E_i^0 (\cos \theta_i x - \sin \theta_i z),$$

$$h_i = E_i^2 y + E_i^0 (-\cos \theta_i x + \sin \theta_i z),$$

where $E_i^p$ and $E_i^s$ are the electric field magnitudes for the $p$ and $s$ polarized incident wave, respectively.

Suppose that the Weyl metamaterial slab along the $z$ axis is located at: (a) and (b) [figure 2(a)], then the reflected EM wave is obtained

$$E_r = \int_{-\infty}^{\infty} e_i \Phi (s_x) \exp [if^r (x, z)] \, dk_x,$$

$$H_r = \sqrt{\varepsilon_0 \mu_0} \int_{-\infty}^{\infty} h_i \Phi (s_x) \exp [if^r (x, z)] \, dk_x,$$

$$f^r = k_x (x - x_0) - k_z (z + z_0 - 2a),$$

where

$$e_r = E_r^2 y + E_r^0 (\cos \theta_i x + \sin \theta_i z),$$

$$h_r = E_r^2 y + E_r^0 (\cos \theta_i x + \sin \theta_i z).$$

And the transmitted wave through the Weyl metamaterial slab into air is formulated as

$$E_t = \int_{-\infty}^{\infty} e_i \Phi (s_x) \exp [if^t (x, z)] \, dk_x,$$

$$H_t = \sqrt{\varepsilon_0 \mu_0} \int_{-\infty}^{\infty} h_i \Phi (s_x) \exp [if^t (x, z)] \, dk_x,$$

$$f^t = k_x (x - x_0) + k_z (z - b) + k^W_{\text{Weyl}} (b - a) + k_z (a - z_0),$$

where

$$e_t = E_t^2 y + E_t^0 (\cos \theta_i x - \sin \theta_i z),$$

$$h_t = E_t^2 y + E_t^0 (-\cos \theta_i x + \sin \theta_i z),$$

$\theta_t$ is the transmission angle. The continuity of the tangential electric and magnetic fields at the planar interface of air and Weyl metamaterial slab gives rise to the following relations between the incident, reflected, and transmitted waves as

$$\begin{vmatrix}
E_i^2 \\
E_i^0 \\
0 \\
0 \\
\end{vmatrix}
= \begin{vmatrix}
Q_{11} & Q_{12} & Q_{13} & Q_{14} \\
Q_{21} & Q_{22} & Q_{23} & Q_{24} \\
Q_{31} & Q_{32} & Q_{33} & Q_{34} \\
Q_{41} & Q_{42} & Q_{43} & Q_{44} \\
\end{vmatrix}
\begin{vmatrix}
E_r^2 \\
E_r^0 \\
0 \\
0 \\
\end{vmatrix},$$

$$\begin{vmatrix}
E_t^2 \\
E_t^0 \\
0 \\
0 \\
\end{vmatrix},$$

where $Q = D_1 D_2^{-1} P_1 D_2 D_1^{-1}$, $P_2 = \begin{pmatrix}
e^{ik_{\text{Weyl}} d}, e^{ik_{\text{Weyl}} d}, e^{ik_{\text{Weyl}} d}, e^{ik_{\text{Weyl}} d} \end{pmatrix}$, and $d$ is the length of Weyl metamaterial slab. $D_1$ and $D_2$ are the matrices of tangential electric and magnetic field components of air and Weyl metamaterial, respectively. The transmission and reflection coefficients can be numerically derived by extracting values from the $Q$ matrix,

$$\begin{vmatrix}
E_t^2 \\
E_t^0 \\
0 \\
0 \\
\end{vmatrix}
= \begin{vmatrix}
r_{ss} & r_{sp} & r_{tp} & r_{pp} \\
r_{ps} & r_{pp} & r_{tp} & r_{ss} \\
r_{ps} & r_{pp} & r_{tp} & r_{ss} \\
r_{ps} & r_{pp} & r_{tp} & r_{ss} \\
\end{vmatrix}
\begin{vmatrix}
E_i^2 \\
E_i^0 \\
0 \\
0 \\
\end{vmatrix},$$

and

$$\begin{vmatrix}
E_r^2 \\
E_r^0 \\
0 \\
0 \\
\end{vmatrix}
= \begin{vmatrix}
t_{ss} & t_{sp} & t_{tp} & t_{pp} \\
t_{ps} & t_{pp} & t_{tp} & t_{ss} \\
t_{ps} & t_{pp} & t_{tp} & t_{ss} \\
t_{ps} & t_{pp} & t_{tp} & t_{ss} \\
\end{vmatrix}
\begin{vmatrix}
E_i^2 \\
E_i^0 \\
0 \\
0 \\
\end{vmatrix}.$$
Appendix B

The core of finite element method implemented in Comsol [51] is the weak form. Here we briefly summarize the formula of the weak form for bi-anisotropic media. Firstly, the general bi-anisotropic constitutive relation can be expressed as

\[
\vec{D} = \varepsilon \vec{E} + \xi \vec{H},
\]

\[
\vec{B} = \mu \vec{H} + \zeta \vec{E},
\]

where \(\varepsilon = \varepsilon_0 \varepsilon_r\) and \(\mu = \mu_0 \mu_r\). Plugging equation (21) into the Maxwell’s equations equation (4) gives the wave equation

\[
\nabla \times \left( \frac{1}{\mu_r} \nabla \times \vec{E} \right) - i \omega \nabla \times \frac{1}{\mu_r} \vec{E} - \omega^2 \left( \varepsilon - \varepsilon_0 \frac{1}{\mu_r} \right) \vec{E} = 0.
\]

Multiply equation (22) by a test function \(\vec{E}\), which is a set of localized basis functions to approximate the exact solution, and perform a volume integral, the resulting integrand can be obtained as

\[
- i \omega \mu_0 \nabla \times \vec{E} - i \omega \nabla \times \frac{1}{\mu_r} \vec{E} + \omega^2 \left( \mu_0 \varepsilon_0 \vec{E} - \frac{1}{\mu_r} \zeta \vec{E} \right) \vec{E} \cdot \vec{E} = 0 \quad (23a)
\]

with

\[
\vec{H} = \frac{1}{i \omega \mu} \nabla \times \vec{E} - \frac{1}{\mu} \zeta \vec{E} \quad (23b)
\]

In practice, say, in the RF module of Comsol, we just simply modify the default weak form by adding the additional terms highlighted in red color shown in equation (23), where the test function can be expressed by the default script in Comsol as \(\vec{E}_x = \text{test}(E_x)\), and so on.

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