2D ARRAYS OF JOSEPHSON NANOCONTACTS AND NANOGRANULAR SUPERCONDUCTORS

ABSTRACT

By introducing a realistic model of nanogranular superconductors (NGS) based on 2D arrays of Josephson nanocontacts (created by a network of twin-boundary dislocations with strain fields acting as insulating barriers between hole-rich domains), in this Chapter we present some novel phenomena related to mechanical, magnetic, electric and transport properties of NGS in underdoped single crystals. In particular, we consider chemically induced magnetoelectric effects and flux driven temperature oscillations of thermal expansion coefficient. We also predict a giant enhancement of the nonlinear thermal conductivity of NGS reaching up to 500% when the intrinsically induced chemoelectric field (created by the gradient of the chemical potential due to segregation of hole producing oxygen vacancies) closely matches the externally produced thermoelectric field. The estimates of the model parameters suggest quite an optimistic possibility to experimentally realize these promising and important for applications effects in non-stoichiometric NGS and artificially prepared arrays of Josephson nanocontacts.

1. INTRODUCTION

Inspired by new possibilities offered by the cutting-edge nanotechnologies, the experimental and theoretical physics of increasingly sophisticated mesoscopic quantum devices, heavily based on Josephson junctions (JJs) and their arrays (JJAs), is becoming one of the most exciting and rapidly growing areas of modern science (for reviews on charge and spin effects in mesoscopic 2D JJs and quantum-state engineering with Josephson devices, see, e.g., Newrock et al. 2000, Makhlin et al. 2001, Krive et al. 2004, Sergeenkov 2006, Beloborodov et al. 2007). In particular, a remarkable increase of the measurements technique resolution made it possible to experimentally detect such interesting phenomena as flux avalanches (Altshuler and Johansen 2004) and geometric quantization (Sergeenkov and Araujo-Moreira 2004) as well as flux dominated behavior of heat capacity (Bourgeois et al. 2005) both in JJs and JJAs.

Recently, it was realized that JJAs can be also used as quantum channels to transfer quantum information between distant sites (Makhlin et al. 2001, Wendin and Shumeiko 2007) through the implementation of the so-called superconducting qubits which take advantage of both charge and phase degrees of freedom.

Both granular superconductors and artificially prepared JJAs proved useful in studying the numerous quantum (charging) effects in these interesting systems, including Coulomb
blockade of Cooper pair tunneling (Iansity et al. 1988), Bloch oscillations (Haviland et al. 1991), propagation of quantum ballistic vortices (van der Zant 1996), spin-tunneling related effects using specially designed SFS-type junctions (Ryazanov et al. 2001, Golubov et al. 2002), novel Coulomb effects in SINIS-type nanoscale junctions (Ostrovsky and Feigel’man 2004), and dynamical AC reentrance (Araujo-Moreira et al. 1997, Barbara et al. 1999, Araujo-Moreira et al. 2005).

At the same time, given a rather specific magnetostrictive (Sergeenkov and Ausloos 1993) and piezomagnetic (Sergeenkov 1998b, Sergeenkov 1999) response of Josephson systems, one can expect some nontrivial behavior of the thermal expansion (TE) coefficient in JJs as well (Sergeenkov et al. 2007). Of special interest are the properties of TE in applied magnetic field. For example, some superconductors like \( \text{Ba}_{1-x}\text{K}_x\text{BiO}_3 \), \( \text{BaPb}_x\text{Bi}_{1-x}\text{O}_3 \) and \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) were found (Anshukova et al. 2000) to exhibit anomalous temperature behavior of both magnetostriction and TE which were attributed to the field-induced suppression of the superstructural ordering in the oxygen sublattices of these systems.

The imaging of the granular structure in underdoped \( \text{Bi}_{2}\text{Sr}_{2}\text{CaCu}_2\text{O}_{8+\delta} \) crystals (Lang et al. 2002) revealed an apparent segregation of its electronic structure into superconducting domains (of the order of a few nanometers) located in an electronically distinct background. In particular, it was found that at low levels of hole doping (\( \delta < 0.2 \)), the holes become concentrated at certain hole-rich domains. (In this regard, it is interesting to mention a somewhat similar phenomenon of "chemical localization" that takes place in materials, composed of atoms of only metallic elements, exhibiting metal-insulator transitions, see, e.g., Gantmakher 2002.) Tunneling between such domains leads to intrinsic nanogranular superconductivity (NGS) in high-\( T_c \) superconductors (HTS). Probably one of the first examples of NGS was observed in \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) single crystals in the form of the so-called "fishtail" anomaly of magnetization (Daeumling et al. 1990). The granular behavior has been related to the 2D clusters of oxygen defects forming twin boundaries (TBs) or dislocation walls within \( \text{CuO} \) plane that restrict supercurrent flow and allow excess flux to enter the crystal. Indeed, there are serious arguments to consider the TB in HTS as insulating regions of the Josephson SIS-type structure. An average distance between boundaries is essentially less than the grain size. In particular, the networks of localized grain boundary dislocations with the spacing ranged from 10nm to 100nm have been observed (Daeumling et al. 1990) which produce effectively continuous normal or insulating barriers at the grain boundaries. It was also verified that the processes of the oxygen ordering in HTS leads to the continuous change of the lattice period along TB with the change of the oxygen content. Besides, a destruction of bulk superconductivity in these non-stoichiometric materials with increasing the oxygen deficiency parameter \( \delta \) was found to follow a classical percolation theory (Gantmakher et al. 1990).

In addition to their importance for understanding the underlying microscopic mechanisms governing HTS materials, the above experiments can provide rather versatile tools for designing chemically-controlled atomic scale JJs and JJAs with pre-selected properties needed for manufacturing the modern quantum devices (Sergeenkov 2001, Araujo-Moreira et al. 2002, Sergeenkov 2003, Sergeenkov 2006). Moreover, as we shall see below, NGS based phenomena can shed some light on the origin and evolution of the so-called paramagnetic Meissner effect (PME) which manifests itself both in high-\( T_c \) and conventional superconductors (Geim et al. 1998, De Leo and Rotoli 2002, Li 2003) and is usually
associated with the presence of \(\pi\)-junctions and/or unconventional \((d\text{-wave})\) pairing symmetry.

In this Chapter we present numerous novel phenomena related to the magnetic, electric, elastic and transport properties of Josephson nanocontacts and NGS. The paper is organized as follows. In Section 1, a realistic model of NGS is introduced which is based on 2D JJAAs created by a regular network of twin-boundary dislocations with strain fields acting as an insulating barrier between hole-rich domains (like in underdoped crystals). In Section 2, we consider some phase-related phenomena expected to occur in NGS, such as Josephson chemomagnetism and magnetoconcentration effect. Section 3 is devoted to a thorough discussion of charge-related polarization phenomena in NGS, including such topics as chemomagnetoelectricity, magnetocapacitance, charge analog of the "fishtail" (magnetization) anomaly, and field-tuned weakening of the chemically-induced Coulomb blockade. In Section 4 we present our latest results on the influence of an intrinsic chemical pressure (created by the gradient of the chemical potential due to segregation of hole producing oxygen vacancies) on temperature behavior of the nonlinear thermal conductivity (NLTC) of NGS. In particular, our theoretical analysis (based on the inductive model of 2D JJAAs) predicts a giant enhancement of NLTC reaching up to 500% when the intrinsically induced chemoelectric field \(E_{\mu} = \frac{1}{2e} \nabla \mu\) closely matches thermoelectric field \(E_T = S_T \nabla T\). And finally, by introducing a concept of thermal expansion (TE) of Josephson contact (as an elastic response of JJ to an effective stress field), in Section 5 we consider the temperature and magnetic field dependence of the TE coefficient \(\alpha(T, H)\) in a small single JJ and in a single plaquette (a prototype of the simplest JJA). In particular, we found that in addition to expected field oscillations due to Fraunhofer-like dependence of the critical current, \(\alpha\) of a small single junction also exhibits strong flux driven temperature oscillations near \(T_C\). The condition under which all the effects predicted here can be experimentally realized in artificially prepared JJAAs and NGS are also discussed. Some important conclusions of the present study are drawn in Section 6.

2. MODEL OF NANOSCOPIC JOSEPHSON JUNCTION ARRAYS

As is well-known, the presence of a homogeneous chemical potential \(\mu\) through a single JJ leads to the AC Josephson effect with time dependent phase difference \(\partial \phi / \partial t = \mu / \hbar\). In this Section, we will consider some effects in dislocation induced JJ caused by a local variation of excess hole concentration \(c(x)\) under the chemical pressure (described by inhomogeneous chemical potential \(\mu(x)\)) equivalent to presence of the strain field of 2D dislocation array \(\epsilon(x)\) forming this Josephson contact.

To understand how NGS manifests itself in non-stoichiometric crystals, let us invoke an analogy with the previously discussed dislocation models of twinning-induced superconductivity (Khaikin and Khlyustikov 1981) and grain-boundary Josephson junctions (Sergeenkov 1999). Recall that under plastic deformation, grain boundaries (GBs) (which are the natural sources of weak links in HTS), move rather rapidly via the movement of the grain boundary dislocations (GBDs) comprising these GBs. At the same time, observed (Daeumling et al. 1990, Lang et al. 2002, Yang et al. 1993, Moeckley et al. 1993) in HTS single crystals regular 2D dislocation networks of oxygen depleted regions (generated by the dissociation of \(<110>\) twinning dislocations) with the size \(d_0\) of a
few Burgers vectors, forming a triangular lattice with a spacing \( d \geq d_0 \) ranging from 10nm to 100nm, can provide quite a realistic possibility for the existence of 2D Josephson network within \( CuO \) plane. Recall furthermore that in a \( d \)-wave orthorhombic \( YBCO \) crystal TBs are represented by tetragonal regions (in which all dislocations are equally spaced by \( d_0 \) and have the same Burgers vector \( a \) parallel to \( y \)-axis within \( CuO \) plane) which produce screened strain fields (Gurevich and Pashitskii 1997) \( \epsilon(x) = \epsilon_0 e^{-|x|/d_0} \) with \( |x| = \sqrt{x^2 + y^2} \).

Though in \( YBa_2Cu_3O_{7-\delta} \) the ordinary oxygen diffusion \( D = D_0 e^{-U_d/k_BT} \) is extremely slow even near \( T_C \) (due to a rather high value of the activation energy \( U_d \) in these materials, typically \( U_d \sim 1eV \)), in underdoped crystals (with oxygen-induced dislocations) there is a real possibility to facilitate oxygen transport via the so-called osmotic (pumping) mechanism (Girifalco 1973, Sergeenkov 1995) which relates a local value of the chemical potential (chemical pressure) \( \mu(x) = \mu(0) + \nabla \mu \cdot x \) with a local concentration of point defects as follows \( c(x) = e^{-\mu(x)/k_BT} \). Indeed, when in such a crystal there exists a nonequilibrium concentration of vacancies, dislocation is moved for atomic distance \( a \) by adding excess vacancies to the extraplane edge. The produced work is simply equal to the chemical potential of added vacancies. What is important, this mechanism allows us to explicitly incorporate the oxygen deficiency parameter \( \delta \) into our model by relating it to the excess oxygen concentration of vacancies \( c_v \equiv c(0) \) as follows \( \delta = 1 - c_v \). As a result, the chemical potential of the single vacancy reads \( \mu_v = \mu(0) = -k_BT \log(1 - \delta) \simeq k_BT \delta \). Remarkably, the same osmotic mechanism was used by Gurevich and Pashitskii (1997) to discuss the modification of oxygen vacancies concentration in the presence of the TB strain field. In particular, they argue that the change of \( \epsilon(x) \) under an applied or chemically induced pressure results in a significant oxygen redistribution producing a highly inhomogeneous filamentary structure of oxygen-deficient nonsuperconducting regions along GB (Moekley et al. 1993) (for underdoped superconductors, the vacancies tend to concentrate in the regions of compressed material). Hence, assuming the following connection between the variation of mechanical and chemical properties of planar defects, namely \( \mu(x) = K \Omega_0(x) \) (where \( \Omega_0 \) is an effective atomic volume of the vacancy and \( K \) is the bulk elastic modulus), we can study the properties of TB induced JJs under intrinsic chemical pressure \( \nabla \mu \) (created by the variation of the oxygen doping parameter \( \delta \)). More specifically, a single \( SIS \) type junction (comprising a Josephson network) is formed around TB due to a local depression of the superconducting order parameter \( \Delta(x) \propto \epsilon(x) \) over distance \( d_0 \) producing thus a weak link with (oxygen deficiency \( \delta \) dependent) Josephson coupling \( J(\delta) = \epsilon(x)J_0 = J_0(\delta)e^{-|x|/d_0} \) where \( J_0(\delta) = \epsilon_0 J_0 = (\mu_v/K \Omega_0)J_0 \) (here \( J_0 \propto \Delta_0/R_n \) with \( R_n \) being a resistance of the junction). Thus, the present model indeed describes chemically induced NGS in underdoped systems (with \( \delta \neq 0 \)) because, in accordance with the observations, for stoichiometric situation (when \( \delta \sim 0 \), the Josephson coupling \( J(\delta) \approx 0 \) and the system loses its explicitly granular signature.

To adequately describe chemomagnetic properties of an intrinsically granular superconductor, we employ a model of 2D overdamped Josephson junction array which is based on the well known Hamiltonian

\[
\mathcal{H} = \sum_{ij}^N J_{ij}(1 - \cos \phi_{ij}) + \sum_{ij}^N \frac{q_i q_j}{C_{ij}} \tag{1}
\]
and introduces a short-range interaction between \( N \) junctions (which are formed around oxygen-rich superconducting areas with phases \( \phi_i(t) \)), arranged in a two-dimensional (2D) lattice with coordinates \( x_i = (x_i, y_i) \). The areas are separated by oxygen-poor insulating boundaries (created by TB strain fields \( \epsilon(x_{ij}) \)) producing a short-range Josephson coupling \( J_{ij} = J_0(\delta)e^{-|x_{ij}|/d} \). Thus, typically for granular superconductors, the Josephson energy of the array varies exponentially with the distance \( x_{ij} = x_i - x_j \) between neighboring junctions (with \( d \) being an average junction size). As usual, the second term in the rhs of Eq.

\[
\mathcal{H} = \frac{1}{2} \sum_{ij} J_{ij} \cos(\phi_{ij} - \phi_{ij}^0)
\]

accounts for Coulomb effects where \( q_i = -2e n_i \) is the junction charge with \( n_i \) being the pair number operator. Naturally, the same strain fields \( \epsilon(x_{ij}) \) will be responsible for dielectric properties of oxygen-depleted regions as well via the \( \delta \)-dependent capacitance tensor \( C_{ij}(\delta) = C[\epsilon(x_{ij})] \).

If, in addition to the chemical pressure \( \nabla \mu(x) = K\Omega_0 \nabla \epsilon(x) \), the network of superconducting grains is under the influence of an applied frustrating magnetic field \( B \), the total phase difference through the contact reads

\[
\phi_{ij}(t) = \phi_{ij}^0 + \frac{\pi w}{\Phi_0}(x_{ij} \wedge n_{ij}) \cdot B + \frac{\nabla \mu \cdot x_{ij} t}{\hbar},
\]

where \( \phi_{ij}^0 \) is the initial phase difference (see below), \( n_{ij} = X_{ij} / |X_{ij}| \) with \( X_{ij} = (x_i + x_j)/2 \), and \( w = 2\lambda_L(T) + l \) with \( \lambda_L \) being the London penetration depth of superconducting area and \( l \) an insulator thickness which, within the discussed here scenario, is simply equal to the TB thickness (Sergeenkov 1995).

To neglect the influence of the self-field effects in a real material, the corresponding Josephson penetration length \( \lambda_J = \sqrt{\Phi_0 / 2\pi \mu_0 j_e w} \) must be larger than the junction size \( d \). Here \( j_e \) is the critical current density of superconducting (hole-rich) area. As we shall see below, this condition is rather well satisfied for HTS single crystals.

Within our scenario, the sheet magnetization \( M \) of 2D granular superconductor is defined via the average Josephson energy of the array

\[
< \mathcal{H} > = \int_0^\tau \frac{dt}{\tau} \int \frac{d^2 x}{s} \mathcal{H}(x, t)
\]

as follows

\[
M(B, \delta) \equiv -\frac{\partial < \mathcal{H} >}{\partial B},
\]

where \( s = 2\pi d^2 \) is properly defined normalization area, \( \tau \) is a characteristic Josephson time, and we made a usual substitution \( \frac{1}{s} \sum_{ij} A_{ij}(t) \rightarrow \frac{1}{s} \int d^2 x A(x, t) \) valid in the long-wavelength approximation (Sergeenkov 2002).

To capture the very essence of the superconducting analog of the chemomagnetic effect, in what follows we assume for simplicity that a stoichiometric sample (with \( \delta \approx 0 \)) does not possess any spontaneous magnetization at zero magnetic field (that is \( M(0, 0) = 0 \)) and that its Meissner response to a small applied field \( B \) is purely diamagnetic (that is \( M(B, 0) \approx -B \)). According to Eq.

\[
\phi_{ij}^0 = 2\pi m
\]

for the initial phase difference with \( m = 0, \pm 1, \pm 2, ... \)

Taking the applied magnetic field along the \( c \)-axis (and normal to the \( CuO \) plane), we obtain finally

\[
M(B, \delta) = -M_0(\delta) \frac{b - b_\mu}{(1 + b^2)(1 + (b - b_\mu)^2)}
\]
for the chemically-induced sheet magnetization of the 2D Josephson network. Here \( M_0(\delta) = J_0(\delta)/B_0 \) with \( J_0(\delta) \) defined earlier, \( b = B/B_0 \), and \( b_\mu = B_\mu/B_0 \simeq (k_B T \tau /\hbar) \delta \) where \( B_\mu(\delta) = (\mu_v \tau /\hbar)B_0 \) is the chemically-induced contribution (which disappears in optimally doped systems with \( \delta \simeq 0 \)), and \( B_0 = \Phi_0/wd \) is a characteristic Josephson field.

![Fig. 1: The susceptibility as a function of applied magnetic field for different values of oxygen deficiency parameter: \( \delta \simeq 0 \) (solid line), \( \delta = 0.05 \) (dashed line), and \( \delta = 0.1 \) (dotted line).](image)

![Fig. 2: The oxygen deficiency induced susceptibility for different values of applied magnetic field (chemomagnetism).](image)

Fig. 1 shows changes of the initial (stoichiometric) diamagnetic susceptibility \( \chi(B, \delta) = \partial M(B, \delta)/\partial B \) (solid line) with oxygen deficiency \( \delta \). As is seen, even relatively small values
of $\delta$ parameter render a low field Meissner phase strongly paramagnetic (dotted and dashed lines). Fig. 2 presents concentration (deficiency) induced susceptibility $\chi(B, \delta)/\chi(0)$ for different values of applied magnetic field $b = B/B_0$ including a true chemomagnetic effect (solid line). According to Eq. (5), the initially diamagnetic Meissner effect turns paramagnetic as soon as the chemomagnetic contribution $B_\mu(\delta)$ exceeds an applied magnetic field $B$. To see whether this can actually happen in a real material, let us estimate a magnitude of the chemomagnetic field $B_\mu$. Typically (Daeumling et al. 1990, Gurevich and Pasitskii 1997), for HTS single crystals $\lambda_L(0) \approx 150\text{nm}$ and $d \approx 10\text{nm}$, leading to $B_0 \approx 0.5T$. Using $\tau \approx \hbar/\mu_v$ and $j_c = 10^{10} A/m^2$ as a pertinent characteristic time and the typical value of the critical current density, respectively, we arrive at the following estimate of the chemomagnetic field $B_\mu(\delta) \approx 0.5B_0$ for $\delta = 0.05$. Thus, the predicted chemically induced PME should be observable for applied magnetic fields $B \approx 0.5B_0 \approx 0.25T$ which are actually much higher than the fields needed to observe the previously discussed piezomagnetism and stress induced PME in high-$T_c$ ceramics (Sergeenkov 1999). Notice that for the above set of parameters, the Josephson length $\lambda_J \approx 1\mu m$, which means that the assumed here small-junction approximation (with $d \ll \lambda_J$) is valid and the so-called "self-field" effects can be safely neglected. So far, we neglected a possible field dependence of the chemical potential $\mu_v$ of oxygen vacancies. However, in high enough applied magnetic fields $B$, the field-induced change of the chemical potential $\Delta \mu_v(B) \equiv \mu_v(B) - \mu_v(0)$ becomes tangible and should be taken into account. As is well-known (Abrikosov 1988, Sergeenkov and Ausloos 1999), in a superconducting state $\Delta \mu_v(B) = -\mathbf{M}(B)B/n$, where $\mathbf{M}(B)$ is the corresponding magnetization, and $n$ is the relevant carriers number density. At the same time, within our scenario, the chemical potential of a single oxygen vacancy $\mu_v$ depends on the concentration of oxygen vacancies (through deficiency parameter $\delta$). As a result, two different effects are possible related respectively to magnetic field dependence of $\mu_v(B)$ and to its dependence on magnetization $\mu_v(\mathbf{M})$. The former is nothing

![Fig. 3: Magnetic field dependence of the oxygen vacancy concentration (magnetoconcentration effect).](image)
else but a superconducting analog of the so-called *magnetoconcentration* effect which was predicted and observed in inhomogeneously doped semiconductors (Akopyan et al. 1990) with field-induced creation of oxygen vacancies $c_v(B) = c_v(0) \exp(-\Delta \mu_v(B)/k_B T)$, while the latter results in a "fishtail"-like behavior of the magnetization. Let us start with the magnetoconcentration effect. Fig. 3 depicts the predicted field-induced creation of oxygen vacancies $c_v(B)$ using the above-obtained magnetization $M(B, \delta)$ (see Fig. 1 and Eq. (5)). We also assumed, for simplicity, a complete stoichiometry of the system in a zero magnetic field (with $c_v(0) = 1$). Notice that $c_v(B)$ exhibits a maximum at $\delta(0) \approx 0.23$ for applied fields $B = B_0$ (in agreement with the classical percolative behavior observed in non-stoichiometric $YBa_2Cu_3O_{7-\delta}$ samples (Daeumling et al. 1990, Gantmakher et al. 1990, Mocekly et al. 1993). Finally, let us show that in underdoped crystals the above-discussed osmotic mechanism of oxygen transport is indeed much more effective than a traditional diffusion. Using typical $YBCO$ parameters (Gurevich and Pashitskii 1997), $\epsilon_0 = 0.01$, $\Omega_0 = a_0^3$ with $a_0 = 0.2nm$, and $K = 115GPa$, we have $\mu_v(0) = \epsilon_0 K \Omega_0 \approx 1meV$ for a zero-field value of the chemical potential in HTS crystals, which leads to creation of excess vacancies with concentration $c_v(0) = e^{-\mu_v(0)/k_B T} \approx 0.75$ (equivalent to a deficiency value of $\delta(0) \approx 0.25$) at $T = T_C$, while the probability of oxygen diffusion in these materials (governed by a rather high activation energy $U_d \approx 1eV$) is extremely low under the same conditions because $D \propto e^{-U_d/k_B T_C} \ll 1$. On the other hand, the change of the chemical potential in applied magnetic field can reach as much as (Sergeenkov and Ausloos 1999) $\Delta \mu_v(B) \approx 0.5meV$ for $B = 0.5T$, which is quite comparable with the above-mentioned zero-field value of $\mu_v(0)$. Let us turn now to the second effect related to the magnetiza-

![Image](https://via.placeholder.com/150)

Fig. 4: A "fishtail"-like behavior of magnetization in applied magnetic field in the presence of magnetoconcentration effect (with field-induced oxygen vacancies $c_v(B)$, see Fig.3) for three values of field-free deficiency parameter: $\delta(0) \approx 0$ (solid line), $\delta(0) = 0.1$ (dashed line), and $\delta(0) = 0.2$ (dotted line).
exhibits a field-induced "fishtail"-like behavior typical for underdoped crystals with intragrain granularity. The extra extremum of the magnetization appears when the applied magnetic field $B$ matches an intrinsic chemomagnetic field $B_\mu(\delta(B))$ (which now also depends on $B$ via the above-discussed magnetoconcentration effect). Notice that a "fishtail" structure of $m_f$ manifests itself even at zero values of field-free deficiency parameter $\delta(0)$ (solid line in Fig.3) thus confirming a field-induced nature of intrinsic nanogranularity (Lang et al. 2002, Daeumling et al. 1990, Yang et al. 1993, Gurevich and Pashitskii 1997, Moekley et al. 1993). At the same time, even a rather small deviation from the zero-field stoichiometry (with $\delta(0) = 0.1$) immediately brings about a paramagnetic Meissner effect at low magnetic fields. Thus, the present model predicts appearance of two interrelated phenomena, Meissner paramagnetism at low fields and "fishtail" anomaly at high fields. It would be very interesting to verify these predictions experimentally in non-stoichiometric superconductors with pronounced networks of planar defects.

3. MAGNETIC FIELD INDUCED POLARIZATION EFFECTS IN 2D JJAs

In this Section, within the same model of JJAs created by a regular 2D network of twin-boundary (TB) dislocations with strain fields acting as an insulating barrier between hole-rich domains in underdoped crystals, we discuss charge-related effects which are actually dual to the above-described phase-related chemomagnetic effects. Specifically, we consider a possible existence of a non-zero electric polarization $P(\delta, B)$ (chemomagnetoelastic effect) and the related change of the charge balance in intrinsically granular non-stoichiometric material under the influence of an applied magnetic field. In particular, we predict an anomalous low-field magnetic behavior of the effective junction charge $Q(\delta, B)$ and concomitant magnetocapacitance $C(\delta, B)$ in paramagnetic Meissner phase and a charge analog of "fishtail"-like anomaly at high magnetic fields along with field-tuned weakening of the chemically-induced Coulomb blockade (Sergeenkov 2007).

Recall that a conventional (zero-field) pair polarization operator within the model under discussion reads (Sergeenkov 1997, 2002, 2007)

$$ p = \sum_{i=1}^{N} q_i x_i $$

(6)

In view of Eqs.(1), (2) and (6), and taking into account a usual "phase-number" commutation relation, $[\phi_i, n_j] = i\delta_{ij}$, it can be shown that the evolution of the pair polarization operator is determined via the equation of motion

$$ \frac{dp}{dt} = \frac{1}{i\hbar} [p, \mathcal{H}] = \frac{2e}{\hbar} \sum_{ij}^{N} J_{ij} \sin \phi_{ij}(t)x_{ij} $$

(7)

Resolving the above equation, we arrive at the following net value of the magnetic-field induced longitudinal (along $x$-axis) electric polarization $P(\delta, B)$ and the corresponding effective junction charge

$$ Q(\delta, B) = \frac{2eJ_0}{\hbar \tau d} \int_{0}^{\tau} dt \int_{0}^{t} dt' \int \frac{d^2x}{S} \sin \phi(x, t') x e^{-|x|/d}, $$

(8)
where \( S = 2\pi d^2 \) is properly defined normalization area, \( \tau \) is a characteristic time (see below), and we made a usual substitution \( \frac{1}{N} \sum_{ij} A_{ij}(t) \to \frac{1}{S} \int d^2 x A(x, t) \) valid in the long-wavelength approximation (Sergeenkov 2002).

To capture the very essence of the superconducting analog of the chemomagnetoelectric effect, in what follows we assume for simplicity that a stoichiometric sample (with \( \delta \approx 0 \)) does not possess any spontaneous polarization at zero magnetic field, that is \( P(0, 0) = 0 \). According to Eq.(8), this condition implies \( \phi_{ij}^0 = 2\pi m \) for the initial phase difference with \( m = 0, \pm 1, \pm 2, ... \)

Taking the applied magnetic field along the \( c \)-axis (and normal to the \( CuO \) plane), we obtain finally

\[
Q(\delta, B) = Q_0(\delta) \frac{2\tilde{b} + b(1 - \tilde{b}^2)}{(1 + b^2)(1 + \tilde{b}^2)^2}
\]

for the magnetic field behavior of the effective junction charge in chemically induced granular superconductors. Here \( Q_0(\delta) = e\tau J_0(\delta)/\hbar \) with \( J_0(\delta) \) defined earlier, \( b = B/B_0 \),

\[
\tilde{b} = b - b_\mu, \quad \text{and} \quad b_\mu = B_\mu/B_0 \simeq (k_B T \tau /\hbar) \delta \quad \text{where} \quad B_\mu(\delta) = (\mu_0 \tau /\hbar) B_0 \quad \text{is the chemically-induced contribution (which disappears in optimally doped systems with} \delta \simeq 0, \text{and} \quad B_0 = \Phi_0/wd \quad \text{is a characteristic Josephson field.}
\]

Fig. 5 shows changes of the initial (stoichiometric) effective junction charge \( \Delta Q(\delta, B) = Q(\delta, B) - Q(\delta, 0) \) (solid line) with oxygen deficiency \( \delta \). According to Eq.(9), the effective charge \( Q \) changes its sign at low magnetic fields (driven by non-zero values of \( \delta \)) as soon as the chemomagnetic contribution \( B_\mu(\delta) \) exceeds an applied magnetic field \( B \). This is nothing else but a charge analog of chemically induced PME. At the same time, Fig. 6 presents a variation of the chemomagnetoelectric effect with concentration (deficiency) for different values of the applied magnetic field. Notice that a zero-field contribution (which is a true chemoelectric effect) exhibits a maximum around \( \delta_c \simeq 0.2 \), in agreement with
the classical percolative behavior observed in non-stoichiometric $YBa_2Cu_3O_{7-\delta}$ samples (Gantmakher et al. 1990).

It is of interest also to consider the magnetic field behavior of the concomitant effective flux capacitance \( C \equiv \tau dQ(\delta, B)/d\Phi \) which in view of Eq.\((9)\) reads

\[
C(\delta, B) = C_0(\delta) \frac{1 - 3b\tilde{b} - 3\tilde{b}^2 + b\tilde{b}^3}{(1+b^2)(1+b\tilde{b})^3},
\]

where \( \Phi = SB \), and \( C_0(\delta) = \tau Q_0(\delta)/\Phi_0 \).

Fig.\(7\) depicts the behavior of the effective flux capacitance \( \Delta C(\delta, B) = C(\delta, B) - C(\delta, 0) \) in applied magnetic field for different values of oxygen deficiency parameter: \( \delta \approx 0 \) (solid line), \( \delta = 0.1 \) (dashed line), and \( \delta = 0.2 \) (dotted line). Notice a decrease of magnetocapacitance amplitude and its peak shifting with increase of \( \delta \) and sign change at low magnetic fields which is another manifestation of the charge analog of chemically induced PME (Cf. Fig.\(5\)). Up to now, we neglected a possible field dependence of the chemical potential \( \mu_v \) of oxygen vacancies. Recall, however, that in high enough applied magnetic fields \( B \), the field-induced change of the chemical potential \( \Delta \mu_v(B) \equiv \mu_v(B) - \mu_v(0) \) becomes tangible and should be taken into account (Abrikosov 1988, Sergeenkov and Ausloos 1999). As a result, we end up with a superconducting analog of the so-called magnetococoncentration effect (Sergeenkov 2003) with field induced creation of oxygen vacancies \( c_v(B) = c_v(0) \exp(-\Delta \mu_v(B)/k_BT) \) which in turn brings about a "fishtail"-like behavior of the high-field chemomagnetization (see Section 2 for more details). Fig.\(8\) shows the field behavior of the effective junction charge in the presence of the above-mentioned magnetococoncentration effect. As it is clearly seen, \( Q(\delta(B), B) \) exhibits a "fishtail"-like anomaly typical for previously discussed (Sergeenkov 2003) chemomagnetization in underdoped crystals with intragrain granularity. This more complex structure of the effective charge appears when the applied magnetic field \( B \) matches an intrinsic chemomagnetic field.
Fig. 7: The effective flux capacitance as a function of applied magnetic field for different values of oxygen deficiency parameter: $\delta \simeq 0$ (solid line), $\delta = 0.1$ (dashed line), and $\delta = 0.2$ (dotted line).

Fig. 8: A "fishtail"-like behavior of an effective charge in applied magnetic field in the presence of magnetoconcentration effect (with field-induced oxygen vacancies $\delta(B)$) for three values of field-free deficiency parameter (from top to bottom): $\delta(0) \simeq 0$ (solid line), $\delta(0) = 0.1$ (dashed line), and $\delta(0) = 0.2$ (dotted line).

$B_{\mu}(\delta(B))$ (which now also depends on $B$ via the magnetoconcentration effect). Notice that a "fishtail" structure of $Q(\delta(B), B)$ manifests itself even at zero values of field-free deficiency parameter $\delta(0)$ (solid line in Fig.8) thus confirming a field-induced nature of intrinsic granularity. Likewise, Fig.9 depicts the evolution of the effective flux capacitance $\Delta C(\delta(B), B) = C(\delta(B), B) - C(\delta(0), 0)$ in applied magnetic field $B$ in the presence of magnetoconcentration effect (Cf. Fig.7).
Thus, the present model predicts appearance of two interrelated phenomena dual to the previously discussed behavior of chemomagnetism (see Section 2), namely a charge analog of Meissner paramagnetism at low fields and a charge analog of “fishtail” anomaly at high fields. To see whether these effects can be actually observed in a real material, let us estimate an order of magnitude of the main model parameters.

Using typical for HTS single crystals values of $\lambda_L(0) \approx 150\,\text{nm}$, $d \approx 10\,\text{nm}$, and $j_c \approx 10^{10}\,\text{A/m}^2$, we arrive at the following estimates of the characteristic $B_0 \approx 0.5\,\text{T}$ and chemomagnetic $B_\mu(\delta) \approx 0.5B_0$ fields, respectively. So, the predicted charge analog of PME should be observable for applied magnetic fields $B < 0.25\,\text{T}$. Notice that, for the above set of parameters, the Josephson length is of the order of $\lambda_J \approx 1\,\mu\text{m}$, which means that the small-junction approximation assumed in this paper is valid and the “self-field” effects can be safely neglected.

Furthermore, the characteristic frequencies $\omega \approx \tau^{-1}$ needed to probe the effects suggested here are related to the processes governed by tunneling relaxation times $\tau \approx h/J_0(\delta)$. Since for oxygen deficiency parameter $\delta = 0.1$ the chemically-induced zero-temperature Josephson energy in non-stoichiometric YBCO single crystals is of the order of $J_0(\delta) \approx k_B T_C \delta \approx 1\,\text{meV}$, we arrive at the required frequencies of $\omega \approx 10^{13}\,\text{Hz}$ and at the following estimates of the effective junction charge $Q_0 \approx e = 1.6 \times 10^{-19}\,\text{C}$ and flux capacitance $C_0 \approx 10^{-18}\,\text{F}$. Notice that the above estimates fall into the range of parameters used in typical experiments for studying the single-electron tunneling effects both in JJs and JJAs (Makhlin et al. 2001, van Bentum et al. 1988) suggesting thus quite an optimistic possibility to observe the above-predicted field induced effects experimentally in non-stoichiometric superconductors with pronounced networks of planar defects or in artificially prepared JJAs. It is worth mentioning that a somewhat similar behavior of the magnetic field induced charge and related flux capacitance has been observed in 2D
electron systems (Chen et al. 1994).

And finally, it can be easily verified that, in view of Eqs. (6)-(8), the field-induced Coulomb energy of the oxygen-depleted region within our model is given by

\[ \mathcal{E}_C(\delta, \mathbf{B}) \equiv \left\langle \sum_{ij} \frac{q_i q_j}{2C_{ij}} \right\rangle = \frac{Q^2(\delta, \mathbf{B})}{2C(\delta, \mathbf{B})} \] (11)

with \( Q(\delta, \mathbf{B}) \) and \( C(\delta, \mathbf{B}) \) defined by Eqs. (9) and (10), respectively.

A thorough analysis of the above expression reveals that in the PME state (when \( B \ll B_{\mu} \)) the chemically-induced granular superconductor is in the so-called Coulomb blockade regime (with \( E_C > J_0 \)), while in the "fishtail" state (for \( B \geq B_{\mu} \)) the energy balance tips in favor of tunneling (with \( E_C < J_0 \)). In particular, we obtain that \( E_C(\delta, \mathbf{B} = 0.1B_{\mu}) = \frac{\pi}{2} J_0(\delta) \) and \( E_C(\delta, \mathbf{B} = B_{\mu}) = \frac{\pi}{8} J_0(\delta) \). It would be also interesting to check this phenomenon of field-induced weakening of the Coulomb blockade experimentally.

4. GIANT ENHANCEMENT OF THERMAL CONDUCTIVITY IN 2D JJA

In this Section, using a 2D model of inductive Josephson junction arrays (created by a network of twin boundary dislocations with strain fields acting as an insulating barrier between hole-rich domains in underdoped crystals), we study the temperature, \( T \), and chemical pressure, \( \nabla \mu \), dependence of the thermal conductivity (TC) \( \kappa \) of an intrinsically nanogranular superconductor. Two major effects affecting the behavior of TC under chemical pressure are predicted: decrease of the linear (i.e., \( \nabla T \) - independent) TC, and giant enhancement of the nonlinear (i.e., \( \nabla T \) - dependent) TC with \( \left[ \kappa(T, \nabla T, \nabla \mu) - \kappa(T, \nabla T, 0) \right] / \kappa(T, \nabla T, 0) \) reaching 500% when chemolectric field \( E_{\mu} = \frac{1}{2e} \nabla \mu \) matches thermoelectric field \( E_T = S_T \nabla T \). The conditions under which these effects can be experimentally measured in non-stoichiometric high-\( T_C \) superconductors are discussed.

There are several approaches for studying the thermal response of JJs and JJAs based on phenomenology of the Josephson effect in the presence of thermal gradients (see, e.g., van Harlingen et al. 1980, Guttmann et al. 1997, Deppe and Feldman 1994, Sergeenkov 2002, Sergeenkov 2007 and further references therein). To adequately describe transport properties of the above-described chemically induced nanogranular superconductor for all temperatures and under a simultaneous influence of intrinsic chemical pressure \( \nabla \mu(x) = K \Omega_0 \nabla \epsilon(x) \) and applied thermal gradient \( \nabla T \), we employ a model of 2D overdamped Josephson junction array which is based on the following total Hamiltonian (Sergeenkov 2002)

\[ \mathcal{H}(t) = \mathcal{H}_T(t) + \mathcal{H}_L(t) + \mathcal{H}_\mu(t), \] (12)

where

\[ \mathcal{H}_T(t) = \sum_{ij} J_{ij} [1 - \cos \phi_{ij}(t)] \] (13)

is the well-known tunneling Hamiltonian,

\[ \mathcal{H}_L(t) = \sum_{ij} \frac{\Phi_{ij}^2(t)}{2L_{ij}} \] (14)
accounts for a mutual inductance $L_{ij}$ between grains (and controls the normal state value of the thermal conductivity, see below) with $\Phi_{ij}(t) = (\hbar/2e)\phi_{ij}(t)$ being the total magnetic flux through an array, and finally

$$\mathcal{H}_\mu(t) = \sum_{i=1}^{N} n_i(t)\delta \mu_i$$

(15)

describes chemical potential induced contribution with $\delta \mu_i = x_i\nabla \mu$, and $n_i$ being the pair number operator.

According to the above-mentioned scenario, the tunneling Hamiltonian $\mathcal{H}_T(t)$ introduces a short-range (nearest-neighbor) interaction between $N$ junctions (which are formed around oxygen-rich superconducting areas with phases $\phi_i(t)$), arranged in a two-dimensional (2D) lattice with coordinates $\mathbf{x}_i = (x_i, y_i)$. The areas are separated by oxygen-poor insulating boundaries (created by TB strain fields $\epsilon(x_{ij})$) producing a short-range Josephson coupling $J_{ij} = J_0(\delta)e^{-|x_{ij}|/d}$. Thus, typically for granular superconductors, the Josephson energy of the array varies exponentially with the distance $x_{ij}$ between neighboring junctions (with $d$ being an average grain size). The temperature dependence of chemically induced Josephson coupling is governed by the following expression,

$$J_{ij}(T) = J_{ij}(0)F(T)$$

(16)

and $J_{ij}(0) = |\Delta(0)/2|(R_0/R_{ij})$ with $\Delta(T)$ being the temperature dependent gap parameter, $R_0 = \hbar/4e^2$ is the quantum resistance, and $R_{ij}$ is the resistance between grains in their normal state.

By analogy with a constant electric field $\mathbf{E}$, a thermal gradient $\nabla T$ applied to a chemically induced JJA will cause a time evolution of the phase difference across insulating barriers as follows (Sergeenkov 2002)

$$\phi_{ij}(t) = \phi_{ij}^0 + \omega_{ij}(\nabla \mu, \nabla T)t$$

(17)

Here $\phi_{ij}^0$ is the initial phase difference (see below), and $\omega_{ij} = 2e(\mathbf{E}_\mu - \mathbf{E}_T)x_{ij}/\hbar$ where $\mathbf{E}_\mu = 2e\nabla \mu$ and $\mathbf{E}_T = S_T \nabla T$ are the induced chemolectric and thermolectric fields, respectively. $S_T$ is the so-called thermophase coefficient (Sergeenkov 1998a) which is related to the Seebeck coefficient $S_0$ as follows, $S_T = (l/d)S_0$ (where $l$ is a relevant sample’s size responsible for the applied thermal gradient, that is $|\nabla T| = \Delta T/l$).

We start our consideration by discussing the temperature behavior of the conventional (that is linear) thermal conductivity of a chemically induced nanogranular superconductor paying a special attention to its evolution with a mutual inductance $L_{ij}$. For simplicity, in what follows we limit our consideration to the longitudinal component of the total thermal flux $Q(t)$ which is defined (in a q-space representation) via the total energy conservation law as follows

$$Q(t) \equiv \lim_{q \to 0} \left[ i q \hat{\mathcal{H}}_q(t) \right],$$

(18)

where $\hat{\mathcal{H}}_q = \partial \mathcal{H}_q/\partial t$ with

$$\mathcal{H}_q(t) = \frac{1}{s} \int d^2x e^{iqx} \mathcal{H}(x, t)$$

(19)
Here \( s = 2\pi d^2 \) is properly defined normalization area, and we made a usual substitution
\[
\sum_{ij} A_{ij}(t) \rightarrow \frac{1}{s} \int d^2 x A(x, t)
\]
valid in the long-wavelength approximation (\( q \to 0 \)).

In turn, the heat flux \( \mathbf{Q}(t) \) is related to the linear thermal conductivity (LTC) tensor \( \kappa_{\alpha\beta} \) by the Fourier law as follows (hereafter, \( \{\alpha, \beta\} = x, y, z \))
\[
\kappa_{\alpha\beta}(T, \nabla \mu) \equiv -\frac{1}{V} \left[ \frac{\partial < Q_\alpha >}{\partial (\nabla_\beta T)} \right]_{T=0},
\]
where
\[
< Q_\alpha > = \frac{1}{\tau} \int_0^\tau dt < Q_\alpha(t) > \quad (21)
\]
Here \( V \) is sample’s volume, \( \tau \) is a characteristic Josephson tunneling time for the network, and \( < ... > \) denotes the thermodynamic averaging over the initial phase differences \( \phi_{ij}^0 \)
\[
< A(\phi_{ij}^0) > = \frac{1}{Z} \int_0^{2\pi} d\phi_{ij}^0 A(\phi_{ij}^0) e^{-\beta H_0}
\]
with an effective Hamiltonian
\[
H_0[\phi_{ij}^0] = \int_0^\tau dt \int \frac{d^2 x}{s} H(x, t)
\]
(23)
Here, \( \beta = 1/k_B T, \) and \( Z = \int_0^{2\pi} \prod_{ij} d\phi_{ij}^0 e^{-\beta H_0} \) is the partition function. The above-defined averaging procedure allows us to study the temperature evolution of the system.

Taking into account that in JJAs (Eichenberger et al. 1996) \( L_{ij} \propto R_{ij} \), we obtain \( L_{ij} = L_0 \exp(|x_{ij}|/d) \) for the explicit \( x \)-dependence of the weak-link inductance in our model. Finally, in view of Eqs.(12)-(23), and making use of the usual “phase-number” commutation relation, \( [\phi_i, n_j] = i\delta_{ij} \), we find the following analytical expression for the temperature and chemical gradient dependence of the electronic contribution to linear thermal conductivity of a granular superconductor
\[
\kappa_{\alpha\beta}(T, \nabla \mu) = \kappa_0 [\delta_{\alpha\beta} \eta(T, \epsilon) + \beta_L(T) \nu(T, \epsilon)f_{\alpha\beta}(\epsilon)]
\]
(24)
where
\[
f_{\alpha\beta}(\epsilon) = \frac{1}{4} [\delta_{\alpha\beta} A(\epsilon) - \epsilon_\alpha \epsilon_\beta B(\epsilon)]
\]
(25)
with
\[
A(\epsilon) = \frac{5 + 3\epsilon^2}{(1 + \epsilon^2)^2} + \frac{3}{\epsilon} \tan^{-1} \epsilon
\]
(26)
and
\[
B(\epsilon) = \frac{3\epsilon^4 + 8\epsilon^2 - 3}{\epsilon^2(1 + \epsilon^2)^3} + \frac{3}{\epsilon^3} \tan^{-1} \epsilon
\]
(27)
Here, \( \kappa_0 = N d^2 S_T \Phi_0/V L_0, \beta_L(T) = 2\pi I_C(T)L_0/\Phi_0 \) with \( I_C(T) = (2e/\hbar) J(T) \) being the critical current; \( \epsilon \equiv \sqrt{\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2} \) with \( \epsilon_\alpha = E_{\mu\alpha}/E_0 \) where \( E_0 = h/2ed\tau \) is a characteristic...
field. In turn, the above-introduced ”order parameters” of the system, \(\eta(T, \epsilon) \equiv < \phi_{ij}^0 >\) and \(\nu(T, \epsilon) \equiv < \sin \phi_{ij}^0 >\), are defined as follows

\[
\eta(T, \epsilon) = \frac{\pi}{2} - 4 \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left[ \frac{I_{2n+1}(\beta_{\mu})}{I_0(\beta_{\mu})} \right] \tag{28}
\]

and

\[
\nu(T, \epsilon) = \frac{\sinh \beta_{\mu}}{\beta_{\mu} I_0(\beta_{\mu})}, \tag{29}
\]

where

\[
\beta_{\mu}(T, \epsilon) = \frac{\beta J(T)}{2} \left( \frac{1}{1+\epsilon^2} + \frac{1}{\epsilon} \tan^{-1} \epsilon \right) \tag{30}
\]

Here \(J(T)\) is given by Eq.(17), and \(I_n(x)\) stand for the modified Bessel functions.

Turning to the discussion of the obtained results, we start with a more simple zero-pressure case. The relevant parameters affecting the behavior of the LTC in this particular case include the mutual inductance \(L_0\) and the normal state resistance between grains \(R_n\). For the temperature dependence of the Josephson energy (see Eq.(17)), we used the well-known (Sergeenkov 2002) approximation for the BCS gap parameter, valid for all temperatures, \(\Delta(T) = \Delta(0) \tanh \left( \gamma \sqrt{T_{C} - T} \right)\) with \(\gamma = 2.2\). Despite a rather simplified nature of our model, it seems to quite reasonably describe the behavior of the LTC for all temperatures. Indeed, in the absence of intrinsic chemical pressure \((\nabla \mu = 0)\), the LTC is isotropic (as expected), \(\kappa_{\alpha\beta}(T, 0) = \delta_{\alpha\beta} \kappa_L(T, 0)\) where \(\kappa_L(T, 0) = \kappa_0[\eta(T, 0) + 2\beta_L(T)\nu(T, 0)]\) vanishes at zero temperature and reaches a normal state value \(\kappa_n \equiv \kappa_L(T_{C}, 0) = (\pi/2)\kappa_0\) at \(T = T_{C}\). Fig.10 shows the temperature dependence of the normalized LTC \(\kappa_L(T, 0)/\kappa_n\) for different values of the so-called SQUID parameter \(\beta_L(0) = 2\pi I_C(0)L_0/\Phi_0\) (increasing from the bottom to the top) and for two values of the resistance ratio \(r_n = R_0/R_n = 0.1\) and \(r_n = R_0/R_n = 1\). First of all, with increasing of the SQUID parameter, the LTC evolves from a flat-like pattern (for a relatively small values of \(L_0\)) to a low-temperature maximum (for higher values of \(\beta_L(0)\)). Notice that the peak temperature \(T_p\) is practically insensitive to the variation of inductance parameter \(L_0\) while being at the same time
strongly influenced by resistivity $R_n$. Indeed, as it is clearly seen in Fig.10 a different choice of $r_n$ leads to quite a tangible shifting of the maximum. Namely, the smaller is the normal resistance between grains $R_n$ (or the better is the quality of the sample) the higher is the temperature at which the peak is developed. As a matter of fact, the peak temperature $T_p$ is related to the so-called phase-locking temperature $T_J$ (which marks the establishment of phase coherence between the adjacent grains in the array and always lies below a single grain superconducting temperature $T_C$) which is usually defined via an average (per grain) Josephson coupling energy as $J(T_J, r_n) = k_BT_J$. Indeed, it can be shown analytically that for $T_J < T < T_C$, $T_J(r_n) \approx r_n T_C$.

Turning to the discussion of the LTC behavior under chemical pressure, let us assume, for simplicity, that $\nabla \mu = (\nabla_x \mu, 0, 0)$ with oxygen-deficiency parameter $\delta$ controlled chemical pressure $\nabla_x \mu \simeq \mu_v(\delta)/d$, and $\nabla T = (\nabla_x T, \nabla_y T, 0)$. Such a choice of the external fields allows us to consider both parallel $\kappa_{xx}(T, \nabla \mu)$ and perpendicular $\kappa_{yy}(T, \nabla \mu)$ components of the LTC corresponding to the two most interesting configurations, $\nabla \mu \parallel \nabla T$ and $\nabla \mu \perp \nabla T$, respectively. Fig.11 demonstrates the predicted chemical pressure dependence of the normalized LTC $\Delta \kappa_L(T, \nabla \mu) = \kappa_L(T, \nabla \mu) - \kappa_L(T, 0)$ for both configurations taken at $T = 0.2T_C$ (with $r_n = 0.1$ and $\beta_L(0) = 1$). First of all, we note that both components of the LTC are decreasing with increasing of the pressure $E_\mu/E_0 = \mu_v(\delta) \tau/\hbar$. And secondly, the normal component $\kappa_{yy}$ decreases more slowly than the parallel one $\kappa_{xx}$, suggesting thus some kind of anisotropy in the system. In view of the structure of Eq.(25), the same behavior is also expected for the temperature dependence of the chemically-induced LTC, that is $\Delta \kappa_L(T, \nabla \mu)/\kappa_L(T, 0) < 0$ for all gradients and temperatures. In terms of the absolute values, for $T = 0.2T_C$ and $E_\mu = E_0$, we obtain $[\Delta \kappa_L(T, \nabla \mu)/\kappa_L(T, 0)]_{xx} = 90\%$ and $[\Delta \kappa_L(T, \nabla \mu)/\kappa_L(T, 0)]_{yy} = 60\%$ for attenuation of LTC under chemical pressure. Let us turn now to the most intriguing part of this Section

![Fig. 11: The dependence of the linear thermal conductivity on the chemical pressure for parallel $(\nabla \mu \parallel \nabla T)$ and perpendicular $(\nabla \mu \perp \nabla T)$ configurations.](image)

and consider a nonlinear generalization of the Fourier law and very unusual behavior of
the resulting nonlinear thermal conductivity (NLTC) under the influence of chemical pressure. In what follows, by the NLTC we understand a $\nabla T$-dependent thermal conductivity $\kappa_{\alpha\beta}^{NL}(T, \nabla \mu) \equiv \kappa_{\alpha\beta}(T, \nabla \mu; \nabla T)$ which is defined as follows

$$\kappa_{\alpha\beta}^{NL}(T, \nabla \mu) \equiv -\frac{1}{V} \left[ \frac{\partial \langle Q_\alpha \rangle}{\partial (\nabla T)} \right]_{\nabla T \neq 0} \quad (31)$$

with $\langle Q_\alpha \rangle$ given by Eq.(21).

Repeating the same procedure as before, we obtain finally for the relevant components of the NLTC tensor

$$\kappa_{\alpha\beta}^{NL}(T, \nabla \mu) = \kappa_0 \left[ \delta_{\alpha\beta} \eta(T, \epsilon_{eff}) + \beta_L(T) \nu(T, \epsilon_{eff}) D_{\alpha\beta}(\epsilon_{eff}) \right], \quad (32)$$

where

$$D_{\alpha\beta}(\epsilon_{eff}) = f_{\alpha\beta}(\epsilon_{eff}) + \epsilon_T^2 g_{\alpha\beta\gamma}(\epsilon_{eff}) \quad (33)$$

with

$$g_{\alpha\beta\gamma}(\epsilon) = \frac{1}{8} \left[ \delta_{\alpha\beta} \epsilon_\gamma + \delta_{\alpha\gamma} \epsilon_\beta + \delta_{\beta\gamma} \epsilon_\alpha \right] B(\epsilon) + 3 \epsilon_\alpha \epsilon_\beta \epsilon_\gamma C(\epsilon) \quad (34)$$

and

$$C(\epsilon) = \frac{3 + 11\epsilon^2 - 11\epsilon^4 - 3\epsilon^6}{\epsilon^4 (1 + \epsilon^2)^4} - \frac{3}{\epsilon^5} \tan^{-1} \epsilon \quad (35)$$

Here, $\epsilon_{eff}^\alpha = \epsilon_\mu^\alpha - \epsilon_T^\alpha$ where $\epsilon_\mu^\alpha = E_\mu^\alpha / E_0$ and $\epsilon_T^\alpha = E_T^\alpha / E_0$ with $E_T^\alpha = S_T \nabla \alpha T$; other parameters ($\eta$, $\nu$, $B$ and $f_{\alpha\beta}$) are the same as before but with $\epsilon \rightarrow \epsilon_{eff}$. As expected, in the limit $E_T \rightarrow 0$ (or when $E_\mu \gg E_T$), from Eq.(32) we recover all the results obtained in the previous section for the LTC. Let us see now what happens when thermoelectric

![Fig. 12: The dependence of the nonlinear thermal conductivity on the chemical pressure for different values of the applied thermal gradient $\epsilon_T = S_T \nabla T / E_0$ ($\epsilon_T = 0.2, 0.4, 0.6, 0.8,$ and $1.0$, increasing from bottom to top).](/images/fig12.png)
field $\mathbf{E}_T = S_T \nabla \mathbf{T}$ becomes comparable with chemolectric field $\mathbf{E}_\mu$. Fig[12] depicts the resulting chemical pressure dependence of the parallel component of the NLTC tensor $\Delta \kappa_{xx}^{NL}(T, E_\mu) = \kappa_{xx}(T, E_\mu) - \kappa_{xx}(T, 0)$ for different values of the dimensionless parameter $\epsilon_T = E_T/E_0$ (the other parameters are the same as before). As is clearly seen from this picture, in a sharp contrast with the pressure behavior of the previously considered LTC, its \textit{nonlinear} analog evolves with the chemolectric field quite differently. Namely, NLTC strongly \textit{increases} for small pressure values (with $E_\mu < E_m$), reaches a pronounced maximum at $E_\mu = E_m = \frac{3}{2} E_T$, and eventually declines at higher values of $\mu$ (with $E_\mu > E_m$). Furthermore, as it directly follows from the very structure of Eq.(32), a similar \textit{"reentrant-like"} behavior of the \textit{nonlinear} thermal conductivity is expected for its temperature dependence as well. Even more remarkable is the absolute value of the pressure-induced enhancement. According to Fig[12] it is easy to estimate that near maximum (with $E_\mu = E_m$ and $E_T = E_0$) one gets $\Delta \kappa_{xx}^{NL}(T, E_\mu)/\kappa_{xx}(T, 0) \simeq 500\%$.

To understand the above-obtained rather unusual results, let us take a closer look at the chemolectric field induced behavior of the Josephson voltage in our system (see Eq.(17)). Clearly, strong heat conduction requires establishment of a quasi-stationary (that is nearly zero-voltage) regime within the array. In other words, the maximum of the thermal conductivity under chemical pressure should correlate with a minimum of the total voltage in the system, $V(\nabla \mu) \equiv \left( \frac{h}{2e} \right) \left< \frac{\partial \phi(t)}{\partial t} \right> = V_0(\epsilon - \epsilon_T)$ where $\epsilon \equiv E_\mu/E_0$ and $V_0 = E_0 d = h/2e\tau$ is a characteristic voltage. For linear TC (which is valid only for small thermal gradients with $\epsilon_T \equiv E_T/E_0 \ll 1$), the average voltage through an array $V_L(\nabla \mu) \simeq V_0(E_\mu/E_0)$ has a minimum at zero chemolectric field (where LTC indeed has its maximum value, see Fig.11) while for nonlinear TC (with $\epsilon_T \simeq 1$) we have to consider the total voltage $V(\nabla \mu)$ which becomes minimal at $E_\mu = E_T$ (in a good agreement with the predictions for NLTC maximum which appears at $E_\mu = \frac{3}{2} E_T$, see Fig[12]).

To complete our study, let us estimate an order of magnitude of the main model parameters. Starting with chemolectric fields $E_\mu$ needed to observe the above-predicted nonlinear field effects in nanogranular superconductors, we notice that according to Fig[12] the most interesting behavior of NLTC takes place for $E_\mu \simeq E_0$. Using typical \textit{YBCO} parameters, $\epsilon_0 = 0.01$, $\Omega_0 = a_0^3$ with $a_0 = 0.2 nm$, and $K = 115GPa$, we have $\mu_v = \epsilon_0 \kappa \Omega_0 \simeq 1 meV$ for an estimate of the chemical potential in HTS crystals, which defines the characteristic Josephson tunneling time $\tau \simeq h/\mu_v \simeq 5 \times 10^{-11} s$ and, at the same time, leads to creation of excess vacancies with concentration $c_v = e^{-\mu_v/k_B T} \simeq 0.75$ at $T = 0.2 T_C$ (equivalent to a deficiency value of $\delta \simeq 0.25$). Notice that in comparison with this linear defects mediated channeling (osmotic) mechanism, the probability of the conventional oxygen diffusion in these materials $D \propto e^{-U_d/k_B T}$ (governed by a rather high activation energy $U_d \simeq 1 eV$) is extremely low under the same conditions ($D \ll 1$).

Furthermore, taking $d \simeq 10 nm$ for typical values of the average ”grain” size (created by oxygen-rich superconducting regions), we get $E_0 = h/2ed\tau \simeq 5 \times 10^5 V/m$ and $|\nabla \mu| = \mu_v/d \simeq 10^6 eV/m$ for the estimates of the characteristic field and chemical potential gradient (intrinsic chemical pressure), respectively. On the other hand, the maximum of NLTC occurs when this field nearly perfectly matches an ”intrinsic” chemolectric field $E_T = S_T \nabla \mathbf{T}$ induced by an applied thermal gradient, that is when $E_\mu \simeq E_0 \simeq E_T$. Recalling that $S_T = (l/d)S_0$ and using $S_0 \simeq 0.5 \mu V/K$ and $l \simeq 0.5 mm$ for an estimate of the \textit{linear} Seebeck coefficient and a typical sample’s size, we obtain $\nabla \mathbf{T} \simeq E_0/S_T \simeq 2 \times 10^6 K/m$.
for the characteristic value of applied thermal gradient needed to observe the predicted here giant chemical pressure induced effects. Let us estimate now the absolute value of the linear thermal conductivity governed by the intrinsic Josephson junctions. Recall that within our model the scattering of normal electrons is due to the presence of mutual inductance between the adjacent grains $L_0$ which is of the order of $L_0 \approx \mu_0 d \approx 1 fH$ assuming $d = 10nm$ for an average ”grain” size. In the absence of chemical pressure effects, the temperature evolution of LTC is given by $\kappa_L(T,0) = \kappa_0[\eta(T,0) + 2\beta_L(T)\nu(T,0)]$ where $\kappa_0 = N\eta S_T\Phi_0/V L_0$. Assuming $V \approx N\eta^2 l$ for the sample’s volume, using the above-mentioned expression for $S_T$, and taking $\beta_L(0) = 5$ and $r_n = 0.1$ for the value of the SQUID parameter and the resistance ratio, we obtain $\kappa_L(0,2T_C,0) \approx 1W/mK$ for an estimate of the maximum of the LTC (see Fig.10).

And finally, it is worth comparing the above estimates for inductively coupled grains (Sergeenkov 2002) with the estimates for capacitively coupled grains (Sergeenkov 2007) where the scattering of normal electrons is governed by the Stewart-McCumber parameter $\beta_C(T) = 2n I_C(T)C_0R_n^2/\Phi_0$ due to the presence of the normal resistance $R_n$ and mutual capacitance $C_0$ between the adjacent grains. The latter is estimated to be $C_0 \approx 1aF$ using $d = 10nm$ for an average ”grain” size. Furthermore, the critical current $I_C(0)$ can be estimated via the critical temperature $T_C$ as follows, $I_C(0) \approx 2\pi k_B T_C/\Phi_0$ which gives $I_C(0) \approx 10\mu A$ (for $T_C \approx 90K$) and leads to $\beta_C(0) \approx 3$ for the value of the Stewart-McCumber parameter assuming $R_n \approx R_0$ for the normal resistance which, in turn, results in $q \approx \Phi_0/R_n \approx 10^{-19}C$ and $E_C = q^2/2C_0 \approx 0.1eV$ for the estimates of the ”grain” charge and the Coulomb energy. Using the above-mentioned expressions for $S_0$ and $\beta_C(0)$, we obtain $\kappa_L \approx 10^{-3}W/mK$ for the maximum of the capacitance controlled LTC which is actually much smaller than a similar estimate obtained above for inductance controlled $\kappa_L$ (Sergeenkov 2002) but at the same time much higher than phonon dominated heat transport in granular systems (Deppe and Feldman 1994).

5. THERMAL EXPANSION OF A SINGLE JOSEPHSON CONTACT AND 2D JJA

In this Section, by introducing a concept of thermal expansion (TE) of a Josephson junction as an elastic response to an effective stress field, we study (both analytically and numerically) the temperature and magnetic field dependence of TE coefficient $\alpha$ in a single small junction and in a square array. In particular, we found (Sergeenkov et al. 2007) that in addition to field oscillations due to Fraunhofer-like dependence of the critical current, $\alpha$ of a small single junction also exhibits strong flux driven temperature oscillations near $T_C$. We also numerically simulated stress induced response of a closed loop with finite self-inductance (a prototype of an array) and found that $\alpha$ of a $5 \times 5$ array may still exhibit temperature oscillations if the applied magnetic field $H$ is strong enough to compensate for the screening induced effects.

Since thermal expansion coefficient $\alpha(T,H)$ is usually measured using mechanical dilatometers (Nagel et al. 2000), it is natural to introduce TE as an elastic response of the Josephson contact to an effective stress field $\sigma$ (D’yachenko et al. 1995, Sergeenkov 1998b, Sergeenkov 1999). Namely, we define the TE coefficient (TEC) $\alpha(T,H)$ as follows:

$$\alpha(T,H) = \frac{d\epsilon}{d\Phi}$$  

(36)
where an appropriate strain field $\epsilon$ in the contact area is related to the Josephson energy $E_J$ as follows ($V$ is the volume of the sample):

$$\epsilon = -\frac{1}{V} \left[ \frac{dE_J}{d\sigma} \right]_{\sigma=0}$$

(37)

For simplicity and to avoid self-field effects, we start with a small Josephson contact of length $w < \lambda_J$ ($\lambda_J = \sqrt{\Phi_0/\mu_0 d_j}$ is the Josephson penetration depth) placed in a strong enough magnetic field (which is applied normally to the contact area) such that $H > \Phi_0/2\pi\lambda_J d$, where $d = 2\lambda_L + t$, $\lambda_L$ is the London penetration depth, and $t$ is an insulator thickness.

The Josephson energy of such a contact in applied magnetic field is governed by a Fraunhofer-like dependence of the critical current (Orlando and Delin 1991):

$$E_J = J \left( 1 - \frac{\sin \varphi}{\varphi} \cos \varphi_0 \right),$$

(38)

where $\varphi = \pi \Phi/\Phi_0$ is the frustration parameter with $\Phi = H w d$ being the flux through the contact area, $\varphi_0$ is the initial phase difference through the contact, and $J \propto e^{-t/\xi}$ is the zero-field tunneling Josephson energy with $\xi$ being a characteristic (decaying) length and $t$ the thickness of the insulating layer. The self-field effects (screening), neglected here, will be considered later for an array.

Notice that in non-zero applied magnetic field $H$, there are two stress-induced contributions to the Josephson energy $E_J$, both related to decreasing of the insulator thickness under pressure. Indeed, according to the experimental data (D’yachenko et al. 1995), the tunneling dominated critical current $I_C$ in granular high-$T_C$ superconductors was found to exponentially increase under compressive stress, viz. $I_C(\sigma) = I_C(0)e^{\kappa \sigma}$. More specifically, the critical current at $\sigma = 9$kbar was found to be three times higher its value at $\sigma = 1.5$kbar, clearly indicating a weak-links-mediated origin of the phenomenon. Hence, for small enough $\sigma$ we can safely assume that (Sergeenkov 1999) $t(\sigma) \simeq t(0)(1 - \beta \sigma/\sigma_0)$ with $\sigma_0$ being some characteristic value (the parameter $\beta$ is related to the so-called ultimate stress $\sigma_m$ as $\beta = \sigma_0/\sigma_m$). As a result, we have the following two stress-induced effects in Josephson contacts:

(I) amplitude modulation leading to the explicit stress dependence of the zero-field energy

$$J(T, \sigma) = J(T, 0)e^{\gamma \sigma/\sigma_0}$$

(39)

with $\gamma = \beta t(0)/\xi$, and

(II) phase modulation leading to the explicit stress dependence of the flux

$$\Phi(T, H, \sigma) = H w d(T, \sigma)$$

(40)

with

$$d(T, \sigma) = 2\lambda_L(T) + t(0)(1 - \beta \sigma/\sigma_0)$$

(41)

Finally, in view of Eqs.(36)-(41), the temperature and field dependence of the small single junction TEC reads (the initial phase difference is conveniently fixed at $\varphi_0 = \pi$):

$$\alpha(T, H) = \alpha(T, 0) [1 + F(T, H)] + \epsilon(T, 0)\frac{dF(T, H)}{dT}$$

(42)
where
\[ F(T, H) = \left[ \frac{\sin \varphi}{\varphi} + \frac{\xi}{d(T, 0)} \left( \frac{\sin \varphi}{\varphi} - \cos \varphi \right) \right] \] (43)

with
\[ \varphi(T, H) = \frac{\pi \Phi(T, H, 0)}{\Phi_0} = \frac{H}{H_0(T)} \] (44)
\[ \alpha(T, 0) = \frac{d \epsilon(T, 0)}{dT} \] (45)

and
\[ \epsilon(T, 0) = -\left( \frac{\Phi_0}{2\pi} \right) \left( \frac{2\gamma}{V\sigma_0} \right) I_C(T) \] (46)

Here, \( H_0(T) = \Phi_0/\pi wd(T, 0) \) with \( d(T, 0) = 2\lambda_L(T) + t(0) \). For the explicit temperature dependence of \( J(T, 0) = \Phi_0 I_C(T)/2\pi \) we use the well-known (Meservey and Schwartz 1969, Sergeenkov 2002) analytical approximation of the BCS gap parameter (valid for all temperatures), \( \Delta(T) = \Delta(0) \tanh \left( 2.2 \sqrt{\frac{T_c - T}{T}} \right) \) with \( \Delta(0) = 1.76k_B T_C \) which governs the temperature dependence of the Josephson critical current
\[ I_C(T) = I_C(0) \left[ \frac{\Delta(T)}{\Delta(0)} \right] \tanh \left[ \frac{\Delta(T)}{2k_B T} \right] \] (47)

while the temperature dependence of the London penetration depth is governed by the two-fluid model:
\[ \lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - (T/T_C)^2}} \] (48)

From the very structure of Eqs.(36)-(44) it is obvious that TEC of a single contact will
exhibit field oscillations imposed by the Fraunhofer dependence of the critical current $I_C$. Much less obvious is its temperature dependence. Indeed, Fig.13 presents the temperature behavior of the contact area strain field $\Delta \epsilon(T, f) = \epsilon(T, f) - \epsilon(T, 0)$ (with $t(0)/\xi = 1$, $\xi/\lambda_L(0) = 0.02$ and $\beta = 0.1$) for different values of the frustration parameter $f = H/H_0(0)$. Notice characteristic flux driven temperature oscillations near $T_C$ which are better seen on a semi-log plot shown in Fig.14 which depicts the dependence of the properly normalized field-induced TEC $\Delta \alpha(T, f) = \alpha(T, f) - \alpha(T, 0)$ as a function of $1 - T/T_C$ for the same set of parameters.

To answer an important question how the neglected in the previous analysis screening effects will affect the above-predicted oscillating behavior of the field-induced TEC, let us consider a more realistic situation with a junction embedded into an array (rather than an isolated contact) which is realized in artificially prepared arrays using photolithographic technique that nowadays allows for controlled manipulations of the junctions parameters (Newrock et al. 2000). Besides, this is also a good approximation for a granular superconductor (if we consider it as a network of superconducting islands connected with each other via Josephson links). Our goal is to model and simulate the elastic response of such systems to an effective stress $\sigma$. For simplicity, we will consider an array with a regular topology and uniform parameters (such approximation already proved useful for describing high-quality artificially prepared structures, see, e.g., Sergeenkov and Araujo-Moreira 2004).

Let us consider a planar square array as shown in Fig.15. The total current includes the bias current flowing through the vertical junctions and the induced screening currents circulating in the plaquette (Nakajima and Sawada 1981). This situation corresponds to the inclusion of screening currents only into the nearest neighbors, neglecting thus the mutual inductance terms (Phillips et al. 1993). Therefore, the equation for the vertical
contacts will read (horizontal and vertical junctions are denoted by superscripts $h$ and $v$, respectively):

$$
\frac{\hbar C}{2e} \frac{d^2 \phi_{i,j}^v}{dt^2} + \frac{\hbar}{2eR} \frac{d \phi_{i,j}^v}{dt} + I_c \sin \phi_{i,j}^v = \Delta I_{i,j}^s + I_b
$$

(49)

where $\Delta I_{i,j}^s = I_{i,j}^s - I_{i-1,j}^s$ and the screening currents $I^s$ obey the fluxoid conservation condition:

$$
- \phi_{i,j}^v + \phi_{i,j+1}^v - \phi_{i,j}^h + \phi_{i+1,j}^h = 2\pi \frac{\Phi_{ext}}{\Phi_0} - 2\pi \frac{LI_{i,j}^s}{\Phi_0}
$$

(50)

Recall that the total flux has two components (an external contribution and the contribution due to the screening currents in the closed loop) and it is equal to the sum of the phase differences describing the array. It is important to underline that the external flux in Eq. (50), $\eta = 2\pi \Phi_{ext}/\Phi_0$, is related to the frustration of the whole array, i.e., this is the flux across the void of the network (Araujo-Moreira et al. 1997, Araujo-Moreira et al. 2005, Grimaldi et al. 1996), and it should be distinguished from the previously introduced applied magnetic field $H$ across the junction barrier which is related to the frustration of a single contact $f = 2\pi H dw/\Phi_0$ and which only modulates the critical current $I_C(T, H, \sigma)$ of a single junction while inducing a negligible flux into the void area of the array.

For simplicity, in what follows we will consider only the elastic effects due to a uniform (homogeneous) stress imposed on the array. With regard to the geometry of the array, the deformation of the loop is the dominant effect with its radius $a$ deforming as follows:

$$
a(\sigma) = a_0(1 - \chi \sigma/\sigma_0)
$$

(51)

As a result, the self-inductance of the loop $L(a) = \mu_0 a F(a)$ (with $F(a)$ being a geometry dependent factor) will change accordingly:

$$
L(a) = L_0(1 - \chi_g \sigma/\sigma_0)
$$

(52)

The relationship between the coefficients $\chi$ and $\chi_g$ is given by

$$
\chi_g = (1 + a_0 B_g) \chi
$$

(53)
where $B_g = \frac{1}{F(\sigma)} \left( \frac{dF}{d\sigma} \right)_{\sigma_0}$. It is also reasonable to assume that in addition to the critical current, the external stress will modify the resistance of the contact:

$$R(\sigma) = \frac{\pi \Delta(0)}{2eI_C(\sigma)} = R_0 e^{-\chi\sigma/\sigma_0}$$  \hspace{1cm} (54)

as well as capacitance (due to the change in the distance between the superconductors):

$$C(\sigma) = \frac{C_0}{1 - \chi\sigma/\sigma_0} \approx C_0 (1 + \chi\sigma/\sigma_0)$$  \hspace{1cm} (55)

To simplify the treatment of the dynamic equations of the array, it is convenient to introduce the standard normalization parameters such as the Josephson frequency:

$$\omega_J = \sqrt{\frac{2\pi I_C(0)}{C_0 \Phi_0}}$$  \hspace{1cm} (56)

the analog of the SQUID parameter:

$$\beta_L = \frac{2\pi I_C(0) L_0}{\Phi_0},$$  \hspace{1cm} (57)

and the dissipation parameter:

$$\beta_C = \frac{2\pi I_C(0) C_0 R_0^2}{\Phi_0}$$  \hspace{1cm} (58)

Combining Eqs. (49) and (50) with the stress-induced effects described by Eqs. (54) and (55) and using the normalization parameters given by Eqs. (56)-(58), we can rewrite the 

Fig. 16: Numerical simulation results for an array $5 \times 5$ (solid line) and a small single contact (dashed line). The dependence of the normalized TEC on the frustration parameter $f$ (applied magnetic field $H$ across the barrier) for the reduced temperature $T/T_C = 0.95$. The parameters used for the simulations: $\eta = 0$, $\beta = 0.1$, $t(0)/\xi = 1$, $\xi/\lambda_L = 0.02$, $\beta_L = 10$, $\gamma_b = 0.95$, and $\chi_g = \chi = 0.01$. 
equations for an array in a rather compact form. Namely, the equations for vertical junctions read:

\[
\frac{1}{1 - \chi \sigma / \sigma_0} \ddot{\phi}_{i,j}^v + \frac{e^{-\chi \sigma / \sigma_0}}{\sqrt{\beta_C}} \dot{\phi}_{i,j}^v + e^{\chi \sigma / \sigma_0} \sin \phi_{i,j}^v = \gamma_b + \\
\frac{1}{\beta_L (1 - \chi \sigma / \sigma_0)} \left[ \phi_{i,j-1}^v - 2\phi_{i,j}^v + \phi_{i,j+1}^v + \phi_{i-1,j}^h - \phi_{i-1,j}^v + \phi_{i+1,j-1}^v - \phi_{i,j-1}^v \right] \tag{59}
\]

Here an overdot denotes the time derivative with respect to the normalized time (inverse Josephson frequency), and the bias current is normalized to the critical current without stress, \( \gamma_b = I_b / I_C(0) \).

The equations for the horizontal junctions will have the same structure safe for the explicit bias related terms:

\[
\frac{1}{1 - \chi \sigma / \sigma_0} \ddot{\phi}_{i,j}^h + \frac{e^{-\chi \sigma / \sigma_0}}{\sqrt{\beta_C}} \dot{\phi}_{i,j}^h + e^{\chi \sigma / \sigma_0} \sin \phi_{i,j}^h = \\
\frac{1}{\beta_L (1 - \chi \sigma / \sigma_0)} \left[ \phi_{i,j-1}^h - 2\phi_{i,j}^h + \phi_{i,j+1}^h + \phi_{i-1,j}^v - \phi_{i-1,j}^h + \phi_{i+1,j-1}^v - \phi_{i,j-1}^v \right] \tag{60}
\]

Finally, Eqs. (59) and (60) should be complemented with the appropriate boundary conditions (Binder et al. 2000) which will include the normalized contribution of the external flux through the plaquette area \( \eta = 2\pi \Phi_{\text{ext}} / \Phi_0 \). It is interesting to notice that Eqs. (59) and (60) will have the same form as their stress-free counterparts if we introduce the stress-dependent renormalization of the parameters:

\[
\tilde{\omega}_J = \omega_J e^{\chi \sigma / 2\sigma_0} \tag{61}
\]

\[
\tilde{\beta}_C = \beta_C e^{-3\chi \sigma / \sigma_0} \tag{62}
\]

\[
\tilde{\beta}_L = \beta_L (1 - \chi \sigma / \sigma_0) e^{\chi \sigma / \sigma_0} \tag{63}
\]

\[
\tilde{\eta} = \eta (1 - 2\chi \sigma / \sigma_0) \tag{64}
\]

\[
\tilde{\gamma}_b = \gamma_b e^{-\chi \sigma / \sigma_0} \tag{65}
\]

Turning to the discussion of the obtained numerical simulation results, it should be stressed that the main problem in dealing with an array is that the total current through the junction should be retrieved by solving self-consistently the array equations in the presence of screening currents. Recall that the Josephson energy of a single junction for an arbitrary current \( I \) through the contact reads:

\[
E_J(T, f, I) = E_J(T, f, I_C) \left[ 1 - \sqrt{1 - \left( \frac{I}{I_C} \right)^2} \right] \tag{66}
\]
Fig. 17: Numerical simulation results for an array $5 \times 5$. The influence of the flux across the void of the network $\eta$ frustrating the whole array on the temperature dependence of the normalized TEC for different values of the barrier field $f$ frustrating a single junction for $\gamma_b = 0.5$ and the rest of parameters same as in Fig.16.

The important consequence of Eq. (66) is that if no current flows in the array’s junction, such junction will not contribute to the TEC (simply because a junction disconnected from the current generator will not contribute to the energy of the system).

Below we sketch the main steps of the numerical procedure used to simulate the stress-induced effects in the array:
(1) a bias point $I_b$ is selected for the whole array;

(2) the parameters of the array (screening, Josephson frequency, dissipation, etc) are selected and modified according to the intensity of the applied stress $\sigma$;

(3) the array equations are simulated to retrieve the static configuration of the phase differences for the parameters selected in step 2;

(4) the total current flowing through the individual junctions is retrieved as:

$$I_{i,j}^{v,h} = I_C \sin \phi_{i,j}^{v,h}$$

(67)

(5) the energy dependence upon stress is numerically estimated using the value of the total current $I_{i,j}^{v,h}$ (which is not necessarily identical for all junctions) found in step 4 via Eq.(67);

(6) the array energy $E_A^J$ is obtained by summing up the contributions of all junctions with the above-found phase differences $\phi_{i,j}^{v,h}$;

(7) the stress-modified screening currents $I_{i,j}^s(T, H, \sigma)$ are computed using Eq.(50) and inserted into the magnetic energy of the array $E_M^A = \frac{1}{2L} \sum_{i,j} (I_{i,j}^s)^2$;

(8) the resulting strain field and TE coefficient of the array are computed using numerical derivatives based on the finite differences:

$$\epsilon^A \approx \frac{1}{V} \left[ \frac{\Delta \left( E_M^A + E_J^A \right)}{\Delta \sigma} \right]_{\Delta \sigma \rightarrow 0}$$

(68)

$$\alpha(T, H) \approx \frac{\Delta \epsilon^A}{\Delta T}$$

(69)

The numerical simulation results show that the overall behavior of the strain field and TE coefficient in the array is qualitatively similar to the behavior of the single contact. In Fig.16 we have simulated the behavior of both the small junction and the array as a function of the field across the barrier of the individual junctions in the presence of bias and screening currents. As is seen, the dependence of $\alpha(T, f)$ is very weak up to $f \approx 0.5$, showing a strong decrease of about 50% when the frustration approaches $f = 1$.

A much more profound change is obtained by varying the temperature for the fixed value of applied magnetic field. Fig.17 depicts the temperature behavior of $\alpha(T, f)$ (on semi-log scale) for different field configurations which include barrier field $f$ frustrating a single junction and the flux across the void of the network $\eta$ frustrating the whole array. First of all, comparing Fig.17(a) and Fig.14 we notice that, due to substantial modulation of the Josephson critical current $I_C(T, H)$ given by Eq.(38), the barrier field $f$ has similar effects on the TE coefficient of both the array and the single contact including temperature oscillations. However, finite screening effects in the array result in the appearance of oscillations at higher values of the frustration $f$ (in comparison with a single contact). On the other hand, Fig.17(b-d) represent the influence of the external field across the void
\( \eta \) on the evolution of \( \alpha(T, f) \). As is seen, in comparison with a field-free configuration (shown in Fig.17(a)), the presence of external field \( \eta \) substantially reduces the magnitude of the TE coefficient of the array. Besides, with \( \eta \) increasing, the onset of temperature oscillations markedly shifts closer to \( T_C \).

6. SUMMARY

In this Chapter, using a realistic model of 2D Josephson junction arrays (created by 2D network of twin boundary dislocations with strain fields acting as an insulating barrier between hole-rich domains in underdoped crystals), we considered many novel effects related to the magnetic, electric, elastic and transport properties of Josephson nanocontacts and nanogranular superconductors. Some of the topics covered here include such interesting phenomena as chemomagnetism and magnetoelectricity, electric analog of the "fish tail" anomaly and field-tuned weakening of the chemically-induced Coulomb blockade as well as a giant enhancement of nonlinear thermal conductivity (reaching 500\% when the intrinsically induced chemoelectric field \( E_\mu \propto |\nabla \mu| \), created by the gradient of the chemical potential due to segregation of hole producing oxygen vacancies, closely matches the externally produced thermoelectric field \( E_T \propto |\nabla T| \)). Besides, we have investigated the influence of a homogeneous mechanical stress on a small single Josephson junction and on a plaquette (array of 5 \( \times \) 5 junctions) and have shown how the stress-induced modulation of the parameters describing the junctions (as well as the connecting circuits) produces such an interesting phenomenon as a thermal expansion (TE) in a single contact and two-dimensional array (plaquette). We also studied the variation of the TE coefficient with an external magnetic field and temperature. In particular, near \( T_C \) (due to some tremendous increase of the effective "sandwich" thickness of the contact) the field-induced TE coefficient of a small junction exhibits clear temperature oscillations scaled with the number of flux quanta crossing the contact area. Our numerical simulations revealed that these oscillations may actually still survive in an array if the applied field is strong enough to compensate for finite screening induced self-field effects.

The accurate estimates of the model parameters suggest quite an optimistic possibility to experimentally realize all of the predicted in this Chapter promising and important for applications effects in non-stoichiometric nanogranular superconductors and artificially prepared arrays of Josephson nanocontacts.

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