The Investigation of Breakage Probability of Irregularly Shaped Particles by Impact tests

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Abstract

The modelling of experimental distributions of breakage energy by compression and impact was carried out. In terms of our model, the part of the particle that is directly contacted with the stressing tool is admitted into the equation as a hemispherical asperity with known breakage energy distribution. The main contribution of stressing energy is accumulated by this hemispherical asperity that is responsible for crack generation and particle breakage. The breakage probability distribution of particles is calculated as a superposition of the breakage probabilities of asperities. Based on geometrical similarity, one can assume the same normalized log-normal size distribution of asperities for all tested particles of a given material. As a result, all experimental distributions of normalized breakage energy can be fitted with the same log-normal function for all particle sizes.

Keywords: particle breakage, compression, impact, breakage probability

1. Introduction

A complete description of physical phenomena that occur during particle breakage is not yet available. Especially difficult is prediction of the stressing and breakage parameters of irregularly shaped particles. For example, breakage is history-dependent, i.e. the number of micro-cracks and dislocations increases due to previous stressing events. As a result, the mechanical properties and breakage parameters vary by testing the geometrically similar particles of the same size. The breakage probability that depends on the particle properties and stressing parameters was introduced to describe the breakage behaviour of particles. Two approaches are commonly applied to obtain the breakage probability as a percentage of the number of broken particles.

The first one is an energy approach that uses the particle fracture energy to calculate the breakage probability. The results obtained by compression, impact and shear tests can be compared using the mass-related breakage energy.

The second one is the stress or breakage force approach. This approach operates by applying stress to individual particles. The stress approach includes material properties such as stiffness, strength or hardness. The breakage probability depends on the resulting force acting on the contact area, i.e. three-dimensional particle contact stress distribution. However, it is not necessary to consider the mechanical behaviour in detail before breakage.

The relationship between breakage energy distribution and force distribution by compression test was investigated by Aman et al. Every distribution was normalized by a mean arithmetic value of breakage energy or force, respectively. The dimensionless normalized distributions were fitted with log-normal functions. A correlation between the force distributions and energy distributions was observed. That means that the fit function of the normalized force distribution can be transformed into the fit function of the normalized energy distribution and vice versa. The equations that connect the parameters of both fit distribution functions are linear. The coefficients in these equations are constant. They do not depend on particle size and material properties. This invariant transformation of distribution can be based on the existence of two universal normalized distributions.
force and energy. Both normalized distributions are independent of particle size and material. Consequently, the equations that connected the parameters of both fit functions remain the same.

Before the compression test one puts the particle on the plate. In all probability, an irregularly shaped particle takes an initially stable position on this plate that corresponds to the equilibrium state of the particle. In this case, the orientation of the particle to the stressing piston is not random. The top of the particle with its low cross-section will make contact with the piston first, see Fig. 1. The cross-section of the particle increases with distance from the contact area, i.e. the internal compressive stress or pressure is reduced with distance to the contact plane. That means that not all the volume of the particle is effectively involved in the initiation of fracture. The universal form of both distributions observed by compression test may be caused by this orientation of the particle. In this context, it is interesting to investigate the impact loading where the stressing occurs by random orientation of the irregular particle to the target, i.e. the stressing tool. Its basic assumptions of the developed model are valid not only for compression but also for impact loading, then the normalization by mean energy leads to a universal form of breakage energy distribution.

Generally speaking, when comparing the impact and compression loading, one has to take into account that the deformation and breakage performance can be dependent on the strain rate. However, the strain rate effects are often associated with the propagation of elastic waves within the particle during quick loading. However, Gildmeister and Schönert demonstrated that wave effects do not play any significant role in the brittle particle breakage behaviour at breakage rates up to 100 m/s. Tavares and King used the quasi-static approach to calculate the breakage probability by using double impact test equipment.

On the other hand, Salman and Gorham investigated the variation forms of particle damage depending on particle diameter and impact velocity. At the lowest velocities, fractures occur mainly due to a brittle-elastic response, with typical Hertzian ring and cone crack systems. At the higher impact velocities, the deformations become more inelastic. This leads to characteristic patterns of fragmentation due to radial, lateral and median cracks.

The breakage behaviour of individual particles with given mechanical properties and shapes by impact and compression can be calculated using sophisticated numerical methods such as DEM (Discrete Element Method). However, the practical application of any computer simulation to a large assembly of particles without any coupling with statistical methods is restricted due to a difference of particle properties and shapes. In this case, the complex statistical nature of particle fracture and the large difference in particle properties result in an enormous increase of calculation time.

According to this fact, it is interesting to investigate the breakage energy distribution by impact loading. A universal normalized distribution function will be found and applied to reduce the calculation time by the characterisation of breakage behaviour of particles in impact grinding machines.

2. Hemispherical Approach

In particle mechanics, one scaling problem deals with the effect of the particle size and form on its breakage probability. In terms of irregularly shaped particles, the theoretical investigation of scaling is restricted due to complex stress distribution in the contact area and within the particles. However, ac-
The similarity of breakage patterns at constant stressing intensity parameter $a_R$ was successfully tested by the compression and impact of glass beads, quartz and limestone, and cement clinker. The particle size distribution $Q_v(d)$ after breakage, the so-called breakage function, was found to be a size-independent function of parameter $W/d^2$ that depends only on material properties. The detailed investigation of the relationship between fracturing energy and resulting fragment size distribution was carried out by Baumgardt et al.

Based on Rumpf’s similarity principle, Weichert introduced the Weibull statistics to the field of comminution to describe the breakage probability of brittle spheres. The breakage probability $P$ can be described as a function of mass-related energy $W_m$:

$$P = 1 - \exp(-cd^2W_m^z)$$

(3)

with constants $c$ and Weibull exponent $z$. For the glass spheres with 4 mm diameter, $z = 2.8$ and $c = 5.57 \times 10^3$ (kg/J)$^{1/2}$ m$^{-2}$.

Antonyuk et al. performed single granule impacts to study the breakage behaviour of granules. The deformation behaviour was explained with the help of the particle caps contact model. The breakage probability was approximated by the use of Weichert’s approach. The results conform to two-dimensional discrete element simulations of the particle deformation by impact.

Salman et al. conducted many research projects on the breakage rate of particles due to impact. He found a simple function of the impact velocity $v$ that is based on a Weibull statistic and that provided a very close fit to most of his experimental data:

$$P = 1 - \exp\left(-\left(\frac{v}{v_0}\right)^m\right)$$

(4)

with fit parameters $v_0$ and $m$. For aluminium oxide particles, $v_0 = 19.5$ m/s and $m = 7.4$.

The Eqs. (3-4) were originally introduced to calculate the breakage probability of spherical particles. For irregularly shaped particles, a distribution function of breakage probability is not precisely defined due to its sensitivity to particle shape. However, Tavares and King investigated the influence of particle shape on the fracture characteristics of quartz particles tested in the impact load cell. They demonstrated that the particle strength and stiffness decrease significantly as the particle shape becomes more irregular, whereas particle fracture energy is not significantly affected. In this context, it is interesting to test the fracture energy as a possible variable to calculate the breakage probability for irregularly shaped particles.

According to May, Baumgart, Buss and Schubert, the breakage probability distribution of the irregularly shaped particles can be fitted with the log-normal distribution function:

$$P(X) = \frac{1}{\sigma_{\ln,X} \cdot \sqrt{2 \cdot \pi}} \cdot \int_0^\infty \exp \left(-\frac{1}{2} \left(\frac{\ln t - \mu_{\ln,X}}{\sigma_{\ln,X}}\right)^2\right) dt$$

(5)

In terms of breakage work $X=W_m, break$, where $W_m, break$ is the mass-related breakage work and $\mu_{\ln,X}$ and $\sigma_{\ln,X}$ are distribution parameters.

The breakage probability is a dimensionless function from physical variables. To calculate the breakage probability, it is necessary to represent the physical variables in a dimensionless form. For example in Eq. (3), the mass-related energy is divided by a combination of parameters with the unit of [J/kg]. A similar normalization procedure of impact velocity is presented in Eq. (4). It is necessary to represent the breakage probability as a function of the dimensionless breakage energy, i.e. in a form similar to Eqs. (3-4). Consequently, a relevant characteristic parameter with units of energy will be introduced. Let us consider the relationship between the characteristic breakage parameters in Eqs. (3-4) and the mean breakage energy. We will show that the arithmetic mean of breakage energy $W_{m,mean}$ can be used as a relevant characteristic energy for those equations.

The mean value $W_{m,mean}$ of mass-related breakage energy can be calculated from probability distribution as follows:

$$W_{m,mean} = \int_0^\infty \left(\frac{dP}{dW_m}\right) W_m dW_m$$

(6)

By using Weichert’s approach, the breakage probability is calculated according to Eq. (3):
3. Simulation of Breakage Probability Distribution

The elastic-plastic contact behaviour of particles is modelled by means of hemispherical asperities or caps\(^2,23\). In terms of this model, the part (cap) of the irregularly shaped particle that is directly contacted with the stressing tool is admitted into the equation as a hemispherical asperity. This “directly contacted” particle cap is mainly stressed. Consequently, it is responsible for crack generation by stressing\(^3\). This fact can be used to calculate the breakage probability of particles based on equations formerly introduced for spherical particles. The breakage behaviour of \(N\) irregularly shaped particles of size \(d\) is seen statistically as common breakage behaviour of a combination of \(N\) hemispherical asperities of sizes \(d_{r,i}\). The appearance frequency of asperities can be calculated from the asperity size distribution, i.e. the shape distribution of the particles. The breakage probability of particles of size \(d\) will be found as the sum of breakage probabilities of individual asperities divided by the number \(N\). The irregularly shaped particles will show a similar contact and breakage behaviour for samples with a similar shape distribution. We will show that the breakage probability distribution is strongly affected by the size distribution of asperities.

It is difficult to find the relevant mass of stressed caps (asperities) to calculate the mass-related breakage energy. Based on this detail, we prefer to use the breakage energy (not mass-related) as an essential variable to describe the deformation and breakage behaviour of particles.

In terms of irregularly shaped particles, the breakage behaviour of \(N\) particles of a given size \(d\) will be simulated statistically as the breakage behaviour of a combination of \(N\) hemispherical asperities of sizes \(d_{r,i}\). It was assumed that the sizes of asperities are distributed between \(d_{\text{min}}\) and \(d\). The lower size limit of \(d_{\text{min}} = (0.1)^{1/3}d\) was chosen due to the definition of breakage. According to the definition, the breakage occurs as soon as the mass loss of the particle is more than 10\% of its initial mass\(^2\). The masses of small asperities of size \(d_{r,i} < d_{\text{min}}\) are under this limit. They cannot satisfy the breakage condition. Consequently, the small asperities were not taken into account by simulation.

The size distribution of asperities corresponds to the size distribution of particle caps directly contacted by the stressing tool. It is expected that the particles are self-similar regarding the size scale. In other words, the caps of the particles are geometrically similar to the whole particles. Usually, the size distribution of irregular particles is the log-normal distribution\(^2\). Consequently, one can assume that the size distribution of caps, i.e. of asperities, is the log-normal distribution function.

The breakage probability of irregularly shaped particles of size \(d\) will be simulated by means of the following method. All we need to do is follow the five simple steps listed below:

1. The sizes of \(N\) asperities were generated as log-normal distributed numbers \(d_{r,i} \in (d_{\text{min}},d)\) with \(i < N\) and \(d_{\text{min}} = (0.1)^{1/3}d\).
2. The mean mass-related breakage energy \(W_{\text{mean}}\) was calculated for spherical particles of diameter \(d\), see Eq. (7).
3. The mean breakage energy was found from the mean mass-related breakage energy \(W_{\text{mean}}\):

\[
W_{\text{mean}} = W_{\text{mean}} \cdot \pi \cdot d^3 \cdot \rho / 6
\]

(11)

For the future simulation, we have to establish the interval of stressing energy \(W\) that is applied to the particle. In our calculations, the stressing energy \(W\) varies in range from 0 to 3\(W_{\text{mean}}\) with step of 0.001\(W_{\text{mean}}\).

\[
W_{\text{mean}} = \left( \frac{1}{c d^2} \right)^{\frac{1}{3}} \Gamma \left( \frac{1}{z} + 1 \right)
\]

(7)
The breakage probabilities of the asperity of size \(d_{r,i}\) were calculated according to Eq. (3) at every value of \(W\) (3000 values). To obtain the mass-related energy of asperity \(W_m\), the stressing energy \(W\) was divided by the mass of a spherical particle (asperity) of size \(d_{r,i}\). It is assumed that all the stressing energy applied to the particle of size \(d\) will be stored in the asperity.

To obtain the breakage probability distribution of irregular particles, the breakage probabilities of individual asperities were summarized at every value of kinetic energy and divided by the number of asperities.

\[
P(W) = \frac{1}{N} \sum_{i=1}^{N} \left[ 1 - \exp \left( - \left( 0.9 \cdot \frac{W \cdot d^3}{W_{\text{mean}} \cdot d_{r,i}^3} \right) \right) \right] \quad (12)
\]

\(P(W)\) is the breakage probability distribution, \(N\) is the number of asperities, \(d\) is the particle size, \(d_{r,i}\) is the asperity size, \(W\) is the stressing energy, \(W_{\text{mean}}\) is the mean stressing energy, and \(z\) is the Weibull exponent.

**Fig. 2.** shows the log-normal size distribution of asperities that corresponds to the distribution of asperities that describe the form of the irregularly shaped particles of size \(d=6\) mm. Only the asperities in a size range from 2.78 mm to 6 mm were taken into account by the breakage simulation.

The breakage energy distribution normalized by \(W_{\text{mean}}\) was calculated for the spherical particles of size \(d=6\) mm, see **Fig. 3.** According to Weichert\(^{18}\), the Weibull exponent \(z\) varies between 1.5 and 2.7 for relevant materials. Based on this fact, the three values of \(z\) (lower - 1.5, middle - 2, and upper - 2.7) were used for the simulation. One can see that the distribution of breakage probability \(P\) is strongly affected by exponent \(z\). The influence of exponent \(z\) is especially significant at the lower energy value. As mentioned above, see Eq.(10), the dependence on the particle size \(d\) disappears due to normalization of the breakage energy. In terms of normalized breakage energy, the calculated dependence on \(z\) reflects only the variance in particle material properties.

The resulting distribution after calculation of the breakage probability, see Eq. (12), is affected by the size distribution of the asperities. Especially at low energy, the influence of size distribution becomes significant. Indeed, at a given stressing energy, the fine asperities exhibit a higher breakage probability compared to large ones. One can assume that at a given energy \(W\), the breakage probability \(P\sim 1\) for all asperities of size:

\[
d_{r,i} \leq d \cdot (W/W_{\text{mean}})^z
\]

Consequently, the breakage probability at low stressing energy will be proportional to the number of all particles of a size lower than \(d_{r,i}\). That means that the probability distribution at low energy values is similar to the size distribution of asperities. However, the size distribution of asperities doesn’t depend on exponent \(z\). Indeed, the influence of exponent \(z\) on the probability distribution becomes less significant for irregular particles compared to single particles, see **Fig. 4.** The distribution obtained due to simulation is shifted to low energy values.

Another interesting result of simulation is that the distribution can be fitted by a log-normal distribution function. **Fig. 5.** shows the log-normal fit of distribution obtained for a combination of asperities at \(z=2.7\). The coefficient of determination \(R^2\) (R-square) is about 0.9996. This result is in agreement with the fact that many authors prefer the log-normal distribution to fit the breakage probability\(^{1,27,30}\).

To test the relevance of the asperities model, the
normalized breakage distributions of different particles will be compared. We will show that in terms of irregularly shaped particles, the normalized breakage probability distribution can be fitted with the log-normal distribution function. If the proposed contact model and simulation are correct, the parameters of the log-normal fit function remain the same for all tested particle sizes.

4. Impact Set-up and Tested Materials

A small-scale pneumatic cannon was designed to carry out the impact tests, see Fig. 6. The acceleration of the moving piston with particle occurs inside a 900-mm-long hard aluminium tube with a core diameter of 12 mm. For particles in the size range of 1.00 – 6.33 mm, velocities up to approximately 35 m/s were reproducibly obtained. The driving pressure of compressed air varies from 0.5 to 3 bar. The charge of the particle into the moving piston occurs at the end of the acceleration tube. The permanent magnet allows the piston with particle to be put in the start position.

The impact velocity was determined using the signals from two Hall sensors mounted along the tube and a vibration sensor mounted on the target. Two time intervals were used to measure the velocity. The first time interval corresponds to the movement of the piston with magnet and particle between two Hall sensors. The velocity of the piston was derived from this time interval. By stopping the piston at the end of the acceleration tube, the particle continues its movement up to collision with the target. The particle collision with the target was detected by the vibration sensor. Consequently, the particle velocity was measured by using the time interval between the passage of the second Hall sensor and the start of target vibrations caused by particle collision. The velocities of particle and piston were compared. The difference in velocities can be caused by attrition between the particle and the piston during deceleration of the piston. When the difference was greater than 5%, the test was not taken into account to exclude the possible particle fragmentation due to friction caused by the particle movement inside the piston during its deceleration.

In order to provide statistically significant data, about 300 particles were provided for every impact test. The mass of each particle was measured before and after impact. If a mass reduction of more than 10% occurs, then the particle was classified as “broken”.

The impact test was carried out with various size fractions of tested materials: NaCl from the Dead Sea, sugar, basalt and granules of Al2O3. All particles were irregularly shaped, see Fig. 7a, Fig. 7b, Fig. 7c. The samples were sieved to obtain particles in the given size ranges, see Table 1. The size range of the tested particles was wide enough to permit selection of a representative probe.

5. Mean Energy Versus Particle Size

The cumulative experimental distribution of breakage probability \( P \) was obtained by variation of the impact energy \( W \) in the relevant range that corresponds to increasing the breakage probability from 0 up to about 0.9. The mean value of breakage energy \( W_{\text{mean}} \) was calculated by numerical integration of the experimental distribution according to Eq. (6).
Fig. 8. represents the dependence of the normalized mean impact energy $w_{\text{mean}, n}$ on the particle of NaCl of size $d_n$. The mean energy was normalized by its maximal value of 7.0 mJ, and the particle size was normalized by 5.65 mm. The increase in mean of impact energy $W_{\text{mean}}$ with particle size $d$ can be described as:

$$
\left( \frac{W_{\text{mean}}}{7\text{mJ}} \right) = \left( \frac{d}{5.65\text{mm}} \right)^k
$$

(14)

The data were fitted by a power function with exponent $k=3.02$. The coefficient of determination $R^2$ (R-square) is about 0.97.

A similar dependence was observed for sugar par-
In the case of sugar particles, the exponent $k = 3.42$ and R-square was about 0.96. Based on experimental data, one can conclude that the mean breakage energy is a power function of particle size.

6. Breakage Probability Versus the Normalized Impact Energy

The breakage behaviour of irregularly shaped NaCl particles was tested first. The cumulative experimental distributions of the normalized impact energy of NaCl particles are shown in Fig. 9. The particle size varies from 1 mm to 6.3 mm. Within the frame of experimental data accuracy, there is no difference in breakage probability behaviour for particles of different sizes. All data are fitted by the same log-normal function.

According to our simulation, the breakage probability can be fitted with a log-normal distribution function depending on the normalized energy. Within the frame of measurement accuracy, the parameters of the log-normal fit function will remain the same as for the tested particle sizes and materials.

The second sequence of experiments was carried out to investigate the breakage behaviour of sugar particles, see Fig. 11. The particle size varies in the same range as in the case of NaCl particles. Both materials – NaCl from the Dead See and the sugar – show a similar breakage behaviour. The log-normal function with the same parameters was applied to fit all experimental distributions.

The third tested material was basalt. Unlike the above two materials, it can be characterized as a brittle material of high strength. However, like the NaCl and sugar particles, basalt exhibits a similar relationship between normalized kinetic energy and breakage probability, see Fig. 11. The parameters of the fit function remain the same as for the above materials.

Finally, it was interesting to apply the developed method to spherical granules of $\gamma Al_2O_3$. The granules of $Al_2O_3$ can show a different impact behaviour compared with the above-tested irregularly shaped solid particles. $Al_2O_3$ granules in two size ranges – $2.5 \text{ mm} < d < 3.15 \text{ mm}$ and $1.6 \text{ mm} < d < 2 \text{ mm}$ – were tested. The breakage probability of $Al_2O_3$ granules can be fitted with the log-normal function in a similar way to the NaCl, sugar, and basalt particles, see Fig. 12. The parameters of the fit function remain the same for all above tested materials. However, at the low normalized energy, the experimental data of granules show a small systematic deviation from the fit curve. It can be caused by the spherical form of the granules.

7. Conclusions

The impact breakage of irregularly shaped particles of Dead Sea salt, sugar, basalt, and granules of $\gamma Al_2O_3$ was tested. The breakage behaviour of the tested particles can be described in terms of a model of hemispherical asperities. According to this model, the cap of a particle directly in contact with the target is admitted into the equation as a hemispherical asperity. The main part of elastic energy that is stored
in the particle will be accumulated by this particle cap. This “directly contacted” particle cap is responsible for crack generation by stressing. Consequently, the breakage of particles is affected by the size distribution of those caps, i.e. asperities.

Usually, the size distribution of irregularly shaped particles is a log-normal function. The geometrical similarity of particle and caps was assumed. That means that not only particles but also caps, i.e. asperities, exhibit log-normal size distribution.

Simulation of the breakage probability distribution was carried out based on the contact model and the log-normal size distribution of asperities. The results of the simulation show the following specific features of the resulting breakage probability distribution:

a) the distribution can be fitted by a log-normal function,

b) due to the cumulative effect of log-normal distributed asperities, the specific material properties tend to disappear in simulated breakage probability distribution,

c) the distribution of breakage probability can be represented in simple universal form. In this representation, the breakage depends on the normalized breakage energy only. It does not depend on the particle size and material.

The results of experiments show a good agreement with the model for brittle irregularly shaped particles. In terms of elastic spherical granules of low stiffness, the model needs improvement.

Acknowledgments

This research project I-875-106.10/2005 was supported by a grant from the G.I.F., the German-Israeli Foundation for Scientific Research and Development.

Legend

- $a_R$: stressing intensity parameter according to Rumpf, J/m²
- $d_r$: diameter of hemispherical asperity, mm
- $c$: constant, (kg/J)$^1/2$ mm²
- $\Gamma$: gamma function
- $d$: particle size, mm
- $d_{sa}$: normalized particle size
- $d_{min}$: minimal size of asperities particle, mm
- $W$: kinetic energy by particle impact, J
- $P$: parameter
- $v$: particle velocity, m/s
- $v_0$: constant, m/s
- $v_{50}$: velocity that corresponds to breakage of 50% of particles, m/s
- $W_m$: mass-related kinetic energy of spherical particles, J/kg
- $W_m$, $W_{mean}$, $W_{mean,n}$: threshold energy, J/kg, normalized kinetic energy, arithmetic mean of kinetic energy, J
- $W_{mean,n}$: normalized arithmetic mean of kinetic energy
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