Coherent state of a nonlinear oscillator and its revival dynamics

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Abstract

The coherent state of a nonlinear oscillator having a nonlinear spectrum is constructed using Gazeau–Klauder formalism. The weighting distribution and the Mandel parameter are studied. Details of the revival structure arising from different time scales underlying the quadratic energy spectrum are investigated by the phase analysis of the autocorrelation function.

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1. Introduction

The nonlinear differential equation

\[ (1 + \lambda x^2) \ddot{x} - (\lambda x) \dot{x}^2 + \alpha^2 x = 0, \quad \lambda > 0 \]  

was studied by Mathews and Lakshmanan in [1, 2] as an example of a nonlinear oscillator and it was shown that the solution of (1) is

\[ x = A \sin(\omega t + \phi) \]  

with the following additional restriction linking frequency and amplitude:

\[ \omega^2 = \frac{\alpha^2}{1 + \lambda A^2} \]  

It has been shown in [1] that equation (1) is obtainable from the Lagrangian density

\[ L = \frac{1}{2} \left( \frac{1}{1 + \lambda x^2} \right) (\dot{x}^2 - \alpha^2 x^2) \]  

Recently, in a series of papers [3–5] this particular nonlinear system has been generalized to higher dimensions and various properties of this system have been studied thoroughly. In [6], the Schrödinger equation corresponding to this nonlinear oscillator has been solved exactly as a Sturm–Liouville problem and \( \lambda \)-dependent eigenvalues and eigenfunctions were obtained for both \( \lambda > 0 \) and \( \lambda < 0 \). The following one to be noted:

(1) This \( \lambda \)-dependent system can be considered as a deformation of the standard harmonic oscillator in the sense that for \( \lambda \to 0 \) all the characteristics of the linear oscillator are recovered.

(2) In Lagrangian (4), the parameter \( \lambda \) is present not only in the potential \( (x^2/(1 + \lambda x^2)) \) but also in the kinetic term. So this nonlinear oscillator may also be considered as a particular system with position-dependent mass \( m(x) = 1/((1 + \lambda x^2)) \).

The Schrödinger equation with a position-dependent mass has found applications in the field of material science and condensed matter physics, such as semiconductors [7], quantum wells and quantum dots [8, 9], 3He clusters [10], graded alloys and semiconductor heterostructures [11–19], etc. It has also been found that such equations appear in very different areas. For example, it has been shown that the constant mass Schrödinger equation in curved space and those based on deformed commutation relations can be interpreted in terms of position-dependent mass [20, 21]. This has generated a great deal of interest in this field, and during the past few years various theoretical aspects of the position-dependent mass Schrödinger equation have been studied widely [22–39].

On the other hand, coherent states have attracted considerable attention in the literature [40, 41]. Coherent states are generally constructed by (i) using the displacement operator technique or defining them as (ii) minimum uncertainty states or (iii) annihilation operator eigenstates. However, even when such operators do not exist, different approaches [42–47] have been utilized to construct coherent states corresponding to different quantum mechanical potentials [48–74]. Coherent states of systems possessing nonlinear energy spectra are of particular interest as their temporal evolution can lead to revival and fractional revival.
leading to Schrödinger cat and cat-like states. For the constant mass Schrödinger equation, potentials such as Pöschl–Teller, Morse and Rosen-Morse lead to nonlinear spectra. Time evolution of the coherent states for these potentials [75–83] is a subject of considerable current interest as they can produce Schrödinger cat and cat-like states.

In the present paper, we shall construct coherent states for the nonlinear oscillator (1) and study their revival dynamics. The motivation comes from the fact that the study of the temporal evolution of a free wave packet with position-dependent mass inside an infinite well [84] has revealed that revival and partial revivals are not only different from the constant mass case but also very much dependent on the mass function \( m(x) \). So it will be interesting to study the role of the mass parameter \( \lambda \) of the nonlinear oscillator (1) on the temporal evolution of its coherent state.

The paper is organized as follows. In section 2, the Gazeau–Klauder coherent state [45, 46] for the nonlinear oscillator is constructed and is shown to satisfy conditions such as continuity of labeling, resolution of unity, temporal stability and action identity. The revival dynamics of the coherent state is studied in section 3. In section 4, we summarize our results.

2. Gazeau–Klauder coherent state for a nonlinear oscillator

The position-dependent mass Schrödinger Hamiltonian \( H \) for the nonlinear oscillator (1) is given by

\[
H = \left[ -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \lambda \frac{\partial}{\partial x} \right) + \frac{\lambda^2}{2m} \left( \frac{\partial}{\partial x} - \frac{1}{\lambda} \right)^2 \right] = E \psi. \tag{5}
\]

The parameter \( \lambda \) may be positive or negative. However, for \( \lambda > 0 \) the discrete energy spectrum can be shown to be finite and consequently for completeness property the continuum has to be taken into account. On the other hand, for \( \lambda < 0 \) there are only discrete energy states and henceforth we shall consider this choice. Replacing the parameter \( g \) in equation (5) by \( ma^2 + \lambda \hbar \alpha \) as was done in [6], the Schrödinger equation is given by

\[
\left[ -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \lambda \frac{\partial}{\partial x} \right) + \frac{\lambda^2}{2m} \left( \frac{\partial}{\partial x} - \frac{1}{\lambda} \right)^2 + \frac{1}{\alpha} \frac{\partial}{\partial \alpha} \left( \alpha + \frac{\hbar}{m} \right) \frac{\lambda^2}{1 + \lambda^2} \right] \psi = E \psi. \tag{6}
\]

Under the transformation \( x = \sqrt{\hbar/m} \alpha \gamma; \lambda = (m \alpha / \hbar) \Lambda \), the above equation transforms to

\[
\left[ \frac{1}{1 + \lambda^2} \frac{\partial^2}{\partial y^2} + \lambda y \frac{\partial}{\partial y} \frac{\partial^2}{\partial y^2} + \frac{1 + \lambda^2}{1 + \lambda^2} + 2 \gamma^2 \right] \psi = 0. \tag{7}
\]

where \( E = e(h\alpha) \). For \( \Lambda < 0 \), the eigenvalues \( e_n \) and the eigenfunctions \( \psi_n \) of the above transformed Hamiltonian are given by [6]

\[
\psi_n = \left( 1 - |\Lambda|^2 \right)^{\frac{n}{2}} H_n(y, \Lambda),
\]

\[
e_n = \left( n + \frac{1}{2} \right) + \frac{1}{2} |\Lambda|^2, \quad n = 0, 1, 2, \ldots, m, \ldots, \tag{8}
\]

where \( H_n(y, \Lambda) \) are called A-deformed Hermite polynomials [6]. So the eigenvalues of the Hamiltonian \( H^1 = H - h\alpha/2 \) are

\[
E_n^1 = E_n - \frac{\hbar\alpha}{2} = \hbar \alpha \left[ \frac{1}{2} |\Lambda|^2 + n \right],
\]

\[
= \frac{\hbar\alpha}{2} \left( n + \frac{1}{2} \right), \quad n = 0, 1, 2, \ldots, \tag{9}
\]

where \( \mu = 2/|\Lambda| \). The Gazeau–Klauder coherent state [45, 46] for this system is given by

\[
|J, \gamma \rangle = \frac{1}{N(J)} \sum_n \frac{(J)^{n/2} \exp(-i\gamma \alpha_n^2)}{\sqrt{\rho_n}} |n\rangle, \tag{10}
\]

where \( \gamma = \alpha t \). The normalization constant \( N(J) \) is given by

\[
N(J) = \left[ \sum_n^{\rho_n} \rho_n \right]^{1/2}, \quad 0 < J < R = \lim_{n \to \infty} \rho_n^{1/n}, \tag{11}
\]

where \( R \) denotes the radius of convergence and \( \rho_n \) denotes the moments of a probability distribution \( \rho(x) \):

\[
\rho_n = \int_0^R x^n \rho(x) \, dx = \sum_{i=1}^n \rho_i, \quad \rho_0 = 1. \tag{12}
\]

For the coherent state (10)

\[
\rho_n = \prod_{i=1}^n \frac{\mu}{\mu} \frac{(i + \mu)}{(i + \mu)} = \frac{\Gamma(\mu + 1)\Gamma(\mu + 1 + \mu)}{\mu \Gamma(1 + \mu)} \rho_{\mu} \left( \frac{2}{\sqrt{\mu}} \right), \tag{13}
\]

so that \( R \) is infinite and

\[
\rho(J) = \frac{2\mu(J\mu)^{1/2}}{\Gamma(1 + \mu)} I_{\mu} \left( \frac{2}{\sqrt{\mu}} \right). \tag{14}
\]

\( K_{\mu}(cx) \) being the modified Bessel function [85].

Also

\[
N(J) = \frac{\Gamma(1 + \mu)}{(J\mu)^{1/2}} I_{\mu} \left( 2\sqrt{\mu} \right), \tag{15}
\]

where \( I_{\mu}(cx) \) is the modified Bessel function [85].

So, the coherent state (10) finally becomes

\[
|J, \gamma \rangle = \frac{\sqrt{\Gamma(1 + \mu)}}{N(J)} \sum_{n=0}^{\infty} \frac{(J\mu)^{(n/2)} \exp(-i\gamma \alpha_n^2/\mu)}{\sqrt{n! \Gamma(\mu + 1 + \mu)}} |n\rangle. \tag{16}
\]

Below, we shall see that the coherent state (10) satisfies the following conditions:

1. Continuity of labeling: from definition (10) it is obvious that \( J, \gamma \) \( \rightarrow \) \( (J', \gamma') \) \( \rightarrow \) \( |J', \gamma' \rangle \rightarrow |J, \gamma \rangle \).

2. Resolution of unity:

\[
\int |J, \gamma \rangle \langle J, \gamma | \, d\mu(J, \gamma) = \int |J, \gamma \rangle \langle J, \gamma | \, d\mu(J, \gamma) \quad \int \kappa(J)|J, \gamma \rangle \langle J, \gamma | \, dJ, \tag{17}
\]

where \( \kappa(J) \) is defined by

\[
\kappa(J) = N(J)^2 \rho(J) \geq 0, \quad 0 \leq J < R = \rho(J) \equiv 0, \quad J > R. \tag{18}
\]
For the coherent state (10)

\[ k(J) = 2\mu |J\rangle \langle J| \left( 2\sqrt{J\mu} \right) K\mu \left( 2\sqrt{J\mu} \right), \]  

so that the resolution of unity is satisfied:

\[ \int |J, \gamma\rangle \langle J, \gamma| d\mu(J, \gamma) = 1. \]  

3. Temporal stability:

\[ e^{-iHt/\hbar} |J, \gamma\rangle = |J, \gamma + \alpha t\rangle. \]  

4. Action identity:

\[ \langle J, \gamma| H^\dagger |J, \gamma\rangle = \hbar \alpha J. \]  

The overlapping of two coherent states is

\[ \langle J', \gamma'| J, \gamma \rangle = \frac{\Gamma(\mu + 1)}{N(J)N(J')} \sum_{n=0}^{\infty} \frac{(J'\mu)^{n/2} e^{i\gamma' \gamma} c_n^2}{n! \Gamma(n + 1 + \mu)}. \]  

If \( \gamma = \gamma' \), the overlapping is reduced to

\[ \langle J', \gamma | J, \gamma \rangle = \frac{1}{\sqrt{I_\mu(2\sqrt{J\mu}) I_\mu(2\sqrt{J'\mu})}} I_\mu(2(JJ'\mu^2)^{1/4}). \]  

3. Revival dynamics

In this section, we shall study the revival dynamics of the coherent state (10). To demonstrate the role of the mass parameter \( \lambda \) on the revival dynamics, all the figures below are drawn for a fixed value of \( J = 10 \) and two different values of \( \mu \), which is inversely proportional \( (\mu = 2/|\lambda|) \) to the mass parameter \( \lambda \). For a general wave packet of the form

\[ |\psi(t)\rangle = \sum_{n=0}^\infty c_n e^{-iE_n/t/\hbar} |n\rangle \]  

with \( \sum_{n=0}^\infty |c_n|^2 = 1 \), the concept of revival arises from the weighting probabilities \( |c_n|^2 \). For the coherent state (10), the weighting distribution is given by

\[ |c_n|^2 = \frac{(J\mu)^{n^{1/2}}}{n! \Gamma(n + 1 + \mu) I_\mu(2\sqrt{J\mu})}. \]  

Since the weighting distribution \( |c_n|^2 \) is crucial for understanding the temporal behavior of the coherent state (10), we show the curves of \( |c_n|^2 \) for \( J = 10 \) and different \( \mu \) in figure 1.

The Mandel parameter \( Q \) is defined by

\[ Q = \frac{(\Delta n)^2}{\langle n \rangle} - 1, \]  

where

\[ \langle n \rangle = \sum_{n=0}^{\infty} \frac{n J^n}{N(J)^2 \rho_n}, \]  

\[ \Delta n = [(n^2) - \langle n \rangle^2]^{1/2} \]

\[ Q = \sqrt{J\mu} \left[ \frac{I_{\mu+1}(2\sqrt{J\mu})}{I_\mu(2\sqrt{J\mu})} - \frac{I_{\mu+1}(2\sqrt{J\mu}^3)}{I_\mu(2\sqrt{J\mu}^3)} \right]. \]  

In figure 2, we plot the Mandel parameter \( Q \) for \( J = 10 \) and \( \mu = 28, 80 \). It is evident from the figure that the Mandel parameter is sub-Poissonian and it has been observed that it remains so for all values \( \mu \).

Now, assuming that the spread \( \Delta n = [(n^2) - \langle n \rangle^2]^{1/2} \) is small compared to \( \langle n \rangle \approx \bar{n} \), we expand the energy \( E_\lambda \) in a Taylor series in \( n \) around the centrally excited value \( \bar{n} \):

\[ E_\lambda^1 \approx E_\lambda^1 + E_\lambda^1 (n - \bar{n}) + \frac{1}{2} E_\lambda^2 (n - \bar{n})^2 + \frac{1}{6} E_\lambda^3 (n - \bar{n})^3 + \cdots. \]  

\[ (n) = \sqrt{J\mu} \left[ \frac{I_{\mu+1}(2\sqrt{J\mu})}{I_\mu(2\sqrt{J\mu})} \right]. \]  

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where each prime on \( E_n^k \) denotes a derivative with respect to \( n \). These derivatives define distinct time scales, namely the classical period \( T_{cl} = 2\pi \hbar/|E_n^k| \), the revival time \( t_{rev} = 2\pi \hbar/1/2|E_n^k| \) and so on. For \( E_n^1 \) in equation (9), \( T_{cl} = 2\pi \hbar/\alpha(2n + \mu) \) and \( t_{rev} = 2\pi \mu/\alpha \). There is no super-revival time here because the energy is a quadratic function in \( n \). It is convenient to describe the wave packet dynamics by an autocorrelation function
\[
A(t) = \langle \psi(x, 0) | \psi(x, t) \rangle = \sum_{n \geq 0} |c_n|^2 e^{-iE_n^1 t/\hbar}. \tag{32}
\]
For the coherent state (10) the autocorrelation function (32) is given by
\[
A(t) = \langle J, 0 | J, \alpha t \rangle = \frac{\Gamma(1+\mu)}{N(J)^2} \sum_{n \geq 0} \frac{(J\mu)^n}{n! \Gamma(n+1+\mu)} e^{-i\alpha(n+\mu)/\mu} t. \tag{33}
\]
From (33) it follows that \( |A(t - t_{rev})|^2 = |A(t)|^2 \) so that the autocorrelation function is symmetric about \( t_{rev}/2 \). In other words, whatever happens in \([0, t_{rev}/2]\) is repeated subsequently. We note that \( 0 \leq |A(t)|^2 \leq 1 \) and it denotes the overlap between the coherent state at \( t = 0 \) and the one at time \( t \). So, the larger the value of \( |A(t)|^2 \), the greater the coherent state at time \( t \) will resemble the initial one. In figures 3(a)–(c), we plot the squared modulus of the autocorrelation function (33) in units of \( t_{rev} \) for \( J = 10 \) and \( \mu = 1, 2, 80 \), respectively. Figure 3(a) clearly shows the full revival. The sharp peaks in figures 3(b) and (c) arise due to fractional revivals.

We can see from figures 3(a) to (c) that different values of \( \mu \) lead to qualitatively different types of motion of the coherent wave packet.

Now, we shall study the mechanism of the fractional revival by phase analysis [86, 87]. For this, we use the time scale determined by the period of complete revival \( t_{rev} = 2\pi \mu/\alpha = 1 \). So, the phase of the \( n \)th stationary state is
\[
\phi_n(t) = 2\pi (\mu n + n^2) t, \tag{34}
\]
where \( \mu = 2/|\lambda| \) is assumed to be an integer.

At arbitrary moments of time, phases (34) of individual components of the packet are uniformly mixed so that it is not possible to make any definite conclusion about the value of the autocorrelation function. However, at specific moments, the distribution of phases can gain some order; for example, the phases can split into several groups of nearly constant values. The fractional revival of \( q \)th order is defined as the time interval during which the phases are distributed among \( q \) groups of nearly constant values.

To consider fractional revival of order \( q \), in the vicinity of time \( t = 1/q \), it is convenient to write \( n \) as \( n = kq + \Delta \), where \( k = 0, 1, 2, \ldots \) and \( \Delta = 0, 1, 2, \ldots, q - 1 \). Then the
The autocorrelation function can be written as

$$A(t) = \sum_{\Delta=0}^{q-1} P_\Delta(t)$$

and

$$P_\Delta(t) = \sum_k c_{kq+\Delta} e^{-\phi_{kq+\Delta}(t)},$$

where

$$c_{kq+\Delta} = \frac{(J\mu)^{kq+\Delta+\mu/2}}{(kq + \Delta)!(kq + \Delta + 1 + \mu)} \Gamma(2\sqrt{J\mu})$$

and \(\phi_{kq+\Delta}(t)\) is given by (34) replacing \(n\) by \(kq + \Delta\). For quantum numbers that are multiples of the revival order, i.e. \(n = kq\), we have

$$\phi_{kq}(\frac{1}{q}) = 2\pi(k\mu + k^2q) = 0 \pmod{2\pi}.$$  

When \(\Delta \neq 0\), the phases of the corresponding states are

$$\phi_{kq+\Delta}(\frac{1}{q}) = 2\pi(\mu + \Delta^2)/q,$$

$$P_\Delta(\frac{1}{q}) = \exp[-2\pi i(\mu\Delta q^{-1} + \Delta^2q^{-1})] \sum_k c_{kq+\Delta},$$

where in obtaining (37) and (38) we have made use of the result

$$2\pi(k(\mu + 2\Delta) + k^2q) = 0 \pmod{2\pi}.$$  

Thus from (38) and (39) it follows that, around time \(t = 1/q\), the Gazeau–Klauder coherent state (10) splits into \(q\) packet fractions such that the \(\Delta = 0\) packet fraction has zero phase, while relative to this the other packet fractions have a constant phase \((\mu \Delta q^{-1} + \Delta^2q^{-1})\). It must be mentioned that the way in which the packet is divided into fractions is determined only by the order of the revival analyzed. Figures 4 and 5 give the time dependence of the survival functions \(|P_\Delta(t)|^2\) in units \(t_{\text{rev}}\) for \(\Delta = 0, 1, 2, 3\) during the whole period of the complete revival for fixed \(J = 10\) and \(\mu = 28, 80\), respectively. It is to be noted that fractions with odd and even \(\Delta\) differ by their revival periods.

The intensity of the fractional revival can be defined as the value of the wave packet survival function at the moment of revival. This value is composed of two terms: the sum of survival functions for the packet fractions (the diagonal term) and the interference term describing the interaction of the packet fractions:

$$S\left(\frac{1}{q}\right) = \sum_{\Delta=0}^{q-1} \left| P_\Delta\left(\frac{1}{q}\right) \right|^2 + \sum_{\Delta=0}^{q-1} \sum_{\Delta \neq \Delta} \left| P_\Delta\left(\frac{1}{q}\right) P_\Delta^*\left(\frac{1}{q}\right) \right|^2.$$  

In figures 6(a) and (b), we present the time dependence of the diagonal terms in units of \(t_{\text{rev}}\) for fixed \(J = 10\) and \(\mu = 28, 80\), respectively. It is seen from the figures that the diagonal term related to the individual packet fractions takes positive values only near the moments of fractional revival.

Figures 7(a) and (b) show the time dependence of the interference terms in units of \(t_{\text{rev}}\) for fixed \(J = 10\) and \(\mu = 28, 80\), respectively. It is seen from the figures that the interference term plays a constructive, destructive or indifferent role near the moments of fractional revival.

4. Summary

In this paper, we have constructed the coherent states for a nonlinear oscillator via Gazeau-Klauder formalism [45, 46].
Figure 5. Plot of the survival functions $|P_\Delta(t)|^2$ given in (37) for $\Delta = 0, 1, 2, 3$ and $J = 10, \mu = 80$.

Figure 6. Plots of the diagonal term $S_1(t) = \sum_{\Delta=0}^{3} |P_\Delta(t)|^2$ of the survival function for (a) $J = 10, \mu = 28$ and (b) $J = 10, \mu = 80$. 
These coherent states are shown to satisfy the requirements of continuity of labeling, resolution of unity, temporal stability and action identity. The plots of the weighting distribution for these coherent states are almost Gaussian in nature. The Mandel parameter $Q$ is sub-Poissonian, which indicates that the coherent state (10) exhibits squeezing for all values of $\mu$. The fractional revivals of the coherent states are evident from figures 3(b) and (c) of the squared modulus of the autocorrelation function. This is in contrast to the result obtained in [84], where the wave packet revival in an infinite well for the Schrödinger equation with position-dependent mass [84] was studied. In [84], it was found that although full revival takes place, there is no fractional revival in the usual sense. Instead, a very narrow wave packet is located near one wall of the well, when the mass is higher. In the present paper, the times of appearance, the spectral compositions and the intensities of the fractional revivals are determined by phase analysis.

References

[1] Mathews P M and Lakshmanan M 1974 Q. Appl. Math. 32 215
[2] Lakshmanan M and Rajsekhar S 2003 Nonlinear Dynamics, Integrability, Chaos and Patterns: Advanced Texts in Physics (Berlin: Springer)
[3] Carinena J F et al 2004 Nonlinearity 17 1941
[4] Carinena J F et al 2005 Regul. Chaotic Dyns. 10 423
[5] Carinena J F et al 2007 Phys. At. Nuclei 70 505
[6] Carinena J F et al 2001 Ann. Phys. NY 322 434
[7] Bastard G 1988 Wave Mechanics Applied to Semiconductor Heterostructures (Les Ulis, France: Les Editions de Physique)
[8] Serra L I and Lipparini E 1997 Europhys. Lett. 40 667
[9] Harrison P 2000 Quantum Wells, Wires and Dots (New York: Wiley)
[10] Barranco M et al 1997 Phys. Rev. B 56 8997
[11] Gora T and Williams F 1969 Phys. Rev. 177 1179
[12] Marrow R A 1985 Phys. Rev. B 27 2294
[13] Marrow R A 1987 Phys. Rev. B 36 4836
[14] Trzeciakowski W 1987 Phys. Rev. B 38 4836
[15] Galbraith I and Duggan G 1988 Phys. Rev. B 38 10057
[16] Young K 1989 Phys. Rev. B 39 13434
[17] Enevoll G T et al 1990 Phys. Rev. B 42 3485
[18] Enevoll G T 1990 Phys. Rev. B 42 3497
[19] Weisbuch C and Vinter B 1983 Quantum Semiconductor Heterostructures (New York: Academic)
[20] Quesne C and Tkachuk V M 2004 J. Phys. A: Math. Gen. 37 4267
[21] Bagchi B et al 2005 J. Phys. A: Math. Gen. 38 2929
[22] Dekar L et al 1998 J. Math. Phys. 39 2551
[23] Dekar L et al 1999 Phys. Rev. A 59 107
[24] Bagchi B et al 2004 Czech. J. Phys. 54 1019
[25] Bagchi B et al 2004 Mod. Phys. Lett. A 19 2765
[26] Yu J et al 2004 Phys. Lett. A 322 290
[27] Yu J and Song S H 2004 Phys. Lett. A 325 194
[28] Ganguly A et al 2006 Phys. Lett. A 360 228
[29] Samani K and Loran F 2003 arXiv: quant-ph/0302191
[30] Bagchi B et al 2005 J. Phys. A: Math. Gen. 38 2929
[31] Koc R et al 2002 J. Phys. A: Math. Gen. 35 L1
[32] Quesne C and Tkachuk V M 2004 J. Phys. A: Math. Gen. 37 4267
[33] Plastino A R et al 1999 Phys. Rev. A 60 4398
[34] Gönül B et al 2002 Mod. Phys. Lett. A 17 2057
[35] Quesne C 2006 Ann. Phys. 321 1221
[36] Ganguly A and Nieto L M 2007 J. Phys. A: Math. Gen. 40 7265

[37] Roy B and Roy P 2002 J. Phys. A: Math. Gen. 35 3961

[38] Koc R and Koca M 2003 J. Phys. A: Math. Gen. 36 8105

[39] Chetouani L et al 1995 Phys. Rev. A: Math. Gen. 52 82

[40] Klauder J R and Skagerstam B S 1985 Coherent States: Applications in Physics and Mathematical Physics (Singapore: World Scientific)

[41] Zhang W M et al 1990 Rev. Mod. Phys. 62 867

[42] Perelomov A M 1986 Generalized Coherent States and Their Application (Berlin: Springer)

[43] Barut A O and Girardello L 1971 Commun. Math. Phys. 21 41

[44] Nieto M M and Simmons L M Jr 1978 Phys. Rev. Lett. 41 207

[45] Klauder J R 1996 J. Phys. A: Math. Gen. 29 L293

[46] Gazeau J P and Klauder J R 1999 J. Phys. A: Math. Gen. 32 123

[47] Shreecharan T et al 2004 Phys. Rev. 69 012102

[48] Benedict M G and Molnar B 1999 Phys. Rev. A 60 R1737

[49] Majumdar P and Sharatchandra H S 1997 Phys. Rev. A 56 R3322

[50] Dong S H 2002 Can. J. Phys. 80 129

[51] Roy B and Roy P 2002 Phys. Lett. A 296 187

[52] Chenaghlou A and Fakhri H 2002 Mod. Phys. Lett. A 17 1701

[53] Chenaghlou A and Fakhri H 2004 Mod. Phys. Lett. A 19 2619

[54] Jellal A 2002 Mod. Phys. Lett. A 17 671

[55] Fakhri H and Chenaghlou A 2003 Phys. Lett. A 310 1

[56] Shapiro E A et al 2003 Phys. Rev. Lett. 91 237901

[57] Nieto M M 1978 Phys. Rev. A: Math. Gen. 17 1273

[58] Nieto M M and Simmons L M Jr 1979 Phys. Rev. D 20 1332

[59] Nieto M M and Simmons L M Jr 1979 Phys. Rev. D 20 1342

[60] Crawford M G A and Vrscay E R 1998 Phys. Rev. A 57 106

[61] Kinani A H and Daoud M 2001 Phys. Lett. A 283 291

[62] Nieto M M 2001 Mod. Phys. Lett. A 16 2305

[63] Dayi O F and Jellal A 2001 Phys. Lett. A 287 349

[64] Chenaghlou A and Faizy O 2007 J. Math. Phys. 48 112106

[65] Chenaghlou A and Faizy O 2008 J. Math. Phys. 49 022104

[66] Fukai T and Aizawa N 1993 Phys. Lett. A 180 308

[67] Balantekin A B et al 1999 J. Phys. A: Math. Gen. 32 2785

[68] Balantekin A B et al 2002 J. Phys. A: Math. Gen. 35 9063

[69] Angelova M and Hussin V 2008 J. Phys.A: Math. Gen. 41 304016

[70] Tavassoly M K 2008 J. Phys. A: Math. Gen. 41 285305

[71] Fakhri H and Dehghani A 2008 J. Math. Phys. 49 042101

[72] Fakhri H and Dehghani A 2008 J. Math. Phys. 49 052101

[73] Cruz S et al 2008 Phys. Lett. A 372 1391

[74] Fernandez D J et al 2007 J. Phys. A: Math. Gen. 40 6491

[75] Averbukh I Sh and Perelman N F 1989 Phys. Lett. A 139 449

[76] Banerji J and Agarwal G S 1999 Phys. Rev. A 59 4777

[77] Hall M J W et al 1999 J. Phys. A: Math. Gen. 32 8275

[78] Kaplan A E et al 2000 Phys. Rev. A: Math. Gen. 61 032101

[79] Bluher R et al 1996 Am. J. Phys. 64 944

[80] Doncheski M A and Rabinett R W 2004 Phys. Rep. 392

[81] Loinaz W and Newman T J 1999 J. Phys. A: Math. Gen. 32 8889

[82] Roy U et al 2005 J. Phys. A: Math. Gen. 38 9115

[83] Antoine J P et al 2001 J. Math. Phys. 42 2349

[84] Schmidt A G M 2006 Phys. Lett. A. 353 459

[85] Abramowitz M and Stegun I A 1965 Handbook of Mathematical Functions (New York: Dover)

[86] Vetchinkin S I et al 1993 Chem. Phys. Lett. 215 11

[87] Eryomin V V et al 1994 J. Chem. Phys. 101 10730