Reconsideration of the $B \rightarrow K^*$ Transition Form Factors within the QCD Light-Cone Sum Rules

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In this paper, we study the $B \rightarrow K^*$ transition form factors (TFFs) within the QCD light-cone sum rules (LCSR) approach. Two correlators, i.e. the usual one and the right-handed one, are adopted in the LCSR calculation. The resultant LCSRs for the $B \rightarrow K^*$ TFFs are arranged according to the twist structure of the $K^*$-meson light-cone distribution amplitudes (LCDAs), whose twist-2, twist-3 and twist-4 terms behave quite differently by using different correlators. We observe that the twist-4 LCDAs, though generally small, shall have sizable contributions to the TFFs $A_{1/2}$, $V$ and $T_1$, thus the twist-4 terms should be kept for a sound prediction. We also observe that even though different choices of the correlator lead to different LCSRs with different twist contributions, the large correlation coefficients for most of the TFFs indicate that the LCSRs for different correlators are close to each order, not only for their values at the large recoil point $q^2 = 0$ but also for their ascending trends in whole $q^2$-region. Such a high degree of correlation is confirmed by their application to the branching fraction of the semi-leptonic decay $B \rightarrow K^* \mu^+\mu^−$. Thus, a proper choice of correlator may inversely provide a chance for probing uncertain LCDAs, i.e. the contributions from those LCDAs can be amplified to a certain degree via a proper choice of correlator, thus amplifying the sensitivity of the TFFs, and hence their related observables, to those LCDAs.

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I. INTRODUCTION

The heavy-to-light $B$-meson decay provides an excellent platform for testing the CP-violation phenomena and for seeking new physics beyond the Standard Model (SM). The heavy-to-light transition form factors (TFFs) are key components in those studies, which however are non-trivial due to the fact that for practical values of the momentum transfer and the $b$-quark mass ($m_b$), the soft contributions are always numerically important and are often dominant.

The Shifman-Vainshtein-Zakharov (SVZ) sum rules [1, 2], which provides an important step forward for studying those non-perturbative hadron phenomenology. It is a method of expanding the correlation function (correlator) into the QCD vacuum condensates with subsequent matching via dispersion relations. The vacuum condensates are non-perturbative but universal, whose contributions follow from the usual power-counting rules at the large $q^2$-region and the first several ones are enough to achieve the required accuracy. Many successful hadron properties have been achieved since its invention, and the SVZ sum rules becomes a useful tool for studying the hadron phenomenology.

Following its strategy, one has to deal with the two-point correlator for the heavy-to-light transition form factors (TFFs) [3–5], which however will meet specific problems such as the breaking of power-counting and the contamination of sum rules by “non-diagonal” transitions [6], severely restricting the precisions and applicabilities of the SVZ sum rules.

To avoid the problems of the two-point SVZ sum rules, the QCD light-cone sum rules (LCSR) has later been suggested to deal with the heavy-to-light TFFs [7–12]. Its main idea is to make a partial resummation of the operator product expansion (OPE) to all orders and reorganize the OPE expansion in terms of the twists of relevant operators rather than their dimensions. The vacuum condensates of the SVZ sum rules are then substituted by the light-meson’s light-cone distribution amplitudes (LCDAs) of increasing twists. The LCDA, which relates the matrix elements of the nonlocal light-ray operators sandwiched between the hadronic state and the vacuum, has a direct physical significance and provides the underlying links between the hadronic phenomena at small and large distances.

Generally, contributions from the LCDAs suffer from the power counting rules basing on the twists, i.e. the high-twist LCDAs are usually powered suppressed to the lower twist ones in large $Q^2$-region, and the first several LCDAs shall usually provide dominant contributions to the LCSR. Since its invention, the LCSR approach has been widely adopted for studying the $B \rightarrow$ light meson decays. In the paper, we shall concentrate our attention on its application to the $B \rightarrow K^*$ decays, which is helpful for studying the $K^*$-meson LCDAs.

How to “design” a proper correlator is a tricky problem for the LCSR approach. By choosing a proper correlator, one can not only study the properties of the hadrons but also simplify the theoretical uncertainties effectively. Usually, the correlator is constructed by using the currents with definite quantum numbers, such as those with

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definite $J^P$, where $J$ is the total angular momentum and $P$ is the parity of the bound state. Such a direct way of constructing the correlator is not the only choice adopted in the literature, e.g. the chiral correlator with a chiral current in between the matrix element has also been suggested so as to suppress part of the hazy contributions from the uncertain LCDAs [13–18].

The LCDAs of the $K^*$-meson have a much complex structure than that of the light pseudo-scalar mesons. It contains two leading-twist (or twist-2) LCDAs $\phi_{2;K^*}$, and $\phi_{3;K^*}$, seven twist-3 LCDAs $\phi_{3;K^*}$, $\psi_{3;K^*}$, $\Phi_{3;K^*}$, $\Phi_{3;K^*}$, $\Phi_{3;K^*}$, $\phi_{4;K^*}$, $\psi_{4;K^*}$, and $\Phi_{4;K^*}$, and twelve twist-4 LCDAs $\phi_{5;K^*}$, $\psi_{5;K^*}$, $\Phi_{5;K^*}$, $\Phi_{5;K^*}$, $\Phi_{5;K^*}$, $\Phi_{5;K^*}$, $\phi_{4;K^*}$, $\psi_{4;K^*}$, $\Phi_{4;K^*}$, and $\Phi_{4;K^*}$. By taking the usual correlator, we shall show that at the twist-4 accuracy, the LCSRs of the $B \to K^*$ TFFs shall contain almost all of the mentioned LCDAs, where the twist-3 and twist-4 LCDAs shall have sizable contributions to the $B \to K^*$ TFFs. At the present, some of the high-twist LCDAs have been studied within the QCD sum rules [19, 20, 21], which, however, are still of large errors.

As an attempt, we have suggested to use a chiral correlator with a right-handed chiral current to deal with the $B \to K^*$ TFFs that to suppress the uncertainties from high-twist LCDAs themselves are effectively suppressed. In previous discussions [21], some of the terms that are proportional to high-twist LCDAs have been omitted due to the $\delta$-power counting rule. In the paper, as a sound prediction, we shall keep all of them in our present calculations; as will be shown later, those terms shall provide sizable contributions for certain TFFs.

It is interesting to show whether the LCSRs under different choices of the correlator are consistent with each other. In the paper, as a step forward, we shall compare the LCSRs for the TFFs under the usual correlator and the right-handed chiral correlator with the help of the correlation coefficient $\rho_{XY}$ [22].

The remaining parts of the paper are organized as follows. In Sec.II, we present the calculation technology for the $B \to K^*$ TFFs within the LCSR approach, where the results for the usual correlator are presented. The results for the right-handed chiral correlator are presented in the Appendix. In Sec.III, we make a comparative study on various LCSRs for the $B \to K^*$ TFFs, and their application for the branching fraction $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ is also presented. Sec.IV is reserved for a summary.

II. THE $B \to K^*$ TFFS WITHIN THE LCSR APPROACH

The $B \to K^*$ TFFs, $V(q^2)$, $A_{0,1,2}(q^2)$ and $T_{1,2,3}(q^2)$, are related with the matrix elements $\langle K^*|\bar{s}\gamma^\mu b|B\rangle$, $\langle K^*|\bar{s}\gamma^\mu \gamma^5 b|B\rangle$, $\langle K^*|\bar{s}\sigma^{\mu\nu} q_\nu b|B\rangle$ and $\langle K^*|\bar{s}\sigma^{\mu\nu} \gamma^5 q_\nu b|B\rangle$ via the following way [12],

\[
\langle K^*(p, \lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle = -ie_\mu^{(\lambda)}(m_B + m_{K^*})A_1(q^2) + ie_\mu^{(\lambda)} \cdot q \frac{2m_{K^*}}{m_B + m_{K^*}} \langle A_3(q^2) - A_0(q^2) \rangle + i\epsilon_{\mu\rho\sigma}e^{(\lambda)\nu} q^\rho p^\beta \frac{2V(q^2)}{m_B + m_{K^*}},
\]

and

\[
\langle K^*(p, \lambda)|\bar{s}\sigma_{\mu\nu} q^\nu(1+\gamma_5)b|B(p+q)\rangle = 2i\epsilon_{\mu\rho\sigma}e^{(\lambda)\nu} q^\rho p^\beta T_1(q^2) + e_\mu^{(\lambda)}(m_B^2 - m_{K^*}^2)T_2(q^2) \]

\[\quad - (2p + q)_\mu e^{(\lambda)} \cdot q \bar{T}_3(q^2) + q_\mu e^{(\lambda)} \cdot q T_3(q^2)\]

where $p$ is the momentum of $K^*$-meson and $q = p_B - p$ is the momentum transfer, $e^{(\lambda)}$ stands for the $K^*$-meson polarization vector with $\lambda$ being its transverse ($\perp$) or longitudinal (||) component, respectively. The following relations are helpful,

\[
T_3(q^2) = \frac{m_B^2 - m_{K^*}^2}{q^2} \bar{T}_3(q^2) - T_2(q^2),
\]

\[
A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2)
\]

and $A_0(0) = A_3(0)$ and $T_1(0) = T_2(0) = \bar{T}_3(0)$.

To derive the LCSRs for the $B \to K^*$ TFFs, we introduce the following correlator

\[
\Pi_{\mu}^{\text{III}}(p, q) = -i \int d^4x e^{iq \cdot x} \langle K^*(p, \lambda)|T(\bar{J}_W^{\text{III}}(x), j_B(0))|0\rangle,
\]

where the currents $\bar{J}_W^{\text{III}}(x) = \bar{s}(x)\gamma_\mu(1-\gamma_5)b(x)$ and $J_B^{\text{III}}(x) = \bar{s}(x)\sigma_{\mu\nu} q^\nu(1+\gamma_5)b(x)$. The current $J_B^{\text{III}}(x)$ is usually chosen as $im_0\bar{b}(x)\gamma_5 q(x)$, which has the same quantum state as the pseudoscalar $B$-meson with $J^P = 0^-$. For simplicity, we call its corresponding LCSR as LCSR-$U$. As mentioned in the Introduction, the current $J_B^{\text{III}}(x)$ can also be chosen as a chiral current, e.g. the right-handed chiral current $im_0\bar{b}(x)(1+\gamma_5)q(x)$. We call
its corresponding LCSR as LCSR-\( R \). The calculation technology for the LCSR are the same for both cases, and we take \( j_B^I(x) = im_b \gamma(x) \gamma_5 q(x) \) as an explicit example to show how to derive the LCSRs for the \( B \to K^* \) TFFs up to twist-4 accuracy.

The correlator (5) is analytic in the whole \( q^2 \)-region. In the time-like region, one can insert a complete series of the intermediate hadronic states in the correlator and obtain its hadronic representation by isolating out the pole term of the lowest pseudoscalar \( B \)-meson. More explicitly, the correlator \( \Pi_H^{(I)} \) can be written as

\[
\Pi_H^{(I)}(p, q) = \frac{\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) b | B \rangle \langle B | \bar{b} i m_b \gamma_5 q_1 | 0 \rangle}{m_B^2 - (p + q)^2} + \sum_H \frac{\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) b | B \rangle \langle B | \bar{b} i m_b \gamma_5 q_1 | 0 \rangle}{m_B^2 - (p + q)^2} + \Pi_H^{(I)}(e^{*\mu}(\lambda) \cdot q)(2p + q)_\mu + i\Pi_4^{(I)}(e^{*\mu}(\lambda) \cdot q) q_\mu + i\Pi_4^{(I)}(e^{*\mu}(\lambda) \cdot q) q_\mu,
\]

where the matrix element \( \langle B | \bar{b} i m_b \gamma_5 q_1 | 0 \rangle = m_B^2 f_B \), where \( f_B \) is the \( B \)-meson decay constant. By replacing the contributions from the high resonances and continuum states with the dispersion relations, the invariant amplitudes can be written as

\[
\Pi_H^{(I)} = \frac{m_B^2 f_B (m_B + m_{K^*})}{m_B^2 - (p + q)^2} \tilde{A}_i(q^2) + \int_{s_0}^{\infty} \frac{\rho_i^{(I)}}{s - (p + q)^2} ds + \cdots,
\]

where \( i = (1, \cdots, 4) \), \( s_0 \) is the threshold parameter, and the ellipsis stands for the subtraction constant or the fi-

The correlator \( \Pi_H^{(I)}(p, q) \) can be treated via a similar way. With the help of the analytic property of the correlator in different \( q^2 \)-region, the LCSRs for the \( B \to K^* \) TFFs are ready to be derived.

As a further step, we apply the usual Borel transformation to the sum rules, which removes the subtraction terms in the dispersion relation and effectively suppresses the contributions from the unknown excited resonances and continuum states heavier than \( K^* \) meson. After applying the Borel transformation, our final LCSRs read
\[ A_{1L}^2(q^2) = \frac{m_K \cdot m_b (m_B + m_K^*)}{2 f_B m_B^2} \int_0^1 du e^{(m_B^2 - s(u))/M^2} \left\{ \frac{m_K \cdot f_{K^+ \rightarrow K^0} \cdot C}{2u^2 m_K^2} \Theta(c(u, s_0)) \phi_{2, K^*}^\perp (u) - \frac{m_K \cdot f_{K^+ \rightarrow K^0} \cdot \Theta(c(u, s_0))}{2u} \right\} \]

\[ \times \psi_{3, K^*}^\perp (u) + \frac{m_b f_{K^* \rightarrow \pi^+} \cdot \Theta(c(u, s_0))}{u} - m_K^* \cdot f_{K^*}^\perp \left[ \frac{m_K^2 C}{8u^4 M^4} \Theta(c(u, s_0)) + \frac{C - 2m^2 - \Theta(c(u, s_0))}{8u^3 M^2} \right] - \frac{1}{8u^2} \Theta(c(u, s_0)) \phi_{3, K^*} (u) - \frac{m_b m_K^* \cdot f_{K^*} \cdot \Theta(c(u, s_0))}{u^2} \]

\[ - \frac{m_b m_K^* \cdot f_{K^*} \cdot \Theta(c(u, s_0))}{u^2} + \frac{m_b m_K^* \cdot f_{K^*} \cdot \Theta(c(u, s_0))}{u^2} \left[ \frac{m_K^2 C}{8u^4 M^4} \Theta(c(u, s_0)) + \frac{C - 2m^2 - \Theta(c(u, s_0))}{8u^3 M^2} \right] - \frac{1}{u^2} \]

\[ \times \Theta(c(u, s_0)) \right]\]

\[ + \int_0^1 du \int_0^1 \int_0^1 dD \]

\[ e^{(m_B^2 - s(u))/M^2} \frac{m_B m_K^* \cdot f_{K^*} \cdot \Theta(c(u, s_0))}{u^2} \left[ \frac{m_K^2 C}{8u^4 M^4} \Theta(c(u, s_0)) + \frac{C - 2m^2 - \Theta(c(u, s_0))}{8u^3 M^2} \right] - \frac{1}{u^2} \]

\[ \times \Theta(c(u, s_0)) \right]\]

\[ + 12F_{3, K^*}^\perp (\alpha) \left( 4v - 2 \Phi_{4, K^*}^\perp (\alpha) + 2 \Phi_{4, K^*}^\perp (\alpha) \right) \left( m_B^2 - m_{B^*}^2 + 2um_{B^*}^2 \right) + 2m_b m_K^* \cdot f_{K^*} \cdot \phi_{3, K^*}^\perp (\alpha) \right) \]
\[ T_1^I(q^2) = \frac{m_b m_K^*}{2m_B^2 M_B} \int_0^1 du (m_B^2 - s(u))/M^2 \left\{ \frac{m_b m_K^* f_{K^*}}{u m_K} \Theta(c(u,s_0)) \frac{\phi_{2,K^*}^*}{\psi_{3,K^*}^*} (u) + f_{K^*}^\parallel \left[ \frac{\mathcal{E}}{4u^2 M^2} \Theta(c(u,s_0)) + \frac{1}{4u} \right] \times \Theta(c(u,s_0)) \right\} \] 

\[ \times \sum_{c=1}^{4} \left( 2m_b m_K^* f_{K^*}^\parallel \left[ \Psi_{4,K^*}^+(x) - 2 \left( \Psi_{4,K^*}^+ + 2 \Phi_{4,K^*}^{(+1)}(u) - 2 \Phi_{4,K^*}^{(+2)}(u) \right) \right] + f_{K^*}^\parallel \right) \left[ \frac{\frac{m_b m_K^* f_{K^*}^\parallel}{u^2 M^2}}{12 f_B m_B^2} \right] \] 

\[ \times \left[ \frac{m_b m_K^* f_{K^*}^\parallel}{u^2 M^2} \left[ \Phi_{3,K^*}^+(x) - 12 \left( \Psi_{4,K^*}^+ + 2 \Phi_{4,K^*}^{(+1)}(u) - 2 \Phi_{4,K^*}^{(+2)}(u) \right) \right] + f_{K^*}^\parallel \right] \left[ \frac{\frac{m_b m_K^* f_{K^*}^\parallel}{u^2 M^2}}{12 f_B m_B^2} \right] \] 

\[ \times \left[ \frac{m_b m_K^* f_{K^*}^\parallel}{u^2 M^2} \left[ \Phi_{3,K^*}^+(x) - 12 \left( \Psi_{4,K^*}^+ + 2 \Phi_{4,K^*}^{(+1)}(u) - 2 \Phi_{4,K^*}^{(+2)}(u) \right) \right] + f_{K^*}^\parallel \right] \left[ \frac{\frac{m_b m_K^* f_{K^*}^\parallel}{u^2 M^2}}{12 f_B m_B^2} \right] \] 

\[ (14) \]
where the superscript $\mathcal{U}$ indicates those LCSRs are for the usual correlator with $j_B^i(x) = im_B(x)\gamma_5q(x)$. The LCDAs are generally scale-dependent, and for convenience we have implicitly omitted the factorization scale $\mu$ in the LCDAs. $\int dD = \int d\alpha_1 d\alpha_2 d\alpha_3 (1 - \sum_{i=1}^{3} \alpha_i)$. $\mathcal{H} = q^2/(m_B^2 - m_2^2)$, $\mathcal{E} = m_u^2 - u^2m_2^2 + q^2$, $\mathcal{Q} = m_u^2 + u^2m_2^2 - q^2$, $\mathcal{F} = m_u^2 - u^2m_2^2 - q^2$, $c(\phi, s) = g_{s0} - m_2^2 + \bar{q}q - \bar{q}m_2^2$, and $s(\phi) = [m_2^2 - \bar{q}(q^2 - \bar{q}m_2^2)]/\bar{q}$ ($\phi = u$) with $\bar{q} = 1 - \phi$. $\Theta(c(u, s_0))$ is the usual step function. $\tilde{\Theta}(c(u, s_0))$ come from the surface terms $\delta(c(u_0, s_0))$ and $\Delta(c(u_0, s_0))$, whose explicit forms have been given in Ref.[21].

The reduced functions $I_L(u)$, $H_3(u)$, $A_K(u)$, $B_K(u)$, and $C_K(u)$ are defined as

$$I_L(u) = \int_0^u dv \int_0^v dw \left[ \phi^{\|}_{3;K}(w) - \frac{1}{2} \phi^{\perp}_{2;K}(w) \right],$$

$$H_3(u) = \int_0^u dv \left[ \psi^{\|}_{4;K}(v) - \frac{1}{2} \psi^{\perp}_{2;K}(v) \right],$$

$$A_K(u) = \int_0^u dv \left[ \phi^{\|}_{2;K}(v) - \frac{1}{2} \phi^{\perp}_{3;K}(v) \right],$$

$$B_K(u) = \int_0^u dv \phi^{\|}_{4;K}(v),$$

$$C_K(u) = \int_0^u dv \left[ \psi^{\|}_{4;K}(v) + \phi^{\|}_{2;K}(v) \right].$$

By using the same correlator and keeping only the first term of the $b$-quark propagator (9), Ref.[11] calculated the LCSRs for the $B \to \rho$ TFFs $A_1$, $A_2$ and $V$, and Ref.[23] calculated the LCSRs for the $B \to K^*$ TFFs $A_1$, $A_2$, $A_3 - A_0$, $V$, $T_1$, $T_2$ and $T_3$. All those LCSRs are given up to twist-3 accuracy. As a cross-check, we find if keeping the terms up to the same twist-3 accuracy and transforming to the same definitions for the form factors, we return to the same expressions listed in Refs.[11, 23].

Up to twist-4 accuracy, we present the required $K^*$-meson LCDAs in Table I. All of those LCDAs are emerged in the LCSRs [11, 12, 13, 14, 15, 16, 17]. The accuracy of the LCSRs thus depend heavily on how well we know those LCDAs.

In general cases, the contributions from the twist-4 terms are numerically small, thus the uncertainties from the twist-4 LCDAs themselves are highly suppressed. We shall directly adopt the twist-4 LCDAs derived by applying the conformal expansion of the matrix element [20] to do the numerical calculation.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
& twist-2 & twist-3 & twist-4 \\
\hline
1, $\gamma$, $\sigma_{\mu\nu}$ & $\phi^{\|}_{3;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ & $\phi^{\|}_{3;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ & $\phi^{\|}_{3;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ \\
\hline
$\gamma$, $\gamma$, $\gamma$ & $\phi^{\|}_{2;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ & $\phi^{\|}_{2;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ & $\phi^{\|}_{2;K}$, $\psi^{\|^\perp}_{4;K}$, $\phi^{\|^\perp}_{4;K}$, $\psi^{\|}_{4;K}$, $\Psi^{\|^\perp}_{4;K}$ \\
\hline
\end{tabular}
\caption{I. Following the idea of Ref.[12], we rewrite the $K^*$-meson LCDAs with different T-structures in the non-perturbative hadronic matrix elements.}
\end{table}

\footnote{In those two references the surface terms have not be taken into consideration, and because only the 2-particle terms have been kept in the matrix elements, the twist-3 LCDAs involving 3-particle contributions have also been missed in the LCSRs.}
The twist-3 contributions are generally suppressed by certain δ-powers (δ = mK*/mb ≈ 0.17) and 1/M2-powers to the leading twist-2 terms. For examples, the twist-3 contributions from the LCDAs φ∥3,K∗, ψ∥3,K∗, φ⊥3,K∗, and ϕ∥3,K∗ are suppressed by δ1 and the twist-3 contributions from the LCDAs φ∥3,K∗, ψ∥3,K∗, and ϕ∥3,K∗ are suppressed by δ2. However, the twist-3 contributions are sizable and important in certain kinematic region, a special effect should be paid for a precise prediction.

On the one hand, one may use more accurate twist-2 LCDAs to predict the twist-3 contributions. This can be achieved by applying the relations among the twist-2 and twist-3 LCDAs. For example, under the Wandzura-Wilczek approximation [24], the 2-particle twist-3 LCDAs ψ⊥3,K∗, ψ∥3,K∗, and ϕ∥3,K∗ can be related to the twist-2 LCDAs φ2,K∗ and ϕ2,K∗ via the following relations [11]

\[ψ⊥3,K∗(u) = 2 \left[ \bar{u} \int_0^u dv φ2,K∗(v) - u \int_u^1 dv φ2,K∗(v) \right],\]

\[φ∥3,K∗(u) = \frac{1}{2} \left[ \int_0^u dv φ2,K∗(v) + u \int_u^1 dv φ2,K∗(v) \right],\]

\[ψ∥3,K∗(u) = 2 \left[ \bar{u} \int_0^u dv φ2,K∗(v) - u \int_u^1 dv φ2,K∗(v) \right],\]

\[ϕ∥3,K∗(u) = (1 - 2\bar{u}) \left[ \int_0^v dv φ2,K∗(v) - \int_u^1 dv φ2,K∗(v) \right],\]

where \(\bar{u} = 1 - u\) and \(\bar{v} = 1 - v\). The contributions from the remaining three 3-particle twist-3 LCDAs to the \(B \rightarrow K^∗\) TFFs are numerically small, thus as the same as the twist-4 LCDAs, we shall directly take them as the ones from Ref.[20].

On the other hand, it has been suggested that by using the improved LCSR approach [13, 14] and by taking a chiral correlator, the less certain high-twist contributions could be highly suppressed. Ref.[21] has shown that by taking a right-handed correlator with \(f_B(x) = \im\bar{b}(x)(1 + \gamma_5)q(x)\), the twist-3 LCDAs, φ∥3,K∗, ψ∥3,K∗, φ⊥3,K∗, and even the twist-2 LCDA φ∥2,K∗ disappear in the LCSRs. Thus the uncertain twist-3 contributions can be highly suppressed.

Following the standard LCSR procedures, and by keeping all the terms that contribute to the LCSRs up to twist-4 accuracy, we recalculate the \(B \rightarrow K^∗\) TFFs for the right-handed chiral correlator. Our final LCSRs are presented in the Appendix.

The hadronic representation of the chiral correlator contains an extra resonance \(J^P = 0^+\) in addition to the usual one with \(J^P = 0^-\), introducing extra uncertainty to the LCSR-R. The LCSR-R eliminates the large uncertainties from the twist-2 and twist-3 structures which are at the \(δ^1\)-order and we can also suppress its pollution by a proper choice of continuum threshold \(s_0\), thus it is worthwhile to use a chiral correlator. Numerically, we confirm our previous observation that the final LCSRs have slight \(s_0\) dependence [21], thus the uncertainties from \(J^P = 0^+\) resonance are small.

### III. NUMERICAL ANALYSIS

#### A. Basic inputs

In doing the numerical calculation, we take the \(K^∗\)-meson decay constants \(f_{K^∗} = \frac{1}{\sqrt{3}}(9)\) GeV and \(f_{K^∗} = 0.220(5)\) GeV [20], the b-quark mass \(m_b = 4.80\pm 0.05\) GeV, the \(K^∗\)-meson mass \(m_{K^∗} = 0.892\) GeV, the \(B\)-meson mass \(m_B = 5.279\) GeV [22], and the \(B\)-meson decay constant \(f_B = 0.160\pm 0.019\) GeV [25]. The factorization scale \(μ\) is set as typical momentum of the heavy b-quark, i.e., \(μ ≃ (m_B^2 - m_{b}^2)^{1/2} \sim 2.2\) GeV [26, 27], and we predict its error by taking \(\Delta μ = ±1.0\) GeV.

The choices of twist-3 and twist-4 LCDAs have been explained in last subsection. As for the twist-2 LCDAs, we adopt the model, following the idea of Wu-Huang model for the pion LCDA [28], to do the calculation [21]

\[φ_2,K∗(x) = \frac{A_2,K∗}{8π^3/2 f_2,K∗} \left[ 1 + B_2,K∗C_1(ξ) + C_2,K∗C_2(ξ) \right] \exp \left[ -b_2,K∗^2 \frac{\bar{x} \bar{m}_q^2 + x m_q Y^2}{x \bar{x}} \right],\]

where \(λ = |∥| \perp\), \(f_{K^∗} = f_{K^∗} / \sqrt{3} \) and \(f_{K^∗} = f_{K^∗} / \sqrt{5}\) are reduced decay constants, \(ξ = 2x - 1\), \(Y = \frac{m_u + m_d}{m_u + m_d}\), the error function, \(\text{Erf}(x) = \frac{2}{\sqrt{π}} \int_0^x e^{-t^2} dt\). The model cooperates the transverse momentum dependence with

the longitudinal one under the Brodsky-Huang-Lepage prescription [29] and the Wigner-Melosh rotation [30–32]. Such a cooperation of transverse effect in the light meson wavefunction is helpful for an effective suppression of the
end-point singularity for high-energy processes involving light mesons, cf. a review [33].

The model parameters $A_{2,K^*}^\parallel$, $B_{2,K^*}^\parallel$, $C_{2,K^*}^\parallel$, and $b_{2,K^*}^\parallel$ can be fixed by applying the criteria,

- The normalization condition of the twist-2 LCDA, i.e. $\int \phi_{2,K^*}^\parallel(x)dx = 1$;
- As shown by Ref.[21], the average of the squared transverse momentum $(k_T^2)^{1/2}$ could be determined from the light-cone wavefunction which is related to the LCDA by integrating out its transverse momentum dependence. To fix the parameter, we adopt $(k_T^2)^{1/2} = 0.37(2)\text{GeV}$ [28, 34].
- Generally, the twist-2 LCDA can be expanded as a Gegenbauer polynomial,

$$
\phi_{2,K^*}^\parallel(x) = 6x\bar{x} \left( 1 + \sum_{n=1,2,\cdots} a_n^\lambda C_n^{3/2}(\xi) \right),
$$

(24)

whose Gegenbauer moment $a_n^\lambda$ can be calculated via the following way due to the orthogonality of the Gegenbauer functions, i.e.

$$
a_n^\lambda = \frac{\int_0^1 dx \phi_{2,K^*}^\parallel(x)C_n^{3/2}(\xi)}{\int_0^1 6x\bar{x}C_n^{3/2}(\xi)^2}.
$$

(25)

Generally, the behavior of the twist-2 LCDA is dominated by its first several terms. We adopt the first two Gegenbauer moments derived from the QCD sum rules [20] to fix the parameters, i.e. $a_1^\parallel(1\text{GeV}) = 0.04(3)$ and $a_2^\parallel(1\text{GeV}) = 0.10(8)$ for $\phi_{2,K^*}^\parallel$, and $a_1^\parallel(1\text{GeV}) = 0.03(2)$ and $a_2^\parallel(1\text{GeV}) = 0.11(9)$ for $\phi_{2,K^*}^\parallel$. This way, we get the LCDA at the scale of 1 GeV, and its behavior at any other scale can be achieved via the renormalization group evolution [35].

| $a_1^\parallel$ | $a_2^\parallel$ | $b_{2,K^*}^\parallel$ | $C_{2,K^*}^\parallel$ | $A_{2,K^*}^\parallel$ | $b_{2,K^*}^\parallel$ |
|----------------|----------------|--------------------|---------------------|---------------------|---------------------|
| 0.03           | 0.11           | -0.007             | 0.178               | 26.645              | 0.629               |
| 0.01           | 0.11           | -0.029             | 0.180               | 26.777              | 0.630               |
| 0.05           | 0.11           | -0.014             | 0.176               | 26.519              | 0.628               |
| 0.03           | 0.20           | -0.008             | 0.275               | 24.256              | 0.599               |
| 0.03           | 0.02           | -0.001             | 0.078               | 27.530              | 0.642               |

TABLE II. Parameters of the twist-2 LCDA $\phi_{2,K^*}^\parallel$, determined for $a_1^\parallel(1\text{GeV}) = 0.03(2)$ and $a_2^\parallel(1\text{GeV}) = 0.11(9)$.

The parameters of $\phi_{2,K^*}^\parallel$ for $a_1^\parallel(1\text{GeV}) = 0.04(3)$ and $a_2^\parallel(1\text{GeV}) = 0.10(8)$ have been given in Ref.[21]. We present the parameters of $\phi_{2,K^*}^\parallel$ for $a_1^\parallel(1\text{GeV}) = 0.03(2)$ and $a_2^\parallel(1\text{GeV}) = 0.11(9)$ in Table II, and the corresponding LCDA behavior in Fig.1.

B. Criteria for the LCSRs

The Borel parameter $M^2$ and the continuum threshold $s_0$ are determined by the criteria,

- The continuum contribution, which is the part of the dispersive integral from $s_0$ to $\infty$, should not be too large. We take it to be less than 50% of the total LCSR, $\frac{\int_{s_0}^\infty ds \rho^{\text{cont}}(s)e^{-s/M^2}}{\int_{m_c^2}^\infty ds \rho^{\text{cont}}(s)e^{-s/M^2}} \leq 50\%$.
- All high-twist LCDAs’ contributions are less than 35% of the total LCSR, qualitatively ensuring the usual power counting of twist contributions.
- The derivatives of the TFFs with respect to $1/M^2$ give the LCSRs for $m_B$. We require all predicted $B$-meson masses to be full-filled with high accuracy, e.g. $|m_B^{\text{LCSR}} - m_B^{\exp}| / m_B^{\exp} \leq 0.1\%$.

The determined continuum threshold $s_0$ and the Borel parameter $M^2$ for various $B \rightarrow K^* TFFs$ at the large recoil point $q^2 = 0$ are listed in Table III.

C. Properties of the LCSRs

We present the sum rules for the $B \rightarrow K^*$ TFFs $T_{1,2,3}(q^2)$, $A_{0,1,2}(q^2)$ and $V(q^2)$ for the right-handed chi-
eral correlator (LCSR-\(\mathcal{R}\)) and for the usual correlator (LCSR-\(\mathcal{U}\)) in Figs.\(\text{(2, 3)}\), in which the solid line stands for its central value and the shaded band is the theoretical error. The error is squared average of errors caused by all the mentioned error sources, e.g. we adopt \(\Delta M_{\mathcal{R}/\mathcal{U}}^2 = \pm0.5\text{GeV}^2\) and \(\Delta \psi_{0,\mathcal{R}/\mathcal{U}} = \pm0.5\text{GeV}^2\).

Figs.\(\text{(2, 3)}\) indicate that all the TFFs increase with the increment of \(q^2\). We present the LCSR together with their errors at the large recoil region \(q^2 \rightarrow 0\) in Table \(\text{IV}\). As a comparison, the Ball and Zwicky (BZ) prediction [12], the AdS/QCD prediction [36], and the LCSR prediction [38] are also presented. Those TFFs are consistent with each other within errors.

A smaller Borel parameter indicates a larger \(M^2\)-dependence due to a weaker convergence over \(1/M^2\), and a larger \(M^2\)-uncertainty could be observed. This explains why a larger \(M^2\)-uncertainty than our present one is observed in Ref.[38], whose Borel parameter is taken as \(M^2 = 1.00 \pm 0.25\text{GeV}^2\) by using a rough scaling relation, \(M^2 \sim 2m_q \tau \sim 1\text{GeV} [1, 2, 39]\), which is much smaller than the \(M^2\)-values shown by Table \(\text{III}\).

We present the contributions from the \(K^*\text{-meson LCDAs up to twist-4 in Table V}\). For the LCSR-\(\mathcal{U}\) of the usual correlator, the relative importance among different twist LCDAs follows the trends, twist-2 \(>\) twist-3 \(>\) twist-4; For the LCSR-\(\mathcal{R}\) of the right-handed chiral correlator, we have, twist-2 \(>\) twist-3 \(\sim\) twist-4. The dominance of the twist-2 term indicates a more convergent twist-expansion could be achieved by using the chiral correlator. In Table \(\text{V}\), a somewhat larger twist-4 contribution is observed for \(A_{1,\mathcal{R}/\mathcal{U}}^2\) and \(T_{1,\mathcal{R}/\mathcal{U}}^2\), which comes from the twist-4 LCDA \(\psi_{1,K}^*\), in the reduced function \(H_3 = \int_0^\infty dv \left[\psi_{1,K}^* (v) - \phi_{2,\mathcal{K}}^* (v)\right]\); because of large suppression from the twist-2 LCDA \(\phi_{2,\mathcal{K}}^*\), the net contribution of \(H_3\) is small, which is about 0.5% of the twist-2 ones. Except for \(H_3\), the remaining twist-4 contri-

\(\text{FIG. 2. The } B \rightarrow K^+ \text{ TFFs } T_{1,2,3}(q^2) \text{ for the right-handed chiral correlator (Lower ones) and the usual correlator (Upper ones), respectively. The solid lines are central values and the shaded bands are their errors.}\)

\(\text{FIG. 3. The } B \rightarrow K^* \text{ axial-vector and vector TFFs } A_{0,1,2}(q^2) \text{ and } V(q^2) \text{ for the right-handed chiral correlator (Lower ones) and the usual correlator (Upper ones), respectively. The solid lines are central values and the shaded bands are their errors.}\)

---

\(^2\) The TFFs changes very slightly by taking \(\Delta M_{\mathcal{R}/\mathcal{U}}^2 = \pm0.5\text{GeV}^2\), which is still \(\sim 3\%\) by setting \(\Delta M_{\mathcal{R}/\mathcal{U}}^2 = \pm1.0\text{GeV}^2\). Thus our predictions are consistent with the usual flatness criterion for determining the Borel window [37].
The $B \to K^*$ TFFs at $q^2 = 0$. As a comparison, the Ball and Zwicky (BZ) prediction [12], the AdS/QCD prediction [36], and the LCSR prediction [38] are also presented. $\mu = 2.2\text{ GeV}$.

|                  | $A_1(0)$ | $A_2(0)$ | $V(0)$ | $T_1(0)$/$T_2(0)$ | $T_3(0)$ |
|------------------|----------|----------|--------|------------------|----------|
| LCSR- $\mathcal{R}$ | $0.310^{+0.030}_{-0.037}$ | $0.260^{+0.055}_{-0.054}$ | $0.322^{+0.051}_{-0.051}$ | $0.254^{+0.046}_{-0.049}$ | $0.152^{+0.039}_{-0.043}$ |
| LCSR- $\mathcal{U}$ | $0.306^{+0.032}_{-0.028}$ | $0.257^{+0.028}_{-0.026}$ | $0.307^{+0.024}_{-0.023}$ | $0.254^{+0.028}_{-0.024}$ | $0.145^{+0.020}_{-0.020}$ |
| LCSR [38]        | $0.29^{+0.16}_{-0.10}$     | $0.23^{+0.19}_{-0.10}$     | $0.36^{+0.23}_{-0.12}$    | $0.31^{+0.18}_{-0.10}$    | $0.22^{+0.17}_{-0.10}$    |
| BZ [12]          | $0.292 \pm 0.028$          | $0.259 \pm 0.027$          | $0.411 \pm 0.033$        | $0.333 \pm 0.028$        | $0.202 \pm 0.018$        |
| AdS [36]         | $0.249$                    | $0.235$                    | $0.277$                  | $0.255$                  | $0.155$                  |

The LCSR predictions are still about 10% of the twist-2 ones for $A_{1/2}^{3/2}$, $V^{3/2}$ and $T_1^{3/2}$, thus the twist-4 terms are important and should be kept for a sound prediction.

|                  | $\mu = 1.2\text{ GeV}$ | $\mu = 2.2\text{ GeV}$ | $\mu = 3.2\text{ GeV}$ |
|------------------|-------------------------|-------------------------|-------------------------|
| $A_1^{\mathcal{R}}$ | $0.307$                 | $0.310$                 | $0.310$                 |
| $A_1^{\mathcal{U}}$ | $0.301$                 | $0.308$                 | $0.308$                 |
| $A_2^{\mathcal{R}}$ | $0.266$                 | $0.260$                 | $0.260$                 |
| $A_2^{\mathcal{U}}$ | $0.253$                 | $0.257$                 | $0.257$                 |
| $V^{\mathcal{R}}$  | $0.329$                 | $0.332$                 | $0.332$                 |
| $V^{\mathcal{U}}$  | $0.302$                 | $0.307$                 | $0.307$                 |
| $T_1^{\mathcal{R}}$ | $0.254$                 | $0.254$                 | $0.254$                 |
| $T_1^{\mathcal{U}}$ | $0.247$                 | $0.251$                 | $0.252$                 |
| $T_3^{\mathcal{R}}$ | $0.152$                 | $0.152$                 | $0.153$                 |
| $T_3^{\mathcal{U}}$ | $0.142$                 | $0.145$                 | $0.145$                 |

The LCSR predictions are valid when the $K^*$-meson energy has large energy in the rest-system of the $B$-meson, $E_{K^*} > \Lambda_{QCD}$: using the relation, $q^2 = m_B^2 - 2m_B E_{K^*}$, one usually adopts $0 \leq q^2 \leq 14\text{GeV}^2$. We adopt the simplified series expansion (SSE) to extrapolate the TFFs to all physically allowable $q^2$-region, i.e. the TFFs $F_i(q^2)$ are expanded as $[40]$

$$F_i(q^2) = \frac{1}{1-q^2/m_{R,i}^2} \sum_{k=1,2} a_k^i [z(q^2) - z(0)]^k,$$

where $F_i$ stands for $A_{0,1,2}(q^2)$, $V(q^2)$ and $T_{1,2,3}(q^2)$, respectively. The function

$$z(t) = \frac{\sqrt{t_+} - t - \sqrt{t_+ - t_0}}{\sqrt{t_+} + \sqrt{t_+ - t_0}}$$

with $t_\pm = (m_B \pm m_{K^*})^2$ and $t_0 = t_+ (1 - \sqrt{1 - t_-/t_+})$. The resonance masses $m_{R,i}$ have been given in Ref.[40]. The coefficients $a_0^i = F_i(0)$, $a_1^i$ and $a_2^i$ are determined such that the quality of fit ($\Delta$) is around several percents. The quality of fit is defined as $[22]$

$$\Delta = \sum_i \left| \frac{F_i(t) - F_i^{\mathrm{fit}}(t)}{\sum_i |F_i(t)|} \right| \times 100,$$

where $t \in [0, \frac{1}{2}, \cdots, \frac{27}{14}] \text{GeV}^2$. We put the determined parameters $a_{1,2}^i$ in Table IV, in which all the LCSR parameters are set to be their central values.

We present the extrapolated $B \to K^*$ TFFs in Fig.(4) and Fig.(5), where the AdS/QCD prediction [36] and the Lattice QCD prediction [41] are also given as a comparison. Figs.(4, 5) show the sum rules of LCSR- $\mathcal{R}$ and LCSR- $\mathcal{U}$ are close in shape. We adopt the correlation coefficient $\rho_{XY}$ to show to what degree those LCSR are correlated. The correlation coefficient is defined as $[22]$

$$\rho_{XY} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}.$$
TABLE VII. The fitted parameters $a^r_1, a^r_2, \Delta_R, \Delta_U$ for the $B \to K^*$ TFFs, where all the LCSR parameters are set to be their central values. $\Delta_R$ and $\Delta_U$ are the qualities of fits for the right-handed correlator and the usual correlator, respectively.

|     | $A_1$ | $A_2$ | $V$  | $A_0$ | $T_1$ | $T_2$ | $T_3$ |
|-----|-------|-------|------|-------|-------|-------|-------|
| $a^r_1, R$ | 1.058 | -0.382 | -1.025 | 1.477 | -0.900 | 0.730 | -0.714 |
| $a^r_2, R$ | 0.130 | -5.008 | 0.318 | 14.238 | -3.330 | -0.399 | -3.715 |
| $\Delta_R$ | 0.9 | 1.0 | 0.03 | 1.5 | 0.4 | 2.8 | 2.2 |
| $a^r_1, U$ | 1.059 | -0.275 | -0.531 | -0.019 | -0.136 | 0.719 | -0.294 |
| $a^r_2, U$ | 0.031 | -1.339 | -0.115 | -0.169 | -0.708 | -0.205 | -1.144 |
| $\Delta_U$ | 0.1 | 0.2 | 0.3 | 1.2 | 0.2 | 0.3 | 0.1 |

The magnitudes of the covariance for most of the TFFs are larger than 0.5, implying those TFFs are consistent with each other, or significantly correlated, even though they are calculated by using different correlators. In the LCRSs, the twist-2, the twist-3, and the twist-4 terms behave differently for different correlators. A larger $\rho_X$ shows the net contributions for LCSR-$R$ and LCSR-$U$ from various twists are close to each order, not only for their values at the large recoil point $q^2 = 0$ but also for their ascending trends in whole $q^2$-region.

E. The branching fraction of $B \to K^* \mu^+ \mu^-$

As an application, we adopt the present TFFs to calculate the branching fraction of the semi-leptonic decay $B \to K^* \mu^+ \mu^-$. We adopt the differential branching fraction derived in Ref.[23] as our starting point, where the relations among the coefficients to the TFFs have also been presented.

We present the branching fraction $dB/dq^2$ of the semi-leptonic decay $B \to K^* \mu^+ \mu^-$ in Fig.(6), where the Belle data [43] and the LHCb data [44–46] are presented. The branching fractions for $B^+ \to K^+ \mu^+ \mu^-$ ($B^+$-type) and $B^0 \to K^{*0} \mu^+ \mu^-$ ($B^0$-type) are shown separately. Fig.(6) shows the differential branching fractions from LCSR-$U$ and LCSR-$R$ are close in shape, both of which are consistent with the LHCb data. Numerically, we find the correlation coefficient for the branching fractions for the channels $B^+ \to K^+ \mu^+ \mu^-$ and $B^0 \to K^{*0} \mu^+ \mu^-$ by using the LCSR-$U$ and the LCSR-$R$ are the same, both of which have a significant covariance with $\rho_{XY} = 0.64$. This is due to the fact that the TFFs $A_1$ and $A_2$ dominate the branching fraction, whose correlation coefficients, as shown by Table VIII, are large.
which lead to different light-cone sum rules for the TFFs, the LCSR-\textit{R} and twist-4 terms behave quite differently by using shaded bands are their errors. The AdS/QCD prediction $B_{\text{LCSR}}$ are for the Belle data $[49]$, and the LHCb data $[44–46]$ are presented as a comparison.

\section*{IV. SUMMARY}

In the paper, we have studied the $B \rightarrow K^*$ TFFs under the LCSR approach by applying two correlators, i.e. the usual one with $j^A_I(x) = \imath m_b \bar{b}(x)\gamma_5 q(x)$ and the right-handed chiral one with $j^R_I(x) = \imath m_b \bar{b}(x)(1 + \gamma_5)q(x)$, which lead to different light-cone sum rules for the TFFs, i.e. LCSR-\textit{U} and LCSR-\textit{R}, respectively. The LCSRs for the $B \rightarrow K^*$ TFFs are arranged according to the twist structure of the $K^*$-meson LCDA, whose twist-2, twist-3 and twist-4 terms behave quite differently by using different correlators.

The 2-particle and 3-particle LCDA up to twist-4 accuracy have been kept explicitly in the LCSRs. For the LCSR-\textit{U}, almost all of the LCDA come into the contribution, and the relative importance among different twists follows the usual trends, twist-2 $\gg$ twist-3 $>\$ twist-4. For the LCSR-\textit{R}, only part of the LCDA are emerged in the TFF, the uncertainty from the unknown high-twist LCDA are thus greatly suppressed; Moreover, the relative importance among different twists changes to twist-2 $\gg$ twist-3 $\sim$ twist-4. The dominance of the twist-2 term indicates a more convergent twist-expansion could be achieved by using a chiral correlator. Two exceptions for the power counting rule over twists are caused by the twist-4 LCDA $\psi_{\perp K^*};$ however it contributes to the TFFs via the reduced function $H_{3} = \int_{0}^{u} dv \left( \psi_{\perp K^*}(v) − \phi_{\perp K^*}(v) \right)$, whose net contribution is negligible. Except for $H_{3}$, the remaining twist-4 contributions are about 10\% of the twist-2 ones for the TFFs $A_{1/2}^R/\mu$, $V$ and $T_{1/2}^R/\mu$, thus the twist-4 terms should be kept for a sound prediction.

We have observed that different LCSRs for the $B \rightarrow K^*$ TFFs, i.e. LCSR-\textit{U} and LCSR-\textit{R}, are consistent with each other even though they have been calculated by using different correlators. As shown by Table VIII, large correlation coefficients for most of the TFFs show the net twist-contributions for LCSR-\textit{R} and LCSR-\textit{U} are close to each order, not only for their values at the large recoil point $q^2 = 0$ but also for their ascending trends in the whole $q^2$-region. The high correlation of those LCSRs is further confirmed by their application to the branching fraction of the semi-leptonic decay $B \rightarrow K^*\mu^+\mu^−$, i.e. they are significantly correlated with $\rho_{\chi Y} = 0.64$.

The $K^*$-meson LCDA contribute differently in the LCSRs by using different correlators. The consistency of different LCSRs therefore provide a suitable platform for probing unknown or uncertain LCDA, i.e. the contributions from those LCDA to the TFFs can be amplified to a certain degree via a proper choice of correlator, thus amplifying the sensitivity of the TFFs, and hence their related observables, to those LCDA.

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Appendix A: The relations between the LCDAs and the nonlocal matrix elements

The nonlocal matrix elements in the right-hand-side of the above equation can be reexpressed by the LCDAs of various twists [12, 20], i.e.

\[ \langle K^*(p, \lambda)|\bar{s}(x)q_1(0)|0\rangle = -i f_{K^*} \int_0^1 du e^{ip \cdot x} \psi_{3,K^*}(u), \]  \hspace{1cm} (A1)

\[ \langle K^*(p, \lambda)|\bar{s}(x)\gamma_5 q_1(0)|0\rangle = \frac{1}{4} \epsilon_{\lambda \beta} m_K f_{K^*} \int_0^1 du e^{ip \cdot x} \psi_{3,K^*}(x), \]  \hspace{1cm} (A2)

\[ \langle K^*(p, \lambda)|\bar{s}(x)\gamma_5 q_1(0)|0\rangle = m_K f_{K^*} \int_0^1 du e^{ip \cdot x} \left\{ \frac{\epsilon^{*\lambda}(x)}{p \cdot x} + \frac{e^{*\lambda}(x)}{p \cdot x} \frac{m_K^2}{16} \phi_{4,K^*}(u) \right\}. \]  \hspace{1cm} (A3)

\[ \langle K^*(p, \lambda)|\bar{s}(x)\sigma_{\mu\nu} q_1(0)|0\rangle = -i f_{K^*} \int_0^1 du \epsilon^{(p-x)} \left\{ \frac{\epsilon^{*\lambda}(x)}{p \cdot x} - \frac{e^{*\lambda}(x)}{p \cdot x} \right\} \left\{ \phi_{3,K^*}(u) + \frac{m_K^2}{16} \phi_{4,K^*}(u) \right\} \]  \hspace{1cm} (A4)

\[ \langle 0| \tilde{\psi}(0)|g G_{\mu\nu}(vx)s(x)|K^*(P, \lambda)\rangle = i f_{K^*} m_K^2 \left[ \frac{\epsilon^{*\lambda}(x)}{p \cdot x} - \frac{e^{*\lambda}(x)}{p \cdot x} \right] \Phi_{4,K^*}(v, px) \]  \hspace{1cm} (A5)

\[ \langle 0| \tilde{\psi}(0)|g G_{\mu\nu}(vx)\gamma_5 s(x)|K^*(P, \lambda)\rangle = i f_{K^*} m_K^2 \left[ \frac{\epsilon^{*\lambda}(x)}{p \cdot x} - \frac{e^{*\lambda}(x)}{p \cdot x} \right] \Phi_{4,K^*}(v, px) \]  \hspace{1cm} (A6)

\[ \langle 0| \tilde{\psi}(x)\gamma_5 g \tilde{G}_{\alpha\beta}(vx)s(x)|K^*(P, \lambda)\rangle = p_{\mu} \left( \frac{\epsilon^{*\lambda}(x)}{p \cdot x} - \frac{e^{*\lambda}(x)}{p \cdot x} \right) \right\} \frac{m_K^2}{16} \left[ \phi_{3,K^*}(u) + \frac{m_K^2}{16} \phi_{4,K^*}(u) \right] \]  \hspace{1cm} (A7)

\[ \langle 0| \tilde{\psi}(x)i \gamma_\mu g \tilde{G}_{\alpha\beta}(vx)s(x)|K^*(P, \lambda)\rangle = p_{\mu} \left( \frac{\epsilon^{*\lambda}(x)}{p \cdot x} - \frac{e^{*\lambda}(x)}{p \cdot x} \right) \right\} \frac{m_K^2}{16} \left[ \phi_{3,K^*}(u) + \frac{m_K^2}{16} \phi_{4,K^*}(u) \right] \]  \hspace{1cm} (A8)

\[ \langle 0| \tilde{\psi}(x)\sigma_{\alpha\beta} G_{\mu\nu}(vx)s(x)|K^*(P, \lambda)\rangle = f_{K^*} m_K^2 \frac{\epsilon^{*\lambda}(x)}{2(p \cdot x)} \left[ p_{\alpha} p_{\mu} g_{\beta\nu} - p_{\alpha} p_{\beta} g_{\mu\nu} - p_{\beta} p_{\mu} g_{\nu\alpha} + p_{\beta} p_{\nu} g_{\alpha\mu} \right] \Phi_{4,K^*}(v, px) \]  \hspace{1cm} (A9)
Here \( f^p_{K^*} \) and \( f^\parallel_{K^*} \) are \( K^* \)-meson decay constants, which are defined as \( \langle K^*(P, \lambda) | \bar{s}(0) \gamma_\mu q(0) | 0 \rangle = f^\parallel_{K^*} e_\mu^{(\lambda)} \) and \( \langle K^*(P, \lambda) | \bar{s}(0) \gamma_\mu q(0) | 0 \rangle = i f^p_{K^*} (e_\mu^{(\lambda)} p_\nu - e_\nu^{(\lambda)} p_\mu) \).

Appendix B: LCSRs for the \( B \to K^* \) TFFs by using the right-handed chiral correlator

We list the LCSRs for the \( B \to K^* \) TFFs by using the right-handed chiral correlator in the following equations:

\[
A^R_1(q^2) = \frac{m_b m_B^2 f^p_{K^*}}{f_B m_B^2 (m_B + m_{K^*})} \left\{ \int_0^1 du \frac{e(m^2_B - s(u))/M^2}{u} \left\{ \frac{C}{um_{K^*}} \Theta(c(u, s_0)) \phi_{2;K^*}(u, \mu) + \Theta(c(u, s_0))(u) \right\} \right. \\
\times \left. \psi_{3;K^*}^{\parallel} - \frac{1}{4} \left[ \frac{m_b^2 C}{u^3 M^4} \Theta(c(u, s_0)) + \frac{C - 2 m_b^2}{u^2 M^2} \Theta(c(u, s_0)) - \frac{1}{u} \Theta(c(u, s_0)) \right] \phi_{4;K^*}(u) \right\} + 12 \int_0^1 dv \int_0^1 du \right.
\times \int_0^1 dv e(m^2_B - s(u))/M^2 \frac{f^p_{K^*} m_B m_{K^*}^2}{12 f_B m_B^2 (m_B + m_{K^*})} \left[ \bar{\Theta}(c(u, s_0)) \right] \\
+ \left. 2 \Phi_{4;K^*}^{(1)}(\alpha) + 2 \Phi_{4;K^*}^{(2)}(\alpha) + \Psi_{4;K^*}^{(1)}(\alpha) \right] \right) (m_b^2 - m_B^2 + 2 m_{K^*}^2),
\]

\[
A^R_2(q^2) = \frac{m_b m_B^2 + m_{K^*} m_B f^p_{K^*}}{f_B m_B^2} \left\{ \int_0^1 du \frac{e(m^2_B - s(u))/M^2}{u} \left\{ \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2;K^*}(u, \mu) - \frac{1}{M^2} \right\} \right. \\
\times \left. \Theta(c(u, s_0)) \psi_{3;K^*}^{\parallel} (u) - \frac{1}{4} \left[ \frac{m_b^2 C}{u^3 M^4} \Theta(c(u, s_0)) + \frac{C - 2 m_b^2}{u^2 M^2} \Theta(c(u, s_0)) \right] \phi_{4;K^*}^{(1)}(u) \right] + 12 \int_0^1 dv \int_0^1 du \int_0^1 dv \int_0^1 du \right.
\times \int_0^1 dv e(m^2_B - s(u))/M^2 \frac{f^p_{K^*} m_B m_{K^*}^2}{12 f_B m_B^2 (m_B + m_{K^*})} \left[ \bar{\Theta}(c(u, s_0)) \right] \\
+ \left. 2 \Phi_{4;K^*}^{(1)}(\alpha) + 2 \Phi_{4;K^*}^{(2)}(\alpha) - \Psi_{4;K^*}^{(1)}(\alpha) \right].
\]
\[
A_3^R(q^2) - A_3^L(q^2) = \frac{me_{\pi K^*}}{2BM_{B}^*} \left\{ \int_0^1 \frac{du}{u} e^{(m_{\pi}^2 - s(u))/M^2} \left\{ -\frac{1}{(2u)^4} \Theta(c(u, s_0)) \phi^{+}_{4;K^*}(u) - \frac{2}{uM^2} \right. \left. \times \Theta(c(u, s_0)) \right\} \right. \\
- \left( \Theta(c(u, s_0)) - \frac{2}{(2u)^2} \Theta(c(u, s_0)) \right) + \left( \frac{4m_b^2}{u^2M^2} \Theta(c(u, s_0)) + \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) \right) \right\} I_{L}(u) \\
+ 12 \left( \Phi_{4;K^*}^{\perp(1)}(\alpha) + 2 \Psi_{4;K^*}^{\perp(2)}(\alpha) - \Psi_{4;K^*}^{\perp(1)}(\alpha) \right),
\]

\[
T_1^R(q^2) = \frac{m_b^2m_{\pi K^*}}{m_b^2f_B^*} \int_0^1 \frac{du}{u} e^{(m_{\pi}^2 - s(u))/M^2} \left\{ \frac{1}{M^2} \Theta(c(u, s_0)) \phi^{+}_{4;K^*}(u) - \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) \phi^{+}_{4;K^*}(u) \right. \\
- \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) I_{L}(u) \left. - \frac{1}{M^2} \tilde{\Theta}(c(u, s_0)) \right\} H_{3}(u) \right\} \right. \\
+ 12 \left( \Phi_{4;K^*}^{\perp(1)}(\alpha) - 2 \Phi_{4;K^*}^{\perp(2)}(\alpha) + \Psi_{4;K^*}^{\perp(1)}(\alpha) \right),
\]

\[
T_2^R(q^2) = \frac{m_b^2m_{\pi K^*}}{m_b^2f_B^*} \int_0^1 \frac{du}{u} e^{(m_{\pi}^2 - s(u))/M^2} \left\{ \frac{1}{M^2} \Theta(c(u, s_0)) \phi^{+}_{4;K^*}(u) - \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) \phi^{+}_{4;K^*}(u) \right. \\
- \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) I_{L}(u) \right\} \right. \\
+ \left. \left( \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) + \frac{4}{u^2M^2} \tilde{\Theta}(c(u, s_0))(m_b^2 - m_{\pi K^*}^2) \right) \right\} I_{L}(u) \right. \\
+ \left. \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) H_{3}(u) \right\} \right. \\
+ 12 \left( \Phi_{4;K^*}^{\perp(1)}(\alpha) - 2 \Phi_{4;K^*}^{\perp(2)}(\alpha) \right)
\]

\[
T_3^R(q^2) = \frac{m_b^2m_{\pi K^*}}{m_b^2f_B^*} \int_0^1 \frac{du}{u} e^{(m_{\pi}^2 - s(u))/M^2} \left\{ \frac{1}{M^2} \Theta(c(u, s_0)) \phi^{+}_{4;K^*}(u) - \frac{m_b^2}{4u^2M^2} \tilde{\Theta}(c(u, s_0)) \phi^{+}_{4;K^*}(u) \right. \\
- \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) + \frac{4}{u^2M^2} \tilde{\Theta}(c(u, s_0))(m_b^2 - m_{\pi K^*}^2) \right\} I_{L}(u) \right. \\
+ \left. \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) H_{3}(u) \right\} \right. \\
+ 12 \left( \Phi_{4;K^*}^{\perp(1)}(\alpha) - 2 \Phi_{4;K^*}^{\perp(2)}(\alpha) \right)
\]

\[
V^R(q^2) = \frac{m_b(m_{b} + m_{K^*})f_{K^*}^2}{f_{B}^2 f_{B}^*} \int_0^1 \frac{du}{u} e^{(m_{\pi}^2 - s(u))/M^2} \left\{ \Theta(c(u, s_0)) \phi^{+}_{4;K^*}(u, \mu) - \frac{m_b^2}{u^2M^2} \tilde{\Theta}(c(u, s_0)) \right. \\
+ \frac{1}{uM^2} \tilde{\Theta}(c(u, s_0)) \right\} \right. \\
+ \left. \left( \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) + \frac{4}{u^2M^2} \tilde{\Theta}(c(u, s_0))(m_b^2 - m_{\pi K^*}^2) \right) \right\} I_{L}(u) \right. \\
+ \left. \frac{2}{uM^2} \tilde{\Theta}(c(u, s_0)) H_{3}(u) \right\} \right. \\
+ 12 \left( \Phi_{4;K^*}^{\perp(1)}(\alpha) - 2(v - 1)\Phi_{4;K^*}^{\perp(2)}(\alpha) \right)
\]

To compare with previous LCSRs given by Ref. [21], in the above formulas, we keep all the three-particle twist-4 terms in the LCSRs.

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