A Novel String Derived $Z'$ With Stable Proton, Light–Neutrinos and R–parity violation

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Abstract

The Standard Model indicates the realization of grand unified structures in nature, and can only be viewed as an effective theory below a higher energy cutoff. While the renormalizable Standard Model forbids proton decay mediating operators due to accidental global symmetries, many extensions of the Standard Model introduce such dimension four, five and six operators. Furthermore, quantum gravity effects are expected to induce proton instability, indicating that the higher energy cutoff scale must be above $10^{16}$ GeV. Quasi–realistic heterotic string models provide the arena to explore how perturbative quantum gravity affects the particle physics phenomenology. An appealing explanation for the proton longevity is provided by the existence of an Abelian gauge symmetry that suppresses the proton decay mediating operators. Additionally, such a low–scale $U(1)$ symmetry should: allow the suppression of the left–handed neutrino masses by a seesaw mechanism; allow fermion Yukawa couplings to the electroweak Higgs doublets; be anomaly free; be family universal. These requirements render the existence of such $U(1)$ symmetries in quasi–realistic heterotic string models highly non–trivial. We demonstrate the existence of a $U(1)$ symmetry that satisfies all of the above requirements in a class of left–right symmetric heterotic string models in the free fermionic formulation. The existence of the extra $Z'$ in the energy range accessible to future experiments is motivated by the requirement of adequate suppression of proton decay mediation. We further show that while the extra $U(1)$ forbids dimension four baryon number violating operators it allows dimension four lepton number violating operators and $R$–parity violation.

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1 Introduction

The Standard Model of particle physics successfully accounts for all observations in the energy range accessible to contemporary experiments. Despite this enormous success the Standard Model can only be viewed as an effective low energy field theory below a higher energy cutoff. In the least, the existence of a Landau pole in the hypercharge sector, albeit at an enormously high scale, unequivocally demonstrates the formal inconsistency of the Standard Model. In this regard, the renormalizability of the Standard Model is an approximate feature and effects of nonrenormalizable operators, suppressed by powers of the high scale cutoff, must be considered.

The high precision analysis of the Standard Model parameters, achieved at LEP and other particle physics experiments, indicates that the Standard Model remains an approximate renormalizable quantum field theory up to a very large energy scale. Possibly the grand unification scale, or the Planck scale. The logarithmic evolution of the Standard Model parameters is in agreement with the available data, and is compatible with the notion of unification at a high energy scale in the gauge and heavy matter sectors of the Standard Model. Preservation of the logarithmic evolution in the scalar sector necessitates the introduction of a new symmetry between bosons and fermions, dubbed supersymmetry.

Perhaps the most important observation indicative that the Standard Model cutoff scale is a very high scale is the longevity of the proton. Renormalizability insures that baryon and lepton violating operators are absent in the perturbative Standard Model. Hence, in the renormalizable Standard Model baryon and lepton numbers are accidental global symmetries. However, at the cutoff scale dimension six operators are induced and the proton is in general expected to decay. The observed proton lifetime implies that the cutoff scale is of order $10^{16}$GeV. The problem is exacerbated in supersymmetric extensions of the Standard Model that allow dimension four and five baryon and lepton violating operators [1]. Indeed, one would expect proton decay mediating operators to arise in most extensions of the Standard Model. In the Minimal Supersymmetric Standard Model one imposes a global symmetry, $\text{R}$–parity, which forbids the dimension four baryon and lepton number violating operators. The difficulty with dimension five operators can only be circumvented if one further assumes that the relevant Yukawa couplings are suppressed. However, as global symmetries are not expected to survive quantum gravity effects [2], the proton lifetime problem becomes especially acute in the context of theories that unify the Standard Model with gravity. This question has been examined extensively in the context of quasi–realistic heterotic string models. In this context, the most appealing suggestion is that the suppression of the proton decay mediating operators is a result of a gauged $U(1)$ symmetry, under which the undesired nonrenormalizable dimension four and five operators are not invariant. If the $U(1)$ symmetry remains unbroken down to sufficiently low scales the problematic operators will be suppressed by at least the VEV that breaks the additional $U(1)$ symmetry over the cutoff scale.
The free fermionic heterotic string models are among the most realistic string models constructed to date [3, 4, 5, 6, 7, 8]. The issue of proton stability was sporadically explored in these models [9, 10, 11, 12, 13], as well as explorations of possible $U(1)$ symmetries that can ensure proton longevity [9, 11, 12, 13]. However, non of the current proposals is satisfactory. The $U(1)$ symmetry of ref. [9] is the $U(1)$ combination of $B-L$ and $T_{3r}$ which is embedded in $SO(10)$ and is orthogonal to the electroweak hypercharge. However, this $U(1)$ symmetry in general needs to be broken to allow for the suppression of the left-handed neutrino masses by a seesaw mechanism. Similarly, the $U(1)$ symmetries studied in ref. [11, 12, 13], that arise in the string models from combinations of the $U(1)$ symmetries that are external to $SO(10)$ are flavour dependent $U(1)$ symmetries that in general must be broken near the string scale to allow for generation of fermion masses. In ref. [12] it was concluded that non of the symmetries suggested in ref. [11] can remain unbroken down to low energies and provide for the suppression of the proton decay mediating operators. Furthermore, a family non–universal $U(1)$ symmetry is restricted by constraints on flavour changing neutral currents, and cannot exist in energy range accessible to forthcoming experiments.

The proton longevity, together with the Standard Model multiplet structure, therefore provide the most important clues for the origin of the Standard Model particle spectrum. These favour the embedding of Standard Model in a Grand Unified Theory, possibly broken to the Standard Model at the string level. The GUT embedding of the Standard Model, and its supersymmetric extension, leads to proton decay mediating operators. The most robust and economical way to suppress the dangerous operators is by the existence of an additional Abelian gauge symmetry which is broken above the electroweak scale and does not interfere with the other phenomenological constraints. Such a $U(1)$ symmetry should fulfill the following requirements:

- Forbid dimension four, five and six proton decay mediating operators.
- Allow suppression of left–handed neutrino masses by a seesaw mechanism.
- Allow the fermion Yukawa couplings to electroweak Higgs doublets.
- Be family universal.
- Be anomaly free.

This list of requirements render the existence of such a $U(1)$ symmetry in string models highly nontrivial. For example, in models with an underlying $SO(10)$ GUT embedding the $U(1)_{B-L}$ symmetry is gauged. It forbids the dimension four baryon and lepton number violating operators, but not the dimension five operator. Furthermore, suppression of left–handed neutrino masses by a seesaw mechanism in general necessitates that the symmetry is broken near the GUT scale. Hence, it cannot
remain unbroken down to low energies, and in general fast proton decay from dimension four operators is expected to ensue. Similarly, the $U(1)_A$ symmetry external to $SO(10)$ in $E_6 \to SO(10) \times U(1)_A$ is anomalous in many of the quasi–realistic string models constructed to date [14] and is broken by a generalised Green–Schwarz mechanism. The additional $U(1)$’s investigated in refs. [11, 12, 13] are either flavour non–universal or constrain the fermion Yukawa mass terms and must therefore be similarly broken near the Planck scale. Thus, of all the extra $U(1)$’s investigated to date non seems to remain viable down to low energies, and to provide the coveted proton protection symmetry.

In this paper we therefore explore further the possibility that quasi–realistic string models give rise to Abelian gauged symmetries that can play the role of the proton lifetime guard. We demonstrate the existence of a $U(1)$ symmetry satisfying all of the above requirements in the class of left–right symmetric string–derived models of ref. [7]. The key to obtaining the $U(1)$ symmetry satisfying the above requirements is the $SO(10)$ symmetry breaking pattern particular to the left–right symmetric models [7]. The key distinction is that in these models the $U(1)_A$, which is external to the unbroken $SO(10)$ subgroup, is anomaly free, and may remain unbroken down to low energies. It is does not restrict the charged fermion mass terms, and it allows for the suppression of the left–handed neutrino masses by a seesaw mechanism. Its existence at low energies is motivated by the longevity of the proton lifetime. Furthermore, as we discuss below, while it forbids the supersymmetric dimension four and five baryon number violating operators, it allows the dimension four lepton number violating operator. Hence, while proton decay from dimension four operators does not ensue, lepton number and $R$–parity violation do arise. This observation has far reaching implications in terms of the phenomenology and collider signatures of the models.

2 The structure of the free fermionic models

In this section we describe the structure of the quasi–realistic free fermionic models and the properties of the proton protecting $U(1)$ symmetry. The free fermionic formulation the 4-dimensional heterotic string, in the light-cone gauge, is described by 20 left–moving and 44 right–moving two dimensional real fermions [16]. The models are constructed by specifying the phases picked up by the world–sheet fermions when transported around the torus non-contractible loops. Each model corresponds to a particular choice of fermion phases consistent with modular invariance that can be generated by a set of basis vectors $v_i, i = 1, \ldots, n, v_i = \{\alpha_i(f_1), \alpha_i(f_2), \alpha_i(f_3)\} \ldots$. The basis vectors span a space $\Sigma$ which consists of $2^N$ sectors that give rise to the string spectrum. The spectrum is truncated by a Generalised GSO (GGSO) projections [16].

The $U(1)$ charges, $Q(f)$, with respect to the unbroken Cartan generators of the four dimensional gauge group, which are in one to one correspondence with the $U(1)$
currents $f^*f$ for each complex fermion $f$, are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \quad (1)$$

where $\alpha(f)$ is the boundary condition of the world-sheet fermion $f$ in the sector $\alpha$. $F(f)$ is the fermion number operator counting each mode of $f$ once (and if $f$ is complex, $f^*$ minus once). For periodic fermions, $\alpha(f) = 1$, the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion $f$ there are two degenerate vacua $|+\rangle, |−\rangle$, annihilated by the zero modes $f_0$ and $f_0^*$ and with fermion numbers $F(f) = 0, −1$, respectively.

The two dimensional world-sheet fermions are divided in the following way: the eight left-moving real fermions $\psi^{1,2}$ and $\chi^{1,\ldots,6}$ correspond to the eight Ramond–Neveu–Schwarz fermions of the ten dimensional heterotic string in the light–cone gauge; the twenty–four real–fermions $\{g^i, \omega^i | \bar{g}^i, \bar{\omega}^i\}, i = 1, \ldots, 6$ correspond to the fermionized internal coordinates of a compactified manifold in a bosonic formulation; the complex right–moving fermions $\bar{\eta}^{1,\ldots,8}$ generate the rank eight hidden gauge group; $\bar{\psi}^{1,\ldots,5}$ generate the $SO(10)$ gauge group; $\eta^{1,2,3}$ generate the three remaining $U(1)$ generators in the Cartan sub-algebra of the observable rank eight gauge group. A combination of these $U(1)$ currents will play the role of the proton lifetime guard.

The free fermionic models are defined in terms of the basis vectors and one–loop GGS0 projection coefficients. The quasi–realistic free fermionic heterotic–string model are typically constructed in two stages. The first stage consists of the NAHE–set, $\{1, S, b_1, b_2, b_3\}$ [17, 18]. The gauge group at this stage is $SO(10) \times SO(6)^3 \times E_8$, and the vacuum contains forty–eight multiplets in the 16 chiral representation of $SO(10)$. The second stage consists of adding three or four basis vectors to the NAHE–set, typically denoted by $\{\alpha, \beta, \gamma\}$. The additional basis vectors reduce the number of generations to three, with one arising from each of the basis vectors $b_1$, $b_2$, and $b_3$. Additional non–chiral generations may arise from the basis vectors that extend the NAHE–set. This distribution of the chiral generations is particular to the class of quasi–realistic free fermionic models that has been explored to date, and other possibilities may exist [15]. Additionally, the basis vectors that extend the NAHE–set break the four dimensional gauge group. The $SO(10)$ symmetry is broken to one of the subgroups: $SU(5) \times U(1)$ [3]; $SO(6) \times SO(4)$ [5]; $SU(3) \times SU(2) \times U(1)^2$ [6]; $SU(3) \times SU(2)^2 \times U(1)$ [7]; or $SU(4) \times SU(2) \times U(1)$ [8]. The three generations from the sectors $b_1$, $b_2$ and $b_3$ are decomposed under the final $SO(10)$ subgroup. The flavour $SO(6)^3$ groups are broken to products of $U(1)^n$ with $3 \leq n \leq 9$. The $U(1)^{1,2,3}$ factors arise from the three right–moving complex fermions $\bar{\eta}^{1,2,3}$. Additional $U(1)$ currents may arise from complexifications of right–moving fermions from the set $\{\bar{g}, \bar{\omega}\}^{1,\ldots,6}$.

The $U(1)$ symmetry that will serve as the proton lifetime guard is a combination of the three $U(1)$ symmetries generated by the world–sheet complex fermions $\bar{\eta}^{1,2,3}$. The states from each of the sectors $b_1$, $b_2$ and $b_3$ are charged with respect to one of these $U(1)$ symmetries, i.e. with respect to $U(1)_1$, $U(1)_2$ and $U(1)_3$, respectively.
Hence the $U(1)$ combination

$$U(1)_\zeta = U_1 + U_2 + U_3$$

(is family universal. In the string derived models of ref. [3, 4, 5, 6] $U(1)_{1,2,3}$ are anomalous. Therefore, also $U(1)_\zeta$ is anomalous and must be broken near the string scale. In the string derived left–right symmetric models of ref [7] $U(1)_{1,2,3}$ are anomaly free, and hence also the combination $U(1)_\zeta$ is anomaly free. It is this property of these models which allows this $U(1)$ combination to remain unbroken.

Subsequent to constructing the basis vectors and extracting the massless spectrum the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V^I_1 V^I_2 V^b_3 \cdots V^b_N \rangle,$$

where $V^I_i$ ($V^b_i$) are the fermionic (scalar) components of the vertex operators, using the rules given in [19]. Generically, correlators of the form (3) are of order $O(g^{N-2})$, and hence of progressively higher orders in the weak-coupling limit. Typically, one of the $U(1)$ factors in the free-fermion models is anomalous, and generates a Fayet–Iliopoulos term which breaks supersymmetry at the Planck scale [20]. The anomalous $U(1)$ is broken, and supersymmetry is restored, by a non–trivial VEV for some scalar field that is charged under the anomalous $U(1)$. Since this field is in general also charged with respect to the other anomaly-free $U(1)$ factors, some non-trivial set of other fields must also get non–vanishing VEVs $\mathcal{V}$, in order to ensure that the vacuum is supersymmetric. Some of these fields will appear in the nonrenormalizable terms (3), leading to effective operators of lower dimension. Their coefficients contain factors of order $\mathcal{V}/M \sim 1/10$. Typically the solution of the D– and F–flatness constraints break most or all of the horizontal $U(1)$ symmetries.

## 3 The proton lifeguard

In this section we discuss the characteristics of $U(1)_\zeta$ in the left–right symmetric string derived models [7], versus those of $U(1)_A$ in the string derived models of refs. [3, 4, 5, 6]. We note that both $U(1)_\zeta$ as well as $U(1)_A$ are obtained from the same combination of complex right–moving world–sheet currents $\bar{\eta}_{1,2,3}$, i.e. both are given by a combination of $U_1$, $U_2$, and $U_3$. The distinction between the two cases, as we describe in detail below, is due to the charges of the Standard Model states, arising from the sectors $b_1$, $b_2$ and $b_3$, under this combination. The key feature of $U(1)_\zeta$ in the models of ref. [7] is that it is anomaly free. To study the characteristics of the proton protecting $U(1)$ symmetry it is instructive to examine in combinatorial notation the vacuum structure of the chiral generations from the sectors $b_1, b_2, b_3$. The vacuum of the sectors $b_j$ contains twelve periodic fermions. Each periodic fermion gives rise to a two dimensional degenerate vacuum $|+\rangle$ and $|-\rangle$ with fermion numbers
0 and −1, respectively. The GSO operator, is a generalised parity operator, which selects states with definite parity. After applying the GSO projections, we can write the degenerate vacuum of the sector $b_1$ in combinatorial form

$$
\left[ \left( \begin{array}{c} 4 \\ 0 \end{array} \right) + \left( \begin{array}{c} 4 \\ 2 \end{array} \right) + \left( \begin{array}{c} 4 \\ 4 \end{array} \right) \right] \left\{ \left( \begin{array}{c} 2 \\ 0 \end{array} \right) + \left( \begin{array}{c} 5 \\ 2 \end{array} \right) + \left( \begin{array}{c} 5 \\ 4 \end{array} \right) \right\} \left( \begin{array}{c} 0 \\ 1 \end{array} \right)
+ \left( \begin{array}{c} 2 \\ 2 \end{array} \right) \left[ \left( \begin{array}{c} 5 \\ 1 \end{array} \right) + \left( \begin{array}{c} 5 \\ 3 \end{array} \right) + \left( \begin{array}{c} 5 \\ 5 \end{array} \right) \right] \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \right\}
$$

(4)

where $4 = \{ y^3y^4, y^5y^6, \bar{y}^3\bar{y}^4, \bar{y}^5\bar{y}^6 \}$, $2 = \{ \psi^\mu, \chi^{12} \}$, $5 = \{ \bar{\psi}^{1,\ldots,5} \}$ and $1 = \{ \bar{\eta}^1 \}$. The combinatorial factor counts the number of $|\rangle$ in the degenerate vacuum of a given state. The first term in square brackets counts the degeneracy of the multiplets, being eight in this case. The two terms in the curly brackets correspond to the two CPT conjugated components of a Weyl spinor. The first term among those corresponds to the 16 spinorial representation of $SO(10)$, and fixes the space–time chirality properties of the representation, whereas the second corresponds to the CPT conjugated anti–spinorial $\bar{16}$ representation. Similar vacuum structure is obtained for $b_2$ and $b_3$. The periodic boundary conditions of the world–sheet fermions $\bar{\eta}^j$ entails that the fermions from each sector $b_j$ are charged with respect to one of the $U(1)_j$ symmetries. The charges, however, depend on the $SO(10)$ symmetry breaking pattern, induced by the basis vectors that extend the NAHE–set, and may, or may not, differ in sign between different components of a given generation. In the models of ref. [3, 6, 5] the charges of a given $b_j$ generation under $U(1)_j$ is of the same sign, whereas in the models of ref. [7] they differ. In general, the distinction is by the breaking of $SO(10)$ to either $SU(5) \times U(1)$ or $SO(6) \times SO(4)$. In the former case they will always have the same sign, whereas in the later they may differ. This distinction fixes the charges of the Standard Model states under the $U(1)$ symmetry which safeguards the proton from decaying, while not obstructing the remaining constraints listed above.

In the free fermionic standard–like models the $SO(10)$ symmetry is broken to$^4$

$$SU(3) \times SU(2) \times U(1)_C \times U(1)_L.$$

The weak hypercharge is given by

$$U(1)_Y = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L,$$

(5)

and the orthogonal $U(1)_{Z'}$ combination is given by

$$U(1)_{Z'} = U(1)_C - U(1)_L.
$$

(6)

The three twisted sectors $b_1$, $b_2$ and $b_3$ produce three generations in the sixteen representation of $SO(10)$ decomposed under the final $SO(10)$ subgroup. In terms of

$^4U(1)_C = 3/2U(1)_{B-L} ; U(1)_L = 2U(1)_{T_{3R}}$
the $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$ decomposition they take the values
\[
E \equiv [(1, \ 3/2); (1, \ 1)]; \\
U \equiv [(3, -1/2); (1, -1)]; \\
Q \equiv [(3, \ 1/2); (2, \ 0)]; \\
N \equiv [(1, \ 3/2); (1, -1)]; \\
D \equiv [(3, -1/2); (1, \ 1)]; \\
L \equiv [(1, -3/2); (2, \ 0)].
\] (7)

In terms of the $SO(6) \times SO(4)$ Pati–Salam decomposition [21] the Standard Model fermion fields are embedded in the
\[
F_L \equiv (4, 2, 1) = Q + L; \\
F_R \equiv (4, 1, 2) = U + D + E + N,
\] (8)
representations of $SU(4) \times SU(2)_L \times SU(2)_R$. In terms of the left–right symmetric decomposition of ref. [7] the embedding is in the following representations:
\[
Q_L = (3, 2, 1, 1/2), \\
Q_R = (3, 1, 2, -1/2) = U + D, \\
L_L = (1, 2, 1, -3/2), \\
L_R = (1, 1, 2, 3/2) = E + N,
\] (9–12)
of $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_C$. The Higgs fields in the later case are in a bi–doublet representation
\[
h = (1, 2, 2, 0) = \begin{pmatrix} h_u^+ & h_d^- \\ h_u^0 & h_d^0 \end{pmatrix}.
\] (13)

Using the combinatorial notation introduced in eq. (4) the decomposition of the 16 representation of $SO(10)$ in the Pati–Salam string models is
\[
\{ [\binom{3}{0} + \binom{3}{2}] \left[ \binom{2}{0} + \binom{2}{2} \right] \} + \{ [\binom{3}{1}] [\binom{3}{2}] \} 
\] (14)
The crucial point is that the Pati–Salam breaking pattern allows the first and second terms in curly brackets to come with opposite charges under $U(1)_j$. This results from the operation of the GSO projection operator, which differentiates between the two terms. Thus, in models that descend from $SO(10)$ via the $SU(5) \times U(1)$ breaking pattern the charges of a generation from a sector $b_j \ j = 1, 2, 3$, under the corresponding symmetry $U(1)_j$ are either $+1/2$, or $-1/2$, for all the states from
that sector. In contrast, in the left–right symmetric string models the corresponding charges, up to a sign are,

\[ Q_j(Q_L; L_L) = +1/2; Q_j(Q_R; L_R) = -1/2, \]  

(i.e. the charges of the \( SU(2)_L \) doublets have the opposite sign from those of the \( SU(2)_R \) doublets. This is in fact the reason that in the left–right symmetric string models [7] it was found that, in contrast to the case of the FSU5 [3], Pati–Salam [5] and standard–like [6], string models, the \( U(1)_j \) symmetries are not part of the anomalous \( U(1) \) symmetry [7].

It is therefore noted that the

\[ U(1)_\zeta = U_1 + U_2 + U_3 \]  

combination is a family–universal, anomaly free, \( U(1) \) symmetry, and allows the quark and lepton fermion mass terms

\[ Q_L Q_R h \quad \text{and} \quad L_L L_R h. \]

The two combinations of \( U(1)_1, U(1)_2 \) and \( U(1)_3 \), that are orthogonal to \( U(1)_\zeta \), are family non–universal and may be broken at, or slightly below, the string scale.

The left–right symmetric heterotic string models of ref. [7] provide explicit quasi–realistic string models, that realize the charge assignment of eq. (15). Furthermore, the dimension four and five baryon number violating operators that arise from

\[ Q_L Q_L Q_L Q_L \rightarrow QQQL \]  
\[ Q_R Q_R Q_R Q_R \rightarrow \{UDDN, UUDE\} \]  

are forbidden, while the lepton number violating operators that arise from

\[ Q_L Q_R L_L L_R \rightarrow QDLN \]  
\[ L_L L_L L_R L_R \rightarrow LLEN \]

are allowed.

The crucial observation is the opposite charge assignment of the left and right–handed fields under \( U(1)_\zeta \). This is available in models that descend from the Pati–Salam symmetry breaking pattern of the underlying \( SO(10) \) GUT symmetry. In this case the left– and right–moving fields carry opposite sign under the GSO projection operator, induced by the basis vector that breaks \( SO(10) \rightarrow SO(6) \times SO(4) \). An additional symmetry breaking stage of the Pati–Salam models [5], or left–right symmetric models [7], can be obtained at the string level or in the effective low–energy

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5We note that there may exist string models in the classes of [3, 5, 6] in which \( U(1)_\zeta \) is anomaly free. This may be the case in the so called self–dual vacua of ref. [15]. Such quasi–realistic string models with an anomaly free \( U(1)_\zeta \) have not been constructed to date.
field theory by the Higgs fields in the representations \( \{ Q_H, \bar{Q}_H \} = \{(4, 1, 2), (4, 1, 2)\} \) or \( \{ L_H, \bar{L}_H \} = \{(\bar{1}, 1, 2, \frac{3}{2}), (1, 1, 2, -\frac{3}{2})\} \). The breaking can be achieved at the string level, while preserving the desired charge assignment, as long as a basis vector of the form \( 2\gamma \) of refs. [6], or \( b_6 \) of ref. [5], are not introduced. The boundary condition assignments in these basis vectors entails that the \( N = 4 \) vacuum that we start with factorizes the gauge degrees of freedom into \( E_8 \times E_8 \) or \( SO(16) \times SO(16) \). The consequence of this is that all the states from the twisted matter sectors \( b_j \) carry the same charge under \( U(1)_j \). Thus, this result is circumvented by not including the vectors \( 2\gamma \) of [6], or \( b_6 \) of [5] in the construction. In effect, such models are descending from a different \( N = 4 \) underlying vacuum [7, 8]. Being \( SO(16) \times E_7 \times E_7 \) in the models of ref. [7], which explicitly realize the desired breaking pattern in a class of quasi–realistic string models. We assume below that the \( SU(2)_R \) symmetry is broken directly at the string level in which case the remnant \( U(1)_{Z'} \) given in eq. (6) has to be broken by the Higgs fields \( \{ N_H, \bar{N}_H \} = (1, 1, 0, 5/2), (1, 1, 0, -5/2) \) under \( SU(3) \times SU(2) \times U(1)_Y \times U(1)_{Z'} \).

4 An effective string inspired \( Z' \) model

Inspired by the \( U(1) \) charge assignment in the left–right symmetric string derived models [7], we present an effective field theory model incorporating these features. At this stage our aim is to build an effective model that can be used in correspondence with experimental data, rather than a complete effective field theory model below the string scale, which is of further interest and will be discussed in future publications. The charges of the fields in the low energy effective field theory of the string inspired model are given by

| Field | \( U(1)_Y \) | \( U(1)_{Z'} \) | \( U(1)_\zeta \) | \( U(1)_{\zeta'} \) |
|-------|---------------|---------------|----------------|----------------|
| \( Q^i \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) |
| \( L^i \) | \( -\frac{1}{2} \) | \( \frac{3}{2} \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) |
| \( U^i \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) |
| \( D^i \) | \( \frac{1}{2} \) | \( -\frac{1}{2} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} \) |
| \( E^i \) | \( 1 \) | \( -\frac{1}{2} \) | \( 1 \) | \( -\frac{1}{2} \) |
| \( N^i \) | \( 0 \) | \( \frac{2}{2} \) | \( 0 \) | \( \frac{2}{2} \) |
| \( \phi^i \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( \phi^0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |
| \( H^U \) | \( \frac{1}{2} \) | \( -1 \) | \( 0 \) | \( -\frac{1}{2} \) |
| \( H^D \) | \( -\frac{1}{2} \) | \( 1 \) | \( 0 \) | \( \frac{1}{2} \) |
| \( N_H \) | \( 0 \) | \( \frac{1}{2} \) | \( 0 \) | \( \frac{1}{2} \) |
| \( \bar{N}_H \) | \( 0 \) | \( -\frac{1}{2} \) | \( 0 \) | \( -\frac{1}{2} \) |
| \( \zeta_H \) | \( 0 \) | \( 0 \) | \( 1 \) | \( 1 \) |
| \( \bar{\zeta}_H \) | \( 0 \) | \( 0 \) | \( -1 \) | \( -1 \) |

(22)
with \( i = 1, 2, 3 \). The \( U(1)_{\zeta'} \) symmetry is the combination of \( U(1)_{Z'} \) and \( U(1)_\zeta \) left unbroken by the vevs of \( N_H \) and \( \bar{N}_H \). The fields \( \zeta_H \) and \( \bar{\zeta}_H \) are needed to break the residual \( U(1)_{\zeta'} \) symmetry. States with the required quantum numbers in (22) exist in the string models [7]. The fields \( \phi^i \) are employed in an extended seesaw mechanism.

Using the superpotential terms

\[
L_i N_j H U^, N_i \bar{N}_H \phi_j, \phi_i \phi_j \phi_k.
\]

The neutrino seesaw mass matrix takes the form

\[
\begin{pmatrix}
\nu_i & N_k & \phi_m \\
(kM_U)_{ij} & 0 & M_\chi \\
0 & M_\chi & O(M_\phi)
\end{pmatrix}
\begin{pmatrix}
\nu_j \\
N_l \\
\phi_n
\end{pmatrix},
\]

with \( M_\chi \sim \langle \bar{N}_H \rangle \) and \( M_\phi \sim \langle \phi_0 \rangle \). The mass eigenstates are mainly \( \nu_i, N_k \) and \( \phi_m \) with a small mixing and with the eigenvalues

\[
m_{\nu_j} \sim M_\phi \left( \frac{kM_u}{M_\chi} \right)^2, m_{N_j}, m_\phi \sim M_\chi.
\]

A detailed fit to the neutrino data was discussed in ref [22]. We emphasize, however, that our aim here is merely to demonstrate that the extra \( U(1)_{\zeta'} \), introduced below, is not in conflict with the requirement of light neutrino masses. Alternatively, the VEV of \( \langle \bar{N}_H \rangle \) induces heavy Majorana mass terms for the right–handed neutrinos from nonrenormalizable terms

\[
N_i N_j \bar{N}_H \bar{N}_H.
\]

The effective Majorana mass scale of the right–handed neutrinos is then \( M_\chi \sim \langle \bar{N}_H \rangle^2 / M \), which for \( \langle \bar{N}_H \rangle \sim 10^{16} \text{GeV} \) gives \( M_\chi \sim 10^{14} \text{GeV} \). The VEV of \( \langle N_H \rangle \) may induce unsuppressed dimension four baryon and lepton number violating interactions

\[
\eta_1 QDL + \eta_2 UDD
\]

from the nonrenormalizable terms given in eqs. (19) and (20). Therefore, if the VEV of \( N_H \) is of the order of the GUT, or intermediate, scale, as is required in the seesaw mass matrix in eq. (24), then unsuppressed proton decay will ensue. However, this VEV leaves the unbroken combination of \( U(1)_{Z'} \) and \( U(1)_\zeta \) given by

\[
U(1)_{\zeta'} = \frac{1}{5} U(1)_{Z'} - U(1)_\zeta.
\]

The induced dimension four lepton number violating operator that arises from eq. (20) is invariant under \( U(1)_{\zeta'} \), whereas the induced dimension four baryon number violating operator that arises from eq. (19) is not. Hence, to generate an unsuppressed dimension four baryon number violating operator we must break also \( U(1)_{\zeta'} \). Therefore, if \( U(1)_{\zeta'} \) remains unbroken down to low energies, it suppresses proton decay from dimension four operators. Similarly, the dimension five baryon and lepton number violating operators given in eqs. (18) and (19) are not invariant under \( U(1)_{\zeta'} \) and hence suppressed if \( U(1)_{\zeta'} \) remains unbroken down to low energies.
5 Estimate of the $U(1)_{\zeta'}$ mass scale

The dimension four and five proton decay mediating operators are forbidden by the $U(1)_{Z'}$ and $U(1)_{\zeta}$ gauge symmetries. These symmetries are broken by some fields and we can estimate the required symmetry breaking scale in order to ensure sufficient suppression. In turn this will indicate the possible mass scale of the additional $Z_{\zeta'}$ vector boson, and whether it may exist in the range accessible to forthcoming experiments. The dimension four operators that give rise to rapid proton decay, $\eta_1 UDD + \eta_2 QLD$, are induced from the non–renormalizable terms of the form

$$
\eta_1 (UDDN)\Phi + \eta_2 (QLDN)\Phi'
$$

(28)

where, $\Phi$ and $\Phi'$ are combinations of fields that fix gauge invariance and the string selection rules. The field $N_H$ can be the Standard Model singlet in the 16 representation of $SO(10)$, or it can be a product of two fields, which effectively reproduces the $SO(10)$ charges of $N_H$ [12]. We take the VEV of $N_H$, which breaks the $B-L$ symmetry, to be of order of the GUT scale, i.e. $\langle N_H \rangle \sim 10^{16}$GeV. This is the case as the VEV of $\bar{N}_H$ induces the seesaw mechanism, which suppresses the left–handed neutrino masses. The VEVs of $\Phi$ and $\Phi'$ then fixes the magnitude of the effective proton decay mediating operators, with

$$
\eta_1' \sim \frac{\langle N_H \rangle}{M} \left( \frac{\langle \phi \rangle}{M} \right)^n ; \quad \eta_2' \sim \frac{\langle N_H \rangle}{M} \left( \frac{\langle \phi' \rangle}{M} \right)^{n'} .
$$

(29)

We take $M$ to be the heterotic string unification scale, $M \sim 10^{18}$GeV. Similarly, the dimension five proton decay mediating operator $QQQL$ can effectively be induced from the nonrenormalizable terms

$$
\lambda_1 QQQL(\Phi'')
$$

(30)

The VEV of $\phi''$ then fixes the magnitude of the effective dimension five operator to be

$$
\lambda_1' \sim \lambda_1 \left( \frac{\langle \phi'' \rangle}{M} \right)^{n''}
$$

(31)

The experimental limits impose that the product ($\eta_1'\eta_2'$) $\leq 10^{-24}$ and ($\lambda_1'/M$) $\leq 10^{-25}$. Hence, for $M \sim M_{\text{string}} \sim 10^{18}$GeV we must have $\lambda_1' \leq 10^{-7}$, to guarantee that the proton lifetime is within the experimental bounds. The induced dimension four lepton number violating operator is invariant under $U(1)_{\zeta'}$. Hence, we can take $n' = 0$. The dimension five baryon number violating operator is not invariant under $U(1)_{\zeta'}$. Hence we must have at least $n'' = 1$. We assume that the dimension four baryon number violating operator in eq. (26) is induced at the quintic order. The corresponding nonrenormalizable term in eq. (28) contain one additional field that breaks the
proton protecting $U(1)_{\zeta'}$ at intermediate energy scale $\Lambda_{\zeta'}$. Hence, we have $n = 1$ in eq. (29), and

$$ (\eta_1^1 \eta_2^2) \sim \left( \frac{\langle N \rangle}{M} \right)^2 \left( \frac{\Lambda_{\zeta'}}{M} \right) $$

Taking $\langle N \rangle \sim 10^{16}\text{GeV}$ and $M \sim 10^{18}\text{GeV}$, we obtain the estimate $\Lambda_{\zeta'} \lesssim 10^{-2}\text{GeV}$, which is clearly too low. Taking $\langle N \rangle \sim 10^{13}\text{GeV}$ yields $\Lambda_{\zeta'} \lesssim 10^4\text{GeV}$. We also have that in this case $\lambda_1/M < 10^{-14}$. Hence, the baryon and lepton number violating dimension five operator is adequately suppressed. On the other hand, we have $\eta_2^2 \sim 10^{-5}$. This may be too small to produce sizable effects in forthcoming collider experiments, but may have interesting consequences for neutralino dark matter searches.

6 Conclusions

The Standard Model gauge and matter spectrum clearly indicates the realization of grand unification structures in nature. Most appealing in this respect is the structure of unification in the context of embedding the Standard Model chiral spectrum into spinorial representations of $SO(10)$. In this case each Standard Model generation together with the right-handed neutrino fits into a single $SO(10)$ spinorial representation. While this can be a mirage, it is the strongest hint from the available experimental data, accumulated over the past century. On the other hand, grand unified theories, and many other extensions of the renormalizable Standard Model, predict processes that lead to proton instability and decay. Proton longevity is therefore another key ingredient in trying to understand the fundamental origin of the Standard Model matter spectrum and interactions. A model that provides a robust explanation for these two key observations, while not interfering with other experimental and theoretical constraints, may indeed stand a good chance to pass further experimental scrutiny.

String theory provides a viable framework for perturbative quantum gravity, while at the same time giving rise to the gauge and matter structures that describe the interactions of the Standard Model. In this respect string theory is unique and enables the development of a phenomenological approach to the unification of the gauge and gravitational interactions. Heterotic-string theory has the further distinction that by giving rise to spinorial representations in the massless spectrum it also enables the embedding of the Standard Model chiral spectrum in $SO(10)$ spinorial representations. The free fermionic models provide examples of quasi-realistic three generation heterotic-string models, in which the chiral spectrum arises from $SO(10)$ spinorial representations. These models therefore admit the $SO(10)$ embedding of the Standard Model matter states. They satisfy the two pivotal criteria suggested by the Standard Model data. These models are related to $Z_2 \times Z_2$ orbifolds at special points in the moduli space. Other classes of quasi-realistic perturbative heterotic-string
models have also been studied on unrelated compactifications and using different techniques [23].

Perhaps the most appealing explanation for the stability of the proton is the existence of additional gauge symmetries that forbid the proton decay mediating operators. However, such gauge symmetries should not interfere, or obstruct, the other phenomenological requirements that must be imposed on any extension of the Standard Model. Therefore, they should allow for generation of fermion masses and suppression of neutrino masses. They should be anomaly free. Gauge symmetries that may be observed in forthcoming collider experiments should also be family universal.

In this paper we examined the question of such an additional $U(1)$ gauge symmetry in the free fermionic models. While in most cases the additional gauge symmetries that arise in the string models do not satisfy the needed requirements, we demonstrated the existence of a $U(1)$ symmetry in the class of models of ref. [7] that indeed does pass all the criteria. The existence of this $U(1)$ symmetry at low energies is therefore motivated by the fact that it protects the proton from decaying, and it may indeed exist in the range accessible to forthcoming experiments. It is noted that although we investigated the additional $U(1)$ in the context of the free fermionic string models, the properties of the $U(1)$ symmetry, and the charges of the Standard Model state under it, rely solely on the weight charges of the string states under the rank 16 gauge symmetry of the ten dimensional theory. A $U(1)$ symmetry with the properties that we extracted here may therefore arise in other classes of string compactifications. We emphasize that the characteristics of the extra $U(1)$ that we extracted from a particular class of free fermionic models, do not depend on the specific string compactification. It ought to be further noted that compactifications that yielded the $U(1)$ and the peculiar Standard Model charges under it, are not decendent from the $E_8 \times E_8$ heterotic string in 10 dimensions. This is because a $U(1)$ symmetry which descends from the $E_8 \times E_8$ (or $SO(16) \times SO(16)$) will necessarily have an embedding in $E_6$ and as we demonstrated here the Standard Model $U(1)$ charges derived in this paper do not possess an $E_6$ embedding, and do not descend from $E_8$. The properties of this $U(1)$ symmetry therefore differ from those that have been predominantly explored in the literature, which are inspired from compactifications of the $E_8 \times E_8$ heterotic string. The investigation of the phenomenological characteristics of this additional $U(1)$ is therefore of further interest and we shall return to it in future publications.

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