d-dimensional SYK, AdS Loops, and 6j Symbols

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IGST, Copenhagen, Aug. 2018
Strange metals
No quasiparticles

AdS/CFT

2d Gravity

Quantum Chaos

New Quantum Field Theories

SYK

Metals

2d String Theory?
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SYK in the infrared is a solvable large N CFT. One can compute all correlation functions, by summing the large N dominant Feynman diagrams.
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SYK in the infrared is a solvable large N CFT. One can compute all correlation functions, by summing the large N dominant Feynman diagrams.

One can also sum the same diagrams in d dimensions.
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In fact, this talk will be centered around the 6j symbol.

We will discuss the appearance of the 6j symbol in CFT, as the crossing kernel, and the appearance of the 6j symbol in AdS loop diagrams. We will compute the 6j symbol.
Quantum Mechanics

Addition of two spins

Addition of three spins

Clebsch-Gordan

Two ways of adding
Quantum Mechanics

Addition of two spins

\[ \uparrow \quad \uparrow \quad\text{Clebsch-Gordan} \]
Quantum Mechanics

Addition of two spins

Addition of three spins

Clebsch-Gordan
Quantum Mechanics

Addition of two spins

Addition of three spins

Two ways of adding

Clebsch-Gordan
Addition of three spins

6j symbols: products of 4 Clebsch-Gordan coefficients, summed over $m_i$

edges: spins

vertex: Clebsch-Gordan
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|-------|-----------|
| angular momentum J | dimension Δ, spin J |
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| SU(2)                  | SO(d+1,1)            |
|------------------------|----------------------|
| angular momentum J     | dimension Δ, spin J  |
| z-component angular    | position x           |
| momentum               |                      |
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SU(2)                     SO(d+1,1)

angular momentum J        dimension Δ, spin J

z-component angular       position x
momentum

Clebsch-Gordan            3-pt function

\langle \mathcal{O}_{\Delta_1,J_1}(x_1) \mathcal{O}_{\Delta_2,J_2}(x_2) \mathcal{O}_{\Delta_3,J_3}(x_3) \rangle
Conformal 6j symbol

\[
\begin{align*}
= \int \frac{d^d x_1 \cdots d^d x_6}{\text{vol}(\text{SO}(d + 1, 1))} \langle \tilde{O}_1 \tilde{O}_2 \tilde{O}_5 \rangle^a \langle \tilde{O}_5 \tilde{O}_3 \tilde{O}_4 \rangle^b \langle \tilde{O}_3 \tilde{O}_2 \tilde{O}_6 \rangle^c \langle \tilde{O}_6 \tilde{O}_1 \tilde{O}_4 \rangle^d
\end{align*}
\]
Where does the 6j symbol appear?
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A CFT four-point function can be expanded in terms of conformal blocks.
Where does the $6j$ symbol appear?

A CFT four-point function can be expanded in terms of conformal blocks.

One can expand in either the s-channel blocks or the t-channel blocks.
Where does the 6j symbol appear?

A CFT four-point function can be expanded in terms of conformal blocks.

One can expand in either the s-channel blocks or the t-channel blocks.

Conformal bootstrap: equality between the two expansions gives constraints on the OPE coefficients.
The 6j symbol is the overlap between the s-channel and the t-channel conformal partial waves (crossing kernel)
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Summary:
The 6j symbol is a group-theoretic quantity. It is clearly important to know it. It appears in the bootstrap as the crossing kernel.

We compute the 6j symbol in dimensions 1,2,4, for external scalars. I will describe the computation later.
In fact, the conformal 6j symbol also appears in an entirely different, and *dynamical* context.
Consider summing the following Feynman diagrams, in a CFT.
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These appear as the planar diagram contribution of the three-point function of bilinears in SYK.
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The melons are not important here. What is important is that we are gluing three 3-point functions.
In the notation from before, with each vertex denoting a three-point function,
The functional form of a three-point function is fixed by conformal symmetry. We can extract the coefficient by contracting with a (bare) three-point function of shadow operators,
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\[
\begin{pmatrix}
\begin{array}{c}
\end{array}
\end{pmatrix}
, 

\begin{pmatrix}
\begin{array}{c}
\end{array}
\end{pmatrix}
= 

\begin{pmatrix}
\begin{array}{c}
\end{array}
\end{pmatrix}
\]

This is a tetrahedron: a 6j symbol
The overlap of two partial waves - a group theoretic quantity - and the planar Feynman diagrams in an SYK correlation function - a dynamical quantity - are just two different ways of splitting a tetrahedron.
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\end{array} \right) \\
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\end{align*}
\]
AdS

There is a third context in which the 6j symbol appears: loop diagrams in AdS.
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The preamplitude for the triangle diagram is a 6j symbol

\[ A^{\text{loop}}_{123}(x_i) = \int dy_1 \int dy_2 \int dy_3 K_{\downarrow 1}(x_1, y_1) K_{\downarrow 2}(x_2, y_2) K_{\downarrow 3}(x_3, y_3) G_{\downarrow 4}(y_1, y_2) G_{\downarrow 5}(y_2, y_3) G_{\downarrow 6}(y_3, y_1) \]
Outline

1. SYK and SYK-like models, and all point correlation functions.

2. Computation of 6j symbols

3. AdS triangle diagram.
\[ S = \int d\tau \left( \frac{1}{2} \chi_i \partial_\tau \chi_i + \frac{1}{4!} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \right) \]

\[ \overline{J_{ijkl}^2} = 3! \frac{J^2}{N^3} \]

Sachdev & Ye
Kitaev
**SYK**

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\overline{J_{ijkl}}^2 = 3! \frac{J^2}{N^3}
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Sachdev & Ye
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**Tensor Model**

Different symmetry,
same leading large N diagrams

\[
S = \int d\tau \left( \frac{1}{2} \psi_{abc} \partial_\tau \psi_{abc} + \frac{1}{4} g \psi_{abc} \psi_{ade} \psi_{fbc} \psi_{fde} \right)
\]

\[
O(N)^3
\]

Gurau; Witten; Carrozza & Tanasa
Klebanov & Tarnopolsky
Large N Models

- Vector model: \( \phi_a \)
  \[ (\vec{\phi} \cdot \vec{\phi})^2 \equiv \phi_a \phi_a \phi_b \phi_b \]
  
  Easy

bubbles
Large N Models

- Vector model: \( \phi_a \)
  \[ (\vec{\phi} \cdot \vec{\phi})^2 \equiv \phi_a \phi_a \phi_b \phi_b \]

  Easy

- Matrix model: \( \phi_{ab} \)
  \[ \text{Tr}(\phi^4) \equiv \phi_{ab} \phi_{bc} \phi_{cd} \phi_{da} \]

  Hard

  all planar

bubbles
Large N Models

- Vector model: \( \phi_a \) \( (\vec{\phi} \cdot \vec{\phi})^2 \equiv \phi_a \phi_a \phi_b \phi_b \)

  Easy

- Matrix model: \( \phi_{ab} \) \( \text{Tr}(\phi^4) \equiv \phi_{ab} \phi_{bc} \phi_{cd} \phi_{da} \)

  Hard

- Tensor model: \( \phi_{abc} \) \( \phi_{abc} \phi_{ade} \phi_{fbe} \phi_{fde} \)

  Medium

Questions about my TALK

- Include more papers, or emphasize several?
- Cite more of my own papers?

Feb 1: THINK more about long term plan

This is a diagram showing the relationships between different models and their corresponding complexity levels. The vector model is the easiest, followed by the matrix model, and the tensor model is the medium level. The melons and bubbles are visual representations of the complexity of these models.
We can compute all large N correlation functions in SYK by summing all Feynman diagrams.
We can compute all large $N$ correlation functions in SYK by summing all Feynman diagrams.

2-pt: Melons $\rightarrow$ Conformal in IR

Sachdev & Ye
We can compute all large N correlation functions in SYK by summing all Feynman diagrams.

2-pt: Melons -> Conformal in IR

4-pt: Ladders: geometric sum

(the lines on the higher-point functions are really dressed propagators)
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8-pt: glue 4-pt functions

(the lines on the higher-point functions are really dressed propagators.)
Higher Dimensions

\[ \mathcal{L} = \frac{1}{2} (\partial \phi_i)^2 + \frac{1}{q!} J_{i_1 i_2 \ldots i_q} \phi_{i_1} \phi_{i_2} \ldots \phi_{i_q} \]

Bosonic d-dimensional model would give same diagrams, but has negative directions. It is only formally defined.
Higher Dimensions

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One can construct a well-defined two-dimensional supersymmetric SYK model. 

Murugan, Stanford, Witten
Higher Dimensions

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One can construct a well-defined two-dimensional supersymmetric SYK model. Murugan, Stanford, Witten

Perhaps a bosonic higher dimensional model will be found Giombi, Klebanov, Popov, Prakash, Tarnopolsky
Fishnet theory

Talks by Basso, Gromov, Kazakov

A generalization of SYK diagrams. Can have $n$ parallel lines, corresponding to a two-point function of, schematically, $\text{tr}(\phi^n)$
SYK 6-pt function of fundamentals (3-pt of bilinears)

The first diagram is what we call the contact diagram, and the second is planar.
Let us look at the planar diagram. As we said before, it corresponds to three 3-pt functions glued together.

\[
\langle O_{\Delta_1,J_1}(x_1) O_{\Delta_2,J_2}(x_2) O_{\Delta_3,J_3}(x_3) \rangle_2 = \int d^d x_a d^d x_b d^d x_c \langle O_{\Delta_1,J_1}(x_1) \phi(x_a) \phi(x_b) \rangle_{\text{amp}}
\]

\[
\langle O_{\Delta_2,J_2}(x_2) \phi(x_c) \phi(x_a) \rangle_{\text{amp}} \langle O_{\Delta_3,J_3}(x_3) \phi(x_b) \phi(x_c) \rangle_{\text{amp}}
\]
We extract the coefficient by contracting with the functional form of a shadow three-point function.

\[
\int \frac{d^d x_1 d^d x_2 d^d x_3 d^d x_a d^d x_b d^d x_c}{\text{vol}(\text{SO}(d + 1, 1))} \langle \mathcal{O}_{\Delta_1, J_1}(x_1) \phi(x_a) \tilde{\phi}(x_b) \rangle \langle \mathcal{O}_{\Delta_2, J_2}(x_2) \phi(x_c) \tilde{\phi}(x_a) \rangle \\
\langle \mathcal{O}_{\Delta_3, J_3}(x_3) \phi(x_b) \tilde{\phi}(x_c) \rangle \langle \tilde{\mathcal{O}}_{\Delta_1, J_1}(x_1) \tilde{\mathcal{O}}_{\Delta_2, J_2}(x_2) \tilde{\mathcal{O}}_{\Delta_3, J_3}(x_3) \rangle^b.
\]

A 6j symbol, as advertised earlier
One can derive a simple formula for the diagrams appearing in the higher point functions.

\[ \int_C \frac{dh}{2\pi i} \tilde{\rho}(h) c_{h_1} h_2 h c_{h_3} h_4 h \mathcal{F}_{h_i}^h(\tau_i) \]

4-pt function fundamentals (sum of ladders)

3-pt function bilinears

Conformal Block

In 1d, \( h \) is dimension
\[
\int \frac{dh}{2\pi i} \rho(h) \mathcal{F}_\Delta^h(\tau_i) \quad \rho(h) \sim \frac{k(h)}{1 - k(h)}
\]

- h-space is to the conformal group, SL(2,R), what Fourier space is to translations

\[
\int \frac{dp}{2\pi} f(p) e^{ipx}
\]
\[
\int_C \frac{d h_a}{2 \pi i} \tilde{\rho}(h_a) \int_C \frac{d h_b}{2 \pi i} \tilde{\rho}(h_b) c_{h_1 h_2 h_a} c_{h_a h_3 h_b} c_{h_b h_4 h_5} \mathcal{F}^{h_a, h_b}_{h_i}(\tau_i)
\]
These are simple rules for summing an infinite number of diagrams. It doesn’t matter that the four-point function is made up of ladders. These apply to any four-point functions.

\[\int C \frac{dh}{2\pi i} \tilde{\rho}(h) c_{h_1 h_2 h} c_{h_3 h_4 h} F^h_{i}(\tau_i)\]
These are simple rules for summing an infinite number of diagrams. It doesn’t matter that the four-point function is made up of ladders. These apply to any four-point functions.

This is not just an OPE expansion. The $c_{h_1 h_2 h_3}$ are the analytically extended OPE coefficients of the single-trace operators. The four-point function is a sum of conformal blocks of single-trace operators and double-trace operators. This emerges upon closing the contour.
Computing the 6j Symbol

\[
\left( \Psi_{\Delta, J}^{\tilde{\Delta}_1, \tilde{\Delta}_2, \tilde{\Delta}_3, \tilde{\Delta}_4}, \Psi_{\Delta', J'}^{\Delta_3, \Delta_2, \Delta_1, \Delta_4} \right)
\]

\[
= \int \frac{d^d x_1 \cdots d^d x_4}{\text{vol}(\text{SO}(d + 1, 1))} \Psi_{\Delta, J}^{\tilde{\Delta}_1, \tilde{\Delta}_2, \tilde{\Delta}_3, \tilde{\Delta}_4}(x_1, x_2, x_3, x_4) \Psi_{\Delta', J'}^{\Delta_3, \Delta_2, \Delta_1, \Delta_4}(x_3, x_2, x_1, x_4),
\]

\[
\Psi_{\Delta, J}^{\Delta_i}(x_i) = \int d^d x_5 \langle \mathcal{O}_{\Delta_1}(x_1) \mathcal{O}_{\Delta_2}(x_2) \mathcal{O}_{\Delta, J}(x_5)^{\mu_1 \cdots \mu_J} \rangle^a \langle \tilde{\mathcal{O}}_{\Delta, J; \mu_1 \cdots \mu_J}(x_5) \mathcal{O}_{\Delta_3}(x_3) \mathcal{O}_{\Delta_4}(x_4) \rangle
\]
The conformal partial wave is a sum of a conformal block and the shadow block.
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In 1d, the conformal block is the hypergeometric function $\binom{2}{1}$ of a single cross ratio. One can evaluate the integral for the 6j symbol directly, to find a $\binom{4}{3}$. 

It turns out one can do this, by an appropriate analytic continuation of the contour into Lorentzian signature.
The conformal partial wave is a sum of a conformal block and the shadow block

In 1d, the conformal block is the hypergeometric function $\ _2F_1$ of a single cross ratio. One can evaluate the integral for the 6j symbol directly, to find a $\ _4F_3$

In higher dimensions, the integral is harder. One would like to somehow make the integral factorize, into a product of one-dimensional integrals. It turns out one can do this, by an appropriate analytic continuation of the contour into Lorentzian signature.
In fact, one can apply Caron-Huot's Lorentzian inversion formula to our integral, which is a special case, with a four-point function that is a conformal partial wave.
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First, recall that one can expand any four-point function in terms of partial waves

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4) \rangle = \sum_{J=0}^{\infty} \int_{\frac{d}{2}}^{\frac{d}{2}+i\infty} \frac{d\Delta}{2\pi i} \frac{I_{\Delta,J}}{n_{\Delta,J}} \Psi^{\Delta_1,\Delta_2,\Delta_3,\Delta_4}_{\Delta,J}(x_i)$$
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\]

Trivially, using the orthogonality of the partial waves, one can invert this

\[
I_{\Delta,J} = \left( \langle O_1 \cdots O_4 \rangle, \Psi^{-i}_{\Delta,J} \right) = \int \frac{d^d x_1 \cdots d^d x_4}{\text{vol}(\text{SO}(d+1, 1))} \langle O_1 \cdots O_4 \rangle \Psi^{-i}_{\Delta,J}(x_i)
\]
Deforming the contour gives Caron Huot’s Lorentzian inversion formula

\[ I_{\Delta,J} = \alpha_{\Delta,J} \left[ (-1)^J \int_0^1 \int_0^1 \frac{d\chi d\bar{\chi}}{(\chi \bar{\chi})^d} |\chi - \bar{\chi}|^{d-2} \hat{G}^{\tilde{\Delta}_i}_{J+d-1, \Delta-d+1}(\chi, \bar{\chi}) \right. \]

\[ \times \left. \frac{\langle [O_3, O_2][O_1, O_4] \rangle}{T^{\Delta_i}} \right] \]

\[ + \int_{-\infty}^0 \int_{-\infty}^0 \frac{d\chi d\bar{\chi}}{(\chi \bar{\chi})^d} |\chi - \bar{\chi}|^{d-2} \hat{G}^{\tilde{\Delta}_i}_{J+d-1, \Delta-d+1}(\chi, \bar{\chi}) \frac{\langle [O_4, O_2][O_1, O_3] \rangle}{T^{\Delta_i}} \]
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\[ + \left. \int_{-\infty}^0 \int_{-\infty}^0 \frac{d\chi d\bar{\chi}}{(\chi \bar{\chi})^d} |\chi - \bar{\chi}|^{d-2} \hat{G}_{J+d-1,\Delta-d+1}(\chi, \bar{\chi}) \frac{\langle [\mathcal{O}_4, \mathcal{O}_2][\mathcal{O}_1, \mathcal{O}_3]\rangle}{T^{\Delta+i}} \right] \]

Caron-Huot
Simmons-Duffin, Stanford, Witten

Applying this we find the 6j symbols in 2d and 4d. It is expressed in terms of a product of two 4\(F_3\)’s
\[ J_4 = (-1)^J K_{\Delta_1, \Delta_4}^{\Delta_1, \Delta_4 \alpha \Delta, J} \left( \Theta(\Delta' + \tilde{J}') \Omega_{\Delta + J, \Delta' + J', \Delta_2 + \Delta_3 - 1}^{\Delta_2 / 2, \Delta + J, \Delta', J', 2} \Omega_{\Delta_2 + \Delta_3 - 1}^{\Delta_2 / 2, \Delta + J, \Delta', J', 2} \right) \]

\[ \Omega_{h, h', p}^{\Delta_1, \Delta_2} = \int_0^1 \frac{d\chi}{\chi^2} \left( \frac{\chi}{1 - \chi} \right)^p \chi^{h_{13}} k_{2h}^{h_{h1}, h_{h2}, h_{h3}, h_{h4}} (\chi) k_{2h'}^{h_{h3}, h_{h2}, h_{h1}, h_{h4}} (1 - \chi) \]

\[ K_{[\Delta_3, J]}^{\Delta_1, \Delta_2} = (-1)^J \frac{\pi i^2}{2} \frac{\Gamma(\Delta_3 - \frac{d}{2}) \Gamma(\Delta_3 + J - 1) \Gamma(\Delta_3 + J) \Gamma(\Delta_3 + J + 1)}{\Gamma(\Delta_3 + 1) \Gamma(d - \Delta_3 + J) \Gamma(\Delta_3 + 1 - \Delta_3 + J) \Gamma(\Delta_3 + 1 - \Delta_3 + J + 1)} \]

\[ \Theta(x) = \frac{4 \pi^2}{\Gamma(\Delta_3 + \Delta_4 - x) \Gamma(1 - \Delta_3 + \Delta_4 - x) \Gamma(\Delta_1 + \Delta_4 - x) \Gamma(1 - \Delta_1 + \Delta_4 - x)} \]

\[ \alpha_{\Delta, J} = -\frac{t_0}{2} \left( \frac{2\pi}{d-2} \right)^{d-2} \frac{\Gamma(J + 1) \Gamma(\Delta - \frac{d}{2}) \Gamma(\Delta + \Delta + \Delta) \Gamma(\Delta_2 + \Delta + \Delta) \Gamma(\Delta_3 + \Delta + \Delta) \Gamma(\Delta + \Delta + \Delta) \Gamma(\Delta + \Delta + \Delta)}{\Gamma(J + \frac{d}{2}) \Gamma(\Delta - 1) \Gamma(J + \Delta) \Gamma(J + d - \Delta)} \]
Triangle Loop in AdS
operators to be scalars; the generalization to external spins will follow trivially. The external legs

We sometimes use

Figure 7: The AdS three-point triangle,

\[ \Omega_\nu(y_1, y_2) = \frac{i\nu}{2\pi}(G_\nu(y_1, y_2) - G_{-\nu}(y_1, y_2)) \]

\[ \Omega_\nu(y_1, y_2) = \frac{\nu^2}{\pi} \int dx_1 K_\nu(x_1, y_1) K_{-\nu}(x_1, y_2) \]

\[ \Delta = \frac{d}{2} + i\nu \]

Finally, do the three bulk integrals. This leaves an integral involving three CFT 3-point functions.
Summary

• We gave three contexts in which the conformal 6j symbol appears: the crossing kernel, Feynman diagrams in SYK, and a Witten loop diagram in AdS

• We computed the 6j symbol in d=1, 2, 4

• We gave a simple formula for all-point correlation functions in SYK, by summing all Feynman diagrams.