Experimental review of CKM sides at $B$ factories

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Abstract. An overview of the measurements of the Cabibbo-Kobayashi-Maskawa matrix elements at BaBar and Belle experiments is given in this talk. Recent results of $|V_{ub}|$ and $|V_{cb}|$ from exclusive and inclusive semileptonic $B$ decays and of the ratio $|V_{td}/V_{ts}|$ from radiative $B$ decays are presented.

1. Introduction
In the Standard Model (SM) the transition between quarks of different flavours is described by the Cabibbo-Kobayashi-Maskawa CKM matrix [1]:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (1)$$

The elements of this matrix are proportional to the weak decay amplitudes of the quarks involved in the processes, and since they are fundamental parameters of the theory, they have to be experimentally determined. In the SM the CKM matrix is unitary and one can construct an Unitary Triangle (UT) underlying in the relation $V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0$. Overconstrained measurements of the vertex angles and side lengths of the UT triangle could probe the presence of New Physics if they are inconsistent. The sides of this triangle are proportional to the ratio of the $|V_{ub}|$ and $|V_{cb}|$ elements and to the ratio of $|V_{td}|$ and $|V_{ts}|$ elements, quantities which can be measured at $B$ factories. $|V_{ub}|$ and $|V_{cb}|$ elements are measured with high precision at BaBar and Belle from exclusive and inclusive $B$ semileptonic decays. The ratio $|V_{td}/V_{ts}|$ is measured primarily at hadron colliders, but it can also be measured at $B$ factories studying radiative $B$ decays. In the next sections the status of these measurements are given.

2. Measurements at $B$ factories
$B$ factories have recorded about 1200 fb$^{-1}$ at the center of mass (cm) energy of the $\Upsilon(4S)$ resonance ($\sim 10.5$ GeV), thanks to the high luminosity provided by the PEP-II and KEK $e^+ - e^-$ machines. At this energy the $\Upsilon(4S)$ decays into $B\bar{B}$ pairs produced almost at rest. Analysis techniques to measure the signal decay channel properties are based on the total or partial reconstruction of the $B$ decay products. Usually one distinguishes between tagged and untagged analyses. In the former one of the $B$ meson is fully reconstructed in hadronic or semileptonic decay modes, making known the flavour of the $B$ and its kinematics. The signal mode is search then from the other $B$. This technique leads to high purity but reduced efficiency. Another
technique does not require the tag of one $B$ (untagged method), and the signal is searched directly from both $B$ decays, resulting in high efficient but less pure analyses.

Common variables used in $B$ decay analyses using the tag method are the beam energy-substituted mass (or beam-constrained mass) $M_{ES} = \sqrt{s/4 - \vec{p}_B^2}$ and the energy difference $\Delta E = E_B^0 - \sqrt{s}/2$, $\sqrt{s}$ being the cm energy of the $e^+e^-$ collision, and $\vec{p}_B$ and $E_B^0$ the momentum and energy of the $B$ meson reconstructed in the same frame. In the case of the $\Upsilon(4S)$ decaying into $B\bar{B}$ pairs and one $B$ meson reconstructed, $m_{ES}$ peaks at the mass of the other $B$ meson and $\Delta E = 0$. Since semileptonic decays of $B$ mesons involve neutrinos another usual variable is the missing mass squared, $m_{miss}^2 = (\sqrt{s} - p_{tag} - p_e - \sum p_{signal~mesons})^2$ which peaks at zero for the signal decay and has larger values for background coming from non-reconstructed particles in the event. Another feature of $\Upsilon(4S) \to B\bar{B}$ decays which allows to suppress continuum background is the event shape. Since $B\bar{B}$ pairs decay almost at rest, the distribution of the decay tracks are spherical in the detector, while it is gathered in jets if the $\Upsilon(4S)$ decays into light quarks $(u,d,s,c)$ or $\tau^+\tau^-$ pairs.

3. $|V_{cb}|$ from exclusive semileptonic decays

$|V_{cb}|$ can be obtained by studying the differential decay width of the exclusive processes $B \to D\ell\nu$ and $B \to D^*\ell\nu$, as function of the product of the four-velocities of the initial and final mesons:

$$w = v_B v_{D(\ell)} = (M_B^2 + M_{D(\ell)}^2 - q^2)/(2M_B M_{D(\ell)})$$. Here $M_B$ and $M_{D(\ell)}$ are the masses of the $B$ and $D$ mesons respectively, and $q^2 = (p_e + p_\nu)^2$. The $B \to D^*\ell\nu$ branching fraction ($\sim 5\%$) is larger than for the $B \to D\ell\nu$ decay mode ($\sim 2\%$), and since the differential decay width for the later is suppressed at high $q^2$ the former is preferred to measure $|V_{cb}|$.

The differential decay width can be expressed as:

$$\frac{d\Gamma(B \to D^{(*)}\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3\hbar} F^2(w) K(w) |V_{cb}|^2$$

(2)

Here $G_F$ is the Fermi constant, $K$ is a known kinematic function and $F$ is the form factor describing the hadronic contribution in the process. The normalization of the form factor at the point of zero recoil (where $q^2$ is maximum and the $D^{(*)}$ is produced at rest, $w = 1$) has been calculated by Lattice-QCD [2, 3]. The values are $G(1) = 1.074 \pm 0.018 \pm 0.016$ for $B \to D\ell\nu$ ($F(w)$ is called $G(w)$ for this channel) and $F(1) = 0.921 \pm 0.013 \pm 0.020$ for $B \to D^*\ell\nu$. The shape of the form factor is parameterized as function of form factor slopes, $\rho_D$ or $\rho_{D^*}$, and in the case of the $B \to D^*\ell\nu$ channel two additional form factor ratios $R_1(1)$ and $R_2(1)$. They are experimentally determined. Measuring the differential decay width and the form factor shape, one can extrapolate to the point of zero recoil and extract $|V_{cb}|$.

Both BABAR and Belle experiments have analysed $B \to D\ell\nu$ and $B \to D^*\ell\nu$ data. For $B \to D\ell\nu$ the BABAR measurements are the most precise since the analyzed statistic are much higher as compared to Belle and previous experiments. A recent result from BABAR [4] selects leptons from 417 fb$^{-1}$ data. $D^0$ mesons from the signal side are reconstructed into several channels. $B_{tag}$ mesons are fully reconstructed into hadrons. By reconstructing the missing mass squared as explained in Section 2 the signal yield can be extracted. The signal yield is fitted in 10 $w$ bins and a $\chi^2$ fit is performed to extract $\rho_D^2$ and $G(1)|V_{cb}|$.

To measure $|V_{cb}|$ from the $B \to D^*\ell\nu$ decay channel an untagged method is often used to analyze the data. Signal candidates correspond to a lepton, a soft $\pi$ coming from the $D^{*1}$ and a $D^0$ meson which is reconstructed into several decay channels. The angle between the $B$ meson and $D^*\ell$ candidates is defined as $\cos \theta_{B,D^*\ell} = (2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2)/(2 |\vec{p}_B||\vec{p}_{D^*\ell}|)$. This variable allows to discriminate between signal and background events since the background

1 $D^*$ decays into $D^0\pi$. Since $D^*$ and $D$ mesons have similar mass, the pion momentum is very small.
populates unphysical values while the signal peaks in the [-1,1] range. In this case the decay rate depends on four kinematic variables: \( w, \cos \theta _{\ell}, \cos \theta _{V} \) and \( \chi \), and inclusive information of the event is used to measure them. Fits are performed to these distributions to determine the form factor slope \( \rho _{D*} \) and form factor ratios \( R_1 \) and \( R_2 \) and extract \( \mathcal{F}(1)|_{V_{cb}} \). A recent measurement from BABAR [5] uses 79 fb\(^{-1}\) and it is quoted in Figure 1 (bottom) labeled as BABAR (excl). Preliminary results from the Belle experiment using 207 fb\(^{-1}\) were presented at the ICHEP08 conference [6] (they are not quoted in Figure 1).

Preliminary results have also been shown by the BABAR experiment using a new technique in which both \( B \rightarrow D^{*} \ell \nu \) and \( B \rightarrow D \ell \nu \) channels are measured [7]. This analysis uses 207 fb\(^{-1}\). \( D^{0} \ell \) and \( D^{+} \ell \) candidates are exclusively reconstructed and three kinematic variables are used to discriminate between \( B \rightarrow D^{*} \ell \nu \) and \( B \rightarrow D^{*} \ell \nu \) channels: the momentum of the lepton and of the \( D \) in the \( \Upsilon(4S) \) frame, and the cosine of the angle of the \( B \) meson and the \( D \ell \) candidate. A 3D global fit is performed and the form factor slopes and branching ratios are obtained for both channels. Results are quoted in Fig. 1 (top and bottom, labeled as Global Fit).

Averaging the results of the different measurements for the \( B \rightarrow D \ell \nu \) channel gives \( \mathcal{G}(1)|_{V_{cb}} = (42.4 \pm 1.56) \times 10^{-3} \). Using the value of \( \mathcal{G}(1) \) given in Section 2, the value of \( |V_{cb}| \) results in \( |V_{cb}| = 39.5 \pm 1.5 \exp \pm 0.9 \text{lat} \) where the first error is experimental and the second comes from the uncertainty of \( \mathcal{G}(1) \) from Lattice calculations. Averaging the results for the \( B \rightarrow D^{*} \ell \nu \) channel gives \( \mathcal{F}(1)|_{V_{cb}} = (35.41 \pm 0.52) \times 10^{-3} \). Using the value of \( \mathcal{F}(1) \) given in Section 2, the value of \( |V_{cb}| \) from this decay gives \( |V_{cb}| = 38.4 \pm 0.6 \exp \pm 0.9 \text{lat} \). Results from different channels agree between them inside uncertainties. Errors are dominated by systematics due to the tracking efficiency, radiative corrections and the limited knowledge of \( D^{**} \) states, which is one of the main backgrounds in these analysis. According to the Heavy Quark Effective Theory (HQET) [8] these mesons are orbitally excited charm states: two doublets of \( j_{Q} = 1/2 \) and \( j_{Q} = 3/2 \) of broad and narrow resonances, \( (D_{0}^{0}, D_{1}^{'}) \) and \( (D_{1}^{+}, D_{2}^{0}) \), respectively. Measurements of the \( B \) sl branching fraction into \( D^{**} \) states gives 1 – 2\% which, considering the branching fractions for the \( B \rightarrow D \ell \nu \) and \( B \rightarrow D^{*} \ell \nu \) channels, does not fulfill the inclusive semileptonic branching fraction of \( B \) decays. The BABAR and Belle experiments have exclusively reconstructed these decays ([9],[10]). Results for narrow resonances are in agreement. Concerning the broad states the Belle experiment does not observe the \( D_{1}^{0} \) state, while BABAR does. In addition, experimental results are in contradiction with the theoretical predictions [11].

4. \( |V_{cb}| \) from inclusive semileptonic decays

The most precise determination of the CKM matrix element \( |V_{cb}| \) relies in measuring parameters entering in the theoretical expression on the inclusive semileptonic decay width of \( B \) hadrons. According to the Operator Product Expansion (OPE), the inclusive decay width can be expressed as a product of the decay width for a free quark decay times an expansion in terms of the inverse of the \( b \) quark mass and \( \alpha _{s} \), which takes into account the hadronic contributions. This expansion starts at order \( 1/m_{b}^{2} \) and depends on non-perturbative parameters related with expected values of the \( b \)-quark properties inside the hadron, such as the kinetic energy \( (\mu _{b}) \), the spin, etc... Since some inclusive observables like the lepton energy spectrum or the mass distribution in \( B \) semileptonic decays, or the photon spectrum in radiative \( B \) decays depend on the same parameters, by measuring the moments of these inclusive observables one can extract the OPE parameters. BABAR and Belle experiments have performed measurements of 52 moments. Recent results on moments of the hadronic mass distribution and lepton energy spectrum using hadronic \( B \) tags can be found at [12],[13],[14]. The hadronic mass is reconstructed from the signal side once the lepton is selected, and it is event by event calibrated. The lepton energy has to be unfolded to extract the original distribution. Several moments are measured as function of a cut on the lepton momentum in the cm frame. The photon spectrum has also been measured by Belle [15]. High energy photons are selected and the background is subtracted...
from off-peak data. Photons from $\pi^0$ and $\eta$ are vetoed. The spectrum is unfolded and moments are obtained as function of the photon energy. Results of a combined fit to 27 moments from BABAR, 25 moments from Belle and 12 moments from other experiments (DELPHI, CLEO and CDF) are shown in Fig. 2. These plots show the projections of the fit in the $m_B$-$|V_{cb}|$ and $m_B$-$\mu/\pi$ planes. The result of the fit gives $|V_{cb}| = (41.67 \pm 0.43_{stat} \pm 0.08_{syst} \pm 0.58_{lifetime}) \times 10^{-3}$, where the first error is given by the fit, the second is due to the $B$ meson lifetime and the third is coming from uncertainties of higher order terms in the theoretical expansion. As compared with the

\[ \Delta \chi^2 = 1 \]

\[ \text{CLEO} \]

\[ \text{Belle} \]

\[ \text{BABAR (Global Fit)} \]

\[ \text{BABAR (D^0)} \]

\[ \text{BABAR (excl)} \]

\[ \text{OPAL (excl)} \]

\[ \text{OPAL (partial reco)} \]

\[ \text{DELPHI (partial reco)} \]

\[ \text{BELLE (excl)} \]

\[ \text{DELPHI (excl)} \]

\[ \text{BABAR (D*0)} \]

\[ \text{AVERAGE} \]

\[ \text{HFAG ICHEP08} \]

\[ \chi^2/dof = 1.2/8 \ (CL = 99.7 \%) \]

\[ \chi^2/D^* \rho = 0.5 \]

\[ \mu/\pi = 0.5 \]

\[ \Delta \chi^2 = 1 \]

\[ \text{HFAG ICHEP08} \]

\[ \chi^2/dof = 39/21 \ (CL = 0.01 \%) \]

\[ \chi^2/D^* \rho = 0.5 \]

\[ \mu/\pi = 0.5 \]

\[ \Delta \chi^2 = 1 \]

\[ \text{HFAG ICHEP08} \]

\[ \chi^2/dof = 39/21 \ (CL = 0.01 \%) \]

\[ \chi^2/D^* \rho = 0.5 \]

\[ \mu/\pi = 0.5 \]
result from exclusive decays, there is a $2.5\sigma$ difference which should be understood. In addition, the fit gives $m_B = 4.601 \pm 0.034 \text{ GeV}$ and $\mu^2 = 0.440 \pm 0.040 \text{ GeV}^2$.

**Figure 2.** Results of the world average for $m_B$ vs $|V_{cb}|$ and $\mu^2$ vs $m_B$. The contours correspond to $\Delta \chi^2 = 1$ Details and references for the measurements can be found at [http://www.slac.stanford.edu/xorg/hfag/semi/ichep08/home.shtml](http://www.slac.stanford.edu/xorg/hfag/semi/ichep08/home.shtml)

5. $|V_{ub}|$ from exclusive semileptonic decays

The $|V_{ub}|$ element can be extracted from exclusive charmless semileptonic $B$ decays ($B \to (\pi, \eta, \eta', \rho, \omega)(\ell \nu)$ in a similar way to the exclusive determination of $|V_{cb}|$, with the disadvantage of much smaller branching fractions and much larger background coming from $b \to c$ decays. The most relevant channel for the $|V_{ub}|$ measurement is $B \to \pi \ell \nu$. Experiments measure this branching fraction using tagged and untagged methods. The extraction of $|V_{ub}|$ relies on the shape and normalization of the form factor which is available as function of $q^2$ using several theoretical approaches. The preferred ones are based on light cone sum rules ($q^2 < 16 \text{GeV}^2$) [16] and lattice QCD calculations ($q^2 > 16 \text{GeV}^2$) [17].

The most precise measurement of the $B \to \pi \ell \nu$ branching fraction has been obtained by BaBar [18] using an untagged method. This analysis is based on $206 \text{fb}^{-1}$ and uses a novel loose neutrino reconstruction technique. The signal yield is extracted in 12 bins of $q^2$ which allow to fit the form factor parameterization. Analyses using hadronic and semileptonic tags have been performed at BaBar and Belle experiments ([19],[20],[21]). Results and the world average are given in Fig. 3 (left) together with the $|V_{ub}|$ extraction (right) using several theoretical calculations. The dominant uncertainty in $|V_{ub}|$ comes from the theory. Branching fractions for other exclusive channels ($B \to (\eta, \eta', \rho, \omega)(\ell \nu)$ have also been measured in these analyses with less precision. The motivation is to crosscheck the theoretical approaches and to understand the composition of inclusive charmless $B$ semileptonic decays.

6. $|V_{ub}|$ from inclusive semileptonic decays

The inclusive determination of $|V_{ub}|$ relies as for $|V_{cb}|$ on OPE calculations. The main problem here resides in the fact that experiments need to suppress the high background coming from $b \to c$ decays (a factor 50 larger) by applying tight kinematic cuts, and reducing the total
rate to 20%-60%. This fact immediately implies problems from the theoretical point of view: OPE breaks down and requires the introduction of a shape function (SF). A big effort has been done on the theoretical determination of the SF [BLNP], [BLL], [GGOU]. A more independent approach relates the SF in $b \rightarrow \gamma$ and $b \rightarrow u\ell\nu$ decays [LLR]. Other methods are based on QCD resummation and no SF is employed [DGE], [ADFR]. References for the models are given in [22].

Experimentally, one measures the partial branching ratio of $B \rightarrow X_u \ell\nu$ using tagged or untagged analyses. The idea is to use kinematic variables ($E_\ell$, $q^2$, $m_X$, $P_\ell = E_X - |p_X|$) to suppress as much as possible the charm background but keeping the theoretical calculations reliable. Measurements with tagged $B$ mesons are preferred, where one has access to all kinematic variables for the signal side. $B_{\bar{A}}$ has recently measured the partial branching ratio with 347 fb$^{-1}$ [23]. Preliminary results of a new multivariate analysis keeping 90% of the total rate was presented at the CKM2008 workshop. Other analyses from $B_{\bar{A}}$ are based on the measurements of moments of the full mass spectrum [24] and in the [LLR] method [25].

Results of the world average measurements for the inclusive $|V_{ub}|$ determination using different theoretical approaches are shown in Fig. 4. These values are systematically higher than the values obtained using exclusive decays or in global fits of the Unitary Triangle, requiring more data and further theoretical improvements to understand it.

7. $|V_{td}/V_{ts}|$ from radiative decays

The best way to measure the $|V_{td}|$ and $|V_{ts}|$ elements is at hadron colliders from $B - \bar{B}$ oscillations, which are mediated by box diagrams with top quarks and the top quark decays almost entirely into a $b$ quark and a $W$ boson. Nevertheless independent measurements can be performed at $B$ factories by measuring radiative penguins $b \rightarrow d\gamma$ and $b \rightarrow s\gamma$. They are affected by different theoretical and experimental uncertainties and can serve as a crosscheck of the $B - \bar{B}$ oscillations measurements. Results have been provided by $B_{\bar{A}}$ and Belle experiments using exclusive
Figure 4. Results of the average measurements for inclusive $|V_{ub}|$ using different theoretical approaches. Details and references can be found at http://www.slac.stanford.edu/xorg/hfag/semi/ichep08/home.shtml

Table 1. Results of the $|V_{td}/V_{ts}|$ ratio. First error is experimental and the second comes from theoretical uncertainties.

| Measurement     | $|V_{td}/V_{ts}|$         |
|-----------------|--------------------------|
| CDF (B mixing)  | $0.206 \pm 0.001 \pm 0.008$ |
| BaBar (excl)    | $0.233 \pm 0.025 \pm 0.022$ |
| Belle (excl)    | $0.195 \pm 0.020 \pm 0.015$ |
| BaBar (incl)    | $0.177 \pm 0.043 \pm 0.001$ |

($B \rightarrow (\rho, \omega)\gamma$, $B \rightarrow K^*\gamma$) and inclusive decays ([26],[27]) and they are shown in Table 1. The precision at hadron colliders (CDF result) is dominated by the theoretical uncertainty coming from Lattice-QCD calculations of the ratio of the $B_d$ and $B_s$ decay constants and bag parameters. At $B$ factories, the inclusive result give small theoretical uncertainties while exclusive measurements are affected by theoretical errors coming from uncertainties in form factors ratio determinations. Even if at present errors are large due to the reduced statistics, a SuperB factory of high luminosity will allow to determine this ratio with high precision.

8. Summary

$\text{BaBar}$ and Belle experiments have measured the $|V_{cb}|$ and $|V_{ub}|$ elements with high precision using exclusive and inclusive $B$ semileptonic decays. At present $|V_{ck}|$ is known with 2% accuracy from the inclusive decay width, and the dominant error is coming from the theory. $|V_{ub}|$ is also measured with 8% accuracy, being the inclusive method the most precise one. Nevertheless discrepancies between the exclusive and inclusive determinations and the dispersion between several models require further improvements from the theory.
and Belle have also contributed to independent determinations of other CKM elements. This is the case for the $|V_{td}/V_{ts}|$ ratio from radiative $B$ decays, or other precise measurements concerning $\tau$ and charm decays which have not been covered in this talk.

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