Abstract

Cosmological inflation models with modifications to include recent cosmological observations has been an active area of research after WMAP 3 results, which have given us information about the composition of dark matter, normal matter and dark energy and the anisotropy at the 300000 years horizon with high precision. We work on inflation models of Guth and Linde and modify them by introducing a doublet scalar field to give normal matter particles and their supersymmetric partners which result in normal and dark matter of our universe. We include the cosmological constant term, as the vacuum expectation value of the stress energy tensor, as the dark energy. We calibrate the parameters of our model using recent observations of density fluctuations. We develop a model which consistently fits with the recent observations.

1 Introduction

With new observations about the universe about the dark matter content and its nature, dark energy and density fluctuations at the 300000 year observable horizon, it has become essential to revisit the standard cosmological models and incorporate these observations.

In the pre-inflationary epoch, there have been diverse developments in Loop Quantum Gravity, Strings & Brane world models, Quantum Gravity et al. Same is the situation in the post-inflationary epoch where we have new theories being added in areas of matter-antimatter models, Baryogenesis, Quark-Gluon models et al. We focus on the inflationary era and take a fresh look at the standard models, in particular those of Guth [1] and Linde [2] in the light of these new findings and the recent cosmological observations from WMAP.

The WMAP results [3] strongly supports the non-baryonic nature of dark matter, hypothesis about the cosmological constant being the best candidate for dark energy and the fixing of density fluctuations at $10^{-5}$. These results also suggest that the $\frac{1}{2}m^2\phi^2$ model is a better candidate for inflation than the $\frac{1}{4}\Lambda\phi^4$ model [3].
Rapid progress has been seen in incorporating these elements of observation into theoretical models. Diverse attempts are being made in this regard as can be seen from the following references of [6][7][8][9][10]. We start with the standard models and modify them on the basis of the following three assumptions,

- A doublet scalar field drives inflation producing normal matter particles and their supersymmetric partners which result in dark matter after they acquire mass from the Higgs and Higgsino Fields.
- Cosmological constant as the vacuum expectation value of the stress-energy tensor is the source of dark energy.
- The observed density fluctuations are due to quantum fluctuations in the primal scalar fields.

We use the observational values of parameters about dark matter, dark energy and the density fluctuations to fix the parameters in our model. Since we deal with an epoch which is not within the directly observable horizon, we expect the results of our model to match with the initial requirements of the post-inflationary epoch which predicts the observed results at the 300000 year horizon.

2 Inflationary Cosmology

In the standard model of the universe with Big-Bang, we start with a singularity, i.e as time $t \to 0$, the temperature $T \to \infty$. However we do not yet have a physical framework to deal with such extreme temperatures and also at such large energy scales and small spacial scales, quantum gravitational effects become prominent and our classical theories fail. Therefore we have to resort to new models to understand the evolution of the universe beyond the planck energy scale (Planck mass) of $M_P = \frac{hc}{\sqrt{G}}$. In units where $h$ and $c$ are 1, $M_P = \frac{1}{\sqrt{G}} \simeq 1.22 \times 10^{19}GeV$.

2.1 Problems with Classical Big-Bang Model

Since we cannot deal with extreme temperatures, we start with a temperature lower than the temperature corresponding to Planck energy scale, $T_o \simeq 10^{17}GeV$ in our standard model and assume that the initial universe by this time to be homogeneous, isotropic and at thermal equilibrium at temperature $T_o$. Thus, under these assumptions we can use the Robertson - Walker model to understand the universe,

$$d\tau^2 = dt^2 - a^2(t)[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)]$$ (1)
where \(a(t)\) is the scale factor and the three normalized values of \(k\), \(-1, 0, +1\) correspond to open, critical and closed universe models respectively. The equations of motion for Friedmann-Robertson-Walker universe can be written as,

\[
\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a 
\]

\[
H^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho 
\]

where \(\rho\) and \(p\) are the energy density and pressure terms and \(H\) is the Hubble parameter,

\[H = \frac{\dot{a}}{a}\]

Now, we use the assumption that the universe is filled with massless particles [1] which are at thermal equilibrium at temperature, say \(T\), to get the equations for the state of matter:

Energy Density,

\[
\rho = 3p = \frac{\pi^2}{30}[N_b(T) + \frac{7}{8}N_f(T)]T^4 
\]

Entropy,

\[
s = \frac{2\pi^2}{45}[N_b(T) + \frac{7}{8}N_f(T)]T^3 
\]

Particle number density,

\[
n = \frac{1.20206}{\pi^2}[N_b(T) + \frac{3}{4}N_f(T)]T^3 
\]

where \(N_b(T)\) and \(N_f(T)\) are the number of bosonic and fermionic particles in the universe at that temperature. From now, on we represent \([N_b(T) + \frac{7}{8}N_f(T)]\) by \(N_1\) and \([N_b(T) + \frac{3}{4}N_f(T)]\) by \(N_2\).

Now, we substitute \(\rho\) from Eq. (4) into Eq. (3) and express \(H\) as \(\left(\frac{\dot{T}}{T}\right)\) to get

\[
\left(\frac{\dot{T}}{T}\right)^2 + \frac{k}{a^2T^2}T^2 = \frac{8\pi}{3}G\frac{\pi^2}{30}N_1T^4 
\]

i.e,

\[
\left(\frac{\dot{T}}{T}\right)^2 + \epsilon(T)T^2 = \frac{4}{45}\pi^3GN_1T^4 
\]

where \(\epsilon(T) = \frac{k}{a^2T^2}\).

This can be expressed as,

\[
\epsilon(T) = k\left|\frac{1}{a^3T^3}\right|^2 
\]

i.e,

\[
\epsilon(T) = k\left|\frac{2\pi^2 N_1}{45\pi^2 N_1T^3}\right|^2 
\]
Using Eq. (5), this becomes

\[ \epsilon(T) = k\left[\frac{2\pi^2 N_1}{45a^3s}\right]^\frac{2}{3}. \]  

(11)

But \(a^3s\) is the total entropy in a volume of radius \(a\), the scale factor,

\[ S = a^3s. \]  

(12)

Therefore

\[ \epsilon(T) = k\left[\frac{2\pi^2 N_1}{45S}\right]^\frac{2}{3}. \]  

(13)

### 2.1.1 Adiabatic Universe Approximation

This approximation uses the conservation of entropy in the early universe and gives the expression for the energy density of the universe as,

\[ \left|\frac{\rho - \rho_{cr}}{\rho}\right| = 45\frac{M_P^2}{4\pi^3 N_1T^2} \left|\epsilon\right|. \]  

(14)

where \(\rho_{cr}\) is the critical energy density of the universe.

To explain the evolution of the universe to its current state, we require the total entropy, \(S > 10^{86}\). This condition gives

\[ \left|\epsilon(T)\right| < k\left[\frac{2\pi^2 N_1}{45 \times 10^{86}}\right]^\frac{2}{3}. \]  

(15)

This constrains the energy density equation as

\[ \left|\frac{\rho - \rho_{cr}}{\rho}\right| < k' 10^{-58} N_1^{-\frac{1}{2}} \frac{M_P^2}{T^2} \]  

(16)

where \(k'\) is another constant. Such a small value in the RHS implies that the energy density of the universe was incredibly close to the critical energy density of the universe, at the beginning of Big-Bang, in order to explain the current evolution of the universe. This remarkable fine-tuning of energy density necessary, is known as the flatness problem.

Now, let us consider how causally well connected the early universe was. We represent the radius of the physical horizon, i.e of the region which could be causally connected, by \(l(t)\). The radius of the universe required to evolve into its current state is represented by \(L(t)\). The ratio of the corresponding volumes comes out to be,

\[ \frac{l^3}{L^3} \approx 4 \times 10^{-89} N_1^{-\frac{1}{2}} \left(\frac{M_P}{T}\right)^3. \]  

(17)

where from entropy conservation, we require \(L^3s = L_P^3s_P\), \(L_P\) and \(s_P\) being present radius and entropy.

Now, the RHS of Eq. (17) being so small, implies that very small portion of the early universe was causally connected. Thus the energy density could not
have undergone redistribution and hence would be non-uniform. This would further imply that the cosmic microwave background radiation should have been highly non-uniform which contradicts the observations. This is the horizon problem of classical Big-Bang model.

2.1.2 Non-Adiabatic Early Universe

Now, let us remove the adiabatic assumption and allow for entropy generation in early universe given by

\[ S_P = Z^3 S_0 \]  

(18)

where \( S_0 = a^3 s \) is the initial entropy and \( Z \) is the scale factor for entropy generation. With this, Eq. (15) becomes

\[ |\epsilon(T)| < k \left[ \frac{2\pi^2 N_1}{45 \times 10^{97}} \right]^{\frac{1}{2}} \]  

(19)

This modifies Eq. (16) to

\[ \left| \frac{\rho - \rho_{cr}}{\rho} \right| < k^2 Z^2 10^{-58} N_1^{-\frac{1}{4}} \left( \frac{M_P}{T} \right)^2 \]  

(20)

For \( \left| \frac{\rho - \rho_{cr}}{\rho} \right| \) to be within the current observational range, \( Z \) needs to be of the order of \( 10^{27} \).

3 Classical Inflation

In our non-adiabatic early universe approximation, we saw that a large value for the scale factor of entropy production solves the flatness and horizon problem. Classical inflation [1] provides such a model where such large entropy generation is possible.

3.0.3 Inflationary Universe

We start with the assumption that the equations for state of matter in early universe exhibit a first order phase transition at some critical temperature, \( T_c \). Then just like any phase diagram of thermodynamics would suggest, we can
expect bubbles of low temperature phase, say phase-2 to nucleate and grow in a
universe of initial state, phase-1. Now, if the nucleation rate for this transition
is very low, the universe will not undergo a transition at constant temperature
but instead cools further due to expansion. It will then be supercooled phase-2
bubbles in high-temperature phase-1 universe. If this supercooled temperature,
\( T_s \) is many orders below \( T_c \), then when phase-transition finally takes place at
\( T_s \) the latent heat released will be huge relative to \( T_s \) since latent heat is a
characteristic of the energy scales at \( T_c \), the critical temperature of transition.
This reheats the universe to some temperature \( T_r \) comparable to \( T_c \). This causes
an entropy increase by a factor of \( (\frac{T_r}{T_s})^3 \) while the scale factor of the universe, \( a \)
remains unchanged. Thus in this model, we have

\[
Z = \frac{T_r}{T_s}
\] (23)

For this model to solve the flatness problem and the horizon problem, \( Z \) needs to
be of the order of \( 10^{27} \) and hence the universe should supercool by 27 exponential
orders or more magnitude of temperature below \( T_c \).

3.1 Evolution of Universe in Classical Inflation

Classical inflation explains the supercooling of the early universe by proposing
a model in which the early universe was in a state of false vacuum characterized
by a scalar field in a local minimum of its potential energy function. In such
a model, as the temperature drops and \( T \to 0 \), the universe cools not towards
true vacuum but towards the false vacuum state of energy density \( \rho_o \), which is
greater than the true vacuum energy density.

Therefore Eq. (4) for energy density becomes

\[
\rho(T) = \frac{\pi^2}{30} N_1 T^4 + \rho_o.
\] (24)

Therefore Eq. (3) now takes the form

\[
\left( \frac{\dot{T}}{T} \right)^2 + \epsilon(T) T^2 = \frac{4 \pi^3}{45} G N_1 T^4 + \frac{8 \pi}{3} G \rho_o.
\] (25)

Based on the value of \( \epsilon \) this equation will have two types of solution. If
\( \epsilon > \epsilon_o \), where

\[
\epsilon_o = \frac{8 \pi^2}{45} \sqrt{30 G \sqrt{N_1 \rho_o}}
\] (26)

Then the universe will have an expanding phase which halts when temperature
reaches \( T_{min} \) given by

\[
T_{min}^4 = \frac{30}{\pi^2 \rho_o} \left[ \frac{\epsilon}{\epsilon_o} + \left( \frac{\epsilon}{\epsilon_o} \right)^2 - 1 \right]^2
\] (27)

After this, the universe starts to contract.
For the case when $\epsilon < \epsilon_o$, we can consider it to mainly be $\epsilon < 0$ since $\epsilon_o \simeq 0$. In this case we get

$$T(t) = \text{Const.} e^{-\chi t},$$

(28)

where

$$\chi^2 = \frac{8\pi}{3} G \rho_o.$$  

(29)

But since $aT = \text{constant}$, we get

$$a(t) = \text{Const.} e^{+\chi t}.$$  

(30)

Since this will be the universe which will reach observable sizes, we are interested in the second solution which has an inflationary phase of exponential growth. Thus we get a universe which is exponentially expanding in a false vacuum state of energy density $\rho_o$. For considerable flattening of the universe, we need an exponential expansion of at least order 60.

### 3.1.1 Shortcomings of Classical Inflation

Classical Inflation is based on several assumptions which do not come naturally in the model. The nucleation rate needs to be very slow compared to the expansion rate of the universe in order for $Z$ to be of the order $10^{27}$. Also, the randomness of bubble formation in analogy with thermodynamic situations should have led to inhomogeneities in the universe.

Also, the state of false vacuum would be stable in classical theory since there would be no energy available to allow the scalar field to cross the potential barrier that separates it from the lower energy states. However this state would decay through quantum mechanical tunneling and thereby ending inflation. Since this decay of false vacuum due to quantum tunneling is random, it would lead to large inhomogeneities in the universe.

Further, when classical inflation was proposed, the dark-matter and dark-energy problems were not linked to cosmological model. However it has now become necessary for inflation to answer the puzzles about the production and nature of dark-matter as well as dark energy.

### 4 Chaotic Inflation

Chaotic Inflation was proposed to solve the problems faced by classical inflation [2]. It assumes that the initial state of the universe was in a state of quantum chaos. If this chaotic state can produce a large enough scalar field $\phi$ then inflation can occur and bring about the same effects which classical inflation does, as the scalar field relaxes from its initial state.
4.1 Evolution of Universe in Chaotic Inflation - $\lambda \phi^4$ model

Here we consider a scalar field potential [4] given by

$$V(\phi) = \frac{1}{4} \lambda \phi^4.$$  \hspace{1cm} (31)

The energy density will be proportional to $(\frac{\partial \phi}{\partial x^\mu})^2$, with $x^\mu$ representing the four co-ordinates.

A classical description of the universe is possible only after the energy density becomes smaller than $M_P^4$, i.e after planck time of $\frac{1}{M_P}$, where $M_P = \frac{1}{\sqrt{\alpha}}$. This constraint implies that the potential at $t = \frac{1}{M_P}$ should satisfy

$$V(\phi) \leq M_P^4.$$  \hspace{1cm} (32)

i.e

$$- \left( \frac{4}{\lambda} \right)^2 M_P \leq \phi \leq + \left( \frac{4}{\lambda} \right)^2 M_P.$$  \hspace{1cm} (33)

Before this time, the universe is assumed to be in a chaotic quantum state.

Let us now consider a locally homogeneous region in such early universe. This part is treated as expanding De-Sitter space with scale factor $a(t) = a_0 e^{Ht}$ with

$$H = \sqrt{\frac{8\pi GV}{3}}.$$  \hspace{1cm} (34)

From the equations of motion in De-Sitter space, we have

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} = -\frac{\partial V}{\partial \phi} = -\lambda \phi^3.$$  \hspace{1cm} (35)

Assuming that $\frac{M_P^4}{6\pi} \approx \phi^2$, we write the RHS as,

$$- \lambda \phi^3 = \lambda \phi(-\phi^2) \approx \lambda \phi(M_P^2 - \phi^2)$$  \hspace{1cm} (36)

$$- \lambda \phi^3 \approx \frac{\lambda \phi M_P^2}{6\pi} - \lambda \phi^3.$$  \hspace{1cm} (37)

And the LHS can be written as

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial t^2} + 3 \sqrt{\frac{2\pi \lambda}{3 M_P}} \phi \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (38)

i.e

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial t^2} + \sqrt{\frac{6\pi \lambda}{M_P^2}} \phi^2 \frac{\partial \phi}{\partial t}.$$  \hspace{1cm} (39)

Equating the LHS and the RHS, we get

$$\frac{\partial^2 \phi}{\partial t^2} + \sqrt{\frac{6\pi \lambda}{M_P^2}} \phi^2 \frac{\partial \phi}{\partial t} = \frac{\lambda \phi M_P^2}{6\pi} - \lambda \phi^3.$$  \hspace{1cm} (40)
Equating the second term on both sides gives both derivatives of $\phi$ in terms of $\phi$,
indicating an exponential behaviour of $\phi$

$$\sqrt{\frac{6\pi\lambda}{M_P^2}} \phi^2 \frac{\partial \phi}{\partial t} = -\lambda \phi^3$$

(41)

which gives the solution as

$$\phi = \phi_o e^{-\sqrt{\frac{\lambda M_P}{6\pi}}}$$

(42)

Putting this in $\frac{\partial^2 \phi}{\partial t^2}$, we can verify that

$$\frac{\partial^2 \phi}{\partial t^2} = \phi_o \left( \sqrt{\frac{\lambda M_P}{6\pi}} \right)^2 e^{-\sqrt{\frac{\lambda M_P}{6\pi}}}$$

(43)

i.e,

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{\lambda \phi M_P^2}{6\pi}.$$  

(44)

Thus we take $\phi = \phi_o e^{-\sqrt{\frac{\lambda M_P}{6\pi}}} t$ as a solution. Here we used the assumption
of $\phi^2 \gg \frac{1}{\sigma \phi}$ which is valid only when $\lambda \ll 1$ and $V(\phi) \leq M_P^4$.

From our solution we see that $\phi$ has a time constant of $t_c = \frac{\sqrt{6\pi}}{M_P \sqrt{\lambda}}$. Therefore
in one time constant the universe expands by

$$a(t_c) = a_o e^{H t_c}$$

(45)

$$a(t_c) = a_o e^{\phi_o^2 e^{-2} \sqrt{\frac{M_P^2}{3M_P^2}}} \sqrt{\frac{M_P}{M_P^4}}$$

(46)

$$a(t_c) \simeq a_o e^{\frac{2\pi \phi_o^2}{M_P^2}}.$$  

(47)

Now, for inflation to flatten the universe considerably in this period we require an expansion of at least 60 e-foldings, i.e $a(t_c) \geq a_o e^{60}$. Therefore

$$\frac{2\pi \phi_o^2}{M_P^2} \geq 60$$

(48)

i.e,

$$\phi_o \geq 3.09 M_P.$$  

(49)

Now considering the two constraints which we derived in our model,

$$\phi \leq \left( \frac{4}{\lambda} \right)^{\frac{1}{4}} M_P$$

(50)

and

$$\phi_o \geq 3.09 M_P$$

(51)
we see that $\lambda$ should be very small for both the constraints to be satisfied.

Taking $\phi \approx \phi_o$, we approximate $\lambda$ as

$$3.1 M_P \leq \left( \frac{4}{\lambda} \right)^{\frac{1}{4}} M_P$$

$$\Rightarrow \lambda < 0.04 \quad (53)$$

The rapid exponential rate suggests that once such a patch of homogeneous and isotropic field is formed, it expands rapidly to dominate the physical volume of the universe. This naturally accounts for the homogeneity of the observed universe.

The energy density at the end of this initial inflationary phase will be

$$\rho_o \approx \frac{1}{4} \lambda \phi_o^4 > 22.82 \lambda M_P^4$$

Therefore for $\lambda \approx 0.04$ we have

$$\rho_o \approx 0.91 M_P^4 < M_P^4$$

thereby satisfying our constraint on the energy density.

### 4.2 Evolution of Universe in Chaotic Inflation - $m^2 \phi^2$ model

Since the recent WMAP results [3] have favoured the $\frac{1}{2} m^2 \phi^2$ model, we now consider a potential of the form

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

as the starting potential in the early universe [4]. We use $V(\phi) \leq M_P^4$ to get

$$- \left( \frac{2}{m^2} \right)^{\frac{1}{4}} M_P^2 \leq \phi \leq \left( \frac{2}{m^2} \right)^{\frac{1}{4}} M_P^2.$$  

(57)

Proceeding in the same way as we did for the $\lambda \phi^4$ model, we get the equation of motion as

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} = - \frac{\partial V}{\partial \phi} = -m^2 \phi.$$  

(58)

Substituting for $H$ as

$$H = \sqrt{\frac{8\pi GV}{3}} = \sqrt{\frac{8\pi G m^2 \phi^2}{6}}$$

we get

$$\frac{\partial^2 \phi}{\partial t^2} + \sqrt{12\pi G m \phi} \frac{\partial \phi}{\partial t} = -m^2 \phi$$

which can be solved to get $\phi$ as

$$\phi = \phi_o - \frac{m}{\sqrt{12\pi G}} t.$$  

(61)
Now, since we have \( a(t) = a_0 e^{Ht} \), we calculate how much the universe has expanded by the time the scalar field relaxes to 0 as

\[
a(t) = a_0 e^N
\]

where the exponent \( N \) is given by

\[
N = \int_{\phi=\phi_0}^{\phi=0} H(t) dt = 2\pi G \phi_0^2.
\]

Since we require the universe to have undergone an expansion of at least 60 e-foldings, we require \( N \geq 60 \). Therefore

\[
\phi_0 \geq \sqrt{\frac{60}{2\pi G}}.
\]

We compare Eq. (57) and Eq. (64) by setting \( \phi \approx \phi_0 \) to get

\[
\sqrt{\frac{60}{2\pi G}} \leq \left( \frac{2}{m^2} \right)^{\frac{3}{2}} M_P^2
\]

giving the limit on \( m \) as

\[
m \leq \sqrt{\frac{4\pi}{60G}}.
\]

Thus we again have a theory which allows for an inflationary phase in the evolution of the universe. However we see here that the condition on \( m \) is not as stringent as it was on \( \lambda \) in the \( \lambda \phi^4 \) model.

5 Inflationary Model Consistent with \( \Lambda \)-CDM & WMAP

After WMAP results [3 - Section 6] have been published, revisions of inflationary models are being actively published. Some of the attempts inspect the inclusion of dark matter and dark energy based on WMAP results [14][6]. Our model also contrasts with [6][8] in which the goal of including dark matter, dark energy and inhomogeneity is addressed in a different way. We develop a model consistent with the recent cosmological observations and based on the hypothesis from \( \Lambda \)-CDM model.

5.1 Overview of modifications to standard models

We work with a pair \( \phi_1, \phi_2 \) of primal scalar fields which transform to \( \phi_1, \phi_2 \) by

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta_{SS} & \sin \theta_{SS} \\
-\sin \theta_{SS} & \cos \theta_{SS}
\end{pmatrix} \begin{pmatrix}
\phi_o \\
\phi_o
\end{pmatrix}
\]

such that the action is defined by the potential,
\[ V = V_o - \frac{1}{2} \mu^2 \phi^\dagger \phi + \frac{1}{4} \lambda (\phi^\dagger \phi)^2 \]  

(68)

where

\[ \phi^\dagger \phi = \left( \phi_o^+ \phi_o \right) \left( \phi_o \phi_o^+ \right) = \left( \phi_1^* \phi_2^* \right) \left( \phi_1 \phi_2 \right) \]  

(69)

Since this is dependent only on even power terms, invariance of even power terms after transformation will ensure that the universe evolves in an identical way.

In this formulation, we have exact SUSY for \( \theta_{SS} = \frac{\pi}{4} \). As \( \theta_{SS} \to 0 \) or \( \frac{\pi}{2} \), we have broken SUSY with one or the other field preferred. The correct \( \theta_{SS} \) resulting in partially broken SUSY will be obtained from matching the derived results with observations.

In this model, we intend to show that normal matter and radiation are generated by \( \phi_2 \) and the dark matter by \( \phi_1 \). [This step is guided by numerous examples in physics where we have doubling and mixing of particles and fields such as \( (\kappa_o, \bar{\kappa}_o), (\nu_e, \nu_\mu) \), W-S model, particle-antiparticle et al due to the breaking of some underlying symmetry.] Here, the \( \phi_1 - \phi_2 \) doublet due to broken SUSY will have different masses, numbers and lifetimes of their particles. So \( \phi_2 \) gives rise to normal matter particles and \( \phi_1 \) to their supersymmetric partners. This model has a \( \theta_{SS} \) parameter in addition to \( \mu \) and \( \lambda \) with which the observed ratio of normal matter to dark matter, 1 : 5 and the production rate of supersymmetric particles is handled.

The dark energy problem is tackled by identifying it with the cosmological constant \( \Lambda \) of the universe which is taken as the vacuum expectation values of the stress-energy tensor. Further, \( \langle T_{\mu\nu} \rangle_{\text{vac}} \equiv \Lambda g_{\mu\nu} \) for the \( p = -\rho \) equation of state for dark energy as they are the only locally lorentz covariant form consistent with the recently found accelerating phase of the universe [12][13].

The classical cosmological models use a constant field \( \phi \) for homogeneous, isotropic models of the universe. However, the fluctuation in the scalar field, \( \phi \to \phi + \delta \phi \) may give rise to anisotropy and inhomogeneity as observed in COBE and WMAP data viz \( \frac{dT}{T} \approx \frac{dp}{p} \approx \frac{d\rho}{\rho} \approx 10^{-5} \). To do this classically we need a stochastic inflation model. But as \( E \) evolves from \( 10^{19} \) GeV after \( 10^{-43} \) s from Big-Bang to \( 10^3 \) GeV corresponding to the QGP baryogenesis at \( 10^2 \) s after Big-Bang, we have \( E \approx pc \) for most particles and hence \( \lambda_{DB} \approx \frac{h}{p} \approx \frac{h}{E} \). So \( \lambda_{DB} \) is comparable to the size of the universe during inflation. Hence quantum effects should dominate and the fluctuations should be calculated quantum mechanically viz for any observable \( A \),

\[ \Delta \hat{A} = [\langle A^2 \rangle - \langle A \rangle^2]^{\frac{1}{2}} \]  

(70)

For our purpose, we will be handling the average values for the number operator and the hamiltonian. Hence the observed fluctuations at 3,00,000 year observable horizon must arise from the ratio \( \frac{\Delta N}{N} \) and \( \frac{\Delta H}{H} \) found. [These fluctuations are correlated with the large scale structure formation of galaxies, voids and cluster filaments as seen in early universe. These fluctuations are also expected...
to be the remnant effects of gravity wave spectrum that arose from Big-Bang
and is to be confirmed by the Planck satellite].

Some of the parameters in our model, namely $\mu, \lambda$ and $\theta_{SS}$ can also be fixed
using this observed and calculated fluctuations. Our model also contrasts with
\cite{6}\cite{8} in which the goal of including dark matter, dark energy and inhomogeneity
is addressed in a different way.

5.2 The Scalar Field Doublet Model for Inflation

We start with a complex pair of primal scalar fields $\phi_o, \bar{\phi}_o$ which could be the
remnant of the pre-inflationary epoch. We then define a doublet \(|\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}\) as the
driving fields for inflation and generated by a transform that mixes $\phi_o, \bar{\phi}_o$ as
inflation proceeds,

\[
\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{SS} & \sin \theta_{SS} \\ -\sin \theta_{SS} & \cos \theta_{SS} \end{pmatrix} \begin{pmatrix} \phi_o \\ \bar{\phi}_o \end{pmatrix}. \tag{71}
\]

Now calculating the energy terms for these new fields, we find that

\[
\phi_1^\dagger \phi_1 = \begin{pmatrix} \cos \theta_{SS} & \sin \theta_{SS} \end{pmatrix} \begin{pmatrix} \phi_o \\ \bar{\phi}_o \end{pmatrix} \begin{pmatrix} \cos \theta_{SS} & \sin \theta_{SS} \end{pmatrix} \begin{pmatrix} \phi_o \\ \bar{\phi}_o \end{pmatrix} = \phi_o \bar{\phi}_o. \tag{72}
\]

which reduces to

\[
\phi_1^\dagger \phi_1 = (\cos^2 \theta_{SS} + \sin^2 \theta_{SS})\phi_o \bar{\phi}_o = \phi_o \bar{\phi}_o. \tag{73}
\]

Similarly we have for $\phi_2$

\[
\phi_2^\dagger \phi_2 = \phi_o \bar{\phi}_o. \tag{74}
\]

Thus we see that this transform ensures that the energy terms remain the
same as the original complex field ensuring that inflation proceeds in the same
way as it would in a model without such a symmetry breaking, but now with a
doublet scalar field \(|\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}\).

Starting with these two fields, we go on to define the potential in the early
universe as

\[ V_1 = \frac{1}{4} \lambda_1 \phi_1^4 \tag{75} \]
\[ V_2 = \frac{1}{4} \lambda_2 \phi_2^4. \tag{76} \]

These potentials when used in the standard $\lambda \phi^4$ inflationary model give the
solution as

\[
\phi_1 = \phi_{1_0} e^{-\sqrt{\frac{\lambda_1 M_P^2}{\phi_{1_0}}} t} \tag{77}
\]

and

\[
\phi_2 = \phi_{2_0} e^{-\sqrt{\frac{\lambda_2 M_P^2}{\phi_{1_0}}} t}. \tag{78}
\]
respectively. Here we observe that the rate at which fields decay is proportional to $\sqrt{\lambda}$, i.e.
\[
\frac{\partial \phi_i}{\partial t} \propto \sqrt{\lambda_i} \quad i = 1, 2.
\]  

[Case:2]
\[
V_1 = \frac{1}{2} m_1^2 \phi_1^2
\]  
\[
V_2 = \frac{1}{2} m_2^2 \phi_2^2.
\]

Again using these potentials in the standard $m^2 \phi^2$ model, we obtain the solutions
\[
\phi_1 = \phi_{1o} - \frac{m_1}{\sqrt{12\pi G}} t
\]  
and
\[
\phi_2 = \phi_{2o} - \frac{m_2}{\sqrt{12\pi G}} t
\]
respectively. Here we have the rate at which fields decay proportional to $m$, i.e.
\[
\frac{\partial \phi_i}{\partial t} \propto m_i \quad i = 1, 2.
\]

This decay of the field results in the creation of massless particles that are produced during the inflationary phase. Accordingly, in our model we have $\phi_2$ generating the normal matter particles and $\phi_1$ giving rise to their supersymmetric partners. Based on recent observations and the $\Lambda$-CDM model, we go by the hypothesis that the dark matter content in our universe is predominantly due to massive supersymmetric partners, in particular neutralino which is the lightest and most stable of the supersymmetric particles. We can therefore relate the decay rate of $\phi_1$ and $\phi_2$ to the creation rate, $\Gamma$ of dark matter (DM) and normal matter (NM) respectively,
\[
\Gamma_{DM, NM} \propto \frac{\partial}{\partial t} \phi_{1, 2}.
\]

Now, in order to fix the values of the coupling coefficients in our model, we calculate the predicted density fluctuation and use the observed value of $\approx 10^{-5}$ from the recent WMAP results. These fluctuations can be evaluated using either stochastic models, second quantization technique or by using the path integral ensemble average.

We use the second quantization approximation for the density fluctuation, $\delta H$ to obtain a limit on these coefficients [31],
\[
\delta H \approx \frac{H^2}{2\pi \phi}
\]
Since we require the density fluctuation for normal matter, we evaluate it for $\phi_2$. For the $\lambda \phi^4$ model, we have
\[ \dot{\phi}_2 = -\frac{\lambda_2 \phi_2^3}{3H} \]  

Therefore

\[ \delta_H \approx \frac{3H^3}{2\pi \lambda_2 \phi_2^2} \]  

which reduces to

\[ \delta_H \approx \frac{1}{2\pi} \sqrt{\frac{\lambda_2}{3}} \left( \frac{2\pi \phi_2^2}{M_P^2} \right)^2 \approx 10^{-5} \]  

With \( \phi_2 \approx \phi_2^* \), we see that the term \( \left( \frac{2\pi \phi_2^2}{M_P^2} \right)^2 \) is the exponent in Eq. (47) and therefore needs to be \( \geq 60 \). This gives the limit for \( \lambda_2 \) as

\[ \lambda_2 \leq \frac{12\pi^2 \times 10^{-10}}{60^3} \]  

\[ \lambda_2 \leq 54.83 \times 10^{-15}. \]  

Similarly, for the \( m^2 \phi^2 \) model, we get

\[ \dot{\phi}_2 = -\frac{m_2 \phi_2^3}{3H} \]  

which gives the density fluctuation as

\[ \delta_H \approx \frac{6m_2}{M_P \pi^2} \left( \frac{\pi}{3} \frac{2\pi G \phi_2^2}{M_P^2} \right)^2 \]  

Again we have \( 2\pi G \phi_2^2 \geq 60 \) which constraints \( m_2 \) as

\[ m_2 \leq 2.5583 \times 10^{-7} M_P. \]  

Now, we use our hypothesis that \( \phi_1 \) generates supersymmetric particles that constitute dark matter and \( \phi_2 \) generates normal matter particles. These particles produced during inflation acquire mass because of their interaction with the Higgs Field which gives mass to normal particles and the Higgsino Field which gives mass to supersymmetric particles, in the post-inflationary epoch. We take the standard normal matter particle as a proton which has an energy of 1 GeV and the standard supersymmetric particle as the neutralino with energy around 100 GeV. We now estimate the ratio of mass of dark matter to the mass of normal matter as

\[ \frac{M_{DM}}{M_{NM}} \propto \frac{\Gamma_{DM} \times M_{\text{neutralino}}}{\Gamma_{NM} \times M_{\text{proton}}} \]  

For \( \lambda \phi^4 \) model, this becomes

\[ \frac{M_{DM}}{M_{NM}} \propto \sqrt{\frac{\lambda_1 \times 100 \text{ GeV}}{\sqrt{\lambda_2} \times 1 \text{ GeV}}} \]  

\[ \text{(95)} \]  

\[ \text{(96)} \]
Recent observations of WMAP give us the $M_{DM}:M_{NM}$ ratio as 22:4. Therefore

$$\sqrt{\lambda_1} = \frac{22}{400}\sqrt{\lambda_2} \quad (97)$$

Using Eq. (91), this limits $\lambda_1$ as

$$\lambda_1 \leq 16.586 \times 10^{-17} \quad (98)$$

Similarly for $m^2\phi^2$ model, we get

$$\frac{M_{DM}}{M_{NM}} \propto \frac{m_1}{m_2} \times \frac{100 \text{ GeV}}{1 \text{ GeV}} \quad (99)$$

Therefore we get from Eq. (94)

$$m_1 \leq 1.407065 \times 10^{-8} M_P. \quad (100)$$

Now that we have fixed the limits for $\lambda_1, \lambda_2 / m_1, m_2$ in our model, we have a model involving a doublet scalar field, driving inflation and resulting in the production of normal particles and their supersymmetric partners in the right ratio to agree with the observed ratio of dark matter to normal matter. The remnant of these fields can be considered to transmute into the Higgs and Higgsino fields after the inflationary epoch when the energy scale of the universe falls below a critical value.

5.2.1 Source of Density Fluctuations

As the universe evolves from the scale of $10^{-35}$ m of initial size at $10^{-43}$ s, we see that such small scales inevitably lead to quantum effects dominating the evolution of the scalar field. Therefore we expect quantum fluctuations in the scalar field leading to fluctuations in the Hamiltonian which gets expressed as the fluctuations in the number density of the particles produced during inflation. When these particles get mass in the post-inflationary epoch, these fluctuations get manifested as density fluctuations.

Since we are dealing with massless particles, we take only the potential in evaluating the expectation value of the Hamiltonian, $\langle H \rangle$.

For the $\lambda \phi^4$ model,

$$\langle H \rangle = \int D[\phi] \frac{1}{4} \lambda \phi^4 e^{-\frac{1}{2} \lambda \phi^4} \quad (101)$$

Making the substitution $\frac{1}{2} \lambda \phi^4 = y$, we can reduce the integral to

$$\langle H \rangle = \frac{\lambda}{16} \frac{4}{\lambda} \frac{1}{2} \Gamma \left(1 + \frac{1}{4}\right). \quad (103)$$
Similarly, we get $\langle H^2 \rangle$ by making similar substitution and simplifying the integral,

$$\langle H^2 \rangle = \int D[\phi] \frac{1}{16} \lambda^2 \phi^8 e^{-\frac{1}{4} \lambda \phi^4}$$  \hspace{1cm} (104)

$$\langle H^2 \rangle = \frac{\lambda^2}{64} \left( \frac{4}{\lambda} \right)^{\frac{3}{2}} \Gamma\left(1 + \frac{5}{4}\right)$$  \hspace{1cm} (105)

Now we use $\langle \Delta H \rangle = [\langle (H^2)^2 \rangle - ( \langle H \rangle )^2]^{\frac{1}{2}}$ to obtain the value of $\langle \Delta H \rangle / \langle H \rangle$ as

$$\frac{\langle \Delta H \rangle}{\langle H \rangle} = \frac{1}{4^{\frac{3}{4}}} \sqrt{5.5163 \lambda^\frac{3}{4} - 4^{\frac{3}{4}}}.$$  \hspace{1cm} (106)

This allows us to limit the value of $\lambda$ in the same way as second quantization scheme.

Similarly for $m^2 \phi^2$ model, we evaluate $\langle H \rangle$ as

$$\langle H \rangle = \int D[\phi] \frac{1}{2} m^2 \phi^2 e^{-\frac{1}{2} m^2 \phi^2}$$  \hspace{1cm} (107)

Making the substitution $\frac{1}{2} m^2 \phi^2 = y$, we can reduce the integral to

$$\langle H \rangle = \frac{m^2}{4} \left( \frac{2}{m^2} \right)^{\frac{3}{2}} \Gamma\left(1 + \frac{1}{2}\right)$$  \hspace{1cm} (108)

and similarly $\langle H^2 \rangle$ becomes

$$\langle H^2 \rangle = \frac{m^4}{8} \left( \frac{2}{m^2} \right)^{\frac{3}{2}} \Gamma\left(2 + \frac{1}{2}\right)$$  \hspace{1cm} (109)

We then calculate $\frac{\langle \Delta H \rangle}{\langle H \rangle}$ to obtain

$$\frac{\langle \Delta H \rangle}{\langle H \rangle} = 2.3936 m - 1.$$  \hspace{1cm} (110)

The $\delta T \approx 10^{-5}$ should be given by $\frac{\langle \Delta H \rangle}{\langle H \rangle}$. From this we get $m$ by using the density fluctuation values from observations.

5.3 Inflation with cosmological constant as dark energy

The current observations show that our universe is under a late acceleration phase attributed to dark energy which has an equation of state $p = -\rho$, the only locally lorentz covariant form consistent with the observed late acceleration phase of the universe. This requires us to include dark energy into cosmological models. We propose here a model where along with the false vacuum energy state towards which the universe cools as $T \to 0$, we have an additional potential in the universe because of the vacuum fluctuations in space-time. We take the expectation value of the stress-energy tensor, $\langle T_{\mu\nu}\rangle_{\text{vac}} \equiv \Lambda g_{\mu\nu}$ to be the cosmological constant which results in an addition of $\Lambda$ to energy density.
Therefore Eq. (4) for energy density becomes

\[ \rho(T) = \frac{\pi^2}{30} N_1 T^4 + \rho_o + \frac{\Lambda}{8\pi G} \]  
(111)

Therefore Eq. (3) now takes the form

\[ \left(\frac{\dot{T}}{T}\right)^2 + \epsilon(T) T^2 = \frac{4\pi^3}{45} GN_1 T^4 + \frac{8\pi}{3} G \rho_o + \frac{\Lambda}{3} \]  
(112)

Calculations similar to that shown for classical inflation we get two solutions based on the value of \( \epsilon \). If \( \epsilon > \epsilon_o \), where

\[ \epsilon_o = \frac{8\pi^2}{45} \sqrt{30G} \sqrt{N_1 (\rho_o + \frac{\Lambda}{8\pi G})} \]  
(113)

Then the universe will have an expanding phase which halts when temperature reaches \( T_{\text{min}} \) given by

\[ T_{\text{min}}^4 = \frac{30}{\pi^2} \left( \rho_o + \frac{\Lambda}{8\pi G} \right) \left[ \frac{\epsilon}{\epsilon_o} + \left\{ \left( \frac{\epsilon}{\epsilon_o} \right)^2 - 1 \right\}^{\frac{3}{2}} \right]^\frac{2}{3} \]  
(114)

After this, the universe starts to contract.

For the case when \( \epsilon < \epsilon_o \), we again consider it to mainly be \( \epsilon < 0 \) since \( \epsilon_o \approx 0 \). In this case we get

\[ T(t) = \text{Const.} e^{-\chi t} \]  
(115)

where we now have

\[ \chi^2 = \frac{8\pi}{3} G \rho_o + \frac{\Lambda}{3} \]  
(116)

and

\[ a(t) = \text{Const.} e^{\sqrt{\frac{8\pi}{3} G \rho_o + \frac{\Lambda}{3}}} t \]  
(117)

In early universe, the \( \rho_o \) term dominates resulting in inflation. Whereas in late universe, it is the cosmological constant term which has a role to play resulting in slow acceleration phase of the universe which we are observing.

5.3.1 Evolution of universe in the new model

In Guth’s original work with classical inflation, the initial false vacuum state of energy density \( \rho_o \) drives the inflation of the early universe. However, as the universe evolves \( \rho_o \) relaxes and decreases in value whereas the density of the universe increases bringing inflation to a halt when the acceleration due to false vacuum energy density is overcome by the deceleration due to the density increase of the universe.

However, in our model, initially since the value of the cosmological constant is very small, its the \( \rho_o \) term which dominates and guides the expansion. Again as \( \rho_o \) decreases and density increases, inflation comes to a halt. But here, as time
evolves though the false vacuum state decays to zero, the cosmological constant
term still remains though very small. When the density of the universe falls
below certain critical value due to the expansion of the universe, the $\Lambda$ term
again begins to dominate and drives the universe to an accelerating phase.
But the small value of $\Lambda$ implies that this acceleration needs to be very small
compared to that during the initial inflationary phase of the early universe.

The current observations about our universe suggest exactly similar behavior
about our universe and this strongly prompts us to consider a cosmological
model with the cosmological constant included.

6 Results and Findings

The constants and coefficients in our inflationary model using doublet scalar
fields are calibrated using the observational values about

- The composition of normal and dark matter in the universe.
- The mass ratio of neutralino and proton.
- The WMAP results for density fluctuations in early universe.

We obtain the following constraints on our parameters in our model:

1. $\lambda_2 \leq 54.83 \times 10^{-15}$ for the scalar field giving rise to Normal Matter in $\lambda\phi^4$ model.
2. $\lambda_1 \leq 16.586 \times 10^{-17}$ for the scalar field giving rise to Dark Matter in $\lambda\phi^4$ model.
3. $m_2 \leq 2.5583 \times 10^{-7}M_P$ for the scalar field giving rise to Normal Matter in $m^2\phi^2$ model.
4. $m_1 \leq 1.407065 \times 10^{-8}M_P$ for the scalar field giving rise to Dark Matter in $m^2\phi^2$ model.

Further, we see that the remnant scalar field doublet at the end of inflationary
epoch provides a starting platform for the appearance of Higgs and Higgsino
fields in the post-inflationary era. Also the Higgs field potential,

$$V = V_0 - \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

(118)
can be generated by a linear combination of the two potentials discussed with
$m^2 \rightarrow -\mu^2$. The mass of particles will be proportional to the mass of the Higgs
boson and the term $\sqrt{\frac{\mu^2}{\lambda}}$.

Thus we have a model which consistently provides some of the initial ele-
ments required in the post inflationary epoch.
7 Conclusions

In this paper, we have undertaken a detailed study of standard and modern inflationary models. Analysis of recent cosmological observations and the WMAP results have created a need to revise and develop improved models for inflation which gives an integrated answer to the questions of dark-matter, dark energy and the cosmological constant problem.

We propose here a model built on simple assumptions which allow us to include supersymmetry into inflationary models and hence provide an explanation for the observed dark matter to normal matter ratio. We go by the hypothesis that supersymmetric partners of normal matter particles are the best candidates for dark matter and proceed to show how inflation asymmetrically produces normal and supersymmetric particles during the inflationary phase.

We fix the limits on the coefficients in our model by calculating the density fluctuation predicted by our model and matching it with the observed value of $10^{-5}$. Further, we include the cosmological constant in classical inflation model as a correction term to the false vacuum energy state which drives inflation. Because the value of cosmological constant is small, this correction term is negligible in early universe when both the false vacuum state as well as the energy density of the universe were large. But as the density of the universe falls due to the expansion of the universe, we have the cosmological constant term dominating the evolution of our universe and hence resulting in a slow late acceleration of the universe.

Though we develop our model on the assumptions of a doublet primal scalar field, supersymmetric particles as the best dark matter candidate and cosmological constant as dark energy, all these assumptions are supported by similar symmetry existing in related branches of high energy physics and the recent observational evidences about the evolution of universe.

However, the nature of connection between the scalar fields of inflation and the Higgs field is an open question which might need LHC Higgs mass and astroparticle physics data and models. More precise data is expected from the PLANCK mission as well as the LHC project which will help in further narrowing the range of reasonable cosmological models.

We thank HBCSE, TIFR for giving us the opportunity to work on NIUS project.

References

[1] A. H. Guth, Phys. Rev. D 23, 347 (1981).
[2] A. D. Linde, Phys. Lett. B 108, 389 (1982).
[3] D. N. Spergel et al., astro-ph/0603449.
[4] A. H. Guth, Phys.Rept. 333, 555-574 (2000).
[5] A. H. Guth. astro-ph/0101507.

[6] Jun-Quing Xia et al., JCAP 0609, 015 (2006).

[7] Robert J. Nemiroff & Bijunath Patla. astro-ph/0409649

[8] S. Capozziello et al., Phys.Lett. B 632, 597-604 (2006).

[9] S. F. King, hep-ph/0110385.

[10] G. Lazarides & C. Pallis, hep-ph/0406081

[11] A. Albrecht and P. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[12] Varun Sahni, astro-ph/0403324

[13] Varun Sahni, Lecture Notes

[14] Laura Covi et al., Phys. Rev. D 74, 083509 (2006).

[15] Bernard F Schutz - A first course in General Relativity

[16] Robert M Wald - General Relativity

[17] A. Guth - Phys. Rev. D23(1981) p347-356

[18] A. Guth - Phys. Rev. Lett 49(1982) p1110-1113

[19] A. Guth - Phys. Rev D32(1985) p1899-1920

[20] A. Linde - Phys. Lett. 108B(1982) p389-393

[21] A. Linde - Phys. Lett. 129B(1983) p177-181

[22] P. Steinhardt, M. Turner Phys. Rev. D29(1984) p2162-2171

[23] J. S. Townsend - Modern Approach to Quantum Mechanics

[24] Greiner , Reinhardt - Field Quantization

[25] Leonard I Schiff - Quantum Mechanics

[26] Richard P Feynman - Feynman Lectures in Physics

[27] Goldstein, Poole and Safko - Classical Mechanics

[28] J J Sakurai - Modern Quantum Mechanics

[29] Schwabl F - Advanced Quantum Mechanics

[30] A. Zee - Quantum Field Theory in a nutshell

[31] John A. Peacock - Cosmological Physics

[32] Ajay Patwardhan - quant-ph/0305150 hep-th/0606080 hep-th/0406049