Probing for cosmological parameters with LAMOST measurement

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Abstract. In this paper we study the sensitivity of the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) project for the determination of cosmological parameters, employing the Monte Carlo Markov chains method. For comparison, we first analyze the constraints on cosmological parameters from current observational data, including those from the Wilkinson Microwave Anisotropy Probe, Sloan Digital Sky Survey and SNIa (type Ia supernovae). We then simulate the 3D matter power spectrum data expected from LAMOST, together with the simulated cosmic microwave background data for PLANCK and the SNIa data from the five-year Supernovae Legacy Survey. With the simulated data, we investigate the future improvement of cosmological parameter constraints, emphasizing the role of LAMOST. Our results show the potential of LAMOST in probing for the cosmological parameters, especially in constraining the equation of state of the dark energy and the neutrino mass.

Keywords: surveys of galaxies, cosmological simulations

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1. Introduction

The measurement of the large scale galaxy clustering has been an important probe in constraining cosmological models. The large scale structure (LSS) measurements have made remarkable progress from observational efforts such as 2dFGRS and the Sloan Digital Sky Survey (SDSS), which have provided an accurate measurement of the galaxy power spectrum and given a robust constraint on cosmological parameters [1, 2].

The LAMOST [3] project is a 4 m quasi-meridian reflecting Schmidt telescope laid down on the ground. It has a 5° field of view, and may accommodate as many as 4000 optical fibers, and the light from 4000 celestial objects will be led into a number of spectrographs simultaneously. Thus the telescope will possess the highest spectrum acquiring rate in the world. The spectroscopic survey, which contains the information about the radial positions of galaxies, can probe the 3D distribution of galaxies effectively. In this paper, we study the sensitivity of LAMOST for the determination of cosmological parameters with the simulated galaxy power spectrum. In our analysis, we also consider the simulated observations for the future CMB and SNIa measurements from PLANCK and the five-year Supernovae Legacy Survey (SNLS), which are presumably to be conducted during the same time period as the LAMOST survey. Our results indicate that LAMOST has great potential in probing for the cosmological parameters, especially in constraining the equation of state (EoS) of the dark energy and the neutrino mass.

The paper is organized as follows. In section 2 we describe the method of fitting and the simulation technique. In section 3, we present the results and discussion. Section 4 contains a summary.

2. Methodology

In this section, we introduce the method and the fitting procedure. For the dynamical dark energy model, we choose the parametrization given by [4]

$$w(a) = w_0 + w_a(1-a),$$  (1)

where $a = 1/(1+z)$ is the scale factor and $w_a = -dw/da$ characterizes the ‘running’ of the EoS (Run $w$ henceforth). For the ΛCDM model, $w_0 = -1$ and $w_a = 0$. 
When using the Monte Carlo Markov chains (MCMC) global fitting strategy to constrain cosmological parameters, dark energy perturbations should be taken into account properly, especially for models with a time evolving EoS of dark energy. This issue has been realized by many researchers including the WMAP group [5]–[8]. However, when the parameterized EoS crosses $-1$, one cannot handle the dark energy perturbations on the basis of quintessence, phantom, k-essence and other non-crossing models. By virtue of the quintom [9], the perturbations at the crossing points are continuous. Thus we have proposed a technique for treating dark energy perturbations in the whole parameter space.

In this study, we have modified the publicly available Markov chain Monte Carlo package CosmoMC [10] to include the dark energy perturbations. For handling the parametrization of the EoS getting across $-1$, firstly we introduce a small positive constant $\epsilon$ to divide the full range of the allowed value of the EoS $w$ into three parts: (1) $w > -1 + \epsilon$; (2) $-1 + \epsilon \geq w \geq -1 - \epsilon$; and (3) $w < -1 - \epsilon$. Working in the conformal Newtonian gauge, the perturbations of DE can be described by

\begin{align}
\dot{\delta} &= -(1 + w)(\theta - 3\dot{\Phi}) - 3H(c_s^2 - w)\delta, \\
\dot{\theta} &= -H(1 - 3w)\theta - \frac{\dot{w}}{1 + w}\theta + k^2\left(\frac{c_s^2\delta}{1 + w} + \Psi\right).
\end{align}

Neglecting the entropy perturbation, for the regions (1) and (3), the EoS does not cross $-1$ and the perturbation is well defined by solving equations (2), (3). For the case (2), the perturbation of the energy density $\delta$, the divergence of the velocity, $\theta$, and the derivatives of $\delta$ and $\theta$ are finite and continuous for the realistic quintom DE models. However for the perturbations of the parameterizations, there is clearly a divergence. In our study for such a regime, we match the perturbations in region (2) to the regions (1) and (3) at the boundary and set

\begin{align}
\dot{\delta} &= 0, \\
\dot{\theta} &= 0.
\end{align}

In our numerical calculations we limit the range to be $|\Delta w = \epsilon| < 10^{-5}$ and find our method to be a very good approximation to the multi-field quintom. More detailed treatments can be found in [5, 6].

Furthermore, we assume purely adiabatic initial conditions and a flat universe. The parameter space that we begin with for the numerical calculation is

\begin{equation}
P \equiv (\omega_b, \omega_c, \Theta_b, \tau, w_0, w_a, n_s, \ln(10^{10} A_s)),
\end{equation}

where $\omega_b \equiv \Omega_b h^2$ and $\omega_c \equiv \Omega_c h^2$ with $\Omega_b$ and $\Omega_c$ being the baryon and cold dark matter densities relative to the critical density, respectively, $\Theta_b$ is the ratio (multiplied by 100) of the sound horizon at decoupling to the angular diameter distance to the last scattering surface, and $\tau$ is the optical depth. In equation (5), $A_s$ and $n_s$ characterize the power spectrum of primordial scalar perturbations. For the pivot scale of the primordial spectrum we set $k_* = 0.05$ Mpc$^{-1}$.

In our calculations, we take the total likelihood to be the product of the separate likelihoods ($\mathcal{L}_i$) of CMB, LSS and SNIa. Defining $\chi^2_{L,i} = -2 \log L_i$, we then have

\begin{equation}
\chi^2_{L,\text{total}} = \chi^2_{L,\text{CMB}} + \chi^2_{L,\text{LSS}} + \chi^2_{L,\text{SNIa}}.
\end{equation}
If the likelihood function is Gaussian, $\chi^2_L$ coincides with the usual definition of $\chi^2$ up to an additive constant corresponding to the logarithm of the normalization factor of $L$.

The data used for current constraints include the three-year WMAP (WMAP3) \textsuperscript{5} temperature–temperature ($TT$) and temperature–polarization ($TE$) power spectra \cite{13}–\cite{16} as well as the data from the smaller scale experiments, including Boomerang-2K2 \cite{17}, CBI \cite{18}, VSA \cite{19} and ACBAR \cite{20}, the SDSS luminous red galaxy (LRG) sample \cite{1} and 2dFGRS \cite{2}, and the recently released ESSENCE (192 sample) data \cite{22, 23}. For the LSS power spectrum, we only use the data in the linear regime up to $k \sim 0.1 h$ Mpc$^{-1}$.

In the calculation of the likelihood from SNIa we have marginalized over the nuisance parameter \cite{21}. Furthermore, we make use of the Hubble space telescope (HST) measurement of the Hubble parameter $H_0 \equiv 100 h$ km s$^{-1}$ Mpc$^{-1}$ \cite{24} by multiplying the likelihood by a Gaussian likelihood function centered around $h = 0.72$ and with a standard deviation $\sigma = 0.08$. We also impose a weak Gaussian prior on the baryon density $\Omega_b h^2 = 0.022 \pm 0.002$ (1$\sigma$) from the big bang nucleosynthesis \cite{25}, and a cosmic age top-hat prior as $10$ Gyr $< t_0 < 20$ Gyr.

For the future data, we consider the measurements of LSS from LAMOST, the CMB from PLANCK \cite{26} and the SNIa from five-year SNLS \cite{27}.

For the simulation of LAMOST, we mainly simulate the galaxy power spectrum. We consider two sources of statistical errors in the power spectrum measurements: the sample variance and the shot noises which are due to the limited number of independent wavenumbers sampled from a finite survey volume and the imperfect sampling of fluctuations by the finite number of the galaxies respectively \cite{28},

$$\left(\frac{\sigma_P}{P}\right)^2 = 2 \times \frac{(2\pi)^3}{V} \times \frac{1}{4\pi k^2 \Delta k} \times \left(1 + \frac{1}{\bar{n}P}\right)^2,$$

where $V$ is the survey volume and $\bar{n}$ is the mean galaxy density. From the more conservative estimation, we know that the redshift distribution of the main sample of LAMOST is between 0 and 0.6 and the mean redshift is around 0.2. So for simplicity, in our study, we simulate the power spectrum of the galaxies at $z = 0.2$ that can be obtained from these galaxies. The survey area is 15 000 deg$^2$ and the total number of galaxies within the survey volume is $10^7$ \cite{3}. We only consider the linear regime; namely the maximum $k$ we consider is $k \sim 0.1 h$ Mpc$^{-1}$. As we know that the galaxy power spectrum $P(k)$ in equation (7) is

$$P(k) = b^2 p_m(k),$$

where $p_m(k)$ is the linear matter power spectrum, and here we take $b$ as a constant $b = 1$ when simulating the data and when using the galaxy power spectrum to constrain cosmological parameters, we take $b$ as a free parameter and marginalize over it.

For the simulation with PLANCK, we follow the method given in our previous paper \cite{29}. We mimic the CMB $TT$, $EE$ and $TE$ power spectra, assuming a certain fiducial cosmological model. For the detailed techniques, please see our previous companion paper \cite{29}. We have also simulated 500 SNIa according to the forecast distribution of the SNLS \cite{30}. For the error, we follow reference \cite{31} which takes the magnitude dispersion

$\textsuperscript{5}$ In view of the new release of five-year WMAP data \cite{11, 12}, we have checked that the new data will not significantly change the results.
0.15 and the systematic error \( \sigma_{\text{sys}} = 0.02 \times z/1.7 \). The whole error for each data is given by

\[
\sigma_{\text{maga}}(z_i) = \sqrt{\sigma^2_{\text{sys}}(z_i) + \frac{0.15^2}{n_i}},
\]

where \( n_i \) is the number of supernova of the \( i \)th redshift bin.

As pointed out in our previous works [32]--[34], the cosmological parameters are highly affected by the dark energy models due to the degeneracies among the EoS of DE and other parameters. Therefore, in our study of this paper, we choose two fiducial models with different dark energy properties: the \( \Lambda \)CDM model (fiducial model I henceforth) and dynamical dark energy (fiducial model II henceforth) with time evolving EoS. The parameters of the two sets of fiducial models are obtained from the current observational data.

### 3. Results and discussion

In table 1, we present the numerical results for the constraints on the cosmological parameters from the current data and the error forecast from the simulated data. To show the importance of LAMOST, we compare the two sets of results, one from PLANCK + SNLS, the other from PLANCK + SNLS + LAMOST. As we know, the matter power spectrum is directly related to the horizon size at matter–radiation equality; in turn the matter power spectrum will make accurate measurements of \( \Omega_m h \). On the other hand, there are degeneracies between \( \Omega_m h \) and the other cosmological parameters, e.g. \( \Omega_{\Lambda} \), \( H_0 \), \( w_0 \), \( w_a \) and so on; hence the tight constraint on \( \Omega_m h \) will be helpful for breaking these degeneracies and improve the constraints on these cosmological parameters. For example, in table 1, one can find that the constraints on \( \Omega_{\Lambda} \) and \( H_0 \) are tightened a lot by including LAMOST. Also from figure 1, one can see that the constraints on the age of the universe also shrink obviously; this is because the age is directly related to the Hubble constant and \( \Omega_m \).

In figure 1, we plot the 2D cross correlation and 1D probability distribution for some of the basic cosmological parameters in equation (5) and also some of the reduced parameters. The black solid lines are given by fitting with the simulated PLANCK and SNLS, and the red solid lines are provided by including the simulated LAMOST data. From the comparison, we find that LAMOST has great potential for constraining cosmological parameters, such as the EoS of dark energy, the dark energy density budget \( \Omega_{\Lambda} \), the age of the universe, \( \sigma_8 \) and the Hubble constant.

In order to see explicitly the effect of LAMOST on dark energy constraints, in figure 2, we plot the 2\( \sigma \) confidence level contours on \( w_0 \) and \( w_a \). The black solid line is given by the current constraints and the red solid line is given by fitting with the simulated data from PLANCK and SNLS five-year data with fiducial model II, while the red dashed line is given by including the simulated LAMOST data. This comparison shows clearly that LAMOST will contribute significantly in tightening the constraints on the EoS of dark energy. Numerically we find the best fit model with the current data is given by the dynamical quintom model with the EoS across \(-1\); however the cosmological constant is within the 1\( \sigma \) confidence level. The future PLANCK measurement and five-year SNLS SNIa will be able to distinguish the cosmological constant from the dynamical model at
Table 1. Constraints on cosmological parameters from the current observations and the future simulations. For the current constraints we have shown the mean values $1\sigma$ (mean) and the best fit results together. For the future mimicked data we list the standard deviation (SD) of these parameters with fiducial model I (FMI) and fiducial model II (FMII). In order to highlight the contribution from LAMOST, we compare the results with/without LAMOST.

| Parameter | Current for $\Lambda$CDM | Future (SD with FMI) | Current for Run $w$ | Future (SD with FMII) |
|-----------|--------------------------|----------------------|---------------------|-----------------------|
|           | Best fit | Mean | PLANCK | PLANCK + SNLS | PLANCK | PLANCK + SNLS |
| $w_0$     | $-1$     | $-1$  | 0.118  | 0.100         | $-1.16$ | $-1.03^{+0.15}_{-0.15}$ |
|           |          |       |        |               | 0.968   | 0.405^{+0.562}_{-0.587} |
| $w_a$     | $0$      | $0$   | 0.522  | 0.417         | $0.756$ | $0.760^{+0.017}_{-0.018}$ |
|           |          |       |        |               | 0.0125  | 0.00460          |
| $\Omega_{\Lambda}$ | $0.760$ | $0.762^{+0.015}_{-0.015}$ | 0.0115 | 0.00547 |
| $H_0$     | 73.1     | $73.3^{+1.6}_{-1.7}$ | 1.594  | 0.828         | 70.3    | $71.2^{+2.3}_{-2.3}$ |
| $\sigma_8$ | $0.769$ | $0.755 \pm 0.031$ | 0.0223 | 0.0174 |
|           |          |       |        |               | 0.634   | $0.675 \pm 0.068$ |
|           |          |       |        |               | 0.0299  | 0.0220          |
Figure 1. One-dimensional distributions and two-dimensional 68\% and 95\% limits on the cosmological parameters. The black solid lines are obtained with the simulated PLANCK + SN Ia and the red solid lines are from PLANCK + SNIa + LAMOST.

2\sigma confidence level, while LAMOST can improve this sensitivity significantly at 3.3\sigma. The blue solid lines and blue dashed lines show the comparison between the results with and without LAMOST for fiducial model I.

On the other hand, for the parameter related to the inflation models, such as \( n_s \) and \( A_s \), the constraints are mainly from PLANCK, as pointed out in our previous paper [29]. Adding in LAMOST cannot further tighten the constraints. We have also done another analysis with the additional parameters \( \alpha_s \) and \( r \), and obtained a similar conclusion, where \( \alpha_s \) characterizes the running of the primordial power spectrum index and \( r \) is the ratio of tensor to scalar perturbations.

Now we study the cosmological constraint on the neutrino mass by adding in a new parameter \( f_\nu \) in equation (5). The parameter \( f_\nu \) is the dark matter neutrino fraction at present, namely,

\[
f_\nu \equiv \frac{\rho_\nu}{\rho_{DM}} = \frac{\sum m_\nu}{93.105 \text{ eV} \Omega_c h^2},
\]

where \( \sum m_\nu \) is the sum of the neutrino masses. In this study, the mimicked data that we use are generated by assuming a massless neutrino, i.e. \( f_\nu = 0 \), in the fiducial models.
Figure 2. 2D joint 68% and 95% confidence regions of the parameters $w_0$ and $w_a$ for a flat universe. The black solid line is given by the current constraints, the red solid line comes from the simulated data of PLANCK and five-year SNLS data with fiducial model II, while the red dashed line is obtained by combining the simulated LAMOST data. The blue solid line and blue dashed line are the results with the fiducial model I with/without LAMOST respectively.

Table 2. Constraints on the neutrino mass from the current observations and the future simulations. We have shown the $2\sigma$ upper limits. In order to highlight the contribution from LAMOST, we compare the results with/without LAMOST.

|                | Future          | Current       | PLANCK + SNLS | PLANCK + SNLS + LAMOST |
|----------------|-----------------|---------------|---------------|------------------------|
| $\Lambda$CDM   |                 |               |               |                        |
| $\Sigma m_\nu$ | $< 0.958$ eV    | FMI           | $< 0.957$ eV  | $< 0.377$ eV           |
| $w$            | $< 1.59$ eV     | FMII          | $< 0.915$ eV  | $< 0.346$ eV           |

Consequently the constraints on $f_\nu$ should be regarded as the upper limits of the neutrino mass which the future observations will be sensitive to. It is well known that the massive neutrinos modify the shape and amplitude of the matter power spectrum, and also the epoch of matter–radiation equality, and angular diameter distance to the last scattering surface. Thus they leave imprints on the observations of CMB and LSS. In table 2, we provide the constraints on neutrino mass from the current observations and the future simulated data. For the current data\(^6\), within the framework of the $\Lambda$CDM model, we get $\Sigma m_\nu < 0.958$ eV (95%) which is consistent with the result of [1]. For the time evolving EoS of the dark energy model, this limit is relaxed to $\Sigma m_\nu < 1.59$ eV (95%), due to the degeneracy between the dark energy parameters and the neutrino mass [33, 36].

\(^6\) Usually Lyman-\(\alpha\) data will give stringent constraints on the neutrino mass; however, the systematics are quite unclear currently. In our global analysis, to be conservative, we have not included these data [12, 35].
Figure 3. One-dimensional constraints on the neutrino mass. The black solid line is given by the current data, the red solid line is given by fitting with the simulated PLANCK + SNLS data with fiducial model II and the red dashed line is given by including the simulated LAMOST data. The blue solid line and blue dashed line are the results obtained with the fiducial model I.

With the simulated data, in figure 3, we illustrate the one-dimensional probability distribution of the total neutrino mass $\sum m_\nu$. The black solid line is given by the current constraints, the red solid line is given by fitting with the simulated PLANCK + SNLS data with fiducial model II and the red dashed line is given by including the simulated LAMOST data. The blue solid line and blue dashed line are the results obtained with the fiducial model I. Our results show that the LAMOST can provide a more stringent constraint on the neutrino mass. For example, the $2\sigma$ neutrino mass limit is changed from 0.957 to 0.377 eV by including the simulated LAMOST data with the fiducial model I.

4. Summary

In this paper we have studied the sensitivity of LAMOST project for the determination of the cosmological parameters. With the simulated 3D matter power spectrum of LAMOST, in combination with the future PLANCK data and five-year SNLS data, we have obtained constraints on the various parameters by employing the MCMC method. Our results show the potential for LAMOST in constraining the cosmological parameters, especially the EoS of dark energy and the neutrino mass.

We have performed our analysis in a flat universe; however, if we take the curvature $\Omega_k$ into consideration, namely if we make $\Omega_k$ free in the global analysis, the basic conclusion will not change. That is to say, we can also find the potential of the future LAMOST data in determining cosmological parameters; however, the specific contours of each cosmological parameters will be enlarged for the additional degree of freedom, and more relevant discussion can be seen in our previous paper [34], in which we have implemented the global fitting with the observational data for a non-flat universe.
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References

[1] Tegmark M et al, 2006 Phys. Rev. D 74 123507 [SPIRES]
[2] Cole S et al, 2005 Mon. Not. R. Astron. Soc. 362 505
[3] http://www.lamost.org/
[4] Chevallier M and Polarski D, 2001 Int. J. Mod. Phys. D 10 213 [SPIRES]
[5] Zhao G B, Xia J Q, Li M, Peng B and Zhang X, 2005 Phys. Rev. D 72 123515 [SPIRES]
[6] Xia J Q, Zhao G B, Feng B, Li H and Zhang X, 2006 Phys. Rev. D 73 063521 [SPIRES]
[7] Spergel D N, 2007 Astrophys. J. Suppl. 170 377
[8] Yeche C, Ealet A, Refregier A, Tao C, Tilquin A, Virey J M and Yvon D, 2006 Astron. Astrophys. 448 831 [SPIRES]
[9] Feng B, Wang X L and Zhang X M, 2005 Phys. Lett. B 607 35 [SPIRES]
[10] Lewis A and Bridle S, 2002 Phys. Rev. D 66 103511 [SPIRES]
[11] Hinshaw G et al, 2008 arXiv:0803.0732
Nolta M R et al, 2008 arXiv:0803.0593
Dunkley J et al, 2008 arXiv:0803.0586
[12] Komatsu E et al, 2008 arXiv:0803.0547
[13] Spergel D N et al, 2007 Astrophys. J. Suppl. 170 377
[14] Page L et al, 2007 Astrophys. J. Suppl. 170 335
[15] Hinshaw G et al, 2007 Astrophys. J. Suppl. 170 288
[16] Jarosik N et al, 2007 Astrophys. J. 170 263
[17] MacTavish C J et al, 2006 Astrophys. J. 647 799 [SPIRES]
[18] Readhead A C S et al, 2004 Astrophys. J. 609 498 [SPIRES]
[19] Dickinson C et al, 2004 Mon. Not. R. Astron. Soc. 353 732
[20] Kuo C l et al, 2004 Astrophys. J. 600 32 [SPIRES]
[21] Di Pietro E and Claeskens J F, 2003 Mon. Not. R. Astron. Soc. 341 1299
[22] Miknaitis G et al, 2007 arXiv:astro-ph/0701043
[23] Davis T M et al, 2007 arXiv:astro-ph/0701510
[24] Freedman W L et al, 2001 Astrophys. J. 553 47 [SPIRES]
[25] Burles S, Nollett K M and Turner M S, 2001 Astrophys. J. 552 L1 [SPIRES]
[26] Planck Collaboration, 2006 arXiv:astro-ph/0604069
[27] The SNLS Collaboration, 2006 Astron. Astrophys. 447 31 [SPIRES]
[28] Feldman H A, Kaiser N and Peacock J A, 1994 Astrophys. J. 426 23 [SPIRES]
[29] Xia J-Q, Li H, Zhao G-B and Zhang X, 2007 arXiv:0708.1111
[30] Yeche Ch et al, 2006 Astron. Astrophys. 448 831 [SPIRES]
[31] Kim A G, Linder E V, Miquel R and Mostek N, 2004 Mon. Not. R. Astron. Soc. 347 909
[32] Xia J Q, Zhao G B, Feng B and Zhang X, 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)015 [SPIRES]
[33] Xia J Q, Zhao G B and Zhang X, 2007 Phys. Rev. D 75 103505 [SPIRES]
[34] Zhao G B, Xia J Q, Li H, Tao C, Virey J M, Zhu Z H and Zhang X, 2007 Phys. Lett. B 648 8 [SPIRES]
[35] Fogli G L et al, 2008 arXiv:0805.2517
[36] Hannestad S, 2005 Phys. Rev. Lett. 95 221301 [SPIRES]