Magnetic moment of the pentaquark $\Theta^+$ state

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Abstract

Although the $\Theta^+$ has been listed as a three star resonance in the 2004 PDG, its existence is still not completely established. Whether the $\Theta^+$ exist or not, but it is still of interest to see what QCD has to say on the subject. The baryon magnetic moment is a fundamental observable as its mass which encodes information of the underlying quark gluon structure and dynamics. Assuming a conventional correlated perturbative chiral quark model (CPQCM), we suggest that the $\Theta^+$ baryon is a bound state of two vector diquarks and a single antiquark, the spatially wave function of these diquarks has a $p$-wave and a $s$-wave in angular momentum in the first and second version of our model respectively, as the result of these considerations we construct the orbital colour - flavour - spin symmetry of $q^3q$ contribution of quarks. Then we calculate the $\Theta^+$ magnetic moment in our model.

1. Introduction

The year 2003 may be remembered as a renaissance of hadron spectroscopy, at the early's of that year (LEPS) collaboration T. Nakano et al. reported the first evidence of a sharp resonance $Z$ renamed to $\Theta^+$ at $M \approx 1.54 \pm 0.01\text{GeV}$ with a width smaller than $\Gamma_\Theta < 25\text{ MeV}$.

The experiment performed at the Spring-8 facility in Harima Japan and this particle was identified in the $K^-\Lambda$ invariant mass spectrum in the photo - production reaction $\gamma p \rightarrow K^+ + \Theta^+$ which was induced by a Spring-8 tagged photon beam of energy up to 2.4 GeV.

The existence of $\Theta^+$ was soon confirmed by various groups in several photo nuclear reactions including $\gamma \Lambda$ T. Nakano et al. reported the first evidence of a sharp resonance $\Theta^+$.

This discovery has triggered an intense experimental and theoretical activity to understand the structure of the state. Such states are believed to belong to a multiplet of states where the possible observability of the other members has to be worked out.

With the conventional constituent quark model, the conservation rules guarantee that it has a strangeness $S=1$, baryon number $B=1$ and charge $Q=1$, thus the hypercharge is $Y = B + S = 2$ and the third component of isospin is $I = 0$, no corresponding $\rho K^-$ ($I=1$) state is observed at the same mass, due to absence of a $\rho K^{-}$ in the $\gamma p \rightarrow K^- + \rho^0 - \pi^0$ channel and thus the isospin of $\Theta^+$ is the same I=0 which also seems important that no $S=1$ baryon states have been observed below the NK threshold and this state seems to be the ground state.

All known baryons with $B=1$ carry negative or zero strangeness, a baryon with strangeness $S=1$, it should contain at least one s, cannot consist of three quarks, but must contain at least four quarks and an antiquark; in other words, must be a pentaquark or still more complicated object. Now it's called $\Theta^+$ pentaquark in literature.

From the charge and the strangeness, $u \bar{d} s$ is a possibility as the content of $\Theta^+$ which called the minimum quark content, such state is exotic, in general states with the $q$ having different flavour than the other four quarks and their quantum numbers cannot be defined by 3 quarks alone are called exotics, thus we have an exotic $\Theta^+$.

The mass and the width of $\Theta^+$ and other exotic pentaquark baryons has predicted by several hadron models, its width $\Gamma < 10\text{ MeV}$ is exceptionally narrow as for a hadron resonance located at 110 Mev a bove the NK threshold usually referred to narrow width puzzle.

There is no direct measurement of its spin $S$ and isospin I and its angular momentum $J$ and parity $P$ are different in various theoretical works, however most of them postulated its angular momentum $J=1/2$ but the possibility of $J=3/2$ and $S=1/2$ and $P=\pm$ is rather plausible.

2. Pentaquark As A Bound State Of Two Vector Diquarks And One Anti-quark

Assuming a conventional correlated perturbative chiral quark model (CPQCM), we suggest that the $\Theta^+$ baryon is a bound state of two diquarks and a single antiquark, the spatially wave function of each diquark leads to $[2\bar{2}]$ spin symmetry for $q^3q$, the $[2\bar{2}]$ spin symmetry of each diquark leads to $[2\bar{2}]$ and $[3\bar{1}]$ spin symmetry for $q^3q$ in the first and second version of our model respectively.

The colour symmetry of each diquark is $[11]$ and for the first version of our model we assume $[2\bar{2}]$ for one of the diquark pairs this leads to $[21\bar{1}]$ color symmetry for $q^3q$.

The orbital symmetry of each diquark is $[2\bar{2}]$ and for the first version of our model we assume $[11]$ for one of the diquark pairs, this leads to $[31\bar{1}]$ and $[4\bar{1}1]$ orbital symmetry for $q^3q$.

Briefly the spin - flavour - colour and parity of our model for the first version and second one are as follows:

$$[QQ^{I}_f,\lambda_\lambda,\mu_\mu,\frac{f}{2} \frac{f}{2} \ell \ell]$$

$$[QQ^{I}_f,\lambda_\lambda,\mu_\mu,\frac{f}{2} \frac{f}{2} \ell \ell]$$

We have considered $[4\bar{1}1]$ and $[31\bar{1}]$ for the spin - flavour - colour and parity of our model for the first and second version of our model respectively, this leads to $[5111\bar{1}]$, and $[4211\bar{1}]$ for the spin - flavour - colour representations of $q^3q$, if one assume $J=1/2$ for the four quarks $q^6$ there are several allowed $SU(6)$ representations for $q^6$ which are $[5111\bar{1}]$, $[4211\bar{1}]$, $[3111\bar{1}]$, and $[3211\bar{1}]$ based on $[4\bar{1}1]^J$, $[31\bar{1}]^J$, $[22\bar{1}]^J$ and $[211]^J$ for $SU(6)$ representations of $q^6$ respectively.
The magnetic moment is an internsic observable of particles which may encode important information of its quark gluon structure and underlying dynamics. Different magnetic moments will affect both the total and different cross sections in the photo- or electro production of pentaquarks. Hence, knowledge of the pentaquark magnetic moments will help us unveil the mysterious curtain over the pentaquarks at present and deepen our understanding of the underlying quark structure and dynamics.

The pentaquark magnetic moments in several typical models have been calculated now we calculate it for our model.

For the magnetic moment of a particle we have:

$$\vec{\mu} = g \vec{S}$$  \hspace{1cm} (3)

Where $\vec{\mu}$ is magnetic moment, $g$ is gyromagnetic ratio and $\vec{S}$ is spin operator, this leads to $\vec{\mu}_{i} = g_{i} \vec{S}_{i}$, for the quarks we have:

$$g_{q} = g_{q}^{I} + 2g_{q}^{G} = 2 \frac{Q_{q}}{2m_{q}} m_{q}$$  \hspace{1cm} (4)

in which $\mu_{q}$ is quark magneton, and $Q_{q}$, $m_{q}$ are quarks charge and mass respectively.

If the particle has angular momentum between $s$-quark and diquarks, we have:

$$\vec{\mu}_{s} = \langle \psi_{s} | \sum_{i} g_{i} S_{i}^{j} \cdot \vec{L}_{i}^{j} | \psi_{s} \rangle$$  \hspace{1cm} (5)

in which $\psi_{s}$ is the flavour- spin wave function of the pentaquark.

For the second term of (6) we have:

$$\vec{\mu}_{s} = \langle \psi_{s} | \sum_{i} g_{i} S_{i}^{j} \cdot \vec{L}_{i}^{j} | \psi_{s} \rangle = \langle \psi_{s} | \sum_{i} g_{i} S_{i}^{j} \cdot \vec{L}_{i}^{j} | \psi_{s} \rangle^{\text{diquark1}}$$

$$+ \langle \psi_{s} | \sum_{i} g_{i} S_{i}^{j} \cdot \vec{L}_{i}^{j} | \psi_{s} \rangle^{\text{diquark2}} = \langle \psi_{s} | \sum_{i} g_{i} S_{i}^{j} \cdot \vec{L}_{i}^{j} | \psi_{s} \rangle^{\text{diquark1}}$$

The contribution of the $\vec{\mu}_{s}$ term would be zero and for the first and second term we have:

$$\mu_{l} = \frac{m_{1} \mu_{1} + m_{2} \mu_{2}}{m_{1} + m_{2}}$$

Where $m_{1}, \mu_{1}$ and $m_{2}, \mu_{2}$ are the masses and magnetic moments for the first and second diquarks respectively.

For the first version of our model in which the two diquarks are in $1 P$ wave we have:

$$\mu_{l} < \vec{l}_{s}^{\text{relative}} = \frac{\mu_{l}}{2} \left( \frac{11}{27}, \frac{1}{2}, \frac{1}{2} \right)$$

But for the second version of our model in which the two diquarks are in $0 S$ wave we have:

$$\mu_{l} < \vec{l}_{s}^{\text{relative}} = 0$$

The contribution of $\vec{\mu}_{s}$ in the first term of Eq (6) is:

$$\mu_{s} < \vec{l}_{s}^{\text{relative}} = \left( \frac{11}{27}, \frac{1}{27}, \frac{11}{27} \right)$$

and for the first version of our model we have:

$$\mu_{s} < \vec{l}_{s}^{\text{relative}} = \left( \frac{11}{27}, \frac{1}{27}, \frac{11}{27} \right)$$

for the contribution of $s$ in the first term of Eq (6) due to relative angular momentum between $s$-quark and diquarks.

References

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