Clapping modes in unconventional superconductors.

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We consider a superconducting state with a mixed symmetry order parameter components, e.g. \( \Delta_0 \) or \( \Delta_1 \) or \( d + id' \), with \( d' = d_{xy} \). We argue for the existence of the new orbital magnetization mode which corresponds to the oscillations of relative phase \( \phi \) between two components around an equilibrium value of \( \phi = \frac{\pi}{2} \). It is similar to the so-called “clapping” mode in superfluid \( ^3He - A \). We estimate the frequency of this mode \( \omega_0(B,T) \) depending on the field and temperature for the specific case of magnetic field induced \( d' \) state. We find that this mode is tunable with an applied magnetic field with \( \omega_0(B,T) \propto B\Delta_0 \), where \( \Delta_0 \) is the magnitude of the d-wave order parameter. We argue also that similar field induced clapping mode should be present in an organic p-wave superconductors.

**Clapping mode in High-Tc superconductors.** The order parameter in high-Tc superconductors is widely believed \cite{1} to be of a \( d_{x^2-y^2} \) symmetry. However more careful consideration indicates that the symmetry of the state in high-Tc might be lower in a number of cases. The symmetry allows the secondary components to be generated whenever there is a perturbing field. These secondary components (\( d+id' \)) generation has been addressed in recent literature for the case of inhomogeneity due to wall scattering \cite{2} or due to vortex texture \cite{3}. Similarly, the generated \( id' \) component near magnetic impurity and spontaneous violation of time reversal symmetry with global \( d + id' \) has also been discussed \cite{4}. Another possibility for a global \( d + id' \) phase has been pointed out \cite{5,6} where the external magnetic field applied to two-dimensional \( d \)-wave superconductor generates \( id' \).

We present here \cite{7} an excitation which constitutes a direct test of the induced component of the order parameter. This excitation is uniquely tied to the existence of the secondary \textit{out of phase} component of the order parameter. Consider the most general situation of a \textit{complex} order parameter which can be generally written as \( \Delta_0 + i\Delta_1 \) where \( \Delta_0 \) is the original \( d_{x^2-y^2} \) component and \( \Delta_1 \) is the induced \( s \) or \( d_{x^y} \) component which is orthogonal to the initial \( d_{x^2-y^2} \) state. Define the global (Josephson) phase of the order parameter \( \nu \) and the relative phase \( \phi \) as:

\[
\Delta = |\Delta_0| + exp(i\phi(\mathbf{r}))(\Delta_1) |exp(i\nu(\mathbf{r}))
\]

If the order parameter \( \Delta \) is to be defined at the Fermi surface, then the functions \( \Delta_i, i = 0,1 \) implicitly contain the angular dependence. The global phase \( \nu(\mathbf{r}) \) can be position dependent and even singular as is the case for the vortex configuration. We will focus on the relative phase \( \phi \). Its dynamics by definition is related to the appearance of the secondary component \( \Delta_1 \). The dispersion of this mode has a gap similar to the Larmor frequency in case of spins. In the sense of a magnetic excitation, that can respond to a time dependent magnetic field in a resonant manner, this mode is also comparable to the longitudinal NMR in \( ^3He-A \) \cite{8}. The details of the dynamics are clearly model dependent.

To be specific we will focus on the field induced \( \Delta_1 = id_{xy} \) secondary component in the bulk. In this case quasiparticle spectrum is fully gapped in the field with the minimal gap vanishing at zero field. The \( d + id' \) state breaks time reversal symmetry and has a finite magnetic moment \( M_z \) perpendicular to the superconducting plane \cite{9,10}. The mode discussed above in this case will corre-
spond to the longitudinal oscillations of the condensate magnetic moment around its equilibrium value. Below we focus on a 2d superconductor at the fields $H \simeq H_{c2}(T)$ and therefore we will ignore small ($O(H - H_{c2})$) difference between induction $B$ and applied field $H$. 

We find the resonance frequency for the relative phase oscillations of $\phi$. It turns out to be a gapped propagating mode with:

$$\omega^2(B, k) = \omega_0^2(B, T) + s_{ij}^2(B, T)k_i k_j,$$

$$\omega_0(B) \simeq \frac{\eta B}{N(0)} \Delta_0(B, T) \quad (2)$$

$$s_{ij}^2 = \delta_{ij} s^2$$

Here $s = (a + bB^2)\Delta_0^2$ is the mode velocity, $a, b$ are some constants and $\eta$ is a constant, a measure of the strength of the interaction, which we discuss in more detail below; $N(0)$ is the Density of States (DOS) at the Fermi level; both $i, j$ refer to in-plane coordinates $x, y$. This mode is a longitudinal magnetization oscillation $\delta M_z(t, r) \propto \delta M_z \exp(i\omega(B, k)t - ikr)$ and is tunable by the external field. There are, in effect, two consequences arising from a non-zero $\eta$ which may be used to estimate its magnitude. In the presence of a secondary order parameter $\Delta_1 = i\delta_{xy}$, the temperature dependent upper critical field $H_{c2}(T)$ has an additional contribution which is quadratic in $B = \frac{1}{T}$. This results in an upward curvature with a scaling field which depends on $\eta$.

We now will prove the existence of the “clapping mode” for $d + id'$ state. The free energy has the standard form:

$$F = F_0 + F_1 + \frac{B^2}{8\pi}$$

$$F_0 = \frac{a_0}{2} |\Delta_0|^2 + \frac{b_0}{4} |\Delta_0|^4 + \frac{K_{ij}}{2} |D_1 \Delta_0 D_j \Delta_0^*|$$

$$F_1 = \frac{a_1}{2} |\Delta_1|^2 + \frac{K_{ij}'}{2} |D_1 \Delta_1 D_j \Delta_1^*|$$

$$F_{int} = \frac{\eta}{2} [D_x, D_y] \Delta_0 \Delta_1^*$$

where $\Delta_0, \Delta_1$ are $d$ and $d'$ components of the order parameter, $a_0 = \alpha_0(T/T_0 - 1)$, $T_0$ is the ordering temperature for $\Delta_0$. The corresponding $a_1 > 0 = N_0$ is always positive, as are $b_0 > 0$ and $K_{ij}, K_{ij}' > 0$. $D_1 = \partial_1 - i\frac{\pi}{2}\Delta_1$, with $B = \nabla \times A$. The interaction term in this form has been proposed earlier [5]. The interaction term, using $[D_x, D_y] = ieB$ can be written as

$$F_{int} = ieB \frac{\eta}{2} \Delta_0 \Delta_1^* + h.c. \quad (8)$$

which corresponds to a coupling of the magnetic field with an intrinsic orbital moment along $z$-axis: $< M_z > = \frac{\eta}{2} \Delta_0 \Delta_1^* + h.c.$.

Here we will assume that the amplitude for the secondary $\Delta_1$ has been developed and we will look at the relative phase $\phi$ oscillations only. It is convenient to introduce the respective phases of each component:

$$\Delta_0 = |\Delta_0| \exp(i\phi_0), \quad \Delta_1 = |\Delta_1| \exp(i\phi_1)$$

which are related to the introduced above $\nu = \phi_0, \phi_1 = \phi_1 - \phi_0$. Derivation of the Eq.(2) then proceeds as follows. We use the Josephson relations for each component:

$$\phi_i = \frac{\partial F}{\partial \bar{N}_i} = \mu_i, \quad \bar{N}_i = -\frac{\partial F}{\partial \phi_i}, i = 0, 1$$

(9)

Where $\mu_i$ are chemical potentials for particles in $\Delta_0, \Delta_1$ condensate and similarly, $\bar{N}_i$ are conjugated number of particles. In the equilibrium,
Figure 2. The $F_{int}$ profile as a function of the relative phase angle $\phi$ between $\Delta_0$ and $\Delta_1$ is shown. Although the minimum is reached at $\phi = \frac{\pi}{2}$ the finite stiffness for $\phi$ oscillations leads to the finite frequency mode at $\omega_0(B, T)$. We have ignored the dependence on the sign of $B$ in the figure.

where $\mu_0 = \mu_1 = \mu$, we find for a relative phase motion:

$$\dot{\phi} = -i \frac{\partial \mu_0}{\partial N_0} + \frac{\partial \mu_1}{\partial N_1} - 2 \frac{\partial \mu_0}{\partial N_1} \frac{\partial F}{\partial \phi}$$  \hspace{1cm} (10)

where in the above we have taken into account the fact that although $\mu_0 = \mu_1$ their derivatives are different. The term in the brackets is on the order of $N(0)$: $\left[\frac{\partial \mu_0}{\partial N_0} + \frac{\partial \mu_1}{\partial N_1} - 2 \frac{\partial \mu_0}{\partial N_1}\right] = \rho^{-1} \approx N(0)$. For the general values of the relative phase $\phi$ the terms in free energy $F$ that contribute to the derivative in Eq.(10) are $F_{int}$, Eq.(8) and terms with gradients $K_{ij}$, $K_{ij}'$.

$$F_{int} = \eta B_2 |\Delta_0| |\Delta_1| \sin \phi$$  \hspace{1cm} (11)

with minimum $F$ reached at $\phi = -\pi/2sgn(B_z)$. In general, there is an additional term proportional to the squares of the two (the bare and the induced) order parameters. However it contributes little new to the physics of the problem. Its effects are largely quantitative. We stress here that the phase $\phi_0$ of $\Delta_0$ is not constant in the external field in the mixed state. Nevertheless the $F_{int}$ in Eq.(11) depends on the relative phase only.

We find:

$$\dot{\phi} = -\eta |\Delta_0| |\Delta_1| B_z \cos \phi - s^2 \nabla^2 \phi$$  \hspace{1cm} (12)

Here $s$ is given by Eq.(3). Minimizing the free energy Eq.(9) with respect to the magnitude from this equation, Eq.(12) follows immediately. The velocity $s$ is field dependent and also can be used in experiments to detect the presence of $id'$ component.

We can estimate the magnitude of the energy gap up to a $O(1)$ prefactors:

$$\omega_0(B, T) \approx |\Delta_1(B, T)| \approx \frac{\eta B}{N(0)} \Delta_0(B, T)$$  \hspace{1cm} (13)

as one can easily see from minimization of the total $F$ with respect to $\Delta_1$. For estimate for the coupling constant $\eta$ and magnitude of $\Delta_1$ see [7]. We estimate thus

$$\omega_0(B, T) \approx \frac{\eta B}{N(0)} \Delta_1(B, T)$$  \hspace{1cm} (14)

This result turns out to be very similar to the “clapping mode” in $^3He-A$, where frequency was found to scale with the gap in the whole temperature range as well [8]. We also find asymptotics in $H$:

$$\omega_0 \sim \left\{ \begin{array}{ll} \frac{H}{H_{c2}} & H \leq H_{c2} \\ \sqrt{H_{c2} - H} & H \to H_{c2} \end{array} \right.$$  \hspace{1cm} (15)

and we note immediately that the mode frequency $\omega_0 \ll \Delta_0$ Therefore this mode will be sharp. The damping coming from the low-lying quasiparticles in the nodes of d-wave gap will not affect this mode because the phase space for the decay will be small.

We assumed that once the field induced component is present in GL region it will persist to a lower temperatures. We present Eq.(15) as an order of magnitude estimate only. From Eq.(14) it follows that the gap $\omega_0$ can be made in the range of $0.1 - 1K(2 - 20GHz)$ in the field $H = 1 - 10T$ similar to the $\Delta_1$ estimates [7].

The longitudinal oscillations of magnetization $M_z$ will also lead to the resonance at $\omega_0(B, T)$ in the AC susceptibility of the superconducting state. Experimentally the proposed clapping mode can be observed in NMR in the field. For any of the resonance techniques used to search for the “clapping mode” resonance at $\omega_0(B, T)$ Eq.(14) the vortex cores contribution would be the biggest source of background.

**Clapping mode in p-wave superconductor.** Here we consider the case of quasi 2-
4 dimensional p-wave superconductor in an external field. We identify here $\Delta_0 \propto p_x$ and $\Delta_1 \propto p_y$. Assume that the zero field state has real order parameter that transforms as $\Delta_0 \propto p_x$. We argue then that external magnetic field will induce secondary component $\Delta_1 = ip_y$. The state $\Delta_0 + \Delta_1$ will have a finite magnetic moment $< M_z >$ that can couple to magnetic field. The free energy term driving the secondary component is given by Eq.(8) and we find $\Delta_1 \propto B\Delta_0$ in case of p-wave superconductor. The results for the clapping mode are similar to the case of d-wave superconductor and will be presented in more details elsewhere [10]. We note that the similar clapping mode for the p-wave state that violates time reversal in zero field, e.g. $p_x + ip_y$, was considered in [11].

In conclusion, we considered the relative phase oscillation mode that can be used to detect the secondary component in the time reversal violating superconducting state, such as $d + id'$ or $p + ip'$. Two components of the order parameter are characterized by respective amplitudes and phases. The relative phase between two components can oscillate around its equilibrium value $\phi = \pm \frac{\pi}{2}$. This mode is very similar to the “clapping mode” in superfluid $^3$He - A. For a specific model we choose the case of field induced $id'$ (for singlet pairing) or $ip'$ (for triplet) component. We show how the relative phase mode frequency is governed by external magnetic field and d-wave gap magnitude $\omega_0(B,T) \propto B\Delta_0$ and is therefore tunable by external magnetic field. Among other probes this mode could be experimentally detected by investigating the ac magnetic susceptibility and possibly by ultrasound attenuation in the in the mixed state as a function of applied field $H$.

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