An Efficient Data Driven Model for Generation Expansion Planning with Short Term Operational Constraints

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Abstract—Generation expansion planning (GEP) models have been useful aids for long-term planning. Recent growth in intermittent renewable generation has increased the need to represent the capability for non-renewables to respond to rapid changes in daily loads, leading research to bring unit commitment (UC) features into GEPs. Such GEP+UC models usually contain discrete variables which, along with many details, make computation times impractically long for analysts who need to develop, debug, modify and use the GEP for many alternative runs. We propose a GEP with generation aggregated by technology type, and with the minimal UC content necessary to represent the variations on generation to respond to rapid changes in demand, with variation limits estimated from historical data on maximum rates of change of each generation type. We illustrate with data for the province of Ontario in Canada; the GEP is a large scale linear program that solves in less than one hour on modest computing equipment, with credible solutions.

Index Terms—Capacity Expansion, Unit Commitment, Data Driven Model, Ramping Constraints, Uncertainty

I. INTRODUCTION

Optimization models for electricity generation expansion planning (GEP) have been proposed and used since the 1950s. By the early 1970s, Anderson [1] could report on a large number of models, some of which were in “constant use” by the electric power industry in several countries. Anderson discussed the “considerable advantages” of linear programming (LP) over nonlinear or dynamic programming, and he described some techniques to reduce LP computing times “by a factor of 2 or more”. This concern with computing times continues to the present, in spite of the enormous advances in computer hardware and LP software, because the models have grown in size, in order to represent more planning periods, more regions, stochastic scenarios, and generally more detail.

It is sometimes suggested that computing time for a GEP model is not of much concern, because it only needs to be solved a few times to complete a planning study for generation expansion that will be implemented months or years in the future; see, e.g., [2] which described a large stochastic GEP model with 4, 5 or 6 investment periods that solved in six hours, three days and more than one week, respectively. However, we believe that for a model to be in “constant use” for modifications with new data or policies, sensitivity analyses, debugging, etc., computing times of several hours or days could be a serious impediment to its adoption by a planning authority.

Starting in the early 1970s, GEP models also appeared as sub-models representing investments in the electricity sector of a larger, energy system optimization model (ESOM) that encompassed all energy sectors; see, e.g., [3] which describes the electricity part of the U.S. Department of Energy’s model Project Independence Evaluation System. See also [4] for a recent review of such ESOMs, with a particular emphasis on the GEP components. With these models, computation time is also an important consideration.

Recently, in response to growth in intermittent renewable generation which requires the other generators in the system to change their power outputs to counterbalance renewable fluctuations, a considerable literature has grown up around the inclusion of short term operations that are common in unit commitment (UC) models (e.g., ramping limits, startup/shutdown costs, minimum power output, minimum up/down times) in long term models. As Poncelet [4] notes, many ESOMs incorporate ad hoc features to approximate the degree to which different types of generation can respond to changes in demand (e.g., by forcing nuclear to produce power at a constant value over each year, but allowing change from one year to the next). However, these features are either too approximate (e.g., nuclear can ramp up or down, to some degree), or they require the estimation of parameters that are hard to measure. We are interested instead in bringing UC features explicitly into the planning model, and so our review focuses on previous efforts of this type.

A mixed integer linear programming model is presented in [5] to incorporate all of the very detailed UC requirements directly in each representative day of a GEP model. The model has only one 24-hour representative day for each month over 17 years. There is no mention of computing time, but there is an indication of computing difficulty – an “integrality gap of 1%” – which may explain the lack of some variation in representative days within each month.

A clustered UC presented in [6] is incorporated into a GEP model in [7]. The clustered UC model aggregates similar generation units, with, e.g., an integer variable to represent the
total number of similar units committed instead of separate binary variables for each unit, which improves computational efficiency without deviating very far from the exact UC model. Their example solves for investment in only one target year, but with all hours of all days of the year. However, some model runs exceeded 2.5 days.

Stiphout, et. al. [8], also used a clustered UC in a GEP, but they relaxed the integer variables, for a considerable improvement in computing time: with just one target year for investments, and all hours of the year (like [7]), computing times ranged from 10 minutes to two hours.

A single target year GEP model is developed in [9] with most of the usual UC features (except minimum up/down times), and compared it to a reduced version which retained only the ramping constraints, but no binary variables of the UC features. The reduced version is a linear program, and it solves dramatically faster than the GEP with the full array of UC features; in one set of comparisons, computation times were reduced from several hours to about 20 seconds. Interestingly, the authors report that the reduced version, with only ramping constraints, produced investments that were very close to those from the more computationally intensive version. The large, multi-year stochastic GEP model presented in [2] also contains only the ramping constraints in its UC features.

In this paper, we present a multi-year generation expansion model that represents the most important constraints on short term operations, and that is solvable in a short enough time to be useful in practice. Based on the findings in [9], the model contains no binary variables for the on/off status of generators during each day; thus, the UC features of startup/shutdown costs, minimum up/down time limits, and minimum run limits are absent from the model. However, the model does include constraints to approximate capacity limits, variation limits in the representation of daily operations. Like [2] and [10] but unlike [9] our model does not track individual generation units with binary variables for built/not-built; instead, generation investment and operation is tracked by aggregated groups (e.g., nuclear, gas, hydro, wind, solar, biofuel). For each group, the hourly variation of power output is limited by maximum variation rates, up and down, that are estimated from actual data on maximum up and down variations for the groups. Thus, with no discrete variables, and having all linear constraints and objective function, the model is a linear program. Many representative days are included for each year, to include typical seasonal and daily variations in demand and availability of renewables, as well as some scenarios for extremes of demand and variations in the generation output at the first hour of each representative day. The model is illustrated with an example based on data for the province of Ontario in Canada.

The practical utility of a model depends in part on two factors: the ability to solve the model in a short enough time to be useful to analysts; and the possibility of estimating credible values for parameters. Therefore, it is important to explain, for the Ontario illustration, our process to define representative days, seasons, etc., which determine the model size and computation time, and it is equally important to explain how parameters are estimated. Therefore, we begin with the Ontario demand data analysis in Section II, to determine the definitions of representative days, seasons and scenarios. Section II also discusses the estimation of some generation parameters, including the variation rate limits; we note that the variation rate estimation method implies an interpretation of the ramping constraints that is somewhat different from the usual understanding of them in unit commitment models.

In Section III, the formulation of the linear programming model of the problem is presented, along with explanations of methods to estimate some other parameters, including the variation rate limits. More specifically, we define a concept of variation in generation level of each group of generators with the same technology which includes not only the total aggregated ramp rates but also changes in generation due to shut down or startups as well. The remaining parameter estimates, and the performance of the model are discussed in Section IV. Also in Section IV, there is a discussion of the impact of including variation constraints on the optimal levels of generation capacities. Section V summarizes the paper and suggests avenues for further research.

II. CATEGORIES AND SOME PARAMETERS FOR ONTARIO MODEL

The LP model of Sections III and IV requires demand and supply data of the types in the parameter list of Appendix A. Most of the data are indexed by one or more of: generation technology type $k$ (e.g., nuclear, gas), years covered by the model $t$ (1 to 20 in Section IV), seasons of each year $ss$ (e.g., winter, etc.), 24 hours $h$ of each representative day, type of representative day $i$ (e.g., weekday versus weekend), and scenarios $s$ (e.g., high, medium or low demand, and hour-zero status of each generator type in each representative day). We use public data from the Independent Electricity System Operator (IESO) for Ontario [11] on hourly demand and supply1 over the years 2003 to 2017 to guide us to precise definitions of the indices, and to estimate most of the parameters. Our goal is to cluster the historical data, and carry the resulting patterns into the future covered by the model, such that the resulting model is as small as possible, for a short computing time, but still represents realistic future demand and supply variability that drives the need to invest in ramping capability as well as generation capacity.

In the following subsections, we describe our use of the IESO data to define the types of generation $k$, the types of representative day $i$, the seasons $ss$, and the scenarios $s$; we also describe the estimation of all demand parameters and some supply parameters.

A. Definitions of Indices $k$, $i$, $ss$, $s$

The IESO supply data are organized in six categories, and we adopt these distinctions for generation technology type $k$, i.e., nuclear, gas, hydro, wind, solar and biofuel.

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1 Because the historical IESO data and our model have one hour as the smallest time unit, the numerical value of energy demand over an hour (in MWh) is the same as the average power demand over the same hour (in MW, for one hour).
The IESO demand data confirms the common observation that the weekday and weekend daily load curves are quite different. Therefore, the representative day index $i$ has two values for weekday and weekend, where the weekend category includes all weekday holidays.

For each day of the year (e.g., the 147th day), and separately for the instances when that day is a weekday or weekend, we calculate the average power for that day over all hours of the day and all the years of data. This measure, when graphed, roughly indicates the seasons $ss$, which we define in Table 1.

### Table 1: Definition of Seasons

| Seasons | Months                          |
|---------|--------------------------------|
| Winter  | 2nd half of Nov, Dec, Jan, Feb, Mar |
| Spring  | Apr, May, and 1st half of Jun   |
| Summer  | 2nd half of Jun, July, Aug, and 1st half of Sep |
| Fall    | 2nd half of Sep, Oct, and 1st half of Nov |

Within each combination of season $ss$ and representative day $i$, there are variations in demand among the days of the $ss$-$i$ combination in the historical data. We need to represent this variation, as part of our definition of the scenario index $s$, for three reasons: (i) generation capacity must be capable of serving the highest demand; (ii) optimal investment in different types of generation $k$ depends in part on operating costs, which depend not just on the highest demands, but also on lower demands; and (iii) investment in different generation types depends in part on their differing ramping capabilities, and the system requirement for ramping is set by the rate of change of demand (up or down), which seems likely to be larger during high demand days. Therefore, we define three daily load curves (low/most likely/high) for each $ss$-$i$ combination, based on the historical demand data.

In the model of Section III, the initial generation, i.e., “hour zero” power output, (which define to be measured at midnight of the previous day) of generators influences how much power can be produced in the next few hours, because generators must ramp up or down, and ramping limits limit the ability to ramp. The hour zero data of generators varies over the seasons (but there is very little difference between weekends and weekdays). From the IESO data for 2003-2017, for each season $ss$, and each generator type $k$, we measure initial generation as a fraction of installed capacity, and then construct a 2-bin histogram (high/low initial generation) for each $ss$-$k$ combination. We find that only one bin is adequate for solar (because there is no sun at midnight) and for biofuel (which is usually zero at midnight); this leaves two bin histograms only for nuclear, gas, hydro and wind. The definition of scenarios $s$ combines demand variations and initial generation levels. With four generation types that each have two hour-zero states (high or low), and three daily load types (high, medium, low), the number of scenarios $s$ is $2^4 \times 3 = 48$, for each combination of season $ss$ and representative day $i$.

### B. Demand Parameters and Some Supply Parameters

$T(i,ss)$ is the number of days in a year during season $ss$ of type weekend/weekday; it is used as a weight for operating costs in the objective function.

$\text{Prob}(ss,i,s)$ is the probability of scenario $s$, measured as the relative frequency of the demand/initial generation combination in the IESO data, for season $s$ and representative day $i$.

$D(ss,i,h,s)$ represents the daily load curve for scenario $s$ during season $ss$ and representative day $i$, as discussed above; when multiplied by $\text{GRW}(i)$ we get the forecast of the load curve for year $i$. We assume values of $\text{GRW}(i)$ based on a growth rate of 0.3% per year; this is slightly less than the middle of the range of possible future demands for Ontario, in the IESO report [12].

$\text{IG}(ss,i,s,k)$ is the initial generation level, as estimated from the 2-bin approximation procedure discussed above; it is a fraction that is multiplied by total installed capacity of type $k$, to give the hour-zero power output.

$\text{Cap}(k,ss,h)$ is a fraction which multiplies total installed capacity of type $k$ to give the model’s estimate of capability, during season $ss$ and hour $h$ of the day, which is the upper limit on power output. We estimated the capability factor from the capability data of the IESO for 2017 only, in order to have the most recent status, divided by the installed capacity.

$\text{VV}(k,ss)$ and $\text{VD}(k,ss)$ are the variation up and down limits, respectively, for generation type $k$, as a fraction of total capability. It would be possible to estimate each variation limit parameter based on the engineering estimate of the ramp limit of an individual unit, expressed as a fraction of the unit’s capacity (see, e.g., Liu et al., 2018), but instead, we choose to estimate the variation limit parameters based on actual data on power outputs of generation groups. In comparison to individual ramping rates for generators, we noticed a significant difference between the individual parameter values and the values obtained from our estimation on the total groups. This means that using the engineering, single-unit parameters could be very misleading for a model like ours that aggregates generations into groups. In order to estimate, from IESO data, the maximum variation up $\text{VV}(k,ss)$ and variation down $\text{VD}(k,ss)$ rates per effective capacity for each generation technology type in each season, we calculated the rates of change of generation output for each generation group over the hours of each season and considered the maximum positive rate of change as the maximum variation up and the maximum of absolute negative rate of change as the maximum variation down parameters. Then considering the capability of each generation group in each season, the maximum variation up and down can be estimated as a fraction of their capable capacity. This method of estimation of variation limits means that the variation constraints in the model are different from ramping constraints in unit commitment models, although both have the same mathematical form (see Section III). In our model, these constraints represent the limits of a group of generators to change its total power output, and these limits are based in part on the observed behavior of system operators who decide on
individual units’ ramping up and down (in the unit commitment sense), and in addition, startup and shutdown.

III. THE INTEGRATED SHORT TERM OPERATIONS WITH GENERATION CAPACITY EXPANSION PROBLEM (ISO-GEP)

In this Section, we present the mathematical expressions for the constraints and the objective function of our ISO-GEP model. We also explain our methods to estimate some more parameters; estimation of the remaining parameters is in Section IV. Please refer to Appendix A for the complete list of symbols for indices, parameters and variables.

We distinguish the existing capacity $XE(k,t)$, a parameter that is taken from the IESO (2016) outlook [12], and the new capacities $XN(k,t)$ which are defined by the variables for investments $x(k,t')$ in years $t'$ before year $t$, considering the depreciation factor as follows:

$$XN(k,t) = \sum_{t'=t}^{} x(k,t')[1 – dep(k)(t-t')]$$  

(3)

The depreciation parameters for the new installed generators are estimated from the life span of each generation technology and the straight line method.

We use a cost minimization objective function, in which the cost of construction, annual fixed cost and expected variable and reserve costs are defined. To calculate the expected values of hourly variable and reserve costs, we use the probabilities of scenarios developed in the previous Section:

$$\min \sum_{k} \sum_{t} \left\{ IV(k,t)x(k,t) + \sum_{t'=t}^{} FC(k,t)x(k,t') \right\} + \sum_{k} \sum_{s} \sum_{i} \sum_{s'} \sum_{s''} Prob(ss,i,s) \left\{ \sum_{h} \sum_{t} T(i,ss,H)g(k,t,ss,i,h) + CV(k,t)r(k,t,ss,i,h,s) \right\}$$

(4)

All cost parameters depend on the year $t$ because costs are discounted to make the objective a present worth, and to allow for real cost increases; see Section IV. Constraints (5) and (6) limit the generation output of each type of generation $(k)$ to the maximum effective capacity of the generators in each year, season, representative day, hour and scenario, using the capability factors $Cap(k,ss,h)$, estimated as discussed in Section II.A. We consider the gas generators separately, because they are used as reserve capacities in our model, and therefore part of their capacity should be put aside for providing reserve ancillary services in each year:

$$g(k, t, ss, i, h, s) – Cap(k,ss,h)(XE(k,t) + XN(k,t)) \leq 0 \ \forall k \neq GAS, t, ss, i, h, s$$  

(5)

$$g(k, t, ss, i, h, s) + alpha_{RES} \times GRW(t) D(ss, i, h, s) – Cap(k,ss,h)(XE(k,t) + XN(k,t)) \leq 0 \ \forall k = GAS, \forall t, ss, i, h, s$$  

(6)

The following two constraints define the variation variables in the objective function:

$$g(k, t, ss, i, h, s) – g(k, t, ss, i, h-1, s) – r(k, t, ss, i, h, s) \leq 0 \ \forall k, t, ss, i, h, s$$  

(7)

$$g(k, t, ss, i, h-1, s) – g(k, t, ss, i, h, s) – r(k, t, ss, i, h, s) \leq 0 \ \forall k, t, ss, i, h, s$$  

(8)

The following constraints represent the variation up and down constraints for each generation technology type in each scenario:

$$g(k, t, ss, i, h, s) – g(k, t, ss, i, h-1, s) – VU(k,ss) Cap(k,ss,h)(XE(k,t) + XN(k,t)) \leq 0, \ \forall k \in \{Nuc, Gas, Hydro, Biofuel\}, \forall t, ss, i, h, s$$  

(9)

$$g(k, t, ss, i, h-1, s) – g(k, t, ss, i, h, s) – VD(k,ss) Cap(k,ss,h)(XE(k,t) + XN(k,t)) \leq 0, \ \forall k \in \{Nuc, Gas, Hydro, Biofuel\}, \forall t, ss, i, h, s$$  

(10)

The initial status of generation at hour 0 is fixed based on the scenarios developed for each year, season and day:

$$g(k, t, ss, i, 0, s) = IG(k, ss, i, 0, s)(XE(k,t) + XN(k,t)), \ \forall k, t, ss, i, s$$  

(11)

In order to satisfy demand in each scenario, we define the following constraint in which the total supply and demand is balanced in every hour of each combination of season, representative day and scenario. Demand in each year is approximated using a growth parameter $GRW(t)$, as discussed in Section II.B.

$$GRW(t) D(ss, i, h, s) – \sum_{k} g(k, t, ss, i, h, s) \leq 0 \ \forall t, ss, i, h, s$$  

(12)

Because governments maintain different generation target mechanisms, such as reduction of coal or gas generation and promotion of other sources, we further add constraints to impose bounds on generation related to each technology type. For instance, there might be a regulation set by government to produce at least 20% of generation by hydro capacities. We define the parameters $alpha_{H}(k,t)$ and $alpha_{L}(k,t)$ as maximum and minimum share of each technology group in the total generation capacity. For our first illustration in Section IV, the share of each generation technology is first calculated from the long term capacity plan of the IESO, and then $alpha_{H}(k,t)$ and $alpha_{L}(k,t)$ are estimated as 5% above and below the IESO plan, respectively. Section IV also contains some other illustrations, including $alpha_{H}(k,t) = 1$ and $alpha_{L}(k,t) = 0$, to represent no share limits. The constraints on the share of generation technologies are as follows:

$$XE(k,t) + XN(k,t) – alpha_{H}(k,t) \sum_{k} [XE(k,t) + XN(k,t)] \leq 0, \ \forall k, t$$  

(13)

$$XE(k,t) + XN(k,t) – alpha_{L}(k,t) \sum_{k} [XE(k,t) + XN(k,t)] \geq 0, \ \forall k, t$$  

(14)

Note that $\sum_{i} Prob(ss,i,s) = 1$. 

4
Many other types of constraints are possible - e.g., constraints on emissions of CO₂ or a more detailed treatment of reserves - but the model as described above is sufficient to investigate whether inclusion of variation alone can direct investment to generation that can respond to rapid changes in demand, while keeping computation time short enough to be practical in use.

IV. RESULTS

In this Section, the model of Section III is applied with parameters that are realistic for Ontario, as discussed in Sections II and III, with some further parameter settings discussed below. Our purpose is not to present a careful policy analysis or forecast of future generation in Ontario, but instead to demonstrate that the model of Section III produces credible results in a short enough time to be useful to policy analysts, and in particular, that the variation constraints (9) and (10) have the desired effect of directing investment towards generation that can respond to rapidly changing demand.

We have been unable to find cost related data on the IESO website, so we have used the cost data from the U.S. Energy Information Administration to estimate approximate costs in Canadian dollars, as summarized in Table 2 ([13] and [14]). These are taken as steady real costs, discounted at 3% per year over the planning horizon.

In order to compare the solution of our model with the 20-year capacity plan of IESO which is from 2016 to 2035 we consider the planning horizon from 2016 to 2035. The life spans of new installed generation units presented in Table 2 are used to calculate the depreciation rate of new installed generators using straight line depreciation and the variation costs are assumed to be 10% of variable costs. The column “Invest for 20 years” illustrates how we allocate a portion of the investment cost, depending on how much of the new capacity’s lifetime falls within the model’s planning horizon. For example, new nuclear installed in year 1 of the model is assigned an investment cost of 1/3 of the total investment cost because the 20 years of the planning horizon is 1/3 of the 60 year life span; however, new nuclear installed in year 11 would be assigned an investment cost of 1/6 of the total investment cost, because the remaining 10 years of the planning horizon represents 1/6 of the life span. All parameter data used in this article are available in [15] for interested readers.

Table 2. Approximate costs based on year 2016

| Life | Invest | Invest | Fixed | Variable |
|------|--------|--------|-------|----------|
| Span (Yrs) | ($/MW) for lifetime | ($/MW) for 20 years | O&M ($/MWh) | cost ($/MWh) |
| NUC | 60 | 7,371,800 | 2,457,267 | 124,347 | 2.852 |
| Hydro | 80 | 7,101,108 | 1,775,277 | 155,000 | 0 |
| GAS | 30 | 1,290,344 | 860,229 | 13,640 | 6.3364 |
| Wind | 30 | 2,327,480 | 1,551,653 | 49,228 | 0 |
| Solar | 30 | 3,244,253 | 2,162,836 | 29,016 | 0 |
| Biofuel | 25 | 6,181,400 | 4,945,120 | 52,204 | 5.208 |

One reason for the difference between the supply capacities of our proposed model and the IESO is the inclusion of storage capacities and demand response mechanisms in the ISO-GEP model, while it is not considered in our proposed model. For example, according to IESO_GEP model, in 2035, storage and demand response mechanisms contribute 1416 MW in supply capacities. Another reason for the observed difference between the two models is the assumptions about forecasting the future demand. Nevertheless, we can say that our results are credible, i.e. reasonably similar to the long-term capacity plan of the IESO.

A. Performance of the Proposed Model (ISO-GEP)

The proposed model is a large scale linear program with 829,560 variables and 5,012,400 constraints. However, because of linearity it is solved efficiently by the CPLEX solver in GAMS in a reasonable time. The optimal solution is obtained in 3035 seconds (51 minutes) as the total execution time of CPLEX on a 9-year old server computer, which is reasonably efficient for such a large scale model. A newer and faster computer, together with some care in reducing the number of scenarios (see, e.g., some new ideas in [10] and [16]) could reduce the execution time considerably, making the model a useful tool for policy analysis.

Figure 1 summarizes the main results obtained by our model and compares them with the long-term capacity plan of Ontario.
the assumed data on costs, the hydro, wind and solar investments are too costly. In another experiment, we dropped most of the share constraints (13) and (14), but kept the lower limit constraints (14) for wind, solar and biofuel generation; the investments in new capacity were closer to the results of the full ISO-GEP model, but again there was no investment in new hydro.

B. The impact of variation constraints and costs on capacity investments

In this Section, we investigate the impact of including short term variation costs and constraints (9) and (10) in the long term capacity planning model. To do so, we examine the solution of the ISO-GEP model of Section IV.A to see whether and when the variation constraints are binding, and we test the performance, in daily operations, of investments determined by models that have no variation constraints and/or variation costs. For the latter, we modify the ISO-GEP model by removing the variation costs and/or constraints (the “no-variation” versions), and then re-run the ISO-GEP model with generation investments fixed at the no-variation solution, but with variation costs and constraints imposed. We also construct a conventional generation expansion model, with demand highly aggregated into three blocks (base, medium and peak) in each year-season-scenario combination; then we run the ISO-GEP model with variation constraints and costs, and with generation investments fixed at the conventional model’s levels.

B.1 Binding variation constraints in the ISO-GEP model

In order to check the impact of variation constraints in the ISO-GEP model, we search through the dual variables of the variation constraints for nonzero values. Variation up constraints are binding mostly in hours 6, 7, or 8 and 18 in almost all seasons and all years, mostly in high demand scenarios. Power demand starts rising in the morning hours 6, 7, and 8 and also hour 18 (which is the peak hour) and so the variation up constraints are mostly binding in these hours. However, the variation down constraint is binding mostly in early morning hours when the power demand starts declining and it happens mostly in the low and medium demand scenarios, in almost all seasons and years.

Because only a small percentage of variation constraints are binding, in times and scenarios that are understandable, it might be possible to reduce the computing time by removing all variation constraints except in times and scenarios that experienced judgement suggests they are needed. As an extreme version of this, we removed all variation constraints that are not binding in the optimal solution, and we ran the model again; however, there was only a small reduction in computing time, from 51 to 50 minutes.

B.2 Testing investments determined by ISO-GEP without variation constraints or costs

We do more investigation on the impact of variation constraints by solving the ISO-GEP model in three cases: no variation constraints and no variation cost, no variation constraints but with variation costs in the objective function, and variation constraints but no variation cost. The total capacity of each generation technology including the existing and new installed capacities is presented in Figure 2, for just the first 10 years, to save space. For comparison, Figure 2 also presents the total capacities of the ISO-GEP model of Section IV.A.

It can be seen that without variation constraints, there is much less investment in capacity, e.g., in nuclear in particular. The explanation is that the variation constraints (9) and (10) relate the variation limit to the total capacity, so one reason to invest in new capacity is to have a supply system that can respond to the variations in demand.

The two cases of no variation constraints as well as no variation cost, and no variation constraints but considering variation cost in the objective function provide the same solution. In the full ISO-GEP model (which includes variation constraints and costs), we then fix the capacities determined above and run the fixed-capacities model to see if the solution is feasible, and if so, at what cost. The solution of no variation constraints and no variation cost as well as no variation constraints but considering variation cost is infeasible in the fixed-capacities.

In contrast, the solution of the model with variation constraints but no variation cost is feasible in the fixed-capacities. The difference between the model with just variation constraints and the full model with variation constraints and cost is small, but in terms of the total cost the full model performs a little better than the model with just variation constraints, as expected.

B.3 Testing investments determined by a conventional generation expansion model

The conventional generation expansion model reorders the demands over a year into the annual load duration curve in which hourly demands are sorted in descending order and then approximated by several blocks. Figure 3 illustrates the load duration curve for Ontario in 2017, and a three-block approximation. Because of the reordering and approximation, the conventional model cannot represent hour-by-hour variations in demand and supply, so variation constraints cannot be represented.

We have extended the conventional generation expansion model by defining separate three-block approximations for each year-season-scenario combination as defined in the ISO-GEP model. We also include the reserve requirement, and the share constraints. The extended conventional generation expansion model is described in more detail in Appendix C.

As in the previous subsection, we determine investments with the extended conventional model, and then run the full ISO-GEP model with the investments fixed at the extended conventional model’s values. Once again, the result is infeasibility.

B.4 Summary of impacts of variation constraints and costs on generation investment

We conclude that the variation constraints play an important role in the investment decisions by the ISO-GEP model, because of the numerous instances of binding variation constraints in the optimal solution, and because the variations of the ISO-GEP model without variation constraints produce
investments that are infeasible, i.e., unable to follow the variations in demands over many of the representative days. Similarly, investment decisions from the extended conventional generation expansion model also produce investments that are infeasible. In contrast, the variation costs have a minor effect on the investment decisions by the ISO-GEP model.

![Figure 2. Total capacities in different cases on variation constraints and cost](image)

V. SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

Conventional generation capacity planning models are highly aggregated in two important ways, for the sake of computing times that are sufficiently short to be useful in policy analysis: similar generation units are grouped together in broad categories, e.g., nuclear, gas-fired, etc., with no distinction of individual units; and short-term, e.g., hourly, operational limits and costs are ignored due to aggregation of time units into year- or season-long basic time units. Thus, conventional capacity planning models do not account for the need for a mix of generation units that not only provide sufficient capacity to meet peak demands and reserve requirements, but also provide the ability of the supply system to deal with the sometimes rapid fluctuations in demand during daily operations.

This paper presents a model that keeps the aggregation of generation units but has 24 hours in each of many representative days for every year in the model. Because there is no distinction of individual generation units, the model has no binary variables for investment. There are also no binary variables for starting, stopping or on/off status of generators. Limitations on the ability of the supply system to respond to hourly changes in demands are represented by variation up and down constraints that are applied to the aggregated groups of generation. The resulting model is a linear program which solves in a sufficiently short time to be useful for policy analysis.

Short computation time is one requirement for a practical tool for policy analysis. Another requirement is that it must be possible to make credible estimates of parameters for the model. This paper shows that such estimation is possible, through an illustration for the province of Ontario in Canada. In particular, the limits on variation capabilities of the aggregated groups of generation can be estimated from the maximum rates of change (up and down) of the groups in recent historical operational statistics, together with the assumption that the total limit (up or down) is proportional to the total installed capacity (in history and in the future years represented in the model).

In future research, the model could easily be extended to include investment in storage for daily arbitrage and for use as reserves to deal with uncertainties in supply from wind and solar. Further features could also be included easily, such as emission limits or taxes, and a more detailed treatment of reserves. Computation time could be further reduced by a more judicious selection of representative days and scenarios; see, e.g., [10] and [16].
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APPENDIX A: Nomenclature

Indices:

\( k \) Index of firm’s technology, \( k = 1, 2, 3, 4, 5, 6 \)

\( t \) Year, \( t = 1, 2, \ldots, 20 \)

\( ss \) Seasons, \( ss = 1, 2, 3, 4 \)

\( s \) Scenarios, \( s = 1, 2, \ldots, 48 \)

\( h \) Hours of representative days, \( h = 1, 2, \ldots, 24 \)

\( i \) Index for representative day, \( i = 1, 2, \) (1: weekday and 2: weekend)

Parameters:

\( T(i, ss) \) Number of representative days \( i \) at season \( ss \)

\( C(k, t) \) Operation cost of new firm of technology \( k \) at year \( t \) (S/MWh)

\( VC(k, t) \) Variation cost of technology \( k \) at year \( t \) (S/MW)

\( IV(k, t) \) Investment cost of technology \( k \) at year \( t \) (S/MW)

\( FC(k, t) \) Fixed cost per year of technology \( k \) at year \( t \) (S/MW)

\( XE(k, t) \) Existing capacity of technology \( k \) at year \( t \) (MW)

\( dep(k) \) Depreciation factor of new capacity for generation group \( k \)

\( VD(k, ss) \) The maximum possible variation up of generation group \( k \) at season \( ss \) in fraction of capable capacity

\( D(ss, i, h, s) \) The maximum possible variation down of generation group \( k \) at season \( ss \) in fraction of capable capacity

\( IG(ss, i, s, k) \) Initial generation level of group \( k \) of generators at the beginning of day \( i \) in season \( ss \) under scenario \( s \) (as fraction of total capacity of \( k \))

\( Cap(k, ss, h) \) Capability of generation group \( k \) at season \( ss \) and hour \( h \) based on derates, outages, and (for solar and wind) weather (fraction of total capacity)

\( \alpha_H(k, t) \) Maximum proportion of generation group \( k \) in generators mix at year \( t \)

\( \alpha_L(k, t) \) Minimum proportion of generation group \( k \) in generators mix at year \( t \)

\( Prob(ss, i, s) \) Probability of scenario \( s \) for season \( ss \) and day \( i \)

\( GRW(t) \) Growth factor of year \( t \) to modify the demand and initial generation levels based on forecasted growth rate

Variables:

\( g(k, t, ss, i, h, s) \) Generation amount of technology \( k \) at period \( t \), season \( ss \), representative day \( i \), hour \( h \) under scenario \( s \) (MWh)

\( r(k, t, ss, i, h, s) \) Variation up/down amount of technology \( k \) at period \( t \), season \( ss \), representative day \( i \), hour \( h \) under scenario \( s \) (MW)

\( x(k, t) \) New capacity of technology \( k \) should be installed in year \( t \) (MW)

\( XN(k, t) \) Total new available capacity of technology \( k \) installed up to year \( t \) (MW)

REFERENCES

[1] D. Anderson, "Models for Determining Least-Cost Investments in Electricity Supply," Bell Journal of Economics and Management Science, vol. 3, no. 1, pp. 267-299, 1972.

[2] Y. Liu, R. Sioshansi and A. J. Conejo, "Multistage Stochastic Investment Planning with Multiscale Representation of Uncertainties and Decisions," IEEE Transactions on Power Systems, vol. 33, no. 1, pp. 781-791, 2018.

[3] A. L. Soyster and R. T. Eynon, "The conceptual basis of the electric utility sub-model of project independence evaluation system," Applied Mathematical Modeling, vol. 3, no. 4, pp. 242-248, 1979.

[4] K. Poncelet, "Long-term energy-system optimization models," Ph.D dissertation. Faculty of Engineering Science, Katholieke Universiteit Leuven, Belgium, 2018.

[5] N. E. Koltsaklis and M. C. Georgiadis, "A multi-period, multi-regional generation expansion planning model incorporating unit commitment constraints," Applied Energy, vol. 158, no. 15, pp. 310-331, 2015.

[6] B. Palminitier and M. Webster, "Heterogeneous Unit Clustering for Efficient Operational Flexibility Modeling," IEEE Transactions on Power Systems, vol. 29, no. 3, pp. 1089-1098, 2014.

[7] B. Palminitier and M. Webster, "Impact of Operational Flexibility on Generation Planning With Renewable and Carbon Targets," IEEE Transactions on Power Systems, vol. 7, no. 2, pp. 672-684, 2016.

[8] A. V. Stiphout, K. D. Vos and G. Deconinck, "The Impact of Operating Reserves of Renewable Power Systems," IEEE Transactions on Power Systems, vol. 32, no. 1, pp. 378-388, 2017.

[9] S. Jin, A. Botterud and S. M. Ryan, "Temporal Versus Stochastic Granularity in Thermal Generation Capacity Planning with Wind Power," IEEE Transactions on Power Systems, vol. 29, no. 5, pp. 2033-2041, 2014.

[10] Y. Liu, R. Sioshansi and A. J. Conejo, "Hierarchical Clustering to Find representative Operating Periods for Capacity-Expansion Modeling," IEEE Transactions on Power Systems, vol. 33, no. 3, pp. 3029-3039, 2018b.

[11] [Online]. Available: http://www.ieso.ca/power-data/data-directory.

[12] [Online]. Available: http://www.ieso.ca/sector-participants/planning-and-forecasting/ontario-planning-outlook.

[13] [Online]. Available: https://www.eia.gov/electricityannual/html/epa_08_04.html.

[14] [Online]. Available: https://www.eia.gov/outlooks/aeo/electricity_generation.php.

[15] M. Pirnia, " https://www.mansci.uwaterloo.ca/~mpirnia/codes.html," [Online].

[16] K. Poncelet, H. H'oschle, E. Delarue, A. Virag and W. D’haeseleer, "Selecting representative days for capturing the implications of integrating intermittent renewables in generation expansion planning problems," IEEE Transactions on Power Systems, vol. 32, pp. 1936-1948, 2017.

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APPENDIX B: Detailed Discussion for Section II (not intended for published paper, but available here for reviewers; to be available to readers on a website)

B.1. Clustering of power demand and supply data

Many earlier studies distinguish weekend days from weekdays, so first we check the data to see whether the power demands of Saturdays and Sundays are sufficiently similar to be considered as the same type of day. To do so we define the average hourly demand of each day by dividing the sum of total power demand of a day by 24 hours. As shown in Figure B1, the average hourly demand is approximately the same for Saturdays and Sundays over the years 2003 to 2017. Therefore, we group the demand on Saturdays and Sundays in one representative day as weekends.

\[
\text{Average hourly demand of day } t = \frac{\sum \text{Demand}(y,t,h)}{N(t)},
\]

The averaging formula (B2) is applied for each individual day of the year. In order to analyze the average daily demand of weekdays and weekends we first divide the days into two segments of weekdays and weekends and then apply (B2) on each day in each segment. \(N(t)\) is the number of years that day \(t\) is a weekday (weekend day).

Figure B3 visualizes the weekdays’ average daily demand over the years 2003 to 2017, without yet excluding weekday holidays in the averaging, in order to show the importance of putting weekday holidays in the weekend category. The fixed-date holidays of January 1, July 1 (Canada’s national day), and December 25, together with nearby dates which are often taken in addition, appear as low demand outliers in Figure B3.

As shown by Figure B3, we see seasonal patterns in the average hourly power demand of the days in a year. Our definition of seasons is different than calendar based seasons. We defined the seasons visually, roughly based on the observed average pattern, as in Table B1.

| Seasons | Months                          |
|---------|--------------------------------|
| Winter  | 2nd half of Nov, Dec, Jan, Feb, Mar |
| Spring  | Apr, May, and 1st half of Jun    |
| Summer  | 2nd half of Jun, July, Aug, and 1st half of Sep |
| Fall    | 2nd half of Sep, Oct, and 1st half of Nov |

Based on our findings on the power demand data, we propose a clustering design including four seasons as defined above and two representative days including weekdays and weekends (including weekday holidays), for eight clusters altogether.

We have also applied the k-means algorithm to cluster the data to see how different is our proposed clustering compared to a classic algorithmic clustering in literature. We ran the k-means algorithm [B1] on the average daily demand of Figure B3 in Scikit-learn (sklearn.cluster) in Python and optimized the number of clusters by Silhouette score analysis through sklearn.metrics.silhouette_score in Python [B2] which ended up with just two clusters: winter and summer times grouped in one cluster and spring and fall seasons in another cluster, which makes sense when considering only demand as in Figure B3. However, in another part of the model, we need to consider also the availability of wind and sunlight, which vary considerably
between winters and summers (see below). Therefore, we define the eight clusters based on our visual analysis.

According to the IESO [B3], the capability (in MW) is “the maximum potential output” of a nuclear, hydro, gas or biofuel generator in a given hour, after deducting derates and outages, whether for maintenance or unplanned problems; for wind and solar generators, exactly the same definition is given the name available capacity, and it is noted that the estimate of actual deliverable power from wind and solar is further affected by the availability of wind and sun. For this paper, we use the single term capability to mean the maximum potential output of any type of generation; in particular, for wind and solar, the availability of wind and sun is part of capability.

In the model of sections III and IV, the capability factor \( Cap(k,ss,h) \) multiplies the total installed capacity of type \( k \) to give the model’s estimate of capability. We estimated the capability factor from the capability data of the IESO for 2017 only, in order to have the most recent status, divided by the availability of wind and sun. For this paper, we use the single term capability to mean the maximum potential output of any type of generation; in particular, for wind and solar, the availability of wind and sun is part of capability.

In the model of sections III and IV, the capability factor \( Cap(k,ss,h) \) multiplies the total installed capacity of type \( k \) to give the model’s estimate of capability. We estimated the capability factor from the capability data of the IESO for 2017 only, in order to have the most recent status, divided by the installed capacity. Figure B4 illustrates the average capability factor of all generator types over all the days of 2017.

From a visual analysis on the capability data of Figure B4, we can say the capability factor varies over the seasons. We averaged the capability factors for each season, each generation type, and each hour of the day to estimate \( Cap(k,ss,h) \). Table B2 shows the result for the early 5 hours of a day as a sample.

There may be many reasons for decreasing demand despite population increase in Ontario. One of the main reasons is the Ontario government’s promotion of replacing inefficient electrical equipment and appliances with the more energy saving ones. It is unclear whether the decreasing trend can continue in the future, but applying the time series forecasting techniques such as regression would keep the decreasing trend. Therefore, we prefer to consider the 18 months ahead power demand forecast provided by the IESO [B4] and obtain the future demands based on this trend by simple regression; in Section IV, this leads to a forecast of demand increasing at 0.3% per year.

We distinguish the seasons and representative days (weekdays and weekends/holidays) to forecast hourly demand data, but we face a range of variation of power demand of each
hour over any particular season and representative day -- see Figure B6 for the range of hourly demands for weekdays and weekends over the years 2003 to 2017. Some of the variation is due to seasonal effects, but charts for each season still show considerable variation for each hour.

Figure B6. The range of hourly demand on weekdays and weekends over the years 2003 to 2017

In order to cover the stochastic variation of hourly power demand on any representative day and season, we consider the scenario approach to forecast demand data. To do so, we start with a detailed histogram of demand data at each hour of a representative day at each season and then convert the histogram to an approximate histogram with three bins. Then the median of each bin is considered as one scenario and the probability of that scenario is calculated by considering the frequencies. Therefore, we will have three different scenarios (low, most likely, high) with their associated probability for the power demand of each hour on a representative day of each season.

Figure B7 illustrates the histogram of demand in hour 18:00 for weekdays of winter and spring over the years 2003 to 2017 as an example and Figure B8 represents the equivalent 3-bin histograms. The equivalent 3-bin histogram can be easily constructed in the Python Mathpolite package by determining the number of bins in the function of histogram.

Figure B7. Histogram of demand on hour 18 for weekdays of winter and spring over 2003-2017

Figure B8. Equivalent 3-bin histogram of demand on hour 18 for weekdays of winter and spring over 2003-2017

Details of the scenario based approach are presented in the next Section.

B.3. Scenario-Based Approach

In the model of Section III, the initial “hour zero” power output (which we define to be measured at midnight) of generators influences how much power can be produced in the next few hours. However, the hour zero data of generators varies for any combination of representative day and season. Therefore, we next extend our definition of scenarios to include variations in initial generation level, and then combine these two to form the scenarios of the problem with their associated probabilities.

B.3.1. Initial generation level of generators

For the unit commitment 24-hour day ahead problem, the initial generation levels of generators at hour zero (midnight) are parameters of the model. In the IESO data, the initial generation level is the generation amount of hour 24 of the previous day. As mentioned earlier, we aggregate the generators by technology type $k$, i.e., nuclear, gas, hydro, wind, solar and biofuel. Since there is very little difference in power generation between weekdays and weekends at hour 24:00, the scenarios of initial status for weekdays and weekends are the same. As an example, Figure B9 shows the histograms of generation levels at hour 24:00 in summer days of 2017. To keep the number of scenarios as small as possible, while still representing some variation in initial generation levels, we consider approximating the histograms with 2-bin histograms.

Figure B9. Generators output on hour 24:00 of summer days over 2017
In order to formulate the initial generation levels parameter in a way that allows for total capacities of the different types \( k \) to change over the model’s horizon, we define the initial generation level-parameter \( IG(\mathbf{s}, \mathbf{i}, \mathbf{s}, k) \) as a fraction of total installed capacity for each generation technology type. In other words, after approximating the generators output on hour 24:00 by a 2-bin histogram, the generation amounts are divided by the related total capacities to calculate the ratios. Then these ratios are considered in the model to be multiplied by the total capacity at each year to determine the initial generation level scenarios.

After analyzing the histograms of the generation level at hour 24:00 for all generator group, for all four seasons, we consider the following number of scenarios for initial status of each type of technology:

| Scenario | Nuclear | Hydro | Gas | Wind | Solar | Biofuel |
|----------|---------|-------|-----|------|-------|---------|
| # of scenarios | 2 | 2 | 2 | 2 | 1 | 1 |

Due to non-availability of sun at hour 24:00 only one scenario of zero level generation with probability of one is considered for solar technology. Also the reason to define one scenario for biofuel generation level is because the approximated 2-bin histogram for biofuel includes a bin with high probability while the other bin has a very low probability and can be ignored here in order to reduce the size of model without loss of generality. Table B4 shows the calculated ratios of initial generation levels over the summer 2017. In other words, Table B4 presents the approximated two scenarios of initial generation level ratios of total installed capacity for each generation technology type.

Considering the number of scenarios presented in Table B3 the total number of scenarios for initial status of generators becomes \( 2^4 \times 1 \times 1 = 16 \), as detailed in Table B5, where “1” and “2” indicate the higher power and lower power bins, respectively.

Due to non-availability of sun at hour 24:00 only one scenario of zero level generation with probability of one is considered for solar technology. Also the reason to define one scenario for biofuel generation level is because the approximated 2-bin histogram for biofuel includes a bin with high probability while the other bin has a very low probability and can be ignored here in order to reduce the size of model without loss of generality. Table B4 shows the calculated ratios of initial generation levels over the summer 2017. In other words, Table B4 presents the approximated two scenarios of initial generation level ratios of total installed capacity for each generation technology type.

| Scenario | Nuclear | Hydro | Gas | Wind | Solar | Biofuel |
|----------|---------|-------|-----|------|-------|---------|
| Generation technology | 0.9066 | 0.4106 | 0.2065 | 0.3429 | 0 | 0.0691 |
| 2 | 0.8242 | 0.3422 | 0.1032 | 0.1143 | - | - |

The probability of a bin for a generator’s histogram is estimated directly from the relative frequency as given by the histogram; the probability of any one of the 16 scenarios of Table B5 is estimated as the product of the probabilities of the generator’s bins that make up the scenario.

### B.3.2. Scenarios of Hourly Power Demand

In order to define scenarios for hourly demand, 3 scenarios of low (\( L \)), most likely (\( M \)), and high (\( H \)) demand as explained in Section II.B is considered for each hour. As illustrated in Figure B10, if a day has \( L, M, \) or \( H \) level of demand in the beginning, it usually continues to have the same pattern of hourly demand for the rest of the day. Using this assumption, the number of daily demand scenarios (representing demand level in each hour) will be reduced to 3 scenarios from \( 3^24 \) – i.e., each day either has low or most likely or high demand in every hour of the day.

In order to find \( L, M, \) or \( H \) level of demand for each hour of the day, we consider the histogram of each hour on any representative day of each season and convert it to an equivalent histogram with three bins and find the associated frequencies; see Section II.B for an example.

### B.3.3. Combination of All Scenarios

Now, considering the 16 scenarios for the generators’ initial status and 3 scenarios for hourly demand, in total we construct 48 scenarios for each representative day and each season, as shown in Table B6.
GRW(t)D(ss, h, s) - \sum g(k, t, ss, h, s) \leq 0 \quad \forall t, ss, h, s \quad (C4)

XE(k, t) + XN(k, t) - a_H(k, t) \sum_{k'} [XE(k', t) + XN(k', t)] 
\leq 0, \forall k, t \quad (C5)

XE(k, t) + XN(k, t) - a_L(k, t) \sum_{k'} [XE(k', t) + XN(k', t)] 
\geq 0, \forall k, t \quad (C6)

In this extended conventional model there are just 3 scenarios of L, M, and H (not including the scenarios related to initial generation levels) and the index h takes three values of B, M, and P as base, medium, and peak time periods.

References

[B1] A. K. Jain and R. C. Dubes, Algorithms for Clustering Data, NJ: Prentice-Hall, 1989.

[B2] [Online]. Available: http://scikit-learn.org/stable/modules/generated/sklearn.metrics.silhouette_score.html.

[B3] [Online]. Available: http://www.ieso.ca/sector-participants/planning-and-forecasting/18-month-outlook.

APPENDIX C: DETAILED DISCUSSION FOR SECTION IV.B.3

(NOT INTENDED FOR PUBLISHED PAPER, BUT AVAILABLE HERE FOR REVIEWER)

The extended conventional capacity expansion model is defined as follows:

\[
\begin{align*}
\text{min} & \quad \sum_k \sum_{t} [IV(k,t)x(k,t) + \sum_{s,t'} FC(k,t)x(k,t')] \\
& + \sum_k \sum_{s,t} \sum_{s'} \sum_s \text{Prob}(ss,s) \left[ \sum_h T(h) | C(k,t)g(k, t, ss, h, s) \right] \\
\text{s.t.} & \\
g(k, t, ss, h, s) - \text{Cap}(k, ss, h) (XE(k, t) + XN(k, t)) \leq 0, \forall k \neq \text{GAS}, t, ss, h, s \\
g(k, t, ss, h, s) + a_{GAS}(t) \text{GRW}(t)D(ss, h, s) \\
- \text{Cap}(k, ss, h) (XE(k, t) + XN(k, t)) \leq 0, k = \text{GAS}, \forall t, ss, h, s
\end{align*}
\]