Universal Singlets, Supergravity and Inflation

Lotfi Boubekeur \(^1\) and Gianmassimo Tasinato \(^2\)

SISSA-Isas, Via Beirut 4, I-34013 Trieste, Italy.
and
INFN Sezione di Trieste, Trieste, Italy.

Abstract

In supersymmetric theories, the occurrence of universal singlets is a delicate issue, because they usually induce tadpoles that destabilize the hierarchy. We study the effects of these tadpoles in supersymmetric hybrid inflation models. The resulting scenario is generically modified, but it is still possible to achieve inflation in a natural way. It is argued that singlets, despite the problems associated with their presence, can lead to interesting cosmological consequences.

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\(^1\) lotfi@he.sissa.it
\(^2\) tasinato@he.sissa.it
1 Introduction

In Particle Physics, the introduction of singlet fields has been invoked in many models to solve various problems. This is done for instance in the Standard Model to give masses to neutrinos with the see-saw mechanism, or in the so-called NMSSM for other purposes. In other cases, their presence is actually unavoidable, like in theories that require compactification from higher dimensions. However, it has been pointed out that the presence of these fields induces generically new quadratic divergences at one (or more) loop(s), in particular tadpoles (terms linear in the singlet) that destabilize dramatically the hierarchy \cite{12,14,15}. Some efforts have been done to show how to 'tame' these divergences in supergravity, exploiting them to solve some notorious problems \cite{17,18}.

Also in Cosmology, singlets have been shown to be very useful. For example, it was pointed out that singlets can be useful to provide a strong first order phase transition essential for a successful baryogenesis in the NMSSM \cite{22}. In some inflationary models their presence, even if less stressed, is required. However, the tadpole contributions have never been taken into account in the cosmological context. Due to their particular properties, singlets are sensitive to the Planck scale physics. Since Cosmology is the study of the early stages of the universe (just after the Planck era), it is perfectly legitimate to ask whether their presence lead to some consequences. In this paper, we will consider the modifications required by the presence of these tadpoles in the hybrid inflationary scenario.

By now, it is well established that the inflationary paradigm provides a successful and elegant solution to three essential questions of standard Cosmology: the horizon, the flatness and the monopole problem \cite{1,2}. It is also widely hoped that successful inflationary models could emerge naturally from pure Particle Physics considerations \cite{3,4}, in the sense that any consistent particle model may have a built-in sector that ensures inflation. Supersymmetric hybrid inflation models appear to be the most promising to achieve this task. Such models (and their extensions) have been constructed and studied extensively \cite{5}. Typically, they are based on superpotentials of the form $W_{\text{inflation}} = \kappa S(\Phi \bar{\Phi} - \mu^2)$, where $\kappa$ is a dimensionless coupling constant, $S$ is a singlet superfield and $\Phi$, $\bar{\Phi}$ are superfields that are conjugate under some non trivial representation of a group $G$. At a certain time, inflation is dominated by the $F$-term of the singlet field ($V_0 = \mu^4$), and this explains the presence of the linear term in the previous superpotential. Usually $\Phi$ and $\bar{\Phi}$ are taken to be the Higgs fields that break the GUT gauge symmetry so that $\mu \sim M_{\text{GUT}}$. The resulting scalar potential is the prototype of hybrid inflation \cite{6} except for the mass term for $S$, which is essential to drive the inflaton to its minimum. Such a slope can however be generated, independently from soft breaking mass terms, by the one loop corrections to the scalar potential along the inflationary trajectory \cite{8}. This model succeed in reproducing the correct values of density perturbation and the spectral index at the price of a small coupling constant ($\kappa \sim 10^{-3} - 10^{-4}$). The generic problem of
inflationary models is the stability of the potential. In other words: how to keep the inflaton potential flat enough to achieve successful inflation? Generally, without $D$-term contribution, supergravity gives new terms to the effective potential of the inflaton that usually destroy the flatness of the potential. However, it is argued that these corrections can be brought under control via a judicious choice of the Kähler potential and the superpotential \cite{10,5}. Models of inflation in which $D$-term contributions are considered have been studied \cite{7}, showing that it is possible to evade the problems associated with supergravity corrections (See however \cite{23}).

As we have seen, many characteristics of supersymmetric models have been largely used in building inflation models. In fact, the singlet nature of the inflaton is a crucial feature, since it protects it from acquiring a too large mass, that will ruin inflation. However, the particular properties of singlets have not been explored yet in inflation, and this is the main purpose of this paper, at least in a specific example. We will see, in a particular model, that the presence of singlet fields provide a Particle Physics realization of a specific version of hybrid inflation, the so called mutated hybrid inflation \cite{11}.

The paper is organized as follows. In Section 2, we briefly review the properties of singlets in supergravity. In Section 3, we will focus our discussion on the case of the superpotential of supersymmetric hybrid inflation, showing that the presence of tadpoles generically changes the scalar potential that drives inflation. In Section 4, without analyzing in full details the consequences of these modifications, we notice that, in a certain regime, the modified scalar potential can provide a realization of the mutated hybrid inflation scenario. Section 5 is devoted to the study of the stability of the potential under one-loop and supergravity corrections. Finally, in Section 6, we give our conclusions.

2 Universal Singlets in Supergravity

In Particle Physics models, universal singlets are fields that do not transform under any gauge symmetry of the Lagrangian. Therefore, roughly speaking, in non supersymmetric models containing a scalar singlet field $s$, nothing will forbid the appearance in the Lagrangian of terms such as $a\Lambda^3 s + b\Lambda^2 s^2 + c\Lambda s^3 + \text{h.c.}$ with $a, b, c \sim \mathcal{O}(1)$. Moreover, the natural value for $\Lambda$ is $M_P^\frac{1}{4}$ so singlets will get masses and vev’s of $\mathcal{O}(M_P)$. If not coupled to light fields, they will decouple from the low energy theory. Instead, if they are coupled to light fields, they will communicate to them their large vev, destabilizing dramatically the hierarchy.

One could think that invoking supersymmetry will ameliorate the things, but the situation remains the same also in SUSY models \cite{12}. Indeed, it has been shown that, if a supergravity model contains singlets, they can destabilize the mass hierar-

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\footnote{Throughout the paper, $M_P$ stands for the reduced Planck scale, namely $M_P = M_{\text{Planck}}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.}
For concreteness, let us consider a supersymmetric model with a visible sector containing an universal singlet superfield $S = s + \theta^2 F_s$, and a hidden sector, whose fields are denoted generically with $\Sigma = \sigma + \theta^2 F_\Sigma$, responsible for supersymmetry breaking. Following [16], tadpoles arise due to terms like

$$\delta K = \left[1 + \frac{c}{M_P} (S + S^\dagger)\right] \Sigma \Sigma^\dagger$$

in the Kähler potential. The higher order term, proportional to $c$, is allowed by all the gauge symmetries, and it is generically present in the Kähler potential just because $S$ is a universal singlet.

The low-energy Lagrangian contains the following $D$-term contribution [16]

$$\mathcal{L}_D = \int d^2 \theta d^2 \bar{\theta} \ e^{K/M_P^2} K,$$

where $K$ here is the Kähler potential written in terms of superfields. After integrating out the hidden fields, the effective potential coming from the tadpole is given by [16, 17]

$$\Delta V_{\text{tadpole}} = \frac{\gamma M^4}{M_P} (s + s^\dagger) + (\alpha \beta F_s M_f^2 + \text{h.c.})$$

where $\alpha$ is a parameter (See [17, 18]) related to the SUSY breaking in the hidden sector, and $\beta$ and $\gamma$ are loop factors that are less than one. The mass $M_f$ stands for the scale of breaking of supersymmetry in the hidden sector, i.e. $\langle F_\Sigma \rangle = M_f^2$. The loop factors and $\alpha$ will be an essential ingredient for our discussion. They are related to $c$, to the number of hidden fields and to the detailed structure of the Kähler potential; their typical value is in the range $\mathcal{O}(1 - 10^{-4})$. In the usual gravity mediated supersymmetry breaking models, one arranges for $M_f^2 \sim m_{3/2} M_P$, where the “gravitino mass” is chosen $m_{3/2} \lesssim \mathcal{O}(\text{TeV})$, to solve the hierarchy problem.

The full scalar potential will include, in addition to standard terms, the tadpole contribution (cf. Eq. (3)). In terms of auxiliary field $F_s$ it reads [17]

$$V_{F_s} = (\beta \alpha M_f^2 F_s + \text{h.c.}) - |F_s|^2 - \left(F_s \frac{\partial W}{\partial S} + \text{h.c.}\right).$$

\[2\]The expression (2) comes from a full supergravity computation, see [14, 15] for more details.

\[3\]The values of $\alpha$, $\beta$ and $\gamma$ are model-dependent. We consider them as free parameters in their respective allowed range.
Since the auxiliary fields $F_s$ are non dynamical, they can be eliminated using their equation of motion

$$F_s^f = -\frac{\partial W}{\partial S} + \alpha\beta M_f^2. \quad (5)$$

At this point, to continue the discussion, we must consider a specific form of the superpotential. In the next section, we will consider the typical superpotential for supersymmetric hybrid inflation.

### 3 The model

Within the model of the previous section, let us plug in the superpotential of supersymmetric hybrid inflation i.e.

$$W_{\text{inflation}} = \kappa S(\Phi \bar{\Phi} - \mu^2). \quad (6)$$

$\kappa$ is a dimensionless coupling constant, $S$ is the singlet chiral superfield, while $\Phi$ and $\bar{\Phi}$ are chiral superfields, belonging to the visible sector, that are conjugate under a non trivial representation of some group $G$. One can always impose an appropriate $R$-symmetry such that the superpotential (3) is the most general renormalizable one. We do not specify the form of the superpotential for the hidden sector.

At tree level, the scalar potential is readily computed. It is

$$V(\varphi, \bar{\varphi}, s) = \kappa^2|\varphi\bar{\varphi} - \mu^2|^2 + \kappa^2|s|^2(|\varphi|^2 + |\bar{\varphi}|^2) + D-\text{terms}. \quad (7)$$

where $s$, $\varphi$ and $\bar{\varphi}$ are the scalar components of $S$, $\Phi$ and $\bar{\Phi}$. We will restrict ourselves to the $D$-flat direction $|\varphi| = |\bar{\varphi}|$. Minimizing the potential, one finds that there are two sets of minima. The first is the supersymmetric one, it is located at $|\varphi| = \mu$ and $s = 0$. The second one breaks SUSY, for $s > s_c = \mu$ and $|\varphi| = 0$. Inflation in this scenario proceeds by assuming chaotic initial conditions for the fields $s$ and $\varphi$. That is, the inflaton field $s$ rolls from $s \gg s_c$ towards the true minimum ($s = 0$), while the ”auxiliary” field $\varphi$ is held at the origin. The universe undergoes an exponential expansion phase (inflation) since its energy density is then dominated by the false vacuum one ($V = \kappa^2\mu^4$). But this will not last forever; as soon as $s$ reaches the critical value $s_c$, all the fields rapidly adjust to their SUSY vacuum values restoring supersymmetry, and inflation finishes.

Let us include the tadpole contributions to the scalar potential. Using Eqts. (4) and (5), one obtains the scalar potential as a function of the two fields $\varphi$ and $s$

$$V = \alpha^2\beta^2M_f^4 + \gamma\frac{M_f^4}{M_P}(s + s^\dagger) + 2\kappa^2|s|^2|\varphi|^2 - 2\kappa\alpha\beta M_f^2(|\varphi|^2 - \mu^2) + \kappa^2(|\varphi|^2 - \mu^2)^2. \quad (8)$$

\footnote{Notice the presence of the extra piece in the $F$-term of $s$, which is due to the tadpole; the effect of the tadpole is to shift the vev of $F_S$ by the amount $\alpha\beta M_f^2$.}

\footnote{These symmetries are global, they are likely to be broken by gravitational interactions, so at the end $S$ will not carry any quantum number.}
Clearly, due to the presence of the linear term in $s$, the minimum for $s$ is no more at the origin, but it is now given by

\[ s = -\frac{\gamma M_P^4}{2\kappa^2 M_P |\phi|^2}. \]

(9)

The supersymmetric minimum is recovered when $\gamma = 0$. This corresponds to choose $e$ exactly zero in the expression of the Kähler potential (1). However, a priori, we have no obvious reason to enforce it to this value.

The result is that the values of $s$ and $|\phi|$ are now correlated, and while $s$ rolls down along the inflationary trajectory, $\phi$ moves away from the origin. The usual scenario for hybrid inflation is modified, but the new characteristics of the model can still be used in an inflationary context. For simplicity, we will set the scale $\mu$ to zero in the scalar potential. The scale $M_f$, in our case, can take any value below the Planck scale ($M_f \leq M_P$), since we do not aim to provide a phenomenologically acceptable scenario for supersymmetry breaking. We imagine that this is achieved by some other sector of the model.

The resulting potential, with $\mu$ set to zero, looks similar to another realization of hybrid inflation, the mutated hybrid inflation. Indeed, some years ago, Stewart proposed a new version of hybrid inflation based on a potential of the form

\[ V(\phi, \psi) = V_0 \left(1 - \frac{\psi}{M}\right) + \frac{\lambda}{2} \psi^2 \phi^2. \]

(10)

The inflationary trajectory is obtained by minimizing on $\psi$. Along this trajectory, both $\psi$ and $\phi$ roll. The potential, as a function of $\phi$, reads

\[ V = V_0 \left(1 - \frac{V_0}{2\lambda M^2 \phi^2}\right). \]

(11)

Stewart argued that such a potential can arise from an effective superpotential due to non perturbative effects such as gaugino condensation. In the next two sections, we will see that the addition of singlet tadpoles will provide a new particle physics motivation to this model.

4 Inflating with tadpoles

Let us proceed to analyze our potential. Minimizing with respect to $s$, we end with the scalar potential for the inflaton field $\phi$

\[ V = M_f^4 \alpha^2 \beta^2 \left(1 - \frac{\gamma^2 M_f^4}{2\kappa^2 \alpha^2 \beta^2 |\phi|^2 M_P^2}\right) + \kappa^2 |\phi|^4 - \kappa \alpha \beta M_f^2 (\phi^2 + \phi^4) \]

(12)

The potential (12) looks very similar to the one of mutated hybrid inflation, except for the two last terms. In order to ignore them we must impose
$\xi \ll \left( \frac{\beta \alpha}{\kappa} \right)^{1/2}$, \hspace{1cm} (13)

where we have defined $\varphi = \xi M_f$. Furthermore their first and second derivatives must also be negligible with respect to the derivatives of the first term that is supposed to drive inflation. These requirements translate into the following condition

$\xi \ll \left( \frac{\gamma M_f}{\kappa^2 M_P} \right)^{1/3}$ \hspace{1cm} (14)

To satisfy the slow roll conditions

$\epsilon = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1$ and $|\eta| = M_P^2 \left| \frac{V''}{V} \right| \ll 1$, \hspace{1cm} (15)

we must have

$\xi \gg \left( \frac{\gamma}{\beta \alpha \kappa} \right)^{1/2}$ \hspace{1cm} (16)

The number of $e$-folds is given by

$N = \frac{1}{M_P^2} \int d\varphi \, \frac{V}{V'} \simeq \frac{1}{4} \xi^4 \left( \frac{\beta \alpha \kappa}{\gamma} \right)^2 \hspace{1cm} (17)$

The COBE density perturbation normalization corresponds to

$\frac{V^{3/2}}{M_P^3 V'} \simeq 2 \sqrt{2} N^{3/4} \left( \frac{\kappa}{\gamma} \right)^{1/2} (\alpha \beta)^{3/2} \left( \frac{M_f}{M_P} \right) = 6 \times 10^{-4}$, \hspace{1cm} (18)

and for $N \approx 60$, we obtain the following expression for $M_f$

$M_f \simeq 10^{-5} \left( \frac{2}{\kappa} \right)^{1/2} \frac{M_P}{(\alpha \beta)^{3/2}} \hspace{1cm} (19)$

As in the usual mutated hybrid inflation [11], the spectral index of density perturbations is given by

$n \simeq 1 - 6 \epsilon + 2 \eta \simeq 1 - \frac{3}{2N}$ \hspace{1cm} (20)

For $N \approx 60$, it gives $n \simeq 0.975$.

Combining Eqts. (19), (14) and (16) one ends with

$\kappa \ll 10^{-5} \hspace{1cm} (21)$

This constraint is not surprising. In fact the smallness of the coupling constant $\kappa$ is a typical prediction of hybrid inflation models [5].
Eventually, combining Eqts. (17) and (19), we obtain

\[ M_f \simeq 10^{-5} \frac{\xi}{\beta \alpha} M_P. \]  

To achieve inflation, the parameters of the model must obey various constraints. However, it is possible to fulfill them in a natural way. As an example, we take \( \alpha \) and \( \beta \) to their maximal value i.e. \( \alpha, \beta \sim 1 \): this choice allows to avoid fine tuning for the other parameters. Taking \( \kappa \sim 10^{-6} \), one can consider the loop factor \( \gamma \) in the allowed range \( \gamma \sim 10^{-1} - 10^{-4} \). Consequently, the range for \( \xi \) is \( 10 \ll \xi \ll 10^3 \). We get a scale of SUSY breaking of the order \( M_f \simeq 10^{14} - 10^{17} \) GeV, and the lower one \( (M_f \simeq 10^{14} \) GeV) is the typical scale for a model of mutated hybrid inflation.

Usually, inflation finishes when the slow roll conditions are no more valid. This happens generally before the inflaton reaches the true minimum. There the inflaton begins oscillating coherently reheating the universe. Also in our model, the inflation ends when the slow roll conditions, represented by formula (16), break down. Actually, the inflaton field energy lies between the two scales given by equations (16) and (14): this means that nor the inflaton \( \varphi \) nor the singlet \( s \) reach the true minimum of the scalar potential at the end of inflation.

5 Stability of the potential

The tree level scalar potential usually receives corrections due to loop effects and to supergravity contributions. Such corrections, in our case\(^6\), are dangerous because they can destabilize the inflationary trajectory.

The one-loop corrections, as in usual supersymmetric theories, depend on the mass splitting between the members of the supermultiplet, induced by the supersymmetry breaking. More precisely, the Coleman-Weinberg one-loop effective potential \(^20\) shows that these corrections are proportional to the fourth power of the mass splitting. In our case, it is easy to see that this quantity, being proportional to the tiny coupling constant \( \kappa \) (See Eq. (21), is small enough to render these corrections negligible during the inflationary era.

Unfortunately, the situation with supergravity corrections is much more delicate. Although tadpole contributions, which are an essential ingredient for our model, come from a \( D \)-term, our scenario is actually an \( F \)-term inflationary one. Consequently the scalar potential receives the usual supergravity corrections to \( F \)-terms.

As clearly explained in \(^11\), these corrections are generically non negligible\(^7\), and
one should expect new contributions to the scalar potential in Eq. (8), proportional to $M_f^4(|\varphi|^2 + |s|^2)/M_\phi^2$. In our case, due to the fact that the scale $M_f$ is so large, these corrections are potentially important. Hopefully, other contributions, in a more refined version of our model, would cancel or keep under control such dangerous terms. However we will not consider this issue since it is out of the scope of the paper (See [24, 25, 26] for interesting ideas in this direction).

6 Conclusions

The presence of singlets in supergravity is a problematic issue, because they usually destabilize the hierarchy. Only in the past few years, it has been realized that their properties can provide interesting phenomenological models in Particle Physics [21]. Singlet fields, in the past, have also been used in Cosmology. For example, it was pointed out in [22] that singlets can be useful to provide a strong first order phase transition essential for a successful baryogenesis in the NMSSM, and moreover they are extensively used in inflationary models.

In this paper, we have shown that these fields can have other cosmological applications, and in a supergravity framework. Indeed, we have shown that due to the presence of the tadpole contributions, the usual hybrid inflation scenario is generically modified. We point out that it is possible to use singlet tadpoles in a simple way to provide a new realization for a different scenario of hybrid inflation: the so-called mutated hybrid inflation. In this framework, we have shown that it is possible to obtain an inflationary regime for a natural choice of the parameters.

There is no doubt, despite the unavoidable problems associated to their presence, that singlets tadpoles can lead to interesting cosmological implications.

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References

[1] A. H. Guth, Phys. Rev. D 23 (1981) 347.
[2] A. Linde, *Particle Physics And Inflationary Cosmology*, Chur, Switzerland: Harwood (1990) 362 p. (Contemporary concepts in physics, 5).

[3] D. H. Lyth and A. Riotto, Phys. Rept. 314 (1999) 1, [hep-ph/9807278](https://arxiv.org/abs/hep-ph/9807278).

[4] G. Dvali and A. Riotto, Phys. Lett. B 417 (1998) 20, [hep-ph/9706408](https://arxiv.org/abs/hep-ph/9706408).

[5] G. Lazarides, [hep-ph/0011130](https://arxiv.org/abs/hep-ph/0011130) and references therein.

[6] A. Linde, Phys. Rev. D 49 (1994) 748, [astro-ph/9307022](https://arxiv.org/abs/astro-ph/9307022).

[7] P. Binetruy and G. Dvali, Phys. Lett. B 388 (1996) 241, [hep-ph/9606342](https://arxiv.org/abs/hep-ph/9606342). E. Halloyo, Phys. Lett. B 387 (1996) 43, [hep-ph/9606423](https://arxiv.org/abs/hep-ph/9606423).

[8] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. 73 (1994) 1886, [hep-ph/9406319](https://arxiv.org/abs/hep-ph/9406319).

[9] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49 (1994) 6410, [astro-ph/9401011](https://arxiv.org/abs/astro-ph/9401011).

[10] E. D. Stewart, Phys. Rev. D 51 (1995) 6847, [hep-ph/9405389](https://arxiv.org/abs/hep-ph/9405389). See also [hep-ph/9408302](https://arxiv.org/abs/hep-ph/9408302).

[11] E. D. Stewart, Phys. Lett. B 345 (1995) 414, [astro-ph/9407040](https://arxiv.org/abs/astro-ph/9407040).

[12] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B 124 (1983) 337; A. B. Lahanas, Phys. Lett. B 124 (1983) 341.

[13] A. Linde and A. Riotto, Phys. Rev. D 56 (1997) 1841, [hep-ph/9703209](https://arxiv.org/abs/hep-ph/9703209).

[14] J. Bagger and E. Poppitz, Phys. Rev. Lett. 71 (1993) 2380, [hep-ph/9307317](https://arxiv.org/abs/hep-ph/9307317).

[15] J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B 455 (1995) 59, [hep-ph/9505214](https://arxiv.org/abs/hep-ph/9505214).

[16] H. P. Nilles and N. Polonsky, Phys. Lett. B 412 (1997) 69, [hep-ph/9707248](https://arxiv.org/abs/hep-ph/9707248).

[17] C. Kolda and N. Polonsky, Phys. Lett. B 433 (1998) 323, [hep-ph/9805240](https://arxiv.org/abs/hep-ph/9805240).

[18] C. Kolda, S. Pokorski and N. Polonsky, Phys. Rev. Lett. 80 (1998) 5263, [hep-ph/9803310](https://arxiv.org/abs/hep-ph/9803310).

[19] G. F. Giudice and A. Masiero, Phys. Lett. B 206 (1988) 480.

[20] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.

[21] U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B 492 (1997) 21, [hep-ph/9611251](https://arxiv.org/abs/hep-ph/9611251).
[22] A. T. Davies, C. D. Froggatt and R. G. Moorhouse, Phys. Lett. B 372 (1996) 88, hep-ph/9603388; S. J. Huber and M. G. Schmidt, hep-ph/0003122.

[23] J. R. Espinosa, A. Riotto and G. G. Ross, Nucl. Phys. B 531 (1998) 461, hep-ph/9804214; D. H. Lyth and A. Riotto, Phys. Lett. B 412 (1997) 28, hep-ph/9707273.

[24] M. Bastero-Gil and S. F. King, Nucl. Phys. B 549 (1999) 391, hep-ph/9806477.

[25] J. A. Casas and G. B. Gelmini, Phys. Lett. B 410 (1997) 36, hep-ph/9706439.

[26] M. K. Gaillard, H. Murayama and K. A. Olive, Phys. Lett. B 355 (1995) 71, hep-ph/9504307.