Mediatrix method for filamentation of objects in images: application to gravitational arcs

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Abstract

We present the Mediatrix filamentation method, a novel iterative procedure that decomposes elongated objects in filaments along their main direction over their intensity peak. From this decomposition, the method measures the object’s length and thickness. This technique is applied in preliminary tests to arc-shaped objects (simulated gravitational arcs) to recover their curvature center.

Keywords: image processing, astronomy.

1. INTRODUCTION

Shape analysis and detection are fundamental issues in image processing. For curved shapes, it is of interest to define a curvature, with an associated center and radius. We may find this kind of shape, among others, in astronomical images. The quintessential example in this context is that of gravitational arcs. These objects are the result of the Strong Lensing effect \cite{1, 2} which occurs when a distant source object is aligned with some intervening distribution of matter in the trajectory of light towards us. This matter distribution distorts the space-time, acting like a lens. The image of astronomical objects is distorted and magnified, forming the curved shapes mentioned above. We have developed a novel technique to decompose images of elongated objects in a set of filaments and measure their length, width, and assign a curvature center.

This paper is organized as follows: In section 2, we introduce the Mediatrix Filamentation method explaining its basic elements and procedure. Section 3 presents measurements derived from the Mediatrix method. In

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Section 4, we apply the method on simulated arcs and discuss the results. In Section 5, we present our concluding remarks and future perspectives.

2. MEDIATRIX FILAMENTATION METHOD

In the following we will describe the method to decompose elongated objects into a set of line segments. For concreteness we will consider the object to be composed by a set of pixels with given intensity, as for a digital image. However, in principle, the method can be applied to any intensity distribution, even if not pixelated. The only requirement is that the object should have a clearly defined boundary, in other words, the intensity must be zero outside the object.

The Mediatrix method was originally designed to search for gravitational arcs. It was inspired on a basic geometrical property of the perpendicular bisector of pairs of points on a circle. Namely that these lines, for any set of pairs of points, intersect at the circle center. Therefore if an elongated object can be decomposed into a set of points along its longer direction, and if this object has a shape close to an arc segment the perpendicular bisectors of pairs of these points will intersect in nearby points (i.e. close to the center of curvature).

It turns out, however, that this method can be used to assign segments along the longer direction of any elongated object, i.e. to “filament” the object, or to determine its “spine”, regardless of the presence of curvature. The key procedure to filament the object is to recursively obtain the perpendicular bisector of pairs of points on the image.

Given the points \( P_1 = (x_1, y_1) \) and \( P_2 = (x_2, y_2) \), the perpendicular bisector is a straight line \( y = mx + b \) perpendicular to \( P_1P_2 \) that crosses the middle point of this segment and whose coefficients given by:

\[
m = -\frac{x_2 - x_1}{y_2 - y_1}, \tag{1}
\]

\[
b = \frac{(y_1 + y_2) - m(x_1 + x_2)}{2}. \tag{2}
\]

The Mediatrix method is a recursive method that operates on several iteration steps. Each step is a new Mediatrix level and the method can be
performed up to an arbitrary level \( n \). In the following, we describe the first few levels as an example.

In the first step we determine the extreme points, \( E_1 \) and \( E_2 \) of the object. Several methods have been considered to determine the extreme points of an object (see, e.g., ref \[3\]). Here we use the “farthest-of-farthest” method, by which \( E_1 \) is defined as the most distant point from a reference point on the object (e.g., the brightest pixel on the image or its geometrical center), whereas \( E_2 \) is defined as the pixel on the object farthest from \( E_1 \). Next, the first perpendicular bisector of these two points is calculated. The first Mediatrix point \( M^1 \) is defined as the brightest pixel of the object along the perpendicular bisector. In practice, we take the brightest pixel located at a distance \( d \leq \alpha \Delta p \) from the perpendicular bisector, where \( \Delta p \) is the pixel scale and \( \alpha \) is a free parameter chosen as \( \alpha = \frac{\sqrt{2}}{2} \). The first Mediatrix Point \( M^1 \) is shown in Fig. 1(A) for an arc-shaped object (more specifically, an ArcEllipse \[4\]).

In the second step the perpendicular bisector is now calculated with respect to \( E_1 \), \( M^1 \) and \( M^1 \), \( E_2 \). These two perpendicular bisectors now define two other Mediatrix Points: \( M^2_1 \) and \( M^2_2 \) using the same criteria we used to define \( M^1 \) (Fig 1B). The upper index refers to the iteration level and the lower index is a label to identify the points. Proceeding to the third step, presented in Fig 1(C), we have the set of Mediatrix Points \( M^1 \), \( M^2_1 \), \( M^2_2 \) and the two extremes \( E_1 \) and \( E_2 \). Those points are used to define new Mediatrix Points \( M^3_i \) calculated by picking the highest intensity pixel near to the perpendicular bisector between two neighboring points. The algorithm continues defining new Mediatrix Points \( M^j_i \), corresponding to the \( i \)-est point in the \( j \)-est iteration level, in higher iteration levels until reaching a specified final step \( n \). In Fig 1(D), we present the last step for \( n = 3 \) (as in the previous panels some points were omitted not to crowd the figure). The collection of Mediatrix Points together with the two extreme points are then named keydots. From the keydots, the object is decomposed in \( N = 2^n \) segments or filaments. Each segment connects a keydot to its neighbors. The algorithm outputs a set of vectors \( \vec{n}_j \), where \( j \) varies from 1 to \( N \). Those vectors are perpendicular to the segment which connects a keydot to its neighbor with origin in the middle point of its segment and norm equal to the length of this segment. This is shown in Fig. 1(D) for \( \vec{n}_7 \), where \( |\vec{n}_7| = |M^3_4 M^2_2| \).
3. MEASUREMENTS DERIVED FROM THE MEDIATRIX METHOD

Using the outputs of the proposed algorithm, the object length, $L$, is defined as:

$$L = \sum_{j=1}^{N} |\vec{n}_j|.$$  (3)

The points $\vec{F}_j^1$ and $\vec{F}_j^2$, represented in Fig. 1(D) for $j = 7$, are defined as the two extreme pixels from the set of points along the perpendicular bisector associated to $\vec{n}_j$. Using those points, it is possible to measure the average thickness $W$:

$$W = \frac{1}{N} \sum_{j=1}^{N} |\vec{F}_j^1 - \vec{F}_j^2|.$$  (4)

For arcs constructed from circle segments, all perpendicular bisectors intercept at the center of curvature. Therefore, we may define as the object center of curvature the average of the points $C_{ik} = (x_{ik}, y_{ik})$ generated by the intersection of the lines in the $\vec{n}_i$ and $\vec{n}_k$ directions. The center coordinates $C_a = (x_a, y_a)$ are given by:

$$x_a = \frac{(N-2)!}{N!} \sum_{i=1}^{N} \sum_{k \neq i}^{N} x_{ik}$$  (5)

$$y_a = \frac{(N-2)!}{N!} \sum_{i=1}^{N} \sum_{k \neq i}^{N} y_{ik}$$  (6)

An alternative way to define a curvature center is using the median intercept point $C_m = (x_m, y_m)$, where $x_m$ is defined as the median of the values of $x_{ik}$ and $y_m$ is given by the median of $y_{ik}$.

From these two “center of curvature” definitions we may assign a curvature radius $R_a$ and $R_m$, associated to $C_m$ and $C_a$, as the distance from the first Mediatrix filamentation point $M^1$ to $C_m$ or $C_a$ respectively, i.e., $R_a = |M^1C_a|$ and $R_m = |M^1C_m|$. 

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The Mediatrix filamentation method was implemented in the Python programming language \cite{5}. The code was developed as part of the SLtools library\cite{3}.

4. APPLICATION TO SIMULATED GRAVITATIONAL ARCS

As an example of application of the Mediatrix filamentation method, we consider 3 arc-shaped objects produced through the gravitational lensing effect (see Fig. 22). These arcs were generated using the AddArcs \cite{6} pipeline to simulate gravitational arcs.

AddArcs is a code that simulates realistic gravitational arcs using the galaxy cluster abundance (i.e. the lenses) provided by cosmological simulations and background galaxies (the sources) with morphological parameters and redshift distribution obtained from the Hubble Ultra Deep Field Survey. The source brightness distribution is modeled by Sérsic profiles \cite{7}, which in principle extend to infinity, with elliptical isophotes. Given the input models for the source and the lens, AddArcs controls the software gravlens \cite{8} to perform the gravitational lensing calculations. The segmentation of the simulated image (i.e. the definition of borders) is carried out with the SExtractor \cite{9} software, in a procedure similar to the one described in \cite{4}.

To perform the Mediatrix filamentation we used the function run\_mediatrix\_decomposition\_on\_stamp from the mediatrix\_decomposition module in SLtools, which was developed to apply the method to an image matrix with one isolated arc. The points \((x_{ik}, y_{ik})\) are shown for the tested arcs in Fig. 3. In order to avoid lines that do not intercept or are almost parallel to each other, the algorithm ignores combinations of \(\vec{n}_i\) and \(\vec{n}_k\) which are almost in the same direction, more specifically, we discard pairs with \(|\tan\theta_i - \tan\theta_k| \leq 10^{-3}\), where \(\theta_i\) and \(\theta_k\) are the angles determined by \(\vec{n}_i\) and \(\vec{n}_k\) directions with respect to the \(x\) axis. The center of the mass distribution that generated the the lensing effect, the lens center \(C_l = (x_l, y_l)\) is given by AddArcs. Although from gravitational lensing theory there is no need for the curvature center and the center of mass to coincide, they are usually close. We define the arc radius \(R\) as the distance from the first Mediatrix

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\(^2\)This pipeline is part of SLtools, a library for image processing, catalog manipulation, and strong lensing applications \cite{10}, and is available at http://che.cbpf.br/sltools/.
|   | \( \Delta R_m \) | \( \Delta R_a \) | \( L \) (pixels) | \( L/W \) |
|---|---|---|---|---|
| Arc A | 4 | 0.036 | 0.062 | 294.7 | 5.6 |
|     | 8 | 0.153 | 0.126 | 295.3 | 5.6 |
|     | 16 | 0.144 | 0.092 | 295.7 | 5.6 |
|     | 32 | 1.09 | 0.230 | 296.8 | 5.7 |
| Arc C | 4 | 0.100 | 0.096 | 362.4 | 12.6 |
|     | 8 | 0.211 | 0.68 | 363.1 | 12.7 |
|     | 16 | 0.166 | 0.170 | 363.3 | 12.7 |
|     | 32 | 0.307 | 0.168 | 363.9 | 12.7 |
| Arc C | 4 | 0.101 | 0.097 | 192.9 | 5.5 |
|     | 8 | 0.051 | 0.093 | 193.1 | 5.5 |
|     | 16 | 0.085 | 0.126 | 193.5 | 5.6 |
|     | 32 | 0.198 | 0.209 | 194.7 | 5.6 |

Table 1: Results of Mediatrix measurements for 3 simulated arcs A, B and C. The first column is the number of segments the arc was divided into. The second and third are the ratio between the distance from the lens center to the center of curvature — calculated using the average \( \Delta R_a \) and median \( \Delta R_m \) respectively — and the curvature radius \( R \). The fourth column is the arc length and the fifth is the \( L/W \) ratio.

filamentation point \( M^1 \) to \( C_l \), so \( R = |M^1C_l| \). The distance between \( C_l \) and the measured centers of curvature \( C_m \) or \( C_a \) is \( \Delta R_m \) and \( \Delta R_a \) respectively. We present the results for \( R_a \), \( R_m \), discrepancy in \( R \) using \( R_a \), \( R_m \), \( L \), \( L/W \) and curvature \( R/L \) in Table I for the 3 arcs.

### 4. CONCLUSIONS AND FUTURE APPLICATIONS

The Mediatrix filamentation method is a technique that decomposes and measures elongated shapes, and may be used in the detection of shapes such as gravitational arcs. The technique can also define an expected center for curved objects. Previous definitions of the curvature center in the gravitational arc characterization problem assumed that the arcs are circular [11]; with the Mediatrix method there is no need to use this hypothesis.
The $\vec{n}_j$ vectors can be used as input for other morphological estimators for arc-shaped objects. For example, in ref. [4] the Mediatrix output is used as a starting point for a method that fits the brightness distribution of arcs using analytical templates to derive structural parameters of the arcs. Another application is to use parameters derived from the Mediatrix decomposition as input for a neural network to classify objects [12].

In the current case, the extreme points were used as the first Mediatrix step but other definitions for the first step can be used, e.g. object edges, in order to filament more complex objects not necessarily with a single preferred direction.

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First Step: Define the extreme points and the first Mediatrix Point.

Second Step: Define new Mediatrix Points between each pair of neighbors.

Step n: Define new Mediatrix Points between each pair of neighbors Mediatrix Points from all previous Steps.

Final Step: The algorithm assigns a set of vectors perpendicular to the segments that connect two neighboring keydots.

Figure 1: Steps in the Mediatrix Filamentation method. After $n$ iterations, the method determines a set of $2^n$ points defined by the maximum of intensity along the $2^n$ perpendicular bisector and the $2^n$ vectors perpendicular to neighboring points with magnitude given by the distance between these points. For clarity, only some points are shown on the figure, which illustrates the steps for $n = 3$. 
Figure 2: Arcs A, B, and C respectively used as input for the Mediatrix method.
Figure 3: Straight lines defined by the vectors $\vec{n}_j$ and their intercepts, for $N = 4$. From top to bottom: arcs A, B and C.