Spontaneous CP violation model with flavor symmetry in large extra dimensions

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Abstract

We construct the minimal SUSY model that causes spontaneous CP violation with an abelian flavor symmetry in the context of the large extra dimensions and show that various phenomenological problems can be solved by introducing only scales below the low fundamental scale. We also realize the realistic size of the CP violation and the small masses of the neutrinos. The strong CP problem can be solved by the axion scenario and the axion can be made invisible by introducing an additional large extra dimension.

1 Introduction

There is a large hierarchy among the fermion masses that have been observed. This hierarchy cannot be explained within the standard model and thus tackling this problem gives crucial clues to high energy physics beyond the standard model. One of the solutions to this problem is introducing a flavor symmetry, which acts on fermions in a flavor dependent way.

Another problem that has not been solved is the origin of the CP violation. CP symmetry is a very good symmetry, but it has been observed that CP is violated in the neutral K meson system by a small amount. This smallness of the CP violation is also a mystery in particle physics. One of the convincing solutions to this problem is the spontaneous CP violation (SCPV). This idea is very attractive since it can control the small amount of the observed CP violation quite naturally.

Recently the impact of the existence of the large extra dimensions, which is indicated by the string theory, are discussed in many papers. The main impact of this possibility is that the fundamental scale of the theory $M_*$ can be lowered from the Planck scale ($\sim 10^{19}$ GeV) to a much lower scale. This argument is assumed that only gravitons can propagate into the large extra dimensions and the standard model particles are confined to a four-dimensional hypersurface, such as a D3-brane. In the case that the gauge bosons of the standard model (and some of the matter particles) also propagate into the extra dimensions, we can lower the grand unification scale $M_{\text{GUT}}$ to the TeV region due to the power-law running of the gauge couplings.

Another implication of the large extra dimensions is the volume factor suppression of the couplings between the bulk fields and boundary fields, which is used to

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explain the smallness of the neutrino masses in Ref. 8. This suppression was also used to explain the hierarchy among the masses of quarks and leptons 9, 10.

In this paper, we discuss the minimal model in which the fermion mass hierarchy is realized by an abelian flavor symmetry and the smallness of the CP violation is controlled by the SCPV scenario in the context of the large extra dimensions. Then we will show that various phenomenological problems can be solved by introducing only scales below the low fundamental scale (1000 TeV in our scenario).

The paper is organized as follows. In the next section, we will introduce our model and realize the realistic hierarchy among fermion masses. In Section 3, we will estimate CP violation parameters and show that they are consistent with the experimental values, and other phenomenological implications are discussed in Section 4. Section 5 is devoted to conclusions.

2 Model

2.1 Flavor symmetry and Yukawa hierarchy

We will introduce an abelian global symmetry $U(1)_A$ as the flavor symmetry. Here we will deal with the 4-Higgs-doublet (4HD) model because it is the minimal model that causes SCPV as is explained later. Thus some symmetry is required to suppress the dangerous flavor changing neutral current (FCNC) and we will introduce the $Z_2$ symmetry, which distinguishes the “standard” Higgs doublets that mainly give masses to fermions from the “extra” Higgs doublets that are prevented from having vacuum expectation values (VEVs). Since the fundamental scale $M_*$ is around 1000 TeV in our scenario, the supersymmetry (SUSY) is assumed to avoid the naturalness problem. R-parity is assumed here. We will consider the case that there is a large extra dimension, which is compactified by the radius $R$, besides the usual four-dimensional space-time and some fields feel the fifth dimension while the others are confined to a four-dimensional boundary. The field contents of our model and their charges of the $U(1)_A$ and $Z_2$ parity are listed in Table 1 and Table 2. Here $Q_i$, $\bar{U}_i$, $\bar{D}_i$, $L_i$ and $\bar{E}_i$ represent the superfields of the $i$-th generation of the left-handed quark doublet, right-handed up-type quark singlet, right-handed down-type quark singlet, left-handed lepton doublet and right-handed charged lepton singlet, respectively. $A_\mu^a (a = 1, 2, 3)$ are the gauge superfields and $H_i (i = 1, 2, 3, 4)$ are the superfields of the Higgs doublets. They are all the description from the viewpoint of the four-dimensional theory, and the superfields listed in Table 2 should be interpreted as the Kaluza-Klein zero modes for the extra dimension. $\Phi$, $\Phi'$ and $\Phi''$ denote the five-dimensional gauge-singlet scalar fields and their superpartners.

Here we will give a brief explanation of the volume factor suppression. Let us denote $\Psi(x, y)$ as a five-dimensional bulk field, where $y$ represents the coordinate of the extra dimension. If we Fourier expand

$$\Psi(x, y) = \sum_0^\infty \frac{1}{\sqrt{2\pi R}} \psi_m(x) e^{i(m/R) y},$$

then we can regard $\psi_m(x)$ as a four-dimensional field corresponding to the $m$-th Kaluza-Klein mode.

On the other hand, the boundary fields are localized at the four-dimensional wall whose thickness is of order $M_*^{-1}$ so a coupling involving at least one boundary

1 Recently, however, it is shown that this possibility seems to be disfavored in the context of the TeV-strings because the Kaluza-Klein modes of the gluons generate dangerous flavor and CP-violating interactions.

2 We consider the hard brane case, in which the brane tension is of order $M_4^{1}$.
where $Z$ charges. The numbers in the parentheses are charges. The numbers in the parentheses are 

Table 2: The fields that can propagate into the extra dimension and their $U(1)_A$ charges. The numbers in the parentheses are $Z_2$ parity.

| Field | $U(1)_A$ | $U(1)_L$ | $U(1)_R$ | $U(1)_H$ | $U(1)_H'$ | $U(1)_H''$ |
|-------|-----------|----------|---------|---------|---------|---------|
| $D_1$ | 1 (+)     | 1 (+)    | 0 (+)   | 0 (+)   | 3 (+)   | 0 (+)   |
| $D_2$ | 1 (+)     | 1 (+)    | 0 (+)   | 0 (+)   | 3 (+)   | 0 (+)   |
| $D_3$ | 1 (+)     | 1 (+)    | 0 (+)   | 0 (+)   | 3 (+)   | 0 (+)   |

field is suppressed by a factor of $(1/\sqrt{2\pi M_* R})^k$, where $k$ is a number of bulk fields included in the coupling $\tilde{g}$.

With the above charge assignment, we have the Yukawa couplings with the following structure after the scalar fields $\Phi$ and $\Phi''$ obtain the VEVs,

$$W_{\text{yukawa}} = -h_{ij}^d Q_i \tilde{D}_j H_1 + h_{ij}^u Q_i \tilde{U}_j H_2 - h_{ij}^e L_i \tilde{E}_j H_1$$

$$-h_{ij}^d Q_i \tilde{D}_j H_3 + h_{ij}^u Q_i \tilde{U}_j H_4 - h_{ij}^e L_i \tilde{E}_j H_3,$$

$$h_{ij}^u \approx \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \quad h_{ij}^d \approx \frac{1}{\sqrt{2\pi M_* R}} \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix},$$

$$h_{ij}^e \approx \frac{1}{\sqrt{2\pi M_* R}} \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad h_{ij}^{u'} \approx \tilde{c} h_{ij}^u, \quad h_{ij}^{d'} \approx \tilde{c} h_{ij}^d, \quad h_{ij}^{e'} \approx \tilde{c} h_{ij}^e,$$  \quad \text{(2)}

where

$$\epsilon \equiv \left| \frac{\langle \Phi \rangle}{M_*^{3/2}} \right| = \left| \frac{v}{M_*} \right|^{3/2}, \quad \tilde{c} \equiv \left| \frac{\langle \Phi'' \rangle}{M_*^{3/2}} \right| = \left| \frac{v''}{M_*} \right|^{3/2}.$$  \quad \text{(3)}

Here $\langle \Phi \rangle \equiv v^{3/2}$ and $\langle \Phi'' \rangle \equiv v''^{3/2}$. Note that the bulk fields $\Phi$ and $\Phi''$ have the mass dimension of $3/2$.

Eq. (3) suggests that $\sqrt{2\pi M_* R} \approx m_t/m_b \approx 40$, so that $M_* \approx 250 R^{-1}$. If $(v/M_*)^{3/4} \approx 1/15$, i.e., $v \simeq M_*/6$, the realistic hierarchy between fermion masses can be realized.

The value of $R^{-1}$ is constrained from Ref. [12, 13] to be greater than 2 TeV, so we will set $R^{-1} = 4$ TeV throughout this paper. Therefore $M_* \approx 1000$ TeV, $v \approx 160$ TeV and $\epsilon \approx 1/15$.

It can easily be shown that the runnings of Yukawa couplings between $M_{\text{GUT}}$ and $R^{-1}$ do not destroy this hierarchy.

The relation between the fundamental scale $M_*$ and the Planck scale $M_p$ in the case of the $n$ extra dimensions is $M_p^2 = (2\pi)^{n} M_*^{2+n} R_1 R_2 \cdots R_n$, where $R_i$ is the radius of the $i$-th extra dimension. We have assumed the existence of more extra dimensions that are irrelevant to our discussion to satisfy this relation.
2.2 Dangerous FCNC

We can expand the bulk field $\Phi$ around its VEV in terms of four-dimensional fields $\phi_n$ as follows.

$$\Phi(x, y) = \frac{v^3}{2} + \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \phi_n(x)e^{i(n/R)y},$$

(4)

where $x$ and $y$ represent the four-dimensional coordinates and the coordinate of the extra dimension respectively.

These fluctuation fields $\phi_n$ are thought to have masses of order $v$ if $n$ is not very large. Since these are much lighter than the counterpart of the usual Froggatt-Nielsen mechanism, tree-level processes exchanging these light $\phi_n$ seem to cause the disastrously large FCNC at first sight. However, their couplings to the light quarks and leptons are largely suppressed by the large power of the volume factor $1/\sqrt{2\pi M_\ast R} \simeq 1/40$, so there are no dangerous FCNC processes arising from $\phi_n$-exchange.

There are another flavor changing interactions that might give rise to too large FCNC. In the case of an abelian flavor symmetry, flavor changing 4-fermi interactions such as $(q_1 \bar{d}_2)(\bar{q}_1 d_2)$, where $q_i$ and $\bar{d}_i$ are quark fields, are allowed. In our case, however, taking into account the volume factor suppression, this flavor changing interaction terms are

$$L_{FC} = \frac{a}{M_G^2(2\pi M_\ast R)}(q_1 \bar{d}_2)(\bar{q}_1 d_2) + \cdots,$$

(5)

where $a$ is a dimensionless $O(1)$ constant.

Thus an effective cut-off $\Lambda$ becomes $\Lambda \sim M_\ast \sqrt{2\pi M_\ast R} \sim 4 \times 10^4$ TeV and the FCNC arising from these interactions do not exceed the experimental bounds. (See Table 1 in Ref.\[4\].)

2.3 Grand Unification

In the case that the gauge bosons of the standard model propagate into the large extra dimensions, the grand unification scale $M_{GUT}$ is significantly lowered. The new GUT scale is expected to be $20 \sim 30R^{-1}$ from Ref.\[7\]. Unfortunately, the naive unification of the gauge couplings does not occur in our model, but the cross points of the running of the gauge couplings are not so far each other due to their power-law runnings. Taking this fact into account, the grand unification of the gauge couplings might be realized with the help of the threshold effect at $M_{GUT}$. In this paper, we will assume such a situation and use the term “GUT” in this sense.

Note that $M_{GUT} \simeq 20 \sim 30R^{-1}$ is the same order as $v$. This suggests that $\Phi$ obtains the VEV at $M_{GUT}$. It seems quite natural for $\Phi$ to acquire the VEV at $M_{GUT}$ because at the same scale some scalar fields obtain non-zero VEVs and break the GUT gauge group into the standard model group: $SU(3)_C \times SU(2)_L \times U(1)_Y$.

3 Spontaneous CP violation

3.1 Higgs sector

We will assume that the bulk scalar fields $\Phi$, $\Phi'$ and $\Phi''$ have complex VEVs at $M_{GUT}$ and thus CP is violated spontaneously. The effective $\mu$-terms are generated after the bulk scalars obtain the VEVs.

$$W_{\mu{\text{term}}} =$$
\[
\lambda_{12} v \left( \frac{v}{M_*} \right)^2 \left( \frac{v'}{M_*} \right)^{3/2} H_1 H_2 + \lambda_{14} v \left( \frac{v}{M_*} \right)^2 \left( \frac{v'}{M_*} \right)^{3/2} \left( \frac{v''}{M_*} \right)^{3/2} H_1 H_4 \\
+ \lambda_{32} v \left( \frac{v}{M_*} \right)^2 \left( \frac{v'}{M_*} \right)^{3/2} \left( \frac{v''}{M_*} \right)^{3/2} H_3 H_2 + \lambda_{34} v \left( \frac{v}{M_*} \right)^2 \left( \frac{v'}{M_*} \right)^{3/2} \left( \frac{v''}{M_*} \right)^{3/2} H_3 H_4,
\]

(6)

where \( \lambda_{ij} \) are dimensionless \( O(1) \) real couplings.

From now on, we will set \( v \) to real by using the \( U(1)_A \) symmetry and assume that \( |v| \approx |v'| \approx 40 R^{-1} = 160 \text{ TeV} \). In this case,

\[
|v \left( \frac{v}{M_*} \right)^2 \left( \frac{v'}{M_*} \right)^{3/2}| \approx 0.07 R^{-1} \approx 300 \text{ GeV},
\]

(7)

so the weak-scale \( \mu \)-parameters are generated.

As a result, the \( \mu \)-terms of the superpotential have the following structure.

\[
W_{\mu\text{term}} = \mu_{12} e^{i \alpha} H_1 H_2 + \hat{\epsilon} \mu_{14} e^{i \beta} H_1 H_4 + \hat{\epsilon} \mu_{32} e^{i \beta} H_3 H_2 + \mu_{34} e^{i \alpha} H_3 H_4,
\]

(8)

where \( \mu_{ij} \) are weak-scale order and real, and the phases \( \alpha \) and \( \beta \) are of order one. The value of \( \hat{\epsilon} \equiv |v''/M_*|^{3/2} \) is determined later.

### 3.2 soft SUSY breaking parameters

We will apply the Scherk-Schwarz mechanism \(^{[15]}\) to break the supersymmetry. In this case the SUSY breaking scale is identified with the compactification scale \( R^{-1} \). According to the charge assignment of the \( R \)-parity, the particles have the following masses below the SUSY breaking scale \( R^{-1} \) \(^{[16]}\).

(i) **SM particles (except Higgs bosons)**: massless

(ii) **Higgs bosons**: obtain masses through one-loop

(iii) **gaugino, \( \tilde{d} \) and \( \tilde{l} \)**: \( O(R^{-1}) \) (\( \sim 2 \text{ TeV} \))

(iv) **the other sfermions**: \( O(\sqrt{\alpha_1} R^{-1} \sim \sqrt{\alpha_3} R^{-1}) \) (several hundred GeV)

where \( \tilde{d} \) and \( \tilde{l} \) are the scalar components of \( \tilde{D} \) and \( \tilde{L} \) respectively.

The mass (squared) matrices of the sfermions satisfy the degeneracy and proportionality conditions, which are required to suppress the dangerous FCNC \(^{[17]}\).

In particular, the mass parameters in the Higgs sector are

\[
m_i^2 \simeq (\text{a few hundred GeV})^2,
\]

\[
m_{ij}^2 \simeq \mu_{ij} \alpha R^{-1}.
\]

Thus the Higgs potential \( V \) has the following structure,

\[
V = m_1^2 H_1^\dagger H_1 + m_2^2 H_1^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_4^2 H_4^\dagger H_4
\]

\[
+ (m_{12}^2 H_1 H_2 + \text{h.c.}) + (\hat{\epsilon} m_{14}^2 H_1 H_4 + \text{h.c.})
\]

\[
+ (\hat{\epsilon} m_{32}^2 H_3 H_2 + \text{h.c.}) + (m_{34}^2 H_3 H_4 + \text{h.c.})
\]

\[
+ (\hat{\epsilon} m_{34}^2 H_3^\dagger H_4 + \text{h.c.}) + (\hat{\epsilon} m_{42}^2 H_4^\dagger H_4 + \text{h.c.}) + V_D,
\]

where \( V_D \) represents the D-term and \( m_i^2 \) and \( m_{ij}^2 \) are about \((100 \sim 300 \text{ GeV})^2\). Here \( m_{ij}^2 \) are complex and their phases cannot be removed by the redefinition of the Higgs fields and one phase is left. In the MSSM or the NMSSM, all the phases of the Higgs mass parameters can be absorbed by field redefinitions and thus SCPV
cannot occur without the appearance of a too light Higgs boson \cite{18, 19, 20, 21}. This is the reason for our model to be the minimal SCPV model.

These situations are the same as in Ref. \cite{22} and as discussed there the potential Eq. (10) has the vacuum with the following structure.

\begin{align}
    v_1, v_2 &= O(v_w), \quad \text{real up to } \epsilon^2, \\
    v_3, v_4 &= O(\epsilon v_w), \quad \text{arbitrary phases.}
\end{align}

Here we have taken the basis on which $m_{13}^2 = m_{24}^2 = 0$ and $m_{32}$ and $m_{34}$ are real \cite{22}. On this basis, Yukawa couplings are

\begin{align}
    h_u, h_d, h_e : \text{real up to } \epsilon^2, \\
    h'_u, h'_d, h'_e : \text{arbitrary phases.}
\end{align}

The hierarchical structure does not change by these field redefinitions.

Thus the CKM matrix is real up to $\epsilon^2$.

### 3.3 K physics

The CP-violation parameter $\epsilon_K$ can be written in terms of the mass matrix element of the neutral K meson in the $K^0$-$\bar{K}^0$ basis $M_{12}$,

$$ |\epsilon_K| \simeq \frac{1}{2\sqrt{2}} \frac{\text{Im} M_{12}}{\text{Re} M_{12}}. $$

The dominant contribution to Re$M_{12}$ can come from the standard box diagram and the tree-level diagram with the neutral-Higgs exchange depicted in Fig. 1 and Fig. 2.

The contribution of Fig. 1 is calculated as follows \cite{23}.

$$ M_{12}^{\text{box}} = \frac{G_F}{2\sqrt{2}} \frac{\alpha}{4\pi \sin^2 \theta_W} (\cos \theta_c \sin \theta_c)^2 \frac{m_Z^2}{m_W^2} \frac{\langle K^0 | \bar{d}_L \gamma_\mu s_L \bar{d}_L \gamma^\mu s_L | \bar{K}^0 \rangle}{m_K}. $$
\[
\simeq 10^{-13} \cdot \frac{(K^0|\bar{d}_L\gamma_\mu s_L\bar{d}_L\gamma_\mu s_L|\bar{K}^0)}{m_K} (\text{GeV}^{-2}),
\]
where \(m_c, m_W\) and \(m_K\) are the masses of the c quark, W boson and the K meson respectively, and \(G_F\) and \(\alpha\) are the Fermi constant and the fine structure constant. \(\theta_W\) and \(\theta_c\) are the Weinberg angle and the Cabibbo angle respectively.

On the other hand, the contribution of Fig. 2 is calculated as
\[
M_{12}^{\text{tree}} = \frac{h_{21}^d h_{21}^{d*}}{m_{H^0}^2} \frac{\langle K^0|\bar{d}_L s_R\bar{d}_R s_L|\bar{K}^0\rangle}{m_K},
\]
where \(m_{H^0}\) is the neutral Higgs mass.

Then,
\[
\left| \frac{M_{12}^{\text{tree}}}{M_{12}^{\text{box}}} \right| \simeq 10^{13} (\text{GeV})^2 \frac{h_{21}^d h_{21}^{d*}}{m_{H^0}^2} \frac{\langle K^0|\bar{d}_L s_R\bar{d}_R s_L|\bar{K}^0\rangle}{\langle K^0|d_L\gamma_\mu s_L d_L\gamma_\mu s_L|K^0\rangle}.
\]
According to Ref. [22],
\[
\frac{\langle K^0|\bar{d}_L s_R\bar{d}_R s_L|\bar{K}^0\rangle}{\langle K^0|d_L\gamma_\mu s_L d_L\gamma_\mu s_L|K^0\rangle} \simeq 7.6,
\]
and we obtain
\[
\left| \frac{M_{12}^{\text{tree}}}{M_{12}^{\text{box}}} \right| \simeq 7.6 \times 10^{13} (\text{GeV})^2 \cdot \frac{1}{40} \frac{1}{40} \cdot 150 \varepsilon \simeq 150 \varepsilon^2.
\]

Here we have assumed that \(m_{H^0} = 300\) GeV. If we set \(|v'| \simeq |v| \simeq 160\) TeV, \(\bar{\varepsilon} \simeq \varepsilon \simeq 1/15\) and thus \(|M_{12}^{\text{tree}}/M_{12}^{\text{box}}| = O(1)\). In such a case, the value of \(\varepsilon_K\) becomes too large. Then we will assume \(|v'| \simeq |v|/2 \simeq 80\) TeV. In this case, \(\bar{\varepsilon} \simeq 1/44\) and we obtain \(|M_{12}^{\text{tree}}/M_{12}^{\text{box}}| = 0.08\). Therefore Re\(M_{12}\) is dominated by the box diagram.

On the contrary, the main contribution to Im\(M_{12}\) comes from the Fig. 3 since the KM phase is greatly suppressed by the factor \(\bar{\varepsilon}^2\) while the “extra” Yukawa couplings \(h_{ab}^{d}\) have arbitrary phases.

Hence we can estimate the absolute value of \(\varepsilon_K\) at
\[
|\varepsilon_K| \simeq \frac{1}{2\sqrt{2}} \left| \frac{M_{12}^{\text{tree}}}{M_{12}^{\text{box}}} \right| \sim 10^{-2}.
\]

Taking into account the fact that there are \(O(1)\) ambiguity in the hierarchical structure of the Yukawa matrices and the mixing among the neutral Higgs fields, we can conclude that the above value of \(\varepsilon_K\) has an ambiguity about one order of magnitude. As a result, we can estimate the value of \(\varepsilon_K\) at
\[
|\varepsilon_K| \simeq 10^{-3} - 10^{-1},
\]
and this is consistent with the experimental value.

Next we will consider the value of \(\varepsilon'_K/\varepsilon_K\). \(\varepsilon'_K\) can be written as
\[
|\varepsilon'_K| \simeq \frac{1}{\sqrt{2}} \left| \frac{A_2}{A_0} \right| t_0, \quad (t_0 = \frac{\text{Im}A_0}{\text{Re}A_2})
\]
where \(A_i\) is the decay amplitude of a \(K^0\) into two pions of isospin \(i\) and \(|A_2/A_0| \simeq 1/22\) from the experiments.

The candidates of the main contributions to \(t_0\) are the standard penguin diagram, the penguin diagrams with chargino and stop and tree-level diagrams with charged Higgs. When the charged Higgs is relatively light, for example \(m_{H^+} \simeq 150\) GeV, the third-type diagrams give the main contribution. According to Ref. [22], \(\varepsilon'_K/\varepsilon_K\) can be estimated at around \(10^{-5}\) in the case of \(m_{H^+} \sim 1\) TeV. Then we can estimate \(\varepsilon'_K/\varepsilon_K \sim 10^{-3}\) with \(m_{H^+} \simeq 150\) GeV. This value is also consistent with the experimental value.
3.4 EDM and B physics

For the same reason as discussed in [22], the values of the electric dipole moments (EDM) of the neutron and the electron are below and close to the experimental upper bounds in our model.

Finally, we will comment that all the CP asymmetries in the B decays are small enough to be neglected because the KM phase and the phases of the “standard” Yukawa couplings are of order $\tilde{\epsilon}^2$, and the “extra” Yukawa couplings and the VEVs of the “extra” Higgs fields are both suppressed by $\tilde{\epsilon}$.

4 Other phenomenological implications

4.1 Neutrino

In our model, it does not seem that the see-saw mechanism can be applied at first sight since the fundamental scale $M_\ast \,(\simeq 1000 \,\text{TeV})$ is much lower than the scale required in the usual see-saw mechanism. However, the volume factor suppression enables the neutrinos to have the desirable small masses.

Suppose that the right-handed neutrinos $\nu_{Ri}$ ($i = 1, 2, 3$) are the bulk fields propagating into the extra dimension. Then the neutrino Yukawa couplings $h^\nu_{ij}$ are suppressed by the volume factor: $1/2\pi M_\ast R \simeq 1/1600$. Therefore it is possible to obtain the small neutrino masses of order eV range even in the case that the Majorana mass scale of the right-handed neutrinos $M_{\nu R}$ is around the low fundamental scale $M_\ast \simeq 1000 \,\text{TeV}$. For example, if we assume $\nu_{Ri}$ not to have the $U(1)_A$ charges, we can estimate the Dirac mass of the neutrino at

$$m_{\nu} \simeq \frac{(\epsilon v_w)^2}{M_{\nu R}} \frac{1}{(2\pi M_\ast R)^2} \sim 10^{-1} \,\text{eV},$$

(22)

where $v_w = 174 \,\text{GeV}$. This is consistent with the value of the mass of $\nu_\tau$ estimated from the neutrino-oscillation experiments.

Further, we can gain more suppression by assigning non-zero $U(1)_A$ charges to $\nu_{Ri}$.

4.2 Strong CP problem

We have a Nambu-Goldstone (NG) boson associated with the breaking of the $U(1)_A$ symmetry since $\Phi$ and $\Phi'$ have VEVs at $M_\text{GUT}$ [2]. This NG boson will behave like an axion, which can set the $\theta$-parameter to be zero at low energy.

The axion field $\varphi$ couples to the quarks and leptons such as

$$\frac{\lambda}{M_\text{GUT}} \partial_\mu \varphi \bar{u} \gamma_\mu u,$$

(23)

where $\lambda$ is a dimensionless $O(1)$ coupling and $u$ is the u-quark field. Since $M_\text{GUT} \sim 10^5 \,\text{GeV}$ here, this coupling seems too strong to realize an invisible axion.

Then we will introduce a new extra dimension whose radius is denoted by $R_2$ and new fields $\psi$ and $\bar{\psi}$ that feel this new dimension. To illustrate the situation, let us consider the toy model with the couplings such as $\xi \varphi \bar{u} \bar{q} \gamma^\mu q$, where $\xi$ and $\eta$ are dimensionless couplings and $q$ denotes the quark field, and the bulk fields $\psi$ and $\bar{\psi}$ couple to only $\varphi$. Then the renormalization group equation (RGE) of $\xi$ is

$$4\pi \frac{d\xi}{dt_2} = 2\pi t_2 \xi^3, \quad \left( t_2 = t_2(\Lambda) \equiv \frac{1}{2\pi} \left( \frac{\Lambda}{R_2} \right)^2 \right)$$

(24)

$^5$To be exact, we have a pseudo-NG boson because $U(1)_A$ is anomalous.
Solving this equation,
\[ \xi^2(A) = \frac{1}{1/\xi^2(R_{1}^{-1}) - 1/2 \cdot (t_2^2 - 1/4\pi^2)}. \]  
(25)

If we will set \( t_2(M_{\text{GUT}}) \sim 10^5 \), i.e., \( R_{1}^{-1} \sim 100 \ \text{MeV} \) and \( \xi(M_{\text{GUT}}) = O(1) \), we can obtain greatly suppressed coupling at low energy \( \xi(R_{1}^{-1}) \sim 10^{-5} \).

On the other hand, the RGE of \( \eta \) is
\[ 4\pi \frac{d\eta}{dt_2} = \eta \left( 2\pi C_\xi t_2 \xi^2 - C_e g_e^2 \right), \]  
(26)
where \( C_\xi \) and \( C_e \) are \( O(1) \) constants and \( g_e \) is the electro-magnetic gauge coupling. We have neglected the terms involving small Yukawa couplings.

Solving this equation,
\[ \eta(A) \simeq C \cdot \frac{\xi C_\xi(A)}{t_2^{C_e - \alpha}}, \]  
(27)
where \( C \) is a constant determined by the initial condition.

In the case of \( C, C_\xi = 1 \), we will obtain very small coupling \( \eta(R_{1}^{-1}) \simeq \xi(R_{1}^{-1}) \sim 10^{-5} \).

This suppression of \( \eta \) comes from the power-law running of the field renormalization factor of \( \varphi \) [10]. Thus the axion-quark coupling in our model \( \lambda \) can receive the similar suppression and make the axion invisible at low energy.

### 4.3 Higgs mass

Our model becomes a supersymmetric 4-Higgs-doublet (4HD) standard model below the compactification scale \( R^{-1} \), so the lightest Higgs boson must be lighter than 130 GeV, which is the same upper bound as the MSSM case [25]. Note that there is no large contribution to the Higgs mass bound coming from the existence of the “extra” Higgs particles appearing at two-loop level discussed in Ref. [25] in spite of the low cut-off scale \( M_\star \). This is because the “extra” Yukawa couplings are suppressed by \( \tilde{\epsilon} \) in our model.

### 5 Conclusions

Here we considered the minimal SUSY model in which the fermion mass hierarchy is realized by an abelian flavor symmetry \( U(1)_A \) and the smallness of the CP violation is controlled by the spontaneous CP violation in the context of the large extra dimensions. In our model various physical scales such as the symmetry breaking scales of \( U(1)_A, Z_2 \)-parity that guarantees the natural flavor conservation in the Higgs sector, CP symmetry and Peccei-Quinn symmetry that is identified with \( U(1)_A \) are unified into the same scale (\( \sim 100 \ \text{TeV} \)), at which the grand unification of the gauge couplings might be occur. This is a quite attractive feature that should be possessed by the theory beyond the standard model. In models with the low fundamental scale \( M_\star \) like theories with large extra dimensions, one must explain various phenomenological problems only with scales that is below \( M_\star \). Our model satisfies this requirement.

Our model becomes the 4HD SUSY standard model whose parameters are strongly controlled by the high energy physics, and we showed that it can realize the realistic size of the CP violation.

The small masses of the neutrinos can also be obtained by an assumption that the right-handed neutrinos are the bulk fields.
Furthermore, the strong CP problem can be solved by the presence of the NG boson associated with the $U(1)_A$ breaking, which is regarded as the axion and can be made invisible by introducing a new large extra dimension and new fields which couple to only the axion and feel this new dimension.

Finally, we comment about the hierarchy between the large VEVs of $\Phi, \Phi'$ and $\Phi''$ ($\sim 100$ TeV) and the small VEVs of the Higgs fields ($\sim 100$ GeV). It is naturally understood by the fact that the physics on the boundary can be regarded as the fluctuation of the physics in the bulk and thus VEVs induced on the boundary are generically much smaller than VEVs in the bulk.

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