Entropic Corrections to Einstein Equations

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Considering the general quantum corrections to the area law of black hole entropy and adopting the viewpoint that gravity interprets as an entropic force, we derive the modified forms of MOND theory of gravitation and Einstein field equations. As two special cases we study the logarithmic and power-law corrections to entropy and find the explicit form of the obtained modified equations.

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I. INTRODUCTION

The discovery of the black holes thermodynamics in 1970's, implies that there should be some deep connection between the laws of thermodynamics and gravity. According to the black hole thermodynamics a black hole has an entropy proportional to its horizon area and a temperature proportional to its surface gravity, and the entropy and temperature together with the mass of the black hole satisfy the first law of thermodynamics [1]. Jacobson was the first who revealed the deep connection between thermodynamics and gravity by deriving the Einstein field equation from the first law of thermodynamics [2]. Following Jacobson, however, several recent investigations have shown that there is indeed a deeper connection between gravitational dynamics and horizon thermodynamics. It has been shown that the gravitational field equations in a wide variety of theories, when evaluated on a horizon, reduce to the first law of thermodynamics and vice versa. This result, first pointed out in [3], has now been demonstrated in various theory including f(R) gravity [4], cosmological setups [5–10], and in braneworld scenarios [11, 12]. For a recent review on the thermodynamical aspects of gravity and complete list of references see [13]. The deep connection between horizon thermodynamics and gravitational dynamics, help us to understand why the field equations should encode information about horizon thermodynamics. These results prompt people to take a statistical physics point of view on gravity.

An interesting new proposal on the statistical mechanical origin of the thermodynamic nature
of gravity was recently suggested by Verlinde [14] who interpreted gravity as an entropic force caused by the changes in the information associated with the positions of material bodies. It is notable that, difficulty of unification of gravity with quantum mechanics is one of the basic motivations that leads Verlinde to propose this new pattern. Verlinde made several interesting observations. First, with the assumption of the entropic force together with the Unruh temperature, he derived the second law of Newton. Second, by taking into account the entropic force together with the holographic principle and the equipartition law of energy he obtained the Newton’s law of gravitation. A relativistic generalization of this argument directly leads to the Einstein equations. Similar discoveries about statistical origin of gravity and combination of the equipartition law for horizon degrees of freedom with the Smarr formula [15] were also studied in [16, 17]. This may imply that the entropy is to link general relativity with the statistical description of unknown spacetime microscopic structure when the horizon is present.

Inspired by Bekenstein’s entropy bound, Verlinde proposed that when a test particle with mass $m$ approaches a holographic screen from a distance $\Delta x$, the change of entropy on the holographic screen is

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x.$$  \hfill (1)

The entropic force can arise in the direction of increasing entropy and is proportional to the temperature, $F = T\Delta S/\Delta x$. Verlinde’s derivation of Newton’s law of gravitation at the very least offers a strong analogy with a well understood statistical mechanism. Therefore, this derivation opens a new window to understand gravity from the first principles. The study on the entropic force has raised a lot of attention recently (see [18–24] and references therein).

According to Verlinde’s discussion, the number of bits on holographic screen can be specified as $N = 4S = \frac{4c^3}{G\hbar}$. Indeed, the derivation of Newton’s law of gravity as well as Einstein equations depend on the entropy-area relationship $S = A/4\ell_p^2$, where $A = 4\pi R^2$ represents the area of the horizon and $\ell_p^2 = G\hbar/c^3$ is the Planck length. However, the entropy-area relation can be modified from the inclusion of quantum effects. Consider the general corrections to area law, we write down the general entropy-corrected relation as

$$S = \frac{A}{4\ell_p^2} + s(A),$$  \hfill (2)

where $s(A)$ stands the general correction terms. Throughout this paper we set $k_B = 1$ for simplicity. Two well-known quantum corrections to the area law have been introduced in the literatures, namely, logarithmic and power-law corrections. Logarithmic corrections, arises from loop quantum
gravity due to thermal equilibrium fluctuations and quantum fluctuations 

\[ S = \frac{A}{4\ell_p^2} - \beta \ln \frac{A}{4\ell_p^2} + \frac{\ell_p^2}{A} + \text{const}, \]

where \( \beta \) and \( \gamma \) are unknown dimensionless constants. The issue of the value of \( \beta \) and \( \gamma \) is highly disputationary and one can find different interpretations in the literature. By using trace anomaly, tunneling method, path integral or other quantum and semiclassical type approaches, many authors has been discussed about the value of the parameter \( \beta \) for the Schwarzschild black hole (see \[29-31\] for more details). In addition, one can find different values for \( \beta \) and \( \gamma \) in literature \[32-36\]. Regardless of different reported value for parameters \( \beta \) and \( \gamma \), in this paper we can derive the modified gravitational field equations. One can find that these distinctions do not affect on our discussion due to our general analysis.

Another form of correction to area law, namely the power-law correction, appears in dealing with the entanglement of quantum fields in and out the horizon. The entanglement entropy of the ground state obeys the Hawking area law. Only the excited state contributes to the correction, and more excitations produce more deviation from the area law \[37\] (also see \[38\] for a review on the origin of black hole entropy through entanglement). The power-law corrected entropy is written as \[39, 40\]

\[ S = \frac{A}{4\ell_p^2} \left[ 1 - K_\alpha A^{1-\alpha/2} \right], \]

where \( \alpha \) is a dimensionless constant whose value ranges as \( 2 < \alpha < 4 \) \[39\]. Here

\[ K_\alpha = \frac{\alpha(4\pi)^{\alpha/2-1}}{(4-\alpha)\ell_p^2}, \]

where \( \ell_p \) is the crossover scale. The second term in Eq. \[41\] can be regarded as a power-law correction to the area law, resulting from entanglement, when the wave-function of the field is chosen to be a superposition of ground state and exited state \[39\].

**II. ENTRISTIC CORRECTIONS TO MOND THEORY**

Modified Newtonian dynamics (briefly abbreviated as MOND) is a hypothesis that proposes a modification of Newton’s law of gravity to explain the galaxy rotation problem. When the uniform velocity of rotation of galaxies was first observed, it was unexpected because Newtonian theory predicts that objects that are further out will have lower velocities.

In 1983, M. Milgrom \[41\] suggested the MOND theory which appears to be highly successful for explaining the observed anomalous rotational-velocity. But unfortunately MOND theory lacks
theoretical support. In fact, the MOND theory is (empirical) modification of Newtonian dynamics through modification in the kinematical acceleration term ‘$a$’ (which is normally taken as $a = v^2/r$) as effective kinematic acceleration $a_{\text{eff}} = a\mu(\frac{a}{a_0})$, wherein the $\mu$-function is identical to one for usual-values of accelerations but equals to $\frac{a}{a_0}(\ll 1)$ if the acceleration ‘$a$’ is extremely low, lower than a critical value $a_0 \ (10^{-10} \text{ m/s}^2)$. At large distance at the galaxy out skirt, the kinematical acceleration ‘$a$’ is extremely small, smaller than $10^{-10} \text{ m/s}^2$, i.e., $a \ll a_0$, hence the function $\mu(\frac{a_{0}}{a}) = \frac{a}{a_0}$. Consequently, the velocity of star on circular orbit from the galaxy-center is constant and does not depend on the distance; the rotational-curve is flat, as it observed.

In this section, by applying the modified entropy-area relation (2), we reproduce the corrected MOND theory following the methods of [42]. We adopt the assumption that MOND theory can be viewed as an entropic force. We suppose that below a critical temperature, the cooling of the holographic screen is not homogeneous. Also we assume that the fraction of bits with zero energy is given by [42]

$$\frac{N_0}{N} = 1 - \frac{T}{T_c}, \quad \text{(6)}$$

where it is a second order phase transitions theory and usual relation of the critical phenomena. Since we consider $T_c$ is a critical temperature in which, below this, the zero energy phenomenon for some bits starts to occur, we keep $N_0 = 0$ for $T \geq T_c$. In other words for temperature $T < T_c$, the number of bits with nonzero energy is

$$N - N_0 = N \frac{T}{T_c}, \quad \text{(7)}$$

and the generalized equipartition law of energy is

$$E = \frac{1}{2}(N - N_0)T. \quad \text{(8)}$$

Substituting Eq. (7) in the generalized equipartition law of energy, we obtain

$$E = \frac{1}{2}N \frac{T}{T_c}T. \quad \text{(9)}$$

One can combine Eq. (9) with $E = Mc^2$ to obtain temperature

$$T^2 = \frac{2Mc^2T_c}{N}. \quad \text{(10)}$$

Using the Unruh temperature formula, $T = \frac{1}{2\pi} \frac{\hbar a}{c}$, as well as Eq. (10), we get

$$a^2 = \frac{8\pi^2c^2}{\hbar^2} \frac{Mc^2T_c}{N}. \quad \text{(11)}$$
According to statistical mechanics the entropy of a system is proportional to the number of bits. Therefore, considering the general entropy-corrected relation (2), we can write the relation between $A$ and $N$ as

$$N = 4S = \frac{1}{\ell_p^2} \left[A + 4\ell_p^2 s(A)\right]. \tag{12}$$

Here, we use Eq. (12) and the fact that $A = 4\pi R^2$, to rewrite Eq. (11) in the following form

$$a^2 \left(\frac{4\pi R^2}{\ell_p^2}\right) \left[1 + \frac{\ell_p^2}{\pi R^2} s(A)\right] = \frac{8\pi^2 c^2}{h^2} Mc^2 T_c. \tag{13}$$

After some algebraic manipulations and using $(1 + \alpha)^n \approx 1 + n\alpha$ for $\alpha \ll 1$ (where $\alpha = \frac{\ell_p^2}{\pi R^2} s(A)$ and $n = -1$), we derive the entropic corrections to MOND theory as

$$a \left(\frac{a}{a_0}\right) = \frac{GM}{R^2} \left[1 - \frac{\ell_p^2}{\pi R^2} s(A)\right], \tag{14}$$

or equivalently

$$a \left(\frac{a}{a_0}\right) = \frac{GM}{R^2} \left[1 - \frac{\ell_p^2}{\pi R^2} s(A)\right], \tag{15}$$

where $a_0 = 2\pi c h^{-1} T_c$. Now, we want to calculate the explicit form of quantum corrections to this formula. In order to do this we need to use Eqs. (3) and (4), which lead to the following relations

$$a \left(\frac{a}{a_0}\right) = \frac{GM}{R^2} \left[1 + \frac{\beta \ell_p^2}{\pi R^2} \ln \frac{\pi R^2}{\ell_p^2} - \frac{\gamma \ell_p^4}{4\pi^2 R^4}\right], \tag{16}$$

$$a \left(\frac{a}{a_0}\right) = \frac{GM}{R^2} \left[1 + \frac{\alpha}{(4 - \alpha)} \left(\frac{r_c}{R}\right)^{\alpha-2}\right]. \tag{17}$$

Eqs. (16) and (17) are the corrected MOND theory corresponding to the logarithmic and the power-law entropy corrections, respectively.

### III. ENTRISTIC CORRECTIONS TO EINSTEIN EQUATIONS

Finally, we reach to our main task in this paper, namely considering the general entropic corrections to Einstein field equations. In particular, we investigate the influence of the number of bits on the holographic screen on the Einstein field equation. In general, the entropy is proportional to the number of bits living on the holographic screen. Therefore, we propose the number of bits-entropy relation has the following form

$$N = 4S. \tag{18}$$
Consider the general entropy-corrected relation (2), and using Eq. (12), we can write the relation between $A$ and $N$ in differential form

$$dN = \frac{1}{e_p^2} \left[ 1 + 4e_p^2 \frac{\partial s}{\partial A} \right] dA.$$  

(19)

Employing the more general equipartition law we have

$$M = \frac{1}{2} \int T dN,$$  

(20)

where $T = \frac{\hbar}{2\pi c^3 e^2} e^\phi N^b \nabla_b \phi$ (see [14] for more details). Substituting $T$ and $dN$, we obtain

$$M = \frac{\hbar}{4\pi c^3 e^2} \int e^\phi \left[ 1 + 4e_p^2 \frac{\partial s}{\partial A} \right] \nabla \phi dA.$$  

(21)

Using the relation between $\phi$ and killing vector $\xi^a$ (44) as well as the Stokes theorem, and following the same logic of [14], we can obtain

$$\int e^\phi \nabla \phi dA = \int R_{\mu\nu} N^\mu \xi^\nu dV,$$  

(22)

or equivalently

$$M = \frac{\hbar}{4\pi c^3 e^2} \left[ \int R_{\mu\nu} N^\mu \xi^\nu dV + 4e_p^2 \int e^\phi \left( \frac{\partial s}{\partial A} \right) \nabla \phi dA \right].$$  

(23)

On the other hand, $M$ can be expressed as an integral over the enclosed volume of certain components of stress energy tensor $T_{ab}$

$$M = 2 \int \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) N^\mu \xi^\nu dV,$$  

(24)

Equating Eqs. (23) and (24), with $e^2 c^3 / \hbar = G$, we find

$$2 \int \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) N^\mu \xi^\nu dV = \frac{1}{4\pi G} \int \left[ R_{\mu\nu} N^\mu \xi^\nu dV + 4e_p^2 \int e^\phi \left( \frac{\partial s}{\partial A} \right) \nabla \phi dA \right].$$  

(25)

One can rewrite Eq. (25) in the following form

$$\int \left[ R_{\mu\nu} - 8\pi G \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \right] N^\mu \xi^\nu dV = -4e_p^2 \int e^\phi \left( \frac{\partial s}{\partial A} \right) \nabla \phi dA$$  

(26)

This is the entropic correction Einstein equations. The right-hand-side of Eq. (26) is an additional term compared with Verlinde’s result [14]. This term is caused by the correction to the number of bits on the holographic screen (or correction to area law) which brings a surface correction to the Einstein field equations. In order to simplify the integral of right-hand-side, the functional form of the correction terms $s(A)$ as well as the value of $e^\phi \nabla \phi$ should be specified. For example, when we consider power-law correction to entropy, the function $s(A)$ is

$$s(A) = -\frac{1}{4e_p^2} K_\alpha A^{2-\alpha/2}.$$  

(27)
Taking the power-law correction to entropy and assuming that the spacetime is vacuum static spherically symmetric, in which describes with the Schwarzschild metric, we reach

\[
\int e^\phi \left( \frac{\partial s}{\partial A} \right) \nabla \phi \, dA = -\frac{\alpha - 4}{\alpha - 2} K_\alpha GM \frac{A^{1-\alpha/2}}{4\ell_p^2},
\]

(28)

Inserting this into Eq. (26) and using (24) we find

\[
\int \left[ R_{\mu\nu} - 8\pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \right) - 2 \left( \frac{\alpha - 4}{\alpha - 2} \right) \frac{K_\alpha G}{A^{\alpha/2-1}} \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \right] N^\mu \xi^\nu \, dV = 0.
\]

(29)

Thus, the explicit form of Einstein equation with power-law correction term is obtained as

\[
R_{\mu\nu} = 8\pi G \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \left( 1 + M_\alpha \right),
\]

(30)

where

\[
M_\alpha = \left( \frac{\alpha - 4}{\alpha - 2} \right) \frac{K_\alpha}{4\pi A^{\alpha/2-1}}.
\]

(31)

represents the correction term to Einstein equation. In the absence of power-law correction \((\alpha = 0 = K_\alpha)\), one recovers the standard Einstein equation in general relativity. For large horizon areas, the power-law correction becomes small and the standard Einstein equation is recovered.

On the other side, if we take the logarithmic correction to entropy, the function \(s(A)\) becomes

\[
s(A) = -\beta \ln \frac{A}{4\ell_p^2} + \gamma \ell_p^2 A + \text{const}.
\]

(32)

In this case the right hand side of Eq. (26) can be written as

\[
-4\ell_p^2 \int e^\phi \left( \frac{\partial s}{\partial A} \right) \nabla \phi \, dA = -4\ell_p^2 GM \left( \frac{\beta}{A} + \frac{\gamma \ell_p^2}{2 A^2} \right).
\]

(33)

It is matter of calculation to show that for the logarithmic entropy correction, Eq. (26) reduces to

\[
\int \left[ R_{\mu\nu} - 8\pi G \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) + 8\ell_p^2 G \left( \frac{\beta}{A} + \frac{\gamma \ell_p^2}{2 A^2} \right) \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \right] N^\mu \xi^\nu \, dV = 0.
\]

(34)

Therefore, we can find the modified Einstein equation as

\[
R_{\mu\nu} = 8\pi G \left( T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) \left( 1 - \Gamma \right),
\]

(35)

where

\[
\Gamma = \frac{\ell_p^2}{\pi} \left( \frac{\beta}{A} + \frac{\gamma \ell_p^2}{2 A^2} \right).
\]

(36)

We see that the Einstein equation will modified accordingly with the logarithmic correction in the entropy-area expression. It is clear that without correction terms \((\beta = \gamma = 0)\), Eq. (35) reduces to the familiar Einstein equation. Again, for large horizon areas, \(A\), the correction becomes small and the standard Einstein equation is recovered.
IV. CONCLUSION

The entropy of a black hole is proportional to its horizon area and obeys the well-known Bekenstein-Hawking area law \( S = A/4\ell^2 \). However, the area law is a semi-classical result and there is no reason to believe it to be the complete answer conceivable from a correct quantum gravity theory. Therefore, it is imperative for any approach to quantum gravity to go beyond area law and provide generic subleading corrections. Logarithmic corrections to area law, arises from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations. Another type of correction, usually called power-law correction, appears in dealing with the entanglement of quantum fields in and out the horizon \[39\]. The entanglement entropy of the ground state obeys the Bekenstein-Hawking area law. However, a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. In other words, the excited state contributes to the power-law correction, and more excitations produce more deviation from the area law \[37\].

In this paper, we considered the general corrections to the area law and studied the modification of several equations related to gravity theory accordingly. We followed the Verlinde’s viewpoint and proposed that gravity is a kind of entropic force caused by the changes in the information associated with the positions of material bodies. Following the logic of \[14\], we derived the correction terms to MOND theory of gravitation and Einstein field equations. As two special cases we considered the logarithmic and power-law corrections to entropy and found the explicit form of the obtained modified equations. The obtained results in this paper are quite general and can be applicable to any kind of correction terms to entropy which may be find in the future.

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