A dynamic model of unsupported pit traversal by a vehicle with 6x6 wheel arrangement

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Abstract. The paper deals with the problem of unsupported pit traversal by a vehicle with 6x6 wheel arrangement. The paper contains passages from an unsupported pit traversal test report of “Korsak” wheel chassis. Typical test episodes are described. A design schematic is provided. Assumptions are provided. For the first time, a mathematical model is provided describing a dynamical pit traversal under recognition of the soil parameters, mass and inertia chassis specifications, as well as the travel speed. A conclusion is made that the obtained model is a more generalized case of existing models describing profile obstacle negotiation by wheel vehicles.

1. Introduction

While crossing complicated terrains it is important for a vehicle to retain sufficient mobility to succeed in its functional operations. The mobility is an integral operational feature of a Special Purpose Vehicle (SPV) determining its performance specifications with optimum adaptivity to the operation conditions and the technical state of order of the vehicle itself, that is the vehicle’s capability of coping with external and internal factors impeding the mission progress [1, 2].

While crossing in spring and in summer terrains susceptible to flooding, also in floodplains after the water level has gone back, vast complicated landscapes may be observed formed by numerous brooks and representing a challenge for an SPV. As a rule, the resulting profiled obstacle may from the point of view of SPV mobility classified as pits, scarps and counterscarps. Thereby, all these obstacles are unsupported and crumbling. The most complicated case is a traversal of unsupported pit [3]. The mobility of an SPV at unsupported pit traversals was the subject of earlier papers of the authors hereof [3-5]. That papers describe pit wall crumbling patterns depending upon the parameters of the soil and the chassis specifications. Also, the dimensions of the pit to overcome correlate with the vehicle’s dynamics. Therefore, we further consider a dynamical model of a pit traversal by a vehicle of 6x6 wheel arrangement.

2. Assumptions

We consider a model of pit traversal by a multi-axle vehicle. The equation of the chassis motion obtains the following appearance:

$$\sum_i F_{pi} - \sum_j F_{fj} = m \frac{dv}{dt},$$

Thereby, $\sum_i F_{pi}$ is the sum of traction on the chassis wheels, $\sum_j F_{fj}$ is the sum of resistances, $m$ is the chassis weight, $\frac{dv}{dt}$ is the acceleration.

Simultaneously, during the pit traversal, the chassis shall follow a rotation trajectory while nosediving into the pit. The equation will then obtain the following appearance:
\[ \sum M = J \frac{da}{dt}, \]

Thereby, \( \sum M \) is the sum of the moments of forces acting on the chassis, \( J \) is the moment of inertia, \( \frac{da}{dt} \) is the angular acceleration.

While driving with a variable speed, the diving motion of the front axle of the chassis is determined by the dimensions of both the vehicle and the pit, the values of the soil, as well as by the driving speed. Important thereby are the axle load distribution and the location of the centre of gravity, the wheel arrangement over the wheel base and the number of wheels. Depending upon these parameters, the pit traversal diagram of a multi-axle vehicle will vary.

Thereby we introduce a number of assumptions, a part of which is aligned with the design parameters of the test chassis unit.

1. The axle load distribution between the left and the right wheel of one axle is even.
2. When solving the pit traversal problem, we assume that the resultant of the forces occurring due to the wheel-terrain interaction is concentrated in the centre line of the wheel passing via its centre of rotation.
3. The location of the centre of gravity does not vary during the vehicle motion.
4. The positions of the rotation axes of the wheels relative to the chassis centre of gravity remains stable.
5. The influence of the tire stress which changes at dynamic loads occurring during the pit traversal and the re-distribution of the reactions over the drivers are neglected or assumed constant.
6. We also assume a constant pit traversal speed.

3. Pit traversal diagram

We consider based on an example of a 6x6 chassis a pit traversal diagram and the typical components of the motion. The figure 1 (left) demonstrates the episodes of a test carried out October 21, 2018 with “Korsak” SPV with 6x6 wheel arrangement. The relevant diagrams are shown in the right column.

Fig. 1 shows typical pit traversal stages. Considered in detail, Fig. 1 a shows the vehicle starting the pit traversal crushing the front pit wall. The load is distributed over all the wheels of the vehicles. The diagram is shown in Fig. 1 b.

Fig. 1 c shows subsequent vehicle motion. A forward nosedive of the vehicle takes place. Thereby, the nosedive amplitude will depend upon the weight, the dimensions, the inertia of the vehicle, on the speed and on the pit dimensions. The chassis motion diagram is shown in Fig. 1 d. Thereby, the locomotion of the vehicle can be described by equations of forward and rotary motion.

Fig. 1 e shows vehicle’s thrusting against the crushed pit edge, thereby a significant vehicle tilt is obvious. Fig. 1 f provides a related diagram. The pit traversing capacity of the vehicle will be determined by the diagram shown in the figure. The locomotion condition will be a case when the sum of torques CW is higher than that CCW. A critical motion condition is the equality of the torques. Thereby, the critical pit width can be determined.

Fig. 1 g shows the moment the chassis has climbed the opposite pit wall. Fig. 1 h shows the diagram. Further, Fig. 1 i shows the pit left behind by the wheels of the 1- and the 2- axle. Fig. 1 j shows the relevant diagram. In principle, a sort of “taillive” may occur thereby. But for a chassis with forward-shifted centre of gravity such phenomenon is deeming negligible at a pit traversal, the condition whereof is assumed in accordance with Fig 1 f. This thesis is based on the fact that as a rule multi-axis chassis are similar to the one considered, with the centre of gravity displaced to the front. It should be stressed, that the approach to the calculation of the maximum width of traversable pit for vehicles with the centre of gravity displaced aft from the middle axle is similar to that set forth above.

It should be noted, that pendular chassis swings are inevitable during the pit traversal due to fluctuations of different types. This phenomenon is not accounted for in the presented model.
Figure 1. Characteristic pit traversal episodes: photos of terrain vehicle test (left) and design schematic (right)
4. Mathematical model

In accordance with the assumptions made, we consider in a detailed manner the traversal diagram of a pit with critical width (Fig. 2).

![Diagram of the critical width of unsupported pit](image)

Figure 2. Calculation diagram of the critical width of unsupported pit

We now consider thoroughly the diagram in Fig. 2.

The nosedive calculation is carried out in accordance with the rotary motion equation. In a general case, the latter will look like:

\[ J \ddot{\varepsilon} = G_a l_b - F_w h_w - F_{kp} h_{kp} - m_a a h_g, \]  

(1)

Thereby, \( J \) is the moment of inertia of the chassis, \( \varepsilon \) is the rotary acceleration, \( G_a \) is the machine weight, \( l_b \) is the moment arm, \( l_b = l_2 \cos \alpha \), \( l_2 \) is the distance of the centre of gravity from the axis of the 2nd axle, \( \alpha \) is the nosedive angle, \( F_w \) is the air resistance, \( h_w \) is the height of the centre of sail, \( F_{kp} \) is the resistance of the hook load, \( h_{kp} \) is the hook load application height, \( m_a \) is the vehicle’s mass, \( a \) is the acceleration of the chassis, \( h_g \) is the height of the centre of gravity.

In case of a steady low-speed locomotion at small nosedive amplitude, we can assume that:

\[ \varepsilon = \frac{G_a l_2}{J}. \]  

(2)

The nosedive amplitude may vary depending on the pit width: it will be determined in accordance with the following motion equation.

\[ \alpha = \alpha_0 + \omega_0 t + \frac{e t^2}{2}, \]  

(3)

Thereby, \( \alpha_0 \) is the initial angle, \( \alpha_0 = 0 \), \( \omega_0 \) is the initial angular speed, \( \omega_0 = 0 \), \( t \) is the pit traversal time till the opposite wall contact, \( t = \frac{S}{V_1} \), \( S \) is the pit width measured from the crushed front to the contact with the wall.

Thus, the nosedive amplitude can be calculated in accordance with the following equation:

\[ h_r = (l_1 + l_2) \sin \alpha, \]  

(4)

Thereby, \( l_1 \) is the distance of the centre of gravity to the 1st chassis axle.

We consider the correlation of parameters in accordance with the diagram in Fig. 2.

\[ G_a = R_{z1} + R_{z2}, \]  

(5)

\[ \frac{R_{z1}}{R_{z2}} = \frac{l_a}{l_b}, \]  

(6)

\[ l_a = l_1 \cos \alpha + r_{\Omega} \sin \beta. \]  

(7)

Thereby, \( r_{\Omega} \) is the dynamic wheel radius.

The angle \( \beta \) will be determined based on the following dimension:
\[
\beta = \min\left[90 - (\varphi + \gamma) ; \arccos(1 - r_n h_k^{-1}) \right],
\]

Thereby, \(\varphi\) is the repose angle of the soil, \(\gamma\) is the inclination angle of the pit wall [6].

For evaluation of the motion options, we consider the chassis motion equation. We project the forces on the OX, OY axes and consider the equality of the moments in relation to the contact point of the front wheel with the pit wall.

\[
\begin{align*}
X: F_{T_1} \cos \beta - F_{f_1} \cos \beta - N \sin \beta + F_{T_2} - F_{f_2} - F_w - F_{kp} &= m \frac{dv}{dt} \\
Y: F_{T_1} \sin \beta - F_{f_1} \sin \beta - N \cos \beta + R_{x_2} - G_a &= 0 \\
M: -G_a l_a + R_{z_2} (l_a + l_b) + (R_{x_2} - F_{f_2}) \left(h_k - (r_n - r_a \cos \beta)\right) + \\
+ ma \left(h_g - (r_n - r_a \cos \beta)\right) - F_w \left(h_w - (r_n - r_a \cos \beta)\right) &= \frac{d \omega}{dt}
\end{align*}
\]

Thereby, \(F_{T_1}\) is the employed traction force on the wheels of the \(i\)-th axis, \(F_{f_1}\) is the resistance on the wheels of the \(i\)-th axis, \(N\) is the normal response of the pit wall to the wheel.

The maximum width of a traversed pit will correspond to a traction force on the wheels not in exceed of the cohesion force. Thus, the traction force can be calculated as follows:

\[
F_{f_1} = (c_{iu} A + N_i (\cot \varphi_{iw})) k_n + (c A + N (\cot \varphi)(1 - k_n)),
\]

Thereby, \(k_n\) is the coefficient of pattern area saturation, \(c_{iu}\), \(\varphi_{iw}\) are the cohesion modulus and the friction angle of the tire tread with the soil, \(c\) is the inherent soil cohesion.

Thus, using the formulae (1) – (10) the maximum traversed width of the unsupported pit by a vehicle with 6x6 wheel arrangement.

5. Conclusion and future work
A study of profile passability was carried out, specifically, a traversal of an unsupported pit by a vehicle with 6x6 wheel arrangement. Terrain experiments were conducted with a “Korsak” SPV with 600 kgs weight.

Typical test episodes were illustrated. A diagram was developed for mathematical modelling of a pit traversal by a wheel-type vehicle.

A mathematical model was developed for calculation of an unsupported pit width dynamically traversed by a multi-axle wheel-type vehicle.

The analysis of the obtained dependencies has demonstrated the following:
- the dependencies obtained are valid for a dynamic traversal of an unsupported pit.
- if the pit is of hard rock type then the obtained dependencies are transformed to the problem of hard pit traversal.
- if the problem is considered statically then the formulae are transformed to dependencies for climbing of a hard step.

Thus, the obtained dependencies are not contradicting the dependencies obtained earlier by other authors [3-5] being a more general case describing the overcoming of threshold type obstacles.

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