Maximal neutrino oscillation solution to the solar neutrino problem

R. Foot and R. R. Volkas

Research Centre for High Energy Physics, School of Physics, University of Melbourne, Parkville, 3052 Australia.

Abstract

We discuss a simple predictive solution to the solar neutrino problem based on maximal vacuum neutrino oscillations. The solution can be motivated by the exact parity symmetric model which predicts that the neutrino mass eigenstates are maximal mixtures of ordinary and mirror weak eigenstates (if neutrinos are massive). We show that this proposed solution to the solar neutrino problem is in reasonable agreement with the experiments, and that in the near future this scheme may be either ruled out or tested more precisely as statistics improve for the SAGE and GALLEX experiments. Predictions are also given for the upcoming Superkamiokande, SNO and Borexino experiments.
The evidence for new neutrino physics from the comparison between the measurements of the flux of solar neutrinos\cite{1, 2, 3, 4} and theoretical models of the sun\cite{5} has been steadily accumulating. Apparently strong evidence for a deficit of atmospheric muon neutrinos has also emerged\cite{6}.

An interesting possibility is that these discrepancies are due to neutrino oscillations. For the solar neutrino problem, a very popular model is based on the matter enhanced MSW oscillations\cite{7, 8}, which can significantly enhance the conversion of electron neutrinos to another neutrino flavour for a range of parameters. This is an interesting idea, however the main drawback of this possibility (in our opinion) is its lack of predictivity. It can accommodate the data, but it does not give any definite predictions for any of the existing experiments (although given the existing experimental results it does give significant predictions for forthcoming experiments such as SNO, Superkamiokande and Borexino). For example, if a deficit had been observed for the gallium experiments and not for the chlorine experiment then this scenario could still have been interpreted in terms of MSW oscillations with a different range of parameters\cite{1}.

Another solution to the solar neutrino problem assumes the existence of long wavelength vacuum neutrino oscillations (sometimes called “just so” neutrino oscillations)\cite{10} which involves large angle vacuum oscillations with oscillation length about equal to the distance between the earth and the sun (which corresponds to $\delta m^2$ in the range $10^{-11} < \delta m^2/eV^2 < 10^{-10}$). While this is an interesting possibility, we do not find it compelling because of its lack of predictivity for any of the known experiments and the limited range of parameters required. (In particular it would be a lucky occurrence if the distance of electron neutrino oscillations just happened to be approximately the same as the distance between the earth and the sun.)\cite{2}

We would like to discuss an alternative solution to the solar neutrino problem based on maximal neutrino oscillations. We will focus on maximal electron neutrino - sterile neutrino oscillations (however our discus-

\footnote{1}Also note that if the LSND experiment\cite{9} is correct then the muon neutrino should have a mass of about 1 eV and hence the MSW solution in terms of $\nu_e - \nu_\mu$ oscillations is not possible.

\footnote{2}The MSW enhancement of solar neutrinos is also only available for a limited range of parameters, but not as limited as the long wavelength vacuum oscillation solution. Since solar neutrinos (with energies relevant for the experiments) can only encounter a resonance for parameters in a limited range: $10^{-8} < \delta m^2/eV^2 < 10^{-4}$ and $\sin 2\theta < 10^{-2}$.
sion will be relevant to maximal electron neutrino - active neutrino oscillations as well). As is well known, for a large range of parameters (i.e. $10^{-10} eV^2 \gtrsim \delta m^2 \gtrsim 10^{-3} eV^2$) vacuum maximal oscillations implies that the flux of electron neutrinos from the sun will be reduced by a factor of two for all neutrino energies relevant to the solar neutrino experiments. We will call this scenario the “maximal vacuum oscillation solution”. We believe this to be a very simple and predictive scheme which can either be ruled out or tested more stringently with the existing experiments. Importantly, it also makes definite predictions for the new experiments, SNO, Superkamiokande and Borexino. [We stress that this solution is distinct from the long wavelength vacuum oscillation solution[10], which has near maximal oscillations of electron neutrinos with another species and $\delta m^2 \simeq 10^{-10} eV^2$. The main difference is that the long wavelength vacuum oscillation solution is a best fit for the solar neutrino experiments (assuming all these to be correct), whereas the maximal oscillation solution assumes that the mixing is exactly maximal and that $\delta m^2 \gtrsim 10^{-10} eV^2$ so that the flux of solar neutrinos is reduced by a factor of two and thus a definite prediction results.] As far as we are aware, this simple idea has never been discussed in detail, although it has been put forward as a solution[12, 13, 14]. Our interest in this scheme is motivated by the exact parity symmetric model (see [14] for a review of this model). This model predicts that ordinary neutrinos will be maximally mixed with mirror neutrinos.

3 Note that the $10^{-3}$ upper bound comes by requiring that the electron neutrino does not oscillate over distances relevant to the atmospheric neutrino experiment. There is a laboratory bound of about $10^{-2}$ (see Ref.[11])

4 Note that if the oscillations are maximal then the flux of electron neutrinos will be reduced by a factor of two independently of whether there is any significant effect due to MSW oscillations in the sun. This is because the vacuum neutrino oscillations from the sun to the earth will mix the electron neutrinos so that there are equal components of electron neutrinos and sterile states. However, if the electron neutrino mixes slightly with the muon or tau neutrinos as well (and such mixing would be expected) then there could be additional effects due to MSW oscillations due to this intergenerational mixing. However, assuming that this mixing is small, the region of parameter space where no significant effects occur (e.g. $\delta m^2 \gtrsim 10^{-4} eV^2$) is much larger than the region of parameter space where there is significant enhancement of the neutrino fluxes.
neutrinos (which are essentially sterile) if neutrinos have mass. If we make the assumption that mixing between generations is small (as it is in the quark sector) then the parity symmetric model predicts that the three known neutrinos will each be (to a good approximation) maximal mixtures of two eigenstates. This model thus nicely explains the atmospheric neutrino anomaly which can be explained if the muon neutrino is maximally mixed with a sterile neutrino (with $\delta m^2 \approx 10^{-2} eV^2$).

One nice feature of the parity symmetric model is that its explanation of the solar and atmospheric neutrino anomalies in terms of maximal mixing between ordinary and mirror neutrinos (which is a necessary consequence of the parity symmetry) is that it is predictive and hence testible. This is unlike other proposed explanations which involve parameter fitting. Another nice feature is that it solves the atmospheric neutrino anomaly and the solar neutrino deficit by essentially the same mechanism: maximal vacuum oscillations. This seems more appealing than the use of two different mechanisms.

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5 It has been argued that the muon neutrino oscillation with a sterile (mirror) neutrino with parameters $\delta m^2 = 10^{-2} eV^2, \sin^2 2\theta_0 \approx 1$ (necessary to solve the atmospheric neutrino anomaly) is inconsistent with standard Big Bang Nucleosynthesis. This is because oscillations with these parameters should bring the sterile neutrino into equilibrium with the known neutrinos. However this result depends critically on the assumption that the relic neutrino asymmetries are small. Recently M.J. Thomson and ourselves have shown that large neutrino asymmetries can be created by neutrino oscillations, and that these bounds can be evaded. This result can solve part of the potential cosmological problem posed by the mirror particles. Note that one still needs to postulate some new physics at high temperatures (or a $t = 0$ boundary condition) to bring about a temperature difference between the mirror particles and the ordinary particles.

6 Other models featuring maximal mixing have been discussed in the literature within the context of neutrino anomalies. For instance, Ref. discusses a model which has both electron and muon neutrinos maximally mixed with two associated sterile species. In addition they postulate that $\nu_e - \nu_\mu$ mixing occurs in the MSW region. This last feature distinguishes their scenario from ours. The papers in Ref. discuss a one-generation model with maximally mixed electron and sterile neutrinos. These works in addition invoke a large phenomenological neutrino magnetic moment. Maximal mixing between electron neutrinos and other active species has also been investigated (see for instance Ref.). Most of the above models use the idea of pseudo-Dirac neutrinos. Therefore, even those models above which have maximally-mixed active-sterile systems are distinct from the exact parity symmetric model from the point of view of their construction. Our discussion of the solar and atmospheric neutrino problems will also be relevant to Ref. Our discussion of the solar neutrino problem only will be relevant to those models which have $\nu_e$ maximally mixed with either $\nu_\mu$ or $\nu_\tau$.
Despite the simplicity of the maximal mixing solution to the solar neutrino deficit, no detailed study of its comparison with existing experiments has been made (so far as we know). The purpose of this letter is to make such a study using the most recent data. We will show that the maximal mixing vacuum oscillation solution is not excluded by existing experiments. We will show that improved data from the existing gallium experiments (SAGE and GALLEX) will be able to better test or exclude the maximal mixing vacuum oscillation solution.

There are a number of theoretical calculations for the solar neutrino flux. The uncertainties in the theoretical calculations are largest for the $^8B$ solar neutrino flux$^7$. However, for the flux of $pp$, $pep$ and $^7Be$ neutrinos there is good agreement among most (all?) theoretical models. Thus instead of using the (possibly unreliable) theoretical calculation for the $^8B$ neutrino flux, the empirical result from the Kamiokande experiment can be used$^{[23, 24]}$. To use this result in the other experiments we need to assume that the shape of the energy spectrum of boron neutrinos as determined in laboratory experiments with terrestrial sources is the same as in the sun$^{[25]}$. This will hold unless some effect occurs to distort the spectrum, like MSW oscillations$^{[25]}$. Maximal vacuum oscillations will not alter the spectrum. It should thus be possible to make reliable predictions for the other three experiments.

The most recent result of the Kamiokande experiment$^2$ is

\[ \phi_K(^8B) = (2.73 \pm 0.17 \pm 0.34(syst)) \times 10^6 cm^{-2}s^{-1}. \]  

(1)

The expected capture rate in the chlorine experiment just from $^8B$ neutrinos (which we denote as $R(^8B; ^{37}Cl)$) as extrapolated from the Kamiokande experiment (assuming the energy spectrum is unchanged) is$^{[24]}$

\[ R(^8B; ^{37}Cl) \geq 2.78 \pm 0.4 \text{ SNU}, \]  

(2)

where 1SNU $\equiv 10^{-36}$ captures per target atom per second. Using the theoretical predictions for the other major sources of neutrinos $^7Be$, $pep$, $pp$

$^7$For example, in the model of Dar and Shaviv$^{[21]}$, they obtain a theoretical prediction for Kamiokande of $2.77 \times 10^6 cm^{-2}s^{-1}$ while in the model of Bachall and Pinsonneault$^{[22]}$ they obtain a prediction of $6.6_{-1.1}^{+0.9} \times 10^6 cm^{-2}s^{-1}$.

$^8$In Ref.$^{[24]}$, they used the Kamiokande flux measurement of $(2.89 \pm 0.22 \pm 0.35(syst)) \times 10^6 cm^{-2}s^{-1}$, and calculated the Chlorine capture rate of $R(^8B; ^{37}Cl) \geq 2.94 \pm 0.40 \text{ SNU}$. However, the latest measurement of the Kamiokande experiment has measured a flux slightly smaller than the value used in Ref.$^{[24]}$. Using this latest value (Eq.(1)), we obtain the expected capture rate for chlorine given in Eq.(2).
and CNO, we obtain the following expectations for the capture rates in the chlorine and gallium experiments respectively:

\[ R(^{37}Cl) = 4.5 \pm 0.5 \text{ SNU} \]
\[ R(^{71}Ga) = 123^{+8}_{-6} \text{ SNU} \] (3)

where we have combined the various errors in quadrature (we combine the errors in quadrature to get some idea of the total error; this procedure is not strictly valid and the reader should be aware that the true error may be different). These predictions are summarised in table 1

| Chlorine          | Gallium          |
|-------------------|------------------|
| \(^8B\) : 2.78 ± 0.4 | \(^8B\) : 7^{+3.5}_{-3.5}\) |
| \(pep\) : 0.22 ± 0.01 | \(pp + pep\) : 74 ± 1 |
| \(^7Be\) : 1.1 ± 0.1   | \(^7Be\) : 34 ± 4   |
| \(CNO\) : 0.4 ± 0.2   | \(CNO\) : 8.0 ± 2.0  |
| **Total** : 4.5 ± 0.5 | **Total** : 123^{+8}_{-6} |

Table 1: Expectation for the chlorine and gallium experiments using the \(^8B\) flux as extrapolated from the Kamiokande experiment and using the theoretical predictions from a range of solar models for the other sources of neutrinos.

The experimental measurement for the chlorine experiments is:

\[ Cl : 2.78 \pm 0.35 \text{ SNU} \] (4)

For \(^7Be\), \(pep\) and \(pp\) neutrinos we have used the values listed in Ref.[26], which have been obtained by examining 10 different solar models. For CNO neutrinos we have taken a range of theoretical models (from table 18 of Ref.[27]) and derived the error from the range of predictions.

Note that the average over the period of operation (1970-1993) of the chlorine experiment is 2.55 ± 0.25 SNU. However, some of the early data has been criticised (see for example Ref.[28]) on the basis that there are quite significant fluctuations, which are not apparent in the more recent Homestake data. Also, since we are using the data from the Kamiokande experiment as a measurement of the boron flux, it is appropriate to use data collected at the same time. For these reasons we use the data collected in the period 1986-1993 and it is this data which is given in Eq.(4).
While for the two gallium experiments\cite{3,4} the most recent measurements are

\begin{align*}
\text{GALLEX} &: \quad 77 \pm 8 \text{ (stat)} \pm 5 \text{ (syst)} \text{ SNU}; \\
\text{SAGE} &: \quad 69 \pm 11 \text{ (stat)} \pm 6 \text{ (syst)} \text{ SNU}.
\end{align*}

(5)

The weighted average of these two gallium measurements is $75 \pm 9 \text{ SNU}$ (where the systematic and statistical errors have been combined in quadrature). Comparing the theoretical predictions from table 1 with the above experimental measurements, we see that there is a very significant discrepancy for both the chlorine experiment and the gallium experiments. While it is conceivable that the discrepancy with the chlorine experiment could be due to some unaccounted systematic error (for example in the neutrino-chlorine absorption cross section), the discrepancy with the gallium predictions seems very robust. (The radiodetection technique has been checked by GALLEX by exposing their detector to a calibrated man-made low energy neutrino source using neutrinos emitted in the electron capture decay of $^{51}\text{Cr}$\cite{29}).

How well does this maximal mixing vacuum oscillation solution work for the solar neutrino problem? If $\delta m^2 > 10^{-10} \text{ eV}^2$ then the effect of maximal vacuum oscillations is simply to reduce the predicted flux of all of the solar neutrinos by a factor of two. First we note that the Kamiokande experiment is in good agreement with the predictions of most theoretical solar models if the flux is halved\cite{11}. However, as before, due to the larger uncertainties for $^{8}\text{B}$ neutrinos, we will use the Kamiokande experiment as a measurement of the $^{8}\text{B}$ flux of neutrinos, and use the theoretical predictions for the fluxes of the other neutrinos. The effect of the maximal mixing is to simply reduce all of these predictions by a factor of two. Thus, following the same steps as before but this time dividing the theoretical fluxes for the $\text{pp, pep, } ^7\text{Be, and CNO}$ neutrinos by a factor of two, we find the following theoretical expectations for the fluxes assuming maximal oscillations which we summarize in table 2:\cite{12}

\footnote{For example, the 1995 analysis of Bahcall and Pinsonneault\cite{22} predicts a rate of $6.62^{+0.42}_{-0.47} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ for Kamiokande. Dividing this by two produces a rate of $3.31^{+0.45}_{-0.55} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}$ which is in good agreement with the observed rate Eq.(1). Note that for the case where $\nu_e$ is maximally mixed with an active neutrino, one should divide the prediction by a number slightly less than two (because Kamiokande is sensitive to neutral currents). Good agreement is still obtained because of the theoretical and experimental error ranges.}

\footnote{Note that this analysis also holds if $\nu_e$ is maximally mixed with an active species,}
Table 2: Expectation for the chlorine and gallium experiments using the $^8B$ flux as extrapolated from the Kamiokande experiment and the theoretical predictions for the other fluxes assuming maximal vacuum oscillations (which have the effect of halving the predicted fluxes).

We see that the prediction for the chlorine experiment is significantly reduced by comparison with the minimal standard model (see table 1). The most dramatic effect occurs for the gallium experiment. We summarise the results and compare with the experimental measurements in table 3 below:

Table 3: Summary of the predictions for the chlorine and gallium experiments assuming 1) standard electro-weak theory (i.e. no new physics) 2) that the electron neutrino oscillates maximally into a sterile state (maximal mixing model) and 3) the experimental measurements.

The prediction of the maximal mixing model for the gallium experiments is obviously in excellent agreement with the data. The discrepancy between the maximal mixing prediction for the chlorine experiment and the measurement is $0.86 \pm 0.4 \pm 0.35$ ($= 0.86 \pm 0.53$ if the errors are added in quadrature). because the chlorine and gallium experiments cannot detect neutral current processes. However the capture rate $R(^8B; ^{37}Cl)$ is a bit smaller (about $2.4 \pm 0.5$ SNU) because part (1/6 to 1/7) of the Kamiokande measurement would be due to neutral current effects of the active $\nu_\mu$ or $\nu_\tau$. 

|            | Chlorine                      | Gallium                     |
|------------|-------------------------------|-----------------------------|
| $^8B$      | $2.78 \pm 0.4$               | $7^{+5}_{-3.5}$            |
| $pep$      | $0.11 \pm 0.005$             | $pp + pep : 37.0 \pm 0.5$  |
| $^7Be$     | $0.55 \pm 0.05$              | $^7Be : 17 \pm 2$          |
| $CNO$      | $0.2 \pm 0.1$                | $CNO : 4.0 \pm 1.0$        |
| $Total$    | $3.64 \pm 0.4$               | $Total : 65^{+7}_{-4}$      |

| Prediction/Expt | Chlorine | Gallium    |
|-----------------|----------|------------|
| Standard Electro-weak theory | 4.5 ± 0.5 | 123$^{+8}_{-6}$  |
| Maximal mixing model | 3.64 ± 0.4 | 65$^{+7}_{-4}$  |
| Experiment    | 2.78 ± 0.35 | 75 ± 9     |
This discrepancy is only about one and a half $\sigma$. We therefore conclude that a simple factor-of-two reduction of all the neutrino fluxes (as predicted by the maximal mixing vacuum oscillation scheme) is sufficient to bring about a reasonably good reconciliation between theory and experiment.

The discrepancy with the chlorine experiment is suggestive of energy dependence to the solar neutrino deficit. Indeed, the relatively low flux observed by Homestake is the prime reason why an energy dependent reduction of neutrino fluxes (as given by the MSW solution and “just so” vacuum oscillation solution) has received much attention. However, within the context of the maximal mixing vacuum oscillation solution, the low Homestake result becomes less than a $2\sigma$ effect. A discrepancy of less than $2\sigma$ in an experiment like Homestake should not be considered compelling evidence for an energy dependence to the neutrino deficit. It could, instead, be due to an unaccounted-for systematic error, or simply to not enough statistics.

Note that our prediction will become more precise as more data is collected from Kamiokande. It will be interesting to see if this discrepancy is reduced as this data is collected. A more important test will be the gallium experiment. At present there is good agreement between the prediction of the maximal mixing solution and the measurements. The error on the measurements will be reduced as more data is collected. A final measurement above about 82 SNU would make the maximal mixing vacuum oscillation solution an unlikely explanation, while a final measurement below about 72 SNU would strongly support it.

What does the maximal mixing solution to the solar neutrino problem predict for the forthcoming experiments: the Sudbury Neutrino Observatory (SNO) experiment, Superkamiokande and Borexino? Superkamiokande and SNO will be able to measure the shape of the energy spectrum of the neutrinos from $^8B$. The maximal vacuum oscillation solution predicts the same energy spectrum as measured in the laboratory (MSW predicts a significant deviation). The maximal mixing solution also predicts no day/night effect. This effect could only potentially occur if $\delta m^2 \sim 10^{-6} \text{ eV}^2$. However, since any regeneration of electron neutrinos in the earth will be compensated by a depletion of electron neutrinos (as there are equal amounts of $\nu_e$ and $\nu_s$ due to the maximally mixed oscillations during the propagation of neutrinos from the sun to the earth) there should be no significant day/night effect if the maximal oscillation solution is correct.

SNO will also be able to distinguish neutral current interactions from
charged current interactions and thus test whether neutrinos oscillate into active neutrinos or sterile neutrinos. Thus the prediction of the maximal oscillation solution depends on whether the maximal oscillations occur between active and sterile neutrinos (as in the parity symmetric model) or between active and active neutrinos. Thus the maximal mixing solution with neutrinos oscillating maximally into sterile neutrinos predicts that both neutral and charged currents will be reduced by a factor of two. In other words the ratio of charged to neutral current events will be the same as the minimal standard model with no oscillations.

Superkamiokande and SNO will be able to search for seasonal effects. Note that the maximal oscillation solution does not predict any significant seasonal effect beyond that due to the inverse square law (as distinct from the long wavelength “just so” oscillation scenario).

Finally, Borexino will be able to give a precise measurement of the $^7\text{Be}$ line. The maximal oscillation solution predicts a reduction of 1/2 compared to the standard model. This experiment would be another good test of the maximal oscillation solution [as the current expectation from the MSW and long wavelength (“just so”) solutions is that there will be a much greater reduction in the $^7\text{Be}$ signal].

The maximal oscillation solution to the solar neutrino problem was motivated by the parity symmetric model. Even if this solution turns out to be incorrect the parity symmetric model may still be connected to the solar neutrino problem. For example the large wavelength vacuum oscillation solution can be fit with maximal oscillations if $\delta m^2 \approx 10^{-10} \text{ eV}^2$. Alternatively, one can have MSW type oscillations between the electron neutrino and the muon neutrino together with maximal oscillations of the electron neutrino and the sterile neutrino (this scenario was discussed in the model of Ref.[17].). However, in our opinion the maximal oscillation solution is a more likely solution.

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