Impact of semi-annihilations on dark matter phenomenology – an example of $Z_N$ symmetric scalar dark matter

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Abstract

We study the impact of semi-annihilations $x_i x_j \leftrightarrow x_k X$, where $x_i$ is any dark matter and $X$ is any standard model particle, on dark matter phenomenology. We formulate minimal scalar dark matter models with an extra doublet and a complex singlet that predict non-trivial dark matter phenomenology with semi-annihilation processes for different discrete Abelian symmetries $Z_N$, $N > 2$. We implement two such example models with $Z_3$ and $Z_4$ symmetry in micrOMEGAs and work out their phenomenology. We show that both semi-annihilations and annihilations involving only particles from two different dark matter sectors significantly modify the dark matter relic abundance in this type of models. We also study the possibility of dark matter direct detection in XENON100 in those models.

1 INTRODUCTION

The origin of dark matter of the Universe is not known. In popular models with new particles beyond the standard model particle content, such as the minimal supersymmetric standard model, an additional discrete $Z_2$ symmetry is introduced [1]. As a result, the lightest new $Z_2$-odd particle, $x$, is stable and is a good candidate for dark matter. The phenomenology of this type of models has been studied extensively.

The discrete symmetry that stabilises dark matter must be the discrete remnant of a broken gauge group [2], because global discrete symmetries are broken by gravity. The most natural way for the discrete symmetry to arise is from the breaking of a $U(1)_X$ embedded in a larger gauge group, e.g. $SO(10)$ [3]. The latter contains gauged $B - L$ as a part of the symmetry, and the existence of dark matter can be related to the neutrino masses, leptogenesis and, in a broader context, to the existence of leptonic and baryonic matter [4–6].

Obviously, the discrete remnant of $U(1)_X$ need not to be $Z_2$ – in general it can be any $Z_N$ Abelian symmetry. The possibility that dark matter may exist due to $Z_N$, $N > 2$, is a known [7–15], but much less studied scenario [4,8]. Model independently, it has been pointed out in Ref. [15] that in $Z_N$ models the dark matter annihilation processes contain new topologies with different number of dark matter particles in the initial and final states – called semi-annihilations –, for example $xx \leftrightarrow x^* X$, where $X$ can be any standard model particle. It has been argued that those processes may significantly change the predictions for the dark matter relic abundance in thermal freeze-out. Furthermore, an
enlarged discrete symmetry group makes it possible to have more than one dark matter candidate. In this case, annihilation processes involving only particles from the dark sectors, leading to the assisted freeze-out mechanism, can also influence the relic abundance of both dark matter candidates [16,17]. The assisted freeze-out mechanism in the case of a $Z_2 \times Z_2$ symmetry was discussed in [17]. However, no detailed studies have been performed that compare dark matter phenomenology of different $Z_N$ models. This is difficult also because presently the publicly available tools for computing dark matter relic abundance do not include the possibility of imposing a $Z_N$ discrete symmetry instead of a $Z_2$.

The aim of this work is to formulate the minimal scalar dark matter model that predicts different non-trivial scalar potentials for different $Z_N$ symmetries and to study their phenomenology. In particular we are interested in quantifying the possible effects of semi-annihilation processes $xx \leftrightarrow x^*X$ as well as of annihilation processes involving particles from two different dark sectors on generating the dark matter relic abundance. In order to perform quantitatively precise analyses we implement minimal $Z_3$ and $Z_4$ symmetric scalar dark matter models that contain one singlet and one extra doublet in micrOMEGAs [18,19]. Using this tool we show that, indeed, the semi-annihilations and the annihilations between two dark sectors affect the dark matter phenomenology and should be taken into account in a quantitatively precise way in studies of any particular model.

2 $Z_N$ LAGRANGIANS

2.1 $Z_N$ symmetry

Under an Abelian $Z_N$ symmetry, where $N$ is a positive integer, addition of charges is modulo $N$. Thus the possible values of $Z_N$ charges can be taken to be $0, 1, \ldots, N-1$ without loss of generality. A field $\phi$ with $Z_N$ charge $X$ transforms under a $Z_N$ transformation as $\phi \rightarrow \omega^X \phi$, where $\omega^N = 1$, that is $\omega = \exp(i2\pi/N)$.

A $Z_N$ symmetry can arise as a discrete gauge symmetry from breaking a $U(1)$ gauge group with a scalar, whose $X$-charge is $N$ [24]. For larger values of $N$, the conditions the $Z_N$ symmetry imposes on the Lagrangian approximate the original $U(1)$ symmetry for two reasons. First, assuming renormalisability, the number of possible Lagrangian terms is limited and will be exhausted for some small finite $N$, though they may come up in different combinations for different values of $N$. Second, if the $Z_N$ symmetry arises from some $U(1)_X$, the $X$-charges of particles cannot be arbitrarily large, because that would make the model nonperturbative. If $N$ is larger than the largest charge in the model, the restrictions on the Lagrangian are the same as in the unbroken $U(1)$.

We shall see below that in spite of the large number of possible assignments of $Z_N$ charges to the fields, the number of possible distinct potentials is much smaller.

2.2 Field content of the minimal model

In order to study the impact of different discrete $Z_N$ symmetries on dark matter phenomenology, the example model must contain more than one neutral particle in the dark sector. The minimal dark matter model with such properties contains, in addition to the standard model fermions and the standard model Higgs boson $H_1$, one extra scalar doublet $H_2$ and one extra complex scalar singlet $S$ [5]. In the case of $Z_2$ symmetry, as proposed in [5], those new fields can be identified with the well known inert doublet $H_2$ [20,23] and the complex singlet $S$ [24,28]. The phenomenology of those models is well studied. However, when both the doublet and singlet are taken into account, qualitatively new features concerning dark matter phenomenology, electroweak symmetry breaking and collider phenomenology occur [5,6,29,31]. The field content of the minimal scalar $Z_N$ model is summarised in Table 1.
Table 1: Scalar field content of the low energy theory with the components of the standard model Higgs $H_1$ in the Feynman gauge. The value of the Higgs VEV is $v = 246$ GeV.

| Field | $SU(3)$ | $SU(2)_L$ | $T^3$ | $Y/2$ | $Q = T^3 + Y/2$ |
|-------|---------|-----------|------|-------|-----------------|
| $H_1 = \left( \frac{g^+}{v + h + iG^0} \right)$ | 1 | 2 | $\left( \frac{1}{2} \right)$ | $\frac{1}{2}$ | $\left( \frac{1}{2} \right)$ |
| $H_2 = \left( \frac{-iH^+}{\sqrt{2}} \right)$ | 1 | 2 | $\left( \frac{1}{2} \right)$ | $\frac{1}{2}$ | $\left( \frac{1}{2} \right)$ |
| $S = \frac{\text{SU}^{iS}}{\sqrt{2}}$ | 1 | 1 | 0 | 0 | 0 |

### 2.3 Constraints on charge assignments

The assignments of $Z_N$ charges have to satisfy

\[
\begin{align*}
X_S &> 0, \\
X_1 &\neq X_2, \\
-X_\ell + X_1 + X_e &= 0 \mod N, \\
-X_q + X_1 + X_d &= 0 \mod N, \\
-X_q - X_1 + X_u &= 0 \mod N.
\end{align*}
\]

The first and second conditions arise from avoiding the $|H_1|^2 S$ term and Yukawa terms for $H_2$, respectively, and the rest from requiring Yukawa interactions between $H_1$ and standard model fermions. The choice of $Z_N$ charges for standard model fermions, the standard model Higgs $H_1$, the inert doublet $H_2$ and the complex singlet $S$ must be such that there are no Yukawa terms for $H_2$ and no mixing between $H_1$ and $H_2$: only annihilation and semi-annihilation terms for $H_2$ and $S$ are allowed. While we will see below that there are many assignments that satisfy Eq. (1), in each case it was possible to find an assignment with the charges of standard model fields set to zero: $X_{q,\ell,u,d,e,1} = 0$.

All possible scalar potentials contain a common piece because the terms where each field is in pair with its Hermitian conjugate are allowed under any $Z_N$ and charge assignment. We denote it by $V_c$ (the ‘c’ stands for ‘common’):

\[
V_c = \lambda_1 \left( |H_1|^2 - \frac{v^2}{2} \right)^2 + \mu_1^2 |H_2|^2 + \lambda_2 |H_2|^4 + \mu_2^2 |S|^2 + \lambda_3 |S|^4 + \\
+ \lambda_{S1} |S|^2 |H_1|^2 + \lambda_{S2} |S|^2 |H_2|^2 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1).
\]

### 2.4 The $Z_2$ scalar potential

There are 256 ways to assign the possible $Z_2$ charges 0, 1 to the standard model and dark sector fields. Of these, 8 satisfy Eq. (1); among them, there are 2 different assignments to the dark sector fields:
\( X_S = X_1 = 1, X_2 = 0 \) and \( X_1 = 0, X_2 = X_S = 1 \). Both give rise to the unique scalar potential

\[
V = V_c + \frac{\mu_s^2}{2} (S^2 + S^{'2}) + \frac{\lambda_5}{2} \left[ (H_1 H_2)^2 + (H_2^+ H_1)^2 \right] \\
+ \frac{\mu_{SH}}{2} (S^t H_1^t H_2 + S H_2^t H_1) + \frac{\mu_{SH}^2}{2} (S^t H_1^t H_2 + S H_2^t H_1) \\
+ \frac{\lambda_1}{2} (S^t + S^t H_1^t + S^t H_2^t H_1) \\
+ \frac{\lambda_0}{2} |H_1|^2 (S^2 + S^{'2}) + \frac{\lambda_0}{2} |H_2|^2 (S^2 + S^{'2}).
\] 

(3)

2.5 \( Z_3 \) scalar potentials and particle content

There are 6561 ways to assign 0, 1, 2 to the fields. Of these, 108 satisfy Eq. (1), among them, there are 12 different assignments to the dark sector fields, giving rise to 2 different scalar potentials. The example potential we choose to work with (given by e.g. \( X_1 = 0, X_2 = X_S = 1 \)) is

\[
V_{Z_3} = V_c + \frac{\mu_s^2}{2} (S^3 + S^{'3}) + \frac{\lambda_{S12}}{2} (S^t H_1^t H_2 + S H_2^t H_1) \\
+ \frac{\mu_{SH}}{2} (S H_2^t H_1 + S H_1^t H_2),
\]

which induces the semi-annihilation processes we are interested in. The second one is obtained from Eq. (1) by changing \( S \rightarrow S^t \) (with \( \mu_{SH} \rightarrow \mu_{SH}^t \) and \( \lambda_{S12} \rightarrow \lambda_{S21} \)).

The following conditions are sufficient to have the global minimum of potential at electroweak vacuum with \( \langle S \rangle = 0, \langle H_2 \rangle = 0 \):

\[
\lambda_1, \lambda_2, \lambda_S, \lambda_{S1}, \lambda_{S2} > 0, \\
\lambda_3 + \lambda_4 > 0, \\
4\lambda_{S1} \lambda_{S2} > \lambda_{S12}^2, \\
\frac{\mu_s^2}{\lambda_3} + \frac{\mu_{SH}^2}{\lambda_3 + \lambda_4} < 4 \mu_s^2.
\]

(5) (6) (7) (8)

We use these conditions for our benchmark points.

The last term in Eq. (4) induces a mixing between the down component of \( H_2 \) and \( S \). In terms of the mass eigenstates \( x_1, x_2 \), we have

\[
H_2 = \begin{pmatrix} -i H^+ \\ x_1 \sin \theta + x_2 \cos \theta \end{pmatrix}, \quad S = x_1 \cos \theta - x_2 \sin \theta.
\]

(9)

The dark sector of this model consists of 3 complex particles \( x_1, x_2 \), and \( H^+ \) with the \( Z_3 \) charge of 1. Taking the masses of \( x_1, x_2 \) and the mixing angle \( \theta \) as free parameters of the model, we get the following relations

\[
\mu_s^2 = M_{x_2}^2 \sin^2 \theta + M_{x_1}^2 \cos^2 \theta - \lambda_{S1} \frac{v^2}{2}, \\
\mu_{SH} = -4(M_{x_2}^2 - M_{x_1}^2) \frac{\cos \theta \sin \theta}{\sqrt{2} v}, \\
\mu_2^2 = -(\lambda_4 + \lambda_3) \frac{v^2}{2} + M_{x_1}^2 \sin^2 \theta + M_{x_2}^2 \cos^2 \theta.
\]

(10) (11) (12)

The \( \lambda_1 \) and the mass of \( H^+ \) can be presented by formulas

\[
\lambda_1 = \frac{1}{2} \frac{M_{x_2}^2}{v^2}, \\
M_{H^+} = \sqrt{\mu_2^2 + \lambda_3 \frac{v^2}{2}}.
\]

(13) (14)
where $M_h$ is mass of SM Higgs.

## 2.6 $Z_4$ scalar potentials and particle content

There are 65536 ways to assign 0, 1, 2, 3 to the fields. Of these, 576 satisfy Eq. (1); among them, there are 36 different assignments to the dark sector fields, giving rise to 5 different scalar potentials. Among those the only potential that contains semi-annihilation terms is

$$V_{14}^J = V_c + \frac{\lambda_S}{2} (S^4 + S^4) + \frac{\lambda_5}{2} \left[ (H_1^4 H_2^4 + (H_1^4 H_2^4)^2 \right]$$

$$+ \frac{\lambda_{S12}}{2} (S^2 H_1^4 H_2 + S^2 H_2^4 H_1) + \frac{\lambda_{S21}}{2} (S^2 H_1^4 H_1 + S^2 H_1^4 H_2),$$

invariant under e.g. the assignment of $Z_4$ charges $X_1 = 0, X_2 = 2, X_S = 1$.

The following conditions are sufficient to have global minimum of potential at electroweak vacuum with $\langle S \rangle = 0, \langle H_2 \rangle = 0$:

$$\lambda_1, \lambda_2, \lambda_{S1}, \lambda_{S2} > 0, \quad (\lambda_{S12})^2 < \lambda_{S1} \lambda_{S2}.$$ (16)

Note that the complex scalar $S$ does not mix with $H_2$ because these fields have different $Z_4$ charges. As a result this model contains two dark sectors, the first one with the complex scalar $S$ (the $Z_4$ charge is 1), the second one comprising the complex scalar $H^+$ and the real scalars $H^0$ and $A^0$ (the $Z_4$ charge is 2). Any of the neutral particles with a non-zero $Z_4$ charge can be a dark matter candidate. We will consider the masses of the neutral scalar particles, $M_S, M_{H^0}$ and $M_{A^0}$, as independent parameters, then

$$\mu_S^2 = M_S^2 - \frac{\lambda_{S1} v^2}{2},$$ (21)

$$\lambda_5 = \frac{M_{H^0}^2 - M_{A^0}^2}{v^2},$$ (22)

$$\mu_2^2 = M_{H^0}^2 - (\lambda_3 + \lambda_4 + \lambda_5) \frac{v^2}{2},$$ (23)

$$M_{H^+} = \sqrt{\frac{M_{A^0}^2 + M_{H^0}^2}{2} - \lambda_4 \frac{v^2}{2}},$$ (24)

$$\lambda_1 = \frac{1}{2} \frac{M_h^2}{v^2}. \quad (25)$$
3 RELIC DENSITY IN CASE OF THE $Z_3$ SYMMETRY

3.1 Evolution equations

Consider the $Z_3$-symmetric theory. The imposed $Z_3$ symmetry implies, as usual, just one dark matter candidate. This is because the $Z_3$ charges $1$ and $-1$ correspond to a particle and its anti-particle. The new feature is that processes of the type $xx \rightarrow x^*X$, where $X$ is any standard model particle, also contribute to dark matter annihilation. The equation for the number density reads

$$\frac{dn}{dt} = -\langle v \sigma_{xx \rightarrow x^*X} \rangle (n^2 - n^2) - \frac{1}{2} \langle v \sigma_{xx \rightarrow x^*X} \rangle (n^2 - n \bar{n}) - 3Hn,$$

(26)

where we use $n = n_{eq}$, $H$ is the Hubble rate, and angular brackets mean thermal averaging. We define

$$\sigma_v \equiv \langle v \sigma_{xx \rightarrow x^*X} \rangle + \frac{1}{2} \langle v \sigma_{xx \rightarrow x^*X} \rangle$$

and

$$\alpha = \frac{1}{2} \sigma_v,$$

(27)

which means that $0 \leq \alpha \leq 1$. Here and in the following we use the notation, $\sigma_v \equiv \langle v \sigma_{xx \rightarrow x^*X} \rangle$.

In terms of the abundance, $Y = n/s$, where $s$ is the entropy density, we obtain

$$\frac{dY}{dt} = -s \sigma_v \left( Y^2 - \alpha YY - (1 - \alpha)Y^2 \right)$$

(28)

or, using the entropy conservation condition $ds/dt = -3Hs$,

$$3H \frac{dY}{ds} = \sigma_v \left( Y^2 - \alpha YY - (1 - \alpha)Y^2 \right).$$

(29)

where $Y = Y_{eq}$ is the equilibrium abundance. We use standard formulae for $H(T)$ and $s(T)$ [32] that allow to replace the entropy evolution with the temperature one. To solve this equation we follow the usual procedure [18,32]. Writing $Y = \bar{Y} + \Delta Y$ we find the starting point for the numerical solution of this equation with the Runge-Kutta method using

$$3H \frac{d\bar{Y}}{ds} = \sigma_v \left( Y^2 - \alpha Y\bar{Y} - (1 - \alpha)\bar{Y}^2 \right),$$

(30)

where $\Delta Y \ll Y$. This is similar to the standard case except that $\Delta Y$ increases by a factor $1/(1 - \alpha/2)$. Furthermore, when solving numerically the evolution equation, the decoupling condition $Y^2 \gg \bar{Y}^2$ is modified to

$$Y^2 \gg \alpha Y\bar{Y} + (1 - \alpha)\bar{Y}^2.$$  

(31)

This implies that the freeze-out starts at an earlier time and lasts until a later time as compared with the standard case. This modified evolution equation is implemented in micrOMEGAs [19,33]. Although semi-annihilation processes can play a significant role in the computation of the relic density, the solution for the abundance depends only weakly on the parameter $\alpha$, typically only by a few percent. This means in particular that the standard freeze-out approximation works with a good precision.

3.2 Numerical results with micrOMEGAs

Using the scalar potential defined in Eq. (4) we have implemented in micrOMEGAs the scalar model with a $Z_3$ symmetry. The scalar sector contains an additional scalar doublet and one complex singlet. The neutral component of the doublet mixes with the singlet, the lightest component $x_1$ is therefore the dark matter candidate, while the heavy component $x_2$ can decay into $x_1h$, where $h$ is the standard model-like Higgs boson. Because $h$ can decay into light particles, $x_2$ is unstable even if the mass difference between $x_1$ and $x_2$ is small. Note that the doublet component of DM has a vector interaction.
with the $Z$. This interaction is determined by the $SU(2) \times U(1)$ gauge group and leads to a large direct detection signal in conflict with exclusion limits, for example from XENON100 \cite{34}. The only way to avoid this constraint is to consider a DM with a very small doublet component, namely we have to assume that the mixing angle

$$\theta \leq 0.025.$$  \hfill (32)

In the limit of small mixing, annihilation processes such as $x_1 x_1^* \rightarrow XX$ where $X$ stands for $W, Z, h$, are dominated by the $\lambda_{S1}|S|^2 |H_1|^2$ term. The semi-annihilation process $x_1 x_1 \rightarrow x_1^* h$ is mainly determined by a product of $\mu_S^p$ and $\lambda_{S1}$ arising from the terms $\mu_S^p (S^3 + S^\dagger S^\dagger)^2 / 2$ and $\lambda_{S1} |S|^2 |H_1|^2$ in Eq. 2 and Eq. 4. To illustrate a scenario where semi-annihilation channels contribute significantly and which predicts reasonable values for the relic density and the direct detection rate, we choose a benchmark point with the following parameters

$$
\begin{array}{c|c|c|c|c|c}
\lambda_2 & 0.1 & \lambda_S & 0.2 & \lambda_{S12} & 0.1 & M_{x_1} & 150 \text{ GeV} \\
\lambda_3 & 0.1 & \lambda_{S1} & 0.05 & M_h & 125 \text{ GeV} & M_{x_2} & 400 \text{ GeV} \\
\lambda_4 & 0.1 & \lambda_{S2} & 0.1 & \mu_S^q & 80 \text{ GeV} & \sin \theta & 0.025 \\
\end{array}
$$

Table 2: Benchmark point for $Z_3$.

For this point, the relic density is $\Omega h^2 = 0.105$. The dominant contribution to $(\Omega h^2)^{-1}$ is from semi-annihilation (54% for $x_1 x_1 \rightarrow h x_1^*$) while the annihilation channels $x_1 x_1^* \rightarrow WW, ZZ, hh$ give a relative contribution of 22%, 13%, and 10% respectively. Fig. 1 illustrates the dependence of the relic density on the DM mass as compared to the relic density when semi-annihilation is ignored, $(\Omega h^2)^{\text{ann}}$. Here all other parameters are fixed to their benchmark values. When $M_{x_1} = 110$ GeV, semi-annihilation with a Higgs in the final state is kinematically forbidden at low velocities. If $M_{x_1}$ increases, semi-annihilation plays an important role and $\Omega h^2$ decreases rapidly due to the contribution of the channel $x_1 x_1 \rightarrow h x_1^*$.

Note that $(\Omega h^2)^{\text{ann}}$ also decreases when $M_{x_1}$ is such that the channel $x_1 x_1^* \rightarrow hh$ is allowed. When $M_{x_1}$ approaches $M_{x_2} / 2$, $\Omega h^2$ falls again because the semi-annihilation channel is enhanced due to $x_2$ exchange near resonance.

The spin independent (SI) scattering cross section on nuclei as a function of the DM mass is illustrated in Fig. 1 (right panel). Here we average over dark matter and anti-dark matter cross sections assuming that they have the same density. The main contribution comes from the $Z$-exchange diagram because there is a $x_1 x_1^* Z$ coupling\footnote{In the inert doublet model with a $Z_2$ symmetry \cite{20, 22}, a $\lambda_S$ term splits the complex doublet into a scalar and a pseudoscalar, when the mass splitting is small such coupling leads to inelastic scattering.}. Furthermore, one can easily show that the scattering amplitudes are not the same for protons and neutrons, with $f_p = (4 \sin^2 \theta_W - 1) f_n = -0.075 f_n$. Since the current experimental bounds on $\sigma_{xN}^{\text{SI}}$ are extracted from experimental results assuming that the couplings to protons ($f_p$) and neutrons ($f_n$) are equal and the same as the couplings of $x_1^*$ to protons ($\bar{f}_p$) and neutrons ($\bar{f}_n$), we define the normalised cross section on a point-like nucleus \cite{33}:

$$\sigma_{xN}^{\text{SI}} = \frac{2}{\pi} \left( \frac{M_N M_{x_1}}{M_N + M_{x_1}} \right)^2 \left( \frac{|Z f_p + (A - Z) f_n|^2}{A^2} + \frac{|Z \bar{f}_p + (A - Z) \bar{f}_n|^2}{A^2} \right).$$  \hfill (33)

This quantity can directly be compared with the limit on $\sigma_{xN}^{\text{SI}}$. 

7
4 RELIC DENSITY IN CASE OF THE $Z_4$ SYMMETRY

4.1 Evolution equations

In the case of a $Z_4$ symmetry all particles can be divided into 3 classes, $\{0,1,2\}$ according to the value of their $Z_4$ charges modulo 4. We can choose SM particles to have $X_{SM} = 0$. We will use the notation $\sigma^{abcd}_v$ for the thermally averaged cross section for reactions $ab \to cd$ where $a, b, c, d = 0, 1, 2$ represent any particle with given $X$-charge. Let $M_{x_1}$ and $M_{x_2}$ be the masses of the lightest particles of classes 1 and 2 respectively. The lightest particle of class 1 is always stable and therefore a DM candidate. The lightest particle of class 2 is stable and can be a second DM candidate if $M_{x_2} < 2M_{x_1}$. Note that if $M_{x_2} > 2M_{x_1}$, then $x_2$ will decay before the freeze-out of $x_1$ and the relic density can be computed following the standard procedure.

The equations for the number density of particles 1 and 2 read

$$\frac{dn_1}{dt} = -\sigma^{1100}_v (n_1^2 - \bar{n}_1^2) - \sigma^{1120}_v (n_1^2 - \bar{n}_1^2 \bar{n}_2^2) - \sigma^{1122}_v (n_1^2 - \bar{n}_1^2 \bar{n}_2^2) - 3Hn_1,$$

$$\frac{dn_2}{dt} = -\sigma^{2200}_v (n_2^2 - \bar{n}_2^2) + \frac{1}{2} \sigma^{1120}_v (n_1^2 - \bar{n}_1^2 \bar{n}_2^2) - \frac{1}{2} \sigma^{1210}_v (n_1 n_2 - n_1 \bar{n}_2) - \sigma^{2211}_v (n_2^2 - n_1^2 \bar{n}_2^2) - 3Hn_2,$$

where we use $\bar{n}_i$ to designate the equilibrium number density of particle $x_i$. In $\sigma_v^{abcd}$ all annihilation and coannihilation processes are taken into account. Here the semi-annihilation processes include all those, where 2 DM particles annihilate into one DM and one standard particle, specifically $\sigma^{1120}_v$ and $\sigma^{1210}_v$. These two cross sections are also described by the same matrix element. However, there is no simple relation between these two cross sections because one process is in the $s$-channel and the other

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3 We take into account that $3 = -1 \mod 4$, so the particle with $X$-charge 3 is the antiparticle of a particle with $X$-charge 1.
in the t-channel. In terms of the abundance, \( Y_i = n_i/s \),

\[
3H \frac{dY_1}{ds} = \sigma_v^{1100} \left( Y_1^2 - Y_2^2 \right) + \sigma_v^{1120} \left( Y_2^2 - Y_2^2 \frac{Y_1^2}{Y_2^2} \right) + \sigma_v^{1222} \left( Y_1^2 - Y_2^2 \frac{Y_1^2}{Y_2^2} \right),
\]

\[
3H \frac{dY_2}{ds} = \sigma_v^{2200} \left( Y_2^2 - Y_2^2 \right) - \frac{1}{2} \sigma_v^{1120} \left( Y_1^2 - Y_2^2 \frac{Y_1^2}{Y_2^2} \right) + \frac{1}{2} \sigma_v^{1210} Y_1 \left( Y_2 - Y_2 \right)
+ \sigma_v^{2211} \left( Y_2^2 - Y_2^2 \frac{Y_1^2}{Y_2^2} \right).
\]

Solving these equations we use standard formulas for entropy \( s(T) \) and the Hubble rate \( H(T) \) temperature dependence [32] that allow to replace the dependence on entropy with one on temperature. The thermally averaged cross section involving particles of different sectors can be expressed as

\[
\sigma_v^{IJKL}(T) = \frac{T}{64\pi^5 s^2 \bar{Y}_I(T) \bar{Y}_J(T)} \int \frac{ds}{\sqrt{s}} K_1 \left( \frac{\sqrt{s}}{T} \right) p_{\text{in}} p_{\text{out}}
\sum_{a \in I, b \in J, c \in K, d \in L, \text{pol.}} |M_{ab \rightarrow cd}(\sqrt{s}, \cos \Theta)|^2 \cos \theta,
\]

\[
\bar{Y}_I(T) = \frac{T}{2\pi^2 s} \sum_{i \in I} g_i m_i^2 K_2 \left( \frac{m_i}{T} \right),
\]

where \( M_{ab \rightarrow cd} \) is the matrix element for the \( 2 \rightarrow 2 \) process and \( K_1, K_2 \) are modified Bessel functions of the second kind. For reactions which are kinematically open at zero relative velocity, \( \sigma_v \) depends slowly on temperature. Otherwise there is a strong \( \exp(-\Delta M/T) \) temperature dependence, where \( \Delta M \) is the difference between the sums of the masses of outgoing and incoming particles. Equation (38) leads to relations between different cross sections

\[
Y_I Y_J \sigma_v^{IJKL} = Y_K Y_L \sigma_v^{KLIJ}.
\]

In particular it implies that, \( \sigma_v^{0211} = \sigma_v^{1120} Y_1^2 / Y_2 \), where the abundance of incoming SM particles \( Y_0 = 1 \).

Introducing \( \Delta Y_i = Y_i - \bar{Y}_i \), Eqs. (36) and (37) take a simple form

\[
3H \frac{\Delta Y_i}{ds} = -C_i + A_{i} (T) \Delta Y_j + Q_{ijk} (T) \Delta Y_j \Delta Y_k,
\]

where

\[
C_i = 3H \frac{d\bar{Y}_i}{ds},
\]

\[
A = \begin{pmatrix}
2(\sigma_v^{1100} + \sigma_v^{1122} + \sigma_v^{1210}) \bar{Y}_1 & -2(\sigma_v^{1120} + 2\sigma_v^{1212}) \frac{\bar{Y}_1}{Y_2} \\
-2(\sigma_v^{1120} \bar{Y}_1 - 2\sigma_v^{1122} \bar{Y}_1) & 0
\end{pmatrix},
\]

\[
Q_1 = \begin{pmatrix}
\sigma_v^{1100} + \sigma_v^{1122} + \sigma_v^{1212} & 0 \\
0 & -\sigma_v^{2211}
\end{pmatrix},
\]

\[
Q_2 = \begin{pmatrix}
-\sigma_v^{1120} - \sigma_v^{1122} & 0 & \frac{1}{2} \sigma_v^{1210} \\
0 & \sigma_v^{2210} + \sigma_v^{2211}
\end{pmatrix}.
\]
At large temperatures we expect the densities of both DM components to be close to their equilibrium values. In general in micrOMEGAs\cite{36} the equation for the abundance is solved numerically starting from large temperatures. However, this procedure poses a problem for Eq. (41). The step of the numerical solution is inversely proportional to $A(T)$ and as long as $A(T)$ is not suppressed by the Boltzmann factor included in $\bar Y$, the step is too small and the numerical method fails.

To avoid this problem, we use the fact that at large temperatures one can neglect the $Q$ term in Eq. (41) and write the explicit solution for the linearised equation. The approximate solution in the case of large $A$ is

$$\Delta Y_i(s) = A_{ij}^{-1}(s)C_j(s).$$

(47)

One can use Eq. (47) to find the lowest temperature where $\Delta Y_i \approx 0.05 Y_i$ and start solving numerically Eq. (41) from this temperature. In the general case it gives a reasonable step for the numerical solution $\delta s/s \approx 0.1$, where $s$ is the variable of integration. This method can, however, lead to some numerical problems if the masses of the two dark matter particles are very different. Let us call the light particle $l$ and the heavy particle $h$. We have to start the numerical solution at a temperature above the freeze-out temperature of the heaviest DM,

$$T_{fo} \approx M_h/25.$$  

(48)

At this temperature,

$$\frac{Y_l}{\bar Y_h} \approx \exp \frac{M_h - M_l}{T_{fo}},$$

(49)

and the step in the numerical solution of the two component equations will be suppressed by a factor $\exp(-M_h - M_l)/T_{fo}$. This small step size is problematic when solving numerically the equation with the Runge-Kutta method. This occurs when $M_h/M_l < 2$. In this case the equation for the heavy component must be solved independently assuming that the light component has reached its equilibrium density. If $M_h/M_l < 2$, the Runge-Kutta procedure can be used to successfully solve the thermal evolution equations \[41\].

The abundances $Y_1$ and $Y_2$ will be modified by the interactions between the two dark matter sectors. Thus the new terms in Eq. \[36\] will simply add to the standard annihilation process with SM particles and will contribute to decrease the final abundance $Y_1$. After $x_2$ freezes-out, interactions of the type $22 \rightarrow 11$ lead to an increase of $Y_2$. When $M_{x_1} \ll M_{x_2}$, the evolution of $Y_2$ will be strongly influenced by the first sector since at its freeze-out temperature $Y_1$ is large. Following the same argument as above the new annihilation terms in Eq. \[37\] will contribute to a decrease in the final abundance $Y_2$. Furthermore, the semi-annihilation process $12 \rightarrow 10$ which is always kinematically open means that $x_1$ acts as a catalyst for the transformation of $x_2$ into SM particles. Thus the light component forces the heavy one to keep its equilibrium value, resulting in a significant decrease of the relic density of $x_2$. When both DM particles have similar masses, the interplay between the two sectors is more complicated, in particular the rôle of the interactions of the type $20 \rightarrow 11$ will depend on the exact mass relation between the two DM particles. For example, this interaction can lead to an increase of the abundance of $x_2$ if $Y_1$ is large enough for the reverse process to give the largest contribution.

4.2 Numerical results

The scalar model with a $Z_4$ symmetry contains two dark sectors. In sector 1 the DM candidate is a complex singlet, $S$, the main contribution to $\sigma^{1100}_v$ comes from annihilation into Higgs pairs and is determined by the term $\lambda_{S1}|S|^2|H|^2$. Sector 2 is similar to the Inert Doublet Model (IDM). The DM candidate can be either the scalar $H^0$ or the pseudoscalar $A^0$. Annihilation of DM into SM particles is

\footnote{Note that $Y_1$ and $Y_2$ correspond to the abundances of the particles with a given $Z_4$ charge. The relative size of the masses of the DM particles depend on the choice of parameters in a given model.}
usually dominated by gauge boson pair production processes, while annihilation into fermion pairs as well as co-annihilation processes can also contribute. Furthermore, for a DM mass at the electroweak scale, it was shown in [37] that annihilation into 3-body final states via a virtual \( W \) can be important below the \( W \) threshold. To avoid this complication we will consider a DM with a mass above masses of the \( W, Z, \) and \( h \). Under this condition, the DM annihilation into SM particles in sector 2 is driven by \( SU(2) \times U(1) \) gauge interactions and leads typically to a value of \( \Omega h^2 < 0.1 \), except for a DM heavier than about 500 GeV. The co-annihilation of \( H^0, A^0, H^+ \) states increases \( \Omega h^2 \).

We will consider a benchmark point where both DM candidates \( S \) and \( H^0 \) have a mass near 350 GeV. Other parameters are chosen so that semi-annihilation processes play an important role, while both components have comparable relic density and \( \Omega h^2 = \Omega_1 h^2 + \Omega_2 h^2 = 0.1 \). In particular to have \( \Omega_2 h^2 \approx 0.05 \) requires the contribution of coannihilation processes – we therefore impose a small mass splitting \( M_{H^0} \approx M_{A^0} \), meaning that \( \lambda_5 \) will be small, see Eq. (22). Furthermore, a small value of \( \lambda_4 \) also leads to a small mass splitting with the charged Higgs. Note that for small \( \lambda_5 \) and \( \lambda_4 \) the positivity condition on the potential, Eqs. (215) is easily satisfied.

\[
\begin{array}{ccccccc}
\lambda_2 & 0.1 & \lambda_{S1} & 0.1 & \lambda_S & 0.1 & M_A & 341 \text{ GeV} \\
\lambda_3 & 0.1 & \lambda_{S2} & 0.3 & \mu_S & 100 \text{ GeV} & M_H & 339 \text{ GeV} \\
\lambda_4 & 0.01 & \lambda_{S12} & 0.13 & M_h & 125 \text{ GeV} & M_S & 350 \text{ GeV} \\
\lambda_S & 0.1 & \lambda_{S21} & 0.13 & & & & \\
\end{array}
\]

Table 3: Benchmark point for \( Z_4 \).

The results of the calculation of the relic density when including different terms in Eq. (36,37) is presented in Table 4. When only (co-)annihilation into SM particles are taken into account, the relic density of \( S \) is too high, while annihilation is much more efficient in Sector 2. Adding the interactions of the type of 1, 1 \( \leftrightarrow \) 2, 2 brings the value of \( \Omega_1 h^2 \) and \( \Omega_2 h^2 \) closer to each other. In our example the DM in sector 1 has weak interactions with SM particles, therefore \( \Omega_1 h^2 \) is large when sector 2 is neglected. As a result of interactions with sector 2 particles the value for \( \Omega_1 h^2 \) is significantly reduced. This effect was also observed for a DM model with a \( Z_2 \times Z_2 \) symmetry [17] and was called the assisted freeze-out mechanism. Finally, when semi-annihilation processes are included, both \( \Omega_1 h^2 \) and \( \Omega_2 h^2 \) decrease. Note that for this benchmark point, the cross section for DM elastic scattering on proton and neutron is \( 1.5(1.8) \cdot 10^{-9} \) pb for the DM in sector 1 and 2, respectively. This is well below current exclusion limits of XENON100 [34], as will be discussed at the end of this section.

To examine more closely the interplay between the two DM sectors as well as the role of semi-annihilation in determining the DM abundance, we let \( M_S \) vary in the range 200-600 GeV and solve for the relic density by including new terms one by one. All other parameters are fixed to the value for the benchmark in Table 3.

\[
\begin{array}{ccc}
\text{included terms of Eq. (36,37)} & \Omega_1 h^2 & \Omega_2 h^2 \\
\sigma_v^{1100}, \sigma_v^{2200} & 0.24 & 0.041 \\
\sigma_v^{1100}, \sigma_v^{2200} \text{ and } \sigma_v^{1122} & 0.079 & 0.064 \\
\text{All} & 0.050 & 0.051 \\
\end{array}
\]

Table 4: Relic density of DM particles for the \( Z_4 \) benchmark point.

and neutron is \( 1.5(1.8) \cdot 10^{-9} \) pb for the DM in sector 1 and 2, respectively. This is well below current exclusion limits of XENON100 [34], as will be discussed at the end of this section.

To examine more closely the interplay between the two DM sectors as well as the role of semi-annihilation in determining the DM abundance, we let \( M_S \) vary in the range 200-600 GeV and solve for the relic density by including new terms one by one. All other parameters are fixed to the value for the benchmark in Table 3.

First we consider only the impact of annihilation processes, the results are displayed in Fig. 2 (left). When solving the evolution equation for the two DM independently, \( \Omega_1 h^2 \) rises rapidly with \( M_S \) while \( \Omega_2 h^2 \) remains constant. Note that for the model under consideration we have that \( \sigma_v^{1100} < \sigma_v^{2200} \) hence
Figure 2: Effect of interactions between the two dark matter sectors (left) and of semi-annihilation (right) on $\Omega_1 h^2$ (solid) and $\Omega_2 h^2$ (dashed) as a function of $M_S$. Left panel – Including only $\sigma_{1100}^v$ and $\sigma_{2200}^v$ (black) as well as $\sigma_{1122}^v$, $\sigma_{2211}^v$ (red). Right panel – Including only $\sigma_{1210}^v$ (green), only $\sigma_{1120}^v$ (red) as well as all semi-annihilations (blue), as a reference in black $\Omega_1 h^2$ (solid) and $\Omega_2 h^2$ (dot) with only standard annihilation terms. Note that $\sigma_{1210}^v$ does not change $\Omega_1 h^2$.

$\Omega_1 h^2 > \Omega_2 h^2$. The impact of $\sigma_{1122}^v$ and $\sigma_{2211}^v$ on the relic density depends on the relative masses of the scalar and doublet DM. The heavier DM candidate freezes out at a larger temperature than the lighter one, $T_{f0h} > T_{f0l}$. If the mass difference is large, this happens when the light one is at its equilibrium value. Thus the contribution of $\sigma_{1122}^v$ just adds to $\sigma_{1100}^v$, leading to a decrease of the heavy DM abundance. Furthermore, after the light DM freezes out, interactions such as $hh \rightarrow ll$ give an additional source of light DM, while the reverse reaction is suppressed by a Boltzmann factor. This effect can, however, be small when the heavy particles have a low density at this point. Thus the interactions between DM sectors 1 and 2 lead altogether to a decrease of the abundance of the heavy component and an increase of the light component. This is observed in the left panel of Fig. 2. In the region where $M_S < M_{H^0} = 350$ GeV, $\Omega_2 h^2$ decreases while in the region $M_S > M_{H^0}$, $\Omega_2 h^2$ increases and vice-versa for $\Omega_1 h^2$. Note that for large values of $M_S$, interactions with SM particles are weak so $\sigma_{1122}^v \gg \sigma_{1100}^v$, leading to a large decrease in $\Omega_1 h^2$. When there is a small difference between the two DM particles, the freeze-out temperatures of both component are similar. The density of the heavy DM component has not yet decreased to its final value at the time the light component freezes out, thus the effect of $hh \leftrightarrow ll$ interactions in increasing the abundance of the light component is more important. This is particularly noticeable when looking at the curve for $\Omega_2 h^2$ in the region, where $M_S$ is just above $M_{H^0} = 350$ GeV in Fig. 2 (left). This discussion, where we ignore the semi-annihilation terms, applies to models with $\lambda_{S12} = \lambda_{S21} = 0$. In this case the $Z_4$ symmetry is replaced with a $Z_2 \times Z_2$ symmetry.

Next we consider the impact of semi-annihilation processes, ignoring the annihilation of pairs of particles from sector 1 to 2. The $\sigma_{1210}^v$ term does not affect $\Omega_1 h^2$ and works as a catalyst for $2 \rightarrow \text{SM}$ transitions. This term has an effect only after the freeze-out of $H^0$ and its effect is stronger when $Y_1$ is large, see Eq. 37. Thus in the region $M_S > M_{H^0}$ where the freeze-out of $S$ occurs first (at a higher temperature), we find a roughly constant factor of suppression of $\Omega_2 h^2$. As $M_S$ decreases, its abundance $Y_1$ at the freeze-out of $H^0$ ($T_{f0H^0}$) will increase, thus the suppression of $\Omega_2 h^2$ is more important, see Fig. 2. Note that the suppression of $\Omega_2 h^2$ for $M_S > M_{H^0}$ is significantly larger than for the other semi-annihilation processes that we will discuss below. This is because the $\sigma_{1210}^v$ term in
Figure 3: Temperature evolution of $Y_1$ (solid) and $Y_2$ (dashed) with standard terms and the contribution of $\sigma_{1120}^v$ for $M_S = 260$ GeV. Temperature evolution of $Y_2$ with only standard terms (green/dashed), $T$ is in GeV.

Eq. 37 depends on $Y_1^2$, which is large in this approximation.

The second type of semi-annihilation process, $11 \rightarrow 20$ (or its reverse $20 \rightarrow 11$) leads to variations in the relic density of both DM components. If $M_S > M_{H^0}$, the impact of $\sigma_{1120}^v$ is very similar to the one discussed above for $\sigma_{1122}^v$. For $S$, the heavy component, the overall annihilation cross section is increased, leading to a decrease in $\Omega_1$, illustrated by the blue curve in Fig. 2. For $H^0$, the relic density increases because the process $11 \rightarrow 20$ is an additional source of sector 2 particles. This increase is even more important when both particles have similar masses – see the blue dashed curve in Fig. 2 when $M_S = 260-350$ GeV. To examine more closely the impact of the semi-annihilation in the region where the mass of both DM particles are similar, we compute the temperature evolution of $Y_1$ and $Y_2$, choosing $M_S = 260$ GeV. The result is displayed in Fig. 3, in particular comparing the evolution of $Y_2$ with and without the contribution of $\sigma_{1120}^v$. For this choice of masses, the freeze-out of $H^0$ occurs when the abundance $Y_1 = Y_1^* \rightarrow Y_1^*$ is large, this means that the term

$$-rac{1}{2} \sigma_{1120}^v \left(Y_1^2 - \frac{Y_2}{Y_2^*} Y_1^2 \right) = +\frac{1}{2} \sigma_{1120}^v \left(Y_2 Y_1 - 1 \right) \ldots$$

in Eq. [37] forces $Y_2$ to follow its equilibrium value. Thus $Y_2$ is further reduced by semi-annihilation at large temperatures. After the freeze-out of $S$, when $Y_1 \gg Y_1^*$, the same interaction leads to an increase of $Y_2$. Thus the overall effect is an increase in the abundance of class 2 particles as compared with the case where only standard interactions are considered.

Finally, when $M_S < 260$ GeV, the cross section $\sigma_{1120}^v$, which consists of processes of the type $SS \rightarrow H^0h$ is small because of a lack of phase space, thus $\Omega_1 h^2$ is the same as when only standard annihilation terms were included. At the same time the reverse process, $20 \rightarrow 11$ drives the depletion of class 2 particles and $\Omega_2 h^2$ drops to very small values. Note that when $M_{H^0} > 2M_S$ we expect that the class 2 DM will decay into pairs of class 1 particles since they are allowed by the $Z_4$ symmetry. However, in this example, the effect of $\sigma_{1210}^v$ and $\sigma_{1120}^v$ terms already leads to very small values of $\Omega_2 h^2$ for low values of $M_S$, so that the decays are irrelevant. In summary, the combined effect of semi-annihilation processes is for this example close to the result of only including $\sigma_{1120}^v$, see Fig. 2.

The result for $\Omega_1 h^2$ and $\Omega_2 h^2$ including all annihilation and semi-annihilation processes is displayed in Fig. 4. The semi-annihilation mechanisms dominate for $M_S < M_{H^0}$ while the assisted freeze-out
机械过程是主导效应当 $M_S > M_{H^0}$. 总的暗物质丰度大约在10%的WMAP测量值在考虑的整个质量范围。而我们所描述的特征是通用的，不同暗物质的衰变和半衰变过程的重要性是模型相关的，取决于各种截面的大小。

最后，我们计算了S和$H^0$在XENON100核上的弹性散射的spin-independent截面。如上所述，基准点 $\sigma_{SI} = 1.5(1.8) \times 10^{-9}$ pb for the DM in sector 1 and 2 respectively. We then compute the number of events that should be expected in XENON100 in the interval $8.4 \text{ keV} < E < 44.6 \text{ keV}$ after an exposure of 1171 kg·day. The number of events is directly proportional to the DM local density and we assume that the fraction of each DM component locally is the same as in the early universe, $\rho_i = \rho \Omega_i / \Omega_{tot}$ where $\rho = 0.3$. For $S$ the cross section is largest for small masses, furthermore $S$ contributes maximally to the DM density, hence the maximum predicted number of events, see Fig. 4. The cross section for $H^0$ scattering on nuclei is clearly independent of $M_S$, the variation of the number of events is simply due to the variation in the density of the second DM.

5 conclusions

我们已经构建了零粒子内容的暗物质模型，在这模型中暗物质稳定性是由于离散$Z_N$对称性与$N > 2$. 已经，极简模型包含一个额外的标量丛和双线已经非平凡的暗物质现象学。特别是，与标准模型粒子$X$是任意标准模型粒子，改变暗物质冷凝过程，必须被考虑在内，当计算暗物质丰度时。此外，模型与两个暗物质候选者，衰变过程只影响两个不同的暗物质粒子。我们已经进行了实验研究的半衰变在两个标量暗物质模型基于$Z_3$和$Z_4$对称性。我们实施了这些模型的micrOMEGAs，并研究了半衰变和
of the interactions between the dark sectors on the generation of dark matter relic abundance at the early Universe and the predictions for dark matter direct detection relevant for the presently running XENON100 experiment. We conclude that in this type of models both semi-annihilations and dark sector interactions may significantly affect the dark matter phenomenology compared to the well studied $Z_2$ models, and, therefore, must be taken into account in precise numerical analyses of dark matter properties.

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