Facility Layout Planning with Continuous Representation Considering Temporal Efficiency*

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Facility layout planning (FLP) is one of the most important stages in the design of manufacturing systems. A major approach is to define an evaluation index based on distance and find a layout which minimizes it. In this approach, temporal efficiency is not considered in this stage but in the stage of production scheduling performed after completing FLP. The resultant temporal efficiency may not be optimal enough, since the scheduling is performed under the fixed layout. From this point of view, integration of FLP and production scheduling have been discussed in some works. However, detailed position and size of facilities were not considered. This paper provides an integrated method using continuous representation by which those factors can be dealt with. The problems of FLP and production scheduling are formulated as a mixed integer programming and a 0-1 integer programming, respectively. By assuming that transportation speed is constant, these problems are correlated by an equation. This correlation enables formulation of FLP considering temporal efficiency as a mixed integer programming. This method was applied to an example, and it was shown that facilities which are significant for production scheduling are located close enough and better temporal efficiency can be achieved.

1. Introduction

In order to convert data of an artifact created in product design into an entity and provide it to a customer as a product with high cost-performance, it is necessary to optimally design, properly manage and efficiently operate a manufacturing system. Facility layout planning (FLP) is one of the most important stages for the optimal design of a manufacturing system and has been a topic of discussion for a long time[1–3]. In FLP research, evaluation indices based on distance such as total travel distance and total material handling cost are usually defined, and optimization based on them is performed by mathematical optimization (quadratic assignment problem (QAP)[4] and mixed integer programming (MIP)[5]) or meta-heuristics (tabu search[6], simulated annealing[7], genetic algorithm[8,9], etc.). Those indices do not include temporal efficiency and it is considered in the stage of production scheduling[10–12] performed after completing the FLP stage. In other words, optimization is independently performed in each of the two stages. However, this may result in inadequate optimization result from the point of view of the whole system. For example, optimization in terms of total travel distance may cause locating some facilities unnecessarily closer than they are required from the scheduling point of view and therefore other facilities which are required to be located as close as possible are located apart. Therefore, it is necessary to take temporal efficiency into account in FLP stage.

There are two reasons of performing the independent optimization. One is that a production schedule is prerequisite for evaluating temporal efficiency and therefore this evaluation cannot be performed in FLP stage. The other is that, unlike a production schedule, it is difficult to change a facility layout depending on changes in manufacturing situations such as demand fluctuation. For these decades in which manufacturing style has transitioned from low-mix high-volume manufacturing to high-mix low-volume manufacturing, flexibility and robustness of manufacturing systems have been a key concept. These concepts have driven development of methodologies of robust FLP[13,14] where an optimal facility layout considering demand uncertainty is designed based on the indices that do not include temporal efficiency, and robust production scheduling[15] where an optimal schedule considering uncertainty in processing times, demand, etc. is discussed. These methodologies would provide solutions for the problem concern-
ing to the latter reason.

This research aims to develop a methodology for FLP considering temporal efficiency which overcomes both of the above two problems. As the first step, this paper focuses on the problem concerning to the former reason and deals with FLP considering temporal efficiency for fixed manufacturing conditions. For this issue, several methods have been proposed [16–18]. However, they are for allocation of facilities to pre-given sites and detailed position and size of facilities are not taken into consideration. This paper provides a method for FLP with continuous representation, by which those factors are dealt with, considering temporal efficiency. In the next section, problems of FLP which those factors are dealt with, considering temporal efficiency is formulated as an MIP problem and a 0-1 integer programming, respectively. In section 3, those two problems are then merged by focusing on required times for transportation between two facilities, and eventually FLP considering temporal efficiency is formulated as an MIP problem. This method is applied to an example and its effectiveness is evaluated in section 5. Section 5 provides conclusions.

2. Mathematical Formulation of FLP and Production Scheduling

This research deals with job-shop production with J jobs and M machines. Job \( j \in \mathcal{J} \equiv \{1, \ldots, J\} \) needs \( O_j \) operations. Operation \( o \in O_j \equiv \{1, \ldots, O_j\} \) of job \( j \) is processed by machine \( m_jo \in \mathcal{M} \equiv \{1, \ldots, M\} \). Width and depth of machine \( m \) are \( w_m \) and \( d_m \). This section provides mathematical formulations of FLP and production scheduling, from which formulae of FLP considering temporal efficiency is derived in section 3.

2.1 Formulation of FLP

There are two well-known ways of formulating FLP problem. One is formulation as a QAP and the other is as an MIP. In QAP formulation, the production area is divided into a set of cells and each machine is assigned to one of those cells. Therefore, it is unsuitable for FLP dealing with detailed position of unequal-sized machines. For this reason, this paper adopts the MIP formulation. Positive continuous decision variables \( x_m, y_m \) and 0–1 decision variables \( s_m \) are introduced, which stand for centroid and orientation of machine \( m \), respectively. The FLP problem is formulated as an MIP as follows:

subject to:
\[
\begin{align*}
\frac{1}{2} w_m s_m + d_m (1 - s_m) & \leq x_m \leq W - \frac{1}{2} w_m s_m + d_m (1 - s_m), \forall m \in \mathcal{M} \\
\frac{1}{2} d_m s_m + w_m (1 - s_m) & \leq y_m \leq D - \frac{1}{2} d_m s_m + w_m (1 - s_m), \forall m \in \mathcal{M} \\
\text{max} \left\{ \frac{X_{lm}}{2}, \frac{Y_{lm}}{2} \right\} & \geq 0, \forall l \neq m \in \mathcal{M}
\end{align*}
\]

where \( X_{lm} \) and \( Y_{lm} \) are defined by \( |x_l - x_m| \) and \( |y_l - y_m| \), respectively, and \( f_{jim} \) is the quantity of material \( j \) transported from machine \( t \) to \( m \). \( W \) and \( D \) are the width and depth of the production area. Equation (1) describes the total travel distance for the production area. Inequality (2) and (3) assure every machine is included in the area, and (4) assures no machines overlap each other.

2.2 Formulation of Production Scheduling

In this paper, time is discretized with unit time \( u \) and the planning period is composed of \( T \) periods whose length is \( u \). 0–1 decision variables \( z_{jont} \) are introduced for describing a production schedule mathematically. This variable takes 1 if operation \( o \) of job \( j \) is processed by machine \( m \) at period \( t \) in \( T \equiv \{1, \ldots, T\} \). The production scheduling problem for achieving the minimum makespan is formulated as a 0-1 integer programming as follows:

subject to:
\[
\begin{align*}
\text{minimize:} & \quad \frac{1}{\tau_{jo}} \sum_{t=1}^{T} \left( t - \left\lfloor \frac{1}{2} \right\rfloor \right) z_{jont} + \frac{\tau_{jo}}{2} \\
\text{subject to:} & \quad \sum_{m=1}^{M} \sum_{t=1}^{T} z_{jont} = \tau_{jo}, \forall j \in \mathcal{J}, \forall o \in O_j \\
& \quad \sum_{j=1}^{J} \sum_{o=1}^{O_j} z_{jont} \leq 1, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \\
& \quad \frac{1}{\tau_{jo}} \sum_{m=1}^{M} \sum_{t=1}^{T} \left( t - \left\lfloor \frac{1}{2} \right\rfloor \right) z_{j(o+1)ont} - \frac{\tau_{j(o+1)}}{2} + 1 \\
& \quad > \frac{1}{\tau_{jo}} \sum_{m=1}^{M} \sum_{t=1}^{T} \left( t - \left\lfloor \frac{1}{2} \right\rfloor \right) z_{jont} + \frac{\tau_{jo}}{2} + h_{jo}, \forall j \in \mathcal{J}, \forall o \in \{1, \ldots, O_j - 1\}
\end{align*}
\]
where $\tau_j$ is the required period for processing the operation $o$ of job $j$, $h_{jo}$ is required time for transporting the material of job $j$ to the machine which performs operation $(o+1)$ after completing operation $o$. Equation (5) describes maximum of finish period of the last operation of each job, and minimizing this value results in minimization of makespan of the schedule. Equation (6) assures processing period for each operation equals to the given required period. Inequality (7) constrains a machine from performing multiple operations at the same time. Inequality (8) is for operation order constraints and equation (9) avoids interruption of an operation.

### 3. FLP Considering Temporal Efficiency

FLP and production scheduling relate each other by transportation of materials. The FLP problem and the production scheduling problem given in the previous section can be merged by formulating relationship between position of machines and transportation times. By assuming that the transportation speed $v$ is constant and by ignoring transportation routes and collisions, the relationship can be described by the following equation.

$$h_{jo} = \frac{|x_{mJO} - x_{mJO}| + |y_{m(J+1)} - y_{mJO}|}{vu}$$  \hspace{1cm} (11)

By using this equation, FLP considering temporal efficiency can be formulated as the following optimization problem which can be equivalently transformed into a MIP by introducing artificial variables and using the “big-M” method.

subject to:

$$\frac{w_m s_m + d_m (1 - s_m)}{2} \leq x_m \leq W - \frac{w_m s_m + d_m (1 - s_m)}{2}, \forall m \in M \hspace{1cm} (13)$$

$$\frac{d_m s_m + w_m (1 - s_m)}{2} \leq y_m \leq D - \frac{d_m s_m + w_m (1 - s_m)}{2}, \forall m \in M \hspace{1cm} (14)$$

max \left\{ \frac{X_{lm}}{2} \right\} \hspace{1cm} (15)

minimize:

$$\max_{j \in J} \left\{ \frac{1}{\tau_j} \sum_{t=1}^{T} \left( t - \frac{1}{2} \right) z_{jO_t, m} + \frac{\tau_j O_t}{2} \right\}$$  \hspace{1cm} (12)

subject to:

$$\sum_{m=1}^{T} \sum_{t=1}^{T} z_{jO_t, m} = \tau_j, \forall j \in J, \forall o \in O_j \hspace{1cm} (16)$$

$$\sum_{j=1}^{\sum_{o=1}^{O_j}} \sum_{m=1}^{T} z_{jO_t, m} \leq 1, \forall m \in M, \forall t \in T \hspace{1cm} (17)$$

$$\frac{1}{\tau_j} \sum_{m=1}^{T} \sum_{t=1}^{T} \left( t - \frac{1}{2} \right) z_{jO_t, m} - \frac{\tau_j O_t}{2} + 1 \geq 0$$

$$\frac{1}{\tau_j} \sum_{m=1}^{T} \sum_{t=1}^{T} \left( t - \frac{1}{2} \right) z_{jO_t, m} + \frac{\tau_j O_t}{2} + h_{jo}$$

$$\forall j \in J, \forall o \in \{1, \ldots, O_j - 1\} \hspace{1cm} (18)$$

$$\tau_{jo} (z_{jO_{t+1}} - z_{jO_t}) \leq \sum_{t=1}^{T} z_{jO_t, m} \hspace{1cm} (19)$$

$$\forall j \in J, \forall o \in O_j, \forall m \in M, \forall t \in \{1, \ldots, T - \tau_{jo}\} \hspace{1cm} (20)$$

$$h_{jo} = \frac{|x_{mJO} - x_{mJO}| + |y_{m(J+1)} - y_{mJO}|}{vu}$$  \hspace{1cm} (21)

### 4. Numerical Example

The proposed method was applied to an example where $W = 45[m]$, $D = 45[m]$, $T = 5$, $O_1 = O_2 = \ldots = O_7 = 7$, $M = 7$, $u = 0.5[min]$, $T = 60$ and $v = 15[m/min]$. Dimensions of the machines were given as shown in Table 1. The machine processing each operation was given as Table 2, and its required time and period were given as Table 3.

![Table 1 Dimensions of machines (m)](image)

| Machine m | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----------|---|---|---|---|---|---|---|
| Width ($w_m$) | 2 | 3 | 4 | 5 | 6 | 4 | 2 |
| Depth ($d_m$) | 2 | 2 | 3 | 4 | 3 | 2 | 2 |

![Table 2 Machine sequence $m_{jo}$](image)

| Job j | Operation o | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|-------------|---|---|---|---|---|---|---|
| 1     | 2           | 4 | 2 | 4 | 3 | 6 | 4 | 8 |
| 2     | 3           | 6 | 1 | 2 | 4 | 8 | 4 | 3 |
| 3     | 1           | 2 | 4 | 2 | 4 | 4 | 2 | 4 |
| 4     | 3           | 4 | 2 | 4 | 8 | 3 | 6 | 1 |
| 5     | 2           | 4 | 1 | 2 | 4 | 8 | 3 | 6 |

### Table 3 Required processing time (m) and periods $\tau_{jo}$

| Job j | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|---|---|---|---|---|---|
| 1     | 2(4) | 3(6) | 2(4) | 3(6) | 2(4) | 4(8) | 3(6) |
| 2     | 3(6) | 1(2) | 3(6) | 2(4) | 4(8) | 3(6) | 2(4) |
| 3     | 1(2) | 2(4) | 3(6) | 2(4) | 4(8) | 3(6) | 1(2) |
| 4     | 3(6) | 2(4) | 4(8) | 3(6) | 1(2) | 3(6) | 2(4) |
| 5     | 2(4) | 1(2) | 4(8) | 3(6) | 1(2) | 3(6) | 2(4) |

The optimizations were performed with a generic workstation (Dell Precision T7400, Intel Xeon E5430@...
Fig. 1 Facility layout generated by the conventional method

Fig. 2 Production schedule generated by the conventional method

Fig. 3 Facility layout generated by the proposed method

Fig. 4 Production schedule generated by the proposed method

Fig. 5 Disjunctive graph of the schedule obtained by the conventional method. Thick arrows stand for the critical path. The values with no underline between two nodes on the critical path denote periods for processing the job and the underlined values denote periods for transporting the material.

Fig. 6 Disjunctive graph of the schedule obtained by the proposed method. Thick arrows stand for the critical path. The values with no underline between two nodes on the critical path denote periods for processing the job and the underlined values denote periods for transporting the material.
2.66GHz, 20GB RAM) and a commercial solver (ILOG CPLEX Optimization Studio 12.5). Figs. 1, 2 show the layout and the schedule obtained by the conventional method in which the two optimization problems in Section 2. are solved individually, and Figs. 3, 4 show those obtained by the proposed method given in Section 3., respectively. Figs. 5, 6 show disjunctive graphs of the two schedules. In these figures, thick arrows show the critical paths. The values without underlines beside those arrows are required periods for performing the operations $\tau_{jo}$ given in Table 3 and those with underlines are for transporting the materials $[h_{jo}]$ ([·] stands for the ceiling function) which are shown in detail in Tables 4, 5. In Fig. 5, required periods for transportation in the critical path was not minimized enough and it takes 2 periods to transport the material of job 5 from the 3rd operation (performed by machine 5) to the 4th operation (performed by machine 6). On the other hand, in Fig. 6, all the required periods for transportation in the critical path were minimized and the shorter makespan was achieved as a result.

Table 4 Required period for transportation $[h_{jo}]$ obtained by the conventional method

| Job | Operation $o$ |
|-----|---------------|
| $j$ | 1→2 | 2→3 | 3→4 | 4→5 | 5→6 | 6→7 |
| 1   | 1   | 1   | 1   | 1   | 2   |     |
| 2   | 1   | 1   | 1   | 1   | 1   |     |
| 3   | 1   | 1   | 1   | 2   | 1   |     |
| 4   | 1   | 2   | 1   | 1   | 1   |     |
| 5   | 1   | 1   | 2   | 1   | 1   | 1   |

Table 5 Required period for transportation $[h_{jo}]$ obtained by the proposed method

| Job | Operation $o$ |
|-----|---------------|
| $j$ | 1→2 | 2→3 | 3→4 | 4→5 | 5→6 | 6→7 |
| 1   | 1   | 1   | 1   | 1   | 1   |     |
| 2   | 1   | 1   | 2   | 1   | 1   |     |
| 3   | 1   | 1   | 2   | 2   | 1   |     |
| 4   | 1   | 1   | 1   | 1   | 1   |     |
| 5   | 1   | 1   | 1   | 1   | 1   | 1   |

5. Conclusion

In this paper, a method for FLP with continuous representation considering temporal efficiency has been presented. FLP has been formulated as an MIP, by which detailed position and size of facilities can be taken into consideration, and production scheduling as a 0-1 integer programming, respectively. Assumption of constant transporting speed correlated those two problems, and then FLP considering temporal efficiency has been formulated as an MIP. An example showed that the proposed method generates the layout which includes information on detailed position and size of facilities, so that required transportation times in the critical path of the schedule are minimized enough and achieves better temporal efficiency than the conventional method in which the above two problems are solved individually.

Although the potential of the proposed method was proven, there are two problems to be resolved in future works. One is concerned with formulation. This method is based on the formulation of the production scheduling problem as a 0-1 integer programming. In this formulation, it is necessary to define a variable for each period in the planning period. Therefore, setting the unit time smaller and dividing the planning period into smaller periods for higher accuracy make the number of the variables huge, and it is impossible to solve the optimization problem, though it has been possible to solve a 0-1 integer programming problem in a reasonably short time because of recent advances in computer technology and solvers. This point also makes it impossible to apply the proposed method to actual manufacturing where a large number of jobs and operations are required. This problem may be resolved by applying another formulation of production scheduling such as all integer programming or MIP to this method. The other problem is concerned with definition of distance among facilities. In this paper, distance among facilities has been evaluated based on the Manhattan distance. This is not reasonable from the point of view of actual manufacturing where materials are transported on complicated routes, and distance should be evaluated considering traveling routes. These two points are the reasons why the improvement of temporal efficiency in the example was small, and resolving these two problems in future works would prove effectiveness of the proposed method more apparently.

In addition, it is also necessary to enhance this method from the point of view of robustness for taking various situation into consideration and applying the method to actual manufacturing, as described in Section 1. In robust FLP, the robustness of a layout is typically defined as deviation in the evaluation index for the optimal solution for each product demand scenario or as that the degree of deviation is within a prespecified percentage. These definition will be applied to the evaluation index of the proposed method and methods for solving the robust problem will be studied in future works.

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