1 Introduction

The present workshop is devoted to ways in which the bewildering pattern of fermion masses and mixings might be understood. The purpose of this talk is to describe the allowed parameter space of mixings and how it may be expected to shrink as a result of improved experiments and theory. A more detailed account may be found in Ref. [1]; we take the opportunity to update some of the numbers presented there. Some of the latest developments since this talk was presented will be mentioned, but are not included in the fits to data.

We set out the frameworks for the discussion in Section 2. The determination of CKM parameters is described in Section 3. A long digression on the top quark is contained in Section 4. Electroweak tests lead primarily to a correlation between the top quark and Higgs masses, as discussed in Section 5. Returning to the CKM matrix itself, we note in Section 6 several ways to obtain improved information on magnitudes and phases of its elements. Among these is the study of $CP$ violation in the decays of neutral $B$ mesons. Recent progress in identifying the flavor of neutral $B$ mesons is reported in Section 7. We conclude in Section 8.

1Invited talk published in Proceedings of the 2nd IFT Workshop on Yukawa Couplings and the Origins of Mass, Gainesville, FL, 11-13 February 1994, edited by P. Ramond (International Press, 1996), pp. 273–293.
2 Frameworks

2.1 The CKM Matrix

The weak charge-changing interactions lead primarily to the transitions \( u \leftrightarrow d, \ c \leftrightarrow s, \ t \leftrightarrow b \) between left-handed quarks \( (u, \ c, \ t) \) of charge \( 2/3 \) and those \( (d, \ s, \ b) \) of charge \(-1/3\). However, as noted by Cabibbo \[2\] and Glashow-Iliopoulos-Maiani \[3\] for two families of quarks and by Kobayashi and Maskawa \[4\] for three, additional transitions of lesser strength can be incorporated into this framework in a universal manner. The charge-changing transitions then connect \( u, \ c, \ t \) not with \( d, \ s, \ b \) but with a rotated set \( (d', \ s', \ b') = V(d, \ s, \ b) \), where \( V \) is a unitary \( 3 \times 3 \) matrix now known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The elements of \( V \) are as mysterious as the quark masses, and are intimately connected with them since the matrix arises as a result of diagonalization of the quark mass matrices (see, e.g., Ref. \[1\]). Moreover, the phases in the matrix are candidates for the source of \( CP \) violation as observed in the decays of neutral kaons. We shall assume that to be the case in the present analysis.

2.2 Precise electroweak tests

The top quark plays an indirect role in the extraction of CKM parameters from data on \( B - \bar{B} \) mixing and \( CP \)-violating \( K \bar{K} \) mixing. Thus, it is important to know its mass. At this time this talk was given, the best source of information on the top quark was its indirect effects on the \( W \) and \( Z \) bosons’ self-energies. Some updated information may be found at the end of Section 4.

In addition to diagrams involving top quarks, \( W \) and \( Z \) self-energies can be affected by Higgs bosons and by various new particles which can appear in loop diagrams. In conjunction with measurement of the top quark mass, precise electroweak tests then will be able to shed first light on these contributions.

3 Determination of CKM parameters

We turn now to a description of the CKM matrix elements. More details on the measurement of the elements \( V_{cb} \) and \( V_{ub} \) may be found in Refs. \[5, 6\].

3.1 Parametrization

We adopt a convention in which quark phases are chosen \[7\] so that the diagonal elements and the elements just above the diagonal are real and positive. The parametrization we shall introduce and employ is one suggested by Wolfenstein \[8\].

The diagonal elements of \( V \) are nearly 1, while the dominant off-diagonal elements are \( V_{us} \approx -V_{cd} \approx \sin \theta \equiv \lambda \approx 0.22 \). Thus to order \( \lambda^2 \), the upper \( 2 \times 2 \) submatrix of \( V \) is
already known from the Cabibbo-GIM four-quark pattern:

\[
V \approx \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]  

(1)

The empirical observation that \( V_{cb} \approx 0.04 \) allows one to express it as \( A\lambda^2 \), where \( A = \mathcal{O}(1) \). Unitarity then requires \( V_{ts} \approx -A\lambda^2 \) as long as \( V_{td} \) and \( V_{ub} \) are small enough (which they are). Finally, \( V_{ub} \) appears to be of order \( A\lambda^3 \times O(1) \). Here one must allow for a phase, so one must write \( V_{ub} = A\lambda^3(\rho - i\eta) \). Finally, unitarity specifies uniquely the form \( V_{td} = A\lambda^3(1 - \rho - i\eta) \). To summarize, the CKM matrix may be written

\[
V \approx \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
\]  

(2)

We shall note below that \( V_{cb} = 0.038 \pm 0.005 \), so that \( A = 0.79 \pm 0.09 \). The measurement of semileptonic charmless \( B \) decays \([9, 10]\) gives \( |V_{ub}/V_{cb}| \) in the range from 0.05 to 0.11, where most of the uncertainty is associated with the spread in models \([11, 12]\) for the lepton spectra. Taking 0.08 ± 0.03 for this ratio, we find that the corresponding constraint on \( \rho \) and \( \eta \) is \((\rho^2 + \eta^2)^{1/2} = 0.36 \pm 0.14\).

The form (2) is only correct to order \( \lambda^3 \) in the matrix elements. For certain purposes it may be necessary to exhibit corrections of higher order to the elements.

The unitarity of \( V \) implies that the scalar product of any row and the complex conjugate of any other row, or of any column and the complex conjugate of any other column, will be zero. In particular, taking account of the fact that \( V_{ud} \) and \( V_{tb} \) are close to 1, we have

\[
V_{ub}^* + V_{td} \approx A\lambda^3
\]

or, to the order of interest in small parameters,

\[
\rho + i\eta + (1 - \rho - i\eta) = 1.
\]

(3)

(4)

The point \((\rho, \eta)\) forms the apex of a triangle in the complex plane, whose other vertices are the points \((0, 0)\) and \((1, 0)\). This “unitarity triangle” \([3]\) and its angles are depicted in Fig. 1.

The main indeterminacy in the CKM matrix concerns the magnitude of \( V_{td} \), for which only indirect evidence exists. Correspondingly, we are still quite uncertain about Arg \( V_{ub}^* = \arctan(\eta/\rho) \). Most of our effort will be devoted to seeing how these quantities can be pinned down better.

### 3.2 Measuring the Cabibbo-GIM submatrix

The elements of the \( 2 \times 2 \) submatrix connecting the quarks \( u, c \) of charge \( 2/3 \) with those \( (d, s) \) of charge \( -1/3 \) are described satisfactorily by the single parameter \( \lambda \) in the form \([3]\). The only lingering question is whether \( |V_{ud}|^2 + |V_{us}|^2 \) really is 1 (up to corrections of order \( |V_{ub}|^2 \), which are negligible), and present data appear to be consistent with this \([4, 14]\).
3.3 Measuring $V_{cb}$

In this subsection and the next we give a cartoon version of a discussion which is set forth much more completely in Ref. [6].

The decay of a $b$ quark to a charmed quark $c$ and a lepton pair offers the best hope for determining $V_{cb}$. One would use the $b$ quark lifetime and the branching ratio for the process $b \to c\ell\nu$ to estimate the rate for the process, which would then be proportional to a known kinematic factor times $|V_{cb}|^2$. Even if $b$ and $c$ were free, we would have to know their masses accurately in order to make a useful estimate.

Since the $b$ quark and charmed quark are incorporated into hadrons such as a $B$ meson and a $D$ meson, the problem becomes one of estimating hadronic effects. There are several ways to do this.

3.3.1 Free quarks

A good deal of indeterminacy of the rate for $b \to c\ell\bar{\nu}_\ell$ is associated merely with uncertainty in quark masses. However, the uncertainty in the predicted decay rate can be reduced by constraints on the mass difference $m_b - m_c$ from hadron spectroscopy [15, 16]. Taking $m_b = 5.0 \pm 0.3$ GeV/$c^2$, $m_b - m_c$ ranging from 3.34 to 3.40 GeV/$c^2$, $B(B \to \text{charm} + \ell + \bar{\nu}_\ell) = 10.5\%$, and $\tau_B = 1.49$ ps, we obtained $V_{cb} = 0.038 \pm 0.003$.

3.3.2 Free quarks and QCD

One can take account of the effects of the light quarks by means of Fermi momentum and can apply QCD corrections to the decay of the free $b$ quark [11]. The result should be an average over the excitation of individual final states of the charmed quark and the spectator antiquark.

3.3.3 Models for final states

One can calculate $B$ semileptonic decay rates to specific final states, such as $D\ell\bar{\nu}_\ell$, $D^*\ell\bar{\nu}_\ell$, and so on [12]. It is then necessary to include all relevant states, so an important question is what charmed states besides $D$ and $D^*$ play a role.
3.3.4 Use of the “zero-recoil” point

When the lepton pair has its maximum invariant mass, the $b$ quark decays to a charmed quark without causing it to recoil [17]. Thus, hadronic effects are kept to a minimum. The limitation on this method is mainly one of statistics at present.

3.3.5 Averages

When the various methods are combined, one gets an idea of the spread in theoretical approaches. In Ref. [1] we quoted the value $V_{cb} = 0.038 \pm 0.005$ obtained in Ref. [5] on the basis of such averages. That is the value which we will use in the present analysis, corresponding to $A = 0.79 \pm 0.09$. More recently Stone [6] estimates $V_{cb} = 0.038 \pm 0.003$, in accord with our original free-quark value and corresponding to an error $\Delta A = 0.06$.

3.4 Measuring $V_{ub}$

In order to see the effects of the process $b \rightarrow u\ell\bar{\nu}_{\ell}$, one has to study leptons beyond the end point for charm production. As a result, one sees only a very small part of the total phase space for the process of interest. The question then becomes one of how the decay populates this small region of phase space. The final $u$ quark can combine with the initial $\bar{u}$ or $\bar{d}$ in the decaying $B$ meson to form a nonstrange hadron such as $\pi$, $\rho$, $a_1$, .... One can either describe this recombination in an average sense [11, 18] or employ models for excitation of individual resonances [12].

The range of theoretical approaches allows values of $|V_{ub}/V_{cb}|$ between 0.05 and about 0.11 when the more recent CLEO data are used [10]. Somewhat larger values (up to a factor of 2, in some models) are implied by earlier ARGUS data [9]. These results correspond to a fraction of $b$ decays without charmed particles of between 1 and 2%. For present purposes we shall take $|V_{ub}/V_{cb}| = 0.08 \pm 0.03$. For comparison, Stone [6] quotes $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$.

3.5 Arg $V_{ub}$

The phase of $V_{ub}$ is one of the least well known parameters of the CKM matrix. For it, we must rely upon indirect information.

3.5.1 $B^0 - \bar{B}^0$ mixing

The original evidence for $B^0 - \bar{B}^0$ mixing came from the observation [19] of “wrong-sign” leptons in $B$ meson semileptonic decays. The diagrams of Fig. 2 give rise to a splitting between mass eigenstates

$$\Delta m \sim f_B^2 |m_t|^2 |V_{td}|^2$$

(5)
times a slowly varying function of $m_t$. (See, e.g., Ref. [1] for detailed expressions.) Here $f_B$ is the “$B$ meson decay constant,” which expresses the overlap of a $b\bar{q}$ state at zero separation with the wave function of the $B$ meson. Information on $f_B$ is improving, but still is a major source of indeterminacy. The top quark mass $m_t$ is becoming better
known, as we shall see at the end of Section 4. The CKM element $V_{td}$ has a magnitude proportional to $|1 - \rho - i\eta|$, which is what we would like to learn.

The average value of data used for the present analysis gives $\Delta m/\Gamma = 0.66 \pm 0.10$. The resulting constraint on $(\rho, \eta)$ for fixed values of $f_B$ and $m_t$ is a circular band with radius approximately 1 and center at the point (1,0). Uncertainty in $f_B$ and, to a lesser extent, $m_t$, is a source of spread in this band, whose shape is illustrated by the dashed arcs in Fig. 3.
Figure 5: Contours of 68% (inner curve) and 90% (outer curve) confidence levels for regions in the \((\rho, \eta)\) plane. Dotted semicircles denote central value and \(\pm 1\sigma\) limits implied by \(|V_{ub}/V_{cb}| = 0.08 \pm 0.03\). Plotted point corresponds to minimum \(\chi^2 = 0.17\), while (dashed, solid) curves correspond to \(\Delta \chi^2 = (2.3, 4.6)\).

3.5.2 \(CP\)-violating \(K^0 - \bar{K}^0\) mixing

The box diagrams shown in Fig. 4 give rise to a \(CP\)-violating term in the matrix element between a \(K^0\) and a \(\bar{K}^0\),

\[
\text{Im} \mathcal{M}_{12} \sim f_K^2 \text{Im} \ (V_{td}^2) \sim \eta(1 - \rho) \ ,
\]

so that the constraint in the \((\rho, \eta)\) plane is a band bounded by hyperbolae with focus at the point (1,0), as illustrated by the example of the solid lines in Fig. 3. Here, again, the top quark mass enters. There are small corrections (not completely negligible) from charmed quarks in the loop. Neglecting these, however, one can take the quotient of the constraints (5) and (6) to find a constraint on \(\text{Arg} \ V_{td}\). As we shall see, such a constraint is useful in predicting the expected asymmetry in certain \(CP\)-violating decays of \(B\) mesons.

3.6 Allowed region of parameters

When the constraints of Eqs. (5) and (6) are combined with that on \(|V_{ub}/V_{cb}|\) [shown as the circular band bounded by the dotted arcs in Fig. 3], one gets the allowed region of parameters shown in Fig. 5 and described by the first line in Table 1.

The parameters taken for the present analysis are those chosen in Ref. [1], and include the choices \(m_t = 160 \pm 30\) GeV, \(f_B = 180 \pm 30\) MeV, \(|V_{ub}/V_{cb}| = 0.08 \pm 0.03\), and \(A = 0.785 \pm 0.093\). The allowed region at 90% c.l. has \(-0.4 \leq \rho \leq 0.5\) for \(\eta \approx 0.3\),
Table 1: Effects of changing parameters in fits to CKM matrix elements from “nominal” values described in text. In all fits there is one degree of freedom

| Parameters   | $\rho$ ($\chi^2_{\text{min}}$) | $\eta$ ($\chi^2_{\text{min}}$) | $\chi^2_{\text{min}}$ | $\rho$ range $^a$ |
|--------------|-------------------------------|-------------------------------|------------------------|------------------|
| “Nominal”    | 0.13                          | 0.37                          | 0.17                   | $-0.37$ to $0.47$ |
| $m_t = 160$ GeV | 0.13                          | 0.37                          | 0.17                   | $-0.31$ to $0.45$ |
| $m_t = 190$ GeV | 0.22                          | 0.34                          | 0.25                   | $-0.19$ to $0.50$ |
| $B_K = 0.80 \pm 0.02$ $^b$ | 0.13                          | 0.38                          | 0.23                   | $-0.28$ to $0.45$ |
| $V_{cb} = 0.038 \pm 0.002$ | 0.11                          | 0.38                          | 0.20                   | $-0.37$ to $0.45$ |
| $f_B = 180 \pm 10$ MeV | 0.15                          | 0.37                          | 0.22                   | $-0.28$ to $0.43$ |

$^a$ bounded by solid lines in Fig. 4

$^b$ $m_t$ fixed at 160 GeV

while for $\rho \simeq 0$ one has $0.2 \leq \eta \leq 0.6$. For a broad range of parameters, CKM phases can describe $CP$ violation in the kaon system. The question is whether this explanation of the observed $CP$ violation is the correct one. A partial answer may be obtained by acquiring improved information about the top quark mass or about CKM elements. (See also Refs. [21, 22].)

The choice of $m_t$ mentioned above was based on an analysis of electroweak data parallel to that presented in Ref. [23] and reaching the same conclusions. We shall give more details in Section 5. The results of fixing the top quark mass at 160 or 190 GeV are shown in Table 1. The allowed region is shrunk only slightly, and there is not much difference between the two cases. The favored value of $\rho$ increases by 0.03 for each 10 GeV increase in $m_t$.

A parameter known as $B_K$ describes the degree to which the diagrams of Fig. 4 actually dominate the $CP$-violating $K^0 - \bar{K}^0$ mixing. In the fits described so far we took the nominal value of $B_K = 0.8 \pm 0.2$. If we take $m_t = 160$ GeV/c$^2$ and reduce the error on $B_K$ to 0.02, we obtain the result shown in Table 1. The reduced errors on $B_K$ are clearly not of much help. The major errors remaining are those of $f_B$, $V_{cb}$, and $|V_{ub}/V_{cb}|$.

We next tried reducing the error on $V_{cb}$ to 0.002, keeping other parameters as in the original fit. Again, there is little shrinkage of the allowed parameter space. Reduction of the error on $f_B$ from 30 to 10 MeV helps a little. The results of these exercises are shown in Table 1. The shapes of the allowed regions change very little. The conclusion is that one needs simultaneous reduction in the errors of several observables to significantly narrow down the range of CKM parameters. We explore these possibilities in Section 6.

First, however, we concentrate on the top quark.

### 4 The top quark
4.1 Indirect evidence

Indirect evidence for the top quark has been around for a long time. The neutral-current couplings of the $b$ quark (both flavor-conserving and the absence of flavor-violating ones) have persuaded us that the left-handed $b$ quark is a member of a doublet ($t,b$)$_L$ of weak SU(2), while the right-handed $b$ is a singlet of weak SU(2) [24].

The expectation that the top quark is relatively heavy is more recent. A value of $m_t$ of at least 70 GeV was needed in order to understand the unexpectedly large magnitude of $B^0 - \bar{B}^0$ mixing [19]. Even a higher lower bound is required to understand the size of $CP$-violating $K^0 - \bar{K}^0$ mixing [25]. The branching ratio of the $W$ to (lepton) + (neutrino) of about 1/9 is compatible with there being no contribution from $W \rightarrow t + \bar{b}$, indicating that $m_t > M_W - m_b$.

An upper limit on the top quark mass is provided by its effects on $W$ and $Z$ self-energies. In the lowest-order electroweak theory, a measurement of the $Z$ mass implies a specific value of $M_W$. The $W$ and $Z$ self-energies are affected by top quark and Higgs masses, so that now $M_W/M_Z = f(m_t, m_{Higgs})$. This function is quadratic in $m_t$ but only logarithmic in $M_{Higgs}$. When the Higgs boson mass is allowed to range up to 1 TeV (above which the theory should dynamically generate a mass of 1 – 2 TeV in any case), the observed values of $M_W$ and of many other electroweak observables allow one to conclude that $m_t \leq 200$ GeV/$c^2$.

4.2 Direct searches

The signature for top quark pair production in $\bar{p}p$ collisions is the simultaneous decay $t \rightarrow W^+ + b$, $\bar{t} \rightarrow W^- + \bar{b}$. One channel with little background involves the decay of one $W$ to $e\nu$ and the other to $\mu\nu$. As of this workshop, the CDF Collaboration had identified two $e\mu$ candidates and the D0 Collaboration had observed one. On this basis, all that were quoted were lower limits on the top quark mass. Using a sample in which hadronic decays of one of the two $W$’s were also searched for, D0 quoted a lower limit [26] of 131 GeV/$c^2$.

4.3 Postscript: evidence

Since this workshop, the CDF Collaboration has presented evidence for the production of a top quark [27] with $m_t = 174 \pm 10 \pm 13$ GeV/$c^2$. The cases we chose of $m_t = 160$ and 190 GeV/$c^2$ are compatible with this value. The main impact of this measurement is felt less on the determination of CKM parameters than on the interpretation of electroweak results, which we discuss next.

5 Impact of electroweak tests

In Fig. 4 we show the electroweak prediction for $M_W$ as a function of $m_t$ for various values of Higgs boson mass $M_H$. Also shown is the latest 1σ range of $M_W$, corresponding to the average over many experiments [28]. (The latest results have been presented by the CDF and D0 collaborations.) Even without a direct observation, one sees the upper bound
of about 200 GeV/c² quite clearly. The recent (post-workshop) observation corresponds to a data point lying in the allowed range. Greater precision on both $M_W$ and $m_t$ will be needed to distinguish among possibilities for Higgs boson masses.

A fit to the electroweak observables cited in Table 2 has been performed. In each case a prediction is made for the “nominal” values of $m_t = 140$ GeV and $M_H = 100$ GeV/c². The Higgs boson mass is held fixed, while the top quark mass is allowed to vary in such a way as to minimize the $\chi^2$ of the fit. The results are shown in Fig. 7 and Table 3.

A slight preference is shown for a light Higgs boson. This is driven in part by the low value of $\sin^2 \theta_W^{\text{eff}}$ obtained at SLC/SLD.

The range of top quark masses we chose to discuss at the workshop is compatible both with the results of Table 3 and with the recently announced observation. Present errors on $m_t$ do not allow a conclusion to be drawn yet about the Higgs boson mass.

6 Improved CKM Information

6.1 Meson decay constants

As we mentioned earlier, uncertainty in $f_B$ is a major source of indeterminacy in extracting $|V_{td}|$ from $B^0 - \bar{B}^0$ mixing. Early compilations of predictions are contained in Refs. 37. More recent information on meson decay constants has been provided by
Figure 7: Behavior of $\chi^2$ as function of top quark mass for Higgs boson masses of 100, 300, and 1000 GeV (labels on curves)

Table 2: Electroweak observables described in fit

| Quantity       | Experimental value | Nominal value | Experiment/Nominal |
|----------------|--------------------|---------------|--------------------|
| $Q_W$ (Cs)     | $-71.0 \pm 1.8$ a) | $-73.2$ b)   | $0.970 \pm 0.025$   |
| $M_W$ (GeV/c²) | $80.22 \pm 0.14$ c) | $80.174$ d)  | $1.001 \pm 0.002$  |
| $\Gamma_{\ell}(Z)$ (MeV) | $83.82 \pm 0.27 e)   | $83.6$ f)   | $1.003 \pm 0.003$  |
| $\Gamma_{\text{tot}}(Z)$ (MeV) | $2489 \pm 7$ e)    | $2488 \pm 6$ f) | $1.000 \pm 0.004$ |
| $\sin^2 \theta^\text{eff}_W$ | $0.2318 \pm 0.0008$ g) | $0.2322$ f) | $0.998 \pm 0.003$ |
| $\sin^2 \theta^\text{eff}_W$ | $0.232 \pm 0.009$ h) | $0.2322$ | $0.999 \pm 0.039$ |
| $\sin^2 \theta^\text{eff}_W$ | $0.2287 \pm 0.0010$ i) | $0.2322$ | $0.985 \pm 0.004$ |

a) Weak charge in cesium. From Ref. 29
b) From Ref. 30, incorporating corrections of Ref. 31
c) Average of direct measurements from Ref. 28 and indirect information from neutral/charged current ratio in deep inelastic neutrino scattering 32
d) As calculated in Ref. 33
e) LEP average as of August, 1993 23
f) As calculated in Ref. 30
g) From asymmetries at LEP, containing corrections of Ref. 34
h) From $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering 33
i) From left-right asymmetry in annihilations at SLC 36, containing corrections of Ref. 34
lattice gauge theory \cite{38}, QCD sum rules \cite{39}, and direct quark-model calculations \cite{40} which make use of spin-dependent electromagnetic mass splittings in charmed and $B$ mesons to estimate the wave function of a light-heavy system at zero interquark separation. One can expect the reliability of the lattice and QCD sum rule calculations to improve as they are tested on a wide range of properties of charmed and $b$-flavored hadrons, while the quark model estimate would be helped by a precision measurement of isospin splittings in $B$ and $B^*$ mesons. Modest improvements of recent measurements of the decay constant $f_{_{D_s}}$ \cite{41} will allow one to check these schemes.

6.2 Rare kaon decays

Several rare kaon decays can provide information on CKM parameters. Here we discuss the decays of kaons to a pion and a lepton pair.

The rate for the process $K^+ \to \pi^+ \nu \bar{\nu}$ (summed over neutrino species) is sensitive to $|V_{td}|^2$. Very roughly, for a nominal top quark mass of 140 GeV/$c^2$, a branching ratio of less than $10^{-10}$ favors $\rho > 0$ while a branching ratio of greater than $10^{-10}$ favors $\rho < 0$. The branching ratio is an increasing function of top quark mass. Details have been given in Refs. \cite{1, 21}.

The present experimental limit \cite{42}, $B(K^+ \to \pi \nu \bar{\nu}) < 5 \times 10^{-9}$ (90\% c.l.) is a factor of 50 above the expected level, but further improvements in data collection are foreseen.

[Postscript: Here information on $m_t$ is very welcome; the predictions were quoted at the workshop for $m_t = 100$, 140, and 180 GeV/$c^2$. An updated set of predictions may be found in Ref. \cite{22}. It now seems more likely that the branching ratio will be at least $10^{-10}$.

The decays $K_L \to \pi^0 \ell^+ \ell^−$ are expected to proceed mainly through $CP$ violation \cite{13}, while key $CP$-conserving backgrounds to this process (see, e.g., Ref. \cite{14}) are absent in $K_L \to \pi^0 \nu \bar{\nu}$. Present 90\% c.l. upper limits on the branching ratios for these processes are shown in Table 4. The expected branching ratios \cite{13} are about $10^{-11}$.

6.3 $CP$ violation in decays of neutral kaons

One can search for a difference between the $CP$-violation parameters $\eta_{\pi^-} = \epsilon + \epsilon'$ and $\eta_{\pi^0} = \epsilon - 2\epsilon'$ in the decays of neutral kaons to pairs of charged and neutral pions. A non-zero value of $\epsilon'/\epsilon$ would confirm predictions of the CKM origin of $CP$ violation in the kaon system, and has long been viewed as one of the most promising ways to disprove a “superweak” theory of this effect \cite{51, 52}.
Table 4: Upper limits on branching ratios for decays of neutral kaons to neutral pions and a lepton pair

| Process          | 90% c.l. upper limit | Reference |
|------------------|----------------------|-----------|
| $K_L \rightarrow \pi^0 e^+ e^-$ | $1.8 \times 10^{-9}$ | [46, 47, 48] |
| $K_L \rightarrow \pi^0 \mu^+ \mu^-$ | $5.1 \times 10^{-9}$ | [49] |
| $K_L \rightarrow \pi^0 \nu \bar{\nu}$ | $2.2 \times 10^{-4}$ | [50] |

The latest estimates by A. Buras and collaborators [53] are equivalent to $\left| \epsilon' / \epsilon \right|_{\text{kaons}} = (1/2 \text{ to } 3) \times 10^{-3} \eta$, with smaller values for higher top quark masses. The Fermilab E731 Collaboration [54] measures $\left| \epsilon' / \epsilon \right| = (7.4 \pm 6) \times 10^{-4}$, leading to no restrictions on $\eta$ in comparison with the range (0.2 to 0.6) we have already specified. The CERN NA31 Collaboration [55] finds $\left| \epsilon' / \epsilon \right| = (23 \pm 7) \times 10^{-4}$, consistent only with $\eta > \sim 1/2$ and a light top quark. [Postscript: this scenario now appears less likely in view of the result of Ref. [27].] Both groups are preparing new experiments, for which results should be available around 1996.

6.4 Rare $B$ decays

The rate for the purely leptonic process $B \rightarrow \ell \bar{\nu}_\ell$ provides information on the combination $f_B |V_{ub}|$. One expects a branching ratio of about $10^{-4}$ for $\tau \bar{\nu}_\tau$ and $(1/2) \times 10^{-6}$ for $\mu \bar{\nu}_\mu$. A suggestion was made [56] for eliminating $f_B$ by comparing the $B \rightarrow \ell \bar{\nu}_\ell$ rate with the $B^0 - \bar{B}^0$ mixing amplitude, and thereby measuring the ratio $|V_{ub}/V_{td}|$ directly. While such a measurement is unlikely to tell whether the unitarity triangle has nonzero area (and thus whether the CKM phase is the origin of $CP$ violation in the kaon system), it can help resolve ambiguity regarding the value of $\rho$.

Another interesting ratio [57] is the quantity $\Gamma(B \rightarrow \rho \gamma)/\Gamma(B \rightarrow K^* \gamma)$, which, aside from small phase space corrections, should just be $|V_{td}/V_{ts}|^2 \simeq 1/20$.

6.5 $B_s - \bar{B}_s$ mixing

The mixing of $B_s$ and $\bar{B}_s$ via diagrams similar to those in Fig. 2 involves the combination $f_{B_s}^2 |V_{ts}|^2$ instead of $f_B^2 |V_{td}|^2$. Since we expect $|V_{ts}| \approx |V_{cb}| \approx 0.04$, the main uncertainties in $x_s \equiv (\Delta m / \Gamma)|\_{B_s}$ are associated with $f_{B_s}$ and $m_\tau$. A range of 10 to 50 is possible for this quantity. Alternatively, one can estimate the ratio $f_{B_s}/f_B$ using models for SU(3) symmetry breaking, and one finds [1] $x_s = (19 \pm 4)/[(1 - \rho)^2 + \eta^2]$. With the values of $\rho$ and $\eta$ suggested by present fits to data, the most likely value of $x_s$ seems to be around 20. This corresponds to many oscillations between $B_s$ and $\bar{B}_s$ over the course of a $B_s$ lifetime (about 1.5 ps), and represents a strong experimental challenge.
Figure 8: Contours of ratios of asymmetries \( R \equiv A(\pi^+\pi^-)/A(J/\psi K_S) = \frac{\sin(2\alpha)}{\sin(2\beta)} \) (labels on curves) in the \((\rho, \eta)\) plane

### 6.6 \textit{CP} violation in \(B\) systems

Asymmetries in the rates for certain decays of \(B\) mesons can provide direct information about the angles in the unitarity triangle of Fig. 1. These decays involve final states which are eigenstates of \(CP\), so that they can be reached both from an initial \(B^0\) and from an initial \(\bar{B}^0\).

We may define a time-integrated rate asymmetry \(A(f)\) as

\[
A(f) \equiv \frac{\Gamma(B_{t=0}^0 \to f) - \Gamma(\bar{B}_{t=0}^0 \to f)}{\Gamma(\bar{B}_{t=0}^0 \to f) + \Gamma(B_{t=0}^0 \to f)} .
\]  

The angles \(\beta\) and \(\alpha\) in Fig. 1 are related to the asymmetries in decays to \(J/\psi K_S\) and \(\pi^+\pi^-\) final states:

\[
A(J/\psi K_S) = -\frac{x_d}{1 + x_d^2} \sin 2\beta ,
\]

\[
A(\pi^+\pi^-) = -\frac{x_d}{1 + x_d^2} \sin 2\alpha ,
\]

where \(x_d \equiv (\Delta m/\Gamma)|_{B^0}\), and we have neglected lifetime differences between eigenstates. Contours of the expected values of these asymmetries have been quoted in Refs. [1, 21] and will not be reproduced here. The ratios of these asymmetries can be useful in cancelling certain common (and sometimes hard-to-estimate) “dilution factors” associated with identification of the flavor of the decaying neutral \(B\) meson. Contours \[56\] of the ratio \(R \equiv \frac{\sin 2\alpha}{\sin 2\beta}\) are shown in Fig. 8. As long as \(\rho^2 + \eta^2 < 1\), which certainly is true, a value of \(R > 1\) signifies \(\rho < 0\), while \(R < 1\) signifies \(\rho > 0\).
7 Recent results on tagging neutral B mesons

7.1 Why neutral B mesons?

The observation of a $CP$-violating asymmetry between the rate for a process and its charge-conjugate requires some sort of interference. Two examples serve to illustrate the major possibilities [58].

7.1.1 Self-tagging modes

The rates for such processes as $B^+ \rightarrow K^+\pi^0$ and $B^- \rightarrow K^-\pi^0$ can differ from one another. Under charge-conjugation, weak phases change sign but strong phases do not. One can see a $CP$-violating rate difference, but only if strong phases differ in the $I = 1/2$ and $I = 3/2$ channels. Interpretation of an effect requires knowledge of this final-state phase difference. [Postscript: with the help of SU(3) symmetry and some simplifying assumptions, it is possible to extract CKM phases from rates of self-tagging modes alone [59].]

7.1.2 Decays to a $CP$ eigenstate

Final-state phase information is not needed if one compares the rates for a state which is produced as a $B^0$ and a state which is produced as a $\bar{B}^0$ to decay to an eigenstate $f$ of $CP$. The relevant interference leading to a rate asymmetry occurs between amplitudes for decay and $B^0 - \bar{B}^0$ mixing. As mentioned above, decay rate asymmetries can directly probe the angles of the unitarity triangle, as long as a single amplitude contributes to each transition $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$. To make use of this method, one must identify the flavor of the decaying particle at the time of production: was it a $B^0$ or a $\bar{B}^0$?

7.2 Identifying neutral B’s

7.2.1 At the $\Upsilon(4S)$ resonance

A peak in the cross section for $e^+e^- \rightarrow B^0\bar{B}^0$ occurs just above threshold at the $\Upsilon(4S)$ resonance. If one “tags” the flavor of the decaying state by observing the semileptonic decay of the “other” $B$, the existence of $B^0 - \bar{B}^0$ mixing and the correlation of the $B^0$ and $\bar{B}^0$ in a state of negative charge-conjugation lead to an asymmetry proportional to $\sin(t_1 - t_2)$, where $t_1$ is the proper time of the decay to the $CP$ eigenstate and $t_2$ is the proper time of the tagging decay. This asymmetry vanishes when integrated over all decay times, so one needs information on $t_1 - t_2$ such as might be provided by an asymmetric $B$ factory.

7.2.2 Away from the $\Upsilon(4S)$ resonance

In any reaction in which a $b\bar{b}$ pair is produced at high energy, such as a hadronic collision or the decay of the $Z^0$, the flavor of a neutral $B$ meson decaying to a $CP$ eigenstate can be tagged by looking at the flavor of the $b$-flavored particle produced in association
Figure 9: Quark graphs describing correlation between flavor of neutral $B$ meson and charge of leading pion in fragmentation

with it. Such a particle might be another neutral nonstrange or strange $B$ (in which case mixing would cause a dilution of tagging efficiency), or it could be a charged $B$ or a $b$-flavored baryon. Here one has to find the tagging particle among the debris of the collision, and estimates of tagging efficiency are likely to be model-dependent.

7.2.3 Tagging using associated hadrons

A neutral $B$ meson produced in a high-energy collision is likely to be accompanied by other hadrons as a result of the fragmentation of the initial quark or as a result of cascades from higher resonances. This feature could be useful for tagging the flavor of a produced $B$ meson \[60, 61, 62\].

The correlation is easily visualized with the help of the quark diagrams shown in Fig. 9. By convention (the same as for kaons), a neutral $B$ meson containing an initially produced $\bar{b}$ is a $B^0$. It also contains a $d$ quark. The next charged pion down the fragmentation chain must contain a $\bar{d}$, and hence must be a $\pi^+$. Similarly, a $\bar{B}^0$ will be correlated with a $\pi^-$. The same conclusion can be drawn by noting that a $B^0$ can resonate with a positive pion to form an excited $B^+$, which we shall call $B^{**+}$ (to distinguish it from the $B^*$, lying less than 50 MeV/$c^2$ above the $B$). Similarly, a $\bar{B}^0$ can resonate with a negative pion to form a $B^{**-}$. The combinations $B^0\pi^-$ and $\bar{B}^0\pi^+$ are exotic, i.e., they cannot be formed as quark-antiquark states. No evidence for exotic resonances exists.

7.3 Results from simulation

We have asked the authors of a fragmentation Monte Carlo program to see if the correlation between pions and neutral $B$ mesons is evident in their work. The result, based on $10^6$ events generated using ARIADNE and JETSET \[63\], shows a slight excess of the “right-sign” combinations $B^0\pi^+$ over the “wrong-sign” combinations $B^0\pi^-$. The ratio $(\text{right} - \text{wrong})/(\text{right} + \text{wrong})$ varies from 0.17 for $M(B\pi) = 5.5$ GeV/$c^2$ to 0.27 for $M(B\pi) = 5.8 - 6.2$ GeV/$c^2$ (where there are fewer events). No explicit resonances were put into the simulation; their inclusion would strengthen the correlation.
Table 5: P-wave resonances of a $b$ quark and a light ($\bar{u}$ or $\bar{d}$) antiquark

| $J^P$ | Mass (GeV/$c^2$) | Allowed final state(s) |
|-------|------------------|------------------------|
| $2^+$ | $\sim 5.77$     | $B\pi, B^*\pi$        |
| $1^+$ | $\sim 5.77$     | $B^*\pi$              |
| $1^+$ | $< 5.77$        | $B^*\pi$              |
| $0^+$ | $< 5.77$        | $B\pi$                |

7.4 $B^{**}$ resonances

The existence of a soft pion in $D^* \to D\pi$ decays [14] has been a key feature in tagging the presence of $D$ mesons since the earliest days of charmed particles. The mass of a $D^*$ is just large enough that the decays $D^* \to D\pi$ can occur (except in the case of $D^0 \to \pi^-D^+$). In contrast, the $B^*$ is only 46 MeV above the $B$, so it cannot decay via pion emission. The lightest states which can decay to $B\pi$ and/or $B^*\pi$ are P-wave resonances of a $b$ quark and an $\bar{u}$ or $\bar{d}$. The expectations for masses of these states [62, 65], based on extrapolation from the known $D^{**}$ resonances, are summarized in Table 5.

The known $D^{**}$ resonances are a $2^+$ state around 2460 MeV/$c^2$, decaying to $D\pi$ and $D^*\pi$, and a $1^+$ state around 2420 MeV/$c^2$, decaying to $D^*\pi$. These states are relatively narrow, probably because they decay via a D-wave. In addition, there are expected to be much broader (and probably lower) $D^{**}$ resonances: a $1^+$ state decaying to $D^*\pi$ and a $0^+$ state decaying to $D\pi$, both via S-waves.

The expected spectrum of nonstrange charmed meson resonances is shown in Fig. 10 [65, 66], as calculated in the potential of Ref. [67]. For strange states, one should add about 0.1 GeV, while for $B$’s one should add about 3.32 GeV. The predicted narrow $B^{**}$ resonances lie at 5767 MeV ($2^+$) and 5755 MeV ($1^+$). These are the states in which the light quark spin $s = 1/2$ and the orbital angular momentum $L = 1$ combine to form a total light-quark angular momentum $j = 3/2$. One also expects broad $1^+$ and $0^+$ $B^{**}$ states with $j = 1/2$, decaying via S-waves.

7.5 The question of coherence

As mentioned, one expects the $B^0\bar{B}^0$ pair produced at the $\Upsilon(4S)$ resonance to be in a state of charge-conjugation eigenvalue $C = -1$. The particle which decays at a time $t_1$ to a $CP$ eigenstate is then a coherent superposition of $B^0$ and $\bar{B}^0$ when the particle produced in association decays at a time $t_2$ to a tagging final state (e.g., to $D^*e^-\bar{\nu}_e$). The fact that the time-dependent rate asymmetry is an odd function of $t_1 - t_2$ is why one has to stretch out the decay region using asymmetric kinematics.

On the other hand, in the high energy associated production of pairs of $b$-flavored hadrons, one usually assumes no coherence between $B^0$ and $\bar{B}^0$ on one side of the reaction when tagging on the other. Thus, the decaying state is assumed to be an incoherent
mixture of $B^0$ and $\bar{B}^0$ with specific probabilities of each.

M. Gronau and I [61] have proposed a way to test for coherence using a density-matrix formalism. One can measure the elements of the density matrix using time-dependences of appearance of specific final states.

We work in a two-component basis labeled either by $B^0$ and $\bar{B}^0$ or, more conveniently, by mass eigenstates. In this last basis, in which components of the density matrix are labeled by primed quantities, we can denote an arbitrary coherent or incoherent state by the density matrix
\begin{equation}
\rho = \frac{1}{2} (1 + Q' \cdot \sigma) \tag{10}
\end{equation}
where $\sigma_i$ are the Pauli matrices. The intensities for decays to states of identified flavor can be written
\begin{equation}
I \left( \frac{B^0}{\bar{B}^0} \right) = \frac{1}{2} |A|^2 e^{-rt} \left[ 1 \pm Q'_\perp \cos(\Delta mt + \delta) \right] \tag{11}
\end{equation}
where
\begin{equation}
Q'_1 = Q'_\perp \cos \delta \quad , \quad Q'_2 = Q'_\perp \sin \delta \tag{12}
\end{equation}
In order to measure $Q'_3$ one needs also to see decays to $CP$ eigenstates, such as $J/\psi K_S$. One then learns not only the components of the density matrix, but also one of the angles of the unitarity triangle (such as $\beta$). Then, one can look at other final states to learn other angles. For example, if penguin diagrams are not important in the decays $B \rightarrow \pi\pi$, the final state $\pi^+\pi^-$ provides information on the angle $\alpha$. 

Figure 10: Predicted spectrum of nonstrange charmed meson resonances. Observed states are labeled by a check mark.
8 Conclusions

The present knowledge about magnitudes and phases of CKM matrix elements allows lots of “wiggle room” for inventive schemes. Choosing among these schemes will require progress on many fronts. Among these, we have discussed improved knowledge of meson decay constants, decays of neutral kaons, rare $B$ decays, mixing of strange neutral $B$ mesons with their antiparticles, and the observation of $CP$ violation in decays of neutral $B$ mesons. The identification of the initial flavor of a neutral $B$ meson may profit from the study of hadrons produced in association with it, and we have described ways in which our knowledge of such correlations may be improved.

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