Local Black Hole Scaling Relations Imply Compton Thick or Super Eddington Accretion

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ABSTRACT

A recent analysis of black hole scaling relations, used to estimate the local mass density in black holes, has indicated that the normalization of the scaling relations should be increased by approximately a factor of five. The local black hole mass density is connected to the mean radiative efficiency of accretion through the time integral of the quasar volume density. The correspondence between this estimate of the radiative efficiency and that expected theoretically from thin-disk accretion has long been used as an argument that most of the growth in black holes occurs via luminous accretion. The increase of the mass density in black holes pushes the mean observed radiative efficiency to values below that expected for thin-disk accretion for any value of the black hole spin, including retrograde accretion disks. This can be accommodated via black hole growth channels that are intrinsically radiatively inefficient, such as super-Eddington accretion, or via growth channels that are intrinsically radiatively efficient but for which few of the photons are observed, such as Compton thick accretion. Measurements of the 30 keV peak in the X-ray background indicate a significant population of Compton thick sources which can explain some, but not all, of the change in the local black hole mass density. If this result is taken as evidence that super-Eddington accretion is common, it greatly reduces the tension associated with the growth timescale of observed $z = 7$ quasars compared to the age of the universe at that redshift.

Key words: galaxies: active – galaxies: bulges – quasars: supermassive black holes

1 INTRODUCTION

Super-massive black holes at the centers of galaxies have long been known to be closely related to the properties of the galaxies that they inhabit (Richstone et al. 1998; Gebhardt et al. 2000; Ferrarese & Merritt 2000; Tremaine et al. 2002; Haring & Rix 2004; Novak et al. 2004; Gültekin et al. 2009; McConnell & Ma 2013). These scaling relations provide the best estimate of the local mass density in black holes (Yu & Tremaine 2002; Shankar et al. 2009). The radiative efficiency of the accretion process onto super-massive black holes can be estimated by connecting the time integral of the quasar luminosity function and the local black hole mass function. For many years, the mean radiative efficiencies estimated in this way (Solari 1982; Yu & Tremaine 2002; Shankar et al. 2009) corresponded roughly to the expected radiative efficiencies for a thin, radiatively efficient accretion disk (Shakura & Sunyaev 1973; Novikov & Thorne 1973), indicating that most of the mass that falls into black holes over the lifetime of the universe does so in this fashion. This is to be contrasted with the possibility that the flows are radiatively inefficient because the accretion rates are a small fraction of the Eddington rate (Narayan & Yi 1994), radiatively inefficient because the accretion rates are greater than the Eddington rate (Begelman 1979; Abramowicz et al. 1980; Paczynski & Wiita 1980), radiatively efficient but difficult to observe because the accretion is heavily obscured, or that the present mass density in black holes is dominated by the initial mass density of seeds (see Volonteri et al. 2008, for a discussion of black hole seeds). Paczynski (1982) argues that it was this very correspondence between the estimates of the observed mean radiative efficiency and that expected efficiency for luminous radiatively efficient Eddington-limited accretion that led to diminished interest in radiatively inefficient, super-Eddington accretion models for many years.

Analyses of the X-ray background (XRB) have raised the possibility of a significant population of Compton-thick sources. If confirmed, these sources contribute to the local black hole mass density without contributing to the
population of observed active galactic nuclei (AGN). A recent comprehensive analysis of black hole mass measurements and scaling relations (Kormendy & Ho 2013) concluded that the normalization of the black hole mass—bulge mass relation increased from the previously accepted value of $M_{BH} = 0.1\% M_{bulge}$ to $M_{BH} = 0.5\% M_{bulge}$ with a weak dependence on mass. That is to say that the missing black hole mass density implied by the XRB may have been found in the form of revised measurements of local black holes. In fact, the revision to the local black hole mass density may be so large that Compton-thick sources are insufficient to explain it. In this case, radiatively inefficient, possibly super-Eddington accretion flows may be required. Evidence that super-Eddington accretion flows are common would help to alleviate the difficulty in explaining how the observed $z = 7$ quasars (Mortlock et al. 2011) grew to their estimated masses given the short time allowed.

The Soltan (1982) argument can be formulated as a continuity equation for the mass density of black holes in the universe:

$$\rho_\bullet = \rho_0 - \rho_{GW} + \int (\dot{\rho}_{UO} + \dot{\rho}_{OB} + \dot{\rho}_{CT} + \dot{\rho}_{RI}) \ dt$$  \hspace{1cm} (1)$$

where $\rho_\bullet$ is the present day mass density in black holes, $\rho_0$ is the mass density in black hole seeds, $\rho_{GW}$ is the mass equivalent of the energy radiated as gravitational waves during black hole mergers, $\dot{\rho}_{UO}$ refers to unobscured accretion, $\dot{\rho}_{OB}$ to obscured accretion, $\dot{\rho}_{CT}$ to Compton-thick accretion, and $\dot{\rho}_{RI}$ to unobscured but radiatively inefficient accretion. The distinction between obscured and Compton-thick accretion is that the former is visible in X-rays while the latter is difficult to detect even in X-rays.

The contribution of black hole seeds to the total mass budget is expected to be small because nearly all of the proposed models produce seeds well below the mass at which the contribution to the local black hole mass density peaks, $\sim 10^7 M_\odot$. Black hole mergers cause individual black holes to grow but reduce the mass density in black holes due to the production of gravitational radiation. The energy in gravitational waves is expected to be a few percent of the initial rest mass energy of the two black holes (Baker et al. 2001; Lousto et al. 2010), so this correction is expected to be small.

Equation (1) can be rewritten in the form of luminosity densities and radiative efficiencies:

$$\rho_\bullet c^2 = \rho_0 c^2 - \rho_{GW} c^2 + \sum_i \int \frac{(1 - \epsilon_i) \ dL_i}{\epsilon_i f_i} \ dV \ dt$$  \hspace{1cm} (2)$$

where $i \in \{UO, OB, CT, RI\}$, $\epsilon$ refers to the intrinsic radiative efficiency and $f$ to the fraction of those photons that we observe. This is done to separate the cases of obscured or Compton-thick accretion, which are intrinsically radiatively efficient but difficult to observe, from true radiatively inefficient accretion flows. We may surmise that $\epsilon_{UO} \sim \epsilon_{OB} \sim \epsilon_{CT} \gg \epsilon_{RI}$ while $f_{UO}, f_{RI} \sim 1$ and $f_{OB}, f_{CT} \ll 1$. The Soltan argument is essentially that the only term that contributes significantly to the left hand side of this equation is the first one inside the integral, corresponding to unobscured accretion.

2 LOCAL BLACK HOLE MASS DENSITY

The correlation between black hole mass and bulge mass was one of the relationships between black holes and galaxy properties to be studied, and value of the proportionality constant 0.001 endured until recently (Kormendy & Richstone 1991; McLure & Dunlop 2001; Marconi & Hunt 2003; Haring & Rix 2004). These black hole mass relations are converted into a local black hole mass density by assuming that every galaxy hosts a black hole well-described by the aforementioned relations, and that there are not many massive black holes outside the centers of galaxies, ejected for example by recoil from gravitational waves (Redmount & Rees 1989; Volonteri 2007) or by three-body interactions following galaxy mergers (Hut & Rees 1992; Xu & Ostriker 1994; Volonteri et al. 2003).

It is very important to realize that the scatter in the black hole scaling relations has a large effect on the black hole mass density. Because the galaxy mass function falls steeply at the high mass end, the population of $10^5 M_\odot$ black holes (for example) contains many objects with black holes that are over-massive than under-massive. This means that the typical $10^5 M_\odot$ black hole lives in a galaxy smaller than one would expect from simply applying the mean relation without scatter, hence there are more $10^5 M_\odot$ black holes than one would expect from the mean relations, therefore increasing the scatter tends to increase the mass density of black holes. This effect can be significant, and careful studies of the black hole population such as Shankar et al. (2003) and Merloni & Heinz (2008) take it into account.

Recently the proportionality constant between black hole mass and bulge mass has risen by a factor of approximately five owing to systematic effects in the measurement of black hole masses, in particular the inclusion of more realistic dark matter models (Kormendy & Ho 2013). As long as the scatter in the scaling relation remains the same, as it has in the updated (Kormendy & Ho 2013) relation, effect of the change in the normalization of the black hole mass scaling relation by a factor of five is to increase the local mass density in black holes by the same factor of five.

A number of other correlations between black hole mass and various galaxy properties have been proposed, with varying degrees of success (e.g. Graham et al. 2004; McLure & Dunlop 2002; Marconi & Hunt 2003; Bettoni et al. 2003), and one may wonder which of these should be used in estimating the black hole mass density. The choice of which correlation is used to estimate the black hole mass density makes a difference (Shankar et al. 2003) and thus follow-up work is necessary to assess the sensitivity of the local black hole mass density to these changes in the normalization of the black hole scaling relations.

3 AGN LUMINOSITY FUNCTION

In order to characterize the history of massive black hole accretion in the universe, it is necessary to include all sources, regardless of obscuration or the waveband in which the source is observed. Therefore the function that we wish to construct is most properly named the active galactic nucleus luminosity function (AGNLF). For this purpose,
AGN come in approximately three flavors depending on the difficulty of observing them: 1) Unobscured in the optical, with column density of hydrogen below approximately $N_H < 10^{21} \text{cm}^{-2}$, 2) Obscured in the optical but visible in X-rays, with $10^{24} \text{cm}^{-2} < N_H < 10^{24} \text{cm}^{-2}$, and 3) Compton thick, where the gas surrounding the AGN is optically thick to Compton scattering, $N_H > 10^{24} \text{cm}^{-2}$. The last class of objects is difficult to observe in any band, except perhaps the reprocessed thermal infrared emission, which suffers from degeneracies with non-AGN sources such as extreme starbursts.

Saltman (1982) used the observed quasar luminosity function to infer the mass density in “dead quasars,” ie, black holes, but he lacked an independent estimate of the black hole mass density with which to compare. Black hole scaling relations have provided the latter tool and [Yu & Tremaine (2002)] arrived at $\rho_s = 2.5 \pm 0.4 \times 10^7 (h/0.65)^2 \left(\frac{M_\odot}{\text{Mpc}^{-3}}\right)$

This gives mean radiative efficiencies between 0.1 and 0.2 depending on the mass of optically bright quasars. Merloni & Heinz (2008) carried out a similar study allowing for various modes of black hole accretion at a broad range of Eddington fractions and found smaller radiative efficiencies, $\epsilon = (0.0675 \pm 0.025) (\rho_s/4.3 \times 10^7 M_\odot/\text{Mpc}^{-3})^{-1}$.

Shankar et al. (2009b) found $\rho_s = 3 - 5.5 \times 10^7 M_\odot/\text{Mpc}^{-3}$ and on which scaling relation one uses to compute the mass density. They also found low mean radiative efficiencies: $\epsilon = 0.065$ using their estimate of the AGNLF, or $\epsilon = 0.09$ for the Shankar et al. (2009b) luminosity function. These authors also allowed for the possibility of Compton-thick accretion in their model and found that it was subdominant.

4 X-RAY BACKGROUND

The most direct observational constraint on Compton thick accretion comes from the XRB. Fabian et al. (1990) pointed out that the observed peak in the XRB at 30 keV could be naturally explained by a Compton reflection spectrum, implying a significant population of Compton thick objects. Such a large column of obscuring gas suggests extreme objects and therefore high accretion rates, thus a possibly significant contribution to the local black hole mass density.

Estimates of the fraction of Compton thick AGN vary wildly. Risaliti et al. (1999) argued that the 30 keV peak requires twice as many Compton thick sources as obscured sources so that Compton thick AGN are approximately half of the total population of AGN. Gilli et al. (2007) found that the XRB implies approximately equal numbers of unobscured, obscured, and Compton thick AGN. These results imply that unobserved black hole accretion is from 30% to a factor of two larger than observed accretion. However, Treister et al. (2009) were able to fit the XRB with a negligible Compton thick fraction, and Akylas et al. (2012) found that the Compton thick fraction can be anywhere from 5% to 50% depending on the modeling. NuSTAR is the first focusing X-ray telescope covering the band from 3 to 79 keV (Harrison et al. 2013) and may be able to answer this question by resolving the 30 keV XRB peak into point sources. In their first results, they found ten sources, equally divided between obscured and unobscured, with no Compton thick sources. Their 90% upper limit on the fraction of Compton thick AGN is 30% (Alexander et al. 2013).

While Compton-thick accretion could bring the mean observed radiative efficiency into line with that expected for thin-disk accretion, present observational constraints to not seem to permit sufficient Compton-thick accretion to allow this explanation.

5 RADIATIVE EFFICIENCY OF ACCRETION

The previous sections have discussed the observational constraints on the mean radiative efficiency of accretion. In this section I review the theoretical expectations for the radiative efficiency. Standard theory for radiatively efficient, geometrically thin, optically thick accretion disks says that radiative efficiency (defined to be $\epsilon = L/mc^2$) is equal to the binding energy of a particle at the innermost stable circular orbit (ISCO) [Shakura & Sunyaev (1973), Novikov & Thorne (1973)]. This ranges from $1 - \sqrt{25/27} \approx 0.037$ for a maximally rotating black hole with a retrograde accretion disk to $1 - \sqrt{7/3} \approx 0.423$ for a maximally rotating black hole with a prograde accretion disk. For a non-rotating Schwarzschild black hole, the value is $1 - \sqrt{8/9} \approx 0.057$.

Figure 1 shows the radiative efficiency as a function of the dimensionless black hole spin parameter for radiatively efficient thin-disk accretion along with the mean radiative efficiencies implied by various values of the present day black hole mass density. Negative values of the black hole spin parameter correspond to retrograde accretion disks. The above
estimates assume that the stress on the gas is zero in the so-called plunging region inside the ISCO. Recent numerical simulations have indicated that this assumption may not be valid, and stresses inside the plunging region lead to dissipation that may boost the radiative efficiency by as much as at 16% (Noble et al. 2011).

The radiative efficiencies found by Merloni & Heinz (2008) and Shankar et al. (2009) using the previously accepted black hole scaling relations correspond to those expected for black holes with prograde accretion disks and low to moderate spins, naturally strengthening the widely held view that most material that enters black holes does so via a luminous thin-disk mode. Given the difficulty in arranging for accreted material enter the black hole in a predominantly retrograde fashion, the lower reasonable bound for the mean radiative efficiency is the value for non-spinning black holes, 5.7%. Therefore even a moderate increase in the local black hole mass density, not to mention the factor of five increase in the black hole mass density implied by the Kormendy & Ho (2013) normalization, pushes the mean radiative efficiency to values that cannot be reasonably explained in terms of luminous thin-disk accretion. This implies that most of the material that falls into black holes over the history of the universe must do so in a heavily obscured phase or via a radiatively inefficient super-Eddington mode.

At accretion rates substantially below the Eddington rate, a variety of radiatively inefficient accretion models may be relevant (Narayan & Yi 1994; Blandford & Begelman 1998; Ono et al. 2000), but it is difficult to significantly change the final mass density in black holes precisely because of the low accretion rates. Radiatively inefficient accretion is also expected to occur at accretion rates significantly above the Eddington rate, owing to the fact that the flow becomes sufficiently optically thick to advect photons into the black hole (Begelman 1979; Abramowicz et al. 1980; Paczyński & Wiita 1980).

The other possible explanation for a low observed radiative efficiency is that it is only the apparent radiative efficiency that is low: most of the photons are emitted but do not reach us. This is the case for obscured AGN and for Compton-thick AGN. In both cases, the intrinsic radiative efficiency is high (~ 0.1), but the fraction of those photons reaching us is low.

6 BLACK HOLE SPINS

Given that the theoretically expected radiative efficiency depends on the black hole spin and the orientation of the accretion disk, it is crucial to know whether black holes are spinning rapidly or not. In this section I discuss theoretical expectations and observational constraints on black hole spins. Angular momentum is expected to be a major barrier to black hole accretion given the large range in length scales that must be traversed in order to arrive at the event horizon. Given the coherent large-scale angular momentum of many galaxies, one may expect black holes to be rapidly spinning and to be aligned with the angular momentum of the gas that the black hole is consuming.

Perfectly coherent gas accretion is not necessary for black holes to have large spins. Physical processes such as the Bardeen & Petterson (1973) effect may be able to align the black hole spin with the angular momentum of the infalling gas. For thin-disk accretion the accretion disk is expected to warp at the radius at a few hundred gravitational radii where Lense-Thirring precession becomes important, providing a large lever arm for the infalling gas to exert torques on the black hole. While changing the magnitude of a black hole’s spin requires the black hole to of order double its mass, changing the direction of the spin vector requires the black hole to accrete only a few percent of its mass (Perego et al. 2009). On the other hand, a picture has emerged recently where accretion is fully chaotic: gas falls into the black hole in discrete events with isotropic angular momenta (King & Pringle 2008). In this case black hole spins will be low, with radiative efficiencies of approximately 5%.

In the case of black hole mergers, the angular momentum of the system is dominated by the orbital angular momentum for the most interesting case of approximately equal mass black holes. The final black hole is expected to be rapidly spinning (Berti & Volonteri 2008; Lousto et al. 2010), but its spin direction may not be aligned with the angular momentum of the gas in the galaxy leading to complicated subsequent evolution. The consequences of chaotic versus coherent gas accretion and black hole mergers in the context of cosmological structure formation have been studied in detail (Dotti et al. 2013).

Measurements of black hole spins will be enormously helpful in constraining the radiative efficiencies. While there are a growing number of spin measurements available, differences in the way different groups model the X-ray spectra of the same object give values for the spin that differ at high significance (Brenneman & Reynolds 2004; de La Calle Pérez et al. 2010; Patrick et al. 2011; Brenneman et al. 2011; Patrick et al. 2013).

7 DISCUSSION

Thin-disk accretion predicts mean radiative efficiencies of between 3.7% (for retrograde accretion disks) and 42% (for prograde accretion disks), while 5.7% is the value associated with non-spinning black holes. Given the difficulty in arranging for accretion to be on average retrograde, the value for non-spinning black holes is the minimum reasonable value for the mean efficiency if black hole accretion occurs predominantly via this mode.

If the angular momentum of accreted material is coherent over long timescales, the black hole spin will eventually become large and aligned with the angular momentum of the incoming material. The requirements for the coherence time are relaxed somewhat if physical processes such as the Bardeen & Petterson (1973) effect exert sufficient torque on the black hole to align its spin with the incoming material. In either case, black hole spins and radiative efficiencies are expected to be large. Conversely, chaotic accretion models predict that black holes will have low spin values, leading to mean radiative efficiencies around 5.7%.

Comparison of the local black hole mass function and the AGN luminosity density over the history of the universe permits a measurement of the mean radiative efficiency of accretion. Careful studies using the previously accepted black hole scaling relation of $M_{\text{BH}} = 0.1\% M_{\text{bulge}}$ arrived at
mean efficiencies of around 6.5%, strengthening the Soltan (1982) argument that the majority of material that falls into black holes through the history of the universe does so via a luminous thin-disk mode.

However, Kormendy & Ho (2013) have recently argued that the normalization of the local black hole scaling relations should be increased by a factor of five to $M_{\text{BH}} = 0.5\%M_{\text{bulge}}$. This increases the local mass density in black holes by a factor of five and decreases the mean radiative efficiency by the same factor. The Shankar et al. (2009) estimate goes from $\epsilon = 0.065$ to 0.013. This revision puts the mean radiative efficiency far below the minimum reasonable value of approximately 5.7%, implying that most of the material that falls into black holes over the history of the universe does so in a way that does not produce light that reaches us.

The low mean radiative efficiency might be because the accretion is intrinsically radiatively efficient but heavily obscured, i.e. Compton thick. The 30 keV peak in the XRB has been taken by some researchers as an indication of significant Compton-thick accretion—nearly enough to explain the low radiative efficiencies implied by the Kormendy & Ho (2013) result. However, the most recent data seem to indicate that it will be difficult to accommodate the full factor of five via Compton thick accretion.

Low mean radiative efficiencies may also result from intrinsically radiatively efficient accretion flows. These are often discussed in the context of low accretion rates—below 1% of the Eddington rate—but it is difficult to affect the mass budget for black holes via the behavior of accretion flows at low accretion rates. If the explanation is intrinsically radiatively inefficient flows, then it seems more likely to be evidence for super-Eddington accretion, where the radiative efficiency is again expected to fall compared to thin-disk accretion. Evidence that super-Eddington accretion is common would help to alleviate tension associated with the fact that the growth time for the black holes powering the observed $z = 7$ quasars is approaching the age of the universe at that redshift if one assumes they grow by radiatively efficient accretion.

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