Robustness of the Pairwise Kinematic Sunyaev–Zeldovich Power Spectrum Shape as a Cosmological Gravity Probe

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Abstract

We prove from the modified gravity (MG) galaxy/halo mock catalogs that the shape of the pairwise kinematic Sunyaev–Zeldovich (kSZ) power spectrum $P_{\text{kSZ}}$ has constraining power on discriminating different gravity theories on cosmological scales. By varying the effective optical depth $\tau_T$ as a free parameter, we verify that the $\tau_T f$ (the linear growth rate) degeneracy in the linear theory of $P_{\text{kSZ}}$ is broken down by the nonlinear structure growth and the scale dependence of $f$ in some MG theories. Equivalently speaking, the shape of $P_{\text{kSZ}}$ alone could be used to tightly constrain the MG theories on cosmological scales. With good knowledge of galaxy density biases, we verify that a combination of the next-generation galaxy spectroscopic redshift and CMB surveys, e.g., BOSS+CMB-S4 or DESI+CMB-S4, could potentially discriminate $f(R)$ models from the general relativity at the $\sim 5\sigma$ level using the shape of the galaxy pairwise kSZ dipole $P_{\text{kSZ},1}$ alone, when $f_{R0} = 10^{-4}$.

Unified Astronomy Thesaurus concepts: Dark energy (351); Dark matter (353); Cosmological parameters from large-scale structure (340); Sunyaev-Zeldovich effect (1654)

1. Introduction

The cosmological test of gravity theories has gained much more attention since the discovery of cosmic acceleration. Profound progress has been made, though there is still room to explore for the next generation of cosmological surveys, which will collect at least one order of magnitude more data than all the existing ones in the current literature. Therefore, many next-to-leading-order cosmological/astrophysical effects will be detected at a high confidence level and will significantly benefit the cosmological gravity test. The kinematic Sunyaev–Zeldovich (kSZ) effect (Sunyaev & Zeldovich 1970, 1972, 1980; Ostriker & Vishniac 1986) is one of these.

The kSZ effect describes one of the secondary cosmic microwave background (CMB) temperature anisotropies, induced by the CMB photons Thomson-scattering off a bunch of free electrons with bulk motion. It is a measure of both the cosmological velocity field and baryon distribution in the universe, and the induced CMB temperature change:

$$\delta T_{\text{kSZ}}(\vec{n}) = -T_0 \int dl \sigma_T n_e \left( \frac{v_c \cdot \hat{n}}{c} \right),$$

(1)

$$\delta T_{\text{kSZ}}(\vec{n}_i) = -T_0 \sigma_T v_i \cdot \hat{n}_i,$$

(2)

Here $T_0 \approx 2.73$ is the averaged CMB temperature, $\sigma_T$ is the Thomson-scattering cross-section, $c$ is the speed of light, $\hat{n}$ is the unit vector along the line of sight (LOS), $n_e$ is the physical free electron number density, $v_i$ is the peculiar velocity of free electrons, and the integration $\int dl$ is along the LOS given by $\hat{n}$. Equation (2) assumes that the CMB photons scatter off only one cloud of moving free electrons surrounding the $i$th halo until they reach the observer, and $\tau_{T,i} = \int dl \sigma_T n_e$ is the optical depth of the $i$th halo. We adopt this assumption throughout this article. Furthermore, we assume a constant $\tau_T$ for all halos for simplicity; thus $\tau_T$ could be singled out in all calculations.

The velocity field of free electrons is believed to be a good tracer of the dark matter velocity field and hence a promising testbed of cosmological models. Several works have detected the kSZ effect in the literature, although the signal-to-noise ratio $(S/N)$ has not reached $5\sigma$ (Hand et al. 2012; Hill et al. 2016; Planck Collaboration et al. 2016b; Schaan et al. 2016; Soergel et al. 2016; De Bernardis et al. 2017; Sugiyama et al. 2018; Planck Collaboration et al. 2018). By combining the next generation of galaxy redshift and CMB data, the significance of kSZ detection could potentially reach $20\sigma–100\sigma$, through varying scenarios (Flender et al. 2016; Sugiyama et al. 2018). This high detection significance will heavily benefit the constraint on the DE properties and the MG theories (Sugiyama et al. 2017).

Though promising, this field always suffers from our poor understanding of $\tau_T$. The calculation of $\tau_T$ is based on various complicated astrophysical processes, and could be only possible in hydrodynamical simulations, which still suffer from numerous uncertainties. As Equation (2) shows, $\tau_T$ is degenerated with the amplitude of the velocity field. Therefore, the uncertainty of $\tau_T$ will heavily degrade the constraining power of the kSZ effect on cosmological models. Additionally, several systematic errors in the kSZ detection, such as the miscentering bias and scatter-in-mass bias, will systematically decrease the overall amplitude of the kSZ signal (Flender et al. 2016) and induce systematic biases if we rely on the amplitude of the kSZ signal to constrain cosmology. Consequently, the conservative choice is to merely use the shape of the kSZ signal for cosmological analysis. This work will prove the robustness of this idea using high-resolution MG N-body simulations.

2. The Pairwise kSZ Power Spectrum

Halos containing free electrons tend to move toward each other, due to mutual gravitational attractions. This peculiar kinematic pattern leaves a distinct feature in the CMB map, which can be captured by the pairwise kSZ estimator in Fourier
space (Ferreira et al. 1999; Sugiyama et al. 2018),

$$P_{kSZ}(k) = \frac{V}{N^2} \sum_{ij} \delta T_{kSZ}(\hat{n}_i) \cdot \delta T_{kSZ}(\hat{n}_j) e^{-ik \cdot s_{ij}}$$

$$\simeq \left( \frac{T_0 \tau_T}{c} \right) P_{\nu}(k),$$

(3)

$$P_{\nu}(k) = \left( \frac{V}{N^2} \sum_{ij} [v_i \cdot \hat{n}_i - v_j \cdot \hat{n}_j] e^{-ik \cdot s_{ij}} \right).$$

(4)

$P_{kSZ}$ is the pairwise kSZ power spectrum in redshift space, $P_{\nu}$ is the galaxy LOS pairwise velocity power spectrum in redshift space, $V$ is the survey volume, $N$ is the number of galaxies, and $s_{ij} = s_i - s_j$ is the galaxy separation vector in redshift space.

In a series of papers Sugiyama et al. (2016, 2018, 2017) proved that, assuming the global plane-parallel approximation $\hat{n}_i \sim \hat{n}_j \sim \hat{n}$ and the amplitude of velocity field is proportional to $f$, we could drive $P_{\nu}(k)$ from the redshift space galaxy density power spectrum $P_s(k)$ in the following way:

$$P_{\nu}(k) = \left( \frac{aHf}{k \cdot \hat{n}} \right) \frac{\partial}{\partial f} P_s(k),$$

(5)

where $a$ is the scale factor, $H$ is the Hubble parameter at redshift $z$. This relation holds for any object such as dark matter particles, halos, galaxies, and galaxy clusters with anisotropic clustering property (in redshift space).

Instead of detailed modeling of $P_{kSZ}$ (Sugiyama et al. 2016; Okumura et al. 2014), we provide here a toy model to qualitatively understand the numerical results. We introduce a simple RSD model, with a linear Kaiser term and a Gaussian Finger-of-God (FoG) term:

$$P(k, \mu) = (b + f \mu^2) P_{\text{lin}}(k) \exp(-k^2 \mu^2 \sigma_v^2 / H^2).$$

(6)

Here $b$ is the linear galaxy bias, $\mu$ denotes the cosine of the angle between $k$ and the LOS, and $P_{\text{lin}}(k)$ is the linear dark matter power spectrum at redshift $z$. In linear theory, the LOS velocity dispersion $\sigma_v = \int f^2 H^2 P_{\text{lin}}(k) dk / 6\pi^2$. By substituting Equation (6) into Equation (5), we derive the corresponding formula for $\Delta_{kSZ}(k, \mu) = k^2 P_{kSZ}(k, \mu) / 2\pi^2$:

$$\Delta_{kSZ}(k, \mu) = \frac{k^3}{2\pi^2} \left( \frac{T_0 \tau_T}{c} \right) 2iuHf \mu (b + f \mu^2) P_{\text{lin}}(k) k \times S(k, \mu) \exp(-k^2 \mu^2 \sigma_v^2 / H^2),$$

(7)

$$S(k, \mu) = 1 - (b/f + \mu^2)k^2 \sigma_v^2 / H^2.$$  

(8)

Equation (7) characterizes two circumstances in which the $\tau_T-f$ degeneracy could be broken. One is when the MG theory predicts a scale-dependent $f$, which will change the shape of $\Delta_{kSZ}$ in a characteristic way and break the $\tau_T-f$ degeneracy. The other is nonlinear structure growth, which generates higher-order terms such as the shape kernel $S(k, \mu)$ and the FoG term. The combination of the term $\tau_Tfb$ and these higher-order terms will also break the $\tau_T-f$ degeneracy.

To illustrate the robustness of above two mechanisms in breaking the $\tau_T-f$ degeneracy, and also to quantify the robustness of the $\Delta_{kSZ}$ shape as a cosmological gravity probe, we will measure and compare the $\Delta_{kSZ}$ dipoles from high-resolution general relativity (GR) and MG simulations in the following sections.

### 3. The MG Simulations and Mock Catalogs

We study two representative MG theories in this work, the $f(R)$ gravity (De Felice & Tsujikawa 2010) and the normal branch of DGP (nDGP) gravity (Dvali et al. 2000). The $f(R)$ gravity acquires a scale-dependent $f$ and we adopt the Hu & Sawicki (HS) functional form of $f(R)$, where the deviation from GR is characterized by the free parameter $|f_{R0}| = \Omega f / \Omega_{R0}$. The nDGP model predicts a scale-independent $f$, and this model has one parameter $r_t$, of length dimension, below which gravity becomes four-dimensional.

$f(R)$ simulations are run by the ECOSMOG code (Li et al. 2012; Bose et al. 2017) and nDGP simulations are run by the ECOSMOG-V code (Li et al. 2013; Barreira et al. 2015). The simulation box size is $1024\text{Mpc}/h$ and the particle number is $1024^3$. Three $|f_{R0}| = 10^{-4}$, $10^{-5}$, $10^{-6}$ values are chosen for $f(R)$ gravity simulations, denoted as F4, F5, and F6. Two $H_0r_t = 1.0, 5.0$ values are adopted for nDGP gravity simulations, named N1 and N5. The levels of deviation from GR are in the sequence of $F4 > F5 > F6$, and $N1 > N5$. All simulations have the same background expansion quantified by the WMAP9 cosmology (Hinshaw et al. 2013),

$\{\Omega_b, \Omega_{\text{CDM}}, h, n_s, \sigma_8\} = \{0.046, 0.235, 0.697, 0.971, 0.82\}$,

and we run five realizations for each gravity model.

The $z = 0.5$ and $z = 0.8$ snapshots are analyzed in this work, mimicking two galaxy catalogs, respectively the CMASS sample of the BOSS survey (Manera et al. 2013) and the LRG galaxy sample of the DESI survey from $z = 0.65$ to $z = 0.95$ (DESI Collaboration et al. 2016; Sugiyama et al. 2018). The dark matter halo catalogs are generated using ROCKSTAR (Behroozi et al. 2013). At $z = 0.5$, by tuning the HOD parameters suggested in Zheng et al. (2007), the mock galaxy catalogs are generated. Different HOD parameters are applied to different MG simulations, and all “constrained” galaxy catalogs are guaranteed to have an identical galaxy number density $n_s$ and projected correlation functions $w_p(r_p)$. This effectively fixes the uncertainties from the complicated galaxy density biases, therefore we could focus solely on the physical deviations induced by the MG theories at cosmological scales. The HOD parameters of the GR simulations are the best-fit HOD parameters from the CMASS data (Manera et al. 2013). At $z = 0.8$, we simply select all dark matter halos with $M > 10^{13} M_\odot / h$ to represent the LRG galaxies of the future DESI survey from $z = 0.65$ to $z = 0.95$, for the purpose of a rough S/N estimation. Detailed description of the simulations and catalogs could be found in Hernández-Aguayo et al. (2019).

For real to redshift space, we use the following formula to move the positions of the mock galaxies/halos:

$$s = r + \frac{v \cdot \hat{n}}{a(z)H(z)} \hat{n},$$

(9)

where $r$ is the real space position and $s$ is the redshift space position.

Besides the mock catalogs, the S/N prediction also depends on the specific survey parameters. We choose a CMB-S4 like survey as our CMB survey baseline (Carlstrom et al. 2019). We choose two galaxy survey setups, the BOSS-like and DESI-like surveys as redshift survey baselines. The detailed survey specifications are shown in Table 1.
4. The Pairwise kSZ Dipole

We study the dipole of $\Delta_{\text{kSZ}}(k, \mu)$ in this section. The first step is to calculate $P_{\nu}(k, \mu)$. For the computational convenience, we adopt an estimator equivalent to Equation (4), namely (Sugiyama et al. 2016)

$$(2\pi)^3 \delta_D(k + k') P_{\nu}(k) = \langle p_{\nu}(k) \delta_{\nu}(k') - \delta_{\nu}(k) p_{\nu}(k') \rangle,$$  

(10)

where $p_{\nu}(s) = [1 + \delta_{\nu}(s)][s\cdot \hat{n}]$ and $\delta_{\nu}(s)$ is the momentum field and the density fluctuation field in redshift space. $p_{\nu}(k)$ and $\delta_{\nu}(k)$ are the corresponding Fourier counterparts. We sample the $p_{\nu}(s)$ and $\delta_{\nu}(s)$ fields on 1024$^3$ regular grids using the nearest-grid-point (NGP) method and calculate $p_{\nu}(k)$ and $\delta_{\nu}(k)$ fields by the fast Fourier transform (FFT) method.

By choosing the AP filter radius $\theta_z$ and maximizing the signal-to-noise, we derive the average $\tau_S$ of two target galaxy catalogs. At $z = 0.5$, $\tau_S = 4.5 \times 10^{-3}$, $\theta_z = 1'37'$, and at $z = 0.8$, $\tau_S = 6.0 \times 10^{-3}$, $\theta_z = 1'12'$. Therefore, Equation (3) gives $P_{\text{kSZ}}(k, \mu)$. Using the Legendre polynomials $P_l(\mu)$, the multipole of $P_{\text{kSZ}}(k, \mu)$ is defined as

$$P_{\text{kSZ}}(k, \mu) = \frac{2l + 1}{2} \int_{-1}^{1} d\mu P_{\text{kSZ}}(k, \mu) P_l(\mu).$$  

(11)

The measured dimensionless dipoles $\Delta_{\text{kSZ,}l=1} = i^{l} k^{l} P_{\text{kSZ,}l=1}(k)/2\pi^2$ of CMB-S4+BOSS-like and +DESI-like mock galaxies are shown in the top panels of Figure 1.\(^1\)

We see that all simulations give a $\Delta_{\text{kSZ,}l=1}$ with a similar trend, with a turnover at around $k \sim 0.08 h$/Mpc. However, different gravity theories predict $\Delta_{\text{kSZ,}l=1}$ of different amplitudes and shapes, and the MG simulations are more distorted than that of GR. The deviations of F4 and N1 simulations from the GR case are larger than that of the F5/F6 and N5 simulations, as expected.

Instead of making careful comparisons between Equation (7) and Figure 1, here we generally discuss the impact of the shape kernel $S(k, \mu)$ on the dipole shape. $S(k, \mu)$ is physically a consequence of the competition between Kaiser and FoG effects, which makes it a decreasing function ranging from unity to $-\infty$ as $k$ increases. Together with the FoG term, $S(k, \mu)$ induces the turnover of $\Delta_{\text{kSZ,}l=1}$ at around $k \sim 0.08 h$/Mpc.

\(^1\) We also calculate and compare the octopoles of $P_{\text{kSZ}}$, but find negligible differences ($<1\sigma$) of $P_{\text{kSZ,}l=1}$ between different gravity simulations.

\(^2\) $k_{s=0} \propto (f/b + f^2 \mu^2)^{1/2}$, thus gravity-generating larger $f$ will predict smaller $k_{s=0}$. Galaxies have $b > 1$, thus their $k_{s=0}$ will be smaller than that of dark matter. Both predictions are consistent with Figure 1. Moreover, it might be possible to use $k_{s=0}$ to constrain $f$, which is beyond the scope of this paper.
nDGP dipoles are in general larger than those of \( f(R) \) models and in the other way around at small scales (e.g., \( k \gtrsim 0.1h/\text{Mpc} \)).

We calculate the diagonal elements of the covariance matrix of the dipole measurements by Equation (A2). Other than the shot noise of the galaxy distribution, the primary CMB anisotropies and the detector noise of the CMB experiment are considered as well. Then we evaluate the S/N of the differences between MG and GR dipoles, where

\[
\frac{S}{N} = \sqrt{\frac{\chi^2}{}}, \quad \chi^2 = \sum \frac{\left[ \Delta_{kSZ,f-1}(k) - \Delta_{GR_{kSZ,f-1}}(k) \right]^2}{\sigma_{\Delta kSZ}^2(k)}, \tag{12}
\]

\( k_{\text{min}} = 0.035h/\text{Mpc}, \quad k_{\text{max}} = 0.195h/\text{Mpc} \) and \( \Delta k = 0.01h/\text{Mpc} \).

The estimated S/N are listed in the bottom panels of Figure 1. We find that, combined with the CMB-S4 like CMB survey, both BOSS- and DESI-like could discriminate the F4 model from GR at a \( \sim 5\sigma \) level using \( \Delta_{kSZ,f-1} \). The DESI-like survey, due to its large survey volume, in general has smaller error bars and higher S/N. The pairwise kSZ power spectrum could help with improving the capacity of a gravity test of next-generation CMB and galaxy surveys.

5. \( \tau_f \) Degeneracy Breaking

In a realistic data analysis, it is difficult to obtain accurate knowledge of \( \tau_f \) in advance, thus the amplitude of the kSZ power spectrum could not be used to constrain cosmology. The key question to ask is without knowing \( \tau_f \), can we still discriminate kSZ signals predicted by different gravity theories? The answer is yes, we could discriminate them using the shape of the galaxy pairwise kSZ dipole.

In order to illustrate this point, we implement the following test. We fix the GR \( \Delta_{kSZ,f-1} \) as it is in the previous section. Then in calculating the MG dipoles, we replace \( \tau_f \) by \( n\tau_f \), and vary \( n \) between [0.5,1,5] to fit the GR dipole. Consequently, we obtain the best-fitted \( n \), and the corresponding S/N of the GR-MG dipole differences are calculated by Equation (12). We consider this S/N as the constraining power of the gravity model from the shape of the galaxy pairwise kSZ dipole, and equivalently, it illustrates how much the \( \tau_f \) degeneracy is broken by the \( \Delta_{kSZ,f-1} \) shape. The results are plotted in Figure 2.

(1) In nDGP cases, the amplitudes of \( \Delta_{kSZ,f-1} \) change by up to \(-10\%\), in \( f(R) \) cases, the amplitudes change by up to \( \sim 4\% \).

(2) The S/N decreases compared with the fixed \( \tau_f \) case, but with a minor level. This discrepancy illustrates that (i) nonlinear structure growth already breaks the \( \tau_f \) degeneracy in the kSZ signal, and (ii) the scale dependence of \( f \) in \( f(R) \) models further breaks the \( \tau_f \) degeneracy, as discussed previously.

Therefore, we prove that using the \( \Delta_{kSZ,f-1} \) shape alone, we could discriminate F4 gravity from GR at the \( \sim 5\sigma \) level, by a BOSS+ or DESI+CMB-S4-like combination. Moreover, it could contribute to the constraint of F5-, N1-, and N5-like gravity theories in the future. This conclusion overcomes the obstacles in the kSZ cosmology originating from the poor understanding of \( \tau_f \) and will have implications for next-generation galaxy and CMB surveys, complementing other cosmic probes in constraining dark energy and gravity theories.

In our analysis, we choose a moderate \( k_{\text{max}} = 0.195h/\text{Mpc} \), which corresponds to a comoving scale around \( 5.1h/\text{Mpc} \sim 32.2h/\text{Mpc} \). Therefore, our conclusions are immune to the systematics affecting the small-scale kSZ signal, e.g., complicated small-scale astrophysical processes.

6. Discussions

In conclusion, we verify that the shape of the galaxy pairwise kSZ dipole has potential constraining power on gravity models at cosmological scales. This constraining ability does not depend on the small-scale kSZ signal, thus it is immune to complicated small-scale astrophysical processes. This probe is in particular useful for a self-consistent test of GR from cosmological data, where our main target is to falsify GR rather than to determine the “true” gravity model. We find that, with good knowledge of galaxy density biases, a BOSS+ or DESI+CMB-S4-like survey combination could discriminate F4 gravity models from GR at the \( \sim 5\sigma \) level, and could also contribute to the constraint of F5-, N1-, and N5-like gravity theories in the future, illustrating the promising potential of the \( \Delta_{kSZ} \) shape in the future.

As a proof-of-concept paper, this work is simplified and could be improved in several aspects. (1) It would be beneficial...
We assume that we have good knowledge of galaxy density bias in this work, while in reality the biases or HOD parameters fitted from $w_p$ will have uncertainties. (3) Current forecasts assume a Gaussian gas profile and a top-hat aperture photometry filter. The signal could be larger than current predictions if the real gas profile is more compact than the Gaussian distribution and if we adopt a matched filtering technique (Alonso et al. 2016). (4) To obtain better forecasts of next-generation galaxy and CMB surveys, realistic kSZ catalogs matching their survey designs would be necessary.

In a given galaxy sample, we assume that all halos have the same optical depth, and that there is no redshift dependence. This is a rough assumption that as long as the optical depths of halos do not correlate with each other on the scales that we are interested in, this assumption is robust, at least on the leading order and we can consider the effective optical depth as the average of the optical depths in a halo-mass bin.

We work with a single snapshot with a fixed redshift in this work. A real data analysis requires a robust estimation of an "effective" redshift of the galaxy sample if we assume a redshift-independent growth function in our data analysis. Given that reasonably large volume surveys are considered and the growth function will vary along each line of sight, this is an important topic in both galaxy RSD analysis and kSZ analysis. Without this, we would introduce systematic error into our final growth function estimation.

For comparison, the SZ tomography technique (Smith et al. 2018; Pan & Johnson 2019) incorporates the halo-mass dependence of optical depth and redshift-dependent growth function by dividing the given galaxy sample into fine enough redshift bins and halo-mass bins. The same spirit has been adopted in the galaxy RSD analysis (Zheng et al. 2019) and could be applied in our future kSZ analysis.

Finally, we would like to use this work to motivate more theoretical and observational kSZ studies on cosmological model constraints in the future.

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Appendix

**$\tau_T$ and Covariance Matrix Modeling**

We adopt the following formula to calculate $\tau_T$ (Sugiyama et al. 2018):

$$\tau_T = \frac{\sigma_T f_{gas} M_{avg}}{\mu_e m_p D_h(z_{eff})} \int d\ell \int d\mu \mathcal{L}_{\ell}(\mu) N(\ell) B(\ell).$$  \hspace{1cm} \text{(A1)}$$

Here $f_{gas}$ is the gas-mass fraction, $f_{gas} = f_b$ and $f_b = \Omega_b/\Omega_m = 0.155$ is the universal baryon fraction (Planck Collaboration et al. 2016). $M_{avg}$ is the averaged halo mass in the targeted halo-mass bin, calculated from simulations. $\mu_e = 1.17$ is the mean particle weight per electron, and $m_p$ is the proton mass. $D_h(z_{eff})$ is the angular diameter distance at the effective redshift. $U(\ell_{\ell})$ is the Fourier transform of the AP filter (Alonso et al. 2016; Sugiyama et al. 2018) and $\theta_c$ is the AP filter radius. $N(\ell)$ is the Fourier transform of the projected gas profile, which we assume is Gaussian. $B(\ell)$ is the Fourier transform of the Gaussian Planck beam function.

Considering only the diagonal Gaussian terms, the covariance matrix of $P_{\text{kSZ},\ell}$ is (Sugiyama et al. 2017)

$$\text{Cov}(P_{\text{kSZ},\ell}(k), P_{\text{kSZ},\ell}(k)) = \frac{1}{N_{\text{mode}}(k)} \int d\varphi \int d\mu \mathcal{L}_{\ell}(\mu) \mathcal{L}_{\ell}(\mu) \times \left( \frac{T_0}{c} \right)^2 \left[ 2P^{(11)}(k) - 2P^{(10)}(k) \right] + (1 + R_{T0}^2) \frac{\sigma^{(2)}}{n}. \hspace{1cm} \text{(A2)}$$

Here $N_{\text{mode}}(k)$ is the number of $k$ modes in a given $k$ bin, $\mathcal{L}_{\ell}(\mu)$ is the Legendre polynomials. $n$ is the number density of mock galaxies or halos. $\sigma^{(2)}$ is the dispersion of the density-weighted velocity of mock galaxies or halos, measured from simulations. In redshift space, $P^{(11)}$ is the momentum-density cross-power spectrum, $P^{(10)}$ is the momentum-momentum autopower spectrum, and $P$ is the density-density autopower spectrum. All three power spectra are calculated from simulations.

In particular, we define the inverse $S/N$ $R_S$ as

$$R_S^2(\theta) = \frac{\sigma_{\text{kSZ}}(\theta)}{\sigma_{\text{kSZ}}(\theta)}, \hspace{1cm} \text{(A3)}$$

with the signal

$$\sigma_{\text{kSZ}}(\theta) = \left( \frac{T_0}{c} \right) \sigma_v, \hspace{1cm} \sigma_v = \sqrt{\sigma^{(2)}}, \hspace{1cm} \text{(A4)}$$

and the noise

$$\sigma^2(\theta) = \frac{\sum}{\ell} 2f_{\text{em}} + 1 \frac{4\pi}{4\pi} \left( C_{\ell}^\text{obs} \right) U(\ell) U(\ell_c). \hspace{1cm} \text{(A5)}$$

The ensemble average of the observed CMB angular power spectrum $C_{\ell}^\text{obs} = B_{\ell}^2 C_{\ell}^{\text{the}} + N_{\ell}$, with a Gaussian beam function $B$, theoretical prediction of the CMB power spectrum $C_{\ell}^{\text{the}}$, and detector noise $N_{\ell}$.

We choose the $\theta$ that minimizes $R_S$ and calculate the corresponding $\tau_T$ and Cov($P_{\text{kSZ},\ell}(k), P_{\text{kSZ},\ell}(k))$.

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