Mirror stability conditions and SYZ conjecture for Fermat polynomials

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Abstract
Calabi-Yau Fermat varieties are obtained from moduli spaces of Lagrangian connect sums of graded Lagrangian vanishing cycles on stability conditions on Fukaya-Seidel categories. These graded Lagrangian vanishing cycles are stable representations of quivers on their mirror stability conditions.

1 Introduction
For a projective variety $X$, Strominger-Yau-Zaslow conjectured that we can obtain the space $X$ as a moduli space of special Lagrangians with $U(1)$ connections when we have a mirror pair of $X$ and argued locally and physically. The author recommends [2] for a recent illustration on the subject and [15] for a general reference.

In this paper, we study the conjecture categorically in the framework of homological mirror symmetry of Kontsevich [21] and stability conditions of Bridgeland [8]. The latter notion, which was inspired by Douglas’ so-called $\Pi$-stabilities in superstring theory [11, 12], categorizes King’s $\theta$-stabilities [20] ([10, Section 5.3], [1, Section 7.3.3], [6]). In particular, for stability conditions of quivers without relations, when deformed on the stability manifold if necessarily, stable representations solve self-dual equations twisted by characters of general linear groups [20, Section 6].

On derived categories defined in terms of graded Lagrangian vanishing cycles over morsifications of Fermat polynomials $X_n := x_1^n + \ldots + x_n^n : \mathbb{C}^n \to \mathbb{C}$ [33, 3], we put pairs of stability conditions which are named mirrors in Definition 2.1 and defined on quivers with commuting relations with the following properties (see Theorem 4.7). For one in each pair, with framing in Definitions 4.1 and 4.2 in terms of a finite group of tensor products of Auslander-Reiten transformations, we have a moduli space of stable Lagrangian connect sums of graded

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Lagrangian vanishing cycles; this moduli space via the Serre-de Rham functor in Definition 4.5 gives the Calabi-Yau Fermat variety in $\mathbb{P}^{-1}$ defined by the zero locus of $X_n$. For the other one in the pair, graded Lagrangian vanishing cycles are stable.

Stable objects above are stable representations of quivers even when we forget relations. In particular, graded Lagrangian vanishing cycles trivially satisfy self-dual equations. In this paper, connections are over transverse intersections of graded Lagrangian vanishing cycles and connect trivial vector bundles on graded Lagrangian vanishing cycles; so, they are maps between vector spaces and consistency with grading make them representations of quivers with relations.

2 Mirror stability conditions

Let us recall the notion of stability conditions. For a triangulated category $\mathcal{T}$, each stability condition is determined by a bounded $t$-structure and a stability function $Z$, which gives central charges of elements of the Grothendieck group of $\mathcal{T}$, on the heart of the $t$-structure.

We go on with explicit examples for our later use. Let $A_{n-1}$ denote the A-type Dynkin quiver of vertices 0, . . . , $n - 1$ and arrows $0 \to \ldots \to n - 1$ and let $A_{n-1}^\otimes n$ denote the $n$-fold tensor product of $A_{n-1}$ (see [23] for the general definition of tensor products of quivers).

Each vertex $a$ of $A_{n-1}^\otimes n$ can be labelled by $n$-tuples $a^1, \ldots, a^n$ for $0 \leq a^i \leq n - 1$. We have an arrow $a \to b$ when $b^i - a^i = 0$ for all $i$ but at most one $j$ such that $b^j - a^j = 1$; we let $\lambda(a, b) = j$ if we have such $j$ or $\lambda(a, b) = 0$ if $a = b$. We have commuting relations on arrows. For our convenience in this paper, for each vertex $a$ of $A_{n-1}^\otimes n$, we call the number $\lambda(a) := \sum a^i$ the index of $a$.

For example, the following figure shows commuting arrows on rectangles which connect vertices of indices 0, 1, 2 for $A_4^\otimes 5$.

![Diagram](image1.png)

Figure 1: A part of $A_4^\otimes 5$
For the triangulated category $\text{D}^b(\text{mod} \ A_n^{\otimes n})$ and the heart mod $A_n^{\otimes n}$ of the bounded $t$-structure, we have a stability function which maps simple representations into the upper-half plane of the complex plane. We mean by $Z_n$ a stability function on the heart mod $A_n^{\otimes n}$ with the following conditions: slopes of simple representations, which are one-dimensional representations over vertices, strictly decreases as their indices increases and central charges of simple representations of the same indices are the same. Choices of such stability functions gives the open submanifold of the stability manifold of $\text{D}^b(\text{mod} \ A_n^{\otimes n})$.

The following figure shows one of such stability functions on mod $A_4^{\otimes 5}$. Arrows indicate increases of coordinates of the complex plane. Central charges of simple representations put dots. For example, central charges of simple representations of vertices with the index 1 put the second left-most dot.

![Figure 2: A stability function $Z_5$ on mod $A_4^{\otimes 5}$](image)

For the stability function above, we have the stability condition of $\text{D}^b(\text{mod} \ A_4^{\otimes 5})$ with the stability function $-\bar{Z}_5$ on the same heart mod $A_4^{\otimes 5}$.

![Figure 3: The stability function $-\bar{Z}_5$ on mod $A_4^{\otimes 5}$](image)

For our convenience in this paper, we call the stability condition of $\text{D}^b(\text{mod} \ A_n^{\otimes n})$.
determined by the stability function $-\bar{Z}$ on the heart mod $A_n^\oplus$ is mirror to the stability condition determined by the stability function $Z$ on the same heart mod $A_n^\oplus$. Between these mirrors, we have wall-crossing paths on the stability manifold of $D^b(mod A_n^\oplus)$. We put the formal definition as follows.

**Definition 2.1.** Let $A$ be the heart of a bounded $t$-structure of a triangulated category $T$ such that $A$ is isomorphic to the category of representations of a quiver with or without relations. Let $Z$ be a stability function on the heart such that central charges of simple representations are in the upper-half plane of the complex plane. We call the stability function $-\bar{Z}$ on $A$ mirror to the stability function $Z$, and the stability condition with the stability function $Z$ on $A$ mirror to the other.

For a nontrivial directed acyclic graph such as of $A_n^\oplus$, we have a stability condition with stable objects being simple representations, and we have the mirror stability condition with a representation of a nontrivial support being a stable object.

We can state notions in Definition 2.1 in a more general way; however, without a finiteness property on the heart such as [8, Proposition 2.3], even if we assume the existence of a stability condition, the existence of the mirror stability condition is obscure. Examples of mirror stability conditions in terms of spherical functors or wall-crossings can be found in [4, 7, 9, 16, 22, 24, 27, 29, 30].

### 3 Recap of homological mirror symmetry

In this paper, derived equivalence $\cong$ means that on both sides of the equivalence, we have compact generating objects with $A_\infty$-isomorphic $A_\infty$ enhancements; this is an example of so-called derived Morita equivalence.

In [28], we have seen that $X_n$ is self-dual up to the equivariance with respect to the group $H_n$ which consists of $n$-tuples of $n$-th roots of the unity modulo diagonals and which acts on coordinates. For example, we have $D^b_{H_n} (\text{Coh } X_n) \cong F^S(X_n)$ for the $H_n$-equivariant bounded derived category of coherent sheaves of the Calabi-Yau Fermat variety and the Fukaya-Seidel category of $X_n$ (see [14] for a different account).

We have a morsification of $X_n$ and an $A_\infty$ algebra of graded Lagrangian vanishing cycles in terms of Lagrangian intersection theories such that the $A_\infty$ algebra is $A_\infty$-isomorphic to the Yoneda (Ext) algebra of simple representations of $A_n^\oplus$. For simple representations $S_a$ and $S_b$ of vertices $a$ and $b$ of $A_n^\oplus$ with an arrow $a \to b$, we have an one-dimensional $\text{Ext}^1(S_a,S_b)$ morphism and these morphisms along arrows anti-commute.

For each $0 \leq i \leq n-1$, let $O_n^i$ denote the object $\Omega_{\mathbb{P}^{n-1}}^{n-1-i}(n-1-i)[i]$ restricted on the Calabi-Yau Fermat variety in $\mathbb{P}^{n-1}$. Bases of $\text{Ext}^1(O_n^i,O_n^{i+1})$ can be given by morphisms $dx_{n,i}^j$ for $1 \leq j \leq n$ such that we have anti-commuting relations $dx_{n,i}^j dx_{n,i-1}^{j'} = -dx_{n,i}^{j'} dx_{n,i-1}^j$ for $1 \leq j, j' \leq n$. Putting (weighted) copies of
\[dx^j_{n,i} \text{ on arrows } a \to b \text{ such that } \lambda(a,b) = j \text{ and } \lambda(a) = i, \text{ it is easy to check that } H_n\text{-equivariant objects of } O^b_n \text{ give also the Yoneda algebra.}\]

For the dual statement of \(D^b_{\hat{H}_n} \text{ Coh}(X_n) \cong \text{ FS}(X_n)\), enhancing the argument above, we have defined the \(\hat{H}_n\) quotient \(\text{FS}^{\hat{H}_n}(X_n)\) of \(\text{FS}(X_n)\) as the perfect derived category of the dg \(\hat{H}_n\)-orbit category (see \([19]\)) of a dg algebra \(A\) which is \(A_\infty\)-isomorphic to the \(A_\infty\) algebra of objects in \(\hat{H}_n\)-orbits of graded Lagrangian vanishing cycles so that the dg \(\hat{H}_n\)-orbit category is a dg enhancement of the Yoneda algebra of objects \(O^b_n\). This is a formal application of Pontryagin duality on categories of dg categories, and \(D^b(\text{Coh } X_n) \cong \text{ FS}^{\hat{H}_n}(X_n)\). As for the existence of such \(A\), we can take the dg algebra of \(\hat{H}_n\)-graded matrix factorizations of \(O^b_n\) in terms of the dg category of graded matrix factorizations of \(X_n\) \([31]\). We have \(\text{FS}(X_n) \cong D^b(\text{mod } A_{n-1}^{\otimes n}) \cong D^b(\text{mod } A_{n-1})^{\otimes n}\) in terms of dg tensor products (see \([18]\)). We have tensor products of Auslander-Reiten transformations \(\tau\) of \(D^b(\text{mod } A_n)\) such that \(\tau^n \cong [2]\). Actions of the finite group consisting of \(\tau^1 \otimes \cdots \otimes \tau^t\) such that \(\sum t_i = 0\) coincides with those of \(\hat{H}_n\) above and the index of each simple representation of \(A_{n-1}^{\otimes n}\) stays the same by the actions.

### 4 Main statement

Directly, it is highly nontrivial to discuss stability conditions and moduli spaces on complexes of \(O^b_n\) with nontrivial \(A_\infty\) structures to take into account (see \([13]\)), even with motivating realizations in terms of coherent sheaves or Lagrangians.

By taking equivariance, we have stability conditions on \(\text{mod } A_{n-1}^{\otimes n}\), and on \(D^b_{\hat{H}_n} (\text{Coh } X_n)\). Still, an issue we face to consider stability conditions on \(D^b(\text{Coh } X_n)\) with stability conditions on \(\text{mod } A_{n-1}^{\otimes n}\) is that such stability conditions are not invariant under \(\hat{H}_n\): because, not all objects which consist of the \(\hat{H}_n\)-orbit of a simple representation of \(A_{n-1}^{\otimes n}\) are representations of \(A_{n-1}^{\otimes n}\). If they were invariant, then we would have taken advantages of the paper \([35]\) by Polishchuk and the paper \([25]\) by Macr`ı-Mehrotra-Stellari.

We overcome the issue by taking the notion of framed \(\hat{H}_n\)-invariance on stability conditions in Definition \([4,1]\) and representations and morphisms in Definition \([4,2]\); instead of full products of general linear groups over vertices, we take ones which commutes with \(\hat{H}_n\) actions restricted on simple representations of \(A_{n-1}^{\otimes n}\).

**Definition 4.1.** We say that a stability function \(Z\) on \(\text{mod } A_{n-1}^{\otimes n}\) is framed \(\hat{H}_n\)-invariant, if central charges of simple representations of \(A_{n-1}^{\otimes n}\) of each index are the same.

For example, stability functions \(Z_i\) and their mirrors in Section \([2]\) are framed \(\hat{H}_n\)-invariant. To define framed \(\hat{H}_n\)-invariant representations, for each representation \(E\) of \(A_{n-1}^{\otimes n}\), let \(E_{a,b} : E_a \to E_b\) denote commuting linear maps along arrows \(a \to b\); in particular, \(E_{a,a}\) are identity maps on \(E_a\).
**Definition 4.2.** We say that a representation $E$ of $A_{n-1}^{\otimes n}$ is framed $\hat{H}_n$-invariant, if for vertices $a, b$ of $A_{n-1}^{\otimes n}$ with the same indices, we have framing isomorphisms $\phi_{a,b}: E_a \rightarrow E_b$ with the following conditions. For vertices $a, b, c, a', c'$ such that $a \rightarrow a'$ and $c \rightarrow c'$ with $\lambda(a, a') = \lambda(c, c')$ and $\lambda(a) = \lambda(b) = \lambda(c)$, we have $\phi_{a',c'}E_{a,a'} = E_{c,c'}\phi_{b,c}\phi_{a,b}$; i.e., we have the following commuting diagram such that squig arrows indicate isomorphisms and plain arrows indicate maps of a representation.

\[
\begin{array}{ccc}
a & \sim & b \\
\downarrow & \circlearrowleft & \downarrow \\
\quad & c \quad & \\
\end{array}
\]

For representations $E$ and $F$ of $A_{n-1}^{\otimes n}$ with framed $\hat{H}_n$-invariance, we say that a morphism $f : E \rightarrow F$ of $A_{n-1}^{\otimes n}$ is framed $\hat{H}_n$-invariant, if for vertices $b, b'$ such that $\lambda(b) = \lambda(b')$, we have $\phi_{b,b'}f_\lambda E = f_\lambda E$.

Let $\text{fmod } A_{n-1}^{\otimes n}$ denote the category of framed $\hat{H}_n$-invariant representations which consists of framed $\hat{H}_n$-invariant representations and morphisms of $A_{n-1}^{\otimes n}$.

In Definition 4.2 by putting $a = c$ and $a' = c'$, we see that $\phi_{b,c}\phi_{a,b} = \phi_{a,a}$, by putting $a = c$, we have $\phi_{b,a}\phi_{a,b} = \phi_{a,a}$, and by putting $a = b = c$, we have $\phi_{a,a} = 1_{E_a}$. We have uniqueness of framing isomorphisms in the following sense.

**Lemma 4.3.** Let $E$ be a framed $\hat{H}_n$-invariant representation of $A_{n-1}^{\otimes n}$ supported over vertices $0, \ldots, n-1$. For vertices $a_i$ of $A_{n-1}^{\otimes n}$ such that $\lambda(a_i) = i$ for $0 \leq i \leq n-1$, we have a framed $\hat{H}_n$-invariant isomorphism $t^E : E \rightarrow E'$ such that $E'$ is a framed $\hat{H}_n$-invariant representation of $A_{n-1}^{\otimes n}$ with trivial framing isomorphisms and $t^E_a = \phi_{a,a_\lambda(a)}$ for vertices $a$ of $A_{n-1}^{\otimes n}$.

**Proof.** For vertices $b_i$ of $A_{n-1}^{\otimes n}$ with $\lambda(b_i) = i$, we let $E'_b := E_{a_i}$, and for arrows $b_i \rightarrow b_{i+1}$, we let $E'_{b_i,b_{i+1}} := \phi_{b_i,b_{i+1}}E_{b_i,b_{i+1}}\phi_{a_i,b_i}$. For vertices $b'_i, b'_{i+1}$ with $\lambda(b'_i, b_{i+1}) = \lambda(b'_i, b_{i+1})$ and $\lambda(b_i, b'_i) = \lambda(b_{i+1}, b_{i+1})$, we have $E'_{b'_i,b_{i+1}}E'_{b_i,b_{i+1}} = E'_{b'_i,b_{i+1}}E_{b_i,b_{i+1}}$. For arrows $b_i \rightarrow b_{i+1}$, we have $t^E_{b_i,b_{i+1}}E_{b_i,b_{i+1}} = E_{b'_i,b_{i+1}}t^E_{b_i,b_{i+1}}$. For vertices $b'_i$ with $\lambda(b'_i) = \lambda(b'_i)$, we have $\phi_{b'_i,b'_i}t^E_{b'_i,b'_i} = t^E_{b'_i,b'_i}$. \qed

Let us recall the quiver $B_{n-1}$, called the Be˘ılinson quiver of $\mathbb{P}^{n-2}$ [5, 29], which has $n$ vertices $0, \ldots, n-1$ and $n-1$ arrows from $i$ to $i+1$ with commuting relations. For vertices $i, i+1$, we label arrows by $s$ for $1 \leq s \leq n$ so that for maps $E_{i,i+1}^s$ on labelled arrows $i \rightarrow i+1$, we have $E_{i,i+1}^sE_{i,i+1}^t = E_{i,i+1}^tE_{i,i+1}^s$.

**Proposition 4.4.** The full subcategory of $\text{fmod } A_{n-1}^{\otimes n}$ consisting of representations supported over vertices with indices $0, \ldots, n-1$ is equivalent to mod $B_{n-1}$.

**Proof.** For each representation $E$ of $B_{n-1}$, we put the representation $F(E)$ of $A_{n-1}^{\otimes n}$ as follows. For vertices $a, b$ of $A_{n-1}^{\otimes n}$ with arrows $a \rightarrow b$, we put linear maps $F(E)_{a,b} := E_{\lambda(a),\lambda(b)}^E$ from $F(E)_a := E_{\lambda(a)}$ to $F(E)_b := E_{\lambda(b)}$. For each
morphism \( f : E \to E' \) of representations \( E \) and \( E' \) of \( B_{n-1} \), we put \( F(f) : F(E) \to F(E') \) by \( F(f)_a := f_{\lambda(a)} \) for each vertex \( a \).

To obtain an inverse of \( G \), let \( a_i \) be vertices of \( A_{n-1}^{\otimes n} \) such that \( \lambda(a_i) = i \) for \( 0 \leq i \leq n - 1 \). For \( E \) of \( \text{mod} \ A_{n-1}^{\otimes n} \) supported over vertices with the indices, we put \( G(E)_i := E_{a_i} \), and for arrows \( \alpha \to \beta \) such that \( \lambda(\alpha) = \lambda(\beta) \), we put the linear map \( G(E)_{\lambda(\alpha),\lambda(\alpha+1)} := \phi_{b_{\alpha,a_{\alpha+1}}} E_{a_{\beta},b_{\alpha,a_{\alpha}}} \), which is independent for choices of such arrows \( \alpha \to \beta \). For \( 1 \leq c,c' \leq n - 1 \) and \( 0 \leq i - 1 \leq n - 3 \), let us note that we have a vertex \( b_{i-1},b_i,b_{i+1} \) of \( A_{n-1}^{\otimes n} \) with \( \lambda(b_{i-1}) = i - 1 \), \( \lambda(b_{i-1},b_i) = \lambda(b_{i+1}) = c \), and \( \lambda(b_{i-1},b_i) = \lambda(b_{i+1}) = c' \); so, we have \( G(E)_{i+1,j+1} G(E)_{j+1,i} = G(E)_{i+1,j+1} G(E)_{j+1,i} \). For a framed \( \hat{H}_n \)-invariant morphism \( f : E \to E' \), we put \( G(f) : G(E) \to G(E') \) such that \( G(f)_a = f_{\lambda(a)} \); we have \( G(E')_{i+1} G(E)_{i+1} = G(f_{i+1} G(E')_{i+1} \).

For the functor \( FG \), a framed \( \hat{H}_n \)-invariant morphism \( f : E_1 \to E_2 \), and \( t^{E_1},t^{E_2} \) in the notation of Lemma 4.3, we have \( t^{E_1} f_a = \phi_{a,a_{\lambda(a)}} f_a = f_{a_{\lambda(a)}} \phi_{a,a_{\lambda(a)}} = FG(f)_a t^{E_2} \) for each vertex \( a \) of \( A_{n-1}^{\otimes n} \). For the functor \( GF \), we take the identity functor on \( \text{mod} \ A_{n-1}^{\otimes n} \).

Similar statements to the one in Proposition 4.3 can be obtained by taking other supporting vertices. For \( n = 3 \) and 4, let us mention that derived categories \( D^b(\text{Coh} \, \mathbb{P}^{n-2}) \), which is derived equivalent to \( D^b(\text{mod} \, B_{n-1}) \), have been described in terms of graded Lagrangian vanishing cycles and Lagrangian intersection theories in [3]. With Serre twists in use, let us define the *Serre-de Rham functor* \( SdR \) as follows.

**Definition 4.5.** For each representation \( E \) of \( \text{mod} \, B_{n-1} \), we put the chain \( SdR(E) \) of morphisms \( \sum_{1 \leq j \leq n} E_{i+1}^j \otimes d_{i+1}^j : E_i \otimes O_n \to E_{i+1} \otimes O_{n+1}^j \) for \( 0 \leq i \leq n - 2 \). For each morphism \( f : E \to F \) in \( \text{mod} \, B_{n-1} \), we put the chain map \( SdR(f) : SdR(E) \to SdR(F) \) such that \( f_i \otimes id_{O_n} : E_i \otimes O_n \to F_i \otimes O_n \) for \( 0 \leq i \leq n - 1 \).

The following is in order.

**Proposition 4.6.** For each representation \( E \) of \( \text{mod} \, B_{n-1} \), we have that \( SdR(E) \) is a complex and \( SdR \) is a functor from \( \text{mod} \, B_{n-1} \) to \( D^b(\text{Coh} \, X_n) \).

**Proof.** Each \( E \) satisfies commuting relations among maps. This translates into zero compositions of consecutive morphisms. Morphisms between objects of \( \text{mod} \, B_{n-1} \) become morphisms of complexes. \( \square \)

For a given framed \( \hat{H}_n \)-invariant stability function \( Z \) on \( \text{mod} \, A_{n-1}^{\otimes n} \), we define the stability function \( Z' \) on \( \text{mod} \, B_{n-1} \) by putting \( Z'(a) := Z(a) \) for a vertex \( a \) of \( A_{n-1}^{\otimes n} \) such that \( \lambda(a) = i \). In the following, we write \( Z' \) on \( \text{mod} \, B_{n-1} \) also as \( Z \) on \( \text{mod} \, B_{n-1} \).

**Theorem 4.7.** For a stability function \( Z_n \) on the category of framed \( \hat{H}_n \)-invariant representations of \( A_{n-1}^{\otimes n} \), we have a moduli space of stable Lagrangian connect sums of graded Lagrangian vanishing cycles for a morrsification of \( X_n \).
such that the Serre-de Rham functor localizes the moduli space into the Calabi-Yau Fermat variety in \( \mathbb{P}^{n-1} \). For the mirror of \( Z_n \) on the category of representations of \( A_{n-1}^n \), each graded Lagrangian vanishing cycle is stable.

Proof. For a stability function \( Z_n \) on \( \text{mod} \ B_{n-1} \) and stable representations \( E \) of \( B_{n-1} \) with the dimension vector \( (1, \ldots, 1) \), commuting relations and the indecomposable property of the representation give nonzero \( k_i^E \) such that \( k_i^E E_{i,i+1}^s = E_{i-1,i}^s \) for \( 1 \leq s \leq n \) and \( 0 \leq i \leq n-1 \).

If objects \( O_i^E \) were unrestricted on the Calabi-Yau Fermat variety, then \( \text{SdR}(E) \) would have been Koszul resolutions of skyscraper sheaves of points in \( \mathbb{P}^{n-1} \). For non-isomorphic representations \( E \) of \( \text{mod} \ B_{n-1} \), objects \( \text{SdR}(E) \) of \( D^b(\text{Coh} \ X_n) \) are non-isomorphic objects supported over distinct points of the Calabi-Yau Fermat variety, unless isomorphic to the zero object outside.

For the mirror of \( Z_n \) on \( \text{mod} \ A_{n-1}^{\otimes n} \), each stable object is isomorphic to a simple object of \( \text{mod} \ A_{n-1}^{\otimes n} \).

Stable objects of Theorem 4.7 in terms of the mirror \( Z_n \) are simple representations of the quiver \( A_{n-1}^{\otimes n} \). We may as well take the mirror of \( Z_n \) on the category of framed ones of \( A_{n-1}^{\otimes n} \); in this case, we obtain polysimple representations of \( A_{n-1}^{\otimes n} \) consisting of graded Lagrangian vanishing cycles.

For projective spaces, and for Calabi-Yau hypersurfaces of \( \mathbb{P}^{n-1} \) represented as \( x_n^1 + \ldots + x_n^n + \psi \cdot x_1 \cdots x_n : \mathbb{C}^n \to \mathbb{C} \) for \( \psi \in \mathbb{C} \), we may obtain similar statements to Theorem 4.7.

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