Detection of pulse signal in chaotic noise background using extreme learning machine

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Abstract: It is still a challenge to detect the useful signals under chaotic noise background with effective methods. Difficults such as suppression of useful signals, large computation and low sensitivity commonly exist the new method based on the nonlinear characteristics of signals, such as neural network method, and traditional methods. On the contrast, The extreme learning machine has advantages of strong nonlinear approximation, simple structure, high precision, fast learning and training speed, etc. On this basis, this paper proposes a method of utilizing extreme learning machine based on least square to get the output weight, in order to train the model to detect the weak pulse signal.

1. Introduction
In many scientific applications, it is hard to detect the weak signal which is non-stationary or submerged in chaotic noise or with low amplitude [1]. Its application includes such as seismic signal detection, oil exploration, medical treatment, early fault diagnosis of large equipment, etc.

Traditional weak signal detection is mostly based on linear method, which amplifies both useful signal and background noise [2]. For the sampling integral method based on time domain, its detection accuracy and threshold value depend on the length of the sampling integral time. This leads to poor real-time performance of the detection signal.

Fourier transforms and the wavelet transformation based on time frequency analysis, are valid for detecting non-stationary weak signal [3, 4]. However, the signal extraction under the background of strong noise has limitations, which leads to inaccurate extraction results, low reliability, and the wavelet basis function in the wavelet transform has no choice standard.

With the research of nonlinear system, some new weak signal detection methods appear. Haykin et al.[5] firstly introduced chaos theory and phase space reconstruction technique to research weak signal detection. Subsequent methods, such as Duffing vibration subsystem and Birkhoff- Shaw vibration subsystem, are both effective to detect weak signals in chaotic background, but most of these methods are low adaptable, heavy calculation burden, and not sensitive to initial values [6, 7].

More and more scholars apply nonlinear model to detect weak pulse signal under chaotic noise such as neural network and support vector machine [8, 9]. However, the prediction accuracy of neural network has been restricted by its own disadvantages, and support vector machine is difficult to implement for large-scale training samples.

Extreme learning machine was proposed by Huang et.al. [10] is a simple feedforward network with only one hidden layer, which is composed of input layer, hidden layer and output layer. The input weights (connect the input layer and the middle layer) are randomly generated and are not adjusted in the whole learning process of the network. The extreme learning machine maps the input variable to the high-dimensional space through the hidden layer activation function, and then carries on the linear
representation of the high-dimensional space state by learning the output weight, and finally approximates the output variable.

2. Detection of weak pulse signal under chaotic noise background

2.1 Weak pulse signal detection

Abstract the problem of detecting weak pulse signals in chaotic noise background into the following hypothesis testing problem:

\[ H_0: x(t) = c(t) + n(t) = \tilde{n}(t) \]
\[ H_1: x(t) = c(t) + s(t) + n(t) = \tilde{n}(t) + s(t) \]

where \( x(t) \) is the observed signal, \( c(t) \) is the chaotic noise, \( s(t) \) is the weak pulse signal, \( n(t) \) is white noise of zero mean, \( \tilde{n}(t) \) represents the sum of \( c(t) \) and \( n(t) \).

Because the weak pulse signal \( s(t) \) is submerged in the chaotic noise \( c(t) \), the chaotic noise needs to be estimated firstly. In this paper, the ELM is applied to complete the estimation.

2.2 An extreme learning machine detection model based on phase space reconstruction

2.2.1 Phase space reconstruction

A certain phase point in the reconstructed phase space can be expressed as \( X(t) = (x(t), x(t-\tau), \ldots, x(t-(m-1)\tau))^T \), \( t = n_1, n_1 + 1, \ldots, n_1 + n_1 = 1 + (m-1)\tau \). For each point in the reconstructed phase space, there is a smooth mapping \( f: R^m \rightarrow R \) \( X(t+1) = f(X(t))(t = n_1, n_1 + 1, \ldots, n_1 + n - 1) \). If the approximate mapping \( f^\prime \) can be obtained, the system can be restored and the next point \( x(t+1) \) can be predicted. In this paper, the complex autocorrelation method is used to solve the delay time \( \tau \), Cao is used to solve the embedding dimension \( m \).

2.2.2 Single output extreme learning machine detection model

For samples input \( X = (X_1, X_2, \ldots, X_N)^T \), the dimension of a single sample \( X(t) \) is determined by the embedded dimension of the reconstruction \( X = (x(t), x(t-\tau), \ldots, x(t-(m-1)\tau )) \).

Fig.1 shows the structure diagram.

Figure 1. The structure of the extreme learning machine based on reconsitution

Its mathematical expression is shown as follows:

\[ x(t+1) = w^T F(w^{in} \cdot X + b) + \varepsilon \]

where \( x(t+1) \) is the output corresponding to the extreme learning machine,
Input weights $w^{in}$ and bias $b$ are randomly initialized, thus making necessary only the optimization of the weights of the output layer. When a sample inputs into the network, the activation function and connection weight are used to approximate the target value of $N$ samples, then the weights of the output layer are computed by

$$w = G'T, \quad w = (G'G)^{-1}G'T,$$

where, $\dagger$ is the generalized inverse operation, $T$ is the target output.

2.3 Process of detection signal implementation

If we consider all points of error distribution, it would inevitably affect the effectiveness of the detection. For this case, we propose to design a threshold value $\varepsilon_0$, in order to reduce the difference in the observed signal whether exist weak pulse signal in this hypothesis testing.

$$\varepsilon_0 = \frac{\overline{\varepsilon}}{2},$$

where $\overline{\varepsilon} = \frac{1}{n} \sum \varepsilon_i$, $\varepsilon_i$ is the value of err which $\varepsilon_i > \frac{\varepsilon_{max}}{2}$.

The steps of detection process signal are shown in Fig. 2.

![Figure 2. Flow chart of signal detection process](image)

Step.1: Obtain the observed signal data.
Step.2: Solve the delay time and the embedding dimension, then reconstruct the phase space.
Step.3: To establish ELM model, and calculate the prediction error $e(t + 1) = x(t + 1) - f_i(X(t))$.
Step 4: Using the prediction error to detect whether the weak pulse signal exist.

3. Test results and analysis

In this part, three experiments are carried out to verify the feasibility and effectiveness of the proposed model. SNR is defined as the signal-to-noise ratio of the detected signal. To evaluate the performance of the model degree, accuracy $ac$, sensitivity $se$, precision $pr$ are introduced. If the pulse signal exist at a certain point, it is positive or, it is negative. TP is true positive, indicating the correct judgment
that there exist an impulse signal. TN is true negative, indicating the correct judgment that there not exist pulse signal. FP is false positive, indicating that there not exist pulse signal, but it is wrongly judged. FN is false negative, indicating that there exist impulse signal, but it is wrongly judged. 

\[ n(t) \]

is white noise with a mean of 0 and a variance of 0.5.

\[
ac = \frac{TP + TN}{TP + TN + FP + FN}, \quad se = \frac{TP}{TP + FN}, \quad pr = \frac{TP}{TP + FP}, \quad SNR = 10 \log \left( \frac{\sigma_a^2}{\sigma_c^2 + \sigma_n^2} \right)
\]  

\( \sigma_a^2 = \frac{1}{n} \sum_{i=1}^{n} (x(t) - \bar{x}(t))^2, \quad \sigma_c^2 = \frac{1}{n} \sum_{i=1}^{n} (c(t) - \bar{c}(t))^2 \), \( \sigma_n^2 = 0.5 \)

where

The chaotic time series generated by Lorenz chaotic equation is used as chaotic noise.

\[
\dot{x} = a(y - x), \quad \dot{y} = xy - bz, \quad \dot{z} = (c - z)x - y
\]

The demonstration of Lorenz chaotic system is shown in Fig.3. Set parameters \( a = 10, \quad b = 3/8, \quad c = 28 \), the initial condition \( x(0) = y(0) = z(0) = 1.0 \). The \( x \) component is used as the chaotic noise background \( c(t) \) as shown in Figure 5.

Figure 3. (a) Bifurcation diagram of Lorenz system; (b) the phase diagram of Lorenz chaotic system

Figure 4. Chaotic background noise \( c(t) \)
**Experiment 1: Detection performance of monocycle pulse signal with the proposed model**

The periodic signal \( s(t) = k_i s_i(t) \), where \( k_i \) is the coefficient to control SNR,

\[
    s_i(t) = \begin{cases} 
        1, & t = 450, 900, 1350, \ldots \\
        0, & \text{others}
    \end{cases}
\]

(6)

When \( k_i = 6.5 \), the SNR is -68dB. \( X(t) \) is the input of the extreme learning machine and \( x(t+1) \) is its output. The demonstration of experiment 1 is shown in Fig.5

![Figure 5](image)

**Figure 5. The result of experiment 1**

Figure 5 (a) shows the pulse signal, the figure 5 (b) shows observation signals, figure 5 (c) shows the prediction error. The figure 5 (c) shows the result of detection, combined with hypothesis testing, can judge observation signal exists weak pulse signal.

By changing the value of \( k_1 \), the experiment was carried out to detect the pulse signal under different SNR conditions. The results are shown in Table 1.

As can be seen from Table 1, among the three dimensions of accuracy, sensitivity and accuracy, the weak monocycle pulse signal detected by the model in this paper performs well. However, when SNR reaches -73.37, the sensitivity and accuracy are significantly affected.

| \( k_1 \) | SNR/dB | ac  | se  | pr  |
|-----------|--------|-----|-----|-----|
| 7.0       | -64.53 | 100%| 100%| 100%|
| 6.5       | -66.02 | 100%| 100%| 100%|
| 6.0       | -67.62 | 100%| 100%| 100%|
| 5.5       | -69.36 | 99.97%| 100%| 88.89%|
| 5.0       | -71.26 | 99.97%| 100%| 88.89%|
| 4.5       | -73.37 | 99.82%| 100%| 53.33%|
| 4.0       | -75.73 | 99.50%| 100%| 28.57%|
| 3.5       | -78.40 | 98.84%| 100%| 14.81%|
| 3.0       | -81.48 | 95.96%| 100%| 4.76%|

**Experiment 2: Detection performance of different models**

In this paper, neural network with a single hidden layer and linear regression without phase space reconstruction are selected to compare with the proposed model under different SNR. The results are shown in Table 2.
Table 2. Comparison of detection performance of different models under the same SNR

| Model               | The proposed model | Neural network | Linear regression |
|---------------------|--------------------|----------------|--------------------|
| SNR/dB              | -66.02             | -71.26         | -73.37             |
| ac                  | 100%               | 99.97%         | 98.82%             |
| se                  | 100%               | 100%           | 100%               |
| pr                  | 100%               | 88.89%         | 53.33%             |

As can be seen from Table 2, the single hidden layer neural network has unstable detection performance and long training time for pulse signals under different SNR. It can be seen that the proposed model and the linear regression model are better than the single hidden layer neural network.

**Experiment 3: Comparison in the superposition periodic signal detection performance**

The pulse signal is consisted of two periodic signals $S(t) = k_1s_1(t) + k_2s_2(t)$, $k_1$ and $k_2$ are the coefficient to control SNR.

$$s_1(t) = \begin{cases} 1, & 450,900, \ldots \\ 0, & \text{others} \end{cases}, \quad s_2(t) = \begin{cases} 1, & 600,1200, \ldots \\ 0, & \text{others} \end{cases}$$

Table 3. Comparison of the two models for superimposed periodic signals

| $k_1$ | $k_2$ | SNR  | The proposed model | Linear regression model |
|-------|-------|------|--------------------|------------------------|
|       |       |      | ac | se | pr | ac | se | pr |
| 8.0   | 7.0   | -59.07 | 100% | 100% | 100% | 100% | 100% | 100% |
| 7.0   | 6.0   | -61.91 | 100% | 100% | 100% | 99.90% | 80% | 80.00% |
| 6.0   | 5.0   | -65.23 | 100% | 100% | 100% | 99.85% | 75.00% | 75.00% |
| 5.0   | 4.0   | -69.78 | 99.90% | 91.67% | 78.60% | 99.50% | 75.00% | 34.62% |
| 4.0   | 3.0   | -74.15 | 99.24% | 91.67% | 27.50% | 98.26% | 66.67% | 10.96% |
| 3.5   | 2.5   | -77.16 | 97.96% | 91.67% | 12.09% | 97.75% | 75.00% | 9.47% |

It can be seen from the experimental data in Table 3 that when the SNR is -59.07, the detection performance of the two models has little difference, and the impulse signals can be accurately detected. As the increasing of SNR, the detection performance of the two models is obviously different, and the detection performance of the model in this paper is better.

4. Conclusion

In this paper, our main motivation is to combine the phase space reconstruction with extreme learning machine and applied to the detection of weak pulse signal under chaotic background. The model proposed in this paper for weak periodic pulse signal detection in chaotic noise background, when SNR is between -64.54 to -67.62, the pulse signal points can be detected accurately, but with the SNR increases gradually reached 69.36, the detection accuracy also gradually reduce. When the SNR is greater than -72 and the detection accuracy drops. When detecting the signal with high SNR, this method needs to be improved. At the same SNR, the detection effect of the model in this paper is more stable than the single hidden layer neural network, which not only improves the accuracy, sensitivity and accuracy, but also can detect signals quickly. The comparison between single period pulse signal detection and non-reconstructed linear model is not obvious. It is found through experiments that with the increase of SNR, the detection performance of the model in this paper is better.

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