DETERMINING NUMBERS OF COLORING $\lambda$-BACKBONE ON SPLIT GRAPH

Fatanur Baity Tsulutsya$^{1}$, Evawati Alisah$^{2}$, and Lailiy Kurnia Ilahi$^{3}$

$^{1}$Engineering Faculty of Trunojoyo University, Jl. Raya Telang PO BOX 2 Kamal, Bangkalan, 69162, Indonesia
$^{2}$Department of Mathematics, Faculty of Sains and Technology, UIN Maulana Malik Ibrahim Malang, Indonesia
$^{3}$Department of English, Faculty of Letters, Universitas Negeri Malang, Indonesia

fatanurbaity.math@gmail.com

ABSTRACT

Vertex coloring on a graph $G = (V(G), E(G))$ is giving color for each point on the graph so that there are no two connected directly the same color. A vertex coloring $f$ from graph $G$ is called coloring Backbone - $\lambda$ of $(G, H)$ if fulfilled $|f(u) - f(v)| \geq \lambda$. The smallest number $k$ where there is backbone coloring $f : V \rightarrow \{1, 2, 3, ... k\}$ is called several Backbone coloring - $\lambda$ and denoted by $B_{\lambda}(G, H)$. A graph used in this research is a split graph.

This paper presents the process or steps to determine coloring number $\lambda$-backbone on a split graph. As for the steps is as follows: determine split graphs, give 1 example of Spanning subgraph (Backbone) containing subgraphs complete maximum of split graphs and contains Hamilton trajectories and give a point coloring on the Hamilton trajectory backbone of the split graph.

Keyword: Graph, Split Graph, Vertex coloring, Coloring Backbone - $\lambda$, Path Hamilton, clique, Number chromatic

INTRODUCTION

One branch of science mathematics that is useful in everyday life is graph theory. When this graph theory is increasingly growing and interesting because of its uniqueness and much once applied. The uniqueness of graph theory is the simplicity of the subject he learned because it can be presented as a point (vertex) and side (edge).

Split graphs are $G$ graphs whose set of points can be partitioned into a clique and free set, the split graph is a subpart of the graph perfect so that split graphs meet $\chi(G) = \alpha(G) = k$. Graph perfect as a graph with clique numbers and chromatic numbers the same. The clique number is defined as the maximum order of a complete subgraph on graph $G$. While chromatic numbers are defined as the smallest number of colors given at points in graph $G$ such that for every two points that are directly connected get different colors. In graph theory, there are several types of coloring, one of them is vertex coloring. Vertex coloring on a graph $G = (V(G), E(G))$ is giving color to each point on the graph such that no two vertexes are directly connected to the same color.

A vertex coloring $f$ from graph $G$ is called coloring Backbone - $\lambda$ of $(G, H)$ if fulfilled $|f(u) - f(v)| \geq \lambda$. The smallest number $k$ where there is backbone coloring $f : V \rightarrow \{1, 2, 3, ... k\}$ is called several Backbone coloring - $\lambda$ and denoted by $B_{\lambda}(G, H)$. 
1. Graph

Graph $G$ is defined as a pair of sets $(V(G), E(G))$ where $V(G)$ is a non-empty set of elements called points (vertices) and $E(G)$ is a set of unsorted pairs, $(u, v)$ from points $u$ and $v$ different at $V(G)$ which is called side (edge). For example, e.g $V(G) = \{v_1, v_2, v_3, v_4\}$ and $E(G) = \{v_1v_2, v_2v_3, v_2v_4, v_3v_4\}$. Then $G$ can be described as follows:

![Graph G consisting of vertex and edge](image1)

2. Hamilton Graph

The connected graph $G$ is a Hamiltonian (Hamiltonian) graph if there is a which contains every point in $G$. This kind of cycle is called the Hamiltonian cycle. Hamilton's path from graph $G$ is an open road that passes each point is exactly once and does not return to its original point. For example:

![A Graph That Has a path Hamilton](image2)

3. Perfect Graph

A perfect graph is a graph that has chromatic numbers and the same clique number ($\chi(H) = \omega(H)$). The clique number is denoted by $\omega(G)$ and is defined as the order of the maximum complete subgraph that can be formed from graph $G$. Chromatic numbers a graph $G$ is denoted by $\chi(G)$ is defined as a minimum amount the colors needed to color the points on the graph $G$ such so that each of the vertexes that are connected directly gets a color different.

Here is an example of a perfect graph:

![Perfect Graph](image3)
A complete subgraph of $K_4$ is:

\[ K_1 = \]
\[ K_2 = \]
\[ K_3 = \]
\[ K_4 = \]

Figure 3. perfect graph

4. Graph coloring

Graph coloring is a color rendering of one of its elements (points or sides) so that the elements are interconnected immediately get a different color. There are three types of graph coloring i.e. vertex coloring, edge coloring, and region coloring.

RESULT AND DISCUSSION

In this chapter, the author will examine how to determine numbers backbone coloring - $\lambda$ on split graphs. As for the steps determine the backbone coloring number - $\lambda$ in a split graph, namely: determine split graphs, Give 1 example of Spanning subgraph (Backbone) containing subgraphs complete maximum of split graphs and contains Hamilton trajectories and give a point coloring on the Hamilton trajectory backbone of the split graph.

A. Split Graph

Split graphs are $G$ graphs whose set of points can be partitioned into a clique and free set. Split graph shapes are subsections of the perfect graph section. Therefore the split graph satisfies $\chi(G) = \omega(G)$.

B. Backbone Path Hamilton

The next step the author gives 1 example of spanning subgraph (backbone) which loads $K_5$ and load the Hamilton path from the following split graph:
The backbone of the split graph above loading $K_5$ with a set of points $\{v_2, v_4, v_5, v_6, v_8\}$.

Consider the following picture:

In Figure 7 above the thick line shows Hamilton's path from a split graph because it is an open path that passes every point exactly one time and not returning to the original point. The path Hamilton is $\{v_9, v_8, v_7, v_6, v_5, v_4, v_3, v_2, v_1\}$ because the points are connected directly and past each point exactly once and the sides between points $v_1$ and $v_5$ are removed because it is not traversed by path Hamilton's.

**C. Backbone coloring - $\lambda$ on Split Graph.**

The next step, the writer will give a point of coloring to the graph split. Backbones that are taken are those that have the maximum complete subgraph.
Figure 8. Split graph

Because the dot \( \{v_9, v_8, v_7, v_6, v_5, v_4, v_3, v_2, v_1\} \) directly connected and pass every point exactly once. Then the minimum coloring given is 5, so \( \chi(P) = 5 \) and can be colored in different colors. If P has coloring \(-k\), then P can be colored \(-k\). The Chromatic number \( P \) denoted by \( \chi(P) = 5 \) is the smallest number \( k \) indicating that P can be colored \(-k\). The following is an example of point coloring in the backbone path hamilton from the graph \( K_5 \).

Figure 9. Vertex coloring at backbone path Hamilton

In the picture above illustrates the 5-color. With \( \chi(P) \geq 5 \) is because P has a 5-color coloring so \( \chi(P) = 5 \). Because there is 1 point which has a degree of 5 is \( v_8 \) which is connected directly with 4 points the other is \( v_7, v_5, v_2, v_4, v_6 \) so the colors of the points are different and color on the vertex \( v_8 \) connected with 5 different colors, while the dots are others are outside \( v_8 \) can be colored according to the color of the dots \( v_9, v_7, v_6, v_5, v_1 \) whether the point is directly connected or not directly connected. If the point these are not directly connected can be colored the same color for example at the vertex \( \{v_9, v_7, v_6, v_1\} \).

Conclusions

Based on the discussion, it can be concluded that to determine the Backbone coloring number \( \lambda \)-based obtained the steps as follows:

1. Determine split graphs
2. Give 1 example of Spanning subgraph (Backbone) containing subgraphs complete maximum of split graphs and contains Hamilton trajectories.
3. Give a point coloring on the Hamilton trajectory backbone of the split graph. Based on the steps above, the split graph construction requires clique and free set because a split graph is formed by both.
Suggestions

The discussion in this thesis only focuses on the problem of determination coloring number \( \lambda \)-backbone on a split graph. So the authors suggest examining the problem of determining the \( \lambda \)-backbone coloring number in a graph others such as cube graphs using a higher backbone.

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