A possible explanation of Galactic Velocity Rotation Curves in terms of a Cosmological Constant

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Abstract

This paper describes how the non-gravitational contribution to Galactic Velocity Rotation Curves can be explained in terms of a negative Cosmological Constant ($\Lambda$).

It will be shown that the Cosmological Constant leads to a velocity contribution proportional to the radii, at large radii, and depending on the mass of the galaxy. This explanation contrasts with the usual interpretation that this effect is due to Dark Matter halos.

The velocity rotation curve for the galaxy NGC 3198 will be analysed in detail, while several other galaxies will be studied superficially.

The Cosmological Constant derived experimentally from the NGC 3198 data was found to be: $|\Lambda|_{Exp} = 5.0 \times 10^{-56} cm^{-2}$. This represents the lowest experimental value obtained from the set of galaxies studied and compares favourably with the theoretical value obtained from the Large Number Hypothesis of: $|\Lambda|_{Theory} = 2.1 \times 10^{-56} cm^{-2}$.

It will be shown that the Cosmological Constant, in the Weak Field Approximation, leads to a correction term for the Newtonian po-

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tential and the corresponding acceleration of a test particle. The Cosmological term leads to a modified Newton equation given by: \( F_{m_1} = m_1 \left[-\frac{G m_0}{r^2} + G \Lambda r \right]. \)

Here the force experienced by a mass \( m_1 \) is given by the sum of the gravitational fields produced by \( m_0 \) and \( \Lambda \). For a non-zero Cosmological Constant the modification force represents a fifth fundamental force, gravitational in nature, and proportional to the distance.

The Extended LNH is then used to define other cosmological parameters: gravitational modification constant, energy density, and the Cosmological Constant in terms of a fundamental length.

A speculative theory for the evolution of the Universe is outlined where it is shown how the Universe can be defined, in any particular era, by two parameters: the fundamental length and the energy density of the vacuum for that epoch (GUT, electroweak, Quark - Hadron Confinement).

The theory is applied to the time evolution of the universe where a possible explanation for the \( \rho_{\text{Planck}} / \rho_{\Lambda} \approx 10^{120} \) problem is proposed.

Finally, the nature of the "vacuum" is reviewed along with a speculative approach for calculating the Cosmological Constant via formal M-theory. The experimentally and theoretically derived results presented in this paper support a decelerating Universe, in contrast with recent results obtained from Type 1a Supernovae experiments, suggesting an accelerating Universe.
1 Cosmological Constant, Dark Matter and Velocity Rotation Curves

1.1 Short History of the Cosmological Constant

There has been considerable historical interest in the Cosmological Constant from the time Einstein added it to his Gravitational Field Equations. His aim was to artificially induce a stationary solution in order to support a static unchanging universe. In doing this he neglected to predict the Expanding Universe, a natural consequence of his theory, which was later developed by Hubble from astronomical observation.

In every decade since its inception the Cosmological Constant has been used to support the prevailing theory of the time. Particle physicists would like it to be identically zero in order to support the Standard Model, while cosmologists have taken negative, zero or positive values in order to predict either a contracting, expanding or accelerating universe.

Two excellent pedagogical reviews of the Cosmological Constant have been written by Cohn [1] and Carroll & Press [2], while Turner [17] has reviewed the cosmological parameters in light of the latest experimental data.

Recent analysis [18] of type Ia supernovae has led to the prediction of an Accelerating Universe. The study and fundamental understanding of the Cosmological Constant has again become a cause célèbre. It lies at the epicentre of several lines of fundamental research, namely: Astronomy, Cosmology, String Theory, Inflation Theory, High Energy and Particle Physics.

If the Universe is accelerating or decelerating, it suggests that it is being driven apart or forced together by an exotic new form of energy. A non zero Cosmological Constant, representing in some way the energy of the vacuum, would produce such a force. The sign of the Cosmological Constant will determine if the force acts as a kind of antigravity leading to an accelerating universe, or a new type of gravitational force, supplementing Newtonian gravity, resulting in a decelerating Universe. In either case it would be expected that there would be a deviation from the inverse square law.

1.2 “Dark Matter” – Energy Deficit of the Universe

In 1844, Friedrich Bessel was the first to infer the existence of non-luminous Dark Matter from gravitational effects on positional measurements of Sirius and Procyon. He inferred that each was in orbit with a mass comparable to its own (See Trimble [19] for a pedagogical review of Dark Matter).

In 1933, Zwicky concluded that the velocity dispersion in Rich Clusters of galaxies required 10 to 100 times more mass to keep them bound than could be accounted for by luminous galaxies themselves.

The old problem of ”missing” matter is today referred to as the Energy Deficit of the Universe. The present consensus is that the energy of the universe [17], assuming a present day flat Universe \( k = 0 \), is comprised of two key components: matter,
Baryonic and Non-Baryonic, and Dark Energy which is associated with the energy of the vacuum represented by the Cosmological Constant.

Baryonic and Non-Baryonic Matter represent approximately, 5% and 35% of the critical mass of the Universe while the Cosmological Constant (vacuum energy) accounts for 60%.

The need for Non-Baryonic Matter arises mainly because Baryonic Matter is limited to approximately 5% of the Critical Mass of the Universe. This limitation is set by Big Bang Nuclear Synthesis considerations [17].

There are many candidates for the Non-Baryonic components of matter: two presently of interest are [20]: WIMPs, Weakly Interacting Massive Particles, and Axions - postulated to explain the lack of CP violation in the strong interaction.

1.3 Velocity Rotation Curves

The experimental determination of Galactic Velocity Rotation Curves has been one of the mechanisms to estimate the "local" mass (energy) density of the Universe. In a paper reviewing Dark Matter Trimble [19] noted that many Velocity Rotation Curves remained flat or even rise to large radii, well outside the radius of the luminous astronomical object. This non-Newtonian behaviour has lead to the development of "Dark Matter" theories in order to explain the missing mass of the Universe.

The majority of the Velocity Rotation Curves are described, at large radii, in terms of some kind of "dark matter" component, results are often described in terms of "Dark Matter" halos (hollow halos) dominating the gravitational potential at some nominally large distance. However none of the present theoretical models successfully explain the observational data [7, 36].

In this paper several Galactic Velocity Rotational Curves data sets will be analysed and the results presented in a later section. The case will be made that the Cosmological Constant represents a constant energy term which contributes to Galactic Velocity Rotation Curves.
2 Experimental Results for Galactic Velocity Rotation Curves

2.1 Approach

This section will discuss and analyse in detail the velocity rotation curve data for galaxy NGC 3198 [25]. The galaxy has been studied extensively resulting in several sets of experimental data that are suitable for testing theoretical predictions. Data for other randomly selected Galaxies: NGC 2403, NGC 4258, NGC 5033, NGC 5055, NGC 2903, NGC 6503, will also be analysed in order to establish a general experimental trend.

The theory presented in the next section of this paper predicts that velocity rotation curves have two main components: gravitational mass and effective mass due to the Cosmological Constant. Following this approach, the gravitational contribution to the velocity rotation curve will be subtracted from the observational data. The resulting graphs are predicted to be straight lines whose gradients are proportional to the Cosmological Constant at large radii.

2.2 NGC 3198 Galaxy

NGC 3198 is found in the constellation of Ursa Major and is classified as a Sc spiral galaxy. The galaxy has little or no prominent bulge and has a normal exponential disk. The observed rotation curve rises gradually near the centre and flattens out at approximately 150 Km/s in the outer region.

It has been exhaustively studied, van Albada et.al., van Albada & Sancisi, Kent, Begeman, [25, 9] and others. For illustrative purposes the extended velocity rotation curve of NGC 3198, shown in figure 1, is taken from Albada et.al. (figure 7). It extends to eleven scales lengths which corresponds to approximately 50 Kpc.

The high quality data has allowed detailed theoretical analysis of the rotation curve. The digitised data, shown in Figure 1, was used to determine the gravitational and Cosmological Constant contributions to the velocity rotation curves.

Drawing a straight line through the data points in Figure 2 yields a value of:

\[ |\Lambda_{NGC3198}^{Exp}| = 5.0 \times 10^{-56} \text{cm}^{-2}, \tag{1} \]

for the Cosmological Constant.

2.3 NGC 2403, 3198, 157, 2841 Galaxies

The velocity rotation curve data for galaxies, chosen at random: NGC 2403 [3], NGC 3198 [4], NGC 157 [5], NGC 2841 [6], were subjected to a crude gravitational and Cosmological Constant decomposition analysis. The gravitational contribution was subtracted from the full rotation curve data, and in all cases good straight lines were obtained for the Cosmological Constant contribution.
| Galaxy    | Radius | Value of Cosmological Constant | Galactic Density        |
|-----------|--------|-------------------------------|--------------------------|
| NGC 2403 | 20Kpc  | $|\Lambda|_{NGC2403} = 3.6 \times 10^{-56} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 3.9 \times 10^{-28} g cm^{-3}$ |
| NGC 4258 | 50 Kpc | $|\Lambda|_{NGC4258} = 5.5 \times 10^{-55} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 5.9 \times 10^{-28} g cm^{-3}$ |
| NGC 3198 | 50Kpc  | $|\Lambda|_{NGC3198} = 5.0 \times 10^{-56} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 5.4 \times 10^{-29} g cm^{-3}$ |
| NGC 5033 | 40 Kpc | $|\Lambda|_{NGC5033} = 1.0 \times 10^{-55} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 1.1 \times 10^{-28} g cm^{-3}$ |
| NGC 5055 | 50 Kpc | $|\Lambda|_{NGC5055} = 1.4 \times 10^{-55} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 1.5 \times 10^{-28} g cm^{-3}$ |
| NGC 2903 | 30Kpc  | $|\Lambda|_{NGC2903} = 3.8 \times 10^{-55} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 4.1 \times 10^{-28} g cm^{-3}$ |
| NGC 6503 | 30 Kpc | $|\Lambda|_{NGC6503} = 4.6 \times 10^{-55} cm^{-2}$ | $\rho_{\Lambda}^{Exp} = 4.9 \times 10^{-28} g cm^{-3}$ |

Table 1: The equations for all the Cosmological parameters are given in the next section.

For the galaxies studied, the radii of the galaxies and the value for the Cosmological Constant and the associated densities, are shown in table 1.

It is quite striking that for such a relatively crude analysis the experimental values for the Cosmological Constant are so consistent and lie within the allowed cosmological constant range [10].

2.4 Review of Experimental results and Comparison to Theory

2.4.1 Review of Experimental Data

The experimental difficulties in obtaining galactic rotation curve data are acute [13]. Apart from the instrumental use and calibration difficulties, assumptions have to be made concerning the different contributions to the galactic baryonic mass from the stellar disk, warm ionised gasses, atomic and molecular hydrogen.

These types of problems are well considered in van Albada et. al. [25] study of NGC 3198. Several types of mass models were used in conjunction with the experimental data in order to obtain the "Dark Matter" component. Figure 7 from their paper leads to a value of the Cosmological Constant of:

$$|\Lambda|_{NGC3198} = 5.0 \times 10^{-56} cm^{-2},$$

while figure 4, with a greater model mass contribution gives:

$$|\Lambda|_{NGC3198} = 1.0 \times 10^{-55} cm^{-2}.$$

The analysis detailed in this paper applies to velocity rotation curves that extend to many radii, at which stage the rotation velocity is constant or slightly rising.
2.4.2 Comparison with Theoretical Predictions

The gradient determined from the experimental galactic velocity rotation curve data is the only information needed to determine the Cosmological Constant for the present day Universe. It is remarkable that for such a crude analysis the experimental results are so consistent.

A representative experimental value for the Cosmological Constant taken from table 1 is $|\Lambda_{Exp}| \sim 2.0 \times 10^{-55} cm^{-2}$. This compares favourably to the theoretical value predicted by the Extended LNH of $|\Lambda_{Theory}^{QH}| = 2.1 \times 10^{-56} cm^{-2}$.

Again, the agreement between experimental and theory is striking. The factor of 10 between the two results maybe significant. However, it will not be possible to comment further until a more fundamental and systematic analysis of galactic velocity rotation curve data has been completed.

Finally, an effective mass density contribution to the Universe due to the Cosmological Constant can be determined (see table 1 above). The values given for experiment and theory are: $\rho_{\Lambda}^{Exp} \sim 4 \times 10^{-28} g cm^{-3}$ and $\rho_{\Lambda}^{Theory} \sim 1.2 \times 10^{-29} g cm^{-3}$ respectively. These can be compared with the best estimates for galactic Baryonic mass density which are in the range $[38, 37]$: $10^{-29} > \rho_B > 2 \times 10^{-31} g cm^{-3}$.

2.4.3 Negative Cosmological Constant

Recently there has been great interest in the Type Ia Supernovae results of Perlmutter et al [18] which suggest that the universe is accelerating.

In this section we will show that the Weak Field Approximation coupled with galactic velocity rotation curve data inevitably lead to a negative Cosmological Constant.

The equation for the VRC is given by

$$-\frac{v^2}{r} \approx -\frac{Gm}{r^2} + \frac{c^2 \Lambda}{3} \frac{1}{r}$$

(this expression is derived in section 3.3). We note that Eq.(2) is only strictly true for small and large radii, however it will serve to illustrate our arguments.

At small galactic radii the velocity versus radius contribution is well known and follows Newtonian physics. For large radii, a negative Cosmological Constant gives a positive contribution to the VRC, which is what is actually observed. On the other hand, the effect of a positive Cosmological Constant would be to lower the rotation curve below that due to matter alone.

The above simple argument, based on observational astronomy, allows only a negative Cosmological Constant as a possible explanation for the galactic velocity rotation
curve data. This is in clear disagreement with the Type Ia supernovae results \[18\]. However, given the uncertainties in the determination of the deceleration parameter, \(q_0\), derived from supernovae data \[18\] the conclusions outlined above have certain merits worth consideration.

In summary these are: the Cosmological Constant is determined from direct measurement unlike the Supernovae results, the experimentally determined value is the correct order of magnitude as that required from cosmological constraints, and lastly a negative Cosmological Constant is consistent, and indeed a natural physical explanation, for the observed galactic velocity rotation curve data.

Finally, observations of global clusters of stars constrain the age of the universe and consequently place an observational limit on a negative Cosmological Constant \[26\] of:

\[
|\Lambda| \leq 2.2 \times 10^{-56} \text{ cm}^{-2}.
\]  

This upper limit value for the Cosmological Constant derived from global cluster constraints is in agreement with the experimentally determined value derived from galactic velocity rotation curve data.

### 2.5 Review of Experimental Results and Energy Deficit of the Universe

A systematic and general study of galactic velocity rotation curves needs to be undertaken in order to confirm the results presented in this paper, and to establish that there are no other ("Dark Matter") components that contribute to the velocity rotation curves.

Superficial results (NGC 3198) suggest that the Cosmological Constant could be a candidate for the "dark energy" which is smoothly distributed and contributes approximately 60% to the critical density of the Universe \[17\]. This assertion is supported by the successful application of the "Cold-Dark Matter" model \[7, 14, 15\] in predicting mass and galactic structures formation.

The ranges in the values for the density contributions of the Cosmological Constant and Baryonic matter, make it difficult to comment further on the larger problem of the Energy Deficit of the Universe.

The cosmological parameters are very sensitive to small changes in the values of the String length and the experimentally derived Cosmological Constant. This sensitivity in conjunction with the rapidly improving accuracy of astronomically derived data \[9\] will combine to provide a powerful tool for probing the Universe.

Finally, the experimental results presented in this paper predict a fifth force, gravitational in nature, proportional to the distance between bodies, leading to a decelerating Universe.
3 Theoretical Framework

3.1 Extended Large Number Hypothesis, Duality, Weak Field Approximation

The starting point for the work presented in this paper was a firm belief in two key points: that nature has a fundamental length and that the Cosmological Constant represents in some way a macroscopic quantum mechanical parameter.

These guiding objectives will be explored using String Theory, the Extended Large Number Hypothesis and the Weak Field Approximation. Duality arising out of String Theory points to a fundamental length, the Extended Large Number Hypothesis show how macroscopic Cosmological Parameters can be related to quantum mechanical origins, and facilitates the estimation of their values, and finally the Weak Field Approximation allows the Cosmological Constant to be defined in terms of an effective mass.

3.1.1 Large Number and Extended Large Number Hypothesis

Extending the work of Eddington [32] Dirac wrote a paper titled, “A new basis for Cosmology” [27], where he considered the strange coincidence between the ratio of Cosmological and Atomic Constants to the approximate age of the Universe, namely:

$$\frac{hc}{Gm_pm_e} \sim \frac{m_pc^2}{Hh} \sim 10^{41},$$

Here the masses represent that of the proton and electron and $H$ is Hubble’s Constant.

Dirac’s proposal became known as the Large Number Hypothesis (LNH) [27, 28, 29]. He firmly believed that these “coincidences” represented an as yet unknown fundamental theory linking the Quantum Mechanical origin of the Universe to the large scale cosmological parameters.

Zel’dovich [30], in an extraordinary paper, extended the LNH to include an expression for the Cosmological Constant, given by

$$|\Lambda| = \frac{8\pi G^2 m_p^6}{h^4}$$

He went on to show that the Cosmological Constant in empty space produces the same gravitation field as when the space contains matter, and that these terms enter as fully fledged terms, in the presence of ordinary matter, in the Gravitational Field Equations.

Starting from a theory for elementary particles, Zel’dovich then showed that the vacuum energy for non-interacting particles was identically equal to zero. However when the interactions between particles were taken into account, it resulted in a non-zero value for the vacuum energy.

Zel’dovich interpreted the gravitational energy of the vacuum in terms of the interaction of virtual particles, the distance between them being defined by $\lambda =$
Here the self energy is exactly equal to zero, but the gravitational interacting of neighbouring particles caused the vacuum energy to have a non-zero value, given by:

$$\epsilon_{\text{vac}} = \frac{Gm_p^6c^4}{h^4}$$

(6)

Sakharov [31] further extended LNH by proposing a gravitational theory or justification of the equations of General Relativity theory based on the considerations of vacuum fluctuations. He stressed the importance of the hypothesis that there is a fundamental length or corresponding limiting momentum, less than which the theory is not valid. He proposed this limit as: $$k_{\text{max}} \sim 10^{33}\text{cm}^{-1}$$, leading to:

$$L^2 = \frac{Gh}{c^3}$$

(7)

Zel’dovich concluded his paper by proposing that the Cosmological Constant could be interpreted as follows:

“there exists a theory of elementary particles which would give (in accordance with a mechanism which is still unknown at present) an identically vanishing energy, provided that this theory was applicable without limit, up to arbitrary large momenta; there exists a momentum $$p_{\text{max}}$$, beyond which the theory is invalid.”

This theory is interpreted as the present day String Theory.

Finally, Matthews demonstrated in an inspirational article [33], that the mass, defined in terms of the mass of a Proton, introduced by Dirac in an arbitrary way, should be replaced by the effective (quantum) mass associated with the various phase transition energies of the Universe (GUT, electroweak, Quark-Hadron).

He showed that using the value for the vacuum energy for the last phase transition, Quark-Hadron, leads to the present observed cosmological parameters, and that current cosmic dynamics were seemingly determined by the aftermath of the most recent quantum vacuum state transition.

He also pointed out that the energy density required to support inflation was that associated with the GUT phase transition.

### 3.1.2 Fundamental Length and Duality

#### 3.1.2.1 Duality – Fundamental Length Justification

Zel’dovich and Sakharov understood the need for a theory of elementary particles that inevitably leads to a concept of a fundamental length. The closest, and indeed, the only theory that is a candidate today is String theory, it provides a consistent framework in which one can study quantum gravity and its unification with the three gauge forces [43].

One of the most profound quantum symmetries of String theory, with no field theory analogy, is (target space) Duality (for a review see Giveon et al [44]). In its simplest form, Duality shows that a (closed) string moving on a circle of radius $$R$$, is equivalent to one that moves on a circle of radius $$R \rightarrow \alpha'/R$$ when the momentum modes ($n/R$) are interchanged with winding modes $$mR/\alpha'$$ and $$\alpha' \simeq (10^{-32}\text{cm})^2$$.
controls the tension of the string. The above symmetry can be extended to more complicated situations when background fields are present in six-dimensional compactifications of string on orbifolds or Calabi-Yau manifolds. There are some interesting consequences of Duality. One is the prediction of a fundamental minimal observable length scale of order $\sqrt{\alpha'}$ in Nature that leads to the natural generalisation of the Heisenberg Uncertainty and Equivalence principles [48]. Another is that it restricts the possible form of scalar (or super) potentials and determines some characteristics of non-perturbative supersymmetry-breaking and CP violation [45, 46].

Duality indicates that Einstein’s action will be modified drastically at short distances, and the approach offered by Duality may help to resolve problems associated with Einstein’s Cosmology, including the initial singularity problem.

The generalised Uncertainty Principle becomes:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}$$

where

$$\Delta x \geq L_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}}.$$  

(9)

For a recent discussion on the theoretical attempts to derive the second term in equation (8) we refer the reader to [47]. The need for a fundamental length was confirmed by Veneziano et al [48] who, when studying fixed angle high energy scattering processes, found that strings were not applicable to distances shorter than the string length. This effect was also confirmed by Gross & Mende [49] who studied the high-energy behaviour of string scattering amplitudes to all order in perturbation theory.

In conclusion, the fundamental length implied by Duality is related beautifully by Sakharov and Zel’dvich in their proposal where they envisioned a theory of fundamental length before the advent of String Theory.

3.2 Relationship between Quantum Mechanics and General Relativity

One of the fundamental aims in the 20th Century has been the formulation of a theory relating Quantum Mechanics, a possible candidate for the creation of the Universe, to General Relativity, the theory of physics for macroscopic masses and distances.

The approach taken by Zel’dovich [30] was to suggest that gravitational interactions of virtual particles in vacuum would endow empty space with an effective energy and pressure, which he interpreted to be the Cosmological Constant term, $\Lambda$. This led Sakharov [31] to suggest that there should be a functional relationship between the quantum nature of the vacuum and the curvature of space. He introduced a momentum cut off of $k_0$ for the virtual particles, which he stated should be given, a priori, by a microscopic field theory. Target-space duality invariant, String Theory,
is the microscopic theory that leads to a fundamental length scale or momentum $k_0$, which in turn determines the curvature of space-time and the Cosmological Constant. The Planck compactification radii are preferred by Duality, and moreover, coincide with the value of $k_0 \sim 10^{33} \text{cm}^{-1}$, which gives the correct Newtonian gravitational constant. This result is regarded as an important evidence for the relevance of Duality in Nature.

### 3.3 Effective Mass due to Cosmological Constant within the Weak Field Approximation

#### 3.3.1 Effective mass due to the presence of the Cosmological Constant

This section derives the contribution to the galactic rotation due to the Cosmological Constant within the Weak Field Approximation.

Following Zel’dovich, the Einstein Field equations can be rewritten as:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8 \pi G}{c^4} \left( T_{\alpha\beta} + \frac{c^4 \Lambda}{8 \pi G} g_{\alpha\beta} \right).$$

(10)

The assumption, $\Lambda \neq 0$, denotes that empty space produces the same gravitational field as when the space contains matter defined as:

$$\rho_\Lambda = \frac{c^2 \Lambda}{8 \pi G}; \quad \epsilon_\Lambda = \frac{c^4 \Lambda}{8 \pi G}; \quad P_\Lambda = -\epsilon_\Lambda,$$

(11)

where $\rho_\Lambda, \epsilon_\Lambda, P_\Lambda$ represents the effective mass density, energy density and pressure due to the presence of the Cosmological Constant. Here the energy density and pressure have been formulated in such as way as to leave the theory relativistically invariant.

Following the approach of Ohanian & Ruffini [26], the Weak Field approximation to the Field Equations gives rise, in the absence of matter, to a differential equation for the Newtonian potential described as:

$$\nabla^2 \Phi = -\frac{c^2 \Lambda}{c^4 \rho}.$$

(12)

Comparing this with Newton’s equation:

$$\nabla^2 \Phi = 4 \pi G \rho,$$

(13)

the $\Lambda$ is recognised to correspond to a uniform effective mass density given by

$$\rho^{\text{eff}}_\Lambda = -\frac{c^2 \Lambda}{4 \pi G}.$$

(14)

It is important to realize that the Cosmological Constant obeys the equation of state given by:

$$P_\Lambda = -c^2 \rho_\Lambda,$$

(15)
Taking the Newtonian limit in the absence of matter, \( T_{\mu\nu} = 0 \), the differential equation for the static Newtonian potential becomes

\[ \nabla^2 \Phi = -c^2 \Lambda \]  

leading to:

\[ \rho_{\text{eff}} = \rho_\Lambda + \frac{3P_\Lambda}{c^2} = -2\rho_\Lambda. \]  

Therefore the factor of two and the change in sign is due to the fact that the Cosmological Constant obeys (15) and that in the linearised theory the source term for \( h_{00} \) is \( T_{00} - \frac{1}{2} T \text{Tr} T_{\mu\nu} \) [26].

By arbitrarily setting \( \Phi = 0 \) at the origin, and using spherical polar coordinates, the solution for \( \Phi \) becomes:

\[ \Phi = -\frac{\Lambda c^2 r^2}{6} \]  

The potential indicates that between any two particles for \( \Lambda > 0 \) and \( \Lambda < 0 \), there acts a repulsive force or attractive force which is proportional to \( r \). This force represents a new fundamental force, a fifth force, which is gravitational in nature.

In the presence of matter and using the Weak Field Approximation the modified Newtonian potential is given by:

\[ \phi_{MN} = -\left[ \frac{Gm_0}{r} + G_{\Lambda} r^2 \right]. \]  

The acceleration experienced by a test particle following a geodesic path created by the gravitational fields produced by \( m_0 \) and \( \Lambda \), is given by:

\[ \ddot{x} = -\frac{Gm_0}{r^2} + G_{\Lambda} r \]  

or when applied to galactic rotation curves becomes:

\[ -\frac{v^2}{r} \approx -\frac{Gm_0}{r^2} + G_{\Lambda} r, \]  

where \( m_0 \) represents the mass of the galaxy, and \( G \) and \( G_{\Lambda} \) are the gravitational and anti-gravitational constants.

The above equation is an approximation that is only strictly correct for small and large radii. In addition to this the second term, arising from the Weak Field Approximation, is valid only when the effective density due to \( \Lambda \) is much greater than the gravitational density. This condition is only usually satisfied at many galactic radii.

The two terms should not be equated in order to provide estimates for a critical radii where effects due to Newtonian gravity and the Cosmological Constant are in balance as proposed by Bergstrom \[13, 14\] following the first version of this paper.
This procedure leads to the incorrect conclusion that effects due to the Cosmological Constant are not significant on galactic scales. To use the approach of Bergstrom a Strong Field Approximation would have to be developed.

[Note: The equation for the acceleration experience by a body is only strictly true for a test particle. The mass of the body itself produces curvature that affects the original geodesic. The gravitational interaction of two massive bodies is not directly addressed by Einstein’s theory of General Relativity; however approximate methods were developed in the 1980’s (Damour & Deruelle, 1986)\cite{12}].

3.3.2 Valid Ranges for Weak Field Approximation

The Weak Field approximation is valid for the following range of values\cite{26}:

\[ \sqrt{\frac{1}{\Lambda}} \gg r \gg \frac{GM}{c^2}. \tag{22} \]

This approximation is strictly valid for astronomical masses and distances in the range below:

\[ \sqrt{\frac{2}{\Lambda_{Exp}}} \sim 1447.3 \text{Mpc}, \quad \sqrt{\frac{2}{\Lambda_{Theory}}} \sim 2233.2 \text{Mpc}, \quad \frac{GM}{c^2} \sim 47.7 \text{pc} \tag{23} \]

Here the mass of a Supercluster\cite{10} has been used, along with the experimental and theoretical derived values for the Cosmological Constant. The approximation defined above is found to be valid for all astronomical objects of interest.

3.4 Definition of Cosmological Constants in terms of a Fundamental Length

Listed below are the expressions for the Cosmological Parameters that have been defined in terms of a fundamental length and the Cosmological Constant. [Note: When defining the Cosmological Constant within the Entended LNH we have followed Dirac\cite{27,28,29} and Matthews\cite{33} in using $h$ instead of $\bar{h}$. Also in order to be consistent with general relativity we have used Kardashev’s\cite{13} expression for the Cosmological Constant which includes a $8\pi$ multiplying term.]

Gravitational Constant:

\[ G(L_s) = \frac{c^3L_s^2}{h}. \tag{24} \]

(Throughout this paper the experimental value of the Gravitational Constant will be used to define the fundamental length. The value is given by $L_s = 1.6 \times 10^{-33}\text{cm}$).
**Cosmological Constant:**

\[ |\Lambda(L_s)| = \frac{8\pi c^6 m_0^6 Q_H}{h^6} L_s^4, \]  \hspace{1cm} (25)

Here the Proton mass has been replaced by an effective mass (quantum energy scale) given by the Quark-Hadron phase transition vacuum energy.

**Gravitational Modification Constant:**

\[ G_\Lambda(L_s) = \frac{c^2 \Lambda}{3}. \]  \hspace{1cm} (26)

Mass density (due to the presence of the Cosmological Constant):

\[ \rho_{\Lambda}^{eff}(L_s) = -\frac{c^2 \Lambda}{4\pi G}. \]  \hspace{1cm} (27)

**Modified Newton Equation:**

Associating the effective mass density, \( \rho_{\Lambda} \), with a cosmological modification force, Newton’s equation can be modified and written as:

\[ F_{m_1}(L_s) = m_1 \left[ \frac{-G m_0}{r^2} + G_\Lambda r \right], \]  \hspace{1cm} (28)

where \( G \) and \( G_\Lambda \) are the Gravitational and Gravitational Modification Constants respectively.

A positive value of \( \Lambda \) leads to a repulsive or anti-gravitation force, whereas a negative value leads to an attractive and contracting force.
4 Speculative Theory for the evolution of the Universe

4.1 Evolutionary Universe

In this section of the paper a speculative theory for the evolution of the Universe will be developed. It will build upon Dirac’s belief that the present day cosmological parameters are historically connected to the quantum mechanical origin of the Universe; the connection being the Cosmological Constant, a macroscopic quantum mechanical parameter.

As the Universe evolves through the various quantum vacuum phase transitions: GUT, Electro-Weak, Quark-Hadron, the value of the Cosmological Constant changes. The value is uniquely determined by the energy density of the vacuum for that epoch (epoch energy scale (EES)).

The relationship between the Cosmological Constant and the quantum mechanical effective mass is shown below:

\[ |\Lambda_{EES}| = \frac{8\pi c^6 m_{EES}^6}{h^6 L_s^4}. \]  
(Note: The Cosmological Constant is not identical to the vacuum energy scale but rather to the gravitational interaction energy of the vacuum for that epoch. The distinction and clarification of these terms will be discussed later).

This equation relates the quantum mechanical origin of the Universe, in terms of an effective mass, to the macroscopic evolution of the Universe via the Cosmological Constant and Einstein’s Field Equations.

Matthews [33] inferred that the Universe’s evolution was determined by the symmetry breaking vacuum phase transition. He suggested replacing the mass of the proton in the LNH by an effective mass determined by the Planck, GUT, Electro - Weak or Quark- Hadron energy scales.

4.2 The Two Parameter Universe

Einstein had a philosophical belief in the simplicity of the laws of Nature and the Universe. This requirement for simplicity is supported by a theory where the Universe can be uniquely described by two parameters. These parameters, within the Extended LNH, are: a fundamental length and an equivalent effective mass for the vacuum energy density for that epoch.

The following sections review the various epochs of the Universe.

4.3 The Present Epoch - Quark-Hadron Era

This section will derive the values for the cosmological parameters for the present epoch. It will be assumed that the Universe to a first approximation is flat, the only curvature being that due to the Cosmological Constant.
The Universe that we see and observe today, defined by the present day cosmological parameters, is determined in great part by the energy density (scale) of the last phase transition - Quark-Hadron.

The values for the String length and effective mass of the Quark-Hadron phase transition \(^{33}\) are taken to be:

\[
L_s = 1.6 \times 10^{-33} \text{cm}, \quad m_{QH} \sim 0.15 \text{ GeV} \equiv 2.5 \times 10^{-25} \text{g},
\]

which in turn lead to values for the Cosmological Constant and energy density of:

\[
|\Lambda_{QH}^{Theory}| = \frac{8\pi c^6 m_{QH}^6}{h^6} L_s^4 = 2.1 \times 10^{-56} \text{cm}^{-2}, \quad \rho_{\Lambda}^{Theory} = 1.2 \times 10^{-29} \text{g cm}^{-3}.
\]

These values compare favourably to the experimental derived values of:

\[
|\Lambda_{QH}^{Exp}| \sim 2.0 \times 10^{-55} \text{cm}^{-2}, \quad \text{and} \quad \rho_{\Lambda}^{Exp} = 4.0 \times 10^{-28} \text{g cm}^{-3}, \quad \text{respectively and the generally accepted galactic Baryonic matter density which is in the range } 1 \times 10^{-29} \geq \rho_{\text{Baryonic}} \geq 2 \times 10^{-31} \text{g cm}^{-3}^{[37]}.
\]

The theoretical and experimental values for the cosmological parameters agree within a factor of ten. This agreement is good considering the magnitude of the parameters being used and the fact that the effective mass has been arbitrarily set to the vacuum energy scale of the epoch. Another explanation could be that there is another contribution to the missing ”smooth dark energy” of the Universe other than the Cosmological Constant.

4.4 Planck, GUT, Electro-Weak and Quark-Hadron Epochs

4.4.1 Field & String Theories

A non-zero vacuum energy presents present day Field & String Theories with a major problem, essentially the energies are too high to explain physical phenomena.

In general vacuum energies in quantum field theories are described by:

\[
u_{vac} = \int_0^{E_{max}} d^{d-1}k \, \frac{\omega_k}{(2\pi)^{d-1}}
\]

where \(\omega_k^2 = k \cdot k + m^2\). Here the vacuum energy density is defined as the sum of the zero-point energies of the modes of the field, and \(k\) represents the wave number. The vacuum energy provides a source term in Einstein’s Field Equations: the Cosmological Constant, which should lead to observational effects.

In four dimensions we find the well known expression for the energy density,

\[
u_{vac} \sim (E_{max})^4/(hc)^3 J/m^3
\]

Listed in table 2 below are the equivalent mass densities for the various epochs that will be used for comparative purposes.
Energy Scale | Density
---|---
Planck $\sim 10^{19}$ GeV | $\rho_{\text{Planck}}^{\text{Theory}} \sim 0.93 \times 10^{34} \text{gcm}^{-3}$
GUT $\sim 10^{16}$ GeV | $\rho_{\text{GUT}}^{\text{Theory}} \sim 0.93 \times 10^{78} \text{gcm}^{-3}$
Electro-Weak $\sim 300$ GeV | $\rho_{\text{EW}}^{\text{Theory}} \sim 0.93 \times 10^{22} \text{gcm}^{-3}$
Quark-Hadron $\sim 0.15$ GeV | $\rho_{\text{QH}}^{\text{Theory}} \sim 4.7 \times 10^{11} \text{gcm}^{-3}$

Table 2: mass densities at various epochs using the QFT result.

String theories, using here Bosonic String theory for illustrative reasons, can be described in terms of the following spacetime action \[ S = \left(\frac{1}{2k_0}\right) \int d^Dx (-G)_{1/2} e^{-2\Phi} \left[ -\frac{2(D-26)}{3\alpha'} + R - \frac{1}{12} H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_{\mu}\Phi \partial^{\mu}\Phi + O(\alpha') \right] \] \hspace{1cm} (34)

Here, $\Phi$ represents the dilaton, $G_{\mu\nu}$ the metric of D dimensional space-time, $\alpha'$ is related to the string tension, $H_{\mu\nu\lambda}$ the 3 index field strength related to the anti-symmetric tensor $B_{\mu\nu}$ and $R$ the Ricci tensor.

The one-loop vacuum energy density in Bosonic String theory is non-zero and of the order $(10^{18}\text{GeV})^4$, since $O(\alpha') \sim 10^{18}\text{GeV}$ which corresponds to the String scale.

Even in softly broken supersymmetric string theories, it is expected that $\rho_\Lambda$ should be of the order $\rho_{\text{Electro-Weak}} \sim 10^{23} \text{gcm}^{-3}$, which is approximately 52 orders of magnitude larger than the present observable value.

As pointed out by Polchinski[50], "the Cosmological Constant is telling us that there is something we do not understand in Field and String theories about the vacuum. It constitutes one of the best clues for a unified = theory with gravity !”

4.4.2 Extended Large Number Hypothesis

The value for the Cosmological Constant and the density will have varied depending on the particular evolutionary era of the Universe. These are shown in table 3 below which relates, energy scales, cosmological parameters and densities to the associated epochs.

The mystery of the $\frac{\rho_{\Lambda_{\text{Planck}}}^{\text{Planck}}}{\rho_{\Lambda_{\text{QH}}}^{\text{QH}}} \approx 10^{120}$, suggested by Weinberg [41] as the “the worst failure of an order-of-magnitude estimate in the history of science” , can now be understood easily. The ratio is indeed correct, but clearly it compares energy densities for different epochs. (The value found from data taken from table 3 gives $\frac{\rho_{\Lambda_{\text{Planck}}}^{\text{Planck}}}{\rho_{\Lambda_{\text{QH}}}^{\text{QH}}} \approx 8.8 \times 10^{118}$).
| Energy Scale       | Cosmological Constant | Density       |
|-------------------|-----------------------|---------------|
| Planck \(\sim 10^{19}\) GeV | \(\Lambda_{\text{Planck}}^{\text{Theory}} \sim 1.9 \times 10^{45} \text{cm}^{-2}\) | \(\rho_{\text{Planck}} \sim 1.0 \times 10^{91} \text{gcm}^{-3}\) |
| GUT \(\sim 10^{16}\) GeV  | \(\Lambda_{\text{GUT}}^{\text{Theory}} \sim 1.9 \times 10^{45} \text{cm}^{-2}\) | \(\rho_{\text{GUT}} \sim 1.0 \times 10^{72} \text{gcm}^{-3}\) |
| Electro - Weak \(\sim 300\) GeV | \(\Lambda_{\text{EW}}^{\text{Theory}} \sim 1.4 \times 10^{-36} \text{cm}^{-2}\) | \(\rho_{\text{EW}} \sim 7.4 \times 10^{-10} \text{gcm}^{-3}\) |
| Quark - Hadron \(\sim 0.15\) GeV | \(\Lambda_{\text{QH}}^{\text{Theory}} \sim 2.1 \times 10^{-56} \text{cm}^{-2}\) | \(\rho_{\text{QH}} \sim 1.2 \times 10^{-29} \text{gcm}^{-3}\) |

Table 3: Cosmological Constant and mass densities at various epochs in the ELNH model.

The values of the cosmological parameters for the GUT scale are consistent with those required to drive inflation [33] but with a positive value for the Cosmological Constant. A possible mechanism to explain a changing sign for \(\Lambda\) will be discussed later in the paper.

4.4.3 Comparison of Field & String Theories with Extended Large Number Hypothesis

It is also noted that while fundamental theories of Particle Physics such as the Standard Model, Quantum Field Theory and String Theory have many major predictive successes they all have problems with a high vacuum energy density.

On the other hand, while the Extended LNH is formulated from a naive theory [30], it appears to predict correctly the magnitude of the vacuum energy density and other cosmological parameters.

The ELNH seems to indicate that the gravitational self energy plays the key and central role in determining the magnitude of the vacuum energy. This could be an important clue towards the theory of Quantum Gravity and suggests a starting point for a more formal approach.
5 In the Search for a Theory of Quantum Gravity

5.1 The Vacuum

The concept of a vacuum (state) has been invoked since the time of Faraday where it was used in relation to the ether. Over time it has come to mean different things to the various practitioners in the different fields of physics.

Questions concerning the vacuum could be:

- What is the vacuum? How is it defined?
- What is the connection between the “real” world and the vacuum? Are they the same?
- How do we determine the energy of the vacuum?
- How does the vacuum “communicate” with the “real world”?

These questions are of particular interest when considering a theory of gravity, and will be discussed in this section.

It is well known that the vacuum has certain properties: it represents the lowest energy state, it is Lorentz invariant and has a zero four-momentum. It may also carry quantum numbers like: isospin, parity and strangeness etc. [51], and is often considered as a “medium”, analogous with a dielectric material, of infinite extent [30].

Ziman [53] defined a “true” vacuum state and then went on to show that not only is the state hypothetical, not physically accessible, but there is no mathematical description of it in terms of excited states (problem of infinities).

The vacuum or vacuum energy density is of paramount importance in cosmology and is often described in terms of scalar fields. Extreme lengths are taken to justify its inclusion.

"... the advent of a constant homogenous scalar field, over all space simply represents the restructuring of the vacuum, in some sense, space filled with a constant scalar field does not carry a preferred reference frame with it, it does not disturb the motion of objects passing through space that it fills, and so forth. But when a scalar field appears, there is a change in the vacuum energy density... If there were no gravitational effects, this change in energy would go unnoticed [52]."

At present, there is no experimental justification for the existence of scalar fields. However, from a theoretical standpoint, scalar fields play an important role, creating masses in the theory of elementary particle (Higgs mechanism), and scalar particles such as the Dilaton are always present in supersymmetric String theories.

For Cosmological reasons, among others, the vacuum energy density is associated with the energy scales of the Planck, GUT, Electro-Weak and Quark-Hadron eras. The question is: are these vacuum energy densities reflecting the “real world”,

18
and if not, what is the difference? How does this hypothetical vacuum energy density lose energy of the order of $10^{120}$ in transitioning from the Planck to the Quark-Hadron era?

These and other difficulties 51, 52 lead to the questioning of the usefulness of a hypothetical vacuum state.

In conclusion, it is clear that the concept of a vacuum state has lead to real confusion and difficulties in calculating “real” world physical quantities. An analogous definition of a vacuum state will be proposed in the following sections.

5.2 Origin of the ”Effective Mass” - Gravitational Self Energy

Zel’doovich tried to relate a theory of elementary particles to the gravitational interaction energy associated with the Cosmological Constant. He argued intuitively that the vacuum energy density should be given by the gravitational interaction energy of virtual particles whose self-energy was identically equal to zero.

The objective of the next section will be to relate the Cosmological Constant, a macroscopic quantum mechanical parameter, with an effective mass, and a microscopic theory of nature, this can be shown as:

$$\Lambda_{\text{Epoch Energy Scale}}(\text{Macroscopic Parameter}) \leftrightarrow m_{EES}^6 \leftrightarrow \epsilon_{\text{Vac}}^{EES}(\text{Microscopic Theory})$$

A speculative microscopic theory of gravity will be outlined in the following sections.

5.3 Quantum Gravity

5.3.1 Theoretical Approach

The approach will be to attempt to apply String theory to the Cosmological Constant problem. Recently there have been advances in the non-perturbative regime of the theory, the most interesting developments have been in the fields of M and D-brane theories.

M-Theory 54 is a unique theory whose moduli space connects the five perturbative ten-dimensional String theories with 11-dimensional supergravity. It can be described in terms of 2 ten-dimensional worlds where gauge and matter fields are localised, the connection between the two being a line segment or orbifold. In this model (see figure 3) gravity propagates in the eleven-dimensional space of the bulk.

The low energy limit of M-theory is equivalent to 11-D supergravity and corresponds to the strong coupling limit of a $E_8 \times E_8$ 10-d heterotic String theory. On compactification, this gives rise to 2 four-dimensional worlds separated by a line segment where gravity propagates in the bulk 5-d space-time. The theory predicts unification of gauge couplings at $10^{16}$GeV.

D-branes 50 are defined as hypersurfaces (dynamical objects) on which open strings end, here the D stands for Dirichlet. The coordinates of the attached strings...
satisfy Dirichlet boundary conditions in the directions normal to the brane and Neumann conditions in the directions tangent to the brane. A D-brane extending over p-flat spatial dimensions is described by the boundary conditions, given by,

$$\partial_\perp X^{\alpha=0,\cdots,p} = X^{m=p+1,\cdots,9} = 0$$

In the discussion of target-space duality in section 3.1.2 the spectrum of closed strings was shown to be invariant under the transformation \( R \to \alpha'/R \). As \( R \to 0 \) the momentum states become massive but the winding modes approach a continuum (become very light), indicating the formation of a non-compact dimension.

D-branes appear in the \( R \to 0 \) limit of open string theory [50], where compactified on a small torus is equivalent to compactification on a large torus but with the open string end points restricted to lie on the hypersurfaces. D-branes are solitonic objects whose masses are given by \( m_S/g_s, m_s = 1/\sqrt{a'} \), where \( g_s \) is the String coupling constant.

Figure 4 shows 2 D-branes connected by an open string. Figure 5 shows the interaction of two D-branes through the exchange of a closed String (graviton).

When the separation between branes is of a comparable length to the Planck scale, the solitons can interact with each other via the exchange of open strings. It is this property of D-brane dynamics that makes them good probes for the small-scale structure of space-time and which may lead to a formal derivation of the second term in the modified Heisenberg Uncertainty Principle equation [17].

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \frac{\Delta p}{\hbar}.$$  \hspace{1cm} (37)

### 5.3.2 Vacuum State - Analogue Definition

It was suggested previously that difficulties arise when trying to calculate physical quantities from a vacuum state. In this section we will associate the lowest energy or ground state of the Universe in terms of the product of the lowest energy states of the real and compactified worlds.

Here, in a leap of speculative imagination, the compactified world will be considered to act as a kind of vacuum, unseen but affecting the real world through the interaction of gravity. The difference is that the hidden world is a physical entity, unlike the vacuum, which will allow the computation of physical quantities. Presently these calculations are not feasible due to technical difficulties, however there is no a priori reasons why they are not possible.

The main justification for the approach is taken from analogy with M-Theory where it is suggested that our four-dimensional world is a brane embedded in a higher dimensional bulk space-time and where all the matter fields (quarks, leptons, etc) together with the three gauge interactions live exclusively on the brane. Again, gravity is the only interaction that lives in the bulk space.
5.3.3 Model - Approach for Calculating the Cosmological Constant

Outlined below is a speculative approach for calculating the Cosmological Constant for the different epochs of the Universe. The Cosmological Constant will be associated with the gravitation interaction energy - Casimir effect. This can be interpreted as the energy due to vacuum fluctuations of open strings stretched between two D-branes (figures 4,5). The tree-level expression for the gravitational interaction is given by [50]:

\[
\epsilon(r, T) = -\frac{V(p)}{2} \int \frac{d^{p+1}k}{(2\pi)^{p+1}} \int_0^\infty \frac{dt}{t} \text{Str} e^{-\pi t(k^2 + M(T)^2)/2},
\]

(38)

\[
\epsilon(r, T) = -2 \times \frac{V(p)}{2} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-(p+1)/2} e^{-r^2 t/2} \left( -1/2 \right) \sum_{s=2,3,4} \left( -\right)^s \frac{\theta^4(0|\frac{it}{2})}{\eta^2(\frac{t}{2})} F(T, \alpha', r, t)
\]

(39)

The second equation is derived from the first for the particular example shown in figure 5, the \( \theta_i \)'s are the Jacobi theta functions and \( \eta(t) \) is the Dedekind eta function.

There is an expectation that the gravitational interaction energies for the various epoch (Planck, GUT, Electro-Weak, Quark-Hadron), namely the Cosmological Constants, will be recovered at some distance and temperature.

Interestingly, in the high temperature limit of string theory the Hagedorn temperature leads to a phase transition which is analogous to the Quark-Hadron phase transition [55].

5.3.4 Negative \( \Lambda \) and Brown-Teitelboim mechanism

The mechanism suggested by Brown and Teitelboim [15] has the appealing feature that a small negative Cosmological Constant compatible with experimental results can be reconciled with a large positive Cosmological Constant at an earlier period and thus with Inflation. In this mechanism, a Cosmological Constant is neutralized by nucleation of membranes associated with antisymmetric fields. However, in order that their mechanism leads to a realistic value for the vacuum energy density the tension of the brane and its coupling constant are severely constrained. Recently, Bousso and Polchinski use [14] more than one four-forms (the field strength of the three-form) to create a small \( \Lambda_{\text{eff}} \). As has been emphasized by the authors an associated difficulty with this approach is the stabilization of the compact dimensions [1] which is a main issue in string theory. The mechanism of [14] combined with a string theory approach can in principle lead to a negative effective Cosmological Constant compatible in magnitude and sign with that needed to explain the Galactic Velocity Rotation Curves.

\[3\text{In principle extreme regions of the string moduli can lead to small charges and brane tensions, however it is unclear how a moduli potential can be minimised in such regions.}\]
5.3.5 Summary

In this part of the paper a speculative connection has been made between the vacuum and the hidden world of M-theory, and the Cosmological Constant and the gravitational interaction, energy given by String Theory. A suggestion has been made as how, at specific distances and temperatures, the different energy scales of the epochs could be recovered.

Note that in order to arrive at a non-zero value for the gravitational interaction an assumption has made that there is soft supersymmetry-breaking (the expression for the gravitational energy is identically equal to zero for exact supersymmetry) and higher-order effects are suppressed.

The application of String theory to the determination of cosmological parameters will be of considerable interest in the future.
6 Discussion

This paper describes how the non-gravitational contribution to Galactic Velocity Rotation Curves can be explained in terms of a negative Cosmological Constant (Λ).

It is shown that, within the Weak Field Approximation, the experimental values for the Cosmological Constant can be determined from galactic velocity rotation curves. A representative value has been determined to be of the order $|\Lambda_{Exp}| = 2.0 \times 10^{-55} \text{cm}^{-2}$. This value compares favourably with the theoretical value, derived from the Extended LNH, of $|\Lambda_{Theory}| = 2.1 \times 10^{-56} \text{cm}^{-2}$.

The Extended LNH was used to predict values for other cosmological parameters such as: Gravitational, Gravitational Modification Constants, and the effective mass density. The Weak Field Approximation was used to derive the modified Newton Equation, and String Theory via Duality, was used to establish the requirement for a fundamental length in nature.

The experimental results presented support the ideas of a long range fifth force, gravitational in nature, leading to a decelerating Universe, and that the Cosmological Constant is a good candidate for providing the missing “Dark Energy” of the Universe.

Matthews [33] pointed out that, in the Extended Large Number Hypothesis, replacing the mass of the Proton by the effective mass of the vacuum energy density of the relevant epoch lead to agreement with experimental observation. The presently observed Universe, seen in terms of its cosmological parameters, is well described by the Quark-Hadron energy scale. However, the effective mass parameter has been introduced in an arbitrary way. The validity of this term in determining cosmological parameters will need to be justified in terms of a more fundamental microscopic quantum theory.

A speculative theory for the evolution of the universe was outlined. It was shown that within the Extended LNH only two parameters are required to fully specify the cosmological parameters for that epoch: fundamental length and the vacuum energy density.

It was suggested that the concept of a vacuum leads to confusion and the inability to calculate “real” world quantities. A speculative analogous physical alternative was proposed for the vacuum.

It was suggested that the application of String Theory to the problem of the Cosmological Constant could lead to a theory of quantum gravity.

Finally, Dirac’s belief in an as yet unknown fundamental theory linking the quantum mechanic origin of the Universe to large-scale cosmological parameters may yet come true.

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Figure 1: Reproduced with the permission of Corbelli & Salucci.
Figure 2: Cosmological Constant contribution to the NGC-3198 rotation curve
Figure 3: M-theory picture of the world

Figure 4: Open strings stretched between D-branes and with both ends on the same defect
Figure 5: Two D-branes interacting through the exchange of a closed string