An organizing principle for two-dimensional strongly correlated superconductivity

L. Fratino, P. Sémon, G. Sordi & A.-M. S. Tremblay

Superconductivity in the cuprates exhibits many unusual features. We study the two-dimensional Hubbard model with plaquette dynamical mean-field theory to address these unusual features and relate them to other normal-state phenomena, such as the pseudogap. Previous studies with this method found that upon doping the Mott insulator at low temperature a pseudogap phase appears. The low-temperature transition between that phase and the correlated metal at higher doping is first-order. A series of crossovers emerge along the Widom line extension of that first-order transition in the supercritical region. Here we show that the highly asymmetric dome of the dynamical mean-field superconducting transition temperature $T_{c_0}$, the maximum of the condensation energy as a function of doping, the correlation between maximum $T_{c_0}$ and normal-state scattering rate, the change from potential-energy driven to kinetic-energy driven pairing mechanisms can all be understood as remnants of the normal state first-order transition and its associated crossovers that also act as an organizing principle for the superconducting state.

Model and Method

The two dimensional Hubbard model on a square lattice reads

$$H = -\sum_{ij\sigma} t_{ij} c_{ij\sigma}^\dagger c_{ij\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_{i\sigma}$$

where $c_{ij\sigma}^\dagger$ and $c_{ij\sigma}$ operators create and destroy an electron of spin $\sigma$ on site $i$, $n_{i\sigma} = c_{ij\sigma}^\dagger c_{ij\sigma}$ is the number operator, $\mu$ is the chemical potential, $U$ the onsite Coulomb repulsion and $t_{ij}$ is the nearest neighbor hopping amplitude. Neglecting second-neighbor hopping, necessary to capture the correct Fermi surface, minimizes the Monte-Carlo sign-problem and does not alter our main findings (see supplementary Fig. S7). Unless specified, the lattice spacing, Planck's constant, Boltzmann’s constant and $t$ are unity.

We solve this model using cellular dynamical mean-field theory (CDMFT) on a $2 \times 2$ plaquette immersed in an infinite self-consistent bath of non-interacting electrons. This plaquette is the minimal cluster that includes all two-dimensional short-range charge, spin and superconducting dynamical correlations. We do not take into account long-range charge-density waves in light of the recent experimental results where this transition...
We use two recent algorithmic improvements to speed up the calculations: a fast rejection algorithm with skip-list data structure and four point updates that are necessary for broken symmetry states like d-wave superconductivity.

Let us first consider the superconducting phase diagram. We then discuss features of the normal state that determine its shape.

**Superconducting dome.** Previous studies show that both at half-filling and at finite doping the metallic state close to the Mott insulator is unstable to d-wave superconductivity in previous work. Since we are interested in large values of U, i.e. a doped Mott insulator, the most appropriate method to solve the impurity (cluster plus bath) problem is the hybridization expansion continuous-time quantum Monte Carlo method. Sign problems prevent the study of large U with alternate quantum Monte Carlo methods. We use two recent algorithmic improvements to speed up the calculations: a fast rejection algorithm with skip-list data structure and four point updates that are necessary for broken symmetry states like d-wave superconductivity.

As a function of U, $T_c^d$ changes from finite to zero discontinuously at the first-order Mott metal-insulator transition (red shaded region in panel a). Superconductivity appears in the metastable metallic state near the Mott insulator, never in the Mott insulator itself (panels a, b). As a function of doping, $T_c^d$ forms a dome as long as U is larger than the critical value necessary to obtain a Mott insulator at half-filling (panels c–g). In our previous studies we left opened two possibilities: as a function of $U_1$ the actual $T_c^d$ is ($T_c^d(δ)$) plummets with decreasing $δ$ or of competing long range order or increased (because of pairing through long wavelength antiferromagnetic fluctuations) but $T_c^d$ still remains a useful quantity marking the region where Mott physics and short-range correlations produce pairing.

The superconducting dome is highly asymmetric. $T_c^d(δ)$ is zero at $δ = 0$, initially rises steeply with further doping and then declines more gently with further doping. The global maximum $T_c^d(δ)$ in the $U - T - δ$ space occurs just above $U_{MIT}$ and at finite doping $δ_{opt}$. Further increase of U leads to a decrease in $T_c^d(δ)$ as expected if $T_c^d(δ)$ scales with the superexchange energy $J = 4t^2/U$ for large enough $U > 10.26$. As a function of U, the optimal doping $δ_{opt}$ departs from $δ = 0$ for $U > U_{MIT}$ increasing with U and saturating around $δ = 0.04$ for large U (see also supplementary Fig. S2).

The range of doping where superconductivity occurs at the lowest temperature is consistent with results obtained with CDMFT at $T = 0$. The asymmetric superconducting dome with an abrupt fall of $T_c^d$ with decreasing $δ$ is also consistent with dynamical cluster approximation results on larger clusters. In the latter calculations, the increased accuracy in momentum space leads to a $T_c^d$ that vanishes before half-filling.

**Superconducting order parameter.** To analyse the shape of the superconducting phase we turn to the superconducting order parameter $Φ$, whose magnitude is color-coded in Fig. 1 (the raw data is in Fig. S1). While $T_{c_0}^{max}$ occurs at finite doping, the overall maximum $Φ_{max}$ is found in the undoped model close to the Mott insula-
Superconductivity and pseudogap. Understanding the normal state has long been considered a prerequisite to a real understanding of high-temperature superconductivity. This comes out clearly from our results. Previous normal-state CDMFT studies show that for $U > U_{\text{MIT}}$ and small $\delta$, large screened Coulomb repulsion $U$ and the emergent superexchange $J$ lead at low $T$ to a state with strong singlet correlations. That phase has the characteristics of the pseudogap phase. The fall of the Knight shift as a function of temperature is usually associated with the pseudogap onset $T^\ast$ (blue circles in Fig. 2a–c) and is thus $T^\ast$ of the underlying normal state. Color corresponds to the magnitude of the scattering rate $\Gamma$, estimated from the zero-frequency extrapolation of the imaginary part of the $(\pi, 0)$ component of the cluster self-energy. The Widom line is known as $T_W$ (orange squares) and separates the superconducting and normal states, for $T/t = 1/50, 1/100$ (full and dashed line, respectively). Shaded bands give standard errors. The loci where the condensation energy is largest are shown in the upper panels as green filled squares. They follow $T_W(\delta)$ and $\Phi_{\text{max}}(\delta)$. But as a function of doping, for $U > U_{\text{MIT}}, \Phi$ forms a dome that reaches a peak at $\delta_{\text{max}}$. At our lowest temperature, $\delta_{\text{max}}$ increases with increasing $U$, and saturates around $\delta \approx 0.1110$ for large values of $U$. Notice that $\delta_{\text{max}}$ at our lowest temperature does not coincide with $\delta_{\text{opt}}$, i.e. the doping that optimizes $T_c$. Hence, $T^\ast(\delta)$ does not scale with $\Phi(\delta, T \to 0)$. Instead, the locus of the maxima of $\Phi$ in the $\delta - T$ plane at fixed $U$ traces a negatively sloped line within the superconducting dome (lines with blue triangles) that separates the superconducting dome in two regions. The sharp asymmetry of the superconducting dome is thus linked to this negatively sloped line, which in turn is related to the phase transition between pseudogap and correlated metal in the underlying normal state, as we discuss below.

![Figure 2](https://example.com/fig2.png)

Figure 2. (a–c) Temperature versus hole doping phase diagram for $U/t = 6.2, 7$ and $9$, respectively. Superconductivity is delimited by $T_c^\ast$ (line with blue filled circles). Beneath the superconducting dome, the normal-state coexistence region across the first-order transition between a pseudogap and a correlated metal appears in (a) as red shaded area. It is delimited by the jumps in the electron density as a function of chemical potential and collapses at the critical endpoint ($T_c^\ast, \delta_p$). The Widom line $T_W$ emerging from the endpoint is estimated by the maxima of the charge compressibility along paths at constant $\delta$ (line with red triangles), and the pseudogap onset $T^\ast$ is computed by the maximum of the spin susceptibility (line with orange circles). The loci of $\Phi_{\text{max}}(\delta)$ are shown by blue triangles and follow $T_W$ of the underlying normal state. Color corresponds to the magnitude of the scattering rate $\Gamma$, estimated from the zero-frequency extrapolation of the imaginary part of the $(\pi, 0)$ component of the cluster self-energy. (d–f) Difference in kinetic, potential and total energies (blue, red and green lines respectively) between the superconducting and normal states, for $T/t = 1/50, 1/100$ (full and dashed line, respectively). Shaded bands give standard errors. The loci where the condensation energy is largest are shown in the upper panels as green filled squares. They follow $T_W(\delta)$ and $\Phi_{\text{max}}(\delta)$.
Condensation energy. The superconducting state clearly has a lower free energy than the normal state out of which it is born. In the ground state, the energy difference between both states is known as the condensation energy. The origin of the condensation energy is unambiguous only within a given model\cite{45,46}. In the BCS model, superconductivity occurs because of a decrease in potential energy. The kinetic energy increase due to particle-hole mixing in the ground state is not large enough to overcome the potential energy drop. In the cuprates, analysis of inelastic neutron scattering\cite{35} has suggested that superconductivity is kinetic-energy driven in the underdoped regime\cite{34,35,41-43}. Analysis of ARPES\cite{36} and optical data\cite{37-40} in the context of the Hubbard model has suggested that superconductivity is kinetic-energy driven in the underdoped regime\cite{34,35,41-43}.

In the lower panels of Fig. 2 plot, for the Hubbard model Eq. 1, the difference in kinetic and potential energies between the superconducting and normal states (\(\Delta E_{\text{kin}}\) and \(\Delta E_{\text{pot}}\); blue and red lines respectively) as a function of doping. The results for the two different temperatures are close enough to suggest we are close to ground state values. The net condensation energy, shown by the green line, is always negative, as expected. The doping dependence of \(\Delta E_{\text{kin}}\) and \(\Delta E_{\text{pot}}\) on the other hand shows two striking features: it is non monotonic and can display a sign change. For \(U = 6.2, 7\), the superconducting kinetic-energy driven at small doping and potential energy driven, as in BCS theory, at large doping. For \(U = 9\), the superconductivity is kinetic energy driven for all dopings, although the potential energy difference \(\Delta E_{\text{pot}}\) can change sign.

Previous investigations\cite{23,39,44} have revealed a complex behavior that remained to this day a puzzle, with \(\Delta E_{\text{kin}}\) going from negative to positive depending on \(T\) and \(U\). What has been missing to make sense of this complexity is the existence of the normal state first-order transition and its associated supercritical crossovers. By considering different values of \(U\), we provide a unified picture of a host of apparently contradictory results. For all \(U\) considered, the largest condensation energy (see green line in bottom panels of Fig. 2 and green squares in top panels of Fig. 2) is concomitant with the largest superconducting order parameter \(\Phi(\delta)\) (but not with the maximum \(T^d_c\)) and hence correlates with the normal-state pseudogap-to-correlated metal first-order transition, and its associated supercritical crossovers. For all \(U\), the sign changes are also close to the maximum condensation energy and hence also correlated with the same normal-state features. The influence of Mott and superexchange physics extends unambiguously all the way to the normal-state first-order transition terminating at the critical endpoint, from which supercritical crossovers emerge\cite{31}. This reflects itself in the superconducting state in a decisive manner: the changes in sign of the different sources of condensation energy occur for dopings similar to those where the normal-state transition occurs.

Source of condensation energy. Bottom panels of Fig. 2 (see also Fig. S5) show that in the underdoped region, the kinetic-energy change in the superconducting state is close to minus twice the potential energy change. This is what is expected if superexchange\cite{47} drives superconductivity there\cite{26}. The decrease with \(U\) of the maximum \(T^d_c\), of the magnitude of the individual kinetic and potential energy contributions to condensation energy, and of the maximum value of the \(T = 0\) order parameter\cite{8-10,18}, are all consistent with the importance of \(j\) in the effective model that arises from the Hubbard model at large \(U\). The BCS-like behavior in the overdoped regime for \(U = 6.2, 7\) probably arises from leftover of the weak-coupling long-wavelength antiferromagnetic spin-wave pairing mechanism\cite{46}, although the effect of the self-consistent rearrangement of the spin-fluctuation spectrum in the superconducting state has not been studied yet.

Discussion

Our findings further broaden our understanding of the CDMFT solution of the Hubbard model in the doped Mott insulator regime by showing how and to what extent the organizing principle for both the normal state and the superconducting state is the finite-doping first-order transition that determines the shape and the properties of both phases, even though the transition itself is invisible in the superconducting state. In the \(T = \delta\) plane, the loci of the maximum order parameter, of the extremum condensation energy, of the maximum normal state scattering relative to the maximum \(T^d_c\), all correlate with crossover lines of the underlying normal state that is unstable to d-wave superconductivity.

We speculate that the application of a magnetic field strong enough to suppress \(T^d_c\) and pressures large enough to remove density waves may reveal the underlying transition. We also speculate that sound anomalies associated with the large compressibility in the underlying normal state above the critical endpoint could appear, in analogy with what is observed near the half-filled Mott transition in layered organics\cite{47-52}. The appearance of large electronic compressibility near the normal state first-order transition suggests that further studies of ubiquitous bond-density waves should be undertaken with the same set of methods.
References

1. Keimer, B., Kivelson, S. A., Norman, M. R., Uchida, S. & Zaanen, J. From quantum matter to high-temperature superconductivity in copper oxides. *Nature* **518**, 179–186 (2015).
2. Tremblay, A.-M. S. Strongly correlated superconductivity. In Pavarini, E., Koch, E. & Schollwöck, U. (eds.) *Emergent Phenomena in Correlated Matter Modeling and Simulation*, vol. 3, chap. 10 (Verlag des Forschungszentrum, 2013).
3. Kotliar, G. et al. Electronic structure calculations with dynamical mean-field theory. *Rev. Mod. Phys.* **78**, 865 (2006).
4. Maier, T., Jarrell, M., Pruschke, T. & Hettler, M. H. Quantum cluster theories. *Rev. Mod. Phys.* **77**, 1027–1080 (2005).
5. Georges, A., Kotliar, G., Krauth, W. & Rozenberg, M. I. Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions. *Rev. Mod. Phys.* **68**, 13 (1996).
6. Sordi, G., Sémon, P., Haule, K. & Tremblay, A.-M. S. Pseudogap temperature as a Widom line in doped Mott insulators. *Sci. Rep.* **2**, 347 (2012).
7. Cyr-Choinière, O. et al. Suppression of charge order by pressure in the cuprate superconductor YBa$_2$Cu$_3$O$_y$; Restoring the full superconducting dome. *ArXiv e-prints* 1503.02033 (2015).
8. Sénéchal, D., Lavertu, P.-L., Marois, M.-A. & Tremblay, A.-M. S. Competition between antiferromagnetism and superconductivity in high-T$_c$ cuprates. *Phys. Rev. Lett.* **94**, 156404 (2005).
9. Capone, M. & Kotliar, G. Competition between d-wave superconductivity and antiferromagnetism in the two-dimensional Hubbard model. *Phys. Rev. B* **74**, 054513 (2006).
10. Kancharla, S. S. et al. Anomalous superconductivity and its competition with antiferromagnetism in doped mott insulators. *Phys. Rev. B* **77**, 184516 (2008).
11. Gull, E. et al. Continuous-time monte carlo methods for quantum impurity models. *Rev. Mod. Phys.* **83**, 349–404 (2011).
12. Sémon, P., Yee, C.-H., Haule, K. & Tremblay, A.-M. S. Lazy skip-lists: An algorithm for fast hybridization-expansion quantum monte carlo. *Phys. Rev. B* **90**, 075149 (2014).
13. Sémon, P., Sordi, G. & Tremblay, A.-M. S. Ergodicity of the hybridization-expansion monte carlo algorithm for broken-symmetry states. *Phys. Rev. B* **89**, 165113 (2014).
14. Sordi, G., Sémon, P., Haule, K. & Tremblay, A.-M. S. Strong coupling superconductivity, pseudogap, and mott transition. *Phys. Rev. Lett.* **108**, 216401 (2012).
15. Maier, T., Jarrell, M., Pruschke, T. & Koller, J. d-wave superconductivity in the hubbard model. *Phys. Rev. Lett.* **85**, 1524–1527 (2000).
16. Lichtenstein, A. I. & Katsnelson, M. I. Antiferromagnetism and d-wave superconductivity in cuprates: A cluster dynamical mean-field theory. *Phys. Rev. B* **62**, R9239–R9286 (2000).
17. Kyung, B. & Tremblay, A.-M. S. Mott transition, antiferromagnetism, and d-wave superconductivity in two-dimensional organic conductors. *Phys. Rev. Lett.* **97**, 046402 (2006).
18. Aichhorn, M., Arrigoni, E., Potthoff, M. & Hanke, W. Antiferromagnetic to superconducting phase transition in the hole- and electron-doped hubbard model at zero temperature. *Phys. Rev. B* **74**, 024508 (2006).
19. Balzer, M., Hanke, W. & Potthoff, M. Importance of local correlations for the order parameter of high-T$_c$ superconductors. *Phys. Rev. B* **81**, 144516 (2010).
20. Maier, T. A., Jarrell, M., Schulthess, T. C., Kent, P. R. C. & White, J. B. Systematic study of d-wave superconductivity in the 2d repulsive hubbard model. *Phys. Rev. Lett.* **95**, 237001 (2005).
21. Haule, K. & Kotliar, G. Strongly correlated superconductivity: A plaquette dynamical mean-field theory study. *Phys. Rev. B* **76**, 104509 (2007).
22. Gull, E., Parcollet, O. &Millis, A. J. Superconductivity and the pseudogap in the two-dimensional hubbard model. *Phys. Rev. Lett.* **110**, 216405 (2013).
23. Gull, E. & Millis, A. J. Energetics of superconductivity in the two-dimensional hubbard model. *Phys. Rev. B* **86**, 241106 (2012).
24. Emery, V. J. & Kivelson, S. A. Superconductivity in bad metals. *Phys. Rev. Lett.* **74**, 3253–3256 (1995).
25. Beal-Monod, M. T., Bourbonnais, C. & Emery, V. J. Possible superconductivity in nearly antiferromagnetic itinerant fermion systems. *Phys. Rev. B* **34**, 7716–20 (1986).
26. Kotliar, G. & Liu, J. Superconducting instabilities in the large-U limit of a generalized hubbard model. *Phys. Rev. Lett.* **61**, 1784–7 (1988).
27. Alloul, H., Mendels, P., Casalot, H., Marucco, J. F. & Arasaki, J. Correlations between magnetic and superconducting properties of Zn-substituted YBa$_2$Cu$_3$O$_{y+}$, *Phys. Rev. Lett.* **67**, 3140–3143 (1991).
28. Sordi, G., Sémon, P., Haule, K. & Tremblay, A.-M. S. c-axis resistivity, pseudogap, superconductivity, and widom line in doped mott insulators. *Phys. Rev. B* **87**, 041103 (2013).
29. Sordi, G., Haule, K. & Tremblay, A.-M. S. Mott physics and first-order transition between two metals in the normal-state phase diagram of the two-dimensional Hubbard model. *Phys. Rev. B* **84**, 075161 (2011).
30. Xu, L. et al. Relation between the Widom line and the dynamic crossover in systems with a liquid liquid phase transition. *Proc. Natl. Acad. Sci. USA* **102**, 16558–16562 (2005).
31. Sordi, G., Haule, K. & Tremblay, A.-M. S. Finite Doping Signatures of the Mott Transition in the Two-Dimensional Hubbard Model. *Phys. Rev. Lett.* **104**, 226402 (2010).
32. Haule, K. & Kotliar, G. Avoided criticality in near-optimally doped high-temperature superconductors. *Phys. Rev. B* **76**, 092503 (2007).
33. Chester, G. V. Difference between normal and superconducting states of a metal. *Phys. Rev. 103*, 1693–1699 (1956).
34. Leggett, A. A. “midinfrared” scenario for cuprate superconductivity. *Proceedings of the National Academy of Sciences* **96**, 8365–8372 (1999).
35. Scalapino, D. J. & White, S. R. Superconducting condensation energy and an antiferromagnetic exchange-based pairing mechanism. *Phys. Rev. B* **58**, 8222–8224 (1998).
36. Norman, M. R., Randeria, M., Jankó, B. & Campuzano, J. C. Condensation energy and spectral functions in high-temperature superconductors. *Phys. Rev. B* **61**, 1472–14750 (2000).
37. Molegraaf, H. J. A., Presura, C., van der Marel, D., Kes, P. H. & Li, M. Superconductivity-induced transfer of in-plane spectral weight in Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$. *Science* **295**, 2239–2241 (2002).
38. Deutschge, G., Mantendorf-ryo, A. F. & Bontemps, N. Kinetic energy change with doping upon superfluid condensation in high-temperature superconductors. *Phys. Rev. B* **72**, 092504 (2005).
39. Carbene, E. et al. Doping dependence of the redistribution of optical spectral weight in Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$. *Phys. Rev. B* **74**, 064510 (2006).
40. Giannetti, C. et al. Revealing the high-energy electronic excitations underlying the onset of high-temperature superconductivity in cuprates. *Nature Communications* **2**, 353 (2011).
41. Anderson, P. W. The theory of Superconductivity in the High Tc cuprates (Princeton University Press, Princeton, 1997).
42. Hirsch, J. & Marsiglio, F. Where is 99% of the condensation energy of Tl$_2$Ba$_2$CuO$_y$ coming from? *Physica C: Superconductivity* **331**, 150–156 (2000).
43. Demler, E. & Zhang, S.-C. Quantitative test of a microscopic mechanism of high-temperature superconductivity. *Nature* **396**, 733–735 (1998).
44. Maier, T. A., Jarrell, M., Macridin, A. & Slezak, C. Kinetic energy driven pairing in cuprate superconductors. *Phys. Rev. Lett.* **92**, 027005 (2004).
45. Fazekas, P. Lecture Notes on Electron Correlation and Magnetism (World Scientific, Singapore, 1999).
46. Scalapino, D. The case for $d_{x^2−y^2}$ pairing in the cuprate superconductors. Phys. Rev. Lett. 90, 329–365 (1995).
47. Fournier, D., Poirier, M., Castonguay, M. & Truong, K. D. Mott transition, compressibility divergence, and the $P−T$ phase diagram of layered organic superconductors: An ultrasonic investigation. Phys. Rev. Lett. 90, 127002 (2003).
48. Hassan, S. R., Georges, A. & Krishnamurthy, H. R. Sound velocity anomaly at the mott transition: Application to organic conductors and $V_{O}$, Phys. Rev. Lett. 94, 036402 (2005).
49. Rozenberg, M. I., Chitra, R. & Kollmar, G. Finite temperature mott transition in the hubbard model in infinite dimensions. Phys. Rev. Lett. 83, 3498–3501 (1999).
50. Furukawa, T., Miyagawa, K., Taniguchi, H., Kato, R. & Kanoda, K. Quantum criticality of Mott transition in organic materials. Nature Physics 3, 221 (2015).
51. Terletskaya, J., Vucicevic, D., Tanaskovic, & Dobrosavljevic V. Phys. Rev. Lett. 107, 026401 (2011) [http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.107.026401]
52. Hebert, C.-D., Semon, P., & Tremblay, A.-M. S. Phys. Rev. B. 92, 195112 (2015) [http://journals.aps.org/prb/abstract/10.1103/PhysRevB.92.195112]

Acknowledgements
We acknowledge D. Sénéchal, L. Taillefer, C. Bourbonnais and H. Alloul for useful discussions. This work was partially supported by the Natural Sciences and Engineering Research council (Canada), and by the Tier I Canada Research Chair Program (A.-M.S.T.). Simulations were performed on computers provided by CFI, MELS, Calcul Québec and Compute Canada.

Author Contributions
L.F. obtained and analysed the data. P.S. wrote the main codes. G.S. and A.-M.S.T. supervised the project and wrote the manuscript, and all authors discussed the results and commented on the manuscript.

Additional Information
Supplementary information accompanies this paper at http://www.nature.com/srep

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Fratino, L. et al. An organizing principle for two-dimensional strongly correlated superconductivity. Sci. Rep. 6, 22715; doi: 10.1038/srep22715 (2016).

This work is licensed under a Creative Commons Attribution 4.0 International License. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder to reproduce the material. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/