Entanglement in Quantum Field Theory is restricted to spacelike separations to the order of the Compton wavelength $\hbar/mc$ (e.g., S. J. Summers and R. Werner, *J. Math. Phys.*, **28**, 10,2440-2447, (1987)). Yet spin entanglement of electrons across macroscopic distances has been observed by Hensen *et al.* (*Nature*, **526**, doi:10.1038/nature/15759, (2015)). The parametrized relativistic quantum mechanics of Feynman and Stueckelberg admits spin singlets, across arbitrary separations, by providing a single covariant wave equation for tensor products of two Dirac spinors (A. F. Bennett, *Ann. Phys.* **345**, 1-16 (2014)). The formalism is extended here from quantum electrodynamics to the electroweak interaction. A relativistic Bell’s inequality for Dirac spinors is extended here to weak isospin.
I. INTRODUCTION

The parametrized relativistic quantum mechanics (hereafter PM) of R. Feynman \cite{1} and C. Stueckelberg \cite{2} can with one exception represent every phenomenon of Quantum Electrodynamics \cite{3,4,5,6}. The sole exception is anti-bunching in quantum optics at very low intensity \cite{7,8}. On the other hand PM, unlike Quantum Field Theory (QFT), admits electron spin entanglement across macroscopic proper distances as has now been observed \cite{9}. PM is extended here to the electroweak interaction. The following sections include a brief statement of the Standard Model (SM) freely referring to a standard modern monograph \cite{10} both for detail and for notation. The PM representation of the SM is identical to QFT, except that (i) the dependent variables are c-valued wavefunctions rather than fields of operators on Fock space, and (ii) the Feynman-Stueckelberg parameter is introduced into the Higgs-fermion couplings. The parameter $\tau$ has physical reality since it explains quantum interference in local coordinate time \cite{11}. Finally, the Pauli-Lubanski matrices are used to extend the relativistic Bell’s inequality for Dirac spinors \cite{6} to weak isospin.

II. THE ELECTROWEAK LAGRANGIAN

A. the Weinberg-Salam Lagrange density

The representation here of the Standard Model is in the form of parametrized relativistic quantum mechanics. The wavefunctions for the fermions all depend upon the same real parameter $\tau$ having the range $-\infty < \tau < \infty$. For example, a Dirac 4-spinor wavefunction becomes $\psi(x, \tau)$, where $x_\mu$ (for $\mu = 0, 1, 2, 3$) or simply $x$ is an event in spacetime. The Lorentz metric $g^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is restricted to $\mathbb{R}^4$, that is, it does not include $\tau$.

The gauge bosons $W_\mu(x), B_\mu(x)$ and the Higgs field $\Phi(x)$ are classical fields, all of which are independent of $\tau$. Their Lagrange densities $\mathcal{L}_G(x)$ and $\mathcal{L}_{HG}(x)$ are identical to those of the SM. See \cite[10, p60]{10}. The notation therein is conventional and is closely followed here. For example, the Higgs self-interaction in $\mathcal{L}_{HG}$ is

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$  \hspace{1cm} (1)

The fermion wavefunctions $\Psi(x, \tau)$ are, as indicated, dependent upon $\tau$. Their Lagrange densities $\mathcal{L}_F(x, \tau)$ are identical to those of the SM, see again \cite[10, p60]{10}. The fermion La-
grangians include averaging over $\tau$, as well as summing over spacetime exactly as in the SM. That is,

$$L_{WS}(x) = L_G(x) + L_{HG}(x) + \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( L_F(x, \tau) + L_{HF}(x, \tau) \right) d\tau. \quad (2)$$

It follows immediately from (2) that the gauge fields and the Higgs boson are supported by $\tau$-averaged fermion currents.

As will be seen in (4) below, the only departure from the SM is in the Lagrange density $L_{HF}$ for the Higgs-fermion couplings. The Higgs ground state operator $-i\partial_\tau$ replaces the Higgs ground state c-number $v/\sqrt{2} = \mu/\sqrt{2\lambda}$, in SM notation [10, p63] [12]. The c-number is not so replaced elsewhere.

### B. gauge bosons

The gauge bosons $W_\mu$ and $B_\mu$, being independent of $\tau$, are Standard. Their contributions to the mass Lagrangian $L_{\text{mass}}$ are Standard, as in [10, p62]. In particular, the bare mass $M_W$ of the charged vector bosons $W^{\pm}$ has the Standard value $M_W = g_2 \mu/2\sqrt{\lambda}$ where $g_2$ is the $SU(2)_L$ coupling constant. The weak mixing angle $\theta_W$ is Standard ($\tan \theta_W = g_1/g_2$, where $g_1$ is the $U(1)$ coupling constant), as are the masses for the photon $A_\mu$ ($M_\gamma = 0$) and the massive neutral boson $Z_\mu$ ($M_Z = M_W/\theta_W$) [10, p62-63].

### C. fermions

The Lagrange density for Higgs couplings to the first generation of quarks and leptons is $L_{HF} = L_{Hq} + L_{HL}$. For up and down quarks,

$$L_{Hq} = -g_u \bar{q}_L \tilde{\Delta} u_R - g_d \bar{d}_L \Delta d_R + \text{h.c.}, \quad (3)$$

where

$$\Delta = \begin{pmatrix} 0 \\ -i\partial_\tau \end{pmatrix} + \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad (4)$$

where $\varphi^+$ and $\varphi^0$ are spin-zero Higgs wavefunctions “with electric charge assignments as indicated”. [10], and where $\tilde{\Delta} = i\tau_2 \Delta^*$. Here, unfortunately, $\tau_2$ represents one of the three
isospin matrices. Again, the Higgs vacuum operator $-i\partial_\tau$ in (4) replaces the Higgs vacuum c-number $\vartheta \equiv v/\sqrt{2}$ of the SM. The coupling constants $g_u$ and $g_d$ are related to the rest masses of particles by $m_u = g_u\vartheta$ and $m_d = g_d\vartheta$ respectively. Combining $L_F$ and $L_{Hq}$ establishes that a free up-quark $u$, for example, obeys the parametrized Dirac wave equation

$$\left(\slashed{\partial} + g_u\partial_\tau\right)u = 0.$$  (5)

For a plane wave $u(x, \tau) = u(0, 0) \exp[i(p \cdot x + \varpi_p \tau)]$, the dispersion relation is $p \cdot p = -g_u^2 \varpi_p^2$. It is now assumed that a fermion is “on shell” if $\varpi_p = \vartheta = v/\sqrt{2}$. Hence a free on-shell up-quark $u(x, \tau)$ satisfies the Dirac equation

$$(i\slashed{\partial} - m_u)u = 0,$$  (6)

where $m_u = g_u\vartheta$ is the up-quark mass. Details of the free wavefunctions, discrete symmetries and influence functions may be found in [3]. The normalization factor here for the spinor amplitudes of a free electron wavefunction is $\sqrt{(E_p + m_p)/2g_em_p}$ where $E_p = p^0$ and $m_p = g_em_p$. A free particle on mass shell propagates on mass shell. If an initial particle in a scattering process is on mass shell ($\varpi_p = \vartheta = v/\sqrt{2}$) then, as a consequence of the scattering field being independent of $\tau$, the final particle is also on mass shell.

For any same-generation fermion doublets $\Psi(x, \tau)$ and $\Upsilon(x, \tau)$ satisfying

$$\left(\slashed{D} + g\slashed{\tau}\right)\Psi(x, \tau) = \left(\slashed{D} + g\slashed{\tau}\right)\Upsilon(x, \tau) = 0,$$  (7)

it may be shown that

$$\frac{\partial}{\partial x^\nu}(\Upsilon\gamma^\nu\Psi) + \frac{\partial}{\partial \tau}\Upsilon g\Psi = 0.$$  (8)

In the case of the first quark generation, for example, $g = g^{(1)}$ is the diagonal matrix diag$(g_u, g_d)$. It follows from (8) that the invariant bilinear form $\int \Upsilon g \Psi d^4x$ is independent of $\tau$. The parametrized wave equation for two fermion doublets is

$$\left(\slashed{D}_x \otimes 1 + 1 \otimes \slashed{D}_y\right)\Theta(x, y, \tau) + \partial_\tau \left(g^{(1)} \otimes g^{(2)}\Theta(x, y, \tau]\right) = 0,$$  (9)

where the wavefunction $\Theta(x, y, \tau)$ is in general a sum of tensor products of doublets such as the wavefunctions $\Psi(x, \tau)$ and $\Upsilon(y, \tau)$. There are two coordinate times in (9), namely $x^0$ and $y^0$. If the two-particle wavefunction $\Theta$ and hence (9) had not been parametrized, then integration with respect to either $x^0$ or $y^0$ would violate covariance. The parametrization of $\Theta$, and integration of (9) with respect to the parameter $\tau$ preserves covariance.
III. WEAK ISOSPIN ENTANGLEMENT

The relativistic Bell’s inequality for a singlet of Dirac spinors is an elementary paraphrase of the non-relativistic development for a singlet of Pauli spinors [13, §12.2]. The covariant spin operator for Dirac spinors is $\gamma_5/\alpha$, where $\alpha_\mu = -1$, replacing the Pauli spin operator $\sigma_j \sigma_j$ where $\sigma_j \sigma_j = 1$. The relativistic Bell’s inequality for weak isospin requires only the definition of a covariant operator for weak isospin. The definition is as follows.

The isospin doublets $\Psi(x, \tau)$ and $\Upsilon(x, \tau)$ for same-generation fermions must respect their spin-statistics, that is, a two-doublet wavefunction $\Theta$ must be of the form

$$\Theta(x, y, \tau) = \frac{1}{\sqrt{2}} \left( \Psi(x, \tau) \otimes \Upsilon(y, \tau) - \Upsilon(x, \tau) \otimes \Psi(y, \tau) \right).$$

(10)

The two-doublet state is manifestly entangled in spacetime, and the parametrized wave equation (9) in no way restricts the separation $x - y$ of this entanglement. The $SU(2)$ spinor basis is $\tau^\mu = \{1, \tau_1, \tau_2, \tau_3\}$ where again the $\tau_j$ for $j = 1, 2, 3$ are the Pauli isospin matrices. The dual basis is $\hat{\tau}^\mu = \{1, -\tau_1, -\tau_2, -\tau_3\}$. An invariant inner product is provided, again, by $\int \overline{\Psi} g \Upsilon d^4 x$ for all positive-energy isospin doublets $\Psi(x, \tau)$ and $\Upsilon(x, \tau)$. The two Dirac spinor components of the doublets transform independently and covariantly in the usual way [14, Ch2], while the doublets transform in the $SU(2)$ representation. Next, the Dirac matrices $\gamma^\mu$ are replaced with the Pauli-Lubanski matrices $X^\mu(p) = iL^{\mu\nu}p_\nu$, where the objects $L^{\mu\nu} = (i/4)(\tau^\mu \hat{\tau}^\nu - \tau^\nu \hat{\tau}^\mu)$ generate the $SU(2)$ representation of the Lorentz group [15]. The isospin operator is then $-(2/m_p)a_\mu X^\mu(p)$, for any $a$ such that $p \cdot a = 0$ and $a \cdot a = -1$.

The spatial extent of a static vector boson is its Compton wavelength and so the feasibility of a “weak Stern-Gerlach apparatus” seems remote, but parametrized relativistic quantum mechanics does admit unrestricted entanglement of left-handed fermion doublets.

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