Abstract

We study the dynamics of Josephson junction arrays with positional disorder and driven by an external current. We consider weak magnetic fields, corresponding to a frustration $f = n + 1/25$ with $n$ integer. We find that above the critical current $i_c$ there is a plastic flow of vortices, where most of the vortices are pinned and only a few vortices flow through channels. This dynamical regime is characterized by strong fluctuations of the total vorticity. The number of the flow channels grow with increasing bias current. At larger currents there is a dynamical regime characterized by the homogeneous motion of all the vortices, i.e. a flux flow regime. We find a dynamical phase transition between the plastic flow and the flux flow regimes when analyzing voltage-voltage correlation functions.

*in Proceedings of the Workshop on Josephson Junction Arrays, ICTP, Trieste, Italy (August 1995), edited by S. R. Shenoy. To appear in Physica B (1996).
1 Introduction

The problem of transport in a medium with quenched disorder leads to very rich physics in a great variety of systems \[1\]. Examples include charge density waves, Wigner solids, magnetic bubble arrays, fluid invasion in porous media and pinning phenomena in type II superconductors \[2, 3, 4, 5, 6, 7\]. All these systems have in common a competition between interaction forces and randomness under an external driving force. For very large driving forces the randomness is believed to be irrelevant, and the dynamics is interaction dominated. The system shows homogeneous flow, and all the internal degrees of freedom move as a whole. On the other hand, for low driving forces the dynamics is dominated by the disorder. The system shows a very inhomogeneous flow, where the motion breaks up into pieces moving with different velocities or not moving at all. It has been recently discussed that a dynamical phase transition may exist between these two kinds of flow \[3, 7\].

In the last years, a great deal of attention has been devoted to the study of vortex pinning and dynamics in type II superconductors as an example of this kind of problem \[3, 5, 6\]. It has been shown that for all but very weak pinning, dissipation starts at the critical current via channels of flow of vortices. Experiments by Bhattacharya and Higgins \[6\] have studied the different dynamical regimes of plastic flow and flux flow motion in superconductors. Koshelev and Vinokur \[7\] have discussed a dynamical melting transition from a moving vortex lattice at large currents to a fluid flow of vortices at lower currents.

Most of the experimental systems mentioned above have the difficulty that there is no control of the nature and amount of disorder. Therefore the study of disordered Josephson junction arrays (JJA) \[8, 9, 10\] becomes particularly promising here, since they can be specifically fabricated with controlled randomness \[8\]. Also, the recent development of imaging techniques in JJA can allow for a direct observation of the different spatio-temporal dynamical regimes expected in these systems \[11, 12\]. In this report we consider JJA with positional disorder \[9\] under an external magnetic field. In this case the effective amount of randomness can be changed by increasing the magnetic flux per plaquette by integer multiples of the quantum of flux. We have previously shown that in the limit of very large disorder (gauge glass) this system has a dynamical phase transition at the critical current \[13\]. Also, the dynamical phase diagrams for different amounts of frustration and bias
currents were discussed in [14]. In this contribution, we consider the case when the average number of quantum of flux per plaquette is \( f = n + 1/25 \), with \( n \) integer. In the absence of disorder, this magnetic field corresponds to a ground state consisting in a periodic vortex lattice. Here we will study the transition between the regime of plastic or inhomogeneous flow and the regime of flux flow.

2 Dynamical equations

The dynamical equations of current driven JJA can be obtained from considering the resistively shunted junction model for each Josephson junction plus current conservation at each node [15], [13]. One obtains the following set of dynamical equations for the superconducting phases \( \theta(\mathbf{r}) \):

\[
\frac{d\theta(\mathbf{r})}{dt} = -\frac{2\pi R}{\Phi_0} \sum_{\mathbf{r}'} G(\mathbf{r}, \mathbf{r}') \{ I^{\text{ext}}(\mathbf{r}) - \sum_{\pm \hat{\mu}} I_0 \sin(\theta(\mathbf{r}' + \hat{\mu}) - \theta(\mathbf{r'}) - 2\pi f_{\hat{\mu}}(\mathbf{r'})) \},
\]

where \( \theta(\mathbf{r}) \) is the phase of the superconducting wave function at site \( \mathbf{r} \), \( \hat{\mu} = \hat{x}, \hat{y} \); \( f_{\hat{\mu}}(\mathbf{r}) = \frac{1}{\Phi_0} \int_{\mathbf{r}}^{\mathbf{r} + \hat{\mu}} A \cdot d\mathbf{l} \) is the line integral of the vector potential with \( \Phi_0 \) the flux quanta. The array has \( N \times N \) sites, and \( a \) is the lattice constant. \( I_0 \) is the critical current of each junction, assumed to be independent of disorder. All the effect of the randomness is taken in \( f_{\hat{\mu}}(\mathbf{r}) \). In the Landau gauge \( f(\mathbf{r}, \mathbf{r}') = -\frac{H}{\Phi_0} \frac{(r'_x - r_x)(r'_y + r_y)}{2} \), with \( H \) the applied magnetic field. We take random displacements of the sites, \( \mathbf{r}/a = (n_x + \delta^x, n_y + \delta^y) \), with \( n_x, n_y \) integers, and \( \delta^x, \delta^y \) a random uniform number in \([-\Delta/2, \Delta/2]\). Noting that \( \langle \sum_{\hat{\mu}} f_{\hat{\mu}}(\mathbf{r}) \rangle = Ha^2/\Phi_0 = f \), we consider here \( f = n + 1/25 \). The effective amount of disorder is \( W = n\Delta \) [4], so one can increase the disorder by increasing \( n \) (i.e. the magnetic field). In the absence of randomness (\( \Delta = 0 \)), \( f = n + 1/25 \) is equivalent to \( f = 1/25 \). \( R \) is the shunted resistance of each junction and \( G(\mathbf{r}, \mathbf{r}') \) is the two dimensional lattice Green’s function. The boundary conditions are periodic along the \( x \)-direction. At the bottom (top) of the array the external current is injected (taken out) with \( I^{\text{ext}}(r_y = 0) = I \), \( I^{\text{ext}}(r_y = Na) = -I \), and \( I^{\text{ext}}(\mathbf{r}) = 0 \) otherwise. We evaluate Eq. (1) with the same fast algorithm as in Ref. [15]. The time integration is done with a
fourth order Runge-Kutta method, with integration step $\Delta t = 0.01 - 0.1\tau_J$ ($\tau_J = \frac{\phi_0}{2\pi R J_0}$), for time intervals of $t = 5000\tau_J$, after a transient of $2000\tau_J$.

Magnitudes of interest are the voltage drops along the direction of the current,

$$v_y(\vec{r}, t) = \tau_J \left( \frac{d\theta(\vec{r} + \hat{y})}{dt} - \frac{d\theta(\vec{r})}{dt} \right)$$

(2) and the distribution of vortices

$$n(\vec{R}, t) = -\sum \text{nint} \left[ \frac{\theta(\vec{r} + \hat{\mu}) - \theta(\vec{r})}{2\pi} - f_{\hat{\mu}}(\vec{r}) \right],$$

(3) with nint($x$) the nearest integer to $x$, and the sum runs around the plaquette $\vec{R}$. We will study time averaged quantities such as the voltage $v = \frac{1}{N_y(N_y-1)} \sum_{\vec{r}} v_y(\vec{r}, t)$, with $\overline{v}$ the average over time. The total vorticity $n_T(t) = \frac{1}{(N-1)^2} \sum_{\vec{R}} n(\vec{R}, t)$ satisfies $\overline{n_T(t)} = f$, because of vortex number conservation, but it can have temporal fluctuations $(\delta n_T)^2 = \overline{n_T(t)^2} - \overline{n_T(t)}^2$.

3 Transition to the flux flow regime

If the frustration is $f = n + p/q$ and $p/q \ll 1$ there is a dilute vortex configuration in the array. Here we consider $p/q = 1/25$ in arrays of size $50 \times 50$. In the absence of disorder ($\Delta = 0$), the vortices form a periodic lattice in the ground state. For intermediate values of disorder $W \sim 0.2 - 0.5$, the equilibrium configuration of vortices at zero bias $I = 0$ is a random array of vortices, with no crystalline order. (We find that a distorted vortex lattice exists only for very weak disorder $W < 0.1$). Here we consider the particular case of $\Delta = 0.05$ and $n = 4$ ($W = 0.2$). A full diagram of the different dynamical regimes as a function of $W$ and bias current $I$ was given in Ref. [14]. When solving numerically Eq. (1), we find that well below a critical current $I_c$ all the vortices are pinned by the disorder potential and there is no dissipation. Close but below $I_c$ some vortices are depinned and move during a transient time until they are pinned again in a new stable configuration. At $I_c$ there is one channel of vortices that move without being pinned again. Close and above $I_c$ only few vortices move (the others remain pinned) following certain channels of flow. An example of this is shown in Fig. 1(a). As the current is increased further, more vortices are moving and
therefore more channels of flow are opened [Fig. 1(b)]. When increasing
the current even more, eventually all the vortices can move but still with
a random, fluid-like, inhomogeneous motion [Fig. 1(c)]. For this range of
currents the voltage-current curve (see Fig. 2) shows a nonlinear increase of
the voltage $v$ from the critical current $i_c$ (normalized by the Josephson current
$i_c = I_c/I_0$). This regime of inhomogeneous vortex motion is usually called
plastic flow, and it has been studied in disordered type II superconductors
[5]. In disordered JJA, it can be characterized by fluctuations of the total
vorticity $\delta n_T > 0$ [8, 14], i.e. by the existence of flux noise. At high
currents, above a characteristic current $i_p$, above which $\delta n_T = 0$, the voltage
$v$ grows linearly with the current with a slope proportional to the vortex
concentration $p/q$. This is the so called flux flow regime. In this case all the
vortices move with almost the same velocity.

A quantitative understanding of the transition from the plastic flow to the
flux flow regime can be obtained by analyzing the correlations of the average
voltages. Let us consider the voltages along the direction of the current,$v_y(\vec{r}, t)$, which are a measure of the vortex velocities. The homogeneity of the
vortex motion can be studied from the time averaged voltages $\bar{v}_y(\vec{r}) = \bar{v}_y(\vec{r}, t)$.  
We define the correlation function

$$
\Gamma_{vx}(x) = \frac{1}{N_x(Ny - 1)} \sum_{\vec{r}} \bar{v}_y(\vec{r})\bar{v}_y(\vec{r} + x\hat{e}_x) - v^2,
$$

and similarly along the $y$ direction, $\Gamma_{vy}(y)$. In Fig. 3 we show the normalized
correlation $C_x(x) = \Gamma_{vx}(x)/\Gamma_{vx}(0)$. We obtain that the voltage correlation
can be approximated by the form

$$
\Gamma_{vx}(x) \sim \Gamma_{vx}(0) \exp\left(-x/\xi_{vx}\right).
$$

Therefore, the voltage correlation length $\xi_{vx}$ is a measure of how inhomogeneous is the flow along the $x$ direction. In other words, it is a measure of
the typical distance that takes a vortex to turn from a straight path perpen-
dicular to the current, so that $\xi_{vx} = \infty$ (i.e. $C_x(x) = 1$) means a perfectly
straight motion. We see in Fig. 3 that there is a dynamical phase transition
at the characteristic current $i_p$ (in this particular case $i_p = 0.405$) where
$\xi_{vx}$ diverges. We obtain that for $i > i_p$, in the flux flow regime, $C_x(x) = 1
(\xi_{vx} = \infty)$, i.e. all the vortices move in a straight path, with the same velocity
along the $x$ direction [this is also confirmed by analyzing the voltages $\bar{v}_x(\vec{r})$].
On the other hand, the voltage-voltage correlations along the $y$ direction, $\Gamma_{vv}(y)$, never reach complete homogeneity, therefore above $i_p$ the voltages $\bar{v}_y(\vec{r})$ have a dependence on $y$ only. This dependence consists on small fluctuations of $\bar{v}_y(\vec{r}) = v_y(y)$ along the $y$ direction around the mean value $v$. This means that in the flux flow regime, $i > i_p$, the effect of the disorder potential becomes irrelevant only along the direction of the Lorentz force (the $x$ direction), but along the $y$ direction the disorder is still important. Another interesting result is that for low currents just above $i_c$, the length $\xi_{vx}$ is almost constant, as it can be seen in Fig. 3. This corresponds with the regime in currents where some of the vortices flow in channels while some other vortices remain pinned. At high currents, of the order of the Josephson current $I_0$, vortex-antivortices are induced in the JJA. A transition from the flux flow regime to dynamical regimes dominated by the vortex-antivortex excitations appear at these very high currents (involving again plastic flow and homogeneous flow, but of vortex-antivortex pairs). These dynamical regimes were discussed in [14].

4 Discussion

In conclusion, we have shown that the transition from the plastic flow regime to a flux flow regime seems to occur as a dynamical critical phenomenon with a diverging correlation length. These could be studied experimentally in JJA with controlled disorder at low temperatures using recently developed vortex imaging techniques [11]. Another possible test of this transition from vortex random motion to vortex straight motion could be obtained from measurements of the fluctuations in the Hall voltage [10].
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Figure Captions

**Figure 1:** Diagrams of vortex flow in 50 × 51 Josephson arrays with frustration \( f = 4 + 1/25 \) and disorder \( \Delta = 0.05 \). (a) \( i = 0.205 \) (\( i_c = 0.203 \)), (b) \( i = 0.21 \), (c) \( i = 0.35 \), (d) \( i = 0.6 \) (\( i_p = 0.405 \)). The black squares represent the position of the vortices at the present time, and the gray squares the positions where the vortices have been previously. Therefore the gray lines indicate the paths of the vortices.

**Figure 2:** Voltage-current curve for a 50 × 51 Josephson junction array with frustration \( f = 4 + 1/25 \) and disorder \( \Delta = 0.05 \). Currents are normalized by \( I_0 \) and voltages by \( R I_0 \).

**Figure 3:** Left: plot of the voltage-voltage correlation function \( C_x(x) \) for different currents. Right: plot of the correlation length \( \xi_{vx} \) extracted from \( C_x(x) \) as a function of current.