Large Lepton Asymmetry for Small Baryon Asymmetry and Warm Dark Matter

Pei-Hong Gu
Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

We propose a resonant leptogenesis scenario in a $U(1)_{B-L}$ gauge extension of the standard model to generate large lepton asymmetries for cosmological baryon asymmetry and dark matter. After $B - L$ number is spontaneously broken, inflaton can pick up a small vacuum expectation value for the mass splits of three pairs of quasi-degenerately heavy Majorana neutrinos and the masses of three sterile neutrinos. With thermal mass effects of sphalerons, the observed small baryon asymmetry can be converted from large lepton asymmetries of individual flavors although total lepton asymmetry is assumed zero. The mixing between sterile and active neutrinos is elegantly suppressed by the heavy Majorana neutrinos. Before the active neutrinos start their strong flavor conversions, the sterile neutrinos as warm dark matter can be produced by resonant active-sterile neutrino oscillations to reconcile X-ray and Lyman-$\alpha$ bounds. Small neutrino masses are naturally realized by seesaw contributions from the heavy Majorana neutrinos and the sterile neutrinos.

PACS numbers: 98.80.Cq, 95.35.+d, 14.60.Pq, 12.60.Cn, 12.60.Fr

I. INTRODUCTION

Sterile neutrinos can provide the dark matter relic density through their oscillations with active neutrinos. Because of the mixing with the active neutrinos, the sterile neutrinos can decay at tree level and loop orders. In particular, the decays of a sterile neutrino into an active neutrino and a photon at one-loop order will produce a narrow line in the X-ray background. The X-ray constraints put an upper bound on the sterile neutrino mass. Furthermore, the analysis on the Lyman-$\alpha$ data shows a low bound on the sterile neutrino mass. The X-ray bound will be in conflict with the Lyman-$\alpha$ bound if the sterile neutrino is produced by the non-resonant oscillations between the active and sterile neutrinos. In the presence of a large neutrino asymmetry, the resonant active-sterile oscillations can reconcile the two bounds. Such a large lepton asymmetry seems inconsistent with the small baryon asymmetry of the universe because the lepton and baryon asymmetries are usually enforced to be at a same order by sphalerons. This problem can be evaded in three ways: (1) the lepton asymmetry is generated below the electroweak scale; (2) the sphaleron transition doesn’t work; (3) one type of lepton asymmetry is canceled by an opposite lepton asymmetry of other flavors. These possibilities have been proposed and studied in many works.

On the other hand, observations of solar, atmospheric, reactor and accelerator neutrino oscillations have established the massive and mixing neutrinos. The cosmological bound shows the neutrino masses should be in the sub-eV range. The smallness of neutrino masses can be naturally explained in the seesaw extension of the standard model (SM). The seesaw essence is to make the neutrino masses tiny via a suppressed ratio of the electroweak scale over a high scale. Most popular seesaw schemes need lepton number violation as the neutrinos are assumed to be Majorana particles. In the seesaw context, the cosmological baryon asymmetry can be understood via leptogenesis, where a lepton asymmetry is first produced and then is partially converted to a baryon asymmetry through the sphaleron transition. In particular, the so-called resonant leptogenesis models with quasi-degenerately decaying particles can induce a CP asymmetry of the order of unit. This allows the production of a large lepton asymmetry if we don’t take the observed small baryon asymmetry into account.

In this paper, we show that by the resonant leptogenesis the large neutrino asymmetries of individual flavors can be generated for the production of the sterile neutrino dark matter. The total lepton asymmetry is assumed zero so that the baryon asymmetry can arrive at a correct value through the sphaleron processes with thermal mass effects. We demonstrate this possibility in a $U(1)_{B-L}$ gauge extension of the SM. There are two singlet and a doublet Higgs scalars with lepton numbers besides the SM one. One Higgs singlet drives the spontaneous symmetry breaking of the $U(1)_{B-L}$. The other one is responsible for the chaotic inflation. After the $B - L$ number is spontaneously broken, the inflaton can pick up a seesaw suppressed vacuum expectation value (VEV) due to the small ratio of the $B - L$ breaking scale over its heavy mass. In the fermion sector, we introduce three types of SM singlet fermions including usual right-handed neutrinos. Through the large VEV for the $B - L$ symmetry breaking, the right-handed neutrinos can mix with one type of additional singlets, which gets small Majorana masses from the VEV of the inflaton. So, we can naturally have three pairs of quasi-degenerately heavy Majorana neutrinos for the resonant leptogenesis. The other singlet fermions obtain small masses through their Yukawa couplings with the inflaton. They can play the role of the sterile neutrinos as their mixing with the active neutrinos is seesaw suppressed. The resonant active-sterile oscillation with neutrinos is induced by a posi-
tive neutrino asymmetry while that with antineutrinos is induced by a negative neutrino asymmetry. Although the total neutrino asymmetry is vanishing, the resonant sterile-active oscillations are still available as they occur much earlier than the beginning of the strong flavor conversions of the active neutrinos. As for the neutrino masses, they are generated in a seesaw scenario, where the heavy Majorana neutrinos and the light sterile neutrinos give a dominant and a negligible contribution, respectively.

II. THE MODEL

The field content of our model is summarized in Table I, where \((q_L,d_R,u_R)\) and \((\psi_L,l_R)\), respectively, are the SM quarks and leptons, \(\varphi\) is the SM Higgs doublet, \(\nu_R\) denotes usual right-handed neutrinos, \(\xi_R\) and \(\zeta_R\) are additional right-handed singlet fermions, \(\chi\) and \(\sigma\) are two Higgs singlets, \(\eta\) is a new Higgs doublet. Note the new fermions \(\nu_R,\xi_R\) and \(\zeta_R\) all have three generations so that the model can be free of gauge anomaly. The full Lagrangian should be \(SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}\) invariant. For simplicity we only write down the part relevant for our discussions,

\[
\mathcal{L} \supset - y_\nu \bar{\nu}_L \nu_R - y_\chi \bar{\chi} \chi_R - \frac{1}{2} h_\xi \sigma^\dagger \sigma \xi_R - \frac{1}{2} h_\zeta \eta^\dagger \eta \zeta_R - \frac{1}{2} h_\chi \eta \sigma \chi_R - \frac{1}{2} \kappa \sigma \chi \varphi \eta - \rho \sigma \chi^2 + \text{H.c.}
\]

\[
- m_\sigma^2 |\sigma|^2 - \lambda |\sigma|^4. 
\]

Table I: Quantum number assignments. Here \((q_L,d_R,u_R)\) and \((\psi_L,l_R)\), respectively, are the SM quarks and leptons, \(\varphi\) is the SM Higgs doublet, \(\nu_R\) denotes usual right-handed neutrinos, \(\xi_R\) and \(\zeta_R\) are additional right-handed singlet fermions, \(\chi\) and \(\sigma\) are two Higgs singlets, \(\eta\) is a new Higgs doublet. All of the fermions have three families.

| \(SU(3)_c \times SU(2)_L \times U(1)_Y\) | \(U(1)_{B-L}\) |
|---|---|
| \(q_L\) | \((3,2,+\frac{1}{6})\) | \(+\frac{1}{3}\) |
| \(d_R\) | \((3,1,-\frac{1}{3})\) | \(+\frac{1}{3}\) |
| \(u_R\) | \((3,1,\frac{2}{3})\) | \(+\frac{1}{2}\) |
| \(\psi_L\) | \((1,2,\frac{1}{2})\) | \(-1\) |
| \(l_R\) | \((1,1,-1)\) | \(-1\) |
| \(\varphi\) | \((1,2,-\frac{1}{2})\) | \(0\) |
| \(\nu_R\) | \((1,1,0)\) | \(-1\) |
| \(\xi_R\) | \((1,1,0)\) | \(+\frac{1}{2}\) |
| \(\zeta_R\) | \((1,1,0)\) | \(-\frac{1}{2}\) |
| \(\chi\) | \((1,1,0)\) | \(+\frac{1}{2}\) |
| \(\sigma\) | \((1,1,0)\) | \(+1\) |
| \(\eta\) | \((1,2,-\frac{1}{2})\) | \(-\frac{3}{2}\) |

where \(f\) is diagonal and real, i.e. \(f = \text{diag}\{f_1,f_2,f_3\}\). For \(f_\chi \gg h_\xi v_\sigma\), we can perform the following rotation,

\[
\nu_{Ri} \simeq \frac{1}{\sqrt{2}} \left( N_{Ri}^+ - i N_{Ri}^- \right), 
\]

\[
\xi_{Ri} \simeq \frac{1}{\sqrt{2}} \left( N_{Ri}^+ + i N_{Ri}^- \right),
\]

to obtain the diagonal masses,

\[
\mathcal{L} \supset - \frac{1}{2} \sum_{i=1}^{3} m_{N^+_i} N_{Ri}^+ N_{Ri}^- \text{H.c.} + \text{H.c.}
\]

with

\[
m_{N^+_i} \simeq f_1 v_\chi + \frac{1}{2} h_{\xi_{i1}} v_\sigma,
\]

\[
m_{N^-_i} \simeq f_1 v_\chi - \frac{1}{2} h_{\xi_{i1}} v_\sigma.
\]
It is then convenient to define the Majorana fermions,

\begin{align}
N^+_i &= N^+_{R_i} + N^{+c}_{R_i}, \\
N^-_i &= N^-_{R_i} + N^{-c}_{R_i}.
\end{align}

Clearly, \( N^+_i \) and \( N^-_i \) are quasi-degenerate as their mass split is much smaller than their masses.

We now derive the Yukawa couplings of the heavy Majorana fermions \( N^\pm \). For convenience, we first define

\[ \varphi = c\phi_1 + s\phi_2, \quad \eta = -s\phi_1 + c\phi_2 \tag{9} \]

with

\[ c \equiv \cos \theta, \quad s \equiv \sin \theta, \quad \theta = \frac{1}{2} \arctan \frac{2\kappa v}{\mu^2_{\phi} - \mu^2_{\eta}}. \tag{10} \]

Here \( \mu^2_{\phi} \) and \( \mu^2_{\eta} \) are the mass terms of \( \varphi \) and \( \eta \), respectively. The mass terms of \( \phi_1 \) and \( \phi_2 \) would be

\begin{align}
\mu^2_{\phi_1} &= \frac{1}{2} \left\{ \mu^2_{\varphi} + \mu^2_{\eta} - \left[ \left( \mu^2_{\varphi} - \mu^2_{\eta} \right)^2 + 4\kappa^2 v_\sigma^2 \right] \right\}^{\frac{1}{2}}, \\
\mu^2_{\phi_2} &= \frac{1}{2} \left\{ \mu^2_{\varphi} + \mu^2_{\eta} + \left[ \left( \mu^2_{\varphi} - \mu^2_{\eta} \right)^2 + 4\kappa^2 v_\sigma^2 \right] \right\}^{\frac{1}{2}}.
\end{align}

At least one of \( \mu^2_{\varphi} \) and \( \mu^2_{\eta} \) should be negative to guarantee the electroweak symmetry breaking, i.e.

\[ v = \sqrt{v^2_{\varphi} + v^2_{\eta}} \approx 174 \text{ GeV} \quad \text{with} \]

\[ v_{\varphi} = \langle \varphi \rangle > \frac{m_{\varphi}}{\sqrt{4\pi}} \approx \frac{171.2 \text{ GeV}}{\sqrt{4\pi}} \approx 48 \text{ GeV}, \quad v_{\eta} = \langle \eta \rangle \tag{12} \]

For example, we can take \( \mu^2_{\varphi} < 0 < \mu^2_{\eta} \) and then obtain \( \mu^2_{\phi_1} < 0 < \mu^2_{\phi_2} < \mu^2_{\phi_N} \). By inserting Eqs. \[5, 8\] and \[9\] to the first and second terms of Eq. \[1\], we eventually obtain

\[ \mathcal{L} \ni - \left( y^a_{\pm} \right)_{\alpha \beta} \bar{\psi}_{La} \phi_a N^\pm_i + \text{H.c.} \tag{13} \]

with

\begin{align}
y^+ &= \frac{1}{\sqrt{2}} \left( cy_\nu - sy_\xi \right), \quad y^- = -\frac{i}{\sqrt{2}} \left( cy_\nu + sy_\xi \right), \\
y^{+}_1 &= \frac{1}{\sqrt{2}} \left( sy_\nu + cy_\xi \right), \quad y^{-}_1 = -\frac{i}{\sqrt{2}} \left( sy_\nu - cy_\xi \right). \tag{14a}
\end{align}

Clearly, the heavy Majorana fermions \( N^\pm \) play the same role with the heavy Majorana neutrinos in the usual seesaw model. We thus refer to \( N^\pm \) as the heavy Majorana neutrinos.

IV. RESONANT LEPTOGENESIS

From Eq. \[12\], it is straightforward to see the heavy Majorana neutrinos \( N^+_i \) and \( N^-_i \) have a very small mass split compared to their masses. This means the Yukawa interaction \[13\] is probably ready for the resonant leptogenesis if other conditions are satisfied. Because of the special texture of the Yukawa couplings \[14\], the decays of \( N^\pm_i \) can not generate a nonzero lepton asymmetry if the two Higgs doublets \( \phi_1, \phi_2 \) both appear in the final states. This could be easily understood in the base with \( (\nu_{R_i}, \xi_{R_i}) \) and \( (\varphi, \eta) \). Since we have ignored the small Majorana mass term \( h_\phi v_\sigma \) for giving the rotation \[3\] and then the Yukawa couplings \[14\], both \( \nu_{R_i} \) and \( \xi_{R_i} \) only has one decay channel, i.e. \( \nu_{R_i} \rightarrow \psi_L + \varphi^* \), \( \xi_{R_i} \rightarrow \psi_L + \eta^* \). In the presence of the \( \varphi - \eta \) mixing \[9\], we further have the decay channels, \( \nu_{R_i} \rightarrow \psi_L + \eta^* \), \( \xi_{R_i} \rightarrow \psi_L + \varphi^* \). Clearly, the decays into \( \psi_L \varphi^* \) and \( \psi_L \eta^* \) will produce an equal but opposite lepton asymmetry stored in the lepton doublets if the CP is not conserved. For a successful leptogenesis, we thus need one of \( \phi_1, \phi_2 \) to be heavier than the lightest pair of \( N^+_1, N^-_1 \). For example, we choose \( \phi_2 \) to be heavier than \( N^+_1 \). The other heavy Majorana neutrinos \( N^\pm_{2,3} \), which are assumed much heavier than \( N^+_1 \), have flexibilities to be heavier or lighter than \( \phi_2 \). Therefore a final lepton asymmetry would be produced by the two-body decays of \( N^\pm_i \), i.e.

\[ N^+_i \rightarrow \psi_{L_a} + \phi^+_1, \quad \psi^c_{L_a} + \phi_1. \tag{15} \]

Following the standard method \[24\] of the resonant leptogenesis, we can calculate the electron, muon and tau types of lepton asymmetries from the decays of per \( N^+_i \),

\[ \sigma \nu_{N^+_i} = \frac{L_a}{N^+_i} = \frac{1}{2} \frac{L_a}{N^+_i} \]

\[ \frac{1}{2} \sum_a \left[ \Gamma(N^+_i \rightarrow \psi_{L_a} + \phi^+_1) + \Gamma(N^+_i \rightarrow \psi^c_{L_a} + \phi_1) \right] \]

\[ \approx \frac{sc}{16\pi A_{N^+_1}} \left\{ \left[ \left| y_{\nu_{v_{a1}}} \right|^2 - s^2 \left| y_{\xi_{a1}} \right|^2 \right] \text{Im} \left( y^c_{\nu_{v_{a1}}} y_{\nu_{v_{a1}}} \right) \right. \\
+ \left. \left[ \left| y_{\nu_{v_{a1}}} \right|^2 - 2 \left| y_{\nu_{v_{a1}}} y_{\xi_{a1}} \right|^2 \right] \text{Im} \left( y^c_{\nu_{v_{a1}}} y_{\nu_{v_{a1}}} \right) \right\} \times \frac{r_{N^+_i}}{r_{N^+_1} + \frac{1}{4\pi} A_{N^+_1}} \tag{16} \]

with

\[ A_{N^+_1} = \frac{1}{2} \left\{ s^2 \left| y^c_{\nu_{v_{11}}} y_{\nu_{v_{11}}} \right|^2 + c^2 \left| y^c_{\nu_{v_{11}}} y_{\nu_{v_{11}}} \right|^2 \right\} + sc \left[ \left| y^c_{\nu_{v_{11}}} + y^c_{\nu_{v_{21}}} \right|^2 \right] \]

\[ = \frac{1}{2} \left\{ \sum_a \left[ s^2 \left| y_{\xi_{a1}} \right|^2 + c^2 \left| y_{\nu_{a1}} \right|^2 \right] \right. + \left. 2sc \left[ \left| y_{\nu_{a1}} \right| \left| y_{\xi_{a1}} \right| \cos \delta_{a1} \right] \right\}. \tag{17} \]

Here the parameter \( r_{N^+_i} \) describes the mass split between \( N^+_1 \) and \( N^-_1 \),

\[ r_{N^+_i} = \frac{m^2_{N^+_1} - m^2_{N^-_1}}{m_{N^+_1} m_{N^-_1}} = \frac{2h_\xi v_\sigma}{f_i v_\chi}. \tag{18} \]
The baryon and lepton asymmetries are determined by the $B-L$ asymmetry in the presence of sphalerons [12]. In the present model, we have [29]

\[
B = \frac{28}{79} \left( B - \sum \alpha L_\alpha \right) = -\frac{28}{79} \sum \alpha L_\alpha = -\frac{56}{79} \sum \alpha L_{\nu_\alpha}.
\]  

(19)

The masses and interactions will give corrections to the above formula [29].

\[
\Delta B = -A \frac{6}{13\pi^2} \sum \alpha \frac{\bar{m}_\alpha^2(T)}{T^2} \left( L_\alpha - \frac{1}{3} B \right)
\]

\[
= -A \frac{6}{13\pi^2} \sum \alpha \frac{\bar{m}_\alpha^2(T) L_\alpha}{T^2}
\]

\[
= -A \frac{12}{13\pi^2} \sum \alpha \frac{\bar{m}_\alpha^2(T) L_{\nu_\alpha}}{T^2}.
\]  

(20)

Here the coefficient $A \simeq 1$ [22]. In the case that the sphaleron is still active after a weakly first-order electroweak phase transition [33], one finds [29]

\[
\frac{\bar{m}_\alpha^2(T)}{T^2} = \frac{1}{6} f_\alpha^2 + \frac{1}{3} f_\alpha^2 \left( \frac{v(T)}{T} \right)^2 \leq \frac{1}{2} f_\alpha^2
\]  

(21)

for $B-\Sigma \alpha L_\alpha = 0$. Here $f_\alpha$ denotes the Yukawa couplings of the electron, muon and tau to the SM Higgs.

With the thermal mass effects of the sphaleron processes, it is possible to generate a small observed baryon asymmetry from large lepton asymmetries of individual flavors [10]. For this purpose, we take

\[
\text{Im} \left[ \left( y_{\nu_1} y_{\nu_i} \right)_{11} \right] = \sum \alpha \left| y_{\nu_1} \right| \left| y_{\nu_\alpha} \right| \sin \delta_{\alpha 1} = 0
\]  

(22)

to give a zero total lepton asymmetry. Under this assumption, the CP asymmetry [10] can be simplified by

\[
\frac{\xi_{\nu_\alpha}}{N^{\pm}_{\nu}} = \frac{\xi_{\nu_\alpha}}{N^{\pm}_{\nu}} = \frac{1}{2} \xi_{N^{\pm}_{\nu}}
\]

\[
= \frac{sc}{16\pi A_{N_{\nu}^{\pm}}} \left[ c^2 \left( y_{\nu_1} y_{\nu_\alpha} \right)_{11} - s^2 \left( y_{\nu_1} y_{\nu_\alpha} \right)_{11} \right] \sin \delta_{\alpha 1}
\]

\[
\times \left| y_{\nu_1} \right| \left| y_{\nu_\alpha} \right| \frac{r_{N^{\pm}_{\nu}}}{N_{\nu}^{\pm} + \frac{1}{16\pi^2} A_{N_{\nu}^{\pm}}.}
\]  

(23)

In the weak washout region where the out-of-equilibrium condition is described by the quantity,

\[
K_{N^{\pm}_{\nu}} = \frac{\Gamma_{N^{\pm}_{\nu}}}{2H(T)} |_{T=m_{N^{\pm}_{\nu}}} < 1
\]  

(24)

with the decay width,

\[
\Gamma_{N^{\pm}_{\nu}} \simeq \sum \alpha \left[ \Gamma (N^{\pm}_{\nu} \rightarrow \psi L_\alpha + \phi_{1}^\ast) + \Gamma (N^{\pm}_{\nu} \rightarrow \psi_\alpha^\ast + \phi_{1}) \right]
\]

\[
= \frac{1}{8\pi} \left( y_{\nu_1} y_{\nu_1} \right)_{11} m_{N^{\pm}_{\nu}} = \frac{1}{8\pi} A_{N^{\pm}_{\nu}} m_{N^{\pm}_{\nu}}
\]  

(25)

and the Hubble constant

\[
H(T) = \left( \frac{8\pi^3 \alpha}{90} \right)^{1/2} \frac{T^2}{M_{pl}},
\]  

(26)

the final neutrino asymmetry can be approximately given by [34]

\[
\eta_{\nu_\alpha} = \frac{n_{\nu_\alpha} - n_{\bar{\nu_\alpha}}}{s} \simeq \frac{\xi_{\nu_\alpha}}{g_\ast}.
\]  

(27)

Here $g_\ast \simeq 112$ is the relativistic degrees of freedom (the SM fields plus $\zeta_{R_{1,2,3}}$) while $M_{pl} \simeq 1.22 \times 10^{19}$ GeV is the Planck mass. The final baryon asymmetry then should be

\[
\eta_B = \frac{n_B - n_{\bar{B}}}{s} \simeq -\frac{6}{13\pi^2} \sum \alpha f_\alpha^2 \eta_{\nu_\alpha}.
\]  

(28)

Note the lepton number violating processes mediated by the heavier $N^{\pm}_{\nu}$ should be decoupled before the leptogenesis epoch $T = M_{N^{\pm}_{\nu}}$ to give the solution (27).

The neutrino asymmetry $\eta_{\nu_\alpha}$ is related to the neutrino chemical potential $\mu_{\nu_\alpha}$ by

\[
\eta_{\nu_\alpha} = \frac{15}{4\pi^2 g_\ast S(T)} \xi_{\nu_\alpha} + O \left( \xi_{\nu_\alpha}^3 \right)
\]

\[
\simeq \frac{15}{64\pi^2 \xi_{\nu_\alpha}} \text{ with } \xi_{\nu_\alpha} = \frac{\mu_{\nu_\alpha}}{T_{\nu_\alpha}}.
\]  

(29)

Here we have taken $g_\ast S(T) = 16$ (photon, three neutrinos, electron, positron plus three sterile neutrinos). The electron neutrino asymmetry is tightly constrained by Primordial Big-Bang Nucleosynthesis (BBN) [35].

\[
\eta_{\nu_e} \in (-0.9, 1.7) \times 10^{-3} \text{ for } \xi_{\nu_e} \in (-0.04, 0.07).
\]  

(30)

The above bound also applies to the muon and tau neutrino asymmetries because the neutrino oscillations will begin at 10 MeV to achieve strong flavor conversions before BBN [36]. On the other hand, the five-year observations of the WMAP collaboration precisely measured the baryon asymmetry as [10]

\[
\eta_B = \frac{1}{7.04} \times (6.225 \pm 0.170) \times 10^{-10}
\]

\[
= (0.884 \pm 0.024) \times 10^{-10}.
\]  

(31)

V. STERILE AND ACTIVE NEUTRINOS

During the evolution of the universe, the electroweak symmetry breaking will happen when the Higgs doublet

\[
H = \frac{v}{\sqrt{2}} \phi + \frac{v}{\sqrt{2}} \eta
\]  

(32)
develops its VEV. The mass terms \( \mathcal{L} \) then should be extended to
\[
\mathcal{L} \supset -y_{\nu} v_{\nu} \bar{\nu}_{L} \nu_{R} - y_{\ell} v_{\ell} \bar{\nu}_{L} \xi_{R} - f_{\chi} \bar{\nu}_{R} \xi_{R} - \frac{1}{2} h_{\xi} v_{\sigma} \bar{\xi}_{R} \xi_{R} - \frac{1}{2} h_{v_{\nu}} v_{\sigma} \bar{v}_{R} \xi_{R} - \mu_{\xi} \bar{\xi}_{R} \xi_{R} + \text{H.c.}.
\] (33)

For convenience, we rewrite the above mass terms to be
\[
\mathcal{L} \supset -\frac{1}{2} (\bar{\nu}_{L}, \bar{\xi}_{R}) \mathcal{M} (\nu_{R}, \xi_{R})^{T} + \text{H.c.},
\] (34)
where the mass matrix \( \mathcal{M} \) is defined by
\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & y_{\nu} v_{\varphi} & y_{\nu} v_{\eta} \\
0 & h_{\nu} v_{\sigma} & 0 & \mu \\
y_{\varphi} v_{\varphi} & 0 & f_{\nu} v_{\chi} & \mu^{T} \\
y_{\eta} v_{\eta} & f^{T} v_{\chi} & h_{\nu} v_{\sigma} & 0
\end{pmatrix}.
\] (35)

Clearly, the heavy mass matrix \( \mathcal{M} \) will give us the quasi-degenerate heavy Majorana neutrinos \( N_{\pm} \). As for the light mass matrix \( \mathcal{M} \), it can also accommodate the seesaw if its diagonal element \( h_{\nu} v_{\sigma} \) is much bigger than other elements. In this seesaw scenario, the neutrino masses should be
\[
\mathcal{L} \supset -\frac{1}{2} \bar{\nu}_{L} m_{\nu \nu} \nu_{R} + \text{H.c.} = -\frac{1}{2} \bar{\nu}_{L} (m_{\nu \nu}^{N} + m_{\nu \nu}^{S}) \nu_{R} + \text{H.c.}
\] (38)
with
\[
m_{\nu \nu}^{N} = y_{\nu} \frac{1}{f^{T}} h_{\nu} \frac{1}{f} y_{\nu} \frac{1}{f} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}} \\
\quad - (y_{\nu} \frac{1}{f^{T}} y_{\nu} + y_{\nu} \frac{1}{f^{T}} y_{\nu} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}} - f_{\nu} v_{\chi}, \quad (39a)
\]
\[
m_{\nu \nu}^{S} = y_{\nu} \frac{1}{f^{T}} \mu^{T} \frac{1}{h_{\nu} \frac{1}{f^{T}} y_{\nu} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}}}. \quad (39b)
\]
The above neutrino mass matrices are expected to explain the neutrino oscillation experiments [18],
\[
\Delta m_{21}^{2} = 7.65^{+0.23}_{-0.20} \times 10^{-5} \text{eV}^{2}, \quad \sin^{2} \theta_{12} = 0.304^{+0.022}_{-0.016},
\]
\[
|\Delta m_{31}^{2}| = 2.44^{+0.12}_{-0.11} \times 10^{-3} \text{eV}^{2}, \quad \sin^{2} \theta_{23} = 0.50^{+0.07}_{-0.06},
\]
\[
\sin^{2} \theta_{13} = 0.01^{+0.06}_{-0.011}. \quad (40)
\]

For \( f_{\nu} \gg y_{\nu} v_{\varphi}, y_{\nu} v_{\eta}, h_{\nu} v_{\sigma} \) and \( \mu \), we can make use of the seesaw formula to diagonalize the above mass matrix \( \mathcal{M} \) in two blocks,
\[
\mathcal{L} \supset -\frac{1}{2} (\bar{\nu}_{R}, \bar{\xi}_{R}) \mathcal{M} (\nu_{R}, \xi_{R})^{T} - \frac{1}{2} (\bar{\nu}_{L}, \bar{\xi}_{R}) \mathcal{M} (\nu_{R}, \xi_{R})^{T} + \text{H.c.},
\] (36)
where the mass matrices \( \mathcal{M} \) and \( \mathcal{M} \) are given by
\[
\mathcal{M} = \begin{pmatrix}
0 & f_{\nu} v_{\chi} \\
0 & h_{\nu} v_{\sigma}
\end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix}
\begin{pmatrix}
y_{\nu} \frac{1}{f^{T}} h_{\nu} \frac{1}{f} y_{\nu} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}} - (y_{\nu} \frac{1}{f^{T}} y_{\nu} + y_{\nu} \frac{1}{f^{T}} y_{\nu} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}} - f_{\nu} v_{\chi} \\
\end{pmatrix} \\
\begin{pmatrix}
-\mu^{T} \frac{1}{f^{T}} y_{\nu} \frac{v_{\sigma}^{2}}{v_{\chi}^{2}} \end{pmatrix}
\end{pmatrix}.
\] (37)

On the other hand, \( \zeta_{R} \) plays the role of the sterile neutrinos, i.e.
\[
\mathcal{L} \supset -\frac{1}{2} h_{\nu} v_{\sigma} \bar{\zeta}_{R} \zeta_{R} + \text{H.c.} = -\frac{1}{2} m_{S} S S^{T},
\] (41)
where we have rotated \( h_{\nu} = \text{diag}(h_{\nu 1}, h_{\nu 2}, h_{\nu 3}) \) and then defined
\[
S_{i} = \zeta_{R_{i}} + \zeta_{R_{i}}^{*} \quad \text{with} \quad m_{S_{i}} = h_{\nu} v_{\sigma}.
\] (42)
The active-sterile mixing angle is
\[
\theta_{a i}^{2} = \frac{\left| y_{\nu_{i} j} \right|^{2}}{m_{S_{i}}^{2}} \quad \text{and} \quad \sin^{2} \theta_{a i} = 4 \sum_{\alpha} \theta_{a i}^{2}.
\] (43)
The sterile neutrinos can be produced through the active-sterile neutrino oscillations. Specifically, the \( \nu_{\alpha} \rightarrow S_{i} \) or \( \bar{\nu}_{\alpha} \rightarrow \bar{S}_{i} \) oscillation is determined by the sterile neutrino mass \( m_{S_{i}} \), the active-sterile mixing angle \( \theta_{a i} \) and the neutrino asymmetry \( \eta_{\nu_{\alpha}} \). In order to compare with other works, we define the following function [4, 37],
\[
c_{\alpha \alpha} = \sqrt{2} G_{F} \left[ 2 \eta_{\nu_{\alpha}} + \sum_{\beta \neq \alpha} \eta_{\nu_{\beta}} + (1 + 2 \sin^{2} \theta_{W}) \eta_{\nu_{\alpha}} \right]
\]
\[
- (1 - 2 \sin^{2} \theta_{W}) \sum_{\beta \neq \alpha} \eta_{\nu_{\beta}} + 2 \sin^{2} \theta_{W} \sum_{\beta} \eta_{\nu_{\beta}} \right]
\]
\[
= 3 \sqrt{2} G_{F} \eta_{\nu_{\alpha}}.
\] (44)
Here we have taken $\eta_{\nu_e} = \eta_{\nu_\mu} = \eta_{\nu_\tau} = 0$ and $\eta_{\bar{\nu}_\mu} = 0$ into account. A positive neutrino asymmetry $\eta_{\nu_e} > 0$ can induce a MSW resonant behavior in the neutrino oscillation $\nu_\alpha \rightarrow S_i$ whereas a negative one $\eta_{\bar{\nu}_e} < 0$ can enhance the antineutrino oscillation $\bar{\nu}_\alpha \rightarrow S_i$. It has been pointed out that a large neutrino asymmetry can reconcile the contradiction between the X-ray and Lyman-$\alpha$ bounds. For example, in the work where the lepton asymmetry is flavor blind, i.e. $\eta_{\nu_e} = \eta_{\nu_\mu} = \eta_{\nu_\tau} = \eta_{\bar{\nu}_e}$, the authors show with the parameter choice,

$$m_{S_i} \simeq 8 \text{ keV}, \quad \sin^2 2\theta_1 \simeq 10^{-12}, \quad \eta_{\nu_e} = 7 \times 10^{-5}, (45)$$

the resonant $\nu_\alpha \rightarrow S_i$ oscillation can produce adequate $S_i$ for the dark matter relic density. We should keep in mind that the resonant active-sterile conversion happens at a temperature of the order of 100 MeV for the sterile neutrino mass $m_{S_i}$ being a few keV. This means the electron neutrino asymmetry $\eta_{\nu_e}$ and the opposite muon and tau neutrino asymmetries $\eta_{\nu_{\mu e}}$ can survive for the resonant enhancement either in the neutrino oscillations or in the antineutrino oscillations since the active neutrino oscillations driven by $\Delta m_{31}^2$ and $\Delta m_{21}^2$ begin at lower temperatures $T \sim 10$ MeV and $T \sim 3$ MeV, respectively.

VI. PARAMETER CHOICE

We now take reasonable choice of parameters to give the observed small cosmological baryon asymmetry from large lepton asymmetries for the dark matter production. Firstly, the heavy Majorana neutrinos $N_i^{\pm}$ couple to the gauge boson $Z_{B-L}$ associated with the $U(1)_{B-L}$ symmetry. For $m_{N_1^\pm} = \mathcal{O}(10^5 \text{ GeV})$, the Higgs singlet $\chi$ should have a VEV bigger than $\mathcal{O}(10^8 \text{ GeV})$ to guarantee the departure from equilibrium of $N_1^{\pm}$ at $T \simeq M_{N_1^{\pm}}$, unless the corresponding gauge coupling $g_{B-L}$ is fine tuned very small. So, we take

$$v_\chi = \mathcal{O}(10^8 \text{ GeV}) \quad (46)$$

and then

$$m_{N_1^\pm} = \mathcal{O}(10^5 \text{ GeV}) \quad \text{for} \quad f_1 = \mathcal{O}(10^{-3}), \quad (47)$$
$$m_{N_{2,3}^\pm} = \mathcal{O}(10^6 \text{ GeV}) \quad \text{for} \quad f_{2,3} = \mathcal{O}(10^{-2}).$$

Secondly, the Higgs singlet $\sigma$ is expected to realize the nice picture of the chaotic inflation. This suggests

$$m_{\sigma} = \mathcal{O}(10^{13} \text{ GeV}), \quad \lambda = \mathcal{O}(10^{-13}). \quad (48)$$

We then conveniently set

$$\rho = v_\chi \quad (49)$$

in Eq. (3) to induce a small VEV,

$$v_\sigma = \mathcal{O}(10 \text{ MeV}). \quad (50)$$

In consequence, the mass splits of the heavy Majorana neutrinos $N_i^{\pm}$ can be determined by

$$r_{N_i} = \mathcal{O}(10^{-12}) \quad (47)$$

The sterile neutrinos $S_i$ also obtain small masses:

$$m_{\nu_i} = \mathcal{O}(1 - 100 \text{ keV}) \quad \text{for} \quad h_{\xi_{i1}} = \mathcal{O}(10^{-4} - 10^{-2}) \quad (52)$$

Thirdly, we consider

$$- [\mathcal{O}(10^2 \text{ GeV})] = \mu_\varphi^2 < 0 < \mu_\sigma^2 = [\mathcal{O}(10^5 \text{ GeV})]^2, \quad (53)$$

to derive the mixing angle:

$$\vartheta \simeq - \frac{\kappa v_\varphi v_\sigma}{\mu_\sigma^2} = 10^{-4} \quad \text{for} \quad \kappa = \mathcal{O}(1) \quad (54)$$

and then determine the VEVs:

$$v_\varphi \simeq 174 \text{ GeV}, \quad v_\eta \simeq - \frac{\kappa v_\varphi v_\sigma}{\mu_\sigma^2} \simeq v_\varphi \vartheta. \quad (55)$$

We now consider the following sample of the Yukawa couplings,

$$| y_{\nu_{\tau 1}} | = 1.15 \times 10^{-6}, \quad | y_{\tau_{01}} | = 10^{-3}, \quad \sin \delta_{\pm 1} = 1, \quad (49)$$
$$| y_{\nu_{\mu 1}} | = 1.15 \times 10^{-7}, \quad | y_{\mu_{01}} | = \frac{10^{-2}}{2}, \quad \sin \delta_{\mu 1} = -1, \quad (50)$$
$$| y_{\nu_{e 1}} | = 1.15 \times 10^{-7}, \quad | y_{e_{01}} | = \frac{10^{-2}}{2}, \quad \sin \delta_{e 1} = -1. \quad (51)$$

to fulfill the assumption. We then read

$$K_{N_1^+} = K_{N_1^-} = 0.135. \quad (52)$$

By further fixing $r_{N_1} = 10^{-12}$, we can obtain the CP asymmetry,

$$\varepsilon_{N_1^+} = \varepsilon_{N_1^-} = 2.03 \times 10^{-3}. \quad (53)$$

In consequence, the large lepton asymmetries should be

$$\eta_{\nu_e} = 3.63 \times 10^{-5} = \frac{6.99 \times 10^{-5}}{1 + 4 \sin^2 \theta_W}, \quad (54)$$

which is in agreement with the observations. Accordingly a desired baryon asymmetry is induced by

$$\eta_B \simeq \frac{6}{13 \pi^2} \frac{f_\mu^2}{1 + x} \eta_{\nu_e} = 0.888 \times 10^{-10} \quad \text{for} \quad x = \frac{| y_{\nu_{\tau 1}} | | y_{\tau_{01}} | \sin \delta_{\pm 1}}{| y_{\nu_{e 1}} | | y_{e_{01}} | \sin \delta_{\eta 1}} = 1. \quad (55)$$
We further take
\[ y_{\nu_{e 2}} = y_{\nu_{e 3}} = 100y_{\nu_{e 1}}, \quad \mu_{ij} = 20 \text{ keV}, \quad (61) \]
and then perform
\[ 4\theta_r^2 = 10^{-12}, \quad 4\theta_{\mu 1}^2 = 4\theta_{\tau 1}^2 = 10^{-14}. \quad (62) \]
So the \( \nu_e \to S_1 \) oscillation is dominant. Compared with the fitting by \[39, 40\], we see the sterile neutrino \( S_1 \) and then perform the mass
\[ m_{S_1} \simeq 8 \text{ keV} \quad (63) \]
can be a good candidate for the dark matter. The other sterile neutrinos \( S_{2,3} \) are also at the keV scale. They could leave a significant relic density if their mixing with the active neutrinos \( \theta_{\alpha i}^2 \) (\( i = 2, 3 \)) are comparable with \( \theta_{\alpha 1}^2 \). Alternatively, their relic density could be negligible if \( \theta_{\alpha i}^2 \) (\( i = 2, 3 \)) are much smaller than \( \theta_{\alpha 1}^2 \). This could be achieved by taking appropriate \( \mu_{ij} (i = 2, 3) \) in Eq. \[43\].

For the above parameter choice, the seesaw contribution from the sterile neutrinos to the neutrino masses is of the order of \( \theta_{\alpha i}^2 m_{S_1} \lesssim O(10^{-9} \text{eV}) \), which is too small to explain the neutrino oscillation data. Furthermore, the contribution from the heavy Majorana neutrinos \( N_{1}^{\pm} \) is also too small (\( \sim O(10^{-5} \text{eV}) \)). Fortunately, the other heavy Majorana neutrinos \( N_{2,3}^{\pm} \) can give a desired contribution. Specifically, we can take the Yukawa couplings \( y_{\kappa i} \lesssim O(1)/(i = 2, 3) \) in the neutrino mass matrix which is dominated by the second term of Eq. \[39a\]. We also check the lepton number violating processes mediated by \( N_{2,3}^{\pm} \) have been decoupled before the leptonogenesis epoch since their reaction rate \[43\],
\[ \Gamma_A = \frac{1}{\pi^3} \frac{1}{v^4} \text{Tr}(m_{\nu}^2 m_{\nu}) T^3 = \frac{1}{\pi^3} \frac{1}{v^4} \sum m_{\nu}^2 T^3, \quad (64) \]
is smaller than the Hubble constant at the temperature \( T = M_{N_{1}^{\pm}} \).

**VII. SUMMARY**

We have shown the resonant leptogenesis in the seesaw context for the observed small baryon asymmetry can allow large neutrino asymmetries of individual flavors for the production of the sterile neutrino dark matter due to the thermal mass effects of the sphaleron processes. In our model, the inflaton can nicely obtain a small VEV as a result of the seesaw suppressed ratio of the \( B - L \) breaking scale over its heavy mass. The quasi-degenerate mass spectrum of the heavy Majorana neutrinos for the resonant leptogenesis is naturally given by the scale of the \( B - L \) symmetry breaking and the small VEV of the inflaton. The sterile neutrinos obtain small Majorana masses through their Yukawa couplings with the inflaton. Their mixing with the active neutrinos are suppressed by the seesaw mechanism. Thus one or more sterile neutrinos can act as the warm dark matter in the presence of the resonant active-sterile neutrino oscillation. Due to the small active-sterile mixing, the sterile neutrinos only have a negligible contribution to the neutrino masses. Instead, the seesaw contribution from the heavy Majorana neutrinos is responsible for generating the desired neutrino masses.

For an appropriate parameter choice, we may realize the \( \nu \)MSM model \[3, 13, 40\] with a pair of quasi-degenerate heavy Majorana neutrinos and a sterile neutrino in our model with three pairs of quasi-degenerate heavy Majorana neutrinos and three sterile neutrinos. In the \( \nu \)MSM model, the processes for generating the lepton asymmetry can keep working below the sphaleron freeze-out temperature at which there is no conversion of the lepton asymmetry to the baryon asymmetry. In this case, we need not resort to the thermal mass effects of the sphaleron processes.

**Acknowledgement:** I thank Manfred Lindner for hospitality at Max-Planck-Institut für Kernphysik. This work is supported by the Alexander von Humboldt Foundation.
