On Masses of Equilibrium Configurations

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Abstract

Proceeding from the new gravitation equations (Phys.Lett.A, v.156, p.404 (1991) ) we argue that the theory in principle allows equilibrium stable configurations of a degenerate electron or neutron gas with very large masses.

Proceeding from the newtonian gravity law and Einstein’s equations it is considered that masses of equilibrium configurations cannot go over several Sun masses. In paper [1] new vacuum gravitational equations in flat space-time was proposed, which have no physical singularity for the spherically symmetric field. If the distance \( r \) from an attractive mass \( M \) is much larger than the Shwarzshild radius \( \alpha \), then their physical consequences coincide with the ones in Einstein’s theory. However, they are quite different at \( r \) of the order \( \alpha \) or less than that. There is no events horizon at \( r = \alpha \). The gravitational force affecting the a test particle of the mass \( m \) in rest is given by

\[
F = -\frac{GmM}{r^2} \left( 1 - \frac{\alpha}{f} \right),
\]

where \( G \) is the gravitational constant, \( \alpha = 2GM/c^2 \), \( c \) is the speed of light, \( f = (\alpha^3 + r^3)^{1/3} \). Fig. 1 shows the plot of the function \( F_1 = -(1/2 \, \tau^2)(1 - \alpha/f) \) against the distance \( \tau = r/\alpha \).
Fig. 1 The plot of the function $F_1$ against the $\tau = r/\alpha$

It follows from Fig. 1 that the $|F|$ reaches its maximum at $r$ of the order of $\alpha$ and tends to zero at $r \to 0$. It would therefore be interesting to know what the limiting masses of the equilibrium configurations the gravitational force $F(r)$ (1) can admit. To answer this question we start from the equation

$$\frac{dp}{dr} = -\frac{G\rho M}{r^2} (1 - \alpha / f)$$

In this equation $\rho$ is the pressure, $M = M(r)$ is the matter mass inside of a sphere of the radius $r$, $\rho = \rho(r)$ is the matter density at the distance $r$ from the center, $\alpha$ and $f$ is the function of $M(r)$.

Suppose the equation of state is $p = K\rho^\Gamma$, where $K$ and $\Gamma$ are constants. For numerical estimates we shall use their values [2]:

For a degenerated electron gas:
$\Gamma = 5/3 \quad K = 1 \cdot 10^{13}$ SGS units at $\rho \ll \rho_0$, where $\rho_0 = 10^6 \text{ gm/cm}^3$,
$\Gamma = 4/3 \quad K = 1 \cdot 10^{15}$ SGS units at $\rho \gg \rho_0$.

For degenerated neutron gas:
$\Gamma = 5/3 \quad K = 5 \cdot 10^9$ SGS units at $\rho \ll \rho_0$, where $\rho_0 = 5 \cdot 10^{15} \text{ gm/cm}^3$,
$\Gamma = 4/3 \quad K = 1 \cdot 10^{15}$ SGS units at $\rho \gg \rho_0$. 

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for rough estimates we replace $dp/dr$ by $-p/r$, where $p$ is the average matter pressure and $R$ is its radius. Under the circumstances we obtain from eq.(2)

$$\frac{p}{\rho c^2} = \frac{\alpha}{2R}(1 - \alpha/f).$$

(3)

If $R \gg \alpha$, then the term $\alpha/f$ is negligible. Setting $M(R) \approx \rho R^3$ we find the mass of equilibrium states as a function of $\rho$:

$$M = (K/G)^{3/2} \rho^{(\Gamma-4/3)(3/2)}. $$

(4)

It follows from eq.(4) that there is the maximal mass $M = (K/G)^{3/2}$ at $\rho \gg \rho_0$.

However, according to eq.(3), there are also equilibrium configurations at $R < \alpha$. In particular, at $R \ll \alpha$ we find from eq.(3) that the masses of the equilibrium configurations are given by

$$M = c^{9/2} 10^{-1} K^{-3/4} G^{-3/2} \rho^{-(\Gamma-1)(3/4)}. $$

(5)

These are the configurations with very large masses. For example, the following equilibrium configurations can be found:

the nonrelativistic electrons: $\rho = 10^5 gm/cm^3$, $M = 1.3 \cdot 10^{42} gm$, $R = 2.3 \cdot 10^{12} cm$,

the relativistic electrons: $\rho = 10^7 gm/cm^3$, $M = 2.3 \cdot 10^{40} gm$, $R = 1.3 \cdot 10^{11} cm$,

the nonrelativistic neutrons: $\rho = 10^{14} gm/cm^3$ $M = 3.9 \cdot 10^{35} gm$, $R = 1.6 \cdot 10^{7} cm$.

The reason of the two types of configurations existence can be seen from Fig. 2, where for $\rho = 10^{15}$ $gm/cm^3$ the plots of right-hand and left-hand sides of Eq.(3) against the mass $M$ are given.
Fig2 The plot of right-hand \((W_2(M))\) and left-hand \((W_1(M))\) sides of Eq.(3) against \(M\).

The following conclusions can be made after considering the plots of the above kind:

1. There are no equilibrium configurations whose the density is larger than a certain value \(\rho_{\text{max}} \sim 10^{16}\text{gm/cm}^3\).
2. For each value of \(\rho < \rho_{\text{max}}\) there are two equilibrium states (with \(R > \alpha\) and \(R < \alpha\)).

Are the configurations with large masses stable?

The total energy of the degenerate gas is \(E = E_{\text{int}} + E_{\text{gr}}\), where \(E_{\text{int}}\) is the intrinsic energy and \(E_{\text{gr}}\) is the gravitational energy. The gravitational energy of a sphere is

\[
E_{\text{gr}} = \int_{\infty}^{R} dM(r) \, \chi(r) \, M(r),
\]

(6)

where

\[
\chi(r) = \int_{\infty}^{r} dr' \, (r')^{-2}(1 - 1/f),
\]

\[
\alpha = 2GM(r)/c^2, \quad f = (\alpha(r)^3 + (r')^3)^{1/3},
\]

\[
M(r) = 4\pi \int_{0}^{r} dr' \, \rho \, (r')^2.
\]
The function $\chi(r)$ is approximately

$$\chi(r) = \frac{1}{r}(1 - \exp(-r/\alpha)).$$

(7)

Therefore, at $p = \text{const}$ up to a constant of the order one

$$E_{gr} = -\frac{GM^2}{R}(1 - \exp(-R/\alpha)).$$

(8)

The intrinsic energy $E_{int} = \int u \, dM$, where $u$ is the energy per mass unit. For the used equation of state $u = K(\Gamma - 1)^{-1} \rho^{\Gamma - 1}$. Thus, up to constants of the order of one

$$E = KM \rho^{\Gamma - 1} - GM^{5/3} \rho^{1/3}[1 - \exp(-QM^{-2/3} \rho^{-1/3})],$$

(9)

where $Q = c^2/2G$. As an example, Fig.3 and Fig.4 show the plot of the function $E = E(\rho)$ for the nonrelativistic neutron gas of the mass $M = 10^{36} \, gm$ and $M = 10^{33} \, gm$ (neutron stares) correspondingly.

Fig. 3 The plot of the function $E = E(\rho)$ for the neutron configuration of the mass $M = 10^{36} \, gm$. 

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The analysis of such plots show that the function $E = E(\rho)$ has the minimum. Thus, the above equilibrium states of large masses are stable.

The gravitational potential on the surface of a stable massive configuration of the degenerate fermion gas of the order of

$$V = (GM/R)[1 - \exp(-R/\alpha)]$$

(10)

It follows from the virial theorem that the above objects of large masses (their $R \ll \alpha$) are the ones with low temperatures. They, probably, refer to "dark" matter of the Universe. If their luminosity are caused by an accretion, then the Eddington limit of luminosity is approximately

$$\mathcal{L} = \mathcal{L}_{Edd}^0[1 - \exp(-R/\alpha)],$$

(11)

where $\mathcal{L}_{Edd}^0 = 1 \cdot 10^{39} M \text{ erg/s}$. Hence, if $R/\alpha \ll 1$, their luminosity $\mathcal{L} \ll \mathcal{L}_{Edd}^0$.

**References**
[1] L.V. Verzub, Phys. Lett. A, v. 156, 404 (1991)

[2] S.L. Shapiro and Teukolsky, Black Holes, White Dwarfs and Neutron Stars, (1983)

[3] S. Chandrasekhar, Mon. Not. Roy. Astron. Soc. v. 95, 226 (1935)