Fully-Bayesian stacking in the presence of confusion

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ABSTRACT
Multi-wavelength astronomical studies brings a wealth of science within reach. One way to achieve a cross-wavelength analysis is via ‘stacking’, i.e. combining precise positional information from an image at one wavelength with data from one at another wavelength in order to extract source-flux distributions and other derived quantities. For the first time we extend stacking to include the effects of confusion. We develop our algorithm in a fully Bayesian framework and apply it to the Square Kilometre Array Design Study (SKADS) simulation in order to extract galaxy number counts. Previous studies have shown that recovered source counts are highly biased high when confusion is non-negligible. With this new method, source counts are returned correctly. We also describe a novel estimator for quantifying the impact of confusion on stacking analyses. This method is an essential step in exploiting scientific return for upcoming deep radio surveys, e.g. MIGHTEE on MeerKAT.

Key words: methods: data analysis – methods: statistical – galaxies: evolution – radio continuum: galaxies – radio continuum: general

1 INTRODUCTION

Measurements of radio-source counts provided some of the earliest tests of cosmology (Ryle 1961; Ryle & Clarke 1961; Longair 1966). Later on, it turned out that the evolution of radio sources is too strong to draw robust conclusions about the cosmological model describing the Universe. Today, source counts can be used to identify new extragalactic populations and study galaxy evolution (including star formation rates and luminosity functions) when combined with redshift information from panchromatic ancillary surveys.

In 1957, Scheuer (Scheuer 1957) first showed that using a ‘probability of deflection’, or \( P(D) \), analysis allows us to statistically measure the differential number counts from an image bearing strong confusion. \(^1\) The idea of this method is to extract source-count information from the histogram of the pixel-fluxes of the image. The performance of this method is highly dependent on the level of confusion. The first application (Hewish 1961) was to the 4C data; the \( P(D) \) method has most recently been used to measure number counts down to 1\( \mu \)Jy at 3 GHz (Condon et al. 2012a; Vernstrom et al. 2014).

Over the past decade many authors have also used ‘stacking’ methods to extract the average properties of populations of extragalactic sources selected in a different waveband (e.g. Zwart et al. 2015b, hereafter Z15, developed the technique proposed by Mitchell-Wynne et al. 2014). Zwart et al. (2015a) and Z15 give detailed overviews of the relative merits of the \( P(D) \) and stacking analyses in the age of the Square Kilometre Array (SKA) precursors.

However, even the most sophisticated stacking analyses have not to date incorporated the effects of the Point Spread Function (PSF) and confusion. The effect have not until now been dominant contribution for current surveys, such as the Very Large Array (VLA), but they must be inescapably accounted for in order to carry out near-threshold analyses with forthcoming radio-continuum surveys, most noticeably MIGHTEE (Jarvis et al. 2017), MWA’s GLEAM (Franzen et al. 2016) and LOFAR’s MSSS (Heald et al. 2015). Indeed, outside the radio-continuum community, Elson et al. (2016) have already sounded the alarm about confusion in \( \text{HI} \) stacking experiments.

When applied to simulated data based on SKADS (Wilman et al. 2010a), the fully-Bayesian algorithm in Z15 led to source counts that were biased by confusing radio sources for a minimum injected source flux of 0.01 \( \mu \)Jy, i.e. the SKADS-\( S^5 \) limiting flux; such a bias arises because the stacking algorithm in Z15 assumes that the contribution to the total flux comes from only one stacked galaxy plus noise, while in this case the flux contributions from other

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\(^{1} \text{We will quantify the limitations of using a } P(D) \text{ analysis jointly with stacking analyses in Sec.4} \)
sources are non-negligible. The stacking counts became consistent with the true, input model only when the confusing source density in the images was decreased by raising the minimum injected source flux. A hard confusion ‘wall’ presents itself in any stacking analysis, its depth being solely a function of the telescope resolution and the underlying source counts. The former is usually outside our control, and the latter is at best a target quantity and at worst a nuisance term ripe for marginalization.

What is therefore needed is a general framework for stacking analyses in the presence of confusion. In this work, we still cast the problem in a fully Bayesian framework rather than in a purely maximum-likelihood one and the Bayesian evidence is used for selecting between multi-power-law models.

In Section 2, we give a short prescription for the derivation of the stacking method in the presence of confusion. In Section 4, we discuss the detectability of any confusion contribution. In Section 5 we give some details of the simulations undertaken as well as our Bayesian framework, including the models, priors and likelihoods. We present our results in Section 6 and Section 7. Finally, we discuss and conclude in Section 8 and 9.

2 STATISTICAL DESCRIPTION

In this section, we first review the state-of-the-art stacking method (see Z15), and the \( P(D) \) analysis. After that we give a full prescription for the derivation of the probability of finding a stacking pixel of a given flux, giving full consideration to the effects of confusion.

2.1 Stacking without confusion

For completion, we now briefly review the earlier Bayesian stacking method. Z15 assumed that the measured flux \( S \) for a given galaxy could be decomposed as,

\[
S = S_1 + \mathcal{N}
\]

(1)

where \( S_1 \) is the underlying intrinsic flux of the stacking galaxy and \( \mathcal{N} \) is the noise of the image which is assumed to follow a Gaussian distribution with zero mean and variance \( \sigma^2_\mathcal{N} \).

The probability of finding one stacking galaxy in a pixel \( \Omega_{pix} \) with measured flux in the interval \( [S, S + dS] \) is

\[
\mathcal{P}_{stk}(S) dS = \int_{-\infty}^{\infty} dS_1 \frac{d^2 N_t}{dS dt}(S_1) \frac{\Omega_{pix}}{\Upsilon_{pix}} \exp\left[-\frac{(S - S_1)^2}{2\sigma^2_\mathcal{N}}\right] dS_1 d\Omega_{pix}
\]

(2)

where \( \bar{n}_t \) is the mean number of stacking sources per unit solid angle,

\[
\bar{n}_t = \int_{-\infty}^{\infty} dS_1 \frac{d^2 N_t}{dS dt}(S_1)
\]

(3)

and the lower indices \( S \) stand for “stacking galaxy” and \( stk \) stand for “stacking method”.

The probability of finding \( k \) stacking pixels in the image \( \Omega_{image} \) with measured flux in the interval \( [S, S + dS] \) obeys a Poisson distribution with a mean value \( \mathcal{P}_{stk}(S) \bar{n}_t \Omega_{image} dS \).

The Z15 stacking method works well when the confusion effect is small compared to the shot noise of the stacking pixel fluxes histogram (see section 4 for a quantitative discussion). However, for the next-generation deep radio surveys, this will not always be the case. On one hand, the large sky coverage will allow us to have more stacking sources which shrinks the statistical uncertainty. On the other hand, the confusion noise will increase substantially as surveys reach ever greater depths.

2.2 \( P(D) \) analysis

For completion, we now give a short review of the \( P(D) \) method. Our derivation and notations follow the original \( P(D) \) analysis paper (Scheuer 1957).

Considering an observation at a random position on the sky, a point source of true flux \( S \) at position \((x, y)\) \(^1\) is observed with a smoothed flux \( X \).

\[
X = B(x, y)S,
\]

(4)

where \( B(x, y) \) represents the point-spread function (PSF). We choose a two-dimensional Gaussian PSF function.

The mean number of total sources within the image \( R_t \) observed with a smoothed flux \( X \) inside the PSF is given by (Condon 1974):

\[
R_t(X) \, dX = \int_{\Omega_{map}} \frac{d^2 N_t}{dS dt}(X) \frac{dX}{B(x, y)} \, dxdy,
\]

(5)

where the lower indices \( t \) stand for “total”, and the extra factor \( B(x, y) \) in the denominator is due to a change of variables from \( X \) to \( S \), and \( \frac{d^2 N_t}{dS dt} \) is the differential number count of the total galaxy population within the image field, no matter if it is detected or not.

The mean number of sources inside the PSF integrated over all possible fluxes is:

\[
\bar{n}_t = \int_{R_t} R_t(X) \, dX.
\]

(6)

The probability \( G(n) \) of having \( n \) sources inside the PSF is a Poisson distribution of mean \( \bar{n}_t \),

\[
G(n) = \frac{(\bar{n}_t)^n}{n!} e^{-\bar{n}_t};
\]

(7)

Since the observed flux of each image pixel is a convolution of the PSF with the flux of galaxies inside the PSF plus instrumental noise, the probability of an image pixel having a flux \( S \) to \( S + dS \) (i.e. the so-called \( P(D) \) where ‘D’ is for deflection) can be split according to the number of sources inside the PSF.

\[
\mathcal{P}_D(S) dS = \sum_{n=0}^{\infty} G(n) P_n(S) dS.
\]

(8)

where \( P_n(S) \) is the probability of having a pixel with flux \( S \)

\(^1\) One is free to choose \((\theta, \phi)\) coordinates, and the result will remain the same. It is always reasonable to use the flat-sky approximation inside the PSF.
Bayesian stacking with confusion noise

knowing there are $n$ galaxies inside the PSF centred at the pixel position.

Now, considering the first term in the summation, $n = 0$, i.e. no galaxy inside the PSF,

$$P_0(S) = \mathcal{N}(S),$$

(9)

where $\mathcal{N}(S)$ is the normalized noise probability density function (PDF), here a Gaussian PDF. Compared to first term, the second term $n = 1$ adds one confusing galaxy into the PSF:

$$P_1(S) = \frac{1}{n!} \int_{-\infty}^{+\infty} dX_1 R_t(X_1) \mathcal{N}(S - X_1),$$

(10)

where we extend the integration range to infinity, and set the differential number count function to be zero for the flux $S > S_{\text{max}}$ or $S < S_{\text{min}}$. Similarly, the third term $n = 2$ is,

$$P_2(S) = \frac{1}{n!} \int_{-\infty}^{+\infty} dX_1 R_t(X_1) \int_{-\infty}^{+\infty} dX_2 R_t(X_2) \mathcal{N}(S - \sum_{j=1}^{2} X_j),$$

(11)

For $n$ galaxies we have

$$P_n(S)dS = \frac{1}{n!} \int_{-\infty}^{+\infty} dX_1 R_t(X_1) \cdots \int_{-\infty}^{+\infty} dX_n R_t(X_n) \mathcal{N}(S - \sum_{j=1}^{n} X_j),$$

(12)

Using the convolution theorem, we see that the Fourier transform of this term can be simplified as

$$\mathcal{F}\{P_n(S)\} \equiv \int_{-\infty}^{\infty} e^{-2\pi i w S} P_n(S)dS$$

(13)

$$= \exp(-2\pi\sigma^2 w^2) \left(\frac{r_1(w)}{\bar{n}}\right)^n,$$

(14)

where $r_1(w)$ is the Fourier transform of $R_t(X)$, i.e. $r_1(w) \equiv \mathcal{F}\{R_t(X)\}$, and $\exp(-2\pi\sigma^2 w^2)$ is the Fourier transform of the Gaussian noise with standard deviation $\sigma$.

Therefore, the probability of obtaining a total flux $S$ in a PSF is

$$P_D(S)dS = \mathcal{F}^{-1}\left\{\sum_{n=0}^{\infty} G(n)\mathcal{F}\{P_n(S)\}\right\}dS$$

(15)

$$= \mathcal{F}^{-1}\left\{\exp(-2\pi\sigma^2 w^2) \exp(r_1(w) - \bar{n})\right\}dS,$$

where we used the Taylor expansion of exponential function. This is the PDF of $P(D)$ analysis used in the previous papers such as (Condon et al. 2012a; Vernstrom et al. 2014).

### 2.3 Stacking with confusion - low density case

The stacking method outlined in section 2.1 is based on the assumption that there is no contribution from other sources inside the point spread function (PSF). However, this is not always true: The confusion effect must in general be taken into account.

In this subsection, we consider the confusion effect, where the total noise is not Gaussian, but rather follows the confusion amplitude distribution $P(D)$. To distinguish a stacking galaxy from a non-stacking galaxy, we introduce lower indices to the differential number count $d^2 N_s / dS d\Omega$, with $s$ for stacking galaxies and $o$ for non-stacking galaxies.

Considering the fact that the pixels we use for the stacking analysis are always centered at the positions of stacking galaxies, the stacking galaxy flux is not modified by the PSF (in the limit of small positional uncertainties). The mean number $R_s$ of stacking galaxies observed with a total flux $S$ inside the unitary solid angle is

$$R_s(S) dS \equiv \frac{d^2 N_s}{dS d\Omega} (S) dS.$$  

(16)

The mean number of stacking galaxies inside the unitary solid angle $\bar{n}_s$ is the same as we defined in Eq. 3

The mean number $R_o$ of non-stacking galaxies observed with a flux $X$ inside a PSF is similarly,

$$R_o(X) dX = \int \frac{d^2 N_o}{dS d\Omega} \left(\frac{X}{B(x, y)}\right) dX dxdy.$$  

(17)

The mean number of non-stacking galaxies inside a PSF is then an integral over all fluxes,

$$\bar{n}_o = \int_0^\infty R_o(X) dX.$$  

(18)

we consider the special case that the total differential number counts is dominated by the non-stacking galaxies, not the stacking galaxies(e.g., the stacked population is sparse).

The conditional probability $P_{\text{stk}}(S) dS$ of a stacking pixel having a flux $S$ to $S + dS$ can be split according to the number of non-stacking galaxies inside the PSF, i.e.

$$P_{\text{stk}}(S) dS = \sum_{n=0}^{\infty} G^n(n) P_s^n(S) dS,$$

(19)

where $G^n(n)$ is the probability of having $n$ sources inside the PSF following a Poisson distribution of mean $\bar{n}_o$.

Now, considering the $n = 0$ case in Eq. 19, there is only one stacking galaxy with noise,

$$P_0(S) = \int_{-\infty}^{+\infty} R_o(X_1) \mathcal{N}(S - X_1) dX_1,$$

(20)

where $\frac{R_o(X_1)}{\bar{n}_o}$ gives the probability of a stacking galaxy have flux $X_1$. Comparing with the previous case, the $n = 1$ case adds one non-stacking galaxy:

$$P_s^1(S) = \frac{1}{\bar{n}_s \bar{n}_o} \int_{-\infty}^{+\infty} dX_1 R_s(X_1)$$

(21)

$$\times \int_{-\infty}^{+\infty} dX_2 \mathcal{N}(S - X_1 - X_2) R_o(X_2).$$

Thus,
$P_{o}^*(S) = \frac{1}{\tilde{n}_t \tilde{n}_o} \int_{-\infty}^{+\infty} dX_1 R_o(X_1)$ (22)

$\int_{-\infty}^{+\infty} dX_2 R_o(X_2) \cdots \int_{-\infty}^{+\infty} dX_n R_o(X_n) N(S - \sum_{j=1}^{n+1} X_j)$,

Similarly to the $P(D)$ derivation, the convolution theorem can simplify the calculation. Following this route, we find that the Fourier transform of $G''(n)\bar{P}_n^*(S)$ is

$\mathcal{F}\{G''(n)\bar{P}_n^*(S)\} = G''(n)\bar{p}_n^*(w)$

$= \left\langle \tilde{n}_o \right\rangle^n \exp(-\tilde{n}_o) \frac{r_o(w)}{\tilde{n}_o} \exp(-2\pi \sigma^2 w^2) \left\{ \frac{r_o(w)}{\tilde{n}_o} \right\}^n,$

where $\bar{p}_n^*(w) \equiv \mathcal{F}\{P_n^*(X)\}$, and $r_o(w)$ is the Fourier transform of $R_o(X)$, i.e. $r_o(w) \equiv \mathcal{F}\{R_o(X)\}$. Therefore, the probability of observing a stacking pixel with total flux $S$ is

$\mathcal{P}_{stk}(S) = \mathcal{F}^{-1}\left\{ \sum_{n=1}^{\infty} G''(n)\bar{p}_n^*(w) \right\}$

$= \mathcal{F}^{-1}\left\{ \frac{r_o(w)}{\tilde{n}_o} \exp(r_o(w) - \tilde{n}_o - 2\pi \sigma^2 w^2) \right\}$.

This is also the convolution of the stacking probability with the $P(D)$, e.g. replacing the Gaussian noise in the stacking equation by the $P(D)$ equation.

### 2.4 Stacking with confusion - high density case

As the density of stacking sources increases, it is getting difficult to measure the non-stacking source differential number count. In this section, we derive a PDF of having a stacking pixel with a certain flux, using the total galaxies differential number counts(including the stacking population).

The conditional probability $\mathcal{P}_{stk}(S)$ for a stacking pixel to have a flux $S$ to $S + dS$ can be considered as a sum of joint probabilities of having a stacking pixel of flux $S$ contributed from $n_t \in N$ galaxies inside the PSF.

$\mathcal{P}_{stk}(S)dS = \sum_{n_t=1}^{\infty} G''(n)\bar{P}_n^*(S)dS,$

where $G''(n)$ is the conditional probability of having $n$ total number of galaxies inside a PSF, given the fact that the stacking galaxy is sitting at the center of PSF. And $P_n^*(S)$ is the probability of having a stacking pixel of flux $S$, knowing that the total number of galaxies inside the PSF is $n$. It is reasonable to assume that having $n$ galaxies inside the PSF and their total flux being $S$ are statistically independent. Thus, the joint probability is simply the product $G''(n)\bar{P}_n^*(S)$.

Since we do not consider the clustering effect in this paper, knowing the stacking galaxy is sitting at the center of PSF do not increase the probability of finding other galaxies inside the same PSF. Therefore, the probability $G''(n)$ is described by the renormalized Poisson distribution function,

$G''(n) = \frac{\tilde{n}_t^n}{n!} e^{-\tilde{n}_t} \frac{e^{-\tilde{n}_t} \sigma^2}{\tilde{n}_o \tilde{n}_t} \left\{ \frac{r_o(w)}{\tilde{n}_o} \right\}^{n-1},$ (26)

where in the renormalization we exclude the $n = 0$ case, and $\tilde{n}_t$ is the mean number of galaxies(including stacking and non-stacking galaxies) inside the PSF,

$\tilde{n}_t = \int_{0}^{\infty} R_t(X) dX,$

where

$R_t(X) dX = \int d^2N \left( \frac{X}{B(x, y)} \right) \frac{dX}{B(x, y) dzdy}.$ (28)

It is important to point out that $R_t(X)$ dX gives the probability of having a total of galaxies inside the whole PSF, and it has already counted the galaxies sitting at the center of the PSF.

Next, we consider $P_n^*(S)$ where $n = 1$ case:

$P_1^*(S) = \frac{1}{\tilde{n}_t \tilde{n}_o} \int_{-\infty}^{+\infty} dX_1 R_o(X_1)$

$\int_{-\infty}^{+\infty} dX_2 N(S - X_1 - X_3) R_t(X_2),$ (30)

where the ratio $R_t(X) dX/\tilde{n}_o$ gives the probability of having PSF averaged flux(or pixel flux) $X$.

Similarly, for the $n_t = 3$ case,

$P_3^*(S) = \frac{1}{\tilde{n}_t \tilde{n}_o} \int_{-\infty}^{+\infty} dX_1 R_o(X_1)$

$\int_{-\infty}^{+\infty} dX_2 R_t(X_2) \int_{-\infty}^{+\infty} dX_3 N(S - \sum_{j=1}^{3} X_j) .

Hence for the $n$-th case we have that

$P_n^*(S) = \frac{1}{\tilde{n}_t \tilde{n}_o} \int_{-\infty}^{+\infty} dX_1 R_o(X_1)$

$\int_{-\infty}^{+\infty} dX_2 R_t(X_2) \cdots \int_{-\infty}^{+\infty} dX_n R_t(X_n) N(S - \sum_{j=1}^{n} X_j),$

Using the convolution theorem, we see that the Fourier transform $p_n^*(w) \equiv \mathcal{F}\{P_n^*(X)\}$ is

$p_n^*(w) = \frac{r_o(w)}{\tilde{n}_o} \exp(-2\pi \sigma^2 w^2) \left\{ \frac{r_o(w)}{\tilde{n}_o} \right\}^{n-1},$

where $r_o(w)$ is the Fourier transform of $R_o(X)$, i.e. $r_o(w) \equiv \mathcal{F}\{R_o(X)\}$. Therefore, the probability of observing a stacking pixel of total flux $S$ is

$\mathcal{P}_{stk}(S) = \mathcal{F}^{-1}\left\{ \sum_{n_t=1}^{\infty} G''(n)\bar{p}_n^*(w) \right\}$

$= \mathcal{F}^{-1}\left\{ \tilde{n}_t \frac{r_o(w)}{\tilde{n}_o} \exp(-2\pi \sigma^2 w^2) \left[ \exp(r_o(w)) - 1 \right] \right\}.$

In section 4, we use this PDF to shed light on the bias in the number counts found by the simulations of Z15, where the stacking sources in the image constitute the dominant population. While, we also use it to estimate the stacking number counts in section 6.2, where 40% of the total injected galaxies are stacking galaxies.

MNRAS 000, 1–12 (2017)
3 METHODOLOGY

We would like to constrain the differential number counts of the stacking galaxies close to the confusion limit. When the number of stacking galaxies is large, it becomes difficult to measure the non-stacking galaxies differential number counts isolated because masking the stacking galaxies in the map (which usually requires a patch of around $3-\sigma$ of the PSF) will remove most pixels in the image.

When the number density of the stacking galaxies is small compared to the non-stacking galaxies, the measured total galaxies differential number counts via the $P(D)$ analysis from the full image can be considered as a good approximation of the non-stacking galaxies differential number counts via the $P(D)$ analysis. Then we use the best-fit galaxy differential number counts as an input, and calculate the stacking likelihood via Eq.24 OR Eq.33 for different stacking differential number counts models.

Our measurements are two histograms of pixel fluxes. One is for the full image pixels, the other is for the stacking pixels only. The statistical uncertainty of each histogram bin is the square root of the total number of pixels belonging to the bin. Since, the total number of pixels in the image field is larger than the number of stacking pixels in the same image, the statistical uncertainty of the full image pixel fluxes histogram is smaller than stacking pixel fluxes histogram. The inferred number counts from the full image pixel fluxes histogram should be tighter constrained as well.

We assume the non-stacking (or total) galaxies differential number counts can be tightly constrained by the full image, and the statistical uncertainty of the stacking pixel fluxes histogram is the dominant error. Under this approximation, we simplify our evaluation by neglecting the uncorrelated flux errors. Under this approximation, the statistical uncertainty of the stacking pixel fluxes histogram is the dominant error. Under this approximation, we simplify our evaluation by neglecting the uncorrelated flux errors.

Establish the best-fit differential number counts parameters.

(iii) Associate the best-fit differential number counts model with the non-stacking galaxies inside the image.

(iv) Generate a histogram of fluxes extracted only from the positions of the source population to be stacked.

(v) Fit the stacking galaxies differential number counts model for the stacked population using Eq.24.

3.1 Algorithm

The recipe for application of our methods is summarized as follows:

I. Low density method:

(i) From a radio image, extract the full pixel fluxes histogram.

(ii) Fit the differential number counts model to these noisy data using the $P(D)$ likelihood given in this section. Establish the best-fit differential source counts parameters.

(iii) Associate the best-fit differential number counts model with the non-stacking galaxies inside the image.

(iv) Generate a histogram of fluxes extracted only from the positions of the source population to be stacked.

(v) Fit the stacking galaxies differential number counts model for the stacked population using Eq.24.

II. High density method:

(i) Same as Low density method

(ii) Same as Low density method

(iii) Associate the best-fit differential number counts model with the entire galaxies inside the image.

(iv) Generate a histogram of fluxes extracted only from the positions of the source population to be stacked.

(v) Fit the stacking galaxies differential number counts model for the stacked population using Eq.33.

3.2 Sampling

Sampling parameter spaces is often slow, especially when evidence integrations, which is required for model selection, are carried out. Nested sampling (Skilling 2004) was introduced specifically for the purpose of cutting the computational cost of this. However, it is an inescapable fact that the evidence integrations are exponential in the number of model parameters, in practice limiting that number to $\lesssim 100$. The many advantages of nested sampling — compared to MCMC methods — are discussed elsewhere (see e.g. Z15).

The de facto implementation of nested sampling is MULTINEST (Feroz & Hobson 2008; Feroz et al. 2009), which has a PYTHON wrapper (Buchner et al. 2014). We deploy MULTINEST on, typically, 48–96 processors, for as many as $10^5$ likelihood calculations in total.

3 To consider correlations, it may be easier to use a Gaussian likelihood, and plug in the covariant matrix from simulation as in Vernstrom et al. (2014)

4 Our stacking likelihood calculation program can be download from https://github.com/phychensong/ConfusIuS.
4 QUANTIFYING CONFUSION-INDUCED BIAS IN STACKED COUNTS

Using the equations derived in Section 2.4, we can further study the upward bias in the differential number counts of a stacking population found by Z15. The convolution theorem allows us to isolate an effective noise function from the stacking PDF:

$$P_{\text{stk}}(S) = \int_{-\infty}^{+\infty} \frac{R_{\nu}(S)}{n_{\nu}} N_{\text{eff}}(S - S_{\nu}) dS_{\nu}. $$

The density of stacking galaxies used by Z15 was high, so we employ Eq. 33. The effective-noise function is

$$N_{\text{eff}}(S) = \int_{-\infty}^{+\infty} \frac{R_{\nu}(S)}{n_{\nu}} \frac{\exp(-2\pi \sigma_{\nu}^2 w^2) \exp(\bar{n}_t w) - 1}{\exp(\bar{n}_t) - 1} [\exp(r_{\nu}(w)) - 1] dS_{\nu}.$$  

This effective noise function is essentially the convolution of Gaussian image noise with the pure confusion effect. In order to see the difference between the ordinary stacking method and the stacking method including confusion, we define an excess term $E_{\text{eff}}$ that traces the extra contribution from confusion,

$$E_{\text{eff}}(S) dS \equiv (N_{\text{eff}} - N) \bar{N}_{\text{stk}} dS,$$

where $\bar{N}_{\text{stk}}$ is the total number of stacking pixels inside the image. We then compare this excess to the uncertainty of the measurement, i.e. the Poisson noise of the bins of the stacking pixel-flux histogram. This is just the square root of the number of stacking pixels falling into a certain flux range, and is proportional to the stacking PDF (i.e. Eq.33).

$$\sqrt{P_{\text{stk}}(S) \bar{N}_{\text{stk}} dS} = N_{\nu}(S) dS$$

If the excess is larger than the uncertainty, $N_{\nu}$, of the measurement, then we can detect this small difference between the Gaussian image noise and the convolution of the Gaussian image noise with the confusion effect from a given measurement. As a result of this ‘detection’, the fitted stacking population number counts using ordinary stacking will be shifted upwards from the true number counts so as to compensate for this extra confusion contribution.

The same idea can be adopted to describe a $P(D)$ analysis as well. The excess $E_{\text{eff}}^p$ for the $P(D)$ analysis is

$$E_{\text{eff}}^p(S) dS \equiv \bar{N}_{\nu}(P_{\text{D}}(S) - N) dS,$$

where $\bar{N}_{\nu}$ is the total number of pixels in the image. The Poisson noise of the bins of the $P(D)$ pixel fluxes histogram is

$$\sqrt{P_{\text{D}}(S) \bar{N}_{\nu} dS} = N_{\nu}^p(S) dS$$

If the the excess $E_{\text{eff}}^p(S)$ is larger than $N_{\nu}^p(S)$, then we can detect the confusion contribution to the histogram. In other words, a $P(D)$ analysis implemented on this data set has sufficient discriminatory power to extract number counts from the confusing population.

In order to compare with the previous results from Z15, our calculation proceeds on the following basis: We set the PSF resolution to be $6''$ and the survey area to be 1deg$^2$. We use the SKADS-S$^3$ simulation $\frac{\Delta N_{\nu}}{\Delta S_{\nu}}(S)$, and bound the source flux between $S_{\text{min}} = 0.1$ and $1.0$ $\mu$Jy and $S_{\text{max}} = 85 \mu$Jy. The stacking sources include all the simulation sources above the limiting flux $S_{\text{min}}$.

Figure 1 shows the excess $E_{\text{eff}}$ behaviour for two different minimum flux conditions, $S_{\text{min}} = 0.1 \mu$Jy and $S_{\text{min}} = 1 \mu$Jy. For the case $S_{\text{min}} = 1 \mu$Jy, the key point is that $E_{\text{eff}}$ has been overtaken by the shot noise over the FULL flux range. This suggests that we would not have a high enough signal-to-noise ratio to ‘detect’ the confusion contribution and, as a result, the reconstructed number counts will not be much affected by confusion. This argument supports the previous findings of Z15, where in the $S_{\text{min}} = 0.1 \mu$Jy image the reconstructed $\frac{\Delta N_{\nu}}{\Delta S_{\nu}}(S)$ function is significantly biased, but for $S_{\text{min}} = 1 \mu$Jy it is not.

From Eq.33, we see the stacking PDF $P_{\text{stk}}(S)$ contains three components: (i) the stacking galaxy number count, (ii) the Gaussian image noise and (iii) the ‘confusion effect’. If we multiply the number of stacking galaxies by a factor of $N$, and keep the image noise the same, then the effective noise excess $E_{\text{eff}}$ is amplified by $N$, but the shot noise is only amplified by $\sqrt{N}$. For the above case, when we change

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5 In Z15, all the galaxies in the image were selected as stacking galaxies.
Bayesian stacking with confusion noise

Table 1. Confusion noise \( \sigma_c \) values for different simulations.

| \( S_{\text{min}} \) | \( \sigma_c/\mu Jy \) |
|-----------------|-----------------|
| 1.0             | 3.718           |
| 0.1             | 3.724           |
| 0.02            | 3.724           |

S_{\text{min}} from 1\mu Jy to 0.1\mu Jy, the number of stacking galaxies is amplified by a factor of 4.

The shot noise \( N_s \) from the stacking pixel-flux histogram can also be changed by varying the Gaussian image noise or the PSF resolution. However, the underlying relations are not straightforward to quantify. Numerical calculation of the above equations, which will not be discussed here, would be required.

We also measured the rms \( \sigma_c \) of the observed image pixel-fluxes, and estimated the rms \( \sigma_c \) of the noise-free source confusion (Condon et al. 2012b) via:

\[
\sigma_c = \sqrt{\sigma_g^2 - \sigma_n^2}. \quad (43)
\]

The results are shown in Table 1. Notice that the rms of the noise-free source confusion is not very sensitive to the minimum source flux \( S_{\text{min}} \). In contradiction to what has previously been assumed in the community, \( \sigma_c \) is a poor estimator of the eventual bias in the inferred stacking number counts. (We see this because the bias in the inferred number count changes even when \( \sigma_c \) is roughly constant.)

The key issue to the stacking method is whether the effect noise distribution is significant different from Gaussian distribution, rather than from the thermal map noise beaten down by the noise-free source confusion. This difference is compared to the shot noise \( N_s \) from the stacking histogram. When the difference is overtaken by the shot noise, we can use the ordinary stacking method from Z15, otherwise we can not.

Finally, comparing \( E_{\text{eff}}(S) \) with \( N_P(S) \), we can describe the more general limitations of the ordinary stacking method. We assume that the shape (not the amplitude) of the stacking-population number counts does not change much with respect to the SKADS-S^2 simulation. For a 1 – deg^-2, 6-arcsec-resolution, 16.2 \( \mu Jy \) noise image whose galaxies fluxes lie in the range 0.02 \( \mu Jy < S < 85 \mu Jy \), we found that the upper limit to the surface density of sources for adopting the ordinary stacking method is 10,000 stacking galaxies (i.e. \( E_{\text{eff}}(S) \) touches \( N_P(S) \)). For the cases that the density of stacking galaxies is \( > 10,000 \), the stacking method with confusion must be used for extracting unbiased counts.

5 SIMULATION

The Square Kilometre Array Design Studies SKA Simulated Skies (SKADS-S^2) simulation (Wilman et al. 2008, 2010b) is a semi-empirical model of the extragalactic radio-continuum sky covering an area of 400 deg^2, from which Z15 extracted a 1deg^2 catalogue at 1.4 GHz for the purposes of testing their method. The simulation, which is the most recent available, incorporates both large- and small-scale clustering and has a flux limit of about 0.01 \( \mu Jy \). We undertook several tests as follows.

For the present study, we adopt the same VLA-like mock survey strategy as Z15, i.e. a 1.4-GHz, 1-deg^2 survey with a gaussian noise with \( \sigma = 16.2 \mu Jy \) and a 6-arcsec gaussian FWHM synthesized beam/PSF. This setup follows the VLA-VIRMOS observations by Bondi et al. (2003).

In total 374,061 sources have been injected into the simulated image taken from SKADS, whose minimum injected source flux is 0.02 \( \mu Jy \). We further set a source flux cutoff at 85 \( \mu Jy \), while we assume the sources with flux larger than this value can be masked out. The flux cutoff setting will not affect our finally result.

To match with ancillary optical/infra-red data (e.g. the VIDEO survey: Jarvis et al. 2013), we select the star forming galaxies whose K-magnitude < 24 in the simulated catalogue. In total there are 149,516 stacking sources left under this selection (hereafter ‘selection-H’).

However, 149,516 stacking galaxies out of 374,061 are way too large to use the low density stacking method. For this reason, we create another stacking selection by adopting a further constraint on redshift 0.5 < z < 1.0. After the redshift selection, only 18,110 galaxies are left (hereafter ‘selection-L’). This selection fulfills the requirements of the low density stacking method.

It is also important to decide the size of image pixel. In order to simplify the analysis, we expect the bins of the image pixel fluxes histogram to be independent from each other, i.e. we want to remove the pixel-to-pixel correlation. This requires that the pixel size is comparable to the PSF resolution (Vernstrom et al. 2014). However, we also want to reduce the offset of the stacking galaxy from the center of its own pixel. In our simulation, we choose the pixel size to be 1 arcsec.

5.1 Modelling differential number count

There are many ways of modelling the differential number-count function \( d^2N/(dSd\Omega) \). From the mathematical point of view, the most straightforward model is the polynomial in log-log space (see e.g. Bridges et al. 2009; Vernstrom et al. 2014). However, the \( d^2N/(dSd\Omega) \) function is over-sensitive to the higher-order parameters in the polynomial model, which need a lot of attention in the sampling process, as well as the prior range.

The pole/node-based model (see e.g. Vernstrom et al. 2014) fixes the position in \( \log_{10}(S) \) of a fixed number of nodes, and we fit for the node amplitudes. Between the nodes the count is interpolated in log space to ensure a continuous function. In this method, the node amplitudes depend not only on the underlying source count but also on the number, or spacing, of the nodes, and also the type of interpolation used between the nodes. Therefore, given a differential number count, the choice of node number and positions has considerable impact on the fit results.

Similar to the pole/node-based model, the multi-power-law model is also based on a small number of break points.
connected by power law segments. Unlike the node-based model, the flux positions of the break points are not fixed, and the physical meaning of each fitting parameter is obvious, but the parameters of the model are highly correlated and high signal-to-noise features tend to attract the free break positions.

Overall, we adopt the multi-power-law model in the following analysis for simplicity. We choose two cases: one-break power-law model and two-breaks power-law model (Models B and C from Z15). Model A in Z15 does not have enough features to characterize the shape of the SKADS differential source count, and model D has too many correlated parameters. The one-break power-law model is defined as

$$\frac{d^2N}{dSd\Omega} = \begin{cases} CS^\alpha & S_\text{min} < S < S_0 \\ C S_0^{-\beta} S^\beta & S_0 < S < S_\text{max} \\ 0 & \text{otherwise} \end{cases}$$

with a parameter vector $\Theta_B = \{C, \alpha, \beta, S_0, S_{\text{min}}, S_{\text{max}}\}$. The two-breaks power-law model incorporates another break in the power law, so $\Theta_C = \{C, \alpha, \beta, \gamma, S_0, S_1, S_{\text{min}}, S_{\text{max}}\}$. Priors on the different model parameters are discussed in section 5.2.

5.2 Priors

Our conservative priors are listed in Table 2. Note that we have assumed equiprobable models a priori, i.e. $Z_1(\mathbf{D}|H_1) = Z_2(\mathbf{D}|H_2)$.

| Parameter | Prior |
|-----------|-------|
| $C/\text{sr}^{-1}\text{Jy}^{-1}$ | log-uniform in $[10^{-5}, 10^9]$ |
| $\alpha_j$ | uniform in $[-2.5, -0.1]$ |
| $S_{\text{min}}/\mu\text{Jy}$ | uniform in $[0.001, 1.5]$ |
| $S_{\text{max}}/\mu\text{Jy}$ | uniform in $[1.5, 5.0]$ |
| $S_0,1$ | uniform in $[S_{\text{min}}, S_{\text{max}}]$ |
| $S_0,1$ | further require $S_0 < S_1$ |
| $\sigma$ | uniform in $[0.5, 2.0] \sigma_{\text{survey}}$ |
| $Z_i$ | equiprobable |

6 RESULTS

We now test the methods developed in the previous sections. We set about extracting binned source counts at the positions of the selected sources, as described in Section 5, for the two selections (i.e. Selection-L and Selection-H) from the same image. Additionally, in order to estimate the contribution to the inferred differential number count from confusion, we also generated a histogram of all image pixel fluxes as the input for a $P(D)$ modeling of the whole image.

It is worth pointing out that the image noise is relatively independent of the parameters of the number-count model, and is very well constrained by the pixel-flux histogram. In order to speed up computation, therefore, we have not fitted simultaneously for the image noise, but our software does already have this capability (as in Z15).

The one-break power-law posterior probability distribution for the total galaxy differential number counts via the $P(D)$ analysis is shown in Fig. 2. We clearly see that the break flux $S_0$ has two preferred values, centering at 30$\mu$Jy and 80$\mu$Jy, which are the two breaks in the differential number counts of the SKADS-S8 simulation. The relative evidence, $\Delta \log_{10} Z = 0.63 \pm 0.27$ indicates that this model is preferred to a two-breaks power-law model at the $\approx 3$-$\sigma$ level.

Using the above posterior probability distribution, we reconstructed the total galaxy differential number counts (Fig. 3). These follow the mock number counts tightly down to 1$\mu$Jy. It is looks like the number counts at faint fluxes $> 1\mu$Jy can be fitted better by allowing more breaks in the number counts model. However, the fitting results from the two-breaks power-law model show that the break are attracted to high signal-to-noise region which is around the bright end (see Fig. 3). A larger image with finer flux resolution and lower noise may help to improve the fitting at fluxes $S < 1\mu$Jy by increasing the signal-to-noise ratio in this flux range.

Summarizing a posterior probability distribution is challenging but a necessary aspect of our two-step approach (see also section 7 below). Ideally, one would importance-sample from the $P(D)$ posterior distribution so as to propagate its morphology (i.e. uncertainties, correlations, degeneracies, multimodalities, skirts, wings and any other non-Gaussianities). Nonetheless, for simplicity, rather than considering each parameter on a case-by-case basis, we do try to describe the posterior distribution using a single statistic, noting especially that this will not propagate the uncertainties from the $P(D)$ part of the analysis. We adopt the maximum-likelihood parameter estimates, which fit the mock number counts better than the median parameter, and for reference these values are given in Table 3.

6.1 Stacking analysis (Selection-L)

For stacking Selection-L, we assume the above reconstructed number-count function from the full image to be a good approximation to the non-stacking number counts. Note that the non-stacking galaxies represent about 95 per cent of the total number of galaxies.

Assuming the parameters from Table 3 as the prescription for the number counts of the non-stacking galaxies,
Bayesian stacking with confusion noise

we implemented the low-density stacking method (Section 3.1) in order to extract the number counts of the stacked population. We compared the fits of the one-break power-law stacking model and the two-breaks power-law stacking model via the Bayesian evidence (Table 4). The model evidences are fully consistent with each other within uncertainties; we choose to adopt the two-breaks power law as the marginal winner since strictly its evidence is the higher. The posterior probability distribution for two-breaks power-law stacking model is shown in Fig. 4.

From this we reconstructed the number counts of the stacked population (Fig. 5). The reconstructed counts are fully consistent with the mock counts of the selection-L sources within the 95 per cent confidence level, except for a slight offset at the very faint end ($S < 1\mu Jy$).

Table 4. Selection-L (low-density method): Nested Sampling Global log$_{10}$-evidence for the different power-law models.

| Breaks | Parameters | log$_{10}$-evidence |
|--------|------------|---------------------|
| 1      | 6          | $-118.0604 \pm 0.1740$ |
| 2      | 8          | $-118.0598 \pm 0.1745$ |

Fig. 6 shows a comparison of the different available stacking methods. The ordinary method gives a significantly-biased reconstruction (in agreement with the findings of Z15). The low-density and high-density methods give almost identical results, and fit the stacked number counts reasonably well.

6.2 Stacking analysis (selection-H)

For stacking selection-H, non-stacking galaxies represent about 60 per cent of the total population, so that the number counts of the total population is no longer a good approximation to those of the non-stacking galaxies. Under these circumstances the low-density method is unsuitable, and could yield biased reconstructed number counts. Hence we use the high density method, Table 5 giving the relative evidences for different power-law source-count models fitted to the data. The data prefer the two-breaks model, whose posterior probability distribution is shown in Fig. 7.
Reconstructing the source counts of the stacked population from the posterior probability distribution (Fig. 8), at the 95-per-cent level these are fully consistent with the mock counts for the selection-H sources, except for a slight offset at the very faint end, \( S < 1 \mu \text{Jy} \).

In Fig. 9, the reconstructed differential number counts using the ordinary method (solid blue line) is biased again, for the same reason of Z15. Besides, the low density method (black line) is below the mock number counts in the range \( S > 1 \mu \text{Jy} \). This is probably due to the fact that we have overestimated the non-stacking source count using an inappropriate approximation.

As a check, we calculated the difference between the mock number counts and the different reconstructed number counts, via \( \chi^2 \). We define \( \chi^2 \) as

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\text{reconstructed dnds} - \text{mock dnds}}{\frac{1}{2} \times 95\% \text{ CI}} \left(45\right) \right)^2 ,
\]

where \( N \) is the total number of the number counts bins. The \( \chi^2 \) values in Table 6 indicate the high density method gives the closest reconstructed number counts.

### Table 6. \( \chi^2 \) values for the reconstructed different number counts with respect to the mock number counts (for 28 bins with \( S > 1 \mu \text{Jy} \)).

| Method             | \( \chi^2 \) |
|--------------------|--------------|
| Ordinary (Z15)     | 43.32        |
| Low-density        | 3.44         |
| High-density       | 1.92         |

This simplified joint likelihood assumes independence between the total pixel-flux histogram and stacking pixel-flux histogram. Under this assumption, we tested the joint method with our earlier simulation. The fitting results of are shown in Fig. 10 and Fig. 11 for the reconstructed full-image source count and the stacking-population count respectively. The joint fitting successfully recovers both the total-galaxy differential number counts and the stacking-population differential number counts.

From the posterior probability distribution (Fig. 12), we see that significant correlations solely exist internally to the parameters of the two number-count models. The correlations between the number counts models are less significant. The \( P(D) \) analysis better constrains \( C, \alpha \) and \( S_{\text{max}} \) than does stacking. \( C \) and \( \alpha \) are strongly correlated parameters and together they fix the general shape of the number count. Since there are more pixels for the \( P(D) \) analysis than the stacking analysis, a stronger constraint on the total galaxy number counts is expected. However, not all of the \( P(D) \) parameters are better constrained. The stacking analysis gives tighter constraints on \( \beta, S_{\text{min}} \) and \( S_0 \), which may be because the actual number of ‘breaks’ for stacking-population model is less than for the total-galaxy model. In another words, the actual stacking-population number counts are closer to a single-break power-law model.

### 8 DISCUSSION

The reconstructed number counts \( S_{\text{min}} \) in Fig. 5 and Fig. 8 are far from the \( S_{\text{min}} \) of the stacked number counts. The posterior probability distribution figures show a strong correlation between the faint slope \( \alpha \) and the minimum flux \( S_{\text{min}} \). The faint slope parameter \( \alpha \) is driven by the slope at brighter fluxes (high signal-to-noise ratio). As a result, the \( S_{\text{min}} \) parameter is over estimated. Adding one extra break to the number counts model does not solve the problem. Because the extra break is always attracted to the high signal-to-noise flux region. To improve this situation, we need to...
increase the signal-to-noise ratio at faint flux, such as reducing the instrumental noise.

It is also worth mentioning that the statistical uncertainty in the reconstructed stacking number counts is underestimated whenever the stacking-pixel fluxes are not statistically independent. This happens when the surface density of stacking galaxies is so high that the stacking-pixel values are correlated via the beam (or PSF). Selection-H has 149,516 sources, and the synthesized-beam resolution is 6 arcsec. On average, each source maintains a 5.3-arcsec radius circle. This radius is 2–3σ of the PSF, which corresponds to a weight of 1 per cent in Gaussian PSF. In principle, the uncertainty in the selection-H stacking results are underestimated because the actual number of independent pixels must be fewer than 149,516, but this effect is negligible.

9 CONCLUSIONS

(i) We have extended the ‘ordinary’ Bayesian stacking technique to include the full effects of source confusion, an enhancement that is highly applicable to data from forthcoming confusion-limited radio-continuum surveys such as MIGHTEE.

(ii) We have derived two core probability density functions (Eq.24 and Eq.33) that describe the stacking analysis including confusion. One PDF uses the non-stacking galaxy number counts, and the other uses the number counts for the total galaxy population.

(iii) We applied these two new stacking methods to synthesized images based on the SKADS simulation. With the new methods, the reconstructed number counts are fully consistent with the injected number counts, while number counts reconstructed via the ordinary stacking method had been biased at the 95-per-cent confidence level.

(iv) While it had previously been assumed in the literature that the confusion contribution to a stacking experiment was non-negligible if the ‘confusion noise’ was much less than the map thermal noise. The conventional confusion
noise rms ($\sigma_c$) was shown to be a poor estimator for quantifying the impact of confusion on stacking analyses. The key issue is whether the total noise is significantly different from Gaussian. We provide a new heuristic that fulfils this role.

(v) A joint analysis of $P(D)$ and stacking, assuming independence, has allowed us to study the interplay between the parameters of the total galaxy number-count and the stacking-population number-count. We found significant correlations solely exist internally to the parameters of the two number-count models. The correlations between the number-count models are less significant.

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Figure 7. Posterior probability distribution for a high-density stacking fit to the differential number count of the stacking population using the two-breaks power-law model. The red dots shows the maximum-likelihood parameters and the 68 and 95 per cent confidence limits are respectively indicated by the dark and light shaded regions.
Figure 8. Reconstructed differential number counts for the Selection-H population using the high-density method with two-breaks power-law model (best-fit: solid blue line). The blue area represents the 95 per cent confidence interval. The red crosses indicate the mock Selection-H stacking-population number counts. The blue dot line shows the noise rms, and brown dot line shows the minimum stacking pixel fluxes histogram bin width (resolution).

Figure 9. Reconstructed selection-H stacking differential number counts compared to the ordinary stacking method that ignores confusion (best-fit: solid blue line). The blue area represents the 95 per cent confidence interval. The red crosses show the mock stacking-population number counts, with the two-breaks power-law reconstructions using the low-density method (section 3.1) are shown with a black solid line and the high-density method with green circles.

Figure 10. Reconstructed differential number counts for the total galaxies using the joint method with single-break power-law model (best-fit: solid red line). The blue area represents the 95 per cent confidence interval. The green solid line indicate the mock number counts.

Figure 11. Reconstructed differential number counts for the selection-H stacking population using the joint method with one-break power-law model (best-fit: black solid line). The blue area represents the 95 per cent confidence interval. The red crosses indicate the mock stacking-population number counts.
Bayesian stacking with confusion noise

Figure 12. Posterior probability distribution for a joint fit to the differential number counts of the stacking population and total galaxies. We used the one-break power-law models for the two number counts. The red dots show the maximum-likelihood parameters and the 68 and 95 per cent confidence limits are respectively indicated by the dark and light shaded regions.