Models to support students’ understanding of measuring area of circles

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Abstract. Many studies showed that enormous students got confused about the concepts of measuring area of circles. The main reason is because mathematics classroom practices emphasized on memorizing formulas rather than understanding concepts. Therefore, in this study, a set of learning activities were designed as an innovation in learning area measurement of circles. The activities involved two models namely grid paper and reshaping which are respectively as a means and a strategy to support students’ learning of area measurement of circles. Design research was used as the research approach to achieve the aim. Thirty-eight of 8th graders in Indonesia were involved in this study. In this study, together with the contextual problems, the grid paper and reshaping sectors, which used as the models in this learning, helped the students to gradually develop their understanding of the area measurement of circles. The grid papers plays important role in comparing and estimating areas. Whereas, the reshaping sectors might support students’ understanding of the circumference and the area measurement of circles. Those two models could be the tool for promoting the informal theory of area measurement. Besides, the whole activities gave important role on distinguishing the area and perimeter of circles.

1. Introduction
Area measurement is an essential part of mathematics contents [1–3]. Cavanagh [2] stated that area measurement is a crucial part in middle-school mathematics curriculum because of two main reasons. First, it is because there are various applications of area concepts in daily lives’ activities such as painting, gardening, tiling, and any tasks involving covering a two dimensional surface. Second, it is because area concepts are needed to introduce many other mathematical ideas. Furthermore, area measurement is also the foundation to understand volume measurement in solid geometry [3].

However, it is not only students, but also teachers experience difficulties with this topic. The main difficulty is a confusion between the concepts of area and perimeter [4,5]. For instance, several studies found that some Indonesian students had counted perimeters to answer the questions about areas [6]. In other words, the students used the formula of area and perimeter incorrectly. Previous studies show that the reasons behind students’ difficulties is because learning experiences provided in school give more focus on memorizing formulas, rather than understanding concept [7].

Related to the difficulties in learning area measurement, some studies have been conducted to support students’ understanding in the concepts [8]. However, these studies talked about the area of all two dimensional figures. There is not yet a particular investigation about area of circles. Moreover, based on an analysis of Indonesian mathematics textbooks for the 8th grade of secondary school [9–13], the books
have provided various contexts on the topic of area of circles. However, in these cases, there is still a gap from the informal level to the formal level, from the concrete situation to the abstract situation [14]. In the case of area measurement of circles, contexts are used only as attributes to introduce the shape of a circle. Afterwards, the lesson is directly presented in formal level. Therefore, an innovation is needed to promote students’ learning, particularly in the topics of area and area measurement of circles.

The innovation is not only about an implementation of a way approach of learning mathematics, but also a new way of thinking about the purpose and practices of school mathematics. In this case, Realistic Mathematics Education (RME) is an approach which provides a new way of learning mathematics and a new way of thinking about the purpose and practices of school mathematics [15]. In this study, we designed a set of learning activities based on PMRI approach. This is based on the five tenets of RME, namely phenomenological exploration, using models and symbols for progressive mathematization, using students’ own construction, interactivity, and intertwinemen [16].

There were several contexts which aimed at guiding students’ thinking to the correct orientation of area. Two models were used to promote students’ understanding of area measurement of circles. Models are various vertical instruments used as bridge connected the informal level to the formal level [16]. It could be schemas, diagrams, symbols, strategies, and so forth.

The first model is grid paper. The used of the model is based on the theory which described that the area could be determined by counting or approximating the number of units covering a shape, dependent on the length, width, and configuration, can be calculated using a rule or formula, and determined by the sum of small parts [5]. In this study, grid paper was used as a model to support students understanding of the area of a circle and the way to measure it by estimation. The grids are representation of the units of area.

The second model is reshaping sectors. It is based on the theory about area conservation. The area of a shape equals the sum of the area of the parts, two shapes form a new shape when a part of the boundaries are put together and the area of the new shape is the sum of the original area [5]. In this study, reshaping the sectors of a circle into a rectangle and other two dimensional figures help students to derive the formula to measure the area of a circle.

Considering the issues about students’ confusion about the area of circles and the need of an innovation to promote students’ learning about area measurement of circles, this study aims to contribute to a local instructional theory which can support the development of students’ understanding of the area measurement of circles. However, this paper will only discuss about how grid papers and reshaping sectors as models used in the learning activities support students’ understanding of area measurement of circles.

2. Methods
Design research was chosen as the research approach used in this study. We first designed a set of learning activities and a Hypothetical Learning Trajectories (HLT) for the learning of area measurement of circles. Afterwards, a pilot experiment was conducted. The results is used to adjust the content of the learning activities and the sequence of the activities which have been established. Then, second cycle was conducted as the actual teaching process in which the sequence of activities was implemented in a natural classroom environment.

Thirty-eight 8th graders of Junior High School were involved in this study. Six students participated in the pilot experiment and thirty-two students contributed in the teaching experiment. Students worked in group with various level of achievement which were seen from the results of pre-test conducted before the experiment.

The data were collected through students’ written work, video registration, students’ interview, and field notes during the teaching experiment. Those collections of information helped us in interpreting and enable us to make data triangulation. All the data were analyzed qualitatively and the analysis focused on comparing the actual learning and the HLT.
3. Hypothetical Learning Trajectory
The following table describe the refined HLT which is implemented in the teaching experiment. The revision was based on the findings in the pilot experiment and the interview with the teacher. As a result, there were several activities which were conducted in four lessons [17].

| Activity                  | Main Goals                                                                 | Description of Activity                                                                 | Conjectures of Students’ thinking                                                                 |
|---------------------------|-----------------------------------------------------------------------------|------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Comparing and Tiling      | Students are able to compare the size of figures and measure the area of plane figures using measurement units in daily life. | - Students are doing an activity of ordering the five biggest islands in Indonesia starting by a contextual problem of covering the miniature of islands by grass. | - Some students might compare and order based on the shape of the islands.                         |
|                           |                                                                             | - Some students might continue the tiling activity by drawing the tiles one by one and counting the tiles to determine the number of tiles needed to cover. | - Some students might use ruler and calculate the area of the islands.                            |
|                           |                                                                             | - Some students might overlap the islands, then do cutting and pasting.                   | - Some students might overlap the islands, then do cutting and pasting.                          |
| Measuring Figures         | Students are able to measure the area of a plane figure by using a grid paper or by making their own grids. | - Students are doing an activity of comparing two lakes on a grid paper.                  | - Some students might use ruler to continue the tiling activity and determine the number of tiles by using formula. |
| Using A Grid Paper        |                                                                             | - The students are doing an activity of estimating the grass needed to cover a circular garden by making grids. | - Some students might reshape the figures of the lakes and disregard the grid paper.            |
|                           |                                                                             | - Some students might compare those two lakes by counting the grids inside.                | - Some students might compare those two lakes by counting the grids inside.                       |
|                           |                                                                             | - The students are doing an activity of estimating the grass needed to cover a circular garden by making grids. | - Some students might say that the area of the circular garden is \( \frac{3}{4} \) of the area of the square land. |
|                           |                                                                             | - Some student might determine the area as greater than \( 3\pi^2 \) and less than \( 4\pi^2 \). | - Some students might directly count the grids then draw bigger grids to make their work easier or make smaller grids to make their result more accurate. |
Reshaping A Circle into A Rectangle

Students are able to determine the area of a circle which is reshaped into a rectangle.

- The students are doing activities of reshaping the sectors of a circle into a parallelogram, then into a rectangle and determining the area of the rectangle.
- Some students might reshape the sectors of a circle into a parallelogram easily, but they might find difficulty to reshape the parallelogram into a rectangle.
- Some students might reshape the parallelogram into a rectangle easily and said that the area of the rectangle could be found by multiplying its length and width.
- Some students might say that the area of the rectangle is the radius times half of the circumference of a circle because they relate the parts of the rectangle with the part of the circle.

Reshaping A Circle into Other Plane Figures

Students are able to reshape the sectors of a circle into other plane figures and explain the relations between the formula of the area of a circle and the formulas of the area of those plane figures.

- The students are doing activities of reshaping the sectors of circle into several kinds of plane figures and determining the area.
- Some students might arrange the sectors of circle only into a parallelogram and a rectangle and determine the area.
- Some students might reshape the sectors into a rectangle, a parallelogram, a triangle and a trapezoid and determine the area.

4. Results

4.1. Grid Paper

The grid paper was used in the second activity which is measuring figures using a grid paper. There were two activities in this lesson. The first activity was comparing two lakes on a grid paper and the second activity was estimating the grass needed to cover a circular garden on a square land by making grids. The students worked in the group with various level of mathematics performances. The group where students worked was remain the same as the previous lesson.

In the first activity, most students counted the grids to justify the larger lake. As the students did in dealing with the incomplete tiles in the previous lesson, they combined the incomplete grids into one grids and added them with the complete grids. However, some students applied different strategy which was reshaping strategy as shown in this following figure.
The written work shows that the students cut the second figures and reshaped it into more similar figure with the first one. Afterwards, they compared those two figures and justified the larger lake by looking at the remaining parts. However, the class discussion for this activity emphasized the role of grids in a comparing activity.

In the second activity, the students were asked to estimate the amount of grass needed to cover a circular garden on a square land. The expectation was that the students could imitate the strategy they applied in the first activity to solve the problem in this activity. And, as it was expected in the HLT, most students estimate the grass needed to cover the area of the circular land by counting the complete grids and estimate the incomplete grids. None of students estimated by considering the square land, as forementioned in the HLT.

Although the idea was the same which was counting the grids inside the circle, the students’ written works showed various way to determine the grass needed to cover the circular garden. Interestingly, all worksheet showed that students also applied formula to determine the area inside the circle. The following figure shows an example of students’ written work.

Generally, they tried to determine the number of grids using their own way which is counting the complete grids and estimate the incomplete grids. However, what they do by applying the formula of
the area of a circle indicates that they wanted to make sure about the accuracy of their estimation. Thus, in the class discussion, the students could see that the results of their work and the results of the formula were about the same. Furthermore, it could be a starting point for exploration about the formula of the area of a circle in the next lessons.

4.2. Reshaping Sectors

The mathematical idea in this activity is that the area of a circle equals the radius times half of its circumference. There was only one main activity in this meeting namely reshaping three circles which were cut into different numbers of sectors into rectangles and determining the way to find its area. However, to make the students get the idea of reshaping the sectors of a circle into a rectangle, firstly there was an activity of reshaping a circular garden into a shape resembles parallelogram which was guided by the teacher. In the second problem, the students were given three circle which are divided into 8, 12, and 16 sectors, respectively. The students were asked to reshape the circle into a parallelogram, then into a rectangle. The students worked in the same group as they did in the previous lessons.

After reshaping all three circles, the students could see that the shapes the formed resemble the shape of parallelogram. However, some students did not get the idea how to reshape the parallelogram into a rectangle. Hence, a discussion which lead by the teacher help them to get the shape of rectangles.

As it was expected in the HLT, the students could determine that the area of the rectangle which is length times width. In this activity, they can clearly see that the width of the rectangle is the radius of the circle. However, it was difficult for some students to see the relation between the length of the rectangle and parts of the circle. Hence the teacher and the researcher gave them kinds of guidance as shown in this following transcript.

Transcript 1. Discourse about the problem of circumference

\[ R : \text{How many sectors is this circle being cut?} \]
\[ S1 : \text{Twelve.} \]
\[ R+S : \text{One-two-three-four-five-six.} \]
\[ (\text{The researcher is pointing the arcs of the sectors of the circle which are formed a rectangle).} \]
\[ R : \text{How about this one? Which part of a circle is this?} \]
\[ S2 : \text{Circumference.} \]
\[ R+S : \text{One-two-three-four-five-six.} \]
\[ (\text{The researcher is pointing the arcs of the sectors of the initial circle).} \]
\[ S2 : \text{Half of its circumference.} \]

The teacher asked the students to look at the initial circle and to see that the length of all arcs of the twelve sectors is the circumference of the circle. Afterwards, together with the researcher, the students pointed out the arcs of six sectors and a students realized that it is half of the circumference. This fragment indicates that the students could see that the length of the rectangle equals half of the circumference of a circle. However, there were no depth discussion to check whether all students really understand this concept after a student came up with this idea.

In the end of the lesson, the teacher asked a group to share their work in front of class. There were two important conclusions in this lesson. Firstly, the students could conclude that the more number of sectors formed from the initial circle, the more similar the rectangle they formed. Secondly, the area of the rectangle can be determined by multiply the length and the width. The length is eight times the length of arcs and the width is the radius of the circle. In this case, the students who share their work in front of class did not state the length of the rectangle as half of circumference. Hence, the teacher posed a question to the classroom as described in Transcript 2.
Transcript 2. Classroom discussion about the problem of circumference

R: Your friend said that the length of the rectangle is eight times the length of each arc, do you all agree with his statement?
S: Yes (the students answered together, yet some of them just mumbling because the other said yes).
R: If you say yes, can you explain how to determine the length of the arcs?
S: Mmm, firstly we find the length of an arc, then… (The student get stuck)
R: How about the other? Who have another idea?
S2: The length of the rectangle is half of its circumference Miss, because there are eight arcs and the total is sixteen.

Based on the fragment and the students’ written works, we may conclude that some students can determine the area of the rectangle as radius times half of circumference. However, some students still confuse to determine the length of the rectangle and think that it is eight times the length of the arcs without any ideas to find the length of the arcs. However, the discussion help them to understand about this concept.

5. Concluding Discussion

Based on the analysis of several mathematics textbook for junior high school [9–13], the contexts provided in those books are only as attributes. It is because there is still a gap from the informal level to the formal level. In other words, in this case, contexts do not play essential role and can be dropped without affecting the mathematics required [5]. Therefore, the use of models plays important roles in bridging the informal level and the formal level [18]. Furthermore, based on the previous study [19], most of textbook that used in Indonesia contain mainly the set of rules and algorithm which is already formal and lack of applications which are needed by students in order to make the concept be real for them. The learning process only emphasized the procedures to solve the problems and there is insufficient attention for students’ reasoning in order to develop their understanding.

In this study, together with the contextual problems, the grid paper and reshaping sectors, which used as the models in this learning, helped the students to gradually develop their understanding of the area of a circle. The grid papers plays important role in not only in estimating areas as expected in the HLT, but also in comparing figures in the beginning activity. This is in line with the theory of calculation using unit of measurement [5]. Moreover, it could be the tool for promoting the informal theory of area measurement [20].

Whereas, the reshaping sectors may support students understanding of the circumference and the area measurement of circles. It is in line with the theory of area conservation and could be the means to support the informal theory of circumference and area measurement [5,20].

However, the students are sometimes coming back to the formal way in determining the area of a circle. Therefore, this study can be done in the very beginning students learn about the area of a circle which is in the sixth grade of elementary school. Yet, we should consider the level of difficulties on the problems designed for the elementary school students.

Moreover, the contexts which used in the problems on each activity were familiar for the students, especially garden and tiling context. However, it is necessary to expand the variation of the context so that the students do not limit the application of the area of a circle just in certain context. Thus, the other suggestion for the next study is give some more contexts as the enrichment material to the students.

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