Strong CP, Up-Quark Mass, and the Randall-Sundrum Microscope

Hooman Davoudiasl* and Amarjit Soni†
Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA

In the Randall-Sundrum model, setting the ratio of up and down quark masses \( m_u/m_d \ll 1 \), relevant to the strong CP problem, does not require chiral symmetry or fine-tuning, due to exponential bulk fermion profiles. We point out that such geometric suppression of the mass of a fermion magnifies the masses of its corresponding Kaluza-Klein (KK) states. In this sense, these KK states act as “microscopes” for probing light quark and lepton masses. In simple realizations, this hypothesis can be testable at future colliders, like the LHC, by measuring the spectrum of level-1 KK fermions. The microscope can then provide an experimental test for the vanishing of \( m_u \) in the ultraviolet, independently of non-perturbative determinations, by lattice simulations or other means, at hadronic scales. We also briefly comment on application of our microscope idea to other fermions, such as the electron and neutrinos.

The Standard Model (SM) has been extremely successful in describing observed electroweak and strong phenomena. In the SM, the Higgs condensate, \( \langle H \rangle \simeq 250 \text{ GeV} \), sets the electroweak scale and QCD dynamics generates a scale \( \Lambda_{\text{QCD}} \simeq 1 \) GeV for strong interactions. The only other known interaction is gravity that is assumed to be governed by the Planck scale \( \mathcal{M}_P \sim 10^{19} \text{ GeV} \). Despite its remarkable success, the SM leaves a number of intriguing questions unanswered. Of these, three long-standing ones are the hierarchy problem, the flavor puzzle, and the Strong CP Problem (SCPP). The Randall-Sundrum model was initially proposed to resolve the first of the above problems. Further work showed that the second problem can also be addressed in this model.

A vanishingly small up-quark mass \( m_u \), an economical solution of the SCPP, can be naturally accomodated in the RS background. Our central observation is that, within the RS framework, suppressing the mass of a zero mode (SM) fermion magnifies the masses of its KK states. We will refer to this phenomena as the “RS microscope”. Remarkably, the simplest realizations of this RS resolution of the SCPP can be tested at multi-TeV-scale colliders, such as the LHC. We note that this simple solution seems to be disfavored by the recent lattice results, taken at face value, such as those in Ref. [4]. However, since establishing \( m_u \approx 0 \) at the weak-scale is of crucial importance, a direct weak scale collider test can be very useful. This will avoid the subtle non-perturbative effects inherent in lattice and non-lattice results. We will also outline how the microscope mechanism applies to other light fermions, such as the electron and neutrinos, and can give us insight into the RS flavor structure.

Before outlining the RS formalism relevant to our proposal, we would like to clarify some issues regarding the SCPP and \( m_u \). To do this, we will first give a brief description of the SCPP. In the SM, CP violation in the strong sector is controlled by the parameter

\[
\bar{\theta} = \theta + \arg(\det M_q)
\]

that receives contributions from two seemingly unrelated sources: \( \theta \) which characterizes the gauge invariant QCD vacuum, and the quark mass matrix \( M_q \). Therefore, absent any compelling physical principle, the generic expectation is \( \bar{\theta} \sim 1 \). However, this parameter enters in the expression for the neutron electric dipole moment \( \bar{\theta} \) and experimental bounds require \( \bar{\theta} \lesssim 10^{-10} \). The reason for the smallness of this parameter remains unknown, which is referred to as the SCPP. Physics beyond the SM, such as the Peccei-Quinn mechanism may explain smallness of \( \bar{\theta} \). However, such proposals often require high scale model-building whose key components cannot be directly probed at colliders in the foreseeable future.

Remarkably, there is a well-known solution to the SCPP problem that does not require any physics beyond the SM, as suggested by Eq. [4]: if there is a massless quark, \( \bar{\theta} \) can be rotated away [10]. In fact, it suffices that the quark mass \( m_q \) is such that \( 3m_q \lesssim 10^{-12} \) GeV [15]. Given the spectrum of the SM quarks, the only potential choice is \( q = u, \) the up-quark. Obviously, this could only be viewed as a resolution if one can explain why \( m_u \) is vanishingly small. The traditional way of addressing this question is to assume that there is a chiral symmetry that prevents \( u \) from obtaining mass. However, this symmetry will be anomalous under QCD interactions and hence is not really a symmetry. One is then forced to think of this symmetry as accidental and introduce the necessary structure at scales far above the SM that will result in the desired outcome. However, as we will explain below, the RS scenario accommodates a natural alternative mechanism for getting very small fermion masses.

At this point, we would like to address how setting the ratio of up and down quark masses \( m_u/m_d \ll 1, \) required to resolve SCPP, can be reconciled with the expectation

\[1 \quad \text{This is also the case for the Peccei-Quinn solution.}\]
that \( m_u/m_d \lesssim 0.3 - 0.5 \) at low energies of order \( \Lambda_{\text{QCD}} \). To see this, note that to solve the SCPP, it is sufficient to have \( m_u \to 0 \) at some scale above \( \Lambda_{\text{QCD}} \). Below this scale, there is an additive non-perturbative contribution to the real part of \( m_u \) \cite{12,13,14}, due to QCD instantons, that generates \( m_u \neq 0 \) at \( \Lambda_{\text{QCD}} \), even if we set \( m_u = 0 \) at the cutoff scale \( \Lambda \sim 1 \) TeV. This contribution has a form similar to that of the Kaplan-Manohar ambiguity \cite{14}, but is physically of a different origin \cite{13,14}. We have

\[
\frac{m_u}{m_d} \simeq \frac{1.15 m_s}{12} \int_0^{\rho_0} d\rho \left( \frac{8\pi^2}{g(\rho)^2} \right)^6 e^{-\frac{8\pi^2}{g(\rho)^2}},
\]

(2)

where \( m_s \) is the strange quark mass, \( g(\rho) \) is the strong coupling constant at scale \( 1/\rho \), and \( \rho \) is the instanton size. In the above formula, the integration has an infrared cutoff at \( 1/\rho_0 \). For 3 flavors, the 1-loop expression

\[
g^2/(8\pi^2) = -1/[9 \ln(\rho \Lambda_{\text{QCD}})]
\]

(3)

is assumed in the above.

Recent lattice results, like those of Ref. \cite{6}, have constrained the efficiency of the above mechanism for non-perturbative generation of \( m_u \). Ref. \cite{6} gives, at 2 GeV, \( m_u/m_d = 0.43(0)(1)(8) \), where the errors are from statistics, simulation systematics, and electromagnetic effects, respectively. If we take \( m_u = 100 \) MeV, \( \Lambda_{\text{QCD}} = 350 \) MeV, and \( 1/\rho_0 = 2 \) GeV, then we find \( m_u/m_d \simeq 2 \times 10^{-3} \), which is far from the lattice result. However, if we choose \( m_u = 140 \) MeV, \( \Lambda_{\text{QCD}} = 500 \) MeV, and \( 1/\rho_0 = 1.5 \) GeV, we find \( m_u/m_d \simeq 8 \times 10^{-2} \), which is only a factor of 5 away from the lattice result.

We do not advocate the latter values of parameters as more motivated; we only use them to show the sensitivity of the instanton result for \( m_u/m_d \) to low energy parameters. Also note that the instanton density used in the above estimates can be subject to corrections by factors of \( O(1) \), due to non-trivial QCD vacuum structure \cite{3,12}. Finally, we point out that the lattice results are reported at a scale which is well above the charm mass \( m_c \approx 1.3 \) GeV. However, the lattice simulations do not include charm, a light state at 2 GeV. We do not speculate on the effect of including charm in the simulations. Nonetheless, we expect that this omission is a source of systematic uncertainty and may change the current lattice estimates.

Clearly Ref. \cite{6} is a very serious and careful lattice study of several of the important low energy constants and quark masses. We want to stress that our concern in here is only with regard to \( m_u \). The notable point for \( m_u \) is that \( m_u/\Lambda_{\text{QCD}} \approx 5 \times 10^{-3} \). Thus even small systematic effects may be quite relevant; charm quark above is just one example. In addition, use of NNLO-type chiral perturbation theory (rather than the full NNLO) fits, e.g., may also be another source of small systematics that could be relevant to \( m_u \). We note also that in fact authors of Ref. \cite{6} themselves were careful enough to caution that for this delicate issue verification by another discretization method is desirable. However, from our perspective it is just not clear that methods on \cite{12}, or off \cite{19}, the lattice have all the rigor that is needed to decide this critical issue.

Furthermore, it also appears that any extraction of \( m_u/m_d \) based on hadronic spectra is bound to be relevant at energies below 1 GeV. If we set \( 1/\rho_0 = m_\rho \approx 0.77 \) GeV, in Eq. (2), using \( m_s = 100 \) MeV, \( \Lambda_{\text{QCD}} = 350 \) MeV, we find \( m_u/m_d \approx 0.34 \), which is a reasonable value. Given the sensitivity of theoretical results from the instanton calculations and possibility of small uncontrollable systematic effects, and given also that the possibility of \( m_u \sim 0 \) is of fundamental importance, having a completely independent handle on \( m_u/m_d \) at the weak scale, where it could resolve the SCPP, is very desirable. With all this in mind, in what follows, we will outline a possible collider test of this hypothesis, in the framework of RS resolution of the gauge and flavor hierarchies.

The RS model explains the ratio \( (H)/M_P \sim 10^{-17} \), by gravitationally red-shifting the 5-d fundamental scale \( M_5 \sim M_P \) down to the TeV-scale, along the warped 5th dimension. This geometry is based on a slice of AdS_5, truncated by flat 4-d boundaries often referred to as the Planck (UV) and TeV (IR) branes. The RS metric is given by

\[
d^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2,
\]

(4)

where \( \sigma = k r_c |\phi| \), \( k \) is the 5-d curvature scale, \( r_c \) is the radius of compactification, \( -\pi \leq \phi \leq \pi \), and a \( Z_2 \) orbifolding of the 5th dimension is assumed.

To solve the hierarchy problem, the Higgs is assumed to be localized near the TeV-brane, where the reduced metric “warps” \( (H)_{5} \sim M_5 \) down to the weak scale: \( \langle H \rangle_4 = e^{-kr_c}\sigma \langle H \rangle_5 \). For \( kr_c \approx 12 \), we then get \( (H)_{\text{SM}} \approx \langle H \rangle_4 \sim 1 \) TeV. Originally, it was proposed that all SM content resides at the IR-brane. However, as the cut-off scale in the 4-d effective theory is also red-shifted to near the weak-scale, this would lead to unsuppressed higher dimensional operators that result in large violations of experimental bounds on various effects, such as those on flavor-changing neutral currents. This problem can be solved by realizing that points along the warped 5th dimension correspond to different effective 4-d scales. In particular, by localizing first and second generation fermions away from the IR-brane, the effective scale that suppresses higher dimensional operators made up of these fields is pushed to much higher scales \cite{6}. In the process of suppressing the dangerous operators, this setup also leads to a natural mechanism for obtaining small fermion masses.

The above localization is achieved by introducing a 5-d mass term in the bulk for each fermion field \cite{6}. Let \( c = \mu/k \), where \( \mu \) is the 5-d mass of the fermion. Each 5-d fermion \( \Psi \) has left- and right-handed components \( \Psi_{L,R} \) which can be expanded in Kaluza-Klein (KK) modes

\[
\Psi_{L,R}(x, \phi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)}(x) e^{2\sigma n} f_{L,R}^{(n)}(\phi).
\]

(5)
The KK wavefunctions \( f^{(n)}_{L,R} \) are orthonormalized
\[
\int d\phi e^{\sigma} f^{(m)}_{L,R} f^{(n)}_{L,R} = \delta_{mn}.\tag{6}
\]

One can then show that the \( n \neq 0 \) modes are given by
\[
f^{(n)}_{L,R} \propto e^{\sigma/2} Z_{nL,R}^{\pm}(z_{nL,R}) \theta,\]
where \( Z_{nL,R}^{\pm}(z_{nL,R}) \) is a linear combination of Bessel functions of order \( a_{nL,R} \) is a linear combination of Bessel functions of order \( a, z_{nL,R} \equiv (m_{n}/k)e^{\sigma} \) and \( m_{n} \) is the KK mass. The zero-mode wavefunction is given by
\[
f^{(0)}_{L,R} = \frac{e^{\mp\sigma}}{N_{L,R}^{D,S}}.\tag{7}
\]

with the normalization
\[
N_{L,R}^{D,S} = \left[ \frac{e^{kr_{c}}(1 \mp 2\epsilon) - 1}{kr_{c}(1/2 + \epsilon)} \right]^{1/2}.\tag{8}
\]

In the SM, all \( SU(2) \) doublets are left-handed, while the singlets are right-handed. Hence, one has to impose a \( Z_{2} \) parity on bulk fermion fields so that only the doublets have left-handed zero modes and only the singlets have right-handed zero modes. However, both the doublets and singlets have left- and right-handed higher KK modes. Note that in our convention, the singlet (right-handed) and doublet (left-handed) zero mode wavefunctions are defined with opposite signs for mass parameters \( c^{S} \) and \( c^{D} \), respectively. This is consistent with our normalization conventions for the wavefunctions.

The above profiles provide a mechanism for suppression of higher dimensional operators involving light fermions, as mentioned before. For example, consider a 4-fermion operator \( O_{4\Psi} \) made up of \( \Psi_{i}, i = 1, \ldots, 4 \), in the 5-d theory
\[
O_{4\Psi} = \frac{\Psi_{1}\Psi_{2}\Psi_{3}\Psi_{4}}{M_{5}^{4}},\tag{9}
\]
with a coefficient of unity. After dimensional reduction, the component of \( O_{4\Psi} \) that is made up of the zero modes (SM fields) is given by
\[
O_{4\Psi} = \frac{\psi_{1}\psi_{2}\psi_{3}\psi_{4}}{\Lambda_{4}},\tag{10}
\]
where the 4-d cutoff scale \( \Lambda_{4} \) is given by
\[
\Lambda_{4} = \sqrt{\frac{M_{5}^{2} k r_{c}^{2} (4 - \sum_{a} a_{a} c_{a}) \prod_{i} N_{i}^{W}}{2 c^{(4 - \sum_{a} a_{a} c_{a}) k r_{c}}} - 1}.\tag{11}
\]

In deriving the above, the wavefunctions given by Eq. (10) have been used, and \( a_{a} = \pm 1 \) for \( (L, R) \) choices, respectively. The scale \( \Lambda_{4} \) can be much higher than the weak scale for light SM fermions.

The above fermion profiles also lead to a natural scheme for SM fermion masses \( m_{u,d,s} \). To see this, we will next examine the Yukawa interactions between fermions and the Higgs field. We will assume that the Higgs is on the IR-brane; this is a very good approximation since the Higgs must be highly IR-localized. Then, a typical Yukawa term in the 5-d action will take the form
\[
S_{Y}^{5} = \int d^{5}x \ 2 \sqrt{-g} \lambda_{5}^{D,S} H(x)\Psi_{L}^{D,S} \Psi_{R}^{D,S} \delta(\phi - \pi),\tag{12}
\]
where \( \lambda_{5}^{D,S} \) is a dimensionless 5-d Yukawa coupling and \( \Psi_{L}^{D,S} \) are doublet left- and singlet right-handed 5-d fermions, respectively. After the rescaling \( H \rightarrow e^{\sqrt{kr_{c}} \sigma} H \), the 4-d action resulting from Eq. (12) is
\[
S_{Y}^{4} = \int d^{4}x \ 2 \sqrt{-g} \lambda_{4} H(x)\psi_{L}^{D} \psi_{R}^{S} + \ldots,\tag{13}
\]
where the 4-d Yukawa coupling for the corresponding zero-mode SM fermion is given by
\[
\lambda_{4} = \frac{\lambda_{5}^{D,S}}{\Lambda_{4}^{2}},\tag{14}
\]
with \( c^{D,S} \) denoting the 5-d mass parameters for \( \Psi_{L}^{D,S} \). Thus, in the quark sector, there are, in general, 9 different values for \( c^{S} \): 3 for the doublets and 6 for the singlets. One can see that the exponential form of the effective Yukawa coupling \( \lambda_{4} \) can accommodate a large hierarchy of values without the need for introducing unnaturally small 5-d parameters.

Thus, the RS background provides a geometric alternative to the requirement of a chiral symmetry: fermion masses far below the weak scale can be naturally obtained, without the need for tiny 5-d Yukawa couplings. In fact, this mechanism was originally proposed as a way of obtaining realistic neutrino masses \( \lambda_{5}^{D} \). If the \( SU(2) \) singlet 5-d up-quark, corresponding to the right-handed up-quark of the SM, is localized near the UV-brane, the resulting zero mode mass \( m_{u} \) will be highly suppressed. We will see that setting \( m_{u} \lesssim 10^{-12} \) GeV at the weak scale in this context only amounts to choosing \( c^{S} \sim 1 \) and does not require fine-tuning of any underlying 5-d parameters. Therefore, it is natural to have \( \theta \lesssim 10^{-10} \) in the RS model with bulk SM content!

A very interesting feature of the above scenario for obtaining \( m_{u} \ll \text{MeV} \) is that it is testable at multi-TeV scale colliders, like the LHC. The values of light quark masses \( (u, d, s) \), as well as those of light leptons, are not observable in high energy collider experiments. However, in the simplest models based on the above RS flavor scheme, the spectrum of the KK states associated with SM fermion of flavor \( \alpha \) encodes the information about the bulk mass parameter \( c_{\alpha} \), at every KK level. Hence, in principle, measuring all the first KK fermion masses, and perhaps a few other quantities, will provide a complete map of the bulk mass parameters \( c_{\alpha} \). This, in turn can lead to a determination of the relative sizes of the “hard” masses of zero mode quarks, at the weak scale and, in particular, the Lagrangian parameter \( m_{u} \). We will see in the following that a generic outcome of this picture is a weak-singlet \( u \)-quark first KK state that is substantially heavier than those associated with the other quarks.
In passing, we point out that the SCPP can also be generated by higher dimensional operators in the RS context. To see this, consider the 5-d operator

$$\mathcal{O}_5 = \frac{HH^1 \varepsilon^{LMNPQ} G_{LM} G_{NP} \hat{Q} |_{\phi=\pi}}{\Lambda_{5}^2},$$

(15)

with $\hat{Q} = (0, 0, 0, 0, 1)$ and $G_{MN}$ the 5-d gluon field strength. $\mathcal{O}_5$ can be written at $\phi = \pi$, since 5-d Lorentz invariance is broken by the IR-boundary. In the 4-d effective theory, the above operator yields

$$\mathcal{O}_{\text{4CP}} \sim \frac{HH^1 G_{\mu\nu} \tilde{G}^{\mu\nu}}{\Lambda^2},$$

(16)

where $\tilde{G}^{\mu\nu}$ is the dual gluon field strength tensor, leading to an effective $\theta$ parameter after electroweak symmetry breaking, $(H) \neq 0$: $\mathcal{O}_{\text{4CP}} \rightarrow \theta_{eff} G_{\mu\nu} \tilde{G}^{\mu\nu}$. For $\Lambda \lesssim 10$ TeV, we get $\theta_{eff} \gtrsim 6 \times 10^{-4}$ which is at least 7 orders of magnitude too big.

The 5-d SM scenario in the RS background is subject to various experimental constraints, including those from precision electroweak and flavor data. A number of models with custodial $SU(2)_L \times SU(2)_R$ bulk symmetry have been proposed to address these constraints [18, 20, 21]. In the following, we will limit the scope of our study to SM fermion masses without specifying a particular framework for such constraints. We note that since we will treat $u_R$ and $d_R$ of the SM differently, models in which the weak singlets $(u, d)$ form an SU(2)$_R$ doublet cannot be used to solve SCPP, as proposed in our work. However, models with split doublets, such as that of Ref. [19] can be used in our setup. We will not discuss the phenomenology of the extra exotics in these models, as they do not change the qualitative picture for the SM KK partners that we present here. This will suffice to demonstrate our key observations. For a study of possible light exotic quarks in some warped scenarios see Ref. [22].

For a detailed analysis of flavor physics in warped models with bulk custodial symmetry see Ref. [23]. Here, we also note that for $m_{KK} \sim 3$ TeV and for generic $O(1)$ phases in the RS bulk, the resulting neutron EDM is $O(20)$ too large [23, 24]. This feature may require additional model building or tuning at the $O(10^{-1})$ level. However, we only focus on the $O(10^{-10})$ tuning (SCPP) that is posed at the SM level, even if we decouple new physics.

We will adopt a simple setup in which the Higgs field and the right-handed top quark are localized on the IR-brane. For simplicity and in order to have a transparent treatment of key features, we will assume a flavor-digonal bulk mass matrix. We will then choose the profile parameters $c_\alpha$, $\alpha = 1, 2, \ldots, 8$, to get a realistic SM fermion mass spectrum. To fix the warp factor at a specific value, we will assume that $\langle H \rangle_5 = M_P \approx 1.2 \times 10^{19}$ GeV. The relation $\langle H \rangle_4 = e^{-kr_c} \langle H \rangle_5$ then yields $kr_c \simeq 12.2$.

In Table I, we provide a reference set of values for $c_\alpha$ that result in realistic SM quark masses. We have assumed a somewhat large value for the 5-d top Yukawa coupling, $\lambda_5^t = 3.1$. All other 5-d Yukawa couplings are set to unity, as expected in a “natural” model. We only use the values of $c_u$ as a guide for our analysis. In practice, there are modest modulations in the profiles and the values of $\lambda_5^t$, depending on the details of a warped scenario. However, we do not expect large deviations from these typical numbers in generic cases. We have also ignored the effect of doublet-singlet KK-fermion Yukawa couplings, which result in a small $O(\langle H \rangle/m_{KK})$ shift in the masses [25]. We will comment on this later. We will not present a detailed numerical analysis, as we are only interested in the most general aspects of this scenario for addressing the SCPP. Next, we will outline the collider phenomenology of SM KK fermions in our proposal.

In the following, we will focus on KK quarks. However, one expects that these signals are accompanied by others, for example from the discovery of the KK gluon [26] or the KK graviton [27] of the RS model. Thus, the value of $kr_c$ for the warped background can in principle be assumed to be known. Since the objects in which we are interested have color charge, we will consider the quark-sector coupling to the gluon-sector first. In the rest of this work, by “KK mode” we mean the first level mode, unless otherwise specified.

In our setup, which we take as typical, the zero modes couple as in the SM, by gauge invariance. The same is true for KK-quark coupling to the SM gluon. However, the coupling of quarks to KK gluons is modified compared to the SM. The 5-d action for the coupling of the bulk fermion $\Psi$ to the gauge field $A_M$ is given by

$$S_{\Phi A} = g_5 \int d^4x \, d\phi V \left[ V_i^M \bar{\Psi} \gamma^i A_M \Psi \right],$$

(17)

where $g_5$ is the 5-d gauge coupling, $V$ is the determinant of the finnebein $V_i^M$, with $l = 0, \ldots, 3$, $V_3^M = e^\phi \delta_3^M$, and $V_4^l = -1; \gamma^i = (\gamma^\lambda, i\gamma^5)$. The $A_M$ field can be expanded

| Quarks | $c^D$ | $c^S$ | $m_u$(SM) (GeV) | $m_u^{KK}/m_u$ |
|--------|-------|-------|-----------------|-----------------|
| $u$    | 0.5   | -1.4  | $(3.5 \times 10^{-14})$ | 1.0, 1.5        |
| $d$    | 0.5   | -0.8  | $(4.8 \times 10^{-3})$ | 1.0, 1.1        |
| $c$    | 0.5   | -0.53 | 1.2             | 1.0             |
| $s$    | 0.5   | -0.61 | 0.11            | 1.0             |
| $t$    | 0.4   | -0.52 | 170.6           | 1.0, 1.0        |
| $b$    | 0.4   | -      | 4.1             | 1.0             |
in KK modes as

\[ A_\mu = \sum_{n=0}^{\infty} A^{(n)}_\mu(x) \chi^{(n)}(\phi) / \sqrt{r_c}. \]  

(18)

The gauge field KK wavefunctions are given by \( \chi^{(n)}_A \propto e^{iZ_1(z_n)} \) \( \mathbb{R}^3 \), subject to the orthonormality condition

\[ \int_0^\infty d\phi \chi^{(n)}(\phi) \chi^{(m)}(\phi) = \delta_{mn}. \]  

(19)

Dimensional reduction of the action (17) then yields the couplings of the fermion and gauge KK towers. With our conventions, the 4-d gauge coupling is given by \( g_4 = g_5/\sqrt{2\pi r_c} \).

Because of UV-brane localization, the coupling of two zero mode singlets to a KK gluon is suppressed, by approximate volume suppression of order \( (kr_c)^{-1} \). The singlet b-quark also has suppressed coupling to KK-gluons, due to “accidental” orthogonality, since for \( c \approx 0.5 \) the overlap integral is approximately of the form in Eq. (19), with one fermion replacing a constant gauge zero mode. This feature also applies to the zero mode doublets (less so for doublet \( b \)), as they have \( c \approx 0.5 \) as well. The coupling to the singlet top-quark is substantial, since it is an IR-brane field. The couplings of light quark singlet zero modes to their KK counter parts and a KK gluon is also suppressed, since the KK-modes are UV-localized, but the singlets are IR-localized. This same coupling is \( \mathcal{O}(1) \) for the singlet b-quark and possibly for the light quark doublets, as they may not be highly UV-localized.

Let us now consider the coupling of the quarks to the Higgs. Zero mode quarks couple to the Higgs with the usual Yukawa strength which is mostly negligible, except for the top quark. Next, we examine the couplings of a KK quark to a zero mode quark and the Higgs. The strength of this coupling can affect the branching ratios of the KK quarks. The coupling of a singlet zero mode to a KK doublet and the Higgs is suppressed to \( \mathcal{O}(10^{-2}) \) or less, due to the UV-localization of the zero mode. This coupling is, however, \( \mathcal{O}(10^{-1}) \) for the b-flavor. The coupling of the singlet KK mode to a doublet zero mode and the Higgs is also \( \mathcal{O}(10^{-1}) \), since none of these fields are UV-localized.

Finally, we expect that coupling of 2 KK-quarks to the Higgs to be \( \mathcal{O}(1) \), for the same reason. Numerically, using the values in Table I, we find this to be the case. Here, we note that this coupling generates an off-diagonal element in the KK mass matrix, which leads to the mixing, as well as the splitting \( \Delta_q \simeq \sqrt{H}/\sqrt{2} \), of the doublet and singlet KK quarks \( \mathbb{R}^3 \). The lower bound from precision data on gauge KK mode masses is in the 2-3 TeV range \( \mathbb{R}^3 \). Given that the quark KK modes are expected to be heavier than the gauge modes, the splitting \( \Delta_q \) is a \((5-10)\%\) effect. Also note that our key signal, a heavy (up-flavor) singlet KK mode, gives a 50\%, or larger, effect. Hence, the off-diagonal couplings do not affect our conclusions significantly. In the following discussion, we will then use the interaction basis, not the mass basis. For 5-d Yukawa couplings smaller than unity, the goodness of this approximation improves.

Given the above analysis, we generically expect that the KK quarks are produced in pairs via \( q\bar{q} \) annihilation. This production mechanism can provide a multi-TeV reach for the KK quark modes, as deduced from collider studies of gluino production in supersymmetric models \( \mathbb{R}^3 \). However, depending on the model, other production mechanisms may also be available.

In order to formulate a search strategy, we must consider the most likely decay modes of each KK quark. The doublet KK modes, as well as the singlet b KK mode, decay into a zero mode counterpart and a KK gluon with relatively significant branching fractions. This is possible since all KK quarks are more massive than the KK gluon, and the emission of zero mode quark is a negligible kinematic constraint. However, such decay modes will be followed by the subsequent decay of the KK gluon, which will then dominantly decay into right-handed top quarks, in our setup. Thus, for KK doublet quark \( q_{KK}^D \), we get the decay chain

\[ q_{KK}^D \rightarrow g_{KK} \rightarrow t_{R}\bar{t}_R \, q^D, \]  

(20)

where \( q^D \) is the zero mode doublet. The above \( t_{R}\bar{t}_R + j \) signal can be used to reconstruct the mass of the original KK quark. Note that the decay mode \( q_{KK}^D \rightarrow H \, q^S \) is not relevant here, due to its suppressed rate.

The decay mode in (20) will not be relevant for measuring the singlet KK quark masses due to its small branching fraction. Since we would like to use the RS “microscope” to get a handle on \( m_{u} \), we need to find a signal for the decay of a singlet KK quark \( q_{KK}^s \). Here, the dominant mode is

\[ q_{KK}^s \rightarrow q_{KK}^D H, \]  

(21)

where we generally expect the KK singlets to be heavier than the KK doublets and the above decay is allowed on-shell. However, this mode will lead to a subsequent decay chain given in (20), which will require a complicated event reconstruction. Hence, we may consider the subdominant, but more transparent decay mode

\[ q_{KK}^s \rightarrow H \, q^D. \]  

(22)

Then, one can use the \( H + j \) signal to measure the mass of the parent KK singlet. As noted before, we expect the corresponding mode to be suppressed for \( q_{KK}^D \) and hence this is a signal for the singlet KK modes. Based on our scenario for getting \( m_{u} \ll \text{MeV} \), we then expect that one of the reconstructed KK masses in this channel will be significantly larger than the rest, as can be seen from Table I.

To test the localization hypothesis, one must get a handle on the relation between the KK masses and the zero mode masses. For example, by measuring the masses of
the doublet and singlet KK $b$-quark, as outlined above, one may infer the profile parameters $c_{\text{s},\text{D}}$. Note that since the relevant decay modes for KK $b$-quarks in our setup are \( b_{\text{KK}}^D \to t_{\text{R}} \bar{t}_L b^D \) and \( b_{\text{KK}}^S \to H^D b^D \), $b$-tagging can single out the parent KK particle. As another reference point, we can take advantage of the microscopic effect in the lepton sector. In particular, the electron is light enough that the effect for its first KK mode may be significant. Assuming $c^D \simeq 0.5$ for the doublet electron, we need $c^D \simeq -0.76$ to achieve a realistic electron mass. In this case, the ratio of the first KK masses of the doublet and the singlet electrons to that of a KK gauge field will be 1.0 and 1.14, respectively.

Such a mass difference is likely measurable. This measurement may, however, require a multi-TeV $e^+e^-$ collider. As in the quark sector, we expect that the $e_{\text{KK}}^D$ will decay into a zero mode doublet $e$ and a KK gauge field, i.e., the KK mode of a photon: \( e_{\text{KK}}^D \to A_{\text{KK}} e^D \). On the other hand, for the singlet KK mode, $e_{\text{KK}}^S \to H e^D$ is likely a reasonable decay mode to study. Quantitative statements about the above measurements in both the lepton and the quark sectors is outside the scope of this work.

Agreement of the above KK masses with the corresponding zero mode localization, as inferred from Eq. (4), will provide a test of the RS microscope and flavor generation mechanism. For the light quark flavors, in the simple small-doublet-singlet-mixing scenario we are considering, the events that reconstruct to $H + j$ will correspond to the singlets, while the doublets will decay into $t_{\text{R}} \bar{t}_R j$ and tightly cluster around a mass value corresponding to $c \simeq 0.5$. Once this is established, the observation of a singlet KK quark (in the $H + j$ channel) with a substantially larger mass than the rest, will point to extreme localization for that flavor.

This will provide collider evidence for a very small up quark mass, as it is the only flavor for which this assumption is feasible. Since, in our scenario, the required mass difference between $u_{\text{KK}}^S$ and the other KK masses is typically very large, the deduced $c_u$ parameters can yield the approximate size of the $m_u$ in the effective Lagrangian. If the deduced mass is sufficiently small, it will provide evidence for a resolution of the SCPP based on an “accidentally” tiny $m_u$. This will be a probe of the high scale value of $m_u$, independently of the low energy lattice results.

In the above, we discussed the key collider phenomenology of the setup presented in our work. However, we expect that the extreme localization of the singlet $u$-flavor will also have other possible ramifications. For example, higher dimensional operators that contain the $u_R$ of the SM will be highly suppressed in this scenario, since the localization acts as an effective chiral symmetry. We will not analyze here the consequences of the suppression of these higher dimensional operators for low energy experiments.

Before closing, we would like to outline how the microscope mechanism can also be useful in other sectors; we will consider neutrino physics as an example. Currently, neutrino oscillation experiments require at least two massive neutrinos, with $\delta m^2_{\text{ atm}} \sim 10^{-3}$ eV$^2$ and $\delta m^2_{\text{sol}} \sim 10^{-4}$ eV$^2$ [22], from solar and atmospheric data, respectively. However, the third neutrino is still allowed to be massless, according to present data. It would be very challenging to settle this question by future low energy measurements of the neutrino mass scale. For example, the Katrin $\beta$-decay experiment will be sensitive down to $m_\nu \sim 0.2$ eV [23]. We will next consider how the RS microscope can help.

Let us assume that neutrinos are Dirac fermions. In that case, a natural way to suppress neutrino masses would be by localizing singlet neutrino $\nu_R$ fields close to the UV-brane, as first suggested in Ref. [2]. In order to address the data, we at least need to generate $m_\nu \sim \sqrt{\delta m^2_{\text{ atm}}} \sim 5 \times 10^{-2}$ eV. This can be accommodated by choosing $c^D \simeq 0.5$ for the lepton doublet ($\nu_L, \ell$), with $\ell = e, \mu, \tau$, and $c_{1,2}^S \sim -1.2$; the 5-d Yukawa interaction is $(\lambda_5/k) H \ell \nu_R \nu_R$ with $\lambda_5 = 1$. However, if the lightest state is substantially less massive, say, by a factor of $O(m_\nu/m_\tau)$, we will need to consider $c_{3}^S \sim -1.45$. In this case, the level-1 singlet KK modes corresponding to the heavy eigenstates are roughly 10% lighter than the corresponding mode for the lightest neutrino. This is a mass difference large enough that collider experiments can likely measure it.

We do not enter here into a detailed discussion of model-building or collider phenomenology of the neutrino sector. However, we point out that KK modes of the singlet right-handed neutrinos can have substantial couplings to the SM, via their Yukawa interactions. Also, in models with bulk custodial symmetry, the right-handed KK neutrinos can be produced through their interactions with the $SU(2)_R$ gauge sector. These interactions can make it possible for the $\nu_R$ KK states to be produced at the LHC or a multi-TeV $e^+e^-$ collider. Thus, the microscope effect within the RS scenario can in principle shed light on the mass scale of neutrinos, which can be challenging for low energy experiments. A more detailed study of the neutrino microscope is needed for more quantitative conclusions and lies outside the scope of this work.

In summary, the RS model provides a natural setting for vanishingly small $m_\nu$, by fermion localization in the bulk. This can be a potential solution to the SCPP. An interesting feature of this scenario is that it is testable at a multi-TeV-scale collider. This is because the KK quark corresponding to the right-handed up-quark in the SM will get a substantially larger mass than the other KK quarks, at the same level. In principle, one can measure the spectrum of the level-1 KK quarks, say, at the LHC and deduce that one of the light quarks has a very tiny mass. This would be a probe of $m_u/m_d$ that is complementary and independent of the ongoing non-perturbative lattice simulations. The enhancement of KK state masses can also shed light on the spectrum of other light fermions, such as a Dirac neutrino which...
is much lighter than the other two. Here again, collider data can provide a new handle on the neutrino mass scale, independently of low energy $\beta$-decay experiments.

Acknowledgments

We would like to thank M. Creutz, J. Hewett, T. Krupovnickas, W. Marciano, G. Perez, and F. Petriello for discussions. This work was supported in part by the United States Department of Energy under Grant Contracts DE-AC02-98CH10886.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].
[2] Y. Grossman and M. Neubert, Phys. Lett. B 474, 361 (2000) [arXiv:hep-ph/9912408].
[3] T. Gherghetta and A. Pomarol, Nucl. Phys. B 586 (2000) 141 [arXiv:hep-ph/0003129].
[4] There are a variety of other models that discuss $\text{SCP}$ within the RS model. However these models do not consider the vanishing of $m_u$ and rely on extra structure to address the $\text{SCP}$. See, for example Ref. [5].
[5] H. Collins and R. Holman, Phys. Rev. D 67, 105004 (2003) [arXiv:hep-ph/0210110].
[6] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].
[7] Y. Grossman and M. Neubert, Phys. Lett. B 586, 251 (2000) [arXiv:hep-ph/9912408].
[8] T. Gherghetta and A. Pomarol, Nucl. Phys. B 586 (2000) 141 [arXiv:hep-ph/0003129].
[9] There are a variety of other models that discuss $\text{SCP}$ within the RS model. However these models do not consider the vanishing of $m_u$ and rely on extra structure to address the $\text{SCP}$. See, for example Ref. [5].
[10] C. Aubin et al. [MILC Collaboration], Phys. Rev. D 70, 114501 (2004) [arXiv:hep-lat/0407028].
[11] H. Collins and R. Holman, Phys. Rev. D 67, 105004 (2003) [arXiv:hep-ph/0210110].
[12] M. Creutz, Phys. Rev. Lett. 92, 101602 (2004) [arXiv:hep-ph/0308024].
[13] T. Flacke, B. Gripaios, J. March-Russell and D. Maybury, JHEP 0701, 061 (2007) [arXiv:hep-ph/0611278].
[14] C. Aubin et al. [MILC Collaboration], Phys. Rev. D 70, 114501 (2004) [arXiv:hep-lat/0407028].
[15] V. Baluni, Phys. Rev. D 19, 2227 (1979); R. J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. B 88, 123 (1979) [Erratum-ibid. B 91, 487 (1980)].
[16] K. Kawarabayashi and N. Ohta, Prog. Theor. Phys. 66, 1789 (1981); M. Pospelov and A. Ritz, Phys. Rev. Lett. 83, 2526 (1999) [arXiv:hep-ph/9904483].
[17] H. Georgi and I. N. McArthur, HUP 81/A011 (1981).
[18] H. Leutwyler, Nucl. Phys. B 337, 108 (1990).
[19] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP 0308, 050 (2003) [arXiv:hep-ph/0308036].
[20] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005) [arXiv:hep-ph/0412089].
[21] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B 641, 62 (2006) [arXiv:hep-ph/0605341].
[22] C. Dennis, M. K. Ueln, G. Servant and J. Tseng, arXiv:hep-ph/0701158.
[23] K. Agashe, G. Perez and A. Soni, Phys. Rev. D 71, 016002 (2005) [arXiv:hep-ph/0408134].
[24] K. Agashe, G. Perez and A. Soni, Phys. Rev. Lett. 93, 201804 (2004) [arXiv:hep-ph/0406101].
[25] K. Agashe, A. E. Blechman and F. Petriello, Phys. Rev. D 74, 053011 (2006) [arXiv:hep-ph/0606021].
[26] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, arXiv:hep-ph/0612015; B. Lillie, L. Randall and L. T. Wang, arXiv:hep-ph/0701166.
[27] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. 84, 2080 (2000) [arXiv:hep-ph/9909255]; H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. D 63, 075004 (2001) [arXiv:hep-ph/0006041]; A. L. Fitzpatrick, J. Kaplan, L. Randall and L. T. Wang, arXiv:hep-ph/0701150; K. Agashe, H. Davoudiasl, G. Perez and A. Soni, arXiv:hep-ph/0701186.
[28] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. B 473, 43 (2000) [arXiv:hep-ph/9911262]; A. Pomarol, Phys. Lett. B 486, 153 (2000) [arXiv:hep-ph/991129].
[29] M. Carena, E. Ponton, J. Santiago and C. E. M. Wagner, arXiv:hep-ph/0701055.
[30] H. Baer, C. h. Chen, M. Drees, F. Paige and X. Tata, Phys. Rev. D 59, 055014 (1999) [arXiv:hep-ph/9809223].
[31] H. Baer, C. Balazs, A. Belyaev, T. Krupovnickas and X. Tata, JHEP 0306, 054 (2003) [arXiv:hep-ph/0304303].
[32] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.
[33] K. Valerius [KATRIN Collaboration], PoS HEP2005, 166 (2006).