Abstract

It is argued that substantial portions of both Newtonian particle mechanics and general relativity can be viewed as relational (rather than absolute) theories. I furthermore use the relational particle models as toy models to investigate the problem of time in closed-universe canonical quantum general relativity. I consider thus in particular the internal time, semiclassical and records tentative resolutions of the problem of time.
1 Introduction

I consider relational models \[1, 2, 3, 4, 5\] for the universe. These have two features. 1) Temporal relationalism: that there is no physically meaningful primitive notion of time for the universe as a whole. 2) Spatial relationalism: that each notion of space possesses a transformation group \(G\) which does not alter the physical content of the universe. 1) is implemented by considering actions which are invariant under reparametrization of ‘label time’ \(\lambda\) by being homogeneous linear in this. 2) is implemented by these actions being constructed out of objects natural to the configuration space \(Q\) in question and furthermore being corrected by auxiliary variables corresponding to the independent infinitesimal transformations of \(G\). Both of these implementations lead to constraints. In the former this is through the \(n\) momenta subsequently being homogeneous of degree 0 in the velocities and hence functions of at most \(n - 1\) independent ratios of velocities so that the momenta must have at least 1 relation between them (which is by definition a primary constraint).

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In the latter case, one may worry that one is giving further objective existence to the magnitude of relative position variables only.

In Sec 2, I consider \(Q\) to be the space \(Q(N)\) of \(N\)-particle positions and \(G\) to be the group of translations and rotations, \(\text{Eucl.}\). This relational particle model (RPM) is a reformulation of the zero angular momentum portion of Newtonian mechanics, for which I furthermore provide a direct implementation of spatial relationalism. In Sec 3, I explain that general relativity (GR) arises if one considers each of these auxiliaries produces one secondary constraint, which uses up two degrees of freedom, so that one ends up on the quotient space \(Q/G\) of equivalence classes of \(Q\) under \(G\) motions, so \(G\) is indeed rendered physically irrelevant by this indirect means.

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2 Relational Particle Models

Consider a reparametrization-invariant action for particle mechanics with the Euclidean group of motions of flat space rendered irrelevant by passing from particle position coordinates\(^1\) \(q^I\) to \(q^I - \alpha^I - \epsilon^I_{\alpha} b_{\beta} q^I\):

\[
S[q^I, \dot{q}^I, \dot{\alpha}^I, \dot{b}^I] = 2 \int d\lambda \sqrt{T(E-V)}, \quad T = \sum_{I=1}^{N} m_I \delta^\alpha_{\beta}(\dot{q}^I - \dot{\alpha}^I - \epsilon^I_{\alpha} b_{\beta} q^I)(\dot{q}^I - \dot{\alpha}^I - \epsilon^I_{\beta} \dot{b}^I q^I). \tag{1}
\]

This is a re-interpretation of Barbour and Bertotti’s work \[1\]. Then, the momenta are

\[
p^{\alpha I} = \sqrt{\frac{E-V}{T}} m_I \delta^\alpha_{\beta}(\dot{q}^I - \dot{\alpha}^I - \epsilon^I_{\beta} \dot{b}^I q^I). \tag{2}
\]

Reparametrization invariance leads to

\[
H = \sum_{I=1}^{N} \frac{1}{2 m_I} \delta^\alpha_{\beta} p^{\alpha I} p^{\beta I} + V = E \quad \text{(energy constraint)} \tag{3}
\]

as a primary constraint via

\[
\sum_{I=1}^{N} \frac{1}{2 m_I} \delta^\alpha_{\beta} p^{\alpha I} p^{\beta I} = \sum_{I=1}^{N} \frac{1}{2 m_I} \delta^\alpha_{\beta} \sqrt{\frac{E-V}{T}}(\dot{q}^I - \dot{\alpha}^I - \epsilon^I_{\beta} \dot{b}^I q^I) \sqrt{\frac{E-V}{T}}(\dot{q}^I - \dot{\alpha}^I - \epsilon^I_{\beta} \dot{b}^I q^I) = \frac{E-V}{T} = E - V,
\]

while \(\alpha^I\) and \(b^I\) variation lead to secondary constraints

\[
P^\alpha = \sum_{I=1}^{N} p^{\alpha I} = 0, \quad L^\alpha = \sum_{I=1}^{N} \epsilon^I_{\alpha \beta} q^I p^{\beta I} = 0 \quad \text{(0 total momentum and 0 total AM constraints)} \tag{4}.
\]

Furthermore, this RPM is a reformulation of a portion of Newtonian mechanics, one restriction being \(L_0 = 0\).

As elimination of \(\alpha^I\) and \(b^I\) from the Lagrangian form of \(P^\alpha\) and \(L^\alpha\) is possible for this example, the indirectness of the above implementation of spatial relationalism (resolution of absolute versus relative motion debate) furthermore turns

\(^1\)Particle positions are indexed by capital letters running from 1 to \(N\). Relative coordinates are indexed by lower-case letters running over 1 to \(N - 1\). Spatial indices are lower-case Greek letters. \(E\) and \(V\) are the total and potential energies of each universe model. \(V\) depends on the magnitude of relative position variables only.
out to be unnecessary. Using relative Jacobi coordinates $R_i^\alpha$, which are interparticle (cluster) separations and which have the useful properties of automatically accounting for $P_\alpha = 0$ and preserving the form of all else in the above expressions (just swap $q_i^\alpha$ for $R_i^\alpha$ throughout), a direct implementation is (see [3] for the derivation and further discussion):

$$S(R_i^\alpha, \dot{R}_j^\beta) = 2 \int d\lambda \sqrt{|E - V|} T, \quad T(R_i^\alpha, \dot{R}_j^\beta) = \sum_{i=1}^{N-1} \frac{M_i R_i^2}{2} - \frac{1}{2} \alpha \beta L_\alpha \left( \Gamma^{-1} \right) \alpha \beta L_\beta,$$

(5)

where $I_{\alpha \beta}$ and $L_\alpha$ are the barycentric inertia tensor and angular momentum respectively:

$$b I_{\alpha \beta} (R_i^\alpha, \dot{R}_j^\beta) = \sum_{i=1}^{N-1} M_i \left( |R_i|^2 \delta_{\alpha \beta} - R_\alpha R_\beta \right) \quad \text{and} \quad b L_\alpha (R_i^\alpha, \dot{R}_j^\beta) = \epsilon_\alpha \beta \gamma \sum_{i=1}^{N-1} M_i R_\beta \dot{R}_\gamma.$$

(6)

### 3 GR as a Relational Theory

A slight re-interpretation [3] of the Baierlein–Sharp–Wheeler [7] action is a relational formulation for spatially compact without boundary GR

$$S = \int dt \int d^3 x \sqrt{\gamma} \sqrt{(\Lambda + R) T_{GR}}, \quad T_{GR} = (\gamma^\alpha \gamma^\beta \gamma^\gamma - \gamma^\alpha \gamma^\beta \gamma^\delta)(\dot{\gamma}_{\alpha \beta} - \mathcal{L}_{\gamma} \gamma_{\alpha \beta})(\dot{\gamma}_{\gamma} - \mathcal{L}_{\gamma} \gamma_{\gamma}).$$

(7)

For, note that this is reparametrization-invariant and is built using not $\gamma_{\alpha \beta}$ but $\gamma_{\alpha \beta} - \mathcal{L}_{\gamma} \gamma_{\alpha \beta}$ where $\mathcal{L}_{\gamma}$ is the Lie derivative with respect to the 3-diffeomorphism auxiliary $s_1$. (In fact, this action emerges as one of only a few consistent options upon considering far more general actions built from these relational first principles on the configuration space of 3-metrics on a compact without boundary spatial 3-manifold [3].) Now, the gravitational momenta are

$$\pi_{\gamma^\delta} = \sqrt{\frac{\Lambda + R}{\gamma}} (\gamma^\alpha \gamma^\beta \gamma^\gamma - \gamma^\alpha \gamma^\beta \gamma^\delta)(\dot{\gamma}_{\alpha \beta} - \mathcal{L}_{\gamma} \gamma_{\alpha \beta}).$$

Then

$$\mathcal{H} = \frac{1}{\sqrt{\gamma}} \left( \gamma^\alpha \gamma^\beta \gamma^\gamma - \frac{1}{2} \gamma_{\alpha \beta} \gamma_{\gamma} \right) \pi_{\gamma^\delta} - \sqrt{\gamma} R = 0 \quad \text{(Hamiltonian constraint)}$$

(8)

follows as a primary constraint by working closely related to that displayed in the previous section, and

$$\mathcal{H}_\alpha = -2D_\beta \pi_{\alpha \beta} = 0 \quad \text{(momentum constraint)}$$

(9)

follows from $s_\alpha$-variation.

Note the close parallels between this and the previous section: energy $E$ and cosmological constant $\Lambda$ play the same role, both actions are of square root form, leading to quadratic constraints ($H$ and $\mathcal{H}$ respectively), and in each case variation with respect to auxiliaries leads to linear constraints ($P_\alpha$, $L_\alpha$, and $\mathcal{H}_\alpha$, respectively). Elimination from the Lagrangian form of the linear constraints is a significant procedure in both cases: in Sec 2 it provides an explicit direct resolution of the absolute versus relative motion debate, while in Sec 3 it now constitutes the well-known thin sandwich conjecture. The relative configuration space quotient of the Q(N)/Eucl parallels the superspace quotient $Riem(\Sigma)/\text{Diff}(\Sigma)$ in being curved and stratified. Furthermore, one can then attempt relative configuration space quantization much as one can attempt superspace quantization.

### 4 Problems with Time and Closed Universes in GR

My interest in the above similarities stems from conceptual and technical problems which one encounters in canonical GR with the quantum form of $\mathcal{H} = 0$ [3] [10] [2] – in the configuration representation, this gives what is prima facie a time-independent Schrödinger equation $\mathcal{H}\Psi = 0$ rather than one which is dependent on some notion of time, $\tau$: $\mathcal{H}\Psi = i\hbar \frac{\partial \Psi}{\partial \tau}$. I hope that light will be shed on the conceptual viability of various suggested resolutions of this by considering the RPMs’ analogous yet technically simpler quantum equation $(H - E)\Psi = 0$.

One of these resolutions is [3] [10] that there is really a time hidden within $\mathcal{H}$ itself. This is based on the hope that there exists a canonical transformation which separates out four embedding variables and two true degrees of freedom of GR from the six 3-metric variables. This would produce $\mathcal{H}_{true} = (a \text{ linear combination of embedding momenta})$, which clearly yields a time-dependent Schrödinger equation upon quantizing in the new configuration representation. The York time approach (see e.g. [11]) is one such attempt. Ignoring the solution of the momentum constraint for simplicity of

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[2] Here, $\gamma_{\alpha \beta}$ is the spatial 3-metric with determinant $\gamma$, covariant derivative $D_\alpha$, and Ricci scalar $R$. $\Lambda$ is the cosmological constant.
The idea is then that \( H \) is proportional to the square root of the moment of inertia of the system. A canonical transformation can then be applied so that \( \tau \) is now a momentum. Then the Hamiltonian constraint is replaced by \(-\hat{H}^{true} = \sqrt{\gamma} = \chi^6\), for \( \chi \) the solution of the conformally-transformed \( H \)

\[
8\nabla^2 \chi = \frac{\pi^2}{6} \chi^6 + R\chi - \pi_{\alpha\beta} \pi^{\alpha\beta} \chi^{-7} \quad \text{(Lichnerowicz–York equation).} \tag{10}
\]

Then quantization gives

\[
\frac{i\hbar}{\delta \gamma} = \hat{H}^{true}\Psi. \tag{11}
\]

The obstruction to this particular resolution is that how to solve the complicated quasilinear elliptic equation is not in practice generally known, so the functional dependence of \( \hat{H}^{true} \) on the other variables is not known, so the quantum ‘true Hamiltonian’ \( \hat{H}^{true} \) cannot be explicitly defined.

Other resolutions consider time not to exist fundamentally, but rather to be an emergent concept. I consider two such approaches to quantum gravity: the semiclassical approach and the records approach. Both have been associated with splits into heavy and light modes, \( h_{A'} \) and \( l_{A''} \) respectively. The semiclassical approach then uses the WKB ansatz for the wavefunction of the universe:

\[
\Psi = e^{iM_{hW} (h_{A'}) / \hbar} = (h_{A'}, l_{A''}) \quad \text{(WKB ansatz for the wavefunction)}, \tag{12}
\]

where \( W \) is the principal function. Substituting this into \( \hat{H} \Psi = 0 \), one can peel off the Hamilton–Jacobi equation as the leading order terms, and moreover keeps the derivative cross term to the next order of approximation. It is this that supplies the emergent WKB time \( \tau_{WKB} \). In the case of ‘heavy gravitational modes’ supplying WKB time to ‘light minimally coupled matter modes’ \( \phi \) (which contribute additive portions \( \mathcal{H}^{\phi} \) and \( \mathcal{H}_{\alpha}^{\phi} \) to \( \mathcal{H} \) and \( \mathcal{H}_{\alpha} \) respectively), one has\(^3\)

\[
\frac{i\hbar}{\delta \tau_{WKB}} = \left( \int d^3x (\alpha \mathcal{H}^{\phi} + \beta^\alpha \mathcal{H}_{\alpha}^{\phi}) \right) \psi \quad \text{(emergent WKB time-dependent Schrödinger equation of GR)} \tag{13}
\]

Records approaches treat particular sorts of instanton configurations as primary and then attempt to reconstruct a semblance of dynamics/history from these. The main problems of these schemes are justifying the WKB ansatz and not being fully worked out respectively.

### 5 Investigation using RPM’s as Toy Models

Via a carefully-ordered approach using Jacobi coordinates, a range of simple RPM’s can be treated in good part using the usual mathematics employed in QM. This means that, at the simplest level, absolutism has not misled the conceptual development of QM. This range of simple RPM’s also permits the study of some simple features of closed universes: truncations and gaps in the eigenspectrum (which can go away with increase in complexity), and the ‘limited resource’ effect of a fixed and finite energy for the whole universe and subsystem angular momentum balance effects (which do not go away with increase in complexity but would seem likely to evade notice in large universes in which only small subsystems are ever studied in practice).

Further investigation reveals that these RPM’s have an ‘Euler’ internal time

\[
\tau_\varepsilon \equiv \sum_{i=1}^{N} q_{i\alpha} p_{i\alpha}. \tag{14}
\]

For this to be a time, it is important that this is monotonic; this follows for a number of substantial cases from the Lagrange–Jacobi identity:

\[
\tau_\varepsilon = 2T + kV = 2E + (k - 2)V, \tag{15}
\]

for \( V \) is homogeneous of degree \(-k\). \( \tau_\varepsilon \) is conjugate to the scale variable \( \sigma \) (which is the logarithm of a quantity proportional to the square root of the moment of inertia of the system) the rest of the variables are shapes and their momenta. A canonical transformation can then be applied so that \( \tau_\varepsilon \) indeed becomes a coordinate and \( \sigma \) its momentum.

The idea is then that \( H = E \) is to be interpreted as equation for \( \sigma \). I have done this e.g for simple 3-particle models in

\(^3\)Here, \( \alpha \) is a lapse (proper time elapsed) e.g. emergent from the reparametrization-invariant form of the action. The shift \( \beta_\alpha \) is the same notion as \( \delta_\alpha \).
1-d in 12. Then, the scale variable is \( \sigma = \ln \sqrt{R_1^2 + R_2^2} \) and there is one shape variable, which may be chosen to be of the form \( S = \text{Arctan}(R_1/R_2) \). Then

\[
E \equiv H(\tau_E, S; -\sigma, P_S) = e^{-2\sigma} \left( \frac{\tau_E^2}{2\mu} + P_S^2 \right) + V(\sigma, S) .
\]

(16)

For a number of standard potentials, this is explicitly soluble as \( \sigma = \sigma(\tau_E, S, P_S) \equiv -H^{true}(\tau_E, S, P_S) \) For each of these one may then pass to an explicit configuration representation Euler internal time dependent Schrödinger equation

\[
i\hbar \frac{\partial \Psi}{\partial \tau_E} = H^{true}(\tau_E, S, P_S) \Psi
\]

(17)

Thus, after stripping Newtonian mechanics bare of absolute space and absolute time, I find that some portions of it nevertheless have an internal time hidden within! Having got round the not explicitly constructible impasse which plagues GR, there is some value in investigating next whether these internal time dependent Schrödinger equations are quantum mechanically well-defined and what properties their solutions have, so as to infer how sound internal time approaches are.

As regards the emergent time resolutions, I begin by setting up the heavy–light split for the relative Jacobi coordinates \( R_{\alpha} \) of the RPM’s. Consider RPM’s for which the \( R_{\alpha} \) subdivide into heavy \( h_{\alpha} \) and light \( l_{\alpha} \) coordinates according to the magnitudes of the associated masses (which are the reduced masses of clusters of the original particle-position masses) being such that \( M_h \gg M_l \). This is possible e.g. for two \( h \) particles of similar mass \( M \) and one \( l \) particle of mass \( m \), whence there is then one heavy Jacobi coordinate \( h_{\alpha} \) and one light one \( l_{\alpha} \) with

\[
M_h \approx M/2 >> m = M_l .
\]

(18)

The semiclassical approach then involves the WKB ansatz

\[
\Psi = e^{iM_h W(h_{\alpha})/\hbar} \varphi(h_{\alpha}, l_{\alpha}) .
\]

(19)

Substitute this into \( (\hat{H} - E) \Psi = 0 \) and \( \hat{L} \Psi = 0 \), keeping the cross-term proportional to \( \partial W / \partial h_{\alpha} \partial l_{\alpha} \). Then at zeroth order the corresponding Hamilton–Jacobi equation appears, while to first order\(^4\)

\[
i\hbar \frac{\partial \varphi}{\partial \tau_{WKB}} = \left( \hat{a} \hat{\mu}(l_{\text{part}}) - b^\alpha l_{\alpha}^{(l_{\text{part}})} \right) \varphi
\]

(20)

eventually arises 12, which is a simple analogue of the emergent WKB time-dependent Schrödinger equation of GR 13. While I addressed a number of objections 13 specific to the semiclassical approach of RPM’s in 6, a remaining objection in general to the semiclassical approach is justifying the WKB-type ansatz in the context of whole universes. I have not resolved this foundational issue, but hope that RPM’s will be a useful arena in which to investigate whether this can be justified.

A records approach built on the above \( h-l \) set-up follows along the lines of 11 which considers one \( h \) particle moving through a medium of \( l \) particles which it disturbs into motion. Subsequent instants consist of the particles’ positions and momenta. It is these instants which are the records, and the motion or history of the large particle can then be reconstructed (perhaps to some approximation) from them. This approach has the complicating feature that a potential including \( h-l \) coupling is required, and the simplifying feature that the environment of \( l \) particles need not be populous. This notion of record can be adapted to e.g. a 1-d 3-particle RPM as follows. Consider the 2 \( h \) and 1 \( l \) particle situation of 15. In relational terms, this situation is the motion of a \( h \) and a \( l \) interparticle (cluster) separations (Jacobi coordinates). If these have coupled potentials so that the \( h \) separation disturbs the \( l \) separation into motion, subsequent record-instants consist of inter-particle (cluster) separations and their momenta with respect to label time. An additional issue to investigate is: what in nature causes the selection of records rather than instants from which a semblance of dynamics/past history cannot be reconstructed. Barbour’s approach 2 speculates that the asymmetry of the underlying curved stratified quotient configuration space causes concentration of the wavefunction on records rather than other instants. As RPM’s share these features, they may serve to investigate this possibility.

Investigating the schemes of the last two paragraphs in detailed particular examples involves further work of greater complexity than in my current work 12. RPM’s may then be a promising arena to investigate, in cases in which two or more of the above resolutions of the problem of time exist, whether these are identical, approximate, or entirely distinct resolutions.

\(^4\)Here \( \hat{a} \) is a ‘lapse’ emerging from the reparametrization-invariant action used. \( \hat{b}^\alpha \) is as in Sec 2.
Acknowledgments

Don Page, Julian Barbour, Niall Ó Murchadha and Gary Gibbons for discussions, Claire Bordenave for accommodation and help with the computer, the Killam Foundation and then Peterhouse for funding.

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