On the Transfer of Metric Fluctuations when Extra Dimensions Bounce or Stabilize

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In this report, we study within the context of general relativity with one extra dimension compactified either on a circle or an orbifold, how radion fluctuations interact with metric fluctuations in the three non-compact directions. The background is non-singular and can either describe an extra dimension on its way to stabilization, or immediately before and after a series of non-singular bounces. We find that the metric fluctuations transfer undisturbed through the bounces or through the transients of the pre-stabilization epoch. Our background is obtained by considering the effects of a gas of massless string modes in the context of a consistent ‘massless background’ (or low energy effective theory) limit of string theory. We discuss applications to various approaches to early universe cosmology, including the ekpyrotic/cyclic universe scenario and string gas cosmology.

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I. INTRODUCTION

The idea that the universe is comprised of any number of extra dimensions, in addition to the three non-compact spatial dimensions that we observe, goes back all the way to the work of Kaluza and Klein in the 1920’s, where extra dimensions were first proposed as a manner in which one could unify particle interactions with gravity. Since then, theories with extra dimensions have evolved from being a mathematical curiosity, albeit one with remarkable consequences for particle physics, to a theoretical necessity with the advent of string theory. Since the famous anomalies cancellation calculation of Green and Schwarz, the challenge has remained for cosmologists and phenomenologists alike to account for the presence of extra dimensions in physics. From a pessimist’s point of view, one might view these extra dimensions as an unwanted surplus that has to be swept under the rug at presently accessible energies. In its most unimaginative form, one could view this as the gist of the moduli problem. From an optimist’s point of view however, one could view the requirement of extra dimensions for the consistency of string theory as a new window of opportunity, through which one might be able to resolve some of the outstanding problems of particle physics and cosmology.

Taken as an essential part of the radically new picture of space-time that string theory offers us, extra dimensions, with strings and branes of various dimensions propagating through them, could offer us a potentially non-anthropic explanation for the observed dimensionality of space-time. In addition to offering us potentially non-singular cosmologies, they could very well be at the heart of the resolution of the dark matter problem. Recently, several alternative stringy cosmologies have emerged as possible successors to the standard big bang model of the universe, such as the pre-big bang model, brane/string gas cosmology, and the cyclic/ekpyrotic scenarios, all of which have the ultimate aim of becoming complete and testable models of the early universe.

A central question that arises in determining whether or not these models of the early universe reproduce observations is, how metric fluctuations in the presence of extra dimensions evolve in the backgrounds that these cosmologies propose. The importance of understanding this question cannot be understated in the context of the abundance of available experimental data, against which we must compare our eventual predictions. For instance, in the ekpyrotic/cyclic scenario the effects of branes colliding in a 5-dimensional bulk are explored.

Key to the success of this program is being able to follow the evolution of metric fluctuations through the bounces of the extra dimension. The eventual goal of this is to be able to explain the observed inhomogeneities of the cosmic microwave background as being seeded by radion fluctuations generated through the motion of the branes towards each other, rather than through the quantum fluctuations of some putative inflaton field.

Similarly, in the context of the pre-big bang scenario, where an expanding and a contracting phase of the universe’s evolution are naturally related to each other by the scale factor duality symmetry inherent to low energy effective string theory, a universe undergoing a big bang-big crunch transition is naturally implemented. In this context, the evolution of metric fluctuations through the bounce becomes a question of central importance in understanding the way the big bang phase of our universe’s evolution happened the way it did. As it turns out, although at first seemingly unrelated, an associated and as yet unexamined issue is how any tentative solution of the moduli problem will modify the spectrum of the metric fluctuations which we observe. Given that the universe is
likely to contain many extra dimensions whose shape and volume moduli are dynamical in the early universe, one might expect that the dynamics of these moduli fields will lead to a highly nontrivial modification of any initial pre-stabilization spectrum.

It is the goal of this report to show that, in the context of a 5-d universe where an extra dimension undergoes non-singular bounces, or is en route to stabilization, the final spectrum for the Bardeen potential corresponding to (long wavelength) fluctuations of the scale factor for the non-compact dimensions, is identical to the initial spectrum of the five-dimensional fluctuations (in a sense which will be specified later). Specifically, if we had an initial scale invariant spectrum for the five-dimensional fluctuations, we would end up with a scale invariant spectrum for the Bardeen potential after the bounce, or after the transients of the stabilization had settled down. Furthermore, we show that the variable corresponding to metric fluctuations of the compact dimensions decays, which corresponds to the stability of the radion degree of freedom to fluctuations in our non-singular setup.

One application of our work concerns the evolution of fluctuations in ekpyrotic/cyclic type models. In the context of a four space-time dimensional effective field theory toy model of this scenario, the dynamics of perturbations has been investigated in detail. The initial analyses yielded the result that a scale-invariant spectrum before the bounce transfers to a scale-invariant spectrum after the bounce. These analyses, however, were questioned in a serious complicating factor turned out to be the fact that the proposed background evolution was singular, thus requiring the use of “matching conditions” (such as those derived in the context of an expanding cosmology undergoing a sudden phase transition) to compute the post-bounce spectrum of fluctuations. As discussed in [31], there is a very sensitive dependence of the final result on the choice of the matching surface. Independently, there has been recent work on the evolution of fluctuations through a nonsingular bounce in four space-time dimensional cosmologies, in which the bounce is constructed by adding extra terms to the standard Lagrangian. These analyses yield results showing a sensitive dependence on the nature of the bounce. The bottom line of this work is that a correct analysis of fluctuations in the ekpyrotic/cyclic scenario needs to be done in a five space-time dimensional context, a context in which the nature of the bounce is unambiguous. A first important step in this direction was taken in [41], confirming the result that a scale-invariant spectrum passes through the bounce (in the five-dimensional context, the bounce means that the radius of the extra spatial dimension bounces, not that the four space-time dimensional scale factor bounces) without change in the spectral index. However, the analysis of [41] was done in the context of a singular background and assuming specific matching conditions for fluctuations applied at a point when the perturbations in fact blow up. Thus, the results are open to doubt. In this work, we study the transfer of fluctuations through a cosmology in which two boundary branes approach each other and bounce without encountering a singularity (see also [20] for previous work done in the context of a particular nonsingular ekpyrotic-type bounce proposed in [11]).

We begin by introducing a non-singular bouncing model of a 5-dimensional universe where one dimension is compactified on a circle or an orbifold. As we shall see shortly in detail, our non-singular background is obtained by considering the effects of gas composed of massless string modes on the dynamics of space-time, in the context of a consistent low energy effective theory limit, or ‘massless background’, of string theory. We then set up the framework for studying cosmological perturbations in this model and study how these transfer through the various bounces that the extra dimension undertakes. Although our setup is seemingly specific to string gas cosmology, the essence of our framework is that we have a non-singular bounce/stabilization mechanism that is affected by degrees of freedom that become massless at a certain point. Hence, the hope is that the results obtained here can be generalized to other settings, an issue we will discuss in detail when we consider applications to different approaches to stringy cosmology. We now commence our paper with a few preliminaries.

**II. THE BACKGROUND – \( \mathbb{R}^4 \times S^1 \)**

Consider a five dimensional space-time with the topology of \( \mathbb{R}^4 \times S^1 \), described by the metric

\[
g_{AB} = \text{diag}(-1, a^2(t), a^2(t), a^2(t), b^2(t)),
\]

from which we derive the following components of the Einstein tensor:

\[
G^t_t = -3\frac{\dot{a}}{a}(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}),
\]

\[
G^{\alpha\beta}_{\alpha\beta} = -\delta^i_j \left[ 2\frac{\ddot{a}}{a} + \frac{\dot{b}}{b} + \left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{b}}{b} \frac{\dot{a}}{a} \right],
\]

\[
G^y_y = -3\frac{\dot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2.
\]

Here, the indices \( i \) and \( j \) run over the three large spatial dimensions and \( y \) denotes the extra dimension. The form of the energy-momentum tensor which will couple to the Einstein tensor is given by

\[
T^A_B = \text{diag}(-\rho, p, p, p, r).
\]

We can recast the Einstein equations \( G^A_B = 8\pi G T^A_B \), where \( G \) is the five-dimensional Newton’s constant, in the form:

\[
\frac{\ddot{a}}{a} + H(2H + \mathcal{H}) + \frac{8\pi G}{3} [r - \rho] = 0,
\]

\[
\frac{\ddot{b}}{b} + 3H\dot{b} + 8\pi G b \left[ \frac{2r - \rho}{3} - \frac{\rho}{3} \right] = 0,
\]

\[
\rho = \frac{3}{8\pi G} H(H + \mathcal{H}),
\]
where \( H = \dot{a}/a \) and \( \mathcal{H} = \dot{b}/b \). The energy-momentum tensor for a string gas in this toroidally compactified background was derived in [12] to which we refer the reader if any of what follows is unfamiliar. It was found to be

\[
\rho_{n,w} = \frac{\mu_0}{\epsilon_{n,w}} \left( \frac{2}{\sqrt{-g}} \right)_{n,w}, \quad (9)
\]

\[
p_{n,w} = \frac{\mu_0}{\epsilon_{n,w} \sqrt{-g}} \left( \frac{p_{n.c.}^2}{3} \right), \quad (10)
\]

\[
r_{n,w} = \frac{\mu_0}{\epsilon_{n,w} \sqrt{-g}} \left( \frac{n^2}{b^2} - w^2 b^2 \right), \quad (11)
\]

where \( g \) denotes the determinant of the metric, \( \tilde{b} := b/\sqrt{\alpha'} \) (with \( 2\pi\alpha' \) being the inverse string tension) and the subscripts \( n, w \) refer to the momentum and winding quantum numbers of a closed string along the 5th dimension respectively, on which all of the subscripted quantities will depend. The density \( \mu_0 \) is the number density of the string gas with the metric dependence factored out \( (\mu(t) = \mu_0/\sqrt{-g}) \), while \( \epsilon_{n,w} \) is the energy of a single closed string in this background

\[
\epsilon_{n,w} = \sqrt{p_{n.c.}^2 + \left( \frac{n}{b} + \frac{w b}{\alpha'} \right)^2} + \frac{4}{\alpha'}(N - 1), \quad (12)
\]

where \( p_{n.c.} \) denotes the center of mass momentum along the three non-compact directions, and \( N \) is the number of right-moving string vibrational modes. For reasons made clear in [12] and [13], string states that are massless at special symmetry points (i.e. the self-dual radius) should play a very distinguished role in any string gas cosmology (see also [14] [15] [16]). There are several reasons for this, among which are prominently the desire to obtain a viable late time phenomenology and a robust stabilization mechanism for the radion. However, the main motivation is that the low energy effective theory limits of string theory (such as general relativity and dilaton gravity) are also consistent backgrounds on which we can study the propagation of massless (and only massless) strings. For these reasons we do not further justify the focus on massless string modes, and refer the reader to [12] [13] [14] for more details. For a general review of string gas cosmology, we refer the interested reader to [12] [13] [14].

We wish to emphasize that the framework within which we chose to work assumes nothing other than the fact that nature is described by a string theory at high energies, and that the degrees of freedom that are likeliest to be excited (namely massless string modes) will be excited. The two particular geometries that we consider for the extra dimensions (i.e. a toroidal geometry in this section and an orbifold geometry in the next section), are necessitated by the fact that they are the unique compact backgrounds on which one stays within the approximation of the low energy effective limit of string theory, when considering extra dimensions that are similar in size to the string scale. This is because in general, were the metric to depend on the extra dimension (as is the case for warped extra dimensions), one will introduce curvatures that approach the string scale when the extra dimensions themselves approach the string scale, and hence the low energy approximation will break down. We discuss this point further in the next section.

The particular states we are interested in are those for which \( n = -w = \pm 1, N = 1 \). That these states are massless at the self-dual radius \( (\tilde{b} = \sqrt{\alpha'}) \) is easily checked from (12). These states imply that (7) becomes

\[
\frac{3}{\tilde{b}} + 3\frac{\dot{\tilde{b}}}{\tilde{b}} + \frac{8\pi G \mu_0}{a^3 \alpha'} \frac{1}{\tilde{b}^3} + \frac{2}{3} \frac{2}{\tilde{b}^3} = 0. \quad (13)
\]

We will see that this background stabilizes around the self-dual radius \( (\tilde{b} = 1) \), whilst the non-compact directions persistently expand as they would in a radiation dominated universe. Before we get to this, however, we wish to discuss a generalization of this background to the situation where the extra dimension is compactified on an orbifold.

A. Extension to \( \mathbb{R}^4 \times S^1/\mathbb{Z}_2 \)

The consistency of the framework that we propose, namely that we remain within the low energy effective theory (or massless background) limit of string theory, requires the background we consider to satisfy the condition

\[
R[g] \ll \frac{1}{\alpha'}. \quad (14)
\]

That is, the Ricci scalar should be considerably bounded from above by the string tension. This requirement translates into the statement that the metric should not change very much on the string scale, which ensures that this background remains a consistent background for the propagation of massless string modes. Conversely, were one to consider backgrounds which do not satisfy (14), not only would the approximations made in deriving the low energy limit of string theory break down, one would also expect massive modes to be created [15]. For our purposes, the requirement of (14) means that we were to consider compatifications of the extra dimension on scales comparable to the self-dual radius, then there should be absolutely no dependence of the metric along the compactified direction. This is a remarkable feature of using a toroidal compactification: one can in fact study string scale processes without invalidating the approximations inherent in the low energy limit of string theory. It was shown in [13] and [17] that the background we are about to derive, does in fact satisfy (14) throughout its dynamics.

Returning to the problem at hand, we see that since we are not allowed to consider metric dependencies along the extra dimension (by homogeneity and isotropy of the
non-compact dimensions, the metric can then only depend on time), the components of the Einstein tensor \( \mathcal{E}_{ij} \) are unchanged after orbifolding the extra dimension. The only place where orbifolding might make a difference is in the energy momentum tensor of the string gas. However, because the massless states that we have focused on are in the so-called untwisted sector \([18]\) (in general, twisted states are localized at the orbifold fixed points), it turns out that there is no difference in the energy-momentum tensor either \([19, 20]\). This is a consequence of the ‘inheritance’ principle of orbifold theories \([18]\), and permits us to use the framework just presented in either situation, provided we only use the modes that we have indicated and we do not consider any variations in any metric quantities along the extra dimension. However, there is a caveat to this in that two orbifold fixed planes are present, which might have matter confined to them. The requirement not to induce any metric variations along the extra dimension translates into, via the Israel junction conditions \([49, 50]\), the condition (among others)

\[
\left[ \frac{da}{dy} \right] \propto 8\pi G \rho_{\text{brane}},
\]

where \([da/dy]\) is the jump of the derivative of the scale factor \(a\) along the extra dimension, evaluated around either of the orbifold fixed planes. Hence, we see that if we require there to be no dependence of the metric along the extra dimensions, then any matter localized on the branes must be sufficiently dilute to render the branes to behave as test branes. Bearing all of this in mind, we can now proceed to derive the background solution within our framework.

III. BACKGROUND SOLUTION IN 5D

A. The three large dimensions

We first note from \([7]\) that any form of matter which satisfies a radiative equation of state \((p = \rho/3, r = 0)\) for all times (as opposed to our wound string states which become massless, hence radiative, only at the self-dual radius) drops out of the driving term in the equation of motion for \(b\) \([7]\). Hence, we can safely consider a situation where we have a radiation gas in addition to our gas of wound strings, which dominates the evolution of the Hubble factor \(H\) for the non-compact dimensions through \([3]\) after the extra dimension has stabilized, or is close to stabilization. In fact, massless unwound closed string states (gravitons: \(n = w = 0, N = 1\)) can provide exactly such a bath. Henceforth, it is natural to include such a bath in our setup. We do not consider this matter further, simply taking as a given in what follows that the scale factor \(a\) for the non-compact dimensions expands at the background level as it would in a radiation dominated universe, that is

\[
a(t) \propto t^{1/2}.
\]

B. The extra dimension

Let us go back to the equation of motion \([13]\) for \(\tilde{b}\), which reads

\[
0 = \ddot{\tilde{b}} + 3\frac{\dot{\tilde{b}}}{a} \frac{\dot{b}}{b} + 8\pi G \mu_0 \alpha \frac{1}{a^3 \alpha'} \sqrt{\frac{1}{b} + \frac{q}{a} \Omega_{\text{n.c.}} - \frac{1}{b} \left(\frac{1}{a} - \frac{1}{b}\right)^2} = 0,
\]

where \(\Omega_{\text{n.c.}} = q/a\) with \(q = \text{const}\) is the center of mass momentum along the three large dimensions. Introducing the characteristic time scale

\[
t_0^{-1} := \sqrt[3]{\frac{64\pi G \mu_0}{3\alpha' \alpha^{3/2} a_0^2 q}}
\]

and defining the dimensionless time variable

\[
\eta := 2 \sqrt{\frac{t}{t_0}},
\]

such that \(a = a_0 \eta\), we can simplify \([17]\) to

\[
\ddot{\tilde{b}}' + \frac{2}{\eta} \dot{\tilde{b}}' + \frac{3}{8} \frac{1}{\eta} \frac{\dot{b}}{b} \frac{\dot{b}}{\eta} = 0,
\]

where a prime denotes a derivative with respect to \(\eta\) and we also introduced the free dimensionless parameter

\[
l := \frac{a_0}{\sqrt{\alpha' q}}.
\]

This equation can easily be integrated numerically, but it will turn out to be useful to have a simple analytic approximation.

If we are close to the self-dual radius, that is \(\tilde{b} = 1 + \varepsilon\) with \(\varepsilon \ll 1\), we can expand the driving term in \([20]\) so that

\[
\varepsilon'' + \frac{2}{\eta} \varepsilon' + \varepsilon = 0
\]
IV. SCALAR PERTURBATIONS IN 5D

We will first perturb the metric, focusing only on scalar metric perturbations. The most general scalar perturbation in “generalized” longitudinal gauge \[21, 51, 52\] can be characterized by four scalar functions, \(\Phi, \Psi, W\) and \(\Gamma\). These functions can be viewed as a basis of gauge invariant variables. In this gauge, the metric is given by

\[
ds^2 = -(1 - 2\Phi(t, x)) dt^2 + a(t)^2 (1 - 2\Psi(t, x)) dx^2 + b(t)^2(1 + 2\Gamma(t, x)) dy^2 - 2W(t, x) dt dy, \tag{24}
\]

where the signs in front of the perturbations are a mere convention. For reasons discussed in sections \(11\) and \(14\), we do not consider any \(y\) dependence, that is we assume homogeneity in the extra dimension (see also \(14\) for a discussion of this and other assumptions of SGC). The corresponding Einstein tensor can be computed to be \[21, 52\] (in the following \(\Delta\) denotes the Laplacian of the three large spatial dimensions):

\[
\begin{align*}
\delta G^z_{x_i} &= \frac{1}{a^2} \left[ \partial_{x_i} \partial_{x_j} \left( -\Gamma + \Phi \right) + 2 \left( \frac{b}{a} - 3\frac{a}{a} + \partial_t \right) \partial_t \Psi 
- 2 \left( \frac{b}{a} + 2\frac{\dot{a}}{ab} + \frac{\dot{a}^2}{ab^2} + 3\frac{b}{2b} \partial_t + \frac{\dot{a}}{a} \partial_t \right) \Phi 
- \left( \frac{2\dot{a}}{a} + 2\frac{\ddot{b}}{b} + \partial_t \right) \partial_t \Gamma \right], \\
\delta G^y_{y} &= \left[ -\frac{2}{a^2} \Delta + 3 \left( \partial_t + \frac{4\dot{a}}{a} \right) \partial_t \right] \Psi + \left[ -\frac{1}{a^2} \Delta - 6 \left( \frac{a^2}{a^2} + \frac{\dot{a}}{a} + \frac{\dot{a}}{2a} \right) \right] \Phi, \\
\delta G^t_{t} &= \left[ 3 \left( \frac{\dot{b}}{b} \partial_t + 2\frac{\dot{a}}{a} \partial_t \right) - \frac{2}{a^2} \Delta \right] \Psi - \left( \frac{\dot{a} b}{a b} + \frac{\dot{a}^2}{a^2} \right) \Phi - \left( \frac{3\dot{a}}{a} \partial_t - \frac{1}{a^2} \Delta \right) \Gamma, \\
\delta G^\delta_{i} &= \partial_{x_i} \left[ \frac{2}{a^2} \partial_t \Psi - \frac{1}{a^2} \left( \frac{\dot{b}}{b} + 2\frac{\dot{a}}{a} \right) \Phi - \frac{1}{a^2} \left( \frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right) \right]. \tag{29}
\end{align*}
\]

One can check that the equations of motion involving \(W\) decouple from the other ones for the matter content \[5\] we consider. Since \(W\) would appear only squared in a four dimensional effective theory, we will not need to compute it at all \[64\].

To write down the perturbed Einstein equations \[55\], we also need the perturbed energy-momentum tensor \(\delta T^A_B\). It will include the thermal bath of radiation (denoted by the subscript \(r\)), and the stringy matter sources denoted by a tilde. To be specific, we have

\[
(\delta^{(r)} T^A_B)^{\delta \rho(r)} = \begin{pmatrix}
\delta \rho(r) \\
(\rho(r) + p(r))V_i \\
- (\rho(r) + p(r))V_{,i} \\
0
\end{pmatrix} \begin{pmatrix}
\partial_\rho(r) \delta \rho(r) \\
\rho(r) + p(r) \delta V_{,i} \\
0 \\
0
\end{pmatrix},
\]

where \(\rho(r)\) and \(p(r)\) are the radiation energy density and pressure, respectively, and \(V\) is the radiation three velocity potential and

\[
(\delta T^A_B^\delta)^{\delta \tilde{\rho}} = \begin{pmatrix}
\delta \tilde{\rho} \\
(\rho(r) + p(r))V_{,i} \\
- (\rho(r) + p(r))V_{,i} \\
0 \\
\end{pmatrix} \begin{pmatrix}
\delta \tilde{\rho} \\
\delta V_{,i} \\
0 \\
\delta \tilde{\rho}
\end{pmatrix},
\]

where \(\tilde{\rho}\) and \(\tilde{\rho}\) are the string gas energy density and pressure, respectively, and \(V\) is the string gas three velocity potential. Note that anisotropic stress does not feature in our setup (see appendix \[A\]), but we will keep \(V\) around for the time being. We will also focus on adiabatic perturbations of the radiation fluid only, that is
\( \delta \rho_{(r)} = \delta \rho_{(s)} / 3 \), even though two ideal fluids are present so that iso-curvature perturbations could arise. The reason for neglecting those is simplicity. The \( x_i - x_j \) \( (i \neq j) \) Einstein equations yield immediately

\[
\Gamma = \Psi + \Phi, \tag{32}
\]

and, after introducing

\[
\xi := \Psi - \Phi \tag{33}
\]

\[
N := \frac{\delta \mu_0}{\mu_0}, \tag{34}
\]

we infer from (39)-(41), that the perturbed energy momentum tensor is of the form

\[
\delta \rho = (N + \Gamma + 2\xi)\rho, \tag{35}
\]

\[
\delta \rho = (N + \Gamma + 2\xi)\rho, \tag{36}
\]

\[
\delta \tilde{r} = (N + \xi)\tilde{r} - 2\Gamma \left( \frac{1}{b^2} + \tilde{b}^2 \right) 3\rho \frac{a^2}{q^2\alpha^\prime}, \tag{37}
\]

where the background quantities are given by

\[
\tilde{\rho} = \frac{\mu_0}{a^\prime} \frac{1}{a b} \sqrt{\frac{q^2 a^\prime}{a^2}} \left( \frac{1}{b^2} - \tilde{b}^2 \right), \tag{38}
\]

\[
\tilde{\rho} = \frac{\mu_0}{a^\prime} \frac{1}{a b} \sqrt{\frac{q^2 a^\prime}{a^2}} \left( \frac{1}{b^2} - \tilde{b}^2 \right), \tag{39}
\]

\[
\tilde{\rho} = \frac{\mu_0}{a^\prime} \frac{1}{a b} \sqrt{\frac{1}{b^2} - \tilde{b}^2}, \tag{40}
\]

In performing this calculation, we started with the source action for this string gas [12], and obtained the perturbed energy momentum tensor through direct calculation. The string gas energy momentum tensor only depends on the metric, the number density of strings \( \mu_0 \), and the center of mass momentum of these strings. However, the latter quantity is not perturbed, because for long wavelength perturbations (compared to the string scale), which we are restricted to if we are to remain within the limits of low energy effective theory, the center of mass motion of a string propagating on a perturbed spacetime is unaffected to first order. Hence the only quantities left to perturb are the metric dependencies, and the number density of the string gas.

Next, we can write the perturbed zero component of the conservation equation \( \nabla^A T_{AB} = 0 \) for the stringy matter as

\[
0 = \delta \dot{\rho} + 3H(\delta \dot{\rho} + \delta \tilde{r}) + \mathcal{H}(\delta \dot{\rho} + \delta \tilde{r}) \tag{41}
\]

\[
-3 \xi \dot{\rho} + \frac{1}{2} \tilde{\Gamma}(\dot{\rho} - 3\rho + 2\tilde{r}) + \frac{\Delta \dot{V}}{\alpha^2} (\dot{\rho} + \tilde{r}).
\]

For simplicity we will set \( \dot{V} = 0 \) from now on, that is we neglect the scalar velocity potential of the string gas, consistent with the arguments of appendix A. Combining the diagonal Einstein equations yields

\[
\frac{1}{3M_5^2} (2\delta \tilde{r} + \delta \rho - 3\delta \tilde{p}) = \tilde{\Gamma} + \tilde{\Gamma}(3H + \mathcal{H}) - 2\mathcal{H} \xi - \frac{\Delta \tilde{\Gamma}}{\alpha^2} \tag{42}
\]

\[
(\Gamma - \xi)(3H\mathcal{H} + \mathcal{H}^2 + \tilde{\mathcal{H}}),
\]

\[
\frac{1}{3M_5^2} (3\delta \rho - \delta \tilde{p}) = \dot{\xi} + \dot{\xi}(2\mathcal{H} + 5\mathcal{H}) = \mathcal{H} \dot{\Gamma} - \frac{\Delta \xi}{3a^2} \tag{43}
\]

\[
+(\xi - \Gamma)(4\mathcal{H}^2 + 3\mathcal{H}\mathcal{H} + \mathcal{H}^2 + \tilde{\mathcal{H}} + 2\dot{\mathcal{H}}).
\]

The last three equations (41)-(43) are the dynamical ones for the perturbation variables \( \mathcal{H}, \xi, \) and \( N \). The \( (t-t) \) equation gives the radiation fluid perturbation \( \delta \rho_{(r)} \) and the \( (x_i-t) \) equation gives the scalar velocity potential \( V \) of the radiation fluid in terms of the other variables.

These equations simplify if we write them in terms of \( \eta \) defined in (19) and make use of the background equation \( a = a_0\eta \). Introducing \( h := \tilde{b}/\tilde{b} \) we get

\[
\Gamma_k'' + \Gamma_k' \left( \frac{h}{2} + \frac{1}{\eta} \right) - \xi_k' 2h + (\xi_k - \Gamma_k) \left( \frac{2h}{\eta} + h^2 + h' \right)
\]

\[
\frac{k^2}{4} \Gamma_k = \eta^2 C (2\delta r^* + \delta r^* - 3\delta p^*) \tag{44}
\]

\[
\xi_k'' + \xi_k' 2 \left( \frac{h}{2} + \frac{1}{\eta} \right) - \xi_k' h (\xi_k - \Gamma_k) \left( \frac{2h}{\eta} + h^2 + h' \right)
\]

\[
\frac{k^2}{12} \xi_k = \eta^2 C (3\delta p^* - \delta \rho^*) \tag{45}
\]

\[
\delta \rho^* + \frac{3}{\eta} \xi_k' (\delta \rho^* + \delta p^*) + h(\delta \rho^* + \delta p^*) - \frac{3}{2} \xi_k' (\delta p^* + p^*)
\]

\[
- \frac{1}{2} \xi_k' (\rho^* - 3p^* + 2r^*) = 0 \tag{46}
\]

Here, we made the transition to Fourier space and we performed the rescaling \( a^* := a/a_0, \ k^* := k t_0/a_0 \). All other starred quantities are defined via

\[
f^* := f_k a_0^3 a'^{\prime} \frac{\mu_0}{\rho_0}. \tag{47}
\]

The two dimensionless constants left are

\[
C := \frac{\tilde{t}_0^2}{M_5^5} \frac{1}{12 a_0^3} \mu_0 a'^{\prime}, \tag{48}
\]

\[
q^* := \frac{q v}{\sqrt{\alpha'^{\prime} / a_0}}, \tag{49}
\]

which are not independent but related via

\[
\frac{2^5 C}{q^*} = 1. \tag{50}
\]

### A. Analytic late time solution

After \( h = h' = 0 \) got approached (that is after the extra dimension got stabilized or, in the language of the ekpyrotic/cyclic scenario, the two approaching branes came
to a halt), we can simplify the equations of the previous section to
\[ \Gamma_k'' + \frac{2}{\eta} \Gamma_k' + \frac{k^* + \bar{k}^2}{4} \Gamma_k = 0, \quad (51) \]
\[ \xi_k'' + \frac{4}{\eta} \xi_k' + \frac{k^* + \bar{k}^2}{12} \xi_k = 0, \quad (52) \]
where we introduced the constant
\[ \bar{k}^2 := \frac{2^5 C}{q^5} = 1. \quad (53) \]

We note that the source term for $\xi_k$ vanishes because of the background equation of state for the gas of massless string modes. Also, the equations for $\xi_k$ and $\Gamma_k$ decouple at late times, just as they should do. The solutions are given by
\[ \Gamma_k = \frac{1}{\eta} \left( C_1 \cos(\omega_1 \eta) + C_2 \sin(\omega_1 \eta) \right), \quad (54) \]
\[ \xi_k = \frac{1}{\eta^3} \left( C_3 (k^* \cos(\omega_2 \eta) - 2\sqrt{3} \sin(\omega_2 \eta)) \right) \]
\[ + C_4 (k^* \sin(\omega_2 \eta) + 2\sqrt{3} \cos(\omega_2 \eta)) \), \quad (55) \]
with $C_i$ constant and
\[ \omega_1^2 := \frac{k^* + \bar{k}^2}{4}, \quad (56) \]
\[ \omega_2^2 := \frac{3k^*}{36}. \quad (57) \]

We would like to evaluate the spectrum
\[ p_k := k^3 \xi_k^2 \propto k^{n_s-1} \quad (58) \]
when a long wavelength mode enters the Hubble radius again at $k^* \eta_r = 2$. If we Taylor expand (55) for small $k^*$ we see that there is an approximately constant mode present for $\xi_k$. On the other hand, $\Gamma_k$ from (54) is oscillating and decaying $\propto 1/\eta$. Therefore, whatever the spectrum for $\xi_k$ was at the initial time, it should persist till re-entry and determine the spectrum for $\Phi_k$ because, by neglecting $\Gamma_k$, we have $\Phi_k \approx -\Psi_k \approx -\xi_k/2$. This is the same result one would conclude in a simple four dimensional universe, dominated by radiation. To be specific, we can approximate
\[ |\Phi_k(\eta_r)| \approx \left| -\frac{\xi_k(\eta)}{2} + \frac{\Gamma_k(\eta)}{\eta r} \cos\left( \frac{k \eta_r}{2 + \beta} \right) \right|, \quad (59) \]
\[ \approx \left| -\frac{\xi_k(\eta)}{2} \right|, \quad (60) \]
where $\beta$ is some irrelevant phase. Of course this holds true only, if the transient stabilizing epoch leaves no strong imprints onto the spectrum; hence, we will examine this crucial issue in the next section.

Before we do that, let us have a quick look at the physics after a mode entered the Hubble radius: $\xi_k$ will start to oscillate with a decaying amplitude $\propto 1/\eta^2$. If one waits long enough, $\Gamma_k$ will catch up and become larger than $\xi_k$. Now $\Phi_k \approx \Psi_k \approx \Gamma_k/2$ and thus $\Gamma_k$ will determine the spectrum, because for $\eta \gg \eta_r$
\[ |\Phi_k(\eta)| \approx \left| \frac{\Gamma_k(\eta_r) \cos\left( \frac{k \eta_r}{2 + \beta} \right)}{2\eta \eta_i \cos\left( \frac{k \eta_i}{2 + \beta} \right)} \right|. \quad (61) \]
There is of course a transition region where neither approximation holds. Nevertheless, this only occurs after Hubble radius crossing and shall not concern us in the following.

**B. Numerical solution and contact with late times**

The analytic solution of the previous section is valid after $\hat{b}$ got stabilized (or the branes came to a halt). However, the main question we would like to address is the following: what effect on the perturbations, if any, does the process of trapping the modulus $\hat{b}$ have? In order to address this question we have to solve (44)-(46) numerically. As we shall see, the late time solution is approached rapidly and long wavelength perturbations are not affected in any significant way. Hence, the naive intuition that the value of $\Phi_k$ for modes outside the Hubble radius stay frozen is indeed valid, with important consequences for cosmological model building, be it in the framework of the ekpyrotic/cyclic scenario, or within string gas cosmology (SGC) – we will focus on concrete models in the next section.

Firstly, let us specify constants and initial conditions: the only free parameter in our setup is $q^*$ defined in (69), describing the initial momentum of the string gas in the three large dimensions. Being a dimensionless parameter, we choose the most natural value
\[ q^* := 1. \quad (62) \]
Our results are not very sensitive to its exact value [66]. We start close to the self-dual radius with some arbitrary velocity (once again, its exact value does not affect our results - see Fig.10, that is
\[ \hat{b}(\eta_i) = 1.1, \quad (63) \]
\[ \hat{b}'(\eta_i) = -0.1. \quad (64) \]
Next, we have to specify initial conditions for $\xi_k, \Gamma_k$ and $N_k$ at $\eta_i = 1$ [67]. Since we did not compute how the universe came close to the self-dual radius (e.g. within the ekpyrotic/cyclic setup or a SGC setup), we have no way of deriving those. However, we can make an educated guess: if the universe underwent some phase of inflation in the three dimension before (we will make this point more concrete in the next section), one should expect similar values for all metric perturbations; hence we will set
\[ \xi_k(\eta_i) = \Gamma_k(\eta_i) =: \epsilon_k, \quad (65) \]
with some small $\epsilon_k$. For simplicity, we will use $\epsilon_k = 0.01$ for all $k$. Furthermore, since the long wavelength modes we are interested in should be frozen once they cross the Hubble radius, we set

$$\xi_k'(\eta_i) = \Gamma_k' (\eta_i) = 0.$$  \hfill (66)

The initial string density perturbation $N_k(\eta_i)$ will be set to zero, because these massless modes just got produced. Naturally, $N_k$ will get sourced by $\Gamma_k$ and $\xi_k$. No conclusion in the following is sensitive to the chosen initial conditions, hence we chose the most simple ones above (other ones were of course also tested).

The only thing left to specify is the wave-number $k^*$: We are interested in long wavelength modes, that is modes outside the Hubble radius with small $k^*$. In addition, we shall require $k^* \ll k$, so that we can compare our results with \cite{58}. With all initial conditions specified, we can use the analytic solution \cite{20} to solve \cite{11}-\cite{14} numerically.

Let us first compare $\Phi_k(\eta) = (\Gamma_k(\eta) - \xi_k(\eta))/2$ for different values of $k^*$, Fig. 2 (a)-(c): There $\Phi_k$ is plotted both numerically (grey) and analytic (black). We see that the analytic late time solution is approached pretty fast – in fact, the small difference at the beginning is not visible in this plot, but only if $\xi_k$ is plotted alone as in Fig. 5 (a). The visible decaying oscillation of $\Phi_k$ is the decaying mode of $\Gamma_k$ form \cite{22} with frequency $\omega$, which is plotted in Fig. 5 (b) (compare with Fig. 2 (b)). In addition, there are strongly damped oscillations on top of the constant mode of $\xi_k$, that carry the same frequency as $b(\eta)$, Fig. 4. As mentioned before, these are not visible in Fig. 2 but in Fig. 5 (a) where $\xi_k$ is plotted alone. These oscillations are the impact of trapping the modulus $b$, an
impact that can safely be ignored in the long wavelength regime: if one decreases $k^*$, all of these effects become less pronounced.

An other feature visible in Fig. 2 is a bending of the lower curve in comparison to the analytic solution (59). This bending is the beginning of an oscillation of $\xi_k$ and hence an expected feature, since the constant mode is actually the beginning of an oscillation with a very small frequency due to $\omega_\xi$. All features are not sensitive to the initial values we choose for $\Gamma_k$ and $\xi_k$.

Last but not least, $N_k$ is plotted in Fig. 5 (c): it gets sourced quickly by $\Gamma_k$ and $\xi_k$ and oscillates with a nearly constant amplitude thereafter, with all frequencies entering. However, there is no mentionable back-reaction of $N_k$ on the metric perturbations. Hence, one could give $N_k$ a non-zero initial value without changing the evolution of $\Gamma_k$ and $\xi_k$ in any significant way.

V. CONSEQUENCES FOR EKPYROTIC/CYCLIC MODELS AND SGC

In the end, we are interested in the spectrum of $\Phi_k$. So far, there seems to be little to no impact of the transient stabilizing era of $b$ onto $\Gamma_k$ and $\xi_k$. This translates directly to the spectrum, where $\Phi_k$ should approach a horizontal line for small $k^*$. This is indeed the case, as can be seen in Fig. 3 (c), where the spectrum is evaluated at horizon crossing: the oscillations that are present for relatively large $k^*$ get damped once the long wavelength regime is approached. This is expected, because long wavelength modes enter the Hubble radius later and henceforth, the decaying modes responsible for the oscillations in the spectrum get damped more. The frequency in the spectrum of $\Phi_k$ is proportional to $1/k^*$, in accordance with $\eta_r \sim 1/k^*$ and equation (59).

Note that no shift of the spectral index is induced, leaving the overall index unaltered. The oscillations in
\[ \xi_k \text{ in Fig. 2(a) are the main impact of the transient era of a dynamical } b. \text{ They are clearly negligible, because the tiny amplitude gets even smaller with decreasing } k^*. \]

To summarize, the main conclusion of our numerical study is that long wavelength metric perturbations quickly approach their asymptotic solution, leaving no trace of the many bounces \( b \) experiences on its way to stabilization. More specifically, we have shown that the initial spectrum of the non-decaying mode of \( \xi_k \) is preserved, i.e. equals the spectrum of \( \xi_k \) at late times. At late times, the spectrum of \( \xi_k \) equals the spectrum of the Bardeen potential \( \Phi_k \), the potential which is relevant for late time observations. Speaking more loosely, we have shown that the initial pre-bounce spectrum of bulk perturbations is transferred to the late time cosmological perturbations without a change in the spectral index.

We saw in the previous section that there is no significant effect of trapping the modulus \( b \) on the spectrum of long wavelength perturbations in the sense that the final spectrum of the Bardeen potential \( \Phi \) equals the initial value of the non-decaying mode of the bulk perturbation \( \xi \). This comes about since the value of \( \xi \) for small \( k^* \) modes which have a wavelength larger than the Hubble radius remains frozen during the phase of the trapping of \( b \), and since the final spectrum of \( \xi \) coincides with the spectrum of \( \Phi \).

Even though our result agrees with a “naive” intuition coming from the analysis of fluctuations in expanding four space-time dimensions, it could be viewed as an unexpected result: The “naive” intuition turned out to be wrong in the case of bouncing cosmologies in four space-time dimensions, for example in pre-big-bang models where it was shown [57] that the growing mode of \( \Phi \) in the contracting phase couples almost exclusively to the decaying mode of \( \Phi \) in the expanding phase, leading to the result that there is a large change in the spectral index of the dominant mode between the contracting and the expanding phase. Other analyses performed within the context of four-dimensional general relativity (see [36, 37, 38, 39] and references therein) yielded a similar result. It is now also generally accepted that the four space-time dimensional toy models which were proposed to describe the ekpyrotic/cyclic models have the same feature, namely that the dominant mode of \( \Phi \) in the contracting phase matches predominantly to the decaying mode of \( \Phi \) in the expanding phase (see e.g. [24]). Our analysis, however, confirms the analysis of [40], which showed, in the context of a singular bounce, that in the case of a bounce of boundary branes in five space-time dimensions the spectrum of \( \Phi \) could be preserved.

Our result is an important step in constructing a viable alternative to standard scalar field-driven inflationary models (next to the remarkable but highly nontrivial KKLMMT construction, [50] and follow-up papers) in the context of cosmologies with extra dimensions. We will now outline two possible proposals which seem promising to us in that regard:

Firstly, one could envisage a modification of the ekpyrotic/cyclic scenario. In the ekpyrotic proposal [14], a test brane moves slowly through the bulk towards the boundary brane of an orbifold (the Horava-Witten setup). In the cyclic scenario [14], it is the two boundary branes that approach each other. In either case, a scale-invariant spectrum of perturbations can be generated during the contracting phase given a suitable potential for the modulus field which describes the interbrane distance [22]. During the collision of the branes (a singular event in the ekpyrotic/cyclic scenarios) a hot universe is supposed to emerge on the boundary brane we live on today. Our background construction can be viewed as a regularized version of the ekpyrotic/cyclic scenarios (ekpyrotic in the context of a single bounce, cyclic in the case we consider the evolution until \( b \) has stabilized). Our work shows that in the context of such a regularized scenario, an initial scale-invariant spectrum could pass through the bounces and thus survive from the initial contracting phase to the final phase of expansion of our three spatial dimensions (this scenario will be developed further in a follow-up paper). In our scenario, the branes do not actually hit each other, but come to a halt as a consequence of the appearance of new massless modes that get produced explosively at a certain brane-separation (that is at a certain value of \( b \)). Our results show that such dynamics does not spoil a scale invariant spectrum generated during the initial brane movement. In this framework, one can still reheate the Universe, since the “kinetic energy” of the branes will get transferred to the stabilizing massless modes and other light modes which act like radiation on the brane (corresponding to the radiation bath that we already included in our setup).

Secondly, an incorporation of inflation into brane/string gas cosmology will rely on the stabilization mechanism described in this article: if any mechanism of inflating three dimensions is found, the internal dimensions will most likely have to deflate in some way. For example, one could employ the idea of anisotropic inflation (studied in the context of vacuum
solutions of higher dimensional general relativity in \[58\] and \[59\]). Once the internal dimensions get close to the self-dual radius, certain string modes become massless and get produced explosively along with a radiation bath, in close analogy to reheating after standard scalar field-driven inflation. These modes will then stabilize the internal dimensions, curing the graceful exit problem of anisotropic inflation, while the spectrum of fluctuations produced during inflation in the metric degrees of freedom remains unaltered. A concrete realization of this proposal is in preparation by the authors of this article.

VI. CONCLUSIONS

In this report, we studied how radion, matter and metric fluctuations interact in a universe that exhibits a transient stabilizing epoch of its extra dimension, which could also be viewed as a series of bounces of the extra dimension. We were primarily interested in the imprints of this epoch on the spectrum of the Bardeen potentials, motivated by the hope of combining an earlier phase, generating a scale invariant spectrum (e.g. in the framework of a modified ekpyrotic scenario, or via an incorporation of inflation within string gas cosmology), with a successful late time stabilizing mechanism, provided by a gas of massless string states.

We found that the spectrum of long wavelength perturbations remains unaltered by this epoch, which has important consequences for various approaches to stringy models of the early universe. In particular, specific realizations of the ekpyrotic/cyclic scenario or an incorporation of inflation into string gas cosmology become possible – we provided two proposals in this report, but many variations are viable.

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APPENDIX A: ANISOTROPIC STRESS

In this appendix, we will provide arguments as to why no anisotropic stress arises in the setup of \[12\], which we examined at the perturbative level in this article. In the following \(a, b, \ldots\) denote string world-sheet coordinates.

First note that our unperturbed background is undergoing FRW expansion in the non-compact directions while the compact direction is stabilized at the self dual radius by a string gas. As a preliminary, we wish to show that long wavelength perturbations of the metric do not affect the motion of strings to first order in the perturbation variables. We begin with the Polyakov action for a closed string

\[
S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \partial_a X^A \partial_a X^B g_{AB}(X),
\]

where we work in conformal gauge from the outset. Consider now perturbations around the background: \(g_{AB} = \delta g_{AB}^0 + h_{AB}\). It was shown in the appendix of \[12\] how one can inherit the string spectrum and the constraint algebra provided the derivatives of the background metric \(\delta g_{AB}^0\) (i.e. the Hubble factors) are several orders of magnitude smaller than the string energy scales. We want to make sure that we stay within this regime while studying the dynamical compactification of the extra dimension. Thus, when we consider the equations of motion for a closed string in this perturbed spacetime, we must neglect all derivatives of the unperturbed metric compared to the worldsheet derivatives of the string co-ordinates, as these will be of the order of the string scale whereas the metric derivatives are constrained to be much smaller \[12\].

Consider the equation of motion for a closed string

\[
\partial_a \left( \partial^a X^A \gamma_{CA}(X) \right) = \frac{1}{2} \partial_a X^A \partial^a X^B \partial_C g_{AB}(X),
\]

from which follows

\[
\frac{1}{2} \partial_a X^A \partial^a X^B \partial C h_{AB} = \partial_a \partial^a X^C + h_{AB}^C \partial_a \partial^a X^A + \partial_a X^A \partial^a X^B \partial_A h_{BD} g_0^{DC}.
\]

Now we expand \(X^A\) as \(X^A = X_0^A + \delta X^A\), where \(X_0^A\) is the solution to the unperturbed equation of motion \(\partial_a \partial^a X_0^A = 0\), see appendix of \[12\]. Our perturbed equation of motion then becomes

\[
\frac{1}{2} \partial_a X_0^A \partial^a X_0^B \partial C h_{AB} = \partial_a \partial^a \delta X^C + \partial_a X_0^A \partial^a X_0^B \partial_A h_{BD} g_0^{DC}.
\]

Notice that there are two vastly different scales in the equation above: the worldsheet derivatives and the spacetime derivatives. If we imagine expanding \(h_{AB}\) in terms of fourier modes, the derivatives would bring down a factor of the 4-d wave-vector. Realizing that we need to stay within the domain of validity of our setup, namely
that we cannot introduce fluctuations in the metric that would invalidate the conclusion in the appendix of [12] concerning the string spectrum, we need these fluctuations to vanish on energy scales several orders of magnitude smaller than the string scale. This still allows for metric fluctuations that are considerably short distance in nature, but nevertheless above the string scale. With this in mind, we can neglect all the terms in (A2) that are multiplied by derivatives of the metric perturbation, since these will be orders of magnitude smaller than the other ones. Henceforth, we obtain the result

$$\partial_a \delta^a \delta X^A = 0,$$  

that is, the string motion is unchanged by long wavelength perturbations.

Ordinarily this would signal the end of any hopes to study perturbative physics; this is not the case for us, since we are primarily interested in perturbing around a background which consists of a gas of such strings. As a result, if we consider the perturbed energy-momentum tensor for such a gas, it will contain no anisotropic stress. To see this, consider the off-diagonal spatial components which can be written as

$$\delta T_{ij} \propto \langle p_i \delta p_j \rangle + \langle \delta p_i \delta p_j \rangle,$$  

where $i \neq j$ and $\langle ... \rangle$ indicates an ensemble (gas) average. These components vanish, since we just determined that for individual strings $\delta p^0 = 0$, which means that there is no anisotropic stress to deal with in our setup.

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[65] Note that, as stressed in [21], none of these alternatives at the present stage solve the homogeneity and flatness problems of standard big bang cosmology without invoking a period of inflation (or something which effectively acts as inflation).

[66] The metric can always be cast into this form, such that t corresponds to cosmic time.

[67] Note that this series of bounces differs from the one in e.g. the cyclic scenario, since ϵ is small.

[68] In our notation Φ = −Ψ would be called the Newtonian potential in a four dimensional description.

[69] That the equations of motion for W decouple from the other ones is a direct consequence of the homogeneity in the y-direction.

[70] Note that fluctuations in SGC (before dilaton stabilization) were considered in [53, 54].

[71] Decreasing q induces a phase shift in Φq and an increase in Nq, while an increase in q leads to a decrease in Nq and no phase shift; however, the metric degrees of freedom decouple form Nq quickly and Nq itself is of no interest to us.

[72] We start at η = 1, because this is the characteristic
time scale of the background oscillations. This means, no oscillation could occur before $\eta_i$. 