Abstract—A cyclic prefix (CP) for an orthogonal frequency division multiplexing (OFDM) transmission shorter than the maximum delay spread of the channel results into inter-symbol interference (ISI) and inter-carrier interference (ICI). This paper shows that an appropriate frequency-domain multi-antenna (MIMO) precoding for OFDM can asymptotically cancel out ISI/ICI as the number of antennas goes to infinity. The method is based on introducing time-delay selectivity over the channel taps from which a conventional frequency-domain precoding method can be based on. This time-selectivity enables to asymptotically remove undesired delayed signals, while the frequency-selectivity in the precoders is preserved to suit the conventional MIMO-OFDM air interface where precoding is applied on a subcarrier level. Analysis reveals an optimization trade-off in the precoding design between interference removal and multi-path diversity gain. The resulting optimized precoders are shown to provide subsequent gains in asymptotic achievable rate with infinite number of antennas, as well as in symbol-error-rate performance of a moderate finite-antenna system.

Index Terms—Large-scale MIMO, OFDM, insufficient CP.

I. INTRODUCTION

In order for the upcoming 5G cellular networks to enable new use cases such as ultra-reliable low latency communications (URLLC), the ongoing 3GPP 5G NR standardization [1] recently introduced several new orthogonal frequency division multiplexing (OFDM) numerologies with larger subcarrier spacing (SCS) than the one defined in the 4G LTE standard [2]. A larger SCS implies a shorter OFDM symbol time duration, and thus a reduced transmission time. For sub-6GHz spectrums, 30 and 60 kHz SCSs are for example supported by NR in addition to the legacy 15 kHz SCS of LTE. The normal cyclic prefix (CP) duration is accordingly downscaled so that the CP overhead is maintained to about 7% of an OFDM symbol. The CP with these new SCSs can thus be much shorter than the delay spread of channels that are supported by the 15 kHz SCS numerology; and this will result into inter-symbol and inter-carrier interference (ISI and ICI) at the receiver. In order to cope with this, 5G NR introduced in addition an extended CP for 60 kHz SCS [1] which however then imposes a high 25% rate overhead.

Multi-input multiple-output (MIMO) systems make use of multiple antennas at transceiver nodes to improve the signal-to-noise-ratio (SNR) of the transmission link via precoding gains. The virtue of MIMO-OFDM is that, if the CP is longer than the delay spread of the channel, the multipath MIMO channel is transformed into multiple orthogonal flat MIMO channels, and subcarrier-level MIMO precoders can be easily designed accordingly. Conventional frequency-selective MIMO precoders are thus typically based on the Fourier transform of the channel impulse response evaluated at the targeted subcarrier frequencies.

As the number of antennas goes to infinity in a MIMO system, a well-known channel hardening effect occurs and the effective precoded channels on each subcarrier tend to a constant [3]. In other words, the frequency-selectivity of the effective channels from frequency-selective precoding is vanishing if the CP is longer than the delay spread. Nevertheless, the time-dispersion of the multi-path channel as observed by the receiver before dropping the CP remains; and in [4] it is shown that without CP and conventional frequency-selective precoders, the ISI/ICI power does not vanish with an infinite number of antennas.

In this paper, it is shown that an appropriately-modified subcarrier-level precoding can nonetheless average out the ISI and ICI in OFDM with insufficient CP as the number of antennas goes to infinity. To achieve this, we propose a precoding method that exploits the spatial selectivity offered by large antenna array to construct MIMO precoders which are time-delay selective in addition to their conventional frequency-selectivity. The precoders are based on the Fourier response of a truncated channel impulse response (CIR) of about the CP length, and thus combination of only certain channel paths that fall within the supported delay range. We show that finding the truncation threshold that asymptotically maximize the subcarriers’ signal-to-interference-noise-ratios (SINR) and resulting achievable rate leads to an optimization trade-off between interference cancellation and multi-path diversity gain. The benefit of the method is shown via asymptotic rate analysis with an infinite number of antennas, and then confirmed by numerical symbol error rate (SER) evaluation with a moderate finite array system of 64 antennas.

II. TRUNCATED CHANNEL IMPULSE RESPONSE

We start by discussing the general idea of the proposed design, illustrated in Fig. [1] Consider the transmission of an OFDM signal via a time-domain multi-antenna CIR of L taps

\[
h[k] = \sum_{p=0}^{L-1} h_p\delta_{k-p}\]  

(1)

where \( h_p \in \mathbb{C}^{1 \times N_t} \) is the channel path coefficient at delay \( p \) (in samples), \( N_t \) is the number of transmitter antennas, and \( \delta_k \)
is the Kronecker delta function. The entries of \( h_p \) are assumed independently and identically distributed (i.i.d.) according to the complex normal distribution \( CN(0, E_p) \).

In OFDM, data symbols are by design transmitted in the frequency-domain, through different frequency-flat channels. It is therefore desirable to have subcarrier-specific precoders. Subcarrier-level, i.e. frequency-selective, precoding enables a high precoding gain when combined with OFDM. The precoders are conventionally constructed according to the frequency response of the CIR

\[
\hat{h}_i = \sum_{k=0}^{N_{in}-1} h[k] e^{-j2\pi \frac{ik}{N_{in}}} = \sum_{p=0}^{L-1} h_p e^{-j2\pi \frac{ip}{N_{in}}} \tag{2}
\]

where \( i \) is the subcarrier index, \( N_{in} \) is the size of the inverse fast Fourier transform (IFFT) in the OFDM modulation.

A MIMO channel behaves asymptotically as a spatial filter with a large number of antennas: Only transmitted signals collinear with the channel will have its multiple paths that will coherently combine at the receiver or otherwise would average out and be totally attenuated. Following this observation, in order to remove ISI/ICI in the case of an insufficient CP, we propose to precode only according to a combination of certain channel taps having delays in the range of the CP.

For a single-user transmission with one receive antenna, the maximum-ratio-transmission (MRT) precoder \( \hat{w}(\tau_{tr}) \) on the \( i \)th subcarrier \( w_i \in \mathbb{C}^{N_t \times 1} \) adapted to the truncated channel is therefore

\[
\hat{w}_i(\tau_{tr}) = \frac{\hat{h}_i^H(\tau_{tr})}{\|\hat{h}_i(\tau_{tr})\|} \tag{5}
\]

\[
= \frac{1}{\omega_i(\tau_{tr})} \sum_{p=0}^{\tau_{tr}-1} \|h_p e^{j2\pi \frac{ip}{N_{in}}} \| \tag{6}
\]

where \( \hat{h}^H \) is the Hermitian transpose, and \( \omega_i(\tau_{tr}) = \|\hat{h}_i(\tau_{tr})\| \) is the normalization constant. If \( \tau_{tr} = L \), the CIR is not truncated and this falls back to conventional MRT precoding. We will focus on this single-user scenario in the rest of the paper but we briefly mention that the method can be directly extended to a multi-user channel: A multi-user MIMO channel is the concatenation of the frequency responses of each single-user channel. Without loss of generality, different thresholds can be applied individually to each user’s CIR to obtain the frequency-response of a truncated multi-user channel. As shown in [4] in the case of no CP, multi-user interference in the large antenna array regime is canceling out and does not play a role in the saturation of the SINR that follows from an insufficient CP.

### III. System Model

We consider a downlink transmission from a base station with \( N_t \) antennas transmitting to a user equipment with a single antenna. An OFDM modulation is defined by a subcarrier spacing \( \Delta_f \), an IFFT size \( N_{in} \), and a CP length \( N_{cp} \). An OFDM symbol (without CP) has then a time duration of \( T_s = 1/\Delta_f \) with sampling period \( T_{sp} = T_s / N_{in} \). We consider that a total of \( N_{sc} \) consecutive subcarriers are allocated by i.i.d. data symbols taken from the same constellation with average power \( P \).
A. Precoded Multi-Antenna OFDM Transmission

The data symbol $x_{b,i}$ for the $i$th subcarrier of the $b$th OFDM block is spatially precoded with $w_l(\tau_l) \in \mathbb{C}^{N_{tx} \times 1}$, and modulated as

$$s_b[k] = \frac{1}{\sqrt{N_{sc}}} \sum_{l=0}^{N_{tx}-1} w_l(\tau_l) x_{b,i} e^{j2\pi \frac{mk}{N_m}} \tag{7}$$

for $-N_{cp} \leq k \leq (N_{it} - 1)$, and $s_b[k] = 0$ otherwise. OFDM blocks are then consecutively transmitted in the signal

$$s[k] = \sum_b s_b[k - b(N_{cp} + N_{it})] \in \mathbb{C}^{N_{tx} \times 1}, \tag{8}$$

later convolved with the channel $h[k] \in \mathbb{C}^{1 \times N_t}$ given in (1). Assuming that the maximum delay of the channel $L$ is less than a block length, interference for one symbol will only depends of the previous block. Then for simplicity, we can limit the analysis to a OFDM signal of two consecutive blocks as

$$s[k] = s_0[k] + s_{-1}[k + N_{it} + N_{cp}] \tag{9}$$

for $-N_{it} - 2N_{cp} \leq n \leq (N_{it} - 1)$.

B. Received and Demodulated Signal

The received signal is

$$r[k] = \sum_{m=0}^{k+N_{cp}} h[m]s[k-m] + z[k] \tag{10}$$

where $z[k] \sim \mathcal{CN}(0, \sigma_z^2)$ is a zero-mean additive white Gaussian noise (AWGN) with variance $\sigma_z^2$. After CP removal, for $0 \leq k \leq (N_{it} - 1)$ we have

$$r[k] = \sum_{m=0}^{k+N_{cp}} h[m]s_0[k-m] + \sum_{m=k+N_{cp}+1}^{k+N_{it}-1} h[m]s_{-1}[k-m+N_{it}+N_{cp}] + z[k]$$

and then by substitution of (2)

$$r[k] = \frac{1}{\sqrt{N_{sc}}} \sum_{l=0}^{N_{sc}-1} \left( \sum_{m=0}^{k+N_{cp}} h[m] e^{-j2\pi \frac{m(k+N_{it})}{N_m}} \right) w_l(\tau_l) x_{0,i} e^{j2\pi \frac{mk}{N_m}} \tag{11}$$

$$\quad + \frac{1}{\sqrt{N_{sc}}} \sum_{l=0}^{N_{sc}-1} \left( \sum_{m=k+N_{cp}+1}^{k+N_{it}-1} h[m] e^{-j2\pi \frac{m(k+N_{it})}{N_m}} \right) w_l(\tau_l) x_{-1,i} e^{j2\pi \frac{m(k+N_{it})}{N_m}}$$

$$\quad + z[k].$$

The received signal is then demodulated by FFT which gives the demodulated symbol for the $i$th subcarrier

$$y[i] = \frac{\sqrt{N_{sc}}}{N_{it}} \sum_{k=0}^{N_{sc}-1} r[k] e^{-j2\pi \frac{N_{it}k}{N_m}} \tag{11}$$

$$= \frac{1}{\sqrt{N_{sc}}} \sum_{l=0}^{N_{sc}-1} \left( \sum_{m=0}^{N_{tx}-1} h[m] e^{-j2\pi \frac{m(k+N_{it})}{N_m}} \right) w_l(\tau_l) x_{0,i} + \frac{1}{\sqrt{N_{sc}}} \sum_{l=0}^{N_{sc}-1} \left( \sum_{m=0}^{N_{tx}-1} h[m] e^{-j2\pi \frac{m(k+N_{it})}{N_m}} \right) w_l(\tau_l) x_{-1,i} + n[i]$$

where

$$h_{0,i,i}(\tau_l) = \frac{1}{N_{it}} \sum_{k=0}^{N_{sc}-1} h[m] w_l(\tau_l) e^{-j2\pi \frac{m(k+N_{it})}{N_m}} \tag{11}$$

is the ICI channel coefficient from the $l$th subcarrier,

$$h_{-1,i,i}(\tau_l) = \frac{1}{N_{it}} \sum_{k=0}^{N_{sc}-1} \sum_{m=k+N_{sc}+1}^{L-1} h[m] w_l(\tau_l) e^{-j2\pi \frac{m(k+N_{sc}+1)}{N_m}} \tag{11}$$

is the ISI channel coefficient from the $l$th subcarrier, and

$$n[i] = \frac{1}{\sqrt{N_{sc}}} \sum_{k=0}^{N_{sc}-1} z[k] e^{-j2\pi \frac{N_{it}k}{N_m}}$$

is the post-processed AWGN with variance $\sigma_n^2 = \frac{N_{sc}}{N_{it}} \sigma_z^2$.

C. SINR and Achievable Rate

Accordingly, the SINR on the $i$th subcarrier is

$$\text{SINR}_i(\tau_l) = \frac{|h_{0,i,i}(\tau_l)|^2}{\sum_{l=0}^{N_{sc}-1} |h_{0,i,i}(\tau_l)|^2 + \sum_{l=0}^{N_{sc}-1} |h_{-1,i,i}(\tau_l)|^2 + 1/\text{SNR}_{\tau}} \tag{12}$$

with $\text{SNR}_{\tau} = P/\sigma_n^2$; and an achievable rate for the given channel realization is expressed in $\text{bps/Hz}$ by

$$R(\text{SNR}; \tau_l) = \frac{N_{it}}{(N_{it} + N_{cp} + N_{sc})} \sum_{l=0}^{N_{sc}-1} \log_2(1 + \text{SINR}_i(\tau_l)). \tag{13}$$

IV. OPTIMIZED TIME-FREQUENCY SELECTIVE PRECODING

For a given channel realization, we seek to find a threshold in the precoding design (3) that maximize the rate as

$$\tau_{\text{max}}(\text{SNR}) = \arg \max_{\tau_l} R(\text{SNR}; \tau_l). \tag{14}$$

A. Asymptotic Optimization in the Large Antenna Regime

The above optimization can be simplified in the asymptotic regime $N_t \to \infty$ with $P/\sigma_n^2 \to 0$ such that the operational SNR, $\text{SNR}_{\tau} = N_t \text{SNR} = PN_t/\sigma_n^2$, is fixed. A similar asymptotic regime is e.g. considered in [7] for energy efficiency. Our motivation for this regime here is more related to performance under a fixed quality of service with a finite-size symbol constellation such as quadrature amplitude modulation (QAM). Indeed with a fixed constellation size, the SNR needed to reach e.g. a certain SER level is shifting in the low SNR-regime with the precoding gain $10 \log_{10} N_t$ (dB) as $N_t$ increases. We have the following result whose derivation details are in appendix.

**Proposition 1.** With time-frequency selective precoding (5), $\text{SINR}_i(\tau_l) \to \text{SINR}_i^\infty(\tau_l)$ as $N_t \to \infty$ with $P/\sigma_n^2 \to 0$ such that $\text{SNR}_{\tau} = PN_t/\sigma_n^2$ is fixed, where

$$\text{SINR}_i^\infty(\tau_l) = \frac{\beta_{\tau_l}}{\beta_{\tau_l}^2 + \frac{P_t}{N_t} \sum_{l=0}^{N_{sc}-1} \frac{|\rho_{\tau_l}(\tau_l)|^2}{2N_t^2 \sin^2(\frac{\pi l}{N_{sc}})}} + 1/\text{SNR}_{\tau} \tag{15}$$

with $\alpha_{\tau_l} = \sqrt{\frac{\sum_{l=0}^{N_{sc}-1} E_p}{\sum_{l=0}^{N_{sc}-1} |\rho_{\tau_l}(\tau_l)|^2}}$, $\beta_{\tau_l} = \frac{1}{\sum_{p=0}^{N_{sc}} \rho_{\tau_l}(\tau_l) E_p (p - N_{sc})}$, and

$$\rho_{\tau_l}(\eta) = \frac{1}{\alpha_{\tau_l}^2} \sum_{p=0}^{N_{sc}} \rho_{\tau_l}(\tau_l) E_p (p - N_{sc}) \left( e^{j\pi \frac{(p-N_{sc})^2}{N_m}} - 1 \right).$$

The asymptotic form $\text{SINR}_i^\infty(\tau_l)$ is dependent of the subcarrier index $i$. This is because not all subcarriers in the IFFT are
occupied, and as a result, subcarriers in the middle of the band receive more interference from neighboring subcarriers, and thus have a lower SINR than the edge subcarriers. If all subcarrier are occupied \((N_{\text{sc}} = N_{\text{fft}})\), without cyclic prefix \((N_{\text{cp}} = 0)\), and without time-selectivity in the precoder \((\tau_{\text{tr}} = L)\), we recover the expression derived in [4]. Here however, unlike with conventional-frequency precoding, we observed that with an appropriate truncation threshold in the precoder design, the system is not necessarily interference-limited.

**Corollary 1.** If \(\tau_{\text{tr}} \leq N_{\text{cp}} + 1\) then \(\text{SINR}_{\text{tr}}^\infty(\tau_{\text{tr}}) = \alpha^2_{\tau_{\text{tr}}} \text{SNR}_{\text{op}},\) i.e., ISI and ICI vanish as \(N_{\text{f}} \to \infty\).

Meanwhile, the term \(\alpha^2_{\tau_{\text{tr}}}\) represents a SNR loss from the maximum received SNR, \(\alpha^2_{\text{SNR}_{\text{op}}},\) with full multipath diversity. A threshold larger than the CP might thus increase the SINRs, and as a result a trade-off appears between interference mitigation and multipath diversity gain for rate maximization. A threshold maximizing the asymptotic achievable rate at a given operational SNR is expressed as

\[
\tau_{\text{max}}^\infty(\text{SNR}_{\text{op}}; \tau_{\text{tr}}) = \arg \max_{\tau_{\text{tr}}} R^\infty(\text{SNR}_{\text{op}}; \tau_{\text{tr}}) \tag{16}
\]

where

\[
R^\infty(\text{SNR}_{\text{op}}; \tau_{\text{tr}}) = \frac{N_{\text{fft}}}{N_{\text{fft}} + N_{\text{cp}}} \sum_{i=0}^{N_{\text{sc}} - 1} \log_2(1 + \text{SINR}_i^\infty(\tau_{\text{tr}})). \tag{17}
\]

If all subcarriers were occupied, they would have the same \(\text{SINR}_i^\infty(\tau_{\text{tr}})\) value and \(\tau_{\text{max}}^\infty(\text{SNR}_{\text{op}}) = \arg \max_{\tau_{\text{tr}}} \text{SINR}_i^\infty(\tau_{\text{tr}})\).

With \(N_{\text{sc}} < N_{\text{fft}},\) optimization based on one subcarrier’s SINR may not return the optimal solution for the rate, but intuitively an optimization from a middle subcarrier with one of the lowest SINRs can be expected to well-approximate the rate optimization.

This asymptotic optimization only depends of the system parameters, the operational SNR, and the average channel energy of the taps (which can be approximated from a single channel acquisition in the large antenna regime as \(\|h_i\|^2/N_{\text{f}} \to E_p\) with \(N_{\text{f}} \to \infty\)). The optimum threshold, which may not be unique, can be found by exhaustive search. The search only needs to be started from \(\tau_{\text{tr}} = N_{\text{cp}} + 1\) as any threshold inside the CP would be suboptimal. Then as \(\tau_{\text{tr}}\) increased, the rate/SNR expression will only change when a non-zero channel tap is included in the precoder, and thus the size of the search follows from the number of non-zero taps rather than time samples. As \(\text{SNR}_{\text{op}} \to 0,\) the interference term becomes negligible in \([15]\) and \(\tau_{\text{max}}^\infty \to L.\) On the other hand as \(\text{SNR}_{\text{op}} \to \infty\) the interference dominates and \(\tau_{\text{max}}^\infty \to N_{\text{cp}} + 1.\)

**B. Numerical Optimization at a Given SNR**

We now optimized the precoder for practical channel models. We consider \(N_{\text{sc}} = 600\) subcarriers separated by \(\Delta_f = 60\) kHz, \(N_{\text{fft}} = 2048,\) and a normal CP of \(N_{\text{cp}} = 144 \approx 7\% N_{\text{fft}}.\) For comparison, we also consider an extended CP of \(N_{\text{cp}} = 512 = 25\% N_{\text{fft}}\) as defined in 5G NR specification [11].

1) **ETU channel:** We first consider the Extended Typical Urban (ETU) channel model whose normalized power delay profile (PDP) is displayed on the upper part of Fig. 2(a). As it can be seen, with a 60 kHz SCS and a normal CP length, 3 channel taps are escaping the CP. The lower part of Fig. 2(a) shows the asymptotic rate \(R^\infty(\text{SNR}_{\text{op}}; \tau_{\text{tr}})\) with the proposed time-frequency selective precoding as a function of the truncation threshold \(\tau_{\text{tr}}\) and assuming an operational SNR of 25 dB. The rate is optimized for any \(\tau_{\text{max}}^\infty \in [62, 63, \ldots, 197]\) which corresponds to precode only according to the channel paths inside the CP and thus here \(\tau_{\text{max}}^\infty = N_{\text{cp}} + 1\) maximize the rate.

The middle part of Fig. 2(a) shows similarly the asymptotic SINR of the edge subcarrier \(\text{SINR}_0^\infty(\tau_{\text{tr}})\) and of the middle subcarrier \(\text{SINR}_i^\infty(\tau_{\text{tr}})\). The optimization of \(\text{SINR}_0^\infty(\tau_{\text{tr}})\) returns the same optimized threshold than for the rate, while \(\text{SINR}_i^\infty(\tau_{\text{tr}})\) would be maximized by including an additional channel tap, i.e. with any \(\tau_{\text{tr}} \in [198, 199, \ldots, 283]\).
2) TDL-C channel (2 μs delay scaling): We now apply the same analysis to the more frequency-selective TDL-C channel \[8\]. This channel model has a tunable delay scaling which we select to be very large, 2 μs, for the purpose of illustrating the optimization trade-off in the precoder. The resulting PDP is shown on the upper part of Fig. 2(b). The asymptotic rate \( R_{\infty}(\text{SNR}_{\text{op}}; \tau_{\text{op}}) \) with \( \text{SNR}_{\text{op}} = 20 \) dB shown on the lower part of Fig. 2(b) is maximized here for any \( \tau_{\text{op}} \in [203, 204, \ldots, 229] \) which corresponds to precoding according to the first 11 taps of which 6 of them are outside the CP. In this case, we have \( \tau_{\text{max}} > N_{\text{cp}} + 1 \) and the gain in rate from the optimization is noticeable compared to selecting \( \tau_{\text{F}} = N_{\text{cp}} + 1 \). Again, the middle part of Fig. 2(b) shows that maximizing the SINR on a middle subcarrier leads to the same optimized truncation threshold than maximizing the rate.

C. Optimized Asymptotic Rate

By performing similar optimizations, we obtain the asymptotic rate \( R_{\infty}(\text{SNR}_{\text{op}}; \tau_{\text{max}}(\text{SNR}_{\text{op}})) \) of optimized time-frequency selective precoding (labeled TF-precoding) on Fig. 3. This is compared to the asymptotic rate with conventional frequency-selective precoding (labeled F-precoding) with normal and extended CP. As expected, optimized time-frequency selective precoding always performed the best and provides notable improvements in the high-SNR region where the system is interference-limited. Using an extended CP converts interference to useful power and thus improve the SINR in the high-SNR regime. However, this solution imposes a high rate penalty since a large proportion of the transmission-time is only used for CP. Time-frequency selective precoding does not have this increased rate penalty while is also able to remove or mitigate interference.

V. SER SIMULATIONS IN THE FINITE-ANTENNA REGIME

Finally, we use the asymptotic optimization in the large antenna regime from the previous section and verify by numerical SER evaluation that the proposed method provides also performance gain for a finite multi-antenna system with \( N = 64 \), shown on Fig. 4. Block-fading Rayleigh channels are assumed with independent realizations every subframe of 14 OFDM blocks. A unique truncation threshold per constellation is used in the time-frequency precoding to obtain the SER curves of Fig. 4. Since the regime of operation of a finite constellation is a rather small SNR region, the optimization can indeed be performed only for a unique SNR value where e.g. the constellation is expected to reach a sufficiently-low error rate.

First, we consider a 64 QAM transmission with ETU channel. With a sufficient CP, this constellation would reach a \( 10^{-3} \) SER in an operational SNR of about \( \text{SNR}_{\text{op}} = 25 \) dB. We therefore use the optimized threshold \( \tau_{\text{max}} = N_{\text{cp}} + 1 \) from Fig. 2(a). As shown on the figure, the proposed time-frequency selective precoding can provide here 4 dB SNR gain at \( 10^{-3} \) SER.

For the TDL-C channel model with 2 μs delay scaling, we consider a 16 QAM transmission which is expected to reach a \( 10^{-3} \) SER in an operational SNR of about \( \text{SNR}_{\text{op}} = 20 \) dB. From the optimization in Fig. 2(b) the rate of the transmission with time-frequency selective precoding is maximized in this case with \( \tau_{\text{max}}(20 \text{ dB}) = 203 > N_{\text{cp}} + 1 \). Using \( \tau_{\text{F}} = \tau_{\text{max}}(20 \text{ dB}) \) in the precoder leads to 3 dB SNR gain at \( 10^{-3} \) SER compared to conventional frequency-selective precoding. As an alternative, the figure also shows the performance of time-frequency selective precoding with \( \tau_{\text{F}} = N_{\text{cp}} + 1 \). With such value of \( \tau_{\text{F}} \) in the precoder design, more interference is removed and the performance is improved in the high-SNR region compared to conventional frequency-selective precoding. However, using \( \tau_{\text{F}} = N_{\text{cp}} + 1 \) here is too time-selective so that a high portion of the multi-path diversity is lost, leading to a constant SNR loss compared to the optimized precoding with \( \tau_{\text{max}} \).
VI. CONCLUSION

We proposed a novel time-frequency selective MIMO precoding design for OFDM system with insufficient CP. The method is based on a truncation of the channel impulse response in order to reduce the time dispersion of the effective precoded channel. An optimum threshold in the high-SNR regime is set equal to the CP length in order to achieve an interference-free transmission as the number of antennas goes to infinity. In a medium SNR regime, an optimized threshold not necessarily equal to the CP can further increase the system performance by accumulating additional multi-path diversity gain at the cost of limited interference.

APPENDIX

Prop. 1 generalizes [4, Prop. 1] and can be derived similarly. Using the precoder [5] and the Heaviside step function

\[ w[n] = \begin{cases} 1, & 0 \leq n \\ 0, & \text{otherwise} \end{cases} \]

as in [9], we can reformulate the ICI/ISI channels as

\[ \mathcal{H}_{0,i,i} = \sum_{k=0}^{N_t-1} \sum_{m=0}^{L-1} \sum_{p=0}^{N_r-1} h[m] h^H[p] e^{-j 2 \pi \frac{(m-p)k(l+i)}{N_m}} w[k - m - N_c_p] \]

\[ \mathcal{H}_{-1,i,i} = \sum_{k=0}^{N_t-1} \sum_{m=N_t+1}^{L-1} \sum_{p=0}^{N_r-1} h[m] h^H[p] e^{-j 2 \pi \frac{(m-N_c_p-p)k(l+i)}{N_m}} \times \left( 1 - w[k - m - N_c_p] \right) \]

As \( N_t \to \infty \), \( h[m] h^H[p] / N_t \to E_p \delta_{m-p} \) and as a by product \( \omega(\tau_{0})/\sqrt{N_t} \to \alpha_{\tau_{0}} \). We then have

\[ \mathcal{H}_{0,i,i} / \sqrt{N_t} \to \frac{1}{\alpha_{\tau_{0}} N_t} \sum_{p=0}^{N_r-1} E_p \sum_{k=0}^{N_t-1} e^{j 2 \pi \frac{k(l+i)}{N_m}} w[k - N_c_p - p] \]

\[ \mathcal{H}_{-1,i,i} / \sqrt{N_t} \to \frac{e^{j 2 \pi \frac{N_c_p}{N_m}}}{\alpha_{\tau_{0}} N_t} \sum_{p=N_t+1}^{N_r-1} E_p \sum_{k=0}^{N_t-1} e^{j 2 \pi \frac{k(l+i)}{N_m}} (1 - w[k - p + N_c_p]). \]

Two cases need to be considered. If \( l = i \)

\[ \mathcal{H}_{0,i,i} / \sqrt{N_t} \to \frac{1}{\alpha_{\tau_{0}} N_t} \left( \sum_{p=0}^{N_r-1} E_p N_{t+p} + \sum_{p=N_r}^{N_t-1} E_p (N_{t+p} + N_c - p) \right) = \left( \alpha_{\tau_{0}} - \beta_{\tau_{0}} \right) N_{t} \]

\[ \alpha_{\tau_{0}} \beta_{\tau_{0}} \]

and similarly

\[ \mathcal{H}_{-1,i,i} / \sqrt{N_t} \to \frac{e^{j 2 \pi \frac{N_c_p}{N_m}}}{\alpha_{\tau_{0}} N_t} \beta_{\tau_{0}}. \]

If \( l \neq i \), using \( \sum_{k=b}^{a} e^{j \omega \cdot k} \) for \( r \neq 1 \), the expressions simplify to

\[ \mathcal{H}_{0,i,i} / \sqrt{N_t} \to \frac{1}{\alpha_{\tau_{0}} N_t} \sum_{p=0}^{N_r-1} E_p \sum_{k=0}^{N_t-1} e^{j 2 \pi \frac{k(l+i)}{N_m}} \]

\[ + \sum_{p=N_t+1}^{N_r-1} E_p \sum_{k=p-N_t}^{N_t-1} e^{j 2 \pi \frac{k(l+i)}{N_m}} \]

\[ = \frac{1}{\alpha_{\tau_{0}} N_t} \sum_{p=0}^{N_r-1} E_p e^{j 2 \pi \frac{l(i-i)}{N_m}} \frac{1}{1 - e^{j 2 \pi \frac{l(i-i)}{N_m}}} \]

\[ = \rho_{\tau_{0}} (l - i) \]

\[ \alpha_{\tau_{0}} \beta_{\tau_{0}} \]

and similarly

\[ \mathcal{H}_{-1,i,i} / \sqrt{N_t} \to -e^{j 2 \pi \frac{N_c_p}{N_m}} \rho_{\tau_{0}} (l - i) \frac{1}{1 - e^{j 2 \pi \frac{N_c_p}{N_m}}}. \]

The final result then follows by substitutions in [12] and further simple simplifications.

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