Evaluation of the Primary Energy of UHE Photon-induced Atmospheric Showers from Ground Array Measurements

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Abstract

A photon induced shower at $E_{\text{prim}} \geq 10^{18}$ eV exhibits very specific features and is different from a hadronic one. At such energies, the LPM effect \cite{6, 8} delays in average the first interactions of the photon in the atmosphere and hence slows down the whole shower development. They also have a smaller muonic content than hadronic ones. The response of a surface detector such as that of the Auger Observatory to these specific showers is thus different and has to be accounted for in order to enable potential photon candidates reconstruction correctly. The energy reconstruction in particular has to be adapted to the late development of photon showers. We propose in this article a method for the reconstruction of the energy of photon showers with a surface detector. The key feature of this method is to rely explicitly on the development stage of the shower. This approach leads to very satisfactory results ($\approx 20\%$). At even higher energies ($5 \times 10^{19}$ eV and above) the probability for the photon to convert into a pair of $e^+e^-$ in the geomagnetic field becomes non negligible and requires a different function to evaluate the energy with the proposed method. We propose several approaches to deal with this issue in the scope of the establishment of an upper bound on the photon fraction in UHECR.

1 General framework

This study is aimed to analyze the response of a surface detector (SD) of extensive atmospheric showers induced by primary photons of ultra high energy (more than $10^{18}$ eV). Particular applications will be made for the Auger Observatory, where the Surface Detector is an array of water Cherenkov tanks, at an average altitude of 1400 m a.s.l. \cite{10}.

2 Energy reconstruction of extensive air showers with the Auger surface detector using a Monte Carlo based calibration

Measuring the primary energy $E_{\text{prim}}$ of an extensive atmospheric shower from measurements in a sparse ground array is not straightforward. Classical evaluations for showers induced by protons or nuclei use a relation between $E_{\text{prim}}$ and the signal interpolated at a given distance $r_0$ from the shower axis \cite{2}:

$$S(r_0) = E_{\text{prim}}^\alpha f(\theta) \quad (\theta : \text{zenith angle})$$

The exponent $\alpha$ is slightly less than 1 (typically 0.95), to account for the longitudinal stretching of the shower, increasing with $E_{\text{prim}}$. The function $f(\theta)$ includes essentially the following ingredients:
• the longitudinal evolution of the shower: the size at ground level depends on the slant depth $X = X_{\text{ground}}/\cos \theta$, where $X_{\text{ground}}$ is the vertical thickness at ground level (practically constant for a given detector site).

• the dependence of the signal to the incidence angle of the shower particles, determined by the type and the geometry of the detector.

• the sensitivity of the detector to different particles: a thin scintillator counts the charged particles (mainly electrons and positrons for moderately inclined showers), while a Cherenkov water tank sees the photon by their internal cascading, and has an enhanced sensitivity to muons: $f(\theta)$ will be more sensitive to the muonic profile, which is different from the electromagnetic one.

In usual conditions, for a detector at moderate altitude, the maximum $X_{\text{max}}$ of the longitudinal profile is above or around the ground level ($X_{\text{max}} \lesssim X_{\text{ground}}$): inclined showers hit the ground in their decreasing phase; moreover, the acceptance does not increase with $\theta$, so that $f(\theta)$ is a decreasing function.

To test these features, we used samples of showers generated with AIRES [13] from both protons and photons, in the energy range $10^{18}$ to $10^{20.5}$ eV, and we apply a standard detector simulation for the Auger array, and an event reconstruction procedure to evaluate the signal in each tank and the interpolated signal $S(1000)$. As can be seen for protons simulations on fig. 1, the factorization is approximately valid for showers initiated by protons or nuclei, because the shower-to-shower fluctuations of the longitudinal profile and of the muon/electromagnetic ratio are not too weak. A bias is expected as a function of the primary mass (mass number $A$) and the primary energy, but studies on simulated events suggests than the above formula can give a precision of the order of 20 % on $E_{\text{prim}}$ if $S(r_0)$ is precisely known, and if, of course, modelling errors in shower simulations may be neglected (see [2]).

![Proton Energy Scale](image)

Figure 1: Relation between primary energy and signal at 1000 m from the core, for simulated proton showers (AIRES), at various zenith angles and energies. Energies are taken in EeV and $S(1000)$ is given in VEM. The exponent of the energy $P$ is there set at 0.96, as indicated. For a given value of $\cos \theta$, the points are shifted to make the plot more readable.
The situation is expected to be quite different for photon induced showers. First, their muonic content is very low compared to a shower initiated by a proton or a nucleus. Secondly, their longitudinal development is slower, especially for $E_{\text{prim}} \gtrsim 10^{19}$ eV, where the LPM effect delays the first steps of the electromagnetic cascade. Simulations indicate that the average value of $X_{\text{max}}$ exceeds 900 g/cm$^2$ at $E_{\text{prim}} = 10^{19}$ eV, and increases rapidly with $E_{\text{prim}}$ (see fig. 2). Then, nearly vertical showers reach the ground before their maximum, and for a given primary energy, inclined showers may have a larger density than vertical ones. The factorization is no more valid, and $f(\theta)$ should be replaced by a function strongly dependent of $E_{\text{prim}}$. This is clear on Fig. 3 for the same energy range as in Fig. 1.

Figure 2: Average depth of shower maximum $X_{\text{max}}$ versus energy simulated for primary photons, protons and iron nuclei. Depending on the specific particle trajectory through the geomagnetic field, photons above $\approx 5 \times 10^{19}$ eV can create a preshower: as indicated by the splitting of the photon line, the average $X_{\text{max}}$ values then do not only depend on primary energy but also arrival direction. For nuclear primaries, calculations for different hadronic interaction models are displayed (QGSJET01 [9], QGSJETII [12], SIBYLL2.1 [11]). Also shown are experimental data (for references to the experiments, see [5]).

Moreover, above a threshold energy depending on the strength of the geomagnetic field ($5 \times 10^{19}$ eV for the southern site of the Auger Observatory, [1]), the photon may undergo a conversion into an $e^+e^-$ pair with a non-negligible probability before entering the atmosphere [7]; the electrons themselves radiate hard synchrotron photons: this causes a electromagnetic “pre-showering” and the particles entering the atmosphere are mainly below the LPM threshold [4], [1]. Then the atmospheric shower will have a much faster development than the one induced by an unconverted photon at the same primary energy (similar to showers from a photon in the range $10^{18} - 10^{19}$ eV). This point will be developped in section 4.
3 The “universal profile” picture

This picture is based on the fact that the “remote electromagnetic profile” (evolution of the density of photons, electrons, positrons with the depth, at a large distance from the core) follows closely the global profile (total number of charged particles in the shower), with a delay due to the lateral diffusion (about 150 g/cm$^2$ at 1000 m from the axis). If we neglect the contribution of the muons to the signals, and if we suppose that the detector response varies smoothly with the incidence angle, we expect that the ratio $S(r_0)/E_{prim}$, when expressed as a function of $X = X_{ground}/\cos \theta$, will behave as a delayed profile, which, in first approximation, is an universal function of $X - X_{max}$, with a shape similar to the Gaisser-Hillas function commonly used to describe the global profile.

Actually, plotting the ratio $S(1000)/E_{prim}$ as a function of $\Delta X = X_{ground}/\cos \theta - X_{max}$ for simulated photon showers at various energies and zenith angles (see Fig. 4), we obtain a overall curve with a maximum around 150 g/cm$^2$.

It is remarkable that showers from magnetically converted photons follow a very similar law, in spite of the large differences in $X_{max}$; moreover, their dispersion is reduced, because they give a superposition of subshower at lower energies. The observed profile is not exactly represented by a Gaisser-Hillas function, because of the small muonic fraction, the detector shape effects, and the fact that the end of the descent is less steep at large distance than in the global profile. We use a empirical parametrization:

$$S(1000)/E_{prim} = 1.4 \frac{1 + \Delta X - 100}{1 + \left(\frac{\Delta X - 100}{340}\right)^2} \frac{\text{VEM}}{\text{EeV}}$$

where the depths are in g/cm$^2$; by convention VEM represents the signal given by a vertical muon.

To fully exploit this relation we have to know the value of $X_{max}$; for events seen by the ground detector only, $X_{max}$ is not measured, then we can use an average dependence on energy,

![Figure 3: Relation between primary energy and signal at 1000 m from the core, for photon induced showers (without pre-showering in the geomagnetic field), with the same conventions as in fig. 1.](image-url)
Figure 4: Ratio $S(1000)/E_{\text{prim}}$ as a function of $X_{\text{ground}}/\cos \theta - X_{\text{max}}$ for photon induced showers. Solid (black): unconverted photons; dash-dotted (red): magnetically converted photons. The curve is a fit on the unconverted ones.

Deduced from the simulations (for photons without pre-showering):

$$X_{\text{max}} = 856 + 141 \log_{10}(E_{\text{prim}}) \quad \text{with} \quad E_{\text{prim}} \text{ in EeV}$$

If we suppose here that both $S(1000)$ and $\theta$ (hence $X$) are reliably measured, even if the primary is a photon, then an iterative procedure may be applied, starting with a rough estimation of the energy (e.g., twice the value computed in the proton hypothesis, as we are expecting a clear underestimation of a factor 2 to 4 of the energy as we use the classic reconstruction):

- estimate $X_{\text{max}}$ from the above formula
- estimate $E$ from $S(1000)$ and $X - X_{\text{max}}$

In most cases the convergence is fast; however, if the value of $X - X_{\text{max}}$ is in the beginning of the ascending phase of the profile, the iteration may be problematic: a small value of $S(1000)/E$ gives a large value of $E$, possibly larger than the previous estimation, pushing down $X - X_{\text{max}}$, then one finds a smaller $S(1000)/E$, and so on. If this positive feedback is large, the fluctuations of $X_{\text{max}}$ produce large fluctuations on $E$, and sometimes the iteration diverges. On the contrary, in the descending phase, the feedback is negative. For these reasons, we discard the left tail on the profile by setting a minimum value of -50 g/cm$^2$ to $X - X_{\text{max}}$. As a consequence, we may underestimate the energy of nearly vertical showers with a late development. Fig. 5 shows that the resolution on energy can be significantly improved, compared to a “proton-like” evaluation with a simple power law. The resolution achieved with this method is roughly 20% even up to $10^{20}$ eV. Nevertheless, distribution tails shown here would degrade gradually at higher energies because of shower to shower important fluctuations, as discussed just below. It may be safe to
put a lower cut on the zenith angle (at $\simeq 35^\circ$ for example) to avoid the problem of showers seen well before their maximum (let us note that anyway the trigger acceptance us suppressed for such showers).

Figure 5: Resolution on photon energy. Black: “proton-like” method (factorization); red: this method. Zenith angles are taken between $35^\circ$ and $60^\circ$. Simulated energies from $10^{19}$ eV to $10^{19.4}$ eV have been taken there.

4 Handling magnetically converted photons

For the southern site of Auger, the conversion of the primary photon in a $e^+e^-$ pair as it enters the geomagnetic field becomes non negligible for energies above $5.10^{19}$ eV (Fig. 7). This phenomenon results in an electromagnetic preshower entering the atmosphere. As a consequence, the particles of the preshower carry individually less energy than the initial photon, and the shower in the atmosphere is less affected by the LPM effect in case of conversion than in case of non conversion. This results in an earlier development of the photon shower (actually the $X_{max}$ of a converted shower never exceeds largely 1000g/cm$^2$ whereas the non converted showers have a typical $X_{max}$ above this value).

The conversion probability depends on the energy and $B_\perp$, the projection of the geomagnetic field perpendicular to the incoming photon direction (see [1]). These considerations lead to the conversion probability maps shown on the figure Fig. 6. These maps show clearly that the conversion process has to be considered for $E_{prim} > 50$ EeV.
As the development of a converted photon shower is earlier compared to that of an unconverted one (with the same characteristics), this additional effect has an impact on the method proposed herein: for converted showers, $X_{\text{max}}$ does not depend on $E_{\text{prim}}$ in the same way than for unconverted photons. If we don’t know if the photon is converted or not, the method described cannot be applied as we don’t know which relation should be applied. Using some average dependence between converted and unconverted ones to give the relation $S(1000)/E_{\text{prim}}$ as a function of $\Delta X$ (e.g. using a combination of the two solutions weighted by the probability of converting or not, accounting for the direction and approximate the energy of the event) can be thought of. Nevertheless, this is not a satisfactory solution: because of the presence of a maximum in the distributions, the average between two identical profiles at different positions, like the ones shown on fig.3 is not a profile at some intermediate position. Even when correcting for such a bias, the resulting resolution in energy would be poor. We have nevertheless several possibilities to deal with this issue and perform studies toward the setting of upper bound on the flux of photons.

A possible way out is to define, for each energy, regions in the sky where the conversion is, either negligible, or almost sure. As a matter of facts, when applying the energy algorithms with both $X_{\text{max}}$ dependencies to candidate events, one can face different situations:
1. the event is consistent with a surely converted photon, that is: when applying the algorithm with the conversion hypothesis, the direction is within the region of almost sure conversion at the estimated energy

2. the event is consistent with a surely unconverted photon (same criterion).

3. the event is consistent with both.

For the cases 1 and 2 we can build an analysis with a firm hypothesis on the conversion and choose correctly the energy converter we have to apply and the simulations we have to refer to for photon characterization purposes. As a consequence, one can build a sophisticated analysis that differs on the different regions of the sky. These regions change with the energy range considered (see Fig. 6, and pick up the lightest and the darkest regions for each energies, corresponding to case 1 and case 2 respectively). A conservative evaluation consists in counting cases 1 to 3 as candidates (twice for case 3, at two different energies), accounting for the acceptance in the angular regions defined above, as a function of energy.

One can stretch in addition that this scheme of analysis would bring another major improvement on the point of the energy resolution for the highest energy events: the LPM effect lowers the photon/air cross-section which results in the already mentioned average delay in the first interactions but also obviously in fluctuations on the development. These fluctuations are becoming larger as the energy grows and the relation between $S(1000)/E_{prim}$ and $X_{max}$ becomes poorly defined at highest energies for unconverted photons. This could lead to serious problems with the iterative procedure proposed here for the energy reconstruction. The analysis pattern proposed here would thus have an extra advantage: with a single analysis, we rely more and more on converted photons as the energy grows. As converted photon showers suffer much less fluctuations as they are less affected by the LPM effect, the use of an energy converter depending on the development stage remain secure and accurate even at highest energies.

To perform a first analysis, we can even think of a simpler scheme than the one proposed just above. With 2 years of Auger data taking, one has to consider that the statistics of events
available at the energies relevant for the conversion occurrence is very small, especially if we
cut the most vertical showers that would be seen well before their maximum and could lead to
misreconstructions (indicated on Fig. 6 by the inner circle). We could for a first step neglect
the conversion, as we are mainly using events below the relevant energies for this phenomenon
to happen. We can make the hypothesis that the potential photon candidates we might find
would be all unconverted, and reconstruct them with the method and function described in
section 3. Under this hypothesis, if there were converted photons, they would be obviously
misreconstructed. Nevertheless, a conservative approach can be adopted to correct for this fact
if we consider we have "lost" all the converted photons for the analysis when taking the upper
bound on the photon flux. In the energy range $10^{18.5} \text{ eV} - 10^{19.2} \text{ eV}$, were we have already
large enough statistics to perform a relevant analysis, this assumption will lead to correct and
conservative results. Note that even if we set integrated upper limits, the corrections made on
the upper bound will be very small in this energy range, due to the expected steepness of the
spectrum of cosmic rays photons and anyway leading to a conservative result. This solution is a
simplistic approach, but can enable to set a robust first result on photon flux using the surface
detector of Auger.

5 Conclusion

The reconstruction of the energy of photon induced showers with a surface detector can’t be
perform with a simple power law, as usually done for showers of proton or nuclear origin because
of their late development. Parametrizations of the signals used for showers initiated by a proton
or any other nucleus (which assume a factorization of the dependences on energy and zenith
angle) are no longer valid. The explicit use of the development stage in the energy determination
enables to build an efficient reconstruction method that leads to a similar resolution in energy
for photon than the one we can currently expect for proton showers on Auger. An estimation
of the $X_{\text{max}}$ of the shower has nevertheless to be done as an input to this method.

With a first guess for the energy of the primary, a precise evaluation may be obtained from
a relation between the interpolated signal at a given distance from the core (e.g. $S(1000)$) and
the stage of development expressed through $X - X_{\text{max}}$ (the "universal profile"). The energy
of the primary is then reconstructed iteratively. Note that, to some extent, the value of $X_{\text{max}}$
could even be inferred from ground observables (e.g. time shape of the signals, curvature of
the front) [3], but these observables are also used to discriminate photon candidates from 30
showers, then it would be delicate to disentangle the discrimination from the energy estima-
tion. We have proposed here an iterative algorithm based on the universal profile and on the
averaged dependence of $X_{\text{max}}$ on $E_{\text{prim}}$ for unconverted photons, which improves the resolution.

At energies above the magnetic conversion threshold, the mixing of unconverted and con-
verted photons makes the situation more complex. A conservative upper bound may be obtained
by defining angular regions (depending on energy) where the probability of conversion is close to
either 0 or 1, and to evaluate the acceptance accordingly; of course, a measurement of the pho-
ton fraction as a function of the energy would be more delicate. The current set of Auger data
has still very poor statistics at energies where conversion occurs, so we can propose a simpler
temporary analysis that relies on the unconverted photons and choose to loose efficiency as we
would be unable to reconstruct converted photons. Here again the acceptance will have to be
computed accordingly. This first analysis will obviously lead to a conservative result. One has
nevertheless to keep in mind that all these algorithms (as well as analysis for the discrimination
of photons in itself) relies on computations of electromagnetic processes at ultra high energies
and implicitly assume that QED may be used safely at these energies.
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