We investigate, both experimentally and theoretically, possible routes towards Anderson-like localization of Bose-Einstein condensates in disordered potentials. The dependence of this quantum interference effect on the nonlinear interactions and the shape of the disorder potential is investigated. Experiments with an optical lattice and a superimposed disordered potential reveal the lack of Anderson localization. A theoretical analysis shows that this absence is due to the large length scale of the disorder potential as well as its screening by the nonlinear interactions. Further analysis shows that incommensurable superlattices should allow for the observation of the cross-over from the nonlinear screening regime to the Anderson localized case within realistic experimental parameters.

Disordered systems have played a central role in condensed matter physics in the last 50 years. Recently, it was proposed that ultracold atomic gases may serve as a laboratory for disordered quantum systems and allow for the experimental investigation of various open problems in that field. Some of these problems concern strongly correlated systems, the realization of Bose or Fermi glasses, quantum spin glasses and quantum percolation. This letter addresses one of the most important issues, namely the interplay of Anderson localization (AL) and repulsive interactions. This interplay may lead to the creation of delocalized phases both for fermions and bosons. The possible occurrence of AL has also been investigated theoretically for weakly interacting Bose-Einstein condensates (BEC), and in this case it was shown that even moderate nonlinear interaction counteracts the localization. As a main result of this letter we show that despite this difficulty there exists an experimentally accessible regime where Anderson-like localization can be realized with present day techniques.

Several methods have been proposed to produce a disordered, or quasi-disordered potential for trapped atomic gases. They include the use of speckle radiation, incommensurable optical lattices, impurity atoms in the sample and the disorder that appears close to the surface of atom chips. Recently, first experiments searching for effects of disorder in the dynamics of weakly interacting BECs were realized.

In this letter we shed new light on the interplay between disorder and interactions by studying trapped BECs under the influence of a disordered potential and a one dimensional (1D) optical lattice (OL). The OL creates a periodic potential and the randomness of the disordered potential leads to AL for noninteracting particles. We study how the presence of interactions affects nontrivial localization in our necessarily finite system.

Our experiments were performed with $^{85}$Rb Bose-Einstein condensates in an elongated magnetic trap (MT) with axial and radial frequencies of $\omega_x = 2\pi \times 14$ Hz and $\omega_y = 2\pi \times 200$ Hz, respectively. Further details of our experimental apparatus were described previously. The number of condensed atoms $N$ was varied between $1.5 \times 10^4$ and $8 \times 10^4$. The OL was provided by a retroreflected laser beam at $\lambda = 825$ nm superimposed on the axial direction of the magnetic trap. The depth of the OL was typically set to $6.5 E_r$, where the recoil energy is given by $E_r = h^2 k^2 / 2m$. For this configuration the peak chemical potential varied between 0.25 $E_r$ and 0.5 $E_r$. The disorder potential (DP) was produced by projecting the image of a randomly structured chrome substrate onto the atoms giving rise to a spatially varying dipole potential along the axial direction of the cloud. Due to the resolution of the imaging system the minimal structure size of the DP was limited to 7 $\mu$m. We define the depth of the DP as twice the standard deviation of the dipole potential, analogously to the combined potential allowed for the first realization of an ultracold disordered lattice gas.

![FIG. 1: Typical absorption images of a BEC with $N = 7 \times 10^4$ released from the combined MT plus DP (left column) and MT plus OL plus DP (right column). The second row shows the column density and the third row shows the result of a 1D simulation. The lattice depth was 6.5 $E_r$ and the DP had a depth of 0.2 $E_r$.](image-url)
formed the following experimental sequence: We first ramped up the OL potential over 60 ms, then the DP was ramped up over another 60 ms, followed by a hold time of 20 ms. Finally all potentials were switched off and the atomic density distribution was measured after 20 ms of ballistic expansion using absorption imaging. Alternatively we performed the same experiment without the OL.

Figure 1 shows typical absorption images for the case of DP only and for the case of combined DP and OL. The obtained density distributions show two characteristic features. On one hand they display pronounced fringes and on the other hand the axial size of the central peak is modified with respect to the case without DP. We extract the axial size of the peak by fitting the density with a parabolic distribution. The resulting sizes are shown as a function of the atom number in Fig. 2. The DP in the experiments changes on a scale much larger than the lattice spacing and the condensate healing length, \( l = 1/\sqrt{8\pi a} \), where \( n \) is the condensate density and \( a \) the atomic scattering length. This suggests the applicability of the so-called effective mass analysis [21].

We determine the ground state solution of the stationary GPE in the form \( \phi_0(x) = \sqrt{N}f(x)u_0(x) \), where \( u_0(x) \) is the Bloch function corresponding to the ground state of the OL potential, \( f(x) \) is an envelope function and \( N \) is a constant chosen such that \( \phi_0 \) is normalized to unity. This leads to an effective GPE where the OL potential is eliminated but the mass of a "particle" and the interaction strength become modified. For the experimental parameters the effective mass is \( m^* = 2.56 \) and the renormalized interaction strength for \( N = 7 \cdot 10^4 \) is \( g^* = 2498 \).

Due to the large value of \( g^* \) we may use the TF approximation and obtain the envelope function in the form \( |f(x)|^2 = (\mu^* - x^2/2 - V_{\text{dis}}(x))/g^* \), where \( \mu^* \) is determined from the condition \( \int |f(x)|^2dx = 1 \). The squared overlap of the obtained \( \phi_0 \) with the exact ground state of the GPE is 0.999 which implies that the effect of the lattice potential is reduced to a modification of the coupling constant for the TF profile of the combined MT plus DP. Thus, similarly to the experiments performed in the absence of an OL [18] we observe a fragmentation of the BEC induced by the DP but this fragmentation does not correspond to Anderson-like localization.

To enter the localized regime, it is therefore necessary
to introduce a disorder that changes on a length scale comparable to the lattice spacing. Due to the limited resolution of the DP imaging optics this poses considerable experimental difficulties. Alternatively one may use a pseudorandom potential obtained with the help of two, or more additional optical lattices with incommensurable frequencies [23]. However, even the realization of such a fine scale disorder is not necessarily sufficient for the observation of non-trivial localization. Indeed, for a solution $\phi_0$ of the stationary GPE the nonlinear term $g|\phi_0(x)|^2$ may be treated as an additional potential. When the atoms accumulate in the wells of the random potential, the nonlinear term in the GPE effectively smoothes the potential modulations [13]. For typical experimental parameters the term $g|\phi_0(x)|^2$ dominates over $V_{\text{dis}}(x)$ and consequently the randomness necessary for localization is lost.

This picture is confirmed by analyzing the dependence of the superfluid fraction on the coupling constant $g$ shown in Fig. 3. To calculate the superfluid fraction we have numerically solved the 1D GPE in a box with periodic boundary conditions in the presence of an OL and a pseudorandom potential created by two additional optical lattices at 960 nm and 1060 nm with depths of 0.2 $E_r$. The superfluid fraction is defined as $f_s = \langle |\phi_0(x)|^2 \rangle$, with $\phi_0(x)$ the ground state wavefunction which is characterized by an exponential localization $|\phi_0(x)|^2 \propto \exp(-|x-x_0|/l)$, with the localization length $l \approx 0.027$. For such a non-interacting system, there might exist several localized single particle states with an energy close to the ground state. For finite observation times, condensation could occur into several of these low energy states, and several "small" condensates with different condensate wavefunctions could coexist. Figure 4 suggests that the condensate wavefunction becomes a combination of these localized states due to nonlinear interactions.

Increasing $g$ causes the ground state to contain a larger number of localization centers. However, the localization length in these cases hardly deviates from the non-interacting case. When $g$ is of the order of 500 one can no longer distinguish individual localized states and the clear signature of non-trivial localization vanishes. This is consistent with the appearance of a significant superfluid fraction in Fig. 3. For $g = 256$ correspond to axial and radial frequencies of $2\pi \times 4$ Hz and $2\pi \times 40$ Hz, respectively and $N = 10^4$. In this case the simulation shows characteristic features of Anderson-like localization while these parameters are within experimental reach. The scenario of a cross-over from the Anderson to the screening regime, presented

FIG. 3: Superfluid fraction as a function of the coupling constant $g$ obtained from a 1D GPE simulation for a pseudorandom potential. Full (open) symbols correspond to a trap frequency of $2\pi \times 14$ Hz ($2\pi \times 4$ Hz).

FIG. 4: Ground states of the GPE (note the varying logarithmic scales) for a condensate in the combined potential of the MT, OL and pseudorandom potential. The depth of the OL is 6.5 $E_r$, while the depths of the additional lattices forming the pseudorandom potential are 0.2 $E_r$. The coupling constants $g$ for the panels are: 0.5 (a), 8 (b), 256 (c). Oscillator units corresponding to a trap frequency of $2\pi \times 4$ Hz are used.
here, is one of the most important results of our analysis.

Our theoretical investigation also shows that the detection of the onset of Anderson-like localization using a measurement of the density distribution after ballistic expansion might be difficult. We have calculated the atomic density profiles after 20 ms of ballistic expansion corresponding to the parameters of Fig. 4. Despite a striking difference in the ground state wavefunction, the width of the envelope of the zero-momentum peak, which is related to the localization length $l$, does not vary significantly as shown in Fig. 5. In addition Fig. 5 shows that the expansion is dominated by the interaction for experimentally accessible values of $g$ within our 1D model. Future experiments on the detection of localization may rather rely on a measurement of the superfluid fraction (see [24]) in an accelerated optical lattice.

In conclusion, we have presented a detailed analysis of non-trivial localization for slowly varying potentials and in pseudorandom potentials in the presence of interactions. We have shown the absence of localization in the experimental case and explained this effect using an effective mass approach. For a truly random potential a suppression of Anderson-like localization due to the screening by nonlinear interactions was found. An analysis for small interactions and a pseudorandom potential reveals the characteristic features of Anderson-like localization. The transition from the localized to the screened delocalized regime may be detected via an analysis of the superfluid fraction. This work paves the way towards the observation of Anderson-like localization in an experimentally accessible regime.

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