Dual-BRST symmetry: 6D Abelian 3-form gauge theory

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Abstract: Within the framework of Becchi-Rouet-Stora-Tyutin (BRST) formalism, we demonstrate the existence of the novel off-shell nilpotent (anti-)dual-BRST symmetries in the context of a six (5 + 1)-dimensional (6D) free Abelian 3-form gauge theory. Under these local and continuous symmetry transformations, the total gauge-fixing term of the Lagrangian density remains invariant. This observation should be contrasted with the off-shell nilpotent (anti-)BRST symmetry transformations, under which, the total kinetic term of the theory remains invariant. The anticommutator of the above nilpotent (anti-)BRST and (anti-)dual-BRST transformations leads to the derivation of a bosonic symmetry in the theory. There exists a discrete symmetry transformation in the theory which provides a thread of connection between the nilpotent (anti-)BRST and (anti-)dual-BRST transformations. This theory is endowed with a ghost-scale symmetry, too. We discuss the algebra of these symmetry transformations and show that the structure of the algebra is reminiscent of the algebra of de Rham cohomological operators of differential geometry.

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1 Introduction

The study of the higher dimensional \((D > 4)\) and higher \(p\)-form \((p \geq 2)\) gauge theories is important because these higher \(p\)-form \((p = 2, 3, \ldots)\) fields appear in the excitations of the quantized versions of (super)strings and related extended objects (see, e.g., [1-3] for details). Furthermore, it is now well-established that the consistent quantum theories of these extended objects do live in dimensions much higher than the ordinary 4D spacetime. Thus, the modern developments in (super)string (and related extended objects) have spurred the interest of theorists to study field theories that are connected with the higher \(p\)-form gauge theories in higher \((D > 4)\) dimensions of spacetime. In some sense, these field theories are generalizations of the usual field theories in the ordinary 4D spacetime. With this background in mind, our present endeavor is an attempt to explore the interesting symmetries associated with the free Abelian 3-form gauge theory in six dimensions of spacetime.

In addition to the above motivation, there is another argument in favor of our interest in the study of higher \(p\)-form \((p \geq 2)\) gauge theories. For instance, it has been shown [4-6] that a 4D Abelian 1-form gauge field acquires a mass without any recourse to the Higgs mechanism when it is merged with an Abelian 2-form \([B^{(2)} = \frac{1}{2!}(dx^\mu \wedge dx^\nu)B_{\mu\nu}]\) antisymmetric gauge field \((B_{\mu\nu})\) through the celebrated topological \((B \wedge F)\) term. Attempts to generalize this model to the topologically massive 4D non-Abelian gauge theory have been made in the past [7-10]. Furthermore, these models have also been studied within the framework of Becchi-Rouet-Stora-Tyutin (BRST) formalism in our earlier works [11-14]. In exactly above fashion, it has been shown that the merging of Abelian 2-form and 3-form gauge fields, through a topological term in the 6D spacetime, leads to the mass generation of the Abelian 2-form gauge field (see, e.g., [15]). Thus, it is important to explore the details of the higher \(p\)-form \((p \geq 2)\) gauge theories because these provide an alternative to the Higgs mechanism as far as the mass generation of gauge fields is concerned.

There is yet another reason behind our interest in the present investigation. In a very recent paper [16], we have claimed that there always exists a set of proper (anti-)dual-BRST symmetries, for an Abelian \(p\)-form gauge theory in \(D = 2p\) dimensions of spacetime, within the framework of BRST formalism. We have already shown in our earlier works on 2D Abelian 1-form gauge theory [17-19] and 4D Abelian 2-form gauge theory [20-22] that the above nilpotent and absolutely anticommuting (anti-)co-BRST symmetries do exist. The purpose of our present investigation is to corroborate the above claim by demonstrating that the proper (i.e. nilpotent and absolutely anticommuting) (anti-)dual-BRST symmetries do exist for the 6D Abelian 3-form gauge theory under which the total gauge-fixing term, owing its origin to the co-exterior derivative of differential geometry [23-25], remains invariant.

In our present investigation, we demonstrate that there exist six continuous symmetries and one discrete symmetry in the theory which obey the algebra of the de Rham cohomological operators of differential geometry. In particular, an elegant interplay between the discrete and continuous symmetry transformations provides a physical realization of the relationship between the exterior and co-exterior derivatives of differential geometry. As a consequence, our present model is a field theoretic realization of the Hodge theory because all the de Rham cohomological operators and their relationships are realized in the language of symmetry transformations on the relevant fields of our present theory.

Our present paper is organized as follows. In Sec. 2, we recapitulate the bare essen-
tials of our earlier work [26] to set up the notations and conventions corresponding to the (anti-)BRST symmetry transformations. Our Sec. 3 is devoted to the discussion of the (anti-)dual-BRST symmetry transformations. In Sec. 4, we deal with the derivation of the bosonic symmetry from the anticommutator of the (anti-)BRST and (anti-)co-BRST symmetry transformations. We discuss the existence of ghost-scale and discrete symmetries of the theory in Sec. 5. In the next Sec. 6, we deduce the algebraic structures of all the above symmetry transformations. Finally, we make some concluding remarks in Sec. 7.

2 Preliminaries: (anti-)BRST symmetries

Let us begin with the following coupled (but equivalent) Lagrangian densities for the free 6D Abelian 3-form gauge theory* [26]

\[
\mathcal{L}_B = \frac{1}{24} H_{\mu\nu\kappa} H_{\mu\nu\xi} + B^{\mu\nu} \left( \partial^\eta A_{\eta\mu\nu} + \frac{1}{2} [\partial_\mu \phi^{(1)}_\nu - \partial_\nu \phi^{(1)}_\mu] \right) - \frac{1}{2} B_{\mu\nu} B^{\mu\nu} \\
+ (\partial_\mu \bar{C}_{\nu\rho} + \partial_\rho \bar{C}_{\nu\mu} + \partial_\eta \bar{C}_{\mu\nu})(\partial^{\rho\eta} C^{\mu\nu}) + (\partial \cdot \phi^{(1)}) B_1 - \frac{1}{2} B_1^2 - B B_2 \\
- (\partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu)(\partial^{\mu\nu} \beta^{(1)}) + (\partial_\mu \bar{C}_{\mu\nu} + \partial^\nu \bar{C}_1) f_\nu - (\partial_\mu C^{\mu\nu} - \partial^\nu C_1) \bar{F}_\nu \\
+ \partial_\mu \bar{C}_2 \bar{\phi}^\mu C_2 + (\partial \cdot \beta) B_2 - (\partial \cdot \bar{\beta}) B,
\]

\[
(1)
\]

\[
\mathcal{L}_B = \frac{1}{24} H_{\mu\nu\kappa} H_{\mu\nu\xi} - \bar{B}^{\mu\nu} \left( \partial^\eta A_{\eta\mu\nu} - \frac{1}{2} [\partial_\mu \phi^{(1)}_\nu - \partial_\nu \phi^{(1)}_\mu] \right) - \frac{1}{2} \bar{B}_{\mu\nu} \bar{B}^{\mu\nu} \\
+ (\partial_\mu \bar{C}_{\nu\rho} + \partial_\rho \bar{C}_{\nu\mu} + \partial_\eta \bar{C}_{\mu\nu})(\partial^{\rho\eta} C^{\mu\nu}) + (\partial \cdot \phi^{(1)}) \bar{B}_1 - \frac{1}{2} \bar{B}_1^2 - B \bar{B}_2 \\
- (\partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu)(\partial^{\mu\nu} \beta^{(1)}) - (\partial_\mu \bar{C}_{\mu\nu} + \partial^\nu \bar{C}_1) \bar{f}_\nu + (\partial_\mu C^{\mu\nu} - \partial^\nu C_1) \bar{F}_\nu \\
+ \partial_\mu \bar{C}_2 \bar{\phi}^\mu C_2 + (\partial \cdot \beta) \bar{B}_2 - (\partial \cdot \bar{\beta}) \bar{B},
\]

\[
(2)
\]

where \( H_{\mu\nu\kappa} = \partial_\mu A_{\nu\kappa} - \partial_\nu A_{\kappa\mu} + \partial_\kappa A_{\mu\nu} - \partial_\lambda A_{\mu\nu\lambda} \) is the totally antisymmetric curvature tensor derived from the 4-form \( H^{(4)} = (1/4!)(dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\kappa) \) \( H_{\mu\nu\kappa} = dA^{(3)} \). The latter is obtained from the operation of the ordinary exterior derivative \( d = dx^\mu \partial_\mu \) (with \( d^2 = 0 \)) on the connection 3-form \( A^{(3)} = (1/3!)(dx^\mu \wedge dx^\nu \wedge dx^\eta) A_{\mu\nu\eta} \) which defines the totally antisymmetric tensor Abelian 3-form gauge field \( A_{\mu\nu\eta} \). The gauge-fixing term \( (\partial^\eta A_{\eta\mu\nu}) \) owes its origin to the co-exterior derivative \( \delta = - \ast d \ast \) because the 2-form \( \Delta A^{(3)} = (1/2!)(dx^\mu \wedge dx^\nu)(\partial^\eta A_{\eta\mu\nu}) \) captures it. In the above discussion, the exterior and co-exterior derivatives are related to each-other by the Hodge duality * operation. It should be noted that one has the freedom to add/subtract a 2-form \( (F^{(2)}) \) in the gauge-fixing term \( (\partial^\eta A_{\eta\mu\nu}) \). This can be easily done by applying an exterior derivative \( d = dx^\mu \partial_\mu \) on a 1-form \( \phi^{(1)} = dx^\mu \phi^{(1)}_\mu \). This is precisely the reason that we have incorporated the vector field \( \phi^{(1)}_\mu \) from

*We adopt here the convention and notations such that the 6D spacetime background Minkowskian manifold is endowed with a flat metric with signatures \((+1, -1, -1, -1, -1)\) and corresponding totally antisymmetric Levi-Civita tensor \( \varepsilon^{\mu\nu\kappa\rho\sigma\lambda} = -6! \varepsilon^{\mu\nu\kappa\rho\sigma\lambda} \) obeys the relations: \( \varepsilon^{\mu\nu\kappa\rho\sigma\lambda} = -6! \varepsilon^{\mu\nu\kappa\rho\sigma\lambda} = -5! \delta^\rho_\lambda, \) etc., with \( \delta^{012345} = +1 \) and the Greek indices \( \mu, \nu, \eta, ... = 0, 1, 2, 3, 4, 5 \) correspond to the 6D spacetime directions. For the sake of brevity, we also use the notation \( A \cdot B = A_\mu B^\mu = A_\mu B_\mu = A_0 B_0 - A_i B_i \) where the Latin indices \( i, j, k, ... = 1, 2, 3, 4, 5 \) correspond to only the space directions.
In the above equations (1) and (2), \((\bar{C}_{\mu
u}, C_{\mu
u})\) are the fermionic antisymmetric \((\bar{C}_{\mu
u} = -\bar{C}_{\nu\mu}, C_{\mu
u} = -C_{\nu\mu})\) (anti-)ghost fields with ghost numbers \((-1)+1\), \((\bar{\beta}_{\mu})\beta_{\mu}\) are the bosonic ghost-for-ghost (anti-)ghost Lorentz vector fields with ghost numbers \((-2)+2\), \((\bar{C}_{2}C_{2}\) are the fermionic ghost-for-ghost-for-ghost (anti-)ghost Lorentz scalar fields with ghost numbers \((-3)+3\), respectively. Furthermore, we have antisymmetric Nakanishi-Lautrup type bosonic auxiliary fields \((B_{\mu\nu}, \bar{B}_{\mu\nu})\) as well as the Lagrange multiplier fields \(B, B_{1}\) and \(B_{2}\) in the theory which have been invoked for the purpose of linearization of some specific terms. Similarly, we have \((f_{\mu})\bar{f}_{\mu}\) and \((\bar{F}_{\mu})F_{\mu}\) in our present theory as the fermionic auxiliary (anti-)ghost fields with ghost numbers \((-1)+1\), respectively. In the complete theory, we have fermionic (anti-)ghost fields \((\bar{C}_{1})C_{1}\) with ghost numbers \((-1)+1\) and bosonic auxiliary ghost fields \(B_{2}\) and \(B\) (which carry the ghost numbers \((-2)+2\), respectively).

It can be checked that, under the following off-shell nilpotent \((s_{b}^{2} = 0)\) BRST symmetry transformations \((s_{b})\) (see, e.g. [26] for details):

\[
\begin{align*}
& s_{b}A_{\mu\nu}\eta = \partial_{\mu}C_{\eta\nu} + \partial_{\nu}C_{\eta\mu} + \partial_{\eta}C_{\mu\nu}, \quad s_{b}C_{\mu\nu} = \partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu}, \quad s_{b}\bar{C}_{\mu\nu} = B_{\mu\nu}, \\
& s_{b}\bar{B}_{\mu\nu} = \partial_{\mu}f_{\nu} - \partial_{\nu}f_{\mu}, \quad s_{b}\bar{\beta}_{\mu} = \bar{F}_{\mu}, \quad s_{b}\beta_{\mu} = \partial_{\mu}C_{2}, \quad s_{b}F_{\mu} = -\partial_{\mu}B, \\
& s_{b}\bar{C}_{2} = B_{2}, \quad s_{b}\bar{C}_{1} = B_{1}, \quad s_{b}C_{1} = B_{1}, \quad s_{b}\phi_{\mu}^{(1)} = f_{\mu}, \quad s_{b}\bar{f}_{\mu} = \partial_{\mu}B_{1}, \\
& s_{b}\left[ C_{2}, f_{\mu}, \bar{F}_{\mu}, B, B_{1}, B_{2}, B_{\mu\nu}, H_{\mu\nu\eta\kappa}\right] = 0,
\end{align*}
\]

the Lagrangian density \(\mathcal{L}_{B}\) transforms to a total spacetime derivative as given below

\[
\begin{align*}
& s_{b}\mathcal{L}_{B} = \partial_{\mu}\left[(\partial^{\nu}C_{\eta\nu} + \partial^{\nu}C^{\mu\eta} + \partial^{\nu}C^{\eta\mu})B_{\nu\eta} + B_{\mu\nu}f_{\nu} - (\partial^{\mu}\beta_{\nu} - \partial^{\nu}\beta_{\mu})\bar{F}_{\nu} \\
& \quad + B_{1}f_{\mu} - BF_{\mu} + B_{2}\partial^{\mu}C_{2}\right].
\end{align*}
\]

In an exactly similar fashion, under the following off-shell nilpotent \((s_{ab}^{2} = 0)\) anti-BRST symmetry transformations \((s_{ab})\) (see, e.g. [26] for details)

\[
\begin{align*}
& s_{ab}A_{\mu\nu}\eta = \partial_{\mu}\bar{C}_{\eta\nu} + \partial_{\nu}\bar{C}_{\eta\mu} + \partial_{\eta}\bar{C}_{\mu\nu}, \quad s_{ab}\bar{C}_{\mu\nu} = \partial_{\mu}\bar{\beta}_{\nu} - \partial_{\nu}\bar{\beta}_{\mu}, \quad s_{ab}C_{\mu\nu} = \bar{B}_{\mu\nu}, \\
& s_{ab}\bar{B}_{\mu\nu} = \partial_{\mu}\bar{f}_{\nu} - \partial_{\nu}\bar{f}_{\mu}, \quad s_{ab}\bar{\beta}_{\mu} = F_{\mu}, \quad s_{ab}\beta_{\mu} = \partial_{\mu}\bar{C}_{2}, \quad s_{ab}\bar{F}_{\mu} = -\partial_{\mu}B_{2}, \\
& s_{ab}\bar{C}_{2} = B_{1}, \quad s_{ab}\bar{f}_{\mu} = -\partial_{\mu}B_{1}, \quad s_{ab}C_{1} = -B_{1}, \quad s_{ab}\bar{C}_{1} = -B_{1}, \quad s_{ab}\phi_{\mu}^{(1)} = \bar{f}_{\mu}, \\
& s_{ab}\left[ \bar{C}_{2}, \bar{f}_{\mu}, F_{\mu}, B, B_{1}, B_{2}, B_{\mu\nu}, H_{\mu\nu\eta\kappa}\right] = 0,
\end{align*}
\]

the Lagrangian density \(\mathcal{L}_{B}\) transforms to a total spacetime derivative as illustrated below:

\[
\begin{align*}
& s_{ab}\mathcal{L}_{B} = \partial_{\mu}\left[-(\partial^{\nu}C^{\eta\nu} + \partial^{\nu}C^{\mu\eta} + \partial^{\nu}C^{\eta\mu})B_{\nu\eta} + B_{\mu\nu}\bar{f}_{\nu} - (\partial^{\mu}\beta^{\nu} - \partial^{\nu}\beta^{\mu})F_{\nu} \\
& \quad + B_{1}\bar{f}_{\mu} + B_{2}F_{\mu} - B\partial^{\mu}\bar{C}_{2}\right].
\end{align*}
\]

As a consequence, the action integral \((S = \int d^{4}x\mathcal{L}_{\{B, \bar{B}\}})\) remains invariant for the well-defined physical fields (incorporated in the theory) which vanish at infinity.

We close this section with the following comments. First, the above off-shell nilpotent BRST and anti-BRST symmetry transformations absolutely anticommute \((s_{b}s_{ab} + s_{ab}s_{b} = 0)\) with each other only when the following CF-type restrictions [26,27]

\[
\begin{align*}
& f_{\mu} + F_{\mu} = \partial_{\mu}C_{1}, \quad \bar{f}_{\mu} + \bar{F}_{\mu} = \partial_{\mu}\bar{C}_{1}, \quad B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_{\mu}\phi_{\mu}^{(1)} - \partial_{\nu}\phi_{\nu}^{(1)},
\end{align*}
\]
are satisfied. Second, the Lagrangian densities $\mathcal{L}_B$ and $\mathcal{L}_{\bar{B}}$ are equivalent (see, e.g. [26]) and both of them respect the (anti-)BRST symmetry transformations ($s_{(a)b}$) on a constraint hypersurface described by the CF-type restrictions given in (7). Third, the kinetic term $[(1/24)H^{\mu\nu\rho}H_{\mu\nu\rho}]$, owing its origin to the exterior derivative $d = dx^{\mu}\partial_{\mu}$, remains invariant under the (anti-)BRST symmetry transformations. Thus, one of the (anti-)BRST symmetry transformations provides a physical realization of $d$. Finally, the above (anti-)BRST symmetry transformations and CF-type restrictions have been derived from the superfield formalism discussed in [27] and they satisfy the key requirements of the nilpotency and anticommutativity properties which are very sacrosanct in the context of discussion of any arbitrary $p$-form ($p = 1, 2, 3,\ldots$) gauge theory within the framework of BRST formalism.

3 (Anti-)dual-BRST symmetries

As we have linearized the gauge-fixing term by introducing the Nakanishi-Lautrup field $B_{\mu\nu}$ (and/or $\bar{B}_{\mu\nu}$) and a vector field $\phi_1^{(1)}$ from the 2-form $F^{(2)} = d\phi^{(1)} = (1/2)(dx^\mu \wedge dx^\nu)(\partial_\mu \phi^{(1)}_1 - \partial_\nu \phi^{(1)}_1)$, similarly the kinetic term $[(1/24)H^{\mu\nu\rho}H_{\mu\nu\rho}]$ can be linearized by invoking the auxiliary antisymmetric field $B_{\mu\nu}$ (and/or $\bar{B}_{\mu\nu}$) and $\phi^{(2)}_\mu$ from another 2-form $\bar{F}^{(2)} = d\bar{\phi}^{(1)} = (1/2)(dx^\mu \wedge dx^\nu)(\partial_\mu \phi^{(2)}_1 - \partial_\nu \phi^{(2)}_1)$. Thus, the Lagrangian densities (1) and (2) can be linearized to the following

$$\mathcal{L}_{(B,\bar{B})} = \frac{1}{2} B_{\mu\nu} B^{\mu\nu} - B^{\mu\nu} \left( \frac{1}{4!} \epsilon_{\mu\nu\rho\sigma\tau} H^{\rho\sigma\tau} + \frac{1}{2} [\partial_\mu \phi^{(2)}_1 - \partial_\nu \phi^{(2)}_1] \right) - \frac{1}{2} B^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{L}_{(\bar{B},\bar{B})} = \frac{1}{2} \bar{B}_{\mu\nu} \bar{B}^{\mu\nu} + (\bar{B}^{\mu\nu} \left( \frac{1}{4!} \epsilon_{\mu\nu\rho\sigma\tau} H^{\rho\sigma\tau} - \frac{1}{2} [\partial_\mu \phi^{(2)}_1 - \partial_\nu \phi^{(2)}_1] \right) - \frac{1}{2} B^{\mu\nu} B_{\mu\nu}$$

It should be noted that the novel gauge-fixing term [i.e. $- (\partial \cdot \phi^{(2)}) B_3 + \frac{1}{2} B_3^2$], corresponding to the additional vector field $\phi^{(2)}_\mu$ has been incorporated through the Nakanishi-Lautrup type auxiliary field $B_3$ (in the above coupled Lagrangian densities) so that the theory, in its full blaze of glory, could become complete in all respects.

The following off-shell nilpotent ($s_a^2 = 0$) dual-BRST symmetry transformations ($s_a$):

$$s_a A_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma\tau} \partial_\tau \bar{C}^{\rho\sigma}, \quad s_a \bar{C}_{\mu\nu} = \partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu, \quad s_a \bar{\beta}_\mu = \partial_\mu \bar{C}_2$$

$$s_a \bar{C}_1 = -B_2, \quad s_a \bar{\beta}_\mu = -f_\mu, \quad s_a C_1 = -B_3, \quad s_a \phi^{(2)}_\mu = \bar{F}_\mu, \quad s_a C_2 = B,$$
\[ s_d C_{\mu\nu} = \mathcal{B}_{\mu\nu}, \quad s_d \tilde{f}_\mu = \partial_\mu B_2, \quad s_d \tilde{\mathcal{B}}_{\mu\nu} = \partial_\mu \tilde{F}_\nu - \partial_\nu \tilde{F}_\mu, \quad s_d F_\mu = \partial_\mu B_3, \]

\[ s_d \left[ \partial^n A_{\eta\mu\nu}, \phi^{(1)}_\mu, B_{\mu\nu}, \mathcal{B}_{\mu\nu}, B, B_1, B_2, B_3, \tilde{C}_2, f_\mu, \tilde{F}_\mu \right] = 0, \tag{10} \]

leave the Lagrangian density \( \mathcal{L}_{(B,B)} \) quasi-invariant. This is due to the fact that the latter transforms to a total spacetime derivative, under \( s_d \), as illustrated below:

\[ s_d \mathcal{L}_{(B,B)} = -\partial_\mu \left[ (\partial^\mu \tilde{C}^{\eta\nu} + \partial^\nu \tilde{C}^{\eta\mu} + \partial^n \tilde{C}^{\mu\nu}) \mathcal{B}_{\nu\eta} + B \partial^\mu \tilde{C}_2 + f_\mu B_2 + \mathcal{B}^{\mu\nu} \tilde{F}_\nu \right. \]

\[ + \tilde{F}_\mu B_3 - (\partial^\mu \tilde{\beta}^{\nu} - \partial^\nu \tilde{\beta}^\mu) f_\nu \]. \tag{11} \]

As a consequence, the action integral \( (S = \int d^6 x \mathcal{L}_{(B,B)}) \) remains invariant for the physically well-defined fields of the theory which vanish at infinity.

Corresponding to the symmetry transformations (10), we have a set of anti-co-BRST (i.e. anti-dual-BRST) symmetry transformations \( (s_{ad}) \):

\[ s_{ad} A_{\mu\nu\eta} = \frac{1}{2} \epsilon_{\mu\nu\eta\rho\sigma} \partial^\rho C^{\sigma\rho}, \quad s_{ad} C_{\mu\nu} = -(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu), \quad s_{ad} \beta_\mu = -\partial_\mu C_2, \]

\[ s_{ad} C_1 = B, \quad s_{ad} \tilde{C}_{\mu\nu} = \tilde{\mathcal{B}}_{\mu\nu}, \quad s_{ad} \tilde{\beta}_\mu = \tilde{f}_\mu, \quad s_{ad} \tilde{C}_2 = -B_2, \quad s_{ad} \tilde{C}_1 = B_3, \]

\[ s_{ad} \phi^{(2)}_\mu = F_\mu, \quad s_{ad} \tilde{F}_\mu = -\partial_\mu B_3, \quad s_{ad} \tilde{f}_\mu = -\partial_\mu B, \quad s_{ad} \tilde{B}_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu, \]

\[ s_{ad} \left[ \partial^n A_{\eta\mu\nu}, \phi^{(1)}_\mu, \tilde{\mathcal{B}}_{\mu\nu}, \tilde{\mathcal{B}}_{\mu\nu}, B, B_1, B_2, B_3, C_2, \tilde{f}_\mu, F_\mu \right] = 0. \tag{12} \]

The decisive features of the (anti-)dual-BRST [i.e. (anti-)co-BRST] symmetry transformations are as listed below:

- Both are off-shell nilpotent of order two (i.e. \( s_{(a)d}^2 = 0 \)).
- Both symmetries leave the total gauge-fixing terms:

\[ B^{\mu\nu}\left( \partial^n A_{\eta\mu\nu} + \frac{1}{2} \left[ \partial_\mu \phi^{(1)}_\nu - \partial_\nu \phi^{(1)}_\mu \right] \right) - \frac{1}{2} B^{\mu\nu} B_{\mu\nu}, \]

\[ -\tilde{B}^{\mu\nu}\left( \partial^n A_{\eta\mu\nu} - \frac{1}{2} \left[ \partial_\mu \phi^{(1)}_\nu - \partial_\nu \phi^{(1)}_\mu \right] \right) - \frac{1}{2} \tilde{B}^{\mu\nu} \tilde{B}_{\mu\nu}, \tag{13} \]

invariant as \( s_{(a)d} (\partial^n A_{\eta\mu\nu}) = 0, s_{(a)d} (\phi^{(1)}_\mu) = 0, s_{(a)d} (B_{\mu\nu}) = 0 \) and \( s_{(a)d} (\tilde{B}_{\mu\nu}) = 0 \).
- Both symmetries are absolutely anticommuting (i.e. \( s_d s_{ad} + s_{ad} s_d = 0 \)) on the constrained hypersurface defined by the following field equations:

\[ \mathcal{B}_{\mu\nu} + \tilde{\mathcal{B}}_{\mu\nu} = \partial_\mu \phi^{(2)}_\nu - \partial_\nu \phi^{(2)}_\mu, \quad f_\mu + F_\mu = \partial_\mu C_1, \quad \tilde{f}_\mu + \tilde{F}_\mu = \partial_\mu \tilde{C}_1, \tag{14} \]

on the 6D Minkowskian flat spacetime manifold. In the above, the first entry has emerged from (8) and (9) due to the Euler-Lagrange equations of motion.
- The gauge-fixing term \( (\partial^n A_{\eta\mu\nu}) \), for the Abelian 3-form field \( A_{\mu\nu\eta} \), owes its origin to the co-exterior derivative \( \delta = -\ast d\ast \) because \( \delta A^{(3)} = (1/2!) (dx^\mu \wedge dx^\nu) (\partial^n A_{\eta\mu\nu}) \) produces it on the 6D spacetime manifold. This term remains invariant under \( s_{(a)d} \).
• The Lagrangian density $\mathcal{L}(\bar{B}, B)$ transforms, under $s_{ad}$, as:

$$s_{ad}\mathcal{L}(\bar{B}, B) = -\partial_\mu \left[ (\partial^{\mu} C^{\nu} + \partial^{\nu} C^{\mu} + \partial^{\mu} C^{\nu}) \bar{B}_{\nu\eta} + \bar{B}^{\mu\nu} F_\nu + B_2 \partial^{\mu} C_2 + F^{\mu} B_3 \right]$$

which is the analogue of (11). As a consequence, the action integral of the theory remains invariant under the transformations $s_{ad}$. Out of the (anti-)co-BRST transformations, at least, one is certainly the analogue of the co-exterior derivative.

• Under the (anti-)BRST symmetry transformations, the total kinetic term of (8) and (9) remains invariant because $s_{(a)b} B_{\mu\nu} = 0, s_{(a)b} \bar{B}_{\mu\nu} = 0, s_{(a)b} H_{\mu\nu\eta\kappa} = 0, s_{(a)b} \phi^{(2)} = 0$. Thus, there is a distinct difference between $s_{(a)b}$ and $s_{(a)d}$.

### 4 Bosonic symmetry

It is obvious, from the preceding sections, that we have four nilpotent ($s_{(a)b}^2 = 0, s_{(a)d}^2 = 0$) symmetries in the theory. We have also shown that the (anti-)BRST and (anti-)co-BRST symmetries anticommute (i.e. $s_b s_{ab} + s_{ab} s_b = 0, s_d s_{ad} + s_{ad} s_d = 0$) separately and independently. Furthermore, the following anticommutation relations $\dagger$

$$\{s_b, s_{ad}\} = 0, \quad \{s_{ab}, s_d\} = 0,$$  

(16)

are true up to a $U(1)$ vector gauge transformation. The remaining non-vanishing anticommutation relations define the bosonic transformations $(s_\omega, s_\bar{\omega})$ in the operator form. These transformations are succinctly expressed as:

$$\{s_b, s_d\} = s_\omega, \quad \{s_{ab}, s_d\} = s_\bar{\omega}.$$  

(17)

It turns out that the above transformations are symmetry transformations of the theory.

Let us first take the transformations $s_\omega$. Under this continuous symmetry transformations $s_\omega$, the fields of the theory transform as:

$$s_\omega A_{\mu\nu\eta} = \frac{1}{2} \varepsilon_{\mu\nu\rho\kappa\sigma} \partial^\rho B^{\sigma\eta} + (\partial_\mu B_{\nu\eta} + \partial_\nu B_{\mu\eta} + \partial_\eta B_{\mu\nu}), \quad s_\omega B_2 = \partial_\mu B_2,$$

$$s_\omega C_{\mu\nu} = - (\partial_\mu \bar{F}_\nu - \partial_\nu \bar{F}_\mu), \quad s_\omega \bar{C}_{\mu\nu} = \partial_\mu \bar{F}_\nu - \partial_\nu \bar{F}_\mu, \quad s_\omega \beta_\mu = \partial_\mu B_2,$$

$$s_\omega [B, B_1, B_2, B_3, C_1, \bar{C}_1, C_2, \bar{C}_2, \phi^{(1)}_\mu, \phi^{(1)}_\mu, \phi^{(2)}_\mu, \bar{f}_\mu, \bar{f}_\mu, \bar{F}_\mu, \bar{F}_\mu, B_{\mu\nu}, \bar{B}_{\mu\nu}, \bar{B}_{\mu\nu}] = 0.$$  

(18)

$\dagger$The algebra (16) is true for all the fields except $\beta_\mu, \bar{\beta}_\mu, \phi^{(1)}_\mu$ and $\phi^{(2)}_\mu$. It can be verified that $\{s_b, s_{ad}\} \beta_\mu = \partial_\mu (B_1 - B_3), \{s_b, s_{ad}\} \bar{\beta}_\mu = \partial_\mu (B_1 + B_3), \{s_b, s_{ad}\} \phi^{(1)}_\mu = - \partial_\mu B, \{s_b, s_{ad}\} \phi^{(2)}_\mu = - \partial_\mu B, \{s_b, s_{ad}\} \phi^{(2)}_\mu = \partial_\mu B_2$ and $\{s_{ab}, s_d\} \phi^{(2)}_\mu = - \partial_\mu B_2$. Thus, it is clear that the transformations $s_b$ and $s_{ad}$ are anticommuting only up to the $U(1)$ vector gauge transformations. Similarly, the nilpotent transformations $s_d$ and $s_{ad}$ are also anticommuting only up to a $U(1)$ vector gauge transformation. The absolute anticommutativity property (i.e. $\{s_b, s_{ad}\} = 0, \{s_d, s_{ab}\} = 0$) is true for the rest of the fields of the theory. In other words, $s_b$ and $s_{ad}$ (as well as $s_d$ and $s_{ab}$) are independent transformations up to a $U(1)$ gauge transformation. Furthermore, the anticommutators $\{s_d, s_{ab}\}$ and $\{s_b, s_{ad}\}$ do not define the bosonic transformations like $s_\omega$ and $s_{ad}$. 

7
In the above, we have taken into account only the fields of the Lagrangian density (8). The latter itself transforms, under $s_\omega$, as follows:

$$
s_\omega \mathcal{L}_{(B,\bar{B})} = \partial_\mu \left[ (\partial^\mu \mathcal{B}^{\nu}) + \partial^\nu \mathcal{B}^{\mu} + \partial^\nu \mathcal{B}^{\mu
u} ) \mathcal{B}_{\nu\eta} - (\partial^\mu \mathcal{B}^{\nu\eta} + \partial^\nu \mathcal{B}^{\mu\eta} + \partial^\mu \mathcal{B}^{\mu\nu\eta} ) \mathcal{B}_{\nu\eta} 
+ B_2 \partial_\mu \mathcal{B} - B \partial^\mu \mathcal{B}_2 + (\partial^\mu \mathcal{F}^{\nu} - \partial^\nu \mathcal{F}^\mu ) \mathcal{F}_\nu + (\partial^\mu \mathcal{F}^{\nu} - \partial^\nu \mathcal{F}^\mu ) f_\nu \right],
$$

which establishes the fact that the action integral $(\int d^6x \mathcal{L}_{(B,\bar{B})})$ remains invariant under the transformations $s_\omega$. As a consequence, $s_\omega$ is truly a symmetry transformation.

The transformations for the fields of the Lagrangian density $\mathcal{L}_{(\bar{B},\bar{B})}$ [cf. (9)] are as follows under the bosonic symmetry transformation ($s_{\bar{\omega}}$):

$$
s_{\bar{\omega}} A_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial^\rho \mathcal{B}^{\sigma\eta} + \left( \partial_\mu \mathcal{B}_{\nu\eta} + \partial_\nu \mathcal{B}_{\eta\mu} + \partial_\eta \mathcal{B}_{\mu\nu} \right), \quad s_{\bar{\omega}} \beta_\mu = -\partial_\mu \mathcal{B},
$$

$$
s_{\bar{\omega}} C_{\mu\nu} = - (\partial_\mu \mathcal{F}_\nu - \partial_\nu \mathcal{F}_\mu), \quad s_{\bar{\omega}} \bar{C}_\mu = \partial_\mu \bar{f}_\nu - \partial_\nu \bar{f}_\mu, \quad s_{\bar{\omega}} \bar{\beta}_\mu = -\partial_\mu \mathcal{B}_2,
$$

$$
s_{\bar{\omega}} [B, B_1, B_2, B_3, C_1, C_2, \bar{C}_1, \bar{C}_2, \phi^{(1)}_\mu, \phi^{(2)}_\mu, f_\mu, \bar{f}_\mu, F_\mu, \bar{F}_\mu, B_{\mu\nu}, B_{\mu\nu}, B_{\mu\nu}, B_{\mu\nu}] = 0.
$$

It can be verified that the Lagrangian density $\mathcal{L}_{(\bar{B},\bar{B})}$ transforms, under the above transformations $s_{\bar{\omega}}$, to a total spacetime derivative, as given below:

$$
s_{\bar{\omega}} \mathcal{L}_{(\bar{B},\bar{B})} = -\partial_\mu \left[ (\partial^\mu \mathcal{B}^{\nu}) + \partial^\nu \mathcal{B}^{\mu} + \partial^\nu \mathcal{B}^{\mu
u} ) \mathcal{B}_{\nu\eta} - (\partial^\mu \mathcal{B}^{\nu\eta} + \partial^\nu \mathcal{B}^{\mu\eta} + \partial^\mu \mathcal{B}^{\mu\nu\eta} ) \mathcal{B}_{\nu\eta} 
+ B_2 \partial_\mu \mathcal{B} - B \partial^\mu \mathcal{B}_2 + (\partial^\mu \mathcal{F}^{\nu} - \partial^\nu \mathcal{F}^\mu ) \mathcal{F}_\nu + (\partial^\mu \mathcal{F}^{\nu} - \partial^\nu \mathcal{F}^\mu ) \bar{f}_\nu \right],
$$

which shows that the action integral $(\int d^6x \mathcal{L}_{(\bar{B},\bar{B})})$ remains invariant under $s_{\bar{\omega}}$.

We wrap this section with the remark that, even though, the bosonic symmetry transformations $s_\omega$ and $s_{\bar{\omega}}$ look different in their appearance, actually, they are connected with each-other. In fact, it is straightforward to note that, if we exploit the CF-type restrictions of (7) and (14), it can be readily verified that

$$
s_\omega + s_{\bar{\omega}} = 0 \quad \implies \quad (s_\omega + s_{\bar{\omega}}) \Phi = 0,
$$

where the generic field $\Phi = A_{\mu\nu}, C_{\mu\nu}, \bar{C}_{\mu\nu}, \beta_\mu, \bar{\beta}_\mu$ corresponds to all the fields that transform non-trivially under (18) and (20). We conclude that, out of the two bosonic symmetry transformations $s_\omega$ and $s_{\bar{\omega}}$, only one bosonic symmetry transformation is independent.

## 5 Ghost and discrete symmetries

The Lagrangian densities $\mathcal{L}_{(B,\bar{B})}$ and $\mathcal{L}_{(\bar{B},\bar{B})}$ have a part that is dependent on the (anti-) ghost fields and the other part contains fields that have ghost number equal to zero. The ghost part of the Lagrangian densities respects the following global scale transformations:

$$
C_{\mu\nu} \rightarrow e^{+\Omega} C_{\mu\nu}, \quad \bar{C}_{\mu\nu} \rightarrow e^{-\Omega} \bar{C}_{\mu\nu}, \quad \beta_\mu \rightarrow e^{+2\Omega} \beta_\mu, \quad \bar{\beta}_\mu \rightarrow e^{-2\Omega} \bar{\beta}_\mu, \quad C_2 \rightarrow e^{+3\Omega} C_2,
$$

$$
C_2 \rightarrow e^{-3\Omega} C_2, \quad C_1 \rightarrow e^{+\Omega} C_1, \quad C_1 \rightarrow e^{-\Omega} C_1, \quad B \rightarrow e^{+2\Omega} B, \quad B_2 \rightarrow e^{-2\Omega} B_2,
$$

$$
f_\mu \rightarrow e^{+\Omega} f_\mu, \quad \bar{f}_\mu \rightarrow e^{-\Omega} \bar{f}_\mu, \quad F_\mu \rightarrow e^{+\Omega} F_\mu, \quad \bar{F}_\mu \rightarrow e^{-\Omega} \bar{F}_\mu,
$$

$$
\tag{23}
$$
where the numbers \((\pm 1, \pm 2, \pm 3)\) in the exponents denote the ghost numbers of the corresponding ghost and anti-ghost fields and \(\Omega\) is a spacetime independent global scale parameter. It should be pointed out that, under the ghost-scale transformations, the rest of the fields of \(\mathcal{L}_{(B,B)}\) and \(\mathcal{L}_{(B,B)}\) transform as

\[
A_{\mu \nu \eta} \rightarrow A_{\mu \nu \eta}, \quad B_{\mu \nu} \rightarrow B_{\mu \nu}, \quad B_{\mu \nu} \rightarrow B_{\mu \nu}, \quad B_1 \rightarrow B_1, \quad B_3 \rightarrow B_3,
\]

\[
B_{\mu \nu} \rightarrow B_{\mu \nu}, \quad B_{\mu \nu} \rightarrow B_{\mu \nu}, \quad \phi^{(1)}_{\mu} \rightarrow \phi^{(1)}_{\mu}, \quad \phi^{(2)}_{\mu} \rightarrow \phi^{(2)}_{\mu}.
\]  

(24)

This is due to the fact that the ghost number for the above fields is zero. The infinitesimal version of the ghost scale transformations (23) (for \(\Omega = 1\)) is

\[
s_g C_{\mu \nu} = +C_{\mu \nu}, \quad s_g \bar{C}_{\mu \nu} = -\bar{C}_{\mu \nu}, \quad s_g \beta_{\mu} = +2\beta_{\mu}, \quad s_g \bar{\beta}_{\mu} = -2\bar{\beta}_{\mu},
\]

\[
s_g C_2 = +3C_2, \quad s_g \bar{C}_2 = -3\bar{C}_2, \quad s_g C_1 = +C_1, \quad s_g \bar{C}_1 = -\bar{C}_1, \quad s_g B = +2B,
\]

\[
s_g B_2 = -2B_2, \quad s_g f_{\mu} = +f_{\mu}, \quad s_g \bar{f}_{\mu} = -\bar{f}_{\mu}, \quad s_g F_{\mu} = +F_{\mu}, \quad s_g \bar{F}_{\mu} = -\bar{F}_{\mu},
\]  

(25)

and the infinitesimal version of (24) is

\[
s_g \Phi = 0, \quad \Phi = A_{\mu \nu \eta}, B_{\mu \nu}, B_{\mu \nu}, B_{\mu \nu}, \phi^{(1)}_{\mu}, \phi^{(2)}_{\mu}, B_1, B_3.
\]  

(26)

These infinitesimal transformations (25) and (26) would play very important roles in the derivation of the algebraic structures that we shall discuss in the next section.

In addition to the above ghost-scale transformations, there exists the following discrete transformations for the non-ghost sector of the Lagrangian densities (8) and (9):

\[
A_{\mu \nu \eta} \rightarrow \pm \frac{i}{3!} \epsilon_{\mu \nu \eta \rho \sigma} A^{\rho \sigma}, \quad B_{\mu \nu} \rightarrow \pm iB_{\mu \nu}, \quad B_{\mu \nu} \rightarrow \pm iB_{\mu \nu}, \quad B_{\mu \nu} \rightarrow \pm iB_{\mu \nu},
\]

\[
B_{\mu \nu} \rightarrow \pm iB_{\mu \nu}, \quad \phi^{(1)}_{\mu} \rightarrow \pm i\phi^{(2)}_{\mu}, \quad \phi^{(2)}_{\mu} \rightarrow \pm i\phi^{(1)}_{\mu}, \quad B_1 \rightarrow \pm iB_3, \quad B_3 \rightarrow \pm iB_1.
\]  

(27)

under which, the non-ghost part of the Lagrangian densities \(\mathcal{L}_{(B,B)}\) and \(\mathcal{L}_{(B,B)}\) remains invariant. Furthermore, under the discrete transformations given below:

\[
C_{\mu \nu} \rightarrow \pm i\bar{C}_{\mu \nu}, \quad \bar{C}_{\mu \nu} \rightarrow \pm iC_{\mu \nu}, \quad \beta_{\mu} \rightarrow \pm i\bar{\beta}_{\mu}, \quad \bar{\beta}_{\mu} \rightarrow \mp i\beta_{\mu}, \quad C_2 \rightarrow \pm i\bar{C}_2,
\]

\[
\bar{C}_2 \rightarrow \pm iC_2, \quad C_1 \rightarrow \mp i\bar{C}_1, \quad \bar{C}_1 \rightarrow \mp iC_1, \quad B \rightarrow \mp iB_2, \quad B_2 \rightarrow \pm iB,
\]

\[
f_{\mu} \rightarrow \pm i\bar{F}_{\mu}, \quad \bar{F}_{\mu} \rightarrow \pm iF_{\mu}, \quad \bar{f}_{\mu} \rightarrow \pm i\bar{f}_{\mu}, \quad F_{\mu} \rightarrow \pm i\bar{F}_{\mu},
\]  

(28)

the ghost part of the Lagrangian densities \(\mathcal{L}_{(B,B)}\) and \(\mathcal{L}_{(B,B)}\) remains invariant. The above discrete transformations (27) and (28) would play very important roles in the determination of the algebraic structures amongst the symmetry transformations and its connection with the algebra of de Rham cohomological operators of differential geometry (cf. Sec. 6).

6 Algebraic structures

It is clear, from the preceding sections, that there exist six continuous symmetries and one discrete symmetry in the theory. The operator form of the continuous symmetry transformations obey the following algebraic structures, namely:

\[
s_{(a)b}^2 = 0, \quad s_{(a)d}^2 = 0, \quad \{s_b, s_{ab}\} = 0, \quad \{s_d, s_{ad}\} = 0, \quad \{s_b, s_{ad}\} = 0,
\]

\[
[s_g, s_b] = +s_b, \quad [s_g, s_{ab}] = -s_{ab}, \quad [s_g, s_d] = -s_d, \quad [s_g, s_{ad}] = +s_{ad},
\]

\[
\{s_{ab}, s_d\} = 0, \quad \{s_b, s_d\} = s_\omega = -\{s_{ab}, s_{ad}\}, \quad [s_\omega, s_r] = 0, \quad r = g, b, ab, d, ad.
\]  

(29)
In the above, the infinitesimal version of transformations (3), (5), (10), (12), (18), (25) and (26) play crucial roles. A close look at the above algebra shows that this algebra is the algebra of de Rham cohomological operators \((d, \delta, \Delta)\) of differential geometry.

To corroborate the above statement, let us recall that the set of de Rham cohomological operators of differential geometry obey the following algebra

\[
d^2 = 0, \quad \delta^2 = 0, \quad \{d, \delta\} = \Delta = (d + \delta)^2, \quad [\Delta, \delta] = 0, \quad [\Delta, d] = 0, \quad \delta = -\ast d\ast, \tag{30}
\]

where \(*\) is the Hodge duality operator and \(\Delta\) is the Casimir operator for the whole algebra. The (co-)exterior derivatives (lower)raise the degree of a form by one on which they operate. In contrast, the degree of a form remains intact when it is operated upon by \(\Delta\).

An accurate comparison of (29) and (30) establishes the fact that the sets \((s_b, s_d, s_\omega)\) as well as \((s_{ab}, s_{ad}, -s_\omega)\) are the analogue of the de Rham cohomological operators \((d, \delta, \Delta)\).

Finally, we make statements on the analogue of the Hodge duality \(*\) operator in the language of the symmetry transformations. It turns out that the discrete symmetry transformations listed in (27) and (28), combined together, lead to the derivation of the following relationship between the continuous symmetry transformation \(s_{(a)b}\) and \(s_{(a)d}\), namely;

\[
s_{(a)d} = \pm \ast s_{(a)b} \ast, \tag{31}
\]

where \(\ast\) corresponds to the transformations listed in (27) and (28). The \((+)-\) signs, in the above, are dictated by the two successive operations of the discrete transformations (27) and (28) on a generic field \(\Phi\) of the theory. This statement can be mathematically expressed in a succinct form as follows (see, e.g. [28] for details on duality):

\[
\ast (\ast \Phi) = \pm \Phi. \tag{32}
\]

One can explicitly check that the \(+\) sign, in the above, is true only for the four fields. These fields are \(\Phi = \beta_\mu, \bar{\beta}_\mu, B, B_2\). Rest of the fields correspond to \((-\) sign in (32).

7 Conclusions

In our present investigation, we have shown the existence of local, off-shell nilpotent and absolutely anticommuting (anti-)co-BRST symmetry transformations in the context of a 6D Abelian 3-form gauge theory. One of the decisive features of the above symmetry transformations is the fact that the total gauge-fixing term, owing its origin to the co-exterior derivative, remains invariant. Thus, one of the (anti-)co-BRST symmetry transformations is an analogue of the co-exterior derivative. On the contrary, it is the kinetic term that remains invariant under the nilpotent (anti-)BRST symmetry transformations. Thus, the off-shell nilpotent BRST transformation is an analogue of the nilpotent exterior derivative because the BRST invariant kinetic term has its origin in this cohomological operator.
We have demonstrated that a suitable anticommutator of the above (anti-)BRST and (anti-)co-BRST symmetry transformations leads to the definition of a single independent bosonic symmetry in the theory, under which, the (anti-)ghost fields transform to the $U(1)$ vector as well as tensor gauge transformations [cf. (18)]. We have also shown that the bosonic symmetry ($s_\omega$) commutes with all the rest of the symmetry transformations of the theory. As a consequence, it is the Casimir operator for the whole algebra and it behaves like the Laplacian operator of differential geometry. Thus, in our present work, we have clearly provided the physical realizations of the cohomological operators ($d, \delta, \Delta$).

One of the novel observations of our present endeavor is that the anticommutators $\{s_b, s_{ad}\}$ and $\{s_{ab}, s_d\}$ are not absolutely zero for the specific fields of theory. In fact, as it turns out, the absolute anticommutativity of the above operators is true only upto the $U(1)$ gauge transformations. This observation is totally different from our earlier works on the free 4D Abelian 2-form [20,21] as well as 2D (non-)Abelian 1-form (free as well as interacting) gauge theories [18,19,17] where we have absolute anticommutativity between $s_d$ and $s_{ab}$ as well as $s_b$ and $s_{ad}$. We very strongly feel that such kind of novel observations will appear in the context of BRST analysis of the higher dimensional ($D > 4$) and higher $p$-form ($p \geq 3$) gauge theories that are important in the context of (super)string theories.

In our present endeavor, we have discussed only the symmetry transformations. We plan to calculate all the generators for the continuous symmetry transformations of the present theory and establish that the present theory is a model for the Hodge theory. Further, we also wish to study its topological features. We feel that the present theory might turn out to be a cute field theoretic model for a topological field theory as we have been able to demonstrate in the case of the free 2D (non-)Abelian 1-form gauge theories [17]. These are some of the issues that are presently under intensive investigation and a longer version of our present work will be reported in our future publications. In this context, it is gratifying to state that we have already accomplished some of the above goals in [29].

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