Canonical Quantization of Photons in a Rindler Wedge

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Abstract

Photons and thermal photons are studied in the Rindler Wedge employing Feynman's gauge and canonical quantization. A Gupta-Bleuler-like formalism is explicitly implemented. Non thermal Wightman functions and related (Euclidean and Lorentzian) Green functions are explicitly calculated and their complex time analytic structure is carefully analyzed using the Fulling-Ruijsenaars master function. The invariance of the advanced minus retarded fundamental solution is checked and a Ward identity discussed. It is suggested the KMS condition can be implemented to define thermal states also dealing with unphysical photons. Following this way, thermal Wightman functions and related (Euclidean and Lorentzian) Green functions are built up. Their analytic structure is carefully examined employing a thermal master function as in the non thermal case and other corresponding properties are discussed. Some subtleties arising dealing with unphysical photons in presence of the Rindler conical singularity are pointed out. In particular, a one-parameter family of thermal Wightman and Schwinger functions with the same physical content is proved to exist due to a remaining (non trivial) static gauge ambiguity. A photon version of Bisognano-Wichmann theorem is investigated in the case of photons propagating in the Rindler Wedge employing Wightman functions. Despite of the found ambiguity in defining Rindler Green functions, the coincidence of $\left(\beta = 2\pi\right)$-Rindler Wightman functions and Minkowski Wightman functions is proved dealing with test functions related to physical photons and Lorentz photons.

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I. INTRODUCTION

As well-known, the Rindler wedge $W_R$ is defined by the inequality $x > |t|$ in a fixed system of rectangular coordinates $(t,x,y,z)$ in Minkowski space-time. $W_R$ is a globally hyperbolic submanifold of Minkowski space-time. In this paper, we shall consider $W_R$ as an open set. A global coordinate frame $(\tau, \rho, y, z)$ on $W_R$ is obtained by setting

$$x = \rho \cosh \tau \quad \text{and} \quad t = \rho \sinh \tau ,$$

for $\rho > 0$, so that $x^2 - t^2 = \rho^2$. Notice that any line $\rho = \rho_0$, $y = y_0$, $z = z_0$ are the trajectory of a uniformly accelerated particle, with proper acceleration $a = \rho_0^{-1}$ and proper time $s = a\tau$ along the trajectory. Furthermore, the surfaces $\tau = \text{constant}$ are Cauchy surfaces of $W_R$. The Minkowski metric takes the form of the Rindler metric

$$ds^2 = -\rho^2 d\tau^2 + d\rho^2 + dx_t^2 ,$$

with $\rho > 0$ and $x_t = (y, z)$.

As well-known, the Minkowski metric admits the timelike Killing field $K = \partial_\tau$ in $W_R$. This vector generates the isometry $\tau \to \tau + \tau_0$. The hypersurface $\rho = 0$, i.e., $x^2 - t^2 = 0$ is a Killing event horizon which bifurcates in the transverse two-plane $x = t = 0$.

We remind that the Rindler metric approximates the metric near the horizon of a Schwarzschild black hole. In this sense, the physics in the Rindler wedge is a toy model of the physics around a black hole. Thus, we expect that some result of this paper can be extended to the Schwarzschild black hole case.

In the present paper we shall study the quantum field theory in the Rindler region $W_R$ in the case of a photon field by building up its Fock representation over the Fulling vacuum $|\mathcal{F}>$ which is invariant under $\tau-$translations. Other authors have studied photon field or thermal photons in the Rindler wedge, directly employing the strength field $F_{\mu\nu}$ instead of the vectorial field $A_\mu$, thus avoiding gauge related problems, or they have analyzed particular problems only. In this paper we shall develop a more mathematical and systematic studying using the field $A_\mu$.

In Section I, we shall implement a canonical approach to quantization of the vectorial photon field using Feynman’s gauge, taking care to correctly deal with the arising unphysical photons. In fact, a Gupta-Bleuler-like formalism will be explicitly implemented and the non-Hilbertian structure of the quantum state space analyzed.

In Section II, the Wightman functions will be built up within the frame-work of a three-smeared distributional formalism. The whole analytic structure of these functions, the related Schwinger function, Feynman propagator and the advanced-minus-retarded function will be analyzed employing a Rindler-time complex master function introduced by Fulling and Ruijsenaars for the scalar case. In particular, the expected invariance of the advanced-minus-retarded function will be proved. Finally, a Ward identity will be discussed.

In Section III, we shall propose a definition of thermal states in terms of Wightman functions which uses the KMS condition also dealing with unphysical photons. We shall see that this definition agrees with all the expected physical requirements. The thermal Wightman functions will be explicitly built up employing the sum over images method and a thermal master function analysis will be implemented. We shall see that some gauge ambiguities remain in the definition of these Green functions. In fact we shall find an one-parameter class of physically equivalent master (Schwinger, Wightman) functions which, differently from the scalar case, are defined away from the conical singularity. This is due to static non trivial unphysical terms which affect all the thermal Green functions. In the case $\beta = 2\pi$ (absence of conical singularities) only one particular Green function defined on the whole Euclidean manifold will arise.
from the above-mentioned class. The Wightman functions, related by analytic continuation to this special Schwinger function, will give rise to the local coincidence of the Minkowski vacuum with the $\beta = 2\pi$ thermal Rindler state. This coincidence generalizes, in terms of Wightman functions, the Bisognano-Wichmann theorem [7, 10, 11] including the photon field. This local vacua identity, considered as a Wightman functions identity, will be proved employing physical or Lorentz test wavefunctions.

II. PHOTON FIELD QUANTIZATION AND GUPTA-BLEULER FORMALISM IN A RINDLER WEDGE

A. INDEFINITE SCALAR PRODUCT

The first step to quantize a (quasi-)free field theory in a globally hyperbolic space-time consists of the definition of an appropriate conserved indefinite scalar product with respect to spatial Cauchy 3-surfaces of space-time; this inner product does not depend on the particular choice of a Cauchy surface [1, 13]. We shall suppose to work in $W_R$ using coordinates $(x^0, x^1, x^2, x^3) = (\tau, \rho, y, z)$ defined above. It is convenient to represent the inner product on the $x^0 = \tau = \text{constant}$ spatial surfaces, they being Cauchy surfaces. A natural choice of a quantum vacuum (Fulling vacuum in our case) is obtained by decomposing the field over normalized modes which are imaginary exponentials in the chosen time [14, 15]. Thus, the creation and destruction operators related with these modes define both the quantum vacuum and the corresponding Fock representation.

Let us define the canonical conjugate momentum of the real field $A_\mu$ in the case of Feynman gauge as $^0$)

$$\Pi^\mu_\nu := \frac{\partial \mathcal{L}}{\partial \partial_\nu A_\mu} = \sqrt{-g} \left[ \nabla_\nu A^\mu - \nabla^\mu A_\nu - g^{\mu\alpha} \nabla_\alpha A_\nu \right] = \sqrt{-g} \left( F^\mu_\nu - g^{\mu\nu} \nabla_\alpha A_\alpha \right),$$

where $\nabla_\mu$ is the covariant derivative and

$$\mathcal{L} := -\sqrt{-g} \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (\nabla^\nu A_\nu)^2 \right] = -\frac{\sqrt{-g}}{2} \left[ \nabla_\mu A_\nu \nabla^\mu A^\nu + \text{(Cov. Deriv.)} \right] \quad (2)$$

is the Lagrangian of the photon in the Feynman gauge, $g$ standing everywhere for the determinant of the complete metric. Classically, one has to impose the Lorentz condition

$$\nabla_\mu A_\mu = 0, \quad (3)$$

as constraint on the solutions of the vectorial Klein-Gordon equation produced by the the Lagrangian (2) as motion equations

$$\nabla_\mu \nabla^\mu A_\nu (x) = 0 \quad (4)$$

Dealing with generally complex photon wavefunctions, the usual canonical conserved indefinite scalar product reads $^8$:

$$(A, B) = i \int_\Sigma dS \sqrt{h} \ n_\mu \left( A_\nu \Pi^\mu_\nu - B_\nu \Pi^\mu_\nu \right) \frac{1}{\sqrt{-g}}, \quad (5)$$

where $dS := dx^1 dx^2 dx^3$, $\Sigma$ is a $x^0$ constant Cauchy surface and $n = -dx^0 / \sqrt{-(dx^0, dx^3)}$ is its normalized, positive time oriented, normal vector. Finally, $h$ is the determinant of the Euclidean 3-metric $h_{ij}$ induced on $\Sigma$.

A simpler conserved scalar product follows from the Fermi Lagrangian obtained by dropping
the classically unimportant total derivative term in Eq.(2). The motion equations remain Klein-Gordon Equations (4). Dealing with as in the previous case, we obtain a new conserved inner product:

\[ (A, B)' = -i \int_{\Sigma} dS\sqrt{h} \, n^{\mu} A_\nu^* \nabla_{\mu} B^{\nu}, \]

where \( f \nabla_{\mu} g := f \nabla_{\mu} g - g \nabla_{\mu} f \). The relation between the two scalar products \( C \) coincide. As in the case of a scalar field \([1, 15]\), it might be possible to build up the whole specify further details. We suppose to deal with the scalar product \((\, , \, )\) with wavefunctions containing both positive and negative frequencies \([1, 15]\). For the time being, trivially coincide. However, this choice (at least dealing with the spinless case) requires to deal both positive and negative frequencies and thus we shall re-consider the identity \((\, , \, ) = (\, , \, )'\) with wavefunctions with spatial compact support containing the classically unimportant total derivative term in Eq.(2). The motion equations remain Klein-Gordon Equations (4). Dealing with as in the previous case, we obtain a new conserved inner product:

\[ (A, B)' = -i \int_{\Sigma} dS\sqrt{h} \, n^{\mu} A_\nu^* \nabla_{\mu} B^{\nu}, \]

where \( f \nabla_{\mu} g := f \nabla_{\mu} g - g \nabla_{\mu} f \). The relation between the two scalar products \( C \) coincide. As in the case of a scalar field \([1, 15]\), it might be possible to build up the whole specify further details. We suppose to deal with the scalar product \((\, , \, )\) with wavefunctions containing both positive and negative frequencies \([1, 15]\). For the time being, trivially coincide. However, this choice (at least dealing with the spinless case) requires to deal both positive and negative frequencies and thus we shall re-consider the identity \((\, , \, ) = (\, , \, )'\).

**B. CANONICAL FORMALISM**

Proceeding to the quantization of photon field in \( W_R \), using coordinates \((\tau, \rho, x, t)\), we have to look for a decomposition of the real field \( A_\mu \) as

\[ A_\mu(x) = \int_{R^2} dk_t \int_0^{+\infty} d\omega \, \sum_{\lambda=0}^3 \{ a_{(\lambda,\omega,k_t)} A_{\mu}^{(\lambda,\omega,k_t)}(x) + C.C. \}. \]  

The positive frequency modes

\[ A_{\mu}^{(\lambda,\omega,k_t)}(x) = A_{\mu}^{(\lambda,\omega,k_t)}(\rho, x_t) e^{-i\omega\tau} \]

must be linearly independent solutions of Klein-Gordon equations (4). We require the following normalization of the modes with respect to the scalar product \((\, , \, )\):

\[ (A^{(\lambda,\omega,k_t)}, A^{(\lambda',\omega',k_t')}) = \eta^{\lambda\lambda'} \delta(k_t - k_t') \delta(\omega - \omega'), \]  

\[ (A^{* (\lambda,\omega,k_t)}, A^{* (\lambda',\omega',k_t')}) = -\eta^{\lambda\lambda'} \delta(k_t - k_t') \delta(\omega - \omega'), \]  

\[ (A^{* (\lambda,\omega,k_t)}, A^{(\lambda',\omega',k_t')}) = 0, \]

where \( \eta_{\mu\nu} \equiv \eta^{\mu\nu} \equiv \text{diag} (-1, 1, 1, 1) \). From Eq.(7), it arises:

\[ (A, A') = \int_{R^2} dk_t \int_0^{+\infty} d\omega \, \sum_{\lambda=0}^3 a_{(\lambda,\omega,k_t)}^* a_{\lambda (\lambda',\omega,k_t)} \eta^{\lambda\lambda'}, \]
where $A_\mu$ and $A'_\mu$ are (generally complex) positive frequency photon wavefunctions. The Fourier coefficients $a(\lambda, \omega, k_t)$ are such that the corresponding positive frequency wavefunction $A$ results to be smooth $^{(0)}$. These coefficients, understood as functions of the variables $\omega$ and $k_t$, define one-particle quantum states $|\Psi_A\rangle$. Holding the normalization relations (7), (8) and (9), it simply follows from Eq.(6):

$$a(\lambda, \omega, k_t) = \eta_{\lambda\lambda'}(A^{(\lambda', \omega', k'_t)}(1, A),$$

$$a^*(\lambda, \omega, k_t) = -\eta_{\lambda\lambda'}(A^{*(\lambda', \omega', k'_t)}(1, A).$$

We have to interpret these coefficients as operators to quantize. As usually, the equal time canonical commutations rules of the operator $A$ and its conjugate momentum $\hat{\Pi}$ imply the bosonic commutations rules of the operators $\hat{a}$ and $\hat{a}^\dagger$. They read respectively:

$$[\hat{A}_\mu(x), n_\alpha \hat{\Pi}^{\alpha\nu}(x')]|_{\tau=\tau'} = i\delta^\mu_\nu \delta(\rho - \rho') \delta(x_t - x'_t) I$$

([The remaining (independent) equal time commutators vanish]

where $n := -d\tau/\sqrt{-(d\tau, d\tau)}$, and

$$[\hat{a}(\lambda, \omega, k_t), \hat{a}^\dagger_{\lambda', \omega', k'_t}] = \eta_{\lambda\lambda'}(k_t - k'_t) \delta(\omega - \omega') I$$

([The remaining (independent) commutators vanish]

We expect that not all the one-particle quantum states are representable by smooth positive frequency wavefunctions. This should hold only for states belonging in a linear manifold $\mathcal{M}$ supposed to be dense (in some topology) in the whole one-particle quantum states space. We expect that one-particle quantum states space $\mathcal{H}$ can be represented as an algebraic tensorial product $^{(5)} \mathcal{H} := \mathcal{G}^4 \otimes \mathcal{H}_0$ where $\mathbb{R}_+ := [0, +\infty)$, $\mathcal{H}_0$ being $L^2(\mathbb{R}^2 \times \mathbb{R}_+)$ or a proper closed subspace of this. We can write:

$$|\Psi\rangle = (\Psi_0(k_t, \omega), \Psi_1(k_t, \omega), \Psi_2(k_t, \omega), \Psi_3(k_t, \omega)) \quad k_t \in \mathbb{R}^2, \quad \omega \in \mathbb{R}_+$$

Finally, $\mathcal{H}$ has to be endowed with a scalar product compatible with the above-written commutation relations. This reads:

$$<\Psi|\Psi'> := \sum_{\lambda, \lambda'} \eta_{\lambda\lambda'} \int_{\mathbb{R}} dk_t \int_{\mathbb{R}_+} d\omega \: \Psi_\lambda^*(k_t, \omega)\Psi_{\lambda'}(k_t, \omega).$$

The space $\mathcal{H}$ cannot be considered a properly defined Hilbert space due to presence of the indefinite matrix $\eta$. This matrix appears due to the unphysical degrees of freedom represented by non transverse photons necessary in order to deal with a gauge constraint explicitly covariant as the Lorentz condition Eq.(3). The problem is the same as in Minkowski coordinates quantization. The positive frequency wavefunction related with a generic vector $|\Psi\rangle \in \mathcal{M}$ is the positive frequency smooth function:

$$A_\mu(x) = \int_{\mathbb{R}} dk_t \int_{\mathbb{R}_+} d\omega \: \sum_{\lambda=0}^3 \Psi_\lambda(k_t, \omega)A^{(\lambda, \omega, k_t)}(x),$$

and thus the coincidence of the two scalar product $(A, B)$ and $<\Psi_A|\Psi_B>$ results to be evident. The inverse formula of Eq.(14), following from the normalization relations of the modes Eqs (7), (8) e (9), holds in the same space $\mathcal{M}$:

$$\Psi_\lambda(k_t, \omega) = -\eta_{\lambda\lambda'}(A^{(\lambda, \omega, k_t)}(1, A) =$$

$$-i \sum_{\mu} dS \sqrt{h} n_\mu \left[ A^\alpha_{\nu}(\lambda, \omega, k_t) (F^\mu_{\alpha\nu} - g^{\mu\nu} \nabla_\alpha A^\alpha) - A_{\nu}(F^\mu_{\alpha\nu}(\lambda, \omega, k_t) - g^{\mu\nu} \nabla_\alpha A^{*(\lambda, \omega, k_t)}\right]$$
or, provided $A$ vanish on the edge of $\Sigma$, it reads:

$$\Psi_\lambda(k_t, \omega) = -(A^{(\lambda, \omega, k_t)}, A)' = i \int_{\Sigma} dS \sqrt{h} \; n^\mu A^{s(\lambda, k_t, \omega)}_\mu A'' .$$

The relations (12) have to be more correctly understood as:

$$[\hat{a}_\Psi, \hat{a}_\Psi^\dagger] = \langle \Psi | \Psi' > \mathbb{I} = (A_{\Psi}, A_{\Psi'}) \mathbb{I} \quad \text{if} \quad |\Psi >, |\Psi' > \in \mathcal{M}$$

(The remaining (independent) commutators vanish)

where $\hat{a}_\Psi$ and $\hat{a}_\Psi^\dagger$, when $|\Psi > \in \mathcal{M}$, are interpreted as

$$\hat{a}_\Psi A = (A, \hat{A}) \quad \text{(antilinear in A)},$$

$$\hat{a}_\Psi^\dagger A = -(A, \hat{A}) \quad \text{(linear in A)} .$$

$\hat{A}$ has to be interpreted as an operator valued distribution working on smooth positive frequency photon wavefunctions $A$.

These identities make sense dealing with an appropriate invariant linear manifold dense (in some topology) in the symmetrized Fock-like space $\mathcal{F}(\mathcal{H})$, algebraically built up as a normal symmetrized Fock space, $\hat{a}$ and $\hat{d}^\dagger$ being destruction and creation operators. The Fulling vacuum $|F >$ is defined as:

$$\hat{a}_\Psi |F > = 0 \quad |\Psi > \in \mathcal{H} .$$

By the normalization relations and, in particular, because of the trivial time dependence of the modes, the Rindler Hamiltonian of the photons results to be diagonal if written in terms of $\hat{a}_{(\lambda, \omega, k_t)}$ and $\hat{a}_{(\lambda', \omega', k'_t)}^\dagger$, the spectral parameter being $\omega$. Using the normal order prescription, we have:

$$\hat{H} := \int_{\Sigma} dS \sqrt{h} n_\sigma \hat{\Pi}^\nu n_\lambda \partial^\lambda \hat{A}_\nu - \hat{\mathcal{L}} := \int d\omega k_t d\omega \sum_{\lambda=0}^3 \eta^{\lambda\lambda} \hat{a}_{(\lambda, \omega, k_t)}^\dagger \hat{a}_{(\lambda, \omega, k_t)} .$$

Thus we can consider the quanta generated by $\hat{a}_{(\lambda', \omega', k'_t)}^\dagger$ as defined Rindler-energy particles. However, there arise particles of negative norm and energy due the anomalous commutation rule of $\hat{a}_{\lambda=0}$ and $\hat{a}_{\lambda=0}^\dagger$ as in the Minkowskian case (f). We expect that a Gupta-Bleuler-like formalism (see [16] for example) can be used in order to deal more correctly with the Feynman gauge.

C. NORMAL MODES AND ONE-PARTICLE SPACE

Let us seek a set of normal modes solutions of Klein-Gordon equation and satisfying the constraints in Eq.s (7), (8), (9). We report the results and some comments here. All the calculations are contained in Appendix A.

We start with the independent modes suggested by Higuchi, Matsas and Sudarsky in [8].

$$A_{\mu}^{(s, \lambda, \omega, k_t)} = C^{(s, \lambda, \omega, k_t)}(0, 0, k_z \phi, -k_y \phi) , \quad (15)$$

$$A_{\mu}^{(II, \lambda, \omega, k_t)} = C^{(II, \lambda, \omega, k_t)}(\rho \partial_\rho \phi, -i \frac{\omega}{\rho} \phi, 0, 0) , \quad (16)$$

$$A_{\mu}^{(G, \omega, k_t)} = C^{(G, \omega, k_t)}(-i \omega \phi, \partial_\rho \phi, i k_y \phi, i k_z \phi) \equiv C^{(G, \omega, k_t)} \partial_\mu \phi , \quad (17)$$

$$A_{\mu}^{(L, \omega, k_t)} = C^{(L, \omega, k_t)}(0, 0, k_y \phi, k_z \phi) , \quad (18)$$

where the coefficients $C$ are normalization constants, and the field $\phi = \phi(\omega, k_t)(x)$ is the mode solution of scalar Klein-Gordon equation in $W_R$:

$$\phi^{(\omega, k_t)}(x) = K_{\omega}(k_{\perp} \rho) e^{i(k_{\perp} x_1 - \omega t)} . \quad (19)$$
\( K_\nu(z) \) is a well-known MacDonald function of imaginary index \([7]\), \( k_\perp := \sqrt{k_y^2 + k_z^2} \) and \( k_t x_t := k_y y + k_z z \). The Klein-Gordon equation reads:

\[
(-\frac{1}{\rho} \partial^2_t + \partial_\rho \rho \partial_\rho + \rho \nabla^2_\perp) \phi = 0 ,
\]

and the solution \( \phi = \phi(\omega, k_t) \) also satisfies:

\[
\partial^2_t \phi = -\omega^2 \phi \quad \text{and} \quad \nabla^2_\perp \phi := \sum_{\alpha=y,z} \partial^2_{\alpha} \phi = -k^2_\perp \phi .
\]

It can be simply proved that the modes \( I \) and \( II \) satisfy the Lorentz condition \([3]\).

The mode \( G \) is proportional to \( \partial_\mu \phi \) and thus it is a pure gauge mode; note that this also satisfies the Lorentz condition because \( \phi \) is a solution of scalar Klein-Gordon equation. The mode \( L \) does not satisfy the gauge condition. Using the inner product \((\ , \ )\), one finds the mode \( I \) to be normal to the mode \( II \), furthermore the linear space spanned by the unphysical modes \( G \) and \( L \) results to be normal to the modes \( I \) and \( II \). Departing from the work \([8]\), we follow a different choice of unphysical modes in order to have a complete set of normal to each other modes. We define new modes, considering two convenient linear combinations of unphysical modes \( G \) and \( L \):

\[
A^{(1, \omega, k_t)}_\mu \equiv C^{(1, \omega, k_t)}(0, 0, k_z \phi, -k_y \phi) , \quad (20)
A^{(2, \omega, k_t)}_\mu \equiv C^{(2, \omega, k_t)}(\rho \partial_\rho \phi, -i \omega \phi, 0, 0) , \quad (21)
A^{(3, \omega, k_t)}_\mu \equiv C^{(3, \omega, k_t)}(-i \omega \phi, \partial_\rho \phi, 0, 0) , \quad (22)
A^{(4, \omega, k_t)}_\mu \equiv C^{(4, \omega, k_t)}(0, 0, i k_y \phi, i k_z \phi) . \quad (23)
\]

Following the calculations of Appendix A we may define normalized modes \( A^{(\lambda, \omega, k_t)}_\mu \):

\[
A^{(0, \omega, k_t)}_\mu \equiv \sqrt{\sinh \frac{\pi \omega}{2 \pi^2 k_\perp}} (-i \omega \phi, \partial_\rho \phi, 0, 0) \equiv A^{(G, \omega, k_t)}_\mu - i A^{(L, \omega, k_t)}_\mu , \quad (24)
A^{(1, \omega, k_t)}_\mu \equiv \sqrt{\sinh \frac{\pi \omega}{2 \pi^2 k_\perp}} (0, 0, k_z \phi, -k_y \phi) , \quad (25)
A^{(2, \omega, k_t)}_\mu \equiv \sqrt{\sinh \frac{\pi \omega}{2 \pi^2 k_\perp}} (\rho \partial_\rho \phi, -i \omega \phi, 0, 0) , \quad (26)
A^{(3, \omega, k_t)}_\mu \equiv \sqrt{\sinh \frac{\pi \omega}{2 \pi^2 k_\perp}} (0, 0, i k_y \phi, i k_z \phi) \equiv i A^{(L, \omega, k)}_\mu . \quad (27)
\]

Using these modes, the normalization relations Eq.s \([7], [8], [8]\) are satisfied.

To conclude, we are able to suggest a possible definition of the space \( \mathcal{M} \) and the one-particle space \( \mathcal{H} \). However, we shall not study this topic in deep. Let us consider the set \( S \) of the \( C^\infty \) real wavefunctions solutions of the vectorial K-G equation with spatial compact support at \( |r| < +\infty \) and such that their transverse Fourier transform vanishes with order \( |k_t|^n \ n \geq 1 \) as \( k_t \to 0 \). They are, for example, transverse coordinate Laplacians of \( C^\infty \) compact support K-G solutions. The required condition, passing to the Fourier decomposition through Eq.\((24)\) (see below) cancels against the divergent factor \( k_\perp^{-1} \) in the modes and assures a finite \( L^2 \) norm (see below) \( h^) \). The following decomposition arises (note the presence of negative frequencies):

\[
A_\mu(x) = \int_{\mathbb{R}} dk_t \int_{\mathbb{R}_+} d\omega \sum_{\lambda=0}^{3} \Psi_\lambda(k_t, \omega) A^{(\lambda, \omega, k_t)}_\mu(x) + \sum_{\lambda=0}^{3} \Psi_\lambda^*(k_t, \omega) A^{*(\lambda, \omega, k_t)}_\mu(x) , \quad (28)
\]

where

\[
\Psi_\lambda(k_t, \omega) = -(A^{(\lambda, \omega, k_t)}, A) = -(A^{(\lambda, \omega, k_t)}, A)' , \quad (29)
\]
because the boundary terms vanish due to compact support of \( A \). Employing the previously found normal modes and standard properties of MacDonald's functions \[17\] it is possible to prove that these scalar products are finite. Similarly it is possible to obtain \( (\lambda = 0, 1, 2, 3) \):

\[
\int_{\mathbb{R}^2 \times \mathbb{R}_+} |\psi_{\lambda}(k_t, \omega)|^2 dk_t d\omega < +\infty ,
\]

when \( A \in S \). We define \( S := S > \mathbb{R} \), i.e., the real linear space spanned by the set \( S \). We can define \( M \) as the complex linear space \( M := \langle M > \mathbb{C} \), where \( M \) is the set of states defined by the left hand side of Eq.(29) when \( A \in S \). The positive frequency wavefunctions of the state \( |\psi > \in M \) is written in Eq.(14). The one photon Hilbert space is thus defined as \( H = M = \mathbb{C}^4 \otimes \mathcal{H}_0 \). The closure as well as the topological tensorial product will be defined employing a certain topology specified in the following section.

**D. GUPTA-BLEULER FORMALISM IN A RINDLER SPACE**

Following the Minkowskian theory, the quantum states space of the whole theory must be formally defined as the space of vectors \( |\psi > \in \mathcal{F}(\mathcal{H})_s \) satisfying the Lorentz constraint:

\[
\nabla^\mu \hat{A}_\mu^+(x) |\psi > = 0 ,
\]

where \( \hat{A}_\mu^+ \) is the part of the operator \( \hat{A} \) containing only destructor operators. Eq.(30) is equivalent to:

\[
(\hat{a}_{(3,k_t,\omega)} - \hat{a}_{(0,k_t,\omega)}) |\psi > = 0 \quad \text{for all} \quad k_t, \omega .
\]

This equation defines the physical quantum states exactly as in the case of the Minkowski theory. Furthermore, it can be simply proved that:

\[
\nabla^\mu A_{\mu}^{(\alpha,\omega,k_t)} = 0 \quad \text{for } \alpha = 1, 2 .
\]

These modes defines real particles, endowed with a positive norm and a positive energy. They are the transverse photons, namely the two physical degrees of freedom of Rindler photons. The Gupta-Bleuler formalism can be employed as in flat coordinates. Eq.(31) reads in a non formal representation (where the indices of destructor operators are referred to the notation in Eq.(13)):

\[
(\hat{a}_{(\Phi,0,0,0,0)} - \hat{a}_{(0,0,0,0,\Phi)}) |\psi > = 0 \quad \text{for all} \quad \Phi \in \mathcal{H}_0 .
\]

The space of these vectors will be termed \( \mathcal{F}(\mathcal{H}_L)_s \). The one particle space \( \mathcal{H}_L \) can be defined introducing the operators:

\[
\hat{\alpha}_\Phi := \frac{1}{\sqrt{2}}(\hat{a}_{(\Phi,0,0,0,0)} - \hat{a}_{(0,0,0,0,\Phi)}) \quad \text{and} \quad \hat{\beta}_\Phi := \frac{1}{\sqrt{2}}(\hat{a}_{(\Phi,0,0,0,0)} + \hat{a}_{(0,0,0,0,\Phi)})
\]

Thus Eq.(33) reads simply:

\[
\hat{\alpha}_\Phi |\psi > = 0 \quad \text{for all} \quad \Phi \in \mathcal{H}_0 ,
\]

and \( \mathcal{H}_L \) contains one-particle states defined by the creation operators \( \hat{a}_1^+, \hat{a}_2^+, \hat{a}_3^+ = \hat{\beta}^* \) only (the adjoint \( \hat{\beta}^* \) is defined below).

One can trivially obtain the form of the wavefunction of \( |\psi > \in \mathcal{H}_L \) passing to the base of the modes \( A^{(1)}, A^{(2)}, A^{(G)}, A^{(L)} \) previously introduced. This reads:

\[
A_L^x(x) = \int_{\mathbb{R}^2} dk_t \int_{\mathbb{R}_+} d\omega \sum_{\lambda=1,2,G} \psi_{\lambda}(k_t, \omega) A_{\mu}^{(\lambda,\omega,k_t)}(x) .
\]
These wavefunctions satisfy the Lorentz condition in Eq. (1).
We define the physical Fock space $F(\mathcal{H}_P)_s$ by requiring the total absence of unphysical photons:
$$\hat{a}_{(\Phi,0,0,0)}|\Psi> = \hat{a}_{(0,0,0,\Phi)}|\Psi> = 0 \quad \text{for all } \Phi \in \mathcal{H}_0.$$ 

The wavefunctions of the states of this space read:
$$A^P_{\mu}(x) = \int_{\mathbb{R}^2} dk_t \int_{\mathbb{R}_+} d\omega \sum_{\lambda=1,2} \Psi_{\lambda}(k_t,\omega)A_{\mu}^{(\lambda,\omega,k_t)}(x)$$

Obviously, it holds $F(\mathcal{H})_s \supset F(\mathcal{H}_L)_s \supset F(\mathcal{H}_P)_s$.

It is possible to define a metrical topology compatible with the physics of the system. Following the Minkowski Gupta-Bleuler formalism, we define a new, positive defined, scalar product of $|\Psi>$ and $|\Psi'> \in \mathcal{H}$ by

$$<\Psi/\Psi'> := \sum_{\lambda,\lambda'=0}^3 \delta^{\lambda\lambda'} \int_{\mathbb{R}} dk_t \int_{\mathbb{R}_+} d\omega \, \Psi^{\dagger}_{\lambda}(k_t,\omega)\Psi_{\lambda'}(k_t,\omega).$$

We shall call this unphysical scalar product the Euclidean scalar product. Employing this we may make $\mathcal{H} = \mathcal{T} \otimes \mathcal{H}_0$ a correctly defined Hilbert space. $\mathcal{H}_0$ results to be $L^2(\mathbb{R}^2 \times \mathbb{R}_+)$ or a proper closed subspace of this obtained imposing $\mathcal{M} = \mathcal{C}^\infty \otimes T \mathcal{H}_0$. We can correctly define the Fock space $F(\mathcal{H})_s$ as a Hilbert space, too. Finally, we can define the spaces $F(\mathcal{H}_L)_s$ and $F(\mathcal{H}_P)_s$ as closed subspaces by taking the topological closure of the corresponding algebraically defined linear manifolds. Using the norm related with the Euclidean scalar product, i.e., the Euclidean norm, we can also define topological tools on the space of operators, in particular we can define the adjoint (including its domain) of an operator $\hat{O}$ represented by the symbol $\hat{O}^\dagger$.

Then we introduce on $F(\mathcal{H})_s$ the limited operator $M$ as the only operator satisfying $0$:

$$M|F> = |F>, \quad MM = I, \quad M^* = M$$

and furthermore:

$$M\hat{a}_{\lambda} = \eta^{\lambda\lambda'}\hat{a}_{\lambda}M \quad \text{(and thus } M\hat{a}_{\lambda}^\dagger = \eta^{\lambda\lambda'}\hat{a}_{\lambda}^\dagger M).$$

Now, we can define the the physical scalar product, by the continuous sesquilinear form:

$$<\Psi/\Psi'> := <\Psi/M/\Psi' >.$$  \hspace{1cm} (34)

Employing this definition, one has to define the adjoint with respect to the physical scalar product (including the definition of the domain of the adjoint) as

$$\hat{O}^\dagger = M\hat{O}^*M.$$  

In particular, it holds:

$$\hat{a}_{\lambda}^\dagger = \eta^{\lambda\lambda'}\hat{a}_{\lambda}^\dagger, \quad \hat{a}^\dagger = \hat{\beta}^*, \quad \hat{\beta}^\dagger = \hat{a}^* \quad \text{and } M^\dagger = M^* = M.$$ 

Starting from formulae obtained above, it can be simply proved that the following statements hold:

- if $|\Psi>, |\Psi' \in F(\mathcal{H}_P)_s$ then $<\Psi/\Psi'> = <\Psi/\Psi' >$,
- if $|\Psi>, |\Psi' \in F(\mathcal{H}_L)_s$ then $<\Psi/\Psi'> = <\Psi/P_P/\Psi' >$,

where $P_P$ is the normal projector onto the (closed) physical Fock space $F(\mathcal{H}_P)_s$. Working in the space $F(\mathcal{H}_L)_s$, only the physical part of the state contributes to the physical scalar products. Thus, we have found exactly the same features which appear in Minkowski theory \cite{16}. In particular, a necessary condition to consider an operator $\hat{O}$ physically sensible, i.e., a gauge invariant observable, consists of the requirement

$$<\Psi/\hat{O}/\Psi'> = <\Psi/P_P/\hat{O}/P_P/\Psi' >,$$

for all the vectors $|\Psi>, |\Psi' \in F(\mathcal{H}_L)_s$ which make sensible the left hand side of this identity.
Substituting this in the left hand side of Eq. (35), it arises holding when 

\[
\langle F| (A, \hat{A})(A', \hat{A})|F > =
\]

\[
= \int_S dS \sqrt{h(x)} n^\nu \int_S dS' \sqrt{h(x')} n'^\nu' \ A^\mu(x) A'^\mu'(x') \hat{\nabla}_\nu \hat{\nabla}_{\nu'} \langle F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F>
\]

(35)

where \( A \) and \( A' \) belong to \( \mathcal{S} \). Notice that, employing such wavefunctions, \((, , ) = (, , )'\). We shall use only the notation \((, , )\) for sake of simplicity. We stress that the left hand side of Eq. (35) is defined independently on the right hand side: \(< F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F >\) is defined just to make sensible Eq. (35) in a distributional-like sense.

Alternatively one could try to define the Wightman functions by using a *four smeared* formalism (see Appendix C). In this paper we prefer to use the *three smeared formalism* based on Eq. (35). Eqs. (15) and (13) lead us to

\[
(A, \hat{A}) = \hat{a}_{\Psi_A} - \hat{a}_{\Psi_A}^\dagger,
\]

holding when \( A, A' \in \mathcal{S} \) are *real* and thus \(|\Psi_A >, |\Psi_A' >| \in \mathcal{M}\) are obtained employing Eq. (29). Substituting this in the left hand side of Eq. (35), it arises \(j\)

\[
= - \int_{\mathbb{R}^3} dk_i \int_{\mathbb{R}^3} dk'_i \int_{\mathbb{R}^3} d\omega \sum_{\lambda \lambda'} \eta^{\lambda \lambda'} (k_i, \omega) \Psi_{\lambda A}^* (k_i, \omega) =
\]

\[
= - \sum_{\lambda \lambda'} \eta^{\lambda \lambda'} \int_{\mathbb{R}^3} dk_i \int_{\mathbb{R}^3} d\omega \int_{\Sigma} dS \sqrt{h} n^\mu A^\mu(k_i, \omega) \hat{\nabla}_\mu A'^\nu \int_{\Sigma} dS' \sqrt{h} n'^\nu' A'^\nu'(k_i, \omega) \hat{\nabla}_{\nu'} A'^\nu'.
\]

It is possible to change the order of the integrals in the right hand side of the equation written above by introducing an \(\varepsilon-\)prescription in the time variable appearing into normal modes. Thus we obtain in a distributional sense (namely taking the limit \( \varepsilon \to 0^+ \) at the end of calculation):

\[
< F|(A, \hat{A})(A, \hat{A}')|F > =
\]

\[
= - \int_{\Sigma} dS \sqrt{h(x)} n^\nu \int_S dS' \sqrt{h(x')} n'^\nu' \ A^\mu(x) A'^\mu'(x') \hat{\nabla}_\nu \hat{\nabla}_{\nu'} \langle F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F > ,
\]

where:

\[
< F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F > (=< F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F >_\varepsilon) =
\]

\[
:= \int_{\mathbb{R}^3} dk_i \int_{\mathbb{R}^3} d\omega \sum_{\lambda \lambda'} \eta^{\lambda \lambda'} (k_i, \omega, k_i) A^\mu_{\lambda \omega, k_i}(\rho, \tau, x_i) A'^\nu'_{\lambda \omega', k_i}(\rho', \tau', x'_i) e^{-i\omega(\tau - \tau' - \varepsilon)} .
\]

Using less rigor, we can obtain the same result by expanding the field operators which appear in the formal object \(< F| \hat{A}_\mu(x) \hat{A}'_{\mu'}(x')|F >\) over the normal modes and introducing the \(\varepsilon-\)prescription by hand.
Using the modes in Eq.s (24), (25), (26) and (27), the expression (19) of the field \( \phi \) and substituting all them in the equation above, one finds the following integral decomposition:

\[
W_{\mu \nu}^+(x, x') = W_{\mu \nu}^+(x, x')_{\infty} := < F | \hat{A}_\mu(x) \hat{A}_\nu(x') | F > = \\
= \frac{1}{4\pi^4} \int_{\mathbb{R}^2} d^2k \frac{e^{ik_t(x_t-x'_t)}}{k^2_t} D_{\mu \nu} \int_0^{+\infty} d\omega \sinh \pi \omega K_{\omega}(k_{\perp} \rho)K_{\omega}(k_{\perp} \rho') e^{-i\omega(\tau-\tau'-i\varepsilon)},
\]

the operator \( D_{\mu \nu} \) is defined as

\[
D_{\tau \tau'} := -\partial_\tau \partial_{\tau'} + \rho \partial_\rho \partial_{\rho'}, \\
D_{\rho \rho'} := \frac{1}{\rho \rho'} [\partial_\tau \partial_{\rho'} - \rho \partial_{\rho} \partial_{\rho'}], \\
D_{\tau \rho'} := \frac{1}{\rho} [\partial_\tau \rho \partial_{\rho'} - \partial_{\tau'} \rho \partial_{\rho}], \\
D_{\rho \rho} := D_{\rho \rho} = k^2_t (= k^2_0)
\]

all the remaining terms vanish.

We shall explicitly calculate the integrals written above employing an indirect method (details are reported in Appendix B).

We needs some preliminary definitions and results.

Let us define the quantity \( \alpha \) by:

\[
\cosh \alpha(\rho, \rho', x_t - x'_t) = \frac{\rho^2 + \rho'^2 + |x_t - x'_t|^2}{2 \rho \rho'} = \frac{1}{2} \left(\frac{\rho}{\rho'} + \frac{\rho'}{\rho} + \frac{|x_t - x'_t|^2}{\rho \rho'}\right)
\]

and let us remember the form of the Wightman function on the Fulling vacuum of a massless scalar field propagating in \( W_R \) (see [12] and Ref.s therein):

\[
W^+(x, x') = W^+(\tau - \tau', \rho, \rho', x_t - x'_t) = \\
= \frac{1}{4\pi^4} \int_{\mathbb{R}^2} d^2k e^{ik_t(x_t-x'_t)} \int_0^{+\infty} d\omega \sinh \pi \omega K_{\omega}(k_{\perp} \rho)K_{\omega}(k_{\perp} \rho') e^{-i\omega(\tau-\tau'-i\varepsilon)} = \\
= \frac{1}{4\pi^2} \frac{\alpha}{\rho \rho' \sinh \alpha} \frac{1}{\alpha^2 - (\tau - \tau' - i\varepsilon)^2}.
\]

The integrand in the latter formula differs from the integrand in Eq.(36) due to the absence of the factor \( k_{\perp}^{-2} \) and the operator \( D_{\mu \nu} \), only. Remind that it formally holds in \( \mathbb{R}^2 \):

\[
2\pi \ln \frac{|x_t|}{\mu_0} = \int_{\mathbb{R}^2} d^2k \frac{e^{ik_t x_t}}{k^2_t} \quad (\text{where } \mu_0 := \int_0^1 du \frac{1 - J_0(u)}{u} - \int_1^{+\infty} du \frac{J_0(u)}{u},)
\]

This is the Fourier decomposition of a well-known Green function of the two-dimensional Laplace operator. This distributional Fourier decomposition works when the logarithm in the \( x_t \) space acts as an integral kernel on a \( L^1 \) test function \( f(x_t) \), provided this remain in \( L^1 \) when multiplied with the logarithm and have a Fourier transform \( \hat{f}(k_t) \) vanishing at \( k_t = 0 \). In this case the following integrals exist and it holds:

\[
\int_{\mathbb{R}^2} d^2k \frac{\hat{f}(k_t)}{k^2_t} = 2\pi \int_{\mathbb{R}^2} dx_t \ln \frac{|x_t|}{\mu_0} f(x_t)
\]
In particular \( \hat{f}(0) = 0 \) trivially holds when the function \( f(x_t) \) is a Laplacian of a function which decays opportune as \( |x_t| \to +\infty \). Thus, we expect that the right hand side of Eq. (36) can be written as, in the cases of \( D_{\tau\tau'} \), \( D_{\rho\rho'} \), \( D_{\tau\rho'} \):

\[
W^{+\mu\nu}_{\mu\nu}(x, x') = \frac{1}{2\pi} \int_{\mathbb{R}^2} dx_{i}'' \ln |x''_{\mu}| \mu_{0} D_{\mu\nu} W^{+}(\tau - \tau', \rho, \rho', |x''_{\mu} - (x''_{\mu} - x_{\mu})|),
\]

provided the function \( D_{\mu\nu} W^{+}(\tau - \tau', \rho, \rho', x'') \) be a Laplacian of a function which decays as required. Notice also that the integrand belongs to \( L^1(\mathbb{R}^2) \) as one can verify by a direct calculation from the asymptotic behaviour of \( D_{\mu\nu} W^{+}(x) \) by Eq. (38).

Let us consider the action of the operator \( D_{\tau\tau'} = -\rho \rho' D_{\rho\rho'} \) and thus the explicit expressions of the functions \( W^{+\tau\tau'}_{\tau\tau'}(x, x') \) and \( W^{+\rho\rho'}_{\rho\rho'}(x, x') \).

In Appendix B we shall prove the following remarkable identity:

\[
D_{\tau\tau'} W^{+}(\tau - \tau', \rho, \rho', x_{\mu}) = \rho \rho' \nabla^2 \cosh \alpha (\rho, \rho', x_{\mu}) W^{+}(\tau - \tau', \rho, \rho', x_{\mu}).
\]

Using the fact that \( \ln |x_t| \) is a Green function \(^m\) of \( \nabla^2_x \), i.e.

\[
\frac{1}{2\pi} \int_{\mathbb{R}^2} dx_{i} \ln |x_t| \nabla^2_x g(y_t - x_t) = \frac{1}{2\pi} \int_{\mathbb{R}^2} dx_{i} \ln |y_t - x_t| \nabla^2_x g(x_t) = -g(y_t)
\]

and reminding Eq. (39), it arises:

\[
W^{+\tau\tau'}_{\tau\tau'}(x, x') = -\frac{W^{+\rho\rho'}_{\rho\rho'}(x, x')}{\rho \rho'} = -\rho \rho' \cosh \alpha (\rho, \rho', x_{\mu} - x_{\mu}') W^{+}(\tau - \tau', \rho, \rho', x_{\mu} - x_{\mu}').
\]

Let us consider the action of the operator \( D_{\tau\rho'} \) and thus the explicit expressions of the functions \( W^{+\tau\rho'}_{\tau\rho'}(x, x') \) and \( W^{+\rho\rho'}_{\rho\rho'}(x, x') \).

It is possible to prove another remarkable identity, (see Appendix B) namely:

\[
D_{\tau\rho'} W^{+}(\tau - \tau', \rho, \rho', x_{\mu}) = -\rho (\tau - \tau') \frac{\sinh \alpha}{\alpha} W^{+}(\tau - \tau', \rho, \rho', x_{\mu} - x_{\mu}').
\]

and thus, dealing with as in the previously considered case:

\[
W^{+\tau\rho'}_{\tau\rho'}(x, x') = -\frac{\rho W^{+\rho\rho'}_{\rho\rho'}(x, x')}{\rho'} = \rho (\tau - \tau') \frac{\sinh \alpha (\rho, \rho', x_{\mu} - x_{\mu}')}{\alpha} W^{+}(\tau - \tau', \rho, \rho', x_{\mu} - x_{\mu}').
\]

The cases of \( D_{yy'} \) and \( D_{zz'} \) are very trivial. In fact, the action of these operators on the integrand in Eq. (36) cancels against the term \( k^2_{\perp} \). Thus we have:

\[
W^{+\tau\tau'}_{yy'}(x, x') = W^{+\tau\tau'}_{zz'}(x, x') = W^{+\tau\tau'}(x, x').
\]

Summarizing, the following Wightman functions calculated on Fulling vacuum state arise:

\[
W^{+\tau\tau'}_{\tau\tau'}(x, x') = -\rho \rho' W^{+\rho\rho'}_{\rho\rho'}(x, x') = -\frac{\alpha \coth \alpha}{4\pi^2 \alpha^2 - (\tau - \tau' - i\varepsilon)^2},
\]

\[
W^{+\rho\rho'}_{\rho\rho'}(x, x') = -\rho' \rho^{-1} W^{+\tau\tau'}_{\tau\tau'}(x, x') = -\frac{\tau - \tau'}{4\pi^2 \rho \alpha^2 - (\tau - \tau' - i\varepsilon)^2},
\]

\[
W^{+\tau\tau'}_{yy'}(x, x') = W^{+\tau\tau'}_{zz'}(x, x') = W^{+\tau\tau'}(x, x') = \frac{\alpha}{4\pi^2 \rho \rho' \sinh \alpha \alpha^2 - (\tau - \tau' - i\varepsilon)^2}.
\]

Notice that \( W^{+\tau\tau'}_{\mu\nu}(x, x') = (W^{+\mu\nu}_{\tau\tau'}(x, x'))^* \).

Let us consider the analytic structure of the Wightman functions \( W^+ \) and \( W^- \) extended to the whole complex \( \tau - \tau' \) plane, when \( \rho, \rho' \in (0, +\infty), x_t, x'_{t} \in \mathbb{R}^2 \) are fixed. We have to consider \( \varepsilon = 0 \) and \( \tau - \tau' \to z = \tau - \tau' + i(s - s') \) \( (\tau, \tau', s, s' \in \mathbb{R}) \) a generally complex number.

The structure is the same as in the scalar and massless case \(^[4]\). It is possible to extend both the
functions on the time complex plane except for the possible appearance of simple poles (instead of branch points) situated at
\[ z = \tau - \tau' = \alpha(\rho, \rho', x_t, x'_t) \] namely \((x - x')^2 + |x_t - x'_t|^2 - (t - t')^2 = 0\).

Then, the poles appear just in case of light-like related arguments. Hence, the extended functions \(W^+_{\mu\nu}'\) and \(W^-_{\mu\nu}'\) result to be holomorphic on the whole remaining complex \(z\) plane and both determine the same analytic continuation on just one Riemann sheet. In other terms each of these functions is an analytic continuation of the other.

We will term the shared extended function the master function \(\mathcal{G}_{\mu\nu'}(\rho, \rho', x_t, x'_t, z)\) where \(z \in \mathcal{A}\). This reads:
\[
\mathcal{G}_{\mu\nu'}(\rho, \rho', x_t, x'_t, z) = V_{\mu\nu'}(\rho, \rho', x_t, x'_t, z)\mathcal{G}(\rho, \rho', x_t, x'_t, z),
\]
where \(\mathcal{G}\) is the master functions, built up dealing with the same method, of a massless scalar field:
\[
\mathcal{G}(\rho, \rho', x_t, x'_t, z) := \frac{\alpha}{4\pi^2 \rho \rho' \sinh \alpha \alpha^2 - z^2},
\]
and the non vanishing bi-vectors \(V_{\mu\nu'}\) are:
\[
V_{\tau\tau'}(\rho, \rho', x_t, x'_t, z) = -\rho \rho' V_{\rho\rho'}(\rho, \rho', x_t, x'_t, z) = -\rho \rho' \cosh \alpha,
\]
\[
V_{\rho\rho'}(\rho, \rho', x_t, x'_t, z) = -\rho \rho'^{-1} V_{\tau\rho'}(\rho, \rho', x_t, x'_t, z) = -\rho' (\tau - \tau') \frac{\sinh \alpha}{\alpha},
\]
\[
V_{\rho'\rho'}(\rho, \rho', x_t, x'_t, z) = V_{zz'}(\rho, \rho', x_t, x'_t, z) = 1.
\]
The functions \(W^+_{\mu\nu'}\) and \(W^-_{\mu\nu'}\) are then obtained from \(\mathcal{G}_{\mu\nu'}\) by restricting the complex argument \(z\) to the real axis avoiding the poles from the lower or the upper \(z\) complex semiplane respectively. This approach to the \(z\) real axis is represented by the \(\varepsilon\) prescription (also in a distributional sense).

**B. PROPAGATOR, SCHWINGER FUNCTION, ADVANCED MINUS RETARDED FUNCTION**

It is possible to define the photon Feynman propagator \(G_F(x, x')_{\mu\nu'}\) by evaluating \(\mathcal{G}_{\mu\nu'}\) on the imaginary \(z\) axis followed by an anticlockwise rigid rotation of the domain from the imaginary axis to the real axis \(\square\). Equivalently one can write down \(\square\):
\[
\imath G_F(x, x')_{\mu\nu'} := \theta(\tau - \tau') W^+_{\mu\nu'}(x, x') + \theta(\tau' - \tau) W^-_{\mu\nu'}(x, x').
\]

Employing the Klein-Gordon equations which are satisfied by the Wightman functions above written, remembering that the derivative of a theta function is a delta function and moreover, using the canonical commutation relations Eq.(11), we obtain also
\[
g_{\alpha\beta}(x) \nabla_2^\alpha \nabla_2^\beta G_F(x, x')_{\mu\nu'} = g_{\mu\nu'}(x) (-g(x))^{1/2} \delta(x, x').
\]

This equation holds for test functions with support inside of \(W_R\) (considered as an open set). Thus, that propagator is a proper Green function of vectorial and massless K-G equation as we expected.

Finally, let us define the two-point Schwinger function as (there is no summation over repeated indexes)
\[
S_{\mu\nu'}(\rho, \rho', x_t, x'_t, s - s') := s(\mu) s(\mu') \mathcal{G}_{\mu\nu'}(\rho, \rho', x_t, x'_t, i(s - s')).
\]


where \( \rho, \rho' \in (0, +\infty) \), \( x_t, x'_t \in \mathbb{R}^2 \) and \( s, s' \in \mathbb{R} \) and we defined:

\[
\begin{align*}
    s(\sigma) &:= -i \quad \text{if} \quad \sigma = 0 \quad (\equiv \tau) \\
    s(\sigma) &:= 1 \quad \text{if} \quad \sigma = 1, 2, 3 \quad (\equiv \rho, x_t).
\end{align*}
\]

This Euclidean function is real and decays as \( |s - s'| \to \infty \). We can write \( S_{\mu\nu}(x, x') \) as

\[
S_{\mu\nu}(x_E, x'_E) = V_{\mu\nu}(x_E, x'_E) S(x_E, x'_E)
\]

where \( x_E := (s, \rho, x_t) \) and the Euclidean bi-vectors \( V_{\mu\nu} \) are trivially defined.

\[
S(x_E, x'_E) := \frac{\alpha}{4\pi^2 \rho \rho' \sinh \alpha (s - s')^2 + \alpha^2}
\]

is the well-known scalar Schwinger function in the Rindler wedge \([15, 9, 14]\) satisfying:

\[
g^{E}_{\alpha\beta}(x_E) \nabla^{\alpha}_E \nabla^{\beta}_E S(x_E, x'_E) = -(g^E(x_E))^{-1/2} \delta(x_E, x'_E) .
\]

Starting from the latter equation, some calculations lead us to \(^n\)

\[
g^{E}_{\alpha\beta}(x_E) \nabla^{\alpha}_E \nabla^{\beta}_E S_{\mu\nu}(x_E, x'_E) = -g^{E}_{\mu\nu}(x_E) (g^E(x_E))^{-1/2} \delta(x_E, x'_E) ,
\]

where \( g^{E}_{\mu\nu} := \text{diag}(+\rho^2, 1, 1, 1) \) is the Euclidean metric associated with the initial Rindler metric and the covariant derivative is defined with respect to this metric \( g^E_{\mu\nu} \). Thus \( S_{\mu\nu} \) is an Euclidean Green function decaying as \( |s - s'| \to \infty \) of the vectorial K-G equation on test functions with support in \( \{ \rho \in (0, +\infty), x_t \in \mathbb{R}^2, s \in \mathbb{R} \} \). Note the points with \( \rho = 0 \) are singular points of the Euclidean manifold. We have defined our Euclidean manifold in order to exclude these points. All that we have found is very similar to the case of a scalar field propagating in the whole Minkowski manifold as well as in the Rindler Wedge \([13, 14]\).

Finally, let us consider the advanced minus retarded fundamental solution namely, the fields operators commutator. We shall deal with in contravariant components.

\[
E^\mu\nu(x, x') := W^+\mu\nu(x, x') - W^-\mu\nu(x, x') = [\hat{A}^\mu(x), \hat{A}^\nu(x')] .
\]

We have from Eqs \(^{12}, (43), (44)\)\(^o\):

\[
\begin{align*}
E^{\tau\tau'}(x, x') &= -\frac{1}{\rho \rho'} E^{\rho\rho'}(x, x') = \frac{i \alpha \coth \alpha}{2\pi \rho \rho'} \frac{\sinh(\tau - \tau')}{\delta(\alpha^2 - (\tau - \tau')^2)} , \\
E^{\rho\tau'}(x, x') &= -\frac{\rho}{\rho'} E^{\tau\rho'}(x, x') = \frac{i(\tau - \tau')^2}{2\pi \rho \rho'} \frac{\sinh(\tau - \tau')}{\delta(\alpha^2 - (\tau - \tau')^2)} , \\
E^{\rho\rho'}(x, x') &= E^{zz'}(x, x') = \frac{-i\alpha}{2\pi \rho \rho' \sinh \alpha} \frac{\sin(\tau - \tau')}{\delta(\alpha^2 - (\tau - \tau')^2)} .
\end{align*}
\]

It is possible to prove that the vectorial advanced minus retarded fundamental solution reduces to the Minkowskian one, when the domain of test functions is restricted to the Rindler Wedge. We shall just sketch a proof of this in the following.

The advanced minus retarded solution of a photon field propagating in the whole Minkowski space results to be written \(^\rho\) in our initial Minkowskian coordinates \((x^0, x^1, x^2, x^3) \equiv (t, x, x_t) \equiv (t, \mathbf{x})\) as

\[
E_M^{\mu\nu}(x, x') = \eta^{\mu\nu} \frac{-i}{2\pi} \frac{\sinh(\tau - \tau')}{\delta((t - t')^2 - |x - x'|^2)} .
\]
Remaining in a Minkowskian base of the tangent space, but passing to Rindler coordinates as far as the arguments of the functions are concerned, employing standard distributional manipulations, one can also write down $E_{\mu\nu}^\prime(x, x')$ in $W_R$ as $q$

$$E_{\mu\nu}^\prime(x, x') = \eta_{\mu\nu} \frac{-i \alpha}{2\pi \rho \rho'} \sinh \alpha \; \text{sign}(\tau - \tau') \; \delta(\alpha^2 - (\tau - \tau')^2),$$

(50)

Starting from $E_{\mu\nu}$ expressed in Rindler coordinates by Eq.s (17), (18) and (49) and coming back to Minkowski tetrad, we find just the right hand side of Eq.(50). Take into account that, because of the presence of a delta function in Eq.s (17), (18) and (49), it is possible to change $|\tau - \tau'|$ with $\alpha$ (and so on) during calculations.

C. A WARD IDENTITY

Let us consider the identity (where primed derivatives works on primed arguments):

$$g_{\mu\nu}(x) \nabla^\nu G_F^{\mu\nu}(x, x') - \nabla^\nu G_F(x, x') = 0,$$

(51)

where $G_F$ is the scalar massless Feynman propagator. Reminding that equal time evaluated fields operators commute and the definition of Feynman propagator in terms of Wightman functions, the formula written above results to be equivalent to:

$$\nabla^\mu W^\pm_{\mu\nu}(x, x') = -g_{\nu\chi}(x') \nabla^\chi W^\pm(x, x').$$

(52)

The identity in Eq.(51) is very important because it is a Ward identity for the photon field in the Feynman gauge obtained (in Minkowskian coordinates) by a path integral quantization and imposing the BRST invariance [20].

It is possible to prove Eq.(52) by explicitly calculating both sides through the formulae obtained above. This proof does not contain interesting comments and we do not report on this here. Conversely, we shall report a less rigorous but physically more interesting proof of Eq.(52). This “proof” points out the role of physical and unphysical modes in Ward’s identity. Holding Eq.(52), it is necessary to prove only that (the proof for the case of $W^-$ is identical):

$$\nabla^\mu \int d\omega \; dk_l [A_{\mu}^{(3, \omega, k_l)}(x)A^{(3, \omega, k_l)}(x') - A_{\mu}^{(0, \omega, k_l)}(x)A_{\mu}^{(0, \omega, k_l)}(x')] = -\partial_{\mu'} W^+(x, x').$$

Employing the modes $A^G$ and $A^L$ which appear in Eq.s (24) and (27), and noticing that $\nabla^\mu A^G_{\mu} = 0$, the identity above written reduces to:

$$i \int d\omega \; dk_l \nabla^\mu A_{\mu}^{(L, \omega, k_l)}(x)A_{\mu}^{(G, \omega, k_l)}(x') = -\partial_{\mu'} W^+(x, x').$$

Expanding the covariant derivative in the integrand and evaluating the modes $A^G$ and $A^L$ in terms of the field $\phi$ by Eq.s (17) and (13), the identity to be proved reads

$$-\int d\omega \; dk_l \frac{\sinh \pi \omega}{4\pi^2} \phi^{(\omega, k_l)}(x)\partial_{\mu'}\phi^{(\omega, k_l)}(x') = -\partial_{\mu'} W^+(x, x').$$

This holds by definition of $W^+$.

IV. THERMAL GREEN FUNCTIONS AND SUBTLETIES WITH GAUGE INVARINENCE

A. PHOTON KMS STATES

Dealing with static coordinates $(x, t)$ in a spatially finite static region of a space time, the scalar thermal Wightman functions are defined as: (see for example [14])

$$W^+_{\beta}(x, x', t - t') = Z_{\beta}^{-1} Tr \{e^{-\beta H} \hat{\phi}(x, t)\hat{\phi}(x', t')\},$$

(53)
\[ W^{-}_\beta(x, x', t - t') = Z^{-1}_\beta \text{Tr}\{ e^{-\beta H} \hat{\phi}(x', t') \hat{\phi}(x, t) \} , \]  

where \( Z_\beta := e^{-\beta H} \) is the partition function of the field at temperature \( \beta \). These formulae have to be opportunely mathematically interpreted due to “operator” \( \hat{\phi} \) which is not a (trace class) bounded operator. However we shall not discuss on this here, because our discussion has to be understood just in an heuristic sense (for details see [9, 11] and Ref.s therein). By extending the thermal Wightman functions defined above to the complex time, we can recover the KMS condition \cite{22} due to cyclic property of the trace \cite{4} \cite{14}:

\[ W^\pm_\beta(x, x', t - t' \mp i\beta) = W^\mp_\beta(x, x', t - t') , \]

Provided appropriate mathematical conditions hold \cite{1}, these Wightman functions can be continued into an analytic function, the thermal master function \( G_\beta(x, x', z) \), defined in the time complex plane \( z = t - t' + i(s - s') \), periodic in the imaginary time \( s - s' \) with period \( \beta \). This function results to be defined on the whole \( z \) plane except for cuts on the real axis (periodically repeated along the imaginary axis, see figures in \cite{1}) corresponding to light-related arguments. The cuts terminate on branch points which become simple poles in the case of a massless field. The Wightman functions \( W^\pm_\beta \) and \( W^\mp_\beta \) result to be defined by approaching the real axis respectively from the lower semiplane and the upper semiplane (following the \( \varepsilon \)-prescription). The discontinuity crossing the cuts gives rise to the coincidence of the difference of the two Wightman functions and the \( (\beta \) independent) advanced minus retarded fundamental solution.

In case of fields propagating inside of an infinite spatial volume the partition function defined as a trace does not exist. However other possible definitions follow from path integral (and \( \zeta \) function or heat-kernel methods) but this is not our case. Following \cite{22} (see also Ref.s \cite{1} \cite{11}) the (quasifree) scalar thermal states can be defined, by an algebraic approach in terms of \( *- \), Weyl, \( C^* \)- and Von Neuman algebras as functionals on the algebra of the field. In this case, provided appropriate mathematical requirements be satisfied \cite{1}, the thermal Wightman functions are (positive) integral kernels bi-solutions of the motion equations which satisfy the KMS condition written above, having the analytic structure previously pointed out and producing the advanced minus retarded fundamental solution by difference. Hence, one can use the integral kernels to built up the (quasifree) state as a positive functional on the \( (*- \) etc.) algebra generated by the field.

The algebraic way to define thermal Wightman functions and thermal states agrees with the naive procedure (based on Eqs. \cite{53} and \cite{54}) whenever that can be implemented in some sense. In particular, when the naive method is correctly used in a finite box with convenient boundary conditions and the box walls are moved away to infinity in the end \cite{1} \cite{11}.

Other remarkable facts are also important. It is possible to prove that the thermal master function \( G_\beta(x, x', z) \) evaluated on the imaginary time axis, the Schwinger function \( S_\beta(x, x', s) := G_\beta(x, x', is) \), coincides with an imaginary time periodic Green function of the Euclidean Laplace operator. This operator is defined in the imaginary time periodic Euclidean section of the manifold with period \( \beta \). In this way, the previously written KMS conditions directly follow from the imaginary time periodicity of the manifold.

Another important point is the sum over images method. It is well-known \cite{3} that the above considered extended periodic Green functions can be obtained from the non thermal ones by the sum:

\[ G_\beta(x, x', z) = \sum_{n \in \mathbb{Z}} G(x, x', z + in\beta) , \]

where \( G(z) \) is the analytic extension to the complex time of the non thermal master function. Furthermore it can be proved \cite{4} \( G_\infty(z) = G(z) \), where \( \infty \) denotes the limit as \( \beta \to +\infty \). All these topics have been more or less rigorously implemented in the scalar and spinorial case in
different manifolds and, in particular, in the Rindler wedge for massless fields, also in relation to the cosmic string theory (see e.g. Refs. [6, 9, 12, 23] and refs therein).

Let us consider the case of a photon field in Feynman’s gauge. Obviously, it is possible to directly define strength field $F_{\mu\nu}$ Wightman functions avoiding unphysical particles and gauge related problems. However, this is not a completely satisfactory way because, for instance, implementing an interaction theory one must use directly the field $A_{\mu}$ in dealing with the minimal coupling.

We shall start by supposing to work within a finite box in order to have a well defined partition function and to be able to use the naive formalism. The following discussion is just heuristic, no mathematical rigor is used.

The hardest problem is due to the presence of unphysical degrees of freedom. Such a difficulty has been pointed out by Bernard [24] dealing with the Euclidean path integral formalism to define the photon partition functions in an arbitrary gauge. He proved that the correct definition, not depending on the gauge, is the trace over the physical degrees of freedom only:

$$Z_{\text{phys}}^{\beta} = \sum_{\Psi_n \text{phys.}} \langle \Psi_n | e^{-\beta \hat{H}} | \Psi_n \rangle$$

Successively, this definition has to be re-written as a path integral in the chosen gauge by a Faddeev-Popov ghost procedure in such a manner to include the unphysical modes in the functional integral. Following this way, we can start by formally define in a large box in the Rindler wedge:

$$W^{\text{phys.}+}_{\beta \mu \nu'} (\rho, \rho', x_t, x'_t, \tau - \tau') :=$$

$$= Z^{-1}_{\text{phys.} \beta} \sum_{\Psi_n \text{phys.}} \langle \Psi_n | e^{-\beta \hat{H}} \hat{A}^{\text{phys.}}_{\mu}(\rho, x_t, t) \hat{A}^{\text{phys.}}_{\mu'}(\rho', x'_t, t') | \Psi_n \rangle,$$  \hspace{1cm} (55)

and

$$W^{\text{phys.}-}_{\beta \mu \nu'} (\rho, \rho', x_t, x'_t, \tau - \tau') :=$$

$$= Z^{-1}_{\text{phys.} \beta} \sum_{\Psi_n \text{phys.}} \langle \Psi_n | e^{-\beta \hat{H}} \hat{A}^{\text{phys.}}_{\mu}(\rho', x'_t, t') \hat{A}^{\text{phys.}}_{\mu'}(\rho, x_t, t) | \Psi_n \rangle,$$  \hspace{1cm} (56)

where $\hat{A}^{\text{phys.}}_{\mu}$ contains only the transverse modes, i.e., $\lambda = 1$ and 2 and $| \Psi_n \rangle$ denotes the eigenvector of $\hat{H}$ with eigenvalue $E_n$.

Now we can add to these Wightman functions an unphysical part related to the considered gauge choice, Feynman gauge in the present case. This part has to vanish when the Wightman functions act on physical wavefunctions. Such a procedure must not affect the Wightman functions calculated by the strength field $F_{\mu\nu}$.

Our proposal consists of the formal definition (the definition of $W^-$ being obvious):

$$W^{\text{Feynman}+}_{\beta \mu \nu'} (\rho, \rho', x_t, x'_t, \tau - \tau') :=$$

$$= Z^{-1}_{E \beta} \sum_{n} \langle \Psi_n / e^{-\beta \hat{H}} | \hat{A}_{\mu}(\rho, x_t, t) \hat{A}_{\mu'}(\rho', x'_t, t') / \Psi_n \rangle,$$  \hspace{1cm} (57)

where

$$Z_{E \beta} = \text{Tr}_E e^{-\beta \hat{H}} := \sum_{n} \langle \Psi_n / e^{-\beta \hat{H}} / \Psi_n \rangle,$$
the index $E$ denotes the use of the Euclidean scalar product in calculating the trace above. Notice that $\hat{H}$ is non-negative employing the Euclidean scalar product, in fact we have:

$$
\hat{H} = \int dk \, d\omega \, \sum_{\lambda=0}^{3} \eta^{\lambda\lambda} \hat{a}_{(\lambda,\omega,k_{l})}^{\dagger} \hat{a}_{(\lambda,\omega,k_{l})} = \int dk \, d\omega \, \sum_{\lambda=0}^{3} \delta^{\lambda\lambda} \hat{a}_{(\lambda,\omega,k_{l})}^{\dagger} \hat{a}_{(\lambda,\omega,k_{l})},
$$

and thus no problem on the divergence of the trace arises. It is quite simply proved that formally:

$$
W_{\text{Feynman}}^{+} = W_{\text{phys.}}^{+} + Z_{\text{unph.}}^{-1} \sum_{\Psi_{n}} <\Psi_{n}/e^{-\beta \hat{H}} \hat{A}_{\mu}^{\dagger} A_{\mu}/\Psi_{n}>,
$$

where the vacuum state is included in the sum over unphysical states. Notice that the second term vanishes employing physical test wavefunctions. Using such wavefunctions the Wightman functions are also positive defined by construction. We also stress, by the definition Eq.(57) the difference of the two Wightman functions does not depend on $\beta$ and reproduces the non thermal commutator. Finally, the thermal strength Wightman functions calculated as derivative of the Wightman functions defined in Eq.(55) and (56) coincide with those obtained by the derivatives of the physical Wightman functions defined in Eq.(57). This is just a trivial consequence of $F^{(G)}_{\mu\nu}(x) = F^{(L)}_{\mu\nu}(x) = 0$.

Our definition trivially satisfies K-G equations and maintains the KMS condition due to cyclic property of the trace involved in Eq.(57). Following the way employed in [9], we expect to find also the analytic structure previously pointed out. Furthermore, the Ward identity in Eq.(51) can be formally proved employing the same method. Finally, one finds the non thermal Wightman functions as the limit $\beta \rightarrow +\infty$.

We stress that different proposals of definition involving, in Eq.(57), the physical scalar product defined in Eq.(34), instead of the Euclidean one, do not maintain the KMS condition. This is due to the presence of the operator $M$ which does not permit to take advantage of the cyclic property of the trace.

**B. THERMAL WIGHTMAN FUNCTIONS AND RELATED THERMAL GREEN FUNCTIONS**

Taking account of the heuristic discussion performed above, we shall define the thermal Wightman functions of the photons in the Feynman gauge by requiring they are bi-solutions of K-G equations, satisfy the KMS condition, take on the analytic structure of the scalar Wightman functions and produce the advanced minus retarded fundamental solution of Eq.(46) by the usual difference.

We shall try to built up such Wightman functions by a thermal master function obtained by summing over images. Remind the series:

$$
\sum_{n \in \mathbb{Z}} \frac{1}{(a+n)^{2} - b^{2}} = \pi \frac{2}{2b} \left\{ \cot \left[ \pi (a - b) \right] - \cot \left[ \pi (a + b) \right] \right\},
$$

absolutely convergent, for $a, b \in \mathbb{C}$ such that both sides are defined and

$$
\sum_{n \in \mathbb{Z}} \frac{a + n}{(a+n)^{2} - b^{2}} = \frac{\pi}{2} \left\{ \cot \left[ \pi (a - b) \right] + \cot \left[ \pi (a + b) \right] \right\},
$$

convergent in the sense of the principal value (namely, $\lim_{N \rightarrow +\infty} \sum_{|n|<N}$), for $a, b \in \mathbb{C}$ such that both sides are defined. Then, let us consider the thermal master function defined as

$$
G_{\beta\mu\nu}(\rho, \rho', x_{t}, x'_{t}, z) := \sum_{n \in \mathbb{Z}} G_{\mu\nu}(\rho, \rho', x_{t}, x'_{t}, z + i n \beta),
$$

(58)
where $G_{\mu\nu}(\rho, \rho', x_t, x_t, z)$ was defined in Eq. (45). The convergence is understood as punctual convergence in the sense of the principal value at least.

Employing the results above as well as trivial calculations we find (re-arranging the result in a convenient form for future reference):

$$G_{\beta \tau\tau'}(z) = -\frac{1}{4\pi\beta} \sinh \left( \frac{2\pi}{\beta} \alpha \right) \frac{\sinh \left( \frac{2\pi}{\beta} (\tau - \tau') \right) \sinh \alpha + \sinh \left( \frac{2\pi}{\beta} - 1 \right) \alpha}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cosh \left( \frac{2\pi}{\beta} z \right)} - \frac{1}{4\pi\beta}, \quad (59)$$

$$G_{\beta \rho\rho'}(z) = -\frac{1}{4\pi\beta} G_{\beta \tau\tau'}(z), \quad (60)$$

$$G_{\beta \rho\tau'}(z) = -\frac{1}{4\pi\beta} \sinh \left( \frac{2\pi}{\beta} \alpha \right) \frac{\sinh \left( \frac{2\pi}{\beta} (\tau - \tau') \right)}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cosh \left( \frac{2\pi}{\beta} z \right)}, \quad (61)$$

$$G_{\beta \tau\rho'}(z) = -\frac{\rho'}{\rho} G_{\beta \rho\tau'}(z), \quad (62)$$

$$G_{\beta \rho\tau}(z) = G_{\beta \tau\tau}(z) = G_{\beta \tau\tau'}(z) = G_{\beta}(z). \quad (63)$$

$G_{\beta}(z)$ is the thermal master function of a massless scalar field obtained summing over images the previously calculated nonthermal master function. Notice the periodicity $\beta$ in the imaginary time. We can consider $z = \tau - \tau' \pm i\varepsilon$ to obtain the thermal Wightman functions. We report $W^{+}_{\beta \mu\nu'}$ only, $W^{-}_{\beta \mu\nu'}$ is obtained by a complex conjugation of the former.

$$W^{+}_{\beta \tau\tau'}(x, x') = -\frac{1}{4\pi\beta} \sinh \left( \frac{2\pi}{\beta} (\tau - \tau') \right) \frac{\sinh \left( \frac{2\pi}{\beta} \alpha \right) \sinh \alpha + \sinh \left( \frac{2\pi}{\beta} - 1 \right) \alpha}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cosh \left( \frac{2\pi}{\beta} (\tau - \tau' - i\varepsilon) \right)} - \frac{1}{4\pi\beta}, \quad (64)$$

$$W^{+}_{\beta \rho\rho'}(x, x') = -\frac{1}{4\pi\beta} W^{+}_{\beta \tau\tau'}(x, x'), \quad (65)$$

$$W^{+}_{\beta \tau\rho'}(x, x') = -\frac{1}{4\pi\beta} \sinh \left( \frac{2\pi}{\beta} (\tau - \tau') \right) \frac{\sinh \left( \frac{2\pi}{\beta} \alpha \right)}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cosh \left( \frac{2\pi}{\beta} (\tau - \tau' - i\varepsilon) \right)}, \quad (66)$$

$$W^{+}_{\beta \rho\tau'}(x, x') = -\frac{\rho'}{\rho} W^{+}_{\beta \tau\rho'}(x, x'), \quad (67)$$

$$W^{+}_{\beta \rho\tau}(x, x') = W^{+}_{\beta \tau\rho}(x, x') = W^{+}_{\beta \tau\tau}(x, x') = W^{+}_{\beta}(x, x'). \quad (68)$$

$W^{+}_{\beta}(x, x')$ is the well-known Rindler thermal Wightman function of a massless scalar field [12, 19]:

$$W^{+}_{\beta}(x, x') = \frac{1}{4\pi\beta} \sinh \alpha \left[ \frac{\sinh \left( \frac{2\pi}{\beta} \alpha \right)}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cosh \left( \frac{2\pi}{\beta} (\tau - \tau' - i\varepsilon) \right)} \right].$$

The vectorial Wightman functions written above trivially satisfy the KMS condition and the related thermal master function has the required analytic structure. In particular, there are not branch points but a pair of simple poles periodically repeated in the imaginary time with period
\( \beta \). The two poles on the real time axis correspond light-like related arguments \(^1\).

Moreover, some calculations involving standard distributional properties prove the difference \( W^{\beta, \mu \nu}(x, x') - W^{\beta, \mu \nu}(x, x') \) coincides with the advanced minus retarded solution defined in Eq. (46). We might prove that the obtained vectorial Wightman functions define a positive bi-functional working on physical test wavefunctions. We shall prove this in the case \( \beta = 2\pi \) only.

Some comments on the obtained functions are necessary. First let us evaluate the thermal master function along the time imaginary axis. We obtain the thermal Schwinger function. \((x^0_E, x^1_E, x^2_E, x^3_E) \equiv (s, \rho, y, z)\)

\[
S_{\beta \, 00'}(x_E, x'_E) = \frac{1}{4\pi \beta \sinh \alpha} \frac{\cos \left( \frac{2\pi}{\beta} (s - s') \right) \sinh \alpha + \sinh \left( \left( \frac{2\pi}{\beta} - 1 \right) \frac{2\pi}{\beta} \right) \alpha}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cos \left( \frac{2\pi}{\beta} (s - s') \right)} + \frac{1}{4\pi \beta},
\]  
(69)

\[
S_{\beta \, 11'}(x_E, x'_E) = \frac{1}{\rho \rho'} S_{\beta \, 00'}(x_E, x'_E),
\]  
(70)

\[
S_{\beta \, 01'}(x_E, x'_E) = -\frac{1}{4\pi \beta \rho} \frac{\sin \left( \frac{2\pi}{\beta} (s - s') \right)}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cos \left( \frac{2\pi}{\beta} (s - s') \right)},
\]  
(71)

\[
S_{\beta \, 10'}(x_E, x'_E) = -\frac{\rho'}{\rho} S_{\beta \, 01'}(x_E, x'_E),
\]  
(72)

\[
S_{\beta \, 01'}(x_E, x'_E) = S_{\beta \, 10'}(x_E, x'_E) = S_\beta(x_E, x'_E).
\]  
(73)

This bi-vectorial function trivially defines a periodic vectorial Laplacian Green function in the Euclidean section of the manifold with imaginary time period \( \beta \). This follows from Eq. (58) when one supposes \( z = is \), considers the series as a series of distributions and reminds that the non thermal Schwinger function is a Green function in the non periodic Euclidean manifold. The function

\[
S_\beta(x_E, x'_E) = \frac{1}{4\pi \beta \rho \rho' \sinh \alpha} \frac{\sinh \left( \frac{2\pi}{\beta} \alpha \right)}{\cosh \left( \frac{2\pi}{\beta} \alpha \right) - \cos \left( \frac{2\pi}{\beta} (s - s') \right)}
\]

is a Rindler thermal Schwinger function for a massless scalar field (see also \([19]\) where a different nomenclature is used) obtained, for instance, by the sum over images method. Few words on this function in relation to the vectorial found ones are necessary. The corresponding test functions of the scalar Scwinger function have support in \( \{ s \in [0, \beta), \rho \in [0, +\infty), y, z \in \mathbb{R}^2 \} \) (where \( 0 \equiv \beta \)). Differently from the case \( \beta = +\infty \) (i.e. the non thermal case), the scalar thermal Schwinger function is defined also when \( \rho' \rightarrow 0 \) and \( \rho > 0 \) (and vice versa), namely, when one of the arguments stays on the tip of the Euclidean Rindler cone. Remind that the Euclidean Rindler manifold is diffeomorphic to \( C_\beta \times \mathbb{R}^2 \) where the first factor is a cone of angular deficit \( 2\pi - \beta \). There, \( s \) is the angular variable and \( \rho \) the radial one. We have:

\[
S_0(x_E, x'_E) := S_\beta(x_E, x'_E)|_{\rho' \rightarrow 0} = \frac{1}{2\pi \beta (\rho^2 + |x_t - x'_t|^2)},
\]

and \( \nabla^2 E S_0(x_E, x'_E) = 0 \) whenever \( \rho > 0 \).

Employing carefully second Green’s identity, one can quite simply prove \(^a\):

\[
\int_{C_\beta \times \mathbb{R}^2} d^4 x_E \sqrt{g_E(x_E)} S_0(x_E, x'_E) \nabla^2_E f(x_E) = -f(s, \rho', x'_t)|_{\rho' = 0}.
\]
Thus, we see that, in the massless scalar case, the Schwinger function is a Green function for the whole Euclidean manifold whenever $0 < \beta < +\infty$. The case of the vectorial field is quite different.

In order to study that case it is convenient to write the vectorial Schwinger functions in a unitary normalized base of the cotangent space (a tetrad). This avoids troubles related to an anomalous normalization of coordinate base vectors in the limit $\rho \to 0$. This vectorial Schwinger function, by normalizing the base $ds, dp, d\tau$, takes on a factor \( \rho^{-1} (\rho')^{-1} \) for each $0 - (0') -$ component. The $11'$ component, and the transverse ones $yy', zz'$ do not change. Except for the case $\beta = 2\pi$ which we shall study later, the limit as $\rho' \to 0$ ($\rho$ fixed) produces vanishing or infinite non transverse components of the vectorial Schwinger function, depending on the sign of $\beta - 2\pi$. Such an anomalous behaviour for the vectorial case seems related to the presence of the conical singularity on the tip of a cone, which does not permit to unambiguously define the tangent (cotangent) space and the metric tools there.

Let us directly consider the found vectorial thermal Wightman functions. We notice that these reduce to the correct non thermal limit Eqs. (12)-(13) and (14) in the case $\beta \to +\infty$. Furthermore, one can prove by direct calculations that both K-G equations and the Ward identity holding in the non thermal case Eq.(52) are satisfied. We do not further report on this here because the proof does not involve interesting comments.

C. SUBTLETIES WITH GAUGE INVARIANCE

Let us consider the strange static term $\delta W_{\beta \mu \nu'}(x, x')$ added to the $\tau \tau'$ and $\rho \rho'$ transversal thermal Wightman functions (see Eqs. (64) and (65)). In components it reads:

\[
\delta W_{\beta \tau \tau'}(x, x') = -\frac{1}{4\pi \beta}, \quad \delta W_{\beta \rho \rho'}(x, x') = \frac{1}{4\pi \beta \rho \rho'}
\]

(all the remaining components vanish).

This term does not contribute to the zero temperature limit because this vanishes as $\beta \to +\infty$. Furthermore, this term is responsible for an apparently bad behaviour of the thermal Wightman functions as $|x_t - x'_t| \to +\infty$ when $\beta < +\infty$. In fact, the thermal Wightman functions do not vanish in this limit. However, considering thermal states, the requirement of a vanishing large distance fields correlation is not so strictly necessary. Anyhow, we shall see that in the present case the physical correlations do vanish in the considered limit, because the terms in Eq.(74) do not contribute to the “physical part” of Wightman functions.

Also notice that, because of the form of $\delta W_{\beta \mu \nu'}(x, x')$, this term does not affect the Wightman functions calculated by the strength field operator $\hat{F}_{\mu \nu}(x)$. In fact, the contribution to the strength field thermal Wightman functions reads

\[
\delta \langle \hat{F}_{\mu \nu}(x) \hat{F}_{\mu' \nu'}(x') \rangle_{\beta} =
\]

\[
\nabla_{\mu} \nabla'_{\nu'} \delta W_{\beta \mu \nu'}(x, x') - \nabla_{\mu} \nabla'_{\nu'} \delta W_{\beta \mu \nu'}(x, x') - \nabla_{\mu} \nabla'_{\nu'} \delta W_{\beta \nu \nu'}(x, x') - \nabla_{\mu} \nabla'_{\nu'} \delta W_{\beta \nu \nu'}(x, x')
\]

In order to obtain some non vanishing term in this sum, it must be $\mu = \nu = \rho$ and $\mu' = \nu' = \rho'$. In such a situation the four terms cancels each others and the final result vanishes.

Let us prove $\delta W_{\beta \mu \nu'}(x, x')$ contains gauge terms only, has a vanishing covariant divergence and satisfies vectorial Klein-Gordon equations.

In particular, the vanishing covariant divergence implies that the term $\delta W_{\beta \mu \nu'}(x, x')$ can be omitted (or that we can use a different value $\beta' \neq \beta$) in checking the previously discussed Ward identity.

We can write

\[
\delta W_{\beta \mu \nu'}(x, x') = \nabla_{\mu} \nabla'_{\nu'} \Phi(x, x') \quad \text{where} \quad \Phi(x, x') := \frac{-1}{4\pi \beta} (\tau \tau' - \ln \rho \ln \rho')
\]

(75)
Hence, only gauge terms appear in \( \delta W_{\beta \mu \nu}(x, x') \).
\( \delta W_{\beta \mu \nu}(x, x') \) has a vanishing covariant divergence because \( \nabla^\mu \nabla_\mu \Phi(x, x') = 0 \) due to Eq.\((74)\). Furthermore, due the commutativity of covariant derivative inside of a flat manifold, we find also \( \nabla_\sigma \nabla^\sigma \delta W_{\beta \mu \nu}(x, x') = 0 \).

Finally, let us prove that the considered term produces no contribution to the value of thermal Wightman functions when they act on, at least one, physical test wavefunction. More generally, we shall prove:

\[
\int_\Sigma dS n^\mu \sqrt{h} A_\nu(x) \nabla_\mu \delta W_{\beta \mu \nu}(x, x') = 0 , \tag{76}
\]

where \( A \in S \) satisfies also \( \nabla_\mu A^\mu = 0 \). This includes the wavefunctions built up employing physical modes \( A^{(1)}, A^{(2)} \) as well as the gauge modes \( A^{(G)} \) (see Eqs.\((24), (25), (26) \) and \( (27) \)), namely physical and Lorentz wavefunctions. An analogous proof can be produced out by employing a four smeared formalism introduced in Appendix C and, in that case, the constraint \( \nabla_\mu A^\mu = 0 \) becomes \( \nabla_\mu F^\mu = 0 \) where \( F^\mu(x) \) is a four smeared test function.

The left hand side of Eq.\((76)\) can be written, due to Eq.\((77)\) (omitting the unimportant second argument of \( \Phi \) and its covariant derivative):

\[
\int_\Sigma dS n^\mu \sqrt{h} A_\nu(x) \nabla_\mu \Phi(x) = \int_\Sigma dS n^\mu \sqrt{h} \nabla^\nu [A_\nu(x) \nabla_\mu \Phi(x)] ,
\]

where we used the vanishing covariant divergence of the wavefunction. We can expand the integrand by adding and subtracting a convenient term, obtaining:

\[
\int_\Sigma dS n^\mu \sqrt{h} A_\nu(x) \nabla_\mu \Phi(x) = \int_\Sigma dS n^\mu \sqrt{h} \nabla_\mu [A_\nu(x) \nabla^\nu \Phi] + \int_\Sigma dS n^\mu \sqrt{h} \nabla_\nu [A^\mu \nabla^\nu \Phi - \Phi \nabla^\mu A^\mu] =
\]

\[
= \int_\Sigma dS n^\mu \sqrt{h} \nabla_\nu G^{\mu \nu} + \int_\Sigma dS n^\mu \sqrt{h} F^{\mu \nu} \nabla_\nu \Phi .
\]

We defined \( G^{\mu \nu} := A^\nu \nabla_\mu \Phi - A^\mu \nabla_\nu \Phi, \ F^{\mu \nu} := \nabla^\nu A^\mu - \nabla^\mu A^\nu \) and used \( \nabla_\nu \nabla^\mu A^\nu = \nabla^\nu \nabla_\nu A^\nu = 0 \) due to the flatness of the space. Notice that, due to Klein-Gordon equations, \( \nabla_\nu F^{\mu \nu} = 0 \). Thus, integrating by parts in the latter integral and re-introducing the second arguments \( x' \) with its covariant derivative, we can write:

\[
\int_\Sigma dS n^\mu \sqrt{h} A_\nu(x) \nabla_\mu \delta W_{\beta \mu \nu}(x, x') = \nabla^\nu \int_\Sigma dS n^\mu \sqrt{h} \nabla_\nu \left( G^{\mu \nu}(x, x') - F^{\mu \nu}(x) \Phi(x, x') \right) .
\]

The integral in the right hand side, due to the antisymmetry of the integrand tensor, reduces to

\[
\int dx^1 dx^2 dx^3 \sum_{i=1,2,3} \partial_i \left[ \sqrt{-g} \left( G^{0i} - \Phi F^{0i} \right) \right] .
\]

This vanishes due to the compactness of the spatial support of \( A \).

We may conclude the static term \( \delta W_{\beta \mu \nu}(x, x') \) represents a remaining static gauge ambiguity which does not affect the physical part of the theory. We can omit this term in \( W_{\beta \tau \tau}(x, x') \) and \( W_{\beta \rho \rho}(x, x') \) or conversely, we can change the value \( \beta \) appearing in \( \delta W_{\beta \mu \nu}(x, x') \) into a “wrong” variable value \( \beta' \neq \beta \) without to affect the physics. This determines an one parameter class of possible thermal (and non thermal in the limit \( \beta \to +\infty \)) Wightman functions carrying the same physical content. These changes can be implemented directly in the thermal master
function in Eqs. (59) and (60) or in the thermal Schwinger functions in Eq. (69) and (70) where we have:

$$\delta S_{\beta', \mu\nu}(x_E, x'_E) = \nabla_{\mu} \nabla'_{\nu} \Phi(x_E, x'_E) \quad \text{where} \quad \Phi(x, x') := \frac{1}{4\pi \beta'}(ss' + \ln \rho \ln \rho')$$

(77)

All these Euclidean time static terms are solutions of Laplace equation away from the conical singularity. Thus, the resulting Schwinger functions remain Euclidean Green functions of the Laplacian away from the conical tip. No choice of the value $\beta', \beta' \to +\infty$ included, produces a vectorial Green function on the whole manifold if the period of the manifold $\beta \neq 2\pi$. This is due to the bad behaviour as $\rho (\rho') \to 0$ of the terms $S_{\beta, 01'}$ and $S_{\beta, 10'}$ non depending on $\delta S_{\beta', \mu\nu}(x, x')$. When the period of the manifold $\beta$ takes the value $2\pi$, no conical singularity appears and this selects just one Schwinger function. This Schwinger function is the only Green function of the Laplacian in the class previously considered defined in the whole Euclidean manifold. This corresponds to the Schwinger function with $\beta' \to +\infty$, i.e., dropping $+1/4\pi \beta$ in Eq. (69) and the corresponding added static term in $S_{\beta, 11'}(x, x')$.

D. COINCIDENCE OF QUANTUM PHOTON VACUA

In the case of a scalar field, the Wightman functions of Minkowski vacuum restricted inside of a Rindler wedges coincide with the thermal Wightman functions with $\beta = 2\pi$ calculated with respect to the Fulling vacuum. This is the content of the Bisognano-Wichmann theorem in terms of Wightman functions \([13, 11]\). This property can be extended on the quantum state by GNS theorem and similar. This property also holds for spin 1/2 in terms of Wightman functions at least (see for example Refs. [12, 23]). In the case of photons, despite of the gauge ambiguity in defining Rindler Green functions we have found, the coincidence of Wightman functions holds dealing with test wavefunction corresponding to physical photons and also for photons carrying modes $A^{(G)}$. Thus, in the case of photons belonging to the Lorentz space $\mathcal{H}_L$. Notice also that the positivity of the Wightman functions, working with physical (Lorentz) states, results to be trivially proved due the positivity of the Minkowski Wightman functions in the case $\beta = 2\pi$.

Following an algebraic approach, one can try to build up a minimal $*-algebra$ generated through the field operators when they act on physical wavefunctions and/or Lorentz wavefunctions. In this background, one should try to implement a GNS reconstruction to extend to the “physical part” of the quantum states the local coincidence of Wightman functions. However, we do not consider these topics in this paper.

One can verify the coincidence of the above considered Wightman functions inside of the open Rindler wedge employing the following way. First one considers the thermal Wightman functions defined in Eqs. (64), (65), (66), (67) and (68), dropping all the static terms $\delta W_{\beta, \mu\nu}(x, x')$, namely, by considering the limit as $\beta' \to +\infty$ in the one-parameter Wightman functions class previously discussed. This omission does not affect the final result by dealing with test wavefunctions corresponding to states belonging to $\mathcal{H}_L$. Then, one has to translate the obtained functions in Minkowski coordinates. The resulting functions represent just the (non thermal) Minkowski Wightman functions in Feynman’s gauge:

$$W^{\pm\mu\nu}(x, x') = \frac{1}{4\pi^2} \frac{\eta^{\mu\nu}}{|x - x'|^2 - (t - t' \mp i\varepsilon)^2}.$$

We report just a technical comment. In order to prove the considered identity using three smeared distributions, it is convenient to work on the Rindler Cauchy surface at $\tau(=\tau') = t(=t') = 0$. This is a part of a Minkowski Cauchy surface. Then, one has to prove the coincidence of the Wightman functions dealing with wavefunctions with a spatial compact support in $W_R$ (hence, non containing points with $\rho, \rho' = 0$) employing the usual indefinite scalar product. The
result follows noticing that, on the considered Cauchy surface $\partial_t = \rho^{-1}\partial_r$, and the following three smeared distributional identities hold there (i.e., at $\tau = \tau' = t = t' = 0$):

$$\frac{1}{2\rho\rho'[\cosh \alpha - \cosh(\tau - \tau' + i\varepsilon)]} = \frac{1}{|x - x'|^2 - (t - t' \mp i\varepsilon)^2},$$

$$\partial_\tau \left( \frac{1}{2\rho\rho'[\cosh \alpha - \cosh(\tau - \tau' + i\varepsilon)]} \right) = \partial_t \left( \frac{1}{|x - x'|^2 - (t - t' \mp i\varepsilon)^2} \right),$$

$$\partial_\tau \partial_\tau' \left( \frac{1}{2\rho\rho'[\cosh \alpha - \cosh(\tau - \tau' + i\varepsilon)]} \right) = \partial_t \partial_\nu \left( \frac{1}{|x - x'|^2 - (t - t' \mp i\varepsilon)^2} \right).$$

Similar results arise also dealing with Schwinger functions. However, in that case an important geometrical difference arises. The Euclidean Rindler coordinates, as the Euclidean Minkowski coordinates, cover the whole Euclidean section of Minkowski spacetime. Thus, we expect to find a coincidence of Rindler Schwinger functions and Minkowski Schwinger functions everywhere. The transformation law from Euclidean Rindler coordinates $(s, \rho, y, z)$ to Euclidean rectangular coordinates $(r^1, r^2, r^3, r^4)$ reads:

$$r^1 = \rho \cos s, \quad r^2 = \rho \sin s \quad \text{and} \quad r^3 = y, \quad r^3 = z,$$

In the present case $\beta = 2\pi$, the Rindler Schwinger function of Eqs. (39), (70), (71), (72) and (73), more generally containing $\beta'$, $0 < \beta' < +\infty$, in the static term of Eq. (71), are Green functions of the Laplace operator in the manifold $\mathbb{R}^4 - \{(0, r^2, r^3, 0) \mid r^2, r^3 \in \mathbb{R}\}$ endowed with the usual flat Euclidean metric. If $\beta = 2\pi$ no conical singularity appears and thus no problem arises in defining Laplacian Green functions on the whole Euclidean manifold. One may build up the only Green function defined on the whole $\mathbb{R}^4$ which decays:

$$S(x_E, x'_E)^{\alpha\alpha'} := \frac{1}{4\pi^2} \frac{\delta^{\alpha\alpha'}}{\delta_{\mu\nu}(r^\mu - r'^\mu)(r^\nu - r'^\nu)},$$

This everywhere defined Green function coincide both with the only photon Minkowski Schwinger function in the Feynman gauge which decays as $|r^4| \to +\infty$ and the Rindler Schwinger function containing no Rindler static terms pointed out in the previous section.

V. SUMMARY

In this paper we proved that it is possible to build up a mathematically consistent canonical theory for a quasi-free photon field propagating in the Rindler wedge, based on a generalization of the Gupta-Bleuler formalism in the Rindler wedge and also considering thermal photons. We employed a three-smeared formalism, however generalizations to a four-smeared formalism should be straightforward. We proved that the Fulling-Ruijsenaars formalism based on a (thermal) master function can be extended to include the vectorial photon field recovering properties similar to those in the massless scalar case.

We proved also that the gauge invariance needs more care than in the Minkowskian case, in particular dealing with the thermal case (KMS conditions) and studying the generalization of the Bisognano-Wichmann theorem for photons in terms of Wightman functions. In fact, a Rindler non static gauge ambiguity coupled with the presence of the conical singularity appears when $\beta \neq 2\pi$. Such a gauge ambiguity is not removed also imposing the validity of the Ward identity which arises from BRST invariance.

In the case $\beta = 2\pi$, we saw that the theory produces the expected coincidence of of the thermal Rindler Wightman (Schwinger) functions with the Minkowski vacuum Wightman (Schwinger) functions as far as the “physical part” of those function is concerned.
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APPENDIX A

In this appendix we shall find the normalization coefficients \( C^{(\alpha,\omega,k_t)} \) of the modes in Eqs. (20), (21), (22), (23) using the scalar product \( ( , ) \) defined in Eq. (3). It can be simply proved that:

\[
(A^3, A^4) = 0 ,
\]

The remaining scalar products of different modes vanish as specified in section III B. Thus the modes appearing in Eqs. (20), (21), (22), (23) define a set of normal to each other modes.

Let us normalize them as required by Eq.s (7), (8), (9).

The normalized \( A^{(2,\omega,k_t)}_\mu \) reads:

\[
A^{(2,\omega,k_t)}_\mu = \frac{\sqrt{\sinh\pi\omega}}{2\pi^2 k_\perp} (\rho \partial_\rho \phi, -i\frac{\omega}{\rho}, 0, 0) .
\]

Let us prove this. Employing the definition of \( ( , ) \) we obtain:

\[
(A^{(2,\omega,k_t)}, A^{(2,\omega',k'_t)}) = \int dx_1 \frac{d\rho}{\rho} C^{(2,\omega,k_t)} C^{(2,\omega',k'_t)} \left\{ \frac{1}{\rho} \partial_\tau \phi^{*}(\omega,k_t)(\partial_\rho \rho \phi^{*}(\omega',k'_t)) - \frac{1}{\rho} \partial_\rho \phi^{*}(\omega,k_t) \partial_\rho \rho \phi^{*}(\omega',k'_t) \right\} +
\]

\[
= -\frac{1}{\rho} \partial_\tau \phi^{*}(\omega,k_t)(\partial_\rho \rho \phi^{*}(\omega,k_t)) =
\]

\[
= -i \int dx_1 \frac{d\rho}{\rho} C^{(2,\omega,k_t)} C^{(2,\omega',k'_t)} (\partial_\tau \phi^{*}(\omega,k_t) \nabla^2 t \phi^{*}(\omega',k'_t) - \partial_\tau \phi^{*}(\omega',k'_t) \nabla^2 t \phi^{*}(\omega,k_t)) =
\]

\[
= \int \frac{d\rho}{\rho} C^{(2,\omega,k_t)} C^{(2,\omega',k'_t)} (2\pi)^2 \delta(k_t - k'_t) (\omega + \omega') k_\perp^2 K_{\omega}(k_t \rho) K_{\omega'}(k_t \rho) =
\]

\[
= C^{(2,\omega,k_t)} C^{(2,\omega',k'_t)} (2\pi)^2 \delta(k_t - k'_t) (\omega + \omega') k_\perp^2 \int_0^{+\infty} \frac{d\rho}{\rho} K_{\omega}(k_t \rho) K_{\omega'}(k_t \rho) .
\]

Reminding the relation:

\[
\int_0^{+\infty} d\rho \rho^{-1} K_{\omega}(k_\perp \rho) K_{\omega'}(k_\perp \rho) = \frac{\pi^2}{2\omega \sinh \pi \omega} \delta(\omega - \omega') ,
\]

and choosing the coefficient \( C^2 \) as real, we find the form (78) of the mode \( A^2 \) producing the required “delta” normalization.

In the case of \( C^1 \) we find:

\[
\frac{1}{C^{(1,\omega,k_t)} C^{(1,\omega',k'_t)}} (A^{(1,\omega,k_t)}, A^{(1,\omega',k'_t)}) = i \int dx_1 \frac{d\rho}{\rho} \sum_{a=y,z} A^{(1,\omega,k_t)} \partial_\tau A^{(1,\omega',k'_t)} ,
\]

where we used also the fact that the only non vanishing Christoffel symbols which appear in our coordinates are \( \Gamma^\tau_\rho = \Gamma^\tau_\rho = 1/\rho \) and \( \Gamma^\rho_\tau = \rho \) (\( \tau \) is the Rindler time and \( \rho \) is the non trivial
space-like Rindler coordinate: they are not generic indexes).
Reminding the form of \( A^{(\beta, \omega, k_t)} \) as function of \( \phi \) given in Eq. (19), we obtain:

\[
\frac{1}{C^{(1, \omega, k_t)} C^{(1, \omega', k'_t)}} (A^{(1, \omega, k_t)}, A^{(1, \omega', k'_t)}) = \\
= (\omega + \omega') (2\pi)^2 k^2_\perp e^{i(\omega - \omega') \tau} \delta(k_t - k'_t) \int \frac{d\rho}{\rho} K_{i\omega}(k_\perp \rho) K_{i\omega'}(k'_\perp \rho). \tag{80}
\]

Using the relation in Eq. (79) and choosing the simplest phase, it arises:

\[
C^{(1, \omega, k_t)} = \sqrt{\frac{\sinh \pi \omega}{2\pi^2 k^2_\perp}} = C^{(2, \omega, k_t)}.
\]

Finally, we have, by inserting this result in Eq. (20):

\[
A^{(1, \omega, k_t)} \equiv \sqrt{\frac{\sinh \pi \omega}{2\pi^2 k^2_\perp}} (0, 0, k_y \phi, -k_x \phi).
\]

Employing similar calculations, we obtain also:

\[
C^{(4, \omega, k_t)} = \sqrt{\frac{\sinh \pi \omega}{2\pi^2 k^2_\perp}} = C^{(2, \omega, k_t)}
\]

and thus

\[
A^{(4, \omega, k_t)} \equiv \sqrt{\frac{\sinh \pi \omega}{2\pi^2 k^2_\perp}} (0, 0, ik_x \phi, ik_y \phi). \tag{81}
\]

Calculations for the case \( C^{(3, \omega, k_t)} \) are more complicated.
Let us start noting that, from Eqs. (22), (17) and (18):

\[
A^{(3, \omega, k_t)} = C^{(3, \omega, k_t)} \left[ A^{(G, \omega, k_t)} - i A^{(L, \omega, k_t)} \right].
\]

Note also that \( (A^{(G, \omega, k_t)}, A^{(G, \omega', k'_t)}) = 0 \) because \( F^{\mu \nu}_{A^G} = 0 \) and \( \nabla_\mu A^G \mu = 0 \).
Then, choosing: \( C^3 = C^{*3} = C^G = C^{*G} = C^L = C^{*L} \) we find:

\[
A^{(3, \omega, k_t)} = \left[ A^{(G, \omega, k_t)} - i A^{(L, \omega, k_t)} \right],
\]

and thus (omitting obvious indexes):

\[
(A^{(3, \omega, k_t)}, A^{(3, \omega', k'_t)}) = -i(A^G, A^L) + i(A^L, A^G) + (A^L, A'^L) =
\]

\[
= i(A^G, A'^L) + i(A^L, A'^G) + \frac{C^3 C^{*3}}{C^4 C^{*4}} (A^4, A'^4),
\]

where, as we found, \( C^{(4, \omega', k'_t)} = \sqrt{\frac{\sinh \pi \omega}{2\pi^2 k^2_\perp}} \).
It follows expanding the formula above \( (i = \rho, y, z \text{ and there is understood a summation over repeated indexes})\):

\[
\frac{1}{C^{(3, \omega, k_t)} C^{(3, \omega', k'_t)}} (A^{(3, \omega, k_t)}, A^{(3, \omega', k'_t)}) =
\]

\[
= \frac{1}{C^{(3, \omega, k_t)} C^{(3, \omega', k'_t)}} \int d\rho \frac{d\rho}{\rho} (i A^G F_{\tau \bar{\tau}} - i A^G \nabla_\mu A'^{\mu \bar{\mu}}). \]

\]
Executing the integrals and using Eq. (79) we obtain the final result:

$$\frac{1}{C(3,\omega,k_1)C(3,\omega',k'_1)} \langle A(3,\omega,k_1), A(3,\omega',k'_1) \rangle =$$

$$= (-\frac{2}{C(3,\omega,k_1)C(3,\omega',k'_1)} + \frac{1}{C(4,\omega,k_1)C(4,\omega',k'_1)}) \delta(k_t - k'_t) \delta(\omega - \omega').$$

We shall take $C(3,\omega,k_1) = C(4,\omega,k_1)$ and thus we have the following normalization relation:

$$\langle A(3,\omega,k_1), A(3,\omega',k'_1) \rangle = -\delta(\omega - \omega') \delta(k_t - k'_t),$$

where

$$A(3,\omega,k_1) = \frac{\sqrt{\sinh \pi \omega}}{2\pi^2 k_\perp} (-i\omega \phi, \partial_\rho \phi, 0, 0).$$

We have found the normalization constant of all the modes:

$$C(1,\omega,k_1) = C(2,\omega,k_1) = C(3,\omega,k_1) = C(4,\omega,k_1) = C(L,\omega,k_1) = C(G,\omega,k_1) = \frac{\sqrt{\sinh \pi \omega}}{2\pi^2 k_\perp}.$$  

**APPENDIX B**

In this appendix we shall prove Eq. (11) and Eq. (12).

Let us start with Eq. (11). Note that, because of the trivial dependence on $\tau$ and $\tau'$ of the function $W^+$, we can redefine $D_{\tau\tau'}$, when it acts on $W^+$, as

$$D_{\tau\tau'} = \frac{1}{2}(\partial_{\tau}^2 + \partial_{\tau'}^2) + \rho \partial_{\rho} \partial_{\rho'}.$$

Working on a solution of the scalar K-G equation like $W^+$ which is a scalar K-G solution in both arguments also in the present case $\epsilon > 0$, it arises:

$$\frac{1}{\rho} \partial_{\tau}^2 = \partial_{\rho} \partial_{\rho'} + \rho \nabla^2_t,$$

where we posed $\nabla^2_t = \sum_{i=y,z} \partial_i^2$. Thus we may write down $D_{\tau\tau'}$ as

$$D_{\tau\tau'} = \frac{1}{2} [\rho^2 \nabla_t^2 + \rho^2 \nabla_t'^2 + (\partial_{\ln \rho} + \partial_{\ln \rho'}^2)] = \frac{1}{2} [\rho^2 \nabla_t^2 + \rho^2 \nabla_t'^2 + 4 \partial_{\ln \rho \rho'}]. \quad (82)$$

In the latter term we considered as independent variables $u := \ln(\rho \rho')$ and $v := \ln(\rho \rho'^{-1})$. These variables appear in the expression defining $\alpha$, Eq. (33), and $\alpha$ appears in $W^+$ as precised by Eq. (28).

Let us consider the action on $W^+$ of the last term in the equation written above.

$$\partial_{\ln \rho \rho'} W^+(\tau - \tau', u, v, x_t) = -W^+ \frac{\partial W^+}{\partial \alpha} \frac{\partial \alpha}{\partial \ln \rho \rho'} =$$

$$= -W^+ \frac{\partial W^+}{\partial \alpha} \frac{1}{\sinh \alpha} \frac{\partial \cosh \alpha}{\partial \ln \rho \rho'} = -W^+ \frac{\partial W^+}{\partial \alpha} \frac{1}{\sinh \alpha} \frac{x_t^2}{2 \rho \rho'}, \quad (83)$$
where we used the formula of simple proof:
\[
\frac{\partial \cosh \alpha}{\partial \ln \rho \rho'} = -\frac{x_t^2}{2\rho \rho'}.
\]
Notice that:
\[
x_t^2 \frac{\partial W^+}{\partial x_t^2} = \frac{\partial W^+}{\partial \alpha} \frac{x_t^2}{\sinh \alpha} \frac{\partial \cosh \alpha}{\partial x_t^2} = \frac{\partial W^+}{\partial \alpha} \frac{1}{\sinh \alpha} \frac{x_t^2}{2 \rho \rho'} .
\]
Comparing Eq.(83) with Eq.(84) it arises:
\[
\partial \ln \rho \rho' W^+ + \partial x_t^2 W^+ = - (1 + x_t^2 \frac{\partial}{\partial x_t^2}) W^+ = -(1 + \frac{1}{2} |x_t| \frac{\partial}{\partial |x_t|}) W^+ .
\]
(85)
Iterating the process by considering that \(\partial \ln \rho \rho'\) and \(|x_t| \partial_{|x_t|}\) *commute*, we obtain
\[
\partial \ln \rho \rho' W^+ = W^+ + |x_t| \partial_{|x_t|} W^+ + \frac{x_t^2}{4} \nabla_t^2 W^+ ,
\]
where we used the independence of \(W^+\) on the angular variable of 2-vector \(x_t\).
Substituting the this expression in Eq.(82), we find just: Eq.(40)
\[
D_{\tau' \rho'} W^+ = \frac{1}{2} \left( \rho^2 \nabla_t^2 + \rho' \nabla_t^2 + x_t^2 \nabla_t^2 \right) W^+ + 2 W^+ + 2 |x_t| \partial_{|x_t|} W^+ = \rho \rho' \cosh \alpha \nabla_t^2 W^+ + 2 W^+ + 2 |x_t| \partial_{|x_t|} W^+ = \rho \rho' \nabla_t^2 (\cosh \alpha W^+) ,
\]
where we used the formulas following from Eq.(37):
\[
\partial_{|x_t|} \cosh \alpha (\rho, \rho', x_t) = \frac{2 |x_t|}{\rho \rho'}
\]
and
\[
\nabla_t^2 \cosh \alpha (\rho, \rho', x_t) = \frac{2}{\rho \rho'} .
\]
In order to prove Eq.(41) notice that, because of the dependence of \(W^+\) on \(\tau - \tau'\), the operator \(D_{\tau' \rho'}\) acting on \(W^+\) can be written down as
\[
D_{\tau' \rho'} = - \frac{1}{\rho' \rho} (\rho \partial_\rho + \rho' \partial_{\rho'}) \partial_\tau = - \frac{2}{\rho'} \partial_{\ln \rho \rho'} \partial_\tau = -2 \rho \partial_\rho \partial_\tau .
\]
We considered \(U := \rho \rho'\) and \(V := \rho / \rho'\) as independent variables above.
Furthermore, from the definition of \(W^+\), Eq.(88), we obtain also:
\[
\partial_\tau W^+ = -\frac{2(\tau - \tau') W^+}{(\tau - \tau' - i \varepsilon)^2 - \alpha^2} .
\]
And thus we have, posing \(T := \tau - \tau' - i \varepsilon\):
\[
D_{\tau' \rho'} W^+ = 2 \rho \partial_{\rho'} \left[ \frac{2 T W^+}{T^2 - \alpha^2} \right] = \frac{- \rho T}{\pi^2} \partial_{\rho'} \left[ \frac{\alpha}{\rho \rho' \sinh \alpha (T^2 - \alpha^2)^2} \right] = \frac{- T \rho}{\pi^2} \left\{ \frac{1}{\rho^2 \rho'^2 \sinh \alpha (T^2 - \alpha^2)^2} + \frac{\alpha}{\rho \rho'} \left[ \frac{1}{\sinh \alpha (T^2 - \alpha^2)^2} \right] \right\} =
\]
\[
\begin{align*}
&= \frac{T \rho}{\pi^2} \left\{ \frac{1}{\rho^2 \rho' \sinh \alpha} \frac{1}{(T^2 - \alpha^2)^2} + \frac{x_i^2}{\rho^2 \rho' \sinh \alpha} \frac{\partial}{\partial x_i^2} \left[ \frac{1}{(T^2 - \alpha^2)^2} \right] \right\},
\end{align*}
\]
where we used the formula:
\[
\frac{\partial}{\partial \rho'} f \left( \frac{x_i^2}{\rho \rho'} \right) = - \frac{x_i^2}{\rho \rho'} \frac{\partial}{\partial x_i^2} f \left( \frac{x_i^2}{\rho \rho'} \right).
\]

We have
\[
D_{\tau \rho'} W^+ = \frac{T \rho}{\pi^2 \rho^2 \rho'^2} \left\{ \frac{\alpha}{\sinh \alpha} \frac{1}{(T^2 - \alpha^2)^2} + \frac{x_i^2}{\rho^2 \rho'^2 \sinh \alpha} \frac{\partial}{\partial x_i^2} \left[ \frac{1}{(T^2 - \alpha^2)^2} \right] \right\} = \tag{88}
\]
\[
= \frac{T \rho}{\pi^2 \rho'^2 \rho'^2} \frac{\partial}{\partial x_i^2} \left[ \frac{x_i^2}{(T^2 - \alpha^2)^2} \frac{\alpha}{\sinh \alpha} \right].
\]

Notice that it holds:
\[
\frac{\alpha}{(T^2 - \alpha^2)^2 \sinh \alpha} = \rho \rho' \frac{\partial}{\partial x_i^2} \frac{1}{T^2 - \alpha^2}.
\]

Substituting this in the latter line we have:
\[
D_{\tau \rho'} W^+ = \frac{T}{\pi^2 \rho'} \frac{\partial}{\partial x_i^2} \left[ \frac{x_i^2}{T^2 - \alpha^2} \frac{1}{\sinh \alpha} \right] = \frac{T}{4\pi^2 \rho' |x_i|} \frac{\partial}{\partial |x_i|} \left[ \frac{|x_i|}{\sinh \alpha} \right] =
\]
\[
= \frac{T}{4\pi^2 \rho'} \nabla_i^2 \frac{1}{T^2 - \alpha^2} = - \rho (\tau - \tau') \nabla_i^2 \left( \frac{\sinh \alpha}{\alpha} W^+ \right).
\]

Thus Eq.(88) has been proved.

**APPENDIX C**

We shall introduce the definition of Wightman functions based on a _four smeared formalism_, (see for example [1] for the scalar case on a curved space). Wightman functions
\[
<F | \hat{A}_\mu(x) \hat{A}_{\nu'}(x') | F >
\]
are defined within this formalism by imposing:
\[
<F | \hat{A}(F) \hat{A}(F') | F > =
\]
\[
= \int_{W_R} d^4x \sqrt{-g(x)} \int_{W_R} d^4x' \sqrt{-g(x')} F^\mu(x) F'^{\nu'}(x') \ < F | \hat{A}_\mu(x) \hat{A}_{\nu'}(x') | F > ,
\]
where \( F_\nu(y) \in C_0^\infty(W_R) \) for \( \nu = 0, 1, 2, 3 \) and we defined:
\[
<F | \hat{A}(F) \hat{A}(F') | F > : = < F | (A_F, \hat{A})(A_{F'}, \hat{A}) | F > ,
\]

The functions \( A_F \) are solutions of K-G equation carrying a compact support on Cauchy surfaces obtained from functions \( F \) as
\[
A_F^\mu(x) = \int_{W_R} d^4y \sqrt{-g(y)} E(x, y)^{\mu\nu} F_\nu(y) \tag{89}
\]
\( E(x, y) \) is the “advanced minus retarded” fundamental solution of K-G equation (see section III.B). Formally speaking (see Ref.s [1] [13] for the scalar case):
\[
E(x, y)_{\mu\nu} := [\hat{A}_\mu(x), \hat{A}_{\nu}(y)] .
\]
Because of the independence of the quantum state of that function, we expect to find, employing test functions with support inside of the open set $W_R$:

$$E(x, y) = E(x, y)_M, \quad (90)$$

The latter two-point function being the Minkowski advanced minus retarded fundamental solution. We have proved this statement in section III.B.

Notice that $E(x, y)_M$ is (distributionally) vanishing outside of the light cone at $y$ and this assures the compactness of the spatial support of the functions $A_F$ whenever the functions $F$ belong to $C_0^\infty$.

Another important property which can be simply proved employing Minkowskian coordinate through Eq.s (89) and (90) is:

$$\nabla_{\mu}A^\mu_F(x) = \int_{W_R} d^4y \sqrt{-g(y)} E(x, y)_S \nabla_{\nu}F^\nu(y),$$

where $E(x, y)_S$ is the scalar advanced minus retarded fundamental solution.

Finally, notice that $\nabla_{\nu}F^\nu = 0$ implies $\nabla_{\mu}A^\mu_F = 0$.

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a) We are employing the signature $(-1, 1, 1, 1)$ and thus some sign results to be changed with respect to Ref.[8] where they used the opposite signature.

b) Remind also the identity $\Gamma^\mu_{\nu\mu} = \partial_{\nu}\{\ln \sqrt{-g}\}$.

c) This is not the metric in Rindler coordinates which is $g_{\mu\nu} \equiv \text{diag} (-\rho^2, 1, 1, 1, )$.

d) As precised above, the stronger assumption of edge-vanishing test wavefunctions permits to drop boundary terms arising by changing the scalar product $(, )$ with $(, )'$. Thus, in the weak sense, one can drop similar boundary terms also in Eq.s (5), (8), (9) and re-write these in terms of $(, )'$.

e) We shall indicate by $\otimes_T$ the topological (Hilbertian) tensorial product.

f) The energy is negative being $<F|\hat{a}_0\hat{H}\hat{a}_0^\dagger|F> < 0$. However, the Hamiltonian eigenvalues of the quanta generated by $\hat{a}_0^\dagger$ are positive.

$g)$ Remind that the spatial surfaces do not contain the points with $\rho = 0$ because $W_R$ is an open set. The spatial support of the considered solution can contain points with $\rho = 0$ (the horizons) only as $|\tau| \to +\infty$.

h) The $d\rho$ integration, due to the factor $K_{k_0}(k_\perp, \rho)$, produces a logarithmically divergent function as $k_\perp \to 0$ which does not affect this result.

i) These formulae hold on the linear manifold $D$, dense in the considered topology, containing all the Fock states carrying whatever finite number of particles. $D$ results to be invariant under the action of $\hat{a}$, $\hat{a}^\dagger$ as well as $M$.

j) Due to $M = H$, we can approximate all the scalar product of the states in $H$ by complex linear combinations of Wightman functions with $A, A' \in S$.

k) See for example Ref.[18] in part II section 6.5, changing the hypotheses of the example h) and using the same proof.
l) If \( f(x_t) = \nabla_t^2 g(|x_t|) \), it is sufficient that \( \frac{\partial g(x_t)}{\partial \ln |x_t|} \to 0 \) as \( |x_t| \to +\infty \). This holds in both the examined cases below.

m) In order to use the following formula it is sufficient, if \( g(x_t) = g(|x_t|) \), that \( g(x_t) \to 0 \) and \( \ln |x_t| \frac{\partial g(x_t)}{\partial \ln |x_t|} \to 0 \) as \( |x_t| \to \infty \). This holds in the present case (as well as in the next one) where we have (as \( |x_t| \to +\infty \)) \( g(|x_t|) = \cosh \alpha W^+ \sim (\ln |x_t|)^{-1} \) and \( g(|x_t|) = \frac{\sinh \alpha}{\alpha} W^+ \sim (\ln |x_t|)^{-2} \).

n) In particular, notice that \( V_{\mu \nu}(x_E, x'_E)|_{x_E = x'_E} = g^{\nu E}_{\mu E}(x_E) \).

o) Use standard identities as \( \frac{1}{x \pm i \epsilon} = \text{PV} \frac{1}{x} \mp i \pi \delta(x) \).

p) This follows trivially from the Green functions calculated in Ref.s [14, 13].

q) We use in particular the identity following directly from Eq.s [1]. \( 2\rho \rho' (\cosh(\tau - \tau') - \cosh \alpha) = |x - x'|^2 - (t - t')^2 \) and \( \text{sign}(t - t') = \text{sign}(\tau - \tau') \) holding for test functions of the variable \( x \) with support inside of the closed light cone at \( x' \).

r) Notice that the scalar propagator \( G_F \) coincides with the ghost propagator.

s) The “local” temperature \( T \) measured by an observer situated at a fixed spatial point is related to \( T_0 \) by the Tolman relation \( T = T_0/\sqrt{-g_{00}} \) see Ref. [2].

t) The condition \( \cosh \left( \frac{2\pi}{\sqrt{\rho}} \alpha \right) - \cosh \left( \frac{2\pi}{\sqrt{\rho}} (\tau - \tau') \right) = 0 \) is equivalent to \( \cosh \alpha - \cosh(\tau - \tau') = 0 \) or, employing Minkowskian coordinates, \( |x - x'|^2 - (t - t')^2 = 0 \).

u) Notice that we must define the smooth test functions requiring also \( f(s, \rho, x_t)|_{\rho = 0} = f(s', \rho, x_t)|_{\rho = 0} \) whatever \( s, s' \in (0, \beta) \) and \( x_t \in \mathbb{R}^2 \). In order to prove the following formula note also \( (x_t \in \mathbb{R}^2) \frac{\rho^2}{\rho^2 + |x_t - x'|^2} \to \pi \delta(x_t - x') \) as \( \rho \to 0^+ \).

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