Research article

Theory and implementation of sub-model method in finite element analysis

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A R T I C L E   I N F O

Keywords:
Beam element
Sub-model
Theoretical derivation
Stress field

A B S T R A C T

Through finite element analysis (FEA) software to study mechanical performance of a bridge structure is currently a commonly used method. The keys to obtain accurate results are to select the appropriate element type and establish a refine mesh model. With the construction of a large number of kilometer-level long-span bridges in practical projects, the time cost of establishing and analyzing a fine mesh solid finite element model (FEM) of a long-span bridge with complex structure can not be ignored. In order to find the balance between accuracy and efficiency, sub-modeling technique can be used to analyze the bridge structure. It is often thought that the sub-modeling technique is only applicable to shell and solid elements, but in fact it is also applicable to plane frame models. Based on two-dimensional (2-D) beam element model, the theory of sub-model was theoretically deduced. Meanwhile the sub-modeling technique was applied and verified by an example of plane frame model. Furthermore, based on the three-dimensional (3-D) solid FEM of a skew bridge, the influence of mesh size on the calculation accuracy was illustrated. Based on sub-modeling technique of nodal displacements, the results of the global model and the sub-model for the skew bridge were compared and studied in terms of stress field and plastic damage of concrete. It is found that the sub-model technique based on nodal displacements is suitable for solid FEM.

1. Introduction

Although most bridges are slender structures, the internal structures of some bridges are complex. The stress field distribution inside the bridge cannot be effectively obtained by establishing the plane frame model. Based on the finite element (FE) method to analyze the mechanical properties of the bridge structures, accuracy and efficiency are two key factors that researchers need to be paid attention to [1, 2, 3]. For a finite element model (FEM) with smaller geometric scale, the use of a denser mesh would not lead to the problem of inefficiency. The analysis of a large bridge with dense meshes will not only greatly increases the amount of calculation, but also sometimes the scale of grid elements of FEM exceeds the computing power of a computer. However, the accuracy of partial analysis is poor in FEM with sparse mesh. At this point, the following two methods can be used to balance accuracy and efficiency in calculation.

The first method is to refine mesh of the focus area in the global model. The analysis method is based on the global model, and the operation is simple without considering the simulation of local boundary conditions. However, the large global model will reduce the computational efficiency.

The second method is to calculate the global model with sparse mesh, and then take out the focus area of refine mesh with the local boundary data in the global calculation results. This method with complicated operation is computationally efficient.

Therefore, it is unrealistic to build a more refined global model to analyze the complex stress distribution area. Better partial analysis method can greatly accelerate the analysis of the problem [4]. However, the boundary condition simulation for local structure is the difficulty and key point of partial analysis, which is also one of the important factors affecting the results of analysis.

Sub-modeling technique for partial analysis is the most commonly used method. Tischert et al. [5] adopts sub-model technique for FEM simulations of power electronic housings under power cycling condition. Romera et al. [6] deals with the edge effects in angle-ply composite laminates subjected to tensile test and uses sub-modeling technique to achieve numerical results. Rajasekaran and Newell [7] uses an example
of a partial slip Cattaneo-Mindin problem to highlight the implication of selecting different contact algorithms in solving a contact problem by a sub-modeling approach. Lippert et al. [8] mandate the use of advanced physical submodels in computing the physical processes occurring during cold starting of diesel engines. Sub-model method is applied to the force analysis of truck side frame and the partial analysis of bridge deck [9, 10]. The multi-scale partial FE modeling method for arch foot connection structure was also studied, which can better simulate the complex boundary conditions and stress distribution of the connection part [11]. The application skills of sub-modeling technique in concrete box girder cantilever plate were studied, which shows the applicability in the partial analysis of cantilever slab of concrete box girder [12].

Sub-model technique is widely used in many engineering fields, such as mechanical engineering, thermal engineering, geotechnical engineering and so on [13, 14, 15, 16, 17, 18, 19, 20]. Many scholars have proved that sub-modeling technique is applicable to shell element and solid element [21, 22, 23, 24, 25, 26, 27, 28]. In recent years, an improved multi-boundary interpolation method for beam element is proposed, which can further improve the calculation accuracy for small distance between the inner and outer cutting boundary [29, 30, 31, 32].

The work of the paper was based on the FE of plane frame structure to theoretically derive the sub-model method. Using the general FE software ABAQUS [33], the modeling strategy of sub-modeling technique was introduced. Finally, taking a 47 degree skew bridge as an example, the stress state of its diaphragm beam was analyzed by sub-model method.

2. Theoretical derivation of sub-model technique based on nodal displacement

Fig. 1 shows a continuous beam with external loads. The FE method of frame model was used to study the stress state of the beam, and the beam can be divided into a structure with n nodes and (n − 1) elements.

The original stiffness equation (Eq. (1)) can be obtained from the basic stiffness equation \([ F ] = [ K ] [ \delta ]\).

\[
\begin{bmatrix}
F_1 \\
\vdots \\
F_i \\
\vdots \\
F_n
\end{bmatrix} = \begin{bmatrix}
F_1 \\
\vdots \\
F_D - (F_F^{i-1} + F_F^{n}) \\
\vdots \\
F_n
\end{bmatrix} = \begin{bmatrix}
k_{11} & k_{12} & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & k_{ii-1} & k_i^{i-1} + k_i^{i} & k_i^j & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}\begin{bmatrix}
\Delta_1 \\
\cdots \\
\Delta_i \\
\cdots \\
\Delta_n
\end{bmatrix}
\]  

(1)

where, \(F_D\) is nodal load of \(i\) node; \(F_F^{i-1}\) is fixed end force of \(i\) node of \(i - 1\) element under non nodal load; \(F_F^{n}\) is fixed end force of \(i\) node of \(i\) element under non nodal load.

In structural mechanics, the load transfer effect of \(i-j\) segment of the global model is taken as shown in Fig. 2. In order to achieve the equivalence of load transfer in the finite element analysis (FEA), beam elements \(i-(i-j)\) are taken out for partial analysis and the action of boundary elements \(i-1\) and \(j\) are introduced. The calculation equations for end forces of elements \(i-1\) and \(j\) are expressed as Eqs. (2) and (3).

\[
F^{i-1} = \begin{bmatrix}
F_F^{i-1} \\
\vdots \\
F_F^{j-1}
\end{bmatrix} + \begin{bmatrix}
k_{ii-1} & k_{i(i-1)} & k_{i(j-1)} \\
\vdots & \vdots & \vdots \\
k_{jj-1} & k_{ij} & k_{i(j+1)} \\
\vdots & \vdots & \vdots \\
0 & \cdots & 0
\end{bmatrix}\begin{bmatrix}
\Delta_{i-1} \\
\Delta_i \\
\Delta_j \\
\Delta_{j+1}
\end{bmatrix} = \begin{bmatrix}
F_F^{i-1} + k_{ii-1} \Delta_{i-1} + k_{i(j-1)} \Delta_j \\
\vdots \\
F_F^{j-1} + k_{jj-1} \Delta_{j-1} + k_{ij} \Delta_j + k_{i(j+1)} \Delta_{j+1}
\end{bmatrix}
\]  

(2)

\[
F = \begin{bmatrix}
F_F^j \\
\vdots \\
F_F^{j+1}
\end{bmatrix} + \begin{bmatrix}
k_{jj} & k_{j(j+1)} \\
\vdots & \vdots \\
k_{jj+1} & k_{j(j+1)}
\end{bmatrix}\begin{bmatrix}
\Delta_j \\
\Delta_{j+1}
\end{bmatrix} = \begin{bmatrix}
F_F^j + k_{jj} \Delta_j + k_{j(j+1)} \Delta_{j+1} \\
\vdots \\
F_F^{j+1} + k_{jj+1} \Delta_j + k_{j(j+1)} \Delta_{j+1}
\end{bmatrix}
\]  

(3)

The end force of the element \(i - 1\) to the node \(j\) is \(F^{i-1}_{ii} + k_{ii-1} \Delta_{i-1} + k_{i(j-1)} \Delta_j\), that is, the force of element \(i - 1\) acting on the partial analysis model. Similarly, it can be concluded that the end force of the element \(j\) to the node \(i\) is \(F^j_{jj} + k_{jj} \Delta_j + k_{j(j+1)} \Delta_{j+1}\), that is, the force of the element \(j\) to the partial analysis model.
The effects of boundary elements \( i - 1 \) and \( j \) are brought into the original stiffness matrix of the elements \( i - (j - 1) \).

\[
\begin{bmatrix}
F_i \\
F_{i+1} \\
\vdots \\
F_{j-1} \\
F_j
\end{bmatrix} = 
\begin{bmatrix}
F_{Di} - (F_{Fi} + F_{Fi+1}) & F_{Fi+1} - (F_{Fi} + F_{Fi+1}) \\
\vdots & \vdots \\
F_{D_{j-1}} - (F_{F_{j-1}} + F_{F_{j-1}+1}) & F_{F_{j-1}+1} - (F_{F_{j-1}} + F_{F_{j-1}+1}) \\
F_{Dj} - (F_{Fj} + F_{Fj+1}) & F_{Fj+1} - (F_{Fj} + F_{Fj+1})
\end{bmatrix}
\begin{bmatrix}
\Delta_{i-1} \\
\Delta_i \\
\vdots \\
\Delta_{j+1} \\
\Delta_{j+1}
\end{bmatrix}
\]

Expanding the node load matrix in Equation (4) to obtain Equation (5), and then the node load matrix expansion Equation (5) was brought into the elements \( i - (j - 1) \) stiffness matrix of partial analysis model, and further decomposed into Equation (6).

\[
\begin{bmatrix}
F_i \\
F_{i+1} \\
\vdots \\
F_{j-1} \\
F_j
\end{bmatrix} = 
\begin{bmatrix}
F_{Di} - F_{Fi} & 0 \\
F_{Di+1} - (F_{Fi} - F_{Fi+1}) & F_{Fi+1} - (F_{Fi} + F_{Fi+1}) \\
\vdots & \vdots \\
F_{D_{j-1}} - (F_{F_{j-1}} + F_{F_{j-1}+1}) & F_{F_{j-1}+1} - (F_{F_{j-1}} - F_{F_{j-1}+1}) \\
F_{Dj} - F_{Fj} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta_{i-1} \\
\Delta_i \\
\vdots \\
\Delta_{j+1} \\
\Delta_{j+1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
F_i \\
F_{i+1} \\
\vdots \\
F_{j-1} \\
F_j
\end{bmatrix} = 
\begin{bmatrix}
F_{Di} - F_{Fi} & 0 \\
F_{Di+1} - (F_{Fi} - F_{Fi+1}) & F_{Fi+1} - (F_{Fi} + F_{Fi+1}) \\
\vdots & \vdots \\
F_{D_{j-1}} - (F_{F_{j-1}} + F_{F_{j-1}+1}) & F_{F_{j-1}+1} - (F_{F_{j-1}} - F_{F_{j-1}+1}) \\
F_{Dj} - F_{Fj} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta_{i-1} \\
\Delta_i \\
\vdots \\
\Delta_{j+1} \\
\Delta_{j+1}
\end{bmatrix}
\]

When \( i - (j - 1) \) beam elements are taken for partial analysis, the node displacement \( \Delta_i \) and \( \Delta_j \) are applied to the node \( i \) and \( j \), respectively. The stiffness Equation (7) for partial analysis can be expressed as following.

\[
\begin{bmatrix}
F_i \\
F_{i+1} \\
\vdots \\
F_{j-1} \\
F_j
\end{bmatrix} = 
\begin{bmatrix}
F_{Di} - F_{Fi} & 0 \\
F_{Di+1} - (F_{Fi} - F_{Fi+1}) & F_{Fi+1} - (F_{Fi} + F_{Fi+1}) \\
\vdots & \vdots \\
F_{D_{j-1}} - (F_{F_{j-1}} + F_{F_{j-1}+1}) & F_{F_{j-1}+1} - (F_{F_{j-1}} - F_{F_{j-1}+1}) \\
F_{Dj} - F_{Fj} & 0
\end{bmatrix}
\begin{bmatrix}
\Delta_{i-1} \\
\Delta_i \\
\vdots \\
\Delta_{j+1} \\
\Delta_{j+1}
\end{bmatrix}
\]

It can be seen from the equations above that the original stiffness matrix equation acting by adding the boundary elements \( i - 1 \) and \( j \) is equivalent to original stiffness matrix equation of applying the node displacements \( \Delta_i \) and \( \Delta_j \) at the node \( i \) and \( j \), respectively. That is, the actions of the boundary elements \( i - 1 \) and \( j \) is equivalent to the application of the node displacement \( \Delta_i \) at the node \( i \) and the node displacement \( \Delta_j \) at the node \( j \). The internal forces, displacements, and stresses solved by two methods are equal to the solution results of the overall stiffness matrix.

According to the theoretical deduction of the sub-model analysis based on beam element, the implementation process of partial analysis by using FEA software is as follows: firstly, the whole model is established for analysis; secondly, the sub-model is taken out and the boundary data calculated by whole model is applied; finally, the solution is obtained.

It is worth noting that although the theoretical derivation of partial analysis is aimed at FE of plane frame structure, the equivalent equation based on FE of shell and solid can be deduced by modifying the basic equation appropriately.

3. Implementation of sub-model theory of nodal displacements based on a plane frame model

The implementation of the sub-model theory of point displacements was verified through an example.

A mid-span concentrated load of 240 kN (see Fig. 3a) and a uniform load of 10.5 kN/m (see Fig. 3b) are applied to a simply supported beam with a length of 20 m, a height of 2.5 m and a width of 1 m. The beam is made of concrete C50, and the properties of concrete C50 are elastic modulus of 34.5 GPa, Poisson’s ratio of 0.2. The models are established by general FE software ABAQUS. A 5 m long beam segment from left support is taken for the sub-model analysis (see Fig. 3). B23 is used to divide global model and sub-model, which is a Timoshenko second-order beam element. The cross section of B23 element remains perpendicular to the axis of the beam in the process of deformation. Therefore, the application of cubic beam element to simulate the structure of slender members is more effective. The nodal displacement and section rotation at 5 m of the beam end calculated by global model were applied to the boundary of the sub-model. In the case, the boundary conditions of the sub-model are simply
supported at left end and displacement constraints at another end. The vertical displacements, section rotation and normal stresses at distances of 5 m (P1), 3.75 m (P2), 2.5 m (P3) and 1.25 m (P4) from the left support are shown in Tables 1 and 2.

As shown in Tables 1 and 2, the results of both global model and partial model at the four positions on the beam are consistent, which confirms the correctness of theoretical deduction of sub-model theory based on FE of plane frame structure.

4. Implementation of sub-model method of nodal displacements based on a FE of solid element model

As known, the frame model of a bridge can only reflect the overall mechanical properties, but can not reflect the stress distribution of the components.

Sub-modeling technique based on nodal displacements analyzes nodal displacements by replacing the global model with the partial model, which takes nodal displacements as the driving variable. It is suitable for the analysis of solid-to-solid, shell-to-shell and shell-to-solid models. Herein, the sub-modeling technique based on nodal displacements was illustrated by taking the displacement transformation of the driving node of shell-to-solid as an example. Fig. 4 represents the displacements conversion of driving nodes from shell to solid, which can be expressed as Eqs. (8) and (9).

\[ u^A = u^A + \phi^A \times D \]

\[ D = X^A - X^A' \]
where, $A$ is driving node; $AI$ is the point formed by driving node image to center surface of the shell; $t$ is shell thickness; $u^A$ is the specified displacement of driving node $A$; $\varphi^A$ is the interpolation displacement of the image node $AI$; $\Phi^A$ is the interpolation angle of the image node; $D$ is the vector that connects the image node $AI$ to the driving node $A$; $X^A$ is the location of the driving node $A$ in global model; $X^AI$ is the location of the image node $AI$ in global model.

4.1. Engineering background

For a box girder bridge, the stress distribution of the diaphragms is complex, and the plane frame model can not effectively evaluate the state of stress of the diaphragms. Herein, based on the sub-model method of nodal displacements, a FEM of a skew bridge was established to analyze the stress state of the diaphragm.

A three-span concrete rigid frame continuous box girder has a span of $(25 + 30 + 25)$ m, as shown in Fig. 5a. The bridge deck width is 10.9 m (see Fig. 5b). The beam height at the middle pier is 3.15 m, and the beam height at the mid-span is 2.15 m (see Figs. 5c & 5d). The oblique angle between the skew bridge and the road is 47 degrees. A vertical downward uniform load of 1.5 kN/m² acts along the longitudinal direction of the bridge. The beam is made of concrete C50 and steel strand $\Phi1860$. The properties of concrete C50 are elastic modulus of 34.5 GPa, Poisson’s ration of 0.2. The properties of steel strand $\Phi1860$ are elastic modulus of 195 GPa, Poisson’s ration of 0.3.

4.2. The global model and sub-model for the skew bridge

The global model and sub-model of the skew bridge were established by software ABAQUS (see Fig. 6). The boundary conditions are the fixed of pier bottom and foundation, two side spans and abutments simply supported constraints. Two models were meshed by utilizing appropriate element types available for concrete and steel strand. The concrete analysis is carried out by using C3D8R element. C3D8R is a 3D element with 8 nodes and is a reduced integral first-order entity unit, which is available to analyze the concrete stress and can avoid volume self-locking in elastic-plastic problem. Steel strand analysis is carried out by using T3D2 element. T3D2 is a 3D truss element with 2 nodes and is used for normal reinforced and steel strand, which can reflect the stress of normal reinforced accurately. Embedded technology is employed in coupling of the degree of freedom between concrete and steel strands. Embedded technology is usually applied to insert molding parts, which saves the time of drawing nodes and merging parts. Embedded constraint can be established when there is interference between two parts, which can embed one part into another without considering whether the embedded position structure is empty. When creating embedded constraints, special attention should be paid to the fact that the exterior tolerance generally does not need to be set. The default value of 0.05 can be used. The global model and sub-model have been discretized by 34,152 solid elements and 6, 520 solid elements, respectively. The location of sub-model in the global model is shown in Fig. 6a, and Fig. 6b shows sub-model of the skew bridge.

4.2.1. Reasonable mesh size of sub-model

In order to find a reasonable mesh size to analyze the maximum stress in the diaphragm. The mesh size of the diaphragm in the sub-model is divided into 0.05, 0.08, 0.10, 0.15, 0.20, 0.30 and 0.40 m, respectively. When the sub model and the global model have the same mesh size, the calculation results are the same. For example, when the mesh size is 0.1 m, the maximum tensile stress calculated by two models is 0.986 MPa. The relation between the maximum principle tensile stress and the mesh size in the stress concentration area of the diaphragm was calculated as shown in Fig. 7.

From Fig. 7, it can be seen that the principal tensile stress gradually decreases with the increase of mesh size. Comparative analysis shows that the maximum stress value of the sub-model with mesh size of 0.1 m is close to the convergence value 4.05 MPa. Therefore, the mesh size of the sub-model is taken as 0.1 m in the later analysis.

4.2.2. Stress distribution in the diaphragm

The skew bridge is a full prestressing structure, and the diaphragms are compressed in the longitudinal direction. In the global model, the displacement and rotation values of the two end sections of the sub-model were queried, and then applied to the two end sections of the sub-model as constraints. Therefore, the normal stress in the longitudinal direction was not analyzed. The maximum transverse normal stress, vertical normal stress and principal tensile stress in the stress concentration area of diaphragm calculated by the sub-model and the global model were compared and analyzed, as shown in Table 3. Figs. 8a and 8b show the principle tensile stress cloud diagrams of the global model and the sub-model, respectively.

As can be seen from Table 3, the maximum stresses calculated by the global model in the stress concentration area are much smaller than that calculated by the sub-model, and the differences are more than 60%. From Fig. 8, the stress field reflected by sub-model is more detailed in manhole area. In the elastic analysis of sub-model, the principle tensile stresses in the manhole area exceed the ultimate tensile stress of concrete C50.
Table 3. Stress comparison of sub-model and global model.

| Stress                        | Sub-model | Global model | Difference/% |
|-------------------------------|-----------|--------------|--------------|
| Stress in transverse direction/MPa | 1.775     | 0.463        | 73.95        |
| Stress in vertical direction/MPa | 3.399     | 0.716        | 78.92        |
| Principle tensile stress/MPa  | 3.403     | 1.206        | 64.56        |

Generally speaking, the sub-model analysis can not only calculate the stresses more accurately, but also reflect the stress area which cannot be obtained due to the rough mesh of the global model more carefully.

4.2.3. Plastic damage of concrete in diaphragm

Based on the elastic analysis results, it can be seen that the principal tensile stress in manhole area of diaphragm calculated by the sub-model exceeds the ultimate tensile stress of concrete C50, which obviously does not exist in the actual structure. Hence it is necessary to carry out plastic damage analysis of concrete for diaphragm. Figs. 9a and 9b show the plastic damage cloud diagrams of concrete for the global model and the sub-model, respectively.
From Figs. 8 and 9, it can be seen that the principle tensile stresses calculated by the global model in the manhole area are small, and there is no plastic damage of concrete in diaphragm. The concrete stresses in manhole area of the diaphragm have larger principle tensile stresses calculated by sub-model. Moreover, the concrete in the manhole area of the diaphragm has plastic damage, and the maximum damage factor is 0.7825.

On the premise of guaranteeing computational efficiency, the sub-modeling technique can also be used to obtain accurately stress values and more detailed stress distribution, which is more meaningful for the design guidance.
5. Conclusions

In this paper, the sub-model technique based on nodal displacements is studied, and the following conclusions are obtained:

First, based on the plane frame model, the theory of sub-model is theoretically deduced;

Second, using finite element software ABAUQS, the sub-model technique is applied to the plane frame model. So the sub-modeling technique is not only applicable to shell and solid elements, but also to beam element.

Third, the sub-model technique based on nodal displacements is applied to the three-dimensional solid finite element model of a skew bridge. Two FE models were established for the skew bridge. One model is used for the global analysis and obtains the displacement and rotation values at both ends of the sub-model. The displacement and rotation values obtained from the global model are applied as constraints at both ends of the other solid element sub-model.

Declarations

Author contribution statement

Shan Chang: Performed the experiments; Wrote the paper.
Kui Liu: Analyzed and interpreted the data.
Ming Yang: Conceived and designed the experiments.
Ling Yuan: Contributed reagents, materials, analysis tools or data.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

Data included in article/supp. material/referenced in article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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