Dynamics research of Fangzhu’s nanoscale surface

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Abstract

In this paper, we mainly focus on a fractal model of Fangzhu’s nanoscale surface for water collection which is established through He’s fractal derivative. Based on the fractal two-scale transform method, the approximate analytical solutions are obtained by the energy balance method and He’s frequency–amplitude formulation method with average residuals. Some specific numerical experiments of the model show that these two methods are simple and effective and can be adopted to other nonlinear fractal oscillators. In addition, these properties of the obtained solution reveal how to enhance the collection rate of Fangzhu by adjusting the smoothness of its surfaces.

Keywords

Fangzhu oscillator, He’s fractal derivative, two-scale transform method, energy balance method, frequency–amplitude formulation method

Introduction

Fangzhu (方诸) is the oldest nano device in ancient China which can collect water from the air.1–4 In Ref. 1, He et al. explained the operating principle of the Fangzhu and its many possible applications in the future. As we know, the nanoscale surface of Fangzhu device plays an outsize role in water collection performance, the super-hydrophobic surface is used to absorb water molecules in the air, and the super-hydrophilic surface is well situated for transmitting the absorbed water molecules to the water collector (see Figure 1).1–3 The Fangzhu oscillator can be expressed by the following form

$$\begin{align*}
\frac{\partial^2 u}{\partial t^2} + \frac{2pa}{ma^{2q+1}} + \frac{2qa(2q + 1)}{mH^{2q+2}} u - \frac{2qa}{mH^{2q+1}} &= 0
\end{align*}$$

(1.1)

in which \(a, p, q\) are constants associated with the surface properties and \(m\) is the mass of the molecule and \(H\) is the distance between the hydrophilic surface and hydrophobic surface.

Usually, Fangzhu oscillator is considered in a smooth space. Therefore, in order to investigate the potentially hidden properties of Fangzhu, a fractal modified model is introduced, which can disproportionately affect the movement of water molecules on the Fangzhu’s surface, and will significantly improve the water collection performance of the Fangzhu. However, the traditional derivative cannot describe the observed phenomena of non-smooth and discontinuous surfaces. So, He’s fractal derivative has to be used, which was proposed by a Chinese mathematician, Dr. Ji-Huan He,5–9 it is a wonderful mathematical tool to deal with discontinuous problems and can describe the physical problems in the fractal space.10–21 The modified fractal Fangzhu oscillator is given by
\[ \frac{\varepsilon}{\mu} \delta^2 u + \frac{2pa}{mu^{2\varepsilon+1}} + \frac{2qa(2q + 1)}{mH^{2q+2}} u - \frac{2qa}{mH^{2q+1}} = 0 \]  

(1.2)

where \( \varepsilon \) denotes the He's fractal derivative given as follows:

\[ \frac{\varepsilon}{\mu} \delta^2 u |_{t = t_0} = \Gamma(1 + \delta) \lim_{\Delta t \to 0} \frac{u(t) - u(t_0)}{(t - t_0)\delta} \]  

(1.3)

In order to simplify the calculation process, equation (1.2) becomes

\[ \frac{\varepsilon}{\mu} \delta^2 u + \frac{\varepsilon_1}{u^{2\varepsilon+1}} + \frac{\varepsilon_2(2q + 1)}{H^{2q+2}} u - \frac{\varepsilon_2}{H^{2q+1}} = 0 \]  

(1.4)

where \( \varepsilon_1 = 2pa/m \) and \( \varepsilon_2 = 2qa/m \).

Equation (1.4) is a fractal Duffing-like oscillator, and we call it as fractal Fangzhu oscillator. Wang constructed the variational principle for the fractal oscillator by the semi-inverse transform method and found the approximate analytical solution through the two-scale transform method and He’s frequency formula. If \( \delta = 1 \), equation (1.4) is equivalent to equation (1.1), which represents the motion of water molecules on the smooth surface of Fangzhu oscillator. Energy balance method was first proposed by He, in this method, a variational principle of the nonlinear oscillator is obtained, next a Hamiltonian is acquired, from which the angular frequency can be readily constructed. Subsequently, this method was used to solve many nonlinear oscillations. To solve nonlinear oscillators, Prof. Ji-Huan He proposed another simple but effective frequency–amplitude formulation. He’s frequency–amplitude formulation was used by many scholars with great success. Moreover, there are many methods to obtain the approximate analytical solution, such as Taylor series method, Homotopy perturbation method, Variational iteration method, He–Laplace method, and so on. As to the fractal nonlinear oscillator, these common methods are more complex to obtain the solution. Here, we employ two valid and simple methods to acquire the approximate analytical solution of the fractal nonlinear Fangzhu oscillator equation, which are called energy balance method and frequency–amplitude formulation method with average residuals.

In this paper, we focus on the fractal nonlinear Fangzhu oscillator equation. In Fractal Two-Scale Transform Method, we recall the two-scale transform method. In The Description of Two Methods, energy balance method and frequency–amplitude formulation method with average residuals are briefly introduced. The numerical experiments of two methods are given in the The Application of the Two Methods. These examples indicate that the two methods are impactful and convenient for solving the fractal problem.

**Fractal two-scale transform method**

In this part, we will simply describe the two-scale transform method. This method was introduced by He and his copartner, which is an effective and powerful method to research fractal problems. Its most direct application is to approximately transform the fractal space into the continuous dimension. On a smaller scale, due to the presence of
hydrophilic and hydrophobic surfaces, the surface of Fangzhu is discontinuous. On the other hand, the larger scales
show a smooth Fangzhu’s surface. The fractal two-scale transform is an approximate one to transform a fractal space
on a small scale into a smooth space with a large scale. Since it was proposed, it has gradually become a hot topic in
research field.

To ease the discussion of the fractal two-scale transform method, we consider the general fractal problem as follows

\[
\frac{\partial}{\partial \tau} \frac{\partial^\beta u(x,y,z,t)}{\partial \tau^\beta} + f(u(t)) = 0
\]

In the above model, \( \frac{\partial^\beta u(x,y,z,t)}{\partial \tau^\beta} \) represent He’s fractal derivatives.

According to the fractal two-scale transform method, we have

\[
T = t^\delta, X = x^\delta, Y = y^\delta, Z = z^\delta
\]

Then, we apply equation (2.2) to equation (2.1), equation (2.1) can be written as

\[
\frac{\partial}{\partial \tau} \frac{\partial^\beta u(X,Y,Z,T)}{\partial \tau^\beta} + \frac{\partial^\beta u(X,Y,Z,T)}{\partial X^\delta} + \frac{\partial^\beta u(X,Y,Z,T)}{\partial Y^\delta} + \frac{\partial^\beta u(X,Y,Z,T)}{\partial Z^\delta} + L(F(X,Y,Z,T)) = 0.
\]

The more details about the fractal two-scale transform method, see Refs. 5–9.

**The description of two methods**

We analyze the general nonlinear oscillator with the following fractal derivative

\[
\frac{\partial^2 u}{\partial \tau^2} + f(u(T)) = 0
\]

with subject to the initial conditions

\[
u(0) = A, \frac{\partial u(0)}{\partial \tau} = 0
\]

where \( f \) is the nonlinear function about \( u \).

The variational principle of equation (3.1) can be expressed as the following form

\[
J(u) = \int \left( -\frac{1}{2} \left( \frac{\partial u}{\partial \tau} \right)^2 + F(u) \right) d\tau
\]

With \( F(u) = \int f(u) du \).

**Energy balance method**

According to the above variational principle, we can get the following Hamiltonian formulation

\[
\Lambda = \frac{1}{2} \left( \frac{\partial u}{\partial \tau} \right)^2 + F(u) = F(A)
\]

or

\[
R(T) = \frac{1}{2} \left( \frac{\partial u}{\partial \tau} \right)^2 + F(u) - F(A)
\]
We assume equation (3.1) has the following solution with angular frequency \( \omega \)

\[ u = A \cos(\omega T) \]  

(3.6)

Substituting equation (3.6) into equation (3.5) yields

\[ R(T) = \frac{1}{2} \omega^2 A^2 \sin^2(\omega T) + F(A \cos(\omega T)) - F(A) = 0 \]  

(3.7)

Equation (3.1) is only the approximation of the exact solution at \( \omega T = \pi/4 \) when \( R = 0 \), which gives

\[ \omega_{EBM} = \sqrt{\frac{4(F(A) - F(A/\sqrt{2}))}{A^2}} \]  

(3.8)

**Frequency–amplitude formulation method with average residuals**

We also select a same suitable solution in the form

\[ u = A \cos(\omega T) \]  

(3.9)

in which \( \omega \) is the frequency which will be determined by the following step. Substituting equation (3.9) into equation (3.1) yields

\[ R(T) = -A\omega^2 \cos \omega T + f(A \cos(\omega T)) \]  

(3.10)

The average residual is given as

\[ \bar{R} = \frac{4}{T} \int_0^T R \cos(\omega T) d\tau \]  

(3.11)

With \( T = 2\pi/\omega \).

Choosing the specific two trial frequencies, \( \omega_1 = 1, \omega_2 = 2 \), and the residuals are, respectively, presented as

\[ \bar{R}_1 = \frac{4}{T_1} \int_0^{\pi/2} R_1 \cos(\omega_1 \tau) d\tau, \bar{R}_2 = \frac{4}{T_2} \int_0^{\pi/2} R_2 \cos(\omega_2 \tau) d\tau \]  

(3.12)

The frequency–amplitude formulation can be written as

\[ \omega = \sqrt{\frac{\omega_2^2 \bar{R}_1 - \omega_1^2 \bar{R}_2}{\bar{R}_1 - \bar{R}_2}} \]  

(3.13)

**The application of the two methods**

Considering the following oscillator equation

\[ \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^{2q+1}} - \frac{\partial^2 u}{\partial x^{2q+1}} = 0 \]  

(4.1)

with the initial conditions

\[ u(0) = A \partial_t^q u(0), \quad \partial_t^q u(0) = 0 \]  

(4.2)

By using the fractal two-scale transform method, equations (4.1) and (4.2) become the following form
\[ \frac{\varepsilon \hat{e}^2 u}{\mu \hat{e}^2 T^2} + \frac{\varepsilon_1}{u^{2p+1}} + \frac{\varepsilon_2 (2q + 1)}{2H^{2q+2}} u - \frac{\varepsilon_2}{H^{2q+1}} = 0 \]  

(subject to the corresponding initial conditions)

\[ u(0) = A, \frac{\partial u(0)}{\partial T} = 0 \]

The application of the energy balance method

Now, we adopt the energy balance method to construct the approximate analytical solution of the governing equation. On the basis of equation (3.3), the variational formulation can be written as

\[ J(u) = \int \left( \frac{1}{2} \left( \varepsilon \frac{\hat{e}u}{\hat{e}T} \right)^2 - \frac{\varepsilon_1}{2p u^{2p}} + \frac{\varepsilon_2 (2q + 1)}{2H^{2q+2}} u^2 - \frac{\varepsilon_2}{H^{2q+1}} u \right) dt \]

The Hamiltonian formulation is given as follows

\[ \Lambda = \frac{1}{2} \left( \varepsilon \frac{\hat{e}u}{\hat{e}T} \right)^2 - \frac{\varepsilon_1}{2p u^{2p}} + \frac{\varepsilon_2 (2q + 1)}{2H^{2q+2}} u^2 - \frac{\varepsilon_2}{H^{2q+1}} u \]

or

\[ R(T) = \frac{1}{2} \left( \varepsilon \frac{\hat{e}u}{\hat{e}T} \right)^2 - \frac{\varepsilon_1}{2p u^{2p}} - \frac{1}{A^{2p}} + \frac{\varepsilon_2 (2q + 1)}{2H^{2q+2}} \left( u^2 - A^2 \right) - \frac{\varepsilon_2}{H^{2q+1}} (u - A) \]

(Substituting equation (3.6) into equation (4.7), and we have)

\[ R(T) = \frac{A^2 \omega^2 \sin(T\omega)^2}{2} - \frac{A^2 (\varepsilon_2 + 2 \varepsilon_2 q)}{2H^{2q+2}} + \frac{A \varepsilon_2}{H^{2q+1}} + \frac{\varepsilon_1}{2p A^{2p}} \]

\[ \frac{\varepsilon_1}{2p (A \cos(T\omega))^{2p}} \frac{A \varepsilon_2 \cos(T\omega)}{H^{2q+1}} + \frac{A^2 \cos(T\omega)^2 (\varepsilon_2 + 2 \varepsilon_2 q)}{2H^{2q+2}} \]

Let \( R(T) = 0 \), we have

\[ \omega = \sqrt{2} \frac{\sqrt{\varepsilon_1}}{A \sin(T\omega)} \sqrt{\frac{1}{2p (A \cos(T\omega))^{2p}}} + \frac{A^2 \varepsilon_2 (1 + 2q)}{2H^{2q+2}} \Xi. \]

where \( \Xi = (A^2 - (A \cos(T\omega))^2) - \frac{\varepsilon_1}{2p (A \cos(T\omega))^{2p}} (A - A \cos(T\omega)) \).

When we let \( \omega = \pi/4 \), we obtain

\[ \omega_{EBM} = \frac{2}{A} \sqrt{\frac{\varepsilon_1}{2p (A^{2p} - 2^{-p} A^{2p})} + \frac{A^2 \varepsilon_2 (1 + 2q)}{4H^{2q+2}} - \frac{\varepsilon_2}{H^{2q+1}} (A - \frac{\sqrt{2}}{2} A)} \]

and then by substituting equation (4.10) into equation (3.6), we have the final result.

In order to conveniently show the movement mechanism of the approximate analytical solution, we take \( A = 1, H = 0.02, a = 0.001, m = 2, p = 0.04, q = 0.25 \).
The application of the frequency–amplitude formulation method with average residuals

According to equation (3.10), we have

\[ R(T) = -A\omega^2 \cos\omega T + \frac{\epsilon_1}{(A \cos(\omega T))^{2q+1}} + \frac{\epsilon_2(2q+1)}{H^{2q+2}} \frac{A \cos(\omega T) - \epsilon_2}{H^{2q+1}} \]  

(4.11)

Then, the average residual is given by

\[ \bar{R} = \frac{T}{4} \int_0^T \left( -A\omega^2 \cos\omega T + \frac{\epsilon_1}{(A \cos(\omega T))^{2q+1}} + \frac{\epsilon_2(2q+1)}{H^{2q+2}} \frac{A \cos(\omega T) - \epsilon_2}{H^{2q+1}} \right) \cos(\omega T) dT \]  

(4.12)

If we take \( \omega_1 = 1, \omega_2 = 2 \), then the residual formulations can be written as

\[ \bar{R}_1 = \frac{2}{\pi} \int_0^{\pi/2} \cos(T) \left( \frac{\epsilon_2}{H^{2q+1}} - \frac{\epsilon_1}{(A \cos(T))^{2q+1}} + A\omega^2 \cos(T) - \frac{A\epsilon_2 \cos(T)(2q+1)}{H^{2q+2}} \right) dT \]

\[ \bar{R}_2 = \frac{4}{\pi} \int_0^{\pi/4} \cos(2T) \left( \frac{\epsilon_2}{H^{2q+1}} - \frac{\epsilon_1}{(A \cos(2T))^{2q+1}} + A\omega^2 \cos(2T) - \frac{A\epsilon_2 \cos(2T)(2q+1)}{H^{2q+2}} \right) dT \]  

(4.13)

Substituting equation (4.13) into equation (3.13), the final frequency \( \omega \) of the approximate analytical solution is obtained. In order to clearly present the movement mechanism of the approximate analytical solution, we also take \( A = 1, H = 0.02, a = 0.001, m = 2, p = 0.04, q = 0.25 \).

Figures 2 and 3 demonstrate the approximate solution of the fractal Fangzhu oscillator with different values of \( \delta \). It can be clearly seen from the figures that when \( \delta \) increases gradually, the wave moves faster while \( \delta \) denotes the fractal dimension. It can be seen that when \( \delta \) gradually decreases, the water movement on the Fangzhu’s surface is very fast.

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**Figure 2.** The movement mechanism of the approximate analytical solution of equations (4.1)–(4.2) by energy balance method with \( \delta = 0.25, 0.5, 0.75, 1 \), respectively.
However, the movement of water molecules on the Fangzhu’s surface is very slowly if \( \delta \) becomes larger. From the above investigation, it can be found that the velocity of water molecules on the Fangzhu’s surface depends on the fractal dimension \( \delta \). This property can be applied to design a device for collecting water in the air or the surface of a waterproof object in outdoor.

**Conclusion**

In this paper, we investigate a fractal model of Fangzhu’s nanoscale surface morphology for water collection by He’s fractal derivative. The two-type approximate analytical solutions of this fractal model are obtained based on the two-scale transform method by the energy balance method and the frequency–amplitude formulation with average residuals, respectively. From the Figures 2 and 3, it is easy to find that as the \( \delta \) increases, the frequency \( \omega \) of movement for the curve becomes smaller, so the fluctuation of the motion track becomes more frequent. In addition, the curves of the approximate analytical solutions obtained by these two methods are very similar, which proves the effectiveness of our two methods. The obtained solutions can reveal how to enhance the collection rate of water by increasing smoothness of the Fangzhu’s surfaces. These simple and effective methods can be used to analyze other nonlinear oscillators with the fractal derivative.

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Notations

\(a, p, q\) constants associated with the surface properties
\(m\) the mass of the molecule
\(H\) the distance between the hydrophilic surface and hydrophobic surface
\(f \delta^\mu u / \mu \delta^\mu t^\delta\) He’s fractal derivative
\(\delta\) fractal dimensional