Radiative neutrino mass in an alternative $U(1)_{B-L}$ gauge symmetry

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Abstract

We propose a neutrino model in which neutrino masses are generated at one loop level and three right-handed fermions have non-trivial charges under $U(1)_{B-L}$ gauge symmetry in no conflict with anomaly cancellation. After the spontaneously symmetry breaking, a remnant $Z_2$ symmetry is induced and plays a role in assuring the stability of dark matter candidate.

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I. INTRODUCTION

Radiatively induced neutrino mass models are attractive candidate to explain the smallness of neutrino masses. In such models, neutrino masses are not allowed at the tree level by some symmetries and they are generated at loop level. Moreover dark matter (DM) candidate can easily be accommodated as a particle propagating inside a loop diagram generating the masses of neutrinos. Based on these ideas, one loop induced neutrino models have widely been studied by a lot of authors; for example, see refs. [1–92]. In addition, refs. [93–96] discuss the systematic analysis of (Dirac) neutrino oscillation, charged lepton flavor violation, and collider physics in the framework of neutrinophilic and inert two Higgs doublet model (THDM), respectively.

In many models, an additional discrete $Z_2$ symmetry has to be imposed in order to forbid tree level masses of neutrinos and to guarantee the stability of DM. However $U(1)_{B-L}$ gauge symmetry can play such a role by taking non-trivial charge assignment of SM gauge singlet fermions as shown in ref. [97], where the lightest neutral particle with non-trivial $U(1)_{B-L}$ charge can be a DM candidate. In this case, its stability is assured by a remnant $Z_2$ symmetry after the spontaneous $U(1)_{B-L}$ symmetry breaking. Thus it is interesting to construct a radiative neutrino mass model based on the alternative charge assignment of $U(1)_{B-L}$.

In this paper, we construct and analyze a model of $U(1)_{B-L}$ with alternative charge assignment, in which neutrino masses are generated at one loop level by introducing some exotic scalar fields. We provide formulas of neutrino mass matrix, decay ratio of lepton flavor violating process and relic density of our DM candidate. Then numerical global analysis is carried out to search for parameter sets that can fit the neutrino oscillation data and satisfy experimental constraints.

This paper is organized as follows. In Sec. II, we show our model, and formulate the neutral fermion sector, boson sector, lepton sector, and dark matter sector. Also we analyze the relic density of DM without conflict of direct detection searches, and carry out global analysis. Finally We conclude and discuss in Sec. III.
II. MODEL SETUP AND PHENOMENOLOGIES

In this section, we introduce our model. First of all, we impose an additional $U(1)_{B-L}$ gauge symmetry with three right-handed neutral fermions $N_{R_i}$ ($i=1-3$), where the right-handed neutrinos have $U(1)_{B-L}$ charge $-4$, $-4$ and 5. Then all the anomalies we have to consider are $U(1)^2_{B-L}$ and $U(1)_{B-L}$, which are found to be zero \[97\]. On the other hand, even when we introduce two types of isospin singlet bosons $\varphi_1$ and $\varphi_2$ in order to acquire nonzero Majorana masses after the spontaneous symmetry breaking of $U(1)_{B-L}$, one cannot find active neutrino masses due to the absence of Yukawa term $\bar{L}_L H N_{R_i}$. Thus we introduce an isospin singlet and doublet inert bosons $s$ and $\eta$ with nonzero $U(1)_{B-L}$ charges, and neutrino masses are induced at one-loop level as shown in Fig. I. Also the stability of DM is assured by a remnant $Z_2$ symmetry after the spontaneous breaking. Field contents and their assignments for fermions and bosons are respectively given by Table I and II. Under these symmetries, the renormalizable Lagrangian for lepton sector and Higgs potential are respectively given by

$$-\mathcal{L}_L = (y_\ell)_{ao} \bar{L}_{La} e_{Ra} H + (y_{\nu})_{ai} \bar{L}_{La} \bar{\nu}_{Bi} N_{R_i} + y_{N_{R_i}} \tilde{N}_{R_i}^C N_{R_j} \varphi_1^* + y_{\lambda_{N}} \tilde{N}_{R_i}^C N_{R_j} \varphi_2^* + \text{c.c.} \quad (II.1)$$

$$V = \mu_H H^\dagger H + \mu_\eta \eta^\dagger \eta + \mu_s s^\dagger s + \mu_{\varphi_1} \varphi_1^* \varphi_1 + \mu_{\varphi_2} \varphi_2^* \varphi_2$$

$$+ \mu (s^\dagger \varphi_2^* + \text{c.c.}) + \lambda_0 (H^\dagger \eta \varphi_1^* + \text{c.c.})$$

$$+ \lambda_H (H^\dagger H)^2 + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_s (s^\dagger s)^2 + \lambda_{\varphi_1} (\varphi_1^* \varphi_1)^2 + \lambda_{\varphi_2} (\varphi_2^* \varphi_2)^2 + \lambda_H \eta (H^\dagger H) (\eta^\dagger \eta)$$

$$+ \lambda_H' (H^\dagger \eta) (\eta^\dagger H) + \lambda_H s (H^\dagger H) (s^\dagger s) + \lambda_H \varphi_1 (H^\dagger H) (\varphi_1^* \varphi_1) + \lambda_H \varphi_2 (H^\dagger H) (\varphi_2^* \varphi_2)$$

$$+ \lambda_{\eta_2} (s^\dagger s) + \lambda_2 \eta_1 (\eta^\dagger \eta) (\varphi_1^* \varphi_1) + \lambda_2 \eta_2 (\eta^\dagger \eta) (\varphi_2^* \varphi_2) + \lambda_s \varphi_1 (s^\dagger s) (\varphi_1^* \varphi_1)$$

$$+ \lambda_s \varphi_2 (s^\dagger s) (\varphi_2^* \varphi_2) + \lambda_{\varphi_1 \varphi_2} (\varphi_1^* \varphi_1) (\varphi_2^* \varphi_2) \quad (II.2)$$

| Fermions   | $Q_L$ | $u_R$ | $d_R$ | $L_L$ | $e_R$ | $N_{R_1}$ | $N_{R_2}$ | $N_{R_3}$ |
|------------|-------|-------|-------|-------|-------|-----------|-----------|-----------|
| SU(3)$_C$  | 3     | 3     | 3     | 1     | 1     | 1         | 1         | 1         |
| SU(2)$_L$  | 2     | 1     | 1     | 2     | 1     | 1         | 1         | 1         |
| U(1)$_Y$   | $\frac{1}{6}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | $-1$ | 0         | 0         | 0         |
| U(1)$_{B-L}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $-1$ | $-1$ | $-4$       | $-4$       | 5         |

TABLE I: Field contents of fermions and their charge assignments under $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where flavor indices are abbreviated.
where $\tilde{H} \equiv (i\sigma_2)H^*$ with $\sigma_2$ being the second Pauli matrix, $(a,b)$ runs over 1 to 3, and $(i,j)$ runs over 1 to 2.

**Scalar sector:** The scalar fields are parameterized as

$$H = \left[ \begin{array}{c} w^+ \\ z^+ \\ \sqrt{2} \end{array} \right], \quad \eta = \left[ \begin{array}{c} \eta^+ \\ \eta_R + i\eta_I \\ \sqrt{2} \end{array} \right], \quad s = \frac{s_R + is_I}{\sqrt{2}}, \quad \varphi_i = \frac{\varphi'_i + \varphi_{R_i} + iz'_\varphi}{\sqrt{2}}, \quad (i = 1, 2),$$

(II.3)

where $w^+$ and $z$ are absorbed by the SM gauge bosons $W^+$ and $Z$, and one of the massless CP odd boson after diagonalizing the matrix in basis of $(z'_{\varphi_1}, z'_{\varphi_2})$ with nonzero VEVs is absorbed by the $B - L$ gauge boson $Z'$. As a result, one physical massless CP-odd Goldstone boson (GB) is induced, which is due to a global symmetry in the scalar potential associated with $\varphi_{1,2}$. Note that we have freedom to identify which component of $(z'_{\varphi_1}, z'_{\varphi_2})$ is the GB, and we choose $G \equiv z'_{\varphi_1}$ to be GB in our analysis. Also we assume that coupling between $G$ and SM Higgs is negligibly small by choosing parameters in scalar potential, and GB does not affect phenomenology; the contribution to relativistic degrees of freedom by GB is also small since it decouples in early stage of the universe due to small interactions. Inserting tadpole conditions, the CP even matrix in basis of $(\varphi_{R_1}, \varphi_{R_2}, h_{R})$ with nonzero VEVs is given by

$$M_R^2 \equiv \begin{bmatrix} 2v_1^2\lambda_{\varphi_1} & v_1'v_2'\lambda_{\varphi_1\varphi_2} & vv_1'\lambda_{H\varphi_1} \\ v_1'v_2'\lambda_{\varphi_1\varphi_2} & 2v_2^2\lambda_{\varphi_2} & vv_2'\lambda_{H\varphi_2} \\ vv_1'\lambda_{H\varphi_1} & vv_2'\lambda_{H\varphi_2} & 2v^2\lambda_H \end{bmatrix},$$

(II.4)

where we define the mass eigenstate $h_i$ ($i = 1 - 3$), and mixing matrix $O_R$ to be $m_{h_i} = O_R M_R^2 O_R^T$ and $(\varphi_{R_1}, \varphi_{R_2}, h_{R})^T = O_R^T h_i$. Here $h_3 \equiv h_{SM}$ is the SM Higgs, therefore, $m_{h_3} = 125$ GeV. In addition, we assume mixing among SM Higgs and other CP-even scalars are small to avoid experimental constraints for simplicity. In the inert sector, we assume the mixing

| Bosons | $H$ | $\eta$ | $s$ | $\varphi_1$ | $\varphi_2$ |
|--------|-----|--------|-----|-------------|-------------|
| $SU(3)_C$ | 1 | 1 | 1 | 1 | 1 |
| $SU(2)_L$ | 2 | 2 | 1 | 1 | 1 |
| $U(1)_Y$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 |
| $U(1)_{B-L}$ | 0 | $-3$ | 4 | 1 | 8 |

TABLE II: Boson sector.
between $s_{R(I)}$ and $\eta_{R(I)}$ is tiny so that we can work the mass insertion approximation below. Thus each of component is given by

$$m_s^2 \equiv m_{s_{R(I)}}^2 \approx \frac{v'^2}{2} \lambda_{\varphi_1} S + v^2 \lambda_{HS} + \lambda_{\varphi_2} S v_2'^2 + 2 \mu_s^2,$$

(II.5)

$$m_{\eta_0}^2 \equiv m_{\eta_{R(I)}}^2 \approx \frac{v'^2}{2} \lambda_{\varphi_1} \eta + v^2 (\lambda_{H\eta} + \lambda'_{H\eta}) + v'_2 \lambda_{\varphi_2} \eta + 2 \mu_{\eta}^2.$$

(II.6)

A. Fermion Sector

The mass matrix for the neutral fermions in basis of $N_{R_1,2,3}$, and given by

$$M_N = \frac{1}{\sqrt{2}} \begin{bmatrix} y'_{N_{11}} v_2' & y'_{N_{12}} v_2' & y_{N_{13}} v'_1 \\ y'_{N_{12}} v_2' & y'_{N_{22}} v_2' & y_{N_{23}} v'_1 \\ y_{N_{13}} v'_1 & y_{N_{23}} v'_1 & 0 \end{bmatrix},$$

(II.7)

where we define this matrix is diagonalized by 3 by 3 orthogonal matrix $V_N$ as $m_{\psi_i} \equiv (V_N M_N V_N^T)_{ij} i = 1 \sim 3$, where $m_{\psi_i}$ is the mass eigenvalue. The mass eigenstates are given by $\psi_i = (V_N)_{ij} N_{R_j}$.

B. Lepton sector and lepton flavor violations

The charged lepton masses are given by $m_\ell = y_\ell v / \sqrt{2}$ after the electroweak symmetry breaking, where $m_\ell$ is assumed to be the mass eigenstate. The neutrino mass matrix is induced at the one-loop level as shown in Fig. 1 and its mass-insertion-approximation form.
is given by
\[
(M_\nu)_{\alpha\beta} = \frac{(\lambda_0 v^\prime)^2 \mu_{\nu A}}{4\sqrt{2}(4\pi)^2} \frac{(Y_\nu)_{\alpha i} M_\psi (Y^{\dagger}_\nu)_i}{m_s^6} F_\nu(r_\eta, r_\psi),
\]
\[
F_\nu(r_1, r_2) = \frac{(1 - r_1)(1 + r_1 - 2r_2)(r_1 - r_2)(1 - r_2) - (1 - r_2)^2[r_2 + r_1(-2r_1 + r_2)] \ln[r_1] + (1 - r_1)^3 r_2 \ln[r_2]}{2(1 - r_1)^3(1 - r_2)^2(r_1 - r_2)^2},
\]
where \( r_i \equiv (m_i/m_s)^2 \), \( (Y_\nu)_{\alpha i} \equiv \sum_{j=1}^3 \frac{(y_\nu)_{\alpha j} (Y^{\dagger}_\nu)_{ji}}{\sqrt{2}} \). Once we define \( D_\nu \equiv U_{MNS} M_\nu U^{T}_{MNS} \equiv U_{MNS} (Y_\nu R^{Y^T}_{\nu}) U^{T}_{MNS} \), \( Y_\nu \) can be rewritten in terms of observables and several arbitral parameters as:
\[
Y_\nu = U_{MNS}^1 D_0^{1/2} OR^{-1/2}, \quad R \equiv \frac{(\lambda_0 v^\prime)^2 \mu_{\nu A} M_\psi}{4\sqrt{2}(4\pi)^2 m_s^6} F_\nu(r_\eta, r_\psi),
\]
where \( O \equiv O(\theta_1, \theta_2, \theta_3) \), satisfying \( OO^T = 1 \), is an arbitral 3 by 3 orthogonal matrix with complex values, and \( U_{MNS} \) and \( D_\nu \) are measured in [98].

**Lepton flavor violations (LFVs):** LFV processes \( \ell \to \ell'\gamma \) are induced from the neutrino Yukawa couplings at one-loop level, and their forms are given by
\[
\text{BR}(\ell_\alpha \to \ell_\beta\gamma) \approx \frac{4\pi^3 \alpha_{em} C_{\alpha\beta}}{3(4\pi)^4 G_F^2} \left| \sum_{i=1}^3 (Y^{\dagger}_\nu)_i \frac{(Y^{\dagger}_\nu)_{\beta i}}{(Y^{\dagger}_\nu)_i} F_{lfv}(\psi_i, \eta^\pm) \right|^2 ,
\]
where \( \alpha_{em} \approx 1/137 \) is the fine-structure constant, \( G_F \approx 1.17 \times 10^{-5} \) GeV\(^{-2} \) is the Fermi constant, and \( C_{21} \approx 1, C_{31} \approx 0.1784, C_{32} \approx 0.1736 \). Experimental upper bounds are found to be [99, 100]:
\[
\text{BR}(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13}, \quad \text{BR}(\tau \to e\gamma) \lesssim 3.3 \times 10^{-8}, \quad \text{BR}(\tau \to \mu\gamma) \lesssim 4.4 \times 10^{-13},
\]
where we define \( \ell_1 \equiv e \), \( \ell_2 \equiv \mu \), and \( \ell_3 \equiv \tau \). Notice here that muon \( g - 2 \) is negatively induced that conflicts with the current experimental data.

**C. Dark matter**

In our scenario, we will focus on the lightest inert fermion \( \psi_1 \) as the DM candidate, defining \( \psi_1 \equiv X \) and \( M_{\psi_1} \equiv M_X \). Note also that we can have bosonic DM candidate
although we omit the discussion in this paper \(^1\). Firstly, we assume contribution from the Higgs mediating interaction is negligibly small and DM annihilation processes are dominated by the gauge interaction with \(Z'\); we thus can easily avoid the constraints from direct detection searches such as LUX \([101]\).

Relic density: We have annihilation modes with Yukawa and kinetic terms to explain the relic density of DM: \(\Omega h^2 \approx 0.12 \,[102]\), and their relevant Lagrangian in basis of mass eigenstate is found to be

\[
-\mathcal{L} = (Y_\nu)_{a1} \bar{\nu}_a P_R X (\eta_R - i \eta_I) + \sqrt{2} (Y_\nu)_{a1} \bar{\ell}_a P_R X \eta^- + \text{c.c.}, \tag{II.13}
\]

\[
+ i \frac{g_{BL}}{2} [-4 + 9 (V_N^\ast)_{13} (V_N^T)_{31}] \bar{X} \gamma^\mu \gamma^5 X Z'_\mu
\]

\[
+ g_{BL} Q_{B-L}^f \bar{f}_{SM} \gamma^\mu f_{SM} Z'_\mu - i g_{BL} \bar{\nu} \gamma P_L \nu Z'_a \tag{II.14}
\]

where we have used the unitary condition; \(V_N^\dagger V_N = 1\) in the central line above, \(g_{BL}\) is \(B-L\) gauge coupling, \(Q_{B-L}^f\) is the charge of \(B-L\) symmetry, and \(f_{SM}\) is all the fermions of SM. Then the squared amplitude for the process \(X \bar{X} \to \ell_a \bar{\ell}_b\), which consists of \(s, t, u\)-channels, is given by

\[
|\mathcal{M}(X \bar{X} \to \ell_a \bar{\ell}_b)|^2 \approx 4 \left| \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{t - m_{\eta^\pm}^2} \right|^2 (p_1 \cdot k_1) (p_2 \cdot k_2) + 4 \left| \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{u - m_{\eta^\pm}^2} \right|^2 (p_1 \cdot k_2) (p_2 \cdot k_1)
\]

\[
- \frac{1}{8} \left[ \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{t - m_{\eta^\pm}^2} \frac{(Y_\nu^\dagger)_{1a} (Y_\nu b_1)}{u - m_{\eta^\pm}^2} \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{t - m_{\eta^\pm}^2} \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{u - m_{\eta^\pm}^2} \right] M_X^2 (k_1 \cdot k_2)
\]

\[
+ 2 \delta_{ab} \left[ \frac{g_{BL}^2}{s - m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}} \right]^2 \left[ \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{t - m_{\eta^\pm}^2} \frac{(Y_\nu^\dagger)_{1a} (Y_\nu b_1)}{u - m_{\eta^\pm}^2} \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{t - m_{\eta^\pm}^2} \frac{(Y_\nu)_{a1} (Y_\nu^\dagger)_{1b}}{u - m_{\eta^\pm}^2} \right] M_X^2 (k_1 \cdot k_2) 
\]

\[
G(M_X, m_{Z'}, \{p_i, k_i\}) \equiv (p_1 \cdot k_1) (p_2 \cdot k_2) + (p_1 \cdot k_2) (p_2 \cdot k_1) + (M_X^2 - p_1 \cdot p_2) (k_1 \cdot k_2 - s) - \frac{s}{2} (p_1 \cdot p_2) \left( \frac{2 m_{Z'}^2 s}{m_{Z'}^2} \right) [(p_1 \cdot k_1) + (p_2 \cdot k_2) + (p_1 \cdot k_2) + (p_2 \cdot k_1) - s], \tag{II.16}
\]

where \(p_{1,2}(k_{1,2})\) denote initial(final) state momentum, \(\Gamma_{Z'}\) is the total decay width of \(Z'\), and masses of charged leptons are neglected. The inner products of momentum such as \((p_1 \cdot p_2)\) are straightforwardly obtained where the details are found in Ref. \([103]\). The decay width

\(^1\) The bosonic DM candidate has been discussed in ref. \([97]\).
of \( Z' \) is given by

\[
\Gamma_{Z'} = \frac{g_{BL}^2 m_{Z'}}{12\pi} \sum_f N_c^f C_f (Q_{B-L}^f)^2 \left( 1 + \frac{2m_f^2}{m_{Z'}^2} \right) \sqrt{1 - \frac{4m_f^2}{m_{Z'}^2}},
\]

where we assume \( Z' \) decays into only SM fermions \( f \), \( N_c^f \) is color factor, and \( C_f = 1/2 \) for neutrino while \( C_f = 1 \) for the other fermions. Note also that \( Z' \) mass is given by

\[
m_{Z'} = g_{BL} \sqrt{(v_1')^2 + (8v_2')^2}.
\]

Similarly we obtain the squared amplitude for the process \( XX \to \ell_a \bar{\ell}_b \) as follows:

\[
|\tilde{M}(XX \to \nu_a \bar{\nu}_b)|^2 \approx 4 \left| \frac{(Y_\nu)_{a1} (Y_{\nu}^\dagger)_{b1}}{t - m_{\nu_0}^2} \right|^2 (p_1 \cdot k_1)(p_2 \cdot k_2) + 4 \left| \frac{(Y_\nu)_{a1} (Y_{\nu}^\dagger)_{b1}}{u - m_{\nu_0}^2} \right|^2 (p_1 \cdot k_2)(p_2 \cdot k_1)
- \frac{1}{8} \left( \frac{(Y_\nu)_{a1} (Y_{\nu}^\dagger)_{b1}}{t - m_{\nu_0}^2} \frac{(Y_{\nu}^\dagger)_{a1} (Y_\nu b_1)}{u - m_{\nu_0}^2} \right) M_X^2 (k_1 \cdot k_2)
+ 2\delta_{ab} g_{BL}^2 \left[ \frac{-4 + 9(V_N^*)_{13} (V_N^T)^*_{31}}{s - m_{Z'}^2 + im_{Z'} \Gamma_{Z'}} \right]^2 G(M_X, m_{Z'}, \{p_i, k_i\}),
\]

where we also assume massless final states for neutrinos. In addition the squared amplitude for the process \( XX \to q_a \bar{q}_a \), which consists of \( s \)-channel, is given by

\[
|\tilde{M}(XX \to q_a \bar{q}_a)|^2 \approx \frac{2}{3} \left| \frac{g_{BL}^2 \left[ -4 + 9(V_N^*)_{13} (V_N^T)^*_{31} \right]}{s - m_{Z'}^2 + im_{Z'} \Gamma_{Z'}} \right|^2 G(M_X, m_{Z'}, \{p_i, k_i\}),
\]

The relic density of DM is then given by

\[
\Omega h^2 \approx \frac{1.07 \times 10^9}{\sqrt{g_\ast(x_f) M_{Pl} J(x_f)[GeV]}},
\]

where \( g_\ast(x_f) \approx 25 \) is the degrees of freedom for relativistic particles at temperature \( T_f = M_X/x_f \), \( M_{Pl} \approx 1.22 \times 10^{19} \text{ GeV} \), and \( J(x_f) \equiv \int_{x_f}^{\infty} dx \left( \frac{\sqrt{s}}{x^2} \right) \) is given by

\[
J(x_f) = \int_{x_f}^{\infty} dx \left[ \int_{4M_X^2}^{\infty} ds \frac{\sqrt{s - 4M_X^2} W(s) K_1 \left( \sqrt{s}/M_X \right)}{16M_X^2 x [K_2(x)]^2} \right],
\]

\[
W(s) \approx \frac{1}{16\pi} \sum_a \int_0^\pi \sin \theta \left| \tilde{M}(XX \to \nu_a \bar{\nu}_a) \right|^2 + \left| \tilde{M}(XX \to \ell_a \bar{\ell}_a) \right|^2
+ \left| \tilde{M}(XX \to q_a \bar{q}_a) \right|^2 \sqrt{1 - \frac{4m_{\nu_0}^2}{s}},
\]

where we implicitly impose the kinematical constraint above. Notice that we relax the allowed range of relic density of DM; \( 0.1 \lesssim h^2 \Omega \lesssim 0.15 \), instead of the exact value in the numerical analysis below.
FIG. 2: The correlation between $m_{Z'}$ and $g_{BL}$ for the allowed sampling points.

D. Numerical analysis

Here we show the global analysis to satisfy the neutrino oscillation data, LFVs, and the correct relic density of DM. In our analysis, we generate 5 million sampling points by randomly selecting the following input regions:

\[
\begin{align*}
\{v_1', v_2'\} &\in [5.0, 15] \text{ TeV}, \quad \mu \in [10, 100] \text{ GeV}, \quad \lambda_0 \in [0.001, 0.1], \\
\theta_{1,2,3} &\in [0.1i, \pi + 10i], \quad (M_N)_{ab} \in [1.0, 10] \text{ TeV}, \quad m_{Z'} \in [0.995 \times 2M_X, 1.005 \times 2M_X], \\
(m_{\eta_0}, m_s) &\in [0.5, 5.0] \text{ TeV}, \quad m_{\eta^\pm} \in [m_{\eta_0}, m_{\eta_0} + 60] \text{ GeV},
\end{align*}
\]

where we assume $2M_X \simeq m_{Z'}$ to obtain the correct relic density \(^2\), and the last range comes from the S-T parameter of the isospin doublet boson \([106]\). The selected parameter sets are applied to our formulas of neutrino mass matrix, LFVs and relic density, and we search for the sampling points which satisfy all the constraints. Note that we also impose the perturbative limit: $Y_\nu \lesssim \sqrt{4\pi}$ and $1.2M_X \lesssim m_{\eta_0}, m_s$ in order to evade the coannihilation processes for simplicity. As a result, we obtain 152 parameter sets which can fit neutrino oscillation data and relic density while satisfying constraints from LFVs.

The fig. 2 shows the correlation between $g_{BL}$ and $m_{Z'}$ in our allowed parameter sets. We find that $Z'$ with $m_{Z'} \lesssim 300 \text{ GeV}$ is preferred to explain DM relic density since s-wave contribution is suppressed. The $Z'$ production cross section is marginal of current constraints from the LHC experiments in the values of $\{g_{BL}, m_{Z'}\}$ in the figure. For example the cross

\(^2\) Even when the wider range of $m_{Z'}$ is taken, the situation does not change much.
section for the process \( pp \rightarrow Z' \rightarrow \ell^+\ell^- (\ell = e, \mu) \) is estimated to be around 0.01 pb for \( g_{BL} \sim 0.006 \) and \( m_{Z'} \sim 300 \text{ GeV} \) using CalcHEP \([107]\), which is close to the current limit \([108, 109]\). Thus our \( Z' \) will be tested in near future at the LHC with more integrated luminosity.

III. CONCLUSION

We have proposed a model providing the neutrino mass and mixing at one loop-level with a nontrivial \( U(1)_{B-L} \) gauge symmetry based on the model proposed by \([97]\), in which the remnant \( Z_2 \) symmetry still be there even after the spontaneous symmetry breaking of \( U(1)_{B-L} \), and a fermionic DM candidate has been discussed instead of bosonic one. Then we have given formulas for neutrino mass matrix, branching ratio of \( \ell \rightarrow \ell'\gamma \) and relic density of DM. Notice that, in our model, a physical GB appears, which is the consequence of two kinds of bosons \( \varphi_1 \) and \( \varphi_2 \) to break \( U(1)_{B-L} \) and obtain neutrino mass. However since it does not couple to any SM fields as well as DM directly, GB does not affect our phenomenology.

Numerical global analysis has been carried out to search for parameter sets which satisfy all the relevant constraints such as neutrino oscillation data, lepton flavor violations, and the relic density of DM. In our scenario, the dominant contribution to DM annihilation process for the relic density arises from the gauge interaction of \( Z' \) where the condition \( m_{Z'} \sim 2m_{DM} \) is required to realize sufficiently large annihilation cross section. On the other hand, the Yukawa contribution could not be dominant due to the constraints of lepton flavor violations.

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