Abstract: This note emphasizes some flaws of the Panzar–Rosse H statistic to address market power. First, I show that it is more related to the pass-through rate than to a market power measure, which implies reconsidering its interpretation and use. Second, I show that the inclusion of other strategic variables rather than prices or quantities may lead to contradictory predictions about market power. Therefore, competition authorities should refrain from using the H statistic to rule out significant market power because it is not a well-behaved measure of the degree of competition.

Keywords: H statistic, market power, Panzar–Rosse, pass-through rate

JEL Codes: L11, L12, L41

1 Introduction

The Panzar–Rosse model builds a competition indicator (H statistic) that provides a quantitative assessment of the competitive nature of a market. The H statistic is the sum of equilibrium gross revenue elasticities with respect to each input price. Panzar and Rosse (1987) showed that this measure systematically varies with different modes of conduct (monopoly, oligopoly, etc.). Following Panzar and Rosse (1987), a positive H statistic has been considered sufficient to rule out market power. In fact, there is a widespread belief that monopolists always have a non-positive H statistic, see (Noman et al. 2017), or (Camino-Mogro and Armijos-Bravo 2018). However, Shaffer (2002) found empirical evidence that highlighted the
possibility of positive H statistics in banking monopolies. More recently, Shaffer and Spierdijk (2015) have computed the H statistic in different competitive environments (Stackelberg, Cournot, and Bertrand games) and have concluded that it is possible to find either positive and negative H statistics in theoretical models, which questions the idea of using the sign of the H statistic as a test to rule out significant market power. In this regard, to the best of my knowledge, the only work that has addressed a theoretical relationship between the H statistic and a measure of market power is Shaffer (1983), who proves that it is possible to derive the Lerner index as a function of the H statistic. However, this work supports the original view that the H statistic may be a well-behaved measure of market power. The goal of this note is to provide a rationale for those conflicting results regarding the sign of the H statistic.

First, following Shaffer (1983), I show that, depending on whether the monopoly maximizes profits over quantities or prices, the relationship of the H statistic with the Lerner index seems to be different. This difference arises because the H statistic is a pass-through rate, which expression depends on whether we pay attention to marginal revenues or marginal units. Although in equilibrium, the H statistic is the same independently of whether the monopoly maximizes profits over quantities or prices, it has no stable relationship with the Lerner index. Second, to prove the previous point, I extend the Shaffer’s model to consider both price and non-price variables. I find that the non-price dimension may lead to rejecting the hypothesis of significant market power in monopolistic frameworks. These results contrast with the widespread belief that the monopolist always has a non-positive H statistic and provide a rationale for Shaffer (2002)’s results, in which monopolies may have positive H statistics. Therefore, the policy conclusion is to avoid using the H statistic as a market power measure. We cannot rule out the monopoly case just because we find a positive H statistic, a significant market power is compatible with positive H statistics.

2 The Panzar–Rosse H Statistic and the Lerner Index

Panzar and Rosse (1987) develop a very general test for monopoly that is suitable to investigate the performance of markets using firm and industry-level data. They defined the H statistic as the sum of equilibrium gross revenue elasticities with respect to each input price. Formally,
\[
\psi = \frac{w}{R(w, \rho)} \frac{\partial R(w, \rho)}{\partial w} + \frac{\rho}{R(w, \rho)} \frac{\partial R(w, \rho)}{\partial \rho}
\]

where, \(w\) and \(\rho\) are input prices, and \(R(w, \rho)\) denotes total revenues. Intuitively, \(\psi\) measures the impact of a proportional increase in all factor prices. Suppose all factor prices rise by 1%. We know from duality theory that a 1% increase in all factor prices leads to a 1% upward shift in all of the firm’s cost curves. Panzar and Rosse highlight that in a long-run perfect competition framework, such a 1% increase in costs leads to a 1% increase in revenues, which leads to \(\psi = 1\). In the monopoly scenario, they claim that the impact of such an increase in all factor prices may reduce revenues \((\psi \leq 0)\). Therefore, the H statistic answers the question what will happen to the revenues if the costs increase. The attractiveness of the H statistic precisely relies on requiring less data than the Lerner index to estimate. In fact, it only requires data on revenue and factor prices, in contrast with the Lerner index, that requires cost data too, and given that only the monopoly has \(\psi \leq 0\), it could be used as a competition indicator. Panzar and Rosse (1987) established that, in a monopoly, the \(\psi\) is non-positive. Therefore, if hypothesis \(\psi \leq 0\) is rejected, it rules out the monopoly model.

Interestingly, we can establish a relationship between the H statistic and the Lerner index. To show this relationship, let us consider the classical monopoly case in which the firm faces an inverse demand \(p(q)\), and has a marginal cost \(c\). Let \(R(q) = p(q)q\) be the revenue of a monopoly and \(C(q, w, \rho)\) be its total costs, where \(p\) is the price, \(q\) the output, \(w\) the wage rate, and \(\rho\) the rental rate of capital. In this case, the Lerner index is the inverse of the price elasticity \((\epsilon)\), \(L = \frac{p-c}{p} = \frac{1}{|\epsilon|}\). Therefore, we can rewrite Eq. (1) as,

\[
\psi = \frac{w}{R(w, \rho)} \frac{\partial R(w, \rho)}{\partial q} \frac{\partial q}{\partial w} + \frac{\rho}{R(w, \rho)} \frac{\partial R(w, \rho)}{\partial q} \frac{\partial q}{\partial \rho}
\]

Using the first-order and second-order conditions for profit maximization, Shaffer (1983) shows that \(\frac{\partial q}{\partial w} = \frac{c_{qw}}{\pi_{qq}}\) and \(\frac{\partial q}{\partial \rho} = \frac{c_{q\rho}}{\pi_{qq}}\). By substituting these expressions into Eq. (2), and making use of the homogeneity of degree 1 of the cost function, Shaffer proves that the H statistic can be expressed as

\[
\psi = \frac{(R_q)^2}{R\pi_{qq}}
\]

1 Given that the monopoly will never be on the inelastic part of the demand curve.
2 This proof is extracted from (Shaffer 1983).
3 We assume the cost function is homogeneous of degree 1. Therefore, we keep the original assumptions of (Panzar and Rosse 1987; Shaffer 1983) throughout the paper.
To establish a relationship with the Lerner index, we need to derive the monopoly’s marginal revenue and the derivative of the marginal profit. The marginal revenue is $R_q = p + q \frac{dp}{dq} = p\left(1 - \frac{1}{e}\right)$ and the marginal profit is $\pi_q = p\left(1 - \frac{1}{e}\right) - C_q = 0$, the derivative of which is $\pi_{qq} = L(L - 1)p + \frac{p}{e} \frac{d}{dq} - c_{qq}$, where $L$ is the Lerner index. If we substitute in $\psi$ and assume that the price elasticity of demand and marginal costs are constant, we have

$$\psi = \frac{p}{q\pi_{qq}} (1 - L^2) = 1 - \frac{1}{L} \tag{4}$$

This simple relationship shows that if the Lerner index tends to one (monopoly), $\psi$ tends to zero, and when the Lerner index tends to zero (perfect competition), the $\psi$ tends to $-\infty$. Interestingly, Shaffer shows that the H statistic monotonically increases in the market power measured by the Lerner index. Given a monopolistic framework, it will never be possible to find a positive H statistic under the assumption of constant marginal costs and elasticities. This result highlights the idea of Panzar and Rosse (1987) that a monopolist must have a non-positive $\psi$. Traditionally, it has been considered that rejecting $\psi \leq 0$ was enough to rule out the monopoly case, and higher H statistics were associated with more competitive markets, see (Noman et al. 2017), or (Camino-Mogro and Armijos-Bravo 2018). Currently, even the World Bank shares this view. As Shaffer and Spierdijk (2015) point out, it is widely believed that $\psi > 0$ is inconsistent with significant market power.

### 3 Issues with the Interpretation of the Panzar–Rosse H Statistic

Let us keep the previous assumption of the classical monopoly case, but this time we consider prices as the strategic variable instead of quantities. Therefore, the monopoly faces a well-behaved demand function $q(p)$. In this case, the Lerner index is also the inverse of the price elasticity ($e$), $L = \frac{\text{P} \cdot \text{C}}{p} = \frac{1}{\text{|e|}}$, so we can focus on the derivation of the H statistic. Following Shaffer (1983), let $R(q) = pq(p)$ be the revenue of a monopoly and $C(q(p), w, \rho)$ be its total costs. Therefore, we rewrite Eq. (1) as
Using the first-order and second-order conditions for profit maximization,

\[
\frac{\partial P}{\partial w} = \frac{C_{P,w}}{\Pi_{PP}} \quad \text{and} \quad \frac{\partial P}{\partial \rho} = \frac{C_{P,\rho}}{\Pi_{PP}}
\]

By substituting these expressions into Eq. (5), and making use of the homogeneity of degree 1 of the cost function, the \( H \) statistic is

\[
\psi = \left( \frac{R_p}{\Pi_{PP}} \right)^2
\]

This expression differs from Shaffer (1983) because in Eq. (7), \( \frac{\partial R}{\partial Q} \) occurs instead of \( \frac{\partial R}{\partial P} \) and \( \Pi_{PP} \) instead of \( \Pi_{QQ} \). Assuming that elasticities and marginal costs are constant, the \( H \) statistic is

\[
\psi = L - 1
\]

Interestingly, the relationship between the Lerner index and the \( H \) statistic seems to depend on whether the monopoly chooses prices or quantities. Note that, only if \( L = 1 \), the \( H \) statistic is zero, independently of whether the firm chooses prices or quantities. However, this apparent difference is a consequence of the nature of the \( H \) statistic as a pass-through rate. To understand this point, we have to look at Eqs. (2) and (5). Note that, in Eq. (5), \( \frac{\partial p}{\partial w} \) is the price pass-through rate and, in Eq. (2), \( \frac{\partial q}{\partial w} \) is the quantity pass-through rate. The difference between both expressions is because we consider two different measures. In Eq. (5), we pay attention to marginal revenues, and in Eq. (2) to marginal units. This difference between quantity and price pass-through rates is also found in other works, such as Weyl and Fabinger (2013), and Eqs. (2) and (5) highlight the pass-through nature of the \( H \) statistic. Therefore, this result points out that we should primarily interpret the \( H \) statistic as a pass-through rate.

However, the difference between Eqs. (2) and (5) does not imply that both measures are different in equilibrium, in fact, they must be equal. This difference implies that there is no stable relationship between the \( H \) statistic and market

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5 Both expressions are related by \( \frac{\partial p}{\partial w} = p'(q) \frac{\partial q}{\partial w} \), but with opposite signs. On the other hand, note that \( \frac{\partial R(w,\rho)}{\partial p} \) and \( \frac{\partial R(w,\rho)}{\partial q} \) have opposite signs. Formally, \( \frac{\partial R}{\partial q} = p \left( 1 - \frac{1}{|\epsilon|} \right) \) and \( \frac{\partial R}{\partial p} = q \left( 1 - |\epsilon| \right) \).

6 A property that also has the pass-through rate, see (Weyl and Fabinger 2013).
power. As Bulow and Pfleiderer (1983) show, the pass-through rate can also be re-stated in terms of the elasticity of demand \( \left( \frac{\partial p(q)}{\partial c} = \frac{e}{e+1+\frac{e^2}{2}} \right) \), and under constant elasticity of demand and marginal costs, the pass-through rate is equal to 1/2. By substitution, it is straightforward to show that the elasticity of demand must be unitary, which is the same condition that makes Eqs. (4) and (8) equal. However, this result does not mean that the elasticity must always be equal to one, but rather that the pass-through rate and the H statistic are fixed regardless of the elasticity (or the Lerner index) when we assume constant elasticities and marginal costs. Therefore, this relationship with the pass-through rate implies that the H statistic is not a well-behaved measure of market power as the pass-through literature shows, see (Weyl and Fabinger 2013), but it also shows that we should only interpret the H statistic as a pass-through rate focused on revenues.

As a summary, the apparent differences between the H statistic when it is computed in terms of prices or quantities are a consequence of its nature as a pass-through rate. The pass-through rate has no stable relationship with market power because different factors influence it, such as the shape of the demand or cost functions, see (Weyl and Fabinger 2013), and the H statistic suffers the same problems. In the following section, to prove that the sign H statistic should not be used to address market power, we address how different factors lead to contradictory predictions about market power.

### 4 Issues with the Sign of the Panzar–Rosse H Statistic

Let us assume a monopoly that faces a linear demand, \( P = a - bQ \) with \( a, b > 0 \) and maximizes its profits with respect to prices. Formally

\[
\max_{P} \pi = (P - c)Q(P)
\]

which leads to \( P^* = \frac{a+c}{2} ; \ Q^* = \frac{a-c}{2b} \). In terms of the H statistic, given that this case violates the constant elasticity assumption of Eqs. (4) and (8), let us use Eq. (7) or Eq. (3) to compute the H statistic. In this case, we have, \( \psi = \frac{2c^2}{-a^2+b^2} \), which is negative.

Let us extend the previous model, but this time, the firm sets prices and a non-price variable (\( \delta \)) that also influences the demand positively, \( Q(P, \delta) \). Additionally, the non-price variable \( \delta \) has a twice differentiable cost, \( F(\delta) \). In this case, and following the same procedure as Shaffer (1983), we can use the H statistic of Eq. (5), but considering the effects of the non-price variable, \( \frac{\partial R(w,\rho)}{\partial w} = \frac{\partial R(w,\rho)}{\partial P} \left( \frac{\partial P}{\partial w} + \frac{\partial P}{\partial \delta} \frac{\partial \delta}{\partial w} \right) \).
and \( \frac{\partial R(w, \rho)}{\partial \rho} = \frac{\partial R(w, \rho)}{\partial P} \left( \frac{\partial P}{\partial \rho} + \frac{\partial P}{\partial \delta} \frac{\partial \delta}{\partial \rho} \right) \). This time, using the first-order and second-order conditions for profit maximization,

\[
\begin{align*}
\left( \frac{\partial P}{\partial w} + \frac{\partial P}{\partial \delta} \frac{\partial \delta}{\partial w} \right) &= \frac{C_{P, w}}{\Pi_{PP}} \\
\left( \frac{\partial P}{\partial \rho} + \frac{\partial P}{\partial \delta} \frac{\partial \delta}{\partial \rho} \right) &= \frac{C_{P, \rho}}{\Pi_{PP}}
\end{align*}
\]

(9)

Interestingly, the inclusion of new strategic variables may modify the sign of the \( H \) statistic. In the previous section, we have \( \frac{\partial P}{\partial w} \), which is positive. But this time, we have an extra parameter, the cross-effect of the non-price variable on prices, \( \frac{\partial P}{\partial \delta} \frac{\partial \delta}{\partial w} \), which may be negative.\(^7\) If those cross-effects are larger than the direct effect of input prices on output prices, the sign of \( \frac{C_{P, w}}{\Pi_{PP}} \) may change, which changes the sign of the \( H \) statistic.

However, the inclusion of new variables does not only alter the \( H \) statistic via prices, but it also has a direct impact. In fact, the \( H \) statistic can be written as follows,

\[
\psi = \frac{w}{R} \left[ \frac{\partial R}{\partial \delta} \frac{C_{\delta, w}}{\Pi_{\delta \delta}} + \frac{\partial R}{\partial P} \frac{C_{P, w}}{\Pi_{PP}} \right] + \rho \left[ \frac{\partial R}{\partial \delta} \frac{C_{\delta, \rho}}{\Pi_{\delta \delta}} + \frac{\partial R}{\partial P} \frac{C_{P, \rho}}{\Pi_{PP}} \right]
\]

(10)

by using the property of homogeneity of degree 1 \( (wC_{\delta, w} + \rho C_{\delta, \rho} = C_{\delta}, wC_{P, w} + \rho C_{P, \rho} = C_P) \), and the equilibrium condition \( R_P = C_P \) and \( R_\delta = C_\delta \), I rewrite Eq. (10),

\[
\psi = \left( \frac{\partial R}{\partial \delta} \right)^2 \frac{1}{R \Pi_{\delta \delta}} + \left( \frac{\partial R}{\partial P} \right)^2 \frac{1}{R \Pi_{PP}}
\]

(11)

The sign of each element in Eq. (11) depends on the direct and indirect effects, which may alter the sign of the \( H \) statistic. This especially relevant when we consider a multi-product firm. If, for example, we assume the following model, \( \Pi(P, v, \delta) = (P - c)Q(P, \delta) + \sum_{j=1}^{n} (v_j - d_j)q_j(Q, v, \delta) - F(\delta) \), where \( v=(v_1, \ldots, v_n) \), Eq. (11) becomes,

\[
\psi = \left( \frac{\partial R}{\partial \delta} \right)^2 \frac{1}{R \Pi_{\delta \delta}} + \left( \frac{\partial R}{\partial P} \right)^2 \frac{1}{R \Pi_{PP}} + \sum_{j} \left( \frac{\partial R}{\partial v_j} \right)^2 \frac{1}{R \Pi_{v_j v_j}}
\]

(12)

As before, the inclusion of new items should consider the direct and cross-relationships among them, which may modify the sign of the \( H \) statistic. To

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\(^7\) Intuitively, if we consider that the non-price variable is a quality factor, an increase in input prices reduces the quantity of quality supplied, \( \frac{\partial \delta}{\partial w} < 0 \). On the other hand, an increase in quality may increase the price of the item, \( \frac{\partial R}{\partial \delta} > 0 \).
illustrate the influence of these new effects, let us consider a linear demand framework, and let us introduce a costly non-price variable ($\delta$), where $F(\delta) = \delta$ is the cost. In this case, the demand that faces the firm becomes $Q = \frac{a}{b} - \frac{Pb}{b_0}$, and we keep the assumption of a price-setter firm. Formally,

$$\max_{P,\delta} \pi = (P - c)Q(P, \delta) - \delta$$

Intuitively, this case may represent a monopoly that uses persuasive advertising ($\delta$). The monopoly can invest in advertising to modify the price perception, in this way, the monopoly can “hide” increases in prices. In this case, the optimal price and quantity are $P^* = \frac{ac}{2\sqrt{a^2 - 4b}} + \frac{c}{2}$; $Q^* = \frac{a - \sqrt{a^2 - 4b}}{2b}$. Interestingly, in contrast with the case without the non-price variable, the optimal demand is not influenced by the marginal cost. The intuition is the following. With persuasive advertising, the monopoly may use it to mitigate the impact that such marginal costs have on the price by making consumers less price-sensitive. Therefore, the price rises, but demand remains equal. However, although the revenues increase, the costs also increase because producing the item and advertising are costly activities.

Let us compute the $H$ statistic. For simplicity’s sake and without loss of generality, instead of using Eq. (11), which is analytically complex in this case, let us compute the $H$ statistic as

$$\psi = \frac{MC}{R} \frac{\partial R}{\partial MC}$$

where, $R$ is the total revenues and $MC$ the marginal costs. The sign of the $H$ statistic depends on $\frac{\partial R}{\partial MC}$, which is strictly positive in our example. Therefore, we have a monopoly with a positive $H$ statistic. A contradiction with the common belief that a positive $H$ statistic rules out monopoly power.

However, other factors may lead to a positive $H$ statistic in monopolist frameworks as well. For example, demand curvature plays a key role in the pass-through rate. Therefore, the presence of convex demands may lead to systematic positive $H$ statistics. This point can be set out graphically. Following Weyl and Fabinger (2013), in Figure 1, we compare a linear and a convex demand. The initial profit-maximizing monopoly outcome is equal for both demand curves, $p^*$ and $q^*$. At this point, their slopes are also equal; therefore, their marginal revenues are also equal. Suppose that there is a cost shock that shifts quantity from $q^*$ to $q^{**}$. In the linear demand case, at this output level, the optimal price will be $p^{***}$, but under a convex demand, the price will be $p^{***}$. Thus, revenues may increase after a cost

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8 This expression is equivalent to Eq. (1) as Shaffer and Spierdijk (2015) proved, but it is more tractable for our purposes in this example.
shock, given that with convex demands, revenues attenuate more slowly as output declines, which may lead to positive H statistics.

Throughout this note, we have addressed the monopoly case but, if we consider oligopoly models, the problem of measuring the degree of competition may worsen as a consequence of the pass-through nature of the H statistic. For example, Weyl and Fabinger (2013) show that the relevance of the demand curvature in the pass-through rate depends on the conduct parameter. The curvature is irrelevant in perfect competition, and it increases its importance as the competition is reduced. It implies that the influence of the curvature is not constant, and it may vary depending on the different modes of conduct, which complicates the assessment of the degree of competition. In other words, in oligopolistic settings, the larger the market power of oligopolists, the larger the influence of demand curvature on the pass-through rate, and the more likely that that the H statistic does not reflect market power.

Another example of the disconnection between market power and the pass-through rate is provided by Zimmerman and Carlson (2010). They show that in a differentiated Cournot competition with symmetric costs, the firm-specific pass-through rate is increasing in the number of competitors, which follows the original idea of Panzar and Rosse that, as we move towards the perfect competition, costs tend to be passed through to prices. However, Zimmerman and Carlson (2010)

Figure 1: Linear and convex demands.
prove that, with asymmetric costs, the pass-through rate is initially decreasing in the number of firms, but rises after hitting a critical minimum value. But in the case of differentiated Bertrand competition with asymmetric costs, the firm-specific pass-through rate is monotonically decreasing in the number of firms. Thus, given the pass-through nature of the H statistic, we cannot expect a stable relationship between different modes of conduct and degree of competition measured by the H statistic.

Altogether, a priori, it is not recommended to rule out the monopoly case just because we observe \( \psi > 0 \). Extracting any conclusion about competition from the sign of the H statistic would be incorrect. These results, in combination with those of Shaffer and Spierdijk (2015) and Weyl and Fabinger (2013), should discourage the use of the H statistic to address the presence of market power. In the best-case scenario, the use of the H statistic should be limited to an indicator of the impact of factor prices on revenues. In other words, as a marginal revenue pass-through rate.

5 Policy Implications

The H statistic is used as a measure of the degree of competition. It measures the elasticity of revenues relative to input prices. It is widely believed that a positive H statistic is enough to rule out significant market power. In this note, I show two problems of using the H statistic as a market power measure. First, I show that it is more related to a pass-through rate than to a market power measure. It could be argued that the H statistic is a marginal revenue pass-through rate, which limits its capacity to address market power. Second, I extended the H statistic to account for non-price strategic variables, and I show that positive H statistics can be found in monopolistic frameworks. In this situation, any assessment of market power based on the sign of the H statistic would be incorrect. Thus, although an interesting theoretical tool, the H statistic seems to be unfit to address market power empirically. Therefore, it is discouraged to enforce any kind of competition policy based on the H statistic. We should avoid using the H statistic as a market power measure, or more precisely, as a tool to rule out monopoly power. Nonetheless, its use as a marginal revenue pass-through rate could be interesting, and re-interpreting previous results under these new lenses could lead to new learnings about the market environment, but those should not be confused with the market power.

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