1 Introduction

The first reported meson was a charged Kaon, observed with a cloud chamber in cosmic rays in 1944 [1], three years before the discovery of the charged pion [2]. The neutral Kaon was also discovered in 1947 [3]. In 1955, Gell-Mann and Pais [4] predicted that the $K_0^0$ is a two-state particle with a non-exponential decay law. This was confirmed in 1956 [5] with 20 $K_0^0$ decays showing a life time at least 10 times longer than that of the dominant $K_0^0 \rightarrow \pi^+\pi^-$ decays. In the same year, Lee and Yang [6] concluded that weak decays violate P symmetry since the charged Kaon decays into 2 $\pi$ and 3 $\pi$ states with opposite parity. P violation was confirmed in two experiments [7, 8] one year later. In 1964, Chistenson et al. [9] discovered that also CP symmetry is violated, either in decays of the long-living $K_0^0$ state $K_L$ (CP=-1) into $\pi^+\pi^-$ or in $K_0^0\overline{K}_0^0$ transitions with mass eigenstates which are not CP eigenstates. A 1967 experiment [10] proved with

$$\Delta L_e = \frac{N(K_L \rightarrow \pi^-e^+\nu) - N(K_L \rightarrow \pi^+e^-\nu)}{N(K_L \rightarrow \pi^-e^+\nu) - N(K_L \rightarrow \pi^+e^-\nu)} = (2.24 \pm 0.36) \times 10^{-3}$$

that CP symmetry is violated in $K_0^0\overline{K}_0^0$ transitions.

2 Phenomenology of $K_0^0\overline{K}_0^0$ Transitions

Following Weisskopf and Wigner [11], the evolution of the two-state neutral Kaon $|\Psi\rangle = \psi_1|K_0^0\rangle + \psi_2|\overline{K}_0^0\rangle$ is given by the effective Schrödinger equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = H_{eff} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{12}^* & m_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (2)$$

Owing to arbitrary phases of the states $|K_0^0\rangle$ and $|\overline{K}_0^0\rangle$, the phases of $m_{12}$ and $\Gamma_{12}$ are unobservable. Their difference, the phase of $\Gamma_{12}/m_{12}$, is an observable. In total, the equation has 7 real observable parameters: $m_{11}$, $m_{22}$, $\Gamma_{11}$, $\Gamma_{22}$, $|m_{12}|$ and $|\Gamma_{12}|$ in addition to $\phi(\Gamma_{12}/m_{12})$. Two solutions of Eq. [2] have exponential decay laws,

$$K_S^0(t) = [(1 + \epsilon + \delta) \cdot K^0 + (1 - \epsilon - \delta) \cdot \overline{K}_0^0 \cdot e^{-im_{12}t - \Gamma_{st}/2}/\sqrt{2}],$$

$$K_L^0(t) = [(1 + \epsilon - \delta) \cdot K^0 - (1 - \epsilon + \delta) \cdot \overline{K}_0^0 \cdot e^{-im_{12}t - \Gamma_{st}/2}/\sqrt{2}]. \quad (3)$$
They are normalized to 1 at $t = 0$ and they are in general not orthogonal,

$$\langle K_S | K_L \rangle = 2 \Re \epsilon - 2i \Im \delta .$$

(4)

The 7 observable parameters of the solutions, following unambiguously from the 7 parameters in Eq. 2, are $m_S$, $m_L$, $\Gamma_S$, $\Gamma_L$, $\Re \epsilon$, $\Re \delta$ and $\Im \delta$. The additional parameter $\Im \epsilon$ is unobservable, and Eqs. 3 are approximations in the limits $|\Re \epsilon| \ll 1$ and $|\delta| \ll 1$, well fulfilled experimentally. The relations between the parameter sets in Eqs. 3 and 2 are well approximated by

$$m_L = m + \Delta m/2 , \quad m_S = m - \Delta m/2 , \quad \Gamma_L = m + \Delta \Gamma/2 , \quad \Gamma_S = m - \Delta \Gamma/2 ,$$

$$m = (m_{11} + m_{22})/2 , \quad \Gamma = (\Gamma_{11} + \Gamma_{22})/2 ,$$

$$\Delta m = 2|m_{12}| , \quad \Delta \Gamma = 2|\Gamma_{12}| \times \text{sign}[\Re(\Gamma_{12}/m_{12})] ,$$

$$\Re \epsilon = \frac{\Im(\Gamma_{12}/m_{12})}{4 + |\Gamma_{12}/m_{12}|^2} , \quad \delta = \frac{(m_{22} - m_{11}) - i(\Gamma_{22} - \Gamma_{11})/2}{2\Delta m - i\Delta \Gamma} .$$

(5)

The choice $\Delta m > 0$ is a convention, the sign of $\Delta \Gamma$ is given by $\Gamma_{12}/m_{12}$. CPT symmetry requires $\delta = 0$, T symmetry $\Re \epsilon = 0$, i.e. $\phi(\Gamma_{12}/m_{12}) = 0$ or $\pi$, and CP symmetry $\Re \epsilon = \delta = 0$. T symmetry requiring $\Im (\Gamma_{12}/m_{12}) = 0$ has the same origin as requiring $\Im (A_{M1}/A_{E2}) = 0$ in atomic transitions with vanishing E1 amplitude, as shown by Lloyd in 1951 [12].

### 3 Motion-Reversal Symmetry in Classical Mechanics

Fig. 1 shows the orbit of a ball with small velocity on the surface of the earth. The motion can be recorded together with a clock showing the time from the start to the end of the orbit. A video recorder allows to replay the recorded movie in the backward direction, as shown in Fig. 2. Even if the motion of the ball looks familiar to the viewer, the clock

![Figure 1: Video recording of a motion with velocity $\vec{v}_F$ in the endpoint F together with a clock showing the running time.](image1)

![Figure 2: Playback of the recorded video in the reversed time-direction.](image2)

![Figure 3: The time-reversed motion with velocity $\vec{v}_I = -\vec{v}_F$ in the new start point I together with a clock showing the forward-running time.](image3)
tells him that this process does not take place in his world. The replayed movie shows an 
unobservable process; time never runs backwards. However, it is possible to observe the 
reversed motion in the real world: we have to start it at the end of the original orbit with 
a velocity vector equal to the opposite of the final velocity in the original motion, as shown 
in Fig. 3. This observable motion with forward-running time is called “reversed motion”. 
The operation with starting at the end point and reversing the velocity is called “motion 
reversal”, and the comparison of Figs. 3 and 4 shows “motion-reversal symmetry”. 

The T operation, time reversal, is defined by the transformation $t \rightarrow -t$ in the equation 
of motion, here

$$m \frac{d^2 \vec{r}}{dt^2} = m \vec{g}.$$ 

(6)

This equation is invariant if $t$ is replaced by $-t$, the equation is T-symmetric. Motion-
reversal symmetry is a consequence of T symmetry. Friction in the air leads to an equation 
of motion such as

$$m \frac{d^2 \vec{r}}{dt^2} = m \vec{g} - \eta \frac{d \vec{r}}{dt},$$ 

(7)

which is not invariant under the T transformation if $\eta \neq 0$. Consequently, motion-reversal 
symmetry is violated, known since many centuries from the asymmetric orbits of cannon 
balls. There are two ways to test T symmetry: either by “direct” observation of motion-
reversal violation (Move the cannon and fire from the target position!) or by “indirect” 
observation of T violation (Measure the parameter $\eta$ in Eq. 7).

4 Time Reversal in Quantum Mechanics

The T operation in quantum mechanics was introduced by Wigner in 1932 [13]. Here follows 
a short summary of the concept and its implications: Together with the commutation 
relation $[x_i, p_j] = i \delta_{ij}$, the operation $t \rightarrow -t$ leads to

$$T \ i \ T^{-1} = -i, \ T = U \ K,$$ 

(8)

where $U$ is an arbitrary unitary transformation, $U U^\dagger = 1$, and $K$ is complex conjugation, 
$K = K^{-1} = z^*$, leading to

$$T |\psi\rangle = U |\psi^*\rangle.$$ 

(9)

$T$ is antiunitary, $T^\dagger \neq T^{-1}$, and antilinear, $T(c_1 |\psi_1\rangle + c_2 |\psi_2\rangle) = c_1^* |\psi_1^*\rangle + c_2^* |\psi_2^*\rangle$, with

$$\langle \psi_1 T |\psi_2 T \rangle = \langle \psi_2 |\psi_1 \rangle.$$ 

(10)

The dynamics of transitions and decays is described by operators $H$, $S$ and $D$ with

$$S = \lim e^{-iHt} = 1 + iD, \ SS^\dagger = 1, \ TDT^{-1} = D^\dagger.$$ 

(11)

States of one or more particles with momenta $p_i$ and spins $s_i$ transform like

$$T |p_i, s_i\rangle = e^{i\phi} | - p_i, - s_i\rangle$$ 

(12)

with an arbitrary phase $\phi$. This leads to

$$T \langle p_f, s_f |D|p_i, s_i\rangle = e^{i(\phi_1 - \phi_f)} \langle -p_i, - s_i |D| - p_f, - s_f \rangle^*,$$ 

(13)
and T symmetry requires “motion-reversal symmetry”,
\[
|\langle pf, sf | D | pi, si \rangle|^2 = |\langle -pi, -si | D | -pf, -sf \rangle|^2.
\] (14)

When motion-reversal symmetry is observed to be violated, e.g. in the $K^0$ system with $p = s = 0$,
\[
|\langle K^0 | D | K^0 \rangle|^2 \neq |\langle K^0 | D | \bar{K}^0 \rangle|^2,
\] (15)
then the T symmetry of $H$, $S$ and $D$ in the dynamics of $K^0\bar{K}^0$ transitions is violated, i.e. Re $\epsilon \neq 0$.

I would like to add an important side remark: T violation in the Hamiltonian of an interaction is different from the omnipresent “arrow of time”. The entropy increase of an ensemble of $10^8$ $K^0$ mesons in $10^{-8}$ s is huge and indistinguishably equal for the two cases without or with a small T violation.

5 CPT

The CPT operation is defined as $CPT = CP \times T$, and for bosonic systems we have $T^2 = +1$; therefore
\[
CP = CPT \times T.
\] (16)

When CP symmetry is broken, CPT or T or both must also be broken. The case with CP conservation and violation of both CPT and T is not possible in $K^0\bar{K}^0$ transitions since its dynamics, see Eq. 2, contains only 7 parameters. The 1967 CP violation in Eq. 1 can have two contributions, T violation with Re $\epsilon \neq 0$ and CPT violation with $\delta \neq 0$. Two experimental ways have been used to determine the two contributions:

- “Direct” measurements of T violation by motion reversal comparing
  \[
  |\langle K^0 | U(t) | \bar{K}^0 \rangle|^2 \text{ and } |\langle K^0 | U(t) | K^0 \rangle|^2,
  \]
  and of CPT violation by comparing
  \[
  |\langle K^0 | U(t) | K^0 \rangle|^2 \text{ and } |\langle \bar{K}^0 | U(t) | \bar{K}^0 \rangle|^2,
  \]
  where $U(t) = e^{-iHt}$,

- “Indirect” determinations of Re $\epsilon$ and $\delta$ using Bell-Steinberger’s unitarity relation.

The direct way has been used by CPLEAR in 1998 [14, 15], the indirect way in 1970 by Schubert et al. [16]. Both lead to the same results as discussed in the following sections.

6 Unitarity Relations

For an unstable single-state particle with the wave function $\psi(t) = e^{-\Gamma t/2}$, unitarity (conservation of probability including all observable decay states $f_i$) requires
\[
|\psi(t)|^2 + \sum_{i=1}^{N} |f_i(t)|^2 = 1, \quad d|\psi|^2/dt + \sum_{i=1}^{N} d|f_i(t)|^2/dt = 0.
\] (17)

At $t = 0$, we have $d|\psi|^2/dt = -\Gamma$ and $d|f_i(t)|^2/dt = |\langle f_i | D | \psi \rangle|^2$ and
\[
\Gamma = \sum_{i=1}^{N} |\langle f_i | D | \psi \rangle|^2.
\] (18)
This unitarity relation connects the sum of all decay rates to the inverse mean life of the unstable particle.

For the two-state particle $\Psi(0) = \psi_1 K^0 + \psi_2 \bar{K}^0 = \alpha K_S + \beta K_L$, we obtain three unitarity relations. As presented e.g. in Ref. [17], unitarity in the space of states $K_S, K_L, f_1 \ldots f_N$ leads to the three relations

$$\Gamma_S = \sum_{i=1}^{N} |\langle f_i | D | K_S^0 \rangle|^2, \quad \Gamma_L = \sum_{i=1}^{N} |\langle f_i | D | K_L^0 \rangle|^2, \quad \frac{(\Gamma_S + \Gamma_L)}{2} + i\Delta m \langle K_S^0 | K_L^0 \rangle = \sum_{i=1}^{N} \langle f_i | D | K_S^0 \rangle^* \langle f_i | D | K_L^0 \rangle.$$ (19)

Using the result for $\langle K_S^0 | K_L^0 \rangle$ in Eq. 2, the third relation can be written as

$$\text{Re} \epsilon - i \text{Im} \delta = \frac{\sum_{i}^{N} \langle f_i | D | K_S^0 \rangle^* \langle f_i | D | K_L^0 \rangle}{\Gamma_S + \Gamma_L + 2i \Delta m}.$$ (20)

It was derived in 1966 by Bell and Steinberger [18] and allows to determine the two $K^0\bar{K}^0$ transition parameters $\text{Re} \epsilon$ and $\text{Im} \delta$ using measurable decay properties, as described in the following.

The dominant final states are $\pi^+\pi^-$ and $\pi^0\pi^0$ with the well-known CP-violation parameters

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | D | K_L^0 \rangle}{\langle \pi^+\pi^- | D | K_S^0 \rangle} = \epsilon_W + \epsilon', \quad (21)$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | D | K_L^0 \rangle}{\langle \pi^0\pi^0 | D | K_S^0 \rangle} = \epsilon_W - 2 \epsilon'.$$ (22)

Note that the $\eta$ parameter for $\pi\pi$ with isospin 0 is called $\epsilon_W$ here (W for Wolfenstein) in order to distinguish it from the $K^0\bar{K}^0$ transition parameter $\epsilon$. The $\eta$ parameter $\epsilon_W$ has an observable phase, $\epsilon$ has not. In the first four years after the discovery of CP violation, $|\eta_{+-}|, \phi(\eta_{+-})$ and $|\eta_{00}|$ were determined precisely enough for being used in a Bell-Steinberger unitarity analysis, but there was no result on $\phi(\eta_{00})$ before 1970.

The Bell-Steinberger relation may be written as

$$\text{Re} \epsilon - i \text{Im} \delta = \frac{\epsilon_W + \sum_{i \neq \pi \pi} \alpha_i}{1 + \Gamma_L/\Gamma_S + 2i \Delta m/\Gamma_S} \times BF(K_S \rightarrow f_i), \quad (23)$$

where $BF$ is the branching fraction. Since $\Gamma_L/\Gamma_S \ll 1$, the phase $\phi(\epsilon_W)$ of $\epsilon_W = (2 \eta_{+-} + \eta_{00})/3$ shows the two contributions of CP violation in $K^0\bar{K}^0$ transitions if $|\sum \alpha_i| \ll |\epsilon_W|$.
• CP violation with CPT symmetry, $\delta = 0$, $\Rightarrow \phi(\epsilon_W) = \arctan(\Delta m/\Gamma_S) \approx 45^\circ$,

• CP violation with T symmetry, $\text{Re } \epsilon = 0$, $\Rightarrow \phi(\epsilon_W) = \arctan(-\Delta m/\Gamma_S) \approx 135^\circ$.

If $\phi(\epsilon_W)$ is neither $45^\circ$ nor $135^\circ$, both T and CPT violation contribute. In 1968, only $\phi(\eta_{00})$ was missing for a determination of the two contributions, if $\sum \alpha_i$ for $i \neq \pi\pi$ is well enough estimated.

7 First Measurement of $\phi(\eta_{00})$ and First Observation of T Violation

The group of J. M. Gaillard at CERN [20] determined in 1970 the time dependence of the decay rate $|\langle \pi^0\pi^0|D|\Psi(t)\rangle|^2$ after a copper regenerator with $\psi(0) = K_L + \rho K_S$ using $\sim 200,000$ photographs from a setup of optical spark chambers with a scintillation-counter trigger. The result is shown in Fig. 5 and the best fit to the data gives

$$|\eta_{00}| = (3.3 \pm 0.7) \times 10^{-3}, \quad \phi(\eta_{00}) = (51 \pm 30)^\circ.$$ (24)

These two results have been used in the same year by the same group [16] for the first Bell-Steinberger analysis together with the results $2\Delta m/\Gamma_S = 0.983 \pm 0.0030$, $|\eta_{+-}| = (1.92 \pm 0.05) \times 10^{-3}$, $\phi(\eta_{+-}) = (44 \pm 5)^\circ$, $\alpha_{3\pi,I=1} = (0.3 \pm 1.7 + i(0.8 \pm 2.5)) \times 10^{-4}$, $\alpha_{\pi\nu+\pi\mu\nu} = [-2.4 \pm 2.1 + i(0.9 \pm 3.4)] \times 10^{-4}$, and well-motivated zero $\alpha$ values for $3\pi, I = 3$, $\pi\pi\gamma$ and $\gamma\gamma$. The analysis results in

$$\text{Re } \epsilon = (1.68 \pm 0.30) \times 10^{-3},$$
$$\text{Im } \delta = (-0.30 \pm 0.45) \times 10^{-3},$$ (25)

T violation in $K^0\bar{K}^0$ transitions is established with $5 \sigma$, and CPT symmetry is found to be valid within errors. The analysis cannot determine $\text{Re } \delta$, only the CPT-violating quantity

$$\text{Re } (\delta + \tilde{\alpha}_0) = (0.07 \pm 0.43) \times 10^{-3},$$ (26)

where $\tilde{\alpha}_0$ describes CPT violation in $K^0 \to \pi\pi_{I=0}$ decays.

8 Updates of Bell-Steinberger Analyses

The following list of updates is far from being complete. It gives some milestones, each time driven by more precise data.
Table 1: Selected milestones of Bell-Steinberger analyses. Comments are given below the Table. For the Re δ results, see Section 9.

| Year | Reference                 | Re ϵ [10^{-3}] | Im δ [10^{-3}] | Re δ [10^{-3}] |
|------|---------------------------|----------------|----------------|----------------|
| 1970 | Schubert et al. [16]      | 1.68 ± 0.30    | −0.30 ± 0.45   |                |
| 1980 | Cronin [21]               | 1.61 ± 0.20    | −0.08 ± 0.17   |                |
| 1983 | ITEP Moscow [22]          | 1.62 ± 0.05    | −0.11 ± 0.10   |                |
| 1999 | CPLEAR [23]               | 1.649 ± 0.025  | −0.02 ± 0.05   | 0.24 ± 0.27    |
| 2006 | KLOE [24]                 | 1.596 ± 0.013  | 0.004 ± 0.021  | 0.23 ± 0.27    |
| 2012 | Particle Data Group [25, 26]| 1.611 ± 0.005  | −0.007 ± 0.014 | 0.24 ± 0.23    |

The main progress in the 1980 analysis originates from the new result \( \phi(\eta_{00}) = (56 \pm 6)^° \) obtained by Christenson et al. [27]. The 1983 analysis profits from new data on \( \pi^+\pi^-\pi^0 \) decays and on the first determination of \( \eta_{000} = \langle \pi^0\pi^0\pi^0 | D|K_S \rangle / \langle \pi^0\pi^0\pi^0 | D|K_L \rangle = (-0.08 \pm 0.18) + i(-0.05 \pm 0.27) \) by Barmin et al. [28]. The 1999 analysis uses the rich CPLEAR data set of \( 10^8 K^0 \) decays into \( \pi^+\pi^-, \pi^0\pi^0, \pi e\nu, \pi^+\pi^-\pi^0 \) and \( 3\pi^0 \). Its first result on Re δ is obtained from the determination of all \( \pi e\nu \) decay parameters in combination with Bell-Steinberger unitarity [23]. The 2006 analyses uses the even larger KLOE data set with \( 2.5 \times 10^9 e^+e^- \rightarrow \Phi \rightarrow K^0\bar{K}^0 \) events with neutral-Kaon decays into essentially all final states.

The 2012 analysis of Antonelli and D’Ambrosio uses the latest PDG averages for all inputs including NA48 and KTeV results on \( 2\pi \) decays to be discussed in Section 12. With the definition of δ in Eq. 5, the obtained values for real and imaginary part of δ determine the CPT-violating parameters in the Schrödinger equation Eq. 2,

\[
\delta m = m_{22} - m_{11} = m(K^0) - m(K^0), \\
\delta \Gamma = \Gamma_{22} - \Gamma_{11} = \Gamma(K^0) - \Gamma(K^0). \\
\]

The results are shown in Fig. 6. For the mass difference, two different results may be given, with no constraint on \( \delta \Gamma \) (first line) and with \( \delta \Gamma = 0 \) (second line):

\[
-2 \times 10^{-18} < \delta m < +6 \times 10^{-18} \text{ GeV}, \\
-4 \times 10^{-19} < \delta m < +4 \times 10^{-19} \text{ GeV}. \\
\]

Note that the assumption \( \delta \Gamma = 0 \) also implies Re δ \( \approx \) Im δ, since \( \Delta m \approx \Delta \Gamma/2 \).
9 πℓν Decay Amplitudes

The four decay amplitudes, all understood to be integrals over the three-particle phase spaces,

\[ \langle \pi^+ \ell^- \nu | D | K^0 \rangle, \quad \langle \pi^+ \ell^- \nu | D | \bar{K}^0 \rangle, \quad \langle \pi^- \ell^+ \nu | D | K^0 \rangle, \quad \langle \pi^- \ell^+ \nu | D | \bar{K}^0 \rangle, \]

define six real parameters \( a, y, \text{Re} \, x_+, \text{Im} \, x_+, \text{Re} \, x_- \) and \( \text{Im} \, x_- \). When \( a \) is much larger than the other five parameters, which is well fulfilled experimentally, they are defined as

\begin{align*}
\frac{a}{2} &= \frac{\langle \pi^+ \ell^- \nu | D | K^0 \rangle + \langle \pi^+ \ell^- \nu | D | \bar{K}^0 \rangle}{2 a}, \\
\frac{y}{2 a} &= \frac{\langle \pi^- \ell^+ \nu | D | K^0 \rangle - \langle \pi^- \ell^+ \nu | D | \bar{K}^0 \rangle}{2 a}, \\
\frac{x_+}{2 a} &= \frac{\langle \pi^- \ell^+ \nu | D | K^0 \rangle + \langle \pi^- \ell^+ \nu | D | \bar{K}^0 \rangle}{2 a}, \\
\frac{x_-}{2 a} &= \frac{\langle \pi^- \ell^+ \nu | D | K^0 \rangle - \langle \pi^- \ell^+ \nu | D | \bar{K}^0 \rangle}{2 a}.
\end{align*}

The parameters \( a \) and \( y \) obey the so-called “\( \Delta Q = \Delta S \)” rule, the two \( x \) parameters violate it, \( a \) obeys and \( y \) violates CPT. \( x_+ \) obeys and \( x_- \) violates CPT. The real parts of \( x_+ \) and \( x_- \) obey and the two imaginary parts violate T. \( \text{Im} \, x_- \) is a very special quantity. If non-zero, it violates all three symmetries CP, T and CPT.

Measuring the time dependences of the four decay rates in Eq. 29 CPLEAR obtained in 1998 from 1.3 \( \times \) 10^6 \( \pi \ell \nu \) events

\begin{align*}
\text{Re} \, x_+ &= (-1.8 \pm 6.1) \times 10^{-3}, \quad \text{Im} \, x_+ = (1.2 \pm 2.1) \times 10^{-3}, \\
\text{Im} \, x_- &= (-0.8 \pm 3.5) \times 10^{-3}, \quad \text{Re} \, \delta = (0.30 \pm 0.34) \times 10^{-3},
\end{align*}

where the values are taken from the 2003 CPLEAR summary report [29] and the quoted errors combine statistics and systematics.

When the time dependences are combined with Bell-Steinberger unitarity [23] and with the 1999 world average for the asymmetry \( \Delta L_e \) as already defined in Eq. 1, the CPLEAR fits improve the sensitivity on \( \text{Re} \, \delta \) and become sensitive to the CPT-violating parameters \( y \) and \( \text{Re} \, x_- \) independent of any additional assumptions,

\begin{align*}
\text{Re} \, \delta &= (0.24 \pm 0.27) \times 10^{-3}, \quad y = (0.3 \pm 3.1) \times 10^{-3}, \quad \text{Re} \, x_- = (-0.5 \pm 3.0) \times 10^{-3}.
\end{align*}

The error on \( \text{Re} \, \delta \) improves, no CPT violation is seen in \( K^0 \bar{K}^0 \) transitions. In addition, no violation of the “\( \Delta Q = \Delta S \)” rule is seen, the two \( x \) parameters are compatible with zero. Therefore, there is no visible T violation in the \( \pi \ell \nu \) decay amplitudes. Any CP violation therein could only come from CPT violation. Since \( y \) is also compatible with zero, the decay amplitudes are symmetric under CPT, T and CPT.

10 Transverse Muon Polarisation in \( K^0 \rightarrow \pi \mu \nu \) Decays

T symmetry of the Hamiltonian led to the result in Eq. 13

\[ \langle p_f, s_f | D | p_i, s_i \rangle = e^{i(\phi_i - \phi_f)} \langle -p_f, -s_f | D^\dagger | -p_i, -s_i \rangle. \]
If decays are only influenced by weak interactions (Standard and weaker), i.e., if stronger final-state interactions (FSI) are absent, unitarity of the $S$ operator, $S = 1 + iD$, $SS^\dagger = 1$ leads to

$$D - D^\dagger = iD^\dagger D \approx 0, \quad D^\dagger \approx D,$$  \hspace{1cm} (34)

since the second-order interaction $D^\dagger D$ is much smaller than $D$. Consequently,

$$\langle p_f, s_f|D|p_i, s_i\rangle \approx e^{i(\phi_i - \phi_f)} \langle -p_f, -s_f|D|-p_i, -s_i\rangle,$$  \hspace{1cm} (35)

which is called $\hat{T}$ symmetry in the textbook of Branco, Lavoura and Silva [17]. For decays of $K$ mesons in their rest frame this means in very good approximation

$$|\langle p_f, s_f|D|0, 0\rangle| = |\langle -p_f, -s_f|D|0, 0\rangle|.$$  \hspace{1cm} (36)

For the polarization triple product in decays $K_L^0 \to \pi^- \mu^+ \nu$ and $K^+ \to \pi^0 \mu^+ \nu$ this requires

$$\vec{s} \cdot [\vec{p}_\pi \times \vec{p}_\mu] = -\vec{s} \cdot [\vec{p}_\pi \times \vec{p}_\mu] = 0$$  \hspace{1cm} (37)

at any point $(|\vec{p}_\mu|, \theta_{\mu\nu})$ of the Dalitz plot. This can be parametrised as

$$\frac{\vec{s} \cdot [\vec{p}_\pi \times \vec{p}_\mu]}{|\vec{p}_\pi \times \vec{p}_\mu|} = \text{Im} \left( \frac{m_\mu}{m_K} f(|\vec{p}_\mu|, \theta_{\mu\nu}) \right).$$  \hspace{1cm} (38)

Since there is only one hadron in the final state, no strong interaction can mimic $T$ violation, and electromagnetic FSI are estimated to produce $\text{Im} \xi \approx 0.008$ [30]. A BNL experiment in 1980 [31] finds $\text{Im} \xi = 0.009 \pm 0.030$ for $K_L^0 \to \pi^- \mu^+ \nu$, and a 2006 KEK experiment [32] finds $\text{Im} \xi = -0.005 \pm 0.008$ for $K^+ \to \pi^0 \mu^+ \nu$. There is no $T$ violation within one standard deviation. Both experiments test $T$ violation in New Physics, Standard Model estimates expect $\text{Im} \xi \approx 10^{-7}$ without FSI [33].

11 “Direct” Tests of $T$ and CPT Violation in $K^0\bar{K}^0$ Transitions

The explicit time dependences for the appearance of $\bar{K}^0$ ($K^0$) states from initial $K^0$ ($\bar{K}^0$) states can easily be derived from Eqs. 3; they are found to be

$$P(K^0 \to \bar{K}^0) = \left(\frac{1}{4} - \text{Re} \, \epsilon \right) \left( e^{-\Gamma s t} + e^{-\Gamma l t} - 2 \cos \Delta mt \cdot e^{-\Gamma t} \right),$$

$$P(\bar{K}^0 \to K^0) = \left(\frac{1}{4} + \text{Re} \, \epsilon \right) \left( e^{-\Gamma s t} + e^{-\Gamma l t} - 2 \cos \Delta mt \cdot e^{-\Gamma t} \right).$$  \hspace{1cm} (39)

They result in a motion-reversal asymmetry

$$A_T(t) = \frac{P(\bar{K}^0 \to K^0) - P(K^0 \to \bar{K}^0)}{P(\bar{K}^0 \to K^0) + P(K^0 \to \bar{K}^0)} = 4 \text{ Re } \epsilon,$$  \hspace{1cm} (40)

which is time-independent and given by the only $T$-violating parameter in $K^0\bar{K}^0$ transitions, Re $\epsilon$. Observation of this asymmetry requires preparation of the initial states and
with those from the earlier Bell-Steinberger analyses.

and from a two-parameter fit the two T-violating quantities

two results in Ref. [14], the T-violating asymmetry average for

amplitudes. In general, 

This is equal to \( A_T \) in Eq. [40] if there is no CPT and no T violation in the \( \pi e \nu \) decay amplitudes. In general,

Assuming CPT symmetry in the decay amplitudes with \( y = x_+ = 0 \), CPLEAR presents two results in Ref. [14], the T-violating asymmetry average for \( \tau_S < t < 20 \tau_S \),

\[ 4 \text{Re} \epsilon + \langle f(t) | \text{Im} x_+ \rangle = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3} , \]

and from a two-parameter fit the two T-violating quantities

\[ 4 \text{Re} \epsilon = (6.2 \pm 1.4 \pm 1.0) \times 10^{-3} \text{ and } \text{Im} x_+ = (1.2 \pm 1.9 \pm 0.9) \times 10^{-3} . \]

Both results violate T symmetry with 4 to 5 \( \sigma \), and \( \text{Re} \epsilon \) in Eq. [44] is in perfect agreement with those from the earlier Bell-Steinberger analyses.

The time dependences for the survival of \( K^0 (\bar{K}^0) \) states from initial \( K^0 (\bar{K}^0) \) states are obtained from Eqs. [3] as well; they are found to be

\[ P(K^0 \to K^0) = (\frac{1}{4} + \text{Re} \delta)e^{-\Gamma_S t} + (\frac{1}{4} - \text{Re} \delta)e^{-\Gamma_L t} + (\frac{1}{2} \cos \Delta mt - 2 \text{Im} \delta \sin \Delta mt)e^{-\Gamma_S t} , \]

\[ P(\bar{K}^0 \to K^0) = (\frac{1}{4} - \text{Re} \delta)e^{-\Gamma_S t} + (\frac{1}{4} + \text{Re} \delta)e^{-\Gamma_L t} + (\frac{1}{2} \cos \Delta mt + 2 \text{Im} \delta \sin \Delta mt)e^{-\Gamma_S t} . \]

The resulting CPT asymmetry

\[ A_{CPT}(t) = \frac{P(\bar{K}^0 \to K^0) - P(K^0 \to K^0)}{P(K^0 \to K^0) - P(\bar{K}^0 \to K^0)} \]

is time-dependent, sensitive to both \( \text{Re} \delta \) and \( \text{Im} \delta \), and is equal to \( 4 \text{Re} \delta \) for large times, \( t \gg \tau_S \). CPLEAR has determined this asymmetry [15], again using preparation of the initial state by associate production in \( p\bar{p} \) annihilation and detection of the final state by a \( \pi e \nu \) decay. The experimental CPT asymmetry

\[ A_{CPT}^{exp}(t) = \frac{P(\bar{K}^0 \to K^0) - P(K^0 \to K^0)}{P(K^0 \to K^0) + P(\bar{K}^0 \to K^0)} \]

is equal to \( 4 \text{Re} \delta + 2 y + 2 \text{Re} x_- \) for \( t \gg \tau_S \). By using their own \( \pi^+ \pi^- \) decay data, CPLEAR determines a modified asymmetry \( A_\delta \) for eliminating the dependence on \( y \) and \( x_- \),

\[ A_\delta = \frac{N(K^0 \to K^0) - N(K^0 \to K^0)(1 + 4 \text{Re} \eta_{+-})}{N(K^0 \to K^0) + N(K^0 \to K^0)(1 + 4 \text{Re} \eta_{+-})} \]

\[ + \frac{N(\bar{K}^0 \to K^0) - N(\bar{K}^0 \to K^0)(1 + 4 \text{Re} \eta_{+-})}{N(\bar{K}^0 \to K^0) + N(\bar{K}^0 \to K^0)(1 + 4 \text{Re} \eta_{+-})} = 8 \text{Re} \delta + f(t) , \]
with $\text{Re} \eta_{+-} = \text{Re} \epsilon - \text{Re} \delta$ and $f$ vanishing for $t \gg \tau_S$. For large values of $t$, this quantity is strictly independent of all parameters with the exception of $\text{Re} \delta$, i.e. $A_\delta = 2 A_{\text{CPT}}$. CPLEAR’s result is compatible with zero: CP violation in $K^0 \overline{K}^0$ transitions is T-violating and not CPT-violating.

12 $\pi \pi$ Decay Amplitudes

After many years of effort the two experiments NA48 at CERN [34] and KTeV at FNAL [35] have determined final results for $\text{Re} (\epsilon'/\epsilon_W)$; see Eqs. 21 and 22 and Fig. 4 for the used notation. Including a scale factor, the present average [25] is

$$\text{Re} (\epsilon'/\epsilon_W) = (1.66 \pm 0.23) \times 10^{-3}.$$  \hfill (49)

KTeV determines also the phases of $\eta_{+-}$ and $\eta_{00}$ leading to the imaginary part of $\epsilon'/\epsilon_W$,

$$\phi_{00} - \phi_{+-} = (0.30 \pm 0.35)^\circ,$$
$$\text{Im} (\epsilon'/\epsilon_W) = (1.7 \pm 2.0) \times 10^{-3}. \hfill (50)$$

Since the phase of $\epsilon_W$ is well measured, $\phi(\epsilon_W) = (43.9 \pm 0.6)^\circ$ [35], real and imaginary part of $\epsilon'/\epsilon_W$ determine the phase of $\epsilon'$. Using the amplitudes

$$A_I = \langle \pi \pi, I | D | K^0 \rangle = a_I \times e^{i \delta_I},$$
$$\overline{A}_I = \langle \pi \pi, I | D | \overline{K}^0 \rangle = \overline{a}_I \times e^{i \delta_I}, \hfill (51)$$

where $a_I$ and $\overline{a}_I$ are the weak amplitudes and $\delta_I$ the final-state scattering phases for isospin $I = 0$ and 2, CPT symmetry requires $\overline{a}_I = a_I^*$ and [36, 17]

$$\epsilon' = \frac{1}{\sqrt{2}} \frac{p a_2 - q a_0^* e^{i \delta_2}}{p a_0 + q a_2 e^{i \delta_0}} = \frac{i}{\sqrt{2}} \text{Im} \frac{a_2}{a_0} e^{i (\delta_2 - \delta_0)}. \hfill (52)$$

With $\delta_2 - \delta_0 = (-45 \pm 6)^\circ$ from $\pi \pi$ scattering [37], we obtain

$$\phi_{\text{CPT}} (\epsilon'/\epsilon_W) = \pi / 2 + \delta_2 - \delta_0 + n \pi = 45^\circ \pm 6^\circ \text{ or } 225^\circ \pm 6^\circ. \hfill (53)$$

T symmetry requires $(p a_2 - q \overline{a}_0)/(p a_0 + q \overline{a}_2)$ to be real, i.e. $\phi (\epsilon') = \delta_2 - \delta_0 + n \pi$,

$$\phi_T (\epsilon'/\epsilon_W) = -89^\circ \pm 6^\circ \text{ or } 91^\circ \pm 6^\circ. \hfill (54)$$

The two regions for the ratio of the predicted $\epsilon'$ and the measured $\epsilon_W$ are shown in Fig. 7. The measured $\epsilon'$ is CPT-symmetric and violates T symmetry with about 6 $\sigma$. The observation that the imaginary part of $a_2/a_0$ is non-zero may be called “direct T violation”.

Figure 7: $\Delta \chi^2 = 1$ contour for $\epsilon'/\epsilon_W$ measured by KTeV [35]. The two bands marked CPT and T are the allowed regions ($\pm 1 \sigma$) for CPT symmetry and T symmetry of $\epsilon'$.
13 Summary

Let me collect here the main results on T violation in the $K^0$-meson system:

- CP violation is observed in $K_L^0 \rightarrow \pi^+\pi^-$ decays (1964). When CP is violated, either CPT or T or both must also be violated.
- The origin is CP violation in $K^0\bar{K}^0$ transitions (1967).
- A unitarity analysis proves that the rate for $K^0 \rightarrow K^0$ is larger than that for $K^0 \rightarrow \bar{K}^0$, i.e. T violation (1970).
- Unitarity analyses with increased data precision determine the transition parameters $\text{Re} \epsilon = (161.1 \pm 0.5) \times 10^{-5}$, $\text{Im} \delta = (-0.7 \pm 1.4) \times 10^{-5}$, $\text{Re} \delta = (24 \pm 23) \times 10^{-5}$ (2012). $|m(\bar{K}^0) - m(K^0)| < 6 \times 10^{-18}$ or $< 4 \times 10^{-19}$ GeV depending on assumptions.
- T violation in $K^0\bar{K}^0$ transitions is Standard-Model physics owing to three-family quark mixing (GLCKM). This was confirmed in 2001 when large CP and T violation were found in B-meson decays. There is no deviation from this conclusion with more and more B-meson data until today.
- There is only one more observed T violation in the Kaon system: The complex decay-amplitude ratio $\epsilon'$ violates T with about $6 \sigma$ (2011), T violation in $\pi\pi_{I=2}$ decays.
- T violation in $\pi\pi_{I=0}$ decays is completely given by Re $\epsilon$ owing to unitarity (1970).
- Because of $\Delta Q = \Delta S$, no T violation is seen in semileptonic decay amplitudes (1999).
- No T violation is seen in the transverse muon polarisation of $K \rightarrow \pi\mu\nu$ decays (2006).

Some more details on the physics of this presentation and more references may be found in a recent review [38].

Acknowledgements

I would like to thank Adrian Bevan and his co-organisers of the Conference for the occasion to present this historical review and for their hospitality. I also thank Hans-Jürg Gerber and Thomas Ruf for many helpful discussions on T violation and Philipp Schubert for critically reading the manuscript.

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