Satellite disturbance detection and rejection with control moment gyros

Jung-Hyung Lee, Joon-Yong Lee and Hwa-Suk Oh*
1Korea Aerospace University, Goyang, Korea

*E-mail: hsoh@kau.ac.kr

Abstract. Uncertain external disturbances either secular or oscillating make the satellite attitude be deviated or oscillating from the mission attitude. These disturbances should be detected in short and rejected with proper counter torques of actuators. Nonlinear second order sliding mode observer is one of the good algorithms that can be used for the detection of disturbances’ magnitude and frequencies. With a control law of the integration and dipole rejection filters based on the identified disturbance information, the actuator torques can compensate the disturbances. The spacecraft attitude is then stabilized at the mission attitude within a tolerable error. In this paper, the performances of detection and rejection algorithms are checked respectively in computer simulations. The applicability is proved on a satellite ground simulator recently developed in Korea Aerospace University. The results show that the algorithms work properly with small error, which is due to the low grade of sensors and actuators.

1. Introduction
Satellite on low earth orbit is perturbed by secular and/or oscillating disturbances like aerodynamic or gravity gradient torques, etc. The disturbance torques make the spacecraft attitude be biased and oscillating from the mission attitude.[1] Although the spacecraft attitude is stabilized about an equilibrium point by a feedback control, the attitude oscillates on the disturbances frequencies in steady state. The external disturbances thus need to be rejected so properly by counter torques of actuators that the attitude oscillation may be decreased. The disturbance rejection filters against the oscillating disturbance have demanded the exact knowledge of disturbance frequencies.[2,3] Observers have been suggested as disturbance detector. Sliding mode observer, a model-based observer, is one of such detectors.[4-6] In this paper, a continuous second order sliding mode observer is applied for the detection of disturbances.[7] It is followed by FFT for the frequency identification.[8] Rejection filter based on the frequency information can be used for the rejection of oscillating disturbance and a PID type control law for the bias error compensation, respectively.[9] The spacecraft attitude is then stabilized at the mission attitude with much smaller error in the cost of oscillating actuator signal. Spacecraft Control Laboratory in Korea Aerospace University(KAU) has recently developed a satellite ground simulator to check the feasibility of the new algorithms. The simulator is actuated by four control moment gyros(CMGs) and equipped with sun sensor and gyros for attitude measurement. It is supported by air-bearing and free to rotation to emulate the satellite rotational motion. For the completion of contents, the back ground theories of the developed detection and rejection algorithms introduced in references in [7] and [9] are repeated again in the next section and followed by hardware simulations at next sections.
2. Spacecraft model on the external disturbances

Consider the motion of the satellite with four-single gimbal CMGs, which is governed by the following equations:

\[
I \ddot{\omega} + \omega \times I \dot{\omega} + \omega \times h = u_c + u_d \\
D \dot{\sigma} = -u_c 
\tag{1}
\]

Vector \( \omega \) represents the satellite angular velocity and \( u_c \) is the control torque generated by steering the CMG’s gimbal angle \( \sigma \). The matrix \( D \) is composed of the output torque vectors \( d_i \)'s of the i-th CMG as \( D = [d_1 \ d_2 \ d_3 \ d_4] \), which varies with the changes of \( \sigma_i \)'s. The momentum vector \( h \) is the sum of each CMG’s wheel momentum. The speed of momentum wheel is normally assumed constant. The vector \( u_d \) is the external disturbance torque to be identified and rejected. Four CMGs can be used as compensating devices for the disturbance rejection as well as attitude control actuators in normal missions. They are installed in a pyramid configuration, as shown in figure 1. A conventional PD type attitude control law of quaternion \( q \) and angular velocity \( \omega \) is applied for the stabilizing attitude and a singular-robust(SR) steering law for the gimbal steering, respectively as Eqs.(2).[10]

\[
u_c = \hat{\omega}^T I \dot{\omega} + \omega \times h - K_D \omega - K_P q
\]

\[
\dot{\sigma} = D^+ u_c
\tag{2}
\]

![Figure 1. Four CMGs on a pyramid configuration.](image)

3. Disturbance detection

3.1. Disturbance observer design

When the external disturbance is exerted on the satellite body, the satellite attitude is then affected immediately. As a monitoring tool of the disturbance, a nonlinear sliding observer [7] is used here as

\[
\hat{\dot{\omega}} + \hat{\omega} \times \hat{I} \dot{\omega} + \hat{\omega} \times h = u_c - \hat{h}
\tag{3}
\]

where \( \hat{\omega}, \hat{I} \) are the estimate of \( \omega \) and \( I \), respectively. The vector \( v \) is the observer-switching vector to be designed. If the state estimate error is defined as \( e = \omega - \hat{\omega} \), then

\[
\dot{e} = f + I^{-1} u_d + v
\tag{4}
\]

where \( f \) is the system error due to \( e \). The switching term \( v \) is designed by the second order sliding mode.[4-8] Let \( S \) be a sliding surface satisfying

\[
\ddot{S} + z_o \dot{S} = \dot{e} + ce
\tag{5}
\]

where \( z_o \) and \( c \) are gains making \( S \rightarrow 0 \). Consider a Lyapunov candidate \( V \) as

\[
V = \frac{1}{2} (\dot{S}^T \dot{S} + S^T I S)
\tag{6}
\]

By choosing \( v \) as \( v = \dot{z}_o \dot{S} - ce - I \dot{S} - d \dot{S} \), then

\[
\dot{V} = \dot{S}^T [f + I^{-1} u_d - d \dot{S}] \leq \| S \| \| f + I^{-1} u_d \| - d \| \dot{S} \|
\tag{7}
\]
With a proper large gain $d$, $\dot{V} \leq 0$ until $\|\tilde{S}\| = \|f + I^{-1}u_D\| = 0$ and $V \leq V_U$. That is, $V$ and $\dot{S}$ are bounded narrowly with a small initial $S_0$, such that $e \to 0$. Finally, the disturbance is estimated as $\hat{u}_D \to -I(f + v)$.

### 3.2. Disturbance detection feasibility

We check the feasibility of the detection algorithm on a computer simulation first. Assume that at time $t = 20$ sec, the secular disturbance $u = [0.1 0 0]$ Nm acts on the $x$-axis of a satellite and the oscillating disturbance $u = [0 \ 0.01\sin 2\pi fDt 0]$ Nm on the $y$-axis, simultaneously. Here, the external disturbance frequency is set as $f_D = 1$ Hz. The observer works every 0.005 sec, i.e., 200 times faster than the disturbance frequency. As shown in the results of figures 2(a) and (b) below, the roll attitude is stabilized but with some error from origin due to the secular disturbance, and the pitch angular velocity oscillates due to the oscillating pitch disturbance. The disturbance is thus well detected and the magnitude and frequency can be estimated exactly as in (c) and (d). Since the non-zero secular control torque is required for the attitude stabilization at the steady state as shown in (e), it thus demands the constant actuator torque as in (f) and eventually induces the momentum saturation. Although the average attitude is stabilized around the equilibrium point by the feedback control, the attitude oscillates at the disturbances frequencies in steady state. The external disturbances thus need to be rejected so properly by counter torques of actuators that the attitude is maintained as near as possible to the mission attitude.

![Figure 2](image.png)

**Figure 2.** Responses to secular and oscillating disturbances.

### 4. Disturbance rejection
4.1. Secular disturbance rejection design
When a PD type attitude control law as Eq.(2) is applied on the satellite in the existence of secular external disturbance, there exists a steady state error in attitude deviated from the mission attitude as shown in figure 2(a). The integral type feedback can be used to eliminate this steady state error.[9] A PID control law is considered here as follows:

\[ u_c = \omega^2 \int (\omega - \omega_0)^2 - K_p \omega - K_v q - K_i \int q dt \]  

(8)

4.2. Oscillating disturbance rejection design
For the rejection of oscillating disturbance, a dipole type filter can be used with the knowledge of disturbance frequency as suggested in Refs.[2-3,9]. In this paper, the disturbance magnitude and frequency are identified exactly from the disturbance observer and FFT. Then the dipole filter implemented as Eq.(9) is applied in addition to the PID control law Eq.(8).

\[ \dot{x}_1 = x_2 \\
\dot{x}_2 = -\omega_p^2 x_1 + Ku_c \\
u_{ef} = x_1 + K_r r^2 u_c \] 

(9)

where \( r \equiv \left( \frac{\omega_p}{\omega_c} \right) \) the filter frequency ratio and \( K \) is the function of the filter ratio and root locus gain \( K_r \).

4.3. Disturbance rejection feasibility
On the existence of secular and oscillating disturbance from initial time, the rejection filter (9) is applied on time \( t = 60 \) sec and the PID control law (8) on time \( t = 100 \) sec respectively in order to see their effects separately. The results are shown in figure 3. The oscillating angular velocity is diminished immediately after applying the rejection filter at time \( t = 60 \) as shown in (d), and the attitude error has been vanished after applying PID control law at time \( t = 100 \) sec as shown in (a). These actions are the effects of the counter torque applied as in (f) which is generated by moving gimbals as in (h).
5. Verification on satellite ground simulator

5.1. Simulator structure
We now verify the performance of the detection and rejection algorithms on the satellite hardware ground simulator, which has been recently designed and manufactured at Spacecraft Control Laboratory in Korea Aerospace University as figure 4. The simulator has a table-top configuration capable of floating 100kg-class platform, with MOI of $I = \text{diag}\{20,15,15\}$, 1/10 scale model for a typical mid-class satellite. Attitude is measured by reference sensors: a sun-sensor and an accelerometer, which simulate the sun vector and earth vector, respectively. The angular velocity is measured by gyros as shown in figure 5. All sensors and actuators are not for the space flight model, but for the ground vehicle models with lower grade. CMGs are developed in SCL KAU as figure 6. Each CMG has a momentum wheel of 2/3 Nms angular momentum and a gimbal with maximum speed of 3 rad/sec.
5.2. Secular disturbance detection
Firstly, a 10g dummy weight are positioned on the edge of the simulator top at about time 20sec to assign a secular disturbance of 0.05 Nm on the simulator. Since no attitude control is assigned, the body starts to move immediately and diverges soon as figure 7. The estimated magnitude of disturbance decreases as the moment arm becomes shortened as expected.

![Figure 7. Estimation of secular disturbance.](image)

5.3. Oscillating disturbance detection
Since the oscillating disturbance is hard to generate externally, it is substituted by internal disturbance. The yaw oscillating disturbance is thus generated by oscillating the speed of momentum wheel of CMG 1 at about time 50sec. Yaw disturbance is estimated clearly as shown in figure 8(c) while the attitude oscillates.

![Figure 8. Estimation of oscillating disturbance.](image)

5.4. Secular disturbance rejection
To prove the performance of the suggested secular rejection algorithm, with a 10g dummy weight laid on the top, the PID control law (8) is applied on the simulator. CMGs are actuated to compensate the external secular disturbance as shown in figure 9(d)(e). The attitude are sustained at the nominal state well initially, but from near 90sec, the CMGs momentums become saturated and the attitude cannot be sustained any more as expected. The saturation of CMG system can be confirmed by figure 9(f), the singularity index figure.

Figure 9. CMG acting for the rejection of secular disturbance.

5.5. Oscillating disturbance rejection
In this case the pitch oscillating disturbance is generated by oscillating the wheel speed of MWA4, and the rejection is performed by remaining CMGs 1, 2 and 3. The attitude is oscillated by the oscillating disturbance at the beginning, but the magnitude of oscillating decreases after applying the rejection filter near times 200sec as shown in figure 10. Since only three among four CMGs actuates, the singularity condition becomes severer than normal four actuator case.
6. Conclusions
A continuous nonlinear second order sliding mode observer suggested for the detection of external disturbances exerted on satellites works well either in secular or oscillating disturbances. The conventional PID control law and the dipole type disturbance rejection filter against the corresponding secular and the oscillating disturbances have been implemented based on the information obtained from observer data. The performance of the rejection algorithms is proved in the satellite hardware ground simulator in KAU. The spacecraft attitude is well stabilized at the mission attitude with tolerable error after applying the rejection filter and PID control law.

Acknowledgments
This study was supported by a research project entitled Analysis of Control Moment Gyro Development for LEO High Agility Satellite (2017-15-009) funded by LIG Nex1 in Korea.

References
[1] M. Sidi, Spacecraft Dynamics and Control (Cambridge University Press, 1997)
[2] C. Hiberg, D. Bailey, and B. Wie, “Precision spacecraft pointing using single-gimbal control moment gyroscopes with disturbance,” J. of Guidance, Control, and Dynamics, 23,1(2000)
[3] J. Lau, et al, “Investigation of periodic disturbance identification and rejection in spacecraft,” J. of Guidance, Control, and Dynamics, 29,4(2006)
[4] J. Slotine and W. Li, Applied Nonlinear Control (Prentice Hall, 1991)
[5] W. Chen and M. Saif, “Robust fault detection in uncertain nonlinear systems via a second order sliding mode observer,” Proceedings of the 40th IEEE Conference on decision and control, Orlando, Fl. (2001)
[6] T. Jiang and K. Khorasani, “Fault detection, isolation, and reconstruction strategy for a satellite’s attitude control subsystem with redundant wheels,” IEEE (2007).
[7] H. Oh and J. Kim, “The Detection and Identification of Control Moment Gyros Faults,” IAA-AAS-DyCoSS2-14-13-05, Rome, Mar.(2014)
[8] H. Oh and D. Cheon, “Precision measurements of reaction wheel disturbances with frequency compensation process,” J. of Mechanical Science and Technology, 19,1(2005)
[9] H. Oh and J. Kim, “Disturbance Rejection based on the Sliding Mode Observer Data,” ICEAS, Hokkaido(2014)
[10] H. Oh and S. Vadali, “Feedback Control and Steering Laws for Spacecraft using Single Gimbal Control Moment Gyros,” J. of Astronautical Science, 39(1991)