ABOUT THE MAGNETIC FLUCTUATION EFFECT ON THE PHASE TRANSITION TO SUPERCONDUCTING STATE IN Al

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Abstract

The free energy and the order parameter profile near the phase transition to the superconducting state in bulk Al samples are calculated within a mean-field-like approximation. The results are compared with those for thin films.

1. Introduction

In this letter we discuss in details the fluctuation-induced weakly-first order phase transition in type I superconductors known as Halperin-Lubensky-Ma (HLM) effect [1]. Our numerical results for the free energy and the order parameter profile are presented for Al which is the best substance for an experimental observation of the effect. Three dimensional (3D), i.e., bulk Al samples are considered. The results are compared with those for quasi-2D (two dimensional) Al films [2]. The possibility for an experimental observation of the effect in Al is briefly discussed. The paper is intended to establish by a quantitatively precise evaluation of measurable physical quantities, the difference in the magnitude of the effect in three dimensional (3D) and quasi-2D samples. It seems interesting to justify the experimental search of the effect in suitable films of type I superconductors where the same effect is relatively strong and can be observed experimentally as predicted in Ref. [2]. This task includes an entire investigation of the quasi-2D and 3D cases as well as
a detailed description of the difference between them. Here we shall compare our results
with those in Refs. [1 2].

2. Model and Results

Let us remember that in 3D type I superconductors the magnetic fluctuations \( \delta \vec{H} = \nabla \times \delta \vec{A} \) at the critical point \( T_{c0} \) corresponding to zero mean magnetic field \( \vec{H}_0 = (\vec{H} - \delta \vec{H}) \) [3 4] produce a very small latent heat evaluated in Ref. [1] and a jump of the superconducting order parameter \( \psi \) which is calculated for the first time in the present report; \( \delta \vec{A} = (\vec{A} - \vec{A}_0) \) is the fluctuation part of the vector potential \( \vec{A} \) of the magnetic field \( \vec{H} \), whereas \( \vec{A}_0 \) corresponds to the mean value \( \vec{H}_0 \). The same type of fluctuation-induced first-order phase transition is predicted in the scalar electrodynamics, early universe theories, and in liquid crystals. Moreover, it is generally believed that the same transition should occur in any system, described by a gauge invariant interaction between a scalar field such as the superconducting order parameter \( \psi(\vec{x}) \) and a vector gauge field as the vector potential \( \vec{A}(\vec{x}) \) of the magnetic field \( \vec{H}(\vec{x}) = \nabla \times \vec{A}(\vec{x}) \) in superconductors, provided the characteristic lengths in the system satisfy certain conditions; see, e.g., Refs. [2 4 5]. These notes justify the significance of the effect and the importance of its investigation.

For our aims we shall use the “mean-field-like approximation” explained in Refs. [1 2 4]; the renormalization group treatment of the effect has been recently reviewed in Ref. [5]. Using the notations from Ref. [3] the Ginzburg-Landau free energy of a superconductor can be written in the form

\[
F = \int d^3x \left[ a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\hbar^2}{4m} \left| \left( \nabla - \frac{2ie}{\hbar c} \vec{A} \right) \psi \right|^2 + \frac{1}{16\pi} \sum_{i,j=1}^{3} \left( \frac{\partial A_i}{\partial x_j} - \frac{\partial A_j}{\partial x_i} \right)^2 \right].
\]  

(1)

Within the mean-field-like approximation [1], the fluctuations of the superconducting order parameter are neglected. Having in mind that in type I superconductors the only stable ordered phase is the Meissner phase, we shall consider the equilibrium order parameter as spatially independent: \( \psi(\vec{x}) \approx \psi_0 \); other details of this approximation and its limitations are given in Refs. [2 4]. For \( \vec{H}_0 = 0 \) we set \( \vec{A}_0 = 0 \) and, therefore, \( \vec{A} = \delta \vec{A} \).

The integration of the vector potential fluctuations \( \delta \vec{A}(\vec{x}) \) in the partition function is made with the help of a loop-like expansion [2]. The next step is the accomplishment of the Landau expansion of the free energy in power series of the magnitude \( |\psi_0| \) of the order parameter \( \psi_0 \). In this way, within the Landau expansion to order \( |\psi| \) we obtain the effective free energy \( f_{\text{eff}} = F_{\text{eff}}/V \), where V is the volume of the superconductor, and

\[
 f_{\text{eff}} = \tilde{a}|\psi|^2 + \frac{\tilde{b}}{2}|\psi|^4 + q|\psi|^3 + c|\psi|^6.
\]  

(2)

The above expression contains new Landau parameters,

\[
\tilde{a} = a + \frac{\rho_0 k_B T}{2\pi^2}, \quad \tilde{b} = b + \frac{\rho_0^2 k_B T}{2\pi^2\Lambda}.
\]  

(3)
renormalized by the fluctuation effects, and additional new parameters

\[ q = -\frac{\rho_0^{3/2}k_B T}{6\pi}, \quad c = -\frac{\rho_0^2k_B T}{18\pi^2\Lambda^3}, \quad (4) \]

which are also generated by the magnetic field fluctuations. In Eqs. (3) and (4), \( \rho_0 = (8\pi e^2/mc^2) \), and \( \Lambda = (\pi/\xi_0) \), where \( \xi_0 = (h^2/4m\alpha_0T_{c0}) \) is the so-called zero-temperature coherence length \( \xi \); \( \alpha_0 \) is related to the Landau parameter \( a \) through \( a = \alpha_0T_{c0}t \). Here \( t = (T - T_{c0})/T_{c0} \) is the reduced temperature distance from the critical temperature \( T_{c0} \). Note, that the \( |\psi|^6 \) term is derived for the first time. Moreover, the \( \rho_0 \)-term in \( \tilde{b} \), see Eqs. (3), which was neglected in Ref. [6] (as mentioned for the first time in Ref. [1]), is also calculated for the first time in the present investigation. As we shall see, the \( \rho_0 \)-contribution to \( \tilde{b} \) is small (\( \sim 0.1b \)) for 3D-systems, but it becomes relatively bigger (\( \sim b \)) in quasi-2D films [2] and generally cannot be omitted. In all other respects the Eqs. (2) - (4) are consistent with the results in the preceding papers [1][6].

The \( |\psi|^3 \)-term in Eq. (2) describes a first order transition (see, e.g., Ref. [4]). The theory shows [4] that this fluctuation-induced first order transition should be well established in type I superconductors with a small Ginzburg-Landau parameter \( \kappa = \lambda_0/\xi_0 \), where \( \lambda_0 \) is the so-called zero-temperature London penetration depth [3][4]. The Al is the type I superconductor with a minimal parameter \( \kappa \sim 10^{-2} \) and therefore, this substance is convenient for a discussion of the HLM effect, as mentioned for the first time in Ref. [1]. Using the experimental values for the critical temperature \( T_{c0} = 1.19K \), the zero-temperature coherence length \( \xi_0 = 1.6 \times 10^{-4} \) cm, and the zero-temperature critical magnetic field \( H_c(0) = 99 \) G, as well as applying certain relations between \( \alpha_0 \) and \( \xi_0 \), and between \( b \) and \( H_c(0) \) (see Ref. [3]), it is easy to calculate the effective free energy for Al:

\[ f_{\text{eff}}(\varphi) = 389.21\left\{2\left[t + 0.972 \times 10^{-4}(1 + t)\right] \varphi^2 + 1.117(1 + t)\varphi^4 - 0.7053 \times 10^{-2}(1 + t)\varphi^3 - 31.1(1 + t)\varphi^6\right\}, \quad (5) \]

where \( \varphi = |\psi_0|/|\psi_{00}| \) is the dimensionless order parameter; \( |\psi_{00}| \) denotes \( |\psi_0| \) at \( T = 0 \). The contribution of the \( \varphi^6 \)-term is very small for 3D Al samples and we shall neglect this term in our further discussion.

The free energy (5) is shown in Fig. 1 for five values of \( t \), and for \( c\varphi^6 \approx 0 \). The Fig. 1 gives for the first time a graphical image of the weakly-first order phase transition predicted in Ref. [1] for 3D superconductors of type I. All curves have a trivial minimum at \( \varphi = 0 \) which describes the normal state. The positive minima of the free energy \( f_{\text{eff}}(\varphi) \), \( f_{\text{eff}}^{\text{min}} > 0 \), corresponding to \( \varphi > 0 \) describe the superconducting Meissner phase which is metastable for the respective temperatures. The negative minima, \( f_{\text{eff}}^{\text{min}} < 0 \), corresponding to \( \varphi > 0 \) represent the free energy of stable superconducting states and occur in certain narrow temperature interval, approximately evaluated for the first time in Ref. [1]. The curve marked with circles (\( \circ \)) exhibits a minimum \( f_{\text{eff}}^{\text{min}}(\varphi = 0.00316) = 0 \) which is equal to the free energy of the normal phase. Therefore, this minimum corresponds to the equilibrium phase transition temperature.
Figure 1: Curves representing the free energy function (5) for $c \equiv 0$ and five values of $t$: $t = 9 \times 10^{-6}$ (□), $t = 6.263 \times 10^{-6}$ (+), $t = 5.567 \times 10^{-6}$ (○), $t = 4.800 \times 10^{-6}$ (◇), $t = 3.000 \times 10^{-6}$ (−).

Figure 2: Order parameter profile near $T_{c0}$. The vertical line at $t = 5.567 \times 10^{-6}$ indicates the equilibrium jump of the order parameter.
The standard analysis [4] of the equations $f_{\text{eff}}(\varphi) = 0$ and $(\partial f_{\text{eff}}/\partial \varphi) = 0$ gives the properties of the order parameter $\varphi$. This important quantity is depicted in Fig. 2 as a function of the reduced temperature $t$. Fig. 2 shows that the stable superconducting states, corresponding to a negative effective free energy occur for temperatures defined by $0 < t < t_{\text{eq}} = 5.567 \times 10^{-6}$, i.e. for $T_{c0} < T < T_{\text{eq}} = (1 + 5.567 \times 10^{-6})T_{c0}$. At the equilibrium transition temperature ($T = T_{\text{eq}}$) the normal and the superconducting phases are equally stable, and from $T > T_{\text{eq}}$ up to the temperature $T^* = (1 + 6.262 \times 10^{-6})T_{c0}$ defined by $t^* = 6.262 \times 10^{-6}$, the superconducting phase is metastable. Above $T^*$ the equation $f_{\text{eff}}(\varphi) = 0$ has no solutions of type $\varphi > 0$, and therefore the 3D Al does not possess any superconducting states. At the equilibrium phase transition point $T_{\text{eq}}$ the order parameter $\varphi$ undergoes an equilibrium jump from the value $\varphi = 0.00316$ to zero, provided quite special circumstances do not ensure an overheating of the superconducting states. The latter possibility is usual for first order transitions, where metastable states above the equilibrium phase transition temperature are possible.

The calculated value of the equilibrium jump of the order parameter is very small for an experimental observation by transport experiments. This is so because of the weakness of the HLM effect for 3D. As pointed out in a recent study [2, 7], the same effect is much better pronounced in quasi-2D Al and, therefore, transport experiments in such films could be successful.

3. Conclusion

For a better comparison between the 3D and quasi-2D Al superconductors, let us note that the order parameter jumps for thin Al films of thicknesses 0.1$\mu$m and 1$\mu$m are $\varphi = 0.032$ and $\varphi = 0.013$, respectively [7]. Besides, the temperature differences ($T_{\text{eq}} - T_{c0}$) and ($T^* - T_{c0}$) are about $10^3$ times bigger than the respective differences for 3D Al discussed above. These results imply that the expected specific heat capacity and latent heat of the first order phase transition in quasi-2D Al films are much bigger than the respective quantities in 3D Al samples. So, the thermodynamic experiments done with Al films may prove the HLM as well.
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