Vector boson production in association with KK modes of the ADD model to NLO in QCD at LHC

M. C. Kumar\textsuperscript{a} \, Prakash Mathews\textsuperscript{b} \, V. Ravindran\textsuperscript{a} \, Satyajit Seth\textsuperscript{b}

\textsuperscript{a} Regional Centre for Accelerator-based Particle Physics
Harish-Chandra Research Institute, Chhatnag Road, Jhunsi,
Allahabad 211 019, India

\textsuperscript{b} Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India

Abstract

Next-to-leading order QCD corrections to the associated production of vector boson ($Z/W^\pm$) with the the Kaluza-Klein modes of the graviton in large extra dimensional model at the LHC, are presented. We have obtained various kinematic distributions using a Monte Carlo code which is based on the two cut off phase space slicing method that handles soft and collinear singularities appearing at NLO level. We estimate the impact of the QCD corrections on various observables and find that they are significant. We also show the reduction in factorization scale uncertainty when QCD corrections are included.

Key words: Large Extra Dimensions, NLO QCD

\textsuperscript{1}mckumar@hri.res.in
\textsuperscript{2}prakash.mathews@saha.ac.in
\textsuperscript{3}ravindra@hri.res.in
\textsuperscript{4}satyajit.seth@saha.ac.in
1 Introduction

With the onset of the Large Hadron Collider (LHC) era, unique opportunity to probe the realm of new physics in the TeV scale has begun. Models with extra spatial dimensions and TeV scale gravity, are proposed to address the large hierarchy between the electroweak and Planck scale and are expected to provide a plethora of new and interesting signals.

The extra dimension model proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) \cite{1}, was the first extra dimension model in which the compactified dimensions could be of macroscopic size and consistent with present experiments. A viable mechanism to hide the extra spatial dimension, is to introduce a 3-brane with negligible tension and localise the Standard Model (SM) particles on it. Only gravity is allowed to propagate in the full 4 + \(\delta\) dimensional space time. As a consequence of these assumptions, it follows from Gauss Law that the effective Planck scale \(M_P\) in 4-dimensions is related to the 4 + \(\delta\) dimensional fundamental scale \(M_S\) through the volume of the compactified extra dimensions \cite{1}. The extra dimensions are compactified on a torus of common circumference \(R_S\). The compactified extra dimensions are flat, of equal size and could be large. The large volume of the compactified extra spatial dimensions would account for the dilution of gravity in 4-dimensions and hence the hierarchy. Current experimental limits on deviation from inverse square law \cite{2}, constraint the number of possible extra spatial dimensions \(\delta \geq 2\). The space time is factorisable and the 4-dimensional spectrum consists of the SM confined to 4-dimensions and a tower of Kaluza-Klein (KK) modes, of the graviton propagating in the full 4 + \(\delta\) dimensional space time.

The interaction of the KK modes \(G_{(\vec{n})}^{\mu\nu}\) with the SM fields localised on the 3-brane is described by an effective theory given by \cite{3, 4}

\[
\mathcal{L}_{\text{int}} = -\frac{1}{M_P} \sum_{\vec{n}=0}^{\infty} T^{\mu\nu}(x) \, G_{\mu\nu}^{(\vec{n})}(x),
\]

where \(T^{\mu\nu}\) is the energy-momentum tensor of the SM fields on the 3-brane and \(M_P = M_P/\sqrt{8\pi}\) is the reduced Planck mass in 4-dimensions. The relation between the 4-dimensional coupling, the volume of the extra dimensions and the fundamental scale \(M_D\) in 4 + \(\delta\)-dimensions

\[
\overline{M}_P^2 = R_D^\delta M_D^{\delta+2}.
\]

The size of the extra dimension \(R_S\) is related to the radius \(R_D\), \(R_S = 2\pi R_D\). The fundamental scales in 4 + \(\delta\) dimensions, as defined in \cite{3} \(M_D\) is related to \(M_S\) \cite{4} as: \(8\pi M_D^{\delta+2} = M_S^{\delta+2} S_{\delta-1}\), where \(S_{\delta-1} = 2\pi^{\delta/2}/\Gamma(\delta/2)\) is the surface area of a unit sphere in \(\delta\) dimensions.

The zero mode corresponds to the usual 4-dimensional massless graviton and higher massive KK modes are labeled by \(\vec{n} = (n_1, n_2, \cdots, n_\delta)\). The masses of the individual KK modes are \(m_{\vec{n}}^2 = \vec{n}^2/R_D^2\) and the mass gap between adjacent KK
modes is $\Delta m = R_D^{-1}$. For not too large $\delta$ the discrete mass spectrum could be replaced by a continuum, with the density of KK states

$$\rho(m_{\vec{n}}) = \frac{1}{2} S_{\delta-1} R_D^4 m_{\vec{n}}^{\delta-2}. \quad (3)$$

For an inclusive cross section at the collider, we have to sum over all accessible KK modes and hence cross section for the production of an individual KK mode of mass $m_{\vec{n}}$ has to be convoluted with the density of states $\rho(m_{\vec{n}})$. The discrete sum $\sum_{\vec{n}}$ can be replaced by $\int dm^2 \rho(m_{\vec{n}})$, and hence the inclusive cross section for the tower of KK modes is

$$d\sigma = S_{\delta-1} \frac{M_P^2}{M_D^{2+\delta}} \int dm^2 m^{\delta-1} d\sigma_D^{(m)}, \quad (4)$$

where $d\sigma_D^{(m)}$ is the cross section to produce a single KK mode. The cross section for an individual KK mode is suppressed by the coupling factor $(2M_P^2)^{-1}$, the high multiplicity of accessible KK modes at the collider would compensate, leading to the exciting possibility of observing low scale quantum gravity effects at the LHC. The additional $1/2$ factor is due to the definition of the sum of polarisation of the KK modes $[3]$. The $\sum_{\vec{n}}$ is kinematically constrained to those KK modes which satisfy $m_{\vec{n}} = |\vec{n}|/R_D < \sqrt{s}$, where $\sqrt{s}$ is the partonic center of mass energy or as the case may be the available energy to produce the KK modes.

Viable signatures of the ADD scenario at the LHC are possible by the exchange of virtual KK modes between the SM particles, leading to an enhanced cross section or by the emission of real KK modes from the SM particles, leading to a missing energy signal. Various such processes have been extensively studied in this model, most of which have been considered only up to leading order (LO) in QCD $[3, 4, 5, 6, 7]$. These LO approximations at the hadron colliders suffer from large factorisation and renormalisation scale dependence which for some processes could be as large as a factor of two. These issues go beyond normalisation of a cross section by a $K$-factor as the shapes of distributions may not be modeled correctly and in addition the LO cross sections are strongly dependent on the factorisation scale. It is hence essential to evaluate the next-to-leading order (NLO) corrections to the process of interest to provide quantitatively reliable predictions. NLO QCD corrections to extra dimension models have been studied for dilepton $[8]$, boson pair $[9, 10]$ productions, and real graviton production processes such as graviton plus jet $[11]$ and graviton plus photon $[12]$. Searches at the Tevatron using the single photon or jet with missing transverse energy have been used to put bounds on extra dimensional scale $M_D$ for different number of extra dimensions $[13, 14]$. The same signal has been simulated for the LHC at the ATLAS detector $[15]$, discovery limit and the methods to determination of the parameters of the extra dimensional models are discussed.

In this paper we consider the graviton production in association with a vector boson at the hadron colliders at NLO in QCD. Z-boson process to LO had been
considered at LEP [16] and simulation studies for the $Z + G_{KK}$ modes production at the LHC was studied to LO [17] as a complement to the more conventional channels.

2 Vector bosons in association with KK modes

The associated production of $Z$-boson with the KK modes of the ADD model leads to missing energy signals at the hadron colliders. We begin by discussing the neutral weak gauge boson ($Z$) production in association with the KK modes of the ADD large extra dimensional model to NLO in QCD and would consider the charged weak gauge bosons ($W^\pm$) towards the end.

At the hadron collider, the associated production $P P \rightarrow Z G_{KK} X$ at LO proceeds via the quark, anti-quark annihilation process $q \bar{q} \rightarrow Z G_{KK}$. There are four diagrams that would contribute to this process, which corresponds to the KK modes of the graviton being emitted of a fermion leg, $Z$-boson or the $q \bar{q} Z$ vertex. The Feynman rules to evaluate the matrix elements are given in [3, 4] and for the vector boson, unitary gauge ($\xi \rightarrow \infty$) is used. Summation of the polarisation tensor of the KK modes is given in [3]. It can be seen that the terms proportional to the inverse powers of KK mode mass $m^2$ vanish on expressing the matrix element square in terms of independent variable and is an useful check.

The NLO calculation presented here uses both analytical and Monte Carlo integration methods and hence is flexible to incorporate the experimental cuts and can generate the various distributions unlike a fully analytical computation. Our code is based on the method of two cutoff phase space slicing [18] to deal with various singularities appearing in the NLO computation of the real diagrams and to implement the numerical integrations over phase space. The analytical results are evaluated using the algebraic manipulation program FORM [19]. The real and virtual corrections have been evaluated in the massless quark limit. We use dimensional regularisation with space time dimensions $d = 4 + \epsilon$. To deal with $\gamma_5$ in $d$-dimensions, we use the completely anti-commuting $\gamma_5$ prescription [20].

The order $O(\alpha_s)$ corrections to the associated production of the $Z$-boson and KK modes of the graviton come from the following $2 \rightarrow 3$ real diagrams (a) $q \bar{q} \rightarrow Z G_{KK}$, (b) $q(\bar{q}) g \rightarrow q(\bar{q}) Z G_{KK}$. There are 14 diagrams that contribute to the quark antiquark annihilation process and can be classified into diagrams where the KK modes couples to the (i) fermion legs, (ii) $Z$-boson leg, (iii) gluon leg, (iv) $q \bar{q} Z$ vertex and (v) $q \bar{q} g$ vertex. The unitary gauge is used for the $Z$ boson propagator and Feynman gauge for the gluon propagator. Note that the gauge parameters influence the coupling of KK modes to the SM fields [4]. For the sum over the polarisation vectors we have retained only the physical degrees of freedom.

The real diagrams involving gluons and massless quarks would be singular in soft and collinear regions of the 3-body phase space integration. Two small slicing parameters $\delta_s$ and $\delta_c$ are introduced to isolate regions of phase space that are
sensitive to soft and collinear singularities. Rest of the region is finite and can be
evaluated in 4-dimensions. Phase space integrations in the mutually exclusive soft
and collinear regions are performed not on the full matrix element but in the leading
pole approximation of soft and collinear regions in $4 + \epsilon$ dimensions. The soft
and collinear poles now appear as poles in $\epsilon$ and in addition the soft part would depend
logarithmically on the soft cut-off $\delta_s$ while the collinear part would depend on the
both $\delta_s$ and $\delta_c$. All positive powers of the small cut-off parameters are set to be zero.
The phase space degrees of freedom that remain, correspond to a 2-body process
and can now be combined with the virtual diagram.

The virtual corrections to the annihilation process to order $\mathcal{O}(\alpha_s)$ can be obtained
by considering the gluonic virtual corrections to the vertex and wave function renor-
malisation for the process $q \bar{q} \rightarrow Z$ and then attaching the KK modes to all possible
legs and vertex as allowed by the Feynman rules [3, 4]. This would generate 27, one
loop $2 \rightarrow 2$ diagrams which on multiplication with the $q \bar{q} \rightarrow Z G_{KK}$ at LO would
give all the virtual contributions for the annihilation process. Performing the loop
integrals in $4 + \epsilon$ dimensions would lead to poles in $\epsilon$ in the soft and collinear regions
and combining this with the real diagrams would lead to the cancellation of the soft
singularities. The remaining collinear singular terms which appear as po les in $\epsilon$ are
systematically removed by collinear counter terms in the \( \overline{MS} \) factorisation scheme,
at an arbitrary factorisation scale $\mu_F$. The resultant expression is now finite but
depends on the cut-off parameters $\delta_s$ and $\delta_c$ logarithmically. Combining this $2 \rightarrow 2$
part with the finite $2 \rightarrow 3$ part and performing the phase space integration for the
various distributions, it is expected that the results be independent of the choice of
the slicing parameters $\delta_s$ and $\delta_c$.

The $q(\bar{q}) g \rightarrow q(\bar{q}) Z G_{KK}$ process begins at $\mathcal{O}(\alpha_s)$ so does not get any virtual
corrections to this order and can be obtained from the $2 \rightarrow 3$ annihilation diagrams
by using the crossing symmetry. The analytical results of LO $2 \rightarrow 2$, finite part of
NLO virtual corrections and full $2 \rightarrow 3$ matrix elements that go into our numerical
code will be presented in the longer version [21].

3 Numerical results

In this section we present the numerical results for the associated production of the
$Z$-boson with the KK modes at the LHC ($\sqrt{S} = 14$ TeV) to NLO in QCD. The
mass of the $Z$-boson and the weak mixing angle are taken to be $m_Z = 91.1876$ and
$\sin^2\theta_w = 0.2312$ respectively. The fine structure constant is taken to be $\alpha = 1/128$.
Through our computation we have used CTEQ6 L/M parton density sets [22]
and $n_f = 5$ light quark flavors. The two loop running strong coupling constant in
the $\overline{MS}$ scheme has been used with the corresponding $\Lambda_{QCD} = 0.226$ GeV. Unless
mentioned otherwise, both the renormalization and the factorization scales are set
to $\mu_R = \mu_F = p_T^Z$, the transverse momentum of the $Z$-boson. Further, the following
cuts have been implemented in our numerical code:

\[ p_T^Z, \ p_T^{\text{miss}} > p_T^{\text{min}} \ \text{and} \ |y^Z| < 2.5 \] (5)

where \( y^Z \) is the rapidity of the \( Z \)-boson, \( p_T^{\text{min}} = 400 \) GeV and the missing transverse momentum \( p_T^{\text{miss}} \) is given by

\[ p_T^{\text{miss}} = p_T^Z (p_T^G) \ \text{for} \ p_T^{\text{jet}} < 20 ( > 20) \ \text{GeV} \] (6)

At LO, the missing transverse momentum \( p_T^{\text{miss}} \) is the same as \( p_T^Z \). The additional jet at NLO can be soft or hard; For the soft jet the \( p_T^{\text{miss}} \) is the same as \( p_T^Z \) while for hard jet \( p_T^{\text{miss}} \) is \( p_T^G \) of the KK modes (Eq. (6)). In addition to the above, we have put a cut on the pseudo rapidity of the jet \( |\eta^{\text{jet}}| < 2.5 \) for \( p_T^{\text{jet}} > 20 \) GeV.

![Variation with \( \delta_s \)](image)

Figure 1: Dependence of the order \( \alpha_s \) contribution to the transverse momentum distribution of the \( Z \)-boson at the LHC, on the slicing parameter \( \delta_s \) (top) with \( \delta_c = \delta_s / 100 \) and for \( M_D = 3 \) TeV and \( \delta = 4 \). Below the variation the sum of 2-body and 3-body contributions is contrasted against the value at \( \delta_s = 10^{-3} \).

We check the stability of our results against the variation of the slicing parameters \( \delta_s \) and \( \delta_c \) introduced in the slicing method. In Fig. \( \Pi \) we present the dependency
of both the 2-body and 3-body pieces of the $O(\alpha_s)$ contribution on $\delta_s$, keeping the ratio $\delta_s/\delta_c = 100$ fixed. These results are obtained for a specific choice of the ADD model parameters $M_D = 3$ TeV and $\delta = 4$. It can be seen from the figure that the sum of these two pieces is positive and is fairly stable for a wide range of $\delta_s$. The positive sum implies that the QCD corrections do enhance the leading order predictions. In the rest of our work, we choose $\delta_s = 10^{-3}$ and $\delta_c = 10^{-5}$.

In Fig. 2 we present the transverse momentum distribution of the $Z$-boson at the LHC to NLO in QCD and its dependence on the number of extra dimensions $\delta$ for $M_D = 3$ TeV. In each of the distributions corresponding to $\delta = 2, 4, 6$ the QCD effects are found to have increased the leading order predictions considerably. In this transverse momentum distribution of the $Z$-boson, the $K$-factor, defined as the ratio of NLO cross sections to the LO ones, is found to increase with $p_T^Z$ and vary from 1.1 to 1.46 depending on the number of extra dimensions $\delta$. In a similar way, the missing transverse momentum distribution is shown in the left panel of Fig. 3.

The ADD model which is an extension of the SM to address the hierarchy problem is an effective theory and the UV completion of the TeV scale gravity has to be quantified. For the real KK mode production process, the kinematical constraint discussed earlier, provides a natural UV cutoff on the integration over the n-sphere, but the hard scattering scales involved at the LHC energies can be close to the fundamental scale $M_D$. To study the sensitivity of our results close to the UV region, we choose $\delta_s = 10^{-3}$ and $\delta_c = 10^{-5}$.

![Graph of Transverse momentum distribution](image)

Figure 2: Transverse momentum distribution of the $Z$-boson (left) and the corresponding K-factors (right) for the associated production of the $Z$-boson and the KK mode at the LHC, for $M_D = 3$ TeV and $\delta = 2, 4, 6$. 

The ADD model which is an extension of the SM to address the hierarchy problem is an effective theory and the UV completion of the TeV scale gravity has to be quantified. For the real KK mode production process, the kinematical constraint discussed earlier, provides a natural UV cutoff on the integration over the n-sphere, but the hard scattering scales involved at the LHC energies can be close to the fundamental scale $M_D$. To study the sensitivity of our results close to the UV region, we choose $\delta_s = 10^{-3}$ and $\delta_c = 10^{-5}$. 

![Graph of Transverse momentum distribution](image)
we compare at LO and NLO the $p_T$ distribution of $Z$-boson wherein the invariant mass of the KK mode and $Z$-boson $Q_{ZG}$ is (a) computed only when $Q_{ZG} < M_D$ (truncation) and (b) computed for all possible values of $Q_{ZG}$ (un-truncation). In the right panel of Fig. 3 we compare the results for the ADD model parameters, $M_D = 5 \text{ TeV}$ and $\delta = 4$ at LO and NLO for the $p_T$ distribution. As compared to the un-truncated distribution, the percentage difference is tabulated below for LO and NLO for a few values of $p_T^Z$.

| $p_T^Z$ (GeV) | LO | NLO |
|--------------|----|-----|
| 500          | 11 | 7.8 |
| 1000         | 23.5 | 20.2 |
| 1500         | 44.9 | 41.8 |

The difference between the un-truncated and truncated results become larger with (a) increase in $p_T^Z$, (b) increase in the number of extra dimensions $\delta$ and (c) decrease in the fundamental scale $M_D$. Further it is seen that the NLO QCD corrections do decrease the difference between the un-truncated and truncated results.

Finally, we study the dependence of both LO and NLO cross sections on the factorization scale $\mu_F$ by varying it from 0.2 $p_T^Z$ to 2.0 $p_T^Z$. One of the motivations for the computation of the QCD corrections is to minimise the scale uncertainties.
Figure 4: Scale uncertainty in the transverse momentum distribution of the $Z$-boson at the LHC (left), for a variation of the factorization scale $\mu_F$ in the range $[0.2, 2.0] \ p_T^Z$ and for the choice of $M_D = 3$ TeV and $\delta = 4$. The transverse momentum distribution of the $Z$-boson for $\sqrt{S} = 7$ TeV at the LHC (right).

by computing the cross sections to higher orders in the perturbation theory. As expected, the scale uncertainties in the leading order predictions are considerably decreased after incorporating the one-loop QCD corrections to the associated production of the $Z$-boson and the KK modes at the LHC. The results are shown in the Fig. 4 (left) for the case of transverse momentum distribution of the $Z$-boson, for $M_D = 3$ TeV and $\delta = 4$. In Fig. 4 (right) we have also plotted the $p_T$ distribution for the current LHC energies of $\sqrt{S} = 7$ TeV. The K-factor ranges from 1.05 to 1.13 for $500 < p_T^Z < 1000$ GeV. The contribution of $q(\bar{q}) g$ subprocess for the $\sqrt{S} = 7$ TeV is much smaller than the contribution at $\sqrt{S} = 14$ TeV and that accounts for the much lower K-factor.

It is interesting to note here that a similar analysis goes through for the associated production of $W^\pm$ and the KK modes in the large extra dimensional model at the LHC. The difference lies both in the couplings of the quarks to the weak bosons and in the respective parton fluxes to be convoluted with the partonic cross sections. In Fig. 5 we have plotted the $W^-\bar{W}$ distribution for $\sqrt{S} = 14$ TeV at LO and NLO. For the $p_T^W$ distribution the K-factor is in the range 1.25 - 1.37 for $500 < p_T^W < 1000$ GeV. A complete analysis of the associated production of $W^\pm$ in association with KK modes of the ADD model will be presented in the longer version [21].
4 Conclusion

To conclude, we have computed the NLO QCD corrections to the associated production of the vector boson at the LHC, using semi-analytical two cutoff phase space slicing method. We have presented results for $\sqrt{S} = 14$ and 7 TeV. Our results are checked for the stability against the variation of the slicing parameters $\delta_s$ and $\delta_c$. We have studied the truncated as well as the un-truncated transverse momentum distributions of the $Z$-boson, together with the missing transverse momentum distribution and their dependence on the number of extra dimensions $\delta$. The NLO QCD corrections are found to have not only enhanced the LO cross sections considerably, with the K-factors ranging from 1.1 to 1.46 depending on the $\delta = 2, 4, 6$ for $M_D = 3$ TeV, but also decreased their factorization scale uncertainties significantly.

Acknowledgments

The work of V.R. and M.C.K. has been partially supported by funds made available to the Regional Centre for Accelerator based Particle Physics (RECAPP) by the Department of Atomic Energy, Govt. of India. We would like to thank the cluster computing facility at Harish-Chandra Research Institute where part of computational work for this study was carried out. S.S. would like to thank UGC, New Delhi for financial support. S.S. would also like to thank RECAPP center for his
References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429 (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436 (1998) 257; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D59 (1999) 086004.

[2] D. J. Kapner et al. Phys. Rev. Lett. 98 (2007) 021101.

[3] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. B544 (1999) 3.

[4] T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D59 (1999) 105006.

[5] E. A. Mirabelli, M. Perelstein and M. E. Peskin, Phys. Rev. Lett. 82 (1999) 2236.

[6] Prakash Mathews, Sreerup Raychaudhuri, K. Sridhar, Phys. Lett. B450 (1999) 343; Phys. Lett. B455 (1999) 115; JHEP 0007 (2000) 008.

[7] X. G. Wu and Z. Y. Fang, Phys. Rev. D78 (2008) 094002.

[8] Prakash Mathews, V. Ravindran, K. Sridhar and W. L. van Neerven, Nucl. Phys. B713 (2005) 333; Prakash Mathews, V. Ravindran, K. Sridhar, JHEP 0510 (2005) 031; Prakash Mathews, V. Ravindran, Nucl. Phys. B753 (2006) 1; M.C. Kumar, Prakash Mathews, V. Ravindran, Eur. Phys. J. C49 (2007) 599.

[9] M.C. Kumar, Prakash Mathews, V. Ravindran, Anurag Tripathi, Phys. Lett. B672 (2009) 45; Nucl. Phys. B818 (2009) 28.

[10] Neelima Agarwal, V. Ravindran, V. K. Tiwari, Anurag Tripathi, Nucl. Phys. B830 (2010) 248; Phys. Lett. B686 (2010) 244; [arXiv:1003.5445 [hep-ph]]; [arXiv:1003.5450 [hep-ph]].

[11] S. Karg, M. Karämer, Q. Li, D. Zeppenfeld, [arXiv:0911.5095].

[12] X Gao, C S Li, J Gao and J Wang, Phys. Rev. D81 (2010) 036008.

[13] T. Aaltonen et al. CDF Collaboration, Phys. Rev. Lett. 101 (2008) 181602.

[14] V. M. Abazov et al. D0 Collaboration, Phys. Rev. Lett. 101 (2008) 011601.

[15] L. Vacavant and I. Hinchliffe, J. Phys. G27 (2001) 1839.

[16] Kingman Cheung and Wai-Yee Keung, Phys. Rev. D60 (1999) 112003.

[17] Stefan Ask, Eur. Phys. J. C60 (2009) 509.
[18] B.W. Harris, J.F. Owens, Phys. Rev. D 65 (2002) 094032.

[19] J. A. M. Vermaseren, math-ph/0010025

[20] M. Chanowitz, M. Furman and I. Hinchliffe, Nucl. Phys. B159 (1979) 225.

[21] M. C. Kumar, Prakash Mathews, V. Ravindran, Satyajit Seth in preparation.

[22] J. Pumplin et. al., JHEP 07 (2002) 012.