Isolating New Physics Effects from Hadronic Form Factor Uncertainties in $B \to K^{*}\ell^{+}\ell^{-}$

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The discovery of New Physics, using weak decays of mesons is difficult due to intractable strong interaction effects needed to describe it. We show how the multitude of “related observables” obtained from $B \to K^{*}\ell^{+}\ell^{-}$, can provide many new “clean tests” of the Standard Model. The hallmark of these tests is that several of them are independent of the unknown form factors required to describe the decay using heavy quark effective theory. We derive a relation between observables that is free of form factors and Wilson coefficients, the violation of which will be an unambiguous signal of New Physics. We also derive other relations between observables and form factors that are independent of Wilson coefficients and enable verification of hadronic estimates. We find that the allowed parameter space for observables is very tightly constrained in Standard Model, thereby providing clean signals of New Physics. The relations derived will provide unambiguous signals of New Physics if it contributes to these decays.

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Indirect searches for New Physics often involve precision measurement of a single quantity. The most well known example is the muon magnetic moment. Unfortunately, even though muon is a lepton, estimating hadronic contributions turn out to be the limiting factor in the search for New Physics. We show how certain B meson decays offer instead a multitude of related observables that enable handling intractable strong interaction effects that hamper the discovery of New Physics. It is hoped that flavor changing neutral current transitions of the $b$ quark will be altered by physics beyond the Standard Model (SM) and their study would reveal possible signal of New Physics (NP) if it exists. However, understanding the hadronic flavor changing neutral current decays requires estimating hadronic effects that are difficult to compute accurately. Experiments seem to indicate that new physics does not show up as a large and unambiguous effect in flavor physics. This has bought into focus the need for theoretically cleaner observables, i.e. observables that are relatively free from hadronic uncertainties. In the search for new physics, it is therefore crucial to effectively isolate the effect of new physics from hadronic uncertainties that contribute to the decay.

One of the modes that is regarded as significant in this attempt is $B \to K^{*}\ell^{+}\ell^{-}$, an angular analysis of which is known to result in a multitude of observables [1, 2], that enable testing the SM and probing possible NP contributions [2–6]. Experimentally, angular analysis of $B \to K^{*}\ell^{+}\ell^{-}$ decays has already been studied and a few of the observables already measured [7–10]. The decay $B(p) \to K^{*}(k)\ell^{+}(q_{1})\ell^{-}(q_{2})$ with $K^{*}(k) \to K(k)\pi(k_{2})$ on the mass shell, is completely described by four independent kinematic variables. These are the lepton-pair invariant mass squared $q^{2} = (q_{1} + q_{2})^{2}$, the angle $\phi$ between the decay planes formed by $\ell^{+}\ell^{-}$ and $K\pi$ respectively and the angles $\theta_{K}$ and $\theta_{\ell}$ of the $K$ and $\ell^{-}$ respectively with the $z$-axis defined in the respective rest frames of $K^{*}$ and $\ell^{+}\ell^{-}$, assuming that $K^{*}$ has momentum along the $+z$ in $B$ rest frame. The differential decay rate is,

$$
\begin{align*}
\frac{d^{4} \Gamma(B \to K^{*}\ell^{+}\ell^{-})}{d^{2}q^{2} d^{2} \Omega} &= \frac{9}{32\pi} \left[ (I_{1}^{+} + I_{2}^{+}\cos 2\theta_{\ell}) \sin^{2} \theta_{K} \\
+ (I_{1}^{-} + I_{2}^{-}\cos 2\theta_{\ell}) \cos^{2} \theta_{K} + I_{3} \sin^{2} \theta_{K} \sin^{2} \theta_{\ell} \cos 2\phi \\
+ I_{4} \sin 2\theta_{K} \sin 2\theta_{\ell} \cos \phi + I_{5} \sin 2\theta_{K} \sin \theta_{\ell} \cos \phi \\
+ I_{6}^{+} \sin^{2} \theta_{K} \cos \theta_{\ell} + I_{7} \sin 2\theta_{K} \sin \theta_{\ell} \sin \phi \\
+ I_{8} \sin 2\theta_{K} \sin 2\theta_{\ell} \sin \phi + I_{9} \sin^{2} \theta_{K} \sin^{2} \theta_{\ell} \sin 2\phi\right].
\end{align*}
$$

(1)

where, $d^{2} \Omega = d \cos \theta_{\ell} d \cos \theta_{K} d \phi$ and $I_{1}^{(c,a)}$ are the coefficients that can be easily measured by studying the angular distribution. In the absence of CP-violation the conjugate mode $B \to \bar{K}^{*}\ell^{+}\ell^{-}$ has an identical decay distribution except that $I_{5,6,8,9} \to -I_{5,6,8,9}$ in the differential decay distribution as a consequence of CP properties [2]. In our discussions we neglect the lepton and $s$-quark masses. We also ignore the very small CP violation arising within the SM and exclude studying the resonant region. Under these approximations all the Wilson coefficients and form factors contributing to the decay are real. This results in only six of the $I_{i}$‘s being non-zero and independent, allowing only six independent observables to be measured. Any observable that is chosen may eventually be expressed in terms of six real transversity amplitudes $A_{\lambda}^{L,R}$, where $L, R$ denote the chirality of the lepton $\ell^{-}$ and $\lambda = \{\perp, \|, 0\}$ are the helicity that contribute. The six observables we choose are the helicity fractions and angular asymmetries that can be easily measured by studying the angular distributions. We define the observables $F_{\lambda} = (|A_{\lambda}^{L}|^{2} + |A_{\lambda}^{R}|^{2}) \Gamma_{\ell}$, $\Gamma_{\ell} \equiv \sum_{\lambda}(|A_{\lambda}^{L}|^{2} + |A_{\lambda}^{R}|^{2})$. The longitudinal helicity fraction $F_{0}$ is often referred to as $F_{L}$ in literature and we henceforth use $F_{L}$ to denote the longitudinal helicity fraction. The forward–backward asymmetry is defined as [11]

$$
A_{FB} = \frac{\int_{-1}^{0} d \cos \theta_{\ell} \frac{d^{2}\Gamma(\ell^{+}\ell^{-})}{d^{2}q^{2} d \cos \theta_{\ell}}}{\int_{-1}^{1} d \cos \theta_{\ell} \frac{d^{2}\Gamma(\ell^{+}\ell^{-})}{d^{2}q^{2} d \cos \theta_{\ell}}},
$$

(2)
where $I_D \equiv \int_0^1 - \int_0^1$. Two more asymmetries can be defined as follows:

$$A_4 = \frac{\int_{D_{LR}} d\phi \int d\cos \theta K \int d\cos \theta \lambda d^4(x - f)}{\int_0^1 d\phi \int_{D_{LR}} d\cos \theta K}$$

$$A_5 = \frac{\int_{D_{LR}} d\phi \int d\cos \theta K \int_{D_{LR}} d\cos \theta \lambda d^4(x + f)}{\int_{D_{LR}} d\phi \int d\cos \theta K}$$

(3) (4)

where $I_D \equiv \int_{\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2}$. $A_4$, $A_5$ and $A_{FB}$ isolate terms proportional to $I_4$, $I_5$ and $I_6$ respectively. The $I_\alpha$’s are expressed in terms of amplitudes $A_{\perp, \parallel, 0}$ as:

$$I_1^\perp = I_2^\perp = \frac{3}{2} [|A_{\perp, 2}|^2 + |A_{\parallel, 2}|^2 + (L \to R)]$$

$$I_1^\parallel = -I_2^\parallel = [|A_{\perp, 2}|^2 + (L \to R)]$$

$$I_3 = \frac{1}{2} [|A_{\perp, 2}|^2 - |A_{\parallel, 2}|^2 + (L \to R)]$$

$$I_4 = \frac{1}{\sqrt{2}} [\text{Re}(A_{\perp, 2}^* A_{\parallel, 2}^*) + (L \to R)]$$

$$I_5 = \frac{\sqrt{2}}{\sqrt{2}} [\text{Re}(A_{\perp, 2}^* A_{\parallel, 2}^*) - (L \to R)]$$

We note that even though the helicity amplitudes $A_{\perp, \parallel}$ are functions of $q^2$, for simplicity we have suppressed the explicit dependence on $q^2$.

The six transversity amplitudes can be written in the most general form as follows:

$$A_{\perp, \parallel} = C_{\perp, \parallel} F_{\perp, \parallel} - G_{\perp, \parallel}$$

(5)

where to leading order, $C_{\perp, \parallel} = C_{\perp, \parallel}^{\text{eff}} = C_{\perp, \parallel}^{\text{eff}} + C_{\perp, \parallel}^{\text{eff}} + C_{\perp, \parallel}^{\text{eff}}$ and $C_{\perp, \parallel}^{\text{eff}}$ are the Wilson coefficients that represent short distance corrections. $F_{\perp, \parallel}$ and $G_{\perp, \parallel}$ are defined in terms of $q^2$-dependent QCD form factors that parameterize the $B \to K^*$ matrix element [3] and are suitably defined to include both factorizable and non-factorizable contributions at any given order. The hadronic form factors have been calculated using QCD sum rules on the light cone and in the heavy quark limit using QCD factorization [13] and soft-collinear theory [14] that is valid for small $q^2$ (large recoil of $K^*$) and using operator product expansion [15] which is valid for large $q^2$ (low recoil). In this letter we limit our discussion to large recoil region for numerical estimates. This is sufficient to demonstrate the merits of our approach, even though our approach is valid for all $q^2$ [12].

In the large recoil limit the next to leading order effects can be parametrically included by replacements $C_{\perp} \to C_{\perp}^{\text{eff}}$ and $G_{\perp} \to C_{\perp}^{\text{eff}} G_{\perp} + \cdots$, with the dots representing the next to leading and higher order terms. We note that even at leading order it is impossible to distinguish between $C_{\perp}^{\text{eff}}$ and $G_{\perp}$. Hence the Wilson coefficient and form factor can be lumped together into a single factor $G_{\perp}$. We also emphasize that our analytic conclusions do not depend (for most of the part) on the explicit expression of the form factors.

The six observables or equivalently the amplitudes $A_{\perp, \parallel}$ are cast in Eq. (5) in terms of six form factors $F_{\perp, \parallel}$, $G_{\perp, \parallel}$ and two Wilson coefficients $C_9$ and $C_{10}$. If two additional inputs can be used all the parameters can be solved in terms of observables. Fortunately, advances in our understanding of these form-factors permit us to make two reliable inputs in terms of ratios of form-factors $F_{\perp} / F_{\parallel}$ and $G_{\perp} / G_{\parallel}$ which are well predicted at next to leading order in QCD corrections and free from form factors $\xi_{\parallel}$ and $\xi_{\perp}$ in heavy quark effective theory. In this letter we will make an additional assumption of the ratio $R = C_9 / C_{10}$ which has also been reliably calculated in the standard model; this allows us to make predictions of yet unmeasured observables. Deviations of these observables from our predicted values would be an unambiguous sign of new physics or a failure of understanding hadronic effects that are considered most reliably estimated. We derive several important relations between observables, Wilson coefficients and form factors. We find that the six observables are not independent as there exists one constraint relation that involves observables alone. A relation that is derived based entirely on the assumption that the amplitudes have the form in Eq. (5), but which is never-the-less independent of form factors and Wilson coefficients would provide an unambiguous test of the standard model relying purely on observables. We present such a relation as one of the central results in this letter and Ref. [12].

We begin by considering only three of the observables mentioned above which can be expressed in our notation as follows:

$$F_{\parallel} \Gamma_y = 2(C_9^2 + C_{10}^2) F_{\parallel}^2 + 2G_{\parallel}^2 - 4C_9 F_{\parallel} G_{\parallel}$$

$$F_{\perp} \Gamma_y = 2(C_9^2 + C_{10}^2) F_{\perp}^2 + 2G_{\perp}^2 - 4C_9 F_{\perp} G_{\perp}$$

$$A_{FB} \Gamma_y = 3C_{10}(F_{\perp} G_{\perp} + F_{\parallel} G_{\parallel}) - 6C_9 C_{10} F_{\perp} F_{\parallel}$$

(6) (7) (8)

The solution to the Wilson coefficients is easily obtained by defining intermediate variables $r_y = \hat{G}_{\parallel, \perp} / F_{\parallel, \perp} - C_9$. We briefly sketch the procedure: Eqs. (6), (7) and (8) get written as

$$F_{\parallel} \Gamma_y = 2F_{\parallel}^2 (r_y^2 + C_9^2)$$

$$F_{\perp} \Gamma_y = 2F_{\perp}^2 (r_y^2 + C_{10}^2)$$

$$A_{FB} \Gamma_y = 3C_{10} F_{\perp} F_{\parallel} (r_y^2 + r_{\perp}^2)$$

(9) (10) (11)

We next define ratios of form factors $P_1$ and $P_1'$ as follows:

$$P_1 = \frac{F_{\perp}}{F_{\parallel}} = -\frac{\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}}{(m_B + m_{K^*})^2} \sqrt{\lambda(a^2 + b^2 + c^2)}$$

$$P_1' = \frac{G_{\perp}}{G_{\parallel}} = -\frac{\sqrt{\lambda(m_B^2, m_{K^*}^2, q^2)}}{(m_B - m_{K^*})^2} \sqrt{\lambda(a^2 + b^2 + c^2)}$$

(12) (13)

where $m_B$ and $m_{K^*}$ are the masses of the $B$ meson and $K^*$ meson respectively; $\lambda(a, b, c) = a^2 + b^2 + c^2 - \lambda(a, b, c)$. 

2(ab + bc + ac). $V(q^2)$, $A_1(q^2)$, $T_1(q^2)$ and $T_2(q^2)$ are the well known $B \to K^*$ hadronic form-factors defined in Ref. [13], where the complete expressions for the "effective photonic form-factors" $T_1, T_2$ up to NLO are also given. In the large recoil limit the form factors reduce to the simple form $A_1(q^2)/V(q^2) = 2E m_B/(m_B + m_K)^2$ and $T_1(q^2)/T_2(q^2) = m_B/(2E)$. Notice that $P_1$ and $P'_1$ are independent of universal form factors $\xi || (q^2)$ and $\xi \perp (q^2)$ in the effective theory.

One easily solves for $r_\parallel + r_\perp$ to be,

$$r_\parallel + r_\perp = \pm \frac{\sqrt{V_{1f}}}{\sqrt{2F_B}} \left[ P_1^2 F_\perp + F_\perp \pm P_1 \sqrt{4F_F F_\perp - \frac{16}{9}A_B^2} \right] \mp \frac{1}{2} \quad (11)$$

Notice Eq. (11) implies that for $A_B = 0$, one must have $r_\parallel + r_\perp = 0$, which reduces the sign ambiguity in the solution of $r_\parallel + r_\perp$. The choice of the sign inside the square bracket is determined by the fact that $P_1$ is negative. In fact, for $A_B = 0$, the vanishing of the square bracket implies that we must have the exact equality,

$$P_1 = -\frac{\sqrt{F_F}}{\sqrt{2F_B}} \bigg|_{A_B = 0}$$

enabling a measurement of $P_1$ in terms of the ratio of helicity fractions. If zero crossing were to occur it would provide an interesting test of our understanding of form factors.

It is now easy to derive the following relations for $C_9$, $C_{10}$ and $\tilde{g}_||$:

$$C_9 = \frac{1}{\sqrt{2F_B}} \left[ (F_\parallel P_1 P'_1 - F_\perp) - \frac{1}{2}(P_1 - P'_1)Z_1 \right]$$

$$C_{10} = \frac{1}{\sqrt{2F_B}} \left[ (F_\perp P_1 P'_1 - F_\parallel) + \frac{1}{2}(P_1 - P'_1)Z_1 \right]$$

$$\tilde{g}_|| = \frac{1}{\sqrt{2}} \left[ (F^2_\perp - F_\parallel) \right] = \frac{1}{\sqrt{2}} \left[ F_\perp (P_1 - P'_1) \right]$$

where, $Z_1$ is defined as $Z_1 = \sqrt{4F_F F_\perp - \frac{16}{9}A_B^2}$ and the same sign in the remaining sign ambiguity must be chosen for all $C_9$, $C_{10}$ and $\tilde{g}_||$. We note that our solutions of the Wilson coefficients depend explicitly on the assumption that $A_B \neq 0$, hence, Wilson coefficients can be determined at any $q^2 \neq 0$. Since the Wilson coefficients are expected to be real constants independent of $q^2$ away from the resonant region, $Z_1$ must be real, implying the constraint

$$F_\parallel F_\perp \geq \frac{4}{9}A_B^2$$

purely in terms of observables. The violation of this constraint will be a clean signal of new physics.

![Figure 1](image-url)  
**FIG. 1.** Left: The(blue) triangular region constrains $F_L - A_{FB}$ parameter space. The solid (blue) lines correspond to contours from Eq. (16) for $|\Gamma| = 0.42 \times 10^{-7}$. Also plotted is $F_\perp$ for $R = -1$. For LHCb [10] data with 1 GeV$^2 \leq q^2 \leq 6$ GeV$^2$ we obtain $|C_{10}| = 3.81 \pm 0.58$ and $F_L = 0.21 \pm 0.05$. Right: The constraints on $F_L$ and $F_\perp$ arising from Eq. (21). For $R = -1$ the allowed values lie on the diagonal solid (blue) line and for $R = -10$ between the two dashed lines. Also indicated is the $|A_{FB}|$ domain implied by $Z^2_1 > 0$. The shaded gray area is forbidden by $F_L + F_\perp + F_\parallel = 1$. The $R = -1$ line approximately divides the domain into regions fixing the sign of $A_{FB}$ relative to $C_9/C_{10}$ and $C_7/C_{10}$ independent of $R$. 

Eq. (16) implies the following bound on the form factor $P_1$ in terms of observables alone:

$$P_1 \leq \frac{4F_F F_\perp - \frac{16}{9}A_B^2}{F_\parallel^2} \forall F_F F_\perp \leq \frac{2}{7} \left( \frac{4A_{FB}}{3} \right)^2$$

The above bound is obtained by straightforward extremization of $P_1^2$ with respect to all non-observables. Eqs. (15) and (16) result in the following:

$$E_1 = \frac{C_9}{C_{10} A_{FB}} \frac{3(F_\parallel P_1 P'_1 - F_\perp) - (P_1 - P'_1)Z_1}{2(P_1 - P'_1)}$$

which can be inverted to express $A_{FB}$ in terms of $P_1$, $P'_1$ and $R$ as:

$$A_{FB} = 3 \left( \frac{RX - \frac{1}{2}Y(P_1 + P'_1)^2(1 + R^2) - X^2}{4(P_1 - P'_1)^2(1 + R^2)} \right)$$

where, $X = 2(F_\parallel P_1 P'_1 - F_\perp)$ and $Y = 4F_F F_\perp$. Since $F_L$ has been measured and $F_L + F_\perp + F_\parallel = 1$, all our conclusion throughout the paper can be re-expressed in terms of just two helicity fractions $F_L$ and $F_\perp$. The usefulness of Eq. (21) is shown in Fig. (1), where we have depicted the allowed parameter space consistent with real $A_{FB}$. The reader will note that the rigorous constraint imposed on $F_\perp$ depending on the value of $F_L$ indicated in the figure.

We now derive some useful relations that involve $C_7$ and are hence valid only at the leading order. Eqs. (16) and (17) can be re-expressed in this limit as:

$$E_2 \equiv \frac{C_{7eff}}{C_{10} A_{FB}} = \frac{3F_\parallel (P_1^2 F_\perp - F_\parallel)}{2 \tilde{g}_|| (P_1 - P'_1)}$$
We emphasize that $C_{7}^{\text{eff}}/C_{10}$ is not as clean as $C_{9}/C_{10}$, which is expressed in Eq. (20) in terms of observables and ratio’s of two form factors which are predicted exactly in heavy quark effective theory. $C_{7}^{\text{eff}}/C_{10}$ on the other-hand depends on $F_{\parallel}/G_{\parallel}$ which in turn depends on the heavy quark effective theory form factor $\xi_{\perp}$. It may nevertheless be noted that the sign of $F_{\parallel}/G_{\parallel}$ is quite accurately predicted to be negative [12].

Eq. (20) together with Eq. (22) can be rewritten in a form,

$$\frac{2}{3}E_{2}P_{1}^{*} - \frac{4}{3}E_{1}P_{1} = (P_{2}^{*}F_{\parallel} + F_{\perp} + P_{1}Z_{1}) > 0,$$

(23)

where $P_{1}^{*} = (G_{\parallel}/F_{\parallel})(P_{1} + P_{1}^{*}) > 0$ since each of $(G_{\parallel}/F_{\parallel})$, $P_{1}$ and $P_{1}^{*}$ are always negative; $(P_{2}^{*}F_{\parallel} + F_{\perp} + P_{1}Z_{1})$ is easily seen to be always positive by an (infinite) series expansion in $A_{FB}$ where every terms is positive. In SM, $C_{7}^{\text{eff}}/C_{10} > 0$ and $C_{9}/C_{10} < 0$, hence the sign of $E_{2}$ ($E_{1}$) will be same (opposite) to that observed for $A_{FB}$. If for any $q^{2}$ we find $A_{FB} > 0$, Eq. (23) cannot be satisfied unless the contribution from the $E_{2}$ term exceeds the $E_{1}$ term, or the sign of the $E_{2}$ term is wrong in SM. In the SM the $E_{2}$ term dominates at small $q^2$, hence, $A_{FB}$ must be positive at small $q^2$ to be consistent with SM. If $A_{FB} < 0$ is observed for all $q^2$, i.e. no zero crossing of $A_{FB}$ is seen, one can convincingly conclude that $C_{7}^{\text{eff}}/C_{10} < 0$ in contradiction with SM. However, if zero crossing of $A_{FB}$ is confirmed with $A_{FB} > 0$ at small $q^2$ it is possible to conclude that the signs $C_{7}^{\text{eff}}/C_{10} > 0$ and $C_{9}^{\text{eff}}/C_{10} < 0$ are in conformity with SM.

The relations derived above depend only on three observables: $F_{\parallel}$, $F_{\perp}$ and $A_{FB}$. Similar relations can be derived using the three other nonzero observables $F_{L}$, $A_{4}$ and $A_{5}$: It is easy to derive these relations which are identical except for the replacements: $F_{\parallel} \rightarrow F_{L}$, $A_{FB} \rightarrow \sqrt{2}A_{5}$, $F_{\perp} \rightarrow F_{0}$ and $G_{\parallel} \rightarrow G_{0}$ in Eqs. (15), (16) and (17). We obtain $C_{9}/C_{10}$ and $C_{7}^{\text{eff}}/C_{10}$ ratios as

$$\frac{C_{9}}{C_{10}} \sqrt{2}A_{5} = \frac{3(F_{L}P_{2}F_{\parallel} - F_{\perp})}{2(P_{2} - P_{2}^{*})} Z_{2},$$

(24)

$$\frac{C_{7}^{\text{eff}}}{C_{10}} \sqrt{2}A_{5} = \frac{3F_{0}(P_{2}^{*}F_{L} - F_{\perp})}{2G_{0}(P_{2} - P_{2}^{*})},$$

(25)

where $Z_{2} = \sqrt{4F_{L}F_{\perp} - \frac{22}{9}A_{5}^{2}}$ and we have defined $P_{2} \equiv F_{\perp}/F_{0}$ and $P_{2}^{*} \equiv G_{\parallel}/G_{0}$.

It is straight forward to derive relations analogous to Eqs. (18), (19), (21) and (23) in terms of $F_{L}$, $F_{\perp}$, $A_{5}$, $P_{2}$ and $P_{2}^{*}$. We present here only one of the interesting possible relation

$$4(1 - F_{\perp})F_{\parallel} \geq \frac{16}{9}(A_{FB}^{2} + 2A_{5}^{2}),$$

(26)

which bounds $A_{FB}^{2}$ and $A_{5}^{2}$ in terms of $F_{\perp}$ alone, and is obtained by generalizing the constraints obtained from $Z_{1} > 0$ and $Z_{2} > 0$. $P_{2}$ and $P_{2}^{*}$ are obtained in terms of observables and $P_{1}$, $P_{1}^{*}$ by comparing the expressions for $C_{7}/C_{10}$ in Eqs. (20) and (24) and for $C_{7}/C_{10}$ in Eqs. (22) and (25). We find,

$$P_{2} = \frac{2P_{1}A_{FB}F_{\perp}}{\sqrt{2}A_{5}(2F_{\perp} + Z_{1}P_{1}) - Z_{2}P_{1}A_{FB}}$$

(27)

The expression for $P_{5}$ is some what more complicated and hence we refrain from presenting it here. We note that $P_{2}$, $P_{2}^{*}$ depend on form factors $\xi_{\perp}$ and $\xi_{\parallel}$, and are hence not regarded as clean parameters. Nevertheless, we have shown that it is possible to estimate both $P_{2}$ and $P_{2}^{*}$ in terms of observable and $P_{1}$ and $P_{1}^{*}$ which are clean and independent of the form factors. An astute reader will realize that the expressions for $P_{2}$ and $P_{2}^{*}$ are valid beyond leading order. There is yet one more set of relations for $C_{7}/C_{10}$ and $C_{9}/C_{10}$ that can be derived involving $A_{4}$ with the substitution: $F_{\parallel} \rightarrow F_{0} + F_{\perp} + \sqrt{2\pi}A_{4}$, $A_{FB} \rightarrow A_{FB} + \sqrt{2}A_{5}$, $F_{\perp} \rightarrow F_{0} + F_{\perp}$ and $G_{\parallel} \rightarrow G_{\parallel} + G_{0}$, $P_{1} \rightarrow P_{3} = P_{1}P_{2}/(P_{1} + P_{2})$ and $P_{2}^{*} \rightarrow P_{3}^{*} = P_{2}^{*}P_{2}/(P_{1} + P_{2}^{*})$. Akin to the relation for $P_{2}$, $P_{3}$ can also be obtained in terms of $P_{1}$ and observables. However, $P_{3}$ is related to $P_{1}$ and $P_{2}$. This results in an relation for $A_{4}$:

$$A_{4} = \frac{8A_{5}A_{FB}}{9\pi F_{\perp}} + \sqrt{2} \sqrt{F_{L}F_{\perp} - \frac{8}{9}A_{5}^{2}} \sqrt{F_{\perp}F_{\parallel} - \frac{8}{9}A_{FB}^{2}}.$$
[1] R. Sinha, [hep-ph/9608314].
[2] F. Kruger, L. M. Schgal, N. Sinha and R. Sinha, Phys. Rev. D61, 114028 (2000). [hep-ph/9907386].
[3] W. Altmannshofer, P. Ball, A. Bharucha et al., JHEP 0901, 019 (2009). [arXiv:0811.1214 [hep-ph]].
[4] C. Bobeth, G. Hiller and G. Piranishvili, JHEP 0807, 106 (2008) [arXiv:0805.2525 [hep-ph]].
[5] U. Egede, T. Hurth, J. Matias, M. Ramon and W. Reece, JHEP 0811, 032 (2008) [arXiv:0807.2589 [hep-ph]].
[6] C. Bobeth, G. Hiller, D. van Dyk, JHEP 1007, 098 (2010). [arXiv:1006.5013 [hep-ph]].
[7] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 79, 031102 (2009) [arXiv:0804.4412 [hep-ex]].
[8] J. T. Wei et al. [BELLE Collaboration], Phys. Rev. Lett. 103, 171801 (2009) [arXiv:0904.0770 [hep-ex]].
[9] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 106, 161801 (2011). [arXiv:1101.1028 [hep-ex]].
[10] R. Aaij et al. [LHCb Collaboration], arXiv:1112.3515 [hep-ex].
[11] Our sign of $A_{FB}$ agrees with [1–3] but differs with most others. The sign depends on the definition of $\theta_\ell$; We have chosen $\theta_\ell$ as the angle of $\ell^-$ with z-axis, instead of $\ell^+$. 
[12] Diganta Das and Rahul Sinha, to be submitted to Phys. Rev. D.
[13] M. Beneke and T. Feldmann, Nucl. Phys. B 592, 3 (2001); M. Beneke, T. Feldmann, D. Seidel, Nucl. Phys. B612, 25-58 (2001); Eur. Phys. J. C41, 173-188 (2005).
[14] A. Ali, G. Kramer, G. -h. Zhu, Eur. Phys. J. C47, 625-641 (2006).
[15] B. Grinstein, D. Pirjol, Phys. Rev. D70, 114005 (2004); M. Beylich, G. Buchalla, T. Feldmann, Eur. Phys. J. C71, 1635 (2011).