Transition process from nucleation to high-speed rupture propagation: scaling from stick-slip experiments to natural earthquakes

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SUMMARY
The process of earthquake generation is governed by a coupled non-linear system consisting of the equation of motion in elastodynamics and a fault constitutive relation. On the basis of the results of stick-slip experiments we constructed a theoretical source model with a slip-dependent constitutive law. Using the theoretical source model, we simulated the transition process numerically from quasi-static nucleation to high-speed rupture propagation and succeeded in quantitatively explaining the three phases observed in stick-slip experiments, that is very slow (1 cm s\(^{-1}\)) quasi-static nucleation preceding the onset of dynamic rupture, dynamic but slow (10 m s\(^{-1}\)) rupture growth without seismic-wave radiation, and subsequent high-speed (2 km s\(^{-1}\)) rupture propagation. Theoretical computation of far-field waveforms with this model shows that a slow initial phase preceding the main P phase expected from a classical source model is radiated in the accelerating stage from the slow dynamic rupture growth to the high-speed rupture propagation. On the assumption that the physical law governing rupture processes in natural earthquakes is essentially the same as that in stick-slip events, we scaled the theoretical source model explaining the stick-slip experiments to the case of natural earthquakes so that the scaled source model explains the observed average stress drop, the critical nucleation-zone size, and the duration of the slow initial phase well. The physical parameters prescribing the source model are the weak-zone size \(L\), the critical weakening displacement \(D_9\), the breakdown strength drop \(\tau_b\), and the rigidity \(m\) of the surrounding elastic medium. In scaling these parameters, we held a non-dimensional controlling parameter \(\mu = (m\mu D_9)/(\tau_b L)\) in numerical simulation constant. From the results of scaling we found the following fundamental relations between the source parameters: (1) the critical weakening displacement \(D_9\) is in proportion to the weak-zone size \(L\), but (2) the breakdown strength drop \(\tau_b\) is independent of \(L\).

Key words: critical weakening displacement, fundamental scaling relations, slow initial phase, transition process, weak-zone size.

1 INTRODUCTION
It is now widely accepted that the source of earthquakes is brittle shear fracture occurring in the Earth’s interior. In the classical theory based on linear elasticity, the brittle fracture is described as crack extension with sudden stress drop. The sudden stress drop necessarily leads to physically unreasonable stress singularities at crack tips. The stress singularities can be resolved by introducing a cohesive force acting between crack surfaces. The concept of the cohesive force was first introduced by Barenblatt (1959) to represent molecular cohesion in brittle tensile fracture and was later extended to the case of shear fracture by Ida (1972) and Palmer & Rice (1973). The essential property of the cohesive force is that it is a decreasing resistant force with the progress of failure. The existence of such a resistant force in brittle shear fracture has been confirmed by Okubo & Dieterich (1984) and Ohnaka, Kuwahara & Yamamoto (1987) through careful stick-slip laboratory experiments. According to these experimental studies, with the progress of fault slip, shear stress acting on the fault plane first...
There have been many studies that modelled the earthquake constitutive relation between shear stress \( \tau \) and slip \( \gamma \) (Tse & Rice 1986; Okubo 1989; Dieterich 1992; Rice 1993). The results of these studies indicate that the rate and state-dependent friction law can be reduced to the slip-dependent friction law in form (Matsu’ura et al. 1992). In the present study, we use the slip-dependent friction law because our present concern is the transition process from nucleation to high-speed rupture propagation.

In the present study, first, we briefly review the basic equations governing the process of earthquake generation. Next, we demonstrate that the transition process observed in stick-slip experiments can be quantitatively explained by a theoretical source model with a slip-dependent friction law. Then, on the assumption that the physical law governing rupture processes in natural earthquakes is essentially the same as that in stick-slip events, we scale the theoretical source model explaining the stick-slip experiments to the case of natural earthquakes. Finally, on the basis of the results of scaling, we derive the fundamental relations amongst the physical parameters prescribing the source model.

2 Basic Equations Governing Earthquake Generation Processes

We represent frictional interaction between fault surfaces by a slip-dependent constitutive relation and use it as the fundamental law governing the entire process of earthquake rupture. This means that the rupture at a certain point on the fault plane must proceed irreversibly along the constitutive relation curve at this point. Then, incorporating the constitutive relation into the equation of equilibrium (in a quasi-static case) or the equation of motion (in a dynamic case), we can treat the process of earthquake generation as a kind of boundary-value problem in elastodynamics.

We take a Cartesian coordinate system \((x, y, z)\) in an infinite elastic medium and consider 2-D, anti-plane shear faulting on the \(x-z\) plane. The directions of fault slip and fault extension are taken to be parallel to the \(z\)-axis and the \(x\)-axis, respectively. The constitutive relation between shear stress \(\tau(x)\) and fault slip \(w(x)\), which is generally a function of position \(x\), is expressed as

\[ \tau(x) = g[w(x); x]. \]  

When we apply a uniform shear stress \(\tau_0\) externally to this system, first, the quasi-static nucleation proceeds in the weakest
portion of the fault. In this case the equilibrium condition for traction on a faulting region, \(-c \leq x \leq c\), is given by

\[
-\frac{\mu}{2\pi} \int_{-c}^{c} \frac{1}{(x-x')} \frac{\partial w(x')}{\partial x'} dx' + \tau_0 = \tau(x),
\]

(2)

where \(\mu\) is the rigidity of the surrounding elastic medium. In the above equation, which was first obtained by Eshelby, Frank & Nabarro (1951) on the basis of elastic-dislocation theory, the first term represents the stress changes produced by the fault slip \(w(x)\). Incorporating the fault constitutive relation (1) into the equilibrium condition (2), we obtain a coupled non-linear system governing the quasi-static nucleation process. This coupled non-linear system can be solved numerically by using the method developed by Matsu’ura et al. (1992).

When the external shear stress exceeds a critical level, the fault system becomes unstable and dynamic rupture starts. In this case, we incorporate the fault constitutive relation into the equation of motion and obtain a coupled non-linear system governing the dynamic process of rupture propagation. Just before the initiation of dynamic rupture, the shear stress and the fault slip are in critical states, \(\tau_0(x)\) and \(w_0(x)\), which can easily be found by successively solving the coupled quasi-static equations, (1) and (2), increasing the external shear stress stepwise. We take these critical states as the reference states to measure changes in stress and fault slip, \(\Delta\tau(x, t)\) and \(\Delta w(x, t)\), during the dynamic rupture process. Then, the total stress \(\tau(x, t)\) and the total fault slip \(w(x, t)\) are given by

\[
\tau(x, t) = \tau_0(x) + \Delta\tau(x, t),
\]

(3)

\[
w(x, t) = w_0(x) + \Delta w(x, t).
\]

(4)

In the case of a 2-D, anti-plane fault, solving a boundary-value problem in elastodynamics, Kostrov (1966) obtained a relation between the incremental stress \(\Delta\tau(x, t)\) and the incremental fault slip \(\Delta w(x, t)\):

\[
\Delta w(x, t) = \frac{2\beta}{\pi\mu S} \int_{S} \frac{\Delta\tau(x', t')}{\sqrt{S'(t-t')^2 - (x - x')^2}} dx'dt',
\]

(5)

with

\[
S; \beta^2(t-t')^2 - (x-x')^2 \geq 0 \quad \text{and} \quad 0 \leq t' \leq t,
\]

(6)

where \(\beta\) indicates the S-wave velocity. We assume that the dynamic rupture process is governed by the same fault constitutive relation as in the case of quasi-static nucleation; that is, if the slip-velocity is positive,

\[
\tau(x, t) = g[w(x, t); x] \quad \text{for} \quad \frac{\partial w(x, t)}{\partial t} > 0,
\]

(7)

but if the slip-velocity is zero,

\[
\tau(x, t) \leq g[w(x, t); x] \quad \text{for} \quad \frac{\partial w(x, t)}{\partial t} = 0.
\]

(8)

We couple the equations (3)–(5) and (7) or (8) and obtain a non-linear system governing the dynamic process of rupture propagation. This coupled non-linear system can be solved numerically by using the boundary-integral method developed by Andrews (1985).

### 3 Interpretation of Transition Processes in Stick-Slip Events

#### 3.1 Transition processes observed in stick-slip experiments

Ohnaka & Kuwahara (1990) carried out stick-slip laboratory experiments with a 28 cm × 28 cm × 5 cm rock sample of Tsukuba granite, which has a pre-cut fault aligned along the diagonal of the sample (Fig. 1). The rigidity and the S-wave velocity of the rock sample are measured as 20 GPa and 2.9 km s\(^{-1}\), respectively. Fig. 2 is an example of experimental results, which shows the development of the breakdown zone after the onset of dynamic rupture. The hatched portion indicates the breakdown zone, in which shear stress decreases from a peak stress \(\sigma_p\) to a constant frictional stress \(\sigma_f\), as shown in Fig. 3. From Fig. 2 it is seen that the rupture front propagates very slowly (50 m s\(^{-1}\)) from CH5 to CH4 at first. Then, after about 0.6 ms from the start of rupture, the dynamic rupture is rapidly accelerated to a terminal velocity (2 km s\(^{-1}\)). The duration of the accelerating stage from slow dynamic rupture growth to high-speed rupture propagation is about 0.06 ms. In Fig. 4(a) we show the changes in the shear stress acting on the fault plane with time after the onset of dynamic rupture. These are the original records from which the diagram in Fig. 2 was constructed. The dynamic rupture starts at a point between CH5 and CH6 and propagates very slowly at first with a gradual stress drop. After about 0.6 ms, the dynamic rupture accelerates to a terminal velocity and propagates outwards with a rapid stress drop. Fig. 4(b) shows changes in fault slip (thick line) and slip velocity (thin line) with time. At CH6 and CH5, located inside the nucleation zone, the fault slip increases very slowly at first. Here, we call the region in which slip weakening proceeds quasi-statically the nucleation zone. After about 0.6 ms, with the onset of the high-speed rupture propagation, the fault slip increases rapidly. At CH1, where

![Figure 1. Configuration of a rock sample with strain-gauge sensors in position in stick-slip experiments (after Ohnaka & Kuwahara 1990).](https://academic.oup.com/gji/article-abstract/132/1/14/2730925)
the rupture propagation has accelerated to a terminal velocity, the fault slip increases abruptly with the onset of dynamic rupture, so the slip velocity has a very high and narrow peak.

Kuwahara *et al*. (1986) also examined how the transition from quasi-static nucleation to dynamic rupture propagation proceeds through stick-slip laboratory experiments. Fig. 5 shows the relation between the rupture growth rate normalized by the S-wave velocity of rock samples and the rupture growth distance normalized by the upper corner wavelength \( \lambda_u \) of the power spectrum of the surface topography. Kuwahara *et al*. found that the normalized data for various experiments with different \( \lambda_u \) lie almost in a single curve. This suggests that the upper corner wavelength \( \lambda_u \), and thus the roughness of rock surfaces, is a key parameter for scaling the transition process. From Fig. 5 it is seen that the process of rupture growth consists of three different phases: quasi-static nucleation (phase I), dynamic but slow rupture growth (phase II) and high-speed rupture propagation (phase III). It should be noted that the rupture growth rate in phase I depends strongly on the applied strain rate \( \dot{\varepsilon} \).

As shown in Fig. 3, the slip-dependent constitutive behaviour is essentially prescribed by the breakdown strength drop \( \tau_b \), which is defined by the difference between the peak stress \( \sigma_p \) and the dynamic frictional stress \( \sigma_f \) and the critical weakening displacement \( D_c \). Ohnaka & Kuwahara (1990) measured the values of these physical parameters directly at each channel. Their results are summarized in Figs 6(a) and (b) for the breakdown strength drop \( \tau_b \) and the critical weakening displacement \( D_c \), respectively. Here, we take the centre of the nucleation zone to be at 1.5 cm to the left of CH5. These diagrams show that both \( \tau_b \) and \( D_c \) take relatively small values in the nucleation zone.

### 3.2 Theoretical interpretation of the transition process

We now demonstrate that the transition process observed in stick-slip experiments can be quantitatively explained by a theoretical source model with the laboratory-based fault constitutive relation. Taking a dynamic frictional stress \( \sigma_f \) as the reference to measure shear stress \( \tau(x) \), we may generally express the slip-dependent constitutive relation in the following form as a function of position \( x \) (Matsu’ura *et al*. 1992):

\[
\tau(w; x) = \tau_b(x) \left[ \frac{5w}{D_c(x)} \right] \exp \left[ 1 - \frac{5w}{D_c(x)} \right]. \tag{9}
\]

On the basis of the experimental results in Fig 6, we determine the concrete functional forms of \( \tau_b(x) \) and \( D_c(x) \), as shown in Table 1. It should be noted here that the spatial variable \( x \) is normalized by the weak-zone size \( L \), and the shear stress \( \tau \) and the fault slip \( w \) are normalized, respectively, by the breakdown strength drop \( \tau_b \) and the critical weakening displacement \( D_c \) in the normal-strength region extending outside the weak zone. In the present case, the weak-zone size \( L \), the breakdown strength drop \( \tau_b \), and the critical displacement \( D_c \) are 10 cm, 1.2 MPa, and 3 \( \mu \)m, respectively.

In Fig. 7 we schematically represent a 2-D fault with the constitutive relation curves reconstructed from the experimental results. The fault has a broad weak zone, characterized by a relatively low peak stress and a small critical-weakening displacement. Outside the weak zone, faulting regions with normal strength extend in both directions to a distance of about 1.4\( L \) from the centre of the weak zone. The faulting

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region bounded by strong barriers corresponds to the pre-cut fault in laboratory experiments.

With the source model reconstructed from experimental results, we numerically compute the entire process from quasi-static nucleation to high-speed dynamic rupture propagation. In the numerical computation, in addition to the normalization of \( \tau, w, \) and \( x \) by \( \tau_b, D_c, \) and \( L, \) the time variable \( t \) is also normalized by a characteristic time \( L/\beta \) of the system. Corresponding to these normalizations of variables, we should replace the S-wave velocity \( \beta \) in the eqs (5) and (6) with unity and the rigidity \( \mu \) in eqs (2) and (5) with

\[
\mu' = (\mu D_c)/(\tau_b L).
\]

Here it should be noted that the non-dimensional quantity \( \mu' \) is the only parameter which essentially controls the quasi-static and dynamic behaviour of the non-linear coupled system in numerical simulations. In the case of the stick-slip experiments performed by Ohnaka & Kuwahara (1990), taking the observed values of \( \mu = 23 \text{ GPa}, \tau_b = 1.2 \text{ MPa}, D_c = 3 \text{ \mu m}, \) and \( L = 10 \text{ cm}, \) we obtain \( \mu' = 0.57. \)

Fig. 8(a) shows changes in shear stress during the quasi-static nucleation process. With an increase of the external stress \( \tau_n, \) at first the shear stress acting on the fault surface rises uniformly over the whole region. When the shear stress reaches a certain level (\( \tau_n = 0.72 \text{ MPa}, \)) quasi-static nucleation starts at the weakest portion and develops outwards with a gradual stress drop (phase I). The gradual stress drop in the nucleation zone brings about a weak stress concentration in the surrounding regions. When the externally applied stress reaches a critical level (\( \tau_n = 0.86 \text{ MPa}, \)) the system becomes unstable and dynamic rupture starts at the end zone. Fig. 8(b) shows the change in shear stress with time after the onset of dynamic rupture. The stress curve at \( t = 0 \) in Fig. 8(b) corresponds to that in the final state of the quasi-static nucleation process shown in Fig. 8(a). The dynamic rupture grows very slowly at first (phase II). Then, after about 0.2 ms, the rupture growth is suddenly accelerated and high-speed dynamic rupture propagates outwards with a rapid stress drop (phase III).

In phase III, high stress concentration appears on the rupture fronts.

Table 1. The values of the parameters prescribing the slip-dependent fault constitutive relation.

| Section | \( \tau_n(\xi)/\tau_b \) | \( D_c(\xi)/D_c \) |
|---------|------------------------|--------------------|
| \( |\xi| \leq 0.35 \) | \( 0.6 + 0.4 \sin(\pi|\xi|/2) \sin(\pi|\xi|/0.7) \) | \( 2/3 \) |
| \( 0.35 < |\xi| \leq 0.5 \) | \( 0.6 + 0.4 \sin(\pi|\xi|/2) \) | \( 2/3 \) |
| \( 0.5 < |\xi| \leq 1 \) | \( 0.6 + 0.4 \sin(\pi|\xi|/2) \) | \( [2 + \sin^2(\pi(|\xi| - 0.5)/3)] \) |
| \( 1 < |\xi| \leq 1.375 \) | \( 1 \) | \( 1 \) |
| \( 1.375 < |\xi| \leq 1.5 \) | \( 1 + 2 \sin^2[4\pi(|\xi| - 1.375)] \) | \( 1 + 2 \sin^2[4\pi(|\xi| - 1.375)] \) |
| \( 1.5 < |\xi| \) | \( 3 \) | \( 3 \) |

\( \xi = x/L; \) normalized distance along the fault.
In Fig. 9(a) we show the changes in shear stress with time at five representative points, CH2–CH6. The corresponding changes in incremental fault slip, \( \Delta \varepsilon \) (thick line), and slip velocity, \( \dot{\varepsilon} \) (thin line), are shown in Fig. 9(b). The locations of the observation points correspond to those in Fig. 8. At CH5 and CH6, located inside the nucleation zone, shear stress drops gradually with time after the onset of dynamic rupture. After about 0.2 ms, high-speed dynamic rupture starts at the end of the nucleation zone and extends outwards with a rapid stress drop. Inside the nucleation zone, the fault-slip motion is very slow at first and then gradually accelerates with the onset of high-speed rupture propagation. Therefore, the slip velocity at CH5 and CH6 has a low and broad peak. At CH4, located just outside the nucleation zone, the slip stress increases fairly rapidly with the onset of dynamic rupture, so the corresponding slip velocity has a high peak with a long tail. At CH2 and CH3, where the rupture propagation has accelerated to a terminal velocity, the shear stress drops monotonically with the increase in slip velocity. At CH2 and CH3, where the rupture propagation has accelerated to a terminal velocity, the shear stress velocity state changes along an oval trajectory. These theoretical results are also in accord with the experimental results of Ohnaka & Kuwahara (1990).

After about 0.2 ms, high-speed dynamic rupture starts at the stress drop. Inside the nucleation zone, the fault-slip motion is very slow at first and then gradually accelerates with the onset of high-speed rupture propagation. Therefore, the slip velocity at CH5 and CH6, located inside the nucleation zone, has a low and broad peak. At CH4, located just outside the nucleation zone, the shear stress increases fairly rapidly with the onset of dynamic rupture, so the corresponding slip velocity has a high peak with a long tail. At CH2 and CH3, where the rupture propagation has accelerated to a terminal velocity, the fault slip increases abruptly with the onset of dynamic rupture, so the slip velocity has a very high and narrow peak. Comparing the experimental results in Fig. 4 and the theoretical results in Fig. 9, we can see that the present theoretical model successfully describes the changes in shear stress and fault slip during the transition process observed in stick-slip experiments.

Fig. 10 shows the relations between shear stress and slip velocity during the breakdown process at the five points CH2–CH6. At CH5 and CH6, located inside the nucleation zone, the shear stress drops monotonically with the increase in slip velocity. At CH2 and CH3, where the rupture propagation has accelerated to a terminal velocity, the shear-stress–slip-velocity state changes along an oval trajectory. These theoretical results are also in accord with the experimental results of Ohnaka & Kuwahara (1990).
Fig. 11 shows the development of the breakdown zone with time in the transition process from quasi-static nucleation (b) to dynamic rupture propagation (a). Here, the breakdown zone is defined as the region where the process of slip weakening is ongoing. The rupture growth rate in the quasi-static nucleation process (Fig. 11b) is in proportion to the rate of increase in the externally applied stress (strain). In the present case, for comparison with the experimental results, we take the increase rate of externally applied strain to be $10^{-6}$ s$^{-1}$. The quasi-static nucleation begins about 1.4 s before the onset of dynamic rupture. For the quasi-static nucleation (phase I) the average rupture growth rate is about $1.4$ cm s$^{-1}$. When the strain rate is $10^{-5}$ s$^{-1}$, the nucleation begins about 0.14 s before the onset of dynamic rupture. In either case the critical size $L_c$ of the nucleation zone is 4.2 cm. The dynamic rupture process shown in Fig. 11(a) does not depend on the applied strain rate. The dynamic rupture grows very slowly ($12$ m s$^{-1}$) at first before the initiation of dynamic rupture. In the case of stick-slip experiments, the normalization factors, $t_b$ (breakdown strength drop), $D_c$ (critical weakening displacement), and $L$ (weak-zone size), are taken to be 1.2 MPa, 3 μm, and 10 cm, respectively. The duration $T_s$ of phase II and the breakdown time $T_b$ in phase III expected from the theoretical model are somewhat shorter than those observed in the laboratory experiments. This may be due to the difference in the mode of fracture between the theoretical calculation (mode III) and the laboratory experiments (mode II).

The duration of slow dynamic rupture growth (phase II) depends on the magnitude of a perturbation given for triggering dynamic rupture. If the magnitude of the given perturbation is larger, the duration of phase II becomes shorter. In stick-slip experiments it is frequently observed that the main dynamic rupture is triggered by the occurrence of a small AE event in a critical stress state. Therefore, in our numerical simulation, we search the critical stress state by increasing the external shear stress with a very small step ($t_b \times 10^{-5}$), and then start the dynamic analysis by applying a very small perturbation at the edge of the nucleation zone.

From a cohesive zone model it is expected that the breakdown time $T_b$ is nearly equal to the width of the slip-acceleration pulse, so its inverse gives a rough estimate of the cut-off frequency $f_s^{\max}$ of the slip-acceleration spectrum (Ohnaka & Yamashita 1989). Fig. 12 shows changes in fault slip, slip velocity, slip acceleration and shear stress with time at a point located in the middle of the normal-strength region, where the dynamic rupture propagates at the terminal velocity. From Fig. 12 the breakdown time and the width of the slip-acceleration pulse is found to be about 0.01 ms. From the slip-acceleration spectrum in Fig. 13, on the other hand, the cut-off frequency $f_s^{\max}$ is found to be about 100 kHz. Therefore, the approximate relation obtained by Ohnaka & Yamashita (1989),

$$f_s^{\max} \approx 1/T_b,$$  \hfill (11)

is valid for the present theoretical model.

Finally, we show a plot of the rupture growth rate versus the half-length of the rupture zone in Fig. 14. Here, the applied strain rate is assumed to be $10^{-6}$ s$^{-1}$, and the rupture growth rate is normalized by the S-wave velocity. In quasi-static nucleation (phase I), the rupture growth rate is very small. In the early stage of slow dynamic rupture growth (phase II), the
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Figure 9. Changes in shear stress, fault slip, and slip velocity with time after the onset of dynamic rupture at five representative points, CH2–CH6. (a) Shear stress. (b) Fault slip (thick line) and slip velocity (thin line). The locations of CH2–CH6 correspond to those in Fig. 8. For CH6, the slip-velocity curve amplified 100 times is also shown for reference.

Rupture growth rate increases rapidly with the length of the rupture zone. In the last stage of phase II, the rupture growth accelerates to the S-wave velocity. The rapid growth of rupture in the early stage of phase II expected from the theoretical model is in accordance with the experimental results in Fig. 5.

4 SCALING FROM STICK-SLIP EXPERIMENTS TO NATURAL EARTHQUAKES

4.1 Basic assumptions in scaling

In the previous section we demonstrated that the transition process observed in stick-slip experiments can be completely described by the theoretical model with the laboratory-based fault constitutive relation. As shown in Fig. 5, the experimental data for various cases with different upper-corner wavelengths \( \lambda_c \) lie almost in a single rupture-velocity versus rupture-length curve, if the rupture length is normalized by \( \lambda_c \). On the other hand, as theoretically demonstrated by Matsu’ura et al. (1992), the critical weakening displacement \( D_c \) has a linear \( \lambda_c \) dependence. These experimental and theoretical results suggest that the critical weakening displacement \( D_c \) has a linear scale dependence. This means that the ratio of the critical weakening displacement \( D_c \) to the weak-zone size \( L \) is nearly constant, so the value of \( \mu' = (\mu D_c)/L \) is independent of the size of stick-slip events, since the weak-zone size \( L \) is the only parameter characterizing the system dimension in the numerical simulation, and \( \mu \) and \( \tau_0 \) are essentially scale-independent parameters. Therefore, we may conclude that the result of numerical simulation presented here will have some generality, although it is only a special case of \( \mu' = 0.57 \).
In the present section, on the hypothesis that the physical law governing the rupture process in natural earthquakes is essentially the same as that in stick-slip events, we scale the theoretical source model explaining the stick-slip experiments to the case of natural earthquakes so that the scaled source model explains the observed average stress drop, the critical nucleation-zone size and the duration of a slow initial phase well. Since the rigidity, $\mu$, of the surrounding elastic medium can be regarded as nearly constant, key parameters in the scaling are the weak-zone size, $L$, the critical weakening displacement, $D_c$, and the breakdown strength drop, $\tau_b$. In scaling these parameters, as a natural extension of the conclusion for stick-slip events, we assume that the non-dimensional quantity $\mu' = (\mu D_c)/(\tau_b L)$ remains constant (0.57) in the case of natural earthquakes.

4.2 Theoretical relations between observable quantities and source parameters

From the results of numerical simulation given in Section 3.2, we can derive some theoretical relations between seismologically observable quantities, such as the average stress drop, the critical nucleation-zone size and the duration of a slow initial phase, and the physical parameters prescribing the source model. As shown in Fig. 8, the difference between the stress levels just before and just after the occurrence of dynamic rupture is 0.86 MPa over the faulting region, except for in the neighbourhood of the nucleation zone. The stress drop of...
0.86 MPa during the dynamic rupture, which roughly corresponds to the average stress drop estimated from seismological observations, is about 0.7 times as large as the breakdown strength drop $\xi_b (= 1.2 \text{ MPa})$ at the normal-strength region. Therefore, we obtain the following approximate relation between the average stress drop $\Delta \bar{\tau}$ and the breakdown strength drop $\xi_b$:

$$\Delta \bar{\tau} = 0.7 \xi_b.$$  

From Fig. 11, the critical size $L_c$ of the nucleation zone is found to be 4.2 cm. This value is about 0.4 times as long as the weak-zone size $L (= 10 \text{ cm})$. Therefore, we obtain the following approximate relation between the critical nucleation-zone size $L_c$ and the weak-zone size $L$:

$$L_c = 0.4 L.$$  

The critical size of nucleation zones for natural earthquakes can be roughly estimated from the extent of immediate foreshock activity in the hypocentral area of the main event (Ohnaka 1993; Shibazaki & Matsu'ura 1995).

Another important observable quantity is the duration of a slowly rising initial phase preceding the main $P$ phase. In Fig. 15 we show an example of the far-field waveforms computed from a rupture nucleation model. In the computation, the 1-D slip-time function $\Delta \bar{\tau}(x, t)$ obtained by the numerical simulation is simply extended to the 2-D slip-time function $\Delta \bar{\tau}(r, t)$ on a circular fault. The factors of radiation pattern and geometrical spreading are omitted. From this diagram it is seen that seismic waves are hardly radiated during the process of slow dynamic rupture growth (phase II). After a silent period, the far-field velocity waveform rises very slowly. This slowly rising initial phase corresponds to the accelerating stage from phase II to phase III (high-speed rupture propagation) in Fig. 11. The slowly rising initial phase cannot be explained by classical source models, in which rupture is presumed to expand from a point at a constant velocity with an instantaneous stress drop. In the case of classical source models, the far-field velocity waveform rises linearly with time after the arrival of the initial phase (Sato & Hirasawa 1973). From Fig. 11 the duration of the accelerating stage from phase II to phase III is found to be $4 \times 10^{-3} \text{ s}$. This value is about 1.2 times as long as the characteristic time $L/\beta (= 3.3 \times 10^{-5} \text{ s})$ of the system. Therefore, we obtain the following approximate relation between the duration $T_s$ of the slow initial phase and the characteristic time $L/\beta$ of the system:

$$T_s = 1.2 L/\beta.$$  

The slow initial phases have actually been observed for many natural earthquakes (Umeda 1990; Iso 1992, 1995; Ellsworth & Beroza 1995).

From the relations (13) and (14), eliminating the source parameter $L$, we obtain the following relation between two seismologically observable quantities, the duration $T_s$ of the slow initial phase and the critical size $L_c$ of the nucleation zone:

$$T_s = 3 L_c/\beta.$$  

Here it should be noted that the above theoretically predicted relation holds, irrespective of the size of events, if our basic assumption in scaling, $$(\mu D_p)/(\xi_b L) = 0.57,$$ is satisfied, that is we can use the relation (15) in order to check the validity of our assumption in scaling.

4.3 Scaling to natural earthquakes

We now scale the theoretical source model explaining the transition process observed in stick-slip experiments to the cases of two large earthquakes which occurred in Japan: the 1995 Hyogoken Nambu earthquake ($M7.2$) and the 1983 Central Japan Sea earthquake ($M7.7$). For these earthquakes, both the foreshock activity just before the occurrence of the main shock and the existence of the slow initial phase have been precisely documented.
4.3.1 The 1995 Hyogoken Nanbu earthquake

From the inversion analysis of far-field waveform data, the average stress drop $\Delta \tau$ of the Hyogoken Nanbu earthquake has been estimated as 10 MPa (Kikuchi 1997). Substituting this value into the relation (12), we obtain $\tau_b = 14$ MPa as the estimate of the breakdown strength drop, which is about 12 times as large as that in the case of stick-slip experiments.

In the case of this earthquake, four foreshocks were observed in the hypocentral area just before the mainshock. Fig. 16 shows the hypocentre distributions of these foreshocks, together with the main shock (Nakamura, personal communication, 1996). The largest foreshock ($M_{3.5}$) occurred 12 hours before the main shock. From this diagram it is seen that the foreshocks occurred on the fault plane of the main shock, which has a strike C–D. Therefore, we may regard these foreshocks as the local dynamic rupture of asperities in the weak zone associated with the nucleation of the main rupture (Shibazaki & Matsu’ura 1995). From the extent of the foreshock activity around the hypocentre of the main shock, we can roughly estimate the critical nucleation-zone size $L_c$ of this earthquake to be 0.5 km.

Fig. 17 shows the initial parts of the velocity seismograms of the Hyogoken Nanbu earthquake, recorded by a JMA-87 strong-motion seismograph at Okayama station with an epicentral distance of $D = 103$ km. The arrival time of the first weak signal is indicated by $P_1$ on the seismograms. The duration $P_1-P_2$ of this initial phase is found to be about 0.6 s.

Shibazaki & Yoshida (1995) have deconvolved the high-gain velocity seismogram of the main shock at Sumoto station, which is very close to the hypocentre of the main shock, by using the seismogram of the largest foreshock ($M_{3.5}$) as the empirical Green’s function. The deconvolved displacement and velocity functions (Figs 18a and b) clearly show the existence of a slowly rising initial phase. From these diagrams the duration $T_i$ of the slow initial phase is found to be 0.6 s.

The S-wave velocity $\beta$ of the Earth’s crust is $3.5 \text{ km s}^{-1}$ on average, so substitution of $L_c = 0.5$ km into the right-hand side of eq. (15) yields $T_i = 0.43$ s. This value is nearly equal to the observed value of $T_i = 0.6$ s. Thus, our assumption in scaling is valid for the Hyogoken Nanbu earthquake. Then, substituting $T_i = 0.6$ s into relation (14), we estimate the weak-zone size $L$ of this earthquake to be 1.7 km. Furthermore, from eq. (16), taking the rigidity $\mu$ of the Earth’s crust to be 35 GPa, we find that the critical weakening displacement $D_c$ must be scaled to 0.4 m in order to hold the value of $(\mu D_c)/(\tau_b L)$ constant (0.57). This value of $D_c$ is about $1.3 \times 10^5$ times as large as that in the case of stick-slip experiments.

4.3.2 The 1983 Central Japan Sea earthquake

From the inversion analysis of tsunami data, the average stress drop $\Delta \tau$ of the Central Japan Sea earthquake has been estimated as 7 MPa (Satake 1985). Substituting this value into relation (12), we obtain $\tau_b = 10$ MPa as the estimate of the breakdown strength drop, which is about eight times as large as that in the case of stick-slip experiments.

In the case of this earthquake, remarkable foreshock activity was observed in the hypocentral area just before the main shock (Hasegawa 1983). Fig. 19 shows the

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**Figure 16.** Hypocentre distributions of the foreshocks that occurred just before the main shock of the 1995 Hyogoken Nanbu earthquake. The horizontal distribution of four foreshocks is shown together with two vertical cross-sections along A–B and C–D, which are perpendicular and parallel to the strike of the main fault. The hypocentre of the main shock is indicated by the largest square. The dashed line shows the fault plane of the main shock.
epicentre distributions of foreshocks (a) and aftershocks (b) of this earthquake. The star indicates the epicentre of the main shock. The largest foreshock (M4.9) occurred 12 days before the main shock within the 2 km epicentral distance (Umino et al. 1985). The extent of subsequent foreshock activity around the hypocentre of the main shock is about 5 km. On the basis of these observations we can roughly estimate the critical size $L_c$ of the nucleation zone as 5 km.

Fig. 20 shows the initial parts of the seismograms of the Central Japan Sea earthquake [(a) medium-period displacement and (b) long-period displacement] recorded at Dodaira station with an epicentral distance $\Delta = 480$ km (Tsujiura 1988). The arrival time of the first weak signal, confirmed by a short-period seismogram at the same station, is indicated by the arrows $P$ and $P_1$ on the seismograms. The duration $P_1$–$P_2$ of the weak signal preceding the main $P$ phase is 5–6 s. In Fig. 21 we show the initial parts of JMA strong-motion seismograms recorded at two stations, Akita ($\Delta = 113$ km) and Morioka ($\Delta = 193$ km), near the hypocentre of the main shock. The arrows indicate the arrival time of the first weak signal, confirmed by high-sensitivity seismograms at the same stations. These records support the existence of a slow initial phase preceding the main $P$ phase. For this earthquake Umeda (1990) also reported the existence of a slow initial phase with a duration of 5–6 s, based on the analysis of JMA-59 seismograms recorded at various stations with different epicentral distances ($\Delta = 187$–294 km).

Substitution of $L_c = 5$ km and $\beta = 3.5$ km s$^{-1}$ into the right-hand side of eq. (15) yields $T_i = 4.3$ s. This value is nearly equal to the observed value of $T_i = 5–6$ s. Thus, our assumption in scaling is valid for the Central Japan Sea earthquake as well as for the Hyogoken Nanbu earthquake. Then, substituting $L_c = 5$ km into relation (13), we can estimate the weak-zone size $L$ of this earthquake as 12 km. Furthermore, from eq. (16) we find that the critical weakening displacement $D_c$ must be scaled to 1.9 m in order to hold the value of $(\mu D_c)/(\xi b L)$ constant (0.57). This value of $D_c$ is about $6 \times 10^4$ times as large as that in the case of stick-slip experiments.

4.4 Fundamental scaling relations

In Section 4.2 we theoretically derived relation (15), $T_i = 3L_c/\beta$, 

![Figure 17](image1.png) Velocity seismograms (E–W component) of the Hyogoken Nanbu earthquake, recorded by the JMA-87 strong-motion seismograph at Okayama station with an epicentral distance $\Delta = 103$ km. The slowly rising part $P_1$–$P_2$ of the seismogram in (a) is magnified in the lower diagram (b).

![Figure 18](image2.png) The displacement function (a) and the velocity function (b) of the Hyogoken Nanbu earthquake, obtained from the deconvolution of the high-gain velocity seismogram at Sumoto station with an epicentral distance $\Delta = 31$ km (after Shibazaki & Yoshida 1995).
between the duration $T_i$ of the slow initial phase and the critical size $L_c$ of the nucleation zone on the basic assumption (16),

$$(\mu \bar{D}_c)/(\bar{r}_b L) = 0.57,$$

in scaling. Fig. 22 is a plot of observed $T_i$ versus observed $L_c$ for two natural earthquakes, the Hyogoken Nanbu earthquake ($M7.2$) and the Central Japan Sea earthquake ($M7.7$), and one stick-slip event. From this diagram, it is seen that the theoretically predicted relation between $T_i$ and $L_c$, which is indicated by the thick line, is roughly realized, irrespective of the sizes of the events. This means that our estimation of source parameters based on the basic assumption in scaling is valid for the natural earthquakes.

In Table 2 we summarize the values of the physical parameters prescribing the theoretical source model (a) and the seismologically observable quantities (b) for the Hyogoken Nanbu earthquake and the Central Japan Sea earthquake, together with those for the stick-slip event. Since the rigidity $\mu$ and the S-wave velocity $c$ are scale-independent in any sense, the essential parameters prescribing the source model are the breakdown strength drop $\bar{r}_b$, the critical weakening displacement $\bar{D}_c$ and the weak-zone size $L$. From Table 2(a) we find the following fundamental relations between the source parameters:

$$\bar{D}_c = \gamma L \quad (\gamma \approx 10^{-4}),$$

$$\bar{r}_b = c \quad (c \approx 10 \text{ MPa}).$$

That is, the critical weakening displacement $\bar{D}_c$ increases linearly with the weak-zone size $L$, but the breakdown strength drop $\bar{r}_b$ is nearly constant, irrespective of the size of events. From these fundamental scaling relations, defining the fracture surface energy by $\bar{G}_c = \bar{r}_b \bar{D}_c/2$, we can obtain the secondary scaling relation

$$\bar{G}_c = \kappa L \quad (\kappa \approx 10^2 \text{ J/m}^2).$$

Given the physical parameters prescribing the source model (Table 2a), we can theoretically compute the seismologically observable quantities, the average stress drop $\Delta \tau$, the critical nucleation-zone size $L_c$, the duration of the initial phase $T_i$ and the cut-off frequency of the slip-acceleration spectrum $f_{s\text{max}}$ for each case. All of the results are summarized in Table 2(b), together with the observed values. Here it should be noted that the cut-off frequency of the slip-acceleration spectrum $f_{s\text{max}}$ is nearly in proportion to the inverse of the sizes of the events. This means that our estimation of the breakdown strength drop $\Delta \tau$ is nearly constant, irrespective of the size of events.

From these fundamental scaling relations, defining the fracture surface energy by $\bar{G}_c = \bar{r}_b \bar{D}_c/2$, we can obtain the secondary scaling relation

$$\bar{G}_c = \kappa L \quad (\kappa \approx 10^2 \text{ J/m}^2).$$

Given the physical parameters prescribing the source model (Table 2a), we can theoretically compute the seismologically observable quantities, the average stress drop $\Delta \tau$, the critical nucleation-zone size $L_c$, the duration of the initial phase $T_i$ and the cut-off frequency of the slip-acceleration spectrum $f_{s\text{max}}$ for each case. All of the results are summarized in Table 2(b), together with the observed values. Here it should be noted that the cut-off frequency of the slip-acceleration spectrum $f_{s\text{max}}$ is nearly in proportion to the inverse of the sizes of the events. This means that our estimation of the breakdown time $T_b$ in the stage of high-speed rupture propagation, as demonstrated in Section 3.2. The breakdown time $T_b$ is scaled by the characteristic time $L/\beta$ of the system:

$$T_b = 0.3L/\beta,$$

so, substituting this equation into eq. (11), we obtain the approximate relation

$$f_{s\text{max}} = 3/\beta L.$$

Furthermore, from relations (13) and (21), eliminating the source parameter $L$, we obtain the following relation between the cut-off frequency $f_{s\text{max}}$ and the critical nucleation-zone size $L_c$:

$$f_{s\text{max}} = 1.3/\beta L_c.$$

For the Hyogoken Nanbu earthquake, the cut-off frequency of 4–5 Hz has been obtained from the spectral analysis of the acceleration seismogram recorded at Kobe University, which is very close to the earthquake fault (Kamae, personal communication, 1996). This value is in accord with the theoretical estimate (6 Hz). In the case of the Central Japan Sea earthquake, from the analysis of a strong-motion seismogram.

























































































































































































































































































































































































































































































































































































































































































































































































































































Transition from nucleation to rupture propagation

Figure 20. Seismograms of the Central Japan Sea earthquake, recorded at Dodaira station with the epicentral distance $\Delta = 480$ km (after Tsujiura 1988). (a) Medium-period displacement. (b) Long-period displacement. The arrows P in (a) and $P_1$ in (b) indicate the arrival time of the first weak signal identified on a short-period seismogram at the same station.

Figure 21. JMA strong-motion seismograms of the Central Japan Sea earthquake, recorded at Akita ($\Delta = 113$ km) and Morioka ($\Delta = 193$ km). The arrows indicate the arrival time of the first weak signal confirmed by high-sensitivity seismograms at the same stations.

Figure 22. A plot of the duration of slow initial phases versus the critical size of nucleation zones for the Central Japan Sea earthquake, the Hyogoken Nanbu earthquake, and the laboratory stick-slip event. The thick line indicates the theoretically predicted relation between the duration $T_i$ of the slow initial phase and the critical size $L_c$ of the nucleation zone.

5 DISCUSSION OF RESULTS

Using a fault model with the slip-dependent constitutive law reconstructed from the results of stick-slip experiments, we numerically simulated the transition process from quasi-static nucleation to high-speed rupture propagation and succeeded in quantitatively explaining the three phases observed in stick-slip experiments: very slow quasi-static nucleation preceding the onset of dynamic rupture, dynamic but slow rupture growth without seismic-wave radiation and subsequent high-speed rupture propagation. As experimentally confirmed by Dieterich (1979, 1981), the frictional stress of rocks depends not only on fault slip but also on slip rate. The success in quantitatively explaining the experimental results with the slip-dependent constitutive law indicates that the rate effect has...
Table 2. Comparison of the physical parameters prescribing the source model (a) and the seismologically observable quantities (b) for the stick-slip event, the Hyogoken Nanbu (HKN) earthquake, and the Central Japan Sea (CJS) earthquake.

(a) Physical parameters prescribing the source model.

| Event         | $L$ (km) | $D_i$ (m) | $\tilde{\tau}_b$ (MPa) | $\mu$ (GPa) | $\beta$ (km s$^{-1}$) |
|---------------|----------|-----------|-------------------------|-------------|-----------------------|
| Stick-slip event | $1.0 \times 10^{-4}$ | $3.0 \times 10^{-6}$ | 1.2                     | 23          | 3.0                   |
| HKN earthquake (scaled) | 1.7       | 0.4       | 14                      | 35          | 3.5                   |
| CJS earthquake (scaled) | 12        | 1.9       | 10                      | 35          | 3.5                   |

(b) Seismologically observable quantities.

| Event         | $\Delta\tau$ (MPa) | $L_c$ (km) | $T_i$ (s) | $f_{\text{max}}$ (Hz) |
|---------------|---------------------|------------|-----------|-----------------------|
| Stick-slip event | 0.7–0.9            | $3\times10^{-5}$ | $6 \times 10^{-5}$ | $5 \times 10^{5}$ |
| computed      | 0.86                | $4.2 \times 10^{-5}$ | $4 \times 10^{-5}$ | $1 \times 10^{5}$ |
| HKN earthquake | observed          | 10         | 0.5       | 0.6                   | 4–5          |
| predicted     | 10                  | 0.7        | 0.6       | 6                     |
| CJS earthquake | observed          | 7–8        | 4–5       | 5–6                   | 4–6          |
| predicted     | 7.2                 | 5.0        | 4         | 1                     |

only secondary significance as far as the transition process from nucleation to dynamic rupture propagation is concerned. In fact, Okubo & Dieterich (1986) have suggested that the rate effect has a high-speed cut-off. In the present numerical simulation, the slip velocity during slow dynamic rupture growth is of the order of $10^{-4}$–$10^{-2}$ m s$^{-1}$, which is above the average high-speed cut-off velocity ($2 \times 10^{-4}$ m s$^{-1}$) estimated by them.

Computation of far-field waveforms with the theoretical source model shows that a slowly rising initial phase preceding the main $P$ phase is radiated in the accelerating stage from slow dynamic rupture growth to high-speed rupture propagation (Fig. 15). The actually observed slow initial phases (e.g. Fig. 18) are, however, not as simple as theoretically predicted, but contain several small pre-events. These pre-events can be considered as the local brittle rupture of small asperities distributed in the nucleation zone of the main rupture. Recently, Shibazaki & Matsu'ura (1995) have succeeded in explaining the occurrence of foreshocks and pre-events associated with the nucleation of large earthquakes by considering the distribution of locally strong parts (asperities) in the broad weak zone on a fault plane. In the present study, we considered a simple weak-zone model for scaling, because the overall features of nucleation are essentially controlled by the frictional properties of the weak zone with the largest scale. In this sense we may regard the weak-zone size $L$ as the largest wavelength characterizing strength variation along the fault.

On the hypothesis that the physical law governing the rupture process in natural earthquakes is essentially the same as that in stick-slip events, we scaled the theoretical source model explaining the stick-slip experiments to the cases of the Hyogoken Nanbu earthquake and the Central Japan Sea earthquake, so that the scaled source model explains the observed average stress drop, the critical nucleation-zone size and the duration of the slow initial phase well. The essential parameters prescribing the source model are the weak-zone size $L$, the critical weakening displacement $D_i$, and the breakdown strength drop $\tilde{\tau}_b$. In scaling these parameters, we assumed that the non-dimensional quantity $\mu' = (\mu D_i)/\tilde{\tau}_b L$ is kept constant (0.57) irrespective of the size of the events. The validity of this basic assumption in scaling was confirmed by checking the theoretically predicted relation (15) between two seismologically observable quantities, the duration $T_i$ of the slow initial phase and the critical size $L_c$ of the nucleation zone. From the results of scaling we found the following fundamental relations between the source parameters: the critical weakening displacement $D_i$ increases linearly with the weak-zone size $L$, but the breakdown strength drop $\tilde{\tau}_b$ is nearly constant irrespective of the size of events. These fundamental relations lead directly to the secondary scaling relation: the fracture surface energy, $\widetilde{G}_i = \tilde{\tau}_b D_i/2$, increases linearly with the weak-zone size $L$.

As a consequence of the fundamental scaling relations, it can theoretically be expected that the duration $T_i$ of the slow initial phase is in proportion to the weak-zone size $L$; that is, taking the value of $\beta$ to be 3 km in eq. (14),

$$T_i = 0.4L \quad (T_i \text{ in s and } L \text{ in km}). \quad (23)$$

From the analysis of far-field seismograms for large and intermediate earthquakes, on the other hand, Umeda (1992) has obtained an empirical relation between the duration $T_i$ of slow initial phases and the magnitude $M$ of earthquakes:

$$\log T_i = 0.5M - 3.4 \quad (T_i \text{ in s}). \quad (24)$$

Recently, from the analysis of near-field seismograms, Ellsworth & Beroza (1995) have also confirmed the existence of a similar relation over a wide range of magnitude.
\( M = 2.6–8.1 \). Using the well-established empirical relation
\[ \log S = M - 4.0 \quad (S \text{ in km}^2) \]  
(25)
between the earthquake fault area \( S \) and the magnitude \( M \) (Utsu & Seki 1955; Kanamori & Anderson 1975), we can rewrite relation (24) as
\[ \log T_i = \log L_i - 1.4 \quad (T_i \text{ in s and } L_i \text{ in km}), \]  
(26)
where \( L_i \) indicates the fault dimension defined by \( L_i = S^{1/2} \). This means that the duration \( T_i \) of slow initial phases increases linearly with the fault dimension \( L_i \). From eqs (23) and (26), eliminating \( T_i \), we finally obtain the following equation relating the weak-zone size \( L \) to the fault dimension \( L_i \):
\[ L = 0.1L_i. \]  
(27)
Therefore, we may read the linear \( L \)-dependence of \( \bar{D} \) and \( \bar{G} \) stated in Section 4.4 as the linear \( L_i \)-dependence of \( \bar{D} \) and \( \bar{G} \). Aki and co-workers determined the source parameters, including \( \bar{D} \) and \( \bar{G} \), of major Californian earthquakes by the application of a specific barrier model (Papageorgiou & Aki 1983; Aki 1992). Their results also indicate the linear \( L_i \)-dependence of \( \bar{D} \) and \( \bar{G} \).

6 CONCLUSIONS
The transition process from nucleation to high-speed rupture propagation observed in stick-slip events was quantitatively explained by a theoretical model with a slip-dependent friction law. Slip dependence of fault constitutive behaviour is essential as far as the transition process is concerned. On the assumption that the physical law governing rupture processes in natural earthquakes is essentially the same as that in stick-slip events, we scaled the theoretical source model explaining the stick slip experiments to the case of the 1995 Hyogoken Nanbu earthquake (\( M7.2 \)) and the 1983 Central Japan Sea earthquake (\( M7.7 \)) so that the scaled source model explains the observed average stress drop, the critical nucleation-zone size and the duration of the slow initial phase well. From the results of scaling, we found the following fundamental relations between the source parameters: (1) the critical weakening displacement \( \bar{D} \) is proportional to the weak-zone size \( L \), but (2) the breakdown strength drop \( \bar{G} \) is independent of \( L \).

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