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Medium-modified average multiplicity and multiplicity fluctuations in jets

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Abstract: The energy evolution of average multiplicities and multiplicity fluctuations in jets produced in heavy-ion collisions is investigated from a toy QCD-inspired model. In this model, we use modified splitting functions accounting for medium-enhanced radiation of gluons by a fast parton which propagates through the quark gluon plasma. The leading contribution of the standard production of soft hadrons is enhanced by a factor $\sqrt{N_s}$ while next-to-leading order (NLO) corrections are suppressed by $1/\sqrt{N_s}$, where the parameter $N_s > 1$ accounts for the induced-soft gluons in the medium. Our results for such global observables are cross-checked and compared with their limits in the vacuum.

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Recent experiments at the Relativistic Heavy Ion Collider (RHIC) have established a phenomenon of strong high-transverse momentum hadron suppression [1], which supports the picture that hard partons going through dense matter suffer a significant energy loss prior to hadronization in the vacuum (for recent review see [2]).

Predictions concerning multi-particle production in nucleus-nucleus collisions can be carried out by using a toy QCD-inspired model introduced by Borghini and Wiedemann in [3]; it allows for analytical computations and may capture some important features of a more complete QCD description. In this model, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting functions $q \to g\bar{q}$ and $g \to gg$ [4] of the QCD evolution equations were distorted so that the role of soft emissions was enhanced by multiplying the infra-red singular terms by the medium factor $N_s$. The model [3] was further discussed and used on the description of final states hadrons produced in heavy-ion collisions [5].

Within the model, we make predictions for the medium-modified average multiplicity $N_A$ in quark and gluon jets ($A = q, g$) produced in such reactions, for the ratio $r = N_g/N_q$ and finally for the second multiplicity correlators $\langle N_A(N_A - 1)/N_A^2 \rangle$, which determines the width of the multiplicity distribution.

The starting point of our analysis is the NLO or Modified-Leading-Logarithmic-Approximation (MLLA) master evolution equation for the generating functional [4] which determine the jet properties at all energies together with the initial conditions at threshold at small $x$, where $x$ is the fraction of the outgoing jet energy carried away by a single gluon. Their solutions with medium-modified splitting functions can be resummed in powers of $\sqrt{\alpha_s/N_s}$ and the leading contribution can be represented as an exponential of the medium-modified anomalous dimension which takes into account the $N_s$-dependence:

$$N_A \simeq \exp \left\{ \int_Y^{\infty} \gamma_{\text{med}}(\alpha_s(Y)) \, dY \right\},$$

where $\gamma_{\text{med}}(\alpha_s)$ can be expressed as a power series of $\sqrt{\alpha_s/N_s}$ in the symbolic form:

$$\gamma_{\text{med}}(\alpha_s) \simeq \sqrt{N_s} \times \sqrt{\alpha_s} \left( 1 + \frac{\alpha_s}{N_s} + \mathcal{O}\left( \frac{\alpha_s}{N_s} \right) \right).$$

Within this logic, the leading double logarithmic approximation (DLA, $\mathcal{O}(\sqrt{N_s\alpha_s})$), which resums both soft and collinear gluons, and NLO (MLLA, $\mathcal{O}(\alpha_s)$), which resums hard collinear partons and accounts for the running of the coupling constant $\alpha_s$, are complete. The choice $dY = d\Theta/\Theta$, where $\Theta \ll 1$ is the angle between outgoing couples of partons in independent partonic emissions, follows from Angular Ordering (AO) in intra-jet cascades [4]. In order to obtain the hadronic spectra, we advocate for the Local Parton Hadron Duality (LPHD) hypothesis [6]: global and differential partonic observables can be normalized to the corresponding hadronic observables via a certain constant $\mathcal{K}$ that can be fitted to the data, i.e. $N_{g,q}^h = \mathcal{K} \times N_{g,q}$.

The evolution of a jet of energy $E$ and half-opening angle $\Theta$ involves the DLA anomalous dimension $\gamma_0$ related to the coupling constant $\alpha_s$ through $\gamma_0^2 = 2N_c\alpha_s/\pi$, with $\alpha_s = 2\pi/4N_c\beta_0(Y + \lambda)$, where $Y = \ln(Q/Q_0)$ ($Q = E\Theta$ is the hardness or maximum transverse momentum of the jet), $\lambda = \ln(Q_0/\Lambda_{\text{QCD}})$ is a parameter associated with hadronization ($Q_0$ is the collinear cut-off parameter, $k_T > Q_0$, and $\Lambda_{\text{QCD}}$ is the intrinsic QCD scale) and $\beta_0 = \frac{1}{4N_c} \left( \frac{11}{3}N_c - \frac{4}{3}T_R \right)$, where $T_R = n_f/2$, $n_f$ being the number of active flavors. At MLLA, as a consequence of angular ordering in parton cascading, the average multiplicity inside a gluon and a quark jet, $N_{g,q}$, obey the system of two-coupled evolution equations [7]
integration over DGLAP splitting functions:

\[ N_{gY} = \int_{0}^{1} dx \gamma_{0}^{2} \left[ \Phi_{g}^{\prime} (N_{g}(x) + N_{g}(1 - x) - N_{g}) + n_{f} \Phi_{g}^{Y} (N_{q}(x) + N_{q}(1 - x) - N_{g}) \right] , \]  

\[ N_{qY} = \int_{0}^{1} dx \gamma_{0}^{2} \left[ \Phi_{q}^{\prime} (N_{g}(x) + N_{q}(1 - x) - N_{g}) \right] , \]

which follow from the MLLA master evolution equation for the generating functional; \( N_{g,q} \equiv N_{g,q}(Y) \), \( N_{g,q}(x) \equiv N_{g,q}(Y + \ln x) \), \( N_{g,q}(1 - x) \equiv N_{g,q}(Y + \ln(1 - x)) \). \( \Phi_{g}^{Y} \) denotes the medium-modified DGLAP splitting functions:

\[ \Phi_{g}^{\prime}(x) = \frac{N_{s}}{x} - (1 - x)[2 - x(1 - x)] , \]

\[ \Phi_{g}^{Y}(x) = \frac{1}{4N_{c}}[x^2 + (1 - x)^2] , \]

\[ \Phi_{q}^{Y}(x) = \frac{C_{F}}{N_{c}} \left( \frac{N_{s}}{x} + 1 + \frac{x}{2} \right) , \]

which accounts for parton energy loss in the medium by enhancing the singular terms like \( \Phi \approx N_{s}/x \) as \( x \ll 1 \) as proposed in the Borghini-Wiedemann model [3]. Thus, when \( N_{s} \) increases the DLA becomes dominant and energy-momentum conservation plays a less important role.

For \( Y \gg \ln x \sim 1 - (1 - x) \), \( N_{g,q}(x) \) \( N_{g,q}(1 - x) \) can be replaced by \( N_{g,q} \) in the hard partonic splitting region \( x \sim 1 - x \sim 1 \) (non-singular or regular parts of the splitting functions), while the dependence at small \( x \ll 1 \) is kept in the singular term \( \Phi(x) \approx N_{s}/x \) as done in the vacuum. Furthermore, the integration over \( x \) can be replaced by the integration over \( Y(x) = \ln \left( \frac{x^{Eg_{s}}}{Q_{0}} \right) \). Thus, one is left with the approximate system of two-coupled equations:

\[ \frac{d^2}{dY^2} N_{g}(Y) = \gamma_{0}^{2} \left( N_{s} - a_{1} \frac{d}{dY} \right) N_{g}(Y) , \]

\[ \frac{d^2}{dY^2} N_{q}(Y) = \frac{C_{F}}{N_{c}} \gamma_{0}^{2} \left( N_{s} - \tilde{a}_{1} \frac{d}{dY} \right) N_{g}(Y) , \]

with the initial conditions at threshold \( N_{A}(0) = 1 \) and \( N_{A}^{\prime}(0) = 0 \) and the hard constants:

\[ a_{1} = \frac{1}{4N_{c}} \left[ \frac{11}{3} N_{c} + \frac{4}{3} T_{R} \left( 1 - 2 \frac{C_{F}}{N_{c}} \right) \right] , \quad \tilde{a}_{1} = 3/4. \]

The quantum corrections \( \propto a_{1} \), \( \tilde{a}_{1} \) in [5] arise from the integration over the regular part of the splitting functions, they are \( \mathcal{O}(\sqrt{\alpha_{s}/N_{s}}) \) suppressed and partially account for energy conservation as happens in the vacuum.

These equations can be solved by applying the inverse Mellin transform to the self-contained gluonic equation [5], which leads to

\[ N_{g}^{\ell}(Y) \simeq \mathcal{K} \times \int_{C} \frac{d\omega}{2\pi i} \omega^{a_{1} - 2} \exp \left[ \omega(Y + \lambda) + \frac{N_{s}}{\beta_{0}\omega} \right] , \]

where the contour \( C \) lies to the right of all singularities of \( N_{g}(\omega) \) in the complex plane. Since we are concerned with the asymptotic solution of the equation as \( Y \gg 1 \) \( (E\Theta \gg Q_{0}) \), that is the high-energy limit, the inverse Mellin transform (5) can be estimated by the steepest descent method. Indeed, the large
Figure 1: MLLA medium-modified average multiplicity as a function of $Q = E/\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

The parameter is $Y$ and the function in the exponent presents a saddle point at $\omega_0 = \sqrt{N_s/\beta_0(Y + \lambda)}$, such that the asymptotic solution reads

$$N_h(Y) \simeq K \times (Y + \lambda)^{-\sigma_1} \exp \left( \frac{4N_s}{\beta_0}(Y + \lambda) \right), \quad (8)$$

where $\sigma_1 = \frac{\partial^2}{2} - \frac{\bar{a}_1}{3}$. The constant $\sigma_1$ is $N_s$-independent because it resums vacuum corrections. Therefore, the production of soft gluons in the medium becomes $\exp \left[ 2(\sqrt{N_s} - 1)/(Y + \lambda) / \beta_0 \right]$ higher than the standard production of soft gluons in the vacuum [4]. From (8) and (1) one obtains the medium-modified MLLA anomalous dimension $\gamma_{med} = \frac{dN_g}{dY} = \frac{\gamma_0}{\sqrt{N_s} + O(\gamma_2)}$, which is nothing but the MLLA rate of multi-particle production with respect to the evolution-time variable $Y$ in the dense medium. In Fig. 1, we display the medium-modified average multiplicity (8) with predictions in the vacuum ($N_s = 1$) in the range $10 \leq Q(\text{GeV}) \leq 200$; we set $Q_0 = \Lambda_{QCD} = 0.23$ GeV in the limiting spectrum approximation [7], and take $K = 0.2$ from [7].

The values $N_s = 1.6$ and $N_s = 1.8$ in the medium may be realistic for RHIC and LHC phenomenology [3, 5]; the jet energy subrange $10 \leq Q(\text{GeV}) \leq 50$ displayed in Fig. 1 has been recently considered by the STAR collaboration, which reported the first measurements of charged hadrons and particle-identified fragmentation functions from $p+p$ collisions [8] at $\sqrt{s_{NN}} = 200$ GeV. Finally, the whole jet energy range in the same figure, in particular for those values at $Q \geq 50$ GeV, will be reached at the LHC, i.e., $Q = 100$ GeV is an accessible value in this experiment (see [3] and references therein).

We find, as expected, that the production of soft hadrons increases as $N_s > 1$: the available phase space for the production of harder collinear hadrons is restricted as the model itself states. The medium-modified MLLA gluon to quark average multiplicity ratio, $r = N_g/N_q = N_g^h/N_q^h$, following from (8) and (3) reads

$$r = r_0 \left[ 1 - r_1 \frac{\gamma_0}{\sqrt{N_s}} + O \left( \frac{\gamma_2}{N_s} \right) \right], \quad r_0 = \frac{N_c}{C_F}, \quad (9)$$

where we introduced the coefficient $r_1 = a_1 - \bar{a}_1$ in the term suppressed by $\gamma_0/\sqrt{N_s}$ as $N_s > 1$. Therefore, if compared with its behavior at $N_s = 1$, we check, as expected from the model [3], that $r$ becomes closer to its asymptotic DLA limit $r_0 = N_c/C_F = 9/4$, as depicted in Fig. 2. Setting $N_s = 1$ in (8), one recovers the appropriate limits in the vacuum [4, 9, 10]. Finally, the gluon jets are still more...
Figure 2: MLLA ratio $r$ as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

active than the quark jets in producing secondary particles and the shape of the curves are roughly the same.

The normalized second multiplicity correlator $A_2 = \langle N_A(N_A - 1) \rangle / N_A^2$ defines the width of the multiplicity distribution and is related to its dispersion by the formula $D_A^2 = (A_2 - 1)N_A^2 + N_A$ [9]. These moments, which are less inclusive than the average multiplicity, prove to be $K$-independent and therefore provide a pure test of multiparticle production. The medium-modified system of two-coupled evolution equations for this observable follows from the MLLA master equation for the azimuthally averaged generating functional [4] and can be written in the convenient form

$$
\frac{d}{dY}(N_g^{(2)} - N_g^2) = \int_{0}^{1} dx \gamma_0^2 \Phi_g^2 \left[ N_g^{(2)}(Y + \ln x) + \left( N_g^{(2)}(Y + \ln(1 - x)) - N_g^{(2)}(Y) \right) \right] \\
+ \left( N_g(Y + \ln x) - N_g(Y) \right) \left( N_g(Y + \ln(1 - x)) - N_g(Y) \right) \\
+ n_f \int_{0}^{1} dx \gamma_0^2 \Phi_g^2 \left[ 2 \left( N_q^{(2)}(Y + \ln x) - N_q^{(2)}(Y + \ln x) \right) - \left( N_q^{(2)}(Y) - N_q^{(2)}(Y) \right) \right] \\
+ \left( 2N_q(Y + \ln x) - N_q(Y) \right) \left( 2N_q(Y + \ln(1 - x)) - N_q(Y) \right),
$$

which proves to be more suitable for obtaining analytical solutions in the following. We use a new method to compute solutions at MLLA by replacing $N_A^{(2)} = A_2N_A^2$ on both sides of the expanded equations at $x \sim 1 - x \sim 1$. The notations in (10-11) follow the same logic as those in (5-6). Applying the analysis that led to the system (5-6), we obtain from (10-11)

$$
\frac{d}{dY}^2 \left( N_g^{(2)} - N_g^2 \right) = \gamma_0^2 \left( N_s - a_1 \frac{d}{dY} \right) N_g^{(2)} + (a_1 - b_1) \gamma_0^2 \frac{d}{dY} N_g^2,
$$

$$
\frac{d}{dY}^2 \left( N_q^{(2)} - N_q^2 \right) = \frac{C_F}{N_c} \gamma_0^2 \left( N_s - \tilde{a}_1 \frac{d}{dY} \right) N_q^{(2)},
$$

where

$$
b_1 = \frac{1}{4N_c} \left[ \frac{11}{3} N_c - 4 \frac{T_R}{N_c} \left( 1 - 2 \frac{C_F}{N_c} \right) \right].
$$
The constant $N_s$ only affects the leading double logarithmic term of the equations. The terms proportional to $a_1$, $(a_1 - b_1)$ and $\tilde{a}_1$ are hard vacuum corrections, which partially account for energy conservation, indeed $\gamma_0^2 N_s \approx \sqrt{N_s} \gamma_0^3$ and the relative correction to DLA is $O(\sqrt{\alpha_s/N_s})$.

Setting $N_s^{(2)} = G_2^2 N_s^2$ in (12) and making use of (8), the system can be solved iteratively by taking terms up to $O(\alpha_s)$ into consideration. The analytical solution reads,

$$G_2 - 1 = \frac{1 - \left(\frac{2}{3} a_1 + 2 b_1\right) \frac{\gamma_0}{\sqrt{N_s}} + O\left(\frac{\gamma_0^2}{N_s}\right)}{3 - (4a_1 - \beta_0) \frac{\gamma_0}{\sqrt{N_s}} + O\left(\frac{\gamma_0^2}{N_s}\right)}, \quad (14)$$

while its expansion in the form $1 + \gamma_0/\sqrt{N_s}$ leads to

$$G_2 - 1 \approx \frac{1}{3} - c_1 \frac{\gamma_0}{\sqrt{N_s}} + O\left(\frac{\gamma_0^2}{N_s}\right), \quad (15)$$

where the linear combination of color factors can be written in the form

$$c_1 = \frac{1}{4N_c} \left(\frac{55}{9} - \frac{4 T_R}{N_c} + \frac{112 T_R C_F}{9 N_c N_c} - \frac{32 T_R C_F^2}{3 N_c N_c^2}\right). \quad (16)$$

We use (13) and (3) and substitute $N_s^{(2)} = G_2^2 N_s^2$ into (13) such that the solution reads

$$Q_2 - 1 \approx \frac{N_c}{C_F} \left(\frac{1}{3} - \tilde{c}_1 \frac{\gamma_0}{\sqrt{N_s}}\right) + O\left(\frac{\gamma_0^2}{N_s}\right), \quad (17)$$

where we obtain the combination of color factors

$$\tilde{c}_1 = \frac{1}{4N_c} \left(\frac{55}{9} + \frac{4 T_R C_F}{9 N_c N_c} - \frac{8 T_R C_F^2}{3 N_c N_c^2}\right). \quad (18)$$

Setting $N_s = 1$ in (13) and (17) we get a perfect agreement with the vacuum results [9]. In Fig. 1 and Fig. 2, we compare our results for the medium-modified second multiplicity correlators (15) and (17) with predictions in the vacuum ($N_s = 1$) [9] in the limiting spectrum approximation inside the typical range $10 \leq Q$(GeV) $\leq 200$ for RHIC and LHC phenomenology. Similarly to the MLLA ratio $r(N_s)$, Eq. (8), the hard corrections $O(\gamma_0)$ are suppressed by a factor $1/\sqrt{N_s}$. As expected from the model,
we check that these results approach their DLA limits when $N_s$ increases; moreover, the multiplicity fluctuations of individual events must be larger for quark jets as compared to gluon jets just like in the vacuum [9]. Another interesting feature of these observables concerns the shape of the curves. They are roughly identical and prove not to depend on the medium parameter $N_s$. Moreover, there exists evidence for a flattening of the slopes as the hardness of the jet $Q = E\Theta$ increases for $N_s \geq 1$ (vacuum and medium). This kind of scaling behavior is known as the Koba-Nielsen-Olsen (KNO) scaling [11]: it was discovered by Polyakov in quantum field theory [12] and experimentally confirmed by $e^+e^-$ measurements [13] for the second and higher order multiplicity correlators.

In this paper we have dealt with the medium-modified average multiplicity and the medium-modified second multiplicity correlator in quark and gluon jets at RHIC and LHC energy scales. The starting point of our calculations is based on the Borghini-Wiedemann work [3], which models parton energy loss in a nuclear medium. The average multiplicity is found to be enhanced by the factor $\sqrt{N_s}$ acting on the exponential leading contribution (8); this leads in particular to the rescaling of the anomalous dimension $\gamma \rightarrow \gamma_{med} \approx \sqrt{N_s}\gamma(0)$ or equivalently, to the enhancement of the in medium coupling constant. Since hard corrections are suppressed by the extra factor $1/\sqrt{N_s}$, it is straightforward to check that $r$, $G_2$ and $Q_2$ approach the asymptotic DLA limits $r_0 = N_c/C_F$, $G_2 = 4/3$ and $Q_2 = 1 + N_c/3C_F$ [4] when $N_s$ increases. The previously mentioned KNO-scaling experienced by $G_2$ and $Q_2$ proves no special sensibility to the model and should normally hold like in the vacuum.

Finally, since these results are model-dependent, they may still be improved in the future, specially after the $N_s$-dependence of the non-singular parts of the splitting functions (8) has been exactly computed.

**Perspective:** Many experimental characterizations of the medium-modified intrajet structure in heavy-ion collisions at RHIC and at the LHC require a soft momentum cut-off $p_T^{cut}$, with $Q > p_T^{cut}$ to remove the effects of the high multiplicity background. In [3], the soft background was subtracted by integrating the single inclusive differential distribution $dN/d\ln p_T$ (“hump-backed plateau”) over the range $Q \geq p_T \geq p_T^{cut}$, with $p_T^{cut} > \Lambda_{QCD}$. Accordingly, the equivalent computation should be performed for the second multiplicity correlator by integrating the double differential inclusive distribution (two-particle correlation) $d^2N/d\ln p_{1,T}d\ln p_{2,T}$ over $p_{i,T}$, with the lower bounds of integration $p_{i,T}^{cut} > \Lambda_{QCD}$ ($i = 1, 2$). Imposing such a cut-off in our calculations will affect the normalization rather than the behavior and the shape of these observables as a function of $N_s$ and the jet energy scale of the process $Q$ [14].
References

[1] K. Adcox et al. (PHENIX Collab.), Phys. Rev. Lett. 88 (2002) 022301; 
   S.S. Adler et al. (PHENIX Collab.), Phys. Rev. Lett. 91 (2003) 072301; 
   C. Adler et al. (STAR Collab.), Phys. Rev. Lett 89 (2002) 202301.

[2] F. Arleo, hep-ph/0810.1193 and references therein; 
   S. Peigné & A.V. Smilga, hep-ph/0810.5702.

[3] N. Borghini & U.A. Wiedemann, hep-ph/0506218.

[4] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller & S.I. Troyan, Basics of Perturbative QCD, Editions Frontières, Paris (1991).

[5] S. Sapeta & U.A. Wiedemann, Eur. Phys. J. C55 (2008) 293.

[6] Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, Z. Phys. C 27 (1985) 65; Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, J. Phys. G 17 (1991) 1585.

[7] I. M. Dremin, V. A. Nechitaîlo, Mod. Phys. Lett. A 9 (1994) 1471; JETP Lett. 58 (1993) 945.

[8] M. Heinz for the STAR Collaboration, nucl-exp/0809.3769.

[9] E. D. Malaza & B. R. Webber, Phys. Lett. B 149 (1984) 501; E. D. Malaza & B. R. Webber, Nucl. Phys. B 267 (1986) 702.

[10] V. A. Khoze & W. Ochs, Int. J. Mod. Phys. A 12 (1997) 2949.

[11] Yu.L. Dokshitzer, Phys. Lett. B 305 (1993) 295.

[12] A.M. Polyakov, Sov. Phys. JETP 32 (1971) 296.

[13] HRS Coll., Phys. Rev. D 34 (1986) 3304; AMY Coll., Phys. Rev. D 42 (1990) 737; DELPHI Coll., Z. Phys. C-Particles and Fields 50 (1991) 185.

[14] N. Borghini & R. Perez-Ramos, in preparation.