A New Coupling Potential for the Scattering of Deformed Light Heavy-Ions

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This letter introduces a new coupling potential to explain the experimental data over wide energy ranges for a number of systems. Within the coupled-channels formalism, this letter first shows the limitations of the standard coupled-channels theory in the case where one of the nuclei in the reaction is strongly deformed and then, demonstrates that a global solution to the problems of light-heavy-ion reactions such as $^{12}\text{C} + ^{12}\text{C}$, $^{16}\text{O} + ^{28}\text{Si}$ and $^{12}\text{C} + ^{24}\text{Mg}$ can be found using a new second-derivative coupling potential in the coupled-channels formalism. This new approach consistently improves the agreement with the experimental data for the elastic and inelastic scattering data as well as for their excitation functions using constant or slightly energy-dependent parameters.

In this letter, building on a previous paper [1], we introduce a new type of coupling potential that explains the scattering observables of the $^{16}\text{O} + ^{28}\text{Si}$, $^{12}\text{C} + ^{24}\text{Mg}$ and $^{12}\text{C} + ^{12}\text{C}$ systems in a consistent way over wide energy ranges. These reactions have been extensively studied and found to present many problems to which no consistent solution has been provided yet using the standard coupled-channels theory.

Over the last 40 years, a large body of experimental data has been accumulated from the systematic studies of these reactions [2, 3]. Although the theoretical models, based on dynamical models or purely phenomenological treatments, provide reasonably good fits [2, 3, 4], no unique model has been proposed that explains consistently the elastic and inelastic scattering data over wide energy ranges without applying any ad-hoc procedures.

Consequently, the following problems continue to exist for light heavy-ion reactions: (1) explanation of anomalous large angle scattering data; (2) reproduction of the oscillatory structure near the Coulomb barrier; (3) the out-of-phase problem between theoretical predictions and experimental data; (4) simultaneous fits of the individual angular distributions, resonances and excitation functions (for the $^{12}\text{C} + ^{12}\text{C}$ system in particular); (5) the magnitude of the mutual-2$^+$ excited state data in the $^{12}\text{C} + ^{12}\text{C}$ system is unaccounted for; (6) the deformation parameters ($\beta$ values). previous calculations require $\beta$ values that are at variance with the empirical values and are physically unjustifiable.

In the next section, we introduce the standard and new coupled-channels models. Sections [I, II] and [V] are devoted to the application of the new coupling potential to the analyzes of the $^{12}\text{C} + ^{12}\text{C}$, $^{16}\text{O} + ^{28}\text{Si}$ and $^{12}\text{C} + ^{24}\text{Mg}$ reactions. Finally, we conclude in section [V].

I. THE MODEL

The three systems we study ($^{16}\text{O} + ^{28}\text{Si}$, $^{12}\text{C} + ^{24}\text{Mg}$ and $^{12}\text{C} + ^{12}\text{C}$) are quite different in many ways but they share two common features: (1) the elastic scattering data suggest that there is weak absorption in the entrance channel in each case and (2) each reaction involves at least one nucleus which is highly deformed.

A. The Standard Coupled-Channels Calculations

In the standard coupled-channels calculations, we describe the interaction between two nuclei with a deformed optical potential. For all the reactions considered, the real potential is assumed to have the square of a Woods-Saxon shape. The parameters of the real potential for the $^{16}\text{O} + ^{28}\text{Si}$ and $^{12}\text{C} + ^{24}\text{Mg}$ systems are fixed as a function of energy and are not changed in the present calculations. For the $^{12}\text{C} + ^{12}\text{C}$ system, it is slightly energy-dependent. The parameters are shown in table [I].

The imaginary part of the potential is the standard Woods-Saxon shape. Only the depth or radius increased linearly with energy.

Since the target nuclei $^{28}\text{Si}$, $^{24}\text{Mg}$ and $^{12}\text{C}$ are strongly deformed, it is essential to treat their collective excitations explicitly in the framework of the coupled-channels formalism. We assume that the target nuclei have static quadrupole deformations, and that their excitations can be described in the framework of the collective rotational model. The empirical values for the deformation parameters ($\beta$), derived from the known B(E2) values, are used in the present calculations.

Using this standard coupled-channels method, we found, as other authors had found, that it was not possible to find a consistent solution over wide energy ranges to the above-mentioned problems (see the discussions of results and figures below).

B. New Coupling Potential

The limitations of the standard coupled-channels method, on the one hand, and the resulting shape of the compound nucleus created by the projectile and target nuclei, on the other hand, have motivated us to use a second-derivative coupling potential. If we consider two $^{12}\text{C}$ nuclei approaching each other, the double-folding model will generate an oblate potential which is correct at large distances. When these two nuclei come close enough, they create the compound nucleus $^{24}\text{Mg}$ which is a prolate nucleus in its ground state. In the standard method, the folding model yields an oblate (attractive) potential for such a configuration. It is not clear how well the double-folding model describes a prolate nucleus...
with this oblate potential and this may be the reason why the earlier calculations using a double-folding model in the coupled-channels method were unable to provide a consistent solution to the problems of such reactions. In order to describe the above-mentioned configuration, the coupling potential must be oblate (attractive) when two $^{12}$C nuclei are at large distances and must be prolate (repulsive) when they are at short distances [1].

II. $^{12}$C+$^{12}$C

The first system we consider is the $^{12}$C+$^{12}$C, which has been studied extensively over the last 40 years. Our analysis consists of a simultaneous investigation of the elastic scattering, single-2$^+$ and mutual-2$^+$ excitation inelastic scattering data from $E_{Lab}=20.0$ MeV to 126.7 MeV. In such a large energy range, we also consider the 90$^\circ$ elastic and inelastic excitation functions.

The conventional folding model potentials fail to reproduce certain aspects of the data such as the reproduction of the gross structure in the 90$^\circ$ elastic scattering excitation function and a simultaneous consistent description of the elastic, single-2$^+$ and mutual-2$^+$ states data. So far, no model has been able to predict the magnitude of the mutual-2$^+$ state data correctly over a wide energy range. The standard coupled-channels model using double-folding potentials underestimates its magnitude by a factor of 3 to 10 and the single-2$^+$ state results are too oscillatory with respect to experimental data [4,7–10]. These problems have remained unsolved so far. Clearly, the $^{12}$C+$^{12}$C system has numerous problems to which no consistent global solution has been provided yet.

We have also observed such results in our standard coupled-channels calculations, as shown only for the mutual-2$^+$ case in figure 2 with dashed lines. Varying the parameters and changing the shape of the real and imaginary potentials and some other attempts do not provide a complete solution to the problems of this reaction [2].

III. $^{16}$O+$^{28}$Si

The second system we consider is $^{16}$O + $^{28}$Si, which shows anomalous large angle scattering (ALAS). In the present work, we consider an extensive simultaneous investigation of the elastic and inelastic scattering of this system at numerous energies from $E_{Lab}=29.0$ to 142.5 MeV. In this wide energy range, the excitation functions for the ground and single-2$^+$ states are also analyzed [1]. Several ad-hoc models have been proposed to explain the experimental data, but no satisfactory microscopic

![Figure 1](image1.png)

**FIG. 1.** For $^{16}$O+$^{28}$Si, the comparison of the standard coupling potential which is the first derivative of the central potential and our new coupling potential which is parameterized as the 2$^{nd}$ derivative of Woods-Saxon shape.

![Figure 2](image2.png)

**FIG. 2.** The $^{12}$C+$^{12}$C system: The results of the single and mutual-2$^+$ states. The solid lines are the results of the new coupling potential, while the dashed lines are the results of the standard coupled-channels model.
models have been put forward yet. The most satisfactory explanation proposed so far is that of Kobos and Satchler who attempted to fit only the elastic scattering data with a microscopic double-folding potential. However, these authors had to use some small additional ad-hoc potentials, which create a deepening in the surface region of the potential, to obtain a good agreement with the experimental data. Without the additional small potentials, they could not reproduce the experimental data. We have shown that this deepening of the real potential in the surface region takes into account the coupling effect in an ad-hoc way.

We have also attempted to overcome these problems by considering: (i) changes in the real and imaginary potentials, (ii) inclusion of 6+ excited state, (iii) changes in the $\beta_2$ value and some other attempts failed to solve the problems. We were unable to get an agreement with the elastic and the 2+ inelastic data as well as the 180° excitation functions simultaneously within the standard coupled-channels formalism.

However, as shown in figure, the new coupling potential has solved the out-of-phase problem for the 180° excitation functions and fits the ground state and 2+ state data simultaneously. To our knowledge, this has not been achieved over such a wide energy range (see ref. for all the results).

IV. $^{12}$C+$^{24}$Mg

The final example we consider is the $^{12}$C + $^{24}$Mg system. The angular distributions oscillate strongly near the Coulomb barrier and the data manifest ALAS. Our analysis consists of a simultaneous investigation of the elastic and inelastic scattering data at numerous energies from $E_{Lab}=16.0$ to 24.0 MeV. The most extensive study for this system was carried out by Sciani et al. The authors could only fit these data with $Q$-value dependent potential parameters in a rather ad-hoc fashion. Without $Q$-dependent potentials, they observed that the theoretical predictions and the experimental data for the elastic and inelastic scattering data were completely out-of-phase.

In the present calculations, we have studied the data of ref. as well as some inelastic scattering data of ref. In the standard coupled-channels calculations, we also found the out-of-phase problem which was in conformity
with the findings of Sciani et al. However, using our new coupling potential, we obtained excellent agreement with the experimental data. Some of the individual angular distribution fits for the ground and $2^+$ states are shown in figure 4 (see ref. [12] for all the results).

V. DISCUSSION and SUMMARY

The effect of this new coupling potential on the scattering has been probed through the total inverted potential, i.e. the dynamical polarization potential (DPP) plus the bare potential, obtained by inversion of the $S$-matrix. The most effective and sensitive regions of the new coupling potential have been identified. We have precisely determined the effect of the inclusion of the excited states on both the real and imaginary potentials as well as the effect of the new coupling potential [13].

We have observed that for the standard coupling potential case, the added attraction is almost the same as the added absorption. On the other hand, for the new coupling potential case, the added attraction is much greater than the added absorption. Therefore, as shown in figure 4, the new coupling potential creates a deepening in the surface region of the real potential [13]. Thus, it increases the interaction radius.

![FIG. 5. The comparison of the bare potentials (solid lines) and the total inverted potentials (dashed lines), found by inverting the $S$-matrix of the new coupled-channels calculations at $E_{Lab}=126.7$ MeV for the $^{12}$C+$^{12}$C system.]

This deepening can be related to the superdeformed structures of the compound nuclei [13]. It is well known that the light heavy-ion reactions are very sensitive to the surface region of the potential and this deepening created by the new coupling potential clearly makes a major improvement in explaining the experimental data over such large energy ranges for different reactions. This deepening has been included by many authors in an artificial way in their calculations and they managed to explain the elastic scattering data successfully (see [15,16] and references therein)

To summarize, while these three systems show quite different properties and problems, a unique solution has come from a new coupling potential. The importance of the new approach should be underlined here because it does not only fit the present experimental data, but it also leads to other novel and testable predictions [17]. To our knowledge, this has not been yet achieved over such wide energy ranges for many different reactions simultaneously. Our work reveals that there is no reason for the coupling potential to have the same energy dependence as the central term and studies using this new coupling potential may lead to new insights into the formalism and also a new interpretation of such reactions. Therefore, this work represents an important step towards a new understanding of the elastic and inelastic scattering of deformed light heavy-ion reactions.

VI. REFERENCES

[1] I. Boztosun and W.D.M. Rae, Phys. Rev. C 63 (2001) 054607.
[2] P. Braun-Munzinger and J. Barrette, Phys. Reports 87 (1982) 209.
[3] W. Sciani et al., Nucl. Phys. A620 (1997) 91.
[4] R.G. Stokstad et al, Phys. Rev. C 20 (1979) 655.
[5] M.E. Brandan and G.R. Satchler, Phys. Reports 285 (1997) 143.
[6] A.M. Kobos and G.R. Satchler, Nucl. Phys. A427 (1984) 589.
[7] P.E. Fry, DPhil Thesis, Oxford University, 1997.
[8] W.D.M. Rae et al, Nuovo-Cimento 110A (1997) 1001
[9] Y. Sakuragi et al, Proceedings of the 7th International Conference on Clustering Aspects of Nuclear Structure and Dynamics, Edited by M Koroliija, Z Basrak and R Caplar, World-Scientific-2000 (138).
[10] R. Wolf et al, Z. Phys. 305A (1982) 179.
[11] I. Boztosun and W.D.M. Rae, in preparation.
[12] I. Boztosun and W.D.M. Rae, in preparation.
[13] I. Boztosun and R.S. Mackintosh, in preparation.
[14] J. Carter et al, Nucl. Phys. A273 (1976) 523.
[15] R.S. Mackintosh and A.M. Kobos, Phys. Lett. 116B (1982) 95.
[16] C.E. Ordoñez et al, Phys. Lett. 173B (1986) 39.
[17] C.A. Brenner et al, in preparation.

TABLE I. The parameters of the central and coupling potentials for the reactions studied.

|                | Central | Coupling |
|----------------|---------|----------|
|                | $V$, $r_0$, $a$ | $V$, $r_0$, $a$ |
| $^{16}$O+$^{28}$Si | 750.5, 0.749, 1.4 | 155.0, 0.748, 0.81 |
| $^{12}$C+$^{24}$Mg | 427.0, 0.865, 1.187 | 185.0, 0.710, 0.62 |
| $^{12}$C+$^{12}$C | $\sim290.0$, $\sim0.8$, $\sim1.28$ | 210.0, $\sim$0.67, $\sim$0.68 |