Radio-frequency-driven motion of single Cooper pairs across the superconducting single-electron transistor with dissipative environment

A. B. Zorin, S. V. Lotkhov, S. A. Bogoslovsky and J. Niemeyer

Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany

We report on the effect of the frequency-locked transfer of single Cooper pairs in a superconducting single-electron Al transistor embedded in a dissipative environment (on-chip Cr resistor of $R \approx 40 \, \text{k}\Omega$). The transistor was dc voltage biased, and the harmonic signal of frequency $f$ of several MHz was applied to the gate. Due to the substantial rate of relaxation, the unidirectional transfer of single pairs occurred in each junction once per clock cycle and the current plateaus at $I \approx 2e_f$ were developed in the transistor’s $I-V$ curves. The mechanisms (supercurrent, Landau-Zener tunneling, quasiparticle tunneling, etc.) deteriorating the phase-locking regime are discussed.

PACS numbers: 74.50.+r, 73.23.Hk, 85.30.Wx

I. INTRODUCTION

In the last few years, the attention of experimenters has been attracted to superconducting systems of small tunnel junctions whose behavior benefited from an interplay of Coulomb and Josephson effects. Besides the interesting physical aspects, such as the charge-phase duality, Cooper-pair solitons, Bloch-band structure, the effect of dissipation, etc., the systems of small Josephson junctions may also be useful for applications like Cooper-pair electrometers, pumps, charge qubit, etc. In the light of fundamental metrology and quantum computation, the researchers are particularly interested in the controlled transfer of individual pairs across a superconducting circuit.

In this paper we focus on the single pair transport in the two-junction circuit with a gate, or, as it is sometimes called, the Bloch transistor. The behavior of this system is determined by the relationship between the Coulomb energy $E_c$ and the Josephson coupling energy $E_J$. In particular, if the Coulomb energy dominates, $E_c \gg E_J$, the transfer of pairs is a resonance process. The motion of pairs across the transistor can be realized in three different ways: (i) by means of the through-supercurrent at the bias voltage $V \approx 0$ and the gate-induced charge on the island $Q \approx \pm e$, (ii) by Josephson tunneling or an Andreev reflection process in one junction accomplished by quasiparticle tunneling in another junction at $V \neq 0$ and at certain values of $Q$ (see Refs. [1, 17, 18] and [19], respectively), and (iii) by means of autonomous periodic tunneling of single pairs (Bloch oscillations [20, 21]). This regime being possible in a very high-ohmic electromagnetic environment $(R \gg R_Q \equiv h/4e^2 \approx 6.5 \, \text{k}\Omega)$, or, in other terms, at the dc current bias.

As long as the phase across the transistor in the voltage-bias regime is meaningful, the process (i) yields the current with the meaninglessness number of pairs tunneling per unit time across the device. Either of the mechanisms (ii) gives the random number of the particles transferred. Although the current-bias regime (iii) should ensure the coherent motion of pairs, in the experiment, the resulting current is strongly affected by fluctuations.

The goal of this paper is to realize a regime of the gate-driven unidirectional motion of single Cooper pairs in the voltage bias transistor. Since Cooper pair tunneling is by its nature an elastic, and, hence, reversible process, the desired regime, as we will show below, can be achieved by introduction of finite damping. Endorsing this idea we fabricated an Al transistor with on-chip resistor of $R \sim$ several $R_Q$, connected in series to the transistor. In the $I-V$ curves of this device we observed remarkable current plateaus at $I = 2e_f$, reflecting the transfer of one Cooper pair per cycle of harmonic signal of frequency $f$ applied to the gate.

II. CIRCUIT PARAMETERS

The diagram of the electric circuit comprising two small Josephson junctions with a capacitive gate and a resistor attached is depicted in Fig. 1. The system parameters are assumed to be:

$$E_c \gg E_J \gg k_B T,$$

(1)

where $E_c = e^2/2C_S$ with $C_S = C_1 + C_2 + C_g$, denoting the total capacitance of the transistor island including the capacitances of individual junctions $C_{1,2} = C_T$ and the gate capacitance $C_g \ll C_T$. The Josephson coupling energy in either junction is $E_J = (\Phi_0/2\pi)I_c = (R_Q/2R_T)\Delta$, where $\Phi_0 = h/2e \approx 2.07 \, \text{mWb}$ is the flux quantum, $I_c$ and $R_T$ are the critical current and the tunnel resistance of the individual junctions, respectively, $\Delta$ is the superconductor energy gap and $T$ the temperature.

The effect of the external resistor (which is characterized by the dimensionless parameter $z = R/R_Q$) can be described as follows: First, the resistor "hampers" the...
tunneling of pairs in a particular junction. As a result, tunneling occurs when the change of the energy associated with this tunneling is positive, \( E > 0 \); the energy \( E \) dissipates in the resistor. In the limit of \( \lambda \equiv E_3/E_c \to 0 \), the rate of the one-pair tunneling \([22, 23]\) is

\[
\Gamma(E) = \frac{\pi}{2\hbar} E^2 P(E),
\]

where \( P(E) \) is a peak-shaped "environment" function obeying the normalization condition \( \int_{-\infty}^{+\infty} P(E) dE = 1 \) and the relation \( P(E) \propto E^{2\zeta-1} \), at small positive \( E \). The parameter \( \zeta' = 1/2 \hat{z} \), where the factor of \( 1/4 = (C_T/C_\Sigma)^2 \) accounts for the reduction of the damping effect due to the capacitance of the neighboring junction connected in series to the resistor \( R \) (see circuit analysis in Refs. [23, 24]).

Secondly, the resistor suppresses through-tunneling across the whole transistor or, in other terms, the total supercurrent \( I_s \). (Note, at \( R = 0 \) and \( \lambda \to 0 \), the total critical current \( I_c = I_{c0}/2 \) \([23]\).) That is to say that for the most intensive (resonance) tunneling occurring at \( Q \approx \pm \pi \), the supercurrent is expressed as

\[
\frac{I_s(V)}{I_c} = \frac{\pi}{8} E_3 P_{tr}(2eV) \approx 0.02\lambda \left( \frac{4V}{V_0} \right)^{2\zeta-1},
\]

where \( P_{tr}(E) \) is the environment function for the whole transistor and \( V_0 = e/(C_\Sigma) \). The second relation in Eq. (3) is valid for small \( V \), viz., \( 0 < V \ll V_0/4 \) and \( z > 1/2 \). In order to ensure sufficient suppression of the supercurrent and, at the same time, to have sufficiently large \( \Gamma(E) \) at small \( E \), the compromise values for \( R \) of about several \( R_0 \) should be chosen.

Finally, we assume that the rate of single electron (quasiparticle) tunneling across the transistor junctions is negligibly small. The factors which lead to its rate being suppressed are: a high pregap resistance \( R_{qp}(V) \gg R_T \) at voltages \( |V| \leq 2\Delta/e \), the presence of a finite resistance \( R \) in the external circuit \([23]\) and, at \( E_c < \Delta \) the even-odd parity effect \([25]\).

III. OPERATING PRINCIPLE

The circuit is dc voltage biased and the harmonic signal \( V_0(t) \) of frequency \( f \), amplitude \( V_0 \) and dc offset \( V_{g0} \) is applied to the gate, which leads to the ac polarization of the island. Figure 1a shows the working cycle in the \( \{V_0, V\} \) plane. The cycle trajectory hits the boundaries corresponding to the resonant tunneling of pairs (at \( R = 0 \)) through the first and second junctions. At these boundaries the values of the electric energy with and without one extra pair on the island are equal, i.e.,

\[
E_{n_1,n_2} = E_{n_1,n_2 + 1} \quad \text{and} \quad E_{n_1,n_2} = E_{n_1,n_2 \pm 1}.
\]

(4)

Here \( n_1 \) (\( n_2 \)) denotes the number of pairs having traversed the first (second) junction; the energy, including the work done by the voltage source, reads \([10]\)

\[
E_{n_1,n_2} = 4E_c \left( v_g + n_1 - n_2 \right)^2 - (n_1 + n_2)v \text{,}
\]

where \( v_g = C_g V_g/2e = Q/2e \) and \( v = V/V_0 \).

Owing to the finite Josephson coupling, the charge states in the vicinity of resonances, Eq. (4), are quantum-mechanically mixed. As a result, e.g., for the states \( (n_1, n_2) \) and \( (n_1 + 1, n_2) \), the total energy of the system assumes the values \([20]\):

\[
E^{(0,1)} = \text{const} \pm \sqrt{\left( E_{n_1,n_2} - E_{n_1+1,n_2} \right)^2 + E_1^2/4},
\]

(6)

where the lower \( E^{(0)} \) and upper \( E^{(1)} \) Bloch-band states are separated by the energy gap of width \( E_1 \). Thus, our system is mapped onto the two-level crossing problem.

In the absence of dissipation \( (R = 0) \), a slow passage of the avoided crossing region along either branch, Eq. (4), in either direction leads to one pair passing in a certain direction; these processes are reversible. At a larger rate of motion, the interband transitions \( E^{(0)} \to E^{(1)} \) and \( E^{(1)} \to E^{(0)} \) may occur; they are not accompanied by pair transfer. The probability of such an event is given by the Landau-Zener formula \([24]\):

\[
p_{\text{LZ}} = \exp(-\gamma) \quad \text{with} \quad \gamma = \frac{\pi E_1^2}{2\hbar E},
\]

(7)

where the derivative \( \dot{E} = \frac{d}{dt}(E_{n_1,n_2} - E_{n_1+1,n_2}) \propto 1 \). If the cycling frequency \( f \) is low enough, \( \exp(-\gamma) \ll 1 \) and the probability of the Cooper pair transfer \( p_{\text{CP}} = 1 - p_{\text{LZ}} \approx 1 \).
FIG. 2: The cycle trajectory (thick solid line) on the \(\{V_g, E_{n_1,n_2}\}\) plane. The arrows indicate the direction of motion. The inset shows in detail the avoided crossing region; two processes are schematically shown: (1) relaxation \(E^{(1)} \rightarrow E^{(0)}\), occurring without transfer of charge (open circles in Fig.1b) and (2) motion along the lower branch \(E^{(0)}\), accompanied by single Cooper pair tunneling (solid circles in Fig.1b).

In the case of finite dissipation and at sufficiently low temperature \(T\), relaxation from state \(E^{(1)}\) to \(E^{(0)}\) may occur during the passage of the avoided crossing region starting in the upper branch. For the appreciable strength of dissipation, according to the theory of Ao and Rammer [27] describing the dynamics of the two-state system in a dissipative environment, the probability of relaxation is

\[
p_{\text{rel}} = 1 - \exp(-\gamma) + \exp(-2\gamma) \approx 1
\]

with \(\gamma\) given by the second expression in Eq. (7). In our notations this result is valid for \(z' > 1\) (or \(R > 4R_Q\)) and \(k_B T < \hbar/RC_2\). Note that with the motion within the lower branch the probability of the transition \(E^{(0)} \rightarrow E^{(1)}\) due to the Landau-Zener mechanism remains unchanged (see Eq. (7)), i.e., the probability \(p_{\text{CP}}\) is \(\approx 1\) [27].

Figure 2 shows the cycle trajectory in the energy representation: the system glides along the pieces of parabolas, Eq. (6). The inset is a blow-up of the avoided crossing region in which first the relaxation (giving the zero charge transferred) and then the crossing of the energy maximum (giving the transfer of charge of \(2e\)) occur. As a result of the back-and-forth sweep, one pair traverses both junctions once per cycle, carrying the average current \(I = 2ef\). The energy dissipation occurring in the electric circuit (in resistor \(R\)) is equal to \(2eV\) per cycle.

The tunnel structures of type Al/AlO\(_x\)/Al (of nominal dimensions 40 nm by 40 nm) with Cr microstrip resistor (with sizes 0.1 \(\mu m \times 7 \mu m \times 7 \mu m\)) were fabricated by the three angle evaporation technique described elsewhere [28, 29]. The tunnel resistance of the junctions \(R_T\) was around 35 k\(\Omega\) and, assuming the experimental value of \(\Delta_{Al} = 175\mu eV\), this yielded \(E_3 \approx 16\mu eV\).

From the measurements of the \(I - V\) and \(I - V_g\) characteristics in the normal state (namely, in magnetic field of induction \(B = 1\) T), the capacitances of the junctions \(C_T\) and of the gate \(C_g\) were found to be about 160 aF and 12 aF, respectively. The total capacitance of the island \(C_I\) and the charging energy \(E_c\) were about 350 aF and 270 \(\mu eV\), respectively. The ratio of the characteristic energies in the sample thus was equal to \(\lambda \approx 0.06\). The resistance of the Cr strip was evaluated at \(R = 40\) k\(\Omega\), so the dimensionless parameter \(z \approx 6\).

The dc and rf measurements were carried out in a dilution fridge at the bath temperature \(T = 10\) mK. The recorded \(I - V\) curves are shown in Fig. 3. The Coulomb-blockade type of the autonomous characteristics showed, in particular, that the critical current of the transistor was dramatically suppressed. Under the action of the rf signal applied to the gate, the \(I - V\) curve changed its shape: the blockade was lifted and the characteristic plateaus were formed close to the current levels \(I = \pm 2ef\). The largest plateaus were recorded at the lowest frequencies of the drive, i.e., at \(f = 2\) and 3 MHz.

FIG. 3: The \(I - V\) curves of the sample without rf drive and driven by harmonic signals of frequency \(f = 2, 3, 4\), and 5 MHz with amplitude \(V_A \approx e/C_g \approx 13\) mV. The dc component of the gate voltage \(V_{g0}\) was tuned to maximize the sizes of the plateaus. The right axis converts the current values into the frequency units.

IV. EXPERIMENT
V. DISCUSSION

The electric current in this transistor is obviously associated with the gate-driven motion of individual Cooper pairs. Furthermore, this preliminary experiment even demonstrates some superiority of the device characteristics over those of the three-junction Cooper pair pumps [1, 2]. On the other hand, there are several factors which affect the shape and the height of the plateaus. These factors are related to the sample parameters and can be improved when new samples are fabricated.

First, the smeared shape of the highest plateaus in Fig. 3 is associated with the insufﬁcient speed of the device. This behavior of the sample is understood: For \( R = 40 \) kΩ yielding the value \( 2(\zeta - 1) \approx 2 \), the tunneling rate at small \( E \) was rather low, \( \Gamma(E) \propto E^2 \). That is why a reliable (relatively large value of the charging energy, \( E \)) was possible only at significant values of \( E > E_c \). This is why a reliable (relatively large value of the charging energy, \( E \)) was possible only at significant values of \( E > E_c \). When \( E \) is why a reliable (relatively large value of the charging energy, \( E \)) was possible only at significant values of \( E > E_c \), achieved at voltages \( |V| \geq V_0/4 \approx 170 \) µV, and a rather low frequency of the drive \( f \). At somewhat lower resistance values, viz. \( R = 20 - 30 \) kΩ yielding \( 2(\zeta - 1) \approx 1 \) and, hence, \( \Gamma_1(E) \propto E \), correct operation should be possible at voltages, \( 170 \) µV \( \geq |V| \geq E_1/e \approx 16 \) µV. As a result, wider and flatter plateaus might appear. A room for increase in the operating frequency is also available: The rate of the Landau-Zener process leading to the quasiparticle tunneling [25] was not possible in this sample.

Secondly, the rate of quasiparticle tunneling during operation was still noticeable. One reason of this was the relatively large value of the charging energy, \( E_c > \Delta_{Al} \), with the result that the even-odd parity blockade of quasiparticle tunneling [23] was not possible in this sample. Furthermore, at the bias voltage \( V \sim \pm E_c/e \), the voltages across individual junctions clearly exceeded the threshold values at which the quasiparticle-current onset at the double-gap voltage dramatically increased the tunneling rate. The quasiparticle tunneling was, in our opinion, the main mechanism leading to somewhat smaller values of the average current on the plateaus. We interpret this behavior as a result of the sporadic tunneling of charge \( \pm e \), shifting the position of the operation trajectory along the \( V_g \) axes and leading to "the empty" cycles.

Although an Al-Cr sample with a smaller \( E_c \) value might in this case be advantageous, a substantial improvement of the shape of the plateaus appears to be possible in, e.g., Nb-Cr samples. In such a sample \( E_c \) can be increased up to \( \sim \Delta_{Nb} \approx 1.4 \) meV (the energy gap of niobium), keeping the quasiparticle tunneling totally suppressed. In this case, the errors associated with the supercurrent (see Eq.(2)) can also be reduced as low as \( \sim 10^{-8} \) at \( |V| \approx 0.06V_0 \approx 80 \) µV even if the electron temperature in the resistor is as high as 100-200 mK.

To conclude, we predicted and experimentally demonstrated the behavior of the dissipative system (the Al Bloch transistor with attached Cr resistor) experiencing the adiabatic energy-level crossing. Based on a strong dissimilarity in the probabilities to pass the avoided crossing region along the lower and upper energy branches, the remarkable unidirectional traversing of single Cooper pairs across the transistor was realized when the gate voltage was cycled. Similar circuits made from Nb (instead of Al) may probably transfer single pairs with a relative accuracy of \( 10^{-8} \) and, hence, meet the requirement of fundamental metrology aiming at the construction of current and capacitance standards [14, 30].

VI. ACKNOWLEDGMENTS

This work is supported in part by the EU (Project COUNT).

[1] L. J. Geerligs, S. M. Verbrugh, P. Hadley, J. E. Mooij, H. Pothier, P. Lafarge, C. Urbina, D. Esteve and M. N. Devoret, Z. Phys. B: Condens. Matter 85, 349 (1991).
[2] M. Matters, W. J. Elion, and J. E. Mooij, Phys. Rev. Lett. 75, 721 (1995).
[3] D. B. Haviland and P. Delsing, Phys. Rev. B 54, R6857 (1996).
[4] L. S. Kuzmin, Yu. A. Pashkin, D. S. Golubev, and A. D. Zaikin, Phys. Rev. B 54, 10074 (1996).
[5] D. J. Flees, S. Han, and J. E. Lukons, Phys. Rev. Lett. 78, 4817 (1997).
[6] Y. Nakamura, C. D. Chen, and J. S. Tsai, Phys. Rev. Lett. 79, 2328 (1997).
[7] A. J. Rimberg, T. R. Ho, Ç. Kurdak, J. Clarke, K. L. Campman, and A. C. Gossard, Phys. Rev. Lett. 78, 2632 (1997).
[8] V. Bouchiat, D. Vion, P. Joyez, D. Esteve and M. H. Devoret, Phys. Scr. T76, 165 (1998).
[9] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) 398, 768 (1999).
[10] J. S. Penttilä, U. Parts, P. J. Hakonen, M. A. Paalanen, and E. B. Sonin, Phys. Rev. Lett. 82, 1004 (1999).
[11] S. V. Lotkhov, H. Zangerle, A. B. Zorin, T. Weimann, H. Scherer, and J. Niemeyer, IEEE Trans. Appl. Supercond. 9, 3664 (1999); A. B. Zorin, S. V. Lotkhov, Yu. A. Pashkin, H. Zangerle, V. A. Krupenin, T. Weimann, H. Scherer, and J. Niemeyer, J. Supercond.
[12] A. B. Zorin, S. A. Bogoslovsky, S. V. Lotkhov, and J. Niemeyer, Preprint, available at http://arXiv.org/abs/cond-mat/0012177.
[13] R. Lindell, J. Penttilä, M. Paalanen, and P. Hakonen, Bull. Am. Phys. Soc., Ser. II, 46, 358 (2001).
[14] M. W. Keller, Preprint 2000, to be published.
[15] D. V. Averin, Solid State Commun. 105, 659 (1998).
[16] D. V. Averin and K. K. Likharev, in Mesoscopic Phenomena in Solids, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).
[17] T. A. Fulton, P. L. Gammel, D. J. Bishop, L. N. Dunkleberger, and G. J. Dolan, Phys. Rev. Lett. 63, 1307 (1989).
[18] A. Maassen van den Brink, G. Schön, and L. J. Geerligs, Phys. Rev. Lett. 67, 3030 (1991).
[19] R. J. Fitzgerald, S. L. Pohlen, and M. Tinkham, Phys. Rev. B 57, R11073 (1998).
[20] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. 59, 347 (1985).
[21] L. S. Kuzmin and D. B. Haviland, Phys. Rev. Lett. 67, 2890 (1991); D. B. Haviland, Yu. A. Pashkin, and L. S. Kuzmin, Physica B 203, 347 (1994).
[22] D. V. Averin and A. A. Odintsov, Phys. Lett. A 140, 251 (1989); G. Falci, V. Bubanja, and G. Schön, Z. Phys. B 85, 451 (1991).
[23] G. L. Ingold and Yu. V. Nazarov, in Single Charge Tunneling, edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992), Chap. 2.
[24] G. L. Ingold, P. Wyrowski, and H. Grabert, Z. Phys. B 85, 443 (1991).
[25] M. T. Tuominen, J. M. Hergenrother, T. S. Tighe and M. Tinkham, Phys. Rev. Lett. 69, 1997 (1992); A. Amar, D. Song, C. J. Lobb and F. C. Wellstood, Phys. Rev. Lett. 72, 3234 (1994).
[26] L. D. Landau, Phys. Z. Sowjetunion 1, 89 (1932); C. Zener, Proc. R. Soc. London, Ser. A 137, 696 (1932).
[27] P. Ao and J. Rammer, Physica B 165&166, 953 (1990); Phys. Rev. B 43, 5397 (1991).
[28] A. B. Zorin, S. V. Lotkhov, H. Zangerle, and J. Niemeyer, J. Appl. Phys. 88, 2665 (2000).
[29] S. V. Lotkhov, S. A. Bogoslovsky, A. B. Zorin, and J. Niemeyer, Appl. Phys. Lett. 78, 946 (2001).
[30] K. Flensberg, A. A. Odintsov, F. Liefrink, and P. Teunissen, Int. J. Mod. Phys. B 13, 2651 (1999).