Theory of time-like baryon form factors near thresholds

Yu.A.Simonov
ITEP, Moscow, Russia

(A dated:)

A new mechanism of baryon-antibaryon production via nonperturbative double pair creation in intermediate mesons is proposed and the theory contains no fitting parameters. It is shown, that near-threshold resonances are responsible for enhancements in electroproduction cross sections: $\psi(4S)$ for $\Lambda_c^+\Lambda_c^-$, $\Upsilon(6S)$ for $\Lambda_b\bar{\Lambda}_b$. An admixture of intermediate $D$-wave resonances produces angular dependence in differential cross section and can explain unusual behavior of the ratio $G_E/G_M$ for the proton.

PACS numbers: 12.38.Lg;13.25Gv;13.60Rj
Keywords: baryon electroproduction, form factors, vector mesons

I. INTRODUCTION

The topic of time-like baryon form factors and barion-antibaryon ($BB$) production is being actively studied experimentally for the last 30 years, for reviews and references see [1, 2]. As a result an impressive amount of data on $pp, \Lambda\Lambda, \Sigma\Sigma, \Lambda_c\Lambda_c$ is obtained, signalling strong enhancements in all cases near the corresponding thresholds. Moreover, in some cases the angular dependence of cross sections was obtained [3, 4], which allows to distinguish between $G_E$ and $G_M$ and obtain its ratio, different from unity near threshold. These features are well established and call for explanation. On theoretical side two approaches are most popular. In the first one exploits perturbative approach with model assumptions about distribution amplitude functions, appropriate for higher energies [5]. A quark-diquark model with more parameters was given in [6] and a more phenomenological approach suggested in [7].

In this letter we propose a new mechanism of $BB$ production and structure formation in time-like form factors, which is of universal character and is free from fitting parameters. This mechanism is based on the detailed description of the string breaking with creation of two light quark pairs, e.g. $(u\bar{u}), (d\bar{d})$. The relativistic theory of string breaking with one pair ($q\bar{q}$) creation was given in [9] and depends on the only parameter – string tension $\sigma$.

In case of double pair creation the initial state is again a meson ($Q\bar{Q}$) as shown in Fig.1 and the string breaking vertex contains in addition to $\sigma$ the fundamental parameter – the vacuum correlation length $\lambda$, which was measured both on the lattice and analytically, $\lambda \approx (0.1 \div 0.2)$ fm [10]. The $BB$ electroproduction cross section was calculated in [11] in the case, when $(Q\bar{Q})$ is in the $(n^3S_1)$ - state and its wave function $\psi_n(r)$ is represented by SHO functions with SHO parameter $\beta_n$, whereas for baryons the lowest hyperspherical mode of gaussian form was used, $\Psi_B \sim \exp \left(-\frac{\xi^2 + \eta^2}{R_0^2}\right), \quad R_0^2 = \frac{1}{8} \langle r_B^2 \rangle, (\xi, \eta$ are normalized Jacoby coordinates, see [12] for details). The resulting

FIG. 1: Electroproduction of $BB$ by double string breaking mechanism

*Electronic address: simonov@itep.ru
FIG. 2: The cross section \( \sigma(e^+e^- \to \Lambda_c^+ \Lambda_c^-) \) in \( nb \) with \( \lambda = 1 \) GeV as function of of \( M(\Lambda_c^+ \Lambda_c^-) = E/c^2 \) (solid line) experimental points are from [12], dashed line is the best normalization fit of Eq. (2) with \( \lambda = 0.17 \) fm.

FIG. 3: \( \Lambda_b \bar{\Lambda}_b \) electroproduction cross section in \( nb \) near threshold as a function of total energy \( E \).

electroproduction cross section can be written as a double sum over \( Q\bar{Q} \) and its excited states,

\[
\sigma(e^+e^- \to \bar{B}\bar{B}) = \frac{12\pi\alpha^2 p^4}{E^3} \sum_Q e_Q \sum_n \psi_n(0) \eta_{BQ} J_{nBB}(p) \left| \frac{\psi_n(p)}{E_n - E - i\frac{\Gamma_n}{2}} \right|^2
\]

where \( E \) is the total c.m. energy , \( p \) — baryon c.m. momentum and \( J_{nBB}(p) \) is the overlap integral of \( \psi_n(r) \) and \( \bar{B}\bar{B} \) wave functions, given in Eq. (29) of [11]. Here \( \eta_{BQ} \) is a spin recoupling coefficient \( \eta_{\Lambda_c} = 1, \eta_{pu} = \frac{2}{3} \).

In case, when only one \( (n^3S_1) \) state is close to the threshold for a given pair \( (Q\bar{Q}) \) and the only pair \( (Q\bar{Q}) \) is dominant, as in the case of \( \Lambda_c \bar{\Lambda}_c \) or \( \Lambda_b \bar{\Lambda}_b \) production, the behavior of \( \sigma \) can be written as

\[
\sigma(e^+e^- \to \bar{B}\bar{B}) = C_n \frac{p}{E^3} \frac{\exp(-p^2 R_0^2 \bar{c})}{(E - E_n)^2 + \frac{1}{4}}.
\]

where the constants \( C_n, \bar{c} \) are calculable through parameters of baryons and \( (n^3S_1) \) state. The latter are easily calculated in the relativistic Hamiltonian, comprising the quarks with current masses \( m_u = m_d \approx 0; m_s \approx 0.17 \) GeV, \( m_c = 1.42 \) GeV, \( m_b = 4.83 \) GeV, the QCD string between \( Q \) and \( \bar{Q} \) with \( \sigma = 0.18 \) GeV\(^2\) and \( \alpha_s \) with asymptotic freedom at small and freezing at large distances. For high excited states also the coupling to decay channels is important, which is taken into account by the so-called flattening potential. The calculated parameters for charmonia [13], bottomonium [14] and light mesons are given in the Table, together with PDG data [15].

The resulting behavior of cross section for \( e^+e^- \to \Lambda_c \bar{\Lambda}_c \) is shown in Fig. 2, together with experimental data from [16]. Theoretical prediction used \( 4^3S_1 \) charmonium state with parameters shown in Table and \( \lambda = 0.2 \) fm, \( \langle r_\Lambda^2 \rangle = (0.8 \text{ fm})^2 \). A more accurate fit to the data (shown in Fig. 2 by broken line) requires \( \lambda \) to be 15\% smaller.

The case of \( \Lambda_b \bar{\Lambda}_b \) production is treated in the same way, with \( 6^3S_1 \) \( \Upsilon(11.02) \) as an intermediate state. Using parameters from the Table, and the same \( \lambda \) and \( R_0 \), one obtains the curve, shown in Fig.3. One can see again the threshold enhancement, produced by the form of Eq. (2). A similar analysis of \( \Lambda \bar{\Lambda}, pp \) electroproduction should take into account angular distribution, which was measured in [17] and [4] respectively. To this end one should include angular dependence due to \( D \) wave admixture in a vector \( ^- \) resonance with the wave function [18]

\[
\Psi_n = \frac{\xi}{\sqrt{8\pi}} v^+ (\sigma_i R_{nS}(r) s_{nS} + p_{ik} \sigma_k R_{nD}(r)) v_n,
\]
where \( P_{ik} = \frac{1}{\sqrt{2}}(n_in_k - \frac{1}{3} \delta_{ik}) \), and \( |\xi_n\rangle^2 + |\xi_D\rangle^2 = 1 \). Insertion of (3) into \( J_{nBB}(p) \) yields a \( D \)-wave part of the latter, \( J_{nBB}(p) \rightarrow J^{(S)}_{nBB} + J^{(D)}_{nBB} \), and the resulting differential cross section can be written as

\[
\frac{d\sigma(e^+e^- \rightarrow BB)}{d\Omega} = \alpha^2 p^2 \frac{P}{E^3} e^{-\nu^2 R^2} \left\{ \frac{1}{2} |\Xi_S|^2 + \frac{1}{\sqrt{2}} Re(\Xi_S \Xi_D^*) + \frac{5}{4} |\Xi_D|^2 - \cos^2 \theta \left( \frac{3}{4} |\Xi_D|^2 + \frac{3}{2} Re(\Xi_S \Xi_D^*) \right) \right\},
\]

with

\[
|\Xi_A = \sum_n \xi_{nA} \psi_{nA}(0) = \frac{N_A(p)}{E - E_n + \frac{m_A^2}{2}}, \quad A = S, D
\]

and \( Q_n(p) \) depends on \( R_0, \beta_n \) and is a polynomial in \( p^2 \)

\[
\rho^2 = 96\pi^3/\epsilon_Q^2 \eta_{BQ}\lambda^2, \quad \psi_{nS}(0) = \frac{1}{\sqrt{4\pi}} R_{nS}(0), \quad \psi_{nD}(0) = \frac{5R_{nD}(0)}{4\sqrt{8\pi}\omega_Q^2}.
\]

Here \( \omega_Q = (\sqrt{p^2 + m_Q^2})_n \). Note, that at small \( p \) the polynomial \( Q_n(p) = O(p^2) \), hence \( \Xi_D(p \rightarrow 0) \sim 0(p^2) \). Comparison to the standard definition of (modified) Sachs form factors \( G_{ML}^\rho(E) \) and \( G_{LE}^\rho(E) \), immediately yields

\[
\left| \frac{2M}{E} G_E^\rho \right|^2 = \rho^2 e^{-\nu^2 R^2} \left\{ |\Xi_S|^2 + 2\sqrt{2} Re(\Xi_S \Xi_D^*) + 2|\Xi_D|^2 \right\},
\]

\[
|G_M^\rho|^2 = \rho^2 e^{-\nu^2 R^2} \left\{ |\Xi_S|^2 - \sqrt{2} Re(\Xi_S \Xi_D^*) + \frac{1}{2} |\Xi_D|^2 \right\}.
\]

where \( M \) is the proton mass. It is important, that due to vanishing of \( \Xi_D \), both form factors coincide at the threshold, as it should be (by definition), and all difference in \( \left| \frac{2MGE^\rho}{EGM_L^\rho} \right|^2 - 1 \) is due to \( D \)-wave admixture.

It is interesting, that in the region \( \sim 200 \text{ MeV} \) above threshold the experimental ratio of \( \left| \frac{G_B^\rho}{G_M^\rho} \right| \), \( B = \Lambda, p \), shows a striking peak, which in our mechanism can be provided by the interference of \( 3S \) and \( 2D \) resonances. In both systems, \( \rho/\omega \) and \( \phi \), theory predicts a combination of a wide \( 3S \) resonance and a more narrow \( 2D \) resonance, see Table for \( \rho/\omega \) parameters. Experimentally wide resonances in both cases are not well established, moreover they are accompanied by even wider \( 4S \) resonances some \( 250 \text{ MeV} \) higher. Therefore to simplify calculations the BW denominator for the \( 3S \) state of \( \rho \) was taken as \( E - E_{3S} + \frac{m_{3S}}{2} \rightarrow 0.15 \text{ GeV} \), and we have increased the ratio \( \frac{Re(\Xi_S \Xi_D^*)}{|\Xi_D|^2} \) by a factor of \( \sim 3 \) to take into account possible \( S - D \) mixing effect on \( \psi_D(0) \). The resulting ratio \( \left| \frac{G_B^\rho}{G_M^\rho} \right|^2 \) is given in Fig. 4 and agrees

**TABLE I:**

| \( \bar{Q} \bar{Q} \) | \( bb \) | \( cc \) | \( u\bar{u} \pm \bar{d}d \) |
|---|---|---|---|
| nS | 6S | 4S | 3S | 2D |
| \( E_n(\text{GeV}) \) | 11.04 | 4.426 | 1.9 | 1.99 |
| \( \beta_n(\text{GeV}) \) | 0.38 | 0.13 | 0.13 | 0.0275 |
| \( \Gamma_n(\text{MeV})[15] \) | 79 \pm 16 | 62 \pm 20 | 130 \div 160 \sim 150 |
| \( C_n(nb) \) | 14.1 | 7.7 | 2.5 | 2.5 |
| \( \epsilon R^2_\rho \) (GeV\(^{-2}\)) | 2.5 | 2.5 | 2.5 | 2.5 |
well with experimental points from \[4\]. For the case of $\Lambda\bar{\Lambda}$ electroproduction one can use the same strategy, however here the $2D$ resonance $\phi(2175)$ is even narrower, and the ratio drops faster. We now turn to the so-called effective form factor, defined for proton as

$$|F_p|^2 = \rho^2 e^{-\rho^2 R^2 e} \frac{3}{2} \frac{E^2}{E^2 + 2M^2} (|\Xi_s|^2 + |\Xi_D|^2). \quad (9)$$

Substituting into $\Xi_S, \Xi_D$ the same combination of $3S$ and $2D$ resonances, one obtains a slowly decreasing function with $|F_p(0)| = 0.4, \lambda = 0.18$ fm. At $E = 1.95$ GeV one obtains $|F_p| = 0.384$ and a wide plateau in the near-threshold region. This agrees with experimental data from \[4\], where the average value in the region ($1.88 \leq E \leq 1.975$) GeV is around 0.4. However, two lowest energy points in \[3, 4\] display a narrow ($\sim 20$ MeV) enhancement, which can be explained by the $p\bar{p}$ final state interaction \[19\]. A fast decrease of all baryon form factors and transition to the quasiperturbative behavior \[3\] at large $E$ requires an explicit derivation of energy dependence of double-pair creation vertex, which is now under study.

Summarizing, we have applied the theory, developed before in \[11\], to the cases of $B\bar{B}$ electroproduction with $B = \Lambda_c, \Lambda_b, \Lambda, p$. The theory does not contain fitting parameters and exploits parameters of intermediate vector mesons and baryons, taken from calculations and experiment. The peaks above threshold are predicted for heavy baryons $\Lambda_c, \Lambda_b$ and are in agreement with experiment for $\Lambda\Lambda$ production \[10\]. A nontrivial angular dependence of electroproduction is shown to occur due to interference of $3S_1$ and $3D_1$ vector meson states, which also explains a nontrivial behavior of the ratio $\left| \frac{G_{B}(\pi)}{G_{B}(\rho)} \right|$ observed earlier in \[4\] for $B = p$ and in \[17\] for $B = \Lambda$. The absolute values of approximately calculated form factors and cross sections near threshold are in general agreement with experiment.

The author is grateful to B.O.Kerbikov for useful discussions, to Yu.S.Kalashnikova and G.V.Pakhlova for discussions, useful suggestions and help, and to A.M.Badalian, who provided results of calculations for heavy and light vector mesons, given in the Table.

![FIG. 4: The ratio $\left| \frac{G_{B}(\pi)}{G_{B}(\rho)} \right|$ due to $3S - 2D$ interference as a function of $E$. Experimental data are from \[4\].](image)

[1] V.P.Druzhinin, S.I.Eidelman, S.I.Serednyakov and E.P.Solodov, arXiv:1105.4975 [hep-ex].
[2] K.K.Seth, arXiv:0712.0356 [hep-ex].
[3] G.Bardin et al. (PS 170 Collaboration), Nucl. Phys. B\textbf{411}, 3 (1994).
[4] B.Aubert et al. (BABAR Collaboration), Phys. Rev. D \textbf{73}, 012005 (2006).
[5] V.L.Chernyak and A.R.Zhitnitsky, JETP Lett \textbf{25}, 510 (1977); G.P.Lepage, S.J.Brodsky, Phys. Rev. D \textbf{22}, 2157(1980); J.Bolz et al. Z.Phys. C\textbf{66}, 267 (1995); T.Hyer, Phys. Rev. D \textbf{47}, 3875 (1993).
[6] P.Kroll, Th.Pilsner, M.Schuermann, W.Schweiger, Phys. Lett. B \textbf{316}, 546 (1993); arXiv: hep-ph 9305251 P.Kroll, Nucl. Phys. Proc. Suppl. \textbf{56A}, 33 (1997).
[7] F.Ischello and Q.Wan, Phys. Rev. C \textbf{69}, 055204 (2004).
[8] E. van Beveren, X.Lin, R.Coiimbra et al., Europhys. Lett. \textbf{85}, 61002 (2009); M.Abud, F.Buccella and F.Tramontano, Phys. Rev. D \textbf{81}, 074018 (2010); G.Cotugno, R.Faccini, A.D.Polosa et al., Phys. Rev. Lett. \textbf{104},132005 (2010).
[9] Yu.A.Simonov, Phys. Rev. D \textbf{84}, 065013 (2011).
[10] Yu.A.Simonov and V.I.Shevchenko, Adv. HEPh, 873051 (2009), arXiv: 0902.1405; Yu.A.Simonov, arXiv: 1003.3608.
[11] Yu.A.Simonov, arXiv: 1109. 5545 [hep-ph].
[12] Yu.A.Simonov, Phys. Atom. Nucl. \textbf{66}, 338 (2003); Phys. Rev.D \textbf{65}, 116004 (2002).
[13] A.M. Badalian, B.L.G. Bakker, I.V. Danilkin, Phys. Atom. Nucl. 72, 638 (2009), arXiv:0805.2291 [hep-ph].
[14] A.M. Badalian, B.L.G. Bakker, I.V. Danilkin, Phys. Atom. Nucl. 73, 138 (2010), arXiv:0903.3643 [hep-ph].
[15] K. Nakamura et al (PDG), J. Phys. G: Nucl. Part. Phys. 37, 075021 (2010).
[16] G.V. Pakhlova et al. (Belle Collaboration), Phys. Rev. Lett. 101, 172001 (2008).
[17] B. Aubert et al. (BABAR Collab.), Phys. Rev. D76, 092006 (2007).
[18] V.A. Novikov, L.B. Okun, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Phys. Rept. 41C, 1 (1978).
[19] I.L. Grach, B. Kerbikov and Yu.A. Simonov, Phys. Lett. B 208, 309 (1988);
    G.Y. Chen, H.R. Dong, J.P. Ma, Phys. Rev. D78, 054022 (2008); O.D. Dalkarov, P.A. Khakhulin and A.Yu. Voronin, Nucl. Phys. A833, 104 (2010).