Efficient adiabatic hydrodynamical simulations of the
high-redshift intergalactic medium

Prakash Gaikwad1⋆, Tirthankar Roy Choudhury1, Raghunathan Srianand2,
and Vikram Khaire1,3

1National Centre for Radio Astrophysics, Tata Institute of Fundamental Research, Pune 411007, India
2Inter-University Centre for Astronomy and Astrophysics (IUCAA), Post Bag 4, Pune 411007, India
3Department of Physics, University of California, Santa Barbara, CA 93106, USA

ABSTRACT
We present a post-processing tool for gadget-2 adiabatic simulations to model various observed properties of the Ly\textalpha forest at 2.5 ≤ z ≤ 4 that enables an efficient parameter estimation. In particular, we model the thermal and ionization histories that are not computed self-consistently by default in gadget-2. We capture the effect of pressure smoothing by running gadget-2 at an elevated temperature floor and using an appropriate smoothing kernel. We validate our procedure by comparing different statistics derived from our method with those derived using self-consistent simulations with gadget-3. These statistics are: line of sight density field power spectrum, flux probability distribution function, flux power spectrum, wavelet statistics, curvature statistics, HI column density (N\textsubscript{HI}) distribution function, linewidth (b) distribution and b versus log N\textsubscript{HI} scatter. For the temperature floor of 10\textsuperscript{4} K and typical signal-to-noise of 25, the results agree well within 20 percent of the self-consistent gadget-3 simulation. However, this difference is smaller than the expected 1σ sample variance for an absorption path length of ∼ 5.35 at z = 3. Moreover for a given cosmology, we gain a factor of ∼ N in computing time for modelling the intergalactic medium under N ≫ 1 different thermal histories. In addition, our method allows us to simulate the non-equilibrium evolution of thermal and ionization state of the gas and include heating due to non-standard sources like cosmic rays and high energy γ-rays from Blazars.

Key words: cosmology: large-scale structure of Universe - methods: numerical - galaxies: intergalactic medium - quasars: absorption lines

1 INTRODUCTION
The Ly\textalpha forest seen in the spectra of distant background QSOs trace the distribution of neutral hydrogen (HI) in the universe at mildly non-linear overdensities (Δ ≲ 10, Miralda-Escudé et al. 1996; Bi & Davidsen 1997; Croft et al. 1997). Observed properties of the Ly\textalpha forest are sensitive to fluctuations in the cosmic density and velocity fields and physical conditions like the temperature, turbulence and ionizing radiation prevailing in the intergalactic medium (IGM; Cen et al. 1994; Zhang et al. 1995; Miralda-Escudé et al. 1996; Hernquist et al. 1996). As a result, Ly\textalpha forest has been used in the literature to constrain cosmological parameters such as Ω\textsubscript{m}, Ω\textsubscript{b}, σ\textsubscript{8}, n\textsubscript{s} (see, e.g., Viel et al. 2004a,b; McDonald et al. 2005), the neutrino mass (Palanque-Delabrouille et al. 2015a,b; Yee et al. 2017), mass of warm dark matter particles (Narayanan et al. 2000; Viel et al. 2005, 2013a) and astrophysical parameters such as the IGM temperature T\textsubscript{HI} at cosmic mean density and slope γ of the temperature (T) - density (Δ) relation (T = T\textsubscript{HI}Δ\textsuperscript{γ−1}, hereafter TDR; Schaye et al. 1999, 2000; Zaldarriaga et al. 2001; McDonald et al. 2001; Theuns & Zaroubi 2000; Lidz et al. 2010; Becker et al. 2011; Boera et al. 2014) and HI photo-ionization rate (Γ\textsubscript{HI}, Rauch et al. 1997; Cooke et al. 1997; Meiksin & White 2004; Becker & Bolton 2013; Kollmeier et al. 2014; Shull et al. 2015; Gaikwad et al. 2017a,b; Viel et al. 2016; Gurvich et al. 2017).

Usually constraining these parameters involves comparing different properties of the Ly\textalpha forest derived from observed spectra with those from the simulated ones. Early simulations of Ly\textalpha forest based on lognormal (Bi et al. 1992; Bi & Davidsen 1997; Gnedin & Hui 1996; Choudhury et al. 2001) or the Zeldovich approximation (Doroshkevich & Shandarin 1977; McGill 1990), although fast and capture
the basic picture, failed to reproduce the quasi- and nonlinear density fields accurately (Viel et al. 2002) or washed out the small scale structures in the Lyα forest (Gnedin & Hui 1998).

By using cosmological N-body simulations (Hernquist & Katz 1989; Springel 2005; O’Shea et al. 2005), the Lyα forest has been modelled in the past using (i) dark matter only simulations where baryons are assumed to follow the dark matter, and the temperature is assigned to the baryons assuming a power-law TDR (Muecket et al. 1996), (ii) smoothed particle hydrodynamic (SPH) codes (Cen et al. 1994; Zhang et al. 1995; Hernquist et al. 1996; Theuns et al. 1998; Davé et al. 1999; Viel et al. 2004a) like GADGET-2 and GADGET-3 (Springel et al. 2001; Springel 2005; Bolton et al. 2006), (iii) grid based adaptive mesh refinement (AMR) code Enzo (Smith et al. 2011; Shull et al. 2012; Bryan et al. 2014) and (iv) hybrid methods such as Lyman Mass Association Scheme (LyMAS) in which moderate resolution dark matter only simulation is used after the calibration using high resolution but small volume hydrodynamic simulations (Peirani et al. 2014; Sorini et al. 2016).

The main drawback of the dark matter only simulations is that it does not account for the smoothing of the baryonic density field due to finite pressure of the baryons, while these effects are self-consistently accounted for in the SPH and AMR based simulations. Interestingly, the Lyα forest flux statistics from SPH and AMR simulations are shown to agree with each other to within 10 per cent accuracy (Regan et al. 2007). These simulations can, in principle, incorporate different complex astrophysical processes such as the radiative heating, cooling, shocks, starbursts and AGN induced feedback processes (Kollmeier et al. 2006; McDonald et al. 2006; Davé et al. 2010; Schaye et al. 2010; Viel et al. 2013b).

While the current state of the art hydrodynamical simulations are extremely useful for probing the physical properties of the IGM, the computational expenses severely limit their usage for constraining the unknown model parameters and their associated errors. Various approaches have been introduced to keep the computational expense within manageable limits while exploring the large parameter space. For example, Viel & Haehnelt (2006); Viel et al. (2006) begin by choosing a “best-guess” model and expand the statistical quantities under consideration (e.g., the flux power spectrum) in a Taylor series around this model. Their method requires calculating a limited number of derivatives which can be achieved by running only a few simulations around the best-guess model. The method of McDonald et al. (2005) involves running simulations on a carefully chosen grid in the parameter space and then interpolating between these runs. Other methods include deriving scaling relations between different parameters from a limited number of hydrodynamical simulations which are useful for studying parameter degeneracies (Bolton et al. 2005; Bolton & Haehnelt 2007; Faucher-Giguère et al. 2008). Since many of the parameters, particularly those related to the thermal state of the IGM, are poorly understood, obtaining robust constraints would require exploring a sufficiently wide range of parameter values. It is thus useful to develop newer methods of simulating the high-z IGM that are efficient, flexible and at the same time sufficiently accurate. This forms one of the main motivation of this work.

In Gaikwad et al. (2017a), we have developed a “Code for Ionization and Temperature Evolution” (CITE) to estimate the temperature of the SPH particles in the post-processing step of GADGET-2 by taking care of radiative cooling and heating effects. CITE allowed us to place good constraints on $\Gamma_{HI}$ while efficiently exploring different thermal histories at low-z ($z \leq 0.5$). While CITE works well for the low resolution simulation (gas particle mass $\delta m = 1.26 \times 10^7 h^{-1} M_{\odot}$ and pixel size $\delta x = 48.86 h^{-1}$ ckpc) as shown in Gaikwad et al. (2017a), the dynamical evolution of SPH particles at finite pressure is an important effect when we consider high resolution simulations (e.g., gas particle mass $\delta m = 1.01 \times 10^5 M_{\odot}$ and pixel size $\delta x = 9.77 h^{-1}$ ckpc). In this article, we present a method to account for this effect by smoothing (in 3 dimensions) the density and velocity fields over a local Jeans scale. We explore the consistency of our method with that from GADGET-3 (Springel 2005, in which the thermal effects on the hydrodynamical evolution of baryonic particles are taken care of in a self-consistent manner) by comparing different Lyα flux statistics frequently used in the literature. Our method (though approximate) is computationally less expensive and accurate enough to constrain physical parameters through a detailed exploration of possible parameter space. Our code is also flexible enough to incorporate effects such as non-equilibrium evolution of the ionization state of the gas and heating by non-standard sources like Blazars or cosmic rays etc.

This paper is organized as follows. In §2, we describe the GADGET-2 and GADGET-3 simulations used in this study. We discuss the method of simulating Lyα forest in §3. We show the consistency of our method with GADGET-3 by comparing 8 different statistics in §4. We summarize our results in §5. We use flat $\Lambda$CDM cosmology with parameters $(\Omega_{m}, \Omega_{\Lambda}, h, n_s, \sigma_8, Y) \equiv (0.31, 0.73, 0.674, 0.83, 0.24)$ consistent with Planck Collaboration et al. (2016). The H I photoionization rate ($\Gamma_{HI}$) expressed in units of $10^{-12}$ s$^{-1}$ is denoted as $\Gamma_{12}$. Unless mentioned all the distances are expressed in comoving co-ordinates.

2 SIMULATION

We use the publicly available GADGET-2 (Springel 2005) to perform smoothed particle hydrodynamical simulations used in this study. The initial conditions are generated at $z = 99$ using the publicly available 2LPT$^3$ code (Scoccimarro et al. 2012). We use 1/30$^3$ of the mean inter-particle distance as the gravitational softening length. The GADGET-2 simulation does not include radiative heating and cooling of the SPH particles internally. As a result, the unshocked gas particles (in the low density regions) are evolved at very

1 GADGET-3 is not publicly available. However, Volker Springel has provided this code through private communication for our studies. GADGET-3 has been frequently used for Lyα forest studies (see e.g., Becker et al. 2011; Viel et al. 2016)

2 http://wwwmpa.mpa-garching.mpg.de/gadget/

3 http://cosmo.nyu.edu/roman/2LPT/
Efficient adiabatic simulations of the high-z IGM

Figure 1. Panels (a) and (b) compare the line of sight density and velocity fields respectively from gadget-3 (black dashed curve) and gadget-2 (red solid curve) simulations for a low resolution simulation box at $z = 2.5$ (box size $L = 50 h^{-1}$ Mpc, gas particle mass $\delta m = 1.26 \times 10^7 h^{-1} M_\odot$ and pixel size $\delta x = 48.8 h^{-1}$ ckpc). Panels (c) and (d) are same as panels (a) and (b) respectively except that these are obtained from high resolution simulation boxes at $z = 2.5$ (box size $L = 10 h^{-1}$ Mpc, gas particle mass $\delta m = 1.01 \times 10^5 M_\odot$ and pixel size $\delta x = 9.77 h^{-1}$ ckpc) used in this paper. gadget-2 models for low and high resolution boxes are performed with the temperature floor of $\sim 100$ K.

Figure 2. Schematic diagram showing main steps adopted in our post-processing method of obtaining Ly$\alpha$ forest spectra from gadget-2 taking into account radiative cooling and heating effects externally. The basic steps involved in our method are: (1) We calculate the temperature of each particle at each redshift using CITE and obtain the thermal history parameters $T_0$ and $\gamma$. (2) Given $T$ and $\Delta$ of particles, we apply pressure smoothing to get new $\Delta_{\text{new}}$ and $v_{\text{new}}$ on grids for a simulation box at a redshift of interest. (3) For this new $\Delta$ on grid points, we apply power-law TDR using thermal history parameters $T_0$ and $\gamma$ obtained in the previous step. (4) We calculate Lya optical depth from the simulation box using our routine GLASS.
low temperature (the default value is 100 K in gadget-2) and pressure. However, the simulation allows one to set the minimum allowed gas temperature (referred as temperature floor) to higher values. In this work, we perform two simulations of gadget-2: (i) G2-LTF with low temperature floor of $T = 100$ K and (ii) G2-HTF with high temperature floor of $T = 10000$ K (corresponding to typical IGM temperatures due to photoheating). An unique identification number is assigned to each particle in gadget-2 and is used for tracing its density and temperature evolution.

We also perform a gadget-3 simulation (a modified version of the publicly available gadget-2 code, see Springel 2005) with the same initial conditions as the gadget-2 simulations discussed above. Unlike gadget-2, the gadget-3 simulation includes radiative heating and cooling of SPH particles internally for any given metagalactic UV background (UVB). We use Haardt & Madau (2012, hereafter HM12) UVB assuming ionization equilibrium in gadget-3. To speed up the calculations, we run the simulations with QUICK_LYALPHA flag that converts particles with $\delta m > 1000$ and $T < 10^5$ K into stars (Viel et al. 2004a) and removes them from subsequent calculations. None of our simulations (i.e., gadget-2 or gadget-3) include AGN feedback, stellar feedback or outflows in the form of galactic wind. The details of our simulations are listed in Table 1.

### 3 METHOD

The Ly$\alpha$ optical depth is calculated by evaluating the overdensity ($\Delta$), temperature ($T$) and velocity ($v$) on grid points along a given sightline in the simulation box. Unlike gadget-3, the TDR obtained in gadget-2 is not realistic as the radiative heating and cooling terms are not incorporated. At moderate to low resolution, the overdensity and velocity fields from gadget-2 matches well with those from gadget-3 as shown in panel (a) and (b) of Fig. 1. This resolution (gas particle mass $\delta m = 1.26 \times 10^7 h^{-1} M_\odot$, pixel size $\delta x = 48.8 h^{-1}$ ckpc) is appropriate for low-$z$ ($z < 0.5$) Ly$\alpha$ forest studies with instruments like the HST-COS (Gaikwad et al. 2017a,b). However gadget-2 does not capture the effect of finite gas pressure in the hydrodynamical evolution of the photoionized gas. This effect becomes important at smaller scales probed well in high resolution resolution equation assuming HM12 UVB to calculate various ion fractions of H and He. We use cite to calculate the last two terms on right hand side of Eq. 1 as they are not self-consistently computed in gadget-2. The actual implementation is as follows.

(i) At the initial redshift (taken to be $z_1 = 6.0$ in this work), we assume a given power-law TDR. In this paper, we choose $T_0 = 7920$ K and $\gamma = 1.52$ in order to match those obtained in gadget-3 at the same redshift for HM12 UVB. We then compute the actual temperature of a gas particle using following prescription: If a particle is shock heated in recent times (i.e., within a time scale corresponding to $\delta z = 0.1$), then the temperature of the particle will not be updated by cite. Otherwise we assume the particle temperature to be following the above mentioned power-law TDR. At the initial redshift, we solve equilibrium ionization evolution equation assuming HM12 UVB to calculate various ion fractions of H and He.

(ii) Given the ion fractions and the temperatures, it is straightforward to calculate last two terms on the right hand side of Eq. 1 for subsequent time steps. For this, we use the photo-heating rates of HM12 UVB model.

(iii) To obtain the temperature of the particles in the next time step ($z_2 = z_1 + \Delta z$), we first check if the particle is shock heated in recent times (i.e., within a time scale corresponding to $\delta z = 0.1$). If the particle is not shock heated, then we neglect the third term on the right hand side of Eq.

\[
\frac{dT}{dt} = -2HT + \frac{2T}{\Delta t} + \frac{dT_{\text{shock}}}{dt} + \frac{dT_{\text{HE}}}{dt} + \frac{dT_{\text{other}}}{dt}. \quad (1)
\]

\[
\frac{dT}{dt} = -2HT + \frac{2T}{\Delta t} + \frac{dT_{\text{shock}}}{dt} + \frac{dT_{\text{HE}}}{dt} + \frac{dT_{\text{other}}}{dt}.
\]

\[
\frac{dT}{dt} = -2HT + \frac{2T}{\Delta t} + \frac{dT_{\text{shock}}}{dt} + \frac{dT_{\text{HE}}}{dt} + \frac{dT_{\text{other}}}{dt}.
\]

\[
\frac{dT}{dt} = -2HT + \frac{2T}{\Delta t} + \frac{dT_{\text{shock}}}{dt} + \frac{dT_{\text{HE}}}{dt} + \frac{dT_{\text{other}}}{dt}.
\]
Figure 3. TDR of the SPH particles from GADGET-3 (left panel), G2-LTF (middle panel) and G2-HTF (right panel) at $z = 2.5$. The temperature in the G2-LTF and G2-HTF models are obtained in the post-processing step of GADGET-2 using cite (see §3). The magenta dashed vertical lines show bins in log $\Delta$. We calculate median $T$ (black stars) in each of these $\Delta$ bins and fit a power-law, $T = T_0 \Delta^{-\gamma}$, to obtain $T_0$ and $\gamma$. The resulting TDR is shown by black dashed line. In the case of GADGET-3 we use \textsc{quick2alpha} flag under which gas particles with $T < 10^5$ K and $\Delta > 1000$ are converted into stars and got removed from subsequent calculations. No such star formation criteria is applied in G2-LTF and G2-HTF models (see Appendix A for more details). The colour scheme represents density of points in logarithmic unit.

Figure 4. Comparison of redshift evolution of the thermal history parameters ($T_0$ and $\gamma$) from our G2-HTF with GADGET-3 (gray stars) simulations and that of Puchwein et al. (2015, magenta up-triangles for non-equilibrium and blue down-triangles for equilibrium ionization evolution). cite is started at $z = 6.0$ with initial conditions $T_0 = 7920$ K and $\gamma = 1.52$ same as those obtained in GADGET-3 at that redshift (see §3 for details). For G2-HTF simulations we run cite using equilibrium (red filled circles) and non-equilibrium (green diamonds) ionization condition influenced by the same UVB. Note that the default version of GADGET-3 solves equilibrium ionization evolution equation.
Table 1. Details of our simulations described in §2

| Model         | GADGET-3 | G2-LTF | G2-HTF |
|---------------|----------|--------|--------|
| N-body code   | GADGET-3 | GADGET-2 | G2-HTF |
| Initial redshift | 99       | 99     | 99     |
| Box size (h⁻¹ c Mpc) | 10       | 10     | 10     |
| Number of particles | 2 × 512³ | 2 × 512³ | 2 × 512³ |
| UVB²           | HM12     | HM12   | HM12   |
| Ionization evolution² | Equilibrium | Equilibrium | Equilibrium |
| T and ∆ evolution² | Internal | Post-process (cite) | Post-process (cite) |
| SFR Criteria³  | QUICK_LYALPHA | – | – |
| Output redshifts | 6.0, 5.9, · · ·, 2.0 | 6.0, 5.9, · · ·, 2.0 | 6.0, 5.9, · · ·, 2.0 |
| Temperature floor⁴ | – | 100 K | 10000 K |
| Smoothing kernel type⁵ | SPH | Modified | Modified |
| Gas particle mass (δm)⁶ | 1.01 × 10⁶ h⁻¹ M⊙ | 1.01 × 10⁶ h⁻¹ M⊙ | 1.01 × 10⁶ h⁻¹ M⊙ |
| Pixel size (δx)⁷ | 9.77h⁻¹ ckpc | 9.77h⁻¹ ckpc | 9.77h⁻¹ ckpc |

1. Otherwise we solve the same Eq. 1 accounting all the five terms.
   (iv) For redshift z₂, we solve equilibrium (or non-equilibrium, if desired) ionization evolution equations to calculate various ion fractions.
   (v) We repeat the steps (ii)-(iv) to obtain the temperature of the particle at subsequent redshifts.

Fig. 3 shows comparison of TDR of SPH particles obtained from GADGET-3 (left panel), G2-LTF (middle panel) and G2-HTF (right panel) simulations at z = 2.5. Qualitatively, the TDR from G2-LTF and G2-HTF (after processing through cite) is remarkably similar to that from GADGET-3. The differences at ∆ > 1000 and T < 10⁵ K can be attributed to the QUICK_LYALPHA flag employed in GADGET-3 (see Appendix A for more details). For each model, we calculate median temperature (black star points) in log ∆ bins with centres at −0.375, −0.125, 0.125, 0.375 and bin width 0.125 (indicated by magenta dashed vertical lines). We then fit power law relation T = T₀ ∆γ⁻¹ to obtain the best fit T₀ and γ (Hui & Gnedin 1997; McDonald et al. 2005). The fitted TDR is shown by black dashed line in each panel. The values of T₀ and γ are also indicated in each panel. It is clear that they are similar within 2.5 percent.

Fig. 4 shows the redshift evolution of best fit T₀ (top panel) and γ (bottom panel) for G2-HTF, GADGET-3 and Puchwein et al. (2015) models for equilibrium and non-equilibrium ionization evolution cases. The evolution of T₀ and γ obtained from cite for the equilibrium ionization case is remarkably similar to those obtained from the GADGET-3 run and Puchwein et al. (2015)⁵. As mentioned earlier, we can also solve for non-equilibrium ionization evolution equation using cite. The T₀ and γ evolution for non-equilibrium case from Puchwein et al. (2015, magenta dashed curve) is also consistent with those from G2-HTF with the maximum difference being less than 2.5 per cent (at z ∼ 3.5). Since the default version of GADGET-3 solves the ionization evolution equation under equilibrium conditions, hereafter we restrict our discussions to the models with equilibrium ionization as we will use GADGET-3 as our reference. While cite reproduces the T₀ and γ evolution well, the issues related to small scale density and velocity field (demonstrated in Fig. 1) still need to be addressed.

3.2 Jeans length of SPH particle in GADGET-2:

In this section, we explore the possibility of using local pressure smoothing in the GADGET-2 simulations to reduce the shortcomings highlighted in panels (c) and (d) of Fig. 1. We choose to smooth the density field in G2-LTF or G2-HTF on the scales of Jeans length of the particles to account for the pressure smoothing. Assuming the Lyα absorbers to lie in local hydrostatic equilibrium, Schaye (2001) has shown that the Jeans length can be obtained by equating dynamical time with sound crossing time and is given by,

$$\frac{L}{1 \text{kpc}} \sim 0.52 \left[ \frac{T}{10^{4} \text{K}} \frac{1 - Y}{0.76} \frac{f_{g}}{0.16} \frac{1 \text{ cm}^{-3}}{n_{H}} \frac{0.59}{\mu} \right]^{1/2}$$ (2)

⁵. The differences between the values of T₀ and γ calculated from G2-LTF and G2-HTF are less than 0.1 per cent.
where, $T$ is temperature, $n_H$ is number density of H, $Y$ is He fraction by mass, $\mu = 4/(8 - 5Y)$ is the mean molecular weight and $f_g$ is fraction of total mass in gas phase. For the scales of interest here $f_g$ is close to its universal value $\Omega_b/\Omega_m \sim 0.16$. It should be emphasized that the Jeans length depends on the density and temperature and hence is different for different particles. For the same reason, it is different for the same particle at different epochs. The above equation is not valid for Ly\a absorbers with characteristic densities smaller than the cosmic mean ($\Delta \sim 1$, Schaye 2001). Hence we ignore the pressure smoothing for such particles and retain only the SPH smoothing. We now explain how the effect of pressure smoothing is incorporated in G2-LTF or G2-HTF by modifying the SPH kernel.

### Smoothing kernel:

The estimate of a quantity $f$ at any grid point $i$ in the SPH formulation (Monaghan 1992; Springel 2005) is given by,

$$f_i = \sum_j f_j \frac{m_j}{\rho_j} S_{ij}$$

(3)

where the summation is performed over all particles. The quantities $m_j, \rho_j, f_j$ are the mass, density and value of the quantity $f$ of $j^{th}$ particle, respectively. The quantity $f$ could be over-density ($\Delta$), temperature ($T$) or any component of the velocity ($\mathbf{v}$). The smoothing kernel, $S_{ij}$, has units of inverse of volume and in general depends on the distance ($r_{ij}$) between $i^{th}$ grid point and $j^{th}$ particle. It is necessary for $S_{ij}$ to satisfy the following normalization condition in order to conserve the quantity $f$ (in particular mass) in SPH formulation (Monaghan 1992),

$$\int_V S_{ij} \, dV = 1$$

(4)

where the integration is over volume $V$.

We use the following smoothing kernels for various simulations,

$$S_{ij} \equiv \begin{cases} W(r_{ij}, h_j), & \text{For GADGET-3} \\ W'(r_{ij}, h_j, 1 \times L_j), & \text{For G2-LTF} \\ W'(r_{ij}, h_j, 0.66 \times L_j), & \text{For G2-HTF} \end{cases}$$

(5)

where $h_j$ and $L_j$ are smoothing length and Jeans length (given by Eq. 2) of the $j^{th}$ particle respectively.

The smoothing kernel used for GADGET-3 is same as SPH kernel given in Springel (2005) and has following form,

$$W(r, h) = W_0 \begin{cases} 1 - 6 \left( \frac{r}{h} \right)^2 + 6 \left( \frac{r}{h} \right)^3, & 0 \leq \frac{r}{h} \leq \frac{1}{2} \\ 2 \left( 1 - \frac{r}{h} \right)^3, & \frac{1}{2} \leq \frac{r}{h} \leq 1 \\ 0, & \frac{r}{h} > 1 \end{cases}$$

(6)

where $W_0 = 8/\pi h^3$ is normalization constant of SPH kernel.

The pressure smoothing can be well approximated by a Gaussian (Gnedin & Hui 1998; Kulkarni et al. 2015). Hence we modify the smoothing kernel by convolving SPH kernel with Gaussian kernel of pressure smoothing

$$W'(r, h, \sigma) = \int d^3x_1 W(r_1, h) G(|r - x_1|, \sigma)$$

(7)

where the Gaussian kernel is assumed to be isotropic and is given by

$$G(|r - x_1|, \sigma) = \frac{1}{(2\pi \sigma^2)^{3/2}} \exp\left[-\frac{|r - x_1|^2}{2\sigma^2}\right] = \frac{1}{(2\pi \sigma^2)^{3/2}} \exp\left[-\frac{(r^2 + r_1^2 - 2 r r_1 \mu)}{2 \sigma^2}\right]$$

(8)

with $\mu$ being the cosine of the angle between $r$ and $x_1$ and $\sigma$ the width of the Gaussian which in turn depends on the Jeans length. At this point let us highlight some of the key properties of $W'(r, h, \sigma)$ which are relevant for our calculations:

- Both $W(r, h)$ in Eq. 6 and $W'(r, h, \sigma)$ in Eq. 7 satisfy the normalization condition given in Eq. 4.
- The kernel in Eq. 7 does not have a closed form analytic solution, hence we need to calculate it numerically (See Appendix B for more details).
- Unlike $W(r, h)$, $W'(r, h, \sigma)$ does not have a compact support as the Gaussian is non-zero at large distances. Hence we put a cut-off such that if distance between particle and grid is more than $h + 3\sigma$, the contribution of $W'(r, h, \sigma)$ is zero. Mathematically,

$$W'(r, h, \sigma) = \begin{cases} W'(r, h, \sigma), & 0 \leq r \leq (h + 3\sigma) \\ 0, & r > (h + 3\sigma) \end{cases}$$

(9)

We find that this cut-off does not have any significant effect on the density, velocity or temperature estimates as long as it is taken to be $\geq h + 3\sigma$.

- The amount of pressure smoothing in Eq. 7 is decided by the width $\sigma$ of the Gaussian. The SPH particles in G2-HTF are evolved at relatively high temperature ($T \sim 10^4$ K) and pressure as compared to G2-LTF ($T \sim 100$ K). It can be shown that the additional pressure smoothing length required in G2-HTF model is factor $\sim 0.66$ times the smoothing length for the model G2-LTF (see Appendix C for details).

- This way of modifying smoothing kernel and estimating quantities along sightlines allow us to account for two important effects: (i) the variation in pressure smoothing for different particles at any epoch and (ii) the evolution of pressure smoothing scale for any particle at different epochs. Note that the pressure smoothing experienced by a particle in the GADGET-3 simulation depends on the whole thermal history and not only on the present temperature as we do in our case (Lukić et al. 2015; Kulkarni et al. 2015). However, as we will discuss later, running the GADGET-2 with high temperature floor captures (on an average) the pressure broadening arising from thermal history effects reasonably well.

### 3.3 Estimation of the temperature field on a grid:

After calculating the overdensity ($\Delta$) and velocity field ($\mathbf{v}$) on grids along a given sightline using Eqs 3-8, we can also estimate the temperature ($T$) along the same sightline using the same equations. However, the resultant TDR is not efficient adiabatic simulations of the high-z IGM
a power-law any more. This is because the temperature of the particle from cite in the first step is calculated using GADGET-2 density field that does not incorporate the pressure smoothing. Hence we need to recalculate the temperature corresponding to the new density field with the pressure smoothing incorporated. In principle, we can again use cite on the new smoothed density field and calculate the temperature. However, we find that this is computationally expensive because we need to calculate the smoothed density field on the grid along the sightline for all redshifts i.e. $z = 6$ to 2 with a $\Delta z = 0.1$. Hence we adopt a simplified approach of applying power-law TDR (Hui & Gnedin 1997; Choudhury et al. 2001)

$$T = \begin{cases} 
T_0 \Delta^{-1}, & \Delta \leq 10 \\
T_0 10^{-1}, & \Delta > 10 \\
T_{\text{shock}}, & T_{\text{shock}} > T 
\end{cases}$$

(10)

where $T_0$ and $\gamma$ are obtained from fitting the TDR for particles in our simulation box at the redshift of our interest as explained in Step (1) (also see Fig. 4). The last relation implies that if a particle is shock heated (or has temperature higher than that predicted by the TDR) then its temperature is not updated. We have confirmed that this approach produces consistent results with those obtained by running cite on the new density field.

### 3.4 Ly$\alpha$ transmitted flux:

We have developed a module for “Generating Ly-Alpha forest Spectra in Simulations” (GLASS) to calculate the Ly$\alpha$ transmitted flux that has signal-to-noise ratio (SNR) and spectral resolution similar to the typical observational data used in the Ly$\alpha$ forest studies. The basic steps involved in GLASS (Choudhury et al. 2001; Padmanabhan et al. 2015; Gaikwad et al. 2017a) are as follows:

(i) We determine the Hi number density ($n_{\text{HI}}$) at any grid point from the baryonic density field ($\Delta$) assuming the gas to be optically thin and in photoionizing equilibrium with the UVB. The Hi photoionization rate ($\Gamma_{\text{HI}}$) is a free parameter. Throughout this paper we consider models with a fixed value $\Gamma_{\text{HI}} = 10^{-12} \text{s}^{-1}$ (Becker & Bolton 2013) for simplicity.

(ii) We calculate the Ly$\alpha$ optical depth ($\tau$) along a line of sight from $n_{\text{HI}}$ field by accounting for peculiar velocity, thermal and natural broadening effects.

(iii) The Ly$\alpha$ transmitted flux is given by $F = e^{-\tau}$.

(iv) When comparing with observations, the Ly$\alpha$ flux field is linearly interpolated to match the wavelength sampling of observations.

(v) The Ly$\alpha$ flux field is then convolved with line spread function (LSF) of the spectrograph used in the observation. In this work we assume that the LSF is a Gaussian with a full width at half maximum, FWHM $\sim 7 \text{ km s}^{-1}$, typical of UVES or HIRES spectra.

(vi) Finally we add Gaussian random noise corresponding to a typical SNR=25 similar to what has been frequently achieved in echelle spectrographic observations with VLT and KECK that are used for Ly$\alpha$ forest studies.

A comparison of slices (having a width of 10 ckipc) of the overdensity (log $\Delta$), line of sight velocity (along x axis, $v_x$) and temperature (log T) fields on grids from a simulation boxes at $z = 2.5$ are shown in Fig. 5. The top, middle and bottom rows show slices from GADGET-3, G2-LTF and G2-HTF simulations respectively. The log $\Delta$, $v_x$ and log T fields are sharper in the G2-LTF model (in particular in low density regions) as compared to those of GADGET-3 model. On the other hand the log $\Delta$, $v_x$ and log T fields from G2-HTF model resembles those to those of GADGET-3. We shoot a sightline through each of these slices as shown by horizontal dashed line and extract the log $\Delta$, $v_x$ and log T fields as shown in panel (a), (b) and (c) of Fig. 6 respectively. The line of sight log $\Delta$, $v_x$ and log T fields from G2-LTF and G2-HTF are very similar to those from GADGET-3. However, in general the variations in these fields for G2-LTF model are slightly more compared to those of GADGET-3 and G2-HTF models. Visually the Ly$\alpha$ transmitted fluxes from different models are similar, despite subtle differences seen in log $\Delta$, $v_x$, log T fields between these models. The Ly$\alpha$ transmitted flux shown in this example is not convolved with LSF and is free of noise.

To perform a quantitative comparison of the Ly$\alpha$ forest spectra extracted from different models, we identify eight statistics that are frequently used in the literature. We shoot random sightlines through the simulation and splice together the lines of sight in such a way that it covers a redshift path $z \pm 0.05$, where $z = 2.5, 3.0, 3.5, 4.0$ are redshifts of the simulation box$^6$. Each Ly$\alpha$ forest spectrum has a path length of $\sim 50 \text{ cMpc}$. Following Rolinde et al. (2013); Gaikwad et al. (2017a,b), we generate a mock sample of $N_{\text{spec}} = 20$ Ly$\alpha$ forest spectra for the GADGET-3, G2-LTF and G2-HTF models. Each mock sample covers path length of $\sim 1000 \text{ cMpc}$ (corresponding dimensionless absorption path length is $X \sim 5.35$ at $z = 3$)$^7$. This path length is similar to the path length covered in the Ly$\alpha$ forest studies by Becker et al. (2011, see their Table 3). We repeat the procedure by choosing different random sightlines and generate $N = 100$ such mock samples. The collection of N mock samples constitute a “mock suite” that consists of $N \times N_{\text{spec}} = 2000$ simulated spectra. Thus total path length covered in mock suite is $\sim 10^5 \text{ cMpc}$. We estimate the covariance matrix for different statistics using the simulated spectra.

### 4 RESULTS

We now compare different properties of the Ly$\alpha$ forest generated from G2-LTF, G2-HTF and GADGET-3 simulations using eight statistics, namely, (i) the line of sight baryonic density field ($\delta = \Delta - 1$) power spectrum (DPS), (ii) the flux probability distribution function (FPDF), (iii) the flux power spectrum (FPS), (iv) the wavelet statistics, (v) the curvature statistics, (vi) the column density distribution function (CDFDF), (vii) the line width (b) distribution function and (viii) the b vs log $N_{\text{HI}}$ scatter plot. The statistics

---

$^6$ We do not splice together the lines of sight for FPS estimation.

$^7$ The dimensionless absorption path length is defined as $dX = dz (1 + z)^2 \frac{H(z)}{H_0}$ where $H(z)$ is hubble parameter at $z$ (Bahcall & Peebles 1969).
Figure 5. Slices from a simulation box having a width $\sim 10$ ckpc at $z = 2.5$ for GADGET-3 (top), G2-LTF (middle) and G2-HTF (bottom). Left, middle and right panels in each row show overdensity ($\log \Delta$), velocity component ($v_x$) along $x$ axis and temperature ($\log T$) field respectively. The colour scheme represents density of points in logarithmic unit. We shoot a sightline parallel to $x$ axis through simulation box in each model as shown by horizontal dashed line in each panel. The extracted $\log \Delta$, $v_x$ and $\log T$ along these sightlines are plotted in Fig. 6.
Figure 6. Comparison of line of sight overdensity (panel (a), log $\Delta$), velocity (panel (b), $v_x$ in km s$^{-1}$), temperature (panel (c), log $T$) and Ly$\alpha$ transmitted flux (panel (d), $F$) for GADGET-3 (black solid line), G2-LTF (blue dotted line) and G2-HTF (red dashed line) from a simulation box at $z = 2.5$ shown in Fig. 5. The Ly$\alpha$ transmitted flux is not convolved with any LSF and no noise is added to the flux.

(i)-(v) are obtained assuming Ly$\alpha$ transmitted flux to be a continuous field whereas, the statistics (vi)-(viii) are based on parameters derived using Voigt profile decomposition of Ly$\alpha$ forest. For this purpose we use our automatic Voigt profile fitting code Viper described in full detail in Gaikwad et al. (2017b).

4.1 Line of sight density power spectrum (DPS)

The density field power spectrum is not a directly measurable quantity but it influences all the observable quantities of Ly$\alpha$ forest. We calculate the power spectrum of the 1D density fluctuations along the line of sights using sightlines of comoving length equal to the simulation box size $10 h^{-1}$ cMpc. This is done by computing the Fourier transform $\delta(k)$ of the density field $\delta(x)$, the corresponding power is simply given by $P_\delta(k) \propto |\delta(k)|^2$. We normalize the DPS (Zhan et al. 2005) as,

$$\sigma^2_{P_\delta} = \int \frac{dk}{2\pi} P_\delta(k),$$

(11)

where $\sigma^2_{P_\delta}$ is variance of the 1D density field. We bin the DPS in 20 equispaced logarithmic bins in the range $\log k = 0.301$ to 2.466 with bin width of $\Delta \log k = 0.114$ (Kim et al. 2004).

Following Rollinde et al. (2013) and Gaikwad et al. (2017a), we take the average of all DPS along different sightlines in a mock sample (consisting of 20 lines of sight). Then we calculate the mean DPS and the associated errors from the mock suite (which consists of $N = 100$ mock sample). Let $P_{\delta,n}(k_i)$ denotes the value of DPS in $i^{th}$ bin of $n^{th}$ mock sample, then the average DPS in $i^{th}$ bin is given by,

$$P_\delta(k_i) = \frac{1}{N} \sum_{n=1}^{N} P_{\delta,n}(k_i).$$

(12)

The covariance matrix element $C(i,j)$ between the $i^{th}$ and $j^{th}$ bins is given by,

$$C(i,j) = \frac{1}{N - 1} \sum_{n=1}^{N} [P_{\delta,n}(k_i) - P_{\delta,n}(k_j)][P_{\delta,n}(k_j) - P_{\delta,n}(k_j)]$$

(13)

where, $i$ and $j$ can take values from 1 to the number of bins. The above analysis assumes a mock sample path length of $1000 h^{-1}$ cMpc (i.e., the mock sample consisting of 20 spectra, corresponding dimensionless absorption path length is $X \sim 5.35$ at $z = 3$). We have done the similar analysis for the $5000 h^{-1}$ cMpc mock sample path length (i.e., the mock sample consisting of 100 spectra, corresponding to $X \sim 26.75$ at $z = 3$). In this case we find that the covariance matrix elements are similar to those from mock samples with $1000 h^{-1}$ cMpc path length for all the statistics (see the discussion...
in Appendix D). Hereafter unless mention the results are presented for mock sample path length of 1000 h^{-1} cMpc.

The top left panel of Fig. 7 shows the DPS for the GADGET-3 (black circles), G2-LTF (blue squares) and G2-HTF (red stars) models at z = 2.5. The grey shaded region is the 1σ uncertainty coming from sample variance (i.e., variation in DPS along different sightlines) in the GADGET-3 DPS. The bottom left panel shows the residual fraction (hereafter residual for simplicity) between G2-LTF (blue squares) and G2-HTF (red stars with errorbars) model with respect to GADGET-3 model. The residual between G2-HTF (or G2-LTF) and GADGET-3 model is defined as follows,

\[ R_{G2-HTF} = 1 - \frac{P_{G2-HTF}}{P_{GADGET-3}}. \]  

(14)

The errors on G2-HTF residuals in bottom left panel of Fig. 7 correspond to grey shaded region in top left panel (sample variance). Other panels in Fig. 7 are similar to left most panels but for different redshifts. The residuals are within 12 and 18 percent of GADGET-3 model at scales k < k_{cutoff}. The other panels show similar comparison at 4 different redshifts that are identified in each panel.

Figure 7. Top left panel shows the comparison of the line of sight density field power spectrum obtained from GADGET-3 (black circle), G2-LTF (blue squares) and G2-HTF (red stars) models at z = 2.5. The grey shaded region represents the 1σ uncertainty (diagonal elements of covariance matrix given in Eq. 13) on the DPS from GADGET-3. The Fourier modes with k > k_{cutoff} (k_{cutoff} varies with redshift e.g., k_{cutoff}(z = 3) = 101 h Mpc^{-1}, magenta dashed vertical line) are not be probed by current observations due to limited velocity resolution (∼7 km s^{-1}) of the spectra. The residuals (R, see Eq. 14) between the G2-LTF, G2-HTF models with respect to the GADGET-3 model at z = 2.5 are shown in bottom left panel. The error-bars shown in this panel represent the 1σ uncertainties generated from a path length of 1000 h^{-1} cMpc (corresponding dimensionless absorption path length is X ∼ 5.35 at z = 3) which correspond to the grey shaded region in the top left panel. The DPS from G2-LTF and G2-HTF models are within 12 and 18 percent of GADGET-3 model at scales k < k_{cutoff}. The other panels show similar comparison at 4 different redshifts that are identified in each panel.

We notice at k > k_{cutoff}, the power in G2-HTF model is smaller as compared to GADGET-3 model. This is due to the fact that the minimum temperature before applying cite (irrespective of the density) in G2-HTF model is ∼ 10^4 K. However in GADGET-3 model, the particles with Δ < 1 are at temperature smaller than 10^4 K (see Fig. 3). Thus higher temperature for Δ < 1 particles in G2-HTF model leads to an additional pressure smoothing, thus the power on scales k > k_{cutoff} is smaller than that from GADGET-3. However it is important to note that (for reasons mentioned above) the mismatch between G2-HTF and GADGET-3 model at

\[ k > k_{cutoff} \]
4.2 Flux probability distribution function (FPDF)

The FPDF is one of the flux statistics that is relatively straightforward to calculate from observations as well as simulations (Jenkins & Ostriker 1991; McDonald et al. 2000; Kim et al. 2007; Desjacques et al. 2007; Rollinde et al. 2013; Gaikwad et al. 2017a). Note that we have added the noise to flux corresponding to the SNR of 25. Unless mentioned, hereafter all the results are presented for SNR=25. We calculate the FPDF in 21 equally spaced bins with bin centres in the range \( F = 0.05 \) to 1.0 and bin width \( \Delta F = 0.05 \) (consistent with Kim et al. 2007). The pixels with \( F < 0 \) (\( F > 1 \)) are included in the first (last) bin. Let \( P_n(F_i) \) denote the value of FPDF in the \( i^{th} \) bin of \( n^{th} \) mock sample then average FPDF in the \( i^{th} \) bin (denoted as \( \bar{P}(F_i) \)) is given by Eq. 12 where we replace \( P_i,n(k_i) \) with \( P_n(F_i) \). Similarly the covariance matrix element \( C(i,j) \) between the \( i^{th} \) and \( j^{th} \) bins is obtained from Eq. 13 by replacing \( P_i,n(k_i) \) with \( P_n(F_i) \) respectively.

Fig. 8 shows the comparison of FPDF obtained from gadget-3, G2-HTF and G2-LTF models. Various symbols and line styles are same as those used in Fig. 7. Note that for all the three models we use \( \Gamma_{12} = 1 \), SNR=25 and a path length of 1000h\(^{-1}\) cMpc (\( X \approx 5.35 \) at \( z = 3 \)) for the mock sample. We calculated the mean FPDF, the covariance matrix and residuals (\( \mathcal{R} \)) using Eq. 12, 13 and 14 respectively with \( P_i \) replaced by \( P_n \). The 1\( \sigma \) uncertainty in top (gray shaded region) and bottom left panels (red stars with errorbars) are contributed by the uncertainty in FPDF along different sightlines (sample variance) and finite SNR of the spectra. In order to separate out the statistical error arising purely from the assumed SNR and that from the sample variance, we calculate the FPDF for noise free spectra (i.e. SNR= \( \infty \) but same \( \Gamma_{12} \) and mock sample path length) for gadget-3 model. The green shaded region in bottom panels of Fig. 8 represents sample variance from gadget-3 model. The size of this sample variance is comparable to the errors on FPDF from gadget-3 model (red stars with errorbars) suggesting that the errors are dominated by sample variance. The mean flux in gadget-3 model (shown by green dashed dot vertical line) differs by less than 0.78 percent with that from G2-HTF and G2-LTF models at all \( z \).

The flux near the normalized continuum (in bins with \( F > 0.9 \)) is usually affected by two observational systematics, (i) the continuum fitting of the spectra\(^8\) and (ii) the noise property of the spectra. On the other hand, flux near saturated region (\( F < 0.1 \)) depends on accurate background sky subtraction. Thus while comparing observed FPDF and the simulated FPDF, one usually compares them in the range \( 0.1 \leq F \leq 0.9 \) (Gaikwad et al. 2017a). In Fig. 8, we compare the FPDF from three models within the range \( 0.1 \leq F \leq 0.9 \) shown by magenta dashed vertical lines.

It is clear from the bottom panels of Fig. 8 that the difference between FPDF in the range \( 0.1 \leq F \leq 0.9 \) is less than 15 and 18 percent respectively at all redshifts. Note that sample variance is typically of the order of 13 percent in the range \( 0.1 \leq F \leq 0.9 \).

G2-HTF and G2-LTF (with respect to gadget-3 model) is less than 15 and 18 percent respectively at all redshifts. We also see that the sample variance in FPDF is smaller as compared to DPS as noted by Zhan et al. (2005). This is because the transformation (logarithmic suppression) between baryon density and flux is non-linear.

4.3 Flux power spectrum (FPS)

Like the DPS, the FPS is a two point correlation function between pixels of the Ly\( \alpha \) transmitted flux (Croft et al. 1998; McDonald et al. 2000; McDonald 2003; Kim et al. 2004; Zhan et al. 2005; Arinyo-Prats et al. 2015). The FPS is known to be sensitive to the astrophysical parameters such as \( \Gamma_{12} \), \( T_0 \) and \( \gamma \) (Zaldarriaga et al. 2001; Zaldarriaga 2002; Viel et al. 2004a) in addition to the cosmological parameters. The procedure for calculating the FPS is identical to that of the DPS. If we denote the value of FPS in the \( i^{th} \) bin of \( n^{th} \) mock sample as \( P_{r,n}(k_i) \) then the average FPS in the \( i^{th} \) bin is obtained from Eq. 12 by replacing \( P_{i,n}(k_i) \) with \( P_{r,n}(k_i) \). In similar vein, the covariance matrix elements \( C(i,j) \) are obtained from Eq. 13. The \( \chi^2 \) is calculated using the full covariance matrix.

In Fig. 9 we compare the FPS between different models. Note that at the redshift of interest the astrophysical parameters \( \Gamma_{12} \), \( T_0 \) and spectral properties such as SNR, resolution and mock sample path lengths are same for different models. The FPS for different models behave in a way similar to the DPS. The FPS obtained from G2-HTF model is consistent within 1\( \sigma \) and 5 percent accuracy with that from the gadget-3 model at all redshifts. However, G2-LTF models at \( z = 3.5 \) and 4.0 have slightly excess power (but still within 5 percent) at scales in the range \( 20 \rightarrow 100 \) km s\(^{-1}\) (\( k \approx 0.06 \rightarrow 0.28 \) km s\(^{-1}\)). Similar to DPS, this excess power in the G2-LTF model can be attributed to the contributions in the thermal history of the particles. We also see that the sample variance in FPS is smaller as compared to DPS as noted by Zhan et al. (2005). This is because the transformation (logarithmic suppression) between baryon density and flux is non-linear.

4.4 Wavelet statistics

The wavelet statistic has been used in the past to constrain \( T_0 \) and \( \gamma \) of the IGM (Theuns & Zaroubi 2000; Theuns et al. 2002; Zaldarriaga 2002; Lidz et al. 2010; Garzilli et al. 2012). Wavelets have finite support in both real and Fourier space and thus can be used to estimate the power at scales of interest. This is necessary because large scales (small \( k \)) are not sensitive to \( T_0 \), \( \gamma \) variation whereas small scales (large \( k \)) are contaminated by noise and metal lines in observations (Lidz et al. 2010). We use the “Morlet” wavelet, usually a sine (or

| \( z \) | gadget-3 | G2-LTF | G2-HTF |
|---|---|---|---|
| 2.5 | \( -2.93 \pm 0.12 \) | \( -2.91 \pm 0.12 \) | \( -2.92 \pm 0.12 \) |
| 3.0 | \( -2.91 \pm 0.12 \) | \( -2.87 \pm 0.12 \) | \( -2.91 \pm 0.12 \) |
| 3.5 | \( -2.85 \pm 0.13 \) | \( -2.76 \pm 0.12 \) | \( -2.86 \pm 0.13 \) |
| 4.0 | \( -2.76 \pm 0.15 \) | \( -2.57 \pm 0.13 \) | \( -2.77 \pm 0.14 \) |

\( k > k_{\text{sat}} \) does not have a significant effect on the Ly\( \alpha \) flux statistics presented later.
a cosine) function damped by Gaussian, which has the form

$$\Psi(x) = A \exp(-i k_0 x) \exp\left(-\frac{x^2}{2 \sigma^2}\right)$$

(15)

where $s_n = 35 \text{ km s}^{-1}$, and $k_0 = s_n / 2\pi$ is the scale over which power is extracted. As shown by Lidz et al. (2010), this scale is sensitive to $T_0$ and $\gamma$ variations. $A$ is a normalization constant fixed by,

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 \, dx = 1 \ .$$

(16)

The wavelet coefficients are obtained by convolving the Ly$\alpha$ flux ($F$) with Morlet as,

$$a_n(x) = \int_{-\infty}^{\infty} F(x') \, \Psi(x - x') \, dx'$$

(17)

The wavelet power is then given by $A_n(x) = |a_n(x)|^2$. Following Lidz et al. (2010), we smooth the wavelet power on scales of $L = 1000 \text{ km s}^{-1}$ to avoid noisy excursions in wavelet power

$$A_{L,n}(x) = \frac{1}{L} \int_{-\infty}^{\infty} \Theta(|x - x'|; L/2) \, A_n(x') \, dx'$$

(18)

Figure 8. Top left panel and bottom left panels are similar to the corresponding panels in Fig. 7 but for the FPDF statistics. Unlike Fig. 7, the uncertainty in top (gray shaded region) and bottom (red stars with error bars) left panel has contribution from the sample variance and finite noise added to the spectra. The green shaded region in the bottom panels represents sample variance for GADGET-3 model calculated using noise free spectra (i.e., SNR= $\infty$). The FPDF is compared in the range $0.1 \leq F \leq 0.9$ (shown by magenta dashed vertical lines) since the flux near continuum and saturated region is usually affected by observational systematics (see section 4.2 for details). The green dashed dot vertical line shows the mean flux for GADGET-3 model. We present the results for 4 redshifts whose values are mentioned in the corresponding panel. All the results are presented for SNR = 25, $\Gamma_{12} = 1$ and for mock sample path length of 1000 $h^{-1}$ cMpc (corresponding to $X \sim 5.35$ at $z = 3$).

Figure 9. Comparison of FPS obtained from GADGET-3, G2-LTF and G2-HTF models. The symbols and line styles are same as in Fig. 8. The FPS from G2-LTF and G2-HTF model is consistent within 5 percent with that from GADGET-3 model (see section 4.3 for details). The magenta dashed vertical line shows the wavelet scale used in section 4.4.
where \( \Theta([x - x']; L/2) \) is the top-hat filter. It is important to note that wavelet power is anti-correlated with \( T_0 \), i.e., the wavelet power is smaller for higher \( T_0 \) and vice-versa.

Fig. 10 shows the comparison of the PDF of the smoothed wavelet power \( (A_{L,n}) \) (hereafter wavelet PDF) from the three models. As the TDR parameters \( T_0 \) and \( \gamma \) evolve with redshift, the peak and amplitude of the wavelet PDF also evolves accordingly. The bottom panels in Fig. 10 show that the wavelet PDF for G2-HTF model is within sample variance (green shaded region) and within 18 percent (red stars with errorbars) to that from the GADGET-3 model at all redshifts. In contrast, the wavelet PDF is systematically shifted to larger values at higher redshifts for the G2-LTF model as compared to the GADGET-3 model even if we use a factor \( \sim 0.5 \) (corresponding to best fit value \( A_{L,n} \)) (hereafter wavelet PDF) for G2-LTF model as compared to gadget-3. However, wavelet power in G2-LTF model is consistently lower than that from GADGET-3 model at higher redshifts (\( z \geq 3.5 \), difference \( \sim 7 \) percent). This is because the wavelet scale used in our analysis (\( s_n = 35 \text{ km s}^{-1} \)) corresponds to \( k \sim 0.18 \text{ s km}^{-1} \) shown by magenta dashed vertical line in Fig. 9. At this scale, the G2-LTF FPS has larger power as compared to GADGET-3 FPS due to the difference in density evolution and thermal history of the particles. Thus corresponding wavelet power is also larger for G2-LTF model as compared to GADGET-3 model. Note that the wavelet power is still large in G2-LTF model even if we use a factor \( \sim 2 \) higher \( T_0 \) (corresponding to best fit value for G2-LTF model see section 4.9).

Due to such systematics, the inferred \( T_0 \) from G2-LTF model (or models in which Jeans smoothing effect from thermal history are not accounted for) would be larger at higher redshift and may lead to a misinterpretation of earlier He II reionization. On the other hand G2-HTF model though approximate in computing the Jeans smoothing does produce consistent results with that from GADGET-3. Thus the inferred \( T_0 \) from G2-HTF model doesn’t seem to be skewed by any systematic discussed above.

### 4.5 Curvature statistics

Similar to the wavelet analysis, Becker et al. (2011) introduced a curvature statistics to measure the amount of small-scale structure in the Lyα forest. The curvature \( \kappa \) is defined as,

\[
\kappa \equiv \frac{F''}{1 + (F')^2}^{3/2}
\]

where \( F' \) and \( F'' \) are first and second derivative of Lyα transmitted flux respectively. This statistics is suitable for obtaining the IGM temperature at characteristic overdensity which is found to be an almost one-to-one function of the mean curvature regardless of \( \gamma \) (Becker et al. 2011; Boera et al. 2014; Padmanabhan et al. 2014, 2015; Upton Sandebeck et al. 2016). Following the earlier works, Padmanabhan et al. (2015) have shown that the mean and the percentiles of the curvature distribution function can be used to obtain constraints on the TDR. The comparisons of curvature PDF from three models are shown in Fig. 11. The curvature PDF from G2-HTF model is within 10 percent to that from GADGET-3 model at all redshifts. On the other hand, curvature is systematically more at higher redshifts for G2-LTF models than those of GADGET-3 models and the resid-

---

**Table 3.** Median curvature in 4 different redshifts bins for the GADGET-3, G2-LTF and G2-HTF models.

| \( z \) | GADGET-3 | G2-LTF | G2-HTF |
|---|---|---|---|
| 2.5 | \(-3.35 \pm 0.57\) | \(-3.35 \pm 0.58\) | \(-3.35 \pm 0.58\) |
| 3.0 | \(-3.32 \pm 0.60\) | \(-3.29 \pm 0.61\) | \(-3.31 \pm 0.60\) |
| 3.5 | \(-3.29 \pm 0.63\) | \(-3.22 \pm 0.65\) | \(-3.27 \pm 0.64\) |
| 4.0 | \(-3.28 \pm 0.69\) | \(-3.17 \pm 0.72\) | \(-3.26 \pm 0.70\) |

---

**Figure 10.** Comparison of wavelet PDF obtained from GADGET-3, G2-LTF and G2-HTF models. The symbols and line styles are same as in Fig. 8. The wavelet PDF from G2-HTF model is within 16 percent with that from GADGET-3 model while the wavelet PDF from G2-LTF is significantly different at \( z \geq 3 \) than GADGET-3 model.
uals are as high as $\sim 120$ percent. The median curvature values are summarized in Table 3. The median curvature in G2-LTF model is systematically higher than that from G2-HTF model at $z = 4$. This is because of small scale fluctuations in density field (and hence flux) are larger for the G2-LTF model (see Fig. 9) which affects the curvature measurement. Thus similar to wavelet analysis, the $T_0$ inferred from G2-LTF model using curvature statistics would be larger at higher redshifts ($z \geq 3.5$). Whereas $T_0$ inferred from G2-HTF model would be consistent with that from GADGET-3 model.

### 4.6 Column Density Distribution function (CDDF)

The next three statistics we will discuss treat the Ly$\alpha$ forest as a composition of discrete clouds. Each Ly$\alpha$ line is fitted with multiple Voigt profiles each having 3 free parameters column density ($N_{HI}$), linewidth ($b$) parameter and line center ($\lambda_c$). We used “Voigt profile Parameter Estimation Routine” (VIPER) to decompose the Ly$\alpha$ forest into multi-component Voigt profiles. More details can be found in (Gailkward et al. 2017b).

The CDDF, $f(N_{HI}, z)$, is a bivariate distribution that describes the number of absorption lines with column density in range $\log N_{HI}$ to $\log N_{HI} + d\log N_{HI}$ and redshift in the range $z$ to $z + dz$. The CDDF is sensitive to $\Gamma_{HI}$ (Schaye 2001; Shull et al. 2012; Kollmeier et al. 2014; Shull et al. 2015; Gailkward et al. 2017b; Viel et al. 2016; Gurvich et al. 2017). Fig. 12 shows the comparison of CDDF obtained from GADGET-3, G2-LTF and G2-HTF models. We fit the noise free spectra from GADGET-3 model and calculate the sample variance as shown by green shaded region in bottom panels. We have not accounted for the incompleteness of the sample in the calculation of redshift path length. This affects the shape of the CDDF at low $\log N_{HI}$ end. The CDDF from G2-HTF models agree within sample variance (1.25σ) and consistent (within 18 percent) with that from GADGET-3 at all redshifts. For $\log N_{HI} > 13.5$, the G2-LTF model is consistent within 10 percent at $z \leq 3.5$ except at $z = 4$ where the differences are large $\sim 40$ percent. In addition, as expected the G2-LTF model predicts more number of lines at lower column densities i.e., $\log N_{HI} < 13.5$. This is because the features arising from small scale density fluctuations of G2-LTF model (as seen in Fig. 6) is identified and fitted by VIPER as narrow lines with smaller column densities (for example see the region between 8 – 9 cMpc in Fig. 6). However, like other statistics the CDDF from G2-HTF model is in good agreement with that from GADGET-3 model.

### Table 4. Median $b$ parameter (with 68 percentile interval) in 4 different redshifts bins for the GADGET-3, G2-LTF and G2-HTF models.

| $z$ | GADGET-3 | G2-LTF | G2-HTF |
|-----|----------|--------|--------|
| 2.5 | $25.03^{+20.51}_{-9.54}$ | $22.53^{+23.46}_{-9.80}$ | $22.76^{+24.44}_{-8.84}$ |
| 3.0 | $26.61^{+30.14}_{-10.02}$ | $23.22^{+22.12}_{-9.65}$ | $24.69^{+27.01}_{-9.58}$ |
| 3.5 | $30.15^{+34.69}_{-11.96}$ | $25.69^{+25.23}_{-10.62}$ | $28.86^{+32.21}_{-11.54}$ |
| 4.0 | $35.43^{+37.28}_{-15.13}$ | $29.22^{+28.12}_{-12.83}$ | $34.21^{+36.47}_{-14.51}$ |

### 4.7 Linewidth ($b$ parameter) distribution function

The top panels in Fig. 13 show the linewidth distribution, which is sensitive to thermal history, pressure smoothing and unknown turbulent motions in the IGM (Schaye et al. 1999, 2000; McDonald et al. 2001; Davé & Tripp 2001; Gailkward et al. 2017b; Viel et al. 2016), from GADGET-3 (black circles), G2-LTF (blue squares) and G2-HTF (red stars) models for different redshift bins. The $b$ parameter distribution and residuals plotted in Fig. 13 is calculated from all the lines in sample with relative error in $b$ parameter smaller than 0.5. Again, unlike the G2-LTF model the linewidth distribution from G2-HTF model is consistent (within 18 percent uncer-
The incompleteness of the sample is not accounted for in the calculation of CDDF. The CDDF from G2-HTF is consistent within 18 percent with that from GADGET-3 (red stars with errorbars).

Figure 13. Comparison of $b$ parameter distribution obtained from GADGET-3, G2-LTF and G2-HTF models. The symbols and line styles are as in Fig. 8. Each panel is similar to that from Fig. 8 except the comparison is shown for $b$ parameter distribution from GADGET-3 (black circle), G2-LTF (blue squares) and G2-HTF (red stars) models. The $b$ parameter distribution and residuals are plotted from all the lines in sample with relative error in $b$ parameter less than 0.3 (the lines below completeness limit are also included). The green shaded region in the bottom panels show the sample variance on $b$ parameter distribution from GADGET-3 model. The $b$ parameter distribution is within ~18 percent agreement (at $b < 60$ km s$^{-1}$) with that from the GADGET-3 model.

The median $b$ increases from $z = 2.5$ to $z = 4.0$ in all models. The 68 percentile intervals around median are asymmetric because the $b$ distribution is skewed. Within 1σ errorbars the $b$ distribution from G2-LTF and G2-HTF model is consistent with that from GADGET-3 model.

4.8 $b$ versus log $N_{\text{HI}}$ scatter

The top panels in Fig. 14 show the $b$ versus log $N_{\text{HI}}$ scatter for GADGET-3 model. The color scheme represents density of points in logarithmic units. One way to assess the goodness of fit is to match the lower-envelope in $b$ versus log $N_{\text{HI}}$ plot. The lower-envelope in the $b$ versus log $N_{\text{HI}}$ plot has been...
used in the past to constrain the thermal history parameters $T_0$ and $\gamma$ (Schaye et al. 1999, 2000; McDonald et al. 2001). Following Garzilli et al. (2015), we obtain the lower-envelope by calculating the 10$^{th}$ percentile of $b$ values in log $N_{\text{HI}}$ bin. The middle and bottom panels show the comparison of the lower-envelope obtained from GADGET-3, G2-HTF and G2-LTF model. The lower envelope in $b$ versus log $N_{\text{HI}}$ plot from G2-HTF (red stars) is within sample variance and in 18 percent agreement with GADGET-3 (black circles). On the other hand, at $z = 3.5$ and 4.0, the lower-envelope from G2-LTF (blue stars) is consistently smaller at log $N_{\text{HI}} < 13.5$ than that from GADGET-3 model. This can again be attributed to extra absorption line features with smaller log $N_{\text{HI}}$ identified by the VIPER. Because of the systematically smaller lower-envelope in G2-LTF model at log $N_{\text{HI}} < 13.5$, $\gamma$ derived from G2-LTF model would be systematically larger than that from GADGET-3 model at $z \geq 3.5$.

To summarize the results presented in sections 4.1–4.8, we find that the Ly$\alpha$ statistics derived from G2-HTF model are within 20 percent (except for $b$ parameter distribution) to that from GADGET-3 model and within the sample variance for a path length of $\sim 1000 h^{-1}$ cMpc ($X \sim 5.35$ at $z = 3$).

4.9 $\chi^2$ analysis

The main motivation of this work is to develop the method to simulate the Ly$\alpha$ forest in order to efficiently explore the parameter space. Hence, it is important to show the accuracy of the method in recovering the astrophysical parameters. In this section we present the $\chi^2$ analysis and show the accuracy of our method in recovering the H$\text{I}$ photoionization rate $\Gamma_{\text{HI}}$.

The differences we see between the FPDF and the FPS from different models will have direct consequence in the derived parameter values like $\Gamma_{\text{HI}}$. To study this, we treat GADGET-3 as the reference model and see how the value of $\Gamma_{\text{HI}}$ is recovered when we use the G2-LTF and G2-HTF models. Note that we use the noise (corresponding to SNR=25) added Ly$\alpha$ transmitted flux in all the models. We vary $\Gamma_{12}$ in G2-HTF (or G2-LTF) model and calculate the FPDF and FPS. The $\chi^2$ between the FPDF / PS calculated from...
GADGET-3 and that from G2-HTF (or G2-LTF) model can be written in the matrix form as (for similar method see Gaikwad et al. 2017a),
\[ \chi^2(\Gamma_{12}) = \{P(\Gamma_{12}) - P_{\text{fid}}\} C^{-1} \{P(\Gamma_{12}) - P_{\text{fid}}\}^T \] (20)
where \( P_{\text{fid}} \) and \( P(\Gamma_{12}) \) is flux statistics (either FPDF or PS) from GADGET-3 and G2-HTF (or G2-LTF) model respectively. \( C \) is the covariance matrix as given in Eq. 13. Note that we use full covariance matrix for \( \chi^2 \) estimation.

\( \Gamma_{12} \) recovery: The panels in Fig. 15 show reduced \( \chi^2 \) as a function of \( \Gamma_{12} \) from G2-HTF (red stars) and G2-LTF (blue squares) model for four different redshifts. The black dashed vertical lines show the statistical uncertainty in \( \Gamma_{12} \) for G2-HTF model\(^9\). The \( \Gamma_{12} \) is recovered within 1\( \sigma \) (\( \Delta \Gamma_{12} \sim \pm 0.05 \), within 5 percent accuracy) in G2-HTF model at all redshifts whereas G2-LTF model fails to recover the \( \Gamma_{12} \) within 1\( \sigma \). The \( \Gamma_{12} \) recovered from G2-LTF model at \( z = 3.0 \) and \( z = 4.0 \) is higher by a factor of 1.7 and 2 respectively. The minimum \( \chi^2_{\text{min}} \) for G2-HTF model is also close to 1 indicating the goodness-of-fit. Note that the analysis above is done assuming HM12 UVB, SNR = 25 and also close to 1 indicating the goodness-of-fit. Note that we calculate the reduced \( \chi^2 \) respectively. The minimum \( \chi^2_{\text{min}} \) for G2-LTF model can also be written in the matrix form as (for similar method see Gadget-3) and that from G2-HTF (or G2-LTF) model can also be written in the matrix form as (for similar method see Press et al. 1992).

\( \Gamma_{12} \) recovery: The panels in Fig. 15 show reduced \( \chi^2 \) as a function of \( \Gamma_{12} \) from G2-HTF (red stars) and G2-LTF (blue squares) model for four different redshifts. The black dashed vertical lines show the statistical uncertainty in \( \Gamma_{12} \) for G2-HTF model\(^9\). The \( \Gamma_{12} \) is recovered within 1\( \sigma \) (\( \Delta \Gamma_{12} \sim \pm 0.05 \), within 5 percent accuracy) in G2-HTF model at all redshifts whereas G2-LTF model fails to recover the \( \Gamma_{12} \) within 1\( \sigma \). The \( \Gamma_{12} \) recovered from G2-LTF model at \( z = 3.0 \) and \( z = 4.0 \) is higher by a factor of 1.7 and 2 respectively. The minimum \( \chi^2_{\text{min}} \) for G2-HTF model is also close to 1 indicating the goodness-of-fit. Note that the analysis above is done assuming HM12 UVB, SNR = 25 and also close to 1 indicating the goodness-of-fit. Note that we calculate the reduced \( \chi^2 \) respectively. The minimum \( \chi^2_{\text{min}} \) for G2-LTF model can also be written in the matrix form as (for similar method see Press et al. 1992).

4.10 Effect of different thermal history

The comparison between different models discussed in sections 4.1-4.9 has been performed for the ionization and heating rates from HM12 UVB model. It is, however, important to validate our method for different UVB models where the thermal history is significantly different from that in the case of the HM12 UVB. In Figs. E2-E9, we validate our method (for G2-HTF model) for a UVB models in which \( T_0 \) is increased by a factor of \( \sim 2 \) while \( \gamma \) remains same at all redshifts (see Appendix E for details). We find that the statistics derived from the G2-HTF model is again consistent within 20 percent to that from GADGET-3 model for such a different thermal history. Thus for a range of physically motivated photo-heating rates from UVB calculations (such as Khaire & Srianand 2015a,b), we can easily probe the \( T_0 - \gamma \) parameter space and calculate Ly\( \alpha \) flux in G2-HTF model without performing full GADGET-3 simulation. Therefore our method can be a good first step to narrow down the parameter space before confirming the best fit parameter with GADGET-3 simulation.

4.11 Computational gain

We now highlight the advantages of using our method for simulating Ly\( \alpha \) forest:

- **Efficiency**: Table 6 summarizes the CPU time consumption in various parts of the code. Significant fraction of time is spent in evolution of \( \Delta, v \) and \( T \) in both codes. However, unlike GADGET-3 we need to evolve \( \Delta, v \) and \( T \) in G2-HTF (or G2-LTF) only once. To vary astrophysical parameters in G2-HTF, we just need to vary UVB in cite. This allows one to probe \( T_0 \) and \( \gamma \) parameter space efficiently. For example, the time (per core) required to simulate Ly\( \alpha \) forest for 10 different UVB in GADGET-3 is \( \sim 67 \) days whereas for GADGET-2 is \( \sim 8 \) days.

- **Accuracy**: We have shown that our method (running GADGET-2 with a high temperature floor and post-process using cite) produces statistical distributions that are consistent within sample variance (calculated from mock sample path length of 1000h\(^{-1}\) cMpc or \( X \sim 5.35 \) at \( z = 3 \)) and within 20 percent to those obtained with GADGET-3. In particular, our method is accurate within 5 percent with that from GADGET-3 in recovering \( \Gamma_{12} \).

- **Flexibility**: In addition to HM12, it is straightforward to incorporate other UVB such as Faucher-Giguère et al. (2009); Khaire & Srianand (2015a,b) in cite and evolve the temperature without performing full hydrodynamic simulation. cite can be run in either equilibrium or non-equilibrium ionization evolution mode. It is easy to incorporate cooling due to metals in cite by changing cooling rate tables (Wiersma et al. 2009; Gaikwad et al. 2017a, for similar analysis)\(^{11}\).

Thus our method (though approximate) is efficient, flexible and sufficiently accurate to explore a large parameter space which otherwise would be more time consuming with self-consistent simulations like GADGET-3. However in practice, while constraining astrophysical parameters from observations, we propose to use the method in 3 steps (i) use

---

\(^9\) Under the assumption of normal distribution, the statistical uncertainty corresponds to \( \chi^2 = \chi^2_{\text{min}} + \Delta \chi^2 \) where \( \Delta \chi^2 = 1 \) (Press et al. 1992).

\(^{10}\) The \( \chi^2_{\text{min}} \) increases by \( \sim 60 \) percent for SNR=\( \infty \) as compared to SNR= 15 due to smaller errorbars in earlier.

\(^{11}\) http://www.strw.leidenuniv.nl/WSS08/
Figure 15. Different panels show the recovery of $\Gamma_{\rm HI}$ for different redshift (given in each panel) using FPDF and FPS statistics. The combined (for FPDF and PS) reduced $\chi^2$ as a function of $\Gamma_{12}$ for G2-LTF (blue squares) and G2-HTF (red stars) model is shown in each panel. GADGET-3 is used as the reference model with $\Gamma_{12} = 1$. The $\chi^2$ is calculated between statistics from GADGET-3 and G2-LTF or G2-HTF models (see Table 5). The 1σ statistical uncertainty on the recovered $\Gamma_{12}$ for G2-HTF model is indicated by black dashed vertical lines. The above analysis is done for SNR = 25 and for mock sample path length of 1000$h^{-1}$ cMpc (corresponding to $X \sim 5.35$ at $z = 3$).

Table 5. Reduced $\chi^2$ between G2-LTF, G2-HTF model and reference model GADGET-3 for different statistics.

| Statistics\(^1\)       | $z = 2.5$ | $z = 3.0$ | $z = 3.5$ | $z = 4.0$ |
|-------------------------|-----------|-----------|-----------|-----------|
|                         | G2-LTF    | G2-HTF    | G2-LTF    | G2-HTF    |
| Density Power spectrum ($\delta$) | 0.41 | 0.98 | 0.67 | 0.79 | 0.82 | 0.60 | 1.21 | 0.58 |
| FPDF                    | 1.26      | 0.76      | 1.41      | 0.80      | 1.79      | 0.76      | 1.87      | 0.81 |
| FPS                     | 0.50      | 0.36      | 0.68      | 0.25      | 1.82      | 0.22      | 3.23      | 0.45 |
| Wavelet PDF             | 0.20      | 0.42      | 0.60      | 0.17      | 4.46      | 0.27      | 12.84     | 0.74 |
| Curvature PDF           | 0.30      | 0.19      | 1.34      | 0.49      | 6.81      | 0.82      | 17.07     | 0.78 |
| CDDF                    | 2.84      | 1.20      | 2.85      | 1.07      | 4.51      | 0.47      | 4.48      | 0.41 |
| $b$ parameter distribution | 4.38      | 1.04      | 6.09      | 0.75      | 4.56      | 0.42      | 2.18      | 0.73 |
| $b$ vs log $N_{\rm HI}$ correlation | 0.70      | 0.54      | 1.27      | 0.46      | 1.35      | 0.44      | 1.55      | 0.62 |

\(^1\) For a given redshift, all the astrophysical parameters ($T_{\alpha}, \gamma_{\rm HI}$) are same for G2-LTF, G2-HTF and GADGET-3 models. Reduced $\chi^2$ is calculated using full covariance matrix for FPDF and FPS. However, for other statistics we used diagonal elements of the covariance matrix as off diagonal elements are noisy.

G2-HTF model to explore large parameter space and obtain the best fit parameters with corresponding statistical uncertainty, (ii) run GADGET-3 simulation with best fit parameters (and also for parameters with 1σ deviation) and (iii) check if the statistics derived from data are consistent with those derived from GADGET-3 model with best fit parameters.

Thus our method, while not a substitute for the full hydrodynamical simulation like GADGET-3, provide an efficient and reasonable accurate tool to explore a large parameter space which otherwise require large resources and computational time. A possible way to make use this method would be to narrow down the parameter space in the first step before confirming the best fit parameters with a full GADGET-3 simulation.

5 SUMMARY

With the advent of high quality observations, an efficient method to simulate the Ly$\alpha$ forest would be useful for parameter estimation. Current state-of-art simulations like GADGET-3, though reproduce observational properties of Ly$\alpha$ forest very well, are computationally expensive for large parameter space exploration. As part of our ongoing effort, we have developed a post processing module for GADGET-2 called “Code for Ionization and Temperature Evolution” (CITE). In Gaikwad et al. (2017a), we have shown that the predictions of our low redshift simulations match well with other existing hydrodynamical simulations and estimated $\Gamma_{\rm HI}$ at $z < 0.5$ and associated uncertainties using extensive exploration of the parameter space.

For the resolution used in the above study (gas particle mass $\delta m = 1.26 \times 10^7 h^{-1} M_{\odot}$, pixel size $\delta x \sim 48.8 h^{-1}$ ckpc), the pressure smoothing of baryons may not be a major issue. However, for studying the high-$z$ Ly$\alpha$ forest one usually uses higher resolution echelle data. When we use appropriate high resolution (gas particle mass $\delta m = 1.01 \times 10^8 h^{-1} M_{\odot}$, pixel size $\delta x \sim 9.77 h^{-1}$ ckpc) simulation boxes, we notice that the density ($\Delta$) and velocity ($v$) fields are smoother for GADGET-3 as compared to those from GADGET-2. This is because the temperature and ionization state of the SPH particles in GADGET-2 is not calculated self-consistently (photo-heating and radiative cooling terms are not accounted for). In this work we show that by running a GADGET-2 simulation with elevated temperature floor (i.e., $T \sim 10^4$ K) and using local Jeans smoothing we are able to appreciably overcome the above mentioned shortcomings of our method in the high resolution simulations.

The basic idea is to apply additional smoothing in GADGET-2 by a local Jeans length at the epoch of our in-
Table 6. Consumption of CPU time (in hours) per core for various tasks of the code for a cosmological run from $z = 99$ to $z = 2.0$

| Step | Description* | gadget-3 | G2-HTF |
|------|---------------|----------|--------|
| 1    | $\Delta V$ and T Evolution | 156      | 108    |
| 2    | cite (T Evolution)$^b$ | –        | 3.5    |
| 3    | Grid calculation$^c$ | 3.5      | 4      |
| 4    | GASS$^d$      | 1        | 1      |
| -    | Total time to run | 1695     | 193    |
| 10 UVB model$^e$ | (67 days) | (8 days) |

* The analysis is done using 256 core on IUCAA PERSEUS cluster.

$^b$ cite evolves the temperature of the SPH particles from $z = 6.0$ to $z = 2.0$. Temperature is evolved internally in gadget-3.

$^c$ We used modified smoothing kernel for G2-HTF or G2-LTF as given in Eq. 5. The time is given for 10240 random sightlines through simulation box.

$^d$ We apply TDR as given in Eq. 10 for G2-HTF and G2-LTF models. The numbers are given for total $10 \times 2048$ simulated Ly$\alpha$ forest spectra. We splice 5 sightline to cover redshift path length for a single spectra.

$^e$ The total time required to run 10 UVB model for gadget-3 is sum of time consumed by steps 1, 3 and 4 (i.e. $160.5 \times 10$ hours). Unlike gadget-3, step 1 is performed only once for G2-HTF or G2-LTF models. For different UVB models, we follow step 2-4 in the post-processing stage. Hence the total time required to run 10 UVB model for G2-LTF or G2-HTF model is (108 hours + 8.5 hours $\times 10 = 193$ hours).

However, it is well known that the smoothing in gadget-3 is not only decided by the instantaneous density and temperature of the particles but also to some extent by the thermal history of the particles. To understand this, we perform three high resolution simulations (gas particle mass $\delta m = 1.01 \times 10^5 h^{-1} M_{\odot}$, pixel size $\delta x \sim 0.77 h^{-1} \text{ ckpc}$) with same initial conditions (i) G2-HTF: gadget-2 with low temperature ($T \sim 100$ K) floor in which local Jeans length is decided by instantaneous density and temperature and (ii) G2-HTF: gadget-2 with high temperature ($T \sim 10^4$ K) floor in which even the unshocked gas is evolved at with a pressure appropriate for a photoionized gas at $T = 10^4$ K and (iii) gadget-3: a reference model for comparison with G2-LTF and G2-HTF model.

For G2-LTF and G2-HTF models, we first estimate the temperature of SPH particles in gadget-2 using our code cite. We modify the smoothing kernel to account for pressure smoothing and estimated the density, velocity, and temperature on grids. We find that the line of sight density and velocity from our method matches well with that from gadget-3. We then compare our method for G2-LTF and G2-HTF models with that from gadget-3 simulation. The main results of our analysis are as follows:

- We obtain the evolution of thermal history parameters $T_0$ and $\gamma$ by estimating the temperature of the SPH particles from cite. We show that the redshift evolution of $T_0$ and $\gamma$ from G2-HTF and G2-LTF are in very good agreement with that from gadget-3. cite also provides us with enough flexibility to solve the non-equilibrium ionization evolution equation. The $T_0$ and $\gamma$ evolution for non-equilibrium case is considerably different ($T_0$ is larger by $\sim 60$ percent and $\gamma$ is smaller by 15 percent at $z = 3.7$) than that for equilibrium case. We show that the redshift evolution of $T_0$ and $\gamma$ for non-equilibrium case from our method is consistent with that from Puchwein et al. (2015, difference less than 2.5 percent).

- We generate the Ly$\alpha$ forest spectra by shooting random sightlines through simulation box in all the 3 models. The resulting Ly$\alpha$ forest spectra along sightline are remarkably similar in the G2-HTF and gadget-3 methods. However the Ly$\alpha$ forest spectra in G2-LTF model show more variation as compared to that from gadget-3. We compare the G2-LTF and G2-HTF with the gadget-3 model using 8 different statistics, namely: (i) 1D density field PS, (ii) FPDF, (iii) FPS, (iv) wavelet PDF, (v) curvature PDF, (vi) column density distribution function, (vii) linewidth distribution and (viii) $b$ vs log $N_{HI}$ correlation, at four different redshift $z = 2.5, 3.0, 3.5$ and $4.0$. Treating the gadget-3 model as the reference, we demonstrate that the H$\alpha$ photoionization rate ($\Gamma_{HI}$) can be recovered, using FPDF and FPS statistics, well within $1\sigma$ statistical uncertainty using the G2-HTF model. We find that the G2-HTF model is in general very good agreement (within $20$ percent and within $1\sigma$ sample variance calculated from $1000 h^{-1}$ cMpc or $X \sim 5.35$ at $z = 3$) with gadget-3 model at all redshifts. On the other hand G2-LTF model overestimates the $\Gamma_{HI}$ by a factor of $\sim 2$ at $z \geq 3.5$.

- Using enhanced HM12 photo-heating rates, we obtain a thermal history such that $T_0$ is increased by a factor of $\sim 2$. We show that our method for such significantly different thermal history is also consistent (in $1\sigma$) with gadget-3 simulation.

Our method to simulate the Ly$\alpha$ forest is computationally less expensive, flexible to incorporate changes in UVB, metallicity, non-equilibrium ionization evolution etc. and accurate (in recovering $\Gamma_{HI}$) to within $5$ percent. This method can be used in future more effectively to explore $T_0$, $\gamma$ and $\Gamma_{HI}$ parameter space and to simultaneously constrain these quantities from observations.

ACKNOWLEDGEMENT

All the computations are performed using the PERSEUS cluster at IUCAA and the HPC cluster at NCRA. We like to thank Volker Springel, Aseem Pananjape and Ewald Puchwein for useful discussion. We also thank the anonymous referee for improving this work and the manuscript.

REFERENCES

Abramowitz M., Stegun I. A., 1972, Handbook of Mathematical Functions

Arinyo-i-Prats A., Miralda-Escudé J., Viel M., Cen R., 2015, J. Cosmology Astropart. Phys., 12, 017

Babcock J. N., Peebles P. J. E., 1969, ApJ, 156, L7

Becker G. D., Bolton J. S., 2013, MNRAS, 436, 1023

Becker G. D., Bolton J. S., Haehnelt M. G., Sargent W. L. W., 2011, MNRAS, 410, 1096

Bi H., Davidsen A. F., 1997, ApJ, 479, 523

Bi H. G., Boerner G., Che Y., 1992, A&A, 266, 1

Boera E., Murphy M. T., Becker G. D., Bolton J. S., 2014, MNRAS, 441, 1916

Bolton J. S., Haehnelt M. G., 2007, MNRAS, 382, 325

Bolton J. S., Haehnelt M. G., Viel M., Springel V., 2005, MNRAS, 357, 1178
APPENDIX A: STAR FORMATION CRITERIA

To speed up the calculations in GADGET-3, we use QUICK_LYALPHA flag that converts gas with $\Delta > 1000$ and $T < 10^4$ K into stars. In order to study its effect on our method, we apply the same criteria to the G2-LTF model. The left, middle and right panels in Fig. A1 show the TDR for G2-LTF and G2-HTF do not employ star formation criteria. To speed up the calculations in gadget-3, we use the same criteria to the G2-HTF model.

APPENDIX B: CONVOLUTION OF SPH KERNEL WITH GAUSSIAN KERNEL

In this section, we show that the convolution integral in Eq. 7 can be recast in to an analytical form that is fast and easy to implement on computers. Let $W(r, h)$ be SPH kernel and $G(r, \sigma)$ be Gaussian kernel of pressure smoothing. Let $\tilde{W}(k, h)$ and $\tilde{G}(k, \sigma)$ be the Fourier transforms of $W(r, h)$ and $G(r, \sigma)$ respectively. The convolution of $W(r, h)$ with $G(r, \sigma)$ is given by,

\[
\tilde{W}(k, h) \times \tilde{G}(k, \sigma) \approx \int \frac{d^3k}{(2\pi)^3} \tilde{S}(k, h, \sigma) e^{i k \cdot r} \tilde{W}(k, h) \times \tilde{G}(k, \sigma) e^{i k \cdot r}
\]

Using the convolution theorem,

\[
W'(r, h, \sigma) = \int d^3x W(r_1, h) G(|r - x_1|, \sigma)
\]
LYALPHA setting of

\[ I_n(l_1, l_2) = \int_{l_1}^{l_2} t^n e^{-t^2} \, dt \]
\[ I_0(l_1, l_2) = \frac{\sqrt{\pi}}{2} \left[ \text{erf} (t) \right]_{l_1}^{l_2} \]
\[ I_1(l_1, l_2) = \left[ -\frac{e^{-t^2}}{2} \right]_{l_1}^{l_2} \]
\[ I_2(l_1, l_2) = \frac{1}{2} I_0(l_1, l_2) - \frac{1}{2} \left[ t e^{-t^2} \right]_{l_1}^{l_2} \]
\[ I_3(l_1, l_2) = I_1(l_1, l_2) - \frac{1}{2} \left[ t^2 e^{-t^2} \right]_{l_1}^{l_2} \]
\[ I_4(l_1, l_2) = \frac{3}{2} I_2(l_1, l_2) - \frac{1}{2} \left[ t^3 e^{-t^2} \right]_{l_1}^{l_2} \] (B7)

These integrals involve error function and hence need to be evaluated numerically. To speed up the calculations we used error function approximation of the following form (Abramowitz & Stegun 1972)

\[ \text{erf}(x) = 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-x^2} + \epsilon(x) \] (B8)

where,

\[ t = \frac{1}{1 + p x} \quad \text{and} \quad |\epsilon(x)| \leq 1.5 \times 10^{-7} \] (B9)

The small value of \( |\epsilon(x)| \) indicates that the uncertainty in error function approximation is negligible. The values of the constants are

\[ p = 0.3275911 \quad a_1 = 0.254829592 \]
\[ a_2 = -0.284496736 \quad a_3 = 1.42143741 \]
\[ a_4 = -1.453152027 \quad a_5 = 1.061405429 \]

Eq. 7 can also be solved numerically using a 3D FFT based method. However, we find that this method is computationally expensive for large number of particles. Fig. B1 shows a comparison of semi-analytical approximations (red stars, given in Eq. B4) with FFT based method (black dashed curve) for a particle. The SPH kernel and Gaussian kernel for this particle are shown by blue solid curve and red dash dot curves respectively. For visual purpose the Gaussian kernel is rescaled to fit the graph. Our method of approximation is accurate within 2 percent of FFT based method.

**APPENDIX C: JEANS LENGTH IN G2-HTF MODEL**

The G2-LTF and G2-HTF models are \textsc{gadget}-2 simulations with temperature floor \( \sim 100 \) K and \( \sim 10000 \) K re-
respectively. For G2-HTF model (high temperature floor) the density field would be smoother as compared to the G2-LTF model (low temperature floor). Hence pressure smoothing scale for G2-HTF model would be smaller than that for G2-LTF. Using Eq. 2, we can quantify the factor by which this pressure smoothing scale is smaller, \( L \propto T^{\frac{1}{2}} \) \( (C1) \)

where we have assumed that \( T \propto \Delta^{-1} \) (valid for \( \Delta \leq 10 \)). The median GADGET-2 temperature (i.e., before applying the cited) in the overdensity range \( \Delta \leq 10 \) for G2-LTF and G2-HTF model is \( T_{\text{G2-LTF}} \sim 4000 \) K and \( T_{\text{G2-HTF}} \sim 14000 \) K respectively. The ratio of pressure smoothing scale in G2-LTF and G2-HTF model is given by (assuming \( \gamma \sim 1.6 \)),

\[
\frac{L_{\text{G2-HTF}}}{L_{\text{G2-LTF}}} = \left( \frac{T_{\text{G2-HTF}}}{T_{\text{G2-LTF}}} \right)^{\frac{1}{2}} \approx 0.66 \tag{C2}
\]

It is clear from the above expression that if we use \( L_j \) as pressure smoothing scale for the G2-LTF model in Eq. 5 then we need to use 0.66 \( \times \) \( L_j \) for the G2-HTF model. Note that we do not modify this value (0.66) for a different thermal history corresponding to different UVB (see Appendix E).

**APPENDIX D: EFFECT OF DIFFERENT PATH LENGTH AND SNR**

**Effect of path length:** The analysis presented in the main paper assumes a default path length for the mock sample as 10000 km s\(^{-1}\) (cMpc) (corresponding to \( X \sim 5.35 \) at \( z = 3 \)). With the advent of surveys like KODIAQ (O’Meara et al. 2015, 2017, ~100 QSO around \( z \sim 2-3.5 \)), XQ-100 (López et al. 2016), the path length of Ly\( \alpha \) forest covered in QSO absorption spectroscopy is likely to increase by factor of 5. To study the effect of increase in path length, we generated a mock sample of 100 spectra at redshifts \( z = 2.5, 3.0, 3.5, 4.0 \). We repeated the procedure and generated 100 such mock samples. Thus in all we generated 5 \( \times \) 100 \( \times \) 100 = 50000 spectra at each redshift (see section 3) and followed the same procedure to calculate 8 different statistics and the associated uncertainty (see section 4). We find that the 1\( \sigma \) uncertainty (similar to grey shaded region in Fig.7 to Fig. 14) is decreased by \( \sim 12 \) percent but it is still dominated by sample variance. The residuals are typically less than 20 percent for G2-HTF model. We are also able to recover the \( \Gamma_{\text{HI}} \) (using G2-HTF model) within accuracy of \( \sim 5 \) percent although the reduced \( \chi^2 \) is slightly large (\( \sim 1.2 \)) in this case due to smaller errorbars. Thus increase in path length does not affect the results of the work presented earlier.

**Effect of SNR:** Table D1 shows the recovery of the \( \Gamma_{\text{12}} \) within 1\( \sigma \) statistical uncertainty from G2-HTF model assuming GADGET-3 as a fiducial model (fiducial \( \Gamma_{\text{12}} = 1 \)) for different SNR. The path length of mock sample is 10000 km s\(^{-1}\)cMpc (see section 3 for details). The values in brackets of Table D1 indicate the reduced \( \chi^2 \) corresponding to best fit \( \Gamma_{\text{12}} \). At all redshifts, the G2-HTF model is able to recover the \( \Gamma_{\text{12}} \) within 1\( \sigma \) statistical uncertainty. However, the \( \chi^2_{\text{of}} \) is large for high SNR. This is because the differences between GADGET-3 and G2-HTF model are significant as SNR increases. But even in the case of high SNR, the reduced \( \chi^2 \) is close to 1 indicating the goodness of fit.

**APPENDIX E: Ly\( \alpha \) FLUX STATISTICS COMPARISON FOR DIFFERENT THERMAL HISTORY**

In order to explore the effect of difference in thermal history, we follow Becker et al. (2011) and modify the photo-heating rates of species \( i = \text{HI}, \text{He} \text{I}, \text{He} \text{II} \) as \( \epsilon_i = a \times \phi_{\text{HM12}} \) where, \( \phi_{\text{HM12}} \) is HM12 photo-heating rates of specie \( i \). We choose \( a = 2.933 \) such that the \( T_0 \) is increased by factor of \( \sim 2 \) while \( \gamma \) remains same at all redshifts. With this updated photo-heating rates we perform a GADGET-3 (with QUICKLYALPHA flag) and G2-HTF simulation with the initial conditions same as described in §2. It is important to emphasize here that we do not perform a GADGET-2 simulation again, rather we only modify the HM12 photo-heating rates while running CITE in the post-processing stage on the same simulation run earlier. Note that at the initial redshift \( z = 6 \), we use \( T_0 = 14543 \) K and \( \gamma = 1.51 \) in CITE consistent with GADGET-3 for enhanced HM12 photo-heating rates at that redshift. Fig. E1 shows comparison of Ly\( \alpha \) flux from GADGET-3 and G2-HTF model for enhanced HM12 photo-heating rates. The flux from the two models match very well with each other. We also calculate the line of sight DPS, FPDF, FPS, wavelet PDF, curvature PDF, CDDF, b parameter distribution and b vs log \( N_{\text{HI}} \) distribution for these models.

Figs. E2-E9 show comparison of different statistics for GADGET-3 and G2-HTF model with enhanced HM12 UVB. Since the gas ionized by the enhanced HM12 UVB is at higher temperature (by a factor of \( \sim 2 \)) as compared to the models using HM12 UVB radiation, one would expect to see the differences in the statistics. We notice that the G2-HTF model residuals for FPDF are slightly large in enhanced HM12 UVB (\( \sim 20 \) percent, see Fig. E3) as compared to those from HM12 UVB (\( \sim 15 \) percent, see Fig. 8). Similar to HM12 G2-HTF model FPS (R \( \sim 5 \) percent, see Fig. 8), we also see the mismatch in enhanced HM12 G2-HTF model (R \( \sim 5 \) percent, Fig. E4) at scales in the range 220–650 kpc (k \( \sim 30 – 10^{4} \) Mpc\(^{-1}\)). The G2-HTF FPS in enhanced HM12 UVB is consistent within 1\( \sigma \) of the sample variance. Waveslet and curvature measurements are anti-correlated with temperature of the IGM. As expected, the comparisons of Fig. 10 with Fig. E5 and Fig. E6 with 11 show that the wavelet PDF and curvature PDF are consistently smaller for enhanced HM12 UVB (higher temperature) respectively. On the other hand the b parameters in enhanced HM12 UVB model (Fig. E8) are consistently larger as compared to that in HM12 UVB model (see Fig. 13) at a given redshift. This trend is furthermore clear from Table E1 (compare with Table 2, 3 and 4) where we tabulated the median value of wavelet, curvature and b parameter from GADGET-3 and G2-HTF model with enhanced HM12 UVB. The wavelet and curvature PDF in enhanced HM12 UVB for G2-HTF models are in agreement (1.1\( \sigma \)) with the sample variance. Similarly the CDDF, b parameter distribution and b vs log \( N_{\text{HI}} \) lower envelope from G2-HTF model are within sample variance with that from GADGET-3 model (see Fig. E7, E8 and E9).

We also calculated the reduced \( \chi^2 \) for different statistics. The reduced \( \chi^2 \) for these statistics are \( \sim 0.33, 0.67, 0.34, 0.41, 0.59, 0.53, 0.47 \) and 0.58 respectively. Using this model we are also able to recover \( \Gamma_{\text{12}} \).
Table D1. Recovery of $\Gamma_{12}$ within statistical uncertainty $d\Gamma_{12}$ for different SNR for G2-HTF model. The path length of mock sample is $1000h^{-1}$ cMpc ($X \sim 5.35$ at $z = 3$). GADGET-3 with $\Gamma_{12} = 1$ is assumed to be fiducial model.

| SNR | $z = 2.5$ | $z = 3.0$ | $z = 3.5$ | $z = 4.0$ |
|-----|-----------|-----------|-----------|-----------|
| 15  | 0.97 ± 0.07 (0.96) | 1.02 ± 0.05 (0.96) | 1.00 ± 0.05 (0.70) | 0.98 ± 0.05 (0.93) |
| 25  | 0.98 ± 0.07 (1.09) | 1.01 ± 0.05 (0.96) | 1.00 ± 0.05 (0.84) | 0.99 ± 0.05 (0.98) |
| 35  | 0.99 ± 0.07 (1.22) | 1.02 ± 0.05 (1.07) | 0.99 ± 0.05 (0.87) | 0.97 ± 0.04 (1.03) |
| 50  | 0.98 ± 0.07 (1.21) | 1.01 ± 0.05 (1.01) | 1.01 ± 0.05 (1.04) | 0.97 ± 0.04 (1.10) |
| 100 | 1.02 ± 0.08 (1.33) | 1.02 ± 0.05 (1.11) | 1.00 ± 0.05 (0.95) | 0.96 ± 0.04 (1.30) |
| Infinite | 1.04 ± 0.07 (1.61) | 1.02 ± 0.05 (1.12) | 0.98 ± 0.05 (1.13) | 0.95 ± 0.04 (1.30) |

Figure E1. Line of sight comparison of Ly$\alpha$ flux ($F$) for GADGET-3 (black solid line) and G2-HTF (red dashed line) simulation boxes at $z = 2.5$ along two different sightlines as shown in top and bottom panels. GADGET-3 simulation is performed with an enhanced photo-heating rates (see §4 for details). For G2-HTF model, we used enhanced HM12 photo-heating rates in cite. The Ly$\alpha$ flux $F$ along the sightline match very well for the two models. The Ly$\alpha$ flux is not convolved with any LSF and no noise is added to the flux.

within 1σ from FPDF and FPS statistics. This shows that G2-HTF model is consistent (within 20 percent) with GADGET-3 model for a significantly different thermal history.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.
Figure E2. Each panel is same as Fig. 7 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details).

Figure E3. Each panel is same as Fig. 8 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details).

Figure E4. Each panel is same as Fig. 9 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details).
Figure E5. Each panel is same as Fig. 10 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details). Median wavelet values for GADGET-3 and G2-HTF model are tabulated in Table. E1.

Figure E6. Each panel is same as Fig. 11 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details). Median curvature values for GADGET-3 and G2-HTF model are tabulated in Table. E1.

Table E1. Comparison of median wavelet power, curvature and $b$ parameters for the GADGET-3 and G2-HTF model with enhanced HM12 UVB. The errorbars correspond to 68 percentile around the median.

| Redshift | Median wavelet power GADGET-3 | Median wavelet power G2-HTF | Median curvature GADGET-3 | Median curvature G2-HTF | Median $b$ parameter GADGET-3 | Median $b$ parameter G2-HTF |
|----------|-------------------------------|-------------------------------|---------------------------|---------------------------|-------------------------------|-------------------------------|
| $z = 2.5$ | $-2.96 \pm 0.13$ | $-2.96 \pm 0.13$ | $-3.43 \pm 0.55$ | $-3.42 \pm 0.57$ | $32.97 \pm 32.36$ | $29.50 \pm 24.09$ |
| $z = 3.0$ | $-3.03 \pm 0.14$ | $-3.03 \pm 0.14$ | $-3.41 \pm 0.57$ | $-3.40 \pm 0.58$ | $34.22 \pm 33.99$ | $30.44 \pm 26.23$ |
| $z = 3.5$ | $-3.12 \pm 0.15$ | $-3.12 \pm 0.16$ | $-3.39 \pm 0.60$ | $-3.37 \pm 0.61$ | $36.15 \pm 36.64$ | $32.23 \pm 28.65$ |
| $z = 4.0$ | $-3.21 \pm 0.16$ | $-3.22 \pm 0.16$ | $-3.39 \pm 0.64$ | $-3.35 \pm 0.66$ | $36.76 \pm 37.21$ | $33.36 \pm 30.92$ |
Figure E7. Each panel is same as Fig. 12 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details).

Figure E8. Each panel is same as Fig. 13 except the comparison is shown for GADGET-3 and G2-HTF models with enhanced HM12 UVB (see the text for details). Median $b$ parameter values for GADGET-3 and G2-HTF model are tabulated in Table E1.
Figure E9. Each panel is same as Fig. 14 except the comparison in middle and bottom panel is shown for gadget-3 and G2-HTF models with enhanced HM12 UVB (see the text for details).