Dark matter and $B$-meson anomalies in a flavor dependent gauge symmetry

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A possibility of explaining the anomalies in the semileptonic $B$-meson decay $B \to K^*\mu\bar{\mu}$ has been explored in the framework of the gauged $U(1)_{\mu-\tau}$ symmetry. Apart from the muon anomalous magnetic moment and neutrino sector, we formulate the model starting with a valid Lagrangian and consider the constraints from the neutral meson mixings, the bounds on direct detection and the relic density of the bosonic dark matter candidate augmented to collider constraints. We search the parameter space, which accommodate the size of the anomaly of the $B \to K^*\mu\bar{\mu}$ decay, to satisfy all experimental constraints. We found the allowed region on the plane of the dark matter and $Z'$ masses is rather narrow compared to the previous analysis.

I. INTRODUCTION

A flavor dependent gauge symmetry is one of the promising candidates for new physics to describe the anomalies and other phenomenologies related with the flavor physics as well as to ensure the dark matter (DM) stability. In particular, the model of $U(1)_{\mu-\tau}$ provides several phenomenological prescriptions to resolve, namely, muon anomalous magnetic moment [1], experimental anomalies of semileptonic $B$-meson decay [2-3], neutrino sector [4-15], and other related topics [16-17]. Among them, the $B$-meson decay anomaly is a very challenging topic due to some indications of new physics have been suggested in the $B$ physics. For example, the angular observable $P_\ell$ in the decay of the $B$ meson $B \to K^*\mu^+\mu^-$ [28] has been measured with deviation of $3.4\sigma$ from the integrated luminosity of $3.0$ fb$^{-1}$ at the LHCb [18] which confirms the previous result with deviation of $3.7\sigma$ [19]. In addition, the same observable were measured by Belle collaboration [20, 21] with the deviation of $2.1\sigma$. Furthermore, an anomaly in the measurement of the ratio of branching fraction $R_K = BR(B^+ \to K^+\mu^+\mu^-)/BR(B^+ \to K^{+}\pi^+\pi^-)$ [22, 23] at the LHCb indicates a deviation of $2.6\sigma$ from the lepton universality predicted in the standard model (SM) [24].

Recently, the LHCb collaboration has also measured the ratio of $R_{K^*} = BR(B \to K^{\ast}\mu^+\mu^-)/BR(B \to K^{\ast}\pi^+\pi^-)$ which is found to be deviated from the SM prediction by $\sim 2.4\sigma$ as $R_{K^*} = 0.660^{+0.118}_{-0.070} \pm 0.024(0.685^{+0.113}_{-0.069} \pm 0.047)$ for $(2m_{\mu}^2) < q^2 < 1.1$ GeV$^2$ $(1.1$ GeV$^2 < q^2 < 6$ GeV$^2$) [25].

In previous study of Ref. [2], we have proposed the flavor dependent gauge symmetry. This has successfully explained the anomaly of $B \to K^*\ell^+\ell^-$ decay through generating the flavor violating $Z'$ boson interactions at one loop level. In the model, the only Wilson coefficient $C_9$ with $\mu$ and $\tau$ in the $B$ decays are generated via extra $Z'$ boson exchange, but this is explicitly not applicable for $B \to K^*e\bar{e}$ process [28]. We have also included a bosonic DM candidate and the vectorlike exotic quarks $Q'_u$, which are needed to generate the Wilson coefficient $C_9$ at one loop level. Thus the DM relic density [29] can be explained the measured anomalies in decay of $B \to K^{\ast}\ell^+\ell^-$ via s-channel process mediated by $Z'$ boson exchange [30, 31], where $Z'$ boson exchange can avoid a conflict with the constraints from the spin independent DM direct detection searches such as experiments of LUX [32] and XENON1t [33].

In this paper, we adopt the flavor dependent gauge symmetry model with more complete manner. We then perform a more detailed analysis in which we take into account the decay width of the $Z'$ boson which is related with the relic density of the DM, the constraints from the spin independent DM-nucleon elastic scattering cross section mediated by the vectorlike quarks $Q'_u$, which are needed to generate the Wilson coefficient $C_9$ at one loop level. Thus the DM relic density [29] can be explained the measured anomalies in decay of $B \to K^{\ast}\ell^+\ell^-$ via s-channel process mediated by $Z'$ boson exchange [30, 31], where $Z'$ boson exchange can avoid a conflict with the constraints from the spin independent DM direct detection searches such as experiments of LUX [32] and XENON1t [33].

This letter is organized as follows. In Sec. III we briefly introduce a valid Lagrangian of our model including the Higgs potential with the inert conditions, the $B$-meson anomaly, the collider physics, and the neutral meson mixings. A brief comment on how to directly produce the exotic quarks, and the experimental constraints of the DM are also presented. In Sec. III we present our numerical analysis results. Finally Sec. IV is devoted to the summary of our results and conclusions.

II. MODEL SETUP AND CONSTRAINTS

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1 Recently, a stringent constraint of the neutrino-trident process gives narrower parameter spaces of the extra gauge coupling ($g'$) and mass ($m_{Z'}$).
In this section, we present a formulation of our model. We briefly introduce a gauged $U(1)_{\mu-\tau}$ symmetry with three families of the vectorlike isospin doublet quarks $Q_a$, an isospin singlet inert complex boson $\chi$, and singlet boson $\varphi$ with nonzero vacuum expectation value (VEV) which is denoted by $\langle \varphi \rangle \equiv v_{\varphi}/\sqrt{2}$, where $H$ is the SM Higgs and its VEV is denoted by $\langle H \rangle \equiv v_H/\sqrt{2}$. The charge assignments of these new fermion and boson fields are summarized in Tables I and II, respectively.

A relevant Lagrangian under these symmetries is defined by

$$\mathcal{L} = y_{ij}\bar{L}_i H e_{Rj} + f_{ij} \bar{Q}_i H^\dagger Q_j + \lambda e_{ij} Q_i Q_j + h.c.,$$

where $a = 1, 2, 3$, and $i, j = e, \mu, \tau$ are generation indices. The quark sector $Q_L$ is same as the SM. Note that the charged-lepton sector is diagonal due to the $U(1)_{\mu-\tau}$ symmetry.

**Higgs potential** is given by

$$V = m_H^2 |H|^2 + m_{\varphi}^2 |\varphi|^2 + m_{\chi}^2 |\chi|^2 + \frac{1}{4} \lambda_H |H|^4 + \frac{1}{4} \lambda_{\varphi} |\varphi|^4 + \frac{1}{4} \lambda_{\chi} |\chi|^4 + \lambda_{H\varphi} |H|^2 |\varphi|^2 + \lambda_{H\chi} |H|^2 |\chi|^2 + \lambda_{\varphi\chi} |\varphi|^2 |\chi|^2,$$

where each field is defined to be

$$H = \left( \begin{array}{c} v_H \cos \theta_H \\ v_H \sin \theta_H \\ i z \end{array} \right), \quad \varphi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_\varphi + \varphi_R + i z' \end{array} \right),$$

where $h^+$, $z$, and $z'$ are respectively absorbed by the gauged bosons; $W^+$, $Z$, and $Z'$. After inserting the tadpole conditions for $H$ and $\varphi$, the CP-even mass matrix is obtained as

$$M_{\text{even}}^2 = \begin{pmatrix} v_H^2 \lambda_H & 2 v_H v_{\varphi} \lambda_{H\varphi} \\ 2 v_H v_{\varphi} \lambda_{H\varphi} & v_{\varphi}^2 \lambda_{\varphi} \end{pmatrix}.$$  

After the diagonalization, the mass eigenvalues and eigenstates are respectively given by

$$OM_{\text{even}}^2 O^T = \begin{pmatrix} c_\theta - s_\theta & s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} v_H^2 \lambda_H & 2 v_H v_{\varphi} \lambda_{H\varphi} \\ 2 v_H v_{\varphi} \lambda_{H\varphi} & v_{\varphi}^2 \lambda_{\varphi} \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} = \text{diag}(m_0^2, m_H^2),$$

where $A = v_H^2 \lambda_H + v_{\varphi}^2 \lambda_{\varphi}$, $\sin \theta(\cos \theta) = c_\theta(s_\theta)$, and $\tan \theta = t_\theta$ satisfies the following relation:

$$2 v_H v_{\varphi} \lambda_{H\varphi} t_\theta = 2 v_{\varphi}^2 \lambda_{\varphi} - 2 v_H v_{\varphi} \lambda_{H\varphi} t_\theta = 0.$$  

The mass eigenvalue of $\chi$ is given by

$$m_\chi^2 = v_{\varphi}^2 \lambda_{\varphi} + v_H^2 \lambda_{H\chi} + 2 m_\varphi^2.$$  

The inert conditions for $\chi$ are given by

$$\lambda_{H\chi} \lambda_{H\varphi} > 0, \quad m_{H\varphi}^2 > 0, \quad \lambda_{H\chi} \lambda_{H\varphi} > 4 \lambda_{H\varphi}^2.$$  

**$Z'$ boson**: We have $Z'$ boson from $U(1)_{\mu-\tau}$ gauge symmetry. After $\varphi$ develops its VEV, the mass of $Z'$ boson is generated as

$$m_{Z'} = q_{g'} g' v_{\varphi},$$

where $g'$ is a gauge coupling for $U(1)_{\mu-\tau}$. The gauge interactions among $Z'$ and fermions are given by

$$\mathcal{L} \supset g' Z'' \mu L_L \gamma^\mu L_L - \bar{L}_L \gamma^\mu L_L + \bar{\nu}_R \gamma^\mu \mu_R - \tau_R \gamma^\mu \tau + q_{g'} \sum_{a=1}^3 \bar{Q}_a^\dagger \gamma^\mu Q_a^\mu.$$  

Note that we ignore the kinetic mixing effects between $U(1)_{\mu-\tau}$ and $U(1)_Y$ by assuming the contributions is relatively very small.

**Explanation of the anomaly in $B \to K^{(*)}\ell^+\ell^-$ decay**: In our case, we have a Wilson coefficient $c_9$, which is associated with...
Since we consider no interactions between $mW$ we assume matrix, respectively, and 3-2 elements of the Cabibbo-Kobayashi-Maskawa (CKM) where $\Delta$ with Wilson coefficient $\Delta C_{9}^{\mu \mu}$ is given by $[2, 23]$

$$\Delta C_{9}^{\mu \mu} = \frac{q_{s}g^{2}}{(4\pi)^{2}m_{Z}^{2}} \sum_{a=1}^{3} f_{a}^{\dagger} f_{a} \int [dx] \ln \left( \frac{\Delta[M_{a}, M_{a}]}{\Delta[m_{\chi}, M_{a}]} \right),$$

which are shown in Fig. 2 and their formulas be lower than the experimental uncertainty if the Yukawa coupling $f$ is taken $O(0.1)$ and the constraint from $\Delta m_K$ tends to be stronger.

**Constraint from $K$-meson decay:** Here we discuss a constraint from $K$-meson decay. Considering a small effect of the charge parity (CP) violation emerges from new physics, the strongest bound is derived by the effective operator $\bar{c}c_{\mu}(1 - \gamma_{5})d(d).$ From this constraint, the effective mass bound suggests $30$ TeV $\lesssim \Lambda,$ where its effective operator is given by $\frac{q_{s}}{g'}(\bar{e}e_{\mu}P_{L}q)(\bar{\nu}\nu_{\mu}P_{L}l).$ We then can easily estimate the bound on $m_{Z'}$ in the effective operator analysis. Similar to the case of $b \to s\bar{\mu}\mu,$ our effective operator is defined as

$$\mathcal{L} = \frac{q_{s}g^{2}}{(4\pi)^{2}m_{Z}^{2}} \sum_{a=1}^{3} f_{a}^{\dagger} f_{a} \int [dx] \ln \left( \frac{xM_{a}^{2} + (y + z)m_{\chi}^{2}}{xM_{a}^{2} + (y + z)m_{\chi}^{2}} \right),$$

where $q = (u, d).$ It implies that we have the following constraint from the LHC:

$$\frac{m_{Z'}}{g'} \gtrsim 30 \left[ \frac{q_{s}}{(4\pi)^{2}} \sum_{a=1}^{3} f_{a}^{\dagger} f_{a} \int [dx] \ln \left( \frac{xM_{a}^{2} + (y + z)m_{\chi}^{2}}{xM_{a}^{2} + (y + z)m_{\chi}^{2}} \right) \right]^{1/2} \times \text{TeV}.$$ 

(18)

When we take a degenerate mass for $M_{a}$ as $1000$ GeV and $m_{\chi} = 100$ GeV, the constraint then gives

$$\frac{m_{Z'}}{g'} \gtrsim 0.45 \left[ \sum_{a=1}^{3} f_{a}^{\dagger} f_{a} \right]^{1/2} \text{TeV}.$$ 

(19)

This shows that the constraint can be easily avoided while the coupling $f_{a}$ is not too large.

**M – $\overline{M}$ mixing:** The neutral meson mixings also give the constraints of the parameter space. The neutral meson mixings are shown in Fig. 3 and their formulas be lower than the ex-
Constraints from direct production of $Q'$s: The exotic quarks $Q'$s can be produced in pair via QCD processes at the LHC. Each $Q'$ will then decay into $Q' \to q_i \chi$, where $q_i$ represents a quark with flavor $i$. Hence, searching for "${tt, bb, tj, bj, jj}$" signals will constrain our present model. The branching ratios for a particular quark flavor $i$ depend on the relative sizes of the Yukawa couplings, $f_{ij}$ and $f_{aj}$ with $a = 1, 2$. We then roughly estimate the lower limit on the mass of $Q'$ using the current LHC data for the s-quark searches $^{41,41}$, which indicates the mass should be larger than $\sim 0.5$-1 TeV which depends on the mass difference between $Q'$ and $\chi$. In our following analysis, we simply take the value of $M_a > 1$ TeV in order to satisfy this constraint.

Dark matter: In our model scenario, a complex scalar $\chi$ is considered as a DM candidate, since we have a remnant $Z_2$ symmetry after the $\mu - \tau$ symmetry breaking. The DM candidate and $Q'_i$ are odd under the $Z_2$ symmetry and the other particles are even. If the $M_a$ is heavier than the $m_\chi$, the DM candidate is a stable particle. The DM dominantly annihilates into the SM leptons via $\chi \chi \to Z' \to \mu^+ \mu^- (\tau^+ \tau^-)$, so that the DM in our model is naturally leptophilic. The relic density of the DM is given by

$$\Omega h^2 \approx \frac{1.07 \times 10^9}{\sqrt{g_*}(x_f)M_{Pl}J(x_f)[GeV]}, \quad (25)$$

where $g^*(x_f) \approx 25$, $M_{Pl} \approx 1.22 \times 10^{19}$, and $J(x_f)(\equiv \int_x^\infty \frac{d\xi}{\xi} (\sigma v_{rel}))$ is given by

$$J(x_f) = \int_{x_f}^\infty \frac{d\xi}{\xi} \frac{s - 4m_\chi^2(\sigma v_{rel})K_1(\frac{\xi}{m_\chi})}{16m^2_\chi [K_2(\xi)]^2},
$$

$$\Gamma_{Z' \to \mu^+ \mu^-} \approx \frac{g^4 m_{Z'}^2}{4\pi} + \frac{|q_i q_j|^2}{16\pi m_{Z'}^2}(m_{Z'}^2 - 4m_\chi^2)^{1/2}.$$  

With $s$ is a Mandelstam variable, and $K_{1,2}$ are the modified Bessel functions of the second kind of order 1 and 2, respectively. We expect $Z'$ decays into $\mu \mu, \tau \tau$, and $\chi \chi^*$ pairs. In our numerical analysis, we use the current experimental range for the relic density at $3\sigma$ confidential level $^{29}$: $0.11 \leq \Omega h^2 \leq 0.13$.

Direct detection of DM: The dominant elastic scattering cross section arises from the $Q'$ exchange process in Fig.3 and its effective Lagrangian of the component level is given by

$$\mathcal{L} \approx -i \sum_{i=1}^{3} \sum_{a=1}^{3} \frac{f_{ia} f_{ai}^*}{2M_a^2} \{[\bar{q}_i \gamma^\mu P_L q_i][\chi^* \frac{\partial}{\partial \mu} \chi], \quad (27)$$

where we use the following assumptions of the four transferred momentum $q^2 = (p_1 - p_2)^2 << M_a^2$ and the nucleus of a target almost stops (at rest frame): $\chi^*(p_1)(p_1 - p_2)\mu \chi(k_1) = \frac{i}{2} \chi^*(p_1)(P_1 + (k_1 - k_2))\mu \chi(k_1) \approx \frac{i}{2} \chi^*(p_1)(p_1 + k_1)\mu \chi(k_1) = \frac{i}{2} \chi^* \frac{\partial}{\partial \mu} \chi$, where the right and left sides correspond to the operators in momentum space and spacetime, respectively. We then straightforwardly define the

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3 We thank Alexander Lenz to bring up a new value of $f_{Ba}$, which includes a bag parameter dependence.

4 Without decay width of $Z'$, the cross section at around the pole of $2m_\chi = m_{Z'}$ is too large and the relic density would be underestimated.
DM-nucleon elastic scattering operator as follows:

\[ \mathcal{L}_N \approx -i \sum_{i,a=1}^{N} \sum_{i,j} f_{ia}^{\dagger} f_{ai} \left[ F_{q_i}^N \langle N(k_2) | \bar{N} \gamma^\mu N | N(p_2) \rangle - F_{A}^{q_i} \langle N(k_2) | \bar{N} \gamma^\mu \gamma_5 N | N(p_2) \rangle \right] \times (\chi^{+} \partial_{\mu} \chi). \]  

(28)

where we assume that this process is an elastic scattering, then the four transferred momentum is expressed by \( q^2 \approx 0 \), where \( p_1 \) and \( p_2 \) are the four momentum of the \( \chi^{+} \) and \( q_{i0} \sigma N \), respectively, and \( F_{A}^{q_i} \) are the form factors, which is taken from Ref. [43]. The squared matrix element is given by

\[ |M|^2 = \frac{1}{16} \sum_{i,a=1}^{N} \sum_{i,j} f_{ia}^{\dagger} f_{ai} \left( F_{q_i}^N \langle N(k_2) | \bar{N} \gamma^\mu N | N(p_2) \rangle - F_{A}^{q_i} \langle N(k_2) | \bar{N} \gamma^\mu \gamma_5 N | N(p_2) \rangle \right) \times |(\chi(p_1) | \chi^{+} \partial_{\mu} \chi | \chi(k_1))|^2 \approx 2m_N^2 m_i^2 \sum_{i=1}^{3} \sum_{a=1}^{3} \left( \left( f_{ia}^{\dagger} f_{ai} \right)^2 M_a^2 + \left( f_{ia}^{\dagger} f_{ai} F_{A}^{q_i} \right)^2 M_a^2 \right). \]  

(29)

Finally, the complete form of the DM-nucleon elastic scattering cross section is expressed by

\[ \sigma \approx \left( \frac{m_N m_i}{m_N + m_i} \right)^2 \frac{|M|^2}{32 \pi m_N^2 m_i^2} = \frac{1}{16 \pi} \left( \frac{m_N m_i}{m_N + m_i} \right)^2 \sum_{i=a=1}^{3} \left( \left( f_{ia}^{\dagger} f_{ai} \right)^2 M_a^2 + \left( f_{ia}^{\dagger} f_{ai} F_{A}^{q_i} \right)^2 M_a^2 \right), \]  

(30)

where \( \Sigma F_1 = 3 \) corresponds to the effective operator \((\bar{N} \gamma^\mu N)(\chi^{+} \partial_{\mu} \chi)\), \( \Sigma F_A = 0.49 \) corresponds to the effective operator \((\bar{N} \gamma^\mu \gamma_5 N)(\chi^{+} \partial_{\mu} \chi)\), and \( m_N \approx 0.939 \) GeV. The current experimental upper bounds for the cross section of the spin independent DM-nucleon elastic scattering are respectively \( \sigma_{exp} \leq 2.2 \times 10^{-46} \) \( \text{cm}^2 \) at \( m_N \approx 50 \) GeV for the LUX data [32], and \( \sigma_{exp} \leq 4.1 \times 10^{-47} \) \( \text{cm}^2 \) at \( m_N \approx 30 \) GeV for the XENON data [33]. In our numerical analysis, we conservatively restrict the LUX/XENON1T bounds for the whole range of the DM mass.

### III. NUMERICAL ANALYSIS

In this section, results for our numerical analysis are presented. In our analysis, we fix a parameter \( |q_{i0}| = 1 \) for simplicity. The ranges of the input parameters are set as follows:

\[ g' \in [0.1, 1], \quad f \in [0.001, \sqrt{4\pi}], \quad m_{Z'} \in [10, 300] \text{[GeV]}, \quad m_{\chi} \in [10, 150] \text{[GeV]}, \quad M_{\alpha} \in [1000, 3000] \text{[GeV]}, \]  

(31)

where the lowest DM mass 10 GeV is an assumption to satisfy a condition \((m_{\chi}/m_N)^2 < 1 \) in the cross section of relic density. In this calculation, we assume \( M_1 < M_2 < M_3 \), and \( m_{Z'} > m_{\chi} \). We then search the allowed region using the range of the input parameters listed above to satisfy all constraints, namely, \( M - M' \) mixing, the measure of the relic density of the DM, the spin independent DM-nucleon scattering cross section via \( Z' \) boson exchange, and the constraint of the LHC as well as to explain the anomaly of \( b \to s\bar{\mu}\mu \) decay.

Fig. 4 shows the allowed region on the plane of \( m_{\chi} \) and \( m_{Z'} \), where the blue, red, and green dots represent respectively the region corresponding to no constraint on \( \Delta C_0 \), i.e. \( -1.29 \leq \Delta C_0 \leq -0.87 \), and \( 3\sigma \) range \( -1.67 \leq \Delta C_0 \leq -0.39 \). Note that the lowest point of the maximum absolute value of \( f \) is of the order 0.1 for \( 1/(3\pi) \). The correlation between \( m_{\chi} \) and \( m_{Z'} \) in Fig. 4 comes from the closed resonant point of the relic density of the DM, when \( m_{\chi} \) is heavier. In the lighter region of \( m_{\chi} \), the allowed region becomes wider due to the larger cross section. At around the resonant region of 10 GeV \( \leq m_{\chi} \leq 40 \) GeV, there are no allowed region since the corresponding cross section is too large to satisfy the relic density. This clearly indicates that the mass ranges of the DM and \( Z' \) are respectively 10 GeV \( \leq m_{\chi} \leq 146 \) GeV and 10 GeV \( \leq m_{Z'} \leq 295 \) GeV, where the specific ranges mainly originate from the constraint of the relic density, although the lowest bound of DM mass 10 GeV comes from the lowest input parameter of the DM mass. The anomaly of the decay of \( b \to s\bar{\mu}\mu \) is well explained for the whole allowed region on the plane of \( m_{\chi} \) and \( m_{Z'} \).

In Fig. 5 we clearly show that the allowed regions on the planes of \( \Delta m_K - \Delta m_{B_s} \) (the left panel) and \( \Delta m_{B_d} - \Delta m_{D} \) (the right panel), where the color representation is similar as in Fig. 4. Fig. 5 indicates the allowed region for explaining the anomaly of \( b \to s\bar{\mu}\mu \), which tends to lie in the range of the experimental bounds in our parameter space. A branching ratio of BR \( (b \to s\gamma) \) is restricted to be 4.02 \times 10^{-4}, but the typical value is at most of the order 10^{-10}. Therefore, the present model clearly satisfies this constraint. Additionally, we found that the LHC constraint tends to be weaker than...
FIG. 4: The allowed region of $\Delta C_9$ on the plane of $m_\chi$ and $m_Z'$ satisfy the relic density $0.11 \sim 0.13$ and the neutral meson mixings, $b \to s\gamma$. The blue, red, and green dots are respectively the region of $\Delta C_9 \leq -5, -1.29 \leq \Delta C_9 \leq -0.87$ at 1$\sigma$, and $-1.67 \leq \Delta C_9 \leq -0.39$ at 3$\sigma$.

Apart from the muon anomalous magnetic moment and neutrino sector, we have formulated a model starting with a valid Lagrangian by considering the Higgs potential with the inert conditions, the Wilson coefficient for the decay of $B \to K^* \mu \bar{\mu}$, the collider physics, the neutral meson mixings, the bound on direct detection, and the relic density of a bosonic dark matter candidate. We have searched the parameter space, which explain the size of the anomaly of $B \to K^* \mu \bar{\mu}$, satisfying all constraints. We found that the allowed region on the plane of the DM and $Z'$ masses is narrower compared to the previous analysis in the heavier DM mass. This is expected due to the decay width of $Z'$ is rather large. On the other hand, in the lighter region of $m_\chi$, the allowed region becomes to be still wider because the cross section is larger. Moreover, there are no allowed region at around the resonant region of $10 \text{GeV} \lesssim m_\chi \lesssim 40 \text{GeV}$, since the corresponding cross section is too large to satisfy the relic density. The meson mixing of $\Delta C_9$ can be well tested in the future experiments, since the structure of the Yukawa couplings $f$ is the same as the one of $\Delta C_9$ of $B \to K^* \mu \bar{\mu}$. The absolute parameter of $f$ can naturally be estimated to explain $\Delta C_9$, and its minimum absolute is at most of the order $\sim 0.1$.

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IV. SUMMARY AND CONCLUSIONS

We have explored the possibility of explaining the experimental anomalies in the semileptonic decay of the $B$-meson, $B \to K^* \mu \bar{\mu}$, in the framework of the gauged $U(1)_{\mu - \tau}$ symmetry. With our present model, which is built in a more complete manner than previous model, we have performed a more detailed analysis by searching the allowed region from several present experimental constraints.

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FIG. 5: The allowed region on the $\Delta m_K - \Delta m_B^u$ plane (the left panel) and $\Delta m_{B_d} - \Delta m_{BD}$ plane (the right panel), where the color representations are similar as in Fig. [3].

FIG. 6: The nucleon-DM elastic scattering generated by the parameter space scanned. The current upper bound by XENON-1T is represented by the solid line.

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