Cross fingerings and associated intonation anomaly in the shakuhachi

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Abstract: Acoustical differences between normal and cross fingerings of the shakuhachi with five tone holes are investigated on the basis of the pressure standing wave along the bore and the input admittance. Cross fingerings in the shakuhachi often yield pitch sharpening in the second register, which is contrary to our conventional understanding of pitch flattening by cross fingerings and is called intonation anomaly. It is essential to identify and discriminate the input admittance spectra between the upper and lower bores on the basis of the standing-wave patterns. Spectrum (or mode) switching between both types of bores is a clue to the cause of the intonation anomaly. This is illustrated by considering stepwise shifts of tone holes while keeping the hole-to-hole distances fixed and by comparing the resulting switches in input admittance spectra. When spectrum switching occurs, docking of the upper and lower bores makes up a higher resonance mode throughout the whole bore and then leads to the intonation anomaly. This spectrum switching on the cross fingering is generalized as the diabatic transition (the Landau–Zener effect) in physics.

Keywords: Cross fingering, Intonation anomaly, Spectrum switching, Diabatic transition

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1. INTRODUCTION

Cross fingerings in woodwind instruments are very significant in musical expressions created by instrument players [1]. Tonal pitch, volume, and timbre are appreciably changed by cross (or fork) fingerings from those given by normal fingerings made of a lattice of open tone holes. However, as modern Western instruments have many tone holes (whose numbers in the modern clarinet, oboe, and flute are 24, 23, and 13, respectively), the attraction of cross fingerings is beginning to be lost. As a result, good opportunities to explore the acoustics of cross fingerings in woodwind instruments have unfortunately almost been missed. Our conventional understanding of the acoustics of cross fingerings is based on Benade’s and Nederveen’s textbooks [2,3] and on the work of Wolfe and Smith [4].

On the other hand, the Japanese longitudinal bamboo flute, shakuhachi, has only five tone holes (four on the front and one on the back). This means a decisive importance of cross fingerings in the playing of the shakuhachi. See Ref. [5] for a concise explanation of this instrument. When tone holes are successively opened from the bottom, D–F–G–A–C–D tones are emitted from a shakuhachi one shaku and eight sun (about 54 cm) long. Cross fingerings are used for A♭, B♭, etc.

A Japanese physicist, Torahiko Terada (1878–1935), first carried out an accurate measurement of the intonation of the shakuhachi [6]. He carefully measured pitch frequencies in the first and second registers for 32 fingerings, and directed attention to the octave balance. If his intonation table is extensively examined, it is known that there are many cases where cross fingerings cause pitch sharpening instead of pitch flattening. Unfortunately, his shakuhachi research ended with the measurement. The pitch sharpening due to cross fingerings is the reverse of our conventional understanding above [2–4]. Therefore, it may be called an intonation anomaly in the present paper.

Nederveen [3] briefly considered this pitch sharpening due to cross fingering in the second register on an old-model flute for A♯₅ with the fingering (●●●●●●), in which three holes were closed below the one opened for sounding the A₅. He also mentioned that a similar phenomenon could be observed on a modern Boehm flute. However, such a phenomenon in modern flutes has never been treated in scientific publications [5]. He explained this intonation anomaly by calculating the input admittance of a model tube (see Fig. A6.3 in Ref. [3]). Another familiar example of the intonation anomaly is D♯₆ on an alto recorder, which

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The shakuhachi used for measurement in a Japanese-style room (at about 23°C) was manufactured by one of the master makers, Ranpo Yokoyama (1911–1988), and was played by the first author. The tonal pitch is usually adjusted by changing the angle between the player’s head and the instrument. However, such pitch adjustment was not used and normal angle was kept in our measurement.

Long tones of a note were played about 10 times and recorded with a linear PCM recorder (Sony PCM-D50). Stable waveforms with a duration of about 1 s were recorded. The intonation anomaly was observed in the third register (symbol $\phi$ indicates the thumb hole that is slightly opened).

The aim of this study is to explore the mechanism of how the intonation anomaly occurs or does not occur upon cross fingering in the shakuhachi. An experimental approach of measuring the internal standing-wave patterns along the bore is introduced. However, prior to that measurement, the situations where the intonation anomaly occurs should be first described by measuring the blown frequencies, as presented in Sect. 2.

### Table 1: Playing frequencies of fingerings A to G. The underlined frequencies denote intonation anomalies.

| Fingering name | Resonance mode (Register) |
|---------------|---------------------------|
| A (Chi)       | 444 Hz, 898 Hz, 1,322 Hz, 1,903 Hz |
| B (Wu13)      | 433 Hz, 853 Hz, 1,475 Hz, 1,856 Hz |
| C (Wu3)       | 426 Hz, 820 Hz, 1,273 Hz, 1,834 Hz |
| D (Ha)        | 587 Hz, 1,170 Hz, (E$^b_6$) |
| E (Ha4)       | 569 Hz, 1,108 Hz, 1,928 Hz, (B$^6$) |
| F (Ha245)     | 591 Hz, 1,244 Hz, 1,852 Hz, (A$^6_3$) |
| G (Ha5)       | 600 Hz, 1,207 Hz, 1,828 Hz, (A$^6_3$) |

The intonation anomaly occurred in the third register as well as in the second register when cross fingering C was used. However, at the same time, fingering C gave a flat third register tone (E$^b_6$) and the intonation anomaly yielded $G^5_6$ instead of $E^6_6$. Cross fingering B gave the intonation anomaly in the third register, while it gave normal pitch lowering in the second register. The result on the first register of fingerings A, B, and C well reflected the conventional understanding of cross fingerings. In the fourth register, normal fingering A gave $A^6_6$ and cross fingerings B and C gave the same $A^6_6$ with lower frequencies. On fingerings D to G, more anomalies were observed. The intonation anomaly occurred even in the first register, although it was very weak and gave tones of $D_3$. The second register brought various degrees of intonation anomaly to all cross fingerings E, F, and G. It was difficult to play the third register on fingering D. However, it may
be assumed that the third register on normal fingering D produces $A_6$ of about 1,760 Hz (this is the case shown in Ref. [7]). Therefore, $A'_6$ in the third register on cross fingerings F and G may be considered as the intonation anomaly (tone $B_6$ on fingering E is related with another third mode at around 1950 Hz on fingering D, and the $B_6$ is not an anomaly) (see Ref. [7] and Sect. 3.2). Interestingly, the frequency (1,852 Hz) of this $A'_6$ in the third register on fingering F was very close to that (1,856 Hz) in the fourth register on fingering B, and the same relation held between fingerings G (1,828 Hz) and C (1,834 Hz). Such an interrelation mainly comes from the specific positions of the third and fifth tone holes as explained in Sect. 2.

3. MEASUREMENT OF INTERNAL STANDING-WAVE PATTERNS

3.1. Setup

A system for measuring the pressure standing waves in the same shakuhachi as used in the playing frequency measurement is sketched in Fig. 2. The shakuhachi was resonated by an external driver consisting of a loudspeaker and an exponential horn that was designed to have its cutoff frequency at about 200 Hz. The horn output end with an inner diameter of 16 mm and an outer diameter of 22 mm was placed in front of the instrument bore bottom (17.8 mm in inner diameter) with a clearance of over twice the end correction. A sinusoidal signal with a resonance frequency was produced by a function generator (NF Electronic Instruments 1930 Wide Function Synthesizer) and amplified by an audio amplifier (Sansui AU-a507MRV). Resonance frequencies of a fingering were measured prior to the standing-wave measurement. The acoustic pressure in the bore was detected by a probe microphone (Brüel & Kjær type 4182) connected to an amplifier (B&K type 2609). The probe tube had a length of 570 mm. It was made of a commercial SUS304 seamless tube (Kuroiwa Stainless Co., G-16; inner diameter of 1.25 mm and outer diameter of 1.61 mm). This long tube was fitted to a B&K probe tube with a length of 50 mm and an outer diameter of 1.24 mm.

Although the shakuhachi bore is usually bent below the first tone hole, that of this instrument is almost straight. Therefore, the long probe tube above was easily applied to the measurement. The embouchure hole was almost stopped by inserting a urethane board (10 mm thick), leaving a small opening near the embouchure edge to mimic the actual playing situation. A tiny hole was opened at the center of this board and the probe tube was introduced into the bore. The acoustic pressure of the standing wave was measured in steps of 10 mm from the embouchure end ($x = 0$ cm) to the position ($x = 55$ cm) 10 mm from the bore bottom ($x = 54$ cm). When closing a tone hole, a sheet of rubber with appropriate hardness and thickness covered the hole and then it was secured by winding vinyl plastic tape around the instrument outer surface.

A brief check of the receiving level of our long probe tube was carried out at $x = 3$ cm in the bore by comparing with the level of a normal B&K probe tube of 50 mm. The receiving level was decreased by about 2, 4, 6, and 8 dB at 400, 1,000, 1,400, and 2,000 Hz, respectively. The additional attenuation when using our probe tube was not so strong, and then this probe tube was successfully applied to measure the standing-wave patterns at room temperature of 26.7–27.0°C. The room for the measurement was treated to reduce reverberation and noise.

3.2. Results for Fingerings D, E, F, and G

Let us first consider (and define) the resonances produced by a bore with a single open tone hole (cf. fingerings C and G in Fig. 1) before describing the results. The open tone hole partly divides the bore into two, the upper and lower bores, each of which has an open end and an open tone hole. These bores may be regarded as passive resonance systems, and may produce the upper-bore and lower-bore resonance modes, respectively. Moreover, as the frequency is greatly increased (above the cutoff frequency defined in Sect. 3.4), the internal acoustic pressure cannot move the mass at the open tone hole [4]. In this situation, the tone hole virtually operates as a closed one, and whole-bore resonances are produced. Hence, three kinds of resonances can be produced by a bore with a single open tone hole. Each of these resonances can form the pressure nodes (or minima) near both open ends, just as seen in a simple open pipe [2–5]. It should also be noted that the driving mechanism controls which resonances are actually produced. Table 1 gives the upper-bore and whole-bore resonances when the shakuhachi is blown at the top end. In the context of intonation anomaly, the three types of resonances above will be discussed below.

Measurement results are shown in Fig. 3 on fingerings D, E, F, and G. The left and right columns indicate the upper-bore and lower-bore resonance modes, respectively. The whole-bore modes are drawn in panels of the upper-bore modes. The influence of the external drive used in our measurement should be considered, as explained below.
Fig. 3 Standing-wave patterns on fingerings D to G. The measured acoustic pressure $p(x)$ is normalized by the acoustic pressure $p_0$ at the fifth tone-hole position. Left and right columns represent the upper-bore and lower-bore resonance modes, respectively. The solid and dashed lines represent the modes numerically calculated and not numerically calculated, respectively.
Also, subscripts such as ‘+‘, ‘ ‘ ‘++‘, ‘ ‘ and ‘ ‘12‘ ‘ are used to lower bore. However, Fig. 3(b). These modes have higher amplitudes in the normal fingering D. Modes assumed that \( f_n \) (cf. Sect. 4).

Calculated and are not numerically calculated, respectively as input admittance peaks [see Fig. 4(b)]. Note that Therefore, all modes measured on fingering E are calculated as antinodes below the open 5th tone hole. However, these modes tend to form the nodes near the bore top and bottom. Therefore, modes \( f_3 \) and \( f_{3a} \) are regarded as the whole-bore resonance modes.

Field on the other hand, \( f'_1 \), \( f'_{12} \), \( f'_{2++} \), and \( f'_3 \) are shown in Fig. 3(b). These modes have higher amplitudes in the lower bore. However, \( f'_1 \) (536 Hz) and \( f'_{12} \) (888 Hz) show increasing amplitudes (toward the upper bore) at the open 5th hole, and violate the resonance condition there (mode \( f'_{12} \) shows the standing-wave pattern and the frequency value intermediate between \( f'_1 \) and \( f'_3 \)). Mode \( f'_1 \) (1,598 Hz) shows the pressure maximum near the bore bottom, and violates the resonance condition there as well. Only \( f'_{2++} \) (1,416 Hz), which is much higher than \( f'_2 \) (not measured) and sufficiently lower than \( f'_3 \), satisfies the resonance conditions at the open 5th hole and at bore bottom.

Since cross fingering E closes the 4th tone hole, the internal pressure takes the maximum near this tone hole, as shown in Figs. 3(c) and 3(d). As a result, the nodes or local minima (kinks) are formed near the open 5th tone hole, and the resonance condition is satisfied there for all modes drawn in Figs. 3(c) and 3(d). Additionally, the upper-bore modes satisfy the resonance condition at the embouchure end, and the lower-bore modes satisfy it at the bore bottom. Therefore, all modes measured on fingering E are calculated as input admittance peaks [see Fig. 4(b)]. Note that cross fingering E cannot produce the 3rd mode (\( f_3 \approx 3f_1 \)) of the upper bore. This is because, in the 3rd mode, the pressure must be made minimum near the 4th tone hole, as shown in Figs. 3(e) and 3(g). Cross fingering E then gives \( f_{3+} \) (1,951 Hz), which is considerably higher than \( f_3 \) of cross fingerings F and G at 1,860 and 1,846 Hz, respectively. The upper-bore 2nd mode \( f_2 \) (1,316 Hz) gives the intonation anomaly (cf. Table 1). Although mode \( f_{3+} \) (1,951 Hz) may be considered as a whole-bore mode, a local pressure maximum was measured near the bore bottom.

Cross fingering F makes reasonable standing-wave patterns of modes \( f_1 \) (602 Hz), \( f_2 \) (1,259 Hz), and \( f_3 \) (1,860 Hz) along the upper bore, as shown in Fig. 3(e), with some distortions near the closed 3rd and open 2nd tone holes. These modes give intonation anomalies (cf. Table 1). Also, these modes seem to form the 2nd, 4th, and 6th modes along the whole bore. Although the modes of the lower bore were measured at \( f'_1 \) (552 Hz), \( f'_{2'} \) (826 Hz), \( f'_2 \) (1,100 Hz), and \( f'_{3'} \) (1,714 Hz), the \( f'_{3'} \) (552 Hz) mode cannot be calculated because it violates the resonance condition at the open 5th tone hole [see Figs. 3(f) and 3(c)].

Cross fingering G yields beautiful patterns of modes \( f_1 \) (615 Hz), \( f_2 \) (1,234 Hz), and \( f_3 \) (1,846 Hz) along the upper bore, as shown in Fig. 3(g). These three modes give intonation anomalies (cf. Table 1) and form the 2nd, 4th, and 6th modes along the whole bore. However, mode \( f_{3+} \) (1,955 Hz), which is slightly higher than \( f_{3+} \) (1,951 Hz) on cross fingering E, violates the resonance condition at the bore end. Also, the lower bore yields beautiful patterns of modes \( f'_{1} \) (454 Hz), \( f'_{2} \) (887 Hz), and \( f'_{3} \) (1,505 Hz) along the lower bore, as shown in Fig. 3(h), and these three modes form the 2nd, 3rd, and 5th modes along the whole bore. However, mode \( f'_{12} \) (543 Hz) violates the resonance conditions at the open 5th hole and bore bottom.

As explained above, our measurement result in Fig. 3 is consistent with the results in Table 1 obtained from playing the shakuhachi very well, in terms of the intonation anomaly. In addition, we may sum up the results as follows:

1. The intonation anomaly by cross fingerings occurs if the \( n \)th mode of the upper bore forms the \( (n + \theta) \)th mode of the whole bore (the whole-bore modes are produced even below the cutoff frequency) and if normal fingering consisting of an open-tone-hole lattice forms the \( [n + (n - 1)] \)st mode of the whole bore. This is exemplified by \( n = 2 \) on cross fingering E and \( n = 1, 2, 3 \) on cross fingerings F and G.

2. The intonation anomaly at higher frequencies probably occurs above the cutoff frequency (around 1,300 Hz) of the open-tone-hole lattice (cf. Sect. 3.4). This is exemplified by \( n = 3 \) (\( f_{3} \) on fingerings F and G).

3. Results for Fingerings A, B, and C

Unfortunately, page limitation does not allow us to explain the results for fingerings A, B, and C in detail. See Ref. [7] for a more detailed explanation. The following may be stated by summing up the results:

1. The intonation anomaly due to cross fingerings occurs if the \( n \)th mode of the upper bore forms the \( [n + (n - 1)] \)st mode of the whole bore. This is exemplified...
by \( n = 2 \) and \( 3 \) on cross fingering \( B \) [see \( f_{2+}(1,038 \text{ Hz}) \) and \( f_{3+}(1,485 \text{ Hz}) \) in Ref. [7]] and \( n = 2 \) on cross fingering \( C \) [see \( f_2(928 \text{ Hz}) \) in Ref. [7]].

(2) However, the intonation anomaly does not occur even if the \( n \)th mode of the upper bore forms the \([n + (n-2)]\)nd mode of the whole bore. This is exemplified by \( n = 3 \) and \( 4 \) on cross fingering \( B \) [see \( f_3(1,301 \text{ Hz}) \) and \( f_4(1,880 \text{ Hz}) \) in Ref. [7]] and \( n = 3 \) on cross fingering \( C \) [see \( f_3(1,293 \text{ Hz}) \) in Ref. [7]].

### 3.4. Cutoff Frequency of an Open-Tone-Hole Lattice

The cutoff frequency \( f_c \) of an open-tone-hole lattice is defined as

\[
f_c = 0.11 c (b/a)(1/s/\lambda)^{1/2}, \quad (1)
\]

where \( a \) denotes the bore radius, \( b \) the tone-hole radius, \( c \) the speed of sound in the bore, \( s \) half the hole-to-hole distance, and \( l \) the acoustical length of a tone hole [2,4,5]. If averaged values are used to these quantities of our shakuhachi \( [a = 8.5 \text{ mm}, \ b = 5 \text{ mm}, \ c = 346.5 \text{ m/s} \text{ at room temperature, } s = 20 \text{ mm}, \text{ and } l = 15.5 \text{ mm} \text{ including open-end corrections at both ends (1.5b)}], \ f_c \) is given as 1,270 Hz.

However, it should be noted that \( f_c \) of Eq. (1) is calculated under the assumption of an infinite bore length and equally spaced open tone holes [4]. Therefore, Eq. (1) may not be applied to cross fingerings. Nevertheless, \( f_c \) of about 1,300 Hz serves to discriminate the modes reflected near the top open hole (mostly yielded below 1,300 Hz) from the modes penetrating it [cf. \( f_3 \) (1,759 Hz) in Fig. 3(a), \( f_3 \) (1,846 Hz) in Fig. 3(g), etc.].

### 4. NUMERICAL CALCULATIONS

The external drive of the instrument tends to cause the modes that violate the resonance conditions, as indicated in Fig. 3 by the dashed line. Numerical calculation of the input admittance can help discriminate such violating modes from the not-violating modes. Moreover, internal pressure distributions (standing-wave patterns) along the bore are essential to discriminate the modes of the upper bore from those of the lower bore. This discrimination (mode identification) is almost impossible using only the limited information of input admittances.

#### 4.1. Bore Model

The inner bore of the shakuhachi used for the standing-wave measurement is modeled as a tube consisting of ten cylindrical elements, two divergent conical elements, and two convergent conical elements (see Table 2), based on the image from the CT scan. The slight bend near the bore bottom is neglected. Note that the position \( x \) along the bore axis is taken from the bore bottom to start with acoustic radiation at the open end (radius \( a_0 \)) of the bore and work up the air column toward the embouchure according to the transmission matrix (TM) method [8–13]. The 13th convergent conical element from the embouchure end \( (x = 540 \text{ mm}) \) to the position \( x = 490 \text{ mm} \) is commonly seen in classical shakuhachis, although its length and conicity differ individually [14].

The end correction \( \Delta E \) at the embouchure hole is incorporated in the 14th cylindrical element with bore diameter \( 2a = 20.3 \text{ mm} \). The length \( \Delta E \) is determined so that the first-mode frequency \( f_1 \) given by numerical calculation matches that given by the standing-wave measurement in the previous section. Also, the tone-hole central positions and geometries are indicated in Table 2. The estimated values of \( \Delta E \) were 37.1, 43.1, 33.9, 44.7, 48.4, 35.5, and 41.6 mm on fingerings A to G, respectively. These \( \Delta E \) values appear reasonable in comparison with \( \Delta E = 42 \text{ mm} \) used for the design of the modern flute by T. Boehm, though these \( \Delta E \) values tend to lower higher modes [5].

#### 4.2. Calculation Method Applied to Tone-Hole System

The transmission matrix (TM) method, which has been developed and applied to various engineering problems such as acoustical filters and mufflers [15,16], is considered to be sufficiently established to apply to woodwind instrument bores with tone holes for calculating the input impedance or admittance [8–13,16]. Keefe’s method [5,8,9] defines a tone hole as a T-section network consisting of a series impedance \( Z_a \) and a shunt impedance \( Z_s \). The TM method can estimate both the input admittance and the standing-wave pattern starting from the radiation impedance and moving up to the embouchure (by multiplying the matrices corresponding to acoustical elements such as the bore and tone hole) [8–13]. For the detailed mathematical expressions in the TM method, readers are directed to Refs. [8–13,16] for the sake of page saving, except the following brief comments on related matters.

If the bore bottom corresponds to the radiation end and there is no tone hole, \( p_1 \) at the input side of the first cylindrical element with length \( L_1 \) is given as [17,18]

\[
p_1 = [\cosh(\gamma L_1) + (Z_c/Z_{\text{rad}}) \sinh(\gamma L_1)] p_{\text{rad}}, \quad (2)
\]

where \( p_{\text{rad}} \) denotes the pressure at the radiation end and \( Z_{\text{rad}} \) the radiation impedance. The \( \gamma = (\alpha + io/c) \) is the complex propagation wave number, where the attenuation constant \( \alpha \) includes the effects of visco-thermal losses at the bore boundary layer and is approximated as \( \alpha = 3 \times 10^{-5} f^{1/2}/a (\text{m}^{-1}) \) [5] \( (o \) denotes the angular frequency), and \( Z_c \) is the characteristic impedance. For the calculation of \( Z_a \) and \( Z_s \), newly improved equations given in Ref. [10] are used instead of conventional ones [5,8,9].

Equation (2) is the starting point of our calculation. Although Fletcher and Rossing [5] consider that the
presence of the baffle has a relatively small effect (except $ka_0 \ll 1$) on $Z_{\text{rad}}$, the fundamental frequency of musical instruments is usually in the range of $ka_0 \ll 1$. Lefebvre and Scavone [10] proposed the radiation impedance of an unflanged tone hole at low frequencies. Their Eq. (10) is adapted to our case as follows:

$$Z_{\text{rad}} = Z_c [0.25 (ka_0)^2 + ik(0.7a_0)],$$

where their tone-hole end correction $0.61b$ is replaced by $0.7a_0$ assuming that the shakuhachi’s bore end made of the bamboo root operates as an intermediate baffle.

Therefore, if $p_{\text{rad}}$ is adequately assumed, the relative distribution of the internal pressure can be calculated from Eq. (2). Similarly, the input admittance is calculated by multiplying the bore transmission matrices and the tone-hole matrices from the bottom to the embouchure end correction $\Delta E$ [5,8–13,16,17]. Room temperature is assumed to be the average (26.9° C) in the measurement.

### 4.3. Results of Input Admittances

The absolute magnitudes of the input admittances $|Y_{IN}|$ on fingerings D, E, F, and G are shown in Fig. 4. In general, cross fingerings change the input admittance spectra (almost harmonic) of normal fingerings to inharmonic. This is due to the acoustic characteristics below the top open tone hole. As a result, the upper-bore modes are mixed with the lower-bore modes, and spectrum identification is required. Note that this spectrum identification is almost impossible without the knowledge of internal standing-wave patterns given by the measurement described in the previous section or by the calculation.

Even in normal fingering D, two small resonant modes of the lower bore $f_2^1$ (1,084 Hz) and $f_2^2$ (1,354 Hz) appear in the input admittance spectra, as shown in Fig. 4(a). In addition, a small peak $f_3^+$ (1,884 Hz) appears above $f_3$ (1,679 Hz) of the upper bore. Such identification is carried out through comparison with standing-wave patterns shown in Figs. 3(a) and 3(b). The situation is the same in Figs. 4(b), 4(c), and 4(d) for cross fingerings E, F, and G, respectively. Interestingly, spectra of the upper and lower bores appear one after the other in Fig. 4(d) for cross fingering G.

Since we do not have enough space to show the results for fingerings A to C, see Refs. [13] and [18] for their input admittance spectra.

### 4.4. Results of Internal Standing-Wave Patterns

The calculation results on internal standing-wave patterns show very good agreement with the measurement results for fingerings A to G [18]. Of course, it is impossible to calculate the measured modes violating the resonance condition near the bore bottom (cf. the modes shown by the dashed line in Fig. 3). In this subsection, results for fingerings A to C (note that the embouchure end correction $\Delta E$ was adjusted to yield the same resonance frequency as the measured frequency) are displayed for the respective mode to show the intonation anomaly from a different viewpoint.

Figure 5(a) is on the first mode, where the pressure along the lower bore below the open 3rd tone hole becomes higher as the 2nd and 1st tone holes are closed in succession in fingerings B and C. Also, a weak kink of the pressure magnitude is seen at the open tone hole. These patterns well illustrate the typical effect of cross fingerings.

On the other hand, fingering C produces a very deep trough near the closed 2nd tone hole, as shown in Fig. 5(b) for the 2nd mode. At this time, the 3rd mode is formed along the whole bore and the intonation anomaly is induced. It may then be understood that the lower bore is almost completely coupled (docked) with the upper bore instead of being separated at the top open tone hole, because the pattern only indicates a negligible kink (phase change) there. The whole-bore mode is thus formed.

Although the 3rd modes form the 4th modes along the whole bore, as shown in Fig. 5(c), the intonation anomaly does not occur, as noted in the measurement result [cf. (2) in Sect. 3.3]. In this case, all patterns indicate an appreciable kink (phase change) at the top open tone hole. The bore docking mentioned above does not occur.

However, cross fingerings B and C easily yield the higher 3rd mode $f_3^+$, as shown in Fig. 5(d) (cf. Table 1). This mode forms the 5th mode along the whole bore and

| Table 2 Bore and tone-hole geometries of the shakuhachi. |
|---|---|---|
| Position $x$ | Diameter $2a$ | Bore/Tone hole (inner dia., length) |
| 0 (bottom) | 17.8 mm | 1st element (divergent conical) |
| 30 mm | 16.2 mm | 2nd element (cylindrical) |
| 119 mm | 16.2 mm | 3rd element (cylindrical) |
| 171 mm | 16.2 mm | 4th element (convergent conical) |
| 200 mm | 17.0 mm | 5th element (cylindrical) |
| 220 mm | 17.0 mm | 6th element (cylindrical) |
| 284 mm | 17.0 mm | 7th element (cylindrical) |
| 320 mm | 17.6 mm | 8th element (cylindrical) |
| 380 mm | 18.4 mm | 9th element (cylindrical) |
| 390 mm | 18.4 mm | 10th element (cylindrical) |
| 430 mm | 18.8 mm | 11th element (divergent conical) |
| 460 mm | 18.0 mm | 12th element (cylindrical) |
| 490 mm | 18.0 mm | 13th element (convergent conical) |
| 540 mm | 20.3 mm | 14th element (cylindrical) |
| 540 + $\Delta E$ | 20.3 mm | (end correction at the embouchure) |
Fig. 4 Calculated input admittances of fingerings D to G.

Fig. 5 Standing-wave patterns for fingerings A, B, and C.
the intonation anomaly occurs. Both patterns display the continuity (no phase change) at the top open tone hole as well as the violence of the resonance condition (the pressure minimum) there. The docking between the upper and lower bores is then much stronger than that in the 2nd mode on fingering C. This is probably because the open 3rd tone hole does not function as an open tone hole above \( c \) or other mechanisms are involved.

5. DEPENDENCE OF INTONATION ANOMALY ON TONE-HOLE POSITION

5.1. Baroque Flute

Our calculation method of the internal standing waves can be applied to Nederveen’s example of pitch sharpening by cross fingering [3] mentioned in Sect. 1. The result is illustrated in Fig. 6, where the absolute magnitude of the input admittance \( |Y_{IN}| \) is calculated and plotted in Fig. 6(a). This \( |Y_{IN}| \) is almost the same as that in his Fig. A6.3 [3] [we used 0.61\( a_0 \) here instead of 0.7\( a_0 \) in Eq. (3)]. His compound tube consists of a left-hand tube with a length of 379 mm, an open hole with a radius of 9.5 mm and an acoustical length of 22 mm, and a right-hand tube whose length \( L_R \) is increased stepwise, starting with 100 mm (curve A) with steps of 25 mm (curves B, C, D) to the final length of 200 mm (curve E). This final configuration that gives curve E is close to that of our fingering C. The plots are shifted vertically for better visibility.

Considering only \( |Y_{IN}| \), Nederveen insisted that the third peak would finally yield the second mode resonance as the second peak moves to lower frequencies and diminishes in height, as displayed in Fig. 6(a). However, actually, the first mode \( f'_1 \) of the right-hand tube cuts in between the first mode \( f_1 \) and the second mode \( f_2 \) of the left-hand tube, as shown by curves D and E in Fig. 6(a). This \( f'_1 \) mode appears between the third mode \( f_3 \) and the fourth mode \( f_4 \) in admittance curve E, and it moves down to lower frequencies, as indicated by arrows. It is impossible to judge this \( f'_1 \) as an upper-bore (second) mode because the resonance condition for the upper-bore mode is violated at the open hole, as clearly shown in Fig. 6(b). In Figs. 6(b), 6(c), and 6(d), the standing-wave patterns respectively obtained from the admittance spectra of curves C, D, and E are plotted. The acoustic pressure \( p(x) \) along the tube is normalized by that at the open tone hole \( p_0 \).

The \( f'_1 \) mode has a frequency higher than that of the \( f_2 \) mode, as known from the shorter wavelength in Fig. 6(b). However, the \( f'_1 \) mode has a frequency lower than that of the \( f_2 \) mode, as known from the longer wavelengths in Figs. 6(c) and 6(d). Therefore, the small second peak on curve E indicates not the second mode of the left-hand tube but the first mode of the right-hand tube. Similarly, the third peak on the curves does not always mean the third mode of the left-hand tube and is the third mode only on curve A. It changes to the first mode \( f'_2 \) of the right-hand tube on curves B and C, and further changes to the second mode \( f_2 \) on curves D and E. The final third peak never corresponds to the third mode of the left-hand tube. Also, the second mode \( f'_2 \) of the right-hand tube cuts in between \( f_3 \) and \( f_4 \) of the left-hand tube, as shown in curves D and E of Fig. 6(a), and it is understood from Figs. 6(c) and 6(d). Note that the frequency of \( f'_2 \) is almost twice that of \( f'_1 \) on curve E and the two modes have smaller amplitudes.

It should be correctly regarded that docking between the left-hand and right-hand tubes is established without any appreciable phase change as if the open tone hole does not exist [compare smooth changes in \( f_2 \) at the tone hole in Figs. 6(c) and 6(d) with the kink there in Fig. 6(b)]. As a result, the 3rd mode is formed along the whole tube and the intonation anomaly occurs. It should be recognized that our correct identification of admittance peak spectra cannot be
accomplished without the knowledge of standing-wave patterns [19].

5.2. Overall Tone-Hole Shift in the Shakuhachi

The correlation between the tone-hole position and the resulting intonation anomaly was previously briefly discussed [7]. In this subsection, the effect of overall tone-hole shift (while keeping the hole-to-hole distances unchanged) is considered [19]. Figure 7(a) displays the frequency change of the 2nd mode when fingerings D, F, and G are used. The TM method is applied to the calculation. The positions of all tone holes are shifted upward and downward in steps of 5 mm. The original position of the 5th tone hole is indicated by the dashed line at \( x = 220 \) mm.

The intonation anomaly observed at the original position is kept between \( x = 200 \) and 230 mm. Note that a large (almost discontinuous) change in resonance frequency is followed by switching between the modes of the upper and lower bores. For example, on fingering D, the locus of \( f_2 \) switches to \( f_2' \) at \( x = 230 \) mm and to \( f_2'' \) at \( x = 200 \) mm. On fingering G, the locus of \( f_2 \) switches to \( f_2' \) at \( x = 255 \) mm and to \( f_3' \) at \( x = 190 \) mm. These mode switches between the upper and lower bores are confirmed by checking the internal standing-wave patterns involved (not shown here), as inferred from Fig. 6.

Cross fingerings F and G maintain the intonation anomaly given at the original tone-hole configuration in the range of about \( x = 220 \pm 20 \) mm in a stable fashion, as shown in Fig. 7(a). Also, cross fingerings B and C maintain the intonation anomaly given at the original tone-hole configuration in a much wider range centered at \( x = 320 \) mm [19]. Such a stable frequency characteristic of the intonation anomaly by cross fingering seems to be a merit of the tone-hole system of the shakuhachi because the intonation anomaly is inevitably necessary for actual playing of the shakuhachi.

5.3. Non-Adiabatic Transition Phenomena in Physics

Very interestingly, the spectrum (or mode) switching demonstrated in Figs. 6 and 7 seems to be an example of the non-adiabatic transition at the crossing of the energy level in quantum mechanical systems (see Fig. 8) [20]. This non-adiabatic transition is one of the fundamental mechanisms of the state or phase changes and is observed in various fields of physics and chemistry [20–22].

A conceptual sketch is depicted in Fig. 8. The abscissa \( x \) denotes the parameter controlling an interaction between two energy states, e.g., the molecular configuration in chemical reactions. The ordinate denotes the energy of the two-level system. The dashed lines indicate the unperturbed energies \( \omega_1 \) and \( \omega_2 \) in frequency units, while the solid lines indicate the perturbed energies \( \omega_A \) and \( \omega_B \), where the mode switching observed in Fig. 7(a) is seen as the spectrum switching in the input admittance spectra shown in Fig. 7(b). This spectrum switching on fingering G occurs in the first and third modes as well as in the second mode. The spectrum order in the original tone-hole configuration (\( x = 220 \) mm) is \( f_1' \), \( f_1 \), \( f_2' \), \( f_2 \), \( f_3' \), and \( f_3 \). However, when all tone holes are shifted by 50 mm to the bore end (\( x = 270 \) mm), this order is switched to \( f_1 \), \( f_1' \), \( f_2 \), \( f_2' \), \( f_3 \), and \( f_3' \). In this configuration, the 2nd mode on cross fingering G does not bring about the intonation anomaly, as known from Fig. 7(a).

Cross fingerings F and G maintain the intonation anomaly given at the original tone-hole configuration in the range of about \( x = 220 \pm 20 \) mm in a stable fashion, as shown in Fig. 7(a). Also, cross fingerings B and C maintain the intonation anomaly given at the original tone-hole configuration in a much wider range centered at \( x = 320 \) mm [19]. Such a stable frequency characteristic of the intonation anomaly by cross fingering seems to be a merit of the tone-hole system of the shakuhachi because the intonation anomaly is inevitably necessary for actual playing of the shakuhachi.
degeneracy at the crossing is broken by an interaction that couples the levels.

The avoided crossing region is passed by remaining on the same branch when an adiabatic transition takes place. However, a non-adiabatic (diabatic) transition yields a jump to another branch across the avoided crossing, as shown by the arrow in Fig. 8. The probability of this transition is given by the famous Landau–Zener formula [23,24] in quantum mechanics.

Figure 7(a) (e.g., the green line of fingering G) clearly demonstrates the jump in the diabatic transition. The embouchure-to-fifth tone hole distance $x$ is now interpreted as the parameter controlling the interaction between the upper and lower bores. The spectrum switching on the cross fingering revealed in Figs. 6 and 7 well reflects the diabatic transition. It is surprising that the cross fingering and the associated intonation anomaly in the shakuhachi, which are minor topics in musical acoustics, are characterized by the fundamental diabatic transition in quantum and classical physics.

Very recently, Adachi [25] has proposed a simplified model to explain the mechanism of the intonation anomaly by considering Nederveen’s example discussed in Sect. 5.1. However, his model is restricted to the conventional adiabatic transition, such as that seen in two strings coupled with a bridge [5], and cannot be applied to the diabatic transition characterized by the jump from one branch to another. Also, as pointed out in Sect. 5.1, his $f_{2+}$ mode (higher than the $f_2$ mode at first) cannot be recognized as a second mode of the left-hand bore and this mode should be defined as $f_2''$, as illustrated in Fig. 6(b). A more relevant model should incorporate mode switches such as that observed between $f_2$ and $f_2''$ (Fig. 6) and mode jumps such as those observed between $f_2$ and $f_2'$ and between $f_2$ and $f_3$ (Fig. 7). The mode identification, which has been carried out very carefully in this paper, is essential for creating a theoretical model applicable to cross fingerings and the associated intonation anomaly in the near future.

6. CONCLUSIONS

The acoustics of cross fingerings was explored in the shakuhachi through the measurement and calculation of pressure standing waves and the calculation of input admittances. Standing waves and input admittances were calculated by the transmission matrix method with the tone-hole matrix formulation. A particular interest was focused on the intonation anomaly due to cross fingerings in the second and third registers. The results of the measurement and calculation displayed good agreement concerning the acoustical characteristics of cross fingerings and their associated intonation anomalies.

Since the cross fingering tends to divide the instrument bore (air column) into the upper and lower bores at the top open tone hole, it was essential to discriminate the upper-bore resonances from the lower-bore resonances. Switching of the input admittance spectra between an upper-bore mode and a lower-bore mode was observed when the associated tone hole positions were varied while keeping the hole-to-hole distances fixed.

The spectrum switching caused the intonation anomaly upon cross fingerings, as shown in Figs. 6 and 7. Also, when the intonation anomaly occurred, docking between the upper and lower bores was established as if the top open tone hole did not exist. As a result, a higher mode was formed along the whole bore, as illustrated in Figs. 3, 5, and 6. Such strong bore docking is never expected for usual cross fingerings. The spectrum switching is a good example of the diabatic transition widely seen in physics.

Cross fingering yielded very complicated spectra of the input admittance, and it was difficult to correctly identify the upper-bore and lower-bore spectra without the knowledge of internal standing-wave patterns, the importance of which has not been fully discussed up to now in the framework of woodwind acoustics.

Also, the cutoff frequency of the open-tone-hole lattice seemed to play a significant role in producing the intonation anomaly upon cross fingering, though it was difficult to exactly define the cutoff frequency of cross fingerings. Below the assumed cutoff frequency, the intonation anomaly was caused by the resonance in the lower bore, which produced a higher whole-bore resonance mode.

Above the cutoff frequency, the top open tone hole did not function as an open tone hole, and the resonating pressure wave in the upper bore penetrated into the lower bore without significant reflection at the open tone hole. However, since this pressure wave was reflected at the bore bottom, a standing-wave pattern of a much higher whole-bore resonance mode was produced, as shown in Figs. 3 and 5. Such a standing-wave pattern causes the intonation anomaly above the assumed cutoff frequency.

It seems that the acoustics of cross fingerings has just come into new phase with the associated intonation anomaly in the shakuhachi. Relevant physical modeling of the spectral (or mode) switching that causes the intonation anomaly is expected in the near future.

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