Research on real-time pricing strategy for load serving entity based on simultaneous perturbation stochastic approximation

Xing Lu1, Hongzhi Zhang1,3 and Dongyi Zhang2

1 Institute of Management Science, Guangdong Power Grid Co. Ltd., Guangzhou, China;
2 Center for Emergency and Risk Management, Guangdong Power Grid Co. Ltd., Guangzhou, China
3 Email: glandchang@gmail.com

Abstract. As one of the main participant in the electricity market, load serving entities (LSE) purchase electricity in the wholesale market, and provide different types of electricity package to users in the retail market. LSE’s profit mainly comes from the difference between the cost of purchasing electricity and the benefit of selling electricity. To reduce the cost, LSE can optimize electricity consumption characteristics of various types of users through real-time price adjustment, which can help to improve the shape of electricity purchase curve and implement demand side response. In this paper, a real-time pricing strategy based on synchronous perturbation stochastic approximation algorithm (SPSA) is proposed. This method considers the difference between diverse users synthetically. Depending on the historical electricity consumption and historical price data, the user side response function is constructed, and the real-time electricity price is formulated with the objective of minimizing load curve’s peak-valley difference. Finally, the effectiveness and advantages of the proposed method are verified by comparing the load curve and the power purchase cost before and after the price adjustment.

1. Introduction

In the electricity market, the profit of the LSE mainly comes from the difference between the cost of purchasing electricity in the wholesale market and the electricity sales profits from the retail market. In order to realize rational allocation of demand-side resources in competitive electricity market, price signals and incentive mechanism can be utilized to adjust users’ load characteristics [1]. By optimizing load declaration curve through demand response (DR), LSEs can reduce the cost of purchasing electricity, ensure the maximization of their own benefits, and optimize resource deployment. In the literature [2], cost-benefit of implementing DR is analyzed, and the decision process of users participating in DR is described in detail.

In recent years, many research achievements have been recorded in real-time pricing and DR, especially in cost-benefit model construction, demand-side price elasticity analysis and optimization algorithm application. Reference [3] proposed a future smart power scenario where subscribers and energy providers can communicate through smart meters, in this case, a real-time pricing algorithm is formulated to optimize subscribers’ consumption while minimizing energy providers’ cost, the utility functions of subscribers are selected from the perspective of microeconomics. Furthermore, an online algorithm aimed at minimizing the load variance while maximizing users’ satisfaction is presented in [4], user’s utility and cost, grid load smoothing, dynamic pricing, and energy provisioning cost are all
considered in the process of modeling. Based on the concept of microeconomics, the utility functions of residents, commercial and industrial subscribers are given respectively in [5], the real-time electricity price is obtained by Lagrange dual algorithm, and simulation results show the effectiveness of the method proposed in improving all subscribers’ welfare benefits. Because enhanced differential evolution and teacher learning–based optimization are easy to implement and have less computational complexity, these two methods are adopted in [6] to solve the optimization problem, the objective is alleviating electricity cost and peak to average ratio in the residential area while decreasing users’ waiting time. Reference [7] summarized current research on time-varying electricity prices and presented a generalized analysis framework of tariff setting decision in the face of behavioral biases. For reducing peak-to-average ratio (PAR), a two-stage optimization model using SAPC (Simulated annealing based price control) algorithm is utilized in [8] to maximize users’ electricity quality with minimum payments. Samadi et al presented two iterative algorithms FDPS and SPSS to minimize the PAR. SPSS can achieve a similar performance while needs less number of measurements, compared to FDPS. Besides, an approximate dynamic programming scheme is proposed to simulate users’ response on the premise of protecting users’ privacy [9]. A five stage dynamic Stackelberg game model is established to simulate users’ behaviors in [10], and the real-time price is settled by using reverse induction method based on Nash Equilibrium theory. Game theory and expected utility theory are adopted in the price negotiation scheme to determine the price range for bilateral contracts [11]. In reference [12], the grid is regarded as a cyber-physical system and a switched Markov chain model is applied to express the power consumption.

Considering the large number of users and the incomplete information-based degree, it is difficult for the LSEs to obtain the specific analytical expression of the optimization model. At the same time, the optimization process of real-time electricity price involves the processing of high-dimensional variables, which are difficult to solve by traditional optimization algorithm. In view of the problems above, SPSA algorithm is introduced for asymptotically unbiased estimation in this paper, which effectively addresses the problem of multi-variable optimization in real-time pricing. In the simulation analysis, the user's time-sharing load curve before and after the real-time tariff setting is compared. The feasibility and superiority of the proposed method are analyzed from the perspective of user utility and social welfare.

2. Real-time price optimization model based on minimum peak-valley difference

2.1. Demand response based on user utility

Utility function refers to the degree of satisfaction that users get when they consume a certain amount of electricity. Based on the theory of microeconomics, the utility functions of resident users, commercial users and industrial users are given respectively in [13, 14].

\( a \) resident users

\[
U(x, \omega) = \begin{cases} 
ax - \frac{S_u}{2}x^2, & 0 \leq x \leq \frac{\omega}{S_u} \\
\frac{\omega^2}{2S_u}, & x > \frac{\omega}{S_u}
\end{cases}
\]

(1)

Where \( U(x, \omega) \) represents the utility function of resident users, \( x \) is the corresponding power consumption, \( S_u \) and \( \omega \) are the shape characteristic parameters. The whole curve increases at first and then remains stable, which means that the utility of residential users will gradually increase with the increase in electricity consumption in the initial stage, and reach saturation when the electricity consumption reaches a certain level.

\( b \) commercial users

\[
B(y, \omega) = \begin{cases} 
S_u \log(\omega y + 1), & y > 0 \\
0, & y = 0
\end{cases}
\]

(2)
Where \( B(y, \omega) \) represents the utility function of commercial users, \( y \) is the corresponding power consumption, \( S_b \) and \( \omega \) are the shape characteristic parameters.

**c) industrial users**

\[
I(z, \omega) = \begin{cases} 
S \log(\omega z + 1), & z > 0 \\
0, & z = 0 
\end{cases}
\]  

(3)

Where \( I(x, \omega) \) represents the utility function of industrial users, \( z \) is the corresponding power consumption, \( S_i \) and \( \omega \) are the shape characteristic parameters respectively.

Assuming that the contract prices signed by the LSE with resident users, commercial users and industrial users are \( p_U, p_B \) and \( p_I \), the welfare functions of the three types of users can be expressed as follows:

\[
\begin{align*}
W_U(x, \omega) &= U(x, \omega) - p_U \cdot x \\
W_B(y, \omega) &= B(y, \omega) - p_B \cdot y \\
W_I(z, \omega) &= I(z, \omega) - p_I \cdot z
\end{align*}
\]  

(4)

Considering that users will adjust their electricity consumption to get maximum welfare, to a given set of price vector \( (p_U, p_B, p_I) \), there exists a unique set of optimal electricity consumption result \( (x_o, y_o, z_o) \):

\[
\begin{align*}
\frac{\partial U(x, \omega)}{\partial x} \bigg|_{x_o} &= p_U \\
\frac{\partial B(y, \omega)}{\partial y} \bigg|_{y_o} &= p_B \\
\frac{\partial I(z, \omega)}{\partial z} \bigg|_{z_o} &= p_I
\end{align*}
\]  

(5)

### 2.2. Objective function

The operation day of the whole electricity market is divided into \( T \) periods, the contract prices provided by LSE to different users in time \( t \) is \( (p_{Ut}, p_{Bt}, p_{It}) \), \( t \in [1, 2, \ldots, T] \). According to formula (5), the real-time power consumption of different users can be calculated as \( (x_{ot}, y_{ot}, z_{ot}) \), the expected purchasing electricity \( l_{ot} \) at time \( t \) equals \( x_{ot} + y_{ot} + z_{ot} \).

As mentioned in the introduction, the profit of the LSE comes from the difference between the electricity purchasing cost and the electricity sales profits. Therefore, there are two separate ways to improve LSE’s profits. The first is to reduce the cost of purchasing electricity in the spot market, the other is to increase the profits while this practice will seriously harm the interests of users. In this case, improving the benefits of LSE is essentially transformed into reducing the purchasing cost. In electricity spot market, node price updates every settlement cycle, LSE can’t predict the trend of electricity price, but in general, electricity price during peak load period is higher than the price during valley load period. Under this circumstances, the objective of increasing LSE’s profits can be achieved by shifting the load from on-peak to off-peak hours, which is reducing peak to valley load difference (PVD). On the other hand, stable load is helpful to ensure the safe operation of power grid and relieve the pressure of peak and frequency regulation. On account of above reasons, real-time price optimization model is expressed as:

\[
P: \quad \min(l_{o_{\text{max}}} - l_{o_{\text{min}}})
\]  

(6)

Where \( l_{o_{\text{max}}} \) and \( l_{o_{\text{min}}} \) are the maximum and minimum load respectively.

In practice, because of the large number of users and their different behaviors, it is difficult to determine the values of the parameters in formula (1) ~ (3), which makes it difficult to obtain the
analytic expression of the objective function directly. Therefore, it is necessary to adopt a new algorithm to obtain the optimal solution of the model.

2.3. Constraint conditions
After the real-time pricing strategy is applied, the total tariff of users can not increase. Real-time tariff formulation strategy is working by changing the time distribution characteristics of load, reducing the cost of power purchasing of LSEs. The corresponding mathematical expression is shown in formula (7):

\[
\begin{align*}
\int x_{at} \cdot p_{Ut} \, dt + \int y_{at} \cdot p_{Bt} \, dt + \int z_{at} \cdot p_{It} \, dt - C_{\text{rebate}} & \leq C_{\text{org}} \\
\int (x_{at} + y_{at} + z_{at}) \, dt - Q_{\text{org}} & \leq C_{\text{org}} \\
C_{\text{aft}} - C_{\text{rebate}} & \leq C_{\text{org}} \\
C_{\text{rebate}} & \geq 0
\end{align*}
\]  

(7)

Where \( x_{at}, y_{at}, z_{at} \) indicate the real-time electricity consumption of residential, commercial and industrial users after price adjustment, respectively. \( p_{Ut}, p_{Bt}, p_{It} \) are the adjusted real-time price of different users. \( C_{\text{org}} \) and \( Q_{\text{org}} \) are the total electricity charges and total electricity consumption before adjustment. \( C_{\text{rebate}} \) is the rebate from LSEs to the users who participate in the load price adjustment. \( C_{\text{aft}} \) is the total electricity charges after adjustment.

2.4. Real-time price optimization based on SPSA algorithm
In the process of real-time pricing, it is hard to determine the analytic expression of the objective function because the parameters cannot obtain directly. However, according to the historical electricity consumption and historical price data of users, LSEs can observe the changes of the total electricity consumption of various users at different price. In this case, SPSA algorithm is adopted for real-time price Optimization.

SPSA algorithm was first proposed by Professor Pall of APL Laboratory of Hopkins University [15]. Based on Kiefer-Wolfowitz random approximation algorithm, SPSA can reduce the number of loss function measurements used in each gradient approximation process, improve the convergence efficiency and promote accuracy in algorithm. As a very effective stochastic optimization method, SPSA can efficiently deal with the optimization or control problems with large-scale variables, and does not want to hear the form of system loss function in the process. It has been extensively used in image recognition, wind turbine control and other fields [16, 17]. For example, in reference [16], SPSA is used to estimate the image Jacobian matrix based on its advantages in high dimensional parameter optimization. Because SPSA has good anti-noise ability and is easy to implement, it is used for tracing the maximum power in [17].

The basic idea of SPSA algorithm is given as follows, assuming that the loss function \( l(p) \) is a convex function, if \( l(p) \) is differentiable, then the optimal solution \( p^* \) satisfies \( g(p^*) = dl(p)/dp_{\infty} = 0 \). If \( l(p) \) is not differentiable or its detailed expression cannot be determined, then the approximate gradient can be used to simulate it.

\[
\begin{align*}
\tilde{p}_k &= \tilde{p}_{k-1} \cdot a_k \cdot \hat{g}_k(\tilde{p}_{k-1}) \\
\hat{g}_k(\tilde{p}_{k-1}) &= \frac{h_k p_{k-1} + c_k \Delta_k}{2c_k} - \frac{h_k p_{k-1} - c_k \Delta_k}{2c_k} \\
\Delta_{k+1}^i &= \frac{\Delta_k^i}{a_k} \\
\Delta^T_{k+1} &= \begin{bmatrix} \Delta^i_{k+1} \\ \vdots \\ \Delta^n_{k+1} \end{bmatrix} \\
a_k &= \frac{1}{A + k + 1} \\
c_k &= \frac{1}{(k + 1)^\gamma}
\end{align*}
\]  

(8)
Where \( \hat{p}_k \) is the optimal control vector in step \( k \), \( a_k \) is the searching step size, \( \hat{g}_k(\hat{p}_{k-1}) \) is the estimated perturbation gradient, \( l(\hat{p}_{k-1} \pm c_i \Delta_i) \) is the synchronous disturbance measurement value of the loss function, \( c_k \) is the perturbation step size, \( a, c, A, \alpha, \gamma \) are all pre-determined nonnegative coefficients, \( \Delta_k \) is a N-dimensional random perturbation vector, \( \Delta_i \in \{1, -1\} \), \( i = 1, 2, \ldots, N \).

Using the idea of SPSA, the real-time price can be determined and the specific implementation steps are as follows:

**Figure 1. Flow Chart of Real-time Pricing Strategy**

Step 1: Set the value of the relevant parameter \( (a, c, A, \alpha, \gamma) \) in formula (8). According to reference [14], \( \alpha = 0.602 \), \( \gamma = 0.101 \). It is recommended to choose \( A \) in the reference [14] such that it is much less than the maximum number of iterations allowed or expected.

Step 2: Update the value of searching step \( a_k \) and disturbance step \( c_k \).

Step 3: Use Monte Carlo method to generate random disturbance vector \( \Delta_k \).
Step 4: Based on the historical data of users and the fluctuation of historical electricity price, calculate the value of synchronous disturbance measurement index $l(\hat{p}_{k-1} \pm c_k \Delta_k)$.

$$l(\hat{p}_{k-1} \pm c_k \Delta_k) = \max(l_{o,t}) - \min(l_{o,t})$$

$$= \max(x_{o,t} + y_{o,t} + z_{o,t}) - \min(x_{o,t} + y_{o,t} + z_{o,t})$$

(9)

Where $\max(l_{o,t})$ and $\min(l_{o,t})$ indicate the maximum and minimum daily electricity purchase of LSE under price fluctuations of $\Delta_k$.

$\hat{p}_{k-1} \pm c_k \Delta_k$ represents the price control vector obtained in step $k-1$.

Step 5: Substitute the value of $l(\hat{p}_{k-1} \pm c_k \Delta_k)$ into formula (8), calculate disturbance gradient $\nabla \nabla k g_k p$ and updating price control vector $\hat{p}_k$.

Step 6: Calculate the convergence coefficient $\hat{p}_k - \hat{p}_{k-1}$, if $|\hat{p}_k - \hat{p}_{k-1}| < \varepsilon$ or the number of iterations exceeds the preset iterations, terminate the cycle and set the real-time tariff. Otherwise, go back to step 2 and enter the next iteration cycle. The detailed flow chart is shown in Figure 1.

Suppose the optimized price control matrix is $p_{opt}$, its structure is shown in formula (10), the column vectors $(P_{U_1}, P_{U_2}, \cdots, P_{U_T}), (P_{B_1}, P_{B_2}, \cdots, P_{B_T})$ and $(P_{I_1}, P_{I_2}, \cdots, P_{I_T})$ are the real-time price for residential, commercial and industrial users.

$$p_{opt} = \begin{pmatrix}
P_{U_1} & P_{B_1} & P_{I_1} \\
P_{U_2} & P_{B_2} & P_{I_2} \\
\vdots & \vdots & \vdots \\
P_{U_T} & P_{B_T} & P_{I_T}
\end{pmatrix}$$

(10)

3. Simulation verification and conclusion

According to the current industrial and commercial tariff standard of a city in Guangdong, China, the electricity tariffs (including taxes) of 220 kV industrial users in different periods are 55.84 cent/kWh for flat period (8:00~14:00, 17:00~19:00, 22:00~24:00), 27.92 cent/kWh for valley period (0:00~8:00) and 92.14 cent/kWh for peak period (14:00~17:00, 19:00~22:00). The time sharing nodal price matrix of LSE is [17.2, 17.2, 16.1, 40.6, 38.7, 39.0, 40.1, 45.0, 47.7, 48.4, 48.8, 48.6, 48.3, 48.7, 49.4, 49.6, 23.9, 18.6, 18.4, 18.5, 18.9, 19.0, 18.8, 18.7].

The standardized typical daily load curve of an industrial user is shown in Figure 2. In the simulation process, the values of the parameters in the formula (8) are set as follows: $a = 2$, $c = 1$, $A = 50$, $\alpha = 0.602$, $\gamma = 0.101$, $k_{max} = 50$. Import the historical data of load and price into the SPSA algorithm. The final optimized results with the objective of minimizing PVD are shown in the following figure.
The standardized typical daily load curve of an industrial user is shown in Figure 2. After real-time price adjustment, the industrial user’s daily load curve is shown in Figure 3, the standardized load fluctuates between 0.486 and 0.496. The simulation results demonstrate that the load curve after adopting the real-time tariff strategy is more stable, standard deviation decreases from 0.3593 to 0.0023 after adjustment. It can also be observed in Figure 4 and Figure 5 that the overall time distribution characteristics of tariff remain basically unchanged after adopting real-time pricing strategy, the peak period still locates at 15:00-19:00, and 21:00-23:00. The diurnal maximum electricity price changes from 92.14 cent/kWh to 109.5 cent/kWh, while the daily minimum electricity price changes from 27.92 cent/kWh to 18.45 cent/kWh.

To protect the benefit of users, the total electricity consumption should remain unchanged at 640.64, since the total electricity purchasing cost on the user side increases from 573.99 to 640.65 after adjustment, according to formula (8), the scale of rebate from LSE to user should be 66.66 at least.
4. Conclusions
For the purpose of optimizing load declaration curve and minimizing electricity purchasing cost, LSEs will be disposed to implement DR with the gradual operation of Guangdong electricity spot market. Consider the difficulties in ascertaining the demand response model of each customer through the traditional utility function, the SPSA algorithm is utilized in this paper to extract the correlation between historical electricity consumption change and historical price fluctuation. The real-time tariff of all kinds of users is established with the objective of minimizing PVD. The simulation results verify the feasibility and validity of the proposed method.

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