We perform large shell model calculations for Calcium isotopes in the full fp shell by using the realistic Kuo–Brown interaction. The Calcium isotopes are especially interesting because the nearest–neighbour spacing distribution $P(s)$ of low–lying energy levels shows significant deviations from the predictions of the Gaussian Orthogonal Ensemble of random–matrix theory. This contrasts with other neighbouring nuclei which show fully chaotic spectral distributions. We study the chaotic behaviour as a function of the excitation energy. In addition, a clear signature of chaos suppression is obtained when the single–particle spacings are increased. In our opinion the relatively weak strength of the neutron-neutron interaction is unable to destroy the regular single–particle mean–field motion completely. In the neighbouring nuclei with both protons and neutrons in valence orbits, on the other hand, the stronger proton-neutron interaction would appear to be sufficient to destroy the regular mean–field motion.

1 Introduction

The nearest neighbour spacing distribution of energy levels provides a good signature of the chaoticity of a quantum system. In fact, the fluctuation properties of quantum systems with underlying classical chaotic behaviour and time–reversal symmetry agree with the predictions of the Gaussian Orthogonal Ensemble (GOE) of random matrix theory, whereas quantum analogues of classically integrable systems display the features of the Poisson statistics.

The nuclear shell model, with a realistic interaction and large configuration space, is one of the best theoretical approaches to the study of nuclear spectra.
The model provides large sets of exact energy levels and wave functions in truncated space, and their statistical analysis can give information on the features and borderlines of the transition from regular to chaotic dynamics in nuclei. The statistical analysis of shell–model energy spectra and wave functions has mainly concentrated on the sd shell region and chaotic behaviour has been found for these nuclei both near the yrast line and at higher energies.

In this paper we extend our recent statistical analysis of the shell–model energy levels in the fp shell, in order to study whether an order to chaos transition can be observed, and how it depends on variables such as excitation energy, angular momentum and single–particle energy spacings.

2 Calculations in the fp shell

The nuclear shell–model Hamiltonian, in second–quantization notation, can be written as

\[ H = \sum_\alpha \epsilon_\alpha a_\alpha^{\dagger} a_\alpha + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} <\alpha\beta|V|\delta\gamma> a_\alpha^{\dagger} a_\beta^{\dagger} a_\gamma a_\delta, \]  

(1)

where the labels denote the accessible single–particle states, \( \epsilon_\alpha \) is the corresponding single–particle energy, and \( <\alpha\beta|V|\delta\gamma> \) is the two–body matrix element of the nuclear residual interaction.

We follow the standard approach to obtain the eigenvalues of the shell–model Hamiltonian. Exact calculations for several nuclei are performed in the \((f7/2, p3/2, f5/2, p1/2)\) configuration space, assuming a \(^{40}\text{Ca}\) inert core. The construction and diagonalization of large shell–model matrices are performed using a modified version of the computer code ANTOINE. For a fixed number of valence protons and neutrons we calculate the energy spectrum for total angular momentum \( J \) and total isospin \( T \). The interaction is a minimally modified Kuo–Brown realistic force with monopole improvements. Coulomb effects are not included.

To analyze the energy level fluctuations, it is necessary to consider only levels which have the same symmetries. In our case this means the same number of nucleons and the same total angular momentum, parity and isospin. The level spectrum is then mapped onto unfolded levels with quasi–uniform level density. The suitable unfolding procedure depends on the region of the spectrum to be analyzed. For the low–lying levels, we use an unfolding procedure based on the constant temperature formula, where the mean level density can be assumed to be of the form

\[ \bar{\rho}(E) = \frac{1}{T} \exp\left[\frac{(E - E_0)}{T}\right], \]  

(2)
Table 1: Brody parameter $\omega$ for the nearest neighbour level spacing distribution for $0 \leq J \leq 9$, $T = T_z$ states up to 4, 5 and 6 MeV above the yrast line in the analysed nuclei.

| Energy | $^{46}$V+$^{46}$Ti+$^{46}$Sc | $^{48}$Ca+$^{48}$Ca+$^{30}$Ca |
|--------|-----------------------------|-----------------------------|
| $\leq$ 4 MeV | 0.92 | 0.56 |
| $\leq$ 5 MeV | 0.93 | 0.60 |
| $\leq$ 6 MeV | 0.95 | 0.61 |

with $T$ and $E_0$ fitting parameters. We have compared this unfolding method with the standard local unfolding method. Provided that the number of energy levels is not too small, the two procedures give similar results in the low-energy region, but the constant temperature level density is smoother and is preferable in the ground-state region. When the analysis includes many levels or the full spectrum, we use the local unfolding because, as is well known, due to the finite size of the shell–model basis, the eigenvalues are generally Gaussian distributed.

3 Analysis of spectral statistics

The $P(s)$ distribution of the nearest–neighbour spacings $s_i = \bar{N}(E_{i+1}) - \bar{N}(E_i)$ of the unfolded levels is the best spectral statistics to study the fluctuations of the short range correlations. To quantify the chaoticity of $P(s)$ in terms of a parameter, we compare it to the Brody distribution,

$$P(s, \omega) = \alpha(\omega + 1)s^\omega \exp(-\alpha s^{\omega+1}),$$  \hspace{1cm} (3)

with

$$\alpha = \left(\Gamma\left[\frac{\omega + 2}{\omega + 1}\right]\right)^{\omega + 1}. \hspace{1cm} (4)$$

This distribution interpolates between the Poisson distribution ($\omega = 0$) of regular systems and the Wigner distribution ($\omega = 1$) of chaotic ones (GOE). The parameter $\omega$ can be used as a simple quantitative measure of the degree of chaoticity.

We analyze the energy spectra looking at the lowest energy region, up to a few MeV above the yrast line. The main problem in this region is that the number of energy levels of the same symmetry is too small for a reliable statistical analysis. But having at our disposal the whole energy spectrum, we can use a sufficiently large set of levels to determine the secular behaviour of the level density and perform the unfolding procedure for each symmetry class of states. Moreover, in order to obtain more meaningful statistics, after the
unfolding we can combine the level spacings of different $J$ in a nucleus, or even of different nuclei, to calculate the $P(s)$ distribution.

A good estimate of the Brody parameter is obtained combining spacings of different nuclei. Table 1 shows the results for $^{46}\text{V}+^{46}\text{Ti}+^{46}\text{Sc}$ and $^{46}\text{Ca}+^{48}\text{Ca}+^{50}\text{Ca}$, up to 6 MeV above the yrast lines. The number of level spacings is now sufficiently large to yield meaningful statistics and we see that Ca isotopes are not very chaotic at low energy, in contrast to other nuclei in the same region.

The combined data of $^{46}\text{Ca}+^{48}\text{Ca}+^{50}\text{Ca}$ show that the chaoticity increases slightly with excitation energy up to 6 MeV. It is generally believed that states at higher energy, in the high density region, should be much more chaotic in nature. Thus, calcium isotopes offer the possibility to study the energy dependence of the chaoticity in the framework of the shell model.

We have calculated the $P(s)$ distribution and the Brody parameter up to a fixed value of the excitation energy above the yrast lines from 6 to 20 MeV. We use the local unfolding method and include all levels with $J = 0$–11. We find that the chaoticity increases rather smoothly with energy. This is illustrated in Figure 1 for $^{48}\text{Ca}$. It is necessary to include levels up to about 14 MeV to

Figure 1: Energy dependence of the Brody parameter $\omega$ in $^{48}\text{Ca}$. All levels up to an energy $E$ above the yrast line are included for each $J$ value.
Table 2 shows the Brody parameter $\omega$ for the whole spectrum of the analysed Ca isotopes, which range from $^{44}\text{Ca}$ to $^{50}\text{Ca}$. We see that the lightest Ca isotopes are not fully chaotic even when the whole energy spectrum is taken into account.

For the heavier calcium isotopes the number of states is very large, e.g. 17,276 for $^{50}\text{Ca}$. Therefore, it is possible to analyze separately the spectra for different $J$ values with good statistics. We do not find any significant dependence on angular momentum, except for $J = 0$ where we find $\omega \simeq 0.8$ both for $^{48}\text{Ca}$ and $^{50}\text{Ca}$. This is probably related to the fact that the pairing interaction, which preserves the seniority quantum number, is more effective for the $J = 0$ states.

It is very interesting to study the effect of the one–body Hamiltonian on the $P(s)$ distribution. The single–particle motion in the spherical mean field is regular, while the nuclear two–body residual interaction is strongly non–linear. Figure 2 shows how the fluctuation properties of nuclear energy levels in $^{44}\text{Ca}$ change when the single–particle spacings are changed. We consider three cases: i) the $f_{7/2}$, $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ single–particle spacings are all degenerate; ii)
Table 2: Brody parameter $\omega$ for the nearest neighbour level spacing distribution for Ca isotopes. All levels for each $J$ value are included.

| Ca   | 44Ca | 45Ca | 46Ca | 47Ca | 48Ca | 49Ca | 50Ca |
|------|------|------|------|------|------|------|------|
| 0.69 | 0.75 | 0.99 | 0.98 | 0.95 | 1.00 | 0.87 |

the single–particle spacings are the experimental ones; iii) the single–particle spacings are multiplied by a factor of two. The realistic residual interaction is the same in all cases. The $P(s)$ distribution includes all the nuclear levels, separately unfolded for each $J$ value. For degenerate single–particle levels Figure 2 clearly shows chaotic dynamics in $^{44}$Ca, but when the single–particle spacings are increased, there is a transition towards regularity. The same kind of chaos to order transition is observed for the other Calcium isotopes.

4 Conclusions

The Ca isotopes are especially interesting because the nearest–neighbour spacing distribution $P(s)$ of low–lying energy levels shows significant deviations from GOE predictions, in contrast to other neighbouring nuclei which show fully chaotic spectral distributions. The analysis of level spacings up to a given excitation energy shows that the chaoticity of Ca isotopes increases smoothly with the excitation energy end point. However, even when the whole energy spectrum is included, the lighter isotopes $^{44}$Ca and $^{45}$Ca are not fully chaotic, and the order to chaos transition is progressive as the number of active particles increases.

In the heavier Ca isotopes studied, the number of energy levels is very large and it is possible to analyse with good statistics the energy levels of a single $J$ value. For example, in $^{50}$Ca the dimension of many Hamiltonian matrices exceeds one or two thousand for fixed $J$. We do not find significant differences in the $P(s)$ distribution for different $J$ values, except perhaps for $J = 0$, for which we obtain a Brody parameter somewhat smaller than for other $J$ values. This is probably related to the pairing component of the interaction and the approximate conservation of the seniority quantum number.

Finally, a clear chaos to order transition is observed as the single–particle spacings are increased. In the lighter Ca isotopes we obtain chaotic behaviour for degenerate single–particle states and quasi–regular motion when the single–particle spacings are twice the experimental values. Thus, it seems that the main reason for the substantial deviations from chaoticity obtained in Ca isotopes is that the relatively weak strength of the neutron-neutron interaction is not able to destroy the regular single–particle mean–field motion completely.
But in the neighbouring nuclei with both protons and neutrons in valence orbits, the stronger proton-neutron interaction seems to be sufficient to destroy the regular mean-field motion.

References

1. M.C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer–Verlag, Berlin, 1990).
2. W.E. Ormand and R.A. Broglia, Phys. Rev. C 46, 1710 (1992); V. Zelevinsky, M. Horoi, and B.A. Brown, Phys. Lett. B 350, 141 (1995); V. Zelevinsky, B.A. Brown, N. Frazier, and M. Horoi, Phys. Rep. 276, 85 (1996).
3. E. Caurier, J.M.G. Gómez, V.R. Manfredi and L. Salasnich, Phys. Lett. B 365, 7 (1996); J.M.G. Gomez, V.R. Manfredi, L. Salasnich, in *New Perspectives in Nuclear Structure*, vol. 5, pp. 225–233, Ed. A. Covello (World Scientific, Singapore, 1996).
4. E. Caurier, computer code ANTOINE, C.R.N., Strasbourg (1989); E. Caurier, A. P. Zuker and A. Poves: in *Nuclear Structure of Light Nuclei far from Stability: Experiment and Theory*, Proceedings of the Obernai Workshop 1989, Ed. G. Klotz (C.R.N, Strasbourg, 1989).
5. E. Caurier, A.P. Zuker, A. Poves and G. Martínez–Pinedo, Phys. Rev. C 50, 225 (1994).
6. J.F. Shriner, Jr., G.E. Mitchell, and T. von Egidy, Z. Phys. A 338, 309 (1991).
7. V.R. Manfredi, Lett. Nuovo Cimento 40, 135 (1984).
8. T.A. Brody, Lett. Nuovo Cimento 7, 482 (1973).