Reconstructing thawing quintessence with multiple datasets

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In this work we model the quintessence potential in a Taylor series expansion, up to second order, around the present-day value of the scalar field. The field is evolved in a thawing regime assuming zero initial velocity. We use the latest data from the Planck satellite, baryonic acoustic oscillations observations from the Sloan Digital Sky Survey, and Supernovae luminosity distance information from Union2.1 to constrain our models parameters, and also include perturbation growth data from WiggleZ. We show explicitly that the growth data does not perform as well as the other datasets in constraining the dark energy parameters we introduce. We also show that the constraints we obtain for our model parameters, when compared to previous works of nearly a decade ago, have not improved significantly. This is indicative of how little dark energy constraints, overall, have improved in the last decade, even when we add new growth of structure data to previous existent types of data.

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I. INTRODUCTION

Dark energy is one of the great discoveries of the 20th century, and remains one of the greatest scientific puzzles of the 21st century. While the observational evidence does not favour any particular physical mechanism beyond a cosmological constant, there are many ideas for mechanisms other than the simplest concordance ΛCDM model [1]. The possibility remains that the observed acceleration is caused by a scalar field, quintessence (see Refs. [2, 3] and references therein). The quintessence field affects both the background expansion history of the Universe, and the rate at which matter over-densities grow. A key area in current and future dark energy (DE) observations is the combination of data on the background expansion history and the structure-growth history [4]. The relation between these two histories is sensitive to the properties of the dark energy mechanism, and could allow stronger model constraints than either method separately.

In this work, assuming quintessence is a valid description of observations, we derive new constraints on the quintessence self-interaction potential and dynamical evolution, based on the latest observational data on baryon acoustic oscillations encoded in the galaxy distribution, the cosmic microwave background, and Type Ia supernovae. We also include galaxy survey data measuring the growth of matter perturbations. Earlier work has not included growth-of-structure data (e.g. Refs. [5–8], though some recent works have placed constraints on inverse power-law quintessence models using new growth-of-structure data [9, 10]). Our work is distinct in that it uses a direct modelling of the scalar field potential as a Taylor expansion and the field dynamics, rather than relying on indirect parameterizations. We restrict ourselves to the case of a ‘thawing’ quintessence [11] field with zero initial velocity at early times.

The coming decade will see a series of observational projects, such as the Dark Energy Survey [12], BOSS [13], BigBOSS [14], WFIRST [15], Planck [16], HETDEX [17], DESI [18], Euclid [19] and LSST [20], all of which will probe the growth of structure by means of different techniques. Some of the prospects for constraining quintessence models with these are discussed in Refs. [1, 4, 21].

II. FORMALISM

A. Cosmological model

We follow the approach pioneered in Refs. [5, 6]. We briefly review the set-up here, which is fairly straightforward. We assume that the quintessence field φ has a self-interaction potential V(φ), that we expand in a series about the present value of the field which is taken (without loss of generality) to be zero. The quintessence field obeys the equation

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi},$$

with the Hubble parameter $H$ given by the Friedmann equation

$$H^2 = \frac{8\pi G}{3} (\rho + \rho_\phi).$$

Here $\rho$ represents all the usual material components: dark matter, baryons, neutrinos, and radiation. We will use $\rho_m$ to give the matter density (dark matter plus
baryons) and $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ the quintessence density. We assume spatial flatness throughout (as motivated by cosmic microwave background (CMB) measurements and the inflationary paradigm), though the generalization to the non-flat case would be straightforward. Our computations start at a redshift of one million.

An important quantity, which determines the cosmological effects we consider from the quintessence field, is the equation of state

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{2} - V(\phi)$$

In order to have $w_\phi$ close to $-1$, mimicking $\Lambda$CDM, the evolution of the field should be potential dominated, which is usually called the slow-roll regime. We focus on thawing quintessence, where the field is initially frozen due to high damping from the Hubble friction term, meaning its energy density is fairly constant at early times. However, at late times, the field starts thawing and slowly rolling on its potential towards larger values of $w_\phi$.

### B. Parameterizations and priors

#### 1. Power series

We will use a Taylor expansion power series to model the potential $V(\phi)$, as

$$V(\phi) = V_0 + V_1 \phi + V_2 \phi^2 + \ldots$$

and work in units of the reduced Planck mass, $M_\text{P} = 1/\sqrt{8\pi G}$, defining $\phi$ in dimensionless units. To have viable flat models, we set $V_0 = \rho_0^\phi / \rho_\text{crit} = \Omega_\phi$, which will be derived from the present-day energy density of matter, $\Omega_\text{m}$, due to the spatial flatness condition. Note that $\Omega_\text{m}$ is the sum of the present-day cold dark matter density and baryon density divided by the present-day critical density, $\Omega_\text{c}$ and $\Omega_b$ respectively. $\Omega_\text{m}$ also includes the contribution of massive neutrinos, for which we fix $\sum m_\nu = 0.06 \text{ eV}$.

For all our models we have set $\dot{\phi} = 0$ at the initial redshift, $z_i$, so that the scalar field starts at rest. We perform a simple binary search to determine the right $\phi(z_i) \equiv \phi_i$ that allows us to recover a flat cosmology today. We then rescale the Taylor series parameters of the potential by the value of $\phi(z = 0) \equiv \phi_0$ so that the present-day value of $\phi$ is set to zero. As an example, for the first-order series expansion of $V(\phi)$, this means that

$$\tilde{V}_0 = V_0 + V_1 \phi_i,$$

$$\tilde{V}_1 = V_1,$$

$$\tilde{\phi}_i = \phi_i - \phi_0$$

and so on for higher-order expansions of the potential. Hence, $V_0$ will correspond to the present-day value of the quintessence potential. Effectively, we will be constraining the rescaled parameters, which we indicate with the tilde.

#### 2. Priors

In this work, as detailed before, we expand the quintessence potential in a Taylor series expansion. We limit ourselves to a second-order expansion, which means we will have three dark energy parameters describing our model, apart from the cosmological parameters such as $\omega$ and $\ln(10^{10} A_s)$. For these parameters we have kept the same prior ranges used in the Planck analysis [22], which are default for CosmoMC [23]. We treat CMB foreground parameters identically to Ref. [22].

In the previous section we defined the zeroth-order term as $V_0$. This is constrained by $\Omega_\text{m}$ to guarantee models with a flat cosmology and when only this term is present it corresponds to the present-day energy density of our dark energy component. If the potential is not flat there will be an additional contribution to the present field energy density from its kinetic energy, which data will constrain to be fairly small corresponding to $w_\phi$ close to $-1$.

We have imposed a flat prior on $V_1$ in the range $[-2, 2]$. The reason for this is that this range includes, conservatively, the one considered in previous similar works [5, 6]. Given the time elapsed since those works were done, we expect that our range won’t have a direct impact on the results and should be quite constrained by the data.

Lastly, there is the second-order term, $V_2$. The previous works we refer to were not able to detect significant constraints on this parameter. After some inspection, we believe our results for this parameter are also very prior related, for reasons we will discuss later. Hence, without any particular motivation, we have set its range between $[-10, 10]$.

### III. DATA ANALYSIS

#### A. Observables

The analysis done in this work was performed using the publicly-available CosmoMC code [23]. For that, we adopted the corresponding quintessence module, which we modified accordingly. For our analysis, we have considered the first Planck data release [22] plus the WMAP polarization data [24]. We have also included the baryonic acoustic oscillations data from the seven-year and nine-year release data sets from the Sloan Digital Sky Survey (SDSS) [25, 26]. Furthermore, we use the Union 2.1 580 SNIa catalogue from the Supernova Cosmology Project (SCP) [27], where we conservatively include systematic errors and marginalize over $H_0$ as detailed in the appendix of Ref. [28].
Lastly, we include growth of structure data from the WiggleZ Dark Energy Survey [29]. The two-dimensional power spectrum data from this survey was used to fit for both the redshift-space distortion effect (which measures the rate of growth of structure $f\sigma_8$) and the Alcock–Paczynski distortion of the survey geometry. The module we use compares our predictions against both types of measurement, which were measured by the WiggleZ Dark Energy Survey in four redshift bins. Further details of these measurements can be found in Ref. [30].

B. Parameter estimation

The parameter space we study will be

$$\Theta = \{100\theta_{\text{MC}}, \tau, \omega_c, \omega_b, \ln(10^{10}A_s), n_s, V_1, V_2\}, \quad (5)$$

where the cosmological parameters have the same meaning as in Ref. [22]. The parameter estimation is carried out using an MCMC approach. The posterior probability of the parameters $\Theta$, given the data and a prior probability distribution $\Pi(\Theta)$, is

$$P(\Theta|\text{data}) = \frac{\Pi(\Theta)}{Z} e^{-\chi^2} \left(\chi^2_{\text{SNLX}} + \chi^2_{\text{MBR}} + \chi^2_{\text{BAO}} + \chi^2_{\text{GRO}}\right)/2, \quad (6)$$

where the $\chi^2$ is a measure of the goodness of fit between the model theoretical predictions and the observed values of a certain physical quantity; $Z = \int \mathcal{L}(\text{data}|\Theta)\Pi(\Theta)d\Theta$ is a normalization constant, irrelevant for parameter fitting.

IV. RESULTS

A. Parameter estimation

1. Cosmological constant

In this subsection, we show the marginalized 2-d contour of the Planck constraints assuming a pure cosmological constant in our model. Therefore, we have set $V_i = 0$ for $i \neq 0$. Hence, $V_0 \equiv \tilde{V}_0$ is just a derived parameter defined as the present-day energy density of the scalar field, with a value set to yield a flat cosmology. In Fig. [1] we can see that our results match those from the Planck data release [22], as expected.

2. Linear potential

Here, we present the results for the linearized quintessence potential model, for which we have set $V_2 = 0$. In Fig. [2] we show the marginalized 2-d contours as well as the one-dimensional marginalized probability distributions for all the parameters considered in this model. We show the constraints from the individual data sets as well from their combination.

In the top plot of Fig. [2] we have the constraints placed by the different data sets, individually, on the cosmological parameters, $H_0$ and $\Omega_m$, and the dark energy parameters, $V_0$ and $\tilde{V}_1 \equiv V_1$. The constraints on $V_0$ closely follow those obtained for $\Omega_m$ since, as specified before, $V_0$ is actually a derived parameter, constrained from imposing a flat cosmology in our models. The larger uncertainty in $V_0$ is due to the inclusion of an additional parameter introducing new degeneracies in explaining the data. We note that the Planck data are clearly superior in constraining the traditional cosmological parameters (here $H_0$ and $\Omega_m$), but that this superiority does not extend to the dark energy parameters.

We note that the Planck, BAO and Growth data are not as effective in constraining the DE parameters as the Supernovae data. The growth data in particular are very insensitive to the parameters we are constraining, and hence exhibits the worst constraints, with the prior limits on $\tilde{V}_1$ exhibiting a significantly high likelihood level, higher than that observed for any other dataset. Clearly, the background data performs better in constraining the DE parameters.

The tightest constraints on $\tilde{V}_1$ come from the Supernovae data where the limits on the posterior probability distribution are the smallest, which could be expected since it has the largest volume of data, i.e., the most data points. Combining the four data sets, as seen on the lower plot of Fig. [2] these limits decrease considerably. The confidence limits resulting from the four data sets combined can be seen in Table [1]. Nevertheless, the improvement is not significant when compared to previous works of Refs. [31, 32] since most of the constraining power is coming from the Supernovae alone and the

FIG. 1: The 2-d marginalized Planck constraints over the zeroth-order Taylor series potential model, representing an effective cosmological constant. We recover the results obtained by the Planck analysis for a flat Universe.
FIG. 2: The 2-d contours and one-dimensional probability distributions for the first-order Taylor series potential model parameters. The top plot shows the constraints from the individual data sets (95% contours only), while the bottom figure presents the constraints when all of the four data sets are combined (68% and 95% contours).
TABLE I: 68% confidence limits of the first-order potential model parameters from the combination of the different data sets considered. We present the 95% upper limit on the absolute value of $V_1$ given its reflection symmetry.

| Parameter | Planck+BAO+Union2.1+Growth |
|-----------|----------------------------|
| $H_0$     | $67.3 \pm 1.0$             |
| $\Omega_m$ | $0.309 \pm 0.011$         |
| $\tilde{V}_0$ | $0.669 \pm 0.027$     |
| $|V_1| < 0.75$ (95%) |                               |

![Graph showing 2-d contours in the $w_0$-$dw/da$ plane.](image)

FIG. 3: The 2-d contours in the $w_0$-$dw/da$ plane and the respective marginalized posterior probabilities distributions for the first-order Taylor series potential. $w_a \equiv dw/da$ was computed at $z = 0$.

Table contributions from the other data sets are not particularly enhancing. As expected the posterior probability distribution of $\tilde{V}_1$ is symmetric around zero, the perfect cosmological constant case, which is well within the 68% confidence limits.

It is interesting to see how the individual data sets behave and how they combine, particularly in the $\Omega_m - \tilde{V}_1$ plane. We observe that the constraints from Supernovae and Planck exhibit opposite trends in how the preferred $|V_1|$ changes with $\Omega_m$, overlapping in a small region when compared to the individual contours. In the Supernovae case, a tilted potential compensates a smaller matter energy density in the luminosity distance because a higher tilt corresponds, effectively, to a higher dark energy equation of state (i.e. higher than $-1$). Hence, the dark energy contribution at higher redshift increases, compensating for a smaller $\Omega_m$, leading to the left-sided boomerang shape we observe.

As for the Planck constraints, the distance-related observables considered are the acoustic scale, $l_A$, and the shift parameter $R$. $l_A$ corresponds to the ratio between the angular-diameter distance, $D_A$, and the comoving sound horizon, $r_s$, at photon decoupling. Increasing $\Omega_m$ decreases both quantities, and a tilted potential also decreases $D_A$ due to a higher dark energy effective equation of state, compensating for higher matter energy densities. Lastly, $R$ is the product between $\Omega_m$ and $D_A$. Increasing the matter density is, therefore, compensated by a higher $V_1$. All these effects end up contributing for the right-shaped boomerang contour we observe.

Finally, in Fig. we see the constraints obtained in the $w_0$-$w_a$ plane, where $dw/da \equiv w_a$ was computed at $z = 0$. We note how our results restrict themselves to a thawing regime, with the scalar field becoming free at later times to roll down the potential and increasing the effective equation of state when $V_1 \neq 0$. Hence our constraints are restricted to the $w_0 \geq -1$ and $w_a \geq 0$, with the data preferring the ΛCDM case, as expected.

3. Quadratic potential

In this subsection, we show the results for the quadratic quintessence potential model, where we vary both $V_1$ and $V_2$. In Fig. we have the marginalized 2-d contours as well as the one-dimensional marginalized probability distributions of all the parameters considered. We show the constraints from the individual data sets as well from their combination. The confidence limits of the parameters that define this model can be seen in Table

![Graph showing 2-d contours in the $V_1$-$V_2$ plane.](image)

TABLE II: 1-$\sigma$ limits of the second-order potential model parameters from the combination of the different data sets considered. We show the 95% upper limit for the absolute value of $V_1$.

| Parameter | Planck+BAO+Union2.1+Growth |
|-----------|----------------------------|
| $H_0$     | $67.2 \pm 1.0$             |
| $\Omega_m$ | $0.311 \pm 0.012$         |
| $\tilde{V}_0$ | $0.64 \pm 0.06$     |
| $|V_1| < 2.7$ (95%) |                               |
| $V_2$     | $-4 \pm 5$                |
FIG. 4: As Fig. 2 for the second-order Taylor series potential model parameters.
the positive, upper limit would extend indefinitely, as the data will always accept models with a large \( \dot{V}_2 \) as long as \( \dot{V}_1 \) remains zero. This is because, in our search for the initial conditions, that means the field will remain immobile at the minimum (\( \phi = 0 \)) of the convex potential, producing, effectively, a \( \Lambda \)CDM-like model, which is in strong agreement with the data. Hence, one obtains the narrow region extending to high values of \( \dot{V}_2 \) in the 2-dimensional \( \dot{V}_1 - \dot{V}_2 \) contours.

As for the bifurcated shape in the \( \dot{V}_1 - \dot{V}_2 \) contours when considering the negative region of \( \dot{V}_2 \), particularly for smaller, negative values of it, is due to the fact that it is increasingly difficult to obtain a zero-valued \( \dot{V}_1 \) in these conditions. The concave nature of the potential makes it hard for the field to remain still at \( \phi = 0 \), as the smallest tilt will make the field roll considerably. Hence, the displacement of the field will always produce a non-zero value of \( \dot{V}_1 \), creating the shape we can see in Fig. 4. This could extend indefinitely according to the negative prior limit on \( \dot{V}_2 \), with the increasing displacement of the field producing a widening of the legged shape figure.

Finally, in Fig. 5 we see the constraining contours on the \( w_0 - w_a \) plane for the second-order Taylor expanded potential. We observe something very similar to the linear potential results, with our results constrained to a thawing regime. However, we do have now a very small region of \( w_a < 0 \), which is however not indicative of a traditional freezing regime. This actually is a result of the field rolling up the potential close to \( z = 0 \) after passing its minimum and being slowed down by the slope of the potential. This type of behavior is observed, for instance, in the models considered in Ref. [32].

V. CONCLUSIONS

In this work, we expanded the quintessence scalar field potential in a Taylor series around the field’s present-day value, which we set to zero by a rescaling of the potential expansion parameters. We also limit ourselves to a thawing scenario by setting the field’s initial velocity to zero. We then used CosmoMC to constrain these parameters with the latest available data.

The main addition in this study was the inclusion of growth of structure data, which constrains the models against observations of redshift distortions of objects and measurements of the normalized growth rate, \( f \sigma_8(z) \). We conclude that the growth data performs poorly in constraining the dark energy parameters we have considered and the worst compared to the other three data sets we have used; i.e., presently the growth observables are less sensitive than background history data when constraining the parameters we have considered [30]. The BAO is the next-worst performing data set in constraining the DE parameters.

We have also shown explicitly how the different data sets constrain our models parameters. It is particularly interesting how the Supernovae and the Planck contours overlap in a considerably smaller region than their individual constraining contours in the \( \Omega_m - V_1 \) plane. This could be promising for future surveys, proving to be the ideal combination for constraining dynamical dark energy models due to the observables involved.

Relative to previous works of Refs. [5, 6], the improvement in constraints we observe is marginal. It is true that the model we consider is different by construction, since we restrict ourselves to thawing quintessence models, but nonetheless the constraints on the parameters of the expanded potential did not improve significantly in the many years since those works despite the accumulated data of the past decade, both of existing types and new growth of structure data. A more obvious example one could think of would be the marginal improvement of constraints on the DE equation of state, \( w \), or on the 2-dimensional contours on the \( w_0-w_a \) plane (for instance, we suggest comparing the results from the WMAP 9-year data with the Planck results [22, 24]). Indeed, it has been shown that the \( w_0-w_a \) parameterization describes poorly many reasonable quintessence models [33]. It is therefore not surprising if surveys designed to measure \( w_0-w_a \) provide limited statistical power to constrain quintessence models.

Despite this, we conclude that for the first-order Taylor series expansion of the potential, \( \Lambda \)CDM is well within the preference of the data, with the confidence limits on the parameters we consider being \( \dot{V}_0 = 0.669 \pm 0.027 \) and \( |\dot{V}_1| < 0.75 \) (95%). These constraints get significantly worse when we consider an extra order on the expansion of the potential. The second-order term, \( \dot{V}_2 \), is unconstrained by the data: \( \dot{V}_2 = -4 \pm 5 \). We also conclude that the posterior limits on this parameter extend to the prior range we define for it, possibly indefinitely.
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