Optimization of oil pads on hydrostatic turntable for supporting energy conservation based on particle swarm optimization

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The performance of hydrostatic turntable is strongly influenced by the geometry of oil pads. Properly designed structure of oil pad can reduce the power consumption of hydrostatic turntable and improves its service performance. A structural optimization of oil pads on hydrostatic turntable based on particle swarm optimization is presented to improve the carrying performance and decrease the energy cost. Using finite difference method, the Reynolds equation is resolved to calculate the pressure distribution and flow rate in oil pads. The power cost of oil supply is determined according to turntable working condition, and minim energy consumption is set as optimal solution. Geometrical parameters of oil pad are chosen as individuals which will converge to an optimal result by using PSO.

Keywords: Hydrostatic turntable, Reynolds equation, PSO, Energy conservation

Highlights:
- Performance characteristics, such as pressure distribution and oil flow rate, are determined by the Reynolds equation and FDM.
- The relationship between oil pocket scale and supporting power consumption is determined numerically. Reliability of the algorithm is verified by conducting experiments on a miniature testing table.
- PSO is carried out to solve the optimal pocket size for minim pump power. An efficient and practical optimization method for hydrostatic turntable design is presented.

0 INTRODUCTION

Hydrostatic turntable is a key part of heavy duty computerized numerical control (CNC) machine tools that plays a supporting role and makes the workpiece high precision rotary motion. The external supply system continuously pumps pressurized oil into oil pads. The cost for running a hydrostatic turntable can be reduced by saving the power consumption of oil pads. The analysis and optimization of hydrostatic bearing is a complex work with many numerical computations. The geometrical size of oil pad is accurate to millimeters considering processing feasibility. Therefore, an optimization algorithm with high efficiency and acceptable accuracy is needed to optimize hydrostatic bearing, and Particle swarm optimization (PSO) is an efficient computational method, suitable to efficiently reduce computational load.

Nenzi Wang [1] presented a fast-convergent numerical method to solve the modified Reynolds equation. Udaya P. Singh [2] applied different types of modified Reynolds equation in analyzing different shapes of oil pads. Zhifeng Liu [3] provided the general type of the Reynolds equation of circular and rectangular oil pad respectively. The variation of viscosity caused by temperature changes was ignored in M. El Khalfi [4] study based on the low moving speed working condition. Tamás M. [5] conducted his optimal research by using support vector regression method. Gustafsson T. [6] introduced changing viscosity in the Reynolds equation. Qianqian Yang [7] introduced non-Newtonian characteristics into the Reynolds equation. Masjedi M. [8] solved the Reynolds equation by the efficient numerical method of successive over relaxation (SOR). Getachew A. D. [9] analyzed the lubrication performance of a slider by Finite Different Method. Ligang Cai [10] optimized the oil pad structure to improve the stiffness of hydrostatic guide way. Weißbacher C. [11] reformed the film clearance to improve the support performance of hydrostatic journal bearing. Qiang Cheng [12] applied PSO to diverse types of hydrostatic and aerostatic bearing. S. H. Chang [13] designed a Double-Pad Aerostatic Bearing...
using PSO. Chia-Wen Chan [14] conducted an optimization research on Hybrid Journal Bearings by PSO. Saeed S.M. [15] proved the PSO is more efficient in computing than simulated annealing method by comparison. Yu-xin Zheng [16] used social emotional PSO (SEPSO). Srisha Rao M.V. [17] improved the convergence of optimization algorithm by using vector evaluated PSO (VEPSO). Forrest W. Flocker [18] discussed the global convergancy of PSO in his article. Dunbing Tang [19] applied PSO in energy conservation of a workshop. Wenjian Luo [20] proposed a dynamic optimal solution based on Species-based PSO (SPSO). Zhifeng Liu [21] made a comparison between different optimization methods in a review. All the above studies show that PSO is a broadly applicable optimization method for many circumstances.

A practical optimization method for hydrostatic turntable based on PSO is proposed in this work to improve its service performance. The Reynolds equation of the hydrostatic turntable is modified according to the working condition. Performance characteristics, such as pressure distribution and oil flow rate, are determined by finite difference method (FDM). Geometry size of oil pads is selected as individuals in optimization progress. With minimum film thickness set to constant safety level, the power consumption is chosen as fitness of each individual. Finally, through the PSO algorithm, the optimal solution is determined.

1 HYDROSTATIC TURNTABLE MODELING AND GENERAL EQUATIONS

1.1 Model of hydrostatic turntable

Hydrostatic turntable is an essential part of heavy machine tools, which is supported by several oil pads on the base. The model of hydrostatic turntable is shown in Fig. 1:

![Fig. 1. Model of hydrostatic turntable on heavy machine tools](image)

The supporting oil pad of hydrostatic turntable is circular step pad with concentric oil pocket and there are \( N \) supporting pads on the rotary table. The model of circular oil pad is shown in Fig. 2:

![Fig. 2. The structure of the supporting oil pad](image)

where \( R_0 \) is radius of oil pad, \( R_1 \) is radius of oil pocket, \( H \) is oil film thickness of supporting pad.
In [10], the load-carrying capacity of the bearing and oil film thickness are considered as -3 power relationship. Without maximum pump pressure limit, the bearing ability is infinitely large when film thickness approaches zero. In real working conditions, this assumption leads to inaccurate calculation especially when oil film is thin. Due to factors such as surface roughness, geometry error, and tilting, a thinner oil film means more chance for film collapse which is seriously affecting the service performance of machine tools. Therefore, the oil film of hydrostatic turntable needs to be regulated to a safety level to avoid damages. The working condition of the hydrostatic turntable can be written as:

\[
\begin{cases}
H \geq H_0 \\
\sum_{i=1}^{N} W_i = mg
\end{cases}
\]  

(1)

where \(H_0\) is preset film thickness, \(W_i\) is load-carrying capacity of \(i\)-th oil pad, \(N\) is total number of oil pads, \(m\) is carrying load, \(g\) is the acceleration of gravity.

1.2 The Reynolds equation and FDM solution

The Reynolds equation is widely used in the analysis of hydrostatic systems to describe the pressure distribution of viscous oil film in oil pads. The following dimensionless parameters are defined before solving the Reynolds equation:

\[
\begin{align*}
\bar{p} &= \frac{p}{p_0}, \bar{p}_0 = 1, \bar{r} = \frac{r}{R_0}, \bar{R}_0 = 1, \bar{z} = z \left( \frac{h}{H} \right), \bar{H} = 1, \bar{U}_r = \frac{U_r}{\eta \psi p_0}, \bar{U}_\theta = \frac{U_\theta}{\eta \psi p_0}, \bar{W} = \frac{w}{\psi p_0}, \bar{q} = \frac{q}{\eta \psi p_0}
\end{align*}
\]

where \(p\) is pressure, \(p_0\) is pressure in the oil pocket, \(r\) is radial coordinate, \(h\) is film thickness, \(U_r\) is radial velocity, \(U_\theta\) is circumferential velocity, \(W\) is load-carrying capacity, \(Q\) is the volumetric flow rate of each oil pad, and \(\eta\) is viscosity. Also, \(\bar{p}\) is dimensionless pressure, \(\bar{h}\) is dimensionless film thickness, \(\bar{U}_r\) is dimensionless radial velocity, \(\bar{U}_\theta\) is dimensionless radial velocity, \(\bar{W}\) is dimensionless load-carrying capacity, and \(\bar{q}\) is dimensionless flow rate.

The oil pocket is usually designed to be of 5~10 mm depth, thicker than the boundary layer of the oil flow. Therefore, no pressure loss exists when the oil flows through the oil pocket. The film thickness at oil sealing edge is usually designed of dozens to hundreds of microns, smaller than the boundary layer of oil flow, so the oil sealing edge keeps the pressure unchanged in the oil pocket. In polar coordinate system, the Reynolds equation describing the pressure distribution of oil film in oil sealing edge is written as Eq. 3 [3]:

\[
\frac{\partial}{\partial \bar{r}} \left( \bar{r} \cdot \bar{h} \cdot \frac{\partial \bar{p}}{\partial \bar{r}} \right) + \frac{\partial}{\partial \bar{\theta}} \left( \bar{h} \cdot \frac{\partial \bar{p}}{\partial \bar{\theta}} \right) = 6 \frac{\partial}{\partial \bar{r}} \left( \bar{r} \bar{U}_r \bar{h} \right) + 6 \frac{\partial}{\partial \bar{\theta}} \left( \bar{U}_\theta \bar{h} \right)
\]

(3)

The hydrostatic turntable remains steady in this research, so the velocity can be ignored. The Reynolds equation is modified as Eq.4:

\[
\frac{\partial}{\partial \bar{r}} \left( \bar{r} \cdot \bar{h} \cdot \frac{\partial \bar{p}}{\partial \bar{r}} \right) + \frac{\partial}{\partial \bar{\theta}} \left( \bar{h} \cdot \frac{\partial \bar{p}}{\partial \bar{\theta}} \right) = 0
\]

(4)

The discretization of the Reynolds partial differential equation is performed by adopting the FDM method; dimensionless pressure distribution can be determined by the Gauss-Seidel iteration. Therefore, the discrete Reynolds equation is written as [9]:

\[
\begin{bmatrix}
\bar{p}_{i+1,j} \\
\bar{p}_{i-1,j} \\
\bar{p}_{i,j} \\
\bar{p}_{i,j-1} \\
\bar{p}_{i,j+1}
\end{bmatrix} = 0
\]

(5)
\[
\begin{align*}
A1_{i,j} &= \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} \\
A2_{i,j} &= \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} \\
A3_{i,j} &= -\frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} - \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} + \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} + \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} \\
A4_{i,j} &= \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}} \\
A5_{i,j} &= \frac{\bar{r}_i \Delta L_i^2}{\bar{r}_{\text{step}}}
\end{align*}
\]

where $\bar{r}_{\text{step}}$ is step length for the $r$ coordinate, $\bar{\theta}_{\text{step}}$ is step length for the $\theta$ coordinate, $i$ and $j$ are numerical counts of the elements on $r$ and $\theta$ coordinate respectively. The boundary condition of dimensionless pressure $p_{i,j}$ is chosen as 1 in the oil pocket and 0 at the external edge of the oil pad:

\[
p_{i,j} = \begin{cases} 
1, & r_i < R_1 \\
0, & r_i = 1 
\end{cases}
\]

The calculation process of FDM will be terminated when results match the error tolerance $\text{tol}_{\text{FDM}}$. The dimensionless load-carrying capacity of oil pad can be calculated by integrating the hydrostatic dimensionless pressure distribution, while the dimensionless flow rate is determined by integrating the outer flow speed on the external surface. The load-carrying performance is written as Eq. 8:

\[
\begin{align*}
\bar{w} &= \iint \bar{p} \bar{r} \, d\bar{r} \, d\bar{\theta} \\
\bar{q} &= \int \frac{x^2 - x \cdot \frac{\partial}{\partial x} d\bar{z}}{2x} d\bar{z}, \text{where } \bar{r}_i = 1
\end{align*}
\]

$\bar{w}$ and $\bar{q}$ are important parameters to evaluate the supporting performance of hydrostatic bearings, which varies with the changing of $R_1$ value. For safe operation of machine tool, a minimum film thickness $H_0$ is chosen to avoid damaging the machine tools. The safety film thickness is set as $H \geq H_0$. Assuming the carrying weight is constant, the pump power $P_s$ is [10]:

\[
\begin{align*}
P_0 &= \frac{mg}{N_{\text{at}} R_1^2} \\
Q &= \bar{q} \cdot \frac{\mu^2 \rho_0}{\eta} \\
P_s &= N \cdot P_0 \cdot Q
\end{align*}
\]

where $P_s$ is supporting power consumption, and it only describes the pump power caused in oil pads. To control variables in this optimization, motor efficiency and pressure losses are ignored.

### 1.3 Particle swarm optimization

PSO is an emerging optimization method with high computational and implementation efficiency. The analysis of hydrostatic turntable using FDM technique requires performing many numerical calculations that make the process very time-consuming. Different values of $R_i$ result in different $\bar{w}$ and $\bar{q}$, namely different supporting power consumption. Aiming at the pump power optimization, many calculation loops are required to calculate the optimal value of $R_i$, so PSO is a suitable method to efficiently reduce the calculation time and simplify the optimization process. In this paper, PSO is basically performed by evaluating the fitness of each individual in the domain to search for a fitting range until it matches error tolerance. An inertia weight $c_i$ is introduced to improve local search ability [16]:

\[
\begin{align*}
v_n^{(k+1)} &= \alpha v_n^{(k)} + c_1 (P_{\text{best}_n} - x_n^{(k)}) + c_2 (G_{\text{best}_n} - x_n^{(k)}) \\
x_n^{(k+1)} &= x_n^{(k)} + v_n^{(k+1)}
\end{align*}
\]

where $x_n$ and $v_n$ are position and velocity of the $n^{th}$ individual, respectively, $P_{\text{best}_n}$ is position of the privative best fitness of the $n^{th}$ individual, $G_{\text{best}_n}$ is position of the global best fitness of all individuals, $k$ is loop iteration count.

Every individual represents a value of $R_i$, and make $R_i = x_n \cdot R_0$. FDM can solve $\bar{w}$ and $\bar{q}$ case by case. The fitness of
each individual is evaluated by the supporting power consumption. The minimization of pump power is chosen as optimal target. Computing will be terminated when every single individual as while as the whole swarm matches the error tolerance. Design tolerance of oil pad is generally at millimeter scale, so error tolerance of PSO is set as $10^{-2}$ [13]. The terminating condition is:

$$\begin{cases} 
|x_n^{(k)} - x_n^{(k-1)}| \leq tol_{PSO} \\
\max(x_n^{(k)}) - \min(x_n^{(k)}) \leq tol_{PSO}
\end{cases}$$

(11)

The algorithm presented in this work is mainly composed of three parts: FDM solution of the Reynolds equation, evaluation of supporting power and PSO for minimal power consumption; Fig. 3 illustrates the detailed calculation process [12]:

![Fig. 3. The PSO implementation framework](image)

### 2 NUMERICAL RESULTS AND DISCUSSION

#### 2.1 Influence of dimensionless pocket scale on the load-carrying performance of circular oil pad

The optimal pocket size can be determined by the algorithm shown in Fig. 3 for minimum supporting power consumption. Table 1 lists the values of the main parameters involved in the computational process.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $R_0$     | 0.085 m | $n$       | 5     |
| $m$       | 24800 kg | $\omega$ | 0.2   |
| $N$       | 20     | $c_1$     | 0.4   |
| $\eta$    | 0.06 Pa·s | $c_2$     | 0.9   |
| $tol_{FDM}$ | $1 \times 10^{-7}$ | $tol_{PSO}$ | $6 \times 10^{-3}$ |

The supporting performance of oil pad can be described by dimensionless load-carrying capacity and dimensionless flow rate. Dimensionless load-carrying capacity corresponds to the force output of oil pads, while dimensionless flow rate corresponds to the oil flow required to maintain the film thickness. Changes in oil pocket size will influence both the dimensionless carrying capacity and dimensionless flow rate. The pressure distribution for different oil pocket sizes is shown in Fig. 4:
From Fig.4, the oil pocket is filled with pressurized oil which provides most of the load-carrying capacity of the oil pad. Therefore, larger oil pocket means greater load-carrying force. With dimensionless oil pocket size increases, the slope at oil film sealing edge grows leading to the increase of dimensionless flow rate. Figure 5 shows the dimensionless pocket size effect on the supporting performance of oil pad:

From Fig.5, both the dimensionless carrying capacity \( \bar{w} \) and flow rate \( \bar{q} \) increase with the increase of pocket size \( \bar{R}_1 \). Flow rate \( \bar{q} \) grows more significantly than \( \bar{w} \). Larger dimension of the pocket means greater supply flow rate and lower pump pressure. The supporting power consumption is highly related to supply flow rate and pump pressure. The difference between the influence of the pocket dimension on supply flow rate and the pump pressure shows there is room for improvement. When \( \bar{R}_1 \) varies within the range \((0.1, 0.9)\), the influence of the pocket on supporting power is shown in Fig. 6, and a minimum value of the curve of supporting power \( P_s \) clearly exists:
Fig.6. Supporting power consumption varying $\bar{R}_1$

However, the exhaustive method can only show the trend vaguely thus it is not suitable for solving optimal result. The exact place where $P_s$ achieves its minimum needs to be determined with the help of PSO.

2.2 Experimental verification on a miniature oil pad

Due to cost and safety constraints, it is difficult for hydrostatic turntable to carry out experiments. To verify the theoretical calculations, an experiment table for single oil pad is designed. The carrying performance of a replaceable oil pad can be measured under different bearing situations. Oil is supplied by a pump and the flow rate is adjusted by a flow valve. Film thickness can be changed by a lead screw with very large reduction ratio. The film thickness varies 2 $\mu$m for every turn of screw hand wheel. Three force sensors, two dial indicators, one flowmeter, one manometer and one thermometer is deployed to measure related parameters. Data of each sensor is stored by a collector by real time. The experiment table is shown in Fig.7:

Fig.7. Experimental table to test single oil pad

To verify the reliability of theoretical algorithm, four different oil pads are tested. The structural parameters of them are shown in Table 2:

| Pad | $R_0$  | $R_1$  | $\bar{R}_1$ |
|-----|--------|--------|-------------|
| Pad 1 | 0.033m | 0.028m | 0.848485    |
| Pad 2 | 0.033m | 0.023m | 0.69697     |
| Pad 3 | 0.033m | 0.018m | 0.545455    |
| Pad 4 | 0.033m | 0.013m | 0.393939    |

Several tests are conducted to determine the dimensionless carrying ability and flow rate of oil pad based on Eq.2. Compared with the theoretical result, experimental results of dimensionless carrying ability are smaller and the maximum error is about 18%. Deviation also appears between theoretical and experimental dimensionless flow rate. The maximum error is about 25%. Although there are some errors between the calculation and tests, the varying trend of dimensionless carrying force and flow rate with $\bar{R}_1$ is same as predicted.
According to the result shown in Fig.6, there is an optimal value of $\bar{R}_1$ to achieve minimum supporting power $P_s$. The supporting power is calculated based on the experimental result shown in Fig.8.

As it shown in Fig.9, both theoretical and experimental result shows a minimum $P_s$ exists. Following optimization is carried out by the real hydrostatic turntable parameters shown in Table 1.

2.3 Optimal solution of oil pocket scale based on particle swarm optimization

The initial position and speed of particles $x_n$ are chosen randomly in the domain. Assuming $R_1=x_nR_0$, the oil pocket scale is related with particle positions. The film thickness is set to $H_0$ and supporting performance is solved by FDM to determine the pump power consumption. PSO can determine the position of minimum power consumption by evaluating the fitness of each particle. Fig. 10 shows the iteration process of PSO when $H_0=5\times10^{-5}$m:
Fig. 10. Iterative process of PSO ($H_0=5 \times 10^{-5}$ m)

where OS is optimal solution determined by PSO and $I_1, I_2, \ldots, I_n$ are individuals. From Fig. 7, the algorithm needs 7 loops to meet the error tolerance when the number of particle swarm is chosen as 5. Individuals get close to each other rapidly from distinct positions and converge to $x_n=0.5263$ where $P_s=30.39$. Fig. 11 and Fig. 12 show the iteration process of PSO when $H_0$ is set to $10 \times 10^{-5}$ m and $15 \times 10^{-5}$ m, respectively:

Fig. 11. Iterative process of PSO ($H_0=10 \times 10^{-5}$ m)

The algorithm needs 7 loops to meet the error tolerance. When $H_0=10 \times 10^{-5}$ m, all particles converge to $x_n=0.5467$ where $P_s=242.9$. When $H_0=15 \times 10^{-5}$ m, all particles converge to $x_n=0.5428$ where $P_s=819.7$. The change of $H_0$ value influences the final result $x_n$ no more than 3.9%. However, varying $H_0$ has a strong effect on $P_s$, meaning a thicker film thickness needs a higher supporting power. Taking 14 different samples of $H_0$, the relationship between film thickness, pump power consumption and optimal pocket size is shown in Fig. 13:

Fig. 13. Relationship between film thickness, supporting power and oil pocket scale.
The supporting power consumption raises with the film thickness increase. Optimal pocket scale changes no more than 7% under different film thickness, percentage which could be considered as a computational error. The average optimal pocket size of 14 samples is determined as $\bar{R}_1 = 0.5431$.

3 CONCLUSIONS

In this work, the load-carrying performance of hydrostatic turntable is analyzed by FDM solution of the Reynolds equation. The relationship between oil pocket scale and supporting power consumption is determined numerically. Reliability of the algorithm is verified by conducting experiments on a miniature testing table. PSO is carried out to solve the optimal pocket size for minim pump power. An efficient and practical optimization method for hydrostatic turntable design is presented, and the following conclusions can be drawn:

1. The oil pocket scale strongly influences the supporting power consumption. A properly designed oil pocket reduces the supporting power and improves the service performance of hydrostatic turntable.
2. When the film thickness is set as a constant (to a safety value), optimal pocket size of $\bar{R}_1 = 0.5431$ is calculated. With film thickness increases, the supporting power consumption grows, but optimal pocket size remains nearly same. The optimal oil pocket scale varies no more than 7% under different film thickness.
3. The design tolerances of the oil pad are at a scale to millimeters, and the FDM analysis processes are always very time-consuming. Therefore, the PSO is a suitable optimization method to speed up calculations and overcome such issues.

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5 NOMENCLATURES

- $R_0$ [m]: radius of oil pad
- $R_1$ [m]: is radius of oil pocket
- $H$ [m]: film thickness
- $H_0$ [m]: preset film thickness
- $W_i$ [N]: load-carrying capacity of $i$-th oil pad
- $N$ [1]: total number of oil pads
- $m$ [Kg]: is carrying load
- $g$ [m·s$^{-2}$]: is the acceleration of gravity.
- $p$ [Pa]: pressure
- $P_0$ [Pa]: pressure in the oil pocket
- $U_r$ [m·s$^{-1}$]: is radial velocity
- $U_\theta$ [m·s$^{-1}$]: is circumferential velocity
- $W$ [N]: load-carrying capacity
- $Q$ [m$^3$·s$^{-1}$]: volumetric flow rate
- $\eta$ [Pa·s]: viscosity
- $\bar{p}$ [1]: dimensionless pressure
- $\bar{h}$ [1]: dimensionless film thickness
\( \bar{U}_r \) [1] is dimensionless radial velocity
\( \bar{U}_\theta \) [1] is dimensionless radial velocity
\( \bar{W} \) [1] dimensionless carrying capacity
\( \bar{q} \) [1] dimensionless flow rate
\( A_{1,i,j}A_{2,i,j}\ldots \) [1] coefficient matrix
\( \bar{r}_{\text{step}} \) [1] is step length for the \( r \) coordinate
\( \bar{\theta}_{\text{step}} \) [1] is step length for the \( \theta \) coordinate
\( i \) [1] numerical counts of the elements on \( r \) coordinate
\( j \) [1] numerical counts of the elements on \( \theta \) coordinate
\( x_n \) [1] position of the \( n^{th} \) individual
\( v_n \) [1] velocity of the \( n^{th} \) individual
\( P_{\text{best}} \) [1] position of the privative best fitness of the \( n^{th} \) individual
\( G_{\text{best}} \) [1] position of the global best fitness of all individuals
\( k \) [1] loop iteration count

6 REFERENCES

[1] Nenzi Wang, Kuo-Chiang Cha, Hua-Chih Huang. (2012). Fast Convergence of Iterative Computation for Incompressible-Fluid Reynolds Equation. *Journal of Tribology*, vol. 134(4), 024504.

[2] Udaya P. Singh, Ram S. Gupta, Vijay K. Kapur. (2012). On the Steady Performance of Annular Hydrostatic Thrust Bearing: Rabinowitsch Fluid Model. *Journal of Tribology*, vol. 134(4), 044502.

[3] Zhifeng Liu, Yumo Wang, Ligang Cai, Qiang Cheng, Haiming Zhang. (2016). Design and manufacturing model of customized hydrostatic bearing system based on cloud and big data technology. *The International Journal of Advanced Manufacturing Technology*, vol. 84, p. 261–273.

[4] M. El Khilfi. (2007). Numerical Modeling of Non-Newtonian Fluids in Slider Bearings and Channel Thermohydrodynamic Flow. *Journal of Tribology*, vol. 129, p. 695-699.

[5] Tamás M., Tamás S., Imre K., István P. (2014). Optimization of the Shape of Axi-Symmetric Rubber Bumpers. *Strojniški vestnik - Journal of Mechanical Engineering*, vol. 60(1), p. 61-67

[6] Tom Gustafsson, K.R. Rajagopal, Rolf Stenberg, Juha Videman. (2015). Nonlinear Reynolds equation for hydrodynamic lubrication. *Applied Mathematical Modelling*, vol. 39, p. 5299-5309.

[7] Qianqian Yang, Ping Huang, Yanfei Fang. (2016). A novel Reynolds equation of non-Newtonian fluid for lubrication simulation. *Tribology International*, vol. 94, p.458–463.

[8] Masjedi M, Khonsari MM. (2015). On the effect of surface roughness in point-contact EHL: Formulas for film thickness and asperity load. Tribology International 82, 228-244

[9] Getachew A. D., Prawal S. (2011). THD analysis for finite slider bearing with roughness: special reference to load generation in parallel sliders. *Acta Mechanica*, vol. 222(1-2), p.1-15.

[10] Ligang Cai, Yumo Wang, Zhifeng Liu, Qiang Cheng. (2015). Carrying capacity analysis and optimizing of hydrostatic slider bearings under inertial force and vibration impact using finite difference method (FDM). *Journal of Vibration Engineering*, vol. 17(6), p. 2781-2794.

[11] Christoph Weißbacher, Christian Schellnegger, Alexander John, Thomas Buchgraber, Walter Pscheidt. (2014). Optimization of Journal Bearing Profiles With Respect to Stiffness and Load-Carrying Capacity. *Journal of Tribology*, vol. 136(3), 031709.

[12] Qiang Cheng, Chengpeng Zhan, Zhifeng Liu, Yongsheng Zhao, Peihua Gu. (2015). Sensitivity-based Multidisciplinary Optimal Design of a Hydrostatic Rotary Table with Particle Swarm Optimization. *Strojniški vestnik - Journal of Mechanical Engineering*, vol. 61(7-8), p. 432-447.

[13] S. H. Chang, Y. R. Jeng. (2014). A Modified Particle Swarm Optimization Algorithm for the Design of a Double-Pad Aerostatic
Bearing with a Pocketed Orifice-Type Restrictor. *Journal of Tribology*, vol. 136(2), 021701.

[14] Chia-Wen Chan. (2015). Modified Particle Swarm Optimization Algorithm for Multi-Objective Optimization Design of Hybrid Journal Bearings. *Journal of Tribology*, vol. 137(2), p. 021101.

[15] Saeed Soltani-Mohammadi, MohammadSafa, HadiMokhtari. (2016). Comparison of particle swarm optimization and simulated annealing for locating additional boreholes considering combined variance minimization. *Computers & Geosciences*, vol. 95, p. 146–155.

[16] Yu-xin Zheng, Ying Liao. (2016). Parameter identification of nonlinear dynamic systems using an improved particle swarm optimization. *Optik*, vol. 127, p. 7865–7874.

[17] Srisha Rao M V, G. Jagadeesh. (2010). Vector Evaluated Particle Swarm Optimization (VEPSO) of Supersonic Ejector for Hydrogen Fuel Cells. *Journal of Fuel Cell Science and Technology*, vol. 7(4), 041014.

[18] Forrest W. Flocker, Ramiro H. Bravo. (2016). On Global Convergence in Design Optimization Using the Particle Swarm Optimization Technique. *Journal of Mechanical Design*, vol. 138(8), 081402.

[19] Dunbing Tang, Min Dai, Miguel A. Salido, Adriana Giret. 82–95 (2016). Energy-efficient dynamic scheduling for a flexible flow shop using an improved particle swarm optimization. *Computers in Industry*, vol. 81, p. 82–95.

[20] Wenjian Luo, Juan Sun, Chenyang Bu, Houjun Liang. (2016). Species-based Particle Swarm Optimizer enhanced by memory for dynamic optimization. *Applied Soft Computing*, vol. 47, p. 130–140.

[21] Zhifeng Liu, YumoWang, Ligang Cai, Yongsheng Zhao, Qiang Cheng, Xiangmin Dong. (2017). A review of hydrostatic bearing system: Researches and applications. *Advances in Mechanical Engineering*, vol. 9(10): 1-27.