Determination of Dynamical exponents of Graphene at quantum critical point by holography

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Abstract: We calculate the transport of a theory with two conserved currents by holographic method and compare it with graphene data to determine its dynamical exponents \((z, \theta)\) which characterizes a QCP. As a result, we find that the electric and the thermal conductivity data can be fit much more naturally if we assume \((z, \theta) = (3/2, 1)\) rather than \((1, 0)\). Furthermore, we find that thermoelectric power data at high temperature can be fit if we use \((3/2, 1)\) but not at all by \((1, 0)\). The \(\theta = 1\) result can be interpreted as taking into account the fermionic nature of the electrons and \(z = 3/2\) can be interpreted as the flattened band by the strong interaction.

Keywords: Gauge/Gravity duality, hyper-scaling violation, quantum critical points, transports, graphene
1 Introduction:

The strong correlation is the property of a phase of general matters because even a weakly interacting material can become strongly interacting in some parameter region. It happens when the Fermi surface (FS) is tuned to be small, or when conduction band is designed to be flat. The Coulomb interaction in a metal is small only because the charge is screened by the particle-hole pairs which are abundantly created when FS is large. In fact, any Dirac material is strongly correlated as far as its FS is near the tip of the Dirac cone. This was demonstrated in the clean graphene [1, 2] and the surface of topological insulator [3–5] through the anomalous transports that could be quantitatively explained by a holographic theory [6–8]. In twisted bi-layered graphene [9, 10] flat band appears due to the formation of effective lattice system called Moire lattice, which has larger size than the original lattice. In short, strong correlation phenomena are ubiquitous, where the traditional methods are not working very well, therefore a new method has been longed-for for many decades.

The strongly interacting system (SIS) is hard to be characterized in terms of its basic building blocks and one faces the question how to simplify the system to make a sensible physics with only a few parameters. One possibility is that they become simple at quantum critical point (QCP) by the universality coming from the loss of system information, which is similar to black hole system. In this sense, the SIS and holographic theory are similar by sharing the property of black hole.

The purpose of this paper is to examine the transport data of graphene by holographic theory with two currents [6, 11] to determine its dynamical exponents \((z, \theta)\) which characterizes a QCP. In our previous work [6], we assumed that the theory has a QCP at \((z, \theta) = (1, 0)\) relying on the presence of the Dirac cone, and showed that there must be at least two independently conserved currents. There, we also had to assume that the entropy density is a free parameter and that that can be tuned to fit the data.

In this paper, by choosing the different set of dynamical exponents, we could eliminate the last assumption, namely, the entropy density is determined by other parameters. That
is, the electric and the thermal conductivity data can be fit much more naturally if we assume \((z, \theta) = (3/2, 1)\) rather than \((1,0)\). We also find that thermo-electric power data at high temperature can be fit if we use \((3/2,1)\) but not at all by \((1,0)\). Our work demonstrate that critical exponents together with the ratio of the conserved charges completely determine the transport data of strong correlated system.

2 The model: hyperscaling violating geometry with two currents

We consider a 4 dimensional action with asymptotically AdS metric \(g_{\mu\nu}\), a dilaton field \(\phi\), a gauge fields \(A_\mu\) to support the hyperscale violating geometry, two extra gauge fields \(B^a_\mu\) which are dual to two conserved currents, and the axion fields \(\chi_1, \chi_2\) to break the translational symmetry.

\[
S = \int_M d^4x \mathcal{L}
\]

\[
\mathcal{L} = \sqrt{-g} \left( R + \sum_{i=1}^{2} V e^{\gamma \phi} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} Z_A F^2 - \sum_{a} \frac{1}{4} Z_a G^2_{(a)} - \frac{1}{2} Y \sum_{i} (\partial \chi_i)^2 \right)
\]

where \(F = dA, G_{(a)} = dB_a\). We use ansatz

\[
Z_A = e^{\lambda_1 \phi}, \quad Z_a = \bar{Z}_a e^{\eta \phi}, \quad Y = e^{-\eta \phi}, \quad \chi_i = \beta x_i,
\]

where \(\beta\) denote the strength of momentum relaxation. The equations of motion for 3 Maxwell fields and gravity are

\[
\partial_\mu (\sqrt{-g} g^{\mu\nu} Y \sum_i \partial_\nu \chi_i) = 0, \quad \partial_\mu (\sqrt{-g} Z_A F^\mu) = 0, \quad \partial_\mu (\sqrt{-g} Z_a G^\mu_{(a)}) = 0,
\]

\[
R_{\mu\nu} = \frac{1}{2} \sqrt{-g} g_{\mu\nu} \mathcal{L} + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \frac{Y}{2} \sum_i \partial_\mu \chi_i \partial_\nu \chi_i + \frac{1}{2} Z_A F^\mu_F F^\nu - \sum_a \frac{1}{2} Z_a G^\rho_{(a)} G^\rho_{\mu\nu}
\]

\[
\Box \phi + \sum_i V e^{\gamma \phi} - \frac{1}{4} Z_A'(\phi) F^2 - \frac{1}{4} \sum_a Z_a'(\phi) G^2_{(a)} - \frac{1}{2} Y'(\phi) \sum_i (\partial \chi_i)^2 = 0.
\]

The solution for the dilaton field is given by

\[
\phi(r) = \nu \ln r, \quad \text{with} \quad \nu = \sqrt{(2 - \theta)(2z - 2 - \theta)}.
\]

The gauge couplings \(Z_1\) and \(Z_2\) can be solved to give

\[
Z_A(\phi) = e^{\lambda_1 \phi} = r^{\theta - 4}, \quad Z_a(\phi) = \bar{Z}_a e^{\eta \phi} = \bar{Z}_a r^{2z - \theta - 2}, \quad Y(\phi) = e^{-\eta \phi},
\]

with \(\lambda_1 = (\theta - 4)/\nu, \quad \eta = \nu/(2 - \theta), \quad \gamma = \frac{\theta}{\nu}, \quad V = \frac{z - \theta + 1}{2(z - 1)} q_0^2\).
Finally, the solutions are given by

\[ A = a(r)dt, \quad B_1 = b_1(r)dt, \quad B_2 = b_2(r)dt, \quad \chi = (\beta x, \beta y), \]

\[ ds^2 = r^{-\theta} \left( -r^{2\theta} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 (dx^2 + dy^2) \right), \]

\[ f(r) = 1 - m r^{\theta - \epsilon - 2} - \frac{\beta^2}{(\theta - 2)(z - 2)} r^{\theta - 2z} + \frac{Z_1 q_1^2 + Z_2 q_2^2 (\theta - z) r^{2\theta - 2z - 2}}{2(\theta - 2)}, \]

\[ a(r) = \frac{-q A}{2 + z - \theta (r^{2\theta + z - \theta} - r^{2 + z - \theta})}, \quad b_a(r) = (\mu_a - q_a r^{\theta - \epsilon}), \]

where \( a = 1, 2 \). Here the reader need to remember that \( r \) as well as other physical coordinate in these expressions are dimensionless variables scaled by the AdS scale \( L \).

From the equations of motion for gauge fields \( B \) eq. (2.3), we can obtain the charge density as the constants of integration

\[ Q_a = \sqrt{-g} Z_a G_{(a)}^{tr} = \bar{Z}_a q_a (z - \theta). \]

The entropy density and the Hawking temperature read

\[ s = 4\pi r_H^{2-\theta}, \quad 4\pi T = (z + 2 - \theta) r_H^{\theta - \epsilon} - \frac{\beta^2 r^{\theta - \epsilon}}{2 - \theta} - \frac{1}{2} \left( \frac{Q_1^2}{Z_1} + \frac{Q_2^2}{Z_2} \right) r_H^{2\theta - 2z} (z - \theta)^2, \]

\[ ds^2 = -dr^2 + \frac{du^2}{l_{eff}^2 u^2} + r_H^{2-\theta} (dx^2 + dy^2). \]

### 3 Calculation of DC transport:

Consider following perturbation to compute the transport coefficients:

\[ \delta g_{tx} = h_{tx}(r) + t f_{3x}(r), \quad \delta g_{rx} = h_{rx}(r), \quad \delta B_{ax} = b_{ax} - t f_{ax}, \quad \delta \chi_1 = \varphi_x(r) \]

We choose the functions \( f_i(r) \) as

\[ f_{1x} = -E_1 + \zeta b_1(r) \]
\[ f_{2x} = -E_2 + \zeta b_2(r) \]
\[ f_{3x} = -\zeta U(r) \]

so that the linearised Einstein equations is time-independent. Here, \( E_1, E_2 \) are thermo-electric forces acting on \( J_1, J_2 \) respectively and \( \zeta \) is temperature gradient which is defined as \( \zeta = -(\nabla T / T) \). After the calculation is finished we will set \( E_1 = E_2 \). The transports can be computed at the event horizon. We take the Eddington-Finkelstein coordinates \((v, r)\)
where the background metric is regular at the horizon,
\[ ds^2 = -U dt^2 - 2\sqrt{UV} dv dr + W dx^2 \]  
(3.3)

where \( v = t + \int dr \sqrt{V/U} \). In this coordinates, the metric perturbation can be written as
\[ \delta g_{\mu\nu} dx^\mu dx^\nu = h_{tx} dv dx + \left( h_{rx} - \sqrt{\frac{V}{U}} h_{tx} \right) dr dx. \]  
(3.4)

To guarantee the regularity of the metric with perturbation at the horizon, we require the last term to vanish at the horizon so that
\[ h_{rx} \sim \sqrt{\frac{V}{U}} h_{tx} \text{ at the horizon.} \]  
(3.5)

Also, we can reexpress the gauge fields in the Eddington-Finkelstein coordinates to obtain the regularity condition at the event horizon:
\[ \delta B_{ax} \sim b_{ax} + E_a v - E_a \int dr \sqrt{V/U}, \]  
(3.6)

Then, the full gauge field with perturbation will take the regular form of \( \delta B_{ax} \sim E_a v + \cdots \) in the Eddington-Finkelstein coordinates when we demand
\[ b'_{ax} \sim \sqrt{\frac{V}{U}} E_a \]  
(3.7)

The \((r,x)\)-component of the Einstein equation are given by
\[ \frac{Y \beta^2}{W} h_{rx} - \frac{1}{U} \sum_{a=1,2} \left( Z_a b'_a f_{ax} + f'_{3x} \right) + \frac{W' f_{3x}}{UW} = 0 \]  
(3.8)

The regularity conditions at the horizon yields
\[ h_{tx}|_H = -\frac{1}{\beta^2 Y} \left( s T\zeta + \sum_{i=1,2} Z_i q_i E_i(z - \theta) \right) \]  
(3.9)

The conserved currents are defined by [12]
\[ J_a = \sqrt{-g} Z_a G^{rr}_{(a)}, \]  
(3.10)
\[ Q = \frac{U^2}{\sqrt{UV}} \left( \frac{h_{tx}}{U} \right)' - \sum_{a=1,2} b_a J_a, \]  
(3.11)

where the index \( a = 1,2 \) is for two currents which are dual to the two gauge field \( B \). These currents are radially conserved so that their boundary values can be computed at the horizon [12]. Using the horizon behavior (3.6), (3.9), we can get the boundary current
in terms of the external sources:

\[
J_1 = \left( Z_1 + \frac{Q_1^2}{WY\beta^2} \right) E_1 + \frac{Q_1 Q_2}{WY\beta^2} E_2 + \frac{4\pi T Q_1}{Y\beta^2} \zeta
\]

\[
J_2 = \frac{Q_1 Q_2}{WY\beta^2} E_1 + \left( Z_2 + \frac{Q_2^2}{WY\beta^2} \right) E_2 + \frac{4\pi T Q_2}{Y\beta^2} \zeta
\]

\[
Q = \frac{4\pi T Q_1}{Y\beta^2} E_1 + \frac{4\pi T Q_2}{Y\beta^2} E_2 + \frac{16\pi^2 W T^2}{Y\beta^2} \zeta
\]  

(3.12)

We can write (3.12) in matrix form, \( J_i = \sum_{ij} E_j \), with \( J_3 = Q \) and \( E_3 = \zeta \):

\[
\begin{pmatrix}
\sigma_{11} & \sigma_{12} & \alpha_1 T \\
\sigma_{21} & \sigma_{22} & \alpha_2 T \\
\bar{\alpha}_1 T & \bar{\alpha}_2 T & \bar{\kappa} T
\end{pmatrix} := \Sigma.
\]  

(3.13)

Notice that the matrix \( \Sigma \) is symmetric, which means

\[
\sigma_{12} = \sigma_{21}, \quad \alpha_i = \bar{\alpha}_i
\]  

(3.14)

The heat conductivity \( \kappa \) is defined by the response of the temperature gradient \( \zeta \) to the heat current in the absence of electric currents \( J_1 \) and \( J_2 \): We can express \( E_1 \) and \( E_2 \) in terms of \( \zeta \) by setting \( J_1 \) and \( J_2 \) to vanish in (3.12). Substituting these expressions for \( E_i \) to the last line of (3.12) and taking derivatives with respect to the temperature gradient, we can get

\[
\kappa = \bar{\kappa} - \frac{T \bar{\alpha}_1 (\alpha_1 \sigma_{22} - \alpha_2 \sigma_{12})}{\sigma_{11} \sigma_{22} - \sigma_{12} \sigma_{21}} - \frac{T \bar{\alpha}_2 (\alpha_2 \sigma_{11} - \alpha_1 \sigma_{21})}{\sigma_{11} \sigma_{22} - \sigma_{12} \sigma_{21}}
\]  

(3.15)

with \( \bar{\kappa} = 4\pi s T / Y\beta^2 \). The Seebeck coefficient is defined by

\[
S_i = \sum_j \sigma_{ij}^{-1} \alpha_j
\]  

(3.16)

Then, the transport coefficients for conserved currents can be calculated as followings:

\[
\sigma_{ij} = Z_i \delta_{ij} + \frac{Q_i Q_j}{WY\beta^2}, \quad \alpha_i = \frac{4\pi Q_i}{Y\beta^2}, \quad \bar{\kappa} = \frac{16\pi^2 W T}{Y\beta^2}
\]  

(3.17)

\[
\kappa = \bar{\kappa} + \frac{s Q_i / Z_i}{1 + \sum_i 4\pi Q_i^2 / s Z_i Y\beta^2}, \quad S_i = \frac{s Q_i / Z_i}{1 + \sum_i (Q_i^2 / Z_i)}
\]  

(3.18)

If we define the total electric current as \( J = \sum_i J_i \) and thermo-electric force as \( E_i = E - T \nabla (\mu_i / T) \), the electric conductivity based on total current is given by

\[
\sigma = \frac{\partial J}{\partial E} = \sum_{ij} \sigma_{ij} = Z + \frac{Q^2}{WY\beta^2}
\]  

(3.19)

where \( Q = \sum_i Q_i \) and \( Z = \sum_i Z_i \), showing the additivity of charge-conjugation-invariant
part [13] of the electric conductivity. If we define the heat conductivity due to the $i$-th current by $1/\kappa_i = 1/\bar{\kappa} + Q_i^2/Z_i s^2 T$, then the heat conductivity formula leads us to additivity of dissipative part of the inverse heat conductivity. Therefore

$$D[1/\kappa] = \sum_i D[1/\kappa_i], \quad \bar{D}[\sigma] = \sum_i \bar{D}[\sigma_i], \quad (3.20)$$

where $D[f]$ denotes the dissipative part of $f$ and $\bar{D}[f] = f - D[f]$.

The total Seebeck coefficient $S$ by the two currents is given by

$$S = S_1 + S_2 = \frac{4\pi W (Z_1 Q_2 + Z_2 Q_1)}{Z_1 Q_1^2 + Z_2 Q_2^2 + Z_1 Z_2 W \beta^2} \quad (3.21)$$

Finally the two currents are independently conserved for a short moments but long enough for the hydrodynamic equilibrium to be reached, as it was argued in [6, 14]. In this case, individual charges, the hole and electron charges are separately conserved, therefore

$$Q_i = g_i Q, \quad \text{so that} \quad Q_2 = g Q_1 \quad (3.22)$$

for some $g_1, g_2, g$. Then the experimental data of graphene will be well fit by our two current theory as we will see below.

4 Theory vs experiments

The total electric current $J$ and total number current $J_n$ are defined by $J = J_e + J_h$, $J_n = J_e - J_h$, respectively and their corresponding densities (electric charge densities and number densities) are related by $Q_1 = q_e n_1$ and $Q_2 = -q_e n_2$ with charge of electron $q_e = -1$. The total electric charge density and total number density are defined by $Q = Q_1 + Q_2$ and $Q_n = -Q_1 + Q_2$ which can be connected with the proportionality constant $g_n$ such that $Q_n = g_n Q$. Notice that $\eta = 0$ when $z = (\theta + 2)/2$ so that $Z_a = Z_n$ and $Y = 1$. From

**Figure 1.** Comparision with real experiment : (a) density plot of electric conductivity $\sigma$ and (b) of thermal conductivity $\kappa$. Red circles are for data used in [1, 2] and black curves are for two current model. The region shaded with blue is for the Fermi Liquid which is far from our theory.
now on, we take \( z = 3/2, \theta = 1 \). There are two reasons for choosing this \((z, \theta)\): (i) \( \theta = 1 \) is necessary to encode the fermionic nature of the system. (ii) \( z = 3/2 \) is the optimized dynamical exponent for fitting the experimental results which will be shown later. Then, the total electric conductivity \( \sigma = \frac{\partial J}{\partial E} \) and \( \kappa \) can be expressed in terms of \( Q \) and \( g_n \):

\[
\sigma = \sigma_0 \left( 1 + \frac{Q^2}{Q_0^2} \right), \quad \kappa = \frac{\kappa}{1 + (1 + g_n^2)(Q/Q_0)^2}.
\]

(4.1)

where \( \sigma_0 = 2Z_0 \) and \( Q_0 = \sigma_0 s \beta^2 / 4\pi \)

Notice that in all our formula so far we used dimensionless version of the parameters which was introduced at the level of the equation of motion before we get the solution. However, for the numerical fitting, all the dimension of the parameters should be restored to their original dimensionful version. Following prescription for the restoration of dimensionality is useful.

\[
\beta \to \beta L, \quad T \to \frac{k_B T}{\hbar v_F}, \quad s \to sL^2, \quad Q \to QL^2
\]

(4.2)

where \( v_F \sim c/300 = 1 \times 10^6 m/s \) which is the Fermi velocity in graphene and \( T \) is restored only for (2.14). With such prescription,

\[
\sigma_0 = \frac{e^2}{\hbar} 2Z_0, \quad \frac{\kappa}{T} = \frac{4\pi k_B^2}{\hbar} \frac{s}{\beta^2}, \quad Q_0^2 = \frac{2Z_0 s \beta^2}{4\pi L^4}.
\]

(4.3)

Notice that \( T \) under \( \bar{\kappa} \) in (4.3) is the real temperature which is not relevant to (4.3). To fit the experimental results in figure 1 for transports in graphene, we used four measured values, \( \sigma_0 = 0.338 k\Omega^{-1} \), \( \bar{\kappa} = 7.7 nW/K \). From the curvature of density plot of \( \kappa \), we fix \( g_n = 3.3 \) and assumed charge conjugation symmetry to set \( Z_1 = Z_2 = Z_0 \). Then, the parameters of the theory can be determined: \( L = 0.2 \mu m, 2Z_0 = 1.387, \beta^2 = 96.75/(\mu m)^2 \). In previous work [6], we replaced the horizon area \( 4\pi r_n \) as the entropy density and considered the latter as a free parameter to tune. On the other hand, we use \( r_n \) as a function of other physical parameters such as temperature (\( T \)), charge density (\( Q \)), and impurity density \( \beta^2 \) which comes from (2.14). In that sense, we could fit the data with one less parameter.

One more available data for the graphene is the seebeck coefficient \( S \) given in ref. [15]. \( S \) can be expressed in terms of \( Q \) and \( g_n \):

\[
S = -\frac{k_B}{e} \frac{8\pi Q/2Z_0 \beta^2}{1 + (1 + g_n^2)Q^2/Q_0^2}
\]

(4.4)

If we try to fit the experimental data of graphene, it seems that one current model without dissipation follows the hydrodynamic model. See Figure 2 (a). For the two currents models with \( z = 1, \theta = 0 \) (Figure 2 (b)), the theory curve of \( S/T \) goes to constant at large \(|Q|\) and the height of its peak is independent of the temperature. On the other hand, the experimental curves of \( S/T \) decreases as \(|Q|\) increases and the height of its peak also decreases as the temperature rises. Both of the features are not even close to the qualitative
Figure 2. Theory vs data for Seebeck effect: (a) We take $g_n = 1$ which corresponds to one current model, and we set $\beta = 0$ to compare with the hydrodynamics result (Dashed line). (b) Seebeck coefficient for $z = 1$ and $\theta = 0$. We used the parameters $L = 0.2 \mu m$, $\beta^2 = 1406/(\mu m)^2$, $2Z_0 = 1.387$, and $g_n = 16$. (c) For $z = 3/2$ and $\theta = 1$, Seebeck coefficient at low temperature fits well with experiment. We used the parameters $L = 0.2 \mu m$, $\beta^2 = 1406/(\mu m)^2$, $2Z_0 = 1.387$, and $g_n = 3.3$. Circles are for data used in [15].

feature of the experimental data.

On the other hand, for $z = 3/2$ and $\theta = 1$, the two currents model fits very well with the data when $T$ is low enough. Also, the theory curves have tendency of convergence for large $|Q|$ and the height of its peak lowered as one raises the temperature. It is natural that the model does not fit with the experimental for large $T$ because our theory does not include the phonon effect which is important for large temperature. The reason to take $z = 3/2$ is following: Due to the Null energy condition $(2 - \theta)(2z - 2 - \theta) \geq 0$, we cannot take $z < 3/2$ with $\theta = 1$, and for $z > 3/2$, the bigger value $z$ has, the greater the inconsistency of the theoretical curve with the experimental results becomes. We conclude that the graphene data can be fit with holographic theory if we choose the dynamical exponents $(z, \theta) = (3/2, 1)$.

5 Conclusion

We determined the QCP’s dynamical exponents for the graphene system. The $\theta = 1$ result can be interpreted as taking into account the fermionic nature of the electrons and $z = 3/2$ can be interpreted as the flattened band by the strong interaction.

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