The Weissenberg number, defined as the product of relaxation time and strain rate, reflects the ratio of elastic forces to viscous forces in a viscoelastic flow. Thus, at high strain rate based on a small (micrometer-scale) characteristic length, elastic properties are important for understanding the flow properties of dilute (100 ppm and 10 ppm) and ultra-dilute (1 ppm) aqueous polymer solutions. However, the very small elasticity and high strain rate make it difficult to measure the elastic properties precisely. In this study, flow properties and elastic stresses were investigated for dilute and ultra-dilute polymer solutions flowing through small slits with characteristic lengths of between 122 µm and 1.1 mm. The pressure drop (PD) and jet thrust (JT) for polyethylene oxide (PEO) and polyacrylamide (PAA) solutions were measured from their flow properties and elastic stresses, respectively. The pressure difference upon passing through a small slit and the momentum issuing from the slit at a constant flow rate were measured as the PD and JT, respectively. Although the measured PD of water and silicone oil agreed with the numerical predictions of the Navier–Stokes equation, the resultant PDs for all PEO samples were less than those for water and silicone oil. Similar results were obtained for all PAA samples; the PDs for aqueous solutions of 100 ppm, 10 ppm, and 1 ppm PAA were reduced. The JT values were also smaller than those for water and silicone oil. Even though ultra-dilute solutions were used, it is interesting that the PDs and JTs were reduced. Furthermore, to understand the experimental results, elastic stress was determined from the JT. The calculated elastic stress depended on the mean velocity. The relationship between the flow properties and the elastic properties was clarified, and the effect of surface tension was described by estimating the capillary number.

Key Words: Slit / Dilute polymer solution / Pressure drop / Jet thrust / Elastic stress

1. INTRODUCTION

The rheological properties of complex fluids (e.g., aqueous surfactant and polymer solutions, suspensions, colloid mixtures) and methods for measuring them are important for both fundamental research and practical applications such as plastic injection molding,1) coating materials and techniques,2) flows of blood and other biological fluids in the human body,3) and food science.4) Previous studies have estimated the viscoelastic properties of complex fluids. Boger et al. used the Barus effect to measure the normal stress of dilute polymer solutions,5) and Brown et al. used flow birefringence spectroscopy to investigate the elastic properties of polymer solutions.6) Hidema et al. used an abrupt-contraction syringe-shaped channel to measure elongational viscosity,7) and also analyzed two-dimensional turbulence in several types of dilute polymer solutions, and reported that the laminarization of turbulence due to elongational viscosity was dependent on the flexibility of the polymer chains.8–10) Ogata et al. investigated the effect of wall surface roughness on pressure drop in micro-channel flows of dilute polymer solutions11) and found that the hydrophobic and hydrophilic walls exhibited different pressure drops. Hasegawa et al. measured elastic stresses via the jet thrust (JT) and jet reaction of solutions flowing from small orifices.12,13) The abovementioned studies were concerned with the elasticity of dilute polymer solutions, and the elastic stress was measured from the JT and jet reaction as the solutions flowed from a small orifice. While these methods have both advantages and disadvantages, dilute and ultra-dilute polymer solutions (defined here as 100 or 10 ppm and 1 ppm, respectively) are non-Newtonian fluids and have been studied previously.

For example, the turbulent drag-reduction effect of dilute polymer solutions has been reported in fluids engineering,14–16) as has the use of such solutions for energy conservation.17–19) In these applications, the characteristic length is

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relatively large (> 1.0 mm). Meanwhile, flows through narrow pipes, rectangular tubes, and micro-apertures with relatively small characteristic lengths (< 1.0 mm) have also been investigated. Meng et al. investigated flows of dilute polymer solutions passing through capillaries with an inner diameter of 100-500 µm, and reported that no effect of surface tension or capillary size was observed. However, they reasoned that these effects would appear when the inner diameter was less than 100 µm. Previously, we investigated the flow properties of dilute and ultra-dilute polymer solutions in narrow pipes and found a pseudo-laminarization effect (i.e., laminar flow was maintained in the transition region) even though ultra-dilute (1 ppm) solutions were used. Moreover, we discussed the possibility of elastic properties at high strain rates. This is because the Weissenberg number, defined as the ratio of elastic forces to viscous forces (= relaxation time × strain rate), is relatively high when a small characteristic length leads to a high strain rate even though dilute and ultra-dilute solutions have very small relaxation times. However, investigation of the flow properties (particularly the pressure drop (PD)) of dilute and ultra-dilute polymer solutions with characteristic lengths of less than 100 µm has been limited because of the difficulty of machining capillaries and pressure holes. In the present study, we investigated the flow properties of dilute and ultra-dilute polymer solutions by using small apertures (slits) and measuring the associated PDs and JTs. Moreover, we discuss the elastic stresses to understand the experimental results.

2. TEST FLUIDS

Eight test fluids were used, namely, deionized water (RFD240NC; ADVANTEC Co., Ltd., Japan), silicone oil (KF-96L; Shin-Etsu Chemical Co., Ltd., Japan), and two types of polymer solutions, namely, polyethylene oxide (PEO; average molecular weight: 4.6 × 10^6 (Sumitomo Seika Chemicals Co., Ltd., Japan) and polyacrylamide (PAA; molecular weight: 5.0 × 10^6 (Wako Pure Chemical Industry Co., Ltd., Japan), in dilute (100 ppm and 10 ppm) and ultra-dilute (1 ppm) concentrations. PEO and PAA are non-ionic and water-soluble polymers that are widely used in rheological studies. Two different types of dilute polymer solutions were used in this study to ensure the generality of the experimental results. The main difference between the two is that PEO has strongly non-Newtonian viscosity even at dilute concentrations, whereas the non-Newtonian viscosity of dilute PAA is less pronounced. In addition, the elongational properties of PAA are stronger than those of PEO. Because dilute and ultra-dilute solutions were used, the density of each test fluid was assumed to be ρ = 1.0 × 10^3 kg/m^3 with the exception of silicone oil. We measured the viscosity by using a capillary viscometer (inner diameter: 125-680 µm; wall shear rate: between 1.7 × 10^3 s^-1 and 5.0 × 10^5 s^-1). In Fig. 1, the wall shear stress τ_w is plotted against the wall shear rate SR_w. The test fluids other than PEO 100 ppm, PEO 10 ppm, and PAA 100 ppm exhibited Newtonian viscosity of μ = 1.0 × 10^-3 Pa·s. For PEO 100 ppm, PEO 10 ppm, and PAA 100 ppm, a non-Newtonian viscosity was obtained (i.e., the shear viscosity depended on the wall shear rate), and therefore a power-law model was applied, namely, Eq. (1).

\[ \tau_w = m (SR_w)^n \]  

(1)

where m is the dilatant viscosity and n is the power-law index. We estimated m = 7.40 × 10^-3 Pa·s^n and n = 0.870.
for PEO 100 ppm, $m = 1.81 \times 10^{-3}$ Pa·s$^n$ and $n = 0.956$ for PEO 10 ppm, and $m = 2.21 \times 10^{-3}$ Pa·s$^n$ and $n = 0.947$ for PAA 100 ppm. The silicone oil had a dynamic viscosity $\mu/p$ of $5.0 \times 10^{-6}$ m$^2$/s. Furthermore, the static surface tension $\sigma_s$ and the dynamic surface tension $\sigma_d$ were measured using the du Noüy method (brass ring: inner diameter, 15.0 mm; outer diameter, 15.5 mm) and the droplet method (SUS304 capillary: inner diameter, 0.28 mm; outer diameter, 0.35 mm), respectively.$^{22,23}$ In Fig. 2, the dynamic surface tension $\sigma_d$ and the static surface tension $\sigma_s$ of the test fluids are plotted against the surface age $T_s$. The values of $\sigma_s$ for each of the polymer solutions were approximately 10% smaller than those of water, but the values of $\sigma_d$ for all the polymer solutions agreed with those of water.

3. EXPERIMENTAL PROCEDURES

The experimental apparatus for measuring the PD and JT is shown schematically in Fig. 3. A syringe pump (JP-HPI; Furue Science Co., Ltd., Japan) pushed the test fluid into an acrylic channel (inner diameter, 25 mm; length, 180 mm). Figure 3(a) shows a schematic of the setup used to measure the PD $\Delta p$ with a pressure transducer (SPX-D, SPD-12, Tsukasa Sokken Co. Ltd., Japan). A jet of the same test fluid stored in the acrylic channel flowed out through a small slit with constant flow rate $Q$ (Fig. 3(b)) and was received by a plastic cup connected to an electronic balance (GR-200; A&D Co., Ltd., Japan). The distance between the outlet region and the cup was approximately 20 mm. Figure 3(c) shows a schematic of the slit that was used, where the width $B$ was 122 $\mu$m, 480 $\mu$m, or 1.1 mm. In each case, the thickness $H$ was 100 $\mu$m and the slit length $W$ was 20 mm. Figure 3(d) shows a schematic of the control surface, where $T_m$ is the momentum of the jet and $T_{out}$ is the momentum flowing out of the control surface. The force $F$ detected by the electronic balance is the sum of these two values, namely, Eq. (2).

$$F = T_m + T_{out} \tag{2}$$

When the inner diameter of the cup (= 120 mm) is very large, Eq. (2) becomes Eq. (3).

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Fig. 2 Dynamic surface tension $\sigma_d$ plotted against surface age $T_s$ with various static surface tensions $\sigma_s$ (solid, broken, and dotted lines) for (a) water and silicone oil, (b) PEO, and (c) PAA.
Thus, the JT $T_m$ is detected via momentum measurements.

4. NUMERICAL ANALYSIS

The dimensionless Navier–Stokes equations

$$\frac{\partial u'_x}{\partial x'} + u'_x \frac{\partial u'_x}{\partial x'} + u'_y \frac{\partial u'_x}{\partial y'} = -\frac{\partial p'}{\partial x'} + \frac{1}{Re} \left\{ \frac{\partial^2 u'_x}{\partial x'^2} + \frac{\partial^2 u'_x}{\partial y'^2} \right\}$$  \hspace{0.5cm} (4)

$$\frac{\partial u'_y}{\partial x'} + u'_x \frac{\partial u'_y}{\partial x'} + u'_y \frac{\partial u'_y}{\partial y'} = -\frac{\partial p'}{\partial y'} + \frac{1}{Re} \left\{ \frac{\partial^2 u'_y}{\partial x'^2} + \frac{\partial^2 u'_y}{\partial y'^2} \right\}$$  \hspace{0.5cm} (5)

and the continuity equation

$$\frac{\partial u'_x}{\partial x'} + \frac{\partial u'_y}{\partial y'} = 0$$  \hspace{0.5cm} (6)

were analyzed numerically for slit flows, where the superscript asterisk denotes a dimensionless quantity. The problem is two-dimensional because no quantity varies in the spanwise ($z$) direction. In eqs. (4)-(6), $u'_x$ and $u'_y$ are the dimensionless velocity components in the $x'$ and $y'$ directions, respectively, $p'$ is the dimensionless pressure, and $t'$ is the dimensionless time. For a Newtonian fluid passing through a slit, the finite-volume method was adopted, in which the velocities and pressures were expressed on a staggered grid with a two-dimensional coordinate system. Coupling of the velocity and pressure was achieved via the SIMPLE method. The convective and viscous terms were discretized by using the first-order upwind difference and the second-order central difference, respectively. Figure 4 shows the numerical regions and the boundary conditions required for the numerical analysis. The slit half-width was $0.5B$, the length of the channel both upstream and downstream of the slit part was $40B$, and the upstream and downstream width was $20B$. The value of the thickness ratio $H/B$ was that corresponding to whichever slit was used. The boundary conditions were two-dimensional Poiseuille flow on the inlet boundary, zero velocity on the wall, zero velocity gradient in the $x$ direction on the center line, zero velocity in the $y$ direction on the center line, and free outflow and zero pressure on the outlet boundary. There were 1024 grid points in the $x$ direction and 512 in the $y$ direction. Preliminary simulations showed that the numerical results were independent of the numbers of grid points. Figure 5(a) shows the dimensionless PD $K$ plotted against the Reynolds number $Re$ for $H/B = 0.091, 0.21$, and $0.82$. For a given value of $Re$, $K$ increases with $H/B$. Figure 5(b) shows the numerical results for the velocity profile $u'_x$ plotted against $y'$ at the exit of the control surface for $Re = 1.0 \times 10^5$, where the dimensionless maximum velocity decreases steeply for $y' > 0.42$. The aforementioned results agree closely with those from a previous study.\textsuperscript{24} Also, Fig. 5(c) shows the di-
5. EXPERIMENTAL RESULTS

Figures 6 and 7 show the dimensionless PD $K$ (Eq. (7)) as a function of the Reynolds number $Re$ (Eq. (8)) and the generalized Reynolds number $Re^*$ (Eq. (9)), namely,

$$K = \frac{2\Delta p}{\rho V^2}$$  \hspace{1cm} (7)

$$Re = \frac{\rho BV}{\mu}$$  \hspace{1cm} (8)

$$Re^* = \frac{\rho B^n V^{2-n}}{m \left( \frac{3n+1}{4n} \right)^{n-1}}$$  \hspace{1cm} (9)

where $V$ is the mean velocity passing through the slit:

$$V = \frac{Q}{BW}$$  \hspace{1cm} (10)

For non-Newtonian viscosity, the generalized Reynolds number is reported for capillary flows, but for convenience we use it herein for slit flows. Furthermore, the experimental error is estimated from

$$\frac{\delta K}{K} = \frac{\delta (\Delta p)}{\Delta p} + \frac{\delta \rho}{\rho} + 2 \frac{\delta B}{B} + 2 \frac{\delta W}{W} + 2 \frac{\delta Q}{Q}$$  \hspace{1cm} (11)

where $\delta K/K = 8\%$ (water and silicone oil) or $9\%$ (PEO and PAA) from $\delta (\Delta p)/\Delta p = 1\%$, $\delta Q/Q = 2.5\%$, $\delta B/B = 1\%$, $\delta W/W = 0\%$, and $\delta \rho/\rho = 1\%$ (PEO and PAA). The values of $K$ for water and silicone oil are in excellent agreement with the predicted values. As shown in Fig. 6, the resultant PDs for dilute and ultra-dilute solutions of PEO are less than those of water and silicone oil, and the reduction in PD depends on the concentration. Even for the 1 ppm solutions, the PD is reduced. Figure 7 shows the experimental results for PAA, which are similar to those for PEO. Figures 8 and 9 show the experimental results for the dimensionless JT

$$S = \frac{T_m}{\rho QV}$$  \hspace{1cm} (12)

which is plotted against the Reynolds number $Re$ and generalized Reynolds number $Re^*$. Furthermore, the experimental error is estimated from

$$\frac{\delta S}{S} = \frac{\delta T_m}{T_m} + \frac{\delta \rho}{\rho} + \frac{\delta B}{B} + \frac{\delta W}{W} + 2 \frac{\delta Q}{Q}$$  \hspace{1cm} (13)

where $\delta S/S = 11\%$ (water and silicone oil) or $12\%$ (PEO and PAA) from $\delta T_m/T_m = 5\%$, $\delta Q/Q = 2.5\%$, $\delta B/B = 1\%$, $\delta W/W$.

mensionless JT $S$ plotted against $Re$ for numerical parameter values corresponding to the experimental conditions (namely, $H/B = 0.091, 0.21, \text{and } 0.82$). It was found that the calculated value of $S$ increases with $Re$ and that $S$ is largely independent of the chosen value of $H/B$. 

Fig. 5 Numerical results for (a) PDs, (b) velocity profile at Reynolds number $Re = 1.0 \times 10^3$ (the thickness ratio was used as a parameter in the numerical calculations), and (c) dimensionless JT $S$ as a function of $Re$ for $H/B = 0.091, 0.21, \text{and } 0.82$. 


\[ K = \frac{H}{B} \]

![Graph showing experimental results for dimensionless PD versus Reynolds number](image)

**Fig. 6** Experimental results for dimensionless PD $K$ versus Reynolds number $Re$, generalized Reynolds number $Re^*$ for water, silicone oil, and PEO for slit width $B = (a) 1.1$ mm, (b) 480 $\mu$m, and (c) 122 $\mu$m with thickness $H = 100$ $\mu$m.

$\delta\rho/\rho = 0\%$, and $\delta\rho/\rho = 1\%$ (PEO and PAA). Compared with the PD, a narrower range of Reynolds numbers was used for the JT. At flow rates corresponding to low Reynolds numbers, the JT was very small. Therefore, it was difficult to measure the JT accurately with the present electronic balance because of its limited accuracy. The range of Reynolds numbers is from $7.0 \times 10^1$ to $2.2 \times 10^3$. The experimental results for water and silicone oil agree with the predicted values within the limits of the experimental errors. As shown in Fig. 8, the...
resultant JTs for the dilute and ultra-dilute solutions of PEO are less than those for water and silicone oil, and the behavior of the JTs depends on the concentration. Even for the 1 ppm solutions, anomalies in JTs were obtained. Figure 9 shows the experimental results for PAA, which are similar to those for PEO.
6. DISCUSSION

The PDs and JTs for PEO and PAA are less than those of water and silicone oil. Normally, we would measure the relaxation times of the test fluids and consider them in terms of the Weissenberg number. However, it is very difficult to measure the relaxation times of dilute and ultra-dilute polymer solutions and nearly impossible to do so with existing commercial experimental equipment. In other work, Hidema et al. discussed the effect of the extensional viscosity of dilute polymer solutions, but the minimum concentration used was 0.12 wt% (1200 ppm). Thus, it is extremely difficult to measure the elongational viscosity of dilute and ultra-dilute polymer solutions. Such entanglement of polymer chains in aqueous solutions. 26, 27) Such entanglement is generated with higher overlap concentrations (PEO: 1600 ppm, PAA: 93 ppm28). In our experimental results, the elastic properties are based on the entanglement of the polymer chains near the wall on pressure drop in micro-channel flows.11) Moreover, previous studies have found that the elastic properties are based on the entanglement of the polymer chains in aqueous solutions.26, 27) Such entanglement is strongly expressed (Ca* < 10^6). Additionally, previous studies reported the swelling of jets of polymer solutions flowing out of small pipes (the Barus effect)33–35) and measured the elastic stresses.36, 37) In a previous study, Hasegawa et al. reported that the difference between the JT of water and that of polymer solutions flowing through capillaries corresponded to the first normal stress difference (elastic stress). However, the first normal stress difference could not be calculated in the present case because the velocity profile passing through the slit was very difficult to express mathematically. Instead, we use the apparent elastic stress

\[ ES_{app} = \frac{T_m|_{pre} - T_m|_{exp}}{BW} \]  

(16)
to discuss the experimental results. The apparent elastic stress can be regarded as the average elastic stress when a test fluid passes through the slit. Figure 12 shows ES_{app} plotted against the mean velocity V. Power-law relationships are obtained between ES_{app} and V, namely, ES_{app} ∝ V^{1.84} (PEO 100 ppm), ES_{app} ∝ V^{1.82} (PEO 10 ppm), ES_{app} ∝ V^{1.80} (PEO 1 ppm), ES_{app} ∝ V^{1.89} (PAA 100 ppm), ES_{app} ∝ V^{1.82} (PAA 10 ppm), and ES_{app} ∝ V^{1.57} (PAA 1 ppm). In previous work, the elastic stress (first normal stress difference) of a dilute polymer solution was found to be proportional to V^2 for flow through a small orifice. Hasegawa et al. reported the relationship between ES_{app} and V for PEO 100 ppm and PAA 200 ppm.39) Furthermore, they derived the following equation ((Eq. (17)) by assuming a perfectly elastic fluid passing through small apertures.

\[ ES_{app} = \rho V^2 - \frac{T_r}{A} \]  

(17)

Here, T_r is the jet reaction from small apertures and A is the cross-sectional area of the small apertures. For a polymer solution with high elastic stress (at concentrations on the order of 100 ppm or higher), the jet reaction becomes almost zero due to the Barus effect (first normal stress difference), which means that ES_{app} is proportional to V^2. This is evident from previous studies.12, 13, 38–40) On the other hand, polymer solutions used in this study exhibited a relatively small first normal stress difference due to the dilute concentrations (< 100 ppm). It is therefore presumed that the jet reaction is not zero. ES_{app} is reduced from \( \rho V^2 \) by the amount of T_r. As the mean veloc-
ity increases, $T_r$ becomes greater and is therefore considered not to follow the relationship $ES_{app} = \rho V^2$. In Fig. 12, the power exponent of all dilute polymer solutions is less than 2, indicating a decrease in elastic stress with respect to the mean velocity. Primarily, it is necessary to estimate the jet reaction from the velocity distribution of test fluids passing through slits. However, this is not possible with the present experimental apparatus. Within the limits of experimental error, the present estimations agree with previously reported
characteristic lengths of 1.1 mm, 480 μm, and 122 μm. The following conclusions are drawn. The resultant PDs and JTs of water and silicone oil agreed with the numerical predictions of the Navier–Stokes equations. The experimental results for all the dilute and ultra-dilute polymer solutions were smaller than those for water and silicone oil. The results for the 1 ppm aqueous polymer solutions were notable, although they were below the overlap concentration. We discussed the effect of surface tension and the estimation of elastic stress. We calculated the capillary number, which ranged from $1.0 \times 10^{-3}$ to $1.5 \times 10^0$.

Thus, the effect of surface tension was strongly expressed in the present flows. Moreover, power-law relationships between the estimated apparent elastic stress and the mean velocity passing through the slit were obtained, and these agreed with previous results obtained by another method.

7. CONCLUSIONS

To investigate experimentally the flow properties of dilute (100 ppm and 10 ppm) and ultra-dilute (1 ppm) polymer solutions passing through small apertures, we measured the PDs and JTs of water, silicone oil, and dilute and ultra-dilute aqueous polymer solutions. However, the detailed mechanism involved is not yet clear, and clarifying this is an aim of our future research.

NOMENCLATURE

$A$ : cross-sectional area ($\mu m^2$, mm$^2$)  
$B$ : slit width ($\mu m$, mm)  
$Ca$ : capillary number (--)  
$Ca^*$ : generalized capillary number (--)  
$ES_{app}$ : apparent elastic stress (Pa)  
$F$ : force detected by electronic balance (N)  
$H$ : thickness of slit ($\mu m$)  
$H/B$ : thickness ratio (--)  
$K$ : dimensionless pressure drop (--)  
$m$ : dilatant viscosity (Pa·s$n$)  
$n$ : power-law index (--)  
$Q$ : flow rate (m$^3$/s)  
$Re$ : Reynolds number (--)  
$Re^*$ : generalized Reynolds number (--)  
$S$ : dimensionless jet thrust (--)  
$SR_w$ : shear rate on wall (s$^{-1}$)  
$T$ : temperature ($^\circ$C)  
$T_m$ : jet thrust (N)  
$T_{out}$ : momentum issuing out of control surface (N)  
$T_r$ : jet reaction (N)  
$T_s$ : surface age (s)  
$V$ : mean velocity (m/s)  
$W$ : slit length (mm)  
$\Delta p$ : pressure drop (Pa)  
$\mu$ : Newtonian viscosity (Pa-s)  
$\rho$ : density (kg/m$^3$)  
$\sigma_d$ : dynamic surface tension (mN/m)  
$\sigma_s$ : static surface tension (mN/m)  
$\tau_w$ : wall shear stress (Pa)
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