Macroscopic Quantum Superposition and Entanglement in light reflection from Bose-Einstein Condensates

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The multiphoton quantum superposition generated by a quantum-injected optical parametric amplifier (QI-OPA) seeded by a single-photon belonging to an EPR entangled pair is made to interact with a Mirror-BEC shaped as a Bragg interference structure. The effect can be greatly enhanced if the mirror is part of a cavity structure or is used within an optical interferometer. The overall process will realize an Entangled Macroscopic Quantum Superposition involving a "microscopic" single-photon state of polarization and the coherent "macroscopic" displacement of the BEC structure acting in space-like separated distant places.

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In recent years a great deal of interest has been attracted by the problem of creating a Macroscopic Quantum Superposition of a massive object, e.g. a tiny mirror, by an entangled interaction with a single photon, then realizing a well known 1935 argument by Erwin Schroedinger \cite{1, 2, 3}. The present work considers the adoption of the process of Atom–Photon off resonant coherent scattering to create joint atom-photon states non-separably entangled by momentum conservation. The extension of such property to macroscopic quantum states of light and matter would indeed realize an Entangled Macroscopic Quantum Superposition \cite{4}. Light scattering from Bose–Einstein condensates have been used to enhance their non–linear macroscopic properties in super–radiance experiments \cite{5} and to show the possibility of matter wave amplification \cite{6} and non–linear wave mixing \cite{7}. It has also been shown that it is possible to transform a purely single particle phenomenon like Rabi oscillations into a super–radian Rayleigh scattering process which is intrinsically macroscopic \cite{8}. Indeed Rayleigh scattering experiments have demonstrated the reflection of light from bulk condensates \cite{9}. What we intend to discuss here is the linear coherent scattering, e.g. the reflection by a Bose-Einstein condensate (BEC) \cite{10} of a large assembly of nearly monochromatic photons generated by a high-gain Optical Parametric Amplifier "quantum-injected" in a nonlocal Einstein-Podolsky-Rosen (EPR) configuration, hereafter referred to as QI-OPA \cite{11, 12, 13}. To this aim we need to structure a BEC in order to realize a high reflectivity mirror of photons, the "Mirror-BEC".

Let’s first account for the optical part of the apparatus: Figure\textsuperscript{1}. The adopted QI-OPA operates in the highly efficient collinear regime and amplifies a single photon in a quantum superposition state of polarization (\( \Phi \)), a qubit, injected over the spatial mode \( k \) into a large number of photons (in excess of \( \Phi = 10^6 \)) over the same mode \( k \) and with orthogonal polarizations. The injected photon belongs to a \( \Phi \)–entangled pair realized by an EPR optical configuration in two distant measurement stations, referred to as Alice (A) and Bob (B) in the the jargon of quantum information. In virtue of the EPR nonlocality a measurement over the state of one photon by the Alice’s apparatus, results in a deterministc control at the Bob’s site of the qubit, injected in the QIOPA device. As investigated in previous works, this device realizes the optimal phase-covariant cloning of injected qubits belonging to the equatorial circle of the corresponding Bloch sphere which is orthogonal to the z-axis, the one that connects the states of linear horizontal and vertical polarizations: \(| H \rangle \) and \(| V \rangle \) \cite{12, 13}. Accordingly, the information encoded in the injected quibits \( |\varphi\rangle = 2^{-1/2} (| H \rangle + e^{i\phi} | V \rangle) \), consists of the phase \( \varphi \).

FIG. 1: Layout of the experimental apparatus

The QI-OPA device developed for the present experiment is a well tested, working device whose properties are outlined here in some details \cite{14}. The excitation source (not shown in the Figure) is a Ti:Sa Coherent MIRA mode-locked laser further amplified by a Ti:Sa regenerative REGA device operating with pulse dura-
tion $180 \text{fs}$ at a repetition rate of $100 \text{kHz}$. The output beam, frequency-doubled by second-harmonic generation, provides the excitation beam of UV wavelength (wl) $\lambda_p = 397.5\text{nm}$. The UV beam is split in two beams through a $\lambda/2$ waveplate and a polarizing beam splitter (PBS) and excites two BBO (β-barium borate) nonlinear (NL) crystals cut for type II phase-matching. Crystal 1, excited by the beam $k_p$, is the spontaneous parametric down-conversion (SPDC) source of entangled photon couples of wl $\lambda = 2\lambda_p$, emitted over the two output modes $k_i$ ($i = 1, 2$) in the entangled singlet state $|\Psi^\prime\rangle_{k_1,k_2} = 2^{-1/2}([|H\rangle_{k_1}|V\rangle_{k_2} - |V\rangle_{k_1}|H\rangle_{k_2})$. This EPR device may be thought of as amplifying the vacuum fields associated with the output modes. The photon associated to mode $k_2$ (the trigger mode) is coupled to a single mode (SM) fiber and filtered by a set of $\pi$-analyzing optical devices, namely a Babinet compensator (B) a $\lambda/2 + \lambda/4$ waveplate set, a PBS (and an interference filter (IF) with a $\text{hwhm}$ band $\Delta \lambda \approx 1.5\text{nm}$). At last the trigger photon excites, at the Alice’s site, the single photon module (SPCM) $D_{\text{a}}^2$, which delivers the trigger signal adopted to establish the overall quantum correlations. By a Dichroic Mirror (DM) the single photon generated over the mode $k_p$ is made to merge into the mode $k_1$ together with the UV pump beam associated with mode $k_p$, and then injected into the NL Crystal 2 where stimulates the emission of many photons pairs over the two output $\pi$-modes, $k_1$. By virtue of the nonlocal correlation acting on modes $k_1$ and $k_2$, the injected qubit is prepared in the state $|\varphi\rangle_{k_1} = 2^{-1/2}(|H\rangle_{k_1} + e^{i\varphi}|V\rangle_{k_1})$ by measuring the photon on mode $k_1$ in the appropriate polarization basis, as said. The exact time and space overlapping in Crystal 2 of the pump and injected pulses is assured by means of an adjustable spatial delay ($Z$).

The Crystal 2 is oriented for collinear operation over the two linear polarization modes, $H$ and $V$. The OPA interaction Hamiltonian $\hat{H} = i\hbar (\hat{a}_H^\dagger \hat{a}_V^\dagger + \text{h.c.})$ acts on the single spatial mode $k_1$ where $\hat{a}_\alpha^\dagger$ is the one photon creation operator associated with $\alpha = H, V$ and $\pi$. The main feature of this Hamiltonian is its phase-covariance, i.e. invariance under $U(1)$ transformations for qubits with equatorial polarization leading for them to the optimality of the cloning process, as said. Owing to this property we can then re-write: $\hat{H} = \frac{1}{2} i \hbar \chi e^{-i\varphi} (\hat{a}_H^2 - e^{-2i\varphi}\hat{a}_V^2) + \text{h.c.}$

for $\varphi \in (0, 2\pi)$ where $\hat{a}_\pi^\dagger = 2^{-1/2}(\hat{a}_H^\dagger + e^{i\varphi}\hat{a}_V^\dagger)$ and $\hat{a}_\varphi^\dagger = 2^{-1/2}(e^{-i\varphi}\hat{a}_H^\dagger + \hat{a}_V^\dagger)$. In particular we consider the action over the states $\{|+\rangle, |-\rangle\}$, e.g. with $\pi = 2^{-1/2}(\pi_H \pm \pi_V)$ belonging to the equatorial plane orthogonal to the z-axis. The injected state on mode $k_1$ evolves into the output state $|\varphi\rangle_{k_1} = \hat{U}|\varphi\rangle_{k_1}$ according to the OPA unitary $\hat{U} = \exp \left[-i\hat{H}t/\hbar\right]$, being $t$ the interaction time [16]. The overall amplified output state on the $k_1$ spatial mode is expressed as:

$$|\Phi\rangle^\pm = \sum_{i,j=0}^{\infty} \gamma_{ij} \sqrt{(1 + 2i)(1 + 2j)} \frac{1}{i!j!} |2i + 1\rangle^\pm |2j\rangle^\pm$$

and $\gamma_{ij} = C^{-2}(-\frac{1}{2})^{i+j}$, $C = \cos h g$, $\Gamma \equiv \tanh g$, being $g$ the NL gain [14]. There $|\varphi\rangle$, $|\gamma\rangle$ stands for a state with $2$ photons with polarization $\pi_+ + q$ photons with $\pi_-$ and $\gamma$ the NL gain. The Macro-states $|\Phi^+\rangle, |\Phi^-\rangle$ are orthonormal, i.e. $|\Phi^+\rangle^\dagger |\Phi^-\rangle = \delta_{ij}$. Note that the entangled state $|\Sigma_{k_1,k_2} = 2^{-1/2}(|\Phi^+\rangle_{k_1} |+\rangle_{k_2} - |\Phi^-\rangle_{k_1} |-\rangle_{k_2})$, keeps its singlet character in the multi-particle regime, and expresses the nonlocal correlations between two distant objects: the Microscopic (single particle) system expressed by the trigger state (mode $k_2$) and the Macroscopic (multiparticle) system ($k_1$). It is referred to in the literature as the "Schrödinger Cat State" [cf Schleich [17], Ch.11, pp. 306-319]. At the output of crystal 2 the output beam with with wl $\lambda$ is spatially separated by the pump UV beam by a DM and an interferential filter (IF) with bandwidth 1.5nm $\text{hwhm}$ and finally coupled to a single mode optical fiber (SM). The corresponding output signals registered by any detector coupled to mode $k_1$ are taken in coincidence with a corresponding single-photon signals which, registered by the detector $D_2$ on the Alice’s site, triggers nonlocally the QI-OPA dynamics. Then we deal here with a "conditional experiment" by which each significant event registered at the output of the QI-OPA is conditioned by the actual realization of an EPR pair by the crystal 1 and then by the actual "quantum injection" of the crystal 2. This removes efficiently the noise due to the SPDC in crystal 2. Let us now analyze the output field $k_1$ over the polarization modes $\pi_\pm$ when the qubit $|\varphi\rangle_{k_1}$ is injected. The ensemble average photon number $N_\pm$ over $k_1$ with $\pi_\pm$ is easily evaluated on the basis of Eq. 1 and found to depend on the phase $\varphi$ as follows: $N_\pm(\varphi) = \pm |\Phi^\pm_{\pi_\pm}(\varphi)| \Phi^\pm_{\pi_\pm} = m + \frac{1}{2}(2m + 1)(1 \pm \cos \varphi)$ with $m = \sinh g$, the average value of the number of "squeezed vacuum" photons emitted by the OPA for each polarization mode in absence of quantum injection [13]. In a actual laboratory test a value of the gain in excess of $g = 6$ could be easily obtained. This corresponds to an average number of QI-OPA generated photons per mode in excess of: $m = 80.000$. The photon number difference: $N(\varphi) = [N_+(\varphi) - N_-(\varphi)] = [(2m + 1) \cos \varphi]$ can give rise, as we shall see at once, to an interference (IF) fringing pattern as function of the phase $\varphi$ of both the quantum-injected qubit and of the correlated, far apart "trigger" qubit. In other words, in virtue of the EPR nonlocality, the quantum superposition implied by the single-particle "trigger" qubit measured by Alice in the microscopic domain can manifest itself at Bob’s site in the macroscopic (multiparticle) regime by a first-order coherence interference (IF) fringing pattern [18]. It has been tested experimentally that this interference behavior is independent of the equatorial orthogonal basis chosen to represent the input qubit and that the QI-OPA realizes a genuine quantum superposition rather than a mixture [12]. The IF fringe visibility $\nu^{(1)} = (N_{\text{max}} - N_{\text{min}})/(N_{\text{max}} + N_{\text{min}})$ is dependent on the gain $g$ as follows: $\nu^{(1)}_{\text{th}} = (2m + 1)/(4m + 1)$. For $M \to \infty$, viz. $g \to \infty$, $\nu^{(1)}_{\text{th}}$ of this first-order corre-
ation function attains the asymptotic value \( = 1/2 \) \[18\].

How to detect the Macroscopic quantum superposition? The method successfully adopted in our Laboratory was to separate the two orthogonal polarizations \( \overrightarrow{\Phi} \) by inserting a (PBS) on the output mode \( k_1 \). The corresponding photon numbers \( N_\pm(\varphi) \) were detected by a couple of Burle C31034-A02 Ga-As photomultipliers with \( \eta_{QE} = 13\% \) and the difference of the output currents: \( I(\varphi) = (I_+ - I_-) \propto N(\varphi) \) was registered. Two independent IF fringing patterns obtained by this technique for opposing phases \( \varphi \) and \( \varphi + \pi \) are shown in Figure 2 \[2\]. That result may considered the preparatory stage of the present more sophisticated experiment, sketched in Figure 1.

At the output of the QI-OPA two counteracting optical beams associated with the the Macro-states \( |\Phi\rangle \pm \) and carrying respectively an average of \( N_+(\varphi) \) and \( N_-(\varphi) \) photons, for a total of \( N(\varphi) \approx 10^5 \) spatially separated and focused on the opposed sides of a cigar-shaped Bose Einstein Condensate (BEC) with approximate diameter 10μm. Since a photon exchanges a momentum \( p = 2h\nu/c \) upon a head on collision with an atom, the counteracting momenta of the two beams can be very efficiently transferred to the BEC if this one is conveniently shaped in order to operate as a mechanical mirror, indeed a Mirror-BEC.

Let us illustrate how we intend to realize such a mirror, if we load a BEC from a magnetic trap into a 1–D optical lattice we get an array of disk shaped condensates with a longitudinal size \( R_l \propto s^{-1/4}k^{-1} \) \[19\] and with a transverse size \( R_\perp \) dictated by the strength of the magnetic trap and by the number of atoms \( N_{\perp} \) in each condensate \[20\]. The number \( N_D \) of such disks is also fixed by the strength of the magnetic trap (this time in the longitudinal direction) and by the number of atoms in the original condensate. Typical numbers are \( N_D = 100 \div 200 \), \( N_{\perp} = 100 \div 1000 \) with \( R_\perp = 2 \div 10\mu m \) and \( R_l = 80 \div 300 \) nm \[21\].

By choosing \( s \) it is then possible to prepare an array of disks with a longitudinal size of \( \frac{s}{2} \) spaced by \( \frac{s}{2} \). If we approximate each condensate with a slab of dielectric with index of refraction \( n_B \) then this configuration is the same as that of a Bragg mirror for the wavelength \( \lambda \).

The reflectivity of a Bragg mirror with \( 2N_D \) alternating layers of index of refraction \( n_B \sim (1 + \epsilon) \) is:

\[
R = \left( \frac{n_{D}^{2}N_{\perp} - 1}{n_{D}^{2}N_{\perp} + 1} \right)^{2} \sim N_{D}^{2}c^{2} \text{ with a bandwidth: } \Delta \nu_{a} = \frac{4\nu}{\pi} \arcsin \left( \frac{a}{n_{D}^{2}N_{\perp}} \right) \sim \frac{2\nu}{\pi} \epsilon.
\]

For a 2–level atom very far from resonance we can approximate \( \epsilon \approx \frac{\Delta \nu_{a}}{\nu} \) where \( N = \frac{2}{\pi} \) is the rescaled density, \( \Gamma \) is the atomic linewidth and \( \Delta \) the detuning from resonance. In rubidium \( \Gamma \approx 6 \) MHz and typical densities are \( \frac{N}{\Gamma} = 10^{13} \div 10^{14} \text{ cm}^{-3} \). It should be stressed that in the same approximation the light scattering probability is \( \sim N_{\perp} \frac{\Delta \nu_{a}}{\nu} \). To be more precise the density is given in the Thomas–Fermi approximation by \( n_{TF} = \frac{\Delta \nu_{a}}{\nu} \left( \frac{a}{n_{D}^{2}N_{\perp}} \right) \) where \( a = 5.77 \text{ nm} \) in \( ^{87}\text{Rb} \) is the scattering length and \( a_{ho} \) is the averaged harmonic oscillator length. Combining all the previous equations and assumptions we obtain the following graphics:

From the inset it is clear that around the atomic resonance with a bandwidth of around \( \Delta \nu_{a} = 2GHz \) the reflectivity of the patterned BEC is essentially unity. Since the spectral width of the QI-OPA generated photon beams is \( \Delta \lambda \sim 1.5nm \), corresponding to a linewidth \( \Delta \nu \sim 700GHz \), about the 3% of the incoming photon beams will be totally reflected by the Mirror-BEC. In our experiment this will corresponds to a number of active photons \( N_{\perp}(\varphi) = (\Delta \nu_{a}/\Delta \nu) \times N_{\perp}(\varphi) \) in the range \((150 \div 300)\), having taken into account the limited value of the visibility \( V^{(1)} \approx 13\% \) measured experimentally at the output of the QI-OPA. This effect is easily detectable on the basis of the previous measurement on Rayleigh scattering from a BEC \[22\]. It must be noted that very close to resonance all the above approximations fail, however atomic resonance lines have linewidths of the order...
of a few MHz which means that our pulses will contain around 1 photon at resonance leaving a significant frequency band available for the proposed experiment. A more careful model should take into account a quasi sinusoidal modulation of the refractive index reflecting the atomic density distribution in the Thomas–Fermi approximation. Such a model would result in a more physical smoothing of the reflection coefficient around resonance. However the simplified approach adopted here captures the main physical aspect i.e. a significant enhancement of elastic light reflection around resonance. An alternative approach would be to consider the second quantized model of atom/photon fields interaction. Such a model goes far beyond the scope of the present paper. Let us mention here that it has been introduced in the framework of the "Collective atomic recoil Laser" and already used to predict entanglement between atomic p-states.

It is important to note that, in order to observe recoil effects of the Mirror-BEC, the condensate has to be released from the optical lattice; indeed typical expansion velocities for a BEC are of the order of 1 mm/ps which leaves at least 50 μs before the pattern gets significantly spoiled. Given the time duration of a QI-OPA pulse (order of 1 ps) there is enough time to observe a reflection from the free standing Mirror-BEC. The release of the condensate is triggered by any UV pump laser pulse exciting the QI-OPA. During the free survival time of the the wavy structure of the Mirror-BEC, the two momentum exchanging $N_\pm(\varphi)$ photon pulses emitted from the QI-OPA are injected simultaneously from opposite sides in the BEC chamber, as said. By a fast electro-optic switch placed at the output of the QI-OPA, this p-transfer process could be repeated by any large number of times, say $10^4$. This device, recently developed in our Laboratory, is deterministically driven by the measurement outcomes registered at the Alice’s station and then can enhance by a coherent add-on process the displacement of the Mirror-BEC due to any elementary interaction: $\Delta x(\varphi) \propto (2\hbar / c) \times N_\pm(\varphi) = [2\hbar (\varphi/2\pi + 1)] / c \cos \varphi$. Remind the Micro-Macro entanglement structure of the state $|\Sigma\rangle_{k_1,k_2}$. The realization of the interference fringe pattern $\Delta x(\varphi)$ as function of the phase $\varphi$ of the single photon-measured by the far apart Alice’s apparatus will be a clear demonstration of the entanglement between the Microscopic single-particle realm and the mechanical motion of a Macroscopic, multi-atom BEC. Note that the totally reflected photons bear the exact bandwidth for further total reflection by the same Mirror-BEC. Thus the effect can be enhanced several times by means of a couple of additional mirrors reflecting back the optical beams to the same BEC after the first interaction. This idea bring us straightforwardly to an electromagnetic cavity structure by which the BEC displacing effect can be enhanced by the cavity quality factor $Q$, i.e. in principle by orders of magnitude. An alternative interesting picture would consider the Mirror-BEC as the key part of a Fabry-Perot or a Michelson interferometer. Furthermore the reflected photons, trapped in any Mirror-BEC cavity structure or optical interferometer, will be themselves entangled with the condensate. This would open a rich perspective of novel quantum physics enlightening the nonlocality for Macroscopic systems, a fundamental long sought problem of modern Science.

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