Charged majoron emission in neutrinoless double beta decay

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Abstract

We examine in detail the predictions of the charged majoron model, introduced recently by Burgess and Cline, for $0^+ \rightarrow 0^+ \beta \beta$ transitions. The relevant nuclear matrix elements are evaluated, within the quasiparticle random phase approximation, for $^{76}$Ge, $^{82}$Se, $^{100}$Mo, $^{128}$Te and $^{150}$Nd nuclei. The calculated transition rates turn out to be much smaller than the experimental upper limits on possible majoron emission, except in a small region of the model’s parameter space.

\textsuperscript{*}Work partially supported by the Fundación Antorchas, Argentina, the CONICET from Argentina, the Universidad Nacional de La Plata, NSERC of Canada and FCAR of Québec.

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In 1987 Elliott, Hahn and Moe [1] observed, for the first time, the two-neutrino double beta decay ($\beta\beta_{2\nu}$) of $^{82}$Se into $^{82}$Kr by a direct counting method. Almost simultaneously, Vogel and Zirnbauer [2] showed that, within the quasiparticle random phase approximation (QRPA), it was possible to explain the smallness of the measured $\beta\beta_{2\nu}$ decay rates. Since then, impressive progress has been achieved in the experimental investigation, and the $2\nu$-decay mode has been unambiguously observed in several nuclear isotopes [3]. This process occurs at second-order in the charged-current weak interactions in the Standard Model, and is the slowest process measured so far in nature. As such, it offers a unique opportunity to test the nuclear structure models for half-lives $\gtrsim 10^{20}$ years.

However, the renaissance of interest in $\beta\beta$-decay over the last decade is mostly stimulated by the possibility of observing other decays to which these experiments are also sensitive. The hope is to find a smoking gun for “new physics” from beyond the standard model. The most promising processes of this type are the lepton-number violating neutrinoless decay ($\beta\beta_{0\nu}$), and the decay $\beta\beta_M$, in which the two outgoing electrons are accompanied by a Nambu-Golstone boson, called the majoron. Both processes were predicted [4, 5] by the model introduced by Gelmini and Roncadelli [6]. While this simple and elegant model stimulated many experimental searches, it was subsequently found to be incompatible with the LEP measurement of the invisible width of the $Z$ boson [7, 8].

Thus, to account for an anomalous excess of $\beta\beta$ events for which some experimental groups had preliminary evidence [9], Burgess and Cline recently advocated a new class of “charged majoron” models [10, 11] so-named because their majoron carries the $U(1)$ charge of lepton number, presumed to be unbroken. Refs. [10, 11] estimated the nuclear matrix elements needed for charged majoron emission to account for the anomalous events observed in $^{76}$Ge, $^{82}$Se, $^{100}$Mo and $^{150}$Nd nuclei. In the present work, we compute the transition rate in detail for the particular model given in [11], simplifying their analytic expressions, and evaluating the corresponding nuclear matrix elements for the above-named elements as well as for $^{128}$Te. We use the quasiparticle random phase approximation (QRPA) [12, 13], which has been shown to give good estimates for the $\beta\beta_{2\nu}$ transition probabilities. Finally, the resulting transition probabilities are compared with the present experimental data.

In the charged majoron model (CMM) of ref. [11], the standard model gauge symmetry
group is augmented by a global nonabelian flavour symmetry group, $SU_F(2) \times U_L(1)$, which breaks down to the ordinary lepton number $U_L(1)$ subgroup (see also refs. [14, 15]). To implement this symmetry-breaking pattern, an electroweak-singlet, $SU_F(2)$-doublet scalar field $\Phi$ is introduced, which gets a vacuum expectation value. The model also includes nonstandard electroweak-singlet Majorana neutrinos ($N_+, N_-$), $s_+$ and $s_-$, and the resulting lagrangian density which respects all the symmetries is

$$\mathcal{L} = -\lambda L\bar{H}P_Rs_- - M s_+ P_Rs_- - g_+(NP_Ls_+) \Phi - g_-(NP_Ls_-) \tilde{\Phi} + h.c.$$  (1)

Here $P_L$ and $P_R$ denote the projections onto left- and right-handed spinors, $L$ is the usual lepton doublet, and $\tilde{H}$ is the charge conjugate of the Higgs doublet. After symmetry breaking by the vacuum expectation values $\langle H \rangle = v = 174$ GeV, $\langle \Phi \rangle = u \sim 100$ MeV, the resulting neutrino mass matrix yields a massless neutrino $\nu'_e$ and two heavy Dirac neutrinos $\psi^\pm$ with masses

$$M^2_\pm = \tilde{M}^2 \pm \sqrt{M^4 - g_+^2 u^2(\lambda^2 v^2 + g_+^2 u^2)}; \quad \tilde{M}^2 = \frac{1}{2} \left[ M^2 + \lambda^2 v^2 + (g_+^2 + g_-^2)u^2 \right].$$  (2)

In terms of the mass eigenstates, the electroweak eigenstate is

$$|\nu_e\rangle = c_\theta |\nu'_e\rangle + s_\theta c_\alpha |\psi^-\rangle + s_\theta s_\alpha |\psi^+\rangle$$  (3)

where $s_\theta = \sin \theta$, $c_\theta = \cos \theta$, etc., denote the mixing angles with

$$\tan \theta = \frac{\lambda v}{g_+ u}; \quad \tan 2\alpha = \frac{2M\sqrt{\lambda^2 v^2 + g_+^2 u^2}}{M^2 - \lambda^2 v^2 + (g_+^2 - g_-^2)u^2}. \quad (4)$$

The transition probability for the $0^+ \to 0^+ \beta\beta$-decay has the form [11]

$$\Gamma(\beta\beta) = \frac{(G_F \cos \theta_C)^4}{256\pi^5} |\mathcal{A}(\beta\beta)|^2 \int (Q - \epsilon_1 - \epsilon_2)^3 \prod_{i=1}^{2} k_i \epsilon_i F(\epsilon_i) d\epsilon_i,$$  (5)

in the notation of ref. [11], with the transition amplitude given by

$$\mathcal{A}(\beta\beta) = 2\sqrt{2}(s_2s_2s_\theta)ig_{A}g^2_\mathcal{A} \langle 0^+ | \sum nm \mathbf{Y}_{Rnm} \cdot \hat{r}_{nm} \partial h_\alpha/\partial r_{nm} | 0^+ \rangle.$$  (6)

Here

$$h_\alpha(r_{nm}) = i\mu \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikr_{nm}}}{(k_0^2 - \mu^2 + i\epsilon)} (P_- - P_+)(P_0 - s_\alpha P_+ - c_\alpha P_-);$$

$$P_+ = (k^2 - M^2_+ + i\epsilon)^{-1}; \quad M_0 \equiv 0,$$  (7)
with $r_{mn}$ and $\mu$ being, respectively, the separation in the position between two decaying nucleons and the average excitation energy of the intermediate nuclear states, and

$$Y_{Rnm} = i \left[ (\sigma_n C_m - C_n \sigma_m) + i \left[ \frac{g_V}{g_A} (\sigma_n \times D_m + D_n \times \sigma_m) + \left[ \frac{g_V}{g_A} \right]^2 (D_n - D_m) \right] \right], \quad (8)$$

happens to be same nucleon recoil operator as appears in Doi et al., [5], except for the sign of the second term. (We believe the sign difference is due to an error in ref. [5].)

It is possible to simplify the above expressions for the matrix element. Previously it has been argued that the middle term of eq. (8) is the most important. Within the approximation where the momenta of electrons and majoron are neglected in comparison with those of the nucleon, and because the main contribution involves the spin singlet state of two nucleons, this term gives [3, 10]

$$Y_{Rnm} \cdot \hat{r}_{nm} \simeq \frac{f_R}{2M_N} (\sigma_n \cdot \sigma_m) (\nabla_{nm} \cdot \hat{r}_{nm}), \quad (9)$$

where $M_N$ is the nucleon mass, $f_R \simeq 5.6$ (for $g_A = 1$ [12, 17]) and $\nabla_{nm} = \partial/\partial r_{nm}$. From eq. (8) it is clear that the amplitude is largest if $s_{2\alpha} = 1$, so we shall make that assumption. Furthermore the mixing angle $\theta$ is typically constrained to be small, so we take $s_\theta = \theta$. Then, after performing the $p_0$ integral in eq. (7) and using the identity $\nabla \cdot \hat{r} \partial h_\alpha(r)/\partial r = \nabla^2 h_\alpha(r)$, the amplitude can be written as a difference of two pieces,

$$A(\beta \beta) = \frac{i\theta^2 g_A^2}{\sqrt{2\pi}} M_{CM}; \quad M_{CM} = M_{CM}^+ - M_{CM}^-; \quad (10)$$

where

$$M_{CM}^\pm = \frac{f_R}{2M_N} \langle 0^+ | \sum_{mn} \tilde{h}(r_{mn}; M_\pm) \sigma_n \cdot \sigma_m | 0^+ \rangle, \quad (11)$$

is the nuclear matrix element for charged majoron emission corresponding to the exchange of the neutrino with mass $M_\pm$, and

$$\tilde{h}(r_{mn}; M_\pm) = \frac{1}{M_\pm^2} [h(r_{mn}; M_\pm) - h(r_{mn}; 0)] + \frac{1}{2} \frac{\partial}{\partial M_\pm^2} h(r_{mn}; M_\pm) \quad (12)$$

is the corresponding neutrino potential, with

$$h(r_{mn}; M_\pm) = \int \frac{dk}{2\pi^2} e^{-i k \cdot r_{mn}} \frac{k^2}{\omega_\pm (\omega_\pm + \mu)}; \quad \omega_\pm = (k^2 + M_\pm^2)^{1/2}. \quad (13)$$
The inverse half-life can be now cast in the form

\[
[T(0^+ \rightarrow 0^+)]^{-1} = g_{CM}^2 G_{CM} |\mathcal{M}_{CM}|^2, \tag{14}
\]

where

\[
g_{CM} = g_+ \theta^2 / 2, \tag{15}
\]
is the effective majoron coupling, and

\[
G_{CM} = \frac{(G_F g_A \cos \theta_C)^4}{128 \pi^7 \ln 2} \int (Q - \epsilon_1 - \epsilon_2)^3 \prod_{i=1}^2 k_i \epsilon_i F(\epsilon_i) d\epsilon_i, \tag{16}
\]
is the kinematical factor as defined in ref. \[5\].

To perform the nuclear structure calculation we use the Fourier-Bessel expansion of the charged majoron matrix element. Thus

\[
\mathcal{M}_{CM}^\pm = \sum_{LSJ} m(M; L, S, J^\pi),
\]

where \(L, S, J\) and \(\pi\) are, respectively, the orbital angular momentum, the spin, the total angular momentum and the parity of the intermediate nuclear states. Within the QRPA formulation presented in ref. \[12\] the individual nuclear matrix element is

\[
m(M; L, S, J^\pi) = (-)^S \sum_{\alpha_{pnp}^\prime n'} W_{pn}^{LSJ} W_{p'n'}^{LSJ} R^L(pnp'; M, J^\pi) \Lambda_+^{\alpha_{pnp}^\prime n'}(\alpha, J^\pi) \Lambda_-^{\alpha_{pnp} n}(\alpha, J^\pi).
\tag{17}
\]

The amplitudes \(\Lambda_\pm(\alpha, J^\pi)\) are

\[
\Lambda_+(\alpha, J^\pi) = \sqrt{\rho_p \rho_n [u_p v_n X_{\alpha, J^\pi} + \bar{v}_p \bar{u}_n Y_{\alpha, J^\pi}]} ;
\]

\[
\Lambda_-(\alpha, J^\pi) = \sqrt{\rho_p \rho_n [\bar{v}_p \bar{u}_n X_{\alpha, J^\pi} + u_p v_n Y_{\alpha, J^\pi}]} ,
\]

where the unbarred (barred) quantities indicate that the quasiparticles are defined with respect to the initial (final) nucleus; \(\rho_p^{-1} = u_p^2 + \bar{v}_p^2, \rho_n^{-1} = \bar{u}_n^2 + v_n^2\), and all the remaining notation has the standard meaning \[12\]. The angular momentum and radial pieces in (17) are, respectively,

\[
W_{pn}^{LSJ} = (-)^{\ell_n - \ell_p + L} \sqrt{2} \hat{J} \hat{S} \hat{L} \hat{j}_p \hat{j}_n (\ell_n 0 L 0 | \ell_p 0) \left\{ \frac{\ell_n}{2} \frac{1}{2} \frac{j_n}{j_p} \right\},
\]

The notation as presented in (17).
and

$$R^L(pnp'n' ; M_{\pm}) = \int_0^\infty dk k^2 v(k ; M_{\pm}) R^L_{pn}(k) R^L_{p'n'}(k),$$

with

$$v(k ; M_{\pm}) = \frac{2k^2}{\pi} \left\{ \frac{1}{M_{\pm}^2} \left[ \frac{1}{\omega_\pm(\omega_\pm + \mu)} - \frac{1}{k(k + \mu)} \right] + \frac{1}{2} \frac{\partial}{\partial M_{\pm}^2} \frac{1}{\omega_\pm(\omega_\pm + \mu)} \right\}$$

and

$$R^L_{pn}(k) = \int_0^\infty u_n(r) u_p(r) j_L(kr) r^2 dr,$$

$u(r)$ being the single-particle radial wave functions.

The numerical calculations were performed with the $\delta$-force (in units of MeV fm$^3$)

$$V = -4\pi(v_s P_s + v_t P_t)\delta(r),$$

different strength constants $v_s$ and $v_t$ for the particle-hole, particle-particle and pairing channels. An eleven-dimensional model space was used, including all the single particle orbitals of oscillator shells $3h\omega$ and $4h\omega$, plus the $0h9/2$ and $0h11/2$ orbitals from the $5h\omega$ oscillator shell. The single particle energies, as well as the parameters $v_s$ and $v_t$, have been fixed by the procedure employed in ref. [12] (i.e., by fitting the experimental pairing gaps to a Wood-Saxon potential well).

The dependence of $M_{CM}^{\pm}$ on $M_{\pm}$ is illustrated in Fig. 1 for the $^{76}$Ge$\rightarrow^{76}$Se decay. As one might expect, the matrix element is insensitive to the mass until it starts to exceed the Fermi momentum of the nucleons, around 100 MeV, thereafter giving a $1/M^2$ suppression. Since the matrix element $M_{CM}$ is the difference between the $M_+$ and $M_-$ contributions, one can easily recover the result for an arbitrary pair of masses by taking the difference between the two corresponding matrix elements. Obviously for $M_+ \approx M_-$ there is destructive interference between the matrix elements $M_{CM}$ and $M_{CM}^+$. This happens, for example, with the choice of parameters $M = \lambda v$ and $g_\pm \sim 1$, which from eqs. (2) and (4) yields $M_\pm = g_\pm u\sqrt{1 \pm \theta}$. Since the mixing angle $\theta$ is experimentally constrained to be of the order of 0.1 [4] the calculated values of $|T_{1/2}^{CM}|^{-1}$ turn out to be four to five orders of magnitude smaller than the corresponding experimental upper limits for the majoron emission, due to the additional $\theta^2$ suppression. These limits are displayed in Table 1 for the $^{76}$Ge, $^{82}$Se, $^{100}$Mo, $^{128}$Te and $^{150}$Nd nuclei. For the sake of completeness, the table also shows the measurements for the $\beta\beta_{2\nu}$ decays, and $G_{CM}$ for the effective axial-vector coupling $g_A = 1$ [12, 17].

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1 For a discussion of these experimental constraints, see ref. [10].
Figure 1: Charged majoron matrix element $\mathcal{M}_{CM}^\pm$ (in natural units) for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay, as a function of the heavy neutrino mass $M_\pm$ (in units of MeV).

Figure 1 also shows that the most favorable situation for majoron emission occurs for in limit that the heavier neutrino becomes infinitely massive, thus making no contribution to the total rate. For $M_-$ we will use 100 MeV, since this is the largest value which still gives an unsuppressed amplitude. In Table 1 we compare the theory to the data by showing how large the effective coupling $g_{CM}$ would have to be in this case in order for the CMM rate to be equal to the experimental limit on the majoron-emitting mode of $\beta\beta$ decay. For all the elements, $g_{CM}$ must be $0.1 - 0.2$ to be presently observable. Given the above mentioned experimental constraints $\theta$, from eq. (13) it is clear that one would need a strong coupling in the sterile neutrino sector in order to achieve such a large value of the majoron-emitting rate.
Table 1: The size of the effective coupling $g_{CM}$ which would be needed for the rate of emission of charged majorons to be equal to the experimental limit on this process; $G_{CM}$ and $|\mathcal{M}|_{CM}$ are the corresponding kinematical factors and the nuclear matrix elements, respectively. For the sake of comparison the pertinent experimental data for the two-neutrino processes are also shown.

| Nucleus       | $[T^{2\nu}_{1/2}]_{exp}(yr^{-1})$ | $[T^{\nu M}_{1/2}]_{exp}(yr^{-1})$ | $G_{CM}(yr^{-1})$ | $|\mathcal{M}|_{CM}$ | $g_{CM}$ needed |
|---------------|-----------------------------------|-----------------------------------|-------------------|---------------------|-----------------|
| $^{76}$Ge     | $(6.99 \pm 0.08) \times 10^{-22}$  | $< 6.0 \times 10^{-23}$           | $2.10 \times 10^{-20}$ | $0.28$              | $< 0.19$        |
| $^{82}$Se     | $(9.26^{+0.51}_{-2.23}) \times 10^{-21}$ | $< 6.2 \times 10^{-22}$           | $3.55 \times 10^{-19}$ | $0.28$              | $< 0.15$        |
| $^{100}$Mo    | $(8.69^{+1.51}_{-2.27}) \times 10^{-20}$ | $< 3.0 \times 10^{-21}$           | $7.33 \times 10^{-19}$ | $0.31$              | $< 0.21$        |
| $^{128}$Te    | $(1.30 \pm 0.07) \times 10^{-25}$  | $< 1.3 \times 10^{-25}$           | $5.24 \times 10^{-22}$ | $0.27$              | $< 0.059$       |
| $^{150}$Nd    | $(5.88^{+3.11}_{-3.46} \pm 1.21) \times 10^{-20}$ | $< 1.9 \times 10^{-21}$           | $6.06 \times 10^{-18}$ | $0.13$              | $< 0.077$       |

$^a$) (laboratory data) ref. [18]
$^b$) (laboratory data) ref. [19]
$^c$) (laboratory data) ref. [1]
$^d$) (laboratory data) ref. [20]
$^e$) (laboratory data) ref. [21]
$^f$) (geochemical data) ref. [22]
$^g$) (laboratory data) ref. [23]
$^h$) (laboratory data) ref. [24]

In conclusion, we have found that the rate of majoron-emitting, neutrinoless $\beta\beta$ decay in the charged majoron model is unobservably small unless there is large mixing between exotic sterile neutrinos of mass $\gtrsim 100$ MeV, and strong couplings among the sterile neutrinos. In computing the relevant nuclear matrix elements, we have not considered the effects of finite nucleon size or short-range two-nucleon correlations, which would tend to reduce the calculated matrix elements [12]. On the other hand, the arguments used to simplify the nuclear transition operator from (8) to (9) are not rigorous, and it is also possible that future variants of the CMM considered here might evade the suppression of the rate by the mixing angle $\theta$. Thus, while we believe that ours is the most quantitative analysis of the CMM to date, if the experimental situation should give serious indications
of anomalous $\beta\beta$ decay events in the future, it would become appropriate to undertake a yet more careful evaluation of the model's predictions.

Note added: As we were finishing this work, Hirsch et al. [25] presented results including a somewhat less detailed analysis of the CMM, in which they reached conclusions similar to ours.

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