Double parton scattering of hadron-hadron interaction
and its gluonic contribution

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Abstract

We propose a formalism for calculating the event cross-section of parton interactions. In this formalism, we use the light-front bound-state wave function to expand scattering initial state. This leads to an expression of the cross-section in terms of square of wave functions which can be used to define the parton distribution functions. The probability interpretation of the partonic interactions among hadron-hadron scattering is naturally achieved. We apply this formalism to calculate the 4-particle final state cross-section for single parton, double parton and gluon splitting interactions. We compare the behaviors of these cross sections and propose a method for experiments to differentiate these interactions.

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I. Introduction

Phenomenological calculation of hadron-hadron cross section uses the probability of finding one parton in each scattering hadron multiplied by the parton-level cross section $d\hat{\sigma}$:

$$d\sigma(A + B \rightarrow n \text{ jets + anything}) = \sum_{a,b} \int dx_a dx_b F_a/A(x_a) F_b/B(x_b)$$

$$\times d\hat{\sigma}(a + b \rightarrow n \text{ jets}).$$

In the calculation, one uses universal parton distribution functions for $F$ and uses perturbative QCD to calculate the n-jet single parton cross section $\hat{\sigma}^{2\rightarrow n}$. Since 1982 various experiments studied two-, three- and four-jet events from hadron-hadron scattering [1]. The two- and three-jet cross sections agree well with theoretical calculations [2]. However, the experiments on four-jet events show some inconsistency to the calculations [3]. The UA2 [4] experiment showed a good agreement to the calculations based on single parton interaction. On the other hand the $p\bar{p}$ collision experiments by AFS Collaboration at CERN ISR [5] and the CDF experiments [6] showed a small violation to the calculations. Consequently CDF and AFS collaborators suggested that a small double parton contribution to the QCD $(2 \rightarrow 4\text{jets})$ cross section may provide a better fit to the experimental data.

The double parton interaction happens when two partons in one hadron collide with two partons in another hadron [Fig. 1]. Since this process involves more than one parton in a hadron at one time, it will give more informations on hadronic structure than the single parton scattering can give. Experimentally one can differentiate the double parton events from single parton events by the final jet configurations. First, since the transverse momenta of partons in a hadron are small, the transverse momenta of the two jets produced in each of the parton-parton scattering should be well balanced. Therefore in a double parton scattering, one can find two pairs of jets with transverse momenta, say $(p_1, p_2)$ and $(p_3, p_4)$, such that $|p_1 + p_2|, |p_3 + p_4| << 1\ \text{GeV}$. In the contrast, in an ordinary four-jet event one will have $p_1 + p_2 + p_3 + p_4 \approx 0$, but for any pair of transverse momenta $(p_i, p_j)$, one will normally find $|p_i + p_j| >> 1\ \text{GeV}$. Secondly, for double parton scattering, if we assume that there should not be strong correlation between the two partons in the individual colliding hadron, the distribution of the double parton cross section in the azimuthal angle between the jet pairs should be flat. The CDF collaboration defines two variables which are sensitive to these signals. They define variable $S$ to exploit the tendency of pairwise balance,

$$S(i + j, k + l) = \min \left[ \frac{1}{2} \left[ \frac{|p_i + p_j|^2}{\sqrt{|p_i|^2 + |p_j|^2}} + \frac{|p_k + p_l|^2}{\sqrt{|p_k|^2 + |p_l|^2}} \right] \right],$$

(2)
where $S$ is minimized over the three possible jet pairings $(12, 34)$, $(13, 24)$ and $(14, 23)$ and they define the variable $\Delta S$ to be the azimuthal angle between the two jet pairs which minimize $S$. The double parton events should favor smaller values of $S$ and the single parton events should favor larger ones. Also, the double parton events should be uniformly distributed over $\Delta S$ in the interval $0$ to $\pi$.

In this paper we discuss the theory of the 4-particle final state events from hadron-hadron scattering. Most of these events are in the form of 4-jet production. There is a small fraction of these events which is in the form of di-lepton pair production. It is believed that the hadron fragmentation of the jet final state, which is a soft process, does not change the kinematics of the final particles. Therefore the di-lepton pair production should have a similar kinematics to the jet production. For a theoretical understanding of double parton scattering, we will use the di-lepton process as examples to the formalism. The same formalism can be applied to calculate the jet cross section. We are interested in the experiments with observables $Q_1, Q_2$ of the TeV order, where $Q_1$ and $Q_2$ are the invariant momenta of the final particle pairs. The largest source of these events is the lowest order single parton scattering. Double parton scattering may contribute these events. There is also a possible type of four-particle events that a gluon splits before collision. In this process, one incoming gluon splits into two partons, while the other incoming gluon also splits, then the two pairs of partons collide. One can interpret this process as an alternative form of double parton scattering. Imagining that the parton pairs are produced in time $\Delta t$ before the collision, where $\Delta t$ is long compared to the parton collision time $1/Q$ but short compared to the typical time for partons within a hadron to interact, then the transverse momenta of the partons produced in the gluon splitting process will be in the range of $0 << k << Q_1, Q_2$. We will examine the effect of these large transverse momentum partons on double parton scattering. The paper is organized in the following fashion. In the next section, we introduce the light-front wave functions which are useful for the formalism. In section III, we derive the cross section for the single parton scattering. In section IV, we use the formalism to calculate the double parton cross section. In section V, we discuss the gluon splitting scattering. Finally, in the discussion section we will compare the four-particle final state cross sections of single parton scattering, double parton scattering and from gluon splitting scattering. We propose a method for experiments to differentiate these interactions.
II. Light-front wave functions

In the light-front field theory, one uses a Hamiltonian approach to derive the equation for a bound-state wave function. For this purpose, one uses gauge field theory quantized on light-front planes. That is, one attempts to solve the eigenvalue equation $P_{\text{op}}^{-}\Psi = E\Psi$, where $P_{\text{op}}^{-}$ is the generator of translations from one light-front plane $x^+ = \text{const}$ to the next. Consider a state $|P^+, \mathbf{P}\rangle$ which denotes a hadron with light-front momentum $(P^+, (\mathbf{P}^2 + M^2)/(2P^+), \mathbf{P})$, normalized by $\langle P^+, \mathbf{P}|P'^+, \mathbf{P}'\rangle = (2\pi)^3 2P^+ \delta(P^+ - P'^+) \delta^2(\mathbf{P} - \mathbf{P}')$. We can expand the state vector in the Fock space

$$
|P^+, \mathbf{P}\rangle = \sum_m \sum_{s_1, s_m} \left( \prod_{i=1}^{m} \frac{\int dp_i^+ dp_i^-}{(2\pi)^3} \right) b^+(p_1^+, p_1^-; s_1) \cdots d^+(p_m^+, p_m^-; s_m) |0\rangle 
\times \psi_m(p_1, \ldots p_m; s_1, \ldots s_m) 2P^+ (2\pi)^3 \delta(P^+ - \sum_{i=1}^{m} p_i^+) \delta^2(\mathbf{P} - \sum_{i=1}^{m} \mathbf{p}_i). \quad (3)
$$

The advantage of this framework that one hopes will make the problem tractable is that partons (at least those with zero momentum) cannot appear spontaneously from the vacuum: The internal state of the hadron is given in terms of a set of wave functions $\psi_n$ defined by

$$
\langle 0|b(p_1^+, p_1^-; s_1) d(p_2^+, p_2^-; s_2) \cdots d(p_m^+, p_m^-; s_m)|P^+, \mathbf{P}\rangle
= (2\pi)^3 2P^+ \delta(P^+ - \sum_{i=1}^{m} p_i^+) \delta^2(\mathbf{P} - \sum_{i=1}^{m} \mathbf{p}_i) \psi_n(p_1^+, p_1^-; p_2^+, p_2^-; \ldots; p_m^+, p_m^-; s_1, \ldots s_m), \quad (4)
$$

where $b$’s and $d$’s are the destruction operators for quarks and antiquarks. The indices $s_i$ in the wave functions are the internal symmetry variables which denote helicity $\lambda_i$, isospin $t_i$, and color $c_i$. From the normalization equation we can show

$$
1 = \sum_m \sum_{s_1, \ldots s_m} \left( \prod_{i=1}^{m} \frac{\int dp_i^+ dp_i^-}{(2\pi)^3} \right) (2\pi)^3 2P^+ \delta(P^+ - \sum_{i=1}^{m} p_i^+) \delta^2(\mathbf{P} - \sum_{i=1}^{m} \mathbf{p}_i)
\times |\psi_m(p_1^+, p_1^-; \ldots; p_m^+, p_m^-; s_1, \ldots s_m)|^2. \quad (5)
$$

The normal light-front wave function is good for describing particles moving in the $+z$ direction. In order to describe the bound-state particles moving in the $-z$ direction, we have to do the followings: We expand the state vector for hadron moving in the $-z$ direction in the Fock space by

$$
|P^-, \mathbf{P}\rangle = \sum_n \sum_{s_1, s_n} \left( \prod_{i=1}^{n} \frac{\int dp_i^- dp_i^+}{(2\pi)^3} \right) b^+(p_1^-, p_1^+; s_1) \cdots d^+(p_n^-, p_n^+; s_n) |0\rangle 
\times \psi_n(p_1, \ldots p_n; s_1, \ldots s_n) 2P^- (2\pi)^3 \delta(P^- - \sum_{i=1}^{n} p_i^-) \delta^2(\mathbf{P} - \sum_{i=1}^{n} \mathbf{p}_i). \quad (6)
$$
Define the spinors $\mu(p, s)$ and $\nu(p, s)$ as the spin components of quarks and anti-quarks moving in the $-z$ direction:

$$
\mu(p, 1/2) = \frac{1}{\sqrt{\sqrt{2}p^-}} \begin{pmatrix} p^1 - ip^2 \\ \sqrt{2}p^- \\ 0 \\ m \end{pmatrix}, \quad \mu(p, -1/2) = \frac{1}{\sqrt{\sqrt{2}p^-}} \begin{pmatrix} m \\ 0 \\ \sqrt{2}p^- \\ -p^1 - ip^2 \end{pmatrix},
\nu(p, 1/2) = \frac{1}{\sqrt{\sqrt{2}p^-}} \begin{pmatrix} -m \\ 0 \\ \sqrt{2}p^- \\ -p^1 - ip^2 \end{pmatrix}, \quad \nu(p, -1/2) = \frac{1}{\sqrt{\sqrt{2}p^-}} \begin{pmatrix} p^1 - ip^2 \\ \sqrt{2}p^- \\ 0 \\ -m \end{pmatrix}.
$$

(7)

The normalization of these vectors is:

$$
\sum_s \mu_\alpha(p^-, p, s)\bar{\mu}_\alpha(p^-, p, s) = -\sum_s \nu_\alpha(p^-, p, s)\bar{\nu}_\alpha(p^-, p, s) = \hat{p} + m.
$$

(8)

It can be shown that these spinors are the eigen-states of a helicity operator referring to a Lorentz frame moving in the $+z$ direction near the speed of light, \textit{i.e.}, $v_z = -\tanh(\omega), \omega \to -\infty$ [7]. The helicity operator is

$$
h_\omega(p) \to h_{-\infty}(p) = \frac{1}{\sqrt{2p^-}} \begin{pmatrix} -\sqrt{2}p^- & p^1 - ip^2 & 0 & 0 \\ 0 & \sqrt{2}p^- & 0 & 0 \\ 0 & 0 & -\sqrt{2}p^- & 0 \\ 0 & 0 & p^1 + ip^2 & \sqrt{2}p^- \end{pmatrix}.
$$

(9)

One can check that the spinors listed in Eq. (7) are eigen states of $h_{-\infty}(p)$:

$$
h_{-\infty}(p)\mu(p, \pm 1) = \pm \frac{1}{2} \mu(p, \pm \frac{1}{2}), \quad h_{-\infty}(p)\nu(p, \pm \frac{1}{2}) = \mp \frac{1}{2} \nu(p, \pm \frac{1}{2}).
$$

(10)

We therefore clarify that the physical meaning of the spin index $s$ is the helicity of the quark as measured in a Lorentz frame moving near the speed of light in the $+z$ direction. Similarly, one can define the photon polarization vectors $\epsilon^\mu(\lambda)$ as

$$
\epsilon^\mu(\pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} q^1 \pm iq^2 \\ q^- \end{pmatrix}, q^+, q^-, q^0.
$$

(11)

The physical meaning of the index $\lambda$ is the helicity of the photon as measured in a frame moving the $+z$ direction.
III. The cross section for single parton scattering

In this section we derive the cross section formula for single parton scattering. For simplicity, we use the di-lepton production from unpolarized hadron-hadron scattering as an example. We are interested in the experiments with observables \(Q_1, Q_2\) of the TeV order. The transverse momenta of partons in a hadron are therefore negligible. The differential cross section for this process as shown in Fig. 2 is

\[
\frac{d\sigma_{sp}}{d^3Q_1 d^4Q_2} = \frac{1}{2s} \frac{1}{2} \sum_{S_A} \sum_{S_B} \left( \prod_{i=1}^{4} \frac{d^3l_i}{(2\pi)^3} \right) |M|^2 \delta^4(l_1 + l_2 - Q_1) \delta^4(l_3 + l_4 - Q_1) \\
\times (2\pi)^4 \delta^4(P_A + P_B - \sum_{i=2}^{m} a_i - \sum_{i=2}^{n} b_i - Q_1 - Q_2), 
\]  

(12)

where \(Q_1\) and \(Q_2\) are the invariant 4-momenta of the lepton pairs \((l_1, l_2)\) and \((l_3, l_4)\) and \(|M|\) is the invariant hadronic scattering amplitude which is calculated by using its relation to the scattering matrix

\[
\langle \text{out}(\text{anything}, l_1, l_2, l_3, l_4) | A, B \rangle_{\text{in}} = (2\pi)^4 \delta^4(p_i - p_f) M. 
\]  

(13)

The state \(|A, B\rangle_{\text{in}}\) denotes the initial state for hadron \(A\) moving in the \(+z\) direction and hadron \(B\) moving in the \(-z\) direction. We assume that the state \(|A, B\rangle_{\text{in}}\) can be factorized into \(|A\rangle_{\text{in}}\langle B|_{\text{in}}\). Then we expand the hadron state \(|A\rangle_{\text{in}}\) by Eq. (10) and expand the hadron state \(|B\rangle_{\text{in}}\) by Eq. (11). After applying these expansions we can express the scattering amplitude square as

\[
|M|^2_{sp} = \sum_{m,n} \sum_{s_{a2}, s_{a6}} \left( \prod_{i=2}^{m} \int \frac{d\alpha_i^+ d^2\alpha_i}{(2\pi)^3 2\alpha_i^+} \sum_{s_{b2}, s_{b6}} \left( \prod_{i=2}^{n} \int \frac{db_i^- d^2b_i}{(2\pi)^3 2b_i^-} \right) |m|^2 \\
\times \frac{1}{x_{a1}^2} \psi_{am}^2(x_{a1}, - \sum_{i=2}^{m} \alpha_i, s_{a1}; x_{a2}, a_2, s_{a2}; \ldots; x_{am}, a_m, s_{am}) \\
\times \frac{1}{x_{b1}^2} \psi_{bn}^2(x_{b1}; Q_1 + Q_2 + \sum_{i=2}^{m} \alpha_i, s_{b1}; x_{b2}, b_2, s_{b2}; \ldots; b_n, s_{bn}) \\
\times \int d^2p_a \delta^2(p_a - \sum_{i=2}^{m} \alpha_i). 
\]  

(14)

The wave function \(\psi_{am}(\psi_{bn})\) is a function of the hadron spin \(S_A(S_B)\), but we do not explicitly display this dependence. We inserted an identity \(1 = \int d^2p_a \delta^2(p_a - \sum_{i=2}^{m} \alpha_i)\) in Eq. (14) for future purpose. This insertion serves to define \(p_a\) as the total transverse momentum of the non-interacting partons numbered from 2 to \(m\). The parton momentum fractions are defined by \(x_{ai} \equiv a_i^+ / P_A^+\) and \(x_{bi} \equiv b_i^- / P_B^-\). The momentum fractions of the interacting partons \(a_1\)
and $b_1$ are fixed by the overall delta function. They are

$$x_{a1} = \frac{P_A^+ - \sum_{i=2}^m a_i^+}{P_A^+} = \frac{Q_1^+ + Q_2^+}{P_B^+} + O\left(\frac{M_B^2}{s}, \frac{b_1^2}{s}\right),$$

$$x_{b1} = \frac{P_B^- - \sum_{i=2}^n b_i^-}{P_B^-} = \frac{Q_1^- + Q_2^-}{P_B^-} + O\left(\frac{M_A^2}{s}, \frac{a_1^2}{s}\right).$$

(15)

In this equation $|m|^2_{sp}$ is the invariant partonic scattering amplitude square. Its integration over the final particle phase space denoted by $|T|^2_{sp}$ will be calculated in the appendix. In order to simplify the formula, we collectively express the sets of the non-interacting parton momenta and spins by:

$$V_a = \{(a_2^+, a_3; \ldots ; a_m^+, a_m) | \sum_{i=2}^m a_i^+ = P_A^+ - Q_1^+ - Q_2^+; \sum_{i=2}^m a_i = p_a\},$$

$$V_b = \{(b_2^-, b_3^-; \ldots ; b_n^-, b_n) | \sum_{i=2}^n b_i^- = P_B^- - Q_1^- - Q_2^-; \sum_{i=2}^n b_i = -p_a - Q_1 + Q_2\},$$

$$S_a = \{s_{a2}, \ldots s_{am} | \sum_{i=1}^m s_{ai} = S_A\},$$

$$S_b = \{s_{b2}, \ldots s_{bn} | \sum_{i=1}^n s_{bi} = S_B\}$$

(16)

and we denote the phase space of the non-interacting final partons as

$$\int [dV_a] = \prod_{i=2}^m \int \frac{d a_i^+ d^2 a_i^+}{(2\pi)^3 2 a_i^+} 2P_A^+(2\pi)^3 \delta(P_A^+ - \sum_{i=2}^m a_i^+ - Q_1^+ - Q_2^+) \delta^2(p_a - \sum_{i=2}^m a_i),$$

$$\int [dV_b] = \prod_{i=2}^n \int \frac{d b_i^- d^2 b_i^-}{(2\pi)^3 2 b_i^-} 2P_B^- (2\pi)^3 \delta(P_B^- - \sum_{i=2}^n b_i^- - Q_1^- - Q_2^-) \delta^2(p_a + Q_1 + Q_2 + \sum_{i=2}^n b_i).$$

(17)

With these notations we can rewrite Eq. (12) as

$$\frac{d\sigma}{d^4Q_1 d^4Q_2} = \frac{d\sigma}{d^4Q_1 d^4Q_2} = \frac{d^2Q_T d^2 A Q dQ_1^+ dQ_1^- dQ_2^+ dQ_2^-}{4s^2(2\pi)^6 x_{a1} x_{b1}^2} \sum_{S_A} \int [dV_a] \int [dV_b] \sum_{S_B} \int [dV_a] \int [dV_b]$$

$$\times \sum_{m,n \sum_{S_A, S_B}} |T|^2 \int d^2p_a \int d^2a_1 \delta^2(p_a + a_1) \times \psi_{am}^2(x_{a1}, a_1, s_{a1}; p_a; V_a, S_a) \times \psi_{bn}^2(x_{a1}, Q_T - a_1, s_{b1}; -Q_T + p_a; V_b, S_b),$$

(18)

in which we define $Q_T = Q_1 + Q_2$, $\Delta Q = (Q_1 - Q_2)/2$. For the following discussion we inserted an identity $1 = \int d^2a_1 \delta^2(a_1 + p_a)$ in Eq. (13) The wave function $\psi_{am}$ (or $\psi_{bn}$) in
Eq. (18) is the same wave function in Eq. (14) expressed by the new variables \( a_1, p_a, V_a \) (or \( b_1, p_b, V_b \)). We can use these momenta to define the following variables:

\[
\Delta p_a = x_a a_1 - x_a p_a, \\
A = p_a + a_1, \\
\tag{19}
\]

in which \( x_a \) is the total momentum fraction of the non-interacting partons and \( A \) is the total transverse momentum of hadron A. By using coordinate \( r_{a1} \) of parton \( a_1 \) and the center mass coordinate \( r_a \) of the remaining partons, we can define the following variables which conjugate to momenta \( \Delta p_a \) and \( A \):

\[
\Delta r_a = r_{a1} - r_a, \\
R_a = x_{a1} r_{a1} + x_a r_a, \\
\tag{20}
\]

here \( \Delta r_a \) is the relative transverse distance between parton \( a_1 \) and the center mass of the non-interacting partons and \( R_a \) is the center mass coordinate of hadron A. Similarly, we can define the momentum variables \( \Delta p_b \) and \( B \) and their conjugate coordinates \( \Delta r_b \) and \( R_b \) for hadron B. To express the cross section Eq. (18) in terms of these coordinate variables, we first express the wave functions in terms of \( \Delta p_a \) and \( A \) then Fourier transform these wave functions to the coordinate space by using, for example,

\[
\psi(\Delta p_a, A; V_A) = \frac{1}{2\pi} \int d^2 \Delta r_a \exp(i\Delta p_a \cdot \Delta r_a) \tilde{\psi}(\Delta r_a, A; V_A).
\]

The integration volume \( \int d^2 p_a \int d^2 a_1 \) in Eq. (18) can be replaced by \( \int d^2 p_{a21} \int d^2 A \). It is easy to see from the overall delta function that the total transverse momentum \( Q_T \) is of the order of the parton transverse momentum which is of a few MeV. Since we are interested in the experiments with observables \( Q^+_1, Q^-_1 \) of the TeV order, we can neglect the \( p_{a21}, A \) and \( Q_T \) dependence in \( |T|^2 \) and we can integrate Eq. (18) over these transverse momenta. The integration of \( Q_T \) gives us a delta function which enables us to express the differential cross section in terms of the wave function squares:

\[
\frac{d\sigma}{d^2 \Delta Q dQ^+_1 dQ^-_1 dQ^+_2 dQ^-_2} = \frac{1}{4s^2(2\pi)^2} \frac{1}{x_{a1}^2 x_{b1}^2} \sum_{m,n} \sum_{s_{a1}s_{b1}} |T|^2 \\
\times \frac{1}{2} \sum_{S_a} \sum_{S_b} [dV_a] \int d^2 \Delta r_{aa} |\tilde{\psi}_{am}(x_{a1}, s_{a1}; \Delta r_{aa}; V_a, S_a)|^2 \\
\times \frac{1}{2} \sum_{S_a} \sum_{S_b} [dV_b] \int d^2 \Delta r_{bb} |\tilde{\psi}_{bn}(x_{b1}, s_{b1}; \Delta r_{bb}; V_b, S_b)|^2. \\
\tag{21}
\]

We can define the following function \( f_a \) (and similarly \( f_b \)):

\[
f_a(x_{a1}, s_{a1}) = \frac{1}{(2\pi)^3} \frac{1}{2x_{a1}} \frac{1}{2} \sum_{m} \sum_{S_a} \sum_{s_{a1}} [dV_a] \int d^2 \Delta r_{aa}
\]

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From the normalization equation Eq. (5), one sees that
\[ \sum_{s_{a_1}} \int dx_{a_1} f_a(x_{a_1}, s_{a_1}) = 1. \] (23)

The function \( f_a \) can then be interpreted as the distribution function for parton \( a_1 \) with momentum fraction \( x_{a_1} \) and internal symmetric variable \( s_{a_1} \).

If we assume that for non-polarized hadron the distribution functions of different helicities are equal, i.e., \( f(x, 1) = f(x, -1) = f(x) \), we get the cross section for single parton scattering as

\[
\frac{d\sigma}{d^2\Delta Q d\gamma_1^+ d\gamma_2^+ d\gamma_2^-} = \frac{1}{4s^2} \frac{1}{x_{a_1} x_{b_1}} f_a(x_{a_1}) f_b(x_{b_1}) |T|^2,
\]

where \( y \)'s are the rapidities defined by \( y_i = 1/2 \ln(Q_1^+/Q_1^-) \) and \( |T|^2 \) is calculated by using the (2 \( \rightarrow \) 4 particle) Feynman diagrams.

IV. The cross section for double parton scattering

In this section we derive the cross section formula for double parton scattering. For simplicity, we use the di-lepton production processes in unpolarized hadron-hadron scattering as an example. The process is shown as in Fig. 1. By expanding the initial states as in the previous section, we can express the scattering amplitude square for double parton interaction as

\[
|M|_{dp}^2 = \sum_{m,n} \sum_{s_{a_1}, s_{a_2}} \prod_{i=3}^{m} \int \frac{d\gamma_1^+ d\gamma_i^+ d\gamma_i^-}{(2\pi)^3 2\gamma_i^+} \sum_{s_{b_1}, s_{b_2}} \prod_{i=3}^{n} \int \frac{d\gamma_1^+ d\gamma_i^+ d\gamma_i^-}{(2\pi)^3 2\gamma_i^+} \times \frac{1}{(2\pi)^4 4s^2 x_{a_1}^2 x_{b_2}^2} \int d^2 a_1 \int d^2 a_1' |m|_{dp}^2
\]

\[
\times \psi_{am}(x_{a_1}, a_1, s_{a_1}; x_{a_2}, -a_1 - \sum_{i=3}^{m} a_i, s_{a_2}; x_{a_3}, a_3, s_{a_3}, \ldots; x_{am}, a_m, s_{am})
\]

\[
\times \psi_{bn}(x_{b_1}, Q_1 - a_1, s_{b_1}; x_{b_2}, Q_2 + a_1 + \sum_{i=3}^{m} a_i, s_{b_2}; x_{b_3}, b_3, s_{b_3}, \ldots; x_{bn}, b_n, s_{bn})
\]
We inserted the following identities in Eq. (27) for a more symmetrical expression:

\[
\begin{align*}
&\psi^*_{am}(x_{a1}, a'_1, s_{a1}, x_{a2}, a'_2, -a'_1, -s_{a2}; x_{a3}, a_3, s_{a3}, \ldots; x_{am}, a_m, s_{am}) \\
&\times \psi^*_{bn}(x_{b1}, Q_1 - a'_1, s_{b1}, x_{b2}, Q_2 + a'_1 + a_1 + s_{b2}; x_{b3}, b_3, s_{b3}, \ldots; x_{bn}, b_n, s_{bn}).
\end{align*}
\]

(25)

In this equation, \( |m|_{dp}^2 \) is the partonic scattering matrix square and the integration over its final particle phase spaces, \( |T|_{dp}^2 \), will be calculated in Appendix. To calculate the jet productions from double parton scattering, one can apply Eq. (25) and use the corresponding \( |T|^2 \) for the jet production. The non-interacting partons are now numbered from \( i = 3 \) to \( m \) or \( n \). The momentum fractions of the interacting partons \( a_1, a_2, b_1, b_2 \) are again fixed by the overall delta function. They are

\[
x_{ai} = Q_{i}^+/P_A^+, \quad x_{bi} = Q_{i}^-/P_B^-, \quad i = 1, 2.
\]

(26)

In order to simplify the expression for the cross section formula, we define analogically to the previous section the collective variables \( V_a, V_b, S_a \) and \( S_b \) for the non-interacting partons. The only difference is that there are now \( m - 2 \) or \( n - 2 \) non-interacting partons. Therefore we can rewrite Eq. (25) as

\[
\begin{align*}
\frac{d\sigma}{d^4 Q_1 d^4 Q_2} &= \frac{1}{16s^4(2\pi)^6} \frac{1}{x_{a1}^2 x_{b2}^2} \\
&\times \frac{1}{2} \sum_{s_A} \frac{1}{2} \sum_{s_B} \left[ \int [dV_a] \int [dV_b] \sum_{s_{a1}s_{a2}s'_{a1}s'_{a2}} \sum_{s_{b1}s_{b2}s'_{b1}s'_{b2}} |T|^2 \\
&\times \int d^2 p_a \int d^2 a_1 \int d^2 a_2 \delta^2(p_a + a_1 + a_2) \psi_{am}(Q_1^+, a_1, s_{a1}; Q_2^+, a_2, s_{a2}, p_a; V_a, S_a) \\
&\times \psi_{bn}(Q_1^+, Q_1 - a'_1, s_{b1}; Q_2^+, Q_2 - a_1 - s_{b2}, -p_a - Q_1 - Q_2; V_b, S_b) \\
&\times \int d^2 p'_a \int d^2 a'_1 \int d^2 a'_2 \delta^2(p'_a + a'_1 + a'_2) \psi_{am}(Q_1^+, a'_1, s'_{a1}; Q_2^+, a'_2, s'_{a2}, p'_a; V_a, S_a) \\
&\times \psi_{bn}(Q_1^+, Q_1 - a'_1, s'_{b1}; Q_2^+, Q_2 - a'_2 - s'_{b2}, -p_a - Q_1 - Q_2; V_b, S_b) \\
&\times \delta^2(p_a - p'_a). \quad (27)
\end{align*}
\]

We inserted the following identities in Eq. (27) for a more symmetrical expression:

\[
1 = \int d^2 p'_a \delta^2(p'_a - p_a) \int d^2 a_2 \delta^2(a_1 + a_2 + p_a) \int d^2 a'_2 \delta^2(a'_1 + a'_2 + p'_a).
\]

Define the following transverse momentum variables by using parton momenta \( a_1, a_2 \) and \( p_a \):

\[
\begin{align*}
a_{21} &= \xi_{a1} a_2 - \xi_{a2} a_1, \\
p_{a21} &= x_{a}(a_1 + a_2) - (x_{a1} + x_{a2})p_a, \\
A &= p_a + a_1 + a_2.
\end{align*}
\]

(28)
here we define $\xi_a = x_{a1}/(x_{a1} + x_{a2})$ and $\xi_{a2} = x_{a2}/(x_{a1} + x_{a2})$. The variable $x_a$ is the sum of the momentum fractions of the partons which are not involving in the collision. Therefore we have $\xi_{a1} + \xi_{a2} = 1$, and $x_{a1} + x_{a2} + x_a = 1$. The momentum $A$ is the total transverse momentum of hadron $A$. By using the coordinate $r_{a1}$ of parton $a_1$, the coordinate $r_{a2}$ of parton $a_2$ and the center mass coordinate $r_a$ of the remaining partons, we can define the following variables which conjugate to momenta $a_{21}, p_{a21}$ and $A$:

$$
\begin{align*}
r_{21a} &= r_{a2} - r_{a1}, \\
r_{aa} &= r_a - r_a, \\
R_a &= (x_{a1} + x_{a2})r_a + x_a r_a,
\end{align*}
$$

(29)

here $r_a = \xi_{a1} r_{a1} + \xi_{a2} r_{a2}$ is the center mass coordinate of the two partons in hadron $A$ which are involving in the collision, $r_{21a}$ is the relative transverse distance between these two partons and $r_{aa}$ is the relative transverse distance between the center mass of these two partons and the center mass of the remaining partons. Note that these relative transverse vectors are Lorentz invariant quantities so that their values are the same as their values in a rest hadron. Finally, $R_a$ is the center mass coordinate of hadron $A$. The momentum and coordinate variables can then be replaced by the new variables by the following equations:

$$
\begin{align*}
a_1 &= x_{a1}A + \xi_{a1}p_{21a} - a_{21}, \\
a_2 &= x_{a2}A + \xi_{a2}p_{21a} + a_{21}, \\
p_a &= x_a A - p_{21a}, \\
r_{a1} &= R_a + x_a r_{aa} - \xi_{a2} r_{21a}, \\
r_{a2} &= R_a + x_a r_{aa} + \xi_{a1} r_{21a}, \\
r_a &= R_a - (x_{a1} + x_{a2}) r_{aa}.
\end{align*}
$$

(30)

The Jacobian of these replacements is 1. Similarly we can define the momentum variables $b_{21}, p_{21b}$, and $B$ and their conjugate coordinate variables $r_{21b}, r_{bb}$, and $R_b$ for hadron $B$. To express Eq. (27) in terms of coordinate variables we Fourier transform the wave functions from the transverse momentum space to the coordinate space by using, for example

$$
\psi(a_{21}, p_{21a}; A) = 1/(2\pi)^2 \int d r_{21a} dr_{aa} \exp(i a_{21} \cdot r_{21a} + i p_{21a} \cdot r_{aa}) \tilde{\psi}(r_{21a}, r_{aa}; A).
$$

The integration volume $\int d^2 p_a \int d^2 a_1 \int d^2 a_2$ in Eq. (27) can be replaced by $\int d^2 p_{a21} \int d^2 a_{21} \int d^2 A$. Since we are interested in the experiments with observables $Q_1^+, Q_1^-$ of a few TeV, we can therefore neglect the $p_{a21}$, $a_{21}$, $A$ dependence in $|T|^2$. Integrating Eq. (27) over these transverse momenta we get the differential cross section as:

$$
\frac{d\sigma}{d^2 Q_1 d^2 Q_2} = \frac{d\sigma}{d^2 Q_T d^2 \Delta Q dQ_1^+ dQ_1^- dQ_2^+ dQ_2^-}
$$
using these distribution functions in Eq. (32) we can express the cross section as

\[
\frac{1}{16s^4(2\pi)^8} \frac{1}{x_1^2x_2^2} \frac{1}{x_b^2x_b^2} \sum_{s_1s_2s_1's_2} \sum_{s_1s_2s_1's_2} |T|^2 
\]

\times \frac{1}{2} \sum_{s_A} \sum_{x_1} \frac{1}{2} \sum_{s_B} \sum_{x_2} \int [dV_a] \int [dV_b]

\times \int d^2r_{aa} \int d^2r_{21a} \int d^2r_{bb} \int d^2r'_{21a}

\times \tilde{\psi}_{am}(x_{a1}, s_{a1}; x_{a2}, s_{a2}; r_{aa}; r_{21a}; V_a, S_a) \tilde{\psi}_{bn}(x_{b1}, s_{b1}; x_{b2}, s_{b2}; r_{bb}; r_{21a}; V_b, S_b)

\times \exp(iQ_T[r_{aa} - r'_{aa} + \frac{1}{2}(\xi_{1a} - \xi_{2a})(r'_{21a} - r_{21a})] \exp[i\Delta Q(r'_{21a} - r_{21a})].

(31)

It is again easy to see from the overall delta function that for double parton scattering process, both the momenta $Q_T$ and $\Delta Q$ are of the order of parton’s transverse momenta which are of a few MeV. We can therefore neglect the $Q_T$ and $\Delta Q$ dependence in $|T|^2$ and we can integrate Eq. (31) over $Q_T$ and $\Delta Q$. These integrations give two delta functions

\[\delta^2(r_{21a} - r_{21a})\]

and $\delta^2(r_{aa} - r'_{aa})$. We see from the Appendix that the spin flip process for $|T|^2_{\text{sp}}$ is negligible therefore we can set $s_{ai} = s'_{ai}$, $s_{bi} = s'_{bi}$. These results enable us to express the differential cross section in terms of the square of wave functions:

\[
\frac{d\sigma}{dQ_T^1 dQ_T^2 dQ_T^3} = \frac{1}{16s^4(2\pi)^8} \frac{1}{x_1^2x_2^2} \frac{1}{x_b^2x_b^2} \sum_{s_1s_2s_1's_2} |T|^2 \int d^2r_{21a}

\times \frac{1}{2} \sum_{s_A} \sum_{x_1} \int [dV_a] \int d^2r_{aa} \tilde{\psi}_{am}(x_{a1}, s_{a1}; x_{a2}, s_{a2}; r_{aa}; r_{21a}; V_a, S_a)^2

\times \frac{1}{2} \sum_{s_B} \sum_{x_2} \int [dV_b] \int d^2r_{bb} \tilde{\psi}_{bn}(x_{b1}, s_{b1}; x_{b2}, s_{b2}; r_{bb}; r_{21a}; V_b, S_b)^2. \quad (32)

We can define the following function

\[
f_a(x_{a1}, x_{a2}, r_{21a}, s_{a1}, s_{a2}) = \frac{1}{4x_1x_2(2\pi)^6} \frac{1}{x_b^2x_b^2} \sum_{s_1s_2} \sum_{s_A} \int [dV_a] \int d^2r_{aa}

\times |\tilde{\psi}_{am}(x_{a1}, s_{a1}; x_{a2}, s_{a2}; r_{aa}; r_{21a}; V_a, S_a)|^2. \quad (33)

By the normalization equation Eq. (5) we can show that $f_a$ the distribution function for two partons with momentum fractions $x_{a1}$ and $x_{a2}$ in a relative transverse distance $r_{21a}$ By using these distribution functions in Eq. (32) we can express the cross section as

\[
\frac{d\sigma}{dQ_T^1 dQ_T^2 dQ_T^3} = \frac{d\sigma}{dQ_T^1 dQ_T^2 dQ_T^3} = \frac{1}{s^4} \frac{1}{x_1x_2x_b^2x_b^2} \sum_{s_1s_2s_1's_2} |T|^2

\times \int d^2r_{21a} f_a(x_{a1}, x_{a2}, r_{21a}, s_{a1}, s_{a2}) f_b(x_{b1}, x_{b2}, r_{21a}, s_{b1}, s_{b2}). \quad (34)

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If we assume that partons are weakly correlated and are evenly distributed in a hadron, we can use a flat distribution for the hadron wave function. For example, for hadron $A$ we have:

$$\frac{1}{4x_1x_2(2\pi)^6} \frac{1}{2} \sum_{S_A} \sum_{S_a} \int [dV_a] |\tilde{\psi}(x_{a1}; s_{a1}, x_{a2}; s_{a2}; r_{a1}; r_{a2}; V_a; S_a)|^2$$

$$= f_a(x_{a1}, s_{a1}; x_{a2}, s_{a2}) \Theta((r_{a1} - R_a)^2 - R^2) \Theta((r_{a2} - R_a)^2 - R^2).$$ \hspace{1cm}(35)

By using this in Eq. (34) and by carrying out the $d^2r_{21a}$ integration, we get

$$\frac{d\sigma}{dQ^+dQ^-dQ_2^+dQ_2^-} = \frac{\left(2\pi\right)^8}{s^4} \frac{1}{x_{a1}x_{a2}x_{b1}x_{b2} s_{a1}s_{a2}s_{b1}s_{b2}} \sum |T|_{2p}^2 \frac{\kappa}{\pi R^2} \times f_a(x_{a1}, x_{a2}, s_{a1}, s_{a2}) f_b(x_{b1}, x_{b2}, s_{b1}, s_{b2}).$$ \hspace{1cm}(36)

where $\kappa \approx 0.5$ is a geometrical factor associated with the overlapping area of the two colliding hadrons. For the cross section of the di-lepton production, we can use the result in Appendix A for $|T|_{2p}^2$ and get

$$\frac{d\sigma}{dm_1dy_1dm_2dy_2} \big|_{y_1=y_2=0} = \sum_{a_1,a_2} \frac{2\kappa \hat{e}_{a1}^2 \hat{e}_{a2}^2}{\pi R^2} f(x_{a1}, x_{a2}) f(x_{b1}, x_{b2}) \left( \frac{8\pi \alpha^2}{9m_1 s} \right) \left( \frac{8\pi \alpha^2}{9m_2 s} \right).$$ \hspace{1cm}(37)

where $m_i = \sqrt{Q_i^2} = s x_{ai} x_{bi}$ is the invariant mass square of the lepton pairs and $\hat{e}_{a_i}$ is the fractional charge of parton $a_i$. The summation is to sum over the flavors of parton $a_1$ and $a_2$. The factor 2 in Eq. (37) comes from when we include the Feynman diagram in Fig. 3 and its cross diagram. Equation (37) enables us to calculate the double parton cross section by using two universal double distribution functions and two individual "hard scattering" single parton cross sections.
V. Gluon splitting process

In this section we discuss the 4-final particle cross section from the gluon splitting process. We can use Eq. (24) for this process if we take the interacting partons $a_1$ and $b_1$ as gluon partons. The corresponding partonic scattering amplitude square is shown in Fig. 4. There are two quark loop integrations in this Feynman diagram and one of them is

$$L = \int d^2k \frac{Tr[\gamma^\beta \not{k} \not{\lambda}_0(\not{q} - \not{k})\gamma^\alpha(\not{q} + \not{k} - Q \not{Q} \not{Q})\not{\lambda}_0(Q - \not{k})]}{(2k^+ k^- - k^2 + i\epsilon)[2(k^+ - Q_2^+)(k^- - Q_2^-) - (k - Q_2)^2 + i\epsilon]} \times \frac{1}{2(2k^+ Q_1^+)(k^- - Q_2^-) - (k - Q_2)^2 + i\epsilon]}.$$ (38)

From the overall delta function, if we assume that all the transverse momenta of the partons are negligible, we have

$$a_1 \approx (Q_1^+ + Q_2^+, 0, 0), \quad b_1 \approx (0, Q_1^- + Q_2^-, 0).$$ (39)

We first carry out the integration over $k^-$. When $0 < k^+ < Q_2^+$, only the second $k^-$-pole in the denominator is located on the lower half complex plane. We can close the contour on the lower plane and pick up a pole at

$$k_2^- = Q_2^- + \frac{(k - Q_2)^2}{2(k^+ - Q_2^+)}. \quad (40)$$

When $-Q_1^+ < k^+ < 0$, only the third $k^-$-pole in the denominator is located on the upper half complex plane. We can close the contour on the upper plane and pick up a pole at

$$k_3^- = Q_2^- + \frac{(k - Q_2)^2}{2(k^+ + Q_1^+)}. \quad (41)$$

A plausible prejudgement is that the main contribution to the loop integration comes from when all the momenta in the loop are almost on their mass shell, i.e., when

$$k^- = Q_2^-, \quad k^+ = 0, \quad k^2 = 0, \quad (k - Q_2)^2 = 0.$$ (42)

With this in mind, we can set $k_2^- = k_3^- = Q_2^-$ and get

$$L = \int d^2k \int_{-Q_1^+}^{Q_2^+} dk^+ \frac{2\pi i Tr[\gamma^\beta \not{k} \not{\lambda}_0(\not{q} - \not{k})\gamma^\alpha(\not{q} + \not{k} - Q \not{Q} \not{Q})\not{\lambda}_0(Q - \not{k})]}{(2k^+ Q_2^+ - k^2 + i\epsilon)2(Q_1^+ + Q_2^+)(k - Q_2)^2(-2k^+ Q_1^- - k^2 + i\epsilon)}.$$ (43)

For $Q_1^+$ and $Q_2^+$ much larger than $|k|$ and $|k - Q_2|$, we can approximate the integration range of $k^+$ in Eq. (43) to the whole space then integrate $dk^+$ by closing the contour in the upper
half plane and pick up a pole at \( k^+ = k^2/(2Q_2^2) \). Eq. (38) is then

\[
L = \int d^2k (2\pi i)^2 Tr [\gamma^\beta (Q_2 \gamma^+ - k)\gamma^\alpha (Q_1 \gamma^- - k + Q_2)\gamma^\lambda (Q_2^\gamma^+ - k - Q_2)] \frac{4(Q_1^2 + Q_2^2)(Q_1^2 + Q_2^2) k^2(k - Q_2)^2}{4(Q_1^2 + Q_2^2)(Q_1^2 + Q_2^2) k^2(k - Q_2)^2} \tag{44}
\]

There are many terms contribute to the denominator in Eq. (44). One can show that the terms proportional to \( Q_1^+ Q_1^+ Q_2^- Q_2^- \) are equal to zero. The leading contributions to the denominator in Eq. (44) are terms proportional to \( Q_1^+ Q_1^- k_i(k - Q_2)^j \) and these terms make the integration over \( d^2k \) of Eq. (44) logarithmically divergent. One needs to introduce a cut-off to regulate this divergence. However, the cut-off is lost because of our previous approximation. To mimic the physical cut-off we will use the \( (2 - 2\epsilon) \)-dimensional regularization in the MS scheme. In dimensional regularization, one introduces a factor \( \mu^{2\epsilon} \) to fix the dimensionality. In order to simplify the formula, one can express \( \mu \) in terms of \( \mu_{MS} \) by the relation \( \mu_{MS}^2 = 4\pi \mu^{2\epsilon} (2\gamma - \epsilon) \), in which \( \gamma_E \) is the Euler constant. The result of the \( d^2 \epsilon \)-dimensional integral has a term with \( 1/\epsilon \) pole. The prescription is to drop this pole term, which effectively cuts off the integration at \( |k| \approx \mu_{MS} \). Since the physical cut-off is at \( k \approx Q_2 \), we set \( \mu_{MS} \approx |Q_2| \).

Preceding with this prescription we can get

\[
\int d^2\kappa \frac{k_i(k - Q_2)^j}{k^2(k - Q_2)^2} = \pi\left(\frac{1}{\epsilon} - \ln \frac{Q_2^2}{\mu_{MS}^2} \right) \frac{\delta^{ij}}{2} - \frac{Q_i Q_j}{Q^2}. \tag{45}
\]

After some tedious calculations we get the leading contribution to the scattering matrix square integrated over the 4-lepton final momentum space as

\[
|T|^2 = 2\frac{16\alpha_s^2\alpha^4 \pi^2}{9(2\pi)^6} \frac{1}{Q_1^2 Q_2^2} \frac{1}{(Q_1^2 + Q_2^2)^2(Q_1^2 + Q_2^2)} \tag{46}
\]

We have inserted a color factor \( C_F = T_F^2/N_G = 1/32 \) for Fig. 4 into Eq. (46), in which \( T_F = 1/2 \) comes from \( \delta_{\alpha\beta} T_F = Tr(t_\alpha t_\beta) \) and \( N_G = 8 \) is the number of gluon color configurations. Inserting \(|T|^2 \) to Eq. (44) and assuming that for unpolarized hadrons the gluon distributions function of helicity 1 is equal to that of helicity -1, we get the cross section for the gluon splitting process as

\[
\frac{d\sigma}{d^2\Delta Q dQ_1^+ dQ_1^- dQ_2^+ dQ_2^-} = \frac{d\sigma}{\frac{2}{7} \Delta Q dQ_1^2 dy_1 \frac{1}{7} dQ_2^2 dy_2} \frac{2\alpha_s^2\alpha^4 (2\pi)^6}{9s^2} \frac{1}{x_{a1} x_{b1}} \frac{1}{Q_1^2 Q_2^2} \frac{1}{(Q_1^2 + Q_2^2)^2(Q_1^2 + Q_2^2)} \times f_{ga}(x_{a1}) f_{gb}(x_{b1}). \tag{47}
\]
If we use the result in the Appendix for $|T|_{sp}^2$ and integrate $d^2\Delta Q$ for Eq. (24) and Eq. (37) up to a value $\Delta Q = q_{\text{max}}$ in which $0 << q_{\text{max}} << Q_1^+, Q_1^-, Q_2^+, Q_2^-$, we can see that the cross sections for single parton and gluon splitting processes behave like

$$\frac{d\sigma}{dm_1 dm_2 dy_1 dy_2} = \frac{1}{3} \sum_a \frac{16\beta^4 a^4}{gs^2(2\pi)^2 x_1 x_2} f_a(x_1) f_b(x_2) \frac{\pi q_{\text{max}}^2}{\sqrt{Q_1^+ Q_2^-}} \times \left( Q_1^+ + Q_2^+ - Q_1^- - Q_2^- \right) \left( Q_1^+ Q_2^- + 2Q_1^- Q_2^+ + Q_1^+ Q_2^- \right), \quad (48)$$

$$\frac{d\sigma}{dm_1 dm_2 dy_1 dy_2} = \frac{2\alpha_s^2 \alpha^4 (2\pi)^6}{gs^2 x_1 x_2} f_a(x_1) f_b(x_2) \frac{\pi q_{\text{max}}^2}{\sqrt{Q_1^+ Q_2^-}} \times \left[ \frac{\ln^2(q_{\text{max}}^2/\mu^2) - \ln(q_{\text{max}}^2/\mu^2) + 1}{(Q_1^+ + Q_2^+)(Q_1^- + Q_2^-)} \right], \quad (49)$$

Since in these processes $Q_T \approx 0$, $\Delta Q \approx Q_1 \approx -Q_2$, if we collect all the 4-particle events with transverse momenta $Q_1$ or $Q_2$ less than $q_{\text{max}}$, we should see the cross section of single parton process increases linearly with $q_{\text{max}}^2$ and the cross section of the gluon splitting process increases approximately as $q_{\text{max}}^6$. However, from Eq. (37) we see that the cross section of double parton process keeps at a constant value when $q_{\text{max}}$ increases. Also if we use the HERA result for the quark distribution function $f(x) \propto x^{-1.4}$ and use Eq. (13) and Eq. (26) for the values of $x$'s, when we select the events with $Q_1^+, Q_2^+, Q_1^-, Q_2^+ \approx Q$ we can see that the differential cross section for single parton process behaves like $s^{0.4}Q^{-6.8}q_{\text{max}}^2$ and that for gluon splitting process behaves like $s^{0.4}Q^{-10.8}a_{\text{max}}^6$. For double parton cross section, if we assume that partons are evenly distributed in a hadron with radius $R$ and assume that the double parton distribution function $f(x_1, x_2)$ can be factorized into multiplication of two single parton distribution functions $f(x_1) f(x_2)$, we see the differential cross section in Eq. (37) behaves like $s^{0.8}Q^{-7.6}R^{-2}$. For experiments of searching double parton events we suggest one do experiments at large colliding hadron center mass energy $s$ and collect the events with small $q_{\text{max}}^2$ then check their cross section behavior on $Q$. For experiments of searching gluon splitting process we suggest one collect the events up to a large $q_{\text{max}}^2$ value and select the events with small $Q$.

The significance of the result of the double parton cross sections in this paper are two-fold. We can use Eq. (37) for double parton cross section and fit it with the single parton distributions obtained from the deep inelastic scattering or Drell-Yan process then find the
hadron size $R$ or we can use the double parton experiments data to fit the new double parton distribution functions in a hadron. In both ways they give us a better understanding of the hadron structure.

ACKNOWLEDGMENTS

Special thanks to my advisor Prof. Davison E. Soper. In memory of my father who passed away during the time of my working on this paper.
APPENDIX

In this appendix we calculate $|T|^2$ for single and double parton processes where $|T|^2$ is the partonic matrix square integrated over the 4-final particle phase spaces:

$$|T|^2 = \left( \prod_{i=1}^{4} \int \frac{d^3l_i}{(2\pi)^32l_i^0} \right) \delta^4(l_1 + l_2 - Q_1)\delta^4(l_3 + l_4 - Q_2).|m|^2$$  \hfill (50)

The following result is essential for calculating $|T|^2$:

$$\int \frac{d^3l_1}{2l_1^0} \int \frac{d^3l_2}{2l_2^0} \delta^4(l_1 + l_2 - Q_1)Tr(\gamma_\alpha I_1 \gamma_{\alpha'} I_2) = \frac{2\pi}{3}(-g_{\alpha\alpha'} + \frac{Q_{1a}Q_{1a'}}{Q_1^2})Q_1^2.$$  \hfill (51)

With this result, the leading Feynman diagram to $|T|^2_{sp}$ shown in Fig. 5 can be express as:

$$|T|^2 = \frac{16e^4_{a1}4^4}{9(2\pi)^6}(-g_{\alpha\alpha'} + \frac{Q_{1a}Q_{1a'}}{Q_1^2})(-g_{\beta\beta'} + \frac{Q_{2\beta}Q_{2\beta'}}{Q_2^2})$$

$$\times Tr[(Q_1 - q)^\alpha q^\alpha' (Q_1 - q)^\beta q^\beta'] \frac{Q_1^2Q_2^2(Q_1 - a)^2(Q_1 - a)^2}{Q_1^2Q_1^2Q_1^2Q_1^2}$$

$$+ \text{cross diagram contribution.}$$ \hfill (52)

If we neglect the transverse momenta for partons, we can set $a \approx (Q_1^+ + Q_2^+, 0, 0)$ and $b \approx (0, Q_1^- + Q_2^-, 0)$. By using these in Eq. (52) we get

$$|T|^2_{sp} = \frac{16e^4_{a1}4^4(Q_1^+ + Q_2^+)(Q_1^- + Q_2^-)(Q_1^+ Q_1^- + 2Q_1^+ Q_2^- + Q_2^+ Q_2^-)}{Q_1^2 Q_2^2 Q_2^2 Q_1^2}.$$ \hfill (53)

For double parton interaction, since it is a four-parton process in the feynman diagram, it is possible that the interference terms from the spin flip processes as show in Fig. 5. may occur. The spin flip contribution as shown in Fig. 6 is:

$$|T_1|^2 = \frac{e^2e_q^2 1}{Q_1^2} \frac{2\pi}{3}(-g_{\alpha\alpha'} + \frac{Q_{1a}Q_{1a'}}{Q_1^2})$$

$$\times Tr(Q^{-1}_1, Q_1 - a, -1/2)(-ie_q\gamma^\alpha_{\theta\beta})u_\beta(Q_1^+, a, 1/2)$$

$$\times Tr(Q_1^+, a', 1/2)(ie_q\gamma^\alpha_{\beta\gamma'})u_{\gamma'}(Q_1^-, Q_1 - a', 1/2).$$ \hfill (54)

To calculate $T_1$ we use spinors in Ref. [7] for quarks moving in the $+z$ direction, they are and use spinors in Eq. [7] for quarks moving in the $-z$ direction. The calculation shows that if we neglect the transverse momenta for partons, i.e., if we set $a_1 \approx (Q_1^+, 0, 0), b_1 \approx (0, Q_1^-, 0), a_2 \approx (Q_2^+, 0, 0)$ and $b_2 \approx (0, Q_2^-, 0)$ the spin flip contribution is zero. With this result, we
can calculate $|T|^2$ for double parton process by using

$$
|T_{dp}|^2 = \frac{e_1^4 e_2^4}{(2\pi)^2 Q_1^2 Q_2^2} \frac{1}{3} \left( -g_{\alpha \alpha'} + \frac{Q_{1\alpha} Q_{1\alpha'}}{Q_1^2} \right) \frac{2\pi}{3} \left( -g_{\beta \beta'} + \frac{Q_{2\beta} Q_{2\beta'}}{Q_2^2} \right) \\
\times Tr[\gamma^\alpha \gamma^\alpha'] Tr[\gamma^\beta \gamma^\beta']
$$

+ cross diagram contribution.

We get

$$
|T_{dp}|^2 = \frac{2 \hat{e}_1^2 \hat{e}_2^2 \alpha^4 16^2}{(2\pi)^6}.
$$
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FIGURE CAPTIONS

Fig. 1 Double parton interaction of hadron-hadron scattering.

Fig. 2 Single parton interaction of hadron-hadron scattering. The dot line denotes quark or gluon partons.

Fig. 3 Partonic scattering matrix square of double parton interaction.

Fig. 4 Partonic scattering matrix square of gluon splitting process.

Fig. 5 The leading Feynman diagram for single parton scattering.

Fig. 6 Spin-flip process contribution to double parton interaction.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6