Decoherence induced by electron accumulation in quantum measurement of charge qubits

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In this paper, we study the quantum decoherence induced by accumulation of electron tunnellings during the quantum measurement of a charge qubit. The charge qubit is a single electron confined in coupled quantum dots. The measurement of the qubit states is performed using a quantum point contact. A set of master equations for qubit states is derived within a non-equilibrium perturbation to the equilibrium reservoir due to the electron accumulation between the source and drain of the quantum point contact. The quantum decoherence of the qubit states arose from the electron accumulation during the measurement is explored in this framework, and several interesting results on charge qubit decoherence are obtained.

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I. INTRODUCTION

Quantum decoherence$^1$ is mainly induced by the interaction of a microscopic system coupled with its environment. The system-environment coupling results in a non-unitary evolution of the system, which destroys the purity of quantum states and leads to information loss toward the environment. These issues have attracted much attention in the study of quantum computation and quantum communication in recent years$^1$. Some investigations of quantum decoherence have been focused on the measurement induced decoherence$^2$, the dynamical controls of the decoherence$^3$, and the implementation of realistic quantum information processors, these investigations become the most champion works in the field. In the solid-state quantum computer with charge qubits, quantum measurement of the qubit states can be performed by coupling the charge qubit with a sensitive electrometer such as quantum point contacts (QPCs)$^4$ or a single-electron transistor$^5,6$. In this paper, we shall focus on the charge qubit measurement using a QPC, and study the non-equilibrium dynamical effect of the QPC on the decoherence of the qubit states, here the charge qubit is a single electron confined in coupled quantum dots (CQD).

Literaturely, to study the quantum decoherence induced by quantum measurements, an equilibrium approximation is applied to the reservoir state$^7$. For example, in the study of a charge qubit measured by a QPC (Fig. 1), the electronic reservoir, the source and the drain of the QPC, is assumed to be macroscopic enough in comparison with the CQD. Then the thermal equilibrium of the reservoir is kept continuously through rapid relaxation processes. This condition of a perfect heat bath causes electrons tunneling such that the extra electrons arriving at the drain will flow back rapidly into the source through a close loop of the transport circuit. No extra electron accumulates in the steady reservoir. Practically, however, the condition of the perfect heat bath is not guaranteed at mesoscopic scale of the QPC. The electron accumulation (EA) in the source and the drain of the QPC may destroy the equilibrium of reservoir and influence the outcome of the electrometer. As a result, the effects of EA may become significant to the decoherence of the measured qubit. In this paper, we shall treat approximately this intermediate non-equilibrium effect as a perturbation to the equilibrium reservoir. The quantum decoherence of the charge qubit due to the EA is then studied within this framework.

The paper is organized as follows: The model of the charge qubit measurement and the perturbation scheme of taking the EA effect into account are presented in Sec. II. A set of master equations for the reduced density operator of the qubit is also derived in this section. In Sec. III., the quantum decoherence of the qubit in the thermal equilibrium limit is illuminated according to the resultant master equations. An abnormal dependence of the qubit decay mode to the temperature and bias voltage is obtained. The EA effect on the quantum decoherence of the qubit states is explored in Sec. IV. The dynamics of the EA is analyzed, and the EA induced decoherence of the qubit is extensively discussed. Remarkably, we also found a decoherenceless EA effect in the low temperature regime. Finally, a summary is given in Sec. V.

II. CHARGE QUBIT MEASUREMENT USING QPC

The model we consider here is a single electron confined in a CQD (as a charge qubit) that is measured by the QPC detector, see Fig. 1. The Hamiltonian of the whole
system is given by \[ H = \hat{H}_S + \hat{H}_B + \hat{H}', \] \( \hat{H}_S = E_L \hat{c}_L^+ \hat{c}_L + E_R \hat{c}_R^+ \hat{c}_R + \Omega_0 (\hat{c}_R^+ \hat{c}_L + \hat{c}_L^+ \hat{c}_R), \] \( \hat{H}_B = \sum \varepsilon \hat{a}_l^+ \hat{a}_l + \sum \varepsilon \hat{a}_r^+ \hat{a}_r, \) \[ \hat{H}' = (\Omega - k \Omega R \hat{c}_L^+ \hat{c}_R) \sum (\hat{a}_l^+ \hat{a}_r + \hat{a}_r^+ \hat{a}_l). \]

Here, \( \hat{H}_S \) is the Hamiltonian of the charge qubit in which \( \hat{c}_L^+ (\hat{c}_L) \) and \( \hat{c}_R^+ (\hat{c}_R) \) are the electron creation (annihilation) operators of the electron sited in the two dots labelled by \( L \) and \( R \) as shown in Fig. 1a. They satisfy the condition \( \hat{c}_L^+ \hat{c}_L + \hat{c}_R^+ \hat{c}_R = 1 \). The parameter \( \Omega_0 \) is the electron tunneling amplitude between the two dots. \( \hat{H}_B \) denotes the Hamiltonian of the QPC reservoir, which is decomposed into two terms for the source and the drain, respectively, and \( \hat{a}_l^+ (\hat{a}_l) \) and \( \hat{a}_r^+ (\hat{a}_r) \) are the corresponding electron creation (annihilation) operators. Since the presence of an electron in a dot close proximity to the QPC will cause a variation in barrier of the QPC, \( \Omega \rightarrow \Omega - \delta \Omega \), the interaction Hamiltonian \( \hat{H}' \) describes the coupling of the quantum dots and the QPC in close proximity to the dot \( R \). The information of the qubit-states can then be read out through the QPC current \[ I. \]

There are two channels for electron tunneling in the QPC. One is the forward tunneling processes from the source to the drain, and the other is the backward tunneling processes from the drain to the source, see Fig. 1b. In the high-bias regime, magnitude of the electron tunneling rate for the forward tunneling processes is much larger than the one in the backward tunneling processes. The contribution of the backward tunneling processes could be ignored as treated in \[ 2. \] However, in a nonequilibrium description, both contributions of the forward tunneling processes and backward tunneling processes should be included for an arbitrary bias voltage. The Hilbert space is then spanned by the following basis

\[ \{|\phi_i\rangle \otimes |D(L^n, R^m)\rangle, |\phi_i\rangle \otimes |D(L^m, R^m)\rangle\}, \]

where \( |\phi_i\rangle \) is an electron eigen-state in the CQD (we define \( |\phi_0\rangle = |g\rangle \) and \( |\phi_1\rangle = |e\rangle \)) as the ground and excited states in the eigen-state representation, respectively, \( |D(L^n, R^m)\rangle = \hat{a}_l^t_1 \cdot \hat{a}_l^t_n |B_0\rangle \) is a state with \( n \) electrons accumulated in the drain through the forward tunneling processes, and \( |D(L^m, R^m)\rangle = \hat{a}_r^t_1 \cdot \hat{a}_r^t_m |B_0\rangle \) a state with \( m \) electrons accumulated in the source through the backward tunneling processes. The state \( |B_0\rangle \) is the reservoir (equilibrium) vacuum, and the operator \( \hat{a}_{l/r}(t) \) denotes the annihilation of an electron from \( |B_0\rangle \) (or the creation of a hole below the Fermi energy of the source (drain)), while the operator \( \hat{a}_{l/r}^+ (t) \) denotes the creation of an electron above the Fermi energy of the source (drain). Therefore, an arbitrary state of the whole system can be expressed as

\[ |\psi(t)\rangle = \sum_i |\phi_i\rangle \otimes \left( \sum_n \sum_{L^n R^m} b_{i,L^n R^m}(t) |D(L^n, R^m)\rangle \right) \]

\[ + \sum_m \sum_{L^m R^m} \tilde{b}_{i,L^m R^m}(t) |D(L^m, R^m)\rangle. \]

To study the charge qubit measured by the QPC, the electronic reservoir is usually assumed to be in the reservoir vacuum state (a thermal equilibrium state). The reservoir vacuum state is uniquely determined for a given temperature and chemical potential. The reservoir time correlation function is usually calculated in this state. However, tunneling electrons from the forward tunneling processes will occupy the levels above the Fermi energy of the drain for a short period and disturbs the reservoir vacuum state, see Fig. 1b. This phenomenon, called the electron accumulation (EA) in QPC, should be taken into account, vice versa for the backward tunneling processes. Meanwhile, the EA may also be induced by the impurity of the reservoir or a circuit with a worse transport mobility (the imperfect condition of the reservoir). Literature, the effect of the EA is ignored \[ 3. \]

One assumes that electrons created in drain (for the forward tunneling processes) will be forced into the circuit instantly, no electrons are accumulated in the drain tem-
porarily. Therefore, there is no any variation to the Fermi energy. The read-out current is completely contributed by the created electron of drain through the circuit, as a result in the macroscopic limit. However, for a mesoscopic reservoir, a variation of the Fermi energy can be generated by the EA. It is certainly interesting to study what the influence of the EA effect is on the decoherence of the measured qubit.

In general, the EA should be considered by treating the reservoir as a fully non-equilibrium system. However, the reservoir is indeed an asymptotic stationary state at a short period. Thus, the EA can be described simply by a variation of the chemical potentials in the source and the drain. In the forward tunneling processes, the tunneling of n electrons moved from the source to the drain results in an effective increase of the chemical potential in the drain \( \mu_R \rightarrow \mu_R^t = \mu_R + \delta \mu_R(n, \beta) \) and a corresponding decrease of the chemical potential in the source \( \mu_L \rightarrow \mu_L^t = \mu_L - \delta \mu_L(n, \beta) \), see Fig. 1b. Here, \( \beta = 1/k_BT \) is the inverse temperature. In thermal equilibrium state, the electron distributions obey Fermi-Dirac function \( \bar{F} = \frac{1}{1+\exp(\beta(\epsilon - \mu))} \) for the source and \( F = \frac{1}{1+\exp(\beta(\epsilon - \mu_R^t))} \) for the drain. Obviously, at a given temperature, the chemical potential determines the number density of electrons \( N \) in reservoir. The relation of its inverse function can be solved. Furthermore, because the QPC is a 2-dimensional electron gas, the density of states, \( g_L \), for the source \( (g_R \text{ for the drain}) \) can be assumed to be energy independent. Then, the chemical potential can be expressed as \( \mu_L, R(N, \beta) = \frac{1}{\beta} \ln \left[ \exp(3N/g_L, R) - 1 \right] \). The variation due to the EA is given by \( \delta \mu_R, L(N + n/A, \beta) - \mu_R, L(N, \beta) = \pm \delta \mu_R, L(n, \beta) \), where \( A \) is the area of the QPC. One can find that up to the first order of \( n/A \),

\[
\delta \mu_R, L(n, \beta) = \frac{n}{A} \left( \frac{1 + e^{-\beta \mu_R}}{g_R, L} \right). \tag{7}
\]

As a result, even if the EA disturbs slightly the equilibrium of the reservoir, it may induce decoherence to the measured qubit. This effect can be explored from the master equation of the measured qubit which has the form as

\[
\frac{d}{dt} \hat{\rho}(t) = \frac{1}{i\hbar} \left[ \hat{H}_s, \hat{\rho}(t) \right] - \hat{R} \hat{\rho}(t),
\]

where \( \hat{\rho}(t) \) is the reduced density operator of the qubit

\[
\hat{\rho}(t) = \sum_{n=0}^{\infty} \langle \hat{D}^{(n)}(t) \rangle = \sum_{L^m, R^n} \langle \hat{D}(L^m, R^n) \rangle \hat{D}(L^m, R^n) \rangle + \hat{\rho}_{\text{tot}}(t) \rangle. \tag{11}
\]

\( \hat{\rho}_{\text{tot}}(t) \) in Eq. 11 denotes the total density operator of the whole system, and the conditional partial traces are defined by

\[
\hat{D}(L^m, R^n) \rangle \hat{D}(L^m, R^n) \rangle = \sum_{L^{m'}, R^{n'}} \langle \hat{D}(L^{m'}, R^{n'}) \rangle \hat{D}(L^{m'}, R^{n'}) \rangle.
\]

The exponential decay factor in Eq. 8 comes from the bias effect. The detailed expression of the ER decay rate \( \Gamma_{R, L}(n) \) depends on the imperfect condition of the QPC associated with the bias effect. In general, \( \Gamma_{R, L} \) are functions of the tunneling electron number \( n \) and, are different for the forward tunneling processes and backward tunneling processes. 1/\( \Gamma_{R, L}(n) \) denotes the release time of the ER. The processes repeat continually through the transport circuit which is given by the oscillation function in Eq. 9, where \( \omega_d \) is the frequency of the repeated processes. It can be estimated that \( \omega_d \sim v_d/l \) with \( l \) the length of the QPC in the tunneling direction and \( v_d \) the electron drift velocity in the transport circuit. Thus, Eq. 9 describes properly the stochastic nature of the tunneling processes embedded in the ER process and the cyclic motion of the external transport circuit. Obviously, the magnitude of the chemical potential variation decays to zero as the reservoir approaches to the local equilibrium state, namely, \( \mu_L(t) - \mu_R(t) = V_d \). Similar to the forward tunneling processes, the tunneling of \( m \) electrons moved from the drain to the source for the backward tunneling processes leads to the chemical potential increasing \( \mu_R \rightarrow \mu_R^t = \mu_R + \delta \mu_R(m, \beta, t) \) in the drain and decreasing \( \mu_L \rightarrow \mu_L^t = \mu_L + \delta \mu_L(m, \beta, t) \) in the source. Up to the first order of \( \delta \mu_{R, L}(n, \beta, t) \), the perturbed Fermi-Dirac function is given by

\[
F_{R, L}^+(n, \beta, t) \approx F_{R, L} \pm \delta \mu_{R, L}(n, \beta, t) \frac{\partial F_{R, L}}{\partial \mu_{R, L}}. \tag{9}
\]

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\]
in the Appendix. The qubit decoherence due to the electrical measurement is governed by the dissipation term \(-\dot{R}_D(t)\) in Eq. (10). This term is composed of the reservoir spectrum functions, which are the Fourier transformation of the reservoir time correlation functions defined by \(\text{Tr}_{D^{(n)}}(\hat{D}^{(m)})[\hat{f}(t)\hat{f}(t')\hat{\rho}_{tot}]\) and \(\text{Tr}_{D^{(n)}}(\hat{D}^{(m)})[\hat{f}(t)\hat{f}(t')\hat{\rho}_{tot}]\). The reservoir fluctuation due to the QPC-dots interaction is encoded in the reservoir operators \(f^{(+)}(t)f^{(+)}(t')\) and \(\hat{f}(t)\hat{f}(t')\), where \(\hat{f}_t = \sum_{\ell} e^{i(e_{\ell}-\epsilon_{\ell})t/h}\hat{a}^{\dagger}_\ell\hat{a}_\ell\) is the electron tunneling operator in the interaction picture of the reservoir. The tunneling processes of electrons relative to the EA have also been taken into account in these functions by treating the EA-fluctuated reservoir state as the perturbed Fermi-Dirac function through Eq. (9). As a result, the decoherence of the measured qubit due to the EA effect is manifested in the master equation through the first order perturbation of the reservoir spectrum functions, which is associated with the perturbed term \(\pm\delta\hat{\mu}_{R,L}(n,\beta, t)\partial\hat{\rho}_{tot}/\partial\hat{\mu}_{R,L}\) of the perturbed Fermi-Dirac function in Eq. (9). A perturbation scheme taking the EA effect into account is then given as follows

\[
\hat{\rho}_{tot}(t) = \hat{\rho}_{0,tot}(t) + \xi\hat{\rho}_1(t) + \xi^2\hat{\rho}_2(t) + \cdots, \quad (12)
\]

where \(\xi = U/\bar{\mu}\) is a perturbation parameter characterizing the EA effect with \(U = ((\delta\hat{\mu}_L(n,\beta) + \delta\hat{\mu}_R(n,\beta))/n\) and \(\bar{\mu} = (\mu_L + \mu_R)/2\). The zero order term \(\hat{\rho}_{0,tot}(t)\) in Eq. (12) represents the usual states without the EA. The higher order terms describe the contribution of the EA. By taking partial trace to Eq. (12), we have the perturbation scheme for the reduced density operator of the qubit

\[
\hat{\rho}(t) = \hat{\rho}_0(t) + \xi\hat{\rho}_1(t) + \cdots. \quad (13)
\]

The QPC, as a sensitive charge detector, may be required a large electron transparency and a relatively strong coupling strength to maximize the detector sensitivity in the current experiments. However, we shall focus in this paper on the tunneling transparency regime (the weak dot-QPC coupling). This is because, as Gurvitz has pointed out, the occupation probability of a single electron in the coupled dot (as a qubit) can be traced through the QPC readout current in this regime. To take the weak dot-QPC coupling into account, the second order cumulant expansion technique is used. Here, the Markovian approximation has been introduced in the technique. It should be noted that since the electron tunneling in the QPC is determined by the external bias and the QPC-dots interaction, it results in the qubit dynamics with three time scales: the qubit decoherence time, the ER decay time due to the electron release by the bias, and the correction time due to the QPC-dots interaction associated with the bias, temperature and the electron accumulation. In literature, the validity of the Markovian approximation could depend on the time scale of the correlation time. When the correlation time is much smaller than other time scales (in the Markovian regime), the non-Markovian memory effect in the qubit dynamics can be ignored. The qubit decoherence under the environment effect through the QPC-dots interaction is simply a decay process. Taking the electron accumulation effect into account, an extra qubit relaxation may be induced but the enhancement should not be so significant under the perturbative treatment. Therefore, the Markovian approximation is still applicable for the derivation of the master equations here.

As a result, the master equations for the measured qubit in our perturbation scheme are obtained as follows

\[
\xi^0 : \frac{d}{dt}\hat{\rho}_0(t) = -i\hat{L}_D\hat{\rho}_0(t) - \lambda \left[ \hat{q}, \left[ \left[ \hat{G}^{(0)}, \hat{\rho}_0(t) \right] \right] \right], \quad (14)
\]

\[
\xi^1 : \frac{d}{dt}\hat{\rho}_1(t) = -i\hat{L}_D\hat{\rho}_1(t) - \lambda \left[ \hat{q}, \left[ \left[ \hat{G}^{(0)}, \hat{\rho}_1(t) \right] \right] + \mathcal{F}(t) \left[ \left[ \hat{G}^{(1)}, \hat{N}_0(t) \right] \right] \right]. \quad (15)
\]

Eq. (14) is nothing but the rate equation of the qubit for the reservoir in the thermal equilibrium state without the EA effect. The EA effect on the measured qubit is described by Eq. (15). In Eqs. (14) and (15), \(\hat{L}_D\) is the Liouvillian operator for the qubit, \(\lambda = g_d\mu_d/\hbar\), the double-bracket commutator \[\left[ \hat{A}, \hat{B} \right] \equiv \hat{A}\hat{B} - \left( \hat{A}\hat{B} \right)^\dagger\], and \(\mathcal{F}(t)\) represents the contribution of the bias effect to the EA

\[
\mathcal{F}(t) = \cos(\omega_d t) \frac{e^{-\gamma t} + e^{-\gamma t'}}{1 + r}, \quad r = \frac{g_R (1 + e^{-\gamma \mu_L})}{g_L (1 + e^{-\gamma \mu_R})}. \quad (16)
\]

Also, the operator \(\hat{q}\) and \(\hat{G}^{(k)}\) for \(k = 0, 1\) in Eqs. (14) and (15) are defined as

\[
\hat{q} = \hat{P}_0 - \hat{P}_1 - \hat{P}_2, \quad \hat{G}^{(k)} = \sum_{i=0,1,2} (\hat{G}^{(k)}_{+i} + \hat{G}^{(k)}_{-i})\hat{P}_i, \quad (17)
\]

and the operators \(\hat{P}_{0,1,2}\) are given by

\[
\hat{P}_0 = \left( \Omega - \frac{\Omega \cos \theta}{2} \right) \frac{1 + \Omega \cos \theta}{2} \left[ \langle g \rangle \langle g \rangle - \langle e \rangle \langle e \rangle \right], \quad \hat{P}_1 = \frac{\Omega \sin \theta}{2} \langle e \rangle \langle g \rangle, \quad \hat{P}_2 = \frac{\Omega \sin \theta}{2} \langle g \rangle \langle e \rangle, \quad (18)
\]

where \(\theta = \cos^{-1} [(E_R - E_L)/\gamma]\), \(E_{R,L}\) denote the energy for a single electron state in right and left dot, and \(\gamma = \sqrt{4\Omega^2 + (E_R - E_L)^2}\) is the energy difference of the two eigenstates of the charge qubit. The coupled dots structure is shown by \(\theta\). The operators \(\hat{P}_{1,2}\) are responsible for the inelastic excitation and relaxation of the electron in the dots coupling with the QPC. The coefficients
As we can see, the EA effect is described by Eqs. (15) and (23). Eqs. (14) and (22) are simply the rate equations for the thermal equilibrium state without the EA effect. If the bias current flows from the source to the drain such that electrons accumulate in the drain, the EA induces a negative effective bias voltage $-\delta V_{EA}$. Then, an additional current $I_1 = \frac{\pi}{\hbar} \text{Tr} [\xi N_1(t)]$ is induced from the drain to the source. If the bias current flows from the drain to the source such that electrons accumulate in the source, a positive effective bias voltage $\delta V_{EA}$ is induced and an additional current flows from the source to the drain. Eq. (24) shows a modification of the first order perturbation to the transport current in the QPC. Eqs. (14) and (24) are the main result of the theory, which will be used to explore the quantum decoherence of the measured qubit in the rest of the paper.

III. DEPHASING AND RELAXATION IN THERMAL EQUILIBRIUM LIMIT

As we have discussed in the previous section, the dephasing dynamics can be considered as a perturbative modification of the thermal equilibrium dynamics for the reduced system. Without considering the EA effect, the electronic reservoir is treated as the thermal equilibrium state, in which the ER process is assumed to respond fast and effectively. The measured qubit in this case can be studied by the zero-order perturbation master equation Eq. (14). When the EA effect also becomes significant, the qubit undergoes a local non-equilibrium process initially, and then approaches to a stable state asymptotically due to the bias effect. In this section we will study the quantum decoherence of the qubit in the thermal equilibrium limit. The EA effect on the qubit decoherence will be explored in the next section.

In order to show the intrinsic measurement effect on the decoherence of the qubit, we concentrate first on the qubit with symmetric CQD, namely, both dots have equal energy levels $E_R = E_L$ and the qubit state is characterized by $\theta = \pi/2$. The qubit with asymmetric CQD will be studied later.

There have been several efforts contributed on this issue by using the secular approximation or discussed in the zero temperature limit. Here, an analytic discussion is presented. To discuss the dephasing and the relaxation of the qubit in the measurement processes, a set of matrix elements for the reduced density operator $\hat{\rho}_0$ are introduced: $\rho_{0,ij} = \rho_{i,0j} = \rho_{j,0i} = \rho_{0,ij} + \rho_{0,ji}$ and $i \rho_{0,p} = \rho_{0,pg} = \rho_{pg,0}$, where $\rho_{0,ij} = (i | \rho_0 | j)$, and $| i, j = g \rangle$ ($| i, j = e \rangle$) are the ground (excited) state of the qubit. Eq. (14) can then be expressed as

\[ \dot{\rho}_{0,r}(t) = -\Gamma_{0,r} (\rho_{0,r}(t) - \rho_{0,r}(\infty)), \]
\[ \Gamma_{0,r} = \eta_d G^{(0)}_{-+,a}, \rho_{0,r}(\infty) = \frac{G^{(0)}_{+-,a}}{G^{(0)}_{++}}, \tag{25} \]
\[ \dot{\rho}_{0, d}(t) = \frac{\gamma}{\hbar} \rho_{0, d}(t), \quad \dot{\rho}_{0, p}(t) = -\frac{\gamma}{\hbar} \rho_{0, d}(t) - \Gamma_{0, r} \rho_{0, p}(t), \]

where \( \eta_d = \pi g_L g_R (\delta \Omega)^2 / \hbar \), and

\[ G_{+a}^{(k)} = -\sum_{i=1,2} \left( \frac{G_{+a}^{(k)} + G_{-a}^{(k)}}{2} \right), \]

\[ G_{+b}^{(k)} = \sum_{i=1,2} \left( (-1)^i \frac{G_{+b}^{(k)} + G_{-b}^{(k)}}{2} \right), \]

with \( k = 0, 1 \).

According to Eq. (26), the relaxation rate of the qubit is \( \Gamma_{0, r} \). Applying the previous definitions in Eqs. (28), it leads to

\[ \Gamma_{0, r} = \frac{\eta_d (V_d \sinh \beta V_d - \gamma \sinh \beta \gamma)}{\cosh \beta V_d - \cosh \beta \gamma}, \quad \rho_{0, r}(\infty) = \frac{-\gamma \eta_d}{\Gamma_{0, r}}. \]

A plot of \( \Gamma_{0, r} \) and \( \rho_{0, r}(\infty) \) is shown in Fig. 2. The solid curves denote the symmetric CQD. The relaxation rate \( \Gamma_{0, r} \) is a positive-monotonic-increase function of the temperature and bias voltage. Increasing temperature and bias voltage will enhance qubit relaxation, as we expected. The asymptotic matrix element \( \rho_{0, r}(\infty) \) denotes an asymptotic stable state of the qubit. It is a negative increasing-monotonic function of temperature and bias voltage with values in the range \( -1 \approx 0 \). Because the external bias can be regarded as a macroscopic reservoir (an effective heat bath) with respect to the QPC, it is expected that there exists an analogy between biased QPC and the heat bath \( \hbar \). A limited case is \( \rho_{0, r}(\infty) \rightarrow 0 \) as \( V_d \gg \gamma \). The qubit tends to stay in a completely random mixed state in the high bias limit. This property is similar to the thermal randomness caused by the heat bath. According to \( \rho_{0, r}(\infty) \) in Eq. (28), the ground state occupation of the qubit shows a similar dependence on the temperature effect and bias effect. Furthermore, it can be checked that \( \Gamma_{0, r} \rightarrow \eta_d \gamma \) in the zero bias and zero temperature limit. The relaxation due to QPC can not be stopped by turning off bias. The QPC will exhaust energy and information of the qubit, as pointed out first in \( \hbar \).

The qubit dephasing is governed by the coupled differential equations (26), the solutions of which are a linear combination of two decay modes

\[ c_1 e^{-\Gamma_{0, r}^+ t} + c_2 e^{-\Gamma_{0, r}^- t} \]

with dephasing rates \( \Gamma_{0, r}^\pm = \frac{1}{2} \Gamma_{0, r} \pm \frac{1}{2} \Gamma_{0, r} \times \sqrt{1 - (2\gamma / \hbar \Gamma_{0, r})^2} \), respectively. The mean dephasing rate is \( \Gamma_{0, r} / 2 \). The coefficients \( c_{1, 2} \) in Eq. (28) depend on the initial condition of the qubit. We then have the result

\[ \rho_{0, p}(\infty) \rightarrow 0, \quad \rho_{0, d}(\infty) \rightarrow 0. \]
symmetric CQD. However, in the large bias limit
form to define the decoherence rate like the case in the
set of coupled differential equations. There is no analytic
where

where

FIG. 3: (color online). (a) The dephasing rate $\Gamma_{0,p}$ of the fast
mode of the measured qubit in the equilibrium state. (b) The
dephasing rate $\Gamma_{0,p}$ of the slow mode for the measured qubit.
The figures are plotted with the fixed parameter $\gamma/h\eta_d = \Omega_0$.
as

where $G_+^{(0)} = G_{+0}^{(0)} + G_{-0}^{(0)} = V_d \coth (\beta V_d/2)$. This is a
set of coupled differential equations. There is no analytic
form to define the decoherence rate like the case in the
symmetric CQD. However, in the large bias limit $\gamma \ll V_d$ we can find from the first two master equations the
relation $\dot{\rho}_{0,d}(t)/\dot{\rho}_{0,r}(t) \approx -\cot \theta$. Then the relaxation
rate can be obtained

It can been shown that three decay rates from master
equations (31–33) correspond to the roots of the following
equation

The mean value of these roots is $2\Gamma_{0,r}/3$. $\Gamma_{0,r}$ is also
plotted in Fig. 2b. All curves show a positive-monotonic-
increase with the temperature and bias voltage. This
bias and temperature dependence of the mean decay rate
approaches to be linear in the high bias regime. In the
low temperature limit, it becomes linear for almost the
whole range of the bias voltage. In addition, it can be
found in Fig. 2a that the asymmetric CQD has a longer
decay time than the symmetric CQD. It indicates that
the influence of the device structure on qubit decoherence
can also become significant in the low temperature and
low bias regime.

The asymptotic behavior of Eqs. (31–33) can be solved easily

$\rho_{0,d}^0(\infty) = \rho_{0,p}^0(\infty) \rightarrow 0$,

$\rho_{0,r}^0(\infty) \rightarrow -\gamma (\cosh \beta V_d - \cosh \beta \gamma)

\frac{V_d}{\sinh \gamma V_d - \gamma \sinh \beta \gamma}$.

$\rho_{0,r}^0(\infty)$ implicitly depends on the device structure
through $\gamma$. The plot of $\rho_{0,r}^0(\infty)$ in Fig. 2b shows that
there are two branches corresponding to the symmetric
(solids curves) and the asymmetric (dash curves) CQD,
respectively. At a given bias voltage, the symmetric CQD
is asymptotically forced into a more random mixed state
in comparison with the asymmetric CQD. This is because
the device structure effectively modifies the bias-
dependence of the qubit coherence.

IV. EA EFFECT ON QUBIT

Now, we turn to discuss the properties of the EA and
the decoherence of the charge qubit due to the EA effect.

A. Properties of the EA

The EA can be characterized by the EA induced current
$I_1(t) = \frac{d}{dt} \text{Tr} \left[ \xi \hat{N}_1(t) \right]$. $I_1(t)$ is plotted in Fig. 4
which is calculated according to the resulted master equations.
The result can be studied in detail by the analysis of Eq. (23).
In the right-hand side of Eq. (23), because the traces of
$-i\hat{\mathcal{D}} \hat{N}_1(t)$ and commutators vanish, the terms $\lambda F(t) \left( \tilde{C}^{(1)} \hat{N}_0(t) \hat{q} + \text{H.c.} \right)$ $\equiv \hat{K}$ and

$\lambda \left( \tilde{C}^{(0)} \hat{\rho}_1(t) \hat{q} + \text{H.c.} \right)$ $\equiv \hat{K}_0$ completely determine the
EA current. Also, the factor $F(t)$ of $\hat{K}_1$ is exponential
decay in time, and $\hat{\rho}_1(t_0) \ll \hat{N}_0(t_0)$. Thus, $\hat{K}_1$ is ini-
tially dominant, and then $\hat{K}_0$ becomes significant as $\hat{K}_1$
exponentially decays to zero. The linear increase of the EA current in the beginning is governed mainly by $\hat{K}_1$, see Fig. 4.

Analytically, $\hat{N}_0(t)$ can be solved in the asymptotic limit

$\tilde{n}_{0,gg}(t) = c_{g0} + c_{g1} t$,  $\tilde{n}_{0,ee}(t) = c_{e0} + c_{e1} t$,

$\tilde{c}_{e1} = c_0 \left( G_{+1}^{(0)} + G_{-1}^{(0)} \right)$,  $\tilde{c}_{g1} = c_0 \left( G_{+2}^{(0)} + G_{-2}^{(0)} \right)$,

(38)
where $c_{g0}$ and $c_{e0}$ are simply constants, $c_0 = -\lambda \Omega^2 V_d G_{+a}^{(0)}$, the higher order contributions of $\delta \Omega$ to the coefficient $c_0$ have also been ignored. For the weak interaction coupling, it can be numerically checked that $n_{0,ge}(t) \ll n_{0,gg(ce)}(t)$ in the asymptotic limit. We then obtain

$$\text{Tr} \left[ \hat{K}_1 \right] = -\mathcal{F}(t) \left((2\lambda \Omega^2)^2 V_d t + \text{constant} \right).$$

In the beginning (the $\hat{K}_1$ dominant regime), the ER decay is adiabatic, namely, $\mathcal{F}(t) \approx \text{constant} \equiv \mathcal{F}^*$. It leads to

$$I_1(t) \approx -\mathcal{F}^* (2\lambda \Omega^2)^2 V_d t + \text{constant.}$$

This shows why the EA current is linear increasing in time, as one can see in Fig. 4.

In the crossover regime in Fig. 4 the EA is slowed down by the bias. The maximum amount of accumulated electrons occurs at the valley of the $I_1$-curves, in which the maximum reverse EA current is induced. Obviously, the smaller the ER decay rate $\Gamma_{R,L}$, the stronger the EA as shown in Fig. 4.

Due to the bias, electrons accumulated in the QPC are completely exhausted through the ER process. $\text{Tr} \left[ \hat{K}_1 \right]$ of Eq. (8) vanishes in this stage, and $\text{Tr} \left[ \hat{K}_0 \right]$ becomes dominant. The asymptotic current can then be solved analytically

$$I_1(\infty) = \frac{d}{dt} \text{Tr} \left[ \xi \hat{N}_1(t) \right] = \xi \text{Tr} \left[ \hat{K}_0(\infty) \right]$$

$$= 2\lambda \xi V_d \left(\Omega^2 - \delta \Omega (1 + \rho_{1,r}(\infty) \cos \theta) \right),$$

which where the higher order contribution from $\delta \Omega^2$ has been ignored, and the first-order relaxation shift $\rho_{1,r}(\infty)$ is given in Eq. (13). It is worth noting that the qubit density $\rho_{1,r}(t)$ records the history of the EA effect. Even the accumulated electrons has been exhausted in this stage, $\rho_{1,r}(t)$ in $\hat{K}_0(t)$ performs as an EA background field and affects the electron tunneling in the QPC. Also, the time independent relaxation shift $\rho_{1,r}(\infty)$ leads to a constant transmission probability for the first-order electron tunneling. Finally, the stable behavior of the EA current is reached, see Fig. 4. As a result, with the EA effect, the asymptotic current up to the first order perturbation is

$$I_0(\infty) + I_1(\infty) = 2\lambda (\Omega - \delta \Omega)^2 V_d$$

$$+ \frac{\lambda \delta \Omega^2}{2} \left( V_d^2 - \gamma^2 \right) \sin \beta V_d$$

$$+ 2\lambda \xi V_d \left( \Omega^2 - \delta \Omega (1 + \rho_{0,r}(\infty) \cos \theta) \right),$$

(42)

B. Decoherence induced by EA effect

The qubit relaxation and dephasing due to the EA effect are studied in this subsection. Eq. (13) describes this decoherence process. The terms $\lambda \Omega \mathcal{F}(t) \left[ \left[ \hat{K}_1^{(1)}, \hat{N}_0(t) \right], \theta \right] \equiv \hat{K}_1^{*}$ in Eq. (13) is related to $\hat{K}_1$ of Eq. (23) through the relation $\hat{N}_k = \sum_{n=0}^{\infty} \eta_{k}^{(0)}(t) - \sum_{m=0}^{\infty} \eta_{k}^{(m)}(t)$. This term arises from the EA, and the induced relaxation is suppressed by the bias. To understand this property, let us look at the asymptotic matrix elements $\langle i | \hat{K}_1^*(\infty) | j \rangle$ first. Applying the previous result Eq. (38) and the asymptotic analysis, we obtain, up to the order $\Omega$, $\delta \Omega$,

$$\langle g | \hat{K}_1^*(\infty) | g \rangle = K_{0g},$$

$$\langle e | \hat{K}_1^*(\infty) | e \rangle = -\langle g | \hat{K}_1^*(\infty) | g \rangle,$$

(43)

where

$$K_{0g} \approx \lambda \Omega \delta \Omega \mathcal{F}(t) V_d \left( G_{+a}^{(0)} - G_{+a}^{(0)} G_{+a}^{(1)} + G_{+a}^{(1)} \right)$$

$$G_{+a}^{(0)} G_{+a}^{(1)}$$

and $G_{+a}^{(0)}$ is defined in Eq. (21). It can be checked that $K_{0g}$ is positive because of $(G_{+o}^{+a} - G_{+a}^{(0)}) \leq 0$ and $-1 < (G_{+o}^{(1)} + G_{+o}^{(1)}) \leq 0$. Since $\hat{\rho}_1(t_0) \ll \hat{N}_0(t_0)$,

$$\frac{d}{dt} \langle i | \hat{\rho}_1(t) | i \rangle \approx \langle i | \hat{K}_1^*(t) | i \rangle$$

before the bias effect becomes active (i.e. in the EA effect dominant regime). We then have

$$\langle g | \hat{\rho}_1(t) | g \rangle \approx K_{0gg} x_1 t + \text{constant},$$

$$\langle e | \hat{\rho}_1(t) | e \rangle \approx -K_{0ee} x_1 t + \text{constant.}$$

(44)

This linear time-dependence behavior indeed indicates an extra qubit relaxation due to the EA effect. The numerical results in Fig. 4a (plotted by green and black...
curves) roughly show this linear time-dependence of the relaxation. In Fig. 5a, the solid (dash) lines correspond to the ground (excited) state, respectively. Instructively, a numerical result for $\Gamma_{L,R} = 0$ is plotted in Fig. 5b. A perfect linear character is obtained. The induced relaxation from the pure EA effect obeys the linear time dependence, as shown by Eq. (4).}

![Graph](image)

**FIG. 5:** (color online). The time evolution of the reduced density matrix of the first order perturbation. The symmetric CQD is simulated. The qubit is initially in $|L\rangle$ state. The asymptotic relaxation shift is completely governed by $\hat{\rho}_{\text{tot}}$. After the accumulated electrons exhausted, this excitation is slowed down by the bias because $K_0 \rightarrow 0$ as $V_d \rightarrow 0$. Thus, $K_1'$ in Eq. (14) vanishes, and the EA makes almost no effect on the qubit dynamics. The result is shown by the red curve in Fig. 5a. The qubit only experiences a relaxation process without the excitation one.

Next, we shall calculate the decoherence rate of the qubit. For a low bias voltage, because of $G^{(1)} \rightarrow 0$ as $V_d \rightarrow 0$, the dynamic structure of Eq. (14) and (15) are almost the same. The decay modes of the qubit contributed by the zero order and the first order perturbation are closer. The EA effect can only enhance the qubit relaxation. Dynamically, the decoherence rate of the qubit does not be speeded up by the EA effect. According to Eq. (28), we obtain the total relaxation time $T_{\text{tot},r}$ and the dephasing time $T_{\text{tot},p}$ of the qubit with the EA effect

$$T_{\text{tot},r} = \frac{1}{\Gamma_{\text{tot},r}(V_d \rightarrow 0)} = \frac{\cosh \beta \gamma - 1}{\eta_d \sinh \beta \gamma}, \quad T_{\text{tot},p} = 2T_{\text{tot},r}$$

for the symmetric CQD in the low bias limit.

With bias voltage increasing, the first-order qubit dephasing rate can not be solved exactly. We study the qubit dephasing according to Eq. (15), or in terms of the set of coupled differential equations for the symmetric CQD

$$\dot{\rho}_{1,d}(t) = \frac{\gamma}{\hbar} \rho_{1,p}(t),$$

$$\dot{\rho}_{1,p}(t) = -\frac{\gamma}{\hbar} \rho_{1,d}(t) - \Gamma_{0,r} \rho_{1,p}(t) - \Gamma_{1,r} \mathcal{F}(t) n_{0,p}(t),$$

where

$$\Gamma_{1,r} = -\eta_d G^{(1)},$$

$$\rho_{1,d} = \rho_{1,eg} + \rho_{1,ge}, \quad \rho_{1,p} = \rho_{1,eg} - \rho_{1,ge} \quad \text{and} \quad i\hbar \omega_{0,p} = \eta_{0,ge} = -\eta_{0,eg}. \quad \text{The term } \Gamma_{0,r} \rho_{1,p}(t) \text{ in Eq. (15) denotes the qubit dephasing arisen from the QPC-dots interaction associated with the bias, while the term } \Gamma_{1,r} \mathcal{F}(t) n_{0,p}(t) \text{ is resulted from the EA. Because } \Gamma_{1,r} \text{ is proportional to the}$$
mean chemical potential $\bar{\mu}$ ($= \frac{\bar{\mu}_0 + \mu_d}{2}$), $\Gamma_{1,r}$ can be larger than $\Gamma_{0,r}$ for $\bar{\mu} > V_d \gg \gamma$. The qubit dephasing is mainly determined by the ER decay rate and the mean chemical potential, it does not so sensitively depend on the bias voltage. However, in the low bias limit, especially for $V_d \ll \gamma$, $G^{(1)}_{+,a} \rightarrow 0$ (i.e. $\Gamma_{1,r} \rightarrow 0$). The qubit dephasing rate becomes much smaller. The numerical plot of the time evolution of the qubit off-diagonal matrix element shows the coincident result, as one can see in Fig. 6 and d.

C. Decoherenceless EA effect in the low temperature regime

We have pointed out that the qubit undergoes an extra relaxation under the EA effect. However, this EA induced relaxation can be suppressed in the low temperature regime. The symmetric CQD is used to check this result. The qubit relaxation rate in the first order perturbation can be deduced from Eq. 1.

$$
\Gamma_{1,r}(t) = -\frac{\hat{\rho}_{1,r}(t)}{(\rho_{1,r}(t) - \rho_{1,0}(\infty))} = \Gamma_{0,r} + \mathcal{K}(t) \Gamma_{1,r}
$$

(50)

where $\mathcal{K}(t) = \mathcal{F}(t) n_{0,r}(t) / (\rho_{1,r}(\infty) - \rho_{1,r}(t))$, and $n_{0,r}(\infty) = n_{0} G^{(1)}_{+,b} / G^{(1)}_{+,a} = 0$ has been used ($n_{0} \equiv \text{Tr}[\hat{N}_0(t_0)]$), and $\Gamma_{1,r}$ is defined in Eq. 49. The relaxation rate $\Gamma_{1,r}(t)$ is time dependent. Due to the EA effect, initially, the qubit severely relaxes with the relaxation rate $\Gamma_{1,r}(t) \approx \Gamma_{1,r}\mathcal{K}(t)$ for $\bar{\mu} \gg V_d$. After the EA effect suppressed by the bias, the relaxation rate asymptotically reduces to the zero-order $\Gamma_{0,r}$. Here, $\mathcal{K}(t)$ is a fluctuant factor. The EA induced relaxation rate $\Gamma_{1,r}$ is plotted in Fig. 6. It shows that $\Gamma_{1,r}$ has a very different behavior from $\Gamma_{0,r}$. For a large mean chemical potential ($\bar{\mu} \gg V_d$) in which $\bar{\mu}$ is almost bias voltage independent, the electron source of the EA is excited from the energy levels below the Fermi surface. The variation of the chemical potential $\delta V_{EA}$ induced by the EA is limited by the chemical potential. The $\delta V_{EA}$-dependence of the qubit relaxation rate leads to a bounded phenomenon, as shown in Fig. 6.

The relaxation rate $\Gamma_{1,r}$ also shows an anti-symmetric bias dependence (see Fig. 6). This feature can be understood as follows. There are two kind of electron-tunneling correlations, $G^{(k)}_{+,i}$ and $G^{(k)}_{-,j}$, are involved. $G^{(k)}_{+,i}$ ($G^{(k)}_{-,j}$) corresponds to the electron tunneling that one electron is from the source to the drain at $\tau$ (t) and the other is from the drain to the source at $t$ ($\tau$). Explicitly, $G^{(k)}_{+,i}$ is associated with $\text{Tr}_{D^a}[\hat{f}_{+,i} \hat{f}_+ \hat{\rho}_{1}(t)]$ for forward tunneling processes and $\text{Tr}_{D^a}[\hat{f}_{+,i} \hat{f}_+ \hat{\rho}_{1}(t)]$ for backward tunneling processes. The relaxation rate is related to $G^{(k)}_{+,i}$ and $G^{(k)}_{-,j}$ through the relation $\Gamma_{k,r} = n_d \sum_{i=1,2} \left( G^{(k)}_{+,i} + G^{(k)}_{-,j} \right) / 2$ with $k = 0, 1$. For $k = 0$, the electron-tunneling correlation is simply governed by the QPC-dots interaction and the bias, no EA involved. It can be checked that a relation of the bias symmetry holds

$$
\frac{G^{(0)}_{+,d}(V_d) + G^{(0)}_{-,d}(V_d)}{2} = \frac{G^{(0)}_{+,d}(-V_d) + G^{(0)}_{-,d}(-V_d)}{2}.
$$

(51)

The $\Gamma_{0,r}$ does not be changed by reversing the bias ($V_d \rightarrow -V_d$), due to the fact that the qubit relaxation caused by the electron tunneling in the QPC only depends on the amplitude of the external bias voltage. However, the electron-tunneling correlation under the EA effect ($k = 1$) obeys the relation of the bias anti-symmetry

$$
\frac{G^{(1)}_{+,d}(V_d) + G^{(1)}_{-,d}(V_d)}{2} = -\frac{G^{(1)}_{+,d}(-V_d) + G^{(1)}_{-,d}(-V_d)}{2}.
$$

(52)

To understand this relation, let us look at the forward tunneling processes, in which an extra negative bias voltage is induced by the EA, $V_d \rightarrow V_d - \delta V_{EA}$. According to the result in Sec. IV B, an effective relaxation of the qubit is induced by the EA effect. On the other hand, for a negative bias voltage, an extra negative bias voltage induced by the EA leads to $V_d \rightarrow -V_d - \delta V_{EA}$. The amplitude of bias voltage is increased. An effective excitation of the qubit is induced by the EA effect. Accordingly, Eq. 32 shows that the EA induced relaxation rate of the qubit under the positive bias voltage ($V_d$) is equal to the EA induced excitation rate of the qubit under the negative bias voltage ($-V_d$).

In addition, the relaxation process can be separated into two effective modes

$$
\Gamma_{k,r} = \eta_d \bar{\mu} \left( \frac{R(z_+) + R(z_-)}{2} \right), \quad z_{\pm} = \beta(V_d \pm \gamma) / 2,
$$

(53)

where the function $R(z) = \text{coth}(z) - z \text{csch}(z)^2$ is a step-like function with bounded values $\pm 1$. The function preserves the bias anti-symmetry. As discussed in Sec. III, the relaxation depends on the structure of the CQD. It leads to two effective modes corresponding to $R(z_+)$ and $R(z_-)$. In the low temperature regime, these two effective modes separate. Eq. 52 shows that the qubit...
excitation rate in the $R(z_{-})$ mode cancels the qubit relaxation rate in the $R(z_{+})$ mode in the regime of $V_{d}$ limited by $\pm \gamma$. Then a relaxationless EA effect occurs, as shown in Fig. 6. However, in the high temperature regime, the CQD-structure dependence of the relaxation is suppressed, and two modes $R(z_{\pm})$ become degenerate. No relaxationless EA effect occurs. As a result, $\Gamma_{b,r}(t)$ reduces to $\Gamma_{b,r}^{0}$ (i.e. $\Gamma_{1,r} \rightarrow 0$) in low temperature regime. For the qubit dephasing, it can be checked that the dephasing term $\Gamma_{1,r}F(t)\eta_{0,p}(t) \rightarrow 0$ as $T \rightarrow 0$. The qubit is also EA-dephasingless in the low temperature regime.

V. SUMMARY

We have developed a perturbative theory to study the EA effect on the decoherence of the charge qubit, a single electron confined in CQD, measured by the QPC. The contribution due to the EA is treated perturbatively. A set of master equations for the reduced density matrix of the qubit has been obtained in this perturbation scheme. By solving the resulted master equations, we obtain the decay modes of the dephasing and the relaxation of the qubit in the thermal equilibrium limit, and study the temperature- and bias-dependence of the dephasing and the relaxation rate. We find two kinds of decay modes for the dephasing in the electrical measurement processes: one decay rate increases as the increase of temperature and bias voltage, as expected; the other shows an abnormal dependence. The qubit in the later decay mode preserves the longest decoherence time in the high temperature and high bias limit, its time scale is much longer than that of the usual mode. In addition, the EA properties are studied extensively. The EA current is obtained according to the master equations. We find that the EA current varies linearly with time under the pure EA effect, and is then slowed down into an asymptotic stable state by the bias. The qubit decoherence due to the EA effect is studied based on the analysis. We find that under the EA effect the qubit dephasing and dephasing rate are much larger than the ones in the thermal equilibrium limit, unless the bias voltage is turned off. Also, the master equations show that an extra qubit relaxation is induced by the EA effect initially, and then be suppressed by the bias. Asymptotically, the qubit will be forced into the state with a small relaxation shift which is independent of the ER decay rate. However, in the low bias limit, the qubit will not be affected by the EA. The qubit decoherence rate will not be speeded up by the EA effect. Finally, we find a decoherenceless EA effect in the low temperature regime. A bias anti-symmetry in EA processes suppresses the qubit decoherence. These decoherent behaviors worth being explored in experiments.

APPENDIX: DERIVATION OF MASTER EQUATIONS

The derivation of the master equations [14, 15, 22, 23] is presented in this appendix. The total density operator in the interaction picture of the reservoir satisfies

$$\frac{d}{dt} \hat{\rho}_{I}(t) = \frac{1}{\hbar} \hat{H}_{I} + \hat{H}_{I}^{\dagger}, \quad \hat{H}_{I}^{\dagger} = e^{\frac{i\hat{H}_{I}}{\hbar}} \hat{H}' e^{\frac{-i\hat{H}_{I}}{\hbar}}. \tag{A.1}$$

In the weak coupling regime, the interaction Hamiltonian $\hat{H}'$ can be treated by using the second order cumulant expansion technique [7, 10, 11, 17]. By taking the second order cummulant expansion, the master equation becomes

$$\frac{d}{dt} \hat{\rho}_{f(b)}^{(n)}(t) = -i\hat{L}_{D,f(b)}^{(n)}(t) - \hat{R}_{f(b)}^{(n)}(t), \tag{A.2}$$

$$\hat{R}_{f(b)}^{(n)}(t) = \frac{1}{\hbar} \int_{0}^{t} d\tau \text{Tr}_{D^{(n)}(D^{(n)})} \left\{ \left[ \hat{H}_{f(b)}^{\dagger}(\tau), \left[ \hat{G}^{\dagger}(t, \tau) \hat{H}_{f(b)}^{\dagger}(t, \tau), \hat{\rho}_{I}(t) \right] \right] \right\}, \tag{A.3}$$

where $\hat{\rho}_{f(b)}^{(n)}(t) = \text{Tr}_{D^{(n)}(D^{(n)})}[\hat{\rho}_{tot}(\tau)]$ and $\hat{G}(t, \tau)$ is the propagator of the CQD. The decoherence of the measured qubit is governed by the dissipation term $-\hat{R}_{f(b)}^{(n)}(t)$ in Eq. (A.2).

Note that the master equations (A.2) (A.3) essentially describe a Markovian process. The Markovian approximation has been introduced in this stage. Therefore, the time integration in Eq. (A.3) is replaced by the one be integrated out along the whole time domain. Carrying out the commutator in Eq. (A.3), $\hat{R}_{f(b)}^{(n)}(t)$ can be expressed as

$$\frac{1}{2\hbar^{2}} \int_{-\infty}^{\infty} d\tau \left\{ \hat{q}(\hat{P}_{0} - e^{i\tau}) \hat{P}_{1} - e^{-i\tau} \hat{P}_{2} \right\}$$

$$\times \left( C_{f(b),-}^{n}(t - \tau) + C_{f(b),+}^{n}(t - \tau) \right) \right\} + H.c.$$}

$$-\frac{1}{2\hbar^{2}} \int_{-\infty}^{\infty} d\tau \left\{ \hat{P}_{0} - e^{i\tau} \hat{P}_{1} - e^{-i\tau} \hat{P}_{2} \right\}$$

$$\times \left( C_{f,+}^{n-1}(t - \tau) + C_{f,-}^{n+1}(t - \tau) \right) \hat{q} \right\} + H.c., \tag{A.4}$$

where the reservoir time correlation functions are defined by

$$C_{f,+}^{n}(t - \tau) = \text{Tr}_{D^{(n)}}[\hat{f}_{+}^{\dagger} \hat{f}_{+} \hat{\rho}_{I}(t)], \tag{A.5}$$

$$C_{f,-}^{n}(t - \tau) = \text{Tr}_{D^{(n)}}[\hat{f}_{-}^{\dagger} \hat{f}_{-} \hat{\rho}_{I}(t)],$$

for the forward tunneling processes, and

$$C_{b,+}^{m}(t - \tau) = \text{Tr}_{D^{(m)}}[\hat{f}_{+}^{\dagger} \hat{f}_{+} \hat{\rho}_{I}(t)], \tag{A.6}$$

$$C_{b,-}^{m}(t - \tau) = \text{Tr}_{D^{(m)}}[\hat{f}_{-}^{\dagger} \hat{f}_{-} \hat{\rho}_{I}(t)],$$

for the backward tunneling processes.
for the backward tunneling processes. It can be easily checked from Eqs. (A.3), (A.6) that without the EA effect, the backward tunneling processes is covered in the forward tunneling processes, and the result is the same as that in (4).

Next, the EA effect is taken into account to calculate the reservoir time correlation functions. It should be noted that the excitation rate of an electron occupying in lower energy levels far from the Fermi energy is much smaller than the one near the Fermi energy. That is, most likely, only the electrons occupying near the Fermi energy can tunnel through the QPC. If the QPC has a low transmission, i.e. $n/A, m/A \ll N$, the states $\{|D (L^n, R^n)\}$ contributed by all allowed tunnelings in which electrons occupying near the Fermi energy can tunnel from the source to the drain are almost the same. Therefore, we have the following approximation

$$\text{Tr}_{D^{(n)}}[\hat{f}_i^+ \hat{f}_j \hat{\rho}_f(t)] \approx \rho_f^{(n)}(t) \sum_{L^n R^n} (D^n | \hat{f}_i^+ \hat{f}_j | D^n), \quad (A.7)$$

and $C_{f,+}^n(t - \tau)$ can be expressed as

$$\rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} [\hat{A}_n^+ \hat{f}_j \hat{A}_n \hat{\rho}_B^{(0)}],$$

where $\hat{A}_n = \hat{a}_1^+ \cdots \hat{a}_n^+ \hat{a}_1 \cdots \hat{a}_n$, and $\hat{\rho}_B^{(0)}$ is the reservoir vacuum state. Furthermore, we define $\rho_f^{(n)}(t) = \hat{\rho}_f^{(n)}(t)$ as the electronic reservoir density operator with $n$ electrons created in the drain for the forward tunneling processes. Combine these analysis together, we obtain

$$C_{f,+}^n(t - \tau) = \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_i^+ \hat{a}_i \hat{f}_j \hat{\rho}_B^{(0)} \right] \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_j^+ \hat{f}_j \hat{\rho}_B^{(0)} \right]$$

$$= \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_i^+ \hat{f}_j \hat{\rho}_B^{(0)} \right] \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_j^+ \hat{f}_j \hat{\rho}_B^{(0)} \right]$$

where the EA-fluctuated Fermi-Dirac functions $F_{\tau}^{(n)}$ have been treated perturbatively and shown in Eq. (9). Note that Eq. (9) is valid when $n/A \ll N$ and $m/A \ll N$, which coincides with the approximation used in Eq. (A.7). Other reservoir time correlation functions can be similarly reduced to the following forms

$$C_{f,+}^n(t - \tau) = \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_i^+ \hat{f}_j \hat{\rho}_B^{(0)} \right] \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_j^+ \hat{f}_j \hat{\rho}_B^{(0)} \right]$$

$$= \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_i^+ \hat{f}_j \hat{\rho}_B^{(0)} \right] \rho_f^{(n)}(t) \sum_{L^n R^n} \text{Tr} \left[ \hat{a}_j^+ \hat{f}_j \hat{\rho}_B^{(0)} \right]$$

Obviously, it shows that, in the above time correlation functions, the perturbed terms proportional to $\delta \mu_{RL} (n, \beta, t)$ come from the EA effect. Using the perturbation scheme in Sec. II, we then have

$$\rho_f^{(n)}(t) = \hat{\rho}_f^{(n)}(t) + \xi \hat{\rho}_{1,f}^{(n)}(t) + \xi^2 \hat{\rho}_{2,f}^{(n)}(t) + \cdots$$

for the forward tunneling processes, and

$$\hat{\rho}_{1,b}^{(n)}(t) = \hat{\rho}_{0,b}^{(n)}(t) + \xi \hat{\rho}_{1,b}^{(n)}(t) + \xi^2 \hat{\rho}_{2,b}^{(n)}(t) + \cdots$$

for the backward tunneling processes.

In addition, the time integration in Eq. (A.4) can be calculated by using the results

$$\frac{1}{2\hbar} \int_{-\infty}^{\infty} dt e^{-i\frac{(\tau - \tau')}{\hbar}} C_{f,b}^n(t - \tau) = \pi g_{L} g_{R} \rho_f^{(n)}(t)$$

$$\times \left( g^{(0)}(V_d - \gamma) \pm n d \tilde{g}^{(1)}(V_d - \gamma) \right),$$

$$\frac{1}{2\hbar} \int_{-\infty}^{\infty} dt e^{-i\frac{(\tau - \tau')}{\hbar}} C_{f,b}^n(t - \tau) = \pi g_{L} g_{R} \rho_f^{(n)}(t)$$

$$\times \left( g^{(0)}(-V_d + \gamma) \pm n d \tilde{g}^{(1)}(-V_d + \gamma) \right),$$

and the relation

$$\delta \mu_{L} (n, \beta, t) + \delta \mu_{R} (n, \beta, t) = n \tilde{\mu} \zeta \mathcal{F}(t),$$

the master equations become

$$\frac{d}{dt} \hat{\rho}_{0,f}^{(n)}(t) + \xi \frac{d}{dt} \hat{\rho}_{1,f}^{(n)}(t) = -i \hat{L}_D (\hat{\rho}_{0,f}^{(n)}(t) + \xi \hat{\rho}_{1,f}^{(n)}(t))$$

$$- \lambda \left[ \hat{q} (\hat{A}^{(0)} + \hat{A}^{(0)}) \hat{\rho}_{0,f}^{(n)}(t) - (\hat{A}^{(0)} \hat{\rho}_{1,f}^{(n)}(t) + \hat{A}^{(0)} \hat{\rho}_{0,f}^{(n+1)}(t)) \hat{q} \right]$$

$$+ \xi \left[ \hat{q} (\hat{A}^{(0)} + \hat{A}^{(0)}) \hat{\rho}_{1,f}^{(n)}(t) - (\hat{A}^{(0)} \hat{\rho}_{1,f}^{(n-1)}(t) + \hat{A}^{(0)} \hat{\rho}_{1,f}^{(n+1)}(t)) \hat{q} \right]$$

$$+ \mathcal{F}(t) n \hat{\rho} (\hat{A}^{(1)} + \hat{A}^{(1)}) \hat{\rho}_{0,f}^{(n)}(t) - \mathcal{F}(t) ((n-1) \hat{A}^{(1)} \hat{\rho}_{0,f}^{(n-1)}(t) + (n+1) \hat{A}^{(1)} \hat{\rho}_{0,f}^{(n+1)}(t)) \hat{q} \right] + H.c.$$
\[
d\frac{d}{dt}\hat{\rho}_{b,0}(t) + \xi \frac{d}{dt}\hat{\rho}_{1,0}(t) = -i\hat{L}_D(\hat{\rho}_{b,0}(t) + \xi\hat{\rho}_{1,0}(t)) \\
- \lambda \left\{ \hat{q}(\hat{A}_0^+) + \hat{A}_0^-(\hat{\rho}_{b,0}(t)) - (\hat{A}_0^-(\hat{\rho}_{b,0}(t)) + \hat{A}_0^+(\hat{\rho}_{b,0}(t)))\hat{q} \\
+ \xi \left[ \hat{q}(\hat{A}_1^+ + \hat{A}_1^-)\hat{\rho}_{1,0}(t) - (\hat{A}_1^-(\hat{\rho}_{b,0}(t)) + \hat{A}_1^+(\hat{\rho}_{b,0}(t)))\hat{q} \right] \\
- \mathcal{F}(t)n\hat{q}(\hat{A}_1^+ + \hat{A}_1^-)\hat{\rho}_{b,0}(t) + \mathcal{F}(t)((n-1)\hat{A}_1^-(\hat{\rho}_{b,0}(t)) + (n+1)\hat{A}_1^+(\hat{\rho}_{b,0}(t)))\hat{q} \right\} + \mathcal{O}(\xi^2) + \cdots
\]

for the backward tunneling processes, where \( \hat{A}_k^{(0,1)} = \sum_{i=1,2} G_{i,k} \hat{\rho}_i \). All the definitions of the elements of Eqs. (A11-A12) can be found in Sec. II.

Eqs. (A11-A12) describes the transport properties of electrons in the system we concerned, for example, the transport current and noise spectrum \[\ldots\].

According to Eq. (6) and recalling that \( \hat{\rho}_{f(0)}(t) \) describes the quantum oscillation of the electron in the CQD with the conditions of \( n \) electrons accumulating in the drain (source), the \( k \)-th order perturbation of the qubit oscillation is completely described by \( \hat{\rho}_k(t) = \sum_{n=0}^{\infty}\hat{\rho}_{k,f}^{(n)}(t) + \sum_{m=0}^{\infty}\hat{\rho}_{k,b}^{(m)}(t) \). Accordingly, the \( k \)-th order perturbation of the QPC current operator is given by

\[
d\frac{d\hat{N}_k}{dt} = \sum_{n=0}^{\infty}\hat{\rho}_{k,f}^{(n)}(t) - \sum_{m=0}^{\infty}\hat{\rho}_{k,b}^{(m)}(t),
\]

and the \( k \)-th order perturbation of noise spectrum is defined by \( \hat{W}_k(t) = \sum_{n=0}^{\infty}\hat{\rho}_{k,f}^{(n)}(t) + \hat{\rho}_{k,b}^{(k)}(t) \). Combine with Eqs. (A11-A12), one can obtain the master equations shown in Sec. II straightforwardly.

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\[\ldots\]

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