Effective rate analysis over Fluctuating Beckmann fading channels

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The effective rate of Fluctuating Beckmann (FB) fading channel is analysed. The moment generating function (MGF) of the instantaneous signal-to-noise (SNR) is used to derive the effective rate for arbitrary values of the fading parameters in terms of the extended generalised bivariate Meijer’s-G function (EGBMGF). For integer valued of the multipath and shadowing severity fading parameters, the probability density function (PDF) of the instantaneous SNR is employed. To this end, simple exact mathematically tractable analytic expression is obtained. The Monte Carlo simulations and the numerical results are presented to verify the validation of our analysis.

Introduction: The effective rate (ER) has been proposed to measure the performance of the wireless communication systems under the quality of service (QoS) constraints, such as system delays, that have not been taken into consideration by Shannon [1]. Accordingly, the analysis of this performance metric over fading channels has been given a special attention by several works. For instance, in [2], the ER over Nakagami-μ, namely, Rayleigh, fading channel is analysed using the moment generating function (MGF) of the instantaneous signal-to-noise (SNR).

Recently, several efforts have been achieved to study the ER over different generalised fading channels. This is because these channels include most of the classic fading models such as Nakagami-μ as special cases but with better fitting to the practical measurements. For example, the ER over κ-μ and η-μ fading channel which are used to model the line-of-sight (LoS) and non-LoS (NLoS) communication scenarios are investigated in [3] and [4], respectively. In [5], the expression of the ER over κ-μ fading channel which is composite of κ-μ fading and Nakagami-m distributions is provided in terms of the extended generalised bivariate Meijer’s-G function (EGBMGF) which doesn’t give clear insights about the results against the variation of the fading parameters. The analysis in [6] is carried out over composite α-μ -gamma fading channels using two approximate unified frameworks. However, the derived expressions are also included in the EGBMGF. The Fisher-Snecorde F distribution is used to study the ER over composite multipath-shadowed fading condition in [7]. Although, the expression is given in terms of a single variable Meijer’s-G function, this fading model includes few number of the conventional distributions as special cases.

More recently, the Fluctuating Beckmann (FB) fading channel has been proposed as an extension of the κ-μ shadowed and the classical Beckmann fading models [8]. In addition, it includes as special cases the one-sided Gaussian, Rayleigh, Nakagami-μ, Rican, κ-μ, η-μ, κ-μ -μ, Beckmann, Rican shadowed and the κ-μ shadowed distributions. Accordingly, this letter is devoted to analyse the ER over FB fading channel. To the best of the authors’ knowledge, there is no effort has been dedicated to investigate the aforementioned analysis in the open literature. To this end, novel exact mathematically tractable expression is derived in terms of the EGBMGF using the MGF approach. To gain more insights into the impact of the channel parameters on the ER, the PDF of the effective rate is derived. The Monte Carlo simulations and the numerical results are presented to verify the validation of our analysis.

Effective rate: The normalised ER is evaluated by [4, eq. (1)]

\[
R = \frac{1}{A} \log_2 E \left( (1 + \gamma)^{-A} \right)
\]

where \(E\{\cdot\}\) denotes the expectation, \(\gamma\) is the instantaneous signal-to-noise ratio (SNR), and \(A \equiv \Theta T B / \nu_o\) with \(\Theta, T\), and \(B\) are the delay exponent, block duration, and bandwidth of the system, respectively.

Fluctuating Beckmann fading channel: The MGF of \(\gamma\) over FB fading channel model is expressed as [8, eq. (3)]

\[
\mathcal{M}_\gamma(s) = \frac{1}{(\alpha_1 c_1 c_2)^m} \left( 1 + \frac{\eta}{\Omega} \right)^{\frac{m-\beta}{m}} \left( 1 + \frac{\mu}{\Omega} \right)^{\frac{m-\beta}{m}} \left( 1 + \frac{\eta}{c_1} \right)^{\frac{m-\beta}{c_1}} \left( 1 + \frac{\alpha}{c_2} \right)^{\frac{m-\beta}{c_2}}
\]

where \(\Omega = \frac{\mu(1+\eta)(1+\kappa)}{\mu + \eta}\) is the real extension of the multipath clusters, \(\gamma\) is the average SNR, and \(m\) is the shadowing severity index. Moreover, \(\kappa = \frac{\sigma_\mu^2 + \sigma_\eta^2}{\mu^2 + \eta^2}\), \(\sigma_\mu^2 = E[X_\mu^2],\) \(\sigma_\eta^2 = E[Y_\eta^2],\) and \(\nu_o\) and \(\eta_i\) are real numbers for ith cluster. The parameters \(c_1\) and \(c_2\) are the roots of \(\alpha_1 s^2 + \beta s + 1\) with [8, eqs. (7-8)]

\[
\begin{align*}
\alpha_1 &= \frac{\eta}{\Omega^2} + \frac{\eta (\gamma^2 + \eta)}{m \Omega (1 + \gamma^2)} \\
\beta &= \frac{1 - \frac{1}{\kappa}}{1 + \frac{\eta}{\mu}} \frac{2}{\mu + \kappa}
\end{align*}
\]

(3)

where \(\gamma^2 = \frac{\omega^2}{\mu^2(1 + \kappa)}\)

When \(\mu\) and \(m\) are even and integer numbers, respectively, the PDF of \(\gamma\) is given as [8, eq. (10)]

\[
f_\gamma(\gamma) = \frac{1}{\alpha_1^{m-\beta}} \left( \frac{\eta}{\Omega} \right)^{\frac{m-\beta}{m}} \sum_{i=0}^{\infty} \left( \frac{\gamma^{c_1}}{c_1} \right)^{i} \left( \frac{\gamma^{c_2}}{c_2} \right)^{-i} \frac{A_{ij}}{\mu^i (j-1)!} \gamma^{j-1}
\]

(4)

where \(\omega = [m, m, \frac{\mu}{\gamma} - m, \frac{\mu}{\gamma} - m], \nu = [c_1, c_2, \Omega, \Omega], N(m, \mu) = 2[1 + u(\frac{\mu}{\gamma}, m)], u(\cdot)\) is the unit step function, and \(A_{ij}\) is computed by [7, eq. (5)].

Effective rate using MGF approach: According to [2, eq. (3)], (1) can be computed by the MGF of \(\gamma\) as follows

\[
J = \mathbb{E} \left( (1 + \gamma)^{-A} \right) = \frac{1}{\Gamma(1)} \int_0^{\infty} \frac{A-1 \times e^{-s \cdot A} \times M_\gamma(s)}{s} ds
\]

(5)

where \(\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx\) is the Gamma function.

Substituting (1) in (5) to yield

\[
J = \frac{1}{(\alpha_1 c_1 c_2)^m} \left[ \Gamma(A-1) \frac{1}{\Omega^2} \right] \int_0^{\infty} \left[ 1 + \frac{\gamma}{\Omega^2} \right]^{m-\beta} \left( 1 + \frac{\gamma}{c_1} \right)^{\frac{m-\beta}{c_1}} \left( 1 + \frac{\gamma}{c_2} \right)^{\frac{m-\beta}{c_2}} ds
\]

The following identity [9, eq. (10)] can be used in (6)

\[
(1 + x)^a = \frac{1}{(a-x) G_{1,1} \left[ \frac{1 + a}{0} | x \right]}
\]

(7)

Accordingly, we have

\[
J = \frac{1}{(\alpha_1 c_1 c_2)^m} \left[ \frac{\Gamma(A-1) \frac{1}{\Omega^2} \right] \int_0^{\infty} \left[ 1 + \frac{\gamma}{\Omega^2} \right]^{m-\beta} \left( 1 + \frac{\gamma}{c_1} \right)^{\frac{m-\beta}{c_1}} \left( 1 + \frac{\gamma}{c_2} \right)^{\frac{m-\beta}{c_2}} ds
\]

(8)

Using the integral representation of the Meijer’s-G function [9, eq. (5)] in (8) to yield

\[
J = \frac{1}{(\alpha_1 c_1 c_2)^m} \left[ \frac{\Gamma(A-1) \frac{1}{\Omega^2} \right] \int_0^{\infty} \left[ 1 + \frac{\gamma}{\Omega^2} \right]^{m-\beta} \left( 1 + \frac{\gamma}{c_1} \right)^{\frac{m-\beta}{c_1}} \left( 1 + \frac{\gamma}{c_2} \right)^{\frac{m-\beta}{c_2}} ds
\]

(9)

where \(j = \sqrt{-1}\) and \(R_i\) for \(i = 1, \ldots, 4\) is the \(\nu^{th}\) suitable closed contours in the complex \(\nu\)-plane.

With the help of [10, eq. (1.1.6)], the inner integral of (9)

\[
J = \frac{1}{(\alpha_1 c_1 c_2)^m} \left[ \frac{\Gamma(A-1) \frac{1}{\Omega^2} \right] \int_0^{\infty} \left[ 1 + \frac{\gamma}{\Omega^2} \right]^{m-\beta} \left( 1 + \frac{\gamma}{c_1} \right)^{\frac{m-\beta}{c_1}} \left( 1 + \frac{\gamma}{c_2} \right)^{\frac{m-\beta}{c_2}} ds
\]

(10)
It can be observed that (10) can be expressed in exact closed-form in terms of the EGBMGF as follows
\[
\mathcal{J} = \frac{1}{(\alpha_1 c_1 c_2) m} \Gamma(A) [\Gamma(m) \Gamma(\frac{1}{\mu} - m)]^2 \\
\times \sum_{i=0}^{m-1} \sum_{j=1}^{N(m,\mu)} A_{ij}^m \left[ 1 - A_i + m \frac{\gamma}{\mu} \right] \\
\times \int_0^{\infty} \frac{1}{\Gamma(\frac{1}{\mu} - m)} \left[ 1 - \frac{\gamma}{\mu} \right] d\gamma
\]

One can see that the EGBMGF is not available in MATLAB and MATHEMATICA software packages. Therefore, this function has been calculated in this letter by employing a MATHEMATICA code that is implemented in [12, Table II].

Effective rate using PDF approach: The expectation of (1) can be evaluated by the PDF as follows
\[
\mathcal{J} = \int_0^\infty (1 + \gamma)^{-A} f_\gamma(\gamma)d\gamma
\]

When \( m \) and \( \mu \) are integer and even numbers, respectively, \( \mathcal{J} \) of (12) can be computed by plugging (4) in (12). Thus, this yields
\[
\mathcal{J} = \frac{1}{\alpha_1^{\frac{1}{\mu}} \gamma^{m}} \left( \frac{\eta}{\Omega} \right)^{m - \frac{\mu}{2}} \sum_{i=1}^{N(m,\mu)} \sum_{j=1}^{A_{ij}} \frac{A_{ij}}{(j - 1)!} \\
\times \int_0^\infty \gamma^{j-1} (1 + \gamma)^{-A} e^{-\frac{\eta}{\mu} \gamma} d\gamma
\]

With the aid of [10, eq. (1.3.14), pp. 38], the integral in (13) can be calculated in exact closed-form expression as follows
\[
\mathcal{J} = \frac{1}{\alpha_1^{\frac{1}{\mu}} \gamma^{m}} \left( \frac{\eta}{\Omega} \right)^{m - \frac{\mu}{2}} \sum_{i=1}^{N(m,\mu)} \sum_{j=1}^{A_{ij}} A_{ij}^m U\left( j; j - A - 1: \frac{\eta}{\mu} \right)
\]

where \( U(\cdot) \) is the Tricomi hypergeometric function of the second kind defined in [10, eq. (1.3.15), pp. 38].

Conclusion: The ER over FB fading channel model which includes wide range of the fading distributions has been analysed using two different exact expressions. The MGF approach is employed first to derive the ER for arbitrary values of \( m \) and \( \mu \) where its expressed in terms of the EGBMGF. In the second case, \( m \) and \( \mu \) are assumed to be integer and even numbers, respectively and the PDF is used to obtain simple exact closed-form analytic expression of the ER. The results are provided for different scenarios via utilizing various values of the fading parameters as well as the special cases of the FB fading channel.

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**Fig. 1** Normalised ER against the average SNR for \( m = 1 \), \( K = 1 \), \( \eta = 0.1 \), \( \sigma^2 = 0.1 \), \( A = 2 \) and different values of \( \mu \).

**Fig. 2** Normalised ER against the average SNR for \( m = 1 \), \( K = 1 \), \( \eta = 0.1 \), \( \sigma^2 = 0.1 \), \( A = 2 \) and different values of \( m \).

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