Schrödinger-Newton wave mechanics. The model

Janina Marciak-Kozlowska
Miroslaw Kozlowski¹

Institute of Electron Technology, Al. Lotników 32/46, 02-668 Warsaw, Poland

Abstract

In this paper the Schrödinger equation (SE) with gravity term is developed and discussed. It is shown that the modified SE is valid for particles with mass $m < M_P$, $M_P$ is the Planck mass, and contains the part which, we argue, describes the pilot wave. For $m \to M_P$ the modified SE has the solution with oscillatory term, i.e. strings.

Key words: Schrödinger-Newton equation, Planck time, pilot wave.

¹Corresponding author, e-mail: MiroslawKozlowski@aster.pl
Introduction

When M. Planck made the first quantum discovery he noted an interesting fact [1]. The speed of light, Newton’s gravity constant and Planck’s constant clearly reflect fundamental properties of the world. From them it is possible to derive the characteristic mass $M_P$, length $L_P$ and time $T_P$ with approximate values

$$
L_P = 10^{-35} \text{m}
$$
$$
T_P = 10^{-43} \text{s}
$$
$$
M_P = 10^{-5} \text{g}
$$

Nowadays much of cosmology is concerned with “interface” of gravity and quantum mechanics.

After the Alpha moment – the spark in eternity [1] the space and time were created by “Intelligent Design” [2] at $t = T_P$. The enormous efforts of the physicists, mathematicians and philosophers investigate the Alpha moment. Scholars seriously discuss the Alpha moment - by all possible means: theological and physico-mathematical with growing complexity of theories. The most important result of these investigation is the anthropic principle and Intelligent Design theory (ID).

In this paper we investigate the very simple question: how gravity can modify the quantum mechanics, i.e. the nonrelativistic Schrödinger equation (SE). We argue that SE with relaxation term describes properly the quantum behaviour of particle with mass $m < M_P$ and contains the part which can be interpreted as the pilot wave equation. For $m \to M_P$ the solution of the SE represent the strings with mass $M_P$. 

2
1 Hyperbolic diffusion

1.1 Generalized Fourier law

The thermal history of the system (heated gas container, semiconductor or Universe) can be described by the generalized Fourier equation [3]-[5]

\[ q(t) = \int_{-\infty}^{t} K(t - t') \nabla T(t') \, dt'. \]  (1)

In Eq. (1) \( q(t) \) is the density of the energy flux, \( T \) is the temperature of the system and \( K(t - t') \) is the thermal memory of the system

\[ K(t - t') = \frac{K}{\tau} \exp \left[ -\frac{(t - t')}{\tau} \right]. \]  (2)

where \( K \) is constant, and \( \tau \) denotes the relaxation time.

As was shown in [3]-[5]

\[ K(t - t') = \begin{cases} 
K \lim_{t_0 \to 0} \delta(t - t' - t_0) & \text{diffusion} \\
K = \text{constant} & \text{wave} \\
\frac{K}{\tau} \exp \left[ -\frac{(t-t')}{\tau} \right] & \text{damped wave or hyperbolic diffusion}.
\end{cases} \]

The damped wave or hyperbolic diffusion equation can be written as:

\[ \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau} \frac{\partial T}{\partial t} = \frac{D_T}{\tau} \nabla^2 T. \]  (3)

For \( \tau \to 0 \), Eq. (3) is the Fourier thermal equation

\[ \frac{\partial T}{\partial t} = D_T \nabla^2 T \]  (4)

and \( D_T \) is the thermal diffusion coefficient. The systems with very short relaxation time have very short memory. On the other hand for \( \tau \to \infty \)
Eq. (3) has the form of the thermal wave (undamped) equation, or *ballistic* thermal equation. In the solid state physics the *ballistic* phonons or electrons are those for which \( \tau \to \infty \). The experiments with *ballistic* phonons or electrons demonstrate the existence of the *wave motion* on the lattice scale or on the electron gas scale.

\[
\frac{\partial^2 T}{\partial t^2} = \frac{D_T}{\tau} \nabla^2 T. \tag{5}
\]

For the systems with very long memory Eq. (3) is time symmetric equation with no arrow of time, for the Eq. (5) does not change the shape when \( t \to -t \).

In Eq. (3) we define:

\[
v = \left( \frac{D_T}{\tau} \right)^{\frac{1}{2}}, \tag{6}
\]

velocity of thermal wave propagation and

\[
\lambda = v\tau \tag{7}
\]

where \( \lambda \) is the mean free path of the heat carriers. With formula (6) equation (3) can be written as

\[
\frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\tau v^2} \frac{\partial T}{\partial t} = \nabla^2 T. \tag{8}
\]

### 1.2 Damped wave equation, thermal carriers in potential well, \( V \)

From the mathematical point of view equation:

\[
\frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{D} \frac{\partial T}{\partial t} = \nabla^2 T
\]
is the hyperbolic partial differential equation (PDE). On the other hand Fourier equation

\[
\frac{1}{D} \frac{\partial T}{\partial t} = \nabla^2 T \tag{9}
\]

and Schrödinger equation

\[
i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi \tag{10}
\]

are the parabolic equations. Formally with substitutions

\[
T \leftrightarrow \Psi, \quad t \leftrightarrow it \tag{11}
\]

Fourier equation (9) can be written as

\[
i\hbar \frac{\partial \Psi}{\partial t} = -D\hbar \nabla^2 \Psi \tag{12}
\]

and by comparison with Schrödinger equation one obtains

\[
D_T \hbar = \frac{\hbar^2}{2m} \tag{13}
\]

and

\[
D_T = \frac{\hbar}{2m}. \tag{14}
\]

Considering that \( D_T = \tau v^2 \) we obtain from (14)

\[
\tau = \frac{\hbar}{2mv^2}. \tag{15}
\]

Formula (15) describes the relaxation time for quantum thermal processes.

Starting with Schrödinger equation for particle with mass \( m \) in potential \( V \):

\[
i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi \tag{16}
\]
and performing the substitution (11) one obtains

\[ \hbar \frac{\partial T}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 T - V T \]  

(17)

and

\[ \frac{\partial T}{\partial t} = \frac{\hbar}{2m} \nabla^2 T - \frac{V}{\hbar} T. \]  

(18)

Equation (18) is Fourier equation (parabolic PDE) for \( \tau = 0 \). For \( \tau \neq 0 \) we obtain

\[ \tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} + \frac{V}{\hbar} T = \frac{\hbar}{2m} \nabla^2 T, \]  

(19)

\[ \tau = \frac{\hbar}{2mv^2} \]  

(20)

or

\[ \frac{1}{v^2} \frac{\partial^2 T}{\partial t^2} + \frac{2m}{\hbar} \frac{\partial T}{\partial t} + \frac{2mV}{\hbar^2} T = \nabla^2 T. \]

2 Relaxation, Schrödinger equation, strings

2.1 Model Schrödinger equation

With the substitution (11) equation (19) can be written as

\[ i\hbar \frac{\partial \Psi}{\partial t} = V \Psi - \frac{\hbar^2}{2m} \nabla^2 \Psi - \tau \hbar \frac{\partial^2 \Psi}{\partial t^2}. \]  

(21)

The new term, relaxation term

\[ \tau \hbar \frac{\partial \Psi}{\partial t^2} \]  

(22)

describes the interaction of the particle with mass \( m \) with space-time. The relaxation time \( \tau \) can be calculated as:

\[ \frac{1}{\tau} = \frac{1}{\tau_{e-p}} + \cdots + \frac{1}{\tau_{\text{Planck}}} \]  

(23)
where, for example $\tau_{e-p}$ denotes the scattering of the particle $m$ on the electron-positron pair ($\tau_{e-p} \sim 10^{-17}$ s) and the shortest relaxation time $\tau_{\text{Planck}}$ is the Planck time ($\tau_{\text{Planck}} \sim 10^{-43}$ s).

From equation (23) we conclude that $\tau \approx \tau_{\text{Planck}}$ and equation (21) can be written as

$$i\hbar \frac{\partial \Psi}{\partial t} = V \Psi - \frac{\hbar^2}{2m} \nabla^2 \Psi - \tau_{\text{Planck}} \hbar \frac{\partial^2 \Psi}{\partial t^2}$$

where

$$\tau_{\text{Planck}} = \frac{1}{2} \left( \frac{\hbar G}{c^5} \right)^{\frac{1}{2}} = \frac{\hbar}{2M_Pc^2}.$$  

In formula (25) $M_P$ is the mass Planck. Considering Eq. (25), Eq. (24) can be written as

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi - \frac{\hbar^2}{2M_P} \nabla^2 \Psi + \frac{\hbar^2}{2M_P} \nabla^2 \Psi - \frac{\hbar^2}{2M PC^2} \frac{\partial^2 \Psi}{\partial t^2}. \quad (26)$$

The last two terms in Eq. (6) can be defined as the Bohmian pilot wave

$$\frac{\hbar^2}{2M_P} \nabla^2 \Psi - \frac{\hbar^2}{2M PC^2} \frac{\partial^2 \Psi}{\partial t^2} = 0 \quad (27)$$

i.e.

$$\nabla^2 \Psi - \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = 0. \quad (28)$$

It is interesting to observe that pilot wave $\Psi$ do not depends on the mass of the particle. With postulate (28) we obtain from equation (26)

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi - \frac{\hbar^2}{2M_P} \nabla^2 \Psi \quad (29)$$

and simultaneously

$$\frac{\hbar^2}{2M_P} \nabla^2 \Psi - \frac{\hbar^2}{2M PC^2} \frac{\partial^2 \Psi}{\partial t^2} = 0 \quad (30)$$
In the operator form Eqs. (9) and (20) can be written as

\[ \hat{E} = \frac{\hat{p}^2}{2m} + \frac{1}{2M_Pc^2} \hat{E}^2, \]  

(31)

where \( \hat{E} \) and \( \hat{p} \) denotes the operator for energy and momentum of the particle with mass \( m \). Equation (31) is the new dispersion relation for quantum particle with mass \( m \). From Eq. (21) one can concludes that Schrödinger quantum mechanics is valid for particles with mass \( m \ll M_P \). But pilot wave \( \Psi \) exist independent of the mass of the particles.

For particles with mass \( m \ll M_P \) Eq. (3) has the form

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V\Psi. \]  

(32)

2.2 Schrödinger equation and the strings

In the case when \( m \approx M_P \) Eq. (29) can be written as

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_P} \nabla^2 \Psi + V\Psi \]  

(33)

but considering Eq. (30) one obtains

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_Pc^2} \frac{\partial^2 \Psi}{\partial t^2} + V\Psi \]  

(34)

or

\[ \frac{\hbar^2}{2M_Pc^2} \frac{\partial^2 \Psi}{\partial t^2} + i\hbar \frac{\partial \Psi}{\partial t} - V\Psi = 0. \]  

(35)

We look for the solution of Eq. (35) in the form

\[ \Psi(x, t) = e^{i\omega t} u(x). \]  

(36)

After substitution formula (16) to Eq. (35) we obtain

\[ \frac{\hbar^2}{2M_Pc^2} \omega^2 + \omega \hbar + V(x) = 0. \]  

(37)
with the solution

\[ \omega_1 = \frac{-M_p c^2 + M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar} \]  

(38)

\[ \omega_2 = \frac{-M_p c^2 - M_p c^2 \sqrt{1 - \frac{2V}{M_p c^2}}}{\hbar} \]

for \( \frac{M_p c^2}{2} > V \) and

\[ \omega_1 = \frac{-M_p c^2 + iM_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar} \]  

(39)

\[ \omega_2 = \frac{-M_p c^2 - iM_p c^2 \sqrt{\frac{2V}{M_p c^2} - 1}}{\hbar} \]

for \( \frac{M_p c^2}{2} < V \).

Both formulae (38) and (39) describe the string oscillation, formula (27) damped oscillation and formula (28) overdamped string oscillation.

3 Conclusion

D. Bohm presented the pilot wave theory in 1952, and de Broglie had presented a similar theory in the mid 1920s. It was rejected in the 1950s and the initial rejection had nothing to do with Bohm’s later work.

There is always the possibility that the pilot wave has a primitive, mind like property. That’s how Bohm described it. We can say that all the particles in Universe and even Universe have their own pilot waves, their own information. In this paper, we discuss in very simple nonrelativistic way possible extension of the Schrödinger equation with relaxation process included.
As the first approximation this lead us to the inclusion of the Planck time, i.e. gravity to the quantum description of the processes in the space time. The relaxation time $\tau_{\text{Planck}}$ is the decoherence time [6] or the Ehrenfest time [7] and describes the collapse of the pilot wave after the interaction with the apparatus.
References

[1] Julian Barbour, The End of Time, Oxford University Press, 2000.

[2] Jeffrey F. Addicot, Ohio State Law Journal, 2003.

[3] M. Kozłowski, J. Marciak-Kozlowska, The time arrow in a Planck gas, Foundations of Physics Letters, 10, p. 295, (1997).

[4] M. Kozłowski, J. Marciak-Kozlowska, The smearing out of the thermal initial conditions created in a planck Era, Foundation of Physics Letters, 10, p. 599, (1997).

[5] M. Kozłowski, J. Marciak-Kozlowska, Klein-Gordon thermal equation for Planck gas, Foundations of Physics Letters, 12, p. 93, (1999).

[6] Tamas Geszti, http://lanl.arxiv.org/quant-ph/0401086.

[7] G. P. Berman at al., http://lanl.arxiv.org/quant-ph/0401038