Metrics building of pp waves orbifold geometries

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ABSTRACT: We study strings on orbifolds of $AdS \times S^5$ by $SU(2)$ discrete groups in the Penrose limit. We derive the degenerate metrics of pp wave of $AdS \times S^5/\Gamma$ using ordinary $ADE$ and affine $\hat{ADE}$ singularities of complex surfaces and results on $\mathcal{N} = 4$ CFT$_4$s. We also give explicit metric moduli dependences for both abelian and non abelian orbifolds.

KEYWORDS: Penrose Limit, pp-wave background, $ADE$ singularities, $\mathcal{N} = 2$ CFT$_4$. 
1. Introduction

It is by now possible to derive spectrum of string theory from the gauge theory point of view not only on a flat space but also on plane wave of $AdS_5 \times S^5$ [1]. This is amongst the fruits of the AdS/CFT correspondance [2, 3, 4, 5] relating the spectrum of type IIB string theory on $AdS_5 \times S^5$ to the spectrum of single trace operators in $4D \, \mathcal{N} = 4$ super Yang Mills theory. The idea is based on considering chiral primary operators in the conformal field side and look for their corresponding states in the string side but here on pp-wave backgrounds. For instance, a field operator like $Tr[Z^J]$ with large $J$ is associated with the string vacuum state in the light cone gauge $|0, p_+\rangle_{lc}$ with large momentum $p_+$. Both objects have a vanishing $\Delta - J$ interpreted on the string side as the light cone energy $E_c$ of type IIB string on the pp wave background and on the field side as an anomalous dimension. The correspondance between the whole tower of string states $\left(\prod_{r,s} a^{inr} S^{ism} \right) |0, p_+\rangle_{lc}$ with $E_c = n$ and gauge invariant conformal operators $Tr[O]$ is deduced from the previous trace $Tr[Z^J]$ by replacing some of the $Z$’s by monomials involving the gauge covariant derivative $DZ$ or/and the remaining four transverse scalars $\phi^j$ and fermions $\chi^a$ of the $\mathcal{N} = 4$ multiplet. The BMN correspondence rule between superstring states creation operators $a^\dagger$ and $S^\dagger$ and CFT$_4$ field operators is.
\[
\begin{align*}
a^i &\rightarrow D_i Z \quad \text{for } i = 1, \ldots, 4 \\
a^j &\rightarrow \phi^{j-4} \quad \text{for } j = 5, \ldots, 8 \\
S^a &\rightarrow \chi^a_{J=1/2}.
\end{align*}
\] (1.1)

For more details, see [1]. Soon after this discovery, an intensive interest has been devoted to further explore this issue; in particular the extension of the BMN results to pp wave orbifolds with \( U (N) \) symmetries [6, 19] and orientifolds of D-brane system with an \( Sp (N) \) gauge invariance [7], see also [8, 9]. In [6], the BMN proposal has been extended to type IIB superstring propagating on pp-wave \( \mathbb{Z}_k \) orbifolds. There, it has been shown that first quantized free string theory on such background is described by the large \( N \), fixed gauge coupling limit of \( \mathcal{N} = 2 \ [U (N)]^k \) quiver gauge theory and have proposed a precise map between gauge theory operators and string states for both untwisted and twisted sectors. For \( \Delta - J = 0 \), the BMN correspondence for the lowest string state reads as,

\[
Tr \left[ S^a Z^J \right] \leftrightarrow |0, p^+\rangle_q,
\]
(1.2)

where \( |0, p^+\rangle_q \) is the vacuum in the \( q \)-th twisted sector and where

\[
S = \text{diag} \left( 1, \exp \frac{i 2 \pi}{k}, \ldots, \exp \frac{i (k - 1) \pi}{k} \right).
\]
(1.3)

One may also write down the correspondence for the other states with \( \Delta - J = n > 0 \). Here one has a rich spectrum due to the presence of \( k - 1 \) twisted sectors in addition to the usual one. This analysis remains however incomplete since it concerns only a special kind of \( \mathcal{N} = 2 \) CFT\(_4\) model; the more familiar supersymmetric scale invariant theory one can have in four dimension. In fact there are several others \( \mathcal{N} = 2 \) CFT\(_4\)’s in one to one correspondence with both ordinary and Affine ADE singularities of the ALE space. These models have very different moduli spaces; and then it would be interesting to explore how the BMN correspondence extends for general \( \mathcal{N} = 2 \) CFT\(_4\) models and how the machinery works in general.

The aim of this paper is to further develop the analysis initiated in [6] by considering all possible kinds of abelian and non abelian pp wave orbifolds. Here we will focus our attention on the type IIB string side by deriving explicitely the moduli dependent metrics of all kinds of pp wave orbifolds preserving sixteen supersymmetries. We will put the accent on the way the analogue of the field moduli, of the quiver gauge theory, enters in the game in the string side. In [14], we will give the details concerning its correspondings \( \mathcal{N} = 2 \) CFT\(_4\) side.

The presentation is as follows: In section 2, we recall some aspects of the pp wave geometry in the BMN limit of \( AdS_5 \times S^5 \). In section 3, we study ordinary \( SU (k) \) pp waves
geometry. In section 4, we consider its $SU(k)$ affine analogue and also give the explicit derivation of the moduli dependent metrics. In section 5, we derive the results for Affine $SO(k)$ pp wave geometries and make comments regarding the other kinds of orbifolds.

2. pp waves orbifold geometries

To start recall that the general form of the plane wave metric of the $AdS_5 \times S^5$ in the BMN limit reads as,

$$ds^2 = -4dx^- dx^+ - A_{ij}(x^+) x^i x^j (dx^+)^2 + \sum_{i=1}^8 dx^i dx^i. \quad (2.1)$$

Here the symmetric matrix $A_{ij}(x^+)$ is in general a function of $x^+$. For simplicity, we will take it as $A_{ij} = \mu \delta_{ij}$. In this case, the five form field strength is given by $F = \mu dx^+ \wedge (dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8)$, and the previous metric reduces to,

$$ds^2 = -4dx^- dx^+ - \mu^2 \left( \sum_{i=1}^8 x^i x^i \right) (dx^+)^2 + \sum_{i=1}^8 dx^i dx^i \quad (2.2)$$

The study of type IIB strings on the Penrose limit of $AdS_5 \times S^5$ orbifolds, $AdS_5 \times S^5 / \Gamma$, depends on the nature of the discrete finite group $\Gamma$. Group theoretical analysis [12] on the types of discrete symmetries $\Gamma$ one can have in such kind of situation, shows that $\Gamma$ has to be contained in a specific $SU(2)$ subgroup of the $SO(6)$ R-symmetry of the underlying $N = 4$ CFT$_4$.

$$\Gamma \subset SU(2) \subset SO(6) \quad (2.3)$$

It follows from this constraint eq that $\Gamma$ may be any finite subgroups of the $\widetilde{ADE}$ classification of discrete finite subgroups of $SU(2)$ [12, 13]. These finite groups, which are well known in the mathematical literature, are either abelian as for the usual cyclic $Z_k$ group or non abelian as for the case of binary $\tilde{D}_{2k}$ groups defined by,

$$\widetilde{D}_{2k} = \{a, b | b^2 = a^k; ab = ba^{-1}; a^{2k} = I_{id}\} ,$$

$$\widetilde{E}_6 = \{a, b | a^3 = b^3 = (ab)^3\} , \quad (2.4)$$

together with analogous relations for $\widetilde{E}_7$ and $\widetilde{E}_8$. In what follows, we shall address the question of metrics building of pp wave orbifolds with respect to some of these groups; more details on the way they are involved in the BMN correspondence will be exposed in a subsequent paper [14].

$\text{\footnote{\textit{\widetilde{ADE}} denote \textit{SU}(2) discrete subgroups and should not be confused with the usual notations for ordinary \textit{ADE} and affine \textit{\widetilde{ADE}} Lie algebras.}}$
To derive the moduli dependent metric of orbifolds of the pp wave geometries, we start from eq(2.2) and use the local coordinates \((x; z_1, z_2)\) of the space \(\mathbb{R}^4 \times \mathbb{C}^2 \sim \mathbb{R}^4 \times \mathbb{R}^4\) where \(x = (x^2, x^3, x^4, x^5)\) and where \(z_1 = (x^6 + ix^7)\) and \(z_2 = (x^8 + ix^9)\). In this coordinate system, the metric of the pp wave background has a manifest \(SO(4) \times SU(2) \times U(1) \subset SO(4) \times SO(4)\) isometry group and reads as

\[
\begin{align*}
ds^2 &= -4dx^+ dx^- + d\mathbf{x}^2 \\
&- \mu^2(x^2 + |z_1|^2 + |z_2|^2)(dx^+)^2 \\
&+ |dz_1|^2 + |dz_2|^2.
\end{align*}
\]

(2.5)

where \(|z_i|^2 = z_i \bar{z}_i\). In type IIB closed string theory where these coordinates are interpreted as two dimension world sheet bosonic periodic fields,

\[
x(\sigma^0, \sigma^1 + 2\pi) = x(\sigma^0, \sigma^1)
\]

\[
z(\sigma^0, \sigma^1 + 2\pi) = z(\sigma^0, \sigma^1),
\]

(2.6)

the above relation leads to a very remarkable field action which, in the light cone gauge, is nothing but the action of a system of free and massive harmonic oscillators,

\[
S_{\text{Base}}[x;z] \sim - \int d^2\sigma \left[ \partial_\alpha x \partial^\alpha x + \partial_\alpha z_i \partial^\alpha \bar{z}_i + \nu^2(x^2 + |z_1|^2 + |z_2|^2) \right]
\]

(2.7)

where \(\nu = p^+ \mu\) and where \(\partial_\alpha \partial^{\alpha} = -\partial^2 + \partial_\tau^2\). The field eq of motion \((\partial_\alpha \partial^{\alpha} - \nu^2)x\) and \((\partial_\alpha \partial^{\alpha} - \nu^2)z\) are exactly solved as,

\[
x(\tau, \sigma) = \sum_{n \in \mathbb{Z}} \frac{i}{\sqrt{2\omega_n}} \left( e^{-i\omega_n \tau + i\sigma} a_n - e^{i\omega_n \tau - i\sigma} a_n^\dagger \right),
\]

(2.8)

and similarly for the \(z\)’s and for fermionic partners. In this equation, \(a_n\) and \(a_n^\dagger\) are, roughly speaking, the harmonic annihilation and creation operators of string states and the \(\omega_n\) frequencies are as follows,

\[
\omega_n = \omega_{-n} = \sqrt{n^2 + \nu^2}.
\]

(2.9)

Due to the presence of the background field, these \(\omega_n\)’s are no longer integers as they are shifted with respect to the standard zero mass results.

To study the field theory associated with pp waves orbifolds with \(\text{ADE}\) singularities and their complex deformations, we consider the form (2.5) of the metric and impose

\[
x(\sigma^0, \sigma^1 + 2\pi) = x(\sigma^0, \sigma^1)
\]

\[
z(\sigma^0, \sigma^1 + 2\pi) = Uz(\sigma^0, \sigma^1) U^{-1},
\]

(2.10)
where $U$ is an element of the orbifold symmetry group and where $UzU^{-1}$ elements belong to the same equivalent class as $z$. For the special case of the $Z_k$ abelian discrete symmetry where $UzU^{-1} = z \exp i \frac{2\pi q}{k}$, the analogue of the expansion (2.8) has twisted sectors and reads as

$$z_1(\tau, \sigma) = \sum_{n \in \mathbb{Z}} \frac{i}{\sqrt{2\omega_n(q)}} e^{-in(q)\sigma} \left( e^{-i\omega_n(q)\tau} b_n - e^{i\omega_n(q)\tau} d_{-n}^\dagger \right),$$

(2.11)

with

$$n(q) = n + \frac{q}{k}; \quad 0 \leq q \leq k - 1$$

$$b_n = a_n^6 + i a_n^7; \quad d_n^\dagger = a_n^{6\dagger} + i a_n^{7\dagger},$$

(2.12)

and similar relations for $z_2(\tau, \sigma)$ with the upper indices 6 and 7 replaced by 8 and 9. Note that the metric (2.5) allows to realize manifestly the orbifold group actions and, as we will see, permits to read directly the various types of fundamental and bi-fundamental matter moduli one has on the $\mathcal{N} = 2$ field theory side and too particularly in the CFT$_4$ model we are interested in here.

3. $SU(k)$ pp waves geometry

There are two known kinds of $\mathcal{N} = 2$ supersymmetric CFT$_4$s associated with $A_k$ singularity. This is due to the fact that there are two kinds of $A_k$ singularities one may have at the origin of complex surfaces: the first kind involving the ordinary $SU(k)$ Lie algebra classification and the other implying $SU(k)$ affine one. In this section, we focus our attention on the first case. In the forthcoming section, we consider the affine one. Note in passing that while conformal invariance presents no problem in the second case; there is however extra constraint eqs one should take into account for the first class of models. This feature, which is mainly associated with the inclusion of fundamental matters in addition to bifundamental matter, will be considered in [14].

3.1 Degenerate orbifold Metric

To start recall that the nearby of the singular point $x^6 = x^7 = x^8 = x^9 = 0$ of the real four dimension space $\mathbb{R}^4/\mathbb{Z}_k \sim \mathbb{C}^2/\mathbb{Z}_k$ is just an ALE space with a $SU(k)$ singularity. In other words in the neighbourhood of the singularity, the space $\mathbb{R}^4 \times \mathbb{C}^2/\mathbb{Z}_k$ may be thought of as $\mathbb{R}^4 \times \mathbf{T}^{*}\mathbb{CP}^1$. The cotangent bundle $\mathbf{T}^{*}\mathbb{CP}^1$ is known to have two toric actions [18]:

$$z \rightarrow e^{i\theta} z; \quad w \rightarrow e^{-i\theta} w,$$

(3.1)

and

$$z \rightarrow e^{i\phi} z; \quad w \rightarrow w,$$

(3.2)
where \( z \) is the non compact direction and \( w \) is the coordinate of \( \mathbb{CP}^1 \). If we denote by \( c \) and \( c' \) the two one dimensional cycles of \( T^2 \) corresponding to the action \( \theta \) and \( \phi \), then \( T^*\mathbb{CP}^1 \) may be viewed as \( T^2 \) fibration over \( \mathbb{R}_+^2 \) with coordinates \(|z|\) and \(|w|\). The toric action has three fixed loci:

\[
\begin{align*}
    c - c' &= 0 \iff |w| = 0, \quad \forall |z|, \\
    c' &= 0 \iff \frac{1}{|w|} = 0, \quad \forall |z|,  \\
    c + c' &= 0 \iff |z| = 0, \quad \forall |w|
\end{align*}
\]

(3.3)

At the singular point where the two cycles shrink, the product \( zw \) goes, in general, to zero as

\( zw = \zeta^k, \quad k \geq 2, \)  

(3.4)

with \( \zeta \to 0 \). Eq(3.4) tells us, amongst others, that the local variables to deal with the geometry of the Penrose limit of \( AdS_5 \times S^5 / \mathbb{Z}_k \) are

\[
z_1 = z; \quad z_2 = \frac{\zeta^k}{z}.
\]

(3.5)

Using these new variables as well as the explicit expression of the differential,

\[
dz_2 = k \zeta^{k-1} \frac{\zeta}{z} \frac{d\zeta}{z_2} - \frac{\zeta^k}{z_2} dz,
\]

(3.6)

the metric of the pp wave orbifold geometry (2.5) reads near the singularity as,

\[
ds^2|_{SU(k)} = -4dx^+dx^- + dx^2
\]

\[
- \mu^2 G_{++} dx^+ \bar{dx}^2 + G_{\zeta\bar{\zeta}} |dz|^2 + G_{\zeta \bar{z}} |d\zeta|^2
\]

\[
+ (G_{\zeta \bar{z}} dz d\bar{\zeta} + G_{\bar{\zeta} \zeta} d\bar{\zeta} d\zeta),
\]

(3.7)

where the metric factors \( G_{ij} \) are given by,

\[
G_{++} = x^2 + |z|^{-2} \left( |z|^4 + |\zeta|^{2k} \right),
\]

\[
G_{\zeta \bar{\zeta}} = 1 + \frac{|\zeta|^{2k}}{|z|^4}, \quad G_{\zeta \bar{\zeta}} = k^2 \frac{|\zeta|^{2(k-1)}}{|z|^2},
\]

(3.8)

\[
G_{\zeta \bar{z}} = -k \frac{|\zeta|^{2(k-1)}}{|z|^2} \frac{\zeta}{z}; \quad G_{\zeta \bar{\zeta}} = - \frac{|\zeta|^{2(k-1)}}{|z|^2} \frac{\bar{\zeta}}{\bar{z}}.
\]

This metric has degenerate zeros at \( z = \zeta = 0 \), which may be lifted by deformations of \( G_{ij} \)'s. In what follows, we describe a way to lift this degeneracy by using complex deformations of ALE space singularity.
3.2 Moduli dependent pp wave geometry

To lift degeneracy of eq(3.8), one can use either Kahler or complex resolutions of the $SU(k)$ singularity. In the second case, this is achieved by deformation of the complex structure of the orbifold point of $\mathbb{C}^2/\mathbb{Z}_k$ which amount to replace eq(3.4) by

$$z_1z_2 = \zeta^k + a_1\zeta^{k-1} + ... + a_{k-1}\zeta,$$

(3.9)

where $a_i$ are complex numbers. Note that if all the $a_i$’s are non zero, the degeneracy is completely lifted, otherwise it is partially lifted. Note also that from the field theory point of view, the $a_i$ moduli are interpreted as the vev’s of the hypermultiplets in the bi-fundamental representations of the $\mathcal{N} = 2$ supersymmetric $\prod_i U(N_i)$ quiver gauge theory [15]. The ratio $z_i = \frac{a_i+1}{a_i}$ are the gauge coupling moduli. From this relation, eq(3.5) extends as,

$$z_2 = a_0\zeta^k + \sum_{j=1}^{k} a_j \frac{\zeta^{k-j}}{z},$$

(3.10)

with $a_0 = 1$ and so the holomorphic differential $dz_2$ reads as,

$$dz_2 = \sum_{j=0}^{k} a_j \frac{\zeta^{k-j-1}}{z} \left[ (k-j) d\zeta - \frac{\zeta}{z} dz \right] + \sum_{j=0}^{k} \frac{\zeta^{k-j}}{z} da_j.$$

(3.11)

In eq(3.11), we have also varied the complex moduli; this is what one should do in $\mathcal{N} = 2$ CFT$_4$ with $SU(k)$ geometry which, in addition to gauge fields and bi-fundamental matters, requires the introduction of fundamental matters as well. We shall not consider this aspect here; we will then purely and simply set $da_j = 0$. As such the previous metric of the degenerate $SU(k)$ pp waves geometry reads now as,

$$G_{++} = x^2 + \frac{1}{|z|^2} \left( |z|^4 + \sum_{j=0}^{k} a_j \zeta^{k-j} \right),$$

$$G_{\zeta\overline{\zeta}} = 1 + \frac{1}{|z|^2} \sum_{j=0}^{k} a_j \frac{\zeta^{k-j}}{z^2}, \quad G_{\zeta \overline{\zeta}} = \frac{1}{|z|^2} \sum_{j=0}^{k} (k-j) a_j \zeta^{k-j-1},$$

$$G_{\overline{\zeta} \overline{\zeta}} = -\frac{1}{|z|^2} \sum_{j=0}^{k} (k-j) a_j \zeta^{k-j-1} \frac{\overline{\zeta}}{\overline{z}}, \quad G_{\zeta \overline{\zeta}} = -\frac{1}{|z|^2} \sum_{j=0}^{k} (k-j) a_j \zeta^{k-j} \frac{\overline{\zeta}}{\overline{z}}.$$

(3.12)

By making appropriate choices of the $a_j$ complex moduli, one can cover all the kinds of pp wave geometries involving subgroups of the $SU(k)$ singularity.
4. Affine $\widehat{ADE}$ pp waves geometry

Here we are interested in the Penrose limit of $AdS_5 \times S^5$ orbifolds with affine $\widehat{ADE}$ singularities. We will focus our attention on the metric building of the pp wave geometry with affine $\widehat{A}_k$ singularity. Then we give the results for the other cases. To start recall that affine $\widehat{A}_k$ singularity is given by the following holomorphic eq

$$z_1^2 + z_2^3 + z_2^2 = \zeta^{k+1};$$

(4.1)

describing a family of complex two surfaces embedded in $\mathbb{C}^3$. A tricky way to handle this singularity is to use elliptic fibration over the complex plane considered in [15, 16, 17]. In this method, one considers instead of eq(4.1), the following equivalent one,

$$v(z_1^2 + z_2^3 + \zeta^6 + a z_1 z_2 \zeta) + \zeta^{k+1} = 0,$$

(4.2)

where the parameter $a$ is the torus complex structure. Note that these eqs are homogeneous under the change $(z_1, z_2, \zeta, v) \rightarrow (\lambda^3 z_1, \lambda^2 z_2, \lambda \zeta, \lambda^{k-5} v)$ allowing to fix one of them; say $v = 1$. With this constraint, one recovers the right dimension of the $SU(k)$ geometry near the origin. In what follows, we consider the case $k = 2n - 1, n \geq 2$ and set $v = 1$.

4.1 Degenerate Orbifold metric

Starting from eq(4.2), which we rewrite as

$$z_1^2 - 2z_1 f + g = 0,$$

(4.3)

where $f$ and $g$ are holomorphic functions given by,

$$f = -\frac{a}{2} z_2 \zeta,$$

$$g = z_2^3 + \zeta^6 + \frac{\zeta^{2n}}{v};$$

(4.4)

one can solve eq(4.3) as a homogenous function of $z_2, \zeta$ and $v^2$

$$z_1 = f + \sqrt{f^2 - g}$$

(4.5)

Using this relation, one may compute the differential $dz_1$ in terms of $dz_2, d\zeta$ and $dv$ as follows

$$dz_1 = \left(1 + \frac{f}{\sqrt{f^2 - g}}\right) df - \frac{1}{2\sqrt{f^2 - g}} dg,$$

(4.6)

one may also take the other solution namely $z_1 = f - \sqrt{f^2 - g}$. 

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with
\[ d\chi = dz_2 \partial_{z_2} \chi + d\zeta \partial_{\zeta} \chi + dv \partial_v \chi, \tag{4.7} \]
where \( \chi \) stands for the functions \( f \) and \( g \). In the coordinate patch \( v = 1 \) where \( dv = 0 \), the metric of pp waves background, near the orbifold point with affine \( SU(2n - 1) \) singularity, reads as,
\[ ds^2 = -4d\chi^+ d\chi^- + d\chi^2 \]
\[ -\mu^2 \left[ \chi^2 + \left| f + \sqrt{f^2 - g} \right|^2 + \left| \zeta \right|^2 \right] (d\chi^+)^2 \]
\[ + G_{z\zeta} |dz|^2 + G_{\zeta\zeta} |d\zeta|^2 + (G_{z\zeta} dz d\zeta + hc) \tag{4.8} \]
where
\[ G_{z\zeta} = 1 + \left( 1 + \frac{f}{\sqrt{f^2 - g}} \right) \partial_{z_2} f - \frac{1}{2\sqrt{f^2 - g}} \partial_{z_2} g \right)^2, \]
\[ G_{\zeta\zeta} = \left( 1 + \frac{f}{\sqrt{f^2 - g}} \right) \partial_{\zeta} f - \frac{1}{2\sqrt{f^2 - g}} \partial_{\zeta} g \right)^2, \]
\[ G_{z\zeta} = \left[ \left( 1 + \frac{f}{\sqrt{f^2 - g}} \right) \partial_{z_2} f - \frac{1}{2\sqrt{f^2 - g}} \partial_{z_2} g \right] \times \left[ \left( 1 + \frac{f}{\sqrt{f^2 - g}} \right) \partial_{\zeta} f - \frac{1}{2\sqrt{f^2 - g}} \partial_{\zeta} g \right] \tag{4.9} \]
A more explicit expression of this degenerate metric may be obtained by using eqs(4.4,4.5).

4.2 pp waves metric

To lift the degeneracy of the orbifold metric eq(4.9), we use the following complex deformation of eq(4.2),
\[ 0 = v \left( z_1^2 + z_2^3 + \zeta^6 + a z_1 z_2 \zeta + \zeta^{2n} + \Delta \right), \tag{4.10} \]
with
\[ \Delta = \sum_{i=1}^{n-1} a_{2i} \zeta_i^{2n-2i} + a_{2n} \zeta_2^n + z_1 \sum_{i=0}^{n-2} b_{2i+1} z_2^{2n-2i-3}, \tag{4.11} \]
where \( a_{2i} \) and \( b_{2i+1} \) are complex numbers describing the complex deformations of the singularity. This relation may also be rewritten as,
\[ z_1^2 - 2z_1 F + G = 0, \tag{4.12} \]
where now,

\[
F = -\frac{1}{2} \left( a z_2 \zeta + \sum_{i=0}^{n-2} b_{2i+1} z_i \zeta^{2n-2i-3} \right),
\]

\[
G = z_2^3 + \zeta^6 + \left( \frac{\zeta^{2n}}{\nu} + \sum_{i=1}^{n} a_{2i} z_i \zeta^{2n-2i} \right).
\]

(4.13)

To get the metric of the pp wave geometry, one does the same analysis as we have done in subsection 4.1 for the case where \(a_i = b_i = 0\). The relations one gets are formally similar to those we have obtained before; one has just to replace \(f\) and \(g\) by \(F\) and \(G\) respectively.

5. Conclusion

The analysis we have described above may be applied for the remaining other kinds of pp waves orbifolds with ordinary \(ADE\) and affine \(\hat{ADE}\) singularities. All one has to do is to identify the explicit expressions of the analogue of the holomorphic functions \(F\) and \(G\) and redo the same calculations. For the case of affine \(SO(k)\) pp wave orbifolds, the analogue of eqs(4.13) is given by,

\[
F = -\frac{1}{2} \left( a z_2 \zeta + b_1 \zeta^{2n} + c_1 \zeta^4 z_2^{n-2} \right),
\]

\[
G = z_2^3 + \zeta^6 + \left( b_2 \zeta^{2n+2} + c_2 \zeta^3 z_2^{n} + \sum_{i=0}^{2n-4} a_{2i} z_i \zeta^{4n-2i} \right),
\]

(5.1)

for \(k = 2n\) and

\[
F = -\frac{1}{2} \left( a z_2 \zeta + b_1 \zeta^{2n} + c_1 \zeta^3 z_2^{n-2} \right),
\]

\[
G = z_2^3 + \zeta^6 + \left( b_2 \zeta^{2n+2} + c_2 \zeta^4 z_2^{n-1} + \sum_{i=0}^{2n-4} a_{2i} z_i \zeta^{4n-2i} \right)
\]

(5.2)

for \(k = 2n - 1\). One may also write down the expression of these holomorphic functions \(F\) and \(G\) for the other remaining geometries. More details on these calculations including Kahler deformations method as well as the corresponding CFT\(_4\) models will be exposed in [14].

References

[1] D. Berenstein, J. Maldacena and H. Nastase, “String in flat space and pp-waves from \(N = 4\) Sper Yang Mills”, hep-th/0202021.
[2] J. Maldacena, “The large N limit of uperconformal field theories and supergravity,” Adv. Theor. Phys. 2, 231(1998), hep-th/9711200

[3] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998), hep-th/9802109.

[4] E. Witten, “Anti de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998), hep-th/9802150.

[5] O. Aharony, S. S Gubser, J. Maldacena, H. Oogori and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183(2000), hep-th/9905111

[6] N. Kim, A. Pankiewicz, S. Rey and S. Theisen, “Superstring on pp-wave orbifold from large N quiver gauge theory”, hep-th/0203080.

[7] D. Berenstein, E. Gava, J. Maldacena, K.S. Narain and H. Nastase, “Open strings on plane waves and their Yang Mills duals” hep-th/0203249.

[8] A. Fayyazuddin and M. Spalinski, “Large N superconformal gauge theories and supergravity orientifold”, Nucl. Phy. B535 (1998)219, hep-th/9805096.

[9] O. Aharony, A. Fayyazuddin and J. Maldacena, “The large L limit of N=1,2 from three-branes in F-theory”, JHEP 9807(1998)013, hep-th/9807159.

[10] M. Blau, J. Figueroa O’Farill, C. Hull and G. Papadopoulos, “Penrose limits and maximal supersymmetry”, hep-th/0201081.

[11] M. Blau, J. Figueroa O’Farill, C. Hull and G. Papadopoulos, “A new maximally supersymmetric background of type IIB superstring theory”, JHEP 0201 047(2001) hep-th/0110242.

[12] A. Lawrence, N. Nekrasov and C. Vafa, “On conformal theories in 4 dimension”, Nucl.Phys. B533 (1998) 199-209, hep-th/9803015

[13] B. Feng, A. Hanany, Yang-Hui He and N. Prezas, “Discrete torsion, non abelian orbifolds and the schur multiplier”, hep-th/0010023

[14] E M Sahraoui and E H Saidi, “Type IIB String on pp Wave Orbifolds and ADE Singularities,” UFRHEP/02/10

[15] S. Kachru and E. Silverstein, “4d conformal field theories and strings on orbifolds”, Phys.Rev.Lett. 80 (1998)4855-4858, hep-th/9802183

[16] S Katz, P. Mayr and C. Vafa, “Mirror symmetry and exact solution of 4d N=2 gauge theories I”, Adv.Theor.Math.Phys. 1 (1998)53-114, hep-th/9706110

[17] A Belhaj, A ElFallah and E H Saidi, “Toric geometry, enhanced non simply laced gauge symmetries in superstrings and F-theory compactifications”, CQG 17 (2000) 517, hep-th/0012131.
[18] N.C. Leung, C. Vafa, “Branes and Toric Geometry”, Adv.Theor.Math.Phys. 2 (1998) 91-118, hep-th/9711013.

[19] M. Alishahiha and M.M. Sheikh-jabbari, “The pp wave limits of orbifolded $AdS_5 \times S^5$”, hep-th/0203018.