The statistical relationship between product life cycle and repeat purchase behavior in convenience stores

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The density function of product life cycles in convenience stores is found to follow the Weibull distribution. To clarify the parameters that determine these life cycles, we introduce the conditional market share—defined as the probability that a product is selected by customers only if it had been previously purchased—and the market share without any conditions. The product life cycle is more strongly correlated with the conditional market share of the product than with the latter type of market share.

§1. Introduction

Over the last decade, physicists and economists engaged in the field of econophysics have actively studied financial markets by analyzing large amounts of financial data using techniques of statistical physics.$^{1,2}$ The results of these studies have often been applied to financial businesses. In order to investigate economic phenomena by using such an empirical approach, there is a need for a large number of datasets demonstrating these phenomena. Therefore, experts in the field first attempted to use commonly available financial datasets. Recently, many datasets except those on the financial markets have become available owing to the developments in information technology. We have studied the statistical laws governing spending by an individual in convenience stores by analyzing about 100 million receipts. The expenditure patterns exhibit violent fluctuations observed in nonequilibrium situations.$^{3,4}$ In the past, D. Sornette and F. Deschatres et al. presented an extensive study on the typical patterns of the time series of book sales by using a large dataset of book sales obtained from Amazon.com.$^{5,6}$ Through our study, we explain more general economic phenomena.

In this paper, we analyze scanner data comprising recorded customer history in the convenience store chain of “am/pm Japan Co. Ltd.” Such scanner data are mainly investigated in field of marketing. However, statistical laws governing customer purchase behavior have seldom been investigated scientifically because typically, the focus when using such data is on identifying the marketing strategies companies that can apply to efficiently obtain profits. In this paper, we clarify the statistical laws governing product life cycle and consumer repeat purchase behavior.

B. Klumb et al. outlined a framework for increasing the likelihood of efficient new
product introductions. Using Nielsen/BASES data on 850 new products introduced in the US, they confirmed that repeat customers are important for increasing the sales of a new product. In addition, the dependence on the diffusion of new products on trial-and-repeat purchases was investigated through model analyses. Through our study, we show that there are correlations between the life cycle of each product and the percentage of repeat buyers of a product.

This paper is organized as follows. We first introduce the scanner data. Next, we focus on the statistical laws governing product life cycle in convenience stores. Most products are replaced with new products in only a few months. For example, 484 kinds of rice ball were released in stores from 2004 to 2007. However, about 70% of them disappeared from stores within 2 months. We investigate the product life cycle on the basis of a survival analysis, which deals with the death of biological organisms and failure of mechanical systems; survival analysis is also known as reliability analysis in engineering and duration analysis in economics or sociology. Next, we observe the repeat purchase behavior of customers. We clarify the probability structure determining the continued purchase of a product by a customer. Then, we show that a product life cycle depends on the probability structure. Finally, we discuss why the life cycle depends on repeat purchase behavior.

§ 2. Dataset

In this study, we analyzed the POS database that recorded the customer purchase history for the convenience store chain under consideration, namely, am/pm Japan Co. Ltd. This company operates around 1,000 stores in Japan. The stores typically sell drinks, magazines, prepackaged foods such as rice balls and lunchboxes, toiletries, cigarettes, daily necessities, and miscellaneous goods.

In recent years, the use of electronic money has increased in Japan. One of the most famous modes of making electronic payments in the country is through a service called “Edy,” provided by bitWallet, Inc. Edy is a prepaid smart card that is rechargeable and contactless and can be used to make payments in places such as convenience stores. Every Edy card has a unique number called the Edy ID. When a payment is made using an Edy card, the Edy ID is recorded in a POS database that is stored on a server. By accessing the Edy ID on the database, details of a purchase such as the receipt number, date of purchase, time of purchase, and Japan article number (JAN), which is specific to each product, can be retrieved.

To investigate product life cycles and consumer repeat purchase behaviors, we used 46,312,663 receipts of customers who made purchases using their Edy cards in am/pm Japan Co. Ltd. stores from December 26, 2003, to December 31, 2007.

§ 3. Probability density function of product life cycles

The life cycles of products in convenience stores are surprisingly short. In Fig.1, we present a cumulative probability function of the product life cycles. The vertical axis, \( P(\geq T) \), shows the probability of finding a product with a life cycle longer than \( T \) days. The product life cycles are approximated by the Weibull distribution, as
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Fig. 1. Cumulative probability density functions of the product life cycle. (a) Rice ball and plastic bottle of green tea. (b) Instant noodles, plastic bottle of juice, and sandwiches. The lines show an exponential distribution, \( P(\geq T) = e^{-0.009(T-14)} \), and a Weibull distribution, \( P(\geq T) = e^{-0.23(T-35)^{0.49}} \).

follows:

\[
P(\geq T) = e^{-a(T-b)^c}
\]  

(3.1)

where \( a \) is the scale parameter and \( c \) is the shape parameter of the distribution, and \( b \) represents the minimum life cycle of each product category. The Weibull distribution is often used in the field of life data analysis due to its flexibility in
Table I. AIC for the probability density function of the product life cycle. We approximate the
function of each category by an exponential distribution and a Weibull distribution with coefficient $c$ of Eq.(3-1)

| Category                | AIC of exponential ($c = 1$) | AIC of Weibull  | $c$ |
|-------------------------|------------------------------|------------------|-----|
| Rice ball               | 967                          | 968              | 0.97|
| Plastic bottle of green tea | 745                          | 742              | 0.92|
| Sandwiches              | 1005                         | 848              | 0.49|
| Plastic bottle of juice | 1846                         | 1725             | 0.65|
| Instant noodles         | 2504                         | 2372             | 0.65|

cases such as the investigation of the failure rate of machine parts. If the failure rate decreases over time, $c < 1$. If the failure rate is constant over time, $c = 1$. If the failure rate increases over time, $c > 1$. When $c = 1$, the Weibull distribution becomes an exponential function. For each category, we measure the goodness of fit of an approximated distribution by using Akaike’s information criterion (AIC). We approximate the product life cycles by the exponential distribution $P(\geq T) = e^{-a(T-b)^c}$ with $c = 1$, and the Weibull distribution $P(\geq T) = e^{-a(T-b)^c}$. Table I shows the AICs of the two distributions.

First, with regard to the products considered, we focus on rice balls and plastic bottles of green tea. The product life cycles for each category are approximated using the exponential function as shown in Fig.1(a). The AIC of Weibull distribution shows almost the same value as the AIC of exponential distribution as shown in Table I. Therefore, the life cycles of the categories follow Poisson processes because $c \approx 1$ in Eq.(3-1).

Next, we also investigate instant noodles, plastic bottles of juice, and sandwiches. As shown in Fig.1(b), the product life cycles are approximated by the Weibull distribution with $c < 1$. The AIC of the Weibull distribution is sufficiently smaller than the AIC of exponential distribution as shown in Table I. Hence, $c < 1$ is a significant parameter. We cannot approximate the life cycles of these categories through pure Poisson processes because the probability that a product is discontinued depends on the number of days over which the product has been sold.

§4. Weak correlation between the life cycle and the market share without any conditions

We considered the parameters that determined product life cycles. We first focused on the product market share that is estimated by the probability, $P_k(N_{i,t} = 1)$, that the product $k$ is bought without any conditions in a category that includes the product $k$. Here, $N_{i,t} = 1$ when a customer $i$ buys product $k$ in his $t$-th purchase in the category. Fig.2 displays the time series of the market shares of two rice ball products. The market share of product A was always higher than that of product B. However, product A was discontinued after about 90 days and product B was continued for over 150 days.

We indicate the relationship between the product life cycle and the market share,
Table II. The correlation coefficient between the product life cycle \((\log T)\) and \(P_k(N_{i,t} = 1)\), \((\log T)\), and \(P_k(N_{i,t} = 1|N_{i,t-1} = 1)\). The market share with a condition, \(P_k(N_{i,t} = 1|N_{i,t-1} = 1)\), is defined by the conditional probability that product \(k\) is consistently chosen. Market share, \(P_k(N_{i,t} = 1)\), is defined by probability that product \(k\) is chosen without any conditions.

| Product                        | \(P_k(1)\) | \(P_k(1|1)\) |
|--------------------------------|-------------|---------------|
| Rice ball                      | 0.27        | 0.61          |
| Plastic bottle of juice        | 0.37        | 0.60          |
| Sandwiches                     | 0.38        | 0.54          |
| Plastic bottle of green tea    | 0.38        | 0.44          |
| Instant noodles                | 0.31        | 0.34          |

\(P_k(N_{i,t} = 1)\), in the case of rice balls, as depicted in Fig.3. We estimated the average market share for each life cycle. When a life cycle is longer than about 200 days, it depends on the market share. However, we found that the life cycle is almost independent of the market share in the case of products with short life cycles. About 70% of the products disappear from stores within 2 months of being released for sale. Therefore, for most products, it is difficult to extrapolate the life cycle from the market share. With regard to the several popular kinds of products available in convenience stores in Japan, we present the correlations between the product life cycle and the market share, \(P_k(N_{i,t} = 1)\), by using a cross-correlation function in Table II. Generally, the correlations are weakly independent of the type of product.

5. Analysis of repeat purchase behavior using conditional probability

In order to investigate the difference between products \(A\) and \(B\) in Fig.2, we introduced a conditional market share, \(P_k(N_{i,t} = 1|N_{i,t-1} = 1)\), which expresses the probability that a product \(k\) is selected by a customer \(i\) only if it had been his \((t-1)\)-th purchase. With many products, we find that the conditional market share is much higher than the market share without any conditions, \(P_k(N_{i,t} = 1)\), as shown in Table III. Thus, customers exhibit characteristic behaviors during repeat purchases.

In Table III, we observe the market share with some conditions; this is defined by the probability \(P_k(N_{i,t} = 1|N_{i,t-1} = 1, N_{i,t-2} = 1, \cdots, N_{i,t-\tau} = 1)\) that product \(k\) is chosen by customer \(i\) who has consistently purchased it \(\tau\) times. We do this to clarify the characteristics of repeat purchase behavior. The probability that product \(k\) is chosen increases depending on its \(\tau\)-th successive purchase. This characteristic is found in both products \(A\) and \(B\). In the case of product \(B\), the probability that product \(B\) is chosen without conditions, \(P_B(N_{i,t} = 1)\), is 0.026. However, the conditional probability, \(P_B(N_{i,t} = 1|N_{i,t-1} = 1)\), that a customer who bought product \(B\) the last time \((t-1)\) chooses product \(B\) at time \(t\) is 0.408. Furthermore, the conditional probability of a customer who bought product \(B\) over \(\tau = 4\) successive times, \(P_B(N_{i,t} = 1|N_{i,t-1} = 1, N_{i,t-2} = 1, \cdots, N_{i,t-\tau} = 1)\), is higher than 0.8. This characteristic of the conditional market share indicates that a customer gradually develops a sticky preference for the product by consistent purchase.
Fig. 2. Time series of the market share, $P_k(N_{i,t} = 1)$, of rice ball products A and B. Product A and B were released for sale on February 22, 2005, and December 12, 2006, respectively. Time $= 0$ indicates the release date.

Fig. 3. Relationship between product life cycle and market share, $P_k(N_{i,t} = 1)$. The curve represents the average market share for each life cycle.

§6. Relationship between product life cycle and conditional market share

As shown in Table III, product A with a short life cycle shows a low conditional market share, $P_A(N_{i,t} = 1|N_{i,t-1} = 1) = 0.310$. However, product B with a long life cycle shows a high conditional market share, $P_B(N_{i,t} = 1|N_{i,t-1} = 1) = 0.408$. On the basis of this result, we can point out the possibility that repeat purchase behavior determines the product life cycle. In order to quantify the degree of repeat purchase behavior for each product, we only focused on the conditional market share,
Table III. Conditional market shares of product A and B. Market share with conditions, $P_k(N_{i,t} = 1|N_{i,t-1} = 1, N_{i,t-2} = 1, \ldots, N_{i,t-\tau} = 1)$, expresses the conditional probability that product $k$ is chosen by customer $i$ who has consistently purchased it $\tau$ times. $N_{i,t} = 0$ when customer $i$ does not buy product $k$ in his $t$-th purchase. Here, $P_k(N_{i,t} = 1)$, is the probability that product $k$ is chosen in $t$-th purchase of rice balls without any conditions. The error term is defined by $\frac{1}{\sqrt{n}}$, where $n$ is the number of data points.

|         | $k =$Product A | $k =$Product B |
|---------|----------------|----------------|
| $P_k(1)$ | $0.067 \pm 0.001$ | $0.026 \pm 0.001$ |
| $P_k(1 \mid 1)$ | $0.310 \pm 0.004$ | $0.408 \pm 0.004$ |
| $P_k(1 \mid 11)$ | $0.556 \pm 0.009$ | $0.657 \pm 0.006$ |
| $P_k(1 \mid 111)$ | $0.685 \pm 0.012$ | $0.763 \pm 0.007$ |
| $P_k(1 \mid 1111)$ | $0.763 \pm 0.013$ | $0.823 \pm 0.007$ |
| $P_k(1 \mid 11111)$ | $0.808 \pm 0.014$ | $0.853 \pm 0.007$ |
| $P_k(1 \mid 111111)$ | $0.852 \pm 0.014$ | $0.865 \pm 0.009$ |

$P_k(N_{i,t} = 1|N_{i,t-1} = 1)$. As shown in the example in Table III, the conditional market shares for product $B$ was always higher than the corresponding one for product $A$; thus, we were able to measure the relationship between the conditional market shares of the products by $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$.

Fig.4 displays the relationship between the product life cycle, $T$, and the conditional market share, $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$, in the case of rice balls. A linear relationship was approximated as follows:

$$T = 20.0 \cdot e^{6.04 \cdot P_k(1)}.$$  \hspace{1cm} (6.1)

We estimated the strength of the relationship by using the cross-correlation function for several popular product types, in Table II. We found that the product life cycle is more strongly correlated with the conditional market share of the product, $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$, than with its market share without any conditions, $P_k(N_{i,t} = 1)$.

In Fig.5, we illustrate the probability density function of the product life cycle for each conditional market share, $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$. For example, the solid line in Fig.5(a) represents the probability density function of rice balls when $0.07 < P_k(N_{i,t} = 1|N_{i,t-1} = 1) \leq 0.17$. We found that the function can be approximated by an exponential distribution, and the coefficient of exponential distribution depends on the conditional market share. This result suggests that we can model the life cycle through a Poisson process where a parameter depends on the conditional market share.

We discuss why the life cycle of a product depends on the conditional market share that expresses its repeated purchase. It is well known that the expenses incurred when manufacturers try to attract new customers are much higher than the cost of maintaining loyal customers. For example, product manufacturers often end up spending large amounts of money to attract new customers. Therefore, products that have a low probability of being consistently chosen by customers may quickly face termination.
Fig. 4. Relationship between product life cycle, $T$, and the conditional market share, $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$, in the case of rice balls. The line represents an exponential function, $T = 20.0 \cdot e^{0.04 \cdot P(1)}$.

Fig. 5. Cumulative probability density functions of the product life cycle for each conditional market share, $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$. (a) denotes rice balls, and (b) plastic bottles of juice. The lines represent the function for each value of $P_k(N_{i,t} = 1|N_{i,t-1} = 1)$. 
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§7. Conclusion

In this study, we observed that the probability density function of a product life cycle follows the Weibull distribution or an exponential distribution that is a special case of the Weibull distribution. We introduced the conditional market share—defined as the probability that a product is selected by customers only if it had been purchased previously—and the market share without any conditions. On the basis of this, we found that the product life cycle is more strongly correlated with the conditional market share of the product than with its market share without conditions. Thus, we may be able to predict a product life cycle on the basis of its conditional market share immediately after its release date. We expect that these results will be applied to retail management in the future.

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