Primordial universe with the running cosmological constant

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Abstract

Theoretically, the running of the cosmological constant in the IR region is not ruled out. On the other hand, from the QFT viewpoint, the energy released due to the variation of the cosmological constant in the late universe cannot go to the matter sector. For this reason, the phenomenological bounds on such a running are not sufficiently restrictive. The situation can be different in the early universe when the gravitational field was sufficiently strong to provide an efficient creation of particles from the vacuum. We develop a framework for systematically exploring this possibility. It is supposed that the running occurs in the epoch when the Dark Matter already decoupled and is expanding adiabatically, while baryons are approximately massless and can be abundantly created from vacuum due to the decay of vacuum energy. By using the handy model of Reduced Relativistic Gas for describing the Dark Matter, we consider the dynamics of both cosmic background and linear perturbations and evaluate the impact of the vacuum decay on the matter power spectrum and to the first CMB peak. Additionally, using the combined data of CMB+BAO+SNIa we find the best fit values for the free parameters of our model.

Keywords: Early universe, running cosmological constant, CMB

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1 Introduction

The improving quality of the data of observational cosmology leads to better estimates of the equation of state of the Dark Energy, which is driving the accelerated expansion of the universe. The current data are consistent with the value of $w = -1$, which means the cosmological constant. From the quantum field theory point of view, the cosmological constant is a necessary element of a consistent semiclassical theory [1, 2, 3, 4] and hence it should not be taken as a surprise that it is non-zero. However, the theory can not provide us the ultimate word about what is the real situation. On the other hand, it can not be ruled out that at some moment the analysis of the observational data prove that the density of the Dark Energy changes with time. Does this mean that there is another component of the Dark Energy, besides the cosmological constant? In order to answer this question, one has to first address another one, of whether the cosmological constant can be not exactly a constant. It is a standard assumption that the observable density of the vacuum energy is a sum of the vacuum counterpart and the contribution generated by a symmetry breaking, e.g. at the electroweak and QCD scales. In principle, both vacuum and induced parts can be variable due to quantum effects.

The variation of cosmological “constant” term, because of the quantum effects, can be explored by means of the renormalization group running of this parameter [5, 6]. The simplest version of such a running can be described in the framework of a minimal subtraction scheme in curved space [2, 7] (see also [3]), but this kind of running leads to the inconsistent cosmological model [5]. The standard interpretation is that the “correct” running at low energies (in the IR) should take into account the decoupling of the massive fields. Such decoupling cannot be verified for the cosmological constant case [5], but the non-running can be proved neither [6]. Thus, the situation is such that one can explore the running cosmological constant only in the phenomenological setting. However, it is important to have this setting well-defined. And in this respect, the main point is what happens with the energy when the cosmological constant varies according to the evolution of the Universe and corresponding change of the energy scale.

It is well-known that the quantum or semiclassical corrections to the action of gravity are typically non-local and rather complicated. How can we separate those terms in the vacuum effective action, that can be attributed to the cosmological constant with quantum contributions? The solution that looks reasonable at least for the cosmological applications is that cosmological constant terms in the effective action should scale like the classical cosmological constant under the global transformation $g_{\mu\nu} \rightarrow g_{\mu\nu} e^{2\lambda}$, where $\lambda = \text{const.}$
For instance, terms as

\[
\int d^4 x \sqrt{-g} R \frac{1}{\sigma^2} R, \quad \int d^4 x \sqrt{-g} R_{\mu\nu} \frac{1}{\sigma^2} R^{\mu\nu}, \quad \int d^4 x \sqrt{-g} R_{\mu\nu\alpha\beta} \frac{1}{\sigma^2} R^{\mu\nu\alpha\beta}
\] (1)

and many other similar structures, belong to this group and could be used as toy models for quantum corrections to the cosmological constant. Under constant scaling, the corresponding Lagrangians transform exactly as the cosmological constant term. For the FLRW (homogeneous and isotropic) metric, the difference with the cosmological constant is proportional to the derivatives of the non-constant scaling parameter \(\sigma(t)\), after we replace \(\lambda \to \sigma(t)\). These derivatives form polynomial corrections, which are weaker than the leading exponential dependence. That is why these terms provide cosmological models equivalent to a slowly varying cosmological constant \[9\] (see also further references therein).

Basically, in the literature, one can find two distinct possibilities for the cosmological constant running. The first one assumes the energy exchange between vacuum and matter sectors. The model of cosmological evolution that follows from this assumption has essential technical advantages. In particular, the evolution of the cosmological background can be easily described using elementary functions \[10\], and the analysis of perturbations is also relatively simple \[11\]. For this reason, this model became popular (see, e.g, the review \[12\] and the recent publication \[13\]), regardless of the existing conceptual difficulties, that will be described below. The second model is much more consistent for the low-energy regime, it is based on the conservation law not involving the matter sector, and assumes a mixture between the cosmological constant term and the Einstein-Hilbert action, that means a running of the Newton constant \(G\). This model is more complicated technically, and also the phenomenological restrictions on the unique free parameter \(\nu\) are very weak, at least from the analysis of structure formation \[14\]. In what follows, we shall concentrate on the models of running cosmological constant of the first kind and explore the physical conditions where this model makes sense.

The main problem with the model based on the vacuum-matter energy exchange is that during most of the history of the universe the typical energies of the gravitational degrees of freedom are very small compared to the masses of all known particles \[17\]. For instance, the value of the Hubble parameter today is about \(H_0 \propto 10^{-42}\) GeV, while the lightest neutrino is supposed to have the mass about thirty orders of magnitude greater. Thus, there is only a possibility to create photons and this is not really phenomenologically interesting, since the energy density of such photons would be about \(T^4\), with the temperature \(T \approx H\). Such an energy density is of course much smaller than the energy density of CMB, which is yet about four orders of magnitude smaller compared to the present-day critical density, or to

\[5\]

In compensation, running \(G\) has interesting astrophysical applications (see e.g. \[15\] \[19\]).
the cosmological constant density. This argument represents a serious obstacle to using this model for a late cosmology.

Let us note that the described restrictions do not apply to the early universe, e.g., to the epoch after inflation, where the value of the Hubble parameter is decreasing from about $10^{13} - 10^{11}$ GeV to the values that are comparable to the energy scale of the Minimal Standard Model of elementary particle physics. This is a reheating period, where the creation of particles is very intensive, and there is nothing wrong with assuming that this happens because of the decay of the cosmological constant into the matter. In the next section, we shall explore the model in the high energy domain. The description of quantum effects is based on the running of cosmological constant described in this paper.

At the same time, the application of the running of cosmological constant to the early cosmology requires special care about the description of matter. The matter contents of the Universe consist mainly of baryons and DM. We assume that the DM consists of the GUT remnants and hence has masses that are much greater than the value of $H$. Thus, the DM can be regarded to decouple, in the sense that DM particles are not created from the vacuum. Thus, an appropriate description of DM is an ideal gas of massive particles that is adiabatically expanding and becomes less relativistic with time. In order to describe such a gas, we shall use the simple and convenient Reduced Relativistic Gas (RRG) model, which was originally developed by Sakharov in the classical paper, and recently reinvented in.

The paper is organized as follows. In the next section, we formulate the framework and the model, including the Einstein-Hilbert action with the running cosmological constant and non-running Newton constant, DM described by RRG, decoupled from everything except the standard gravitational interaction, and the baryonic matter, that has the equation of state of radiation and is exchanging energy with the varying cosmological constant sector. In Sec. we describe the perturbations in this model, and derive the observable consequences of the running cosmological constant in Sec. Finally, in Sec. we draw our conclusions and discuss the perspectives for subsequent work.

2 Background solution

We consider a cosmological model with the possibility of particle creation in the primordial universe due to the quantum effects of vacuum. More precisely, we study the potential vacuum energy decay as a result of the renormalization group (RG) equation for the density of the cosmological constant term.

In Refs. it has been shown, from the general arguments based on covariance and dimensions, that the form of these quantum corrections can be defined up to a single
free parameter $\nu$,

$$\rho_\Lambda = \rho_\Lambda^0 + \frac{3\nu}{8\pi G} (H^2 - H^0_0),$$  \hfill (2)

where the subscript 0 means that the quantity is taken at some reference redshift parameter when time is $t_0$ and the conformal factor $a_0$. The main argument of \cite{5,21} (see also \cite{4}) was based on covariance, and looks as follows. The effective action terms which can be classified as quantum contributions to the cosmological constant are certainly non-local, but they are also certainly covariant, see (1) as examples. Making an expansion in the powers of metric derivatives (on flat or even de Sitter background), we arrive at the local expressions and all the terms in these expansions are of the even powers in metric derivatives. The reason is that with the odd powers it is algebraically impossible to provide a scalar, regardless of the complexity of initial non-local action. On the cosmological background, the absence of odd powers of derivatives means that the first terms of the expansion include $H^2$ and $\dot{H}$. Now, the possible $\mathcal{O}(\dot{H})$-actions are surface terms or they reduce to $\mathcal{O}(H^2)$ when substituted into the action\cite{4}. Thus, it is sufficient to explore the cosmological models based on the assumption (2).

In what follows primes indicate derivatives with respect to the redshift parameter

$$1 + z = \frac{a_0}{a}. \hfill (3)$$

In the present paper, we use the normalization with the scale factor at present $a_0 = 1$. The sign of $\nu$ indicates whether bosons or fermions dominate in the running \cite{10}.

The matter contents of the universe include baryonic matter, DM and radiation, according to the current estimate \cite{23}. Here, the DM component is described as a reduced relativistic gas (RRG) of massive particles, which take into account in a simple and useful way the warmness of the fluid\cite{7}. The RRG is a reliable approximation when the interaction between the particles is irrelevant \cite{19}, as it assumes that the gas is composed of particles with equal speed $v = c\beta$.

An elementary consideration \cite{19} (see also \cite{24,25} for alternative derivations) shows that the equation of state of such a gas is

$$P_{dm} = \frac{\rho_{dm}}{3} \left[1 - \left(\frac{mc^2}{\varepsilon}\right)^2\right]^2 = \frac{\rho_{dm}}{3} \left(1 - \frac{\rho_d^2}{\rho_{dm}^2}\right),$$  \hfill (4)

\footnote{Indeed, this argument has no absolute power because this and other terms can emerge on the way from the action to equations of motion. On the other hand, phenomenologically $\mathcal{O}(\dot{H})$-term is also not very relevant \cite{12}.}

\footnote{As we have explained above, for the baryonic matter we assume an ultrarelativistic equation of state with $P_b \approx \frac{1}{3} \rho_b$.}
where $\varepsilon = \frac{mc^2}{\sqrt{1-\beta^2}}$ is the kinetic energy of the individual particle, $\rho_{dm} = n\varepsilon$ and $P_{dm}$ are energy density of the gas, while $\rho_d = nmc^2$ is the density of the rest energy. Consequently, the scaling rule for this quantity is

$$\rho_d(z) = \rho_d^0 (1 + z)^3. \tag{5}$$

Here we consider an early post-inflationary universe, where the DM have already decoupled from the other matter components and satisfies its own continuity equation

$$\rho_{dm}' = \frac{(4 - r)}{1 + z} \rho_{dm}, \tag{6}$$

where we defined the useful function

$$r = r(z) = \frac{\rho_{dm}^2(z)}{\rho_{dm}^2(z)}. \tag{7}$$

In the early Universe, one can restrict the consideration by only the spatially flat FLRW metric. The solution for Eq. (6) can be easily found for a single adiabatically expanding fluid [20]. Then the relative energy density (relative to the critical density) for the relativistic gas representing the DM, is given by the expression

$$\Omega_{dm}(z) = \frac{\Omega_{dm}^0 (1 + z)^3}{\sqrt{1 + b^2}} \sqrt{1 + b^2(1 + z)^2}, \quad \text{where} \quad b = \frac{\beta}{\sqrt{1 - \beta^2}}, \tag{8}$$

and $\Omega_{dm}^0$ is the DM density in the present-day Universe. The parameter $b$ measures the warmness of the matter (DM in our case). In the limit of low warmness $\beta \ll 1$, we have $b \sim \beta$. Thus, $b \approx 0$ means that the matter contents is “cold”. If taken alone, the RRG model provides an interpolation between the radiation ($b \rightarrow \infty$) and matter ($b = 0$) dominated regimes [19]. The model can be used also to describe several fluids that are in thermal contact, exchanging energy [26, 27].

According to our physical setting the running cosmological constant [6] is exchanging energy only with the baryonic matter, and the last has the approximate equation of state of radiation. Then the conservation law has the form

$$\rho_r' - \frac{3(1 + w)}{1 + z} \rho_r = -\rho'_\Lambda, \tag{9}$$

where we left $w$ to be the equation of state parameter for the sake of generality. When starting to deal with the numerical estimates, we shall set $w = 1/3$. Finally, the Hubble parameter is given by the Friedman equation

$$H^2(z) = \frac{8\pi G}{3} \left[\rho_\Lambda(z) + \rho_r(z) + \rho_{dm}(z)\right]. \tag{10}$$
The solution of the system (2), (9) and (10) can be performed following the pattern of [28], since the technical complications related to the presence of DM are not critical. In order to obtain $\Omega_r(z)$ one has to consider the derivative of Eq. (10) and then use (2). After this, we arrive at the equation

$$\rho'_\Lambda = \frac{\nu}{1-\nu}(\rho'_r + \rho'_{dm}).$$

(11)

Using (11) in (9) to eliminate $\rho_\Lambda$, after some simple algebra we obtain the differential equation for $\rho_r(z)$,

$$\rho'_r - \frac{\zeta}{1+z}\rho_r = -\nu \rho_{dm},$$

(12)

where

$$\zeta = 3(1+w)(1-\nu).$$

(13)

Let us stress that the interaction between radiation (remember it is all baryonic matter in this case) and DM, is not direct, but occurs because of the running of the cosmological constant term in Eq. (2), parameterized by $\nu$, and the Friedmann equation (10). This implicit interaction occurs regardless of the DM satisfies separate continuity equation (6).

Using Eq. (8) the solution of (12) can be found in the form

$$\Omega_r(z) = C_0(1+z)^\zeta \frac{\nu \Omega^0_{dm}(1+z)^3}{\sqrt{1+b^2}} \left[ \sqrt{1+b^2(1+z)^2} + \frac{\zeta}{3-\zeta} 2F_1(\alpha, \beta; \gamma; Z) \right],$$

(14)

with

$$C_0 = \Omega^0_r + \frac{\nu \Omega^0_{dm}}{\sqrt{1+b^2}} \left[ \sqrt{1+b^2} + \frac{\zeta}{3-\zeta} 2F_1(\alpha, \beta; \gamma; -b^2) \right].$$

(15)

Here $2F_1(\alpha, \beta; \gamma; Z)$ is the hypergeometric function defined as

$$2F_1(\alpha, \beta; \gamma; Z) = \sum_{k=0}^{\infty} \frac{(\alpha)_k(\beta)_k}{(\gamma)_k} \frac{Z^k}{k!},$$

(16)

where $(\alpha)_k$ is the Pochhammer symbol. In our case

$$\alpha = -\frac{1}{2}, \quad \beta = \frac{3-\zeta}{2}, \quad \gamma = \frac{5-\zeta}{2} \quad \text{and} \quad Z = -b^2(1+z)^2.$$  

(17)

Furthermore, $\Omega_\Lambda(z)$ is directly obtained by integrating (11),

$$\Omega_\Lambda(z) = B_0 + \frac{\nu}{1-\nu} [\Omega_r(z) + \Omega_{dm}(z)],$$

(18)
where

$$B_0 = \Omega_\Lambda^0 - \frac{\nu}{1-\nu} (\Omega_r^0 + \Omega_{dm}^0).$$

(19)

Finally, the Hubble parameter can be found from the Friedmann equation,

$$H(z) = H_0 \sqrt{\Omega_\Lambda(z) + \Omega_r(z) + \Omega_{dm}(z)}.$$

(20)

In order to illustrate the behavior of the model we can consider the total effective equation of state. It can be obtained by means of the second Friedman equation,

$$-2(1+z)H'H + 3H^2 = -8\pi G \rho_t \equiv -8\pi G w_{\text{eff}}(z)\rho_t,$$

(21)

where

$$\rho_t(z) \equiv \rho_\Lambda(z) + \rho_r(z) + \rho_{dm}(z).$$

(22)

Thus,

$$w_{\text{eff}}(z) = \frac{2H'}{3H} - 1.$$

(23)

In Fig. 1 we plot $w_{\text{eff}}(z)$ for the energy balance obtained by the best fit of $\chi^2_{\text{L}}$ and $\chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} + \chi^2_{\text{SNIa}}$ (see Sec. 4.1 and Sec. 4.2). As expected $w_{\text{eff}}(z) \to -1$ when $z \to -1$, while $w_{\text{eff}}(z) \to 0.333$ for $z \to \infty$ [29]. When compared with the $\Lambda$CDM model with the same $\Omega$'s and CMB+BAO+SNIa combined data are used, our model fits better for small $z$ and approaches faster to radiation dominated epoch when $z$ increases.

Additionally, it is worthwhile to mention that the decay of the cosmological term into radiation, which includes relativistic (in the very early universe) baryons, does not affect the nucleosynthesis process, as it can be seen in Fig. 2 where the abundance of the relativistic specie, including baryons is compared with the corresponding data of $\Lambda$CDM case represented by $\nu = 0$. In this plot we take again the best fit values given by using CMB+BAO+SNIa combined data, this is, $\nu = 1.25235 \times 10^{-5}$ for the cosmological constant running parameter and $b = 4.26321 \times 10^{-5}$ for the DM warmness.

3 Including perturbations

The cosmological perturbations in the model described above can be analysed following the approach developed in Refs. [11] and [20]. This implies simultaneous perturbations of metric, energy density and the four-velocities in the co-moving coordinates,

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} , \quad \rho_i \to \rho_i + \delta \rho_i , \quad U^\alpha \to U^\alpha + \delta U^\alpha , \quad V^\alpha \to V^\alpha + \delta V^\alpha.$$
in the synchronous gauge \( h_{0\mu} = 0 \). Here \( U^\alpha \) is the DM velocity and \( V^\alpha \) is the radiation (baryonic matter, in our case) velocity. In the following calculations we use the constraint \( \delta U^0 = \delta V^0 = 0 \).

The perturbation of the DM pressure should be derived from the equation of state (4),

\[
\delta P_{dm} = \frac{\delta \rho_{dm}}{3} \left[ 1 - \left( \frac{mc^2}{\varepsilon} \right)^2 \right]^2 = \frac{\delta \rho_{dm}(1 - r)}{3},
\]

meaning that the perturbations satisfy the same equation of state as the background quantities. Technically, this means that the variations of the energy density \( \delta \rho_{dm} \) and the rest energy density \( \delta \rho_d \) are always proportional. The reason for this restriction is that in the framework of the RRG model one has to provide kinetic energies of all particles to be equal and, therefore, we have no right to change the ratio \( mc^2/\varepsilon \) [24]. The definitions of the perturbations for other densities are straightforward.

Let us introduce the useful notations for the quantities (22),

\[
\begin{align*}
    f_1(z) &= \frac{\rho_\Lambda(z)}{\rho_t(z)}, \\
    f_2(z) &= \frac{\rho_\Lambda(z)}{\rho_t(z)}, \\
    f_3(z) &= \frac{\rho_{dm}(z)}{\rho_t(z)}, \\
    g(z) &= \frac{2\nu}{3H(z)}.
\end{align*}
\]
thus, we arrive at the 00-component of the Einstein equations,

\[ h' - \frac{2h}{1+z} = -\frac{2\nu}{(1+z)g} [(1+3w)f_1 \delta_r - 2f_2 \delta_\Lambda + (2-r)f_3 \delta_{dm}], \quad (27) \]

where \( h = \partial_t(h_{ii}/a^2) \) and

\[ \delta_i = \frac{\delta \rho_i}{\rho_i} \quad (28) \]

are the corresponding density contrasts. Other equations, corresponding to the time and spatial components of the perturbation for the conservation law \( \delta (\nabla_\mu T^{\mu \nu}) = 0 \), have the form

\[ \delta_r' + \left[ f_1' \frac{3(1+w)f_2}{1+z} + \frac{1-r-3w}{1+z} f_3 \right] \delta_r - \frac{1+w}{(1+z)H} \left( \frac{v}{f_1} - \frac{h}{2} \right) = 0, \quad (29) \]

\[ v' + \left[ \frac{3(1+w)f_1 + (4-r)f_3 - 5}{1+z} \right] v = \frac{k^2(1+z)}{(1+w)H} \left( f_2 \delta_\Lambda - w f_1 \delta_r \right), \quad (30) \]

\[ \delta_{dm}' + \left\{ f_3' \frac{3(1+w)f_1 + (r-4)(f_1 + f_2)}{1+z} \right\} \delta_{dm} + \frac{4-r}{3H(1+z)} \left( \frac{h}{2} - \frac{u}{f_3} \right) = 0, \quad (31) \]

\[ u' + \left[ \frac{3(1+w)f_1 + (4-r)f_3 - 5}{1+z} \right] u + \frac{k^2(1+z)f_3}{4-r} \left( \frac{1-r}{H} \right) \delta_{dm} = 0. \quad (32) \]

Here we used the notations \( v = f_1 \nabla_i(\delta V^i) \) and \( u = f_3 \nabla_i(\delta U^i) \) for divergences of the peculiar velocities and we rewrote all the previous perturbation equations in the Fourier space, using

\[ f(x,t) = \int \frac{d^3k}{(2\pi)^3} f(k,t) e^{ik\cdot x}, \quad \text{with} \quad k = |k|. \quad (33) \]
Perturbing the formula (2), one finds
\[ \delta_\Lambda = \frac{g}{f_2} \left( \frac{v}{f_1} - h \right). \]  
(34)

The last equation is not dynamical, representing a constraint that can be replaced into other equations. Using (34) in Eqs. (27), (29) and (30), we arrive at the equations
\[ h' + \frac{2(\nu - 1)}{1 + z} h = \frac{2
\nu}{1 + z} \left[ \frac{2
\nu}{f_1} - (1 + 3w) \frac{f_1}{g} \delta_r - (2 - r) \frac{f_3}{g} \delta_{dm} \right], \]
(35)
\[ \delta'_r + \left[ \frac{f_1}{f_1} - \frac{3(1 + w)f_2}{1 + z} + \frac{(1 - r - 3w)f_3}{1 + z} \right] \delta_r = \frac{1}{f_1} \left( \frac{gh}{2} - \frac{gv}{f_1} \right)' \]
\[ + \frac{1 + w}{1 + z} \left[ 3g + \frac{(4 - r)g f_3}{(1 + w)f_1} - \frac{1}{H} \right] \left( \frac{h}{2} - \frac{v}{f_1} \right), \]
(36)
\[ v' + \left\{ \frac{3(1 + w)f_1 + (4 - r)f_3 - 5}{1 + z} - \frac{k^2 g (1 + z)}{(1 + w) H f_1} \right\} v \]
\[ = - \frac{k^2 g (1 + z)}{2(1 + w) H} \left( h + \frac{2 w f_1}{g} \delta_r \right). \]
(37)

Thus, we arrive at the complete system of perturbation equations, given by Eqs. (31), (32), (35), (36) and (37).

4 Observational tests

The free parameters of the cosmological model of early universe with running cosmological constant and energy exchange between vacuum and matter can be constrained from various observational tests. Thus, the general framework of the model formulated above may have different applications (one can see e.g. [30] for the possibilities in a simpler model without cosmological constant running). As a first step, in the present section we consider essentially the two tests: the position of the first acoustic peak in the CMB power spectrum and CMB+BAO+SNIa combined data.

Let us note that the process of cosmological constant decay into baryons, as discussed in the previous sections, is effective in the primordial universe, that is long before BBN. For this reason, we are allowed to use the transfer function in the usual standard format. However, this process leaves traces for the later epochs of the universe evolution, encoded in the values of parameters \( \nu \) and \( b \). In this way one can use the tests from the late phase of the Universe for exploring the effect of running cosmological constant in the earlier epoch.

The statistical analysis of the data starts with the \( \chi^2 \) functions, constructed according to the general expression
\[ \chi^2(X^j) = \sum_{i=1}^{N} \left[ \frac{\mu_{i}^{\text{obs}} - \mu_{i}^{\text{th}}(X^j)}{\sigma_i} \right]^2, \]  
(38)

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where $N$ is the total number of observational data, $\mu^i_{th}$ are the theoretical predictions depending on free parameters $X^j$, and $\mu^i_{obs}$ represent the observational values with an error bar given by $\sigma_i$. In our case the free parameters are $\nu$, $\Omega^0_{dm}$ and $b$. Let us remember that the first one, $\nu$, defines the hypothetical running of vacuum energy, that is a typical feature of the running cosmological constant models. Also, $\Omega^0_{dm}$ and $b$ describe the DM relative density and warmness. As usual, $\Omega^0_\Lambda = 1 - \Omega^0_{dm} - \Omega^0_b - \Omega^0_r$. It is worthwhile mentioning, that here we are dealing with the late universe, and hence baryon and radiation contents are separated.

The probability distribution function is constructed from $\chi^2$ as

$$P(X^j) = Ae^{-\chi^2(X^j)/2},$$

(39)

where $A$ is a normalization constant.

### 4.1 The first CMB peak

The position of the first peak in the CMB spectrum $l_1$ is related to the acoustic scale $l_A$ by the relation

$$l_1 = l_A (1 - \delta_1), \quad \text{where} \quad \delta_1 = 0.267 \left( \frac{\bar{r}}{0.3} \right)^{0.1},$$

(40)

with $\bar{r} = \frac{\rho_c(z_{ls})}{\rho_m(z_{ls})}$ evaluated at the redshift of the last scattering surface $z_{ls} = 1090$ [31]. The acoustic scale is defined by

$$l_A = \pi \int_0^{z_{ls}} \frac{dz}{H(z)} / \int_{z_{ls}}^{\infty} \frac{c_s(z)}{c} \frac{dz}{H(z)},$$

(41)

where $c_s(z)$ is the sound speed

$$c_s(z) = c \left( 3 + \frac{9}{4} \frac{\Omega^0_b}{\Omega^0_{dm} z} \right)^{-1/2}.$$  

(42)

In Eq. (42) $\Omega^0_b$ and $\Omega^0_{dm}$ stand for the present density parameters of baryons and photons, respectively. The relation (40) does not depend on the dark energy model. Here we consider the estimate $l_1 = 220.6 \pm 0.6$ and we use the values $\Omega^0_b = 2.47 \times 10^{-5}/h^2$, $\Omega^0_\Lambda = 0.022/h^2$ and $\Omega^0_r = 4.18 \times 10^{-5}/h^2$ with the reduced Hubble constant $h = 0.6732$ [32]. Furthermore, we let the free parameters run in the intervals $\nu \in (0, 10^{-4})$, $b \in (0, 10^{-4})$ and $\Omega^0_{dm} \in (0, 0.95)$. The minimization of the $\chi^2$ statistics is done according to

$$\chi^2_{l_1} = \left[ \frac{220.6 - l_1(\Omega^0_{dm}, \nu, b)}{0.6} \right]^2,$$

(43)
Figure 3: The first CMB peak one-dimensional probability distribution, after marginalizing on the other variables.

where this function has a local minimum around

$$\Omega_{dm}^0 = 0.61355, \quad \nu = 1.25235 \times 10^{-5}, \quad b = 4.26321 \times 10^{-5}.$$  \hspace{1cm} (44)

Here we can see that the current DM energy density value $\Omega_{dm}^0$ is higher than expected, indicating the necessity of a more robust observational test in order to get a better fit with respect to standard model of cosmology (see Sec. 4.2). In Fig. 3 one can see the results for the one-dimensional marginalized probability distribution (PDF) for the free parameters of the model. It is easy to see that this test alone cannot constraint too much the parameters. Furthermore, the two-dimensional probability distribution, with both parameters being varied and one is integrated out, is shown in Fig. 4. The regions of
Figure 4: Two-dimensional probability distribution. The brighter regions have higher probabilities.

higher probabilities in these plots are indicated by brighter tons.

4.2 CMB, BAO and SNIa data

In order to find better constraints for our free parameters, in this section it is constructed a more robust test using CMB, BAO and SNIa combined data. Thus, we shall use

$$\chi^2_{total} = \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{SNIa},$$

(45)

where $\chi^2_{CMB}$, $\chi^2_{BAO}$ and $\chi^2_{SNIa}$ are constructed following the reference [33]. The results of this test are summarized in Table I. Note that $\Omega^0_{dm}$ is lower than the previous estimate [44] and therefore looks closer to the respect to recent observational data. Additionally,
we observe just slight variations in $\nu$ and $b$ values. The plots with $1\sigma$ and $2\sigma$ levels for the CMB+BAO+SNIa combined data are shown in Fig. 5.

Let us note that the fact that the most probable values [44] include $\nu \neq 0$ does not constitute proof of the running of the cosmological constant. As usual, the statistics with an extra free parameter, such as $\nu$, always gives the best values for the non-zero parameter, and this is what we observe here. At the same time, it is remarkable that letting cosmological constant run does not lead to dramatic changes in the best fit for other parameters, such as DM relative density $\Omega_{dm}^0$ and the warmness $b$.

### 4.3 Matter power spectrum

The matter power spectrum at $z = 0$ is given by

$$P(k) = |\delta_m(k)|^2 = A k^2 \left( \frac{\bar{g}(\Omega)}{\bar{g}(\Omega^0_m)} \right)^2,$$

where $A$ is a normalization constant of the spectrum. This constant can be fixed from the spectrum of anisotropy of the CMB radiation and

$$\bar{g}(\Omega) = \frac{5\Omega}{2 \left[ \Omega^{4/7} + 1.01(\Omega/2 + 1) - 0.75 \right]},$$

Here we use the Bardeen-Bond-Kaiser-Szalay (BBKS) transfer function [34]

$$T(k) = \frac{\ln (1 + 2.34q)}{2.34q} \left(1 + 3.89q + 16.1q^2 + 5.64q^3 + 6.71q^4\right)^{-1/4},$$

where

$$q(k) = \frac{k}{h\Gamma \text{Mpc}^{-1}} \quad \text{and} \quad \Gamma = \frac{\Omega^0_m h}{\Omega^0_{dm}} \exp \left\{ -\Omega^0_b - \frac{\Omega^0_b}{\Omega^0_{dm}} \right\}.$$
Figure 5: Observational constraints for our three free parameters $\nu$, $b$ and $\Omega^0_{dm}$, for 1$\sigma$ and 2$\sigma$ levels. Here we have used CMB+BAO+SNIa combined data. The marked points are given by $(\Omega^0_{dm}, b) = (0.365962, 4.66025 \times 10^{-5})$, $(\Omega^0_{dm}, \nu) = (0.365962, 4.3078 \times 10^{-6})$ and $(\nu, b) = (4.3078 \times 10^{-6}, 4.66025 \times 10^{-5})$, respectively, in correspondence with best fit values presented above in Table 1.

to construct a set of initial conditions for the system of equations (31), (32), (35)-(37).

In Fig. 4.3 we compare the data from 2dFGRS survey [35] with the matter power spectrum of our model for the energy balance obtained by the best fit of $\chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{SNIa}$ (see Table 1). Compared to more recent surveys (see e.g. [36]), the 2dFGRG data present the advantage of being less contaminated by the standard model used in the calibration.
5 Conclusions

We have implemented the model for the running of the cosmological constant in a early stage of the universe, where the dark matter sector is modeled using the reduced relativistic gas model. At the background level, we proposed and solved our model analytically, considering the interaction between vacuum energy and baryons. The effective equation of state parameter evolves as expected and we find the best fit with respect to the standard model, once the constrained values using CMB+BAO+SNIa combined data are obtained. Additionally, this effective parameter goes to radiation value faster than in the standard model for large redshift $z$. The first CMB peak and the CMB+BAO+SNIa combined data are used for constraining our free parameters, where we met the best correspondence with observations for the latter case.

On the other hand, when perturbations are considered and in order to compute the matter power spectrum, the system of equations for the geometric perturbation, density contrasts, and velocities are found and solved numerically. We compared our results with the ones given by the 2dFRG data, obtaining a better correspondence for small $k$, in
contrast to the standard ΛCDM model.

Our results suggest that a primordial running of the cosmological constant and the possible creation of baryons at this early stage from vacuum energy cannot be ruled out and deserves more attention in the possible future work.

The model which we developed here explores the possibility that the cosmological term decays into the baryonic component in the early universe, when the running of the cosmological constant and the intensity of the gravitational field are sufficiently strong and, on the other hand, baryons can be regarded as ultra-relativistic particles. The parameter $\nu \neq 0$ indicates a non-constant cosmological term and the parameter $b$ parameterizes the warmness of the matter component.

The comparison with observations points to a small deviation from the ΛCDM model as the preferred scenario, even though the strict ΛCDM case, given by $\nu = 0$, is not excluded. It must be remembered also that the running of the cosmological term implies a new free parameter with respect to the standard cosmological model and, therefore, the results can not be interpreted such that the statistical analysis proves that the cosmological constant runs. Furthermore, the warmness of the dark matter component $b \neq 0$ is allowed, with a present-day average speed of the corresponding particles (or indefinite origin, as usual) of the order of $10^{-5}c$.

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