Dual-comb spectroscopy is a rapidly developing technique enabling ultraprecise broadband optical diagnostics of atoms and molecules. This powerful tool typically requires two phase-locked femtosecond lasers, yet it has been shown that it can be realized without any stabilization if the combs are generated from a single laser cavity. Still, unavoidable intrinsic relative phase fluctuations always set a limit on the precision of any spectroscopic measurements, hitherto limiting the applicability of bulk dual-comb lasers for mode-resolved studies. Here, a versatile concept for low-noise dual-comb generation from a single-cavity femtosecond solid-state laser based on intrinsic polarization-multiplexing inside an optically anisotropic gain crystal is demonstrated. Due to intracavity spatial separation of the orthogonally polarized beams, two sub-100 fs pulse trains are simultaneously generated from a 1.05 μm Yb:CNGS (calcium niobium gallium silicate) oscillator with a repetition rate difference of 4.7 kHz. The laser exhibits the lowest relative noise ever demonstrated for a bulk dual-comb source, supporting free-running mode-resolved spectroscopic measurements over a second. Moreover, the developed dual-comb generation technique can be applied to any solid-state laser exploiting a birefringent active crystal, paving the way toward a new class of highly coherent, single-cavity, dual-comb laser sources operating in various spectral regions.

1. Introduction

Dual-comb spectroscopy (DCS) has played a pivotal role in modern optical metrology by making it possible to acquire simultaneously broadband and high-resolution optical spectra at unprecedentedly high speeds.[1,2] Unlike Fourier-transform spectroscopy, DCS does not require any moving parts for interferometry, while in contrast to single-frequency diode lasers conventionally used in solid-state spectrometers, the obtainable spectral coverage in DCS routinely reaches multiple terahertz. The concept of DCS relies on an interaction between two lasers with a unique spectral pattern of narrow, equidistant and mutually locked lines known as optical frequency combs (OFCs). A mismatch of their line spacings enables multi-heterodyne beating on a square-law photodetector resulting in a transfer of the optical comb structure to the radio-frequency (RF) domain. As in the case of all important scientific advancements, dual-comb laser sources are not only used in a single field of spectroscopy but also they have been found to be particularly useful in several other applications including ranging,[3] communication,[4] or microscopy.[5] Although frequency combs can be generated from quantum cascade lasers (QCL)[6] or from single continuous-wave (CW) laser through dual electro-optical modulation[7] and nonlinear Kerr interaction inside a pair of microresonators,[8] arguably the most conventional DCS implementation uses a pair of spectrally overlapping, yet independently running mode-locked lasers. Nevertheless, attempts to perform precise free-running DCS measurements in this configuration without any synchronization loops remained largely unsuccessful. This relates to uncorrelated fluctuations of the combs’ line spacings ($f_{\text{rep}}$) and their starting points defined by a carrier-envelope-offset frequency ($f_{\text{ceo}}$), which induce frequency instabilities of the DCS beating signal referred to as the interferogram (IFG). A loss of the discrete character of the RF comb in many cases makes precise DCS measurements impossible. Frequency instabilities are inherent to all lasers, yet the IFG phase noise can be strongly reduced via active phase-locking, which allows one to achieve long-term coherence sufficient for spectroscopic measurements.[9,10] Although an intermediate solution...
has been recently demonstrated with only a single longitudinal mode of a dual-comb QCL system being stabilized.\cite{11} The traditional full phase-locking requires simultaneous synchronization of $\Delta f_{rep}$ and $\Delta f_{ceo}$ between two lasers, which, despite several years since its first demonstration, still appears to be a challenging task.

An alternative concept stems from the idea to generate two mode-locked pulse trains with different repetition rates from a single laser cavity. The strength of this technique lies in the intrinsic common-mode nature of phase fluctuations for the two combs, which renders their high mutual coherence and virtually eliminates the need for phase-locking electronics. Because the beams share the same cavity, most of the relative phase noise originating from thermal or mechanical perturbations cancels out. Consequently, as long as the uncorrelated phase excursions between the two OFCs remain minor, the combs exhibit sufficient relative stability to enable free-running DCS over second time scales.

Historically, the first single-cavity dual-comb mode-locked laser was a 1.55 µm fiber oscillator operating in a bidirectional configuration.\cite{12,13} Subsequently, similar fiber sources have been demonstrated in various spectral ranges with dual-comb generation based on polarization-multiplexing\cite{14–16} or multiwavelength emission.\cite{17–20} Despite these advances, the main drawback of soliton mode-locked lasers based on all-fiber technology is the limitation of their output power due to excessive nonlinearities, which typically reduces the achievable peak power below the kilowatt level. This in turn prevents their efficient nonlinear frequency conversion to other spectral ranges (e.g., mid-IR), where many important chemical species have their strongest absorption features. Although it has been recently shown that the pulse energy can be enhanced by operating a fiber laser in the normal dispersion regime,\cite{21} it always comes at the expense of chirping the pulse to the picosecond level. The power limitation can be in principle mitigated by exploiting solid-state lasers, often with superior noise properties over their fiberized counterparts.\cite{21} To date, however, only few examples of bulk dual-comb lasers can be found in the literature.

Although dual-wavelength (time-synchronized) Ti:sapphire femtosecond oscillators were demonstrated already in the early 1990s,\cite{23} the first single-cavity dual-comb bulk laser was reported more than 20 years later in 2015. A semiconductor mode-locked integrated external-cavity surface emitting laser (MIXSEL) generated 18 ps long pulses at $\approx$0.97 µm.\cite{24} The simultaneous emission of two mutually coherent combs relied on the presence of an additional birefringent medium in the laser cavity. A pair of orthogonally polarized beams experienced a double refraction inside the calcium carbonate crystal, which resulted in their spatial separation and hence different intracavity path lengths. The same concept was later also applied to a mode-locked picosecond Nd:YAG laser\cite{25} and very recently an Yb:CaF$_2$ oscillator generating sub-200 fs pulses.\cite{26} From an application perspective, MIXSEL dual-comb lasers have already proven their usefulness in molecular spectroscopy of gaseous acetylene at 1.03 µm.\cite{27} The concept exploited spatial separation of two beams enabled by non-common cavity end mirrors\cite{31,32} and/or polarization-splitting.\cite{33} The unprecedented output power levels of several watts achieved for pulse durations on the order of 300 fs promise effective nonlinear frequency conversion of the dual-comb structure toward the mid-IR or UV domains. Nevertheless, neither of these two systems can be termed as single cavity, because in both cases the intracavity beams did not share all the resonator mirrors, which is expected to compromise the mutual coherence between the generated OFCs. Finally, free-running dual-comb laser systems have also been realized in QCL technology, permitting a direct access to long wavelength radiation.\cite{34}

In this paper, we present a novel concept for dual-comb generation from a single-cavity mode-locked solid-state laser based on an optically anisotropic gain crystal. It relies on simultaneous generation of two spatially separated beams and it builds on the idea of polarization-multiplexing originating from the double refraction phenomenon described in ref. \cite{24}. In contrast to the latter work, our scheme does not require any additional components to be added to a standard laser cavity, as it is based on inherent birefringence of the used gain medium. If the oscillating laser beam does not propagate along the optical axis of a uniaxial active crystal, its path will vary depending if it is polarized normal to the principal plane (an ordinary (o)-ray) or in the principal plane (extraordinary (e)-ray). However, if the propagation is also not at 90° to the optical axis, then the two eigen-polarizations will experience a spatial offset. It is possible to achieve simultaneous lasing at both polarizations, if the corresponding rays both overlap with the pump beam inside the active medium. Ideally, this can be realized if the angles of incidence (AOI) on the crystal are different for the o/e-beams, so that the birefringent spatial walk-off is compensated and both Poynting vectors overlap being collinear with the pump. Consequently, the orthogonally polarized laser beams will follow different intracavity optical paths (providing nonzero $\Delta f_{rep}$ in the mode-locked regime), yet they will stay spatially combined inside the gain medium due to the double refraction phenomenon. Note, that the feasibility to control the relative overlap of the beams permits to counterbalance the possible anisotropy of the gain in a birefringent active crystal. Due to the simple single-cavity design with minimized number of components and a single pump source, one can use the presented technique to achieve remarkably low-noise dual-comb generation.

We demonstrate an experimental realization of the described concept in an Yb:Ca$_3$NbGa$_3$Si$_4$O$_{14}$ (calcium niobium gallium sili- cate: Yb:CNGS) mode-locked oscillator. The laser simultaneously emits two spatially separated pulse trains at 1.05 µm with a repetition rate difference of 4.73 kHz around 78.3 MHz. The pulse durations of the combs amount to 88 and 93 fs, making this source...
the first sub-100 fs dual-comb single-cavity ytterbium laser. The performed characterization of the mutual coherence between the combs indicates sub-kilohertz relative linewidth over ten IFG periods, which permits free-running mode-resolved DCS over several milliseconds simultaneously supporting further enhancement of the measurement precision to second timescales by computational phase correction.

2. Experimental Section

The experimental setup of our dual-comb spectrometer is shown in Figure 1. The core of the system is the mode-locked Yb:CNGS laser with a cavity design based on our previous works.\cite{35,36} As a gain medium we used an antireflection (AR)-coated 3 at% Yb:CNGS crystal with a length of 4 mm and an aperture of 3 × 3 mm². It was pumped by a semiconductor laser diode delivering up to 1 W of 979.4 nm radiation at a maximum driving current of 1.7 A (3SP Technologies). The polarization-maintaining (PM) single-mode-fiber-coupled diode provided diffraction-limited pump beam quality and exhibited intrinsically low-noise operation compared to powerful multimode diodes.\cite{37,38} The radiation from the diode was reimaged inside the gain medium (see picture in Figure 1*), they also experience different reflection angles from the concave M₁,₂ mirrors, which enables to align the cavity to be stable for both polarizations in the same time. Due to non-common intracavity paths and crystal dispersion, the mode-locked laser pulse trains corresponding to the o/ε-beams exhibit different repetition rates, which is a prerequisite for DCS. Note that this simple polarization-multiplexing concept relies exclusively on the gain medium birefringence, and it does not require any additional elements to be introduced. Moreover, all the cavity components together with the pump source are common for both beams, enabling one to achieve superior dual-comb coherence compared with systems sharing only a part of the laser resonator.\cite{31–33} While in the described setup Δf_{rep} is
fixed, it can be easily tuned, e.g., by introducing an optical wedge in the intracavity path of only one of the laser beams.

In the laboratory routine, we initially aligned the cavity to achieve lasing of the e-beam, which was collinear with the pump and shared its polarization state. Lasing at orthogonal polarization (e-beam) was initiated by minor fine-tuning of the SESAM and OC horizontal tilt which simultaneously served to control the energy balance between the two beams. The latter is essential for simultaneous dual-polarization output as it allows to balance the gain anisotropy of the uniaxial Yb:CNGS. A subsequent mechanical perturbation of the SESAM initiated stable mode-locked dual-comb operation. The average output power of each beam was equal to 30 mW at an incident pump power of ≈700 mW. Figure 2a shows the optical spectra measured with an optical spectrum analyzer (OSA; AQ6370, Yokogawa) and the corresponding second-harmonic intensity autocorrelation traces (APE PulseCheck). The emission spectrum of the e(o)-beam was centered at 1051 nm (1057 nm) with a 13.8 nm (13.4 nm) full-width-at-half-maximum (FWHM) bandwidth. The corresponding pulse duration amounted to 93 fs (88 fs), which results in a time-bandwidth product of 0.35 (0.32). Figure 2b presents the RF spectrum of the fundamental beat notes separated by Δf_rep = 4.73 kHz and centered around 78.3 MHz (recorded with FSW43, R&S). The additional low intensity peaks are artifacts originating from intermodulation of the f_rep beat notes in the photodetector. The insets show clean 1-GHz-wide RF spectra (RBW = 1 kHz) of the individual beams.

3. Characterization of the Free-Running Dual-Comb Laser

To prove the high mutual coherence between the generated combs and evaluate the feasibility of performing demanding mode-resolved dual-comb measurements, we characterized the laser with respect to timing jitter and relative frequency fluctuations. We will start the discussion with the repetition rate stability analysis in the short- and long-term regime, followed by a focus on relative frequency fluctuations.

3.1. Repetition Rate Noise Characteristics

To characterize the repetition rate noise characteristics of the two combs, we used a commercial phase noise analyzer (FSWP8, R&S). Both polarizations were measured independently in a frequency range from 1 Hz to 1 MHz. As shown in Figure 3, the system shows excellent phase noise performance. At an offset of 100 Hz from the carrier located at ≈78.3 MHz the phase noise spectral density amounts to ≈100 dBc Hz⁻¹. Figure 3b shows the corresponding integrated timing jitter, which amounts to 14.8 and 37.2 ps for the e- and o-beam, respectively. The minor difference between their jitter can be explained by slight differences of the beams overlap with the pump, which was necessary to compensate for the gain anisotropy experienced by the two orthogonally polarized signals.
When compared with other reports on dual-comb lasers including this kind of characterization,\textsuperscript{24,33} the source presented in this work clearly exhibits superior stability. This can be mainly attributed to the different pump source used. In contrast to powerful but spatially multimode diodes used in above-mentioned reports, we used a single-mode laser diode with intrinsically low-noise operation, which further transfers to the phase fluctuations of the oscillator.\textsuperscript{37,38}

To demonstrate the feasibility of prolonged dual-comb lasing and characterize its long-term stability, we recorded a spectrogram of the 100th harmonic of the repetition rates with a temporal resolution of 5 s, and a resolution bandwidth of 50 Hz. The spectra were next interpolated 16 times for accurate peak frequency estimation, and the harmonic frequency axis was normalized to the fundamental. Figure 4 plots the peak frequencies retrieved from the spectrogram recorded over an hour. Although the repetition rates drift during the measurement by \( \approx 275 \) Hz, the difference frequency oscillates by at most 3 Hz with 0.52 Hz standard deviation.

### 3.2. Relative Linewidth Measurement

Our observation that the repetition rates precisely follow each other down to a sub-hertz level over an hour timescale does not necessarily imply suitability for mode-resolved free-running dual-comb spectroscopy. This is because the repetition rate characterization quantifies only the timing noise, while completely ignoring relative offset frequency fluctuations \( \Delta f_{\text{cor}}(t) \). From a practical standpoint, however, it is easier to measure the combined effect of the two unstable frequencies in an experiment with a CW single-frequency laser, which is an established technique for characterizing single-cavity dual-comb lasers.\textsuperscript{12,13} We heterodyned our dual-comb laser with a 1063.6 nm single-frequency laser (D-C-1060, PicoQuant) and recorded the beating signal over 1 s. While this technique allows only for coarse estimation of the combs’ optical linewidths (absolute stability), it is perfect for relative coherence assessments. This is because two beat notes \((f_{b1}, f_{b2})\) are produced as a result of beating between the CW laser and one tooth from each comb. When both are simultaneously recorded, their instantaneous difference frequency can be conveniently accessed and characterized.

Figure 5a plots the 1-s-long spectrogram of the two beat notes \((f_{b1}, f_{b2})\) with 30 µs temporal resolution (50 kHz RBW). One can observe frequency fluctuations with \( \approx 4.1 \) MHz peak-to-peak deviations, which correspond to an upper bound of the optical linewidth of \( \approx 3.1 \) MHz assuming uncorrelated frequency noise between the combs and the CW laser. More importantly, the instantaneous frequency of the two beat notes is visually similar (see zoomed panel), which is expected for high relative coherence. Using a digital difference frequency generation (DDFG) routine,\textsuperscript{40} we obtained the relative frequency fluctuations of the signal and characterized it in the frequency domain with a resolution bandwidth of 710 Hz, which corresponds to ten repetition rate difference frequency periods (2.1 ms). This analysis follows techniques described in detail in our previous report.\textsuperscript{16} The results of the relative coherence analysis are plotted in Figure 5b–f. First, despite the 4 MHz-wide fluctuations of the CW laser beat notes, the difference frequency (Figure 5b) remains almost constant with a 30 kHz drift in total. This indicates a high degree of coherence between the two combs, as the drift within 1 s corresponds to a shift in DCS line position by less than seven comb teeth (\(<500\) MHz of optical resolution).

Fortunately, this number can be greatly improved if one uses frequency-tracking computational DCS phase correction algorithms developed recently.\textsuperscript{40–42} A prerequisite for such algorithms to enable mode-resolved dual-comb measurements is the Nyquist criterion: the relative frequency between the two combs cannot fluctuate by more than half of the repetition rate difference within the duration of a single dual-comb IFG.\textsuperscript{43} In other words, as long as frequency-noisy dual-comb lines drift by less than half the spacing between the lines from IFG shot to shot, they can be digitally shifted back to
their original position. Unfortunately, larger frequency excursions cannot be tracked because they become aliased—the frequency drift is much higher than accurately measurable during a single IFG period. Although an attempt to apply digital phase correction to such noisy systems will improve the look of the dual-comb spectrum and restore its discrete character with sharp lines, optical sample information will leak from a single comb tooth into its neighbors, thus making spectroscopic data inaccurate. This is because the digital algorithm will be blind to drifts exceeding multiple line spacings (shot-to-shot) and will simply lock the position of the drifted lines closest to those already averaged. In practice, a maximally shared laser cavity design along with low-noise pump sources renders relative linewidths compatible with the correctability criterion. Even the most demanding free-running DCS are possible, where a drift by more than one line spacing would have severely affected absorption lineshapes\cite{16,17}. Still, one has to keep in mind that the resolution in such systems always remains limited by the comb optical linewidth.

In our system, the required shot-to-shot relative stability for mode-resolved DCS must be lower than \(\approx 2.3 \text{ kHz} \) over \(1/4.73 \text{ kHz} = 211 \mu\text{s} \) time scale. To characterize it, we measured the FWHM of the relative beat note for all time instances of the data from Figure 5b. Overall, the system shows relative beat notes as narrow as \(\approx 940 \text{ Hz} \) over ten periods of the IFG equal to 2.1 ms (Figure 5c). Nevertheless, rather than focusing on the best case, we performed a statistical linewidth analysis as depicted in Figure 5d,e. The mode of the distribution is 1.2 kHz, while the mean is 2.4 kHz. The empirical cumulative distribution function (Figure 5e) indicates that linewidths narrower than 5 kHz are obtained with 95% probability. Given that this level of fluctuations occurs over timescales \(10\times \) longer than needed for mode-resolved operation, we are confident that the shot-to-shot stability fully complies with requirements for computational frequency tracking. It should be also noted that without computational assistance, the system will be still compatible with mode-resolved operation up to 5 ms, as concluded from the global minimum of the difference frequency Allan deviation (Figure 5f).

4. Dual-Comb Spectroscopy

To validate the results of our stability analysis and prove the spectroscopic capabilities of the dual-comb source, we performed a free-running DCS experiment. Rather than acquiring the DCS IFG with an oscilloscope, we utilized a 16-bit quadrature demodulator (FSW43, R&S), recording complex IFG data centered around 19 MHz for 1 s. As a test medium, we measured a 1-mm thick fused-silica etalon with a dielectric coating increasing its reflectance around 1 µm. The sample was inserted into one of the dual-comb spectrometer arms (c; Figure 1). For spectral normalization purposes, an analogous DCS signal was recorded without the etalon referred to as a reference measurement. Both signals were computationally phase-corrected and coherently averaged. Before proceeding to the recovered transmittance spectra, we focus first on the characteristics of the dual-comb signals shown in Figure 6.

As expected from the stability analysis, the high mutual coherence allows us to perform tooth-resolved measurements over \(\approx 5 \text{ ms} \) with \(\approx 10 \text{ dB} \) carrier-to-noise ratio and percentage
Figure 6. Dual-comb signal analysis. a) Raw and computationally phase-corrected dual-comb spectra. Zoomed panels show a comparison of the corrected spectrum (red) with original unprocessed 1-s-long data (gray) and cropped 5-ms-long acquisition (blue). After the computational correction, the RF comb linewidth reaches an acquisition-time-limited linewidth (anywhere in the spectrum, not only around the center). Without digital enhancement, the system is capable of mode-resolved measurements over 5 ms with ≈10 dB carrier-to-noise ratio. b) Coherently averaged 4742 dual-comb IFGs (without etalon) plotted along with a single-shot unprocessed trace. The traces are normalized to 1 V. The averaging gain reaches 99.4% of the theoretical limit (36.76 dB). The arrows indicate spurious signals originating from nonlinear (NL) coupling between the combs and from intracavity reflections at the gain crystal faces. c) Amplitude Allan deviation analysis of the beat notes located close (0 MHz), and far-away (10 MHz) from the carrier frequency of 19 MHz.

precision. Even though over extended time scales exceeding 1 s (Figure 6a), the DCS lines lose their discrete character, the sufficient mutual coherence between the combs permits to faithfully track the phase of the IFG and its time-varying duration. We have already demonstrated the phase correction algorithm in demanding high-resolution Doppler-limited spectroscopy applications,[16] and here it is also used to unlock the full potential of the system. After digital correction, a boost in the spectral SNR exceeds 36 dB, which agrees well with expectations from the number of averaged frames equal to 4742. The dual-comb spectral linewidth is limited by the acquisition time to ≈1 Hz. Nevertheless, it should be noted that the algorithm corrects only for RF signal fluctuations, while leaving optical frequency fluctuations intact. In other words, the obtainable resolution remains limited by the optical linewidth of the comb on the order of megahertz. The obtainable precision with the system in 1 s of averaging characterized using Allan-Werle deviation analysis[44] is 0.2% for lines around the carrier frequency (strongest), degrading to 1.2% for lines 10 MHz away from the carrier (Figure 6c). The corresponding spectroscopic SNRs are 500 and 83, respectively.

Previous studies have indicated that the generation of dual-comb radiation from a single laser cavity can be accompanied by strong coupling between the generated beams, which will manifest itself in the presence of additional spurious signals in the dual-comb beating.[20,26] This effect is clearly undesirable as it leads to modulation of the spectral envelope and potential spectroscopic baseline issues. Figure 6b shows a comparison of the raw (single-shot) and coherently averaged IFGs. Because the original data are complex and carrier-free (due to complex demodulation), for visualization purposes we frequency-shifted the complex signals to show analogous baseband real IFGs. Raw single-shot (uncorrected) IFGs are spurious-free and already show a relatively high dynamic range of 52 dB, which improves to 89 dB upon averaging. Therefore, the difference between the two traces becomes noticeable only at high vertical scale magnification. Prolonged averaging reveals the existence of very weak bursts (all with power below −60 dB relative to the main peak) surrounding the IFG center-burst. Most of the spurious signals can be clearly attributed to parasitic reflections from the gain crystal faces with imperfect AR-coatings. The only spurious signal that cannot be associated with intracavity reflections appears at approx. 92 µs (Figure 6b) and can indeed be a sign of extremely weak nonlinear coupling between the two combs. Note that the relative amplitude of this peak is below −70 dBc. We believe that the advantageous absence of any significant cross-coupling could have been achieved due to the orthogonal polarizations of the two beams and their short interaction length within the crystal. Moreover, as already mentioned, spatial separation of the beams on the saturable absorber suppresses any additional nonlinear cross-coupling.[26] More details about the presence of the parasitic signals can be found in Section S1.
In the final step, we compared the digitally corrected dual-comb spectra (etalon and reference) with a simultaneous measurement using an optical spectrum analyzer (OSA, AQ6370B Yokogawa), which was used for optical frequency axis calibration. Unlike in previous demonstrations, we performed a measurement in both amplitude and phase. The latter was possible by forcing the carrier phase of the two measurements to be identical. Consequently, the measurement may in general possess a constant phase offset compared to fully CEO-phase-stabilized systems. Fortunately, for many applications involving characterization of optically thin samples, such dispersion spectra are still practically useful. Overall, an excellent agreement is obtained between the DCS and OSA measurement plotted in Figure 7. Slight deviations between the curves occur around the peaks, which may be related to photodetector nonlinearities in DCS and the finite resolution of the OSA (0.01 nm).

As expected from the Allan analysis, the spectroscopic precision in the central part of the spectrum is higher than on the wings, which directly relates to a steep beat note power roll-off above 10 MHz from the spectral center. The etalon effect visible in the transmission and dispersion spectra stems from the aforementioned imperfections of the crystal AR-coatings, as no interferogram windowing was used to improve the spectroscopic performance at the expense of loss in resolution. The DCS spectral coverage reaches $\approx 500$ GHz ($\approx 1.8$ nm around 1052.8 nm), which is merely a limitation of the non-aliased RF bandwidth. The used grating-based tunable optical filter allowed for selection of an arbitrary center wavelength on demand within the available spectral range without reducing the dual-comb coherence.

**5. Conclusions**

We have demonstrated a novel scheme for dual-comb generation from a single-cavity mode-locked solid-state laser. It is based on intrinsic polarization-multiplexing originating from double refraction inside a birefringent gain medium and hence does not require any additional components to be introduced to a standard laser resonator. Despite its simplicity, all the cavity components are common for both beams, enabling to achieve high relative coherence of the free-running combs, a key aspect for DCS.

The proposed concept was validated in a Yb:CNGS laser generating two orthogonally polarized sub-100 fs pulse trains at the central emission wavelength of 1.05 µm with repetition rate offset of 4.73 kHz (at 78.3 MHz). Note that the femtosecond pulse duration is crucial regarding potential transfer of the generated dual-comb radiation to other spectral regions via nonlinear frequency conversion. The developed source was precisely characterized for timing jitter and relative frequency fluctuations. A detailed investigation revealed unprecedented stability with sub-kilohertz relative linewidth over 10 IFG periods (2.1 ms) permitting mode-resolved measurements over several milliseconds. Moreover, the measurement time can be greatly extended by using computational phase correction exploiting the high mutual coherence between the combs. We unveiled the potential of the developed system by presenting mode-resolved transmission and dispersion measurements of a low finesse etalon with permille amplitude precision achievable within 1 second. To the best of our knowledge, our source exhibits the lowest relative noise ever demonstrated for a dual-comb solid-state laser, which may be attributed to its simple single-cavity design and the stability of the used pump source.

What seems particularly important regarding spectroscopic applications is that our straightforward concept can be in...
principle applied to any bulk laser based on an optically anisotropic gain crystal. Consequently, it can be exploited to realize dual-comb laser sources operating in the mid-IR spectral band, a region containing the majority of characteristic ro-vibrational molecular transitions. This can be achieved by developing high-power, femtosecond 2 µm lasers based on Tm3+,[45] Ho1+,[46] or Cr2+ doped[47,48] crystals and its subsequent nonlinear conversion to the longer wavelength range[49] or by direct dual-comb generation from Fe2+ lasers up to 6 µm.[47,50] Consequently, the reported technique paves the way for a new class of single-cavity dual-comb lasers, which, due to their low-noise free-running operation, will enable spectroscopic measurements without any active stabilization in the near future.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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dual-comb lasers, dual-comb spectroscopy, frequency combs, mode-locked lasers

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