Conformal invariance of massless
Duffin–Kemmer–Petiau theory in Riemannian
spacetimes

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Abstract

We investigate the conformal invariance of massless Duffin–Kemmer–Petiau theory coupled to Riemannian spacetimes. We show that, as usual, in the minimal coupling procedure only the spin 1 sector of the theory—which corresponds to the electromagnetic field—is conformally invariant. We also show that the conformal invariance of the spin 0 sector can be naturally achieved by introducing a compensating term in the Lagrangian. Such a procedure—besides not modifying the spin 1 sector—leads to the well-known conformal coupling between the scalar curvature and the massless Klein–Gordon–Fock field. Going beyond the Riemannian spacetimes, we briefly discuss the effects of a nonvanishing torsion in the scalar case.

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1. Introduction

Conformal symmetry was first introduced into physics in the beginning of the last century when Cunningham and Bateman [1] showed that Maxwell’s vacuum equations are also invariant under the larger conformal group, which proves to be the maximal extension of the Poincaré group that leaves the light-cone invariant. A particular feature of the conformal group is that it extends the usual Lorentz group in allowing transformations to frames undergoing constant acceleration. Some aspects of conformal invariance on curved spacetime were studied in [2].

The conformal symmetry of a quantum field theory was intensively studied at the end of 1960s and the 1970s, with the motivation coming independently from the Bjorken scaling
hypothesis [3] and the theory of second-order phase transitions [4]. It turns out that conformal symmetry of a two-dimensional quantum field theory implies the existence of infinitely many conservation laws and strong constraints on the correlation functions. In particular, spectacular results were obtained in statistical mechanical models in two dimensions [5]. The subject of two-dimensional conformal field theories (CFT) has become very fashionable due to its connections with string theory and quantum gravity [6, 7], and it has grown into a separate branch of mathematical physics with its own methods and language [8]. In some four-dimensional theories with supersymmetry, the $\beta$-function vanishes (to lowest order or to all orders) and some features reminiscent of two-dimensional CFT emerge. This subject attracted a lot of interest, enhanced in recent years in connection with the conjecture about AdS/CFT correspondence (see [9] for a review).

The Duffin–Kemmer–Petiau (DKP) theory [10] provides a unified description of scalar and vector fields through a first-order equation, similar to Dirac’s one. Notwithstanding the complete equivalence between the alternative treatments based on DKP and Klein–Gordon–Fock (KGF) and Proca equations in the free-field case, such equivalence does not generally hold when interactions are taken into account [11]. For instance, the massive DKP theory is richer than KGF and Proca theories when minimally coupled [12, 13] to a Riemann–Cartan spacetime [14, 15]. On the other hand, the Harish-Chandra theory for massless DKP fields in Minkowski spacetime [16] was extended in [17, 18] to curved spacetimes via minimal coupling procedure and it proved the complete equivalence between this theory and the KGF and Maxwell ones, both in Riemannian and in Riemann–Cartan spacetimes.

Our aim in this work is to study the properties of the massless DKP field in Riemannian spacetimes under conformal transformations. We start from actions which are invariant under general coordinate transformations and, as is well known, in this case the conformal invariance of the theory can be inferred from its invariance under the so-called Weyl rescalings [7, 19]. The paper is organized as follows. In section 2, we briefly review the massless DKP theory minimally coupled to Riemannian spacetimes, quoting the main results we shall need. In section 3, we set the transformation laws for DKP fields under Weyl transformations and show that only the spin 1 sector of this theory is conformally invariant. We achieve the conformal invariance also of the spin 0 sector through the introduction of a compensating term—which by its turn does not modify the spin 1 sector—in the Lagrangian, thus recovering in a natural way the usual (non-minimal) conformal coupling between the scalar massless KGF field and the scalar curvature. At the end of this section we go beyond the Riemannian spacetimes and briefly discuss the effects of a nonvanishing torsion on the scalar sector. Our conclusions are presented in section 4.

2. Brief review of massless DKP theory

We start with the Lagrangian density for the massless DKP fields in Minkowski spacetime [16, 18]

$$\mathcal{L}_M = i \bar{\psi} \gamma \beta^a \partial_a \psi - i \partial_a \bar{\psi} \beta^a \gamma \psi - \bar{\psi} \gamma \psi,$$

where $\bar{\psi} = \psi^\dagger \eta^0$, $\eta^0 = 2(\beta^0)^2 - 1$, and $\gamma$ and $\beta^a$ are matrices satisfying the massless DKP algebra

$$\beta^a \beta^b \beta^c + \beta^c \beta^b \beta^a = \beta^a \eta^{bc} + \beta^c \eta^{ba},$$

Throughout the text we adopt the convention that Latin indices refer to the Minkowski spacetime with (constant) metric $\eta^{\mu\nu} = (+1, -1, -1, -1)$, while Greek indices refer to a Riemannian spacetime with metric $g^{\mu\nu}(x)$.

We choose a representation in which $\beta^0 = 0$, $\beta^1 = -\beta^3$ and $\gamma^1 = \gamma$. 

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The resulting equation of motion is
\[ i\beta^a \partial_a \psi - \gamma \psi = 0. \] (3)

In fact, equations (1)–(3) describe a set of four free massless gauge fields [16, 20], among them the massless Klein–Gordon–Fock and the Maxwell electromagnetic fields. The remaining two fields are of topological nature and will not be considered here. The theory presented so far is manifestly covariant under Lorentz transformations.

The above equations can be generalized to Riemannian spacetimes through the formalism of tetrads (also called vierbeins), together with the minimal coupling procedure. Here we shall simply quote the main results we shall need; for details we refer to [17, 18] and references therein. In this formalism we consider a Riemannian spacetime \( \mathcal{R} \) with metric \( g_{\mu \nu} \), whose point coordinates are labelled \( x^\mu \). To each point in \( \mathcal{R} \) we consider a tangent Minkowski spacetime \( \mathcal{M} \) with (constant) metric \( \eta_{ab} \), whose point coordinates are labelled \( x^a \). The DKP fields \( \psi \) are Lorentz group representations in this Minkowski space. The projections into \( \mathcal{R} \) of all tensor quantities defined on \( \mathcal{M} \) are done via the tetrad fields \( e^{\mu a}(x) \). These fields and inverses \( E_\mu^a(x) \) satisfy the relations
\[ g_{\mu \nu}(x) = \eta_{ab} e_\mu^a(x) e_\nu^b(x), \] (4a)
\[ g_{\mu \nu}(x) = \eta_{ab} E_{\mu}^a(x) E_{\nu}^b(x), \] (4b)
\[ E_{\mu}^a(x) = g_{\mu \nu}(x) \eta^{ab} e_\nu^b(x), \] (4c)
and
\[ e = \det (E_{\mu}^a) = \sqrt{-g}, \]
where \( g = \det(g_{\mu \nu}) \).

The resulting Lagrangian density for massless DKP fields minimally coupled to \( \mathcal{R} \) is
\[ \mathcal{L}_R = e(i\overline{\psi} \gamma \beta^a e^e_{\mu a} \nabla_{\mu} \psi - i e^{\mu a} \nabla_{\mu} \overline{\psi} \beta^a \gamma \psi - \overline{\psi} \gamma \psi), \] (5)
where \( \nabla_{\mu} \) is the Riemannian covariant derivative associated with the symmetric connection (Christoffel symbol) \( \Gamma^\alpha_{\mu \nu} \). The covariant derivatives of DKP fields are
\[ \nabla_{\mu} \psi = \partial_\mu \psi + \frac{1}{2} \omega_{\mu a b} S^{ab} \psi, \] (6a)
\[ \nabla_{\mu} \overline{\psi} = \partial_\mu \overline{\psi} - \frac{1}{2} \omega_{\mu a b} \overline{\psi} S^{ab}, \] (6b)
where \( S^{ab} = [\beta^{a \nu}, \beta^{b \nu}] \) and \( \omega_{\mu a b} \) is the spin connection, which in Riemannian spacetimes is related to the metric and vierbeins in the following way [21]:
\[ \omega^{ab}_\mu = e^{ib} \left\{ \frac{1}{2} E_{\rho}^{\ a} S^{cd} (\partial_\mu g_{cd} + \partial_c g_{d \mu} - \partial_d g_{c \mu}) - \partial_\rho E^a_{\mu} \right\}. \] (7)
The equation of motion for massless DKP fields in \( \mathcal{R} \) follows from Lagrangian (5)
\[ i\beta^a e_\mu^a \nabla_{\mu} \psi - \gamma \psi = 0. \] (8)

We recall that (5) and (8) are manifestly covariant under Lorentz transformations on the tangent Minkowski spacetime and also under general coordinate transformations on the underlying Riemannian spacetime [22].
2.1. Spin 0 sector

The scalar sector of massless DKP theory can be explicitly worked out by using a five-dimensional representation (see [18]) of DKP algebra (2). In this case, the field $\psi$ is given by a 5-component column vector

$$
\psi = (\varphi, \psi^0, \psi^1, \psi^2, \psi^3)^T,
$$

where $\varphi$ and $\psi^a$ ($a = 0, 1, 2, 3$) behave, respectively, as a scalar and a 4-vector under Lorentz transformations on the Minkowski tangent space. Expressing Lagrangian (5) in this representation, we obtain

$$
L_0 = e \left( i\psi^a e^{\mu a} \nabla_\mu \varphi - i\varphi e^{\mu a} \nabla_\mu \psi^a - \psi^a \varphi \right).
$$

The massless DKP equation (8) implies

$$
\psi^a = i e^{\mu a} \nabla_\mu \varphi,
$$

$$
e^{\mu a} \nabla_\mu \psi^a = 0,
$$

which, together, give

$$
g^{\mu \nu} \nabla_\mu \nabla_\nu \varphi = 0,
$$

which is the massless Klein–Gordon–Fock equation in $\mathcal{R}$. Accordingly, after substituting equation (11a) into Lagrangian (10), we obtain the corresponding massless scalar Klein–Gordon–Fock Lagrangian minimally coupled to $\mathcal{R}$

$$
L_0 = \sqrt{-g} g^{\mu \nu} \nabla_\mu \varphi \nabla_\nu \varphi.
$$

2.2. Spin 1 sector

Similarly, we can work out explicitly the vector sector of the theory by using a representation (see [18]) for the DKP algebra (2). The field $\psi$ is now a 10-component column vector

$$
\psi = (\psi^0, \psi^1, \psi^2, \psi^3, \psi^{23}, \psi^{31}, \psi^{12}, \psi^{10}, \psi^{30})^T,
$$

where $\psi^a$ ($a = 0, 1, 2, 3$) and $\psi^{ab}$ behave, respectively, as a 4-vector and an antisymmetric tensor under Lorentz transformations on the Minkowski tangent space. In terms of these components the Lagrangian (5) is written as

$$
L_1 = e \left( \frac{1}{2} \psi^{ab} (e^{\mu a} \nabla_\mu \psi^b - e^{\mu b} \nabla_\mu \psi^a) - \frac{i}{2} \psi^{ab} (e^{\mu a} \nabla_\mu \psi^b - e^{\mu b} \nabla_\mu \psi^a) + \frac{1}{2} \psi^{ab} \psi_{ab} \right),
$$

while the equations of motion are

$$
\psi_{ab} = -i (e^{\mu a} \nabla_\mu \psi^b - e^{\mu b} \nabla_\mu \psi^a),
$$

$$
e^{\mu b} \nabla_\mu \psi^{ab} = 0.
$$

Together, they give

$$
\nabla_\mu F^{\mu \nu} = 0,
$$

where

$$
F_{\mu \nu} = \nabla_\mu \psi_\nu - \nabla_\nu \psi_\mu = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu.
$$

Correspondingly, turning the equation of motion (16a) into the Lagrangian (15), we obtain the Maxwell Lagrangian minimally coupled to $\mathcal{R}$

$$
L_0 = e \left( \mathcal{R} \right).
$$

We identify $\psi_\mu$ with the electromagnetic field $A_\mu$. 
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\[ \mathcal{L}_1 = -\frac{\sqrt{-g}}{2} g^{\mu\alpha} g^{\nu\beta} F_{\mu\alpha} F_{\nu\beta}. \]  

(19)

3. Conformal invariance

In order to study the properties of massless DKP theory under conformal transformations we construct the action corresponding to the Lagrangian (5),

\[ S_R = \int d^4x \left[ i \bar{\psi} \gamma^\mu e_{\mu a} \partial_\mu \psi - i \bar{\psi} \gamma^\mu e_a \partial_\mu \psi - \bar{\psi} \gamma^\mu \gamma^\nu e_{\mu a} e_{\nu a} \psi \right]. \]  

(20)

As we have already mentioned, this action is manifestly invariant under general coordinate transformations on the Riemannian spacetime. Therefore, the study of conformal invariance of the theory can be reduced to the study of the properties of Lagrangian (5) under the so-called local Weyl rescalings \(^{[7, 21]}\). In this case, invariance under local Weyl rescalings implies conformal invariance.

In the following, as is conventional in the study of local transformations \(^{[19]}\), we first consider the restricted class of global Weyl rescalings in order to get the transformation laws for DKP fields. These laws will be later generalized to the case of local transformations.

3.1. Global Weyl rescaling

The restricted class of global Weyl rescalings of the metric and the vierbeins is given by \(^{[19, 21]}\)

\[ e^{\mu a} \rightarrow e^{-\sigma} e^{\mu a}, \quad E_{\mu a} \rightarrow e^\sigma E_{\mu a}, \quad e \rightarrow e^{2\sigma} e, \quad g_{\mu\nu} \rightarrow e^{2\sigma} g_{\mu\nu}. \]  

(21)

where \( \sigma \), the scale parameter, is an arbitrary real number.

In order to determine the transformation laws for the DKP field, we first note that \( \psi \) is represented by a column matrix \(^{[10]}\) whose components have different canonical dimensions and, accordingly, must have different conformal weights. To deal with this fact, we search for a nonsingular real matrix \( M(\sigma) \), depending on the scale parameter \( \sigma \), such that under (21) \( \psi \) behaves as

\[ \psi \rightarrow M(\sigma) \psi, \]  

(22)

and, accordingly,

\[ \bar{\psi} \rightarrow \bar{\psi} M^T(\sigma) \gamma^0 = \bar{\psi} M(\sigma). \]  

(23)

The 'contracted' spin connection

\[ \omega_{\mu} \equiv \frac{1}{4} \omega_{\mu ab} S^{ab} \]  

(24)

remains invariant under (21), as it can be shown from (7). Accordingly, DKP covariant derivatives transform covariantly, i.e.,

\[ \nabla_\mu \psi \rightarrow M(\sigma) \nabla_\mu \psi, \]  

(25a)

\[ \nabla_\mu \bar{\psi} \rightarrow (\nabla_\mu \bar{\psi}) M(\sigma). \]  

(25b)

The action (20) will be invariant under the above transformations if, and only if, the matrix \( M(\sigma) \) satisfies the following conditions:

\[ e^{2\sigma} M(\sigma) \gamma^\beta \gamma^\alpha M(\sigma) = \gamma^\beta \gamma^\alpha, \]  

(26a)
\[ e^{3\sigma} M(\sigma) \beta^a \gamma M(\sigma) = \beta^a \gamma, \quad (26b) \]
\[ e^{4\sigma} M(\sigma) \gamma M(\sigma) = \gamma, \quad (26c) \]

which, with the aid of algebra (2), can be reduced to

\[ M(\sigma) \beta^a M(\sigma) = e^{-3\sigma} \beta^a, \quad (27a) \]
\[ M(\sigma) \gamma M(\sigma) = e^{-4\sigma} \gamma. \quad (27b) \]

It can be verified that the matrix

\[ M(\sigma) = (1 - \gamma) e^{-\sigma} + \gamma e^{-2\sigma} \quad (28) \]

satisfies all the above requirements and it is the solution we search for. For instance, this matrix can be easily determined from the above conditions by using the explicit representations for DKP algebra given in [18].

Summarizing, the massless DKP theory minimally coupled to Riemannian spacetimes is invariant under global Weyl rescalings. Of course, this holds for both spin 0 and spin 1 sectors of the theory, as can be easily verified.

3.2. Local Weyl rescaling

The local Weyl rescalings can be obtained from the global transformations (21), (22) and (28) by allowing the scale parameter \( \sigma \) to become a smooth function (the scale function) depending on the coordinates, denoted as \( \sigma(x) \). It can be easily verified that conditions (26) still hold in the local case.

From (7) and (24), it can be shown that the spin connection \( \omega_\mu \) now changes according to

\[ \omega_\mu \rightarrow \omega_\mu + e^{ib} E_\mu^{a} S_{ab} \partial_\sigma. \quad (29) \]

Taking into account the explicit form (28) for the matrix \( M(\sigma(x)) \) and the fact that \([M, S_{ab}] = 0\), we find that the first term in Lagrangian (5) changes as

\[ i e \overline{\psi} \gamma \beta^a \nabla_\mu \psi \rightarrow i e \overline{\psi} \gamma \beta^a \nabla_\mu \psi + i e \overline{\psi} (-2\gamma + \gamma \beta^a \beta^a) \beta^b \epsilon^{ab} \partial_\sigma \psi, \quad (31) \]

where we have used the algebraic identity \( \beta^a \beta^b \beta^a = \beta^b \). Similarly, using the identity \( \beta^a \beta^b \beta^c + \beta^b \beta^a \beta^a = 5 \beta^b \), the second term in the Lagrangian changes as

\[ i e \nabla_\mu \overline{\psi} \beta^a \gamma \psi \rightarrow i e \nabla_\mu \overline{\psi} \beta^a \gamma \psi + i e \overline{\psi} (-3\gamma + \gamma \beta^a \beta^a - \beta^b \beta^a + 3) \beta^b \epsilon^{ab} \partial_\sigma \psi, \quad (32) \]

while the third term remains invariant.

Collecting the above results we find that the local Weyl rescalings change the Lagrangian (5) by an amount \( \delta L \), given by

\[ \delta L = i e \overline{\psi} (\beta^a \beta^a - 3 + \gamma) \epsilon^{ab} \beta^b \psi \partial_\sigma \psi. \quad (33) \]

This variation cannot be identically vanishing, because it would imply the condition

\[ \beta^a \beta^a - 3 + \gamma = 0, \quad (34) \]

which is not an algebraic identity. More precisely, though this condition is satisfied restricted to the spin 1 sector of the theory, it is not satisfied in the spin 0 sector, where it is easy to verify\(^7\) that the following relation holds

\[ \beta^a \beta^a - 3 + \gamma = 1 - 2\gamma. \quad (35) \]

\(^7\) By using an explicit representation for the spin 0 sector, for example.
Summarizing, we have shown that only the spin 1 sector of the massless DKP theory minimally coupled to Riemannian spacetimes is conformally invariant. Restricted to this sector, the transformation law (29) can be written as
\[
\omega_\mu \rightarrow M \omega_\mu M^{-1} + M \partial_\mu M^{-1} = \omega_\mu + (1 + \gamma) \partial_\mu \sigma,
\]
and, accordingly, transformation (30) implies that the covariant derivatives transform covariantly,
\[
\nabla_\mu \psi \rightarrow M(\sigma(x)) \nabla_\mu \psi, \quad \nabla_\mu \bar{\psi} \rightarrow (\nabla_\mu \bar{\psi}) M(\sigma(x)).
\]

The above results are in complete agreement with those obtained in the framework of (massless) KGF and Maxwell theories for the spin 0 and spin 1 sectors, respectively. This could be expected from the equivalence between these theories and DKP when a minimal coupling to Riemannian spacetimes is considered [18]. In the following subsection, guided by the form of variation (33), we achieve in a natural way the conformal invariance also of the spin 0 sector, without changing the content of the spin 1 sector.

### 3.3. Conformal invariant theory

We now require the whole theory to be invariant under local Weyl rescalings. This can be achieved, as usual, by adding to the Lagrangian (5) a compensating term, such that its variation cancels the variation (33) [19]. The most natural form for such a term is
\[
-i e \bar{\psi} \beta^a \beta^a - 3 + \gamma) e^\mu_b \beta^b C_\mu \psi,
\]
where \( C_\mu \) must be a compensating field transforming under local Weyl rescalings as
\[
C_\mu \rightarrow C_\mu + \partial_\mu \sigma.
\]

As we do not intend to introduce any new kind of field into the theory, we require that \( C_\mu \) must be given solely in terms of the metric tensor [2]. Because (38) vanishes identically in the spin 1 sector, from now on we shall be concerned only with the changes it causes in the spin 0 sector. Restricted to this sector, and taking into account relation (35), the equation of motion reads
\[
i \beta^{\mu} \nabla_\mu \psi - \gamma \psi - i(1 - 2 \gamma) \beta^{\mu} C_\mu \psi = 0.
\]

By using the scalar representation of [18] this equation gives the following relations among DKP field components:
\[
(\nabla_\mu - C_\mu) \psi^{\mu} = 0, \tag{41a}
\psi_\mu = i(\nabla_\mu + C_\mu) \psi, \tag{41b}
\]
which, together, result in the following equation for the scalar field \( \psi \):
\[
g^{\mu \nu} \nabla_\mu \partial_\nu \psi = C \psi = 0, \tag{42}
\]
where the scalar \( C \) was defined as
\[
C \equiv g^{\mu \nu}(C_\mu C_\nu - \nabla_\mu C_\nu).
\]

Naturally, as \( C_\mu \) is given solely in terms of the metric tensor, the same holds for the scalar \( C \). As a consequence, this scalar must be completely determined as a function of the scalar curvature \( R \) [23]. To precisely determine this function we observe that, under Weyl rescalings,
\[
C \rightarrow e^{-2\sigma}(C - g^{\mu \nu} \nabla_\mu \partial_\nu \sigma - g^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma), \tag{44}
\]
which is precisely the same way that the scalar $\frac{1}{6} R$ behaves under these transformations. This result implies that

$$C = \frac{R}{6}. \quad (45)$$

When this solution is replaced into (42), it gives the well-known $\frac{R}{6} \phi$ coupling among scalar curvature and the massless scalar KGF field.

3.4. Effects of torsion

We now go a step further from Riemannian to Riemann–Cartan spacetimes by discussing the effects of a nonvanishing torsion on the scalar sector. We shall consider the ‘weak’ and ‘strong’ ways of defining the conformal transformations of the torsion (see [25] for a review, for instance). In the weak case, the torsion $Q^\nu_{\mu \lambda}$ (which is the antisymmetric part of the spacetime connection) does not change under Weyl rescalings, i.e.

$$Q^\nu_{\mu \lambda} \rightarrow Q^\nu_{\mu \lambda}. \quad (46)$$

We recall that the spin connection in the presence of torsion is given by [18]

$$\tilde{\omega}^a_{\mu b} = \omega^a_{\mu b} - K^a_{b \mu},$$

where $\omega^a_{\mu b}$ is given by (7) and $K^a_{b \mu} = E^a_{\nu} e^b_{\nu} K^\nu_{\alpha \mu}$, while

$$K^\nu_{\alpha \mu} = Q^\nu_{\alpha \mu} - Q^\nu_{\alpha \mu} - Q^\nu_{\mu \alpha}$$

is the contorsion tensor. Accordingly, in the weak case the spin connection changes as before, as if there were no torsion, namely

$$\tilde{\omega}^a_{\mu} \rightarrow \tilde{\omega}^a_{\mu} + e^{\nu b} E^a_{\mu} S_{ab} \partial \nu \sigma. \quad (47)$$

Therefore, all the analysis previously done for the Riemannian spacetimes still holds in the weak case.

On the other hand, in the strong case the torsion transforms as

$$Q^\nu_{\mu \lambda} \rightarrow Q^\nu_{\mu \lambda} + r_c \left( \delta^\nu_{\lambda} \partial_\nu \sigma - \delta^\nu_{\mu} \partial_\nu \sigma \right), \quad (48)$$

where $r_c$ is an arbitrary parameter. In this case, the spin connection transforms as

$$\tilde{\omega}^a_{\mu} \rightarrow \tilde{\omega}^a_{\mu} + (1 - 2r_c) e^{\nu b} E^a_{\mu} S_{ab} \partial_\nu \sigma, \quad (49)$$

where now appears an extra term depending on the parameter $r_c$,

$$\delta \mathcal{L} = i e \overline{\psi} \left[ \beta_\alpha \beta^\alpha - 3 + \gamma + 2r_c (4 - 3\gamma - \beta_\alpha \beta^\alpha) \right] e^\mu b \beta^b \psi \partial_\mu \sigma. \quad (50)$$

In order to assure the conformal invariance of the theory we must add to the Lagrangian the following compensating term:

$$\mathcal{L}_{ct} = -i e \overline{\psi} \left[ \beta_\alpha \beta^\alpha - 3 + \gamma + 2r_c (4 - 3\gamma - \beta_\alpha \beta^\alpha) \right] \beta^b \epsilon^\mu b \tilde{C}_\mu \psi, \quad (51)$$

where $\tilde{C}_\mu$ is a compensating field transforming under local Weyl rescalings as

$$\tilde{C}_\mu \rightarrow \tilde{C}_\mu + \partial_\mu \sigma. \quad (52)$$

From (35) we observe that the term depending on the parameter $r_c$ within the brackets vanishes in the spin 0 sector. Therefore, the torsion does not affect our previous result on the scalar sector and $\tilde{C}_\mu$ is the same field introduced in the torsionless case. This result could already be expected from the complete equivalence between massless Klein–Gordon–Fock and the scalar sector of the massless DKP theory in Riemann–Cartan spacetime [18], together with the results of [24, 25]. Summarizing, both massless scalar DKP and KGF fields do not couple to the torsion and both of them are conformally invariant in Riemann–Cartan spacetimes with the same conformal coupling $\frac{1}{6} R \phi^2$, exactly as occurs in the Riemannian case.
4. Conclusions

In this paper, we studied the behaviour under conformal transformations of massless DKP fields in Riemannian spacetimes. We started from the massless theory in the Riemannian spacetimes, as obtained from the formalism of vierbein fields and from the standard form of the minimal coupling procedure. Taking advantage of the fact that this theory is already manifestly covariant under general coordinate transformations, we carried out the analysis of conformal transformations by investigating only its invariance properties under the special group of local Weyl transformations. In this case, invariance under this last group of transformations implies the conformal invariance of the theory.

We showed that, while the spin 1 (vector) sector of this theory is invariant under conformal transformations, the spin 0 (scalar) sector is not. These results were in complete agreement with those obtained in the framework of (massless) Klein–Gordon–Fock and Maxwell theories in the context of minimal couplings, as was expected on the grounds of our previous results [18], where we demonstrated the complete equivalence between such theories and DKP when minimal couplings with Riemannian spacetimes are considered. In order to achieve also the conformal invariance of the scalar sector, without modifying the vector sector, we introduced a compensating term into the Lagrangian which, by its turn, carried a compensating field into the theory. By imposing the condition that this field were completely determined from the metric tensor, we naturally obtained the well-known conformal coupling $\frac{1}{6} R \phi^2$ among the massless scalar field and the scalar curvature.

Finally, we extended our analysis beyond the Riemannian case by discussing the effects of torsion on the scalar sector. We obtained, for this case, the same conformal coupling obtained in the Riemannian case. This result was expected on the grounds of [18] and in agreement with [24, 25].

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