Teleporting digital images

Mario Mastriani

Received: 8 August 2022 / Accepted: 4 March 2023 / Published online: 8 April 2023
© The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2023

Abstract
During the last 30 years the scientific community has coexisted with the most fascinating protocol due to Quantum Physics: quantum teleportation, which would have been impossible if quantum entanglement, so questioned by Einstein, did not exist. In this work, a complete architecture for the teleportation of Computational Basis States (CBS) is presented. Such CBS will represent each of the possible 24 classical bits commonly used to encode every pixel of a 3-color-channel image (red–green–blue, or cyan–yellow–magenta). For this purpose, a couple of interfaces: classical-to-quantum and quantum-to-classical are presented with two versions of the teleportation protocol: standard and simplified.

Keywords Interfaces · Quantum communications · Quantum entanglement · Quantum teleportation · Superdense coding

1 Introduction
Since their appearance in the literature (Bennett et al. 1993), both the quantum teleportation protocol and superdense coding have become central components of a key area within quantum information processing (Nielsen and Chuang 2004) known as quantum communications (Pathak 2013; Cariolaro 2015; Mishra 2016; Imre and Gyongyosi 2012). During the last three decades, both protocols have been successfully implemented on different platforms (Bouwmeester et al. 1997; Boschi et al. 1998; Furusawa et al. 1998; Barrett et al. 2004; Riebe et al. 2004; Jacob et al. 2006; Yang 2009; Ma, et al. 2012; Houwelingen et al. 2006; Carlo et al. 2003; Marzolino and Buchleitner 2016; Hedemann 1605; Huo et al. 2018; Mastriani 2018) in order to transmit information through a combination of classical and quantum channels.

The purpose of this study lies in the transmission of a classical (digital) image by means of a quantum configuration. However, the first concomitant step with the aforementioned purpose consists of the internal representation of a classical image in a quantum environment in a practical way. To date, this milestone has not been achieved due to the poor performance in quantum environments of the most famous internal image
representation techniques, which implies a deep negation of the practical problems that these techniques present when implementing them on a real physical platform.

The most famous techniques for the internal representation of the classical image in a quantum environment have been shown to work in simulation environments implemented in programming languages, but not on superconducting physical machines, and even less so on optical tables. This is the case with techniques such as flexible representation of quantum images (FRQI) (Le et al. 2011), novel enhanced quantum representation (NEQR) (Zhang et al. 2013), novel classical-quantum images (NCQI) (Sang et al. 2017), and quantum representation of multi-wavelength (QRMW) (Sahin and Yilmaz 2018), to name a few. The same has happened with the techniques derived from them, some of which have come to work on a simulator of the IBM Q Experience program (IBM Quantum Experience 2022) called Qiskit, but never on one of the physical machines of that program.

A complete study, which shows the serious problems of practical implementation of techniques like NCQI, among others, can be found in Su et al. (2021), in which it is mentioned that this technique has problems being implemented even on a simulator like Qiskit (IBM Quantum Experience 2022). Other works complete a detailed description of all the difficulties of practical implementation of the mentioned techniques, among which a spurious quantum entanglement at the output of these techniques stands out, which complicates obtaining the correct outcomes when measuring (Mastriani 2020, 2017; Mastriani et al. 2021). This curious phenomenon has been observed on superconducting physical machines, optical tables, and some simulators such as Quirk (Algassert 2022), but not in implementations carried out in programming languages.

Therefore, the technique of internal representation of the classical image in a quantum environment is an extremely sensitive point, which is why we have selected the superdense coding protocol as the classical-to-quantum interface in order to represent the constitutive bits of the classical image as the computational basis states in the quantum environment. This protocol has been successfully tested both in implementations on physical machines (Mastriani 2023), and in an optical environment (Williams et al. 2017). However, its use is restricted because it only works with two classical bits. To counteract this limitation, in this study we will use a generalization of this protocol with no limits on the number of bits to be processed (Mastriani 2023). Despite this, in order to simplify the development of the presentation, in this study we will explain the architecture for the quantum transmission of digital images by means of the original superdense coding protocol, that is, of only two bits, using the generalized version without limits of bits processed at the end of the paper.

The originality of this study consists of presenting, for the first time in the literature in the area, a complete architecture for the transmission of classical (digital) images through a quantum configuration, using for this purpose some modules that have been shown to work in real quantum environments. This possible milestone constitutes a real source of motivation for this study since the precedents for quantum transmission of images present in the literature consist exclusively of purely theoretical works such as (Almeida et al. 2006; Ivanov et al. 2022; Almeida and Souto Ribeiro 2003; Gilev and Popov 2019), others implemented exclusively in a programming language (Janani and Brindha 2021), e.g., MATLAB (https://www.mathworks.com/products/matlab.html), and some presenting comparative models of the type known as mimicking teleportation to transmit an image (Chen 2021).

The outline of the paper is as follows: In Sect. 2, both the fundamental protocols and the complete architecture for the development of this study are introduced. Section 3 shows the results obtained in the Quirk platform (Algassert 2022). Section 4 presents a full discussion of the results. Finally, Sect. 5 deals with the general conclusions of this study.
2 Materials and methods

2.1 Preliminaries

Some of the tools that are guidelines for conducting this study are described below. The central idea is that the approach to these tools evolves throughout this study from the most general to the most specific.

Quantum Teleportation: In 1993 a fundamental paper for the history of Physics was published (Bennett et al. 1993), where a protocol for the teleportation of an unknown quantum state was proposed without violating the No-Cloning Theorem (Wootters and Zurek 1982). This protocol uses a strange phenomenon of Quantum Mechanics known as Quantum Entanglement (Nielsen and Chuang 2004; Agrawal 2022), in which two or more particles leave their individualities to become part of something unique and integral. These particles retain the aforementioned attribute even if they are separated into opposite places in the universe (Nielsen and Chuang 2004; Agrawal 2022).

Now, if we measure one of those particles, the entanglement disappears, the joint wave function collapses, and if the measured particle becomes a spin-up, the other instantaneously becomes a spin-down, and vice-versa. This instanteity is given independently of the distance that mediates between both particles, contradicting the very essence of Special Relativity (Einstein et al. 1952), which postulates that nothing can travel faster than light. This curious result was called by Einstein a "spooky action at a distance" and constitutes the center of one of the most famous paradoxes in the history of Physics suggested by Albert Einstein, Boris Podolsky, and Nathan Rosen and is known as the EPR paradox (Einstein et al. 1935). There is a palliative to this controversy which consists of the impossibility of transmitting useful information through an instantaneous link based on entanglement (Bennett et al. 1993). This palliative constitutes a hot border between two of the main pillars of Physics: Special Relativity (Einstein et al. 1952) and Quantum Mechanics (Phillips 2003).

In 1964 John Bell (Bell 1964) tried to solve this controversy by proposing a theorem based on an inequality by which the non-locality of Quantum Mechanics could be established as long as it violated such inequality. Experiments carried out by Aspect et al. (1982a, 1982b, 2007) and others (with and without loopholes) apparently established the non-locality of Quantum Mechanics, contradicting what was manifested by the EPR paradox. Part of the scientific community does not question the non-locality of Quantum Mechanics, since there is an absolute consensus; however, they are hesitant about the quality of the mentioned experiments.

Since 1997, a series of experiments have demonstrated the practical feasibility of Quantum Teleportation protocol (Bouwmeester et al. 1997; Boschi et al. 1998; Furusawa et al 1998; Barrett et al. 2004; Riebe et al. 2004; Jacob et al. 2006; Yang 2009; Ma et al. 2012; Houwelingen et al. 2006), even in the presence of noise (Carlo et al. 2003; Marzolino and Buchleitner 2016; Hedemann 1605; Huo et al. 2018). Thirteen years later, Prof. Hotta (Hotta 2010) demonstrated how energy could be teleported, triggering all kinds of speculations about the possible teleportation of matter, by virtue of the close link between matter and energy starting from the famous Einstein equation, \( E = mc^2 \), for a particle at rest. Currently, a simplified version of the original Quantum Teleportation protocol (Mastriani 2018, 2022) opens up a whole new range of possibilities in Quantum Communications (Pathak 2013; Cariolaro 2015; Mishra 2016; Imre and Gyongyosi 2012), thus completing the arsenal of essential tools for Quantum Technology to be used in the future.
Superdense Coding: A complementary protocol to Quantum Teleportation, called Superdense Coding (Bennett et al. 1993), allows us to send classical bits through a quantum channel. Besides, Superdense Coding is the foundational basis of a couple of interfaces needed when handling Computational Basis States in a Quantum Communications context (Cariolaro 2015). Recently, a new Superdense Coding protocol based on a simplified version of Quantum Teleportation has come out with remarkable results (Mastriani 2022). In fact, the mentioned protocol is presented as a probable basis for secure communication between Earth and Mars, in a future mission to Mars (Mastriani 2023).

Interfaces: The need for Classical-to-Quantum (Cl2Qu) and Quantum-to-Classical (Qu2Cl) interfaces in all branches of Quantum Information Processing is evident (Nielsen and Chuang 2004). However, in no other case as in Quantum Image Processing has it become so evident. Works like Quantum Boolean Image Denoising (Mastriani 2015) highlight the imperative need for an efficient interface between each of the possible 24 classical bits commonly used to encode each pixel of an image with 3 color channels (red, green, blue) and the internal representation that those classical bits must have within the quantum computer, that is, as Computational Basis States. This conversion cannot result from a qubits preparation procedure because the images are too large, this must be done automatically thanks to an interface. For example, for an image of 1920 columns, 1080 rows, 8 bits-by-color, and 3 colors (an image of common size these days), we must prepare 49,766,400 qubits (in this particular case: Computational Basis States). This is highly impractical, since how long would a similar amount of qubit preparations take us in a laboratory? The problem of Quantum Measurement (Busch et al. 2016) has confined the practicality of Quantum Image Processing to the exclusive use of Computational Basis States, as it is exposed in Mastriani (2017). If we wanted to work in Quantum Image Processing with generic qubits, we would find ourselves with a problem that does not exist to date even with Classical-to-Quantum interface for this type of qubit. Besides, it is impossible to recover exactly the Quantum Algorithm result because of Quantum Measurement (Busch et al. 2016), as explained in Mastriani (2017). It happens that the measurement noise due to Quantum Mechanics is greater than that admissible in a standard process of Digital Image Processing (Jain 1989; Gonzalez and Woods 2002; Gonzalez et al. 2004; Schalkoff 1989). The distortion in the recovery of a qubit at the output of a Quantum Algorithm is greater than that normally accepted (Mastriani 2017), being the Heisenberg Uncertainty Principle and the complementarity (Nielsen and Chuang 2004), the only ones responsible for it, as explained in Mastriani (2017). This is the problem that fundamentally affects the internal representation technique known as Flexible Representation of Quantum Images (FRQI) (Le et al. 2011) and all its variants. Besides, another famous technique within Quantum Image Processing is known as Novel Enhanced Quantum Representation (NEQR) (Zhang et al. 2013). The problem with NEQR, as well as all its variants, is that it is not a Classical-to-Quantum Interface; however, they need one. In fact, if we had a Classical-to-Quantum interface, then why would we need NEQR? On the other hand, the reason why both techniques (FRQI and NEQR) are accepted within Quantum Image Processing resides in the fact that all the papers that mention them only involve implementations in a high-level interpreter such as MATLAB® (https://www.mathworks.com/products/matlab.html), and not on an optical table as it should be. Under these circumstances, everything seems to work, but when done on an optical table the real outcome is very different.
2.2 Color decomposition and bit slicing

First, we need to decompose all digital images into their 3 color components (red, green, and blue) (Mastriani 2015). Thus, we will obtain 24 bitplanes (8 bitplanes for every color) thanks to a procedure known as bit slicing (Mastriani 2015). We get 8 bitplanes per color, where the 7th bitplane (the closest to the observer) is called the Most Significant Bit (MSB) being the most morphologically committed bitplane with the original image (Mastriani 2015) (Fig. 1). Conversely, bitplane 0 (the furthest from the observer) is the Least Significant Bit (LSB) and the least morphologically committed bitplane.

![Diagram showing color decomposition and bit slicing](image)

**Fig. 1** Bitplanes of the red component for Angelina were obtained by slicing, with special remarks for the Most Significant Bit (MSB) and the Least Significant Bit (LSB).
with the original image. Below, we provide the MATLAB® code (https://www.mathworks.com/products/matlab.html) necessary for slicing:

```matlab
function lbpp = slicer(l,bpp)
    % Casting of the algorithm:
    % bpp = bit-per-pixel
    % l = Each color component of the image
    % lbpp = l in bpp bitplanes (strictly binary)
    [ROW,COL] = size(l);
    for r = 1:ROW
        for c = 1:COL
            aux = d2b(l(r,c)-1,bpp);
            for b = 1:bpp
                lbpp(r,c,b) = aux(b);
            end
        end
    end
    return;
end

function bvpp = d2b(p,bpp)
    % Casting of algorithm:
    % d = bit depth
    % p = pixel value
    % bvpp = binary vector per pixel
    bvpp = zeros(1,bpp);
    d = 1;
    while p > 0,
        bvpp(d) = mod(p,2);
        p = p/2;
        p = floor(p);
        d = d+1;
    end
    bvpp = rot90(rot90(bvpp));
    return;
end
```
It is evident, from here on, that an element in black in every bitplane is a classical bit equal to 1 and will be represented with a qubit equal to $|1\rangle$, while, an element in white in every bitplane is a classical bit equal to 0 and will be represented with a qubit equal to $|0\rangle$ in a future Cl2Qu interface.

Figure 2 shows the 8 bitplanes of Angelina for the red channel in detail, from MSB (bitplane 7) to LSB (bitplane 0). Let us observe that as we move from MSB to LSB, different bitplanes are increasingly unrecognizable compared to the original image, i.e., Angelina. As we can see, LSB is completely different from the original morphology of Angelina’s picture. This is one reason why the LSB is a Steganography territory (Mas-triani 2016). The other reason is that any change in the LSB does not produce visually detectable changes in the original image.

2.3 Standard quantum teleportation—noiseless analysis

Quantum Teleportation begins with the distribution of an EPR pair between Alice and Bob. We can choose any of the EPRs of the complete set of Bell’s bases:

\[
|\beta_{00}\rangle = |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |\beta_{01}\rangle = |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\
|\beta_{10}\rangle = |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad |\beta_{11}\rangle = |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad (1)
\]

We normally choose $|\beta_{00}\rangle$. This distribution constitutes the entanglement link between Alice and Bob. After that, we continue with the complete sketch of Quantum Teleportation of Fig. 3, where the green line indicates the border between Alice’s and Bob’s sides, that is, both extremes of the entanglement link. In Fig. 3, a single fine line represents a wire carrying one qubit, while a double line represents a wire carrying one classical bit (Nielsen and Chuang 2004), while the classical channel is really a control classical channel for disambiguation purposes (as we will see below through two bits), and the entanglement link is really an entanglement data link. On the other hand, in Fig. 3, the following blocks mean: SPD (single photon detectors), \{\sigma_x, \sigma_z\} are Pauli’s matrices activated by the bits \{b_2, b_1\} respectively (Bennett et al. 1993; Nielsen and Chuang 2004), and EPR is the source of $|\Phi_{+00}\rangle \equiv \Phi_{+00}^{A_0B_0}$ of Eq. (1).

Now, if $|\psi_0\rangle = |\psi\rangle = a|0\rangle + \beta|1\rangle$ is an arbitrary and unknown state to be teleported with $|a|^2 + |\beta|^2 = 1$ and $a, \beta \in \mathbb{C}$ of a Hilbert space, then, the initial state (3-partite state) will be,

\[
|\psi_0\rangle = |\psi\rangle \otimes |\beta_{00}\rangle = |\psi\rangle |\beta_{00}\rangle = (a|0\rangle + \beta|1\rangle) \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}[a|000\rangle + a|011\rangle + \beta|100\rangle + \beta|111\rangle] = [\begin{array}{cccc}a & \sqrt{2} & 0 & 0 \\ \beta & \sqrt{2} & 0 & 0 \end{array}]^T 
\]

(2)

where for simplicity (and from here on) we have adopted $|x\rangle \otimes |y\rangle = |x\rangle |y\rangle$ in a generic form, and this operation is done inside a beam splitter.

Now, a CNOT gate (Nielsen and Chuang 2004) is applied to Eq. (2).
Fig. 2 Angelina and her 8 bitplanes, including the Most Significant Bit (MSB) and the Least Significant Bit (LSB)
In practice, Kronecker’s product and CNOT gate are implemented together on the same beam splitter (Bouwmeester et al. 1997; Boschi et al. 1998; Yang 2009; Ma, et al. 2012; Houwelingen et al. 2006). Then, we apply a Hadamard’s gate to the elements of Eq. (3),

$$|\psi_2\rangle = \frac{1}{2} [ |00\rangle \sigma_x^0 \sigma_z^0 |\psi\rangle + |01\rangle \sigma_x^1 \sigma_z^0 |\psi\rangle + |10\rangle \sigma_x^1 \sigma_z^1 |\psi\rangle + |11\rangle \sigma_x^0 \sigma_z^1 |\psi\rangle ]$$

$$= \frac{1}{2} [ |\Phi^+\rangle \sigma_x^0 |\psi\rangle + |\Phi^-\rangle \sigma_x^1 |\psi\rangle + |\Psi^+\rangle \sigma_z^0 |\psi\rangle + |\Psi^-\rangle \sigma_z^1 |\psi\rangle ]$$

$$= \begin{bmatrix} \alpha/\sqrt{2} & 0 & \beta/\sqrt{2} & 0 & \beta/\sqrt{2} & \alpha/2 & \alpha/2 \end{bmatrix}^T$$

In practice, Kronecker’s product and CNOT gate are implemented together on the same beam splitter (Bouwmeester et al. 1997; Boschi et al. 1998; Yang 2009; Ma, et al. 2012; Houwelingen et al. 2006). Then, we apply a Hadamard’s gate to the elements of Eq. (3),

$$|\psi_2\rangle = \frac{1}{2} [ |00\rangle \sigma_x^0 \sigma_z^0 |\psi\rangle + |01\rangle \sigma_x^1 \sigma_z^0 |\psi\rangle + |10\rangle \sigma_x^1 \sigma_z^1 |\psi\rangle + |11\rangle \sigma_x^0 \sigma_z^1 |\psi\rangle ]$$

$$= \frac{1}{2} [ |\Phi^+\rangle \sigma_x^0 |\psi\rangle + |\Phi^-\rangle \sigma_x^1 |\psi\rangle + |\Psi^+\rangle \sigma_z^0 |\psi\rangle + |\Psi^-\rangle \sigma_z^1 |\psi\rangle ]$$

$$= \begin{bmatrix} \alpha_2 & \beta_2 & -\beta_2 & \alpha_2 & \alpha_2 \end{bmatrix}^T$$

The last rows of Eq. (4) represent Alice’s options inside the Single Photon Detector. Alice randomly selects one of the bases and performs the measurement, transmitting to Bob the corresponding classical bits through a classical channel. Alice’s options within the Single Photon Detector are equally probable and the random choice that she makes of the base has to do with being sure not to clone the original state between her and Bob (Bouwmeester et al. 1997; Boschi et al. 1998).

Table 1 synthesizes the complete process of Quantum Teleportation, where Alice measures two of the possible qubits of the basis of Eq. (1), and therefore, she transmits the corresponding bits $b_1$ and $b_2$ via a classical channel to Bob. The Quantum Measurement process is imperative to make the wave function of the original arbitrary state collapse, since this is necessary to do so as not to violate the No-Cloning Theorem (Wootters and Zurek).
1982). In other words, the Quantum Measurement process destroys the original arbitrary state (Nielsen and Chuang 2004; Busch et al. 2016) eliminating any possibility of cloning.

The last column of Table 1 represents the local operations realized by Bob in $t_3$ and $t_4$ to reconstruct the original state $|\psi\rangle$. At this point, it is important to mention that in literature there are several concerns regarding the implementation of teleportation protocols using a bigger or smaller dimensional commitment but always with two classical bits for disambiguation. An interesting example can be found in Kiktenko et al. (2016a), which shows that the one-qubit teleportation can be considered as a state transfer between subspaces of the whole Hilbert space of an indivisible eight-dimensional system. However, this as well as the rest of the papers that manipulate high dimensional quantum systems for the implementation of Quantum Teleportation protocols do it with two classical bits for disambiguation, except in the case of the new protocol presented here which does not use disambiguation bits.

On Alice’s side, the combination of the modules composed by the following gates: $CNOT$, $H$ (Hadamard) and Quantum Measurement, constitute what is known as the Bell-State-Measurement (BSM) module, while on Bob’s side, its modules are unitary operations necessary for the reconstruction of the teleported state. Alice’s measurement and transmission of the classical bits of disambiguation along with Bob’s unitary operations are the clearest examples of Local Operations and Classical Communications (LOCC) (Kiktenko et al. 2016b).

### 2.4 Standard quantum teleportation—noisy analysis

Starting again from Fig. 3, and considering noise in the EPR pair by a disturbance of the shape

$$|\beta_{00}\rangle_n = A|00\rangle + B|11\rangle$$

where subscript $n$ means noise, and

$$|A|^2 + |B|^2 = 1, \quad \text{with} \quad (A \neq B) \wedge (A \neq i/\sqrt{2}) \wedge (B \neq i/\sqrt{2})$$

(6)

Then, repeating Eq. (2) but with $|\beta_{00}\rangle_n$ instead of $|\beta_{00}\rangle$, we will have,

$$|\psi_0\rangle = |\psi\rangle |\beta_{00}\rangle_n = (\alpha|0\rangle + \beta|1\rangle)(A|00\rangle + B|11\rangle)$$

$$= \alpha A|00\rangle + \beta A|10\rangle + \alpha B|01\rangle + \beta B|11\rangle$$

(7)

Then, a CNOT gate is applied to Eq. (7), resulting in
\[ |\psi_1\rangle = \alpha A|000\rangle + \beta A|110\rangle + \alpha B|011\rangle + \beta B|101\rangle. \] (8)

Then, we apply a Hadamard’s gate to the elements of Eq. (8),

\[
|\psi_2\rangle = \frac{1}{\sqrt{2}} [\alpha A|000\rangle + \alpha A|100\rangle + \beta A|010\rangle - \beta A|110\rangle + \alpha B|011\rangle + \beta B|001\rangle - \beta B|101\rangle]
\]

\[
= \frac{A}{\sqrt{2}} |00\rangle \alpha|0\rangle + \frac{B}{\sqrt{2}} |00\rangle \beta|1\rangle + \frac{A}{\sqrt{2}} |10\rangle \alpha|0\rangle - \frac{B}{\sqrt{2}} |10\rangle \beta|1\rangle + \frac{A}{\sqrt{2}} |01\rangle \beta|0\rangle + \frac{B}{\sqrt{2}} |01\rangle \alpha|1\rangle
\]

\[
+ \frac{B}{\sqrt{2}} |11\rangle \alpha|1\rangle - \frac{A}{\sqrt{2}} |11\rangle \beta|0\rangle = \frac{1}{\sqrt{2}} [|00\rangle (\alpha \alpha|0\rangle + B \beta|1\rangle) + |10\rangle (\alpha \beta|0\rangle - B \beta|1\rangle)
\]

\[
+ |01\rangle (\beta \alpha|1\rangle + |11\rangle (\beta \alpha|1\rangle - A \beta|0\rangle)]
\] (9)

From here on, we will follow a procedure similar to that in Table 1 but taking into account how sensitively the state is affected by noise. In fact, seeing Eq. (9), it is evident that it is almost impossible to recover the state \( |\psi\rangle \) in an exact way (Carlo et al. 2003; Marzolino and Buchleitner 2016; Hedemann 1605; Huo et al. 2018; Mastriani 2018, 2022).

### 2.5 Simplified quantum teleportation—noiseless analysis

Unlike the standard version, the new protocol frees us from the use of a classical channel to transmit the disambiguation bits, and the use of Pauli’s matrices in Bob’s side used to reconstruct the teleported state from the mentioned disambiguation bits. These simplifications are the underlying reasons for the title of this subsection, that is, simplified protocol.

In the new protocol (Fig. 4), the block \( P_h \) does a strict reset of the qubit, while we need to produce \( |\beta_{00}\rangle \otimes |\psi\rangle \) instead of \( |\psi\rangle \otimes |\beta_{00}\rangle \) used in the standard version, thanks to a SWAP gate (Nielsen and Chuang 2004; Mastriani 2018). We must highlight as a fundamental contrast between both versions of Quantum Teleportation (the standard and the simplified) that the Kronecker product “\( \otimes \)” is not commutative (Nielsen and Chuang 2004).

\[ |\psi_0\rangle = \text{SWAP}(|\psi\rangle |\beta_{00}\rangle) = |\beta_{00}\rangle |\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)(\alpha|0\rangle + \beta|1\rangle)
\]

\[
= \frac{1}{\sqrt{2}} [\alpha|000\rangle + \beta|001\rangle + \alpha|110\rangle + \beta|111\rangle]
\] (10)

![Fig. 4 Simplified teleportation protocol using an EPR pair but without classical bits for disambiguation](image-url)
Now, a CNOT gate is applied to Eq. (10), and the result will be present on Alice’s lower branch because in her upper branch $|\beta_{00}\rangle$ will be,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|000\rangle + \beta|001\rangle + \alpha|100\rangle + \beta|101\rangle]$$

(11)

Next, a Hadamard’s gate is applied to Eq. (11), then,

$$|\psi_2\rangle = \frac{1}{2}[\alpha|000\rangle + \alpha|100\rangle + \beta|001\rangle + \beta|101\rangle + \alpha|000\rangle - \alpha|100\rangle + \beta|001\rangle - \beta|101\rangle]
= \frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |10\rangle(\alpha|0\rangle + \beta|1\rangle) + |00\rangle(\alpha|0\rangle + \beta|1\rangle) - |10\rangle(\alpha|0\rangle + \beta|1\rangle)]
= |00\rangle(\alpha|0\rangle + \beta|1\rangle) = (P_h \otimes P_h \otimes I_{2 \times 2})|00\rangle(\alpha|0\rangle + \beta|1\rangle).
$$

(12)

As we can see, Eq. (12) is also the result of applying the two $P_h$ on Alice’s side; therefore, no disambiguation is necessary. Alice blocks her two branches thanks to a $P_h$ pair in order to collapse the wave function, and in this way not to force replication of the same result at both ends of the quantum channel, that is, the No-Cloning Theorem (Wootters and Zurek 1982) is never violated. The application of both $P_h$ is deterministic, although they are non-unitary matrices, which are implemented in an optical circuit as two horizontal polarizers, permanently present at the output of the Bell-State-Measurement (BSM)-module beam-splitter. Both $P_h$ block those components with $|1\rangle$, that is, vertical polarization. The 3 dB drop produced by each $P_h$ is compensated with two amplifiers, in such a way that the new protocol has the same teleportation efficiency as the original version. In this way, it is possible to completely dispense with the disambiguating classical channel used in the original protocol.

Moreover, both $P_h$ blocks are responsible for removing the natural ambiguity of the original quantum teleportation protocol (Bennett et al. 1993), eliminating any notification Alice needs to make to Bob in order to reconstruct the teleported state. We can also see in Fig. 4 that it is not necessary for Bob to apply any unitary transformation. This eliminates the classical channel that is responsible for making teleportation as a whole to be carried out in a time greater than zero, i.e., not being instantaneous.

Although this result seems to contradict the relativistic principle of causality (Einstein et al. 1935), the reality is that this never happens. As we can see in Mastriani (2022), the instantaneity of entanglement is possible without the need to resort to superluminal signaling and without any contradictions between Quantum Mechanics (Phillips 2003) and Special Relativity (Einstein et al. 1952). This last fact then covers the new protocol in a direct and complete way.

### 2.6 Simplified quantum teleportation—noisy analysis

For noisy EPR pairs, we also resorted to Fig. 4 using the same version of Eqs. (5) and (6). Then, repeating Eq. (10) but with $|\beta_{00}\rangle_n$ instead of $|\beta_{00}\rangle$, we will have,

$$|\psi_0\rangle = |\beta_{00}\rangle_n|\psi\rangle = (A|00\rangle + B|11\rangle)(\alpha|0\rangle + \beta|1\rangle)
= A\alpha|000\rangle + B\alpha|110\rangle + A\beta|001\rangle + B\beta|111\rangle
$$

(13)
Now, we apply a CNOT gate to Eq. (13), and the result will be present on Alice’s lower branch again, because in her upper branch $|\beta_{00}\rangle$ will be,

$$|\psi_1\rangle = A|\alpha|00\rangle + B|0|100\rangle + A|\beta|001\rangle + B|1|101\rangle$$

$$= A|00\rangle(|\alpha|00 + \beta|10\rangle) + B|10\rangle(|\alpha|00 + \beta|10\rangle)$$

$$= (A|00\rangle + B|10\rangle)(|\alpha|00 + \beta|10\rangle)$$

$$= (A|00\rangle + B|10\rangle)|\psi\rangle$$

where,

$$C = (A|00\rangle + B|10\rangle)$$

In this case again, the Hadamard gate only involves Alice’s upper branch, then,

$$(I \otimes H)|\beta_{00}\rangle_n = \left(\begin{array}{cc}1 & 0 \\ 0 & 1\end{array}\right) \otimes \frac{1}{\sqrt{2}}\left(\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right)(A|00\rangle + B|11\rangle)$$

$$= \frac{1}{\sqrt{2}}(A|00\rangle + A|10\rangle + B|01\rangle - B|11\rangle)$$

The worst consequence of noise in the new protocol is that the teleported state loses its purity, which means, it would not be on the Bloch sphere, in the more general case, given that $C \neq 1$, even so, the teleported state is recovered without problems or disambiguation. This clearly indicates that the new protocol is much more robust (immune to noise) than the standard, for generic qubits as well as for CBS.

### 2.7 Interfaces

From Superdense Coding emerges an extraordinary set of interfaces for an efficient relationship between classical bits and CBS, and vice-versa. Figure 5 shows the Superdense Coding protocol, which is composed of two well-defined blocks: one which is light blue and the other one pink. The light blue block works as a Cl2Qu interface, while the pink block works as a Qu2Ci interface. All this, of course, is exclusively defined to be used with classical bits, and in consequence, with CBS.

![Superdense Coding protocol and the detail of its constituent interfaces](image)
Next, we will describe how the Superdense Coding protocol works from left to right, which is the correct way of describing how both interfaces (Cl2Qu and Qu2Cl) work. An important detail to highlight before starting is that we can work with 2 classic bits to be transmitted (Fig. 5), a single classic bit \(b_1\) to be transmitted and \(b_2\) which makes ancilla equal to zero, or \(N\) classic bits to be transmitted as a natural extension of the protocol of Fig. 5. This last case will be particularly useful in the practical application of the interfaces for the treatment of digital images, in such a way that \(N=24\) and each application of the protocol implies working with a complete pixel of a color image. For a better development of the idea, we will work with 2 classic bits extracting Cl2Qu from Fig. 5 in order to form Fig. 6. We begin with both possibilities according to \(b_2\), i.e., 0 or 1. If \(b_2=0\), then, in \(t_1\) we will have \(|\beta_{00}\rangle\) again (Fig. 5). But, if \(b_2=1\), then, in \(t_1\) we will have,

\[
(\sigma_z \otimes I)|\beta_{00}\rangle = \begin{pmatrix} 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \quad (17)
\]

If in \(t_2\), \(b_2=0\) and \(b_1=0\), we will obtain \(|\beta_{00}\rangle\). But if \(b_2=0\) and \(b_1=1\), we will have,

\[
(\sigma_z \otimes I)|\beta_{00}\rangle = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ -1/\sqrt{2} \end{pmatrix} \quad (18)
\]

Now, if \(b_2=1\) and \(b_1=1\), we will use the result of Eq. (17) to obtain the state in \(t_2\).

\[
(\sigma_z \otimes I) \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (19)
\]
Obviously, the result in \( t_2 \) is the same as in \( t_3 \) (Fig. 5). The four obtained results, according to the values of \( b_2 \) and \( b_1 \), travel through the optical channel of Fig. 5 under the generic name of \( |\beta_{b_1b_2}\rangle \).

Now, if we apply a CNOT gate to these results, then, for \( b_2 = 0 \) and \( b_1 = 0 \), we will have,

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
0 \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
0 \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
\]  

(20)

For \( b_2 = 0 \) and \( b_1 = 1 \),

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
0 \\
-\frac{1}{\sqrt{2}}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
0 \\
-\frac{1}{\sqrt{2}}
\end{bmatrix}
\]  

(21)

For \( b_2 = 1 \) and \( b_1 = 0 \),

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0
\end{bmatrix}
\]  

(22)

For \( b_2 = 1 \) and \( b_1 = 1 \),

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0
\end{bmatrix}
\]  

(23)

At this point, it is important to clarify that the quantum channel of Fig. 5 inside the Cl2Qu interface is obviously unnecessary (Fig. 6). In fact, the definitive Cl2Qu interface is absolutely compact and henceforth it will be considered as a unified block. Finally, we will apply a Hadamard gate to the last set of equations according to Figs. 6 and 7 (Mastriani 2018, 2022), thus, for \( b_2 = 0 \) and \( b_1 = 0 \), we will have,

\[
(H \otimes I)
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
|0\rangle \\
|1\rangle \\
|0\rangle \\
|0\rangle
\end{bmatrix}
\]  

(24)
for $b_2=0$ and $b_1=1$,

$$
(H \otimes I) \begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
-\frac{1}{\sqrt{2}}
\end{bmatrix}
= \left( \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix}
\frac{1}{\sqrt{2}} \\
0 \\
-\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |1\rangle \otimes |0\rangle = |10\rangle
$$

(25)

for $b_2=1$ and $b_1=0$,

$$
(H \otimes I) \begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
= \left( \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0 \\
1
\end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |0\rangle \otimes |1\rangle = |01\rangle
$$

(26)

for $b_2=1$ and $b_1=1$,

$$
(H \otimes I) \begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
= \left( \begin{bmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix}
0 \\
\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |1\rangle \otimes |1\rangle = |11\rangle
$$

(27)

Here, Cl2Qu interface finalizes. Basically, Cl2Qu interface is a block that performs a transfer of type $\{b_1, b_2\} \rightarrow \{|b_1\rangle, |b_2\rangle\}$ for CBS, i.e., $\{0, 1\} \rightarrow \{|0\rangle, |1\rangle\}$.

The pink block of Fig. 5 constitutes the Qu2Cl interface. In fact, at the entrance of that block, we have $\{b_1, b_2\}$, while at its output we have $\{b_1, b_2\}$, i.e., Quantum Measurement (Busch et al. 2016) is a Qu2Cl interface itself. With this sequence Cl2Qu + Qu2Cl, we complete the standard Superdense Coding (Bennett et al. 1993). In this way, the circle closes, and at least in theory, we should recover the same classical bits that we have entered at the entrance of the light blue block at the output of the pink block. Consequently, we can use a coincidence counter to evaluate the performance of the complete Superdense Coding protocol (Bennett et al. 1993), and in this way, we know which is the level of coupling between both interfaces.

A relevant detail to take into account is that when we measure a CBS, we completely recover its classical counterpart, a situation very different from what happens with a generic qubit, which does not have a classical counterpart and if it existed it would be unattainable to obtain (Nielsen and Chuang 2004; Mastriani 2017, 2015). In fact, it is only necessary to make a measurement on the z-axis of the Bloch sphere (vertical) avoiding the
2.8 Complete architecture to be used

Figure 7 represents a complete architecture for an enhanced Superdense Coding. This figure shows both interfaces where we can appreciate a pair of green blocks labeled Quantum Teleportation between them. Actually, these blocks can represent the two types of teleportations studied: standard and simplified. Besides, these blocks constitute the imaginary boundary between Alice’s and Bob’s sides. It is important to mention that this configuration can be used to transmit a single classic bit at a time, where \( b_2 \) could be considered an ancilla, and then, the architecture of Fig. 7 would have a single block of Quantum Teleportation or horizontal thread.

If we expand the idea of Fig. 7 by involving the steps necessary to decompose a digital color image in their corresponding constituent bits, then, we will arrive to Fig. 8. This is essentially the scheme we will use for the experiments in the next section. Figure 8 shows on top the original image to be teleported, then the three-color channels (red, green and blue) of the image are separated. Later, each color channel is decomposed in 8-bitplanes. The bit of each bitplane is introduced into a Cl2Qu interface. The equivalent qubits are teleported. Bob receives each qubit with a Qu2Cl interface. With the obtained bits, the bitplanes are reconstructed, then the three-color channels are reassembled and finally, the teleported image is obtained. The lower part of Fig. 8 shows a comparison between the acquired bit of the original image and the recovered bit from the complete architecture. Finally, the complete comparison of all individual bits becomes the total comparison between both images, where that comparison between the original image and the teleported one for the purpose of evaluating the level of degradation introduced by the complete procedure used as a metric can be interpreted as a coincidence counter. Notwithstanding the foregoing, in the following section, the intermediate instances of each step of the procedure will also be monitored and analyzed.

2.9 Generalized versions of the superdense coding and teleportation protocols

While in the two previous subsections we worked with the original superdense coding and quantum teleportation protocols, which implies that the former can only process 2 bits at a time, while the latter is only capable of teleporting just one qubit, this time we will use generalized versions of them with no limit on the number of bits and qubits processed, respectively. In this way, thanks to these versions it is possible to work with 24 bits directly, thus we can process all the bits of a pixel of the image to be teleported at the same time.

Figure 9a shows the generalized version of the superdense coding protocol (Mastriani 2023) for 4 bits to be processed simultaneously, where the configuration on the left (underlined in light blue) constitutes what we will call Multi-Classical-to-Quantum interface (Multi-Cl2Qu), while the configuration on the right (underlined in pink) represents the Multi-Quantum-to-Classical interface (Multi-Qu2Cl). Figure 9b represents an extension of the above setting to 24 bits, which is the number of bits that make up one pixel of an 8-bit image for each color channel. In both figures, the bits are incorporated into the Multi-Cl2Qu interface through an arrangement of CNOT gates, with the
exception of the most significant bit, which is incorporated through a Control-Z type gate, where Z is the Pauli phase matrix (Nielsen and Chuang 2004).

Figure 10a represents a configuration for the simultaneous teleportation of 4 qubits which we will refer to as Multi-QTele, that is, it is a natural extension of the simplified version of teleportation in Fig. 4. Figure 10b shows a natural extension of the configuration of Fig. 10a for the case of transmitting 24 qubits simultaneously.

In this way, and thanks to the generalized versions of both protocols, we can arrive at Fig. 11, which implies the possibility of processing and transmitting the full 24 bits of each pixel of the image to be teleported, reusing the configuration of Fig. 8, in which we only have to replace the modules with the original protocols by their respective generalized versions (i.e., Multi-C12Qu, Multi-QTele, and Multi-Qu2Cl).

Fig. 8 Complete teleportation of a digital-3-color-image (Angelina)
Notwithstanding what is stated in this subsection, in the implementations in the Quirk platform (Algassert 2022) that take place in the next section, we will work with the original versions of both protocols, without any loss of generality, since the size of the implementations of both Protocols for 24 bits/qubits would imply figures of such a dimension that it would negatively impact the clarity of the description of the experiments carried out there.

3 Results

3.1 Setup

All experiments of this section consist of implementations on the Quirk® (Algassert 2022) simulator. In the complete architecture for the Quantum Teleportation of Angelina image (Fig. 8), we randomly select 100 of the 1920 × 1080x3 × 8 bits of the original
Fig. 10 Generalization of the quantum teleportation protocol for an arbitrary number of qubits to be teleported: a for 4 qubits, and b a natural extension of the previous one for the case of teleporting 24 qubits.
image, although fewer bits of alternate values (i.e., 0 and 1) and from different locations would be enough.

We will evaluate the performance in the reconstruction of such bits thanks to a coincidence counter, which can be seen in the lower part of Fig. 8.

Finally, we will clarify the number of qubits used in each simulation carried out with the Quirk® simulator, as well as the corresponding circuit lay-out, in order to facilitate the reproduction of all the experiments done here.

3.2 Partial tests

These experiments involve evaluating separately each of the protocols to be used in the final configuration for the Quantum Teleportation of the image, before proceeding with it. Such experiments aim to unmask in an individual way the possible responsible for the collectively incorrect results.

3.2.1 Superdense coding

Based on the protocol of Fig. 5, we will implement the complete Superdense Coding, which is equivalent to the union between Classical-to-Quantum (Cl2Qu) and Quantum-to-Classical (Qu2Cl) interfaces. Figure 12 represents the complete configuration for the Quirk® simulator with the explicit results inside the figure through a series of activation (on) and deactivation (off) flags, from \( \{b_1, b_2\} = \{0,0\} \) to \( \{b_1, b_2\} = \{1,1\} \). The white and yellow rectangle labeled as + \([\mathfrak{t}]\) in the upper left of the four versions of Fig. 12 represents the computational basis states (CBS) generator, which is a built-in module of the Quirk® simulator.

For this experiment, we used 4 qubits on the Quirk® simulator. The total coincidences of the flags (in on or off, at the entrance and exit of the protocol) in the four cases (i.e., 00, 01, 10 and 11) represent the success in the Superdense Coding process. The connection labelled as \textit{send} represents the optical channel in yellow in Fig. 5, although, as it is obvious, within the Cl2Qu interface such a connection does not exist. Finally, 3 points in a row represent the border between both blocks.

The experimental implementation for the transmission of 100 classical bits taken into pairs and their posterior recovering gave us as a result on the Quirk® simulator a complete set of coincidences. It is evident that fewer pairs would have given the same results.
Fig. 12  a Superdense Coding $\equiv \text{Cl2Qu }\cup\text{Qu2Cl}$ on Quirk® for \{b_1,b_2\} = \{0,0\}.  

b Superdense Coding $\equiv \text{Cl2Qu }\cup\text{Qu2Cl}$ on Quirk® for \{b_1,b_2\} = \{1,0\}.  
c Superdense Coding $\equiv \text{Cl2Qu }\cup\text{Qu2Cl}$ on Quirk® for \{b_1,b_2\} = \{0,1\}.  
d Superdense Coding $\equiv \text{Cl2Qu }\cup\text{Qu2Cl}$ on Quirk® for \{b_1,b_2\} = \{1,1\}
3.2.2 Standard quantum teleportation

Figure 13 shows the complete process of Standard Quantum Teleportation (Bennett et al. 1993), which is only implemented thanks to 3 qubits on the Quirk® simulator.

Figure 13a remits us to the case of a generic qubit, which has an arbitrary allocation on the Bloch sphere (Nielsen and Chuang 2004; Kaye et al. 2004). The square block, in the upper left margin of Fig. 13a with a yellow icon inside it similar to the packman symbol, represents a built-in module of the Quirk® simulator intended for the preparation of qubits with capricious orientations on the Bloch sphere.

Figure 13b and c show a pair of particular cases where the qubits are both CBS, i.e., $|0\rangle$ and $|1\rangle$, respectively.
As in the case of Figs. 4 and 5, a single fine line represents a wire carrying one qubit, while a double line represents a wire carrying one classical bit (Nielsen and Chuang 2004; Kaye et al. 2004). The possibility of a fast, precise and graphically expressive simulation on a complete toolbox environment, independently of the pertinent code makes the Quirk® simulator (Algassert 2022) an attractive choice.

Fig. 14  a Simplified Quantum Teleportation for a generic qubit to be teleported. b Simplified Quantum Teleportation for the teleportation of $|0\rangle$. c Simplified Quantum Teleportation for the teleportation of $|1\rangle$. 
3.2.3 Simplified quantum teleportation

Figure 14 shows the Simplified Quantum Teleportation protocol (Mastriani 2018) with identical considerations to the previous case.

3.3 Standard quantum teleportation of a digital image

As we have said, this is the first implementation of a Quantum Teleportation of an image. Figure 15 represents that implementation on the Quirk® simulator (Algassert 2022).

The architecture of Fig. 15 shows the perfect complementation between the Standard Quantum Teleportation and both interfaces, i.e., Cl2Qu and Qu2Cl. This configuration required 8 qubits.

We must bear in mind that at a certain point during the Quantum Teleportation Alice’s side and Bob’s side must be separated a considerable distance. In Fig. 15 that separation has not been incorporated so as not to complicate the graphic.

The 50 pairs of classic bits of type \{00,01,10,11\} were randomly taken from all those that makeup Angelina image, in fact, $1920 \times 1080 \times 3 \times 8$, according to the procedure of Fig. 8. The percentage of coincidences reached 100%.

3.4 Simplified quantum teleportation of a digital image

Figure 16 shows us the economy in the gates utilization of the Simplified version respect to the implementation based on the Standard version of Quantum Teleportation; however, we have obtained similar results in both cases with 8 qubits too.

For this case, we also used 50 pairs of bits randomly selected from the complete set of bits corresponding to the original image of Angelina. Figure 16 represents the four cases of possible pairs that we found during the random exploration of the image. These pairs are the same that we used in the previous case for the Superdense Coding protocol, i.e., \{00, 01, 10, 11\}.

This implementation, like the previous ones, demonstrates the effectiveness and ductility of the Cl2Qu and Qu2Cl interfaces, which can be successfully reused in other areas of Quantum Information Processing (Nielsen and Chuang 2004; Kaye et al. 2004), such as Quantum Computing (Kaye et al. 2004), Quantum Communications (Pathak 2013; Carolaro 2015; Mishra 2016; Imre and Gyongyosi 2012), and the Quantum Internet (Wehner et al. 2018; Castelvecchi 2018; Caleffi et al. 2018, 2020; Kimble 2008; Cacciapuoti et al. 2020a, 2020b; Gyongyosi and Imre 2020, 2019a, 2019b); including undoubtedly, Quantum Cryptography (Sergienko 2006), in general, and Quantum Key Distribution (QKD) (Liao et al. 2017; Bedington et al. 2017; Sibson et al. 2017; Lucamarini et al. 2018), in particular.

4 Discussion

The results of Sect. 3 show the absolute viability of the procedure consisting of the Quantum Teleportation of an image, as well as the complete success of the new interfaces: Cl2Qu and Qu2Cl. Besides, the simplified version of Quantum Teleportation has
Fig. 15  a Standard Quantum Teleportation on Quirk® of a pair \(\{b_1, b_2\} = \{0,0\}\),  
\b Standard Quantum Teleportation on Quirk® of a pair \(\{b_1, b_2\} = \{0,1\}\),  
\c Standard Quantum Teleportation on Quirk® of a pair \(\{b_1, b_2\} = \{1,0\}\),  
\d Standard Quantum Teleportation on Quirk® of a pair \(\{b_1, b_2\} = \{1,1\}\)
demonstrated to have similar results to the standard version, which is plausible considering the fact that it has fewer quantum gates than the standard version. The two Quantum Teleportation versions were successful in both the partial and the collective experiments.

The fact that both integrating experiments begin and end with the binary bits belonging to the pixels of the image to be teleported and the teleported image, respectively, allowed us the use of a coincidence counter as a metric. This constituted an excellent strategy for evaluating the functional quality of the integrating architecture, with the same strictness than those individually used in both Quantum Teleportation protocols and both interfaces.

Another important strategy consisted of the use of the Quirk® simulator in each experiment. It let us know the final outcome following a binary criterion: it works-or-it does not work. This is of great value since knowing the final outcome, whether or not a specific configuration or architecture works, is key before mobilizing a huge number of human resources as well as purchasing and/or replacing expensive laboratory equipment for an eventual optical table.

The findings have deep implications in the context of all type of digital signal transmission, where, such signals can represent streaming, multi and hyper-spectral images, video or future TV broadcasting. Besides, the new interfaces have an excellent projection on Quantum Information Processing (Nielsen and Chuang 2004; Kaye et al. 2004) in general and Quantum Computing (Kaye et al. 2004) in particular, in configurations like CI2Qu-Quantum-Algorithm-Qu2Cl.

Future implementations will be directed to applications outside the laboratory, i.e., practical uses like the Quantum Internet (Wehner et al. 2018; Castelvecchi 2018; Caleffi et al. 2018, 2020; Kimble 2008; Cacciapuoti et al. 2020a, b; Gyongyosi and Imre 2020, 2019a, b). Besides, the projection of the simplified Quantum Teleportation protocol on Quantum Key Distribution (QKD) (Liao et al. 2017; Bedington et al. 2017; Sibson et al. 2017; Lucamarini et al. 2018) is very interesting, since this new protocol does not use classical bits for disambiguation via a classical channel, while increasing considerably the level of security of QKD eliminating an unnecessary exposition on one of the channels commonly used in it.
(a) Simplified Quantum Teleportation on Quirk® of a pair \( \{b_1, b_2\}=\{0,0\} \).

(b) Simplified Quantum Teleportation on Quirk® of a pair \( \{b_1, b_2\}=\{0,1\} \).

(c) Simplified Quantum Teleportation on Quirk® of a pair \( \{b_1, b_2\}=\{1,0\} \).
On the other hand, we must bear in mind that for both integrating architectures we must distribute $1920 \times 1080 \times 3 \times 8/2$ EPR pairs, in other words, a logistical nightmare. Besides, we must consider the following:

- the short half-life of the entanglement due to decoherence (Zurek 2006),
- the short half-life of the qubits (Popkin 2016),
- the type of used qubit (trapped ion, superconductor, topological, etc.) (Linke et al. 2017), and
- the way of distributing the EPR pairs: delivery vs take-out (Mastriani 2022).

Finally, Figs. 12, 13, 14, 15, and 16 were edited specifically to incorporate the lower labels that demarcate each constituent block only for a better understanding of the original Quirk® simulator outcome.

### 5 Conclusions

In this study, a complete configuration for the quantum transmission of digital images has been presented by means of protocols of proven effectiveness on physical machines of superconductors and optical tables.

All the implementations were carried out on a platform called Quirk (Algassert 2022), which was selected due to the following advantages:

1. it has an excellent graphical environment,
2. this allowed us to work in a drag-and-drop way,
3. it is extremely rich in metrics, which allows to quickly evaluate the performance of the implemented protocols, and
4. this is extremely sensitive to the presence of spurious phenomena related to the outcomes of each protocol.

It was precisely this platform that allowed for the first time to expose problems related to the presence of undesirable entanglement (coupling) at the output of internal image representation techniques such as FRQI (Le et al. 2011), NEQR (Zhang et al. 2013) and their derivatives (Sang et al. 2017; Sahin and Yilmaz 2018), where the aforementioned entanglement significantly modifies the result of the measurements between the theoretical versions of these internal image representation techniques and their physical implementations.

Furthermore, in this study a generalization of the superdense coding protocol was used for the first time as a Classical-to-Quantum interface with no limit on the number of bits processed, which allows working with the full 24 bits that represent the total information of each pixel of the digital image to be teleported.

Future work, along the lines of this study, should be applied to digital audio and video transmission, with a particular projection on the future quantum network of networks.
(Wehner et al. 2018; Castelvecchi 2018; Caleffi et al. 2018, 2020; Kimble 2008; Cacciapuoti et al. 2020a, b; Gyongyosi and Imre 2020, 2019a, b).

Acknowledgements M.M. thanks the staff of the Knight Foundation School of Computing and Information Sciences at Florida International University for all their help and support.

Authors’ contributions M.M. conceived the idea and fully developed the theory, wrote the complete manuscript, prepared figures, and reviewed the manuscript.

Funding The author has not disclosed any funding.

Availability of data and materials The experimental data that support the findings of this study are available in ResearchGate with the identifier https://doi.org/10.13140/RG.2.2.17059.53286.

Declarations

Conflict of interest The author declares no competing interests.

Ethical approval Not applicable.

References

Agrawal, P.: Evolution and structure of elementary physical particles. Nat. Sci. 14, 328–342 (2022). https://doi.org/10.4236/ns.2022.148030
Almeida, M.P., Souto Ribeiro, P.H.: Transmission of quantum images through long distances. arXiv: 0312134 (2003)
Almeida, M.P., Caetano, D.P., Souto Ribeiro, P.H.: Manipulation and transmission of quantum images. J. Mod. Opt. 53(5–6), 729–738 (2006). https://doi.org/10.1080/09500340500259839
Aspect, A.: Quantum mechanics: to be or not to be local. Nature 446(7138), 866–867 (2007). https://doi.org/10.1038/446866a
Aspect, A., Grangier, P., Roger, G.: Experimental realization of Einstein–Podolsky–Rosen–Bohm Gedankenexperiment: a new violation of bell’s inequalities. Phys. Rev. Lett. 49(2), 91–94 (1982a). https://doi.org/10.1103/PhysRevLett.49.91
Teleporting digital images

Aspect, A., Dalibard, J., Roger, G.: Experimental test of bell’s inequalities using time-varying analyzers. Phys. Rev. Lett. 49(25), 1804–1807 (1982). https://doi.org/10.1103/PhysRevLett.49.1804

Barrett, M.D., et al.: Deterministic quantum teleportation of atomic qubits. Nature 429, 737–739 (2004). https://doi.org/10.1038/nature02608

Bedington, R., Arrazola, J.M., Ling, A.: Progress in satellite quantum key distribution. NPJ Quantum Inf. 3, 30 (2017). https://doi.org/10.1038/s41534-017-0031-5

Bell, J.: On the Einstein Podolsky Rosen paradox. Phys. Phys. Fizika 1(3), 195–200 (1964). https://doi.org/10.1103/PhysiqueFizika.1.195

Bennett, C.H., et al.: Teleporting an unknown quantum state via dual classic and Einstein–Podolsky–Rosen channels. Phys. Rev. Lett. 70, 1895–1899 (1993). https://doi.org/10.1103/PhysRevLett.70.1895

Boschi, D., et al.: Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein–Podolsky–Rosen channels. Phys. Rev. Lett. 80, 1121–1125 (1998). https://doi.org/10.1103/PhysRevLett.80.1121

Bouwmeester, D., et al.: Experimental quantum teleportation. Nature 390, 575–579 (1997). https://doi.org/10.1038/37539

Busch, P., et al.: Quantum Measurement. Springer, New York (2016)

Cacciapuoti, A.S., Caleffi, M., Cataliotti, F.S., Gherardini, S., Tafuri, F., Bianchi, G.: The quantum internet: networking challenges in distributed quantum computing. IEEE Netw. 34(1), 137–143 (2020a)

Cacciapuoti, A.S., Caleffi, M., Van Meter, R., Hanzo, L.: When entanglement meets classical communications: quantum teleportation for the quantum internet. IEEE Trans. Commun. 68(6), 3808–3833 (2020b)

Caleffi, M., Cacciapuoti, A.S., Bianchi, G.: Quantum internet: from communication to distributed computing. In: NANOCOM ’18: Proceedings of the 5th ACM International Conference on Nanoscale Computing and Communication, Vol. 3, pp. 1–4 https://doi.org/10.1145/3233188.3233224 (2018)

Caleffi, M., Chandra, D., Cuomo, D., Hassanpour, S., Cacciapuoti, A.S.: The rise of the quantum internet. IEEE Comput. 53(06), 67–72 (2020)

Cariolaro, G.: Quantum Communications. Springer International Publishing, N.Y. (2015)

Carlo, G.G., Benenti, G., Casati, G.: Teleportation in a noisy environment: a quantum trajectories approach. Phys. Rev. Lett. 91, 25 (2003). https://doi.org/10.1103/PhysRevLett.91.257903

Castelvecchi, D.: Here’s what the quantum internet has in store: physicists say this futuristic, super-secure network could be useful long before it reaches technological maturity. Nat. News (2018). https://doi.org/10.1038/d41586-018-07129-y

Chen, L.: Quantum discord of thermal two-photon orbital angular momentum state: mimicking teleportation to transmit an image. Light Sci. Appl. 10, 148 (2021). https://doi.org/10.1038/s41377-021-00585-8

Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete. Phys. Rev. 47(10), 777–780 (1935). https://doi.org/10.1103/PhysRev.47.777

Einstein, A., Lorentz, H.A., Minkowski, H., Weyl, H.: The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity. Courier Dover Publications, New York (1952)

Furusawa, A., et al.: Unconditional quantum teleportation. Science 282, 706–709 (1998). https://doi.org/10.1126/science.282.5389.706

Gilev, P.A., Popov, I.Y.: Quantum image transmission based on linear elements. Nanosyst. Phys. Chem. Math. 10(4), 410–414 (2019). https://doi.org/10.17586/2220-8054-2019-10-4-410-414

Gonzalez, R.C., Woods, R.E.: Digital Image Processing, 2nd edn. Prentice-Hall, Englewood Cliffs (2002)

Gonzalez, R.C., Woods, R.E., Eddins, S.L.: Digital Image Processing Using Matlab. Pearson Prentice Hall, Upper Saddle River (2004)

Gyongyosi, L., Imre, S.: Entanglement access control for the quantum internet. Quantum Inf. Process. 18, 107 (2019a)

Gyongyosi, L., Imre, S.: Opportunistic entanglement distribution for the quantum internet. Nat. Sci. Rep. 9, 2219 (2019b)

Gyongyosi, L., Imre, S.: Entanglement accessibility measures for the quantum internet. Quantum Inf. Process. 19, 115 (2020)

Hedemann, S.R.: Noise-Resistant Quantum Teleportation, Ansibles, and the No-Projector Theorem. arXiv:1605.09233v1 (2016)

Hotta, M.: Energy entanglement relation for quantum energy teleportation. Phys. Lett. A 374(34), 3416–3421 (2010). https://doi.org/10.1016/j.physleta.2010.06.058

Hu, M., et al.: Deterministic quantum teleportation through fiber channels. Sci. Adv. 4, 10 (2018). https://doi.org/10.1126/sciadv.aas9401

IBM Quantum Experience https://quantum-computing.ibm.com/. Last accessed 5 April 2022
Imre, S., Gyongyosi, L.: Advanced Quantum Communications: An Engineering Approach. Wiley-IEEE Press, New York (2012)

Ivanov, S.S., Gilev, P.A., Popov, I.Y.: On the efficiency of quantum error correction for quantum image transmission algorithm. Pramana J. Phys. 96, 211 (2022). https://doi.org/10.1007/s12043-022-02454-4

Jain, A.K.: Fundamentals of Digital Image Processing. Prentice-Hall. Englewood Cliffs (1989)

Janani, T., Brindha, M.: secure medical image transmission scheme aided by quantum representation. J. Inf. Secur. Appl. 59, 102832 (2021). https://doi.org/10.1016/j.jisa.2021.102832

Kaye, P., Laffamme, R., Mosca, M.: An Introduction to Quantum Computing. Oxford University Press, Oxford (2004)

Kiktenko, E.O., Fedorov, A.K., Manko, V.I.: Teleportation in an indivisible quantum system. Quantum Meas. Quantum Metrol. 3(1), 1–5 (2016a). https://doi.org/10.1515/qmetro-2016-0003

Kiktenko, E.O., Popov, A.A., Fedorov, A.K.: Bidirectional imperfect quantum teleportation with a single Bell state. Phys. Rev. A 93(6), 062305 (2016b). https://doi.org/10.1103/PhysRevA.93.062305

Kimble, H.J.: The quantum internet. Nature 453, 1023–1030 (2008). https://doi.org/10.1038/nature07127

Le, P.Q., Dong, F., Hirota, K.: A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. Quantum Inf. Process. 10, 63–84 (2011). https://doi.org/10.1007/s11128-010-0177-y

Liao, S.-K., et al.: Satellite-to-ground quantum key distribution. Nature 549, 43–47 (2017). https://doi.org/10.1038/nature23655

Linke, N.M., et al.: Experimental comparison of two quantum computing architectures. PNAS 114(13), 3305–3310 (2017). https://doi.org/10.1073/pnas.1618020114

Lucamarini, M., et al.: Overcoming the rate–distance limit of quantum key distribution without quantum repeaters. Nature 557, 400–403 (2018). https://doi.org/10.1038/s41586-018-0066-6

Ma, X.-S., et al.: Experimental quantum teleportation over a high-loss free-space channel. Opt. Express 20(21), 23126–23137 (2012). https://doi.org/10.1364/OE.20.023126

Marzolino, U., Buchleitner, A.: Performances and robustness of quantum teleportation with identical particles. Proc. Math. Phys. Eng. Sci. 472, 2185 (2016). https://doi.org/10.1098/rspa.2015.0621

Mastriani, M.: Quantum Boolean image denoising. Quantum Inf. Process. 14(5), 1647–1673 (2015). https://doi.org/10.1007/s11128-014-0881-0

Mastriani, M.: Systholic Boolean orthonormalizer network in wavelet domain for SAR image despeckling. WSEAS Trans. Signal Process. 4(3), 5714–6125 (2016)

Mastriani, M.: Quantum image processing? Quantum Inf. Process. 16, 27 (2017). https://doi.org/10.1007/s11128-016-1457-y

Mastriani, M.: simplified protocol of quantum teleportation. J. Quantum Inf. Sci. 8, 107–120 (2018). https://doi.org/10.4236/jqis.2018.83007

Mastriani, M.: Quantum image processing: the pros and cons of the techniques for the internal representation of the image. A reply to: a comment on “quantum image processing?” Quantum Inf. Process. 19, 156 (2020). https://doi.org/10.1007/s11128-020-02653-1

Mastriani, M.: How can a random phenomenon between particles be synchronized instantaneously and independently of the distance between said particles? Opt. Quant. Electron. 54, 235 (2022). https://doi.org/10.1007/s11082-022-03590-2

Mastriani, M.: Quantum key secure communication protocol via enhanced superdense coding. Opt. Quant. Electron. 55, 10 (2023). https://doi.org/10.1007/s11082-022-04303-5

Mastriani, M., Iyengar, S.S., Kumar, L.: Analysis of five techniques for the internal representation of a digital image inside a quantum processor. SN Comput. Sci. 2, 450 (2021). https://doi.org/10.1007/s42979-021-00847-7

MATLAB® Mathworks https://www.mathworks.com/products/matlab.html. Last accessed 5 April 2022

Mishra, V.K.: An Introduction to Quantum Communication. Momentum Press, New York (2016)

Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2004)

Pathak, A.: Elements of Quantum Computation and Quantum Communication. Taylor & Francis Group, Boca Raton (2013)

Phillips, A.C.: Introduction to Quantum Mechanics. Wiley, New York (2003)

Popkin, G.: Scientists are close to building a quantum computer that can beat a conventional one (2016). https://doi.org/10.1126/science.aal0442. Accessed 16 Jan 2023

Quirk Algsassert https://algassert.com/quirk. Last accessed 5 April 2022

Riebe, M., et al.: Deterministic quantum teleportation with atoms. Nature 429, 734–737 (2004). https://doi.org/10.1038/nature02570

Sahin, E., Yilmaz, I.: QRMW: quantum representation of multi wavelength images. Turk. J. Electr. Eng. Comput. Sci. 26, 768–779 (2018). https://doi.org/10.3906/elk-1705-396
Sang, J.Z., Wang, S., Li, Q.: A novel quantum representation of color digital images. Quantum Inf. Process. 16, 14 (2017). https://doi.org/10.1007/s11128-016-1463-0

Schalkoff, R.J.: Digital Image Processing and Computer Vision. Wiley, New York (1989)

Sergienko, A.V.: Quantum Communications and Cryptography. Taylor and Francis, London (2006)

Sherson, J.F., et al.: Quantum teleportation between light and matter. Nature 443, 557–560 (2006). https://doi.org/10.1038/nature05136

Sibson, P., et al.: Chip-based quantum key distribution. Nat. Commun. 8, 13984 (2017). https://doi.org/10.1038/ncomms13984

Su, J., Guo, X., Liu, C., Lu, S., Li, L.: An improved novel quantum image representation and its experimental test on IBM quantum experience. Sci. Rep. 11, 13879 (2021). https://doi.org/10.1038/s41598-021-93471-7

van Houwelingen, J.A.W., et al.: Experimental quantum teleportation with a 3-bell-state analyzer. Phys. Rev. A 74, 11 (2006). https://doi.org/10.1103/PhysRevA.74.022303

Wehner, S., Elkouss, D., Hanson, R.: Quantum internet: a vision for the road ahead. Science 362(6412), eaam9288 (2018). https://doi.org/10.1126/science.aam9288

Williams, B.P., Sadlier, R.J., Humble, T.S.: Superdense coding over optical fiber links with complete bell-state measurements. Phys. Rev. Lett. 118(5), 050501 (2017). https://doi.org/10.1103/PhysRevLett.118.050501

Wootters, W.K., Zurek, W.H.: A single quantum cannot be cloned. Nature 299, 802–803 (1982). https://doi.org/10.1038/299802a0

Yang, J., et al.: Experimental quantum teleportation and multi-photon entanglement via interfering narrowband photon sources. Phys. Rev. A 80(4), 042321 (2009). https://doi.org/10.1103/PhysRevA.80.042321

Zhang, Y., et al.: NEQR: a novel enhanced quantum representation of digital images. Quantum Inf. Process 12, 2833–2860 (2013). https://doi.org/10.1007/s11128-013-0567-z

Zurek, W.H.: Decoherence and the transition from quantum to classical—revisited. In: Duplantier, B., Raimond, J.M., Rivasseau, V. (eds.) Quantum Decoherence Progress in Mathematical Physics, vol. 48. Birkhäuser Basel, New York (2006). https://doi.org/10.1007/978-3-7643-7808-0_1

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.