Simulation of raindrop shapes and its interaction with electromagnetic wave

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Abstract. Radio waves propagating through a rain zone will undergo scattering, depolarization, absorption and delay in time. All these effects on wave propagation are related to the frequency at which the signal is transmitted and as well as the rain parameters, a good knowledge of these hydrometeor parameters is therefore of great importance in the modeling of their effects on radio communication. These parameters can be measured with micro rain radar (MRR). The MRR instrument is a versatile equipment for collecting rain parameter. In this study these parameters were obtained using a MATLAB software to develop a rain radar simulator, this was found necessary because of the inadequacy of micro rain radar instrument in many locations of interest, thus the simulator will facilitate ubiquitous use and application in weather and radio wave propagation and augment the available MRR equipment. The simulation result shows the oblate spheroidal raindrop shape. The axis ratio was obtained and compared with those obtained by other researcher. This raindrop parameter was further used to simulate "the interaction of raindrop with electromagnetic wave and the raindrop reflectivity was obtained. The simulation was validated by comparing the reflectivity obtained with that measured by the MRR equipment.

Keywords: Radiowave, simulation, reflectivity, raindrops

1. Introduction

The demand for bandwidth has increased globally as a result of advancement of telecommunication, remote sensing and other forms of radio communication. This has made frequency reuse attractive. But hydrometeor such as rain has constituted the major impairment to radio wave transmission at microwave band. Therefore there is need for the understanding of rainfall parameters to better model radiowave transmission in this band. Rain parameters which include raindrop shapes, drop size, reflectivity and canting angle, have profound effects on waves propagating at the extremely high frequencies ranging from 5GHz to 100 GHz [1]. The MRR have been used to retrieve these properties of raindrop. This study determined these parameter by simulating the micro radar measurement process by using a Mathlab based programme.

1.1 Raindrop Shape

Raindrop shapes is an important parameters needed for evaluating radio wave propagation effects in rain for both terrestrial and satellite systems operating in the microwave and millimeter wave frequencies [2]. In the case of linearly polarized electromagnetic wave, the specific attenuation for horizontal and vertical polarizations will differ because of the non-spherical shapes of raindrops, the
horizontal being higher than the vertical because the “most probable” shapes have the horizontal dimension larger than the vertical and the drops tend to fall with their symmetry axes almost aligned with the vertical [3]. Initial studies on shapes of raindrop were observed from a meteorological aspect, due to its relevant to various subjects in meteorology, such as fall velocity and drop breakup. A lot of researches have been carried out using the photographic measurements of the drop shape. Various measurement of raindrop made while suspended in the air stream of a vertical wind tunnel and at terminal velocity after falling from a sufficient height (~12 m) in stagnant air revealed that water drops larger than about 1.0 mm in radius were of oblate spheroidal shape with a flattened base [4].

Several researchers have studied the drop shape theoretically by solving the drop surface internal and external pressure balance equation. However due to the difficulty involved in solving this equation, several assumptions or approximations had to be introduced. In 1971, Pruppacher and Pitter introduced some assumptions into the aerodynamic pressure around the surface of a water sphere, and numerically solved the pressure-balance equation.

Figure 1 shows the drop shapes of 13 water drops whose equivolume radii range from 0.25 to 3.25 mm as observed by Pruppacher and Pitter. The shapes observed are in agreement with those derived from measurements made with equipment available as at that time. Recent advancement in technology and the advent of weather radar and other measuring equipment has made it possible for accurate measurement of rainfall parameters. The use of two-dimensional video disdrometers under calm-air conditions by Narekar, & Bhalerao [5] in recent time showed that rain drop shapes is an oblate spherical shape with no concave deformation for large drops as indicate by the Pruppacher–Pitter model.

![Figure 1. The thirteen raindrop sizes and Shapes](image)

### 1.2 Electromagnetic Wave and Raindrop Interaction

Rain influences the propagation of electromagnetic waves. It affects the propagation of electromagnetic waves by absorbing part of the incident energy which is thus converted to heat and by scattering the electromagnetic energy [13]. This strong effect of rain is observed in the microwave
frequency band, this is due to the fact that, raindrops diameter which ranges from 0.5 to 7mm exhibit Mie scattering in the region, the wavelength are close to the diameter of the raindrops [14, 15]. In addition, this frequency band raindrops exhibit a strong absorption characteristics. This dielectric property of Raindrops and its effects on electromagnetic wave can be explained by the Debye’s formula [13].

Apart from the fraction of the incident electric field that rain scatters, rain also absorbs energy which causes attenuation [12]. This has greater effect for higher frequencies electromagnetic waves.

1.3 MATLAB SIMULATION OF RAINDROP

Raindrop of different shape and sizes were created by solving the numerical equation obtained by bread and Chuang’s model for raindrop shape [11] which specified drop shapes as:

\[ r(\theta) = a[1 + \sum_{n=0}^{10} C_n \cos(n\theta)] \] (1)

where \(a\) is the radius of the undistorted sphere, located at the center of mass of the drop, \(\theta = 0^\circ\) pointing vertically downward. The value for \(C_n\) which is the shape coefficient was obtained as in table 1

| \(a\) (mm) | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|------------|----|----|----|----|----|----|----|----|----|----|----|
| 0.5        | -28| -30| -83| -22| -3 | 2  | 1  | 0  | 0  | 0  | 0  |
| 1          | -134| -118| -345| -100| -5 | 17 | 16 | -1 | -3 | -1 | 1  |
| 1.5        | -297| -247| -816| -160| 24 | 52 | 13 | -8 | -8 | -1 | 4  |
| 2          | -481| -359| -1268| -220| 90 | -75| -3 | -15| -21| 7  | 10 |
| 2.5        | -665| -445| -1674| -230| 95 | -80| -4 | -15| -21| 11 | 17 |
| 3.0        | -843| -472| -2040| -240| 100| -90| -21| -73| -20| 25 | 24 |
| 3.5        | -1014| -492| -2164| -250| 111| -90| 40 | -93| -12| 43 | 30 |
| 4.0        | -1087| -482| -2250| -290| 120| -171| -100| -107| 2  | 64 | 32 |

The oblate spherical shape of the raindrop were obtained as in Figure 2. The obtained shape was found to be in agreement with those obtained by the Prauppacher and Pitter model.
Figure 2. Raindrop shape obtained for radius 1mm to 4mm

Furthermore the major and semi major axes of the raindrop shapes were calculated using the equation for the cross sectional area of the distorted ellipsoid obtained as:

$$A = \pi a^2 \left[ 1 + 2C_0 + C_0^2 + \sum_{n=1}^{10} \frac{C_n^2}{2} \right]$$  \hspace{1cm} (2)

where $a$ is the semi-axis along x,y.

The cross sectional area of the undistorted ellipsoid is:

$$A = \pi ab$$  \hspace{1cm} (3)

$b$ is the semi-axis along z.

Equating (2) and (3), we obtain the equation for $b$ as

$$b = a \left[ 1 + 2C_0 + C_0^2 + \sum_{n=1}^{10} \frac{C_n^2}{2} \right]$$  \hspace{1cm} (4)

Table 2 presents the radius of the major and semi major axes as well as the axial ratios obtained using equation 4.

| A    | b     | C_0  | b/a   |
|------|-------|------|-------|
| 1.   | 0.9751| -341 | 0.9751|
| 2.   | 1.8297| -481 | 0.9149|
| 3.   | 2.5431| -843 | 0.8477|
| 4.   | 3.2053| -1087| 0.8013|

The axis ratio obtained with this simulator were compared with those obtained by other researchers as shown in table 3.
| a (mm) | b/a   | Prappacher and Pitter | and | Green | Beard |
|-------|-------|-----------------------|-----|-------|-------|
| 1.0   | 0.98  | 0.98                  | 0.98| 0.98  | 0.98  |
| 2.0   | 0.9149| 0.92                  | 0.92| 0.92  | 0.92  |
| 3.0   | 0.8477| 0.85                  | 0.85| 0.85  | 0.85  |
| 4.0   | 0.8013| 0.77                  | 0.77| 0.77  | 0.77  |

3. SIMULATION OF REFLECTIVITY

The reflectivity, one of the rainfall parameters obtained by the MRR was simulated by adopting a model developed by Schroder [7]. This was done by sending electromagnetic wave represented as the Maxwell equation through raindrops and analyzing the backscattered signal. The input parameter to the simulator which is substitute as the values for the MRR system where applicable are shown in Table 4.

The simulator was developed in modules so as to enable easy bugging and debugging. The simulation algorithm consists of five main functional parts. The first part of the simulator generate a beam resolution cell through which the rain transverse over time by using the MRR radar parameter, the second part uses the rain parameters and the log normal approximation for rain drop size distribution to generate a T-Matrix consisting of the diameter vector and number of drops of various diameters while in the third part of the simulator all the drops generated in the second part are placed randomly in the cell developed in the first part and in the fourth part the coherent Electric field return is calculated. The fifth part calculates all the rain parameters and present the results in plots.

INPUT PARAMETER FOR THE RADAR SIMULATOR

Table 4. Parameters for the Radar Simulator

| Parameters                      | MRR  |
|--------------------------------|------|
| Transmitted power (mW)         | 50   |
| Frequency of operation (GHz)   | 24   |
| Antenna gain (dB)              | 20   |
| Minimum raindrop diameter (mm) | 8    |
| Maximum raindrop diameter (mm) | 0.5  |
| Range (km)                     | 0.5  |
| Wavelength                     | 2.5  |

The performance of the simulator was validated by comparing the reflectivity obtained from the simulator with those measured by the MRR equipment, although the simulator assume a continuous stable rain event, a close relation was found between the simulator reflectivity and the MRR measured reflectivity as shown in Figure 4.
4. Conclusion

In this work, raindrops of different shapes and sizes have been simulated and a radar simulator has been developed using Mathlab software. The characteristics of the micro rain radar system were taken into account in the development of the simulator so as to provide a means for validation. The simulator has proven to be a very useful tool for the investigation of the interaction of electromagnetic wave with raindrops and for the characterization of rainfall parameters.

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