We investigate by numerical simulations the behavior of the power dissipated in a resistive load capacitively coupled to a Josephson flux flow oscillator and compare the results to those obtained for a d.c. coupled purely resistive load. Assuming realistic values for the parameters $R$ and $C$, both in the high- and in the low-$T_c$ case the power is large enough to allow the operation of such a device in applications.
I. INTRODUCTION

Low noise measurements in the sub-millimeter range, e.g., in radioastronomy, require a stable, low noise local oscillator for the receiver. The Flux Flow Oscillator (FFO), a device made with a long Josephson junction, has been proposed as a good candidate for this application \[1\]. Some of the most important features of such a device are the following \[2,3\]: the output power is relatively large \(0.1 - 1 \mu W\), the oscillator can be easily tuned in a wide band \(75 - 500 \text{GHz}\) by varying the bias and a magnetic field, and the emitted signal has a very narrow linewidth \(130 \text{kHz} \text{ at } 70 \text{GHz} \[4\], less than 2.1 \text{MHz} \text{ in the band } 280 - 330 \text{GHz} \[5\]). As the signal generated by a local oscillator has to be coupled to a mixer or to a transmission line, in the literature different couplings to a load have been realized and studied \[3,5–10\]. In some works, in particular, the FFO has been d.c. coupled to a small junction acting as a detector, or, possibly, as a mixer; in this case, the real drawback is that the junctions cannot be biased independently. Capacitive coupling not only overcomes this drawback, but it also allows to increase the power transferred to the load, since it eliminates the d.c. loading of the oscillator. In this paper we investigate and compare these two coupling techniques for an FFO. We note in passing that ours is not the first work to propose the use of a d.c. block between oscillator and load device–see, e.g., \[3\]. However, in contrast with \[3\], which described a complete and detailed integrated subsystem, we focus specifically on the effects of the coupling element on system performance.

II. THE PHYSICAL DEVICE - THE FLUX FLOW OSCILLATOR

The FFO consists of a long Josephson junction biased by a d.c. current \(I_B\) and driven by the effect of a magnetic field \(H_e\), perpendicular to the length \(L\) of the junction and parallel to the barrier, into a dynamical state in which the unidirectional motion of flux quanta (Josephson current vortices, or fluxons) takes place \[11\]. With reference to the configuration sketched in Fig. \[1\] the fluxons continuously penetrate from the left edge of the junction and
propagate to the right; this regime corresponds to the appearance of typical branches in the $I-V$ characteristic. An array of vortices travels in the junction with phase velocity $u = (V_{dc}/\tau d\mu_0 H_e)\tau$, where $V_{dc}$ is the average voltage across the junction, $H_e$ is the external magnetic field applied in the $-y$ direction, $\tau$ is the propagation velocity of electromagnetic waves in the junction and $d$ is the effective magnetic thickness of the barrier. The largest signal in the flux flow regime is obtained biasing the junction at the top of the flux flow branch in the $I-V$ characteristic; the corresponding voltage is determined by the velocity matching condition: the velocity $u$ approaches the Swihart velocity $\tau\mu_0$ and, hence, $V_{dc} \approx (\tau d\mu_0) H_e$. One should take into account that a possible self magnetic field and the focusing effect of the external magnetic field could make $V_{dc}$ larger. We shall assume that in the stationary state the expression for the junction voltage is given by the Fourier series

$$v(x, t) = V_{dc} + \sum_n V_n(x) \exp(jn\omega_{FF} t),$$

(1)

where the coefficients $V_n$ are complex amplitudes and the $\sum_n'$ denotes summation from $n = -\infty$ to $n = +\infty$, but without the term $n = 0$, and $\omega_{FF} = (2\pi/\Phi_0)V_{dc}$ is the fundamental angular frequency of the series. Since the amplitude of the fundamental component of the series is dominant, the frequency of the FFO signal is

$$f_{FF} = V_{dc}/\Phi_0.$$

(2)

This is just the frequency expected from the Josephson relations when a d.c. voltage $V_{dc}$ is applied across a junction.

An important parameter for an oscillator is the frequency range in which it can generate a signal useful for applications, or, in other terms, its effective bandwidth. Since, under the velocity matching condition ($u \to \tau\mu_0$), the frequency of the FFO is proportional to the magnetic field, $f_{FF} = (\tau d\mu_0/\Phi_0)H_e$, the effective bandwidth of the FFO is bounded to the values of the magnetic field within which the flux flow dynamic state is stable. Its lower limit is given by the minimum value of magnetic field needed to have penetration of fluxons into the junction, $i.e.$, by the critical field $H_{e\text{min}} = 2j\lambda J$. The upper limit can be estimated
by supposing the maximum voltage in the flux flow regime to be of the order of the voltage gap (or of the order of $R_nI_c$ in the high-$T_c$ context), as the nonlinear internal dynamics is strongly attenuated above that value. The tuning of the FFO to the desired frequency is achieved by changing the magnetic field to move the flux flow branch and by setting the d.c. current to bias the junction near the top of the branch.

### III. MODEL AND COMPUTATIONAL TECHNIQUES

In Fig. 1 there is a schematic representation of a long Josephson junction, the FFO, that produces a signal coupled to the load through a capacitance. For this investigation the load device is assumed to be purely resistive. This assumption is made with the hypothesis that such a load model is sufficiently accurate to give information on the practical advantages of the capacitive coupling in comparison with simple d.c. coupling; for other loads, an appropriate model could be chosen and analyzed using the same approach.

The mathematical model describing the flux-flow oscillator is the perturbed sine-Gordon equation (PSGE), which in normalized form is

$$\varphi_{xx} - \varphi_{tt} - \sin \varphi = \alpha\varphi_t - \beta\varphi_{xt} - \gamma,$$

with the boundary conditions

$$\varphi_x(0, t) + \beta\varphi_{xt}(0, t) = -\eta,$$

$$\varphi_x(L, t) + \beta\varphi_{xt}(L, t) = -\eta - i_L(t).$$

Here, $\varphi$ is the phase difference between the junction electrodes, $x$ is the spatial coordinate normalized to the Josephson penetration length $\lambda_J$, $t$ is the time normalized to the inverse of the Josephson angular plasma frequency $\omega_J$, the subscripts indicate partial differentiation, the term in $\alpha$ represents shunt loss due to quasiparticle tunneling (here assumed ohmic), the term in $\beta$ represents surface loss in the junction electrodes, $\gamma$ is the distributed bias current $j_B$ normalized to the critical current density $j_c$, $L$ is the normalized junction length, $\eta$ is the
external magnetic field in the plane of junction and perpendicular to its long dimension in units normalized to $j_c \lambda_J$. In the $R-C$ load network $i_L$ is the current normalized to $j_c \lambda_J w$ (where $w$ is the width of the junction), $R_L$ is the load resistance normalized to the (linear) characteristic impedance of the junction $Z_0$, $C_L$ is the coupling capacitance normalized to the capacitance $C_0 = 1/\omega_J Z_0$ and $\omega_L = 1/R_L C_L$ is the load angular frequency normalized to $\omega_J$; accordingly, the equation for the current $i_L$ is

$$\frac{di_L}{dt} = -\omega_L i_L + \frac{\varphi_D(L, t)}{R_L}. \quad (5)$$

IV. NUMERICAL RESULTS

Eqs. (3, 5) with the boundary conditions Eqs. (4a, 4b) have been integrated numerically using a 4th order Runge-Kutta algorithm on a spatially discretized counterpart, varying $R_L$ from 0.1 to $10^3$ and $C_L$ from $10^{-2}$ to 10. In the following we list the most relevant results. We note in passing that Fig. 1 depicts schematically a sandwich-type tunnel junction structure. Although the fabrication technology for high-$T_c$ tunnel junctions is not yet completely mature, significant progress in this direction is being made \[13\]. Accordingly, in what follows we have chosen model parameter values that are aimed in the direction of describing a lightly-hysteretic tunnel junction.

In Fig. 2 we report the Fourier spectrum of the voltage at the right edge of the junction $v(L, t)$ calculated over more than two hundred periods. The spectrum consists of a d.c. component, whose height is the average voltage $V_{dc}$, a fundamental line at the frequency $f_{FF}$, having amplitude $V_{ac}(f_{FF})$, and a number of harmonics, up to the 7th; the small background is due to the finite sampling, to the finite integration time and to numerical noise. We remark that the fundamental frequency turns out to be just equal to what one would calculate from the height of the d.c. component (taking into account the normalization), in full agreement with Eq. (2). Moreover, one sees that the fundamental harmonic is dominant and the amplitude of the other harmonics decreases exponentially with increasing order, as can also
be inferred qualitatively from the inset in the same figure. We did not calculate the voltage spectrum for every value of the load used in our simulations; rather, we performed for most of them a simpler test to check that the series (1) could be truncated to the second term. In fact, we calculated the rms amplitude of the fundamental harmonic $V_{ac}(f_{FF})/\sqrt{2}$ from the signal $v(L,t)$ using the formula for the Fourier series coefficient and the rms value of the a.c. component of the signal $v_{ac}(L,t) \equiv v(L,t) - V$; the comparison shows that they are always equal within 1%, so that in practice all the power is in the fundamental harmonic. In other words, in the case of the $R-C$ load the flux flow signal $v_{ac}(L,t)$ maintains, with good approximation, a sinusoidal form for every value examined of $R_L$ and $C_L$. Therefore, the output power $P_L$ is assumed to be the power of the first harmonic of the signal transferred from the flux flow oscillator to the resistive load $R_L$. For our $R-C$ loading network it is given by

$$P_L = \frac{V_{ac}^2(f_{FF})}{2R_L \left[1 + \left(\frac{\omega_L}{\omega_{FF}}\right)^2\right]}.$$  

(6)

Here $P_L$ is normalized to the Josephson power $P_u = V_J^2/Z_0$, where $V_J = \Phi_0 \omega_J/2\pi$ is the normalizing Josephson voltage.

We compared the output power for the $R-C$ load to that of d.c. coupling to a resistive load, which has been already studied by Zhang [14], to emphasize the different behavior. We have, in our model, the pure-$R$ load case by setting $\omega_L = 0$ in Eq. (5) and Eq. (6). For the sake of consistency, we used the same parameters of Ref. [14], i.e., $\alpha = 0.25$, $\beta = 0.005$, $L = 20$, biasing the junction with $\gamma = 1.25$ on a flux flow step obtained with $\eta = 4$; the results essentially agree with those in [14], differences being attributable to the fact that with purely resistive coupling the output waveform is no longer a clean sinusoid, so the total power differs from the first-harmonic power. The maximum output power dissipated by the a.c. component is $P_{L_{\text{max}}} = 0.76$ for $R_L = 7$, and changes very little in the range 3–10 (we note parenthetically that these numbers are reminiscent of those obtained in an early study [15] of a resonant-fluxon oscillator with pure-$R$ loading). This value is represented in Fig. 3 and Fig. 4 as a horizontal straight line for the sake of comparison with the $R-C$ case.
The $R-C$ case is, at first sight, more complex, as has already been noted by Zhang (see Section 4.4 of [14]). In Fig. 3 we report the output power as a function of the resistance $R_L$ for fixed values of the ratio $\omega_{FF}/\omega_L$. We see that the power increases as the oscillator frequency grows with respect to the load characteristic frequency, and that it is not a monotonic function of the resistance. In fact, maximum output power is obtained for $R_L \simeq 1$, i.e., when the load resistance $R_L$ is close to the junction characteristic impedance $Z_0$. This result is consistent with what one should expect from considering the best matching condition in the framework of (linear) microwave transmission line theory. We remark also that the variation of the output power is negligible for $\omega_{FF}/\omega_L \geq 5$. In Fig. 3 the comparison with the curve with $P_L = 0.76$ shows in which range the $R-C$ loaded FFO is more efficient than the $R$ loaded FFO.

In Fig. 4, in order to provide a more straightforward tool for device design, we plot the output power as a function of the load capacitance for fixed values of the load resistance $R_L$. For a given value of $R_L$, the power is first an increasing function of $C_L$ which, however, quickly saturates. The asymptotic values lie on the curve with $\omega_{FF}/\omega_L = 100$ of Fig. 3, which well approximates the case of infinite capacitance.

Whereas, as mentioned above, the $R-C$ case appears at first to be more complicated than the pure-$R$ case, in fact it is simpler: once the values of $V_{ac}$ and $\omega_{FF}$ in Eq. 6 are established by the junction dynamics, at least for the parameter values that we have studied, the oscillator behaves with respect to the load much as a simple linear oscillator, characterized by a fixed internal voltage and a fixed internal resistance equal to $Z_0$. This operational simplicity is obtained essentially from the elimination of the d.c. loading of the oscillator, which, instead, is present in the pure-$R$ case.

V. DISCUSSION

As we have seen in the previous section, in order to maximize the output power for a given value of the resistance, it is recommendable to increase the load capacitance, because
the best coupling is obtained for $C_L \geq 1$. To check whether this indication can be translated into realistic parameters to design and fabricate a device, we shall estimate what one should expect in two practical cases. In both cases we shall estimate $C_p$ from the formula $C_p = \epsilon_r \epsilon_0 S / d$, with the usual meaning of the parameters. First, we shall consider the high-$T_c$ case, and we shall suppose that the coupling capacitor is a typical silicon monoxide element ($\epsilon_r \cong 5$, $d = 600$ nm and $S = 50\mu m \times 50\mu m$). Since $\omega_J = \sqrt{2\pi j_c / (\Phi_0 C)}$ and $Z_0 = (1/w)\sqrt{\mu_0 d/C}$ where $C$ is the junction capacitance per unit area, the previous definitions give:

$$C_L = \frac{C_p}{wC} \sqrt{\frac{2\pi \mu_0 j_c d}{\Phi_0}}. \quad (7)$$

Assuming from the literature $j_c = 20 \text{ kA/cm}^2$, $w = 200$ nm, $d = 280$ nm, $C = 30 \text{ fF/\mu m}^2$, we finally get $C_L = 15.7$. To evaluate the power dissipated in the load, we can use the same figures, and find that $P_u = 0.13 \mu W$; this gives an idea of the order of magnitude of the power that could be extracted.

Next, let us consider a low-$T_c$ oscillator. In this case, the coupling capacitor can be made by growing a niobium oxide film by anodization ($\epsilon_r \cong 25$, $d = 100$ nm). From the previous formula, assuming typical parameters for Nb-AlO$_x$-Nb Josephson junctions ($j_c = 1 \text{ kA/cm}^2$, $w = 3 \mu m$, $d = 80$ nm, $C = 60 \text{ fF/\mu m}^2$), we find $C_L = 2.3$. Of course, in this case one is left with the problem of fabricating a load resistor that should be perhaps 0.1 ohm (or less), but this is feasible with present-day technology; moreover, our numerical simulations should be considered as being merely indicative, in that the dissipation in a low-$T_c$ junction can be significantly lower than the value considered in this paper, which might bring about qualitative changes in the dynamics. Nevertheless, we think that it is interesting to take into account a design based on the well established niobium technology, and we shall explore this topic in more detail in the future.

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† Electronic address: parment@vaxsa.csied.unisa.it

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FIGURES

FIG. 1. Schematic layout of flux-flow oscillator capacitively coupled to a load.

FIG. 2. Fourier spectrum of the output voltage. The waveform is shown in the inset. The parameters used in this calculation are: \( R_L = 1.5 \), \( C_L = 0.01 \), \( V_{dc} = 4.218 \), \( \eta = 4 \), \( \alpha = 0.25 \), \( \beta = 0.005 \), \( \gamma = 1.25 \), \( L = 20 \). The amplitude is normalized to \( V_J = \Phi_0 \omega_J / 2\pi \); the frequency is normalized to \( f_{FF} = 0.671 \).

FIG. 3. Output power as a function of load resistance for different values of the ratio between the FFO frequency and the \( R - C \) characteristic frequency. Here \( \eta = 4 \), \( \alpha = 0.25 \), \( \beta = 0.005 \), \( \gamma = 1.25 \), \( L = 20 \).

FIG. 4. Output power as a function of load capacitance for different values of the load resistance. Parameters are as in Fig. 3.
The diagram illustrates the relationship between $P_L$ (in normalized units) and $R_L$ (in normalized units) for different values of $\omega_{FF} / \omega_L$. The peak $P_L_{max}$ in the R-load case is indicated by an arrow. The curves represent different values of $\omega_{FF} / \omega_L$: 100, 10, 5, 2, and 1.
$P_L$ (normalized units) vs. $C_L$ (normalized units) for different values of $R_L$: 1.0, 1.5, 2.0, 2.5, and 5.0. The graph shows the maximum $P_L$ in the R-load case ($P_{L_{\text{max}}}$) for each $R_L$ value.