Research of the Sound-Absorbing Structureshape Influence on the SAS Acoustic Efficiency in the Nonlinear Operation Mode

P V Pisarev\textsuperscript{1a} and K A Maksimova\textsuperscript{1b}

\textsuperscript{1}Perm National Research Polytechnic University (PNRPU), Komsomolsky Prospect, 29, Perm, Russia

E-mail: \textsuperscript{a}pisarev85@live.ru (corresponding author), \textsuperscript{b}karina-maksimova0402@yandex.ru

Abstract. This research is focused on the acoustic efficiency of various shape Helmholtz resonators on non-linear operating modes, with an acoustic pressure level of 130 dB. In this work physical and mathematical models for predicting the effective acoustic properties of SAS cells are formulated. Numerical studies have been performed to assess the acoustic efficacy of honeycomb and cone-shaped sound-absorbing structures samples in an interferometer with a normal incidence of a sound wave in a nonlinear operation mode. The validation of the developed mathematical models was carried out. A comparison of the numerical calculation results with the results of laboratory studies has been carried out. According to the results of the conducted computing experiments, the most effective resonators were identified, and recommendations on the SAS design based on the most effective resonators were formulated.

1. Introduction
Resonant sound-absorbing structures (SAS) are widely used to reduce the noise of railway and water transport, highways, construction, industrial equipment, testing grounds, and aircraft engines. Such SAS consist of sets of cells - Helmholtz resonators, located between the power and perforated shells. A large number of works [1–5] are devoted to the numerical simulation of the sound waves propagation in the free volume of Helmholtz resonators and the adjacent channel. First of all, they consider the calculation of the resonant frequency and the influence of the chamber volume and the cavity of the resonator on its acoustic efficiency.

Many works [6–8] are devoted to the improvement of the SAS characteristics, but they are based on the traditional increase in the noise attenuation coefficient due to the use of multilayer SASs (three, four and five layers). However, multilayer SASs have a number of disadvantages, such as mediocre acoustic characteristics; high density; the complexity and high cost of manufacture; large mass and size. In this regard, the design of highly effective SAS demands developing new methods and approaches for the study of acoustic processes in the SAS cells.

Of particular interest are the new ways to control the impedance of cells and SAS. For example, in [9], the authors discovered the influence of the constructive design of the entrance to the resonator (neck geometry) and the shape of the resonator on its resonant frequency, as well as on the value of the acoustic pressure loss coefficient [10]. In [11–15], the authors describe the effect of the mutual influence of Helmholtz resonators of various shapes. In addition, solutions are known in which new
forms of SAS cells are proposed, allowing to achieve a significant increase in the acoustic pressure loss coefficient [16].

Analysis of the literature revealed that the study of the SAS cells acoustic efficiency in the nonlinear operation mode is not enough. At the same time, the study of the resonator shapes influence on the SAS acoustic efficiency in nonlinear operating modes has not been studied until now.

In this work, the results of the acoustic efficiency of the Helmholtz resonators of various shapes on nonlinear operating modes are given, with the acoustic pressure level of 130 dB. In the numerical experiments, the normal propagation of a sound harmonic wave in a cylindrical channel with cone-shaped Helmholtz cell resonators was simulated. The computational domain included the space inside the SAS cell and a part of the interferometer channel with a normal incidence of a 150 mm long acoustic wave. The calculation of the sound absorption coefficient produced by the resonators in the operating frequency range of 500–3000 Hz was carried out on the basis of a numerical solution of the Navier-Stokes equation.

2. Conducting Numerical Studies of the SAS Acoustic Characteristics

To conduct computing experiments two geometric models were built. In the model 1, a single cell resonator is considered, and model 2 is a single cone-shaped resonator. Both models contain a 150 mm long interferometer free volume. The geometric characteristics of the cellular and conical resonators were taken in such a way that the internal volume of the cells was the same and equal to 1622.77 mm$^3$. The perforation thickness is 1 mm, the perforation hole diameter is 2 mm, the perforation percentage is 1.2%. A general view of the geometric models is presented in Figure 1.

![Figure 1. Geometric models: a – with a prismatic resonator; b – with cone-shaped resonators.](image)

For better convergence of the solution and reduction of the errors in the results obtained, a computational grid was used, the cells of which have a shape close to an equilateral tetrahedron. The grid thickened in the neck area of the resonator so that the neck height was 20 cells. With the distance from the neck, an increase in the linear dimensions of the element occurred until the average linear dimension of 2 mm was reached. Additionally, thickening on the wall of 20 layers with a growth factor of 1.2 was used. As a result, a calculated grid of 120,014 elements was obtained. Among other things, when grinding the grid, sharp differences in the geometric dimensions of neighboring cells were avoided – the linear dimensions of neighboring cells do not differ by more than 2 times [17].

To solve this problem, a mathematical model has been formulated that describes the normal propagation of a sound harmonic wave in a cylindrical channel. The simulation of these processes is based on the direct numerical solution of the non-stationary Navier-Stokes equation taking into account compressibility and viscous losses [18]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot \mathbf{\tau},$$

$$\frac{\partial}{\partial t} (\rho \mathbf{E}) + \nabla \cdot (\mathbf{v} (\rho \mathbf{E} + p)) = \nabla \cdot (\mathbf{\chi} \nabla T + \mathbf{\tau} \cdot \mathbf{v}).$$
Viscous stress tensor is defined by the formula:

$$\tau = \eta \left[ (\nabla \ddot{v} + \nabla \ddot{v}^T) - \frac{2}{3} \nabla \cdot \ddot{v} I \right]$$  \hspace{1cm} (2)

The total energy is:

$$E = h - \frac{p}{\rho} + \frac{v^2}{2}$$  \hspace{1cm} (3)

where $\rho$ is the density; $\ddot{v}$ is velocity vector; $p$ is pressure; $T$ is the temperature; $h$ is the enthalpy; $\eta$ is the molecular viscosity; $\chi$ is thermal conductivity; $I$ is the unit vector. To close the system of equations (1) – (3), the perfect gas state equation is used.

Figure 2 presents the boundary conditions scheme.

As the boundary conditions, an unsteady pressure change was set at the input boundary, through which the dynamics was modeled. When calculated using the full Navier-Stokes system in Ansys Fluent, the boundary conditions at the input section $S_1$ are:

$$\ddot{v}\big|_{S_2} = 0; \quad p\big|_{S_1} = P_{rand}(t); \quad T\big|_{S_1} = 300K,$$

where $P_{rand}(t)$ is a time function with a uniform frequency spectrum in a given frequency range of 500 – 3000 Hz. Density is determined by a given pressure and temperature from the perfect gas state equation. Reflected sound waves also exited through this boundary condition. The boundary condition of the rigid wall $S_2$ was used on the wall of the pipe and the sample.

To calculate the impedance, pressure signals were recorded at the control points (microphones) $P_1$, $P_2$. Microphones were located at some distance from the sample. In the calculations, the acoustic pressure was determined on the wall of the interferometer at the points corresponding to the central axis of the microphones. Acoustic characteristics of the SAS were calculated from the recorded pressure signals. The calculations were carried out with a time step of 1/65536 seconds for 0.5 second. Signals obtained in the frequency domain were processed by the transfer function method, similar to experiment.

According to the results of computational experiments, the dependences of the sound absorption coefficient on the frequency were obtained (Fig. 3).
The analysis of dependences showed that the greatest value of sound absorption coefficient is observed for a cone-shaped resonator and is 0.86 at a resonant frequency of 1240 Hz. For a honeycomb resonator, the sound absorption coefficient was 0.81 at a resonant frequency of 1490 Hz. An analysis of the operating frequencies of the studied SAS cells revealed that the honeycomb one is more broadband than cone-shaped. The operating frequency range for a cone-shaped cell was 1200÷1350 Hz, for a honeycomb cell - 1376÷1621 Hz with a sound absorption coefficient of 0.6.

3. Experimental Determination of the SAS Samples Acoustic Characteristics

As part of the validation of the numerical model, the results of computational experiments were compared with the results of laboratory tests conducted on an interferometer with a normal incidence of the sound wave in the nonlinear mode.

Acoustic testing in an interferometer with a normal fall of sound waves is widespread due to the relative simplicity of the experiment. The experimental setup is a tube of circular cross section, at one end of which a SAS sample is located, on the other – a speaker that irradiates the sample with acoustic waves (Fig. 4). Also, at a certain distance from the SAS sample, microphones were installed flush with the wall of the interferometer, which record the acoustic pressure of incident and reflected waves in time (Fig. 5). Next, the recorded pressure is processed by the Fast Fourier Transform algorithm, as a result of which the impedance of the SAS sample is calculated.

Impedance connects the pressure and the normal velocity at the boundary, the acoustic properties of which it characterizes. In most works, the influence of high sound pressure levels of an incident wave is described in terms of the velocity of the particles of a medium in an orifice [19–23]. The basis of most semi-empirical models are the works of Crandall and Melling. Crandall [23] gives an expression for the impedance of a circular hole, and Melling [24] explores the effect of high sound pressure levels, which leads to non-linear behavior of the SAS.

According to the definition, the dimensionless specific acoustic impedance is expressed as the ratio

$$Z = X + iY = \frac{p}{\rho cu_n}$$

(5)

where $p$ is the acoustic pressure; $\rho$ is the air density; $c$ is the speed of sound in the air; $u_n$ is the acoustic speed.

Despite the seeming simplicity of the formula (5), the determination of the impedance of a sound-absorbing structure in practice is a difficult task. To date, preference is given to the experimental determination of the impedance of one or another SAS sample.

Usually, two microphones are used for measurements, since the “transfer function method” used to determine the impedance is the easiest to calculate. At the same time, this option can be used if there is
only a piston mode in the channel, which leads to the dependence of the frequency range of the installation on the channel dimensions of the impedance tube.

Interferometer characteristics:
• sound pressure up to 160 dB;
• frequency range 500-6400 Hz;
• The size of the test sample is 30 mm.

**Figure 4.** Interferometer loaded and ready for operation.

1 – speaker;  
2 – collar fixing the sample;  
3 – SAS sample;  
4 – stock;  
5 – guide sleeve.

**Figure 5.** The internal channel of the interferometer.

For carrying out laboratory experiments, SAS samples were made by 3D printing [25], which represent a cylindrical “patch” (Fig. 6).

**Figure 6.** SAS samples after printing: a – honeycomb, b – cone.

According to the results of experimental studies of the two-configuration SAS samples (honeycomb and cone) acoustic characteristics, the dependences of sound absorption coefficient on the frequency were obtained (Fig. 7).

**Figure 7.** A plot of sound absorption coefficient dependences on the frequency.
The analysis of the obtained dependence revealed that the sound absorption coefficient of the cone-shaped cell is 0.77 at the resonant frequency of 1272 Hz, the honeycomb cell sound absorption coefficient is 0.74 at the resonant frequency of 1512 Hz. Thus, a comparative analysis of the results of numerical calculations with the results of laboratory tests revealed that for both models the discrepancy in sound absorption coefficient is no more than 9%, in frequency - no more than 5%.

4. Conclusion
According to the results of the research, a mathematical and numerical model has been developed to evaluate the acoustic efficiency of sound-absorbing structures with resonators of various shapes. The calculation of the damping effect produced by various shapes resonators in the operating frequency range of 500-3000 Hz has been carried out. The sound absorption coefficient of a cone-shaped cell is 5% higher than that of a honeycomb cell. The influence of the resonator geometry on its acoustic efficiency and resonant frequency with the same volume is revealed. A comparative analysis of the results of numerical calculations with the results of laboratory tests revealed that for both models, the discrepancy in sound absorption coefficient is not more than 9%, in frequency - not more than 5%. At the next stages of the work, it is planned to conduct a study of a group of cone-shaped cells operation.

5. References
[1] Selamet A, Lee I-J 2003 Helmholtz resonator with extended neck J. Acoust. Soc. Am. 113 1975–1985
[2] Selamet A, Xu M B, Lee I-J, Huff N T 2005 Helmholtz resonator lined with absorbing material J. Acoust. Soc. Am. 117 725-733
[3] Tang S K 2005 On Helmholtz resonators with tapered necks J. Sound Vib. 279 1085-1096
[4] Griffin S, Lane S A, Huybrechts S 2001 Coupled Helmholtz resonators for acoustic attenuation J. Vib. Acoust. 123 11-17
[5] Park S H 2013 Acoustic properties of micro-perforated panel absorbers backed by Helmholtz resonators for the improvement of low-frequency sound absorption J. Sound. Vib. 332 4895-4911
[6] Postnov V I, Viakin V N, Veshkin E A 2011 Research and optimization of the choice of sound-absorbing structures VESTNIK of Samara University. Aerospace and Mechanical Engineering 3-3(27) 55-64
[7] Tang X, Yan X 2017 Acoustic energy absorption properties of fibrous materials: A review, Composites: Part A 101 360-380
[8] Zhelezina G F, BeyderE Ya, Raskutin A E, Migunov V P, Stolyankov Yu V 2012 Materials for sound-absorbing structures of aircrafts All Materials. Encylopedic Directory 4 12-16
[9] Pisarev P V 2016 Biconical Cell The certificate on official registration of the computer programs 2016616458
[10] Pisarev P V, Anoshkin A N, Pan’kov A A 2016 Effect of neck geometry of resonance cells on noise reduction efficiency in sound absorbing structures Citation: AIP Conference Proceedings 1770
[11] Pisarev P V, Pankov A A, Anoshkin A N 2015 Study of the effect of the distance between two Helmholtz resonators on the level of acoustic pressure in a model channel Mathematical modeling in the natural sciences 1 354-357
[12] Pisarev P V, Pankov A A, Anoshkin A N 2017 Numerical calculation of the acoustic efficiency of the active emitter and the resonant cell when working together Aerospace engineering, high technologies and innovations 1 212-215
[13] Pisarev P V, Anoshkin A N 2018 Numerical study of the acoustic efficiency of a group of Helmholtz resonators of various configurations MATEC Web of Conferences 243
[14] Pisarev P V, Maksimova K A 2019 Interaction of Resonators with a Normal Fall of a Sound Wave Scientific and Technical Volga Region Bulletin 4 90-92
Acknowledgments
The study was carried out at the Perm National Research Polytechnic University with the support of the Russian Science Foundation (Project No. 18-79-00295).