Difficulties with inflationary initial conditions

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Abstract

We consider the question of deriving initial conditions for scalar fields that can drive both an early and late, quintessence dark energy, inflationary phase. Current notions of quantum cosmology have difficulty in determining suitably displaced scalar fields with vastly differing energy scales. Due to finite causal length constraints the homogeneity of the dark energy field also presents an unresolved uniformity problem.

Some further specific concerns with kinetic, phantom and assisted inflationary models are outlined, especially when used as possible dark energy candidates.

We review the use of the canonical measure in predicting a single phase of inflation and find the negative conclusions of Gibbons and Turok can be allayed by means of a reasonable quantum cosmological input.

Further attempts at incorporating inflationary schemes into bouncing or cyclic models are considered. Without some imposed low-entropy boundary condition at the bounce any subsequent inflationary phase is difficult to countenance.

PACS numbers: 04.20, 98.80 Bp
1.0 Scalar field driven cosmology

Although inflationary cosmology is at present the established model for the early universe and is apparently in agreement with observational evidence from WMAP [1] the underlying assumptions are still far from being understood: essentially as we lack a full quantum gravity description of the phenomena.

There is also much speculation as to the possible cause of the apparent acceleration of the universe. Various explanation have been postulated for this phenomena - see e.g.[2] for reviews. One possible model that has received much attention is to use a further, so-called quintessence, scalar field to drive an inflationary expansion, rather like that postulated for the early universe but now at a vastly reduced energy scale. One advantage of this scheme is that the two inflationary stages are caused by the same basic mechanism. We later at times mention the alternative use of a cosmological constant $\Lambda$ in driving the present acceleration - although problems with obtaining a suitable value for $\Lambda$ are well established [2,3].

The present cosmological paradigm we then consider is outlined in Fig.(1),

$\rightarrow$ Inflation(1) $\rightarrow$ non-Inflation $\rightarrow$ Inflation(2) $\rightarrow$ ?

Fig. 1: Rough schematic of a model of the universe with, at least, two inflationary phases. The preceding and subsequent points of evolution are poorly understood.

where inflation(1) is caused by, say, a scalar field $\phi$ during the early universe and inflation(2) a further field $\Phi$ dominating in the universe today.

The simplest chaotic version of inflation uses a displaced scalar field to violate the strong energy condition [4-6]. During this time the potential $V(\phi)$ dominates over the kinetic and spatial gradient terms. In the simplest FRW model the energy density and pressure are given by e.g.[5,6]

$$\rho = \frac{\dot{\phi}^2}{2} + \frac{(\nabla \phi)^2}{2a^2} + V(\phi)$$

(1)
\[ p = \frac{\dot{\phi}^2}{2} - \frac{(\nabla \phi)^2}{6a^2} - V(\phi) \]  

(2)

If, instead, initially the kinetic energy dominates it decays rapidly as a stiff fluid \( \dot{\phi}^2 \propto a^{-6} \), while a displaced scalar field itself only decays logarithmically slowly \( \phi \propto \ln(t) \): so one can expect an eventual inflationary phase [4]. The spatial gradient \((\nabla \phi)^2\) term falls as \( \sim a^{-2} \) so again any initially displaced and slowly changing field can be expected to eventually dominate. This spatial gradient alone behaves roughly like a perfect fluid \( p = (\gamma - 1)\rho \) with \( \gamma = 2/3 \) so is itself on the verge of inflationary expansion, actually \( a \sim t [7,8] \). A single kinetic term behaves like a perfect fluid with stiff equation of state \( \gamma = 2 \).

Note that in scalar field driven cosmology one might argue that the presence of a potential term \( V(\phi) \) is actually necessary to prevent an inhomogeneous scalar field simply producing an everlasting coasting solution \( a \sim t \).

The field \( \phi \) has a different time dependence when the universe is dominated by a non-inflationary matter source, say with radiation. For a massive scalar field case the solution of the

\[ \ddot{\phi} + 3H \dot{\phi} = -m^2 \phi \]  

(3)

field equation, with \( H = 1/2t \) for radiation, is given in the slow-rolling limit by [9]

\[ \phi \simeq \phi_i \exp(-\frac{1}{3}m^2 t^2) \]  

(4)

The field says approximately constant for times \( t < m^{-1} \). For \( m \sim 10^{-5} \), (the required value for perturbations [5,6]) inflation needs to proceed within the roll down time, now \( 10^5 t_{pl} \) in Planck units, if the field is still to be displaced sufficiently from its minimum. This can impose a fine tuning of the parameters if inflation(2) is also to be driven by a displaced scalar field and is preceded by a long period of non-inflationary behaviour.

Arguments both from classical equipartition reasoning or quantum notions of likelihood have been advanced to justify inflationary conditions during the early universe - see e.g[5,6,10]. For example quantum cosmology might be able to provide an initial large scalar field \( \phi \) over an initial patch of sufficient size \( \sim H^{-1} \), with \( H \) being the corresponding Hubble parameter. This size is required since a negative pressure is susceptible to any positive pressures surrounding it: a large size allows more time for expansion to dominate before any equalization processes can occur [7,11]. These arguments are far from rigorous and few conclusions can be reached about the actual likelihood of inflation happening without many underlying assumptions.
These quantum arguments are often in contraction to the requirements of a late universe inflationary stage. We wish to point out this and a number of further difficulties that then become apparent.

2.0 Initial conditions for two subsequent $V(\phi)$ inflationary stages.

In order for the field $\Phi$ to cause the present apparent inflationary acceleration it must be smooth over a patch size $L > H_o^{-1}$ where $H_o$ is the present Hubble parameter. To justify this uniformity one might like to make use of the earlier inflation(1). This then requires that the field $\Phi$ be present during the earlier inflationary epoch and so displaced from its minimum also beyond the initial patch size $H^{-1}$; otherwise it would simply be red-shifted away just as a spatial gradient. If instead the field $\Phi$ is produced later it cannot be expected, without introducing a further horizon problem, to be homogeneous over the present visible size of the universe, it would have some smaller coherence length over which uniformity could be justified - this would correspond to the particle horizon size commencing from the time the field $\Phi$ was first produced. There are related concerns if a cosmological constant $\Lambda$ is postulated to drive inflation(2) but with the further issue that symmetry breaking contributions during the early universe should have vastly exceeded the required $\Lambda \sim 10^{-120}$ value cf. section (2.6) in ref.[12].

Topological defects might also be expected to form if the field $\Phi$ is spontaneously broken - see also sec. 28.3 in ref.[13] for a related discussion. We ignore the case where inflation might be formed within the cores of topological defects [14] - so allowing instead $L < 1/H$. This might be relevant for inflation(1) but is unlikely to be suitable for inflation(2) - we know for related “universe in a lab” work that the resulting inflationary universe expands not within the existing space but into a new evolving region [15]. Note that we generally try and work within a single Copernican principled model that obeys (near) spatial homogeneity - so allowing the foliation as outlined in Fig.(1).

If the universe evolves from an initial Planck size nugget a quantum cosmological calculation using the Wheeler-DeWitt (WDW) equation, with tunnelling boundary conditions, for a spatially closed model can give an initial factor e.g. [5,6,10]

$$\Psi_T \sim \exp \left( -\frac{1}{V(\phi) + V(\Phi)} \right)$$  \hspace{1cm} (5)
which is peaked at $V(\phi) + V(\Phi) \sim M_{pl}^4$. Such arguments seem unlikely to explain the vast discrepancy in scale between initial values for $V(\Phi)$ and $V(\phi)$. An alternative Hartle-Hawking condition gives a corresponding + sign in the exponent and is seemingly less conducive to inflationary initial conditions [10] - see however ref.[16] and section(3).

If the potential $V(\Phi)$ is taken to be a massive scalar field i.e. $V(\phi) = 1/2m^2\Phi^2$ then the large Compton wavelength of the effective mass $m \sim 10^{-33}eV$ is of the order of the of the present size of the observable universe [2]. This might be inconsistent with the early universe’s limited causal horizon size during inflation(1) cf.[17].

At the least a more rigorous quantum version of the inflationary no-hair property [18,19] would be required to justify if the light field can be sufficiently smoothed during the $\phi$ driven inflationary phase, or indeed, just by simply expansionary behaviour. Note that already at the classical level there are some ways of evading a full no-hair property - see e.g.[20]: the remnant hair from a tilted Bianchi model has been proposed as one way of simulating dark energy [21].

There are some alternative tracker potentials that considerably reduce this initial discrepancy between $V(\phi)$ and $V(\Phi)$ but the requirement that the two inflationary stages should be distinct still imposes the condition that initially $V(\phi) >> V(\Phi)$. This discrepancy argument is therefore fairly immune to the specific form of the quintessence potential - see e.g. [2,22-24] for various examples.

If we take $V(\phi) = m^2\phi^2$ and $m \sim 10^{-5}$ for fluctuations, then to prevent $V(\Phi)$ inflating when inflation(1) is finishing at $\phi \sim 1$ means that $V(\Phi) < \sim 10^{-10}M_{pl}^4$. With the exponential function in eq.(5) this is a large initial discrepancy to overcome. Roughly speaking one is requiring Hartle-Hawking, or suppression, like boundary conditions for the $\Phi$ field and Tunnelling, or enhancing, ones for the high energy $\phi$ field.

Alternatively, one can consider quantum cosmology with compact flat or open cosmologies [25-27] then the exponential suppression is lost and the action $S \propto \sqrt{V(\phi)}$ [27]. Although we presently lack a principle to impose a boundary condition in these cases, a small action suggests that smaller $V(\phi)$ are preferred. It still remains unclear why two fields with vastly different actions are being produced.

There is also a possible complication that quantum fluctuations in the field $\Phi$, having the same value as those in $\phi$ i.e. $\delta\Phi \sim \delta\phi \sim H$, will cause
diffusive behaviour in the field $\Phi$. This can cause the field $\Phi$ to grow to larger values so in turn allowing the potential $V(\Phi)$ to become dominant earlier than expected [28,29]. This has been used as an argument to constrain the amount of inflation allowed, but by doing so it is in danger of counteracting the standard inflationary no-hair property: if too much or too little inflation is a problem then it simply reintroduces a further fine tuning problem. A similar problem would occur if quantum fluctuations are growing during the inflationary phase cf.[30].

The field $\Phi$ also has to be immune to being further jumbled up during the turbulent reheating phase at the end of inflation: so the two fields remaining totally uncoupled.

Alternatively, if one appeals to an infinite universe with random initial conditions it likewise will be difficult to explain the presence of two such fields appearing displaced within one causal patch. Indeed if there were say $N$ fields, as often expected in higher dimensional quantum gravity models, plus $\phi$ present, one might expect the universe to have as some time $N$ subsequent stages of inflation prepared by the first inflationary expansion $\phi$: a surfeit of inflationary stages each at diminishing energy scale.

2.1 Extension to assisted inflation

A closely related form of inflation is assisted inflation where a number of fields $N$, each of which is too steep to cause inflation by itself, can increase the friction so giving an overall inflationary expansion [31]. A quantum cosmology argument would be required to see if such initially displaced fields can be expected. Because the potential of the WDW equation is no longer necessarily isolated from the origin at zero scale factor the usual boundary conditions cannot give the typical $\sim \exp(\pm 1/V(\phi))$ factors - somewhat similar to the previously mentioned flat and open models. This also occurs with a classical signature change when the forbidden region is also absent: the initial measure might then be uniform in $\phi$ [32]. We here ignore the $N$ spatial gradient terms which is a further complication.

There seems a more serious problem, however, if inflation is to be driven by $N$ steep fields driven in concert. During the previous non-inflationary phase the fields still individually roll down the potential, so the corresponding roll down time $t_r$ is still comparatively short for, say large mass scalar fields where $t_r \propto m^{-1}$. For exponential type potentials [33] that have no absolute minimum this would mean that initially the fields are having to have very large potentials, say at energy scale $\rho$ to provide some eventual inflationary behaviour at a smaller energy scale $\rho_0$. But in this case they should have
provided an earlier inflationary phase back at energy scale $\rho$, unless other matter fields present contrive to prevent this. In summary, it appears less likely that assisted inflation can provide inflation(2) due to this more severe fine tuning.

### 2.2 Phantom inflation

A further, and more extreme, type of inflation is caused by phantom matter where the equation of state gives $\gamma < 0$ so that the weak energy condition is also violated. This is a more extreme pole-law expansion that produces a future big-rip singularity: it therefore was generally discounted [34] for inflation(1): one reason being that the fluctuations typically have an unwanted blue spectrum for increasing Hubble parameter towards the impending singularity [34,35].

A simple example of phantom inflation is to switch the sign of the kinetic energy term [39]. However, the corresponding switch in the spatial gradient term causes the spatial gradient to now contribute a potentially dangerous positive pressure term cf. eq.(2). In fact it contributes a term analogous to having a positive curvature $k = +1$ present. The model is therefore more susceptible to collapse than the standard scalar field model where inhomogeneity still contributes a negative pressure e.g.[7]. One might argue that if phantom is only being used for inflation(2), this inhomogeneity can be suppressed by the earlier inflation(1), but with the previous outlined provisos that we presently lack a fully quantum version of the no-hair property.

A further difficulty is that phantom driven inflation requires the presence of a potential that is driven up during the phantom phase e.g.[40]. The starting value on the potential has to be near the minimum so the phantom field climbs up the potential; this is in contrast to usual inflation where a large displaced potential is required. Again if inflation(1) is scalar field driven and inflation(2) phantom these complementary starting points on the various potentials are problematic. If phantom is to be inflation (2) there is again a severe problem with fine tuning: since the energy density of the phantom grows with scale factor the presence of the earlier inflation should have expanded the energy density of the initial phantom, of say effective $\gamma = -1/3$, by the total increase in the scale factor $\sim 10^{50}$ times.\(^2\)

\(^1\)More recently in the limit of $\gamma << 0$ a near scale invariant spectrum can also be obtained [36]; or else an additional scalar might source a slightly red perturbation spectrum [37]. Though for $\gamma < -2/3$ there is possibly an insufficient no-hair property due to a growing classical perturbation mode causing inhomogeneity [38].

\(^2\) We assume that the scale factor grows a factor $\sim 10^{25}$ during each of the inflationary
degree of fine tuning is therefore a factor $\sim 10^{50}$ times more than for the previous example using a massive scalar field with standard inflation.

### 2.3 Kinetic-inflation

A related example is k-inflation for either the early universe [41] or as dark energy [42]. One includes a number of higher order derivative terms: so being in some sense a generalization of the previous phantom case. Some kinetic terms still require negative signs in order to drive an inflationary phase without the need of an explicit potential $V(\phi)$ term. However, instead new arbitrary dimensional constants have to be introduced to compensate dimensionally for the unusual higher order kinetic terms so obviating some of the possible advantages of these models. Also, unlike in standard potential driven inflation the corresponding spatial gradient terms can potentially become the more dominant ones. Initially at small initial scale factor the largest derivative term will dominate. For a fourth derivative term the spatial derivative will be $\pm (\nabla \phi)^4$ which contributes a term $\pm a^{-4}$. Depending on the sign this is either a positive or negative radiation term. A six derivative will give likewise a positive or negative stiff fluid. Negative terms can push the model out of bounds and restrict the generality of the corresponding cosmic no-hair property of such models [43].

One can also consider 2nd order derivative terms by means of an arbitrary function of the D'Alembertian operator - so-called box inflation [44]. This requires a closer analysis to see how spatial gradient terms behave and whether it is compatible with Ostrogradski's theorem - see e.g.[45].

With Born-Infeld type terms an effective square root on the kinetic term is present see e.g.[46]. This causes the corresponding spatial gradient term to potentially only fall off as $\sim a^{-1}$, so simulating a perfect fluid with inflationary $\gamma = 1/3$ equation of state. There is now a danger that a suitably inhomogeneous field would instead cause perpetual spatial gradient driven inflation. Note that in this limit the speed of sound can diverge to infinity $c_s >> 1$ [47], which might have problems with causality [48].

Assuming an homogeneous field we can consider some simplified models of kinetic inflation. The kinetic Lagrangian or pressure $p$ is given by a term [41],

$$p = F(X) \quad (6)$$

with $X = \dot{\phi}^2/2$ for a spatially homogeneous field. For a usual scalar field and non-inflationary phases.
\( p = X \). To take a particular example

\[
p = aX + bX^2
\]  

(7)

In order for the pressure to be somewhere negative one of the constants \( a \) or \( b \) has to be taken negative. Since the energy density is also of the form [41]

\[
\rho = aX + 3bX^2
\]  

(8)

one can also get negative energy densities. In general the quantities are related by an expression \( \rho = 2Xp, X - p \) with comma representing derivative w.r.t \( X \). When \( p, X = 0 \) there is a possible de Sitter solution, \( p = -\rho \). For the simple case \( a = -1 = -b \) this occurs for \( X = 1/2 \). The equation of state now depends on the value of the kinetic energy. For large \( X \) we get in this case a radiation equation of state, while for \( 1/3 < X < 1/2 \) there is phantom like behaviour. For \( X < 1/3 \) the energy density is negative. One can see this change by again looking at the scalar field equation [41,46]

\[
\ddot{\phi} + 3Hc_s^2 \dot{\phi} = 0
\]  

(9)

where the speed of sound is defined as \( c_s^2 = (2X - 1)/(6X - 1) \). Solving this equation the kinetic energy decays with scale factor as \( \dot{\phi}^2 \propto a^{-6c_s^2} \), showing how the kinetic energy behaves less stiff as \( c_s^2 \) is reduced and becoming de Sitter like as \( c_s^2 \to 0 \).

This dependence on \( X \) can be contrasted with usual potential driven inflation where the equation of state depends on the slope of the potential only: especially for an exponential potential. The quantum boundary conditions try to impose a large displacement of the potential, and suppress the corresponding kinetic energy [10]. Roughly speaking the kinetic inflation model above having more variety is less suitable since, for example, large initial \( X \) might allow the universe to re-collapse before the de Sitter value \( X = 1/2 \) is ever approached. In the phantom range, which might be unstable [49], the scalar field will be driven up any scalar potential \( V(\phi) \) present since in the slow roll approximation the field equation takes the form, e.g.[46]

\[
3p, X H \dot{\phi} = -V'(\phi)
\]  

(10)

A very cursory attempt at obtaining the corresponding WDW equation for such a \( F(X) \) finds that the resulting equation is a highly non-linear wave
equation with some aspects of the Boussinesq or equation of transverse vibration e.g. [50]. There will be a number of arbitrary constants to determine and as the solutions are so dependent on the actual value of $X$ it will require a more specific, than in the usual potential driven case, quantum boundary condition proposal to make any real predictions. If kinetic inflation is to provide inflation (2) the kinetic terms provide a extra, and probably unwanted non-inflationary component in the early universe: or generally with equation of state $p = \rho/(2n - 1)$ for a $p = X^n$ term. Again it would be problematic for quantum cosmology to give a displaced potential for inflation (1) together with suitable kinetic terms to later drive inflation (2).

The kinetic driven model, if used for inflation (1), anyway has to be amended since it is attracted to the de Sitter value and would permanently inflate. One tries to introduce functions of the field $\phi$ and allow their evolution to change the energy density of the inflationary phase. Because of gravitational wave constraints the energy density must be below $\sim 10^{-10}M_{pl}^4$ around $\sim 40$ e-folding from the end of inflation [5,6]. A simplified factorized version of this could be $p = K(\phi)F(X)$. For this case the function $K(\phi)$ does not determine the actual equation of state but $K(\phi)$ should be initially displaced from its minimum so further evolution can occur. This again will be difficult to determine with quantum cosmological arguments since $K(\phi)$ is not responsible for violating any energy conditions per se. There is some sleight of hand with this model since it still depends on the values of $\phi$ even when an explicit potential term is excluded: this can be quantified using the canonical measure where a further ambiguity due to $\phi$ will be introduced.

### 3.0 Classical and quantum measures for inflation

The conditions that enable a single inflationary phase to proceed have recently again been disputed. Originally using a classical canonical measure $\omega$ [52,53] the probability of inflation was found to be arbitrary, although the flatness problem could be resolved for potentials unsuitable for inflation: so the flatness problem did not strictly require inflation for its resolution. However, in doing so the measure has to appeal to energy densities vastly exceeding Planck values where the classical equations would be expected to be superseded [54]. The early universe is then dominated by extremely large post-Planckian values of particularly $\dot{a}$. This in turn sets the kinetic energy to be extremely large in order to alone solve the flatness problem and give
a present energy density $\sim 10^{-30} g cm^{-3}$. See also [55,56] for some further issues regarding the validity of this measure.

Gibbons and Turok [57] wish to further resolve the ambiguity as to whether inflation occurs or not. Firstly, they have placed a cut-off for values of the scale factor, or flatness $\Omega \sim 1$ that cannot be distinguished experimentally. This seems to place a rather restrictive selection effect upon the measure. Unlike the simple anthropic principle e.g.[58] observers are now having to decide what they can or cannot measure. More subtle future experiments might overcome this limitation. Indeed one might argue that in order to resolve the flatness problem we should indeed consider the universes arbitrary close to flatness and not simply remove them as being equivalent. Note also that although the canonical measure can solve the flatness problem without inflation in certain closed $k = 1$ cases there is a further ambiguity for bounded potentials: such as in the case of $R^2$ inflation [59,5] when conformally transformed to an effective scalar field model [54,60]. The measure $\omega$ diverges even for a fixed value of the scale factor provided the initial energy density is sufficiently large: above the plateau of the scalar potential. Likewise for the factorized kinetic inflationary model with $p = K(\phi)F(X)$, the measure at fixed $\dot{a}$, so signifying a maximum closed universe before inflationary behaviour can proceed, will involve a term $\omega \propto F(\phi) d\phi$, which without restrictions on the form of $F(\phi)$ can contribute an infinity of solutions due to the redundancy of the $\phi$ variable.

More importantly for their argument [57] that the likelihood of inflation is largely suppressed they evolve backwards from the end of inflation and find the solution actually unstable to kinetic domination: or in general to the “stiffest” matter present.

If we first accept this procedure there are a few ways to evade this conclusion. Firstly, in earlier loop quantum approaches the matter terms are affected by finite size corrections e.g.[61]. Massless scalar field can themselves violate the various energy conditions and become, actually phantom-like, inflationary. Then the solution simply cannot evolve to any non-inflationary behaviour in the past. This stage of loop driven inflation tends to have insufficient duration without choosing arbitrary large parameters and a second conventional phase of inflation was added to the scheme [62]. This conventional phase spoils the chance of evading the Gibbons-Turok argument [63],

\footnote{We ignore the presence of inflation(2) in this section which should also ideally be incorporated as a requirement on the measure.
although in any case the use of a phantom inflationary phase to prime a
standard inflationary phase has other problems of fine tuning [64]. However,
in principle quantum gravity effects can cause all matter sources to behave
inflationary at high densities so avoiding any such problem of stability?

Another way of evading the scheme is in certain kinetic inflationary mod-
els provided the previous ambiguity is first resolved: when the generalized
momentum $\pi = \dot{\phi} p, X$ cannot diverge to infinity i.e. like $\pi \propto a^{-3}$ as the solu-
tion is evolved backwards without pushing the corresponding energy density
negative; or else the universe evolves onto some previously collapsing phase
cf.[65]. In the previous model of section 2.3 this corresponds to taking $b$
negative - if on the contrary the momenta can diverge the Gibbons-Turok
argument holds [66]. It might be argued that both cases suggest inflation
is unlikely but, with $b$ negative, it also prevents the flatness problem from
being solved since an extremely large energy density is then not present to
set the initial value of $\Omega$ arbitrary close to unity.

However it is well known that the inflationary solution is an attractor only
in the forward direction, so the field cannot be expected to evolve gradually
up the potential as the solution is continued backwards. In the forward
direction the inflationary solution is an attractor with the kinetic energy term
decaying exponentially quicker than the value of the scalar field [67]. One
can also see this difference in that particle horizons become event horizons
and vice-versa when evolution is reversed [68]. So a backwards evolving
inflationary solution has a corresponding particle horizon. This result can,
though, be thought consistent with the requirement of inflation that the field
$\phi$ be initially homogeneous over a length scale $L > 1/H$. This is in some sense
a highly ordered low-entropic state that requires some further justification.
Evolving backwards one would expect to obtain, à la Gibbons and Turok,
a high entropy state that would indeed not be compatible with inflationary
behaviour.

If however the universe is assumed to start “small” then the canonical
measure can also give a suitable measure for inflation to proceed, actually
uniform over $\phi$. This can give a highly likely probability of inflation provided
the initial energy density is taken large [53,54]. This can be thought of as
a combination of classical and quantum reasoning: quantum suggesting the
initial small dimensions or action.

The Hartle-Hawking wavefunction also suppresses the potential $\Psi \sim
\exp(1/V(\phi))$ and seems to not give a strong prediction for an inflationary
phase: although if one lets the field take unrestricted values, way beyond
Planckian, one can still get some significant prediction for inflation [16]. Note however that the Hartle-Hawking state also does not give a large value for the kinetic energy that would have produced a singularity as $a \to 0$. Although it seemingly agrees with Gibbons and Turok in that inflation is exponentially suppressed it would not alone be able to provide a solution alone to the flatness problem without an explicit inflationary phase being present. We note that the Hartle-Hawking boundary condition is somewhat ambiguous and might also give big bang like solutions with exponential potentials: having there both singular potential and kinetic energies [69].

There is a further aspect: using a notion of a typical boundary condition Gibbons and Grischuk [70] found that Hartle-Hawking boundary conditions were unfavoured and Tunnelling ones (that give inflation) were actually more typical: an indifference principle was applied at initial Planckian values for the energy density. This is a somewhat surprising result: one might have expected a random state to be the more highly entropic one and so not conducive to inflationary behaviour. It remains to be seen if this result can be upheld, especially with more realistic inhomogeneous models.

The previous notion of a typical boundary condition was an example of an a priori measure. The Hartle-Hawking wavefunction can also be amended by assuming instead an observer’s perspective inside the present Hubble volume. One can firstly insist that $\phi > \phi_*$ for a Lorentzian space to develop [71,10]. Then because of volume weighting, due to a factor $\sim \exp(3N)$ caused by $N$ e-foldings of inflation, more Hubble volumes are produced the longer inflation proceeds. This reasoning can seemingly allow the Hartle-Hawking proposal to produce significant inflation during our previous history [71,72]. But there is a danger that this a posteriori reasoning could correct almost any boundary condition proposal and make the distinction with others e.g. the Tunnelling one irrelevant - so we are not sure if this selection effect should be used in this way.

Given this reasoning the universe is found to bounce from a previous collapsing phase. A closely related boundary condition, roughly that the strong-energy condition be violated, has also been formulated by Page [73], but this is only sufficient for closed models: compact flat and open model would require effectively further energy conditions to be violated. The entropy has to be low for inflation to ensue and so whether the collapsing phase can be allowed, perhaps with its arrow of time reversed to prevent entropy build-up, or one should consider the universe created at the bouncing point is still unclear cf.[72]. There is also the suggestion, using a more general
complexified field, that the Hartle-Hawking proposal can start the universe at its largest possible size, but perturbations apparently still grow during the subsequent collapse [74]. Note also that previously people had introduced a repulsive Planck potential [75] to also produce a bounce but this did not alone explain why a subsequent inflationary stage was also present. We suspect the more numerous “separation like” constants in kinetic inflationary models would allow bouncing like behaviour also in non-closed models, but many unwanted and singular solution would have to be excluded.

4.0 Bouncing or cyclic universes

We can briefly consider further aspects of bouncing cosmologies where the universe first collapses from a previous phase and in turn the possibility of repeatedly using this mechanism to produce a cyclic universe. The idealized model is outlined in Fig.(2).

\[
\text{bounce} \rightarrow \text{Inflation (1)} \rightarrow \text{non-Inflation} \rightarrow \text{Inflation (2)} \rightarrow \text{collapse}
\]

↑

\[\text{← Cyclic?}\]

Fig.(2): Possible extension of previous model to cyclic behaviour by means of a suitable bounce. Can entropy be dissipated on going around the loop?

Consider the Friedmann equation for a FRW model [5,6]

\[
H^2 + \frac{k}{a^2} = \rho
\]  \hspace{1cm} (11)

A FRW bounce is typically described by an equation of the form

\[
H^2 = \frac{A}{a^n} - \frac{B}{a^m}
\]  \hspace{1cm} (12)

A bounce requires \( m > n \) so the stiffer matter component requires the minus sign. For a closed model the curvature plays this role and only the
strong energy has to be violated for a bounce to happen - unlike the general case where more drastic violations are required e.g.[76].

Some approaches to quantum gravity suggest that the Friedmann equation be modified such that

\[ H^2 = \rho - \frac{\rho^2}{\rho_c} \]  \hspace{1cm} (13)

where \( \rho_c \) represents the critical energy scale. This occurs in more recent work in loop quantum gravity [77,78]. Related behaviour might be obtained with brane models with an extra time dimension [79] although this is probably observationally discounted [80]; but see recently [81]. Note that a single negative tension brane is not suitable: it differs from eq.(13) by an overall minus sign on the R.H.S. since starting with a positive 5-dimensional Planck mass the negative tension causes the 4-dimensional Newton’s constant to become negative cf.[82].

If one first tries to work with non-inflationary matter and use say a closed model to re-collapse the universe one finds the bounce size \( a_b \) and maximum size \( a_{\text{max}} \) do not differ sufficiently. For the case of radiation \( a_b^2 = a_{\text{max}} \) so it is difficult to justify the universe becoming so large without arbitrary large constants. To rectify this one would want to add the inflation(1) phase but again we have difficulties in understanding how the strong energy condition becomes violated after the bounce and not before.\footnote{There are some bouncing models e.g.[65] that permanently violate all the energy conditions but these then have as much inflationary contraction as expansion: so not contributing overall to resolving the various problems that inflation is usually invoked for.}

Indeed the previous results of Gibbons and Turok now become relevant for a collapsing universe in that the kinetic energy will increasingly dominate. We doubt therefore that that an “anti-friction” effect can drive the scalar field up the potential cf.[77] so that an inflationary stage can proceed after the bounce. One might instead try to resolve the various cosmological puzzles without an explicit inflationary phase, or else impose some further boundary condition at the bounce itself like the amended Hartle-Hawking one.

Note also that for this modified Friedmann equation \( H \to 0 \) as \( \rho \to \rho_c \) so a large cosmological constant is tending towards a static universe. This incidentally can have some influence on whether quantum fluctuations can produce eternal inflation cf.[5].

It has been noticed that this Friedmann equation prevents a phantom matter source \( \rho \propto a^n \) with \( n > 1 \) from reaching a big rip singularity [83,84].
Instead the universe slows before re-collapsing without the necessity of entering a high curvature phase. With just a phantom matter source it will then approach a super-collapsing phase. Previously there was a related model [85] of the universe that started at the big rip before undergoing super-collapse and eventually bouncing into a standard matter dominated phase. The super-collapsing phase does not alone solve the usual cosmological puzzles, for example the particle horizon goes as:

\[ R_H = a(t) \int_0^t \frac{dt}{a(t)} \propto t \]  \hspace{1cm} (14)

for a collapsing scale factor \( a \propto 1/t \), where \( t = 0 \) represents the start of the collapsing phase. This has the same behaviour as a usual non-inflationary expanding model. Neither does this collapsing phase reduce the entropy by fragmenting the universe which stems from a misuse of horizons and/or, problematically to most people, equating the entropy with the corresponding universe’s size cf. [84].

More crucial is to obtain a generalized second law (GSL) e.g.,[86] of thermodynamics that allows entropy to increase together with a gap between the maximum allowed entropy and that actually present in the matter components [87]. Firstly, it is rather difficult to formulate a GSL, in an expanding model with phantom matter: one apparently has to introduce negative values for the entropy [88] or temperature [89]. Simply setting the entropy zero for the phantom component, like in an analogous superfluid, would allow phantom matter to dissolve black holes upon approaching a big rip in violation of the GSL cf.[90]. Related negative entropy/temperature values have previously been suggested for de Sitter [91,92], although the correct sign of “energy” in the Gibbs equation confuses matters - see e.g.[93].

Incidentally during the super-collapsing phase this problem of horizon entropy is obviated by the lack of an actual event horizon. But as we have previously discussed to obtain an ensuing inflation(1) phase requires a low-entropic state to develop. This is a rather difficult obstacle to overcome since the comoving entropy density would be expected to be growing, or at least remain constant, during the collapsing phase. It therefore appears difficult to obtain the cyclic universe as envisioned in the figure(2).

Other approaches have tried to impose a cyclic structure but superimposed upon an underlying expanding universe. For example the quasi-steady

\[ \text{These large negative entropy/temperature values for the phantom are probably inconsistent with an inflation(1) phase} \]
state model [94] or the cyclic ekpyrotic one [95]. These then attempt to use the cosmic no-hair property in order to dilute entropy production. However, this by sleight of hand introduces an infinity into which we can sweep the problem of excessive entropy production. It also means that all scales eventually originate from sub-Planck sizes of previous stages of the universe [97] and further introduces geodesic-incompleteness problems of constantly expanding models [98].

5.0 Conclusions

The presence of two inflationary stages poses two sorts of problem: i) it shows up the weaknesses in the original arguments that justified a displaced scalar, now being in apparent contradiction with the necessary conditions of the second field; ii) a uniformity issue for the second scalar field is rather similar to that of the original horizon problem in non-inflationary models - so reintroducing a similar puzzle.

The general difficulty is that conditions for inflation(2) has also to be set up before inflation(1) proceeds in order to have homogeneous conditions over the present horizon size. Because of finite particle horizon sizes it cannot simply be caused by evolution from the end of inflation(1). There is a related problem in obtaining sufficient homogeneity for a cosmological constant if it is being used for inflation(2); but with the further difficulty of producing a sufficiently small value while various phase transitions have taken place. Any early universe inflation(1) can be ignored for the sake of adjusting this $\Lambda$ [12]: so the required uniformity is actually analogous to a big bang model with an eventual $\Lambda$ dominated phase: but such a model has a known horizon problem. Any dark energy inflationary stage is at the expense of an unnatural uniformity which then requires a further explanation.

To put this another way, originally inflation was a single assumption, hidden in the murky waters of Planck scale physics, that solved a number of puzzles, but now the second phase of inflation having not yet “solved” anything is itself becoming a puzzle requiring further explanation. This reasoning would become worse the more often the universe stopped and started inflating, i.e. if the [inflation $\rightarrow$ non-inflation] sequence in Fig(1) was ex-

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7 There is a related suggestion of Penrose [96] that envisions an infinite conformal rescaling during the massless phase of a cosmological constant dominated universe to create suitable conditions of low-entropy for a subsequent big bang phase. However, the definition of allowed entropy uses the notion of mass by means of the Planck units incorporated into Newton’s constant $G$ - so this scheme at present appears somewhat inconsistent - unless gravity itself, and not simply mass, can be adjusted.
tended indefinitely in one or both directions. We have tried to ignore more elaborate notions of our universe having branched off from an earlier, or still constantly evolving, inflationary phase so that the actual universe would not have distinct inflationary or non-inflationary stages as in the simple model of Fig.(1). Firstly we do not think it realistic observationally for inflation (2) to have formed this way. Secondly other principles might constrain such branching phenomena, for example in brane models the bulk space might fix the space-time system to prevent quantum branching [99]; there might also be trouble in obtaining a suitable arrow of time in any new quantum dominated universe production [100].

If the inflationary stages have mixed causes, for example one being kinetic driven or phantom, similar concerns are present. Generally speaking the various alternative inflationary models: assisted, phantom, kinetic etc. appear less suited to describe inflation(2), having even more fine tuning concerns when a period of non-inflationary behaviour precedes them.

Although we have considered only scalar field model these problems should persist in many higher derivative gravity models that have been proposed as dark energy candidates [101,2]; like those with a Ricci scalar term $1/R^n$ added to the gravitational action which can usually be transformed to a conformally equivalent scalar field model. Some possible advantages of modifying gravity schemes over the use of a quintessence field have been made in ref.[45]; essentially the subsequent modified gravity inflationary epoch is set \textit{ab initio} into the action so obviating causal constraints on obtaining a homogeneous quintessence field. The problem is then displaced into the explanation of why the action takes its particular form. We would just add though, that such higher derivative gravity theories especially with more general Ricci tensor terms $R_{\mu\nu}R^{\mu\nu}$ or Weyl tensor are known to have more limited cosmic no-hair properties - with possible premature collapse [102] or anisotropic inflationary solutions [103]. This might not be a serious problem for inflation(2) since we do not necessarily want to establish the cosmological principle into the far distant future but it is unattractive if the inflation is of this limit form compared perhaps with inflation(1); or it requires starting conditions that only slightly depart from FRW in order to restrict the effects of these more general higher derivative terms involving combinations of Ricci, Weyl or Riemann tensors. Such modified gravity theories also tend to be strongly constrained by unwanted consequences during the early universe - see e.g.,[104].

To summarize some possible avenues for future study:
• A suitable boundary condition that can give a \textit{a priori} prediction of two distinct stages of inflation, either starting from some creation event or from a previous pre-big bang phase. Presently the usual proposals are too energy density dependent, not amenable to justifying initial conditions at vastly differing energy scales.
• Quantum formulism of cosmological no-hair property to explain possible smoothing of dark energy scalar field, or else the initial non-causal like \textit{uniformity} issue needs to be further resolved.
• Quantization with higher derivative scalar matter terms i.e. kinetic or Box inflation: obtaining solutions of WDW equation together with a justifiable boundary condition that eliminates unwanted solutions.
• Can a entropy sink be incorporated to produce an actual cyclic model. Various ideas of e.g. infinite spatial size, reversing arrow of time in collapsing model, to dilute entropy do not appear realistic. Perpetually expanding models typically have a incompleteness problem: although various counterexamples appear possible \cite{105,106}. Neither do such cyclic models explain why the entropy at any time is not already maximized by the presence of black holes.
• Can modified gravity models provide adequate inflationary stages: both in the early and late universes? Can the specific action be justified from more fundamental principles and what restrictions on initial conditions are still necessary for its implementation?

\textbf{Acknowledgement}

I should like to thank A. Aguirre, O. Corrandi, W. Nelson, D. Page, Yun-Song Piao, A. Vikman and Yi Wang for helpful discussions.
References

1. D.N. Spergel et al, Astrophys. J. Suppl. 170 (2007) p.377.
   arXiv:0603449.[astro-ph]

2. P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. 75 (2003) p.559.
   R.R. Caldwell, Phys. World 17 (2004) p.37.
   S.M. Carroll, preprint astro-ph/0310342.
   T. Padmanabhan, preprint gr-qc/0503107.
   E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) p.1753.
   P. Ruiz-Lapuente, Class. Quant. Grav. 24 (2007) p.R91.

3. S. Weinberg, Rev. Mod. Phys. 61 (1989) p.1.

4. A.D. Linde, Phys. Lett. B 129 (1983) p. 177. *ibid* 175 (1986) p.395.

5. A.D. Linde, “Particle Physics and Inflationary cosmology” (Harwood Press) 1990.

6. E.W. Kolb and M.S. Turner, “The Early Universe” (Addison-Wesley: New York) 1990.

7. D.S. Goldwirth and T. Piran, Phys. Rep. 214 (1992) p.223.

8. M.S. Madsen and P. Coles, Nucl. Phys. B 298 (1988) p. 2757.

9. M.S. Madsen, Class. Quant. Grav. 7 (1990) p.2073.

10. J.J. Halliwell, in “Quantum cosmology and baby universes” eds. S. Coleman et al. (World Scientific: Singapore) 1991.
    D.L. Wiltshire, in “Cosmology the physics of the universe”, eds. B. Robson et al. (World Scientific, Singapore) 1996.
    also as preprint gr-qc/0101003.

11. T. Vachaspati and M. Trodden, Phys Rev. D 61 (2000) p.023502.

12. R. Bousso, arXiv:0708.4231 [hep-th].

13. R. Penrose, “The Road to Reality”, Johnathon Cape: London (2004).
14. A.D. Linde, Phys. Lett. B 327 (1994) p.208.
   A. Vilenkin, Phys. Rev. Lett. 72 (1994) p.3137.

15. S. Blau, E. Guendelman and A.H. Guth, Phys. Rev. D 35 (1987) p.1747.
    E. Fahri and A.H. Guth, Phys. Lett. B 183 (1987) p.149.
    E. Fahri, A.H. Guth and J. Guven, Nucl. Phys. B 339 (1990) p.417.
    W. Fischler, D. Morgon and J. Polchinski, Phys. Rev. D 41 (1990) p.2638.
    A.D. Linde, Nucl. Phys. B 372 (1992) p.421.
    S. Ansoldi and E. I. Guendelman, preprint arXiv:0706.1233.

16. D.N. Page, Phys. Rev. D 56 (1997) p.2065.

17. P.C.W. Davies, arXiv:0708.1783 [gr-qc].
    Da-Ping Du, Bin Wang and Ru-Keng Su, Phys. Rev. D 70 (2004) p.064024.

18. G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15 (1977) p.2738.

19. R.M. Wald, Phys. Rev. D 28 (1983) p.2118.

20. A.K. Raychaudhuri and B. Modak, Class. Quant. Grav. 5 (1989) p.225.
    see also: P. Anninos, R.A. Matzner, T. Rothman and M.P. Ryan, Phys. Rev. D 43 (1991) p. 3821.

21. A.A. Coley, S. Hervik and W.C. Lim, Phys. Lett. B 638 (2006) p.310.

22. P.J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D 59 (1999) p.123504.

23. T. Barreiro, E.J. Copeland and N.J. Nunes, Phys. Rev. D 61 (2000) p.127301.

24. R.R. Caldwell and E.V. Linder, Phys. Rev. Lett. 95 (2005) p.141301.

25. Y.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. 10 (1984) p.135.

26. D.H. Coule and J. Martin, Phys. Rev. D 61 (2000) p.063501.
27. A.D. Linde, JCAP 0401 (2004) p.004.
28. M. Malquarti and A.R. Liddle, Phys. Rev. D 66 (2002) p. 023524.
29. J. Martin and M. Musso, Phys. Rev. D 71 (2005) p.063514.
30. Chun-Hsien Wu, Kin-Wang Ng and L.H. Ford, Phys. Rev. D 75 (2007) p.103502.
31. A.R. Liddle, A. Mazumdar and F.E. Schunck, Phys. Rev. D 58 (1998) p. 061301.
32. J. Martin, Phys. Rev. D 49 (1994) p.5086.
   A. Carlini, D.H. Coule and D.M. Solomons, Mod. Phys. Lett. A 11 (1996) p.1453.
33. F. Luchin and S. Matarrese, Phys. Rev. D 32 (1985) p.1316.
   J.J. Halliwell, Phys. Lett. B 185 (1987) p.341.
   A.B. Burd and J.D. Barrow, Nucl. Phys. B 308 (1988) p.929.
34. D.H. Coule, Phys. Lett. B 450 (1999) p. 48.
35. S. Mollerach, S. Matarrese and F. Lucchin, Phys. Rev. D 50 (1994) p.4835.
36. Yun-Song Piao and E. Zhou, Phys. Rev. D 68 (2003) p.083515.
   M. Baldi, F. Finelli and S. Matarrese, Phys. Rev. D 72 (2005) p.083504.
37. Yun-Song Piao and Yuan-Zhong Zhang, Phys. Rev. D 70 (2004) p.063513.
38. J.C. Fabris and S.V.B. Goncalves, Phys. Rev. D 74 (2006) p.027301.
   K.A. Bronnikov, J.C. Fabris and S.V.B. Goncalves, gr-qc/0611038.
39. R.R. Caldwell, Phys. Lett. B 545 (2002) p.23.
40. V. Faraoni, Class. Quant. Grav. 22 (2005) p.3235.
41. C. Armendariz-Picon, T. Damour and V. Mukhanov, Phys. Lett. B 458 (1999) p.209.
42. C. Armendariz-Picon, T. Damour and V. Mukhanov, Phys. Rev. D 63 (2001) p.103510.
43. A.D. Rendall, Class. Quant. Grav. 23 (2006) p.1557.
44. A. Anisimov, E. Babichev and A. Vikman, JCAP 0506 (2005) p.006.
45. R.P. Woodard, astro-ph/0601672.
46. Wei Fang, H.Q. Lu and Z.G. Huang, Class. Quant. Grav. 24 (2007) p.3799.
47. V. Mukhanov and A. Vikman, JCAP 0602 (2006) p.004.
48. C. Bonvin, C. Caprini and R. Durrer, Phys. Rev. Lett. 97 (2006) p.081303.
   G.F.R. Ellis, R. Maartens and M. MacCallum, Gen. Rel. Grav. 39 (2007) p.1651.
49. L. Raul Abramo and N. Pinto-Neto, Phys. Rev. D 73 (2006) p.063522.
50. A.D. Polyanin and V.F. Zaitsev, Handbook of nonlinear partial differential equations, Chapman and Hall, Boca Raton 2004.
51. H.Q. Lu, Z.G. Huang, W. Fang and P.Y. Ji, hep-th/0701135.
52. G.W. Gibbons, S.W. Hawking and J.M. Stewart, Nucl. Phys. B 281 (1987) p.736.
   see also: V.A. Belinski and I.M. Khalatnikov, Sov. Phys. 66 (1988) p.3.
53. S.W. Hawking and D.N. Page, Nucl. Phys. B 298 (1988) p.789.
54. D.H. Coule, Class. Quant. Grav. 12 (1995) p.455.
55. S. Hollands and R.M. Wald, Gen. Rel. Grav. 34 (2002) p.2043.
   ibid [hep-th/0210001].
56. L. Kofman, A.D. Linde and V. Mukhanov, JHEP 0210 (2002) p.057.
57. G.W. Gibbons and N. Turok, hep-th/0609095v2.
58. J.D. Barrow and F.J. Tipler, The Anthropic Cosmological Principle, Oxford University Press: Oxford 1986.
59. A.A. Starobinsky, Phys. Lett. B 91 (1980) p.99.
60. D.N. Page, Phys. Rev. D 36 (1987) p.1607.
61. M. Bojowald, Living Rev. Rel. 8 (2005) p.11
62. M. Bojowald, Phys. Rev. Lett. 89 (2002) p.261301.
63. C. Germani, W. Nelson and M. Sakellariadou, Phys. Rev. D 76 (2007) p.043529.
64. D.H. Coule, Class. Quant. Grav. 22 (2005) p.R125.
65. L.R. Abramo and P. Peter, astro-ph/0705.2893.
66. Miao Li and Yi Wang, JCAP 0706 (2007) p.012.
67. D.S. Salopek and J.R. Bond, Phys. Rev. D 42 (1990) p.3936.
68. W. Rindler, “Essential Relativity 2nd edn.” (Springer-Verlag: New York) 1977.
69. D.N. Page, in “The Future of Theoretical Physics and Cosmology” eds. G.W. Gibbons, E.P.S. Shellard and S.J. Rankin, Cambridge University Press 2003. 
also [hep-th/0610121]
70. G.W. Gibbons and L.P. Grishchuk, Nucl. Phys. B 313 (1989) p.736.
71. J.B. Hartle, S.W Hawking and T. Hertog, arXiv:0711.4630
72. S.W. Hawking, arXiv:0710.2029
73. D.N. Page, arXiv:0707.2081
74. D. Green and W.G. Unruh, gr-qc/0206068.
75. H.D. Conradi and H.D. Zeh, Phys. Lett. A 154 (1991) p.321. 
H.D. Conradi, Phys. Rev. D 46 (1992) p.612.
76. C. Molina Paris and M. Visser, Phys. Lett. B 455 (1999) p.90.
77. P. Singh, K. Vandersloot and G.V. Vereshchagin, Phys. Rev. D 74 (2006) p. 043510.
78. A. Ashtekar, gr-qc/0702030.
79. Y. Shtanov and V. Sahni, Phys. Lett. B 557 (2003) p.1.
M.G. Brown, K. Freese and W.H. Kinney, astro-ph/0405353.
80. G. Dvali, G. Gabadadze and G. Senjanovic, hep-ph/9910207.
81. I. Quiros, arXiv:0706.2400.
82. C. Barcelo and M. Visser, Phys. Lett. B 482 (2000) p.183.
83. M. Sami, P. Singh and S. Tsujikawa, Phys. Rev. D 74 (2006) p.043514.
84. L. Baum and P.H. Frampton, Phys. Rev. Lett. 98 (2007) p.071301.
P.H. Frampton, astro-ph/0612243.
85. F.G. Alvarenga and J.C. Fabris, Class. Quant. Grav. 12 (1995) p.L69.
see also: A.B. Batista, J.C. Fabris and S.V.B. Goncalves, Class. Quant.
Grav. 18 (2001) p.1389.
86. P.C.W. Davies and T.M. Davis, Found. Phys. 32 (2002) p.1877.
87. J.D. Barrow, New. Astron. 4 (1999) p.333.
88. G. Izquierdo and D. Pavon, Phys. Lett. B 633 (2006) p.420.
J.A.S. Lima and J.S. Alcaniz, Phys. Lett. B 600 (2004) p.191.
I. Brevik, S. Nojiri, S.D. Odintsov and L. Vanzo, Phys. Rev. D 70
(2004) p.043520.
89. P.F. Gonzalez-Diaz and C.L. Sigueza, Phys. Lett. B 589 (2004) p.78.
90. G. Izquierdo and D. Pavon, Phys. Lett. B 639 (2006) p.1.
91. M.D. Pollock and T.P. Singh, Class. Quant. Grav. 6 (1989) p.901.
92. D. Klemm and L. Vanzo, JCAP 0411 (2004) p.006.
93. T. Padmanabhan, Phys. Reports 406 (2005) p.49.
M. Spradlin, A. Strominger and A. Volovich, hep-th/0110007.
94. F. Hoyle, G. Burbidge and J.V. Narlikar, “A different approach to
  cosmology” (Cambridge University Press: Cambridge) 1999.
95. P.J. Steinhardt and N. Turok, Phys. Rev. D 65 (2002) p.126003.
96. R. Penrose, Before the big bang? lecture at The Newton Institute Cambridge 2005. available online: www.newton.ac.uk

97. D.H. Coule, Int. J. Mod. Phys. D 12 (2003) p.963.

98. A. Borde, A.H. Guth and A. Vilenkin, Phys. Rev. Lett. 90 (2003) p.151301.

99. D.H. Coule, Gen. Rel. Grav 36 (2004) p.2095.

100. B. McInnes, preprint [arXiv:0705.4141]

101. S.M. Carroll, V. Duvvuri, M. Trodden and M.S. Turner, Phys. Rev. D 70 (2004) p.043528.

102. A.L. Berkin, Phys. Rev. D 44 (1991) p.1020.

103. J.D. Barrow and S. Hervik, Phys. Rev. D 74 (2006) p. 124017.
     ibid, D 73 (2006) p.023007.
     D. Muller and S.D.P. Vitenti, Phys. Rev. D 74 (2006) p.083516.

104. A.A. Starobinsky, arXiv:0706.2041
     and references therein

105. G. Ellis and R. Maartens, Class. Quant. Grav. 21 (2004) p.223.

106. A. Aguirre and S. Gratton, Phys. Rev. D 65 (2002) p.083507.
     A. Aguirre, arXiv:0712.0571 [hep-th]