Soft Pion Emission from the Nucleon
Induced by Twist-2 Light-Cone Operators

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Abstract

We compute the matrix elements for pion emission from the nucleon induced by twist-two light-cone operators in the kinematics where the pion is close to threshold. It is shown that the results of heavy baryon ChPT are in agreement with the results from the soft-pion theorem approach. Therefore the amplitude of soft-pion emission is directly related to the generalized parton distributions in the nucleon.

1 Introduction

Hard processes are known to provide us with valuable information about the quark and gluon structure of hadrons in terms of parton distributions and distribution amplitudes. The generalized parton distributions (GPDs) [1, 2, 3, 4], entering the QCD description of hard exclusive processes, interpolate, in a sense, between usual parton distributions, distribution amplitudes, and elastic hadron form factors (for a review see e.g. [5]). GPDs are low energy observables and therefore their dependence on the quark mass, small momentum transfer, etc. can be studied with help of the chiral perturbation theory (ChPT).

The main source of information about GPDs are exclusive reactions which can be studied at many experimental facilities [6]. At the same time the existing experiments suffer from an ambiguity due to errors in the identification of the recoiling nucleon. The missing mass technique used for this purpose cannot distinguish between, for instance, one nucleon and a nucleon with a pion produced close to the threshold. An estimate of such a contamination was suggested in [7] using the approach of current algebra and PCAC (= soft-pion theorems). Recently, results of [7] were criticized in [8]. The authors of [8] used the chiral perturbation theory to calculate the same matrix elements of twist-2 operators as in [7]. It was claimed that the two results do not match each other. The aim of our paper is to clarify this situation.

In our presentation we consider for simplicity only twist-2 isovector operators, a generalization to other operators is trivial. We construct the corresponding effective operators in ChPT in Section 2 and calculate their matrix elements for soft pion emission from the nucleon. In Section 3 we present soft-pion theorem (SPT) results and perform a comparison with the ChPT calculations.
1.1 GPDs in the chiral limit

The nonlocal twist-2 light-cone matrix elements are parametrized in terms of the GPDs \( H, E, \tilde{H} \) and \( \tilde{E} \) as

\[
(\bar{P} \cdot n) \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda(P,n)} \langle \bar{P} + \Delta/2 | \bar{\psi}(\lambda n/2) \gamma_\mu \frac{T^3}{2} \psi(-\lambda n/2) | \bar{P} - \Delta/2 \rangle =
\]

\[
\bar{N}(\bar{P} + \Delta/2) \left[ \frac{\not{\bar{q}} H(x, \xi, \Delta^2)}{2M} + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E(x, \xi, \Delta^2) \right] \frac{T^3}{2} N(\bar{P} - \Delta/2),
\]

(1)

\[
(\bar{P} \cdot n) \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda(P,n)} \langle \bar{P} + \Delta/2 | \bar{\psi}(\lambda n/2) \gamma_5 \frac{T^3}{2} \psi(-\lambda n/2) | \bar{P} - \Delta/2 \rangle =
\]

\[
\bar{N}(\bar{P} + \Delta/2) \left[ \frac{\gamma_5 \bar{\psi}(x, \xi, \Delta^2) + \gamma_5 \left( \frac{\Delta \cdot n}{2M} \right) E(x, \xi, \Delta^2) }{2M} \right] \frac{T^3}{2} N(\bar{P} - \Delta/2),
\]

(2)

where \( n \) is a light-cone vector, \( n^2 = 0 \), defining the separation of the fields in the operators; \( \bar{N} \) and \( N \) are nucleon spinors; \( \xi \) denotes the standard skewedness variable

\[
\xi = -\frac{1}{2} \frac{(\Delta \cdot n)}{(\bar{P} \cdot n)}.
\]

The variable \( x \) has the meaning of momentum fraction carried by the quark with respect to the average momentum \( \bar{P} \) of the nucleon.

The following sum rules relate the GPDs to local form factors:

\[
\int_{-1}^{1} dx \; H(x, \xi, \Delta^2) = F_1^p(\Delta^2) - F_1^n(\Delta^2),
\]

(3)

\[
\int_{-1}^{1} dx \; E(x, \xi, \Delta^2) = F_2^p(\Delta^2) - F_2^n(\Delta^2),
\]

(4)

\[
F_1^p(0) = 1, \quad F_1^n(0) = 0, \quad F_2^p(0) - F_2^n(0) = \kappa_p - \kappa_n \approx 3.706,
\]

(5)

\[
\int_{-1}^{1} dx \; \tilde{H}(x, \xi, \Delta^2) = G_A(\Delta^2),
\]

(6)

\[
\int_{-1}^{1} dx \; \tilde{E}(x, \xi, \Delta^2) = G_P(\Delta^2),
\]

(7)

\[
G_A(0) = g_A, \quad G_P(\Delta^2) \approx \frac{4M^2 g_A}{\Delta^2 - m_\pi^2},
\]

(8)

where we use the notations \( M \) and \( m_\pi \) for the masses of the nucleon and the pion, respectively. To get a connection with heavy baryon ChPT we shall perform a non-relativistic expansion of the non-local matrix elements. For this purpose, we choose the Breit frame where

\[
\bar{P} = [M + O(\Delta^2/M)] v, \quad v = (1, 0, 0, 0), \quad (\Delta \cdot v) = 0.
\]

(9)

In this case, the momentum transfer is small, \( \Delta \sim O(\varepsilon) \), where as usual in ChPT \( \varepsilon \) is a small generic momentum. We shall also introduce the standard large mass decomposition for the nucleon spinors:

\[
N(\bar{P} - \Delta/2) = \left( 1 - \frac{\Delta \mu}{4M} \right) N_v + O(\Delta^2), \quad \not{\bar{\psi}} N_v = N_v.
\]

(10)
The power counting for the light-cone variables is as follows:

\[
\begin{align*}
\text{momentum fraction} & \quad x \sim \mathcal{O}(1), \\
\text{momentum transfer} & \quad \Delta \sim \mathcal{O}(\varepsilon), \\
\text{skewedness} & \quad \xi \sim \mathcal{O}(\Delta/M).
\end{align*}
\]

Performing an expansion of (1) and (2) with respect to the small parameter \( \Delta \) we arrive at the non-relativistic expansion of the GPDs. The result can be written as

\[
(\bar{P} \cdot n) \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda(P \cdot n)} (\bar{P} + \Delta/2\bar{\psi}(\lambda n/2)\gamma \frac{\tau^a}{2}\psi(-\lambda n/2)\bar{P} - \Delta/2) = \bar{\psi}(x \cdot n)\bar{N}_v \tau^3 \frac{N_v + \mathcal{O}(\Delta)}{2},
\]

(14)

\[
(\bar{P} \cdot n) \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda(P \cdot n)} (P + \Delta/2\bar{\psi}(\lambda n/2)\gamma_5 \frac{\tau^3}{2}\psi(-\lambda n/2)\bar{P} - \Delta/2) = \Delta \bar{\psi}(x \cdot n)\bar{N}_v (S \cdot n)\tau^3 \frac{N_v + \mathcal{O}(\Delta)}{2},
\]

(15)

where \( \bar{\psi}(x) = H(x, 0, 0) \) and \( \Delta \bar{\psi}(x) = \bar{H}(x, 0, 0) \) are the isovector vector and axial-vector forward distributions in the chiral limit and \( S^\mu = \frac{i}{2} \gamma_5 \sigma_{\mu\nu} v^\nu \) is the covariant spin matrix.

\section{Light-cone operators in the effective theory}

In this section we briefly discuss the form of the isovector operators in terms of effective fields. We consider the \( SU(2) \) flavor sector with the leading order effective Lagrangian

\[
\mathcal{L}_{\text{eff}} = \frac{F^2}{4} \text{Tr}(\partial \mu U \partial^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U) + \bar{H}_v [(iv \cdot D) + g_A (u \cdot S)] H_v,
\]

(16)

where as usual

\[
U(x) = \exp(iv^a(x)\tau^a/F_v), \quad \chi = 2B \text{diag}(m_u, m_d), \quad U = u^2, \quad u = i\bigl[u^\dagger \partial_u u - u \partial_u u^\dagger\bigr],
\]

(17)

(18)

\( D_\mu = \partial_\mu + \Gamma_\mu \) is the chiral covariant derivative and \( H_v \) denotes the nucleon field in the heavy baryon approach. Under the chiral \( SU(2)_L \times SU(2)_R \) transformation the fields change according to

\[
U'(x) = RUL^\dagger, \quad H'_v = K(L, R, U)H_v, \quad u' = RuK^\dagger = KuL^\dagger,
\]

(19)

where the transformation matrix \( K \) depends not only on the group elements \( L \) and \( R \) but also on the pion field \( U \) [9]. For simplicity, we do not consider contributions from spin-\( \frac{3}{2} \) \( \Delta \)-resonances, since they are irrelevant for the subject of our discussion.

Our task is to find the leading-order expressions for the effective operators that correspond to the following light-cone QCD operators:

\[
\begin{align*}
V^a(\lambda) &= \bar{\psi}(\lambda n/2)\gamma^a \frac{\tau^a}{2}\psi(-\lambda n/2), \\
A^a(\lambda) &= \bar{\psi}(\lambda n/2)\gamma_5 \gamma^a \frac{\tau^a}{2}\psi(-\lambda n/2).
\end{align*}
\]

(20)

(21)
We remark that we shall not perform the expansion in local operators because the non-local notation is simpler.

The effective operators have to respect certain properties of their QCD counterparts. First, we note that the QCD operators (20) and (21) are isovectors, and that they transform with respect to parity according to

\[ V^a(\lambda) \rightarrow V^a(\lambda), \quad A^a(\lambda) \rightarrow -A^a(\lambda). \] (22)

Taking these properties into account, we find that the effective operators can be constructed from appropriate building blocks in the following way:

\[ V^a(\lambda) = \frac{(v \cdot n)}{4} C_1(\lambda) \bar{H}_v [u^\dagger \tau^a u + u \tau^a u^\dagger] H_v + \frac{1}{2} C_2(\lambda) \bar{H}_v (S \cdot n)[u^\dagger \tau^a u - u \tau^a u^\dagger] H_v + \text{(pion part)}, \] (23)

\[ A^a(\lambda) = \frac{(v \cdot n)}{4} C_3(\lambda) \bar{H}_v [u^\dagger \tau^a u - u \tau^a u^\dagger] H_v + \frac{1}{2} C_4(\lambda) \bar{H}_v (S \cdot n)[u^\dagger \tau^a u + u \tau^a u^\dagger] H_v + \text{(pion part)}, \] (24)

where \( C_i, i = 1, 2, 3, 4, \) are unknown functions of the variable \( \lambda \) and, for brevity, we do not write explicitly pure pion contributions.

The number of functions can be reduced further if we consider that under global \( SU(2)_L \times SU(2)_R \) rotations the operator combinations \( V^a(\lambda) \pm A^a(\lambda) \) transform as pure right- and left-handed quantities, respectively:

\[ A^a(\lambda) + V^a(\lambda) = \bar{\psi}_R(\lambda n/2)\gamma^a \psi_R(-\lambda n/2), \] (25)

\[ A^a(\lambda) - V^a(\lambda) = -\bar{\psi}_L(\lambda n/2)\gamma^a \psi_L(-\lambda n/2), \] (26)

where we use the definitions \( \psi_{R,L} = \frac{1 + \gamma_5}{2} \psi \). According to the transformation laws given in (19), the same is true for certain combinations of the effective fields. Namely, for an arbitrary \( SU(2)_L \times SU(2)_R \) transformation it turns out that

\[ (\bar{H}_v u^\dagger \tau^a u H_v) = \bar{H}_v u^\dagger R^L \tau^a R u H_v, \] (27)

\[ (\bar{H}_v u \tau^a u^\dagger H_v) = \bar{H}_v u^\dagger L \tau^a L u H_v. \] (28)

Hence the correct transformation behavior of \( V^a(\lambda) \pm A^a(\lambda) \) can be reproduced only if the coefficient functions obey \( C_1 = C_3 \) and \( C_2 = C_4 \).

The remaining two functions are determined by comparing the nucleon matrix elements of the operators in the effective theory with the Fourier transforms of the leading order expressions (14) and (15), which read

\[ \langle \bar{P} + \Delta/2|V^a(\lambda)|\bar{P} - \Delta/2 \rangle = (v \cdot n) Q(\lambda) \bar{N}_v \frac{\tau^a}{2} N_v, \] (29)

\[ \langle \bar{P} + \Delta/2|A^b(\lambda)|\bar{P} - \Delta/2 \rangle = \bar{Q}(\lambda) \bar{N}_v (S \cdot n) \tau^b N_v - \frac{(\Delta \cdot n) g_A}{\Delta^2 - m^2} \bar{N}_v (S \cdot \Delta) \tau^b N_v. \] (30)
Here, the functions $Q$ and $\tilde{Q}$ are Fourier transforms of the corresponding parton distributions:

$$Q(\lambda) = \int_{-1}^{1} dx e^{ix\lambda(P\cdot n)} q(x), \quad (31)$$

$$\tilde{Q}(\lambda) = \int_{-1}^{1} dx e^{ix\lambda(P\cdot n)} \Delta q(x). \quad (32)$$

Consequently, the functions $C_1$ and $C_2$ are fixed as

$$C_1(\lambda) = Q(\lambda), \quad C_2(\lambda) = \tilde{Q}(\lambda). \quad (33)$$

Therefore, the final leading order expressions for the effective operators are

$$V^a(\lambda) = \frac{1}{4}(v \cdot n)Q(\lambda)\bar{H}_v[u^\dagger \tau^a u + u\tau^a u^\dagger]H_v + \frac{1}{2}\tilde{Q}(\lambda)\bar{H}_v(S \cdot n)[u^\dagger \tau^a u - u\tau^a u^\dagger]H_v + \text{(pion term)}, \quad (34)$$

$$A^a(\lambda) = \frac{1}{4}(v \cdot n)Q(\lambda)\bar{H}_v[u^\dagger \tau^a u - u\tau^a u^\dagger]H_v + \frac{1}{2}\tilde{Q}(\lambda)\bar{H}_v(S \cdot n)[u^\dagger \tau^a u + u\tau^a u^\dagger]H_v + \text{(pion term)}. \quad (35)$$

In conclusion, let us emphasize that the number of unknown functions in (23) and (24) was fixed using the behavior of the operators $V^a(\lambda)$ and $A^a(\lambda)$ under chiral transformations. In the paper [8] this was not done and as a result it was claimed that the final expressions for the effective operators contain additional unknown functions. Hence, the conclusion made in [8] about contradictions of the chiral perturbation theory with the soft-pion theorem derived in [7] seems to be incorrect and has to be reconsidered. Below we shall show that at tree level the correct effective operators give the same answer as the soft-pion theorem, as usually expected.

The explicit form of the pion terms in (34) and (35) is not important. In the kinematics of the heavy baryon approach we shall need for actual calculations only contributions arising from the vertex corresponding to local vector and axial-vector currents. Therefore, here we skip the discussion of the structure of such terms. Let us only mention that similar issues in the pure pion sector were discussed in [11]. In particular, it was demonstrated that the chiral perturbation theory and SPTs, derived from the current algebra approach, give identical results.

### 3 Soft pion emission

In this section we evaluate matrix elements of the effective operators (34) and (35) between an initial nucleon state and a final nucleon-pion state. Relevant diagrams are given in Fig.1. A direct calculation of these diagrams gives
Figure 1: Chiral perturbation theory diagrams. The large black blobs denote the vertices of the effective operator. Dashed lines and solid lines with arrows represent the pion and nucleon, respectively.

\[
\langle \vec{P} + \Delta/2, \pi^a(k)|V^3(\lambda)|\vec{P} - \Delta/2 \rangle = -\frac{1}{F_\pi} \bar{Q}(\lambda)\epsilon_{3ab} \bar{N}_v (S \cdot n) \tau^b N_v + \\
g_A \frac{(v \cdot n)Q(\lambda)}{F_\pi (k \cdot v)} \epsilon_{3ab} \bar{N}_v (S \cdot k) \tau^b N_v - g_A \frac{(k - \Delta) \cdot n}{F_\pi \Delta^2 - m^2_\pi} \epsilon_{3ab} \bar{N}_v (S \cdot \Delta) \tau^b N_v,
\]

where \( k \) denotes the small pion momentum and all other notations are the same as before. The first term on the right-hand side of (36) corresponds to the contribution of diagram a) in Fig.1, the second term is the result of the sum of contributions b) and c), and the third term originates from diagram d). Diagrams e), f), and g) do not contribute to the matrix element of the vector operator.

For the axial-vector isovector operator we obtain

\[
\langle \vec{P} + \Delta/2, \pi^a(k)|A^3(\lambda)|\vec{P} - \Delta/2 \rangle = -\frac{1}{2F_\pi} (v \cdot n)Q(\lambda)\epsilon_{3ab} \bar{N}_v \tau^b N_v + \\
ig_A \frac{\bar{Q}(\lambda)}{F_\pi (k \cdot v)} \bar{N}_v [(S \cdot k)(S \cdot n)\tau^a \tau^3 - (S \cdot n)(S \cdot k)\tau^3 \tau^a] N_v + \\
ig_A \frac{1}{F_\pi (k \cdot v) (k + \Delta)^2 - m^2_\pi} \bar{N}_v [(S \cdot (k + \Delta))(S \cdot k)\tau^3 \tau^a - (S \cdot k)(S \cdot (k + \Delta))\tau^a \tau^3] N_v + \\
\frac{1}{2F_\pi (k + \Delta)^2 - m^2_\pi} \epsilon_{3ab} \bar{N}_v \tau^b N_v.
\]

Again, the first term on the right hand side of (37) is due to a), the second is due to b)+c). Further, the third term comes from f)+g) and the fourth is the contribution from e). In the local limit we have \( Q(0) = 1 \) and \( \bar{Q}(0) = g_A \), so formulas (36) and (37) reproduce the correct expressions for the local currents which satisfy the following properties:

\[
\partial_\mu \bar{\psi} \gamma_\mu \frac{\tau^3}{2} \psi = 0, \quad \partial_\mu \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi = O(m^2_\pi).
\]

Our goal is to compare the ChPT results (36) and (37) with the corresponding soft-pion theorems (SPTs). A detailed derivation of the SPT formulas will be given in a
In the derivation of these formulas it is essential to assume that $k$ in (45) and (46) are of order $O(\varepsilon)$, separate publication [10]. In this presentation we shall only discuss the connection of the SPT approach with ChPT.

Before the presentation of the SPT formulas we introduce a set of slightly different notations which are convenient in this case. We shall use

\[ p' = \bar{P} + \Delta/2, \quad p = \bar{P} - \Delta/2, \quad p' - p = \Delta, \]

\[ Q^a_5 = \int d^3x \bar{\psi}(x) \gamma_5 \gamma^a \frac{\tau_3}{2} \psi(x), \]

\[ \langle p'|V^3(\lambda)|p \rangle = \bar{N}(p') \Gamma^3_V(p', p, \lambda) N(p), \]

\[ \Gamma^3_V(p', p, \lambda) \equiv \int_{-1}^{1} dx e^{i \lambda(P-n)} \left[ \gamma H(x, \xi, \Delta^2) + \frac{i \sigma_{\mu\nu} n_\mu \Delta_\nu}{2M} E(x, \xi, \Delta^2) \right] \frac{\tau_3}{2}, \]

\[ \langle p'|A^3(\lambda)|p \rangle = \bar{N}(p') \Gamma^3_A(p', p, \lambda) N(p), \]

\[ \Gamma^3_A(p', p, \lambda) \equiv \int_{-1}^{1} dx e^{i \lambda(P-n)} \left[ \lambda \gamma_5 \bar{H}(x, \xi, \Delta^2) + \lambda \gamma_5 \frac{\Delta \cdot (\Delta - n)}{(2M)^2} E(x, \xi, \Delta^2) \right] \frac{\tau_3}{2}. \]

With these definitions the soft-pion theorems for the matrix elements of the isovector operators read

\[ \langle p', \pi^a(k)|V^3(\lambda)|p \rangle = -\frac{i}{F_\pi} \langle p'|[Q^a_5, V^3(\lambda)]|p \rangle + \frac{g A}{2F_\pi} \bar{N}(p') \left[ \kappa \gamma_5 \tau^a \frac{i(p' + M)}{2(p' \cdot k)} \Gamma^3_V(p' + k, p, \lambda) + \Gamma^3_V(p', p - k, \lambda) \frac{i(p + M)}{-2(p \cdot k)} k \gamma_5 \tau^a \right] N(p) \]

\[ + \frac{g A}{2F_\pi} \bar{N}(p')  \Delta \gamma_5 \tau^b N(p) \frac{i}{\Delta^2 - m_\pi^2} i(n \cdot k) \epsilon_{3ab} + O(\varepsilon), \]  

(45)

\[ \langle p', \pi^a(k)|A^3(\lambda)|p \rangle = -\frac{i}{F_\pi} \langle p'|[Q^a_5, A^3(\lambda)]|p \rangle + \frac{g A}{2F_\pi} \bar{N}(p') \left[ \kappa \gamma_5 \tau^a \frac{i(p' + M)}{2(p' \cdot k)} \Gamma^3_A(p' + k, p, \lambda) + \Gamma^3_A(p', p - k, \lambda) \frac{i(p + M)}{-2(p \cdot k)} k \gamma_5 \tau^a \right] N(p) \]

\[ - \frac{\epsilon_{3ab}}{4F_\pi^2} \bar{N}(p')(\Delta + 2k) \tau^b N(p) \frac{i}{(k + \Delta)^2 - m_\pi^2} i F_\pi (n \cdot (\Delta + k)) + O(\varepsilon). \]

(46)

In the derivation of these formulas it is essential to assume

\[ m_\pi = O(\varepsilon), \quad k = O(\varepsilon), \]

while in contrast to the ChPT counting (11) here it is not necessary to demand $\Delta^2 = O(\varepsilon^2)$. Note that in the particular range where $|\Delta^2| \gg \varepsilon^2$, the explicit pion pole terms in (45) and (46) are of order $k$, so within the overall accuracy of the SPTs they can be neglected. Therefore, in this kinematical region, our results are in agreement with corresponding formulas in [7].

For a comparison with the ChPT results (36) and (37) we have to consider the region $\Delta^2 = O(\varepsilon^2)$, i.e. we have to perform a non-relativistic expansion of the SPTs (45) and
(46). Let us show the procedure in some detail for the vector current formula (45). First, we have to evaluate the commutator \([Q_5^a, V^3(\lambda)]\). Since we are only interested in the leading-order contribution, we can restrict ourselves to the chiral limit where \(Q_5^a\) is time-independent. Then the commutation relations of the fields,

\[
[Q_5^a, \psi] = -\frac{\tau^a}{2} \gamma_5 \psi, \quad [Q_5^a, \bar{\psi}] = -\bar{\psi} \gamma_5 \frac{\tau^a}{2},
\]

can be used to calculate the commutator of \(Q_5^a\) with the light-cone operator \(V^3(\lambda)\). In this way we obtain for the first term on the right-hand side of (45)

\[
-\frac{i}{F_\pi} \langle p' | [Q_5^a, V^3(\lambda)] | p \rangle = -\frac{\epsilon_{3ab}}{F_\pi} \langle p' | A^b(\lambda) | p \rangle
\]

\[
= \frac{\epsilon_{3ab}}{F_\pi} \bar{Q}(\lambda) \bar{N}_v(S \cdot n) \tau^b N_v + \frac{\epsilon_{3ab} (\Delta \cdot n) g^A}{F_\pi \Delta^2 - m_\pi^2} \bar{N}_v (S \cdot \Delta) \tau^b N_v + O(\varepsilon),
\]

where we used (30) to pass from (49) to (50). The first term of this contribution exactly equals that one of diagram a) in (36). The second term in combination with the pion pole in the last line of (45) is identical to diagram d).

Consider now the second line on the right hand side of (45), which represents the contributions of the nucleon poles. In the non-relativistic limit we obtain

\[
\frac{g_A}{2F_\pi} \bar{N}(p') \left[ \not\! \! \! \gamma_5 \tau^a \frac{i(p' + M)}{2(p' \cdot k)} \Gamma_3(p' + k, p, \lambda) + \Gamma_3^\lambda(p', p - k, \lambda) \frac{i(p + M)}{-2(p \cdot k)} \not\! \! \! \gamma_5 \tau^a \right] N(p)
\]

\[
= \frac{g_A}{F_\pi} \frac{(v \cdot n) Q(\lambda)}{(k \cdot v)} \epsilon_{3ab} \bar{N}_v(S \cdot k) \tau^b N_v + O(\varepsilon).
\]

Comparing with (36) we find that the nucleon pole terms (51) correspond to the contributions of diagrams b) and c) in Fig.1.

An analogous analysis can be performed for the axial-vector current as well. Again, the final expression is in agreement with the corresponding ChPT calculation (37). Therefore, we have demonstrated the agreement between results which have been obtained by two technically different ways. We would like to stress that this agreement is expected, because historically speaking, soft-pion theorems represented the first step in the development of ChPT.

## 4 Conclusions

Using the approach of chiral perturbation theory we calculated the soft pion emission from nucleon induced by twist-2 light-cone operators. It was shown that the soft-pion theorem results are in agreement with ChPT as it should be. We have found that the contradictions that were reported in paper [8] originate from the incorrect representation of the effective twist-2 operators given in that paper.
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