Renormalization group running cosmologies - from a scale setting to holographic dark energy

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Abstract.
A scale-dependent cosmological constant $\Lambda$ and the Newton constant $G$ emerge naturally in quantum field theory in a curved space-time background leading to renormalization group running cosmologies. A scale-setting procedure is discussed in these cosmological models and the interpretation of the scale is emphasized. This setup introduces dark energy without invoking quintessence-like fields and can be applied to a variety of problems. The scale-dependent $\Lambda$ and $G$ are also naturally incorporated into the generalized holographic dark energy model, and applied to different aspects of cosmology.

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1. Introduction

The resurrection of the cosmological constant\[1, 2, 3\], - a consequence of the evidence for the acceleration of the universe driven by nongravitating (unclustered) dark energy with negative pressure \[9\], precise measurement of the cosmic microwave background\[7, 8\] - has become one of the most futile playgrounds for the broad spectrum of new investigations \[4, 5\]. On the one hand, there is experimental evidence for a very tiny but positive cosmological constant, which, theoretically can be studied using the powerful tools of general relativity, and from quantum field theory to supersymmetry, superstrings, and branes\[9, 10, 11, 12, 13, 14\]. On the other hand, there is, horribile dictu, a flagrant discrepancy of the 123 orders of magnitude between the theoretical result and the experimental value. In this paper we discuss the basic underlying principles and/or ansätze in the effective quantum field theory on curved space-time, which is one of the possible frameworks used to study the running of the cosmological constant.

We would like to answer the following questions: Do we have a tool/framework which we can use in calculation of the cosmological constant properties without solving the $10^{123}$ discrepancy? Are we able to reconcile general relativity with quantum field theory (QFT) in spite of the fact that such a theory appears to be nonrenormalizable\[1\]? Especially, can one build a reliable theory at low energies (large distances) which can unite an obviously successful general relativity with QFT?

2. Cosmological constant renormalization and the decoupling theorem

By an ‘effective field theory’ (EFT) we understand a full quantum field theory with loops, regularization of divergences, renormalization, etc. It is basically the uncertainty principle that splits the theory in two regimes, so that EFT ‘decouples’ from the high energy sector\[15\]. All effects of heavy particles appear in loops and the short-distance physics is described by a local lagrangian which generally contains infinitely many local terms. The most general local lagrangian $\xi$ describes the high energy behavior of the theory\[16\]. But can we calculate anything with an infinite number of terms in $\xi$?

The second very important ingredient which resolves the problem of infinitely many terms in the theory is the fact that a local lagrangian is an energy expansion, the expansion parameter being the ratio of low energy scale and high energy scale. This reduces an infinite number of terms in lagrangian to the first few terms which can be used in calculation. In the quantized theory there appear loops and ultraviolet divergences

\[§\ I\ have\ again\ perpetrated\ something\ relating\ to\ the\ theory\ of\ gravitation\ that\ might\ endanger\ me\ of\ being\ committed\ to\ a\ madhouse,\ quoted\ as\ in\ [17].\]
can be removed using the counterterms which are exactly of forms which are present in the most general local lagrangian.

A very well-known example of an effective field theory used instead of a complete renormalizable theory is the heavy quark effective theory [18]. The latter is a nonrenormalizable theory but exhibits the properties of heavy quarks explicitly in a transparent way, with the rest of physics left as a few parameters. In the end the theory proved to be a very successful quid pro quo.

Another example which serves as a benchmark for EFT is the chiral perturbation theory (CPT) [19], which is a nonlinear realization of a low-energy limit of QCD. It is a complete QFT, calculated to one- and two-loops and compared with experiment. It appears that some rare processes which are absent at the tree level have contributions from the loops - this answers a sceptical questioning, namely the raison d'être of calculation of loops in a nonrenormalizable theory.

The effective field theory of gravity has been pushed up by Feynman [20], de Witt [21], 't Hooft and Veltman [22], etc., and was focused on high energy aspects of the theory. The program encountered serious difficulties with the divergence structure and did not lead to a satisfactory quantum gravity.

The low energy formulation of quantum field theory on curved space-time [23] [24] starts with the Einstein-Hilbert action plus the gauged matter lagrangian. The theory is quantized in such a way that the background field method preserves the symmetries of general relativity, and still allows to gauge-fix quantum fluctuations. By inspection of the results obtained for the graviton-graviton scattering, one encounters the non-analytic terms in the logarithms - a clear signal of long-distance effects in quantum gravity [15].

The vacuum action necessary to ensure the renormalizability of the gauged scalar (matter) Lagrangian should contain terms: $R_{\mu \nu \rho \sigma}^2, R_{\mu \nu}^2, R^2$, and $\Box R$ [25-26], and a trace anomaly term [27, 28].

The vacuum action is then given as

$$\mathcal{A}_{\text{vac}} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ (R - 2\Lambda) + (a_1 R_{\mu \nu \rho \sigma}^2 + a_2 R_{\mu \nu}^2 + a_3 R^2 + a_4 \Box R) \right] + \mathcal{A}_{\text{anom}}. \tag{1}$$

where $\mathcal{A}_{\text{anom}}$ is a trace anomaly term. Like the anomalous effective action in QCD, which appears as a consequence of chiral anomaly in QCD, the term $\mathcal{A}_{\text{anom}}$ should be included in EFT of gravity coupled to matter, even in zero momentum limit. It is the renormalization of the stress tensor that generates the trace anomaly, and provides us with the possibly important effects at large distances. This is true even if one had a theory with only massive particles - in that case the fluctuations of metric would generate such a term.

All divergences can be removed by renormalization of the matter fields, their masses...
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and couplings, the bare parameters. The matter (scalar) action is given by

$$A_m = \int d^4 x \sqrt{-g} \mathcal{L}_m = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi \partial_\nu \phi) - V(\phi) \right].$$

(2)

$V(\phi)$ can be taken to be, e.g., the following expression:

$$V(\phi) = -\frac{1}{2} m_0^2 \phi^2 - \frac{g_0}{24} \phi^4 - \bar{\Lambda} \phi + \eta_0 \phi,$$

(3)

where $\bar{\Lambda} = (8\pi G)^{-1}\Lambda_0$ is an arbitrary constant, which is basically the same as the cosmological constant in the Einstein lagrangian. Taking, for simplicity, $g_0 = 0$ (free scalar field) and $\eta_0 = 0$, one can calculate the lowest-order vacuum energy. After the renormalization one obtains the physical vacuum energy density given as

$$E = \bar{\Lambda} + \frac{m^4}{4(4\pi)^2} \left[ \ln \frac{m^2}{\mu^2} - \frac{3}{2} \right].$$

(4)

Here, the renormalized and bare CC are related as $\bar{\Lambda}_{\text{bare}} = \mu^d (\bar{\Lambda} + z_{\Lambda} m^4)$, where $z_{\Lambda}$ is a counterterm, $z_{\Lambda} = -\frac{1}{4(4\pi)^2} \frac{1}{\epsilon}$. It is clear that the vacuum energy $E$, as given in (4) at the one-loop level, is independent of the arbitrary renormalization scale $\mu$. The $\mu$-dependence of $\Lambda(\mu)$ cancels the $\mu$-dependence of $\ln \frac{m^2}{\mu^2}$.

It is now easy to derive the renormalization group equations for the CC. Normally, one expects that heavy particles decouple in the theory, according to the Appelquist-Carazzone decoupling theorem [30]. However, an interesting result of nondecoupling of heavy particles is found by Babic et al. [31]. Assume there exist two particles, a heavy one with mass $M$ and a light one with mass $m$. Then, for $m \ll \mu \ll M$ one expects the decoupling of a heavy particle with the suppression factor $\mu^2/M^2$. Instead, one finds the following behavior:

$$(4\pi)^2 \mu \frac{\partial}{\partial \mu} \Lambda(\mu) = \frac{1}{2} a \frac{\mu^2}{M^2} M^4 + \frac{1}{2} m^4.$$  

(5)

Obviously, the suppression factor $\mu^2/M^2$ ($a$ is the number of order $\mathcal{O}(1)$) is not sufficient to suppress the contribution of the heavy scalar, because

$$\mu^2 M^2 \gg m^4.$$  

(6)

The net result of [31, 32] is that one actually has to take into account the whole spectrum of heavy particles (which we do not know).

The question intimately connected with the running parameters is about scale fixing. We may argue, per analogiam with QCD, that the scale should be a typical momentum of the particles involved in a given physical process. In our case, this would be, for example, typical momenta of the gravitons involved in the loop calculation. Actually, in QCD, the running scale is obtained by looking at the scaling properties of Green functions. All momenta are scaled according to $p_i \to \lambda p_i$, where $\lambda$ is a scale parameter. In QFT on curved space-time one is doing exactly the same. The transformation $g_{\mu\nu} \to \lambda^2 g_{\mu\nu}$ implies $p^2 \equiv g_{\mu\nu} p^\mu p^\nu \to g_{\mu\nu} (\lambda p^\mu)(\lambda p^\nu)$, in full analogy with QCD.
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However, there is a serious problem because there are no gravitons attached in the lowest order - therefore, no explicit momentum is determined. Even more, as shown by Gorbar and Shapiro [33], it appears impossible to determine the lowest-order $\beta$-functions if calculation was performed on the flat background. One is therefore forced to rely on a certain intuitive educated guess so as to determine the scale or to reinterpret the scale as an infrared cutoff, which is physically quite well founded and acceptable. We will encounter and discuss this question in the next chapters, too.

3. RG running cosmologies - a scale setting procedure

\textit{Vos calculs sont corrects, mais votre physique est abominable.}

A. Einstein to G. E. Lemaître

The RGE scale setting procedure is far from being obvious and satisfactory. The question arises: is there a certain physical argument that would induce a procedure which might remove arbitrariness and lead to a scale setting?

The class of RGE-based cosmological models have certain common properties. To simplify an argument, let us assume that there is only one universal running scale $\mu$, and the only running quantities are $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ and the Newton constant $G$. This means that we ignore, for example, the eventual mild dependence of the particle masses that appear in the theory, etc. A further assumption is that the ponderable matter and radiation evolve in a standard way; the energy-momentum exchange between these components and the dynamical cosmological term is allowed. However, it is clear that one cannot talk about the conservation of energy and momentum for matter alone [1]. This follows from the vanishing of the covariant derivative of the mixed energy tensor of matter

$$\frac{\partial \mathcal{T}^\mu_\sigma}{\partial x^\mu} - \Gamma^\mu_\sigma\mathcal{T}_\mu = 0,$$

where the energy-momentum density $\mathcal{T}^\mu_\sigma$ is related to the energy-momentum tensor $T^\mu_\sigma$ as $\mathcal{T}_\sigma = T_{\mu\sigma}g^{\mu\rho}\sqrt{-g}$. By exerting forces upon 'matter' the gravitational field transfers energy to it - as is precisely described by the term $\Gamma^\mu_\sigma\mathcal{T}^\mu_\sigma$ in (7).

The scale setting procedure will now be applied to two specific cosmologies: the nonperturbative quantum gravity [35], and the cosmological model derived using quantum field theory on curved space-time [26][31].

The RGE scale setting procedure is far from being obvious and satisfactory. The question arises: is there a certain physical argument that would induce a procedure which might remove certain arbitrariness and lead to a scale fixing?

Our input equation takes a form of the RGE improved Einstein equation

$$G_{\mu\nu} = -8\pi G(\mu)\left[T^m_{\mu\nu} + T^h_{\mu\nu}\right],$$

Gorbar-Shapiro calculation was performed in a physical mass-dependent scheme and the behavior of the higher order terms in $\beta$ function in the infrared regime shows a clear decoupling - much the same behavior which was predicted for $\Lambda/G$ by Babic et al. [31] on intuitive basis. However, as pointed in [33], the absence of the $\beta$-functions for $\Lambda/G$ and $G^{-1}$ is probably an artefact of the perturbative expansion in $h_{\mu\nu}$, and not a fundamental property of the RG in curved space-time.
where
\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \] (9)
and \( T^\Lambda_{\mu\nu}(\mu) = g_{\mu\nu} \rho(\mu) \). Here, \( G_{\mu\nu} \) is the Einstein tensor, \( R_{\mu\nu} \) and \( R \) denote the Ricci tensor and scalar, respectively, while \( T^m_{\mu\nu} \) and \( T^\Lambda_{\mu\nu} \) denote the matter and cosmological constant energy-momentum tensors, respectively. To summarize, the only physical requirements to this equation are: i. its general covariance, ii. the \( \mu \)-dependence of \( G \) and \( T^\Lambda_{\mu\nu} \), and iii. the implicit time dependence of \( \mu \).

The conditions i. - iii. translate into
\[ G(\mu)[\rho_m + \rho(\mu)] + G(\mu) \dot{\rho}(\mu) = 0, \] (10)
where dots denote time derivatives. Assuming the nonvanishing \( \dot{\rho} \) throughout the evolution of the Universe (which seems to be a reasonable assumption) suggests the matter density equation \[ \rho_m = -\rho(\mu) - G(\mu) \frac{d\rho(\mu)}{d\mu} \frac{dG}{d\mu}^{-1}. \] (11)

The r.h.s. of the (11) is a function of the scale factor \( a \) since we assume the canonical behavior of \( \rho_m \), i.e., \( \rho_m = \rho_m,0 \left( \frac{a}{a_0} \right)^{-3(1+w)} \). The r.h.s. is, however, a function of \( \mu \), i.e., (11) has a form \( \rho_m = f^{-1}(\rho_m) \).

It is important to stress that our procedure lacks the first-principle connection to quantum gravity and, therefore, is not a fundamental one. However, as long as \( \rho_m \) in (11) retains its canonical form, the scale is univocally fixed \[34\].

3.1. Nonperturbative quantum gravity

This theory is based on the exact renormalization group approach, applied to quantum gravity \[35\]. The keystone of the theory is the effective average action \( \Gamma_k[g_{\mu\nu}] \) which is basically a Wilsonian coarse-grained free energy \[35, 36\]. The momentum scale \( k \) is then interpreted as an infrared cutoff - for a physical system with a size \( L \), the parameter \( k \propto 1/L \) defines an infrared cutoff. The path integral which defines the effective average action \( \Gamma_k[g_{\mu\nu}] \) integrates only the quantum fluctuations with the momenta \( p^2 \ll k^2 \), thus describing the dynamics of the metric averaged over the volume \( (k^{-1})^3 \). The theory is valid near the scale \( k \) in the sense that for any scale \( k \) there is a \( \Gamma_k \) which is an effective field theory at that scale.

All gravitational phenomena are correctly described at tree level by \( \Gamma_k \) including the contributions of loops with \( p^2 \geq k^2 \). This means that all quantum fluctuations with \( p^2 > k^2 \) are integrated out. This is very similar to the effective QCD, where high-energy quarks and gluons are integrated out. The large-distance metric fluctuations, \( p^2 \leq k^2 \), are not included as expected. However, in the limit \( k \rightarrow 0 \), the infrared cutoff disappears and one recovers the original action \( \Gamma \).

From the physical point of view, the infrared cutoff in nonperturbative quantum gravity corresponds physically to the dimension of the system. Its determination is not
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trivial - in massless theories such as massless QED, its interpretation is clear because $k^{-1}$ is the only mass scale present in the theory. In reality, a variety of mass scales is present and caution is demanded.

The correct way to proceed is to study the RG flaw of the effective action $\Gamma_k[g_{\mu\nu}]$ and identify the infrared cutoff by inspecting the RG evolution. Once the infrared cutoff is fixed, one should solve the Bianchi identities and the conservation laws for matter [37].

One can also start *vice versa*: first, use the Bianchi identities to fix a scale and, then, look for a meaningful physical interpretation of the scale [34].

In nonperturbative quantum gravity the choice $k \propto 1/t$, where $t$ is a cosmological time, i.e., the temporal distance between a given event and the big bang, seems very plausible. In EFT one integrates quantum fluctuations with momenta smaller than $1/t$, since these fluctuations should not play any role yet. This is a physical meaning of the infrared cutoff.

In the QG setting [36], the infrared cutoff plays the role of the general RG scale $\mu$. The application of the scale-setting procedure results in expressing the scale $k$ in terms of cosmological parameters

$$k = (8\pi g_\ast\lambda_\ast^{-1}\rho_m)^{1/4}$$

(12)

where $g_\ast$ and $\lambda_\ast$ are constants related to $G$ and $\Lambda$, respectively, in the following way: $\Lambda(k) = \lambda_\ast k^2$, $G(k) = g_\ast k^{-2}$. The values of $k$ for different scenarios are obtained in [34] and compared with the results of [36].

For $\Omega_K = 0,$ and arbitrary $w,$ our result for $k$ agrees with the result of [36]. The same happens to be true for $w = 1/3$ and arbitrary $\Omega_K.$ However, for $w = 0$ and $\Omega_K \geq 0,$ i.e., for the universe containing nonrelativistic matter only, and having arbitrary positive curvature, one obtains the law of evolution of the scale factor as a function of time.

This result is somewhat different from the result of [36]. In their calculation consistent solutions with $K = +1$ or $K = -1$ exist only for a radiation-dominated universe. On the contrary, our procedure, which is obviously mathematically correct, leads to consistent solutions for a universe having arbitrary curvature and for arbitrary $w$.

Is it possible that our solutions, although mathematically correct, to paraphrase Einstein, are *abominable*?

The answer is largely discussed in [34], and to shorten the argument, we emphasize that the scale-setting procedure always leads to a mathematically correct consistent fixing of $k$. However, at the same time, the physically acceptable choices for the scale $k$ are only those having a geometrical interpretation. As a matter of fact, we argued in [34] that for the universe with small curvature, the scale $k$ is reasonably close (in a regime with the IR fixed point domination) to the scale obtained for a flat universe and, therefore, a satisfactory geometrical interpretation is still possible.
3.2. Models from quantum field theory on curved space-time

Let us consider a generic case when both $\rho$ and $\Lambda$ can be expanded in series in $\mu^2$, where $\mu$ is the RGE scale. These models are studied in the framework of QFT on curved space-time, with a correct treatment of heavy-particle decoupling.

When the scale $\mu$ is smaller than all masses in the theory, the coefficients in the expansion, $C_i$ and $D_i$ can be either calculated or estimated. The expansion is given by

$$\rho \Lambda = \sum_{n=0}^{\infty} C_n \mu^{2n}, \quad G^{-1} = \sum_{n=0}^{\infty} D_n \mu^{2n}.$$  \hspace{1cm} (13)

The application of the scale-setting procedure \cite{34} yields the identification of the scale $\mu$

$$\rho_m = -C_0 + \frac{C_1 D_0}{D_1} + 2 \frac{D_0}{D_1} \left( C_2 - \frac{C_1 D_2}{D_1} \right) \mu^2 + \ldots ,$$  \hspace{1cm} (14)

Generally, in these cosmologies, $C_1 \sim m_{\text{max}}^2$, $C_2 \sim N_b - N_f \sim 1$, $C_3 \sim 1/m_{\text{min}}^2$, etc. Here $m_{\text{max}}$ and $m_{\text{min}}$ are the largest and the smallest mass, respectively.

The same reasoning applies to $G$. The qualitative analysis of (14) shows \cite{34} that the results obtained are at variance with observational bounds. By the 'qualitative analysis' we mean the fact that the coefficients $C_i$ and $D_i$ are determined on dimensional grounds only.

4. Holographic Dark Energy

The holographic principle is based on the assumption that QFT overcounts the true degrees of freedom - therefore some extra nonlocal constraints are necessary to obtain a reliable effective field theory.

The entropy $S$ scales extensively in an effective quantum field theory: for a system of size $L$ with the UV cutoff $\Lambda_{\text{UV}}$,

$$S \sim L^3 \Lambda_{\text{UV}}^3.$$  \hspace{1cm} (15)

However, it is known that, according to Beckenstein \cite{38}, the maximal entropy in a box of volume $L^3$ grows only as the area $A$ of the box. This means that, for example, all information that can be present in a black hole, should be coded on the two-dimensional horizon (surface) in Planckian pixels \cite{39}. The underlying principle was dubbed a holographic principle - a connection to a holographic display of 'a very pretty flirtatious girl' in the Stanford Physics Department was a rather recent revelation \cite{39}. A connection to physics was clear previously - a fact that measuring everything

\footnote{The finiteness of the Universe (the finite age and the finite particle horizon) leads to the upper limit in information (a number of bits) inside the horizon volume - for the universe at present time it amounts to approximately $10^{123}$ \cite{40}. This limit which is basically a consequence of the holographic principle, can have a more profound implications for fundamental physics, as discussed recently by Paul Davies in \cite{41}, see also the references therein.}
on the surface with Planckian resolution, one can reconstruct everything inside the volume - pointed to a very well known notion of hologram. The idea of holography is also intimately related to the ultraviolet/infrared connection \[42\], i. e., to the fact that going to higher and higher UV energies (short distances) one actually probes the long-distance physics - at very high energies, black holes would be created, which would emit long-wavelength quanta.

On the other hand, it was known \[43\] that 3 + 1 QFT’s overcome the the degrees of freedom, and a local QFT cannot describe quantum gravity, because it has too many degrees of freedom in UV.

The exit, suggested by Cohen et al. \[44\], is to limit the volume of the system according to

\[
L^3 \Lambda^3 \leq S_{BH} \equiv \pi L^2 M_P^2 ,
\]

where \(L\) is the size of the box, and \(S_{BH}\) is the entropy of the black hole. Obviously, the black-hole entropy grows as area (\(\sim L^2\)) of the horizon surrounding it.

If inside the volume \(V \sim L^3\) one were able to find a region with an entropy larger than the entropy of a black hole of the same size \(L^3\), but with smaller energy than \(E_{BH}\), this would immediately lead to a violation of the second law of thermodynamics. Namely, by adding an additional matter to a box \(L^3\) one would eventually form a black hole, but with a smaller entropy than the original entropy.

Cohen et al.\[44\] realized that the constraint (16) unavoidably included many states with the Schwarzschild radius larger than the box size \(L\). Therefore, the introduction of an infrared cutoff \(1/L\) which excludes all states that lie within their Schwarzschild radius was a necessity. A new constraint reads

\[
L^3 \Lambda_{UV}^4 \leq L M_P^2 ,
\]

i. e., the entropy in a given volume \(L^3\) should not exceed the energy of a black hole of the same size \(L\). An immediate consequence is that the IR cutoff now scales as \(\Lambda_{UV}^{-2}\) \[44\]. To summarize: It is obvious that the usual quantum corrections to the vacuum energy density (zero-point energy) give a wrong prediction. A holographic principle has, however, an intuitive physical idea behind it - the idea that the fields at the present energy scales do not fluctuate independently over the entire horizon, or even over the universe - the idea we have already encountered in the basis of some theories, such as nonperturbative quantum gravity \[36\]. Actually, if one takes the infrared cutoff to be of the order of \(H^{-1}\), i. e., approximately the size of the present horizon, the value of

\[\begin{footnotesize}
\footnote{Assuming the dominant energy condition, \(\rho + p \geq 0\), Davies\[45\] showed that the cosmological event horizon area of the Friedmann-Lemaître universe never decreases, \textit{per analogiam} with Hawking’s area theorem for black holes\[46\]. Even more, the cosmological event horizon increases also in models in which the radius of event horizon decreases. Actually, the loss of entropy from within the cosmological horizon (due to the matter, radiation and/or black holes crossing the cosmological horizon) is compensated by an increase in cosmological event horizon entropy - quite in agreement with the generalized second law of thermodynamics\[17\], \[18\], \[49\].}
\end{footnotesize}\]
\( \Lambda_{UV}^4 \) is coming down to something of the order of \((meV)^4\) - the right size of the present cosmological constant.

A number of authors have developed what is called a generalized holographic dark energy\[50\] - a model where both the cosmological constant (CC) and \( G \) within QFT on curved space are running. Applying the relation of Cohen \textit{et al.} between the UV and IR cutoff results in an upper bound to the zero-point energy density \( \rho_\Lambda \), given as

\[
\rho_\Lambda \leq \mu^2 G_N^{-1}(\mu).
\]  

This is a generalized dark-energy model, where the RG scale \( \mu \) is promoted to the IR cutoff \[51, 52\]. In this approach, \( \mu = 1/L \) is taken to be an IR cutoff and, therefore, the interpretation of the scale \( \mu \) is restricted to this definition. Again, taking \( L = H_0^{-1} = 10^{28}cm \) leads to the present observed value for the dark-energy density \( \rho_\Lambda = 10^{-47}GeV^4 \).

The precise choice of \( \mu \) is a sensitive question. If \( \rho_\Lambda \) is considered to be the energy density of a noninteracting perfect fluid - then the choice \( \mu = L^{-1} \) fails to recover the equation of state (EOS) for a dark-energy dominated universe, cf. the work of Hsu \[53\]. Even more, choosing \( L = H^{-1} \) always leads to \( \rho_\Lambda = \rho_m \) for flat space, thus hindering a decelerating era of the universe for redshifts \( z > 0.5 \). However, a correct EOS is obtained if one chooses a future event horizon for an infrared cutoff, as pointed by Li\[54\].

Since (18) is derived using the ZPE, the natural interpretation of dark energy in this equation is through the variable, or interacting CC with \( w = -1 \).

The energy transfer between various components in the universe (including a case with \( G \) varying with time) is described by a generalized equation of continuity

\[
\dot{G}_N(\rho_\Lambda + \rho_m) + G_N \dot{\rho}_\Lambda + G_N(\dot{\rho}_m + 3H \rho_m) = 0.
\]  

(19)

Here one should notice that \( \rho_\Lambda \) is affected not only by matter, but also by a time-dependent \( G_N \). In addition, (19) leads to the conservation of the quantity \( G_N T^{\text{total}}_{\mu \nu} \).

The holographic restriction (18) and the generalized equation of continuity (19) were used in \[34\] in order to constrain the parameters of of the RG evolution in QFT in curved-space background. It was assumed that the scale dependence of the CC and \( G \) arose solely from particle field fluctuations, i. e., no quintessence-type scalar fields were present. Again, one assumes the usual RG laws for the RG running scale \( \mu \) below the lowest mass in the theory. It is important to note that the scale \( \mu \) cannot be set from the first principles. Estimating the coefficients \( C_i \) and \( D_i \) on dimensional grounds, and noting that \( C_0 \), i. e., the vacuum ground state of the CC, coincides here with the IR limit of the CC, one is able to give a qualitative analysis. The context is set by fixing the matter density law to be a canonical one, i. e., no energy transfer between matter and other components is allowed. This further reduces (19), which, after insertion of the holographic expression (18), leads to the scale \( \mu \) given as

\[
\mu = -\frac{1}{2} G'_N(\mu) \rho_m.
\]  

(20)

* Sourced Friedmann eqs. with holographic dark energy are studied by Myung \[55\].
Equation (20) shows that once $G_N(\mu)$ is known, the IR cutoff $\mu$ is fixed. For $\mu > 0, G'_N > 0$, then $d/dt G_N > 0$, i.e., $G_N(t)$ increases with increasing cosmic time $t$. This implies that $G_N$ is asymptotically free - the property seen in quantum gravity models at the one-loop level, cf. [56]. This is an interesting phenomenon as the asymptotic freedom is of some interest for galaxy dynamics and rotation curves [57].

A more interesting case is the one with variable $G_N$ and $\rho_{\Lambda}$, and the canonical law for matter. Inserting the expansions in $\mu$ for $\rho_{\Lambda}$ and $G_N$ into the scaling fixing relation (25) leads to the expression for the scale $\mu$. Using the estimates for $D_n$, one obtains

$$\mu^2 \approx \frac{1}{2} m_{\text{min}}^2 (1 - M_{\text{Pl}}^{-1} \rho_m).$$

The following remarks are in order. The value of $\mu$ is marginally acceptable as far as the convergence of the $\rho_{\Lambda}(\mu)$ and $G_N(\mu)$ series is concerned. In addition, since the first time derivative of $G_N$ is negative, one obtains $D_1 \approx C_2 > 0$. Furthermore, (26) shows a very slow variation of the scale $\mu$ with the scale factor $a$ or the cosmic time $t$.

However, once the RG scale crosses below the smallest mass in the theory, it effectively freezes at $m_{\text{min}} \sim H_0 \sim 10^{-33} \text{eV}$. This is the main result of holography - one finds a hint for possible quintessence-like particles in the spectrum. What holography basically does - it expands the particle spectrum to the extremum - the largest possible particle masses approach the Planck mass, and the smallest possible particle masses are around the lowest mass scale in the universe, $m_{\text{min}} \sim H_0$. The present value of the vacuum energy density appears as the product of squared masses of the particles lying on the opposite sides of the spectrum - a hint to a natural solution to the coincidence problem! 

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♯ Further developments along these lines are given in [58].
†† In a different context, Pavon and Zimdahl [59] showed that interaction between dark matter and dark energy could also lead to the solution of the coincidence problem.
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