Magnetic field effects on neutrino oscillations

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Using the exact propagators in a constant magnetic field, the neutrino self-energy has been calculated to all orders in the field strength $B$ within the minimal extension of the Weinberg-Salam model with massive Dirac neutrinos. A neutrino dispersion relation, effective potential and effective mass have been obtained that depend on $B$. The consequences of this effective potential on neutrino oscillations have been explored, and resonance conditions have been obtained for magnetic fields $0 < eB \leq 4m_e^2$, where $e$ is the elementary charge and $m_e$ is the electron mass. The oscillation length has also been obtained, showing that $\nu_e - \nu_\mu$ resonant oscillations in magnetic field are likely to occur in the proximity of most stars generating high magnetic fields.

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I. INTRODUCTION

The study of neutrinos in the presence of a magnetic field is important in astrophysics and cosmology [1, 2]. The neutrino self-energy and its dispersion relation are modified in a magnetic field, and such modifications have been studied extensively in the literature [3–12]. There is a natural magnetic field scale that is required to significantly impact quantum processes, dependent on the electron mass $m_e$ and the elementary charge $e$, $B_e = m_e^2/e = 4.41 \times 10^{13}$ G. Large magnetic fields are present in many astrophysical sites such as supernovae, neutron stars, and white dwarfs. Fields larger than $B_e$ can appear during the explosion of a supernova or the coalescing of a neutron star. Magnetars are young neutron stars that generate even stronger magnetic fields, $10^{14} - 10^{16}$ G [13–15]. Magnetic fields as high as $10^{22} - 10^{24}$ G have been hypothesized to exist during the electroweak phase transition of the early universe [16–18]. Although it is rare for neutrinos to encounter magnetic fields larger than $10^{16}$ G, in many situations neutrinos come upon astrophysical sites where magnetic field strengths are at or around $B_e$.

While the electromagnetic properties of massless neutrinos have been studied extensively and for a long time [3–6], the nontrivial electromagnetic properties of massive neutrinos have only been addressed more recently [19–21]. A recent investigation on the role of a neutrino magnetic moment in flavor, spin and spin-flavor oscillations has produced interesting results [22, 23], but does not consider magnetic effects beyond those linear in $B$. In this paper we will focus on massive Dirac neutrinos within the minimally extended Standard Model and take a rigorous field theoretical approach to calculating their self-energy in a homogeneous magnetic field. We will accomplish this task by using Schwinger’s proper time method [24] and the exact fermion, scalar, and $W$-propagator in a constant magnetic field [6, 7].

In Section II, the notation for the fermion, gauge boson and scalar propagators in magnetic field is reviewed and the one-loop neutrino self-energy is set up. In Section III we calculate the self-energy operator and obtain a simple expression for the self-energy in the limit of $eB < m_W^2$, where $m_W$ is the mass of the $W$-boson, exact up to order $B^2$. In Section IV we use the self-energy operator we obtained to calculate the dispersion relation, effective potential and effective mass for massive Dirac neutrinos in a magnetic field and show that terms in the self-energy of order $B^2$ are often dominant when compared to linear terms such as the magnetic dipole moment term. In Section V we use the effective potential to investigate neutrino oscillation in a magnetic field and obtain resonance condition and oscillation length.

II. PROPAGATORS AND NEUTRINO SELF-ENERGY IN A CONSTANT MAGNETIC FIELD

The metric used in this paper is $g^{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ and the $z$-axis points in the direction of the constant magnetic field $B$. Therefore the electromagnetic field strength tensor $F^{\mu\nu}$ has only two non-vanishing components $F^{12} = -F^{21} = B$. 

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For the purpose of this work, it would seem convenient to work in the unitary gauge where the unphysical scalars disappear. However, the W-propagator is quite cumbersome in this gauge, and we prefer to work in the Feynman gauge, where the W-propagator has a much simpler expression. The following expressions for the charged lepton self-energy operator in the Feynman gauge corresponds to the sum of two diagrams, a bubble diagram with the gauge boson and a bubble diagram with the scalar propagators $D(x', x'')$ in a constant magnetic field have been written using Schwinger’s proper time method:

\[
S(x', x'') = \Omega^*(x', x'') \int \frac{d^4 k}{(2\pi)^4} e^{ik(x' - x'')} S(k) \ ,
\]

\[
G^{\mu\nu}(x', x'') = \Omega(x', x'') \int \frac{d^4 k}{(2\pi)^4} e^{ik(x' - x'')} G^{\mu\nu}(k) \ ,
\]

\[
D(x', x'') = \Omega(x', x'') \int \frac{d^4 k}{(2\pi)^4} e^{ik(x' - x'')} D(k) \ .
\]

In the Feynman gauge, the translationally invariant parts of the propagators are

\[
S(k) = \int_0^\infty \frac{ds}{\cos eBs} \exp \left[ -is \left( m_\ell^2 + k_\parallel^2 + k_\perp^2 \tan eBs \right) \right] \left( m_\ell - k_\parallel \right) e^{-ieBs\sigma_3} - \frac{k_\perp}{\cos eBs} ,
\]

\[
G^{\mu\nu}(k) = \int_0^\infty \frac{ds}{\cos eBs} \exp \left[ -is \left( m_W^2 + k_\parallel^2 + k_\perp^2 \tan eBs \right) \right] [g^{\mu\nu} + (e^{2eFs})^{\mu\nu}] ,
\]

\[
D(k) = \int_0^\infty \frac{ds}{\cos eBs} \exp \left[ -is \left( m_W^2 + k_\parallel^2 + k_\perp^2 \tan eBs \right) \right] ,
\]

where $-e$ and $m_\ell$ are the charge and mass of the charged lepton $\ell$, and $m_W$ is the $W$-mass. It is convenient to use the notation

\[
a_\parallel^\mu = (0, 0, 0, a^3), \quad a_\perp^\mu = (0, a^1, a^2, 0)
\]

and

\[
(ab)_\parallel = -a^0 b^0 + a^3 b^3, \quad (ab)_\perp = a^1 b^1 + a^2 b^2
\]

for arbitrary four-vectors $a$ and $b$. Using this notation we write the metric tensor as

\[
g^{\mu\nu} = g^{\mu\nu}_\parallel + g^{\mu\nu}_\perp .
\]

The $4 \times 4$ matrix $\sigma_3$ that appears in the charged lepton propagator \[4\], is

\[
\sigma_3 = \frac{i}{2} [\gamma^1, \gamma^2] .
\]

When writing the $W$-propagator \[5\], we use the notation

\[
(e^{2eFs})^{\mu\nu} = g^{\mu\nu}_\perp \cos (2eBs) + \frac{F^{\mu\nu}}{B} \sin (2eBs) .
\]

We choose the electromagnetic vector potential to be $A_\mu = -\frac{1}{2} F_{\mu\nu} x^\nu$ and therefore the phase factor which appears in Eqs. \[2\], \[3\] is given by

\[
\Omega(x', x'') = \exp \left( ieBs \int_{x'}^{x''} F_{\mu\nu} x^\nu \right) ,
\]

with $\Omega^*(x', x'')$ in Eq. \[1\] being its complex conjugate.

Perturbatively, the self-energy operator in the Feynman gauge corresponds to the sum of two diagrams, a bubble diagram with the gauge boson and a bubble diagram with the scalar

\[
\Sigma(p) = \Sigma_W(p) + \Sigma_\Phi(p) .
\]
The translationally non-invariant phase factors $\Omega(x',x'')$ are identical for all propagators and the product of phase factors in the two-vertex loop is

$$\Omega^*(x',x'')\Omega(x',x'') = 1, \quad (14)$$

therefore, within the minimally extended version of the standard model of electroweak interactions with an $SU(2)$-singlet right-handed neutrino, the two bubble diagrams can be written as $[6, 25]$

$$\Sigma_W(p) = -i\frac{g^2}{2} R\gamma_\alpha \int \frac{d^4k}{(2\pi)^4} S(p-k)G^{\alpha\beta}(k)\gamma_\beta L, \quad (15)$$

$$\Sigma_\Phi(p) = -i\frac{g^2}{2m_W^2} [m_\ell R - m_\nu L] \int \frac{d^4k}{(2\pi)^4} S(p-k)D(k)[m_\ell L - m_\nu R], \quad (16)$$

where $g$ is the $SU(2)$ coupling constant, $L = \frac{1}{2}(1 - \gamma_5)$ and $R = \frac{1}{2}(1 + \gamma_5)$ are the left-handed and right-handed projectors and neutrino mixing is allowed by taking a non-diagonal neutrino mass matrix $m_\nu$ in Eq. (16).

### III. THE SELF-ENERGY OPERATOR

Inserting the expression for the propagators from Eqs. (4) and (5) into the self-energy, we write $\Sigma_W(p)$ as

$$\Sigma_W(p) = -i\frac{g^2}{2} \int \frac{d^4k}{(2\pi)^4} \int_0^\infty \frac{ds_1}{\cos z_1} \frac{ds_2}{\cos z_2} e^{-i(s_1(m_\ell^2 + q_0^2 + q_1^2 \tan z_1) + s_2(m_\nu^2 + k_1^2 + k_2^2 \tan z_2))} \times$$

$$R\gamma_\alpha \left[ (s_1 - q_1) e^{-iz_1\sigma_3} - \frac{q_1}{\cos z_1} \right] \left[ g^\alpha_\beta + (e^{2F_s})^\alpha_\beta \right] \gamma_\beta L$$

where

$$q = p - k, \quad z_1 = eBs_1, \quad z_2 = eBs_2. \quad (17)$$

We do the straightforward $\gamma$-algebra, change variables from $s_i$ to $z_i$, translate the $k$ variables of integration and do the four gaussian integrals over the shifted variables $k$, to obtain

$$\Sigma_W(p) = \frac{g^2}{(4\pi)^2} \int_0^\infty \int_0^\infty \frac{dz_1 dz_2}{(z_1 + z_2) \sin(z_1 + z_2)} e^{-i(z_1 m_\ell^2 + z_2 m_\nu^2 + P/eB) \times}$$

$$\left[ \frac{z_2}{z_1 + z_2} p_\| e^{is\sigma_3(z_1 + z_2)} + \frac{\sin z_2}{\sin(z_1 + z_2)} p_\perp \right] L + (\text{c.t.})_W \quad (19)$$

where

$$P = \frac{z_1 z_2}{(z_1 + z_2)} p_\|^2 + \frac{\sin z_1 \sin z_2}{\sin(z_1 + z_2)} p_\perp^2 \quad (20)$$

and the appropriate counter-terms (c.t.) are added. Next it is convenient to change integration variables from $(z_1, z_2)$ to $(s, u)$ defined by

$$z_1 = eBs u = zu \quad \text{and} \quad z_2 = eBs(1 - u) = z(1 - u). \quad (21)$$

The result is

$$\Sigma_W(p) = \frac{g^2}{(4\pi)^2} \int_0^\infty \frac{dz}{\sin z} \int_0^1 du e^{-is\Lambda^2} R \left[ (1 - u)e^{iz(2-u)\sigma_3} p_\| + \frac{\sin[z(1 - u)]}{\sin z} p_\perp \right] L + (\text{c.t.})_W \quad (22)$$

where

$$\Lambda^2 = um_\ell^2 + (1 - u)m_\nu^2 + u(1 - u)p_\|^2 + \frac{\sin z u \sin(z - zu)}{z \sin z} p_\perp^2 \quad (23)$$
The scalar-loop contribution to the self-energy is found similarly

$$\Sigma_\Phi(p) = \frac{g_2}{(4\pi)^2} \frac{\delta_1}{2} \int_0^{\infty} \frac{dz}{\sin z} \int_0^1 du \ e^{-is\Lambda^2} R \left[ (1 - u)e^{-i\sigma_3} p_\parallel + \frac{\sin[z(1 - u)]}{\sin z} \rho_\perp \right] L + (c.t.) \Phi$$

where we used

$$\delta_1 = \frac{m_i^2}{m_W^2},$$

and neglected the matrix $m_\nu$ from Eq. (16) whose elements are all of the order of $2\text{eV}$ or less.

At this point we focus on magnetic fields $eB \ll m_W^2$, since magnetic fields $eB \sim m_W^2$ are not found in the universe. The expression of $\Sigma_W(p)$ in the limit of $eB \ll m_W^2$ is readily found by taking $z \ll 1$, and we obtain

$$\Sigma_W(p) = \frac{g_2}{(4\pi)^2} \int_0^{\infty} \frac{ds}{s} \int_0^1 du \ e^{-is\Lambda_0^2 (1 - u)} R \left[ i(2 - u) \frac{z}{m_W^2} \sigma_3 \rho_\parallel + \frac{z^2}{6m_W^2} \rho_\perp - \frac{z^2}{2m_W^2}(2 - u)^2 \rho_\parallel \right]$$

$$+ \frac{z^2}{6m_W^4} u(2 - u) \rho_\perp - i\sigma_3 \frac{z^2}{3m_W^2} u^2(1 - u)^2 \left( \frac{p_\perp^2}{m_W^2} \right) \rho L,$$

where we neglected terms that do not depend on the magnetic field since they will be absorbed by the neutrino wavefunction and mass renormalization, and we used the notation

$$\Lambda_0^2 = (1 - u) + u \frac{m_i^2}{m_W^2} + (1 - u)u \frac{p_\perp^2}{m_W^2}.$$  

Since $p^2 \sim m_\nu^2$, where $m_\nu$ is the mass of the neutrino, we can take $\Lambda_0^2 \simeq (1 - u) + u \delta_1$ and, after doing the $s$ and $u$ integration, we obtain

$$\Sigma_W(p) = \frac{g_2}{(4\pi)^2} R \left[ \frac{3eB}{2m_W^2} \sigma_3 \rho_\parallel + \frac{(eB)^2}{6m_W^4} \left( \frac{5}{2} + \frac{4p_\perp^2}{3m_W^2} \ln \delta_1 \right) \rho_\parallel \right.$$

$$+ \frac{(eB)^2}{2m_W^4} (1 - \ln \delta_1) \rho_\parallel + \frac{(eB)^2}{6m_W^2} (\ln \delta_1) \rho_\perp \left.] L. \right.$$ 

Undertaking similar steps, we find

$$\Sigma_\Phi(p) = \frac{g_2}{(4\pi)^2} \delta_1 R \left[ - \frac{eB}{4m_W^2} \sigma_3 \rho_\parallel + \frac{(eB)^2}{3m_W^4} \left( \frac{1}{4} + \frac{p_\perp^2}{3m_W^2} + \frac{1}{4} \ln \delta_1 \right) \rho_\parallel \right.$$

$$\left. - \frac{(eB)^2}{4m_W^4} \left( \frac{5}{2} + \ln \delta_1 \right) \rho_\parallel + \frac{(eB)^2}{12m_W^2} \left( \frac{3}{2} + \ln \delta_1 \right) \rho_\perp \right] L,$$

where each term is suppressed by a factor of $\delta_1$ when compared to the similar term in $\Sigma_W(p)$, thus making $\Sigma_\Phi(p)$ negligible when compared to $\Sigma_W(p)$. The self-energy operator is therefore $\Sigma(p) \simeq \Sigma_W(p)$.

### IV. Dispersion Relation, Effective Potential and Effective Mass

We intend to interpret the effect of the magnetic field on the self-energy operator as an effective neutrino mass $6^{[26]}$. To do so we must first evaluate the average value of $\Sigma(p)$ over the neutrino spinor in the mass basis, $\langle \Sigma \rangle = \bar{u}_i \Sigma(p) u_i$, since neutrino mass is defined in the mass basis and not in the flavor basis. We find,

$$\bar{u}_i R \sigma_3 \rho_\parallel L u_i = - \frac{m_\nu s_3}{2}$$

where $s_3 = \pm 1$, depending on the neutrino spin orientation relative to the magnetic field and, similarly,

$$\bar{u}_i R \rho L u_i = - \frac{m_i}{2},$$
\[ \bar{u}_i R \hat{p}_\perp L u_i = \chi \bar{u}_i \hat{p}_\perp u_i, \]  

(32)

where \( \chi = \frac{1}{2} (1 - \frac{\vec{p} \cdot \vec{p}}{2m_i^2}) \) in the Dirac representation and \( E \) and \( \vec{p} \) are the neutrino energy and momentum. Indicating with \( U \) the neutrino mixing matrix

\[ \nu_\ell = \sum_{i=1}^{3} U_{\ell i} \nu_i, \]  

(33)

and using Eqs. (30), (31), and (32), we write

\[ \langle \Sigma \rangle = -\frac{g^2}{64\pi^2} \frac{eB}{m_W^2} \sum_\ell U^*_\ell i U_{\ell i} \left[ 3s_3 m_i + \frac{eB}{3m_W^2} \left( \frac{11}{2} + \frac{4p_\perp^2}{3m_W^2} - 2 \ln \delta_\ell \right) m_i + \frac{2eB}{m_W^2} \left( 1 - \frac{4}{3} \frac{\ln \delta_\ell}{\chi} \right) \chi \bar{u}_i \hat{p}_\perp u_i \right], \]  

(34)

where \( \ell = e, \mu, \tau \). Eq (34) agrees with [6] in the absence of mixing and neutrino masses. Notice that transitions of the type \( \nu_i \rightarrow \nu_j \) are suppressed [27] by a GIM type mechanism, and therefore quantities such as \( \bar{u}_j \Sigma(p) u_i \) can be neglected. We obtain the dispersion relation by setting

\[ \hat{p} + m_i - \langle \Sigma \rangle = 0, \]  

(35)

and find the following energy-momentum relation for the massive neutrino in a magnetic field

\[ E^2 = p^2 + 2\Lambda p_\perp^2 + (1 + 2\mu)m_i^2, \]  

(36)

with

\[ \Lambda = \frac{g^2}{32\pi^2} \frac{(eB)^2}{m_W^4} \sum_\ell U^*_\ell i U_{\ell i} \left( 1 - \frac{4}{3} \ln \delta_\ell \right) \chi, \]  

(37)

and

\[ \mu = \frac{g^2}{64\pi^2} \frac{eB}{m_W^2} \sum_\ell U^*_\ell i U_{\ell i} \left[ 3s_3 + \frac{eB}{3m_W^2} \left( \frac{11}{2} + \frac{4p_\perp^2}{3m_W^2} - 2 \ln \delta_\ell \right) \right]. \]  

(38)

Eq. (38) is valid in the case of weak \((eB \ll m_\nu^2)\) and moderate \((m_\nu^2 \ll eB \ll m_W^2)\) magnetic field, while Eq. (37) is valid only in the case of weak magnetic field. In the case of moderate magnetic field \( \Lambda \) becomes [7]

\[ \Lambda = \frac{g^2}{32\pi^2} \frac{(eB)^2}{m_W^4} \sum_\ell U^*_\ell i U_{\ell i} \left[ 1 - \frac{4}{3} \ln \left( \frac{eB}{6m_W^2} + \delta_\ell \right) \right] \chi, \]  

(39)

which is more general than Eq. (37) and renders obsolete the distinction between weak and moderate magnetic field.

The dispersion relation of Eq (36) yields the following effective neutrino mass

\[ m_{eff} = \sqrt{(1 + 2\mu)m_i^2 + 2\Lambda p_\perp^2}. \]  

(40)

Notice that, since \( m_i \leq 2eV \), for \( p_\perp \sim 1MeV \) or higher the \( \Lambda \) term in Eq. (40) dominates over the \( \mu \) term, which can be safely neglected in many situations.

The effective potential is most useful in the flavor basis and, neglecting terms similar to \( \mu \) of Eq. (38) for the reason stated above, we find

\[ V_\ell = \lambda_\ell p_\perp^2 / E, \]  

(41)

with

\[ \lambda_\ell = \frac{g^2}{32\pi^2} \frac{(eB)^2}{m_W^4} \left[ 1 - \frac{4}{3} \ln \left( \frac{eB}{6m_W^2} + \delta_\ell \right) \right] \chi, \]  

(42)

where \( \ell = e, \mu, \tau \).
V. RESONANCE CONDITION AND OSCILLATION LENGTH

Our main results, Eqs. (40) - (42), have significant consequences on neutrino oscillations of the type $\nu_e \leftrightarrow \nu_{\mu,\tau}$. While the original MSW effect [28–30] showed that the presence of a medium alone can produce neutrino oscillations, in this work we will explore the role of a lone magnetic field in neutrino oscillations. The mixing angle $\theta_B$ in a magnetic field is determined by the following

$$\sin^2 2\theta_B = \frac{\sin^2 2\theta_{1i}}{\cos 2\theta_{1i} - \frac{2E(V_e - V_\ell)}{\Delta m^2_{1i}}} + \sin^2 2\theta_{1i}$$  \hspace{1cm} (43)

where $\Delta m^2_{1i}$ and $\theta_{1i}$ are the squared-mass splitting and vacuum mixing angle in the $\nu_e, \nu_\ell$ system, with $\ell = \mu, \tau$ and $i = 2, 3$ for $\ell = \mu, \tau$, and $V_e$ and $V_\ell$ are the $\nu_e$ and $\nu_\ell$ effective potentials in a magnetic field obtained in Eq. (41). Resonant oscillations will occur if

$$\Delta m^2_{1i} \cos 2\theta_{1i} = 2E(V_e - V_\ell)$$  \hspace{1cm} (44)

and, using Eq. (41), we find that the resonance conditions is

$$\Delta m^2_{1i} \cos 2\theta_{1i} \simeq 2(\lambda_e - \lambda_\ell)p^2_{\perp},$$  \hspace{1cm} (45)

where we neglected the smaller term proportional to $\mu$ in (41) and

$$\lambda_e - \lambda_\ell = \frac{g^2}{24\pi^2} \frac{(eB)^2}{m^4_W} \chi \ln \left( \frac{eB + 6m^2_\ell}{eB + 6m^2_e} \right).$$  \hspace{1cm} (46)

We take $\chi \sim 1$, the newest and most accurate values of $\Delta m^2_{1i}$ and $\cos 2\theta_{1i}$ [31], and in Figures 1 and 2 plot the value of $p_{\perp}$ for which resonance occurs as a function of $B$, for $0 < B \leq 4B_e$, where $B_e = m^2_e/e = 4.4 \times 10^{13}$ G. Figure 1 shows our result for $\nu_e$-$\nu_\mu$ oscillations, Figure 2 shows it for $\nu_e$-$\nu_\tau$ oscillations.

Neutrino oscillatory behavior is prominent when $L \sim L^\text{osc}$, where $L$ is the distance travelled inside the magnetic field and $L^\text{osc}$ is the oscillation length. The oscillation length for neutrinos in a magnetic field is

$$L^\text{osc} = \sqrt{\frac{L^\text{osc}_0}{\cos 2\theta_{1i} - \frac{2(\lambda_e - \lambda_\ell)p^2_{\perp}}{\Delta m^2_{1i}}}} + \sin^2 2\theta_{1i}$$  \hspace{1cm} (47)

where $L^\text{osc}_0$ is the vacuum oscillation length, given by

$$L^\text{osc}_0 = \frac{4\pi E}{\Delta m^2_{1i}} = 2.5 \frac{E}{\Delta m^2_{1i}} \text{ meter}$$  \hspace{1cm} (48)

![FIG. 1: Plot of $p_{\perp}$ for which resonance occurs as a function of $B$, in the case of $\nu_e$-$\nu_\mu$ oscillations. $p_{\perp}$ is in GeV and $0 < B \leq 4B_e$.](image-url)
where the neutrino energy $E$ is in MeV and $\Delta m^2_{11}$ is in eV$^2$. The neutrino energy is easily related to $p_\perp$ by assuming that the three components of the neutrino momentum are of similar magnitude, $p_1 \simeq p_2 \simeq p_3$, which leads to $E \sim \sqrt{\frac{3}{2}p_\perp}$. The oscillation length at resonance is

$$L_{R}^{osc} = \frac{L_{0}^{osc}}{\sin 2\theta_{1i}},$$

and, in the case of $\nu_e$-$\nu_\mu$ oscillations when $B = B_e$ and $E \sim 270$ MeV, we find $L_{R}^{osc} \sim 10^4$ Km. For $\nu_e$-$\nu_\mu$ oscillations, $B = 4B_e$, and $E \sim 70$ MeV, the oscillation length is $L_{R}^{osc} \sim 2.5 \times 10^3$ Km. Many astrophysical sites such as supernovae, neutron stars, white dwarfs and magnetars have magnetic fields as large as $B_e$ or larger, even larger by more than one order of magnitude, and therefore $\nu_e$-$\nu_\mu$ resonant oscillations in magnetic field are likely to occur in the proximity of such objects.

[1] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, Chicago, 1996).
[2] C. Giunti and C. W. Kim, *Fundamentals of Neutrino Physics and Astrophysics* (Oxford, UK: University Press, 2007).
[3] A. Erdas, C. W. Kim, and T. H. Lee, Phys. Rev. D 58, 085016 (1998).
[4] J. C. D’Olivo, J. F. Nieves and P. B. Pal, Phys. Rev. D 40, 3679 (1989).
[5] P. Elmfors, D. Grasso and G. Raffelt, Nucl. Phys. B 479, 3 (1996).
[6] A. Erdas and G. Feldman, Nucl. Phys. B 343, 597 (1990).
[7] A. Erdas, Phys. Rev. D 80, 113004 (2009).
[8] A. V. Kuznetsov, N. V. Mikheev, G. G. Raffelt and L. A. Vassilevskaya, Phys. Rev. D 73, 023001 (2006).
[9] G. McKeon, Phys. Rev. D 24, 2744 (1981).
[10] E. Elizalde, E. J. Ferrer and V. de la Incera, Annals Phys. 295, 33 (2002).
[11] E. Elizalde, E. J. Ferrer, and V. de la Incera Phys. Rev. D 70, 043012 (2004).
[12] J. F. Nieves and S. Sahu, Eur. Phys. J. C 78, no. 7, 547 (2018).
[13] R. C. Duncan and C. Thompson, Astrophys. J. 392, L9 (1992).
[14] C. Thompson and R. C. Duncan, Mon. Not. Roy. Astron. Soc. 275, 255 (1995).
[15] C. Thompson and R. C. Duncan, Astrophys. J. 473, 322 (1996).
[16] A. Brandenburg, K. Enqvist and P. Olesen, Phys. Rev. D 54, 1291 (1996).
[17] M. Joyce and M. E. Shaposhnikov, Phys. Rev. Lett. 79, 1193 (1997).
[18] D. Grasso and H. R. Rubinstein, Phys. Rept. 348, 163 (2001).
[19] C. Giunti and A. Studenikin, Rev. Mod. Phys. 87, 531 (2015).
[20] A. Studenikin, PoS EPS -HEP2017, 137 (2017).
[21] C. G. Tarazona, A. Castillo, R. A. Díaz and J. Morales, arXiv:1706.08614 [hep-ph].
[22] A. Popov and A. Studenikin, Eur. Phys. J. C 79, no. 2, 144 (2019).
[23] A. Popov and A. Studenikin, arXiv:1803.05755 [hep-ph].
[24] J. Schwinger, Phys. Rev. 82, 664 (1951).
[25] A. V. Kuznetsov and N. V. Mikheev, arXiv:0605114 [hep-ph].
[26] H. A. Bethe, Phys. Rev. Lett. 56, 1305 (1986).
[27] A. V. Borisov, I. M. Ternov and L. A. Vasilevskaya, Phys. Lett. B 273, 163 (1991).
[28] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
[29] L. Wolfenstein, Phys. Rev. D 20, 2634 (1979).
[30] S. P. Mikheyev and A. Y. Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985) [Yad. Fiz. 42, 1441 (1985)].
[31] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018).