Mathematical Modeling Skills on Solving PISA Problems

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Abstract. This study aimed to describe the mathematical modeling skills of students on solving PISA problems. Participant consisted of 40 students in class VIII - 1 SMP Negeri 13 Palembang. Data collection techniques was a test to determine students’ modeling skills, and interview. Results of tests and interviews were analyzed based on indicators of mathematical modeling skills as follows: (1) Formulate, (2) Employ, (3) Interpret. Results showed that: (1) For indicator 1 the students in good category, the students can identified the significant variables in problems or situation, while in enough category can’t use the variables, and then the students in bad category, the students can’t identified and used the variables, (2) For indicator 2 the students in good category, the students can employed concept, fact, procedure and reasoning. In enough category, the students employed concept and fact but can’t employed procedure and reasoning. In bad category the students can’t employed concept, fact, procedure, and reasoning. (3) For indicator 3 the students in good category, the students can reflected solutions, result and conclusion. In enough category, the students can reflected solution and result, but didn’t give conclusion. While in bad category, the students can’t reflected solution, result, and conclusion.

1. Introduction

PISA (Program for International Student Assessment) is a form of evaluation that measures skills and abilities designed for 15-year-old international students. PISA itself is a project of the Organization for Economic Co-operation and Development (OECD) which was first held in 2000 and was assessed covering the areas of reading, math and science. Since Indonesia has been joint PISA (Program for International Student Assessment) since 2000, Indonesian students were not able to solve HOT problems [1]. The results on PISA in mathematics have been instability. In the year 2000, PISA mathematics Indonesia was ranked 39 of 41 countries in mathematics. Then, in 2003, the rank was 38 of 40 countries. Then the worst result in PISA was in 2015 which is ranked 62 out of 70 in PISA 2015 [2, 3, 4]. Indonesia’s involvement in PISA is to see what extent our country’s education programs are growing compared to other countries in the world. It is important to be seen in the interests of our children in the future in order to have competed with other countries in the era of globalization [5].

PISA problems are consisted of Higher Order Thinking. Higher Order Thinking is thinking on the higher level such as mathematical thinking and reasoning, communication, critical attitude, interpretation, reflection, creativity, generalization and mathematizing [6, 7]. In the framework of PISA 2012 math literacy assessment [8], there are seven mathematical abilities underlying mathematical processes. One of them is modeling. According to De Lange [9] some competencies or
abilities that must be studied and mastered by students during the process of learning mathematics in the class is Modeling. Modeling is inseparable from other mathematical competencies, i.e. reading and communicating, designing and implementing problem-solving strategies or working mathematically such as reasoning, computing, etc. [10].

One of the possibilities that happens is that math becomes more meaningful to the learner [11, 12]. The mathematical model can be considered as a simplification of a complex real-world problem into a mathematical form. This mathematical problem can be solved by using any known technique to obtain a mathematical solution. Then, this solution is interpreted and translated into real form.

Chairman of an international mathematics group for PISA 2012, Kaye Stacey [13], argues that the concept of literacy is closely related to some of the concepts discussed in mathematics education. But the most important is modeling because the mathematical modeling cycle is an important aspect of PISA student conception as an active problem solver. So the ability of a mathematical model for solving a problem becomes very important in learning mathematics. Bliss [14] states that 21st century skills are creativity and innovation, critical thinking and problem-solving, and communication and collaboration which are all achievable through modeling. The importance of preparing students for mathematical literacy is by equipping them with 21st century competencies [15]. Mathematical modeling is one way to achieve this goal [16]. However, in Indonesian, mathematical modeling is not formally introduced at any school level [17]. Whereas mathematical modeling should be started early when the students already have basic competence then modeling can be developed [18, 19]. Even though the 21st century skills are creativity and innovation, critical thinking and problem solving, communication and collaboration which are all accessible via modeling [14, 20, 21, 22].

Mathematical literacy is expected to make the individual really understand the role of mathematics in modern life that he faces in the future in various situations encountered. So Mathematical Literacy is a very important thing [23]. The definition of Mathematical literacy has explicitly explained that there are three main processes in solving Mathematical problems, namely formulate, employ, and interpret.

These three processes provide an important and useful overview in organizing the contextual mathematical process of problems so that they can be solved. The categories to be used as assessments in this research are to formulate the situation mathematically, apply concepts, facts, and procedures, Interpret and evaluate mathematical results [8].

The application of the three processes (formulate, employ, interpret) requires specific and fundamental Mathematical competencies with diverse forms and degrees. So these three processes are cycles of modeling useful in solving a problem [24]. So the real problem can be solved through mathematical problems with the process of formulate, employ, interpret. The formulation in the definition of mathematical literacy, the word formulation refers to one's ability to recognize and identify opportunities for a given problem in some contextual forms, employ refers to one's ability to use concepts, facts, and procedures to solve problems that have been formulated to obtain the mathematical conclusion. In the process of applying, a person performs the mathematical procedures necessary to obtain results and find mathematical solutions (eg: perform arithmetic calculations, solve equations, make deductive reasoning from mathematical assumptions, manipulate symbols, filter information presented in tables and graphs, manipulate inner forms space, and analyzing data). In this process, other mathematical procedures, such as forming order, identify relationships in mathematical unity, the interpretation focuses on one's ability to reflect solutions, results, or mathematical conclusions and interpret them into the context of real-world problems. This can be interpreted as an act of translating reasoning or mathematical solutions into the context of the problem and determining whether the results are reasonable or not.

In the mathematical literacy model, the process of interpreting and evaluating processes is combined with the reasons these two processes involve the ability to build and communicate explanations and arguments in the context of the problem back and make a mathematical argument. In order to make it simple, look at the process in figure 1 below.
2. Experimental

The type of this research is descriptive research. Descriptive research is defined as research that tries to describe a phenomenon/events systematically in accordance with what it is [25]. In this study, researchers will describe the ability to modeling at junior high school students in solving PISA Problems.

This research is analyzed modeling ability of eighth grade students at SMP N 13 Palembang in solving PISA problem. Thus, the descriptions and categorization of modeling ability level is the main target of this research. To achieve these targets, the research procedure is required.

3. Results and Discussion

Preparing For The Experiment

Research subjects are 15 years old junior high school students as many as 40 students who are in SMP N 13 Palembang. Furthermore, the subject of the study followed a PISA test and an interview at a predetermined time. The results of the scores obtained from PISA test answers and interviews are used in determining the level of students' modeling abilities. From the test results and student interviews that include the process formulate, employment, and interpret scores obtained the highest and lowest scores. From the score is made categorization level of modeling ability.

Results

From the results of the tests that have been subjected to research and have been examined based on the assessment rubric, researchers classify students based on the criteria of modeling capabilities presented in table 1:

| Score | Assessment category | Frequency | Percentage |
|-------|---------------------|-----------|------------|
| 81–100| Very Good           | 2         | 5%         |
| 61–80 | Good                | 9         | 22.5%      |
| 41–60 | Enough              | 15        | 37.5%      |
| 21–40 | Bad                 | 8         | 20%        |
| 0–20  | Very Bad            | 6         | 15%        |
|       | **Total**           | **40**    | **100%**   |

5% participants were able to apply the ability of modeling correctly. However, on the high level problem, the participants were not able to solve it yet thus, their scores just reach 81-100 interval. Even for some problems, students were able to fill the indicator of modeling very well, in case on formulating problem by creating an illustration and then applying the fact and giving the conclusion very well. The participants with 22.5% which were categorized as good was the participants who have been solved some problems with the better modeling indicator. The participant with 37.5% which
were categorized as enough was the participants who have been solved problems exactly yet, but they have filled some modeling indicator. The participant with 20% which were categorized as bad was the participants who have not solved the problem correctly and they only can formulate the problems become simple on modeling indicator. The participant with 15% which were categorized as very bad is the participants who have not solved their problems and they only could write back the information which showed on the problems.

Furthermore, it will be described in the test data analysis and interviews and discussion. The achievement of student modeling indicators can be seen in the table below. Modeling ability consists of 3 aspects, namely formulate, employ, and interpret. Here is the frequency of students on each aspect of modeling ability.

Table 2. Achievement of Student Mathematical Modeling Indicators.

| Number | Percentage | Aspect  | Interpret |
|--------|------------|---------|-----------|
|        | Formulate  | Employ  |           |
| 1      | 75%        | 70%     | 22.5%     |
| 2      | 52.5%      | 27.5%   | 30%       |
| 3      | 27.5%      | 70%     | 37.5%     |
| 4      | 15%        | 37.5%   | 50%       |
| 5      | 5%         | 20%     | 17.5%     |
| Average| 34.5%      | 45%     | 31.5%     |

The ability of formulate has achievement as big as 34.5%. This indicator appeared where the participants must able to write the known information and questions and to create the form of math model as certain symbol and variable. Besides those reasons which have been declared previously, it also discovered some factors which cause the participants were not able to solve some modeling correctly that is the participants were not get used to of sampling problems become math model such as using symbol or certain variable. That case is in accordance with wijaya’s [26] research result that shows that the difficulties of students on understanding the problems and changing the real problems into the form of math were the most dominant error of participants on the process of math and math interpretation into the real situation. The problems also could be appeared because the participants were not get used to involved into the activities or experience on solving hot problems [27]. Where PISA problems were the problems which need hard thinking to be solved.

Employ indicator is the indicator with the highest percentage was 45%. It is marked along with the participant’s ability on correcting the facts, concepts, and procedures to get the result or to work mathematically where those results were used for getting the last result of the problems. This indicator was almost reached a half of participant’s success to apply.

Interpret indicator with the lower percentage was 31.5% where the participants have to be able to create valid conclusion based on the information and available evidences. In addition, there were more case where most of students did not write the last conclusion where it can be the confusion because the problems with the real problems have to be returned on the real context. It is in accordance with Lutfianto [28] that says that the failure of students to do PISA problems were on the way earn the result mathematically which was not continued to the fase on interpretation in to the situation or context which is needed.

Table 3. Achievement of SP Modeling Indicator.

| Subject | Modeling Indicator |
|---------|--------------------|
|         | 1                  | 2                  | 3                  | 4                  | 5                  |
| SP1     | Formulate          | Formulate          | Formulate          | Employ             | interpret |
|         | Employ             | Employ             | Employ             |                    | X                  |
Based on the test results obtained, SP1 is a student who obtained the category of good modeling ability. In the results of the interview, SP1 explained that the PISA problem is a pictorial problem that has a celebration with long words so we have to form a model in simplifying it.

Based on the results of tests obtained, SP2 indicates sufficient modeling ability. In the interview results, SP2 explained that the problem of PISA is an interesting problem because each of its always discussed with real-life context.

Based on the test results obtained, SP3 is a student with less or low modeling ability. In the results of interviews, SP3 explained that the problem of PISA is a problem that is categorized as difficult so it takes more understanding to solve it.

|          | Interpret | Employ | Formulate | Interpret | Employ | Formulate |
|----------|-----------|--------|-----------|-----------|--------|-----------|
| SP2      | interpret | Employ  | X         | X         |        |           |
| SP3      | X         | Formulate | Employ   | X         | X      |           |

**Figure 2.** Example of students solution on solving problems about bookshelves.

One of the interesting of the problems, the students, produced solutions in different strategies that were reflecting the reasoning of the students.

P : why did Yanuar use table to get the total of bookshelves
Sp : it will be reduced continuously until it becomes zero value, after that, it will be compared of the things which has been finished first and then it will be counted how much it takes until it reach zero.

P : why do you should do that?
Sp : to get the conclusion
P : so, what is the conclusion ?
Sp : there are 5 bookshelves

In this problem, student used the table to get the solution. However, it was not important to do. Because the student has already done the employ process by dividing the total number of things with the things which are needed and done it by rounding it, so that choosing the smallest number is enough to do as the conclusion. create a table to obtain a solution, but it is not necessary, because after the students do the employment process by dividing the number of goods owned with the required goods and then rounding the value, then by choosing the smallest value as the conclusion is enough. So, this case can be interesting. The Traditional Approach to teaching mathematics is not sufficient for strengthening student problem-solving skills and inadequacy of learning resources supporting the use of context-based mathematical problems and thus can not help students to develop competence in
math and application [29, 30, 31]. so the development of questions with the type of Research Design with the use of context becomes important [32].

4. Conclusion
Students with high modeling ability, based on the results of the test data analysis and the overall interview have been very good, because able to perform formulate process, employ, although there is a problem that has not fulfilled the interpretation process. There is only a lack of reasoning to solve the problem correctly. But overall it is able to meet the modeling indicators.

Students with enough modeling abilities, based on test data analysis and interviews, modeling ability is sufficient. It's just still not able to formulate the problem because it has not been able to such as objects using certain variables or symbols. But it is good enough in solving this matter seen that is enough in applying mathematics in solving a problem seen in problem no. 1,2,3 SP2 can fulfill the employment process well so as to give correct result and through proper interpretation process.

Students with low modeling abilities, based on analysis of test and interview data, modeling capabilities are classified as lacking, seen from how to answer PISA questions that show that they have not been able to formulate, apply mathematics correctly and provide appropriate conclusions. It's just still able to solve the problems at a low level.

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