Self-consistent theory for a plane wave in a moving medium and light-momentum criterion

Changbiao Wang
ShangGang Group, 70 Huntington Road, Apartment 11, New Haven, CT 06512, USA
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A self-consistent theory is developed based on the principle of relativity for a plane wave in a moving non-dispersive, lossless, non-conducting, isotropic uniform medium. Light-momentum criterion is set up for the first time, which states that the momentum of light in a medium is parallel to the wave vector in all inertial frames of reference. By rigorous analysis, novel basic properties of the plane wave are exposed: (1) Poynting vector does not necessarily represent the electromagnetic (EM) power flow when a medium moves, (2) Minkowski light momentum and energy constitute a Lorentz four-vector in a form of single EM-field cell or single photon, and Planck constant is a Lorentz invariant, (3) there is no momentum transfer taking place between the plane wave and the uniform medium, and the EM momentum conservation equation cannot be uniquely determined without resort to the principle of relativity, and (4) the moving medium behaves as a so-called “negative index medium” when it moves opposite to the wave vector at a faster-than-dielectric light speed.

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I. INTRODUCTION

The momentum of light in a medium is a long-lasting controversial question in physics. Abraham and Minkowski, respectively, proposed a formulation of light momentum. Abraham momentum is inversely proportional to the refractive index of medium, while the Minkowski’s is directly proportional to the index. Experiments were claimed to support both Abraham’s [1–4] and Minkowski’s [5, 6]. Barnett and Loudon assert that the early experiments by Walker et al. [2] “provide evidence that is no less convincing in favor of the Abraham form” [7]. But Feigel insists that “as far as we know, there are no experimental data that demonstrate the inverse dependence of the radiation pressure on the refractive index” [8]; in other words, no experimental observations of light momentum are quantitatively in agreement with the formulation given by Abraham. The recent direct fiber-recoiling observation by She et al. [4], which was claimed to support the Abraham momentum, is also thought to be “not uncontroversial” [8].

Light momentum has been widely investigated, even for all different kinds of dielectric materials, including magnetic [9, 10] and dispersive [11], but no agreement has been reached about which formulation is correct. Comprehensive presentations of the Abraham-Minkowski controversy are given in some review papers such as in [12–14], where there are a lot of valuable references collected.

Maxwell equations support various forms of momentum conservation equations, which is a kind of indeterminacy. However it is the indeterminacy that results in the question of light momentum. To find out which formulation of light momentum in a medium is correct, various theories have been proposed.

(1) Laue-Moller theory. Laue and Moller proposed a theory where four-vector covariance is imposed on the electromagnetic (EM) energy velocity in a moving medium [13, 10]. Laue-Moller theory supports Minkowski EM tensor and momentum, because the Minkowski tensor is a real four-tensor while Abraham’s is not, as indicated by Veselago and Shchavlev recently [17]. But Brevik disagrees, criticizing that such a theory is only “a test of a tensor’s convenience rather than its correctness” [12].

(2) Pfeifer-coworkers theory. Pfeifer and coworkers claim that the “division of the total energy-momentum tensor into electromagnetic and material components is arbitrary” [13]. In other words, the EM part and the material part in the total momentum can be arbitrarily distributed as long as the total momentum is kept the same.

(3) Mansuripur-Zakharian theory. Mansuripur and Zakharian argue that for EM radiation waves, Poynting vector represents EM power flow (energy flow) in any system of materials, and they claim that the Abraham momentum is “the sole electromagnetic momentum in any system of materials distributed throughout the free space” [18].

(4) Barnett’s theory. In a recent Letter, Barnett argues that the medium Einstein-box thought experiment (also known as “Balazs thought experiment”) supports Abraham momentum while the photon-atom Doppler resonance absorption experiment supports Minkowski momentum, and claims that both Abraham and Minkowski momentums are correct: one is kinetic, and the other is canonical [19].

Pfeifer-coworkers theory supports “arbitrary” EM mo-
momentums \[13\] while Barnett’s theory supports both Abraham and Minkowski momentums \[19\]. Laue-Møller theory only supports Minkowski momentum \[15, 16\] while Mansuripur-Zakharian theory only supports Abraham momentum \[18\].

Clearly, it is an insufficiency of the Pfeifer-coworkers theory \[13\] that the EM momentum in a medium cannot be uniquely determined. Photons are the carriers of EM momentum for radiation EM waves. According to Pfeifer-coworkers theory, the momentum of a specific photon in a medium could be Abraham’s, Minkowski’s, or even arbitrary; thus leading to the momentum not having a determine value.

In Barnett’s theory \[10\], the argument for supporting Abraham momentum is based on the analysis of Einstein-box thought experience by the “center-of-mass-energy” approach, where global momentum-energy conservation law is employed to obtain Abraham photon momentum and energy in the medium box in lab frame \[5\]. At first sight, such an approach is indeed impeccable. However on second thoughts, one may find that the approach itself has implicitly assumed the Abraham momentum to be the correct momentum; thus leaving readers an open question: Do the Abraham momentum and energy obtained still satisfy the global momentum-energy conservation law in all inertial frames of reference so that the argument is consistent with the principle of relativity?

Laue-Møller theory imposes four-vector covariance on the EM energy velocity in a moving medium, where the energy velocity is defined as Poynting vector divided by EM energy density \[15, 16\]. Obviously, the Poynting vector is assumed to be the EM power flow in the moving medium. In Mansuripur-Zakharian theory, the Poynting vector is also assumed to be the EM power flow in any system of materials \[18\]. The two theories have the same basic assumption, but they result in completely different physical conclusions: Minkowski momentum is the unique momentum for Laue-Møller theory, while Abraham momentum is the unique momentum for Mansuripur-Zakharian theory. From this, one may have every reason to question the justification of the assumption used in their theories: Does the Poynting vector really represent the EM power flow in any system of materials, including the moving medium?

In fact, there is another interesting question in Laue-Møller theory. The Laue-Møller theory assumes Poynting vector as the EM power flow (energy flow). Since the photon is the carrier of the EM energy and momentum, the Minkowski momentum which the theory solely supports is supposed to be parallel to the Poynting vector. However the Minkowski momentum and Poynting vector are not parallel in general in a moving medium [confer Eqs. \[37\] and \[38\] in the present paper]; resulting in a serious contradiction between the basic assumption and conclusion.

From above analysis we can see that there are flaws in the existing theories. Thus the crux of the matter is to set up a self-consistent theory. This theory must be based on a most fundamental postulate, which constitutes an additional condition imposed on physical laws, so that the light momentum can be uniquely determined. Such a postulate is the principle of relativity: The laws of physics are the same in all inertial frames of reference \[20\]. This principle is a restriction but also is a guide in formulating physical theories. According to this principle, there is no preferred inertial frame for descriptions of physical phenomena. For example, Maxwell equations, global momentum and energy conservation laws, Fermat’s principle, and Einstein light-quantum hypothesis are all valid in any inertial frames, no matter whether the medium is moving or at rest, and no matter whether the space is fully or partially filled with a medium.

In this paper, a self-consistent theory is developed based on the principle of relativity for a plane wave in a moving non-dispersive, lossless, non-conducting, isotropic uniform medium, which can uniquely determine the light momentum. By analysis of the plane wave, important unconventional conclusions are obtained, which are outlined below.

- There may be a pseudo-power flow when a medium moves, and the Poynting vector does not necessarily denote the EM power flow. This conclusion explains why the Laue-Møller and Mansuripur-Zakharian theories use the same assumption but result in different physical results.

- Minkowski light momentum and energy constitute a Lorentz four-vector in a form of single photon or single EM-field cell, and Planck constant is a Lorentz invariant. This conclusion has been applied to analysis of Einstein-box thought experiment, revealing why the argument for Abraham momentum in Barnett’s theory is not consistent with the principle of relativity \[21\].

- There is no momentum transfer taking place between the plane wave and the uniform medium, and the EM momentum conservation equation cannot be uniquely determined without resort to the principle of relativity. This conclusion is also supported by the Einstein-box thought experiment analyzed by EM boundary-condition matching approach, where the leading and trailing light pulse edges in a medium box do not produce additional Lorentz force, and both Abraham and Minkowski momentums satisfy the EM boundary conditions on the vacuum-medium interface \[21\].

- The moving medium behaves as a so-called “negative index medium” when it moves opposite to the wave vector at a faster-than-dielectric light speed. This result might help scientists build new kinds of metamaterials by simulating the dielectric properties of moving media.

It should be noted that the application of relativity principle is very tricky, not just manipulating Lorentz
transformations. For example, when applying this principle to Maxwell equations in free space, one may directly obtain the constancy of light speed \[22\]; when applying it to analysis of Abraham photon momentum in Einstein-box thought experiment, one may find that the Abraham momentum must have exactly the same form in all inertial frames \[21\], both without any need of Lorentz transformations.

According to the principle of relativity, the phase function for a plane wave [confer Eq. (3)] has the same form in all inertial frames. From this we can directly obtain an important light-momentum criterion:

- The momentum of light in a medium (including empty space) is parallel to the wave vector in all inertial frames.

The argument for this criterion is given below.

From Einstein light-quantum hypothesis, photons are the carriers of light momentum and energy. Thus the direction of motion of photons is the propagation direction of the light momentum and energy. The phase function defines equi-phase planes of motion (wavefronts), with the wave vector as the normal vector. From one equi-phase plane to another equi-phase plane, the path parallel to the normal vector is the shortest. According to Fermat’s principle, light follows the path of least time. Thus the direction of motion of the photons must be parallel to the wave vector, and so must the light momentum. Since the phase function is invariant in form, this property of light momentum must be valid in all inertial frames.

In some literature, the momentum of light in a medium is defined as the total momentum, namely the sum of EM part and mechanical part \[23\]. In this paper, the light momentum is defined as single photon momentum or EM momentum. According to this definition, the single photon momentum is the direct result of Einstein light-quantized EM momentum.

It should be emphasized that the principle of relativity is the backbone of the theory developed in the present paper. One might insist that the medium should define a preferred inertial frame of reference so that there is no reason why the Fermat’s principle is valid in all inertial frames. For example, Ravndal suggested a preferred Lorentz transformation when a dielectric medium exists \[24\]. However in this paper, the principle of relativity is taken as a fundamental postulate no matter with or without the existence of a medium, and the standard Lorentz transformation \[20\] is assumed to be universal.

The paper is organized as follows. In Sec. II, refractive index, phase velocity (photon velocity), and group velocity are defined for a plane wave in a moving uniform medium. In Sec. III, single photon momentum is analyzed. In Sec. IV, novel basic properties of a plane wave are revealed. Finally in Sec. V, some conclusions and remarks are given.

II. REFRACTIVE INDEX, PHASE VELOCITY, AND GROUP VELOCITY

In this section, invariant forms of refractive index, phase velocity (photon velocity), and group velocity are defined for a plane wave in a moving uniform medium. An unconventional analysis of the relation between the group velocity and Poynting vector is given.

Suppose that the frame \(X'Y'Z'\) moves with respect to the lab frame \(XYZ\) at a constant velocity of \(\beta c\), with all corresponding coordinate axes in the same directions and their origins overlapping at \(t = t' = 0\), as shown in Fig. 1. The Lorentz transformation of the time-space four-vector \((x, ct)\) is given by \[25\]

\[
x = x' + \frac{\gamma - 1}{\beta^2} (\beta' \cdot x') \beta' - \gamma \beta' ct',
\]

\[
ct = \gamma (ct' - \beta' \cdot x').
\]

where \(c\) is the universal light speed, and \(\gamma = (1 - \beta^2)^{-1/2}\) is the time dilation factor.

\[\text{FIG. 1: Two inertial frames of relative motion.} \quad X'Y'Z' \text{ moves with respect to } XYZ \text{ at } \beta c, \text{ while } XYZ \text{ moves with respect to } X'Y'Z' \text{ at } \beta' c \text{ (not shown), with } \beta' = -\beta. \text{ Note: } (\gamma\beta, \gamma) \text{ is the four-vector describing the motion of } X'Y'Z', \text{ while } (\gamma'\beta', \gamma') \text{ with } \gamma' = \gamma \text{ is the four-vector describing the motion of } XYZ; \text{ thus } (\gamma\beta, \gamma) \text{ and } (\gamma'\beta', \gamma') \text{ are not the same four-vector, which is an exception in this primed-unprimed symbol usage.}\

The EM fields \(E\) and \(B\), and \(D\) and \(H\) respectively constitute a covariant second-rank anti-symmetric tensor \(F^{\alpha\beta}(E, B)\) and \(G^{\alpha\beta}(D, H)\), of which the Lorentz transformations can be written in intuitive 3D-vector forms, given by \[24, 20\]

\[
E' = \gamma \left[ \begin{array}{c} E' \\ D' \end{array} \right] + \gamma \beta' \times \left[ \begin{array}{c} B'c_H/c \\ H'c/c \end{array} \right] - \frac{\gamma - 1}{\beta^2} \beta' \cdot \left[ \begin{array}{c} E' \\ D' \end{array} \right] \beta',
\]

\[
B' = \gamma \left[ \begin{array}{c} B' \\ H' \end{array} \right] - \gamma \beta' \times \left[ \begin{array}{c} E'/c \\ D'c/c \end{array} \right] - \frac{\gamma - 1}{\beta^2} \beta' \cdot \left[ \begin{array}{c} B' \\ H' \end{array} \right] \beta'.
\]
with \(E \cdot B, E^2 - (Bc)^2, D \cdot H, (Dc)^2 - H^2\), and \(E \cdot D - B \cdot H\) as Lorentz invariants.

### A. Refractive index and its Lorentz transformation

Suppose that there is a plane wave propagating in the medium-rest frame \(X'Y'Z'\), and the plane wave has a phase function given by \(\Psi(x', t') = \omega t' - n'_d k' \cdot x'\), with \(\omega' (> 0)\) the angular frequency, \(n'_d k'\) the wave vector, \(n'_d \equiv |n'_d k'|/|\omega'/c|\) the refractive index of medium, and \(k' = \omega'/c\). It is seen from Eqs. (3) and (4) that the phase function \(\Psi(x, t)\) for this plane wave observed in the lab frame \(XYZ\) must be equal to \(\Psi'(x', t')\) (confer Sec. IV), namely invariance of phase. Thus we have

\[
\Psi = \omega t - n_d k \cdot x = \omega t' - n'_d k' \cdot x',
\]

where \(n_d k\) is the wave vector in the lab frame, with \(n_d \equiv |n_d k|/|\omega/c|\) the refractive index and \(|k| = |\omega/c|\). Note: \(\omega\) can be negative. 

From the covariance of \((x', ct')\) and the invariance of phase, we conclude that \((n'_d k', \omega'/c)\) must be Lorentz covariant [20]. By setting the time-space four-vector \(X^\mu = (x, ct)\) and the wave four-vector \(K^\mu = (n_d k, \omega/c)\), Eq. (5) can be written in a covariant form, given by \((\dot{\omega} - n_d k \cdot x) = g_{\mu\nu} K^\mu K^\nu\) with the metric tensor \(g_{\mu\nu} = g^{\mu\nu} = \text{diag}(-1, -1, -1, +1)\) [29]. Since \(X^\mu\) must fulfill four-vector Lorentz rule, the invariance of phase and the covariance of \(K^\mu\) are equivalent.

From Eqs. (1) and (2) with \(n'_d k' \rightarrow x'\) and \(\omega'/c \rightarrow ct'\), we obtain \(K^\mu = (n_d k, \omega/c)\). Setting \(\hat{n}' = n'_d k' / |n'_d k'|\) as the unit wave vector we have

\[
\omega = \omega' \gamma(1 - n'_d \hat{n}' \cdot \beta'), \quad \text{(Doppler formula)}
\]

\[
n_d k = (n'_d k') + \frac{\gamma - 1}{\beta^2} (n'_d k') \cdot \beta' \beta' - \gamma \beta' - \gamma \beta' - \beta' \left(\frac{\omega}{c}\right).
\]

Since \(K_\mu K^\mu = g^{\mu\nu} K_\mu K_\nu\) is a Lorentz scalar, we have

\[
g^{\mu\nu} (K_\mu K_\nu - K'_\mu K'_\nu) = 0,
\]

namely

\[
\left(\frac{\omega}{c}\right)^2 - (n_d k)^2 = \left(\frac{\omega'}{c}\right)^2 - (n'_d k')^2.
\]

Eq. (9) indicates that \(\omega^2 (1 - n_d^2) = \omega'^2 (1 - n'_d^2)\) is a Lorentz invariant; thus we have \(n_d > 1 \Rightarrow n'_d > 1\). From Eqs. (10) and (9) we obtain the Lorentz transformation of refractive index, given by

\[
n_d = \sqrt{(n'_d)^2 - 1 + \gamma^2 (1 - n'_d \hat{n}' \cdot \beta')^2} / \sqrt{\gamma (1 - n'_d \hat{n}' \cdot \beta')}.
\]

From above Eq. (10), we can see that the motion of dielectric medium results in an anisotropic refractive index. But in free space with \(n'_d = 1\), we have \(n_d = 1\) holding for any propagation directions of waves, namely the empty space is always isotropic.

### B. Phase velocity and photon propagation velocity

It is seen from Eq. (5) that the phase function is symmetric with respect to all inertial frames, independent of which frame the medium is fixed in; accordingly, no frame should make its phase function have any priority in time and space. From this we can conclude that the definitions of equi-phase plane and phase velocity should be symmetric, independent of the choice of inertial frames. Thus the phase velocity can be defined as

\[
\beta_{ph} c = \frac{\omega}{n_d k} \hat{n} = c / \frac{n'_d}{\gamma} \hat{n} = \beta_{ph} c \hat{n},
\]

leading to

\[
\omega - n_d k \cdot \beta_{ph} c = 0,
\]

where \(\hat{n} = n_d k / n_d k\) is the unit wave vector in the lab frame, and \(\beta_{ph} c\) and \(K^\mu\) are related through \(K^\mu = (n_d k, \omega/c) = \omega (n_d/c)^2 (\beta_{ph} c, c/n'_d)\). Note: the definition of the phase velocity \(\beta_{ph} c\) is based on the wave four-vector \(K^\mu\), while the velocity definition of a massive particle is based on the time-space four-vector \(X^\mu\). Because the phase velocity \(\beta_{ph} c\) is parallel to \(n_d k\), which is a constraint, there is no “phase velocity four-vector”.

One might conjecture that \(\gamma_{ph} (\beta_{ph} c, c)\) could be the “phase velocity four-vector”, with \(\gamma_{ph} = (1 - \beta_{ph}^2)^{-1/2}\); however, by a further examination one can find that it is not true because \(\gamma_{ph} (\beta_{ph} c, c)\) does not fulfill four-vector Lorentz rule.

From Eq. (5), the equi-phase-plane (wavefront) equation of motion is given by \(\dot{\omega} - n_d k \hat{n} \cdot x = \text{const.}\) with \(\hat{n}\) as the unit normal vector of the plane, leading to \(\omega - n_d k \hat{n} \cdot (dx/dt) = 0\). Comparing with Eq. (11), we obtain \(\beta_{ph} c = \hat{n} \cdot (dx/dt) \hat{n}\). Thus we have a physical explanation to \(\beta_{ph} c\): the phase velocity is equal to the changing rate of the equi-phase plane’s distance displacement \(\hat{n} \cdot dx/dt\) over time \(dt\), and it is the photon propagation velocity. Obviously, this photon-velocity definition is consistent with the Fermat’s principle in all inertial frames: Light follows the path of least time.
FIG. 2: Photon real and apparent displacements. The photon propagation velocity is the phase velocity. From Fermat’s principle and the principle of relativity, when a photon together with its associated equi-phase plane moves from $A'$ to $B'$ along the unit wave vector $\hat{n}$ in the medium-rest frame, it moves from $A$ to $B$ observed in the lab frame. However because the time-space coordinates may not reflect its real location, resulting in an illusion, the photon looks like having moved to $C$ in terms of the time-space Lorentz transformation. Thus the photon real displacement $\Delta x_{\text{photon}}$ can only be converted from its apparent displacement $\Delta x$ through $\Delta x_{\text{photon}} = (\hat{n} \cdot \Delta x)\hat{n}$, and the phase velocity is related through $\beta_{\text{ph}} c = d\Delta x_{\text{photon}}/dt = (\hat{n} \cdot u)\hat{n}$, where $u \equiv dx/dt$ is assigned in the medium-rest frame. This conversion is governed by the photon real-apparent $\Delta x$.

In general, $dx/dt$ in the expression $\beta_{\text{ph}} c = \hat{n} \cdot (dx/dt)\hat{n}$ is undetermined unless a definition is given. If $dx'/dt = \beta_{\text{ph}}' c$ is assigned in the medium-rest frame, we call $u \equiv dx/dt$ the photon apparent velocity (“apparent” here means “look like but not necessarily real”). Note: $\gamma u = (1 - u^2/c^2)^{-1/2}$ is a four-vector. Thus we have $\beta_{\text{ph}} c = (\hat{n} \cdot u)\hat{n}$, with $|\beta_{\text{ph}} c| \leq |u|$, and $|\beta_{\text{ph}} c| = |u|$ if $u \parallel \hat{n}$.

It can be shown that, the photon apparent velocity $u$, Poynting vector $S = E \times H$, and EM energy density $W_{\text{em}} = 0.5(D \cdot E + B \cdot H)$ are related through $u = S/W_{\text{em}}$, where $S/W_{\text{em}}$ is the so-called “energy velocity” traditionally $1/c$. Calculations indicate $|u| = (1 - \xi)^{1/2} c \leq c$, where $\xi = (n_d^2 - 1)/[\gamma^2(n_d^2 - \hat{n} \cdot \hat{n}'\cdot\beta^2)] \geq 0$, and thus we have $|\beta_{\text{ph}} c| \leq |u| \leq c$, as expected.

From the equi-phase-plane equation $\omega t - n_d k \cdot x = \text{const} \Rightarrow \omega - n_d k \cdot u = 0 \Rightarrow \beta_{\text{ph}} c = (\hat{n} \cdot u)\hat{n}$, we have introduced the photon apparent velocity $u$. The appearance of $u$ comes from the fact: the photon real velocity is the phase velocity $\beta_{\text{ph}} c$, which is defined based on the wave four-vector $K'$ instead of the time-space four-vector $X'$
. From this it follows that, when using the time-space coordinates to describe the motion of a photon, the space coordinates may not reflect the photon real location, resulting in an illusion. Thus there must be a conversion between the photon apparent and real locations. This conversion is governed by the photon real-apparent velocity equation $\beta_{\text{ph}} c = (\hat{n} \cdot u)\hat{n}$. From it we have $\Delta x_{\text{photon}} = (\hat{n} \cdot \Delta x)\hat{n}$, where $\Delta x_{\text{photon}} = \beta_{\text{ph}} c \Delta t$ is the photon real displacement and $\Delta x \equiv u \Delta t$ is its apparent displacement, as shown in Fig. 2.

Note: $(\Delta x, c \Delta t)$ is a four-vector while $(\Delta x_{\text{photon}}, c \Delta t)$ is not, except for in free space ($n_d = 1$), where the “empty space” is isotropic, and the Poynting vector $S = W_{\text{em}} u$ is always parallel to the wave vector in all inertial frames, leading to $\beta_{\text{ph}} c = u = c\hat{n}$ and $\Delta x_{\text{photon}} = \beta_{\text{ph}} c \Delta t = u \Delta t = \Delta x$.

Now let us check the conservation law of photon Minkowski angular momentum. Photon momentum is given by $h n_d k$ [confer Eq. (10)]. Without loss of generality, suppose that the photon is located at $x = x' = 0$ when $t = t' = 0$. Thus we have $x_{\text{photon}} = \Delta x_{\text{photon}} = (\hat{n} \cdot \Delta x)\hat{n} = (\hat{n} \cdot x)\hat{n}$, and $x_{\text{photon}} \times h n_d k = 0$, namely the photon angular momentum is conservative in all inertial frames.

C. Group velocity and its relation with Poynting vector

The classical definition of group velocity is given by $v_{gr-c} = \partial \omega / \partial (n_d k)$, defined in the normal direction of wave-vector surface $\{20, 26\}$. In this paper we suggest a modified definition, given by $v_{gr} = \hat{n} \partial \omega / \partial (n_d k)$, defined in the wave-vector direction. Obviously, $v_{gr} \hat{n} = v_{gr-c} \hat{n}$ holds between the two definitions.

From Eq. (20), we know that the form-invariant definition of refractive index $n_d = |n_d k|/|\omega/c|$ itself also defines a dispersion equation of $(n_d k)^2 - (n_d \omega/c)^2 = 0$ for the plane wave, where $n_d = n_d k / (\omega/c)$ with $n_d = |n_d|$ is the refractive-index vector $\{20\}$. From Maxwell equations $Bc = n_d \times E$ and $Dc = -n_d \times H$ [confer Eq. (20)], we have $\hat{e} \cdot E c^2 + n_d \times [\hat{\mu}^{-1} \cdot (n_d \times E)] = 0$, which is a system of linear equations for $(E_x, E_y, E_z)$, and where $\hat{e}$ and $\hat{\mu}$ are, respectively, the dielectric vector...
permittivity and permeability tensors, and $\hat{\mu}^{-1}$ denotes the inverse tensor of $\hat{\mu}$. From this, we obtain the (eigen) Fresnel equation $F(n_d, \epsilon_{ij}, \mu_{ij}, \omega) = 0$ \cite{30}, or $n_d = n_d(\epsilon_{ij}, \mu_{ij}, \omega)$, with $\epsilon_{ij}$ and $\mu_{ij}$ the dielectric tensor elements, and $\omega$ the wave frequency. Moving parallel to the wave vector $\mathbf{k}$, and $\hat{\mu}$ does not explicitly contain $\mathbf{k}$. If there is any dispersion, $n_d$ implicitly contains $\omega$ through $\epsilon_{ij}$ and $\mu_{ij}$. Thus, the modified group-velocity definition $v_{gr} = \hat{\nabla} \partial \omega / \partial n_d |_{\omega}$, we obtain

$$v_{gr} = \frac{\beta_{ph} c}{1 + (\omega / n_d)(\partial n_d / \partial \omega)}. \quad (13)$$

Since the dielectric medium is assumed to be non-dispersive for the physical model considered in the present paper, $\partial n_d / \partial \omega = 0$ is valid. Thus we have $v_{gr} = \beta_{ph} c$, namely the group velocity is equal to the phase velocity, parallel to the wave vector.

As we know, for a plane wave in an anisotropic medium the wave vector and Poynting vector usually are not parallel. It has been thought that the group velocity is parallel to the Poynting vector, instead of the wave vector, as shown in the classical electrodynamics textbook by Landau and Lifshitz \cite{30}.

The moving isotropic medium becomes an anisotropic medium, as seen in Eq. (13); however, the group velocity we obtained is $v_{gr} = \beta_{ph} c$, parallel to the wave vector instead of the Poynting vector. Obviously, this is not in agreement with the result in the textbook \cite{30}.

Why do we have to modify the group velocity definition? By careful analysis we find that there is some flaw in the classical definition, of which the argument is given below.

Following the Landau-Lifshitz approach in analysis of a plane wave in an anisotropic lossless medium \cite{30}, with the holding of $(\delta \mathbf{D} \cdot \mathbf{E} - \mathbf{D} \cdot \delta \mathbf{E}) + (\delta \mathbf{B} \cdot \mathbf{H} - \mathbf{B} \cdot \delta \mathbf{H}) = 0$ taken into account for a moving non-dispersive uniform medium, from Maxwell equations we obtain $\partial \omega / \partial n_d = S \cdot \delta(n_d |_{\omega})/W_{em}$, where $\delta(n_d |_{\omega})$ is an arbitrary infinitesimal change in wave vector, $S = \mathbf{E} \times \mathbf{H}$ is the Poynting vector, and $W_{em} = 0.5(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$ is the EM energy density. From the mathematical definition of the gradient $\partial \omega / \partial n_d = v_{gr-c}$, we have $\partial \omega = v_{gr-c} \cdot \delta(n_d |_{\omega})$ for an arbitrary $\delta(n_d |_{\omega})$. Comparing $\partial \omega = S \cdot \delta(n_d |_{\omega})/W_{em}$ and $\partial \omega = v_{gr-c} \cdot \delta(n_d |_{\omega})$, we have $v_{gr-c} = S/W_{em}$, namely the classical group velocity is equal to the “energy velocity” \cite{17}, parallel to the Poynting vector. (Note: $\hat{\epsilon} \cdot \hat{\mu}$ in $D = \hat{\epsilon} \cdot E$ and $\hat{\mu} \cdot H$ are not symmetric in general for a moving medium so that $\partial D \cdot E - D \cdot \partial E = 0$ and $\partial B \cdot H - B \cdot \partial H = 0$ cannot separately hold, unlike in the traditional anisotropic medium where the symmetry of $\hat{\epsilon}$ and $\hat{\mu}$ is assumed \cite{30}.)

However there is a serious flaw for $v_{gr-c} = S/W_{em}$, because $|v_{gr-c}|$ can be greater than the phase velocity $|\beta_{ph} c|$, as shown in Sec. IV, which is not physical for a non-dispersive lossless medium. The modified definition $v_{gr} = \hat{\nabla} \partial \omega / \partial n_d |_{\omega}$, which leads to $v_{gr} = \beta_{ph} c$ for a non-dispersive medium, has removed the above flaw, which is right the reason why the classical definition of group velocity must be modified.

Since the modified group velocity Eq. (13) is always parallel to the wave vector $n_d \mathbf{k}$ instead of Poynting vector, the Poynting vector does not necessarily denote the direction of power flowing; this is clearly confirmed by the strict EM field solutions given in Sec. IV [confer Eq. (35)].

III. FOUR-VECTOR COVARIANCE OF MINKOWSKI PHOTON MOMENTUM AND ENERGY

In this section, single photon momentum in a medium is analyzed based on Einstein light-quantum hypothesis, and it is shown that the Minkowski photon momentum is strongly supported by Lorentz four-vector covariance, and it meets light-momentum criterion. Fizeau running water experiment is re-analyzed as a support to the Minkowski momentum.

For a uniform plane wave, observed in the medium-rest frame the EM fields $E', B', D'$, and $H'$ are related through $B'/n_d = \hat{n}' \times E'$ and $H' = \hat{n}' \times (c/n_d)D'$ (confer Sec. IV). Thus the Minkowski and Abraham EM momentum density vectors can be expressed as

$$g'_M = D' \times B' = \frac{n'_d}{c}(D' \cdot E')\hat{n}', \quad (14)$$

$$g'_A = \frac{E' \times H'}{c^2} = \frac{1}{n'_d c}(D' \cdot E')\hat{n}'. \quad (15)$$

Note: $(E' \times H')/|\hat{n}'| \hat{n}'$ holds in the medium-rest frame, but $(E \times H)/|\hat{n}| \hat{n}$ is not valid in general in the lab frame [confer Eq. (35)]. According to Einstein light-quantum hypothesis, the EM energy density $D' \cdot E'$ is proportional to single photon energy $\hbar \omega / \omega' > 0$, namely $D' \cdot E' = N'_p \hbar \omega / \omega'$ with $N'_p$ the photon number density, and the EM momentum density vectors $g'_M$ and $g'_A$ are, respectively, proportional to single photon momentums $p'_M$ for Minkowski’s and $p'_A$ for Abraham’s, namely $g'_M = N'_p p'_M$ and $g'_A = N'_p p'_A$. Thus from Eqs. (14) and (15), we obtain $p'_M = \hat{n}' n'_d \hbar \omega / c$ and $p'_A = \hat{n}' \hbar \omega / (n'_d c)$.

However the Minkowski photon momentum also can be naturally obtained from the covariance of relativity of wave four-vector, as shown below.

Suppose that the Planck constant $h$ is a Lorentz scalar (confer Sec. IV). From the given definition of wave four-vector, $K^\mu = (n'_d \mathbf{k}', \omega'/c) = (\hat{n}' n'_d \omega'/c, \omega'/c)$ multiplied by $h$, we obtain a “momentum-energy four-vector”, which has exactly the same form as a massive particle’s, given by

$$K^\mu = (n'_d \mathbf{k}', \omega'/c) = (\hat{n}' n'_d \omega'/c, \omega'/c).$$
where \( E' = \hbar \omega' \) is the very photon energy. In terms of the four-vector structure, \( \mathbf{p}' \) must be the momentum; thus we have the photon momentum in a medium, given by \( \mathbf{p}' = \mathbf{n}' \frac{\hbar \omega'}{c} \), that is right the Minkowski photon momentum \( \mathbf{p}'_M \) obtained above from Eq. (13).

Since \( \hbar K''^\mu \) is a four-vector, the Minkowski photon momentum \( \mathbf{p}'_M = \hbar n'_\omega' / c \) is parallel to the wave vector in all inertial frames, and thus it meets light-momentum criterion.

On the other hand, because the Minkowski photon momentum and energy constitute a Lorentz four-vector, the Abraham's must not, otherwise mathematical contradictions would result, except for in free space where Minkowski and Abraham momentums are identical \(^{21}\).

From the principle of relativity, we have the invariance of phase, from which we have the covariant wave four-vector. From the wave four-vector combined with Einstein light-quantum hypothesis, we have the Minkowski vector. From the wave four-vector combined with Einstein light-quantum hypothesis, we have the Minkowski photon momentum, which strongly supports the consistency of Minkowski momentum with the relativity and light-momentum criterion.

In the classical electrodynamics, Fizeau running water experiment is usually taken to be an experimental evidence of the relativistic four-velocity addition rule \(^{22}\). In fact, it should be taken to be a support to the Fizeau experiment. When the water runs along (opposite to) the wave vector direction, we have \( \beta > 0 \) (\( \beta < 0 \)) and the light speed is increased (reduced).

One might argue for a different photon energy in a medium. In the medium-rest frame, the dispersion equation, directly resulting from second-order wave equation, is given by

\[
(n'_d \omega')^2 - (n'_d k')^2 = 0,
\]

which actually is the expression, in the medium-rest frame, of the general definition of form-invariant refractive index \( n_d = |n'_d k'|/\omega' / c \). This dispersion relation was thought to be the characterization of the relation between EM energy and momentum, and the photon energy in a medium was suggested to be \( n'_d \hbar \omega' \) to keep a zero rest energy (see §3.1a of Ref. \(^{26}\), for example). However it should be noted that, although Eq. (11) is Lorentz invariant in form, \((n'_d k', n'_d \omega' / c)\) is not a Lorentz covariant four-vector, since only \( k''^\mu = (n'_d k', \omega'/c) \) is; except for \( n'_d = 1 \). If using \((n'_d k', n'_d \omega' / c)\) to define the photon momentum-energy four-vector, then it is not Lorentz covariant.

Thus it is justifiable to define \( \hbar K''^\mu = (n'_d k', \hbar \omega' / c) \) as the photon momentum-energy four-vector, as done in Eq. (16), because \( \hbar K''^\mu \) is four-vector covariant, with Eq. (19) as a natural result.

### IV. NOVEL PROPERTIES OF A PLANE WAVE IN A MOVING MEDIUM

A plane wave is the simplest strict solution to Maxwell equations \(^{23}\); however, its physics is far from being well understood. In this section, we will expose novel basic properties for a plane wave in a moving non-dispersive, lossless, non-conducting, isotropic uniform medium. Specifically, we will show that (1) Poynting vector does not necessarily represent EM power flow when a medium moves, (2) Minkowski EM momentum and energy constitute a Lorentz four-vector, and Planck constant is a Lorentz invariant, (3) there is no momentum transfer between the plane wave and medium, and the EM momentum conservation equation cannot be uniquely determined without resort to the principle of relativity, and (4) the moving medium may behave as a “negative index medium” when it moves opposite to the wave vector at a faster-than-dielectric light speed.

Suppose that the plane-wave solution in the medium-rest frame \( X'Y'Z' \) is given by

\[
(E', B', D', H') = (E'_0, B'_0, D'_0, H'_0) \cos \Psi',
\]

where \( \Psi' = (\omega' t' - n''_d k' \cdot \mathbf{x}') \), with \( \omega' > 0 \), and \((E'_0, B'_0, D'_0, H'_0)\) are real constant amplitude vectors. \( D' = \epsilon' E' \) and \( B' = \mu' H' \) hold, where \( \epsilon' > 0 \) and \( \mu' > 0 \) are the constant dielectric permittivity and permeability. Required by wave equation, the refractive index \( n'_d = |n'_d k'|/\omega' / c \) is given by \( n'_d = c_0 \sqrt{\epsilon' / \mu'} \), where \( n'_d \geq 1 \) is assumed to hold. Thus \((E', B', \mathbf{n}')\) and \((D', H', \mathbf{n}')\) are, respectively, two sets of right-hand orthogonal vectors, with \( E' = (c/n'_d) \mathbf{B} \times \mathbf{n}' \) and \( H' = \mathbf{n}' \times (c/n'_d) \mathbf{D}' \), resulting from Maxwell equations.
Inserting Eq. (20) into Eqs. (3) and (1), we obtain the plane-wave solution in the lab frame $XYZ$, given by

$$(E, B, D, H) = (E_0, B_0, D_0, H_0) \cos \Psi,$$  

(21)

where $\Psi = (\omega t - n_d k \cdot x)$, and the phase factor $\cos \Psi = \cos \Psi'$ must hold for any time-space points $\Rightarrow \Psi(x, t) = \Psi'(x', t') + 2\pi n$ with $l$ the integer, but $\Psi = \Psi'$ holds when $x = x' = 0$ and $t = t' = 0$, $\Rightarrow l = 0$, or $\Psi = \Psi'$, namely “invariance of phase”; $(E_0, B_0, D_0, H_0)$ are given by

$$\begin{bmatrix} E_0 \\ H_0 \end{bmatrix} = \gamma(1 - n_d' \hat{n} \cdot \beta') \begin{bmatrix} E'_0 \\ H'_0 \end{bmatrix}$$

$$+ \left( \gamma n_d' \hat{n} - \frac{\gamma - 1}{\beta^2} \beta' \right) \begin{bmatrix} \beta' \cdot E'_0 \\ \beta' \cdot H'_0 \end{bmatrix},$$

(22)

$$\begin{bmatrix} B_0 \\ D_0 \end{bmatrix} = \gamma(1 - \frac{1}{n_d} \hat{n} \cdot \beta) \begin{bmatrix} B'_0 \\ D'_0 \end{bmatrix}$$

$$+ \left( \gamma n_d \hat{n} - \frac{\gamma - 1}{\beta^2} \beta \right) \begin{bmatrix} \beta \cdot B'_0 \\ \beta \cdot D'_0 \end{bmatrix}. $$

(23)

Note: The transformations in Eqs. (22) and (23) are “synchronous”; for example, $E_0$ is expressed only in terms of $E'_0$. All field quantities have the same phase factor, no matter in the medium-rest frame or lab frame. It is clearly seen from Eqs. (20)–(23) that, the invariance of phase, $\Psi = \Psi'$, is a natural result. In the following analysis, formulas are derived in the lab frame, because they are invariant in forms in all inertial frames.

Under Lorentz transformations, the Maxwell equations keep the same forms as in the medium-rest frame, given by

$$\nabla \times E = -\partial B/\partial t, \quad \nabla \times H = J + \partial D/\partial t, \quad \nabla \cdot B = 0,$$

(24)

(25)

with $J = 0$ and $\rho = 0$ for the plane wave. From above, we have

$$\omega B = n_d k \cdot E, \quad \omega D = -n_d k \cdot H,$$

(26)

leading to $D \cdot E = B \cdot H$, namely the electric energy density is equal to the magnetic energy density, which is valid in all inertial frames.

From Eqs. (22) and (23), by tedious calculations we can obtain intuitive expressions for examining the space relations of EM fields observed in the lab frame, given by

$$D \cdot \hat{n} = D' \cdot \hat{n}' = 0, \quad B \cdot \hat{n} = B' \cdot \hat{n}' = 0,$$

(27)

$$E \cdot \hat{n} = \frac{\gamma(n_d^2 - 1)(E' \cdot \beta')}{\sqrt{(n_d^2 - 1) + \gamma^2(1 - n_d' \hat{n}' \cdot \beta')^2}},$$

(28)

$$H \cdot \hat{n} = \frac{\gamma(n_d^2 - 1)(H' \cdot \beta')}{\sqrt{(n_d^2 - 1) + \gamma^2(1 - n_d' \hat{n}' \cdot \beta')^2}},$$

(29)

and

$$E \cdot B = E' \cdot B' = 0, \quad D \cdot H = D' \cdot H' = 0,$$

(30)

$$E \cdot H = \gamma^2(n_d^2 - 1)(\beta' \cdot E')(\beta' \cdot H'),$$

(31)

$$D \cdot B = -\gamma^2 \left( 1 - \frac{1}{n_d^2} \right) (\beta' \cdot D')(\beta' \cdot B'),$$

(32)

$$D \cdot E = \gamma^2(1 - n_d' \hat{n}' \cdot \beta') \left( 1 - \frac{1}{n_d} \hat{n} \cdot \beta \right) D' \cdot E',$$

(33)

$$B \cdot H = \gamma^2(1 - n_d' \hat{n}' \cdot \beta') \left( 1 - \frac{1}{n_d} \hat{n} \cdot \beta \right) B' \cdot H',$$

(34)

and from Eq. (20) we have

$$E = (\hat{n} \cdot E) \hat{n} - \beta_{ph} c \hat{n} \times B,$$

(35)

$$H = (\hat{n} \cdot H) \hat{n} + \beta_{ph} c \hat{n} \times D.$$  

(36)

It is seen from above that $E \bot B$, $E \bot \hat{n}$, $B \bot H$, and $B \bot \hat{n}$ hold in the lab frame, but $E \parallel D$, $B \parallel H$, $E \parallel H$, $D \parallel B$, $E \parallel \hat{n}$, and $H \parallel \hat{n}$ usually do not hold any more.

A. Pseudo-power flow due to the motion of medium

As seen in Eq. (10), a moving isotropic uniform medium becomes an anisotropic medium, and as a result, a pseudo-power flow may be incurred, which is shown below.

From Eqs. (20), (35), and (36), we obtain Minkowski EM momentum and Poynting vector, given by

$$D \times B = \left( \frac{D \cdot E}{\omega} \right) n_d k = \left( \frac{n_d}{c} \right)^2 (D \cdot E)v_{gr},$$

(37)

$$E \times H = v_{gr}[v_{gr}(D \times B) - (\hat{n} \cdot H)B - (\hat{n} \cdot E)D],$$

(38)

where $v_{gr}$, with $v_{gr} = v_{gr} \cdot \hat{n}$ and $|v_{gr}| = c/n_d$, is the group velocity obtained from Eq. (13) with no dispersion ($\partial n_d / \partial \omega = 0$) considered, which is equal to the phase velocity $\beta_{ph} c$, as defined by Eq. (11). Note that the Minkowski momentum $D \times B$ has the same direction as the wave vector $n_d k$, while the Poynting vector $E \times H$ has three components: one in $(D \times B)$-direction, one in $B$-direction, and one in $D$-direction; the latter two are perpendicular to the group velocity $v_{gr} = \beta_{ph} c$ or the wave vector $n_d k$.

We can divide the Poynting vector $S = E \times H$ into two parts, namely $S = S_{power} + S_{pseu}$, where

$$S_{power} = v_{gr}^2 (D \times B) = (D \cdot E)v_{gr} = W_{em} v_{gr},$$

(39)

$$S_{pseu} = -v_{gr}[(\hat{n} \cdot H)B + (\hat{n} \cdot E)D],$$

(40)

with $S_{power}$ and $S_{pseu}$ being perpendicular each other $(S_{power} \perp S_{pseu})$. 


From Eq. (33), we find that $\mathbf{S}_{\text{power}}$ carries all the EM energy $W_{em} = 0.5(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H})$ moving at the group velocity $\mathbf{v}_{gr}$, and it is a real power flow. According to the energy conservation law, $\mathbf{S}_{\text{power}}$ should not be responsible for any EM energy transport, and it is a pseudo-power flow. Thus the energy velocity, defined as $\mathbf{S}_{\text{power}}/W_{em}$, is equal to the group velocity $\mathbf{v}_{gr}$ and the phase velocity $\beta_{ph}c$, which is justifiable when considering that the medium is assumed to be non-dispersive and lossless.

In the medium-rest frame, both $\mathbf{E}' \perp (n_d'k')$ and $\mathbf{H}' \perp (n_d'k')$ hold. Thus if $\beta \parallel (n_d'k')$ holds, we have $|\mathbf{E}'| = 0 = |\mathbf{E} \cdot \mathbf{n}|$ from Eq. (23), and $|\mathbf{H}' \cdot \mathbf{n}| = 0$ from Eq. (29). From this, according to Eq. (40) we find that the pseudo-power flow $\mathbf{S}_{\text{pseudo}}$ vanishes only when the medium ($n_d' \neq 1$) moves at $\beta c = -\beta' c$ parallel to the wave vector $n_d'k'$. In other words, $\mathbf{S}_{\text{pseudo}} \neq 0$ is incurred in general in a moving medium ($n_d' \neq 1$), so that the Poynting vector $\mathbf{E} \times \mathbf{H}$ is not parallel to the wave vector $n_dk$.

The physical difference between $\mathbf{S}_{\text{power}}$ and $\mathbf{S}_{\text{pseudo}}$ also can be seen from divergence theorem. The divergence of $\mathbf{S}_{\text{power}}$ is given by

$$\nabla \cdot \mathbf{S}_{\text{power}} = -\frac{\partial W_{em}}{\partial t}$$

$$= -(\mathbf{D}_0 \cdot \mathbf{E}_0) \frac{\partial}{\partial t} c^2 (\omega t - n_d \mathbf{k} \cdot \mathbf{x}) \neq 0$$  \hspace{1cm} (41)$$

holding except for those discrete points, which means that $\mathbf{S}_{\text{power}}$ is responsible for an EM power flowing into and out from a differential box, but the time average $\langle \nabla \cdot \mathbf{S}_{\text{power}} \rangle = 0$, meaning that the powers going in and out are the same on time average, with no net energy left in the box. In contrast, $\nabla \cdot \mathbf{S}_{\text{pseudo}} \equiv 0$ holds resulting from $\nabla \cdot \mathbf{B} = 0$, $\nabla \cdot \mathbf{D} = 0$, $\mathbf{B} \perp (n_d \mathbf{k})$, and $\mathbf{D} \perp (n_d \mathbf{k})$, which means that $\mathbf{S}_{\text{pseudo}}$ is not responsible for a power flowing at any time for any places (otherwise energy conservation would be broken).

Since $\mathbf{S}_{\text{power}}$ and $\mathbf{S}_{\text{pseudo}}$ are perpendicular each other, $|\mathbf{S}/W_{em}| > |\mathbf{S}_{\text{power}}/W_{em}| = |\beta_{ph} c|$ holds for $\mathbf{S}_{\text{pseudo}} \neq 0$. If $\mathbf{S}/W_{em}$ were defined as the group velocity or energy velocity as done in the classical textbooks [16, 30], then the group velocity or energy velocity would be greater than the phase velocity, which is not physical for a non-dispersive lossless medium.

It is seen from above analysis that Poynting vector does not necessarily denote a real EM power flow. However such a phenomenon seems neglected in the physics community, in view of the fact that the Abraham momentum, defined through the Poynting vector, is taken as an EM momentum postulate, as proposed by Mansuripur and Zakharian [18].

In summary, we can make some conclusions for the EM momentums and EM power flows:

- Observed in any inertial frames, the Minkowski EM momentum $\mathbf{D} \times \mathbf{B}$ is parallel to the wave vector [see Eq. (37)], which is completely in agreement with the light-momentum criterion as stated in the Introduction section.

- Observed in the medium-rest frame, the Abraham EM momentum $\mathbf{E} \times \mathbf{H}/c^2$ is parallel to the wave vector; however, observed in general inertial frames, it is not [see Eq. (38)]. Thus the Abraham EM momentum does not meet the light-momentum criterion.

- When a medium moves, the Poynting vector $\mathbf{E} \times \mathbf{H}$ consists of two parts: one is parallel to the wave vector, and it is a real power flow; the other is perpendicular to the wave vector, and it is a pseudo-power flow [see Eq. (38)–Eq. (40)].

B. Four-vector covariance of Minkowski EM momentum and energy, and invariance of Planck constant

We have shown the Lorentz covariance of Minkowski photon momentum and energy from the wave four-vector combined with Einstein light-quantum hypothesis in Sec. III. This covariance suggests us that there should be a covariant EM momentum-energy four-vector, given by

$$\bar{P}^\mu = (\bar{p}_{em}, \bar{E}_{em}/c),$$  \hspace{1cm} (42)$$

where $\bar{p}_{em}$ and $\bar{E}_{em}$ are, respectively, the EM momentum and energy for a single “EM-field cell” or “photon”, given by

$$\bar{p}_{em} = \frac{\mathbf{D} \times \mathbf{B}}{N_p}, \quad \bar{E}_{em} = \frac{\mathbf{D} \cdot \mathbf{E}}{N_p},$$  \hspace{1cm} (43)$$

with $N_p$ the “EM-field-cell number density” or “photon number density” in volume. With $\mathbf{D} \times \mathbf{B} = (\mathbf{D} \cdot \mathbf{E}/\omega)n_d \mathbf{k}$ from Eq. (37) taken into account, the EM momentum-energy four-vector and wave four-vector are related through $\bar{P}^\mu = (\bar{E}_{em}/\omega)K^\mu$, with $(\bar{E}_{em}/\omega) = (\mathbf{D} \cdot \mathbf{E})/(N_p \omega)$ corresponding to Planck constant $h$ physically. Thus what we need to do is to find out the condition for $N_p$ to satisfy for the four-vector covariance of $(\bar{p}_{em}, \bar{E}_{em}/c)$.

The four-vector $\bar{P}^\mu = (\bar{p}_{em}, \bar{E}_{em}/c)$ is required to fulfill four-vector Lorentz rule given by Eqs. (11) and (2), while the EM fields must fulfill the Lorentz rule of second-rank tensors $F^{\alpha \beta}(\mathbf{E}, \mathbf{B})$ and $G^{\alpha \beta}(\mathbf{D}, \mathbf{H})$, given by Eqs. (23) and (4), or Eqs. (22) and (23) for a plane wave. From the four-vector Lorentz transformation of $\bar{P}^\mu = (\bar{E}_{em}/\omega)K^\mu$, we have

$$\frac{N_p \omega}{N_p' \omega'} = \frac{\mathbf{D} \cdot \mathbf{E}}{\mathbf{D}' \cdot \mathbf{E}'} \left( = \frac{W_{em}}{W_{em}'} \right).$$  \hspace{1cm} (44)$$
The above equation has a clear physical explanation that the Doppler factor of EM energy density is equal to the product of the Doppler factors of EM-field-cell density and frequency.

From the second-rank tensor Lorentz transformations given by Eqs. (22) and (23), we obtain the transformation of EM energy density $W_{em} = D \cdot E = (B \cdot H)$, given by

$$\frac{(D \cdot E)}{(D' \cdot E')} = \gamma (1 - n_d' \hat{n} \cdot \beta') \left[ \gamma \left(1 - \frac{1}{n_d'} \hat{n} \cdot \beta' \right) \right],$$

namely Eq. (33). Comparing with Eq. (6), we know that $\gamma (1 - n_d' \hat{n} \cdot \beta')$ is the frequency Doppler factor. In free space ($n_d' = 1$), the above equation is reduced to Einstein’s result [20]: $[|E| = \gamma (1 - \hat{n} \cdot \beta') |E'|]$.

Inserting Eq. (45) into Eq. (44) we obtain the transformation of the EM-field-cell density $N_p$, given by

$$N_p' = \gamma \left(1 - \frac{1}{n_d'} \hat{n} \cdot \beta' \right) N_p.$$  

(46)

So far we have finished the proof of the covariance of $(\hat{p}_{em}, \hat{E}_{em}/c)$ by resorting to a parameter of $N_p$, so-called EM-field-cell density. Actually, we do not have to know what the specific value of $N_p'$ or $N_p$ is, but the ratio of $N_p/N_p'$, and $P^{\mu} = (\hat{p}_{em}, \hat{E}_{em}/c)$ is pure “classical” without Planck constant $\hbar$ involved. However, $N_p$ must be the “photon density” when Einstein light-quantum hypothesis is imposed. In such a case, we have $\hbar = (D \cdot E)/(N_p \omega)$, or $N_p = [(D_0 \cdot E_0)/(\hbar \omega)] \cos^2 \Psi$, namely the photon density $N_p$ is a “wave”.

Mathematically speaking, the existence of the covariance of $P^{\mu} = (\hat{p}_{em}, \hat{E}_{em}/c)$ is apparent. $P^{\mu}$ and $K^{\mu}$ are “parallel”, just different by a factor of $(\hat{E}_{em}/\omega) = (D \cdot E)/(N_p \omega)$ which contains an introduced parameter $N_p$ to make the transformation hold.

It is seen from Eqs. (43) and (44) that $\hat{E}_{em}/\omega)$ holds and it is a Lorentz invariant, and $(\hat{E}_{em}/\omega) = \hbar$ holds when Einstein light-quantum hypothesis is imposed. Thus the Planck constant $\hbar$ must be a Lorentz invariant. In other words, Einstein light-quantum hypothesis requires the Lorentz invariance of Planck constant for a plane wave. Therefore, the construction of photon momentum-energy four-vector, Eq. (16) in Sec. III, is well grounded.

If a volume $dV''_{light}$ in the medium-rest frame moves along the wave vector $n_d' \hat{k}'$ at the light speed $(c/n_d')$, then there are no photons that cross its boundary, and the photon number within $dV''_{light}$ keeps constant. In such a case, the transformation of the moving volume (termed light volume) is given by

$$\frac{dV'_{light}}{dV_{light}} = \gamma \left(1 - \frac{1}{n_d'} \hat{n} \cdot \beta' \right).$$

(47)

Compared with Eq. (46), we find that

$$N_p dV_{light} = N'_p dV''_{light}$$

is a Lorentz invariant, namely the photon number in the light volume is Lorentz invariant. Thus we have the total momentum-energy four-vector in the light volume, given by

$$\hat{P}^{\mu}(N_p dV_{light}) = (D \times B, D \cdot E/c)dV_{light},$$

(49)

or

$$\int_{V_{light}} (\hat{P}^{\mu} N_p) dV = \int_{V_{light}} (D \times B, D \cdot E/c)dV.$$

(50)

The invariance of $N_p dV_{light}$ implies that observed in any inertial frames, all the $N_p dV_{light}$ photons are frozen inside the light volume $dV_{light}$. Thus the light volume can be taken to be an approximate description of practical low-divergence light pulses.

Inserting Eq. (17) and Eq. (9) into Eq. (45), we have

$$\frac{(D \cdot E) dV_{light}}{\omega} = \frac{(D' \cdot E') dV''_{light}}{\omega'},$$

(51)

which is also a Lorentz invariant; namely the light-volume energy and the frequency transform in the same law. This result, which is obtained in the moving medium, is exactly the same as that obtained by Einstein in free-space [20].

From above analysis, we can draw the following conclusions:

- The Minkowski momentum per unit EM-field cell, $N_p^{-1} (D \times B)$, is Lorentz covariant, as the space component of the EM momentum-energy four-vector $P^{\mu} = N_p^{-1} (D \times B, D \cdot E/c)$, just like the Minkowski photon momentum $h n_d \hat{k}$ is Lorentz covariant, as the space component of the photon momentum-energy four-vector $P^{\mu} = (h n_d \hat{k}, h \omega/c)$. When Einstein light-quantum hypothesis $N_p^{-1} D \cdot E = h \omega$ is imposed on the former, the two four-vectors become the same, namely $N_p^{-1} (D \times B, D \cdot E/c) = (h n_d \hat{k}, h \omega/c)$.

- There are two forms of momentum-energy four-vectors: (a) the momentum and energy in a single EM-field cell or photon constitute a four-vector, namely $N_p^{-1} (D \times B, D \cdot E/c)$ or $(h n_d \hat{k}, h \omega/c)$ is a four-vector, and (b) the total momentum and energy in a light volume constitute a four-vector,
Planck constant is a Lorentz invariant, which is not inserted, we have \( \omega t \) where the symmetric Minkowski EM stress tensor is \( \mathbf{T} \) with \( \mathbf{T} \). Since \( \mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H} = \mathbf{E}_0, \mathbf{B}_0, \mathbf{D}_0, \mathbf{H}_0 \cos \Psi \), with \( \Psi = (\omega t - n_d \mathbf{k} \cdot \mathbf{x}) \), \( \nabla \cdot \mathbf{D} = 0 \Rightarrow \mathbf{D} = \mathbf{n} \), \( \nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B} = \mathbf{n} \), and \( \mathbf{D} \cdot \mathbf{E} = \mathbf{B} \cdot \mathbf{H} \), leading to \( \partial (\mathbf{D} \times \mathbf{B}) / \partial t = -2(D_0 \cdot E_0)(\cos \Psi \sin \Psi)(n_d \mathbf{k}) \), we have

\[
\frac{\partial (\mathbf{D} \times \mathbf{B})}{\partial t} = -\nabla \cdot \mathbf{T}_M, \tag{53}
\]

where the symmetric Minkowski EM stress tensor is given by

\[
\mathbf{T}_M = \mathbf{I}(\mathbf{D} \cdot \mathbf{E}) = \mathbf{I}(\mathbf{B} \cdot \mathbf{H}), \tag{54}
\]

with \( \mathbf{I} \) the unit tensor.

Eq. (53) is the Minkowski momentum conservation equation for a plane wave in a moving medium, and it is invariant in form in all inertial frames together with the Maxwell equations.

To understand the physical implication of stress tensor, let us consider the EM momentum in a given dielectric volume \( V \) closed by surface \( S \). From Eq. (53), using divergence theorems for a tensor and a vector, with the stress tensor \( \mathbf{T}_M = \mathbf{I}(\mathbf{D}_0 \cdot \mathbf{E}_0) \cos^2(\omega t - n_d \mathbf{k} \cdot \mathbf{x}) \) inserted, we have

\[
\frac{\partial}{\partial t} \int_V (\mathbf{D} \times \mathbf{B}) dV = -\int_S d\mathbf{S} \cdot \mathbf{T}_M
\]

\[
= -\int_S d\mathbf{S} (\mathbf{D} \cdot \mathbf{E}) = -\int_V \nabla (\mathbf{D} \cdot \mathbf{E}) dV
\]

\[
= -\int_V (\mathbf{D}_0 \cdot \mathbf{E}_0) \sin(2\Psi)(n_d \mathbf{k}) dV, \tag{55}
\]

where \( \int (\mathbf{D} \times \mathbf{B}) dV \) is the total EM momentum in \( V \), and \( d\mathbf{S} \cdot \mathbf{T}_M \) is the momentum element through \( d\mathbf{S} \)-area element per unit time. From Eq. (55), we find \( \int d\mathbf{S} \mathbf{T}_M \neq 0 \) in general while the time average \( < \oint d\mathbf{S} \cdot \mathbf{T}_M >= 0 \), which also hold in empty space. This phenomenon results from the “travelling-wave” attribution of \( \mathbf{T}_M \); namely \( \mathbf{T}_M \) varies with space coordinates, and the total EM momentum flowing in \( (d\mathbf{S} \cdot \mathbf{T}_M < 0) \) and out \( (d\mathbf{S} \cdot \mathbf{T}_M > 0) \) of a given volume \( V \) are usually different at a given instant, but they are equal on time average. In other words, \( \partial / \partial t \int (\mathbf{D} \times \mathbf{B}) dV \) only denotes the change rate of momentum flowing into \( V \) at a given instant, instead of a force given by the medium; thus there is no momentum transfer taking place between the plane wave and the uniform medium, and there is no force acting on the dielectric. This also can be understood through the light-quantized Minkowski EM-field-cell/photon four-vector \( N^{-1}_\gamma (\mathbf{D} \times \mathbf{B} \cdot \mathbf{E}/c) = (h_n \mathbf{k}, \hbar \omega/c) \), which indicates that the momentum of a photon keeps constant during the propagation in a uniform medium. Obviously, this conclusion is applicable to any inertial frames and it can be used to explain why the momentum transfer only takes place on the vacuum-medium interface in the medium Einstein-box thought experiment for a light pulse [21].

It should be pointed out that the conventional EM force definition (see p. 159 of Ref. [27], for example) is questionable because it cannot pass a plane-wave test. For a plane wave in the medium-rest frame \( X'Y'Z' \), the medium is isotropic and uniform \( (\partial \mathbf{e}^t / \partial \mathbf{x}^t = 0 \) and \( \partial \mathbf{r}^t / \partial \mathbf{x}^t = 0 \) \). According to the conventional definition, however, the EM force exerted on a volume is given by \( \mathbf{f}^t / (\partial / \partial \mathbf{x}^t \mathbf{e}^t \times \mathbf{H}^t) \), which implies that there is a momentum transfer between the plane wave and the uniform medium, clearly contradicting the conclusion obtained from above. Thus this conventional definition is flawed.

The construction of stress tensor is flexible; in a sense, it is artificial within the Maxwell-equation frame. For example, \( \nabla \cdot (-\mathbf{D} \mathbf{E} - \mathbf{B} \mathbf{H}) = 0 \) and \( \mathbf{D} \cdot \mathbf{E} = \mathbf{B} \cdot \mathbf{H} \) hold for a plane wave, and the Minkowski tensor can be re-written in an asymmetric form [14]

\[
\mathbf{T}_M = -\mathbf{D} \mathbf{E} - \mathbf{B} \mathbf{H} + \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}), \tag{56}
\]

which does not affect the validity of Eq. (53) and Eq. (55).
Similarly, we can obtain the Abraham momentum conservation equation, given by

$$\frac{\partial}{\partial t} \left( \frac{E \times H}{c^2} \right) = -\nabla \cdot \mathbf{T}_A,$$

(57)

where the Abraham stress tensor is given by

$$\mathbf{T}_A = \beta^2_{ph} [-(\mathbf{E}D + \mathbf{H}B) + \mathbf{I}(\mathbf{D} \cdot \mathbf{E})],$$

(58)

which is not symmetric. By taking advantage of $\nabla \cdot (\mathbf{DE}) = 0$, $\nabla \cdot (\mathbf{BH}) = 0$, and $\mathbf{D} \cdot \mathbf{E} = \mathbf{B} \cdot \mathbf{H}$, the Abraham stress tensor can be re-written in a symmetric form, given by

$$\mathbf{T}_A = \beta^2_{ph} [-(\mathbf{DE} + \mathbf{DE})]$$

$$- (\mathbf{HB} + \mathbf{BH}) + \mathbf{I} \left[ \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}) \right].$$

(59)

Note that in a moving medium, $\mathbf{ED} = \mathbf{DE}$ and $\mathbf{HB} = \mathbf{BH}$ usually are not true because of the anisotropy of the moving medium. However in free space, $\mathbf{ED} = \mathbf{DE}$ and $\mathbf{HB} = \mathbf{BH}$ always hold because the empty space ($\beta^2_{ph} = 1$) is isotropic observed in any inertial frames.

Thus in free space, $\mathbf{T}_M$ given by Eq. (56) and $\mathbf{T}_A$ given by Eq. (59) are identical.

It can be seen from Eq. (58) and Eq. (57) that the momentum conservation equations are all differential equations and they can be converted from each other through Maxwell equations. In fact, Eq. (53) and Eq. (57) can be obtained from a more general momentum conservation equation given by $\partial \mathbf{g}/\partial t + \nabla \cdot \mathbf{T} = 0$, where $\mathbf{g} = a \mathbf{E} + (1-a) \mathbf{A}$ and $\mathbf{T} = a \mathbf{T}_A + (1-a) \mathbf{T}_M$, with $\mathbf{g}_A = (\mathbf{E} \times \mathbf{H})/c^2$, $\mathbf{g}_M = \mathbf{D} \times \mathbf{B}$, $a$ being any constant, $\mathbf{T}_A$ given by Eq. (56), and $\mathbf{T}_M$ given by Eq. (58). We have $\partial \mathbf{g}_M/\partial t + \nabla \cdot \mathbf{T}_M = 0$, namely Eq. (53) for $a = 0$, and $\partial \mathbf{g}_A/\partial t + \nabla \cdot \mathbf{T}_A = 0$, namely Eq. (57) for $a = 1$, while $\mathbf{g}$ and $\mathbf{T}$ are restored to $\mathbf{g} = \mathbf{g}_A = \mathbf{g}_M$ and $\mathbf{T} = \mathbf{T}_A = \mathbf{T}_M$ in free space. Just as indicated at the beginning of the paper, Maxwell equations themselves support various forms of momentum conservation equations, resulting in an indeterminacy of momentum definitions. Thus to identify the correctness of momentum definitions, Fermat’s principle and the principle of relativity are indispensable.

D. Unconventional phenomenon for a superluminal medium

In addition to the negative-frequency appearance, as indicated by Huang [28], a Negative Energy Density (NED) may result for a plane wave when the medium moves opposite to the wave vector direction at a faster-than-dielectric light speed: this phenomenon is called “NED zone” for the sake of convenience.

In the NED zone, the photons possess negative energy from the viewpoint of phenomenological quantum-electrodynamics [31]. The origin of negative EM energy density can be seen from Eq. (49). The energy density Doppler factor is equal to the product of the Doppler factors of frequency and EM-field-cell density. The EM-field-cell density Doppler factor is always positive while the frequency’s is negative in the NED zone ($\beta' \mathbf{n} \cdot \beta' = n_d' |\beta'| > 1$), leading to $\mathbf{D} \cdot \mathbf{E} < 0$ when $\omega < 0$. [Note that no matter whether $\omega < 0$ or $\omega > 0$, Eqs. (57) and (59) can be written as $\mathbf{D} \times \mathbf{B} = \hat{n} \hat{n} (\mathbf{D} \cdot \mathbf{E}) [(n_d/c) \gamma]$ and $\mathbf{S}_{\text{power}} = \hat{n} |\mathbf{D} \cdot \mathbf{E}| [c/(n_d)]$, with $\mathbf{S}_{\text{power}} = [c/(n_d)]^2 (\mathbf{D} \times \mathbf{B})].$]

In the NED zone, $\mathbf{D} |\mathbf{E}$ and $\mathbf{H} |\mathbf{B}$ are valid, and from Eqs. (24) and (28) we have

$$\frac{\mathbf{D}}{\mathbf{E}} = \left( 1 - \frac{|\beta'|/n_d'}{1 - n_d' |\beta'|} \right) \epsilon' < 0,$$

(60)

$$\frac{\mathbf{B}}{\mathbf{H}} = \left( 1 - \frac{|\beta'|/n_d'}{1 - n_d' |\beta'|} \right) \mu' < 0.$$  

(61)

Because of $\omega < 0$ in the NED zone, from Eq. (11) we have $\beta_{ph} c = (\beta_{ph} c) \hat{n}$ with $\beta_{ph} c < 0$. From Eqs. (35) and (40) we have

$$\mathbf{E} = -\beta_{ph} c \hat{n} \times \mathbf{B},$$

(62)

$$\mathbf{H} = +\beta_{ph} c \hat{n} \times \mathbf{D}.$$  

(63)

It follows that the plane wave in the NED zone is a left-hand wave: (1) $\mathbf{E}, \mathbf{B}, \hat{n}$ and $\mathbf{D}, \mathbf{H}, \hat{n}$ follow the left-hand rule, and (2) the phase velocity or group velocity is opposite to the wave vector. In other words, the moving medium in the NED zone behaves as a so-called “negative index medium” [32], where the refractive index is taken to be negative, instead of the frequency here.

As mentioned above, in the NED zone the group velocity $\mathbf{v}_{gr}$ is opposite to the wave vector $n_d \mathbf{k}$, and $\mathbf{D} \cdot \mathbf{E} < 0$ holds. Thus from Eq. (49)-Eq. (51), with $\mathbf{S}_{\text{aux}} = 0$ taken into account, we find that the Poynting vector $\mathbf{S} = \mathbf{S}_{\text{power}} = (\mathbf{D} \cdot \mathbf{E}) \mathbf{v}_{gr}$ also has the same direction as the wave vector $n_d \mathbf{k}$ or the EM momentum $\mathbf{D} \times \mathbf{B}$ for the “negative index medium” effect. This conclusion makes sense physically, because photons are the carriers of EM energy and momentum, and the EM power flow and momentum are supposed to have the same direction.

As we have known, the NED zone results from $\omega < 0$. However how to understand the sign of the frequency has been thought to be a difficult question. Just as Huang indicated [28], “Can we find a convincing explanation of the meaning of negative frequency of waves in any literature?” In fact, from the following analysis we can see that the sign of frequency (EM energy density) is just a reflection of the property of wave propagation.

In the NED zone we have $n_d' \mathbf{k} = (1 - n_d'^{-1} |\beta'| \hat{n}') \gamma (n_d' \mathbf{k}')$. Because of $(1 - n_d'^{-1} |\beta'| \hat{n}') > 0$, $(n_d' \mathbf{k})$ and
\((n'_B k')\) take the same direction. Observed in the lab frame and the medium-rest frame respectively, the both power flows have the same direction. From Eqs. (60) and (61) we see that, the moving medium in such a case physically behaves as a “negative index medium”. Thus the negative-frequency effect denotes a distinct physical phenomenon where the EM wave is a left-hand wave. In other words, the sign of the frequency (EM energy density) only characterizes the propagation property of EM waves. Experimentally, the observed frequency is always positive, and a positive EM energy propagates along the wave vector direction, while the sign of the frequency is determined by examining the property of wave propagation in the moving medium: (-) for the left-hand and (+) for the right-hand.

It is interesting to point out that, in the effect of “negative index medium” analyzed above, the dispersion of the medium material is not required, and all the Poynting vector, EM momentum, and wave vector have the same direction. In contrast, in the traditional effect of negative index medium, which was first analyzed by Veselago, the medium material must be dispersive to support a positive EM energy, and the Poynting vector is directed opposite to the EM momentum or wave vector.[24]

V. CONCLUSIONS AND REMARKS

An electromagnetic plane wave, although not practical, is a simplest strict solution of Maxwell equations, and it is often used to explore most fundamental physics. For example, Einstein used a plane wave to develop his special theory of relativity and derived the well-known relativistic Doppler formula in free space.[20] In this paper, we use the plane wave, which propagates in a moving non-dispersive, lossless, non-conducting, isotropic uniform medium, to identify which formulation of light momentum is correct.

We have shown that (a) Minkowski light momentum and energy constitute a Lorentz four-vector, while Abraham momentum and energy do not, and (b) observed in any inertial frames, Minkowski EM momentum \(g_M = D \times B\) always take the direction of the wave vector \(n_B k\), while the Abraham momentum \(g_A = E \times H/c^2\) does not, unless in free space or when the dielectric medium moves parallel to the wave vector, as shown in Eqs. (67) and (68). The Minkowski momentum is completely consistent with Fermat’s principle and the principle of relativity, and it is the unique correct light momentum.

The photon momentum-energy four-vector \(P^\mu = hK^\mu\) is constructed based on the wave four-vector combined with Einstein light-quantum hypothesis, while the EM momentum-energy four-vector \(P^\mu = N_p^{-1}(D \times B, D \cdot E/c)\) is constructed based on the Lorentz covariance of EM field-strength tensors \(F^{\alpha\beta}(E, B)\) and \(G^{\alpha\beta}(D, H)\) (to keep Maxwell equations invariant in form in all inertial frames) 23, where \(N_p^{-1}D\times B\) and \(N_p^{-1}D\cdot E\) are, respectively, the momentum and energy for a “single EM-field cell”. When Einstein light-quantum hypothesis \(N_p^{-1}D \cdot E = \hbar \omega\) is imposed on the latter, the latter is restored to the former, namely \(N_p^{-1}(D \times B, D \cdot E/c) = (hn_B k, \hbar \omega/c)\), with the “single EM-field cell” becoming “single photon”. Thus the single photon momentum is the direct result of Einstein light-quantized EM momentum. In other words, the monochromatic plane wave is an identical-photon model, in which every photon has the same momentum and energy so that the light-wave momentum has the exact and simplest definition.

As the carriers of EM momentum and energy of the plane wave, all photons move uniformly in any inertial frames at the phase velocity \(\beta_{ph}c = (\omega/|n_B k|)\hat{n}\) (= group velocity = energy velocity). There is no momentum transfer taking place between the plane wave and the uniform medium, and there is no EM force acting on the medium. The photon density \(N_p = W_{em}/(\hbar \omega)\) is a “particle density wave”, and the EM power flow \(S_{power} = W_{em}\beta_{ph} c = |n_B | W_{em}|c/n_B| = |n_B | N_p \omega |c/n_B|\) is also a “wave”, changing with time and space, where \(W_{em} = D \cdot E = B \cdot H = (D_0 \cdot E_0) \cos^2(\omega t - n_B k \cdot x)\) is the EM energy density.

The principle of relativity, Fermat’s principle, and global momentum-energy conservation law are all basic postulates in physics. In the principle-of-relativity frame, it is the Fermat’s principle that requires the correct light momentum and energy to constitute a Lorentz four-vector for a plane wave in a moving uniform medium as shown in this paper, while it is the global momentum-energy conservation law that requires the correct light momentum and energy to constitute a Lorentz four-vector in medium Einstein-box thought experiment as shown in [21]. From this we can conclude that the justification of Minkowski momentum as the correct light momentum is completely required by the basic postulates in physics.

It should be emphasized that, the significance of the resolution of the Abraham-Minkowski debate presented in the paper is not just to show the justification of the Minkowski momentum as the unique correct light momentum. In fact, through seeking the resolution we have clarified and developed some basic concepts and principles in electrodynamics and special relativity, which are outlined below.

(1) We have set up a light-momentum criterion for the first time, which states that, the momentum of light in a medium (including empty space) is parallel to the wave vector in all inertial frames. This criterion is the direct result of the principle of relativity and Fermat’s principle for a plane wave. In a dielectric medium (not including the empty space), Minkowski momentum satisfies the criterion while Abraham momentum does not; thus the Minkowski momentum is the unique correct light momentum. In the empty space, Minkowski and Abraham momentums are equal, and both satisfy the criterion.

(2) In conventional EM wave theory, Poynting vector
\( S = \mathbf{E} \times \mathbf{H} \) as EM power flow has been thought to be a well-established basic concept. In view of the existence of some kind of mathematical ambiguity for this concept, some scientists suggested it to be a “postulate” \( \text{[18]} \), or “hypothesis”, “until a clash with new experimental evidence shall call for its revision” (see p. 135 of Ref. \( [27] \)). However in this paper, we have shown that the Poynting vector may not denote the EM power flow in an anisotropic medium (see Eq. \( 65 \)). This result revises the conventional understanding of Poynting vector, and also explains why Laue-Møller theory \( [12, 16] \) and Mansuripur-Zakharian theory \( [18] \) have the same Poynting-vector assumption, but they have completely different physical results: one supporting Minkowski momentum and the other supporting Abraham momentum.

(3) It is well-accepted conventionally that the force exerted by an electromagnetic field on a unit volume of isotropic dielectric medium is given by \( 12, 27, 53, 36 \)

\[
f = f^M + f^A, \tag{64}\]

where

\[
f^M = \rho \mathbf{E} + \mathbf{J} \times \mathbf{B} - \frac{1}{2} \mathbf{E}^2 \nabla \epsilon - \frac{1}{2} \mathbf{H}^2 \nabla \mu, \tag{65}\]

\[
f^A = \frac{n_d^2 - 1}{c^2} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{H}). \tag{66}\]

It is argued that the \( f^A \)-term “simply fluctuates out when averaged over an optical period in a stationary beam”, but “it is in principle measurable” \( \text{[32]} \), and various ideas were proposed to experimentally measure or identify the \( f^A \)-term \( \text{[34, 33]} \). Unfortunately, as shown in the paper, the above conventional formula cannot pass the test of a plane wave in an isotropic, lossless, uniform medium, and consequently, it is flawed. To directly understand this, inserting \( \rho = 0, \mathbf{J} = 0, \nabla \epsilon = 0, \nabla \mu = 0 \), and \( (\mathbf{E}, \mathbf{H}) = (\mathbf{E}_0, \mathbf{H}_0) \cos(\omega t - n_d \mathbf{k} \cdot \mathbf{x}) \) into Eq. (64), we have \( f^M = 0 \) and

\[
f = \frac{n_d^2 - 1}{c^2} (\mathbf{E}_0 \times \mathbf{H}_0) \frac{\partial}{\partial t} \cos(\omega t - n_d \mathbf{k} \cdot \mathbf{x}) \neq 0 \tag{67}\]

holding (except for those discrete points). According to the physical implication of force, \( f \neq 0 \) implies that there is a momentum transfer taking place between the plane wave and the medium. However this apparently contradicts the fact that in an isotropic uniform medium (\( \nabla \epsilon = 0 \) and \( \nabla \mu = 0 \)), all photons move uniformly and they do not have any momentum exchanges with the medium. Thus this conventional EM force formula Eq. (63) is indeed flawed.

(4) We have shown in \( 21 \) that the total-momentum model proposed by Barnett \( [19] \), which is widely accepted in the community, is not compatible with the principle of relativity and the global momentum-energy conservation law, which are all fundamental postulates in physics. This conclusion comes from the fact that in the Einstein-box thought experiment, the medium-box kinetic momentum and energy always constitute a Lorentz four-vector before and after the photon enters the box because the medium-box is made up of massive particles, while the Abraham photon momentum and energy cannot constitute a Lorentz four-vector after the photon enters the box; resulting in the breakdown of the total momentum-energy conservation law within the principle-of-relativity frame.

In the total-momentum model, as shown by Eq. (7) of Ref. \( [19] \), the total momentum is given by \( p^\text{med} + p^\text{med} = p^\text{med} + p^\text{med}, \) where \( p^\text{med} \) and \( p^\text{med} \) are, respectively, the Abraham and Minkowski light momentums. \( p^\text{med} \) is the medium kinetic momentum (also called “Abraham material momentum” in some literature \( [13] \)), and \( p^\text{med} \) is the medium canonical momentum (also called “Minkowski material momentum” \( [13] \)). According to the relativistic analysis of Einstein-box thought experiment \( [21] \), the total-momentum model should be modified into \( p^\text{kin} + p^\text{kin} = p^\text{total}, \) because the medium-box kinetic momentum-energy and Minkowski photon momentum-energy, respectively, constitute a Lorentz four-vector no matter before and after the photon enters the box. In other words, in a system consisting of massive particles and photons, the momentums and energies of all individual massive particles and photons respectively constitute Lorentz four-vectors no matter whether they have interactions or not.

(5) We have shown that there may be apparent photon velocity and apparent photon displacement in a moving medium (see Fig. \( 2 \)). This conclusion comes from the fact that the photon propagation velocity is the phase velocity, for which there is no “phase velocity four-vector”, and thus the photon does not have a four-velocity like a massive particle. When using the time-space four-vector to describe photon’s motion, the space coordinates may not reflect its real location; thus resulting in the appearance of apparent velocity and displacement. In addition, the fact that the photon does not have a four-velocity also questions the justification of the four-vector covariance imposed on the EM energy velocity in Laue-Møller theory because the photon is the carrier of EM energy.

(6) We have shown that the Planck constant is a Lorentz invariant for a plane wave in a uniform medium (including empty space), which is a strict result of special relativity and Einstein light-quantum hypothesis, while the Planck constant as a Lorentz invariant is an implicit postulate in Dirac relativistic quantum mechanics \( [37] \). This result will make the fine structure constant also a Lorentz invariant, which is explained as follows.

As shown in Sec. 11. 9 of the textbook by Jackson \( [25] \), Maxwell equations \( \nabla \times \mathbf{H} = \partial (\varepsilon \mathbf{D}) / \partial t, \nabla \cdot (\mu \mathbf{D}) = (\mathbf{J}, c\rho) \) and \( \nabla \times \mathbf{E} = \partial (-\mathbf{cB}) / \partial t, \nabla \cdot (-\mathbf{cB}) = (0, 0) \) can be written as \( \partial_{\mu} G^\mu (\mathbf{D}, \mathbf{H}) = J^\nu \) and \( \partial_{\mu} F^{\mu \nu} (\mathbf{B}, \mathbf{E}) = 0. \)
\( G^{\mu\nu}(D, H) \) and \( \mathcal{F}^{\mu\nu}(B, E) \) are assumed to be four-tensor Lorentz covariant to keep Maxwell equations invariant in form in all inertial frames, and thus \((J, cp)\) must be a four-vector. On the other hand, the electron’s moving velocity must be less than the light speed. Accordingly, the electron charge must be a Lorentz invariant, as shown in \[33\], although it is usually taken as an experimental invariant (see p. 555 of Ref. \[22\], for example). Now that light speed \(c\), Planck constant \(h\), and electron charge \(e\) are all Lorentz invariants, the fine structure constant \(\alpha = e^2/\hbar c\) (in CGS unit) is also a Lorentz invariant.

(7) In the relativistic dynamics, there is a well-known “classical mathematical conjecture”, which states:

If \(\Theta^{\mu\nu}(X)\) is a Lorentz four-tensor defined on the domain \(V(X)\), with \(X^\sigma = (x, ct)\), and it is symmetric \((\Theta^{\mu\nu} = \Theta^{\nu\mu})\) and divergence-less \((\partial_\nu \Theta^{\mu\nu} = 0 \iff \partial_\mu \Theta^{\nu\mu} = 0)\), then the time-row (column) integrals at any given time (assumed to be convergent)

\[
P^\nu = \int_V \Theta^{\nu\mu} d^3x \quad \left( \int_V \Theta^{\mu\nu} d^3x \right)
\]

constitute a Lorentz four-vector.

The above conjecture has been thought to be a well-established result of tensor calculus \[22\], and it was used as a starting point in relativistic analysis of Einstein-box thought experiment for resolution of Abraham-Minkowski debate \[19\]. However we have shown in \[33\] that this conjecture is not true.

Why is the conjecture not true? As we know, the correctness of a mathematical conjecture cannot be legitimately affirmed by enumerating specific examples, no matter how many; however, it can be directly negated by finding specific examples, even only one. As shown in \[33\], the classical electron is right the specific example to negate the above conjecture, because the EM stress-energy four-tensor \(T^{\mu\nu}\) for the classical electron is symmetric \((T^{\mu\nu} = T^{\nu\mu})\) and divergence-less \((\partial_\nu T^{\mu\nu} = 0)\), but the time-row (column) integrals \(\int V T^{\mu\nu} d^3x \left( = \int V T^{\nu\mu} d^3x \right)\), or

\[
\left( \int_V \frac{E \times H}{c} d^3x, \int_V \frac{1}{2} (D \cdot E + B \cdot H) d^3x \right)
\]

never constitute a Lorentz four-vector. In other words, the total (Abraham = Minkowski) EM momentum \(\int V [(E \times H)/c^2] d^3x\) and energy \(\int V 0.5(D \cdot E + B \cdot H) d^3x\) carried by the classical electron cannot constitute a four-vector. This conclusion is also clearly supported by the direct calculation result that, in an ideal planar-plate capacitor the total “field’s momentum-energy is seen to behave in a way that is not expected from a four vector”, as claimed by Mansuripur and Zakharian \[18\]. Thus the above “classical mathematical conjecture” is indeed not correct.

(8) We have shown in \[41\] that there exists a new physics of so-called “intrinsic Lorentz violation” within the frame of the two postulates of special relativity (principle of relativity and constancy of light speed). Traditionally, “Lorentz invariance” refers to that all mathematical equations expressing the laws of nature must be invariant in form only under the Lorentz transformation, and they must be Lorentz scalars, four-vectors, or four-tensors ... (see p. 540 of Ref. \[22\], for example). In other words, the Lorentz invariance is a single requirement that combines the two postulates together, and it is equivalent to the two postulates \[42\]. Unfortunately, as shown in \[41\], this is not true for the Doppler effect from a moving point light source, where the Doppler formula cannot be obtained from the Lorentz transformation but it is exactly a result of the two postulates. This phenomenon, called “intrinsic Lorentz violation”, has never been realized in the community.

In fact, there is also an interesting “intrinsic Lorentz violation” for the plane wave in a moving uniform medium. As shown in Sec. II of this paper, there are two velocities associated with the photon. One is the photon velocity, namely the phase velocity \(v_{ph} = \omega / |\mathbf{p}|\), defined based on the wave four-vector \(K^\nu = (n, k, \omega/c)\), which is completely consistent with the two postulates. The other is the apparent photon velocity \(u = (E \times H)/W_{em}\), which is the EM “energy velocity” \[10\] or “group velocity” traditionally \[30\]. The principle of relativity requires that a physical law be invariant in form and, of course, keep the same physical meaning in all inertial frames. \(v_{ph}\) and \(u\) are equal in the medium-rest frame, and they are invariant in form in any inertial frames. \(v_{ph}\) is the photon velocity in any inertial frames and it denotes the “law of EM energy transport”, but \(v_{ph}\) is not a four-vector. In contrast, \(\gamma u(v, c)\) is a four-vector and \(u = (E \times H)/W_{em}\) is the photon velocity in the medium-rest frame, but under the four-vector Lorentz transformation of \(v_{ph}(v, c)\), \(u\) is not the photon velocity any more in general (with physical meaning changed); accordingly, \(u = (E \times H)/W_{em}\) cannot not denote the law of energy transport. From this it follows that, (a) \(v_{ph}\) denotes a physical law, but \(v_{ph}(v_{ph}, c)\) is not a four-vector, and (b) \(u\) does not denote a physical law, but \(\gamma u(v, c)\) is a four-vector. Thus the two cases both break the definition of “Lorentz invariance”, resulting in “intrinsic Lorentz violation”. [Note: \(v_{ph}(\beta_{ph}(c, v), c)\) and \(\gamma u(v, c)\) do not exist in free space where \(\beta_{ph}(\phi c) = |\mathbf{u}| = c\) and both \(v_{ph}\) and \(\gamma u\) \(\rightarrow \infty\).]

It should be indicated that, the intrinsic Lorentz violation exposed in \[41\] and here is essentially different from the “Lorentz violation” presented in \[43\]. The intrinsic Lorentz violation takes place within the frame of the two postulates, and it is completely consistent with the special relativity. In contrast, the Lorentz violation \[43\] describes deviations from the two postulates; for example, there has been a controversy recently about whether there are deviations in the time dilation predicted by special relativity in experiments of high-energy ions \[44\].
In summary, in this paper we have answered the following most fundamental questions in classical physics and quantum physics. When there exist dielectric materials in space:

- Is the principle of relativity still valid?
- Are the Maxwell equations, momentum-energy conservation law, Fermat’s principle, and Einstein light-quantum hypothesis equally valid in all inertial frames of reference?
- Why is the principle of relativity needed for identifying the justification of light-momentum definition?

- Does the Poynting vector always represent EM power flow in any system of materials?
- Is the Planck constant a Lorentz invariant?
- Does the photon have a Lorentz four-velocity like a massive particle?
- Why must the photon momentum and energy constitute a Lorentz four-vector?
- Why is Lorentz invariance not equivalent to the two postulates of special relativity?

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