Steady-state modeling and analysis of six-phase synchronous motor

Arif Iqbal*, G.K. Singh and Vinay Pant

Department of Electrical Engineering, Indian Institute of Technology, Roorkee 247667, India

(Received 12 November 2013; final version received 15 February 2014)

Steady-state analysis is needed for a better understanding of the operational behavior of a six-phase synchronous motor. Therefore, this paper primarily addresses the steady-state analysis of an asymmetrical six-phase synchronous motor, including the development of a mathematical model employing Park’s (dq0) transformation, valid for different terminal conditions. These conditions include the motor operation with input supply at both sets of the three-phase stator winding as well as at only one set of the three-phase stator winding (motor-generator mode). A unified and simplified mathematical treatment along with its experimental results is presented.

Keywords: simulation < analysis; energy and power systems < applications; nonlinear systems < systems

1. Introduction

Area of the analysis of multiphase motor drive system has drawn considerable attention during the last two decades. Research in this field is going on and numerous interesting developments have been reported in various available literatures. This is because the multiphase motor possesses several advantages over the conventional three-phase motor, such as reduction in space and time harmonics, reduction in amplitude and increase in the frequency of torque pulsations, reduction in current per phase without increasing the voltage per phase, lowering the dc link current harmonics, increase in power to weight ratio, and higher reliability etc. Therefore, the use of multiphase motor can be found in many applications such as electric ship propulsion, electric/hybrid electric vehicle propulsion, aircraft, thermal power plant to drive and induce draft fans, nuclear power plant, etc. (Jahns, 1980; Klinghrin, 1983; Levi, 2008; Singh, 2002).

As far as six-phase synchronous motor is concerned, limited literature is available as the field is in its primitive stage. A general modeling of six-phase synchronous machine is available in some papers (Fuch & Rosenberg, 1974; Schiferl & Ong, 1983a; Singh, 2011a, 2011b). The behavior of a six-phase synchronous alternator with its two three-phase stator winding displaced by an arbitrary angle was analyzed by Fuchs and Rosenberg (1974). In their analysis, they have used orthogonal transformation of the phase variables into a new set of (d – q) variables. They have also concluded that the orthogonal transformation eliminates partly the time-dependence of the coefficients of the system of differential equations, and phasor can be employed for the analysis of steady-state behavior of generator with two stator windings. Schiferl and Ong (1983a) have presented the mathematical model of a six-phase synchronous machine, wherein the mutual leakage couplings between two sets of three-phase stator windings have been considered. Steady-state operations with sinusoidal voltage inputs were examined, using phasor diagrams to illustrate the transformer and motor mode of power transfer. Steady-state operations with ac–dc stator connections were also considered. In their companion paper (Schiferl & Ong, 1983b), the authors have derived the relationships of mutual leakage inductances with a winding displacement angle and pitch for a number of practical six-phase winding configurations. They have also proposed a single machine uninterruptible power supply scheme, using a six-phase synchronous machine. Modeling and simulation of high power drives using a double star synchronous motor was presented by Terrien and Benkhoris (1999). They have analyzed the effect of the winding displacement angle on torque ripple and harmonic content. Abuissmais, Arshad, and Kanerva (2008) have given the analysis of voltage source inverter fed direct torque controlled six-phase synchronous machine with emphasis on redundancy, fault conditions, the machine behavior under non-sinusoidal voltage profiles and sensitivity of the design parameters.

Generally, any electrical motor operates at steady-state condition at a particular output load. Therefore, steady-state analysis is needed for a better understanding of the operational behavior of a six-phase synchronous motor under different operating conditions. It will be advantageous to examine its describing equations followed by the development of the phasor diagram. While performing dynamic simulation of motors, these steady-state equations

*Corresponding author. Email: aieerdee@iitr.ac.in

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can also be used for the calculation of steady-state initial values to initialize dynamic simulation. Also, knowledge of steady-state behavior is indispensable for checking the correctness of the simulation results. The steady-state analysis of six-phase synchronous motor is not yet reported to best of the authors’ knowledge. Therefore, the main contribution of this paper is to explore the steady-state operation of six-phase synchronous motor during its normal operating condition as well as the operation with input supply in only one set of the three-phase stator winding (motor-generator mode). Initially, a mathematical model of six-phase synchronous motor has been developed, followed by the development of its phasor diagram in the steady-state operating condition, from which performance of the motor may be directly analyzed. Mathematical treatment has been carried out employing the Park’s (dq0) transformation, and the calculated steady state values have been verified through experimental results.

2. Mathematical modeling

For designing a six-phase motor, it is common strategy to split the stator winding into two sets, each of which is supplied by a separate three-phase electrical source. The two sets of three-phase windings namely abc and xyz, have an angular displacement of $\xi = 30^\circ$, to have an asymmetrical winding. This is because, with $30^\circ$ space shift between two winding sets and with the same electrical phase shift between the supply voltages, many harmonics are eliminated from the airgap flux. Actually, the excitation voltage time harmonics such as 5th, 7th, 17th, 19th, ..., etc. are prevented from contributing to the airgap flux and torque pulsation, while they are contributing in the supply input current.

For developing the motor equations, some simplifying assumptions are made. These are as follows:

- Both the stator windings (abc and xyz) are symmetrical and have a perfect sinusoidal distribution along the air-gap,
- Space harmonics are neglected, flux and magnetomotive are sinusoidal in space,
- Saturation and hysteresis effects are neglected,
- Skin effect is neglected, windings resistances are not dependent on frequency.

A schematic representation of stator and rotor axes is shown in Figure 1(a). Stator consists of two balanced three-phase windings, abc and xyz which are physically displaced by 30 electrical degrees. Rotor is equipped with the field winding $f_r$, damper winding $K_d$ along the $d$-axis and a damper winding $K_q$ along the $q$-axis. By adopting the motor convention, voltage equations for stator and rotor windings can be written. The voltage and electromagnetic torque equations written in machine variables will result in the set of nonlinear differential equations. Nonlinearity increases the computational complexity. Nonlinearity is introduced due to the presence of the inductance term, which is a function of rotor position and is time dependent. Elimination of nonlinearity from motor equations is advantageous and is achieved by using reference frame theory, i.e Park’s (dq0) transformation. This simplifies equations with constant inductance terms. The equations of voltages and flux linkage per second of a six-phase synchronous motor in Park’s variables are (Singh, 2011a, 2011b):

Voltage equation:

$$v_{q1} = r_1i_{q1} + \frac{\omega_b}{\omega_f}\psi_{d1} + \frac{p}{\omega_b}\psi_{q1},$$
$$v_{d1} = r_1i_{d1} - \frac{\omega_b}{\omega_f}\psi_{q1} + \frac{p}{\omega_b}\psi_{d1},$$
$$v_{q2} = r_2i_{q2} + \frac{\omega_b}{\omega_f}\psi_{d2} + \frac{p}{\omega_b}\psi_{q2},$$
$$v_{d2} = r_2i_{d2} - \frac{\omega_b}{\omega_f}\psi_{q2} + \frac{p}{\omega_b}\psi_{d2},$$
$$v_{Kq} = r_{Kq}i_{Kq} + \frac{p}{\omega_b}\psi_{Kq},$$
$$v_{Kd} = r_{Kd}i_{Kd} + \frac{p}{\omega_b}\psi_{Kd},$$
$$v_{fr} = \frac{x_{md}}{r_{fr}}\left(r_{fr}\psi_{fr} + \frac{p}{\omega_b}\psi_{fr}\right),$$

where, all the symbols stands at their usual meaning, $\omega_f$ and $\omega_b$ are the rotor speed (also the speed of the rotating reference frame) and base speed, respectively. Here $p$ denotes the differentiation function with respect to time.

Flux linkage per second:

$$\psi_{q1} = x_{l1}i_{q1} + x_{lm}(i_{q1} + i_{q2}) - x_{ldd}i_{d2} + \psi_{mq},$$
$$\psi_{d1} = x_{l1}i_{d1} + x_{lm}(i_{d1} + i_{d2}) - x_{ldd}i_{q2} + \psi_{md},$$
$$\psi_{q2} = x_{l2}i_{q1} + x_{lm}(i_{q1} + i_{q2}) + x_{ldd}i_{q1} + \psi_{mq},$$
$$\psi_{d2} = x_{l2}i_{d2} + x_{lm}(i_{d1} + i_{d2}) - x_{ldd}i_{q1} + \psi_{md},$$
$$\psi_{Kq} = x_{Kq}i_{Kq} + \psi_{mq},$$
$$\psi_{Kd} = x_{Kd}i_{Kd} + \psi_{md},$$
$$\psi_{fr} = x_{fr}i_{fr} + \psi_{md},$$

where,

$$\psi_{mq} = x_{mq}(i_{q1} + i_{q2} + i_{Kq}),$$
$$\psi_{md} = x_{md}(i_{d1} + i_{d2} + i_{Kd} + i_{fr}).$$

The rotor circuit parameters are referred to one of the stator windings (abc windings). The voltage and flux linkage equations can be represented by an equivalent circuit as shown in Figure 1(b), where $L_{lm}$ and $L_{ldd}$ are known as the common mutual leakage inductance and cross mutual coupling inductance between the $d$ and $q$-axis of the stator,
respectively, and are given by

\[ x_{lm} = x_{lax} \cos(\xi) + x_{lay} \cos \left(\frac{\xi + 2\pi}{3}\right) \]
\[ + x_{laz} \cos \left(\frac{\xi - 2\pi}{3}\right), \]  \hspace{1cm} (17)

\[ x_{ldg} = x_{ldx} \sin(\xi) + x_{ldy} \sin \left(\frac{\xi + 2\pi}{3}\right) \]
\[ + x_{ldz} \sin \left(\frac{\xi - 2\pi}{3}\right). \]  \hspace{1cm} (18)

The common mutual leakage reactance \( x_{lm} \) accounts for the mutual coupling due to the leakage flux between two sets of stator windings occupying the same slot. It depends on the winding pitch and displacement angle between the two stator winding sets and has an important effect on the harmonic coupling between them. However, neglecting this parameter has no noticeable effect on transient effect except some changes in voltage harmonic distortion (Singh, 2011b). It has been explained in detail and a technique for finding the slot reactance is given in Alger (1970). Standard test procedures are available to determine various machine parameters are given in Aghamohammadi and Pourgholi (2008) and Jones (1967).

### 3. Steady-state analysis

It is convenient to use the park’s equations for the derivation of steady-state equations, though other several approaches may be used to describe the balanced steady-state operation of synchronous machine. In this mode of operation, the rotor rotates at a synchronous speed, i.e. the relative speed between the armature field and rotor field is zero. Hence, the time rate change of flux linkage is zero with no current in the short-circuited damper windings. This fact has been incorporated by dropping the differential term in the voltage equation. These equations can then be written as

\[ V_{q1} = r_1 I_{q1} + \frac{\omega e}{\omega h} (x_{l1} + x_{lm} + x_{md}) I_{d1} \]
\[ + \frac{\omega e}{\omega h} (x_{l1} + x_{md}) I_{d2} + \frac{\omega e}{\omega h} x_{ldq} I_{q2} + E_{fr}, \]  \hspace{1cm} (19)

\[ V_{d1} = r_1 I_{d1} - \frac{\omega e}{\omega h} (x_{l1} + x_{lm} + x_{mq}) I_{q1} \]
\[ - \frac{\omega e}{\omega h} (x_{l1} + x_{mq}) I_{q2} + \frac{\omega e}{\omega h} x_{ldq} I_{d2}, \]  \hspace{1cm} (20)

\[ V_{q2} = r_2 I_{q2} + \frac{\omega e}{\omega h} (x_{l2} + x_{lm} + x_{md}) I_{d2} \]
\[ + \frac{\omega e}{\omega h} (x_{l2} + x_{md}) I_{d1} \]
\[ - \frac{\omega e}{\omega h} x_{ldq} I_{q1} + E_{fr}, \]  \hspace{1cm} (21)

\[ V_{d2} = r_2 I_{d2} - \frac{\omega e}{\omega h} (x_{l2} + x_{lm} + x_{mq}) I_{q2} \]
\[ - \frac{\omega e}{\omega h} (x_{l2} + x_{mq}) I_{q1} - \frac{\omega e}{\omega h} x_{ldq} I_{d1}, \]  \hspace{1cm} (22)

\[ E_{fr} = \frac{\omega e}{\omega h} x_{md} I_{fr}. \]  \hspace{1cm} (23)

Steady-state analysis has been carried out under the assumption that each of the stator winding sets \( abc \) and \( xyz \) are supplied by a balanced set of three-phase voltage sources such that the phase voltage can be written as follows:

\[ v_a = \sqrt{2} V_1 \cos \omega_c t, \]  \hspace{1cm} (24)

\[ v_x = \sqrt{2} V_2 \cos (\omega_c t - \alpha), \]  \hspace{1cm} (25)

and the phase current can be written as

\[ i_a = \sqrt{2} I_1 \cos (\omega_c t - \varphi_1), \]  \hspace{1cm} (26)

\[ i_x = \sqrt{2} I_2 \cos (\omega_c t - \varphi_2 - \alpha), \]  \hspace{1cm} (27)

where \( V_1 \) is the rms value of the phase voltage of winding set \( abc \); \( I_1 \) the rms value of the phase current of the winding set \( abc \); \( V_2 \), the rms value of the phase voltage of the winding...
set \(xyz\); \(I_2\), the rms value of the phase current of winding set \(xyz\); \(\phi_1\), the power factor angle of winding set \(abc\); \(\phi_2\) the power factor angle of the winding set \(xyz\); and \(\alpha\) the phase difference between the phase voltage of \(a\) and \(x\).

In the rotor reference frame, \(d - q\) components of stator voltages become constant and can be written as

\[
V_{q1} = V_1 \cos \delta, \quad V_{q1} = V_1 \sin \delta, \quad V_{q2} = V_2 \cos(\delta - \xi - \alpha), \quad V_{q2} = V_2 \sin(\delta - \xi - \alpha).
\]

(28) (29) (30) (31)

The above voltages can be related to synchronously rotating voltage phasors \(V_a\) and \(V_x\) as

\[
V_{ae}^{e^{j\beta}} = V_{q1} - jV_{d1}, \quad V_{xe}^{e^{j(\beta - \xi)}} = V_{q2} - jV_{d2}.
\]

(32) (33)

A similar relation can be written for the stator current as

\[
I_{ae}^{e^{j\beta}} = I_{q1} - jI_{d1}, \quad I_{xe}^{e^{j(\beta - \xi)}} = I_{q2} - jI_{d2}.
\]

(34) (35)

Substitution of Equations (19) and (20) in Equation (32), followed by the mathematical simplification yields the voltage equation in the phasor form as

\[
V_a = \left[ r_1 + j\frac{\omega_e}{\omega_h}(x_{11} + x_{lm} + x_{mq}) \right] I_a + \frac{\omega_e}{\omega_h}[x_{ldq} + j(x_{lm} + x_{mq})]I_x e^{j\xi} + E_q.
\]

(36)

Similarly, for other winding sets, \(xyz\) yields

\[
V_x e^{j\xi} = \left[ r_2 + j\frac{\omega_e}{\omega_h}(x_{12} + x_{lm} + x_{mq}) \right] I_x e^{j\xi} + \frac{\omega_e}{\omega_h}[-x_{ldq} + j(x_{lm} + x_{mq})]I_a + E_q,
\]

(37)

where,

\[
E_q = \left[ \frac{\omega_e}{\omega_h}(x_{md} - x_{mq})(I_{d1} + I_{d2}) + E_{fr} \right] e^{j\delta},
\]

(38)

is the voltage phasor along the \(q\)-axis and the field voltage is given by

\[
E_{fr} = \frac{\omega_e}{\omega_h}x_{md}I_{fr}.
\]

(39)

The above voltage phasors can be used to draw an equivalent circuit as well as the phasor diagram as depicted in Figures 2 and 3. In order to retain the generality of the operation of two winding sets \(abc\) and \(xyz\), the voltage drops and power factor angles are shown to be different for two sets. While evaluating the performance indices during the steady-state, these values are taken to be equal for both the winding sets.

This is because power sharing by each stator winding set is equal.

Three modes of operation can be illustrated, based on the three operating conditions: (1) \(E_q = V_a(= V_x e^{j\xi})\): The first condition appears during no-load condition, if friction and windage losses along with the ohmic drop of stator windings are neglected, then input currents to the stator windings will be zero, i.e. \(I_a = I_x e^{j\xi} = 0\). In this mode of operation, the motor will run at a synchronous speed without absorbing energy from the electrical system. This operation is referred as “floating on the line.” The operation is practically not possible because the motor will absorb a small amount of energy associated with ohmic and friction and windage losses. (2) \(E_q < V_a(= V_x e^{j\xi})\): During this mode of operation, \(I_a(\text{and } I_x e^{j\xi})\) is lagging \(V_a\) and \(V_x e^{j\xi}\), therefore, the motor will absorb the reactive power, hence this appears as an inductor to the system. (3) \(E_q > V_a(= V_x e^{j\xi})\): During this mode of operation, \(I_a(\text{and } I_x e^{j\xi})\) is leading \(V_a\) and \(V_x e^{j\xi}\), hence this appears as a capacitor supplying reactive power to the system.

Power and torque expression:

The total complex power of both sets of stator windings \(abc\) and \(xyz\) is given by

\[
S = P + jQ,
\]

(40)

where, \(P\) and \(Q\) are the active and reactive power components of the motor, respectively. These powers can be expressed as

\[
P = 3[(V_{q1}I_{q1} + V_{d1}I_{d1}) + (V_{q2}I_{q2} + V_{d2}I_{d2})]
\]

(41)

\[
Q = 3[(V_{q1}I_{d1} - V_{d1}I_{q1}) + (V_{q2}I_{d2} - V_{d2}I_{q2})].
\]

(42)

Electromagnetic power developed by the motor is obtained by subtracting from the input power, the losses in stator, which is just the copper loss of stator windings in the model. This can be easily obtained by putting the value of \(d - q\) voltages from Equations (19) to (22) in Equation (41). Suitable mathematical simplification then yields,

\[
P = \text{copper loss} + P_{em},
\]

(43)

where

\[
\text{copper loss} = r_1(I_{q1}^2 + I_{d1}^2) + r_2(I_{q2}^2 + I_{d2}^2),
\]

(44)
is the stator winding copper loss.

\[ P_{em} = E_{fr}(I_{q1} + I_{q2}) + (x_{md} - x_{mq})(i_{d1}i_{q1} + i_{d2}i_{q2}) + (x_{md} - x_{mq})(i_{d1}i_{q2} + i_{d2}i_{q1}) \]

\[ + (x_{md} - x_{mq})(i_{d1}i_{q2} + i_{q1}i_{d2}) \]  \hspace{1cm} (45)

is the developed electromagnetic power by the motor.

Electromagnetic torque expression is obtained by dividing the expression of electromagnetic power by the actual rotor speed. Therefore, we have

\[ T_{em} = \frac{P_{em}}{\omega_r} = \left( \frac{P}{2\omega_b} \right) (E_{fr}(I_{q1} + I_{q2}) + (x_{md} - x_{mq})(i_{d1}i_{q1} + i_{d2}i_{q2}) + (x_{md} - x_{mq})(i_{d1}i_{q2} + i_{q1}i_{d2}) \). \hspace{1cm} (46)

The above equation of torque is valid for steady-state operation, neglecting the stator resistance. However, in a variable-speed drive system at low frequencies, the stator resistance must be considered while evaluating the torque.

Torque expression permits a quantitative description of the nature of the steady-state electromagnetic torque of the six-phase synchronous motor. The first term on the right-hand side of Equation (46) is due to the interaction of the magnetic system produced by the currents flowing in both the winding sets abc and xyz and the magnetic system produced by the current flowing in the field winding. The second and third terms are due to the saliency of the rotor. The second term is named as the “self-reluctance torque,” which is the sum of reluctance torques produced by the two sets of stator windings operating independently. The third term is named as the “mutual-reluctance torque,” which is produced due to the mutual coupling between the two sets of stator windings. The torque produced due to the interaction of stator and field currents is the dominant torque. The reluctance torque is generally a relatively small part of the total torque of a synchronous motor. It is worthwhile to mention here that the reluctance torque (self and mutual-reluctance torque) vanishes in the cylindrical rotor synchronous motor because \( x_{md} = x_{mq} \).

The above analytical approach can be easily extended to other types of multiphase machines with multiple three-phase winding sets \( N = n/3 \) having angular displacement of \( \xi \), where \( N \) is an integer representing the number of winding sets, and takes the value 1 and 2 for 6-phase; 1, 2 and 3 for 9-phase; 1, 2, 3 and 4 for 12-phase; 1, 2, 3 and \( n/3 \) for the \( n \)-phase.
3.1. Supply in one set of winding only

Apart from the above discussed cases of the steady-state operation of the motor with two sets of stator windings \( abc \) and \( xyz \), a case of complete outage of supply in one winding set \( xyz \) is also considered here. As there is only one set of the three phase supply connected to winding set \( abc \), the six-phase synchronous motor in this case will act as a three-phase synchronous motor. Analytical results can be obtained by putting \( i_{q2} = i_{d2} = 0 \), i.e. \( I_2 = 0 \) in the above discussed model of the six-phase synchronous motor. As there is a coupling between the winding sets \( abc \) and \( xyz \) due to sharing of same stator slots by coil sides of \( abc \) and \( xyz \) winding, therefore, a voltage will be generated in the winding set \( xyz \) due to transformer action. Hence, the six-phase synchronous machine in this mode will both act as a motor as well as a generator (motor-generator mode).

The developed model of the machine can be applied easily to evaluate the motor performance under steady-state condition with input in both sets of stator windings as well as input in only one set of stator winding. A unified and simplified step for these operating conditions is shown in the flow chart in Figure 4.

4. Simulation results

The mathematical model developed for the operation of a six-phase synchronous motor at the steady-state condition in the previous section has been investigated via simulation in the Matlab/Simulink environment. A motor of 3.7 kW with its parameters given in Appendix 1 has been simulated for 50 percent of the base/rated torque, operating at power factor of 0.85 (lagging). The input phase voltage of both the winding sets \( abc \) and \( xyz \) has been maintained constant at 160 V where the input phase current was found to be 2.29 A in each winding set. The steady-state phase voltage–current waveforms of phases \( a \) and \( x \) are shown in Figure 5. Under this condition, the calculated active power and reactive power was found to be 1865 W and 1155.8 VAr, respectively, with the load angle \( \delta \) equal to \(-5.76^\circ\). The induced field voltage and current have been maintained at 206.73 V and 33.53 A, respectively.

Simulation results for various other operating conditions during steady-state are shown in Figure 6. Figure 6(a) shows the operating properties at different load conditions (i.e. at different load torque) operating at a power factor of 0.85 (lagging). As expected, the motor current as well as the load angle \( \delta \) (only numerical value shown) proportionately increases with the increase of output load torque. Figure 6(b) shows the operating properties at different operating power factors (shown by different phase angles) operating at 50 percent of base/rated torque. Variation in current and load angle \( \delta \) can be noted at different phase angles (i.e. at different power factor). The variation of the current is basically due to the variation of the reactive component of power, since motor is operating at constant active power to deliver a constant load. Under this condition, variation of load angle, \( \delta \) can also be noted and it decreases when the motor operating mode moves from the lagging to the leading power factor. Simulation results at different operating conditions (different load torque with constant power factor and constant load torque with different power factors) have also been shown in Table A1. Some more results are given in Appendix 2 to give more insight into the motor operation.

4.1. Inclusion of supply asymmetry

For the purpose of simplicity and better understanding, it has been assumed that the input supply in the above analysis is perfectly ideal. Practically, this may not be the case as there is a continuous change of input supply voltage (in magnitude and/or phase) due to the variation of other loads connected to the supply lines, resulting in an unbalance flow of current in the motor. This phenomenon has been identified in a number of available references (Apsley, 2010; Bojoi, Farina, Lazzari, Profumo, & Tenconi, 2003; Bojoi, Lazzari, Profumo, & Tenconi, 2003; Lyra & Lipo, 2003) for multiphase motor. It has been found that the multiphase motor is more sensitive to the supply asymmetries than the machine asymmetry (Apsley, 2010; Bojoi, Farina et al., 2003). Therefore, supply asymmetries have also been incorporated into simulation results similar to the actual motor operation in the following sections.

Following the inclusion of supply asymmetry with the operation of motor at 50 percent of the base/rated torque approximately, the input current in each phase of the six-phase synchronous motor was found to be different. The magnitude of input currents was found to be 3.20, 3.35, 2.87 A and 2.83, 2.35, 2.45 A in the phases of winding sets \( abc \) and \( xyz \), respectively, as shown in Figure 7. The supply voltage at input motor phases was taken as 160 V, with the variation similar to that of the experimental operating condition, as shown in section 5. It has to be noted that the motor torque is supplying the load, accompanied by the production of harmonics, which varies between the peak value of 56.24 and 53.56 Nm. Variation of instantaneous torque can further be noted in the rotor speed, as depicted in Figure 8(b).

4.2. Supply in one winding set (abc) only

Simulation was carried out for the supply in the winding set \( abc \) only, assuming a complete outage of supply in winding set \( xyz \). Therefore, the output power will have to be supplied by winding set \( abc \) only, resulting in the increase of its current magnitude to 5.45 A approximately. Since the machine in this mode is giving the output power both in the form of mechanical (to supply the load torque) as well as electrical (voltage generated in winding set \( xyz \) power, therefore, the machine is acting both as a motor and a generator at a time (motor-generator mode). Generation of voltage in the winding set \( xyz \) will take place due to the transformer action, the
generated phase voltage was found to be 159.67 V approximately for each phase. Computer traces of generated phase voltage \( V_x \) and voltage–current of phase “a” are shown in Figure 9. Harmonic production in this mode of operation was found to decrease considerably as compared to the above. This is due to the reduced sensitivity of the motor
(with three phase winding configuration) towards supply asymmetries; hence, resulting in the flow of relatively more balanced three-phase current into the motor, though with an increased magnitude.

5. Experimental results

In order to investigate the performance of the six-phase synchronous motor, a three-phase, 50 Hz, 36 slots, 3.7 kW synchronous machine was selected. All the 72 end terminals of stator coils were taken out to the terminal box mounted on the top of the machine, so that the various winding schemes with different number of poles and phases could be realized. Six-phase connection of the motor was obtained by employing phase belt splitting to have two sets of three phase winding, namely $abc$ and $xyz$. Angular displacement between the two sets of winding was $30^\circ$ electrical apart, so as to have asymmetrical winding to prevent the airgap contribution due to excitation voltage time harmonics of the lower order. Neutral point of the two three phase sets (winding sets $abc$ and $xyz$) was kept isolated in order to prevent physical fault propagation from one winding set to the other, and also to prevent the flow of triplen harmonics.

As shown in Figure 10(a), the variable three-phase ac supply was obtained by using an inverter and three
winding transformer. This enables one to obtain the variable three-phase ac supply of different magnitudes (and frequency). In order to obtain the two three-phase supplies, having a phase shift of 30° electrical, 10 kVA, 415/415-415 V, Y/Y-Δ, three to six-phase, three winding transformer has been employed in the experimental setup. The three winding transformer was locally manufactured where the magnitude of phase voltages at both secondary were found to be different due to a manufacturing defect. In this experimental setup, a synchronous motor was mechanically coupled to a DC machine of 5 kW, 250 V, 21.6 A, 1300/1750 rpm, which acts as a dc generator. An electrical load in the form of lamp load was connected to the output of the dc generator. Experimental scheme is shown in Figure 10(a), while the general view of the test rig is shown in Figure 10(b).
Hioki 3197 Power Quality Analyzer has been used for the measurement and recording of various waveforms and performance indices. It is important to state that a small variation in the data of various experimental waveforms for the same operating condition was observed. This is due to the time lag in manual recording of the waveform and data.

The steady-state value of input phase current of winding sets $abc$ and $xyz$ was found be 3.23 A (at 160.1 V), 3.32 A (at 162.3 V), 2.81 A (at 163.1 V) and 2.78 A (at 154.9 V), 2.36 A (at 156.3 V), 2.45 A (at 157.3 V), respectively, as shown in Figure 11. As can be seen that the phase voltage is varying in nature, which is a major factor in contributing...
to the asymmetrical condition, hence the unequal flow of current in a winding set.

Supply asymmetries was found to be contributed by the difference in magnitude of the output secondary voltage by the three winding transformer and the continuous change of supply voltage and current due to the variation of other loads connected to the supply line. Motor current is further affected due to the presence of a high inductive element in the form of a three winding transformer in experimental circuitry. It is worthwhile to mention here that in the developed analytical model, saturation and effect of space harmonics have been neglected. Also, in the model, parameter sensitivity has not been considered. These along with the supply asymmetry are the main reasons for small mismatch in the analytical and experimental current waveform.

Motor behavior can further be explored for different operating conditions. Conditions include the operation at different loads, operating voltages or frequencies. Two such operations at different loads and different voltage levels (at 50 percent of motor base/rated torque) have been tabulated in Table A2.

5.1. Motor operation with excitation in one set of winding

This operating condition was experimentally examined for a complete outage of the three phase input supply of one winding set of the motor. In this mode of operation, the input supply was given to only one set of winding (winding set abc). Therefore, the voltage will be generated on the other winding set (winding set xyz) due to the transformer action which was found to be 145, 148, 147 V for phases x, y, z, respectively, as shown in Figure 12 (only phase x voltage is shown). Hence, synchronous machine in this mode, will act both as a motor as well as a generator. In this mode of operation, machine mechanical output power needs to be constant, which has to be supplied by only one winding set abc. Currents of this winding set were found
to be 5.43, 5.23, and 5.08 A for phase a, phase b and phase c, respectively. The steady state phase voltage and current of phase a (winding set abc) along with the generated voltage of phase x (winding set xyz) are shown in Figure 12. Furthermore, a variation of generated voltage in the winding set xyz was observed with the variation of load torque and the input voltage of the winding set abc at constant output load torque (50 percent of rated/base torque). Value of generated voltage was found to decrease proportionately with the increase in load torque whereas it increases proportionately with the increase in magnitude of the input voltage of winding set abc, as depicted in Figure 13. It is worthwhile to mention here that the experimental results are presented for upto 50 percent of mechanical load on motor. This is because the phase current approaches to rated value, when the machine is delivering the power with input supply in
only one winding set $abc$ (only half of the armature is being utilized). Therefore, the motor was derated by 50 percent approximately, to ensure a reliable operation of the system under the safe current limit. Further, it has to be noted that the flow of unbalanced current in the motor is due to the supply asymmetry, which also results in performance degradation. It is evident from Figure 8(a) that indicates the derating requirement of the motor due to the production of harmonics in the motor torque. Calculation of derating of the multiphase motor due to supply asymmetry has been reported in (Apsley, 2010).

Effect of supply asymmetry can be reduced by having a careful selection of components in electrical circuitry. However, it can be effectively reduced by employing a closed-loop current control scheme, as illustrated in Bojoi, Lazzari et al. (2003) and Lyra and Lipo (2003).

6. Conclusion

In this paper, a simple steady-state mathematical model of an asymmetrical six-phase synchronous motor is presented, where the effect of common mutual leakage reactance between the two sets of the three-phase stator winding has been considered. A unified and simplified approach has been adopted for digital simulation, considering the operation at two operating conditions at the input terminals i.e. the input supply at the two three-phase of stator winding and one set of three-phase stator winding. This shows the additional possibilities offered by using the six-phase synchronous motor compared to its three-phase counterpart which indicates its redundancy characteristic making it suitable for the application where reliability is of prime importance. Furthermore, there is a possibility of generation of voltage at one of the three-phase winding set, when other winding sets are supplied by three-phase electrical source. Hence, a single machine acts as motor as well as a generator making it suitable for some special applications. This paper also provides a general methodology of the steady-state analysis using the approach of Park’s ($dq0$) transformation, which can be easily extended to other types of the multiphase machine under different operating conditions.

References

Abuismais, I., Arshad, W. M., & Kanerva, S. (2008). Analysis of VSI-DTC fed six phase synchronous machines. 13th International Power Electronics and Motor control conference, Poznan, pp. 883–888.

Aghamohammadi, M. R., & Pourgholi, M. (2008). Experience with SSSSRF test for synchronous generator model identification using Hook-Jeeves optimization method. International Journal of System Applications, Engineering and Development, 2(3), 122–127.

Alger, P. L. (1970). Induction machine. New York, NY: Gorden and Breach.

Apsley, J. M. (2010). Derating of multiphase induction machines due to supply imbalance. IEEE Transaction on Industrial Applications, 46(2), 798–805.

Bojoi, R., Farina, F., Lazzari, M., Profumo, F., & Tenconi, A. (2003). Analysis of the asymmetrical operation of dual three-phase induction machines. Proceedings of the IEEE International Electric Machine and Drives Conference, Madison Wisconsin, pp. 429–435.

Bojoi, R., Lazzari, M., Profumo, F., & Tenconi, A. (2003). Digital field-oriented control of dual three-phase induction motor drives. IEEE Transaction on Industrial Applications, 39(3), 752–760.

Fuchs, E. F., & Rosenberg, L. T. (1974). Analysis of an alternator with two displaced stator windings. IEEE Transaction on Power Apparatus Systems, 93(6), 1776–1786.

Jahns, T. M. (1980). Improved reliability in solid-state ac drives by means of multiple independent phase drive Units. IEEE Transaction on Industrial Applications, 16(3), 321–331.

Jones, C. V. (1967). The unified theory of electric machine. London: Butterworths.

Klingshirn, E. A. (1983). High phase order induction motor-Part-I: Description and theoretical consideration. IEEE Transaction on Power Apparatus Systems, 102(1), 47–53.

Levi, E. (2008). Multiphase electric machines for variable-speed applications. IEEE Transaction on Industrial Electronics, 55(3), 1893–1909.

Lyra, R. O. C., & Lipo, T. A. (2003). Torque density improvement in a six-phase induction motor with third harmonic current injection. IEEE Transaction on Industrial Applications, 38(5), 1351–1360.

Schiferl, R. F., & Ong, C. M. (1983a). Six phase synchronous machine with ac and dc stator connection, Part-I. IEEE Transaction on Power Apparatus Systems, 102(8), 2685–2693.

Schiferl, R. F., & Ong, C. M. (1983b). Harmonic studies and a proposed uninterruptible power supply scheme, Part-II. Transaction on Power Apparatus Systems, 102(8), 2694–2701.

Singh, G. K. (2002). Multiphase induction machine drive research-A survey. Electric Power Systems Research, 61(2), 139–147.

Singh, G. K. (2011a). A six-phase synchronous generator for stand-alone renewable energy generation: Experimental analysis. Energy, 36(3), 1768–1775.

Singh, G. K. (2011b). Modeling and analysis of six-phase synchronous generator for stand-alone renewable energy generation. Energy, 36(9), 5621–5631.

Terrien, F., & Benkhoris, M. F. (1999). Analysis of double star motor drives for electric propulsion. Conference publication no. 468, IEE, pp. 90–94.
Appendix 1

Parameter of 3.7 kW, 36 slots, 6-poles six-phase synchronous motor is given below.

\[
\begin{align*}
    x_{mq} &= 3.9112 \ \Omega \\
    x_{md} &= 6.1732 \ \Omega \\
    x_{ldq} &= 0.66097 \ \Omega \\
    x_{lm} &= 0.001652 \ \Omega \\
    x_{Kd} &= 1.550 \ \Omega \\
    x_{Kq} &= 2.535 \ \Omega \\
    x_{lf} &= 0.2402 \ \Omega \\
    x_{l1} &= x_{l2} = 0.1758 \ \Omega \\
    r_1 &= 0.210 \ \Omega \\
    r_2 &= 0.210 \ \Omega \\
    r_{fr} &= 0.056 \ \Omega \\
    r_{Kd} &= 140.0 \ \Omega \\
    r_{Kq} &= 2.535 \ \Omega \\
    r_{fr} &= 0.056 \ \Omega \\

\end{align*}
\]

Appendix 2

Table A1. Steady-state performance calculation under (a) different loads at 0.85 power factor (lagging) and (b) different power factor at 50 percentage load.

| Percentage of base torque | Active power (W) | Reactive power (VAr) | Phase current (Amp.) | Field voltage, $E_{fr}$ (V) | Load Angle, $\delta$ (degree) | Field current $I_{fr}$ (Amp.) |
|---------------------------|------------------|---------------------|----------------------|-----------------------------|-------------------------------|-----------------------------|
| (a) 20                    | 746              | 462.34              | 0.73                 | 276.09                      | −1.41                         | 44.72                       |
| 50                        | 1865             | 1155.80             | 1.83                 | 266.68                      | −3.61                         | 43.20                       |
| 80                        | 2984             | 1849.30             | 2.92                 | 258.21                      | −5.90                         | 41.82                       |
| 100                       | 3730             | 2311.60             | 3.65                 | 253.11                      | −7.49                         | 41.00                       |
| (b) 0.4 (lagging)         | 1865             | 4273.3              | 3.88                 | 221.11                      | −3.88                         | 35.81                       |
| 0.8 (lagging)             | 1865             | 1398.8              | 1.94                 | 263.13                      | −3.63                         | 42.62                       |
| 1.0 (unity)               | 1865             | 0.00                | 1.55                 | 283.59                      | −3.52                         | 45.94                       |
| 0.8 (leading)             | 1865             | −1398.8             | 1.94                 | 304.05                      | −3.42                         | 49.25                       |
| 0.4 (leading)             | 1865             | −4273.3             | 3.88                 | 346.11                      | −3.25                         | 56.06                       |

Table A2. Steady-state performance calculation under (a) different load and (b) different voltage.

| Phase Load torque | A Exp. | Sim. | B Exp. | Sim. | C Exp. | Sim. | X Exp. | Sim. | Y Exp. | Sim. | Z Exp. | Sim. |
|-------------------|--------|------|--------|------|--------|------|--------|------|--------|------|--------|------|
| (a) 33.83%        | 2.58   | 2.53 | 2.73   | 2.70 | 2.19   | 2.21 | 2.38   | 2.34 | 1.94   | 1.85 | 1.96   | 2.05 |
| 26.46%            | 2.32   | 2.36 | 2.51   | 2.56 | 1.97   | 2.07 | 1.87   | 1.90 | 1.44   | 1.42 | 1.52   | 1.59 |
| (b) Phase Voltage level | A Exp. | Sim. | B Exp. | Sim. | C Exp. | Sim. | X Exp. | Sim. | Y Exp. | Sim. | Z Exp. | Sim. |
| 180               | 3.55   | 3.52 | 3.74   | 3.70 | 3.11   | 3.19 | 3.27   | 3.22 | 2.89   | 2.80 | 2.83   | 2.85 |
| 200               | 3.87   | 3.86 | 4.24   | 4.28 | 3.61   | 3.68 | 3.82   | 3.82 | 3.32   | 3.25 | 3.24   | 3.20 |