Improvement of wavefront correcting particle swarm optimization algorithm based on metropolis criterion

Shiqi Hao¹, Chenlu Xu¹, Yang Chen², Zehua Liu³

¹State Key Laboratory of Pluse Laser Technology, National University of Defense Technology, Hefei, Anhui 230037, China
²Chinese people’s liberation army unit 94654, Nanjing, Jiangsu 210046, China
³Chinese people’s liberation army unit 95910, Jiuquan, Gansu 210046, China

Corresponding author’s e-mail: liu_hsq@126.com

Abstract. In this paper, the wavefront phase distortion of laser beam caused by atmospheric turbulence is corrected by an adaptive optics sensor-less system. According to the shortcoming that particle swarm optimization algorithm (PSO) is easily falling into local optimum, an improved PSO algorithm based on metropolis criterion is proposed in order to improve the efficiency and effect of wavefront correction. The metropolis criterion is used to judge whether an inferior state is accepted or not, so that the particle swarm optimization algorithm can jump out of the local optimum. Finally, the correcting effect of the improved PSO algorithm is compared by numerical simulation analysis, which verifies that the improved PSO algorithm can improve the correcting speed and effect, and the running time of achieving the same correcting effect is greatly reduced.

1. Introduction

The wavefront distortion[1-2] caused by atmospheric turbulence is usually corrected adaptive optics (AO) system. The AO system has two types, one is sensor AO system with wavefront sensor, and the other is sensor-less AO system without wavefront sensor [3-4]. The AO system with wavefront sensor is complicated, and the atmospheric turbulence would lead to the difficulty of wavefront detection. While for sensor-less AO system, the controller controls the correcting voltage of the deformable mirror through the intelligent optimization algorithm. Since the wavefront distortion affects the light intensity distribution of the laser, the wavefront detector is replaced by a CCD so as to detect the light intensity. When the wavefront distortion is eliminated, the light intensity reaches the maximum value. This system structure is relatively simple, and the application range is more extensive. Therefore, the sensor-less AO system is used to correct the wavefront phase distortion in this paper.

The commonly used intelligent optimization algorithms for controlling the voltage of deformable mirror are hill climbing method [5], stochastic parallel gradient descent (SPGD) algorithm [6], simulated annealing (SA) algorithm [7] and particle swarm optimization (PSO) algorithm [8]. The hill climbing method selects some nodes by inspiration, so as to achieve the purpose of improving efficiency, but it is easy to fall into the local optimum. After falling into the local optimum, the optimal searching direction can’t be determined. The search may be wandering in the two sides of the optimal value, so the searching efficiency would be reduced. The SPGD algorithm has a quick correcting speed, but the shortcoming is that it can’t reach the globally optimal. The simulated annealing algorithm whether accepts a poor solution depending on the probability calculated by the
metropolis criterion. It can overcome the problem of getting into local optimal, but its correcting effect and efficiency are greatly affected by the parameters of the algorithm. The PSO algorithm has fast searching speed and high efficiency, and the algorithm is simple. But it has the problem of falling into local optimum and early convergence. Because the metropolis criterion can get out of the local optimal, this paper improves the PSO algorithm based on the metropolis criterion. The improved algorithm not only has quick searching speed and correcting efficiency, but also overcomes the problem that it is easy to fall into the local optimum, thus to improve the wavefront correcting effect.

2. System Model
The FSO communication system model is shown in Fig.1. The transmitting terminal emits laser beams, which is transmitted through the atmospheric channel. The atmospheric turbulence affects the wavefront of the laser, and the wavefront phase distortion is compensated by the sensor-less AO system. Finally, the lasers are received by the receiver for subsequent signal processing.

![Figure1. Free space laser communication system](image)

The sensor-less AO system is shown in Fig.2. The laser beam is split into two by the beam splitter. One beam is corrected by the deformable mirror, and the other beam is received by the CCD. The CCD detects the light intensity distribution and calculates the objective function. The imaging parameter is sent to the computer controller. The computer changes the correcting voltages of the drive electronics, so as to control the deformable mirror to compensate the distorted wavefront. After the corrected laser beam is reflected by the beam splitter, it is received by CCD. The above processes form the wavefront correction loop.

The distorted wavefront is compensated by the deformable mirror. The deformable mirror with 32-unit is used to correct the wavefront phase distortion.

The influence function of the deformable mirror is:

$$S_j(x,y) = \exp \left\{ \ln \alpha \left[ \frac{1}{d^2} \sqrt{(x-x_j)^2 + (y-y_j)^2} \right] \right\}$$

(1)

Where, \( \alpha \) is the driver cross-connect value, \( d \) is the normalized distance between the drivers, \((x_j, y_j)\) is the coordinate of the jth driver center, and \( \alpha \) is the Gaussian exponent.

The phase compensated by the deformable mirror can be expressed as:
Where, \( v_j \) is the controlling voltage of the \( j \)th driver.

3. Wavefront correcting algorithm

3.1. Basic particle swarm optimization algorithm

The various biological groups in nature have certain group behaviors. The biologist Heppner found that the birds can be synchronized, sometimes scattered, and sometimes gathered. And the birds can maintain the best distance between other birds. Kennedy and Eberhart improved the Heppner model as follows[9]: In order to get the best habitat of the entire bird group, the information exchange between birds and birds are considered. The birds can remember the best habitat they have found so far, and at the same time, they can also remember the best habitat found by the entire flock of birds. The PSO algorithm model is established based on the group behavior of birds. The basic idea of the PSO algorithm is as follows:

At time \( t \), \( X_i = (x_{i1}, x_{i2}, \ldots, x_{iN}) \) is the position of particle \( i \), \( N \) is the dimension of the particle, \( V_i = (v_{i1}, v_{i2}, \ldots, v_{iN}) \) is the velocity of the particle \( i \), \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iN}) \) is the optimal position of the particle \( i \), and the total number of particles is \( M \), \( P_g(t) \) is the optimal position of the group.

Abstracting it as a mathematical problem: there is an objective function \( f(x) \) need to be optimized, the position \( X_i \) is the solution of \( f(x) \), the velocity \( V_i \) is the increasement of \( X_i \), and the optimal position of the particle is the \( X \) corresponding to the best objective function value.

The best position of particle \( i \) can be expressed as:

\[
P_i(t+1) = \begin{cases} P_i(t) & , f(X_i(t+1)) \geq f(P_i(t)) \\ X_i(t+1) & , f(X_i(t+1)) < f(P_i(t)) \end{cases}
\]

The evolution equations [10] of PSO algorithm can be expressed as:

\[
v_i(t+1) = \omega v_i(t) + c_1 r_1(t)[p_i(t) - x_i(t)] + c_2 r_2(t)[p_g(t) - x_i(t)]
\]

\[
x_i(t+1) = x_i(t) + v_i(t+1)
\]

Where \( \omega \) is a non-negative number, which controls the influencing degree of the previous speed on the current speed, \( j \) is the \( j \)th dimension of the particle, \( c_1, c_2 \) are the accelerating factors, \( c_1 \) adjusts the step size of the particle flying to the best position of itself, and \( c_2 \) adjusts the step size of the particle flying to the global best position, \( r_1, r_2 \) are the random numbers between the interval \((0, 1)\).

For sensor-less AO system, the spot imaging quality reflects the correcting effect. The imaging quality can be expressed by Strehl ratio, which is defined as the ratio of the distorted spot peak intensity \( I_{\text{max}} \) to the ideal spot peak intensity \( I_{\text{0max}} \):

\[
\text{SR} = \frac{I_{\text{max}}}{I_{\text{0max}}}
\]

When PSO algorithm is applied to the wavefront correction, the parameters correspondence is shown in Table 1 below:

| Type                          | Corresponding parameters                          |
|-------------------------------|---------------------------------------------------|
| PSO algorithm position of particle | velocity of particle best position of particle objective function |
| Wavefront correction correction voltage increment of correction voltage best correction voltage SR |

The wavefront correcting process is:

1. Generating a set of random initial voltages as the optimal correcting voltages; calculating the objective function values; setting the number of iterations.
(2) Updating the correcting voltages and the increment of voltages according to Eq(5) and Eq(6);
(3) Comparing the values of the objective function SR; updating the optimal correcting voltages of each particle and the entire particle group;
(4) Judging whether the number of iterations or the required objective function value meets the requirement. If yes, stop, otherwise return to step (2).

3.2. metropolis criterion
The metropolis criterion refers to the probability of the particles toward equilibrium at temperature T during solid annealing process is \(\exp(-\Delta E/kT)\). Where \(E\) is the internal energy of the particle at temperature \(T\), \(\Delta E\) is the internal energy variation, and \(k\) is the Boltzmann constant. When the metropolis criterion is applied in optimizing problems, the metropolis criterion is evolved to have the probability to accept poor solutions [4]:

\[ J \] is set as the objective function needed to be optimized. In state \(n\), when \(\Delta J_n > 0\), it indicates that the new state is inferior to the current state, then

\[ \begin{align*}
(1) & \quad \text{If } \exp(-\Delta J_n/T_n) > \text{rand}(1), \text{accept the inferior solution;} \\
(2) & \quad \text{If } \exp(-\Delta J_n/T_n) < \text{rand}(1), \text{refuse the inferior solution.}
\end{align*} \]

Where \(T\) is the controlling parameter, and \(\text{rand}(1)\) represents a random number between 0 and 1.

The variation of \(T\) is an exponential damping process:

\[ T_{n+1} = \lambda T_n \] (7)

Where \(\lambda\) is the annealing coefficient, \(0 < \lambda < 1\).

It can be seen that if \(\Delta J_n > 0\), when \(T\) is large, \(\exp(-\Delta J_n/T_n) \to 1\), the probability of accepting the inferior solution is large; while when \(T\) is close to zero, \(\exp(-\Delta J_n/T_n) \to 0\), the probability of accepting the inferior solution is small. If \(\Delta J_n < 0\), \(\exp(-\Delta J_n/T_n) > 1\), the new state is better than the current state, then the new solution is accepted as the current optimal solution. Therefore, the position of the particle \(i\) can be expressed as:

\[ P_i(t+1) = \begin{cases} 
X_i, & \text{if } \exp(-\Delta J_n/T_n) \cdot \text{rand}(1) \\
(1 - \exp(-\Delta J_n/T_n)) \cdot P_i(t) & \text{if } \exp(-\Delta J_n/T_n) \cdot \text{rand}(1)
\end{cases} \] (8)

3.3. Wavefront Correction PSO Algorithm Based on Metropolis Criterion
Because the metropolis criterion has the probability to accept inferior solution, it can be applied to the wavefront correction so as to overcome the problem of local optimality. We call this algorithm as MPSO algorithm. The process of MPSO algorithm is as follows:

(1) Set the initial controlling parameter \(T_0\), the number of particles \(N\), and the number of iterations \(M\). For different particles, randomly generate a set of initial voltages as the current particle optimal correcting voltages, and compare the value of the objective function to select the global optimum correcting voltages;

(2) In an iterative process, \(T\) remains unchanged, and different particles respectively generate a new set of solutions according to formula (5)(6), then determine whether to accept the new solution according to the metropolis criterion. Thus, the optimal correcting voltages of different particles and the global optimal correcting voltages are selected;

(3) Change \(T\) according to equation (8) and repeat step (2) until the set number of iterations or the set objective function value is reached.

4. Numerical simulations
In this paper, the power spectrum inversion method [11] is used to simulate the wavefront phase distortion caused by atmospheric turbulence during laser propagation in the atmospheric channel, and the sub-harmonic resampling [12] is used to compensate the low frequency energy. Fig.3 compares the wavefront correcting effects of the PSO algorithm and the MPSO algorithm. To overcome the randomness in correcting process, Fig.3 is the average of the multiple correcting processes. The
simulating parameters are shown in Table 2.

Table 2. Simulating parameters of PSO algorithm and MPSO algorithm

| Type of algorithm | Population number | Accelerated factor $c_1$ | Accelerated factor $c_2$ | Power constant $\omega$ | Initial control parameter $T$ | Annealing temperature coefficient $\lambda$ |
|-------------------|-------------------|--------------------------|--------------------------|--------------------------|-------------------------------|----------------------------------|
| MPSO              | 100               | 2                        | 2                        | 0.8                      | 5000                          | 0.99                             |
| PSO               | 100               | 2                        | 2                        | 0.8                      | —                             | —                                |

It can be seen from Fig. 3 that compared with PSO algorithm, the wavefront correcting effect is significantly improved when using MPSO algorithm. After 1000 iterations, the SR value of MPSO is improved by 32.4% compared with PSO algorithm. When PSO algorithm has a SR value of 0.68, the running time is 28s. While when MPSO algorithm has an SR value of 0.68, it only takes 17 iterations and the running time is 0.595s. The running time is greatly reduced.

Figure 3. Comparison of wavefront correction effect between PSO algorithm and MPSO algorithm

Fig. 4 shows the correcting effects of the PSO algorithm and MPSO algorithm under different dynamic constant $\omega$. The simulating parameters are identical to those given in Table 2 except for $\omega$. It can be seen that the MPSO algorithm is not affected by $\omega$, and can achieve global optimization; however, the correcting effect of the PSO algorithm has a close relationship with $\omega$, and when it is large, there exists a local optimal phenomenon. It can also be seen from Fig. 3 that the convergence speed of MPSO algorithm is faster than that of PSO algorithm. When $\omega = 0.6$, both algorithms can achieve global optimization. The PSO algorithm needs 474 iterations to reach the global optimal, which takes 13.272s, but the MPSO algorithm only needs 249 iterations to reach the global optimal, and the time is 8.715s. From this, it can be seen that the MPSO algorithm is superior to the PSO algorithm regardless of the convergence speed or the correcting effect.

Figure 4. Comparison of PSO algorithm and MPSO algorithm under different power constants
5. Conclusions
In order to overcome the problem of local optimality, this paper improves the PSO algorithm based on metropolis criterion and applies MPSO algorithm to wavefront correction. The wavefront phase distortion of the laser beam caused by atmospheric turbulence is simulated by power spectrum inversion and low frequency compensation. The PSO algorithm and MPSO algorithm are used to control the deformable mirror to correct the wavefront distortion. Within the same number of iterations, the objective function value of the MPSO algorithm is 32.4% higher than that of the PSO algorithm, and the wavefront correcting effect is significantly improved. When the same correcting effect is achieved, the number of iterations required by the MPSO algorithm is greatly reduced, and the running time is also greatly decreased. The correcting effects of PSO algorithm and MPSO algorithm under different dynamic constants are compared. It is proved that the MPSO algorithm is superior to the PSO algorithm regardless of the convergence speed or correcting effect.

References
[1] Wu J P, Liu Q, Yu L T. Gamma-Gamma Atmospheric Turbulence Partial Coherent Optical Communication System Performance [J]. Infrared and Laser Engineering, 2017, 46 (3): 322004-322004 (7)
[2] Li Y Q, Wang L G, Wu Z S. Study on intensities, phases and orbital angular momentum of vortex beams in atmospheric turbulence using numerical simulation method[J]. Optik - International Journal for Light and Electron Optics, 2018, 158:S0030402618300329.
[3] Li Z , Cao J , Zhao X , et al. Atmospheric compensation in free space optical communication with simulated annealing algorithm[J]. Optics Communications, 2015, 338:11-21.
[4] Cai Y, Wang H M, Qi B. Simulation and analysis of wavefront-free sensor optimization algorithm for satellite-to-ground laser communication [J].Infrared and Laser Engineering, 2013, 42 (4): 1063-1068.
[5] Jiang W H, Huang S F, Wu X B. Mountain climbing adaptive optical wavefront correction system [J]. China Laser, 1988, 15 (1): 19-23.
[6] Yang H Z, Li X Y, Jiang W H. Simulation and analysis of stochastic parallel gradient descent control algorithm for adaptive optical systems [J]. Journal of Optics, 2007, 28 (8): 205-210.
[7] Liu Y, Ma J Q, He T. Simulated annealing-mountain climbing hybrid algorithm for fast aberration correction without wavefront sensor [J]. Optical Precision Engineering, 2012, 20 (02): 213-219.
[8] Chen Y. Research on Key Technologies of Adaptive Optical Simulation System [D]. Chengdu University of Electronic Science and Technology, 2013.
[9] Kennedy J, Eberhart R. Particle swarm optimization[C]// Icnn95-international Conference on Neural Networks. 2002.
[10] Eberhart R C , Shi Y . Guest editorial special issue on particle swarm optimization.[J]. Evolutionary Computation IEEE Transactions on, 2004, 8(3):201-203.
[11] Mcglamery B L . Restoration of Turbulence-Degraded Images*[J]. Journal of the Optical Society of America, 1967, 57(3):293-296.
[12] RGLane, AGlindemann, JDainty. Simulation of a Kolmogorov phase screen[J]. Waves in Random Media, 1992, 2(3):209-224.