Study of the effect of the tensor correlation on the alpha clustering in $^8$Be with the charge- and parity-projected Hartree-Fock method

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Abstract

We study the effect of the tensor correlation in the alpha clustering in $^8$Be. We take as the wave function of the alpha particle the one calculated by the charge- and parity-projected Hartree-Fock method, which was proposed by us recently. We use the wave function in a cluster-model calculation. The expectation value of the potential energy from the tensor force is almost double of that in the alpha particle. The energy surface as the function of the relative distance of two alpha particles becomes much steeper in an inner region by the inclusion of the tensor correlation in the wave function of the alpha particle. This is caused by the $p$-state mixing induced by the tensor correlation. By superposing the wave function with different relative distances, the reasonable binding energy of $^8$Be is obtained.

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I. INTRODUCTION

The understanding of nuclear structure based on the realistic nuclear force is one of the main issue of nuclear physics. The strong tensor force is a characteristic feature of the nuclear force and is known to play important roles in nuclear structure. In the clustering structure of nuclei, the tensor force is believed to be essential. The reaction matrix (G-matrix) calculation for $^8\text{Be}$ and $^{12}\text{C}$ adopting molecular orbitals as single-particle states showed that the starting energies in the G-matrix equation become smaller when an alpha-clustering structure develops and, as the result, the $G$ matrices in the triplet-even channel become more attractive. This attraction is caused mainly by the tensor force and enhances the alpha-clustering structure. The Argonne-Illinois group performed the variational Monte Carlo and Green’s function Monte Carlo calculations with the realistic nuclear force systematically in light nuclei. They found that $^8\text{Be}$ has a well-developed two-alpha cluster structure. In their result a large attraction energy comes from the one-pion-exchange potential. This result infers the importance of the tensor force in alpha clustering.

For the alpha clustering in the Be isotopes, there are many studies using various models. They shows the importance of the alpha-clustering structure in the Be isotopes. However, the tensor force is not usually treated explicitly there. The effect of the tensor force is included implicitly by renormalizing the central and LS parts of effective interactions appropriate to model spaces. Recently, there are attempts that try to treat the tensor force or the pion explicitly by expanding usual model spaces. We proposed a mean-field-type model (the charge- and parity-projected Hartree-Fock (CPPHF) method), which can treat the tensor force explicitly by mixing parities and charge states in single-particle states. We applied the CPPHF method to the alpha particle and found that the tensor correlation can be treated in our method. Because two parities are mixed in a single-particle state, a $p$-state component appears in the single-particle wave function in addition to an $s$-state component. By performing the parity projection on the total wave function consisting of the single-particle states with parity mixing, the wave function with 2-particle–2-hole correlations ($\langle 0s \rangle^2 \langle 0p \rangle^2$) and 4-particle–4-hole correlations ($\langle 0p \rangle^4$) is obtained. The $p$-state component is induced by the tensor force in the $\langle 0s \rangle^4$ configuration, which is usually assumed as a wave function of the alpha particle. The $p$-state component corresponds to the $D$-state probability and is not treated in usual model calculations explicitly. In fact, in
the \(G\)-matrix calculation in Refs. \cite{1,2}, the effect of the \(p\)-state component is included in the \(G\) matrix as the effective interaction in the model space which essentially consists of the \((0s)^4\) configuration. Therefore it is interesting to study \(^8\text{Be}\) based on the CPPHF method.

In the present paper, we make an alpha-cluster model using the wave function of the alpha particle calculated by the CPPHF method and apply it to \(^8\text{Be}\) to see the effect of the tensor force on the alpha clustering. In Section II we formulate the alpha-cluster model with the wave function of the alpha particle calculated in the CPPHF method and in Section III we apply it to \(^8\text{Be}\). In Section IV we summarize the paper.

II. FORMULATION

In the present study, the wave function of the alpha particle is calculated with the charge- and parity-projected Hartree-Fock (CPPHF) method.\cite{11} In the CPPHF method, the wave function of the alpha particle has the following form,

\[
\Psi_\alpha = P^C(Z)P^P(\pm)\Phi_\alpha.
\] (1)

Here, \(\Phi_\alpha\) is a Slater determinant composed of single-particle states with charge and parity mixing and, therefore, does not have a good parity and a definite charge number. To exploit a wave function having a good parity, positive (+) or negative (−), and a definite charge number \(Z\), the parity-projection operator \(P^P(\pm)\) and the charge-projection operator \(P^C(Z)\) are operated on \(\Phi_\alpha\). In the present study, the wave function of the alpha particle is fixed to the ground state and, then, the parity is positive and \(Z\) is equal to 2. The detail of the CPPHF method is found in Ref. \cite{11}. \(\Phi_\alpha\) can be thought as a kind of an intrinsic wave function. Assuming the spherical symmetry, the intrinsic wave function can be written as

\[
\Phi_\alpha = A \prod_{i=1}^{4} \left( \phi_{s_{1/2};\pi}(r_i)\mathcal{Y}_{0;\frac{1}{2}m_i}(\Omega_i)\zeta_{\frac{1}{2}}(i) + \phi_{s_{1/2};\nu}(r_i)\mathcal{Y}_{0;\frac{1}{2}m_i}(\Omega_i)\zeta_{\frac{1}{2}+\frac{1}{2}}(i) \right)
+ \phi_{p_{1/2};\pi}(r_i)\mathcal{Y}_{1;\frac{1}{2}m_i}(\Omega_i)\zeta_{\frac{1}{2}}(i) + \phi_{p_{1/2};\nu}(r_i)\mathcal{Y}_{1;\frac{1}{2}m_i}(\Omega_i)\zeta_{\frac{1}{2}+\frac{1}{2}}(i) \right).
\] (2)

In the above equation, \(\phi_{j;\sigma}\) is a radial wave function for the component with the total angular momentum \(j\) (\(\sigma = \pi\) for proton and \(\sigma = \nu\) for neutron), \(\mathcal{Y}_{ljm}\) is an eigenfunction of the total spin \(j = l + s\), and \(\zeta_{1/2m_t}\) is an isospin wave function for proton when \(m_t = 1/2\) or for neutron when \(m_t = -1/2\). \(\Phi_\alpha\) has \(p\)-state components, which are induced by the tensor
force. By performing the parity and charge projections on the intrinsic wave function with parity and charge mixing as in Eq. (1), the correlated wave function which have 2-particle–2-hole \( ((os)^2(0p)^2) \) and 4-particle–4-hole \( ((0p)^4) \) components is obtained.\(^8\)\(^1\)\(^1\) We should note that the widths of the \( p \)-state components are narrower compared to those of the \( s \)-state ones.\(^1\)\(^0\)\(^,\)\(^1\)\(^1\)\(^,\)\(^1\)\(^2\) It means that to gain the energy from the tensor force we need to treat high-momentum components, which are not included in a usual mean-field or a shell-model calculation.

As a wave function of \(^8\)Be, we put two alpha particles which have a finite relative distance. Actually \(^8\)Be is not bound but we treat it in the bound-state approximation by fixing the relative distance. By putting the wave functions of the alpha particle along the z axis, the wave function of \(^8\)Be becomes

\[
\Psi_{\text{Be}}(R) = N(R)A_{1-8} \left[ \Psi_\alpha(1, 2, 3, 4; R/2)\Psi_\alpha(5, 6, 7, 8; -R/2) \right].
\] (3)

Here, the integer numbers from 1 to 8 label nucleons before the antisymmetrization, \( R \) is the relative distance between the two alpha particles, and \( N(R) \) is a normalization factor. \( \Psi_\alpha(i, j, k, l; d) \) is the wave function of the alpha particle which consists of the nucleons having particle numbers \( i, j, k, \) and \( l \) and located at \( z = d \). \( \Psi_\alpha(i, j, k, l; d) \) is performed by the parity and charge projections and, therefore, have a good parity \( (+) \) and a definite charge number \( (Z = 2) \) as in Eq. (1). It is important to perform the charge and parity projections on the intrinsic wave functions located at \( z = R/2 \) and \( -R/2 \) respectively. By doing so, the wave function \( \Psi_{\text{Be}}(R) \) is going into the wave functions of two isolated alpha particles when \( R \to \infty \). \( A_{1-8} \) is the antisymmetrization operator for all 8 particles.

The wave function \( \Psi_{\text{Be}}(R) \) is not spherical symmetric but deformed axially symmetrically. Then we need to perform the angular momentum projection to obtain a wave function which has a good angular momentum. In principle we can do such a calculation, but it is too time consuming. Therefore we only subtract the expectation value of rotational energy in the rigid rotor approximation, \( \Delta E_{\text{rot}} \), as in Ref. \(^6\). \( \Delta E_{\text{ROT}} \) is defined as

\[
\Delta E_{\text{ROT}} = \frac{\hbar^2}{2I} \langle \Psi_{\text{Be}} | J^2 | \Psi_{\text{Be}} \rangle
\] (4)

with the momentum of inertia around the \( y \) axis,

\[
I = M \langle \Psi_{\text{Be}} | \sum_{i=1}^{8} (x_i - X_G)^2 + (z_i - Z_G)^2 | \Psi_{\text{Be}} \rangle.
\] (5)
Here, $M$ is the mass of nucleon, $X_G$ and $Z_G$ are the $x$ and $z$ components of the coordinate of the center of mass of all nucleons, and $J$ is the total angular momentum $\sum_{i=1}^{8}(l_i + s_i)$.

For the Hamiltonian, we take the same form as in Ref.\,[11],

$$H = -\sum_{i=1}^{8} \frac{\hbar^2}{2M} \Delta_i + \sum_{1 \leq i < j \leq 8} (V_{ij}^C + V_{ij}^T) - E_G. \quad (6)$$

Here, $E_G$ is the energy of the center of mass motion, $V^C$ is the potential energy from the central force, and $V^T$ is the potential energy from the tensor force. We use the Volkov No. 1 force \,[13] for the central part ($V_C$) and the G3RS force \,[14] for the tensor part ($V_T$). For simplicity, we omit the LS and Coulomb forces in the present study. We expect that the inclusion of these forces does not change the results so much.

The effect of the tensor force is already included in the Volkov No. 1 force, because the binding energy of the alpha particle can be reproduced in the absence of the tensor force with the Volkov No. 1 force. Because the effect of the tensor force is thought to appear as attraction in the triplet even part of the central force, we reduce the attractive part of the Volkov No. 1 force in the triplet even channel by multiplying a numerical factor $x_{TE}$.

In the present study we assume the intrinsic wave function of the alpha particle as spherical symmetric and, then, the CPPHF method can take into account only the coupling between $s_{1/2}$ and $p_{1/2}$ by the tensor force. However, the couplings of $s_{1/2}$ to $p_{3/2}$, $d_{3/2}$, and $f_{5/2}$ are found to be also important to gain the attractive energy from the tensor force in the study of the shell-model-type calculation proposed by Myo et al.\,[12] By considering this, we change the strength of the tensor force by multiplying the $\vec{\tau}_1 \cdot \vec{\tau}_2$ part of the tensor force by a numerical factor $x_T$, which is a dominant part in the tensor force.

### III. RESULT

In the Table\,[II] the results for the properties of the alpha particle in the simple Hartree-Fock (HF), the parity-projected Hartree-Fock (PPHF), and the charge- and parity-projected Hartree-Fock schemes (CPPHF) are summarized. In the PPHF scheme, only the parity projection is performed. We take two values for $x_T$, 1.00 (the normal tensor force case) and 1.50 (the strong tensor force case). For each $x_T$, the value of $x_{TE}$ is determined to reproduce the binding energy of the alpha particle. In the HF case, the single-particle wave function is fixed to the $0s$ harmonic oscillator wave function with the oscillator length $b = 1.37$ fm.
TABLE I: Results for the ground state properties of the alpha particle in the simple Hartree-Fock (HF), the parity-projected Hartree-Fock (PPHF), and the charge- and parity-projected Hartree-Fock (CPPHF) schemes. \( x_T \) and \( x_{TE} \) are the numerical factors multiplied to the \( \vec{r}_1 \cdot \vec{r}_2 \) part of the tensor force and the attractive part of the central force in the triplet-even channel respectively. \( E_{\text{TOT}}, K_{\text{TOT}}, V_{\text{TOT}}, V_C, \text{and} \ V_T \) are the expectation values for the total energy, the kinetic energy, the sum of the potential energies, the potential energy from the central force, and the potential energy from the tensor force, which are given in MeV. \( R_m \) is the matter radius in fm and \( P(D) \) is the \( D \)-state probability in %.

|          | \( x_T \) | \( x_{TE} \) | \( E_{\text{TOT}} \) | \( K_{\text{TOT}} \) | \( V_{\text{TOT}} \) | \( V_C \) | \( V_T \) | \( R_m \) | \( P(D) \) |
|----------|-----------|-----------|-----------------|-----------------|-----------------|-------|-------|-------|-------|
| HF       | 0.00      | 1.00      | -27.92          | 49.72           | -77.64          | -77.64| 0.00  | 1.45  | 0.00  |
| PPHF     | 1.00      | 0.96      | -28.26          | 52.50           | -80.76          | -75.07| -5.68 | 1.45  | 0.68  |
| PPHF     | 1.50      | 0.91      | -28.60          | 57.31           | -85.92          | -72.07| -13.84| 1.42  | 1.76  |
| CPPHF    | 1.00      | 0.92      | -28.74          | 57.80           | -86.54          | -73.23| -13.31| 1.42  | 3.22  |
| CPPHF    | 1.50      | 0.79      | -28.58          | 70.27           | -98.86          | -63.58| -35.28| 1.36  | 8.65  |

Although we set \( x_T \) to 0.00 in the simple HF calculation, the potential energy from the tensor force (\( V_T \)) becomes zero even if we use the finite value for \( x_T \), because the wave function is the simple \((0s)^4\) configuration. From the table, you can see that the charge projection is effective to gain the energy from the tensor force. By performing the charge projection in addition to the parity projection, \( V_T \) becomes almost three-time larger. \( P(D) \) is the \( D \)-state probability, which is defined as the probability of the component with the total spin \( S = 2 \). \[1\]

In Table [11] we show the results for \(^8\)Be. The relative distance \( R \) is fixed to 3 fm. The energy gains from the correlation between alpha particles are around 20 MeV for the central force and several MeV for the tensor force. The increases of the kinetic energy are around 30 MeV. The rotational energies defined in Eq. [11] are about 10 MeV. The increase of the rotational energy for the cases with the strong tensor force, is caused by the two factors. One is the increase of the expectation value of \( J^2 \) due to the mixing of \( p \) state in the wave function of the alpha particle and the other is the decrease of the momentum of inertia \( I \) due to the shrinkage of the radius of the alpha particle. Both are caused by the tensor force.

To see the relative-distance dependence of the total energy, we show the energy surfaces
of $^8\text{Be}$ as the functions of the relative distance $R$ in various schemes in Fig. 1. In the figure, the solid line is the result for the simple HF scheme without the tensor force, the dashed line for the PPHF scheme with the normal tensor force ($x_T = 1.0$), the dotted line for the PPHF scheme with the strong tensor force ($x_T = 1.5$), the dashed-and-dotted line for the CPPHF scheme with the normal tensor force ($x_T = 1.0$), and the dashed-and-double-dotted line for the CPPHF scheme with the strong tensor force ($x_T = 1.5$). The energy minima appear around $R = 3$ fm for the simple Hartree-Fock case and around $R = 3.5$ fm for the PPHF and CPPHF cases. The minimum values are between 52 $\sim$ 53 MeV except for the CPPHF scheme with the strong tensor force. For the CPPHF case with the strong tensor force the energy minimum is around 51 MeV. In comparison with the simple HF case without the tensor force, the energy surfaces become shallower by including the tensor correlation in the wave functions of the alpha particle. A significant effect of the tensor correlation appears in an inner region of the energy surfaces. Here, the energy surfaces for both the PPHF and the CPPHF cases rise sharply. In fact, the total energy becomes near 300 MeV at $R = 0.5$ fm for the CPPHF scheme with the strong tensor force. In the simple HF case, where there is no tensor correlation, such a sharp rise of the energy surface does not show up.

To check the cause of the sharp increase of the total energy, in Fig. 2 the contribution to the total energy from the kinetic energy $K_{\text{TOT}}$ and the potential energy $V_{\text{TOT}}$ are shown separately. The potential energies for all cases become smaller when the relative distance becomes smaller. This tendency continues beyond $R = 2$ fm. In contrast to the potential energy, the kinetic energy increases monotonically. This increase of the kinetic energy is

TABLE II: Results for $^8\text{Be}$ with the relative distance $R = 3$ fm. $x_T$, $x_{\text{TE}}$, $E_{\text{TOT}}$, $K_{\text{TOT}}$, $V_{\text{TOT}}$, $V_C$, $V_T$, and $R_m$ have the same meanings as in the Table I. $\Delta E_{\text{ROT}}$ is the rotational energy defined in Eq. (4) and $E_{\text{TOT}} = K_{\text{TOT}} + V_{\text{TOT}} - \Delta E_{\text{ROT}}$.

|     | $x_T$ | $x_{\text{TE}}$ | $E_{\text{TOT}}$ | $K_{\text{TOT}}$ | $V_{\text{TOT}}$ | $V_C$ | $V_T$ | $\Delta E_{\text{ROT}}$ | $R_m$ |
|-----|-------|-----------------|-----------------|-----------------|-----------------|------|------|-----------------|------|
| HF  | 0.00  | 1.00            | -52.84          | 126.65          | -172.24         | -172.24| 0.00 | 7.25            | 2.22 |
| PPHF| 1.00  | 0.96            | -52.50          | 134.48          | -179.15         | -166.66| -12.48| 7.83            | 2.21 |
| PPHF| 1.50  | 0.91            | -52.12          | 145.95          | -189.41         | -159.47| -29.94| 8.66            | 2.18 |
| CPPHF| 1.00 | 0.92            | -52.90          | 146.90          | -190.96         | -162.05| -28.91| 8.83            | 2.18 |
| CPPHF| 1.50 | 0.79            | -50.03          | 175.65          | -214.66         | -139.94| -74.72| 11.02           | 2.12 |
FIG. 1: The total energies $E_{\text{TOT}}$ of $^8\text{Be}$ as the functions of the relative distance $R$ in various schemes. The solid line is the result for the simple Hartree-Fock (HF) scheme without the tensor force, the dashed line for the parity-projected Hartree-Fock (PPHF) scheme with $x_T = 1.0$, the dotted line for the PPHF scheme with $x_T = 1.5$, the dashed-and-dotted line for the charge- and parity-projected Hartree-Fock (CPPHF) scheme with $x_T = 1.0$, and the dash-and-double-dotted line for the CPPHF scheme with $x_T = 1.5$.

more significantly for the cases with the tensor correlation than for that without the tensor correlation. These facts indicate that the sharp increase of the total energy is caused by the steep rise of the kinetic energy. In the PPHF and CPPHF case, $p$-state components are induced in the wave functions of the alpha particle by the tensor correlation as indicated in Eq. (2). This $p$-state mixing results in finite $P(D)$ value as shown in Table 4. In the simple HF case, the wave function of the alpha particle is in the simple $(0s)^4$ configuration and has no $p$-state component. Furthermore, the $p$-state component induced by the tensor force is compact in size. It implies that the $p$-state component has high-momentum component and large kinetic energy. Therefore, the sharp increase of the kinetic energy in the inner region is thought to be caused by the $p$-state mixing induced by the tensor force. The repulsion between two alpha particles in the small distance is usually thought to come
mainly from the effect of the antisymmetrization. In the simple HF case the repulsion in a small distance is mainly due to the effect of the antisymmetrization and the wave function becomes a deformed-shell configuration when $R \to 0$. Our results for the CPPHF and PPHF cases indicate the correlation by the tensor force is more important for the repulsion.

In Fig. 3 we show the energy contribution to the potential energy from the central force $V_C$ and the tensor force $V_T$ separately. Both the central potential energy and the tensor potential energy become smaller when the relative distance of the alpha particle, $R$, becomes smaller for $R \gtrsim 2$ fm. The decrease of the tensor potential energy is less significant than that of the central potential energy. When $R$ goes down less than about 1.5 fm the potential energies become larger. In fact the tensor potential energy becomes positive for very small $R$. It should be noted that for the small-relative-distance region the approximation of two-alpha clusters for the wave function of $^8$Be would not be valid. The other configurations like deformed alpha clusters and a deformed configuration of $^8$Be as a whole become important. Such configurations are not treated here. The inclusion of these configurations may change energy surface in the inner region.
FIG. 3: The potential energies from the central force $V_C$ and the tensor force $V_T$ of $^8$Be as the functions of the relative distance $R$ in various schemes. The meaning of each line is the same as in Fig. II.

TABLE III: Results of the generator coordinate method (GCM) calculation using the wave functions with the relative distances from 1 fm to 11 fm with the equal separation. $x_T$, $x_{TE}$, $E_{TOT}$, $K_{TOT}$, $V_{TOT}$, $V_C$, $V_T$, $R_m$, and $\Delta E_{ROT}$ have the same meanings as in the Table II.

|        | $x_T$ | $x_{TE}$ | $E_{TOT}$ | $K_{TOT}$ | $V_{TOT}$ | $V_C$ | $V_T$ | $\Delta E_{ROT}$ | $R_m$ |
|--------|-------|----------|-----------|-----------|-----------|-------|-------|------------------|-------|
| HF     | 0.00  | 1.00     | -55.25    | 121.65    | -169.05   | -169.05| 0.00  | 7.85             | 2.46  |
| PPHF   | 1.00  | 0.96     | -56.50    | 123.15    | -170.79   | -159.53| -11.26| 8.86             | 2.71  |
| PPHF   | 1.50  | 0.91     | -57.93    | 130.76    | -178.56   | -151.62| -26.94| 10.13            | 2.89  |
| CPPHF  | 1.00  | 0.92     | -58.54    | 131.71    | -180.00   | -154.17| -25.82| 10.25            | 2.86  |
| CPPHF  | 1.50  | 0.79     | -59.81    | 153.71    | -200.00   | -132.50| -67.50| 13.52            | 3.24  |

Finally, we apply the generator coordinate method (GCM) [16] for the relative distance $R$. The wave functions of several discrete relative distances $R_i$ are superposed as following,

$$\Psi_{^8\text{Be}}^{\text{GCM}}(R) = \sum_i c_i \Psi_{^8\text{Be}}(R_i).$$

(7)
The coefficients $c_i$ are determined by solving the Hill-Wheeler equation for the discrete $R_i$,

$$\sum_j \langle \Psi_{sBe}(R_i) | H | \Psi_{sBe}(R_j) \rangle c_j = E_{GCM} \sum_j \langle \Psi_{sBe}(R_i) | \Psi_{sBe}(R_j) \rangle c_j. \quad (8)$$

We subtract from $E_{GCM}$ the rotational energy $\Delta E_{ROT}$ in Eq. (4) to obtain $E_{TOT}$. We adopt as $R_i$ the six points from 1 fm to 11 fm with the equal separation. We show the results for the various cases in Table III. By superposing the several wave functions with the different relative distance, the reasonable binding energies are obtained for the cases with the tensor correlation. The energy gains are larger for the strong tensor force cases. It is due to the larger expectation value of $J^2$ in $\Delta E_{ROT}$ for the strong tensor force cases. The radii are also larger for the strong tensor force cases due to the shallower energy surfaces. We should note that in this work the expectation value of the rotational energy is subtracted after the variation but the angular momentum projection may change the results.

IV. SUMMARY

We formulate a cluster model with the charge- and parity-projected Hartree-Fock (CP-PHF) method to study the effect of the tensor force on the alpha clustering in nuclei. The wave function of the alpha particle is calculated by the CPPHF method and used in the cluster-model calculation. The wave function of the alpha particle has $p$-state mixing induced by the tensor correlation in addition to the simple $(0s)^4$ configuration, which are usually assumed in cluster-model calculations.

We apply the model to $^8$Be. The correlation energy from the tensor force becomes almost double of the isolated alpha particle. The kinetic energy becomes much larger for the case with the tensor correlation due to the $p$-state mixing in the wave function of the alpha particle. The energy surface becomes much steeper in a small relative distance by the inclusion of the tensor correlation. The sharp rise of the energy surfaces is mainly caused by the large increase of the kinetic energy. It is also due to the $p$-state mixing in the wave function of the alpha particle. In our result its effect is stronger than the effect from the antisymmetrization. The dependence of the potential energy from the tensor force is smaller than that from the central force. By superposing the wave function with the different relative distances, the reasonable binding energies are obtained with the cases with the tensor correlation.
In the present work we only include the $p$-state mixing in the wave function of the alpha particle. The mixing of higher angular-momentum components like $d$-state and $f$-state are probably important. The effect of the dissolution of the alpha particle in the small relative distance region is also important. It is not included in the present calculation. Solving the scattering problem using the wave function of the alpha particle with the tensor correlation is also interesting. The studies of these are under progress.

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