Dynamic Response of a Nonlinear Fabry-Perot Etalon for Various Medium Response Times

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Abstract:
In this work, the switching nonlinear dynamics of a Fabry-Perot etalon are studied. The method used to complete the solution of the differential equations for the nonlinear medium. The Debye relaxation equations solved numerically to predict the behavior of the cavity for modulated input power. The response of the cavity filled with materials of different response time is depicted. For a material with a response time equal to $\tau_R = 50$ ns, the cavity switches after about (100 ns). Notice that there is always a finite time delay before the cavity switches. The switch up time is much longer than the cavity build-up time of the corresponding linear cavity which was found to be of the order of a few round-trip times. The slowing down of the cavity response occurs when the incident intensity is approximately equal to the critical switching intensity. This effect is called critical slowing down. As a result, the response of the cavity is much slower than what could be expected from the steady state analysis. The reflected intensity and the change in round-trip phase have similar dynamic response. In this research, the matlap programs are used to study the switching dynamics of a Fabry-Perot etalon.

Key words: Dynamic response, Response time, Switching dynamics

Introduction:
A Fabry-Perot interferometer has been used extensively in spectroscopy, interferometry, and laser resonators (1). However, the Fabry-Perot in the optical regime has been limited to the observation of the cavity’s static response. Advances in the lasers and optical coatings now offer the opportunity to explore the time response of the Fabry-Perot interferometer. It is of interest to note, however, that this dynamic behavior has been previously observed in a high-Q superconducting cavity with a swept microwave source (2).

Optical bistability has been observed in a number of materials exhibiting the optical Kerr effect (e.g. sodium vapor, carbon disulfide, and nitrobenzene) (3). The response time is large (nanosecond regime) and the power requirement for switching is also high. Semiconductors such as GaAs, InSb, InAs, and CdS exhibit a strong optical nonlinearity due to excitonic effects at wavelengths near the bandgap (4). A bistable device may simply be made of a layer of the semiconductor material with two parallel partially reflecting faces acting as the mirrors of a Fabry-Perot etalon (5).

It has been established that the switch-ON and OFF in optically bistable devices will occur by superimposing an external pulse on a continuous wave holding the system close to one of the critical points of the bistability curve. The role of the external pulse in this process is to generate more carrier in order to change the refractive index and thereby change the optical path length of the beam (6). However, if the difference between the critical power and the holding power is small compared with the power change involved by the switching pulse. This characteristic is pointed out in a numerical study of absorptive bistability in a good cavity limit (where the cavity lifetime is more than the material life time). The switch-ON of intrinsic optical bistable devices with external pulse was demonstrated (7) and to get the switch-OFF time, the input intensity must be reduced below the switch-OFF intensity. Obviously, the switching of the devices in either direction ON and OFF with external pulses and keeping the input intensity constant is very desirable in many applications (8).

In this paper, we present analytical study for simulating a Fabry-Perot bistable etalon filled with a nonlinear optical material (Kerr type) such as semiconductors (InSb) illuminated with a pulse laser. Then the steady state bistable is studied for...
different cavity parameters such as finesse (F) and initial detuning (\(\phi_o\)) to predict the switching point (intensity) for each value of finesse and initial detuning (8).

**Optical Bistability Dynamic Equation**

Optical bistability can be defined as the existence of two stable output optical states for one input optical state. To understand this phenomenon more clearly, first describe the main requirements to achieve it. When a continuous wave (CW) laser beam is incident on an optical cavity either Fabry-Perot etalon or ring cavity, the beam is partially reflected, absorbed and transmitted (8).

In general the dynamic equation for a bistable etalon device is governed by whether or not it is in the good cavity limit or bad cavity limit. In the former, the dynamic process is governed by the field characteristic and cavity relaxation time (8).

An alternative method to study the dynamic or transient behavior of dispersive optical bistability (OB) systems is using the optical field equations coupled with the rate equation. The method with approximations to the optical field equation which gives a phenomenon called “overshoot switching”, the time dependence of the transmitted field is given by (9):

\[
\tau_c \frac{\partial E_T(t)}{\partial t} + [1 - R \cdot e^{i\phi(t)}]E_T(t) = T E_{in} \left( t + \frac{\tau_c}{2} \right)
\]

where \(\tau_c\) is the cavity time constant, \(\phi(t)\) is the round trip phase change, and R is the mirror reflectivity. The second equation for the nonlinear medium obeys a Debye relaxation (10):

\[
\tau_R \frac{\partial \phi}{\partial t} + \phi = \beta_o + \beta E_T^2
\]

where \(\beta_o\) contains all intensity dependent phase shifts, \(\tau_R\) is the medium carrier recombination time, \(\phi\) is the nonlinear phase shift, and \(\beta\) is the intensity dependent phase shift. Eqs.1 and 2 model the time evolution of the output field and cavity round trip phase change for time dependent inputs whose characteristic times of change are small compared to the cavity round trip time.

An alternative representation of the source term \((\beta_o + \beta E_T^2)\) for the nonlinear material is formed by an etalon with \(\tau_R \gg \tau_c\), if we neglect transverse effects, diffusion and nonlinear decay is given by Eq.3 in which the internal intensity \((I_{in})\) is related to the input intensity \((I_{in})\) by (11):

\[
I_{in} \propto \frac{I_{in}}{1 + F \sin^2(\phi + \phi_o)}
\]

where \(\phi\) is the nonlinear phase shift due to the change in cavity tuning induced by the optical nonlinearity, \(\phi_o\) represents the initial detuning, and F is the coefficient of finesse.

\[
F = \frac{4\tau}{\left|1 - R\right|}
\]

When Eq. 3 is used, the dynamical Eq.2 becomes:

\[
\tau \frac{\partial \phi}{\partial t} + \phi = \frac{I_{in}}{1 + F \sin^2(\phi + \phi_o)}
\]

Where \(I_{in}\) is scaled to be in unit of phase. We will now use Eq.5 to represent the dynamic behavior of InSb etalon device, where the equation represents efficiently the dynamic behavior of the system. To study the switching dynamics of a Fabry-Perot etalon, Eqs.1 and 2 are solved numerically to predict the behavior of the cavity for modulated input power.

**Result and Discussion:**

The method used to complete the solution of the differential Eqs.1 and 2, employs variable step size control iteration. It was found that a small mistuning of the cavity will sizably reduce the computation time.

The temporal evolution of the change in round-trip phase \((\Delta \phi = \phi - \phi_o)\) in the cavity and of the reflected intensity is shown in Fig.1a and b respectively. The cavity is initially tuned close to a minimum transmission state (Fig.1a). It is excited by a plane wave of magnitude approximately equal to the minimum state switching intensity (4.07 units). The response of the cavity filled with materials of different response time is depicted. For a material with a response time equal to \(\tau_R = 50\) ns, the cavity switches after about \((100\) ns). Materials with a slower response time take a longer time to switch.

Notice that there is always a finite time delay before the cavity switches. As seen from the graph, the switch up time is much longer than the cavity build-up time of the corresponding linear cavity which was found to be of the order of a few round-trip times. This slowing down of the cavity response occurs when the incident intensity is approximately equal to the critical switching intensity. This effect is called critical slowing down. As a result, the response of the cavity is much slower than what could be expected from the steady state analysis. The reflected intensity and the change in round-trip phase have similar dynamic response.
Figure 1. Dynamic response of a nonlinear F-P. Cavity for various medium response times. (a) Change in round-trip phase, (b) Reflected intensity. The cavity is initially tuned close to a minimum transmission state.

Figure 2 is a similar plot for a cavity which is initially tuned close to a maximum transmission state. Fig. 2a and b shows that the dynamics of the change in round-trip phase is basically the same in both cases and the same delay in switching is observed in both cases. And the critical slowing down occurs when the incident intensity is approximately equal to the critical switching intensity.

The magnitude of the slowing down increases with the medium response time increase. Figure 3 shows a plot of the switch-up time versus the medium response time. It shows that the switch-up time is directly proportional to the medium response time. The cavity is excited by step function input with the same incident intensity but with materials having different response times. The incident intensity is approximately equal to the critical switching intensity.

Figure 2. Dynamic response of a nonlinear F-P. Cavity for various medium response times. (a) Change in round-trip phase, (b) Reflected intensity. The cavity is initial tuned close to a maximum transmission state.

Figure 3. Switch-up time of a nonlinear cavity against the medium response time.

The critical slowing down effect depends on the excess of intensity above the critical switching intensity and can be reduced by increasing the incident intensity. The closer the
incident intensity is to the critical value, the longer the slowing down. Consequently, practical switching devices have to be switched with pulses of magnitude considerably larger than the critical switching intensity. Figure 4 shows the dynamic response of a nonlinear F-P. cavity for different values of the incident intensity. It can be seen that the switching time increases drastically as the incident intensity gets closer to the critical switching intensity.

Figure 4. The dependence of the critical slowing down on the difference between the incident intensity and the critical switching intensity. (a) phase change, (b) reflected intensity.

In the bistable mode of operation, the switch time ($t_s$) may be defined as the total time taken to attain a new steady state after the switching increment is applied or removed. The total time is thus the sum of any switching delay ($t_D$) and the switching rise time ($t_R$). The rise time ($t_R$) is determined by the form of the bistable characteristic and the degree of overdrive in the system.

The switching time has been measured under marginal switching (minimal over switching) for bistable characteristics ranging from critical switching (no hysteresis) to a large bistable loop. For marginal operation where the bistable system is...
operated with switching increments which drive the bistable system less than 1% beyond the switch point, the lowest value of \(t_R\) observed was \(-0.6\,\mu s\). This switch time may be reduced by overdriving the bistable system with a switching increment which goes beyond the switch point.

**Figure 6.** The variation of switching time \(t_s\) with initial detuning under marginal switching conditions. (5 percent hold off, 0.5 percent over switch).

The stand-OFF from the switch point is plotted versus the switch-ON intensity for three different hysteresis loops as shown in Fig.7. It is clear from this figure that the required switching intensity for a critical switching characteristic is less than that for a large bistable loop for a certain stand-OFF.

**Figure 7.** The dependence of the switching intensity on the stand-off from the switch *point for different hysteresis loops.

**Conclusion:**

The analytical study of optical bistability for a Fabry-Perot etalon containing a nonlinear refraction material which gives various nonlinear relation between the input and output intensity, starting from the differential equation for nonlinear medium and solution numerically of the Debye relaxation equation to dynamic switching of optical bistable loops to predict the behavior of the cavity for modulated input power, which leads to different switch ON and OFF points depending on the initial detuning, switching time on the external pulse intensity and the cavity finesse values.

**Conflicts of Interest:** None.

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الأستجابة الديناميكية لمرنان فابري – بيروت اللاخطي لمواد ذات أزمان استجابة مختلفة
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الخلاصة:
في هذا العمل، تم دراسة ديناميكية التحويل اللاخطية أو عملية الفتح والغلق لتجويف فابري بيروت، لأكمال حل المعادلات التفاضلية لوسط اللاخطي حيث تم استخدام معادلة ديباي لدراسة الظاهرة الديناميكية لعملية الفتح والغلق لتجويف المرنان. الغرض من حل معادلة ديباي لاستخلاص عددية هو للتثبيت بالتجويف لطاقة الإدخال المضمنة. يمكن تمثيل استجابة التحويل العددية زمن استخدام معادلة ديباي حيث يتمثل النتيجة بطول التحويل في مرحلة الاستجابة، أما بالنسبة للمواد التي لديها زمن استجابة بحدود 50 نانو ثانية، فإن زمن تحويل التحويل يكون بعد حوالي 100 نانو ثانية. لاحظ أن هناك فترة زمنية محددة (زمن تأخير) قبل تحويل أو فتح وغلق المرنان، حيث يكون وقت التبديل أطول بكثير من وقت تراكم التحويل لمواد المصنفة. حيث يحدث هذا التباطؤ في استجابة التحويل عندما تكون الشدة داخل التحويل مساوية تقريباً لشدة التبديل الحرجة، وهذا التأثير يسمى بالتباطؤ الحرجة. فالشدة المنعكسة والتغير في مرحلة الذهاب والأياب لهما استجابة ديناميكية مماثلة.
وفي هذا البحث، تم استخدام برامجيات الماتلاب لدراسة حركيات التحويل اللاخطية لمرنان فابري – بيروت.

الكلمات المفتاحية: الاستجابة الديناميكية وزمان الاستجابة وديناميكية التحويل.