Can topological defects be formed during preheating?

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We study the dynamics of a scalar field \( \Phi \) with the potential \( g|\Phi|^2 - \eta^2/2 \) (\( g = \) self-coupling constant, \( \eta = \) symmetry breaking scale) after inflation and make clear whether topological defects can ever be formed during preheating. In particular, we pay attention to GUT defects \( (\eta \sim 10^{13}\text{GeV} - 10^{17}\text{GeV}) \), and consider three types of fluctuations. The first one is produced due to parametric resonance, the second is due to the negative curvature of the potential, and the last is created during inflation. We search for the parameter region that nonthermal fluctuations of the scalar field produced through the parametric resonant decay of its homogeneous part do not lead to defect formation. We find that this region is rather wide, and the GUT defects are not produced after inflation. This fact shows that the positiveness of the effective mass square of the field and production of large fluctuations whose amplitude is as large as that of homogeneous mode are not enough conditions for full symmetry restoration.

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I. INTRODUCTION

The inflation models \([1,2]\) were invented to solve several cosmological problems such as the flatness problem and horizon problem. Among many inflation models it seems that the most natural and simplest model is chaotic inflation \([3]\), in which one only needs a flat potential for a scalar field called the inflaton. The quantum fluctuations of the inflaton field during inflationary epoch become the density fluctuations with scale-invariant spectrum which accounts for the structure formation of the universe. The amplitudes of the fluctuations are determined by the self-coupling constant \( \lambda \) of the inflaton field \( \phi \) (for the \( V = (\lambda/4)\phi^4 \) model), which should be \( \sim 10^{-13}\) \([4]\) to explain the anisotropies of Cosmic Microwave Background (CMB) radiation observed by the Cosmic Background Explorer (COBE) \([5]\).

In order that the radiation-dominated universe should be recovered after inflation, the reheating process is necessary for transferring the vacuum energy to relativistic particles. The inflaton field that is oscillating at the bottom of its effective potential decays into other lighter particles due to its coupling to other particles. In the old version of reheating theory \([6]\), the decay rate is estimated by using perturbation theory (Born approximation), and the reheating temperature can be estimated as \( T_{RH} \sim 10^{-1}\sqrt{\Gamma_{\text{tot}} M_p} \) \([7]\), based on the single particle decay. Here \( \Gamma_{\text{tot}} \) is the total decay rate and \( M_p \) the Planck mass.

Recent investigations revealed that the explosive decay of the inflaton field takes place in the first stage of reheating \([8,17]\) due to the effect of parametric resonance. There are three stages in the reheating process: Inflaton field decays drastically into bose particles with broad resonance. This first stage is called preheating \([8]\). After the back reactions of produced particles become significant, the resonant decay goes into the narrower band. At the last stage the scatterings and further decays of created particles occur and the thermal equilibrium is achieved, which is the completion of the reheating process.

Particle production can be described by a Mathieu-type equation or Lamé-type equation, which has an unstable solution in some regions of parameters (instability bands). If the relevant parameters stay in the instability bands long enough, the solution grows exponentially. This means that the number of particles produced becomes exponentially large so that the parametric resonant decay of the inflaton takes place very efficiently. The physical meaning of this phenomenon is considered as an induced effect, in the sense that the presence of produced particles stimulates the inflaton field to decay into those particles \([14]\). This phenomenon is thus peculiar to those particles that obey Bose-Einstein statistics, i.e., bosons. This is the reason that the parametric resonant decay into fermions cannot occur due to Pauli’s exclusion principle.

Bosons produced at the preheating stage are far from thermal equilibrium and have enormously large occupation numbers in the low-energy side of their spectrum. One of surprising results of these nonthermal fluctuations (created particles) during preheating is the symmetry restoration which may lead to the formation of topological defects \([15]\) (see also \([16]\)). Assuming that the inflaton potential is given by \( (\lambda/4)(\phi^2 - \eta^2)^2 \), the inflaton \( \phi \) decays into \( \phi \) particles through parametric resonance and produced particles result in large fluctuations \( \langle \delta \phi^2 \rangle \gg \eta^2 \), the effective potential of the inflaton is changed such as it has no negative mass, i.e., the symmetry is restored. Then, the restored symmetry is spontaneously broken again to produce topological defects. If the symmetry-breaking scale is the grand unified theory (GUT) scale, GUT defects are produced. The GUT monopoles or domain walls created after inflation destroy the advantage of introducing the inflationary universe. On the other
hand, the GUT cosmic string may be another seed for
the large scale structure formation. In Ref. [18], however, the
details of the dynamics of the inflaton field were not studied.

In this paper, we study the dynamics of both real and
complex scalar field which has a flat potential $V(\Phi) = \frac{g}{2}(|\Phi|^2 - \eta^2)^2$ and is responsible for topological defects, using numerical calculations including the effect of the resonant decay and taking into account the back reactions of fluctuations during preheating in an expanding universe. Naively, the topological defects are formed for this type of potential if there are very large fluctuations of $\Phi$ ($|\Phi| \gg |\Phi|$) which implies the full symmetry restoration. As mentioned above, in Ref. [18], it is pointed out that these fluctuations are produced by the effect of parametric resonance. Besides, it will be seen later that the fluctuations also grow efficiently by the effect of the negative curvature of the potential of the field $\Phi$.

However, there is a much more efficient mechanism for producing topological defects. It is due to the facts that the field has small fluctuations at the end of inflationary stage and that the potential of the field has two minima in the radial direction. After inflation, the phase of the field $\Phi$ is almost fixed and it is sufficient to consider only the dynamics of the radial direction, and the field has small fluctuations beyond the horizon produced at the inflationary epoch. See below). In this mechanism, the radial part of $\Phi$ oscillates and settles down to one of two minima ($\pm \eta$) of the potential, and even if there are not so large fluctuations the final values of $\Phi$ may be different in the different regions in the universe, which leads to the formation of topological defects. It is this mechanism that we mainly considered in this paper. We search for the parameter space where the topological defects are produced.

For a real scalar field, we find that the topological
defects are not formed for the GUT scale models ($\eta \simeq 10^{15} - 10^{17}$ GeV), even if the effective mass square of the field $m^2 = 0$ becomes positive. The crucial point is that nonthermal fluctuations cannot be as large as the amplitude of the coherent mode of the field during preheating. In fact they are smaller by two orders of magnitude: $(\delta \phi^2) \sim 0.01 \phi^2$. Furthermore, even though the amplitude of fluctuations becomes the same order as that of homogeneous mode due to the effect of the negative curvature of the effective potential, the field is dragged by its classical motion (homogeneous part) so that the field in the entire universe settles down to one minimum of its potential. Therefore, in the case of GUT topological defects, the field dynamics (the final value of the field $\Phi$) is determined only by the initial value of its coherent mode and the nonthermal fluctuations do not affect decisively. On the other hand, however, if we are allowed to take a breaking scale much smaller than the GUT scale, there is formation of topological defects because of the initial fluctuations due to inflation.

For a complex scalar field, fluctuations in the phase direction grow much more rapidly and the dynamics of homogeneous mode is affected in some extent. This is because the field $\Phi$ does not feel any potential in the phase direction. As a result, the amplitude of the fluctuations are indeed grows as large as that of the homogeneous mode. However, we find that the topological defects are not formed in the wide region of the parameter space for the GUT scale models. Therefore, we can conclude that the defect formation due to the parametric resonance is very unlikely, even if the effective mass square of the field $\Phi$ is positive and the amplitude of fluctuations becomes large. We also find that topological defects are formed if the breaking scale is lower than $10^{14}$ GeV such as axion models [20] because there are enough time for fluctuations to affect the dynamics of homogeneous part due to, say, the effects of the narrow resonance.

In Sec. II, we present our models and formalism which are used in this paper. We study the real scalar field for defects production and find that the GUT defects cannot be formed in Sec. III. Section IV is devoted to the study of the complex field. Finally, we make our conclusions and discussions in Sec. V.

II. MODEL AND FORMALISM

We consider the dynamics of a scalar field $\Phi$. Since this field is responsible for the formation of topological defects, one can write its Lagrangian as

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi^* - \frac{g}{2}(|\Phi|^2 - \eta^2)^2,$$

(1)

where $g$ is self-coupling constant and $\eta$ symmetry-breaking scale. Here the scalar field $\Phi$ is real or complex, and the produced topological defects are domain walls or cosmic strings, respectively. We assume that $g$ is very small, otherwise the field $\Phi$ will reach the minimum of its potential very soon during inflation and no topological defects are formed. (Here we suppose that the inflaton is another field. See below for more discussions.) The equation of motion is

$$\partial_\mu \partial^\mu \Phi + 3 H \Phi + g(|\Phi|^2 - \eta^2) \Phi = 0,$$

(2)

where $H = \dot{a}/a$ is the Hubble constant. When we study the parametric resonant decay of the complex field $\Phi$, it is more convenient to use two real fields $X$ and $Y$ which are defined by $\Phi = X + iY$ with initial conditions $X(0) = |\Phi(0)|$ and $Y(0) = 0$. We take the initial time at the end of inflation. Fluctuations of $X$ and $Y$ can be represented in terms of the annihilation and creation operators

$$\delta X(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k (a_x(k) \delta X_k(t) e^{-i\vec{k} \cdot \vec{x}}.$$
the typical energy of produced particles is estimated as

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where we use

\[ \delta X(t, x) = \frac{1}{(2\pi)^{3/2}} \int \delta^3 k (a_p(t)X_k(t)e^{-i\vec{k}\vec{x}} + a_{-1}^+(k)\delta X_k^*(t)e^{i\vec{k}\vec{x}}), \quad (3) \]

where \( a_i(k) \) and \( a_i^+(k) \) satisfy commutation relations: 

\[ [a_i(k), a_i^+(k')] = \delta(\vec{k} - \vec{k}') \] with \( i = x, y \). If we decompose these fields into a homogeneous mode and fluctuations, Eq. (3) can be rewritten as

\[ \begin{align*}
\ddot{X} + 3H\dot{X} + g(X^2 - \eta^2)X &= -3g((\delta X)^2)X - g((\delta Y)^2)X, \\
\delta\ddot{X}_k + 3H\delta\dot{X}_k + \left[ \frac{k^2}{a^2} - g\eta^2 + 3gX^2 \right] \delta X_k &= -3g((\delta X)^2)\delta X_k - g((\delta Y)^2)\delta X_k, \\
\delta\ddot{Y}_k + 3H\delta\dot{Y}_k + \left[ \frac{k^2}{a^2} - g\eta^2 + gX^2 \right] \delta Y_k &= -3g((\delta X)^2)\delta Y_k - 3g((\delta Y)^2)\delta Y_k,
\end{align*} \]

where \( \langle \cdots \rangle \) denotes average over space. The right-hand side of these equations represent back reactions due to fluctuations and we have used the mean field approximations here (e.g., \( \delta X^3 \approx 3(\langle \delta X \rangle^2)\delta X, \ldots \)). This approximation corresponds to neglecting rescattering of particles \(^\ddagger\), but we can justify it in the following way. Since the typical energy of produced particles is estimated as \( E \sim \sqrt{1/2} \Phi \), the number density is \( n \sim g(\Phi)^4/E \sim g^{1/2} \Phi \). On the other hand, the cross section of scattering is approximately given by

\[ \sigma \sim \frac{g^2}{E^2} \sim \frac{g}{|\Phi|^2}. \]

Then the scattering rate becomes

\[ \Gamma \sim n\sigma \sim g^{3/2}|\Phi|. \]

What we have to compare with this is the hubble constant \( H \sim \lambda^{1/2} \phi^2/M_p \):

\[ \frac{\Gamma}{H} \sim \frac{g^{1/2}|\Phi|}{\lambda^{1/2}\phi} \sim \frac{M_p}{\phi} \sim \frac{g(a(t))}{a(0)}, \]

where we use \( g^{1/2}|\Phi| \sim \lambda^{1/2}\phi \) (see Eq. (24) below). Therefore, \( \Gamma \ll H \) for \( g(a(t)/a(0)) \ll 1 \), which means that the rescattering effect may be neglected before the time when \( a(t)/a(0) \ll g^{-1} \).

Rescaling \( t, X \), and \( Y \) as

\[ \begin{align*}
a\tau &= \sqrt{2}\int X(0)a(0)dt, \\
x &= \frac{X(a)}{X(0)a(0)}, \\
y &= \frac{Y(a)}{X(0)a(0)},
\end{align*} \]

we obtain the rescaled dimensionless equations

\[ \begin{align*}
\ddot{\tau} - \dot{\eta}^2 a^2 X + 3(\delta X^2)X + (\delta Y^2)X &= 0, \\
\delta\ddot{X}_k + \left[ \frac{k^2}{a^2} - \eta^2 a^2 + 3a^2 \right] \delta X_k &= -3(\langle \delta X \rangle^2)\delta X_k - \langle \delta Y^2 \rangle\delta X_k, \\
\delta\ddot{Y}_k + \left[ \frac{k^2}{a^2} - \eta^2 a^2 + a^2 \right] \delta Y_k &= -3(\langle \delta X \rangle^2)\delta Y_k - 3\langle \delta Y^2 \rangle\delta Y_k,
\end{align*} \]

where we define \( \tilde{k} = k/\sqrt{g}X(0) \), \( \tilde{\eta} = \eta/X(0) \), and \( a(0) = 1 \). The prime denotes differentiation with respect to \( \tau \) and we assume that the universe is radiation dominated, which is a reasonable approximation.

If the universe is radiation dominated, the Hubble parameter evolves as

\[ H(\tau) = H(0) \left( \frac{a(0)}{a(\tau)} \right)^2. \]

Since the potential energy of the inflaton field dominates the universe during inflation, the Hubble parameter at the end of inflation is

\[ H(0) = \sqrt{\frac{2\pi}{3}} \frac{\sqrt{\lambda}}{M_p} \phi^2(0), \]

where we assume that the inflaton potential is \( V(\phi) = (\lambda/4)\phi^4 \) \((\lambda = 10^{-13})\). First let us consider the case that the field \( \Phi \) which we are concerned with is the inflaton, i.e., \( \Phi = \phi \) (or \( g = \lambda \)). From Eqs. (17) and (11)

\[ H(\tau) = \sqrt{\lambda} \phi(0)a(0) \frac{a'(\tau)}{a^2(\tau)} = H(0) \left( \frac{a(0)}{a(\tau)} \right)^2. \]

Integrating this equation and use Eq. (18), we get

\[ a(\tau) = \sqrt{\frac{2\pi}{3}} \frac{\sqrt{\lambda}}{M_p} a(0)\tau + a(0). \]

As the inflationary epoch ends at the time when \( \phi = M_p/\sqrt{3\pi} \), and \( a(0) = 1 \) is imposed, the evolution of the scale factor becomes

\[ a(\tau) = \sqrt{\frac{2\pi}{3}} \tau + 1. \]

Next we consider the case that the inflaton field is different from \( \Phi \) field. Then the Hubble parameter becomes

\[ H(\tau) = \sqrt{g}X(0)a(0) \frac{a'(\tau)}{a^2(\tau)} = H(0) \left( \frac{a(0)}{a(\tau)} \right)^2. \]

Integrating this equation, we obtain

\[ a(\tau) = \sqrt{\frac{2\pi}{3}} \frac{\sqrt{\lambda}}{M_p} a(0) \sqrt{gX(0)} \tau + 1, \]

\(^\dagger\)Not all the effect of scatterings are neglected in this approximations. Actually, forward scatterings are taken into account.
where $a(0) = 1$ is assumed again. Since the classical (homogeneous) mode $X$ slowly evolves as

$$X \simeq \left(\frac{\dot{\lambda}}{g}\right)^{1/2} \phi,$$

(24)

in the inflationary epoch, Eq. (23) becomes identical to Eq. (21) using $\phi(0) = M_p / \sqrt{3\pi}$ again. Thus we can use Eq. (23) for a general scalar field $\Phi$.

Now let us consider the initial conditions. The initial condition for the homogeneous mode is $x(0) = 1$ from its definition. The value of the field $\Phi$ is initially almost constant in the whole universe because of the inflation. However, during inflation, the field has small quantum fluctuations, which become seeds for the large scale structures of the universe. We take account of these small fluctuations in the following way. In the inflationary epoch, the scalar field fluctuates with the amplitude $H/2\pi (\sim 10^{-6}(g/\lambda)^{1/2}X(0))$. These fluctuations are stretched beyond the horizon size by inflation and become classical. Therefore, one can regard that the initial value will be slightly changed as $X(0) \rightarrow X(0) = X(0)(1 + \Delta)$, where $\Delta \sim 10^{-6} \times (g/\lambda)^{1/2}$, in each region of the universe $\frac{2t}{H}$ and $X(0)$ is considered as the mean initial value. Then the initial condition for homogeneous mode must be $x(0) = 1 + \Delta$.

Next we consider the initial values of fluctuations. Hamiltonian is a diagonalized operator for a free field, but in the presence of interactions, it is not diagonalized in terms of $a$ and $a^\dagger$. It can be diagonalized at any instant of time by means of Bogoliubov transformations, which relate annihilation and creation operators $a$ and $a^\dagger$ at $t = 0$ to time dependent annihilation and creation operators $b(t)$ and $b^\dagger(t)$:

$$b_X(t) = \alpha_X(t) a_X + \beta_X(t) a_X^\dagger,$$

$$b_X^\dagger(t) = \beta_X(t) a_X + \alpha_X^\dagger(t) a_X^\dagger,$$

(25)

(26)

and $\alpha_X(t)$ and $\beta_X(t)$ can be written as

$$\alpha_X = \frac{\epsilon^{-i\int \Omega_X dt}}{\sqrt{2\Omega_X}} (\Omega_X \delta X_k - i \delta \dot{X}_k),$$

$$\beta_X = \frac{\epsilon^{i\int \Omega_X dt}}{\sqrt{2\Omega_X}} (\Omega_X \delta X_k + i \delta \dot{X}_k),$$

(27)

(28)

where $\Omega_X^2 = k^2/a^2 - gn^2a^2 + g \tilde{X}^2$, and the initial conditions for $\alpha_X$ and $\beta_X$ are

$$|\alpha_X(0)| = 1, \quad \beta_X(0) = 0,$$

(29)

which means that there is no particles at $t = 0$. These conditions correspond to

$$|\delta X_k(0)| = \frac{1}{\sqrt{2\Omega_X(0)}}, \quad i\delta \dot{X}_k(0) = \Omega_X(0) \delta X_k(0).$$

(30)

Similarly, initial conditions for $\delta Y_k$ are

$$|\delta Y_k(0)| = \frac{1}{\sqrt{2\Omega_Y(0)}}, \quad i\delta \dot{Y}_k(0) = \Omega_Y(0) \delta Y_k(0),$$

(31)

where $\Omega_Y^2 = k^2/a^2 - g\eta^2a^2 + g \tilde{X}^2$.

Then if we rescale as Eqs. (31)-(32), we get

$$|\delta x_k(0)| = \frac{1}{\sqrt{2\Omega_X(0)}},$$

$$\delta x_k(0) = \left[h(0) - \frac{\Omega_X(0)}{\sqrt{gX(0)}}\right] \delta x_k(0),$$

$$|\delta y_k(0)| = \frac{1}{\sqrt{2\Omega_Y(0)}},$$

$$\delta y_k(0) = \left[h(0) - \frac{\Omega_Y(0)}{\sqrt{gX(0)}}\right] \delta y_k(0),$$

(32)

(33)

where $h(\tau) = a'(\tau)/a(\tau)$ (from Eqs. (21), $h(0) = \sqrt{2}/3$, and the average of the square of the fluctuations can be written as

$$\langle \delta x^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |\delta x_k|^2,$$

$$\langle \delta y^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |\delta y_k|^2.$$

(34)

(35)

In the following sections, we integrate Eqs. (14)-(10) using relation (21) with initial conditions (22) and $x(0) = 1 + \Delta$ to see whether topological defects are formed or not. We take enough resolution of momentum space to calculate Eqs. (34) and (35) in all cases which we considered.

### III. THE REAL SCALAR FIELD

#### A. The classical dynamics of the field: Only with cosmic expansion

First we consider the evolution of the scalar field without fluctuations after inflation. We expect that the dynamics of the field should be determined entirely classically. As mentioned above, however, the scalar field
fluctuates with the amplitude $H/2\pi$ in the inflationary epoch, and the fluctuation will be $|\delta \Phi/\Phi| \approx 10^{-6} \times (g/\lambda)^{1/2}$ at the end of inflation, which leads to the field in different space points might fall into the different minima of its effective potential. Therefore, we should study the dependence of the field dynamics (the final value of the field) on its initial value. If there are no dependence on those initial values whose differences are as large as $10^{-6} \times (g/\lambda)^{1/2}$, we can conclude that the evolution of the field is completely determined by classical dynamics. Otherwise, we cannot discuss its classical dynamics, to say nothing of the effects of nonthermal fluctuations produced in the preheating epoch which will be considered later.

From Eq. (14), the classical equation of motion without the effect of fluctuations is written as

$$x'' - \eta^2 a^2 x + x^3 = 0,$$

where the initial condition is $x(0) = 1 + \Delta$.

Figure 1 is an example for the time evolution of the field with $\Delta = 0$ (Here we take $x = c = \eta = 10^{15}$GeV). We see that the amplitude of the field $x$ grows larger than the initial value. This is because we rescaled the field as Eq. (14) and the amplitude grows as $\sqrt{1 + \eta^2 a^2}$. Figure 2 shows the numerical result of the critical value of $\Delta$ with $\eta = 10^{12}$ GeV for $g = \lambda = 10^{-13}$. If $\Delta > \Delta_{\text{crit}}$, the initial fluctuations do affect the dynamics of the field. The final value of the field, which results in the formation of topological defects. On the other hand, if $\Delta \lesssim \Delta_{\text{crit}}$, the fluctuation of the field never affects the final value of the field. Therefore, the GUT scale topological defects are not produced after inflation if we neglect the fluctuations of $x$.

Let us consider the evolution of the scalar field $\Phi$ for $g > \lambda$ (i.e., $\Phi$ is not an inflaton). As $g$ grows larger, $\Delta_{\text{inf}}$ grows as $\Delta_{\text{inf}} \propto g^{1/2}$. On the other hand, the potential of the field can be rewritten as $V = \frac{1}{4}[(g^{1/2}\Phi)^2 - (g^{1/2}\eta)^2]^2$. Since we have relation, $g^{1/2}\Phi(0) \approx \lambda^{1/2}\phi(0)$ (cf. eq. (14)), the evolution of the field is equivalent to that for $g = 10^{-13}$ when $\eta$ is larger by the factor $(g/\lambda)^{1/2}$. Therefore, from our calculations (Fig. 3), we find that no topological defect is formed for $\eta \gtrsim 10^{13}$GeV, which is independent of $g$.

We can also obtain this result analytically by comparing the initial fluctuation $|\delta \Phi/\Phi|(\sim 10^{-6} \times (g/\lambda)^{1/2})$ with the change of the amplitude $\Delta A$ of oscillating $\Phi$ due to cosmic expansion in one oscillation time $T$. Since $\Phi \approx -\frac{3}{2}H\Phi$ at the critical epoch (when $A \sim \eta$), $\Delta A$ is given by

$$\frac{\Delta A}{A} \sim HT \sim \frac{\lambda^{1/2}\phi^2}{M_p} (g^{1/2}\eta)^{-1} \sim \frac{\eta}{M_p} \left( \frac{g}{\lambda} \right)^{1/2} \equiv \Delta_{\text{crit}},$$

where we use eq. (24). If $\Delta A/A$ is larger than $|\delta \Phi/\Phi|$, the initial fluctuations do not affect the dynamics of the classical evolution of the field $\Phi$. Since $\Delta_{\text{inf}} \equiv |\delta \Phi/\Phi| \sim 10^{-6} \times (g/\lambda)^{1/2}$, we can obtain the condition that the field $\Phi$ settles down to the definite minimum of the potential in the entire universe:

$$\frac{\eta}{M_p} \gtrsim 10^{-6}.$$

This is the same as what we obtained from our numerical calculations. Notice that this condition is, of course, independent of the coupling constant $g$.

B. Preheating stage: Effect of nonthermal fluctuations

In order to study the production of nonthermal fluctuations and find whether the formation of topological defects occurs or not, we must investigate how fluctuations can be created. In fact, they are produced due to both parametric resonance during preheating and the negative curvature of the effective potential. To investigate how large the non-thermal fluctuations grow, we consider these two effects separately. First, we study the parametric resonance in this subsection. The effect of the negative curvature of the potential will be considered in the next subsection.

We integrate those equations which the breaking terms are omitted from Eqs. (14) and (15), i.e., $\eta = 0$. For the early stage (during preheating), neglecting the breaking terms is quite a reasonable approximation, since the breaking scale is much smaller than the initial amplitude of the field $\Phi$. The result is shown in Fig. 3. The solid line denotes the envelope of $x^2$ and the dotted line $\delta x^2$: fluctuations of $x$, and these lines are extrapolated to larger $a$. Note that the decaying power of homogeneous mode is $\sim -0.26$, which is smaller than the value $-1/3$ estimated in the Ref. [18]. This is because the actual dynamics is much more complicated since $\langle \delta x^2 \rangle$ is also oscillating for example. (Our value $-0.26$ is much smaller than the value $-2/3$ in Ref. [2].) In this reference, however, the calculation has been done for only a short time such as

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††The value $-1/3$ can be estimated with assumptions that $\langle \delta x^2 \rangle$ grows large and dominates the effective mass square of the field and becomes almost constant. Using this naive estimation, we get the same decaying power for the both cases that the evolution of homogeneous mode is written in terms of the elliptic function and the trigonometric function. However, the dynamics of fluctuation obeys Lamé equation, not Mathieu equation, so that the instability band is narrower and the growth rate is much smaller in the case of Lamé equation. Therefore, the decaying power is smaller.
\( \tau \sim 800 \), so that the decaying power in the narrow resonance stage cannot be seen from such a short time evolution.) The time that the field falls into one minimum of its potential is estimated as \( a_r \sim (\eta/X(0))^{-1} \). For the GUT scale \( (\eta \sim 10^{16}\text{GeV}) \), \( a_r \sim 400 \). At this time the amplitude of fluctuations is much less than that of homogeneous mode. Therefore, the fluctuations is too small to produce the GUT defects. On the other hand, we can see that the crossing takes place at the time \( a_c \sim 5 \times 10^5 \).

For \( a > a_c \), the dynamics of the field \( \Phi \) may be fully affected by its fluctuations since the field fluctuates more than \( \mathcal{O}(1) \), which means the topological defects may be developed in the same sense as pointed out in Ref. [13]. Therefore, IF \( \eta \lesssim 10^{13}\text{GeV} \), it is expected that the full symmetry restoration occurs and leads to the formation of topological defects due to the later spontaneous symmetry breaking (if there is any breaking term in the equation).

### C. Postpreheating: Effects of the negative curvature of the potential

Now let us take into account the effects of the negative curvature of the potential. To this end, we integrate equations

\[
\begin{align*}
x'' - \bar{\eta}^2 a^2 x + x^3 + 3\langle \delta x^2 \rangle x &= 0, \quad (39) \\
\delta x''_k + \tilde{k}^2 - \bar{\eta}^2 a^2 + 3x^2 + 3\langle \delta x^2 \rangle \delta x_k &= 0, \quad (40)
\end{align*}
\]

and see how large the amplitude of the fluctuations grows comparing with that of the homogeneous mode. The time evolution of the homogeneous mode and the fluctuations are shown in Figs. 6 and 7, respectively. From these figures we see that the preheating stage ends at \( a \sim 250 \). What we must pay attention to is the fact that the amplitude of nonthermal fluctuations during preheating is two orders smaller than that of homogeneous mode, which is contrary to the estimation of Ref. [13] (It can also be seen from Figs. 3 and 4 and is consistent with Ref. [22]). Moreover, the amplitude grows due to the global instability of negative mass (breaking term), but the maximum amplitude is at most a few factor less than that of the homogeneous mode (at \( a \sim 3000 \)). This is the crucial point. We can say in other words as follows: although the effective mass square of the field becomes positive during preheating (see Fig. 3), the amplitude of the fluctuations is smaller than that of homogeneous one so that no full symmetry restoration takes place, which results in no development of GUT topological defects (domain walls). Notice that the typical momentum of the fluctuations which contributes to fluctuations \( \langle \delta x^2 \rangle \) is larger than \( H \sim \sqrt{\lambda_0} \sim \sqrt{\mathcal{M}} \Phi \) (see Fig. 3). This means that it is a good approximation to take the conformal vacuum for initial value of fluctuations.

### D. Dynamics of the field with the initial fluctuations

As we mentioned in Sec. II, there is much more efficient mechanism for producing topological defects. It is due to the facts that the field has small fluctuations at the end of inflationary stage and that the potential of the field has two minima in the radial direction. In this mechanism, the radial part of \( \Phi \) oscillates and settles down to one of two minima (\( \pm \eta \)) of the potential, and even if there are not so large fluctuations the final values of \( \Phi \) may be different in the different regions in the universe, which leads to the formation of topological defects. In order to study this effect is the main investigation of this paper, and we show it in this subsection.

We consider the dynamics of the field taking into account of the initial fluctuations created during inflation in the following way: As we discussed in Sec. II, we numerically integrate Eqs. (39) and (40) with initial conditions (42) for fluctuations and \( x(0) = 1 + \Delta \) for a homogeneous mode, where \( \Delta \) denotes the amplitude of the initial fluctuations, and see how much the dynamics of the homogeneous mode is affected by those nonthermal fluctuations produced both during preheating due to the parametric resonance and due to the negative curvature of the potential.

We can see little effects of the nonthermal fluctuations, since the difference between Figs. 3 and 4 cannot be seen. Therefore, we conclude that topological defects are not be produced during reheating after inflation. This is the same conclusion as the case without considering the nonthermal fluctuations.

### IV. THE COMPLEX SCALAR FIELD

#### A. Preheating stage: Effect of nonthermal fluctuations

Let us turn our attention to the dynamics of a complex field \( \Phi = X + iY \). As mentioned previously, there are three kind of fluctuations: the nonthermal fluctuations created due to parametric resonance, the one due to the effect of the negative curvature of the potential, and the initial fluctuations produced during inflation. In this subsection, we pay attention to the first one. To this end, omitting the breaking term, we have integrated the following rescaled equations:

\[
\begin{align*}
x'' + x^3 + 3\langle \delta x^2 \rangle x + \langle \delta y^2 \rangle x &= 0, \quad (41) \\
\delta x''_k + \tilde{k}^2 + 3x^2 + 3\langle \delta x^2 \rangle \delta x_k &= 0, \quad (42) \\
\delta y''_k + \tilde{k}^2 + x^2 + 3\langle \delta x^2 \rangle + 3\langle \delta y^2 \rangle \delta y_k &= 0. \quad (43)
\end{align*}
\]

The result is shown in Fig. 3. The solid line denotes the envelope of \( \varphi^2 = x^2 \) and the dotted line fluctuations, i.e., \( \langle \delta \varphi^2 \rangle = \langle \delta x^2 \rangle + \langle \delta y^2 \rangle \), which are extrapolated to larger \( a \). At \( a_r \sim (\eta/X(0))^{-1} \sim 400 \), there are not large enough...
fluctuations to produce the GUT defects. We see that
the crossing time is \( a_c \sim 10^5 \). This means that topologi-
cal defects may be formed for the models with breaking
scale \( \eta \lesssim 3 \times 10^{13}\text{GeV} \). Note that the amplitude of fluct-
uations is larger than that of the real scalar field. This is
because the phase fluctuation \( \delta \theta \) (Goldstone mode) ap-
proximately written as \( \delta y/x \) feels no potential so that it
can grow much faster (Figs. 14 and 15). Therefore, the crossing
time is earlier than the real scalar field case even if the
decaying power \( (z \sim -0.185) \) is smaller due to large back
reaction from the phase fluctuation.

B. Postpreheating: Effects of the negative curvature
of the potential

Now we must include the breaking term in the equa-
tions in order to see the effect of the negative curvature
of the potential on producing fluctuations. We integrate
Eqs. (14) - (18) numerically. The dynamical evolution of the
homogeneous mode with taking account of the effects of non-thermal fluctuations is shown in Fig. 10.
The field decays abruptly at \( a \sim 50 \). This time is ear-
lier than that for the case of the real scalar field.
The reason can be seen in Fig. 12 (Figs. 11 and 12 are the
evolutions of radial and phase fluctuations, respectively,
when \( \Delta = 0 \)). It is reasonable that the amplitude of \( \delta y \)
 grows much faster than that of \( \delta x \) (compare Fig. 11 with
 Fig. 12), since the fluctuation \( \delta y \) has an approximate re-
lation \( \delta \theta \approx \delta y/x \) to the phase fluctuation which can be
identified with a Goldstone mode who feels no potential,
as previously mentioned. Therefore, the energy stored
in the homogeneous mode of the field was quickly trans-
ferred into this Goldstone mode first. This is also seen in
the momentum distribution of \( \delta x \) and \( \delta y \) (Figs. 13 and
14).

We see, however, that nonthermal fluctuations are not
so much produced in the preheating epoch. At the end of
preheating, the amplitudes of nonthermal fluctuations are
\( \langle (\delta x)^2 \rangle \sim 0.01x^2 \), \( \langle (\delta y)^2 \rangle \sim 0.1x^2 \), which are smaller
than those estimated \( \sim x^2 \) in the Ref. 18. There-
fore, topological defects cannot be directly formed due
to the effects of nonthermal fluctuations produced, even
if the effective mass square of the field is positive during
preheating (see Fig. 12). Even though the amplitude of fluc-
tuations becomes as large as that of the homogeneous
mode at \( a \sim 3000 \) due to the effect of the negative curva-
ture of the potential, it is still not enough condition for
the formation of topological defects (It will be verified in
the next subsection).

C. Dynamics of the field with the initial fluctuations

As mentioned many times above, there are fluctuations
produced at the inflationary epoch. Therefore, the ini-
tial value of the field \( \Phi \) is slightly different in each region
which contains many horizons, and we take the initial
condition as \( x(0) = 1 + \Delta \). In order to consider the e-
effects of nonthermal fluctuations to the dynamics of the
homogeneous mode in the presence of this initial fluctu-
tions, we integrate Eqs. (14) - (18) with initial conditions
(13) and (13) for fluctuations and \( x(0) = 1 + \Delta \) for the
homogeneous mode.

Figure 14 is the result for \( g = 10^{-13} \). The evolution
of the homogeneous mode (the final value of the field)
is affected if the breaking scale \( \eta \lesssim 10^{16}\text{GeV} \). Com-
paring with the results without including fluctuations (the
dashed line), the fluctuations considerably affect the dy-
namics of the homogeneous field (the final value of the
field), and it is different from the real scalar field case.
This is because the large phase fluctuations are produced.
There are some unstable region about \( \eta \sim 10^{16}\text{GeV} \). For
\( \eta \sim 10^{16}\text{GeV} \), the critical time when the field settles
down to one of the minimum of the potential is almost
the same time when the radial fluctuations become sig-
ificant. We think that this is the reason for the unstable
behavior of \( \Delta \) around \( \eta = 10^{16}\text{GeV} \).

For \( g > \lambda \), we can study the dynamics of \( \Phi \) by rescaling
\( \Delta_{\text{inf}} \) and \( \eta \) from the result for \( g = \lambda = 10^{-13} \). In Fig. 17
we show the region of the defects formation in the \( g - \eta \)
plane. From this figure it is seen that the topological
defects are not formed for almost all the region of the
GUT scale \( (\eta = 10^{15} - 10^{17}\text{GeV}) \).

Nonthermal fluctuations do actually affect the evolu-
tion when comparing with the results for the classical
evolution without fluctuations studied in the previous
section. Owing to the effects of nontherm fluctuations,
the region where topological defects are produced are
larger than that for the pure classical one. Note that the
GUT symmetry is not fully restored during preheating in
spite of the fact that the effective mass square of the field
becomes positive before the field settles down to the mini-
mum of its effective potential. This is because that the
amplitude of fluctuations do not highly overcome that
of the homogeneous mode, and the fluctuations cannot
disturb the whole dynamics of the field even if its am-
pitude becomes as large as that of homogeneous mode.
The authors in Ref. 18 assert that the criterion of sym-
metry restoration is the sign of the effective mass square
of the field. As we mentioned above, this condition is
not enough to judge the symmetry restoration. They
consider that the amplitude of fluctuations is as large as
that of homogeneous mode at the end of preheating and
the former always overcome the latter due to the nar-
row resonance effects in the following stage. The very
difference from their conclusion is that the fluctuations
of the filed \( \Phi \) are not created so much and hence there
needs much longer period of narrow resonance stage for
the amplitude of fluctuations to be much larger than that
of homogeneous one.

Another viewpoint is insisted by the authors of Ref.
22: the symmetry is always restored if there is a very
large energy density in the initial state enough for the
field to occupy both vacua under the dynamical evolu-
of the fluctuations produced during the reheating stage grows as large as that of the homogeneous mode. This means that the symmetry restoration does not occurred in spite of the positiveness of the effective mass square of the field Φ.

For the complex scalar field, fluctuations do affect the evolution of the field, since the phase fluctuation is produced much more rapidly than that in the radial direction since the field feel no potential in phase direction (Goldstone mode). However, there is no but a tiny region for the formation of topological defects for the GUT scale in the parameter space, even if fluctuations are much produced and the effective mass square of the field becomes positive during preheating. In other words, it is not enough condition for the symmetry restoration that the field has a positive effective mass square and fluctuates at most of order $\mathcal{O}(1)$. On the other hand, topological defects are formed in the model with much lower breaking scale ($\eta \lesssim 10^{14}$GeV) such as axion models.

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FIG. 1. Evolution of the scalar field without nonthermal fluctuations (only classical evolution) for the initial deviation \( \Delta = 0 \) and \( g = 10^{-13} \).

FIG. 2. Critical value of \( \Delta \) as a function of \( \eta \) for \( g = 10^{-13} \). The stars are from numerical calculations. The dashed line shows the analytic estimation of Eq. (33) including some numerical factors, and the dotted line denotes the amplitude of the initial fluctuations produced in the inflationary epoch. If the latter line is above the former one, the dynamics of the field is considerably affected, which results in the formation of defects.

FIG. 3. Long time evolution of the dynamics of the real scalar field with nonthermal fluctuations. The dots and stars denote the amplitude square of homogeneous part \((x^2)\) and fluctuations \((\delta x^2)\), respectively. The solid line stands for the envelope of \(x^2\) and is extrapolated to the large \(a\). Similarly, the dotted line represents the envelope of \((\delta x^2)\).

FIG. 4. Evolution of the homogeneous mode of the real scalar field with nonthermal fluctuations with initial deviation \( \Delta = 0 \). We take \( \eta = 10^{10}\)GeV and \( g = 10^{-13} \).

FIG. 5. Evolution of the fluctuation of the real scalar field with initial deviation \( \Delta = 0 \).

FIG. 6. Evolution of the effective mass square at \( \Phi = 0 \) for the real scalar field with initial deviation \( \Delta = 0 \).

FIG. 7. Power spectrum of the radial fluctuation \( \delta x_k \) for \( a \approx 1000 \), \( b \approx 2000 \), \( c \approx 3000 \) and \( d \approx 3500 \).

FIG. 8. Critical value of \( \Delta \) as a function of \( \eta \). The stars are from numerical calculations. The dashed line shows the analytic estimation without fluctuations (see Fig. 3), and the dotted line denotes the amplitude of the initial fluctuations produced in the inflationary epoch.

FIG. 9. Long time evolution of the dynamics of the complex scalar field with nonthermal fluctuations. The dots and stars denote the amplitude square of homogeneous part \((\phi^2)\) and fluctuations \((\delta\phi^2)\), respectively. The solid line stands for the envelope of \(\phi^2\) and is extrapolated to the large \(a\). Similarly, the dotted line represents the envelope of \((\delta\phi^2)\).

FIG. 10. Evolution of the homogeneous mode of the complex scalar field with nonthermal fluctuations for initial deviation \( \Delta = 0 \) and \( g = 10^{-13} \).

FIG. 11. Evolution of the radial fluctuation \( \delta x \) of the complex scalar field with initial deviation \( \Delta = 0 \).

FIG. 12. Evolution of the phase fluctuation \( \delta y \) of the complex scalar field with initial deviation \( \Delta = 0 \).

FIG. 13. Power spectrum of the radial fluctuation \( \delta x_k \) for \( a \approx 1000 \), \( b \approx 2000 \), \( c \approx 3000 \) and \( d \approx 3500 \).

FIG. 14. Power spectrum of the fluctuation \( \delta y_k \) for \( a \approx 1000 \), \( b \approx 2000 \), \( c \approx 3000 \) and \( d \approx 3500 \).

FIG. 15. Evolution of the effective mass square at \( \Phi = 0 \) for the complex scalar field with initial deviation \( \Delta = 0 \).
FIG. 16. Critical value of $\Delta$ as a function of $\eta$. The stars are from numerical calculations. The dashed line shows the analytic estimation without fluctuations (see Fig. 3), and the dotted line denotes the amplitude of the initial fluctuations produced in the inflationary epoch.

FIG. 17. Parameter space for formation/no formation of topological defects in the $g - \eta$ plane. NA denotes the region where we do not have the result, and OC is the region where the initial value of the field is less than the breaking scale: $|\Phi(0)| < \eta$. In the OC region, the field already settles down to one the minimum of the potential at the end of inflation.
