Vacuum Stability Higgs Mass Bound Revisited with Implications for Extra Dimension Theories

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Abstract

We take the standard model to be an effective theory including higher dimensional operators suppressed by scale $\Lambda$ and re-examine the higgs mass bounds from the requirements of vacuum stability. Our results show that the effects of the higher dimensional operators on the higgs mass limits are significant. As an implication of our results, we study the vacuum stability higgs mass bounds in theories with extra dimensions.
One of the important issues in particle physics is to understand the origin of the electroweak scale. In the standard model, the electroweak symmetry breaking arises from a complex fundamental higgs scalar. However, the theoretical arguments of ”triviality” [1] and ”naturalness” [2], suggest that such a simple spontaneous symmetry breaking mechanism may not be the whole story. This leads to the belief that the higgs sector of the standard model is an effective theory valid below some cut-off scale \( \Lambda \). Direct searches at LEP has put a lower bound on the higgs mass \( m_h \sim 95.5 \) GeV [3] and from a global fit to electroweak precision observables one can put an upper bound on the higgs mass, \( m_h < 250 \) GeV at 95 % C.L [4]. This assumes that the scale of new physics, \( \Lambda \), is high enough and thus new physics does not have significant effects on the electroweak precision observables. Recent studies show that this upper bound on the higgs mass may be relaxed if the scale \( \Lambda \), which suppresses the higher dimensional operators arising from new physics, is around a few TeV [5, 6, 7]. There are also theoretical arguments of triviality and vacuum stability which place bounds on the higgs mass. A lower bound on the higgs mass \( \sim 135 \) GeV is obtained by requiring the standard model vacuum to be stable to the Planck scale [8]. It was shown in Ref [9] that, in the presence of higher dimensional operators, the vacuum stability limit on the higgs mass can also be changed.

There are many proposals for beyond the standard model physics. They have to address the hierarchy problem which is the understanding of why the weak scale \( M_W \sim 100 \) GeV is so much smaller than \( M_{Pl} \approx 10^{19} \) GeV, which is believed to be the fundamental mass scale of gravity. An exciting solution to the hierarchy problem in recent years is provided by the theories with extra dimensions [10]. In such theories space time is enlarged to contain large extra compact spatial dimensions. The possibility of lowering the compactification scale \( \sim \)TeV was discussed in Ref[11]. At distances smaller than the size of these extra dimensions the gravitational force varies more rapidly than the inverse square law, so that the fundamental mass scale of gravity, \( \Lambda \), can be made much smaller than \( M_{Pl} \). The hierarchy problem is avoided if this fundamental mass scale is of the
order of the weak scale.

Below the scale $\Lambda$, the physics can be described by an effective theory where the leading terms in the lagrangian are given by the standard model. The corrections which come from the underlying theory with extra dimensions are described by higher dimension operators,

$$L^{\text{new}} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i,$$

(1)

where $d_i$ are the dimensions of $\mathcal{O}_i$, which are integers greater than 4. The operators $\mathcal{O}_i$ are $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant and contain only the standard model fields. The dimensionless parameters $c_i$, determining the strength of the contribution of operators $\mathcal{O}_i$, are expected to be of $O(1)$ or larger [12].

In this work, we first present a complete analysis of the higgs mass bound from consideration of vacuum stability using the effective lagrangian approach. We improve upon the analysis of Ref[9] by extending the range of the coefficients $c_i$ from $c_i = (-1) - 0.1$ to the range $|c_i| = 1 - 10$ as expected in theories with large extra dimensions. We also extend the scale of new physics $\Lambda$ from 20 TeV to $M_{Pl} \sim 10^{19}$ GeV. Extending the range of $c_i$ and $\Lambda$ requires addressing several issues which are fully explored in the first part of the paper. In particular, we show that for large enough positive values of $c$ the vacuum stability analysis of Ref[8, 9] has to be modified. We also show that, for $c < 0$, even close to $M_{Pl}$ there can be a significant difference between the higgs mass bound obtained in the standard model and the higgs mass bound obtained with the presence of higher dimensional operators. However, for $c > 0$ the higgs mass bound in the effective theory differs from the standard model value by only a few GeV for high $\Lambda$ even for $c = 10$.

Depending on the size and sign of the coupling of the higher dimensional operator, vacuum stability analysis gives a band instead of a single value for the lower bound on the higgs mass for fixed $\Lambda$. In general, higgs mass bound from vacuum stability complement the bounds obtained from precision electroweak observables. For instance, electroweak precision measurements can be used to obtain an upper bound on the higgs mass for a given $\Lambda$ or alternately for a given higgs mass one
can obtain a lower bound on the scale $\Lambda$. Vacuum stability analysis provide a lower bound on the higgs mass for a given $\Lambda$ and alternately for a given higgs mass one can obtain an upper bound on the scale of new physics $\Lambda$.

Next we demonstrate that higher dimensional operators that contribute to the effective potential can naturally arise in extra dimension theories and then we study the implications of vacuum stability analysis for such theories.

Analyses of the higher dimension operators in Eq. (1) have been performed by many authors in the literature[13]. The operator, up to dimension 6, relevant for deriving the lower bound on the higgs mass from vacuum stability[8] is given by

$$L^{\text{new}} = \frac{c}{\Lambda^2} (\Phi^+ \Phi - \frac{v^2}{2})^3. \quad (2)$$

In the presence of the higher dimensional operator in Eq.(2) the tree level Higgs potential can now be written as [14]

$$V_{\text{tree}} = -\frac{m^2}{2} \phi^2 + \frac{1}{4} \lambda \phi^4 + \frac{1}{8} \frac{c}{\Lambda^2} (\phi^2 - v^2)^3, \quad (3)$$

which is corrected by the one-loop term, $V_{\text{1loop}}$,

$$V_{\text{1loop}}(\mu) = \sum_i \frac{n_i}{64\pi^2} M_i^4(\phi) \left[ \log \frac{M_i^2(\phi)}{\mu^2} - C_i \right], \quad (4)$$

where

$$M_i^2(\phi) = k_i \phi^2 - k'_i. \quad (5)$$

The summation goes over the gauge bosons, the fermions and the scalars of the standard model. The values of the constants $n_i, k_i, k'_i$ and $C_i$ can be found in Refs[15, 8]. The full effective potential up to one-loop correction is

$$V = V_{\text{tree}} + V_{\text{1loop}}.$$
Note that the effect of a positive $c$ is to delay the onset of vacuum instability compared to the standard model while the effect of a negative $c$ is to accelerate the onset of vacuum instability.

To obtain a lower bound on the higgs boson mass, in the absence of higher dimensional operators, one can take the location of vacuum instability to be as large as $\Lambda$. However, in our approach, for the low energy theory to make sense\footnote{Effective theory in general will not be valid in the region close to the cutoff scale. One of the examples is the chiral lagrangian of pions where the predictions for processes such as $\pi\pi$ scattering can only be reliable for small momentum transfer relative to the cutoff $\Lambda \sim 4\pi f_\pi \sim 1$ GeV.}, we should require $\phi < \Lambda$. We take the scale of vacuum instability, $\Lambda'$, to be $0.5\Lambda$, so the corrections from operators of dimension greater than six to our result is suppressed by a factor of $\frac{\Lambda'^2}{\Lambda^2} = 0.25$.

Since we are dealing with values of the field $\phi$ larger than $v$, we need to consider a renormalization group improved potential for our analysis \cite{15, 8, 16, 17, 18, 19}. Working with the one-loop effective potential, we consider two-loop running for $\lambda$, the top Yukawa coupling ($g_Y$), gauge couplings and the higgs mass. This procedure resums all next-to-leading logarithm contributions \cite{20}. The various $\beta$ functions to two-loop order can be found in Ref \cite{21}.

After obtaining the running higgs boson mass, the physical pole mass can be calculated. The relevant equation relating the running mass to the pole mass can be found in Ref \cite{8}. The boundary conditions for the gauge couplings and the top quark Yukawa couplings are known at the electroweak scale in terms of the measured values, taking into account the connection between the running top mass and the pole top mass measured at 174 GeV.

In the presence of higher dimensional operators, with the scale of vacuum stability $\Lambda'$, the vacuum stability requirement

$$V(\Lambda') = V(v)$$

provides the boundary condition for $\lambda$ at the scale $\Lambda'$, which is given by

$$\lambda_{\text{eff}}(\Lambda') \approx -\sum_i \frac{n_i}{16\pi^2} k_i^2 (\log k_i - C_i) - \frac{1}{2} \frac{\Lambda'^2}{\Lambda^2} c + \frac{2m^2}{\Lambda^2},$$

\cite{63x707}
where

$$\lambda_{eff}(\Lambda') = \lambda(\Lambda') - \frac{3}{2} c \frac{v^2}{\Lambda^2}. \quad (8)$$

One can then run down the higgs coupling $\lambda$ to obtain the higgs mass that ensures vacuum stability to the scale $\Lambda'$. In general the effective potential increases with $\phi$ for $\phi > v$ and attains a local maximum beyond which it turns around and becomes unstable at the scale $\Lambda'$ where the depth of the potential is the same as for $\phi = v$.

For $c > 0$ the higher dimensional operator tends to stabilize the vacuum and for a given higgs mass the onset of vacuum stability can be delayed and hence the scale of vacuum stability can be raised compared to the standard model value. For large enough values of $c$ the effect of the higher dimensional operator can compensate for the tendency of the standard model higgs potential to
becomes unstable and the instability of the effective potential disappears for all values of $v \leq \phi \leq \Lambda'$. This effect can be demonstrated in a toy model of new physics where a scalar field of mass $M$ is added to the standard model \cite{22}. It was shown in Ref. \cite{22} that, for a given choice of parameters in the effective potential, there is a critical value for the scalar mass $M$, below which the vacuum instability disappears.

In such cases the boundary condition for $\lambda$ at the scale $\Lambda'$, is no longer given by Eq. (6) and one has to numerically search for the minimum higgs mass that ensures

$$V(\phi) \geq V(v)$$

for all $\phi \leq \Lambda'$. In fact if one starts from the boundary condition of Eq. (6) the vacuum becomes unstable much before $\Lambda'$ and the potential attains a second local minimum which is deeper than the minimum at $\phi = v$.

In Fig. 1 we show the renormalization group improved lower bound on the higgs mass versus $\Lambda$. For $c$ we have chosen the values $c = \pm 1$ and $c = \pm 10$ along with $c = 0$ which corresponds to the standard model. In the standard model the higgs self coupling $\lambda$ increases as we run down from the scale $\Lambda$ to lower scales and the lower bound on the higgs mass from vacuum stability increases with $\Lambda$. For $c < 0$ the higher dimensional operator tends to destabilize the vacuum and so for a given $\Lambda$ the lower bound on the higgs mass for vacuum stability is larger than the corresponding standard model value. For $c = -1$ the higgs self coupling $\lambda$ continues to increase as we run down from $\Lambda$ to lower scales and the lower bound on the higgs mass increases with $\Lambda$ till $\Lambda \sim 10^8$ TeV, beyond which it decreases slightly with increasing $\Lambda$. For $c = -10$ one obtains $\lambda(\Lambda') > 1$ from the boundary condition given in Eq. (7). The running of the higgs self coupling in this case is dominated by terms that depend on $\lambda$ and and the higgs self coupling decreases as one goes from a higher scale to a lower scale. The lower bound on the higgs mass, in this case, decreases with increasing $\Lambda$ for most values of $\Lambda$. For $c > 0$ the higher dimensional operator tends to stabilize the vacuum and so for a given $\Lambda$ the lower bound on the higgs mass for vacuum stability is smaller than the corresponding
standard model value. In this case also the lower bound on the higgs mass increases with $\Lambda$ as in the standard model.

An important observation from Fig. 1 is that the lower bound on the higgs mass in the presence of the higher dimensional operator, with $c < 0$, can be quite different from the standard model value even for large $\Lambda$ as high as $M_{Pl}$. For $c > 0$, the lower bound on the higgs mass differs from the standard model value by at most a few GeV for high $\Lambda$ though it differs significantly from the standard model value for low $\Lambda$. One can understand the behavior at large $\Lambda$ from the following consideration. For large $\Lambda$ the effect of the higher dimensional operator begins to become significant beyond some large value of $\phi = \Lambda_1$. If $m_{H1}$ is the higgs mass that ensures vacuum stability to the scale $\Lambda_1$ in the standard model then one would expect the effective potential, with the higher dimensional operator included, to be stable up to $\phi \sim \Lambda_1$ with the higgs mass $m_{H1}$. If $c > 0$ the higher dimensional operator stabilizes the vacuum and the vacuum continues to be stable beyond $\phi = \Lambda_1$ and one can obtain vacuum stability to the scale $\Lambda'$ with the higgs mass $m_{H1}$. However the study of the standard model curve in Fig. 1 shows that the lower bound on the higgs mass increases by smaller amounts as we go to higher values of $\Lambda$. Consequently the difference in the higgs mass, $m_{H1}$, that ensures vacuum stability to the scale $\Lambda'$ in the presence of the higher dimensional operator and the corresponding standard model value is small. Note that the above argument does not work for $c < 0$ because beyond $\phi \sim \Lambda_1$ the vacuum rapidly becomes unstable and one has to significantly increase the higgs mass from $m_{H1}$ to ensure that the vacuum remains stable beyond $\phi = \Lambda_1$ all the way to $\phi = \Lambda'$.

In general, theories with extra dimensions can produce higher dimensional operators that contribute to the effective higgs potential considered above. Even though the underlying theory describing the physics of extra dimensions is unknown, one can construct models where an effective lagrangian describing the coupling of gravity with the standard model fields may be written down. To demonstrate the existence of higher dimensional operators that contribute to the effective higgs
potential we consider a simple model where only gravity is allowed to live in $n$ ‘large’ extra dimensions, while the Standard Model(SM) fields are confined on a 3-D surface or brane. Gravity then becomes strong in the full $4+n$-dimensional space at a scale $M_s$ which may be far below the Planck scale, $M_{Pl} \sim 10^{19}$ GeV. The scales $M_s$ and $M_{Pl}$ are related via Gauss’s Law:

$$M_{Pl}^2 = V_n M_s^{n+2}$$

with $V_n$ being the volume of the compactified extra dimensions. For the simplest case, termed a symmetric compactification, $V_n \sim R^n$. If $M_s$ is in the TeV range then only $n \geq 2$ is allowed. In such a scenario the effects of large extra dimensions arise from the interactions involving the Kaluza-Klein(KK) excitations of the graviton from compactification. At low energies one can construct an effective theory of KK gravitons interacting with the standard model fields [23, 24].

Note that we have not included supersymmetry in our framework. Here we are essentially following the approach of Ref [10, 5] where supersymmetry is not necessary from the low energy point of view for stabilizing the hierarchy between the weak scale and the Planck scale. However supersymmetry may be crucial for the self-consistency of the underlying theory of extra dimension which could well be a superstring theory. Here we are assuming that supersymmetry is broken at some high enough scale above $M_s$ and is therefore irrelevant for our analysis.

The contribution to the effective higgs potential in the one loop approximation can be obtained by calculating the KK loop attached to external higgs legs with zero momenta. The loop diagrams are generated from the fundamental four-point KK-KK-ΦΦ (seagull) vertex. The loop diagram with six external legs would therefore give rise to a higher dimensional operator $\sim \phi^6$. The four-point KK-KK-ΦΦ (seagull) vertex is given by [23]

$$\frac{i}{4} \delta_{ij} \left( C_{\mu\nu,\rho\sigma} m_\phi^2 + C_{\mu\nu,\rho\sigma} |\lambda_{\eta} k_1^\lambda k_2^\eta \right),$$

where $k_1, k_2$ are four-momentum of the scalars, $m_\phi$ is the scalar mass, $C_{\mu\nu,\rho\sigma}$ is defined in Ref [23].
\[ C_{\mu\nu,\rho\sigma} = \frac{1}{2} \left[ \eta_{\mu\lambda} C_{\rho\sigma,\nu\eta} + \eta_{\sigma\lambda} C_{\mu\nu,\rho\eta} + \eta_{\rho\lambda} C_{\mu\nu,\sigma\eta} - \eta_{\lambda\eta} C_{\mu\nu,\rho\sigma} + (\lambda \leftrightarrow \eta) \right], \quad (12) \]

with

\[ \kappa^2 R^n = 16\pi (4\pi)^{n/2} \Gamma(n/2) M_s^{-(n+2)}. \quad (13) \]

The expression for the massive spin 2 KK propagator can be found in Ref [23, 24].

One can now calculate the contribution to the \( \phi^6 \) term in the effective potential

\[ V_{HD} = \frac{m_6^6 \kappa^6 M_s^2}{16\pi^2} \int_0^\infty \frac{B(k)}{(k^2 + m_{\tilde{n}}^2)^3} k^2 dk^2 \frac{\phi^6}{M_s^2} \]

\[ B(k) = \frac{k^{12}}{216m_{\tilde{n}}^{12}} + \frac{5k^{10}}{72m_{\tilde{n}}^{10}} + \frac{13k^8}{36m_{\tilde{n}}^8} + \frac{263k^6}{216m_{\tilde{n}}^6} + \frac{19k^4}{9m_{\tilde{n}}^4} + \frac{61k^2}{36m_{\tilde{n}}^2} + \frac{51}{54}. \quad (14) \]

where we have performed a Wick rotation and

\[ m_{\tilde{n}}^2 = \frac{4\pi^2 n^2}{R^2}. \quad (15) \]

To evaluate the above integral we need to sum over the tower of KK states. Following Ref[23] we make the replacement

\[ \sum_{\tilde{n}} \rightarrow \int_0^\infty \rho(m_{\tilde{n}}) dm_{\tilde{n}}^2 \quad (16) \]

where the KK state density as a function of \( m_{\tilde{n}} \) is given by

\[ \rho(m_{\tilde{n}}) = \frac{R^n m_{\tilde{n}}^{n-2}}{(4\pi)^{n/2} \Gamma(n/2)}. \quad (17) \]

The expression for \( V_{HD} \) above is both ultraviolet and infrared divergent. To evaluate \( V_{HD} \) we introduce the ultraviolet cut-off \( \Lambda \sim M_s \) and the infrared cut-off \( \mu \sim 1/R \). One then obtains

\[ V_{HD} = (16\pi)^3 (4\pi)^n \Gamma(n/2)^2 \left[ \int_0^1 xdx \int_{y_{\text{min}}}^1 y^{n-2} N(x, y) dy \right] \frac{m_6^6 M_s^4}{M_s^6 M_{Pl}^2 M_s^2} \phi^6 \]

\[ N(x, y) = \frac{x^3}{216y^6} + \frac{5x^2}{72y^5} + \frac{13x}{36y^4} + \frac{263}{216y^3} + \left[ -\frac{x^5}{72y^5} - \frac{2x^4}{9y^4} - \frac{35x^3}{27y^3} - \frac{97x^2}{36y^2} - \frac{167x}{72y} - \frac{19}{216} \right] / (x + y)^3. \quad (18) \]
where \( y_{\text{min}} = M_s^2 / M_{Pl}^2 \). The expression above is dominated by the lowest lying KK excitations and one ends up with an unreasonably large coefficient for \( \phi^6 \) if \( M_s \) is in the TeV range. In fact for \( n = 2 \) the leading contribution is

\[
V_{HD} = \frac{512\pi^3 m_6 M_{Pl}^6 \phi^6}{675 M_s^6 M_s^6 M_s^2}
\]

We would expect that in the underlying theory of extra dimensions there are mechanisms that suppress the contributions of the lowest lying KK excitations and a reasonable value for the coefficient for the higher dimensional operators can be obtained. We also note that there are models where the large hierarchy between the lowest KK excitation and \( M_s \) is absent\(^\text{[25]}\). In the absence of the knowledge about the underlying theory of extra dimensions we replace the coefficient of the higher dimensional operator by an unknown coefficient \( c \). However, the above analysis clearly demonstrates that higher dimensional operators that contribute to the effective higgs potential can naturally arise in theories with extra dimensions.

In theories with extra dimensions, phenomenological constraints from CP violations, FCNC and electroweak precision measurements give a bound on \( \Lambda = M_s \sim 10\text{TeV} \)\(^\text{[12, 5]}\). Astrophysical considerations lead to the conservative constraint \( \Lambda > 110\text{TeV} \) for \( n = 2 \)\(^\text{[26]}\). In fact for realistic values of the parameters the bound on \( \Lambda \) can be raised to \( \Lambda > 250 - 350\text{TeV} \). From Fig.1 we see that if corrections from higher dimensional operators are neglected, the astrophysical constraints put limits on the higgs mass \( m_h \geq 103 - 109 \text{GeV} \) for \( n = 2 \). Hence the discovery of the higgs boson at LEP II will make models of extra dimensions with \( n = 2 \) barely viable.

With the inclusion of the higher dimensional operators we obtain a range for the lower bound on the higgs mass, \( m_h \geq 52-137 \text{GeV} \) for \( c = +1 \) to \(-1\) and \( m_h \geq 30-317 \text{GeV} \) for \( c = +10 \) to \(-10\). Thus the discovery of a light Higgs with mass less than 137 GeV will rule out models which produce large negative \( c \). This will have important implications on model building for extra dimension theories and consequently on collider signatures of such theories.

Direct searches from LEP II give \( m_h > 95.5 \text{ GeV} \) at 95% C.L\(^\text{[3]}\). For a higgs mass of \( m_h = 100 \)
GeV the standard model curve in Fig. 1 implies an upper bound on the scale of new physics $\Lambda \sim 50$ TeV. However in the effective theory with $c > 0$ the upper bound on the scale of new physics, $\Lambda$, could be higher than the standard model value by more than a factor of 20-60 for $c = 1 - 10$. Hence the discovery of a light higgs with $m_h \sim 100$ GeV could still allow models of extra dimension physics with $n = 2$ to be consistent with collider and astrophysical constraints.

In summary we have re-examined the lower bound on the higgs mass from vacuum stability using an effective lagrangian with higher dimensional operators included. We have shown that corrections from higher dimensional operators can effect the lower bound significantly. We demonstrated how higher dimensional operators that contribute to the effective higgs potential can naturally arise in models of extra dimension theories. We also studied the implication of vacuum stability analysis for extra dimension theories.

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