Neutral Higgs bosons phenomenology in the compact 341 model and confrontations with the LHC results

M. Djouala*

Laboratoire de Physique Mathématique et Subatomique, Frères Mentouri University, Constantine1, Algeria
* djoualamersiem@gmail.com

N. Mebarki

Laboratoire de Physique Mathématique et Subatomique, Frères Mentouri University, Constantine1, Algeria
nmmebarki@yahoo.fr

Received Day Month Year
Revised Day Month Year

The phenomenology of the neutral scalar bosons in the compact 341 model is discussed. We show that the predictions of this model are fairly good and compatible with the experimental data by calculating the signal strength of $h_1$ for the channels $b\bar{b}$, $WW^*$, $ZZ^*$, $\tau\bar{\tau}$ and $\gamma\gamma$. Moreover, we use the branching ratios to search for the heavy neutral scalar bosons $h_2$ and $h_3$ where we take into account the theoretical constraints to precise the allowed ranges for the fundamental parameters of the scalar potential.

Keywords: Beyond the Standard Model; Higgs physics

PACS numbers:12.60.-i, 14.80.Cp

1. Introduction

The discovery of a new scalar resonance by the ATLAS and CMS at CERN with a mass around 125 GeV in the Run 1 has paved the way for new directions in high energy particle physics. Analyzing the properties of this particle is compatible with the standard model (SM) predictions, that is a strong evidence that it is the Higgs boson of the Standard Model. Currently the combined analysis based on the Run 1 (7 and 8 TeV) LHC data shows that the couplings of the SM Higgs boson with the vector bosons are found to be compatible with those expected from the SM within a $\sim 10\%$ uncertainty, whereas the coupling of the SM higgs boson to the third generation fermions is compatible within an uncertainty of $\sim 15$-$20\%$. Thus, the current status of the Higgs properties still allows to explore new interpretations of the observation coming from new physics of different underlying structures.

*Corresponding author.
The compact 341 model is among many BSM theories\cite{4,5} which is constructed from the gauge group $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$, it predicts the existence of new particles namely the exotic quarks, scalar and the gauge bosons, moreover, the physical scalar spectrum of this kind of models is composed by three quadruplets $\chi, \eta$ and $\rho$ which lead to three neutral CP-even scalars $h_{1,2,3}$ where the lightest neutral scalar boson $h_1$ plays the role of the Standard Model Higgs boson, four simply charged $h_{\pm,\mp}$ with two doubly charged scalar bosons $h^{\pm\pm}$. The compact 341 model has a very specific arrangement of the fermions into generations: for leptons, one has both right and left handed helicities arranged in the same multiplet. In order to make the model anomaly free, the second and third quarks families have to belong to the conjugate 4* fundamental representation of the $SU(4)_L$ gauge group, while the first family transforms as a quadruplet in the fundamental representation or vice versa.

The motivations to study 341 models are various as, for example, explanation of family replication\cite{8,9} electric charge quantization\cite{10,11} strong CP-problem\cite{12,13} incorporation of inflation\cite{15} and others. Moreover, the 341 models are able to deal with the current Higgs physics which has an important role on the LHC to discover new physics. In this work we will study the decay of the neutral scalar bosons in the compact 341 model and confront our results to the experimental data.

The paper is organized as follow: section \ref{sec:2} reviews the fundamental features of the compact 341 model by focusing on its particle content and its scalar potential followed by the mass spectrum. In section \ref{sec:3} we review the theoretical constraints on the scalar potential parameters which found from the tree-level vacuum stability, perturbative unitarity and from the positivity of the scalar boson masses. All couplings (Feynman rules) needed to calculate the BRs and signal strengths of the neutral scalar bosons $h_1, h_2, h_3$ are presented in section \ref{sec:4}. In section \ref{sec:5} we calculate the signal strength of $h_1$ using the various analytical expressions of the partial decays width for different channels $\tau^+\tau^-, b\bar{b}, WW^*, ZZ^*, \gamma\gamma$ and $Z\gamma$ and the branching ratios of the other heavy scalar bosons $h_i$ ($i=2,3$). We present our numerical results in section \ref{sec:6}. We draw our conclusions in section \ref{sec:7}. Finally, the expressions of the theoretical constraints are given in the appendix.

### 2. Brief review of the model

In general, one of the most important parameters to distinguish different 341 models are denoted as $\beta$ and $\gamma$ which define the electric charges of new particles through the following electric charge operator\cite{17}

$$Q = \frac{1}{2} \left( \lambda_3 - \frac{1}{\sqrt{3}} \lambda_8 - \frac{4}{\sqrt{6}} \lambda_{15} \right) + X. \quad (1)$$

Where

$$\lambda_3 = \text{diag}(1, -1, 0, 0), \quad \lambda_8 = \frac{1}{\sqrt{3}} \text{diag}(1, 1, -2, 0), \quad \lambda_{15} = \frac{1}{\sqrt{6}} \text{diag}(1, 1, 1, -3). \quad (2)$$
The particle content of this model is represented in the following table:

| Name        | 341 rep | 331 rep | components                  | F |
|-------------|---------|---------|-----------------------------|---|
| $\psi_\alpha L$ | (1,4,0) | (1,3,1)  | $+(1,1,1)$                  | 3 |
| $Q_{iL} (i=2,3)$ | (3,1,3) | (3,3,0)  | $+(3,1,3)$                  | 2 |
| $Q_{jL} (j=1,i)$ | (3,1,3) | (3,1,3)  | $+(3,1,3)$                  | 2 |
| $d_{jR} (j=1,i)$ | (3,1,3) | (3,1,3)  | $+(3,1,3)$                  | 1 |
| $J_{iR} (i=2,3)$ | (3,1,3) | (3,1,3)  | $+(3,1,3)$                  | 1 |
| $u_{jR} (j=1,i)$ | (3,1,3) | (3,1,3)  | $+(3,1,3)$                  | 1 |
| $\chi$ | (1,4,1) | (1,3,1)  | $+(1,1,0)$                  | 1 |
| $\eta$ | (1,4,0) | (1,3,1)  | $+(1,1,1)$                  | 1 |
| $\rho$ | (1,4,1) | (1,3,1)  | $+(1,1,2)$                  | 1 |

To generate masses for all particles, three quadruplets scalar fields $SU(4)_L$ are introduced as we indicated in table [1] where:

$$\chi^0 = \frac{1}{\sqrt{2}} (v_\chi + R_\chi + iI_\chi),$$

$$\eta^0 = \frac{1}{\sqrt{2}} (v_\eta + R_\eta + iI_\eta),$$

$$\rho^0 = \frac{1}{\sqrt{2}} (v_\rho + R_\rho + iI_\rho).$$

with

$$R_\chi = h_1, \quad R_\eta = \alpha h_2 + \beta h_3, \quad R_\rho = \gamma h_2 + \sigma h_3,$$

where $\alpha$, $\beta$, $\gamma$ and $\sigma$ are real parameters:

$$\alpha = \frac{-\sqrt{X^2 + (Y - \sqrt{X^2 + Y^2})^2}}{\sqrt{4(X^2 + Y^2)}}, \quad \beta = \frac{\sqrt{X^2 + (Y + \sqrt{X^2 + Y^2})^2}}{\sqrt{4(X^2 + Y^2)}},$$

$$\gamma = \frac{(Y + \sqrt{X^2 + Y^2})(\sqrt{X^2 + (Y - \sqrt{X^2 + Y^2})^2})}{X \sqrt{4(X^2 + Y^2)}},$$

$$\sigma = \frac{-(Y - \sqrt{X^2 + Y^2})(\sqrt{X^2 + (Y + \sqrt{X^2 + Y^2})^2})}{X \sqrt{4(X^2 + Y^2)}},$$

with

$$X = \lambda_5, \quad Y = \lambda_1 - \lambda_3.$$
and $I_x$, $I_\eta$ and $I_\rho$ represent the Goldstone bosons.

The most general scalar potential in the compact 341 model is given by

$$V(\eta, \rho, \chi) = \mu_\eta^2 \eta \eta + \mu_\rho^2 \rho \rho + \mu_\chi^2 \chi \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2$$

$$+ \lambda_4 (\eta^\dagger \eta) (\rho^\dagger \rho) + \lambda_5 (\eta^\dagger \eta) (\chi^\dagger \chi) + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_7 (\rho^\dagger \rho) (\eta^\dagger \rho)$$

$$+ \lambda_8 (\chi^\dagger \chi) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \chi) (\rho^\dagger \rho),$$

where $\mu_\eta, \mu_\rho, \mu_\chi$ are the mass dimension parameters and $\lambda_S$ are dimensionless real coupling constants.

In the compact 341 model, the extension of the electroweak group leads to the existence of nine scalar bosons, namely three neutral scalars $h$ bosons acquire their masses:

$$M_{h_1}^2 \approx \lambda_2 v_\rho^2 + \frac{\lambda_3 \lambda_4^2}{\lambda_5^2 - 4 \lambda_1 \lambda_3} v_\rho^2, \quad M_{h_2}^2 \approx c_1 v_\chi^2 + c_2 v_\rho^2 \approx c_1 v_\chi^2, \quad M_{h_3}^2 \approx c_3 v_\chi^2 + c_4 v_\rho^2 \approx c_3 v_\chi^2, \quad M_{h_0}^2 = \frac{\lambda_7}{2} (v_\eta^2 + v_\rho^2), \quad M_{h_1}^2 = \frac{\lambda_8}{2} (v_\eta^2 + v_\rho^2),$$

$$M_{h_\pm}^2 = \frac{\lambda_9}{2} (v_\rho^2 + v_\chi^2), \quad (12) \quad (13) \quad (14)$$

where

$$c_1 = \frac{1}{2} \left( \lambda_1 + \lambda_3 - \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_3^2} \right) v_\chi, \quad (15)$$

$$c_3 = \frac{1}{2} \left( \lambda_1 + \lambda_3 + \sqrt{(\lambda_1 - \lambda_3)^2 + \lambda_3^2} \right) v_\chi. \quad (16)$$

This model contains 15 electroweak gauge bosons, four of them are identified as the SM gauge bosons and the remaining ones are heavy where some of them are electrically charged and the others are neutral: $K^0, K^{\pm}, K_1^\pm, X^\pm, V^{\pm}$ and $Y^+, Z'$ and $Z''$, their masses are found from the following Lagrangian:

$$\mathcal{L} = (D_\mu \eta) \dagger (D^\mu \eta) + (D_\mu \rho) \dagger (D^\mu \rho) + (D_\mu \chi) \dagger (D^\mu \chi), \quad (17)$$

where

$$D_\mu = \partial_\mu + ig W_\mu^{a} \frac{\lambda_6}{2} + i X g_X W_\mu^X, \quad \text{with} \quad a = 1...15. \quad (18)$$

And by using the following combination:

$$W^\pm = \left( \frac{W_\mu^1 \pm i W_\mu^2}{\sqrt{2}} \right), \quad K^0, K^{\pm} = \left( \frac{W_\mu^4 \mp i W_\mu^5}{\sqrt{2}} \right), \quad K_1^\pm = \left( \frac{W_\mu^6 \mp i W_\mu^7}{\sqrt{2}} \right), \quad (19)$$

$$X^\pm = \left( \frac{W_\mu^9 \mp i W_\mu^{10}}{\sqrt{2}} \right), \quad V^{\pm} = \left( \frac{W_\mu^{11} \mp i W_\mu^{12}}{\sqrt{2}} \right), \quad Y^\pm = \left( \frac{W_\mu^{13} \mp i W_\mu^{14}}{\sqrt{2}} \right). \quad (20)$$

\footnote{This scalar potential is invariant under an additional discrete symmetry $Z_2$.}
Neutral Higgs bosons Phenomenology in the compact 341 model and confrontations with the LHC results

We get:

\[ M_{W^\pm}^2 = \frac{g^2}{4} v_\rho^2, \quad M_{K^0, K^0}^2 = \frac{g^2}{4} v_\eta^2, \quad M_{V^\pm}^2 = \frac{g^2}{4} v_\omega^2, \quad M_{X^\pm}^2 = \frac{g^2}{4} v_\chi^2, \quad (21) \]

\[ M_{\bar{\nu}}^{2, \pm} = \frac{g^2}{4} v_\chi^2, \quad M_{\bar{\nu}}^{2, 0} = \frac{g^2}{4} (v_\eta^2 + v_\chi^2), \quad (22) \]

\[ M_{Z^\pm}^2 = \frac{g^2}{4} c_W^2 v_\eta^2, \quad M_{Z^\pm}^{2, \prime} = \frac{g^2}{4} (1 - 4 s_W^2)^2 + b_W^2 \]

\[ M_{Z^\pm}^{2, \prime} = \frac{g^2}{4} v_\eta^2 \left(1 - 4 s_W^2\right)^2 + b_W^2 \]

\[ \lambda_1 + \lambda_3 - \sqrt{\left(\lambda_1 - \lambda_3\right)^2 + \lambda_5^2} > 0, \quad \lambda_1 + \lambda_3 + \sqrt{\left(\lambda_1 - \lambda_3\right)^2 + \lambda_5^2} > 0, \quad (25) \]

3. Constraints

The theoretical constraints on the scalar parameters in the compact 341 model are:

- to ensure the perturbative unitarity, in order to keep the scalar potential bounded from below in the large fields limit for all possible directions in the field space (the vacuum stability) constraints and to provide the vacuum configuration \( \langle \rho \rangle_0, \langle \chi \rangle_0 \) and \( \langle \eta \rangle_0 \) to be a minimum of the scalar potential one needs to maintain the conditions which are given in the appendix A.
- To maintain the perturbativity of the scalar potential, all the scalar quartic couplings \( \lambda_{1...9} \) have to satisfy:

\[ |\lambda_i| \leq 4\pi \quad (24) \]

- The positivity of the scalar bosons masses requires the following conditions:

\[ \lambda_1 + \lambda_3 - \sqrt{\left(\lambda_1 - \lambda_3\right)^2 + \lambda_5^2} > 0, \quad \lambda_1 + \lambda_3 + \sqrt{\left(\lambda_1 - \lambda_3\right)^2 + \lambda_5^2} > 0, \]

\[ \lambda_2 + \frac{\lambda_3 \lambda_5^2 + \lambda_6 (\lambda_1 \lambda_5 - \lambda_4 \lambda_5)}{\lambda_5^2 - 4 \lambda_1 \lambda_3} > 0, \quad \lambda_7 > 0, \quad \lambda_8 > 0, \quad \lambda_9 > 0. \quad (25) \]

- A new constraint over the parameters of the scalar potential is imposing by identifying the lightest scalar \( h_1 \) as the Standard Model Higgs boson, then we get:

\[ \lambda_2 + \frac{\lambda_3 \lambda_5^2 + \lambda_6 (\lambda_1 \lambda_5 - \lambda_4 \lambda_5)}{\lambda_5^2 - 4 \lambda_1 \lambda_3} = \frac{m_{h_1}^2}{v_\rho^2}, \quad (26) \]

where we take \( v_\rho^2 = 246 \) GeV.

- Moreover, the existence of a Landau pole in our model \( \Lambda \) at a scale \( \mu \) all the VEVs and the particle masses \( m_i \) are constrained by:

\[ m_i \text{ and } \text{VEVs} \leq \Lambda \quad (27) \]
4. Feynman rules

From the scalar potential \[^{[11]}\], we can derive all Higgs couplings \(h_j s s\) where \(h_j\) (j=1,2,3) and \(s=h^{1 \pm}, h^{2 \pm}, h^{3 \pm}\) and based on the following Yukawa Lagrangian:

\[
\mathcal{L}_Y = \lambda_{ij}^L \bar{Q}_{iL} \chi J_{1R} + \lambda_{ij}^L \bar{Q}_{iL} \chi^* J_{iR} + \lambda_{ij}^d \bar{U}_{iL} \rho d_{aR} + \lambda_{ij}^u \bar{Q}_{iL} \rho^* u_{aR} + \lambda_{ij}^\nu \bar{U}_{iL} \eta_{1R} + \lambda_{ij}^\nu \bar{U}_{iL} \eta^* D_{J_{iR}} + \frac{\lambda_{ij}^\nu}{\Lambda_2} \rho_{mn} \left( \bar{Q}_{iLm} \rho_m \chi_n \eta_{nR} \right) u_{aR} + h.c.,
\]

(28)

the couplings of the Higgs bosons \(h_j\) with fermions \(h_j ff\) can be determined. Furthermore, the couplings of Higgs bosons \(h_j\) and gauge bosons are contained in the covariant kinetic terms of the Higgs bosons \[^{[17]}\]. Whereas, The triple couplings of three gauge bosons arise from the covariant kinetic Lagrangian of the non-Abelian gauge bosons (Yang Mills Lagrangian):

\[
\mathcal{L} = -\frac{1}{4} W_{\mu\nu} a W^{a\mu\nu},
\]

(29)

with

\[
W^{a\mu\nu} = \partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} + g f^{abc} W^{b\mu} W^{c\nu},
\]

(30)

where \(a, b, c\) go from 1 to 15 and \(f^{abc}\) are the structure constants of the SU(4) group.

The non vanishing Feynman rules are listed in the following tables:\[^{[b]}\]

| Interactions | Couplings | 
|--------------|-----------|
| \(\bar{u}_i u_i h_1\) | \(\frac{A u}{\nu_\rho}\) with \(u_i \equiv u, c, t\) | 
| \(\overline{d}_i d_i h_1\) | \(\frac{A d}{\nu_\rho}\) with \(d_i \equiv d, s, b\) | 
| \(\overline{H} h_1\) | \(\frac{A h}{\nu_\rho}\) | 
| \(W^+ W^- h_1\) | \(\frac{A E}{\nu_\rho}\) | 
| \(K_1^+ K_1^+ h_1\) | \(\frac{A E}{\nu_\rho}\) | 
| \(V^{++} V^{--} h_1\) | \(\frac{A E}{\nu_\rho}\) | 
| \(h^{++} h^{--} h_1\) | \(2\lambda_2 v^\rho\) | 
| \(h^1_1 h^1_1 h_1\) | \(2\lambda_2 v^\rho\) | 
| \(h^1_1 h^2_2 h_1\) | \(\frac{A}{\nu_\rho} (\lambda_0 + \lambda_4)\) | 
| \(Z Z h_1\) | \(\frac{g^2 v^\rho}{4} \left( (C^{21})^2 - \frac{2}{\sqrt{3}} C^{21} C^{22} - \frac{3}{\sqrt{6}} C^{21} C^{23} + \frac{(C^{22})^2}{\sqrt{3}} + \frac{2}{\sqrt{21}} C^{21} C^{22} C^{23} + \frac{1}{6} (C^{22})^2 + \frac{4\sqrt{2}}{3} (C^{24})^2 - 4t C^{24} C^{21} + 4t C^{24} C^{22} + \frac{4t}{\sqrt{6}} C^{24} C^{23} \right)\) | 

\(^{[b]}\)In those tables, we mention only couplings that contribute in our discussion.
Neutral Higgs bosons Phenomenology in the compact $SU(4)$ model and confrontations with the LHC results

Table 3. Higgs $h_2$ interactions.

| Interactions | Couplings |
|--------------|-----------|
| $\eta h_2$  | $\frac{M^2}{v_\chi}(\gamma + \frac{v_\chi}{v_\eta} \alpha)$ with $q \equiv u, s, b$ |
| $\eta h_2$  | $\frac{M^2}{v_\chi} \gamma$ |
| $\eta h_2$  | $\frac{M^2}{v_\chi} \alpha$ |
| $\eta h_2$  | $\frac{M^2}{v_\chi} \eta$ |
| $X^+ X^- h_2$ | $\frac{\sqrt{2}}{2} v_\chi \gamma$ |
| $Y^+ Y^- h_2$ | $\frac{\sqrt{2}}{2} v_\chi \gamma$ |
| $K^{00} K^{00} h_2$ | $\frac{\sqrt{2}}{2} v_\eta \alpha$ |
| $K^- K^+ h_2$ | $\frac{\sqrt{2}}{2} v_\eta \alpha$ |
| $Y^+ Y^- h_2$ | $\frac{\sqrt{2}}{2} (v_\chi \gamma + v_\eta \beta)$ |
| $h_2^+ h_2^- h_2$ | $(\lambda_4 + \lambda_7) v_\eta \alpha + \lambda_6 v_\chi \gamma$ |
| $Z Z h_2$ | $\frac{v_\chi^2}{4} v_\chi \left( \frac{2}{3} (C^{23})^2 + \frac{4}{9} (C^{24})^2 + \frac{1}{18} C^{23} C^{24} \right) + \frac{v_\eta^2}{4} v_\eta \left( \frac{4}{9} (C^{22})^2 + \frac{2}{9} C^{23} C^{24} - \frac{1}{9} C^{22} C^{23} \right)$ |
| $Z' Z' h_2$ | $\lambda_4 v_\eta \alpha + (\lambda_6 + \lambda_9) v_\chi \gamma$ |
| $h_1^+ h_1^- h_2$ | $\frac{\lambda_4}{2} v_\chi \gamma + \frac{\lambda_6}{2} v_\eta \alpha$ |

Table 4. Higgs $h_3$ interactions.

| Interactions | Couplings |
|--------------|-----------|
| $\eta h_3$  | $\frac{M^2}{v_\chi}(\sigma + \frac{v_\chi}{v_\eta} \beta)$ with $q \equiv u, s, b$ |
| $\eta h_3$  | $\frac{M^2}{v_\chi} \sigma$ |
| $\eta h_3$  | $\frac{M^2}{v_\chi} \beta$ |
| $\eta h_3$  | $\frac{M^2}{v_\chi} \eta$ |
| $X^+ X^- h_3$ | $\frac{v_\chi^2}{4} v_\chi \sigma$ |
| $Y^+ Y^- h_3$ | $\frac{v_\chi^2}{4} v_\chi \beta$ |
| $K^{00} K^{00} h_3$ | $\frac{v_\eta^2}{4} v_\eta \beta$ |
| $K^+ K^- h_3$ | $\frac{v_\eta^2}{4} v_\eta \beta$ |
| $Y^+ Y^- h_3$ | $\frac{v_\chi^2}{4} v_\chi \sigma$ |
| $h_2^+ h_2^- h_3$ | $v_\eta \beta (\lambda_4 + \lambda_7) + \lambda_6 v_\chi \gamma$ |
| $h_2^+ h_2^- h_2$ | $\frac{v_\chi^2}{4} (2\lambda_3 + \lambda_5 + \lambda_8) + \frac{\lambda_6 \lambda_8}{2} \beta + \frac{v_\chi^2}{2} (\lambda_5 + \lambda_8 + 2\lambda_1) \frac{v_\chi^2}{4} (2\lambda_3 + \lambda_5 + \lambda_8 + 2\lambda_1) + \frac{v_\eta^2}{4} \eta$ |
| $Z Z h_3$ | $\frac{v_\chi^2}{4} v_\chi \left( \frac{2}{3} (C^{23})^2 + \frac{4}{9} (C^{24})^2 + \frac{1}{18} C^{23} C^{24} \right) + \frac{v_\eta^2}{4} v_\eta \left( \frac{4}{9} (C^{22})^2 + \frac{2}{9} C^{23} C^{24} - \frac{1}{9} C^{22} C^{23} \right)$ |
| $Z' Z' h_3$ | $\lambda_4 v_\eta \alpha + (\lambda_6 + \lambda_9) v_\chi \gamma$ |
| $h_1 h_1 h_3$ | $v_\eta \lambda_4 \frac{\lambda_6}{2} \sigma + \frac{\lambda_6}{2} v_\eta \beta$ |
| $h_2 h_2 h_3$ | $\lambda_4 v_\eta \alpha + (\lambda_6 + \lambda_9) v_\chi \gamma$ |
| $h_2 h_2 h_3$ | $\lambda_4 v_\eta \alpha + \lambda_6 v_\chi \gamma$ |
We mention that 

$$\sin \theta = \frac{g_\gamma - g_A}{2} \frac{1}{c_W},$$

where $c_W \equiv \cos \theta_W$, $s_W \equiv \sin \theta_W$, $T_W \equiv \tan \theta_W$ and $h_W = 3 - 4s_W^2$, where $\theta_W$ is the Weinberg angle.

5. Higgs Decays

The main goal of our work is to study the signal strength of $h_1$ and the branching ratio of the other heavy scalar bosons $h_2$ and $h_3$ for different channels in the compact

### Table 5. Couplings $\frac{2g_{\mu}}{\tau} (g_V - g_A)$ of the $Zq\bar{q}$ vertex.

| Interactions | $g_V$ | $g_A$ |
|--------------|-------|-------|
| $\bar{u}uZ$ | $\frac{2}{3}s_W T_W + C_W$ | $\frac{1}{c_W}$ |
| $\bar{d}dZ$ | $\frac{1}{3}s_W T_W - C_W$ | $\frac{c_W}{c_W}$ |
| $\bar{s}sZ$ | $C_W - \frac{2}{3}s_W T_W$ | $C_W - 3s_W T_W$ |
| $\bar{b}bZ$ | $C_W - \frac{2}{3}s_W T_W$ | $C_W - 3s_W T_W$ |
| $\bar{c}cZ$ | $-C_W + \frac{1}{3}s_W T_W$ | $-\frac{1}{c_W}$ |
| $\bar{t}tZ$ | $-C_W + \frac{1}{3}s_W T_W$ | $-\frac{1}{c_W}$ |
| $\bar{U}_1 U_1 Z$ | $\frac{2}{3}s_W T_W$ | 0 |
| $\bar{T}_1 T_1 Z$ | $\frac{2}{3}s_W T_W$ | 0 |
| $\bar{D}_2 D_2 Z$ | $\frac{1}{3}s_W T_W$ | 0 |
| $\bar{D}_3 D_3 Z$ | $\frac{1}{3}s_W T_W$ | 0 |
| $\bar{T}_2 T_2 Z$ | $\frac{3}{2}s_W T_W$ | 0 |
| $\bar{T}_3 T_3 Z$ | $\frac{3}{2}s_W T_W$ | 0 |

### Table 6. Couplings $ZVV$, where $V$ is the gauge bosons $X^\pm$, $Y^\pm$, $V^\pm$, $K_L^\pm$, $K^0$ and $W^\pm$, while, $\sum_{\alpha,\beta,\mu}(p, k, q) = g_{\alpha\beta}(p - k)_\mu + g_{\beta\mu}(k - q)_\alpha + g_{\mu\alpha}(q - p)_\beta$.

| Interactions | Couplings |
|--------------|-----------|
| $W_\mu^+ W_\mu^- Z_\mu$ | $\frac{2g_{\mu}}{\tau} \sum_{\alpha,\beta,\mu}(p, k, q)$ |
| $K_{L\mu}^\pm K_{L\mu}^- Z_\mu$ | $\frac{-ig}{\sqrt{3}}(C_{22} + \sqrt{3}c_{22}) \sum_{\alpha,\beta,\mu}(p, k, q)$ |
| $X_{\mu}^+ X_{\mu}^- Z_\mu$ | $\frac{ig}{\sqrt{3}}(\sqrt{3}c_{22} + C_{22} + C_{22}) \sum_{\alpha,\beta,\mu}(p, k, q)$ |
| $K_{L\mu}^\pm K_{L\mu}^- Z_\mu$ | $\frac{-ig}{\sqrt{3}}(\sqrt{3}c_{21} + C_{21}) \sum_{\alpha,\beta,\mu}(p, k, q)$ |
| $V_{\mu}^+ V_{\mu}^- Z_\mu$ | $\frac{-ig}{\sqrt{3}}(\sqrt{3}c_{21} + C_{21} + C_{21}) \sum_{\alpha,\beta,\mu}(p, k, q)$ |
| $Y_{\mu}^+ Y_{\mu}^- Z_\mu$ | $\frac{-ig}{\sqrt{3}}(-C_{22} + C_{22}) \sum_{\alpha,\beta,\mu}(p, k, q)$ |

Where

$$C_{21} = C_W, \quad C_{22} = \frac{S_W T_W}{\sqrt{3}}, \quad C_{23} = \frac{4S_W T_W}{\sqrt{6}}, \quad C_{24} = \frac{-S_W T_W}{t}, \quad (31)$$

$$C_{32} = \sqrt{\frac{3 - T_W^2}{3}}, \quad C_{33} = \frac{-4}{\sqrt{3}} \frac{T_W^2}{\sqrt{3 - T_W^2}}, \quad C_{34} = \frac{T_W^2}{t \sqrt{3 - T_W^2}}, \quad (32)$$

$$t = \frac{S_W}{\sqrt{1 - 4S_W^2}}. \quad (33)$$

We mention that $S_W = \sin \theta_W$, $C_W = \cos \theta_W$, $T_W = \tan \theta_W$ and $h_W = 3 - 4S_W^2$ where $\theta_W$ is the Weinberg angle.
5.1. $h_1$ decay

The first purpose of this paper is to study the deviation of our model from the Standard Model by calculating the signal strength $\mu$ of the lightest scalar boson $h_1$ and confront our results to the experimental data for each individual channels $b\bar{b}$, $WW^*$, $ZZ^*$, $\tau\bar{\tau}$ and $\gamma\gamma$.

The signal strength of any process with a giving initial state $i$ producing an Higgs $h$ which decays to the final state $f$ can be written as a product of the Higgs boson production cross section and its branching ratio in units of the corresponding value predicted by the SM:

$$\mu_{xy} = \frac{\sigma_{341}(pp \rightarrow h_1)BR_{341}(h_1 \rightarrow xy)}{\sigma_{SM}(pp \rightarrow h)BR_{SM}(h \rightarrow xy)}, \quad (34)$$

where the superscript SM and 341 refer to the Standard Model and the compact 341 model respectively, while, $x$ and $y$ are any finale state, $\sigma_i$ and $BR_i$ ($i=341, SM$) are the corresponding cross section production taking gluon-gluon fusion (ggF) is the dominant contribution of the Higgs production and the branching ratio respectively.

In the compact 341 model, the coupling $tth_1$ is the same as $tth$ of the Standard Model (SM), hence, the cross section of the light Higgs $h_1$ production process is the same as the SM at the LHC:

$$\sigma_{341}(pp \rightarrow h_1) = \sigma_{SM}(pp \rightarrow h), \quad (35)$$

thus, the signal strength $\mu_{xy}$ becomes the ratio between the branching ratio of the SM and of the compact 341 model:

$$\mu_{xy} = \frac{BR_{341}}{BR_{SM}} = \frac{\Gamma_{341}(h \rightarrow all)\Gamma_{341}(h_1 \rightarrow xy)}{\Gamma_{341}(h \rightarrow xy)}, \quad (36)$$

where the total decay width $\Gamma_{341}(h_1 \rightarrow all)$ turns out to be the same as the one of the SM Higgs boson $\Gamma_{SM}(h \rightarrow all)$:

$$\Gamma_{341}(h_1 \rightarrow all) = \Gamma_{341}(h_1 \rightarrow b\bar{b}) + \Gamma_{341}(h_1 \rightarrow \tau^+\tau^-) + \Gamma_{341}(h_1 \rightarrow WW^*) + \Gamma_{341}(h_1 \rightarrow ZZ^*) + \Gamma_{341}(h_1 \rightarrow \gamma\gamma) + \Gamma_{341}(h_1 \rightarrow gg). \quad (37)$$

All the tree level couplings between the SM-like Higgs boson $h_1$ and all the fermions and the W boson, are the same as the SM, that makes the partial decay widths of $h_1$ into SM particles such as $b\bar{b}$, $\tau^+\tau^-$, $WW^*$ and $gg$ have the same expressions as the SM [20] whereas, the partial decay width of $h_1$ into $ZZ^*$ is given by:

$$\Gamma_{341}(h_1 \rightarrow ZZ^*) = \frac{g^4m_{h_1}}{2048\pi^3}\sigma_{h_1ZZ}^2F\left(\frac{m_Z}{m_{h_1}}\right)\left(\sum_{j=u,d,c,s,b}(g_{jV}^2 + g_{jA}^2)\right) + \sum_{\ell=leptons}(g_{\ell V}^2 + g_{\ell A}^2), \quad (38)$$

$$F\left(\frac{m_Z}{m_{h_1}}\right) = \left(1 + \frac{m_Z^2}{m_{h_1}^2} + \frac{m_Z^4}{m_{h_1}^4}\right)^{-1/2}.$$
with $G_{h_1ZZ} = 4g_{h_1ZZ}/(g^2 v_\rho)$ where $g_{ZZh_1}$ represents the coupling between a pair of $Z$ bosons and $h_1$ (see table 2) and

$$F(x) = -|1 - x^2|(\frac{47}{2} x^2 - \frac{13}{2} + \frac{1}{x^2}) - \frac{3}{2}(1 - 6x^2 + 4x^4) \ln(x) + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}} \arccos(\frac{3x^2 - 1}{2x^2}),$$

(39)

with the parameter $x = \frac{m_Z}{m_{h_1}}$.

The compact 341 model contains new charged particles including fermions, scalars (the new singly and doubly charged scalar bosons) and gauge bosons that will contribute to the decay amplitudes of the decays $h_1 \rightarrow \gamma\gamma$ and $h_1 \rightarrow \gamma Z$ at one loop level. In the case of $h_1$, besides the contribution of $W^\pm$ boson and the top quark, the processes $\Gamma(h_1 \rightarrow \gamma\gamma(\gamma Z))$ receive new contributions that come from new particles (gauge and scalar bosons) such as $V^\pm$, $K^+_1$, $h^+_2$ and $h^\pm$. Furthermore, in our model, the SM-like Higgs boson $h_1$ does not couple to the new heavy charged fermions, hence, the exotic quarks do not contribute to the one-loop decay amplitudes of the processes $h_{2,3} \rightarrow (\gamma\gamma, Z\gamma, gg)$. Thus, the partial decay widths of $h_1$ into $\gamma\gamma$ and $Z\gamma$ are given by

$$\Gamma(h_1 \rightarrow \gamma\gamma) = \frac{\alpha^2 m_{h_1}^3}{1024\pi^3} \left| \sum_V g_{h_1VV} Q_V^2 A_1(\tau_V) + \sum_f \frac{2g_{h_1ff} Q_f^2}{m_f} N_{c,f} A_1^2(\tau_f) \right|^2,$$

(40)

$$\Gamma(h_1 \rightarrow Z\gamma) = \frac{\alpha^2 m_{h_1}^3}{512\pi^3} \left( 1 - \frac{M_Z^2}{M_{h_1}^2} \right)^3 \left| \frac{2}{\upsilon \sin \theta_W} A_{SM} + A_0 \right|^2,$$

(41)

where the symbol $V$, $f$ and $S$ refer to Spin 1, Spin $\frac{1}{2}$ and Spin 0 particles respectively, $Q_V, Q_f, Q_S$ are electric charges of the vectors, fermions and scalars, $g_{h_1SS}, g_{h_1VV}$ and $g_{h_1ff}$ represent the couplings of the Higgs with scalars $S$, gauge bosons $V$ and fermions $f$, $N_{c,f}, N_{c,S}$ are the number of fermion and scalar colors and the loop functions for $V,f$ and $S$ particles $A_1(\tau_V)$, $A_1^2(\tau_f)$ and $A_0(\tau_S)$ are given by

$$A_1(x) = -x^2 \left( 2x^2 - 3 + 3(2x^{-1} - 1)f(x^{-1}) \right),$$

$$A_{1/2}(x) = 2x^2 \left( x^{-1} + (x^{-1} - 1)f(x^{-1}) \right),$$

$$A_0(x) = -x^2 \left( x^{-1} - f(x^{-1}) \right).$$

(42)
The parameter $\tau_i = \frac{4m_i^2}{m_{h_i}}$ where i represents the corresponding particles in the loop and:

$$f(x) = \begin{cases} \arcsin^2 \sqrt{x} & \text{for } x \geq 1, \\ \frac{-1}{4} \left( \ln \left( \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} \right) - i\pi \right)^2 & \text{for } x < 1. \end{cases}$$ (43)

The factors $A$ and $A_{SM}$ represent the contributions of the new particles predicted by our model and the contributions coming from the particles of the Standard Model respectively, their expressions are given by:

$$A = \frac{g_{h,VV}}{m_V^2}g_{ZVV}A_1(\tau_V, \lambda_V) + \bar{N}_{c,f} \frac{2g_{h,ff}}{m_f}2Q_f(g_{Z\ell\ell} + g_{Zrr})A_2(\tau_f, \lambda_f)$$
$$- \bar{N}_{c,s} \frac{2g_{h,SS}}{m_S^2}Q_S g_{ZSS}A_0(\tau_S, \lambda_S),$$ (44)

$$A_{SM} = \cos \theta_W A_1(\tau_W, \lambda_W) + N_C \frac{Q_1(2T_3^2 - 4Q_t \sin^2 \theta_W)}{\cos \theta_W} A_1(\tau_t, \lambda_t),$$ (45)

Where $\lambda_i = \frac{4m_i^2}{m_{h_i}^2}$, $T_3 = \frac{1}{2}$ is the weak isospin of the top quark and $A_i(x, y)$ are the loop functions:19,20

$$A_1(x, y) = 4(3 - \tan^2 \theta_W)I_2(x, y) + \left( (1 + 2x^{-1}) \tan^2 \theta_W - (5 + 2x^{-1}) \right)I_1(x, y),$$

$$A_2(x, y) = I_1(x, y) - I_2(x, y),$$

$$A_0(x, y) = I_1(x, y),$$ (46)

where:

$$I_1(x, y) = \frac{xy}{2(x-y)} + \frac{x^2y^2}{2(x-y)^2} \left( f(x^{-1}) - f(y^{-1}) \right) + \frac{x^2y}{(x-y)^2} \left( g(x^{-1}) - g(y^{-1}) \right),$$

$$I_2(x, y) = \frac{-xy}{2(x-y)} \left( f(x^{-1}) - f(y^{-1}) \right),$$ (47)

with:

$$g(x) = \begin{cases} \sqrt{x^{-1} - 1} \arcsin \sqrt{x} & \text{for } x \geq 1, \\ \frac{\sqrt{1-x^{-1}}}{x^{-1/2}} \left( \ln \left( \frac{1+\sqrt{1-x^{-1}}}{1-\sqrt{1-x^{-1}}} \right) - i\pi \right) & \text{for } x < 1. \end{cases}$$ (48)

where the couplings $g_{h,VV}$, $g_{ZVV}$, $g_{h,ff}$, $g_{Z\ell\ell}$, $g_{Zrr}$, $g_{SS}$, and $g_{ZSS}$ are given in tables 24,5o.

**5.2. The decay of the neutral heavy scalar bosons $h_2$ and $h_3$**

For the neutral heavy scalar bosons $h_2$ and $h_3$, we calculate their branching ratios (BRs) where:

$$\text{BR} = \frac{\Gamma_i}{\Gamma(h \rightarrow all)}.$$ (49)
with $\Gamma(h \rightarrow all) = \sum \Gamma_i$ is the total decay rate that represents the sum of the individual decay rates. The total decay width of $h_2$ is determined by the following channels:

$$\Gamma_{341}(h_2 \rightarrow all) = \Gamma_{341}(h_2 \rightarrow \tau^+ \tau^-, b\bar{b}) + \Gamma_{341}(h_2 \rightarrow ZZ)$$

$$+ \Gamma_{341}(h_2 \rightarrow \gamma Z) + \Gamma_{341}(h_2 \rightarrow \gamma\gamma) + \Gamma_{341}(h_2 \rightarrow gg)$$

$$+ \Gamma_{341}(h_2 \rightarrow h_1h_1).$$  \hspace{1cm} (50)

Furthermore, $h_2$ can also decay to new charged and neutral particles $F$ including fermions, scalar and gauge bosons, if kinematically allowed which would require $m_{h_2} > 2m_F$, therefore, in this case, $\Gamma_{341}(h_2 \rightarrow all)$ has new contributions $\Gamma_{341}(h_2 \rightarrow F)$ where $F$ can be $K^\mp, K^{*0}, X^+, Y^+, V^{\mp\mp}, Z'$ or exotic quarks.

In our model, the heavy scalar bosons $h_{2,3}$ does not couple to the SM top quark, therefore, the fermion contributions come only from exotic quarks in the loop processes $\Gamma(h_2 \rightarrow gg, \gamma\gamma, \gamma Z)$, this result in:

$$\Gamma(h_2 \rightarrow gg) = \frac{\alpha_S^2 M_{h_2}^3}{128\pi^3 m_0^2} \alpha^2 \bigg| \sum_{i=1}^3 A_2(\tau_i) \bigg|^2 + \frac{\alpha_S^2 M_{h_2}^3}{128\pi^3 m_0^2} \alpha^2 \bigg| \sum_{i=1}^3 A_3(\tau_i) \bigg|^2,$$  \hspace{1cm} (51)

where $\alpha_S$ is the strong coupling and $A_2(\tau_f)$ represents the loop function of fermions.

The one loop expressions for $h_{2,3}$ decays into final states including massless bosons $\gamma\gamma$ and $Z\gamma$ can be mediated by new contributions that come from the new charged particles namely $K^\mp, K^{*0}, X^+, Y^+, V^{\mp\mp}, U, J, h_1^\mp, h_3^\mp$ and $h^{\mp\mp}$, we note that the contribution of the $W^\mp$ boson is not included in these amplitudes, since there is no direct coupling between $h_{2,3}$ and the $W^\mp$ boson.

Moreover, it is worth pointing out that the fermions and bosonic decays are similar to the $h_1$ case just one needs to replace the couplings of table 2 by those of table 3 for $h_2$ and by table 4 for $h_3$.

Furthermore, the partial decay widths of $h_2$ into a pair of Higgs like-boson $h_1$, into a pair of exotic quarks ($U$ and $J$), into a pair of gauge bosons $V_i$ ($V_i \equiv X^\mp, V^{\mp\mp}, K^{*0}, K_1^\mp, Y^\mp$) and into a pair of $Z$ and $Z'$ are given respectively by:

$$\Gamma_{341}(h_2 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_2}} (g_{h_2h_1h_1})^2 \bigg( 1 - \frac{4m_{h_1}^2}{m_{h_2}^2} \bigg)^{1/2},$$  \hspace{1cm} (52)

$$\Gamma_{341}(h_2 \rightarrow UU) = \frac{3g^2 m_1^2 m_{h_2}}{32\pi m_W^4} \left( 1 - \frac{4m_{h_1}^2}{m_{h_2}^2} \right)^{3/2} \left( \frac{v_p v_\alpha}{v_q \alpha} \right)^2,$$  \hspace{1cm} (53)

$$\Gamma_{341}(h_2 \rightarrow JJ) = \frac{3g^2 m_1^2 m_{h_2}}{32\pi m_W^4} \left( 1 - \frac{4m_{h_1}^2}{m_{h_2}^2} \right)^{3/2} \left( \frac{v_p v_\gamma}{v_q \gamma} \right)^2,$$  \hspace{1cm} (54)

$$\Gamma_{341}(h_2 \rightarrow V_iV_i) = \frac{k_i}{4\pi M_{h_2}^2 v_p^2} \bigg( 1 - \frac{4M_{h_1}^2}{M_{h_2}^2} \bigg)^{1/2} \bigg( 3 + \frac{1}{4} \frac{M_{h_1}^2}{M_{h_2}^2} - \frac{M_{h_1}^2}{M_{h_2}^2} \bigg) G_{ZZh_2}^2,$$  \hspace{1cm} (55)

$$\Gamma(h_2 \rightarrow ZZ) = \frac{1}{8\pi M_{h_2}^2 v_p^2} \bigg( 1 - \frac{4M_{h_1}^2}{M_{h_2}^2} \bigg)^{1/2} \bigg( 3 + \frac{M_{h_1}^2}{M_{h_2}^2} - \frac{M_{h_1}^2}{M_{h_2}^2} \bigg) G_{ZZh_2}^2,$$  \hspace{1cm} (56)
where the expression of the trilinear coupling $h_2h_1h_1$ is given in Table 3. $m_U$, $m_J$ are the exotic quarks masses, $v_\eta$ is the vacuum expectation value and the $k_i$ coefficients are given in the following table:

| Higgs          | $k_{\chi^+}$ | $k_{\eta^+}$ | $k_{k_{\chi^+}}$ | $k_{k_{\eta^+}}$ | $k_{V^+}$ |
|----------------|---------------|--------------|-------------------|-------------------|-----------|
| $h_2$          | $\frac{v_\chi}{v_\eta}$ | $\frac{v_\chi}{v_\eta}$ | $\frac{v_\chi}{v_\eta}$ | $\frac{v_\chi}{v_\eta}$ | $\frac{1}{v_\eta}(v_\chi + v_\eta)$ |

while the expressions of $G_{ZZh_2}$ and $G_{Z'Zh_2}$ are given by:

$$G_{ZZh_2} = \frac{C_W^2}{2} \left( \frac{3}{2} \frac{v_\chi}{v_p} (C^{23})^2 \gamma + 4(C^{24})^2 \frac{S_W}{1-4S_W^2} v_\chi \gamma + 12 \sqrt{\frac{2}{1-4S_W^2}} \frac{v_\chi}{v_p} C^{23}C^{24} \gamma \right. + \left. \frac{4}{3} (C^{23})^2 \frac{v_\eta}{v_p} + \frac{(C^{23})^2}{6} \frac{v_\eta}{v_p} - \frac{4}{\sqrt{18}} (C^{23} C^{24}) \right),$$

$$G_{Z'Zh_2} = \frac{C_W^2}{2} \left( \frac{3}{2} \frac{v_\chi}{v_p} (C^{33})^2 \gamma + 4(C^{34})^2 \frac{S_W}{1-4S_W^2} v_\chi \gamma + 12 \sqrt{\frac{2}{1-4S_W^2}} \frac{v_\chi}{v_p} C^{33}C^{34} \gamma \right. + \left. \frac{4}{3} (C^{33})^2 \frac{v_\eta}{v_p} + \frac{(C^{33})^2}{6} \frac{v_\eta}{v_p} - \frac{4}{\sqrt{18}} (C^{33} C^{34}) \right).$$

Regarding, the third neutral scalar boson $h_3$, its total decay width is composed by the following decays:

$$\Gamma_{341}(h_3 \rightarrow all) = \Gamma_{341}(h_3 \rightarrow \tau^+\tau^-, b\bar{b}) + \Gamma_{341}(h_3 \rightarrow exotic \ quarks)$$

$$+ \Gamma_{341}(h_3 \rightarrow \gamma\gamma) + \Gamma_{341}(h_3 \rightarrow Z\gamma) + \Gamma_{341}(h_3 \rightarrow gg)$$

$$+ \Gamma_{341}(h_3 \rightarrow h_2h_2) + \Gamma_{341}(h_3 \rightarrow h_1h_1)$$

$$+ \Gamma_{341}(h_3 \rightarrow h_2h_1) + \Gamma_{341}(h_3 \rightarrow VV),$$

where $V$ represents the gauge bosons $X^\pm$, $Y^\pm$, $K^{\pm}$, $K_1^{\pm}$, $Y^\mp$, $Z$ and $Z'$. It is worth to point out that $h_3$ may decay into a pair of $h_1h_1$, $h_2h_2$ and $h_1h_2$ where their decay widths are given by:

$$\Gamma_{341}(h_3 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_1 h_1)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_2) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_2 h_2)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_1) = \frac{1}{16\pi m_{h_3}^2}(m_{h_3} h_2 h_1)^2 \left(\frac{m_{h_3}^4}{m_{h_3}^2} - \frac{2m_{h_3}^2 m_{h_2}^2}{m_{h_3}^2} - 2m_{h_2}^2 + \frac{m_{h_1}^4}{m_{h_3}^2} - 2m_{h_1}^2 \right) + \frac{1}{2},$$

and the third neutral scalar boson $h_3$, its total decay width is composed by the following decays:

$$\Gamma_{341}(h_3 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_1 h_1)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_2) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_2 h_2)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_1) = \frac{1}{16\pi m_{h_3}^2}(m_{h_3} h_2 h_1)^2 \left(\frac{m_{h_3}^4}{m_{h_3}^2} - \frac{2m_{h_3}^2 m_{h_2}^2}{m_{h_3}^2} - 2m_{h_2}^2 + \frac{m_{h_1}^4}{m_{h_3}^2} - 2m_{h_1}^2 \right) + \frac{1}{2},$$

and the third neutral scalar boson $h_3$, its total decay width is composed by the following decays:

$$\Gamma_{341}(h_3 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_1 h_1)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_2) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_2 h_2)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_1) = \frac{1}{16\pi m_{h_3}^2}(m_{h_3} h_2 h_1)^2 \left(\frac{m_{h_3}^4}{m_{h_3}^2} - \frac{2m_{h_3}^2 m_{h_2}^2}{m_{h_3}^2} - 2m_{h_2}^2 + \frac{m_{h_1}^4}{m_{h_3}^2} - 2m_{h_1}^2 \right) + \frac{1}{2},$$

and the third neutral scalar boson $h_3$, its total decay width is composed by the following decays:

$$\Gamma_{341}(h_3 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_1 h_1)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_2) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_2 h_2)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_1) = \frac{1}{16\pi m_{h_3}^2}(m_{h_3} h_2 h_1)^2 \left(\frac{m_{h_3}^4}{m_{h_3}^2} - \frac{2m_{h_3}^2 m_{h_2}^2}{m_{h_3}^2} - 2m_{h_2}^2 + \frac{m_{h_1}^4}{m_{h_3}^2} - 2m_{h_1}^2 \right) + \frac{1}{2},$$

and the third neutral scalar boson $h_3$, its total decay width is composed by the following decays:

$$\Gamma_{341}(h_3 \rightarrow h_1h_1) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_1 h_1)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_2) = \frac{1}{16\pi m_{h_3}}(m_{h_3} h_2 h_2)^2 \left(1 - \frac{4m_{h_3}^2}{m_{h_3}^2} \right) \frac{1}{2},$$

$$\Gamma_{341}(h_3 \rightarrow h_2h_1) = \frac{1}{16\pi m_{h_3}^2}(m_{h_3} h_2 h_1)^2 \left(\frac{m_{h_3}^4}{m_{h_3}^2} - \frac{2m_{h_3}^2 m_{h_2}^2}{m_{h_3}^2} - 2m_{h_2}^2 + \frac{m_{h_1}^4}{m_{h_3}^2} - 2m_{h_1}^2 \right) + \frac{1}{2}.
where $g_{h_3h_2h_1}$, $g_{h_3h_1h_1}$ and $g_{h_3h_2h_2}$ represents trilinear terms $h_3h_2h_1$, $h_3h_1h_1$ and $h_3h_2h_2$ respectively, their expressions are given in table [3].

The partial decay width of $h_3$ into a pair of gauge bosons $V_i$ is given by Eq (55) where in the case of $h_3$, the coefficients $k_i$ are given by:

| Higgs | $k_{XX}$ | $k_{YY}$ | $k_{KK}$ | $k_{KK}$ | $k_{YY}$ |
|-------|---------|---------|---------|---------|---------|
| $h_3$ | $\frac{v_X}{v_P} \sigma$ | $\frac{v_X}{v_P} \sigma$ | $\frac{v_X}{v_P} \beta$ | $\frac{v_X}{v_P} \beta$ | $\frac{1}{v_P} (v_Y \sigma + v_H \beta)$ |

The partial decay width of $h_3$ into a pair of SM $Z$ and $Z'$ gauge bosons are given by the Eqs (56) and (57) respectively only one needs to replace $m_{h_2}$ with $m_{h_3}$ and both $G_{ZZh_3}$ and $G_{Z'Zh_3}$ with $G_{ZZh_3}$ and $G_{Z'Zh_3}$ respectively, where:

$$G_{ZZh_3} = \frac{C_W^2}{2} \left( \frac{3}{2} \frac{v_X}{v_P} (C^{23})^2 \sigma + 4(C^{24})^2 \frac{S_W^2}{1 - 4S_W^2} \frac{v_X}{v_P} + \frac{12}{\sqrt{6}} \frac{S_W}{1 - 4S_W^2} \frac{v_X}{v_P} C^{23} C^{24} \sigma \right) + \frac{4}{3} \left( C^{23} \right)^2 \frac{v_H}{v_P} \beta - \frac{4}{\sqrt{18}} \frac{C^{22} C^{23} v_H \beta}{v_P} \right),$$

$$G_{Z'Zh_3} = \frac{C_W^2}{2} \left( \frac{3}{2} \frac{v_X}{v_P} (C^{33})^2 \sigma + 4(C^{34})^2 \frac{S_W^2}{1 - 4S_W^2} \frac{v_X}{v_P} + \frac{12}{\sqrt{6}} \frac{S_W}{1 - 4S_W^2} \frac{v_X}{v_P} C^{33} C^{34} \sigma \right) + \frac{4}{3} \left( C^{32} \right)^2 \frac{v_H}{v_P} \beta - \frac{4}{\sqrt{18}} \frac{C^{32} C^{33} v_H \beta}{v_P} \right).$$

The partial decay widths of the $h_3$ into gluons and into a pair of exotic quarks are given respectively by:

$$\Gamma(h_3 \to gg) = \frac{\alpha_s^2 M_{h_3}^3}{128 \pi v_H^2} \beta^2 \sum_{i=1}^{3} A_{Ui}^2 (\tau_U)^2 + \frac{\alpha_s^2 M_{h_3}^3}{128 \pi v_X^2} \beta^2 \sum_{i=1}^{3} A_{Ji}^2 (\tau_J)^2$$

$$\Gamma_{341}(h_3 \to UU) = \frac{3g^2}{32 \pi} \frac{m_{h_3}^3}{m_{h_3}^2} \left( 1 - \frac{4m_{Uj}^2}{m_{h_3}^2} \right)^2 \left( \frac{v_H}{v_H} \right)^2,$$

$$\Gamma_{341}(h_3 \to JJ) = \frac{3g^2}{32 \pi} \frac{m_{h_3}^3}{m_{h_3}^2} \left( 1 - \frac{4m_{Jj}^2}{m_{h_3}^2} \right)^2 \left( \frac{v_H}{v_H} \right)^2.$$

6. Numerical Analysis

In this section, we discuss our numerical results of the signal strength of the Higgs like-boson $h_1$ and the branching ration (BR) of the other scalars $h_2$ and $h_3$. All the expressions of the compact 341 model are related to $v_X$, $v_H$, $v_P$, and to the scalar parameters $\lambda_{1,9}$.

In our work, we consider the following scenario:

$v_P=246$ GeV, $v_X = v_H =2$ TeV and $m_{exotic~~quarks}=750$ GeV[21] while, for the scalar parameters $\lambda_{1,9}$, we make random choices where we constrained them using the theoretical constraints that we have discussed in the text.
6.1. The signal strength

Any extension of the SM must possess a scalar with 125 GeV of mass, in our model, we identify $h_1$ as the SM Higgs boson, therefore, the signal strength is discussed to clarify if the lightest scalar boson $h_1$ recovers the SM Higgs boson and to check the validity of our model.

We calculated the signal strength of $h_1$ where we take into account that the most dominant production channel of $h_1$ comes from the gluon-gluon fusion, while, its decay is into several channels namely $b\bar{b}$, $WW^*$, $ZZ^*$, $\gamma\gamma$ and $\tau^+\tau^-$. The fermionic decays and the bosonic decays are possible at the tree-level, whereas we apply one-loop expressions for the decays into final states including massless bosons (that is $gg$, $\gamma\gamma$ and $Z\gamma$).

Table 9 shows the results of our model and the available experimental signal strength values from LHC Run 1 for the combination of ATLAS and CMS, and separately for each experiment, for the combined $\sqrt{s} = 7$ and 8 TeV data for different Higgs boson decay channels. These results are obtained assuming that the Higgs boson production process cross sections at $s = 7$ and 8 TeV are the same as in the SM.

Figure 1 shows the signal strengths for the various decay modes $b\bar{b}$, $WW^*$, $ZZ^*$, $\gamma\gamma$ and $\tau^+\tau^-$ in the compact 341 model with the data reported at ATLAS, CMS and the combined ATLAS+CMS Run1. Our results can fit the current data withing the experimental errors which makes the compact 341 model in perfect agreement with the values measured by ATLAS and CMS. That ensures the viability of the compact 341 model to be an available model in the future work at the LHC.

| Decay channel | ATLAS   | CMS     | ATLAS+CMS | The compact 341 model |
|---------------|---------|---------|-----------|----------------------|
| $\mu^\gamma\gamma$ | $1.14^{+0.27}_{-0.25}$ | $1.11^{+0.25}_{-0.23}$ | $1.14^{+0.19}_{-0.18}$ | 1.03 |
| $\mu^{ZZ}$    | $1.52^{+0.40}_{-0.34}$ | $1.04^{+0.32}_{-0.26}$ | $1.29^{+0.26}_{-0.24}$ | 1.06 |
| $\mu^{WW}$    | $1.22^{+0.23}_{-0.21}$ | $0.90^{+0.23}_{-0.21}$ | $1.09^{+0.18}_{-0.16}$ | 0.99 |
| $\mu^{\tau\tau}$ | $1.41^{+0.36}_{-0.30}$ | $0.88^{+0.30}_{-0.28}$ | $1.11^{+0.24}_{-0.22}$ | 0.99 |
| $\mu^{bb}$    | $0.62^{+0.30}_{-0.37}$ | $0.81^{+0.45}_{-0.43}$ | $0.70^{+0.29}_{-0.27}$ | 0.99 |

6.2. Searches for $h_i = 2,3$

Figure 2 shows BR($h_2 \rightarrow h_1 h_1$) as a function of the heavy Higgs $h_2$ mass for $v_\chi = v_\eta = 2$ TeV where in the allowed parameter space, the mass of $m_{h_2}$ ranges from 500 GeV to 1.3 TeV. Its clear that in the region 500-1300 GeV, the decay channel $h_2 \rightarrow h_1 h_1$ is the most important one with a BR $\geq 90\%$, this happens because the trilinear coupling $h_1 h_2 h_2$ increases about one order of magnitude $v_\eta$. Interestingly,
one can has a new production mechanism for $h_2$ namely $pp \rightarrow h_2 \rightarrow h_1h_1$. This production channel might be useful to increase the signal of the double Higgs production at the LHC.

As the mass becomes larger than 1300 GeV, we can notice that the decay modes of $h_2$ into the heavy gauge bosons $K^0$, $K^\pm$, $Y^\pm$, $V^{++}$, $X^\pm$ and $Z'$ and into exotic quarks are kinetically allowed, in this case, the total decay width contains additional channels $h_2 \rightarrow VV$ and $h_2 \rightarrow QQ$ where $V \equiv K^0$, $K^\pm$, $Y^\pm$, $V^{++}$, $X^\pm$ and $Z'$ and $Q \equiv$ exotic quarks. The maximum value of the branching ratio of the process $h_2$ into gauge bosons $K^0$, $K^\pm$, $V^{++}$ and $X^\pm$ plateaus close to $\sim 0.35$, $\sim 0.26$ as we shown in tables 10 and 11 and in figures 3 and 4, therefore, one may notice that those decays are the most significant decay channel and the decay channel $h_2 \rightarrow h_1h_1$ becomes weak and contributing roughly 20%.

The branching ratios of the remaining decay modes are tabulated in the following tables:

**Table 10.** The branching ratios (BRs) of the heavy scalar $h_2$ for different channels.

| Decay channel | BR where $m_{h_2} \in [500-1300]$ (GeV) | BR where $m_{h_2} \in [1300-3500]$ (GeV) |
|---------------|------------------------------------------|------------------------------------------|
| $b\bar{b}$    | $\sim O(10^{-5})$                        | $[10^{-5} - 10^{-6}]$                    |
| $\tau^+\tau^-$| $\sim O(10^{-7})$                        | $\sim 10^{-7}$                          |
| $\gamma\gamma$| $\sim O(10^{-7})$                        | $\sim 10^{-7}$                          |
| $\gamma Z$    | $\sim O(10^{-3})$                        | $[10^{-3} - 2.6\times10^{-3}]$           |
| $gg$          | $\sim O(10^{-4})$                        | $[10^{-4} - 10^{-5}]$                    |
| $ZZ$          | $\sim O(10^{-32})$                       | $[10^{-32} - 10^{-31}]$                  |
| $Z'Z'$        | $\sim O(10^{-4})$                        | $[10^{-3} - 10^{-2}]$                    |
| $K^0K^*$      | 0                                        | $[10^{-3.26}]$                          |
| $Y^+Y^-$      | 0                                        | $[10^{-4.10^{-2}}]$                      |
| $X^+X^-$      | 0                                        | $[10^{-3.35}]$                          |
| $V^{++}V^{--}$| 0                                        | $[10^{-3.34}]$                          |
Table 11. The branching ratios (BRs) of the heavy scalar $h_2$ for different channels.

| Decay channel | BR where $m_{h_2} \in [500-1300]$ (GeV) | BR where $m_{h_2} \in [1300-3500]$ (GeV) |
|---------------|---------------------------------|---------------------------------|
| $K^{\pm} K^{\mp}$ | 0 | $[10^{-3}-0.26]$ |
| $U \bar{U}$ | 0 | $[10^{-4}-10^{-2}]$ |
| $J \bar{J}$ | 0 | $[10^{-5}-10^{-1}]$ |

Fig. 2. The branching ratio of $h_2 \rightarrow h_1 h_1$ versus the heavy Higgs $h_2$ mass.

Fig. 3. The branching ratio of $h_2 \rightarrow K^{\theta} K^{\bar{\theta}}$ versus the heavy Higgs $h_2$ mass.
Now we turn our discussion to $h_3$ decay. Figure 5 shows $\text{BR}(h_3 \rightarrow ZZ)$ as a function of the heavy Higgs $h_3$ mass for $v_\chi = v_\eta = 2$ TeV where in the allowed parameter space, the mass of $m_{h_3}$ ranges from 3 TeV to 4 TeV. The decay channel $h_3 \rightarrow ZZ$ is the most important one with a BR $\geq 80\%$ which is about $\sim 0.8$, while, the BR in $h_1 h_1$ is the second most significant decay channel contributing roughly
Neutral Higgs bosons Phenomenology in the compact 341 model and confrontations with the LHC results

20% as we shown in figure 6, whereas the lowest value of the branching ratio in that region is just below $\sim 10^{-7}$.

The branching ratios of the other processes are shown in the following table:

| Decay channel | BR where $m_{h_3} \in [3000-4000]$(GeV) |
|---------------|----------------------------------------|
| $b\bar{b}$    | $\sim \mathcal{O}(10^{-5})$           |
| $\tau^+\tau^-$| $[10^{-7} - 10^{-9}]$                 |
| $\gamma\gamma$| $[10^{-7} - 10^{-5}]$                 |
| $gZ$          | $[10^{-3} - 10^{-2}]$                 |
| $gg$          | $[10^{-5} - 10^{-4}]$                 |
| $ZZ$          | $\sim 0.80$                          |
| $Z'Z'$        | $[10^{-7} - 10^{-5}]$                 |
| $K^0K^0$      | $[1.5.10^{-6} - 1.9.10^{-4}]$        |
| $Y^+Y^-$      | $[6.10^{-6} - 5.10^{-5}]$             |
| $X^+X^-$      | $[10^{-6} - 3.10^{-5}]$               |
| $V^+V^-$      | $[10^{-6} - 10^{-5}]$                 |
| $K_1^+K_1^-$ | $[1.5.10^{-6} - 1.9.10^{-4}]$        |
| $U\bar{U}$   | $[5.10^{-2} - 0.25]$                 |
| $J\bar{J}$   | $[5.10^{-2} - 10^{-3}]$               |
| $h_1h_1$     | $\sim 0.20$                          |
| $h_1h_2$     | $[10^{-4} - 10^{-2}]$                 |
| $h_2h_2$     | $[5.10^{-4} - 10^{-3}]$               |

![BR(h_3 \rightarrow ZZ) vs m_{h_3}(GeV)](image)

Fig. 5. The branching ratio of $h_3 \rightarrow ZZ$ as a function of the heavy Higgs $h_3$ mass.

At the end, we give the total decay width for the two CP-even scalars $h_2$ and $h_3$ in figure 7. It is well known that the SM Higgs with a mass of 125 GeV has a very narrow width almost equal to $\Gamma_h \sim 4$ MeV.

In our case, the total width of $h_2$ is in the range $4.1.10^{-5}-1.014$ (TeV), we notice a very narrow width of $h_2$ when the decay of $h_2$ into the heavy gauge bosons and
exotic quarks are closed and it can decay only into a pair of leptons, quarks, scalars and $\gamma\gamma(Z\gamma)$, while, when they opened the total width is large and it achieves the value $\sim 1.014$ TeV. Whereas, the total width of $h_3$ can be located between 0.0649 and 1.488 TeV.

7. Conclusion

The compact 341 model is a gauge extension of the Standard Model with only three quadruplets scalar fields composed by three neutral scalars, four singly charged and two doubly charged scalar bosons. In this work we discussed some phenomenology of the scalar sector.

We have used a set of the theoretical constraints such as: perturbative unitarity, boundedness from below constraints and the positivity of the scalar bosons masses condition to constrain the scalar parameters.

We have calculated the signal strength of $h_1$ for different channels where we
obtained a good fit to the data as can be seen in figure 1. In particular, with $\nu = 2$ TeV, we have shown that the compact 341 model is achieved to a good result that are compatible to the measurement of LHC.

The second focus of our paper is the calculation of the branching ratio of the other heavy scalar bosons $h_2$ and $h_3$, we found that $h_2$ decay preferentially into a pair of Higgs-like particles with a branching ratio $\sim 1$ when $m_{h_2} \in [500,1300]$ GeV, a feature not easily obtained in other extensions of the Standard Model and in the second region where $m_{h_2} \in [1300,3500]$ GeV, $h_2$ prefers to decay into the heavy gauge bosons $X^{\mp}, K^{\mp}_1, K^{\mp}_0, V^{\mp}$ . Moreover, we found that $h_3$ decay preferentially into a pair of Z bosons where $m_{h_3} \in [3000,4000]$ GeV.

Finally, we have studied the total decay width of $h_2$ and $h_3$, we found that $\Gamma_{h_2}$ and $\Gamma_{h_3}$ can achieve 1.014 TeV and 1.488 TeV respectively.

In conclusion, we have studied the scalar sector of the compact 341 model and showed that at a scale of a few TeV this model is a compelling alternative to the SM once, it is able to reproduce the experimental data regarding the signal strength.

The production process $gg \rightarrow h_{2,3}$ followed by the decays $h_{2,3} \rightarrow h_1 h_1$ could be sizeable and could be an important source of double $h_1$ production where it is rather difficult to produce it using the conventional channel.

### Appendix A. Perturbative unitarity and Vacuum stability conditions

The perturbative unitarity conditions are \[18\]
\[
\begin{align*}
\alpha^2 \lambda_4 + \gamma^2 (\lambda_6 + \lambda_9) &< 16\pi, \\
\beta^2 \lambda_4 + \sigma^2 (\lambda_6 + \lambda_9) &< 16\pi, \\
\alpha \beta \lambda_4 + \gamma \sigma (\lambda_6 + \lambda_9) &< 8\pi, \\
& (\alpha^2 + 2 \alpha \beta) \lambda_4 + \gamma^2 \lambda_6 < 32\pi, \\
\lambda_4 + \lambda_6 &< 32\pi, \\
\beta^2 \lambda_4 + \sigma^2 \lambda_6 &< 32\pi, \\
\sigma \lambda_9 + \beta \lambda_7 &< 16\pi, \\
\gamma \lambda_9 + \alpha \lambda_7 &< 16\pi, \\
\gamma \sigma \lambda_6 &< 16\pi, \\
\alpha^2 \lambda_4 + \gamma^2 \lambda_6 + \alpha^2 \lambda_7 &< 16\pi, \\
\beta^2 \lambda_4 + \sigma^2 \lambda_6 + \beta^2 \lambda_7 &< 16\pi, \\
\alpha \beta \lambda_4 + \sigma \gamma \lambda_6 + \alpha \beta \lambda_7 &< 8\pi, \\
2 \alpha^2 \lambda_1 + 2 \gamma \lambda_3 + \lambda_5 (\alpha^2 + \gamma^2) + \lambda_6 (\alpha^2 + \gamma^2 + (\sqrt{2} + 1) \gamma \alpha) &< 32\pi, \\
2 \beta^2 \lambda_1 + 2 \sigma^2 \lambda_3 + \lambda_5 (\beta^2 + \sigma^2) + \lambda_6 (\beta^2 + \sigma^2 + \sigma \beta) &< 32\pi, \\
2 \beta \alpha \lambda_1 + 4 \sigma \gamma \lambda_3 + 2 \lambda_5 (\beta \alpha + \gamma^2) + \lambda_6 (2 \beta \alpha + 2 \sigma \alpha + \beta + \sigma \gamma + \beta \gamma + \sigma \beta) &< 32\pi, \\
\lambda_1 + \lambda_3 + \lambda_5 &< 32\pi,
\end{align*}
\]
\[\begin{align*}
\alpha^4 \lambda_1 + \gamma^2 \lambda_3 &< 32\pi, \\
\beta^4 \lambda_1 + \sigma^4 \lambda_2 + \sigma^2 \beta^2 \lambda_5 &< 32\pi, \\
6\beta^2 \alpha^2 \lambda_1 + 6\sigma^2 \gamma^2 \lambda_3 + \lambda_5 (4\alpha \beta \gamma \sigma + \gamma^2 \alpha^2 + \gamma^2 \beta^2 + \alpha^2 \sigma^2) &< 32\pi, \\
2\alpha^3 \beta \lambda_1 + 2\lambda_3 \gamma^3 \sigma + \lambda_5 (\beta \alpha \gamma^2 + \alpha^2 \gamma \sigma) &< 16\pi, \\
2\beta^3 \alpha \lambda_1 + 2\lambda_3 \sigma^3 \gamma + \lambda_5 (\beta \alpha \sigma^2 + \beta^2 \gamma \sigma) &< 16\pi, \\
\lambda_4 + \lambda_6 + \lambda_9 &< 16\pi, \\
\lambda_4 + \lambda_6 + \lambda_7 &< 16\pi.
\end{align*}\]

(A.1)

The vacuum stability and the minimization conditions are satisfied only if\[8\]
\[\begin{align*}
\lambda_1 &> 0, \quad \lambda_2 > 0, \quad \lambda_3 > 0, \\
\lambda_4 + 2\sqrt{\lambda_1 \lambda_2} &> 0, \\
\lambda_4 + \lambda_7 + 2\sqrt{\lambda_1 \lambda_2} &> 0 \\
-2\sqrt{\lambda_1 \lambda_3} &< \lambda_5 < 2\sqrt{\lambda_1 \lambda_3}, \\
-2\sqrt{\lambda_2 \lambda_3} &< \lambda_6 < 2\sqrt{\lambda_2 \lambda_3}, \\
-2\sqrt{\lambda_1 \lambda_3} &< \lambda_5 + \lambda_8 < 2\sqrt{\lambda_1 \lambda_3}, \\
-2\sqrt{\lambda_2 \lambda_3} &< \lambda_6 + \lambda_9 < 2\sqrt{\lambda_2 \lambda_3}, \\
4(\lambda_4 + \lambda_7)\lambda_3 - 2(\lambda_5 + \lambda_8)(\lambda_6 + \lambda_9) + 2\sqrt{\Lambda_1} &> 0, \\
4\lambda_4 \lambda_3 - 2\lambda_5 \lambda_6 + 2\sqrt{\Lambda_2} &> 0, \\
4\lambda_4 \lambda_3 - 2\lambda_5 (\lambda_6 + \lambda_9) + 2\sqrt{\Lambda_3} &> 0, \\
4\lambda_4 \lambda_3 - 2(\lambda_5 + \lambda_8)(\lambda_6 + \lambda_9) + 2\sqrt{\Lambda_4} &> 0, \\
4\lambda_4 \lambda_3 - 2(\lambda_5 + \lambda_8)\lambda_6 + 2\sqrt{\Lambda_5} &> 0, \\
4(\lambda_4 + \lambda_7)\lambda_3 - 2\lambda_5 \lambda_6 + 2\sqrt{\Lambda_6} &> 0, \\
4(\lambda_4 + \lambda_7)\lambda_3 - 2\lambda_5 (\lambda_6 + \lambda_9) + 2\sqrt{\Lambda_7} &> 0, \\
4(\lambda_4 + \lambda_7)\lambda_3 - 2(\lambda_5 + \lambda_8)\lambda_6 + 2\sqrt{\Lambda_8} &> 0.
\end{align*}\]

(A.2)

where:
\[\begin{align*}
\Lambda_1 &= (4\lambda_1 \lambda_3 - (\lambda_5 + \lambda_8)^2)(4\lambda_2 \lambda_3 - (\lambda_6 + \lambda_9)^2), \\
\Lambda_2 &= (4\lambda_1 \lambda_3 - \lambda_5^2)(4\lambda_2 \lambda_3 - \lambda_6^2), \\
\Lambda_3 &= (4\lambda_1 \lambda_3 - \lambda_8^2)(4\lambda_2 \lambda_3 - (\lambda_6 + \lambda_9)^2), \\
\Lambda_4 &= (4\lambda_1 \lambda_3 - (\lambda_5 + \lambda_8)^2)(4\lambda_2 \lambda_3 - (\lambda_6 + \lambda_9)^2), \\
\Lambda_5 &= (4\lambda_1 \lambda_3 - (\lambda_5 + \lambda_8)^2)(4\lambda_2 \lambda_3 - \lambda_6^2), \\
\Lambda_6 &= (4\lambda_1 \lambda_3 - \lambda_5^2)(4\lambda_2 \lambda_3 - \lambda_8^2), \\
\Lambda_7 &= (4\lambda_1 \lambda_3 - \lambda_5^2)(4\lambda_2 \lambda_3 - (\lambda_6 + \lambda_9)^2), \\
\Lambda_8 &= (4\lambda_1 \lambda_3 - (\lambda_5 + \lambda_8)^2)(4\lambda_2 \lambda_3 - \lambda_6^2).
\end{align*}\]

(A.3)
Neutral Higgs bosons Phenomenology in the compact $341$ model and confrontations with the LHC results  

Acknowledgments  
We are very grateful to the Algerian ministry of higher education and scientific research and DGRSDT for the financial support.

References  
1. G. Aad et al. (ATLAS), *Phys. Lett. B* **716**, 1 (2012).
2. S. Chatrchyan et al. (CMS), *Phys. Lett. B* **716**, 30 (2012).
3. A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka, and L. Rahilib, *JHEP* **04**, 136 (2012).
4. C. S. Aulakh, A. Mello, and G. Senjanovic, *Phys. Rev. D* **57**, 4174 (1998).
5. A. G. Dias, P. R. D. Pinheiro, C. A. de S. Pires, P. S. Rodrigues da Silva, *Ann. Phys.* **349**, 232 (2014).
6. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
7. W. Caetano, C. A. de S. Pires, and P. S. Rodrigues da Silva, *Eur. Phys. J. C* **73**, 2607 (2013).
8. F. Pisano and V. Pleitez, *Phys. Rev. D* **46**, 410 (1992).
9. P. H. Frampton, *Phys. Rev. Lett.* **69**, 2889 (1992).
10. C. A. de Sousa Pires and O. P. Ravinez, *Phys. Rev. D* **58**, 035008 (1998).
11. C. A. de Sousa Pires, *Phys. Rev. D* **60**, 075013 (1999).
12. P. B. Pal, *Phys. Rev. D* **52**, 1659 (1995).
13. A. G. Dias and V. Pleitez, *Phys. Rev. D* **69**, 077702 (2004).
14. A. G. Dias, C. A. de S. Pires, and P. S. Rodrigues da Silva, *Phys. Rev. D* **68**, 115009 (2003).
15. J. G. Ferreira, C. A. de S. Pires, J. G. Rodrigues, and P. S. Rodrigues da Silva, *Phys. Lett. B* **771**, 199 (2017).
16. M. Djouala, N. Mebarki, and H. Aissaoui, arXiv:1911.04887.
17. D. Cogollo, *Int. J. Mod. Phys. A* **30**, 1550038 (2015).
18. M. Djouala and N. Mebarki, arXiv:2002.08758.
19. Marcela Carena, Ian Low, and Carlos E. M. Wagner, *JHEP* **08**, 060 (2012).
20. N. Mebarki, M. Djouala, J. Mimouni, and H. Aissaoui, *J. Phys. Conf. Ser.*, **1258**, 012011 (2019).
21. J. Beringer et al. (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
22. The ATLAS and CMS Collaborations, *JHEP* **08**, 045 (2016).