The two-step ejection of massive stars and the issue of their formation in isolation

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ABSTRACT

In this paper, we investigate the combined effect of massive binary ejection from star clusters and a second acceleration of a massive star during a subsequent supernova explosion. We call this the two-step-ejection scenario. The main results are as follows. (i) Massive field stars produced via the two-step-ejection process cannot in the vast majority of cases be traced back to their parent star clusters. These stars can be mistakenly considered as massive stars formed in isolation. (ii) The expected O-star fraction produced via the two-step-ejection process is of the order of 1–4 per cent, in quantitative agreement with the observed fraction of candidates for isolated-O-star formation. (iii) Stars ejected via the two-step-ejection process can get a higher final velocity (up to 1.5–2 times higher) than the pre-supernova velocity of the massive-star binary.

Key words: binaries: general – stars: formation – stars: kinematics – supernovae: general.

1 INTRODUCTION

Considering pure number counts, massive stars are by far only a tiny minority in the stellar population of galaxies. But they mainly drive galactic evolution due to their dominating chemical and energetic feedback. Although the importance of massive stars for galactic astrophysics has been accepted, the physical circumstances of their formation are still not resolved, i.e. where, why and how they form.

It is currently strongly debated whether the formation of massive stars is entirely restricted to the interior of massive-star clusters or if they can form in isolation in the galactic field.

Indeed, on the basis of a statistical analysis of a sample of galactic O stars, de Wit et al. (2005) conclude that 4 ± 2 per cent of all O-type stars can be considered as formed outside a cluster environment. They further show that this fraction of isolated O stars is expected if the slope of the cluster mass function (CMF) is \( \beta = 1.7 \). This assumed CMF slope in the low-mass star cluster regime is in disagreement with the slope of \( \beta = 2 \) observed in the solar neighbourhood (Lada & Lada 2003).

The definition of an isolated O star in the initial mass function Monte Carlo simulations of de Wit et al. (2005) is restricted to stellar ensembles which contain only one O star. Parker & Goodwin (2007) strengthened the definition of an isolated O star being an O star without B-star companions but allowed the O star to be surrounded by a cluster with a mass of \(< 100 \, M_\odot\). Using this definition, Parker & Goodwin (2007) conclude that an observed CMF slope of \( \beta = 2 \) can quantitatively explain the statistical analysis by de Wit et al. (2005).

The analysis by de Wit et al. (2004, 2005) identifies a small number of O stars which are deemed to be truly isolated in the sense of not being traceable to an origin in a cluster or OB association. However, Gvaramadze & Bomans (2008) reported the existence of a bow shock associated with the O star HD 165319, which is marked in de Wit et al. (2005) as a very likely candidate for an O star formed in isolation.

In a different study Schibach & Röser (2008) conclude that for up to 9 per cent of all O stars the question of their origin in groups is not solved.

In general, two main processes exist for the production of high-velocity O and B stars, as follows. (i) Close encounters between binaries and single stars or binaries and binaries can result in the ejection of massive stars (e.g. Poveda, Ruiz & Allen 1967; Hoffer 1983; Mikkola 1983, 1984; Leonard 1991). The ejection velocity is of the order of the orbital velocity of the binary (Heggie 1980). Thus, tighter binaries can produce larger ejection velocities. (ii) The supernova explosion of one component of the binary leads to a recoil of the other component (e.g. Zwicky 1957; Blaauw 1961; Iben & Tutukov 1997; Tauris & Takens 1998; Portegies Zwart 2000; Hoogerwerf, de Bruijne & de Zeeuw 2001). The supernova ejection scenario only requires the existence of massive binaries, whereas ejection rates in the dynamical ejection scenario depend on the close encounter frequency. This frequency will be increased if massive stars form in compact few-body groups (Clarke & Pringle 1992; Pflamm-Altenburg & Kroupa 2006).

Various studies on the individual ejection processes exist. To our knowledge only one investigation exists combining both processes (Gvaramadze, Gualandris & Portegies Zwart 2008), where the hypothesis is explored that a hyperfast pulsar can be the remnant of a symmetric supernova explosion of a massive O star dynamically ejected from a young massive-star cluster. In this contribution we...
investigate for the first time the combination of the dynamical and supernova ejection process, in which a massive binary is dynamically ejected from a star cluster with subsequent supernova explosion of one binary component with recoil of the other binary component. We refer to this composite ejection scenario as the two-step-ejection process of massive stars.

We start our investigation in Section 2 with the calculation of the velocity spectrum of stars which are released with the same velocity during a supernova from binaries with identical ejection velocities. We then derive the probability that stars released by a supernova from ejected binaries can be traced back to their parent star cluster (Section 3) and discuss the maximum possible velocity which stars can get in the two-step-ejection process in Section 5.

2 COMPOUND VELOCITY SPECTRUM

Due to dynamical interactions during close encounters of stars, binaries can be ejected from star clusters with the ejection velocity, \( v_c \). If one component of the binary explodes in a supernova, the gravitational force acting on the other component decreases rapidly if the expanding supernova shell has passed the orbit of the stellar companion releasing it with nearly its orbital velocity into the galactic field. Depending on the configuration of the pre-supernova binary and the supernova details (e.g. eccentricity, supernova mass loss, asymmetry of the supernova etc.), the supernova remnant may still be bound to its companion. Portegies Zwart (2000) calculated that between 20 and 40 per cent of the supernova runaways have neutron star companions, but less than 1 per cent are detectable as pulsars. Similar results have been obtained earlier by de Donder, Vanbeveren & van Bever (1997). Irrespective of whether or not the supernova remnant (neutron star or black hole) still remains bound to the other massive star or not we speak of binary disintegration throughout this paper. The post-supernova massive-star runaway is released with a release velocity, \( v_r \), with respect to the centre of mass of the pre-supernova binary.

The vectorial sum of both velocities, the ejection and release velocities, is the new velocity of the released star. We call this velocity the compound velocity, \( v_c \). For fixed ejection velocity, \( v_c \), and release velocity, \( v_r \), the compound velocity, \( v_c \), is distributed, because the direction of the release of the star from the disintegrating binary during the supernova is randomly distributed in space. The corresponding spectrum of the compound velocity, \( f_c(v_c) \), defines the number of stars, \( dN(v_c) \), which have a compound velocity, \( v_c \), after they have been released from ejected binaries.

2.1 Calculating \( f_c(v_c) \)

We start the calculation of the resulting compound velocity spectrum with the definition of the compound velocity through vectorial addition:

\[
v_c = v_e + v_r.
\]  

The relation between the absolute values of the velocities follows from the cosine theorem (Fig. 1):

\[
v_c^2 = v_e^2 + v_r^2 - 2v_e v_r \cos \theta.
\]  

Differentiation leads to the relation

\[
\frac{dv_c}{d\theta} = \frac{v_r}{v_c} \sin \theta
\]  

between the compound velocity and the release angle \( \theta \).

![Figure 1](https://academic.oup.com/mnras/article-abstract/404/3/1564/1053814)

**Figure 1.** Illustration of the compound velocity. A binary is ejected with the velocity \( v_c \). One component is then released with the velocity \( v_r \) resulting in the compound velocity \( v_c \) given by the vectorial sum.

The orientations of the binaries are randomly distributed in space. Thus, the released stars are \( 4\pi \) distributed with respect to the centre-of-mass system of the binary. If a set of binaries release \( N \) stars isotropically, the number of stars \( dN(\theta) \) released in a small angle \( d\phi \) is given by the ratio of the area of the small circular stripe, \( dA(\theta) \), to the angle \( \theta \) and the unit sphere, \( 4\pi \) (Fig. 1):

\[
\frac{dN(\theta)}{N} = \frac{dA(\theta)}{4\pi}.
\]  

where the area of the small circular stripe is

\[
dA(\theta) = 2\pi \sin(\theta) d\theta.
\]  

By combining these equations, the number fraction of stars having the compound velocity \( v_c \) is

\[
\frac{dN(v_c)}{dv_c} = \frac{N}{2v_c v_r}.
\]  

As the angle \( \theta \) varies from 0 to 2\( \pi \), the allowed range of the compound velocity can be obtained from the above cosine theorem and is

\[
|v_c - v_r| \leq v_c \leq v_r + v_c.
\]  

The distribution function, \( f_c(v_c) \), of the compound velocity is normalized by

\[
1 = \int f_c(v_c) \, dv_c,
\]  

and is for a constant ejection velocity, \( v_e \), and constant release velocity, \( v_r \),

\[
f_c(v_c) = \frac{1}{2v_e v_r} v_r \Theta(v_r - |v_c - v_r|) \Theta(v_r + v_c - v_r),
\]  

where the \( \Theta \) mapping is defined by

\[
\Theta(x) = \begin{cases} 
1; & x \geq 0 \\
0; & x < 0
\end{cases}
\]  

The form of the compound spectrum can be seen in Fig. 2. Compound velocities at the high speed end are preferred. But the distribution function, \( f_c(v_c) \), flattens with increasing ejection or release velocity. The distribution is symmetric in \( v_c \) and \( v_r \). Note that in equation (8) an additional factor \( 4\pi v_r^2 \), as for example in the Maxwellian distribution calculated from a three-dimensional Gaussian distribution function, is not required, because \( f_c(v_c) \) already refers to an absolute velocity value and is not calculated from spatial integration over a three-dimensional distribution function.
If the velocities of the ejected binaries and of the released stars are distributed according to uncorrelated distribution functions, \( f_e(v_e) \) and \( f_r(v_r) \), then the resulting distribution of compound velocities is calculated by integration over both distributions:

\[
 f_c(v_c) = \frac{v_c}{2} \int \int \frac{f_e(v_e) f_r(v_r)}{v_c v_e} \Theta(v_c - |v_e - v_r|) \Theta(v_c + v_e - v_r) \, dv_e \, dv_r. \tag{11}
\]

### 2.2 Properties of \( f_c(v_c) \)

In the following, we derive some properties of the compound velocity spectrum.

(i) The simplest case we can consider is that if one of the velocities, \( v_e \) or \( v_r \), is zero. If for example \( v_e \) converges against zero, then the velocity spectrum converges against the \( \delta \) distribution:

\[
 f_c(v_c) = \lim_{v_e \to 0} f_c(v_c) = \delta(v_c - v_r). \tag{12}
\]

Because \( f_c(v_c) \) is symmetric in \( v_c \) and \( v_r \), the same result follows for \( v_r \to 0 \). The compound velocity is identical to the ejection or release velocity.

(ii) The released stars are not necessarily faster than the previous binaries. They can also be decelerated. The fraction of stars (\( \mu_{>v_c} \)) which are accelerated, i.e. having a compound velocity greater than the ejection velocity, is given by the integral

\[
 \mu_{>v_c} = \int_{v_c}^{v_e + v_r} f_c(v_c) \, dv_c. \tag{13}
\]

The evaluation of the integral leads to

\[
 \mu_{>v_c} = \frac{1}{4v_c v_e} \left( v_e^2 + 2v_c v_e - \Theta(v_c - |v_e - v_r|) \left( v_e^2 - 2v_c v_r \right) \right). \tag{14}
\]

The fraction of accelerated stars as a function of the ejection velocity for different release velocities can be seen in Fig. 3. Two cases can be distinguished:

\[
 |v_e - v_r| \geq v_c : \mu_{>v_c} = 1 \tag{15}
\]

and

\[
 |v_e - v_r| < v_c : \mu_{>v_c} = \frac{v_c}{4v_e} + \frac{1}{2}. \tag{16}
\]

We define

\[
 v_{\min} = \min\{v_e, v_r\}, \quad v_{\max} = \max\{v_e, v_r\} \tag{18}
\]

and write

\[
 v_e + v_r = v_{\min} + v_{\max} \tag{19}
\]

and

\[
 |v_e - v_r| = v_{\max} - v_{\min}. \tag{20}
\]

Finally, the mean compound velocity can be written as

\[
 \bar{v}_c = \int_{|v_e - v_r|}^{v_e + v_r} v_c f_c(v_c) \, dv_c = \frac{1}{6v_c v_e} \left( \frac{v_{\min} + v_{\max}}{v_{\max}} \right). \tag{21}
\]

It follows that the mean compound velocity is always greater than or equal to the ejection and release velocity:

\[
 \bar{v}_c \geq \frac{1}{3} \frac{3v_{\max}^2}{v_{\max}} = v_{\max}. \tag{22}
\]

For the case that \( v_e = v_r \), which will be important for Section 5, i.e. the ejection velocity is comparable to the orbital velocity of the ejected binary, the mean compound velocity is

\[
 \bar{v}_c = \frac{4}{3} v_e. \tag{23}
\]

### 3 Back-Tracing Probability

If single stars or binaries are ejected from star clusters, it is theoretically possible to calculate their orbits backwards if the force field, through which the objects have moved in time, is given. In such a case the star cluster, where the stars have their origin, can be identified.

If the binary disintegrates due to a supernova, the released component suffers a strong deflection of its previous binary orbit, and the
A binary is ejected from a star cluster and moves along path 1. After the binary has moved the distance $\xi$ from the centre of the cluster $O$, it disintegrates at the position $P$ and one component of the binary is released. The star cluster, from which the star has been ejected, is identified if the extrapolated path of the released star intersects a sphere with the identification radius, $R_i$, round the star cluster. If the released star moves along path 2, the star cluster is identified (extrapolated long dashed line). If the star moves along path 3, the parent star cluster cannot be identified (extrapolated short dashed line). Only stars which have new orbits within the grey-shaded region between the dotted lines can be traced back to their parent star cluster.

At the location of binary disintegration (point $P$), the identification sphere appears under the angle $2\alpha$, i.e. a solid angle of $2\pi(1 - \cos \alpha)$. Because the directions of the released stars are randomly distributed over the full solid angle of $4\pi$, the probability that the star cluster can be identified is given by the ratio of the solid angle of the identification sphere to the full solid angle:

$$P = \frac{1 - \cos \alpha}{2}. \quad (24)$$

The cosine can be expressed by the identification radius, $R_i$, and the disintegration distance, $\xi$:

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \sqrt{1 - \frac{R_i^2}{\xi^2}}. \quad (25)$$

Then the identification probability is

$$P = \frac{1}{2} \left(1 - \sqrt{1 - \frac{R_i^2}{\xi^2}}\right). \quad (26)$$

The resulting probabilities for different ratios of the identification radius and disintegration distance are listed in Table 1.

### 4 OBSERVED STATISTICS OF RUNAWAYS AND APPARENTLY ISOLATED O STARS

Massive stars and massive binaries can only be ejected dynamically from star clusters during the early stage of their life. If they form within compact few-body configurations or trapezia systems (Clarke & Pringle 1992), then the decay time-scale of these few-body systems (< 1 Myr; Pfamm-Altenburg & Kroupa 2006) implies only early ejections of massive stars. The close encounter probability of massive stars and binaries and therefore their ejection rates increase with increasing stellar number density. Due to gas expulsion, young embedded star clusters become super-virial and start to expand and the stellar density decreases rapidly (Kroupa, Aarseth & Hurley 2001). The time-scale of the decrease of the stellar density is comparable to the gas expulsion time-scale, of the order of $\approx 1$ Myr. Thus, the time of flight of ejected binaries is comparable to their maximum lifetime.

Taking a lower velocity cut-off for O-star runaways of 30 km s$^{-1}$ as considered in Gies & Bolton (1986) and a mean lifetime of 5 Myr of O stars implies a disintegration distance $\xi = 150$ pc. For a large identification radius of $R_i = 10$ pc of the star cluster, the back-tracing probability is 1\% (Table 1). Lowering the cut-off velocity of the runaway-star definition to 10 km s$^{-1}$ results in a back-tracing probability of 1 per cent. Thus, ejected massive binaries which are listed in runaway O-star surveys will produce O stars which cannot be traced back to their parent star cluster. But one might expect that in this case, the massive star can be traced back to a supernova shell. However, as single supernova shells disappear on a time-scale of 0.5–1 Myr (Chevalier 1974) it might be possible to identify the parent supernova of the released star only in very rare cases.

The observationally derived runaway fraction among O stars varies widely in the literature (see e.g. table 13 in Gies & Bolton 1986 or table 5 in Stone 1991). Gies & Bolton (1986) identified 15 stars out of a sample of 36 runaway candidates with confirmed peculiar radial velocities of $\geq 30$ km s$^{-1}$. Comparing with their stated total number of about 90 O stars within their sample space, a runaway fraction of 15/90 = 16 per cent results. They further conclude that the true runaway fraction of O stars depends on the adopted velocity cut-off and may lie in the range of 10–25 per cent. Gies & Bolton (1986) also found the binary fraction among runaway O stars in their sample to be about 10 per cent. As explained above, O stars released in a supernova in an ejected massive binary result in field O stars which cannot be traced back to their parent star clusters. Thus, on the basis of the O-star runaway fraction and binary fraction of O-runaway data published by Gies & Bolton (1986), 1–2.5 per cent of O stars cannot be traced back to the star cluster where they have formed. These O stars will appear to have formed in isolation, although they were born in an ordinary star cluster.

The different O-star runaway fractions in the literature have been unified by Stone (1991) by considering true space frequencies. The radial velocity spectrum of O stars is decomposed into two different Gaussian velocity distributions, which correspond to two different Maxwellian space velocity distributions. The high-velocity component has a number fraction of $f_H = 46$ per cent and a velocity distribution of $\sigma_H = 28.2$ km s$^{-1}$. By transforming individual O-star runaway studies, which are based on individual runaway definitions, to true space frequencies based on bimodality in the velocity

| $R_i/\xi$ | 1 | 1/2 | 1/5 | 1/10 | 1/15 | 1/50 |
|-----------|---|-----|-----|------|------|-----|
| $P$ (%)   | 500 | 67 | 10 | 2.5 | 1 | 0.1 |

*Note. $R_i$ is the radius of the identification sphere around the star cluster from which the binary has been ejected. $\xi$ is the distance between the location of the disintegration of the binary and the centre of the identification sphere which coincides with the centre of the star cluster. $P$ is the back-tracing probability in %.*
distribution of O stars, Stone (1991) achieves good agreement between the different individual studies.

From the runaway fraction of 46 per cent derived by Stone (1991), it follows that a fraction of 4.6 per cent of O stars will apparently form in isolation if the runaway binary fraction of 10 per cent by Gies & Bolton (1986) is used.

Thus, the two-step-ejection process predicts a fraction of O stars which have formed apparently in isolation in the range of 1–4.6 per cent. de Wit et al. (2004, 2005) conclude, based on the actually observed positions and velocities of O stars, that 4 ± 2 per cent of O stars can be considered as candidates of massive stars formed in isolation, because such stars cannot be traced back to a young star cluster. Consequently, the process of two-step ejection can quantitatively account for the proposed fraction of massive candidates formed in isolation.

5 MAXIMUM POSSIBLE VELOCITY

The maximum possible velocity in the two-step-ejection process is
\[ v_{\text{max}} = v_{c} + v_{e}, \]
where \( v_{e} \) is the ejection velocity of the single star, and \( v_{c} \) is the release velocity of the ejected star.

If, on the other hand, the companion of the massive star is a low-mass star then the ejection velocity of the binary is equal to the ejection velocity of the single star; thus, we have \( v_{e} = v_{c} \). The maximum possible velocity of the released star is \( v_{\text{max}} = \frac{1}{2} v_{o} \).

If the ejection velocity of the binary is the vectorial sum of the ejection velocity of the single star and the release velocity of the binary, \( v_{r} \), then the max-

6 CONCLUSION

Various theoretical and numerical studies exist on individual ejection processes of massive stars, namely the dynamical ejection scenario and the supernova ejection scenario, but the combined effect of both scenarios for the distribution of massive stars in a galaxy has not been considered yet. In this paper, we investigate for the first time the implications of the combination of the dynamical and the supernova ejection scenario for the O-star population of the Galactic field. We call this combined effect the two-step-ejection process of massive stars. The main results are as follows.

(i) The compound velocity, \( v_{r} \), which is the vectorial sum of the ejection velocity, \( v_{e} \), of the binary from the star cluster and the release velocity, \( v_{c} \), with which a star is released during a supernova can be larger or smaller than the previous ejection velocity. Stars can be both, accelerated and decelerated.

(ii) The mean compound velocity is always greater than each of the initial velocities, \( v_{e} \) and \( v_{c} \) (equation 22).

(iii) It is very unlikely that the parent star cluster of a massive field star produced by the two-step-ejection scenario can be identified. The expected number fraction of such massive field stars which are formed apparently in isolation can account quantitatively for the number of candidates for isolated massive-star formation derived in de Wit et al. (2005).

(iv) Massive stars which are ejected via the two-step scenario can get higher maximum space velocities (up to 1.5 times higher for equal-mass binary components or two times higher for significantly unequal-mass companion masses) than can be obtained by each process (dynamical or supernova ejection) individually.

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