Composite Weak Bosons: a Lattice Analysis

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We present a lattice analysis of a confining Yang-Mills theory without Goldstone boson. We have analytically investigated the model by a strong coupling expansion and by an intensive lattice Monte Carlo simulation using standard lattice QCD methods. We show that this theory is an interesting candidate for describing weak bosons as composite particles.

The Standard Model [1] (SM) describes the strong, weak and electromagnetic interactions by a gauge theory based on the group $G_{SM} = SU(3) \times SU(2) \times U(1)$ which is broken by the Higgs mechanism to $SU(3) \times U(1)$. The theory is essentially determined once the matter fields and their transformation under the local gauge transformations of $G_{SM}$ are specified. The matter fields (leptons and quarks) and the Higgs boson are considered to be elementary. They interact with each other by the exchange of gauge bosons which are also considered to be elementary. The structure of the SM has been phenomenologically confirmed to high accuracy.

In spite of the beautiful corroboration of the SM by experiments a natural question arises: How elementary are the leptons, the quarks, the Higgs bosons and the gauge bosons? The idea that the SM itself is an effective theory of another, more fundamental, theory, where quarks, leptons and bosons are composites of more fundamental fields, is almost as old as the SM itself. The idea of quark and lepton compositeness is motivated by the observed connection between quarks and leptons, by the generation problem and by the existence of too many parameters in the SM. The Higgs compositeness is motivated by the fine tuning problem. The W and Z compositeness is motivated by their relation to a composite Higgs and by the observation that all short-range interactions are residual interactions of a more fundamental long-range interaction.

The substructures, the new fundamental fields, are supposed to carry a new internal quantum number (which we refer to hypercolor) and the quarks, leptons and bosons are hypercolorless composite systems of them. The binding of the substructures due to hypercolor is viewed as an analogy with the color confinement mechanism of QCD. However, since the SM spectrum is different from the hadron spectrum, the hypercolor interaction has to be described by a strongly coupled Yang-Mills theory different from QCD.

Several models treat the quarks, leptons and bosons as composite systems. Today a conspicuous number of theorems exist which have ruled out most of the existing models and radically restricted the possibilities of constructing realistic composite models [2]. One particular model has survived: The Yang-Mills theory without Goldstone bosons [3]. This model is a usual confining Yang-Mills theory with $SU(2)$ local hypercolor gauge group, $SU(2)$ global isospin group and generalized Majorana fermions in the fundamental representation of the local and global symmetry groups.

We consider a gauge theory whose fermion content is represented by a Weyl spinor $F_{\alpha,a}(x)$. Here $\alpha$ denotes the (undotted) spinor index ($\alpha = 1, 2$), $A$ denotes the fundamental representation index of a global $SU(2)$ isospin group ($A = 1, 2$) and $a$ denotes the fundamental representation index of the local $SU(2)$ hypercolor gauge group ($a = 1, 2$). We introduce the generalized Majorana spinor $\psi$ starting from the Weyl spinors $F$ and its conjugate $F^\dagger$

$$\psi(x) = \begin{pmatrix} F(x) \\ QF^\dagger(x) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & Q \end{pmatrix} \varphi(x) \quad (1)$$
and its adjoint
\[ \tilde{\psi}(x) = (F^T(x)Q, F^\dagger(x)) = \varphi^T(x) \begin{pmatrix} Q & 0 \\ 0 & 1 \end{pmatrix}. \] (2)

The matrix \( Q \) represents the antisymmetric matrix in spin, hypercolor and isospin space. Of course the fields \( \psi \) and \( \bar{\psi} \) are not independent fields. The choice of the global isospin group \( SU(2) \) and of the local hypercolor group \( SU(2) \) allows us to write a gauge invariant mass term for the generalized Majorana fermion fields
\[ \bar{\psi}\psi = FQF + F^\dagger QF^\dagger. \] (3)

Note that this choice is unique if one deals with Majorana fermions. Because of the existence of the mass term we can define the Yang-Mills action on the lattice in Euclidean space in the form of a Wilson action. For further details we refer to ref. [5].

To be precise, this model considers the photon to remain elementary and switched off. The weak gauge bosons \( W^\pm \) and \( Z^0 \) then form a mass degenerate triplet. The vector isotriplet bound state of the substructure represents the W-boson triplet. To be viable a composite model of the weak bosons has to reproduce the known weak boson spectrum: the lightest bound states have to be the W-bosons and heavier bound states have to lie in an experimentally unexplored energy range. The only possibility of having a Yang-Mills theory which reproduces the weak boson spectrum is to choose the degrees of freedom in a way that they naturally avoid bound states lighter than the vector isotriplet of the theory which characterizes the W-boson triplet. This is possible if the unwanted light bound states which naturally show up as Goldstone bosons or pseudo Goldstone bosons in many models (like, for example, a pseudoscalar isomultiplet, which would be the pion analogue of QCD) are avoided. The choice of the generalized Majorana fermions in this model avoids the \( SU_A(2) \) global chiral symmetry of the Yang-Mills Lagrangian because left- and right-handed degrees of freedom are not independent. The axial current (which would generate the \( SU_A(2) \) chiral symmetry) does not exist and it is not possible to have a breaking of \( SU_A(2) \) with the related low lying Goldstone bosons. In fact, the pseudoscalar isotriplet vanishes by the Pauli principle (it is a symmetric combination of Grassmann variables).

Because of the confining character of this theory, we need non-perturbative methods to make predictions. It is important that the fermion theory under discussion can be defined by a gauge invariant lattice regularization. A lattice regularization à la Wilson is possible because the choice of the isospin group \( SU(2) \) allows us to replace the Dirac mass term and the Dirac-type Wilson term by a hypercolor gauge invariant Majorana type expression.

An extensive strong coupling expansion analysis of the spectrum of this theory has shown that the spin one isotriplet bound state (the right quantum number to represent the W-boson of the SM) could be the lightest state if the pseudoscalar isosinglet acquires a mass by the chiral anomaly in analogy with the \( \eta' \) in QCD.

We also calculated the spectrum of the lightest bound states by a quenched Monte Carlo simulation and we showed that the vector isotriplet bound state of this theory is the lightest one. We have performed our Monte Carlo simulations using the following standard technique of lattice QCD:

1. We used the quenched approximation. As in QCD we assume that the quenched approximation is reasonable also in our model.

2. To generate the quenched gauge configurations we used the heat-bath and over relaxed updating (1 heat-bath sweep for 6 over relaxed sweeps).

3. We used the symmetrized Peskin’s formula and the cooling algorithm to measure the topological charge of the gauge configurations. Topological non-trivial configurations are needed to evaluate the chiral anomaly contribution to the mass of the pseudoscalar isosinglet bound state.

4. To invert the fermion matrix we used the minimal residual and the conjugate gradient algorithms.
We used the smearing technique (PSI-Wuppertal smearing) to obtain early plateaux in the local masses.

To connect the measured quantities to physics we have to fix the lattice spacing $a$. For setting the lattice spacing $a$ we identify the experimental value of the $W$-boson triplet $M_W = 80$ GeV with the value of the mass of the vector isotriplet determined from the simulations and extrapolated to the chiral limit. We expect that the mass of the substructure is much smaller than a typical binding energy, therefore in analogy with QCD we extrapolated the bound state masses to the chiral limit, determined by the critical hopping parameter. In our model there is no Goldstone boson which can identify the chiral limit. However, the critical hopping parameter can be evaluated by assuming that the pseudoscalar isosinglet behaves like a Goldstone boson when the contribution of the chiral anomaly is switched off and therefore it is massless.

We have performed the simulations on different lattices and with different values of $\beta$ to control finite $a$ and volume effects. In some simulations we have calculated the masses of bound states without the contribution of the chiral anomaly. These simulations confirmed the results of the strong coupling expansion. In other simulations we have used and improved the method of ref. to evaluate the chiral anomaly contribution to the mass of the pseudoscalar isosinglet bound state. This computation required the evaluation of disconnected fermion loops and the generation of topological non-trivial gauge configurations. The mass of the pseudoscalar isosinglet turned out to be larger than the vector isotriplet mass.

As a main result our lattice simulations has shown that the vector isotriplet bound state, which represents the weak boson triplet, is the lightest bound state in our model as it should be for any viable candidate of electroweak composite model. In addition we have also predicted the masses of the first bound states which are heavier than the vector isotriplet representing the weak boson triplet. These bound states are an additional vector isotriplet, a vector isosinglet and a pseudoscalar isosinglet with masses in the range of a few hundred GeV. These predictions open new interesting experimental perspectives at LEPII and LHC.

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