A TIME VARYING STRONG COUPLING CONSTANT AS A MODEL OF INFLATIONARY UNIVERSE

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Abstract

We consider a scenario where the strong coupling constant was changing in the early universe. We attribute this change to a variation in the colour charge within a Bekenstein-like model. Treating the vacuum gluon condensate $< G^2 >$ as a free parameter, we could generate inflation, similar to a chaotic inflation scenarios, with the required properties to solve the fluctuation and other standard cosmology problems. A possible approach to end the inflation is suggested.

Key words: fundamental constants; inflationary models; cosmology; QCD

PACS: 98.80Cq,98.80-k,12.38-t

1 Introduction

The hypothesis that the universe underwent a period of exponential expansion at very early times is the most popular theory of the early universe. The
“old” \[1\] and “new” \[2, 3\] inflationary universe models were able to solve the “horizon”, “flatness” and “structure formation” problems creating, in turn, their own problems, some of which the “chaotic” model \[4\] and its extensions \[5\] could tackle. On the other hand, these kind of cosmological models are favored by recent measurements from the Cosmic Microwave Background (CMB) \[6, 7\] and the WMAP results \[8\]. Usually, inflationary models are all based on the use of new fundamental scalar fields, the “inflatons”, which can not be the Higgs fields of ordinary gauge theories. Later, other possible alternatives to inflationary cosmology were proposed. Rather than changing the matter content of the universe, new concepts were adopted such as non-commutative space-time quantizations \[9\], brane settings of a cycling universe \[10\], and a change in the speed of light in the early universe \[11, 12\] or, more generally, a varying fine structure constant \(\alpha_{em}\) \[13\]. In \[14\], we have generalized Bekenstein model \[15\] for the time variation of \(\alpha_{em}\) to QCD strong coupling constant \(\alpha_S\) and found that experimental constraints going backward till quasar formation times rule out \(\alpha_S\) variability. In this letter, we discuss how our “minimal” Bekenstein-like model for \(\alpha_S\), when implemented in the very early universe, can provide a realization of inflation driven by the trace anomaly of QCD energy momentum tensor. Inflationary models driven by the trace anomaly of conformally coupled matter fields are treated in the literature \[16\] while in our toy model we concentrate on the gauge fields of QCD. Assuming the universe is radiation-dominated at early times and that vacuum expectation values predominate over matter densities, we find, with suitable values of the gluon condensate \(< G^2 >\) far larger than its present value and the Bekenstein length scale \(l\) smaller than the Planck-Wheeler length scale \(L_P\), that our model for inflation is self-consistent with acceptable numerical results to solve the fluctuation and other problems. We find that the QCD lagrangian with a changing strong coupling constant leads naturally to a monomial quadratic potential like the chaotic scenario. However, while the large values of the inflaton matter field plague the latter scenario, they just amount in our model to a reduction of the strong charge by around 10 times during the inflation. We shall not dwell on the possible mechanisms by which the gluon condensate could have decreased to reach its present value, and which might be necessary in order not to have exotic relics. Rather we wish to concentrate on the conditions we should impose on our model to be interesting. We hope this phenomenological approach could prompt further work on a possible connection between time-varying fundamental “constants” and inflationary models.
2 Analysis

We follow the notations of [14]. Therein, we used the QCD Lagrangian with a varying coupling “constant”

\[ L_{QCD} = L_\epsilon + L_g + L_m \]

\[ = - \frac{1}{2l^2} \frac{\epsilon^\mu_{\epsilon,\mu}}{\epsilon^2} - \frac{1}{2} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_f \tilde{\psi}^{(f)} (i\gamma^\mu D_\mu - M_f) \psi^{(f)} \]  

(1)

where the metric used for raising and lowering indices is the R-W metric, \( l \) is the Bekenstein scale length, \( \epsilon(x) \) is a scalar gauge-invariant and dimensionless field representing the variation of the strong coupling “constant” \( g(x) = g_0 \epsilon(x) \), \( D_\mu = \partial_\mu - i g_0 \epsilon(x) A_\mu \) is the covariant derivative and where the gluon tensor field is given by:

\[ G_{\mu\nu}^a = \frac{1}{\epsilon} \left[ \partial_\mu (\epsilon A_{\nu}^a) - \partial_\nu (\epsilon A_{\mu}^a) + g_0 \epsilon^2 f^{abc} A_{\mu}^b A_{\nu}^c \right] \]  

(2)

We assume homogeneity and isotropy for an expanding universe and so consider only temporal variations for \( \alpha_S \equiv \frac{\alpha^2(t)}{4\pi} = \alpha_{S_0} \epsilon^2(t) \). We assume also, rather plausibly in the radiation-dominated early universe, negligence of matter contribution to get the following equations of motion

\[ \left( \frac{G_{\mu\nu}^a}{\epsilon} \right)_{;\mu} - g_0 f^{abc} G_{\nu}^b A_{\mu}^c + \sum_f g_0 \bar{\psi}^{(f)} D_\nu^{(f)} \psi^{(f)} = 0 \]  

(3)

\[ (a^2 \frac{\dot{\epsilon}}{\epsilon}) = \frac{a^3(t)}{2} \langle G^2 \rangle \]  

(4)

where \( a(t) \) is the expansion scale factor in the R-W metric.

Computing the canonical energy-momentum tensor \( \frac{\partial L}{\partial (\partial_\alpha \phi_i)} \partial^\beta \phi_i - g^\alpha\beta L \) we get a non gauge invariant quantity. This may be cured by a standard technique [17] amounting to a subtraction of a total derivative and hence not changing the equations of motion. We subtract the total derivative \( \Delta T^{\alpha\beta} = \partial_\nu \left( - \frac{G^{\alpha\nu}}{\epsilon} \right) A^{\alpha\beta} \) to get, with the use of equation (3), the gauge-invariant energy momentum tensor

\[ T^{\alpha\beta} = G^{\alpha\nu} G_{\nu}^{\alpha\beta} + i \sum_f \bar{\psi}^{(f)} \gamma^{(\alpha D^{\beta})} \psi^{(f)} - \frac{1}{l^2} \frac{\partial^\alpha \epsilon \partial^\beta \epsilon}{\epsilon^2} \]

\[ - g^{\alpha\beta} \left[ - \frac{1}{4} G^{\mu\nu} G^{\alpha\beta}_{\mu\nu} + \sum_f \bar{\psi}^{(f)} (i\gamma^\mu D_\mu - M_f) \psi^{(f)} - \frac{1}{2l^2} \frac{\epsilon_{\mu\nu} \epsilon_{\mu'\nu'}}{\epsilon^2} \right] \]  

(5)
Since we assume radiation dominated era during the very early universe we can concentrate on the gauge fields and neglect the matter fields contribution and so we decompose our energy momentum tensor into two parts: the gauge part and the \( \epsilon \)-scalar field part

\[
T_{\alpha \beta} = T_{\alpha \beta}^g + T_{\alpha \beta}^\epsilon
\]

\[
= \left( G_{\alpha \nu} G_{\beta}^{\nu} - g_{\alpha \beta} \left[ -\frac{1}{4} G_{\mu \nu}^{\alpha} G_{\mu \nu}^{\alpha} \right] \right) + \left( -\frac{1}{l^2} \frac{\partial_{\alpha} \epsilon \partial_{\beta} \epsilon}{\epsilon^2} - g_{\alpha \beta} \left[ -\frac{1}{2l^2} \frac{\epsilon_{\mu} \epsilon_{\mu}}{\epsilon^2} \right] \right)
\]

Here all the operators are supposed to be renormalized and it is essential in the inflationary paradigm that quantum effects are small in order to get small fluctuations in the CMB, which will be seen to be the case in the model.

The contribution of the scalar field to the energy density \( \rho_\epsilon = T_{\epsilon 00}^\epsilon \) and to the pressure \( T_{ij}^\epsilon = g_{ij}; \rho_\epsilon \) can be computed and we get

\[
\rho_\epsilon = -\frac{1}{2} l^2 \left( \frac{\dot{\epsilon}}{\epsilon} \right)^2 = p_\epsilon
\]

On the other hand, the gauge field contribution \( T_{\alpha \beta}^g \) can be decomposed into trace and traceless parts. In such a way, we can write the corresponding energy mass density \( \rho_g = T_{00}^g \) and the pressure \( p_g \), like in “ordinary” QCD, as a sum of two terms:

\[
\rho_g = \rho_g^r + \rho_g^T
\]

\[
p_g = p_g^r + p_g^T
\]

where \( \rho_g^r \) is the density corresponding to the “traceless” part of the gauge field satisfying

\[
\rho_g^r = 3p_g^r
\]

while the trace part of the gauge field energy-momentum tensor is proportional to \( g_{\alpha \beta} \) and behaves like a ‘cosmological constant’ term. Thus, the corresponding equation of state reads:

\[
\rho_g^T = -p_g^T
\]

We shall assume, here, that the trace anomaly relation for \( T_{\mu \nu}^g \) is the same as the “ordinary” QCD trace anomaly. This can be justified because the energy-momentum tensor \( T_{\alpha \beta}^g \) is identical in form to “ordinary” QCD and that the trace anomaly which involves only gauge invariant quantities should, by dimensional analysis, be proportional to \( G^2 \). However, we have checked
that a change in the numerical value of the proportionality factor would not alter our conclusions. Thus we take, up to leading order in the time-varying coupling “constant” \( \alpha_S = \alpha_{S0} \varepsilon^2 \), the relation \[18\]:

\[
T_{\mu}^{\nu} = \rho_g - 3p_g = -\frac{9\alpha_{S0} \varepsilon^2}{8\pi} G_{\mu}^{\mu} G_{\nu}^{\nu}
\]  

Again, neglecting matter contribution during the radiation dominated era, we limit our gluon operator \( G_{\mu}^{\mu} G_{\nu}^{\nu} \) matrix elements to its condensate vacuum expectation value \( <G^2> \), and so we get:

\[
\rho_g^T = -\frac{9\alpha_{S0} \varepsilon^2}{32\pi} <G^2>
\]  

Hence, we can write the total energy mass density as

\[
\rho = \rho_g + \rho_g^T + \rho_c
\]  

and equation (11) would suggest, in analogy to ordinary inflationary models, that the QCD trace anomaly could generate the inflation. For this, let us assume that the “trace-anomaly” energy mass density contribution is much larger than the other densities:

\[
\rho_g^T >> \rho_c, \rho_g \Rightarrow \rho \sim \rho_g^T
\]  

Then, equation (13) tells that the vacuum gluon condensate \( <G^2> \) should have a negative value which is not unreasonable since the inflationary vacuum has “strange” properties. In ordinary inflationary models, it is filled with repulsive-gravity matter turning gravity on its head \[19\]. This reversal of the vacuum properties is reflected, in our model, by a reversal of sign for the vacuum gluon condensate.

Now we seek a consistent inflationary solution to the FRW equations in a flat space-time:

\[
\frac{(\dot{a})^2}{a} = \frac{8\pi G_N}{3} \rho
\]  

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} (\rho + 3p)
\]  

where \( G_N \) is Newton’s constant. The first FRW equation with (13) will give

\[
H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{3\alpha_{S0} \varepsilon^2}{4}} \frac{G_N}{|<G^2>|}
\]
On the other hand, the equation of motion (eq.4) of the scalar field can be expressed in the following way:
\[
\frac{\ddot{\epsilon}}{\epsilon} + 3H \frac{\dot{\epsilon}}{\epsilon} - \left(\frac{\dot{\epsilon}}{\epsilon}\right)^2 = \frac{\ell^2 < G^2 >}{2}
\]
(19)
This equation differs from the ordinary ‘matter’ inflationary scenarios in the term \( (\frac{\dot{\epsilon}}{\epsilon})^2 \). However, for “slow roll” solutions we neglect the terms involving \( \frac{\ddot{\epsilon}}{\epsilon} \) and \( (\frac{\dot{\epsilon}}{\epsilon})^2 \) to get
\[
3H \dot{\epsilon} = \frac{\ell^2 < G^2 >}{2} \epsilon = -V'(\epsilon)
\]
(20)
which is the same as the “slow roll” equation of motion of the inflaton in ordinary scenarios. In our model, the “slow roll” condition can be written as:
\[
\delta \equiv \left| \frac{\dot{\epsilon}}{H\epsilon} \right| = \frac{2}{9\alpha S_0} \left( \frac{l}{L_P} \right)^2 \frac{1}{\epsilon^2} << 1
\]
(21)
This gives us the first hint that Bekenstein hypothesis \((L_P < l)\) might not survive. Now, we set \( \epsilon_f \), the value of \( \epsilon \)-field at the end of inflation \( t_f \), to 1 so that the time evolution of the strong coupling terminates with the end of inflation. We expect for “slow roll” solutions that \( \epsilon_i \), the value of \( \epsilon \) at the initial time of inflation \( t_i \) corresponding to when the CMB modes freezed out, to be of order 1. Taking this into account, and in order that the changes of the Hubble constant and the energy mass density are not very large during the inflation, we assume the gluon condensate value \( < G^2 > \) to stay approximately constant during much of the inflation. Then we get a linear evolution in time for the \( \epsilon \)- scalar field
\[
\epsilon(t) = \epsilon_i - \frac{1}{3^{3/2}(\alpha S_0)^{1/2}} \left( \frac{l}{L_P} \right)^2 G^{1/2} N \left| < G^2 > \right|^{1/2} (t - t_i)
\]
(22)
and we have, as in chaotic scenarios, a simple quadratic potential:
\[
V(\epsilon) = \frac{\ell^2 | < G^2 > |}{4} \epsilon^2
\]
(23)
One can make explicit the correspondence between our model with \( \epsilon \)-scalar field and the chaotic scenario with an \( \phi \)-inflaton matter field. Looking at equation (13) and comparing with \( \rho = V(\phi) \) in ordinary inflationary models, we see that the gluon condensate plays a role of a potential for the “inflaton”-\( \epsilon \) field. Comparing equations (18) and (20) with the corresponding relations in ordinary inflationary models:
\[
H^2 = \frac{8\pi}{3M^2_P} G_N V(\phi)
\]
(24)
\[
3H \dot{\phi} = -V'(\phi)
\]
(25)
and remembering that the dimension of the matter scalar field $\phi$ is one, we have $V \propto \frac{V}{l^2}$ and $\phi \propto \frac{\epsilon}{l}$, and we can find the relations between $(\epsilon, V(\epsilon))$ and $(\phi, V(\phi))$:

$$\phi = \frac{\sqrt{g}}{l} \epsilon \quad \text{with} \quad y = \frac{9\alpha S_0}{8\pi} \quad (26)$$

$$\frac{y}{l^2} V(\epsilon) = V(\phi) = \frac{|l^2 < G^2 > |}{4} \phi^2 \quad (27)$$

### 3 Results and Conclusion

Now, we check that our model is able to fix the usual problems of the standard (big bang) cosmology. First, in order to solve the “horizon” and “flatness” problems we need an inflation $\frac{a(t_f)}{a(t_i)}$ of order $10^{28}$ implying an inflation period $\Delta t = t_f - t_i$ such that

$$H \Delta t \sim 65 \quad (28)$$

Furthermore, it should satisfy the constraint

$$10^{-40} s \leq \Delta t \ll 10^{-10} s \quad (29)$$

so that not to conflict with the explanation of the baryon number and not to create too large density fluctuations [22, 23]. The bound $10^{-10} s$ corresponds to the time, after the big bang, when the electroweak symmetry breaking took place. Presumably, our inflation should have ended far before this time. Thus, from equations (28), (29) and (18) we get the following bounds on $| < G^2 > |$:

$$3 \times 10^7 GeV^2 \ll | < G^2 > |^{1/2} \leq 3 \times 10^{37} GeV^2 \quad (30)$$

In order to determine $\epsilon_i$, we have $\frac{d \ln a}{d \epsilon} = \frac{H}{\epsilon} \simeq -\frac{3H^2}{V(\epsilon)} \simeq -\frac{8\pi \rho^T}{M_{Pl}^2 V}$ which gives, using equations (13) and (20), the relation:

$$65 \sim \ln \frac{a(t_f)}{a(t_i)} = \left( \frac{L_P}{l} \right)^2 \frac{9\alpha S_0}{4} (\epsilon_i^2 - 1) \quad (31)$$

Next, comes the “formation of structure” problem and we require the fractional density fluctuations at the end of inflation to be of the order $\frac{\delta M}{M} |_{t_f} \sim 10^{-5}$ so that quantum fluctuations in the de Sitter phase of the inflationary universe form the source of perturbations providing the seeds for galaxy formation and in order to agree with the CMB anisotropy limits. Within the relativistic theory of cosmological perturbations [24], the above fractional
density fluctuations represent (to linear order) a gauge-invariant quantity and satisfy the equation

$$\frac{\delta M}{M} |_{t_f} = \frac{\delta M}{M} |_{t_i} \frac{1}{1 + \frac{p}{\rho}} |_{t_i}$$

(32)

where $\delta M$ represent the mass perturbations.

The initial fluctuations are generated quantum mechanically and, at the linearized level, the equations describing both gravitational and matter perturbations can be quantized in a consistent way [25]. The time dependence of the mass is reflected in the nontrivial form of the solutions to the mode equations and one can compute the expectation value of field operators such as the power spectrum and get the following result for the initial mass perturbation [24, 25]

$$\frac{\delta M}{M} |_{t_i} = \frac{V'(\Phi) H}{\rho} = \sqrt{y} \frac{V'(|\epsilon|)}{l \rho}$$

(33)

whence

$$10^{-5} \sim |\frac{\delta M}{M} |_{t_f} = \sqrt{y} | \frac{V'H}{l} |_{t_i} \frac{1}{(\rho + p)|_{t_i}}$$

(34)

In order to evaluate $(\rho + p)|_{t_i}$ we use the energy conservation equation:

$$\dot{\rho} + 3(\rho + p) \frac{\dot{a}}{a} = 0$$

(35)

and after substituting $\rho \sim \rho^T_g$ we get

$$(\rho + p)|_{t_i} = \frac{1}{24\pi} \frac{l}{L_P}^2 | < G^2 > |$$

(36)

In fact, the energy conservation equation can be used to solve for $\rho^T_g$ and we could check that

$$\rho^T_g(\dot{\rho}^T_g) \sim \rho_e(\dot{\rho}_e) \sim \delta \times \rho^T_g(\delta \times \dot{\rho}^T_g)$$

(37)

where $\delta \equiv |\frac{\dot{a}}{a}| \sim \frac{1}{8} (\frac{l}{L_P})^2$ and so, when the “slow roll” condition (21) is satisfied, our solution assuming the predominance of the “trace-anomaly” energy mass density is self-consistent. Substituting equation (36) in (35) and using equation (20) we get

$$\frac{l}{L_P} \sim 9 \sqrt{\frac{3\pi}{2}} \alpha_{s_0} 10^5 \epsilon^2 | < G^2 > | \frac{1}{G_N}$$

(38)
Hence, taking $G_N \sim 10^{-38} GeV^{-2}$ we obtain

$$\frac{l}{L_P} \sim \frac{|<G^2>|^{1/2}}{10^{34} GeV^2 \epsilon^2}$$  \hspace{1cm} (39)$$

and combining this last result with (30), we get

$$10^{-27} \ll \frac{l}{L_P} \leq 10^3$$ \hspace{1cm} (40)$$

The “slow roll” condition (21) is consistent with the upper bound, while the lower bound restricts $\epsilon_i$ not to be too large.

On the other hand, it is possible to calculate the spectral index of the primordial power spectrum for a quadratic potential as follows:

$$n - 1 = -4\eta \text{ where } \eta = \frac{M_P^2 V''}{8\pi V} = \frac{2}{9\alpha_0} \left(\frac{l}{L_P}\right)^2 \frac{1}{\epsilon^2} = \delta$$ \hspace{1cm} (41)$$

and we find:

$$n = 1 - \frac{1}{\pi y} \left(\frac{l}{L_P}\right)^2 \frac{1}{\epsilon_i^2}$$ \hspace{1cm} (42)$$

The inflation would end ($\epsilon_f = 1$) when the “slow roll” parameter $\eta = \delta = 1$.

We should evaluate the QCD coupling constant $\alpha_0(\mu) = \frac{4\pi}{\beta_0 \ln(\frac{\mu^2}{\Lambda_{QCD}^2})}$ at an energy scale corresponding to the inflationary period. We take this to be around the GUT scale $\sim 10^{15} GeV$ and $\beta_0 = 11 - \frac{2}{3}n_f = 7$ whereas the weak logarithmic dependence would assure the same order of magnitude for $\alpha_0$ calculated at other larger scales. With $\Lambda_{QCD} \sim 0.2 GeV$ we estimate $\alpha_0 \sim 0.025$, and so we get

$$\left(\frac{l}{L_P}\right)^2 \sim 10^{-1}$$ \hspace{1cm} (43)$$

This is in disagreement with Bekenstein assumption that $L_P$ is the shortest length scale in any physical theory. However, it should be noted that Bekenstein’s framework is very similar to the dilatonic sector of string theory and it has been pointed out in the context of string theories that there is no need for a universal relation between the Planck and the string scale. Furthermore, determining the order of magnitude of $\frac{l}{L_P}$ is interesting in the context of these theories.

From (31), we have $\epsilon_i \sim 11$, and then using (42), we have $n = 0.97$ which is within the range of WMAP results.
Thus, we see that our model reproduces the results of the chaotic inflationary scenario. However, the shape of the potential was not put by hand, rather a gauge theory with a changing coupling constant led naturally to it. Moreover, in typical chaotic models, the inflaton field starts from very large values ($\phi_i \sim 15M_{Pl}$) and ends at around $1M_{Pl}$. One might suspect whether field theory is reliable at such high energies. Nonetheless, this problem is absent in our model since the large values have another meaning in that they just refer to a reduction of the strong coupling by around 10 times during the inflation.

Furthermore, chaotic inflations get a typical reheating of order $T_{rh} \sim 10^{15}GeV$, and one might need to worry about the relic problem. Similarly, equation (39) leads in our model to a gluon condensate $|<G^2>|_i \sim 10^{62}GeV^4$ at the start of inflation. From equation (22), we see that this corresponds to an inflation time interval $\Delta t \sim 10^{-35}s$ satisfying the constraint (29). If the gluon condensate stays constant, as we assumed in our analysis, we will have the same reheating temperature as in chaotic models ($T_{rh} \sim \rho(t_f)^{1/4}$). However, we should compare this value for $<G^2>$ with its present value renormalized at GUT scale $\sim 10^{15}GeV$ which can be calculated knowing its value at $1GeV$ [14] and that the anomalous dimension of $\alpha_S G^2$ is identically zero. We get

$$<G^2(\text{now, } \mu \sim 10^{15}GeV) > \sim 1GeV^4$$

which represents a decrease of 62 orders of magnitude.

This can give us the following possible picture for an exit scenario. Lacking a clear theory for the non-perturbative dynamical gluon condensate, we consider its value $|<G^2>|$ depending on energy, and thus implicitly on cosmological time, as given by the standard RGE which turns it off logarithmically at high energy. However, we can furthermore assume the condensate value to depend explicitly on time during inflation:

$$<G^2(E,t) >=<G^2_0(E(t)) > f(t)$$

where $<G^2_0(E)>$ is the piece determined by the RGE. In order that our model be consistent, the value of $<G^2>$ at the start of inflation should be very huge and negative. The unknown function $f(t)$ should be such that it varies slowly during most of the inflationary era, to conform with an approximately constant value of $<G^2>$, while at the end of inflation it causes a drastic drop of the condensate value $<G^2>$ to around zero. The energy release of this helps in reheating the universe, while reaching the value 0 leads to a minute “trace-anomaly” energy mass density (equation 13) ending, thus, the inflation. The other types of energy density would contribute to give the
gluon condensate its ‘small’ positive value of (44), and the subsequent evolution is just the standard one given by RGE. Surely, this phenomenological description needs to be tested and expanded into a theory where the concept of symmetry breaking of such a phase transition for the condensate $<G^2>$ provides the physical basis for ending the inflation. Nonetheless, with a test function of the form $f(t) = -\beta^2 \tanh^2(\epsilon - 1)$ with $\beta \sim 10^{31}$, one can integrate analytically the equation of motion, and in “slow roll” regime we have $\epsilon = 1 + \text{Arcsinh}(\exp[-\alpha \beta t])$ with $\alpha \beta \sim 10^{12}\text{GeV}$. The graph in Fig. 1 shows the time evolutions of the condensate and the $\epsilon$-field, which agree with the required features. This example is meant to be just an existence example, and the temporal dependence of the condensate could be of completely different shape while the whole picture is still self-consistent. The issue demands a detailed study for the condensate within an underlying theory and we do not further it here. We hope this work will stimulate interest in the subject.

![Graph showing time evolutions of condensate and $\epsilon$-field](image)

**Figure 1:** Temporal evolution of the condensate $|<G^2>|$ (thick line) and the $\epsilon$-field (thin line), for the choice $f(t) = -\beta^2 \tanh^2(\epsilon - 1)$. The $<G^2>$ scale has been adapted so that to visualize both graphs together.

## Acknowledgements

This work was supported in part by CONICET, Argentina. N. C. recognizes economic support from TWAS.
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