Charged Kaon $K^+ \rightarrow 3\pi$ CP Violating Asymmetries vs $\varepsilon'_K / \varepsilon_K$ \(^1\)

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Abstract

We present the next-to-leading order full results in Chiral Perturbation Theory for the charged Kaon $K \rightarrow 3\pi$ slope $g$ CP violating asymmetries. We discuss the constraints that a measurement of these asymmetries would impose on the Standard Model results of $\varepsilon'_K$ and search for new physics. We also study the kind of information that such measurement can provide on $\text{Im } G_8$, $\text{Im } (e^2 G_E)$ and higher order weak couplings.

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1 Introduction and Motivation

Direct CP violation in Kaons has been unambiguously established in $K \to \pi\pi$ decays by KTeV [1] and NA48 [2] through the measurement of $\text{Re} \left( \varepsilon_K'/\varepsilon_K \right)$, which present world average is [1, 2, 3, 4]

$$\text{Re} \left( \varepsilon_K'/\varepsilon_K \right) = (1.67 \pm 0.16) \cdot 10^{-3}.$$ (1)

The theoretical understanding of this quantity within the Standard Model (SM) is not at the same level. We just mention here the most recent advances: the Chiral Perturbation Theory (CHPT) calculation [5, 6] and the isospin breaking corrections [7] are both fully known to next-to-leading order (NLO) and the role of final state interactions (FSI) has also been understood [8] – for a more extensive description of these works and references see [9]. There also have been recent advances on the calculation of the lowest order CHPT couplings $\text{Im} G_8$ and $\text{Im} (\varepsilon G_E)$ [10, 11, 12, 13]. They are not fully under control yet and more work is needed.

Asymmetries in the Dalitz variable slope $g$ of $K \to 3\pi$ offer a very promising opportunity to study direct CP violation in Kaon decays. In fact, there are several experiments, NA48/2 [14] at CERN, KLOE [15] at Frascati and OKA [16] at Protvino which have announced an expected sensitivity to these asymmetries of the order of $10^{-4}$, i.e., one order of magnitude better than at present [17]. On the theory side, though the first NLO in CHPT calculation of $K \to 3\pi$ was done long ago [5], the analytical results were unfortunately not available until recently [18, 19]. $K \to 3\pi$ CP violating asymmetries were therefore predicted just at LO in CHPT plus various estimates of NLO effects [20]. The first full NLO calculation within CHPT for those asymmetries was done in [19].

2 Technique

The effective quantum field theory of the SM at energies below or of the order of 1 GeV is CHPT [21]. Some introductory lectures on CHPT can be found in [22] while some recent reviews in [23].

Recently, the one-loop in CHPT $K \to 3\pi$ calculation was redone by two groups [18, 19] in the isospin limit as mentioned above. All the needed notation and definitions were given there. There are also now available the NLO $K \to 3\pi$ isospin breaking effects from quark masses and electromagnetic interactions [24]. These effects turn out to be very suppressed and certainly much smaller than the contribution of the unknown counterterms, we therefore use the isospin limit results. Notice that some misprints in the first reference in [19] were reported in the third reference in [19].
There appear eleven unknown counterterms at NLO in the isospin limit. The real part of them and the LO couplings $G_8$ and $G'_{27}$ can be fixed from a fit to all available $K \rightarrow \pi\pi$ amplitudes at NLO in CHPT \cite{6} and $K \rightarrow 3\pi$ amplitudes and slopes also at NLO \cite{18, 19}. This was done in \cite{18} and we used the results of the fit as inputs in all the results we report here.

The values we use for Im $(e^2 G_E)$ and Im $G_8$ can be found in \cite{19}. They are taken mainly from \cite{10, 11} but are compatible also with those in \cite{12, 13}.

The imaginary part of the order $p^4$ counterterms, Im $\tilde{K}_i$, are much more problematic to predict. They cannot be obtained from data and there is not a NLO in $1/N_c$ ($N_c$ is the number of QCD colors) available calculation for them. One can still get the order of magnitude and/or signs of Im $\tilde{K}_i$ using several approaches. We followed in \cite{19} a more naive approach but that is enough for the purpose of estimating the effect of those counterterms. Namely, we assumed that the ratio of the real to the imaginary part of the octet couplings is roughly dominated by the same strong dynamics at LO and at NLO in CHPT, i.e.

$$\frac{\text{Im} \tilde{K}_i}{\text{Re} K_i} \simeq \frac{\text{Im} G_8}{\text{Re} G_8} \simeq \frac{\text{Im} G'_8}{\text{Re} G'_8} \simeq (0.9 \pm 0.3) \text{Im} \tau$$

$$= - (0.9 \pm 0.3) \text{Im} \left( \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} \right). \tag{2}$$

### 3 $K \rightarrow 3\pi$ CP Violating Asymmetries

The definition of the CP violating asymmetries in the slope $g$ and analogous asymmetries for the decay rates $\Gamma$ can be found, for instance, in \cite{19}. They start at $\mathcal{O}(p^2)$ in CHPT and at NLO require final state interactions phases of the three-pions at NLO too, i.e., an $\mathcal{O}(p^6)$ calculation.

Though the full $\mathcal{O}(p^6)$ result is not yet available, we calculated analytically the full two-pion cut diagrams contribution to the FSI phases for the charged Kaon decays in \cite{19}. In \cite{25}, we present the full analytic results for the neutral Kaon decays including the three-pion cuts both for the neutral and charged Kaon decays. We also discuss there some applications of this $K \rightarrow 3\pi$ FSI calculation to the Cabibbo’s proposal to measure the $a_0 - a_2$ pion scattering lengths combination \cite{26} from the cusp effect in the $\pi^0\pi^0$ spectrum in $K^+ \rightarrow \pi^+\pi^0\pi^0$ and $K_L \rightarrow \pi^0\pi^0\pi^0$ decays.

Including the calculated NLO FSI and substituting the pion and Kaon masses, Re $G_8$, $G'_{27}$ and the real part of the NLO CHPT couplings, the result we get for the asymmetry $\Delta g_C$ of the slope $g_C$ in $K^+ \rightarrow \pi^+\pi^+\pi^-$ decay is

$$\frac{\Delta g_C}{10^{-2}} \simeq \left[ (0.7 \pm 0.1) \text{Im} G_8 + (4.3 \pm 1.6) \text{Im} \tilde{K}_2 \right.$$  \hspace{1cm} \tag{3}

$$\left. - (18.1 \pm 2.2) \text{Im} \tilde{K}_3 \right] - (0.07 \pm 0.02) \text{Im} (e^2 G_E).$$
Figure 1: $\varepsilon'_K$: Theory vs Experiment. See text for explanation.

And, when values for the imaginary part of the needed couplings are taken as explained in the previous section, one gets

$$\Delta g_C = -(2.4 \pm 1.2) \cdot 10^{-5}.$$

(4)

Results for the rest of the asymmetries can be found in [19].

4 \hspace{1em} K \rightarrow 3\pi \hspace{0.5em} \text{CP Violating Asymmetries vs} \hspace{0.5em} \varepsilon'_K

Including FSI to all orders and CHPT and isospin breaking at NLO [6, 7, 8] one gets within the Standard Model,

$$\text{Re} \left( \frac{\varepsilon'_K}{\varepsilon_K} \right) \simeq - \left[ (1.88 \pm 1.0) \text{Im} G_8 + (0.38 \pm 0.13) \text{Im} (e^2 G_E) \right].$$

(5)

Using this result, the experimental measurement in (1) imposes that Im $G_8$ and Im $(e^2 G_E)$ are constrained to be within the horizontal band in Figure 1. In the same figure we also show the present theoretical predictions at NLO in $1/N_c$ expansion for Im $G_8$ and Im $(e^2 G_E)$: from [10, 11] –rectangle on the right, from [12] –rectangle on the left, and from [13] –vertical band. The circle is the leading in $1/N_c$ prediction.

A measurement of $\Delta g_C$ can have an important impact on constraining what we know on Im $G_8$ and Im $(e^2 G_E)$ from $\varepsilon'_K$. To assess the quality of these constraints, we plot in Figure 2 the dashed horizontal band that one gets using (3) for $\Delta g_C = -3.5 \cdot 10^{-5}$ with a typical $(20 \sim 30)\%$ uncertainty together with the
5 Conclusions

The CP violating asymmetry $\Delta g_C$ is dominated by the value of $\text{Im} G_8$ and its final uncertainty comes mainly from this input. This is the only $K \to 3\pi$ CP asymmetry with uncertainty below 50%. The predictions for the rest of the asymmetries studied can be found in [19].

The eventual measurement of the asymmetry $\Delta g_C$ when combined with the accurate measurement of $\varepsilon'_K$ [1, 2, 3, 4] in (1) offers an opportunity to both check the Standard Model calculations and to obtain more information on possible new physics.

The SM prefers values for $\Delta g_C$ larger than $-0.4 \cdot 10^{-4}$ and an experimental bound of the order or smaller than $-2 \cdot 10^{-4}$ would indicate the presence of new physics – see Figures 2 and 3. For a discussion on possible SUSY implications of a measurement of these asymmetries see [27].

The CP asymmetries $\Delta g_N$ and in the decay rate were also discussed in [19] and we found that they are dominated by the imaginary part of the $O(p^4)$ counterterms. Information on these asymmetries would therefore give very interesting constraints about the size of those imaginary parts.

As a final remark, direct CP violating asymmetries in $K \to 3\pi$ provide extremely interesting and valuable information on the SM and its possible exten-
Figure 3: $\varepsilon'_K$ vs $\Delta g_C$ for $\Delta g_C = -1 \cdot 10^{-5}$.

sions which is complementary to the one obtained from $\varepsilon'_K$. The first experimental result by NA48/2 at CERN was already announced [28]

$$|\Delta g_C| = (0.5 \pm 3.8) \cdot 10^{-4},$$

which uncertainty is expected to be reduced.

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