1. Introduction

Our ultimate goal is the construction of a model for interactions of two nuclei in the energy range between several tens of GeV up to several TeV per nucleon in the centre-of-mass system. Such nuclear collisions are very complex, being composed of many components, and therefore some strategy is needed to construct a reliable model. The central point of our approach is the hypothesis, that the behavior of high energy interactions is universal (universality hypothesis). So, for example, the hadronization of partons in nuclear interactions follows the same rules as the one in electron-positron annihilation; the radiation of off-shell partons in nuclear collisions is based on the same principles as the one in deep inelastic scattering. We construct a model for nuclear interactions in a modular fashion. The individual modules, based on the universality hypothesis, are identified as building blocks for more elementary interactions (like $e^+e^-$, lepton-proton), and can therefore be studied in a much simpler context. With these building blocks under control, we can provide a quite reliable model for nucleus-nucleus scattering, providing in particular very useful tests for the complicated numerical procedures using Monte Carlo techniques.
2. The Universality Hypothesis

Generalizing proton-proton interactions, the structure of nucleus-nucleus scattering should be as follows: there are elementary inelastic interactions between individual nucleons, realized by partonic “half-ladders”, where the same nucleon may participate in several of these elementary interactions. Also elastic scatterings are possible, represented by parton ladders. Although such diagrams can be calculated in the framework of perturbative QCD, there are quite a few problems: important cut-offs have to be chosen, one has to choose the appropriate evolution variables, one may question the validity of the “leading logarithmic approximation”, the coupling of the parton ladder to the nucleon is not known, the hadronization procedure is not calculable from first principles and so on. So there are still many unknowns, and a more detailed study is needed.

Our starting point is the universality-hypothesis, saying that the behavior of high-energy interactions is universal. In this case all the details of nuclear interactions can be determined by studying simple systems in connection with using a modular structure for modeling nuclear scattering. One might think of proton-proton scattering representing a simple system, but this is already quite complicated considering the fact that we have in general already several elementary interactions. It would be desirable to study just one elementary interaction, which we refer to as “semihard Pomeron”, which will be done in the next section.

3. The semihard Pomeron

In order to investigate the semihard Pomeron, we turn to an even simpler system, namely lepton-nucleon scattering. A photon is exchanged between the lepton and a quark of the proton, where this quark represents the last one in a “cascade” of partons emitted from the nucleon. The squared diagram represents a parton ladder. In the leading logarithmic approximation (LLA) the virtualities of the partons are ordered such that the largest one is close to the photon [1, 2]. If we compare with proton-proton scattering, we have ordering from both sides with the largest virtuality in the middle, so in some sense the hadronic part of the lepton-proton diagram represents half of the elementary proton-proton diagram, and should therefore be studied first. In fact such statements are to some extent commonly accepted, but not carried through rigorously in the sense that also for example the hadronization of these two processes is related.

But first we investigate the so-called structure function $F_2$, related to
the lepton-proton cross section via [3]

\[
\frac{d\sigma}{dx dQ^2} = \frac{2\pi \alpha^2}{Q^4x} \left(2 - 2y + \frac{y^2}{1 + R}\right) F_2(x, Q^2)
\]  \tag{1}

with

\[
R = \frac{F_L}{F_2 - F_L}, \quad x = \frac{q^2}{2pq}
\]  \tag{2}

and with the kinematic variables

\[
Q^2 = -q^2; \quad x = \frac{q^2}{2pq}
\]  \tag{3}

where \(q\) and \(p\) are the four-momenta of the photon the proton respectively. \(F_L\) is the longitudinal structure function. \(F_2\) represents the hadronic part of the diagram, and is, using eq. (1), measurable. In lowest order and considering only leading logarithms of \(Q^2\), only two diagrams contribute, which turn out to be [2]

\[
D_0 + D_1 = \sum_j e_j^2 \delta(1 - \frac{x}{\xi}) + \sum_{ij} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} \epsilon_j^2 \frac{\alpha_s}{2\pi} \frac{x}{\xi} P_{ij}(\frac{x}{\xi}), \quad \tag{4}
\]

where \(P_{ij}\) is the Altarelli-Parisi splitting function and \(Q_0\) is some cut-off of the \(Q^2\) integration. The variable \(\xi\) is the momentum fraction of the quark with respect to the proton. Assuming some proton distribution \(f(\xi, Q^2)\) at the “factorization scale” \(Q_0^2\), an incoherent superposition provides [2]

\[
F_2(x, Q^2) = \sum_j e_j^2 x f^j(x, Q^2) \quad \tag{5}
\]

with

\[
f^j(x, Q^2) = f^j(x, Q_0^2) + \sum_{ij} \int_x^1 \frac{d\xi}{\xi} \int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} f^i(\xi, Q'^2) \frac{\alpha_s}{2\pi} P_{ij}(\frac{x}{\xi}). \quad \tag{6}
\]

Iterating this equation obviously represents a parton ladder with ordered virtualities. Strictly speaking, eq. (6) is still useless, because some of the functions \(P_{ij}(z)\) diverge for \(z \to 1\). However this is cured by considering virtual emissions, and this can be conveniently taken into account, by “regularizing” the functions \(P_{ij}\), which amounts to adding terms proportional \(\delta(1 - z)\), which cures the divergence. For Monte Carlo applications it is more useful to proceed differently [4]: one distinguishes between “resolvable” and “unresolvable” emissions. Unresolvable emissions are the virtual ones and
emissions with very small momentum fraction \((\epsilon < \epsilon)\). Then one sums over unresolved emissions which provides a factor
\[
\Delta^i(Q_0^2, Q_1^2) = \exp \left\{ -\sum_j \int_{Q_0^2}^{Q_1^2} \frac{dQ^2}{Q^2} \int_0^{1-\epsilon} d\xi \frac{\alpha_s}{2\pi} P^i_j(\xi) \right\}, \tag{7}
\]
called Sudakov form factor. This can also be interpreted as probability of no resolvable emission between \(Q_0^2\) and \(Q_1^2\). So the procedure amounts to only considering resolvable emissions, but to multiply each propagator with \(\Delta^i\).

Based on the above discussion, we define a so-called QCD evolution function \(E_{\text{QCD}}\), representing the evolution of a parton cascade from scale \(Q_0^2\) to \(Q_1^2\), as
\[
E_{\text{QCD}}^{ij}(Q_0^2, Q_1^2, x) = \lim_{n \to \infty} E_{\text{QCD}}^{(n)ij}(Q_0^2, Q_1^2, x), \tag{8}
\]
where \(E_{\text{QCD}}^{(n)}\) represents an ordered ladder with at most \(n\) ladder rungs. This is calculated iteratively based on
\[
E_{\text{QCD}}^{(n)ij}(Q_0^2, Q_1^2, x) = \delta(1-x) \delta_{ij} \Delta^i(Q_0^2, Q_1^2)
+ \sum_k \int_{Q_0^2}^{Q_1^2} \frac{dQ^2}{Q^2} \int_0^{1-\epsilon} d\xi \frac{\alpha_s}{2\pi} E_{\text{QCD}}^{(n-1)ik}(Q_0^2, Q^2, \xi) \Delta^k(Q^2, Q_1^2) P^{i}_j(\frac{x}{\xi}). \tag{9}
\]
The indices \(i, j, k\) represent parton flavors. The function \(E_{\text{QCD}}\) is calculated initially for discrete values of the variables, and later used via interpolation. In this way we are really sure to use the same QCD evolution for any application, it is the same in deep inelastic scattering as in nuclear interactions.

Next we have to determine the \(x\)-distribution of the first parton of the ladder. We expect for the momentum share of this first (the fastest) parton of the ladder a distribution as \(1/x\), which leads to mostly small values of \(x\), which implies on the other hand a large mass \(M \sim 1/x\) [5]. Such large mass objects are theoretically described in terms of Regge theory, the most prominent object at large masses being the Pomeron (IP). So, as discussed already in [6], the complete diagram is composed of the parton ladder, as discussed, and a soft Pomeron, given as
\[
E_{\text{soft IP}} \sim x^{-\alpha_{\text{IP}}}, \tag{10}
\]
and the coupling between the soft Pomeron and the nucleon, which takes the form
\[
C_{\text{IP}} \sim x^{-\beta_{\text{IP}}}. \tag{11}
\]
We call \(E_{\text{soft IP}}\) also the soft evolution, to indicate that we consider this as simply a continuation of the QCD evolution, however, in a region where
perturbative techniques do not apply any more. We consider quarks and
gluons to be emitted from the soft Pomeron, in case of quarks we have to
split the gluon momentum. So we have the following initial distribution,
\begin{equation}
\varphi^i_{\text{P}}(x) = C_{\text{P}} \otimes E^i_{\text{soft P}},
\end{equation}
with
\begin{equation}
E^i_{\text{soft P}} = \begin{cases} 
    E_{\text{soft P}} & \text{if } i = g \\
    E_{\text{soft P}} \otimes P^q_g & \text{if } i = q, \bar{q}
\end{cases}.
\end{equation}
There is a second contribution, where a Reggeon is involved. The corre-
sponding soft evolution is
\begin{equation}
E^i_{\text{soft R}} \sim x^{-\alpha_{\text{R}}},
\end{equation}
and for the Reggeon–nucleon coupling we take
\begin{equation}
C_{\text{R}} \sim x^{-\beta_{\text{R}}}. 
\end{equation}
So we have the following initial distribution
\begin{equation}
\varphi^i_{\text{R}}(x) = C_{\text{R}} \otimes E^i_{\text{soft R}},
\end{equation}
with
\begin{equation}
E^i_{\text{soft R}} = \begin{cases} 
    E_{\text{soft R}} & \text{if } i = q, \bar{q} \\
    0 & \text{if } i = g
\end{cases}.
\end{equation}
The total initial distribution is obviously
\begin{equation}
\varphi^i(x) = \varphi^i_{\text{P}}(x) + \varphi^i_{\text{R}}(x),
\end{equation}
which is by construction equal to \( f^i(x, Q_0^2) \). The distribution at scale \( Q^2 \) is
\begin{equation}
f^j = \sum_i \varphi^i \otimes E^{ij}_{\text{QCD}}
\end{equation}
The structure function is then calculated as :
\begin{equation}
F_2(x, Q^2) = \sum_j e^2_j x f^j(x, Q^2).
\end{equation}
For \( Q = Q_0 \), the \( \text{P} \)-contribution is a function which peaks at very small
values of \( x \) and then decreases monotonically towards zero for \( x = 1 \), the
\( \text{R} \)-contribution on the other hand has a maximum at large values of \( x \) and
goes towards zero for small values of \( x \). The precise form of \( f \) depends
crucially on the exponent for the Pomeron–nucleon coupling, and we find a
good agreement for \( \beta_{\text{P}} = \frac{1}{2} \).
We are now in a position to write down the expression $G_{\text{semi}}$ for a cut semihard Pomeron, representing an elementary inelastic interaction in $pp$ scattering. We can divide the corresponding diagram into three parts.

We have the process involving the highest parton virtuality in the middle, and the upper and lower part representing each an ordered parton ladder coupled to the nucleon. According to the universality hypothesis, the two latter parts are known from studying deep inelastic scattering, representing each the hadronic part of the DIS diagram

$$f^j = \sum_i \phi^i \otimes E_{\text{QCD}}^{ij},$$

where $\phi^i$ is the distribution at $Q = Q_0^2$. $E_{\text{QCD}}^{ij}$ represents the evolution between $Q_0^2$ and some $Q^2$, $f^j$ is therefore a function of $x$ and $Q^2$. The complete diagram is therefore, for given impact parameter $b$ and given energy squared $s$,

$$G_{\text{semi}} = \sum_{ij} \int d\xi^+ d\xi^- dQ^2 f^i(\xi^+, Q^2) f^j(\xi^-, Q^2) \frac{d\sigma_{\text{Born}}^{ij}}{dQ^2}(\xi^+ \xi^- s, Q^2),$$

This may be written as

$$G_{\text{semi}} = \sum_{IJ} \int dx^+ dx^- \tilde{G}_{\text{semi}}^{IJ}(x^+, x^-),$$

with

$$\tilde{G}_{\text{semi}}^{IJ}(x^+, x^-) = C_I(x^+) C_J(x^-) \sum_{ijkl} \int du^+ du^- dQ^2 E_{\text{soft}}^k I \otimes E_{\text{QCD}}^{kj} E_{\text{soft}}^l J \otimes E_{\text{QCD}}^{lj} \frac{d\sigma_{\text{Born}}^{ij}}{dQ^2}(u^+ u^- x^+ x^- s, Q^2).$$

The variables $I$ and $J$ may take the values $\text{IP}$ and $\text{IR}$. Integrating $G_{\text{semi}}$ over $b$, we obtain the cross section to produce a pair of hard partons, which is measurable since the partons show up as hadron jets and may be reconstructed. This cross section is therefore called jet cross section, given as

$$\sigma_{\text{jet}}(s) = \int d^2 b G_{\text{semi}}(s, b).$$

So, in particular, checking the differential cross section $d\sigma_{\text{jet}}/dt$ against data provides an important consistency check.

In addition to the semihard Pomeron, one has to consider the expression representing the soft Pomeron [7]. The latter one, $G_{\text{soft}}$, is the Fourier
transform of a Regge pole amplitude \( A \sim s^{\alpha(t)} \). So an elementary inelastic interaction in an energy range of say 10 - 10^4 GeV is therefore written as

\[
G_{\text{tot}} = G_{\text{semi}} + G_{\text{soft}}.
\]

### 4. Hadron Production

As discussed in the last chapter, there exist observables which can be calculated without detailed knowledge about hadron production. There exist, however, a huge variety of data concerning hadronic observables such as rapidity and \( p_t \) spectra of different types of hadrons, multiplicity distributions and so on. From a theoretical point of view, this requires some more details to be worked out. In addition, the numerical procedures are getting more complicated, and in fact, the most convenient method is provided by the Monte Carlo technique, which means in this context the following: for a given reaction, one writes the total cross section as a sum over contributions from different configurations \( X \),

\[
\sigma_{\text{tot}} = \sum_{X \in \mathcal{K}} \sigma(X),
\]

such that \( \sigma(X) \) is non negative and can be interpreted as probability. The sum has to be replaced by an integral in case of a continuous configuration space. In general, such a configuration \( X \) represents a sum over a certain class of diagrams, because individual diagrams may be negative. So our work is, besides providing appropriate expressions for the total cross sections, also to provide algorithms to generate configurations \( X \) according to the corresponding distributions.

The first step amounts to generating parton configurations before worrying about hadronisation.

Let us start again with the case of lepton-nucleon scattering. We first have to generate the kinematical variables \( x \) and \( Q^2 \) according to the lepton-proton cross section,

\[
\frac{d\sigma_{lp}}{dx \, dQ^2} = \frac{2\pi \alpha^2}{Q^4 x} \left( 2 - 2y + \frac{y^2}{1 + R} \right) F_2(x, Q^2).
\]

Then a Pomeron or Reggeon type of coupling is chosen, with probabilities \( F_{2P}^P/F_2 \) and \( F_{2R}^R/F_2 \) respectively. We have

\[
F_{2P}^{P/R} = x \sum_{ij} e_j^2 \tau^i_{P/R} \otimes E_{QCD}^{ij}.
\]
$E_{\text{QCD}}$ can be written as

$$E_{\text{QCD}}^{ij}(Q_0^2, Q^2, z) = \delta(1-z) \delta_{ij} \Delta^i(Q_0^2, Q^2) + \tilde{E}_{\text{QCD}}^{ij}(Q_0^2, Q^2, z), \quad (30)$$

where the first term represents no parton emission and the second term at least one emission. The probability of no emission is therefore

$$\frac{1}{F_2} x \sum_j \langle \varphi(x) \Delta^j(Q_0^2, Q^2) \rangle, \quad (31)$$

and a configuration with no emission is generated according to this probability. In case of at least one emission, one generates the flavour $i$ and the fraction $x_0$ of the first parton of the QCD cascade according to

$$\sum_j \left\langle \varphi_i(x_0) \tilde{E}_{\text{QCD}}^{ij}(Q_0^2, Q^2, \frac{x}{x_0}) \right\rangle. \quad (32)$$

We are left with the problem of generating the cascade of partons starting from a parton with fraction $x_0$, flavour $i$, at scale $Q_0$, up to the photon vertex at scale $Q^2$. We have

$$\tilde{E}_{\text{QCD}}^{ij}(Q_0^2, Q^2, z) =$$

$$\sum_k \int \frac{dQ_1^2}{Q_1^2} \int \frac{dz_1}{z_1} \Delta^i(Q_0^2, Q_1^2) \frac{\alpha_s}{2\pi} P_i^k(z_1) E_{\text{QCD}}^{kj}(Q_1^2, Q^2, \frac{z}{z_1}), \quad (33)$$

and therefore $k$, $z_1$, and $Q_1^2$ of the next emission are generated according to

$$\sum_j \frac{c_j^2}{Q_1^2} \Delta^j(Q_0^2, Q_1^2) \frac{1}{z_1} \frac{\alpha_s}{2\pi} P_i^k(z_1) E_{\text{QCD}}^{kj}(Q_1^2, Q^2, \frac{z}{z_1}), \quad (34)$$

and so on. This completed the description of the algorithm to generate parton configurations, based on exactly the same formulas used to calculate $F_2$ as discussed in the previous section.

The next step consists of generating with certain probabilities hadron configurations, starting from a given parton configuration. We cannot calculate those probabilities within QCD, so we simply provide a recipe, the so-called string model. The first step consists of mapping a partonic configuration into a string configuration. For this purpose, we use the colour representation of the parton configuration: a quark is represented by a colour line, a gluon by a colour-anticolour pair. One then follows the colour flow starting from a quark via gluons, as intermediate steps, till one finds an antiquark. The corresponding sequences

$$q - g_1 - g_2 \ldots - g_n - \bar{q} \quad (35)$$
are identified with kinky strings, where the gluons represent the kinks. Such a string decays into hadron configurations with the corresponding probabilities given in the framework of the theory of classical relativistic strings.

The above discussion of how to generate parton and hadron configurations is not yet complete: the emitted partons are in general off-shell and can therefore radiate further partons. This so called timelike radiation is taken into account using standard techniques. The mapping of parton to hadron configurations still works the same way as discussed above.

Let us now discuss the generation of parton configurations for an elementary proton-proton interaction represented by a semihard Pomeron. Based on

\[ G_{\text{semi}} = \sum_{IJ} \int dx^+ dx^- \tilde{G}^{IJ}_{\text{semi}}(x^+, x^-), \]  

(36)

one generates the light come momentum fractions \( x^+, x^- \) of the "Pomeron ends" according to \( \tilde{G} \). This quantity may be written as

\[ \tilde{G}^{IJ}_{\text{semi}}(x^+, x^-) = C_I(x^+) C_J(x^-) \sum_{ij} \int dz^+ dz^- E_{\text{soft}I}^j(z^+) E_{\text{soft}J}^i(z^-) \sigma_{\text{jet}}^{ij}(z^+ z^- x^+ x^- s), \]

(37)

with

\[ \sigma_{\text{jet}}^{ij}(\hat{s}) = \sum_{kl} \int dw^+ dw^- dQ^2 \]

\[ E_{QCD}^{ik}(Q_0^2, Q^2, w^+) E_{QCD}^{jl}(Q_0^2, Q^2, w^-) \frac{d\sigma_{\text{Born}}^{kl}}{dQ^2}(w^+ w^- \hat{s}, Q^2). \]

The integrand serves as probability distribution to generate \( z^+ \) and \( z^- \). The two quantities \( z^+ \) and \( z^- \) are the momentum fractions of the "ladder ends" with respect to the "Pomeron ends". Knowing the ladder mass \( \hat{s} = z^+ z^- x^+ x^- s \), we have to generate the ladder rungs. Generalizing the definition of \( \sigma_{\text{jet}} \), we define

\[ \sigma_{\text{jet}}^{ij}(Q_1^2, Q_2^2, \hat{s}) = \sum_{kl} \int dw^+ dw^- dQ^2 \]

\[ E_{QCD}^{ik}(Q_1^2, Q^2, w^+) E_{QCD}^{jl}(Q_2^2, Q^2, w^-) \frac{d\sigma_{\text{Born}}^{kl}}{dQ^2}(w^+ w^- \hat{s}, Q^2). \]

(39)

and

\[ \sigma_{\text{ord}}^{ij}(Q_1^2, Q_2^2, \hat{s}) = \sum_{k} \int dw^- dQ^2 \]

\[ E_{QCD}^{jk}(Q_2^2, Q^2, w^-) \Delta^i(Q_1^2, Q^2) \frac{d\sigma_{\text{Born}}^{ki}}{dQ^2}(w^- \hat{s}, Q^2). \]

(40)
representing ladders with ordering of virtualities on both sides ($\sigma_{\text{jet}}$) or on one side only ($\sigma_{\text{ord}}$). We calculate and tabulate $\sigma_{\text{jet}}$ and $\sigma_{\text{ord}}$ initially so that we can use them via interpolation to generate partons. The generation of partons is done in an iterative fashion based on the following ladder equation:

$$\sigma_{\text{jet}}(Q_1^2, Q_2^2, \hat{s}) = \sigma_{\text{Born}}(Q_1^2, Q_2^2, \hat{s})$$

$$+ \sum_k \int \frac{dQ^2}{Q^2} \int \frac{d\xi}{\xi} \Delta^i(Q_1^2, Q_2^2) \frac{\alpha_s}{2\pi} P^k_i(\xi) \sigma_{\text{jet}}^{kj}(Q^2, Q^2, \hat{s})$$

$$+ \sigma_{\text{ord}}(Q_1^2, Q_2^2, \hat{s}).$$

Our treatment of generating parton configurations for an elementary $p\bar{p}$ interaction is absolutely compatible with deep inelastic scattering, it is based on the same building blocks, in particular on the evolution functions.

Our procedure can now be used to treat the case of many semihard Pomerons, for $p - p$ as well as nuclear scattering. A detailed discussion will be given in a future publication.

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