A MODEL OF THE HELIOSPHERE WITH JETS

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ABSTRACT

An analytic model of the heliosheath (HS) between the termination shock (TS) and the heliopause (HP) is developed in the limit in which the interstellar flow and magnetic field are neglected. The heliosphere in this limit is axisymmetric and the overall structure of the HS and HP is controlled by the solar magnetic field in the limit in which the ratio of the plasma to magnetic field pressure, \( \beta = 8\pi P/B^2 \), is large. The tension of the solar magnetic field produces a drop in the total pressure between the TS and the HP. This same pressure drop accelerates the plasma flow downstream of the TS into the north and south directions to form two collimated jets. The radii of these jets are controlled by the flow through the TS and the acceleration of this flow by the magnetic field—a stronger solar magnetic field boosts the velocity of the jets and reduces the radii of the jets and the HP. MHD simulations of the global heliosphere embedded in a stationary interstellar medium match well with the analytic model. The results suggest that mechanisms that reduce the HS plasma pressure downstream of the TS can enhance the jet outflow velocity and reduce the HP radius to values more consistent with the Voyager 1 observations than in current global models.

Key words: ISM: jets and outflows – stars: jets – Sun: heliosphere – Sun: magnetic fields

1. INTRODUCTION

The historically accepted shape of the heliosphere is that of a comet-like object with a long tail that is dragged downstream by the flow of the local interstellar medium (LISM) past the Sun (Parker 1961; Baranov & Malama 1993). These early pictures, however, were based on a hydrodynamic description of the solar outflow—the solar magnetic field was assumed to play a negligible role in the overall structure of the heliosphere and its interaction with the LISM. Computational models based on the MHD equations included the solar magnetic field as well as that of the interstellar medium and also produced a heliosphere with a comet-like shape (Opher et al. 2006, 2009; Pogorelov et al. 2007, 2013; Washimi et al. 2011; Opher & Drake 2013).

On the other hand, the measurements of energetic neutral atoms (ENAs) by IBEX and CASSINI produced some surprises. These ENAs travel long distances through the heliosphere without being influenced by the ambient magnetic field and therefore yield information about the large-scale structure of the heliosphere. The CASSINI ENA fluxes from the direction of the nose and the tail were comparable, leading the CASSINI observers to conclude that the heliosphere was “tailless” (Krimigis et al. 2009; Dialynas et al. 2013). The IBEX observations from the tail revealed that the hardest spectrum of ENAs were localized in two lobes at high latitudes, while the softest spectra were at low latitudes (McComas et al. 2013).

Recent MHD simulations using a monopole model for the solar magnetic field, designed to reduce the numerical dissipation of magnetic energy that arises from a conventional dipole model, revealed that the solar magnetic field was strong enough to collimate the solar wind into a pair of jets that flow to the north and south (Opher et al. 2015). These jets bend in the direction of the tail, pushed by the flow of the LISM. The interstellar rather than the solar wind plasma flows between these jets in the equatorial region downstream. Such bent jets have been seen in protostellar systems (Fendt & Zinnecker 1998; Gueth & Guilloteau 1999) and clusters of galaxies (Owen & Rudnick 1976). Astrophysical jets around massive black holes are thought to be driven by centrifugal forces that push the plasma along a rotating helical magnetic field (Blandford & Payne 1982). However, the jets in the case of the heliosphere are driven in the region downstream of the termination shock (TS), as was proposed for the Crab Nebula (Begelman & Li 1992; Chevalier & Luo 1994; Lyubarsky 2002). In this region of subsonic flow, the magnetic tension (hoop) force is strong enough to collimate and drive the wind.

MHD models of the global heliosphere are complex, and the mechanisms that control the shape of the heliopause (HP), the thickness of the heliosheath (HS), and the structure of the heliospheric jets, including the driver for the outflow, remain uncertain. The Voyager 1 observations have revealed that the thickness of the HS is around 30 AU, which is substantially thinner than expected from the global simulations. We present an analytic model of the heliosphere outside of a spherically symmetric TS where we neglect the ambient flow and magnetic field of the LISM. Taking the resulting heliosphere as axisymmetric and the flows within the HS as subsonic, we obtain the pressure and magnetic field structure of the HS along with the radius \( r_{hp} \) of the HP. The overall shape of the HS takes the classic form of an astrophysical jet: the flows through the TS are accelerated to the north and south by the solar magnetic field. The HP radius is determined by continuity: the plasma flow through the TS must balance the outflow through the jets. We present parallel global MHD simulations in the limit of zero magnetic field and flow in the LISM that support the analytic model.

One reason the influence of the solar magnetic field on the structure of the heliosphere is often neglected in the literature is because the pressure of the ambient plasma is large compared to that of the magnetic field—\( \beta = 8\pi n T B^2 \sim 10 \) just downstream of the TS. We show, however, that the total plasma pressure does not control either the flows in or the...
thickness of the HS. The overall pressure in the HS is balanced by the pressure in the LISM. It is the tension force of the HS magnetic field that controls the pressure difference between the TS and HP (Axford 1972). To the north and south there is no tension force, and this same pressure difference drives the axial flow of the heliospheric jets. Thus, it is ultimately the solar magnetic field that controls the large-scale structure of the HS.

2. ANALYTIC MODEL OF THE HS AND HP

We consider a simple axisymmetric system in which there is no LISM flow or magnetic field, and the LISM is specified by its ambient pressure $P_{\text{LISM}}$. We write down the steady-state MHD equations, including continuity, pressure, momentum, and magnetic field:

$$\nabla \cdot nV = 0,$$

$$\nabla \cdot P^{\Gamma} = 0,$$

$$M \nabla \cdot nVV = -\nabla \left( P + \frac{B^2}{8\pi} \right) - \frac{B^2}{4\pi r} \nabla r,$$

$$\nabla \times (V \times B) = 0,$$

where $r$ is the radius in cylindrical coordinates, $\Gamma$ is the ratio of specific heats, and $B$ is in the azimuthal direction. These equations are solved in the HS with boundary conditions on the radius $r_s$ and magnetic $M_{n}$ at the TS. The density is also a constant, $n_s$. Since $P$ is linked to $B$ and $r$ through Equation (10), the constancy of $P_0$ requires that

$$B = B_s \frac{r}{R_s},$$

so that $B$ increases with radius outside of the TS (Axford 1972; Chevalier & Luo 1994). At first order, we include the inertial terms and magnetic field in the momentum equation, which becomes

$$M_n V_0 \cdot \nabla V_0 = -\nabla \left( P + \frac{B^2r^2}{8\pi R_s^2} \right) - \frac{B^2r}{4\pi R_s^2} \nabla r,$$

where $n$ has been replaced by $n_s$ and $V_0 = \nabla \phi \times \nabla \psi/n_s$. Before discussing the flows in the HS, we consider the weak flow limit of Equation (13) so that the inertial forces in the radial direction can be discarded. In this limit, the magnetic tension force in Equation (13) causes the plasma pressure and total pressure to decrease with radius. This limit is artificial for the HS since the flow $V_r$ downstream of the TS is comparable to the Alfvén speed, and the associated radial inertial forces are comparable to the magnetic forces. Nevertheless, this limit illustrates how the pressure in the HS varies. We do not require zero pressure gradient in $z$. The pressure drop from the TS to the LISM is balanced by magnetic tension in the radial direction, but the same pressure drop also develops from the equator to the outflow jets to the north and south. This pressure gradient along $z$ drives the outward flows associated with the jets. Thus, we integrate Equation (13) from the TS outward to obtain an explicit expression for $P_1$:

$$P_1(r, z) = -\frac{B^2}{4\pi R_s^2} \left( r^2 - R_s^2 \sin^2 \theta \right),$$

where $\theta(z)$ is the value of $\theta$ at the TS and is dependent on $z$. Pressure balance across the HP, which requires that $P(n_H) + B^2(n_H)/8\pi = P_{\text{LISM}}$, then yields an explicit
expression for $r_{hp}$:

$$\frac{r_{hp}^2}{R_s^2} = \frac{8\pi \Delta P}{B_s^2} + 2 \sin^2 \theta_s, \quad (15)$$

where $\Delta P = P_i - P_{LISM}$. At this stage in the calculation, the pressure difference $\Delta P$ remains undetermined. We will show, however, that the requirement that the mass flow into the TS across the TS balances out the two jets constrains the pressure difference. In Figure 1, we show 2D plots of the plasma pressure, the magnetic pressure, and the total pressure in the $r$, $z$ plane in the HS. The data are shown for $8\pi \Delta P/B_s^2 = 2$, which, as shown later, is the upper limit on $\Delta P$. The inner boundary of the data shown is the TS and the outer boundary is the HP. The radius of the HP peaks at the midplane and falls off with $z$ until $z > R_s$, where it remains constant, forming the northward jet. The plasma and total pressure decrease with radius $r$, while the magnetic pressure increases with $r$. The total pressure falls off with distance from the equator until it approaches a constant value in the jet. We will show that this pressure difference, which is a consequence of magnetic tension, drives the jet outflow. Along the axis, $P$ remains constant at $P_i$. This shape was obtained by neglecting the radial plasma inertia, an assumption which breaks down where the straight portion of the HP in Figure 1 intersects with the curved portion at $z/R_s = 1$. The sharp kink in the HP is not real and is not seen in the MHD simulations discussed later. In Figure 2, cuts along $r$ of the total, plasma, and magnetic pressures from the data of the Figure 1 are shown at the equator in (a) and across the jet in (b).

We now discuss the flows driven in the HS. We derive a Bernoulli-like equation by taking the dot product of Equation (13) with $V_0$ and integrating along the streamline:

$$\frac{1}{2} M_n V_0^2 + R + \frac{B_s^2 r^2}{4\pi R_s^2} = \frac{1}{2} M_n V_s^2 + \frac{B_s^2 \sin^2 \theta_s}{4\pi}. \quad (16)$$

Unfortunately, this equation does not enable us to calculate $V_0$ throughout the HS because it requires that we know the trajectory of a streamline within the HS to link the local radius $r$ with its position at the TS where $\theta_s$ is known. Along the axis where $\theta_s = 0$, Equation (16) gives $V_0 = V_s$ since the pressure is constant and $B$ is zero. Similarly, the velocity at the jet radius, $r_{jet}$, can be calculated since at that location, $\theta_s = \pi/2$. We find $V_0^2(r_{jet}) = V_s^2 + c_{As}^2$, where $c_{As}^2 = B_s^2/4\pi M_n$. Thus, the increase in the jet velocity above $V_s$ is linked to the Alfvén velocity based on the magnetic field strength $B_s$ at the TS. More generally, we can calculate $V_0(r)$ across the jet radius by noting that within the jet,

$$V_0 = V_0 \sin \theta_s = -\frac{1}{n_r} \frac{\partial \psi}{\partial r}. \quad (17)$$

From Equation (8), $\sin \theta_s$ can be written in terms of $\psi$, so we are left with a single equation for $\psi$ across the jet:

$$\frac{1}{n_r^2 \sin^2 \theta_s} \left( \frac{\partial \psi}{\partial r} \right)^2 = \frac{V_s^2}{c_{As}^2} + 2 c_{As}^2 \left( 1 - \frac{\psi^2}{n_s^2 R_s^4 V_s^2} \right), \quad (18)$$

where $\psi$ varies from $n_s V_s R_s^2$ at the jet axis to 0 at the HP. The equality of the particle fluxes through the TS and jets requires
that Equation (18) produce the requisite jump in $\psi$ across the jet. Equation (18) can be simplified by defining an angle variable $\cos \theta = \psi / n_s V_s R_s^2$:

$$\frac{R_s^4 \sin^2 \theta}{r^2} \left( \frac{\partial \theta}{\partial r} \right)^2 = 1 + \frac{2 c_{As}^2}{V_s^2} \sin^2 \theta,$$  \hspace{1cm} (19)$$

where $\theta$ varies from 0 at the jet axis to $\pi/2$ at the HP. This equation can be integrated directly to obtain the jet radius, $r_{jet}$:

$$r_{jet}^2 = 2 R_s^2 \tan^{-1} \left( \frac{\sqrt{2} c_{As}/V_s}{\sqrt{2} c_{As}/V_s} \right).$$  \hspace{1cm} (20)$$

The jet radius is a maximum for $c_{As} \ll V_s$ when the jet outflow velocity is given by $V_s$. In this limit, the conservation of particle flux reduces to the jet cross-sectional area being equal to the TS area or $r_{jet} = \sqrt{2} R_s$. With increasing $c_{As}$, the outflow velocity of the jet increases and $r_{jet}$ decreases. For $c_{As} \gg V_s$, $r_{jet} \propto R_s \sqrt{V_s/c_{As}}$. An expression for the pressure jump $\Delta P$ between the TS and the LISM can be calculated from Equation (15), which is exact in the jet where $V_{0r} = 0$:

$$\Delta P = 2 B_s^2 \tan^{-1} \left( \frac{\sqrt{2} c_{As}/V_s}{\sqrt{2} c_{As}/V_s} \right).$$  \hspace{1cm} (21)$$

The pressure jump is a maximum when $c_{As}$ is small and decreases with increasing $c_{As}$. The dependence of $r_{jet}$ and $\Delta P$ is shown as functions of $c_{As}/V_s$ in Figure 3. From Equation (15), $r_{hp}$ therefore also decreases with increasing $c_{As}/V_s$. 

![Figure 2.](image1.png)  

![Figure 3.](image2.png)
3. GLOBAL MHD SIMULATIONS

We have carried out MHD simulations of the global heliosphere without an interstellar wind and magnetic field. Our model is based on the 3D multi-fluid MHD code BATS-R-US. It involves one ionized and four neutral H species as well as the magnetic field of the Sun. We used a monopole configuration for the solar magnetic field to eliminate artificial reconnection across the heliospheric current sheet. The basic parameters of the simulation are the same as those described in Opher et al. (2015). The computational grid was ±3000 AU in each direction. Parameters of the solar wind at the inner boundary at 30 AU were \( \rho_{SW} = 417 \text{ km s}^{-1}, n_{SW} = 8.74 \times 10^{-3} \text{ cm}^{-3}, T_{SW} = 1.087 \times 10^8 \text{ K} \), and the Parker spiral magnetic field with a radial component \( B_{SW} = 7.17 \times 10^{-3} \text{ nT} \) at the equator (with an azimuthal component \( B_{\phi} = 0.22 \text{ nT} \)). The solar wind flow at the inner boundary is assumed to be spherically symmetric, and the magnetic axis is aligned with the solar rotation axis. For the LISM, we assume \( T_{ISM} = 6519 \text{ K} \), while the plasma density was raised to 0.483 cm\(^{-3}\) to make up for the absence of pressure associated with the interstellar magnetic field. The number density of H atoms in the interstellar medium is \( n_H = 0.18 \text{ cm}^{-3} \), and the temperature is the same as for the interstellar plasma. The z axis is parallel to the solar rotation axis. The grid has cells ranging from 2.93 AU at the inner boundary to 187.5 AU at the outer boundary. The simulation had a resolution of 3.0 AU between \( z = \pm 750 \text{ AU} \) and \( x = \pm 305 \text{ AU}; y = \pm 400 \text{ AU} \), encompassing the entire HS. The run was stepped forward for 3061 years.

In Figure 4, we show in yellow the surface of the HP as defined by \( \ln T = 13.9 \). The simulation reveals jets to the north and south as in the analytic model. The HP bulges at the equator as in the model. The gray lines are the solar magnetic field. Shown in Figure 5 in the \( x-z \) plane are the plasma pressure in (a), the magnetic pressure in (b), and the speed and streamlines in (c). As in the model, the plasma pressure decreases with cylindrical radius \( r \) away from the jet axis, while the magnetic pressure increases with \( r \) and the strongest magnetic fields are in the equatorial region just upstream of the HP. The streamlines reveal the north- and south-directed outflows that make up the jets. The HP boundary does not reveal the sharp indentation seen in the model. Finally, in Figure 2(c), we show cuts of the total pressure (solid), the plasma pressure (dotted), the magnetic pressure (dashed), and the magnetic field (dotted–dashed) in cuts along \( r \) at the equator from just upstream of the TS to past the HP. The pressures have been normalized to \( B_0^2/8\pi \), \( B \) to \( B_0 \), and \( r \) to \( R_s \) with \( R_s = 135 \text{ AU} \) taken to be the location of the maximum of \( P_{\text{plasma}} \). The cuts are in remarkable agreement with the cuts from the model in (a). In the simulation \( c_s/V_s = 1.1 \), which, from Figure 3, yields \( 8\pi \Delta P/B_0^2 = 1.3 \) compared with the measured value of 1.5 from Figure 2(c). For \( 8\pi \Delta P/B_0^2 = 1.5 \), Equation (15) yields \( r_{sp}/R_s = 1.9 \) at the equator, essentially identical to the HP radius in the cuts, and the ratio of the HP radius at the equator to that in the jet is 0.64 (Equation (15)) compared with the measured value of 0.66. Finally, the measured particle flux through the TS is the same as that out the jets.

4. DISCUSSION AND CONCLUSIONS

We have explored the structure of the HS and HP when the interstellar flow and magnetic field are neglected and the system can be treated as axisymmetric. We show that even in the limit in which \( P \approx B_0^2/8\pi \) in the HS, the magnetic field controls the large-scale structure of the HS and drives northward- and southward-directed jets. To the lowest order, the pressure in the HS is balanced by the pressure in the interstellar medium. The magnetic field controls the pressure variation within the HS and redirects and boosts the flow across the TS to the north and south to form heliospheric jets. The radial distance from the TS to the HP and the jet radii are controlled by the requirement that the plasma flowing into the HS across the TS flows outward in the jets (see also Yu 1974). For very weak magnetic fields, the jet outflow velocity is the same as the velocity \( V_s \) downstream of the TS. In this limit, the total cross-sectional area of the jets is equal to the area of the TS and \( r_{sp} = \sqrt{2} R_s \). With increasing magnetic field strength, the jet outflow velocity increases, and the radii of the HP and the outflow jet decrease (Equation (20) and Figure 3(a)).

The global MHD models of the heliosphere (Malama et al. 2006; Pogorelov et al. 2007; Opher et al. 2009) produce HS thicknesses that are around 50–70 AU, substantially larger than the value of 30 AU determined from Voyager 1’s crossing of the HP in 2012 (Stone et al. 2013). The results here suggest that mechanisms that increase the jet outflows will reduce the HP radius. Pressure reductions in the downstream region associated with thermal conduction or other mechanisms might produce such enhanced flows.
There is evidence from both the present MHD simulations and those carried out earlier (Opher et al. 2015) that the jets are subject to large-scale instabilities. The resulting turbulence might be a driver of anomalous cosmic rays (Stone et al. 2005, 2008). High time-resolution ENA measurements might be able to establish the existence of the heliospheric jets and associated turbulence. For a jet radius of around 140 AU and an Alfvén velocity of around 100 km s$^{-1}$, the Alfvén transit time is around 10 years. The jet turbulence might, of course, cascade to smaller scales, so the relevant timescales could be shorter.

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REFERENCES

Axford, W. I. 1972, NASSP, 308, 609

Baranov, V. B., & Malama, Y. G. 1993, IGR, 98, 15157
Begelman, M. C., & Li, Z.-Y. 1992, ApJ, 397, 187
Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 883
Chevalier, R. A., & Luo, D. 1994, ApJ, 421, 225
Dialynas, K., Krimigis, S. M., Mitchell, D. G., Roelof, E. C., & Decker, R. B. 2013, ApJ, 778, 40
Fendt, C., & Zinnecker, H. 1998, A&A, 334, 750
Gueth, F., & Guilloteau, S. 1999, A&A, 343, 571
Krimigis, S. M., Mitchell, D. G., Roelof, E. C., Hsieh, K. C., & McComas, D. J. 2009, Sci, 326, 971
Lyubarsky, Y. E. 2002, MNRAS, 329, L34
Malama, Y. G., Izmodenov, V. V., & Chalov, S. V. 2006, A&A, 445, 693
McComas, D. J., Dayeh, M. A., Funsten, H. O., Livadiotis, G., & Schwadron, N. A. 2013, ApJ, 771, 77
Opher, M., & Drake, J. F. 2013, ApJL, 778, L26
Opher, M., Drake, J. F., Zieger, B., & Gombosi, T. I. 2015, ApJL, 800, L28
Opher, M., Richardson, J. C., Toth, G., & Gombosi, T. I. 2009, SSRv, 143, 43
Opher, M., Stone, E. C., & Liewer, P. C. 2006, ApJL, 640, L71
Owen, F. N., & Rudnick, L. 1976, ApJL, 205, L1
Parker, E. N. 1961, ApJ, 134, 20
Pogorelov, N. V., Stone, E. C., Florinski, V., & Zank, G. P. 2007, ApJ, 668, 611
Pogorelov, N. V., Suess, S. T., Borovikov, S. N., et al. 2013, ApJ, 772, 2
Stone, E. C., Cummings, A. C., McDonald, F. B., et al. 2005, Sci, 309, 17
Stone, E. C., Cummings, A. C., McDonald, F. B., et al. 2008, Natur, 454, 71
Stone, E. C., Cummings, A. C., McDonald, F. B., et al. 2013, Sci, 341, 150
Washimi, H., Zank, G. P., Hu, Q., et al. 2011, MNRAS, 416, 1475
Yu, G. 1974, ApJ, 194, 187

Figure 5. (a) Plasma and (b) magnetic pressures (Pa) and (c) the plasma speed (km s$^{-1}$) and streamlines. All are in the x-z plane through the center of the heliosphere from the simulation in Figure 4.