String breaking with dynamical Wilson fermions

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We present results of our ongoing determination of string breaking in full QCD with $N_f = 2$ Wilson fermions. Our investigation of the fission of the static quark-antiquark string into a static-light meson-antimeson system is based on dynamical configurations of size $24^3 \times 40$ produced by the $\chi$QCD collaboration. Combining various optimization methods we determine the matrix elements of the two-by-two system with so far unprecedented accuracy. The all-to-all light quark propagators occurring in the transition element are computed from eigenmodes of the Hermitian Wilson-Dirac matrix complemented by stochastic estimates in the orthogonal subspace. We observe a clear signature for level-splitting between ground state and excited potential. Thus, for the first time, string breaking induced by sea quarks is observed in a simulation of 4-dimensional lattice-QCD.

1. INTRODUCTION

The fission of the flux string between two heavy quarks, is a key problem of lattice quantum chromodynamics (QCD). While it is fundamental to a proper understanding of QCD, after nearly 25 years of lattice QCD we are still lacking a convincing demonstration of string breaking through dynamical quarks \[1\].

Of course, extremely precise studies of the potential have been carried out, however, even at separations $r = |R|a$ much larger than the anticipated breaking distance of about 1 fm, still a linearly rising potential is found. This phenomenon is attributed to the insufficient overlap of the Wilson loop with the final state at large $r$.

As first suggested back in 1992 by C. Michael, progress in the conceptual understanding of string breaking on the lattice could be achieved: in Ref. \[2\], QCD was mimicked by the the SU(2)-Higgs model using a two channel setting. The authors could confirm the interpretation of string-breaking as a dynamical level-crossing phenomenon between string-type and meson-type states. They observed a pronounced gap of the energy levels at the location of the crossing.

A main hindrance of a straightforward application of this procedure to 4-dimensional QCD is the computation of all-to-all propagators to achieve a proper signal for the heavy-light contributions. The elements of the $2 \times 2$ mixing matrix—three of which are composites of Wilson lines and fermion propagators— must be averaged over all spatial lattice sites and dimensions. This requires the computation of the inverse of the Hermitian Wilson-Dirac matrix, $Q^{-1}$. As the dimension of $Q$ is about $O(10^7)$, the full inverse cannot be represented even on high-end systems.

In our investigation, we employ various “smearings” like APE smearing, Wuppertal smearing, fat temporal links and a new hopping parameter acceleration. The latter is shown to enhance the low eigenmode dominance. Furthermore, with the truncated eigenmode approximation (TEA) \[3\], rendered exact by stochastic estimates on the orthogonal subspace, all-to-all propagators can be computed efficiently.

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2. MODELLING THE TRANSITION

Close to the location of QCD string breaking, we deal with a mixture of two states, the initial heavy $Q\bar{Q}$-pair at small separation $r$, represented by the standard Wilson-Loop $W(R,T)$, and a final state at large separation $r$, which is given by two heavy-light mesons. The interesting physics, i.e. the fission of the flux-tube, takes place at intermediate $r$.

In the two-channel approach the correlator is represented as the following $2 \times 2$-matrix

$$C(t) = e^{-2m_Q t} \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

$C_{1,1}$ is the correlation function of the static $Q\bar{Q}$ pair, namely the Wilson loop, $C_{2,2}$ is the correlator of the heavy-light pair, $B\overline{B}$, and $C_{1,2}$ the correlator of the transition matrix element. The light quark propagators in the generalized Wilson loops are represented by wavy lines.

The exponential decay of the correlator is found by diagonalization of the real $2 \times 2$ transfer matrix, with the lower eigenvalue being the ground state energy of the system.

As demonstrated in Ref. [21], characteristic signatures are required in order to "prove" string breaking:

1. The energy levels show crossing with the asymptotic ground state reaching the same mass as the system of two separated heavy-light mesons.
2. At the location of crossing, dynamical string breaking manifests itself in a pronounced energy level-splitting.
3. The transition matrix element $C_{12}$ will stay finite for $a \to 0$.

3. SETTING

The basis for our investigation of string breaking are the $N_f = 2$ dynamical configurations produced by the SESAM and TχL groups. SESAM generated 8 ensembles of about 200decorated configurations each on $16^3 \times 32$ lattices at two different $\beta$-values, 5.6 ($\kappa = 0.156, 0.1565, 0.157, 0.1575, 0.1580, 0.56 < 0.085 < 0.83$) and 5.5 ($\kappa = 0.158, 0.159, 0.1596, 0.160, 0.67 < 0.085 < 0.85$). These ensembles have been analyzed for string breaking in order to determine a $\beta - \kappa$ combination with small enough sea quark mass still being unaffected by finite-size effects. We have chosen the pair $\beta = 5.6, \kappa = 0.1575$.

With a Sommer scale of $r_0 = 5.892(27) a$ at $r = 0.5$ fm the lattice spacing is $a = 0.085$ fm or $a^{-1} = 2.32$ GeV. $\frac{m_{\pi}}{m_{\rho}} = 0.704(5)$ suggests a degenerate quark mass close to the strange quark mass. For the $16^3 \times 32$ system, both the pion window of $La \approx 4m_{\pi}^{-1}$ and the string breaking window with $\sqrt{2L/2r_{SB}} \approx 0.9 < 1$ and $\sqrt{3L/2r_B} \approx 1.2 \geq 1$ are not large enough to comfortably accommodate the string breaking experiment. We chose to use our $T\chi L$ configurations generated on $24^3 \times 40$ lattices. Here the windows are $La \approx 6m_{\pi}^{-1}$ and $\sqrt{2L/2r_{SB}} \approx 1.2 > 1$, $\sqrt{3L/2r_B} \approx 1.4 > 1$. 183 decorated configurations were analyzed for Wilson loops, 20 decorated configurations were analyzed for fermion lines.

4. OVERLAP ENHANCEMENTS

We use a set of mutually optimized methods to improve the ground state overlap which reduce the statistical error by at least a factor of 5:

APE smearing. APE smearing is carried out on the gauge fields with $\alpha = 2.0$ and $N = 50$:

$$U_i(x) = P_{SU(3)} \left[ U_i(x) \right]$$

$$+ \frac{1}{2} \sum_{j \neq i} \left( U_i(x) U_j(x + \hat{i}) U^\dagger_j(x + \hat{j}) U^\dagger_i(x \hat{i}) + U^\dagger_i(x \hat{i}) U_j(x \hat{i}) U^\dagger_j(x \hat{j}) \right)$$

Wuppertal smearing with $\delta = 4$ and linear combinations of $N_W = 20, 40, 50$:

$$\phi^{(n+1)}_x = \frac{1}{1 + 6\delta} \left( \phi^{(n)}_x + \delta \sum_{j=\pm 1} U_{x,j} \phi^{(n)}_{x+a_j} \right)$$

Fat temporal links (HYP) à la Hasenfratz and Knechtli can lead to an exponential improvement of the signal quality.
Hopping parameter acceleration (HPA). The $k$th power of the Wilson hopping term, $(\kappa D)^k$, with $M = 1 - \kappa D$, connects sites at distance $k$ and less. Multiplying $M^{-1} = 1 + (\kappa D) + (\kappa D)^2 + (\kappa D)^3 \ldots$ by $(\kappa D)^k$ leads to $M^{-1}(\kappa D)^k = M^{-1} - \sum_{i=1}^{k-1} (\kappa D)^i$. Thus, at distance $k$, the first $k - 1$ terms do not contribute, and the low eigenmode dominance is enhanced, since with $M|\psi\rangle = \lambda|\psi\rangle$ we have $M^{-1}(\kappa D)^k|\psi\rangle = (1 - \lambda)^k|\psi\rangle$.

Furthermore, we improve $Q^{-1}$ by the Truncated Eigenmode Approximation (TEA) [3]. TEA is made exact by means of stochastic estimates (SET) in the orthogonal subspace. In Fig. 1 we compare the standard SET-method (no mode computed via TEA) on the smeared gauge fields with the improvement through HPA and TEA (200 modes).

**5. RESULTS**

Fig. 2 shows intermediate results of our ongoing evaluation of $T\chi L$ configurations that have been obtained with a preliminary form of the fat link static action. Hence the asymptotic ground state is just slightly higher than twice the heavy-light mass. The error bars of the zoomed plot are smaller than the symbols. Level-splitting is achieved at the crossing point for the first time in 4-dimensional QCD with dynamical fermions, demonstrating string breaking.

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