Rankine earth pressure theory considering microstructure of porous materials

Junhu Li¹, Wei Xue²,⁴, Chao Zhang¹, Wenchao Zhang²,⁴, Riqing Xu¹
¹Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou, 310058, China
²Guangzhou Chemical Grouting Engineering Co., Ltd., CAS, Guangzhou, 510650, China
³China Road and Bridge Corporation, Beijing, 100011, China
⁴Guangdong Province Chemical Grouting Engineering and Materials Academician Workstation, CAS, Guangzhou, 510650, China
*E-mail: 285014418@qq.com

Abstract: Soil as an engineering material has very complex properties, such as non-continuous, non-uniformity and nonlinear mechanical. In a certain extent, macroscopic properties of soil are affected by the changes of the microstructure. And microscopic porosity of soft clay and its influencing factors, the relationship between macro and micro porosity, the average contact area rate and its influencing factors are studied. Some mechanics problems were analyzed by using the relationship between macro-porosity and the average contact area rate. Combining soil lateral stress transfer principle, a calculation theory of earth pressure considering soil contact area was got. The possible reason of the differences between earth pressure and the actual monitoring earth pressure was analyzed by the case.

1. Introduction

The soil and water underground interacts with each other via the pore water in soil medium. The existence of the pore water and the pore water pressure is closely related to the permeability of soil. In the soil of strong permeability, because the pore water is interconnected which may cause the buoyancy on soil particles, we use the submerged unit weight of soil in the calculation. In this case, the pore water pressure can be regarded as equal to the hydrostatic pressure. However, as to the soil of weak permeability, such as clay, it is different. The relationship between porosity medium and water is extremely complicated. Thus it’s not easy to cause the buoyancy on the soil particles, and the pore water pressure can’t technically be seen as hydrostatic pressure any more. As is known, in the method of estimating water and earth pressures separately, which is clear on mechanical mechanism and also rigorous in theory, hydrostatic pressure acts on the soil retaining structure directly. This method works out well on the sandy soil of strong permeability. The method of estimating water and earth pressures together can avoid the difficulty in measuring the excess pore water pressure, but as for low permeable clay the pore water pressure can’t be transferred totally, which turns out to be limited.

As for these questions, the scholars domestic and abroad have carried out plenty of qualitative and quantitative research work on microstructure of fine particles based on microscope images in the past few decades[1-5]. CHEN Yu-jiong[6] believed that, as far as the method of estimating water and earth pressures together manages to avoid the puzzle of the pore water pressure by seeing the water and soil as an integral using total stress strength parameters, there is no need to figure out the mechanical
mechanism of the pore water pressure. But LI Guang-xin[7] pointed out the insufficiency of this method and believed that this method makes sense from microcosmic point of view. Besides, he proposed to incorporate the view of microstructure into the study of stress transmission mechanism. On the basis of previous studies, the paper analyses soft clay microstructure and modifies the commonly used formula of earth pressure, followed by a calculation theory considering soil contact area.

2. The micro porosity of the clay
Fig.1 is the microscopic pore figure of some kind of soft clay observed using scanning electron microscope (SEM). We can see from this figure that the gray level of the pore of soft clay decreases from deep to shallow. In the bottom of the pore, the gray level is highest and the color is darkest, which is opposite on the opening of the pore. The sample used in experiments shows the section area is getting bigger from the bottom to opening.

Fig.1 Microscopic pore figure of soft clay
Fig.2 Three-dimensional display of particle surface

In the SEM figure, the pixel represents the area of the figure and the gray level stands for the color depth. Different gray levels can be expressed by different thresholds, of which the values of range from 0 to 255. We set the brightness of the deepest place as the minimum value, and when the threshold goes larger, the brightness varies from deep to shallow.

In order to calculate the volume of the irregular shape, we applied the means of integration. The volume can be got by multiplying the area of each section by the corresponding height. The different thresholds in the SEM figure correspond to the area of pores on different section. Symbol Yi stands for different thresholds. Suppose the area of the smaller loop at the gray level of threshold Yi is $A_i$, the volume of pore between these two thresholds is equal to:

$$\Delta V = (A_{i+1} + A_i) \times (Y_{i+1} - Y_i)$$

(1)

$$dV = \Delta V_i$$

(2)

$$V_{3D} = \sum_{i=1}^{255} (A_{i+1} + A_i) \times (Y_{i+1} - Y_i)$$

(3)

Also, we can get the 3D porosity at arbitrary thresholds of gray level:

$$n_{3D} = \sum_{i=1}^{m} \frac{A_{i+1} + A_i}{2} \times (Y_{i+1} - Y_i)$$

$$n_{3D} = \frac{A_1 + A_m}{2} \times (Y_m - Y_0) \times S_d$$

(4)

In this formula, $m$ is the value of thresholds and $S_d$ is the area of the pixels of selected regions, which is analyzed by Image-Pro Plus (IPP) software. The initial threshold $Y_0$ is equal to 0.
3. The relationship between the contact area and porosity

According to the method mentioned above, we carried out an experiment by scanning the soft clay from three cities (Hangzhou, Ningbo, and Fenghua) and got 225 SEM images analyzed, accomplished by configuring the moisture content of the soil to control the size of the porosity. The rate of the average contact area can be expressed by the statistical mean of 5 SEM images value from each test specimen with different porosity. We can get the relationship of the contact area ($A_c$), pore area ($A_p$) and total area ($A$) as following:

$$R_{CA} = 1 - f(n)$$  \hspace{1cm} (5)

In practical situation, the soil is in a humid environment and the double electric layer on the surface of the clay particle includes the strong and weak bound water. The water film thickness of montmorillonite is 210 Å (or Angstrom), and that of kaolin is 410 Å. The hydrophilic property of illitic soil is somewhere in between both of those, so its water film thickness ranges between 210 Å and 410 Å.

Considering that the water film thickness varies with the properties of the soil, the water film thickness is set as 200 Å, 250 Å, 300 Å, 350 Å, 400 Å, and 450 Å respectively. Statistical analysis is made to build the relationship of the porosity and the rate of average pore area, as shown in fig.5, fig.6 and fig.7.

We can get the conclusion from these figures that the rate of average pore area considering the water film thickness of the three are consistent in changing trends. With the porosity becoming larger, the rate of the average pore area increases. The results indicate that their relationship can be well fitted by a power function. The equation is as follows:

$$R_{CA} = 1 - R_{PA} = 1 - n^{2^{1/2}}$$  \hspace{1cm} (6)
4. The Rankine earth pressure considering the contact area of particles by estimating water and earth pressures separately

Rankine active earth pressure:

\[ p_a' = \gamma' z K_a' - 2c' \sqrt{K_a'} \]
\[ u = u_w + u \]

(7)

Rankine passive earth pressure:

\[ p_p' = \gamma' z K_p' + 2c' \sqrt{K_p'} \]
\[ u = u_w + u \]

(8)

Considering the size of contact area, we analyze the effect of the cohesion with the contact area. When the height of the retaining wall is \( h \), and the influence depth of underground water (or pore water pressure) is \( h_1 \), the effective lateral pressure and pore pressure on the retaining wall on unit length can be calculated as following respectively.

Total Rankine active earth pressure:

\[ E_a = \left( \frac{1}{2} \gamma' h K_a' - 2c' \sqrt{K_a'} \right) h (1 - n^2) \]
\[ U = \frac{1}{2} (u_w + u) h_1^2 n^2 \]

(9)

Total Rankine passive earth pressure:

\[ E_p = \left( \frac{1}{2} \gamma' h K_p' + 2c' \sqrt{K_p'} \right) h (1 - n^2) \]
\[ U = \frac{1}{2} (u_w + u) h_1^2 n^2 \]

(10)

According to the analysis of load, the calculation of pore water pressure can be divided into 4 situations:

1) The pore water pressure is the static water pressure for only.

By the above part of the analysis, when the pore water pressure is the static water pressure for only, considering the size of the particle contact area, the effective lateral pressure on the retaining wall on unit length and the pore pressure are as following:

Total active earth pressure:

\[ E_a = \left( \frac{1}{2} \gamma' h K_a' - 2c' \sqrt{K_a'} \right) h (1 - n^2) \]
\[ U = \frac{1}{2} \gamma' h_1^2 n^2 \]

(11)

Total passive earth pressure:

\[ E_p = \left( \frac{1}{2} \gamma' h K_p' + 2c' \sqrt{K_p'} \right) h (1 - n^2) \]
\[ U = \frac{1}{2} \gamma_0 h_1^2 n^2 \]

(12)

2) The pore water pressure is combination of the static water pressure and the excess pore water pressure.

Considering the excess pore water pressure (\( \bar{u} \)), the pore water pressure is \( u = u_w + \bar{u} \), \( u_w = \gamma_0 h_1 \).

Given \( \sigma' = \gamma' z - u \)

\[ p_a = \gamma' z K_a' - 2c' \sqrt{K_a'} + \gamma_0 z + (1 - K_a') \bar{u} \]
\[ p_p = \gamma' z K_p' + 2c' \sqrt{K_p'} + \gamma_0 z + (1 - K_p') \bar{u} \]

(13)
The first two terms of the equations above are the effective pressure and the last two terms are static water pressure and excess pore water pressure respectively. Assuming the excess pore water pressure is uniformly distributed on the \( h \)-meter-thick aquifer, the effective lateral pressure on the retaining wall on unit length and the pore pressure are as following, considering the contact area of the particles.

Total active earth pressure:

\[
E_a = \left( \frac{1}{2} \gamma^{'h} K_i - 2c' \sqrt{K_i} \right) h(1-n^a) \\
U = \left[ \frac{1}{2} \gamma^{'h_i} (1-K_i) \bar{u} \right] h_n^n^a
\]

Total passive earth pressure:

\[
E_p = \left( \frac{1}{2} \gamma^{'h} K_r - 2c' \sqrt{K_r} \right) h(1-n^a) \\
U = \left[ \frac{1}{2} \gamma^{'h_i} (1-K_r) \bar{u} \right] h_n^n^a
\]

(3) The pore water pressure is the seepage pore water pressure for only considering the steady seepage, we get \( u = u_f \), \( u_f = \gamma_i z - u_i \), where \( u_f \) is the seepage pore water pressure and \( i \) is the hydraulic slope.

Given \( \sigma_i = \gamma_i z - u_f \), we get the equations:

\[
p = \gamma_i z K_i - 2c' \sqrt{K_i} + (1-K_i) u_f \\
p = \gamma_i z K_r + 2c' \sqrt{K_r} + (1-K_r) u_f
\]

Base on the assumption of steady seepage flow in the equation above, the seepage pressure is uniformly distributed on the \( h \)-meter-thick aquifer. Considering the contact area of the particles, the effective lateral pressure on the retaining wall on unit length and the pore pressure are as following:

Total active earth pressure:

\[
E_a = \left( \frac{1}{2} \gamma^{'h} K_i - 2c' \sqrt{K_i} \right) h(1-n^a) \\
U = (1-K_i) u_f h_n^n^a
\]

Total passive earth pressure:

\[
E_p = \left( \frac{1}{2} \gamma^{'h} K_r - 2c' \sqrt{K_r} \right) h(1-n^a) \\
U = (1-K_r) u_f h_n^n^a
\]

(4) The pore water pressure is combination of the seepage water pressure and the excess pore water pressure

Given the combined effects of the seepage water pressure and the excess pore water pressure, we know \( u = u_f + \bar{u} \), where \( u_f \) is the pore water pressure in seepage, \( \bar{u} \) is the excess pore water pressure. And also \( \sigma_i = \gamma_i z - u_f - \bar{u} \):

\[
p = \gamma_i z K_i - 2c' \sqrt{K_i} + (1-K_i) (u_f + \bar{u}) \\
p = \gamma_i z K_r + 2c' \sqrt{K_r} + (1-K_r) (u_f + \bar{u})
\]
the aquifer with the thickness of $h$, considering the contact area of the particles, the effective lateral pressure on the retaining wall per meter and the pore pressure are as following:

Total active earth pressure:

$$E_a = \left(\frac{1}{2} \gamma_w hK_a - 2c' \sqrt{K_a} \right)h(1 - n^2)$$

$$U_{a} = (1 - K_a)x(u + u_j)h \frac{n^2}{3}$$

Total passive earth pressure:

$$E_p = \left(\frac{1}{2} \gamma_w hK_p + 2c' \sqrt{K_p} \right)h(1 - n^2)$$

$$U_{p} = (1 - K_p)x(u + u_j)h \frac{n^2}{3}$$

5. Conclusion

In the regular method to calculate the earth pressure, including estimating water and earth pressure separately and together, the soil is regarded as continuous elastic body. In the method of separate calculation method, the lateral pressure caused by lateral deformation distributes onto the retaining wall using the entire section area, as well as the pore water pressure. However, in that case, it exaggerates to some extent the impact of soil deformation on the lateral pressure, as well as the action area of the pore water pressure, which makes the results on the high side in the three methods. The result of estimating water and earth pressure together above the water level is similar with that of the separate calculation, which likewise enlarges the influence of the lateral deformation. In this method, the saturated soil under water level can be viewed as continuous elastic material using the entire section area as the action area, which sounds somewhat reasonable. But the soil is kind of solid-liquid two-phase media, the force situations of which are very complex, and the assumption that soil is the continuous elastic material can’t well satisfy the real condition. If we regard the soil on the back of the retaining wall as a whole without considering the impact of underground water (when estimating water and earth pressure together), the pore water pressure acting on the wall in the calculation results will be ignored; if we take the soil as two phase materials (when estimating water and earth pressure separately), the influence of submerged unit weight of soil on the lateral pressure will be magnified, as well as the action area of the pore water pressure. Though modifications are proposed to reduce the exaggeration, that is to say, multiply the pore water pressure by the lateral pressure coefficient of soil, yet it’s not allowed to modify the lateral pressure of water because the static water pressure keeps all the same in all directions.

Instead of using the strength reduction method to calculate the pore water pressure, the method put forward in this paper consider the real contact area of the pore from the structural features of clay itself, which corresponds closely to the actual conditions in a sense. But the soil is much more complicated actually, and the distribution of the pore scale may not always be uniform. On account that the influence coefficient of the contact area ($\alpha$) has influencing factors within itself, how to calculate the lateral earth pressure more accurately is remained to be further researched and verified in numerous engineering practices. The rationalization mind of earth pressure on the support structure provides important guarantee for the engineering construction. Moreover, in addition to the safety and economy, more aspects such as multi engineering conditions need to be taken into account comprehensively during the design. Namely it saves investment, satisfies the demands of the engineering structure and really comes to achieve the double winning goal of social and economic efficiency.

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References

[1] Lin B, Cerato A B. Prediction of expansive soil swelling based on four micro-scale properties, J. (2012). *Bulletin of Engineering Geology & the Environment*. 71(1), 71–78.

[2] SHI B, LI S. Quantitative approach on SEM images of microstructure of clay soils, J. (1995). *Science in China, Series B*. 36(8), 741–748.

[3] SHI Bin. Quantitative assessment of changes of microstructure for clayey soil in the process of compaction, J. (1996). *Chinese Journal of Geotechnical and Engineering*. 18(4), 57–62.

[4] XIONG Cheng-ren, TANG Hui-ming, LIU Bao-chen, et al. Using SEM photos to gain the pore structural parameters of soil samples, J. (2007). *Journal of China University of Geosciences: Earth Science*. 32(3), 415–419.

[5] ZHANG Xian-wei, KONG Ling-wei, GUO Ai-guo, et al. Evolution of microscopic pore of structured clay in compression process based on SEM and MIP test, J. (2012). *Chinese Journal of Rock Mechanics and Engineering*. 31(2), 406–412.

[6] Chen Yu-jiong, Wen Yan-feng. Water and earth pressures on the supporting structure around a foundation pit, J. (1999). *Chinese Journal of Geotechnical Engineering*. 21(2), 139-143.

[7] LI Guang-xin. Estimating the water and earth pressures on the supporting structure around a foundation pit separately and together, J. (2000). *Chinese Journal of Geotechnical Engineering*. 22(3), 348-352.

[8] WANG Bao-jun, SHI Bin, CAI Yi, et al. 3D visualization and porosity computation of clay soil SEM image by GIS, J. (2008). *Chinese Journal of Geotechnical Engineering*. 29(1), 251-255.

[9] Xu Xian-zhi, LI Pei-chao, LI Chuan-liang. Principle of effective stress based on porous medium, J. (2001). *Mechanics in Engineering*. 23(4), 42-45.

[10] CHEN Cui-cui, SUN Wei, ZHOU Wei-ling. Quantitative image analysis on inner damage of cementitious material, J. (2010). *Journal of the Chinese Ceramic Society*. 4, 574-580.