Bound states and electromagnetic radiation of relativistically rotating cylindrical wells

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We compute the effect of rigid rotation on the non-relativistic bound states. The energy levels of the bound states increase with the angular velocity of rotation until at certain value of the angular velocity they are completely pushed out into the continuum which corresponds to dissociation of the bound states. When the angular velocity exceeds the critical value at which the ground state disappears into the continuum, no bound state is possible. This effect should have important consequences for the phenomenology of the quark-gluon plasma. One of the ways to study it experimentally is to observe the electromagnetic radiation emitted by a rotating bound state. We compute the corresponding intensity of electromagnetic radiation and show that it strongly depends on the angular velocity of rotation.

I. INTRODUCTION

The study of the properties of rotating systems has a long history. Relatively recent studies focused on thermodynamics and kinetic of rotating quantum systems \(^1\)\(^2\)\(^3\)\(^4\)\(^5\)\(^6\)\(^7\)\(^8\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\)\(^15\)\(^16\)\(^17\)\(^18\)\(^19\)\(^20\)\(^21\). A great interest in relativistic rotating systems was recently spurred by the phenomenology of the relativistic heavy-ion collisions that indicates that the quark-gluon plasma has large local vorticity \(^22\)\(^23\)\(^24\)\(^25\). In fact, the vorticity \(\Omega\) found in numerical simulations approaches the relativistic limit \(\Omega r = 1\), where \(r\) is the radial distance from the rotation axis. The importance of the causal boundary was emphasized by a number of authors \(^11\)\(^12\)\(^27\)\(^28\).

One of the plasma signatures is dissociation of the bound states immersed into it through the mechanism of the Debye screening. The ability of plasma to break up the bound states depends on the relationship between the Debye radius which is a decreasing function of temperature and the linear size of the bound state. On the other hand, the rotating plasma drags the bound state along. As a result, the rotating bound state possesses extra centrifugal energy that makes it more fragile. Thus, both the Debye screening and rotation induce the break up of bound states. The main goal of this paper is to investigate the effect of rotation on the bound state spectrum.

To this end we consider a model consisting of a cylindrical well of radius \(R\) and constant depth \(U_0\), rotating with constant angular velocity \(\Omega\) about an axis passing through its center-of-mass.
We neglect all plasma effects apart from being the source of rotation. Furthermore, we treat the bound state non-relativistically which allows us to retain essential qualitative features while keeping algebra simple. Our results thus apply literally to a heavy quarkonium immersed into and coaxial with a vortex of the rotating plasma. The critical ingredient of our calculation is the causal boundary condition at $r = 1/\Omega$. The results strongly depend on whether $\Omega R$ is larger or smaller than unity. We refer to this two cases as rapid and slow rotation. Our main observation is that the energy levels of the rotating quarkonium increase with $\Omega$ and get pushed out of well at a certain $\Omega$ that is larger for deeper wells. We study these matters in Sec. III.

The dependence of the spectrum on $\Omega$ affects the electromagnetic radiation emitted by the quarkonium. In particular, we found that when $\Omega > 1/R$ the intensity increases by $(\Omega R)^6$ in comparison with the non-rotating well. This is studied it Sec [V]. This means that the life-time of the excited quarkonium states are sharply decrease with $\Omega$. Similar conclusion can be made with regard to the gluon strong and electroweak interactions as well. This observation may be instrumental for the experimental investigation of the rotation effects on the bound states.

II. BOUND STATES OF STATIONARY CYLINDRICAL WELL

First, for the future reference, consider a non-rotating cylindrical well as shown in Fig. [I]. We are interested in the solutions with $E < 0$. The Schrödinger equation inside the well $r < R$, reads

$$H_0\psi_0 = -\frac{1}{2M} \nabla^2 \psi_0 - U_0 \psi_0 = E_0 \psi_0.$$  

The corresponding set of eigenfunctions, regular at the origin and bound in the radial direction, is

$$\psi_0(x) = AJ_m(kr)e^{im\phi}e^{ikz},$$  

where $J_m(z)$ is the Bessel function of the first kind, $A$ is the normalization constant and

$$k = \sqrt{2M(U_0 - k_x^2/2M - |E_0|)}.$$  

Clearly the bound states exist only if $|E_0| \leq U_0 - k_x^2/2M$.

Outside the well the functions that vanish at $r \to \infty$ are

$$\psi_0(x) = BK_m(\kappa r)e^{im\phi}e^{ikz},$$  

where $K_m(z)$ is the modified Bessel function of the second kind and

$$\kappa = \sqrt{2M(k_x^2/2M + |E_0|)}.$$
FIG. 1. Potential cylindrical well rotating with angular velocity $\Omega$. Because of the causality, the wave function must vanish at $r > 1/\Omega$. Left panel: slow rotation, right panel: fast rotation. In the stationary case $\Omega^{-1} \to \infty$.

Eq. (4) satisfy the boundary condition $\psi \to 0$ as $r \to \infty$.

The boundary conditions at $r = R$ are the requirements of continuity of the wave function and its first derivative:

$$AJ_m(kR) = BK_m(\kappa R), \quad (6)$$
$$AJ'_m(kR)k = BK'_m(\kappa R)\kappa. \quad (7)$$

The solution exists only if the determinant vanishes:

$$J_m(kR)K'_m(\kappa R)\kappa = K_m(\kappa R)J'_m(kR)k. \quad (8)$$

This equation along with

$$k = \sqrt{2MU_0 - \kappa^2} \quad (9)$$
gives the spectrum.

III. BOUND STATES OF ROTATING CYLINDRICAL WELL

The Hamiltonian of the rotating system is $H = H_0 - \Omega J_z$. Thus, in cylindrical coordinates, $H\psi = E\psi$ is equivalent to $H_0\psi = (E + m\Omega)\psi$, where $m$ is an integer eigenvalue of the operator $J_z = -i\partial_\phi$. The general solution of the latter equation was found in the previous section, we just need to substitute $E_0 = E + m\Omega$ and apply the new boundary conditions. We proceed by considering separately the slowly and rapidly rotating well.
A. Slow rotation $R < 1/\Omega$

The wave functions inside the well have the same functional form as (2). However, $k$ is now given by

$$k = \sqrt{2M(U_0 - k_z^2/2M - |E + m\Omega|)}, \quad E + m\Omega < 0.$$  (10)

The solution outside the well reads

$$\psi(x) = [BK_m(\kappa r) + CI_m(\kappa r)] e^{im\phi} e^{ik_z z},$$  (11)

where $I_m(z)$ is the modified Bessel function of the first kind and

$$\kappa = \sqrt{2M(k_z^2/2M + |E + m\Omega|)}.$$  (12)

Unlike $\psi_0$ in (4), eigenfunctions (11) cannot extend to infinity. Instead, causality requires that $\psi$ vanishes at $r = 1/\Omega$:

$$BK_m(\kappa/\Omega) + CI_m(\kappa/\Omega) = 0.$$  (13)

Additionally, the boundary conditions at $r = R$ yield

$$AJ_m(kR) = BK_m(\kappa R) + CI_m(\kappa R),$$  (14)
$$AJ'_m(kR)k = BK'_m(\kappa R) + CI'_m(\kappa R)\kappa.$$  (15)

The determinant of the set of linear equations (13), (14), (15) must vanish:

$$I_m(\kappa/\Omega) \left[ -J_m(kR)K'_m(\kappa R)\kappa + K_m(\kappa R)J'_m(kR)k \right]$$
$$- K_m(\kappa/\Omega) \left[ -J_m(kR)I'_m(\kappa R)\kappa + I_m(\kappa R)J'_m(kR)k \right] = 0.$$  (16)

As $\Omega \to 0$ this equation reduces to (8).

Using the convenient notation: $x = kR, y = \kappa R, \lambda = R\Omega, \beta = \sqrt{2MU_0 R^2}$, we can write the boundary condition (16) as:

$$I_m(y/\lambda) \left[ -J_m(x)K'_m(y)y + K_m(y)J'_m(x)x \right]$$
$$- K_m(y/\lambda) \left[ -J_m(x)I'_m(y)y + I_m(y)J'_m(x)x \right] = 0,$$  (17)

where $x$ and $y$ are related to each other as

$$x = \sqrt{\beta^2 - y^2}.$$  (18)

Denoting the roots of (17) as $y_{mn}, n = 1, 2, \ldots$ we obtain the energy spectrum

$$E_{mnk_z} = -\frac{1}{2MR^2} y_{mn}^2 - \frac{m\lambda}{R} + \frac{k_z^2}{2M}.$$  (19)
FIG. 2. Energy levels (19) and (22) with \( m = 0, k_z = 0 \) in wells of different depth: \( \beta = 1 \) (dotted line), \( \beta = 1.5 \) (short-dashed line), \( \beta = 2 \) (long-dashed line), \( \beta = 2.5 \) (dashed-dotted line) and \( \beta = 3 \) (solid line) as a function of \( \lambda = R\Omega \). Notice the discontinuity at \( \lambda = 1 \).

B. Rapid rotation

If the rotation is fast \( R > 1/\Omega \), all the region \( r > 1/\Omega \) is outside of the rotating spacetime. Thus there is only solution inside the well; its functional form is (2) and the boundary condition is

\[
J_m(k/\Omega) = 0. \tag{20}
\]

Denoting zeros of the \( m \)'th Bessel function as \( x_{ma} \), \( a = 1, 2, \ldots \) we obtain \( k = \Omega x_{ma} \). The spectrum of a particle in the rapidly rotating cylindrical well is

\[
E_{mak_z} = \frac{1}{2M} (\Omega^2 x_{ma}^2 + k_z^2) - U_0 - m\Omega, \tag{21}
\]

or, equivalently,

\[
E_{mak_z} = \frac{1}{2MR^2} (\lambda^2 x_{ma}^2 - \beta^2) + \frac{k_z^2}{2M} - \frac{m\lambda}{R}. \tag{22}
\]

Fig. 2 displays the energy level (19), for \( \lambda < 1 \), and (22), for \( \lambda > 1 \), with \( m = 0, k_z = 0 \) in the potential wells of different depths. As the angular velocity of rotation increases, the level increases (i.e. its absolute value decreases) until it reaches \( E = 0 \). The discontinuity at \( R\Omega = 1 \) is an artifact of our model and disappears when the boundary is treated more carefully, see Sec. IV. For the range of \( \beta \) shown in the figure, there is only one level, but in a deeper well there are more energy levels. All of them exhibit similar dependence on \( \Omega \).
C. Shallow and slowly rotating cylindrical well

Consider states with $m = 0$, $k_z = 0$ in the shallow well limit $\beta^2 \ll 1$, we can expand the Bessel functions in (17) at $x, y \ll 1$ which yields

$$1 - \frac{1}{2}(\beta^2 - y^2) \ln \frac{2}{\gamma y} = -\frac{\pi \beta^2}{2} \exp\left(-\frac{2y}{\lambda}\right),$$

(23)

where $\gamma = \exp \gamma \approx 1.78$, with $\gamma$ being Euler-Mascheroni constant. Considering the right-hand-side of (23) a perturbation we can write at the leading “non-rotating” approximation

$$1 - \frac{1}{2}(\beta^2 - y_0^2) \ln \frac{2}{\gamma y_0} = 0.$$  

(24)

The only solution is obtained by neglecting $y_0$ compared to $\beta$:

$$y_0 = \frac{2}{\gamma} \exp\left(-\frac{2}{\beta^2}\right).$$  

(25)

Clearly, $y_0$ is much smaller than any positive power of (small) $\beta$. In usual units

$$E_0 = -\frac{2}{\gamma^2 MR^2} \exp\left(-\frac{2}{MU_0 R^2}\right).$$  

(26)

The second iteration of (23) is obtained by expanding $y = y_0 + y_1$ at small $y_1 \ll y_0$:

$$y_1 = -\pi y_0 \exp\left(-\frac{4}{\lambda \gamma} e^{-\frac{2}{\beta^2}}\right).$$  

(27)

Converting this back to the dimensional quantities gives the correction due to rotation

$$\Delta E = -2\pi E_0 \exp\left(-\frac{4}{MU_0 R^2} e^{-\frac{2}{MU_0 R^2}}\right).$$  

(28)

As $\Omega$ increases, $\Delta E$ also increases with the result that the energy level moves up.

The level crosses into the continuum when $E_0 + \Delta E = 0$. This happens at the value $\Omega_c$ given by

$$\Omega_c = \frac{2\sqrt{2M|E_0|}}{\ln(2\pi)}.$$  

(29)

IV. BOUND STATES OF ROTATING 2D $1/r$ POTENTIAL

Another instructive model of a rotating well with cylindrical symmetry is an attractive $U = -\alpha/r$ potential in two dimensions. Unlike the previous example, this potential does not have a sharp boundary and hence no discontinuity of the energy levels observed in Fig. 2.
As in the previous section we first consider the stationary problem
\[
\left( \nabla^2 + \frac{2M\alpha}{r} - 2M|E_0| \right) \psi_0 = 0. \tag{30}
\]
Its solution, finite at the origin, is
\[
\psi_0 = A_1 \, {}_1F_1(-n + |m| + 1/2, 2|m| + 1, \rho) \rho^{|m|} e^{-\rho/2} e^{im\phi}, \tag{31}
\]
where \( {}_1F_1 \) is the confluent hypergeometric function of the first kind and we defined
\[
\rho = \frac{2\alpha M}{n} r, \quad n = \frac{1}{\sqrt{2|E|}}. \tag{32}
\]
Requiring that at \( r \to \infty \) the wave function \( \psi \) is finite gives the quantization condition \( n = n' - 1/2 \) with \( n' = 1, 2, \ldots \) and \( |m| \leq n' - 1 \).

The boundary condition of the rotating potential requires vanishing of the wave function at \( r = 1/\Omega \), i.e.
\[
\psi \bigg|_{\rho=2\ell/n} = 0, \quad \ell = \frac{\alpha M}{\Omega}. \tag{33}
\]
As a result \( n' \) depends on \( \Omega \) and is not integer anymore. Solving \( \ell = \frac{\alpha M}{\Omega} \) for the given values of \( m \) and \( \ell \) we find the spectrum of \( n' \). The energy levels are then given by
\[
E_{n',m} = -\frac{\alpha^2 M}{2(n'(\Omega) - 1/2)^2} - m\Omega. \tag{34}
\]

Fig. 3 (left panel) shows \( E_{n',0} \) as a function of \( \ell \). The qualitative behavior is similar to Fig. 2.

As the angular velocity of rotation \( \Omega \) increases the energy levels get pushed out of the well until at \( \ell = \ell_{1,0} = 0.72 \) the last azimuthally symmetric level \((m = 0, n' = 1)\) disappears indicating absence of such bound states. The next azimuthally symmetric level \((m = 0, n' = 2)\) disappears at \( \ell = \ell_{2,0} = 3.84 \).

V. ELECTROMAGNETIC RADIATION BY ROTATING CYLINDRICAL WELL

A rotating well binding oppositely charged particles emits electromagnetic radiation. The intensity of radiation can be used as an effective diagnostic tool.

The \( S \)-matrix element describing photon radiation when the system transitions from the initial state \( i \) to the final state \( f \) is
\[
S_{fi} = -ie \int dt \int d^3x \, \mathbf{j}_{fi}(x) \cdot \mathbf{A}^*(x), \tag{35}
\]
\* More precisely, the level that has \( m = 0, n' = 1 \) at \( \Omega = 0 \).
FIG. 3. Left panel: Energy levels $m = 0$, $n' = 1, 2, 3, 4$ (dotted, dashed, dashed-dotted and solid lines respectively) in the two-dimensional potential $U = -\alpha/r$ at different values of $1/\ell$. Right panel: energy levels with given $n'$ (first number) and $m$ (second number) at $\ell = 10$ and $\alpha \ell = 1$.

where the transition current is

$$ j_{fi} = \frac{i}{2M} (\psi_i \nabla \psi_f^* - \psi_f^* \nabla \psi_i) . $$ (36)

We will distinguish the quantum numbers of the final state with the prime, i.e. $i = \{m, k_z, \ldots\}$, $f = \{m', k'_z, \ldots\}$. The well wave functions, including their time dependence, read

$$ \psi(x) = \frac{1}{2\pi} X(r) e^{im\phi} e^{ikz} e^{-iEt} , $$ (37)

where $X(r)$ is an appropriate radial function derived in the previous section. The wave functions are normalized by $\int \psi^* \psi d^3x = 1$ which implies

$$ \int_0^{1/\Omega} drr X^2 = 1 . $$ (38)

The photon wave function is

$$ A^*(x) = \frac{1}{\sqrt{2\omega V}} \epsilon_{p,\lambda}^* e^{-ip\cdot x + i\omega t} , $$ (39)

where $\epsilon_{p,\lambda}^*$ is the polarization vector, $\lambda = \pm 1$.

Since we treat the well in non-relativistic approximation, the leading contribution to the radiation intensity is the electric dipole radiation corresponding to wave lengths much larger than the system size, i.e. $pR \ll 1$. Thus neglecting the phase $p \cdot r$ in the exponent in (39) and substituting it along with (37) into (36) and (35) produces

$$ S_{fi} = \frac{e(2\pi)^2}{2M \sqrt{2\omega V}} \epsilon_{p,\lambda}^* \cdot I_{fi} \delta(E - E' - \omega) \delta_{mm'} \delta_{k_z k'_z} , $$ (40)
where we defined

\[ I_{fi} = I_{r,fi} \hat{r} + I_{\phi,fi} \hat{\phi} + I_{z,fi} \hat{z} \]

\[ = \hat{r} \int_0^{1/\Omega} (X'_r X_i - X_f X'_i) r dr + 2m \hat{\phi} \int_0^{1/\Omega} X_f X_i dr + 2k_z \hat{z} \int_0^{1/\Omega} X_f X_i r dr. \]  

(41)

The total radiation intensity is given by dividing the emission probability \(|S|^2\) by the observation time \(\Delta t\), multiplying by the photon energy \(\omega\) and summing and integrating over the photon phase space:

\[ W_{fi} = \sum_{\lambda} \int \frac{|S|^2}{\Delta t} \frac{d\omega \omega^2 d\omega V}{(2\pi)^3}, \]  

where \(d\omega\) is the solid angle element in the direction of the photon momentum. Substituting (42) into (40) and using

\[ \sum_{\lambda} \left| \epsilon^*_{p,\lambda} \cdot I_{fi} \right|^2 = \frac{|I_{fi} \times \mathbf{p}|^2}{\omega^2}, \]  

we obtain

\[ W_{fi} = \frac{1}{24\pi} \frac{e^2 \omega^2}{M^2} |I_{fi}|^2 \delta_{mm'} \delta_{k_z k'_z}. \]  

(44)

Eqs. (44) and (41) give the intensity of the electromagnetic radiation in the dipole approximation. The photon energy \(\omega\) is fixed by the energy and momentum conservation and depends on the angular velocity \(\Omega\).

A. Radiation by rapidly rotating cylindrical well

Using the model we developed in the previous section we can compute the intensity of electromagnetic radiation of the rapidly rotating cylindrical well \(R > 1/\Omega\). For simplicity we consider a reference frame where \(k_z = 0\). The radial wave functions normalized by (38) and satisfying the boundary condition (20) are

\[ X(r) = \sqrt{2 \Omega \over J_{m+1}(x_{ma})} J_m(x_{ma} r). \]  

(45)

Using these in (41) and plugging into (44) furnishes

\[ W_{fi} = \frac{1}{96\pi} \frac{e^2 \Omega^6}{M^4} (x_{ma}^2 - x_{ma'}^2)^2 \Omega^2 (I_{r,fi}^2 + I_{\phi,fi}^2) \delta_{mm'}, \]  

where the photon energy is, using (22),

\[ \omega = E_{ma0} - E_{ma'0} = \frac{\Omega^2}{2M} (x_{ma}^2 - x_{ma'}^2), \]  

(47)
and the functions

$$
\tilde{I}_{r,fi} = \int_0^1 \frac{2\rho d\rho}{J_{m+1}(x_{ma})J_{m+1}(x_{ma'}')} \left[ J_m(x_{ma'}\rho)J_m(x_{ma'}\rho')x_{ma'} - J_m(x_{ma}\rho)J'_m(x_{ma}\rho)x_{ma} \right],
$$

(48)

$$
\tilde{I}_{\phi,fi} = \int_0^1 \frac{4md\rho}{J_{m+1}(x_{ma})J_{m+1}(x_{ma'}')} J_m(x_{ma'}\rho)J_m(x_{ma})
$$

(49)

are independent of \( \Omega \). They can be expressed in terms of the hypergeometric function.

### B. Radiation by rotating infinite cylindrical well

The calculation of the radiation intensity for slowly rotating cylindrical well \( R < 1/\Omega \) can also be done analytically. However it leads to bulky expressions involving Bessel functions. A more instructive model is an infinite cylindrical well, i.e. the potential \( U = 0 \) if \( r \leq R \) and \( U = \infty \) otherwise. The corresponding wave functions are similar to (45)

$$
X(r) = \frac{\sqrt{2}}{\ell J_{m+1}(x_{ma})} J_m(x_{ma}r/\ell),
$$

(50)

where \( \ell = R \) if \( R < 1/\Omega \) (slow rotation) and \( \ell = 1/\Omega \) otherwise. The intensity is now given by (see (46)):

$$
W_{fi} = \frac{1}{96\pi} \frac{e^2}{M^4\ell^6} (x_{ma}^2 - x_{ma'}^2)^2 (\tilde{I}_{r,fi} + \tilde{I}_{\phi,fi}) \delta_{mm'}.
$$

(51)

Thus, in intensity is \( \Omega \) independent for slow rotation. However, once \( \Omega > 1/R \) the intensity increases rapidly in proportion to \( \Omega^6 \):

$$
W_{fi} = (R\Omega)^6 W_{fi}^{\Omega=0}, \quad \text{if } \Omega > 1.
$$

(52)

While the particular functional form of \( W_{fi}(\Omega) \) depends on the form of the potential \( U \), the observation of the steep growth of the radiation intensity with \( \Omega \) appears to be quite general.

### VI. DISCUSSION

We computed the energy spectrum of a particle in the non-relativistic cylindrical potential well rotating with constant angular velocity \( \Omega \). The energy levels increase with the angular velocity \( \Omega \) until they are pushed out of the well at some critical value \( \Omega_c \) as can be seen in Fig. 2. The particular form of the functional dependence \( E(\Omega) \) is model-dependent. The discontinuity at \( R\Omega = 1 \) is an artifact of the cylindrical well approximation; it disappears for a smoother potential such as the attractive \( 1/r \) potential discussed in Sec. IV.
To investigate the effect of rotation in a phenomenologically interesting case of charmonium one would have to consider rotation of the spherical symmetric potential, which has to be dealt with numerically. Nevertheless, we can make a ballpark estimate using (34). The binding energy of $J/\psi$ is $M(\psi') - M(J/\psi) = \frac{16}{3} \alpha^2 M = 0.59$ GeV. The reduced mass is $M = m_c/2 = 0.63$ GeV which implies that $\alpha = 0.72$. Using the values $\ell_{1,0}$ and $\ell_{2,0}$ listed at the end of Sec. [IV] we derive that $J/\psi$ and $\psi'$ dissociate into the continuum when they get swirled by a vortex rotating with angular velocity $\Omega = 3$ fm$^{-1}$ and $\Omega = 0.6$ fm$^{-1}$ respectively. These estimates of $\Omega$ are somewhat larger than the vorticity achievable at a (relatively) low energy relativistic heavy ion collisions [24]. We stress that at the above values of vorticity, the bound states completely dissolve, however the effect of rotation — the decrease of the binding energy with $\Omega$ — is essential even at sub-critical angular velocities. Needless to say that these estimates should be taken only as a motivation for further investigation with more accurate quarkonium models.

We also computed the intensity of radiation emitted by a rotating bound state and found that when $R\Omega < 1$ it increases by a factor $(R\Omega)^6$. This indicates an increase in transition probability from the excited to the ground state.

Our results indicate that plasma rotation has significant effect on the hadron spectra in relativistic heavy-ion which deserves a dedicated quantitative study.

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[1] A. Vilenkin, “Quantum field theory at finite temperature in a rotating system,” Phys. Rev. D 21 (1980), 2260-2269
[2] F. Becattini and F. Piccinini, “The Ideal relativistic spinning gas: Polarization and spectra,” Annals Phys. 323 (2008), 2452-2473 [arXiv:0710.5694 [nucl-th]].
[3] V. E. Ambruş and E. Winstanley, “Rotating quantum states,” Phys. Lett. B 734 (2014), 296-301 [arXiv:1401.6388 [hep-th]].
[4] B. McInnes, Nucl. Phys. B 887 (2014), 246-264 doi:10.1016/j.nuclphysb.2014.08.011 [arXiv:1403.3258 [hep-th]].
[5] B. McInnes, “Inverse Magnetic/Shear Catalysis,” Nucl. Phys. B 906 (2016), 40-59 [arXiv:1511.05293 [hep-th]].
[6] V. E. Ambrus and E. Winstanley, “Rotating fermions inside a cylindrical boundary,” Phys. Rev. D 93 (2016) no.10, 104014 [arXiv:1512.05239 [hep-th]].

[7] H. L. Chen, K. Fukushima, X. G. Huang and K. Mameda, “Analogy between rotation and density for Dirac fermions in a magnetic field,” Phys. Rev. D 93 (2016) no.10, 104052 [arXiv:1512.08974 [hep-ph]].

[8] B. McInnes, “A rotation/magnetism analogy for the quark–gluon plasma,” Nucl. Phys. B 911 (2016), 173-190 [arXiv:1604.03669 [hep-th]].

[9] Y. Jiang and J. Liao, “Pairing Phase Transitions of Matter under Rotation,” Phys. Rev. Lett. 117 (2016) no.19, 192302 [arXiv:1606.03808 [hep-ph]].

[10] M. N. Chernodub and S. Gongyo, “Interacting fermions in rotation: chiral symmetry restoration, moment of inertia and thermodynamics,” JHEP 01 (2017), 136 [arXiv:1611.02598 [hep-th]].

[11] S. Ebihara, K. Fukushima and K. Mameda, “Boundary effects and gapped dispersion in rotating fermionic matter,” Phys. Lett. B 764 (2017), 94-99 [arXiv:1608.00336 [hep-ph]].

[12] M. N. Chernodub and S. Gongyo, “Effects of rotation and boundaries on chiral symmetry breaking of relativistic fermions,” Phys. Rev. D 95 (2017) no.9, 096006 [arXiv:1702.08266 [hep-th]].

[13] M. Buzzegoli, E. Grossi and F. Becattini, “General equilibrium second-order hydrodynamic coefficients for free quantum fields,” JHEP 10, 091 (2017) [erratum: JHEP 07, 119 (2018)] [arXiv:1704.02808 [hep-th]].

[14] H. Zhang, D. Hou and J. Liao, “Mesonic Condensation in Isospin Matter under Rotation,” Chin. Phys. C 44 (2020) no.11, 111001 [arXiv:1812.11787 [hep-ph]].

[15] M. Buzzegoli, “Thermodynamic equilibrium of massless fermions with vorticity, chirality and electromagnetic field,” Lect. Notes Phys. 987, 59-93 (2021) [arXiv:2011.09974 [hep-th]].

[16] L. Wang, Y. Jiang, L. He and P. Zhuang, “Local suppression and enhancement of the pairing condensate under rotation,” Phys. Rev. C 100 (2019) no.3, 034902 [arXiv:1901.00804 [nucl-th]].

[17] Y. Liu and I. Zahed, “Pion Condensation by Rotation in a Magnetic field,” Phys. Rev. Lett. 120 (2018) no.3, 032001 [arXiv:1711.08354 [hep-ph]].

[18] L. Wang, Y. Jiang, L. He and P. Zhuang, “Chiral vortices and pseudoscalar condensation due to rotation,” Phys. Rev. D 100 (2019) no.11, 114009 [arXiv:1901.04697 [nucl-th]].

[19] N. Sadooghi, S. M. A. Tabatabaei Mehr and F. Taghinavaz, “Inverse magnetorotational catalysis and the phase diagram of a rotating hot and magnetized quark matter,” Phys. Rev. D 104 (2021) no.11, 116022 [arXiv:2108.12760 [hep-ph]].

[20] H. L. Chen, X. G. Huang and K. Mameda, “Do charged pions condense in a magnetic field with rotation?,” [arXiv:1910.02700 [nucl-th]].

[21] A. Palermo, M. Buzzegoli and F. Becattini, “Exact equilibrium distributions in statistical quantum field theory with rotation and acceleration: Dirac field,” JHEP 10 (2021), 077 [arXiv:2106.08340 [hep-th]].

[22] L. Adamczyk et al. [STAR], “Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid,” Nature 548 (2017), 62-65 [arXiv:1701.06657 [nucl-ex]].
[23] L. P. Csernai, V. K. Magas and D. J. Wang, “Flow Vorticity in Peripheral High Energy Heavy Ion Collisions,” Phys. Rev. C 87 (2013) no.3, 034906 [arXiv:1302.5310 [nucl-th]].

[24] W. T. Deng and X. G. Huang, “Vorticity in Heavy-Ion Collisions,” Phys. Rev. C 93 (2016) no.6, 064907 [arXiv:1603.06117 [nucl-th]].

[25] Y. Jiang, Z. W. Lin and J. Liao, “Rotating quark-gluon plasma in relativistic heavy ion collisions,” Phys. Rev. C 94 (2016) no.4, 044910 [erratum: Phys. Rev. C 95 (2017) no.4, 049904] [arXiv:1602.06580 [hep-ph]].

[26] X. L. Xia, H. Li, Z. B. Tang and Q. Wang, “Probing vorticity structure in heavy-ion collisions by local Λ polarization,” Phys. Rev. C 98 (2018), 024905 [arXiv:1803.00867 [nucl-th]].

[27] M. N. Chernodub and S. Gongyo, “Edge states and thermodynamics of rotating relativistic fermions under magnetic field,” Phys. Rev. D 96, no.9, 096014 (2017) [arXiv:1706.08448 [hep-th]].

[28] G. Duffy and A. C. Ottewill, “The Rotating quantum thermal distribution,” Phys. Rev. D 67 (2003), 044002 [arXiv:hep-th/0211096 [hep-th]].