Joint torque calculation of compression sports spats using anisotropic hyperelastic model

Hitoshi Aoki*, Takatsugu Shimana, Hiroki Sato, Ryuma Yabuki, Akihiro Matsuda

* University of Tsukuba, Tennodai 1-1-1, Tsukuba, Ibaraki, 3058573, Japan
b Mizuno Corporation, Nanko-Kita 1-12-35, Suminoe-ku, Osaka, 5598510, Japan

Abstract

In this study, a numerical design method for high-performance sports spats was proposed. The effect of sports spats on the human body during running motions was calculated. An anisotropic hyperelastic model considering stress softening was implemented to reproduce the mechanical characteristics of the sports spats fabric. Also, cyclic tensile loading tests were conducted to evaluate the mechanical characteristics of the sports spats fabric, which was composed of tricot fiber. The stiffness changed with loading direction, and the stiffness was softened depending on previous maximum stretch. The material parameters of the sports spats were identified using the cyclic loading test results. The theoretical calculations showed good agreement with experimental results. The anisotropic hyperelastic model was applied to a 3D-CG human model and running motion data were also introduced into the human model. The proposed numerical design method enabled calculation of the stress induced by the sports spats during running motions. Also, the joint torque generated in each knee was calculated using the stress calculation. The effect of high-performance sports spats on the human body was thus evaluated numerically.

1. Introduction

High-performance sports spats are widely applied in many sport fields. Prevention of injury and performance improvement are afforded by wearing sports spats. Also, it is known that sports spats fabrics exhibit anisotropy and stress softening because they are made from tricot fabric. Stress softening is when the fabric is softened by previously experienced maximum strain. However, these mechanical characteristics are not often considered in the design methods for high-performance sports spats. Therefore, considering anisotropy and stress softening would be useful for improving design of sports spats.

In this study, a three-dimensional stress calculation of sports spats using an anisotropic hyperelastic model considering stress softening is proposed. Cyclic tensile loading tests were conducted to investigate the characteristics of the fabrics. Cyclic loading test results were applied to the proposed anisotropic model. Finally, three-dimensional stress calculations were conducted, and joint torque was calculated to numerically evaluate the effect of sports spats on the human body during running.
2. Anisotropic hyperelastic model considering stress softening

2.1. Anisotropic hyperelastic model considering stress softening

In this study, the anisotropic hyperelastic model was defined by the strain energy function. The strain energy function considering stress softening proposed by Matsuda et al. [1] was introduced to reproduce the mechanical characteristics of the sports spats with stress analysis:

\[
W(C) = W_{iso}(C) + S(t_{4\max}^{(1)}) W_{ani}(C, M^{(1)}) + S(t_{4\max}^{(2)}) W_{ani}(C, M^{(2)})
\]  

(1)

where, \(C\) is the right Cauchy-Green deformation tensor, \(M^{(1)}\) and \(M^{(2)}\) are structural tensors, and \(t_{4\max}^{(1)}\) and \(t_{4\max}^{(2)}\) are maximum stretch experienced previously. The isotropic part \(W_{iso}\) is the Mooney-Rivlin model proposed by Rivlin et al. [2] and anisotropic part \(W_{ani}\) was proposed by Asai et al. [3]. The softening functions \(S(t_{4\max}^{(1)})\) and \(S(t_{4\max}^{(2)})\), proposed by Matsuda et al. [1], were defined as follows:

\[
\begin{align*}
S(t_{4\max}^{(1)}) &= 1 - \alpha_1 \left[ 1 - \exp \left( - \gamma_1 t_{4\max}^{(1)} - 1 \right) \right], \\
S(t_{4\max}^{(2)}) &= 1 - \alpha_2 \left[ 1 - \exp \left( - \gamma_2 t_{4\max}^{(2)} - 1 \right) \right]
\end{align*}
\]

(2)

where \(\alpha_1, \alpha_2, \gamma_1\) and \(\gamma_2\) are the softening parameters for sports spats fabrics. The second Piola-Kirchhoff stress tensor \(S\) is given by the partial differentiation of the strain energy function \(W\) with respect to the right Cauchy-Green tensor \(C\) as follows:

\[
S = 2 \frac{\partial W}{\partial C}
\]

(3)

In addition, the theoretical nominal stress \(\Pi\) is given by the product of a second Piola-Kirchhoff stress tensor and the transpose of the deformation gradient tensor \(F^T\) as follows:

\[
\Pi = S \cdot F^T
\]

(4)

3. Material parameters for the anisotropic hyperelasticity

In this study, cyclic loading tests were conducted to approximate the material parameters of an anisotropic hyperelastic model considering stress softening, and the material parameters were applied to the stress calculations to reproduce the effect of sports spats for analysis.

3.1. Cyclic loading tests

The cyclic loading tests were conducted to evaluate material parameters of the anisotropy and stress softening. The samples were cut directly from sports spats made by MIZUNO Corporation, as shown in Figure 1. The samples were cut using a Super-Dumbbell cutter made by DUMBELL CO., LTD, as shown in Figure 2. The material of the samples was a tricot composed of 71% polyester and 21% polyurethane. Tricot is a kind of knit notable because its fabric forms continuous loops in the warp direction of the yarns. Tricot has excellent elasticity and is comfortable and superior fitting. A magnified image of the structure of tricot is shown in Figure 3. Tricot is anisotropic because of its structure, and is, therefore, useful when different degrees of stiffness are needed according to the directions of the fibers. The samples were 50-mm long, 10-mm wide and 0.5-mm thick. In addition, the fiber orientation angles \((\theta)\) of the samples were 0°, 45° and 90° to evaluate anisotropy. The dimensions of the samples are shown in Figure 4.

For the following tests, a uniaxial testing machine made by SHIMADZU CORPORATION was used. A 500-N load cell was used to measure load. Each sample was consecutively stretched five times to 140%, 160%, 180%, 200% and 220% to evaluate stress softening. The stretch of the samples was measured using a displacement camera. The tensile speed was set at 0.5 mm/s.

The nominal experimental stress \(\bar{\Pi}\) was defined as follows using the measured load \(f\) and initial sectional area \(A\):

\[
\bar{\Pi} = \frac{f}{A}
\]

(5)

Figure 5 shows the cyclic loading test results \((\theta=0°, \theta=45°\) and \(\theta=90°)\) for the first and second cycles. The numeric evaluation of anisotropy and stress softening are displayed in Figure 5.
Fig. 1. Sports spats made by MIZUNO Corporation

Fig. 2. Super-Dumbbell-cutter made by DUMBBELL CO., LTD.

Fig. 3. Structure of tricot fiber

Fig. 4. Dimensions of samples

Fig. 5. Cyclic loading test results (a) $\theta=0^\circ$ (b) $\theta=45^\circ$ (c) $\theta=90^\circ$
3.2. Identification of material parameters

The material parameters of the anisotropic hyperelastic model used in Equations (1) and (2) were identified using the cyclic loading test results. Those of Equation (1) were identified using experimental nominal stress and theoretical nominal stress; those of Equation (2) were identified using stiffness ratios. In this study, approximation with experimental values was determined by calculating the minimum value of the error function using the nonlinear least squares method. The error function was defined as follows:

\[
E = \sum_{i=1}^{n} \left[ \left( \overline{\Pi}^{(i)}(\theta = 0) - \Pi^{(i)}(\theta = 0) \right)^2 + \left( \overline{\Pi}^{(i)}(\theta = 45) - \Pi^{(i)}(\theta = 45) \right)^2 + \left( \overline{\Pi}^{(i)}(\theta = 90) - \Pi^{(i)}(\theta = 90) \right)^2 \right]
\]  

(6)

where \( \overline{\Pi}^{(i)} \) is experimental nominal stress, \( \Pi^{(i)} \) is experimental stretch, \( \theta \) represents the fiber orientation angles of the samples, and \( n \) represents the number of measured data points used for the approximation of the nonlinear least squares method. The nonlinear least squares method was applied to the cyclic loading test results (\( \theta = 0^\circ \), \( \theta = 45^\circ \) and \( \theta = 90^\circ \)), of which the results for \( \theta = 0^\circ \) and \( \theta = 90^\circ \) were insufficient to identify material parameters.

4. Three-dimensional stress and joint torque calculation

The three-dimensional stress calculations for high-performance sports spats were conducted to numerically evaluate their effect on the human body during running. Strain distributions of human skin during running were reproduced by applying running motion data to a 3D-CG human model. Figure 6 exhibits the CG mesh of sports spats based on the node information newly produced by the 3D-CG model, and the sports spats were defined to fit closely to the human body because of their compression. The stress generated by sports spats during running was calculated using the acquired skin stretch data. The running motion data were composed of 76 frames of three-dimensional polygonal data, which was one cycle of motion. Figure 7 shows the Cauchy stress distribution generated during running. The Cauchy stress \( \mathbf{T} \) was calculated using the second Piola-Kirchhoff stress tensor and the deformation gradient tensor as follows:

\[
\mathbf{T} = \frac{1}{J} \mathbf{F} \cdot \mathbf{S} \cdot \mathbf{F}^T
\]

(7)

where \( J \) was calculated by \( J = \det(\mathbf{F}) \).

![Fig. 6. CG mesh of sports spats](image1)

![Fig. 7 Stress visualization of sports spats during running motion](image2)
Joint torque was calculated using the stress data generated by the sports spats as follows:

$$\tau = r \times f$$  \hspace{1cm} (8)

where \( r \) is the position vector between each element of the sports spats from the knee joint and \( f \) is the load vector generated in each element of the sports spats. Joint torque occurred with the load of bordering segments. In this study, the lower body was divided into the pelvis segment, the thigh segment and the crus segment as shown in Figure 8. The positive direction of the knee joint torque is shown in Figure 9. Here, the effect of the load on each segment was defined as the distribution ratio. The distribution ratios for the pelvis segment, the thigh segment and the crus segment are shown respectively in Figure 10.

The relationship between knee joint torque and running motion is shown in Figure 11. Knee joint torque means support of the motion to kick the legs. The torque calculation enabled the new design of sports spats considering the effect of sports spats on the human body.

Fig. 8. Three lower body segments
Fig. 9. Positive direction of knee joint torque
Fig. 10. Distribution ratios for (a) the pelvis segment, (b) the thigh segment, and (c) the crus segment
5. Conclusion

Three-dimensional stress calculations of high-performance sports spats using a 3D-CG human model were reported in this paper. An anisotropic hyperelastic model considering stress softening was introduced to reproduce the mechanical behavior of sports spats. Anisotropy and stress softening were evaluated using cyclic loading tests. The material parameters of the anisotropic hyperelastic model were identified using the cyclic loading test results. The running motion of a 3D-CG model, which reproduces human body motion, was used to apply the stress calculations of sports spats. The knee joint torque was calculated with the stress calculation.

References

[1] Matsuda, A., Tanabe, H., Nagaoka, T., Nakashima, M., Shimana, T., Omori, K., 2013, 3D-CG based stress calculation of competitive swimwear using anisotropic hyperelastic model. The Impact of Technology on Sport V, Vol.60, pp349-354.
[2] Rivlin, R. S., and Saunders, D. W., 1951, Large elastic deformations of isotropic materials VII. Experiments on the deformation of rubber, Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 243, No. 865, pp. 251-288.
[3] Asai, M., Kimura, Y., Sonoda, Y., Nishimoto, Y., Nishino, Y., 2010, Constitutive modeling for texture reinforced rubber by using an anisotropic visco-hyperelastic model. Journal of Structure Mechanics and Earthquake Engineering A, Vol. 66, No. 2, pp. 194-205.