Electroweak Resonant Leptogenesis in the Singlet Majoron Model

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ABSTRACT

We study resonant leptogenesis at the electroweak phase transition in the singlet Majoron model with right-handed neutrinos. We consider a scenario, where the SM gauge group and the lepton number break down spontaneously during a second-order electroweak phase transition. We calculate the flavour- and temperature-dependent leptonic asymmetries, by including the novel contributions from the transverse polarisations of the $W^\pm$ and $Z$ bosons. The required resummation of the gauge-dependent off-shell heavy-neutrino self-energies is consistently treated within the gauge-invariant framework of the Pinch Technique. Taking into consideration the freeze-out dynamics of sphalerons, we delineate the parameter space of the model that is compatible with successful electroweak resonant leptogenesis. The phenomenological and astrophysical implications of the model are discussed.

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1 Introduction

Leptogenesis [1] provides an elegant framework to consistently address the observed Baryon Asymmetry in the Universe (BAU) [2] in minimal extensions of the Standard Model (SM) [3]. According to the standard paradigm of leptogenesis, there exist heavy Majorana neutrinos of masses close to the Grand Unified Theory (GUT) scale $M_{\text{GUT}} \sim 10^{16}$ that decay out of equilibrium and create a net excess of lepton number ($L$), which gets reprocessed into the observed baryon number ($B$), through the ($B + L$)-violating sphaleron interactions [4]. The attractive feature of such a scenario is that the GUT-scale heavy Majorana neutrinos could also explain the observed smallness in mass of the SM light neutrinos by means of the so-called seesaw mechanism [5].

The original GUT-scale leptogenesis scenario, however, runs into certain difficulties, when one attempts to explain the flatness of the Universe and other cosmological data [2] within supergravity models of inflation. To avoid overproduction of gravitinos $\tilde{G}$ whose late decays may ruin the successful predictions of Big Bang Nucleosynthesis (BBN), the reheating temperature $T_{\text{reh}}$ of the Universe should be lower than $10^9 - 10^6$ GeV, for $m_{\tilde{G}} = 8 - 0.2$ TeV [6]. This implies that the heavy Majorana neutrinos should accordingly have masses as low as $T_{\text{reh}} < \sim 10^9$ GeV, thereby rendering the relation of these particles with GUT-scale physics less natural. On the other hand, it proves very difficult to directly probe the heavy-neutrino sector of such a model at high-energy colliders, e.g. at the LHC or ILC, or in any other foreseeable experiment.

A potentially interesting solution to the above problems may be obtained within the framework of resonant leptogenesis (RL) [7]. The key aspect of RL is that self-energy effects dominate the leptonic asymmetries [8], when two heavy Majorana neutrinos happen to have a small mass difference with respect to their actual masses. If this mass difference becomes comparable to the heavy neutrino widths, a resonant enhancement of the leptonic asymmetries takes place that may reach values $\mathcal{O}(1)$ [7,9]. An indispensable feature of RL models is that flavour effects due to the light-to-heavy neutrino Yukawa couplings [10] play a dramatic role and can modify the predictions for the BAU by many orders of magnitude [11,12]. Most importantly, these flavour effects enable the modelling [12] of minimal RL scenarios with electroweak-scale heavy Majorana neutrinos that could be tested at the LHC [13,14] and in other non-accelerator experiments, while maintaining agreement with the low-energy neutrino data. Many variants of RL have been proposed in the literature [15,16], including soft leptogenesis [17] and radiative leptogenesis [18].

In spite of the many existing studies, leptogenesis models face in general a serious restriction concerning the origin of the required CP and $L$ violation. If CP or $L$ violation
were due to the spontaneous symmetry breaking (SSB) of the SM gauge group, a net $L$ asymmetry could only be generated during the electroweak phase transition (EWPT), provided the heavy Majorana neutrinos are not too heavy such that they have not already decayed away while the Universe was expanding.

In this paper we show how RL constitutes an interesting alternative to provide a viable solution to the above problem as well. For definiteness, we consider a minimal extension of the SM with right-handed neutrinos and a complex singlet field $\Sigma$. The model possesses a global lepton symmetry $U(1)_l$, which gets spontaneously broken through the vacuum expectation value (VEV) of $\Sigma$, giving rise to the usual $\Delta L = 2$ Majorana masses. Because of the SSB of the $U(1)_l$, the model predicts a true massless Goldstone boson, the Majoron. Therefore, this scenario is called the singlet Majoron model in the literature [20, 21]. Depending on the particular structure of the Higgs potential, the VEV of $\Sigma$ may be related to the VEV of the SM Higgs doublet $\Phi$. Such a relation, for example, arises if the bilinear operator $\Sigma^* \Sigma$ is small or absent from the Higgs potential. In this case, the breaking of $L$ occurs during the EWPT. For the model under study and given the LEP limit [22] on the SM Higgs boson $M_H > \sim 115$ GeV, the EWPT is expected to be second order and hence continuous from the symmetric phase to the broken one [23].

We should now notice that all SM fermions and right-handed neutrinos have no chiral masses above the EWPT and therefore the generation of a net leptonic asymmetry is not possible. Consequently, in this model successful baryogenesis can result from RL at the EWPT. Although the singlet Majoron model that we will be studying here violates CP explicitly, the results of our analysis can straightforwardly apply to models with an extended Higgs sector that realise spontaneous CP violation at the electroweak scale.

The paper is organised as follows: Section 2 presents the basic features of the singlet Majoron model with right-handed neutrinos, including the interaction Lagrangians that are relevant to the calculation of the leptonic asymmetries in Section 3. Moreover, in Section 3 we consider the novel contributions to the leptonic asymmetries, coming from the transverse polarisations of the $W^\pm$ and $Z$ bosons. In the same context, the resummation of the gauge-dependent off-shell heavy-neutrino self-energies [24, 25] (which remains an essential operation in RL) is performed within the so-called Pinch Technique (PT) framework [26]. In Section 4 we analyse the Boltzmann dynamics of the sphaleron effects on RL and present predictions for the BAU. Section 5 is devoted to the phenomenological and astrophysical implications of the singlet Majoron model. Finally, Section 6 contains our conclusions.

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*An exception to this argument may result from a phase transition that is strongly first order. However, such a scenario is not feasible within the SM with singlet neutrinos [19] (see also our discussion below).
2 The Singlet Majoron Model

Here we describe the basic features of the singlet Majoron model [20,21] augmented with a number $n_R$ of right-handed neutrinos $\nu_{\alpha R}$ (with $\alpha = 1, 2, \ldots, n_R$) that will be relevant to our study. As mentioned in the introduction, the singlet Majoron model contains one complex singlet field $\Sigma$ in addition to the SM Higgs doublet $\Phi$. Although $\Sigma$ is not charged under the SM gauge group $SU(2)_L \otimes U(1)_Y$, it still carries a non-zero quantum number under the global lepton symmetry $U(1)_l$. More explicitly, the scalar potential of the model is given by

$$- \mathcal{L}_V = m_\Phi^2 \Phi^\dagger \Phi + m_\Sigma^2 \Sigma^* \Sigma + \frac{\lambda_\Phi}{2} (\Phi^\dagger \Phi)^2 + \frac{\lambda_\Sigma}{2} (\Sigma^* \Sigma)^2 - \delta \Phi^\dagger \Phi \Sigma^* \Sigma. \quad (2.1)$$

In order to minimise the potential (2.1), we first linearly decompose the scalar fields as follows:

$$\Phi = \left( \begin{array}{c} \frac{v}{\sqrt{2}} + \frac{\phi + iG}{\sqrt{2}} \end{array} \right), \quad \Sigma = \frac{w}{\sqrt{2}} + \frac{\sigma + iJ}{\sqrt{2}}. \quad (2.2)$$

Then, the extremal or tadpole conditions may easily be calculated by

$$T_\phi \equiv -\left\langle \frac{\partial \mathcal{L}_V}{\partial \phi} \right\rangle = v \left( m_\Phi^2 + \frac{\lambda_\Phi}{2} v^2 - \frac{\delta}{2} w^2 \right) = 0, \quad (2.3)$$

$$T_\sigma \equiv -\left\langle \frac{\partial \mathcal{L}_V}{\partial \sigma} \right\rangle = w \left( m_\Sigma^2 + \frac{\lambda_\Sigma}{2} w^2 - \frac{\delta}{2} v^2 \right) = 0. \quad (2.4)$$

If $m_\Phi^2$ or $m_\Sigma^2$ are negative, the tadpole conditions (2.3) and (2.4) imply that the ground state of the scalar potential breaks spontaneously the local $SU(2)_L \otimes U(1)_Y$ and the global $U(1)_l$ symmetries, through the non-zero VEVs $v$ and $w$, respectively.

Expanding the fields $\Phi$ and $\Sigma$ about their VEVs, we obtain three would-be Goldstone bosons $G^\pm$ and $G^0$, which become the longitudinal polarisations of $W^\pm$ and $Z$ bosons, and one true massless Goldstone boson $J$ associated with the SSB of $U(1)_l$. This massless CP-odd field $J$ is called the Majoron in the literature [20,21]. In addition, there are two CP-even Higgs fields $H$ and $S$, whose masses are determined by the diagonalisation of the mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} \lambda_\Phi \frac{v^2}{2} & -\frac{\delta}{2} vw \\ -\frac{\delta}{2} vw & \lambda_\Sigma \frac{w^2}{2} \end{pmatrix}, \quad (2.5)$$

where $\mathcal{M}^2$ is defined in the weak basis $(\phi, \sigma)$. The Higgs mass eigenstates $H$ and $S$ are related to the states $\phi$ and $\sigma$, through the orthogonal transformation:

$$\begin{pmatrix} \phi \\ \sigma \end{pmatrix} = \begin{pmatrix} c_\theta & \frac{v}{w} s_\theta \\ s_\theta & \frac{v}{w} c_\theta \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix}, \quad (2.6)$$
with \( t_\beta = s_\beta / c_\beta = v / w \) and
\[
t_{2\theta} = \frac{2 \delta t_\beta}{\lambda_\Sigma - \lambda_\Phi t_\beta^2}.
\] (2.7)
In the above we used the short-hand notation: \( s_x \equiv \sin x \), \( c_x \equiv \cos x \) and \( t_x \equiv \tan x \).
Moreover, the squared mass eigenvalues of the CP-even \( H \) and \( S \) bosons may easily be calculated from \( M^2 \) in (2.5) and are given by
\[
M^2_{H,S} = \frac{v^2}{2} \left[ \lambda_\Phi + \lambda_\Sigma t_\beta^2 \pm \sqrt{(\lambda_\Phi - \lambda_\Sigma t_\beta^{-2})^2 + \delta^2 t_\beta^{-2}} \right].
\] (2.8)
The requirement that \( M^2_{H,S} \) be positive gives rise to the inequality conditions,
\[
\lambda_\Phi, \lambda_\Sigma > 0 , \quad \lambda_\Phi \lambda_\Sigma > \delta^2 ,
\] (2.9)
for the quartic couplings of the potential. In this context, we note that if \( |m^2_\Sigma| \ll (\delta / \lambda_\Phi) |m^2_\Phi| \) such that \( m^2_\Sigma \) can be completely neglected in the scalar potential, the VEV \( w \) of \( \Sigma \) is then entirely determined by the VEV \( v \) of \( \Phi \) and the quartic couplings \( \lambda_\Sigma \) and \( \delta \), viz.
\[
w = \sqrt{\frac{\delta}{\lambda_\Sigma}} v.
\] (2.10)
This is an interesting scenario, since the ratio \( t_\beta = v / w = \sqrt{\lambda_\Sigma / \delta} \) does not strongly depend on the temperature \( T \), as opposed to what happens to the VEVs \( v \) and \( w \) individually. In fact, as long as \( \lambda_\Phi, \lambda_\Sigma, \delta \ll 1 \), the thermally-corrected effective potential can be expanded, to a very good approximation, in powers of \( T^2 / m^2_\Phi \). In such a high-\( T \) expansion, the quartic couplings of \( \mathcal{L}_V \) turn out to be \( T \)-independent [27] and hence \( t_\beta \) does not depend on \( T \).

We now turn our attention to the neutrino Yukawa sector of the model, which is non-standard. After SSB, it is given in the unitary gauge by
\[
\mathcal{L}_Y = \frac{\Phi}{v} \bar{\nu}_i L (m_D)_{i\alpha} \nu_{\alpha R} + \frac{\sigma + iJ}{2w} \bar{\nu}^C_{\alpha R} (m_M)_{\alpha\beta} \nu_{\beta R} + \text{H.c.}
\] (2.11)
where summation over repeated indices is understood. Hereafter we use Latin indices to label the left-handed neutrinos, e.g. \( \nu_{iL} \), and Greek indices for the right-handed ones, e.g. \( \nu_{\alpha R} \). Observe that the spontaneous breaking of \( U(1)_l \) generates lepton-number-violating \( \Delta L = 2 \) Majorana masses \( (m_M)_{\alpha\beta} \) in addition to the lepton-number-preserving \( \Delta L = 0 \) Dirac masses \( (m_D)_{i\alpha} \).

The model under discussion predicts a number \( (3 + n_R) \) of Majorana neutrinos which we collectively denote by \( n_I \), with \( I = i, \alpha \). Their physical masses are obtained from the diagonalisation of the neutrino mass matrix
\[
\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix},
\] (2.12)
by means of the unitary transformation $U^\nu T \mathcal{M}^\nu U^\nu = \tilde{\mathcal{M}}^\nu$, where $\tilde{\mathcal{M}}^\nu$ is a non-negative diagonal matrix. The neutrino mass eigenstates $(n_I)_R$ and $(n_I)_L$ are related to the states $\nu_iL$, $(\nu_iL)^C$, $\nu_oR$ and $(\nu_oR)^C$ through

$$
\begin{pmatrix}
\nu_L^C \\
\nu_R^C
\end{pmatrix}_I = U^\nu_{IJ} (n_J)_R , \\
\begin{pmatrix}
\nu_L \\
\nu_R
\end{pmatrix}_I = U^{\nu*}_{IJ} (n_J)_L .
$$

(2.13)

Assuming the seesaw hierarchy $(m_D)_{\alpha \beta} / (m_M)_{\alpha \beta} \ll 1$, the model predicts 3 light states that are identified with the observed light neutrinos $(n_i \equiv \nu_i)$, and a number $n_R$ of heavy Majorana neutrinos $(n_\alpha \equiv N_\alpha)$ with masses of order $(m_M)_{\alpha \beta} = \rho_{\alpha \beta} w$, where $\rho_{\alpha \beta} = \rho_{\beta \alpha}$ are the Yukawa couplings of $\Sigma$ to right-handed neutrinos.

To obtain an accurate light and heavy neutrino mass spectrum within the context of models of electroweak RL, it is important to go beyond the leading seesaw approximation. To this end, we need first to perform a block diagonalisation and cast $\mathcal{M}^\nu$ into the form:

$$
\mathcal{M}^\nu \rightarrow \begin{pmatrix}
m^\nu & 0 \\
0 & m^N
\end{pmatrix} .
$$

(2.14)

This can be achieved by introducing the unitary matrix $V$ [28]:

$$
V = \begin{pmatrix}
(1_3 + \xi^* \xi^T)^{-1/2} & \xi^*(1_{n_R} + \xi^T \xi^*)^{-1/2} \\
-\xi^T (1_3 + \xi^* \xi^T)^{-1/2} & (1_{n_R} + \xi^T \xi^*)^{-1/2}
\end{pmatrix} ,
$$

(2.15)

where $\xi$ is an arbitrary $3 \times n_R$ matrix. The expressions $(1_3 + \xi^* \xi^T)^{-1/2}$ and $(1_{n_R} + \xi^T \xi^*)^{-1/2}$ are defined in terms of a Taylor series expansion about the $N \times N$ identity matrix $1_N$. These infinite series converge provided the norm $||\xi||$ is much smaller than 1, where $||\xi|| \equiv \sqrt{\text{Tr}(\xi^2)}$. This condition is naturally fulfilled within the seesaw framework [29]. Block diagonalisation of the matrix $\mathcal{M}^\nu$ given in (2.12) implies that the \{12\} block element of $V^T \mathcal{M}^\nu V$ vanishes, or equivalently that

$$
m_D - \xi m_M - \xi m_D^T \xi^* = 0 .
$$

(2.16)

Equation (2.16) determines $\xi$ in terms of $m_D$ and $m_M$. It can be solved iteratively, with the first iteration given by

$$
\xi = m_D m_M^{-1} - m_D m_M^{-1} m_D^T m_D m_M^{-1} .
$$

(2.17)

Note that the second term on the RHS of (2.17) is suppressed by the ratio of the light-to-heavy neutrino masses and can thus be safely neglected in numerical estimates. Upon block diagonalisation, the block mass “eigen-matrices” are

$$
\begin{align*}
\mathbf{m}^N &= (1_{n_R} + \xi^T \xi)^{-1/2} (m_M + m_D^T \xi^* + \xi^T m_D) (1_{n_R} + \xi^T \xi^*)^{-1/2} , \\
\mathbf{m}^\nu &= - (1_3 + \xi^T \xi)^{-1/2} (m_D \xi^T + \xi m_D^T - \xi m_M \xi^T) (1_3 + \xi^T \xi)^{-1/2} \\
&= - \xi \mathbf{m}^N \xi^T ,
\end{align*}
$$

(2.18)

(2.19)
where we used (2.16) to arrive at the last equality of (2.19). Keeping the leading order terms in an expansion of $m^N$ in powers of $m_D m_M^{-1}$, we find that

$$m^N = m_M + \frac{1}{2} \left( m_D^\dagger m_M^{-1} m_D + m_D^T m_M^{-1} m_D^* \right), \quad m^\nu = -m_D m_M^{-1} m^N m_M^{-1} m_D^T. \tag{2.20}$$

These last expressions are used to calculate the light and heavy neutrino mass spectra of the RL scenarios discussed in Section 4.

In order to calculate the leptonic asymmetries in the next section, we need to know the Lagrangians that govern the interactions of the Majorana neutrinos $n_I$ and charged leptons $l = e, \mu, \tau$ with: (i) the $W^\pm$ and $Z$ bosons; (ii) their respective would-be Goldstone bosons $G^\pm$ and $G$; (iii) the CP-odd Majoron particle $J$; (iv) the CP-even Higgs fields $H$ and $S$. In detail, these interaction Lagrangians are given by [21]

$$\mathcal{L}_{W^\pm} = -\frac{g_w}{\sqrt{2}} W^{-\mu} \bar{l} b_{ll} \gamma_\mu P_L n_I + \text{H.c.}, \quad \tag{2.21}$$

$$\mathcal{L}_Z = -\frac{g_w}{4 \cos \theta_w} Z^\mu \bar{n}_I \gamma_\mu \left( C_{1I} P_L - C_{1J}^* P_R \right) n_J, \quad \tag{2.22}$$

$$\mathcal{L}_{G^\pm} = -\frac{g_w}{2M_W} G^- \bar{l} b_{ll} \left( m_I P_L - m_I P_R \right) n_I + \text{H.c.}, \quad \tag{2.23}$$

$$\mathcal{L}_G = -\frac{i g_w}{4M_W} G \bar{n}_I \left[ C_{1I} \left( m_I P_L - m_J P_R \right) + C_{1J}^* \left( m_J P_L - m_I P_R \right) \right] n_J, \quad \tag{2.24}$$

$$\mathcal{L}_J = -\frac{i g_w}{4M_W} t_\beta J \bar{n}_I \left[ C_{1I} \left( m_I P_L - m_J P_R \right) + C_{1J}^* \left( m_J P_L - m_I P_R \right) \right. \left. + \delta_{IJ} m_I \gamma_5 \right] n_J, \quad \tag{2.25}$$

$$\mathcal{L}_H = -\frac{g_w}{4M_W} \left( c_\theta - s_\theta t_\beta \right) H \bar{n}_I \left[ C_{1I} \left( m_I P_L + m_J P_R \right) + C_{1J}^* \left( m_J P_L + m_I P_R \right) \right. \left. - \frac{i t_\beta}{t_\theta - t_\beta} \delta_{IJ} m_I \gamma_5 \right] n_J, \quad \tag{2.26}$$

$$\mathcal{L}_S = -\frac{g_w}{4M_W} \left( s_\theta + c_\theta t_\beta \right) S \bar{n}_I \left[ C_{1I} \left( m_I P_L + m_J P_R \right) + C_{1J}^* \left( m_J P_L + m_I P_R \right) \right. \left. + \frac{i t_\beta}{t_\theta + t_\beta} \delta_{IJ} m_I \gamma_5 \right] n_J, \quad \tag{2.27}$$

where $P_{L,R} = \frac{1}{2} (1 \mp \gamma_5)$, $g_w$ is the SU(2)$_L$ gauge coupling of the SM and

$$B_{ll} = V_{ik}^l U_{kl}^{\nu*}, \quad C_{IJ} = U_{kJ}^{\nu} U_{kJ}^{\nu*}. \quad \tag{2.28}$$
In (2.28) $V^l$ is a 3-by-3 unitary matrix that occurs in the diagonalisation of the charged lepton mass matrix $\mathcal{M}^l$. Without loss of generality, we assume throughout the present study that $\mathcal{M}^l$ is positive and diagonal, which implies that $V^l = 1_3$. Finally, we comment on the limit of $t_\beta \to 0$. It is easy to see from (2.7) that this limit leads to $t_\theta \to 0$ and the fields $S$ and $J$ decouple from matter; only the Higgs field $H$ couples to Majorana neutrinos and to the rest of the SM fermions (cf. [13]).

3 Leptonic Asymmetries

In this section we calculate the leptonic asymmetries produced by the decays of the heavy Majorana neutrinos during a second-order EWPT. The novel aspect of such a calculation is that, in stark contrast to the conventional leptogenesis scenario, the $W^\pm$ and $Z$ bosons also contribute to the decays and leptonic asymmetries of the heavy Majorana neutrinos. This fact raises new issues related to the gauge invariance of off-shell Green functions which are here addressed within the so-called Pinch Technique (PT) framework [26].

Since sphalerons act on the left-handed SM fermions converting an excess in leptons into that of baryons, we only need to consider the decays of the heavy Majorana neutrinos $N_\alpha$ into the left-handed charged leptons $l^-_L$ and light neutrinos $\nu_{lL}$. In detail, we have to calculate the partial decay width of the heavy Majorana neutrino $N_\alpha$ into a particular lepton flavour $l$,

$$\Gamma_{N_\alpha}^l = \Gamma(N_\alpha \to l^-_L W^+, G^+) + \Gamma(N_\alpha \to \nu_{lL} Z, G, J, H, S). \quad (3.1)$$

To compute $\Gamma_{N_\alpha}^l$, it proves more convenient to first calculate the absorptive part $\Sigma_{\alpha\beta}^{\text{abs}}(\hat{p})$ of the heavy Majorana-neutrino self-energy transition $N_\beta \to N_\alpha$ in the Feynman–’t Hooft gauge $\xi = 1$, where $p^\mu$ is the 4-momentum carried by $N_{\alpha,\beta}$. The Feynman–’t Hooft gauge is not a simple choice of gauge, but the result obtained in the gauge-independent framework of the PT [26], within which issues of analyticity, unitarity and CPT invariance can self-consistently be addressed [24, 25].

Neglecting the small charged-lepton and light-neutrino masses, $\Sigma_{\alpha\beta}^{\text{abs}}(\hat{p})$ acquires the simple spinorial structure:

$$\Sigma_{\alpha\beta}^{\text{abs}}(\hat{p}) = A_{\alpha\beta}(s) \hat{p} P_L + A^*_{\alpha\beta}(s) \hat{p} P_R, \quad (3.2)$$

where $s = p^2$ is the squared Lorentz-invariant mass associated to the self-energy transition $N_\beta \to N_\alpha$. Considering the Feynman graphs shown in Fig. [1] and the interaction
Lagrangians (2.21)–(2.27), the absorptive transition amplitudes $A_{\alpha\beta}(s)$ are calculated to be

$$A_{\alpha\beta}(s) = \frac{\alpha_w}{32} \sum_{l=e,\mu,\tau} \left\{ B_{l\alpha}^* B_{l\beta} \left[ 4 \left( 1 - \frac{M_W^2}{s} \right)^2 \theta(s - M_W^2) + \frac{2 M_Z^2}{M_W^2} \left( 1 - \frac{M_Z^2}{s} \right)^2 \theta(s - M_Z^2) \right] ight. \\
+ \frac{m_{N_{\alpha}^c} m_{N_{\beta}^c}}{M_W^2} B_{l\alpha} B_{l\beta}^* \left[ 2 \left( 1 - \frac{M_W^2}{s} \right)^2 \theta(s - M_W^2) + \left( 1 - \frac{M_Z^2}{s} \right)^2 \theta(s - M_Z^2) + t^2_{\beta} \theta(s) \right. \\
+ \left. (c_{\theta} - s_{\theta} t_{\beta})^2 \left( 1 - \frac{M_H^2}{s} \right)^2 \theta(s - M_H^2) + (s_{\theta} + c_{\theta} t_{\beta})^2 \left( 1 - \frac{M_S^2}{s} \right)^2 \theta(s - M_S^2) \right] \right\} , \quad (3.3)$$

where $\alpha_w = g_w^2/(4\pi)$ is the SU(2)$_L$ fine-structure constant and $\theta(x)$ is the usual step function: $\theta(x) = 1$ for $x > 0$, whilst $\theta(x) = 0$ if $x \leq 0$. In the calculation of $A_{\alpha\beta}(s)$, we used the fact that $B_{l\alpha} = C_{\nu l\alpha} + \mathcal{O}(C_{\nu l\alpha}^2)$, which is an excellent approximation in the physical charged-lepton mass basis.

We should bear in mind that all masses involved on the RHS of (3.3) depend on the temperature $T$, through the $T$-dependent VEVs $v(T)$ and $w(T)$ related to the Higgs doublet $\Phi$ and the complex singlet $\Sigma$, respectively [cf. (4.8) and (4.17)]. In the symmetric phase of the theory, i.e. for temperatures above the electroweak phase transition, these VEVs vanish and the absorptive transition amplitude becomes

$$A_{\alpha\beta}(s) = \frac{\alpha_w}{8} \frac{(m_D^T m_{D}^*)_{\alpha\beta}}{M_W^2} \left( 1 + \frac{t_{\beta}^2}{2} \right). \quad (3.4)$$

Note that this last formula is only valid in the weak basis in which the Majorana mass matrix $m_M$ is diagonal.

To account for unstable-particle-mixing effects between heavy Majorana neutrinos, we follow [7,9] and define the resummed effective couplings $\overline{B}_{l\alpha}$ and their CP-conjugate ones $\overline{B}_{l\alpha}^c$ related to the vertices $W^- l_N \alpha$ and $W^+(l_{\alpha})^C N_{\alpha}$, respectively. For a symmetric
model with 3 left-handed and 3 right-handed neutrinos, the effective couplings $B_{l\alpha}$ exhibit the same analytic dependence on the absorptive transition amplitudes $A_{\alpha\beta}$ as the one found in [9].

\[ B_{l\alpha} = B_{l\alpha} - i \sum_{\beta,\gamma=1}^{3} |\varepsilon_{\alpha\beta\gamma}| B_{l\beta} \]

\[ \times \frac{m_{\alpha}(m_{\alpha}A_{\alpha\beta} + m_{\beta}A_{\beta\alpha}) - iR_{\alpha\gamma}\left[m_{\alpha}A_{\gamma\beta}(m_{\alpha}A_{\alpha\gamma} + m_{\gamma}A_{\gamma\alpha}) + m_{\beta}A_{\beta\gamma}(m_{\alpha}A_{\gamma\alpha} + m_{\gamma}A_{\gamma\alpha})\right]}{m_{\alpha}^2 - m_{\beta}^2 + 2i m_{\alpha}^2 A_{\beta\gamma} + 2i \text{Im}R_{\alpha\gamma}\left(m_{\alpha}^2|A_{\beta\gamma}|^2 + m_{\beta}m_{\gamma}\text{Re}A_{\beta\gamma}^2\right)} , \]

where all transition amplitudes $A_{\alpha\beta}$, $A_{\beta\gamma}$ etc are evaluated at $s = m_{N_3}^2 \equiv m_{l\alpha}^2$ and

\[ R_{\alpha\beta} = \frac{m_{\alpha}^2}{m_{\alpha}^2 - m_{\beta}^2 + 2i m_{\alpha}^2 A_{\beta\gamma}(m_{\alpha}^2)} . \]

Moreover, $|\varepsilon_{\alpha\beta\gamma}|$ is the modulus of the usual Levi–Civita anti-symmetric tensor. The respective CP-conjugate effective couplings $B_{l\alpha}^c$ are easily obtained from (3.5) by replacing the ordinary $W^-$-boson couplings $B_{l\alpha}$ and $A_{\alpha\beta}(s)$ by their complex conjugates. In the decoupling limit of $m_{N_3} \gg m_{N_1,2}$, we recover the analytic results known for a model with 2 right-handed neutrinos [7, 9], where the effective couplings $B_{l1,2}$ are given by

\[ B_{l1} = B_{l1} - i B_{l2} \frac{m_{N_1}\left(m_{N_1}A_{12}(m_{N_1}^2) + m_{N_2}A_{21}(m_{N_1}^2)\right)}{m_{N_1}^2 - m_{N_2}^2 + 2i m_{N_1}^2 A_{22}(m_{N_1}^2)} , \]

\[ B_{l2} = B_{l2} - i B_{l1} \frac{m_{N_2}\left(m_{N_2}A_{21}(m_{N_2}^2) + m_{N_1}A_{12}(m_{N_2}^2)\right)}{m_{N_2}^2 - m_{N_1}^2 + 2i m_{N_2}^2 A_{11}(m_{N_2}^2)} . \]

In all our results, we neglect the 1-loop corrections to the vertices $W^{\pm}l_LN_\alpha$, $Z\nu_LN_\alpha$ etc, whose absorptive parts are numerically insignificant in leptogenesis, but essential otherwise to ensure gauge invariance and unitarity within the PT framework [25].

In terms of the resummed effective couplings $\overline{B}_{l\alpha}$ and $\overline{B}_{l\alpha}^c$ and the absorptive transition amplitudes $A_{\alpha\beta}(s)$, the partial decay widths $\Gamma^l_{N_\alpha}$ and their CP-conjugates $\Gamma^l_{\bar{N}_\alpha}$ are now given by

\[ \Gamma^l_{N_\alpha} = m_{N_\alpha} A_{\alpha\alpha}(m_{N_\alpha}^2; \overline{B}_{l\alpha}) , \quad \Gamma^l_{\bar{N}_\alpha} = m_{N_\alpha} A_{\alpha\alpha}(m_{N_\alpha}^2; \overline{B}_{l\alpha}) , \]

where the dependence of the absorptive transition amplitudes on $\overline{B}_{l\alpha}$ and $\overline{B}_{l\alpha}^c$ has explicitly been indicated. Note that no summation over the individual charged leptons and light neutrinos running in the loop should be performed when calculating $\Gamma^l_{N_\alpha}$ and $\Gamma^l_{\bar{N}_\alpha}$ using (3.3).

\[ \text{Note:}[9] \]

\[ \text{Note:} \]

\[ i \text{Here we eliminate a typo that occurred in [9], where } R_{\alpha\gamma} \text{ in the numerator of the fraction needs be multiplied with } -i. \]
and (3.9). Then, the leptonic asymmetries for each individual lepton flavour are readily found to be
\[
\delta_{N_{\alpha}}^{l} = \frac{\Delta \Gamma_{N_{\alpha}}^{l}}{\Gamma_{N_{\alpha}}} = \frac{|\mathcal{B}_{l_{\alpha}}^{l}|^2 - |\mathcal{B}_{l_{\alpha}}^{c}|^2}{\sum_{l=e,\mu,\tau} (|\mathcal{B}_{l_{\alpha}}^{l}|^2 + |\mathcal{B}_{l_{\alpha}}^{c}|^2)},
\]
with
\[
\Gamma_{N_{\alpha}} = \sum_{l=e,\mu,\tau} \left( \Gamma_{N_{\alpha}}^{l} + \Gamma_{N_{\alpha}}^{c} \right), \quad \Delta \Gamma_{N_{\alpha}}^{l} = \Gamma_{N_{\alpha}}^{l} - \Gamma_{N_{\alpha}}^{c}.
\]
Notice that both $\Gamma_{N_{\alpha}}$ and $\delta_{N_{i}}^{l}$ do in general depend on the temperature $T$, through the $T$-dependent masses, during a second-order electroweak phase transition. More details on this issue will be presented in the next section.

## 4 Electroweak Resonant Leptogenesis

In this section we present the relevant Boltzmann equations (BEs) that will enable us to evaluate the lepton-to-photon and baryon-to-photon ratios, $\eta_{L_{i}}$ and $\eta_{B}$, during a second-order EWPT. In our numerical estimates, we only include the dominant collision terms related to the $1 \leftrightarrow 2$ decays and inverse decays of the heavy Majorana neutrinos $N_{\alpha}$. We also neglect chemical potential contributions from the right-handed charged leptons and quarks [3]. A complete account of the aforementioned subdominant effects may be given elsewhere.

To start with, we first write down the BEs that govern the photon normalised number densities $\eta_{N_{\alpha}}$ and $\eta_{\Delta L_{i}}$ for the heavy Majorana neutrinos $N_{\alpha}$ and the left-handed leptons $l_{L}, \nu_{lL}$, respectively:
\[
\frac{d\eta_{N_{\alpha}}}{dz} = \frac{z D_{N_{\alpha}}}{H(T_{c})} \left( 1 - \eta_{N_{\alpha}} \frac{\eta_{eq_{N_{\alpha}}}}{\eta_{N_{\alpha}}} \right),
\]
\[
\frac{d\eta_{\Delta L_{i}}}{dz} = \frac{z D_{N_{\alpha}}}{H(T_{c})} \left[ \left( \eta_{N_{\alpha}} \frac{\eta_{eq_{N_{\alpha}}}}{\eta_{N_{\alpha}}} - 1 \right) \delta_{N_{\alpha}}^{l} - \frac{2}{3} \mathcal{B}_{N_{\alpha}}^{l} \eta_{\Delta L_{i}} \right].
\]

Although our conventions and notations follow those of [12], there are several key differences pertinent to our EWPT scenario that need to be stressed here. Specifically, we express the $T$-dependence of the BEs (4.1) and (4.2) in terms of the dimensionless parameter $z$:
\[
z = \frac{T_{c}}{T},
\]
where $T_{c}$ is the critical temperature of the EWPT to be determined below [cf. (4.6)]. The parameter $H(T_{c}) \approx 17 \times T_{c}^2/M_{P}$ is the Hubble constant at $T = T_{c}$, where $M_{P} =$
1.2 \times 10^{19} \text{ GeV} \) is the Planck mass. The parameter \( B_{N_{\alpha}}^l \) denotes the branching fraction of the decays of the heavy Majorana neutrino \( N_{\alpha} \) into a particular lepton flavour \( l \), i.e. \( B_{N_{\alpha}}^l = (\Gamma_{N_{\alpha}}^l + \Gamma_{N_{\alpha}}^\ell)/\Gamma_{N_{\alpha}} \). Moreover, \( \eta_{N_{\alpha}}^{\text{eq}} \) is the equilibrium number density of the heavy neutrino \( N_{\alpha} \), normalised to the number density of photons \( n_\gamma = 2T^3/\pi^2 \):

\[
\eta_{N_{\alpha}}^{\text{eq}} = \frac{m_{N_{\alpha}}^2(T)}{2T^2} K_2 \left( \frac{m_{N_{\alpha}}(T)}{T} \right),
\]

where \( K_n(x) \) is the \( n \)th-order modified Bessel function [30]. Finally, \( D_{N_{\alpha}} \) is the \( T \)-dependent collision term related to the decay and inverse decay of the heavy Majorana neutrino \( N_{\alpha} \):

\[
D_{N_{\alpha}} = \frac{\Gamma_{N_{\alpha}}(T)}{n_\gamma} g_{N_{\alpha}} \int \frac{d^3p_{N_{\alpha}}}{(2\pi)^3} \frac{m_{N_{\alpha}}(T)}{E_{N_{\alpha}}(T)} e^{-E_{N_{\alpha}}(T)/T} = \frac{m_{N_{\alpha}}^2(T)}{2T^2} \Gamma_{N_{\alpha}}(T) K_1 \left( \frac{m_{N_{\alpha}}(T)}{T} \right),
\]

where \( E_{N_{\alpha}}(T) = [|p_{N_{\alpha}}|^2 + m_{N_{\alpha}}^2(T)]^{1/2} \) and \( g_{N_{\alpha}} = 2 \) is the number of helicities of \( N_{\alpha} \).

Our next step is to include the effect of the \((B + L)\)-violating sphalerons [4] on the lepton-number densities produced by the decays of \( N_{\alpha} \) during the EWPT. In particular, our interest is to implement the temperature dependence of the rate of \((B + L)\) violation just below the critical temperature \( T_c \), where \( T_c \) is given by [31]

\[
T_c = v \left( \frac{1}{2} + \frac{3g_w^2}{8\lambda_\Phi} + \frac{g'^2}{8\lambda_\Phi} + \frac{h_t^2}{2\lambda_\Phi} \right)^{-1/2}.
\]

In the above, \( g' \) is the U(1)$_Y$ gauge coupling and \( h_t \) is the top-quark Yukawa coupling. We should notice that \( \Phi - \Sigma \) mixing effects have been omitted in (4.6), which is a good approximation for scenarios with \( \delta/\lambda_\Phi \ll 1 \) as the ones to be considered here.

A reliable estimate [32, 33] of the rate of \((B + L)\)-violating sphaleron transitions can be obtained for temperatures satisfying the double inequality

\[
M_W(T) \ll T \ll \frac{M_W(T)}{\alpha_w},
\]

where \( \alpha_w = g_w^2/4\pi \) is the SU(2)$_L$ fine structure constant, \( M_W(T) = g_w v(T)/2 \) is the \( T \)-dependent W-boson mass and

\[
v(T) = v \left( 1 - \frac{T^2}{T_c^2} \right)^{1/2}
\]

is the \( T \)-dependent VEV of the Higgs field. In detail, the rate of \( B + L \) violation per unit volume is [32]

\[
\gamma_{\Delta(B+L)} = \frac{\omega_{\text{rot}}}{2\pi} N_{tr} (N V)_{\text{rot}} \left( \frac{\alpha_w T}{4\pi} \right)^3 \alpha_3^{-6} e^{-E_{\text{sp}}/T} K_\kappa.
\]

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| $\lambda_Φ/g_w^2$ | $\omega_-$ | $N_{\text{rot}}$ | $N_{\text{tr}}$ | $\kappa$ | $A$ |
|-----------------|-----------|-----------------|----------------|-------|-----|
| 0.556           | 1.612×$M_W$ | 11.2            | 7.6            | 0.135 – 1.65 | 1   |

Table 1: Values of the parameters occurring in (4.9) for $\lambda_Φ/g_w^2 = 0.556$, which corresponds to a SM Higgs-boson mass of 120 GeV when $\delta = 0$.

Given the double inequality (4.7), this last expression is valid for temperatures $T \lesssim T_c$. Following the notation of [32], the parameters $\omega_-, N_{\text{tr}}$ and $N_{\text{rot}}$ that occur in (4.9) are functions of $\lambda_Φ/g_w^2$, $V_{\text{rot}} = 8\pi^2$ and $\alpha_3 = \alpha_w T/[2 M_W(T)]$. The quantity $E_{\text{sp}}$ is the $T$-dependent energy of the sphaleron and is determined by

$$ E_{\text{sp}} = A \frac{2 M_W(T)}{\alpha_w} , $$

where $A$ is a function of $\lambda_Φ/g_w^2$ and is $O(1)$, for values of phenomenological interest. The dependence of the parameter $\kappa$ on $\lambda_Φ/g_w^2$ has been calculated in [32, 33], and the results of those studies are summarised in Table 1 for $\lambda_Φ/g_w^2 = 0.556$. This value corresponds to a SM Higgs-boson mass $M_H$ of 120 GeV in the vanishing limit of a $Φ-Σ$ mixing.

Since the SM Higgs-boson mass is $M_H \gtrsim 115$ GeV, it can be shown [23] that the EWPT in the SM is not first order, but continuous from $v(T_c) = 0$ to $v$, without bubble nucleation and the formation of large spatial inhomogeneities in particle densities. Therefore, we use the formalism developed in [12], where the $(B+L)$-violating sphaleron dynamics is described in terms of spatially independent $B$- and $L$-number densities $η_B$ and $η_L$. More explicitly, the BEs of interest to us are [12]:

$$ \frac{dη_B}{dz} = - z \frac{\Gamma_{Δ(B+L)}}{H(T_c)} \left[ η_B + \frac{28}{51} η_L + \frac{v_0^2(T)}{T^2} \left( \frac{75}{187} η_B + \frac{16}{187} η_L \right) \right] , $$

$$ \frac{dη_L}{dz} = \frac{dη_{ΔL}}{dz} + \frac{1}{3} \frac{dη_B}{dz} , $$

where $η_L = Σ_{l=e,μ,τ} η_{L_l}$ is the total lepton asymmetry and

$$ \Gamma_{Δ(B+L)} = \frac{1683}{132 T^3 + 51 T v_0^2(T)} \gamma_{Δ(B+L)} . $$

We observe that in the limit $\Gamma_{Δ(B+L)}/H(T_c) → ∞$ and for $T > T_c$, the conversion of the lepton-to-photon ratio $η_L$ to the baryon-to-photon ratio $η_B$ is given by the known relation [34, 35]:

$$ η_B = - \frac{28}{51} η_L . $$
Likewise, when $1 \lesssim z \lesssim 1.7$ and $\kappa = 1$, it is $\Gamma_{\Delta(B+L)}/H(T_c) \gg 1$ and the baryon-to-photon ratio $\eta_B$ is then related to the total lepton-to-photon ratio $\eta_L$ by

$$\eta_B = -\left(\frac{28}{51} + \frac{16}{187} \frac{v^2(T)}{T^2}\right) \left(1 + \frac{75}{187} \frac{v^2(T)}{T^2}\right)^{-1} \eta_L.$$  \hspace{1cm} (4.15)

For $z \gtrsim 1.7$, sphaleron effects get sharply out of equilibrium and $\eta_B$ freezes out. To account for the $T$-dependent $(B + L)$-violating sphaleron effects, our numerical estimates will be based on the BEs (4.1), (4.2), (4.11) and (4.12).

In the singlet Majoron model, the restoration of the global symmetry $U(1)_l$ will occur for temperatures above a critical temperature $T_l^c$ that could in general differ from $T_c$ of the SM gauge group given in (4.6). For example, in the absence of a doublet-singlet mixing, the critical temperature related to the SSB of $U(1)_l$ is [27]

$$T_l^c = -\frac{6 m_\Sigma^2}{\lambda_\Sigma}.$$  \hspace{1cm} (4.16)

Consequently, the $T$-dependence of $w(T)$ for $T < T_l^c$ will be analogous to $v(T)$ in (4.8), i.e.

$$w(T) = w \left(1 - \frac{T^2}{(T_l^c)^2}\right)^{1/2}.$$  \hspace{1cm} (4.17)

However, if $m_\Sigma^2$ vanishes, the singlet VEV $w(T)$ and the doublet VEV $v(T)$ will be related by an expression very analogous to (2.10), namely

$$w(T) \approx t^{-1}_\beta v(T).$$  \hspace{1cm} (4.18)

As was mentioned after (2.10), the above relation becomes exact in a high-$T$ expansion of the thermally corrected effective potential. Such an expansion is a very good approximation to the level of a few % for perturbatively small quartic couplings [27]. As a consequence, the SM gauge group and the global lepton symmetry $U(1)_l$ will both break down spontaneously via the same second-order electroweak phase transition, with $T_l^c = T_c$. Even though the focus of the paper will be on this class of scenarios, we will comment on possible differences for models with $T_l^c \neq T_c$.

If $T_l^c = T_c$, the heavy neutrino masses $m_{N_\alpha}$, the gauge-boson masses $M_{W,Z}$ and the Higgs masses $M_{H,S}$ all scale with the same $T$-dependent factor, $(1 - T^2/T_c^2)^{1/2}$, for temperatures $T < T_c$ of our interest. Hence, the $T$-dependence drops out exactly in the expression (3.3) of the absorptive transition amplitudes $A_{\alpha\beta}(m_{N_\alpha}^2)$, and likewise in the leptonic asymmetries $\delta^l_{N_\alpha}$ and the branching fractions $B^l_{N_\alpha}$. However, as can be seen from (4.5), the collision terms $D_{N_\alpha}$ exhibit a non-trivial $T$-dependence that needs be carefully implemented in the BEs.
For our numerical estimates of the BAU, we consider the 3-generation flavour scenario of the RL model discussed in [11, 12]. Specifically, the Majorana sector is assumed to be approximately SO(3) symmetric,

\[ m_M = m_N 1_3 + \Delta M_S , \]

where \( \Delta M_S \) are small SO(3)-breaking terms that are of order \( m_D^\dagger m_D / m_N \) as these are naturally expected from (2.20). Plugging (4.19) into (2.20), we find that, to leading order in \( \Delta M_S \), the heavy neutrino mass matrix \( m_N \) deviates from \( m_N 1_3 \) by an amount

\[ \delta m_N = \Delta M_S + \frac{1}{2m_N} \left( m_D^\dagger m_D + m_D^T m_D^* \right) . \]

It is interesting to observe that possible renormalisation-group (RG) running effects from a high-energy scale \( M_X \), e.g. GUT scale, down to \( m_N \) will induce a negative contribution to \( \delta m_N \) [18], i.e.

\[ (\delta m_N)^{\text{RG}} = -\frac{\alpha_w}{8\pi} \frac{m_N}{M_W^2} \left( m_D^\dagger m_D + m_D^T m_D^* \right) \ln \left( \frac{M_X}{m_N} \right) . \]

For \( M_X = M_{\text{GUT}} \sim 10^{16} \) and \( m_N = 80-150 \) GeV, the RG-induced terms are typically smaller by a factor \( \sim 0.1-0.4 \) with respect to the tree-level contribution given in (4.20). Thus, the inclusion of the RG effects are not going to affect the results of our analysis in a substantial manner.

As was mentioned already, the SO(3) symmetry is broken by the Dirac mass terms \( (m_D)_{i\alpha} \), which in our case possess an approximate U(1)-symmetric flavour pattern [11]:

\[ m_D = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & a e^{-i\pi/4} & a e^{i\pi/4} \\ 0 & b e^{-i\pi/4} & b e^{i\pi/4} \\ 0 & c e^{-i\pi/4} & c e^{i\pi/4} \end{pmatrix} + \delta m_D , \]

where the 3-by-3 matrix \( \delta m_D \),

\[ \delta m_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \varepsilon_e & 0 \varepsilon_\mu \\ 0 & \varepsilon_\tau \end{pmatrix} , \]

violates the U(1) symmetry by small terms of order of the electron mass \( m_e \). Instead, the U(1)-symmetric Yukawa couplings \( a \) and \( b \) can be as large as the \( \tau \)-lepton Yukawa coupling \( m_\tau / v \), i.e. of order \( 10^{-2}-10^{-3} \). For successful RL, it was found [11, 12] that the parameter \( c \) needs to be taken of the order of the electron Yukawa coupling \( m_e / v \). It is important to
stress here that the approximate flavour symmetries SO(3) and U(1) ensure the stability of the light- and heavy-neutrino sector under loop corrections [11,13,36].

For our numerical analysis, we fully specify in Table 2 the values of the theoretical parameters for the Higgs and neutrino sectors. The only parameter that we allow to vary is the heavy Majorana mass scale $m_N$. For $50 \text{ GeV} \lesssim m_N \lesssim 200 \text{ GeV}$, the choice of parameters in Table 2 leads to an inverted hierarchical light-neutrino spectrum with the following squared mass differences and mixing angles:

\[ m_{\nu_2}^2 - m_{\nu_1}^2 = (7.5 - 7.7) \times 10^{-5} \text{ eV}^2, \quad m_{\nu_1}^2 - m_{\nu_3}^2 = 2.44 \times 10^{-3} \text{ eV}^2, \]
\[ \sin^2 \theta_{12} = 0.362, \quad \sin^2 \theta_{23} = 0.341, \quad \sin^2 \theta_{13} = 0.047 \]

and $m_{\nu_3} = 0$. The spectrum is compatible with the light-neutrino data at the $3\sigma$ confidence level (CL) [37].

In Fig. 2 we present numerical estimates of the lepton-flavour asymmetries $\eta_{L,e,\mu,\tau}$ and the baryon asymmetry $\eta_B$ as functions of $z = T_c/T$, for a typical electroweak RL scenario with $m_N = 100 \text{ GeV}$. As initial conditions at $T = T_c \approx 133 \text{ GeV}$, we take $\eta_{L,e,\mu,\tau}^\text{in} = 1$ for the heavy neutrino number densities and vanishing lepton-to-photon and baryon-to-photon ratios, i.e. $\eta_{L,e,\mu,\tau}^\text{in} = 0$ and $\eta_B^\text{in} = 0$. The thermal in-equilibrium condition $\eta_{L,e,\mu,\tau}^\text{in} = 1$ is expected, since the heavy neutrinos $N_{1,2,3}$ have no chiral masses when $T > T_c$ and get rapidly thermalised by the sizeable light-to-heavy neutrino Yukawa couplings.

| Table 2: Complete set of the theoretical parameters used for the singlet Majoron model, where $\Delta m_N = 2(\delta m_N)_{23} + i[(\delta m_N)_{33} - (\delta m_N)_{22}].$ |  |

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| Sector | $\lambda_\Phi$ | $\lambda_\Sigma$ | $\delta$ | $\tan \beta$ | $M_H$ [GeV] | $M_S$ [GeV] |
|--------|----------------|------------------|---------|-------------|--------------|--------------|
| Higgs  | 0.238          | $\frac{1}{30} \lambda_\Phi$ | $\frac{1}{15} \lambda_\Phi$ | 1/$\sqrt{2}$ | 121          | 29           |
| Neutrino Sector | $(\delta m_N)_{11}$ | $(\delta m_N)_{12}$ | $(\delta m_N)_{13}$ | $(\delta m_N)_{22}$ | $(\delta m_N)_{23}$ | $(\delta m_N)_{33}$ |
|        | $10^{-5}$      | $-10^{-9}$       | $-4 \times 10^{-10}$ | $4 \times 10^{-9}$ | $(6.8 - 0.6i) \times 10^{-9}$ | $5.2 \times 10^{-9}$ |
|        | $a$            | $b$              | $c$      | $\varepsilon_e$ | $\varepsilon_\mu$ | $\varepsilon_\tau$ |
|        | $\frac{3}{500}$ | $\frac{57}{25000}$ | $2 \times 10^{-7}$ | $\frac{1563}{25000}$ | $\frac{39}{50000}$ | $\frac{-147}{128000}$ |
Figure 2: Numerical estimates of $\eta_B$ (solid), $\eta_{L\tau}$ (dash-dotted), $\eta_{L_e} = \eta_{L\mu}$ (dotted) and $\eta_L$ (dashed) as functions of $z = T_c/T$, for a model with $m_N = 100$ GeV, and $\eta_{N_\alpha}^\alpha = 1$. The model parameters are given in Table 2. The horizontal grey line corresponds to the observed baryon-to-photon ratio $\eta_{B}^{\text{obs}} = 1.65 \times 10^{-8}$, after evolving the latter back to the higher temperature $T = T_c/10$.

$\sqrt{2}(m_D)_{i\alpha}/v \gtrsim 10^{-7}$. As can be seen from Fig. 2, a net baryon asymmetry $\eta_B$ is generated by a non-zero $\tau$-lepton asymmetry $\eta_{L\tau}$. This $L\tau$-excess is created before sphalerons sharply freeze out, i.e. for temperatures $T \gtrsim T_{\text{sph}} \approx 78$ GeV ($z \lesssim 1.7$). Consequently, in the thermal evolution of the Universe, there is a sufficiently long interval $78$ GeV $\lesssim T \lesssim 133$ GeV, where a leptonic asymmetry can be converted into the observed BAU for our scenarios with spontaneous lepton-number violation at the electroweak scale.

Figure 3 exhibits the dependence of the baryon-to-photon ratio $\eta_B$ on $z = T_c/T$ for different values of the heavy Majorana mass scale $m_N$. We notice that the lighter the heavy neutrinos are, the smaller the created baryon asymmetry is. For example, for heavy-neutrino masses $m_N \sim 80$ (50) GeV, $\eta_B$ falls short almost by one order (two orders) of magnitude with respect to the observed BAU $\eta_{B}^{\text{obs}}$. This is a generic feature of our
electroweak RL scenarios based on large wash-out effects due to the relatively large Dirac-neutrino Yukawa couplings \((m_D)_{i\alpha}/v\). If the heavy neutrinos have masses \(m_N < 90\) GeV, their number densities will start decreasing for \(T < m_N\), potentially creating a net lepton asymmetry that can be converted into \(\eta_B^{\text{obs}}\). However, this should happen above the freeze-out temperature \(T_{\text{sph}} \approx 78\) GeV of sphalerons. Thus, successful electroweak RL requires that \(m_N > T_{\text{sph}}\).

\[\begin{align*}
\frac{\eta_B}{\text{obs}} &\sim 10^{-7} \\
120\text{ GeV} &
\end{align*}\]

\[\begin{align*}
100\text{ GeV} &
\end{align*}\]

\[\begin{align*}
80\text{ GeV} &
\end{align*}\]

\[\begin{align*}
50\text{ GeV} &
\end{align*}\]

\[\begin{align*}
z = \frac{T_c}{T}
\end{align*}\]

\[\begin{align*}
\eta_B
\end{align*}\]

\[\begin{align*}
\text{Figure 3: Numerical estimates of } \eta_B \text{ versus } z = T_c/T \text{ for } m_N = 120 \text{ GeV (dashed), 100 GeV (solid), 80 GeV (dash-dotted), 50 GeV (dotted). The meaning of the horizontal grey line is the same as in Fig. 2.}\]

\[\begin{align*}
\text{Recently, a different leptogenesis scenario with } m_N \ll T_{\text{sph}} \text{ was studied in [38], where the BAU is generated by sterile-neutrino oscillations. Such a realisation relies on the assumption that the oscillating sterile neutrinos start evolving from a coherent state and retain their coherent nature within the thermal plasma of the expanding Universe. In the singlet Majoron model we have been studying here however, } t\text{-channel } 2 \leftrightarrow 2 \text{ scattering processes, such as } JJ \leftrightarrow \nu_R^C \nu_R, \text{ that occur before the EWPT } (T \gtrsim T_c) \text{ are strong, with rates } \mathcal{O}[\rho_{\alpha\alpha}^4 T/(8\pi)] \gg H(T)\text{, for Higgs-singlet Yukawa couplings } \rho_{\alpha\alpha} \sim 1. \text{ They can therefore lead to rapid thermalization and loss of coherence of the massless right-handed neutrinos. Shortly after the EWPT, for } z = T_c/T \gtrsim 1.1, \text{ it is } \Gamma_{N_{1,2}}/H \sim 10^9-10^{10} \text{ and } \Gamma_{N_3}/H \sim 1-10, \text{ which again gives rise to an almost instant thermalization of all the heavy neutrino mass eigenstates } N_{1,2,3}.
\end{align*}\]
Finally, it is important to comment on the last condition \( m_N > T_{\text{sph}} \) for scenarios with \( T^l_c \neq T_c \). This condition will still be valid, as long as \( T^l_c > T_{\text{sph}} \). However, for scenarios with \( T^l_c \approx T_c \), the predicted BAU \( \eta_B \) will sensitively depend on the initial values \( \eta^\text{in}_{\text{Le,} \mu, \tau} \) and \( \eta^\text{in}_B \) at \( T = T_c \). Instead, if \( T^l_c \gg T_c \) and \( m_N > \sim 90 \text{ GeV} \), the predictions for the BAU will remain almost unaffected, even if \( \eta^\text{in}_B \sim 10^2 \eta^\text{obs}_B \) at \( T = 10 T_c \) [12].

5 Astrophysical and Phenomenological Implications

It is interesting to discuss the implications of the singlet Majoron model for astrophysics and low-energy phenomenology. To quantify the effects of heavy Majorana neutrinos, we define the new-physics parameters

\[
\Omega_{ll'} = \delta_{ll'} - B^*_{lk} B_{\nu k} = B^*_{l\alpha} B_{\nu \alpha},
\]

where \( l, l' = e, \mu, \tau \). Evidently, in the absence of light-to-heavy neutrino mixings, the parameters \( \Omega_{ll'} \) vanish. LEP and low-energy electroweak data put severe limits on the diagonal parameters \( \Omega_{ll} \) [39]:

\[
\Omega_{ee} \leq 0.012, \quad \Omega_{\mu\mu} \leq 0.0096, \quad \Omega_{\tau\tau} \leq 0.016,
\]

at the 90% CL. On the other hand, lepton-flavour-violating (LFV) decays, such as \( \mu \rightarrow e\gamma \) [40], \( \mu \rightarrow eee \), \( \tau \rightarrow e\gamma \), \( \tau \rightarrow eee \), \( \mu \rightarrow e \) conversion in nuclei [41, 42] and \( Z \rightarrow ll' \) [43], constrain the off-diagonal parameters \( \Omega_{ll'} \), with \( l \neq l' \). The derived constraints strongly depend on the heavy neutrino masses \( m_{N\alpha} \) and the size of the Dirac masses \( (m_D)_{l\alpha} \). However, for models relevant to leptogenesis, with \( (m_D)_{l\alpha} \ll M_W \) [41], we obtain the following limits:

\[
|\Omega_{e\mu}| \lesssim 0.0001, \quad |\Omega_{e\tau}| \lesssim 0.02, \quad |\Omega_{\mu\tau}| \lesssim 0.02,
\]

including the recent BaBar data on LFV \( \tau \) decays [44].

The predictions for LFV decays in models of resonant leptogenesis has been extensively discussed in [12]. Since our results obtained in Section 4 agree well with this earlier analysis, we will not repeat the details of this study here. Here, we only reiterate the fact that successful electroweak RL requires that \( m_N \gtrsim 100 \text{ GeV} \). This latter constraint gives rise to the following upper limits:

\[
\Omega_{ee} \lesssim 2.2 \times 10^{-4}, \quad |\Omega_{e\mu}| \lesssim 8.3 \times 10^{-5}, \quad \Omega_{\mu\mu} \lesssim 3.1 \times 10^{-5},
\]

whereas all remaining parameters \( \Omega_{ll'} \) are \( \mathcal{O}(10^{-8}) \) and so unobservably small. All these limits are deduced by using the model parameters of Table 2.
Figure 4: Loop-induced couplings of the Majoron to charged leptons $l$, $l'$ and quarks $q$.

In the singlet Majoron model under study, there are additional LFV decays for the muon and the tau-lepton that involve the Majoron, i.e. $\mu \to J e$, $\tau \to J e$ and $\tau \to J \mu$. As shown in Fig. 4, these LFV decays are induced by heavy Majorana neutrinos at the 1-loop level. Detailed analytic expressions for the loop-induced couplings $J_{ll'}$ and $J_{qq}$, where $q$ is a quark, may be found in [21]. To leading order in $\Omega_{ll'}$, the prediction for the LFV decay $l^- \to l'^- J$ is

$$ R(l \to l'^- J) \equiv \frac{\Gamma(l^- \to l'^- \nu \bar{\nu})}{\Gamma(l^- \to l^- \nu \bar{\nu})} = \frac{3\alpha_w}{8\pi} \ell^2 |\Omega_{ll'}|^2 \frac{M_W^2}{m_l^2} \frac{\lambda_N^4}{(1 - \lambda_N)^2} \left(1 + \frac{\ln \lambda_N}{1 - \lambda_N} \right)^2, \quad (5.5) $$

where $\lambda_N = m_N^2/M_W^2$. For $\lambda_N = 1$, the prediction for the observable $R(l \to l'^- J)$ takes on the simpler form:

$$ R(l \to l'^- J) = \frac{3\alpha_w}{32\pi} \ell^2 |\Omega_{ll'}|^2 \frac{M_W^2}{m_l^2}. \quad (5.6) $$

The requirement for successful electroweak RL, i.e. $m_N \gtrsim 100$ GeV, gets translated into the following upper bounds:

$$ R(\mu \to e J) \lesssim 2.7 \times 10^{-6}, \quad R(\tau \to e J) \lesssim 4.6 \times 10^{-14}, \quad R(\tau \to \mu J) \lesssim 6.7 \times 10^{-15}. \quad (5.7) $$

On the experimental side, however, the following upper limits are quoted:

$$ R(\mu \to e J) \leq 2.6 \times 10^{-6}, \quad \text{at 90\% CL [45]}; $$
$$ R(\tau \to e J) \leq 1.5 \times 10^{-2}, \quad \text{at 95\% CL [46]}; $$
$$ R(\tau \to \mu J) \leq 2.6 \times 10^{-2}, \quad \text{at 95\% CL [46]}. \quad (5.8) $$

It is interesting to remark that the predicted value for $R(\mu \to e J)$ is close to the present experimental sensitivity, whereas the other decay modes turn out to be very suppressed for the given RL model with inverted light-neutrino hierarchy. Had we chosen a model with normal hierarchy, the decay rates $R(\tau \to e J)$ and $R(\tau \to \mu J)$ would have been enhanced.
by a factor $\sim 10^8$, but they will still be rather small $O(10^{-6})$ to be observed; the predictions generally lie 4 orders of magnitude below the current experimental upper bounds.

Useful constraints on the parameters of the theory are obtained from astrophysics as well [47]. Specifically, observational evidence of cooling rates of white dwarfs implies that the interaction of the Majoron to electrons, $g_{\text{Je}e}J\bar{e}i\gamma_5e$, should be sufficiently weak and the coupling $g_{\text{Je}e}$ must obey the approximate upper bound [48]:

$$|g_{\text{Je}e}| \lesssim 10^{-12}.$$  \hspace{1cm} (5.9)

The above limit gets further consolidated by considerations of the helium ignition process in red giants, leading to the excluded range: $3 \times 10^{-13} \lesssim |g_{\text{Je}e}| \lesssim 6 \times 10^{-7}$. To leading order in $\Omega_{ll}$, the loop-induced coupling $g_{\text{Je}e}$, is given by [21]:

$$g_{\text{Je}e} = \frac{g_w \alpha_w}{16\pi} \frac{m_e}{M_W} t_\beta \lambda_N \left[ \Omega_{\text{Je}e} \frac{\lambda_N}{1 - \lambda_N} \left( 1 + \frac{\ln \lambda_N}{1 - \lambda_N} \right) + \frac{1}{2} \sum_{l=e,\mu,\tau} \Omega_{ll} \right].$$  \hspace{1cm} (5.10)

If $\lambda_N \gg 1$, the expression for the coupling $g_{\text{Je}e}$ simplifies to

$$g_{\text{Je}e} = \frac{g_w \alpha_w}{32\pi} \frac{m_e}{M_W} t_\beta \lambda_N \left( \Omega_{\mu\mu} + \Omega_{\tau\tau} - \Omega_{\text{Je}e} \right),$$  \hspace{1cm} (5.11)

whilst for $\lambda_N = 1$ $g_{\text{Je}e}$ becomes

$$g_{\text{Je}e} = \frac{g_w \alpha_w}{32\pi} \frac{m_e}{M_W} t_\beta \left( \Omega_{\mu\mu} + \Omega_{\tau\tau} \right).$$  \hspace{1cm} (5.12)

Given the limits [5.4] for successful RL, we can estimate that

$$g_{\text{Je}e} \lesssim -3.3 \times 10^{-17},$$  \hspace{1cm} (5.13)

which passes comfortably the astrophysical constraint given in (5.9).

Useful astrophysical constraints may also be obtained from considerations of the cooling rate of neutron stars [47]. Neutron stars will loose energy by Majoron emission through the interaction: $g_{JNN}J \overleftrightarrow{N}i\gamma_5N$, where $N$ is a nucleon, specifically a neutron. The observational limit on $g_{JNN}$ is [49]

$$g_{JNN} \lesssim 10^{-9}.$$  \hspace{1cm} (5.14)

On the other hand, the theoretical prediction for $g_{Jqq}$ at the quark level is

$$g_{Jqq} = \frac{g_w \alpha_w}{32\pi} \frac{m_q}{M_W} t_\beta \lambda_N \left( \Omega_{\text{Je}e} + \Omega_{\mu\mu} + \Omega_{\tau\tau} \right).$$  \hspace{1cm} (5.15)

From naive dimensional analysis arguments, one expects that $g_{JNN} \sim (m_N/m_q)g_{Jqq}$. In this way, one may estimate that

$$g_{JNN} \approx 7 \times 10^{-10},$$  \hspace{1cm} (5.16)
after taking into consideration the limits stated in \((5.4)\).

Cosmic microwave background (CMB) data and BBN put stringent limits on the maximum number of weakly-interacting relativistic degrees of freedom, such as light neutrinos and Majorons \([50, 51]\). In particular, the allowed range obtained for the effective number \(N_\nu\) of left-handed neutrino species is \(N_\nu = 2.70^{+0.91}_{-1.32}\) at the 68\% CL \([51]\). The upper bound on \(N_\nu\) may naively be translated into an upper limit on \(\Delta N_\nu = N_\nu - 3 = 0.61\) of extra effective neutrino species beyond the 3 SM left-handed neutrinos. The singlet Majoron contributes \(\Delta N_\nu = \left(\frac{1}{2} \times \frac{8}{7}\right)^{4/3} \approx 0.474\), if its freeze-out or decoupling temperature \(T_J\) is equal to the corresponding one \(T_\nu\) of the neutrinos. Although this result does not pose by itself a serious limitation on the singlet Majoron model, it can be estimated, however, that \(T_J \gg T_\nu \approx 1\) MeV and the contribution of \(J\) to \(\Delta N_\nu\) becomes even more suppressed. Specifically, the freeze-out temperature \(T_J\) is determined when the annihilation rate of Majorons through the process \(JJ \rightarrow \nu\nu\) becomes smaller than the Hubble expansion rate \(H(T)\) of the Universe. The annihilation process \(JJ \rightarrow \nu\nu\) is mediated by the \(H\) and \(S\) bosons in the \(s\)-channel and by the heavy neutrinos \(N_{1,2,3}\) in the \(t\)-channel. Considering the latter reactions only, one may naively estimate that

\[
\frac{T_J}{T_\nu} \sim \left(\frac{G_F^2 m_N^4}{\Omega_{ee}^2 t_\beta^4}\right)^{1/3} \sim 10^2 - 10^3.
\]

A similar value for \(T_J/T_\nu\) is obtained if the \(S, H\)-boson exchange processes are used for the model parameters of Table 2. Thus, the freeze-out temperature \(T_J\) lies in the range 0.1–1 GeV, namely about the quark-hadron deconfinement phase. In this epoch of the Universe, the effective number of relativistic degrees of freedom is \(g_*(T_J) \approx 66\). Then, the actual contribution of the Majoron to \(\Delta N_\nu\) is reduced with respect to the \(T_J = T_\nu\) case by a factor \((g_*(T_\nu)/g_*(T_J))^{4/3} \approx 0.016\) to the value \(\Delta N_\nu \approx 0.008\), which is far below the present and future observational sensitivity \([51]\).

Finally, singlet Majorons \(J\) and singlet scalars \(S\) may also give rise to interesting collider phenomenology \([52]\) through the singlet-doublet mixing parameter \(\delta\) in the scalar potential \((2.1)\). However, since \(\delta \ll \lambda_\Phi\) (cf. Table 2), the singlet Majoron scenario under study predicts a rather small mixing angle \(s_\theta \approx -0.1\). The production cross section of \(S\), via the process \(e^+e^- \rightarrow ZS\), is then suppressed with respect to the SM one by a factor \(s_\theta^2 \approx 0.01\). Moreover, the so-produced Higgs singlets may decay quasi-invisibly into a pair of Majorons \(J\), which makes difficult to fully rule out such a scenario by LEP2 data or at the LHC. Future high-energy \(e^+e^-\) colliders of higher luminosity will severely constrain the allowed parameter space of this singlet Majoron model.
6 Conclusions

The origin of CP violation in nature still remains an open physics question. If CP violation originates from the SSB of the SM gauge group, the original scenario [1] of GUT-scale leptogenesis will be excluded. Similar will be the fate of all high-scale leptogenesis models, if the source of lepton-number violation is due to the SSB of a global $U(1)_L$ symmetry at the electroweak scale. In this paper we have shown how resonant leptogenesis at the EWPT constitutes a realistic alternative for successful baryogenesis in models with spontaneous lepton-number violation. Specifically, we have considered a minimal extension of the SM, the singlet Majoron model, which includes right-handed neutrinos and a complex singlet field that carries a non-zero lepton number. Depending on the form of the scalar potential, the lepton number can get broken spontaneously through the VEV of the SM Higgs doublet. Taking into consideration the Boltzmann dynamics of sphaleron effects, we have analysed the BAU for different values of the Majorana mass scale $m_N$ within the context of a benchmark scenario whose model parameters are given in Table 2. The generic constraint from having successful electroweak RL is that $m_N \gtrsim T_{\text{sph}}$, where $T_{\text{sph}} \approx 78$ GeV is the freeze-out temperature of the sphalerons.

The singlet Majoron model predicts a massless Goldstone particle, the Majoron $J$. The Majoron can be produced via the LFV decays, $\mu \to J e$, $\tau \to J \mu$ and $\tau \to J e$. Considering the constraints from successful electroweak RL and the astrophysical limits derived from the cooling rate of neutron stars, we have found that the decay mode $\mu \to J e$ is the most promising channel, with sizeable branching fraction that can be looked for in the next-round low-energy experiments.

The predictions obtained for the BAU in this study are limited by the approximations that are inherent in the calculation of the non-perturbative sphaleron dynamics. The predicted values should be regarded as order-of-magnitude estimates, since the $(B + L)$-violating sphaleron transitions crucially depend on the parameter $\kappa$ that varies by a factor of 10 or so. It would therefore be very valuable to go beyond the current approximation methods and improve the computation of the out-of-equilibrium sphaleron dynamics during a second-order electroweak phase transition.

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