Research on tolerance modeling method and 3D tolerance analysis technology based on STD

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Abstract. The description methods of various tolerance zones and kinematic links based on SDT are discussed. The surfacemodels are applied to describe an assembly, and SDT is used to express the relative position between any two surfaces of an assembly; According to the effect of each functional element on the whole functional requirements of products, the 3 -1; dimension-chain is created, and 3-D dimensional and geometrical tolerance analysis for assembly is realized. The engineering example involving 3-D tolerance analysis is given to test the proposed method.

Keywords: small displacement torsos; tolerance modeling; three-dimensional tolerance analysis; surface model.

1. Introduction

At present, 3D CAD software has gradually become the direction and master of product design. When the production is directly guided by a three-dimensional model, tolerance analysis and synthesis must be carried out on it. The original tolerance design technology based on one-dimensional dimension chain has already been developed. It can not meet the requirements of tolerance design in various fields. Studying the new tolerance modeling technology and the theory and method of three-dimensional tolerance design has become the focus of widespread concern. P.Varghese et al ([2]) proposed the analysis of geometric tolerances by vector tolerance, but because geometric tolerances and vector tolerances have completely different tolerance representations, most of the work done is to solve the problem of geometric tolerances and vector tolerance conversions. In order to minimize the tolerance chain, M.Radouani and so on ([3]) also proposed a new method of three-dimensional tolerance analysis, which is based on the homogeneous transformation matrix of vector tolerance. The advantage of 3D tolerance analysis based on vector tolerance is that the uncertain geometric tolerances can be transformed into parametric tolerances, and it is easy to integrate with the CAD system. K.W.Chase et al. [4] put forward the DirectLinearization Method (DLM) method for tolerance analysis of 2D and 3D mechanical assembly. On the basis of the study of the influence of the variation of part size feature, the change of geometric feature and the adjustment of assembly kinematics to the accumulation of assembly tolerance, a two-dimensional and three-dimensional tolerance analysis method, which is based on the variation of geometric features, is proposed. The DLM method has obvious advantages in dealing with location tolerance and dimensional tolerance in the transfer of three-dimensional space, but it lacks the
ability to analyze the analysis of the shape tolerance. In this paper, the theory and method of tolerance modeling and 3D tolerance analysis are studied based on small displacement spinor.

2. 3D tolerance assembly modeling based on STD

Any mechanical product is made up of parts. There are direct contacts between parts, and also through kinematic pairs. It is necessary to establish the quantitative description of the assembly tolerance in the process of assembly tolerance analysis, whether it is the geometric error in the manufacturing process of each part, or the error between the direct contact of different parts or the connection with the motion pair.[1]

2.1. The description of the rotation of various tolerance regions

After the concept of small displacement spinor (SDT) was proposed, it was introduced into the field of tolerance in 1996 by Bourdel P et al. 5. The spinor parameter can describe the variation of a surface or its element relative to its nominal position (tolerance area). Each tolerance area can be expressed in a form of rotation, which represents a small change in the tolerance area, which can be transformed into a constraint inequality. A. Desrochers and so on in the literature [6] lists the rotation description and geometric constraint of the standard tolerance zone. After we have modified it properly, the corresponding relation between the shape of tolerance zone and the corresponding rotation parameter and the constraint inequality is listed in Table 1 [1].

| Area | Describe the area | Spinor | Constraint inequality |
|------|-------------------|--------|-----------------------|
| Ring | ![Ring Diagram](image) | \[0 \quad u \]
\[0 \quad v \]
\[0 \quad 0 \] | \[u^2 + v^2 \leq t^2/4 \] |
| Between two parallel lines | ![Parallel Lines Diagram](image) | \[0 \quad 0 \]
\[0 \quad 0 \]
\[0 \quad v \] | \[-tL \leq \alpha \leq tL \]
\[-t/2 \leq v \leq t/2 \] |
| Between two concentric cylindrical surfaces | ![Cylindrical Surfaces Diagram](image) | \[\alpha \quad 0 \]
\[\beta \quad v \]
\[0 \quad 0 \] | \[-uL \leq \alpha \leq uL \]
\[-uL \leq \beta \leq uL \]
\[u = (u/2) \cos \theta \]
\[v = (u/2) \sin \theta \]
\[u^2 + v^2 \leq t^2/4 \] |
| Between two parallel planes | ![Parallel Planes Diagram](image) | \[\alpha \quad 0 \]
\[\beta \quad 0 \]
\[0 \quad w \] | \[-uL \leq \alpha \leq uL \]
\[-uL \leq \beta \leq uL \]
\[-t/2 \leq w \leq t/2 \] |

In Table 1, the rotation representation and geometric constraint inequalities for several tolerance regions are listed. These geometric constraints restrict the range of the rotation, and integrate the matrix form of the rotation and the geometric constraints. The representation of the small displacement rotation is changed into a form.

\[\tau = (\rho \varepsilon) = \left\{ \begin{array}{ll}
\alpha^- \leq \alpha \leq \alpha^+ & u^- \leq u \leq u^+ \\
\beta^- \leq \beta \leq \beta^+ & v^- \leq v \leq v^+ \\
y^- \leq y \leq y^+ & w^- \leq w \leq w^+
\end{array} \right\}
\left[ \begin{array}{c}
{\alpha^+} \\
{\beta^+} \\
{y^+}
\end{array} \right]
\left[ \begin{array}{c}
{u^+} \\
{v^+} \\
{w^+}
\end{array} \right]
\]

Formula: \(\rho\) and \(\varepsilon\) denote small rotation vectors and small translation vectors respectively, which are small displacement spinor parameters. \([[^+][-]+[\beta^+, \beta^+]+[\gamma^+, \gamma^+]+[u^-][v^-][w^-]]\)
*, $u^+, v^+, w^+$] respectively represent the limit interval of small displacement spinor parameters such as beta, gamma, $u$, $v$ and $W$.

2.2. The rotation description of the connection

Products are divided into functional elements (points, lines and surfaces) of different parts, which may be directly contacted or connected by kinematic pairs. The connection between these parts can be a fixed connection (there is no relative displacement between the two contact surfaces) and the movable connection, while the movable connection has some form of contact, line contact and surface contact. Table 2 lists the connection forms and spinor representation of common parts of mechanical products [2].

**Table 2.** connection and rotation description between parts.

| The form of connection | Spinor | The form of connection | Spinor |
|------------------------|--------|------------------------|--------|
| Fixed connection       | $\begin{cases} \alpha & u \\ \beta & v \\ \gamma & w \end{cases}$ | Spherical pair | $\begin{cases} 0 & u \\ 0 & v \\ 0 & w \end{cases}$ |
| Translation pair       | $\begin{cases} \alpha & 0 \\ \beta & v \\ \gamma & w \end{cases}$ | Screw pair   | $\begin{cases} \alpha & u \\ \beta & v \\ \gamma & w \end{cases}$ |
| Revolute pair          | $\begin{cases} 0 & u \\ \beta & v \\ \gamma & w \end{cases}$ | Planar pair  | $\begin{cases} \alpha & 0 \\ \beta & 0 \\ 0 & w \end{cases}$ |

3. 3D tolerance analysis model based on SDT

3.1. The concept of surface model

The use of surface model to describe assembly is convenient for tolerance analysis. The 3D dimension chain can be represented by graphics. K.LPP and D.C.Gossard[7] have developed a data structure that describes the assembly into components. D.Teissandier and other so forth [8] propose a three-level graphic description method, each of which corresponds to the level of the assembly description, as shown in Figure 1 as an example of a case where an assembly is decomposed into a surface figure.
For the convenience of 3D tolerance analysis, we use "surface graphics" to describe assembly. As shown in Figure 2, a surface identification is made up of two numbers separated by commas. For example, the surface 1 of Part 1 is identified as (1,1), and the part 1 is divided into two surfaces: plane (1,1) and cylindrical surface (1,2). Any surface is a vertex on the "surface figure", and the vertex (1, 0) corresponds to the nominal surface (nominal part) of Part 1. The edge between the vertex (I, 0) and (I, J) represents the tolerance zone associated with the surface (I, J). The edge between the vertex (I, J) and (m, n) (Note: i ≠ m, j ≠ 0, n ≠ 0) represents a connection. Thus, a 3D dimensional chain graph can be obtained sequentially or in parallel by connecting the edges with "surface graphics".

3.2. Creation of three-dimensional dimension chain
The three-dimensional dimensional tolerance analysis of mechanical assembly is first to create a three-dimensional dimension chain, which runs through each function factor size, and these functional elements have an impact on the entire assembly function requirements.

The relative position between any two surfaces of the assembly can be determined by means of a set of spinor. In order to calculate the spinor between any two surfaces, two operation rules need to be defined, i.e., sequential operation and parallel operation. As shown in Figure 3, the sequential operation rules are suitable for sequential connection on the edge of the surface elements.
3.3. Three dimensional tolerance analysis model

Tolerance analysis is to determine the sensitivity coefficient of each component element in the dimension chain to the closed ring, so as to determine the tolerance of the closed ring. We have studied the spinor representation methods for various tolerance areas and kinematic chains, and the problem of three-dimensional tolerance analysis has been simplified by means of the expression of spinor parameters. The mathematical model describing the effect of small displacement on each functional element on assembly function requirements is expressed as [1]

\[
\begin{bmatrix}
\alpha^-, \alpha^+ \\
\beta^-, \beta^+ \\
y^-, y^+ \\
u^-, u^+ \\
v^-, v^+ \\
w^-, w^+
\end{bmatrix}_{FR} =
\begin{bmatrix}
\alpha^-, \alpha^+ \\
\beta^-, \beta^+ \\
y^-, y^+ \\
u^-, u^+ \\
v^-, v^+ \\
w^-, w^+
\end{bmatrix}_{FEI}
\begin{bmatrix}
\alpha^-, \alpha^+ \\
\beta^-, \beta^+ \\
y^-, y^+ \\
u^-, u^+ \\
v^-, v^+ \\
w^-, w^+
\end{bmatrix}_{FEK}
\begin{bmatrix}
\alpha^-, \alpha^+ \\
\beta^-, \beta^+ \\
y^-, y^+ \\
u^-, u^+ \\
v^-, v^+ \\
w^-, w^+
\end{bmatrix}_{FEP}
\]

Formula: It represents the SDT matrix for assembly function requirements. It represents the SDT matrix of the K(k=1,2,…p) function element. P represents the number of matrices. [J_1J_2J_4J_5J_6]_{FEI} : It describes the Jacobian matrix of the geometric
relationship between the K functional elements and the functional requirements of assembly. Among them, \( J_1 J_2 J_3 \) is 3 rotation components, \( J_4 J_5 J_6 \) is 3 translational components, and (2) multiplication uses interval algorithm.[6]

4. An example of tolerance analysis
The following is an example of the assembly of two parts[1]. The dimensions and tolerances of each part are shown in Figure 4. Verify whether the clearance after installation of the left side of the 1,2 part meets the functional requirements.

After determining the Jacobian matrix of each functional element of the assembly, according to formula (2)[1]

\[
\begin{bmatrix}
\alpha^- & \alpha^+ \\
\beta^- & \beta^+ \\
\gamma^- & \gamma^+ \\
u^- & u^+ \\
v^- & v^+ \\
w^- & w^+
\end{bmatrix} =
\begin{bmatrix}
-0.0413 & +0.0413 \\
-0.0141 & +0.0141 \\
-0.0241 & +0.0241 \\
-0.0566 & +0.0566 \\
-0.0712 & +0.0712 \\
-0.1152 & +0.1152
\end{bmatrix}
\]

It can be seen that the clearance between the left side contact surfaces of the assembly parts 1,2 is located in the interval \([-0.0712, +0.1212]\).

5. Conclusion
According to the tolerance modeling theory of new generation GPS based on SDT, the concept of SDT is introduced into the 3D tolerance design model, and the tolerance value or geometric constraint conditions of geometric parameters are integrated into the expression of the rotation quantity by combining the interval algorithm. To create a three-dimensional dimension chain, the dimension chain is usually composed of some functional elements that directly affect the requirements of the mechanical assembly function, including the internal pairs of the parts and the motion pairs between the parts. With the help of the description of tolerance regions and connections of small displacement spinor, a three-dimensional tolerance analysis model including dimensional tolerances and geometric tolerances is proposed. An engineering application example is given to verify the effectiveness of the model.

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