2D gravity without test particles is pointless
(Comment on hep-th/0011136)

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Abstract

Claims of a general Weyl invariance of an arbitrary 2D dilaton
theory are critically discussed.
1 Introduction

The explicit or implicit assumption that generic (matterless) dilaton theories in two dimensions which are related by Weyl (conformal) transformations lead to “equivalent” formulations, has quite a long history [1]. A recent note [2] raises this issue again and contains statements which require critical comment.

Conformal transformations are very useful in many contexts of physics, ranging from classical electrodynamics to string theory. They are especially convenient in particular 2D models, where the absence of an extrinsic scale implies invariance under local non-singular conformal transformations (cf. e.g. [3]). This seems to be the reason why they have also attracted attention of the community studying 2D dilaton theories of the form

\[ S = \int_{\mathcal{M}_2} \sqrt{-\tilde{g}} \left[ X R - U(X) (\nabla X)^2 + V(X) \right]. \]  

In particular, the field-dependent Weyl transformation

\[ g_{\alpha\beta} = \Omega(X)^{-2} \tilde{g}_{\alpha\beta}, \quad \Omega(X) = \exp \left[ -\frac{1}{2} \int X' U(X') dX' \right] \]  

has been used to simplify (1) to

\[ \tilde{S} = \int_{\tilde{M}_2} \sqrt{-\tilde{g}} \left[ X \tilde{R} + \tilde{V}(X) \right] \]  

with \( \tilde{V}(X) = \Omega^2 \tilde{V}(X) \).

However, it should be stressed that for a large class of models\(^1\) \( U(X) \) in (1) and hence the conformal factor (2) are singular at the “origin” (where the curvature singularity is “located”), because \( \lim_{X \to 0} \Omega(X) = 0 \). In addition, on dimensional grounds the last term in (1) must contain a scale. Moreover, the only invariance transformations of (1) are diffeomorphisms. A nontrivial redefinition of the potentials \( U(X) \) and \( V(X) \) goes beyond the usual definition of an invariance. Additionally, the dilaton may carry a conformal weight whenever the 2D model (1) stems from dimensional reduction \( \text{(SRG)} \).

These simple observations show that even at the classical level serious problems are likely to occur when the singularity of that transformation and the scale dependence are not duly taken into account. Indeed this is

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\(^1\)We use the same notation as in [4].

\(^2\)E.g. theories with \( U(X) = aX^{-1} \) with \( a \in \mathbb{R} \), including spherically reduced gravity (SRG) and the CGHS model [5].
confirmed by consideration of the geodesics of test particles and their consequences for the causal structure of spacetime.

Since all the arguments given below appeared already in several papers (reaching back at least half a century [7]) we restrict ourselves to a brief qualitative discussion and refer to the literature for a more detailed analysis.

2 Classical observables

One of the key ingredients of the attempt to prove Weyl invariance in [2] is the premise that the only physical observable is the conserved quantity, which is present in all such theories [8]. It is true that the conserved quantity $C$ is proportional to the ADM mass $M$ in a given conformal frame. But conformal invariance of $C$ does not imply automatically conformal invariance of $M$. Any (local) function of $C$ will be again a conserved quantity and there is no preferred way to relate $C$ with $M$ in different conformal frames. Thus, the physical observable $M$ is in general not invariant under conformal transformations. This has been exploited in detail in [9]. We claim that also the scalar curvature is a classically accessible physical observable, which can be obtained by investigating the geodesics of lightlike and timelike test particles (i.e. no backreactions are involved). Indeed, many textbooks about general relativity use the geodesic deviation equations to motivate the concept of curvature (cf. e.g. [10]). Moreover, every time a conformal diagram is constructed in order to discuss the causal structure the geodesics of test particles are used (at least implicitly). It is a non-negligible difference whether they reach the singularity with finite affine parameter or not. Thus geometry without test particles to probe it has no well-defined meaning.

The rôle of geometric variables in gravity theories is twofold: On the one hand they represent fields, analogous to gauge fields on a fixed background. On the other hand, $g_{\mu\nu}$ is identified as the metric of the twodimensional manifold, which is exploited by the geodesics of test particles calculated from that $g_{\mu\nu}$. As noted correctly in the conclusions of [2] explicit coupling to matter fields breaks Weyl invariance, in general. But already the (at least implicit) inevitable presence of test particles has the same consequence, although they usually are not regarded as “matter” because they have no influence on the metric by assumption.

Even if the action and the equations of motion were Weyl invariant, the geodesics of test particles are not. Since we regard the causal structure of spacetime and the Ricci scalar as geometrical properties (although one needs test particles to probe them), we conclude that the geometry itself is not Weyl invariant.
In fact, this discussion has quite a long history. Already Fierz pointed out that after performing a Weyl transformation one has to transform in addition the geodesics of test particles and that they no longer obey the equation for geodesics calculated with the metric of the transformed geometry \cite{7}. This observation was also made in the original work of Jordan \cite{11} and Brans and Dicke \cite{12} who were the first to consider scalar-tensor theories.

3 Quantum observables

At the quantum level the situation becomes even worse. The field quantization brings in a natural scale, the vacuum energy, that breaks explicitly conformal invariance in those special cases where the classical theory is conformally invariant. Not surprisingly, there is also ample evidence that the flux of Hawking radiation depends on the choice of the conformal frame \cite{14}. This is to be expected because the asymptotic flux is measured at infinity and hence it is a global property that can be changed under a conformal transformation. The issue of Hawking radiation in 2D dilaton theories is not settled completely, although considerable progress in calculating the Hawking flux including backreactions from the conformal anomaly of a scalar field \cite{14} and a dilaton anomaly has been achieved \cite{15} (see also references therein).

Finally, we should stress that within the path integral approach to 2D quantum gravity \cite{16} a field redefinition \cite{2} introduces functional determinants with unmanageable problems following from the inevitable singularities present in such a transformation.

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