Dynamics of the Flows  
Accreting onto a Magnetized Neutron Star  

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Abstract  
Non-stationary column accretion onto a surface of a magnetized neutron star is studied with a numerical code based on modified first-order Godunov method with splitting. Formation and evolution of shocks in the column is modeled for accretion rates ranging from $10^{15} \text{ g s}^{-1}$ to $10^{16} \text{ g s}^{-1}$ and surface magnetic fields ranging from $5 \cdot 10^{11} \text{ G}$ to $10^{13} \text{ G}$. Non-stationary solutions with plasma deceleration at collisionless oscillating shocks are found. The kinetic energy of the accreting flow efficiently transforms into a cyclotron radiation field. Collisionless stopping of the flow allows a substantial part of accreting CNO nuclei to avoid spallation and reach the surface. The nuclei survival fraction depends on the surface magnetic field, being higher at lower magnetic fields.  

keywords: accretion, neutron stars, hydrodynamics  

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1 Introduction

Accretion onto compact objects has been being considered an efficient source of hard emission for about forty years already (see the pioneer papers of Zel’dovich (1964), Salpeter (1964), Pringle and Rees (1972), and Shakura and Sunyaev (1973)).

A number of successful analytical, semi-analytical, and numerical models of accretion onto neutron stars (NSs) and black holes has been developed by now [e.g. see the book of Frank, King, and Raine (2002) and references therein], but several important problems already formulated in the very earliest papers have no clear answers as yet. One of the fundamental points is whether the matter accretes as a gas of interacting particles ("hydrodynamic regime") or as a number of separate non-interacting particles ("freefall regime"). Zel’dovich (1967) and Zel’dovich and Shakura (1969) considered both cases and showed that the spectrum emitted near the surface of a NS critically depends upon the particular regime under consideration. Up to now accretion models exploit only indirect qualitative arguments to give prove to a particular regime in the hope that a realistic model supported by observational evidences may a posteriori justify their choice at least for a certain range of global parametres of an accretion system. Another important point is whether and when collisionless shocks (CSs) appear and evolve in an accreting flow to decelerate the matter on its way to the surface.

Bisnovatyi-Kogan and Fridman (1970) pointed out that a CS can appear in a flow accreting onto a NS if the star possesses a dipole magnetic field \( B \sim 10^8 \) G.

A shock that decelerates the accreting flow in a binary system plays a key role in the model of Davidson and Ostriker (1973). Shapiro and Salpeter (1975), Basko and Sunyaev (1976), Langer and Rappoport (1982), and Braun and Yahel (1984) considered accretion onto a NS under various assumptions on the star’s magnetic field strength and found stationary solutions for similar sets of hydrodynamic equations used to describe the accretion flow. The models of Shapiro and Salpeter (1975) and Langer and Rappoport (1982) postulated the existence of a stationary collisionless shock at a certain height over the star’s surface, the height being a model parameter. Basko and Sunyaev (1976) constructed an accretion model with a radiative shock in the star’s atmosphere. Braun and Yahel (1984) claimed that a stationary collisionless shock can exist over the surface of a magnetized NS only when the accretion rate is low enough (namely, when the rate does not exceed a few percent of the Eddington value).

Detailed two-dimensional non-stationary radiation-dominated super-Eddington models of accretion onto a magnetized NS have been developed by Arons, Klein, and collaborators (see Klein and Arons (1989), Klein et al. (1996) and references therein). The authors of the models showed that if the accretion rate is much higher than the Eddington one, non-stationary radiation-dominated shocks come out and evolve in the accretion column. An essential feature of these models is that the existence of a shock in the column is not postulated a priori, being a result of evolution of the accretion system.

In this paper a numerical model of sub-Eddington one-dimensional non-stationary hydrodynamic (in the above-mentioned sense) accretion onto a magnetized NS is presented. The main feature of the model is a Godunov-type numerical approach that allows to describe discontinuous flows, and in particular, dynamics and evolution of shocks.

The model consistently takes into account kinetics of electron-ion flows in a strong
external magnetic field. The field in the accretion column is fixed, but it plays a key role in various processes of interaction of matter and radiation that govern the flow evolution.

In section 2 a hydrodynamic model of an accretion flow is presented. Based on the modeled flow profiles, destruction probabilities for accreting CNO nuclei are calculated in section 3. The obtained results are summarized in section 4.

2 Accretion Flow Hydrodynamics

2.1 Formulation of the Problem

Evolution of an accretion flow in a magnetic column above a polar cap of a magnetized NS at distances up to several NS radii from the surface is considered.

The accretion flow is assumed to be ”hydrodynamic” as magnetohydrodynamic instabilities grow very fast under typical conditions of an accretion column. As the initial flow is highly anisotropic, multistream instabilities can have growth rates comparable with the ion plasma frequency \( \omega_{pi} \sim 1.2 \cdot 10^{12} n_{18}^{1/2} \text{s}^{-1} \), where \( n_{18} \) is the ion number density measured in \( 10^{18} \text{ cm}^{-3} \). Such growth rates are typical for isotropisation processes in CSs. Note that the cyclotron frequencies are much higher at considered magnetic fields. A typical propagation velocity of MHD-waves is \( V_{\text{Alfvén}} \approx c (1 - \alpha) \), where \( \alpha = 10^{-11} \cdot n_{18} \cdot B_{12}^{-2} \ll 1 \). This scaling formula can be derived from general relations mentioned, e.g., in the book of Libermann and Velikovich (1986). Microscopic modeling of instabilities at NS magnetic fields is not yet feasible, and thus it is assumed that instabilities grow with increments of order \( \omega_{pi} \).

It is assumed that electrons and ions move in the accretion flow with the same mean (flow) velocity but have different temperatures.

The NS is assumed to possess a constant dipole magnetic field on the considered timescales. The accretion column configuration is shown in Fig. 1.

2.2 Main Parameters and Equations of the Model

The main parameters of the model are the mass \( M_{\star} \) and the radius \( r_{\star} \) of a neutron star, as well as the magnetic field strength at its magnetic pole \( B_{\star} \) and the accretion rate per unit area of the base of the accretion column \( \dot{M}/A_0 \).

A basic system of hydrodynamic equations that describe flow evolution can be written as

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0 \quad (1)
\]

\[
\frac{\partial (\rho \mathbf{u}_\alpha)}{\partial t} + \frac{\partial p}{\partial x_\alpha} + \frac{\partial}{\partial x_\beta} (\rho u_\alpha u_\beta) = \mathbf{F}_\alpha \quad (2)
\]

\[
\frac{\partial}{\partial t} \left[ \rho_s (E_s + \frac{u^2}{2}) \right] + \text{div} \left[ \rho_s \mathbf{u} (E_s + \frac{u^2}{2}) + p_s \mathbf{u} \right] = \mathbf{Q}_s \quad (3)
\]

where \( \rho = \rho_e + \rho_i, p = p_i + p_e, \mathbf{F} \) and \( \mathbf{Q}_s \) denote sources of momentum and energy, while \( s = i, e \) denotes the sort of particles.
The system should be completed with equations of state for each sort of particles. The equation of state for an ideal gas is used: \( E_s = p_s / [\rho_s (\gamma_s - 1)] \). For \( x_s = k_B T_s / (m_s c^2) \ll 1 \) the adiabatic index \( \gamma_s \) may be written as \( \gamma_s \approx \gamma_{0e} (1 - x_i) \) (de Groot et al. (1980)), where \( \gamma_{0e} = 5/3 \) is a usual non-relativistic value for particles with three degrees of freedom, while \( \gamma_{0e} = 3 \), as in strong fields under consideration the electrons are quasi-one-dimensional.

The momentum distribution of the electrons is one-dimensional, because the typical relaxation time of electron Landau levels is about \( 10^{-15} B_{12}^{-2} \) s, where \( B_{12} = B / 10^{12} \) G (see e.g. Bussard 1980); this time is the shortest one in the system after the cyclotron time. The radiative decay time of the ion Landau levels is about \( 5 \cdot 10^{-9} B_{12}^{-2} \) s that is much longer than the time of collisionless relaxation of the ions \( \propto \omega_{ci}^{-1} \). Thus, the ions occupy highly excited Landau levels and may be considered three-dimensional and treated quasi-classically.

One-dimensional motion of accreting plasma along dipole lines of the star’s strong magnetic field is considered. Within such a geometry the system (1)-(3) may be rewritten as

\[
r^3 \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r} (r^3 \rho u) = 0 \tag{4}
\]

\[
r^3 \frac{\partial \rho (u^2)}{\partial t} + \frac{\partial}{\partial r} \left[ r^3 (p + \rho u^2) \right] = r^3 \mathcal{F} + 3r^2 \rho \tag{5}
\]

\[
r^3 \frac{\partial}{\partial t} \left[ \rho_s (E_s + \frac{u^2}{2}) \right] + \frac{\partial}{\partial r} \left[ r^3 \rho_s u (E_s + \frac{u^2}{2}) + p_s u^2 \right] = r^3 Q_s. \tag{6}
\]

The system (1)-(6) should be completed by initial and boundary conditions. An accretion column filled with cold free-falling gas is considered as an initial condition for the simulations. The boundary condition in the upper part of the column is an inflow of a cold supersonic stream, while the boundary condition at the star’s surface is the absence of any flows into the star.

### 2.3 Physical Processes in an Accreting Flow

In this section the processes contributing to \( \mathcal{F} \) and \( Q_s \) in the system (1)-(6) are described.

Since a single-velocity flow is considered, the forces acting on ions and electrons are all summed in a single force term:

\[
\mathcal{F} = F^i + F^e, \quad F^i = F^i_{grav} - F_{atm}, \quad F^e = F^e_{grav} - F_{nonres} - F_{res},
\]

where \( F^i_{grav} \) and \( F_{atm} \) denote gravity and viscous friction force (that is substantial only in the thin atmosphere of the star), while \( F^e_{grav}, F_{nonres}, \) and \( F_{res} \) denote gravity, non-resonant, and resonant radiative pressure, respectively.

The force of gravitation acting on a unit volume is

\[
F_{grav} = F_{grav}^i + F_{grav}^e = (n m_i + Z n m_e) \frac{G M_*}{r^2}, \tag{7}
\]

where \( n \) is the ion number density.
To calculate the non-resonant radiative pressure the following formula from the book of Zheleznyakov (1996) is adopted:

\[ F_{\text{nonres}} = n_e \sigma_T \frac{\sigma_{ST} T_\gamma^4}{1 + \tau_T}, \]  

where \( n_e \) is the electron number density, \( \sigma_{ST} \) is the Stefan-Boltzmann constant, \( T_\gamma \) is the local temperature of radiation field, and \( \tau_T \) is the non-resonant optical depth.

An equation of radiation transfer in the cyclotron line is numerically integrated to calculate the resonant radiative pressure force \( F_{\text{res}} = dU_{\text{phot}}/dr \) and obtain \( U_{\text{phot}} \) – the energy density of the photon field.

As the accretion column is optically thick in the cyclotron line, the transfer equation may be written as a diffusion equation:

\[ \nabla \cdot J_{\text{phot}} = S_{\text{phot}} + \frac{1}{3} u \cdot \nabla U_{\text{phot}}, \]  

where \( S_{\text{phot}} \) denotes the sources of cyclotron photons, and \( J_{\text{phot}} = \frac{4}{3} u U_{\text{phot}} - \kappa \nabla U_{\text{phot}} \) is the diffusive flux of photons in the cyclotron line. As diffusion of cyclotron photons across the magnetic field lines is substantially hampered (see e.g. Arons, Klein, and Lea 1987), only the parallel component of (9) is considered, that may be written as

\[ \frac{1}{r^3} \frac{\partial}{\partial r} \left\{ r^3 \left[ \frac{4}{3} U_{\text{phot}} u - \kappa_{||} \frac{1}{r^3} \frac{\partial}{\partial r} (r^3 U_{\text{phot}}) \right] \right\} = S_{\text{phot}} + \frac{1}{3} u \frac{1}{r^3} \frac{\partial}{\partial r} (r^3 U_{\text{phot}}), \]  

where \( \kappa_{||} \) is the diffusion coefficient parallel to the magnetic field. The equation is integrated by a shooting method with the following boundary conditions: cyclotron photons freely leave the upper border of the column, while their number density on the star’s surface is determined by a blackbody spectrum of temperature \( T_{\text{eff}} \). The surface temperature \( T_{\text{eff}} \) varies with the part of flow energy released at the surface.

To calculate the friction force acting on the flow in the star’s atmosphere a standard expression for Coulomb stopping in a dense environment is used:

\[ F_{\text{atm}} = \frac{4 \pi n_a n_i e^4 Z^2 \ln \Lambda}{m_e u^2}, \]  

where \( \Lambda \) is a Coulomb logarithm, \( n_a \) is the electron number density in the atmosphere, and \( u \) is the flow velocity. A similar expression was used in the paper of Bildsten et al (1992) devoted to collisional destruction of CNO nuclei in a NS atmosphere.

The forces acting on the flow perform a certain work on it, and this work is effectively redistributed between the ions and the electrons.

Let an external force \( F^i \) act on the ions and an external force \( F^e \) act on the electrons, then it can derived from the local electroneutrality (if the frequencies of variations of the external forces are much lower that the plasma frequencies) that the fluxes of both types of particles are equal, i.e.
\[(F^e - eE) \frac{n_e}{m_e \nu_{ei}} = (F^i + ZeE) \frac{n_i}{m_i \nu_{ei}},\]

where \(E\) is the ambipolar electric field, and \(\nu_{ei}\) is the effective electron-ion relaxation frequency. Since \(n_e = Zn_i,\)

\[eE = F^e \frac{1}{\xi + 1} - F^i \frac{\xi}{Z(\xi + 1)},\]

where \(\xi = m_e/m_i,\) and the resulting effective force acting of the ions is

\[F^i_{\text{eff}} = (F^i + F^e Z) \frac{1}{\xi + 1},\]

while the resulting effective force acting on the electrons is

\[F^e_{\text{eff}} = (F^i + F^e Z) \frac{\xi}{Z(\xi + 1)}.\]

The temperature of the ions changes in the following processes:

(i) small-angle e-i Coulomb scatterings ("heat exchange"): \(H_{ie}\)
(ii) collisional excitation of electron Landau levels: \(Q_{cyc}\)
(iii) collisional Coulomb relaxation in the star’s atmosphere: \(Q_{\text{relax}}\)
(iv) the work of the effective force: \(F^i_{\text{eff}}u\)

The model also accounts for the cyclotron cooling of the ions. This effect is noticeable if the star’s dipole magnetic field exceeds \(5 \cdot 10^{11} \text{ G}\) and the column is transparent for the photons of the ion cyclotron fundamental line.

The temperature of the electrons changes in the following processes:

(i) small-angle e-i Coulomb scatterings ("heat exchange"): \(H_{ie}\)
(ii) Bremsstrahlung cooling in e-i and e-e collisions: \(Br_{ei}\ \ Br_{ee}\)
(iii) collisional excitation of Landau levels in e-i and e-e collisions: \(Cy_{cie} \ Cy_{cde}\)
(iv) Compton processes: \(Q_{\text{compt}}\)
(v) the work of the effective force: \(F^e_{\text{eff}}u\)

For a detailed description of the terms above, see the Appendix.

### 2.4 The Numerical Approach

A multicomponent accretion flow onto the surface of a NS can have discontinuities, and in particular, shocks. That is why the chosen numerical approach is based on a well-known Godunov method (see, e.g. Godunov 1959; 1964).
The traditional form of Godunov method is only applicable to one-component systems without source terms. To account for the source terms in \[ \text{(4)-(6)} \] a numerical scheme based on the work of LeVeque (1997) is exploited. The system \[ \text{(4)-(6)} \] is split into two parts: one of them consists of flux-conserving terms and is integrated with a modified Godunov method, while the other describes the sources of energy and momentum and is integrated with a Gear method.

Such a way of solving the system is due to its complex structure with two sorts of particles non-linearly interacting with each other, with external magnetic and gravitational fields as well as with the radiation field.

To make the system \[ \text{(4)-(6)} \] dimensionless \( F \) is multiplied by \( C_F = \frac{t_s}{\rho_s u_s^2} \), while \( Q_s \) is multiplied by \( C_Q = \frac{t_s}{\rho_s u_s^2} \), where \( t_s, u_s, \) and \( \rho_s \) are scales of time, velocity, and density, respectively.

Integration of equations \[ \text{(4)-(6)} \] in the accretion column is performed in a combined way that allows to generalize the standard Godunov method for systems with energy and momentum exchange between the components (i.e. account for source term in \[ \text{(4)-(6)} \]). The accretion column is represented by a number of spatial cells. The system is integrated from an initial state at the moment \( t = 0 \) to a current state at a moment \( t \) in a number of time steps \( \Delta t \) each of them consisting of the following stages:

(i) integration of flux-conserving terms without source terms:

\[
\begin{align*}
   r^3 \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(r^3 \rho u) &= 0 \\
   r^3 \frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial r}[r^3(p + \rho u^2)] &= 0 \\
   r^3 \frac{\partial}{\partial t} \left[ \rho_s(\mathcal{E}_s + \frac{u^2}{2}) \right] + \frac{\partial}{\partial r}(r^3[\rho_s u(\mathcal{E}_s + \frac{u^2}{2}) + p_s u]) &= 0
\end{align*}
\]

At this stage the system is simultaneously integrated in all the cells.

(ii) integration of source terms in individual cells:

\[
\begin{align*}
   \frac{\partial (\rho_i u_i)}{\partial t} &= C_F F_i + \frac{3}{r_i} p_i \\
   \frac{\partial}{\partial t} \left[ \rho_s(E_{si} + \frac{u_i^2}{2}) \right] &= C_Q Q_{si},
\end{align*}
\]

where \( q_i \) are cell-averaged quantities in the cell number \( i \).

At each of the two stages the system is integrated over the same time step determined by the Courant condition at the first stage.

The system \[ \text{(15)-(16)} \] is stiff and thus it is integrated with a standard LSODE routine (Hindmarsh 1983) that implements the Gear-B method (see, e.g. Gear 1971). It should

\footnote{For a review of modern multi-component methods see, e.g., Zabrodin and Prokopov (1988).}
be noted that a number of modern methods for integration of stiff systems based on the modified Bulirsch-Stoer method (see Press et al. 1993) require an explicit Jacobian of the integrated system to be supplied. As the system (15)-(16) has a very complex right-hand part with non-analytical terms, the Bulirsch-Stoer methods appear too complicated to implement.

The system (12)-(14) is integrated with a "capacity-differencing" modification of the standard first-order Godunov method suggested by LeVeque (1997).

The essence of "capacity-differencing" is the following. If a conservation law for a physical quantity \(q(x,t)\) has a generalized form

\[
\kappa(x) \frac{\partial q(x,t)}{\partial t} + \frac{\partial f(q(x,t))}{\partial x} = 0,
\]

where \(\kappa(x)\) is a known function of the space coordinate, denoting effective "capacity" (e.g. porosity of the medium) the traditional Godunov value of net function \(q\) at the moment \(t_0 + \Delta t\)

\[
\tilde{q}_i = q_i - \frac{\Delta t}{\Delta x_i} (F_i - F_{i+1}),
\]

where \(F_i\) is the flux of \(q\) from the \(i\)th cell to the \((i-1)\)th cell should be replaced by

\[
\tilde{q}_i = q_i - \frac{\Delta t}{\kappa_i \Delta x_i} (F_i - F_{i+1}),
\]

where \(\Delta x_i\) is the size of the \(i\)th cell, while \(\kappa_i\) is the cell-averaged value of \(\kappa(x)\).

### 2.5 Simulation Results

Evolution of an accreting flow has been simulated according to the scheme described above for a number of sets of global parameters.

Strong shocks have been found to evolve in the column on timescales of about \(10^{-5}\) s. The shocks oscillate around their equilibrium positions with periods of about \(10^{-5}\) s, the oscillations dumping time being about \(10^{-3}\) s. A typical evolution of a shock and of a flow velocity profile is presented in Fig.2.

Modeled flow profiles in Figs. 3 and 5 demonstrate stable and strong shocks that decelerate and heat the accreting flow. At such shocks the ions are heated much more than the electrons as they contain most of the kinetic energy of the flow. However, as the hot ions move towards the surface in the downstream zone, they pass a substantial part of their energy to the electrons. In their turn, the electrons emit the energy as cyclotron and Bremsstrahlung photons and pass it to non-resonant photons in Compton collisions.

In most of the modeled cases the compression ratio at the shocks slightly exceeds 4 (the maximal value for non-relativistic single-fluid shocks) due to weak relativistic changes in the adiabatic index of the ions heated upto a few tens MeV.

An important property of the model is transformation of a substantial part of the ram energy of the flow into cyclotron photons of the optically thick line. Fig.4 demonstrates the dependence of the part on magnetic field strength. The pressure of the trapped cyclotron

\[
\text{(Equation)}
\]

\[
\text{(Equation)}
\]
photons noticeably affects the stopping of the accreting flow. The cyclotron cooling of the ions is significant at magnetic fields exceeding $5 \cdot 10^{11}$ G; in this case, the particular accretion regime significantly depends upon the detailed structure of magnetic fields in the star’s atmosphere about $10^3$ cm from the surface. At such heights the field may be significantly non-dipolar due to the presence of local high-multipole components. The non-uniform structure of the field makes the column transparent for optical and X-ray cyclotron photons emitted by ions and electrons. Accretion regimes for such an optically thin column where the cyclotron emission of the ions freely comes out are demonstrated in Fig.4. An account for cyclotron cooling of the ions at magnetic fields exceeding $5 \cdot 10^{11}$ G leads to a significantly lower position of the shock. In this case the emission spectrum will most probably contain a prominent proton cyclotron line in the optical/UV range. Due to strong deceleration and efficient cooling of the flow in an optically thin part of the column only about half of the flow energy reaches the surface of the star (see Fig.5).

If magnetic field structures are regular at the heights of about $10^3$ cm from the surface, the proton cyclotron line may become optically thick. In this case the accretion regime is analogous to that in Fig.5, as the high-frequency collisionless relaxation recovers the isotropy of the ion distribution much faster than the transverse temperature changes.

It should be also noted that the qualitative difference in flow profiles at low and high values of magnetic field (Fig.6) is due to the fact, that at low fields the electron temperature in the downstream region is lower and the gradient of locked cyclotron photons energy density increases. A large gradient makes the flow to decelerate more efficiently.

### 3 Destruction of CNO Nuclei in an Accretion Column

An important characteristic of an accreting flow is the chemical composition of the matter that reaches the surface of a NS. The characteristic is particularly important for the theory of X-ray bursts, as accreted CNO nuclei may catalyze thermonuclear burning of hydrogen on the surface of a NS (see Lewin, van Paradijs, and Taam 1993, Strohmayer and Bildsten 2003, and references therein).

Most of the bursts (Type I bursts) have been observed in low-mass X-ray binary systems where the NSs are usually thought to possess magnetic fields not exceeding $10^9$ G, while the model presented here treats much stronger fields usually attributed to high-mass X-ray binary systems. Nevertheless, it is instructive to study the dependence of the survival fraction on magnetic field and mass accretion rate in the scenario of collisionless matter stopping at an accretion shock wave described above.

The nuclei spallation process leads to fast destruction of tens MeV/nucleon range nuclei and thus, to a significant suppression of gamma-ray line emission from accreting objects (see e.g. Aharonian and Sunyaev 1984 and Bildsten et al. 1992). Bildsten et al. (1992) considered the destruction scenario for the case when the accretion flow is decelerated by Coulomb collisions in a dense atmosphere of a NS and concluded, that almost all the CNO nuclei would be destroyed before they reach the surface. The authors noted, however, that this conclusion may be not true in the case when the flow decelerates in a collisionless column above the atmosphere as it is in our model.
Indeed, in that case, the depth traversed by a nucleus as it decelerates down to energies of about 10 MeV/nucl can be much smaller than the depth for a pure Coulomb deceleration down to the same energies. Having modeled flow profiles in an accretion column, one is able to say how efficient the destruction of CNO nuclei in an accretion flow is and where they are destroyed.

In order to test the destruction efficiency quantitatively the destruction probability for a carbon nucleus that accretes inside a modeled flow was calculated (the destruction crosssections of nitrogen and oxygen are very similar to that of carbon, so they would be destroyed in the same way).

The carbon nuclei destruction rate [with crosssections given by Read and Viola (1984)] was numerically integrated within the flow to obtain the nuclei survival fractions as a function of the distance from the NS surface.

The dependencies of survival fractions for a carbon nucleus on the distance from the NS surface are presented in Fig.6 for a set of magnetic field values. It follows that the survival fraction of C nuclei drops by an order of magnitude for magnetic fields increasing from $5 \cdot 10^{11}$ G to $5 \cdot 10^{12}$ G. For magnetic fields about $5 \cdot 10^{11}$ G a significant fraction of the nuclei may reach the surface of the star. The survival fraction is not too sensitive to the accretion rate in the interval studied (from $10^{15}$ g s$^{-1}$ to $10^{16}$ g s$^{-1}$). The effect may prevent Type I bursts for NSs in HMXBs with magnetic fields $\gtrsim 3 \cdot 10^{12}$ G, while the bursts are allowed for lower fields, if the hydrodynamic accretion regime considered in the paper realizes.

4 Summary

A numerical model of a non-stationary one-dimensional accretion column over a polar cap of a magnetized neutron star has been constructed. The model describes a two-fluid flow of accreting plasma in a strong dipolar magnetic field. One of the main features of the model is an ability to treat consistently flow discontinuities (including shocks) with a modified Godunov method that made it possible to study temporal evolution of shocks in the accreting flow.

After several free-fall periods a quasi-stationary state of the column with a stable accretion shock is usually reached. At the shock the ion temperature jumps upto $\sim 10^{11}$ K (this value weakly depends upon the magnetic field strength and accretion rate). Depending on the magnetic field strength the electron temperature reaches $(3-5) \times 10^8$ K.

A part of the kinetic energy of the flow is transformed into emission of a thick cyclotron line. The radiation pressure in the line significantly affects the deceleration of the plasma flow. A substantial part of the kinetic energy of the flow is emitted into the optically thin part of the spectrum well before the flow reaches the bottom of the column. It is often assumed that the kinetic energy of the flow and the emission of the optically thin part of the column are transformed into blackbody radiation in the optically thick part of the star’s atmosphere. As a significant part of the flow energy is emitted far above the atmosphere the effective temperature of the polar cap $T_{\text{eff}}$ is reduced and the usual relation $T_{\text{eff}} \propto \dot{M}^{1/4}$ is not true anymore, because the part of flow energy emitted above
the surface is now a complicated function of $\dot{M}$.

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Appendix: Energy Exchange Rates

1. The rates of energy exchange due to small-angle Coulomb collisions $H_{ei}$ (and $H_{ie} = -H_{ei}$) were adopted from the work of Langer and Rappoport (1982):

$$H_{ei} = 2 \sqrt{\frac{2}{\pi}} r_0 n_e n_i \frac{T_i - T_e}{T_e + \xi T_i} Z^2 \sqrt{\frac{m_e c^2}{k_B(T_e + \xi T_i)}} \Lambda,$$  

(17)

where $\Lambda$ is a Coulomb logarithm, while $r_0 = 4\pi r_e^2 m_e c^3$ is a typical energy-loss timescale.

2. To calculate the cooling rates $Q_{cyc}$ and $Cyc_{ei}$ the complete QED crosssection of collisional excitation of electron Landau levels in a strong magnetic field adopted from the work of Langer (1981) was integrated numerically. In order to ensure reasonable accuracy in a wide range of temperatures and magnetic fields, ten Landau levels were taken into account.

3. To calculate $Cyc_{ee}$ the following approximation was adopted from the work of Langer and Rappoport (1982).

$$Cyc_{ee} = 2.04 r_0 n_e^2 B_{12}^{-1/2} \sqrt{\frac{k_B T_e}{\hbar \omega_B}} \times$$

$$\times \exp\left\{-\frac{m_e c^2}{k_B T_e} \left(\sqrt{1 + 0.04531 B_{12}^2} - 1\right)\left(\frac{B_{12}}{5}\right)^{0.5}\right\}.$$  

(18)

4. To calculate the Bremsstrahlung cooling rate $Br_{ee}$ we numerically integrated the crosssection of Haug (1975) and constructed the following approximation:

$$Br_{ee} \approx 2.5410 \cdot 10^{-37} T_e^{1.45811} n_e^2 \cdot g(B, T_e),$$  

(19)

where $g(B, T_e) = (0.409 - 0.0193 B_{12} - 0.00244 B_{12}^2)(k_B T_e/10 \text{keV})^{0.25}$ is a gaunt-factor adopted from the work of Langer and Rappoport (1982).

5. To calculate the Bremsstrahlung cooling rate $Br_{ei}$ we numerically integrated the crosssection of Berestetskii and Landau (1982) for high electron temperatures and adopted the approximation from the work of Langer and Rappoport for low electron temperatures.

$$Br_{ei} \approx \begin{cases} 0.36 \alpha r_0 (T_e/T_e^b)^{0.5} n_e n_i Z^2 g(B, T_e), & \text{when } T_e < T_e^b \\ 0.36 \alpha r_0 (T_e/T_e^b)^{1.2} n_e n_i Z^2 g(B, T_e), & \text{when } T_e \geq T_e^b \end{cases}$$  

(20)

where $T_e^b = 5 \cdot 10^8 \text{K}$.

6. $Q_{relax}$ denotes Coulomb relaxation of the accreting flow on the electrons of the dense and thin atmosphere of a NS. The simplest model of an isothermal atmosphere is used and $Q_{relax}$ is defined as

$$Q_{relax} = -\nu_{ei} \frac{k_B n_i}{\gamma_i - 1} (T_i - T_{atm}),$$  

(21)

where $\nu_{ei}$ is the frequency of Coulomb collisions.
7. $Q_{\text{compt}}$ denotes cooling of the flow electrons in single Compton scatterings. If a scattering photon is not too hard ($\frac{\gamma E}{m_e c^2} \ll 1$), the energy lost by an electron in a single scattering is $\Delta E = -\frac{E^2}{m_e c^2} + \frac{4k_B T_e E}{m_e c^2}$. As $< E_\gamma > = 3k_B T_\gamma$, $< E_{\gamma}^2 > = 12k_B T_\gamma^2$,

$$Q_{\text{compt}} = n_e n_\gamma \langle \sigma_T v_{\text{rel}} \Delta E \rangle H(B, T_\gamma) = 12n_e n_\gamma \sigma_T c k_B T_\gamma k_B \frac{T_e - T_\gamma}{m_e c^2} H(B, T_\gamma),$$

where $H(B, T_\gamma) = (1 + 0.0165(\hbar \omega_B/k_B T_\gamma)^{2.48})/(1 + 0.0825(\hbar \omega_B/k_B T_\gamma)^{2.48})$ is the gaunt-factor adopted from the paper of Arons, Klein, and Lea (1987), while $n_\gamma$ and $T_\gamma$ are local values of photon temperature and energy density.
Figure 1: Accretion column geometry

- Neutron Star
  - $M \sim 1.4 M_{\odot}$
  - $B \sim 10^{12} \text{ G}$

- Supersonic flow
- Subsonic flow
- Magnetic tunnel
- Collisionless shock
- Polar cap

$R \sim 10^6 \text{ cm}$
Figure 2: Evolution of an accreting flow: a) shock height vs time
b) evolution of velocity profile
Figure 3: Flow profiles at different magnetic field values
Figure 4: Flow profiles at different magnetic field values when the column is optically thin for ion cyclotron emission
Flow Profiles at time $= 5.0 \times 10^{-4}$ s

- $\dot{M}=1 \times 10^{15}$ g s$^{-1}$
- $\dot{M}=3 \times 10^{15}$ g s$^{-1}$
- $\dot{M}=6 \times 10^{15}$ g s$^{-1}$

$B = 3 \times 10^{12}$ G

Figure 5: Flow profiles at different accretion rate values
Figure 6: Survival fraction of $^{12}$C nuclei on their way down to the surface
Figure 7: Part of flow energy converted into cyclotron emission at different magnetic field values

\[ \dot{M} = 6 \cdot 10^{15} \text{ g s}^{-1} \]
Figure 8: Temporal evolution of the part of flow energy that reaches the surface

Fraction of the freefall energy that reaches the surface

\[ \dot{M} = 3 \cdot 10^{15} \text{ g s}^{-1} \]

\[ B = 3 \cdot 10^{12} \text{ G} \]

\[ t_{ff} = 5.2 \cdot 10^{-5} \text{ s} \]