Observational appearance of a freely-falling star in an asymmetric thin-shell wormhole

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Abstract It has been recently reported that, at late times, the total luminosity of a star freely falling in black holes decays exponentially with time, and one or two series of flashes with decreasing intensity are seen by a specific observer, depending on the number of photon spheres. In this paper, we examine observational appearances of an infalling star in a reflection-asymmetric wormhole, which has two photon spheres, one on each side of the wormhole. We find that the late-time total luminosity measured by distant observers gradually decays with time or remains roughly constant due to the absence of the event horizon. Moreover, a specific observer would detect a couple of light flashes in a bright background at late times. These observations would offer a new tool to distinguish wormholes from black holes, even those with multiple photon spheres.

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1 Introduction

The event horizon telescope (EHT) collaboration released images of the supermassive black holes M87* [1–8] and Sgr A* [9–14], which provides a new method to test general relativity in the strong field regime. The main feature displayed in these images is a central brightness depression, namely black hole shadow, surrounded by a bright ring. The edge of black hole shadow involves a critical curve in the sky of observers, which is closely related to some unstable bound photon orbits. For static spherically symmetric black holes, unstable photon orbits form photon spheres outside the event horizon. Since light rays undergo strong gravitational lensing near photon spheres, black hole images encode valuable information of the geometry in the vicinity of photon spheres. Therefore, black hole images have been widely studied in the context of different theories of gravity, e.g., nonlinear electrodynamics [15–21], the Gauss–Bonnet theory [22–25], the Chern–Simons type theory [26,27], f(R) gravity [28–30], string inspired black holes [31–34] and other theories [35–46].

On the other hand, testing the nature of compact objects in the universe has been an important question in astrophysics for decades. Although the black hole images captured by EHT are in good agreement with the predictions of Kerr black holes, the black hole mass/distance and EHT systematic uncertainties still leave some room within observational uncertainty bounds for black hole mimickers. An intriguing type of black hole mimickers is exotic compact objects, which are more massive than neutron stars but horizonless (e.g., boson stars, gravastars and wormholes). Among exotic compact objects, those with enough compactness to possess light rings (or photon spheres in the spherically symmetric case) are called ultra compact objects (UCOs) [47]. UCOs are of particular interest since their observational signatures can be quite similar to those of black holes [48–51]. Never-
The document discusses the importance of distinguishing UCOS from black holes, particularly focusing on the use of photon spheres and their role in observational features. The text mentions the detection of photons using baseline interferometry and the simulation of an observer detecting flashes. It also references the study of wormholes and their effects on black hole images.

Key points:
- The importance of distinguishing UCOS from black holes.
- The use of photon spheres in detecting photons and their role in observational features.
- The simulation of an observer detecting flashes.
- The study of wormholes and their effects on black hole images.

The document also includes mathematical expressions and equations, such as:

\[ ds^2 = -f_1(r_i)dt_i^2 + \frac{dr_i^2}{f_1(r_i)} + r_i^2d\Omega^2, \]

where \( f_1 \) and \( r_i \) are quantities in \( M_1 \) and \( M_2 \) respectively.

The rest of the paper is organized as follows. In Sect. 2, we briefly review the asymmetric thin-shell wormhole and introduce our observational settings. Numerical results are presented in Sect. 3. Finally, we conclude with a brief discussion in Sect. 4. We set \( G = c = 1 \) throughout this paper.
In this paper, we study a point-like star freely falling along the radial direction at $\theta_i = \pi/2$ and $\dot{\phi}_i = 0$, which emits photons isotropically in its rest frame. With spherical symmetry, we can confine ourselves to emissions on the equatorial plane. The geodesics on the equatorial plane are described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left[ f_i(r_i) \dot{r}_i^2 + \frac{1}{f_i(r_i)} \dot{\theta}_i^2 + \dot{\phi}_i^2 \right],$$

(6)

where dots stand for derivative with respect to an affine parameter $\tau$. Since the Lagrangian $\mathcal{L}$ does not depend on coordinates $t_i$ and $\phi_i$, the geodesics can be characterized by their conserved energy $E_i$ and angular momentum $l_i$ in $\mathcal{M}_i$,

$$E_i = -p_{\theta_i} = f_i(r_i) \dot{\theta}_i, \quad l_i = p_{\phi_i} = r_i^2 \dot{\phi}_i.$$  

(7)

Note that, according to Eq. (5), one has $E_1 = E_2 = E$ and $l_1 = l_2$.

The Lagrangian of the freely-falling star obeys the constancy $\mathcal{L} = -1/2$ when the affine parameter $\tau$ is chosen as the proper time. Since the star falls radially, its angular momentum $l_i = 0$. Due to the traversability of the wormhole, we consider two scenarios with distinct trajectories of the star. In the scenario I, the star with energy $E_1 = 1/Z$ ($E_2 = 1$) has a nonzero initial velocity at spatial infinity of $\mathcal{M}_1$. So, the star can pass through the throat and travel towards spatial infinity of $\mathcal{M}_2$. With the relation (7), the four-velocities of the star in $\mathcal{M}_1$ and $\mathcal{M}_2$ are given by

$$v^{\mu 1}_e(r_1) = \left( \frac{1}{1 - 2r_1^{-1}}, \frac{\sqrt{R - 2} - (2k - 2)}{\sqrt{R - 2k}}, \frac{2k - 2}{r_1}, 0, 0 \right),$$

$$v^{\mu 2}_e(r_2) = \left( \frac{1}{1 - 2kr_2^{-1}}, \frac{\sqrt{2k}}{r_2}, 0, 0 \right).$$  

(8)

In the scenario II, the star with energy $E_1 = 1$ is initially at rest at spatial infinity of $\mathcal{M}_1$. At first, the star falls freely in $\mathcal{M}_1$, passes through the throat and reaches a turning point in $\mathcal{M}_2$. Then, it moves towards the throat in $\mathcal{M}_2$, returns to $\mathcal{M}_1$ and comes to rest at spatial infinity of $\mathcal{M}_1$. Similarly, the four-velocities of the star in $\mathcal{M}_1$ and $\mathcal{M}_2$ are

$$v^{\mu 1}_e(r_1) = \left( \frac{1}{1 - 2r_1^{-1}}, \frac{\sqrt{2M}}{r_1}, 0, 0 \right),$$

$$v^{\mu 2}_e(r_2) = \left( \frac{1}{1 - 2kr_2^{-1}}, \frac{\sqrt{R - 2k}}{r_2}, \frac{-2k + 2}{r_2}, 0, 0 \right).$$  

(9)

where plus and minus signs correspond outward and inward moving, respectively.

Moreover, null geodesics on the equatorial plane are also governed by the Lagrangian (6) with $\mathcal{L} = 0$, which rewrites the radial component of the null geodesic equations as

$$\frac{\dot{r}_i^2}{L_i^2} = \frac{1}{b_i^2} = V_i, \quad V_i = \frac{1}{b_i^2} - V_{i,\text{eff}}(r_i),$$

(10)

where $b_i \equiv l_i/E_i$ is the impact parameter, and $V_{i,\text{eff}}(r_i) = f_i(r_i)r_i^{-2}$ is the effective potential. Note that the impact parameters of a null geodesic in $\mathcal{M}_1$ and $\mathcal{M}_2$, namely $b_1$ and $b_2$, are related by $b_1 = Zb_2$. A photon sphere in $\mathcal{M}_1$ is constituted of unstable circular null geodesics, whose radius $r_i^\text{ph}$ is determined by

$$V_i,\text{eff}(r_i) = \frac{1}{(b_i^\text{ph})^2}, \quad V_i,\text{eff}(r_i) = 0, \quad V_i,\text{eff}(r_i) < 0,$$

(11)

where $b_i^\text{ph}$ is the corresponding impact parameter. Photons with $b_i \approx b_i^\text{ph}$ are temporarily trapped at the photon sphere and can determine late-time observational appearances of the wormhole. If the throat radius satisfies $\max\{2, 2k\} < R < \min\{3, 3k\}$, the asymmetric thin-shell wormhole can be free of the event horizon and possess two photon spheres, which are located at $r_1^\text{ph} = 3$ and $r_2^\text{ph} = 3k$ in $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively. In this paper, we consider the asymmetric thin-shell wormhole with $k = 1.2$ and $R = 2.6$, whose observational appearance of an accretion disk has been discussed in [66].

We assume that the emitted photons are collected by distant observers distributed on a celestial sphere located at $r_1 = r_o$ in $\mathcal{M}_1$. To trace light rays emitting from the star to a distant observer, one needs to supply initial conditions. For a photon of four-momentum $p_{\mu i}$, the momentum measured in the rest frame of the star with four-velocity $v^{\mu i}_e$ at $r_i = r_e$ is

$$p^i = -v^{\mu i}_e(r_e)p_\theta - v^{\mu i}_e(r_e)p_r,$$

$$p^\theta = -\sqrt{\left[v^{\mu i}_e(r_e)\right]^2 - f_{i-1}(r_e)p_\theta \pm \sqrt{\left[v^{\mu i}_e(r_e)\right]^2 + f_i(r_e)p_r}},$$

$$p^\phi = 0, \quad p^\mu = \frac{p_{\mu i}}{r_e}$$

(12)

where plus and minus signs correspond to negative and positive $v^{\mu i}_e$, respectively. The emission angle $\alpha$ is defined as

$$\cos \alpha = \frac{p^\phi}{p^\mu},$$

(13)

which is the angle between the propagation direction of the photon and the radial direction in the rest frame of the star. In the rest frame, the photon is emitted with proper frequency $\omega_e = -\left(v^{\mu i}_e p_{\mu i}\right)_e = p^\mu$. For a distant static observer with four-velocity $u^{\mu i}_o = (1, 0, 0, 0)$, the photon is observed with frequency $\omega_o = -\left(u^{\mu i}_o p_{\mu i}\right)_o = p^\mu$. With Eqs. (5), (12) and (13), we express the normalized frequency $\omega_{o}/\omega_e$ as a function of the star position $r_e$ and the emission angle $\alpha$ for two scenarios in Table 1. Furthermore, the luminosity of photons is given by $L_k = d\mathcal{E}_k/d\tau_k$, where $\mathcal{E}_k$ is the total energy, $\tau_k$
is the proper time, and $k = e$ and $o$ denote quantities corresponding to the emitter and the observer, respectively. Similar to the normalized frequency, one can define the normalized luminosity

$$L_o = \frac{d\mathcal{E}_o/d\tau_o}{d\mathcal{E}_e/d\tau_e} \approx \frac{o_\omega d\nu_o}{e_\omega d\nu_e} \left(\frac{dt_o}{d\tau_e}\right)^{-1},$$

(14)

where $n_o$ and $n_e$ are the observed and emitted photon numbers, respectively, and we replaced $d\tau_o$ by $dt_o$ since they are almost the same for distant observers.

### 3 Numerical results

In this section, we numerically study observational appearances of a star freely falling radially in the asymmetric thin-shell wormhole in the scenarios I and II. During the free fall of the star, photons are emitted isotropically in the rest frame of the star. Specifically, we assume that the star starts emitting photons at $t_1 = t_2 = 0$ and $r_1 = 30.65$ in $\mathcal{M}_1$, and emits 3200 photons, which are uniformly distributed in the emission angle $\alpha$, every proper time interval $\delta\tau_e = 0.002$. It is worth emphasizing that observational appearances of the freely-falling star, especially late-time appearances, are rather insensitive to the initial position where the star starts emitting. Here, for better comparison with the Schwarzschild black hole case, we simply choose the initial position as $r_1 = 30.65$, which is in agreement with that of [78].

Here, observational appearances of the star are studied for two kinds of observers in $\mathcal{M}_1$. The first kind is observers distributed on a celestial sphere at the radius $r_o = 100$, which refers to collecting photons in the whole sky at fixed radial coordinate $r_o = 100$ in $\mathcal{M}_1$. The measurement by the observers on the celestial sphere would give the frequency distribution and the total luminosity of photons that reach the celestial sphere. The second kind is a specific observer, who is located at $\varphi_o = 0$ on the equator of the celestial sphere. Among all photons collected on the celestial sphere, we select photons with $\cos \varphi > 0.99$ to mimic photons detected by the specific observer. To calculate observed luminosities, the collected photons are grouped into packets of 50 (i.e., $dn_o = 50$) according to their arrival time.

As shown in Fig. 1, the asymmetric thin-shell wormhole with $k = 1.2$ and $R = 2.6$ has a double-peak effective potential, corresponding to one photon sphere in $\mathcal{M}_1$ and one in $\mathcal{M}_2$. Specifically, the photon sphere in $\mathcal{M}_1$ is located at $r_1^{ph} = 3$ with the critical impact parameter $b_1^{ph} = 3\sqrt{3}$, and that in $\mathcal{M}_2$ is located at $r_2^{ph} = 3.6$ with the critical impact parameter $b_2^{ph} = 3.6\sqrt{3}$. To discuss how photons with different impact parameters contribute to the observations of the star, we classify received photons into seven categories according to their impact parameter $b_1$ in $\mathcal{M}_1$.

- $b_1 < 3.579$. Yellow region in Fig. 1 and yellow dots in Figs. 4, 5, 7 and 8.
- $3.579 \leq b_1 < Zb_2^{ph}$. Pink region in Fig. 1 and pink dots in Figs. 4, 5, 7 and 8. In this category, photons emitted inward outside the photon sphere in $\mathcal{M}_2$ can circle around the photon sphere more than once before reaching a distant observer in $\mathcal{M}_1$. For example, a light ray with $b_1 = 3.579$, which has $\Delta \varphi = 2\pi$, is displayed in the upper-left panel of Fig. 2.
- $Zb_2^{ph} < b_1 \leq 3.664$. Brown region in Fig. 1 and brown dots in Figs. 4, 5, 7 and 8. In this category, photons emitted inward would circle around the photon sphere in $\mathcal{M}_2$ roughly with $\Delta \varphi = 2\pi$ before escaping to the celestial sphere in $\mathcal{M}_1$. For example, a light ray with $b_1 = 3.664$, which has $\Delta \varphi = 2\pi$, is displayed in the upper-right panel of Fig. 2.
- $3.664 < b_1 \leq 4.923$. Blue region in Fig. 1 and blue dots in Figs. 4, 5, 7 and 8.
- $4.923 < b_1 < b_1^{ph}$. Orange region in Fig. 1 and orange dots in Figs. 4, 5, 7 and 8. In this category, if photons are emitted inward outside the photon sphere in $\mathcal{M}_1$, they would linger for some time around the photon sphere by orbiting it approximately with $\Delta \varphi \geq 2\pi$. For example, a light ray with $b_1 = 4.923$, which has $\Delta \varphi = 2\pi$, is displayed in the lower-left panel of Fig. 2.

\[ \text{Since } \varphi_1 = \varphi_2 \text{ at the throat, the subscript of } \varphi \text{ is omitted for simplicity.} \]
The effective potential of null geodesics in the asymmetric thin-shell wormhole with $k = 1.2$ and $R = 2.6$. The potential has two peaks at $r_1^{\text{ph}} = 3$ (solid vertical blue line) and $r_2^{\text{ph}} = 3.6$ (dashed vertical blue line), corresponding to a photon sphere with $b_1^{\text{ph}} = 3 \sqrt{3}$ in $M_1$ and another one with $b_2^{\text{ph}} = 3.6 \sqrt{3}$ in $M_2$, respectively. The vertical red line denotes the throat at $r_1 = r_2 = R$. Photons emitted in the pink, brown, orange and purple regions have impact parameters close to the impact parameters of the photon spheres, and hence can be temporarily trapped around the photon spheres. In particular, when photons are emitted towards the throat at $r_2 > r_2^{\text{ph}}$ in the pink region or at $r_1 > r_1^{\text{ph}}$ in the brown, orange and purple regions, they usually orbit the wormhole with $\Delta \psi \geq 2\pi$.

- $b_1^{\text{ph}} < b_1 \leq 5.238$. Purple region in Fig. 1 and purple dots in Figs. 4, 5, 7 and 8. In this category, photons emitted inward outside the photon sphere in $M_1$ usually circle around the photon sphere more than once. For example, a light ray with $b_1 = 5.238$, which has $\Delta \psi = 2\pi$, is displayed in the lower-right panel of Fig. 2.
- $b_1 > 5.238$. Green region in Fig. 1 and green dots in Figs. 4, 5, 7 and 8.

In short, we use the orbit number of light rays emitted at $r_1 = 5$ in $M_1$ or $r_2 = 5$ in $M_2$ to determine the threshold impact parameters separating the seven categories. To sum up, light rays emitted inward at $r_2 = 5$ in the yellow/pink category would circle around the wormhole less/more than once before being received; light rays emitted inward at $r_1 = 5$ would circle around the wormhole less than once before being received in the blue and green categories, or more than once in the brown, orange and purple categories. Note that the orbit number of light rays with a given impact parameter depends slightly on the emitting position. So, light rays connecting the star and the observers circle around the wormhole approximately more than once in the pink, brown, orange and purple categories, and less than once in the yellow, blue and green categories. In other words, photons in the pink, brown, orange and purple categories can be temporarily trapped near the photon spheres.

3.1 Scenario I

In the scenario I, the star with energy $E_1 = 1/Z = \sqrt{3}$ would travel through the throat and move towards spatial infinity of $M_2$. For near-critical photons emitted with the impact parameter very close to those of the photon spheres in $M_1$ (i.e., $b_1 \simeq b_1^{\text{ph}}$) and $M_2$ (i.e., $b_2 \simeq b_2^{\text{ph}}$), their normalized frequencies $\omega_o/\omega_e$ measured by observers on the celestial sphere are plotted against the emitted position $r_e$.

For photons of $b_2 \simeq b_2^{\text{ph}}$, the normalized frequency can noticeably exceed 1 at a large $r_e$ in $M_1$ since the Doppler effect plays a more important role than the gravitational redshift. As the star falls towards the throat, the normalized frequency decreases due to stronger gravitational redshift, and blueshift becomes redshift at $r_e = 4.063$ in $M_1$, where the normalized frequency is 1. When emitted at the throat, the normalized frequency reaches the minimum. After the star enters $M_2$, the normalized frequency increases as...
Fig. 2 Photon trajectories in the asymmetric thin-shell wormhole with \( k = 1.2 \) and \( R = 2.6 \). The red points and circles denote the star and the throat, respectively. The blue solid and dashed circles represent the photon spheres in \( M_1 \) and \( M_2 \), respectively. The upper-left panel shows a photon emitted at \( r_e = 5 \) in \( M_2 \) with \( b_1 = 3.579 \), and the light ray has \( \Delta \varphi = 2\pi \). Other panels show photons emitted at \( r_e = 5 \) in \( M_1 \) with \( b_1 = 3.664, 4.923 \) and \( 5.238 \), and the light rays all have \( \Delta \varphi = 2\pi \). The solid and dashed segments of the light rays correspond to the segments in \( M_1 \) and \( M_2 \), respectively.

As \( r_e \) grows, and observed photons become bluershifted when \( r_e > 12.281 \). For photons of \( b_1 \approx b_1^{\text{ph}} \), the behavior of the normalized frequency is quite similar to those of \( b_2 \approx b_2^{\text{ph}} \) when they are emitted outside the photon sphere in \( M_1 \). When the star emits photons between the two photon spheres, inward-emitted and outward-emitted photons can both be captured by a distant observer after they circle around the photon sphere in \( M_1 \), thus leading to two branches as shown in the inset. The upper and lower branches correspond to photons emitted away from and towards the observer, respectively.

In the left panel of Fig. 4, we display the normalized frequency distribution of photons, which are emitted from the freely-falling star in the scenario I and collected by observers distributed on the celestial sphere at \( r_o = 100 \) in \( M_1 \). At early times, received photons are dominated by those emitted in the green region of Fig. 1, among which inward-emitted photons contribute to the high-frequency observation. When \( t_o > 160 \), photons emitted towards the photon sphere in \( M_2 \) in the blue and brown regions start reaching the observers after orbiting around the photon sphere. Subsequently, the observers receive photons emitted towards the photon sphere in \( M_1 \) in the purple and orange regions. Since time moves faster in \( M_2 \) roughly by a factor of \( 1/Z = \sqrt{3} \) relative to in \( M_1 \), photons circling around the photon sphere in \( M_2 \) arrive earlier. Moreover, the maximum frequency of photons emitted in the blue and brown regions is higher than that of photons emitted in the green, purple and orange regions. This is expected from Fig. 3, which shows that near-critical photons with \( b_2 \approx b_2^{\text{ph}} \) have higher normalized frequency than...
The normalized frequency $\frac{\omega_0}{\omega_e}$ as a function of the emitted position $r_e$ for photons in the scenario I, whose impact parameter is very close to those of the photon spheres in $M_1$ (solid lines) and $M_2$ (dashed lines). The observers are distributed on the celestial sphere at $r_o = 100$ in $M_1$. For a large $r_e$ in $M_1$, inward-emitted and near-critical photons can be blueshifted since the Doppler effect dominates over the gravitational redshift. Due to the relation (5), near-critical photons can also be blueshifted when $r_e$ is large in $M_2$. Photons emitted inward and outward between the two photon spheres can both reach a distant observer after orbiting the photon sphere in $M_1$, which gives two branches of the orange line in the inset. Moreover, the normalized frequency reaches the minimum at the throat, which is located at $r_e = 2.6$.

Fig. 3

these with $b_1 \simeq b_1^{ph}$. Afterwards, the frequency observations are dominated by photons emitted in the orange region, which are trapped at the photon sphere in $M_1$ for a longer time. At late times, the observers mostly receive photons in the yellow and pink regions, which are emitted towards the throat in $M_2$ with a small impact parameter.

The normalized total luminosity of the freely-falling star is displayed in the right panel of Fig. 4, where a dot corresponds to a packet of 50 photons, and the color of the dot is that having most photons in the packet. The luminosity gradually increases until reaching a peak around $t_0 \simeq 145$, and is dominated by photons emitted in the green region roughly before $t_0 = 150$, which is in agreement with the frequency observation. After $t_0 \simeq 160$, photons emitted in the blue region give rise to a noticeable increase of the total luminosity. As the star moves towards spatial infinity of $M_2$, emitted photons can still propagate to the observers in $M_1$ through the throat, and a slight decrease of the total luminosity is displayed at late times. Interestingly, this late-time observation is strikingly different from the black hole case, where the total luminosity has been found to decay exponentially at late times [78,79].

For a specific observer located at $\varphi_o = 0$ and $\theta_o = \pi/2$ on the celestial sphere at $r_o = 100$ in $M_1$, the angular coordinate change $\Delta \varphi$ of light rays connecting the star with the observer is

$$\Delta \varphi = 2n\pi,$$

where $n = 0, 1, 2 \ldots$ is the number of orbits that the light rays complete around the wormhole. To simulate observational appearances of the star seen by the observer, we select photons with $\cos \varphi > 0.99$ from all photons received on the celestial sphere. The frequency observation is presented in the left panel of Fig. 5, which shows a discrete spectrum separated by the received time. The yellow line is formed by photons with $n = 0$, which radially propagate to the observer.

Fig. 4

Fig. 4 The normalized frequency distribution and the total luminosity of the freely-falling star in the scenario I, measured by observers on a celestial sphere at $r_o = 100$ in $M_1$. Left: the observers receive photons with a wide range of frequencies. At the early stage, photons emitted in the green region of Fig. 1 give rise to the frequency observation. Afterwards, photons emitted in the brown, blue, orange and purple regions are observed. In particular, photons with a near-critical impact parameter produce high frequency observations. The late-time frequency observation is determined by photons in the yellow and pink regions, which are emitted at a large $r_e$ in $M_2$. Right: the luminosity is calculated by grouping received photons into packets of 50. An increase of the observed luminosity is caused by photons emitted inward in the blue region, leading to a peak at $t_0 \simeq 168$. At late times, the total luminosity gradually decays with time and is mainly controlled by photons, which are emitted at a large $r_e$ in $M_2$ and travel through the throat to reach the observers.
finite number of photons in our numerical simulation, only 3 Equation (7) leads to

\[ \Delta T_1 \simeq 2\pi b_1^{\text{ph}} \simeq 33 \text{ and } \Delta T_2 \simeq 2\pi Zb_2^{\text{ph}} \simeq 23, \text{ respectively.} \]

On the other hand, the frequency line with \( b_1 \) is roughly the period of circular null geodesics at the photon sphere, which results from the existence of two photon spheres, the number of dominant photons emitted in the yellow region. In contrast, for a black hole with two photon spheres, a series of flashes with decreasing luminosity are observed at late times due to photons orbiting around the hairy black hole different times [79].

3 Equation (7) leads to \( dt/d\phi_{\text{ph}} = b^{-1} V_{\text{eff}}^{-1}(r_{\text{ph}}) = b_{\text{ph}}, \) which gives \( \Delta T \simeq 2\pi b_{\text{ph}}. \)

At early times, the observed frequency of the \( n = 0 \) photons decreases with the received time as the star falls towards the throat. After the star passes through the throat, the observed frequency of the \( n = 0 \) photons increases since the gravitational redshift becomes weaker as the star moves further away from the throat, which results in the dip at \( t_o \simeq 150. \) Owing to the existence of two photon spheres, the \( n = 1 \) photons with impact parameters \( b_1 \geq b_1^{\text{ph}}, \) \( b_1 \lesssim b_1^{\text{ph}} \) and \( b_2 \simeq b_2^{\text{ph}} \) can form three frequency lines, which are highlighted in the inset of Fig. 5. As the star falls towards the throat, the three frequency lines decrease rapidly due to strong gravitational redshift near the throat. After the star passes through the throat, the frequency line with \( b_2 \simeq b_2^{\text{ph}} \) gradually increases. For \( n = 2, \) the frequency lines with \( b_1 \geq b_1^{\text{ph}}, b_1 \sim b_1^{\text{ph}} \) and \( b_2 \simeq b_2^{\text{ph}} \) move closer and are hardly distinguishable from each other. On the other hand, the frequency line with \( b_2 \simeq b_2^{\text{ph}} \) becomes more separate from them since photons spend more time orbiting around the photon sphere in \( \mathcal{M}_1. \) Indeed, it takes \( \Delta T_1 \simeq 2\pi b_1^{\text{ph}} \simeq 33 \) to orbit around the photon sphere in \( \mathcal{M}_1 \) one time, and \( \Delta T_2 \simeq 2\pi Zb_2^{\text{ph}} \simeq 23 \) to orbit around that in \( \mathcal{M}_2. \)

Therefore, for \( b_1 \simeq b_1^{\text{ph}}, \) \( b_1 \lesssim b_1^{\text{ph}} \) and \( b_2 \simeq b_2^{\text{ph}}, \) the time delay between the adjacent \( n = 1 \) and \( 2 \) frequency lines roughly equals to \( \Delta T_1 \ (\Delta T_2). \) For \( n = 3, \) because of the finite number of photons in our numerical simulation, only

\[ \omega_o/\omega_e \simeq T_o/\pi \]

Fig. 5 The normalized frequency and the luminosity of the freely-falling star in the scenario I, measured by a distant observer at \( r_o = 100, \theta_o = \pi/2 \) and \( \phi_o = 0 \) in \( \mathcal{M}_1. \) The colored dots denote photons emitted in the regions with the same color in Fig. 1. Left: received photons form several frequency lines indexed by the orbiting number \( n. \) The inset displays three frequency lines caused by \( n = 1 \) photons with \( b_1 \geq b_1^{\text{ph}}, b_1 \lesssim b_1^{\text{ph}} \) and \( b_2 \simeq b_2^{\text{ph}}. \) The time delay between the adjacent \( n = 1 \) lines formed by photons orbiting around the photon sphere in \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \) is roughly the period of circular null geodesics at the photon sphere, which produces a luminous flash at \( t_o \simeq 170. \) Later, the luminosity is mainly contributed by the \( n = 0 \) photons emitted in the yellow region of \( \mathcal{M}_2 \) and almost declines gradually at late times. In addition, a faint flash, which results from the \( n = 2 \) photons emitted in the orange region, is observed at \( t_o \simeq 200. \)

3.2 Scenario II

In the scenario II, the star starts falling from spatial infinity of \( \mathcal{M}_1 \) and returns to the infinity after going through the throat twice. Similarly, the normalized frequency \( \omega_o/\omega_e \) for near-critical photons is plotted in Fig. 6. Specifically, we focus on photons with \( b_1 \simeq b_1^{\text{ph}} \) emitted in the purple and orange regions and those with \( b_2 \simeq b_2^{\text{ph}} \) emitted in the brown region, which are denoted by solid and dashed lines, respectively.
photons with \( b_1 \simeq b_{1ph} \) emitted outside the photon sphere in \( \mathcal{M}_1 \) (i.e., the purple region) and those with \( b_2 \simeq b_{2ph} \), the normalized frequency has high-frequency and low-frequency branches, corresponding to the star falling away from and towards the observer, respectively. If photons are emitted inside the photon sphere in \( \mathcal{M}_1 \) with \( b_1 \simeq b_{1ph} \), the high-frequency (low-frequency) branch denotes ingoing and outgoing (outgoing and ingoing) emissions from the star falling away from and towards the observer, respectively. For the high-frequency branches, strong gravitational lensing around the photon spheres can cause blueshifts of near-critical photons emitted inward at a large \( r_e \) in \( \mathcal{M}_1 \). In particular, the normalized frequency with \( b_1 \simeq b_{1ph} \) \( (b_2 \simeq b_{2ph}) \) reaches the maximum \( \omega_o/\omega_e = 4/3 \) \( (\omega_o/\omega_e = 1.392) \) at \( r_e = 12 \) \( (r_e = 8.679) \), becomes one at \( r_e = 5.196 \) \( (r_e = 3.6) \), and reaches the minimum \( \omega_o/\omega_e = 0.306 \) \( (\omega_o/\omega_e = 0.139) \) at the throat. In \( \mathcal{M}_2 \), the normalized frequency with \( b_2 \simeq b_{2ph} \) reaches the maximum \( \omega_o/\omega_e = 1 \) at \( r_e = 3.6 \), where the star returns.

The normalized frequency distribution of photons received by observers distributed on the celestial sphere is presented in the left panel of Fig. 7. When \( t_o \lesssim 200 \), a wide range of frequencies is observed for photons emitted in the green region. After near-critical photons emitted in the purple, orange, blue and brown regions start arriving at the observers around \( t_o \simeq 150 \), they come to dominate the high-frequency part of the frequency distribution. This early-stage frequency distribution bears a resemblance to the Schwarzschild black hole case, in which a star falls from spatial infinity at rest [78]. Similar to the scenario I, the maximum frequency of photons emitted inward in the blue and brown regions is greater than that of photons emitted inward in the green, purple and orange regions.

After the star enters \( \mathcal{M}_2 \), the observed frequency of photons emitted in the yellow region starts to increase and reaches a maximum around \( t_o \simeq 220 \), which is associated with the star returning to the throat. Subsequently, photons emitted in the brown and purple regions are observed to have a wide range of frequencies after they circle around the photon sphere in \( \mathcal{M}_1 \) and reach the observers. At late times, the star comes back to \( \mathcal{M}_1 \) and moves towards the observer, and the observers nearly in the radial direction, produce frequency and luminosity peaks around \( t_o \simeq 220 \). Later, near-critical photons with a wide range of frequencies are observed. At late times, the emitted position \( r_e \) is in \( \mathcal{M}_1 \) and large, and therefore the observers would collect most of emitted photons, which leads to a nearly constant total luminosity.
thus the low-frequency distribution is dominated by photons emitted towards the throat with $b_1 \simeq b_1^{\text{ph}}$ and $b_2 \simeq b_2^{\text{ph}}$. On the other hand, photons emitted towards the observers with a small impact parameter produce the high-frequency observation.

The normalized total luminosity of the freely-falling star in the scenario II is displayed in the right panel of Fig. 7. Before $t_0 \simeq 200$, the total luminosity behaves similarly to the Schwarzschild black hole case studied in [78], which is in consistency with the frequency observation. Afterwards, the received blueshifted photons with a small impact parameter dominate the total luminosity, resulting in a peak at $t_0 \simeq 220$. At late times, the total luminosity is maintained around one since most emitted photons can be collected by the observers.

In the left panel of Fig. 8, we exhibit the normalized frequency of photons received by an observer located at $\varphi = 0$ and $\theta = \pi/2$ on the celestial sphere in $M_1$ for the scenario II. The observed frequency of radially emitted photons with $n = 0$ is represented by the yellow line, which displays three periods. In the first and last periods, the photons are emitted when the star moves towards and away from the throat in $M_1$, respectively, and the $n = 0$ frequency line both decreases with the received time; in the intermediate period, the star emits the photons in $M_2$, and the $n = 0$ frequency line increases. There appears a peak and a dip of the $n = 0$ frequency line, which correspond to the star going through the throat the first time and the second time, respectively. Similar to the scenario I, the $n = 1$ frequency lines consist of three lines with $b_1 \gtrsim b_1^{\text{ph}}$, $b_1 \lesssim b_1^{\text{ph}}$ and $b_2 \simeq b_2^{\text{ph}}$, respectively. The $n = 1$ frequency line with $b_2 \simeq b_2^{\text{ph}}$ increases slowly until the maximum and then decreases rapidly in the first period, rises to a peak followed by a steep decline in the intermediate period, and gradually increases in the last period. For the $n = 1$ frequency lines with $b_1 \gtrsim b_1^{\text{ph}}$ and $b_1 \lesssim b_1^{\text{ph}}$, there is a sharp drop after reaching a peak when the star moves away from the observer, and a steady increase when the star moves towards the observer. For $n = 2$, only two frequency lines are visible, namely the $b_1 \lesssim b_1^{\text{ph}}$ (orange dots) and $b_2 \simeq b_2^{\text{ph}}$ (brown dots) lines. Note that the $n = 2$ frequency lines are quite similar to the $n = 1$ counterparts. In addition, only the frequency line with $b_1 \lesssim b_1^{\text{ph}}$ is visible for $n = 3$.

The normalized luminosity of the star in the scenario II measured by the observer is displayed in the right panel of Fig. 8. Similar to the scenario I, the luminosity decreases slowly before $t_0 \simeq 170$, which is dominated by radially emitted photons in the yellow region. Afterwards, photons emitted in the blue region come to control the luminosity observation and lead to a flash around $t_0 \simeq 180$. Subsequently, photons emitted in the yellow region determine the luminosity observation again and produce a peak around $t_0 \simeq 220$. At late times, the star travels towards the observer at a large $r_\infty$ in $M_1$, and hence radially emitted photons would make a dominant contribution to the total luminosity. In particular, the late-time luminosity remains fairly constant, which is greatly different from the black hole case.
4 Conclusions

In this paper, we investigated observational appearances of a point-like freely-falling star, which emits photons isotropically in its rest frame, in an asymmetric thin-shell wormhole connecting two spacetimes, $\mathcal{M}_1$ and $\mathcal{M}_2$. Specifically, two scenarios with different initial velocities of the star were considered. In the scenario I, the star starts with a nonzero velocity at spatial infinity of $\mathcal{M}_1$ and moves towards spatial infinity of $\mathcal{M}_2$. In the scenario II, the star falls at rest from spatial infinity of $\mathcal{M}_1$, reaches a turning point in $\mathcal{M}_2$ and returns to $\mathcal{M}_1$. For the two scenarios, the frequency distribution and luminosity of the star measured by all observers and a specific observer on a celestial sphere were obtained by numerically tracing emitted light rays. Interestingly, it was found that the absence of the event horizon and the presence of two photon spheres play a pivotal role in frequency and luminosity observations.

In [78,79], observational appearances of a star freely falling in black holes with one or two photon spheres were investigated. To compare the wormhole case with the black hole one, we briefly summarize the main findings of [78,79] and this paper as follows.

- **Black holes with a single photon sphere**: The total luminosity of the star fades out with an exponentially decaying tail, which is determined by quasinormal modes at the photon sphere. At late times, the specific observer sees a series of flashes indexed by the orbit number, whose luminosity decreases exponentially with the orbit number. Moreover, the frequency content of received photons contains a discrete spectrum of frequency lines indexed by the orbit number, which decay sharply at late times.

- **Black holes with double photon spheres**: At late times, the total luminosity first rises to a peak and then decreases with an exponentially decaying tail. The sub-long-lived quasinormal modes at the outer photon sphere are responsible for the slowly decaying exponential tail, and the leakage of photons trapped between the inner and outer photon spheres results in the luminosity peak. The specific observer sees two series of flashes, which are mainly determined by photons orbiting outside the outer and inner photon spheres, respectively. Moreover, the specific observer detects a discrete spectrum of frequency lines indexed by the orbit number and the photon sphere that received photons orbit around, which fall steeply at late times.

- **Wormhole**: At late times, the total luminosity first rises to a peak and then gradually decays with time (scenario I) or remains roughly constant (scenario II). The luminosity peak is caused by photons travelling between the two photon spheres (scenario I) or those emitted in $\mathcal{M}_2$ nearly along the radial direction (scenario II). Due to the absence of the event horizon, a considerable number of photons can still reach observers at late times, and hence an exponentially decaying tail would not appear. Similarly, the late-time luminosity measured by the specific observer can be sizable, and therefore he only sees a bright flash and a faint one (scenario I) or two bright flashes (scenario II) due to strong background luminance. Moreover, the specific observer detects frequency lines indexed by the orbit number and the photon sphere that received photons orbit around. The frequency lines produced by photons orbiting around the photon sphere in $\mathcal{M}_1$ decline sharply (scenario I) or grow steadily (scenario II) at late limes; those produced by photons orbiting around the photon sphere in $\mathcal{M}_2$ gradually increase at late limes.

In short, we showed that the absence of the event horizon in wormholes gives rise to significantly different optical appearances of a luminous star accreted onto wormholes at late times. Therefore, these findings can provide us a novel tool to distinguish wormholes from black holes in future observations.

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