Destruction of Long-range Order by Quenching the Hopping Range in One Dimension

Masaki Tezuka, 1 Antonio M. García-García, 2, 3 and Miguel A. Cazalilla 4

1 Department of Physics, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606-8502, Japan
2 Cavendish Laboratory, University of Cambridge, JJ Thomson Avenue, Cambridge, CB3 0HE, UK
3 CFIF, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
4 Department of Physics, National Tsing Hua University, and National Center for Theoretical Sciences (NCTS), Hsinchu City, Taiwan

(Dated: May 14, 2014)

We study the dynamics in a one dimensional hard-core Bose gas with power-law hopping after an abrupt reduction of the hopping range the time-dependent density-matrix renormalization group (t-DMRG) and bosonization techniques. In particular, we focus on the destruction of the Bose-Einstein condensate (BEC), which is present in the initial state. We argue that this type of quench is akin to a sudden reduction in the effective dimensionality of the system (from \( d > 1 \) to \( d = 1 \)). We identify two regimes in the evolution of the BEC fraction. For short times the decay of the BEC fraction is Gaussian while for intermediate to long times, it is well described by a stretched exponential with an exponent that depends on the initial effective dimensionality of the system. These results are potentially relevant for cold trapped-ion experiments which can simulate hard-core bosons (i.e. spins) with tunable long-range interactions.

PACS numbers: 74.78.Na, 74.40.-n, 75.10.Pq

Many physical phenomena occur under non-equilibrium conditions. Small deviations from equilibrium are well understood in the framework of linear response theory. However, the time evolution of far from equilibrium states, ubiquitous in nature and beyond linear response theory, has resisted so far a comprehensive understanding despite its potential to open new avenues of research in several fields including condensed matter, cold atom and cosmology.

A number of recent advances is changing this situation rapidly. In condensed matter, experiments 1–4 that combine femtosecond extreme ultraviolet pulses with very sensitive time and angle resolved photo-emission spectroscopy are able to probe the far from equilibrium dynamics of many-electron systems. Relevant examples include the time evolution of the order parameter 2 characterizing superconductivity or charge density wave (CDW) order 3, 4 or the discovery of transient superconductivity in cuprates 5.

In the context of ultracold atom physics, it has been observed experimentally 6 the momentum distribution of a quasi-one-dimensional (1D) Bose gas that is initially prepared in a far-from-equilibrium state does not exhibit thermalization. More recently, the loss of phase coherence for sufficiently long times, after a sudden decoupling of two phase coherent 1D Bose condensates was investigated experimentally in 6 and theoretically in 7, 8. On the theory side, it has been shown that integrable models in general fail to thermalize to the standard Gibbs ensemble but do it instead to a generalized Gibbs ensemble (GGE) 9, 10. This was first conjectured by Rigol and coworkers in Ref. 9 based on numerical simulations and subsequently analytically shown for a quantum quench in the Luttinger model by one of us 11. Furthermore, quantum quenches from a non-critical to a critical state were first studied by Calabrese and Cardy 12 using a clever mapping to a boundary conformal field theory. They found that the order parameter would decay exponentially after the quench.

Indeed, a quench is a convenient procedure to study non-equilibrium dynamics by making a sudden change of either a coupling constant in the Hamiltonian or an external parameter such as temperature or magnetic field. In some cases, the sudden removal of an external field or a change in a coupling constant can lead to destruction of long-range order in the initial state. In this article, we report an example of this, in which the parameter being quenched is the range of the hopping amplitude (from long to short range) in a one-dimensional (1D) interacting Bose system. As we argue below, this type of quench has some similarities with a sudden change in the effective dimensionality of the system from \( d > 1 \) to \( d = 1 \). We show using both numerical (i.e. DMRG) and analytical (i.e. bosonization) techniques that this type of quench leads to an interesting non-equilibrium regime in the time evolution of a Bose gas at zero temperature.

In order to mimic a higher spatial (i.e. \( d > 1 \)) dimensionality while retaining both the numerical and analytical convenience of one dimension, we assume that the system is a 1D Bose gas with long-range power-law hopping in the initial state. By now there is solid evidence from studies in different physical contexts 13–16 that long-range hopping is an effective way of mimicking higher dimensional effects in one dimensional systems. It is therefore reasonable to expect that power-law hopping in our model will also resemble an effective, not necessarily integer, dimensionality \( d > 1 \), which yields a true BEC 17 in the ground state for sufficiently slow (large) decay (dimensionality). As we explain below, such dimensionality quench can be realized in an optical lattice of coupled 1D gas tubes 18. An additional motivation for this type of quenches stems from the recent realization of spin-chains in trapped ion systems with highly-controllable long-range interactions 19. In this regard, we recall that in spins and bosons are related by an exact mapping 20.

In our calculations, to be described below, we have observed two different regimes in the evolution of the condensate fraction. For short times, the depletion of BEC is consistent with a Gaussian decay, whereas at intermediate to long times,
it appears to cross over to a stretched exponential behavior, both behaviors being predictions of a bosonization \[20\] approach. Consistent with this description, which treats phonons (i.e. the low energy excitations of the post-quench Hamiltonian) as propagating ballistically, we believe this regime corresponds to a prethermalized regime.

The rest of the article is organized as follows: In Sect. I we describe the model of that we have studied. Details about the quench protocol and the way the BEC fraction is measured using DMRG are given in Sect. II. The details of the analytical approach are presented in Sect. III. The conclusions of this work can be found in Sect. IV.

\section{Model and Quench}

The initial state before the quench is described by the following Hamiltonian of hard-core bosons hopping on a 1D lattice:

\[ \hat{H}_i = -\sum_{m, r \geq 1} t_r \hat{b}^\dagger_{m+1} \hat{b}_m + V \sum_m \hat{b}^\dagger_m \hat{b}_{m+1}, \tag{1} \]

in which \( V \) is the strength of the nearest-neighbor density-density interaction, \( \hat{b}_m \equiv \hat{b}^\dagger_m \hat{b}_m \) and

\[ t_r = t_1 \delta_{r, 1} + \frac{f}{\mu}(1 - \delta_{r, 1}) \tag{2} \]

is the hopping amplitude. In Ref. [17], some of us studied a 1D fermionic model with power-law hopping characterized by an exponent \( \alpha = \kappa/2 \) whose low energy behavior is equivalent to the Hamiltonian Eq. (1). In this work, long-range phase coherence was found for \( \kappa = 2/\alpha \leq 3 \).

We note that long-range interactions are of interest in a broad variety of problems: spin-chains with long-range interactions [19], non-interacting weakly disordered systems [13], quantum chaos [14], systems controlled by dipolar interactions [13] and certain types of Mott metal–insulator transitions [16]. In particular, it is also believed that long range interactions provide an effective way of accounting for higher dimensional effects in certain one-dimensional systems. This is because long-range interactions provide a way around the Mermin-Wagner theorem, which forbids the existence of spontaneous breaking of continuum symmetries long range in 1D quantum systems. Thus, as mentioned above, the model (1) exhibits long range phase coherence [17] in its ground state.

Below, we consider a quantum quench at \( t = 0 \), in which the long range tail (i.e. the term proportional to \( f \) in Eq. (2)) is suddenly turned off. In other words, we shall assume that the system is prepared in the ground state of Eq. (1) with a \( \kappa \leq 2 \) such that a bona-fide Bose-Einstein condensate exists. Following the quench, the time evolution of the Hamiltonian is described by the hard-core Bose-Hubbard model,

\[ \hat{H}(t > 0) = \hat{H}_0 = -t_1 \sum_m \left[ \hat{b}^\dagger_m \hat{b}_{m+1} + \hat{b}^\dagger_{m+1} \hat{b}_m \right] + V \sum_m \hat{b}^\dagger_m \hat{b}_{m+1}. \tag{3} \]

Thus, we expect that, since the ground state of \( \hat{H}_0 \) lacks long-range phase coherence (i.e. is not a BEC), but the latter is present in the initial state, the BEC will be destroyed by the time-evolution under \( \hat{H}_0 \). The question that we want address here is how this destruction takes place.

However, before describing the results of the DMRG simulation, it is worth dwelling on the role played by the parameter \( \kappa \) as an effective system dimensionality. To this end, we can draw an analogy with a quantum quench in an array of 1D bosons (labeled by a new index \( i \)). Let us assume that each 1D system is described by a copy of \( \hat{H}_0 \) and is initially coupled to \( z \) nearest neighbors by means of the following Josephson tunneling term:

\[ \hat{H}_1 = -t_\perp \sum_{(i,j)} \sum_m [\hat{b}^\dagger_m(i) \hat{b}_m(j) + \text{H.c.}], \tag{4} \]

where \( \sum_{(i,j)} \) indicates sum over nearest neighbors and \( t_\perp \ll t_1 \). Let \( z \) approach infinity while \( z t_\perp \) remains constant. In this limit, which effectively amounts to infinite dimensions, the mean-field approximation becomes exact and (up to a constant), the Josephson coupling term becomes \( H^{\text{MF}}_1 = \sum_i \hat{H}_1(i) \), in which \( \hat{H}_1(i) = -2t_\perp b_0 \sum_m [\hat{b}^\dagger_m(i) \hat{b}_m(i)] \) with \( b_0 = \langle \hat{b}_m \rangle \). Using bosonization [20], it is possible to map the mean-field Hamiltonian in the continuum limit to a sine-Gordon model [18,20]. In this setup, a quench where the Josephson coupling \( t_\perp \) is suddenly turned off corresponds to suddenly turning off the sine-Gordon term in the continuum model, a problem that has attracted much interest in recent times [13,21,22]. Since the quench in the sine-Gordon model is from a gapped to a critical state, we can rely on the boundary conformal field theory results of Calabrese and Cardy [12], which predict that the order parameter will decay as:

\[ \langle \hat{b}_m(t) \rangle \sim e^{-t/\tau} \tag{5} \]

at asymptotically long times after the quench. In the above expression \( \tau \) is related to the size of the gap in the initial non-critical state (which may be related to the healing length of the initial condensate [20]). This behavior bears some resemblance to the result derived below using bosonization for the model with long-range hopping in Eq. (1), which is a stretched exponential for a generic value of \( \kappa \) (cf. Eq. (14)). Indeed, tentatively, we may assume that the limit \( z \rightarrow +\infty \) corresponding to infinite dimensions is akin to setting \( \kappa = 1 \). Thus, in general, we can regard the role of the parameter \( \kappa \) as controlling the effective dimensionality of the system. This allows for a study of how long-range phase coherence is suppressed by fluctuations after an abrupt change in the dimension of the system.

However, the theory of Ref. [12] presumes that (boundary) conformal field theory is an exact description of the critical state. This ignores the existence of an infinite set of irrelevant operators that are responsible for the breaking of the infinite set of conservation laws that characterize the conformal field theory. Therefore, Eq. (5) is expected to hold at intermediate times, only. For longer times, the system will eventually transition to a different state. However, since \( \hat{H}_0 \) is an integrable model [20], the nature of such a state is not clear at the moment.
of the DMRG is reliable only for times for which the spatial average of the single particle correlation function is a good approximation, phase coherence holds. In Fig. 1 we plot deviations, energy is conserved.

We only display time intervals for which, up to constant of time. We have managed to get converged results retaining a relatively small number, $m \leq 500$, of density matrix eigenstates for up to $L = 80$ lattice sites at half-filling. This is presumably a consequence of the existence of long-range phase coherence in the ground state of the Eq. (1).

We choose $t_1 = 1$ as the unit of energy (and $1/t_1$ as the unit of time in $\hbar = 1$ units) and set $t_r = t r^{-\kappa}$ for $r \geq 2$ in (1). The system size is fixed to $L = 80$ with $L/2 = 40$ the number of bosons in the system. After the quench, converged results up to time $t \sim 10/t_1$ have been obtained for a time step $\Delta t \equiv 0.025$, $m = 500$, $f = 0.1$ and $V = -1.9$. The number of condensed bosons $\chi(t)$ is obtained as the largest eigenvalue of the $L \times L$ matrix representation of the two-point correlation function, $C_s(t) = \langle \Psi(t)|\hat{b}_s^\dagger \hat{b}_s|\Psi(t)\rangle$. In general terms, t-DMRG is reliable only for times for which $\chi(t)$ is significantly above unity. Another useful indicator of the accuracy of the DMRG simulation is the total energy which should be a constant of time. We only display time intervals for which, up to 3% deviations, energy is conserved.

First of all we prepare the initial state and check that, to a good approximation, phase coherence holds. In Fig. 1 we plot the spatial average of the single particle correlation function $C_{s,t}\equiv \langle 0|\langle \hat{b}_s^\dagger \hat{b}_s \rangle t=0\rangle_j$ against the distance $r$ for different values of $\kappa$ obtained by t-DMRG. As expected, long-range phase coherence, which manifest itself in a small variation of the single-particle correlation function, is present for sufficiently small $\kappa$. As described above, the initial state is therefore akin to a Bose gas with short-range hopping and an effective higher dimensionality.

With these technical limitations in mind, we compute $\chi(t)$, which is related to the Bose-Einstein condensate fraction $q(t) = \chi(t)$. In Fig. 2 we plot the time evolution of the condensate fraction $q(t)$ for $V = -1.9$ and $0.6 \leq \kappa \leq 1.8$. The decay is faster as $\kappa$ increases. Qualitatively this is understood as follows: A larger $\kappa$ corresponds to a lower effective initial dimensionality and therefore to a smaller quench since the final state is 1D. The saturation observed for small $\kappa$ and large times is a finite size artifact as the condensate fraction tends to a finite value in a finite size system. Indeed, even for a system in which each boson is in a separate orbital, which resembles a system of non-interacting spinless fermions, $\chi = 1$ so that $q(t) = \chi/N = 2/L$ holds.

### II. NUMERICAL RESULTS

We simulate the quench dynamics using the time-dependent DMRG (t-DMRG) [23]. Although the pre-quench Hamiltonian contains terms involving power-law hopping between distant lattice sites, which make DMRG calculations less efficient, the post-quench Hamiltonian contains only hoppings to nearest-neighbors. Therefore the efficient time evolution algorithm based on the Suzuki-Trotter breakup [23] can be employed. We have managed to get converged results retaining a relatively small number, $m \leq 500$, of density matrix eigenstates for up to $L = 80$ lattice sites at half-filling. This is a consequence of the existence of long-range phase coherence in the ground state of the Eq. (1).

In this section, we study the bosonized form of Eq. (1). To this end, we write $\mathbb{H}_0 = \sqrt{\rho} + \Delta\phi \hat{b}_m e^{i\phi(x_m)} + \cdots$, in which $x_m = m \theta(x)$ (the phase field) and $\partial_x \phi(x)$ (the density field) are canonically conjugate to each other [20]; the mean lattice occupation is $\rho = 2 \kappa (= \tfrac{3}{2})$ in the numerical simulation of Sect. II. In this section, we consider a translationally invariant system with $L \rightarrow +\infty$ and $\rho$ constant. In terms of $\phi(x)$ and $\theta(x)$, the low-energy effective Hamiltonian reads:

$$\hat{H}_t^\text{eff} = \frac{v}{2\pi} \int dx \left[ K(\partial_x \phi(x))^2 + K^{-1}(\partial_x \theta(x))^2 \right]$$

$$- \rho \int_{|x-x'|>d} dx' t(x-x') \cos[\theta(x)-\theta(x')] ,$$

where $v$ is the sound velocity, $K$ the Luttinger parameter, and $t(x-x') = f/|x-x'|^r$ (a $\approx 1$ is a short distance cut-off). The last term is a non-linear function of the phase field, $\theta(x)$, which makes the task of computing the ground state by elementary methods impossible. However, a reasonably good approximation can be obtained using the self-consistent harmonic approximation (SCHA) [17], which amounts to replac-
obtained from Eqs. (14) and (15). Note that the time scale for all is not the same. The flattening of the condensate fraction for longer with an exponent that depends on κ and intermediate times (blue) where the decay is stretched exponentially.

\[ V = f(t) \]

Before the quench and, from top to bottom, κ = 0.6, 0.9, 1.2, 1.5. After the quench still \( V = -1.9 \) but the hopping is restricted to only nearest neighbours. Comparison between td-DMRG results (circles) and the analytical results in the limit of short times (red), where the decay is Gaussian, and intermediate times (blue) where the decay is stretched exponential with an exponent that depends on κ. The analytical prediction is obtained from Eqs. (14) and (15). Note that the time scale for all κ’s is not the same. The flattening of the condensate fraction for longer times is likely a finite size effects related to the fact that \( q(t) > 2/L \).

\[ \frac{1}{2} \int_{|x-x'|=a} dx' T(x-x') [\theta(x) - \theta(x')]^2, \]

where the function \( T(x-x') \sim 1/|x-x'|^a \) must be determined variationally [17]. In order to diagonalize the quadratic Hamiltonian resulting from the CHA we expand the phase field as \( \theta(x) = \theta_0 + \frac{\theta}{L} + \Theta(x) + \Theta^*(x) \) with J the winding number, \( \Theta(x) = \frac{1}{2} \sum_{q \neq 0} (\frac{q}{|q|L})^{1/2} \text{sgn}(q) e^{i q \hat{\theta}_q} \hat{b}_q \) and \( [\hat{b}_q, \hat{b}_q^\dagger] = \delta_{q,q'} \).

The resulting Hamiltonian can be diagonalized by a new set of boson eigenmodes described by \( (\hat{a}_q, \hat{b}_q^\dagger) \), and for \( L \to +\infty \), it reads:

\[ \hat{H}_1^{\text{eff}} \approx E_0 + \sum_{q \neq 0} \omega(q) \hat{a}_q \hat{a}_q^\dagger \]

where the constant \( E_0 \) is the ground state energy and \( \omega(q) = \sqrt{\kappa q^2 + \eta \rho T(q)/K} \sim T^{1/2}(q) \sim |q|^{1/2} \) since \( T(q) \sim |q|^{\kappa - 1} \) for \( \kappa < 3 \).

In the following, we focus on the time-evolution at times \( t > 0 \) of the phase field \( \theta(x) \) under the Hamiltonian \( \hat{H}_1^{\text{eff}} = \sum_{q \neq 0} v(q) \hat{b}_q \hat{b}_q^\dagger \) corresponding to a 1D Bose gas with short-range hopping [20]:

\[ \theta(x, t) = e^{i\hat{H}_1^{\text{eff}t}} \theta(x) e^{-i\hat{H}_1^{\text{eff}t}} \]

\[ = \frac{1}{2} \sum_{q \neq 0} \left( \frac{2\pi}{K|q|L} \right)^{1/2} \text{sgn}(q) e^{i q\theta} \left[ e^{-i\rho(q)\hat{\theta}_q} - e^{i\rho(q)\hat{\theta}_q} \right]. \]

We emphasize that, thanks to the CHA, \( (\hat{a}_q, \hat{b}_q^\dagger) \) and \( (\hat{b}_q, \hat{b}_q^\dagger) \) are the eigenmodes of two quadratic Hamiltonians and therefore they can be related by a canonical transformation [10]:

\[ \hat{b}_q = u_q \hat{a}_q + v_q \hat{a}_q^\dagger, \quad \hat{b}_q^\dagger = v_q \hat{a}_q + u_q \hat{a}_q^\dagger \]

in which \( u_q = \cosh \theta_q \) and \( v_q = \sinh \theta_q \), with \( \theta_q \) satisfying

\[ \tanh 2\theta_q = \frac{2\mu v_q |q|}{u_q^2 + v_q^2} = \frac{\eta \rho T(q)/(|q|L^2)}{v_q^2 + \eta \rho T(q)/K^2}. \]

In order to compute the evolution of the BEC order parameter, \( \langle \hat{b}_m(t) \rangle \approx \rho \langle e^{i\theta_0 t} \rangle = \rho e^{-\frac{1}{2} \langle \theta_0^2(t) \rangle} \) we need to obtain \( \langle \theta^2(0, t) \rangle \), where \( \langle \ldots \rangle \) stands for the average over the ground state of the CHA to the initial Hamiltonian, Eq. (1). Asymptotically, at times \( t \), we find:

\[ \langle \theta^2(0, t) \rangle = \langle \theta^2(0, 0) \rangle \int_{1/t}^{L} T^{1/2}(q) \frac{q^2}{q^2} \sim \int_{1/(vt)}^{L} |q|^{1/2} \sim \frac{(vt)^{1/2}}{K^{1/2}}. \]

Hence, at long times the condensate fraction should decay according to the law:

\[ \langle \hat{b}_m(t) \rangle = A_0 e^{-vn \tanh 2\theta_q}, \]

FIG. 3. (Color online) The condensate fraction \( q(t) \) after a quench of the Hamiltonian Eq. (1). Before the quench \( f = 0.1, V = -1.9 \) and, from top to bottom, \( \kappa = 0.6, 0.9, 1.2, 1.5 \). After the quench still \( V = -1.9 \) but the hopping is restricted to only nearest neighbours. Comparison between td-DMRG results (circles) and the analytical results in the limit of short times (red), where the decay is Gaussian, and intermediate times (blue) where the decay is stretched exponential with an exponent that depends on \( \kappa \). The analytical prediction is obtained from Eqs. (14) and (15). Note that the time scale for all \( \kappa \)'s is not the same. The flattening of the condensate fraction for longer times is likely a finite size effects related to the fact that \( q(t) > 2/L \).
where $\gamma$ and $A_0$ are positive constants that depend on the microscopic details of the initial and final Hamiltonians.

In the opposite limit of short times after the quench, the decay is a gaussian:

$$
\langle e^{i\delta(t)} \rangle \sim q_0 e^{-\beta t^2} \approx q_0 (1 - \beta t^2),
$$

(15)

where the dependence on $\kappa$ appears only through $A_1 > 0$ and $\beta > 0$. The same time dependence in both regimes also holds for the condensate fraction $q(t) = \frac{A_1}{A_0}$.

In Fig. 3, we compare td-DMRG results and the analytical predictions above. We recall that for short times we expect $q(t) = q_0 e^{-\beta t^2}$ with $q_0$ the initial condensate fraction. The parameter $\beta$ cannot be computed by bosonization techniques so it is a fitting parameter in the comparison with the numerical findings.

For intermediate times the theoretical prediction is $q(t) \sim e^{-\gamma t^{\delta}}$, where again $\gamma \propto \frac{1}{\beta}$ is a fitting parameter. With these limitations in mind, the results depicted in Fig 3 show good agreement between analytical and numerical results for the full range of $\kappa$'s considered. The fitting time interval cannot be further extended because numerically the condensate fraction eventually saturates to a value $\sim \frac{2}{L}$ corresponding to the limit of only one boson in the condensed phase in a box of size $L$. This is the likely reason for the flattening of the condensate fraction for sufficiently long times.

Indeed, long-time deviations from the stretched exponential behavior obtained from the bosonization approach described in Sect. III are in any case expected. In fact, in an infinite system, the stretched exponential behavior corresponds to a prethermalized regime [22, 24, 25]. In such a regime, the phonons described by $\hat{b}_q, \hat{\bar{b}}_q$, which are the low-energy elementary excitations of the post-quench Hamiltonian, $H_f$, propagate ballistically. However, at longer times, the phonons will scatter each other, which should lead to a different decay of the BEC fraction [22]. The calculation of this behavior is beyond the scope of this work, as it requires taking into account an infinite number of irrelevant operators that are neglected in the bosonization approach. Furthermore, we have not been able to determine whether the observed deviations from the sub-exponential behavior are due to the approximate treatment of the initial state and to the neglect of the open boundaries and finite-size effects, which are inevitably present in the DMRG simulations.

For even longer times the destruction of the initial BEC will be driven, not only by the phonons, but also by topological excitations known as phase slips. The latter are instantons, i.e., topological defects in imaginary time. Qualitatively we expect the inverse Kibble-Zurek [26, 27] mechanism, similar to the phenomenon studied recently in Ref. [28] in the context of a classical quench in a two-dimensional Bose gas. According to the Kibble-Zurek [26, 27] scenario, after a quench, from a disordered phase, topological fluctuations are expected to survive for a long time in the final ordered phase. In the inverse Kibble-Zurek scenario the quench is from the ordered to disordered phase. The resulting dynamics is characterized [28] by a supercooled phase for short times and a slow proliferation of vortices for longer times that leads to an unexpected resilience of the superfluid state.

IV. CONCLUSION

Using a combination of time-dependent DMRG and bosonization, we have studied the destruction of the Bose-Einstein condensate (BEC) in a one-dimensional Bose gas following a quench in the range of the hopping amplitude. We have argued that this is akin to a dimensionality quench, as the exponent characterizing the range of the hopping in the pre-quench Hamiltonian plays the role of an effective (non-integer) dimensionality $d > 1$.

Our main result is the identification of two distinct regimes for the destruction of Bose-Einstein condensate (BEC) after the quench in effective dimensionality quench described above. At short times, we find a decrease of the BEC fraction of the form $q(t) = q_0 e^{-\beta t^2} \approx q_0 (1 - \beta t^2)$, where $q_0$ is the initial BEC fraction and $\beta > 0$ is a constant. At longer times, this behavior crosses over to a different kind of time dependence. From a bosonization approach (see Sect. III), we find a sub-exponential behavior $q(t) \sim e^{-\gamma t^{\delta}}$ with $\delta = \frac{1}{\beta}$, with $\gamma > 0$ and $\kappa$ the power-law hopping exponent which is directly related to the initial effective dimensionality of the condensate. This behavior appears to be consistent with the behavior of $q(t)$ obtained from the time-dependent DMRG calculations described in Sect. III. However, we find deviations from the sub-exponential behavior at the longer times accessible to DMRG likely due to finite-size effects. In any case, our theoretical approach is expected to break down for sufficiently long times due to the phonon scattering and the proliferations of phase slips.

The authors thank Ippei Danshita and Alejandro Lobos for fruitful discussions. MT appreciates the hospitality of the TCM Group, Cavendish Laboratory, which MT visited under the support from the Kyoto University Global Frontier Project for Young Professionals “John-Mung Program”, and the hospitality of NCTS (Taiwan) where parts of the work were done. AMG was supported by EPSRC, grant No. EP/I004637/1, FCT, grant PTDC/FIS/111348/2009 and a Marie Curie International Reintegration Grant PIRG07-GA-2010-268172. MAC is supported by NSC and a start-up grant from NTHU (Taiwan). Part of numerical computation in this work was carried out at the Supercomputer Center, ISSP, University of Tokyo and Yukawa Institute Computer Facility, Kyoto University.

[1] D. Fausti et al., Science 331, 189 (2011).

[2] C. L. Smallwood et al., Science 336, 1137 (2012).
[3] T. Rohwer et al., Nature 471, 490 (2011).
[4] W. S. Lee et al., Nature Communications 3, 838 (2012).
[5] T. Kinoshita, T. Wenger, and D. S. Weiss, Nature 440, 900 (2006).
[6] S. Hofferberth, I. Lesanovsky, B. Fischer, T. Schumm, and J. Schmiedmayer, Nature (London) 449, 324 (2007).
[7] R. Bistritzer and E. Altman, PNAS 104, 9955 (2007).
[8] A. A. Burkov, M. D. Lukin, and E. Demler, Phys. Rev. Lett. 98, 200404 (2007).
[9] M. Rigol, V. Dunjko, and M. Olshanii, Nature 452, 854 (2008); M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys. Rev. Lett. 98, 050405 (2007).
[10] M. A. Cazalilla, A. Iucci, and M.-C. Chung, Phys. Rev. E 85, 011133 (2012).
[11] M. A. Cazalilla, Phys. Rev. Lett. 97, 156403 (2006); A. Iucci and M. A. Cazalilla, Phys. Rev. A 80, 063619 (2009).
[12] P. Calabrese and J. Cardy, Phys. Rev. Lett. 96, 136801 (2006); J. Stat. Mech. - Theor. and Exp., P06008 (2007).
[13] A. D. Mirlin et al., Phys. Rev. E 54, 3221 (1996).
[14] W. Jiao and A. M. Garcia-Garcia, Phys. Rev. E 79, 036206 (2009); A. M. Garcia-Garcia and J. Wang, Phys. Rev. Lett. 94, 244102 (2005).
[15] L. S. Levitov, Phys. Rev. Lett. 64, 547 (1990).
[16] F. Gebhard and A. E. Ruckenstein, Phys. Rev. Lett. 68, 244 (1992).
[17] A. M. Lobos, M. Tezuka, and A. M. Garcia-Garcia, Phys. Rev. B 88, 134506 (2013).
[18] M. A. Cazalilla, A. F. Ho, and T. Giamarchi, New J. Phys. 8, 158 (2006); A. F. Ho, M. A. Cazalilla, and T. Giamarchi, Phys. Rev. Lett. 92, 130405 (2004).
[19] J. W. Britton, B. C. Sawyer, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Nature 484, 489 (2012); R. Islam, C. Senko, W. C. Campbell, S. Korrenblit, J. Smith, A. Lee, E. E. Edwards, C.-C. J. Wang, J. K. Freericks, and C. Monroe, Science 340, 583 (2013).
[20] M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and M. Rigol, Rev. Mod. Phys. 83, 1405 (2011).
[21] A. Iucci and M. A. Cazalilla, New J. of Phys. 12, 055019 (2010).
[22] A. Mitra, Phys. Rev. Lett. 109, 260601 (2012).
[23] S. R. White and A. E. Feiguin, Phys. Rev. Lett. 93, 076401 (2004).
[24] M. Moeckel and S. Kehrein, Phys. Rev. Lett. 100, 175702 (2008); Ann. Phys. (N. Y.) 324, 2146 (2009).
[25] N. Nessi, A. Iucci, and M. A. Cazalilla, arXiv:1401.1986.
[26] T. W. B. Kibble, J. Phys. A 9, 1387 (1976); Phys. Today 60, 47 (2007).
[27] W. H. Zurek, Nature 317, 505 (1985).
[28] L. Mathey and A. Polkovnikov, Phys. Rev. A 80, 041601(R) (2009); L. Mathey and A. Polkovnikov, Phys. Rev. A 81, 033605 (2010).