Internal Bremsstrahlung Signature of Real Scalar Dark Matter
and Consistency with Thermal Relic Density

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A gamma-ray excess from the galactic center consistent with line emission around 130 GeV was recently found in the Fermi-LAT data. Although the Fermi-LAT Collaboration has not confirmed its significance, such a signal would be a clear signature of Dark Matter annihilation. Until now, there have been many attempts to explain the excess by Dark Matter. However these efforts tend to give too-small cross sections into photons if consistency with the correct thermal relic density of Dark Matter is required. In this letter, we consider a simple Yukawa interaction that can be compatible with both aspects and show which parameters are favored.

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Observations of gamma-rays, cosmic ray positrons, anti-protons and neutrinos are performing to look for excess around 130 GeV. In particular, Fermi-LAT public data has recently been examined in detail, and a gamma-ray excess around 130 GeV from the region of the galactic center has been claimed [1, 2]. The Fermi-LAT Collaboration also found the excess at 135 GeV independently [3], however they found a much lower significance in the re-processed data-set [4]. Many authors have provided models of this excess by monochromatic gamma-rays from DM annihilation or decay, see for example [5–23]. If the source of the gamma-ray excess is DM annihilation, the required cross section into two photons is \( \sigma_{\gamma \gamma} \approx 1.27 \times 10^{-27} \text{ cm}^3/s \) for an Einasto DM density distribution; this value can change for a different DM profile [2]. The process of DM annihilation into two photons is loop-suppressed because DM does not have electric charge. The loop-suppression factor is naively expected to be \( \alpha_{\text{em}}^2 / (4 \pi)^2 \sim 10^{-7} \) compared with the annihilation cross section \( \sigma_{\text{ann}} \sim 10^{-26} \text{ cm}^3/s \) where \( \alpha_{\text{em}} \) is the electromagnetic fine structure constant. This value of \( \sigma_{\text{ann}} \) is needed to achieve the correct relic density of DM. Thus, it seems difficult to be consistent with the thermal relic density of DM unless some enhancement of the cross section is introduced [24]. In other words, if we assume DM is thermally produced, the gamma-ray production cross section is fixed to a value that is too small to explain the excess around 130 GeV.

Another possibility is the explanation via Internal Bremsstrahlung (IB) of Majorana DM [1]. The possibility of explaining the 130 GeV excess with IB has been explored in refs. [1, 25–28]. This process is the radiative correction for the final state charged particles and the intermediate particle. The IB process generates a line-like energy spectrum. The suppression factor compared with \( \sigma_{\text{ann}} \) is roughly \( \alpha_{\text{em}}^2 / \pi \sim 10^{-3} \) which is larger than the monochromatic photon case. Thus the IB process has better prospects than the monochromatic \( \gamma \gamma \) process from this point of view. However, even with the IB process, it seems difficult to be compatible with the thermal relic density of DM. For standard p-wave annihilating neutralino DM, the IB signal is still a factor of a few below the nominally required rate for the observed density.

In this letter, we consider IB for real scalar DM interacting with a fermionic mediator and a light fermion. As we discuss below, the annihilation cross section into a light fermion-anti-fermion pair is expanded with the relative velocity of DM, with a suppressed constant term. As a result, a higher order term of the cross section can be dominant in the early universe, and the cross section into gamma-rays becomes relatively large at present times, thus reconciling the relic density value and the interpretation of the gamma-ray excess by DM annihilation.

We consider a real scalar DM particle \( \chi \) which has the following Yukawa interaction with the electromagnetically charged fermion \( f \) and the fermionic mediator \( \psi \):

\[
\mathcal{L} = y_L \overline{\chi} \psi P_L f + \text{h.c.},
\]

where the fermion \( f \) is typically a light lepton or a quark. The annihilation cross section into \( f f \) is expanded as \( \sigma_{ff} = a + bv^2 + cv^4 + \mathcal{O}(v^6) \) with the DM relative velocity \( v \), and it is calculated under the approximation of \( m_f \ll m_\chi \) as

\[
\sigma_{ff} = \frac{y_L^2}{4\pi m_\chi^2} \frac{m_f^2}{m_\chi^2 (1+\mu)^2} \left( 1 + \frac{2\mu}{6\pi m_\chi^2 m_f^2 (1+\mu)^2} v^2 \right) + \frac{y_L^2}{60\pi m_\chi^2 (1+\mu)} v^4 + \mathcal{O}(v^6),
\]

where the Yukawa coupling \( y_L \) is assumed to be real, and the parameter \( \mu \) is the ratio of masses defined as \( \mu = \frac{m_\psi}{m_\chi} > 1 \). The first and second terms of Eq. (2), which are called the s-wave and p-wave respectively, agree with the appendix of ref. [29]. In addition, the d-wave term which is proportional to \( v^4 \) is easily found to be the leading term in the limit of \( m_f \to 0 \). The s-wave...
and p-wave are suppressed by the factor $m_f^2/m_\chi^2$, thus the d-wave becomes the dominant contribution to the cross section in the early universe when $\mu$ is not large enough. Conversely, the s-wave becomes dominant today even if the mass of particle $f$ is as low as the electron mass. The non-relativistic thermally averaged cross section $\langle \sigma v_f \rangle$ is given by substituting $\langle \psi^2 \rangle \to 6T/m_\chi$ and $\langle \psi^4 \rangle \to 60T^2/m_\chi^2$ where $T$ is the temperature of the universe. This replacement coincides with refs. [30, 31]. The thermally averaged cross section is important to estimate the relic density of DM. The typical value of the temperature that sets the correct relic density is roughly $m_\chi/T \approx 20$–25.

Next, we consider the radiative correction for the above 2-body process, that is $\chi\chi \to f\gamma$ shown in Fig. 1. This process is the IB of the real scalar DM, and the emitted photon can be a gamma-ray signal which is comparable with the Fermi-LAT excess. The amplitude for the total IB is separated to two pieces of Final State Radiation (FSR) and Virtual Internal Bremsstrahlung (VIB). One cannot generally treat the VIB diagram separately in a gauge invariant manner, and it is only the sum of all three diagrams in Fig. 1 that is gauge invariant. Here we define the FSR amplitude as the leading term of the differential cross section in Eq. (4), and the VIB one as

$$\frac{d\sigma^{\text{VIB}}}{dx} = \frac{Q^2\alpha_{\text{em}}^2y_f^2}{4\pi^2m_\chi^2} (1-x) \left[ \frac{2x}{(\mu+1)(\mu+1-2x)} - \frac{x}{(\mu+1-2x)^2} \right]$$

where $\text{Li}_2(z)$ is the dilogarithm function defined by $\text{Li}_2(z) = -\int_0^1 \log(1-zt)/tdt$. The above VIB cross section for real scalar DM is a factor of 8 times larger than that for Majorana DM. Note that the continuum gamma-ray spectrum due to hadronization should be added in Eq. (3) when the final state particles are tauons or light quarks.

The above discussion is valid when the other interactions are sufficiently suppressed. Here we add the interaction with the right-handed component of the fermion $f$, and we estimate how much hierarchy is necessary among the interactions in order for the above scheme to work.
If the interaction with the left and right-handed fermion is
\[ \mathcal{L} = \chi \bar{\psi} (y_L P_L + y_R P_R) f + h.c., \] (7)
the s-wave and p-wave components are not suppressed as in Eq. (2), and they are given by
\[ \sigma v f = \frac{y^2_L y^2_R}{\pi m^2 \chi} \frac{\mu}{(1 + \mu)^2} - \frac{y^2_L y^2_R}{3 \pi m^2 \chi} \frac{\mu + 3 \mu^2}{(1 + \mu)^2} v^2 + \mathcal{O} (v^4). \] (8)
This formula coincides with the appendix of ref. [29]. We can estimate the required condition among the parameters to validate the above discussion. The s-wave component should be suppressed enough compared with the d-wave, leading to the condition:
\[ \left( \frac{y_R}{y_L} \right)^2 \lesssim \frac{v^4}{60 \mu (1 + \mu)^2}. \] (9)
Therefore, \( y_R/y_L \lesssim 0.02 \) is required when \( \mu \sim 1 \) and \( v^2 \sim 0.3 \). For example even if \( y_R \) does not exist at tree level, \( y_R \) is induced at one-loop level from the left-handed Yukawa coupling \( y_L \). When \( \mu \sim 1 \), the right-handed Yukawa coupling \( y_R \) is then
\[ y_R \approx -\frac{y_L^3}{2 (4 \pi)^2} \frac{m_f}{m \chi}. \] (10)
This is sufficiently small compared with \( y_L \), and the required Eq. (9) is satisfied.

We comment about other DM models, such as complex scalar and fermionic DM. In the case of complex scalar DM, the s-wave component of the 2-body cross section is suppressed by the factor \( m^2_f/m^2 \chi \), just like real scalar DM, but the p-wave component remains. Thus this framework discussed above does not work. For Majorana DM, the non-suppressed p-wave term is also present, and we cannot reconcile the DM relic abundance and the explanation of the gamma-ray excess by DM annihilation. For Dirac DM case, the s-wave term exists for the 2-body process, and the FSR process is always larger than the VIB process. Thus we cannot obtain the line-like gamma-ray spectrum from VIB because the VIB signal is swamped by the broad FSR spectrum.

We numerically analyze the consistency between the thermal relic abundance of DM and the gamma-ray excess. In the following calculation, fermion \( f \) is taken to be the electron. The thermal relic density of DM is obtained by solving the Boltzmann equation [30, 31, 39]
\[ \frac{z}{Y_{eq}} \frac{dY}{dz} = -\frac{\Gamma}{H} \left( \frac{Y^2}{Y_{eq}^2} - 1 \right), \] (11)
where \( Y \) is defined as the DM number density \( n_\chi \) divided by the entropy density of the universe, and \( Y_{eq} \) is the value of \( Y \) in thermal equilibrium. The reaction rate \( \Gamma \) is defined as \( \Gamma \equiv \langle \sigma v \rangle n_\chi^2 \) with the number density in thermal equilibrium, \( H \) is the Hubble parameter and \( z \) is a dimensionless parameter defined by \( z = m \chi / T \). The total cross section implies \( \langle \sigma v \rangle = \langle \sigma v f \rangle + \langle \sigma v f \rangle \). In general, we need approximately \( \langle \sigma v \rangle \sim 10^{-26} \text{cm}^3/\text{s} \) in order to get the correct relic abundance of DM which is \( \Omega_\chi h^2 = 0.120 \) observed by Planck [40].

We have only three parameters: \( m_\chi, \mu \) and \( y_L \). We solve the Boltzmann equation numerically with an implicit method. The contours of Yukawa coupling \( y_L \) which satisfy the observed DM relic density are depicted in Fig. 2 in \( m_\chi - \mu \) plane. We can see from the figure that a larger Yukawa coupling is required for larger \( \mu \). Coannihilation between the DM \( \chi \) and the mediator \( \psi \) begins to be effective in the region of \( \mu \lesssim 1.2 \). For example, the process \( \chi \psi \rightarrow fH \) can occur with the interaction of Eq. (1) and the SM Yukawa couplings where \( H \) is the SM Higgs. However, it would be small for the light SM charged particles. If the other interactions lead to effective coannihilation, this should be taken into account, as it may affect the numerical analysis.

The gamma-ray flux coming from DM annihilation for the target region \( \Delta \Omega \) is given by
\[ \frac{d \Phi^\text{DM}}{dE_\gamma} = \frac{r_\odot^2}{8 \pi m^2 \chi} J(\sigma v_f) \frac{dN_s}{dE_\gamma}, \] (12)
where \( r_\odot = 8.5 \text{ kpc} \) is the distance of the earth from the galactic center, and \( \rho_\odot = 0.4 \text{ cm}^3/\text{s} \) is the local DM density [41]. The parameter \( J \) is defined as
\[ J \equiv \frac{1}{\Delta \Omega} \int d b d \ell \cos b \int_{r_\odot}^{\odot} \frac{d s}{r} \left( \frac{\rho (r, b, \ell)}{\rho_\odot} \right)^2, \] (13)
where \( b \) and \( \ell \) are the galactic latitude and longitude of the target region. The integral variable \( s \) is related with the distance from the galactic center \( r \) as \( r(s, b, \ell) = \sqrt{r_\odot^2 + s^2 - 2 r_\odot s \cos b \cos \ell} \). Note that the energy dependence arises only from the energy spectrum \( dN_s/dE_\gamma \), and \( \langle \sigma v_f \rangle dN_{\gamma}/dE_\gamma \) simply corresponds to Eq. (3) at present situation. We use the generalized NFW profile [42] which is written
\[ \rho (r) = \frac{\rho_\odot}{(r/r_s)^\alpha (1 + r/r_s)^{3-2 \alpha}}. \] (14)
It corresponds to the normal NFW profile if \( \alpha = 1 \). The parameter \( \rho_\odot \) is the normalization factor in order to fix to \( \rho (r_\odot) = 0.4 \text{ GeV/cm}^3 \). The parameters \( r_s \) and \( \alpha \) are taken as \( r_s = 20 \text{ kpc} \) and \( \alpha = 1.15 \). This DM profile, as well as the parameter values, are the same as ref. [43] used to fit the excess around 130 GeV in the Fermi-LAT data. We focus on the region of Reg4 in ref. [2, 43] to compare the gamma-ray flux from DM annihilation and the claimed gamma-ray excess. The background of the gamma-ray flux is evaluated by the fitting function [2] in the unit of \( \text{GeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \)
\[ \frac{d \Phi^B}{dE_\gamma} = 2.4 \times 10^{-5} E^{-2.55}. \] (15)
The Yukawa coupling $y_L$ is fixed by the constraint of thermal relic density of DM at each point. From the figure, we can see that the DM abundance and the gamma-ray excess coming from DM annihilation are consistent each other. The favored region in large $\mu$ would be slightly changed if the monochromatic photon induced by the box diagrams is taken into account in the model [44]. The fitting of the gamma-ray excess with the evaluated values is depicted in Fig. 3.

The Yukawa interaction considered here contributes to the anomalous magnetic moment of fermion $f$, thus it may constrain the strength of the interaction. The anomalous magnetic moment of $f$ is calculated from the Yukawa interaction Eq. (1) as [29]

$$\delta a_f = \frac{y_L^2}{(4\pi)^2} \frac{m_f^2}{m^2} \frac{2 + 3\mu - 6\mu^2 + \mu^3 + 6\mu \log \mu}{6(1 - \mu)^3}. \quad (16)$$

The current experimental bound [45, 46] for the electron anomalous magnetic moment is given as $\delta a_e = a_e(\text{SM}) - a_e(\exp) = 1.06 \times 10^{-12}$ [47], while the value at the fitting point is two orders of magnitude lower: $\delta a_e \approx 9.4 \times 10^{-15}$. For the muon, the experimental bound is $\delta a_\mu = 25.5 \times 10^{-10}$ [48, 49] and our value is $4.0 \times 10^{-10}$, one order of magnitude below the bound. However, for example if we have simultaneous Yukawa couplings to both the electron and muon, the Yukawa coupling is extremely constrained by charged lepton flavor violating processes like $\mu \to e\gamma$ unless destructive interference occurs.

We have discussed scalar DM having Yukawa interaction with the left-handed light fermion $f$ and the mediator $\psi$. The annihilation cross section of the DM into $f\bar{f}$ is highly suppressed since the s-wave and p-wave are proportional to the ratio of masses $m_f^2/m^2$. As a result, the d-wave can be dominant in the early universe, and the DM relic abundance is obtained by the d-wave cross section. Simultaneously, the VIB component of the radiative correction for the process that is $\chi \chi \to f\bar{f}\gamma$ has s-wave and it gives the line-like gamma-ray signal. The recently-observed gamma-ray excess is well explained without inconsistency with the thermal relic density of DM. We have three parameters of $m_\chi$, $\mu$ and $y_L$, and obtained the best fit point $m_\chi = 155$ GeV, $\mu = 2.05$ and $y_L = 1.82$ by fitting to the gamma-ray excess with the constraint of the DM thermal relic density.

The framework discussed here works when the other interactions of DM are small enough. The lightest right-handed sneutrino DM in supersymmetric extended models would be a realistic candidate since the chargino plays a role in the mediator $\psi$. To do that, the neutrino Yukawa interaction should be large. Therefore supersymmetric radiative seesaw models such as refs. [50, 51] would be promising concrete models to implement this scheme because the neutrino Yukawa coupling can be order one and tiny neutrino masses are derived without contradiction. In addition, inverse seesaw models with supersymmetry would also be good candidates for the framework [52].
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Note added: An article finding similar results was almost simultaneously published on the arXiv [44].

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