Recovery of the hydrounit performance characteristics via a few observations

A Kovartcev\textsuperscript{1}, A Nazarova\textsuperscript{1}, V Zakharchenko\textsuperscript{2}

\textsuperscript{1}Samara National Research University, Molodogvardeiskaya street 151, Samara, Russia, 443001
\textsuperscript{2}Sensors, Modules, Systems, Ltd., Galaktionovskaya street 7, Samara, Russia, 443020

Abstract. The performance characteristic of hydrounits is used very often in various control systems on hydro power plant for velocity control, power distribution, for displaying at different levels of automation pyramid for technicians, technologists, management and so on. As building of new HPP is very expensive, last years main investments in this sphere were made to reconstruction, modernization and increasing of efficiency of the existing HPPs. Before new or reconstructed hydrounit is put into industrial operation, it should pass a set of tests to check its operability at set parameters of safety operating conditions. In this case variety of hydrounit tests is significantly limited. Resulted from this fact there is a problem of the hydrounits performance characteristics recovery via a few observations. The present paper describes the hydrounit characteristics recovery method based on usage of limited number of experimental data. Results of examination of properties of the method offered are also described.

1. Introduction

According to modern trends of hydraulic power industry development it is supposed that main efforts should not be aimed at new HPPs building but at modernization and reconstruction of existing HPP equipment and at development of modern automation system to control these equipment more precisely and more efficiently [1].

The most important characteristics used in HPP automation system are the turbine performance ones. Usually the turbine performance characteristic is a dependence of efficiency coefficient as a function of the turbine power (active electrical power) and water head. The characteristics are used in different HPP systems: in the turbine speed governor, in active and reactive power joint control systems, in the system for rational control of hydrounits state [3–5] – to calculate power limits, in the hydrounits automatic control system to measure the hydrounit operation time in different modes and in restricted and forbidden operation areas, and for visualization at operator local panels, dispatcher workstations and at sensor panels, dispatcher video walls and visualization boards [2].

Over time the turbine is getting older and its performance characteristic changes its appearance. In the first, the turbine efficiency coefficient decreases averagely from one to three percent, which affect effectiveness of HPP operation control because false information is used. In this case before the complete overhaul, hydrounit tests are often executed in a limited scope. Number of the data received is definitely not enough to develop performance characteristic as provided by the industry standards.
So there is improperly posed problem of the turbine changed characteristic recovery using a few number of observations.

This task of dependencies recovery using a few number of observations has significant practical value but has not been examined in details yet. These tasks include: the one of the model parametric identification using a few number of observations [6], the task of estimation of gas turbine engine project scientific and technical level with the lack of information [7], etc.

The present paper offers original method of the hydroturbine efficiency coefficient two-dimensional dependencies using a few number of observations via functional transformations.

2. Two-dimension dependencies approximation via a few number of observations using non-linear conversions

In general, approximation of non-linear functional dependencies via a few number of observations without additional information is insolvable problem. But in certain circumstances it could be successfully solved. Let’s formulate the following assumptions:

Assumption No. 1. Deviation of restored functional dependency \( f(x, y) \) from the source \( \varphi(x, y) \) in the space of continuous functions \( C_0 \) does not overcome the set value, i.e.

\[
\| f(x, y) - \varphi(x, y) \| < \varepsilon. \tag{1}
\]

Assumption No. 2. The number of these observations is definitely not enough to approximate the function using traditional methods. At the experiment points the functions are estimated with defined accuracy \( \delta \).

Let’s introduce the following set of conversions:

1. Function incrementation

\[
\Phi_1(X) : \tilde{f}(x, y) \rightarrow f(x, y) + \Delta f. \tag{2}
\]

2. Conversion “focus” shift

\[
\Phi_2(X) : \begin{cases} x_0 \rightarrow x_0 + \Delta x, \\ y_0 \rightarrow y_0 + \Delta y. \end{cases} \tag{3}
\]

3. Coordinate axes rotation for the angle \( \alpha \)

\[
\Phi_3(X) : \begin{cases} \tilde{x} = x \cos \alpha + y \sin \alpha, \\ \tilde{y} = -x \sin \alpha + y \cos \alpha \end{cases} \tag{4}
\]

4. Asymmetric stretching/compression - \( \Phi_4(X) \).

Above introduced conversions implement independent variables space conversion via displaying: \( \Phi_4(X) : X \rightarrow X \), where \( X = (x, y, z)^T \). Let’s consider the last conversion in more details \( \Phi_4(X) \).

Stretching/compression operation is executed under control of the pattern shown in the figure 1. Stretching/compression itself is implemented along the beams radiating from the center of symmetric ellipse. Stretching/compression coefficient is defined \( \gamma \) as follows. Knowing coordinates of the point being converted \( X = (x, y)^T \), we can define the beam angle \( \theta = \arctan(y/x) \). The point where the beam \( y = t g \theta \cdot x \) and the ellipse \( x^2/a^2 + y^2/b^2 = 1 \) are crossed we can find by solving set of equations

\[
\begin{cases} x' = \sqrt{1/(\beta^2 + 1/a^2)}, \\ y' = \beta x. \end{cases} \tag{5}
\]

where \( \beta = t g \theta \). As a result stretching/compression coefficient is calculated as follows \( \gamma = \sqrt{x'^2 + y'^2} \).

Really, the ellipse pattern defines the stretching/compression coefficient measurement rules. New position of the source point \( X \) could be found from the following expressions...
\[
\begin{align*}
\tilde{x} &= \gamma \rho \cos \theta + x_0, \\
\tilde{y} &= \gamma \rho \sin \theta + y_0.
\end{align*}
\]  

Above mentioned method is referred to the first quarter of the coordinate system. Analogical functions are generated for the rest quarters of the coordinate grid. Where \(X_0 = (x_0, y_0)^T\) – coordinates of “focus” of stretching/compression operation (asymmetric ellipse center), \(\rho\) – distance to the point \(X\).

![Figure 1. Stretching/compression operation pattern.](image)

Conceptual basis of the offered method of recovery of functional dependency using a few number of observations is concluded in the following idea. It is offered to expose the source known dependency to superimposition of offered non-linear conversions until the conditions of a new deformed dependency will be met at least over some number of observations with set accuracy.

In the present work the following superimposition of the functional dependency scope definition is used

\[
\Phi_2(X) = \Phi_1(\Phi_1(\Phi_1(X)))
\]

And the task of recovery of functional dependency using a few number of observations was considered as the task of constrained optimization.

Let’s represent multitude of observation data resulted from testing of the dependency being restored as a matrix

\[
F = \begin{bmatrix}
x_1 & y_1 & f_1 \\
x_2 & y_2 & f_2 \\
\vdots & \vdots & \vdots \\
x_N & y_N & f_N
\end{bmatrix}
\]

Resulting conversion \(\Phi_2(X)\), as it is seen, depends from 8 parameters: \(p = (\Delta f, \Delta \nu, \Delta y, a, b, c, d, \alpha)^T\) changing which we “fit” the source dependency to the generated one, so the task of recovery of the functional dependency could be represented as a conditional optimization task

\[
\min \left| p - (0, 0, 0, 1, 1, 1, 0)^T \right|, \quad \left| \phi(\Phi_2(X_k)) - f_k \right| - \delta \leq 0, \quad k = 1, N.
\]

For this settlement of the task, assumptions 1 and 2 are used. Really, as recovered dependency in linear metric (1) is not “far” from the source one it seems to be possible to reach desired dependency by the means of non-linear conversions of the coordinate system. In any case, this should occur in the neighborhoods of supplementary data (8).
3. Test bench model

In order to research the offered method of the hydrounit characteristic recovery using a few number of observations there was developed a model of deformation of the source characteristic simulating age hardening of the hydrounit which leads to decrease of the efficiency coefficient of the turbine. As the source characteristic there was chosen performance one of the hydrounit ПЛ-586-ВБ-930 of Voljzskaya HPP, generated with the help of optimal cubic Hermite spline [8].

Considering Assumptions 1-2 estimation of quality of restoring of defected dependency \( \eta_r(N,H) \) was executed in the characteristic operating area, really, near the neighborhoods of the points (8) of tests of hydrounit \( F \). As a proximity criteria of restored and defected characteristics we are going to use standard (root-mean-square) deviation

\[
SD_{\text{var}} = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (\eta_k^i - \eta_{\text{var}}^i)^2},
\]

of one characteristic from another, calculated at the function operating area.

Distortion measure of the source characteristic could be estimated using analog parameter

\[
SD_{\eta} = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (\eta_r(X_k) - \eta^i_\eta)^2}.
\]

Model of the test bench to check the offered method of the hydrounits performance characteristics recovery is shown in the figure 2.

According to structural-functional diagram initially there are used: the source characteristic as the level of deviation of distorted characteristic from the source one \( \Delta \eta \) and the variation range of the distorted characteristic rotation angle \( \pm \Delta \alpha \) (module 1 of figure 2). The module 2 randomly generates rotation angle for source characteristic.

Main deformation of the source characteristic is executed in modules 3 and 4 by the means of efficiency coefficient increment for \( \Delta \eta \) value and rotation of the characteristic longer axis for the angle \( \alpha_i \). Moreover, as the experiments shown, 75–80% of the distortions are at \( \Delta \eta \), and 15–20% of the distortions are generated by its rotation.

In the module 5 preliminary distorted characteristic is processed using radial basis neural network. In this case approximation of the function by discrete limited points multitude brings non-linear distortions of the processed object. And, the source characteristic deformation process is completed by approximation of the distorted function using optimal cubic Hermit spline [8]. Last operation also brings insignificant non-linear distortions into generated characteristic's image, but it is necessary to increase test bench processing speed, as cubic Hermite spline's processing speed is 67 times higher the neural network's one [8].

Having generated turbine distorted characteristic \( \eta_r^i(N,H) \) of the hydrounit we can research effectiveness of the offered method of the hydrounit characteristic recovery using a few number of observations. In this case, distorted characteristic can be considered as some pattern the restored dependency is orienting to. In module 7 some additional information (in real conditions by the means of field tests) is added by portions auto-incremental by 7, 12, 16, 22, 28 and 33 points.

It is not easy to execute tests at operating HPP. For such tests expensive equipment is required, and, besides, the water head \( H \) depends from the water-storage filling level and could not be changed in certain limits. Usually data are grouped into the clusters with similar heads. That is why the first data set consisted from 7 points at \( H=23 \).

Recovery of the distorted characteristic is executed in module 8 via solving the task of conditional optimization (9) using the method of conversions superimposition (7).

Module 7 – module 8 cycle makes possible to research dependency of error of distorted characteristic recovery from the number of additional data (of tests).

Module 2 – module 8 cycle helps to estimate effectiveness of the offered method at different variants of distortion of the source performance characteristic in the scope of chosen hydrounit age-hardening model.
In module 9 there is executed statistic processing of the offered method of restoring of functional dependencies using partial set of data.

Figure 2. Virtual structural and functional diagram of the test bench.

4. Simulation experiments
Research of the offered method of recovering of functional dependencies was executed using above described test bench. There were carried out 4 experiments with the set levels of the hydrounit efficiency coefficient reduction for 1%, 1.5%, 2% and 2.5%, what in operating area of the performance characteristic correspond to distortion root-mean-square levels $SD_\eta$ for 1.04; 1.52; 2.03 and 2.51 percent. Rotation angle of the characteristic longer axis was changed in the limits $\pm 3^\circ$. It is
supposed that field tests results in the observation matrix $F$ are estimated with the accuracy $\delta = 0.15\%$.

In the figure 3 results of research of recovering of the hydroturbine distorted characteristic using a few number of observations at different levels of the source characteristic distortions.

As it can be seen from the figure for all four cases the root-mean-square deviation of distorted characteristic from the source one $SD_{\text{Vos}}$ is in $0.3 - 0.4$ percent, practically independently from number of additional observation points, what proves high effectiveness of the offered method of recovering HPP hydrounit performance characteristic. For illustrative purposes in the figure 3 are shown maximum (max) and minimum (min) $SD_{\text{Vos}}$, reached in the tests. As we can see from the figure, even the worst recovering of the characteristic is insignificantly different from average case.
Figure 5. Recovering of the turbine distorted characteristic by 16 points.

Figure 6. Recovering of the turbine distorted characteristic by 18 points.

For example in the figure 4 there are shown source and distorted characteristics received at the test bench at decreasing level of efficiency coefficient \( \Delta \eta = 1.5\% \) and \( \alpha = -1 \).

In the characteristic operating area root-mean-square deviation \( SD_\alpha \) of the source characteristic from distorted is 1.46%.

In the figure 5 there are shown results of recovering of the turbine characteristic by 16 points. As we can see from the figure there is rather good compliance between generated approximation using a few number of observations with the goal distorted characteristic. In any case, \( SD_{V_{\text{os}}} = 0.3\% \).
If add 12 additional points recovering results become better. In the figure 6 there are shown distorted and recovered characteristics generated by 28 check points. Root-mean-square deviation in the characteristic operating area for this case is $SD_{V_{os}} = 0.19\%$.

5. Conclusion
In the present work there is offered a method of recovering of performance characteristics of the hydroturbine using a few number of observations. Complexity of this task is in lack of required scope of information for correct approximation of the hydrounit performance characteristic using traditional functions approximation methods. To solve this task using a few number of observations additional information is required. As such additional information it is offered to use model of source performance characteristic, generated in factory conditions according to the industry standards. In assumption that “aged” characteristic is not significantly different from the source one it is offered by non-linear (stretching/compression) distortions of independent variables area of the source characteristic to get suitable execution of a new approximation via tests points.

Research of the offered method using virtual test bench has shown principal possibility to use method of non-linear conversion of approximated dependency to solve the task of recovering of the hydrounit performance characteristics via a few observations. Further improvement of this method is planned by the means of adding new non-linear conversions and updating of hydrounit turbine age-hardening model.

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7. References
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