Unusual behavior of $^3\text{He}-^4\text{He}$ mixtures in connected 0D boxes

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Abstract.
We report on measurements of the specific heat of $^3\text{He}-^4\text{He}$ mixtures confined to boxes connected with fill channels 1 $\mu$m wide and 19 nm high. These data were meant as a study of mixtures to test universality of finite-size scaling along the lambda-line. We find instead that the mixtures have very unusual behavior relative to mixtures studied in geometries for 2D and 1D crossover. These latter data can be scaled successfully as long as one does not include data for pure $^4\text{He}$. In contrast, the 0D crossover data do not scale on either side of the bulk transition temperature $T_\lambda$ and have rather different overall behavior: They show a dramatically larger temperature shift of the specific heat maximum $C_{\text{max}}$ relative to data for 2D and 1D, the value of the specific heat at $T_\lambda$ is substantially smaller than that observed for the other dimensionalities, and the value of $C_{\text{max}}$ exceeds the magnitude of the bulk specific heat at the same temperature. This overall behavior is so unusual that it suggests there is a role being played by the connecting channels even though the liquid within them remains in the normal state over the temperature range in which the specific heat is measured.

The addition of $^3\text{He}$ to $^4\text{He}$ has the effect of increasing the correlation length for a given temperature relative to pure $^4\text{He}$. Thus, as one approaches the bulk transition temperature $T_\lambda$ the mixtures will experience finite-size effects sooner due this enhanced correlation length. The addition of $^4\text{He}$ also suppresses the transition temperature of the mixtures below that of pure $^4\text{He}$. However, even though mixtures of $^3\text{He}-^4\text{He}$ differ from $^4\text{He}$, the concept of universality states that the critical exponents should be identical for both. This allows one to use a single spatial confinement and vary the concentration to test universality. In addition, finite-size scaling theory predicts that data taken for mixtures should collapse upon the pure data once the differences in correlation length and specific heat amplitudes are taken into account.

Data for pure $^4\text{He}$ as well as four mixtures of $^3\text{He}-^4\text{He}$ were obtained when helium was confined to an array of 1 $\mu$m$^3$ boxes. These boxes were created by first growing a uniform layer of silicon dioxide on a two inch diameter silicon wafer and then patterning nearly $10^9$ boxes in the oxide such that all three spatial lengths are nominally 1 $\mu$m. A second wafer was prepared by growing a shallow oxide layer of only 18.5 nm and then patterning 1 $\mu$m wide channels separated by 1 $\mu$m of unpatterned oxide. These channels will allow helium to flow from a hole bored through the center of this wafer to the boxes when the two wafers are bonded together to form a closed cell using direct wafer bonding techniques [1]. The height of the fill channels was chosen such that the helium in the channels will remain normal over the temperature range of interest for helium in the boxes.
Figure 1. The specific heat for pure $^4$He and four $^3$He-$^4$He mixtures measured at constant concentration as a function of $t$, the reduced temperature difference from $T_\lambda$. The bulk specific heat is shown as lines for the pure data and one mixture. The short vertical lines are the temperature where the fill channels become superfluid for each concentration.

Figure 2. The specific heat at constant chemical potential difference between $^3$He and $^4$He. The variable $\theta$ is the dimensionless temperature difference from $T_\lambda$ appropriate for $C_{P\phi}$. The bulk data for the highest concentration mixture are shown as the dashed line.
critical exponent and test universality. A plot showing the data plotted according to equation 1.

The specific heat was measured as a function of temperature at constant mixture concentration \( x \). The bulk specific heat at each concentration is constructed by parametrizing available data \([2; 3]\) and interpolating the parameters used in the bulk description at each desired concentration. One sees in figure 1 that the overall magnitude of the specific heat for both the bulk and confined helium decreases with increasing concentration. The temperature where the confined helium’s specific heat exhibits a maximum shifts to lower values as a function of concentration as expected on the basis of finite-size scaling.

True critical behavior for helium mixtures is not expected to occur along the thermodynamic path of constant concentration but along a path where the chemical potential difference between \( ^3\text{He} \) and \( ^4\text{He} \), \( \phi = \mu_3 - \mu_4 \), is held constant \([4; 5]\). Therefore, the mixture data must be converted from the measured \( C_{P_x} \) to \( C_{P\phi} \) according to a procedure outlined in reference \([6]\). This conversion also requires a temperature variable \( \theta = [1 - T/T_\lambda] \) along the thermodynamic path \( \phi = \phi_\lambda \). This process is identical to that performed for helium confined to a planar confinement \([7]\) and one-dimensional channels (see the paper regarding this submitted to LT25 by Mooney, Kimball, and Gasparini).

During the measurement of the mixtures, it became clear that the maximum in the confined \( C_{P_x} \) data at higher concentrations was larger than that of the bulk near the temperature region of the maximum. This is unexpected and becomes more pronounced when \( C_{P_x} \) is converted to \( C_{P\phi} \). This has repercussions when one attempts to scale the data, most obviously for temperatures less than \( T_\lambda \). One can test finite-size scaling by plotting the data according to the equation proposed in reference \([7]\)

\[
\Delta C \left| \frac{\alpha_\phi}{A_\phi(x)} \right| \theta^{\alpha_\phi} = g_2 \left( \frac{L}{\xi(x,t)} \right)^{1/\nu}. \tag{1}
\]

Here \( \Delta C \) is \( C(\theta, \infty) - C(\theta, L) \), \( L \) is the confinement size of 1 \( \mu \)m, and \( \xi(x,t) \) is the correlation length at a given concentration and temperature. The specific heat and correlation-length critical exponents are \( \alpha \) and \( \nu \) respectively. The term \( |\alpha_\phi/A_\phi(x)| \) accounts for the non-universal specific heat amplitude \( A_\phi(x) \) for the mixtures and \( \alpha_\phi \) is included to allow one to vary the critical exponent and test universality. A plot showing the data plotted according to equation 1

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Data above and below \( T_\lambda \) cast in scaling form according to equation 1. Data on both sides of the bulk transition do not collapse upon the pure data. This was also observed for 2D and 1D crossovers. The mixtures also do not collapse amongst themselves unlike data for 2D and 1D crossover.}
\end{figure}
Figure 4. The reduced temperature shift from $T_\lambda$ of the specific heat maximum for helium confined to all three lower dimensionality crossovers. The 2D and 1D data shift with concentration in a similar fashion while the 0D data show a more dramatic shift as a function of concentration.

is seen in figure 3. One sees that the mixture data do not collapse upon the bulk and also do not collapse amongst themselves. This is different from that seen for 2D and 1D crossover where the mixtures collapse upon a single locus that was different from that defined by the pure data.

Another feature of interest in the confined data is the temperature at which the specific heat is maximum. The dimensionless temperature shift of $C_{\text{max}}$ given by $t_{\text{max}} = 1 - T_{\text{max}}/T_\lambda$, as a function of concentration, is shown in figure 4 for 2D, 1D, and 0D crossover. Data at $x = 0$ behave as expected: There is a larger shift as more spatial dimensions are made small, i.e. $t_{\text{max}}$ for 0D crossover is larger than that for 2D. The trend in $t_{\text{max}}$ as a function of concentration is similar for both the 2D and 1D data. Helium confined to 0D boxes, on the other hand, shows a larger shift as a function of concentration compared to either 2D or 1D crossover.

This increased shift in $C_{\text{max}}$ for the 0D data may be responsible for the lack of collapse of the mixture data both above and below $T_\lambda$. It is unclear why the 0D data show a larger shift. Perhaps there is a collective behavior of helium in the boxes that suppresses the magnitude of the specific heat until some temperature close to, but above the point, where the helium in the channels becomes superfluid. New experiments are underway that will probe the behavior of helium in an array of boxes that are separated by shallow films of normal fluid.

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