Kardar-Parisi-Zhang scaling for the Faddeev-Takhtajan classical integrable spin chain

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Abstract

Recent studies report on anomalous spin transport for the integrable Heisenberg spin chain at its isotropic point. Anomalous scaling is also observed in the time-evolution of non-equilibrium initial conditions, the decay of current-current correlations, and non-equilibrium steady state averages. These studies indicate a space-time scaling with \( x \sim t^{2/3} \) behaviour at the isotropic point, in sharp contrast to the ballistic form \( x \sim t \) expected for integrable systems. In our contribution we study the scaling behaviour for the Faddeev-Takhtajan model, which is an integrable classical spin chain with the same symmetries as the quantum Heisenberg model. We report on equilibrium time correlations and the evolution with step initial conditions. Remarkably, the scaling function is identical to the one obtained from Kardar-Parisi-Zhang equation. In addition, we present results for the easy-plane and easy-axis regimes for which, respectively, ballistic and diffusive spin transport is observed, whereas the energy remains ballistic over the entire parameter regime.
I. INTRODUCTION

Classical Hamiltonian systems are usually classified as non-integrable and integrable, depending on whether they have either a small or a macroscopically large number of conserved fields. More precisely, a Hamiltonian system with \( N \) degrees of freedom is called integrable if one can find \( N \) independent constants of motion – otherwise it is referred to as non-integrable. In general one would expect integrable and non-integrable systems to have drastically different transport properties. Let us consider the example of a translation invariant one-dimensional mechanical system with \( Q = \sum_{j=1}^{N} Q_j \) a conserved field satisfying a local conservation law of the form \( \partial Q_j / \partial t = J_j - J_{j+1} \), where \( J_j \) is the corresponding local current. The corresponding dynamical equilibrium correlation function is defined by

\[
C(j, t) = \langle Q_j(t)Q_0(0) \rangle_{eq}^{c},
\]

where the average is over initial conditions chosen from the Gibbs equilibrium distribution and the superscript denotes the connected part of the correlator, defined as \( \langle Q_j(t)Q_0(0) \rangle_{eq}^{c} := \langle (Q_j(t) - \langle Q_0 \rangle_{eq})(Q_0(0) - \langle Q_0 \rangle_{eq}) \rangle_{eq} \). Since \( Q \) is conserved, one expects a scaling form as

\[
C(j, t) = \chi \Gamma^{-\alpha} f \left( \Gamma \Gamma^{-\alpha}(j - ct) \right).
\]

\( \alpha > 0 \) is the scaling exponent, \( c \) a potential systematic shift (the “sound” velocity), \( \Gamma \) a model dependent parameter, and \( f \) the scaling function normalized with total sum equal to 1. Our particular form ensures that \( \sum_j C(j, t) = \chi \) independent of \( t \), and \( \chi \) is the static susceptibility. For integrable systems most commonly \( \alpha = 1 \) and \( c = 0 \), which is the ballistic behavior. The scaling function depends on \( Q \). On the other hand, in non-integrable systems one often observes \( \alpha = \frac{1}{2} \) with a Gaussian scaling function. But also anomalous scaling with \( \alpha = \frac{2}{3} \) has been discovered [1–8]. Such differences between integrable and non-integrable systems are also observed in other transport simulations, for instance in the evolution of non-equilibrium initial conditions and in properties of boundary driven non-equilibrium steady states (NESS). Through generalized hydrodynamics much progress has been accomplished in the understanding of transport in integrable systems [9–11].

A surprising exception to the generic behaviour has been discovered for spin transport in the integrable XXZ Heisenberg spin chain. The quantum XXZ spin \( \frac{1}{2} \) chain is Bethe ansatz solvable for an arbitrary choice of the anisotropy parameter \( \Delta \). The spectrum is gapless for
$|\Delta| \leq 1$ and gapped otherwise. A number of studies find that spin transport in this system is diffusive for $\Delta > 1$, ballistic for $\Delta < 1$ and anomalous at $\Delta = 1$. First indications of this behaviour came from the Drude weight for spin transport [12, 13]. Subsequent evidence was obtained in NESS studies at infinite temperatures [14, 15], in the form of equilibrium correlation functions [16], and in the evolution of quenched initial conditions [17]. There has been some understanding of the un-expected diffusive and anomalous regimes of spin transport using the GHD framework [18].

The standard lattice version of the classical XXZ Heisenberg model is known to be non-integrable and recent work [8] has explored spin and energy correlations. They turn out to be diffusive at high temperatures, while anomalous features emerge at low temperatures. On the other hand, Faddeev and Takhtajan [19] discovered an integrable version of a classical spin chain, having still nearest neighbor interactions and the same symmetries as the Heisenberg spin chain. This model (defined below) has a parameter $\rho$, which plays the role of the anisotropy parameter $\Delta$ in the Heisenberg model, such that $\rho > 0$ corresponds to easy-plane and $\rho < 0$ corresponds to easy-axis, while $\rho \to 0$ is the isotropic case. For this model the equilibrium current correlations were studied in [20]. Quite remarkably, the current correlation shows an exponential decay for easy-axis ($\rho < 0$) and hence a vanishing Drude weight and diffusive transport. Saturation to a non-zero value is observed for easy-plane ($\rho > 0$), implying a finite Drude weight and ballistic transport. For the isotropic model an anomalous decay of the form $\sim t^{-\alpha}$ with $\alpha \approx 0.65$ is found. In our contribution, we study the spin and energy transport in this model by studying the scaling properties of equilibrium space-time correlations. We confirm the expected behavior of the scaling exponent for spin transport in different parameter regimes. In addition, we determine the scaling functions: Gaussian for the case of diffusive transport in easy-axis regime, and remarkably, we find a Kardar-Parisi-Zhang (KPZ) scaling form $f_{\text{KPZ}}$ [described later in equation (13)] in the isotropic regime. The energy transport remains ballistic in all regimes.

Returning to the quantum XXZ chain, in [21], the evolution of an initial step magnetization profile was studied. Not unexpected, it is observed that for $\Delta > 1$ the magnetization profile has diffusive scaling $\alpha = \frac{1}{2}$, while for $\Delta = 1$, the scaling is anomalous with $\alpha = \frac{2}{3}$. For small step height, in both cases the scaling seemed to be well-fitted by an error function. In more recent work [22], the higher precision numerics suggest that the scaling function is related to the stationary KPZ equation. As noted above, in our present study of the clas-
sical integrable Fadeev-Takhtajan chain, we observe a clear KPZ scaling in the correlation functions, and we next investigate whether this KPZ scaling also holds for the evolution of the step-profile. Note that it is not proven and not obvious a priori, that the classical and quantum systems should show exactly the same scaling properties in the hydrodynamic limit, although this is expected. To the best of our knowledge there seems to be no evidence for this in the literature. The pair ‘quantum XXZ’ and ‘classical Fadeev-Takhtajan’ thus offers a unique chance for a serious test. Here we explore this and provide strong evidence for this ‘classical-quantum correspondence’ for the first time.

Our paper is organized as follows. In Sec. II we provide the details of the model, discuss the quantities of interest, and give a brief analysis using linear response theory. We also describe the numerical methods used. Sec. III focuses on analyzing the equilibrium time correlations for all the three cases of the Fadeev-Takthajan chain – isotropic, easy-plane, and easy-axis regimes. In Sec. IV we study the evolution of an initial step magnetization profile and its scaling. We summarize our findings in Sec. V along with an outlook.

II. THE CLASSICAL CHAIN

The Faddeev and Takhtajan spin chain for $N$ spins, $\vec{S}_j$, $j = 1, \ldots, N$, $|\vec{S}_j| = 1$, is defined by the following hamiltonian

$$ H = \sum_{j=1}^{N} h(\vec{S}_j, \vec{S}_{j+1}), $$

where the nearest neighbor interactions are given by

$$ h(\vec{S}, \vec{S}') = -\log \left| \cos(\gamma S^{(z)}) \cos(\gamma S'^{(z)}) + (\cot(\gamma))^2 \sin(\gamma S^{(z)}) \sin(\gamma S'^{(z)}) \right| $$

$$ + \left( \sin(\gamma) \right)^2 G(S^{(z)}) G(S'^{(z)}) \left( S^{(x)} S'^{(x)} + S^{(y)} S'^{(y)} \right), $$

$$ G(x) = (1 - x^2)^{-\frac{1}{2}} \left( \cos(2\gamma x) - \cos(2\gamma) \right)^{\frac{1}{2}}. $$

$\gamma$ is the model parameter which can be either real or purely imaginary. Without loss of generality, we introduce the new parameter $\rho = \gamma^2$, $\rho \in \mathbb{R}$. The boundary conditions will be taken to be either periodic or open, depending on the particular physical situation studied. Apparently, the hamiltonian (3) is the only classical spin chain known to be integrable. Easy-plane corresponds to $\rho > 0$, easy-axis to $\rho < 0$, while in the limit $\rho \to 0$ one obtains
the isotropic interaction,

\[ h(\vec{S}, \vec{S}') = -\log (1 + \vec{S} \cdot \vec{S}'). \]  

(5)

Note that the “−” sign in front of \( h \) corresponds to the ferromagnetic interaction, whereas the positive sign will correspond to an anti-ferromagnetic interaction. In the present work we focus on the ferromagnetic Hamiltonian. For the anti-ferromagnetic case, the potential is not bounded from below and hence there would be equilibration problems at low temperatures. To see this, we note that for the general case with \( h(\vec{S}, \vec{S}') = -J \log (1 + \vec{S} \cdot \vec{S}') \), with \( J > 0 \) \((J < 0)\) corresponding to ferromagnetic (antiferromagnetic) interactions, the equilibrium state is given by,

\[ \prod_j (1 + \vec{S}_j \cdot \vec{S}_{j+1})^{\beta J}. \]  

(6)

For \( J < 0 \) this Boltzmann weight becomes unbounded as two neighboring spins point oppositely and can no longer be normalized once \( \beta J \leq -1 \). Close to that value typically the chain will have long anti-ferromagnetic domains, which slow down the evolution. A trace of this feature is still present at \( \beta = 0 \). Thus we find for \( \beta = 0 \) and \( J = 1 \) that after \( 10^6 \) averages the data are still too noisy to pin down the tail behavior. More precise numerical data are achieved for \( \beta = 1 \), and we use this value for all the simulations presented in this paper.

The dynamics of spins is governed by the Hamilton’s equations of motion,

\[ \frac{d}{dt} \vec{S}_j = \{\vec{S}_j, H\} = \vec{S}_j \times \vec{B}_j, \quad \vec{B}_j = -\nabla_{\vec{S}_j} H. \]  

(7)

As to be expected, \(|\vec{S}_j(t)| = 1\) for all times.

We study transport properties of this model, both equilibrium and nonequilibrium properties.

(i) In the equilibrium simulations we use periodic boundary conditions and compute spin and energy space-time correlators defined by

\[ C_{ss}(j, t) = \langle S_j^{(z)}(t)S_0^{(z)}(0)\rangle^c, \quad C_{ee}(j, t) = \langle e_j(t)e_0(t)\rangle^c \]  

(8)

where \( e_j = h(\vec{S}_j, \vec{S}_{j+1}) \) and the truncated average \( \langle \ldots \rangle^c \) is taken with respect to the equilibrium distribution \( e^{-\beta H}/Z \).

(ii) In the nonequilibrium simulations, we consider an open XXX chain initially prepared with a uniform temperature and a step in the magnetization. More precisely, the initial
state is $Z^{-1} \exp \left[ -\beta \left( H - \sum_j h_j^{(z)} S_j^{(z)} \right) \right]$, where $h_j^{(z)} = -h_0$ for $j \leq 0$ and $h_j^{(z)} = h_0$ for $j > 0$. Of interest is the average magnetization $s(j, t) = \langle S_j^{(z)}(t) \rangle_{h_0}$ at time $t$, where the dynamics is according to $H$ and the index recalls the dependence of the initial state on $h_0$. If the step is small, this average can be expanded in $h_0$. The zeroth order vanishes and, using that $C_{ss}(j, t) = C_{ss}(-j, t)$, to first order one arrives at

$$s(j, t) = \beta \sum_i h_i^{(z)} C_{ss}(j - i, t) = \beta h_0 \left( C(0, t) + 2 \sum_{i=1}^{j} C_{ss}(i, t) \right)$$

(9)

for $j \geq 1$ with $s(-j + 1, t) = -s(j, t)$. As a consequence, if $C_{ss}(j, t)$ scales as in (2) with $c = 0$, then $s(j, t)$ inherits the corresponding scaling. In the continuum form one arrives at

$$s(x, t) = 2\beta h_0 \chi \int_0^{x/(\Gamma t)^{\alpha}} dx' f(x')$$

(10)

for $x \geq 0$. The derivative of the scaling function of $s$ yields the scaling function for $C$. For example, if $C_{ss}$ has Gaussian scaling, then $s(x, t)$ would scale with the error function. Note that the scaling exponent remains unchanged.

**KPZ equation and scaling functions.** KPZ equation describes the surface growth under a random ballistic deposition. The height function $h(x, t)$ is governed by the Langevin equation,

$$\partial_t h = \frac{1}{2} \lambda (\partial_x h)^2 + \nu \partial_x^2 h + \sqrt{D} \eta,$$

(11)

where $\eta$ is normalized space-time white noise. The slope $u(x, t) = \partial_x h(x, t)$ is governed by the stochastic Burgers equation

$$\partial_t u + \partial_x \left( -\frac{1}{2} \lambda u^2 - \nu \partial_x u - \sqrt{D} \eta \right).$$

(12)

In the stationary state the mean of $u$ can be chosen to vanish and $x \mapsto u(x, 0)$ is spatial white noise of strength $\chi = D/2\nu$. As shown in [2] the two-point function of the stationary stochastic Burgers equation is given by

$$\langle u(0, 0) u(x, t) \rangle \sim \chi (\Gamma t)^{-2/3} f_{\text{KPZ}} \left( (\Gamma t)^{-2/3} x \right).$$

(13)

$\Gamma$ determines the non-universal time scale, which in case of the Burgers equation turns out to be $\Gamma = \sqrt{2\lambda}$. The scaling function $f_{\text{KPZ}}(x)$ is positive, symmetric relative to the origin, normalized to 1. It looks like a Gaussian in bulk but has tails which decay as $\exp(-0.295|x|^3)$.
hence faster than a Gaussian.

*Simulation details.* We integrate the evolution equation (7) using adaptive Runge-Kutta method [23]. In some regions in configurations space, the logarithmic interaction potential is very steep, and because of this the fixed step-size Runge-Kutta method turned out to be insufficient, especially at large times. One challenge is to keep the energy and the lengths of individual spins conserved during the numerical integration. Both these quantities dissipate quite a bit with time due to the accumulation of numerical errors. We give the input tolerance in the adaptive algorithm such that at the final time total energy remains conserved up to 4 decimal places and individual lengths of spins up to 5 decimal places. Total magnetization remains conserved well, up to 13 decimal places.

We use Metropolis Monte Carlo sampling to generate the canonical ensemble. Starting from an ordered initial configuration, we allow 5000 Monte Carlo swipes to make sure that the system has reached thermal equilibrium at the desired temperature. Once equilibrium has reached, we drop 500 swipes every time we generate a new thermal configuration to use as the initial condition for the time evolution. Thereby one ensures that the initial conditions used in the time evolution are sufficiently uncorrelated among themselves. All averages are taken over these initial conditions. The step initial profile is generated by equilibrating the system using a step magnetic field of the appropriate size at given temperature. In our study we chose the value of $\beta = 1$. At higher temperatures, the average energy per site increases and the spins access the steeper parts of the inverted log potential (see (4)) and as a result the simulation using the adaptive step size algorithm becomes very slow [see discussion around Eq.(6)]. For the choice $\beta = 1$, the simulation efficiency is reasonable and it is expected that our main results should be valid at other temperatures.

III. SIMULATION RESULTS FOR EQUILIBRIUM DYNAMICAL CORRELATIONS

A. Isotropic regime

This corresponds to the choice $\rho \to 0$ in (4) which leads to the simpler form of the hamiltonian (5). In this regime, spins have no directional bias and lie uniformly on the unit sphere. At infinite temperature these directions don’t have any correlation but at finite
FIG. 1. (Isotropic regime) (a) Plot of the spin-spin correlation $C_{ss}(x,t)$. And the same after a $t^{2/3}$ scaling with a fit to (b) Gaussian and (c) the KPZ scaling functions. In (d) we show the two fits compared to the data in logarithmic $y$–scale. This plot reveals that the KPZ scaling function offers a much better fit to the data. Parameter values: system size = 2048, averaging over $\sim 10^6$ initial conditions and inverse temperature $\beta = 1$.

and low temperatures the correlation grows. In Fig. (1a) we plot the spin-spin correlation function $C_{ss}(x,t)$ for $\beta = 1$. We see a very good $x \sim t^{2/3}$ scaling of the data. In Fig. (1b) we compare the scaled data with a Gaussian distribution while in Fig. (1c) we compare the same data with the KPZ distribution. We first compute the sum $\sum_j C_{ss}(j,t)$, which is independent of time and gives an estimate of the area under the fit curve. This is essentially the value of $\chi$ in (2). Then we find the best fit parameter $\Gamma$ using the NonlinearModelFit function of Mathematica. In particular, we found that $\chi = 0.526698$ and $\Gamma = 1.93609$ for $f_{\text{KPZ}}$ and $1.21582$ for $f_{\text{Gaussian}}$. Although the distinction is not so significant on this scale, we see that a much better fit is obtained with the KPZ distribution. The distinction becomes
FIG. 2. (Isotropic regime) Plot of the energy-energy correlation $C_{ee}(x, t)$ and the ballistic scaling of it. Parameter values: system size = 2048, final time = 320, averaging over $\sim 10^6$ initial conditions and inverse temperature $\beta = 1$.

very prominent in the log plot shown in Fig. (1d). This is because the KPZ scaling function differs from a Gaussian only in the tails. Although spin transport is superdiffusive in this regime of the hamiltonian, energy transport is ballistic. Energy correlations are plotted in Fig. (2) which show a clear ballistic scaling.

Note that in many cases, the diffusive or superdiffusive modes come coupled with the ballistic modes and to see them one needs to subtract the ballistic contributions, which is a difficult task in general [10]. In our case it turns out that for spin transport at the isotropic point the ballistic contribution does not exist and we directly see the superdiffusive mode.

B. Easy-plane regime

This corresponds to the choice $\rho > 0$ in Eq. (4). Spins tend to lie near the $x - y$ plane at finite temperatures. We use the value $\rho = 1$. As shown in Fig. (3), both spin and energy show ballistic scaling in this regime. We however observe that spin transport is slower than the energy transport. In other words, in Fig. (3) the line shapes for spin and energy transport are distinctly different.
FIG. 3. (Easy plane regime) Plot of the spin-spin correlation $C_{ss}(x,t)$ and energy-energy correlation $C_{ee}(x,t)$ in easy-plane regime and corresponding ballistic scalings. Parameter values: system size $= 2048$, averaging over $\sim 4 \times 10^4$ initial conditions and inverse temperature $\beta = 1$.

C. Easy-axis regime

This corresponds to the choice $\rho < 0$ in Eq. (4), i.e. $\gamma$ becomes purely imaginary and the trigonometric functions become hyperbolic functions in the Hamiltonian. For our purpose, we use the value $\rho = -1$. In this regime, spins have the tendency to lie near the $z$-axis at finite temperatures. As shown in Fig. (4), we now observe that spin correlations spread diffusively while energy correlations spread ballistically. In this particular regime we have diffusive transport of spin. In Table. 1, we summarize the transport properties in the Faddeev-Takhtajan chain.
FIG. 4. (Easy axis regime) Spin-spin correlation $C_{ss}(x,t)$ and energy-energy correlation $C_{ee}(x,t)$ in easy-axis regime. In (b) we show the diffusive scaling of spin correlations while in (d) we see the ballistic scaling of energy correlations. Parameter values: system size $= 2048$, averaging over $\sim 4 \times 10^4$ initial conditions and inverse temperature $\beta = 1$.

IV. MAGNETIZATION PROFILE FOR STEP INITIAL CONDITION

We consider now a chain of $N = 512$ spins and prepare it at the inverse temperature $\beta = 1$ using a step magnetic field as described in Sec. II with $h_0 = 0.01$. We average over $8 \times 10^5$ such initial conditions. The resulting step height in the magnetization is $\pm 0.00665$. These step initial conditions are evolved according to the isotropic Hamiltonian (5) and we monitor the average magnetization profile $s(x,t)$ at later times. Magnetization profiles at different times are shown in Fig. (5a) while Fig. (5b) shows the $2/3$ scaling of $s(x,t)$. This is expected from (10) and our previous finding of $2/3$ scaling of $C_{SS}(x,t)$ in the isotropic regime. Although $s(x,t)$ correctly reproduces the scaling exponent, the data is noisy and
FIG. 5. (Isotropic regime) (a) Magnetization profile at different times starting from a step initial condition. (b) Collapse of the data under a $t^{2/3}$ scaling. The dashed line corresponds to the integrated KPZ scaling function \[ 10 \]. Inset shows the fit with integrated Gaussian, namely the Error function. Although the $2/3$ scaling is prominent, we cannot distinguish the (integrated) Gaussian and KPZ here. Parameter values: System size = 512, inverse temperature $\beta = 1$ and averaging over $\sim 8 \times 10^5$ initial conditions.

not accurate enough for us to rule out a fit to an error function (integral of a Gaussian). In Fig. (5b) we show the fit with integral of $f_{\text{KPZ}}$ and, in the inset, we show the fit with the error function. Much more averaging over the initial conditions is required to arrive at smoother data shown here. Here we are essentially dealing with \[ 10 \]. This equation is supposed to be exact in the linear response limit $h_0 \to 0$, and so one should recover the same values of $\Gamma$’s and $\chi$ obtained from $C_{ss}$ data by analyzing the step profile. However, in our simulations we have kept $h_0 = 0.01$ and as a result we observe slight deviations in the $\Gamma$ and $\chi$ values. Here we see $\chi = 0.665$ and $\Gamma = 1.74603$ for both KPZ and Gaussian functions.

V. CONCLUSIONS

From the study of several integrable many-body systems, there seems to be a consensus that their large scale behavior has many common features. In particular, since based on hydrodynamic type arguments, quantum models should not differ from their classical version. We presented the numerical study of the classical integrable Fadeev-Takthajan spin chain and compared with previous studies of the quantum XXZ Heisenberg model. Our findings
are summarized in Table I and support the view that on large scales classical and quantum cannot be distinguished. In the isotropic case, we find that the quantities involving spin show superdiffusive behavior with scaling exponent $2/3$ and the scaling function is KPZ. In the easy-plane regime, we find that the spin transport is ballistic while in the easy-axis regime it is diffusive. The fact that energy transport shows ballistic scaling over the entire parameter range simply results from the energy current itself being a locally conserved field. To probe the KPZ behavior further we also studied the evolution of an initial magnetization step. Again, we find the $t^{2/3}$ scaling but from these data, we are not able to conclusively differentiate between KPZ versus Gaussian scaling.

While the numerical evidence is pointing in the expected direction, strong theoretical arguments are still missing. Of course, a first inclination is to compare the corresponding GHD, which is available for the quantum XXZ model but currently not for its classical version. In addition, KPZ scaling requires a particular nonlinearity and noise, which is beyond conventional GHD.

| Regime             | Spin transport              | Energy transport |
|--------------------|------------------------------|------------------|
| Easy plane ($\rho > 0$) | Ballistic                   | Ballistic        |
| Isotropic ($\rho \to 0$) | Super-diffusive (scaling function: KPZ) | Ballistic        |
| Easy axis ($\rho < 0$)   | Diffusive (scaling function: Gaussian) | Ballistic        |

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