The Progressive Partition Method to Identify Damage of Single-Layer Reticulated Shell

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Abstract. For the damage location in the existing damage identification methods, this paper adopts the progressive partition method in view of the modal strain energy to locate and evaluate the bars’ damage degree in the K8 single-layer reticulated shell model. Through the correlation between damage identification index and damage degree, the location and damage degree of damage element can be precisely determined. Meanwhile, this method can effectively improve the anti-noise performance in the damage identification of structural, and determine the key elements for the overall stability of single-layer reticulated shells under specific loads. The research provides a basis for the diagnosis and reinforcement of damaged structure.

Keywords: reticulated shell; damage identification; stability of damaged structure; element modal strain energy.

1. Introduction

Reticulated shell is a spatial structure system composed of bars arranged regularly according to a certain curved surface. Its main stress type is “membrane force”, and most bars are stressed by axial force [1,2]. Reticulated shell has been widely used in the large-scale public building such as stadium, exhibition center, airport, and station for its uniform stress, long-span capacity, light weight, and hard stiffness [3-5]. However, this kind of structure may inevitably suffer various types of damage during service. For example, the damage caused by natural factors such as earthquake, fire, strong wind, flood, or the damage induced by aging of building materials, defects of design or construction, improper use, etc. These damages will accelerate performance deterioration of reticulated shell, and even lead to structural collapse and other accidents, which will cause huge casualties and economic losses [6-8].

The existing researches on the damaged reticulated shell mainly involve the effect of damage on materials and bars, not the structural performance [9-13]. Therefore, studying the damage identification and stability analysis of the damaged single-layer reticulated shells from the structural level can give a reference for design and engineering application, with far-reaching significance. At present, relatively complete structural damage identification indexes have been put forward, including damage identification methods based on natural frequency, mode shape, strain mode, flexibility matrix, and modal strain energy [14-20]. In these methods, the damage identification method depending on modal strain energy is relatively mature, which has better accuracy and anti-noise performance than only relying on natural frequency, mode shape, or flexibility matrix [21-23]. Nevertheless, the researches on damage identification relying on modal strain energy focus on judging whether the damage occurs, rather than diagnosing the position and degree of structural damage.

To better identify the damage of reticulated shell, this paper will carry out a simulation study on the
progressive partition method based on modal strain energy. The damaged single-layer reticulated shell’s damage identification and stability analysis will be completed through an example of the reticulated shell model established in this paper, whose key bars are damaged. Finally, the validity and engineering applicability of the progressive partition method for damage identification of reticulated shell will be proved by simulation.

2. The Damage Identification of Element Based on Modal Strain Energy

The mechanical properties of the damaged elements are affected after the damage, which can be uniformly manifested as the degradation of element stiffness. The element stiffness degradation will affect the structure’s the frequencies and mode shapes, then impact the element strain energy in different modes.

According to reference [24], assuming that the structure has \( n \) degrees of freedom, the calculation formula of the \( i \)-th modal strain energy of element \( m \) (\( MSE_{im} \)) is as follows:

\[
MSE_{im} = \{\phi_i\}^T[k_m]\{\phi_i\}
\]

Where \([k_m]\) represents the stiffness matrix of element \( m \), expressed in the form of element contribution matrix; \([\phi_i]\) represents the \( i \)-th mode shapes.

Meanwhile, the \( i \)-th order modal strain energy of the damaged element \( m \) (\( MSE_{dim,or} \)) is calculated as follows:

\[
MSE_{dim,or} = \{\phi_{di}\}^T[k_m]\{\phi_{di}\}
\]

Where the element stiffness matrix takes the stiffness without damage, \([\phi_{di}]\) denotes the damaged structure’s the \( i \)-th mode shape.

The strain of element \( m \) in \( i \)-th mode \( \{\varepsilon_{im}\} \) is extracted from the numerical simulation results, and the volume of the element \( m \) is given as \( V \). The modal strain energy before and following damage is obtained by Eq. (3) and Eq. (4), respectively.

\[
MSE_{im} = \frac{1}{2}E\varepsilon_{im}^2V
\]

\[
MSE_{dim} = \frac{1}{2}E\varepsilon_{dim}^2V
\]

For the convenience of comparison, the element modal strain energy’s change rate before and after damage is normalized to get \( NMSE_{im} \) by Eq. (5), i.e., in a certain mode, the ratio of each element’s modal strain energy difference to the maximum value of the element strain energy change rate, so that damage identification results of each mode are between 0 and 1.

\[
NMSE_{im} = \frac{(MSE_{dim} - MSE_{im})}{\max(MSE_{im}(m))}
\]

3. Finite Element Model of Single-Layer Reticulated Shell

Kiewitt reticulated shell is composed of \( n \) (\( n=6, 8, 12, \ldots \)) long main ribs (radial rod), which separate the spherical surface into \( n \) symmetric sectors. These sectors are then divided into a relatively uniform triangular mesh by ring ribs (weft bars) and diagonal bracing (diagonal bars). K8 single-layer reticulated shell refers to the Kiewitt reticulated shell with 8 symmetrical sectors.

The K8 single-layer reticulated shell model is built in ANSYS with the bottom surface diameter of 65m, the arrow height of 10m, the separation frequency of 6, the bar length of 4-6m, and the bar number of 456, as shown in figure 1. All bars are made of hollow hot-rolled round steel tubes, which are divided into three types according to their positions: main rib, ring rib and diagonal bar. The cross sections of main rib and ring rib are \( \Phi180\times5 \), and those of diagonal bar are \( \Phi160\times6 \). All bars adopt Q345 steel with Young’s modulus of \( 2.1 \times 10^5MPa \), shear modulus of \( 8.5 \times 10^4MPa \), yield strength of 345 MPa, and damping ratio of 0.02. Besides, all of them are simulated by beam element, and their joints are treated as rigid connection. The constraint condition is considered as simply supported, where only the node displacement is constrained, but not the joint rotation. The ring ribs are defined from inside to outside as the first ring ring ribs to the sixth ring ring ribs, and the main ribs and
diagonal bars between the \( n \) and the \( n-1 \) ring ring ribs are called the \( n \) ring main ribs and diagonal bars.

![Image](image1.png)

\( \text{a) Top view} \)

![Image](image2.png)

\( \text{b) The finite element modal} \)

**Figure 1.** K8 single-layer reticulated shell.

### 4. The Progressive Partition Method to Identify Damage of Reticulated Shell

The damage identification method relying on the element modal strain energy’s change rate is more precise. However, since the whole single-layer reticulated shell contains plenty of elements, the strain of the element can usually differ by several orders of magnitude. Especially, the modal of the single-layer reticulated shell involves the local buckling and overall buckling. In this condition, if the change rate of all elements’ modal strain energy is calculated indiscriminately, the accuracy of damage identification will be seriously affected. Therefore, this paper partitions the bars according to the element strain in a certain mode, and then identifies the damaged bars in the divided region. By this method, the calculation time can be saved and the accuracy of damage identification can be improved.

#### 4.1. Identification Method

The process of identifying the damaged single-layer reticulated shell by the partition progressive method involves five steps. Firstly, the finite element model of single-layer reticulated shell is created according to the structural data, and the natural vibration characteristics of this model are explored. Secondly, the model’s stability is analyzed, and the key bars are determined. Thirdly, according to the element strain in a certain mode, the bars are zoned. Fourthly, the damage identification results are
obtained after measuring the actual structure, and the damage of elements is assessed by the
 corresponding relation between the damage identification index and the damage degree. Thereafter,
 the damaged model is built. Fifthly, the stability analysis for the damaged reticulated shell is given. A
 K8 single-layer reticulated shell is taken as an example, and its damage identification analysis process
 is as follows.

4.2. Identification Process

4.2.1. Analysis on Natural Vibration Characteristics. In this simulation, the first six modes are
extracted for analysis of the model’s natural vibration characteristics. The frequencies and
 corresponding mode shapes of each mode are shown in figure 2. Owing to the symmetry of K8
reticulated shell, its mode shapes are also symmetric. For better observation, the displacement results
are amplified by 5 times, and the region of local instability is marked. It is found that the first
frequency is 5.197Hz, and the instability region is concentrated in the first main ribs along with the
second and third ring ribs.

4.2.2. Determination of Key Bars. The vertical node load is applied to the model. It is assumed that all
nodes share the sum of vertical loads. The load combination is 1.2 times of dead load + 1.4 times of
live load, in which roof weight, live load, and snow load are 0.4kN/m², 0.3kN/m², 0.15kN/m²,
respectively. Thus, the design value of load is calculated as follows: 

\[ q = 1.2 \times 0.4kN/m^2 + 1.4 \times 0.3kN/m^2 = 0.9 \, kN/m^2. \]

The node loading speed is set as 2.4kN/s. Figure 3 shows the load and constraint conditions.

Figure 2. The first six natural frequencies and mode shapes of K8 single-layer reticulated shell.

(a) The first natural frequency and mode shape. (b) The second natural frequency and mode
shape.
(c) The third natural frequency and mode shape. (d) The fourth natural frequency and mode
shape.
(e) The fifth natural frequency and mode shape. (f) The six natural frequency and mode shape.
Figure 3. Load and constraint conditions.

Figure 4 depicts the axial force of the bars in the limit state before instability failure. The vertical top displacement reaching 1/250 of the span is used as the basis to judge the structure instability, and the corresponding load is the ultimate load. In this example, the equivalent uniform distributed load at the time of instability is 2.16KN/m². Figure 5 depicts the relevance of uniform distributed load and top displacement.

Figure 4. Axial force of K8 single-layer reticulated shell in limit state.

Figure 5. The relationship between uniform load and top displacement.

It transpires from figure 4 that, except for 8 groups of diagonal bars along the radial direction and the sixth ring ribs, the bars are all compression members. The bars with maximum compressive stress
are the first ring main ribs as well as the second and third rings ring ribs, which are also the bars with initial local instability, as shown in State 2 in figure 6. The initial instability mode of the structure is similar to the deformation of the first mode. After the instability of the first ring main ribs and the second and third rings ring ribs, other bars lose stability successively. Thus, it is demonstrated that the key bars are the first ring ring ribs, the first ring 45° main ribs, and the third ring ring ribs. Moreover, the single-layer reticulated shell is less vulnerable to the stress of the rest bars. The analysis of the first ring ring ribs among the key bars is conducted to gain the change of the shell’s ultimate load with the first ring ring ribs’ damage degree, as shown in table 1. It is revealed that when the damage of the first ring ring ribs is up to 90%, the structure can still support the design load of 0.9kN/m².

4.2.3. The Partition Method of Bars. Because the initial instability mode of the structure is consistent with the deformation of the first mode, the partition method based on the first mode strain is adopted to zone the bars. Figure 7 illustrates the element strain of the first-order mode. The element strain ranges from $-2.0 \times 10^{-3}$ to $1.8 \times 10^{-3}$, mainly from $-1.0 \times 10^{-5}$ to $1.0 \times 10^{-5}$. In particular, the identified minimum element strain is $-1.0 \times 10^{-7}$, and the maximum and minimum element strains differ by 4 orders of magnitude. Considering the number of elements, the elements are divided into six regions according to strain, as shown in figure 7. Areas P1 and P6 are the maximum compressive stress and tensile stress areas respectively. Figure 8 plots the distribution of elements in areas P1 and P6.
4.2.4. Damage Identification. Taking the first ring ring ribs as an example, the representative compression element B301 is selected, and the damage of elements is represented by the reduction of Young's modulus. The element B301 with the damage degree of 10% to 90% can be identified, and the damage degree interval is 10%. In the light of the analysis results in Section 3.2.2, only the element strain energy’s change rate in the first mode is calculated. The first ring ring ribs are located in area P1, which can be directly extracted for damage identification. Figure 9 plots the damage identification results of element B301 with 10% and 90% damage degree. Figure 10 shows that as damage degree rises, the change rate of element B301’s modal strain energy increases. Furthermore, the relationship between the element damage degree and ultimate load is shown in figure 11. In the case of damage degree less than 60%, the ultimate load declines monotonously with the degree of damage gradually serious, whereas in the case of damage degree more than 60%, the ultimate bearing capacity of the damaged structure weakens obviously. Therefore, special attention should be paid when the damage degree exceeds 60%.
Figure 9. Recognition result of different damage degree of element B301.

(a) Recognition result of 90% different damage degree of element B301.

(b) Recognition result of 10% different damage degree of element B301.

Figure 10. Relationship between the element strain energy’s change rate and damage degree.

Figure 11. Relationship between element damage degree and the ultimate load.
4.2.5. Stability Analysis of Damaged Structures. The stability of the damaged structure is accessible by the relation between the ultimate load and the element’s damage degree, or by establishing the damaged structure model.

5. Conclusion
In this paper, by using the numerical simulation method, taking the element strain energy’s change rate as the identification index, the damage identification of reticulated shell structure is carried out by using the partition progressive method. The following conclusions can be drawn:

1) The change rate of element strain energy can effectively identify damaged bars. With the degree of damage gradually serious, the modal strain energy’s change rate of key elements increases. For the K8 single-layer reticulated shell, the key bars are the first ring ring ribs, the first ring 45° main ribs, and the third ring ring ribs. The ultimate load of the damaged structure declines evidently with the increase of the key bars’ damage degree.

2) Because the whole structure contains a large number of elements, this paper adopts the partition method based on the first-order overall buckling modal strain energy. The elements are divided into six regions according to the strain, and then damage identification is carried out, which can lessen calculation burden and improve the accuracy of the results.

3) For the analysis process of damaged single-layer reticulated shell, firstly, the structure is loaded, then the key bars are found by the partition progressive method, and finally, the damage degree of the structure is determined according to the relationship between the element strain energy’s change rate and the element damage degree.

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