Neuro-adaptive cooperative tracking control with prescribed performance of unknown higher-order nonlinear multi-agent systems

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ABSTRACT
This paper is concerned with the design of a distributed cooperative synchronisation controller for a class of higher-order nonlinear multi-agent systems. The objective is to achieve synchronisation and satisfy a predefined time-based performance. Dynamics of the agents (also called the nodes) are assumed to be unknown to the controller and are estimated using neural networks. The proposed robust neuro-adaptive controller drives different states of nodes systematically to synchronise with the state of the leader node within the constraints of the prescribed performance. The nodes are connected through a weighted directed graph with a time-invariant topology. Only few nodes have access to the leader. Lyapunov-based stability proofs demonstrate that the multi-agent system is uniformly ultimately bounded stable. Highly nonlinear heterogeneous networked systems with uncertain parameters and external disturbances were used to validate the robustness and performance of the new novel approach. Simulation results considered two different examples: single-input single-output and multi-input multi-output, which demonstrate the effectiveness of the proposed controller.

1. Introduction
The use of collaborative autonomous robotic vehicles allows for greater flexibility and capacity as well as higher performance in areas such as surveillance, inspection, space explorations, communication, sensor deployment and many others. Multi-agent systems (MAS) distribute work in a logical manner and exchange information via self-formed local network, and hence, they are often called nodes. The network is named a communication graph formed by a set of nodes and the communication lines between different nodes are called edges. The graph can be directed or undirected. An undirected graph allows the information to flow in both directions. The connected nodes of such a graph own similar characteristics. On a directed graph or a digraph, the direction of the information flow is fixed. The direction is pointed from one node to another indicating how the information flows from one node to its neighbours. Moreover, the structure of the network can be fixed or variable.

The control of such MAS faces several practical as well as theoretical challenges (see for instance, Olfati-Saber & Murray, 2004). In particular, dynamics of the node can be nonlinear and unknown, the network bandwidth capacity is limited and may suffer from variable delays and loss of packets, the operating environment is changing and complex with presence of noise, the embedded computational resources are limited, etc. In the literature, several studies addressed either cooperative regulation problem, called consensus, or cooperative tracking problem, known as synchronisation (see for example, Fax & Murray, 2004). Recently, several control methods for higher-order non-linear MAS have been proposed. Synchronisation of passive nonlinear systems has been considered in Chopra and Spong (2006) while distributive tracking problem of node consensus has been studied extensively such as Lewis, Zhang, Hengster-Movric, and Das (2013), Olfati-Saber, Fax, and Murray (2007), Liao, Lu, and Liu (2016) and Zhang and Lewis (2012). Work of Das and Lewis (2010) and Cao and Ren (2012) studied cooperative tracking control for single node representing a single-input single-output (SISO) system with high-order dynamics. Due to unknown dynamics, Das and Lewis (2010) and Zhang and Lewis (2012) proposed a neuro-adaptive distributed control for heterogeneous agents connected through a digraph. Das and Lewis (2010) considered single integrator agents, and later on Zhang and Lewis (2012), high-order affine systems described in Brunovsky form and connected through a directed graph have been addressed. The authors assumed that the input function \( g_i(\cdot) \) is equal to one for each agent \( i = 1, \ldots, N. \)
In all previous studies, the input function was assumed to be known. On the other hand, adaptive distributed tracking control of affine systems has been studied assuming unknown input function by Theodoridis, Boutilis, and Christodoulou (2012) and extended in El-Ferik, Qureshi, and Lewis (2014). Also, consensus with saturation and dead-zone was examined in Shen, Shi, Shi, and Zhang (2016) and Shen and Shi (2016). Theodoridis et al. (2012) approximated the unknown nonlinear dynamics and input functions using a neuro-adaptive fuzzy and defined the output membership functions by a set of offline trials. All these previous studies mainly focused on ultimate stability of the error response. Most of the proposed controllers for highly nonlinear systems guarantee that the consensus tracking error is upper bounded due to uncertainties in dynamics and external disturbances. Consensus in error has been proven to be ultimately uniformly bounded and to converge into a residual set having a size that depends on some unknown but bounded sets. However, bounded sets represent uncertainties in dynamics and external disturbances. Therefore, it is almost impossible to make the prediction of transient performance as well as steady-state behaviour analytically (Bechlioulis & Rovithakis, 2008).

On the other hand, designing a cooperative adaptive control for a group of agents satisfying prescribed performance function (PPF) has some advantages. PPF forces the output error to begin within large set and steer systematically into an arbitrarily small set satisfying a known measure (Bechlioulis & Rovithakis, 2008; Hashim, El-Ferik, and Lewis, 2017). Under prescribed performance, the error should display some measure of dynamic features. For instance, convergence rate should obey a predefined value and a maximum value of overshoot or undershoot is not exceeding a given range. In addition to having the error dynamically bounded, prescribed performance-based controller for cooperative adaptive control is capable of reducing the control effort and improving its robustness. Upper and lower bounds of PPF should be defined appropriately in order to provide smooth tracking error with prescribed convergence. Neuro-adaptive control with PPF for strict feedback linearisable systems has been presented by Bechlioulis and Rovithakis (2008). Since then, several papers developed neuro-adaptive control with prescribed performance approximating the unknown nonlineairities and disturbances through linearly parametrised neural network (NN) (see for instance, Bu, Wu, Huang, & Wei, 2016; El-Ferik, Hashim, & Lewis, 2017; Yang, Ge, Wang, Li, & Hua, 2015). A model reference adaptive control with PPF has been proposed to avoid defining neural weights via trial and error methods (Mohamed, 2014). Most of the studies considered only single autonomous systems. However, just recently, Hashim et al. (2017) considered networked graph and proposed an adaptive cooperative control with prescribed performance for a first-order node dynamics with unknown nonlinearities. Zhang, Hua, and Guan (2016) addressed the problem of distributed output feedback consensus tracking control for leader following nonlinear MAS in strict-feedback form with PPF requirement. A similar work has been proposed by Shahvali and Askari (2016).

Indeed, the present proposed control scheme is developed using prescribed performance to satisfy transient and steady-state dynamic performance for each node’s state through synchronisation error. The data exchange between nodes is carried out according to a given directed graph. NN is used to estimate the unknown nonlinear dynamics. In addition, this paper considers the original prescribed performance scheme presented by Bechlioulis and Rovithakis (2008). Hence, the interactions between all nodes are considered in the consensus algorithm to track the leader trajectory and guarantee stable non-oscillatory dynamics.

The rest of the paper is organised as follows. Section 2 presents graph theory preliminaries and math notations. In Section 3, problem formulation, the associated local error synchronisations and prescribed performance characteristics are formulated. Section 4 develops the control law in order to prove stability of the directed connected graph and satisfy prescribed performance characteristics. Section 4 also presents the neural approximation and stability of the control design of distributed agents based on neural approximation. Section 5 illustrates results which guarantee effectiveness and robustness of the proposed control for SISO and multi-input multi-output (MIMO) problems. Finally, conclusion and future directions of research are given in Section 6.

2. Preliminaries

2.1. Mathematical identities

Throughout this paper, the set of real numbers is denoted as $\mathbb{R}$; $n$-dimensional vector space as $\mathbb{R}^n$; the space span by $n \times m$ matrix as $\mathbb{R}^{n \times m}$; identity matrix of order $m$ as $I_m$; absolute value as $|\cdot|$. For $x \in \mathbb{R}^n$, the Euclidean norm is given as $\|x\| = \sqrt{x^\top x}$ and matrix Frobenius norm is given as $\|\cdot\|_F$. For any $x_i \in \mathbb{R}^n$ we have $x_i = [x_{i1}, \ldots, x_{in}]^\top$ for $i = 1, \ldots, N$, and for $x^j \in \mathbb{R}^N$ we have $x^j = [x^j_1, \ldots, x^j_N]$ for $j = 1, \ldots, n$. Trace of associated matrix is denoted as $\text{Tr} (\cdot)$, $\text{diag} (\cdot)$ denotes the diagonal of associated matrix, $\mathcal{N}$ is the set $\{1, \ldots, N\}$ and $1_N$ is a unity vector $[1, \ldots, 1]^\top \in \mathbb{R}^N$. $A$ is said to be positive definite if $A > 0$ for $A \in \mathbb{R}^{n \times n}$; $A \succeq 0$ indicates positive semidefinite; $\sigma (\cdot)$ is the set of singular values of a matrix with
maximum value $\bar{\sigma}(\cdot)$ and minimum value $\underline{\sigma}(\cdot)$. Finally, $\otimes$ denotes the Kronecker product.

### 2.2. Basic graph theory

A graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a nonempty finite set of nodes (or vertices) $\mathcal{V} = \{V_1, V_2, \ldots, V_N\}$, and a set of edges (or arcs) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. $(V_i, V_j) \in \mathcal{E}$ if there is an edge from node $i$ to $j$. Topology of a weighted graph is described by the adjacency matrix $A = [a_{i,j}] \in \mathbb{R}^{N \times N}$ with weights $a_{i,j} > 0$ if $(V_j, V_i) \in \mathcal{E}$; otherwise $a_{i,j} = 0$. Throughout the paper, a directed graph is called diagraph. Also, the topology is fixed where $A$ is time-invariant and the self-connectivity element $a_{i,i} = 0$. A graph can be directed or undirected. The weighted in-degree of a node $i$ is given by the sum of $i$th row of $A$, i.e. $d_i = \sum_{j=1}^{N} a_{i,j}$. Also, the diagonal in-degree matrix is $D = \text{diag}(d_1, \ldots, d_N) \in \mathbb{R}^{N \times N}$ and the graph Laplacian matrix $L = D - A$. The set of neighbours of a node $i$ is $N_i = \{j|(V_j, V_i) \in \mathcal{E}\}$. If node $j$ is a neighbour of node $i$, then node $i$ can get information from node $j$, but not necessarily vice versa. For undirected graph, neighbourhood is a mutual relation. A direct path from node $i$ to node $j$ is a sequence of successive edges in the form $\{ (V_i, V_k), (V_k, V_l), \ldots, (V_m, V_j) \}$. If there is a node such that there is a directed path from one node to every other node in the graph, then the diagraph has a spanning tree. If for any ordered pair of nodes $[V_i, V_j]$ with $i \neq j$, then a diagraph is strongly connected and there is a directed path from node $i$ to $j$ (Ren & Beard, 2008).

### 3. Problem formulation in prescribed performance

Let the nonlinear dynamics of the $i$th node be given by

\[
\begin{align*}
\dot{x}_i^1 & = x^2_i \\
\dot{x}_i^2 & = x^3_i \\
\vdots & \quad \vdots \\
\dot{x}_{i,M_i} & = f_i(x_i) + G_iu_i \\
y_i & = x_i^1
\end{align*}
\]  

(1)

where $x_i^{m_p} \in \mathbb{R}^P$ is the $m_p$th state variable of the leader where $x_0 = [x_0^1, \ldots, x_0^{M_0}]^\top \in \mathbb{R}^{PM_0}$ and the nonlinear function vector $f_i : \mathbb{R}^{P \times M_i} \rightarrow \mathbb{R}^P$ is unknown vector and Lipschitz. The global dynamics of Equation (1) can be described by

\[
\begin{align*}
\dot{x}_1 & = x^2 \\
\dot{x}_2 & = x^3 \\
\vdots & \quad \vdots \\
\dot{x}_M & = f(x) + Gu \\
y & = x_1
\end{align*}
\]  

(2)

where $x^{m_p} = [x_1^{m_p}, \ldots, x_N^{m_p}]^\top \in \mathbb{R}^{PN}$, $u = [u_1^\top, \ldots, u_N^\top]^\top \in \mathbb{R}^{PN}$, $G = \text{diag}(G_i) \in \mathbb{R}^{PN \times PN}$, $y = [y_1, \ldots, y_N]^\top \in \mathbb{R}^{PN}$, $i = 1, \ldots, N$ and $f(x) = [f_1(x_1), \ldots, f_N(x_N)]^\top \in \mathbb{R}^{PN}$. The leader state vector can be time-varying and is noted as $x_0$. It can be considered as an ecosystem defining the desired consensus trajectory. Let us define the leader dynamics by

\[
\begin{align*}
\dot{x}_0^1 & = x_0^2 \\
\dot{x}_0^2 & = x_0^3 \\
\vdots & \quad \vdots \\
\dot{x}_0^{M_p} & = f_0(t, x_0) \\
y_0 & = x_0^1
\end{align*}
\]  

(3)

where $x_0^{m_p} \in \mathbb{R}^P$ is the $m_p$th state variable of the leader where $x_0 = [x_0^1, \ldots, x_0^{M_0}]^\top \in \mathbb{R}^{PM_0}$ and the leader nonlinear function vector $f_0 : [0, \infty) \times \mathbb{R}^{P \times M_p} \rightarrow \mathbb{R}^P$ is piecewise continuous in $t$ and locally Lipschitz. The disagreement variable for node $i$ is $\delta_i^1 = x_i^1 - x_0^1$ and the global disagreement order is

\[
\gamma_{M_p}^i = x_i^1 - x_0^1
\]  

(4)

where $\gamma_{M_p}^i = [\gamma_1^1, \ldots, \gamma_N^1]^\top \in \mathbb{R}^{PN}$, $\delta_0^1 = [x_0^1, \ldots, x_0^1]^\top \in \mathbb{R}^{PN}$. In this paper, local distributed state information is assumed to be known on the communication graph for $i$th node, and the only given information of the neighbourhood synchronisation as in Li, Wang, and Chen (2004) and Khoo, Xie, and Man (2009) is

\[
e_i = \sum_{j \in N_i} a_{i,j} (x_j^1 - x_i^1) + b_{i,i} (x_i^1 - x_0^1),
\]  

(5)

where $e_i = [e_1^1, \ldots, e_N^1]^\top \in \mathbb{R}^P$, $a_{i,j} \geq 0$ and $a_{i,i} > 0$ in case of agent $i$ is directed to agent $j$, $b_i \geq 0$ and $b_i > 0$ for one or more agents $i$ are directed to the leader. $e^\top = [e_1^1, \ldots, e_N^1]^\top \in \mathbb{R}^PN$, $p = 1, \ldots, P$ and $B = \text{diag}(b_i) \in \mathbb{R}^{N \times N}$. The global error dynamics for SISO system can be driven from Equation (5) to be

\[
e = -(L + B) (x_0^1 - x^1) = (L + B) (x^1 - x_0^1)
\]  

(6)
Thereby, the error dynamics of Equation (6) can be written in the global form such as
\[
\dot{e}^1 = e^2 \\
\dot{e}^2 = e^3 \\
\vdots \\
\dot{e}^{M+} = (L + B)(f(x) + Gu - f_0)
\]
(7)

Note that \( f_0 = [f_0(t, x_0), \ldots, f_0(t, x_0)]^T \in \mathbb{R}^N \). The proof of Equation (7) can be found in Lewis et al. (2013).

**Remark 3.1:** Global error dynamics of Equation (6) for MIMO systems in case of \( P > 1 \) becomes
\[
e = -((L+B) \otimes I_P)(x_1 - x^1) \\
= ((L+B) \otimes I_P)(x^1 - x_1)
\]
(8)

and similarly, Equation (7) for the MIMO case can be written as
\[
\dot{e}^1 = e^2 \\
\dot{e}^2 = e^3 \\
\vdots \\
\dot{e}^{M+} = ((L+B) \otimes I_P)\left(f(x) + Gu - f_0\right)
\]
with \( \otimes \) as the Kronecker product and \( I_P \in \mathbb{R}^{P \times P} \) as the identity matrix.

**Remark 3.2:** The networked graph is strongly connected. Therefore, if there is one or more nodes \( i, i = 1, \ldots, N \) such that \( b_i \neq 0 \), then the matrix \( (L + B) \) is an irreducible diagonally dominant M-matrix. Thus, it is nonsingular (Qu, 2009).

For the case of graph is strongly connected, \( B \neq 0 \) and the \( ||e_0|| \) is
\[
||e_0|| \leq \frac{||e||}{\sigma(L+B)}
\]
(10)

such that \( \sigma (L + B) \) denotes the minimum singular value of matrix \( L + B \).

### 3.1 Prescribed performance

The objective of this section is to introduce the PPF into the control algorithm. PPF is a time function enables the tracking error \( e(t) \) to start within a known large set and reduce in a systematic manner to a known narrow set (Bechlioulis and Rovithakis, 2008; Hashim et al., 2017). Providing smooth tracking response with allocated properties and improving the control signal range are classified as distinguished features of the control algorithm with PPF.

Consider the performance function of a single agent system with \( \rho(t) \) is a smooth function includes the error component \( e(t) \) such as \( \rho(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is a decreasing positive function \( \lim_{t \rightarrow \infty} \rho(t) = \rho_{\infty} > 0 \) where \( \rho_{\infty} > 0 \) is a constant and refers to the smaller set upper bound. Now, the general PPF of Equation (5) can be described as
\[
\rho_i^p(t) = (\rho_{i,0}^p - \rho_{i,\infty}^p) \exp( - \delta_i^p t ) + \rho_{i,\infty}^p
\]
(11)

for all \( t \geq 0 \) and \( 0 \leq \delta_i^p \leq 1, i = 1, \ldots, N \) and \( p = 1, \ldots, P \). The control algorithm should consider the interactions between agents’ dynamics which may lead to instability. The systematic convergence of the tracking error \( e_i^p(t) \) between the constraint bounds \( \rho_i^p(t) \) and \( -\delta_i^p \rho_i^p(t) \) or \( -\rho_i^p(t) \) and \( \delta_i^p \rho_i^p(t) \) should obey the transient trajectory of these foregoing bounds as revealed in Figure 1. In fact, Figure 1 illustrates the full idea of prescribed performance such that the error will be tracked systematically from a predefined bigger set to a given smaller set.

A transformed error will be defined to drive the error dynamics from constrained bounds in Equations (12) and (13) into an unconstrained one as follows:
\[
e_i^p(t) = \gamma \left( \frac{\rho_i^p(t)}{\rho_i^p(t)} \right)
\]
(14)

or equivalently,
\[
e_i^p(t) = \rho_i^p(t) F(e_i^p(t))
\]
(15)

where \( e_i^p, F(\cdot) \) and \( \gamma^{-1}(\cdot) \) are smooth functions, \( i = 1, 2, \ldots, N \). For simplification, let us denote \( x = x(t), \rho = \rho(t), e = e(t) \) and \( e_i^p(t) \). \( F(\cdot) = \gamma^{-1}(\cdot) \) and \( F(\cdot) \) satisfy the following properties:

1. \( F(e_i^p) \) is smooth and strictly increasing.
2. \( -\delta_i^p < F(e_i^p) < \delta_i^p, \) if \( e_i^p(0) \geq 0 \)

\[
-\delta_i^p < F(e_i^p) < \delta_i^p, \) if \( e_i^p(0) < 0 \)
\[
\begin{align*}
\lim_{\epsilon_i^p \to +\infty} \mathcal{F}(\epsilon_i^p) &= -\delta_i^p \quad \text{if } \epsilon_i^p (0) \geq 0 \\
\lim_{\epsilon_i^p \to -\infty} \mathcal{F}(\epsilon_i^p) &= \delta_i^p \quad \text{if } \epsilon_i^p (0) < 0
\end{align*}
\]

for \( \delta_i^p, \delta_i^p \in \mathbb{R}_+ \) are known constants. These constants should be defined to satisfy

\[
\mathcal{F}(\epsilon_i^p) = \begin{cases} 
\frac{\delta_i^p \exp(\epsilon_i^p) - \delta_i^p \exp(-\epsilon_i^p)}{\exp(\epsilon_i^p) + \exp(-\epsilon_i^p)}, & \delta_i^p > \delta_i^p \text{ if } \epsilon_i^p (0) \geq 0 \\
\frac{\delta_i^p \exp(\epsilon_i^p) - \delta_i^p \exp(-\epsilon_i^p)}{\exp(\epsilon_i^p) + \exp(-\epsilon_i^p)}, & \delta_i^p > \delta_i^p \text{ if } \epsilon_i^p (0) < 0
\end{cases}
\]

\[\text{(16)}\]

Now, consider the smooth function

\[
\mathcal{F}(\epsilon_i^p) = \delta_i^p \exp(\epsilon_i^p) - \delta_i^p \exp(-\epsilon_i^p) \quad \text{exp}(\epsilon_i^p) + \exp(-\epsilon_i^p)
\]

\[\text{(17)}\]

and the transformed error

\[
\epsilon_i^p = \mathcal{F}^{-1}(\epsilon_i^p / \rho_i^p)
\]

\[
= \begin{cases} 
\ln \frac{\delta_i^p + \epsilon_i^p / \rho_i^p}{\delta_i^p - \epsilon_i^p / \rho_i^p}, & \delta_i^p > \delta_i^p \text{ if } \epsilon_i^p (0) \geq 0 \\
\ln \frac{\delta_i^p + \epsilon_i^p / \rho_i^p}{\delta_i^p - \epsilon_i^p / \rho_i^p}, & \delta_i^p > \delta_i^p \text{ if } \epsilon_i^p (0) < 0
\end{cases}
\]

\[\text{(18)}\]

Therefore, the derivative of the transformed error in Equation (18) will be

\[
\dot{\epsilon_i^p} = \frac{1}{2 \rho_i^p} \left( \frac{1}{\delta_i^p + \epsilon_i^p / \rho_i^p} + \frac{1}{\delta_i^p - \epsilon_i^p / \rho_i^p} \right) \left( \epsilon_i^p - \frac{\epsilon_i^p \dot{\epsilon_i^p}}{\rho_i^p} \right)
\]

\[\text{(19)}\]

where \( \epsilon_i \in \mathbb{R}^p \) and from Equation (19), we define new variable \( r_i^p \) such as

\[
r_i^p = \frac{1}{2 \rho_i^p} \frac{\partial \mathcal{F}^{-1}(\epsilon_i^p / \rho_i^p)}{\partial (\epsilon_i^p / \rho_i^p)}
\]

\[\text{(20)}\]

For further explanations, we define a new component \( E_i \in \mathbb{R}^p \) such that \( E_i \) is a metric error that can be described as

\[
E_i = \left( \frac{d}{dt} + \lambda_i^m \right) e_i^{m_p} , \quad i = 1, \ldots, N \\
M_p = 1, \ldots, M_p
\]

\[\text{(21)}\]

where \( \lambda_i^m \) is a positive constant, alternatively, Equation (21) is equivalent to

\[
E_i = \epsilon_i^{M_p} + \lambda_i^{M_p-1} \epsilon_i^{M_p-1} + \cdots + \lambda_i^1 \epsilon_i^1
\]

\[\text{(22)}\]

One can write the global form of Equation (22) as

\[
\mathbf{E} = e^{M_p} + \lambda^{M_p-1} e^{M_p-1} + \cdots + \lambda^1 e^1
\]

\[\text{(23)}\]
where \( e^m_p = [\varepsilon^1_p, \ldots, \varepsilon^N_p]^T \), \( m_p = 1, \ldots, M_p \). Let us define

\[
\Phi_1 = [\varepsilon^1, \varepsilon^2, \ldots, \varepsilon^{M_p-1}]^T \quad (24)
\]

\[
\Phi_2 = \Phi_1 = [\varepsilon^2, \varepsilon^3, \ldots, \varepsilon^{M_p}]^T
\]

\[l = [0, 0, \ldots, 0, 1]^T \in \mathbb{R}^{M_p-1} \quad (25)
\]

and

\[
\Lambda = \\
\begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 1 \\
-\lambda^1 & -\lambda^2 & -\lambda^3 & \ldots & \lambda^{M_p-2} & -\lambda^{M_p-1}
\end{bmatrix}
\]

such that \( \Lambda \) is Hurwitz; hence, from Equations (24) and (25), one can say

\[
\Phi_2 = \Phi_1 \Lambda^T + E\Lambda^T \quad (26)
\]

and

\[
\Lambda^T M + M\Lambda = -\beta \mathbb{I}_{M_p-1} \quad (27)
\]

where \( \beta \) is a positive constant, \( M > 0 \) and \( \mathbb{I}_{M_p-1} \in \mathbb{R}^{(M_p-1) \times (M_p-1)} \) is the identity matrix. Consider each of Equations (7) and (18), the derivative of the metric error in Equation (21) with respect to time is given by

\[
\dot{\mathbf{E}}_i = \sum_{j=1}^{M_p-1} \left[ \frac{M_p - 1}{j} \right] \lambda_j^{M_p-j} e_i^j + e_i^M_p \quad (28)
\]

with \( \tilde{\lambda} = [\lambda^1, \ldots, \lambda^{M_p-1}]^T \in \mathbb{R}^{P\times N}, \ p = 1, \ldots, P \), and Equation (28) can be written in the global form for SISO systems as

\[
\dot{\mathbf{E}} = e^{M_p+1} + \Phi_2 \tilde{\lambda} \\
= R (L + B) \left( f(x) + Gu - f_d \right) + \Delta + \Phi_2 \tilde{\lambda} \quad (29)
\]

and for MIMO case

\[
\dot{\mathbf{E}} = R ((L + B) \otimes \mathbb{I}_p) \left( f(x) + Gu - f_d \right) + \Delta + \Phi_2 \tilde{\lambda} \quad (30)
\]

where \( \mathbf{E} = [\mathbf{E}_1, \ldots, \mathbf{E}_N]^T \in \mathbb{R}^{PN}, e^{M_p+1} = e^{M_p+1} + \Delta \) and \( \Delta \) is the function of higher orders of \( \rho^p_i, r^p_i \). It should be remarked that the higher orders of \( \rho^p_i, r^p_i \) are vanishing components with time which by the way lead to \( \Delta = 0 \) as \( t \to \infty \). Also, \( R = \text{diag} \{ \Omega \} \in \mathbb{R}^{PN \times PN} \) and

\[
\Omega_i = \\
\begin{bmatrix}
1 & \frac{\partial F^{-1}(\varepsilon^i/\rho^i)}{\partial (\varepsilon^i/\rho^i)} & \cdots & 0 \\
\frac{2\rho^i}{\partial (\varepsilon^i/\rho^i)} & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 1 & \frac{\partial F^{-1}(\varepsilon^p/\rho^p)}{\partial (\varepsilon^p/\rho^p)}
\end{bmatrix}
\]

Note that \( \Omega_i \) is a decreasing diagonal matrix with \( \Omega_i > 0 \). Let us recall the following definitions (see Das & Lewis, 2010)

**Definition 3.1:** The global neighbourhood error \( e(t) \in \mathbb{R}^{PN} \) is uniformly ultimately bounded (UUB) if there exists a compact set \( \Psi \subset \mathbb{R}^{PN} \) so that \( \forall t \geq t_0 \in \Psi \) exists a bound \( B \) and a time \( t_f(B, e(t_0)) \), both independent of \( t_0 \geq 0 \), such that \( \| e(t) \| \leq B \) so that \( \forall t > t_0 + t_f \).

**Definition 3.2:** The control node trajectory \( x_0(t) \) given by Equation (1) is cooperative UUB with respect to solutions of node dynamics (3); if there exists a compact set \( \Psi \subset \mathbb{R}^{PN} \) so that \( \forall \{x_0(t_0) - x_0(t_0)\} \in \Psi \), there exist a bound \( B \) and a time \( t_f \), both independent of \( t_0 \geq 0 \), such that \( \| x(t_0) - x_0(t_0) \| \leq B \), \( \forall t \geq t_0 + t_f \).

4. **Neural approximation and distributed control in prescribed performance**

4.1 **Neural approximations**

NN with linear weights are used to approximate the unknown nonlinear dynamics of local agents in Equation (1) as

\[
f_i(x_i) = W_i^T \phi_i(x_i) + \alpha_i \quad (31)
\]

where \( \phi_i(x_i) \in \mathbb{R}^{n \times 1} \) and \( \dot{v}_i \) is a sufficient number of neurons at each node, \( W_i \in \mathbb{R}^{n \times P} \) and \( \alpha_i \in \mathbb{R}^{P \times 1} \) is the approximated error vector. It should be remarked that based on Hornik, Stinchcombe, and White (1989) and Lewis, Jagannathan, and Yesildirak (1998), nonlinearities could be approximated via variety of sets such as radial basis functions (Poggio & Girosi, 1990), sigmoid functions (Cotter, 1989), etc.

The objective of this work is to use the available information to track local performance behaviour of each node and to compensate unknown nonlinearities. Thereby, the unknown nonlinearities of the local nodes can be approximated by

\[
\hat{f}_i(x_i) = \hat{W}_i^T \phi_i(x_i) \quad (32)
\]
where \( \hat{W}_i \in \mathbb{R}^{n_i \times p} \) and \( \hat{f}_i (x_i) \in \mathbb{R}^p \) approximate the component \( f_i (x_i) \). The description of the global synchronisation of the graph \( G \) could be defined by

\[
f(x) = W^\top \phi(x) + \alpha \tag{33}
\]

where \( \phi(x) = [\phi_1(x_1), \ldots, \phi_N(x_N)]^\top, i = 1, \ldots, N, W = \text{diag}[W_i], \alpha = [\alpha_1, \ldots, \alpha_N]^\top \) and the global estimate of \( f(x) \) is

\[
\hat{f}(x) = \hat{W}^\top \phi(x) \tag{34}
\]

with \( \hat{W}^\top = \text{diag}[\hat{W}_i] \). The error between true and estimated nonlinearities is defined as

\[
\hat{f}(x) = f(x) - \hat{f}(x) = \hat{W}^\top \phi(x) + \alpha \tag{35}
\]

such as \( \hat{W} = W - \hat{W} \).

### 4.2 Neuro-adaptive control design with PPF of distributed agents

There are several assumptions that should be considered (Zhang & Lewis, 2012).

**Assumption 4.1:**

1. NN weights are bounded but otherwise, they are unknown such as \( \|W\| \leq W_M \) with \( W_M \) is a fixed bound.
2. Leader states \( \|x_0\| \leq X_0 \) are bounded with \( X_0 \) as a finite bound.
3. The unknown nonlinear dynamics associated to the leader is bounded by \( \|f(x_0, t)\| \leq F_M \).
4. The activation function is finite such as \( \|\phi\| \leq \phi_M \).

**Lemma 4.1 Qu (2009):** Consider \( L \) is an irreducible matrix with \( (L + B) \) as a nonsingular matrix such as \( B \neq 0 \); hence, there is

\[
q = [q_1, \ldots, q_N]^\top = (L + B)^{-1} \cdot \mathbb{1} \tag{36}
\]

\[
\mathcal{M} = \text{diag}[m_i] = \text{diag}[1/q_i] \tag{37}
\]

Then, \( \mathcal{M} > 0 \) and the matrix \( \mathcal{Q} \) is defined as

\[
\mathcal{Q} = \mathcal{M} (L + B) + (L + B)^\top \mathcal{M} \tag{38}
\]

**Remark 4.1:** For simplification, the control signal and the estimated weights of NN throughout the research paper will be developed for SISO systems. In case of MIMO systems, these developments can be easily modified by introducing the Kronecker product as will be illustrated later. Two different simulation for SISO and MIMO systems will be presented.

Consider a control signal of each local nodes to be

\[
u_i = B_i^{-1} \left( -cE_i - \hat{W}_i^\top \phi_i (x_i) - (d_i + b_i)^{-1} \Omega_i^{-1} (\lambda_i^{M_i} - 1 + \rho_i M_i + \ldots + \lambda_i^{M_i^2} - 1) \right) \tag{39}
\]

where \( c > 0 \) is a control gain and \( \Omega_{N+1} = [1, \ldots, 1]^\top \in \mathbb{R}^{N \times 1} \). The control input is defined by

\[
\nu = G^{-1} \left( -cE - \hat{W}^\top \phi(x) - (D + B)^{-1} R^{-1} \Phi_2 \tilde{\lambda} \right) \tag{40}
\]

Then, the estimated weights of NN is defined by

\[
\dot{\hat{W}}_i = F_i \phi_i E_i^\top m_i \Omega_i (d_i + b_i) - k \hat{f}_i \hat{W}_i \tag{41}
\]

with \( F_i \in \mathbb{R}^{n_i \times n_i}, F_i = \Pi_i \|\phi\|_2 \) and \( \Pi_i > 0 \) are positive gains and \( k > 0 \) is a scalar gain. \( c \) and \( k \) should be selected to satisfy Equation (42).

**Theorem 4.1:** Consider the distributed system in Equation (1) and the leader dynamics in Equation (3). If Assumption 4.1 holds and the distributed control is as in Equation (40) and the NN tuning law as in Equation (41), then the control variable \( c \) should satisfy

\[
c > \frac{1}{\sigma(M) \sigma(R)} \left( \frac{\gamma^2}{k} + \frac{2}{\beta} g^2 + v \right) \tag{42}
\]

\[
\gamma = -\frac{1}{2} \Phi \tilde{\lambda} (\lambda) \Phi \tilde{\lambda} (A), \quad g = -\frac{1}{2} (\sigma(M) + \tilde{\sigma}(M) \tilde{\lambda}(A)) \quad \text{and} \quad v = \tilde{\sigma}(M) \tilde{\lambda}(A) \|\tilde{\lambda}\|, \quad \text{where} \quad \tilde{\lambda} \quad \text{was defined in Equation (27) for} \quad \beta > 0. \quad \text{Hence, the trajectory of} \quad x_0(t) \quad \text{is uniformly ultimate bounded. Also, all nodes steer close to} \quad x_0(t) \quad \text{for all} \quad t \geq 0.
\]

**Proof:** Based on Equation (33), Equation (7) becomes

\[
\dot{e}^M = (L + B)(W^\top \phi(x) + \alpha + Gu - \phi(x_0, t)) \tag{43}
\]

Consider the result in Equations (34) and (35), and one can write Equation (43) as

\[
\dot{e} = (L + B) \left( \hat{W}^\top \phi(x) + \alpha - cE - (D + B)^{-1} R^{-1} \Phi_2 \tilde{\lambda} - \phi(x_0, t) \right) \tag{44}
\]

and from Equations (29) and (44), the transformed error could be obtained as

\[
\dot{\hat{E}} = R (L + B) \left( \hat{W}^\top \phi(x) + \alpha - cE - (D + B)^{-1} R^{-1} \Phi_2 \tilde{\lambda} - \phi(x_0, t) \right) + \Delta + \Phi_2 \tilde{\lambda} \tag{45}
\]
Consider the following Lyapunov candidate function

\[
V = \frac{1}{2} E^T M E + \frac{1}{2} \text{Tr}[\tilde{W}^T F^{-1} \tilde{W}] + \frac{1}{2} \text{Tr} \left\{ \Phi_1 M \Phi_1^T \right\}
\]

\[
= V_1 + V_2 + V_3
\]  

(46)

with \( V_1 = \frac{1}{2} E^T M E \), \( V_2 = \frac{1}{2} \text{Tr}[\tilde{W}^T F^{-1} \tilde{W}] \), \( V_3 = \frac{1}{2} \text{Tr} \left\{ \Phi_1 M \Phi_1^T \right\} \), and \( M \succ 0 \) is as defined in Lemma 4.1 and \( F^{-1} = \text{diag} \{ F_i^{-1} \} \) is a zero matrix with positive components in diagonal as in Equation (41). Then, \( V_1 \) and \( V_2 \) after substitution of Equation (40) are

\[
\dot{V}_1 + \dot{V}_2 = E^T M R (L + B) (\tilde{W}^T \phi(x) + \alpha - c E) - (D + B)^{-1} R^{-1} \Phi_2 \lambda - f(x_0, t) + E^T M (\Delta + \Phi_2 \lambda) + \text{Tr} \left[ \tilde{W}^T F^{-1} \tilde{W} \right]
\]  

(47)

\[
\dot{V}_1 + \dot{V}_2 = -c E^T M R (L + B) E + E^T M R (D + B) \tilde{W}^T \phi(x) - E^T M R A \left( \tilde{W}^T \phi(x) + (D + B)^{-1} R^{-1} \Phi_2 \lambda \right) + E^T M R (L + B) (\alpha - f(x_0, t)) + E^T M \Delta + \text{Tr} \left[ \tilde{W}^T F^{-1} \tilde{W} \right]
\]  

(48)

Let \( T_M = \alpha_M - f_M \) such that

\[
\dot{V}_1 + \dot{V}_2 \leq -\left( c \sigma(R) \sigma(Q) - \frac{\tilde{\sigma}(M) \tilde{\sigma}(A)}{\sigma(D + B)} \| \tilde{\lambda} \| \right) \| E \|^2
\]

\[
- k \| \tilde{W} \|^2_F + \left( - \tilde{\sigma}(A) \phi_M(R) \| \tilde{W} \|_F + \frac{\tilde{\sigma}(A) \| A \|_F \| \Phi_1 \|}{\sigma(D + B)} + \tilde{\sigma}(\Delta) \right)
\]

\[
+ \tilde{\sigma}(R) \tilde{\sigma}(L + B) T_M \| \tilde{\sigma}(M) \| \| E \|
\]

\[
+ k W_M \| \tilde{W} \|_F
\]  

(51)

Now, the derivative of the third Lyapunov term \( V_3 \) is

\[
\dot{V}_3 = \frac{1}{2} \text{Tr} \left\{ \Phi_1 M \Phi_1^T + \Phi_1 M \Phi_1^T \right\}
\]

\[
= \text{Tr} \left\{ \Phi_2 M \Phi_1^T \right\}
\]  

(55)

and substituting Equation (26) in Equation (55) gives

\[
\dot{V}_3 = \text{Tr} \left\{ \Phi_1 \Lambda^T M \Phi_1^T \right\} + \text{Tr} \left\{ E^T M \Phi_1^T \right\}
\]  

(56)

One can write the derivative of the complete form in Equation (47) as

\[
\dot{V}_1 + \dot{V}_2 + \dot{V}_3 \leq -\left( c \sigma(R) \sigma(Q) - \frac{\tilde{\sigma}(M) \tilde{\sigma}(A)}{\sigma(D + B)} \| \tilde{\lambda} \| \right) \| E \|^2
\]

\[
- \frac{1}{2} \beta \| \tilde{\lambda} \|^2
\]

\[
- k \| \tilde{W} \|^2_F - \frac{1}{2} \beta \| \Phi_1 \|^2
\]  

(57)
We have \( \dot{V} \leq 0 \) if \( \|z\| > \eta \), according to Equation (46), we have

\[
\frac{1}{2} z^T \begin{bmatrix}
\tilde{\sigma}(M) & 0 & 0 \\
0 & \tilde{\sigma}(F) & 0 \\
0 & 0 & \tilde{\sigma}(M)
\end{bmatrix} z \leq V
\]

\[
\leq \frac{1}{2} z^T \begin{bmatrix}
\tilde{\sigma}(M) & 0 & 0 \\
0 & \tilde{\sigma}(F) & 0 \\
0 & 0 & \tilde{\sigma}(M)
\end{bmatrix} z
\]  

Define the appropriate variables matched with Equation (63) to write

\[
\frac{1}{2} \dot{z}^T z \leq V \leq \frac{1}{2} \tilde{z}^T \tilde{z}
\]

Accordingly, it is equivalent to

\[
\frac{1}{2} \tilde{\sigma}(\tilde{x}) \|z\|^2 \leq V \leq \frac{1}{2} \tilde{\sigma}(\tilde{x}) \|z\|^2
\]

\[
V > \frac{1}{2} \tilde{\sigma}(\tilde{x}) \frac{\|h\|^2}{\tilde{\sigma}^2(H)}
\]

Hence, based on Theorem 4.18 in Khalil (2002), for any the initial value \( z(t_0) \), there exists \( T_0 \) such that

\[
z(t) < \sqrt{\frac{\tilde{\sigma}(\tilde{x})}{\sigma(\tilde{x})}} \eta, \forall t \geq t_0 + T_0
\]

The time \( T_0 \) can be evaluated by

\[
T_0 = \frac{V(t_0) - \tilde{\sigma}(\tilde{x}) \eta^2}{k}
\]

And according to Equation (64), one can find

\[
\|z\| \leq \sqrt{\frac{2V}{\tilde{\sigma}(\tilde{x})}}, \|z\| \geq \sqrt{\frac{2V}{\tilde{\sigma}(\tilde{x})}}
\]

Therefore, \( \dot{V} \) in Equation (59) can be given by

\[
\dot{V} \leq -\tau_1 V + \tau_2 \sqrt{V}
\]

with \( \tau_1 = \frac{2e(H)}{\tilde{\sigma}(\tilde{x})} \) and \( \tau_2 = \frac{\sqrt{2\|h\|}}{\sqrt{\tilde{\sigma}(\tilde{x})}} \) yield

\[
\sqrt{V} \leq \sqrt{V(0)} + \frac{\tau_2}{\tau_1}
\]

It can be concluded that \( \epsilon \) is \( L_\infty \) for \( t \geq t_0 \) in a compact set \( \mathcal{W}_0 = \{ \epsilon(t) \mid \| \epsilon(t) \| \leq \epsilon_0 \} \). Consequently, \( \epsilon(t) \) will satisfy the prescribed performance for all \( t \geq 0 \) if we start at \( t = t_0 \) within the prescribed functions.
Remark 4.2: The control signal for high-order MIMO systems can be written as

\[ u_i = B_i^{-1} \left( -cE_i - \hat{W}_i^\top \phi_i(x_i) \right. \]
\[ \left. - \left( (d_i + b_i)^{-1} \otimes I_P \right) \Omega_i^{-1} \left( \lambda_i^{M_i-1} \lambda_i^{M_i} + \cdots + \lambda_i^1 \right) \right) \]

(71)

and the estimated weights of NN can be defined by

\[ \dot{\hat{W}}_i = F_i \phi_i E_i^\top m_i \Omega_i ((d_i + b_i)^{-1} \otimes I_P) - kF_i \hat{W}_i \]

(72)

In brief, the proposed algorithm of nonlinear high-order agent dynamics as in Equation (1) can be summarised by

Step 1. Select the setting parameters \( \delta_i^p, \delta_i^r, \rho_i, \varepsilon_i, \Pi_i, k \) and \( c \).
Step 2. Obtain the synchronised local error \( e_i^p \) from Equation (5) or Equation (7).
Step 3. Evaluate the PPF \( \rho_i^p \) from Equation (11).
Step 4. Evaluate \( r_i^p \) from Equation (20).
Step 5. Obtain the transformed error from Equation (20) starting from \( \varepsilon^1 \) to \( \varepsilon^{M_r} \).
Step 6. Evaluate the metric error \( E_i \) from Equation (21) or Equation (22).
Step 7. The control signal \( u_i \) from Equation (39) or Equation (71).
Step 8. Find \( \hat{W}_i \) from Equation (41) or Equation (72).
Step 9. Go to Step 2.

5. Simulation results

Problem 5.1: Consider the connected network in Figure 2 includes a group of five nodes with node (3) is connected to the leader node (0) (Zhang & Lewis, 2012). The connected network is composed of five agents denoted by 1 to 5 with one leader denoted by 0 and the leader node is connected to agents 1 and 5 as in Figure 2.

The SISO dynamics of the graph are high-order nonlinear such as

\[ \dot{x}_i^1 = x_i^1 \]
\[ \dot{x}_i^2 = x_i^2 \]
\[ \dot{x}_i^3 = f_i(x_i) + u_i \]

such that \( i = 1, 2, \ldots, 5 \) with nonlinear dynamics

\[
\begin{align*}
    f_1 &= x_1^1 \sin(x_1^1) + \cos(x_1^1)^2, \\
    f_2 &= - (x_2^1)^2 x_2^2 + 0.01x_2^1 - 0.01 (x_2^1)^3,
\end{align*}
\]

Figure 3. The output performance of high-order nonlinear SISO networked system.
Figure 4. Control signal of high-order nonlinear SISO networked system.

\begin{align*}
    f_3 &= x_3^3 + \sin(x_3), \\
    f_4 &= -3 (x_4 + x_4^3 - 1)^2 (x_4^1 + x_4^2 + x_4^3 - 1) - x_4^1 \nonumber \\
        &\quad + 0.5\sin(2t) + \cos(2t), \\
    f_5 &= \cos(x_5^3) 
\end{align*}

and the leader dynamics is

\begin{align*}
    \dot{x}_0^1 &= x_0^2 \\
    \dot{x}_0^3 &= x_0^3
\end{align*}

The setting parameters in this problem were selected as \( \rho_\infty = 0.03 \times \mathbf{1}_{5 \times 1} \), \( \rho_0 = 4 \times \mathbf{1}_{5 \times 1} \), \( \ell = 0.6 \times \mathbf{1}_{5 \times 1} \), \( \Gamma = 0.05 \mathbb{I}_{n_0 \times n_0} \), \( \delta = 4 \times \mathbf{1}_{5 \times 1} \), \( \delta = 4 \times \mathbf{1}_{5 \times 1} \), \( \nu_i = 6 \), \( c = 30 \times \mathbf{1}_{5 \times 1} \), \( k = 0.1 \), \( x_0(0) = [0.3, 0.3, 0.3]^T \) is the initial vector of the nonlinear leader system, and the values of agents are \( x_i(0) = [-0.2850, -0.0821, -0.2126]^T \).

Figure 5. Error and transformed error of high-order nonlinear SISO networked system.
\[ x_2(0) = [-0.6044, -0.3964, -0.0775]^T, \quad x_3(0) = [-0.2110, -0.4237, -0.3253]^T, \quad x_4(0) = [-0.1501, -0.3986, -0.0050]^T \text{ and } \ x_5(0) = [-0.3281, 0.1618, -0.4160]^T. \]

Figure 6. Nonlinearities compensation for each agent.

This networked system with unknown high nonlinear dynamics achieved consensus in the presence of high nonlinearities and time-varying disturbances. The control effort is depicted in Figure 4. Figure 5 illustrates the systematic convergence of the synchronised error \( e_i \) for \( i = 1, \ldots, 5 \) satisfying the predefined constraints and setting parameters imposed on the system. In fact, Figure 5 shows how the error started from a predefined large set and reduced systematically into the predefined small set.

Figure 7. Output performance of MIMO nonlinear networked system for \( y_1 \) and \( y_2 \).
set prescribed by the value of $\rho_{eo}$. Moreover, Figure 5 presents transformed error $\epsilon_i$ associated with agent $i$. The nonlinear compensation of each agent is bounded and smooth as presented in Figure 6.

**Problem 5.2:** Consider a second-order dynamics of MIMO system with graph similar to Figure 2. Each agent has two inputs and two outputs. The nonlinear dynamics are defined by

$$
\begin{bmatrix}
\dot{x}_1^1 \\
\dot{x}_2^2 \\
\end{bmatrix} = 
\begin{bmatrix}
f_1^1(x_i, t) \\
f_2^2(x_i, t) \\
\end{bmatrix} + \psi_i(t) \begin{bmatrix}
x_1^1 \\
x_2^2 \\
\end{bmatrix} + \begin{bmatrix}
D_1^1(t) \\
D_2^2(t) \\
\end{bmatrix} + \begin{bmatrix}
u_1^1 \\
u_2^2 \\
\end{bmatrix}
$$

where

$$y_i := \begin{bmatrix} x_1^1 \ x_2^2 \end{bmatrix}^T$$

![Figure 8. Control signal of MIMO nonlinear networked system for $u_1$ and $u_2$.](image)

![Figure 9. Error and transformed error of MIMO nonlinear networked system for the first output.](image)
Figure 10. Error and transformed error of MIMO nonlinear networked system for the second output.

$$f_i(x_i) = \begin{bmatrix} \frac{a_i^1 x_i^1 x_i^1 x_i^1 + 0.2\sin (a_i^1 x_i^1 x_i^1)}{2} \\ -a_i^2 x_i^1 x_i^1 x_i^1 - 0.2a_i^2\cos (a_i^2 x_i^1 x_i^1) \end{bmatrix},$$

$$\psi_i = \begin{bmatrix} 3c_i^1 \sin (0.5t) \\ 2c_i^1 \sin (0.4c_i^1 t) \cos (0.3t) \end{bmatrix},$$

$$D_i (t) = \begin{bmatrix} 1 + b_i^1\sin (b_i^1 t) \\ 1.2\cos (b_i^1 t) \end{bmatrix},$$

and

$$a = \begin{bmatrix} a_i^1 \\ a_i^2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix},$$

$$b = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.1 \end{bmatrix},$$

$$c = \begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 1.3 \\ 1.3 \end{bmatrix},$$

The leader dynamics is $x_0 = [0.5\cos(0.8t), 0.6\cos(0.7t)]^T$.

In this problem, the setting parameters are $\rho_0 = \cdots$.

Figure 11. Phase plane plot for all the agents.
6 \times 1_{5 \times 2}, \quad \rho_{\infty} = 0.03 \times 1_{5 \times 2}, \quad \ell = 0.6 \times 1_{5 \times 2}, \quad \Gamma = 0.05 I_{5 \times 2}, \quad \delta = 6 \times 1_{5 \times 2}, \quad \delta = 6 \times 1_{5 \times 2}, \quad c = 300 I_{5 \times 2} \text{ and } k = 0.1 I_{5 \times 2}\). Initial conditions of \( x_1(0) = [0.1956, -0.2307], x_2(0) = [-0.4947, -0.3852], x_3(0) = [-0.1475, -0.4880], x_4(0) = [-0.2947, -0.2203], x_5(0) = [-0.2850, -0.1593] \) and \( x_i(0) = [0, 0]^T, i = 1, \ldots, 5 \). Finally, the number of neurons is \( v_i = 50 \).

The effectiveness of the proposed control algorithm against unknown nonlinearities and time-variant components is examined in this problem. The output performance illustrates the robustness and high tracking capabilities of control algorithm. The output performance for high-order MIMO case is given in Figure 7. The output performance in Figure 7 moved from random initialisation to consensus with the leader softly. The control input of system dynamics in the connected graph is shown in Figure 8. Each of transformed and tracking errors are illustrated in Figures 9 and 10. One can clearly observe from these figures the quality of the control approach and its efficiency in ensuring synchronisation with prescribed performance characteristics. Clearly, errors obeyed the predefined constraints and settings and the error started within a predefined large set to end within a predefined small set. Finally, the phase plane is presented in Figure 11 to show the random initialisation of each agent with systematic consensus up to out destination or desired point.

6. Conclusion

Neuro-adaptive distributed control with prescribed performance of higher-order nonlinear affine MAS with full-state synchronisation has been proposed. NN is employed to estimate unknown nonlinear dynamics of each node. The control signal has been chosen to both respect the digraph and ensure stability. The proposed controller successfully allowed the nodes to synchronise the leader trajectory and satisfy at any point in time the desired performance with small residual errors. Also, the controller guarantees tracking the leaders’ states with a synchronisation error within predefined time-varying constraints. Prescribed performance controller was designed based on robust neuro-adaptive approach. Lyapunov-based stability proofs establish that the synchronisation error of each node is UUB. Simulation examples related to SISO and MIMO consider high-order dynamics with unknown nonlinearities and time-variant components. However, the class of systems addressed in this paper possesses an input function with constant values. More realistic systems with general nonlinear input functions can be considered in future work. In addition, hard actuator nonlinearities and especially saturation input functions could be studied in future research.

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