Automated Feedback Generation for Introductory Programming Assignments

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Abstract

We present a new method for automatically providing feedback for introductory programming problems. In order to use this method, we need a reference implementation of the assignment, and an error model consisting of potential corrections to errors that students might make. Using this information, the system automatically derives minimal corrections to student's incorrect solutions, providing them with a quantifiable measure of exactly how incorrect a given solution was, as well as feedback about what they did wrong.

We introduce a simple language for describing error models in terms of correction rules, and formally define a rule-directed translation strategy that reduces the problem of finding minimal corrections in an incorrect program to the problem of synthesizing a correct program from a sketch. We have evaluated our system on thousands of real student attempts obtained from 6.00 and 6.00x. Our results show that relatively simple error models can correct on average 65% of all incorrect submissions.

1. Introduction

There has been a lot of interest recently in making quality education more accessible to students worldwide using information technology. Several education initiatives such as EdX, Coursera, and Udacity are teaming up with experts to provide online courses on various college-level subjects ranging from computer science to physics and psychology. These courses, also called massive open online courses (MOOC), are typically taken by several thousands of students worldwide, and presents many interesting challenges that are not present in a traditional classroom setting consisting of only a few hundred students. One such challenge in these courses is to provide personalized feedback on practice exercises and assignments to a large number of students. We consider the problem of providing automated feedback for online introductory programming courses in this paper. We envision this technology to be useful in a traditional classroom setting as well.

The two most commonly used methods today for providing feedback on programming problems are: (i) test-cases based feedback and (ii) peer-feedback [7]. In automated test-cases based feedback, the student program is run on a set of test cases and the failing test cases are reported back as feedback to the student. This is also how the 6.00x course (Introduction to Computer Science and Programming) offered by MITX currently provides feedback for the python programming exercises. The provided feedback of failing test cases is however not ideal, especially for beginner programmers, as they find it difficult to map the failing test cases to errors in their code. We found a lot of students posting their submissions on the discussion board seeking help from instructors and other students after struggling for several hours to correct the mistakes themselves. In fact, for the classroom version of the Introduction to Programming course (6.00) taught at MIT, the teaching assistants are required to manually go through each student submission and provide qualitative feedback describing exactly what is wrong with the submission and how to correct it. This manual feedback by teaching assistants is simply prohibitive for the number of students in the online class setting.

The second approach of peer-feedback is being suggested as a potential solution to this problem. In this approach, the peer students who are also taking the same course answer the posts on the discussion boards – this way the problem of providing feedback is distributed across several peer students. Unfortunately, providing quality feedback is a big challenge for experienced teaching assistants, and therefore it presents an even bigger challenge for the peer students who are also beginning to learn programming themselves. From the 6.00x discussion boards, we observed that in many instances students had to wait several hours (or days) to get any feedback, and in many cases the feedback provided was either too general, incomplete or even wrong in a few cases.

In this paper, we present an automated technique to provide feedback for introductory programming assignments. The approach leverages program synthesis technology to automatically determine minimal fixes to the student’s solution that will make it match the behavior of a reference solution written by the instructor. This technology makes it possible to provide students with precise feedback about what they did wrong and how to correct them. The problem of providing automatic feedback appears to be related to the problem of automatic bug fixing, but it differs from it in following two significant respects:

- **The complete specification is known.** An important challenge in automatic debugging is that there is no way to know whether a fix is addressing the root cause of a problem, or simply masking it and potentially introducing new errors. Usually the best one can do is check a candidate fix against a test suite or a partial specification [9]. While providing feedback on the other hand, the solution to the problem is known, and it is safe to assume that the instructor already wrote a correct reference implementation for the problem.

- **Errors are predictable.** In a homework assignment, everyone is solving the same problem after having attended the same lectures, so errors tend to follow predictable patterns. This makes it possible to use a model-based feedback approach, where the potential fixes are guided by a model of the kinds of errors students typically make for a given problem.

These simplifying assumptions, however, introduce their own set of challenges. For example, since the complete specification is known, the tool now needs to reason about the equivalence of the student solution with the reference implementation. Also, in order to take advantage of the predictability of errors, the tool needs to be parameterized with models that describe the classes of errors. And finally, these programs can be expected to have higher density of
errors than production code, so techniques like the one suggested by [18], which attempts to correct bugs one path at a time will not work for many of these problems that require coordinated fixes in multiple places.

Our automated feedback generation technique handles all of these challenges. The tool can reason about the semantic equivalence of student programs and reference implementations written in a fairly large subset of python, so the instructor does not have to learn a new formalism to write specifications. The tool also provides an error model language that can be used to write an error model: a very high level description of potential corrections to errors that students might make in the solution. When the system encounters an incorrect solution by a student, it symbolically explores the space of all possible combinations of corrections allowed by the error model and finds a correct solution requiring a minimal set of corrections.

We have evaluated our approach on thousands of student solutions on programming problems obtained from the 6.00x submissions and discussion boards, and from the 6.00 class submissions. These problems constitute a major portion of first month of assignment problems. Our tool can successfully provide feedback on over 65% of the incorrect solutions.

This paper makes the following key contributions:

- We show that the problem of providing automated feedback for introductory programming assignments can be framed as a synthesis problem. Our reduction uses a constraint-based mechanism to model python’s dynamic typing and supports complex python constructs such as closures, higher-order functions and list comprehensions.

- We define a high-level language EML that can be used to provide correction rules to be used for providing feedback. We also show that a small set of such rules is sufficient to correct thousands of incorrect solutions written by students.

- We report the successful evaluation of our technique on thousands of real student attempts obtained from 6.00 and 6.00x classes, as well as from PEx4FUN website. Our tool can provide feedback on 65% of all submitted solutions that are incorrect in about 10 seconds on average.

2. Overview of the approach

In order to illustrate the key ideas behind our approach, consider the problem of computing the derivative of a polynomial whose coefficients are represented as a list of integers. This problem is taken from week 3 problem set of 6.00x (PS3: Derivatives). Given the input list poly, the problem asks students to write the function computeDeriv that computes a list poly’ such that

\[ poly’ = \begin{cases} 
1 \times poly[1] & |0 < i < \text{len}(poly)| \text{ if } \text{len}(poly) > 1 \\
0 & \text{if } \text{len}(poly) = 1 
\end{cases} \]

For example, if the input list poly is [2, -3, 1, 4] (denoting \( f(x) = 4x^3 - 3x^2 + 2x + 2 \)), the computeDeriv function should return [\(-6, 12, 2\)] (denoting the derivative \( f'(x) = 12x^2 + 2x - 3 \)). The reference implementation for the computeDeriv function is shown in Figure 1. This problem teaches concepts of conditionals and iteration over lists. For this problem, students struggled with many low-level python semantics issues such as the list indexing and iteration bounds. In addition, they also struggled with conceptual issues such as missing the corner case of handling lists consisting of single element (denoting constant function).

A student solution for the computeDeriv problem taken from the 6.00x discussion forum is shown in Figure 2(a). The student posted the code in the forum seeking help and received two responses. The first response asked the student to look for the first if-block return value, and the second response said that the code should return [0] instead of empty list for the first if statement. There are many different ways to modify the code to return [0] for the case \( \text{len}(poly) = 1 \). The student chose to change the initialization of the deriv variable from [ ] to the list [0]. The problem with this modification is that the result will now have an additional 0 in front of the output list for all input lists (which is undesirable for lists of length greater than 1). The student then posted the query again on the forum on how to remove the leading 0 from result, but unfortunately this time did not get any more response.

Our tool generates the feedback shown in Figure 2(b) for the student program in about 40 seconds. During these 40 seconds, the tool searches over more than 10^7 candidate fixes and finds the fix that requires minimum number of corrections. There are three problems with the student code: first it should return [0] in line 5 as was suggested in the forum but wasn’t specified how to make the change, second the if block should be removed in line 7, and third that the loop iteration should start from index 1 instead of 0 in line 6. The generated feedback consists of four pieces of information (shown in bold in the figure for emphasis):

- the location of the error denoted by the line number.
- the problematic expression in the line.
- the sub-expression which needs to be modified.
- the new modified value of the sub-expression.

The feedback generator is parameterized with a feedback-level parameter to generate feedback consisting of different combinations of the four information depending on how much information the instructor is willing to provide to the student.

2.1 Workflow

In order to provide the level of feedback described above, the tool needs some information from the instructor. First, the tool needs to know what the problem is that the students are supposed to solve. The instructor provides this information by writing a reference implementation such as the one in Figure 1. Since python is dynamically typed, the instructor also provides the types of function arguments and return value. In Figure 1, the instructor specifies the type of input argument to be list of integers (poly_list_int) by appending the type to the name.

In addition to the reference implementation, the tool needs a description of the kinds of errors students might make. We have designed an error model language EML, which can describe a set of correction rules that denote the potential corrections to errors that students might make. For example, in the student attempt in Figure 2(a), we observe that corrections often involve modifying the return value and the range iteration values. We can specify this

```
1 def computeDeriv_list_int(poly_list_int):
2     result = []
3     for i in range(len(poly_list_int)):
4         result += [i + poly_list_int[i]]
5     if len(poly_list_int) == 1:
6         return result  # return [0]
7     else:
8         return result[1:]  # remove the leading 0
```

Figure 1. The reference implementation for computeDeriv function.
The architecture of our automated feedback generation tool is shown in Figure 3. The solution strategy to find minimal corrections to a student’s solution is based on a two-phase translation to the Sketch synthesis language.

In the first phase, the Program Rewriter uses the correction rules to translate the solution into a language we call MPy; this language provides us with a concise notation to describe sets of MPy candidate programs, together with a cost model to reflect the number of corrections associated with each program in this set. In the second phase, this MPy program is translated into a sketch program by the Sketch Translator.

In the case of example in Figure 2(a), the Program Rewriter produces the MPy program shown in Figure 4 using the correction rules from Section 2.1. This program includes all the possible corrections induced by the correction rules in the model. The MPy language extends the imperative language miniPython with expression choices, where the choices are denoted with squiggly brackets. Whenever there are multiple choices for an expression or a statement, the zero-cost choice, the one that will leave the expression unchanged, is boxed. For example, the expression choice \{a_0, a_1, \ldots, a_n\} denotes a choice between expressions $a_0, \ldots, a_n$, where $a_0$ denotes the zero-cost default choice.

For this simple program, the three correction rules induce a space of 32 different candidate programs. This candidate space is fairly small, but the number of candidate programs grows exponentially with the number of correction places in the program and with the number of correction choices in the rules. The error model that we use in our experiments induces a space of more than $10^{12}$ candidate programs for some of the benchmark problems. In order to search this large space efficiently, the program is translated to a sketch by the Sketch Translator.

The program requires 3 changes:

- In the return statement return deriv in line 5, replace deriv by [0].
- In the comparison expression (poly[expo] == 0) in line 7, change (poly[expo] == 0) to False.
- In the expression range(0, len(poly)) in line 6, increment 0 by 1.

Based on a two-phase translation to the Sketch synthesis language. In the first phase, the Program Rewriter uses the correction rules to translate the solution into a language we call MPy; this language provides us with a concise notation to describe sets of MPy candidate programs, together with a cost model to reflect the number of corrections associated with each program in this set. In the second phase, this MPy program is translated into a sketch program by the Sketch Translator.

The architecture of our tool is shown in Figure 3. The solution strategy to find minimal corrections to a student’s solution is

```python
1 def computeDeriv(poly):
2     deriv = []
3     zero = 0
4     if {len(poly) == 1}:
5         return deriv
6     for expo in range (0, len(poly)):
7         if (poly[expo] == 0):
8             zero += 1
9         else:
10             deriv.append(poly[expo]*expo)
11     return deriv
```

(a) Student’s solution
(b) Generated Feedback

Figure 2. (a) A student’s computeDeriv solution from the 6.00x discussion board and (b) the feedback generated by our tool on this solution.

The correction rule return $a \rightarrow \text{return } [0]$ states that the expression of a return statement can be optionally replaced by [0]. The error model for this problem that we use for our experiments is shown in Figure 8, but we will use this simple error model for simplifying the presentation in this section. In later experiments, we also show how only a few tens of incorrect solutions can provide enough information to create an error model that can automatically provide feedback for thousands of incorrect solutions.

The tool now needs to explore the space of all candidate programs based on applying these correction rules to the student program, and compute the candidate program that is equivalent to the reference implementation and that requires minimum number of corrections. We use constraint-based synthesis technology [11, 24, 27] to efficiently search over this large space of programs. Specifically, we use the SKETCH synthesizer that uses a sat-based algorithm to complete program sketches (programs with holes) so that they meet a given specification. We extend the SKETCH synthesizer with support for minimize hole expressions whose values are computed efficiently by using incremental constraint solving. To simplify the presentation, we use a simpler language MPy (miniPython) in place of python to explain the details of our algorithm. In practice, our tool supports a fairly large subset of python including closures, higher order functions and list comprehensions.

2.2 Solution Strategy

Figure 3. The architecture of our automated feedback generation tool.

The architecture of our tool is shown in Figure 3. The solution strategy to find minimal corrections to a student’s solution is
The `struct` for encoding dynamic types in Python:

```
struct MultiType{
    int val, type;
    MTList lst;
}
struct MTList{
    int len;
    MultiType[len] lVals;
}
```

**Figure 5.** The definition of `MultiType` struct for encoding dynamic types in Python.

### 2.3 Synthesizing Corrections with Sketch

The Sketch [24] synthesis system allows programmers to write programs while leaving fragments of it unspecified as *holes*; the contents of these holes are filled up automatically by the synthesizer such that the program conforms to a specification provided in terms of a reference implementation. The synthesizer uses the CEGIS algorithm [25] to efficiently compute the values for holes and uses bounded symbolic verification techniques for performing equivalence check of the two implementations.

There are two key aspects in the translation of an MPY program to a Sketch program. The first aspect is specific to the Python language. Sketch supports high-level features such as closures and higher-order functions which simplifies the translation, but it is statically typed whereas MPY programs (like python) are dynamically typed. The translation models the dynamically typed variables and operations over them using struct types in Sketch in a way similar to the union types. The second aspect of the translation is the modeling of set-expressions in MPY using `??` (holes) in Sketch, which is language independent.

The dynamic variable types in MPY language are modeled using the `MultiType` struct defined in Figure 5. The `MultiType` struct consists of a type field that denotes the dynamic type of variables and currently supports the following set of types: `INTEGER, BOOL, TYPE, LIST, TUPLE`. The `val` field stores the integer value or the Boolean value of the variables, whereas the list field of type `MTList` stores the value of list and tuple variables. The `MTList` struct consists of a field `len` that denotes the length of the list and a field `lVals` of type array of `MultiType` that stores the list elements. For example, the integer value 5 is represented as the value `MultiType(val=5, flag=INTEGER)` and the list `[1, 2]` is represented as the value `MultiType(lst=new MTList(len=2, lVals=[new MultiType(val=1, flag=INTEGER), new MultiType(val=2, flag=INTEGER)], flag=LIST)]`.

The second key aspect of this translation is the translation of expression choices in MPY. The Sketch construct `??` denotes an unknown integer hole that can be assigned any constant integer value by the synthesizer. The expression choices in MPY are translated to functions in Sketch that based on the unknown hole values return either the default expression or one of the other expression choices. Each such function is associated with a unique Boolean choice variable, which is set by the function whenever it returns a non-default expression choice. For example, the set-statement `modRetVal0(MultiType a)` is defined as:

```
if(??) return a; // default choice
choiceRetVal0 = True; // non-default choice
MTList list = new MTList(lVals=[new MultiType(val=0, flag=INTEGER)], len=1);
return new MultiType(lst=list, type = LIST);
```

The translation phase also generates a Sketch harness that calls and compares the outputs of the translated student and reference implementations on all inputs of a bounded size. For example in case of the `computeDeriv` function that takes a list as input, with bounds of `n = 4` for both the number of integer bits and the maximum length of the input list, the harness makes sure that the output of the two implementations match for more than `2^{16}` different inputs as opposed to 10 test-cases used in 6.0X. The harness also defines a `totalCost` variable as a function of all choice variables that computes the total number of modifications performed in the original program, and asserts that the value of `totalCost` should be minimized. The synthesizer then solves this minimization problem efficiently using an incremental solving algorithm CEGISMIN described in Section 4.2.

After the synthesizer finds a solution, the Feedback Generator uses the solution to the unknown integer holes in the sketch to compute the choices made by the synthesizer and generates the corresponding feedback. For this example, the tool generates the feedback shown in Figure 2(b) in less than 40 seconds.

### 3. EML: Error Model Language

In this section, we describe the syntax and semantics of the error model language EML. An EML error model consists of a set of rewrite rules that captures the potential corrections for mistakes that students might make in their solutions. We define the rewrite rules over a simple python-like imperative language MPY. A rewrite rule transforms a program element in MPY to a set of weighted MPY program elements. This weighted set of MPY program elements is represented succinctly as an MPY program element, where MPY extends the MPY language with set-exprs (set of expressions) and set-stmts (set of statements). The weight associated with a program element in this set denotes the cost of performing the corresponding correction. An error model transforms an MPY program to an MPY program (representing a set of MPY programs) by recursively applying the rewrite rules. We show that this transformation is deterministic and is guaranteed to terminate on well-formed error models.

### 3.1 MPY and MPY languages

The syntax for the simple imperative language MPY is shown in Figure 6(a) and the syntax of MPY language is shown in Figure 6(b). The purpose of MPY language is to represent a large collection of MPY programs succinctly. The MPY language consists of set-expressions (`??`) and set-statements (`??`) that represent a weighted set of corresponding MPY expressions and statements respectively. For example, the set expression `[(a0,· · · , an)]` represents a weighted set of constant integers where `a0` denotes the default integer value associated with cost 0 and all other integer constants (`a1,· · · , an`) are associated with cost 1. The set of composite expressions are represented succinctly in terms of sets of their constituent sub-expressions. For example, the composite expression `[[a0, a1 + 1], [a0, 0]]` represents 96 MPY expressions.

Each MPY program in the set of programs represented by an MPY program is associated with a cost (weight) that encodes the number of modifications performed in the original program to obtain the transformed program. This cost allows the tool to search for corrections that require minimum number of modifications. The weighted set of MPY programs is defined using the `[]` function shown partially in Figure 7, the complete function definition can be found in [1]. The `[]` function on MPY expressions such as `a` returns a singleton set `{(a, 0)}` consisting of the corresponding expression that is associated with cost 0. On set-expressions of the form `[(a0,· · · , an)]`, the function returns the union of the weighted set of MPY expressions corresponding to the default set-expression (`[a0]`) and the weighted set of expressions corresponding to other set-expressions (`a1,...,an`), where each expression in `[ai]` is
### Syntax of EML

An EML error model consists of a set of correction rules that are used to transform an MPY program to an MPY program. A **correction rule** \( C \) is written as a rewrite rule \( L \rightarrow R \), where \( L \) and \( R \) denote a **program element** in MPY and MPY respectively. A program element can either be a term, an expression, a statement, a method, or the program itself. The left hand side (\( L \)) denotes an MPY program element that is pattern matched to be transformed to an MPY program element denoted by the right hand side (\( R \)). The left hand side of the rule can use free variables whereas the right hand side can only refer to the variables present in the left hand side. The language also supports a special \( t \) (prime) operator that can be used to tag sub-expressions in \( R \) that are further transformed recursively using the error model. The rules use a shorthand notation \( ?a \) (in the right hand side) to denote the set of all variables that are of the same type as the type of expression \( a \) and are in scope at the corresponding program location. We assume each correction rule is associated with cost \( 1 \), but it can be easily extended to different costs to account for different levels of mistakes.

**Example 1.** The error model for the `computeDeriv` problem is shown in Figure 8. The **INDR** rewrite rule transforms the list access indices. The **INITR** rule transforms the right hand side of constant initializations. The **RANR** rule transforms the arguments for the range function; similar rules are defined in the model for other range functions that take one and three arguments. The **COMPR** rule transforms the operands and operator of the comparisons. The **RETR** rule adds the two common corner cases of returning \([0]\) when the length of input list is \( 1 \), and the case of deleting the first list element before returning the list. Note that these rewrite rules define the corrections that can be performed optionally; the zero cost (default) case of not correcting a program element is added automatically as described in Section 3.3.

### Transformation with EML

An error model \( E \) is syntactically translated to a function \( T_E \) that transforms an MPY program to an MPY program. The \( T_E \) function first traverses the program element \( w \) in the default way, i.e. no transformation happens at this level of the syntax tree, and the method is called recursively on all of its top-level sub-term to
obtain the transformed element \( w_0 \in \widetilde{MPY} \). For each correction rule \( C_i : L_i \rightarrow R_i \) in the error model \( E \), the method contains a match expression that matches the term \( w \) with the left hand side of the rule \( L_i \) (with appropriate unification of the free variables in \( L_i \)). If the match succeeds, it is transformed to a term \( w_i \in \widetilde{MPY} \) as defined by the right hand side \( R_i \) of the rule after applying the \( T_E \) method recursively on each of its tagged sub-terms \( w' \). Finally, the method returns the set of all transformed terms \( \{ w_0, \ldots, w_n \} \).

**Example 2.** Consider an error model \( E_3 \) consisting of the following three correction rules:

\[
\begin{align*}
C_1 : v[a] & \rightarrow v\{a - 1, a + 1\} \\
C_2 : a_0 \ op_c \ a_1 & \rightarrow \{a_0' - 1, 0\} \ op_c \ \{a_1' - 1, 0\} \\
C_3 : v[a] & \rightarrow ?v[a]
\end{align*}
\]

The transformation function \( T_{E_3} \) for the error model \( E_3 \) is shown in Figure 9.

\[
T_{E_3}(w : \widetilde{MPY}) = \begin{cases}
let \ w_0 = \mu t \rightarrow T_{E_3}(t) \in (* t : \text{a sub-term of } w *) \\
let \ w_1 = \text{Match } w \text{ with } \\
\hspace{1cm} v[a] \rightarrow v\{a + 1, a - 1\} \in \\
let \ w_2 = \text{Match } w \text{ with } \\
\hspace{1cm} a_0 \ op_c \ a_1 \rightarrow \{T_{E_3}(a_0) - 1, 0\} \ op_c \\
\hspace{2cm} \{T_{E_3}(a_1) - 1, 0\} \in \\
\{ w_0, w_1, w_2 \}
\end{cases}
\]

Figure 9. The \( T_{E_3} \) method for error model \( E_3 \).

The recursive steps of application of \( T_{E_3} \) function on expression \( \{x[i] < y[j]\} \) are shown in Figure 10. This example illustrates two interesting features of the transformation function:

- **Nested Transformations**: Once a rewrite rule \( L \rightarrow R \) is applied to transform a program element matching \( L \rightarrow R \), the instructor may want to apply another rewrite rule on only a few sub-terms of \( R \). For example, she may want to avoid transforming the sub-terms which have already been transformed by some other correction rule. The EML language facilitates making such distinction between the sub-terms for performing nested corrections using the \( \mu \) (prime) operator. Only the sub-terms in \( R \) that are tagged with the prime operator are visited for applying further transformations (using the \( T_E \) method recursively on its tagged sub-terms \( w' \)), whereas the remaining non-tagged sub-terms are not transformed any further. After applying the rewrite rule \( C_3 \) in the example, the sub-terms \( x[i] \) and \( y[j] \) are further transformed by applying rewrite rules \( C_1 \) and \( C_2 \).

- **Ambiguous Transformations**: While transforming a program using an error model, it may happen that there are multiple rewrite rules that pattern match the program element \( w \). After applying rewrite rule \( C_2 \) in the example, there are two rewrite rules \( C_1 \) and \( C_3 \) that pattern match the terms \( x[i] \) and \( y[j] \). After applying one of these rules \( (C_1 \text{ or } C_3) \) to an expression \( v[a] \), we cannot apply the other rule to the transformed expression. In such ambiguous cases, the \( T_E \) function creates a separate copy of the transformed program element \( w_i \) for each ambiguous choice and then performs the set union of all such elements to obtain the transformed program element. This semantics of handling ambiguity of rewrite rules also matches naturally with the intent of the instructor. If the instructor wanted to perform both transformations together on array accesses, she could have provided a combined rewrite rule such as \( v[a] \rightarrow ?v[a + 1, a - 1] \).

**Theorem 1.** Given a well-formed error model \( E \), the transformation function \( T_E \) always terminates.

**Proof.** From the definition of well-formed error model, each of its constituent rewrite rule is also well-formed. Hence, each application of a rewrite rule reduces the size of the syntax tree of terms that are required to be visited further for transformation by \( T_E \). Therefore, the \( T_E \) function terminates in a finite number of steps. \qed

### 4. Constraint-based Solving of MPY programs

In the previous section, we saw the transformation of an MPY program to an MPY program based on an error model. We now present the translation of MPY programs into Sketch programs [24].

### 4.1 Translation of MPY programs to SKETCH

The MPY programs are translated to Sketch programs to perform constraint-based analysis. The main aspects of the translation include the translation of: (i) python-like constructs in MPY to Sketch, and (ii) set-expr choices in MPY to Sketch functions.

**Handling dynamic typing of \( \widetilde{MPY} \) variables** The dynamic typing of MPY is handled using MultiType variable as described in Section 2.3. The MPY expressions and statements are transformed to Sketch functions that perform the corresponding transformations over MultiType. For example, the python statement \( (a = b) \) is translated to assignMT \((a, b)\), where the assignMT function assigns MultiType \( b \) to \( a \). Similarly, the binary add expression \( (a + b) \) is translated to bin0MT \((a, b, \text{ADD.OP})\) that in turn calls the function addMT \((a, b)\) to add \( a \) and \( b \) as shown in Figure 11.

```plaintext
1 MultiType addMT(MultiType a, MultiType b){
  2  assert a.flag == b.flag; // same types can be added
  3  if(a.flag == INTEGER) // add for integers
  4    return new MultiType(val=a.val+b.val, flag = INTEGER);
  5  if(a.flag == LIST){ // add for lists
  6    int newLen = a.lst.len + b.lst.len;
  7    MultiType newLVals = a.lst.lVals;
  8    for(int i=0; i<newLen; i++)
  9      newLVals[i] = b.lst.lVals[i];
 10    return new MultiType(lst = new
 11                        MTList(lVals=newLVals, len=newLen),
 12                        flag=LIST));
 13                      }  

Figure 11. The addMT function for adding two MultiType a and b.
```

**Translation of MPY set-expressions** The set-expressions in MPY are translated to Sketch functions. The function bodies obtained from translation (\( \Phi \)) of some of the interesting MPY constructs are shown in Figure 12. The Sketch construct \( ?? \)(called \( \text{hole} \)) is a placeholder for a constant value, which is filled up by the Sketch synthesizer while solving the constraints to satisfy the given specification.

The singleton sets consisting of an MPY expression such as \( \{a\} \) are translated simply to the corresponding expression itself.
The translation rules (shown partially) for converting set-expressions to corresponding SKETCH function bodies.

A set-expression of the form \[ \{ \{ a_0, \ldots, a_n \} \} \] is translated recursively to the if expression \( \text{if} (??) \Phi(a_0) \text{ else } \Phi(\{a_1, \ldots, a_n\}) \), which means that the synthesizer can optionally select the default set-expression \( \Phi(a_0) \) (by choosing ?? to be true) or select one of the other choices (\( a_1, \ldots, a_n \)). The set-expressions of the form \( \{a_0, \ldots, a_n\} \) are similarly translated but with an additional statement for setting a fresh variable \( \text{choice}_k \) if the synthesizer selects the non-default choice \( a_0 \).

The translation rules for the assignment statements (\( a_0 := a_1 \)) results in if expressions on both left and right sides of the assignment. The if expression choices occurring on the left hand side are desugared to individual assignments. For example, the left hand side expression if (??) \( x := 10 \) else \( y := 10 \) is desugared to if (??) \( x := 10 \) else \( y := 10 \). The infix operators in MPY are first translated to function calls and are then translated to sketch using the translation for set-function expressions. The remaining MPY expressions are similarly translated recursively as shown in the figure.

**Translating function calls** The translation of function calls for recursive problems and for problems that require writing a function that uses other sub-functions is parameterized in three options: 1) use the student’s implementation of sub-functions, 2) use the teacher’s implementation of sub-functions, and 3) treat the sub-functions as uninterpreted functions.

**Generating the driver functions** The SKETCH synthesizer supports checking equivalence of functions whose input arguments and return values are over SKETCH primitive types such as int, bit and arrays. Therefore, after the translation of MPY programs to

\[
T(x[i] < y[j]) \equiv \{ T(x[i]) < T(y[j]) \} \{ (T(x[i]) - 1, 0) \} \}
\]

\[
T(x[i]) \equiv \{ T(x[i]) [x[i + 1, i - 1], y[i]] \}
\]

\[
T(y[j]) \equiv \{ T(y[j]) [y[j + 1, j - 1], x[j]] \}
\]

\[
T(x[i] < y[j]) \equiv \{ (x[i], i) \{ y[j], j \} \}
\]

Therefore, after substitution the result is:

\[
T(x[i] < y[j]) \equiv \{ (x[i], i) \{ y[j], j \} \}
\]

\[
\{ (x[i], i) \{ y[j], j \} \}
\]

\[
\{ (x[i], i) \{ y[j], j \} \}
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\{ (x[i], i) \{ y[j], j \} \}
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\{ (x[i], i) \{ y[j], j \} \}
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\{ (x[i], i) \{ y[j], j \} \}
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\{ (x[i], i) \{ y[j], j \} \}
\]


described in Algorithm 1. The algorithm begins by assigning initial states to the program store (\( \sigma_0 \)) and the guess store (\( \varphi_0 \)). It then enters the main loop which iterates through the sketch program. The loop body consists of two phases: the synthesis phase and the verification phase. The synthesis phase is used to generate the hole values for the sketch program. The verification phase is used to verify the correctness of the generated hole values.

**Algorithm 1 CEGISMIN Algorithm for Minimization**

1. \( \sigma_0 \leftarrow \sigma_{\text{random}} \)
2. \( i \leftarrow 0 \)
3. \( \varphi_0 \leftarrow \emptyset \)
4. \( \Phi_i \leftarrow \text{null} \)
5. \( \varphi_{\text{prev}} \leftarrow \emptyset \)
6. \( \text{while} \ (\text{True}) \)
7. \( \Phi_i \leftarrow \text{Synth} (\sigma_{i - 1}, \Phi_{i - 1}) \)
8. \( i \leftarrow i + 1 \)
9. \( \Phi_i \leftarrow \text{null} \)
10. \( \varphi_{\text{prev}} \leftarrow \emptyset \)
11. \( \sigma_{i - 1} \leftarrow \text{null} \)
12. \( \varphi_{\text{prev}} \leftarrow \emptyset \)
13. \( \Phi_i \leftarrow \Phi_i \cup \{ \text{encode} (\text{minHole} < \text{minHoleVal}) \} \)

We extend the CEGIS algorithm in SKETCH [24] to get the CEGISMIN algorithm shown in Algorithm 1 for efficiently solving sketches that include a minimize hole expression. The input state of the sketch program is denoted by \( \sigma \) whereas the sketch constraint store denoted by \( \Phi \). Initially, the input state \( \sigma_0 \) is assigned a random input state value and the constraint store \( \Phi_0 \) is assigned the constraint set obtained from the sketch program. The variable \( \varphi_0 \) stores the previous satisfiable hole values and is initialized to \( \text{null} \). In each iteration of the loop, the synthesizer first performs the inducive synthesis phase where it shrinks the constraints set \( \Phi_{i - 1} \) to \( \Phi_i \) by removing behaviors from \( \Phi_{i - 1} \) that do not conform to the input state \( \sigma_{i - 1} \). If the constraint set becomes unsatisfiable, it
either returns the sketch completed with hole values from the previous solution if one exists, otherwise it returns UNSAT. On the other hand, if the constraint set is satisfiable, then it first chooses a conforming solution to generate feedback. The tool maps back the values of unknown integer holes to their corresponding expression choices. These expression choices are then mapped to natural language feedback using the messages associated with the corresponding correction rules, together with the line numbers.

5. Implementation and Experiments

We now briefly describe some of the implementation details of the tool, and then describe the experiments we performed with it.

5.1 Implementation and Features

The tool’s frontend that converts a python program to a SKETCH program is implemented in python itself and uses the python ast module for parsing and rewriting ASTs. The backend system that solves the SKETCH program and provides feedback is implemented as a wrapper over the SKETCH system extended with the CEGISMIN algorithm. Error models in our tool are currently written in python in terms of the python AST. The tool also provides a mechanism to assign different cost measure to correction rules in the error model to account for different levels of mistakes.

5.2 Benchmarks

We created our benchmark set with problems taken from the Introduction to Programming course at MIT (6.00) and the EdX version of the class (6.00x). The prodby5sum problem asks to compute the product using the sum operator, the oddTuples problem asks to compute a list consisting of alternate elements of the input list, the evalPoly problem asks to compute the value of a polynomial on a given value, the iterPower (and recurPower) problem asks to compute the exponentiation using multiplication and the iterGCD problem computes the gcd of two numbers. We also created a few AP-level loop-over-arrays and dynamic programming problems on PEXFUN to show the scalability and applicability of our technique to other languages such as C#.

5.3 Experiments

Performance Table 1 shows the number of student attempts corrected for each benchmark problem as well as the time taken by the tool to provide the feedback. The experiments were performed on a 2.4GHz Intel Xeon CPU with 16 cores and 16GB RAM. The experiments were performed with bounds of 4 bits for input integer values and maximum length 4 for input lists. For each benchmark problem, we first removed the student attempts with syntax errors to get the Test Set on which we ran our tool. We then separated the attempts which were correct to measure the effectiveness of the tool on the incorrect attempts. The tool was able to provide appropriate corrections as feedback for 65% of all incorrect student attempts in around 10 seconds on average. The remaining 35% of incorrect student attempts on which the tool could not provide feedback fall in one of the following categories:

- **Completely incorrect solutions**: We observed many student attempts that were empty or performing trivial computations such as printing variables.
- **Big conceptual errors**: A common error we found in the case of eval-poly-6.00x was that a large fraction of incorrect attempts (260/541) were using the list function index to get the index of a value in the list, whereas the index function returns the index of first occurrence of the value in the list. Some other similar mistakes involve introducing or moving program statements from one place to another. These mistakes can not be corrected with the application of a set of local correction rules.
- **Unimplemented features**: Our implementation currently lacks a few of the complex python features such as pattern matching on list enumerate function.
- **Timeout**: In our experiments, we found less than 1% of the student attempts timed out (set as 2 minutes).

**Number of Corrections** The number of student attempts that require different number of corrections are shown in Figure 13(a). We observe from the figure that a large fraction of the problems require 3 and 4 coordinated corrections, and to provide feedback on such attempts, we need a technology like ours that can symbolically encode the outcome of different corrections on all input values.

**Repetitive Mistakes** In this experiment, we validate our hypothesis that students make similar mistakes while solving a given problem. The graph in Figure 13(b) shows the number of student attempts corrected as more rules are added to the error model of the benchmark problems. As can be seen in the figure, adding a single rule to the error model can lead to correction of hundreds of attempts.

**Generalization of Error Models** In this experiment, we check the hypothesis that whether the correction rules generalize across problems of similar kind. The result of running the compute-deriv error model on other benchmark problems is shown in figure graphs(c). As expected, it does not perform as well as the problem-specific error models, but it does suffice to fix a fraction of the incorrect attempts and also provides a good starting point to add more problem-specific rules.

6. Related Work

In this section, we describe several related work to our technique from the areas of automated learning programming tutors, automated program repair, fault localization, automated debugging and program synthesis.

6.1 AI based programming tutors

There has been a lot of work done in the AI community for building automated tutors for helping novice programmers learn programming by providing feedback about semantic errors. These tutoring systems can be categorized into the following two major classes:

- **Code-based matching approaches**: LAURA [2] converts teacher’s and student’s program into a graph based representation and compares them heuristically by applying program transformations while reporting mismatches as potential bugs. TALUS [22]
Figure 13. (a) The number of incorrect problem that require different number of corrections, (b) the number of problems corrected with adding rules to the error models, and (c) the performance of compute-deriv error model on other problems.

| Benchmark       | Total Attempts | Syntax Errors | Test Set | Correct | Incorrect Attempts | Generated Feedback | Average Time (in s) | Median Time (in s) |
|-----------------|----------------|---------------|----------|---------|-------------------|--------------------|-------------------|-------------------|
| prodBySum-6.00x| 1056           | 16            | 1040     | 772     | 268               | 218 (81.3%)        | 2.49s             | 2.53s             |
| oddTuples-6.00x | 2386           | 1040          | 1346     | 1002    | 344               | 185 (53.8%)        | 2.65s             | 2.54s             |
| computeDeriv-6.00x | 144         | 20            | 124      | 21      | 103               | 88 (85.4%)         | 12.95s            | 4.9s              |
| evalPoly-6.00x  | 4146           | 1134          | 3012     | 2094    | 918               | 735 (82.1%)        | 12.42s            | 6.32s             |
| computeDeriv-6.00x | 4698         | 1004          | 3694     | 3153    | 541               | 167 (30.9%)        | 4.78s             | 4.19s             |
| oddTuples-6.00x | 10985          | 5047          | 5938     | 4182    | 1756              | 860 (48.9%)        | 4.14s             | 3.77s             |
| iterPower-6.00x | 8982           | 3792          | 5190     | 2315    | 2875              | 1693 (58.9%)       | 3.58s             | 3.46s             |
| recurPower-6.00x | 8879           | 3395          | 5484     | 2546    | 2938              | 2271 (77.3%)       | 10.59s            | 5.88s             |
| iterGCd-6.00x   | 6934           | 3732          | 3202     | 214     | 2988              | 2052 (68.7%)       | 17.13s            | 9.52s             |

| stock-market-I(#) | 52            | 11           | 41       | 19      | 22                | 16 (72.3%)         | 7.54s             | 5.23s             |
| stock-market-II(#) | 51            | 8            | 43       | 19      | 24                | 14 (58.3%)         | 11.16s            | 10.28s            |
| restaurant rush (＃) | 124           | 38           | 86       | 20      | 66                | 41 (62.1%)         | 8.78s             | 8.19s             |

Table 1. The percentage of student attempts corrected and the time taken for correction for the benchmark problems.

matches a student’s attempt with a collection of teacher’s algorithms. It first tries to recognize the algorithm used and then tentatively replaces the top-level expressions in the student’s attempt with the recognized algorithm for generating correction feedback.

The problem with these approach is that the enumeration of all possible algorithms (with its variants) for covering all corrections is very large and tedious on part of the teacher.

**Intention-based matching approaches:** LISP tutor [8] creates a model of the student goals and updates it dynamically as the student makes edits. The drawback of this approach is that it forces students to write code in a certain pre-defined structure and limits their freedom. MENO-II [26] parses student programs into a deep syntax tree whose nodes are annotated with plan tags. This annotated tree is then matched with the plans obtained from teacher’s solution. PROUST [17], on the other hand, uses a knowledge base of goals and their corresponding plans for implementing them for each programming problem. It first tries to find correspondence of these plans in the student’s code and then performs matching to find discrepancies. CHIRON [23] is its improved version in which the goals and plans in the knowledge base are organized in a hierarchical manner based on their generality and uses machine learning techniques for plan identification in the student code. These approaches require teacher to provide all possible plans a student can use to solve the goals of a given problem and do not perform well if the student’s attempt uses a plan not present in the knowledge base.

Our approach performs semantic equivalence of student’s attempt and teacher’s solution based on exhaustive bounded symbolic verification techniques and makes no assumptions on the algorithms or plans that students can use for solving the problem. Moreover, our approach is modular with respect to error models; the local correction rules are provided in a declarative manner and their complex interactions are handled by the solver itself.

### 6.2 Automated Program Repair

Könighofer et. al. [20] present an approach for automated error localization and correction of imperative programs. They use model-based diagnosis to localize components that need to be replaced and then use a template-based approach for providing corrections using SMT reasoning. Their fault model only considers the right hand side (RHS) of assignment statements as replaceable components. The approaches in [16, 28] frame the problem of program repair as a game between an environment that provides the inputs and a system that provides correct values for the buggy expressions such that the specification is satisfied. These approaches only support simple corrections (e.g., correcting RHS side of expressions) in the fault model as they aim to repair large programs with arbitrary errors. In our setting, we exploit the fact that we have access to the dataset of previous student mistakes that we can use to construct a concise and precise error model. This enables us to model more sophisticated transformations such as introducing new program statements, replacing LHS of assignments etc. in our error model. Our approach also supports minimal cost changes to student’s programs where each error in the model is associated with a certain cost, unlike the earlier mentioned approaches.
Mutation-based program repair [6] performs mutations repeatedly to statements in a buggy program in order of their suspiciousness until the program becomes correct. The large state space of mutants (10^{125}) makes this approach infeasible. Our approach uses a symbolic search for exploring correct solutions over this large set. There are also some genetic programming approaches that exploit redundancy present in other parts of the code for fixing faults [3, 9]. These techniques are not applicable in our setting as such redundancy is not present in introductory programming problems.

6.3 Automated Debugging and Fault localization

Techniques like Delta Debugging [30] and QuickXplain [19] aim to simplify a failing test case to a minimal test case that still exhibits the same failure. Our approach can be complemented with these techniques to restrict the application of rewrite rules to certain failing parts of the program only. There are many algorithms for fault localization [4, 10, 11] that use the difference between faulty and successful executions of the system to identify potential faulty locations. Jose et al. [18] recently suggested an approach that uses a MAX-SAT solver to satisfy maximum number of clauses in a formula obtained from a failing test case to compute potential error locations. These approaches, however, only localize faults for a single failing test case and the suggested error location might not be the desired error location, since we are looking for common error locations. These approaches, however, only localize faults for a single failing test case and the suggested error location might not be the desired error location, since we are looking for common error locations that cause failure of multiple test cases. Moreover, these techniques provide only a limited set of suggestions (if any) for repairing these faults.

6.4 Program Synthesis

Program synthesis has been used recently for many applications such as synthesis of efficient low-level code [21, 25], inference of efficient synchronization in concurrent programs [29], snippets of excel macros [13], relational data structures [14, 15] and angelic programming [5]. The SKETCH tool [24, 25] takes a partial program and a reference implementation as input and uses constraint-based reasoning to synthesize a complete program that is equivalent to the reference implementation. In general cases, the template of the desired program as well as the reference specification is unknown and puts an additional burden on the users to provide them; in our case we use the student’s solution as the template program and teacher’s solution as the reference implementation. A recent work by Gulwani et. al. [12] also uses program synthesis techniques for automatically synthesizing solutions to ruler/compass based geometry construction problems. Their focus is primarily on finding a solution to a given geometry problem whereas we aim to provide feedback on a given programming exercise solution.

7. Conclusions

In this paper, we presented a new technique of automatically providing feedback for introductory programming assignments that can complement manual and test-cases based techniques. The technique uses an error model describing the potential corrections and constraint-based synthesis to compute minimal corrections to student’s incorrect solutions. We have evaluated our technique on a large set of benchmarks and it can correct 65% of incorrect solutions. We believe this technique can provide a basis for providing automated feedback to hundreds of thousands of students learning from online introductory programming courses that are being taught by MITx and Udacity.

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