ADAPTIVE MESH REFINEMENT AND RELATIVISTIC MHD

DAVID NEILSEN, ERIC W. HIRSCHMANN

Department of Physics and Astronomy, Brigham Young University,
Provo, UT 84602, USA

MATTHEW ANDERSON

Department of Physics and Astronomy, Louisiana State University,
Baton Rouge, LA 70803-4001, USA

STEVEN L. LIEBLING

Department of Physics, Long Island University—C.W. Post Campus,
Brookville, NY 11548, USA

We solve the general relativistic magnetohydrodynamics equations using distributed parallel adaptive mesh refinement. We discuss strong scaling tests of the code, and present evolutions of Michel accretion and a TOV star.

1. Introduction

Compact objects combine a wide array of fascinating physics, and gravitational waves may open new ways to probe these objects. We are interested in systems where gravitational and magnetic fields are dynamically important. One challenge in simulating astrophysical compact objects is that these systems require a range of important length and time scales. Adaptive mesh refinement (AMR) thus becomes an increasingly important tool for large scale computations. Furthermore, large computational problems on today’s computers must be able to effectively utilize a large number of distributed processors.

To address some of the challenges in studying compact objects with numerical relativity, we have developed a code to solve the general relativistic magnetohydrodynamics (GRMHD) equations with AMR. We use the HAD infrastructure, a modular code for solving hyperbolic and elliptic differential equations with distributed parallel AMR. HAD uses Berger–Oliger style AMR with sub-cycling in time. Refinement criteria may be problem specific, or a shadow hierarchy allows one to easily estimate the truncation error dynamically for use in specifying refinement criteria. The equations to be solved for a specific problem are isolated in equation modules, which may be used independently or combined with other modules. For example, the MHD and GR equations are in separate modules, which may be used independently or combined for the GRMHD code.

The MHD equations are solved using the Convex Essentially Non-Oscillatory (CENO) method, a third order scheme for smooth fluid flow. Although our AMR driver can accommodate both finite difference and finite volume discretization methods, we choose a finite difference high-resolution shock-capturing method for the fluid equations to simplify the combined GRMHD code. We use hyperbolic divergence cleaning to control the $\nabla \cdot \mathbf{B} = 0$ constraint for the magnetic field. Commu-
cation between coarse and fine grids uses WENO interpolation, a scheme designed for discontinuous functions. Finally, the method of lines is used for the temporal discretization, and we use a TVD-preserving, third-order Runge–Kutta scheme to integrate the equations. In this paper, we briefly summarize some of our results. The details of our method and more extensive tests are presented elsewhere.\textsuperscript{1,2}

2. Results

Astrophysical simulations of compact objects require that a large number of processors can be used efficiently. A rigorous measure of such performance is the strong scaling test, where a model problem of fixed size is run on increasing numbers of processors. These tests indicate that the GRMHD code uses the distributed parallel computing environment relatively efficiently, as our code scales approximately linearly as the number of processors is increased by a factor of ten.\textsuperscript{2} (See Figure 1.) Note, since the problem size is fixed, the scaling can not be linear indefinitely.

To verify that the equations are implemented correctly, we have compared results with exact solutions, and we discuss two of these tests here: the Michel solution, and Tolman–Oppenheimer–Volkoff (TOV) solutions. The Michel solution describes the continuous spherical accretion of a fluid onto a Schwarzschild black hole in the presence of a radial magnetic field. We use in-going Eddington-Finkelstein coordinates in our calculation, and excise a centrally located cubical region of half width 0.3 $M$ to remove the singularity. In this test, the fluid is initially set to the Michel solution for radius $r > 2.5M$ while for $r \leq 2.5M$ a constant pressure and density are chosen. The system is then evolved until a steady state is reached. The refinement criterion is based on the estimation of the truncation error provided by the shadow hierarchy. Figure 1 shows the AMR grid structures at time $t = 50M$.

A stable TOV solution is used for our second test, which we evolve in the Cowling approximation (fixed geometry) for over 400 light-crossing times. The star oscillates as expected, and the oscillations of the density at half the stellar radius ($R/2$) are shown in Figure 2. The equation of state for the initial data is $P = \kappa \rho_0^{\Gamma}$, with $\Gamma = 5/3$ and $\kappa = 4.349$. Similar runs on dynamic backgrounds have also been performed and show similar results, though for slightly shorter periods of time.

Acknowledgments

We are pleased to thank Luis Lehner, Patrick Motl, and Ignacio Olabarrieta for numerous suggestions and discussions during the course of this work. We also thank Bruno Giacomazzo, Carlos Palenzuela, Oscar Reula, Luciano Rezzolla, and Joel Tohline for helpful discussions. This work was supported by the NSF through the grants PHY-0326311, PHY-0244699, PHY-0326378, PHY-0502218, and PHY-0325224.
Adaptive Mesh Refinement and Relativistic MHD

Fig. 1. The left frame shows a strong scaling test of our MHD code using AMR. In this test, thirty iterations were performed on a coarse grid of size $81^3$ and a single level of refinement. The right frame shows the rest density $\rho_0$ and the AMR grid structures for the Michel solution at $t = 50M$ in the $x$-$y$ plane. The domain of simulation is $\{x, y, z\} \in [-15M, 15M]$. The cubical excision region is highlighted in the center of the grid on the left.

Fig. 2. The variation in the density at $R/2$ for a stable TOV solution. The evolution is performed using $129^3$ points on a cubical domain $\{x, y, z\} \in [-11M, 11M]$. The central density is $8.1 \times 10^{-4}$, the stellar radius $R = 9.279$, and mass $M = 0.5659$.

References
1. D. Neilsen, E. W. Hirschmann and R. S. Millward, Class. Quantum Grav. 23, S505 (2006).
2. M. Anderson, E. Hirschmann, S. L. Liebling and D. Neilsen, Class. Quantum Grav. 23, 6503 (2006).