Non-stationary mass transfer of oil-water mixture in reservoirs with a system of horizontal wells

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Abstract. The paper presents a physical and mathematical model of quasi-three-dimensional non-stationary filtration of a water-oil mixture in a system of horizontal wells and the corresponding numerical calculation method. Based on them, a computer program was created, designed to solve a number of scientific and production problems. The developed model and program are aimed at theoretical and numerical study of the processes of mass transfer of the oil-water mixture in the field of analysis, maintenance and development of natural formations with high-viscosity oil deposits. The results of numerical experiments for calculating the two-phase flow of an incompressible fluid in a reservoir are given as an example of a real field, located in the north of Western Siberia. The paper proposes an approach to discretization of the computational domain, based on the construction of a dynamic computational grid in a natural semi-fixed coordinate system. For the first time, the problem of calculating two-phase mass transfer using a semi-fixed natural grid and an analytical expression for the flow rates of phases in current streams of a found shape has been solved. Based on the proposed model, calculation and parametric studies of various modes of field development with a system of parallel horizontal wells were performed.

1. Introduction

In recent decades, physical and mathematical modeling has been the main tool for making decisions on the analysis, maintenance and development of oil and gas fields.

The correct solution of the filtration problem, taking into account the specifics of the deposits in question, allows predicting hydrocarbon production in the long term, localizing residual hydrocarbon reserves, and evaluating the effectiveness of measures to maintain reservoir pressure and other measures to increase oil recovery.

The first steps in the development of the theory of filtration were laid by the French engineer A. Darcy, who published an experimental law in 1865, which corresponded the pressure gradient to the linear filtration rate [9]. This work is considered the beginning of the theory of filtration. To date, many works by domestic and foreign authors are devoted to the development of the theoretical foundations of filtration. The works of many scientists, such as R.I. Nigmatulin [3], G.N. Barenblatt [1], K.S. Basniev [2], studied complex two-phase and three-phase flows in a reservoir and wells.

Taking into account the latest achievements in the field of applied mathematics and a constant increase in the productivity of computing technologies, the modern idea of modeling the processes of developing hydrocarbon deposits is reduced to solving a detailed three-dimensional problem [10, 11].
Despite this, the practice of using hydrodynamic models shows that the capabilities of simulators are far from always meeting the needs that arise when solving important production problems associated with field development [6]. In particular, this applies to modeling development processes in complex unconventional fields. An urgent problem is the construction of computational models and computer programs for calculating the parameters of the development of viscous oil fields, especially near horizontal wells.

2. Physico-mathematical model of mass transfer
The system of equations for calculating the non-stationary filtration of the oil-water mixture includes [1, 2, 3, 7, 8]:
- phase continuity equation:
\[
\frac{\partial (m \rho_i S_i)}{\partial \tau} + \text{div}(\rho_i \vec{v}_i) = 0,
\]
- phase equation (generalized Darcy equation):
\[
\vec{v}_i = \frac{k_{af} f_i}{\mu_i} \nabla P,
\]
- equation of balance of internal energy of a mixture:
\[
\frac{\partial (\rho U)}{\partial \tau} + \text{div} (\bar{q} + \rho U \vec{v}_i) = 0,
\]

where \( \tau \) is time; \( i = 1, 2 \); oil parameters correspond to the index \( i = 1 \); for water \( i = 2 \); \( m \) is porosity; \( S_i \) is the saturation by the \( i \)-th phase (considering \( S_1 + S_2 = 1 \)); \( \rho_i^0 \) is true density; \( k_0 \) is absolute permeability; \( f_i \) is relative phase permeability; \( \mu_i \) is dynamic viscosity; \( \rho U = (\rho_1 c_1 + \rho_2 c_2 + \rho_{rock} c_{rock}) T \) is specific internal energy of the mixture, \( \bar{q} \) is heat flux vector due to heat conduction (\( \lambda \)).

In equations (1–3), the unknowns are velocity (\( \vec{v}_i \)), pressure (\( P \)), temperature (\( T \)), and saturation (\( S_i \)). For the system of equations (1–3), the initial (at the time \( \tau = 0 \)) distribution of parameters in the flow region is set, as well as the conditions on the boundary of the region.

3. Problem statement
The paper considers quasi-three-dimensional isothermal non-stationary processes that occur during the operation of an oil field with a system of horizontal injection and production wells. The system "injection well – reservoir – production well" is considered. The scheme of the problem under consideration is presented in Figure 1.

The following assumptions are used in the work:
1. The three-dimensional problem is reduced to a combination of three interrelated tasks:
   1) quasi-one-dimensional problem of changing pressure, flow rate of water (steam) and temperature (in the case of solving the thermal problem) in the horizontal section of the injection well;
   2) the set \( N_x \) of two-dimensional two-phase filtering problems in vertical layers of \( \Delta Z_i = L_z / N_z \), where \( i = 1, N_z \);
   3) quasi-one-dimensional problem of changing pressure, temperature and flow rates of oil and water (at this stage we restrict ourselves to solving the two-phase filtration problem) in the horizontal section of the well.

The oil-water mixture flows in horizontal sections of production wells along the \( Z \)-axis \( 0 \leq Z < L_z \); like in injection wells, hot water or other coolant flows along the \( Z \) axis, while \( Z \) varies from \( Z = L_z \) to \( Z = 0 \). The projection of the filtration rate along the \( Z \) axis (\( W_z \)) is assumed to be significantly smaller compared to \( W_x \) and \( W_y \), thus, equality (4) holds:
\[
W = \sqrt{W_x^2 + W_y^2}
\]
Figure 1. Schematic representation of a quasi-three-dimensional problem in a system of parallel horizontal wells

Legend in Figure 1: H is the reservoir thickness, \( L_x \) is the length of the horizontal section of the well, \( \Delta Z \) are the dimensions of the model cell along the Z axis, \( W \) is water, \( O + W \) is the water-oil mixture.

2. A quasistationary approximation is considered with a known distribution of parameters at time \( t^n \) and certain external influences over time \( \Delta t \), parameters at time \( t^{n+1} = t^n + \Delta t \), where \( n = 1, N_t \).

The development system (Figure 1), which is used in practice, is described by solving a three-dimensional non-stationary problem of heat and mass transfer in a system of horizontal sections of an injection well - formation - horizontal section of a producing well.

In this work, aimed at improving methods for calculating hydrodynamic parameters, it is assumed that the temperature field in the region of the filtration flow is known.

4. Calculation algorithm

The following iterative method for solving the problem under the above assumptions is proposed, which includes the following blocks:

1) Formation of a computational grid in the reservoir area and in cross sections in wells.
2) Setting the initial data for modeling:
   - geometric dimensions: diameter of the injection \( (d_i) \) and production wells \( (d_p) \); distance between horizontal sections of wells \( (L_x) \); effective length of the horizontal section of wells \( (L_z) \);
   - initial reservoir parameter distributions: porosity \( (m) \), absolute \( (K_0) \) and relative phase permeability \( (f_{o,w}(S_w)) \), water or oil saturation \( (S_o = 1 - S_w) \), pressure \( (P_0) \) and temperature \( (P_0) \) of the reservoir at the initial moment of time \( (t^0) \);
   - initial pressure in the injection \( (P_i) \) and production \( (P_p) \) well or rates \( (G_{o,w}) \) and temperature \( (T_i) \) in the injection well;
   - generalized data on the thermophysical and rheological properties of fluids and rocks: true densities \( \rho_{o,w,rock}(T, P) \), dynamic viscosities \( \mu_{o,w}(T, P) \), thermal conductivity and heat capacity coefficients \( \lambda_{o,w}(T) \), \( C_{o,w}(T) \);
   - calculation parameters: time step \( (\Delta t) \), number of sections \( (N_x) \), number of streamlines and equipotential surfaces, error during iterations over the flows \( (\xi_g) \).
3) The initial distributions of pressure \( P_{i,p}(x, t) \) and temperature \( T_i(x, t) \) in the horizontal section of the injection and production wells along the length \( 0 \leq x \leq L_x \) are calculated; the pressure distribution is expressed from the equation of balance of momentum [7] and is described by equation (5):
\[
P_i(x) = P_i(0) + a_k \rho \frac{v(0)^2 - v(x)^2}{2} + g(z(0) - z(x)) + (l' - l_{fr})
\]

where \( \frac{v(0)^2 - v(x)^2}{2} \) is the change in kinetic energy; \( g(z(0) - z(x)) \) work in the field of gravity; \( l' = -(v - v_c) \frac{\rho}{\rho s} \) is specific work associated with mass transfer through the lateral surface \( S \): \( l_{fr} = \lambda fr \frac{\Delta x v^2}{2} \) is work of friction forces according to the Darcy–Weisbach equation.

4) The method of complex source potentials [4, 5, 7] determines the initial position of streamlines and equipotential surfaces in the planes \( Z_i = \text{const} \) (\( i = \overline{1, N_z} \)), control volumes are formed at the points of intersection of the lines. Thus, we use the natural coordinate system \( (\tilde{l}, \tilde{l}_1) \) [4, 5], where \( l_i = \text{const} \) corresponds to the lines of equal potential \( \varphi_i = \text{const} \) determined in the calculation process and the streamlines \( \psi_j = \text{const} \). The integrated potential for modeling the flow, taking into account the geometric shape of the object of study (Figure 1), taking into account the influence of the boundaries of the reservoir, is presented in the form of a superposition of sources and sinks [7]:

\[
w(z) = \frac{Q_i}{2\pi} \ln \left( \frac{z - \frac{h}{2}}{z + \frac{h}{2}} \right) - \frac{Q_p}{2\pi} \ln \left( \frac{z + \frac{h}{2}}{z - \left( H - \frac{h}{2} \right)} \right) - \frac{Q_p}{2\pi} \ln \left( \frac{z + \left( H - \frac{h}{2} \right)}{z + \left( H + \frac{h}{2} \right)} \right)
\]

5) From the system of quasi-one-dimensional equations of fluid motion in horizontal wells [8], we find the change in velocities, pressures, saturations, and temperatures along the length \( 0 \leq x \leq L_x \).

The pressure distribution along the flow stream will be expressed as

\[
p(l) = p_1 - J_1(l) * G.
\]

The flow rate of the mixture through the flow stream is determined from (7) taking into account \( \tilde{l} = L_j \), where \( L_j \) is the current stream length, \( p = p_2 \) and \( J_1(l) = J_1(L) \):

\[
G = \frac{p_1 - p_2}{J_1(l)}.
\]

The definite integral \( J_1(l) \) has the following form:

\[
J_1(l) = \int_0^l \frac{dt}{k_0 s \left( \frac{\rho f_0 o}{\mu o} + \frac{\rho f_0 w}{\mu w} \right)}.
\]

The distribution of phase velocities along the flow stream \( l \) taking into account (8) and (9):

\[
w_o(l) = \frac{k_0 f_o}{\mu o} \left( \frac{\rho f_0 o}{\mu o} + \frac{\rho f_o w}{\mu w} \right) = \frac{G}{F \left( \rho o_0 + \rho f o f o w \right)}
\]

\[
w_w(l) = \frac{k_0 f_o}{\mu w} \left( \frac{\rho f_0 o}{\mu o} + \frac{\rho f_o w}{\mu w} \right) = \frac{G}{F \left( \rho w_0 + \rho f o f o w \right)}
\]

Oil and water flow rates are in the section \( l = L \) with area \( F(L) \):

\[
\begin{cases}
G_o = w_o(l) \left( \rho o_0 \right) \left( F(L) \right) = \frac{G}{1 + \rho f o f o w_0} \\
G_w = G - G_o.
\end{cases}
\]
\[ S_i^{n+1} = S_i^n + \left[ \left( \rho_{1i}^0 w_1 F_1 \right) - \left( \rho_{2i}^0 w_2 F_2 \right) \right] n \frac{\Delta t}{V_{m*} \rho_i^0} \] (13)

6) With the control volumes found in step 4 in the natural coordinate system, the initial pressure distributions in the current streams and phase flows are determined, by analogy with [8], but taking into account the shape of the streamline;

7) Taking into account the parameters found in step 6, the calculated streamlines (step 4) and the parameters of the control volumes in the planes \( x_i = \text{const} \ i = \overline{1,N_x} \) are refined. Thus, a dynamic natural semi-fixed coordinate system is used to discretize space. To account for flow dynamics and changes in control volumes, lines of equal potentials are fixed, and streamlines are refined taking into account constantly changing parameters of the filtration flow.

8) The iterative procedure according to steps 5 - 7 is repeated until convergence from the conditions:

\[ \frac{\Delta G_{w}^{k+1} - \Delta G_{w}^{k}}{\Delta G_{w}^{k}} \leq \xi_{G} \] (10)

\[ \frac{\Delta G_{o}^{k+1} - \Delta G_{o}^{k}}{\Delta G_{o}^{k}} \leq \xi_{G} \] (11)

where \( k \) is the iteration number, \( \xi_{G} \) is the permissible error during iterations over the flows in the layers \( x_i = \text{const} \ i = \overline{1,N_x} \);

9) The calculation according to steps 5-8 is repeated for time intervals \( (t^n, t^{n+1}) \) for \( n = \overline{2,N_t} \), where \( N_t \) is the number of time steps;

10) Visualization and processing of simulation results;

11) Adaptation of results and calculation and parametric studies.

5. Calculation and parametric studies on the example of a real field with high-viscosity oil

As an example, we used real data from a field in Western Siberia with a similar development system. The relative phase permeability functions were set according to the results of digital stochastic pore-network modeling, a detailed description of the approach is presented in [12]. The initial data for the calculation are summarized in Table 1.

| Parameter                          | Designation | Value |
|-----------------------------------|-------------|-------|
| Reservoir thickness [m]           | \( H_{res} \) | 20    |
| Coefficient of porosity [%]       | m           | 30    |
| Initial oil saturation [%]        | \( S_o \)   | 70    |
| Initial reservoir pressure [MPa]  | \( P_0 \)   | 80    |
| Initial reservoir temperature [°C]| \( T_0 \)   | 20    |
| Downhole pressure of injection well [MPa] | \( P_i \) | 100   |
| Downhole pressure of producing well [MPa] | \( P_p \) | 60    |
| Length of horizontal section of wells [m] | \( L_z \) | 500   |
| Diameter of wells [mm]           | \( d_{p1} \) | 169   |

At the first stage, technological indicators were calculated for various modes of field development (Figure 2).
Figure 2. Technological indicators of development in various modes: a) the dynamics of cumulative oil production, b) the dynamics of cumulative water production

At stage two, a series of isothermal calculations was performed with different initial formation temperatures (Figure 3a). Initial formation temperatures were taken equal to 20, 60 and 80 °C. In this series of experiments, the key factor influencing the result was the dependence of fluid viscosity, surface tension, and the change in the relative permeability diagram. The dependence of the dynamic viscosity and surface tension were obtained from reservoir fluid samples, and the patterns of change in the relative permeability diagrams were justified by the proposed digital core research technology [12].

Figure 3. A series of isothermal calculations with different initial formation temperatures (a); comparison of simulation results for cumulative oil production of the developed program and a commercial simulator (b)
The reliability of the research is provided by the application of the fundamental laws of the mechanics of multiphase systems in conjunction with modern physical and mathematical models and numerical methods for solving problems. The results obtained are substantiated by verification of data with well-known commercial hydrodynamic simulators (Figure 3b).

6. Conclusions

1. The paper for the first time proposed an approach to discretization of the computational domain, based on the construction of a dynamic computational grid in a natural semi-fixed coordinate system.
2. A technique for thinning the shape of current streams as applied to filtering a two-phase mixture with the determination of RPT by the core digital research technology was developed and implemented as a computer program.
3. A new method for calculating quasi-three-dimensional non-stationary isothermal operating modes of a system of parallel horizontal wells has been developed.
4. Based on the proposed model, calculation method, and software implementation of computational algorithms, computational-parametric studies of various development modes with a system of parallel horizontal wells were performed using an example of a real hydrocarbon field.

The proposed calculation methods and their software implementations are the basis for further modeling of heat and mass transfer processes in order to theoretically and numerically study the processes that occur during thermal methods of developing high-viscosity oil fields.

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