Thermo conductivity in melting and freezing processes of the capillary-porous bodies

M V Salnikov*, A V Fedoseev and G I Sukhinin
Institute of Thermophysics SB RAS, Lavrentyev Ave. 1, Novosibirsk 630090, Russia

*E-mail: salnikov@itp.nsc.ru

Abstract. The aim of the research is to present numerical one-dimensional solution for the temperature distribution in spherical capillary porous body, where the melting and freezing processes are taken into account. The solution includes the non-linear temperature distribution for every moment of heating and cooling at given surface temperature, equal to the temperature of the ambient medium.

1. Introduction
In capillary-porous materials, such as soil, clay minerals, wood, food products, etc., the pores are usually filled with moisture, water vapor and air. Moisture content determines the state of the porous body, its mechanical and thermophysical properties, such as density, viscoelasticity, ductility, and the like. Complex processes of heat and mass exchange occur in porous bodies when they are heated or cooled, during drying or freezing [1, 2, 3]. The development of these processes depends essentially on the physicochemical composition and properties of the porous material, the dimensions, porosity and moisture of the body, on the content of water in the pores and its aggregate state, and on the dependence of the effective transport coefficients on temperature and humidity [4, 5].

Samples of clay are often subjected to non-stationary heat treatment to modify its physical properties when the temperature in the environment changes. In this case, the change in moisture content in the clay volume usually occurs at times more slowly than the change in temperature, due to the smallness of the effective diffusion coefficient of moisture content in comparison with the coefficient of thermal diffusivity. In this case, when studying the non-stationary heating or cooling of porous bodies, the calculation of the temperature field in the body can be carried out using a heat exchange model with a fixed dependence of all parameters and transport coefficients on moisture content [6, 7].

In this paper we present a one-dimensional numerical model for calculating the nonlinear temperature distribution in the process of freezing and heating of spherical clay samples at various initial and boundary conditions in the environment.

2. Model
During the porous bodies heating, a mass-exchange occurs between the processing medium and the materials. The effect of the mass diffusion in these materials is usually much smaller than the effect of temperature conductivity. Therefore diffusion of mass in the materials lags significantly from the distribution of heat during the heating or cooling. This allows to amiss in our consideration the exchange of mass and provides the opportunity to study the change in temperature as a result of a
The thermo-exchange process, where the heat is distributed only through thermo-conductivity. Therefore the heat distribution can be described by the equation of heat conduction:

\[ c_{\text{eff}} \rho \frac{\partial T}{\partial \tau} = - \text{div}(\lambda \text{grad}T) + q. \]  

(1)

Equation (1) is correct for any coordinate system and for any processing medium – mobile or immobile. Variables \( c, \rho \) and \( \lambda \) depend on \( T \) and \( u \), i.e. \( c = c(T,u) \), \( \rho = \rho(T,u) \) and \( \lambda = \lambda(T,u) \), where \( u \) is a moisture content. Water contained in porous bodies can be in liquid or solid state. Heat capacity of the non-frozen water at 0°C is equal to 4237 J/kg°C, and ice heat capacity is 2261 J/kg°C, i.e. which is about two times smaller. Therefore the frozen water causes smaller \( c \) than the liquid water.

The ice in porous bodies is formed from the freeze higrosopically bounded or free water. Phase transition of water into ice is expressed with the help of the ice “latent heat”. It worthwhile to include such heat in the effective heat capacity \( c_{\text{eff}} \), which can be determined with the following equation:

\[ c_{\text{eff}} = c + c_{bw} + c_{fw}, \]

(2)

where \( c \) is heat capacity of the porous body, \( c_{bw} \) is bounded water heat capacity, \( c_{fw} \) is free water heat capacity. In the absence of the interior heat source (\( q = 0 \)) and in the case of the isotropic sphere for a body, the equation (1) can be written in the following form in the spherical coordinate system:

\[ c_{\text{eff}} \rho \frac{\partial T}{\partial \tau} = \lambda \left( \frac{\partial^2 T}{\partial r^2} + \frac{\partial \lambda}{\partial T} \left( \frac{\partial T}{\partial T} \right)^2 \right). \]

(3)

This equation also has an initial and boundary conditions:

\[ T(r,0) = T_0, \]

(4)

\[ T(0,t) = T_m, \]

(5)

where \( T_m \) and \( T_0 \) are ambient medium and body initial temperatures, correspondingly. For the purpose of this model being simple it is accepted that conductivity is linear and described with the equation:

\[ \lambda = \lambda_0 (1 + \beta (T - 273.5)). \]

(6)

Experimentally observed that for \( u > u_{sp} \) for the temperature near -2°C temperature experiences jump. Here \( u_{sp} \) is a critical moisture content – the moment after which porous body can’t absorb water anymore. The \( u_{sp} \) value is derived by the equation:

\[ u_{sp} = u_{sp}^{20} - 0.001(T - 293.15), \]

(7)

where \( u_{sp}^{20} \) – critical moisture content of the observed material for temperature 20°C. Its value can be found in the specialized literature. In pair with \( u_{sp} \) the variable \( u_{nfw} \) is also widely used in the present model. It determines the moisture content of non-frozen water as

\[ u_{nfw} = 0.12 + (u_{sp} - 0.12) \exp[0.0567(T - 293.15)]. \]

(8)

Heat capacities from equation (3) are determined by the next set of equations:

1) For the case \( T > 271.15 \) or \( T \leq 271.15 \) but with \( u < u_{nfw} \):

\[ c = \frac{2097u + 826}{1 + u} + \frac{9.92u + 2.55}{1 + u} T + \frac{0.0002}{1 + u} T^2, \]

(9)

\[ u < u_{sp} \]

\[ c = \frac{2862u + 555}{1 + u} + \frac{5.49u + 2.95}{1 + u} T + \frac{0.0036}{1 + u} T^2, \]

(10)

\[ u \geq u_{sp} \]
2) For the case \( T \leq 271.15 \) but with \( u \geq u_{nfw} \)

\[
c = \left( 1.06 + 0.04u + \frac{0.00075(T - 271.15)}{u_{nfw}} \right) \frac{526 + 2.957 + 0.0022T^2 + 2261u + 1976u_{nfw}}{1 + u}, \tag{11}
\]

\[
c_{fw} = 3.34 \cdot 10^5 \frac{u - u_{sp}}{1 + u}, \; 271.15 < T \leq 272.15 \; \& \; u > u_{sp} \tag{12}
\]

\[
c_{bw} = 1.8938 \cdot 10^4 (u_{sp} - 0.12) \frac{\exp[0.0576(T - 271.15)]}{1 + u}, \; u > u_{nfw} \tag{13}
\]

Coefficients in equation (6) is derived by the following formulas:

\[
\lambda_0 = \psi[0.165 + (1.39 + 3.8u)(3.3 \cdot 10^{-7} \rho_b^2 + 1.015 \cdot 10^{-3} \rho_b)], \tag{14}
\]

where \( \rho_b \) is density of the absolutely dry porous body and

\[
v = 0.15 - 0.07u, \; u \leq u_{sp} + 0.1 \tag{15}
\]

\[
v = 0.1284 - 0.013u, \; u > u_{sp} + 0.1 \tag{16}
\]

1) For the case \( T > 271.15 \) or \( T \leq 271.15 \) but with \( u < u_{sp} \)

\[
\gamma = 1 \tag{17}
\]

\[
\beta = (2.05 + 4u)\left( \frac{579}{p_b} - 0.124 \right) \cdot 10^{-3}, \; u \leq u_{sp} + 0.1 \tag{18}
\]

\[
\beta = 3.65\left( \frac{579}{p_b} - 0.124 \right) \cdot 10^{-3}, \; u > u_{sp} + 0.1 \tag{19}
\]

2) For the case \( T \leq 271.15 \) but with \( u \geq u_{sp} \) or when \( T \leq T_{nfw} \) and \( 0,125 \leq u < u_{sp} \)

\[
\gamma = 1 + 0.34[1.15(u - u_{sp})] \tag{20}
\]

\[
\beta = 0.002(u - u_{sp}) - 0.0038\left( \frac{579}{p_b} - 0.124 \right) \tag{21}
\]

\[
T_{nfw} = 271.15 + \frac{\ln \frac{u_{nfw} - 0.12}{u_{sp} - 0.12}}{0.0567} \tag{22}
\]

And, finally, current density of the material is obtained through following empirical equations:

\[
\rho = \rho_b \frac{1 + u}{1 - 9.3 \cdot 10^{-4} \rho_b(u_{sp} - u)}, \; u \leq u_{sp} \tag{23}
\]

\[
\rho = \rho_b (1 + u), \; u > u_{sp} \tag{24}
\]

The model explained above was used to calculate the evolution of temperature distribution of spherically isotropic ball of clay. As it was mention before, it is common that in the case of fast
heating and cooling the moisture content distribution experiences time lag and therefore could be considered as a parameter.

3. Results
In this paper the temperature distribution of a spherically isotropic porous clay ball is considered, with the radius of the ball, unless otherwise stated, being equal to twenty centimeters. The density of dry clay is taken as a parameter and given by a constant equal to $\rho_b = 1800$ kg/m$^3$.

First of all, cooling of a clay ball is considered in this model. The results of this experiments are shown in figure 1. The initial temperature of the prototype is set equal to room temperature $T_0 = 20^\circ$C, and the ambient temperature (freezer compartment) is set equal to $T_m = -20^\circ$C.

As shown in this evolutionary curve, as the moisture content in the sample increases, the gap in which the temperature equal to the constant is broadened. This is due to the fact that the volume of water, present in the pores, turns into ice not instantly, but slowly cools down to the required temperature. The more water content, the more time is needed to all water to be freeze.

Figure 2 depicts the reverse process, i.e. the process of heating a uniformly cooled sample, which is taken out of the refrigerating chamber. In this case, the temperature of the medium-the drying apparatus is set equal to $T_m = 80^\circ$C, with the initial temperature of the sample being equal to $T_0 = -20^\circ$C.

As in figure 1, with the moisture increase, a region of constant temperature values is formed. The more moisture content, the area is wider. However, it should be noted that in figure 1 and figure 2, with the moisture increase, the change in temperature, after overcoming the freezing barrier, is more rapid for higher moisture values, due to the fact that the center of the clay sample is surrounded by the clay with higher temperatures and this areas are actively transmit heat to the center.

The same experiment was carried out for a moisture content equal to 15% for samples of different radii: 20, 40, 50 and 60 centimeters. Figure 3 shows results of these experiments. The left curve is the ordinary data. The right one is the data that has been processed, namely: the values on the temporal abscissa were divided by the square of the ratio of the radii, in other words, the time is normalized.

As a result of this experiment, it was found that, despite significant differences in volume (from 8 to 64 times), sample heating occurs in a similar way. The following experiment was devoted to a cyclic temperature change in the case when the frozen object was first placed in a drying chamber and then placed in a refrigerator.

It is seen that the way in which the temperature changes in the sample is different for heating and cooling, i.e. hysteresis is observed in the time dependence of the temperature that shown in figure 4. The explanation is simple: the temperature changes faster in the case of a larger temperature gradient.
If the temperatures of two elementary volumes of clay adjacent to one another differ significantly, the heat exchange between these volumes is more intense, and therefore, a sharp change in temperature occurs. That’s why the temperature of the edge of the clay ball changes sharply and a large temperature gradient appears in the material itself then the medium temperature changes instantly. This gradient decreases over time, which reduces intensity of heat transfer.

The case demonstrated in figure 5 shows the results of the simulation in which clay is placed in an air environment, where the temperature is determined by the daily weather changes. For the sake of this experiment simplicity, it is suggested that the daily cyclic change in the medium temperature is determined sinusoidally and its amplitude is equal to the ten degrees Celsius.

The fluctuations in the internal temperatures of the ball differ from the external oscillations by a phase shift (the deeper in the material, the larger the phase difference) and the shape. The inner layers are the most twisted - the water there does not have time to freeze and regular regions with a constant temperature are formed.

**Figure 3.** The change in temperature at the center of the sample for different sample sizes ($T_0 = -20^\circ\text{C}, T_m = 80^\circ\text{C}$). On the right is a curve where the time is normalized.

**Figure 4.** The change in temperature at different points of the sample in the case of a cyclic temperature change (first cycle: $T_0 = -20^\circ\text{C}, T_m = 80^\circ\text{C}$; second cycle: $T_0 = 80^\circ\text{C}, T_m = -20^\circ\text{C}$) for moisture content $u = 25\%$.

**Figure 5.** The change in temperature at different points of the sample in the case of a daily temperature change ($T_u = 10^\circ\text{C}$) for moisture content $u = 10\%$. 
4. Conclusion
In this paper, evolutionary temperature curves are obtained in a clay ball for different conditions: for the case of cooling, heating, in the case when the sample is subjected to a cyclic temperature treatment, and also the effect of the daily change of the medium temperatures on the heat exchange is observed. It was determined that the temperatures (especially at the center of the sample) experience the jump due to the non-instantaneous freezing of water, and it is also obtained that, during cyclic or daily cooling, the heating path differs significantly from the cooling path of the material and vice versa. Because of the complexity, this model does not take into account the material expansion due to heating and cooling, but, in reality, such changes in temperatures necessarily entail deformation of the sample, and, if the path through which the temperature varies differs, then the changes in the object are irreversible.

Acknowledgement
This research was funded by the Russian Ministry of Education and Science. Project Identifier: RFMEFI60417X0193.

References
[1] Luikov A V 1997 Mass and Momentum Transfer in Porous Media: A Theory of Drying. Advances in Heat Transfer 13 119–203
[2] Twardowski K, Rychinski S, Traple J 2006 Faculty of Drilling, Oil and Gas (AGHUST, Krakow) 208–12
[3] Deliiski N 2011 Convection and Conduction Heat Transfer 149–76
[4] Kulasiri D, Woodhead I 2005 Mathematical Problems in Engineering 3 275–91
[5] Zhang Z, Yang S, Liu D 1999 Heat Transfer – Asian Research 28(5) 337–51
[6] Ben Nasrallah S, Perre P 1988 International Journal of Heat and Mass Transfer 31(5) 297–310
[7] Murugesan K, Suresh H N, Seetharamu K N, Narayana P A A and Sundararajan T A 2001 International Journal of Heat and Mass Transfer 44(21) 4075–86