Veneziano Ghost Versus Isospin Breaking

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Abstract

It is argued that an account for the Veneziano ghost pole, appearing in resolving the U(1) problem, is necessary for understanding an isospin violation in the $\pi - \eta - \eta'$ system. By virtue of a perturbative expansion around the $SU(2)_V$ ($m_u = m_d$) symmetric Veneziano solution, we find that the ghost considerably suppresses isospin breaking gluon and s-quark matrix elements. We speculate further on a few cases where the proposed mechanism can play an essential role. We discuss the isospin violation in meson-nucleon couplings and its relevance to the problem of charge asymmetric nuclear forces and possible breaking of the Bjorken sum rule. It is shown that the ghost pole could yield the isospin violation of order 2% for the $\pi N$ couplings and 20% for the Bjorken sum rule.

1 Introduction

The isospin symmetry is known to be a good approximation to the real world. The common wisdom claims that one can set $m_u \simeq m_d$ not because of their closeness ($m_d - m_u \sim O(m_u, m_d)$) but rather due to the fact that they are both small at the scale of hadronic masses. For the case of pseudo-goldstone bosons, this scale is set, however, by the quark masses themselves, that makes their physics exceptional in the following sense. Since $\frac{m_d - m_u}{m_d + m_u} \sim O(1)$, one could expect large isospin violations which are not usually observed experimentally. On the other hand, there exist phenomena (e.g. heavy quarkonia decays, isospin symmetry breaking in nuclea, etc.) where these effects may be significant.

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As it has been suggested by Gross, Treiman and Wilczek (GTW) in their seminal paper [1], the reason for small isospin breaking lies in the existence of the axial anomaly. By its virtue, the \( \eta' \) gains a large mass and effectively decouples from mixing in the \( \pi^0 - \eta - \eta' \) system. In this case, an isospin violation is only \( O \left( \frac{m_d - m_u}{m_s} \right) \) due to \( \pi^0 - \eta \) mixing (under assumption on the flavor independence for condensates). However, the \( \eta' \) has a mass comparable to that of the \( \eta \) and sizable mixing with the \( \eta \). Thus, one can conclude that an account for the mass generation for the \( \eta' \) (i.e., a solution of the celebrated U(1) problem) is necessary for quantitative studying the isospin violating effects. (In all likelihood, one can neglect electromagnetic interactions for isospin breaking since \( \left( \frac{m_d - m_u}{m_s} \right)/\alpha \sim 10 \).)

In this paper, we apply the Veneziano scheme of resolving U(1) problem [2,3], combined with a perturbation theory over \( \frac{m_d - m_u}{m_d + m_u} \), to the problem of mixing \( \pi^0 - \eta - \eta' \). In Sect.2 we discuss problems with calculating the pion matrix element of the topological density which plays an important role in the heavy quarkonia decays, see e.g. [5,6]. The Veneziano construction in the \( SU(2)_V \) (\( m_u = m_d \)) limit is reviewed in Sect.3, the presentation being close to that of Ref.[3]. In Sect.4 we develop a perturbation theory around this limit and calculate first order corrections to the wave functions. This allows us to re-estimate the gluon matrix element discussed in Sect.2. (It should be noted that the only known to the author anomaly-based discussion of the role of mixing in the gluon matrix elements [4] has rested on adding "by hands" the mass terms into the effective chiral Lagrangian and seems to contradict in both modulo and the signs to the estimates made in [1,3].) The proposed technique is applied to a calculation of \( s \)-quark matrix elements in Sect.5. We proceed further in Sect.6 to discussing the isospin asymmetry in the \( \pi N \) interaction resulting from the presented mechanism of the mixing. It should be stressed that we only concern with the ghost contribution into the studied phenomena. Nonanomalous mechanism are left beyond the scope of this paper and deserve separate studying. The effects are found to be sizable, of order 2\%, while their sign turns out to be opposite to what is expected for a solution of the so-called Nolen-Schiffer anomaly known in nuclear physics [15,16,23]. A relevance to the proton spin problem and possible violation of the Bjorken sum rule in QCD is discussed in Sect.7. Sect.8 contains some comments and conclusions.

## 2 Matrix element \( \langle 0 | \alpha_s G \tilde{G} | \pi^0 \rangle \)

There are a few reasons to start our presentation with this matrix element. The first one, it is proportional to the isospin breaking parameter \( \frac{m_d - m_u}{m_d + m_u} \) and related to the experimental heavy quarkonia decay ratios like \( \frac{\Gamma(\Psi' \rightarrow \Psi + \pi^0)}{\Gamma(\Psi' \rightarrow \Psi + \eta')} \) [5,6]. The second one, and more important for us, it helps to explain the necessity for an account for mixing for its correct determination. Using the soft pion technique and expanding to the first order in \( L^{ΔI = 1} = \frac{m_d - m_u}{2} (\bar{u}u - \bar{d}d) \), one can write the following chain of equations

\[
\langle 0 | \frac{\alpha_s}{4\pi} \tilde{G} G | \pi^0 \rangle = i \int dy e^{iqy} (\Box_y + m_\pi^2) \langle 0 | T\{φ_{π0}(y)\} | \alpha_s/4\pi \tilde{G} G(0) \} | 0 \rangle
\]

\[
= \int dy | T\{∂_μ J^5_μ(y)\} | \alpha_s/4\pi \tilde{G} G(0) \} | 0 \rangle
\]

\[
= \frac{m_u - m_d}{2\sqrt{2} f_π} \int dz dy | T\{∂_μ J^5_μ(y) (\bar{u}u - \bar{d}d)(z)\} | \alpha_s/4\pi \tilde{G} G(0) \} | 0 \rangle
\]
\[ \text{where on the last step we have used the Ward identity with the canonical commutator } \delta(y_0 - z_0)[J^2_0(y), (\bar{u}u - dd)(z)] = -26(y_0 - z_0)\delta^3(\bar{y} - z)[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d](z), J^5_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d \text{ and the PCAC formula } \phi_{\pi^0}(x) = \frac{1}{\sqrt{2} f_\pi m_s^2} \partial_\mu J^5_\mu(x). \] 

The last correlator can now be extracted from the low energy theorem [7] that yields the result \( f_\pi \simeq 133 \text{ MeV} \)

\[ \langle 0 | \frac{\alpha_\pi}{4\pi} \bar{G}G | \pi^0 \rangle = \frac{1}{\sqrt{2}} f_\pi \frac{m_d - m_u}{m_d + m_u} m_\pi^2 (1 + \zeta), \] (2)

(Here \( \zeta \) is a correction factor which we will discuss later on) that coincides with the answer [1] obtained for this matrix element by a different method. However, the above derivation can (and, in fact, must) be seriously criticized. First, a non-zero mass difference \( m_u - m_d \) induces mixing of the \( \pi_0 \) with \( \eta, \eta' \), so that the second equality in (1) seems rather suspicious. There one may expect an error of order 50% since a small mixing angle \( O(m_d - m_u) \) can be well compensated by a large value of \( \langle 0 | \frac{\alpha_\pi}{4\pi} \bar{G}G | \eta \rangle \sim f_\pi m_\eta^2 \) [8] with \( \frac{m_\pi^2}{m_\eta^2} \). Second, one may wonder whether the account for \( L^{\Delta \ell = 1} \) generates new interactions involving the Veneziano ghost [2] potentially revealing itself in the correlator (1).

Let us address now the original derivation of GTW [1]. They have used the Sutherland theorem [9] for the SU(2) singlet current \( j_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d \), stating that

\[ \lim_{q_\mu \rightarrow 0} q_\mu \int dx e^{iqx} \langle \gamma(k_1)\gamma(k_2) | j_\mu | 0 \rangle = 0 \]

After saturating this correlator by the pion intermediate state and accounting for the anomaly’s contribution, they have arrived at the equation

\[ \langle 0 | \frac{\alpha_\pi}{4\pi} \bar{G}G | \pi^0 \rangle = -\langle 0 | m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d | \pi^0 \rangle \] (3)

To discuss possible corrections to (4), consider the anomaly equation sandwiched in between the vacuum and one pion states

\[ -iq_\mu \langle 0 | j_\mu | \pi^0 \rangle = Af_\pi m_\pi^2 = 2\langle 0 | m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d | \pi^0 \rangle + \langle 0 | \frac{\alpha_\pi}{4\pi} \bar{G}G | \pi^0 \rangle \] \] (5)

where we have defined \( \langle 0 | \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d | \pi^0 \rangle \) \( = i f_\pi Aq_\mu \). The unknown parameter \( A \) has to be of the form \( -\frac{m_u - m_d}{\sqrt{2} m_u + m_d} \zeta \) where \( \zeta \) is some dimensionless function of the mass parameters. Along with reasoning of GTW, one has to expect \( \zeta \sim O(\frac{m_u - m_d}{M_{\pi^0}}) \) that can yet bring in corrections of order 30% due to \( \frac{m_u - m_d}{M_{\pi^0}} \) [1,5] \( (M_{\pi^0} \text{ stands for a scale of the strong interaction}) \). However, such corrections apparently do not show up in eq. (4). This seeming paradox is traced back to the fact that eq. (4) stands for the off-shell matrix elements. While the mixing effects on the matrix element of the isoscalar density in the r.h.s. of eq. (4) are small on the mass shell, this property can be lost for the off-shell case \( q_\mu \rightarrow 0 \), thus bringing back the \( \zeta \) factor into the on-shell version of eq. (4). The two above derivations yield the same answer as long as they are obtained within the same approximation, i.e. they both neglect mixing as well as the on-shellness. As will be argued in Sect.4, there is another class of corrections which is not taken into account in (2),(4). It is the Veneziano ghost [2] that brings a leading large (of order 40%) and negative correction to the GTW formula (2).
3 Ghost pole and the U(1) problem

Let us start with introducing the following currents

$$J^1_{\mu5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d), \quad J^2_{\mu5} = \bar{s}\gamma_\mu\gamma_5 s, \quad J^3_{\mu5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$$

with

$$\partial_\mu J^1_{\mu5} = i\sqrt{2}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d) + \sqrt{2}\frac{G_F}{4\pi}\bar{G}G \equiv P_1 + 2\sqrt{2}Q$$

$$\partial_\mu J^2_{\mu5} = 2im_s \bar{s}\gamma_5 s + 2Q \equiv P_2 + 2Q$$

$$\partial_\mu J^3_{\mu5} = i\sqrt{2}(m_u \bar{u}\gamma_5 u - m_d \bar{d}\gamma_5 d) \equiv P_3$$

This choice of the currents is motivated by the fact that they correspond to the mass eigenstates

$$\pi_1 \sim \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d}{\sqrt{2}}, \quad \pi_2 \sim \bar{s}\gamma_5 s, \quad \pi_3 \sim \frac{\bar{u}\gamma_5 u - \bar{d}\gamma_5 d}{\sqrt{2}}$$

with \( f_1 = f_\pi, \ m_q = \frac{m_u + m_d}{2}, \ \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{q}q \rangle \)

$$m_3^2 = m_1^2 = -\frac{1}{f^2}\frac{4m_q \langle \bar{q}q \rangle + O(m_q^2)}{f^2} \approx 0.02 \text{ Gev}^2$$

$$m_2^2 = -\frac{1}{f^2}\frac{4m_s \langle \bar{s}s \rangle + O(m_s^2)}{f^2} \approx m_{K0}^2 + m_{K^-}^2 - m_{\pi0}^2 \approx 0.47 \text{ Gev}^2$$

in the limit where the both quark mass difference and anomaly are neglected (Clearly, the formulae (9),(10) do not respect the real world with \( m_q \approx 549 \text{ Mev} \) and \( m_{\eta'} \approx 958 \text{ Mev} \)). In the naive chiral limit \( m_q = 0 \) the formula (8) represents the composite goldstone fields related to the spontaneous breaking of the chiral U(3) invariance \( \langle \bar{q}q \rangle \neq 0 \).

Making use of the standard technique [10], one can obtain the following set of the Ward identities (WI) (hereafter we denote \( \langle AB \rangle_q = \int dxe^{iqx} \langle T\{A(x)B(0)\} \rangle \), \( \langle AB \rangle = \lim_{q \to 0} \langle AB \rangle_q \))

$$\langle P_3P_3 \rangle + 2m_u \langle \bar{u}u \rangle + 2m_d \langle \bar{d}d \rangle = 0$$

$$\langle P_3P_1 \rangle + 2m_u \langle \bar{u}u \rangle - 2m_d \langle \bar{d}d \rangle = 0$$

$$\langle P_2P_3 \rangle = 0$$

$$\langle QP_3 \rangle = 0$$

$$\langle P_1P_1 \rangle + 2\sqrt{2} \langle QP_1 \rangle + 2m_u \langle \bar{u}u \rangle + 2m_d \langle \bar{d}d \rangle = 0$$

$$\langle P_1P_2 \rangle + 2\sqrt{2} \langle QP_2 \rangle = 0$$

$$\langle QP_1 \rangle + 2\sqrt{2} \langle QQ \rangle = 0$$

$$\langle P_2P_2 \rangle + 2\langle QP_2 \rangle + 4m_s \langle \bar{s}s \rangle = 0$$

$$\langle QP_2 \rangle + 2\langle QQ \rangle = 0$$

We immediately note that in the \( SU(2)_V \) limit \( m_u = m_d \) the \( \pi_3 \) state decouples from the anomalous WI (14-18), thus simplifying enormously their analysis. Therefore we will first concentrate on this limit and postpone a discussion of isospin breaking corrections until Sect.4.
As one can see from (14),(16), the correlator of the topological density \(Q\) at zero momentum \(\langle QQ\rangle\) must have a non-zero value in order to repair the wrong mass formulae (9). It can be shown from the analysis of the commutation relations that the correlator \(\langle QQ\rangle\) entering the WI (14)-(18) is to be recognized as defined by virtue of the Wick T-product [3]. Moreover, the following relation holds

\[
\langle QQ\rangle^W_q = \langle QQ\rangle^D_q - \frac{\alpha_s}{4\pi} G^2 \left(-iq\mu\right)\langle K_\mu K_\nu\rangle^D_q
\]

(19)

( The symbols D, W stand for the Dyson and Wick T-products ). Here

\[
K_\mu = \frac{\alpha_s}{4\pi} \varepsilon_{\mu\alpha\beta\gamma} A_\alpha^a(\partial_\beta A_\gamma^a + \frac{1}{3} gf^{abc} A_\beta^b A_\gamma^c), \quad Q = \partial_\mu K_\mu
\]

is the gauge-dependent gluon current. Thus, the presence of a pole as \(q^2 \to 0\) in the correlator \(\langle K_\mu K_\nu\rangle\) is necessary for a solution of the U(1) problem [2,3] :

\[
\langle K_\mu K_\nu\rangle^D_q = \text{const} \frac{g_{\mu\nu}}{q^2}
\]

(21)

( It is worth recalling that only the Dyson T-product admits a intermediate states representation ). The residue in this pole can be argued to vanish in the \(m_q \to 0\) limit. As has been shown in [3], the existence of such ghost pole is deeply motivated: it is the consequence of a periodic dependence of the potential energy in QCD on the collective variable \(X = \int d^3\vec{x}K_0(\vec{x}, t)\). Thus, its appearance is a purely nonperturbative phenomenon.

After this physical idea is introduced, mathematics becomes rather simple. One defines [2,3] the ghost propagator

\[
\langle a_\mu a_\nu\rangle_0 = -\frac{q_\mu q_\nu}{q^4}
\]

(22)

with

\[
\langle 0|K_\mu|a^p\rangle = \lambda^2 \varepsilon_\mu^p, \quad \langle 0|Q|a_\mu\rangle = -iq_\mu \lambda^2
\]

( \(\varepsilon_\mu^p\) stands for the polarization vector ), so that

\[
\langle QQ\rangle^W = -\lambda^4 \neq 0
\]

(24)

One introduces further the bare propagators for the \(\pi_{1,2}\) states

\[
\langle \pi_i \pi_j\rangle_0 = \delta_{ij} \frac{1}{m_i^2 - q^2}
\]

(25)

( i.e. those obtained when gluon intermediate states are excluded ) and the point-like transition amplitudes

\[
\langle a_\nu | \pi_i \rangle = -iq_\mu_i
\]

(26)

Then the exact propagators can be found from (22-26) by solving the system of the Dyson equations ( no sum over i )

\[
\begin{align*}
\langle \pi_i \pi_i \rangle &= \langle \pi_i \pi_i\rangle_0(1 + \langle \pi_i | a_\mu \rangle \langle a_\mu a_\nu \rangle \langle a_\nu | \pi_i \rangle \langle \pi_i \pi_i\rangle_0) \\
\langle \pi_i \pi_j \rangle &= \langle \pi_i \pi_i\rangle_0 \delta_{ij} \langle a_\mu a_\nu \rangle \langle a_\nu a_\mu \rangle \langle \pi_i \pi_i\rangle_0 \\
\langle a_\mu a_\nu \rangle &= \langle a_\mu a_\nu \rangle_0 + \sum_i \langle a_\mu a_\mu \rangle_0 \langle a_\mu | a_\nu \pi_i \rangle \langle a_\nu | a_\mu \pi_i \rangle \langle a_\mu a_\nu \rangle_0 \\
\langle a_\mu \pi_i \rangle &= \langle a_\mu a_\nu \rangle \langle a_\nu | \pi_i \rangle \langle \pi_i \pi_i\rangle_0
\end{align*}
\]

(27)
Then, all the masses, wave functions and matrix elements can be calculated from successive saturating the gauge invariant correlators by the contributions of $\pi_1, \pi_2, a_\mu$ and physical $\eta, \eta'$ states, with the parameters $\mu_i, \lambda^2$ being defined with the help of the WI (14-18) (plus e.g. the experimental value for $m_\eta^2 + m_{\eta'}^2$ [2,3]). We just cite the final answers [3]

$$m_{\eta,\eta'}^2 = \frac{1}{2} \left\{ (m_1^2 + m_2^2 + \mu_1^2 + \mu_2^2) \pm \sqrt{(m_1^2 + \mu_1^2 - m_2^2 - \mu_2^2)^2 + 4\mu_1^2\mu_2^2} \right\}$$

$$= \frac{f_1\mu_1}{\sqrt{2}} = f_2\mu_2 = 2\lambda^2, \mu_1^2 \simeq 0.57 \text{ Gev}^2, \mu_2^2 \simeq 0.16 \text{ Gev}^2$$

$$m_\eta^2 \simeq 0.307(\exp.0.301) \text{ Gev}^2, m_{\eta'}^2 \simeq 0.912(\exp.0.917) \text{ Gev}^2$$

$$|\eta'\rangle = \cos \theta |1\rangle + \sin \theta |8\rangle \quad (\theta \simeq -10^0)$$

$$|\eta\rangle = -\sin \theta |1\rangle + \cos \theta |8\rangle$$

and

$$\langle 0|Q|\eta\rangle = \lambda^2 \sqrt{(m_\eta^2 - m_{\eta'}^2)(m_\eta^2 - m_\eta^2)} \simeq 0.010 \text{ Gev}^3$$

$$\langle 0|Q|\eta'\rangle = \lambda^2 \sqrt{(m_{\eta'}^2 - m_\eta^2)(m_{\eta'}^2 - m_\eta^2)} \simeq 0.028 \text{ Gev}^3$$

$$\langle 0|P_1|\eta\rangle = f_1 m_\eta^2 \sqrt{(m_\eta^2 + \mu_1^2 - m_\eta^2)} \simeq 0.0019 \text{ Gev}^3$$

$$\langle 0|P_1|\eta'\rangle = f_1 m_{\eta'}^2 \sqrt{(m_\eta^2 - m_{\eta'}^2 - \mu_2^2)} \simeq 0.0018 \text{ Gev}^3$$

$$\langle 0|P_2|\eta\rangle = -f_2 m_\eta^2 \sqrt{(m_\eta^2 + \mu_2^2 - m_\eta^2)} \simeq 0.056 \text{ Gev}^3$$

$$\langle 0|P_2|\eta'\rangle = f_2 m_{\eta'}^2 \sqrt{(m_\eta^2 - m_{\eta'}^2 - \mu_1^2)} \simeq 0.062 \text{ Gev}^3$$

In the next sections we will see that these are the matrix elements (30)-(35) that determine all the corrections of interest.
4 Isospin breaking as a perturbation

Now we are able to proceed to discussing the isospin breaking effects. The easiest way
to do it is to use a perturbation theory over the isospin breaking Hamiltonian 
\( H^{\Delta I=1} = \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d) \). Let us write down the total Hamiltonian as

\[
H = H_{QCD} + \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) + m_s\bar{s}s - \theta \frac{\alpha_s}{2\pi}(\tilde{\pi}\tilde{H}) + \theta^2 \frac{\alpha_s}{2\pi} \tilde{H}^2 + \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d) = H^0 + H^{\Delta I=1} \tag{36}
\]

where \( \tilde{\pi}^a = \tilde{A}^a + \frac{\alpha_s}{2\pi} \theta \tilde{H}^a \) is the canonical momentum and \( H_{QCD} \) corresponds to the massless theory. The \( \theta \) - term displayed in (36) is just another way of incorporating the Veneziano ghost discussed above, provided \( \langle QQ \rangle^W = -\partial^2 \varepsilon_{\text{vac}}/\partial \theta^2 \) [11,2,3] (in such formulation, the theory contains only gauge invariant states). The Hamiltonian form of the Veneziano construction enables us to build the perturbation theory around the \( SU(2)_V \) symmetric Witten-Veneziano Hamiltonian \( H^0 \). One deals there with the (gauge invariant) states \( |\pi_B\rangle, |\eta_B\rangle, |\eta_B'\rangle \) defined by (8),(9),(28),(29). Then the first order corrections to the wave functions over the perturbation \( H^{\Delta I=1} \) are given by the quantum mechanical formula

\[
|\Psi\rangle' = \sum_{m\neq n} \frac{V_{mn}}{E_n - E_m} \frac{1}{\langle \Psi_m | \Psi_m \rangle} |\Psi_m\rangle = \sum_{m\neq n} \frac{V_{mn}}{2E_m(E_n - E_m)} |\Psi_m\rangle \tag{37}
\]

where \( V_{mn} \) stands for the matrix element of the perturbation

\[
V_{mn} = \langle \Psi_m (\vec{p}, E_m) | H^{\Delta I=1} | \Psi_n (\vec{p}, E_n) \rangle \tag{38}
\]

The only non-vanishing matrix elements \( V_{\pi\eta(\eta')\eta(\eta')} \) can be evaluated in the pion’s rest frame by the soft pion technique [12]:

\[
\langle \pi_B | H^{\Delta I=1} | \eta_B \rangle = \frac{1}{f_\pi} \sum_{m\neq n} \frac{V_{\eta\eta'}}{m_u - m_d} |0 \rangle |\eta_B \rangle \tag{39}
\]

(Using the soft pion theorem in this case seems to be harmless since an expected accuracy is \( O(\frac{m_s^2}{(m_q - m_s)^2}) \approx 10\% \)). Finally, the wave functions formulae read

\[
|\pi\rangle = |\pi_B\rangle + \frac{V_{\pi\eta}}{2m_\eta(m_\pi - m_\eta)} |\eta_B\rangle + \frac{V_{\pi\eta'}}{2m_\eta'(m_\pi - m_\eta')} |\eta_B'\rangle \tag{40}
\]

\[
|\eta\rangle = |\eta_B\rangle + \frac{V_{\pi\eta}}{2m_\pi(m_\eta - m_\pi)} |\pi_B\rangle
\]

\[
|\eta'\rangle = |\eta_B'\rangle + \frac{V_{\pi\eta'}}{2m_\pi(m_\eta' - m_\pi)} |\pi_B\rangle
\]

(Of course, all the masses here are to be calculated in the \( SU(2)_V \) limit. Note also that there is no need for changing the normalizations of the states since these effects are of second order in the perturbation). Numerically, \( \theta_{\pi\eta} = \frac{V_{\pi\eta}}{2m_\eta(m_\pi - m_\eta)} \approx 0.032 \frac{m_u - m_d}{m_d + m_u} \) and the analogous \( \theta_{\pi\eta'} \approx 0.009 \frac{m_u - m_d}{m_d + m_u} \). It should be stressed that these mixing angles are quite different from those discussed usually for the mixing among the massless states [1]. Moreover, any calculation scheme based on treating the total mass term \( L^m = -m_\eta \bar{u}u - \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d) \)
as a perturbation seems to be in troubles when concerns isospin breaking matrix elements. The same mass term is responsible there both for the mixing and for non-vanishing values of matrix elements for "bare" states, thus the problem of double counting arises. In contrast, there is no such difficulty in the present approach. Let us point out also that as long as all the diagonal matrix elements $V_{nn}$ vanish, the mass corrections start only with the second order and, thus, are very small. For example, the contribution of this mechanism into $\pi^+ - \pi^0$ mass difference is only about 0.18 Mev for $m_d - m_u \approx 0.4$.

Now we can, using (30),(31),(39),(40), calculate the pion matrix element of the topological density:

$$\langle 0 \left| \frac{\alpha_s}{4\pi} G \tilde{G} \right| \pi^0 \rangle = 2(\theta_{\pi\eta}(0|Q|\eta_B) + \theta_{\pi\eta'}(0|Q|\eta'_B))$$

This formula can be easily cast into a form comparable with (2). One obtains

$$\zeta = \frac{\mu_1(\mu_1 + \mu_2)}{2(\mu_1^2 + \mu_2^2)} - 1 + O(m_1^2, m_2^2) \simeq 0.6 - 1 + 0.003 \simeq -0.4$$

( the number 0.003 is the contribution of the $O(m_d^2)$ term omitted in (42) ). We see that the naive answer (2) widely used in the literature [5],[6] is reduced by about 40 % . The couplings of the ghost-goldstone interactions $\mu_1, \mu_2$ constitute the hidden large parameters in the problem. Note that $\eta$ and $\eta'$ contribute about equally into (41).

5 Strange quark content of pion

This is a good exercise to get a feeling of a size of effects that the above considered mechanism can bring in. Let us start with the one-pion matrix elements. In this case, non-vanishing values for them are obtained as a sole result of the mixing. One finds from (34),(35),(40)

$$\langle 0|P_2|\pi^0\rangle \simeq \frac{1}{2} \frac{m_d - m_u}{m_d + m_u} m^2 \pi (-0.176 + 0.058) \simeq \frac{m_d - m_u}{m_d + m_u} m^2 \pi \cdot 0.06$$

where the two numbers in the parenthesis stand for the numerical contributions of the $\eta$ and $\eta'$, correspondingly. Let us point out that the value (43) is again about twice less than the original estimate of GTW [1]. The axial current matrix element $\langle 0|\bar{s}\gamma_\mu\gamma_5s|\pi^0\rangle = if_\pi A_q\mu$ can be easily found from (41), (43):

$$A = \frac{1}{f_\pi m^2_\pi} \langle 0|P_2 + 2Q|\pi^0\rangle \simeq \frac{m_d - m_u}{m_d + m_u} \cdot 0.03$$

One may further address the strange quark condensate in the pion. One obtains

$$\langle \pi|\bar{s}s|\pi\rangle = \langle \pi_B|\bar{s}s|\pi_B\rangle + \theta^2_{\pi\eta}(\eta_B|\bar{s}s|\eta_B) + \theta^2_{\pi\eta'}(\eta_B'|\bar{s}s|\eta_B') + 2\theta_{\pi\eta}\theta_{\pi\eta'}(\eta_B|\bar{s}s|\eta_B')$$

(45)

Let us first concentrate on the correction terms in (45). To find $\langle \eta_B|\bar{s}s|\eta_B\rangle$, $\langle \eta_B|\bar{s}s|\eta'_B\rangle$, one can apply the "soft $\eta$ - theorem" ( we neglect here the mixing with the singlet ):

$$\langle \eta_B|\bar{s}s|\eta_B\rangle = -\frac{2}{\sqrt{6}f_\eta m_s} \langle 0|P_2|\eta_B\rangle \simeq 2.1 Gev$$

(46)
\[ \langle \eta_B | \bar{s}s | \eta'_B \rangle = -\frac{2}{\sqrt{6} f_q m_s} \langle 0 | P_2 | \eta'_B \rangle \simeq 2.3 \text{ Gev} \quad (47) \]

One can argue (by comparing (46),(47) with an analog of (49) for \( \eta \leftrightarrow \eta' \)) that the accuracy of these estimates is of order 30\% . The calculation of \( \langle \eta'_B | \bar{s}s | \eta'_B \rangle \) is slightly more involved. Consider the correlation function

\[ \Pi = i \int dx \langle 0 | T \{ P_2 (x) \bar{s}s (0) \} | \eta'_B \rangle \quad (48) \]

The pole approximation applied to (48) can be shown to give the following relation

\[ \frac{1}{m_{\eta'}^2} \langle 0 | P_2 + 2Q | \eta'_B \rangle \langle \eta'_B | \bar{s}s | \eta'_B \rangle + \frac{1}{m_{\eta}^2} \langle 0 | P_2 + 2Q | \eta_B \rangle \langle \eta_B | \bar{s}s | \eta_B \rangle = \frac{1}{m_s} \langle 0 | P_2 | \eta_B \rangle \quad (49) \]

Together with (46),(47) this yields

\[ \langle \eta'_B | \bar{s}s | \eta'_B \rangle \simeq 1.2 \text{ Gev} \quad (50) \]

Thus, the mixing contribution into (45) is estimated as

\[ \frac{\langle \bar{s}s \rangle_\pi}{\langle \bar{u}u + d\bar{d} \rangle_\pi} |_{(\text{mixing})} \simeq 3.3 \cdot 10^{-5} \quad (51) \]

This estimate is to be compared with the first term in (45). It has been recently calculated (for the charged pions) within the chiral perturbation theory [13] and the NJL model [14] :

\[ \frac{|\langle \bar{s}s \rangle_\pi|}{\langle \bar{u}u + d\bar{d} \rangle_\pi} \leq (4 - 6) \cdot 10^{-4} \quad (52) \]

that apparently is of one order larger than the mixing contribution (51). The unpleasant thing with (52) is that this ratio turns out very sensitive to the choice of \( \tilde{m} = 1/2 (m_u + m_d) \) and even changes the sign when \( \tilde{m} \) varies from 5.1 to 5.8 Mev [14]. So, one may wonder whether this value is not exactly zero, up to an accuracy of the methods under consideration. One can argue that this is not the case from the following argument. Let us differentiate the WI (10) (taken in the SU(2)_V limit) over the mass of the strange quark :

\[ -i \langle P_3 \bar{s}s P_3 \rangle = -4m_q \frac{d \langle \bar{q}q \rangle}{dm_s} \quad (53) \]

The dominant contribution into the l.h.s. of (53) is due to the pion intermediate state, so that we arrive at

\[ \langle \pi^0 | \bar{s}s | \pi^0 \rangle = \frac{4m_q}{f_{\pi}^2} \frac{d \langle \bar{q}q \rangle}{dm_s} \quad (54) \]

The quantity \( d \langle \bar{q}q \rangle/dm_s \) (\( \langle \bar{q}q \rangle = \langle \bar{u}u \rangle \)) in the chiral SU(2) limit has been calculated in Ref.[19] within the instanton vacuum model: \( K = d \langle \bar{u}u \rangle/dm_s \simeq -0.085 \text{ Gev}^2 \). Assuming that the correlator \( K \) in the SU(2)_V limit is not very different from this value, one obtains from (54)

\[ \frac{\langle \bar{s}s \rangle_\pi}{\langle \bar{u}u + d\bar{d} \rangle_\pi} \simeq -3 \cdot 10^{-2} \quad (55) \]

We conclude that the mixing only slightly affects the strange quark condensate in the pion. Still, a large discrepancy between two estimates (52) and (55) deserves further studying.
6 Isospin breaking in the πN interaction

Another interesting consequences of the above developed formalism concern isospin symmetry breaking effects in the πN and NN interactions, the long staying problem in nuclear physics. Among them, one frequently discussed phenomenon is the scattering length difference in the $^1S_0$ partial wave ( $|a_{nn}| - |a_{pp}| = 1.5 \pm 0.5$ fm with the Coulomb corrections subtracted ). Another one is the discrepancy of order a few hundreds keV between the measured masses of the mirror nuclei after allowing for the n-p mass difference and the calculated e.m. corrections - this is the so-called Nolen-Schiffer (NS) anomaly [15] ( see [16,23] for review ). The data signals that the nn interaction is more attractive than the pp one, that in turn means $|g_{\pi_0nn}| > g_{\pi_0pp}$ in the framework of the one-pion exchange potential models ( OPEP ). However, the attempts of direct evaluating the coupling $g_{\pi_0pp}$ from a phase shift analysis suffer large uncertainties [16], so that no decisive conclusion can be driven. The situation in the theory is also ambiguous since the various quark model based calculations disagree even in the signs [16].

Let us now appeal to our scheme. For the Yukawa Lagrangian
\begin{equation}
L_{Yuk} = i g_\pi^0 \langle \bar{p} \gamma_5 p - \bar{n} \gamma_5 n \rangle \pi_0^B + i g_\eta^0 \langle \bar{p} \gamma_5 p + \bar{n} \gamma_5 n \rangle \eta_B + (\eta_B \leftrightarrow \eta'_B)
\end{equation}
( the $SU(2)_V$ limit is implied ) one obtains from (40)
\begin{equation}
|g_{\pi^0pp}(\pi^0nn)| = g_\pi^0 \pm (\theta_{\pi\eta} g_\eta^0 + \theta_{\pi\eta'} g_{\eta'}^0)
\end{equation}
As the both mixing angles are positive, it is seen from (57) that the mixing tends to increase the coupling $g_{\pi^0pp}$ and reduce $g_{\pi^0nn}$, i.e. yields a result opposite to what is expected for explaining the isospin asymmetry. Unfortunately, it is not so easy to determine exactly the $SU(2)_V$ limit of the coupling constants $g_\eta; g_{\eta'}$, though some estimates can still be done.

The first one is the $SU(3)$ prediction $g_8 = \sqrt{3} (1 - \frac{4}{3} D/(D + F)) g_\pi$. Substituting $D/(D + F) \simeq 0.6$ from the hyperon decay data [17] and neglecting the octet-singlet mixing, one finds $g_\eta \simeq g_8 \simeq 4.6$. The second estimate comes from the Goldberger-Treiman relation for the $\eta$:
\begin{equation}
2M(\Delta u + \Delta d - 2\Delta s) = \sqrt{6} f_\eta g_\eta
\end{equation}
With $\Delta u + \Delta d - 2\Delta s \simeq 0.68$ [17] and $f_\eta \simeq 0.6 f_\pi$ from the $\eta \rightarrow 2 \gamma$ decay, this yields $g_\eta \simeq 6.3$.

One more estimate for the couplings $g_\eta, g_{\eta'}$ can be obtained under assumption on their slight variation with moving from the $SU(2)$ to $SU(3)$ chiral limit. Let us apply the pole approximation to the matrix element of the topological density over the proton states $\langle p|Q|p \rangle = \lim_{p' \rightarrow p} \langle p'|Q|p \rangle$. Then
\begin{equation}
\langle p|Q|p \rangle = -\frac{\langle 0|Q|\eta \rangle}{m_\eta^2} g_\eta + \frac{\langle 0|Q|\eta' \rangle}{m_{\eta'}^2} g_{\eta'} \bar{p}i\gamma_5 p
\end{equation}
On the other hand, this quantity has been estimated in Ref.[18] by some extension of an argumentation based on the dimensional transmutation phenomenon:
\begin{equation}
\langle p|Q|p \rangle = -\frac{2n_f}{3b} m_p \bar{p}i\gamma_5 p , \quad b = \frac{11}{3} N_c - \frac{2}{3} n_f
\end{equation}
( \( n_f \) stands for the number of massless flavors). Assuming the validity of this formula, one can obtain

\[
0.032g_\eta + 0.031g_{\eta'} = 0.123 \quad (\text{chiral} \ SU(2))
\]

(61)

\[
0.027g_\eta + 0.038g_{\eta'} = 0.102 \quad (\text{chiral} \ SU(3))
\]

(62)

where we have also accounted for the fact that the s-quark carry about one half of the nucleon mass [19,20]. (Note that the first term in the r.h.s. of eq. (59) survives the chiral SU(3) limit.) Then one obtains the estimate

\[
g_\eta \simeq 3.9 \quad , \quad g_{\eta'} \simeq 0
\]

(63)

(Taken literally, the system (61),(62) yields \( g_\eta = 4.01 \), \( g_{\eta'} = -0.17 \).) It is worth noting that estimate (63) is consistent with the claims on a smallness of the \( g_{\eta'} \) coupling [21,22] driven from the spin crisis studies. The value \( g_\eta \simeq 3.9 \) appears to be a lower bound for the \( SU(2)_V \) value \( g_\eta^{(0)} \). Thus, our final estimates are

\[
g_\eta^{(0)} \simeq 5 \pm 1 \quad , \quad g_{\eta'}^{(0)} \simeq 0
\]

(64)

From (57),(64) we find (\( \frac{g_{\eta NN}^2}{4\pi} \simeq 14 \))

\[
\frac{g_\eta^{opp} - |g_\eta^{nn}|}{g_{\pi NN}} \simeq \frac{m_d - m_u}{m_d + m_u} (2.4 \pm 0.5) \cdot 10^{-2} ,
\]

(65)

\[-\alpha \equiv \frac{g_{\eta NN}^{opp} - g_{\eta NN}^{nn}}{g_{\pi NN}^2} \simeq (1.9 \pm 0.4) \cdot 10^{-2} , \quad \frac{m_d - m_u}{m_d + m_u} \simeq 0.4\]

Thus, the long-range part of the charge asymmetric nuclear potential \( V_{CA} = V_{nn} - V_{pp} \) for the \( ^1S_0 \) state can be written as

\[
V_{CA} = \frac{g_{\eta NN}^2 - g_{\pi NN}^2 \cdot m_\pi^2 \cdot e^{-m_\pi r}}{4\pi \cdot 2M_N^2 \cdot r}
\]

(66)

It is known that for explaining the NS anomaly for \( A = 41 \), \( \alpha \) must be larger than 2 \% [23]. The binding energy difference for \( A = 41 \) calculated within the shell model wave functions provides \( E_{11Sc} - E_{11Ca} \simeq -780 \) keV that is the right number in modulus but wrong in the sign! (The answer for the shell model matrix element was kindly presented to us by N.Auerbach.)

A few comments are in order here. From the viewpoint of the \( 1/N_c \) expansion, the presented effect is \( O(N_c^-) \). Thus, it has to be taken on equal footing with other \( O(N_c) \) contributions which can be found e.g. from the the chiral theory of the nucleon or the QCD sum rules. Unfortunately, this can be done within the modern state-of-art only for the strangeless nucleon, whereas the strangeness is effectively appearing in our calculations. We believe, however, that our estimate (66) constitutes the leading contribution into the charge asymmetric nuclear potential as resulting from a strong interaction of the ghost. Another observation is that our conclusion on the sign of \( (g_{\pi NN}^{opp} - g_{\pi NN}^{nn}) \) is not at variance with most quark model based calculations (the only exception is the cloudy bag model) [16]. In contrast to previous works, it is deduced this time from the exact QCD dynamics. In all likelihood, the result (65),(66) means that long-range meson exchange forces cannot explain the isospin violation in the \( \pi N \) and NN interactions. On the other hand, the large effect of (66) has to be taken into account in any scheme of resolving the NS anomaly, e.g. the one based on a partial restoration of the chiral symmetry in the nuclear medium (see e.g. [24]).
Proton spin and violation of Bjorken sum rule

Now we would like to come back to the above mentioned connection of the discussed phenomena with the spin crisis problem (see e.g. [25] for review). The necessity of explaining the data on deep inelastic polarized lepton-nucleon scattering has led to re-examining some usually done assumptions such as e.g. an isoscalarity of the anti-quark sea in the nucleon. In this view, there has been raising interest [26] during a last few years in checking in QCD the Bjorken sum rule [27], relating the isos triplet component of the first moment of the polarized nucleon structure function $g_1(x)$ to the weak coupling $g_A$. The validity of the Bjorken sum rule has been discussed in the recent papers [28,29]. As it is argued in Ref. [28], the Bjorken sum rule is consistent with all the available data at the 12% level after the kinematic and higher-twist power corrections [30] are taken into account. These corrections have been however calculated in the chiral SU(2) limit, thus leaving open the question on a possible violation of the Bjorken sum rule due to isospin breaking. This latter point has been discussed in Ref.[29] under assumption on the validity of the Sutherland theorem in the isoscalar channel [1] which is to be abandoned according to the results of the present work. Thus, it would be interesting to estimate corrections to the Bjorken sum rule resulting from above considered isospin breaking in the meson couplings.

The operator product expansion of the antisymmetric part of the T-product of two electromagnetic currents provides

$$
\int dx g_1^N(x, Q^2) = \frac{1}{2} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right) \left[ \frac{4}{9} \Delta u^N + \frac{1}{9} \Delta d^N + \frac{1}{9} \Delta s^N \right] (Q^2) + O\left( \frac{M^2}{Q^2} \right) \tag{67}
$$

where

$$
\langle N | (q_i \gamma_\mu \gamma_5 q_i) \mu^2 = Q^2 | N \rangle = \Delta q_i(Q^2) s_\mu \tag{68}
$$

and

$$
s_\mu = \bar{u}(p,s)\gamma_\mu \gamma_5 u(p,s) \text{ is the proton spin vector.} \text{ Let us estimate first the s-quark contribution into the nucleon spin. To this end, consider the matrix element}
$$
\langle N(p') | \bar{s} \gamma_\mu \gamma_5 s | N(p) \rangle = \bar{u}(p') \left[ \gamma_\mu \gamma_5 G_1^N (q^2) + q_\mu \gamma_5 G_2^N (q^2) \right] u(p) \tag{69}
$$

where $q = p' - p$. Differentiating this relation and saturating it with the $\pi, \eta, \eta'$ contributions, we find in the limit $q \to 0$

$$
\Delta s^N = G_1^N(0) = \frac{1}{2m_N} \sum_{i=\pi,\eta,\eta'} \frac{\langle 0 | P_2 + 2Q | i \rangle}{m_i^2} g_{iNN} \tag{70}
$$

Then, using (30),(31),(34),(35),(40),(44),(63), we obtain

$$
\Delta s^p \simeq -0.30 , \Delta s^n \simeq -0.28 \tag{71}
$$

(These values refer to a low normalization point $\mu \sim 500 \text{ Mev}$. We neglect, however, a weak logarithmical dependence on $Q^2$ due to the anomalous dimensions.) Note that the dominant contribution into (69) comes from the $\eta$ with $g_{\eta NN}^{ppp} \simeq g^{(0)} \pm 0.66$ (see (40)). One can conclude from (71) that the s-quark fraction of the nucleon spin is close to the isoscalar and thus cannot bring any sizable correction into the Bjorken sum rule. Numerically, the answer for $\Delta s^p$ is somewhat larger than the latest value.
\( \Delta s^p = -0.13 \pm 0.04 \) [28] or the previous world average \( \Delta s^p = -0.20 \pm 0.11 \) [31], which have been obtained within the SU(3) limit. One could expect an accuracy of order 30% for the estimates (70), which is typical in applications of the PCAC technique for the kaons. We believe, however, that the actual accuracy of (70) is in fact better since it is the approximation that has been used systematically in the previous discussion of the Veneziano mechanism.

The fraction of the nucleon spin due to the non-strange quarks can be calculated analogously. It is convenient to decompose the quark part of the operator in the matrix elements into the isovector and isoscalar components (note that we do not separate parton and anomalous contributions into the nucleon spin):

\[
m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d = \frac{m_u - m_d}{2} (\bar{u} i \gamma_5 u - \bar{d} i \gamma_5 d) + \frac{m_u + m_d}{2} (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d)
\]

Then we observe the strong cancellation between different contributions into \( \Delta u^p + \Delta d^p \):

\[
\Delta u^p + \Delta d^p \simeq \frac{1}{m_p} (-0.50 + 0.40 + 0.31 - 0.02) \simeq 0.21,
\]

where the first two terms in the parenthesis are originating from the isovector and isoscalar pion matrix elements, the third and fourth ones are due to the \( \eta \) and \( \eta' \), correspondingly. All the other contributions are small. Similarly, one obtains for the neutron

\[
\Delta u^n + \Delta d^n \simeq \frac{1}{m_n} (0.49 - 0.41 + 0.41 + 0.02) \simeq 0.54
\]

These numbers are to be compared with the ones obtained from the SU(3) symmetric fit under assumption on the validity of the Bjorken sum rule [28]

\[
\Delta u = 0.80 \pm 0.04 , \; \Delta d = -0.46 \pm 0.04
\]

Now we are able to estimate the isospin violating corrections to the Bjorken sum rule. Neglecting negligible (of order \( 10^{-4} \)) isospin breaking in the isovector part of the e/m current and the strange current, we obtain after using the Goldberger-Treiman relation

\[
\int dx (g_1^p(x) - g_1^n(x)) \simeq \frac{1}{2} (1 - \frac{\alpha_s}{\pi}) \left[ \frac{1}{3} g_A + \frac{5}{18} (\Delta u^{p-n} + \Delta d^{p-n}) \right]
\]

\[
\simeq \frac{1}{2} (1 - \frac{\alpha_s}{\pi}) \left[ \frac{1}{3} g_A - 0.09 \right]
\]

Thus, one can expect isospin breaking of order 20% in the Bjorken sum rule. Actually, the formula (76) represents the isospin violation due to the one-meson reducible contributions [29] into the matrix element (68). An interesting possibility is that irreducible (e.g. instanton-induced) contributions turn out also large [29] but of opposite sign, thus reducing the total isospin violation in the Bjorken sum rule. Presumably, the role of the non-resonant contribution into the proton spin could be clarified in the framework of the dispersion approach.
8 Conclusions

Let us summarize the results of this work. We have calculated the mixing in the $\pi^0 - \eta - \eta'$ system basing on the Veneziano solution of the U(1) problem in the $SU(2)_V$ symmetric world. The ghost interacts strongly with the pseudo-goldstone states (or, more precisely, the OZI goldstone modes, in the terminology of Ref.[22]) and must be taken into account before allowing for an isospin violation. As a consequence of this scheme, we have found large corrections to the naive estimations of the isospin breaking matrix elements. In all the cases, the ghost suppresses the isospin violation due to the quark mass difference. Being applied to the charge asymmetry phenomena in nuclear physics, our results indicate essential troubles in attempts of explaining them in terms of meson exchange forces. For the spin crisis problem, the presented mechanism allows one to re-estimate ”phenomenologically” (i.e. in the pole approximation) the quark contribution into the proton spin. At the 30 % level, the obtained values agree with those derived from the SU(3) symmetric fit. It would be interesting to find a correspondence between the present work and the effective chiral Lagrangian technique. We are planning to return to this problem elsewhere.

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