A model of a varying Ghost Dark energy

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Abstract

Motivated by recent developments in Cosmology we would like to assume a possibility of the existence of varying Ghost Dark energy. Ghost Dark energy like other models was introduced recently as a model of dark energy. As accepted in general, in GR dark energy is a possible way to explain accelerated expansion of the Universe reported from different experimental data. From the beginning we would like to stress the fact, that proposed modifications are based on our believe and probably for this reason there is not a well establish physical theory behind the models. Concerning to the origin of varying Ghost dark energy, we can assume an existence of some interaction (and unknown physics behind it), between Ghost Dark energy and a fluid and last component during interaction evaporated completely making sense of the proposed effect. Moreover, we can assume that this was in epochs and scales which are unreachable by nowadays experiments. The same assumptions could be accepted or proposed for the interaction terms.

Introduction

The observations of high redshift type SNIa supernovae [1] reveal the speeding up expansion of our universe. The surveys of clusters of galaxies show that the density of matter is very much less than critical density [2], observations of Cosmic Microwave Background anisotropies indicate that the universe is flat and the total energy density is very close to the critical $\Omega_{\text{tot}} \simeq 1$ [3]. In order to explain experimental data concerning to the nature of the accelerated expansion of the Universe a huge number of hypothesis were proposed. For instance, in General Relativity framework, the desirable result could be achieved by so-called dark energy: an exotic and mysterious component of the Universe, with negative pressure (we thought that the energy density is always positive) and with negative EoS parameter $\omega < 0$. Dark energy occupies about 73% of the
energy of our universe, other component, Dark matter, about 23%, and usual baryonic matter occupy about 4%. The simplest model for a dark energy is a cosmological constant \( \omega_\Lambda = -1 \) introduced by Einstein, but with cosmological constant we faced with two problems i.e. absence of a fundamental mechanism which sets the cosmological constant zero or very small value the problem known as fine-tuning problem, because in the framework of quantum field theory, the expectation value of vacuum energy is 123 order of magnitude larger than the observed value [4]. The second problem known as cosmological coincidence problem, which asks why are we living in an epoch in which the densities of dark energy and matter are comparable? Alternative models of dark energy suggest a dynamical form of dark energy, which at least in an effective level, can originate from a variable cosmological constant [5], or from various fields, such is a canonical scalar field [6] (quintessence), a phantom field, that is a scalar field with a negative sign of the kinetic term [7], [8], or the combination of quintessence and phantom in a unified model named quintom [9] and could alleviate these problems. Finally, an interesting attempt to probe the nature of dark energy according to some basic quantum gravitational principles are the holographic dark energy paradigm [10] and agegraphic dark energy models [11].

Among various models of dark energy, a new model of dark energy called Veneziano ghost dark energy of our interest has been recently proposed, which supposed to exist to solve the \( U(1)_A \) problem in low-energy effective theory of QCD, has attracted a lot of interests in recent years [12]-[14]. Indeed, the contribution of the ghosts field to the vacuum energy in curved space or time-dependent background can be regarded as a possible candidate for the dark energy. It is completely decoupled from the physics sector. Veneziano ghost is unphysical in the QFT formulation in Minkowski spacetime, but exhibits important non-trivial physical effects in the expanding Universe and these effects give rise to a vacuum energy density \( \rho_D = \Lambda_{QCD}^2 H \sim (10^{-3} eV)^4 \). With \( H \sim 10^{-33} eV \) and \( \Lambda_{QCD} \sim 100 eV \) we have the right value for the force accelerating the Universe today. It is hard to accept such linear behavior and it is thought that there should be some exponentially small corrections. However, it can be argued that the form of this behavior can be result of the fact of the very complicated topological structure of strongly coupled QCD. This model has advantage compared to other models of dark energy, that it can be explained by standard model and general relativity. Comparision with experimental data reveal that the current data does not favour it compared to the \( \Lambda \)CDM model, which is not conclusive and future study of the problem is needed. Energy density of Ghost dark energy reads as

\[
\rho_{GDe} = \alpha H, 
\]

where \( H \) is Hubble parameter \( H = \frac{\dot{a}}{a} \) and \( \alpha \) is constant parameter of the model, which should be determined. A generalization of the model [15] also was proposed for which energy density and reads as

\[
\rho_{GDe} = \alpha H + \beta H^2, 
\]
with $\alpha$ and $\beta$ constant parameters of the model. Such kind of fluids could be named as a geometrical fluids, because from the description it is clear that it contains information about geometry of the space-time and metric. In this letter our attention will be focused on a question concerning to the possibility of an existence of a varying Ghost Dark energy and its impact to the later time accelerated expansion of the Universe. Concerning to the origin of varying Ghost dark energy, we can assume an existence of some interaction (and unknown physics behind it), that was between Ghost Dark energy and a fluid and last component during interaction evaporated completely making sense of the proposed effect i.e varying Ghost Dark Energy. Moreover, we can assume that this was in epochs and scales which are unreachable by nowadays experiments. The same assumptions could be accepted or proposed for the interaction terms. This just our opinion and could be very far from the opinions of the readers. Research of the years bring us to the conclusion that we have right to consider fluids for which EoS could not be so nice as was proposed initially or energy density should be inhomogeneous (see for instance [18] and references therein). And the fact of the saying could be considered Chaplyagin gas with its unusual EoS. For instance, we can came up with this assumption just only based on the efforts done to find an appropriate unification of inflation, dark mater and dark energy existing in literature. We are sure that there could be many other proofs that assumptions are correct and we also should accept unusual EoS for fluids (of course if they are fluids or can be accepted as fluids according to the some approximation!). It is very important for the readers, especially who are new in the field to understand that in this step what we do it is just based on the assumptions and final results could be accepted as true-like assumptions based on experimental data, but at the same time we can not be sure about the results, because experience showed us that different couplings between fluids (and many other facts) can provide the same behavior for the Universe for later stages of the evolution and moreover the range of the values of the parameters for the models can be varied between the range of the values, which are fixed and found from experiments. Here, we would like to finish with the critics concerning to subject with the following, that we have right to try any method, model in order to clarify the situation which is very dark, even darker than the Universe. What we are proposing here, maybe at this point has not well establish "physics" behind it, but we do not so exited about it. Our job here will be to solve a set of differential equations and to see about the behavior of the model for different values of the parameters, even in the case, when for some parameters we have very long range of the values established from observations. We hope that we were clear related to the strategy which we adopted in this article. Among variety of the possibilities we would like to consider a model where $\alpha$ and $\beta$ terms presented in energy density are functions of some parameter $\varrho$ as $\alpha(\varrho)$ and $\beta(\varrho)$. For simplicity following to the research line concerning to the initial ideas, for instance, of varying Chaplyagin gas model particularly, we consider that $\varrho$ is a scale factor $a$ and $\alpha(a) = a^{\xi}$. After we suppose a simple model of a Universe in presence of a varying Ghost Dark energy and a barotropic fluid with

$$P_m = \omega \rho_m. \tag{3}$$

EoS equation. Let suppose that our Universe with FRW metric consists of a mixture of described fluids and suppose that energy conservation for the com-
posed fluid reads as

$$\dot{\rho} + 3H(\rho + P) = 0,$$  \hspace{1cm} (4)

with total energy density and pressure of composed fluid given as

$$\rho = \rho_{GDe} + \rho_m \quad \text{and} \quad P = P_{GDe} + P_m.$$  \hspace{1cm} (5)

Field equations according to the metric read as

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3},$$  \hspace{1cm} (6)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P).$$  \hspace{1cm} (7)

We suppose, that $\Lambda = 0$, $G$ and $c$ are constants and $c = 8\pi G = 1$. Interaction starts play a role and enter to the game when we split (4) taking into account

$$\dot{\rho}_m + 3H(\rho_m + P_m) = Q$$  \hspace{1cm} (8)

and

$$\dot{\rho}_{GDe} + 3H(\rho_{GDe} + P_{GDe}) = -Q.$$  \hspace{1cm} (9)

$Q$ denotes the phenomenological interaction term. Usually, three forms of $Q$ are used

$$Q = 3Hb\dot{\rho}_d,$$  \hspace{1cm} (10)

$$Q = 3Hb(\rho_d + \rho_m),$$  \hspace{1cm} (11)

and

$$Q = 3Hb\rho_m.$$  \hspace{1cm} (12)

$b$ is a coupling constant. From the thermodynamical view, it is argued that the second law of thermodynamics strongly favors that dark energy decays into dark matter, which implies $b$ to be positive. These type of interactions are either positive or negative and can not change sign. However, recently by using a model independent metod to deal with the observational data Cai and Su found that the sign of interaction $Q$ in the dark sector changed in the redshift range of $0.45 \lesssim z \lesssim 0.9$. Hereafter, a sign-changeable interaction \cite{10,17,20} were introduced

$$Q = q(\alpha \dot{\rho} + 3\beta H\rho).$$  \hspace{1cm} (13)

where $\alpha$ and $\beta$ are dimensionless constants, the energy density $\rho$ could be $\rho_m$, $\rho_{GDe}$, $\rho_{tot}$. $q$ is the deceleration parameter

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a} = -1 - \frac{\dot{H}}{H^2}.$$  \hspace{1cm} (14)

This new type of interaction, where deceleration parameter $q$ is a key ingredient makes this type of interactions different from the ones considered in literature and presented above, because it can change its sign when our universe changes from deceleration $q > 0$ to acceleration $q < 0$. $\gamma \dot{\rho}$ is introduced from the dimensional point of view. We would like also to stress a fact, that by this way we import a more information about the geometry of the Universe into the interaction term. This fact can be mean that we should consider more general forms for the interaction term. It is obvious that this splitting (as a mathematical
act) can be done for any fluid with any number of components making a linear combination of pressure and energy density. From equations we see that unit of interaction $Q$ should be $time^{-1} \times energy\ density$. Other type of interaction is of the form $Q = \gamma \dot{\rho}$, where for $\rho$ we can say the same as in previous case. Question of $time^{-1}$ here was solved by taking derivative of energy density instead of using Hubble parameter with $time^{-1}$ unit. Combination of these two type of interactions also were considered. Recently other tendency was observed that constants $b$ and $\gamma$ from interaction terms were assumed to be a function of scale factor $b(a) = b_{0}a^{\eta}$ [19]. As a simple model we will consider an Universe with a single component fluid. Then as a generalization of the model we will consider a composed fluid model with different couplings between fluid components interpreted within interaction terms. For each case we investigate different parameters of a model. Parameter value searching criteria is based on to find later time acceleration corresponding to negative deceleration parameter associated with negative EoS parameter. Other steps for proving viability of the models could be comparison with experimental data and for instance, convincing in a validity of generalized second law of thermodynamics. Last is done in this article.

The paper is organized as follows. After introduction, in the next section, as a simple case we consider a model of a Universe with varying GD energy. After, in next sections we will consider composed fluid models. We will start with non interacting case. The main goal is to consider interaction between components, thus we will consider different forms of interactions and provide some conclusion concerning behavior of the model, especially for later stages of evolution. In the last section we will check validity of the Generalized Second Law of Thermodynamics. Discussion of obtained results is also presented in last section.

Varying Ghost Dark Energy fluid Universe

In this section we would like to consider single fluid model and behavior of the Universe. In this case we will describe the model of our fluid as given by the following energy density

$$\rho_{GDc} = \alpha_{0}a(t)^{\xi}H(t) + \beta H(t)^{2}, \quad (15)$$

where $\alpha_{0}$ and $\beta$ are constants, $\xi$ is a number and a new parameter of the model. $H(t)$ is a Hubble parameter. Our job is to find values of parameters $\alpha_{0}, \beta, \xi$. Pressure can be found easily and reads as

$$P_{GDc} = -\frac{\dot{H}}{3} \left[ \frac{a_{0}a^{\xi}}{H} + 2\beta \right] - \frac{(\xi + 3)\alpha_{0}a^{\xi}H}{3} - \beta H^{2}. \quad (16)$$

Taking into account last two equation as well as [7] we will have complete set of equations, which we solve numerically to recover scale factor, therefore Hubble parameter $H$, deceleration parameter $q$ as well as EoS parameter of the fluid $\omega_{GDc} = P_{GDc}/\rho_{GDc}$. Bellow we present profiles of the mentioned functions with some conclusions. In case of $\alpha_{0} = 1.5$ and $\beta = 2.5$ we consider behavior of the model with different values of $\xi$. Within Fig.1 we collect behavior
of $\omega = \frac{P_{GDe}}{\rho_{GDe}}$, deceleration parameter $q$, $P_{GDe}$ and $\rho_{GDe}$. Models with $\xi < 0.5$ give an ever accelerated Universe, with a quintessence fluid, which pressure is negative and energy density $\rho$ decrease approaching to 0 over evolution. We observe that in this case phantom crossing is not possible. Models with $\xi > 0.5$ provide us models of Universes with a possibility of a transition from $q > 0$ to $q < 0$. For the models with $\xi > 1$ we faced with contradiction i.e we have dark energy but $q > 0$. As an example follow behavior of red line. Transitions to accelerated expansion phase observed at early stages of evolution. For the later stages of evolution it seems that Universe composes by the same matter (for different values of $\xi$) however quantitative behavior of $q$ is different for different values of $\xi$. Next we would like discuss a behavior of the Universe, when $\alpha_0 = 1.5, \xi = 0.2$ and $\beta$ takes different values $2.5, 3.5, 4.5$. From Fig. 2 we conclude that for initial state of evolution, we could have either quintessence ($\beta = 2.5$) or phantom fluid ($\beta = 3.4, 4.5$). After, very short period from start of evolution independent on the value of $\beta$ we have quintessence fluid. Then we can observe that for later stages of evolution we have the same behavior for $\omega$ independent of the values of the parameters and $\omega \to -1$ i.e cosmological constant is possible for later stages of evolution. Deceleration parameter $q$ is always negative, which means that we have ever accelerated expansion. For $\beta = 2.5$ it is decreasing function and then it is a constant. For higher values of $\beta$ the behavior of $q$ is increasing function and then for future it becomes $\beta$ independent and stays in constant regime, which an appropriate with observational data. Decreasing in $\omega$ corresponds decreasing in $q$, while increasing in $\omega$ corresponds to increasing $q$. We can also stress that for later stages of evolution we have constant behavior in all $\omega$, $q$, $P$ and $\rho_{GDe}$. Behavior of $\omega$ is consistent with behavior of $\rho_{GDe}$ i.e. fluid is a quintessence: during evolution its energy density is a decreasing

Figure 1: Profiles of $\omega$, $q$, $P$ and $\rho_{GDe}$ against time. 2 parameters of a single fluid model $\alpha_0 = 1.5$ and $\beta = 2.5$ are fixed, while $\xi$ takes values $0.2, 0.7, 1.2$. $1.5, \xi = 0.2$ and $\beta$ takes different values $2.5, 3.5, 4.5$. From Fig. 2 we conclude that for initial state of evolution, we could have either quintessence ($\beta = 2.5$) or phantom fluid ($\beta = 3.4, 4.5$). After, very short period from start of evolution independent on the value of $\beta$ we have quintessence fluid. Then we can observe that for later stages of evolution we have the same behavior for $\omega$ independent of the values of the parameters and $\omega \to -1$ i.e cosmological constant is possible for later stages of evolution. Deceleration parameter $q$ is always negative, which means that we have ever accelerated expansion. For $\beta = 2.5$ it is decreasing function and then it is a constant. For higher values of $\beta$ the behavior of $q$ is increasing function and then for future it becomes $\beta$ independent and stays in constant regime, which an appropriate with observational data. Decreasing in $\omega$ corresponds decreasing in $q$, while increasing in $\omega$ corresponds to increasing $q$. We can also stress that for later stages of evolution we have constant behavior in all $\omega$, $q$, $P$ and $\rho_{GDe}$. Behavior of $\omega$ is consistent with behavior of $\rho_{GDe}$ i.e. fluid is a quintessence: during evolution its energy density is a decreasing
Composed fluid model

In this section we suppose existence of the second fluid. We will consider composed phenomenological two component fluid and investigate cosmological parameters. We will start from non interacting model and then subsequently will include interactions of different form and analyse numerical solutions.

Non interacting case

In this section we will explore a mixture of considered fluids supposing that there is not interaction between them, which means that evolution dynamics of each of them is independent of each other. Then in the next section we will consider several forms of interaction. Non interacting case generally simplifies mathematics concerning to the problem and it is a good starting point to understand general properties of the model. In this case (8) and (9) were reduced to a simple form

\[ \dot{\rho}_m + 3H(\rho_m + P_m) = 0 \]  \hspace{1cm} (17)

and

\[ \dot{\rho}_{GDe} + 3H(\rho_{GDe} + P_{GDe}) = 0 \]  \hspace{1cm} (18)

and according to the setups of our problem they will allow us to determine other parameters like \( \rho_m \) and \( P_{GDe} \). From (18) for a pressure of Ghost Dark energy

Figure 2: Profiles of \( \omega \), \( q \), \( P \) and \( \rho_{GDe} \) against time. 2 parameters of a single fluid model \( \alpha_0 = 1.5 \) and \( \xi = 0.2 \) are fixed, while \( \beta \) takes values 2.5, 3.5, 4.5.
we have

\[ P_{\text{GDe}} = -\frac{\dot{\rho}_{\text{GDe}}}{3H} - \rho_{\text{GDe}}. \]  

Combining last two equations with (7) and taking into account generalized Ghost Dark energy \(^2\) we will recover scale factor \(a\). It is obvious that in the case of interaction between components we need to modify (19)

\[ P_{\text{GDe}} = -\frac{Q - \dot{\rho}_{\text{GDe}}}{3H} - \rho_{\text{GDe}}. \]  

Deceleration parameter \(q\) and EoS parameter of the composed fluid

\[ \omega_{\text{tot}} = \frac{P_m + P_{\text{GDe}}}{\rho_m + \rho_{\text{GDe}}} \]  

are of our interest. First we suppose that the fluid described by (3) could be a dark energy as well, which means that EoS parameter \(\omega\) will take a negative value. Using different values for parameters of the model we observed that, for instance, when \(\omega = -1\) and \(\xi = 0\) we have always accelerated Universe with \(q < 0\) and effective fluid can be understood as a dark energy with negative EoS parameter. For later times of evolution EoS parameter becomes equal to \(-1\). Which could be accepted as true-like history as it is accepted i.e. dark energy is able to explain accelerated expansion of the Universe. We also observer, that this picture is true for any negative \(\xi\), while in positive domain for the parameter we have very narrow domain \(0 \leq \xi < 1\). In Fig.3 we present a behavior of the model. Parameters are chosen to be in the range which is intensively

\[ \omega_{\text{tot}}, q, P_{\text{GDe}} \] and \(\rho_{\text{GDe}}\) against time. \(\beta = 2.5\).

Figure 3: Profiles of \(\omega_{\text{tot}}, q, P_{\text{GDe}}\) and \(\rho_{\text{GDe}}\) against time. \(\beta = 2.5\).

\(^1\)later on investigation shows that the same physics is seen for positive EoS parameter \(\omega\)

\(^2\)equal 0 case corresponds to the Ghost Dark energy with constant parameters
considered in literature, but among such parameters we observe several compositions of the parameters, where Universe has decelerated expansion, but it is filled with an effective dark energy, which for later epochs of evolution becomes a cosmological constant. However, such situation also can not be excluded from the considerations, because several crucial questions concerning to the field are still open. From Fig.(3) we see 3-phase Universe history. For the early times we have phantom-like behavior, while then we have transition to the quintessence-like epoch and finally we can observe that Universe keeping quintessence-like behavior for later stages of evolution has tendency to become a Universe filled with a cosmological constant with $\omega = -1$.

Model and different forms of interactions

In this section we would like to add interaction between components of the fluid and to observe graphically changes arising from interaction. First type of interaction which we will consider reads as

$$Q = 3Hqb\rho + \gamma(a)\dot{\rho}, \quad (22)$$

Profiles of $\omega_{\text{tot}}$, $q$, $P_{\text{tot}}$ and $\rho_{\text{tot}}$ against time present in Fig.(4). Second type of interaction reads as

$$Q = 3Hb(a)\rho + \gamma\dot{\rho}. \quad (23)$$

Profiles of cosmological parameters of our interest with this type of interaction is presented in Fig.(5) For both cases we suppose that $b$ and $\gamma$ are functions of scale factor and for both we will assume $a(t)^{\nu}$ [19]. $q$ is deceleration parameter and it was considered to achieve sign-changeable interaction and $\rho$ is total energy

![Figure 4: Profiles of $\omega_{\text{tot}}$, $q$, $P_{\text{tot}}$ and $\rho_{\text{tot}}$ against time. Interaction is given as $Q = 3Hqb\rho + \gamma(a)\dot{\rho}$](image)

![Figure 5: Profiles of $\omega_{\text{tot}}$, $q$, $P_{\text{tot}}$ and $\rho_{\text{tot}}$ against time. Interaction is given as $Q = 3Hb(a)\rho + \gamma\dot{\rho}$.](image)
Figure 5: Profiles of $\omega_{\text{tot}}$, $q$, $P_{\text{tot}}$ and $\rho_{\text{tot}}$ against time. $Q = 3Hb(a)\rho + \gamma\dot{\rho}$.

density. We suppose that only for one of the components appearing in (22) and (23) will change sign during the evolution. As a last step we also consider interaction which reads as

$$Q = 3Hb(a)q\rho.$$  \hspace{1cm} (24)

where $b(a) = ba(t)\zeta$. For all three types of the interactions it is possible to

Figure 6: Profiles of $P_{\text{tot}}$ and $\rho_{\text{tot}}$ against time. Interaction is given by $Q = 3Hb(a)q\rho$.

find a set of variables, which can provide some reasonable picture concerning to the results of the experimental data i.e we obtained accelerated Universe with a dark energy. But as in case with non interacting components, we found that we can have set of variables, values of which are in scopes of the values discussed in literature, that either we have $q > 0$ for whole evolution but effective fluid is a dark energy with negative EoS parameter or we have transition from accelerated to decelerated Universe, while fluid does not change its behavior i.e it is dark
energy. This was seen in case of (24). Of course, what was done in this research, it is just based on the assumption that the values for parameters of the model can be values from literature, but however to give final conclusion about the model some additional steps should be taken, namely comparing model with experimental data. This will give us a possibility to fix a viable range of values for the parameters, and then we can clarify the picture. Considering different interactions between components proposed in this section, practically do not change the situation: Still we have 3-phase Universe, which starts as it is filled by phantom-like fluid, then fluid behaves as quintessence with $\omega > -1$. Then we observe that for later stages of evolution fluid will start to behave as a cosmological constant, but the time of this transition depends on the values of parameters i.e by different values of parameters we can either accelerate or decelerate transition from quintessence-like epoch to a epoch with cosmological constant. As the last step of the research we will consider of the validity of The Generalized Second Law of Thermodynamics.

**The Generalized Second Law of Thermodynamics**

In this section we are going to deal with the question of the validity of the Generalized Second Law of Thermodynamics. As well as we will consider question of statefinder diagnostics which will be done in this article. For GSL of Thermodynamics we will follow [22] for the writings of this section (references therein are useful for complete introduction), where was considered validity of the Generalized Second Law of Thermodynamics for the Universe bounded by the Hubble horizon

$$R_H = \frac{1}{H}. \quad (25)$$

cosmological event horizon

$$R_E = a \int_1^\infty \frac{dt}{a} \quad (26)$$

and the particle horizon

$$R_P = a \int_0^t \frac{dt}{a}. \quad (27)$$

The contents in the Universe bounded by the event horizons taken as interacting two components of a single scalar field. The foundation of GSL required the Gibb’s equation of thermodynamics is

$$T_X dS_{IX} = P dV_X + dE_{IX} \quad (28)$$

where $S_{IX}$ and $E_{IX} = \rho V_X$, are internal entropy and energy within the horizon, while $V_X = \frac{4}{3} \pi R_X^3$ be the volume of sphere with horizon radius

$$R_X = \left( \sqrt{H^2 + \frac{k}{a^2}} \right)^{-1}.$$  

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3 Recall that in case when $k = 0$ as in our case apparent horizon $R_A = \frac{1}{\sqrt{H^2 + k/a^2}}$ we get the radius of the Hubble horizon [25].
Recall that GSL with First Law for the time derivative of total entropy gives

$$\dot{S}_X + \dot{S}_{IX} = \frac{R_X^2}{G T_X} \left( \frac{k}{a^2} - \dot{H} \right) \dot{R}_X. \tag{29}$$

while in case without First Law used we get

$$\dot{S}_X + \dot{S}_{IX} = \frac{2\pi R_X}{G} \left[ R_X^2 \left( \frac{k}{a^2} - \dot{H} \right) (\dot{R}_X - HR_X) + \dot{R}_X \right]. \tag{30}$$

Under the notations used above we understood that $T_X = \frac{1}{2\pi R_X}$ and $R_X$ is temperature and Radius for a given horizon under equilibrium thermodynamics respectively, $S_X$ is the horizon entropy and $\dot{S}_{IX}$ as the rate of change of internal entropy. It was found that the first and second laws of thermodynamics hold on the apparent horizon when the apparent horizon and the event horizon of the Universe are different, while for consideration of only event horizon these laws breakdown[?]. The Friedmann equations and the first law of thermodynamics ( on the apparent horizon ) are equivalent if the Universe is bounded by the apparent horizon $R_A$ with temperature $T_A = \frac{1}{2\pi R_A}$ and entropy $S_A = \frac{\pi R_A^2}{15}$. Usually, the Universe bounded by apparent horizon and in this region the Bekenstein’s entropy - mass bound ($S \leq 2\pi E R_A$) and entropy - area bound ($S \leq \frac{A}{4}$) are hold.

In order the GSL to be hold it is required that $\dot{S}_X + \dot{S}_{IX} \geq 0$ i.e. the sum of entropy of matter enclosed by horizon must be not be a decreasing function of time. We observed that for all models considered in this paper the GSL of Thermodynamics is satisfied.

**Discussion**

In this article we propose and consider a varying Ghost Dark energy. In base of a generalized Ghost dark energy with energy density $\rho_{GDe} = \alpha \dot{H} + \beta H^2$ we assume that $\alpha$ can be a function of scale factor, for instance, $\alpha(a) = a^5$. As a possibility of this effect we suppose that there was (or in the future will be) an interaction between a Ghost Dark energy and a fluid, which at the end of the interaction will evaporate completely. We consider possibility of this scenario related to the unknown physics and at the same time we argue that our opinion could not coincide with the others’ opinion. We also consider some modifications concerning to interaction term. The same mechanism could be accepted for the origin as was done for energy model. This model can be understood as a toy model. Considering existence of an additional fluid with pressure $P = \omega \rho$ we suppose possibility of having non interaction and interaction. For both cases a set of parameters for the model providing accelerated expansion of the Universe were obtained, assuming that for this an effective dark energy is responsible. A possibility of having decelerated expansion in presence of an effective dark energy with negative EoS parameter also was obtained, while the parameters of the model were considered ones which are discussed in literature. From graphical analysis we observed 3-phase Universe history for all considered cases. For the early times we have phantom-like behavior, while then we have transition to the quintessence-like epoch and finally we can observe that Universe keeping...
quintessence-like behavior for later stages of evolution has tendency to become a Universe filled with a cosmological constant with $\omega = -1$. Including interaction of the considered behavior we are able to control the time of the last epoch transition. As the last step we check validity of the Generalized Second Law of the Thermodynamics. We become proved that with the Universe bounded by Hubble horizon we have GSL validity condition satisfied.

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References

[1] A.G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116 1009 (1998); S Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999); R. Amanullah et al., Astrophys. J. 716, 712 (2010)

[2] A.C. Pope et al. Astrophys. J. 607 655 (2004), astro-ph/0401249

[3] D.N. Spergel et al. Astrophys. J. Supp. 148 175 (2003), astro-ph/0302209

[4] P.J. Steinhardt, Critical Problems in Physics (1997), Prinston University Press

[5] J. Sola and H. Stefancic, Phys. Lett. B 624, 147 (2005); I. L. Shapiro and J. Sola, Phys. Lett. B 682, 105 (2009)

[6] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988); C. Wetterich, Nucl. Phys. B 302, 668 (1988); A. R. Liddle and R. J. Scherrer, Phys. Rev. D 59, 023509 (1999); I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999); Z. K. Guo, N. Ohta and Y. Z. Zhang, Mod. Phys. Lett. A 22, 883 (2007); S. Dutta, E. N. Saridakis and R. J. Scherrer, Phys. Rev. D 79, 103005 (2009); E. N. Saridakis and S. V. Sushkov, Phys. Rev. D 81, 083510 (2010)

[7] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003)

[8] R. R. Caldwell, Phys. Lett. B 545, 23 (2002); S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003); P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003); J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004); V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004); W. Hu, Phys. Rev. D 71, 047301 (2005); M. R. Setare and E. N. Saridakis, JCAP 0903, 002 (2009); E. N. Saridakis, Nucl. Phys. B 819, 116 (2009); S. Dutta and R. J. Scherrer, Phys. Lett. B 676, 12 (2009)

[9] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005); E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Rev. D 70, 043539 (2004); Z. K. Guo, et al., Phys. Lett. B 608, 177 (2005); M.-Z Li, B. Feng, X.-M Zhang,
JCAP, 0512, 002 (2005); B. Feng, M. Li, Y.-S. Piao and X. Zhang, Phys. Lett. B 634, 101 (2006); S. Capozziello, S. Nojiri and S. D.Odintsov, Phys. Lett. B 632, 597 (2006); W. Zhao and Y. Zhang, Phys. Rev. D 73, 123509 (2006); Y. F. Cai, T. Qiu, Y. S. Piao, M. Li and X. Zhang, JHEP 0710, 071 (2007); E. N. Saridakis and J. M. Weller, Phys. Rev. D 81, 123523 (2010); Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, JCAP 0803, 013 (2008); M. R. Setare and E. N. Saridakis, Phys. Lett. B 668, 177 (2008); M. R. Setare and E. N. Saridakis, Int. J. Mod. Phys. D 18, 549 (2009); Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, Phys. Rept. 493 (2010) 1; T. Qiu, Mod. Phys. Lett. A 25, 909 (2010).

[10] S. D. H. Hsu, Phys. Lett. B 594, 13 (2004); M. Li, Phys. Lett. B 603, 1 (2004); Q. G. Huang and M. Li, JCAP 0408, 013 (2004); M. Ito, Europhys. Lett. 71, 712 (2005); X. Zhang and F. Q. Wu, Phys. Rev. D 72, 043524 (2005); D. Pavon and W. Zimdahl, Phys. Lett. B 628, 206 (2005); S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. 38, 1285 (2006); E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D 71, 103504 (2005); H. Li, Z. K. Guo and Y. Z. Zhang, Int. J. Mod. Phys. D 15, 869 (2006); E. N. Saridakis, Phys. Lett. B 660, 138 (2008); E. N. Saridakis, JCAP 0804, 020 (2008); E. N. Saridakis, Phys. Lett. B 661, 335 (2008)

[11] R.G. Cai, Phys. Lett. B 657, 228 (2007); H. Wei and R.G. Cai, Phys. Lett. B 660, 113 (2008); H. Wei and R.G. Cai, Eur. Phys. J. C 59, 99 (2009)

[12] F.R. Urban, A.R. Zhitnitsky, Phys. Rev. D 80, 063001 (2009); F.R. Urban, A.R. Zhitnitsky, JCAP 09, 018 (2009); F.R. Urban, A.R. Zhitnitsky, Phys. Lett. B 688, 9 (2010); F.R. Urban, A.R. Zhitnitsky, Nucl. Phys. B 835, 135 (2010); N. Ohta, Phys. Lett. B 695, 41 (2011); R.G. Cai, Z.L. Tuo, H.B. Zhang, [arXiv:1011.3212]

[13] A. Sheykhi, A. Bagheri, Europhys. Lett. 95, 39001 (2011); E. Ebrahimi, A. Sheykhi, Phys. Lett. B 705, 19 (2011); E. Ebrahimi, A. Sheykhi, Int. J. Mod. Phys. D 20, 2369 (2011); A. Sheykhi, M. Sadegh Movahed, Gen. Relativ. Gravit. [DOI:10.1007/s10714-011-1286-3].

[14] Chao-Jun Feng, Xin-Zhou Li, Ping Xi, Global behavior of cosmological dynamics with interacting Veneziano ghost, JHEP 1205 (2012) 046; Chao-Jun Feng, Xin-Zhou Li, Xian-Yong Shen, Latest Observational Constraints to the Ghost Dark Energy Model by Using Markov Chain Monte Carlo Approach. Phys.Rev. D87 (2013) 023006. Chao-Jun Feng, Xin-Zhou Li, Xian-Yong Shen, Thermodynamic of the QCD Ghost Dark Energy Universe, Mod.Phys.Lett. A27 (2012) 1250182.

[15] R.G. Cai, Z.L. Tuo, Y.B. Wu, Y.Y. Zhao, Phys Rev. D 86, 023511 (2012).

[16] WEI Hao, Cosmological Constraints on the Sign-Changeable Interactions, Common. Theory. Phys. 56 (2011) 972-980.

[17] H.Wei, Nucl. Phys. B 845 (2011) 381.

[18] Shin’ichi Nojiri and Sergei Odintsov, Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models, [arXiv:1011.0544v4], 2011.
[19] Xi-ming Chen, Yungui Gong and Emmanuel n. Saridakis, Time-dependent interacting dark energy and transient acceleration, arXiv:1111.6743v2, 2012.

[20] WEI Hao, Cosmological Constraints on the Sign-Changeable Interactions, Common. Theory. Phys. 56 (2011) 972-980. H.Wei, Nucl. Phys. B 845 (2011) 381. Cheng-Yi Sun, Rui-Hong Yue, New Interaction between Dark Energy and Dark Matter Changes Sign during Cosmological Evolution, arXiv:1009.1214v4, 2012.

[21] Martiros Khurshudyan, A Dark Energy Model interacting with Dark Matter described by an effective EoS, arXiv:1302.1220v3, 2013; Martiros Khurshudyan, Interaction between Variable Chaplygin gas and Tachyonic Matter, arXiv:1301.4990v3, 2013; Martiros Khurshudyan, Interaction between Generalized Varying Chaplygin gas and Tachyonic Fluid, arXiv:1301.1021v4, 2013. Martiros Khurshudyan, A Matter with an effective EoS interacting with a tachyonic field in an accelerating Universe, arXiv:1301.0005v3, 2013.

[22] Arundhati Das, Surajit Chattopadhyay, Ujjal Debnath, Validity of the Generalized Second Law of Thermodynamics in the Logamediate and Intermediate Scenarios of the Universe, Found Phys (2012) 42:266-283.
Appendix: One component fluid Universe

Figure 7: Profiles of $\omega$, $q$, $P$ and $\rho_{GDe}$ against time. 2 parameters of a single fluid model $\xi = 1.2$ and $\beta = 2.5$ are fixed, while $\alpha_0$ takes values 1.5, 2.5, 3.5.
Figure 8: Profiles of $\omega$, $q$, $P$ and $\rho_{GDe}$ against time. 2 parameters of a single fluid model $\xi = 2.5$ and $\alpha_0 = 1.5$ are fixed, while $\beta$ takes values 2.5, 3.5, 4.5.

Appendix: Two components fluid Universe
Figure 9: Two component non interacting model. Profiles of $\omega_{\text{tot}}$, $q$, $P_{GD_e}$ and $\rho_{GD_e}$ against time. Two parameters $\alpha_0 = 1.5$ and $\beta = 2.5$ are given, while couple of parameters $\omega$ and $\xi$ are taking different values.

Figure 10: Profiles of $\omega$, $q$, $P$ and $\rho_{GD_e}$ against time for a different values of parameters. Case corresponds to two component non interacting model.
Figure 11: Profiles of $\omega_{tot}$, $q$, $P_{tot}$ and $\rho_{tot}$ against time. Interaction is given as $Q = 3Hqbp + \gamma(a)\dot{p}$. 