Interaction of Ultrasound with Vortices in Type-II Superconductors

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Abstract

The theory of the ultrasound propagation in the mixed state of type-II superconductors is suggested which takes into account the Magnus force on vortices, the anti-Magnus force on ions, and diamagnetism of the mixed state. The acoustic Faraday effect (rotation of polarization of the transverse ultrasonic wave propagating along vortices) is shown to be linear in the Magnus force in any regime of the flux flow for wavelengths used in the ultrasound experiments now. Therefore, in contrast to previous predictions, the Faraday effect should be looked for only in clean superconductors with a large amplitude of the Magnus force.

Investigation of ultrasound propagation in the mixed state of a type-II superconductor has proved to be an effective method of studying high-$T_c$ superconductors [1] (see also Ref. [2] for ultrasound experiment and theory in low-$T_c$ superconductors). Recently Dominguez et al. [3] have attracted attention to an interesting manifestation of interaction between the ultrasound and vortices: acoustic Faraday effect. The effect results from a force on ions transverse to the ion velocity. Due to this force the velocity of the transverse sound propagation depends on the sign of circular polarization and the plane of linear polarization should rotate when the sound wave propagates along vortices.

However, the system of equations used by Dominguez et al. [3] (see also their later paper [4]) is not Galilean-invariant and does not satisfy the momentum-conservation law for the whole system “ions+electrons” since they missed to include into the ion equation of motion the so-called anti-Magnus force which is especially important for dirty superconductors. In the present work I use dynamic equations which are free from these deficiencies. The correct theory predicts other functional dependences on the physical parameters and different conditions for observation of the Faraday effect. In particular, our analysis does not confirm the conclusion of Dominguez et al. on the strong Faraday effect without the Magnus force in dirty superconductors at high temperatures and predicts a many orders of magnitudes
weaker effect at low temperatures. Finally, for the ultrasound wavelengths used in the
experiments the best conditions for observation of the acoustic Faraday effect are at high
temperatures in clean superconductors. The present work derives the theory of Refs. [1,2]
as a particular limiting case, but generalizes the theory on a much larger domain of physical
parameters: the nonzero Magnus force (clean superconductors), low magnetic fields, where
the diamagnetism of the mixed state is essential, wavelengths short compared to the London
penetration depth.

We adopt the following picture of electron and ion motion induced by the ultrasound.
As usual, the electron liquid consists of two parts: normal and superfluid. The normal
component is effectively clamped to the crystal ions by viscous forces responsible for the
normal resistance: they move together with the ion velocity \( \vec{v}_i = d\vec{u}_i/dt \) (\( \vec{u}_i \) is the ion
displacement). In contrast, the superfluid electrons move with the superfluid velocity \( \vec{v}_s \)
which is different from the ion velocity in general. Such a physical picture holds until the
magnetic field is weak compared to the upper critical magnetic field \( H_{c2} \) and the flux-flow
resistance is much less than the normal resistance. Then the normal current proportional
to the velocity of the normal electrons with respect to ions may be neglected.

Thus our three-component system (ions, normal and superfluid electrons) becomes effect-
ively two-component as in the two-fluid model for superfluids: there is a superfluid with the
velocity \( \vec{v}_s \) and the mass density \( m_e n_s \) and a heavy normal fluid with the velocity \( \vec{v}_i \) and the
mass density \( m_i n + m_e (n - n_s) \approx m_i n \). The charge densities of these two components are \( e n_s \)
and \( -e n_s \). Here \( e \) is the electron charge, \( m_e \) and \( m_i \) are the electron and the ion masses, and
\( n_s \) and \( n \) are the superfluid electron and the total charge number densities respectively. We
can write the equations of motion for the superfluid and the normal fluid immediately using
a close analogy with the two-fluid hydrodynamics for rotating superfluids (modifications due
to presence of the electromagnetic forces are self-evident) [3]:

\[
\frac{\partial \vec{v}_s}{\partial t} = \frac{e}{m_e} \left\{ \vec{E} + \frac{1}{c} [\vec{v}_L \times \vec{B}] \right\} . \tag{1}
\]
\[
\frac{\partial^2 \vec{u}_i}{\partial t^2} - c_t^2 \nabla^2 \vec{u}_i = - \frac{e}{m_i} \frac{n_s}{n} \left\{ \vec{E} + \frac{1}{c} [\vec{v}_i \times \vec{B}] \right. \\
\left. + \frac{1}{c} [(\vec{v}_L - \vec{v}_{sl}) \times \vec{B}] \right\},
\]

Here \(c_t\) is the “bare” sound velocity ignoring interaction with superfluid electrons, the equilibrium magnetic induction \(\vec{B}\) is proportional to the vortex density \(B/\Phi_0\), \(\Phi_0\) is the magnetic flux quantum, \(\vec{v}_L = d\vec{u}_L/dt\) is the vortex velocity, \(\vec{u}_L\) is the vortex displacement, and \(\vec{v}_{sl}\) is the \textit{local} superfluid velocity determined at the points of the vortex line which differs from the \textit{average} superfluid velocity \(\vec{v}_s\) because of deformations of the vortex lattice. If the transverse sound wave propagates along vortices (the axis \(\hat{z}\)), then

\[
\vec{v}_{sl} = \vec{v}_s + \frac{c}{e n_s B} C^*_{44} \left[ \hat{z} \times \frac{\partial^2 \vec{u}_L}{\partial z^2} \right].
\]

The renormalized tilt-modulus \(C^*_{44}\) relates only to the vortex line-tension, without including the elastic energy of the average magnetic field \([\text{Ref}].\) For an isotropic superconductor not very close to \(H_{c1}\) \(C^*_{44} = (\Phi_0 B/4\pi\lambda^2) \ln(a/r_c)\) (for an anisotropic case see, e.g., \([\text{Ref}].\) and references therein). Here \(a \sim \sqrt{\Phi_0/B}\) and \(r_c\) are the intervortex distance and the vortex core radius.

The electrical field \(\vec{E}\) and the magnetic field \(\vec{h}\) generated by the ultrasound wave satisfy the Maxwell equations:

\[
\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{h}}{\partial t}, \quad 4\pi \frac{\vec{j}}{c} = \nabla \times \vec{h},
\]

where \(\vec{j} = e n_s (\vec{v}_s - \vec{v}_i)\) is the average electric current. The superfluid component of the current, the supercurrent \(\vec{j}_s = e n_s \vec{v}_s\), should satisfy the London equation averaged over the vortex array cell:

\[
\frac{4\pi \lambda^2}{c} \nabla \times \vec{j}_s = \vec{b}_v - \vec{h}.
\]

Here \(\lambda = \sqrt{m_e c^2/4\pi e^2 n_s}\) is the London penetration depth and \(\vec{b}_v\) is the a.c. component of the vortex field \([\text{Ref}].\) (the vortex induction in \([\text{Ref}].\)) which coincides with the magnetic field \(\vec{h}\) only in an uniform vortex lattice. For the transverse sound propagating along vortices \(\vec{b}_v = B \partial \vec{u}_L/\partial z\).
We need also the equation of vortex motion which connects three velocities \( \vec{v}_i \), \( \vec{v}_{sl} \), and \( \vec{v}_L \):

\[
- \eta (\vec{v}_L - \vec{v}_i) + \eta' [\hat{z} \times (\vec{v}_L - \vec{v}_i)] \\
= \pi \hbar n_s [\hat{z} \times (\vec{v}_{sl} - \vec{v}_i)] .
\]

Equation (6) is a self-evident generalization of the equation of the vortex motion known for a crystal at rest: the velocities \( \vec{v}_L \) and \( \vec{v}_{sl} \) are replaced by the relative velocities \( \vec{v}_L - \vec{v}_i \) and \( \vec{v}_{sl} - \vec{v}_i \) in accordance with Galilean invariance. The Lorentz force on the right-hand side is balanced by the viscous force \( \propto \eta \) and the Magnus force \( \propto \eta' \). The parameter \( \eta' \) was known to vary from \( \pi \hbar n_s \) for superclean superconductors \([10]\) to 0 for dirty superconductors \([11]\). However, \( \eta' \) may even exceed \( \pi \hbar n_s \), being equal \( \pi \hbar n \), when the Iordanskii force is essential (see discussion and references in Sec. X of Ref. \([5]\)).

Equations (1)-(6) is a closed system of equations averaged over the vortex-array cell. Equation (1) for electrons does not differ from the case of the ion lattice at rest. As for Eq. (2) for ions, I shall show now that the force from vortices in this equation (the last term on the right-hand side) is required by the momentum conservation and the third Newton law.

If there is no external forces on the electron superfluid, the Helmholtz theorem tells that \( \vec{v}_L = \vec{v}_{sl} \) \([4]\). This means that \( \eta = 0 \) and \( \eta' = \pi \hbar n_s \). Bearing this in mind, one can present \( \vec{v}_L \) as \( \vec{v}_L = \vec{v}_{sl} + (\vec{v}_L - \vec{v}_{sl}) \) in the right-hand side of Eq. (1). Then the contribution \( \propto [(\vec{v}_L - \vec{v}_{sl}) \times \vec{B}] \) is an external force due to interaction with the normal fluid. Correspondingly, the same force must appear in the ion (normal fluid) equation (2), but with the opposite sign. In superfluid hydrodynamics this force is called mutual friction force \([3]\). But for superconductors it is better to call it coupling force, since the force may incorporate not only friction, but also elastic pinning (see below). The coupling force, being external for the superfluid component, is internal for the system “ions+electrons” as a whole. Therefore this force does not contribute to the time variation of the total momentum of ions and electrons

\[
\frac{\partial}{\partial t}[m_i n \vec{v}_i + m_e (n - n_s) \vec{v}_i + m_e n_s \vec{v}_s]
\]
\[ \approx m_i n \frac{\partial \vec{v}_i}{\partial t} + m_e n_s \frac{\partial \vec{v}_s}{\partial t} = m_i n c_i^2 \vec{\nabla}^2 \vec{u}_i \]
\[ + \frac{1}{c} [\vec{j}_l \times \vec{B}] = \frac{\partial}{\partial z} \left( m_i n c_i^2 \frac{\partial \vec{u}_i}{\partial z} + \frac{B \vec{h}}{4\pi} + C_{44}^* \frac{\partial \vec{u}_L}{\partial z} \right). \quad (7) \]
Here \( \vec{j}_l = en_s (\vec{v}_{sl} - \vec{v}_i) \) is the local current on the vortex line, in contrast to the average current \( \vec{j} = en_s (\vec{v}_s - \vec{v}_i) \) in the Maxwell equations (4). Equation (7) demonstrates the momentum conservation law for the case when the wave propagates along vortices (the axis z): the time variation of the momentum is determined by the divergence of the momentum flux which consists of (i) the stress tensor of the ion lattice, (ii) the stress tensor of the magnetic field linearized with respect to the a.c. field \( \vec{h} \), and (iii) the vortex-lattice stress tensor given by the tilt-modulus \( C_{44}^* \). In the long-wavelength limit \( \lambda k \to 0 \) (see below) the superfluid component \( \propto \partial \vec{v}_s / \partial t \) of the momentum variation may be neglected, and Eq. (7) has a simpler form:
\[ m_i n \left( \frac{\partial^2 \vec{u}_i}{\partial t^2} - c_i^2 \vec{\nabla}^2 \vec{u}_i \right) = \frac{1}{c} [\vec{j}_l \times \vec{B}] . \quad (8) \]
This equation was used in the analysis of Shapira and Neuringer [2], but they neglected difference between the local and average electric current which is responsible for the diamagnetism of the mixed state.

Replacing the coupling force in the equation of ion motion Eq. (2) with the help of Eq. (6) one obtains:
\[ \frac{\partial^2 \vec{u}_i}{\partial t^2} - c_i^2 \vec{\nabla}^2 \vec{u}_i = - \frac{e}{m_i n} \left\{ \vec{E} + \frac{1}{c} [\vec{v}_i \times \vec{B}] \right\} - \frac{B}{c} \vec{\eta} (\vec{v}_L - \vec{u}_i) - \frac{\alpha_M - 1}{c} (\vec{v}_L - \vec{u}_i) \times \vec{B} \right\} , \quad (9) \]
where \( \vec{\eta} = \eta / \pi \hbar n_s \) and \( \alpha_M = \eta / \pi \hbar n_s \) are the dimensionless amplitudes of the viscous and the Magnus force respectively. The coupling force has a component \( \propto (\alpha_M - 1) \) transverse to the ion velocity which is maximal in a dirty superconductor when the Magnus force vanishes \( (\alpha_M = 0) \). Indeed, suppression of the Magnus force in the dirty superconductor means that impurities produce a force which cancels the Magnus force on vortices, i.e. on superfluid electrons. Then in accordance with the third Newton law one must expect that the force of
opposite sign acts from vortices on impurities, i.e., on crystal ions. This explains the force
\( \propto (\alpha_M - 1) \) which may be called the anti-Magnus force.

Later on we assume that the transverse ultrasound plane wave \( \propto \exp(ikz - i\omega t) \) propa-
gates along the vortices. From Eqs. (3)-(5) one can derive relations connecting the electric
field and the local superfluid velocity with the ion and the vortex velocities and displacements:

\[
\vec{E} = -\frac{m_e \omega^2 \vec{u}_i}{e} - \frac{1}{1 + \lambda^2 k^2} \frac{1}{c} [\vec{v}_L \times \vec{B}] , \tag{10}
\]

\[
\vec{v}_{sl} = \frac{\vec{v}_i}{1 + \lambda^2 k^2} - \frac{c}{e n_s B} C_{44}(k) k^2 [\hat{z} \times \vec{u}_L] . \tag{11}
\]

where now

\[
C_{44}(k) = \frac{B^2}{4\pi} \frac{1}{1 + \lambda^2 k^2} + C_4^* \tag{12}
\]
is the \( k \)-dependent tilt-modulus related to the total energy of deformation. The contribu-
tion \( \propto \omega^2 \) to the electric field \( \vec{E} \) will be neglected later on, since it is not connected with
vortices. It yields a small correction to the sound velocity of the relative order \( m_e n_s/m_i n \)
which is present even in the Meissner state.

Finally, with the help of Eqs. (8), (10), and (11), one can rewrite the equation of the ion
motion, Eq. (2), in the terms of the ion velocity and displacement only:

\[
- \left( \omega^2 - c_i^2 k^2 \right) m_i n \vec{u}_i = \frac{e n_s B}{c} \left\{ f_\parallel \vec{v}_i + f_\perp [\hat{z} \times \vec{v}_i] \right\} . \tag{13}
\]

The longitudinal and the transverse forces on ions from vortices are given by the dimension-
less force parameters:

\[
f_\parallel = \frac{Dk^2}{i\omega} - \frac{1}{(\bar{\eta} - Dk^2/i\omega)^2 + \alpha_M^2} \left\{ \frac{\bar{\eta} - Dk^2}{i\omega} \right\}
\times \left[ \left( \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right)^2 - \left( \frac{Dk^2}{i\omega} \right)^2 \right] + 2\alpha_M \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right\} , \tag{14}
\]

\[
f_\perp = \frac{2\lambda^2 k^2}{1 + \lambda^2 k^2} + \frac{1}{(\bar{\eta} - Dk^2/i\omega)^2 + \alpha_M^2} .
\]
\[ \times \left\{ 2 \left( \frac{\eta - Dk^2}{i\omega} \right) \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \frac{Dk^2}{i\omega} \right\} + \alpha_M \left[ \left( \frac{Dk^2}{i\omega} \right)^2 - \left( \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right)^2 \right] \right\}. \quad (15) \]

Here \( D = cC_{44}/en_sB \).

The previous experimental and theoretical investigations \([1-3]\) dealt with the long-wavelength case \( \lambda k \to 0 \). One may call it the electrodynamic limit since in this case all forces from vortices on ions can be expressed in terms of the electrodynamic parameters: the Ohmic and the Hall conductivities, \( \sigma_O = \eta c^2/\Phi_0 B \), \( \sigma_H = \eta' c^2/\Phi_0 B \), related to the viscous and the Magnus force respectively, and the magnetic permeability

\[ \mu = \frac{B^2/4\pi}{B^2/4\pi + C_{44}^*} = \frac{B^2}{4\pi C_{44}(0)}, \quad (16) \]

which describes the diamagnetism due to circular currents over the vortex-lattice cell \([3]\). Here \( C_{44}(0) \) is the tilt-modulus in the limit \( k \to 0 \) (the Labusch modulus). One can check that Eq. (16) yields the differential magnetic permeability \( \mu = B/H \) for the case when the variation of the magnetic field is normal to the vortices. Here \( H = (1/4\pi)\partial F(B)/\partial B \) is the thermodynamic magnetic field along the equilibrium magnetization curve. In the terms of electrodynamics the equation of vortex motion \([3]\) is simply the Ohm law

\[ \vec{j}_l = \sigma_O \vec{E}_i - \sigma_H [\hat{\varepsilon} \times \vec{E}_i], \quad (17) \]

which connects the local current \( \vec{j}_l = \vec{j}/\mu \) and the electric field \( \vec{E}_i = \vec{E} + (1/c)[\vec{v}_i \times \vec{B}] \) in the coordinate frame moving with the ion velocity. Then Eq. (13) may be derived from Eq. (8) together with the Ohm law Eq. (17) and the relation \( \vec{E} = (4\pi i\omega/c^2k^2)\vec{j} = (4\pi i\omega \mu/c^2k^2)\vec{j}_l \) which follows from the Maxwell equations \([4]\). This yields the following values of the force parameters:

\[ f_\parallel = \frac{B}{en_s c} \left( \frac{c^2 k^2}{4\pi i\omega \mu} \right)^2 \left[ \frac{4\pi i\omega \mu}{c^2 k^2} \right] \left[ \frac{\sigma_O - c^2 k^2/4\pi i\omega \mu}{(\sigma_O - c^2 k^2/4\pi i\omega \mu)^2 + \sigma_H^2} \right], \quad (18) \]
\[
f_\perp = \frac{B}{en_x e} \left( \frac{c^2 k^2}{4\pi i \omega \mu} \right)^2 \frac{\sigma_H}{(\sigma_o - c^2 k^2/4\pi i \omega \mu)^2 + \sigma_H^2}.
\] (19)

One can easily check that Eqs. (18) and (19) coincide with Eqs. (14) and (15) if \(\lambda k \to 0\). In the electrodynamic limit our theory is valid in a wider interval of the magnetic fields than in the general case of arbitrary \(\lambda k\). Since we use the equations averaged over the vortex-array cell, they hold until the wavelength \(2\pi/k\) exceeds the intervortex distance \(a \sim \sqrt{\Phi_0 / B}\). But because of \(\lambda \ll 1/k\), this condition may be satisfied even if \(a \gg \lambda\), i.e., rather close to the lower critical field \(H_{c1}\) where the diamagnetism is important, i.e., \(\mu\) essentially less than unity. On the other hand, in the electrodynamic approach our assumption in the beginning of the paper that the current of normal electrons is negligible is not necessary: one can use the conductivity tensor taking into account this current. Then the theory is valid even close to \(H_{c2}\).

For the sake of simplicity we did not include the elastic pinning force into our analysis explicitly, but it is easy to do simply by replacing the viscous coefficient \(\eta\) by \(\eta - \alpha_P/i\omega\) in all equations. Here \(\alpha_P\) is the bulk pinning coefficient which may, in principle, depend on the frequency as assumed by Dominguez et al. [3]. Thus our analysis holds for any regime of vortex motion, either the non-activated flux flow, or the thermally assisted flux flow (TAFF) with flux jumps over pinning barriers. But different regimes of the flux flow correspond to different expressions for the conductivities.

One can obtain the results of Refs. [1,2] from Eqs. (18) and (19) neglecting the Magnus force and the finite diamagnetism of the mixed state (\(\sigma_H \sim \alpha_M = 0, \mu = 1\)). Note that the magnetic permeability \(\mu\) of Shapira and Neuringer [2] relates to the atomic magnetism, whereas the latter was ignored in the present work and our \(\mu\) is due to the circular currents in the vortex-lattice cell and the difference between the local and average currents as a result of them. However, finally the role of \(\mu\) in the equations is similar in both cases, as one might expect for an electrodynamic theory.

Our analysis seriously differs from that of Dominguez et al. [3]: (i) They missed to take into account the anti-Magnus force in the equation of ion motion. This caused violation of
the momentum conservation law and the wrong prediction for the Faraday effect in the dirty superconductors in which the anti-Magnus force is especially important. (ii) The equation of vortex motion used in Ref. [3] contained the laboratory vortex velocity and thereby was not Galilean-invariant. This means an assumption that the laboratory frame is preferential. But for our problem the only preferential coordinate system is that moves with the ion velocity.

Experiment and theory [1] have shown that in a Bi superconductor there is a crossover between two temperature regions: the low-temperature region of high Ohmic conductivity due to high activation barriers in the TAFF model where \( \omega \tilde{\eta}/Dk^2 = c_\eta/Dk = 4\pi\omega\mu\sigma_O/c^2k^2 > 1 \), and high-temperatures region of low conductivity where \( 4\pi\omega\mu\sigma_O/c^2k^2 < 1 \). The predictions of Dominguez et al. [3] for the Faraday effect must be revised both in the low-temperature and the high-temperature regions. At low temperatures they obtained that the Faraday rotation (the angle of polarization rotation per unit length) is \( d\theta/dz = \sigma_HB^2/2m_i\kappa c^2 \), whereas Eq. (19) yields the Faraday rotation \( d\theta/dz = e\kappa BRef_\perp/2m_i\kappa c \) which is by the factor \( (c^2k^2/4\pi\omega\mu\sigma_O)^2 \) smaller. This factor is of order unity at the crossover temperature \( T \sim 60 \text{ K} \), but at low temperatures the Ohmic conductivity increases proportionally to \( \exp(U/T) \) where the activation barrier is about 500 K for typical magnetic fields [1]. Thus Dominguez et al. overestimated the Faraday effect at low temperatures by many orders of magnitude. At higher temperatures Dominguez et al. obtained the Faraday rotation \( d\theta/dz = eB/2m_i\kappa c \) which did not depend on the Hall conductivity. This means that the Faraday effect is possible in a dirty superconductor without the Magnus force even in the limit \( \lambda k \to 0 \) in disagreement with Refs. [1,2]. Dominguez et al. explained it by the effect of the electromagnetic force \( \propto (\vec{E} + \frac{1}{c}[\vec{v}_i \times \vec{B}]) \) [see Eq. (11) for \( \vec{E} \)] in Eq. (9), which was neglected in Ref. [1]. However, as explained after Eq. (9), this equation should include also the anti-Magnus force which cancels the electrodynamic force in the limit \( \lambda k \to 0 \) restoring the result of Refs. [1]. According to Eq. (15) the Faraday rotation without the Magnus force \( (\alpha_M \sim \sigma_H = 0) \) is given by

\[
\frac{d\theta}{dz} = \frac{en_sB}{2m_i\kappa c} Ref_\perp = \frac{en_sB}{m_i\kappa c} \frac{\lambda^2k^2}{1 + \lambda^2k^2} \frac{1}{1 + (c^2k^2/4\pi\omega\mu\sigma_O)^2}. \tag{20}
\]
For the ultrasound frequencies 3 MHz from Ref. [1] this expression yields at high temperatures $> 60$ K the Faraday rotation at least by the factor $10^{-5}$ smaller than predicted by Dominguez et al. [3]. So the Faraday effect in dirty superconductors is possible only at $\lambda k \geq 1$. However, the frequency of ultrasound must be very high in order to achieve this case (about a few GHz). For the frequencies used in the ultrasound experiments now, the best conditions for observation of the Faraday effect are clean superconductors with large Hall angle at high temperatures. Then the Faraday rotation can be strongly amplified close to the resonances with vortex modes. The resonances correspond to zeroes of the denominator in Eqs. (18) and (19). Weakly damped vortex modes exist in superclean high-$T_c$ superconductors according to recent experimental and theoretical investigations [7,13]. Thus the ultrasound experiment is able to reveal these vortex modes as has been already discussed in Ref. [14].

In summary, the theory of interaction between the ultrasound and the vortices in the type-II superconductors has been suggested. The first time the theory takes into account the Magnus force on vortices, the anti-Magnus force on ions, and the diamagnetism of the mixed state self-consistently. This results in a serious revision of previous predictions concerning the acoustic Faraday effect. Despite this revision, our analysis confirms that possible observation of the acoustic Faraday effect is expected to provide a valuable information on vortex dynamics in type-II superconductors.

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