Abstract—In this paper, we investigate unconstrained and constrained sample-based federated optimization, respectively. For each problem, we propose a privacy preserving algorithm using stochastic successive convex approximation (SSCA) techniques, and show that it can converge to a Karush-Kuhn-Tucker (KKT) point. To the best of our knowledge, SSCA has not been used for solving federated optimization, and federated optimization with nonconvex constraints has not been investigated. Next, we customize the two proposed SSCA-based algorithms to two application examples, and provide closed-form solutions for the respective approximate convex problems at each iteration of SSCA. Finally, numerical experiments demonstrate inherent advantages of the proposed algorithms in terms of convergence speed, communication cost and model specification.

Index Terms—Federated learning, non-convex optimization, stochastic optimization, stochastic successive convex approximation.

I. INTRODUCTION

Machine learning with distributed databases has been a hot research area [1]. The amount of data at each client can be large, and hence the data uploading to a central server may be constrained by energy and bandwidth limitations. Besides, local data may contain highly sensitive information, e.g., travel records, health information and web browsing history, and thus a client may be unwilling to share it. Therefore, it is impossible or undesirable to upload distributed databases to a central server. Recent years have witnessed the growing interest in federated learning, where data is maintained locally during the collaborative training of the server and clients [2].

Data privacy and communication efficiency are the two main advantages of federated learning, as only model parameters or intermediate parameters are exchanged for calculating the gradient before model aggregation steps.

Most existing works for federated learning focus on solving unconstrained optimization problems using mini-batch stochastic gradient descent (SGD) [2]–[6]. Depending on whether data is distributed over the sample space or feature space, federated learning can be typically classified into sample-based (horizontal) federated learning and feature-based (vertical) federated learning [2]. In sample-based federated learning, the datasets of different clients have the same sample space but little intersection on the sample space. Most studies on federated learning focus on this category [3]–[5]. In the existing sample-based federated learning algorithms, the global model is iteratively updated at the server by aggregating and averaging the locally computed models at clients. Data privacy is naturally preserved as the model averaging steps avoid exposing raw data. Specifically, at each communication round, the selected clients download the current model parameters and conduct one or multiple SGD updates to refine the local model. Multiple local SGD updates can reduce the required number of model averaging steps and hence save communication costs. However, they may yield the divergence of sample-based federated learning when local datasets across clients are heterogeneous. The most commonly used sample-based federated learning algorithm is the Federated Averaging algorithm [3]. On the contrary, in feature-based federated learning, the datasets of different clients share the same sample space but differ in the feature space. Feature-based federated learning is more challenging, as a client cannot obtain the gradient of a loss function relying purely on its local data. In the existing feature-based federated learning algorithms [2], [6], intermediate parameters are exchanged for calculating the gradient before model aggregation steps.

SGD has long been used for solving unconstrained stochastic optimization problems or stochastic optimization problems with deterministic convex constraints. Recently, stochastic successive convex approximation (SSCA) is proposed to obtain Karush-Kuhn-Tucker (KKT) points of stochastic optimization problems with deterministic convex constraints [7] and with general stochastic nonconvex constraints [8], [9]. Apparently, SSCA has a wider range of applications than SGD. It has also been shown in [7] that SSCA empirically achieves a higher convergence speed than SGD, as SGD utilizes only first-order information of the objective function. Some recent works have applied SSCA to solve machine learning problems [10]. Nevertheless, SSCA has not been applied for solving federated optimization so far.

In this paper, we focus on designing sample-based federated learning algorithms using SSCA for unconstrained problems and constrained problems, respectively. First, we propose a privacy preserving algorithm to obtain a KKT point of unconstrained sample-based federated optimization using mini-batch SSCA, and analyze its computational complexity and convergence. Such algorithm empirically converges faster (i.e., achieves a lower communication cost) than the SGD-based ones in [3]–[5] and can achieve the same order of computational complexity as the SGD-based ones in [3]–[5]. Then, we propose a privacy preserving algorithm to obtain a KKT point of constrained sample-based federated optimization.
by combining the exact penalty method for SSCA in [9] and
mini-batch techniques, and analyze its convergence. Notice
that federated optimization with nonconvex constraints, which
can explicitly limit the cost function of a model, has not
been investigated so far. Next, we customize the two SSCA-
based algorithms to two application examples, and show that
all updates at each iteration have closed-form expressions.
Finally, numerical experiments demonstrate that the proposed
algorithm for unconstrained sample-based federated optimization
converges faster (i.e., yield lower communication costs)
but that the existing SGD-based ones [3–5], and the proposed
algorithms for constrained federated optimization can more
flexibly specify a training model.

II. SYSTEM SETTING

Consider $N$ data samples, each of which has $K$ features.
For all $n \in \mathcal{N} \triangleq \{1, \ldots, N\}$, the $K$ features of the $n$-th
sample are represented by a $K$-dimensional vector $x_n \in \mathbb{R}^K$.
Consider a central server connected with $I$ local clients, each
of which maintains a local dataset. Specifically, partition $\mathcal{N}$
into $K$ disjoint subsets, denoted by $\mathcal{N}_i$, $i \in I \triangleq \{1, \ldots, I\}$,
where $N_i \triangleq |\mathcal{N}_i|$ denotes the cardinality of the $i$-th subset and
$\sum_{i \in I} N_i = N$. For all $i \in I$, the $i$-th client maintains a local
dataset containing $N_i$ samples, i.e., $x_m, n \in \mathcal{N}_i$. For example,
two companies with similar business in different cities may
diffferent user groups (from their respective regions) but
the same type of data, e.g., users’ occupations, ages, incomes,
deposits, etc. The server and $I$ clients collaboratively train a
model from the local datasets stored on the $I$ clients under
the condition that each client cannot expose its local raw data
to the others. This training process is referred to as sample-
based (horizontal) federated learning [2]. The underlying opti-
mization, termed sample-based federated optimization [2], is
to minimize the following function:

$$F_0(\omega) \triangleq \frac{1}{N} \sum_{n \in \mathcal{N}} f_0(\omega, x_n)$$ (1)

with respect to (w.r.t) model parameters $\omega \in \mathbb{R}^d$. To be
general, we do not assume $F_0(\omega)$ to be convex in $\omega$.

In Section III and Section IV, we investigate sample-
based federated Learning for unconstrained optimization
and constrained optimization, respectively. To guarantee the
convergence of the proposed SSCA-based federated learning
algorithms, we assume that $f_0(\omega, x_n)$ satisfies the following
assumption in the rest of the paper.

Assumption I (Assumption on $f(\omega, x)$): [2], [3]. For any
given $x$, each $f(\omega, x)$ is continuously differentiable, and its
gradient is Lipschitz continuous.

Remark I (Discussion on Assumption [2]): Assumption I
is also necessary for the convergence of SSCA [7–9] and
SGD [5].

III. SAMPLE-BASED FEDERATED LEARNING FOR
UNCONSTRAINED OPTIMIZATION

In this section, we consider the following unconstrained
sample-based federated optimization problem:

$$\min_{\omega} F_0(\omega)$$

where $F_0(\omega)$ is given by (1).

Problem I (Unconstrained Sample-based Federated Opti-
mization):

$$\min_{\omega} F_0(\omega)$$

A. Algorithm Description

The main idea of Algorithm I is to solve a sequence of successively refined convex problems, each of which is
obtained by approximating $F_0(\omega)$ with a convex function
based on its structure and samples in a randomly selected mini-
batch by the server. Specifically, at iteration $t$, we choose:

$$\tilde{F}_0^{(t)}(\omega) = (1 - \rho(t))\tilde{F}_0^{(t-1)}(\omega)
+ \rho(t) \sum_{i \in I} N_i \sum_{n \in \mathcal{N}_i} f_0(\omega, (\omega_n), x_n)$$ (2)

with $\tilde{F}_0^{(0)}(\omega) = 0$ as an approximation function of $F_0(\omega)$,
where $\rho(t)$ is a stepsize satisfying:

$$\rho(t) > 0, \lim_{t \to \infty} \rho(t) = 0, \sum_{t=1}^{\infty} \rho(t) = \infty, \ (3)$$

$\mathcal{N}_i^{(t)} \subseteq \mathcal{N}_i$ is a randomly selected mini-batch by client $i$ at
iteration $t$, $B \leq N_i$ is the batch size, and $f_0(\omega, (\omega_n), x_n)$ is
a convex approximation of $f_0(\omega, x_n)$ around $\omega(t)$ satisfying
the following assumptions. A common example of $f_0$ will be
given later.

Assumption 2 (Assumptions on $f(\omega, \omega', x)$ for Approximating $f(\omega, x)$ Around $\omega'$): [2]

1) $\nabla f(\omega, \omega', x) = \nabla f(\omega, x)$; $2)\]
Algorithm 2 Mini-batch SSCA for Problem 3:
1: initialize: choose any $\omega^0$ and $c > 0$ at the server.
2: for $t = 1, 2, \ldots, T - 1$ do
3:   the server sends $\omega(t)$ to all clients.
4:   for all $i \in I$, client $i$ randomly selects a mini-batch $N_i^t \subseteq N_i$ with batch size $B \leq \bar{N}_i$, computes $q_m \left( \omega(t), (x_n)_{n \in N_i^t} \right)$, $m = 0, 1, \ldots, M$, and sends them to the server.
5:   the server obtains $(\tilde{\omega}(t), s(t))$ by solving Problem 5 and updates $\omega(t+1)$ according to (4).
6: end for
7: Output: $\omega^T$

where $\tau > 0$ can be any constant, and the term $\tau\|\omega - \omega(t)\|^2_2$ is used to ensure strong convexity. Obviously, $f_0$ given by (6) satisfies Assumption 2. Notice that Problem 3 with $f_0$ given by (6) is an unconstrained convex quadratic programming w.r.t. $\omega$ and hence has an analytical solution with the same order of computational complexity as the SGD-based ones in (3)–(5) (which is $O(d)$). The details of the analytical solution will be given in Section IV.

IV. SAMPLE-BASED FEDERATED LEARNING FOR CONSTRAINED OPTIMIZATION

In this section, we consider the following constrained sample-based federated optimization problem:

**Problem 3 (Constrained Sample-based Federated Optimization):**

$$\min_{\omega} f_0(\omega)$$

s.t. $F_m(\omega) \leq 0$, $m = 1, 2, \ldots, M$, where $F_m(\omega)$ is given by (6).

To be general, $F_m(\omega)$, $m = 0, \ldots, M$ are not assumed to be convex in $\omega$. Notice that federated optimization with nonconvex constraints has not been investigated so far. It is quite challenging, as the stochastic nature of a constraint function may cause infeasibility at each iteration of an ordinary stochastic iterative method [9]. In the following, we propose a privacy-preserving sample-based federated learning algorithm, i.e., Algorithm 2 to obtain a KKT point of Problem 3 by combining the exact penalty method [12] for SSCA in [9] and mini-batch techniques.

A. Algorithm Description

First, we transform Problem 2 to the following stochastic optimization problem whose objective function is the weighted sum of the original objective and the penalty for violating the original constraints.

**Problem 4 (Transformed Problem of Problem 3):**

$$\min_{\omega : s} F_0(\omega) + \frac{\tau}{2}\|\omega - \omega(t)\|^2_2,$$

s.t. $F_m(\omega) \leq s_m$, $m = 1, 2, \ldots, M$,
where \( s \triangleq (s_m)_{m=1,\ldots,M} \) are slack variables and \( c > 0 \) is a penalty parameter that trades off the original objective function and the slack penalty term.

At iteration \( t \), we choose \( F_m^{(t)}(\omega) \) given in (8) as an approximation function of \( F_m(\omega) \), and choose:

\[
F_m^{(t)}(\omega) = (1 - \rho^{(t)}) F_m^{(t-1)}(\omega) + \rho^{(t)} \sum_{n \in N_t} \frac{N_t}{BN} f_m(\omega, \omega^t, x_n), \quad m = 1, \ldots, M
\]

with \( F_m^{(0)}(\omega) = 0 \) as an approximation function of \( F_m(\omega) \), for all \( m = 1, \ldots, M \), where \( \rho^{(t)} \) is a stepsize satisfying (3). \( N_t \) is the randomly selected mini-batch by client \( i \) at iteration \( t \), and \( f_m(\omega, \omega^t, x_n) \) is a convex approximation of \( f_m(\omega, x_n)\) around \( \omega^t \) satisfying \( f_m(\omega, x_n) = f_m(\omega^t, x_n) \) and Assumption 2 for all \( m = 1, \ldots, M \). A common example of \( f_m, m = 0, \ldots, M \) will be given later.

Note that for all \( i \in \mathcal{I} \) and any mini-batch \( N_i' \subseteq N_i \) with batch size \( B \leq N_i \), \( \sum_{n \in N_i'} f_m(\omega, \omega', x_n) \), \( m = 0, \ldots, M \), can be written as \( \sum_{n \in N_i'} f_m(\omega, \omega^t, x_n) = p_m (\omega) x_n \in \mathbb{R}^{B} \) and \( \sum_{m=0}^{M} \mathbb{R}^{BK-d} \rightarrow \mathbb{R}^{P_m} \). Assume that the expressions of \( f_m, m = 0, \ldots, M \) are known to the server and \( N \) clients. Each client \( i \in \mathcal{I} \) computes \( q_m (\omega, \omega^t, x_n) \), \( m = 0, \ldots, M \) and send them to the server. Then, the server solves the following approximate problem to obtain \( \omega^t \).

\[
\text{Problem 5 (Convex Approximate Problem of Problem 4):}
\]

\[
(\omega^t, s^t) \triangleq \arg\min_{\omega, s} F_0^{(t)}(\omega) + c \sum_{m=1}^{M} s_m
\]

\[
s.t. \quad F_m^{(t)}(\omega) \leq s_m, \quad m = 1, 2, \ldots, M,
\]

\[
s_m \geq 0, \quad m = 1, 2, \ldots, M.
\]

Problem 5 is convex and can be readily solved. Given \( \omega^t \), the server updates \( \omega^t \) according to (9). The detailed procedure is summarized in Algorithm 2 and the convergence of Algorithm 2 is summarized below. Consider a sequence \{\( c_j \)\}.

For all \( j \), let \((\omega^t_j, s^t_j)\) denote a limit point of \((\omega^t, s^t)\) generated by Algorithm 2 with \( c = c_j \).

**Theorem 2 (Convergence of Algorithm 2):** Suppose that \( f_m, m = 0, \ldots, M \) satisfy Assumption 1, \( f_0 \) satisfies Assumption 2 and \( f_m(\omega, x) = f_m(\omega, x) \) and Assumption 2 for all \( m = 1, \ldots, M \). Consider an \( L \)-class classification problem with a dataset of \( N \) samples \( (x_n, y_n)_{n \in \mathcal{N}} \), where \( x_n \triangleq (x_{n,k})_{k \in \mathcal{K}} \) and \( y_n \triangleq (y_{n,l})_{l \in \mathcal{L}} \) with \( x_{n,k} \in \mathbb{R} \) and \( y_{n,l} \in \{0,1\} \). Consider a three-layer neural network, including an input layer composed of \( K \) cells, a hidden layer composed of \( J \) cells, and an output layer composed of \( L \) cells. We use the swish activation function \( S(z) = z/(1 + \exp(-z)) \) for the hidden layer and the softmax activation function for the output layer. We consider the cross entropy loss function. Thus, the resulting cost functions for sample-based and feature-based federated learning are given by:

\[
F(\omega) \triangleq -\frac{1}{N} \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}} y_{n,l} \log(Q_l(\omega, x_n)),
\]

where \( s_m \geq 0, \quad m = 1, 2, \ldots, M \).
with \( \omega \triangleq (\omega_{1,k}, \omega_{2,l,j})_{k \in K, j \in J, l \in L} \)
and
\[
Q_l(\omega, x_n) = \frac{\exp\left(\sum_{j \in J} \sum_{l \in L} \omega_{2,l,j} S\left(\sum_{k \in K} \omega_{1,k,j} x_{kn}\right)\right)}{\sum_{j \in J} \exp\left(\sum_{l \in L} \omega_{2,l,j} S\left(\sum_{k \in K} \omega_{1,k,j} x_{kn}\right)\right) },
\]
\( l \in L \). \hfill (10)

**A. Unconstrained Federated Optimization**

One unconstrained federated optimization formulation for the \( L \)-class classification problem is to minimize the weighted sum of the cost function \( F(\omega) \) in (9) together with the \( \ell_2 \)-norm regularization term \( \|\omega\|_2^2 \):
\[
\min_{\omega} \quad F_0(\omega) \triangleq F(\omega) + \lambda \|\omega\|_2^2 \hfill \tag{11}
\]
where \( \lambda > 0 \) is the regularization parameter that trades off the cost and model sparsity. We can apply Algorithm 1 with \( f_0(\omega, (\omega^t, x_n)) \) given by (3) to solve the problem in (11). Theorem 1 guarantees the convergence of Algorithm 1 as Assumption 1 and Assumption 2 are satisfied. Specifically, the server solves the following convex approximate problem:

\[
\min_{\omega} \quad F_0^t(\omega) = F(\omega) + 2\lambda(\beta^t)^T\omega
\]

where \( \beta^t = (1 - \rho^t(1) - \rho^t(0)^t) \), and \( \quad \beta^t = (1 - \rho^t(1), \beta^t) = (1 - \rho^t(1), -2\tau\omega^t) \), respectively, with \( \beta^t = 0 \) and \( \beta^t = 0 \). Here, \( \beta^t \) and \( \beta^t \) are given by:
\[
B_{j,k} = \sum_{l \in L} \frac{N_l}{BN} \sum_{n \in N_l(\omega^t)} Q_l(\omega^t, x_n) - y_{nl},
\]
\[
C_{i,j} = \sum_{l \in L} \frac{N_l}{BN} \sum_{n \in N_l(\omega^t)} Q_l(\omega^t, x_n) - y_{nl},
\]

respectively, with \( \beta^t = 0 \) and \( \beta^t = 0 \). Here, \( \beta^t \) and \( \beta^t \) are given by:
\[
B_{j,k} = \sum_{l \in L} \frac{N_l}{BN} \sum_{n \in N_l(\omega^t)} Q_l(\omega^t, x_n) - y_{nl},
\]
\[
C_{i,j} = \sum_{l \in L} \frac{N_l}{BN} \sum_{n \in N_l(\omega^t)} Q_l(\omega^t, x_n) - y_{nl},
\]

respectively, with \( \beta^t = 0 \) and \( \beta^t = 0 \).

Then, in Step 5 of Algorithm 1, the server only needs to compute \( \omega^t \) according to (16) and (17), respectively.

**B. Constrained Federated Optimization**

One constrained federated optimization formulation for the \( L \)-class classification problem is to minimize the \( \ell_2 \)-norm of the network parameters \( \|\omega\|_2^2 \) under a constraint on the cost function \( F(\omega) \) in (9):
\[
\min_{\omega} \quad F_0(\omega) \triangleq \|\omega\|_2^2
\]
\[
\text{s.t.} \quad F_1(\omega) \triangleq F(\omega) - U \leq 0,
\]
where \( U \) represents the limit on the cost. We can apply Algorithm 2 with \( f_0(\omega, (\omega^t, x_n)) \) given by (3) and \( f_m(\omega, (\omega^t, x_n)) \) given by (8) to solve the problem in (13). The convergence of Algorithm 2 is guaranteed by Theorem 2 as Assumption 1 and Assumption 2 are satisfied. Specifically, the server solves the following convex approximate problem:
\[
\min_{\omega, s} \quad \|\omega\|_2^2 + cs
\]
\[
\text{s.t.} \quad F(\omega) + A(\omega) \leq s, \quad s \geq 0,
\]
where \( F(\omega) \) is given by (11) with \( B^{(t)}(\omega), C^{(t)}(\omega) \) and \( A^{(t)} \) updated according to (12) and (13) and
\[
A^{(t)} = (1 - \rho^t(t^{-1} + \rho^t(0))),
\]
respectively, with \( A^{(t)} = 0 \) and \( A^{(t)} = 0 \). Here, \( B^{(t)}(\omega), C^{(t)}(\omega) \) and \( A^{(t)} \) are given by:
\[
\omega^{(t)}_{1,k} = \frac{\nu B^{(t)}_{1,k}}{2(1 + \nu)}, \quad j \in J, \quad k \in K,
\]
\[
\omega^{(t)}_{2,l,j} = \frac{\nu C^{(t)}_{l,j}}{2(1 + \nu)}, \quad l \in L, \quad j \in J,
\]
where
\[
\nu = \left\{ \begin{array}{ll}
\frac{1}{2} \left( \frac{b}{b + 4\tau(U - A^{(t)})} \right)^c, & b + 4\tau(U - A^{(t)}) > 0 \\
0, & b + 4\tau(U - A^{(t)}) \leq 0
\end{array} \right.
\]
(23)

Thus, in Step 5 of Algorithm 2, the server only needs to compute \( \omega^t \) according to (21) and (22), respectively.

**VI. NUMERICAL RESULTS**

In this section, we show the performance of Algorithm 1, Algorithm 2 and the SGD-based algorithms [13–15] in the application examples in Sections V using numerical experiments. We carry our experiments on Mnist data set. For the training model, we choose, \( N = 60000, I = 10, K = 784, J = 128, L = 10 \). For Algorithm 1 and Algorithm 2, we choose \( T = 100, \tau = 0.1, c = 10^3, \rho^t = \alpha^t/10^a \) and \( \gamma^t = \alpha^t/10^{a+0.05} \) with \( \alpha^t = 0.4, 0.6, 0.9, \alpha^t = 0.4, 0.9, 0.9, \alpha = 0.4, 0.3, 0.3 \) for batch sizes \( B = 1, 10, 100 \), respectively.
that the proposed algorithms with larger batch sizes converge versus the iteration index. From Fig. 1 and Fig. 2, we can see that the unconstrained federated optimization, Algorithm 1 converges based algorithms [3]–[5], let $E$ denote the number of local SGD updates, and the learning rate is set as $r = \frac{\hat{\alpha}}{t^\alpha}$, where $\hat{\alpha}$ and $\alpha$ are selected using grid search method. Note that all the results are given by the average over 100 runs.

![Training cost versus iteration index](image1)

**Fig. 1.** Training cost versus iteration index.

![Test accuracy versus iteration index](image2)

**Fig. 2.** Test accuracy versus iteration index.

![Model sparsity versus training cost](image3)

**Fig. 3.** Model sparsity versus training cost.

Fig. 1 and Fig. 2 illustrate the training cost and test accuracy versus the iteration index. From Fig. 1 and Fig. 2, we can see that the proposed algorithms with larger batch sizes converge faster. From Fig. 1(a) and Fig. 2(a), we can observe that for unconstrained federated optimization, Algorithm 1 converges faster than the SGD-based algorithm with $E = 1$ at the same batch size. In addition, Algorithm 1 with $B = 10(100)$ converges faster than the SGD-based algorithm with $B = 5(50)$ and $E = 2$, i.e., Algorithm 1 converges faster that the SGD-based algorithm when the two algorithms induce the same computation load for each client. Fig. 3(a) and Fig. 3(b) show the tradeoff curve between the model sparsity and training cost of each proposed algorithm. From Fig. 3(b) we see that with constrained sample-based federated optimization, one can set an explicit constraint on the training cost to effectively control the test accuracy. Furthermore, by comparing Fig. 3(a) and Fig. 3(b), we can see that Algorithm 2 can achieve a better tradeoff between the model sparsity and training cost than Algorithm 1. The main reason is that the underlying constrained sample-based federated optimization has a convex objective function and the chance for Algorithm 2 to converge to an optimal point is higher.

**VII. CONCLUSIONS**

In this paper, we proposed two privacy preserving algorithms for unconstrained and constrained sample-based federated optimization problems, respectively, using SSA techniques. We also showed that each algorithm can converge to a KKT point of the corresponding problem. It is worth noting that SSA has not been used for solving federated optimization, and federated optimization with nonconvex constraints has not been investigated. Numerical experiments showed that the proposed SSA-based algorithm for unconstrained sample-based federated optimization converges faster than the existing SGD-based algorithms, and the proposed SSA-based algorithm for constrained sample-based federated optimization can obtain a sparser model that satisfies an explicit constraint on the model cost.

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