NONDIFRACTING OPTICAL BEAMS: physical properties, experiments and applications

Zdeněk Bouchal
Department of Optics, Palacký University,
17. listopadu 50, 772 07 Olomouc,
Czech Republic

Abstract

The controversial term "nondiffracting beam" was introduced into optics by Durnin in 1987. Discussions related to that term revived interest in problems of the light diffraction and resulted in an appearance of the new research direction of the classical optics dealing with the localized transfer of electromagnetic energy. In the paper, the physical concept of the nondiffracting propagation is presented and the basic properties of the nondiffracting beams are reviewed. Attention is also focused to the experimental realization and to applications of the nondiffracting beams.

1 INTRODUCTION

Many phenomena observable in our everyday life indicate that light propagates rectilinearly. Rectilinear propagation is one of the most apparent properties of light. It serves as an argument that light is a stream of particles. However, some optical phenomena and experiments indicate that the law of rectilinear propagation of light does not hold. They can be satisfactorily explained only on the assumption that light is a wave. Historically, the diffraction effects are associated with violation of the rectilinear propagation of light. By Sommerfeld, diffraction is defined as any deviation of light from rectilinear propagation, not caused by reflection or refraction. The strong diffraction effects appear if the transverse dimensions of the beam of light are comparable to the wavelength. The diffraction phenomena are best appreciable for long waves such as sound or water waves. In optics, the diffraction effects are less apparent. They are responsible for the beam divergence in the free propagation and for penetration of light into the region of the geometric shadow. In the modern treatment, diffraction effects are not connected with light transmission through apertures and obstacles only. Diffraction is examined as a natural property of wavefield with the nonhomogeneous transverse intensity distribution. It commonly appears even if the beam is transversally unbounded. The Gaussian beam is the best known example.

In optics, the nondiffracting propagation of the beam-like fields can be obtained in convenient media such as waveguide or nonlinear materials. The beams then propagate as waveguide modes and spatial solitons, respectively. In 1987, the term nondiffracting beam appeared also in relation to the propagation in vacuo [1]. The nondiffracting beam was comprehended as the monochromatic optical field whose transverse intensity profile remains unchanged in free-space propagation. In the original Durnin's paper, the beams were examined as exact solutions to the homogeneous Helmholtz equation. They were obtained in the system of the cylindrical coordinates under restriction that their complex amplitude is separable as the product of the functions $R(r)$, $\Phi(\phi)$ and $Z(z)$ depending on the coordinates $r$, $\phi$ and $z$, respectively. The transverse amplitude profile of such beams can be described by the Bessel functions so that they are usually called Bessel beams. Later, the more general types of nondiffracting beams were introduced [2, 3, 4] and the properties by which they become different from the common laser beams were examined. Recently, the method enabling generation with the transverse intensity profile which can be predetermined and controlled has been proposed and examined [5]. The particular attention was focused on the analysis of the admissible amplitude profiles of the nondiffracting beams and on their wavefront properties. The nondiffracting beams originally analysed in the scalar approximation were generalized to the vectorial...
electromagnetic beams exactly fulfilling the Maxwell equations [6, 7, 8, 9]. Propagation invariance of the intensity profile of the nondiffracting beams was explained as a result of the convenient composition of the angular spectrum. It composes the plane waves whose propagation vectors are placed on the conical surface. Mathematically, such angular spectrum can be described by the Dirac delta function \( \delta(\nu - \nu_0) \), where \( \nu_0 \) is the single radial spatial frequency representing the basic beam parameter. The ideal nondiffracting beam then arises as an interference field produced by the coherent superposition of the plane waves whose relative phase differences remain unaltered in the free propagation. By that way, the diffraction effects can be overcome in free propagation of the source-free monochromatic wavefields or pulses. However, the ideal nondiffracting beams possessing the sharp \( \delta \)-like angular spectrum carry an infinite energy. That is a reason, why the diffraction cannot be overcome in real situations, and why the nondiffracting beams cannot be exactly realized. In experiments, only approximations known as the pseudo-nondiffracting beams can be obtained [10, 11]. An idea about properties of the realizable beams with the finite energy can be simply obtained if the ideal nondiffracting beam is bounded by the homogeneously transmitting aperture of finite dimensions or by the Gaussian aperture. The propagation invariance of the transverse intensity profile of the nondiffracting beam impinging on the aperture is lost and the beam behind the aperture propagates with the diffractive divergence. Regardless of that fact, there are important distinctions between propagation properties of the pseudo-nondiffracting and the conventional, for example Gaussian beams. They are usually demonstrated in numerical simulations. Recently, the distinct diffractive divergences of the pseudo-nondiffracting and the conventional beams have been explained and interpreted by means of the uncertainty relations and demonstrated experimentally [12].

Except of the fully eliminated diffractive divergence of the ideal nondiffracting beams and the reduced diffractive spread of the pseudo-nondiffracting beams their further peculiar properties useful for applications were explored. Attention was focused on the robustness of the beams manifested by their resistance against amplitude and phase distortions. It was shown that the ideal nondiffracting beam disturbed by a nontransparent obstacle is able to regenerate its intensity profile to the original form in the free propagation behind the obstacle [13]. In [14], the effect was described for both the nondiffracting and the pseudo-nondiffracting beams and verified by the simple experiments.

The coherent superposition of the nondiffracting modes resulting in the self-imaging effect was examined in [15, 16]. The effect appears if the angular wavenumbers of the nondiffracting modes are conveniently coupled. It represents the spatial analogy with the mode-locking realized in the temporal domain. Due to the interference of the modes, the transverse intensity profile of the beam reappears periodically at the planes of the constructive interference and vanishes at the planes of the destructive interference.

Recently, the self-imaging effect was used for periodical self-reconstruction of the coherent light field with an arbitrary predetermined amplitude profile. Theoretical description of the effect was proposed for both the monochromatic and the nonstationary wavefields [17, 18, 19] and the experimental verification was realized applying the special Fourier filter used in the 4-f optical system [20]. The controllable 3D spatial shaping of the coherent optical fields has also been proposed and examined. That method enables localization of the light energy into the small volume elements with the size comparable to the wavelength [21].

During last decade an increasing attention has been given to the wavefields possessing the line, spiral or combined wavefront dislocations. In optics, such fields are known as the optical vortices [22, 23]. Some types of nondiffracting beams can also belong to the class of optical vortices. The vortex beam is characterized by the topological charge and its phase singularities and the helical wavefront can be visualized by the interferometric methods. The wavefront helicity of the vortex beam is associated with the spiral flow of the electromagnetic energy. That property was successfully applied in experiments testing the transfer of the angular momentum of the electromagnetic field to the microparticles [24]. Recently, the effect of the self-regeneration of the nondiffracting vortex beam appearing after interaction with microparticles has been verified [25, 26].

The basic concept of the nondiffracting propagation has been developed for the fully coherent light. Recently, attention has been focused also to an interesting task to join the problems of the variable-coherence optics with the nondiffracting propagation of light beams. A general description of the partially coherent propagation-invariant fields including the partially coherent nondiffracting beams has been proposed in [27, 28]. Directionality of the partially-coherent Bessel-Gauss beams has been analysed on the
assumption that the beams are produced from a globally incoherent source [29]. Propagation properties of the partially coherent pseudo-nondiffracting beams obtained by the incoherent superposition of identical coherent beams whose propagation axes lie on a conical surface have been examined in [30]. Recently, the nondiffracting beams with the controlled spatial coherence have been introduced and analyzed [31].

The ideal nondiffracting beams can be obtained as a superposition of the plane waves whose radial angular frequencies are restricted by the Dirac delta function to the single value. In the geometrical interpretation, the propagation vectors of the plane wave components of the angular spectrum form the conical surface. In optics, several experiments have been proposed to produce a good approximation of the required composition of the angular spectrum. The original Durnin’s experimental demonstration of the zero-order Bessel beam utilized an annular slit placed at the focal plane of the lens [32]. More efficient methods of generating the required conic wavefront based on the use of the computer-generated holograms [33], the axicon [34, 35] or the programmable spatial light modulators [36] were also suggested. Experimental realization of the nondiffracting Bessel beam due to the spherical aberration of the simple lens [37] and by means of the two-element refracting system [38] has been successfully performed. Experimental methods applicable to generation of the pseudo-nondiffracting beams were reviewed in [39]. Their applications have been proposed in the field of acoustics, metrology and nonlinear optics [40, 41, 42]. Properties of the nondiffracting beams are perspective for the design of the electron accelerators [43] and the optical tweezers [44].

2 COHERENT NONDIFFRACTING BEAMS

2.1 Concepts of nondiffracting propagation

The ideal monochromatic spatially coherent nondiffracting beam propagating along the z-axis is comprehended as the mode-like field whose complex amplitude can be written in the form

\[ U(x, y, z, t) = u(x, y) \exp[i(\omega t - \beta z)], \]

where \( u \) describes the transverse amplitude profile and \( \omega \) and \( \beta \) are the angular frequency and the angular wavenumber, respectively. The slowly varying amplitude \( u \) is then independent of the \( z \)-coordinate so that the intensity of the beam \( I = |U|^2 \) is propagation invariant. The fields (1) are known as waveguide modes or spatial solitons propagating in optical linear and nonlinear materials but Durnin’s original work [1] has excited interest also in their free-space propagation. In that case, the complex amplitudes \( U \) must fulfill the homogeneous wave equation. The temporally independent amplitude

\[ a(x, y, z) = u(x, y) \exp(-i\beta z) \]

then fulfills the Helmholtz equation

\[ (\nabla^2 + k^2) a(x, y, z) = 0, \]

where \( k = \omega/c \) and \( c \) is the light velocity in vacuo. The mathematical description of the ideal monochromatic nondiffracting beam can be based on the differential or on the integral formalism.

2.1.1 Separable solutions to the Helmholtz equation

The spatial evolution of the complex amplitude \( a \) can be described by the transverse and the longitudinal parts depending only on the transverse coordinates \( (x, y) \) and on the \( z \)-coordinate, respectively. Though the homogeneous (source-free) Helmholtz equation can be separated in 11 coordinate systems, the required separability into the transverse and the longitudinal parts is possible only in Cartesian, circular cylindrical, parabolic cylindrical, and elliptical cylindrical coordinates. The particular attention has been focused on the circular cylindrical and the elliptical cylindrical coordinates for which the transverse amplitude profile \( u \) can be expressed by the known functions.
Circular cylindrical coordinates

The circular cylindrical coordinates \((r, \varphi, z)\) are related to the Cartesian coordinates \((x, y, z)\) by \(x = r \cos \varphi, y = r \sin \varphi\) and \(z = z\) where \(r \in (0, \infty)\) and \(\varphi \in (-\pi, \pi]\). The solutions of the Helmholtz equation (3) then can be found only under restricting assumption that the amplitude \(u\) can be expressed as a product of the functions \(R\) and \(\Phi\) depending on the radial coordinates \(r\) and \(\varphi\), respectively. In that case, the complex amplitudes \(a\) are assumed to be of the form

\[
a(r, \varphi, z) = R(r) \Phi(\varphi) \exp(-i\beta z).
\]

The function \(\Phi\) describing dependence of the transverse amplitude profile of the beam must be periodical. Usually we assume that it is of the form

\[
\Phi(\varphi) = \exp(im\varphi), \quad m = 0, 1, 2, \ldots.
\]

Substituting (4) and (5) into the Helmholtz equation (3) we obtain the differential equation for the radial function \(R\). It is known as the Bessel equation

\[
\frac{d^2 R(r)}{dr^2} + \frac{1}{r} \frac{dR(r)}{dr} + \alpha^2 R(r) \left(1 - \frac{m^2}{\alpha^2 r^2}\right) = 0,
\]

where

\[
\alpha^2 = k^2 - \beta^2.
\]

Its general solution can be given as a linear combination of the \(m\)-th order Bessel functions of the first kind \(J_m\) and the \(m\)-th order Neumann functions \(N_m\) [45],

\[
R_m(r) = \mu J_m(\alpha r) + \nu N_m(\alpha r),
\]

where \(\mu\) and \(\nu\) are the weighing coefficients. Usually, the Bessel functions of the first kind are considered as the only physical solutions. The reason why the Neumann functions are considered as the unphysical solutions is that they possess singularities at the zero point if they are used separately. However, as has been shown in [4], in combination with the Bessel functions of the first kind, the Neumann functions have a physical meaning. Two linear combinations yielding the \(m\)-th order Hankel functions are of particular importance. The radial functions obtained as the solutions of the Bessel equation can be denoted as \(R^j \equiv H^j_m, j = 1, 2\), where the Hankel functions \(H^j_m\) are given as

\[
\begin{align*}
H^1_m &= J_m(\alpha r) + iN_m(\alpha r), \\
H^2_m &= J_m(\alpha r) - iN_m(\alpha r).
\end{align*}
\]

They have simple interpretation for the zero-order Hankel function [4]. The real amplitude of the rotationally symmetrical beam can be defined as

\[
\overline{U}(r, z) = \frac{1}{2} \left[ U(r, z) + U^*(r, z) \right],
\]

where

\[
U(r, z) = \frac{1}{2} \left[ H^1_0(\alpha r) + H^2_0(\alpha r) \right] \exp[i(\omega t - \beta z)].
\]

Applying (8) and (9), the amplitude \(\overline{U}\) can be rewritten to the form

\[
\overline{U}(r, z) = \frac{1}{2} \left[ \overline{U}^1_0(r, z) + \overline{U}^2_0(r, z) \right],
\]

where

\[
\begin{align*}
\overline{U}^1_0(r, z) &= J_0(\alpha r) \cos(\omega t - \beta z) + N_0(\alpha r) \sin(\omega t - \beta z), \\
\overline{U}^2_0(r, z) &= J_0(\alpha r) \cos(\omega t - \beta z) - N_0(\alpha r) \sin(\omega t - \beta z).
\end{align*}
\]

The real amplitude of the beam \(\overline{U}\) is now expressed as a superposition of two travelling waves given by the Hankel functions. The amplitude \(\overline{U}^1_0\) describes the rotationally symmetric outgoing wave travelling away from the axis (Fig. 1a) while the amplitude \(\overline{U}^2_0\) represents the incoming wave travelling towards the axis (Fig. 1b). In Fig. 1 the waves are illustrated for the time \(t\) such that \(\exp(i\omega t) = 1\). The singularity of the
included Neumann function can be interpreted as arising from the collapse of the incoming cylindrical wave onto the beam axis which serves, simultaneously, as the source from which the outgoing wave emanates. To satisfy boundary conditions at \( r = 0 \), each cylindrical wave must be the complex conjugate of the other. In that case, the imaginary parts of the Hankel functions composing the Neumann functions cancel out so that the total field is the standing wave represented by the Bessel function (Fig. 1c).

Elliptical cylindrical coordinates

The system of elliptical cylindrical coordinates \((\zeta, \eta, z)\) can be defined as
\[
x = h \cosh \zeta \cos \eta, \\
y = h \sinh \zeta \sin \eta \\
z = z,
\]
where \( \zeta \in (-\infty, \infty) \) and \( \eta \in (-\pi, 2\pi) \), and \( 2h \) represents the distance between the foci of an ellipse placed at the plane \((x, y)\) of the Cartesian coordinate system [46]. Taking into account the separation of the complex amplitude (2) and applying the elliptical cylindrical coordinates the Helmholtz equation (3) can be rewritten to the form [51]
\[
\frac{\partial^2 u(\zeta, \eta)}{\partial \zeta^2} + \frac{\partial^2 u(\zeta, \eta)}{\partial \eta^2} + \frac{\alpha^2 h^2}{2} (\cosh 2\zeta - \cos 2\eta) u(\zeta, \eta) = 0. 
\tag{15}
\]

On the assumption that the complex amplitude \( u \) can be written as the product of the functions depending only on the variables \( \zeta \) and \( \eta \), respectively, the Helmholtz equation (15) can be split into the Mathieu differential equations. Application of their solutions to the description of the optical beams has been proposed in [47]. In [48] the propagation properties of the zero-order Mathieu beam have been examined. Its complex amplitude can be written in the form
\[
U(\zeta, \eta, z, t; q) = U_0 c_0(\zeta; q) c_0(\eta; q) \exp[i(\omega t - \beta z)], 
\tag{16}
\]
where the parameter \( q = \alpha^2 h^2/4 \) carries information about the radial spatial frequency \( \alpha/k \) influencing the beam transverse size and the ellipticity of the coordinate system \( h \). The beam possesses the highly localized intensity distribution along the \( x \)-direction and the sharply peaked quasi-periodic structure along the \( y \)-direction. The simple experimental set-up providing a good approximation of the zero-order Mathieu beam was proposed in [49].

The separable solutions of the Helmholtz equation obtained applying the circular cylindrical and the elliptical cylindrical coordinates represent only special examples of the nondiffracting fields whose transverse intensity profiles can be described by the known functions. The more general nondiffracting fields can be effectively examined applying the integral formalism.

2.1.2 Integral form of the nondiffracting beam

In the integral form, the temporally independent amplitude of the general nondiffracting beam can be conveniently expressed applying the circular cylindrical coordinates \( r \equiv (r, \varphi, z) \)
\[
a(r) = \frac{ik}{2\pi} \int_{-\pi}^{\pi} A(\psi) f(r, \psi) d\psi, 
\tag{17}
\]
where
\[
f(r, \psi) = \exp(-i\beta z) \exp[i\alpha r \cos(\psi - \varphi)],
\]
and \( A \) denotes an arbitrary periodical function. The parameter \( \alpha \) can be interpreted by means of the angular spectrum. The angular spectrum \( F \) is a function of the angular frequencies \( \nu_x \) and \( \nu_y \) and can be obtained as the two-dimensional Fourier transformation of the amplitude \( a \). Applying the radial angular frequency \( \nu \) defined as \( \nu_x = \nu \cos \psi \) and \( \nu_y = \nu \sin \psi \) it can be expressed as
\[
F(\nu, \psi) = A(\psi) \delta(\nu - \nu_0), 
\tag{18}
\]
where
\[
\nu_0 = \frac{\alpha}{2\pi}. 
\tag{19}
\]
The peculiar propagation properties of the ideal nondiffracting beams appear as a consequence of the composition of the angular spectrum. It contains only the single radial frequency $\nu_0$ so that the relative phases of the plane wave components remain unchanged under propagation. In geometrical interpretation the angular spectrum represents a coherent superposition of plane waves whose propagation vectors cover the conical surface with the vertex angle $2\theta_0 = 2\arcsin(\lambda\nu_0)$, where $\lambda$ denotes the wavelength. The amplitudes and relative phases of the superposed plane waves can be arbitrary. They are described by the function $A$. Due to that arbitrariness an infinite number of the nondiffracting beams with different transverse intensity profiles can be obtained. The parameters $\alpha$ and $\beta$ of the nondiffracting beam have a simple geometrical interpretation. They represent projections of the propagation vectors of the plane wave components of the angular spectrum to the transverse plane $(x,y)$ and to the direction of propagation coinciding with the $z-$axis, respectively. They can be expressed as $\alpha = k\sin\theta_0$ and $\beta = k\cos\theta_0$.

2.2 Types of coherent nondiffracting beams

The coherent nondiffracting field can be comprehended as the interference field produced by the interference of plane waves whose propagation vectors create the conical surface. Due to the interference the total field can possess an appreciable beam-like intensity peak so that it is often called the nondiffracting beam. The condition (18) laid on the composition of its angular spectrum represents necessary and sufficient condition of the nondiffracting propagation. The various intensity profiles of the nondiffracting fields can be obtained by manipulations of the amplitudes and phases of the plane wave components of the angular spectrum. Because an arbitrary azimuthal modulation of the angular spectrum is admitted there exists an infinite number of nondiffracting fields. Here, only best known examples are reviewed.

2.2.1 Bessel beams

In the special case when the amplitudes of the plane wave components of the angular spectrum are constant and their phases are azimuthally modulated by $A(\psi) = A_0 \exp(im\psi)$ the integral representation (17) results in the Bessel nondiffracting beams. Their complex amplitude can be written in the form

$$U(r,\varphi,z,t) = A_0 J_m(\alpha r) \exp[i(\omega t + m\varphi - \beta z)],$$

where $J_m$ is the $m-$th order Bessel function of the first kind. If the plane waves are coherently superposed without azimuthal phase modulation ($m = 0$), we obtain the bright beam-like field with the propagation-invariant intensity profile given by

$$I(r,z) = |A_0|^2 J_0^2(\alpha r).$$

The radius of the central intensity spot $r_0$ is given by the first zero-point of the Bessel function $J_0$ and can be written as $r_0 = 2.4/\alpha$. In geometrical interpretation that relation means that the increase of the vertex angle of the conical surface formed by the propagation vectors of the interfering plane waves results in the reduction of the size of the intensity spot of the produced beam. The transverse intensity profile of the zero-order Bessel beam is illustrated in Fig. 2a.

2.2.2 Nondiffracting vortex beams

During last decade an increasing attention has been focused to the wavefields possessing the line, spiral or combined wavefront dislocations. Some types of nondiffracting beams belong to the class of fields with the spiral wavefront dislocations. In optics such fields are known as optical vortices. If the slowly varying complex amplitude $u$ of the nondiffracting beam (1) is rewritten by means of the real amplitude $u_0$ and the phase $\Phi$ in the form

$$u(x,y) = u_0(x,y) \exp[i\Phi(x,y)],$$

(22)
then the point where the phase dislocation appears can be identified by the nonzero value of the integral

$$\oint_L \nabla \Phi \cdot dL,$$

(23)

where the integration is performed along the closed line surrounding the examined point. The integral (23) can result in the values $2\pi m$, $m = 1, 2, \ldots$, where $m$ represents the topological charge of the vortex field. At the singularity point the validity of the relations for real and imaginary parts $\Re(u) = 0$ and $\Im(u) = 0$ can be verified so that we speak about the dark optical vortices. The simplest type of the nondiffracting vortex beam is the Bessel beam (20) represented by the first- or higher-order Bessel function of the first kind. The intensity profile of the first-order Bessel beam is illustrated in Fig. 2b. The helical wavefronts of the Bessel vortex beams are illustrated in Fig. 3a and 3b for the topological charges $m = 1$ and $m = 2$, respectively. The wavefront dislocations of the optical vortices can be also identified experimentally applying the interferometric methods. Interference of the optical vortex possessing the helical wavefront with the spherical wave results in the typical spiral patterns. The numerical simulation for the optical vortex with the topological charges $m = 1$ and $m = 2$ is illustrated in Fig. 4a and 4c. The pattern obtained by interference of the optical vortex with the plane wave has the fork-like form. It is illustrated in Fig. 4b and 4d for the topological charges of the vortex beam $m = 1$ and $m = 2$, respectively.

### 2.2.3 Mathieu beams

The Bessel beams are obtained if the angular spectrum (18) consists of the plane waves whose phases are conveniently modulated and the real amplitudes remain unmodulated. On the contrary, a good approximation to the zero-order Mathieu beams can be obtained if the relative phases of the plane wave components are constant and their real amplitudes are modified by [48]

$$A(\psi) = \exp \left[ - \left( \frac{\nu_0 \cos \psi}{w_0} \right)^2 \right],$$

(24)

where $\nu_0$ is given by (19) and $w_0$ denotes the bandwidth of the Gaussian profile at the plane of spatial frequencies. The change of the parameter $w_0$ results in the change of the form of the transverse intensity profile of the nondiffracting Mathieu beam. In Fig. 5a and 5b the intensity spots are illustrated for $w_0 = 4\nu_0$ and $w_0 = 2\nu_0$, respectively.

### 2.2.4 Caleidoscopic nondiffracting patterns

The function $A$ used in (18) describes the amplitude and phase modulation of the angular spectrum of the nondiffracting beam. Usually we assume that it is a continuous function of the azimuthal angle $\psi$ so that the nondiffracting beam is produced as the coherent superposition of the plane waves whose propagation vectors continuously cover the conical surface. In a special case the nondiffracting field can be obtained as a discrete superposition of $N$ plane waves. In that case the modification of the angular spectrum (18) can be expressed applying the Dirac delta-function

$$A(\psi) = \sum_{j=1}^{N} A_0(\psi) \delta(\psi - \psi_j),$$

(25)

where $N$ is an integer. In that case the integration used in (17) is replaced by the summation and the complex amplitude of the nondiffracting field can be rewritten as

$$a(r, \varphi, z) = \frac{ik}{2\pi} \exp(-i\beta z) \sum_{j=1}^{N} A_0(\psi_j) \exp[iar \cos(\psi_j - \varphi)].$$

(26)
The numerical simulation of the transverse intensity profile of the nondiffracting beam is illustrated in Fig. 6. The azimuthal angles \( \psi_j \) related to the interfering plane waves are chosen as

\[
\psi_j = (j - 1) \Delta \psi,
\]

where \( \Delta \psi = 2\pi/N \) and \( A_0 \) is assumed to be a constant. The intensity spots illustrated in Fig. 6a-d are obtained for \( N = 5, 10, 15 \) and 30, respectively. If the number of interfering plane waves is even \( N = 2N' \) and the angular spectrum is modified by the function possessing property \( A_0(\psi) = A_0(\psi + \pi) \) the complex amplitude of the nondiffracting field (26) can be alternatively expressed as a superposition of the azimuthally rotating cosine gratings

\[
a(r, \varphi, z) = \frac{ik}{\pi} \exp(-i\beta z) \sum_{j=1}^{N'} A_0(\psi_j) \cos[\alpha r \cos(\psi_j) + \alpha y \sin(\psi_j)].
\]

The nondiffracting pattern Fig.6b obtained due to the interference of \( N = 10 \) plane waves possessing the same amplitude and phase can also be created as a coherent superposition of \( N' = 5 \) cosine gratings rotating in the azimuthal direction with the angular increment \( \Delta \psi = \pi/N' \). The cosine grating components and the transverse intensity pattern obtained by their superposition are illustrated in Fig. 7. The nondiffracting field described by the integral (17) can also be interpreted as the superposition of the cosine gratings. In that case the gratings are continuously rotating along the azimuthal angle [12].

3 PARTIALLY-COHERENT NONDIFFRACTING BEAMS

In the case of the fully coherent nondiffracting beam the azimuthal amplitude and phase modulation of the angular spectrum is described by the deterministic function \( A \). As has been shown its changes can be used for the formation of the transverse intensity profile of the beam. The partially-coherent nondiffracting beam is still related to the angular spectrum (18) but the modulation function \( A \) is given as a product of the deterministic amplitude \( A_d \) and the random amplitude \( A_r \),

\[
A(\psi) = A_d(\psi) A_r(\psi).
\]

In that case the partially-coherent nondiffracting beam is described by the second-order cross-spectral density function

\[
W(\mathbf{r}_1, \mathbf{r}_2) = \langle a^*(\mathbf{r}_1) a(\mathbf{r}_2) \rangle, \tag{30}
\]

where \( \mathbf{r}_j \equiv (x_j, y_j, z_j) \) is the position vector. Applying (17), (18) and (29) we obtain

\[
W(\mathbf{r}_1, \mathbf{r}_2) = \left( \frac{k}{2\pi} \right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \gamma(\psi_1, \psi_2) |A_r(\psi_1)| |A_r(\psi_2)| A_d(\psi_1) A_d(\psi_2) \times f^*(\mathbf{r}_1, \psi_1) f(\mathbf{r}_2, \psi_2) d\psi_1 d\psi_2, \tag{31}
\]

where \( \gamma \) is the degree of angular correlation defined as

\[
\gamma(\psi_1, \psi_2) = \frac{\langle A_d^*(\psi_1) A_r(\psi_2) \rangle}{|A_r(\psi_1)| |A_r(\psi_2)|}, \tag{32}
\]

and \( \langle \rangle \) means ensemble average over field realizations. It describes the mutual correlation of the plane wave components of the angular spectrum propagating with the propagation vectors \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \). The propagation vectors create the conical surface with the vertex angle \( \theta_0 \) so that their longitudinal \( z \)-components are constant. The transverse \( x \)- and \( y \)-components depend on the azimuthal angle \( \psi \). The propagation vector \( \mathbf{k}_j \) then possesses components \((|\mathbf{k}_j| \sin \theta_0 \cos \psi_j, |\mathbf{k}_j| \sin \theta_0 \sin \psi_j, |\mathbf{k}_j| \cos \theta_0)\). The simplest type of the partially-coherent nondiffracting field is obtained if the plane wave components of the angular spectrum are superposed incoherently so that \( \gamma \) is given by the Dirac delta function

\[
\gamma(\psi_1, \psi_2) = \delta(\psi_1 - \psi_2). \tag{33}
\]
If the deterministic function $A_d$ modulates only the phase of the angular spectrum the cross-spectral density function (31) becomes

$$W(r_1, r_2) = C \exp[-i\beta(z_2 - z_1)]J_0(\alpha \rho),$$

(34)

where

$$\rho = [(x_1 - x_2)^2 + (y_1 - y_2)^2]^{1/2},$$

and $C$ denotes the constant amplitude. The cross-spectral density function (34) describes the Bessel correlated field with the constant intensity and the sharply peaked transverse-spatial-correlation profile [27]. The more general partially-coherent nondiffracting fields are examined in [31]. They are obtained on the assumption that the degree of angular correlation $\gamma$ can be controlled. The controlled change of the mutual correlation of the plane wave components of the angular spectrum offers a possibility to change the transverse intensity profile of the beam to the desired form and to control the coherence properties of the generated nondiffracting beam. The controlled change of $\gamma$ can be realized experimentally in the illumination chain using the pseudo-thermal spatially incoherent source such as the Gaussian Shell-model source [50]. Analysis of that experimental set-up is presented in [31]. The change of the transverse intensity profile of the partially-coherent nondiffracting beam caused by the change of the degree of angular correlation is illustrated in Fig. 8. The intensity spot in Fig. 8a is obtained for the coherent superposition of the plane wave components ($\gamma = 1$). The intensity patterns in Figs. 8b, 8c and 8d are obtained by the partially-coherent superposition of the plane waves. They are related to the functions $\gamma$ illustrated in Fig. 9 by the curves denoted as ‘o o o’, ‘ooo’, and ‘−’, respectively. The coherent beam with the intensity profile in Fig. 8a represents the nondiffracting vortex of the Bessel type possessing the topological charge $m = 2$. The beam is dark (axial intensity is equal to zero) and its wavefront has the helical form. In the interference experiment using the spherical reference wave the phase singularity is visualized by the spiral interference pattern in Fig. 10a. Due to the change of the spatial coherence the beam becomes bright (axial intensity is nonzero) and the phase dislocation is removed. This is obvious from Figs. 10b-d illustrating the vanishing spiral character of the interference pattern.

### 4 SPATIAL SHAPING OF NONDIFFRACTING FIELDS

The condition of nondiffracting beam propagation (18) puts restrictions only on the propagation directions of the plane wave components of the angular spectrum. Their amplitudes and phases are given by $A(\psi)$ and can be arbitrary. The freedom of the amplitude and the phase modulation of the angular spectrum can be applied to the control of the spatial shaping of the produced nondiffracting patterns. In [5, 21], the azimuthal modulation of the form $A(\psi) = t(\psi)a(\psi)$ is assumed. The function $t$ is used to control the shape of the created nondiffracting spot and $a$ is applied to move it to the required position at the transverse plane. Furthermore, the size of the nondiffracting spot can be changed by the parameter $\alpha$ defined by (19). The shift of the centre of the nondiffracting spot to the point with coordinates $\Delta x$ and $\Delta y$ is achieved if the function $a$ is of the form

$$a(\psi) = W(\Delta x, \Delta y) \exp[i\alpha(\Delta x \cos \psi + \Delta y \sin \psi)],$$

(35)

where $W$ is the weighing function representing the amplitude of the nondiffracting spot at the position $(\Delta x, \Delta y)$. The nondiffracting pattern whose transverse amplitude profile is composed of the continuously shifted nondiffracting spots is then described by the convolution

$$U(r) = \exp[i(\omega t - \beta z)] \int_{-\infty}^{+\infty} W(\Delta x, \Delta y)T(x - \Delta x, y - \Delta y)d\Delta xd\Delta y,$$

(36)

where

$$T(x - \Delta x, y - \Delta y) = \int_{0}^{2\pi} t(\psi) \exp\{-i\alpha[(x - \Delta x) \cos \psi + (y - \Delta y) \sin \psi]\}d\psi.$$  

(37)

By the control of amplitudes, shapes, sizes and shifts of the nondiffracting spots we can create the total field whose transverse amplitude profile approximates the form predetermined by the function $W$. If
the nondiffracting spots are superposed coherently the similarity between the predetermined and the realized amplitude profiles is degraded by the interference effects. It can be significantly improved if the nondiffracting spots are mutually incoherent. In [5], the simple experimental implementation of the controllable spatial shaping of the nondiffracting fields was proposed and realized for both the coherent and incoherent light. The obtained results are shown in Figs. 11 and 12. In Fig. 11a, the predicted transverse intensity profiles of the generated nondiffracting beams obtained with incoherent and coherent light are shown in Figs. 11b and 11c, respectively. In both cases the generated fields are nondiffracting so that the intensity profiles do not change under propagation. The intensity profiles illustrated in Figs. 11d and 11e are again obtained for incoherent light but the parameter $\alpha$ is changed in such a way that the size of the nondiffracting spots is reduced so that their resolution is improved. As is obvious, the similarity between the required and generated intensity profiles is much more for incoherent light than for coherent one. In Fig. 12 the similar situation is illustrated for the required profile resembling the initials of Palacký University. In that case, the transverse intensity profile is obtained as a continuous superposition of the nondiffracting spots.

5 VECTORIAL NONDIFFRACTING BEAMS

The detailed study of the nondiffracting beams including analysis of the polarization states and the flow of the electromagnetic energy requires the vectorial electromagnetic description. The problem is to find the monochromatic electromagnetic field whose vector complex amplitudes exactly fulfill the Maxwell equations and can be written in the form

\[
E(x, y, z, t) = e(x, y) \exp[i(\omega t - \beta z)],
\]

\[
H(x, y, z, t) = h(x, y) \exp[i(\omega t - \beta z)],
\]

where $e$ and $h$ are the propagation invariant amplitudes. In [43] the special type of the vectorial nondiffracting beams was introduced on the assumption that the longitudinal component of the electric field resembles the zero-order Bessel function of the first kind. The model resulted in a radially polarized beam whose radial electric field corresponds to the first-order Bessel function of the first kind. In [8] the representative theorem for the Helmholtz equation was applied to derive $E$ and $H$ from the scalar complex amplitudes $a_m$ exactly fulfilling the scalar Helmholtz equation. The vector complex amplitudes then can be written as

\[
E = -\sum_m (p_m P_m + q_m Q_m) \exp(i\omega t),
\]

\[
H = i\xi \sum_m (p_m Q_m + q_m P_m) \exp(i\omega t),
\]

where

\[
P_m = -\left( s \times \nabla a_m \right),
\]

\[
Q_m = \frac{1}{k} \nabla \times P_m,
\]

and $1/\xi = \sqrt{\mu_0/\epsilon_0}$ is the impedance of the vacuo. It can be shown that the vector amplitudes (40) and (41) exactly fulfill the Maxwell equations and can be expressed in the form (38) and (39). They are constructed from the base of the scalar nondiffracting beams. The complex amplitudes $a_m$ represent the independent scalar nondiffracting solutions to the Helmholtz equation obtained for the same angular wavenumber $\beta$. In the vectorial solution they are applied with the weighing coefficients $p_m$ and $q_m$. The used symbols $\epsilon$, $\mu$ and $s$ denote the permittivity, the permeability and an arbitrary constant vector, respectively. The nondiffracting fields related to the single summation indices represent the nondiffracting modes. The vector nondiffracting fields can then be constructed as the one- or multi-mode fields. Classification and analysis of their properties are presented in [52]. The special types of the vector nondiffracting beams are obtained by an appropriate choice of the weighing coefficients $p_m$ and $q_m$ and the constant vector $s$. For example, the transversal electric (TE) field follows from (40) and (41) used with $p_m \neq 0$, $q_m = 0$ and $s \equiv (0, 0, s_z)$. The transversal magnetic (TM) field is obtained with $p_m = 0$, $q_m \neq 0$ and $s \equiv (0, 0, s_z)$. If
the complex amplitudes of the scalar nondiffracting beams $a_m$ are represented by the Bessel functions the vector nondiffracting beams possessing the azimuthal or the radial polarization can be obtained. In that case, the vector complex amplitudes of the electric and the magnetic fields can be conveniently expressed by the radial, azimuthal and longitudinal components $\mathbf{E} \equiv (E_r, E_\varphi, E_z)$ and $\mathbf{H} \equiv (H_r, H_\varphi, H_z)$. The nondiffracting TE beam possessing the azimuthal polarization of the electric field can be described by the vectors $\mathbf{E} \equiv (0, E_\varphi, 0)$ and $\mathbf{H} \equiv (H_r, 0, H_z)$. Their components can be written as [8]

\begin{align}
E_r &= 0, \\
E_\varphi &= -p_0 \alpha J_1(\alpha r) \exp[i(\omega t - \beta z)], \\
E_z &= 0, \\
H_r &= -p_0 \alpha \xi \beta J_1(\alpha r) \exp[i(\omega t - \beta z)], \\
H_\varphi &= 0, \\
H_z &= ip_0 \xi \alpha J_0(\alpha r) \exp[i(\omega t - \beta z)].
\end{align}

In the case of the TM beam the magnetic field possesses the azimuthal polarization while the electric field is radially polarized. The components of the field vectors can be expressed as

\begin{align}
E_r &= -iq_0 \alpha \beta J_1(\alpha r) \exp[i(\omega t - \beta z)], \\
E_\varphi &= 0, \\
E_z &= -q_0 \alpha \xi^2 J_0(\alpha r) \exp[i(\omega t - \beta z)], \\
H_r &= 0, \\
H_\varphi &= iq_0 \xi \alpha J_1(\alpha r) \exp[i(\omega t - \beta z)], \\
H_z &= 0.
\end{align}

The azimuthally and the radially polarized fields can be considered as the linearly polarized fields with the spatial change of the direction of oscillations. At each point of the transverse plane the field vectors are linearly polarized. For the radially polarized field the direction of oscillations is given by the line connecting that point with the center of the beam while the azimuthally polarized field oscillates along the direction orthogonal to that line. The TE nondiffracting beam with the azimuthal polarization of the electric field and the radially polarized magnetic field is illustrated in Fig. 13. In Fig. 13a the magnitude and the direction of the transverse component of the electric intensity $\mathbf{E}_\perp \equiv (E_r, E_\varphi)$ are illustrated by the arrows at the separate points of the transverse plane. The radially polarized magnetic field is similarly illustrated in Fig. 13b.

6 PROPERTIES OF NONDIFFRACTING BEAMS

The nondiffracting beams exhibit interesting properties by which they differ from the common types of beams, for example Gaussian beams. Recently, an increasing attention was devoted to those properties because they offer many potential applications and their explanation can be important for better understanding of the origin of the diffraction phenomena and of the nature of the electromagnetic field. Some of them are briefly discussed.

6.1 Beam robustness

An important property of the nondiffracting beam is its resistance against amplitude and phase distortions. The transverse intensity profile of the nondiffracting beam disturbed by the nontransparent obstacle regenerates during free-propagation behind that obstacle. The healing effect causes that the initial transverse intensity profile is restored certain distance behind the obstacle. The effect was explained theoretically applying Babinet’s principle and verified experimentally [14]. The results of the experiment
are presented in Fig. 14. The nondiffracting beam with the transverse intensity profile approximately corresponding to the first-order Bessel function $J_1$ is disturbed by the nontransparent rectangular obstacle. During free-propagation of the beam behind the obstacle its transverse intensity profile regenerates. As is obvious from Fig. 14, the initial Bessel-like profile is restored with a very good fidelity.

6.2 Beam energetics

In the framework of the Maxwell theory the energetics of the optical beams is characterized by the density of the flow of the electromagnetic energy denoted as the Poynting vector $\mathbf{S}$ and by the volume density of the electromagnetic energy $w$. Both quantities are spatially and temporally dependent and are mutually coupled in the energy conservation law. In nonconducting media it can be expressed by

$$\nabla \cdot \mathbf{S} + \frac{\partial w}{\partial t} = 0. \tag{55}$$

In real situations the temporally averaged quantities $<\mathbf{S}>$ and $<w>$ are of the particular importance. For the monochromatic nondiffracting electromagnetic beam (38) and (39) they can be written as

$$<\mathbf{S}> = \mathbf{e}^* \times \mathbf{h} + \mathbf{e} \times \mathbf{h}^*, \tag{56}$$
$$<w> = \epsilon (\mathbf{e}^* \cdot \mathbf{e}) + \mu (\mathbf{h}^* \cdot \mathbf{h}). \tag{57}$$

As the vector amplitudes $\mathbf{e}$ and $\mathbf{h}$ of the nondiffracting beams are independent of the $z-$coordinate, the energy conservation law can be simplified to the form

$$\nabla \cdot <\mathbf{S}_\perp> = 0, \tag{58}$$

where $<\mathbf{S}_\perp>$ denotes the transverse part of the Poynting vector. The transverse and the longitudinal components of the Poynting vector fulfil relations $<\mathbf{S}_\perp> \cdot \mathbf{z} = 0$ and $\mathbf{S}_\parallel \times \mathbf{z} = 0$, where $\mathbf{z}$ is the unit vector coinciding with the $z$-axis. The Poynting vector providing the density of the electromagnetic energy flow can be obtained as the vector sum $<\mathbf{S}> = <\mathbf{S}_\perp> + <\mathbf{S}_\parallel>$. The energy conservation law formulated for the nondiffracting beams (58) requires the zero divergence of the transverse component of the Poynting vector but the transverse energy flow itself can be nonzero. The fact that the monochromatic nondiffracting beam whose transverse intensity profile remains unchanged under propagation can exhibit the nonzero energy flow orthogonal to the direction of propagation is surprising. It excited interest in the structure of the transverse component of the Poynting vector. Applying the system of the circular cylindrical coordinates it can be decomposed into the radial and azimuthal components $<\mathbf{S}_\perp> = <\mathbf{S}_r> + <\mathbf{S}_\phi>$. It can be shown that for the one-mode nondiffracting beam the radial flow must be equal to zero and the transverse energy flow can possess only azimuthal component. Due to the superposition with the longitudinal component of the Poynting vector the total flow has the helical character. In a general case of the multi-mode nondiffracting beam the radial energy flow can be nonzero due to the interference of the modes. As is obvious from (58) the points of the transverse plane where the transverse energy flow has the constant magnitude lye on the closed lines. An exhaustive analysis of the energetic properties of the vector nondiffracting beams is presented in [52].

6.3 Orbital angular momentum

The nondiffracting vortex beams carry the angular momentum which can be transferred to atoms and microscopical particles. The mechanical consequence of that interaction is rotation of the particles. The angular momentum has two components - the orbital angular momentum and the spin. The spin depends on the polarization state of the beam and is equal to zero for the linear polarization. If the beam interacts with particles, the spin causes rotation of the particles around their own axis. The orbital angular momentum is a consequence of the spiral flow of the electromagnetic energy and is typical for beams with the helical wavefront. They are known as the vortex beams. Under interaction with the particle, the orbital angular momentum causes its rotation around the center of the vortex nested in the host beam.
The angular momentum \( J \) can be written as a vectorial product of the position vector \( \mathbf{r} \) and the linear momentum \( \mathbf{p} \),

\[
J = \mathbf{r} \times \mathbf{p}.
\]  
(59)

If the beam propagates in vacuo with velocity \( c \), \( \mathbf{p} \) can be expressed by means of the Poynting vector as

\[
\mathbf{p} = \mathbf{S} / c^2.
\]  
(60)

Assuming the beam propagating along the \( z \)-direction, its orbital angular momentum is given by the \( z \)-component of \( \mathbf{J} \),

\[
J_z = \frac{(\mathbf{r} \times \mathbf{S})_z}{c^2}.
\]  
(61)

Applying the circular cylindrical coordinates \( r, \phi, z \), we obtain

\[
J_z = \frac{rS_\phi}{c^2},
\]  
(62)

where \( S_\phi \) is the magnitude of the azimuthal component of the Poynting vector. Performing normalization of the orbital angular momentum by the volume energy density,

\[
j_z = \frac{J_z}{w},
\]  
(63)

the quantity \( j_z \) can be interpreted as a magnitude of the orbital angular momentum carried by the photon of the beam.

The orbital angular momentum of the optical beam can be expressed by the Poynting vector representing the density of the flow of the electromagnetic energy. In the exact vectorial theory, the Poynting vector is given by the vectors of the electromagnetic field (56) and fulfills the energy conservation law (55). In the framework of the scalar approximation the optical beams are described by the scalar complex amplitude \( U \) fulfilling the wave equation. By simple manipulations of the wave equation, the relation resembling the form of the energy conservation law can be obtained

\[
\nabla \cdot \mathbf{S}' + \frac{\partial w'}{\partial t} = 0,
\]  
(64)

where

\[
\mathbf{S}' = - \left( \nabla U^* \frac{\partial U}{\partial t} + \nabla U \frac{\partial U^*}{\partial t} \right),
\]  
(65)

\[
w' = \nabla U \cdot \nabla U^* + \frac{1}{c^2} \left| \frac{\partial U}{\partial t} \right|^2.
\]  
(66)

The quantities \( \mathbf{S}' \) and \( w' \) can be comprehended as approximations to the Poynting vector and the volume energy density of the optical beam, respectively. The orbital angular momentum can then be simply demonstrated on the case of the scalar monochromatic vortex beam. If the vortex is nested at the centrum of the nondiffracting host beam, its complex amplitude can be expressed by means of the circular cylindrical coordinates as

\[
U(r, \phi, z, t) = a(r) \exp[i(\omega t + m\phi - \beta z)],
\]  
(67)

where \( m \) denotes the topological charge of the vortex and \( \beta \) is the angular wavenumber of the host nondiffracting beam. If \( S_\phi \) used in (62) is replaced by its scalar approximation \( S'_\phi \) following from (65), we obtain

\[
J_z = \frac{2m\omega |a|^2}{c^2}.
\]  
(68)

Scalar approximation to the volume energy density can be expressed as

\[
w' = 2\beta^2 |a|^2.
\]  
(69)
The orbital angular momentum carried by the single photon of the monochromatic vortex beam with the angular frequency \( \omega \) is then given by

\[
j_z = \frac{m}{\omega}.
\]

(70)

The orbital angular momentum is proportional to the topological charge of the vortex beam and inversely proportional to its angular frequency. It is a reason why the mechanical influence of the vortex is more appreciable for the microwaves than for the optical waves.

One of the most important tasks in design of MEMS (Micro Electro Mechanical Systems) is to find ways how to power machines that measure only microns across. The promising solution is to rotate them by the blowing of “the light wind”. It can be produced by the optical vortices carrying the orbital angular momentum. In [25, 26], the simple model of the vortex beam interaction accompanied by the exchange of the orbital angular momentum was proposed and analyzed. It was shown that both the phase topology and the local distribution of the orbital angular momentum of the vortex nested in the nondiffracting host beam can revive and regenerate to the initial form after interaction with the 2D object. In the simulation model, the rotationally nonsymmetrical object takes the orbital angular momentum from the beam and rotates. The healing of the spatial distribution of the orbital angular momentum after interaction is illustrated in Fig. 15. It is accompanied by the self-regeneration of the phase topology of the vortex beam. In Figs. 16a and 16b, the vortex helical wavefront of the initial beam is visualized by interference with plane and spherical waves, respectively. After interaction with 2D object, the phase topology is strongly disturbed but during free propagation revives to the initial form (Figs. 16c-16f).

6.4 Transversality of electric and magnetic field

An ideal homogeneous monochromatic plane wave is the transversal electromagnetic (TEM) wave. Its electric and magnetic fields oscillate at the transverse planes so that the projections of the vectors \( \mathbf{E} \) and \( \mathbf{H} \) to the direction of propagation are equal to zero. For the free-space propagation that property follows directly from the Maxwell equations \( \nabla \cdot \mathbf{E} = 0 \) and \( \nabla \cdot \mathbf{H} = 0 \). In real cases the wave is not homogeneous (vectors \( \mathbf{E} \) and \( \mathbf{H} \) depend on the transverse coordinates) so that the electromagnetic transversality cannot be exactly achieved. Nevertheless, the common beams, for example Gaussian beams, can be considered to be nearly transversal with respect to the dominant propagation direction. That means that the longitudinal component of the electric or magnetic vector of the beam is very small in comparison with the transversal one. Applying the concept of the nondiffracting propagation we can prepare quite different situation when the longitudinal component of the electric or magnetic filed is comparable to the transverse component. That property is given by the structure of the spatial spectrum of the nondiffracting beam and depends on the beam spot size. It was verified that the extremely strong longitudinal component of the electric field of the nondiffracting beam can be obtained only if the transverse dimensions of the beam are comparable to the wavelength [8]. The beams with the strong longitudinal field are important for applications and were successfully applied to the design of the electron accelerators [43].

6.5 Self-imaging

The basic parameter of the scalar nondiffracting beam (1) is the angular wavenumber \( \beta \). If we perform the coherent superposition of two nondiffracting modes possessing the different angular wavenumbers \( \beta_1 \) and \( \beta_2 \), the produced beam is not propagation invariant. Its transverse intensity profile depends on the \( z \)-coordinate and reappears periodically in the free-space propagation. In dependence on the propagation coordinate the beam axial intensity changes sinusoidally. Under convenient choice of the angular wavenumbers that effect can be realized also by the coherent superposition of \( m \) nondiffracting beams. The effect represents the spatial analog of the mode-locking realized in the temporal domain. Due to the interference of the modes, the transverse intensity profile of the beam reappears periodically at the planes of the constructive interference and vanishes at the places of the destructive interference. Its complex amplitude \( a \) then fulfills condition

\[
a(x, y, z) = a(x, y, z + L),
\]

(71)
where $L$ is the longitudinal period. The beam axial intensity is significantly nonzero only near the planes of the constructive interference and vanishes elsewhere. The intensity distribution of the field exhibiting the self-imaging effect is illustrated in Fig. 17. It can be obtained as the coherent superposition of the nondiffracting modes whose angular wavenumbers are adapted to the chosen period $L$ as

$$\beta_m = \frac{2m\pi}{L}, \quad m = 0, 1, 2, \ldots, \quad m < L/\lambda.$$  \hspace{1cm} (72)

The width of the peaks of the axial intensity $I(0, 0, z)$ can be decreased if the number of the superposed modes is increased. The width of the transverse intensity profile decreases with the increasing values of the angular wavenumbers of the used nondiffracting modes. The general vectorial treatment of the self-imaging effect and its experimental verification was presented in [16].

### 6.6 Self-reconstruction ability

The beams possessing the self-reconstruction ability belong to the class of fields exhibiting the longitudinal propagation periodicity. The Talbot effect and the self-imaging are also members of that class of fields. To express their distinctions the exact definition of the self-reconstruction effect is necessary. It can be introduced applying the concept of the shape-invariant transformation. The spatial evolution of the complex amplitude of the monochromatic field fulfilling the Helmholtz equation can be expressed by the integral operator $\Gamma$ as

$$a(x, y, z) = \Gamma a_0(x, y, z_0),$$  \hspace{1cm} (73)

where $a_0$ is the complex amplitude at the $z = z_0$ plane. The shape-invariant transformation is achieved if the complex amplitude can be written in the form

$$a(x, y, z) = a_0(x, y, z_0)Z(z_0, z).$$  \hspace{1cm} (74)

If the property (74) is required only for a pair of planes, it can be realized applying the imaging system. Under certain conditions the shape-invariant property can also be achieved in the free-space propagation of the beam behind the plane $z = z_0$. The known examples are the nondiffracting propagation, the Talbot effect and the self-imaging. For the ideal nondiffracting beam the amplitude profile $a_0$ remains propagation invariant. In the cases of the Talbot effect and the self-imaging the initial profile reappears periodically so that (74) is fulfilled for $z = z_0 + mL$, where $L$ is the longitudinal period. The longitudinal periodicity of the Talbot effect requires the lateral periodicity whose period depends on $L$. In the case of the self-imaging the amplitude profiles $a_0$ can be nonperiodical but they cannot be chosen arbitrarily. The fundamental difference of the self-reconstruction effect in comparison with the self-imaging consists in the fact that the amplitude profile $a_0$ to be periodically reconstructed can be predetermined. Taking into account that property the self-reconstruction can be comprehended as the effect by which the field with the predetermined transverse amplitude profile $a_0$ can be converted into the field described by the complex amplitude $a_s$ possessing the following properties:

(a) the complex amplitude $a_s$ is the exact solution to the Helmholtz equation,

(b) in the free-space propagation the transverse profile of the field $a_s$ reappears periodically with the longitudinal period $L$, $a_s(x, y, z) = a_s(x, y, z + mL)$, $m = 0, 1, 2, \ldots$,

(c) at the planes of reconstruction $z = z_0 + mL$ the complex amplitude $a_s$ approximates the predetermined transverse amplitude profile $a_0$, $a_s(x, y, z_0 + mL) \approx a_0(x, y, z_0)$.

The theoretical description of the self-reconstruction was proposed in [17], [52] and [53]. In practise, the transformation of the signal field with the predetermined amplitude profile $a_0$ into the field exhibiting the self-reconstruction ability can be performed applying the spatial filtering in the 4-f optical system. The used spatial filter is the amplitude mask consisting a set of concentric annular rings. After spatial filtering the initial field is represented by the discrete superposition of the nondiffracting modes propagating with the angular wavenumbers fulfilling condition of the self-imaging (72). Experimental verification of the effect was presented in [20].
7 CONTROLLED 3D LIGHT BENDING

Recently, the concept of the spatial shaping of the nondiffracting fields and the self-imaging effect have been adopted to realize the controlled 3D light bending [21]. By that method the light can be confined in the volume elements whose transverse and longitudinal dimensions are comparable to the wavelength. The transverse intensity profile of the light field is created as a coherent superposition of the nondiffracting spots whose position, size and amplitude profile can be controlled. By the spatial shaping of the nondiffracting fields the single nondiffracting spots are centred at the required positions. By the 3D light bending many nondiffracting spots possessing different angular wavenumbers contribute at the same position of the transverse plane. As the spots are superposed constructively at their centra, the resulting spot is strongly peaked in comparison with the single one. By that way the predetermined transverse amplitude profile can be shaped with a high resolution. As the total field is composed of the nondiffracting modes with different angular wavenumbers, it is not nondiffracting. If the angular wavenumbers are conveniently coupled, the self-imaging effect is achieved. The required transverse amplitude profiles then appear periodically along the propagation direction with the controllable period. If the number of contributing modes with different angular wavenumbers is sufficiently large, the field amplitude is strongly peaked also along the direction of propagation. The intensity maxima then appear periodically at the planes where the required transverse amplitude profiles are formed. By that way, the controllable 3D light distribution can be produced. In Fig. 18, the comparison of the spatial shaping of the nondiffracting field and the 3D light bending is presented. The former case is illustrated in Fig. 18a. The required intensity profile is an array of 9 point sources. That profile is replicated in the nondiffracting field in such a way that each source point of the array excites the nondiffracting spot placed at the position depending on the position of the corresponding source. The obtained profile remains invariant under free-space propagation. In Fig. 18b, the 3D light bending is shown. In that case, the required profile is created in such a way that each point source of the array excites many nondiffracting modes localized at the same position of the transverse plane. As they are superposed constructively, the resulting spot is sharpened in comparison with the single mode spot so that the required profile is replicated with a very good fidelity. Due to the self-imaging effect, the transverse amplitude profile is available only at the planes placed periodically along the propagation direction. Between those planes it disappears due to the destructive interference of the nondiffracting modes. Experimental implementation of both the spatial shaping of nondiffracting fields and of the 3D light bending is proposed in [5, 21]. Applications of the 3D light bending can be expected in the design of adaptable optical tweezers enabling 3D manipulation of electrically neutral particles and atoms.

8 PSEUDO-NONDIFFRACTING BEAMS

The nondiffracting beams indicate that the diffraction effects can be overcome if the propagation of the source-free monochromatic field described by the homogeneous Helmholtz equation is considered. However, the ideal nondiffracting beams carry an infinite energy and their transverse intensity profile remains unchanged from $-\infty$ to $+\infty$. This is a reason why the nondiffracting beams cannot be exactly realized. In experiments only their approximations known as the nearly nondiffracting or the pseudo-nondiffracting beams can be obtained. They possess the finite energy and their propagation properties can be approximated by the properties of the nondiffracting beams transmitted through the aperture of finite dimensions or through the Gaussian aperture [11]. The simplest type of the pseudo-nondiffracting beam known as the Bessel-Gauss beam [10] can be obtained directly from the paraxial form of the Helmholtz equation.

8.1 Bessel-Gauss beam
The pseudo-nondiffracting Bessel-Gauss beam represents the spatially modulated Gaussian beam. It can be searched as the exact solution to the paraxial Helmholtz equation whose rotationally symmetrical complex amplitude is assumed as

$$a(r, z) = R[g(z)r^2]e^{iZ(z)}a_g(r, z),$$

(75)

where $a_g$ is the complex amplitude of the conventional Gaussian beam and $R$, $g$, and $Z$ are in the meantime unknown functions. Substituting (75) into the paraxial Helmholtz equation and applying the denotation $t^2 = gr^2$ we can write

$$t^2 \frac{d^2R}{dt^2} + t \frac{dR}{dt} \left[1 - 2ik \frac{1}{2g} \frac{dg}{dz} - 2i^2 \left(\frac{2}{w^2} + i \frac{k}{R_g}\right)\right] + t^2 \frac{2k}{g} \frac{dZ}{dz} R = 0,$$

(76)

where $w$ is the bandwidth of the Gaussian beam and $R_g$ denotes the radius of the curvature of its wavefront. If the functions $g$ and $Z$ fulfil equations

$$i \frac{k}{2g} \frac{dg}{dz} + \frac{2k}{w^2} + i \frac{k}{R_g} = 0,$$

(77)

$$\frac{2k}{g} = 1,$$

(78)

then (76) represents the special form of the Bessel equation. In the examined case, its solution is the function $R$ given by the Bessel function of the first kind and zero order

$$R(t) = J_0(t).$$

(79)

Integrating (77) we obtain

$$\ln|g| + C = i2 \arctan(z/\eta_0) - \ln(w^2 + \eta_0^2),$$

(80)

where $\eta_0$ is the Rayleigh distance of the Gaussian beam. If the integration constant $C$ is rewritten by means of the new constant $K$

$$C = -2\ln|K\eta_0|,$$

(81)

then the searched function $g$ can be expressed as

$$g = \frac{K^2}{(1 - iz/\eta_0)^2}.$$  

(82)

If we substitute $g$ into (78) the function $Z$ is obtained after integration. Applying it the complex amplitude of the Bessel-Gauss beam can be rewritten in the form

$$a(r, z) = J_0\left(\frac{Kr}{1 - iz/\eta_0}\right)\exp\left[i \frac{K^2\eta_0^2}{4k} \left(\frac{2}{kw^2} + i \frac{1}{R_g}\right) (z - i\eta_0)\right]a_g(r, z),$$

(83)

where $a_g$ is the complex amplitude of the Gaussian beam.

### 8.2 Comparison of pseudo-nondiffracting and ideal nondiffracting beams

The nondiffracting beams possess the sharp $\delta$-like angular spectrum represented by a circle at the Fourier plane. The basic parameter of the spectrum is the single radial frequency which can be related to the radius of that circle. The radial frequency is inversely proportional to the transverse dimension of the intensity profile of the beam. Because the ideal nondiffracting beams transfer an infinite energy, they can be realized only approximately in experiments. They are usually called the pseudo-nondiffracting beams. The simplest model of the pseudo-nondiffracting beam can be obtained if the nondiffracting beam is transmitted through the aperture whose transparency is described by the Gaussian function. The energy of the transmitted beam is then finite but its angular spectrum possesses the spread caused by the aperture. As a result, the propagation invariance of the transverse intensity profile is lost but the fundamental differences between propagation properties of the conventional and the pseudo-nondiffracting beams still exist. In [12] they were demonstrated on the composition of the angular spectra of both types of beams. The main results of that analysis can be concluded as follows:
The conventional beam with the spatial bandwidth $2\Delta r$ transmitted through the aperture with the transverse dimension $2\Delta R$ possesses the angular spectrum with the uncertainty $2\Delta \theta$. If $\Delta R \gg \Delta r$, the spread of the angular spectrum depends on the transverse size $2\Delta r$ of the beam impinging on the aperture. This dependence can be written as the uncertainty relation

$$\Delta r \Delta \theta = \text{const.} \quad (84)$$

The pseudo-nondiffracting beam is obtained if the nondiffracting beam with the intensity spot size $2\Delta r$ is transmitted through the aperture with the transverse dimension $2\Delta R$. If $\Delta R \gg \Delta r$, the spread of the angular spectrum of the pseudo-nondiffracting beam is given by the relation

$$\Delta R \Delta \theta = \text{const.} \quad (85)$$

The spread of the angular spectrum of the pseudo-nondiffracting beam depends only on $\Delta R$. If the size of the aperture $2\Delta R$ is constant, $\Delta \theta$ remains unchanged even if the spot size $2\Delta r$ of the beam impinging on the aperture decreases. This property represents the fundamental difference between the conventional and the pseudo-nondiffracting beams. It represents an essence of the pseudo-nondiffracting propagation. In this sense we can speak of the diffraction elimination. The property (85) is graphically illustrated in Fig. 19. The transverse intensity profiles of the nondiffracting beams impinging on the Gaussian aperture are illustrated in Fig. 19a and 19c. The corresponding annular spectra are illustrated in Fig. 19b and 19d. As is obvious, the reduction of the size of the intensity spot of the input beam changes only diameter of the annular ring but its width important for the diffractive divergence of the beam remains unchanged.

For the constant intensity spot size of the impinging nondiffracting beam $2\Delta r$, the spread of the angular spectrum $2\Delta \theta$ and also the diffractive divergence of the transmitted pseudo-nondiffracting beam $2\Delta \theta$ can be reduced if the window is enlarged. As the impinging nondiffracting beam falls to zero in the transverse direction very slowly, the reduction of the diffractive spread is associated with the increase of the energy consumption.

The price payed for the reduction of the diffraction effects is shortening of the longitudinal range of the beam existence $\Delta z$. It is given by the relation

$$\Delta z \Delta \theta = \text{const.} \Delta r. \quad (86)$$

For the constant size of the aperture $2\Delta R$, the longitudinal range $\Delta z$ is shortened if the spot size of the beam impinging on the window decreases.

9 EXPERIMENTS AND APPLICATIONS

The monochromatic coherent nondiffracting beam can be comprehended as an interference field produced by the superposition of plane waves whose propagation vectors form the conical surface. Such interference field can be realized in a good approximation by several methods (see [39] for a review). The simple way how to generate the pseudo-nondiffracting beam whose transverse amplitude profile resembles the zero-order Bessel function of the first kind $J_0$ was proposed by Durnin et al [56]. The used experimental setup is illustrated in Fig. 20. In that experiment the spatially filtered and expanded laser beam illuminates the annular ring mask placed at the front focal plane of the lens. The mask serves as a secondary source generating spherical waves. Each of them is transformed by the lens to the beam of parallel rays propagating with the angle $\theta_0$ with respect to the optical axis. The beam-like interference field with the $J_0$ transverse amplitude profile appears in the interference region behind the lens where the beams intersect. The spot size of the generated beam depends on the angle $\theta_0$. The radius of the beam central spot $r_0$ can be approximated by $r_0 \approx \lambda / \sin \theta_0$. For angles close to $\pi/2$ the light tubes with the size comparable to the wavelength can be obtained. The longitudinal distance $L$ where the beam is available without changes of its transverse intensity profile depends on the angle $\theta_0$ and on the
size of the aperture of the Fourier lens. The distance where the beam of the given spot size propagates without apparent changes of its intensity profiles can be controlled by the size of the lens aperture. The pseudo-nondiffracting beam can be obtained even if an arbitrary amplitude and/or phase modulation is applied at the plane of the annular mask. For example, the Bessel-like beams of higher-order can be realized if the spiral phase plate is adjacent to the annular ring. Production of such phase plate is a complicated technical problem. It can be successfully prepared applying the photolithographic techniques. If the phase plate causes the azimuthal phase modulation given by \( \exp\left(im\phi\right) \), the \( m \)-th order Bessel beam is approximately generated. Its axial intensity is equal to zero so that the beam is dark and exhibits phase properties of the optical vortices. The pseudo-nondiffracting beams can also be realized by the azimuthal amplitude modulation of the annular ring mask. For example, it can be achieved if the annular mask is illuminated by the one-dimensional strip pattern with the Gaussian profile. The obtained beam represents a good approximation of the Mathieu beam. The pseudo-nondiffracting beam can also be obtained as an interference field produced by the discrete superposition of plane waves. In that case the annular ring mask is transparent only at the finite number of points. It can be simply realized by means of the auxiliary amplitude mask. If the annular ring mask is illuminated by the source whose correlation properties can be controlled the partially-coherent nondiffracting beams can be generated. In [31] their properties were examined for the illumination realized applying the Gaussian Shell-model source. The self-imaging effect can be experimentally realized if the amplitude mask consisting of a set of annular rings with the required diameters is used. The experimental verification of that effect was presented in [16]. Another possible way how to obtain an interference field of plane waves whose wave vectors form the conical surface is based on the use of the refractive axicon illuminated by the collimate laser beam [37, 35]. An advantage of that method is the high efficiency with which the power of the common laser beam can be converted into the pseudo-nondiffracting form. In [57, 58] it was shown that the action of the annular mask or the refractive axicon can be alternatively performed by the computer-generated axicon-type hologram. In that way the beam approximating the zero-order Bessel beam was obtained with the relative high conversion efficiency approaching 50% [59]. The dark higher-order Bessel beams were also successfully generated by the holographic means [60, 61]. The simple but efficient method providing a good approximation of the zero-order Bessel beam can be realized applying the centrally obscured lens exhibiting the spherical aberration [37]. In [38] a possibility to convert the Gaussian beam to the zero-order Bessel beam applying the two-element refracting system was examined. In that method the conversion efficiency is good but the system is hardly realizable because the optical elements with the aspheric surfaces are required. The general pseudo-nondiffracting patterns useful for the optical interconnection applications were realized by means of the magneto-optic spatial light modulators [62, 36]. The pseudo-nondiffracting beams can be also generated directly at the laser resonator. The special resonator construction was proposed in [63]. Recently, the experiment enabling generation of the nondiffracting beams with the controllable spatial coherence was proposed and realized [31]. It is based on the use of the pseudothermal, so called Gaussian Shell-model source. In that case the optical scheme in Fig. 20 remains unchanged but the coherent laser beam illuminating the annular ring is replaced by the source shown in Fig. 21. The coherent laser beam is focused to the rotating diffuser where its phase is randomized. The beam spot created at the diffuser serves as a spatially incoherent source illuminating the annular ring. During free propagation between the diffuser and the annular mask the spatial coherence of the beam is increased so that the annular ring is illuminated by the partially coherent light. Its coherence properties can be continuously changed by the change of the beam spot at the rotating diffuser and described mathematically applying the Van Cittert-Zernike theorem. The light illumination can be changed from fully incoherent to nearly fully coherent. The change of the spatial coherence was applied to the optical set-up enabling generation of the nondiffracting beams with the predetermined transverse intensity profile [5]. The optical scheme is shown in Fig. 22. The required amplitude profile of the beam is predicted by the source array. The mask is then illuminated by the light of the controllable spatial coherence. The required amplitude profile is then replicated as the nondiffracting pattern. The method works in such a way that each point of the source array excites the nondiffracting spot whose size and form can be driven by the experiment geometry. The position of the nondiffracting spot is defined by the position of the source point. The nondiffracting pattern is obtained as a superposition of the nondiffracting spots. The fidelity of the replication is highest if the nondiffracting modes are superposed incoherently.

The unique properties of the nondiffracting beams are useful for both the technical and physical applications. The propagation invariance of their transverse intensity profile is applicable in metrology for scanning optical systems [41]. The nondiffracting beams are also suitable for large-scale straitness
measurement and other large size measurement [64] because are much less influenced by atmospheric turbulence than other beams [65]. Attention was concentrated also to the imaging applications of the nondiffracting beams. It was verified that the imaging realized with nondiffracting beams can provide an extremely long focal dept. In [66], a kilometer-long imaging was proposed and examined. An increasing attention is devoted to the nondiffracting beams for their applicability in nonlinear optics. In [42], it was shown that the nondiffracting Bessel beam can be viewed as a light beam with the tunable wavelength. Due to that property, the phase-matched second-harmonic generation at angles usually not suited for phase matching in a KDP crystal was performed. The application of the nondiffracting beams to the third-harmonic generation [67] and to the Čerenkov second-harmonic generation in bulk optical crystals was also proposed [68]. An efficient conical emission of light in Raman scattering stimulated by nondiffracting Bessel beam was examined in [69]. The Bessel pump beam was applied also to the design of the distributed-feedback laser [70]. The nondiffracting beams were applied to increase the sensitivity of the measurement of the nonlinear refractive index by the Z-scan method [71]. The radially polarized nondiffracting beams possess the strong longitudinal component of the electric field. That component can accelerate the particles of the electron beam propagating nearly collinearly with the laser nondiffracting beam [72]. Recently, the application of the nondiffracting beams significantly improved the techniques for manipulations of micrometer-sized particles. The self-reconstruction ability of the nondiffracting beam enables to manipulate ensembles of particles simultaneously in multiple planes [73, 44]. The nondiffracting vortex beams are perspective for the research focused to the transfer of the orbital angular momentum to the particles. The obtained results are promising for realization of the light motors whose rotors can be forced by the laser beam. The nondiffracting beams are also perspective for the atom guiding in their optical potential [74].

10 CONCLUSIONS

In the paper, theoretical concepts, mathematical methods of description and numerical simulations of the nondiffracting propagation were reviewed. The particular attention was focused to the physical properties of nondiffracting beams and to their physical applications such as 3D light synthesis, self-imaging and self-healing effects and the transfer of the orbital angular momentum to the material particles. The experimental realization of nondiffracting beams and their technical applications were also presented.

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Figure captions

Fig. 1
The Bessel beam as a superposition of the travelling waves given by the Hankel functions. The outgoing wave travelling away from the axis (a) and the incoming wave travelling towards the axis (b) create the standing wave represented by the Bessel function (c).

Fig. 2
The transverse intensity profiles of the bright zero-order Bessel beam (a) and the dark first-order Bessel beam (b).

Fig. 3
The helical wavefronts of the nondiffracting vortex beams for the topological charge (a) $m = 1$ and (b) $m = 2$.

Fig. 4
Visualisation of the optical vortices. Interference of the optical vortex $m = 1$ with the spherical wave results in spiral and fork-like interference patterns (a) and (b). Interference of the vortex with the topological charge $m = 2$ is in (c) and (d).

Fig. 5
The transverse intensity profile of the Mathieu beams for parameters (a) $w_0 = 4\nu_0$ and (b) $w_0 = 2\nu_0$.

Fig. 6
The caleidoscopic nondiffracting patterns obtained as a discrete superposition of the finite number of plane waves: (a) $N = 5$, (b) $N = 10$, (c) $N = 15$, and (d) $N = 30$.

Fig. 7
Illustration of the nondiffracting field as a superposition of five cosine gratings with the same period and various orientations.

Fig. 8
Nondiffracting beams with the variable spatial coherence. The fully coherent dark vortex beam (a) is continuously changed to the bright nondiffracting beam (b)-(d) if the spatial coherence is decreased.

Fig. 9
Illustration of the degree of angular correlation of the plane waves creating the partially coherent nondiffracting beam.

Fig. 10
Change of the vortex topology caused by the change of the spatial coherence. The vortex of the coherent beam (a) vanishes if the spatial coherence is decreased (b)-(d).

Fig. 11
Spatial shaping of coherent and incoherent nondiffracting fields. The required intensity profile (a) is replicated in the nondiffracting beam created with incoherent light (b) and coherent light (c). In (d) and (c) the parameter $\alpha$ of the nondiffracting spots is increased so that their size is reduced.

Fig. 12
The same as in Fig. 11 but for the continuous required intensity profile.

Fig. 13
Illustration of the vectorial electromagnetic nondiffracting beams. The azimuthal polarization of the electric field (a) and the radially polarized magnetic field (b). The short arrows illustrate the magnitude and direction of the transverse components of the field vectors at the separate points of the transverse plane.

Fig. 14
Experimental verification of the resistance of the nondiffracting beam against amplitude and phase perturbations. The nondiffracting beam impinging on the nontransparent obstacle is fully revived during free propagation behind the obstacle.

Fig. 15
Healing of the spatial distribution of the orbital angular momentum of the disturbed nondiffracting vortex beam. The initial beam with the spatial distribution of the orbital angular momentum (a) interacts with
the complex object which takes the orbital angular momentum and rotates. After interaction the spatial
distribution of the beam orbital angular momentum is disturbed (b)-(c) but during free propagation is
revived to the nearly initial form (d).

Fig. 16
Healing of the vortex topology after interaction with the complex object accompanied by the exchange
of the orbital angular momentum.

Fig. 17
The self-imaging effect obtained due to the coherent superposition of the nondiffracting modes. The
transverse intensity profile appears periodically along the propagation direction $z$ with the longitudinal
period controllable by the choice of the angular wavenumbers of the nondiffracting modes.

Fig. 18
Comparison of the controllable shaping of the nondiffracting fields (a) with the 3D light bending (b). By
the light bending the required transverse profile appear only at the near vicinity of the planes placed
periodically along the propagation direction so that the light is confined at 3 dimensions. By that method
the light can be localized in the volume elements with dimensions comparable to the wavelength.

Fig. 19
Illustration of spatial spectrum of the pseudo-nondiffracting beam. The pseudo-nondiffracting beam with
intensity profile (a) possesses the spatial spectrum of the form (b). If the intensity profile is rescaled
(c), the spread of the spatial spectrum (width of the ring) remains unchanged (d). On that behavior
the unique properties of the pseudo-nondiffracting beams are based and it cannot be achieved by the
conventional beams.

Fig. 20
Experimental set-up enabling generation of the pseudo-nondiffracting beam.

Fig. 21
Gaussian Shall-model source used for generation of the partially coherent pseudo-nondiffracting beams.

Fig. 22
Optical set-up for generation of the nondiffracting fields with the predetermined intensity profile.
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