The general relation between the weak inclusive decays of bound and free heavy quarks

S. Kotkovsky a, I.M. Narodetskii a, K.A. Ter-Martirosyan a and S. Simula b

aInstitute for Theoretical and Experimental Physics, 117218 Moscow, Russia
bINFN, Sezione Roma III, I-00146 Roma, Italy

We derive a new parton formula for the inclusive $B$ decays and briefly discuss its applications to semileptonic and weak radiative decays of the $B$-meson.

1. INTRODUCTION

The inclusive $B$ decays offer the most direct way to determine the $CKM$ parameters $|V_{cb}|$ and $|V_{ub}|$ and the internal structure of the $B$-meson. Both tasks complement each other: an understanding of the connection between quark and hadron properties is a necessary prerequisite for a precise determination of the $CKM$ matrix.

Weak $B$ decays are particular simple in the limit $m_b \to \infty$, when the decay rate of the $B$-meson is completely determined by the decay rate of the heavy quark itself. The account of the soft degrees of freedom generates important pre-asymptotic contributions due to the binding effects and Fermi-motion of the heavy quark inside the hadron.

The leading non-perturbative correction is described by a shape function $F(x)$ which governs the light-cone (LC) momentum distribution of the heavy quark inside the hadron. Here $x = p^+_b/P^+_B$ is the LC fraction of the heavy-quark momentum in the $B$-meson. The shape function arises as a result of resummation of an infinite set of leading twist corrections in the Heavy Quark Expansion and has been incorporated into phenomenological models of inclusive decays.

The impact of the Fermi motion has also been addressed in the parton models of Refs. [3]-[5]. The approach of these works was based on the hypothesis of quark-hadron duality, which assumes that when a sufficient number of exclusive hadronic decay modes is summed up, the decay probability into hadrons equals the one into the free quarks. The $b$-quark was treated as a virtual particle with mass $m_b^2 = x^2 M_B^2$, where $M_B$ is the $B$ meson mass, while the effects due to the transverse motion of the $b$-quark were neglected.

The final expressions obtained for the semileptonic branching fractions exhibit a close analogy with deep inelastic lepton-nucleon scattering.

The aim of this talk is to discuss the generalization of the work of Refs. [3]-[5]. The new features are the treatment of the $b$-quark as an on-mass-shell particle and the inclusion of the effects of the $b$-quark transverse momenta. The main result is the derivation of a new parton formula for the inclusive width, which is similar to the one derived by Bjorken et al. in case of infinitely heavy $b$- and $c$-quarks:

$$\frac{d\Gamma_B}{dq^2} = \frac{d\Gamma^{(\text{free})}}{dq^2} R(q^2),$$

where $d\Gamma^{(\text{free})}/dq^2$ is the free-quark differential decay rate and the function $R(q^2)$ incorporates the non-perturbative effects. In Eq. (1) $q$ is the 4-momentum of the $W$-boson. The structure of Eq. (1) suggests that in the limit $m_b \to \infty$, $m_c \to \infty$ one has $R(q^2) = 1$, which means that the inclusive width of the $B$-meson is the same as the inclusive width at the free quark level. At the physical $b$-quark masses the corrections due to the bound-
state factor $R(q^2)$ are important and sensitive to the values of quark masses.

2. A NEW PARTON APPROXIMATION

The derivation of Eq. (1) is straightforward. The modulus squared of the $B$-meson decay amplitude is $|\mathcal{M}_B|^2 = (G_F^2/2)|V_{qb}|^2 l_{\alpha\beta}w_{\alpha\beta}$, where $l_{\alpha\beta}$ and $w_{\alpha\beta}$ are the lepton and hadronic tensors, respectively. Following the above assumption the hadronic tensor $W_{\alpha\beta}$ is given through the optical theorem by the imaginary part of the quark models of refs. [10] (cases $A$). We should stress that the nonperturbative ingredient in Eq. (6) is characterized by a quark wave function. The latter ones have been already used to calculate the form factors of heavy-to-heavy and heavy-to-light exclusive transitions [8], which allows to convert the $ET$ wave function into a relativistic $LC$ wave function. The latter one results to a soft Gaussian ansatz, whereas the $ET$ wave function at high momenta. Models $B$ and $C$ correspond to a soft Gaussian ansatz, whereas the $ET$ wave function corresponding to models $D$ and $E$ exhibit high momentum components generated by the one-gluon-exchange part of the interquark potential.

We have calculated Eqs. (1) and (6) for the inclusive decays $B \to X_c(1)\nu\bar{\nu}$ adopting the five models $A$ to $E$. In all cases the nonperturbative effects lead to a suppression with respect to the free-decay rate. The suppression factor varies in the range from 0.69 (case $D$) to 0.95 (case $A$) and it is sensitive both to the width of the $x$-distribution and to the quark masses $m_b$ and $m_q$. We should stress that the $ET$ wave functions are derived from quark models containing constituent quark masses constrained phenomenologically by $B$- and $D$-meson spectroscopy. Thus, for each of the quark model considered we get a
different free-quark result. The values predicted for $\Gamma_{SL}/\Gamma_{B}^{(exp)}$ exhibit a model dependence of about 20%. The average over the various quark model predictions yields $|V_{ub}| = (40.8 \pm 0.7_{\exp} \pm 1.7_{th}) \cdot 10^{-3}$, $\sqrt{BR_{SL}^{(exp)}}/10.43\% \cdot \sqrt{1.57 \; ps/\tau_{B}^{(exp)}}$ (assuming the world average value $BR_{SL}^{(exp)} = (10.43 \pm 0.24)\%$) and $|V_{ub}| = (3.53 \pm 0.44_{\exp} \pm 0.14_{th}) \cdot 10^{-3}$, $\sqrt{BR_{SL}^{(exp)}}/0.16\% \cdot \sqrt{1.57 \; ps/\tau_{B}^{(exp)}}$. Our result for $|V_{ub}|$ is based on the preliminary ALEPH measurement $BR(b \to u\ell\nu_{l}) = (0.16 \pm 0.04)\%$.

4. WEAK RADIATIVE DECAYS

The same approach can be applied to the inclusive radiative decay $B \to X_{s}\gamma$ that is one of the central decays in the rare decays phenomenology. The motion of the $b$-quark inside the $B$-meson leads to a modification both of the inclusive rate and the photon energy spectrum. The LC partonic approximation for $\Gamma(B \to X_{s}\gamma)$ is evaluated using the magnetic photon penguin. The result is $\Gamma(B \to X_{s}\gamma) = \Gamma_{(free)}(b \to s\gamma) R_{\gamma}$, where

$$R_{\gamma} = 2m_{s}^{2} \int_{0}^{M_{B}/2} dq_{0} \int_{x_{min}}^{1} dx f(x, p_{x}^{2})$$

(7)

with $p_{x}^{2} = m_{s}^{2}(x/x_{min} - 1)$, and $x_{min} = 2q_{0}/M_{B}$, the mass of the strange quark being neglected. In Table 1 the coefficients $R_{\gamma}$ obtained using the above ansatz for the LC wave function are collected. We also report the branching fractions $BR_{\gamma} = BR(B \to X_{s}\gamma)$ calculated using the quark models A to E, showing the sensitivity of the theoretical predictions to the different choices of the ET wave functions and constituent quark masses entering the calculation. We use $|C_{\tau}^{(0)}| \approx 0.3$ and $1/\alpha_{EM} = 137$. Averaging over the models one gets $BR_{\gamma} = (3.3^{+0.7}_{-0.9}) \cdot 10^{-4}$ that agrees with the very recent preliminary update from CLEO, $BR_{\gamma} = (2.50 \pm 0.47 \pm 0.39) \cdot 10^{-4}$
[12], and the ALEPH measurement, $BR_{\gamma} = (3.11 \pm 0.80 \pm 0.72) \cdot 10^{-4}$
[13], within one standard deviation.

| Model | A | B | C | D | E |
|-------|---|---|---|---|---|
| $R_{\gamma}$ | 0.96 | 0.94 | 0.88 | 0.88 | 0.88 |
| $BR_{\gamma}$ | 2.4 | 2.8 | 3.8 | 4.0 | 3.5 |

Table 1

The ratio $R_{\gamma}$ (Eq. 7) and the branching fraction $BR_{\gamma}$ (in units $10^{-4}$) calculated within the quark models A to E.

ACKNOWLEDGMENTS

One of the authors (I.M.N.) would like to thank Carlo Caso and Calvin Kalman for organizing an excellent Conference with a stimulating scientific program.

REFERENCES

1. I. Bigi et al., Int. J. Nucl. Phys. A9 (1994) 2467; M. Neubert, Phys. Rev. D49 (1994) 3392
2. G. Altarelli et al., Nucl. Phys. B208 (1982) 519
3. C.H. Jin et al., Phys. Lett. B329 (1994) 364
4. V.L. Morgunov and K.A. Ter-Martirosyan, Phys. of Atom. Nucl. 59 (1996) 1221
5. I.L. Grach et al., Nucl. Phys. B502 (1997) 227
6. J. Bjorken et al., Nucl. Phys. B371 (1992) 111
7. S. Kotkovsky, Pis’ma v ZhETP 65 (1997) 734; S. Kotkovsky et al., hep-ph/9712543
8. F. Coester, Prog. Part. Nucl. Phys. 29 (1992) 1
9. See e.g. N.B. Demchuk et al., Phys. Lett. B409 (1997) 2152 and references therein quoted
10. V.O. Galkin et al., Yad. Fiz. 55 (1992) 2175 (case B); I.M. Narodetskii et al., J. Phys., G18 (1992) 2175 (case C); D. Scora, N.Isgur, Phys. Rev. D52 (1995) 2783 (case D); S. Godfrey, N.Isgur, Phys. Rev., D32 (1985) 185 (case E)
11. ALEPH collaboration: contribution PA05-059 to the XXVIII Int. Conf. on High Energy Physics, Warsaw, Poland, 1996
12. S. Glenn, talk presented at the Meeting of the APS, Columbus, Ohio, 18-21 March 1998
13. R. Barate et al., CERN-EP/98-044