Analysis of the scalar doubly charmed hexaquark state with QCD sum rules

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Abstract In this article, we study the scalar-diquark–
diquark–scalar-diquark type hexaquark state with the
QCD sum rules by carrying out the operator product expan-
sion up to the vacuum condensates of dimension 16. We
obtain a lowest hexaquark mass of $6.60^{+0.12}_{-0.09}$ GeV, which can be confronted with the experimental data in the future.

1 Introduction

In the past years, a number of new charmonium-like states
have been observed; some are excellent candidates for the
exotic states, such as tetraquark states and molecular states,
and the spectroscopy of the charmonium-like states has
attracted much attention [1]. The QCD sum rules play an
important role in assigning those new charmonium-like states
[2–12].

The scattering amplitude for one-gluon exchange is pro-
portional to

$$t_{ij}^{a}t_{kl}^{a} = -\frac{1}{3} (\delta_{ij} \delta_{kl} - \delta_{il} \delta_{kj}) + \frac{1}{6} (\delta_{ij} \delta_{kl} + \delta_{il} \delta_{kj}),$$

where the $t^{a}$ are the generators of the $SU_{c}(3)$ gauge group.
The negative sign in front of the antisymmetric antitriplet
indicates the interaction is attractive, while the positive sign
in front of the symmetric sextet indicates the interaction is
repulsive. The attractive interaction of one-gluon exchange
favors formation of the diquarks in color antitriplet $\bar{3}_{c}$, fla-
vor antitriplet $\bar{3}_{f}$ and spin singlet $1_{s}$ or flavor sextet $6_{f}$
and spin triplet $3_{c}$ [13,14]. The color antitriplet diquarks
$q_{i}^{c} q_{j}^{c}$ have five structures in Dirac spinor space, where
$C_{T} = C_{Y5}, C, C_{Y_{L}Y_{S}}, C_{Y_{L}}$ and $C_{\sigma_{\mu\nu}}$ for the scalar,
psuedoscalar, vector, axialvector and tensor diquarks, respec-
tively. The calculations based on the QCD sum rules indi-
cate that the favored configurations are the $C_{Y5}$ and $C_{Y_{L}}$
diquark states have almost degenerate masses [15,16]. We
can construct the lowest tetraquark states by the $C_{Y_{L}}$
and $C_{Y_{L}Y_{S}}$ diquark states and antidiquark states, for example, the
$Z_{c}(3900)$ can be tentatively assigned to the ground state
$C_{Y_{L}} \otimes \gamma_{5} C = C_{Y_{L}} \otimes \gamma_{5} C$ type tetraquark state [11].
The diquark–antidiquark type tetraquark states have been stud-
ied extensively with the QCD sum rules.

In the QCD sum rules for the four-quark states, the largest
power of the QCD spectral densities $\rho(s) \propto s^{4}$, the integral
$$\int_{\Lambda_{QCD}^{2}}^{\infty} ds \rho(s) \exp\left(-\frac{s}{T^{2}}\right)$$ converges slowly, the pole domi-
nance condition is difficult to satisfy, where the $T^{2}$ is the
Borel parameter. In previous work, we study the energy-scale
dependence of the QCD sum rules for the hidden-charm and
hidden-bottom tetraquark states and molecular states for the
first time, and we suggest a formula,

$$\mu = \sqrt{M_{X/Y/Z}^{2} - (2M_{Q})^{2}},$$

with the effective mass $M_{Q}$ to determine the energy scales
of the QCD spectral densities [11,12,18–20], where the $X, Y, Z$
denote the tetraquark states and molecular states. The formula enhances the pole contributions considerably.

In this article, we extend our previous work to the study of
the scalar hexaquark state $uuudcc$ with the QCD sum rules
in detail. We construct the scalar-diquark–scalar-diquark–
scalar-diquark type current, which is supposed to couple
potentially to the lowest hexaquark state. In the QCD sum
rules for the six-quark states, the largest power of the QCD
spectral densities $\rho(s) \propto s^{7}$, the pole dominance condition
is more difficult to satisfy compared to the QCD sum rules
for the four-quark states. We use the energy-scale formula to
enhance the pole contributions.

The article is arranged as follows: we derive the QCD
sum rules for the mass and pole residue of the scalar doubly
charmed hexaquark state. In Sect. 2; in Sect. 3, we present
the numerical results and discussions; and Sect. 4 is reserved
for our conclusion.

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2 The QCD sum rules for the scalar doubly charmed hexaquark state

In the following, we write down the two-point correlation function $\Pi(p)$ in the QCD sum rules,

$$\Pi(p) = i \int d^4x e^{ip\cdot x} \langle 0|T \left( J(x) J^T(0) \right) |0 \rangle,$$

where

$$J(x) = e^{abc} e^{a} e^{i} j^{i} b^{i} e^m n^m U^T_{i} (x) C \gamma_5 \bar{d}_j (x) U^T_{k} (x) C \gamma_5 c_i (x) \times \frac{d^m T(x) C \gamma_5 C_{n}}{d^m(x)},$$

the $a, b, c, i, j, k, l, m, n$ are color indices, the $C$ is the charge conjugation matrix. We construct the scalar-diquark–scalar-diquark–scalar-diquark type current $J(x)$ to interpolate the lowest hexaquark state $Z_{cc}^{++}$. At the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J(x)$ into the correlation function $\Pi(p)$ to obtain the hadronic representation [21–23], and isolate the ground state contribution,

$$\Pi(p) = \frac{\lambda^2 Z}{M^2_Z - p^2} + \ldots,$$

where the pole residue $\lambda Z$ is defined by $\langle 0|J(0)|Z_{cc}^{++}(p)\rangle = \lambda Z$.

In the following, we briefly outline the operator product expansion for the correlation function $\Pi(p)$ in perturbative QCD. We contract the $u, d$ and $c$ quark fields in the correlation function $\Pi(p)$ with the Wick theorem, and we obtain the result

$$\Pi(p) = -i e^{abc} e^{a} e^{i} j^{i} b^{i} e^m n^m \int d^4x e^{ip\cdot x} \left[ \begin{array}{c} \gamma_5 D_{jj'}(x) \gamma_5 C U^T_{i'i'}(x) C \\ \gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \\ \gamma_5 C_{nn'}(x) \gamma_5 C D^T_{mm'}(x) C \\ -\gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \\ -\gamma_5 C_{nn'}(x) \gamma_5 C D^T_{mm'}(x) C \\ \gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \\ \gamma_5 C_{nn'}(x) \gamma_5 C D^T_{mm'}(x) C \\ -\gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \\ \gamma_5 C_{nn'}(x) \gamma_5 C D^T_{mm'}(x) C \\ -\gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \\ \gamma_5 C_{nn'}(x) \gamma_5 C D^T_{mm'}(x) C \\ \gamma_5 C_{ii'}(x) \gamma_5 C U^T_{kk'}(x) C \end{array} \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{ii'}(x) \gamma_5 U^T_{i'i'}(x) \gamma_5 C D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 U^T_{i'i'}(x) \gamma_5 C U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 C D^T_{jj'}(x) \gamma_5 U^T_{i'i'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 U^T_{i'i'}(x) \gamma_5 C D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 C U^T_{i'i'}(x) \gamma_5 C D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 C U^T_{i'i'}(x) \gamma_5 C D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

$$+ \text{Tr} \left[ \gamma_5 C C^T_{i'i'}(x) \gamma_5 D^T_{jj'}(x) \gamma_5 C C^T_{nn'}(x) \times \gamma_5 U^T_{kk'}(x) \right]$$

where the $U_{ij}(x)$, $D_{ij}(x)$ and $C_{ij}(x)$ are the full $u$, $d$ and $c$ quark propagators, respectively [23, 24], the $U_{ij}(x)$ and $D_{ij}(x)$ can be written as $S_{ij}(x)$,

$$S_{ij}(x) = \frac{i \delta_{ij} \kappa}{2 \pi^2 \kappa^4} - \frac{\delta_{ij} (\bar{q} g_s \sigma G q)}{12} - \frac{i g_s G_a^a T^a_{ij} (\sigma_{ab}) \kappa}{32 \pi^2 \kappa^4} - \frac{1}{8} (\bar{q} J(\sigma_{\mu \nu}) q) \kappa_{\mu \nu} + \ldots,$$

$$C_{ij}(x) = i \left( \frac{2 \pi}{\kappa} \right)^2 \int d^4k e^{-ik\cdot x}$$

$$\left\{ \begin{array}{c} \delta_{ij} - \frac{g_s G_a^a T^a_{ij} (\sigma_{ab}) (k + m_c) + (\bar{k} + m_c) \kappa_{ab}}{4 (k^2 - m_c^2)^2} \\ \frac{g_s^2 (t^{a} t^{b})_{ij} G_a^a G_{b'}^b (f_{ac} \mu \nu + f_{ab} \mu \rho + f_{ab} \nu \rho)}{4 (k^2 - m_c^2)^5} + \ldots \end{array} \right\},$$

and $t^n = \frac{\lambda^n}{\kappa^4}$; the $\lambda^n$ are the Gell-Mann matrix [23]. Then we compute the integrals both in coordinate space and in momentum space, and we obtain the correlation function $\Pi(p)$ at the quark level, therefore the QCD spectral density through dispersion relation. In Eq. (7), we retain the term $\langle \bar{q} g_s \sigma_{\mu \nu} q \rangle$ originates from the Fierz rearrangement of the $(\bar{q} J q)$ to absorb the gluons emitted from other quark lines to form $(\bar{q} j g_s G_a^a T^a_{ij} \sigma_{\mu \nu} q)$ so as to extract the mixed condensate $(\bar{q} g_s \sigma G q)$ and squared mixed condensate $(\bar{q} g_s \sigma G q)^2$, which play an important role in determining the Borel window; see the typical Feynman diagrams shown in Figs. 1 and 2. It is straightforward but very difficult to calculate those diagrams.
In this article, we carry out the operator product expansion to the vacuum condensates up to dimension-16, and we take into account the vacuum condensates which are vacuum expectations of the operators of the orders $\mathcal{O}(\alpha_s^k)$ with $k \leq 1$ consistently. The condensates $\langle \bar{q}q \rangle$, $(\alpha_s^3 GGG)$, $(\alpha_s^2 \sigma GG)$, $(\alpha_s GGG)$ have the dimensions 6, 8, 9, respectively, but they are the vacuum expectations of the operators of the order $\mathcal{O}(\alpha_s^3), \mathcal{O}(\alpha_s^2), \mathcal{O}(\alpha_s)$, respectively, and are discarded. Furthermore, the condensates $(\bar{q}q)(\alpha_s^2 GGG)$, $(\bar{q}q)^2(\alpha_s GGG)$, $(\bar{q}q)^3(\alpha_s GGG)$ have dimensions 7, 10, 13, respectively, and they are the vacuum expectations of the operators of the order $\mathcal{O}(\alpha_s^3)$; however, they play a minor role and are neglected [11,12,18–20].

Once the QCD spectral density is obtained, we can take the quark–hadron duality and perform a Borel transform with respect to the variable $P^2 = -p^2$ to obtain the following QCD sum rule:

$$\lambda_Z^2 \exp \left( \frac{M_Z^2}{T^2} \right) = \int_{4m_e^2}^{\infty} d\rho (s) \exp \left( -\frac{s}{T^2} \right),$$

where

$$\rho(s) = \rho_0(s) + \rho_2(s) + \rho_4(s) + \rho_5(s) + \rho_6(s) + \rho_8(s) + \rho_9(s) + \rho_{10}(s) + \rho_{11}(s) + \rho_{12}(s) + \rho_{13}(s) + \rho_{14}(s) + \rho_{16}(s),$$

for $\rho_0(s)$,

$$\rho_0(s) = \frac{1}{183500800\pi^{10}} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \frac{y(z(1-y-z)^5)}{\lambda^2 \exp \left( \frac{M_Z^2}{T^2} \right)}$$

where

$$\rho_2(s) = \frac{m_e^6}{491520\pi^8} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz (s - \bar{m}_c^2)^4 Y^4(1-y-z)^4$$

and

$$\rho_4(s) = -\frac{m_e^8}{7864320\pi^8} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \left( s - \bar{m}_c^2 \right)^3 \left( 3s - \bar{m}_c^2 \right)$$

and

$$\rho_5(s) = \frac{m_e^{10}}{35389440\pi^8} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \left( 1 - y - z \right)^5 \left( s - \bar{m}_c^2 \right)^3$$

and

$$\rho_6(s) = \frac{m_e^{12}}{47185920\pi^8} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2} + \frac{1}{z^2} \right) (1 - y - z)^5 \left( \frac{s - \bar{m}_c^2}{s - \bar{m}_c} \right)^3$$

and

$$\rho_8(s) = -\frac{m_e^{14}}{183500800\pi^{10}} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2} + \frac{1}{z^2} \right) (1 - y - z)^5 \left( \frac{s - \bar{m}_c^2}{s - \bar{m}_c} \right)^3$$

and

$$\rho_{10}(s) = \frac{m_e^{16}}{35389440\pi^8} \int_{y_f}^{y_i} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2} + \frac{1}{z^2} \right) (1 - y - z)^5 \left( s - \bar{m}_c^2 \right)^3.$$
\[\rho_s(s) = -\frac{91 m_c^2 \langle \bar{q} g s G q \rangle}{4718592 \pi^8} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (s - m_c^2) \left( s - m_c^2 \right)^3 (3s - m_c^2) \]

\[\rho_5(s) = -\frac{91 m_c^2 \langle \bar{q} g s G q \rangle}{4718592 \pi^8} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (s - m_c^2) \left( s - m_c^2 \right)^3 (3s - m_c^2) \]

\[\rho_6(s) = \frac{7 \langle \bar{q} q \rangle^2}{18432 \pi^6} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (1 - y - z) \left( s - m_c^2 \right)^2 (5s - 2m_c^2) \]

\[\rho_7(s) = \frac{119 m_c^2 \langle \bar{q} q \rangle \langle \bar{q} g s G q \rangle}{49152 \pi^6} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (1 - y - z) \left( s - m_c^2 \right)^2 \]

\[\rho_8(s) = -\frac{119 m_c^2 \langle \bar{q} q \rangle \langle \bar{q} g s G q \rangle}{147456 \pi^6} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (1 - y - z) \left( s - m_c^2 \right)^2 \]

\[\rho_9(s) = \frac{m_c^2 \langle \bar{q} q \rangle}{96 \pi^4} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (s - m_c^2) \left( 2s - m_c^2 \right) \]

\[\rho_{10}(s) = \frac{253 \langle \bar{q} g s G q \rangle^2}{393216 \pi^6} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (s - m_c^2) \left( 2s - m_c^2 \right) \]

\[\rho_{11}(s) = -\frac{283 m_c^2 \langle \bar{q} g s G q \rangle \langle \bar{q} q \rangle}{36864 \pi^4} \int_{y_f}^{y_i} dy \int_{z_f}^{z_i} dz \times (1 - y - z) \left( s - m_c^2 \right) \left( 3s - 2m_c^2 \right) \]
\[
\rho_{12}(s) = \frac{\langle \bar{q} q \rangle^4}{864 \pi^2} \int_{y_1}^{y_f} dy \, y(1-y) \times \left( 3s - 2\tilde{m}_c^2 \right) + \frac{m_c^2(m_c)^4}{96 \pi^2} \int_{y_i}^{y_f} dy, \\
\rho_{13}(s) = -\frac{355 m_c(m_c)^2}{110592 \pi^2} \int_{y_i}^{y_f} dy \times 1 - y \left( \frac{3}{2} \right) \left( s - \tilde{m}_c^2 \right) + \frac{313 m_c(m_c)^2}{196608 \pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \times 1 - y \left( \frac{3}{2} \right) \left( s - \tilde{m}_c^2 \right) + \frac{467 m_c(m_c)^2}{884376 \pi^2} \int_{y_i}^{y_f} dy \times 1 - y \left( \frac{3}{2} \right) \left( s - \tilde{m}_c^2 \right), \\
\rho_{14}(s) = -\frac{\langle q \bar{q} \rangle^3 \langle \bar{q} g_s \sigma G q \rangle}{432 \pi^2} \int_{y_1}^{y_f} dy \, y(1-y) \times \left[ 3 + \frac{13}{2} + \frac{5 s}{T^2} \right] s \left( s - \tilde{m}_c^2 \right) + \frac{11 \langle q \bar{q} \rangle^3 \langle \bar{q} g_s \sigma G q \rangle}{27648 \pi^2} \int_{y_1}^{y_f} dy \times \left[ 3 + \frac{13}{2} + \frac{5 s}{T^2} \right] s \left( s - \tilde{m}_c^2 \right) + \frac{185 \langle q \bar{q} \rangle^3 \langle \bar{q} g_s \sigma G q \rangle}{55296 \pi^2} \int_{y_1}^{y_f} dy \, s \left( s - \tilde{m}_c^2 \right), \\
\rho_{16}(s) = \frac{\langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{384 \pi^2} \int_{y_1}^{y_f} dy \, y(1-y) \times \left[ 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} + \frac{5 s^3}{3T^6} \right] \delta \left( s - \tilde{m}_c^2 \right) - \frac{1255 \langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{5308416 \pi^2} \int_{y_1}^{y_f} dy \delta \left( s - \tilde{m}_c^2 \right) + \frac{151 \langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{196608 \pi^2} \int_{y_1}^{y_f} dy \, s \frac{T^2}{T^2} \delta \left( s - \tilde{m}_c^2 \right), \\
\rho_{18}(s) = \frac{\langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{2187 \pi^4} \int_{y_1}^{y_f} dy \, y(1-y) \times \left[ 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} + \frac{5 s^3}{3T^6} \right] \delta \left( s - \tilde{m}_c^2 \right) - \frac{1255 \langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{5308416 \pi^2} \int_{y_1}^{y_f} dy \delta \left( s - \tilde{m}_c^2 \right) + \frac{151 \langle q \bar{q} \rangle^2 \langle \bar{q} g_s \sigma G q \rangle}{196608 \pi^2} \int_{y_1}^{y_f} dy \, s \frac{T^2}{T^2} \delta \left( s - \tilde{m}_c^2 \right). 
\]

3 Numerical results and discussions

We take the standard values of the vacuum condensates \( \langle q \bar{q} \rangle = -0.24 \pm 0.01 \text{GeV}^3 \), \( \langle \bar{q} g_s \sigma G q \rangle = m_0^2 \langle q \bar{q} \rangle \), \( m_0^2 = 0.8 \pm 0.1 \text{GeV}^2 \), \( \langle \bar{q} G q \rangle = (0.33 \text{GeV})^4 \) at the energy scale \( \mu = 1 \text{GeV} \) [21–23, 25], and we choose the mass \( m_c(m_c) = (1.275 \pm 0.025) \text{GeV} \) from the Particle Data Group [1]. Moreover, we take into account the energy-scale dependence of the input parameters,

\[
\langle \bar{q} q \rangle(\mu) = \left( \frac{\mu}{\mu_0} \right)^{\alpha_{\bar{q} q}(\mu)}, \\
\langle \bar{q} g_s \sigma G q \rangle(\mu) = \left( \frac{\mu}{\mu_0} \right)^{\alpha_{\bar{q} g_s \sigma G q}(\mu)}, \\
m_c(\mu) = \left( \frac{\mu}{\mu_0} \right)^{\alpha_{m_c}(\mu)}, \\
\alpha_{\bar{q} q}(\mu) = \frac{1}{b_0 \log t} \left[ 1 - b_1 \log t + b_2 (\log^2 t - \log t - 1) + b_3 \right],
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33 - 2n_f}{12\pi} \), \( b_1 = \frac{153 - 19n_f}{24\pi} \), \( b_2 = \frac{2857 - 903n_f + 25n_f^2}{128\pi^2} \), \( \Lambda = 213, 296 \) and 339 MeV for the flavors \( n_f = 5, 4 \) and 3, respectively [1], and we evolve all the input parameters to the optimal energy scale \( \mu \) to extract the mass of the \( Z_{cc}^{++} \).

In Refs. [11,12,18–20], we study the acceptable energy scales of the QCD spectral densities for the hidden-charm (hidden-bottom) tetraquark states and molecular states in the QCD sum rules for the first time, and we suggest an empirical formula \( \mu = \sqrt{M_{X_{cc}}^2/m_f - (2M_{Q})^2} \) to determine the optimal energy scales. The energy-scale formula enhances the pole contributions remarkably and works well. The energy-scale formula also works well in studying the hidden-charm pentaquark states [26]. In this article, we study the diquark–diquark–diquark type hexaquark state, the basic constituent
The pole contribution of the $Z^{++}_{cc}$ with variation of the Borel parameter $T^2$ are also diquarks, just like in the case of the diquark–antidiquark type tetraquark states [11,12,18]. So we extend our previous work to the study of the hexaquark state by taking the energy-scale formula

$$\mu = \sqrt{\frac{M_{X/Z}}{Y/Z} - (2M_c)^2}$$

with the updated value $M_c = 1.82$ GeV as a constraint [27].

Experimentally, there is no candidate for the doubly charged hexaquark state $Z^{++}_{cc}$ with the symbolic quark structure $uuddcc$. In the scenario of tetraquark states, the QCD sum rules indicate that the $Z_{c}(3900)$ and $Z(4430)$ can be tentatively assigned to the ground state and the first radial excited state of the axialvector tetraquark states, respectively [28], the $Y(3915)$ and $X(4500)$ can be tentatively assigned to the ground state and the first radial excited state of the scalar tetraquark states, respectively [29,30]. The energy gap between the ground state and the first radial excited state of the hidden-charm tetraquark states is about 0.6 GeV. Now we assume the energy gap between the ground state and the first radial excited state of the doubly charmed hexaquark states is about 0.6 GeV, and tentatively we take the continuum threshold parameter to be $\sqrt{s_0} = 7.1 \pm 0.1$ GeV for the energy scale $\mu = 5.5$ GeV, the predicted mass satisfies the energy-scale formula and the continuum threshold parameter satisfies our naive expectation. The pole contribution is about (26–41)%; the pole dominance condition is not satisfied; see Fig. 3. In fact, if we do not use the energy-scale formula, the pole contribution is much smaller. In Fig. 4, we plot the contributions of the vacuum condensates in the operator product expansion with variations of the Borel parameter $T^2$ for the value $\sqrt{s_0} = 7.1$ GeV. From the figure, we can see that the vacuum condensates of dimensions 10, 12, 13, 14, 16 play a minor role in the Borel window, the operator product expansion is well convergent. In calculations, we observe that the integral $\int^{s_0}_{4\pi} dx \rho(s) \exp \left(-\frac{s}{T^2}\right)$ is negative at the region $T^2 < 4$ GeV$^2$ for $\sqrt{s_0} = 7.1$ GeV. Although the vacuum condensates of dimensions 10, 12, 13, 14, 16 play a minor role in the Borel window, they play an important role in determining the Borel window. In Fig. 5, we plot the mass with variation of the Borel parameter $T^2$ by taking into account the vacuum condensates up to dimensions 16 and 10, respectively. From the figure, we can see that the predicted mass decreases monotonously with increase of the Borel parameter $T^2$ for the truncation $n \leq 10$, there appears no platform.

We take into account all uncertainties of the input parameters, and we obtain the values of the mass and pole residue of the $Z^{++}_{cc}$, which are shown explicitly in Figs. 6 and 7,

$$M_Z = 6.60^{+0.12}_{-0.09} \text{ GeV},$$

$$\lambda_Z = 7.64^{+1.17}_{-1.05} \times 10^{-3} \text{ GeV}.$$

(27)
Fig. 5 The mass of the $Z_{cc}^{++}$ with variation of the Borel parameter $T^2$, where the $D = 16$ and $D = 10$ denote the truncations in the operator product expansion.

Fig. 6 The mass of the $Z_{cc}^{++}$ with variation of the Borel parameter $T^2$

4 Conclusion

In this article, we construct the scalar-diquark–scalar-diquark–scalar-diquark type current to interpolate the scalar hexaquark state, and we study it with QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 16. In calculation, we take the energy-scale formula as a constraint to determine the energy scale of the QCD spectral density to extract the mass and pole residue. In the Borel window, the operator product expansion is well convergent, while the pole contribution is about (26–41)%.

We obtain the lowest hexaquark mass $M_Z = 6.60^{+0.12}_{-0.09}$ GeV, which can be confronted with the experimental data in the future, while the predicted pole residue can be used to study the strong decays of the hexaquark state with the three-point QCD sum rules.

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