\( \alpha' \)-Corrections and de Sitter Vacua - a Mirage?

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In this work we analyze the role of \( \alpha' \)-corrections to type IIB orientifold compactifications in Kähler moduli stabilization and inflation. In particular, we propose a model independent scenario to achieve non-supersymmetric Minkowski and de Sitter vacua for geometric backgrounds with positive Euler-characteristic and generic number of Kähler moduli. The vacua are obtained by a tuning of the flux superpotential. Moreover, in the one-modulus case we argue for a mechanisms to achieve model independent slow-roll.

I. INTRODUCTION

It is of great interest to find de Sitter vacua of supergravities and string theory from a phenomenological perspective as a small positive cosmological constant is observed experimentally, as well as from a fundamental theoretical point of view.

No-go theorems such as [1] suggest that those cannot be obtained from the lowest order in \( g_s \) and \( \alpha' \) of the ten and eleven-dimensional supergravity action, i.e. in string or M-theory. Where \( g_s \) is the string coupling constant and \( 2\pi \alpha' = \ell_s^2 \) with \( \ell_s \) being the string length. The recent swampland conjecture [2] may be evaded along those lines. It is thus of great interest to advance beyond the classical results [3]. In this work we analyze the role of the leading order \( \alpha' \)-corrections to the four-dimensional, \( \mathcal{N} = 1 \) effective actions of type IIB orientifold compactifications [4–10] in Kähler moduli stabilization [8, 9, 11]. We argue that due to the presence of the \( \alpha' \)-corrections Minkowski and de Sitter vacua may be obtained for Calabi-Yau orientifolds of positive Euler-characteristic.

The proposed scenario assumes that the complex structure moduli are stabilized beforehand by the flux superpotential [12–14] and additionally requires the vacuum value of the flux superpotential to be tuned to a specific value in relation to other topological quantities of the Calabi-Yau orientifold [15]. Whilst maintaining moderately large values a fine tuning of the flux superpotential engineers a small cosmological constant. Let us stress that the vacua are obtained model independently. However, we do not study explicit geometries in this work, but derive additional constraints on certain topological quantities of the geometric background.

Let us emphasize that the vacua generically are at large volume where higher-order \( \alpha' \) and \( g_s \)-corrections are under control and moreover non-perturbative effects to the superpotential are suppressed.

Secondly, we propose a mechanism in which the \( \alpha' \)-scalar potential exhibits slow-roll inflation, where the Kähler modulus plays the role of the inflaton [16, 17]. We discuss the one-modulus case explicitly and derive model independent results. To stabilize the overall volume in a Minkowski or de Sitter Minimum at the end of inflation we require the flux superpotential to be perturbed from its vacuum expectation value during inflation. Inflation ends as the flux superpotential acquires its vacuum expectation value and the Kähler modulus is stabilized in the resulting minimum discussed in the first part of this work. For simplicity we do not discuss a dynamical model of this process. Two possible alternatives occur to us. Firstly, the fluxes may tunnel from one vacuum configuration to another one close by. Secondly, the fluxes are fixed and the complex structure moduli are perturbed slightly from their minimum. We conclude that one may neglect those dynamical effects if the complex structure modulus perturbed is stabilized at a large complex structure point. Thus the single field approximation is valid and one may infer the observable quantities such as the spectral index \( n_s \) and the tensor to scalar ratio \( r \). For simplicity we work under the assumption that the flux superpotential perturbation is constant during inflation and then acquires its vacuum value rapidly. We focus on the regime \( n \approx 0.965 \) [18, 19]. For sixty e-foldings we find the tensor-to-scalar ratio to be \( r \leq 5.65 \cdot 10^{-4} \). Relaxing this constraint to fifty e-foldings one acquires \( r \leq 1.19 \cdot 10^{-3} \).

Let us shortly outline the structure of this note. In section II we review the relevant leading order \( \alpha' \)-corrections to four-dimensional \( \mathcal{N} = 1 \) super gravity obtained from type IIB orientifolds. We also provide a critical assessment on their status. In section III we argue for the Minkowski and de Sitter vacua in the one-modulus case, followed by section IV in which we show that Minkowski and de Sitter vacua can be obtained for geometric backgrounds with an arbitrary number of Kähler moduli.

Lastly, in section V we propose an inflationary scenario in the one-modulus case.

II. \( \alpha' \)-CORRECTED SCALAR POTENTIAL

Let us set the stage by reviewing the recent progress made in determining \( \alpha' \)-corrections to the four-dimensional \( \mathcal{N} = 1 \) scalar potential of IIB effective actions of string theory. At the time being three different sources are worth noting. The well known Euler-characteristic \( \alpha'^{19} \)-correction to the Kähler potential

\[
\xi = -\frac{\zeta(3)}{4} g_s \frac{-3}{2} \chi(Y_3), \tag{1}
\]
with \( \chi(Y_3) \) Euler-characteristic of the Calabi-Yau background \( Y_3 \). Note that it is of order \( O(\alpha'^3) \) and it depends on the Type IIB string coupling, \( g_s = e^{\phi_d} \) where \( \phi_d \) is the dilaton. It is obtained from the parent \( \mathcal{N} = 2 \) theory arising from compactification of type IIB on Calabi–Yau orientifolds [4, 5]. Secondly, evidence has been provided for the existence of a \( \alpha'^2 \)-correction to the Kähler coordinates utilizing F-theory based on a series of papers [6–8, 20–22]. This formulation of Type IIB string theory with space-time filling seven-branes at varying string coupling [23] captures the string coupling dependence in the geometry of an elliptically fibered Calabi–Yau fourfold with base \( B_3 \). Effective actions of F-theory compactifications have been studied using the duality with M-theory [14, 24]. Note that the type IIB Calabi-Yau threefold is a double cover of the base \( B_3 \) branched along the \( O7 \)-plane. Thus in particular one infers that \( \chi(Y_3) = \chi(B_3) \). The conjectured \( \alpha'^2 \)-correction to the Kähler coordinates [8] is found to be

\[
T_i = K_i + \gamma Z_i \log(V) , \quad \text{with} \quad Z_i = \int_{D_i} C , \quad (2)
\]

with \( T_i \) the Kähler coordinates, i.e. the complex structure on Kähler moduli space, and where \( i = 1, \ldots, h^{1,1}(B_3) \) and \( \gamma = -\frac{1}{2\pi} \). Note that other \( \alpha'^2 \)-corrections to the Kähler potential and coordinates do not lead to a breaking of the no-scale condition [8, 25]. Thus (2) is sufficient for the context of this work. Moreover, we denote the Kähler moduli fields as \( v^i \) and \( C \) is the curve corresponding to the self-intersection of stacks of \( D7 \)-branes and \( O7 \)-planes [7]. Furthermore, \( D_i \subset B_3 \) denotes divisors in the internal space. Note that the orientifold involution in IIB Calabi–Yau threefolds with additional \( O7 \) and \( O9 \)-planes acting on \( Y_3 \) projects the Kähler moduli space to \( H^{1,1}_\mathbb{R}(Y_3) \), i.e. the \((1,1)\)-forms that are even under the isometric involution [24], i.e. \( h^{1,1}(B_3) = h^{1,1}_\mathbb{R}(Y_3) \). Let us speculate on the type IIB origin of a logarithmic correction in the volume such as (2). When four-dimensional minimal supergravity is integrated out at one-loop on the circle the three-dimensional Kähler coordinates receive an analogous logarithmic correction in the radius [26]. One might expect that (2) arises by integrating out certain massive Kaluza-Klein states in a supergravity theory on Calabi–Yau orientifolds.

We summarize the other relevant topological quantities such as intersection numbers \( K_{ijk}, K_{ij}, K_i \) and the volume \( V \) in appendix A. Note that we have defined \( V \) to be dimensionless in units of \( (2\pi \alpha')^3 \), i.e. we measure units of length in units of \( \sqrt{2\pi \alpha'} \). The contribution to the F-term scalar potential of a four-dimensional \( \mathcal{N} = 1 \) supergravity theory is

\[
V_F = e^K \left( K_{ij} D^i W D^j W - 3|W|^2 \right) , \quad (3)
\]

covariant derivative given by

\[
D^i W = \frac{\partial K}{\partial \text{Re}T_i} W + \frac{\partial W}{\partial \text{Re}T_i} , \quad (4)
\]

The Kähler potential then results in

\[
K = -2\log \left( V + \xi \right) + \log \left( g_s \right) , \quad (5)
\]

In this work we consider the \( h^{2,1} \) complex-structure moduli as well as the dilaton to be stabilized by the flux superpotential [12] which in the vacuum then takes the constant value \( |W_0| > 0 \). Thus supersymmetry is broken in the vacuum. A critical assessment of this two step procedure is provided in e.g. [27].

Lastly, one encounters contributions to the four-dimensional scalar potential by considering four-derivative, \( \mathcal{N} = 1 \) supergravity [9] resulting in

\[
\delta V_F = -e^{2K} T_{ijkl} D^i W D^j W D^k W D^l W \quad (6)
\]

with the contribution to the IIB effective four-dimensional action to be

\[
\delta V_F = \frac{3 |W_0|^4 g_s}{4 V^3} \xi_2 \Gamma_i v^i , \quad \Gamma_i = \int_{D_i} c_2 , \quad \xi_2 = \frac{11}{576} \frac{1}{4\pi} g_s \frac{1}{2} , \quad (7)
\]

where \( c_2 \) is the second Chern-form of the Calabi–Yau orientifold which may be written as the square of a covariant object, thus \( \Gamma_i > 0 \). The precise form of (7) was inferred by dimensional reduction of ten-dimensional IIB \( R^2 \)-terms [28] on Calabi–Yau orientifolds in [10].

The scalar potential arising from the \( \alpha' \)-corrections (1), (2), (5) and (7) in the large volume limit then takes the form

\[
V_F = \frac{3 |W_0|^2 g_s}{4 V^3} \left( 3 \xi + \gamma Z_i v^i + \frac{|W_0|^2}{V} \Gamma_i v^i \right) , \quad (8)
\]

where we have defined \( \xi = \xi / 3 \). We refer to the large volume limit to the regime at large volumes \( V \) and weak string coupling such that higher-order \( \alpha' \) and \( g_s \)-corrections as well as non-perturbative instanton effects can be neglected. Let us emphasize that the correction (2) is intrinsically \( \mathcal{N} = 1 \) in contrast to (1) and (7).

Let us comment on the stability of the following scenarios in section III and IV in the light of [29]. The classical correction to the scalar potential vanishes due to the no-scale condition and thus the leading order \( g_s \) and \( \alpha' \)-correction determine the vacuum. Higher-order \( \alpha' \)-corrections are parametrically under control as one stabilizes the internal space at large volumes. Moreover the string coupling constant \( g_s \) may be achieved to be parametrically small thus higher-order string loop corrections can generically be neglected. However, as the perturbative corrections depend on the topological quantities of the geometric background higher-order \( g_s \)-corrections
may become relevant in certain vacua. Non-perturbative effects to the superpotential due to gaugino condensation or Euclidean D3-brane instanton effects play a sub-leading role as those are exponentially suppressed at large volumes. Let us close this section with critical remarks on the status of the $\alpha'$-corrections. A derivation of the well established $\alpha'^2$-correction (1) in a full-fledged $\mathcal{N} = 1$ set-up such as F-theory is absent [30]. Furthermore, the $\alpha'^2$-correction (2) has been conjectured recently in [8], in particular the factor $\gamma$ might be subject to change and thus its ultimate faith is yet to be determined in future work. Concerning, the correction (7) two caveats are to be named. Firstly, the four-derivative $\mathcal{N} = 1$ on-shell supergravity action remains elusive [9]. Moreover as pointed out in [10] relevant eight-derivative terms in ten-dimensions are not established to fix $\hat{\gamma}_2$ precisely. It would be of great interest in particular in the light of this work to investigate those topics.

### III. One Kähler Modulus Vacua

In this section we study the one-modulus case to lay the foundation for the generic scenario discussed in section IV. We show that the overall volume may be stabilized in a non-supersymmetric de Sitter and Minkowski minima by the scalar potential (8) for manifolds with $\chi(Y_3) > 0$. Note that as $\chi(Y_3) = 2h_{1,1} - 2h_{2,1}^3$ the geometric background needs to obey $h_{1,1}^2 > h_{2,1}^3$ i.e. $h_{1,1} < 1$. However, note that the orientifold involution projects the Kähler moduli space to $H_{1,1}^0(Y_3)$. Thus we consider backgrounds with $h_{1,1}^0 = 1$ in this section. In the one-modulus case the scalar potential (8) derives to

$$V_F = \frac{3|W_0|^2}{4V^3} \left( 3\hat{\chi} + 2\chi Z \sqrt[3]{\gamma} + \hat{\gamma}_2 |W_0|^2 \Gamma V^{-\frac{2}{3}} \right).$$

We argue for model independent Minkowski and de Sitter vacua which may be tuned by achieving the fluxes. The superpotential in the vacuum $|W_0|$ is to be fixed at specific values in relation to the topological $Z$, $\chi$ and $\Gamma$. Furthermore, one requires $Z < 0$. The vacuum is obtained for

$$\langle V \rangle_{\Omega=0} = \frac{8|\xi|^3}{\gamma |Z|^3}, \quad \langle V \rangle_{\Omega=1} = \frac{11.3906|\xi|^3}{\gamma |Z|^3},$$

$$|W_0|^2 = \frac{|\xi|^3}{\gamma^2 \sqrt{\frac{2}{\gamma^2} \Omega_2}} \left( 4 + \frac{2\pi}{176} \Omega^2 \right), \quad 0 \leq \Omega^2 \leq 1,$$

where $\Omega$ can be chosen freely. In the case of $\Omega = 0$ one finds a Minkowski minimum in (10), else-wise de Sitter minima. The local maximum of the potential is located at

$$V_{\text{max}}|_{\Omega=0} = \frac{15.3024|\xi|^3}{\gamma |Z|^3}.$$  

In figure 1 we illustrate the scalar potential exhibiting the discussed features. Note that as $\langle V \rangle \sim g_s^{-9/2}$ large volumes can be achieved easily at weak string coupling. Furthermore, the numerical value of $\gamma$ in (2) favours large volumes for moderate values of the topological quantities. Let us stress that the minimum is obtained by tuning the fluxes such that (11) is satisfied. The value of the potential in the minimum is

$$\langle V_F \rangle = f(\Omega) \cdot \frac{|\gamma Z|^7}{\gamma_2 \Gamma |\xi|^3} \sim g_s^9,$$

where on can solve for $f(\Omega)$ analytically. The explicit expressions for $f(\Omega)$ is long and non-illustrative thus we just note that $f(0) = 0$ and $f(1) = 0.000573574$. For weak string coupling small values of the potential in the minimum are natural as one concludes from (13). Let us next compare the gravitino mass with the string and Kaluza-Klein scale [31] for which one finds that

$$m_S \sim \langle V \rangle^{\frac{1}{2}}, \quad m_{KK} \sim \langle V \rangle^{-\frac{2}{3}}, \quad m_{3/2} \sim \frac{|W_0|}{\langle V \rangle}.$$  

One infers from (14) for the present scenario the relations

$$\frac{m_{3/2}}{m_S} = \sqrt{\frac{\gamma Z}{2\gamma_2 \Omega}} \sim g_s^{\frac{1}{2}}, \quad \frac{m_{KK}}{m_S} \sim g_s^3,$$

which follows from the scaling $|W_0| \sim g_s^{-2}$ as one infers from (11). To achieve a gravitino mass smaller than the Kaluza Klein-scale we note that $m_{3/2} / m_{KK} = (|\xi| / \gamma_2 \Gamma)^{1/2} < 1$, thus the topological quantities need to be accordingly as it scales with $\sim g_s^{-1/2}$.

Let us close this section with a short remark on the stability of the found vacua. The Minkowski vacuum constitutes a global minimum and is thus stable whilst the de-Sitter vacua are meta-stable. One can estimate the life time of the de Sitter vacua i.e. the inverse probability for decay in the runaway direction $V \to \infty$, for

FIG. 1: Minkowski and de Sitter vacua. The scalar potential $\langle V \rangle$ for the choice of the string coupling $g_s = 0.2$ and the topological number $\chi = \Gamma = 1$ and $Z = -1$. From below with $\Omega = 0, 0.5, 0.75, 1, 1.2$. The horizontal axis displays the volume $V / (2\pi\alpha')^3$ versus the scalar potential $V_F$ on the vertical axis.
small Ω to be
\[ \tau \sim \frac{1}{g_s} \cdot \text{Exp} \left( \frac{-2g_s^2}{2|\xi| \sqrt{2} \Gamma \Omega^{3/11}} \right) \sim \frac{1}{g_s} \cdot e^{g_s^2/4 \cdot \Omega^{3/11}} . \]  
(16)

From (16) one infers that large life times require a fine tuning of \( \Omega \ll 1 \) and thus of the superpotential i.e. the fluxes in (11). Let us emphasize that the proposed moduli stabilization scenario does not require non-perturbative effects but is achieved solely by the leading order \( \alpha' \)-effects.

IV. GENERIC KÄHLER MODULI SCENARIO

In this section we propose scenarios in which all Kähler moduli might be stabilized in a non-supersymmetric de Sitter and Minkowski minima for manifolds with \( \chi(Y_3) > 0 \). We argue for a model independent extremum and provide sufficient conditions on the topological quantities of the geometric background for the existence of a local minimum. Furthermore, the presented scenarios require a fine tuning of fluxes such that the value of the superpotential in the vacuum \( |W_0| \) exhibits a certain relation w.r.t. the topological quantities \( Z_i \) and \( \chi \). The stabilization is achieved by an interplay of the \( Z_i \)-correction, with the \( \alpha'^3 \) Euler-characteristic correction (1) to the Kähler potential \[4, 5\] and the correction to the scalar potential in the vacuum according to (25) if (20) constitutes an extremum of the scalar potential (8). By analyzing the matrix of second derivatives in the extremum one infers
\[ \left\langle \frac{\partial^2 V_F}{\partial v_i \partial v_j} \right\rangle = \frac{3|W_0|^2|\xi| g_s}{2 \langle \mathcal{V} \rangle^4} \left( \langle \mathcal{K}_i \rangle \langle \mathcal{K}_j \rangle - \mathcal{V} \langle \mathcal{K}_i \rangle + \mathcal{T}_{ij} \right) , \]  
(23)
with
\[ \mathcal{T}_{ij} = \frac{\xi}{\xi^2} \left( \gamma^2 \Lambda^4 Z_i Z_j - \Gamma_{ij} \right) . \]  
(24)

The objective is to argue that (23) is positive definite. Note that the positive definite metric on the Kähler moduli space is proportional to \( \langle \mathcal{K}_i \rangle \langle \mathcal{K}_j \rangle - \mathcal{V} \langle \mathcal{K}_i \rangle \), thus one remains to show the criterium for \( \mathcal{T}_{ij} \). For any vector within the Kähler cone \( v^i \) we find the sufficient condition that
\[ \mathcal{T}_{ij} v^i v^j > 0 \quad \text{if} \quad |Z_i| > \frac{1}{|\gamma| \Lambda^4} \Gamma_i , \]  
(25)
for all \( i = 1, \ldots, h^{1,1}_+(Y_3) \). Thus in geometric backgrounds in which (25) is satisfied one encounters a local Minkowski minimum for all Kähler moduli. One finds
\[ \Lambda^2 = \frac{2}{|\gamma|} \frac{\Gamma_i \langle v^i \rangle}{|Z_i| \langle v^i \rangle} , \]  
(26)
and thus the geometric condition (25) becomes
\[ |Z_i| > \frac{1}{2 \Gamma_i \langle v^i \rangle} \Gamma_i . \]  
(27)

A strong sufficient requirement for (27) to be satisfied is that \( \alpha_{\text{max}} < 2 \cdot \alpha_{\text{min}} \) where we have used that \( \Gamma_i = \alpha(i) |Z_i| \), for a positive vector with components \( \alpha(i) \in \mathbb{Q}_+ \). Thus we conclude that the requirements on
the geometric background are that \( \chi(Y_3) > 0, \ Z_i < 0 \) for all 4-cycles, and that the inequality (40) is valid. Moreover, one requires the existence of fluxes which can be fine-tuned as (21). Thus one infers by using (14) and (22) that
\[
\frac{m_{3/2}}{m_s} \sim \sqrt{\frac{|\gamma|}{2\gamma_2}}, \quad |Z_i|/\Gamma_i \sim g_3/s, \quad \frac{m_{KK}}{m_s} \sim g_3^2. \tag{28}
\]

The gravitino mass is to be smaller than the KK-scale as the effective supergravity theory is derived in a classical reduction. One infers that
\[
m_{3/2} < m_{KK} \quad \text{if} \quad \Gamma_i(v_0^i) > \frac{\hat{\xi}}{\gamma_2}. \tag{29}
\]

Let us conclude that this mechanism may lead to a stabilization of all four-cycles such that the overall volume is stabilized at sufficiently large value thus higher-order \( \alpha' \)-corrections are under control i.e. the vacuum may not be shifted.

\[\text{b. de Sitter vacua.}\quad \text{We henceforth consider a modification of the previous Minkowski vacuum solution such that it constitutes a de Sitter minimum of the scalar potential (8). We choose Kähler cone variables} \ v^i > 0 \ 	ext{such that the Kähler form (17) is positive. Next we argue that one obtains de Sitter vacua for} \ \chi(Y_3) > 0, \ \text{i.e.} \ \xi < 0 \ \text{at all four-cycle volumes} \ K_i \ \text{are stabilized at}
\]
\[
\langle K_i \rangle = \frac{|W_0|^2 \gamma_2}{|\xi|(1 + \Omega^2/8)} \left( \frac{\gamma A^2}{1 - \Omega^2} Z_i + \Gamma_i \right), \tag{30}
\]
\[\Lambda^2 = \frac{1}{|\xi|} \Gamma_i(v^i), \quad Z_i\langle v^i \rangle = \frac{|\xi|}{\gamma} \left( 2 + \frac{11}{8} \Omega^2 \right). \tag{31}
\]

where we have introduced the positive parameter
\[0 < \Omega^2 < \frac{2}{11}. \tag{32}\]

As \( \Gamma_i > 0 \), a sufficient condition for the positivity of the four-cycle volumes is that \( Z_i < 0 \) for all \( i = 1, \ldots, h^{1,1}(Y_3) \). Note that (30) and (31) constitute an implicit definition for \( \langle v^i \rangle \). We next assert that by tuning \( |W_0| \) one may always find a solution. Note that (30) only depends on (31) via a scaling. By using \( \langle v_0^i \rangle = \frac{1}{|W_0|} \langle v^i \rangle \) one infers that
\[K_{ijk}\langle v_0^j \rangle/\langle v_0^k \rangle = \frac{\gamma_2}{|\xi|(1 + \Omega^2/8)} \left( 2\gamma_3 \frac{\Gamma_i(v_0^i)}{Z_i(v_0^i)} Z_i + \Gamma_i \right), \tag{33}\]

with \( \gamma_3 = (1 + \frac{11}{8} \Omega^2)/(1 - \Omega^2) \) where one imposes
\[|W_0| = \frac{2(1 + \frac{11}{8} \Omega^2)|\xi|}{|\gamma| |Z_i|/\langle v_0^i \rangle}. \tag{34}\]

In particular, one infers that (31) can be satisfied by tuning the flux-superpotential according to (34) if (33) exhibits a solution for \( \langle v_0^i \rangle \). Let us next discuss relevant quantities in the de Sitter extremum. We infer
\[
\langle V_F \rangle = \frac{9\Omega^2}{8} \frac{|\xi| |W_0|^2}{(\lambda)^3} \sim g_3^9 > 0, \tag{35}
\]

with the volume in the vacuum
\[
\langle V \rangle = \frac{|W_0|^2 \gamma_2}{|\xi|(1 - \Omega^2)} \Gamma_i(v^i) \sim g_3^9, \tag{36}
\]

where we have used that \( |W_0| \sim g_3^{-2} \) and \( \langle v_0^i \rangle \sim g_3^{1/2} \). One infers from (35) that in the present scenario a small cosmological constant i.e. vacuum value \( \langle V_F \rangle \approx 1 \) is achieved naturally due the weakly coupled string regime \( g_s < 1 \). We expect however that the vacuum stability analysis of section III carries over to the generic moduli case and thus a fine-tuning of the fluxes such that \( \Omega^2 \approx 0 \) is to be required. Let us mention that anti-de Sitter vacua are obtained for \( \Omega^2 < 0 \). Furthermore, the positive overall volume may generically be stabilized at large values as seen from (36). We are next to show that the extremum (30) and (31) at hand is a local minimum. One computes the matrix of second derivatives in the extremum to be
\[
\left\langle \frac{\partial^2 V_F}{\partial v^i \partial v^j} \right\rangle = \frac{3|W_0|^2 |\xi| g_3}{2(\lambda)^4} \left( G_{ij} + T_{ij} \right), \tag{37}
\]
\[G_{ij} = \left( 1 - \frac{35}{8} \Omega^2 \right) \langle K_i \rangle \langle K_j \rangle - \left( 1 + \frac{1}{8} \Omega^2 \right) \langle V \rangle \langle K_{ij} \rangle, \tag{38}\]
\[T_{ij} = \frac{\gamma_2^2}{\xi(1 + \Omega^2/8)} \left( \frac{\gamma \Lambda^2}{(1 - \Omega^2)} Z_i Z_j - \Gamma_i \Gamma_j \right). \tag{39}\]

The objective is to show that (23) is positive definite. It was argued in [33] that \( \langle K_{ij} \rangle \) is of signature \( (1, h^{1,1}(Y_3)) \), i.e. it exhibits one positive eigenvalue in the direction of the vector \( \langle v^i \rangle \). As the pre-factor of \( \langle K_{ij} \rangle \) is negative and moreover \( \langle K_i \rangle \langle K_j \rangle \) is positive semi-definite, we need to show that \( (G_{ij} + T_{ij}) \langle v^i \rangle \langle v^j \rangle > 0 \) which leads to \( \Omega^2 < 2/11 \). Thus one remains to show the criterion for \( T_{ij} \) in generic direction. For any vector within the Kähler cone \( v^i \) we find the sufficient condition that
\[T_{ij} v^i v^j > 0 \quad \text{if} \quad |Z_i| > \frac{1 - \Omega^2}{2 + \frac{11}{8} \Omega^2} |Z_i|/\langle v_0^i \rangle \Gamma_i, \tag{40}\]

for all \( i = 1, \ldots, h^{1,1}(Y_3) \). One infers that a strong sufficient condition for (27) to be satisfied is that \( \alpha_{\text{max}} < (2 + 11 \Omega^2/8)/(1 - \Omega^2) \cdot \alpha_{\text{min}}, \) where we have used that \( \Gamma_i = \alpha(i)|Z_i|, \) for a positive vector with components \( \alpha(i) \in \mathbb{Q}_+. \) It remains to show that the matrix (37) is positive definite on the entire space, i.e. for all vectors \( \langle v^i \rangle + v^i \) where \( \langle K_i \rangle v^i_0 = 0 \) describes the orthogonality. Note that positivity of (39) is granted due to (40). Thus the non-vanishing contribution which could alter the bound on \( \Omega^2 \) however computes to a positive number \(-(1 + \Omega^2/8)\langle V \rangle \langle K_{ij} \rangle v^i_0 v^j_0 > 0, \) as \( \langle K_{ij} \rangle \) admits negative eigenvalues w.r.t. \( v^i_0. \)
Moreover, one concludes assuming (40) that (37) admits one negative eigenvalue, i.e. not stabilized direction and $h^1,1(Y_3) - 1$ positive eigenvalues for

$$\frac{2}{\Pi} < \Omega^2 < \frac{4}{\delta_5}, \quad (41)$$

which will be of relevance in section V. Thus we conclude that the requirements on the geometric background are that $\chi(Y_3) > 0$, $Z_i < 0$ for all 4-cycles, and that the inequality (40) is valid, and moreover the existence of fluxes which can be fine-tuned such that (34) is satisfied.

By using (14) one may compare the gravitino mass with the string and Kaluza-Klein scale. One infers that

$$m_{3/2} \sim g_s^{1/4}, \quad m_{3/2} \sim g_s^{1/2}, \quad m_{KK} \sim m_s \sim g_s^{3/4}. \quad (42)$$

The gravitino mass achieved needs to be smaller than the Kaluza Klein-scale. A concise analysis gives an analogous relation to (29) to be

$$m_{3/2} < m_{KK} \text{ if } \Gamma_i(v_i^0) > \frac{|\hat{\xi}|}{\gamma_2}(1 - \Omega^2). \quad (43)$$

Note that a small cosmological constant implies that $\Omega^2 \approx 0$ and thus (43) reduces to (29).

Let us conclude that all four-cycles are generically stabilized such that the overall volume is at large values if $\chi(Y_3) > 0$ and $Z_i < 0$ for all $i = 1, \ldots, h^1,1(Y_3)$. Thus higher-order $\alpha'$-corrections are under control. Moreover, small string coupling $g_s < 1$ assures correct hierarchies of scales (42) thus the vacuum is to be trusted. Additionally the constraints on the topological invariants (40) and (43) are to be suitable, i.e. this imposes additional restrictions on allowed geometric backgrounds.

V. A TOY MODEL FOR INFLATION

In this section we propose a scenario for inflation based on the one-modulus scalar potential discussed in section III. We noted that a minimum of the potential is obtained by tuning the flux in the vacuum such that (11) is satisfied for a choice of $0 \leq \Omega \leq 1$. However the resulting potential does not admit inflationary dynamics. Intriguingly, for values $\Omega > 1$ the potential exhibits features of slow-roll. To achieve a stable final vacuum state we propose the following scenario. During inflation the flux superpotential is perturbed from it vacuum state such that $\Omega > 1$. When inflation ends the flux-superpotential acquires its vacuum expectation value such that the inflaton is stabilized in the Minkowski or de Sitter minimum, i.e. $\Omega = 0$ and $\Omega \approx 0$, respectively. In other words we will assume that after a sufficient number of e-foldings, i.e. fifty to sixty the flux superpotential obtains its vacuum expectation value which determines the inflation process. In, particular in the model at hand the epsilon parameter stays small at high e-folds especially for $r < 10^{-4}$, thus we need to assume the above mechanism. Let us note that as we do not study the fluxes dynamically we refer to the described mechanism as scenario.

The one Kähler modulus with canonical normalized kinetic term is

$$V = \text{Exp} \left( \frac{\sqrt{3}}{2} \Phi \right), \quad \Phi = \frac{\alpha}{\sqrt{3}} \text{Log}(V), \quad (44)$$

where we have suppressed the $\alpha'$-corrections to the kinetic coupling as those lead to sub-leading effects $\sim 1/\sqrt{V}$. The inflationary potential (9) is thus expressed as

$$V_F = \frac{3}{2} g_s |W_0|^2 \text{Exp} \left( \frac{1}{\sqrt{12}} \Phi \right) \left( 3\xi e^{\sqrt{3} \Phi} + \gamma Z e^{\sqrt{3} \Phi} + \hat{\gamma}_2 |W_0|^2 \Gamma \right),$$

$$|W_0|^2 = \frac{|\hat{\xi}|^3}{\gamma^2 Z^2 \Gamma^2 \hat{\gamma}_2} \left( 4 + \frac{25}{176} (\Omega^2 + \delta) \right), \quad 0 < \delta \ll 1. \quad (45)$$

where $\Omega^2 = 1$ during inflation. Note that the inflationary potential is to good approximation displayed as the $\Omega = 1$ curve in figure 1. Let us next comment on the slow-roll parameters

$$\epsilon = \frac{1}{2} \left( \frac{V''}{V} \right)^2, \quad \eta = \frac{V'''}{V} \frac{V''}{V}, \quad (46)$$

with $V_F = \partial_\phi V_F$, and $V''_F = \partial_\phi \partial_\phi V_F$ the partial derivatives w.r.t. the inflaton. To discuss the slow roll parameters it is convenient to choose coordinates which reflect the minimum and maximum of the potential in the vacuum i.e. $\Omega \approx 0$ given by (10) and (12). In such a variable $\phi = (|\gamma Z|/|\xi|)^3 \cdot \text{Exp}(\sqrt{3} \Phi/2)$ the slow roll parameters (46) of the potential (45) become

$$\epsilon = \frac{1}{24} \left( 11 \hat{\delta} - 27 \hat{\phi}^{2/3} + 8 \hat{\phi} \right)^2, \quad (47)$$

$$\eta = \frac{16}{3} + \frac{19 \hat{\delta} - 17 \hat{\phi}^{2/3}}{4 (\hat{\delta} - 3 \hat{\phi}^{2/3} + \hat{\phi})}, \quad (48)$$

where $\hat{\delta} = 4 + \frac{25}{176} (1 + \delta)$. Note that for $\delta = 0$ (47) vanishes at $\phi = 11.3906$. We encounter for $\delta > 0$ that the point of horizon exit $\phi_e$ is in this region. Furthermore, note that (47) and (48) do not depend on the topological quantities of the geometric background and are thus model independent. The main cosmological observables of interest are the spectral index and the tensor-to-scalar ratio evaluated at horizon exit denoted by the asterisk which are given by

$$n_s = 1 + 2 \eta_s - 6 \epsilon_s \quad \text{and} \quad r = 16 \epsilon_s, \quad (49)$$

respectively, i.e. $\epsilon_s = \epsilon(\phi_e)$. The number of e-foldings in between $\phi_e$ and the end of inflation is

$$N_e = \int_{\phi_e}^{\phi_{end}} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} \quad (50)$$
as we find $\phi_{\text{end}} > \phi_*$. The end of inflation is marked when the $\epsilon$-parameter fails to be much smaller than one \cite{34}. As we do not discuss a dynamic model for the fluxes acquiring their vacuum value, we simply assert that $\delta \to 0$ and $\Omega^2 \to 0$ for Minkowski or for de Sitter some small value $\Omega^2 \ll 1$. We assume this process to be instantaneous to determine inflation such that $N_e = 60$ is obtained, see figure 2. Note that the field displacement is sufficiently small such that $\phi_{\text{end}}$ is well below the maximum of the barrier which forms at $\phi \approx 15.3$ as seen from (12). This constitutes a necessary requirement for stabilization of the modulus in the minimum. Furthermore, the density perturbation amplitude at the horizon exit is to matched with the observed value to give

$$\mathcal{A}_{\text{COBE}}(\phi_*) = \frac{V_*^3}{(V^*_F)^2} = 2.7 \cdot 10^{-3},$$

which evaluated for the potential (8) results in

$$\frac{54 g_* \gamma Z |W_0|^2 V_*^{-\frac{11}{3\sqrt{3}}} (V_*^{\frac{1}{3\sqrt{3}}} - \frac{3\sqrt{3}}{\gamma} |\xi|^2 V_*^{\frac{2}{3\sqrt{3}}} + V_0^2) |^{\frac{3}{2}}}{2.7 \cdot 10^{-3} \left(8 V_*^{\frac{1}{\sqrt{3}}} - \frac{27}{\gamma^2} |\xi|^2 V_*^{\frac{2}{\sqrt{3}}} + 11V_0^2 \right)^2} = 1,$$

where $V_*$ is the volume at the horizon exit point and $V_0 = \delta \cdot |\xi|^2 / \gamma^2 Z^3$. Note that (52) may be satisfied by stabilizing the dilaton in the vacuum accordingly i.e. by the string coupling constant. For the exemplary values $Z = -3$ and $\chi = \Gamma = 1$ one infers from (52) that $g_* \approx 0.27$.

Let us next estimate the possible error resulting from the assumption that the complex structure moduli are perturbed from their minimum of the flux induced potential during inflation. In a large complex structure point generic statements independent of the number of complex structure moduli and the geometry is possible \cite{35,36}. The superpotential to good approximation is given by

$$W \sim K(\tilde{Y}_3)_{\alpha\beta\gamma} z^\alpha z^\beta z^\gamma,$$

$$K = -\log\left(\frac{1}{3} K(\tilde{Y}_3)_{\alpha\beta\gamma} \text{Re} z^\alpha \text{Re} z^\beta \text{Re} z^\gamma \right),$$

where $\tilde{Y}_3$ is the mirror dual Calabi-Yau threefold to $Y_3$ and thus $s = 1, \ldots, h^{2,1}(Y_3)$. The masses of the complex structure moduli in a non super symmetric vacuum were shown to be $\mathcal{O}(m_{3/2})$ \cite{36}. Thus we assert that the complex structure modulus perturbed from its minimum is to admit a mass of the same order as the gravitino mass. With the Hubble scale $H = \sqrt{V F/3}$ during inflation and (14) one computes

$$\frac{H}{m_{3/2}} \approx 0.039 g_* \frac{\frac{1}{3} |\gamma Z|^{\frac{3}{2}}}{|\xi|} \sim g_*^2.$$

One infers from (54) that for weak string coupling and moderate values $Z/\chi$ of the geometric background $H / m_{3/2} \ll 1$. Furthermore, one estimates from (53) for the displacement of $\phi_*$ the complex structure field with canonical kinetic term that $\delta \phi_* / (\phi_*^2) \ll 1$. We conclude that the perturbation of the massive complex structure field from the minimum is small, i.e. the single field inflation approximation is applicable.

Note that in the generic moduli case one encounters an analogous scenario. In the regime (41) a single effective direction in moduli space is not stabilized which plays the role of the inflaton. We again assume that the complex structure moduli are perturbed from their vacuum expectation value in (34) during inflation such that the lower bound (41) is appropriately obtained. When inflation ends the fluxes acquire their vacuum expectation value and all the Kähler moduli are thus stabilized in the resulting minimum $\Omega^2 = 0$ for Minkowski and $\Omega^2 \approx 0$ for de Sitter, as discussed in section IV. A more concise analysis of the generic moduli case is desirable.

Let us conclude with some remarks. Firstly, at the end of inflation the modulus is to be stabilized in the potential. The success of this endeavor depends on details of the inflation process. In particular, on the value of $\phi_{\text{end}}$ and the finite transmission amplitude towards the runaway direction $V \to \infty$, which is expected to be maximal right after inflation ends. Such an analysis might reduce the obtained region of the tensor-to-scalar ratio. Secondly, all the scenarios in this work are crucially bound to the existence of the $\alpha'$-corrections. However, their ultimate faith is still under investigation. Let us end on a positive note. It would be of great interest to realize the present scenarios in explicit models, i.e. geometric backgrounds with suitable topological quantities.
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Appendix A: Details

The intersection numbers of $Y_3$ are given by

\[
K_{ijk} = \int_{B_3} \omega_i \wedge \omega_j \wedge \omega_k , \quad K_{ij} = K_{ijk}v^k,
\]

\[K_i = \frac{1}{2} K_{ijk}v^j v^k , \quad \nu = \frac{1}{3!} K_{ijk}v^j v^k v^k . \tag{A1}\]

where \(\{\omega_i\}\) are the $h^{1,1}(Y_3)$ harmonic $(1,1)$-forms.

[1] J. M. Maldacena and C. Nunez, “Supergravity description of field theories on curved manifolds and a no go theorem,” *Int. J. Mod. Phys.* A16 (2001) 822–855, hep-th/0007018. [182(2000)].

[2] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, “De Sitter Space and the Swampland,” 1806.08362.

[3] M. Cicoli, S. De Alwis, A. Maharana, F. Muia, and F. Quevedo, “De Sitter vs Quintessence in String Theory,” *Fortsch. Phys.* 2018 (2018) 1800079, 1808.08967.

[4] K. Becker, M. Becker, M. Haack, and J. Louis, “Supersymmetry breaking and alpha-prime corrections to flux induced potentials,” *JHEP* 06 (2002) 060, hep-th/0204254.

[5] F. Bonetti and M. Weissenbacher, “The Euler characteristic correction to the Khler potential revisited,” *JHEP* 01 (2017) 003, 1608.01300.

[6] T. W. Grimm, R. Savelli, and M. Weissenbacher, “On $\alpha$ corrections in N=1 F-theory compactifications,” *Phys. Lett.* B725 (2013) 431–436, 1303.3317.

[7] T. W. Grimm, J. Keitel, R. Savelli, and M. Weissenbacher, “From M-theory higher curvature terms to $\alpha'$ corrections in F-theory,” *Nucl. Phys.* B903 (2016) 325–359, 1312.1376.

[8] M. Weissenbacher, “F-theory Vacua and $\alpha'$-Corrections,” 1901.04758.

[9] D. Cuipe, J. Louis, and A. Westphal, “Higher-Derivative Supergravity and Moduli Stabilization,” *JHEP* 10 (2015) 004, 1505.03092.

[10] T. W. Grimm, K. Mayer, and M. Weissenbacher, “Higher derivatives in Type II and M-theory on Calabi-Yau threefolds,” 1702.08404.

[11] A. Westphal, “de Sitter string vacua from Kahler uplifting,” *JHEP* 03 (2007) 102, hep-th/0611332.

[12] S. Gukov, C. Vafa, and E. Witten, “CFT’s from Calabi-Yau four folds,” *Nucl. Phys.* B584 (2000) 69–108, hep-th/9906070. [Erratum: Nucl. Phys.B608,477(2001)].

[13] S. Sethi, “Supersymmetry Breaking by Fluxes,” 1709.03554.

[14] F. Denef, “Les Houches Lectures on Constructing String Vacua,” in *String theory and the real world: From particle physics to astrophysics. Proceedings, Summer School in Theoretical Physics, 8th Session, Les Houches, France, July 2-27, 2007*, pp. 483–610. 2008. 0803.1194.

[15] M. Rummel and A. Westphal, “A sufficient condition for de Sitter vacua in type IIB string theory,” *JHEP* 01 (2012) 020, 1107.2115.

[16] B. J. Broy, D. Cuipe, F. G. Pedro, and A. Westphal, “Starobinsky-Type Inflation from $\alpha'$-Corrections,” *JCAP* 1601 (2016) 001, 1509.00024.

[17] M. Cicoli, D. Cuipe, S. de Alwis, and F. Muia, “$\alpha'$ Inflation: moduli stabilisation and observable tensors from higher derivatives,” *JHEP* 09 (2016) 026, 1607.01395.

[18] Planck Collaboration, N. Aghanim et al., “Planck 2018 results. VI. Cosmological parameters,” 1807.06209.

[19] WMAP Collaboration, E. Komatsu et al., “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,” *Astrophys. J. Suppl.* 192 (2011) 18, 1001.4538.

[20] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “The effective action of warped M-theory reductions with higher derivative terms - part I,” *JHEP* 01 (2016) 142, 1412.5073.

[21] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “The effective action of warped M-theory reductions with higher-derivative terms - Part II,” *JHEP* 12 (2015) 117, 1507.00343.

[22] T. W. Grimm, K. Mayer, and M. Weissenbacher, “One-modulus Calabi-Yau fourfold reductions with higher-derivative terms,” *JHEP* 04 (2018) 021, 1712.07074.

[23] C. Vafa, “Evidence for F theory,” *Nucl. Phys.* B469 (1996) 403–418, hep-th/9602022.

[24] T. W. Grimm, “The $\alpha$-effective action of F-theory compactifications,” *Nucl. Phys.* B845 (2011) 48–92, 1008.4133.

[25] M. Cicoli, J. P. Conlon, and F. Quevedo, “Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications,” *JHEP* 01 (2008) 052, 0708.1873.

[26] D. Tong and C. Turner, “Quantum dynamics of supergravity on $\mathbb{R}^2 \times S^1$,” *JHEP* 12 (2014) 142, 1408.3418.

[27] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, “Stability of flux compactifications and the pattern of supersymmetry breaking,” *JHEP* 11 (2004) 076, hep-th/0411066.

[28] D. J. Gross and E. Witten, “Superstring Modifications of Einstein’s Equations,” *Nucl. Phys.* B277 (1986) 1.

[29] M. Dine and N. Seiberg, “Is the Superstring Weakly Coupled?”, *Phys. Lett.* B212 (1985) 299–302.

[30] R. Minasian, T. G. Pugh, and R. Savelli, “F-theory at order $\alpha'$,” *JHEP* 10 (2015) 050, 1506.06756.

[31] J. P. Conlon, F. Quevedo, and K. Suruliz,
“Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,” *JHEP* 08 (2005) 007, hep-th/0505076.

[32] R. Kallosh and A. D. Linde, “Landscape, the scale of SUSY breaking, and inflation,” *JHEP* 12 (2004) 004, hep-th/0411011.

[33] P. Candelas and X. C. de la Ossa, “Moduli space of Calabi-Yau manifolds,” *Nuclear Physics B* 355 (1991), no. 2, 455 – 481.

[34] P. J. Steinhardt and M. S. Turner, “Prescription for successful new inflation,” *Phys. Rev. D* 29 (May, 1984) 2162–2171.

[35] C. Brodie and M. C. D. Marsh, “The Spectra of Type IIB Flux Compactifications at Large Complex Structure,” *JHEP* 01 (2016) 037, 1509.06761.

[36] M. C. D. Marsh and K. Sousa, “Universal Properties of Type IIB and F-theory Flux Compactifications at Large Complex Structure,” *JHEP* 03 (2016) 064, 1512.08549.

[37] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “On M-theory fourfold vacua with higher curvature terms,” *Phys. Lett.* B743 (2015) 284–289, 1408.5136.

[38]