The possibility of detecting magnetic moment generated by ultrasound in polarized tissues

Yuan Xu, Elena Renzhiglova
Department of Physics, Ryerson University, ON M5B2K3, Canada
E-mail: yxu@ryerson.ca

Abstract. We propose a method to combine focused ultrasound and electromagnetic field to image the permittivity of biological tissues. It will be shown that magnetic moment can be generated in a polarized tissue by propagating ultrasound. The value of this magnetic moment is proportional to the relative permittivity of the tissue. If the magnetic moment is detectable by a pick-up coil, the described method has the potential for imaging the electrical permittivity of biological tissues. In this paper, we derive the expression for the ultrasound-generated magnetic moment and the voltage in the pick-up coil. We also provide an estimation of the voltage signal.

1. Introduction
1.1. Permittivity of biological tissues
Biological tissues possess unique electric properties, which have been successfully used in several imaging modalities. The current modalities, both clinically approved and under study, aim to image current density or conductivity of biological tissues [1, 2, 3]. These modalities can as well provide information on relative permittivity of biological tissues, which can be a reliable contrast mechanism for clinical imaging, due to the high variances of its value between different tissues [4].

Biological tissues have very large values of relative permittivity (on the order of $10^6$) at low frequency range. Their dielectric spectra have several dispersion regions noted as $\alpha$, $\beta$ and $\gamma$. The $\alpha$-dispersion observed at frequencies below 10 kHz may arise from counterion polarization around charged surfaces, such as cellular membranes, and from interfacial polarization on intracellular membrane systems and intercellular gap junctions [5, 6]. The region of $\alpha$-dispersion and below is characterized by the largest values of relative permittivity for a particular tissue.

$\beta$ dispersion occurs at intermediate frequencies and is due to the capacitive charging of cellular membranes, as well as those of membrane-bound intracellular bodies. Lastly, $\gamma$ dispersion is due to the polarization of tissue water.

The electric permittivity can be imaged in Electrical Impedance Tomography (EIT) [2], Magnetic Induction Tomography (MIT) [3] and Magnetic Resonance Electrical Impedance Tomography (MREIT) [2, 7]. At present, the resolution of the existing impedance imaging modalities varies from 10% of the electrode/coil array diameter in EIT/MIT to a few millimeters in MREIT.

In this paper, we consider the possibility of imaging the electric permittivity of the tissues by combining ultrasound and electric fields. We will show in the following section that a magnetic moment can be generated in a polarized dielectric sample (e.g., tissue) with ultrasound.
When the frequency of the electric field applied to the tissue is in the $\alpha$-dispersion range, biological tissues have huge permittivity and the magnetic moments induced by a diagnostic level ultrasound is estimated to be in the same order as the magnetic moment in a normal MRI imaging condition. The proposed method can be a potential mechanism for developing a low-cost high-spatial-resolution imaging modality of permittivity of biological tissues.

1.2. Theoretical formulation of the ultrasound-induced magnetic moment in a polarized tissue

Consider that a DC or low-frequency voltage with amplitude of $U_0$ and frequency $\omega_V$ is applied through a pair of electrodes to a piece of tissue (Fig. 1). The electric field will polarize the tissue and create an electric dipole volume density $P$ pointing to the right. Assume that a focused ultrasound field with frequency $\omega_U$ significantly larger than $\omega_V$ is propagating upward in the sample. The acoustic field causes the electric dipoles in the polarized tissue to vibrate vertically around their equilibrium positions with a velocity $V$ at the ultrasound frequency. As $\omega_U \gg \omega_V$, the electric dipole density $P$ can be assumed constant over the duration of the ultrasonic pulse. The positive charges will produce a current pointing upward, while the negative charges will produce a current with the same magnitude but pointing downward. Two currents pointing to opposite directions with the same amplitude will generate a magnetic moment. Since the dipoles vibrate around their equilibrium positions, the magnetic moment is oscillatory, and, therefore, it can be detected with a pick-up coil.

Now we will express the generated magnetic moment $M$ in terms of electric dipole density $P$ and vibration velocity $V$. For an arbitrary current distribution, the associated magnetic moment can be found as:

$$M = \frac{1}{2} \int_V (\mathbf{r} \times \mathbf{J}) \, d\mathbf{r},$$

where $\mathbf{J}$ is the current density at position $\mathbf{r}$ [2]. For each dipole, there are two current densities $\mathbf{J}_+ = nq_+ \mathbf{v}$ and $\mathbf{J}_- = nq_- \mathbf{v}$ associated with the displacement of the pair of charges $q_+$ and $q_-$ in a dipole, whose positions are $\mathbf{r}_+$ and $\mathbf{r}_-$, respectively. Now, assuming that $\mathbf{J}_- = -\mathbf{J}_+$ because of $q_+ = -q_-$, we have:
\[ M_{\text{tot}} = \frac{1}{2} \int_V (\mathbf{r}_+ \times \mathbf{J}_+ + \mathbf{r}_- \times \mathbf{J}_-) \, d\mathbf{r} = \frac{1}{2} \int_V (\mathbf{r}_+ \times \mathbf{J}_+ - \mathbf{r}_- \times \mathbf{J}_+) \, d\mathbf{r} \]
\[ = \frac{1}{2} \int_V \mathbf{J}_+ \times \mathbf{l}_p \, d\mathbf{r} = \frac{1}{2} \int_V \mathbf{v} \times nq_+ \mathbf{l}_p \, d\mathbf{r} \]
\[ = \frac{1}{2} \int_V \mathbf{v} \times \mathbf{P} \, d\mathbf{r} = \int_M \, d\mathbf{r}, \quad (2) \]

where \( \mathbf{P} = nq_+ \mathbf{l}_p \) is the electric dipole density, \( n \) is the volume number density of positive and negative charges, \( \mathbf{l}_p = \mathbf{r}_+ - \mathbf{r}_- \) is the separation between the positive and negative charges in a dipole and \( \mathbf{M} = \frac{1}{2} (\mathbf{v} \times \mathbf{P}) \) is the magnetic moment density. The magnetic moment \( \mathbf{M} \) can be detected by a pick-up coil, which is similar to the signal detection in MRI. Therefore, we can write the following expression for the voltage signal in our experiment [8]:
\[ U(t) = -\int_{\text{object}} \frac{\partial}{\partial t} \mathbf{M}(\mathbf{r}, t) \cdot \mathbf{B}_r(\mathbf{r}) \, d\mathbf{r} \quad (3) \]

where \( \mathbf{M}(\mathbf{r}, t) \) is the time-varying magnetic field that exists throughout the focused ultrasound beam inside the sample and \( \mathbf{B}_r \) is the magnetic field produced by a unit direct current in the coil.

2. Results

2.1. Simplified expression for the voltage signal

To estimate \( \mathbf{M} \), we assumed that the applied low-frequency voltage creates a homogeneous electric field whose amplitude is simply \( \frac{U_0}{d} \). Here \( U_0 \) is the amplitude of the voltage applied to the sample and \( d \) is the distance between the electrodes. This allows us writing the following expression for the electric dipole density:
\[ P \approx \frac{\epsilon_0 \epsilon_r U_0}{d}, \quad (4) \]

since \( \epsilon_r \gg 1 \) for biological tissue. Here, \( \epsilon_0 \) is the permittivity of vacuum, and \( \epsilon_r \) is the relative permittivity.

2.2. Comparison of magnetization moment density induced by ultrasound with that in MRI

For the estimations, we used the following values: \( \omega = 6.28 \text{ MHz}, \ U = 36 \text{ V}, \ \epsilon_r = 10^7 \) (10 Hz, muscle tissue, longitudinal direction [4]), \( d = 20 \text{ cm} \).

We can calculate magnetization in the sample induced by ultrasound. By substituting Eq. 4 in Eq. 2, for the parameters given above, the magnetization is \( 132 \text{ mA} \cdot \text{m}^2 \). This value can be compared to the value of the magnetization induced in MRI. For a spin system composed of hydrogen nuclei \( ^1\text{H} \) under magnetic field of strength \( B_0 \), the magnetization can be calculated as follows:
\[ M_0 = \frac{B_0 \gamma^2 h^2}{4kT} P_D, \quad (5) \]

where \( \gamma \) is the gyromagnetic ratio in rad \( \text{s}^{-1} \cdot \text{T}^{-1} \), \( h \) is the Planck’s constant, \( k \) is the Boltzmann’s constant, \( T \) is the temperature in \( ^\circ \text{K} \), \( P_D \) is the proton density, i.e. the number of hydrogen atoms in a unit volume [8]. Assuming \( \frac{\gamma}{2\pi} = 42.58 \text{ MHz} \cdot \text{T}^{-1} \), \( B_0 = 1.5 \text{ T}, \ T = 300 ^\circ \text{K} \) and \( P_D \) of \( 1.1 \cdot 10^5 \text{ M} \cdot \text{m}^{-3} \), we get a magnetic moment with a magnitude of \( 30 \text{ mA} \cdot \text{m}^2 \).
3. Discussion
In the described method, same RF coils as in MRI can be used to detect the ultrasound-induced magnetic moment. However, the signal detection described here has an important difference from signal detection in MRI due to the difference in its bandwidth. In MRI, the signals have very narrow bandwidths [8]. Therefore, the coils are working at resonance frequency to increase the sensitivity. In our experiments, to obtain a good spatial resolution, ultrasound signals have broad bandwidth. Therefore, we also need the resonance frequency of the coil to be above the signal spectrum to detect the broad band signal. If the coil’s resonance frequency is below the frequency of the signal, the coil behaves like a capacitor partially shunting the voltage dropping on the inductance. In comparison to EIT, MIT and MREIT, the advantage of our method is that it combines high resolution and low cost. The high resolution is obtained through the use of focused ultrasound to generate magnetic moment in a very localized volume inside the tissue. In addition, our method requires minimal electrical contact with the tissue in the case of signal detection with coil.

4. Conclusion and future work
We showed that an oscillating magnetic moment can be generated in a biological tissue by propagating ultrasound. Its value is proportional to the relative permittivity. We derived the expression for signal in the detection configuration of using a pick-up coil placed above the sample.

References
[1] Hasanov K, Ma A, Nachman A and Joy M 2008 IEEE T. Biomed. Eng. 27 1301
[2] Holder D 2005 Electrical impedance tomography: methods, history and applications (London: IOP) pp xxi, 239
[3] Zou Y and Guo Z 2003 Med. Eng. Phys. 25 79
[4] Miklavcic D, Pavselj N and Hart F 2006 Electric properties of tissues (New York: Wiley Encyclopedia of Biomedical Engineering, John Wiley and Sons, Inc)
[5] Gabriel C 2006 Dielectric properties of biological materials (Taylor Francis Group LLC) p 77
[6] Asami K 2007 J. Phys. D: Appl. Phys. 40 3718
[7] Sekino M Tatara S and Ohsaki H 2008 IEEE T. Magn. 44 4460
[8] Prince J L and Links J M 2006 Medical imaging signals and systems (Upper Saddle River: Pearson Prentice Hall) pp 381-388
[9] Hayder S, Hrbek A and Xu Y 2008 Physiol. Meas. 29 S41