The determining of the coefficient of safety of bearing ability of anisotropic bars in the general case of their complex resistance

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Abstract. The bars of any form made of a uniform anisotropic material are considered. Generally in the cross section of a bar all internal power factors (IPF) – three forces and three moments are other than zero. Values IPF are known from the solution of the corresponding task. The coefficient of a stock of bearing ability of a bar is defined by a way of comparison of known vector IPF \( \overrightarrow{R} \) with the corresponding required vector of durability \( \overrightarrow{R} \) in IPF space.

1. Introduction

In many science courses on resistance of materials [1-9], as a rule, special cases of complex resistance of timber are considered: oblique bending, eccentric compression (extension), bending with torsion. The influence of shear forces on the destruction of the bar under its complex resistance is often neglected, considering this influence as insignificant. As a rule, checking the strength of the bar is being conducted, comparing the greatest normal (or equivalent) voltage \( \sigma_{\text{max}} \), operating in a dangerous point of timber, with dangerous proper voltage \( |\sigma_u| \). In this article the technique of checking strength of anisotropic bars of arbitrary shape in the most general case of complex resistance is given, while remaining in the IPF space. The article is a continuation of work [10].

2. Basic part

Fig.1 shows the IPF of the effective cross-section bar: \( T_{11} \) – normal force; \( Q_{21}, Q_{31} \) – cross-axis forces; \( M_1 \) – torsion torque; \( M_2, M_3 \) – bending torques.

We believe that the vector of the IPF \( \overrightarrow{R} \{ T_{11}^*, Q_{21}^*, Q_{31}^*, M_1^*, M_2^*, M_3^* \} \) is known. The corresponding vector strength \( \overrightarrow{R} \{ T_{11}, Q_{21}, Q_{31}, M_1, M_2, M_3 \} \) and a factor of the bearing capacity of the considered section \( k = \frac{\overrightarrow{R}}{\overrightarrow{R}^*} \) must be defined (1).
On Fig. 2 the limiting surface $\Sigma$ in IPF space (surface strength) is schematically shown where $\overline{OA} = R^*$ is a known vector of IPF; $\overline{OB} = \hat{R}$ – the sought vector strength corresponding to the vector $R^*$. It is obviously, that vectors $R^*$ and $\hat{R}$ should lie on the same straight and be equally directed; their origin should coincide with the beginning of the space coordinates IPF.

Let the surface of strength for anisotropic material in space voltages $\sigma_{ij}(i, j = 1,3)$ be set by the equation

$$
\Phi(\sigma_{ij}) = A\sigma_{11}^2 + L\sigma_{12}^2 + K\sigma_{13}^2 + 2P\sigma_{11}\sigma_{12} + 2S\sigma_{12}\sigma_{13} + 2G\sigma_{13}\sigma_{11} + 2D\sigma_{11} + 2Q\sigma_{12} + 2T\sigma_{13} - 1 = 0
$$

(2)

In [10], based on the equations (2) and using the rigid-plastic model of deformable rigid body, the following parametric equations limit surface in the space IPF for anisotropic bars were received:

$$
T_{11} = \frac{1}{2\Delta} \left[ (\delta_{11}\dot{e}_{11} + 0.5\delta_{12}\dot{e}_{21} + 0.5\delta_{13}\dot{e}_{31})I_1 + (0.5\delta_{13}\dot{e}_{11} - \delta_{11}\dot{e}_{21})I_2 + \right. \\
+ (\delta_{11}\dot{e}_{31} - 0.5\delta_{13}\dot{e}_{11})I_3 \left. \right] \frac{\Delta_1}{\Delta} A_s; \ldots;
$$

(3)

$$
M_3 = \frac{1}{2\Delta} \left[ (\delta_{11}\dot{e}_{11} + 0.5\delta_{12}\dot{e}_{21} + 0.5\delta_{13}\dot{e}_{31})I_1 + 0.5(I_1\delta_{13} - I_5\delta_{12})\dot{e}_{11} - \\
-I_4\delta_{11}\dot{e}_{21} + I_5\delta_{11}\dot{e}_{31} \right] \frac{\Delta_1}{\Delta} S_3
$$

where $I_1 \div I_6$, $S_1$, $S_2$ are integrals on square $A_s$ of cross-section of a bar; $\Delta, \Delta_i, \delta_{ik} = \delta_{ki}(i, k = 1,3)$ – quantities dependent on the equation coefficient (2). Speeds $\dot{e}_{11}, \ldots, \dot{e}_{21}$ are in integrals $I_1 \div I_6$ [9].

The parameters in equations (3) are the relationships of the velocity vector components of generalized displacements $\bar{E}\{\dot{e}_{11}, \dot{e}_{21}, \dot{e}_{31}, \dot{e}_{11}, \dot{e}_{31}, \dot{e}_{21}\}$. In general it is impossible to exclude these options from equations (3), using the exact mathematical methods. Thus, the practical application of the equations (3) is inseparably linked with the use of the computer for calculations. On the given vector of generalized velocities of displacement, using equations (3), it is relatively easy to find the corresponding vector strength (for example, vectors $\overline{E}$ and $\overline{OB}$).

To have an idea about the size and shape of the marginal surface (3), it is to be built on the points of its typical cross-section. These sections may be helpful in determining the safety factor for the bearing capacity of the bars in some cases, of their complex resistance. The range of practical applications of equations (3) would be significantly wider than if it is possible to use them in determining the stock carrying capacity of anisotropic beams in the most general case it difficult resistance. The below method is suggested for solving this problem.

Components of collinear vectors satisfy the following equations:

$$
\frac{T_{11}^*}{T_{11}} = \frac{Q_{21}^*}{Q_{21}} = \frac{Q_{31}^*}{Q_{31}} = \frac{M_1^*}{M_1} = \frac{M_2^*}{M_2} = \frac{M_3^*}{M_3} = k
$$

(4)

Using (4), the following system of equations can be got:
\[ Q_{21}^* T_{11}^* - T_{11}^* Q_{21} = 0; \quad Q_{31}^* Q_{21} - Q_{21}^* Q_{31} = 0; \quad M_1^* Q_{31} - Q_{31}^* M_1 = 0; \]
\[ M_2^* M_1 - M_1^* M_2 = 0; \quad M_3^* M_2 - M_2^* M_3 = 0 \]

(5)

Putting the results from (3) into (5), we get

\[ Q_{21}^* \left( \frac{1}{2\Delta} \left( (\delta_1^i \dot{e}_{11} + \ldots) I_1 + \ldots - \frac{\Delta_1}{\Delta} A_k^i \right) \right) - T_{11}^* \left( \frac{1}{2\Delta} \left( (\delta_2^i \dot{e}_{11} + \ldots) I_1 + \ldots - \frac{\Delta_2}{\Delta} A_k^i \right) \right) = 0; \ldots; \]
\[ M_3^* \left( \frac{1}{2\Delta} \left( (\delta_1^i \dot{e}_{11} + \ldots) I_3 + \ldots - \frac{\Delta_1}{\Delta} S_2 \right) \right) - M_2^* \left( \frac{1}{2\Delta} \left( (\delta_1^i \dot{e}_{11} + \ldots) I_2 + \ldots - \frac{\Delta_1}{\Delta} S_3 \right) \right) = 0 \]

(6)

Since in this case only the direction of the vector \( \overrightarrow{E} \) is essential and its length may be arbitrary, as an additional to the system (6) we will adopt the following equation:

\[ \dot{e}_{11}^2 + \dot{e}_{21}^2 + \dot{e}_{31}^2 + \ddot{e}_{11}^2 + \ddot{e}_{21}^2 = 1 \]

(7)

The algorithm for determining the factor of anisotropic bars is the following:

1°. To define a vector \( \overrightarrow{IFP} \) \( \overrightarrow{R} \{ T_{11}^*, Q_{21}^*, Q_{31}^*, M_1^*, M_2^*, M_3^* \} \) in a dangerous section of the rod as a result of solution of the corresponding problem, for example, the methods of "Resistance of materials".

2°. Solving the set of equations (6), (7) relatively to find the vector associated with the vector \( \overrightarrow{R} \) of the strength of the associated law [11] (Fig.2).

3°. Substituting found in paragraph 2 vector components in equation (3), to identify the components of the strength vector \( \overrightarrow{R} \{ T_{11}, Q_{21}, Q_{31}, M_1, M_2, M_3 \} \).

4°. To check compliance with the conditions (4).

5°. According to the formula (1) determine the factor \( k \) anisotropic bars by method of limiting conditions.

**Some results of calculations.** Take the bar cross section of which is shown Fig. 3.

Here \( \xi_2, \xi_3 \) are the main central axis of the square cross, \( \xi_2', \xi_3' \) - section-arbitrary axis, dimensions are given in meters. The strength properties of material of the bar is described by the equation

\[ 0.2\sigma_{11}^2 + 30\sigma_{12}^2 + 60\sigma_{13}^2 - 0.1\sigma_{12}\sigma_{12} - \sigma_{12}\sigma_{13} - 0.2\sigma_{13}\sigma_{11} - 0.6\sigma_{11}^2 + 14\sigma_{12}^2 + 20\sigma_{13} - 1 = 0 \]

(8)

Here the voltages are attributed to a characteristic value \( \sigma_0 \), having the dimension of stress. In [8] for the timber, using (8) and (3), cross section limit surfaces were built (3) by coordinate planes \( T_{11} O M_1, T_{11} O M_2, T_{11} O M_3, M_1 O M_2, M_1 O M_3, M_2 O M_3, Q_{21} O Q_{31} \).

The table below lists the results obtained for illustrated in Fig. 3 section using the above-proposed algorithm. Here \( T_{11}^*, Q_{21}^*, Q_{31}^*, M_1^*, M_2^*, M_3^* \) are the known values of IFP.
$T_{11}, Q_{21}, Q_{31}, M_1, M_2, M_3$ - relevant components of the strength vector $\vec{R}$, defined by the decision of systems of the equations (6, 7).

| № | $T_{11}$ | $Q_{21}$ | $Q_{31}$ | $M_1$ | $M_2$ | $M_3$ | $k$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.369 | -0.0465 | -0.0330 | 0.000538 | -0.308 | -0.00606 | 0.5 |
| 2 | 0.741 | -0.0932 | -0.0662 | 0.00106 | -0.617 | -0.0118 | 4.5 |

Summary conclusion. The proposed methodology strength check of anisotropic bars on limit states has, in our opinion, the following advantages:

1. The need for separate consideration of private kinds of complex resistance of timber (oblique bending, bending with torsion and the like) becomes unnecessary.

2. Each of the components of the vector $\vec{R}^*$ of IFP has equal rights to participate in the test of strength.

3. only the most general requirements accepted in the theory of rods are imposed on the form and the dimensions of timber.

4. The ratio $|\sigma_\text{u}|/ \max |\sigma|$ is closer to reality assesses the robustness of timber than the ratio $|\sigma_\text{u}|/ \max |\sigma_\text{max}|$ (this remark is especially true if the material of the bar has plastic properties). As noted in [12], "... calculation of limit loads gives significantly more correct understanding of the structural strength, than the calculation of the maximum stress...". The proposed methodology strength check the bars expands the sphere of practical application of equations (3); it can be assumed that it will find application in practice of calculation and design of various products.

3. References

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