Physical and mathematic N-option modeling and optimization of dynamically loaded nonlinear technical systems

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Abstract. The development of modern industries significantly depends on the effective usage of innovative research methods and optimization of technological processes. Modeling methods are one of the science-intensive methods put into practice in research and diagnostics of both the current state of mechanical systems and predicting changes in the state of technological equipment during operation. The authors have developed and successfully adapted fundamental theoretical foundations of physical and mathematical modeling for solving problems of optimization and simulation of dynamically loaded nonlinear technical systems. A method for diagnosing tribo-couplings of helicopter tail drive couplings is proposed.

1. Introduction
The development of modern industries significantly depends on the effective application of innovative research methods, optimization of technological processes, application of advanced methods and processes for the production of structural and consumable materials, and usage of modern technological equipment.

Modeling methods are one of the science-intensive techniques exploited in research and diagnostics, both in the current state of mechanical systems and in predicting changes in the state of technological equipment during operation.

For solving problems of optimization and modeling of dynamically loaded nonlinear technical systems (NTS) the authors have developed and successfully adapted fundamental theoretical foundations of physical and mathematical modeling [1-5]. The methods of physical and mathematical modeling and dynamic monitoring are successfully used in solving tasks of optimizing loaded NTS.

The main advantage of these methods is that the cost of resources for their implementation is significantly less than for setting up and carrying out physical, laboratory, or even full-scale modeling. However, the measure of confidence in the results of a mathematical experiment is interrelated with the degree of resemblance of the model to the physical processes that it describes.

Based on available theoretical and experimental data in the field of friction dynamics, novel research methods have been developed, and innovative approaches have been applied to solve the problems of optimization of NTS and to improve their efficiency and competitiveness. In particular, the interaction and interrelation of dynamic processes in quasilinear and essentially nonlinear NTS subsystems were taken into account. The authors also took into consideration self-similarity of the dynamic processes that occur during the physical and mathematical modeling of NTS in frictional contacts, and provide the correct results due to significant overlapping of full-scale and model tests data.

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2. Method for assessing the elastic-dissipative characteristics of the frictional interaction

The application of n-variant modeling allows drastic reduction of the time required for solving optimization problems by cutting down the volume of full-scale research of NTS. The creation of a physical and mathematical model of a full-scale system in 2, 3 and n-different scales lets us limit the results of volume of full-scale studies of the NTS to fixation of one or two leading parameters.

A new method has been developed for assessing the elastic-dissipative characteristics of the frictional interaction of the spline couplings of the tail transmission of the Mi-26 helicopter on the example of heavily loaded friction pairs of helicopters, in order to increase their reliability and durability when operating at low temperatures (Figure 1).

![Figure 1. The tail shaft of the helicopter Mi-26:](image)

I - splined coupling with flanges in the bearing; 1, 3 - hollow shafts; 2 - bearings of intermediate hearings; 4 - temperature sensors

The operational reliability and efficiency of heavily loaded helicopter couplers is determined not only by speed and load parameters, but also by physico-mechanical, physico-chemical, and tribological parameters of frictional processes in frictional subsystems.

The basic purpose of the tail shaft is to transfer the rotating moment from the main gearbox to the tail rotor by means of series-connected elastic elements having certain masses and moments of inertia. The splined coupling is designed in such a way that the annular thimble rotates in a bearing mounted on the frame (Figure 2).

The analysis of the research has shown that the main faults of the spline couplings of the helicopter transmission are the following:

- the formation of cracks and delamination of the rubber bearing cage;
- leakage of lubricant, which stimulates overheating of the coupling and its bearing;
- deformation and wear products generation of the coupling components;
- the formation of a sideways clearance in the coupling joints;
- misalignment of tail shaft bearings;
- increased outrun of the shaft tube, as well as axle fracture or shaft twisting.

During the launch of the helicopter transmission under extremely low temperatures conditions, the stationary temperature monitoring sensors 4 installed in the couplings (Fig. 1) are unable to inform the pilots in a timely manner about any emerging problems. A more advanced splined joint diagnostics technology is required in order to allow real-time identification of any emergency situations, which would improve the safety of long-distance piloting.
3. Implementing the dynamic monitoring task

The tribological system of the spline joint of the transmission of the Mi-26 helicopter (Fig. 2) refers to systems that are characterized by nonlinear interconnected physico-mechanical, thermophysical, tribochemical, and load-velocity factors, as well as environmental ones. A feature of the coupling tribosystem operation spline joint is in the following. Its operation takes place under the influence of significant vibrations from the main engine and the entire helicopter as a whole. Therefore, modern technical means of measuring physical quantities, digital transmission and signal processing equipment should be used to achieve the goal.

The most effective vibration measuring circuits include two measuring signals about oscillations in the input impact (engine thrust torque) and vibrations of the spline coupling itself. This allows using spectral analysis of data, estimating the ratio of the coherent spectrum to the spectrum of the input impact, and eliminating a significant part of random interference [6, 7].

To identify the processes of dynamic loading of couplings, we used the foundations of the automatic control theory [8] by analyzing the amplitude phase, time, and integral quality criteria. Based on the time-recorded measurement subsystem of the oscillations of the input impact (engine traction torque) and the output coordinates of the coupling movements, the complex transmission coefficient at each harmonic oscillation frequency is estimated as follows:

\[
W(i\omega) = \frac{S_N(i\omega)S_M(-i\omega)}{|S_M(i\omega)|^2} = A(\omega)e^{i\phi(\omega)} = P(\omega) + iQ(\omega),
\]

where \(S_M(i\omega)\) is the power spectral density of the input impact, the traction moment \(M(t)\); \(S_N(i\omega)\) is the power spectral density of the normal displacement force \(N(t)\); \(A(\omega)\) is amplitude spectrum; \(\phi(\omega)\) is phase spectrum; \(P(\omega)\) is real spectrum; \(Q(\omega)\) is an imaginary spectrum.

Expression (1) allows calculating the complex transfer coefficient [8] and the amplitude phase function that characterizes the ratio of elastic-inertial and dissipative components of the frictional interaction [6, 7]. Integrated estimates are known to allow estimating the amount of oscillation damping and deviations of the controlled value in aggregate, without determining them separately [8]. Therefore,
along with the well-known linear and quadratic integral quality criteria of the transient characteristics, we also proposed to evaluate the elastic-inertial and dissipative components by the amplitude phase characteristics for each time point in the given octave (1/3, 1/12 or 1/24-octave) frequency ranges [6, 7]. With a higher degree of reliability, it allows selecting the most informative ranges of natural frequencies, where you can find the influence of physical and mechanical characteristics of the lubricant (hypoid lubricant), as well as the wear of the surfaces of the splines of the coupling joint.

To identify the elastic-dissipative and dynamic characteristics of the couplings, the following integral estimates were used:

- elastic-inertial components of the interaction, conductive to the convergence of the contacting friction surfaces, as well as prompting an increase in contact stresses and temperatures:

\[
I_c(t) = \int_{\omega_0}^{\omega_1} P(\omega) d\omega, \quad \text{given } P(\omega) \geq 0, \tag{2}
\]

where \( t \) is registered operating time;
\( \omega_0, \omega_1 \) are boundary frequencies for octave (fractional octave) spectral analysis;
\( P(\omega) \) is real frequency response characterizing the elastic-inertial components of a complex quantity (1)

\[
P(\omega) = \text{Re}[W(i\omega)] = A(\omega) \cdot \cos[\varphi(\omega)]; \tag{3}
\]

- inertial components of the interaction, contributing to the loss of stability of the friction bond:

\[
I_M(t) = \int_{\omega_0}^{\omega_1} P(\omega) d\omega, \quad \text{given } P(\omega) < 0; \tag{4}
\]

- dissipative components of the interaction, characterizing the resistance forces that are directed oppositely to the resultant sliding velocity vector:

\[
I_{F_c}(t) = \int_{\omega_0}^{\omega_1} Q(\omega) d\omega, \quad \text{given } Q(\omega) \leq 0, \tag{5}
\]

where \( Q(\omega) \) is imaginary frequency response of the complex quantity (1), characterizing the dissipation of energy during dynamic interaction:

\[
Q(\omega) = \text{Im}[W(i\omega)] = A(\omega) \cdot \sin[\varphi(\omega)]; \tag{6}
\]

- dissipative components of the interaction, characterizing the development in the frictional-mechanical subsystem of frictional self-oscillations, that is resistance forces with the vector co-directed with the sliding velocity vector:

\[
I_d(t) = \int_{\omega_0}^{\omega_1} Q(\omega) d\omega, \quad \text{given } Q(\omega) > 0. \tag{7}
\]

It is known that the damping properties of mechanical systems in the linear theory of oscillations are estimated by the dimensionless damping coefficient \( \xi \), as the ratio of the exponent of the amplitude oscillations damping \( n \) to the frequency of free oscillations \( \omega_0 \). It is problematic to determine the coefficient in substantially nonlinear friction systems. The authors proposed an empirical expression for estimating the coefficient \( \xi \) in the form of a certain integral value \( I_{\xi} (t) \) for the current time moment \( t \), which makes it possible to identify the frequency ranges of nonlinear system oscillations where dissipative properties have the greatest influence on the dynamics of the system:

\[
\xi = \frac{n}{\omega_0} = \frac{\beta}{\beta_0}, \quad I_{\xi} (t) \approx \frac{1}{\sqrt{1 + \frac{1}{I_{F_c}(t) + I_d(t)}}}, \tag{8}
\]

where \( n \) is the vibration damping coefficient, \( c \sim 1; \)
\( \omega_0 \) is the frequency of natural vibrations, \( c \sim 1; \)
\( \beta \) is coefficient of resistance to vibrations, \( N \cdot s / m; \)
\( \beta_0 \) is the critical value of the coefficient of resistance to oscillations, at which the oscillatory character is replaced by a monotonically decaying (aperiodic), \( N \cdot s / m; \)

\( I_{F_c}, I_A, I_C, I_m \) are integral assessment of quality [6, 7], calculated on the basis of the analysis of amplitude-phase characteristics and indirectly characterizing the ratio of resistance forces (5), frictional self-oscillations (7), elastic forces (2), and inertial forces (4).

Observing the expectation value, mean squared departure and peak-factor of estimates (2) - (8) in real time observation \( t \), allows solving tasks of identification and time-to-time variation of elastic-dissipative characteristics.
It is more convenient to implement the task of dynamic monitoring of spline couplings using a single criterion that would allow analyzing the generalized characteristics of the friction-mechanical system and predicting, with a certain probability, changes in the trend of characteristics. We have proposed a dynamic quality criterion (9) to be such a characteristic. Its limiting value is equal to one, and corresponds to the "warning" threshold:

$$I_D(t) = \frac{1}{12} \sum_{k=1}^{12} I_k,$$

(9)

where $I_k$ are integral estimates of the estimated parameters of the stationary transmission coefficient (1).

The lower the level of $I_D(t)$, the more stable and balanced the friction system of joint is. Values (9) exceeding 1 correspond to abnormal operating conditions in the form of outruns, resonance, and other forms of deviations from the stationary trajectory of motion. In accordance with the three sigma rule, the "danger" threshold was also set at the values of $I_D(t) \geq 1.15$.

Changes in load-speed operating conditions, ambient temperature and a significant number of side factors have a significant impact on the performance of the splined coupling. Such factors might include the following: high frequency oscillations in the formation of local metal links in the case of fretting-corrosion; a significant change in the rate of relative slip of friction surfaces; convergence or removal of contact surfaces of friction under the influence of inertia forces; deformation of active volumes of friction surfaces, increase of contact stresses. The combination of unfavorable factors, loads and sliding speeds can cause the change of the microgeometry of the friction surfaces, followed by the development of plastic deformations, friction auto fluctuations. All these can lead to increased instability of the tribosystem, athermic or thermal capture, high values of dynamic coefficient of friction (Tolstoi - Push effect).

One can estimate the residual supply of the couture according to the results of long-term tests or during the operation. For this purpose both the method of octave analysis of energy dissipation [sum of calculated estimates (5) and (7)], and the proposed method of dynamic quality criterion with established thresholds of "warning" and "danger" can be used. The nature of the octave spectrum changes is shown in Fig. 3, depending on the hourly operating time.

Observing the values of the amplitudes of the octave spectra during the operating time of the support coupling 5, it is possible to identify more reliably the periods of running-in (0–881 sec) and normal operation (992–1522 sec), when the dissipative energy losses for friction processes are maximum, and the fixed value of the damping coefficient $\xi$ reaches minimum values of 0.11. In addition, monitoring several couplings simultaneously allows establishing statistical levels of the damping coefficient in each frequency range of the fractional octave spectrum: the base level (B), as well as warning (W) and danger (D) thresholds (Figure 3).

To simulate the main dynamic vibrations of the prongs of the spline connection of the tail shaft couplings of the Mi-26 helicopter when the nature of the loading effect changes, a laboratory stand was made to monitor the technical state of the spline model (Figure 4).

Thus, a technique for diagnostics of tribological couplings of the tail drive couplings of Mi-26 helicopters has been proposed. It allows identifying in real time the stability of elastic-inertial and dissipative characteristics of frictional interaction, periods of running-in, normal operation and catastrophic wear using the values of fractional octave spectra.
Figure 3. 1/3-octave spectrum analysis of the damping coefficient depending on the operating time of the splined coupling 5 of the frame support.

Figure 4. Stand for research of the spline connection:
1 - model node; 2 - double lever; 3 - bottom bar; 4 - top bar; 5 - loading springs; 6 - lock nuts.
4. Conclusion
The use of the developed dynamic criterion simplifies the technological process of diagnostics, monitoring, and forecasting. It also allows optimization of significant changes in elastic-dissipative relationships, preventing the occurrence of abnormal operating conditions by informing the pilots about the fixed threshold values of "warning" or "danger".

The proposed technique for tribospectral identification of friction processes and dynamic monitoring of friction systems contributes to an increase in the safety of helicopter operation. It can also be applied to any friction units, which will make it possible to implement a short-term or long-term forecast in the change of the dynamic characteristics of loaded NTS.

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