Born–Infeld phantom gravastars

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Abstract. We construct new gravitational vacuum star solutions with a Born–Infeld phantom replacing the de Sitter interior. The model allows for a wide range of masses and radii required by phenomenology, and can be motivated from low energy string theory.

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1. Introduction

From its inception, the Schwarzschild solution of Einstein’s equations has been the subject of controversy over the possibility of compressing matter within the gravitational radius $2GM$. Given the accumulated evidence for supermassive compact objects ranging from a few $10^6 M_\odot$ to a few $10^9 M_\odot$, the existence of black hole-like objects is beyond doubt [1, 2]. What does remain an issue is whether the Schwarzschild metric correctly describes the physics of the interior. Alternatives to classical black holes have been proposed with no singularities in the interior [3]–[13].

The simplest model proposed for supermassive compact objects at the galactic centres is a self-gravitating degenerate fermion gas composed of, e.g., heavy sterile neutrinos [3, 4], [14]–[16]. However, this scenario cannot cover the whole mass range of supermassive black hole candidates with a single sterile neutrino species [17].

Recently, Chapline et al [5, 6] put forward an interesting proposal based on analogies to condensed matter systems where the effective general relativity was an emergent phenomenon. Specifically, they suggested that the sphere where the lapse function vanished marked a quantum phase transition, the lapse function increasing again at $r < 2GM$. As this required negative pressure, the authors of [6] assumed the interior vacuum condensate to be described by de Sitter space with the equation of state $p = -\rho$.

Subsequently, the idea of gravitational vacuum condensate, or ‘gravastar’, was taken up by Mazur and Mottola [7, 9], replacing the horizon with a shell of stiff matter astride the surface at $r = 2GM$. Visser and Wiltshire [18] and recently Carter [19] also examined the stability of the gravastar using the Israel thin shell formalism [20]. Despite the fact that general relativity is an emergent phenomenon in string theory [21], the gravastar has met with a cool reception. Certainly, the assumption of a de Sitter interior presents a quandary: on the one hand, the quantum phase transition would suggest that the associated cosmological constant is a fundamental parameter; on the other hand, to accommodate the mass range of supermassive black hole candidates, it must vary over some six orders of magnitude. Thus, it seems prudent to explore other possibilities for the gravastar interior.
One obvious extension of the de Sitter equation of state $p + \rho = 0$ would be an equation that satisfies $p + \rho \leq 0$, thus violating the dominant energy condition. The fluid whose equation of state violates the dominant energy condition has been dubbed *phantom energy* [22, 23] and it has recently become a popular alternative to quintessence and to the cosmological constant [24]. The motivation for introducing the phantom energy is that the equation of state $w \equiv p/\rho < -1$ produces a superaccelerated cosmological expansion which seems to be favoured by the combined analysis of the CMB and supernova type I data [25]. However, a superaccelerated expansion may also be obtained without violating the dominant energy condition in scalar–tensor theories of gravity [26] and in models with variable gravitational constant [27] or variable cosmological constant [28]. Some astrophysical aspects of the phantom have recently been discussed, such as phantom energy accretion [29], phantom energy wormholes [30] and a possible relation of the phantom tachyon model to supermassive black holes [31].

In this paper we consider a gravastar interior consisting of a self-gravitating scalar field described by a Born–Infeld-type Lagrangian which yields the Chaplygin gas equation of state [32, 33]. Hence, we look for static solutions of the self-gravitating Chaplygin gas. In particular, we consider static Chaplygin gas configurations in the phantom regime, i.e., when $p + \rho < 0$, and we show that these configurations could provide an alternative scenario for compact massive objects at galactic centres.

The paper is organized as follows. In section 2 we introduce the basics of the model. In section 3 we investigate static solutions using Tolman–Oppenheimer–Volkoff equations. In section 4 we discuss a possible interpretation of galactic centres as Born–Infeld phantom gravastars. A stability analysis is given in section 5 and we conclude the paper with section 6.

### 2. The model

Consider the equation of state

$$ p = -\frac{A}{\rho} \quad (1) $$

in the phantom regime, i.e., when

$$ \rho < \sqrt{A}. \quad (2) $$

Equation (1) describes the Chaplygin gas which, for $\rho \geq \sqrt{A}$, has attracted some attention as a dark energy candidate [32, 33]. Astrophysical objects made of the so-called generalized Chaplygin gas [34] have recently been discussed [35]. The generalized Chaplygin gas has also been exploited in the phantom regime [36]. As we shall shortly demonstrate, static solutions to Einstein’s equations with matter described by (1) with (2) cover the range of masses and radii required to fit the phenomenology of supermassive dark compact objects at the galactic centres. Moreover, equation (1) yields the de Sitter gravastar solution in the limit when the central density of the static solution approaches the value $\sqrt{A}$.

The Chaplygin gas equation of state (1) with the condition $\rho < \sqrt{A}$ may be derived from the Dirac–Born–Infeld-type Lagrangian

$$ L_{\text{DBI}} = -\sqrt{A} \sqrt{1 + X}, $$

where $X = \rho/\rho_0$ and $\rho_0$ is the central density.
Born–Infeld phantom gravastars

where

\[ X = g^{\mu \nu} \partial_\mu \partial_\nu. \] (4)

Clearly, in the limit \( X \to 0 \) this Lagrangian becomes a free scalar field Lagrangian with a ‘wrong sign’ kinetic term, and hence a phantom\(^4\). A phantom Lagrangian of the type (3) has been proposed in the context of the superaccelerated expansion \([38]\). Note that the Lagrangian (3) leads to a perfect fluid with the 4-velocity

\[ u_\alpha = \frac{\partial_\alpha}{\sqrt{X}}, \] (5)

the pressure \( p = \mathcal{L}_{\text{DBI}} \) and the density

\[ \rho = \frac{\sqrt{A}}{\sqrt{1 + X}} \leq \sqrt{A}. \] (6)

Clearly, the pressure and the density obey equation (1) and \( \rho \leq \sqrt{A} \). It is worth noting that the Chaplygin gas cosmological model \([32]\), in which the condition \( \rho > \sqrt{A} \) holds, is described by the Lagrangian (3) with a \((-\) sign in front of \( X \). It is as though instead of the lapse function changing its sign, as it does in the Schwarzschild case for \( r < 2GM \), the scalar kinetic energy changed its sign to become a phantom.

Equivalent to \( \mathcal{L}_{\text{DBI}} \) of equation (3) is

\[ \mathcal{L} = -\frac{\phi^2}{2} X - \frac{1}{2} \left( \phi^2 + \frac{A}{\phi^2} \right), \] (7)

as may be seen by eliminating \( \phi^2 \) through its equation of motion. Now recall that in four dimensions, a 3-form field strength is dual to a scalar:

\[ H^{\mu \nu \alpha} = \phi^2 \epsilon^{\mu \nu \alpha \beta} \partial_\beta. \] (8)

Then the content of the model is

\[ \mathcal{L} = \frac{1}{12\phi^2} H_{\mu \nu \alpha} H^{\mu \nu \alpha} - \frac{1}{2} \left( \phi^2 + \frac{A}{\phi^2} \right). \] (9)

Notably, this is also the content of low energy string theory \([21]\) in the Einstein frame where \( H_{\mu \nu \alpha} \) is the Kalb–Ramond field, \( \phi^2 \) corresponds to the string coupling and the dilaton kinetic term is neglected in lieu of a potential assumed to arise from non-perturbative effects\(^5\). A potential similar to that in (7) and (9) has been considered in the context of polymer scaling and black holes \([39]\).

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\(^4\) It has recently been shown that quantum effects could lead to an effective dark energy equation of state violating the dominant energy condition on cosmological scales even for a scalar field Lagrangian having the correct sign of the kinetic energy \([37]\).

\(^5\) Strictly, the exterior should have \( H_{\mu \nu \alpha} H^{\mu \nu \alpha} = 6A \) and \( \phi^2 = 0 \) to be Schwarzschild.
3. Static solutions

Next, we proceed to solve Einstein’s equations for a static, spherically symmetric configuration with an interior described by (1) and with a Schwarzschild exterior. Our approach is similar to that of Armendariz-Picon and Lim [40]. However, in contrast to us, they consider the classes of Lagrangian with a strictly spacelike gradient of $\phi$, i.e. with $X < 0$, so that in their case the quantity (5) cannot be interpreted as a 4-velocity. As a consequence, their energy–momentum tensor does not have a standard perfect fluid form and the pressure is not isotropic. Here, we consider strictly timelike gradient of $\phi$ with $X > 0$, but we allow phantom-like Lagrangians which violate the dominant energy condition.

For the static, spherically symmetric line element

$$ds^2 = \xi^2(r) dt^2 - \frac{dr^2}{1 - 2GM(r)/r} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(10)

with $T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p)$, Einstein’s equations become [41]

$$M' = 4\pi r^2 \rho,$$

(11)

$$\xi' = G\xi \frac{M + 4\pi r^3 p}{r(r - 2GM)},$$

(12)

while $T_{\mu\nu} = 0$ gives

$$p' = -(\rho + p) \frac{\xi'}{\xi}.$$  

(13)

We focus on the equation of state (1) to close the system and we require the solution to (11) and (12) to be regular at $r = 0$. Since by rescaling $t$ one may set $\xi(0) = 1$, equations (1) and (13) yield

$$\xi(r) = \frac{\rho}{\rho_0} \sqrt{\frac{A - \rho_0^2}{A - \rho^2}}.$$  

(14)

Combining equations (1), (12), and (13), one has

$$\rho' = G \left(1 - \frac{\rho^2}{A}\right) \left(\frac{\rho M - 4\pi Ar^3}{r(r - 2GM)}\right).$$  

(15)

In figures 1, 2 and 3, respectively, we exhibit the resulting $\rho(r)/\rho_0$, $\xi(r)$ and $2GM(r)/r$ for selected values of $\rho_0/\sqrt{A}$. The solutions depend essentially on the magnitude of $\rho_0$ relative to $\sqrt{A}$. In the following we summarize the properties of three classes of solutions corresponding to whether $\rho_0$ is larger, smaller or equal to $\sqrt{A}$.

(i) For $\rho_0 > \sqrt{A}$, the density $\rho$ increases and the lapse function $\xi$ decreases with $r$ starting from the origin up to the black hole horizon radius $R_{\text{bh}}$, where $2GM(R_{\text{bh}}) = R_{\text{bh}}$. In the limit $\rho_0 \to \infty$, a limiting solution exists with a singular behaviour

$$\rho(r) \simeq \left(\frac{7A}{18\pi Gr^2}\right)^{1/3}$$

(16)

near the origin.
Born–Infeld phantom gravastars

Figure 1. Density profile of the Chaplygin star for \( \rho_0/\sqrt{A} = 1.2 \) (short dashed), 0.98 (dotted), 0.9 (long dashed). The limiting singular solution with \( r^{-2/3} \) behaviour at small \( r \) is represented by the dot–dashed and the de Sitter gravastar by the solid line.

(ii) For \( \rho_0 < \sqrt{A} \), both \( \rho \) and \( \xi \) decrease with \( r \) up to the radius \( R_0 \) where they vanish. At that point the pressure \( p \) blows up to \(-\infty\) owing to (1). The enclosed mass \( M \) is always less than \( r/(2G) \), never reaching the black hole horizon, i.e., the radius where \( 2GM(r) = r \).

(iii) For \( \rho_0 = \sqrt{A} \), the density \( \rho \) remains constant, equal to \( \sqrt{A} \), up to the de Sitter radius \( R_{\text{dS}} = 2GM = 8\pi GR_{\text{dS}}^2/3\rho_0/3 \). Hence, the interior is de Sitter, precisely as in Chapline et al [6]. The lapse function is given by

\[
\xi = \left(1 - \frac{r^2}{R_{\text{dS}}^2}\right)^{1/2}
\]

with

\[
R_{\text{dS}} = \sqrt{\frac{3}{8\pi G}} A^{-1/4}.
\]

As \( \rho_0 \rightarrow \sqrt{A} \) from above or from below, solutions (i) or (ii), respectively, converge to (iii) except at the end-point. The lapse function in (iii) joins the Schwarzschild solution outside

\[
\xi(r) = \left(1 - \frac{2GM}{r}\right)^{1/2},
\]

continuously, whereas in (i) and (ii) it happens discontinuously.

As in the case of a de Sitter gravastar, in order to join our interior solutions to a Schwarzschild exterior at a spherical boundary of radius \( R \), it is necessary to put a thin
spherical shell\textsuperscript{6} at the boundary with a surface density and a surface tension satisfying Israel’s junction conditions \cite{20}. In all the solutions discussed above the pressure is isotropic and does not vanish at the boundary. Hence, the pressure at the boundary

\textsuperscript{6} It has recently been demonstrated that the joining can be made continuous without the presence of a thin shell for a gravastar made of the fluid with an anisotropic pressure \cite{42}.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{Lapse function $\xi/\xi_0$ for various solutions as in figure 1.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure3}
\caption{Enclosed mass divided by the radius of the star for various solutions as in figure 1.}
\end{figure}
must be compensated by a negative surface tension of the membrane. We postpone this issue to section 5 where we discuss the stability of the solution.

4. Black holes at galactic centres

Case (ii), together with (iii), is of particular interest as we would like to interpret the supermassive compact dark objects at the galactic centres in terms of phantom energy rather than in terms of a classical black hole. It is natural to assume that the most massive such object is described by the de Sitter gravastar, i.e., solution (iii) (depicted by the solid line in figures 1–3). If we identify the most massive black hole candidate observed at the centre of M87, with mass $M_{\text{max}} = 3 \times 10^9 \, M_\odot$, with the de Sitter gravastar, then $A^{1/8} = 9.7 \, \text{keV}^4$, to be contrasted with the $10^{-3} \, \text{eV}$ values wanted for cosmology [33]. The radius of this object is $R_{\text{ds}}$, equal to the Schwarzschild radius $2GM_{\text{max}}$. Clearly, solutions belonging to class (ii) can fit all masses $M < M_{\text{max}}$. However, for the phenomenology of supermassive galactic centres it is important to find, at least approximately, the mass–radius relationship for these solutions. This may be done in the low central density approximation, i.e., $\rho_0 \ll \sqrt{A}$, which is similar to the Newtonian approximation but, in contrast to the Newtonian approximation case, one cannot neglect the pressure term in equation (12). Moreover, as may be easily shown, in this approximation $M \ll r^3 \rho$, so that the pressure term becomes dominant. Next, neglecting $2GM$ with respect to $r$, as in the usual Newtonian approximation, equation (15) simplifies to $\rho' = 4\pi GA r$, with the solution

$$\rho = \rho_0 \left(1 - \frac{r^2}{R_0^2}\right); \quad R_0^2 = \frac{\rho_0}{2\pi GA},$$

which gives a mass–radius relation

$$\frac{M}{R_0^5} = \frac{16\pi^2}{15}GA = \text{constant}. \quad (21)$$

The mass–radius relationship $M \propto R_0^3$ which phantom gravastars obey offers the prospect of unifying the description of all supermassive compact dark objects that have been observed at the galactic centres as Born–Infeld phantom gravastars with masses ranging from $M_{\text{min}} = 10^6 \, M_\odot$ to $M_{\text{max}} = 3 \times 10^9 \, M_\odot$. Indeed, assuming that the most massive compact dark object, observed at the centre of M87, is a Born–Infeld phantom gravastar near the black hole limit, with $R_{\text{max}} = 2GM_{\text{max}} = 8.86 \times 10^9 \, \text{km} = 8.21 \, \text{lr}$, the compact dark object at the centre of our Galaxy, with mass $M_{\text{GC}} = 3 \times 10^6 \, M_\odot$, would have a radius $R_{\text{GC}} = 2.06 \, \text{lr}$ if the scaling law (21) holds. This radius is well below the distances of closest approach to Sgr A* which the stars SO-2 ($R_{\text{min}} = 17 \, \text{lr} = 123 \, \text{AU}$, [43]) and SO-16 ($R_{\text{min}} = 8.32 \, \text{lr} = 60 \, \text{AU}$ [44]) recently had and beyond which the Keplerian nature of the gravitational potential of Sgr A* is well established.

5. Dynamical stability

None of the solutions discussed in the preceding section will be stable unless there is a membrane, e.g. in the form of a thin shell, placed at the boundary with surface density $\sigma$ and surface tension $\theta$ satisfying Israel’s junction conditions. Following Visser and
Wiltshire [18] we consider a dynamical thin shell allowed to move radially at the boundary of the phantom gravastar and discuss under which conditions a gravastar configuration will be stable. A dynamical thin shell connecting two general static spherically symmetric spacetimes has also recently been considered [45] in a slightly different context.

Israel’s junction conditions read [20]
\[
\left[ [K^b_a - \delta^b_c K^c_a] \right] = 8\pi G S^b_a, \tag{22}
\]
where \( S^b_a = \text{diag}(\sigma, \theta, \theta) \) is the surface stress energy and \( [[f]] \) denotes the discontinuity in \( f \) across the boundary, i.e.,
\[
[[f(r)]] = \lim_{\epsilon \to 0} (f(R + \epsilon) - f(R - \epsilon)). \tag{23}
\]
The tensor \( K_{ab} \) is the extrinsic curvature defined by
\[
K_{ab} = h^c_a h^d_b n_{dc}, \tag{24}
\]
where \( n_a \) is a spacelike unit vector orthogonal to the timelike boundary and \( h_{ab} \) is the induced metric on the shell.

The angular components of the extrinsic curvature may be easily calculated from (24) yielding
\[
K^{\vartheta\vartheta} = K^{\varphi\varphi} = \frac{1}{r} (\Delta + \dot{R})^{1/2}, \tag{25}
\]
precisely as in [18], where the dot denotes the derivative with respect to the proper time and where
\[
\Delta = 1 - \frac{2GM}{r}. \tag{26}
\]
The calculation of the time–time component \( K^{tt} \) is slightly more involved and is most easily done following Israel [20]. By making use of the Gauss normal coordinates and orthogonality from (24) it follows that
\[
K^{tt} = u^a u^b K_{ab} = -n_a u^b u^a;_b, \tag{27}
\]
where \( u^a \) is the 4-velocity of a point on the shell. Its non-vanishing components for the metric (10) are
\[
u^t = \frac{1}{\xi} \left( 1 + \frac{\dot{R}}{\Delta} \right)^{1/2}; \qquad u^r = \dot{R}. \tag{28}
\]
Similarly, the non-vanishing components of \( n_a \) are
\[
n_t = \frac{\xi \dot{R}}{\Delta^{1/2}}; \qquad n_r = \frac{(\Delta + \dot{R}^2)^{1/2}}{\Delta}. \tag{29}
\]
Using these expressions it may be shown that the 4-acceleration \( u^a u^a;_b \) satisfies [20]
\[
n_a u^b u^a;_b = -\frac{1}{(\Delta + \dot{R}^2)^{1/2}} (\dot{R} + \Gamma^r_{ab} u^a u^b). \tag{30}
\]
A straightforward calculation yields
\[
K^{tt} = \frac{1}{(\Delta + \dot{R}^2)^{1/2}} \left[ \dot{R} + \frac{\xi^r}{\xi} (\Delta + \dot{R}^2) + \left( \frac{M}{r} \right)' \frac{\dot{R}^2}{\Delta} \right]. \tag{31}
\]
Then, from (22) using (11) and (12) we find

\[ 4\pi\sigma = -\frac{1}{G} \left[ \sqrt{\Delta + \frac{\dot{R}^2}{R}} \right], \]  
\[ (32) \]

\[ 4\pi(\sigma - 2\theta) = \left[ \frac{R^2\ddot{R} + GM - 4\pi G R^3 \rho}{GR^2 \sqrt{\Delta + \frac{\dot{R}^2}{R}}} \left( \rho + \rho \right) \right]. \]  
\[ (33) \]

To derive the stability condition à la Visser and Wiltshire [18], it is now sufficient to replace \( \dot{R}^2 \) by \(-2V(R)\) and \(\ddot{R}\) by \(-V'(R)\) in equations (32) and (33), where \(V(R)\) is a potential. The shell will be stable against small radial perturbation if there exists an equilibrium position \( \bar{R} \) such that

\[ V(\bar{R}) = 0; \quad V'(\bar{R}) = 0; \quad V''(\bar{R}) > 0. \]  
\[ (34) \]

Then, by choosing a suitable potential \(V(R)\), equations (32) and (33) define a parametric equation of state \(\theta = \theta(\sigma)\) for the shell.

For the static shell and the metric (10) we find

\[ \left( 1 - \frac{2GM}{R} \right)^{1/2} - \left( 1 - \frac{2GM(R)}{R} \right)^{1/2} = -4\pi GR\sigma, \]  
\[ (35) \]

and with the help of equation (12) we obtain

\[ M \left( 1 - \frac{2GM}{R} \right)^{-1/2} - \left( M(R) - \frac{4\pi R^3 A}{\rho(R)} \right) \left( 1 - \frac{2GM(R)}{R} \right)^{-1/2} = 4\pi R^2(\sigma - 2\theta), \]  
\[ (36) \]

where \(M\) is the total mass. Note that if the joining is affected at the point \(R_0\) where \(\xi = 0\) and \(p\) is infinite (the point of naked singularity [42]), the surface density \(\sigma\) is finite, whereas the surface tension \(\theta \to -\infty\). Hence, to avoid the singularity the shell must be placed at some \(R < R_0\).

For the Newtonian gravastars discussed in the preceding section, equations (32) and (33) may be considerably simplified. Using (20), the approximation \(2GM/R \ll 1\) and assuming that the boundary radius \(R\) is close to \(R_0\), i.e.,

\[ y \equiv \frac{R_0 - R}{R_0} \ll 1, \]  
\[ (37) \]

we find

\[ \sigma = 2\pi GAR_0^3 \frac{y^2}{\sqrt{1 - 2V}}, \]  
\[ (38) \]

\[ \theta = -\frac{1}{8\pi GR_0} \sqrt{1 - 2V} - \frac{4\pi GAR_0^3}{y} \frac{yV'}{\sqrt{1 - 2V}} + \frac{\pi GAR_0^4}{1 - 2V}. \]  
\[ (39) \]

In general, the potential \(V\) is a function of both \(R_0\) and \(y\), depending on the chosen equation of state \(\theta = \theta(\sigma)\). The dynamical stability is achieved if for any \(R_0\), \(V\) has a minimum at some point \(\bar{y} = 1 - \bar{R}/R\) which should not depend strongly on \(R_0\) and at which \(V(\bar{y}) = 0\).

Instead of postulating \(\theta = \theta(\sigma)\) we can choose a desirable potential and determine the equation of state \(a posteriori\). Static stability (indifferent balance) is obtained by setting
$V \equiv 0$. Depending on the potential $V(R_0, y)$, this may still be a complicated parametric equation of state. However, for the static shell, i.e. $V \equiv 0$, the equation of state simplifies to

$$\theta = -\left(\frac{AR_0}{32\pi G\sigma}\right)^{1/2}.$$  \hspace{1cm} (40)

This equation describes a two-dimensional generalized anti-Chaplygin gas [46] with the equation of state of the form $p = -\theta = A'/\sigma^\alpha$, with $\alpha = 1/2$.

6. Conclusions

In conclusion, we have shown that replacing the de Sitter interior of the gravastar by a Born–Infeld phantom allows a wide range of gravastar mass and radii related to the central density, or equivalently to the velocity of the phantom scalar. We have demonstrated that if the constant $A$, as the only free parameter of the model, is fixed by assuming that the most massive galactic centre object is the maximal (de Sitter) gravastar, then the model is able to explain all supermassive compact dark objects at the centre of the galaxies. Furthermore, as demonstrated above, the phantom gravastar model can lay claim to a connection with low energy string theory.

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