Optimal portfolio strategies of purchasing electricity for electricity company based on distributional robust CVaR

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Abstract. Considering the uncertainty of probability distribution of electricity price in multi-energy markets and taking the maximum expected profits as optimization object, this paper establishes the optimal power purchasing portfolio model based on distributional robust CVaR. An optimal portfolio problem of purchasing proportion in real-time electricity market, day-ahead electricity market, and mid-long term contract market is transformed into a semi-definite programming problem. The analysis of case study shows the efficiency of the proposed model, which paves a new way for electricity companies to determine the optimal portfolio strategies considering the risk.

1. Introduction
In the electricity market, the electricity company will be an independent retail company which is responsible for purchasing electricity from multiple energy markets and providing services to end users. How to build the optimal portfolio strategies for electricity purchasing and control the risk is an issue with extensive concerns. In China, because of the government regulation, the selling price of the electricity company is fixed in a certain period of time. Therefore, only by optimizing portfolio strategies of purchasing electricity in the multiple energy market can reduce the risk and expand the income. However, the prices of electricity in multiple markets have a volatility due to the relationship of supply and demand, quotation strategy and other conditions. These factors directly affect the power electricity company's decision making. Therefore, when studying the purchasing strategy of electricity companies in multiple markets, the uncertainty due to price fluctuation will be given priority.

To quantify the risk from uncertainty of prices in multiple markets, various risk management methods [1] are introduced into the portfolio strategies of electricity purchasing. These methods are divided into two categories: mean variance based method and conditional risk value (CVaR) based method. Ref. [2] uses the mean variance model to analyse the risk management problems of the electricity companies in multiple markets by taking the variance of mean electricity price as a risk. The mean variance model does not take into account the risk tolerance of the electricity company, and involves the expected income part of the investor in the measurement risk, so it is necessary to further improve the model. Ref. [3] introduces the risk management in the financial field to the portfolio decision of electricity companies in the multiple markets, and sets up a power purchasing combination model of CVaR based on the deterministic probability distribution. CVaR, as a risk measure, reflects the investor's preference for risk, and also reflects the real electricity purchasing loss to some extent. But it is necessary to know the accurate probability distribution of electricity prices in the CVaR.
model, which is difficult in practice. To this end, Ref. [4] uses the weighted CVaR model to coordinate the probability distribution between various probability distributions.

However, the above researches do not consider the uncertainty of the probability distribution of electricity price in the real time market, the day-ahead market and the medium and long term contract market. Since the historical data in different markets often change, the probability distribution established by the long-term data cannot accurately reflect the actual probability distribution represented by the few data obtained in the near future. Therefore, it is necessary to consider the problem that the electricity price data in different markets is limited, and the probability distribution is uncertain.

In this paper, an electricity purchasing portfolio model is derived from the distributional robust conditional value-at-risk (DR-CVaR) concept [5], which can be transformed into a deterministic semi-definite programming problem and easy to be solved by a mature solver. The proposed model and solution approach provide a new method for electricity company decision-making, which can effectively enhance the risk management ability of electricity companies.

2. Preliminaries and relative theories

2.1. VaR and CVaR

Value-at-Risk (VaR) does not meet the conformance axiom and lacks of secondary additivity, and its computational form is equivalent to the chance constrained programming. CVaR model avoids calculating the chance constrained programming and makes up for the lack of the tail loss measurement of the VaR [2].

2.2. Calculation of CVaR

We denote decision variable and random variable by \( x \in X \) and \( y \in R^n \), respectively, and define \( f(x,y) \in R^n \) as loss function, in which \( X \) is a feasible set of portfolio strategies for a certain condition. Assuming that the probability distribution function of \( y \) is known, then \( y \) has a joint probability density function \( p(y) \). With a fixed \( x \), the probability of the loss function \( f(x,y) \) not exceeding the critical value \( \alpha \) is

\[
P(x,\alpha) = \int_{f(x,y) \geq \alpha} p(y)dy
\]  

When the confidence level is \( \beta \in (0,1) \), VaR and CVaR can be expressed as:

\[
VaR_\beta(x) = \min \{ \alpha \in R; P(x,\alpha) \geq \beta \}
\]

\[
CVaR_\beta(x) = E[f(x,y) | f(x,y) \geq VaR_\beta(x)]
\]

\[
= \frac{1}{1-\beta} \int_{f(x,y) \geq VaR_\beta(x)} f(x,y) p(y)dy
\]

Because the VaR function involved in CVaR is difficult to be computed, a relatively simple method is introduced in literature to solve CVaR without calculating VaR.\( F_\beta(x,\alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in R} [f(x,y) - \alpha]^+ p(y)dy \)

where function \( [f(x,y) - \alpha]^+ \) is equal to \( \max(0,f(x,y) - \alpha) \). Eq. (4) is a convex function of \( \alpha \) and is continuously differentiable. And it is proved that \( \min_{x} CVaR_\beta(x) = \min_{x,\alpha} F_\beta(x,\alpha) \)

2.3. Distributional robust conditional value-at-risk

Considering the fact that accurate probability distribution is difficult to be obtained in reality, some information of random variables, such as expectation and variance, is relatively easy to access. In addition, if we use the maximum conditional risk value (DR-CVaR) under all the possible probability
distribution of the same expectation and variance to describe the risk faced by the electricity portfolio decision-making, it will effectively improve the robustness of the electricity purchasing portfolio strategy and enhance the risk aversion of the electricity company.

DR-CVaR can be described as Eq.(6) [5].

\[
\sup_{\rho\in\Gamma} P \cdot F_{\rho}(x) = \inf_{\alpha} \left\{ \alpha + \frac{1}{1-\beta} \sup_{\rho\in\Gamma} \mathbb{E}_{\rho} \left[ (f(x,y) - \alpha) \right] \right\} \tag{6}
\]

where \( P \) denotes the joint probability distribution of \( y \), \( \Gamma \) is a set of all the probability distribution with the same expectation \( \mu \) and covariance matric \( \Sigma \), and \( P \) belongs to \( \Gamma \). Since \( \Gamma \) covers all the possible probability distribution, the maximum CVaR is denoted by \( \sup_{\rho\in\Gamma} P \cdot F_{\rho}(x) \), which can be calculated by Eq. (6).

The electricity company can obtain a more robust electricity purchasing portfolio strategy by minimizing the maximum conditional risk in (6).

2.4. The dual transformation of DR-CVaR

The DR-CVaR method is a combination of stochastic programming and robust optimization. Considering the condition of given expectation and variance, the CVaR value under the most severe distribution affecting the decision results is optimized. Since only the expected value of \( (f(x,y) - \alpha) \) denotes the robustness on the right side of (6), the duality transformation of the distributional robust expectation is given.

Under the condition of distribution uncertainty, the maximum expectation about function \( \Phi(x,y) \) can be expressed as

\[
\inf_{x} \sup_{\rho\in\Gamma} \mathbb{E}_{\rho}(\Phi(x,y)) \tag{7}
\]

When be solved directly, the optimization (7) is a NP hard problem. In the aforementioned literature, we propose to transform the above problem into a semi-definition programming problem by duality principle, which satisfies the strong dual condition [5].

The dual formulation of problem (7) is

\[
\inf_{\alpha} \inf_{\beta} \mathbb{E}_{\rho}(\Phi(x,y)) \tag{8}
\]

where \( M \in \mathbb{R}^{(n+1)(n+1)} \) is an unknown symmetric matrix, \( tr \) denotes the trace, and \( Q = \left[ \Sigma + \mu \mu^T / \mu \right] \).

When \( \Phi(x,y) \) is a linear or quadratic function of \( y \), the dual programming problem (8) can be converted to a semi-definition programming (SDP) through Farkas Lemma or S-lemma.

3. Power purchasing portfolio model for distributional robust CVaR

3.1. Problem setting

It is assumed that terminal price is subject to government regulation and increased by a fixed rate on the basis of the mean value of multiple market electricity prices.

The decision vector \( x^T = [x_1, x_2, \ldots, x_m] \in \mathcal{X} \) denotes the electricity purchasing portfolio strategies in multiple markets, where \( \mathcal{X} \) is the set of feasible solution, and \( x_i (i=1,2,\ldots,m) \) denotes the percentage of the total electricity purchased which satisfies \( x_i \geq 0, \sum_{i=1}^{m} x_i = 1 \). \( y_i (i=1,2,\ldots,m) \) denotes the price of electricity purchased by electricity company in multiple markets, \( y^T = [y_1, y_2, \ldots, y_m] \) denotes the vector of \( y_i \), then the cost of purchasing electricity is \( \sum_{i=1}^{m} y_i x_i \).
We denote the price vector in three markets, real-time electricity market, day-ahead electricity market and mid-long term contract market, by \( y^T = [y_1, y_2, y_3] \), whose mean value and covariance are \( \mu = (\mu_1, \mu_2, \mu_3) \) and \( \sigma^2 = (\sigma_1^2, \sigma_2^2, \sigma_3^2) \), respectively.

In this paper, we consider that the probability distribution of the price in three markets is uncertain, and take the maximum CVaR under the possible probability distribution of the same expectation \( \mu \) and covariance \( \sigma^2 \) as the risk constraint. With a fixed rate \( b \) denoting the yield, \( m = (m_1, m_2, m_3) = (1 + b)(\mu, \mu_2, \mu_3) \) can be used to denote the selling price of electricity companies to end users.

3.2. Construction of power purchasing portfolio model for DR-CVaR

Based on the above setting, per unit return of the power purchasing portfolio is denoted as

\[
    r(x) = \sum_{i=1}^{3} m_i x_i - \sum_{i=1}^{3} y_i x_i
\]

Per unit loss function is

\[
    f(x, y) = \sum_{i=1}^{3} y_i x_i - \sum_{i=1}^{3} m_i x_i
\]

The mean value of per unit return of the power purchasing portfolio is

\[
    R(x) = E(r(x)) = \sum_{i=1}^{3} m_i x_i - \sum_{i=1}^{3} E(y_i) x_i
\]

Referring to the Eq. (6)-(8), the DR-CVaR of \( R(x) \) can be formulated as

\[
    \sup_{\alpha, M} P \cdot F_p(x) = \inf_{\alpha, M} \alpha + \frac{1}{1-\beta} \text{tr}(Q \cdot M) \quad \text{s.t.} \quad M \geq 0
\]

\[
    M = \begin{bmatrix}
    0 & \frac{1}{2} x^T \\
    \frac{1}{2} x - \sum_{i=1}^{3} m_i x_i - \alpha & 1
    \end{bmatrix} \geq 0
\]

where \( f(x, y) = \sum_{i=1}^{3} y_i x_i - \sum_{i=1}^{3} m_i x_i \), \( \sup_{\alpha} P \cdot F_p(x) \) denotes DR-CVaR of per unit loss function.

When we choose DR-CVaR to calculate the risk from loss function \( f(x, y) \) and aim to make \( R(x) \) maximum, the optimization model of power purchasing portfolio is constructed as

\[
    \min[-R(x)] = \min \sum_{i=1}^{3} E(y_i) x_i - \sum_{i=1}^{3} m_i x_i
    \quad \text{s.t.} \quad \alpha + \frac{1}{1-\beta} \text{tr}(Q \cdot M) \leq \omega \|
\]

\[
    M \geq 0, x_i \geq 0, \sum_{i=1}^{3} x_i = 1
\]

\[
    M = \begin{bmatrix}
    0 & \frac{1}{2} x^T \\
    \frac{1}{2} x - \sum_{i=1}^{3} m_i x_i - \alpha & 1
    \end{bmatrix} \geq 0
\]

where \( \omega \) is the upper limit of the risk that the electricity company can tolerate, and satisfies \( \omega \geq 0 \).
4. Case study

The average and variance of purchasing price \( y' = [y_1, y_2, y_3] \) in a power market is obtained through the data of real-time market, day-ahead market and mid-long term contract market for 100 days (seen in table 1). Setting selling price in different markets as \( m_i = (1 + b)E(y_i) \), and \( b = 0.1 \), we can solve the optimization model (13) by the solver named SDPT3 which is a mature solver for SDP problem[6]. The results are shown in table 2, where \( x(i=1,2,3) \) denotes the percentage of the total power purchased in three markets. In addition, \( \omega \) is not listed in table 2 because it is the same as the last column of table 2.

**Table 1.** Price in different markets.

| market            | Real time | Day-ahead | mid-long term contract market |
|-------------------|-----------|-----------|--------------------------------|
| Price data        | \( \mu_1 \) | \( \sigma_1 \) | \( \mu_2 \) | \( \sigma_2 \) | \( \mu_3 \) | \( \sigma_3 \) |
|                   | 53.30     | 18.30     | 46.30                          | 13.50                          | 38.50     | 7.50            |

**Table 2.** Allocation of power purchases and DR-CVaR.

| Confidence level | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( R(x) \) | DR-CVaR/$ |
|------------------|-----------|-----------|-----------|------------|-----------|
| \( \beta = 5\% \) | 0.244     | 0.2954    | 0.4605    | 4.4416     | 4.15      |
|                  | 0.535     | 0.3994    | 0.0658    | 4.953      | 6.92      |
|                  | 0.677     | 0.3229    | 0         | 5.104      | 8.54      |
|                  | 1         | 0         | 0         | 5.33       | 11.05     |
| \( \beta = 10\% \) | 0.252     | 0.2983    | 0.4493    | 4.4562     | 2.08      |
|                  | 0.472     | 0.3770    | 0.1506    | 4.84       | 3         |
|                  | 0.690     | 0.3098    | 0         | 5.11       | 4.5       |
|                  | 1         | 0         | 0         | 5.33       | 7.5       |

In order to observe how the expected return of power purchasing portfolio \( R(x, y) \) varies when DR-CVaR is changed, we change the value of \( \omega \) and solve the optimization problem again and again. Figure 1 shows the efficient frontier of the expected return and DR-CVaR, and all the solutions that satisfies the constraints are on the curve.

In further, when we set the confidence level \( \beta \) as different values, the curves of the efficient frontier of the expected return and DR-CVaR are also changed. In practical decision-making situation, there is a lower bound of the value of \( \omega \), when we set \( \omega \) less than the bound, the optimization problem can not be solved effectively. On the other side, if we set \( \omega \) too large, the expected return \( R(x) \) will not increase with the \( \omega \) becoming larger, and the solution will be \( x = [1 \ 0 \ 0] \) all the time, and meanwhile, the objective function is 5.33. In Figure 1, the points label as 1 and 2 are the maximal value of the expected return under different confidence level, and the points label as 1’ and 2’ denote the minimal value of the expected return under different confidence level.
5. Conclusions
The model proposed in this paper is robust to all possible probability distributions under the same expectation and variance in different market prices. In theory, it is more conservative than the result based on a certain probability distribution, but the necessary conservatism can improve the reliability of the risk management of the electricity company. This paper poses a new way of management in the operation of power market.

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