Majorana Neutrinos
in a Warped 5D Standard Model

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Abstract

We consider neutrino oscillations and neutrinoless double beta decay in a five dimensional standard model with warped geometry. Although the seesaw mechanism in its simplest form cannot be implemented because of the warped geometry, the bulk standard model neutrinos can acquire the desired (Majorana) masses from dimension five interactions. We discuss how large mixings can arise, why the large mixing angle MSW solution for solar neutrinos is favored, and provide estimates for the mixing angle $U_{e3}$. Implications for neutrinoless double beta decay are also discussed.

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1 Introduction

Extra dimensions are an interesting possibility for physics beyond the standard model (SM). They offer an explanation for the weakness of gravity by the large volume of the compactified space \([1]\), or by localization of the graviton at a space-time boundary \([2, 3]\). Gravity becomes strong at the TeV scale, and the large hierarchy between the scale of gravity and the weak scale is eliminated.

We take the fifth dimension to be an \(S_1/Z_2\) orbifold with a negative bulk cosmological constant, bordered by two 3-branes with opposite tensions and separated by distance \(R\). Einstein’s equations are satisfied by the non-factorizable metric \([2]\)

\[
ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \sigma(y) = k|y|
\]

which describes a slice of AdS\(_5\). The 4-dimensional metric is \(\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)\), \(k\) is the AdS curvature related to the bulk cosmological constant and brane tensions, and \(y\) denotes the fifth coordinate. The AdS curvature and the 5d Planck mass \(M_5\) are both assumed to be of order \(M_{\text{Pl}} \sim 10^{19}\) GeV. The AdS warp factor \(\Omega = e^{-\pi ky}\) generates an exponential hierarchy of energy scales. If the brane separation is \(kR \simeq 11\), the scale at the negative tension brane, located at \(y = \pi R\), is of TeV-size, while the scale at the brane at \(y = 0\) is of order \(M_{\text{Pl}}\). At the TeV-brane gravity is weak because the zero mode corresponding to the 4D graviton is localized at the positive tension brane (Planck-brane).

Models with localized gravity open up attractive possibilities for flavor physics. If the SM fermions reside in the 5-dimensional bulk, the hierarchy of quark and lepton masses can be interpreted in a geometrical way \([4, 5]\). Different flavors are localized at different positions in the extra dimension or, more precisely, have different wave functions. The fermion masses are in direct proportion to the overlap of their wave functions with the Higgs field \([6]\). Moreover, bulk fermions reduce the impact of non-renormalizable operators which, for instance, induce rapid proton decay and large neutrino masses \([4, 5]\).

In the proposed framework it is natural to take the SM gauge bosons as bulk fields as well to maintain 5D gauge invariance. The Higgs field has to be localized at the TeV-brane. Otherwise the gauge hierarchy problem reappears \([7, 8]\). Comparison with electroweak data, in particular with the weak mixing angle and gauge boson masses, requires the Kaluza-Klein (KK) excitations of SM particles to be heavier than about 10 TeV \([8, 9]\). If the fermions were confined to the TeV-brane, the KK scale would be even more constrained \([8, 10]\).

In refs. \([12, 13]\) it was demonstrated that small Dirac neutrino masses can be generated by adding sterile neutrinos in the bulk. Both the solar and the atmospheric neutrino anomalies can be accommodated.

\(\text{\footnotesize 3Recently it has been advocated that the third generation of quarks should be confined to the TeV-brane to prevent large contributions to the electroweak } \rho \text{ parameter } [1]. \text{ However, no such conclusion can be drawn for the third generation of leptons.}\)
In this letter we investigate the alternative possibility of Majorana neutrinos. Because of the warped geometry, the usual see-saw mechanism cannot be implemented in a straightforward way. Namely, under the assumption that the large 5D Majorana mass is $y$ independent, the lowest state of the singlet neutrino, it turns out, is localized close to the TeV-brane where it acquires a typical KK scale mass ($\sim$ a few TeV) from the warped geometry. Fortunately, in contrast to the SM, the warped 5D SM admits larger dimension five Majorana masses and, with a judicious choice of parameters these can be employed for resolving the solar and atmospheric neutrino anomalies. Large mixings arise if the SM neutrinos are located at similar positions in the extra dimension. We demonstrate that the large mixing angle MSW is the favored solution to the solar neutrino anomaly, while $\theta_{13}$ can be kept below the observational bound. We also discuss possible experimental signatures to distinguish between the scenarios of Dirac and Majorana neutrinos, such as neutrinoless double beta decay and $\mu \rightarrow e\gamma$. We also comment on the issue of proton decay.

2 Fermions in a Warped Background

To fix the notation let us briefly summarize the properties of fermions in a warped geometrical background. Since the 5D theory is non-chiral, every Weyl fermion of the SM has to be associated with a Dirac fermion in the bulk. The number of fermionic degrees of freedom is doubled. Chirality in the 4D low energy effective theory is restored by the orbifold boundary conditions.

The Dirac equation for a fermion in curved space-time reads

$$E_a^M \gamma^a (\partial_M + \omega_M) \Psi + m_\Psi \Psi = 0,$$

where $E_a^M$ is the fünfbine, $\gamma^a = (\gamma^\mu, \gamma^5)$ are the Dirac matrices in flat space,

$$\omega_M = \left( \frac{1}{2} e^{-\sigma'} \gamma_5 \gamma_\mu, 0 \right)$$

is the spin connection, and $\sigma' = d\sigma/dy$. The index $M$ refers to objects in 5D curved space, the index $a$ to those in tangent space. Fermions have two possible transformation properties under the $Z_2$ orbifold symmetry: $\Psi(-y)_\pm = \pm \gamma_5 \Psi(y)_\pm$. Thus, $\Psi_\pm \Psi_\pm$ is odd under $Z_2$, and the Dirac mass parameter, which is also odd, can be parametrized as $m_\Psi = c\sigma'$. The Dirac mass should therefore originate from the coupling to a $Z_2$ odd scalar field which acquires a vev. On the other hand, $\Psi_\pm \Psi_\mp$ is even. Using the metric (1.1) one obtains for the left- and right-handed components of the Dirac spinor [4,12]

$$[e^{2\sigma} \partial_\mu \partial^\mu + \partial_5^2 - \sigma' \partial_5 - M^2]e^{-2\sigma} \Psi_{L,R} = 0,$$

where $M^2 = c(c \pm 1)k^2 \mp c\sigma''$ and $\Psi_{L,R} = \pm \gamma_5 \Psi_{L,R}$. 


Decomposing the 5d fields as
\[
\Psi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi^{(n)}(x^\mu) f_n(y),
\]
(2.5)
eq(2.4) admits a zero mode solution \[4, 12\]
\[
f_0(y) = \frac{e^{(2-c)\sigma}}{N_0},
\]
(2.6)
and a tower of KK excited states
\[
f_n(y) = \frac{e^{5\sigma/2}}{N_n} \left[ J_\alpha \left( \frac{m_n}{k} e^\sigma \right) + b_\alpha(m_n) Y_\alpha \left( \frac{m_n}{k} e^\sigma \right) \right].
\]
(2.7)
The order of the Bessel functions is \(\alpha = |c \pm 1/2| \) for \(\Psi_{L,R}\). The spectrum of KK masses \(m_n\) and the coefficients \(b_\alpha\) are determined by the boundary conditions of the wave functions at the branes. The normalization constants follow from
\[
\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \, e^{-3\sigma} f_m(y) f_n(y) = \delta_{mn}.
\]
(2.8)
Because of the orbifold boundary conditions, the zero mode of \(\Psi_+ \) (\(\Psi_-\)) is a left-handed (right-handed) Weyl spinor. For \(c > 1/2 \) (\(c < 1/2\)) the zero mode of the fermion is localized near the boundary at \(y = 0 \) \((y = \pi R)\), i.e. at the Planck- (TeV-) brane. The KK states are always localized at the TeV-brane.

3 The See-Saw Mechanism and Warped Geometry

In the see-saw mechanism lepton number is broken by the Majorana mass of heavy right-handed neutrinos, \(\mathcal{L}_M = M_N N N\). Together with a Dirac mass term \(\mathcal{L}_D = \lambda_N N_L H\), where \(H\) denotes the SM Higgs field, a Majorana mass for the SM neutrinos \(\nu_L\) is generated,
\[
M_\nu = \frac{\lambda_N^2 \langle H \rangle^2}{M_N}.
\]
(3.9)
Taking \(M_\nu \sim 50\text{meV}\) (of the order of the atmospheric \(\Delta m^2\)), one finds \(M_N \sim \lambda_N^2 \cdot 6 \times 10^{14} \text{GeV}\). For \(0.01 \lesssim \lambda_N \lesssim 1\) this points to an intermediate scale for the right-handed Majorana mass.

In theories with TeV-scale gravity \(M_N\) is naturally bounded to be below a few TeV. Therefore, very small Yukawa couplings \(\lambda_N \lesssim 10^{-6}\) are required to generate sub-eV neutrino masses. Keeping in mind the large hierarchy of Yukawa couplings
for quarks and charged leptons, this result may not seem “completely unreasonable”. Nonetheless, a deeper understanding of the smallness of neutrino masses is missing.

If the SM and right-handed neutrinos reside in the bulk, the 5D Yukawa coupling \( \lambda_N^{(5)} \) is somewhat less constrained than the 4D coupling we were concerned with in the previous paragraph. The Dirac mass which enters eq. (3.9) depends on the overlap of the neutrino zero modes and the Higgs

\[
m_D \equiv \lambda_N \langle H \rangle = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_N^{(5)} e^{-4\sigma} H(y) f_0^{(\nu)}(y) f_0^{(N)}(y). \quad (3.10)
\]

We assume that the Higgs profile has an exponential form which peaks at the TeV-brane, \( H(y) = H_0 e^{4k(y)} - \pi R \). This shape can be motivated by the equation of motion of a bulk scalar field [14]. Numerically it is equivalent to a delta function-like profile. Using the known mass of the W-boson, we can fix the amplitude \( H_0 \sim M_{\text{Pl}} \) in terms of the 5D weak gauge coupling.

The zero mode of the SM neutrino has an exponential shape (2.6) which for \( c_\nu > 1/2 \) has only a small overlap with the Higgs at the TeV-brane. Sticking to the idea that all input parameters in the 5D theory should be of order unity in natural units, we assume that in the bulk lepton number is broken at \( M_N^{(5)} \sim M_{\text{Pl}} \). Because of this large Majorana mass, the wave function of the lowest state of the right-handed neutrino is modified. Rather than acquiring a mass of order \( M_{\text{Pl}} \), this state gets localized at the TeV-brane where the warped geometry induces a typical KK mass for it, very similar to excited fermionic states. As a result, the overlap of the right-handed neutrino and the Higgs is large. These basic properties of the right-handed “zero mode” are fairly independent of the details of the KK reduction in the presence of a large bulk Majorana mass[4]. They also do not depend on a possible 5D Dirac mass \( c_N \) for the right-handed neutrino.

We now estimate the required value of \( \lambda_N^{(5)} \) to generate a 50 meV neutrino mass. As explained above, the Majorana mass of the zero mode \( M_N \) is close to the KK scale \( M_{KK} \). From comparison with electroweak precision data, \( M_{KK} \) cannot be smaller than about 10 TeV [4]. By localizing \( f_0^{(\nu)} \) close to the Planck-brane (\( c_\nu \gg 1/2 \)), \( m_D \) can be made very small. However, at the same time, the mass of the charged lepton, residing in an SU(2) multiplet with the neutrino, can become very small as well [4, 13]. Since the tau is the heaviest lepton, it has to reside closest to the TeV-brane, and induces the largest \( m_D \). Assuming 5D lepton Yukawa couplings of order unity (i.e. \( \lambda_L^{(5)} \sim g^{(5)} \), the 5D weak gauge coupling), the left-handed tau has to obey

\( c \lesssim 0.57 \) (when the right-handed tau is delocalized, \( c = 1/2 \) [4]. If we take \( M_N = 10 \) TeV, \( \lambda_N^{(5)} = g^{(5)} \) and \( c_\tau = 0.57 \), we find \( m_D = 7.9 \) GeV. A neutrino mass of 50 meV requires \( \lambda_N^{(5)}/g^{(5)} \sim 9 \times 10^{-5} \). In accordance with the discussion in the previous paragraph, we have approximated the right-handed zero mode by the wave function

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4 If one allows for a suitably tuned profile of the Majorana mass in the bulk, the lowest state of the right-handed neutrino can be peaked around the Planck-brane. Its mass is still found to be bounded by the KK scale.
of the first KK state of a fermion \( (2.7) \) without Majorana mass and \( c_N = 1/2 \). In fig. 1 we present the associated wave functions of the \( \nu_\tau \) and \( N \) zero modes, together with the Higgs profile. The bound on the Yukawa coupling only mildly depends on \( c_N \), increasing it from 1/2 to 1 would result in \( \lambda_N^{(5)}/g^{(5)} \sim 7 \times 10^{-5} \). The error arising from our approximation of the right-handed neutrino wave function is of the same level.

The bound on \( \lambda_N^{(5)} \) can be somewhat relaxed if we assume that the right-handed tau is localized at the TeV-brane (let’s say with \( c = -1 \)). Then we can move the left-handed tau and the tau neutrino closer to the Planck brane, \( c \sim 0.63 \), and find \( \lambda_N^{(5)}/g^{(5)} \sim 6 \times 10^{-4} \). However, having the right-handed leptons localized towards the TeV-brane \( (c < 1/2) \) induces large deviations in the electroweak observables \( [9] \). The KK scale is required to be much higher than 10 TeV, and the gauge hierarchy problem gets reintroduced, albeit in a “mild” form. In the case of the muon the constraints on \( \lambda_N^{(5)} \) are an order of magnitude weaker.

If one relies on supersymmetry to solve the gauge hierarchy problem, the Higgs fields can reside in the bulk. Then a different and more standard version of the see-saw mechanism may be implemented: The right-handed neutrinos, having an intermediate mass, are confined to the Planck-brane. This translates into an intermediate see-saw scale in the effective 4D theory. Since the Higgs is spread homogeneously in the bulk, there are no small overlaps anymore. Thus, an explanation of the fermion mass hierarchy is lost. An alternative way of implementing the see-saw mechanism is to assume a \( y \) dependent singlet Majorana mass.

In conclusion, having the right-handed and sterile neutrinos in the bulk reduces the fine-tuning of the neutrino Yukawa coupling in the see-saw approach to some extent. However, a small number of order \( 10^{-4} \) is still needed to generate sub-eV neutrino masses.
4 Dimension Five Neutrino Masses

In the 4D SM one does not have to worry about possible higher-dimensional operators, since they are safely suppressed by powers of the huge Planck mass. Models with a low scale of gravity are very different in this respect. Naively one expects exotic non-renormalizable interactions suppressed by only a few TeV mass scale, unless they are forbidden by symmetries or multiplied by tiny coupling constants. In the warped SM the suppression scale for non-renormalizable operators can be anywhere between a few TeV and the Planck scale, depending on the localization of the fermions \[\text{(1.1)}\].

As previously discussed in refs. \[\text{(1.1)}\], Majorana masses for left-handed neutrinos are generated by the dimension-five operator

\[
\int d^4x \int d\sqrt{-g} \frac{l_{ij}}{M_5^2} H^2 \Psi_{iL} C \Psi_{jL} \equiv \int d^4x \; M_{ij}^{(v)}(y) \Psi_{iL}^{(0)} C \Psi_{jL}^{(0)},
\]

(4.11)

where \(l_{ij}\) are dimensionless couplings constants and \(C\) is the charge conjugation operator. The neutrino mass matrix reads

\[
M_{ij}^{(v)} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \frac{l_{ij}}{M_5^2} e^{-4\sigma(y)} H^2(y) f_{0i}^{(v)}(y) f_{0j}^{(v)}(y).
\]

(4.12)

Again the most stringent constraint on \(l\) comes from the tau neutrino. Assuming the Yukawa coupling of the tau to be \(\lambda^{(5)}_{\tau} \approx g^{(5)}\) and \(c(\tau_R) = 1/2\), we found \(c(\tau_L) = 0.567\). From eq. (4.12) we obtain a tau neutrino mass of \(m_{\nu,\tau} = l \cdot 33\) MeV \[\text{(5)}\]. Bringing the neutrino mass down to 50 meV requires a very small value of \(l \sim 10^{-9}\). Such a small coupling could originate from non-perturbative effects of gravity, especially if there is an extra dimension somewhat larger than \(M_{PL}\) \[\text{(5)}\]. However, it would be attractive to relax the bound on \(l\) within the framework of the effective theory we have considered so far.

We can reduce the neutrino masses by shifting the left-handed leptons closer to the Planck-brane. To maintain the masses of the charged leptons, we either have to move the right-handed leptons closer to the TeV-brane or increase the 5D Yukawa couplings of the leptons \(\lambda^{(5)}\). The first possibility is clearly disfavored by the electroweak precision data, as explained in the previous section. Keeping the positions of the right-handed leptons fixed at \(c = 1/2\), the neutrino masses are roughly proportional to \(1/(\lambda^{(5)})^2\) \[\text{(5)}\]. If we restrict ourselves to \(\lambda^{(5)}_{\nu} = 10g^{(5)}\), in order not to induce a large hierarchy in the 5D couplings, we find \(c(\tau_L) = 0.647\) and \(m_{\nu,\tau} = l \cdot 320\) keV.

The neutrino masses can be further decreased if we take the AdS curvature \(k\) to be smaller than the 5D Planck mass. Approximately, the relation \(m_{\nu} \propto (k/M_5)^2\) holds \[\text{(5)}\]. The assumption \(k < M_5\) is anyway necessary to derive the warped metric \[\text{(1.1)}\] from Einstein equations, neglecting terms with higher derivatives \[\text{(5)}\]. In discussions of the collider phenomenology of the warped model, the parameter range
$M_5/100 < k < M_5$ has been considered [10]. Taking now the very favorable case $\lambda_5^{(5)} = 10g_5^{(5)}$ and $k/M_5 = 0.01$, we obtain $m_{\nu,\tau} = l \cdot 25$ eV. A neutrino mass of 50 meV can therefore be generated by $l = 0.002$, which is only a moderately small number. However, one has to keep in mind that for $k/M_5 = 0.01$ the radius $kR = 8.63$ is already of order $10^3$ in units of the fundamental scale $M_5$.

Thus, the Majorana neutrino masses induced by the dimension five operator (4.11) are quite large if we take the model parameters to be strictly of order unity. In ref. [5] we therefore imposed lepton number to eliminate (4.11) completely. Here we assume that one or a combination of the mechanisms discussed above is at work to bring down the neutrino masses to values which are consistent with experimental observations. In the following we investigate the consequences for neutrino mixing as well as for neutrinoless double beta decay.

5 Neutrino Mixings

In ref. [3] it was found that the dimension five neutrino masses (4.12) lead to small mixing angles for the neutrinos. This result arose from the different locations of the left-handed lepton flavors in the extra dimension in order to explain the hierarchy of charged lepton masses. In the following we demonstrate that large mixing angles naturally arise if the left-handed leptons have the same location in the extra dimension. This result is similar to the case of Dirac masses for neutrinos studied in ref. [13]. We will also show that separating the electron neutrino somewhat from the muon and tau neutrinos helps to keep $U_{e3}$ sufficiently small.

To obtain the neutrino masses and mixings we diagonalize the mass matrix (4.12) with a unitary matrix $M^{(\nu)} = U_{\nu} M^{(\nu)}_{\text{diag}} U_{\nu}^T$. The physical neutrino mixings

$$U = U_l^T U_{\nu},$$

also depend on the rotations of the left-handed charged leptons $U_l$. The charged lepton masses $M^{(l)}$ arise from their couplings to the Higgs field in the same way as the Dirac neutrino masses of eq. (3.10),

$$M^{(l)}_{ij} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda^{(5)}_N e^{-4\sigma} H(y) f^{(l)}_{bi}(y) f^{(e)}_{ji}(y),$$

where we have inserted the charged lepton wave functions and Yukawa couplings $\lambda^{(5)}_N$ [4, 5]. We determine $U_l$ by diagonalizing $M^{(l)}(M^{(l)})^\dagger$. In general, the neutrino and charged lepton rotations are expected to show the same pattern on average since their positions in the extra dimension are tied together by the SU(2) gauge symmetry of the SM. Including CP violation, the mixing matrix $U$ can be written as

$$U = \begin{pmatrix}
c_2c_3 & c_2s_3 & s_2e^{-i\delta} \\
-c_1s_3 - s_2s_1c_3e^{i\delta} & c_1c_3 - s_2s_1s_3e^{i\delta} & c_2s_1 \\
s_1s_3 - s_2c_1c_3e^{i\delta} & -s_1c_3 - s_2c_1s_3e^{i\delta} & c_2c_1
\end{pmatrix},$$

(5.15)
where \( s_i \) and \( c_i \) are the sines and cosines of \( \theta_i \). Like in the CKM matrix, there is a single complex phase \( \delta \) which induces CP violation in the lepton sector. The neutrino mass matrix contains two additional Majorana phases which, however, do not show up in the mixing matrix \( U \).

The atmospheric neutrino data imply \(^{17}\)

\[
1 \cdot 10^{-3} \text{eV}^2 < \Delta m_{\text{atm}}^2 < 5 \cdot 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_1 > 0.85. \tag{5.16}
\]

There are several solutions to the solar neutrino anomaly \(^{18}\)

\[
\begin{array}{ccc}
\Delta m_{\text{sol}}^2 [\text{eV}^2] & \sin^2 2\theta_3 \\
\text{LMA} & 2 \cdot 10^{-5} - 4 \cdot 10^{-4} & 0.3 - 0.93 \\
\text{SMA} & 4 \cdot 10^{-6} - 9 \cdot 10^{-6} & 0.0008 - 0.008 \\
\text{LOW} & 6 \cdot 10^{-8} - 2 \cdot 10^{-7} & 0.89 - 1 \\
\text{VAC} & \sim 10^{-10} & 0.7 - 0.95
\end{array}
\tag{5.17}
\]

The SMA solution gives only a poor fit to the data. The CHOOZ reactor experiment constrains \(|U_{e3}|^2 \equiv s_3^2\) to be at most a few percent \(^{19}\). Nothing is known about the CP violating phase \( \delta \).

Our solution to the neutrino puzzle follows the “neutrino mass anarchy” models \(^{20}\). Mass matrices with randomly chosen entries of order unity have a large probability to fit the neutrino data. Thus, we parametrize the coefficients in (4.12) by

\[
l_{ij} \equiv \Lambda \cdot \tilde{l}_{ij}. \tag{5.18}
\]

In contrast to the 4D realization of “neutrino mass anarchy”, in our scenario the smallness of the neutrino masses is not attributed to a very small overall magnitude \( \Lambda \) of the couplings. Rather, the neutrinos are localized close to the Planck-brane, where the dimension five operator \((\overline{\psi} \psi \gamma_5)\) is suppressed. The supposedly fundamental theory responsible for the effective interaction \((\overline{\psi} \psi)\) is represented through the order unity coefficients \( \tilde{l}_{ij} \). To incorporate the charged lepton rotations we also generate charged lepton mass matrices with random Yukawa couplings of order unity. We choose suitable lepton locations to reproduce the measured lepton masses.

We consider the very favorable case of \( \lambda^{(5)} = 10 g^{(5)} \) and \( k/M_5 = 0.01 \). We take the position of the right-handed tau to be \( c_{\tau,R} = 1/2 \), which means its wave function is delocalized in the extra dimension. To reproduce the tau mass we find the position of the left-handed tau to be \( c_{\tau,L} = 0.72 \). Since different positions for the left-handed leptons tend to produce small neutrino mixing angles \(^{4,13}\), we take

\[
c_{e,L} = c_{\mu,L} = c_{\tau,L} = 0.72. \tag{5.19}
\]

To accommodate the muon and electron masses we use \( c_{\mu,R} = 0.64 \) and \( c_{e,R} = 0.85 \). Taking the order unity coefficients to be homogeneously distributed \( 1/2 < |\tilde{l}_{ij}| < 2 \)
with random phases from 0 to $2\pi$, we find the most favorable value for the overall scale to be $\Delta = 1.9 \cdot 10^{-3}$. We randomly generate parameter sets for $\tilde{l}_{ij}$, calculate the neutrino mass matrix from (4.12) and compute the neutrino masses and mixings. For the charged lepton rotations we take random Yukawa couplings $10g^{(5)}/\sqrt{2} < |[\lambda_l^{(5)}]_{ij}| < 10g^{(5)} \cdot \sqrt{2}$ with phases from 0 to $2\pi$. The charged lepton masses and left-handed rotations are then calculated from the mass matrix eq. (5.14). We require the computed lepton masses to agree up to a factor of $3/2$ with the measured values, which holds for about $50\%$ of the random sets of $\lambda_l^{(5)}$. These sets are the starting point for the investigation of the neutrino properties.

Focusing on the LMA solution of the solar neutrino anomaly, which turns out to be clearly favored, we find the following picture. From our neutrino parameter sets about $70\%$ reproduce $\Delta m^2_{\text{atm}}$. Imposing in addition the constraint from $\Delta m^2_{\text{sol}}$ (5.17), we are left with about $28\%$ of the parameter sets. The solar and atmospheric mixings angles bring this number down to about $6\%$, which is still a considerable fraction given the number of constraints. The most stringent constraint turns out to come from the CHOOZ experiment. Adding the requirement $|U_{e3}|^2 < 0.05$, the fraction of viable parameter sets shrinks to about $0.7\%$. This result is clearly related to $\langle |U_{e3}|^2 \rangle = 0.22$, which is considerably above the experimentally favored value. Here we use the logarithmic average of a quantity $X$

$$
\langle X \rangle = \exp \left( \frac{1}{N} \sum_i \ln(X_i) \right).
$$

(5.20)

If not stated otherwise, we average over the parameter sets which reproduce the correct $\Delta m^2$’s. In fig. 2 we display the distribution of $|U_{e3}|^2$ found in our statistical analysis. We included only parameter sets that reproduce the correct $\Delta m^2$’s. Small values of $|U_{e3}|^2$ are somewhat favored. The corresponding distributions for the solar and atmospheric mixing angles are peaked at maximal mixings. Averaging only over the sets which pass all experimental constraints, we find $\langle |U_{e3}|^2 \rangle = 0.022$. This
means that $|U_{e3}|^2$ is probably close to the experimental bound and can very likely be tested at future neutrino experiments, such as MINOS, which is sensitive down to $|U_{e3}|^2 \sim 0.0025$ [21]. Since we typically start with large phases in the neutrino and charged lepton mass matrices, the complex phase $\delta$ in the mixing matrix $U$ is found to be most likely of order unity. If we ignore the charged lepton rotations, i.e. take $U = U_\nu$, the fraction of parameter sets satisfying all constraints rises somewhat from 0.7 % to 1.7 %. The reason is that cancellations amongst the large mixing angles in $U_l$ and $U_\nu$ make it slightly more difficult to reproduce large solar and atmospheric mixings. These results depend only mildly on the interval chosen for the coefficients $\tilde{l}_{ij}$. If we narrow the range, let’s say to $0.8 < |\tilde{l}_{ij}| < 1.7$, the fit for the $\Delta m^2$’s, the solar and atmospheric mixings angles improves remains unchanged, while the CHOOZ constraint becomes slightly harder to satisfy. We find that now about 0.6 % of the parameter sets meet the constraints from eqs. (5.17) and (5.16) and $|U_{e3}|^2 < 0.05$. The SMA, LOW and VAC solutions to the solar neutrino anomaly can be realized only with severe fine-tuning. The required small values of $\Delta m^2_{\text{sol}}$ are very unlikely to be produced through accidental cancellations in the neutrino mass matrix. For the SMA we find that for the most favorable value of $\Lambda = 1.6 \cdot 10^{-3}$ and $1/2 < |\tilde{l}_{ij}| < 2$, only about 0.04 % of the parameter sets reproduce the correct $\Delta m^2$’s. Including the constraints from the solar and atmospheric mixing angles and the CHOOZ experiment brings the fraction down to less than $10^{-5}$. The LOW and VAC solutions are even more fine-tuned. Under the given assumptions the LMA case is therefore selected as the only realistic solution to the solar neutrino problem. These results, which are summarized in table 1, agree nicely with the 4D model studied in ref. [20]. Note that for the LMA solution the results remain stable if we use real instead of complex mass matrices. However, the SMA, LOW and VAC solutions are more difficult to accommodate with complex mass matrices since the required cancellations to obtain a small $\Delta m^2_{\text{sol}}$ become even more unlikely. For instance, with real mass matrices, the fraction of parameter sets which reproduce the SMA mass squared differences increases from 0.04 % to 0.7 %. So far the constraint from the CHOOZ experiment is the most stringent one. Its inclusion reduces the probability that a parameter set realizes the LMA solution from about 6 % to less than one percent. With all entries in the neutrino mass matrix

|       | $\Delta m^2_{\text{atm, sol}}$ | $+ \sin^2 2\theta_{\text{atm, sol}}$ | $|U_{e3}|^2 < 0.05$ |
|-------|-------------------------------|-----------------------------------|------------------|
| LMA   | 44.4 (28.1)                   | 5.8 (5.9)                         | 5.0 (0.7)        |
| SMA   | 1.3 (0.04)                    | 0.3 (< 0.001)                     | 0.3 (< 0.001)   |
| LOW   | 0.008 (< 0.001)               | 0.002 (< 0.001)                   | 0.002 (< 0.001) |

Table 1: Probability in percent that a randomly generated set of coefficients $1/2 < |\tilde{l}_{ij}| < 2$ satisfies the constraints from $\Delta m^2_{\text{atm, sol}}$ (first column) and $\sin^2 2\theta_{\text{atm, sol}}$ (second column) and $|U_{e3}|^2 < 0.05$ (third column). The results are given for the case $c_{e,L} > c_{\mu,L} = c_{\tau,L}$ ($c_{e,L} = c_{\mu,L} = c_{\tau,L}$).
being of similar magnitude, the ensuing mixing angles are typically large. The fit to the neutrino data improves considerably if the electron neutrino is somewhat separated from the other two neutrino species. Shifting the electron neutrino closer to the Planck-brane induces small elements in the neutrino mass matrix. As a result, small values of $\Delta m^2_{\text{sol}}$ and $|U_{e3}|^2$ become more probable. The neutrino mass matrix (4.12) acquires the following structure

$$M^{(\nu)} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

(5.21)

where $\epsilon \approx f_{0,e}^{(\nu)}(\pi R)/f_{0,\mu}^{(\nu)}(\pi R) \approx \exp(-(c_{e,L} - c_{\mu,L})\pi k R)$. The charged lepton mass matrix does not follow the pattern (5.21) because of the different locations of left- and right-handed leptons. Still $U_l$ and $U_\nu$

We keep the muon and tau neutrinos at the previous locations $c_{\mu,L} = c_{\tau,L} = 0.72$. A separation of these two would make the fit to the atmospheric neutrino data more difficult since the corresponding mixing is reduced. (A small separation, e.g. $|c_{\mu,L} - c_{\tau,L}| \lesssim 0.02$ could, however, be tolerated.) For the electron neutrino the most favorable choice turns out to be $c_{e,L} = 0.79$ leading to $\epsilon = 0.15$. This value is of the order of $\sqrt{m_\mu/m_\tau}$, and also of the Cabbibo angle. However, in our model there is no relation to either of these quantities. The positions of the right-handed leptons are found to be $c_{e,R} = 0.79$, $c_{\mu,R} = 0.62$ and $c_{\tau,R} = 0.49$.

Let us again focus on the LMA solution. Assuming $1/2 < |\tilde{l}_{ij}| < 2$ for the order unity coefficients, we find the best value $\Lambda = 2.6 \cdot 10^{-3}$. The correct values of the $\Delta m^2$‘s are fitted by 44% of the parameter sets. Taking into account also the constraints from the solar and atmospheric mixing angles, this fraction shrinks to 5.8%. This reduction is mostly due to the solar mixing angle which is suppressed if $\epsilon$ is small. Most important, however, the CHOOZ constraint $|U_{e3}|^2 < 0.05$ is now satisfied almost automatically, and we are finally left with 5.0% of the parameter

![Figure 3: Distribution of $|U_{e3}|^2$ for the case $c_{e,L} > c_{\mu,L} = c_{\tau,L}$ and $1/2 < |\tilde{l}_{ij}| < 2$.](image)
sets. Compared to the case of \( \epsilon = 1 \), the probability for a parameter set to satisfy all observational constraints is enhanced by almost an order of magnitude. It turns out that including phases in the mass matrices is very important to obtain this result. With real valued mass matrices there tend to be large cancellations between the \( U_{\nu} \) and \( U_l \) contributions to the solar and atmospheric mixing angles. Large mixings are then very difficult to realize. If we consider the interval for the coefficients to be \( 0.8 < |\tilde{t}_{ij}| < 1.7 \), the fit becomes slightly better because the solar mixing angle gets somewhat enhanced. We find that 5.5% of the parameter sets pass all the constraints.

In fig. 3 we display the distribution of \( |U_{e3}|^2 \) for the case \( 1/2 < |\tilde{t}_{ij}| < 2 \), where we again include only the parameter sets which reproduce the correct \( \Delta m^2 \)'s. As expected, the distribution is now peaked at small values of \( |U_{e3}|^2 \). Averaging only over parameter sets which satisfy all constraints, we find a moderately small value \( \langle |U_{e3}|^2 \rangle = 0.010 \). As a consequence, the next generation neutrino experiments still has a good chance to detect a non-vanishing \( |U_{e3}|^2 \) [21].

Taking even smaller values of \( \epsilon \), it is possible to implement the SMA solution to the solar neutrino problem. The most favorable choice of parameters we find to be
\[
c_{e,L} = 0.96
\]
 corresponding to \( \epsilon = 0.0020 \) and \( \Lambda = 2.6 \cdot 10^{-3} \). The correct values of \( \Delta m^2 \) are reproduced by 1% of the sets of coefficients \( 1/2 < |\tilde{t}_{ij}| < 2 \). The constraints from the solar and atmospheric mixing angles reduce this amount to about 0.3%. The CHOOZ constraint is always satisfied, and we obtain \( \langle |U_{e3}|^2 \rangle = 2.7 \cdot 10^{-6} \) which is much too small to be measurable. The LOW solution is very difficult to implement because of the small solar \( \Delta m^2 \). We take \( c_{e,L} = 0.86 \) corresponding to \( \epsilon = 0.0026 \), and \( \Lambda = 2.4 \cdot 10^{-3} \). The constraints from the \( \Delta m^2 \)'s are satisfied by only about 0.01% of the parameter sets. All the neutrino data are reproduced by a fraction smaller then \( 10^{-5} \). The CHOOZ constraint is again satisfied automatically. Small \( \Delta m^2 \)'s could be accommodated by moving the electron (and muon) neutrino closer to the Planck-brane. Then it becomes however more difficult to obtain large mixings. The VAC solution is even more difficult to realize because of the very small value of \( \Delta m^2_{\text{sol}} \). Thus, the LMA solution is by far the most favored scenario. A collection of our results is given table 1.

Neutrino mass matrices of the type (5.21) have previously been considered in refs. [22, 23]. There the small quantity \( \epsilon \) was attributed to a Froggatt-Nielsen mechanism [24]. Our findings agree very well with the results of these studies.

6 Corrections from KK Neutrinos

So far we have only considered the neutrino zero modes. But eq. (4.11) also induces mixings between the zero modes and the vector-like excited states. In the following we show that this effect does not modify the conclusions we have reached above.
The general neutrino mass matrix takes the symmetric form

$$M_\nu = \begin{pmatrix} m_{L}^{(0,0)} & m_{L}^{(0,1)} & 0 & \cdots \\ m_{L}^{(1,0)} & m_{L}^{(1,1)} & m_{KK,1} & \cdots \\ 0 & m_{KK,1} & m_{R}^{(1,1)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \end{pmatrix} \begin{pmatrix} \nu_{L}^{(0)} \\ \nu_{L}^{(1)} \\ \nu_{R}^{(1)} \\ \nu_{R}^{(1)} \end{pmatrix}$$

(6.22)

Here $m_{L}^{(0,0)} \equiv M^{(\nu)}$ are the Majorana masses for the zero modes from eq. (4.12). The Majorana masses $m_{L}^{(i,j)}$ and $m_{R}^{(i,j)}$ are obtained from eq. (4.12) by inserting the wave functions of the relevant states. $m_{KK,i}$ are the KK masses of the excited states. Since the KK states have a greater overlap with the Higgs, the corresponding Majorana masses are larger than those involving zero modes. We consider again the case $k = 0.01 M_5$ and $\lambda^{(5)} = 10 g^{(5)}$ implying $c_{r,L} = 0.69$. To estimate the magnitude of the effect we restrict ourselves to a single neutrino flavor. For the first excited state we find $m_{L}^{(0,1)} = l \cdot 8.8 \text{ keV}$, $m_{L}^{(1,1)} = l \cdot 3.5 \text{ MeV}$ and $m_{R}^{(1,1)} = l \cdot 20 \text{ keV}$. For higher excited states these masses approximately do not change. If we would have used a delta function-like Higgs profile, $m_{R}^{(i,j)}$ would be zero since the wave functions of $\nu_{R}^{(i)}$ are odd and vanish at the boundaries. Truncating the KK tower at the first excitation, we find for the lowest state (i.e. the zero mode)

$$\nu_1 \approx \nu_{L}^{(0)} + \frac{m_{R}^{(1,1)} m_{L}^{(0,1)}}{m_{KK,1}^2} \nu_{L}^{(1)} - \frac{m_{L}^{(0,1)}}{m_{KK,1}} \nu_{R}^{(1)}$$

(6.23)

$$\approx \nu_{L}^{(0)} + 1.5 \cdot 10^{-18} \nu_{L}^{(1)} - 8 \cdot 10^{-10} l \nu_{R}^{(1)}.$$

In the second line we used numbers from the previous example. These tiny admixtures have only a negligible input on the properties of the zero modes. The tiny correction to the mass of the zero mode, for instance, is given by

$$\delta m_{\nu_1} \approx \frac{m_{R}^{(1,1)} m_{L}^{(0,1)}}{m_{KK,1}^2} (m_{L}^{0,1})^2$$

(6.24)

$$\approx 1.3 \cdot 10^{-14} \text{ eV}.$$

The contributions of the higher KK states are suppressed by larger KK masses and will affect our result by at most a factor of order unity. In a similar way to (6.22) the charged lepton masses (5.14) also couple zero modes and KK states. Here too the large KK masses suppress corrections to charged lepton masses and mixings from the excited states. Thus, the KK states can be safely neglected in the discussion of neutrino mixings.

7 Dirac vs. Majorana Neutrinos

We finally discuss some experimental signatures to distinguish the presented scenario of Majorana neutrinos from the Dirac neutrino scheme discussed in refs. [12, 13].
Figure 4: Fraction of parameter sets $p$ which reproduce the $\Delta m^2$'s as a function of the electron neutrino Majorana mass $m_{ee}$ for $c_L = 0.72$ (in logarithmic scale).

The most direct evidence for a Majorana mass of neutrinos would be the discovery of neutrinoless double beta decay ($0\nu\beta\beta$). The non-observation of this process in the Heidelberg-Moscow experiment implies an upper bound on $|M_{11}^{(\nu)}| \equiv m_{ee} < 0.35$ eV [23]. There are plans to bring this limit down to about 0.01 eV [26]. If all neutrinos are at the same position in the extra dimension, we expect $m_{ee} \sim 0.02$ eV, which is far below the current experimental sensitivity but within reach of future experiments. This result does not depend on what solution to the solar neutrino anomaly is realized. Fig. 4 illustrates the unnaturalness of a large Majorana mass in our scenario. As a function of $m_{ee}$ we show in a logarithmic scale the fraction of parameter sets $p$ which reproduce the $\Delta m^2$'s required for the solar LMA solution and for the atmospheric neutrino anomaly. (Including also the constraints from the mixing angles would reduce the number of viable parameter sets by another order of magnitude.) We assume all neutrinos to be at the same location with $c_L = 0.72$, and take $1/2 < |\tilde{l}_{ij}| < 2$. To tune $m_{ee}$ we set $\tilde{l}_{11} = 1$ and vary $\Lambda$. For small values of $m_{ee}$ (i.e. small $\Lambda$) it becomes impossible to accommodate the atmospheric $\Delta m^2$, leading to $p = 0$. For large Majorana masses $p$ becomes exponentially small since it is highly improbable to accidentally generate $\Delta m^2 \ll m^2$. The peak around $m_{ee} \approx 0.015$ eV corresponds to the fraction of 28% quoted in table I. For larger ranges of $\tilde{l}_{ij}$ the distribution is spread out further.

Once the electron neutrino is localized closer towards the Planck-brane $0\nu\beta\beta$ becomes drastically suppressed proportional to $\epsilon^2$. For the clearly favored LMA scenario we find $m_{ee} \sim 0.001$ eV, which is too small to be detected in the near future. For the SMA, LOW and VAC solutions $m_{ee}$ is even smaller. It is therefore questionable if our model induces $0\nu\beta\beta$ at a detectable level. In any case the Majorana masses we find are far below the recently claimed evidence of about 0.4 eV [27].

Tritium endpoint (beta decay) experiments are sensitive to neutrino masses of Majorana and Dirac type. The MAINZ collaboration plans to look for neutrino
masses down to about 0.3 eV [28], which is, however, still an order of magnitude larger than the typical neutrino masses in the scenario considered.

In refs. [5, 13] we imposed lepton number symmetry to make the proton stable and eliminate Majorana neutrino masses from non-renormalizable operators. If in the current framework we want to keep Majorana masses but still forbid proton decay, lepton parity is an attractive possibility. It still would allow baryon number violating processes, such as neutron anti-neutron oscillations, which might be close to the observational bound [13]. Without some symmetry dimension six operators inducing proton decay have to be multiplied by small couplings of order $10^{-8}$. The proton lifetime then should not be too far above the present experimental constraint.

The scenario with Dirac fermions could manifest itself through lepton flavor violating decays. For instance, the rate for $\mu \rightarrow e\gamma$ is considerably enhanced by the presence of sterile neutrino KK states, which spoil the GIM cancellation of the SM. If the SM neutrinos are confined to the TeV-brane, the branching ratio for $\mu \rightarrow e\gamma$ is above the experimental limit of about $10^{-11}$, unless the KK scale is above 25 TeV [29]. In the case of bulk SM neutrinos $\mu \rightarrow e\gamma$ does not constrain the KK scale anymore, but the branching ratio may still be close to the experimental bound.

## 8 Conclusions

Neutrino masses generated by dimension five interactions in the SM are of magnitude $\lesssim 10^{-5}$ eV, and consequently the atmospheric neutrino anomaly remains problematic. In contrast, such masses are significantly larger in the 5D warped SM, and we have shown that an explanation of the solar and atmospheric neutrino anomalies can be realized, based on the idea of neutrino mass anarchy. We also provide estimates for the mixing angle $U_{e3}$ as well as the parameter $m_{ee}$ that appears in the amplitude for neutrinoless double beta decay. No new particles such as SM singlet neutrinos are needed to implement the scenario. Indeed, because of the warped geometry, the usual seesaw mechanism will not work in its simplest form, without either giving up the resolution of the gauge hierarchy problem, or invoking supersymmetry.

## Acknowledgements

We thank David Emmanuel Costa, Holger Nielsen and Chin-Aik Lee for helpful discussions, and Wilfried Buchmüller for valuable comments on the draft. This work was supported in part by DOE under contract DE-FG02-91ER40626. We thank the Alexander von Humboldt Stiftung for providing the impetus for this collaboration. Q.S. also acknowledges the hospitality of the Theory Group at DESY where this work was completed.

## References
[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 08004; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370.

[3] M. Gogberashvili, hep-ph/9812296.

[4] T. Gherghetta and A. Pomarol, Nucl. Phys. B586 (2000) 141 hep-ph/0003129.

[5] S.J. Huber and Q. Shafi, Phys. Lett. B498 (2001) 256 hep-ph/0010196.

[6] N. Arkani-Hamed, M. Schmaltz, Phys. Rev. D61 (2000) 033005; G. Dvali, M. Shifman, Phys.Lett. B475 (2000) 295;

[7] S. Chang, J. Hisano, H. Nakano, N. Okada and Yamaguchi, Phys. Rev. D62 (2000) 084025 hep-ph/9912498.

[8] S.J. Huber and Q. Shafi, Phys. Rev. D63 (2001) 045010 hep-ph/0005286.

[9] S.J. Huber, C.-A. Lee and Q. Shafi, Phys. Lett. B531 (2002) 112 hep-ph/0111465.

[10] C. Csaki, J. Erlich and J. Terning, hep-ph/0203034.

[11] J.L. Hewett, F.J. Petriello and T.G. Rizzo, hep-ph/0203091.

[12] Y. Grossman and M. Neubert, Phys. Lett. B474 (2000) 361 hep-ph/9912408.

[13] S.J. Huber and Q. Shafi, Phys. Lett. B512 (2001) 365 hep-ph/0104293.

[14] W.D. Goldberger and M.B. Wise, Phys. Rev. Lett. 83 (1999) 4922 hep-ph/9907447.

[15] A.B. Kobakhidze, Phys. Lett. B514 (2001) 131-138 hep-ph/0102323; R. Kallosh, A.D. Linde, D.A. Linde and L. Susskind, Phys. Rev. D52 (1995) 912.

[16] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, Phys. Rev. D63 (2001) 075004 hep-ph/0006041.

[17] T. Toshito [SuperKamiokande Collaboration], hep-ex/0105023.

[18] J.N. Bahcall, M.C. Gonzalez-Garcia and C. Pena-Garay, JHEP 0204 (2002) 007 hep-ph/0111150; V. Barger, D. Marfatia, K. Whisnant and B.P. Wood, hep-ph/0204253; J.N. Bahcall, M.C. Gonzalez-Garcia, C. Pena-Garay, hep-ph/0204314.
[19] M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B466 (1999) 415 [hep-ex/9907037].

[20] L. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. 84 (2000) 2572 [hep-ph/9911341]; N. Haba and H. Murayama, Phys. Rev. D63 (2001) 053010 [hep-ph/0009174].

[21] See e.g. A. Para and M. Szleper, hep-ex/0110032.

[22] J. Sato and T. Yanagida, Phys. Lett. B430 (1998) 127 [hep-ph/9710516]; N. Irges, S. Lavignac and P. Ramond Phys. Rev. D58 (1998) 035003 [hep-ph/9802334].

[23] J. Sato and T. Yanagida, Phys. Lett. B493 (2000) 356 [hep-ph/0009205]; F. Vissani, Phys. Lett. B508 (2001) 79 [hep-ph/0102236].

[24] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 277.

[25] The Heidelberg-Moscow collaboration, H.V. Klapdor-Kleingrothaus el al. in the proceedings of DARK 2000, Heidelberg, Germany, 10-16 July 2000, 520-533 [hep-ph/0103062].

[26] Sixty Years of Double Beta Decay, by H.V. Klapdor-Kleingrothaus, World Scientific, 2001.

[27] H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney and I.V. Krivosheina., Mod. Phys. Lett. A16 (2002) 2409 [hep-ph/0201231].

[28] The MAINZ collaboration, J. Bonn et al., Nucl. Phys. Proc. Suppl., 91 (2001) 273.

[29] R. Kitano, Phys. Lett. B481 (2000) 39 [hep-ph/0002279].