The dependence of the conductivity tensor on electromagnetic waves and laser fields in a quantum well with infinite potential in the case of electrons - optical phonon scattering

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Abstract. There are many studies on low-dimensional semiconductors, including quantum wells. Conductivity tensor is one of the key concepts in materials research. This work studies the dependence of the conductivity tensor on electromagnetic waves and laser fields in a quantum well with infinite potential in the case of electrons - optical phonon scattering. Results are obtained using the quantum kinetic equation for electrons in a quantum well, the particle system is placed in an electromagnetic wave field and a laser field, from solving the quantum kinetic equation yields conductivity tensor expressions. This work also only considers the case of electron scattering - optical phonon and with quantum well there can be infinite potential. The conductivity tensor expression shows its dependence on the electromagnetic wave frequency, laser field frequency, laser field amplitude, and other parameters that characterize the quantum well. A graph of the dependence of the conductivity tensor on electromagnetic wave frequency, laser field frequency, and laser field amplitude will be plotted and examined. The semiconductor investigated here is the GaAs/GaAsAl quantum well.

1. Introduction

In recent years, there are many studies on low-dimensional physics. In low-dimensional semiconductors, because electrons are restricted to move in one direction, two directions, or all three directions, so new effects appear [1-12]. Electron movement limitation leads to the wave function and energy of electrons being altered [8]. Because the wave and energy functions are altered, other physical quantities also change. There are many studies on conductivity tensors [3], which is an important quantity in the study of materials in general and semiconductors in particular [13-20]. Calculating the conductivity tensor will lead to the computation of other quantities in the system. Conductivity tensors in semiconductors depend on many quantities specific to the system, in this work the author studies the dependence of the conductivity tensor on electromagnetic waves and laser fields in a quantum well with infinite potential in the case of electrons - optical phonon scattering. Use quantum kinetic equations to solve problems for electrons in systems that include electromagnetic waves and laser fields in a quantum well. We only consider the case of quantum wells with infinite potential and electrons - optical phonon scattering. The conductivity tensor's expression will show its dependence on the electromagnetic wave field and the laser field. The dependence of the conductivity tensor on the electromagnetic wave frequency,
laser field frequency and laser field amplitude is plotted. The quantum well under investigation is the GaAs / GaAsAl quantum well.

2. Calculating conductivity sensor for the case electrons–acoustic phonon scattering

There are many types of quantum wells, quantum wells are considered here as quantum wells with infinite potential. Here we assume that electrons move freely in the x0y plane and are limited to the quantum well;

\[ e_{n,p} = \frac{p_{z}^{2}}{2m} + \frac{\pi^{2}}{2md}n^{2} \]  

(1)

with \( d \) is the width of the quantum well; \( n = 0,1,2,... \); \( p_{z}^{2} = p^{2} - p_{z}^{2} \); \( \hat{p} \) is momentum of electron; \( m \) is the effective mass of the electron.

In case of electrons–optical phonon scattering, the time-independent component of distribution function of optical phonon:

\[ N_{q} = \frac{k_{B}T}{\hbar \omega_{q}} \]  

(2)

and the electrons-optical phonon interaction constant:

\[ C_{q}^{2} = \frac{2\pi^{2}\alpha_{q}}{V\xi_{q}q^{3}} \left( \frac{1}{\chi_{e}} - \frac{1}{\chi_{0}} \right); \quad \xi_{q} = \frac{10^{-9}}{36\pi} \]  

(3)

\( \chi_{e} \) and \( \chi_{0} \) are high-frequency dielectric constant and the static dielectric constant; \( V \) is the volume of the quantum well; \( \alpha_{q} \) is the frequency of the optical phonon; \( q \) is momentum of phonon.

Assuming carrier system is placed in a laser field:

\[ \frac{\partial f_{0}(\hat{p}_{z},t)}{\partial t} + \left[ \hat{H}(t) + \omega_{n}\left[ \hat{p}_{z},\hat{h}(t) \right] \right] \frac{\partial f_{0}(\hat{p}_{z},t)}{\partial \hat{p}_{z}} = \frac{1}{\hbar} \sum_{n,a,q} M_{n,a}(\hat{q}) \sum_{l_{1},l_{2}} J_{l_{1}}^{2}(\hat{q}) \left[ f_{n}(\hat{p}_{z} + \hat{q},t) - f_{n}(\hat{p}_{z},t) \right] \delta\left( e_{n,\hat{p}_{z} + \hat{q}} - e_{n,\hat{p}_{z}} - \Omega \right) \]  

(6)

here \( f_{0}(\hat{p}_{z},t) \) is distribution function of electrons;

\[ M_{n,a}(\hat{q}) = \left( \frac{2\pi}{\hbar} \right)^{3} \int \hat{1}_{n,a}^{*}(x,y) N_{q} \]

(7)

\[ f_{0}(\hat{p}_{z},t) = f_{0}(e_{n,\hat{p}_{z}}) + f_{0}(e_{n,\hat{p}_{z}}); \quad f_{0}(e_{n,\hat{p}_{z}}) = \theta(e_{0} - e_{n,\hat{p}_{z}}); \quad f_{0}(\hat{p}_{z},t) = -\hat{p}_{z}\hat{\chi}(t) f_{0}(e_{n,\hat{p}_{z}}); \]

(8)

\[ f_{0}(e_{n,\hat{p}_{z}}) = \frac{\partial f_{0}(e_{n,\hat{p}_{z}})}{\partial e_{n,\hat{p}_{z}}}; \quad f_{0}(\hat{p}_{z},t) = f_{0}(\hat{p}_{z}) e^{\text{i}\Omega t} + f_{0}^{*}(\hat{p}_{z}) e^{\text{it} \Omega} \]

(9)

\[ \hat{\chi}(t) = \hat{\chi} e^{\text{i}\Omega t}. \]

(10)
\[
\frac{\partial \mathbf{f}_n (\mathbf{\tilde{p}}_z , t)}{\partial t} = \frac{\partial}{\partial t} \left[ f_n (\epsilon_{n,p_z}) + f_n (\mathbf{\tilde{p}}_z) e^{i\omega t} + f_n^* (\mathbf{\tilde{p}}_z) e^{i\omega t} \right] = -i \omega f_n (\mathbf{\tilde{p}}_z) e^{-i\omega t} + i \omega f_n^* (\mathbf{\tilde{p}}_z) e^{i\omega t}
\]

(11)

\[\mathbf{\tilde{h}}(t) = \frac{\mathbf{H}(t)}{\mathbf{H}}\]

is the unit vector in the magnetic field direction;

\[J^z (\tilde{a}, \tilde{q})\]

is the Bessel function of real argument; \(\tilde{a} = \frac{e \mathbf{F}}{m \mathbf{H}}\); \(\omega_t = \frac{e \mathbf{H}}{m c}\)

(12)

\[\epsilon_{n,p_z+q} = \left( \mathbf{p}_z + q \right)^2 + \frac{\pi^2}{2m^2} n^2
\]

(13)

The simple case: \(l = 0, \pm 1\) \(J_0 \approx 1\); \(J^z = \frac{(\tilde{a}, \tilde{q})^2}{4}\); Form factor with quantum well: \(I_{n^2} = \frac{2}{L_0} \int \psi_n^*(z) \psi_n(z) \exp(iqz) dz\)

(14)

Multiply both sides Equation (6) by \(-\frac{e}{m} \mathbf{p}\delta (\epsilon - \epsilon_{n,p_z})\) and then sum over \(\mathbf{\tilde{p}}_z\), we obtained:

\[-\frac{e}{m} \sum_{n,p_z} \mathbf{\tilde{p}}_z \left[ \frac{\partial f (\mathbf{\tilde{p}}_z , t)}{\partial t} \right] + \left( e \mathbf{E}(t) + e \mathbf{H}(t) \right) \delta (\epsilon - \epsilon_{n,p_z}) =
\]

\[-\frac{e}{m} \sum_{n,n',q} \mathbf{M}_{n,n'} (q) \sum_{p_z} \frac{2 \pi}{\mathbf{\tilde{p}}_z} \int J^z (\tilde{a}, \tilde{q}) \left[ f_{n'} (\mathbf{\tilde{p}}_z + q, t) - f_{n'} (\mathbf{\tilde{p}}_z + q, t) \right] \delta (\epsilon_{n',p_z+q} - \epsilon_{n,p_z} - \mathbf{H}^2) \delta (\epsilon - \epsilon_{n,p_z})
\]

(15)

\[\psi_n (z)\]

is a wave function in state \(n\).

After calculating, we obtain the equation:

\[\mathbf{R} (\epsilon) = \frac{\tau (\epsilon)}{1 - i \omega t} (\mathbf{Q} + \mathbf{S} (\epsilon))
\]

(16)

\[\mathbf{R}^* (\epsilon) = \frac{\tau (\epsilon)}{1 + i \omega t} (\mathbf{Q} + \mathbf{S} (\epsilon))
\]

(17)

with

\[\mathbf{Q} = \frac{e^2}{m} \sum_{n,p_z} \left( \frac{\partial f (\mathbf{\tilde{p}}_z , t)}{\partial p_z} \right) \mathbf{\tilde{p}}_z \delta (\epsilon - \epsilon_{n,p_z})
\]

(18)

\[\mathbf{S} (\epsilon) = -\frac{e}{m} \sum_{n,n',q} \mathbf{M}_{n,n'} (q) \left( \frac{\tilde{a}, \tilde{q}}{4} \right)^2 \sum_{p_z} f_{n'} (\mathbf{\tilde{p}}_z) \times
\]

\[\times \left[ \delta (\epsilon_{n',p_z+q} - \epsilon_{n,p_z} - \mathbf{H}^2) + \delta (\epsilon_{n',p_z+q} - \epsilon_{n,p_z} + \mathbf{H}^2) \right] \times
\]

\[\times \left[ (\mathbf{\tilde{p}}_z + \tilde{q}) \delta (\epsilon - \epsilon_{n,p_z+q}) - \mathbf{\tilde{p}}_z \delta (\epsilon - \epsilon_{n,p_z}) \right]
\]

(19)

At time \(t = 0\), the density of current [1-3]:

\[\mathbf{I}(\mathbf{\tilde{p}}_z) = \frac{e}{m} \left( \frac{\partial f (\mathbf{\tilde{p}}_z , 0)}{\partial p_z} \right) \mathbf{\tilde{p}}_z \delta (\epsilon - \epsilon_{n,p_z})
\]
\[ j(t=0) = \int (\mathbf{R}(\varepsilon) + \mathbf{R}^*(\varepsilon)) \text{d}e = j^0 + j^* \]

\[ = \frac{4e^2n_0}{m} \tau(\varepsilon_p) \left[ \varepsilon_p - \frac{\pi^2}{2md^2}n^2 \right] + \frac{\tau(\Omega)}{1 + \omega^2\pi^2(\Omega)} + \frac{\tau(\varepsilon_p)}{1 + \omega^2\pi^2(\varepsilon_p)} \frac{1}{\lambda} \mathbf{E} \]

(20)

\[ A = \frac{e^2F^2}{2m\Omega} M_{n,n}(2m\Omega) \sqrt{2m\left( \varepsilon_p - \frac{\pi^2}{2md^2}n^2 \right)} \]

(21)

\[ \lambda = \frac{e^2F^2}{2m\Omega} M_{n,n}(2m\Omega) \sqrt{2m\left( \varepsilon_p - \frac{\pi^2}{2md^2}n^2 \right)} \sqrt{2m\left( \Omega - \frac{\pi^2}{2md^2}n^2 \right)} - 1 \]

(22)

\[ n_0 \text{ is particle density; } m \text{ is the effective mass of electron; } e=1.6*10^{-19} \text{C;} \tau(\varepsilon) \text{ is the momentum relaxation time in absence of laser radiation.} \]

with electrons-optical phonon scattering:

\[ M_{n,n}(2m\Omega) = |C_q|^2 I_{n,n}^2 N_q = \frac{Ne^2}{2m_0^2} k_B T \left( \frac{1}{\chi_e} - \frac{1}{\chi_0} \right) |I_{n,n}|^2 \]

(23)

here \( j(t=0) = \sigma \mathbf{E}(t=0) = \sigma 2\mathbf{E} \) [3]

so:

\[ \sigma_n = \frac{2e^2n_0}{m} \tau(\varepsilon_p) \left[ \varepsilon_p - \frac{\pi^2}{2md^2}n^2 \right] \delta_{q_0} + \frac{\tau(\Omega)}{1 + \omega^2\pi^2(\Omega)} \frac{1}{\lambda} \mathbf{E} \]

(24)

with \( \delta_{q_0} \) is the Kronecker symbol.

Expression (24) is the expression for the conductivity tensor in the quantum well with the infinite and only consider the case of electrons - optical phonons scattering.

3. Numerical results and discussion

The parameters used in the calculations are as follows [1, 2]: \( m = 0.0665m_0 \) (\( m_0 \) is the mass of free electron); \( \varepsilon_p = 50 \text{meV} \); and \( \tau(\varepsilon_p) \sim 10^{-11} \text{ s}^{-1} \); \( n_0 = 10^{21} \text{ m}^{-3} \); \( \rho = 5.3 \times 10^3 \text{ kg/m}^3 \); \( \xi_0 = 2.2 \times 10^{-6} \text{ J} \); \( \omega_b = 5 \times 10^{10} \text{ s}^{-1} \); \( \chi_e = 10 \); \( \chi_0 = 12 \); \( \lambda = 100 \text{ Å} \); \( T = 300 \text{ K} \).

Figure 1 shows the dependence of the conductivity tensor on the laser field frequency. At a laser field frequency of \( 10^9 \text{s}^{-1} \) the plot shows that as the laser field frequency increases, the tensor of conductivity decreases. The decrease of the conductivity tensor is very small as the laser field frequency increases, indicating that the conductivity tensor is less dependent on the laser field frequency, the presence of the laser field has little effect on the conductivity tensor of the system. Figure 1 also shows that for three different values of the electromagnetic wave field frequency we see three separate plots, which shows that the electromagnetic wave frequency affects the conductivity tensor when there is a laser field.
Looking at figure 2 we see that as the laser field frequency increases in the range from $4 \times 10^3 \text{s}^{-1}$ to $4400 \text{s}^{-1}$, the figure has a ladder shape. There are ranges of $\Omega$ values for which the conductivity tensor does not change. This shows that as the laser field frequency increases, the dependence of the conductivity tensor on the laser field frequency becomes less and less. The change in conductivity tensor in this frequency range is very small.

**Figure 1.** The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ when $\Omega < 1150$.

**Figure 2.** The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ when $4000 < \Omega < 4400$.

**Figure 3.** The dependence of $\sigma$ on the frequency $\Omega$ with different values of $\omega$ when $\Omega > 10^{14}$.

**Figure 4.** The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $\omega < 10^4$. 


Figure 5. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $10^3 < \omega < 4 \times 10^4$.

Figure 6. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $10^3 < \omega < 4 \times 10^7$.

Figure 7. The dependence of $\sigma$ on the frequency $\omega$ with different values of $\Omega$ when $\omega > 10^2$.

Figure 8. The dependence of $\sigma$ on the amplitude of the laser field.

When the laser field frequency has large values $\Omega > 10^{14} \text{s}^{-1}$, the dependence of the conductivity tensor on the laser field frequency is represented by figure 3. The figure is a straight line parallel to the horizontal axis, the conductivity tensor does not depend on the laser field frequency. The three plots with three different values of the electromagnetic wave frequency now overlap, proving that the electromagnetic wave frequency also does not change the value of the conductivity tensor.

Figure 4 shows the dependence of the conductivity tensor on the electromagnetic wave field frequency at frequencies $\omega < 10^4$. At the figure we see that at this frequency range the conductivity tensor does
not depend on the electromagnetic wave frequency. With three different values for the laser field frequency, we have three overlapping plots, so the effect of the laser field is also almost absent. In the frequency range from $10^4 \text{s}^{-1}$ to $4 \times 10^4 \text{s}^{-1}$ the dependence of the conductivity tensor on the electromagnetic wave frequency is represented by figure 5, the figure has a ladder shape. Conductivity tensor decreases very little as the frequency of electromagnetic waves increases. In the frequency range from $10^5 \text{s}^{-1}$ to $4 \times 10^5 \text{s}^{-1}$ the dependence of the conductivity tensor on the electromagnetic wave frequency is represented by figure 6. In this frequency range, the conductivity tensor also decreases very small when the frequency electromagnetic waves rise.

Figure 7 shows the dependence of the conductivity tensor on the electromagnetic wave frequency when $\omega > 10^4$. The figure shows that the conductivity tensor decreases very quickly as frequency increases. This figure shows the most clearly the dependence of the conductivity tensor on the electromagnetic wave frequency.

The amplitude of the laser field has a great influence on the conductivity tensor, which is shown in figure 8. As the laser field amplitude increases, the conductivity tensor also increases rapidly. The three different values of the electromagnetic wave frequency give three separate plots, which show the role of the electromagnetic wave field in the change in conductivity tensor.

4. Conclusions

In this work we study the dependence of the conductivity tensor on electromagnetic waves and laser fields in a quantum well with infinite potential in the case of electrons - optical phonon scattering. We have computed the expression for conductivity tensor in this case. From the research results, we see that the conductivity tensor is a function of the electromagnetic wave frequency, the laser field frequency, the laser field amplitude, and the parameters characteristic for low-dimensional semiconductor systems. We also investigated and plotted the dependence of conductivity tensors on laser field frequency, laser field amplitude, and electromagnetic wave frequency.

The figures show that the conductivity tensor depends very little on the laser field frequency, when the high frequency the conductivity tensor is almost no longer dependent. For electromagnetic wave frequencies, the conductivity tensor is strongly dependent only when $\omega > 10^4$. For the laser field amplitude the conductivity tensor depends on this amplitude very strongly, as the laser field amplitude increases, the conductivity tensor increases very quickly. We can conclude that the presence of electromagnetic waves and laser fields in quantum wells greatly affects the conductivity tensor of the system.

5. References

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