Reconstructing the dark matter and dark energy interaction scenarios from observations

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We consider a class of interacting dark energy models in a spatially flat and nonflat FLRW universe where the interaction is characterized by the modified evolution of the pressureless dark matter sector as $a^{-3+\delta(a)}$, in which $a$ is scale factor of the FLRW universe and $\delta(a)$ is related to the interaction function. By assuming the most natural and nonsingular parametrization for $\delta(a)$ as $\delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + \ldots$, where $\delta_i$'s ($i = 0, 1, 2, 3, \ldots$) are constants, we reconstruct the expansion history of the universe for three particular choices of the dark energy sector; namely, the vacuum energy, dark energy with constant equation of state parameter, and dark energy having dynamical equation of state parameter using the observational data from cosmic microwave background radiation, supernovae Type Ia, baryon acoustic oscillations distance measurements and the Hubble parameter measurements at different redshifts. Our analyses show that although the observational data seem to allow a very mild interaction in the dark sector but within 68% CL, the non-interacting scenario is recovered. Our reconstruction technique shows that the parameters $\delta_2$ and $\delta_3$ are almost zero for any interaction model and thus the effective scenario is well described by the linear parametrization $\delta(a) \approx \delta_0 + \delta_1 (1-a)$. We further observe that a strong negative correlation between $\delta_0$ and $\delta_1$ exists independent of different types of dark energy fluid and this is independent of the curvature of our universe. Finally, we remark that for the dark energy sector with constant and dynamical equation of state, the current value of the dark energy equation of state has a tendency towards the phantom regime.

PACS numbers: 98.80.-k, 95.36.+x, 95.35.+d, 98.80.Es

I. INTRODUCTION

Observational evidences from a series of astronomical sources firmly point towards an accelerating phase of the universe at the present epoch. Theoretically this accelerating phase is realized either by introducing some dark energy fluid in the context of Einstein gravity or by introducing new gravitational theories different from General Relativity. Concerning the dark energy problem, one of the simplest dark energy candidate is the cosmological constant $\Lambda$. Along with a cold dark matter (CDM), this model accounts for the current cosmological data quite effectively but it suffers from the major problem, the so-called “Fine-tuning” problem [1, 2, 3, 4]. The Fine-tuning problem has to do with the huge mismatch of the theoretically predicted value of $\Lambda$ which is $\sim$ 120 orders of magnitude larger than the value consistent with observations. In order to overpass this major problem, various cosmological models has been proposed such as the introduction of scalar fields, or matter sources with exotic equations of state, modification of Einstein’s General Relativity and many others, see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] and references therein.

In this work we shall deal with cosmological models where an interaction exists between the fluid terms in the dark sector; that is, the dark energy and dark matter do not satisfy the conservation equations independently. The conservation of energy is satisfied only for the combined dark sector. The interaction mechanism became focused and popular in the cosmological regime when such models were found to explain the cosmic coincidence problem [20, 21, 22, 23], another crucial problem of modern cosmology. However, if one looks back into the literature, one can find that the original proposal of an interaction between the matter fields was motivated to explain the lowest value of the cosmological constant [24]. Now, in presence of an interaction, the conservation laws for the cold dark matter (CDM) and the dark energy (DE) can be recast as $\nabla_\nu T^{\mu \nu} = -Q_\nu$ and $\nabla_\nu T^{\mu \nu}_x = Q_\nu$, where $Q_\nu = Q_x = Q$ such that $\nabla_\nu (T^{\mu \nu} + T^{\mu \nu}_x) = 0$. Here, $Q$ is the energy transfer rate or the interaction function which characterizes the strength and direction of energy flow between the dark sectors; $c, x$ stand for the CDM and DE sectors respectively. However, there is no pressing need to choose a particular form of the interaction, except perhaps the requirement that the interaction is...
non-gravitational, but indeed there are ways to choose an ansatz for the interaction rate and estimate the interaction parameters from available data.

The basic motivation for introducing the interaction is that there is no a priori reason for assuming that there is none. The interaction that is considered is certainly not gravitational, but there is hardly any preferred model for the interaction. We shall refer to quite a few of the works already there in the literature in relevant places, but for a very thorough review, we refer two works, one by Bolotin, Kostenko, Lemos and Yerokhin [25] and other work by Wang, Abdalla, Atro-Barandela and Pavón [26].

In a Friedmann-Lemaître-Robertson-Walker universe, if the interaction is not present, then one can easily see that CDM sector, for which the pressure is zero, evolves as \( a^{-3} \) where \( a \) is the scale factor of this universe. Certainly, if we allow an interaction in the dark sector, then the evolution of the CDM will deviate from this usual evolution law.

A small deviation from the CDM evolution can be looked as a modification of the evolution of the matter density as \( a^{-3+\delta} \), where \( \delta \) could be either constant or variable. In the next section we shall show that such evolution of the CDM sector effectively reduces to a particular interaction function. If \( \delta \) is treated as a variable, then one can explore several possibilities and moreover one can reconstruct the interaction using the observational data.

In this work we have considered that dark matter and dark energy interact with each other where the interaction is characterized by the evolution of the CDM sector: \( \rho_c \propto a^{-3+\delta(a)} \) where \( \delta(a) \) is a time varying quantity. Since the dark energy could be anything, hence, we have considered three types of dark energy fluid, namely, the vacuum energy, dark energy with constant equation of state \( w \), and finally dark energy fluid that steers the late time acceleration of the universe. In addition to that, we consider that there is an interaction in the dark sector of the universe. In particular, we consider that the cold dark matter (CDM) and dark energy (DE) are coupled to each other where DE has a barotropic equation of state parameter, that is, \( p_x = w_x \rho_x \), where \( \rho, p \) and \( w \) are the density, pressure and the equation of state parameter respectively, and the subscript \( x \) corresponds to DE.

Due to the coupling between CDM and DE, the total conservation equation can be decoupled as follows

\[
\dot{\rho}_c + 3H \rho_c = -\dot{\rho}_x - 3H (1 + w_x) \rho_x = Q, \tag{1}
\]

where the subscript \( c \) corresponds to CDM and \( Q \) is rate of interaction between the dark sectors already mentioned about in the introduction. Various phenomenological choices for \( Q \) have been widely studied in the literature [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58].

However, the interaction can manifest itself as a deviation from the standard evolution \( a^{-3} \) of CDM as given by

\[
\rho_c = \rho_{c,0} a^{-3+\delta(a)}, \tag{2}
\]

where \( \rho_{c,0} \) is the present value of \( \rho_c \); \( \delta(a) \), that characterizes the effect of the interaction, can in general be varying with evolution and we assume that it is an analytic function of the scale factor. Here the present value of the scale factor is scaled to be unity.

The flow of energy between the dark sectors depends on the sign of \( \delta(a) \). Also, for \( \delta(a) < 0 \), the evolution of the CDM sector is faster, while on the other hand, for \( \delta(a) > 0 \), the evolution of the CDM factor is slower.

In the special case in which \( \delta(a) = 0 \), the standard CDM evolution is recovered, i.e., \( \rho_c \propto a^{-3} \), which indicates that there is no coupling or interaction between the CDM and DE. When \( \delta(a) = \text{constant} \), one recovers the cosmological scenarios discussed in [56, 57, 58]. For a variable \( \delta(a) \), one particular ansatz was introduced in Ref. [59], namely, \( \delta(a) = \delta_0 \left( \frac{2a}{1+a} \right)^2 \), while we think that a detailed analysis will be worth in this direction. The rate of interaction with a varying \( \delta(a) \) can be calculated as

\[
Q = H \rho_c \left( \delta(a) + a \delta'(a) \ln a \right), \tag{3}
\]

where \( \delta'(a) \) is the differentiation of \( \delta(a) \) with respect to the scale factor 'a'. Now, for the evolution of CDM as in eqn. (2), one can write the DE density evolution as
\[ \rho_x = \frac{1}{f(a)} \left[ \rho_{x,0} f(1) - \rho_{c,0} \int_1^a a^{-4+\delta(a)} f(a) \left( \delta(a) + a \delta'(a) \ln a \right) da \right], \quad (4) \]

where the new quantities \( \rho_{x,0}, \rho_{c,0} \) are the current values of \( \rho_x \) and \( \rho_c \) respectively; the functional \( f(a) \) is given by

\[ f(a) = \exp \left( 3 \int \frac{1+w_x}{a} da \right), \]

and \( f(1) \) is the value of \( f(a) \) at \( a = a_0 = 1 \).

We assume that the function \( \delta(a) \) is a smooth function, and can have a Taylor expansion around the present value of \( a = a_0 = 1 \)

\[ \delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + ... \quad (5) \]

where \( \delta_0 = \delta(a)|_{a\to 1}, \delta_1 = \delta'(a)|_{a\to 1}, \delta_2 = \frac{\delta''(a)|_{a\to 1}}{2!} \) etc.

We can reconstruct the \( \delta_i \)‘s \((i = 0, 1, 2,...) \) and consequently the function \( \delta(a) \) using the observational data. Such an approach has been applied in other facets of cosmology such as the reconstruction of the inflationary model [60, 61].

The evolution of the energy density for the DE fluid, for such a function given by in eqn. \( \delta \) takes the form

\[ \rho_x = \frac{1}{f(a)} \left[ \rho_{x,0} f(1) - \rho_{c,0} \int_1^a a^{-4+\delta_0+\delta_1(1-a)} f(a) \left( \delta_0 + \delta_1 \right. \right. \]

\[ \left. \left. - \delta_1 a (1 + \ln a) \right) da \right]. \quad (6) \]

when the Taylor expansion is taken only up to the first order,

\[ \delta(a) = \delta_0 + \delta_1 (1-a). \quad (7) \]

Considering the next higher order terms in \( \delta(a) \), in a similar fashion, one can calculate the evolutions of \( \rho_x \). In this work we consider up to the second and the third order expansion,

\[ \delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2, \quad (8) \]

\[ \delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + \delta_3 (1-a)^3. \quad (9) \]

Now we introduce three equation of state parameters for the dark energy fluid which corresponds to three different scenarios as follows:

1. If CDM interacts with the constant vacuum energy which is characterized by \( w_x = -1 \).
2. When CDM interacts with a dark energy fluid having constant equation of state \( w_x \).
3. Finally, we consider a scenario where the interacting dark energy has a dynamical equation of state as

\[ w_x(a) = w_0 + w_a (1-a), \quad (10) \]

where \( w_0, \ w_a \), are real parameters. This parametrization is well known in the literature as the Chevaller-Polarski-Linder (CPL) parametrization [62, 63].

III. OBSERVATIONAL DATA

In this section we describe the main observational datasets that are employed to constrain the parameters of models of interacting dark energy scenarios.

1. Cosmic Microwave Background data (CMB): Observations from the cosmic microwave background radiation plays a crucial role in constraining the cosmological models. Here we use the CMB data from the Planck’s 2015 measurements [64, 65]. Precisely, we have used the CMB data which in notation is identified as “Planck TTTEEE+lowP”.

2. Supernovae Type Ia (SNIa): The Supernovae Type Ia (SNIa) data were the first observational data that indicated an accelerating phase of the universe at late-time. We take the latest joint light curves (JLA) sample [66] of SNIa containing 740 SNIa in the redshift span \( z \in [0.01, 1.30] \).

3. Baryon acoustic oscillations (BAO) distance measurements: For the BAO data we use the estimated ratio \( r_s/D_V \) as a ‘standard ruler’ in which \( r_s \) is the comoving sound horizon at the baryon drag epoch and \( D_V \) is the effective distance determined by the angular diameter distance \( D_A \) and Hubble parameter \( H \) as \( D_V(z) = \left[ (1+z)^2 D_A(a)^2 + \frac{a}{H(a)^2} \right]^{1/3} \). We consider three different measurements, \( r_s(z_d)/D_V(z = 0.106) = 0.336 \pm 0.015 \) from 6-degree Field Galaxy Redshift Survey (6dFGRS) data [67], \( r_s(z_d)/D_V(z = 0.35) = 0.1126 \pm 0.0022 \) from Sloan Digital Sky Survey Data Release 7 (SDSS DR7) data [68], and finally \( r_s(z_d)/D_V(z = 0.57) = 0.0732 \pm 0.0012 \) from the SDSS DR9 [69].

4. Cosmic chronometers (CC): The cosmic chronometers are some extremely massive and passively evolving galaxies in our Universe. We employ
the recent cosmic chronometers data comprising 30 measurements of the Hubble parameter in the redshift interval $0 < z < 2$ [70]. Here, we determine the Hubble parameter values through the relation $H(z) = -(1/1+z)\,dz/dt$ where the measurement of $dz$ is obtained through the spectroscopic method with high accuracy, and the precise measurement of $dt$ — the differential age evolution of such galaxies. As a result, a precise measurement of the Hubble parameter is obtained and thus, these measurements can be taken as model independent for the cosmological studies. For a detailed reading about the cosmic chronometers we refer to [70].

In order to constrain the interacting scenarios, we use a fastest cosmological package, namely the markov chain monte carlo package cosmomc [71] equipped with the Gelman-Rubin statistics $R - 1$ [72] that determines the convergence of the chains. Moreover, we note that the package cosmomc supports the Planck 2015 Likelihood Code [65] (see http://cosmologist.info/cosmomc/ a publicly available code). The total likelihood for the Code [65] (see http://cosmologist.info/cosmomc/ a publicly available code).

IV. RESULTS OF THE ANALYSIS

In this section we describe the observational constraints that are imposed on different interacting scenarios. In the first half of this section we shall consider the interaction scenarios in a spatially flat FLRW universe whilst in the last half of this section we concentrate on the interaction scenarios in presence of the curvature of the universe. For all the interacting scenarios we use the same dataset: CMB + JLA + BAO + CC.

A. Reconstruction of the interaction rate in a spatially flat universe

In this section, we shall describe the reconstruction of the interacting scenario for three particular types of the dark energy fluids, namely the vacuum energy, dark energy fluid with constant equation of state and finally a dark energy fluid with dynamical equation of state for a spatially flat universe. For the dynamical state parameter in DE, we choose the most used and well known state parameter, namely, the CPL parametrization [62, 63]. The combined observational data for every analysis have been considered to be CMB + JLA + BAO + CC.

1. Interacting vacuum

We have reconstructed this interacting model using three steps. We first constrained the interacting model using $\delta(a)$ given in eqn. (9), that means for $\delta(a) = \delta_0 + \delta_1(1-a)$. The results of this analysis have been summarized in the second and third column of Table I. From the analysis, we see that the mean values of both $\delta_0$ and $\delta_1$ are nonzero while within 68% CL, both $\delta_0$ and $\delta_1$ allow their zero values, as one can see $\delta_0 = -0.1199^{+0.1510}_{-0.1096}$ (at 68% CL) and $\delta_1 = 0.1173^{+0.1069}_{-0.1474}$ (at 68% CL). That means the possibility of both interacting and non-interacting scenarios are allowed within this confidence-region. We also notice that the best fit values of $\delta_0$ and $\delta_1$ are indeed small and close to zero. For a better view of the behaviour of the parameters, in Fig. 1 we show the 68% and 95% CL contour plots and the likelihood analysis for some selected parameters of the model. From Fig. 1 we see that the parameters $\delta_0$ and $\delta_1$ are strongly negatively correlated. Thus, if one of $\delta_0$ and $\delta_1$ increases, the other one decreases leading to result that the interaction is hardly effective.

In the next step we extend the parametrization in (7) to (8) with the allowance of one extra parameter $\delta_2$. To reconstruct this scenario from the observations, we fix the values of $\delta_0$, $\delta_1$ to their corresponding mean values summarized in Table I and constrain the new interaction parameter $\delta_2$. The results of this analysis have been summarized in the fourth and fifth column of Table I. From Table I one can clearly notice that the estimated values of $\delta_2$ (the mean value: $\delta_2 = -0.000046^{+0.001303}_{-0.001184}$ at 68% CL and the best-fit value: $\delta_2 = 0.000555$) are very very small and close to zero. In addition to that, within 68% CL, $\delta_2$ is allowed to have a zero value. In summary, we find that the new interaction parameter, $\delta_2$ does not really contribute much. In Fig. 2 we show the corresponding contour plots and the one dimensional posterior distributions for some parameters.

Finally, we consider the parametrization (9) and repeat the similar analysis. We fix the values of $\delta_0$, $\delta_1$, $\delta_2$ to their corresponding mean values obtained in the previous analyses and perform the fittings with the same set of observational data. We find that the new interaction parameter $\delta_3$ is constrained to be extremely small ($\delta_3 = 0.000050^{+0.001247}_{-0.001197}$ at 68% CL (mean value) and $\delta_3 = 0.000555$ (best-fit value)) and effectively this does not substantially contribute to $\delta(a)$. In a similar fashion, in Fig. 3 we display different contour plots of the parameters and the one-dimensional posterior distributions for different parameters for this particular interaction scenario.

A natural question that one asks is, what happens if we consider the general parametrization of the interaction function in eqn. (9), i.e., $\delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3$, and try to constrain $\delta_i$’s. The dimension of the parameters space is big and one apprehends that the degeneracies of the parameters would increase. Anyway, we also try this and fit this more general version
with the same observational data CMB + JLA + BAO + CC and show the observational constraints on the model parameters in Table III. The contour plots for this analysis have been shown in Fig. 4. The figure clearly shows that the parameters $\delta_2$ and $\delta_3$ are degenerate. That means, according to the present observational data, this interaction scenario with the general parametrization in eqn. (9) cannot be constrained well. That is the reason we made an attempt to constrain these parameters step by step, fixing the lower order parameters to their best fit values. However, an interesting feature that Fig. 4 exhibits is that although the parameters $\delta_2$ and $\delta_3$ are found to be degenerate, $\delta_0$ and $\delta_1$ are not and they are still strongly negatively correlated to each other. However, all the attempts indicate quite strongly that the effective $\delta(a)$ is determined by the first two parameters, that means, $\delta(a) \simeq \delta_0 + \delta_1 (1 - a)$.

2. Interacting DE with constant EoS other than vacuum

As a second interacting scenario we pick up a constant EoS in DE, $w_x$. Our treatment is similar to the previous case, $\delta(a)$ given by eqns. (7), (8) and (9).

We begin the analysis with the simplest case when $\delta(a)$ has form as in eqn. (7). Using the same combined analysis, CMB + JLA + BAO + CC, we constrain the model parameters that are summarized in Table III. From the analysis, we see that the mean values as well as the best-fit values of $(\delta_0, \delta_1)$ are nonzero. In particular, the mean values of $\delta_0$ and $\delta_1$ are as follows:

$\delta_0 = -0.1778 \pm 0.1285$ at 68% CL and $0.1748 \pm 0.1250$ at 68% CL, $0.1748 \pm 0.2342$ at 95% CL. Thus, one can see that within 68% CL both $\delta_0$ and $\delta_1$ are definitely non-zero which is different from the previous scenario in section IV A 1. However, within 95% CL, their zero values are indeed allowed. From the observational constraints on the DE in the phantom scenario is suggested. We find that the mean value of the dark energy state parameter ($w_x = -1.055_{-0.066}^{+0.068}$ at 68.3% CL) and its best-fit value ($w_x = -1.080$) cross the phantom divide line, while in 68% CL, $w_x$ allows its quintessential character. In Fig. 5 we have shown the behaviour of the model parameters through their two-dimensional contour plots. An worthwhile point that one must note is that, here too, the parameters $\delta_0$ and $\delta_1$ are strongly negatively correlated.

Now, we proceed with the next step where we take the second parametrization of $\delta(a)$ given in eqn. (8) containing three parameters $(\delta_0, \delta_1, \delta_2)$. We fix the first two parameters, $(\delta_0, \delta_1)$ to their mean values from the previous analysis and constrain the remaining parameter $\delta_2$ in order to reconstruct the scenario. The constraints on the model parameters have been shown in the fourth and fifth column of Table III and in Fig. 6 we show the 68% and 95% confidence-level contours for various combinations of the model parameters. Our analysis shows that the new interaction parameter $\delta_2$ is extremely tiny with $\delta_2 = -0.000262_{-0.001655}^{+0.001707}$ (at 68% CL) and within 68% CL, $\delta_2 = 0$ is allowed. The dark energy equation of state parameter exhibits its phantom behaviour (both mean and best-fit values cross the phantom boundary $w_x = -1$ but within 68% CL, it may be quintessential as well.

After that we consider the next parametrization (9) having four parameters $(\delta_0, \delta_1, \delta_2, \delta_3)$ following the same procedure as in section IV A 1 that means we fix the first three parameters to their corresponding mean values summarized in the last two columns of Table III and constrain $\delta_3$. We find that this parameter is not significant in the sense that, it is very small taking $\delta_3 = 0.000166_{-0.001609}^{+0.001594}$ (68% CL) which also recovers its null value in 68% CL. Furthermore, the mean and best-fit values of the dark energy state parameter are phantom where $w_x = -1.052_{-0.059}^{+0.064}$ at 68% CL while its quintessential nature is also allowed.

Thus, following the reconstruction mechanism described in the three analyses above, it is clear that for the interacting scenario where dark energy state parameter remains constant, the parameters $\delta_2$ and $\delta_3$ in the parametrization (9) are insignificant.

We now exercise the general parametrization in eqn. (9) and wish to constrain all the four interaction parameters $\delta_i (i = 0, 1, 2, 3)$ by starting with all of them as free. The dimension of the parameters space now becomes eleven. Thus, some of the parameters might be degenerate. We constrained this interaction scenario with the same combined analysis CMB + JLA + BAO + CC. The Table V shows the constraints on the model parameters and in Fig. 8 we show the confidence-level contour plots for different combinations of the model parameters. The figure clearly shows that the last two interaction parameters in the general parametrization (9), i.e., $\delta_2$ and $\delta_3$ cannot be constrained well as expected. The parameters $\delta_0$ and $\delta_1$ are strongly correlated to one another with negative orientation, that means, although we increase the dimension of the parameters space, the relation between these two parameters seems to be unaltered.

3. Interacting DE with dynamical EoS

We now consider the interacting scenario between dark matter and dark energy where the dark energy has a dynamical equation of state. We choose a very widely used dynamical parametrization for DE namely
FIG. 1: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1 (1 - a)$. The combined data for this analysis has been set to be CMB + JLA + BAO + CC.

FIG. 2: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2$ in which we fix the values of $(\delta_0, \delta_1)$ from the previous analysis.
TABLE I: Reconstruction of the interacting vacuum energy scenario for the spatially flat FLRW universe using the observational data CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best-fit | Mean with errors | Best-fit | Mean with errors | Best-fit |
|------------|-----------------|----------|-----------------|----------|-----------------|----------|
| $\Omega_0 h^2$ | $0.1161_{-0.0038}^{+0.0041}$ \pm 0.0075 | 0.1194 | $0.1161_{-0.0013}^{+0.0011}$ \pm 0.0024 | 0.1164 | $0.1162_{-0.0011}^{+0.0014}$ \pm 0.0022 | 0.1164 |
| $\Omega_0 h^2$ | $0.0222_{-0.0008}^{+0.0018}$ \pm 0.0033 | 0.0220 | $0.0222_{-0.0015}^{+0.0016}$ \pm 0.0033 | 0.0224 | $0.0222_{-0.0017}^{+0.0017}$ \pm 0.0034 | 0.0222 |
| $\Omega_{MC}^{100}$ | $1.0410_{-0.00042}^{+0.00044}$ \pm 0.0086 | 1.04073 | $1.0410_{-0.00050}^{+0.00050}$ \pm 0.0057 | 1.04108 | $1.0410_{-0.00050}^{+0.00050}$ \pm 0.0061 | 1.04108 |
| $\tau$ | $0.084_{-0.017}^{+0.017}$ \pm 0.035 | 0.081 | $0.083_{-0.017}^{+0.017}$ \pm 0.032 | 0.092 | $0.082_{-0.018}^{+0.018}$ \pm 0.032 | 0.092 |
| $n_s$ | $0.969_{-0.005}^{+0.006}$ \pm 0.011 | 0.9656 | $0.968_{-0.0040}^{+0.0040}$ \pm 0.0077 | 0.9714 | $0.969_{-0.0041}^{+0.0041}$ \pm 0.0078 | 0.9714 |
| $\ln(10^{10}A_s)$ | $3.099_{-0.024+0.026}^{+0.034+0.036}$ \pm 0.036 | $3.0976_{-0.032}^{+0.032}$ \pm 0.065 | $3.113_{-0.033}^{+0.033}$ \pm 0.064 | $3.113_{-0.033}^{+0.033}$ \pm 0.064 |

$\delta_0 = -0.1199_{-0.0011}^{+0.00046} - 0.1096_{-0.029}^{+0.0242+0.0238} 0.000555$,
$\delta_1 = 0.1173_{-0.0012}^{+0.0147-0.032} - 0.0137$,
$\delta_2 = -0.000046_{-0.0001}^{+0.0003} -0.00184_{-0.00242}^{+0.00238}$.

FIG. 3: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3$ in which the values of ($\delta_0$, $\delta_1$, $\delta_2$) have been fixed from the previous analysis summarized in Table I.

the Chevallier-Polarski-Linder (CPL) parametrization given in eqn. (10). One may notice that the inclusion of CPL parametrization increases the parameters space compared to the other two interacting scenarios. However, the procedure is exactly same as that we adopted in the previous two analyses.

We first consider the parametrization of $\delta(a) = \delta_0 + \delta_1(1-a)$ of eqn. (7) and work out the observational fittings with the same combined analysis. Table V summarizes the mean and the best-fit values of the free and derived parameters of this model (see the second and third column respectively of Table V). From Table V we see that the estimated mean and best-fit values of the parameters $\delta_0$, $\delta_1$ are nonzero and within 68% CL, they are strictly nonzero, so within 68% CL, the observational data allow an interaction in the dark sector, however, within 95% CL, both of them allow their zero values and hence a non-interacting scenario as well. Looking at the current value of the dark energy state parameter, $w_0$, a perfect quintessential scenario is observed from
the mean ($w_0 = -0.892^{+0.151}_{-0.152}$ at 68% CL) and best-fit ($w_0 = -0.907$) estimations while one may notice that, within 68% CL, $-1.044 < w_0 < -0.741$, implying that the phantom scenario is also allowed. In Fig. 9 we have shown the two dimensional contour plots for different combinations of the model parameters and the one dimensional posterior distributions for some parameters as well. Similar to the previous two interaction scenarios, we again observe that the parameters $\delta_0$ and $\delta_1$ are strongly correlated to each other in the negative orientation, and they together yield an insignificantly small value for $\delta$.

As a second step, we consider the next parametrization \([8]\) in this series having three free parameters $\delta_0$, $\delta_1$ and $\delta_2$. We apply the same procedure, fix the parameters $\delta_0$, $\delta_1$ to their corresponding mean values and constrain the interaction scenario aiming to constrain the third interaction parameter $\delta_2$. The results of the analysis have been summarized in Table \([7]\) (fourth and fifth columns). We find that even if we allow the dark energy state parameter to be dynamical, but the interaction parameter $\delta_2$ is

![Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by the most general parametrization $\delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + \delta_3 (1-a)^3$ where the interaction parameters $\delta_i$'s are kept free. One can clearly notice that the parameters $\delta_2$ and $\delta_3$ are degenerate while the other parameters are not.](image)

**TABLE II:** Reconstruction of the interacting vacuum scenario for the spatially flat FLRW universe and for the general parametrization $\delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + \delta_3 (1-a)^3$, using the combined analysis CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit |
|-----------|-----------------|----------|
| $\Omega_m h^2$ | $0.1156^{+0.0038}_{-0.0030}$ | 0.1176 |
| $\Omega_b h^2$ | $0.0223^{+0.0017}_{-0.00034}$ | 0.02218 |
| $100\theta_{MC}$ | $1.0492700^{+0.00044}_{-0.00092}$ | 1.0492700 |
| $\tau$ | $0.06773746$ | $0.06773746$ |
| $n_s$ | $0.9705^{+0.0064}_{-0.0115}$ | 0.9670 |
| $\ln(10^{10} A_s)$ | $3.098^{+0.034}_{-0.065}$ | 3.074 |
| $\delta_0$ | $-0.1415^{+0.1328}_{-0.2743}$ | $-0.0394$ |
| $\delta_1$ | $0.1387^{+0.1298}_{-0.2677}$ | $0.0379$ |
| $\delta_2$ | $0.1472^{+0.7166}_{-0.8528}$ | $0.5627$ |
| $\delta_3$ | $0.0287^{+0.4934}_{-0.1973}$ | $0.6220$ |
| $\Omega_{m0}$ | $0.307^{+0.011}_{-0.026}$ | 0.305 |
| $\sigma_8$ | $0.824^{+0.016}_{-0.032}$ | 0.824 |
| $H_0$ | $67.2^{+0.77}_{-1.52}$ | $67.81$ |
FIG. 5: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant DE state parameter \( w_x \), where the interaction is parametrized by \( \delta(a) = \delta_0 + \delta_1(1 - a) \) and the combined observational analysis is CMB + JLA + BAO + CC.

FIG. 6: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant DE state parameter \( w_x \), where the interaction is parametrized by \( \delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2 \) and the combined observational analysis is CMB + JLA + BAO + CC.
constrain $\delta$ of the energy state parameter (the mean and best-fit values are very small, $\delta_2 = -0.00027 \pm 0.000134$ (68% CL constraint) and the zero value of $\delta_2$ is allowed in this CL. The dark energy state parameter (the mean and best-fit values) shows its quintessential behaviour although within 68% CL, the phantom crossing is allowed.

Now we do the analysis with the last parametrization of $\delta(a)$ as in equation 9 where we fix the interaction parameters $\delta_0$, $\delta_1$, $\delta_2$ from their previous analyses and constrain $\delta_3$ using the same observational data. The observational constraints have been summarized in the last two columns of Table III where we find that $\delta_3 = 0.000207 \pm 0.00006$ at 68% CL and one can see that within this CL, $\delta_3 = 0$ is possible. The current value of the dark energy state parameter is found to be $w_0 = -0.905 \pm 0.142$ (at 68% CL) which shows its quintessential character but phantom nature is still allowed. The allowance of the phantom state parameter in dark energy is further supported from its best fit value which crosses the phantom

| Parameters | Mean with errors | Best fit | Mean with errors | Best fit | Mean with errors | Best fit |
|-----------|-----------------|----------|-----------------|----------|-----------------|----------|
| $\Omega M C$ | 0.115$^{+0.0038}_{-0.0039}$ $0.28$ | 0.115 | 0.115$^{+0.0019}_{-0.0019}$ $0.28$ | 0.1140 | 0.115$^{+0.0013}_{-0.0013}$ $0.28$ | 0.1157 |
| $\Omega b^2$ | 0.0222$^{+0.00032}_{-0.00033}$ $0.28$ | 0.0222 | 0.0222$^{+0.00034}_{-0.00033}$ $0.28$ | 0.02226 | 0.0222$^{+0.00032}_{-0.00032}$ $0.28$ | 0.02235 |
| $\ln(10^{10}A_s)$ | 3.092$^{+0.033}_{-0.022}$ $0.28$ | 3.097 | 3.098$^{+0.024}_{-0.017}$ $0.28$ | 3.119 | 3.097$^{+0.032}_{-0.032}$ $0.28$ | 3.108 |
| $\delta_0$ | $-0.177$ $0.28$ | $-0.1721$ | $-0.00026$ $0.28$ | $-0.00080$ | $-0.00026$ $0.28$ | $-0.00080$ |
| $\delta_1$ | 0.174$^{+0.025}_{-0.025}$ $0.28$ | 0.1684 | 0.174$^{+0.025}_{-0.025}$ $0.28$ | 0.1684 | 0.174$^{+0.025}_{-0.025}$ $0.28$ | 0.1684 |
| $\delta_3$ | $-0.00026$ $0.28$ | $-0.00026$ $0.28$ | $-0.00032$ $0.28$ | $-0.00032$ | $-0.00032$ $0.28$ | $-0.00032$ |
| $\Omega m_0$ | 0.302$^{+0.012}_{-0.012}$ $0.28$ | 0.292 $0.28$ | 0.302$^{+0.010}_{-0.010}$ $0.28$ | 0.298 | 0.301$^{+0.009}_{-0.009}$ $0.28$ | 0.297 |
| $\sigma_8$ | 0.830$^{+0.017}_{-0.017}$ $0.28$ | 0.846 | 0.829$^{+0.014}_{-0.014}$ $0.28$ | 0.833 | 0.831$^{+0.017}_{-0.017}$ $0.28$ | 0.837 |
| $H_0$ | 67.83$^{+1.03}_{-1.05}$ $0.28$ | 68.95 | 67.67$^{+1.04}_{-1.04}$ $0.28$ | 67.82 | 67.84$^{+1.02}_{-1.02}$ $0.28$ | 68.29 |

**TABLE III:** Reconstruction of the interacting DE scenario with constant equation of state of DE, $w_x$, for the spatially flat FLRW universe using the observational data CMB + JLA + BAO + CC.
degenerate similar to the interaction vacuum scenario.

The dimension of the parameters space for this interaction scenario is twelve which is double of the dimension of the parameters space for the non-interacting ΛCDM model. We employ the same combined analysis and show the results of the free parameters in Table VI. And in Fig. 12 we also show the contour plots for different combinations of the free parameters. From the Fig. 12 one can clearly see that in a similar fashion, the parameters δ2, δ3 are degenerate, at least with the present astronomical data and again we come to the same conclusions for δ0 and δ1 already in earlier sections, the connection between these two parameters are not affected with the increase of the dimension of the parameters space.

Similar to the previous two interaction models, we constrain the scenario for the general parametrization of eqn. (9) taking all the interaction parameters δi’s to be free.

**FIG. 8:** Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant EoS in DE, w_x, for the most general parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \) where the interaction parameters \( \delta_i \)'s are kept free. One can clearly notice that the parameters \( \delta_2 \) and \( \delta_3 \) are degenerate similar to the interaction vacuum scenario.

**TABLE IV:** Reconstruction of the interacting DE scenario with constant equation of state of DE, \( w_x \), for the spatially flat FLRW universe and for the general parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \), using the combined analysis CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit |
|------------|-----------------|---------|
| \( \Omega_m h^2 \) | 0.116 \(^{+0.0041}_{-0.0039} \) \(^{+0.0073}_{-0.0076} \) | 0.1136 |
| \( \Omega_b h^2 \) | 0.02223 \(^{+0.00173}_{-0.00034} \) | 0.02206 |
| **1000_{MC}** | 1.04103 \(^{+0.00041}_{-0.00081} \) | 1.04126 |
| \( \tau \) | 0.0813 \(^{+0.017}_{-0.0055} \) \(^{+0.035}_{-0.0118} \) | 0.072 |
| \( n_s \) | 0.9696 \(^{+0.0085}_{-0.0113} \) | 0.9693 |
| \( \ln(10^{10} A_s) \) | 3.095 \(^{+0.032}_{-0.0065} \) \(^{+0.036}_{-0.0087} \) \(^{+0.127}_{-0.130} \) | 3.080 |
| \( w_x \) | -1.055 \(^{+0.065}_{-0.130} \) \(^{+0.133}_{-0.274} \) | -1.051 |
| \( \delta_0 \) | -0.1677 \(^{+0.133}_{-0.274} \) \(^{+0.133}_{-0.264} \) | -0.2882 |
| \( \delta_1 \) | 0.1650 \(^{+0.131}_{-0.258} \) \(^{+0.130}_{-0.268} \) | 0.2829 |
| \( \delta_2 \) | 0.0820 \(^{+0.559}_{-0.918} \) \(^{+0.563}_{-1.082} \) | 0.4414 |
| \( \delta_3 \) | 0.0102 \(^{+0.990}_{-0.990} \) \(^{+0.263}_{-1.016} \) | -0.6387 |
| \( \Omega_{m0} \) | 0.303 \(^{+0.012}_{-0.028} \) \(^{+0.016}_{-0.027} \) | 0.303 |
| \( \sigma_8 \) | 0.830 \(^{+0.017}_{-0.033} \) \(^{+0.016}_{-0.027} \) | 0.817 |
| \( H_0 \) | 67.80 \(^{+0.77}_{-0.10} \) \(^{+0.67}_{-0.26} \) | 67.09 |
FIG. 9: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical DE state parameter $w_\chi(a) = w_0 + w_a(1 - a)$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1 - a)$ and the combined observational analysis is CMB + JLA + BAO + CC.

FIG. 10: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical DE state parameter $w_\chi(a) = w_0 + w_a(1 - a)$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2$ and the combined observational analysis is CMB + JLA + BAO + CC.
we now consider a universe with a nonzero spatial curvature, namely, the CPL parametrization. The combined observational data for all the analyses are, CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit | Mean with errors | Best fit | Mean with errors | Best fit |
|------------|-----------------|----------|-----------------|----------|-----------------|----------|
| $\Omega_{b} h^2$ | 0.1160±0.0038±0.0073 | 0.1193 | 0.1157±0.0022±0.0044 | 0.1164 | 0.1160±0.0022±0.0040 | 0.1154 |
| $\Omega_{c} h^2$ | 0.02225±0.00017±0.00034 | 0.02244 | 0.02224±0.00017±0.00036 | 0.02233 | 0.02225±0.00017±0.00034 | 0.02219 |
| $\Omega_{0M} H_0^{10}$ | 1.04104±0.00042±0.00085 | 1.04091 | 1.04104±0.00034±0.00063 | 1.04082 | 1.04102±0.00035±0.00066 | 1.04113 |
| $\tau$ | 0.86±0.017±0.034 | 0.057 | 0.079±0.016±0.032 | 0.062 | 0.078±0.017±0.034 | 0.076 |
| $n_s$ | 0.9695±0.0060±0.0122 | 0.9682 | 0.9697±0.0047±0.0093 | 0.9707 | 0.9693±0.0046±0.0093 | 0.9710 |
| $\ln(10^{10} A_s)$ | 3.092±0.033±0.066 | 3.047 | 3.090±0.032±0.064 | 3.058 | 3.089±0.032±0.065 | 3.083 |
| $w_0$ | -0.892±0.0152±0.29 | -0.907 | -0.926±0.156±0.272 | -0.942 | -0.906±0.157±0.280 | -1.112 |
| $w_a$ | -0.562±0.504±0.897 | -0.727 | -0.450±0.514±0.736 | -0.379 | -0.520±0.542±0.797 | 0.092 |
| $\delta_0$ | -0.248±0.1342±0.2143 | -0.1930 | - | - | - | - |
| $\delta_1$ | 0.2457±0.1312±0.2679 | 0.1937 | - | - | - | - |
| $\delta_2$ | - | - | -0.0000273±0.0000244±0.0000069 | 0.000069 | - | - |
| $\delta_3$ | - | - | -0.0000027±0.0000148±0.0000009 | 0.000069 | - | - |
| $\Omega_{m0}$ | 0.303±0.013±0.027 | 0.303 | 0.301±0.011±0.021 | 0.309 | 0.303±0.011±0.021 | 0.304 |
| $\sigma_8$ | 0.821±0.017±0.037 | 0.809 | 0.823±0.018±0.035 | 0.804 | 0.822±0.017±0.036 | 0.829 |
| $H_0$ | 67.69±0.97±2.03 | 68.52 | 67.83±1.02±2.01 | 67.18 | 67.78±0.99±2.07 | 67.46 |

TABLE V: Reconstruction of the dark matter-dark energy interaction scenario in a spatially flat universe where dark energy assumes the dynamical state parameter, namely, the CPL parametrization. The combined observational data for all the analyses are, CMB + JLA + BAO + CC.

For the three particular type of the dark energy fluids, we now consider a universe with a nonzero spatial curvature. The model parameters are estimated against the same four observational dataset CMB + JLA + BAO + CC.

B. Reconstruction of the interaction rate in the nonflat universe

In the beginning we consider that the interaction parameter $\delta(a)$ is characterized by its Taylor series expa-
FIG. 12: Spatially flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with varying state parameter in DE for the most general parametrization \( \delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2 + \delta_3(1 - a)^3 \) where the interaction parameters \( \delta_i \)'s are kept free. One can clearly notice that the parameters \( \delta_2 \) and \( \delta_3 \) are degenerate similar to the interaction vacuum scenario.

We constrained this scenario with the same observational data and present the results in the second and third column of Table VII. From the estimations of \( \delta_0 \) and \( \delta_1 \), namely, \( \delta_0 = -0.1348^{+0.1405}_{-0.1766} \) (at 68% CL) and \( \delta_1 = 0.1318^{+0.1741}_{-0.1560} \) (at 68% CL), one can see that an interaction scenario is not really visible, at least for low values of \( z \). Also \( (\delta_0, \delta_1) = (0, 0) \) are also allowed by the data within 68% CL, which means that according to the observational data, the model hardly distinguishes between an interacting and non-interacting scenario. The best fit values of these parameters are also constrained to be small: \( \delta_0 = -0.0662 \) and \( \delta_1 = 0.0631 \). We also estimated the curvature of the universe, \( \Omega_K = -K/(aH)^2 \), at present time, i.e., \( \Omega_{K0} \), for this interaction scenario. Our results show (see Table VII) that both the mean and best-fit values of the curvature parameters are negative, that means \( \Omega_{K0} < 0 \), which further means that \( \Omega_0 < 1 \) (as we know \( \Omega_0 = 1 + \Omega_{K0} \)). So, the possibility of an open universe is apparently suggested. But, within 68% CL, \( \Omega_{K0} \) allows its positive and zero values. Thus, based on the observational constraints, we cannot arrive at a definite conclusion on the possible shape of the universe. However, in both the cases, one can see that, \( \Omega_{K0} \) is a very small quantity and close to zero \( (\Omega_{K0} = -0.0016^{+0.0053}_{-0.0054} \) at 68% CL, i.e., \( -0.00601 < \Omega_{K0} < 0.00533 \)). In Fig. 13 we have clearly displayed the two dimensional contour plots between different variations of the parameters and the likelihood plots for some model parameters. We now point out one of the interesting observations from Fig. 13. From this figure, one can see that the parameters \( (\delta_0, \delta_1) \) are (strongly-) negatively correlated to each other, very similar to the interacting scenarios without the presence of curvature. So, one can realize that the presence of curvature cannot alter the behaviour of these two parameters.
TABLE VI: Reconstruction of the interacting DE scenario with its dynamical EoS for the spatially flat FLRW universe and for the general parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \), using the combined analysis CMB + JLA + BAO + CC.

Following similar trend as in other interaction scenarios described in the previous sections, we now consider the extended parametrization of \( \delta(a) \) given in eqn. [3] that includes one more free parameter \( \delta_2 \). We fix the values of \( \delta_0, \delta_1 \) to their corresponding mean values obtained from the analysis with eqn. [3] and constrain the model with one parameter left, i.e. \( \delta_2 \). The observational summary has been given in the fourth and fifth column of Table VII. Fig. 14 displays the one dimensional posterior distributions for some selected parameters plus two dimensional contour plots. From the analysis, we see that, \( \delta_2 \) is very very small \( \delta_2 = -0.000066^{+0.002243}_{-0.000066} \) at 68% CL) and within 68% CL, \( \delta_2 \) = 0 is allowed. Now, in the estimation of the curvature parameter, we have a slight different result. From the analysis we see that (see the fourth and fifth column of Table VII), the mean value of \( \Omega_K \) is negative — an indication for an open universe, although, within 68% CL, its zero value is still allowed (i.e., spatial flatness), but, the best-fit value of \( \Omega_K \) is positive — an indication for a closed universe. One can clearly notice that \( \Omega_K \) is sufficiently small, \( \Omega_K = -0.00032^{+0.000090}_{-0.000124} \) (at 68% CL) but higher than the order of the radiation density at present \( (\Omega(10^{-4})) \).

We discuss the last parametrization of \( \delta(a) \) in eqn. [3] and fix the parameters \( \delta_i \)'s \( (i = 0, 1, 2) \) to their estimated mean values, see Table VII and wish to constrain the remaining free interaction parameter \( \delta_3 \). The results of the analysis have been summarized in the last two columns of Table VII and in Fig. 15 we present the graphical behaviour of the free parameters of the model. From the analysis one can see that the interaction parameter \( \delta_3 \) is very small with \( \delta_3 = -0.000041^{+0.000228}_{-0.000216} \) (68% CL) where \( \delta_3 = 0 \) is also allowed. The curvature parameter is constrained to be, \( \Omega_K = -0.00024^{+0.00040}_{-0.00030} \) (68% CL) and its best fit value is also negative \( \Omega_K = -0.00305 \). So, one can see that, the mean and best-fit value of the curvature density parameter suggest the possibility of an open universe while the closed universe scenario is allowed within 68% CL. But, overall, the curvature density parameter is close to zero but let us note that, the order of the curvature parameter is higher than that of the order of the radiation density at present \( (\Omega(10^{-4})) \).

We now consider the general parametrization for \( \delta(a) \) given in eqn. [3] and constrain the interaction model using the same datasets. See Table VIII and Fig. 16 for summary. Including the expected degeneracies in the values of \( \delta_2 \) and \( \delta_3 \), all conclusions remain the same.

2. Interacting DE with constant EoS other than vacuum

We now focus on the observational constraints on the interacting scenario when the universe has a nonzero curvature and the dark energy has a constant state parameter \( w_x \).

We start with the parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) \) of eqn. [3] and constrain all the model parameters with the combined observational data that we have already mentioned. The summary of the observational constraints has been given in the second and third column of the Table IX and the graphical behaviour between various parameters have been shown in Fig. 17. From the analysis we see that both \( \delta_0 \) and \( \delta_1 \) predict nonzero values. More specifically, the estimated mean values are as follows: \( \delta_0 = 0.1342^{+0.1751}_{-0.2081} \) at 68% CL \( (0.1342^{+0.3594}_{-0.3594} \) at 95% CL), \( \delta_1 = 0.1342^{+0.1751}_{-0.2081} \) at 68% CL \( (0.1342^{+0.3594}_{-0.3594} \) at 95% CL). However, one can clearly notice that within 68% CL, \( \delta_0 = 0 \) and \( \delta_1 = 0 \) are also allowed by the data. Additionally, the best-fit values of \( \delta_0, \delta_1 \) are also close to zero. Regarding the dark energy state parameter, we see that both the mean and best-fit values of \( w_x \) suggest a phantom dark energy. Precisely, the 68.3% CL constraint on the state parameter is, \( w_x = -1.046^{+0.076}_{-0.059} \) (68% CL) that also includes its quintessential nature and it is close to the cosmological constant boundary. Furthermore, from the curvature parameter, \( \Omega_K = -0.00053^{+0.00057}_{-0.00034} \) (at 68% CL) one may see that since its error bars are bigger compared to its mean value, thus, the possibilities of open, closed and flat universe are all allowed. About the nature of the parameters \( \delta_0 \) and \( \delta_1 \), we have exactly similar conclusion (Fig. 17) as mentioned in other interacting scenarios.

We now consider the second parametrization, i.e., eqn. [3] and fix the first two free parameters \( \delta_0 \) and \( \delta_1 \) to their corresponding mean values obtained from the previous analysis (columns second and fourth of Table IX). We find the parameter \( \delta_2 \) is very small, \( \delta_2 =
FIG. 13: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1 - a)$. The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

FIG. 14: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2$ in which we fix the values of $(\delta_0, \delta_1)$ from the previous analysis but left $\delta_2$ as a free parameter. The combined observational data we set to be CMB + JLA + BAO + CC.
### Table VII: Reconstruction of the dark matter-dark energy interaction scenario for the interacting vacuum universe in the nonflat universe using the combined analysis CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit | Mean with errors | Best fit | Mean with errors | Best fit |
|------------|------------------|----------|------------------|----------|------------------|----------|
| \( \Omega_{m0} \) | 0.00016 \pm 0.00049 \pm 1.01107 \pm 0.00585 \pm 0.01968 | -0.000182 | -0.000032 \pm 0.000390 \pm 0.00820 | 0.0077 | -0.00024 \pm 0.000412 \pm 0.00791 | -0.000305 |
| \( \Omega_{K0} \) | 0.307 \pm 0.012 \pm 0.226 | 0.303 | 0.307 \pm 0.010 \pm 0.221 | 0.309 | 0.306 \pm 0.016 \pm 0.019 | 0.311 |
| \( \sigma_8 \) | 0.825 \pm 0.020 \pm 0.946 | 0.830 | 0.825 \pm 0.017 \pm 0.035 | 0.826 | 0.825 \pm 0.017 \pm 0.033 | 0.838 |
| \( H_0 \) | 67.23 \pm 0.85 \pm 1.67 | 67.66 | 67.21 \pm 0.82 \pm 1.66 | 67.16 | 67.30 \pm 0.82 \pm 1.59 | 0.80 \pm 1.52 |

**FIG. 15:** Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario where the interaction is parametrized by \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \) in which we fix the values of \( \delta_0, \delta_1, \delta_2 \) from the previous analysis but left \( \delta_3 \) as a free parameter. The combined observational data we set to be CMB + JLA + BAO + CC.

Both closed and flat universes are also allowed. We further note that the estimation of the curvature parameter is slightly higher compared to the present day radiation density (\( \mathcal{O}(10^{-4}) \)).

After this we take the last parametrization of this series, i.e., equation \( \delta(a) \) where similar to the previous cases, we fix the first three parameters \( \delta_0, \delta_1 \) and \( \delta_2 \) and constrain \( \delta_3 \). The results are summarized in the last
FIG. 16: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting vacuum scenario for the general parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \). The combined observational data we set to be CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit |
|------------|-----------------|----------|
| \( \Omega_K \) | 0.1515 \( \pm 0.0036 \pm 0.0073 \) | 0.1114 |
| \( \Omega_K^2 \) | 0.0222 \( \pm 0.0017 \pm 0.0033 \) | 0.02215 |
| \( 100\delta_{MC} \) | 1.04109 \( \pm 0.00944 \pm 0.0082 \) | 1.04135 |
| \( \tau \) | 0.0843 \( \pm 0.017 \pm 0.034 \) | 0.100 |
| \( n_s \) | 0.9713 \( \pm 0.0059 \pm 0.0119 \) | 0.9740 |
| \( \ln(10^{10} A_s) \) | 3.098 \( \pm 0.034 \pm 0.066 \) | 3.130 |
| \( \delta_0 \) | -0.1632 \( \pm 0.0176 \pm 0.0340 \) | -0.3795 |
| \( \delta_1 \) | 0.1598 \( \pm 0.1550 \pm 0.3495 \) | 0.3713 |
| \( \delta_2 \) | -0.0422 \( \pm 0.0422 \pm 0.0096 \) | -0.0158 |
| \( \delta_3 \) | 0.404 \( \pm 0.959 \pm 0.9596 \) | 0.8876 |
| \( \Omega_{K0} \) | 0.00036 \( \pm 0.0012 \pm 0.0116 \) | 0.00051 |
| \( \Omega_{m0} \) | 0.305 \( \pm 0.013 \pm 0.026 \) | 0.288 |
| \( \sigma_8 \) | 0.82 \( \pm 0.021 \pm 0.044 \) | 0.831 |
| \( H_0 \) | 67.24 \( \pm 0.86 \pm 1.69 \) | 68.28 |

TABLE VIII: Reconstruction of the dark matter-dark energy interaction scenario for the interacting vacuum universe for the nonflat universe using the general parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \). The combined analysis has been set to be CMB + JLA + BAO + CC.

two columns of Table [X] and the corresponding graphical variations are shown in Fig. [19]. We found that \( \delta_3 \) is very small and the dark energy state parameter \( w_x \), always represents a phantom universe (both mean value: \( w_x = -1.045 \pm 0.067 \) at 68% CL and best fit value \( w_x = -1.072 \), cross the phantom divide line) while from the mean value of \( w_x \), its quintessential character is still allowed within 68% CL. We find that \( |\Omega_{K0}| \) is higher in this case compared to previous two analyses with equations [8] and [7].

Now, as usual, we constrain this interacting scenario for the parametrization \( \delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2 + \delta_3(1-a)^3 \) where all the interaction parameters \( \delta_i \)'s are kept free. The results of the analysis have been presented in Table [X] and the corresponding graphical variations are displayed in Fig. [20]. From this figure, one can clearly see that although \( \delta_i \)'s for \( i = 2, 3 \) are not well constrained by the present data, but, the behavior of \( \delta_0 \) and \( \delta_1 \) are again not altered as significantly.
FIG. 17: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant state parameter in DE, $w_x$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1-a)$. The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

FIG. 18: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant state parameter in DE, $w_x$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1-a) + \delta_2(1-a)^2$, in which we fix the first two interaction parameters ($\delta_0, \delta_1$) from the previous analyses. The combined data for this analysis have been set to be CMB + JLA + BAO + CC.
### TABLE IX: Non-flat universe: Reconstruction of the interacting DE scenario where DE has a constant EoS in DE, $w_x$, using the combined analysis CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit | Mean with errors | Best fit | Mean with errors | Best fit |
|------------|------------------|----------|------------------|----------|------------------|----------|
| $\Omega_h^2$ | 0.1164$^{+0.0039}_{-0.0077}$ | 0.1195 | 0.1162$^{+0.0038}_{-0.0064}$ | 0.1207 | 0.1160$^{+0.0032}_{-0.0066}$ | 0.1126 |
| $\Omega_M h^2$ | 0.02226$^{+0.0017}_{-0.0032}$ | 0.02238 | 0.02226$^{+0.0016}_{-0.0032}$ | 0.02219 | 0.02225$^{+0.0015}_{-0.0031}$ | 0.02220 |
| $1000\theta_{MC}$ | 1.04101$^{+0.00044}_{-0.00088}$ | 1.04076 | 1.04101$^{+0.00039}_{-0.00076}$ | 1.04087 | 1.04101$^{+0.00044}_{-0.00077}$ | 1.04111 |
| $\tau$ | 0.082$^{+0.018}_{-0.034}$ | 0.070 | 0.082$^{+0.017}_{-0.035}$ | 0.079 | 0.081$^{+0.017}_{-0.031}$ | 0.084 |
| $n_s$ | 0.9697$^{+0.0641}_{-0.126}$ | 0.061$^{+0.0058}_{-0.0117}$ | 0.9696$^{+0.0648}_{-0.0111}$ | 0.9645 | 0.9698$^{+0.0559}_{-0.0112}$ | 0.9747 |
| $\ln(10^{10}A_s)$ | 3.096$^{+0.035}_{-0.070}$ | 3.066 | 3.095$^{+0.034}_{-0.067}$ | 3.099 | 3.095$^{+0.033}_{-0.063}$ | 3.103 |
| $w_x$ | $-1.046_{-0.058}^{+0.134}$ | $-1.092_{-0.063}^{+0.128}$ | $-1.043_{-0.065}^{+0.122}$ | $-1.048_{-0.061}^{+0.117}$ | $-1.072_{-0.067}^{+0.110}$ |
| $\delta_0$ | $-0.136_{-0.183}^{+0.365}$ | $-0.0419$ | $-0.0294_{-0.157}^{+0.044}$ | $-0.0426$ | $-0.0234_{-0.151}^{+0.045}$ | $-0.0425$ |
| $\delta_1$ | $-0.1342_{-0.281}^{+0.352}$ | $0.0426$ | $-0.0000249_{-0.00238}^{+0.00418}$ | $-0.0000250$ | $-0.0000259_{-0.00239}^{+0.00416}$ | $-0.00002758$ |
| $\delta_2$ | $-0.00053_{-0.00557}^{+0.01106}$ | $-0.00144$ | $-0.00073_{-0.00910}^{+0.00393}$ | $-0.00498$ | $-0.00101_{-0.00960}^{+0.00452}$ | $-0.00656$ |
| $\Omega_{K0}$ | $0.305_{-0.015}^{+0.028}$ | $0.306$ | $0.304_{-0.012}^{+0.022}$ | $0.314$ | $0.303_{-0.010}^{+0.022}$ | $0.294$ |
| $\sigma_8$ | $0.831_{-0.023}^{+0.044}$ | $0.829$ | $0.832_{-0.020}^{+0.042}$ | $0.821$ | $0.833_{-0.020}^{+0.040}$ | $0.852$ |
| $H_0$ | $67.66_{-1.11}^{+1.20}$ | $68.27$ | $67.69_{-1.06}^{+2.09}$ | $67.64$ | $67.72_{-1.05}^{+2.01}$ | $67.89$ |

### FIG. 19: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant state parameter in DE, $w_x$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1 (1-a) + \delta_2 (1-a)^2 + \delta_3 (1-a)^3$, in which we fix the first three interaction parameters ($\delta_0, \delta_1, \delta_2$) from the previous analyses. The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

3. Interacting DE with dynamical EoS

Finally, we discuss the interacting model in the nonflat FLRW universe when the dark energy equation of state is dynamical, given by the CPL parametrization, eqn. [10].

We begin with the first parametrization $\delta(a) = \delta_0 + \delta_1 (1-a)$ of eqn. [7]. The observational constraints on
FIG. 20: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with constant state parameter in DE, \(w_x\), where the interaction is parametrized by \(\delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2 + \delta_3(1 - a)^3\). The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

the model are shown in the second and third columns of Table XI. In Fig. 21 we present the corresponding graphical analysis. Our analysis reveals as usual that both \(\delta_0\) and \(\delta_1\) are non-null but within 68% CL, both of them allow zero values. The dark energy state parameter shows its quintessential behaviour as, \(w_0\) is \(w_0 = -0.910^{+0.152}_{-0.184}\) (at 68% CL). The best fit value of \(w_0\) is also greater than \(-1\). However, the phantom character of \(w_0\) is still allowed within the 68% CL where \(-1.095 < w_0 < -0.758\). For the curvature parameter, one may conclude that the mean and best-fit values of \(\Omega_{K0}\) suggest an open universe, but indeed, the statistical analysis includes the closed universe since within 68% CL, \(-0.00981 < \Omega_{K0} < 0.00238\), which also includes the flat case \(\Omega_{K0} = 0\). From Fig. 21 we again conclude that, irrespective of the dynamical nature in the DE state parameter, the negative correlation between \(\delta_0\) and \(\delta_1\) exists and such correlation is very very strong.

We then start with the second parametrization of \(\delta(a)\), i.e., equation (8) where we fix the two parameters \(\delta_0\) and \(\delta_1\) to their obtained values, and constrain the interacting scenario with the same astronomical data. The results are summarized in the fourth and fifth columns of Table XI and Fig. 22. Our results show that \(\delta_2\) assumes a very small value. The mean value of \(\delta_2\) is, \(\delta_2 = 0.000350^{+0.002540}_{-0.002261}\) (68% CL) and its best fit value is, \(-0.000517\). Thus, one can see that the contribution of \(\delta_2\) towards the interaction parameter \(\delta(a)\) is insignificant. While on the other hand, the present value of the dark energy equation of state keeps its quintessential character (see Table XI), although the observational data allow it to cross the phantom divide line, since within 68% CL, \(-1.102 < w_0 < -0.828\). For the curvature parameter, our conclusion follows the similar statements made in earlier sections.

After the fittings with first two parametrizations, namely, (7) and (8), we start with the next parametrization (9) and fit the interacting scenario where we fix the first parameters \(\delta_0\), \(\delta_1\), \(\delta_2\) and constrain the last parameter \(\delta_3\) along with other free parameters. The summary is given in the last two columns of Table XI and in Fig. 23 we display the one dimensional posterior distribution.
FIG. 21: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical state parameter in DE, \( w_x(a) = w_0 + w_a(1 - a) \), where the interaction is parametrized by \( \delta(a) = \delta_0 + \delta_1(1 - a) \). The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

FIG. 22: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical state parameter in DE, \( w_x(a) = w_0 + w_a(1 - a) \), where the interaction is parametrized by \( \delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2 \). The combined data for this analysis have been set to be CMB + JLA + BAO + CC.
FIG. 23: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical state parameter in DE, \( w_a = w_0 + w_a (1 - a) \), where the interaction is parametrized by \( \delta (a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \). The combined data for this analysis have been set to be CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit |
|------------|-----------------|----------|
| \( \Omega_{K} h^2 \) | 0.1160 \( ^{+0.0036}_{-0.0036} \) \( \pm 0.0008 \) | 0.1158 |
| \( \Omega_{M} h^2 \) | 0.02224 \( ^{+0.00017}_{-0.00017} \) \( \pm 0.00034 \) | 0.02224 |
| \( \Omega_{b} h^2 \) | 1.04102 \( ^{+0.00042}_{-0.00042} \) \( \pm 0.00082 \) | 1.04119 |
| \( \tau \) | 0.082 \( ^{+0.017}_{-0.017} \) \( \pm 0.035 \) | 0.096 |
| \( a_s \) | 0.9698 \( ^{+0.0059}_{-0.0059} \) \( \pm 0.0119 \) | 0.9716 |
| \( \ln (10^{10} A_s) \) | 3.09 \( ^{+0.034}_{-0.033} \) \( \pm 0.067 \) | 3.122 |
| \( w_a \) | -1.046 \( ^{+0.067}_{-0.067} \) \( \pm 0.117 \) | -1.006 |
| \( \delta_0 \) | -0.1649 \( ^{+0.1463}_{-0.1472} \) \( \pm 0.348 \) | -0.0815 |
| \( \delta_1 \) | 0.1621 \( ^{+0.1906}_{-0.1904} \) \( \pm 0.3081 \) | 0.0782 |
| \( \delta_2 \) | 0.0493 \( ^{+0.5096}_{-0.5094} \) \( \pm 0.507 \) | -0.1826 |
| \( \delta_3 \) | 0.0126 \( ^{+0.0872}_{-0.0867} \) \( \pm 0.3401 \) | -0.1754 |
| \( \Omega_{K0} \) | 0.00013 \( ^{+0.00562}_{-0.00562} \) \( \pm 0.01082 \) | -0.00448 |
| \( \Omega_{m0} \) | 0.303 \( ^{+0.014}_{-0.014} \) \( \pm 0.026 \) | 0.314 |
| \( \sigma_8 \) | 0.82 \( ^{+0.022}_{-0.022} \) \( \pm 0.043 \) | 0.847 |
| \( H_0 \) | 67.72 \( ^{+1.08}_{-1.08} \) \( \pm 2.16 \) | 66.44 |

TABLE X: Reconstruction of the interacting DE scenario where DE has a constant EoS in DE, \( w_a \), in the spatially nonflat universe for the general parametrization \( \delta (a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \) using the combined analysis CMB + JLA + BAO + CC.

along with the two dimensional contour plots for several combinations of the model parameters. The conclusion is that \( \delta_3 \) is very small and hence there is no effective contribution to \( \delta (a) \); the mean value of the dark energy state parameter crosses the ‘−1’ boundary although it may be quintessential in 68% CL; and the curvature parameter assumes the negative value (indication for an open universe) while the postive and zero values are allowed by the data.

As the last example, we take the general parametrization \( \delta (a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \) and constrain the interacting scenario using the same combined analysis employed throughout this work. The results have been summarized in Table XII and the graphical dependence between several free parameters including the one dimensional posterior distributions and the two dimensional contour plots are shown in Fig. 24. From Fig. 24 one can see that the parameters \( \delta_2 \) and \( \delta_3 \) are not properly constrained since from the analysis we cannot put any upper or lower bounds on them. This is not surprising because we already mentioned that the parameters space for this interaction scenario increases considerably compared to the previous interaction models and one may expect such degeneracy in the parameters. From Fig. 24 one can see that the strong negative correlation between \( \delta_0 \) and \( \delta_1 \) does not change. This is probably the most important finding of this work because such nature is independent of the curvature of the universe and with the dimension of the parameters space under consideration.
| Parameters | Mean ± errors | Best fit | Mean ± errors | Best fit | Mean ± errors | Best fit |
|-----------|--------------|----------|--------------|----------|--------------|----------|
| $\Omega_K h^2$ | $0.1163 \pm 0.0034 \pm 0.0037$ | 0.1142 | $0.1161 \pm 0.0031 \pm 0.0057$ | 0.1172 | $0.1165 \pm 0.0035 \pm 0.0057$ | 0.1127 |
| $\Omega_b h^2$ | $0.02227 \pm 0.0018 \pm 0.00055$ | 0.02226 | $0.02222 \pm 0.0017 \pm 0.00032$ | 0.02217 | $0.02222 \pm 0.0016 \pm 0.00033$ | 0.02230 |
| $\Omega_K$ | $1.04104 \pm 0.0041 \pm 0.0008$ | 1.04145 | $1.04104 \pm 0.0041 \pm 0.00078$ | 1.04089 | $1.04096 \pm 0.0043 \pm 0.00071$ | 1.04145 |
| $\tau$ | $0.075 \pm 0.017 \pm 0.032$ | 0.078 | $0.078 \pm 0.016 \pm 0.032$ | 0.089 | $0.079 \pm 0.016 \pm 0.035$ | 0.102 |
| $n_s$ | $0.9701 \pm 0.0057 \pm 0.0138$ | 0.9714 | $0.9694 \pm 0.0055 \pm 0.0112$ | 0.9684 | $0.9694 \pm 0.0055 \pm 0.0117$ | 0.9777 |
| $\ln(10^{10} A_s)$ | $3.088 \pm 0.034 \pm 0.071$ | 3.091 | $3.088 \pm 0.032 \pm 0.064$ | 3.115 | $3.091 \pm 0.032 \pm 0.067$ | 3.131 |
| $w_0$ | $-0.910 \pm 0.015 \pm 0.0348$ | -0.873 | $-0.932 \pm 0.014 \pm 0.0299$ | -0.898 | $-1.005 \pm 0.004 \pm 0.0215$ | -0.982 |
| $w_a$ | $-0.454 \pm 0.033 \pm 0.162$ | -0.531 | $-0.383 \pm 0.056 \pm 0.748$ | -0.180 | $-0.133 \pm 0.032 \pm 0.412$ | -0.140 |

**TABLE XI:** Non-flat universe: Reconstruction of the interacting DE scenario where DE has a dynamical EoS in DE, $w_x(a) = w_0 + w_a(1 - a)$, using the combined analysis CMB + JLA + BAO + CC.

![FIG. 24: Non-flat universe: 68% and 95% CL contour plots for different combinations of the free parameters for the interacting DE scenario with dynamical state parameter in DE, $w_x(a) = w_0 + w_a(1 - a)$, where the interaction is parametrized by $\delta(a) = \delta_0 + \delta_1(1 - a) + \delta_2(1 - a)^2 + \delta_3(1 - a)^3$. The combined data for this analysis have been set to be CMB + JLA + BAO + CC.](image-url)
TABLE XII: Reconstruction of the interacting DE scenario for the dynamical EoS in DE, \( w_a (a) = w_0 + w_a (1 - a) \), where the interaction is parametrized by \( \delta (a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \). The combined analysis is CMB + JLA + BAO + CC.

| Parameters | Mean with errors | Best fit |
|------------|-----------------|----------|
| \( \Omega_M h^2 \) | 0.1161\(^{+0.0036}_{-0.0076} \) | 0.1200 |
| \( \Omega_b h^2 \) | 0.0222\(^{+0.0017}_{-0.0035} \) | 0.02219 |
| 1000\( \Omega_M C \) | 1.0410\(^{+0.0043}_{-0.0084} \) | 1.04079 |
| \( \alpha \) | 0.078\(^{+0.015}_{-0.017} \) | 0.069 |
| \( \sigma_8 \) | 0.970\(^{+0.0359}_{-0.0119} \) | 0.9623 |
| \( \ln (10^3 A_s) \) | 3.085\(^{+0.0340}_{-0.066} \) | 3.066 |
| \( w_0 \) | -0.512\(^{+0.177}_{-0.345} \) | -0.4919 |
| \( w_a \) | -0.471\(^{+0.313}_{-1.335} \) | -0.418 - 1.154 |
| \( \delta_0 \) | -0.1616\(^{+0.1791}_{-0.3711} \) | -0.1691 |
| \( \delta_1 \) | 0.1590\(^{+0.1778}_{-0.3680} \) | 0.1692 |
| \( \delta_2 \) | -0.0415\(^{+0.5347}_{-0.9585} \) | -0.0508 - 0.1455 |
| \( \delta_3 \) | 0.0355\(^{+0.4829}_{-1.0355} \) | 0.0355 |
| \( \Omega_{K_0} \) | -0.0028\(^{+0.0068}_{-0.0117} \) | 0.00171 |
| \( \Omega_{m_0} \) | 0.306\(^{+0.013+0.029} \) | 0.308 |
| \( \sigma_8 \) | 0.828\(^{+0.023+0.044} \) | 0.818 |
| \( H_0 \) | 67.49\(^{+1.18}_{-2.23} \) | 68.12 |

V. SUMMARY AND DISCUSSIONS

In the present work we focus on a specific class of interacting models where we denote the presence of interaction through the deviation in the evolution law of the standard cold dark matter sector; we assume that the evolution of CDM is governed by the law \( \rho_c \propto a^{-3+\delta(a)} \), where \( \delta(a) \) is assumed to evolve and any \( \delta(a) \) apart from its null values, reflects the interaction in the dark sector. The flow of energy can be characterized with the sign of \( \delta(a) \). For instance, \( \delta(a) < 0 \) implies an energy flow from CDM to DE while its positive value indicates the flow of energy from DE to CDM. As the functional form of \( \delta(a) \) is not known, we take a standard approach – the Taylor series expansion of \( \delta(a) \) around the present scale factor \( a_0 = 1 \) as follows

\[
\delta(a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 + \ldots
\]

where \( \delta_i \)'s, \( i = 0, 1, 2, 3, \ldots \), are constants with \( \delta_0 \) as the current value of the parameter \( \delta(a) \). However, the consideration of a large number of free parameters in a cosmological model generally increases the degeneracy amongst the parameters and it is naturally expected that the observational analysis with the entire parametrization might be plagued with this issue. In the present work we have performed two separate reconstructions of the interaction scenarios. To start with, we consider the Taylor series expansion up to its second term, i.e. \( \delta(a) = \delta_0 + \delta_1 (1 - a) \) and constrain the interaction parameters, \( \delta_0 \) and \( \delta_1 \). Next we increase one more term in the Taylor expansion as \( \delta(a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 \), but this time, we fix the free parameters \( \delta_0, \delta_1 \) of this Taylor expansion to their corresponding mean values obtained in the previous analysis with the parametrization \( \delta(a) = \delta_0 + \delta_1 (1 - a) \), and constrain the free parameter \( \delta_2 \). In a similar fashion, we consider the final parametrization in this series, \( \delta(a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \), and similarly we fix the values of \( \delta_0, \delta_1, \delta_2 \) from the previous analyses and constrain the last interaction parameter \( \delta_3 \). We note that, in a similar way, one can continue with more free parametrizations. But from the present analysis we find that the parameters \( \delta_2 \) and \( \delta_3 \) are in fact very small so that effectively the terms containing \( \delta_2, \delta_3 \) might be neglected, and any further generalization is hardly expected to improve the analysis.

Then we consider the full parametrization up to third order, \( \delta(a) = \delta_0 + \delta_1 (1 - a) + \delta_2 (1 - a)^2 + \delta_3 (1 - a)^3 \) where we consider the parameters \( \delta_i \)'s to be free and constrained all the interaction scenarios. The analyses performed with this entire parameterization now reflect that the parameters \( \delta_2 \) and \( \delta_3 \) are indeed degenerate as expected. In the following we briefly mention how the interaction parameters behave with different dark energy models.

We have considered three dark energy models for both spatially flat and curved universe with various parameterization of the interaction. But in all these diverse models, there is a very strong indication that there is hardly any interaction in the dark sector of the universe. Even if there is any, that should have been in a distant past. The present dark matter and dark energy hardly influence upon the independent evolution of each other.

Acknowledgments

W. Yang’s work is supported by the National Natural Science Foundation of China under Grants No. 11705079 and No. 11647153. AP was supported by FONDECYT postdoctoral grant no. 3160121.

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