A ring of instantons inducing a monopole loop

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We consider the superposition of infinitely many instantons on a circle in $\mathbb{R}^4$. The construction yields a self-dual solution of the Yang-Mills equations with action density concentrated on the ring. We show that this configuration is reducible in which case magnetic charge can be defined in a gauge invariant way. Indeed, we find a unit charge monopole (worldline) on the ring. This is an analytic example of the correlation between monopoles and action/topological density, however with infinite action. We show that both the Maximal Abelian Gauge and the Laplacian Abelian Gauge detect the monopole, while the Polyakov gauge does not. We discuss the implications of this configuration.

1 Introduction

Quantum Chromodynamics as the theory of strong interactions exhibits interesting non-perturbative phenomena at low energies, among them chiral symmetry breaking and confinement. While the first of them is explained by instanton related mechanisms, confinement has remained a puzzle and other topological excitations have been proposed
to explain this effect. One of the most popular models is the dual superconductor scenario \[1, 2\]. In this picture, the condensation of magnetic monopoles leads to flux tubes between chromoelectric charges. The fundamental problem of this model is the fact that monopoles are a priori not present in Yang-Mills theories. Nevertheless, it is possible to introduce them by partial gauge fixings called Abelian gauges (AG) \[3\], where the gauge freedom is used to diagonalise an auxiliary operator \(O(x)\) transforming in the adjoint representation. Then monopoles occur as defects of the gauge fixing at points where \(O(x)\) has coinciding eigenvalues.

However, this approach suffers from the weakness that depending on the choice of the operator \(O\) one ends up with different Abelian gauges, the best studied examples being the Maximally Abelian Gauge (MAG), the Laplacian Abelian Gauge (LAG) and the Polyakov gauge (PG), respectively. It is still under discussion whether all AG’s are equally useful or one of them is best suited for solving the confinement problem.

There is a strong hint that instantons and the configurations responsible for confinement are related: Lattice simulations have shown that chiral symmetry breaking and confinement have the same critical temperature \[4\]. In other words, instantons and monopoles\(^1\) may occur on the same footing. Since these objects are both of topological origin, a relation between the monopole charge and the instanton number in Abelian gauges has been derived \[5\]. This may be seen as some gauge-independent content of Abelian gauges.

Moreover, the monopole position is often correlated to lumps of the action/topological charge density. The first observation in this direction was the finding that the high temperature limit of the Harrington-Shepard caloron \[6\] leads to a static BPS monopole \[7\], i.e. a monopole line. In the same spirit we expect a monopole loop to be induced by a configuration with topological charge concentrated on a ring. Two overlapping instantons as well as a finite number of instantons are known to possess this property \[8, 9\]. Pushing this to the limit, our investigation will concern the superposition of infinitely many instantons on a ring. The gauge field constructed this way will come out to be \(U(1)\)-reducible, i.e. effectively Abelian. Therefore there is a natural operator \(O\) to diagonalise and this gauge transformation in fact induces a monopole on the ring. This singularity prevents the configuration from having finite action. We will use it for kinematical aspects, namely as a test for the mentioned popular Abelian gauges. Throughout the paper we will restrict ourselves to gauge group \(SU(2)\).

\section{Construction of the instanton ring}

The general instanton in four dimensional Euclidean space-time can be obtained by the (algebraic and nonlinear) ADHM construction \[10\], while a subclass with restricted relative color orientation of the constituents fits into the ansatz,

\[
A_\mu = A_\mu^a \frac{\sigma_a}{2}, \quad A_\mu^a = -\bar{\eta}^a_{\mu\nu} \partial_\nu \ln \Pi(x),
\]  

\(^1\)and presumably also vortices

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where $\bar{\eta}$ and $\sigma$ are the 't Hooft tensor and the Pauli matrices, respectively. Within this ansatz the self-duality condition $F_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$ reduces to the condition $\Box \Pi / \Pi = 0$ which is solved by the Green’s function of point ‘charges’ at positions $y_k$ with scale parameters $\rho_k$. We will use the ‘t Hooft ansatz \cite{11} and conformal ansatz \cite{12}, in which the scalar potential $\Pi$ for a charge $N$ instanton reads

$$\Pi(x) = 1 + \sum_{k=1}^{N} \frac{\rho_k^2}{(x - y_k)^2}, \quad \Pi(x) = \sum_{k=1}^{N+1} \frac{\rho_k^2}{(x - y_k)^2},$$

(2)

respectively. In both cases, the topological charge density can be written quite simply as $\text{Tr} F_{\mu\nu} F_{\mu\nu} = \Box \Box \ln \Pi$.

The superposition of infinitely many BPST instantons along the time axis leads to the Harrington-Shepard caloron \cite{6} which at high temperature turns into the BPS monopole \cite{7}. In the limit of vanishing distance between the instanton copies this construction amounts to the following potential,

$$\Pi(x) = \int_{-\infty}^{+\infty} dt \frac{\rho^2}{x^2 + (x_4 - t)^2} = \frac{\pi \rho^2}{|x|}.$$  

(3)

Inspired by this construction we consider now the configuration of infinitely many instantons with same size equidistantly placed on a circle. We choose this circle to be in the $x_1 x_2$-plane,

$$y_{k\mu} = R (\cos(2\pi k/N), \sin(2\pi k/N), 0, 0), \quad \rho_k = \rho.$$

(4)

An arbitrary location of the circle can be gained by the action of the symmetry group $SO(4)$. For our calculations, double polar coordinates

$$x_\mu = (r_{12} \cos \varphi_{12}, r_{12} \sin \varphi_{12}, r_{34} \cos \varphi_{34}, r_{34} \sin \varphi_{34})$$

(5)

are most suitable. The limit $N \to \infty$ can be performed as the integral

$$\Pi(x) = a + \frac{N \rho^2}{2\pi} \int_0^{2\pi} d\xi \frac{1}{r_{12}^2 + R^2 - 2R r_{12} \cos(\varphi_{12} - \xi) + r_{34}^2}, \quad \xi \sim \frac{2\pi k}{N}.$$  

(6)

In this limit, the only difference between the two ansätze \cite{2} is the first term which we parametrise by $a = 1$ for the ‘t Hooft ansatz and $a = 0$ for the conformal ansatz, respectively. In order to get a well behaved $\Pi$, the size $\rho^2$ has to decrease to zero when taking $N$ to infinity\footnote{Allowing $N \rho^2$ to diverge automatically leads to the conformal ansatz, since then $a$ can be neglected (and the divergence is multiplicative).}, that is we perform the limit

$$N \rho^2 \xrightarrow{N \to \infty} \text{const} \equiv \lambda^2.$$  

(7)

Then the integral yields,

$$\Pi(x) = a + \frac{\lambda^2}{S(r_{12}, r_{34})}, \quad S(r_{12}, r_{34}) = \sqrt{(r_{12}^2 + r_{34}^2 - R^2)^2 + 4r_{34}^2 R^2}.$$  

(8)
In [9] this formula was found by summability of the finite $N$ case. Aspects of such superpositions from the viewpoint of hyperbolic monopoles have been studied in [13].

$S(r_{12}, r_{34})$ vanishes at the circle $r_{12} = R, r_{34} = 0$. Therefore, $\Pi$ is singular there. Not surprisingly the latter is the Greens function of the Laplacian with a constant charge density on the ring. The ring ansatz together with the limit $N \to \infty$ has lead to a symmetry $SO(2) \times SO(2)$ since $\Pi$ only depends on the two radii.

We note in passing that the scalar potential [9] can be obtained from the one of the single instanton by complexifying the radius $r_{34}$ and computing the absolute value,

$$\Pi(x) = a + \frac{\lambda^2}{r_{12}^2 + (r_{34} \pm iR)^2}.$$  

Seen from far away, the potential behaves as

$$\Pi(x) = a + \frac{\lambda^2}{r^2}, \quad r \equiv \sqrt{r_{12}^2 + r_{34}^2} \gg R.$$  

Thus the ring configuration looks like the ordinary BPST instanton in singular gauge for $a = 1$ and like pure gauge for $a = 0$, respectively. However, the topological charge of these configurations is infinite by construction. The lumpiness of the action density manifests itself in an extreme way, namely as a non-integrable singularity at the ring where both ansätze behave as

$$\text{Tr} \ F_{\mu \nu} F^{\mu \nu} = -32 \frac{R^4}{S(r_{12}, r_{34})^4} \quad \text{as } r_{12} \to R, r_{34} \to 0$$  

We will comment more on this singularity later.

### 3 Reducibility and monopole content

It is straightforward to calculate the field strength of the ring configuration [8]. Since self-duality is still fulfilled, the chromoelectric field is sufficient. For the conformal case $a = 0$ one finds,

$$E_i^a(x) = f_i(x) n^a(x).$$  

This means that all components of the field strength point in the same color direction. This can be made explicit$^3$ by the vanishing of the matrix $M_{ij} = (E_i^a)^2 (E_j^b)^2 - (E_i^a E_j^a)^2$. The direction in color space is given by the normalised vector field $n$,

$$n^a(x) = \frac{1}{S(r_{12}, r_{34})} \begin{pmatrix} 2(-x_1 x_3 - x_2 x_4) \\ 2(x_1 x_4 - x_2 x_3) \\ r_{12}^2 - r_{34}^2 - R^2 \end{pmatrix}.$$  

Such a behaviour is a strong hint for the reducibility of the configuration. Loosely speaking, the configuration is Abelian with a space-dependent embedding $n^a(x)$ of the Abelian direction into the non-Abelian color space. More precisely, the definition of

$^3$DH thanks M. Garcia–Perez for marking this point.
reducibility involves a normalised ‘Higgs’ field $n$ in the adjoint representation being covariantly conserved in the background of $A$,

$$D_\mu n \equiv \partial_\mu n - i[A_\mu, n] \equiv 0.$$  \hfill (12)

A related characterisation of reducibility is the non-trivial stabiliser of $A$, namely the $U(1)$ subgroup $\exp(i\mu n(x))$ with constant $\mu$. From the gauge covariance of $\Box$ follows that on diagonalising $n$, the gauge field $A$ becomes purely diagonal, i.e. Abelian. Accordingly, all components of the field strength become diagonal. This or equivalently the integrability condition $[D_\mu, D_\nu]n = [F_{\mu\nu}, n] = 0$ gives the particular behaviour of the field strength mentioned above. All this can be verified for the ring in the conformal ansatz, but does not hold for the ’t Hooft ansatz. This indicates that the relative orientation of the constituents – which in general is different for the two ansätze as can be seen from the completeness of the conformal ansatz for topological charge 2 – is important for properties like reducibility and the monopole content, which we discuss in a moment. From now on we will consider only the conformal case $a = 0$.

The $f_i$’s are scalar functions of a similar structure as $n$,

$$f_i(x) = \frac{4R^2}{S(r_{12}, r_{34})^3} \left( \frac{2(-x_1x_3 + x_2x_4)}{2(x_1x_4 + x_2x_3)} \frac{r_{12}^2 - r_{34}^2 - R^2}{r_{34}^2} \right),$$  \hfill (13)

which is typical for symmetric configurations.

Reducible gauge fields can be decomposed in a natural way w.r.t. $n$,

$$A_\mu^a = C_\mu n^a - \epsilon_{abc} n^b \partial_\mu n^c, \quad F_{\mu\nu}^a = (\epsilon_{bcd} n^b \partial_\mu n^c \partial_\nu n^d + \partial_\mu C_\nu - \partial_\nu C_\mu)n^a,$$  \hfill (14)

Figure 1: Lines of constant azimuthal angle $\beta$ for the normalised Higgs field $n$ as given by $n_{a=3}(x) = \cos \beta(x)$ and [14, 5], basically the same as in [14, 5].
known as Cho connections [15]. $C_\mu = A_\mu^a n^a$ plays the role of an Abelian gauge field, for the ring it can be written in a compact form as

$$C_\mu = \frac{2}{S(r_{12}, r_{34})} (x_2, -x_1, x_4, -x_3) = 2\eta_\mu^a S(r_{12}, r_{34}) x_\nu.$$  \hfill (15)

Coming back to our original motivation we now investigate whether the ring configuration has a monopole content. Reducibility of the ring configuration provides a natural operator to diagonalise in the spirit of Abelian gauges, namely $n$. Let us repeat that $n$ can be obtained from such a particular gauge field via the field strength (10) or the conservation equation (12). The normalised field $n$ takes values in $S^2 \subset su(2)$ which is exactly the coset $SU(2)/U(1)$ fixed by Abelian gauges.

For further computations it is helpful to parametrise $n$ in terms of Euler angles, $n^a(x) = (\sin \beta(x) \cos \alpha(x), \sin \beta(x) \sin \alpha(x), \cos \beta(x))^T$. In Fig. 1 we depict the azimuthal angle $\beta(x)$. The polar angle $\alpha$ is simply given by $\alpha(x) = \varphi_{12} - \varphi_{34} + \pi$. From this it is easy to see that the $n$-field is a hedgehog around the ring (for fixed worldline variable $\varphi_{12}$), i.e. in the vicinity of the ring it takes on all values once. It behaves like the asymptotic Higgs field in the ‘t Hooft-Polyakov monopole. Thus, the diagonalisation of $n$ introduces a unit charge Dirac monopole with its worldline being the ring.

4 Detection of the monopole in Abelian gauges

With the help of reducibility we have shown that the ring configuration contains a monopole loop in a canonical way. In this section we will test whether the three most popular Abelian gauges recognize the monopole, too.

4.1 Maximally Abelian gauge

In the continuum, the MAG is defined by the gauge condition

$$\partial_\mu A_\mu^\perp - i [A_\mu^3, A_\mu^\perp] = 0,$$  \hfill (16)

where $A_\mu^\perp = A_\mu^1 \sigma_1/2 + A_\mu^2 \sigma_2/2$ represents the off-diagonal gauge field [3]. Using the properties of the $\bar{\eta}$-symbol, it is clear that the instanton ring both in the conformal and ‘t Hooft ansatz satisfies the MAG condition.

In order to come to a unique gauge fixing the above condition has to be restricted further. The gauge condition is endowed by the minimum of the following MAG functional,

$$F_{\text{MAG}}[A_\mu] = \frac{1}{2} \int d^4x \ Tr(A_\mu^\perp)^2 \to \min.$$  \hfill (17)

This can be read as going from a solution of the equation of motion (16) to an actual minimum of the action (17).

\footnote{which means going to the unitary gauge}
The singularity at the ring does not spoil the integrability of this functional for the configuration at hand. It turns out that the functional diverges due to its behaviour at infinity, \( F_{\text{MAG}}[A_\mu] = \int d^4 x \frac{r^2}{S^2} = \infty \). On the other hand, we know from reducibility that there exists a gauge transformation which makes \( A_\mu \) purely diagonal. Accordingly, the MAG-functional takes its smallest possible value, namely zero. This is a general property of reducible configurations/Cho connections \( (14) \).

Observe that the singular gauge transformation bringing the configuration into the MAG is exactly the one which introduces the Dirac monopole. This is supported by an equivalent version of the MAG which defines its (normalised) Higgs field \( n \) as the one which minimizes \( \int d^4 x \text{Tr}(D_\mu n)^2 \). Indeed this alternative MAG-functional vanishes for the \( n \)-field of the ring due to equation \( (12) \). We conclude that the Maximal Abelian gauge indeed recognizes the monopole content of this configuration. However, it is not sensitive to the size of the ring since the functional becomes zero independent of the radius \( R \). Actually, this statement also holds for the ’t Hooft ansatz, where the MAG-functional is finite from the beginning. These findings should be compared to the fact that the monopole loop in the background of a single instanton is suppressed by the MAG-functional \( (14) \).

4.2 Laplacian Abelian gauge

The auxiliary operator \( O(x) \) of the LAG is the ground state \( \phi \) of the gauge covariant Laplacian in the adjoint representation \( (16) \),

\[
- D^2 \phi = E_0 \phi .
\]

Thus this field has to be square integrable. The \( n \)-field of the ring is a zero mode of the Laplacian due to \( (12) \), but being pointwise normalised, \( n^\alpha(x)n^\alpha(x) = 1 \), it is clearly not square integrable. This is a known problem of this gauge when defined over infinite volume manifolds. In order to circumvent it, conformal invariance was used to go onto the four-sphere \( (17) \). Since the covariant Laplacian there reads

\[
- D^2 = - \frac{1}{\sqrt{g}} D_\mu \sqrt{g} g^{\mu\nu} D_\nu ,
\]

\( n \) is again a zero mode due to the same property \( (12) \). Moreover, \( n \) is now square integrable since the problem at infinity is gone. Thus \( n \) is the operator of the LAG and therefore the LAG detects the monopole loop, too.

4.3 Polyakov gauge

The Polyakov gauge is adapted to finite temperature or other situations where at least one of the coordinates is compact. The operator \( O(x) \) in this case is the Wilson loop in this ‘time’ direction,

\[
P(\vec{x}) = \mathcal{P} \exp \left( i \int_0^T A_4(x_4, \vec{x}) \, dx_4 \right) .
\]
It is a group valued variable, and for gauge group \( SU(2) \) the defects occur at \( P(\vec{x}) = \pm 1 \), i.e. \( \text{tr} \, P(\vec{x}) = \pm 2 \) \([18, 19, 20]\). The diagonalising transformation of the Polyakov loop is known explicitly and can be understood as coming close to the Weyl gauge \( A_4 = 0 \) while respecting the holonomy in this direction.

The Polyakov gauge can also be applied to the single instanton on four-dimensional Euclidean space by virtue of the fact that the according unlimited integral in the Polyakov line

\[
P(\vec{x}) = \mathcal{P} \exp \left( i \int_{-\infty}^{\infty} A_4(x_4, \vec{x}) \, dx_4 \right)
\]

exists. The instanton reveals a monopole which is static (by definition), runs through the instanton position and shows a perfect hedgehog behaviour in the spacial directions.

A priori it is not obvious, whether the Polyakov gauge is applicable to the ring in the same spirit. There are essentially two different choices of a ‘time’ direction, \( x_4 \) and \( x_1 \). In the first case a whole monopole loop appears at some instant in ‘time’. In the second case there is a monopole pair creation, a loop ‘evolution’ and finally the annihilation of the pair. The calculation of the Polyakov line in both cases does not involve path ordering when performed at the monopole position. As a matter of fact, the Polyakov line in the first case is traceless, so does not see the monopole as a defect. In the second case one has to compute the following integral

\[
\text{tr} \, P(\vec{x}) = 2 \cos \left( \int_{-\infty}^{\infty} \frac{x_2}{x_1^2 + x_2^2 - R^2} \, dx_1 \right).
\]

For the range of interest \( x_2 \in [-R, R] \) the Cauchy principal value of this integral gives indeed \( \text{tr} \, P = 2 \), but in addition there are infinitely many defects just outside the monopole since the integral diverges as \( |x_2| \to R + 0 \). Altogether the Polyakov gauge is not appropriate in our situation.

**5 Discussion**

We have constructed a gauge field coming from a superposition of infinitely many instantons on a ring. As an overlap effect the ring becomes the worldline of a magnetic monopole. This is a gauge-independent statement since the configuration at hand is \( U(1) \)-reducible, i.e. essentially Abelian. It results in all components of the electric and magnetic field being parallel to each other in color space. Put differently, the configuration is an example of a Cho connection with non-trivial fields \( n \) and \( C \).

We have studied the application of Maximally Abelian gauge, Laplacian Abelian gauge and Polyakov gauge to this configuration. It turns out that the first two of them do recognize the monopole loop. This fact is merely based on reducibility. The Polyakov gauge fails to do so, which we interpret as an effect of the superposition; the Polyakov gauge is questionable in this situation anyhow.

Of particular interest is the MAG-functional since its minimisation is as difficult as a spinglass problem. We propose to study the scalar potential \( \Pi(x) = 1 + r^2 / r^2 + \)
\[ \lambda^2/S(r_{12}, r_{34}), \] that corresponds to a combination of an instanton (in singular gauge) and a ring configuration at radius \( R \). A minimum for \( F_{\text{MAG}} \), in particular for non-trivial values of \( \rho, \lambda \) and \( R \), could give more insight into how the functional weights instantons and monopoles and how they influence each other.

The ring configuration is an analytic example that monopoles come with an excess of topological density as has been observed on the lattice. However, by construction the ring has infinite action\(^5\). This should be compared to the BPS monopole\(^6\) (better the dyon) which has finite energy and is static; therefore its action for an infinite time extension becomes divergent. Since in our case the analogue of time – the angle \( \varphi_{12} \) – is compact, the divergence has to be present already for fixed ‘\( \varphi_{12} \)-slice’ near the ring. Infact, the divergence of the action comes from a non-integrable singularity at the ring: \( S(r_{12}, r_{34}) \) is proportional to the distance \( \bar{r} = \sqrt{(r_{12} - R)^2 + r_{34}^2} \) from the ring and the action density behaves as \( 1/\bar{r}^4 \) which cannot be cured by the measure.

This pathological behaviour is provoked by the complete reducibility of the configuration. A Cho connection around the non-continuity of \( n \) always yields \( A \sim d\bar{n} \sim 1/\bar{r} \) which results in the described divergence for the action density. We have also investigated the possibility of keeping reducibility but relaxing selfduality, that is keeping \( n \) but deforming \( C \) say near the ring. It is possible to soften the singularity in the topological density to an integrable one. However, the action density stays infinite because its leading singularity comes from \( d\bar{n} \) alone.

Therefore a more realistic configuration is expected to be Abelian (reducible) only outside a non-Abelian core. Such a mechanism is at work for calorons with non-trivial holonomy \(^{21, 22} \). Since these are built from a superposition of relatively oriented instantons, one has to use the ADHM construction. The latter will make the ring superposition much more complicated. Observe that also in this situation the alternative MAG-functional will be close to zero, since its integrand \( D_\mu n \) vanishes in a large fraction of space-time.

The relative color orientation of constituents has been shown to be crucial for the instanton approach to confinement \(^{23} \). Our findings that the conformal ansatz induces a monopole loop while the ’t Hooft ansatz does not, supports this fact. More knowledge is needed of how to build instanton clusters or chains in order to obtain a monopole worldline. The percolation of a monopole loop through the volume – as one criterion for confinement – should then be induced by an instanton ensemble.

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\(^5\)there are no strange effects when superposing the constituents, which would make the action somehow finite
\(^6\)actually a conformal transformation connects the two
References

[1] G. ’t Hooft, in: High Energy Physics, Proceedings of the EPS International Conference, Palermo 1975, A. Zichichi, ed., Editrice Compositori, Bologna 1976.

[2] S. Mandelstam, Vortices and quark confinement in non-Abelian gauge theories, Phys. Rep. C23 (1976) 245–249.

[3] G. ’t Hooft, Topology of the gauge condition and new confinement phases in non-Abelian gauge theories, Nucl. Phys. B190 (1981) 455.

[4] J. Kogut, M. Stone, H. W. Wyld, W. R. Gibbs, J. Shigemitsu, S. H. Shenker, and D. K. Sinclair, Deconfinement and chiral symmetry restoration at finite temperature in SU(2) and SU(3) gauge theories, Phys. Rev. Lett. 50 (1983) 393.

[5] O. Jahn, Instantons and monopoles in general Abelian gauges, J. Phys. A33 (2000) 2997–3019, hep-th/9909004.

[6] B. J. Harrington and H. K. Shepard, Periodic Euclidean solutions and the finite temperature Yang-Mills gas, Phys. Rev. D17 (1978) 2122.

[7] P. Rossi, Propagation functions in the field of a monopole, Nucl. Phys. B149 (1979) 170.

[8] M. Garcia Perez, T. G. Kovacs, and P. van Baal, Overlapping instantons, hep-ph/0006155.

[9] Y. Brihaye and J. Kunz, Summable chains of instantons and their symmetries, J. Math. Phys. 30 (1989) 1913–1917.

[10] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld, and Y. A. Manin, Construction of instantons, Phys. Lett. A65 (1978) 185–187.

[11] G. ’t Hooft unpublished (1976).

[12] R. Jackiw, C. Nohl, and C. Rebbi, Conformal properties of pseudoparticle configurations, Phys. Rev. D15 (1977) 1642–1646.

[13] A. Chakrabarti, Construction of hyperbolic monopoles, J. Math. Phys. 27 (1986) 340–348.

[14] R. C. Brower, K. N. Orginos, and C.-I. Tan, Magnetic monopole loop for the Yang-Mills instanton, Phys. Rev. D55 (1997) 6313, hep-th/9610101.

[15] Y. M. Cho, A restricted gauge theory, Phys. Rev. D21 (1980) 1080.

[16] A. J. van der Sijs, Laplacian Abelian projection, Nucl. Phys. B (Proc. Suppl.) 53 (1997) 535, hep-lat/9608041.
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[17] F. Bruckmann, T. Heinzl, T. Vekua, and A. Wipf, Magnetic monopoles vs. Hopf defects in the Laplacian (Abelian) gauge, Nucl. Phys. B593 (2001) 545–561, hep-th/0007119

[18] H. Reinhardt, Resolution of Gauss’ law in Yang-Mills theory by gauge invariant projection: Topology and magnetic monopoles, Nucl. Phys. B503 (1997) 505–529, hep-th/9702049

[19] C. Ford, U. G. Mitreuter, J. M. Pawlowski, T. Tok, and A. Wipf, Monopoles, Polyakov loops and gauge fixing on the torus, Ann. Phys. (N.Y.) 269 (1998) 26, hep-th/9802191

[20] O. Jahn and F. Lenz, Structure and dynamics of monopoles in axial gauge QCD, Phys. Rev. D58 (1998) 085006, hep-th/9803177

[21] K. Lee and C. Lu, SU(2) calorons and magnetic monopoles, Phys. Rev. D58 (1998) 025011, hep-th/9802108

[22] T. C. Kraan and P. van Baal, Periodic instantons with non-trivial holonomy, Nucl. Phys. B533 (1998) 627–659, hep-th/9805168

[23] A. Hart and M. Teper, Instantons and monopoles in the maximally Abelian gauge, Phys. Lett. B371 (1996) 261–269, hep-lat/9511016