Modeling the Effect of Copper on Hardness of Microalloyed Dual Phase Steel through Neural Network and Neuro-fuzzy Systems

S. K. GHOSH, S. GANGULY, P. P. CHATTOPADHYAY and S. DATTA

Dr. M. N. Dastur School of Materials Science and Engineering, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India. 1) Department of Metallurgy and Materials Engineering, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.

(Received on March 30, 2005; accepted on June 21, 2005)

The effects of copper along with some microalloying elements and the processing parameters are modeled with artificial neural network and adaptive neuro-fuzzy inference system. Both the tools are found to be useful for modeling the effect of copper and other alloying additions along with the processing parameters on the hardness of microalloyed DP steels. In case of the neural network, the proposed committee of models is found to be effective in handling the problem of mapping the input-output relation in these steels. The increase in the number of rules is found to improve the predictability of the neuro-fuzzy inference system. The predictions made by both the models substantiate the knowledge of physical metallurgy principles.

KEY WORDS: dual phase steel; microalloying; copper; modeling; neural network; neuro-fuzzy inference system.

1. Introduction

Development of high strength steel with adequate ductility, formability and fracture toughness has drawn interest of several metallurgists. The demand of such steels is mainly in the sectors like automobile, defense and naval applications. Dual phase (DP) steel is one of the prospective members of the family of HSLA steels providing the option for achieving judicious balance among the desired mechanical properties by virtue of its composite microstructure comprising of hard martensite particles dispersed in the soft ferrite matrix. The microstructural constitution of DP steel offers the opportunity of configuring the same in a most flexible manner by varying the volume fraction, morphology and distribution of the constituent phases.

In this regard, copper (Cu) being an element known to strengthen through solid solution strengthening in ferrite as well as through precipitation hardening; a Ti, B, microalloyed steel has been alloyed with Cu to utilize the individual effect of Ti, B and Cu as well as synergistic effect of the combination of Ti, B and Cu for improving the hardenability of austenite which in turn is expected to give rise to a DP microstructure depending on the finish rolling temperature and the subsequent cooling rate.

These alloying additions as well as different processing parameters influence the mechanical property of the steel through different strengthening mechanisms. Quantitative determination of properties of steels in terms of its composition and process parameters has remained inadequate in models due to a complex and nonlinear relationship between the dependent and independent variables. For this type of systems, mapping of inputs to an appropriate output space needs a black box to be put in between the input and output spaces. Artificial neural network (ANN) is such a learning technique that enables to map the hidden input-output relationship accurately. Several attempts have been made to predict the mechanical properties of steel using neural network. In the present work the effect of copper is in microalloyed DP steel have been studied primarily using ANN.

Similar to ANN, fuzzy logic can be a potential black box to serve the same purpose. Since expert knowledge can be very easily used to build fuzzy rules and sets, fuzzy inference systems (FIS) are very much attuned with the real world situation. Considerable improvement in FIS is seemed to be possible if neuro-fuzzy system is used. These systems incorporate adaptive technique in developing a model of the input and output variables. While FIS uses expert knowledge for prediction, the neuro fuzzy systems call for the adaptive learning processes, similar to ANN, for refinement of the results of FIS. Efforts had been made earlier to apply fuzzy system in assessing the effects of compositional variables and the processing parameters on the mechanical properties of aluminium alloys and steel. It was demonstrated that the phenomenon could be described in fuzzy inference systems through some if-then rules.

In the present study an attempt has been made towards the application of neuro-fuzzy inference system in modeling the simple processing route with the predictability of the systems parameters in copper bearing Ti, B microalloyed steels. A comparative study between the two above-mentioned modeling tools has also been made to assess the effect of the different input parameters on the hardness of the steel.
2. Database

The alloy chemistry (C, Mn, Si, Cu, Ti and B), finish rolling temperature (FRT), cooling rate (CR), ageing time and ageing temperature have been taken as input parameters, whereas hardness is designated as the output variable for the ANN model. For the Neuro-fuzzy model the number of input variables was reduced. The data used for the present exercise have been generated in the laboratories. The chemical analyses are done in Optical Emission Spectrometer (SPECTROLAB-M8). Rolling has been made in a laboratory scale two high rolling mill (capacity 5 HP). The hardness testing has been carried out in Brinell cum Vickers hardness testing machine (Model: BV-250 (SPL)) and the Vickers hardness values are measured. Around 150 data have been used for training the networks and 50 data have been used for validation.

The ranges of variables used in the present work are listed in Table 1. Each variable is normalized within the range of 0 to 1 for ANN modeling by the operation given below:

\[ x_N = \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \]  

where \( x_N \) is the normalized value of a variable \( x \), \( x_{\text{max}} \) and \( x_{\text{min}} \) are the maximum and minimum limit of \( x \) respectively. The \( x_{\text{max}} \) and \( x_{\text{min}} \) of all the variables are described in Table 1.

3. The Neural Network Model

3.1. The Technique

The neural network used in the present study is a supervised multilayered feed forward network trained with gradient descent back propagation algorithms. Three compositional and four process variables are defined as input nodes and the property variable is described as output node. The inputs and outputs are connected through hidden units. The inputs \( x_i \) are multiplied by weights \( w_{ji} \) for a hidden node \( h_j \); summation of all the \( w_{ji}x_i \) is then added to a bias value \( b_{ji} \) and finally operated by a suitable transfer function (f). The operation can be written as

\[ h_j = f\left( \sum w_{ji}x_i + b_{ji} \right) \]  

Similar operations are repeated for varying number of hidden layers in order to find out suitable network architecture.

Hidden layers contribute to the output node through a linear operation. The output \( y \) can be written as:

\[ y = \left( \sum w_{j}h_j + b' \right) \]  

where \( w_{j} \) and \( b' \) are new sets of weights and bias values. During the training process, the error of the calculated or predicted output in relation to the actual output is back propagated to adjust all the weight and bias values.

3.2. Optimization of Architecture

The non-linearity of an ANN model depends on the number of transfer nodes as well as number of hidden layers. Greater non-linearity permits the neural network to capture non-linear relationships among the independent and the dependent variables. So to get good approximation of input-output relation optimizations of the number of hidden units and the number of hidden layers in the ANN are necessary.

In the present work, the ANNs were trained with Scaled Conjugate Gradient (SCG) and Lavenberg-Marquardt (LM) algorithm, which were found to be superior to other algorithms for error minimization. The transfer function used was \( \tanh \). Figure 1 shows that the error level decreases with the number of hidden units and layers in the ANN when trained with SCG algorithm. From similar results developed through learning networks with LM algorithm having different architecture, two networks trained with SCG algorithm and one with LM algorithm was found to give better prediction. These three networks were used to develop a three-member committee model, the mean prediction of the three networks was used as the final prediction of the output. Figure 2 shows a typical training pattern of one of the said committee member. Figures 3–5 show the prediction of the trained members of the committee of networks with unknown inputs.

3.3. Effect of the Composition and Process Parameters on the Hardness Property

The effects of individual input variables onto the output variables are predicted by the network when all other input variables are kept at constant values as shown in Figs. 6–8.

Table 1. The minimum, maximum, mean and standard deviation of the variable parameters.

| Variables | Minimum Value | Maximum Value | Mean Value | Standard Deviation |
|-----------|---------------|---------------|------------|--------------------|
| % C       | 0.03          | 0.06          | 0.04       | 0.0033             |
| % Mn      | 1.40          | 1.75          | 1.59       | 0.1008             |
| % Si      | 0.325         | 0.575         | 0.494      | 0.0712             |
| % Ti      | 0             | 0.05          | 0.034      | 0.0135             |
| % B       | 0             | 0.003         | 0.0016     | 0.0007             |
| % Cu      | 0             | 2.2           | 1.34       | 0.4595             |
| Cooling Rate (°C/sec) | 0 | 100 | 41.76 | 43.45 |
| Cold Deformation (%) | 0 | 75 | 21.04 | 21.46 |
| Ageing Temp. (°C) | 0 | 600 | 331.7 | 163.5 |
| Ageing Time (min.) | 0 | 350 | 108.4 | 134.2 |

© 2005 ISIJ
The composition of the steel was 0.04C–1.69Mn–0.569Si–0.032Ti–0.0013B–1.64Cu. The post-rolling cooling rate was 3°C/s, then the steel was 15% cold deformed and aged at 250°C for one hour. The error bars show the limit of prediction of different members of the committee and the curve is drawn through the mean of the three predictions. It is seen that the learning of the neural network is quite good as far as its predictability is concerned. It is observed that increasing copper content beyond a particular value leads to a sharp increase in hardness (Fig. 6). This observation is quite expected in such class of steel, as the copper in excess of solubility leads to precipitation of fine ε-copper. It is seen from the Fig. 6 that the hardness has shown a sharp increase around normalized value of 0.5, which is around 1.1 wt% of copper. It is known that around this amount of copper is required to get effective precipitation hardening.20) The initial rise in the hardness due to copper is due to ferrite strengthening. Increase in hardness due to titanium is also seen to occur from around 0.02 wt% of titanium.

Fig. 2. Behavior of error computed during training with scaled conjugate gradient algorithm for a network with 5 hidden layers having 63 nodes.

Fig. 3. Predicted versus measured values in normalized condition after training the network with scaled conjugate gradient algorithm for a network with 5 hidden layers having 63 nodes.

Fig. 4. Predicted versus measured values in normalized condition after training the network with scaled conjugate gradient algorithm for a network with 6 hidden layers having 64 nodes.

Fig. 5. Predicted versus measured values in normalized condition after training the network with Levenberg–Marquardt algorithm for a network with 6 hidden layers having 64 nodes.
This finding is also not unexpected as the effect of titanium is not considerable if added in an amount less than the above-mentioned quantity.\textsuperscript{21}) Drop in hardness due to addition of titanium higher than around 0.04 wt% cannot be explained properly. Higher amount of titanium producing higher amount of carbides and thus making the matrix lean in carbon may be stated to be a possible clarification. Effect of boron is also found to be noteworthy when added around 0.002 wt% or more.\textsuperscript{22)}

In case of post rolling cooling rate, it is clearly visible that initially the hardness value increases with faster cooling rate (Fig. 7) due to formation of low temperature transformation products like acicular ferrite or bainite. But the hardness value reaches a plateau when the cooling rate is further increased before resulting in a steep rise at still higher cooling rate. This increase can easily be designated to the formation of martensite. When the effect of another deformation (rolling) parameter \textit{i.e.} amount of cold deformation on hardness is considered, it is seen that hardness is initially increased due to dislocation hardening. Increase in dislocation density is also known to increase nucleation sites for precipitates. But due to further increase of cold deformation, no significant improvement in hardness could be visible. It shows that higher amount of cold deformation could not improve the dislocation and/or precipitation hardening. It may be presumed that this increased deformation has resulted in development of textured structure, which is not reflected through the hardness studies.

Coming to the ageing parameters (Fig. 8), it is found that the hardness increases with ageing temperature and after reaching an optimum value it decreases due to over ageing. Variation of ageing time is found to have no significant influence on the hardness property of the steels. This may be attributed to the fact that the age hardening effect of these steels are predominantly due to precipitation of copper, and the kinetics of this precipitation is quite rapid and thus no further improvement could be caused due to higher ageing time.

4. Neuro-fuzzy Modelling Technique

4.1. The Technique

Fuzzy inference system (FIS) facilitates mapping from a set of inputs to an output space using fuzzy logic. The FIS consists of (a) membership function, (b) fuzzy logic operator and (c) if-then rule. A membership function (MF) is a curve that describes the mapping of each point in the input space to a membership value between 0 and 1, called the degree of membership ($m$). There are quite a few types of membership functions, \textit{viz.} triangular, trapezoidal, gaussian, sigmoidal, asymmetrical polynomial \textit{etc.,} of which the gaussian has been used in the present work. There are several types of fuzzy logic operators, of which the Sugeno-type\textsuperscript{23)} is used here. If-then rule statements are used to formulate the conditional statements between the inputs and the outputs. The if-then rule assumes the form.

\[
\text{If } x \text{ is } A \text{ then } y \text{ is } B
\]

where $A$ and $B$ are linguistic values defined by the fuzzy sets on the specific arrays. In the present system, it can be said that if titanium is low then hardness is low (say).

An adaptive neuro-fuzzy inference system (ANFIS) constructs a FIS whose membership function parameters are adjusted using certain learning algorithm. This allows the fuzzy systems to learn from the data they are modeling. A network-type structure resembling that of a neural network then maps the inputs through their membership functions and associated parameters, and finally the membership functions and associated parameters of the output are used to interpret the input/output relations. The parameters associated with the membership functions will change through
the learning process. The modeling approach used by ANFIS is similar to any of the system identification techniques. The primary job is to hypothesize a parameterized model structure (relating inputs to membership functions to rules to outputs to membership functions), then to train the FIS model to emulate the training data presented to it by modifying the membership function parameters according to a chosen error criterion. This type of modeling yields best results if the training data presented to the ANFIS for training membership function parameters is fully representative of the features of the data that the trained FIS is intended to model. So, if a large amount of data is collected, it will contain all the necessary representative features. There are certain other constraints in using ANFIS, e.g., it only supports Sugeno-type systems, and that is also with a single output, which is obtained by using the weighted average defuzzification (linear or constant output membership functions). Moreover, it cannot accept all the customization options that basic fuzzy inference allows. That is, own membership functions and defuzzification functions cannot be used in this system.

The above concept is used for developing two systems, where composition and process parameters are used as inputs and yield strength is used as output. The database is used to train the Sugeno type FIS developed on the basis of some if-then rules relating the composition and process parameters with the hardness of the microalloyed DP steel. A comparative study between the prediction of the FIS itself and the prediction of the FIS after the Neuro-Adaptive learning is done.

4.2. ANFIS 1: Effect of Cold Deformation (Rolling) on Ageing

To develop this model, the hardness data obtained from the alloy with composition (wt%) 0.04C–1.69Mn–0.57Si–1.64Cu–0.03Ti–0.001B has been used. In the first case the relation between input variables i.e., cold deformation, ageing temperature, ageing time with hardness value has been considered to develop the model. Structure of the ANFIS 1 is shown in Fig. 9.

The five rules formulated to define the relations are given below:

1. If (cold-working is low) and (ageing-temp is low) and (ageing-time is low) then (hardness is low)
2. If (cold-working is medium) and (ageing-temp is low) and (ageing-time is low) then (hardness is low-medium)
3. If (cold-working is low) and (ageing-temp is high) and (ageing-time is medium) then (hardness is medium)
4. If (cold-working is low) and (ageing-temp is high) and (ageing-time is high) then (hardness is high-medium)
5. If (cold-working is high) and (ageing-temp is medium) and (ageing-time is low) then (hardness is high)

Error obtained to predict the hardness value from FIS thus developed before training is found 48.7 VHN and after learning the system error is reduced to 22.6 VHN. To improve the performance of the ANFIS, another attempt has been made using seven rules to define the relations between input and output variables. These rules are

1. If (cold-working is low) then (hardness is very-low)
2. If (cold-working is high) then (hardness is medium)
3. If (cold-working is low) and (ageing-temp is low) and (ageing-time is low) then (hardness is low)
4. If (cold-working is low) and (ageing-temp is high) and (ageing-time is medium) then (hardness is low-medium)
5. If (cold-working is high) and (ageing-temp is medium) and (ageing-time is low) then (hardness is very-high)
6. If (cold-working is high) and (ageing-temp is medium) and (ageing-time is medium) then (hardness is high)

![Fig. 9. Structure of the ANFIS 1 with five rules.](image-url)
7. If (ageing-temp is high) and (ageing-time is medium) then (hardness is high-medium)

Error obtained in this case before learning is 18.3 VHN and after learning it is 12.3 VHN. The surfaces generated from the aforesaid rules are shown in Fig. 10.

4.3. ANFIS 2: Relation of all the Composition and Processing Variables with Hardness

In this case role of alloying elements e.g. Ti, B, Cu and the process variables like amount of cold deformation, ageing temperature, ageing time with the hardness values of the alloy have been modeled. The Fig. 11 shows the ANFIS 2 structure. The five rules developed to describe the complication of the highly nonlinear relations are given below.

1. If (titanium is not added) and (boron is not added) and (copper is not added) and (cooling rate is low) and (cold deformation is low) and (ageing temperature is low) and (ageing time is low) then (hardness is low)

2. If (titanium is added) and (boron is added) and (copper is added) and (cooling rate is low) and (cold deformation is low) and (ageing temperature is low) and (ageing time is low) then (hardness is low-medium)

3. If (titanium is added) and (boron is added) and (copper is added) and (cooling rate is high) and (cold deformation is medium) and (ageing temperature is high) and (ageing time is high) then (hardness is medium)

4. If (titanium is added) and (boron is added) and (copper is added) and (cooling rate is high) and (cold deformation is low) and (ageing temperature is medium) and (ageing time is medium) then (hardness is high-medium)

5. If (titanium is added) and (boron is added) and (copper is added) and (cooling rate is high) and (cold deformation is high) and (ageing temperature is low) and (ageing time is low) then (hardness is high)

The error found before training is 66.8 VHN while the error is reduced to 27.0 VHN after learning. The surfaces generated from the model to view the relationship between hardness and the different input parameters are given in Fig. 12.

5. Discussion

It is to be noted that neural network acts a black box and learns whatever data are presented to it. The learning process in this particular architecture does not truly rest on the physical metallurgical reasoning. A number of interactions of various types are operative within the system, viz. titanium and boron synergistically retard the austenite (\(\gamma\))→ferrite (\(\alpha\)) transformation. Unless such expert knowledge is also fed to the system, it cannot learn the real relationship between the inputs and outputs. In spite of this, the network is seen to have learnt satisfactorily in most of the cases except for a few instances where a fair prediction could not be achieved. It is too difficult to map the effect of a single input amidst other variables kept at constant values. It is also difficult to explain the effect of changing a single input parameter in this type of steel, since there are other parameters, which may respond to this change in a complicated manner. Still the predictions made by the network regarding these variations are found to be in conformity with the metallurgical understanding. From the above observations, it may be stated that if learning is affected by imparting some more knowledge from physical metallurgy of HSLA steel, the ANN predictions may be further improved.

To avoid the above limitation, ANFIS modeling is attempted in this work, where the prior knowledge from the metallurgical point of view regarding the system in question could be incorporated through fuzzy rules. The average error value of prediction by ANFIS 1 is found to be more or less acceptable, when the mechanism of Cu precipitation as

![Fig. 11. Structure of ANFIS 2 with five rules.](image)

![Fig. 12. Surface views of the relationship between hardness and (a) ageing time–copper, (b) cold deformation–copper.](image)
a result of ageing with or without cold deformation as found be quite difficult to express through some if-then rules. It seems that the error is decreased with increase in the number of rules. As in ANFIS the number of if-then rules has to be exactly equal to the number of Fuzzy Sets of the input and each of the rules assigned to one of the sets, the disadvantage remains to divide the output range to a large no of sets. Still in the prediction, error is found to be quite acceptable in both the data sets having the hardness value of the steels.

When all the variables were used to model the kinetics of ageing the complication and the non-linearity of the relationship between the independent and dependent variables acted as a barrier to formulate the if-then rules. From the previous experience,\(^{19}\) it can easily be stated that the increase in number of rules could have improved the predictability of the ANFIS. But it has found to be difficult to frame such rules from such a complicated system. The surface views generated through different ANFIS’s are found to comply with the general understanding of materials science as well as the observation made during experimental studies in most cases.

As mentioned earlier, improvement in the predictability of an ANFIS can be made through increase in the number of rules. So the need to gather thorough understanding of the system to be modeled is necessary for successful design of a Neuro-Fuzzy system. On the other hand, dependence on if-then rules \(i.e\). prior understanding of the system act as a limitation for developing such models with large number of input variables. In the present exercise this problem has been found during modeling with all the variables (ANFIS 2).

If we compare the effect of the input parameters on the hardness property as modeled by the two different types of modeling techniques we find that in both cases they are almost comparable. The hardness value as a result of cold working is seen to reach a plateau after initial significant increase in the ANN model. On the other hand the effect is almost similar, as depicted by the surface view, in case of ANFIS models (compare Fig. 7 with Figs. 10(a) and 12 (b)). It is better to mention here that in this case though the graph styles are different we can still compare the graphs generated from ANN with that of ANFIS if we consider the axis of any one input and the axis of output in the ANFIS graphs. In case of the surface views shown by the ANFIS, there is a definite influence of the if-then rules designed on the basis of prior knowledge. So similarity of the nature of curves as predicted by ANN as well as ANFIS clearly show that ANN have the ability to assimilate the inherent natures of the parameters to a certain extent without any knowledge supplied to the system. But this feature is definitely depend-
ed on the suitability of the network architecture as well as the sanctity of the training data. In our case, it is interesting to note that this phenomenon has been observed in most cases. In case of the effect of ageing temperature on hardness both the models show a sharp rise initially, followed by a softening effect, which stabilizes with further increase in temperature (compare Fig. 8 with Fig. 10(b)). The nature of variation of hardness with ageing time is seen to be identical in both cases (compare Fig. 8 with Fig. 10(b)), and it is seen that it does not have any significant effect on the hardness of the material.

6. Conclusions

(1) Artificial neural network is found to be a useful tool for modeling the effect of copper and other alloying additions along with process parameters on the hardness of microalloyed DP steels.

(2) The committee of models for prediction is found to be effective in handling the problem of mapping the input-output relation in these steels.

(3) The increase in the number of rules improves the predictability of the neuro-fuzzy system.

(4) Appropriate design of a neuro-fuzzy system enables to model a complicated system of non-linear input-output relationship.

(5) The predictions made by both the ANN and ANFIS model corroborate with existing knowledge of physical metallurgy, in most cases.

(6) Similarity of both the models points to the fact that suitably designed and trained ANN have the capacity to assimilate the inherent relationship between the variables.

REFERENCES

1) F. B. Pickering: Physical Metallurgy and Design of Steel, Applied Science Pub., London, (1978), 103.
2) M. Takahashi and H. K. D. H. Bhadeshia: Mater. Trans., JIM, 32 (1991), No. 8, 690.
3) J. Y. Koo and G. Thomas: Metall. Trans. A, 8A (1977), 525.
4) S. Kang and H. Kwon: Metall. Trans. A, 21A (1987), 1577.
5) E. J. Czyryca: Development of Low-Carbon, Copper-Strengthened HSLA Steel Plate for Naval Ship Construction, R & D Report DTRC-SME-90/21, David Taylor Research Center, Bethesda, USA (1990).
6) A. Bhattacharya, T. Sakaki and G. J. Weng: Metall. Trans. A, 24A (1993), 301.
7) M. R. Krishnadv, G. J. Sojka, I. Le May, L. McD. Schetky and S. K. Banerji: Proc. Int. Conf. on Recent Developments in Specialty Steels and Hard Materials, ed. by N. R. Comings and J. B. Clark, Pergamon, Oxford, (1983), 195.
8) I. Le May, L. McD. Schetky and M. R. Krishnadv: Proc. Int. Conf. on High Strength Low Alloy Steel, ed. by D. P. Dunne and T. Chandra, University of Wollongong, N. S. W., Australia, (1984), 64.
9) X. P. Shen and R. Prießner: Metall. Trans. A, 21A (1990), 2547.
10) S. K. Ghosh, A. Samanta and P. B. Chattopadhyay: Trans. Indian Inst. Met., 57 (2004), No. 2, 171.
11) N. K. Bose and P. Liang, Neural Network Fundamentals, Tata McGraw Hill, New Delhi, (1996), 407.
12) H. K. D. H. Bhadeshia: ISIJ Int., 39 (1999), 966.
13) H. Fujii, D. J. C. Mackay and H. K. D. H. Bhadeshia: ISIJ Int., 36 (1996), 1773.
14) P. D. Hodgson, L. X. Kong and C. H. J. Davies: Mat. Proc. Tech., 87 (1999), 131.
15) S. Datta and M. K. Banerjee: ISIJ Int., 44 (2004), 846.
16) S. Datta and M. K. Banerjee: Scand. J. Metall., 33 (2004), 310.
17) S. Datta and M. K. Banerjee: Mater. Sci. Eng. A, in press.
18) O. P. Femminella, M. J. Stariak, M. Brown, I. Sinclair, C. J. Harris and P. A. S. Reed: ISIJ Int., 39 (1999), 1027.
19) S. Datta and M. K. Banerjee: Mater. Manuf. Process, in press.
20) I. Le May and L. McD. Schetky: Copper in Iron and Steel, A Wiley-Interscience Pub., John Wiley & Sons, New York, (1982), 23.
21) ASM Metals Hand Book, 1, 10/E: Properties and Selection: Iron, Steels and High Performance Alloys, ASM International, Metals Park, OH, (1995), 408.
22) W. C. Leslie: The Physical Metallurgy of Steels, McGraw-Hill Int. Book Co., Tokyo, (1982), 269.
23) M. Sugeno: Fuzzy Measures and Fuzzy Integrals: A Survey, ed. by M. M. Gupta, G. N. Sardis and B. R. Gaines, Fuzzy Automata and Decision Processes, North Holland, NY, (1977).