Big Bang nucleosynthesis and cosmic microwave background constraints on the time variation of the Higgs vacuum expectation value

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We derive constraints on the time variation of the Higgs vacuum expectation value $\langle \phi \rangle$ through the effects on Big Bang nucleosynthesis (BBN) and the cosmic microwave background (CMB). In the former case, we include the (previously-neglected) effect of the change in the deuteron binding energy, which alters both the $^4$He and deuterium abundances significantly. We find that the current BBN limits on the relative change in $\langle \phi \rangle$ are $-(0.6 - 0.7) \times 10^{-2} < \Delta \langle \phi \rangle / \langle \phi \rangle < (1.5 - 2.0) \times 10^{-2}$, where the exact limits depend on the model we choose for the dependence of the deuteron binding energy on $\langle \phi \rangle$. The limits from the current CMB data are much weaker.

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I. INTRODUCTION

Physicists have long speculated that the fundamental constants might not, in fact, be constant, but might instead vary with time [1]. Among the various possibilities the interaction coupling constants have received the greatest attention (for a recent review, see Ref. [2]). A great deal of attention has been focused on the fine structure constant $\alpha$, for which a time variation has been claimed in observations of quasar absorption lines [3]. Cosmological limits on a time variation in $\alpha$ can be derived from both big-bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) [4, 5, 6, 8, 9, 10]. Comparatively less interest has been inspired by possible spatial variation of the Fermi coupling $G_F$. In this paper, we derive constraints on the time variation of $G_F$, as discussed in Sec. II.

However, improved observational limits on the primordial element abundances now allow us to place stronger limits than those derived, for example in Ref. [13]. Furthermore, these previous studies ignored the effect of changing the Higgs vacuum expectation value, which we have incorporated into our calculations.

The effects of a variation in $\langle \phi \rangle$ on the CMB have been investigated previously in Ref. [14], but this paper made no attempt to derive actual limits based on observational CMB data, which we will do here.

In this paper, we derive constraints on the time variation of $\langle \phi \rangle$ from both BBN and CMB observations. In this regard, the present work is most similar in spirit to that of Avelino et al. [8], who did a comparable calculation for $\alpha$. As noted in Ref. [14], the effect on the CMB of changing the Higgs vacuum expectation value is similar, but not quite identical, to the effect of changing $\alpha$. The effects on BBN, on the other hand, are quite different. We assume that $\langle \phi \rangle$ has the same value at the era of primordial nucleosynthesis $T \sim 10^{10} - 10^9$ K as it has at recombination $T \sim 10^3$ K, but our results can easily be generalized to cases where this is not so.

In the next section, we discuss the effects of $\langle \phi \rangle$ variation on BBN. In Sec. III, we present the effect on recombination and calculate the CMB temperature anisotropy. Finally, we discuss our results and conclusions in Sec. IV. We take $\hbar = c = 1$ throughout.

II. EFFECTS ON BIG BANG NUCLEOSYNTHESIS

A time variation in $\langle \phi \rangle$ affects Big Bang nucleosynthesis in several ways. The variation of $\langle \phi \rangle$ changes $G_F$, as given in equation (1), which alters the weak $n \leftrightarrow p$ interaction rates. Changing $\langle \phi \rangle$ also alters the fermion masses as

$$m_F \propto \langle \phi \rangle.$$  (2)
Consequently, the electron and the quark mass changes. The change in the electron mass alters the \( n \leftrightarrow p \) rates (see below) and affects the evolution of the electron-positron energy density during the epoch of nucleosynthesis. Due to the changes in the \( u \) and \( d \) quark masses, the neutron-proton mass difference changes, and the pion mass varies. The latter affects nucleosynthesis through the change in the binding energy of the deuteron. We have incorporated all of these effects into our calculations.

Consider first the neutron-proton mass difference, \( Q \equiv m_n - m_p \). Following Ichikawa and Kawasaki [5], we take

\[
Q = -0.76 \text{ MeV} + 2.053 \text{ MeV} \frac{\langle \phi \rangle}{\langle \phi \rangle_0},
\]

where \( \langle \phi \rangle_0 \) is the present Higgs vacuum expectation value. This change in \( Q \) alters the ratio of neutron to proton abundances in thermal equilibrium:

\[
\frac{n_n}{n_p} = e^{-Q/kT}.
\]

The effect on the \( ^4\text{He} \) abundance due to the change in \( Q \) (alone) is illustrated in Fig. 1. An increase in \( \langle \phi \rangle \) leads to an increase in \( Q \), giving a smaller equilibrium neutron-proton ratio, which produces a smaller \( ^4\text{He} \) abundance.

The weak interactions which interconvert neutrons and protons are affected by both the change in \( G_F \) and \( m_e \). These interactions are

\[
\begin{align*}
n + \nu_e & \leftrightarrow p + e^-, \\
n + e^+ & \leftrightarrow p + \bar{\nu}_e, \\
n & \leftrightarrow p + e^- + \bar{\nu}_e.
\end{align*}
\]

The total \( n \rightarrow p \) rate is

\[
\lambda(n \rightarrow p) = \frac{4}{\pi^2} n_n G_F^2 \int_{m_e}^{\infty} dE_e \frac{E_e|p_e|}{1 + \exp(E_e/kT)} \left\{ \frac{(E_e + Q)^2}{1 + \exp[-(E_e + Q)/kT]} + \frac{(E_e - Q)^2 \exp(E_e/kT)}{1 + \exp[(E_e - Q)/kT]} \right\}
\]

where the subscripts \( e \) and \( \nu \) denote the quantities associated with the electron and neutrino, respectively. A similar expression can be derived for the \( p \rightarrow n \) rate. (Note that in the actual BBN calculation, the weak rates are scaled off of the neutron lifetime; we have simply rescaled these rates using equation 6).

Note that these rates depend on both \( G_F \) and \( m_e \), both of which are altered by changing \( \langle \phi \rangle \). There is a further small change in BBN produced by the effect on the expansion rate of the total \( e^+e^- \) energy density, which depends on \( m_e \). We have included this effect, although it is small. In Fig. 1, we show the effect on the \( ^4\text{He} \) abundance of changing \( m_e \) and \( G_F \) separately. Note that most of the change from \( m_e \) is due to the change in the weak rates, as we have noted, rather than from the change in the expansion rate. An increase in \( \langle \phi \rangle \) results in a decrease in \( G_F \), leading to earlier freeze-out of the \( n \leftrightarrow p \) reactions, producing more \( ^4\text{He} \). Similarly, an increase in \( \langle \phi \rangle \) results in an increase in \( m_e \), decreasing the \( n \leftrightarrow p \) reaction rates and also producing more \( ^4\text{He} \).

Finally, changing \( \langle \phi \rangle \) alters the deuteron binding energy through the change in the pion mass [11].

\[
m_\pi^2 \propto m_u + m_d \propto \langle \phi \rangle.
\]

Early calculations suggested a linear dependence of \( B_D \) (the deuteron binding energy) on \( m_\pi \), and in previous discussions of BBN with a time-varying \( \langle \phi \rangle \), it was argued that this effect would be negligible and could be ignored [11, 13] (although the effect of changing \( B_D \) was explored in a slightly different context in Ref. 15). Although more recent calculations [16, 17] have not produced a definitive result for the dependence of \( B_D \) on \( m_\pi \), they are in qualitative agreement over our limited range of interest (i.e., very small changes in \( m_\pi \)). In this regime, \( B_D \) is a decreasing function of \( m_\pi \) (the opposite of what was previously assumed [11]). The calculations in both [16] and [17] have very large uncertainties, but within our narrow range of interest, we can safely approximate the change in \( B_D \) as a linear dependence on \( m_\pi \), through

\[
\frac{B_D}{B_{D0}} = (r + 1) - r \frac{m_\pi}{m_{\pi0}}.
\]
where the 0 subscript denotes the values of these quantities at present, and the coefficient \( r \) in the fitting formula is to be derived from Refs. [16, 17]. We estimate the central values for \( r \) from these references to be \( r \approx 6 \) [16] and \( r \approx 10 \) [17]. Although these values for \( r \) are quite different, we will see that they lead to similar constraints on a change in \( \langle \phi \rangle \). On the other hand, our constraint differs sharply from what is derived by ignoring the change in \( B_D \).

The effect on the helium abundance of changing \( B_D \) is shown in Fig. 1 (for \( r = 6 \)). It is clear that this effect is not negligible. Further, (and unlike the other effects we have considered) changing \( B_D \) has a significant effect on the primordial deuterium abundance. This is most evident from our graphs of the allowed region for the primordial deuterium abundance. This is most evident from our graphs of the allowed region for \( \phi \) (see Figs. 2−4 below). An increase in \( \langle \phi \rangle \) leads to an increase in \( m_\pi \), resulting in a decrease in \( B_D \). This leads to a smaller equilibrium deuterium abundance. Thus, the production of \( ^4\text{He} \) begins later, leading to a smaller \( ^4\text{He} \) abundance, as shown in Fig. 1. Somewhat paradoxically, a decrease in the deuteron binding energy leads to an increase in the final deuterium abundance; nucleosynthesis at lower temperatures faces a larger Coulomb barrier, leaving more deuterium behind.

To derive BBN limits on a change in \( \langle \phi \rangle \), we assume two free parameters, the baryon to photon ratio \( \eta \), and \( \langle \phi \rangle / \langle \phi \rangle_0 \). We vary these parameters within the range \( \eta = 10^{-10} − 10^{-9} \) and \( \langle \phi \rangle / \langle \phi \rangle_0 = 0.9 − 1.1 \).

To derive our constraints, we consider only the abundances of deuterium and \( ^4\text{He} \). Recent observations [18] suggest the limits

\[
(D/H) = 3.0^{+1.0}_{-0.5} \times 10^{-5}, \tag{9}
\]

on the ratio of deuterium to hydrogen, and

\[
Y_p = 0.238 \pm 0.005, \tag{10}
\]

on the primordial mass fraction of \( ^4\text{He} \). We ignore the somewhat more uncertain limits on \( ^7\text{Li} \) in determining our constraints, although we have verified that the allowed regions we derive from BBN also produce an acceptable \( ^7\text{Li} \) abundance.

Our limits on shown in Figs. 2−4 where we have translated our constraints on \( \eta \) into limits on \( \Omega_B h^2 \). In Fig. 2 we show (for reference) the effect of neglecting the deuteron binding energy, while Figs. 3 and 4 correspond to our two choices for \( r \). The solid lines give the constraints from the deuterium and \( ^4\text{He} \) abundances, with the shaded area indicating the region producing an acceptable abundance of both.

The importance of including the effect of the deuteron binding energy is obvious from comparing Fig. 2 to Figs. 3−4. In Fig. 2 the regions of allowed deuterium and allowed \( ^4\text{He} \) are almost parallel to the axes, indicating that the deuterium abundance is nearly independent of changes in \( \langle \phi \rangle \); in this approximation, the abundance of deuterium sets the bounds on \( \Omega_B h^2 \), with the \( ^4\text{He} \) abundance then determining the limits on \( \langle \phi \rangle \). In Figs. 3−4 we see a strong dependence of the deuterium abundance on \( \langle \phi \rangle \) (through the deuteron binding energy), while the allowed band for \( ^4\text{He} \) is much narrower. On the other hand, Fig. 3 and Fig. 4 are reasonably similar, allowing us to put useful constraints on the time variation in \( \langle \phi \rangle \) despite the uncertainty in the dependence of \( B_D \) on \( \langle \phi \rangle \).

Including the change in the deuteron binding energy, we find that the BBN constraint on \( \langle \phi \rangle \) variation is ex-
tremely tight. Defining \( \Delta \langle \phi \rangle \equiv \langle \phi \rangle - \langle \phi \rangle_0 \), we get
\[
-0.7 \times 10^{-2} < \Delta \langle \phi \rangle / \langle \phi \rangle_0 < 2.0 \times 10^{-2},
\]
for \( r = 6 \), and
\[
-0.6 \times 10^{-2} < \Delta \langle \phi \rangle / \langle \phi \rangle_0 < 1.5 \times 10^{-2},
\]
for \( r = 10 \).

### III. Effects on the Cosmic Microwave Background Radiation

Since the weak interaction has no effect on the recombination process, the only effect of changing \( \langle \phi \rangle \) arises through the change in the electron mass, \( m_e \). We have neglected any change in the mass of the dark matter particle. For axions, we expect no change, while the change in the mass of a supersymmetric dark matter particle will be model-dependent. The change in \( m_e \) alters both the Thomson scattering cross section and the binding energy of hydrogen. The Thomson scattering cross-section is
\[
\sigma_T = \frac{8 \pi \alpha^2}{3} m_e^{-2},
\]
while the binding energy of hydrogen is given by
\[
B = \frac{\alpha^2}{2} m_e.
\]

The ionization fraction, \( x_e \), is determined by the balance between photoionization and recombination. The evolution of \( x_e \) with a variation in \( \langle \phi \rangle \) is given by
\[
-\frac{dx_e}{dt} = \left( \frac{R^n p x_e^2}{\beta'(1 - x_e)} \right) \exp \left( -\frac{B'_1 - B'_2}{kT} \right),
\]
where \( R, \beta, B_n \), and \( C \) represent the recombination coefficient, the ionization coefficient, the binding energy of the \( n_{th} \) hydrogen atomic level, and the Peebles correction factor, respectively. The primed quantities stand for the modified coefficients following \( \langle \phi \rangle \) variation. Finally, the differential optical depth of the CMB photons is determined by
\[
\tau = x_e n_p \sigma_T.
\]

With these modifications, we use CMBFAST to generate the theoretical power spectra of temperature fluctuations. (See Ref. [14] for a more detailed discussion). Fig. 5 shows the effects of \( \langle \phi \rangle \) variation on the CMB power spectrum. An increase in \( \langle \phi \rangle \) leads to a decrease in the Thomson scattering cross section and an increase in the hydrogen binding energy. The latter effect is the most important, since it shifts the last-scattering surface to a smaller comoving length scale, moving the curve in Fig. 5 to the right.

Using RADPACK, we perform a \( \chi^2 \) analysis for the most recent CMB data: VSA, BOOMERANG, DASI, MAXIMA, and CBI. Assuming a flat universe \( \Omega_B + \Omega_{CDM} + \Omega_{\Lambda} = 1 \), we set the model parameter space \( 0.5 \leq h_0 \leq 0.8 \) and \( 0.3 \leq \Omega_M \leq 0.4 \) (where \( \Omega_M \equiv \Omega_B + \Omega_{CDM} \)), and we take the spectral index \( n_s \) to lie in the range \( 0.7 \leq n_s \leq 1.3 \). The power spectra are obtained by a modified CMBFAST with \( \langle \phi \rangle \) variation. We then marginalize over \( \langle \phi \rangle \) and \( \Omega_{th} h^2 \).

The CMB constraints are shown in Figs. 4-5 (identical in all three figures). The dotted and the dashed lines represent the 95% and the 68% confidence level regions.
of the CMB experiments, respectively. In general, the CMB calculations provide less stringent constraints than BBN; almost the entire region allowed by BBN lies inside the 95% confidence region from the CMB. The small area allowed by BBN and excluded by the CMB represents a tighter bound on $\Omega_B h^2$, but it does not alter the limits on $\langle \phi \rangle$.

IV. DISCUSSION

This work provides new constraints on the possible time-variation of $\langle \phi \rangle$ from both BBN and the CMB. We find that the BBN limits are significantly more stringent than those that can be derived from the CMB. More specifically, we get

$$-(0.6-0.7) \times 10^{-2} < \Delta \langle \phi \rangle/\langle \phi \rangle < (1.5-2.0) \times 10^{-2} , \quad (17)$$

with the exact numbers depending on the assumptions made about the dependence of the deuteron binding energy on $m_e$. These limits are considerably more stringent than those in, e.g., Ref. [12], due to better observational limits on the primordial element abundances, as well as the inclusion of the (previously-neglected) effect of $\langle \phi \rangle$ on the deuteron binding energy.

In contrast, the limits from the CMB are much less stringent. This conclusion is similar to the results obtained for the time-variation in $\alpha$ obtained by Avelino et al. (Note also that the effect of a time-variation in $\alpha$ and a time variation in $\langle \phi \rangle$ are almost completely degenerate [14], but BBN breaks this degeneracy). The chief advantages of considering the CMB limits are that the calculation is much more straightforward (dependent only on the change in the electron mass), and that the CMB limits should improve sharply in the near future as more observational data comes in. Of course, if $\langle \phi \rangle$ varied between the epoch of BBN ($T \sim 10^{10} - 10^{11}$ K) and the epoch of recombination ($T \sim 10^3$ K), then the limits we have derived could be applied separately to constrain $\Delta \langle \phi \rangle/\langle \phi \rangle$ at each epoch.

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