Paired fractional quantum Hall states and the $\nu = 5/2$ puzzle

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Work on the problem of the $\nu = 5/2$ fractional quantum Hall state is reviewed, with emphasis on recent progress concerning paired states.

I. INTRODUCTION

Recently, there has been a resurgence of interest in the quantized Hall state at $\nu = 5/2$ (and $7/2$) in single-layer samples. This has been a long-standing puzzle, because it violates the “odd-denominator rule” satisfied by other observed quantized Hall states. Moreover, the history of the problem exhibits some remarkable twists and turns.

In this paper, I review both early and recent work on this problem, with emphasis on paired quantum Hall states. There are now many definite theoretical predictions, and a clear need for more experimental results. This paper is in three main sections. Section II reviews the history of experimental and theoretical work designed to reveal the nature of the $\nu = 5/2$ state, with the conclusion that it is theoretically expected to be the Moore-Read state. Section III reviews my recent work (with D. Green) which sheds light on the properties of this state and other proposals. Section IV suggests experiments which could pin down the nature of the observed state. Section V is the conclusion.

II. QUANTIZED STATE AT 5/2

A. Experimental discovery

In 1987, Willett and co-workers published results that showed clear evidence of a quantized Hall plateau forming at low temperatures at a filling factor of $5/2$ \[\frac{5}{2}\]. Subsequently, it was shown \[\frac{5}{2}\] that when a parallel component $B_{||}$ of the magnetic field is applied, the dip in $\rho_{xx}$ decreases and disappears at a critical value. Beyond this point, there is no quantized Hall state. This was supported by activation energy gaps from $\rho_{xx}$ \[\frac{5}{2}\]. The exactness of the quantization of $\rho_{xy}$ at low temperature, and the smallness of the activation gaps, has been confirmed in later work, in particular Ref. \[\frac{5}{2}\]. The $5/2$ state was the first even-denominator quantized Hall state observed, and $5/2$ and $7/2$ remain the only ones observed in single-layer samples.

In the early days, it was widely believed that reversed spins were involved in the $\nu = 5/2$ quantized Hall state. To understand this point, we note that we will assume throughout that the lowest (Landau level index $n = 0$) Landau level (LL) is filled with electrons of both spins.

The remainder of the filling factor $\nu = 2 + 1/2$ is made up of electrons in the first excited ($n = 1$) LL. These electrons in the topmost, partially-filled or “valence”, LL may be either fully-polarized and aligned with the Zeeman effect of the magnetic field, or unpolarized, half of them with spin up, half with spin down, or somewhere in between. We will refer to these possibilities as “polarized”, “unpolarized”, and “partially-polarized”, respectively, even though the lowest LL is unpolarized in all cases. (An identical discussion applies to the $7/2$ case, in which the lowest LL is filled with both spins and the $n = 1$ LL is half-filled with holes. This is expected to exhibit similar physics, due to particle-hole symmetry within a LL. Since this expectation appears to hold up experimentally and theoretically, we neglect $7/2$ hereafter.) The main reasons for believing that the ground state was either unpolarized or partially-polarized were (i) the $5/2$ state occurred at relatively small magnetic fields (around $5$ T) where the Zeeman term is relatively small compared with Coulomb interactions, and (ii) the parallel magnetic field increases the Zeeman term, and in partially- or unpolarized states, this could eventually cause a transition to a polarized ground state, which might be an unquantized state, and such a transition was observed \[\frac{5}{2}\]. Also, (iii) many types of not-fully-polarized ground states would have charged excitations with reversed spins relative to the ground state, such that the energy gap would decrease in a characteristic way involving the $g$ factor with $B_{||}$; this agreed approximately with observation \[\frac{5}{2}\]. However, none of these arguments is irrefutable, and we will see in a moment that there are alternative possible explanations of the observations.

The quantized state at $5/2$ may be contrasted with the vicinity of half-filling of other LLs. At $\nu = 1/2$ and $3/2$, an unquantized, compressible, Fermi-liquid-like state is observed \[\frac{5}{2}\]. On the other hand, at half-filling of higher LLs, $\nu = 9/2, 11/2, \ldots$, a highly anisotropic, unquantized, compressible state has been observed \[\frac{5}{2}\]. Such “stripe” states for half-filling LLs of high index were predicted theoretically in advance \[\frac{5}{2}\]. The effect of $B_{||}$ on $\nu = 5/2$ was then reexamined, and it was found that the compressible state at $B_{||}$ greater than the critical value is again an anisotropic or “stripe” state \[\frac{5}{2}\].

While doubts about the unpolarized nature of the $5/2$ state had begun to surface throughout the 90’s, the last results \[\frac{5}{2}\] gave a very strong indication that
the physics of the 5/2 state had not been understood. The parallel field experiments did not directly probe the polarization of the state, yet the transition to a stripe state (and the sensitivity of the spatial orientation of the anisotropy, for various \( \nu \) where it is observed, to the direction of \( B_{||} \)) indicates that the interactions between the electrons are being affected by \( B_{||} \), which is possible because of the finite thickness of the electron wavefunctions in the two-dimensional electron gas. This mechanism is then a plausible alternative to the explanation via the increased Zeeman term, so that the 5/2 state at \( B_{||} = 0 \) might be fully polarized after all.

B. Theory for incompressible states at even denominators

The “odd-denominator” rule, that quantized Hall states are observed only at filling factors with odd denominators, dates from the earliest observations of the fractional quantum Hall effect, and was “explained” by Laughlin’s theory \([12]\) for \( \nu = 1/q \) (\( q \) odd) and its extension to other filling factors by the hierarchy and composite-fermion theories \([13,14]\) (which are essentially equivalent \([15,16]\)); these approaches produce all, and only, odd-denominator fractions, and cannot account for the 5/2 state.

Some early attempts to generalize Laughlin’s results in different directions were put forward by Halperin \([3]\). One idea was that if, for some reason, electrons are bound in pairs, then these pairs are charge 2 bosons (throughout, we give charges in units of that on the electron), and these can in principle form a Laughlin state for bosons. The filling factor for the electrons in such a state is then of the form \( \nu = 4/m \), where \( m \) is even; this gives a sequence of fractions that includes 1/2, 1/4, \ldots. Such a state has Laughlin quasiparticle excitations of charge \( \pm 2/m = \pm \nu/2 \). Such excitations are a common feature of all the paired states we will discuss; note that the quasiparticle charge is fractionalized compared with the usual Laughlin states, which give charge \( \pm \nu \) quasiparticles. Excitations obtained by breaking the electron pairs are here assumed to be very costly in energy. In applying these and the following states to the 5/2 problem, we again assume that the \( n = 0 \) Landau level is filled with both spins, and use the fact that the \( n = 1 \) Landau level can be mapped to the \( n = 0 \) Landau level, so we will describe it as \( \nu = 1/2 \). Note, however, that the interaction Hamiltonian should be that for electrons in the \( n = 1 \) LL (and possibly should include the effect of virtual excitations involving the \( n = 0 \) LL).

After the experiment of Willett \textit{et al.} \([1]\), Haldane and Rezayi (HR) \([15]\) investigated spin-singlet (i.e. unpolarized) states at \( \nu = 1/2 \). They used the pairing idea, but for spin-singlet pairs, which allows two electrons to occupy the same single particle state in a LL. They also used a “hollow core” Hamiltonian, in which the zeroth Haldane pseudopotential \( V_0 \) (which corresponds to the contact interaction of two electrons) is zero, motivated by the reduction of this parameter in the \( n = 1 \) LL. For a hollow core model in which only the first pseudopotential \( V_1 > 0 \) is nonzero, they found a unique exact ground state at \( \nu = 1/2 \), which was argued to be incompressible (similar Hamiltonians and ground states exist for \( \nu = 1/4, 1/6, \ldots \)). The nature of the HR state will be discussed again later.

Subsequently, Moore and Read (MR) \([19]\) pointed out that paired states at \( \nu = 1/2, 1/4, \ldots \), can be interpreted as BCS pairing \([20]\) of composite fermions in zero net magnetic field. In this point of view, the HR state is a spin-singlet d-wave (\( d_{xy} - i d_{yx} \)) pairing state of composite fermions; the spin-singlet nature of the state, which was initially obscure \([18]\), becomes obvious from this point of view \([14]\). Inspired by this picture and by the structure of the HR state, MR constructed another state, a p-wave (\( p_x - i p_y \)) polarized state, which they called the Pfaffian state. Motivated by deep considerations of conformal field theory and its relation to the quantum Hall effect, they argued that the charge 1/2 quasiparticle excitations of the MR state obey nonabelian, rather than ordinary fractional, statistics. We will explain the meaning of this later.

Soon after, Greiter and coworkers \([21]\) considered the MR state, also from the viewpoint of pairing composite fermions, but argued that the statistics are ordinary abelian fractional statistics, based on the Halperin picture of a Laughlin state of charge 2 bosons (the resolution of this issue will be discussed below). They found a three-body Hamiltonian for which the MR state is the unique exact zero-energy eigenstate for the case of charge 1 bosons (instead of electrons) at \( \nu = 1 \) (the generalization to electrons at \( \nu = 1/2 \) in Ref. \([21]\) is incorrect, but was corrected by later authors; for still smaller \( \nu = 1/q \), two-body terms are necessary in addition \([22]\); this Hamiltonian was very useful in later work. Finally, they suggested that the MR state may represent the observed \( \nu = 5/2 \) state, which should therefore be polarized, in conflict with the conventional wisdom at the time.

Numerical work on the \( \nu = 5/2 \) problem absorbed many person-years of labor during the 1990’s. Eventually, results were forthcoming that now strongly indicate that the 5/2 state is expected to be the spin-polarized MR state. The first published work was by Morf \([23]\), who argued that the ground state is spin-polarized, with a large overlap with the MR state in finite size systems. Further, weakening the short range repulsion led to a transition to a compressible state, while strengthening it gave a transition to a Fermi-sea state, like that in the lowest Landau level \([14,24]\). Rezayi and Haldane \([23]\) confirmed Morf's results, using the torus geometry, rather than the sphere, and studied the transitions in detail. In particular, they established that the compressible state
at weak short-range interaction is a stripe state \[25\]. Also, results for realistic potentials, including finite thickness effects, screening by the \(n = 0\) LL, and tilted magnetic field, confirm that at \(B_{||} = 0\), the ground state at \(5/2\) should be the MR state, while \(B_{||}\) drives the system into a stripe state \[23\], as occurs experimentally \[10,11\]. Apparently, the system with \(B_{||} = 0\) lies close enough to the transition to the stripe state that a small change in the interactions due to nonzero \(B_{||}\) (through finite thickness effects) can push it into the stripe phase. From this work, a systematic picture has emerged of how at half-filling of each LL with \(B_{||} = 0\), the ground state evolves successively from Fermi-liquid (for \(n = 0\)), to paired (for \(n = 1\)), to striped (for \(n > 1\)).

There are also Monte Carlo studies that indicate a low energy for the MR state at \(\nu = 5/2\) \[27\]. A recent attempt to show that Cooper pairs of composite fermions form using trial states \[28\] has been criticized \[29\].

### III. BCS PAIRING OF COMPOSITE FERMIONS

#### A. MR state

Now that we have seen strong reasons to believe that the \(\nu = 5/2\) state is the MR state, we are motivated to inquire more deeply into its properties, in search of experimental signatures. In this section, we review recent progress in understanding these properties \[30\].

First, we will simply say that for \(\nu = 1/2\), a composite fermion is an electron bound to two vortices in the wavefunction of the other electrons (see e.g. \[31\]). This object is a fermion, is electrically neutral, and experiences zero effective magnetic field \(B_{\text{eff}}\) — each of these properties holding only at \(\nu = 1/2\). These statements generalize to \(\nu = 1/q\) for electrons bound to \(q\) vortices, \(q\) even; if instead \(q\) is odd, the statements hold, except for the statistics: the object is a composite boson \[32\] \[33\].

Fermions in zero magnetic field can form a ground state represented by a BCS trial wavefunction (formally, we arrive at this by a mean field approximation that yields fermions in zero net magnetic field \[3\], followed by the BCS mean field \[28\] approximation that describes pairing). In the quantum Hall context, such a state is an incompressible fluid that generalizes the Laughlin state, in the following sense (related to that of Ref. \[17\]). The Laughlin state can be viewed as a Bose condensate of composite bosons \[22\] \[23\]. The condensate allows magnetic flux (more accurately, vortices) to be inserted only in quantized amounts that cost a nonzero, finite energy (the Meissner effect for the condensate); the quantum Hall relation between flux and charge implies that these excitations carry a charge \(\pm 1/q\), and since they cost finite energy, the state is incompressible. In the case of the paired states at \(\nu = 1/q\), the Cooper pair condensate carries twice the electric charge, which halves the flux quantum; the fluid is again incompressible, but the elementary charged excitations carry charge \(\pm 1/2q\) \[19\].

The wavefunction written down by MR for one possible spin-polarized state with filling factor \(1/q\) \((q\ even)\) was

\[
\Psi_{\text{MR}}(z_1, \ldots, z_N) = \text{Pf} \left( \frac{1}{z_i - z_j} \right) \prod_{i<j}(z_i - z_j)^{\eta},
\]

where we have used complex coordinates \(z_j = x_j + iy_j\) for the \(N\) electrons \((N\ even)\), omitted the ubiquitous Gaussian factor, and the Pfaffian \(\text{Pf}\) is defined by

\[
\text{Pf} (M_{ij}) = \mathcal{A}(M_{12}M_{34} \ldots M_{N-1,N}),
\]

where \(M_{ij}\) are the elements of an antisymmetric matrix, and \(\mathcal{A}\) denotes the operation of antisymmetrization, normalized such that each distinct term appears once with coefficient 1. As we will see, the Pfaffian is the general structure of the position-space form of the BCS state in the spin-polarized, p-wave case, so the wavefunction \(\Psi_{\text{MR}}\) represents BCS pairing of composite fermions \[13\].

Pairing composite fermions leads us to expect two types of elementary excitations of this ground state. One type are the charged vortices discussed above, with charge \(1/2q\), which according to MR are supposed to obey nonabelian statistics. The other type are the analog of the BCS quasiparticles, which are fermions, and are created (in twos) by breaking pairs; there should be an energy gap for these. These excitations are charge zero, like the underlying composite fermions (since \(\nu = 1/q\)).

To make further progress, we consider (following Ref. \[30\]) BCS theory at the mean field level for p-wave pairing of spinless or spin-polarized fermions \[36\]. We are not interested here in the mechanism for an attractive interaction between composite fermions that gives the pairing, nor in solving the gap equation. Rather we are interested in the physical properties of the resulting ground states, especially those related to the fermionic quasiparticles. At the mean field level, one works with the following effective Hamiltonian for the quasiparticles:

\[
K_{\text{eff}} = \sum_k \left[ \xi_k c_k^\dagger c_k + \frac{1}{2} \left( \Delta_k c_{-k}^\dagger c_k + \Delta_{-k} c_k^\dagger c_{-k}^\dagger \right) \right],
\]

where \(\xi_k = \varepsilon_k - \mu\) and \(\varepsilon_k\) is the single-particle kinetic energy and \(\Delta_k\) is the gap function. For the usual fermion problems, \(\mu\) is the chemical potential, but may not have this meaning in the quantum Hall applications. At small \(k\), we assume \(\varepsilon_k \simeq k^2/2m^*\) where \(m^*\) is an effective mass, and so \(-\mu\) is simply the small \(k\) limit of \(\xi_k\). For complex p-wave pairing, we take \(\Delta_k\) to be an eigenfunction of rotations in \(k\) of eigenvalue (two-dimensional angular momentum) \(l = -1\), and thus at small \(k\) it generically takes the form

\[
\Delta_k \simeq \hat{\Delta}(k_x - ik_y),
\]
where $\hat{\Delta}$ is a constant. For large $k$, $\Delta_k$ will go to zero. The $c_k$ obey \{\$c_k, c_k^\dagger\} = \delta_{kk'}$; for the moment we work in a square box of side $L$.

The diagonalization of this Hamiltonian is a standard exercise. The quasiparticle dispersion relation is

$$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2} \approx \sqrt{\mu^2 + |\Delta|^2 k^2}$$

(5)

as $k, \mu \to 0$. The corresponding ground state wavefunction in position space for $N$ particles is

$$\Psi(r_1, \ldots, r_N) = \text{Pf} \left( g(r_i - r_j) \right),$$

(6)

where $g(r)$ is the inverse Fourier transform of $(\xi_k - E_k)/\Delta_k$.

A transition occurs at $\mu = 0$, where $E_k$ becomes gapless at $k = 0$. The large $r$ behavior of $g$ is different on the two sides of, and at, this transition. For $\mu > 0$, $g(r) \sim 1/z$ as $r \to \infty$; for $\mu = 0$, $\sim 1/(z|z|)$; and for $\mu < 0$, $\sim e^{-\text{const} \cdot |z|}/z$. Because of the long-range behavior of $g$, we call $\mu > 0$ the weak-pairing phase, while we call $\mu < 0$, where the pairs are tightly bound, the strong-pairing phase. Intermediate behavior is found at the transition, $\mu = 0$. We see that the weak-pairing regime has the same asymptotic behavior as the MR state has for all $r$; this $\mu > 0$ case also corresponds to weak attractive coupling. We will argue that this phase generally has the properties associated with the nonabelian statistics of the MR state. The strong-pairing regime corresponds to very strong attractive coupling, which can produce $\mu < 0$, and we will see that the physics there is that of the simple Halperin picture of a Laughlin state of charge 2 bosons.

Next we consider the fermion spectrum in nontranslationally invariant situations, specifically edges and vortices. We allow either $c_k$ or $\Delta_k$ to depend on position. Again, in principle the form of both of these should be found by solving the mean-field equations self-consistently, but we do not do this here; the results we are interested in are generic throughout a phase, and do not change unless a transition is crossed, so self-consistency should not matter. We work close to the transition, where $\mu$ is small, and assume the position dependence is in each case slowly-varying, so that the small $k$ behavior is sufficient. The problem of finding the fermion spectrum is simply the solution of the Bogoliubov-de Gennes equation \[ \hat{\Delta} \] , and in this limit it reduces to a Dirac equation, with $|\Delta|$ as the speed of light \[ \Delta \]. It has reality properties that imply that the quasiparticles are their own antiparticles, and Dirac fermions with this property are known as Majorana fermions.

For the case of an edge, we let $\mu$ depend on $x$, but not $y$ (for an edge parallel to the $y$-axis); $\mu$ becomes large and negative outside the edge. This corresponds to a large, positive potential for electrons outside the edge, which confines them to the interior. Thus outside the system, we would be in the strong-pairing phase, but with the particle density going to zero far outside. If the interior is in the weak-pairing phase, $\mu > 0$, then the edge is effectively a domain wall that separates regions in either phase. It is now described by a Dirac equation with a mass $\mu$ that changes sign. It can be shown that there is a gapless low-energy spectrum of chiral Majorana fermion modes that are bound to, and propagate in one direction along, the domain wall \[ \hat{\Delta} \], whereas the remainder of the spectrum, associated with bulk states, has an energy gap. On the other hand, when the interior is in the strong-pairing phase, there is no such domain wall at the edge, and no gapless chiral modes are present. The chiral Majorana fermions on the edge in the weak-pairing phase agree with the results obtained earlier for the three-body Hamiltonian \[ \hat{\Delta} \], for which the MR state is the exact ground state \[ \hat{\Delta} \]. In addition, the fractional quantum Hall state has gapless chiral density excitations, the usual “edge states”, which are not obtained from the fermion analysis.

Vortices may be thought of as small circular edges, enclosing a half quantum of magnetic flux. When the bulk is in the weak-pairing phase, a similar calculation shows that there is a Majorana zero-energy state associated with each vortex (vortices should occur in even numbers if the boundary conditions at infinity are the same as for no vortices). When the separation of the vortices is finite, these energies are split by amounts that go to zero exponentially fast as the separations go to infinity. Neglecting these splittings, the many-particle states have a degeneracy $2^n$ for $2n$ vortices, or $2^n-1$ if we restrict to a fixed particle number (either even or odd). This number arises because only $n$ creation-annihilation operator pairs can be formed from the $2n$ real (Majorana) fermion operators. This asymptotic degeneracy of many-particle states agrees with that obtained \[ \hat{\Delta} \] for any separation of vortices using the three-body Hamiltonian. Because the number is $2^n$, not $2^{n-1}$, it cannot be viewed as a two-fold degeneracy of each vortex, but instead is somehow nonlocal; the states are shared among the vortices. It is these facts that give rise to nonabelian statistics. In the strong-pairing phase, there are no such zero modes of the BdG equation, and the multivortex states are nondegenerate for fixed positions, even asymptotically.

To summarize, we have found that the weak-pairing phase has properties previously associated with the MR state, while the strong-pairing phase has no such properties. The strong-pairing phase is then left with only the properties it inherits from the condensation of pairs, and is entirely consistent with the physics that follows from Halperin’s picture of a Laughlin state of charge 2 bosons \[^7\]. Nonabelian statistics of the vortices (or quasiparticles) of the MR or weak-pairing phase may now be explained.
When all separations are large, a set of $2n$ quasiparticles has $2^n$ degenerate ground states. When two of them are exchanged adiabatically, the effect can be described as a matrix operation on the space of possible states (this is derived simply in Ref. [41]), rather than just multiplication by a phase as in the more familiar abelian "fractional" statistics. Matrices for different exchanges will not commute, hence the name nonabelian. We note that the order of limits (infinite separation before the limit of low-speed exchange) is important here. In practice, for finite separations, the exchange must not be done too slowly, compared with the exponentially-small splittings. It has been suggested in some quarters that these exchanges could be used to perform computations in a quantum computer.

We will also comment here very briefly on the effects of disorder on the MR state. In the quantum Hall effect, potential disorder can nucleate and localize vortices (quasiparticles): since the latter have finite energy, they can be localized in essentially uncorrelated positions, with some mean density depending on the disorder strength and on the distance in magnetic field (filling factor) from the center of the quantized Hall plateau. In the case of the MR phase, each vortex carries a fermion zero mode, and as we have noted, the degeneracies of many-particle states can be split by tunneling of the fermions from one vortex to another. For a finite density of vortices, there will then be a band of localized low-energy fermion excitations, like an “impurity band”. It has been argued [42] that this has the effect of destroying the properties of the MR phase and replacing it by a disordered version of the strong-pairing or Halperin phase. In particular, the chiral Majorana fermion edge modes will be destroyed, by backscattering and localization via the nearby vortex zero modes in the bulk. However, for a high-quality sample, all of this may be occurring at extremely low energies and large length scales: thus in finite systems at finite temperature, the only effect may be that there is a bath of the quasidegenerate fermion states on the localized vortices. There could be a lot of interesting physics associated with this.

### B. HR state

We will consider here briefly the fate of the HR state. The original HR state had (spinor-valued) wavefunction

$$\Psi_{HR} = \text{Pf} \left( \frac{1}{(z_i - z_j)^2} \right) \prod_{i<j} (z_i - z_j)^q, \quad (7)$$

(where $\uparrow$ means the spin state $\uparrow$ for particle $i$, and the product is the tensor product) which corresponds to spin-singlet (as in the original BCS theory [20]) complex d-wave pairing, and the corresponding gap function would have the form

$$\Delta_k \simeq \hat{\Delta}(k_x - ik_y)^2 \quad (8)$$

for small $k$. We may perform a similar analysis of BCS mean field theory in this case, and there are again weak- and strong-pairing phases separated by transition. However, the behavior of the pairing function $g$ at the transition is $g \sim 1/z^2$ (times the spin singlet factor), that is, the same as in the HR state. This, together with an analysis of the ground states on the torus, suggested that the hollow-core Hamiltonian, and the HR wavefunction, are sitting right at the weak–strong-pairing transition point, and hence the fermionic quasiparticle spectrum in the bulk should be gapless, $E_k \sim k^2$. Since this would be reached in practice by tuning a parameter, it cannot be the generic behavior of a phase—even if the 5/2 state is spin unpolarized. Then earlier results on the edge and quasiparticle properties of the HR state [39,22] are moot. Instead, there is a weak-pairing phase which has abelian statistics, and is equivalent to states obtained in several earlier approaches [50], and also a strong-pairing phase.

### IV. EXPERIMENTS NEEDED

Several difficult experiments, which have been done successfully for some states in the lowest LL, would pin down the nature of the 5/2 state if they could be done. These are (i) measurement of the spin polarization by Knight shift, to see if the valence Landau level is polarized; (ii) shot noise or antidot experiments, to measure the fractional charge of excitations, which should be 1/4 in a paired state at $\nu = 5/2$; (iii) tunneling into the edge, to measure the exponent in the current-voltage relation $I \sim V^\alpha$, which (neglecting the lowest-Landau-level contribution with $\alpha = 1$) should be $\alpha = 3$ in the weak-, but 8 in the strong-pairing phase [33]. Together, positive results for these experiments would show that the state is a spin-polarized weak-pairing phase, which must almost certainly be the MR phase. However, the smallness of the gaps in the 5/2 state [44] make all of these extremely difficult. Of course, other suggestions for ways to probe the physics of the MR state would be welcome.

### V. CONCLUSION

To conclude, there is now plenty of theoretical evidence that the 5/2 state is the MR phase. The latter has a great deal of fascinating physics, including chiral fermion edge excitations, nonabelian statistics due to fermion zero modes on vortices, and resulting effects of disorder. Clearly, more experiments are needed to finally solve the puzzles posed by $\nu = 5/2$, the most surprising of fractional quantum Hall states.
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[1] R.L. Willett, et al., Phys. Rev. Lett. 59, 1779 (1987).
[2] J.P. Eisenstein et al., Phys. Rev. Lett. 61, 997 (1988).
[3] J.P. Eisenstein et al., Surface Sci. 229, 31 (1990).
[4] W. Pan et al., Phys. Rev. Lett. 83, 3530 (1999).
[5] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
[6] M.P. Lilly et al., Phys. Rev. Lett. 82, 394 (1999).
[7] J.P. Eisenstein et al., Phys. Rev. Lett. 61, 997 (1988).
[8] A.A. Koulakov, M.M. Fogler, and B.I. Shklovskii, Phys. Rev. Lett. 76, 499 (1996); M.M. Fogler, A.A. Koulakov, and B.I. Shklovskii, Phys. Rev. B 54, 1853 (1996).
[9] R. Moessner and J.T. Chalker, Phys. Rev. B 54, 5006 (1996).
[10] W. Pan et al., Phys. Rev. Lett. 83, 820 (1999).
[11] M.P. Lilly et al., Phys. Rev. Lett. 83, 824 (1999).
[12] R.B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[13] F.D.M. Haldane, Phys. Rev. Lett. 51, 605 (1983); B.I. Halperin, Phys. Rev. Lett. 52, 1583 (1984).
[14] J.K. Jain, Phys. Rev. Lett. 63, 199 (1989); Phys. Rev. B 40, 8079 (1989); ibid. 41, 7653 (1990).
[15] N. Read, Phys. Rev. Lett. 65, 1502 (1990).
[16] B. Blok and X.-G. Wen, Phys. Rev. B 42, 8133, 8145 (1990); ibid. 43, 8337 (1991).
[17] B.I. Halperin, Helv. Phys. Acta, 56, 75 (1983).
[18] F.D.M. Haldane and E.H. Rezayi, Phys. Rev. Lett. 60, 956, 1886 (E) (1988).
[19] G. Moore and N. Read, Nucl. Phys. B360, 362 (1991).
[20] J. Bardeen, L.N. Cooper, and J.R. Schrieffer, Phys. Rev. 106, 162 (1957); 108, 1175 (1957).
[21] M. Greiter, X.-G. Wen and F. Wilczek, Phys. Rev. Lett. 66, 3205 (1991); Nucl. Phys. B374, 567 (1992).
[22] N. Read and E. Rezayi, Phys. Rev. B 54, 16864 (1996).
[23] R. Morf, Phys. Rev. Lett. 80, 1505 (1998).
[24] E. Rezayi and N. Read, Phys. Rev. Lett. 72, 900 (1994); ibid. 73, 1052 (C) (1994).
[25] E.H. Rezayi and F.D.M. Haldane, Phys. Rev. Lett. 84, 4685 (2000).
[26] E.H. Rezayi, F.D.M. Haldane, and K. Yang, Phys. Rev. Lett. 83, 1219 (1999).
[27] K. Park, V. Melik-Alaverdian, N.E. Bonesteel, and J.K. Jain, Phys. Rev. B 58, 10167 (1998).
[28] V.W. Scarola, K. Park, and J.K. Jain, Nature 406, 863 (2000).
[29] N. Read, cond-mat/0010071.
[30] N. Read and D. Green, Phys. Rev. B 61, 10267 (2000).
[31] N. Read, Semicond. Sci. Technol. 9, 1859 (1994) = cond-mat/9501094.
[32] S.M. Girvin, in The Quantum Hall Effect, edited by R.E. Prange and S.M. Girvin (Second Edition, Springer-Verlag, New York, 1990).
[33] S.M. Girvin and A.H. MacDonald, Phys. Rev. Lett. 58, 1252 (1987).
[34] N. Read, Bull. Am. Phys. Soc. 32, 923 (1987); Phys. Rev. Lett. 62, 86 (1989).