Non-minimal coupling Gauss-Bonnet holographic superconductors

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By employing the gauge/gravity duality, we disclose the effects of the non-minimal coupling between the scalar and Maxwell fields on the holographic superconductors in the probe limit. As the background spacetime, we consider a five dimensional Gauss-Bonnet (GB) black hole with a flat horizon. We find out that the critical temperature decreases for larger values of GB coupling constant, $\alpha$, or smaller values of non-minimal coupling constant, $\lambda$, which means that the condensation is harder to form. In addition, we study the electrical conductivity in the holographic setup. We show that at low frequency regime ($\omega \to 0$), Kramers-Kronig relation connects both parts of conductivity to each other, while at high frequency ($\omega \to \infty$), two parts go up with constant slope. Furthermore, we observe that the gap frequency shifts to larger values for stronger $\alpha$, and becomes flat by increasing $\lambda$.

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I. INTRODUCTION

Condensed matter systems provide an area to test the concepts of high energy theory experimentally. Within this field, there are many strongly coupled systems, which are of significant technological interest but challenging to observe [1]. In addition, there is no analytic way to compute the transport coefficients, for such condensed matter or fluid systems directly. One of transport coefficients of interest is the superconductivity coefficient [1]. Superconductivity is a phase of matter that have no electrical resistance at low temperatures and since its discovery, many researches have been done to understand its physics [2]. The BCS theory proposed by Bardeen, Cooper and Schrieffer is almost an acceptable microscopic theory which addresses the superconductivity as a microscopic effect originates from condensation of Cooper pairs into a boson-like state through the exchange of phonons [3]. However, decoupling of Cooper pairs at high temperatures is one of defects of this theory. The Anti-de Sitter/Conformal Field Theory duality known as AdS/CFT correspondence may consider as an effective approach to describe strong coupling systems through corresponding the strong coupling conformal field theory living on the boundary in $d$-dimensions to a weak coupling gravity in $(d + 1)$-dimensional spacetime in the bulk [4]. One of the fascinating consequence of the holographic approach is that instead of enumerating Boltzmann partition sums for the thermal problem, we can solve a simple black hole problem in the bulk. Within this approach, standard thermodynamical quantities such as the free energy, thermal phase diagrams and the entropy may be calculated [1]. In addition, the frequency-dependent conductivity, which is one of the important observable characterizing the properties of condensed matter systems, can be obtained in this approach [1]. A very instructive example of a quantum phase transition within gauge/gravity duality is obtained by using a magnetic field as the control parameter [5–10].

The holographic duality provides description of thermal properties of matter, Fermi liquids, superfluids and superconductors. An important characteristic of holographic models at finite charge density is that there is a critical temperature below which a new ground state with lower free energy forms. This new ground state corresponds to a condensate [9]. To introduce the temperature in the holographic quantum field theory and to introduce a phase transition, we need to find a black hole that has hair only at low temperatures, and it has no hair at high temperatures [11]. On the other hand, Landau-Fermi liquid theory is a well-understood approach for describing fermions in weakly coupled systems. These systems posess a Fermi surface which contains essential information about the physical properties of strongly coupled systems. AdS/CFT duality provides a way to calculate spectral functions and identifying Fermi surfaces for strongly coupled systems [12]. With time, lots of works have been done in the context of holographic condensed matter. For instance in 2008, Hartnoll and et al. presented the idea of holographic superconductors [13]. Many properties of high and unconventional condensed matter systems such as free energy, entropy, phase transition, electrical, thermal and thermoelectrical conductivities and etc are obtained by applying gauge/gravity duality [14–17]. In spite of intense researches in this subject (see e.g. [18–57]), there are still many open

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problems in strongly coupled systems that each of them can help us to understand the physics of strongly coupled condensed matter systems. Studying the effects of non-minimal coupling on holographic condensed matter systems by considering different forms of coupling is one of this problems. Nowadays, scalarized black holes solutions have been attracted much attention (see e.g. [58-61]). Since the non-minimal couplings can provide an effective mass for the scalar field and lead to the spontaneous scalarization that can be interpreted as the holography phase transition. In [62, 65], Einstein-Maxwell-scalar (EMS) model with a non minimal coupling between the scalar and Maxwell fields were studied.

In this work, we study the effects of non-minimal coupling between scalar and Maxwell fields in GB holographic superconductors. By analyzing equations of motion numerically, we shall investigate the influence of the GB coupling \( \alpha \) and the non-minimal coefficient \( \lambda \) on the critical temperature \( T_c \) and phase transition of the system. In addition, by applying a suitable electromagnetic perturbation on the black hole background, we explore the behavior of real and imaginary parts of conductivity for different values of \( \alpha \) and \( \lambda \).

This paper is organized as follows. In section II, we study holographic setup when scalar and Maxwell fields couple to each other non-minimally in the background of GB gravity. In section III, we analyze the electrical conductivity in holographic context. Finally, in section IV, we summarize our results.

II. THE HOLOGRAPHIC MODEL

In order to study the condensation of the scalar field \( \psi \) with mass \( m \) and charge \( q \) in the background of the AdS black holes with a non-minimal coupling between the Maxwell and scalar fields in the presence of GB gravity, we consider the action as [62, 63, 65]

\[
S = \int d^5x \sqrt{-g} [\mathcal{L}_G + \mathcal{L}_m],
\]

\[
\mathcal{L}_G = R - 2\Lambda + \frac{\alpha}{2} \left[ R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} \right],
\]

\[
\mathcal{L}_m = -\frac{h(\psi)}{4} F_{\mu\nu}F^{\mu\nu} - |\nabla\psi - iqA\psi|^2 - m^2|\psi|^2,
\]

(1)

where \( g, R, R_{\mu\nu} \) and \( R_{\mu\nu\rho\sigma} \) are metric determinant, Ricci scalar, Ricci tensor and Riemann curvature tensor, respectively. \( \Lambda \) represents the negative cosmological constant and equals to \(-6/l^2\) in five dimension with \( l \) as the radius of the AdS spacetime [64]. Here, we choose \( l = 1 \). \( \alpha \) is the GB parameter and in \( \alpha \to 0 \) limit, \( \mathcal{L}_G \) in Eq. (1) turns to Einstein case. In the above action, \( h(\psi) = 1 + \lambda\psi^2 \) represents the non-minimal coupling function with \( \lambda \geq 0 \). We choose this form of \( h(\psi) \) because it gives us more information about wide range of non-minimal coupling constant \( \lambda \) and temperature [65]. By considering \( A_\mu \) as the vector potential, the strength of the Maxwell field is defined by \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \).

As the background spacetime, we consider a five dimensional GB black hole which its geometry is given by the line elements [63]

\[
ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2 + dz^2),
\]

(2)

\[
f(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - 4\alpha \left( 1 - \frac{1}{r^4} \right) } \right],
\]

(3)

where the function \( f(r) \) has the asymptotic behavior \( (r \to \infty) \) as

\[
f(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - 4\alpha} \right].
\]

(4)

We can present the effective radius \( L_{\text{eff}} \) for the AdS spacetime as [22]

\[
L_{\text{eff}}^2 = \frac{2\alpha}{1 - \sqrt{1 - 4\alpha}}.
\]

(5)

Varying action (1) with respect to the scalar field \( \psi \) and the gauge field \( A_\mu \), by choosing \( \psi = \psi(r) \) and \( A_\mu dx^\mu = \phi(r)dt \), yields to equations of motion

\[
\psi''(r) + \left[ \frac{f'(r)}{f(r)} + \frac{3}{r} \right] \psi'(r) + \left[ -\frac{m^2}{f(r)} + \frac{q^2\phi(r)^2}{f(r)^2} + \frac{\lambda\phi'(r)^2}{2f(r)} \right] \psi(r) = 0,
\]

(6)
In this section, we take $\psi(r) = \frac{\psi_+}{r^{\Delta_+}} + \frac{\psi_-}{r^{\Delta_-}}$, where the Breitenlohner-Freedman (BF) bound is given by
\[ m^2 \geq -4, \quad m^2 = m^2 L_{\text{eff}}^2. \] (10)
In this work, we take $m^2 = -3/L_{\text{eff}}^2$. Since we are looking for spontaneous symmetry breaking, we set $\psi_- = 0$ because it plays the role of source. $\psi_+$ is known as expectation value of the order parameter $\langle O_+ \rangle$. Without loss of generality, we set $q = 1$. Shooting method is a practical approach to solve equations of motion numerically. With the help of this method, we find the relation between critical temperature $T_c$ and $\rho^{1/3}$ for different values of $\lambda$ and $\alpha$. The results are listed in table I and show that increasing the effect of GB parameter $\alpha$ makes conductor/superconductor phase transition more difficult. While larger values of $\lambda$ lead to higher critical temperature which is so interesting because of the importance of high temperature superconductors. So, this approach can provide a good theoretical way to explore the trend of these matters at higher temperatures. In addition, the behavior of condensation as a function of temperature for various effects of $\lambda$ and $\alpha$ are shown in figure 1. Larger values of condensation for stronger effect of $\alpha$ and weaker effect of $\lambda$ are pointed the difficulty of phase transition. In five dimension, Hawking temperature $T$ equals to $r_+/\pi$ that $r_+$ defines the horizon location [64]. In this work, for simplicity we fix $r_+ = 1$.

III. CONDUCTIVITY

In this section, we are going to calculate electrical conductivity in holographic setup through turning on an appropriate electromagnetic perturbation as $\delta A_x = A_x e^{-i\omega t}$ on the black hole background which is dual to the boundary electrical current. Turning on this component yields to
\[ A_x''(r) + \left[ \frac{1}{r} + \frac{f'(r)}{f(r)} + \frac{2\lambda \psi(r) \psi'(r)}{f(r)^2 (1 + \lambda \psi(r)^2)} \right] A_x'(r) + \left[ \frac{\omega^2}{f(r)^2} - \frac{2q^2 \psi(r)^2}{f(r)(1 + \lambda \psi(r)^2)} \right] A_x(r) = 0, \] (11)
which behaves asymptotically as
\[ A_x''(r) + \frac{3}{r} A_x'(r) + \frac{\omega^2 L_{\text{eff}}^4}{r^4} A_x(r) = 0, \] (12)
by considering $A^{(0)}$ and $A^{(1)}$ as constant parameters, the asymptotic solution of $A_x$ is
\[ A_x = A^{(0)} + \frac{A^{(1)}}{r^2} + \frac{\omega^2 L_{\text{eff}}^4 \log(\Lambda r)}{2r^2} A^{(0)} + \cdots. \] (13)
FIG. 1: The behavior of the condensation as a function of temperature.

Thus, the electrical conductivity based on holographic approach has the following form

$$\sigma_{xx} = \frac{2A^{(1)}}{i\omega L_{\text{eff}}^2 A^{(0)}} + \frac{i\omega T_{\text{eff}}^2}{2}, \quad (14)$$

By applying the ingoing wave boundary condition, the behavior of conductivity as a function of frequency are shown in figures 2-7. Based on these figures, at low frequency regime $\omega \to 0$, the real and imaginary parts of conductivity are connected to each other with Kramers-Kronig relation by showing a delta behaviour and pole respectively. On the other hand, when $\omega \to \infty$ both parts increase with constant slope which is proportional to $\omega$. Furthermore, superconducting gap is occurred at temperatures below the critical value and becomes sharper at lower temperatures which shows difficulty of conductor/superconductor phase transition. To illustrate the effects of $\alpha$ and $\lambda$ on the gap frequency, we plot the trend of real and imaginary parts at fixed temperature $T = 0.3T_c$ in figures 4-7. It is seen that $\omega_g$ shifts to larger values for stronger effect of GB parameter which proves the results of previous section. In addition, we face with flatter gap for larger $\lambda$ values. Based on the BCS theory $\omega_g \approx 3.5T_c$, but in holographic approach due to strong coupling between the pairs, $\omega_g \approx 8T_c$.

IV. SUMMARY AND DISCUSSION

By applying AdS/CFT duality, we explored the holographic superconductors in the context of GB gravity when the Maxwell and scalar fields couple to each other non-minimally. First, we investigated the effects of different values of $\alpha$ and $\lambda$ on the critical temperature and condensation by solving the equations of motion numerically in the probe limit. We face with lower values of critical temperature $T_c$ and higher values of condensation by increasing $\alpha$ or decreasing $\lambda$ which indicates the difficulty of the conductor/superconductor phase transitions. Next, we analyzed the behavior of real and imaginary parts of conductivity through turning on an appropriate electromagnetic perturbation on black hole back ground. Based on our results, Kramers-Kronig relation connects these two parts at low frequency regime while in $\omega \to \infty$ limit both parts of conductivity go up with a constant slope proportional to $\omega$. By decreasing temperature below the critical value, the gap frequency occurs at about $\omega_g \approx 8T_c$ which shifts to higher frequencies.
by decreasing temperature or increasing the GB coupling constant $\alpha$. Stronger effects of non-minimal coupling term $\lambda$ make this gap flatter.

FIG. 2: The behavior of real parts of conductivity

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FIG. 3: The behavior of imaginary parts of conductivity

FIG. 4: The behavior of real and imaginary parts of conductivity for $\lambda = 0$ in $T/T_c = 0.3$

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FIG. 5: The behavior of real and imaginary parts of conductivity for $\lambda = 10$ in $T/T_c = 0.3$

FIG. 6: The behavior of real and imaginary parts of conductivity for $\alpha = -0.08$ in $T/T_c = 0.3$

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FIG. 7: The behavior of real and imaginary parts of conductivity for $\alpha = 0.08$ in $T/T_c = 0.3$.