We consider tachyon effective theory with Born-Infeld electromagnetic fields and investigate the configurational entropy of the various tachyon kink solutions. We find that the configurational entropy starts at a minimum value and saturates to a maximum value as the negative pressure of pure tachyonic field increases. In particular, when an electric field is turned on and its magnitude is larger than or equal to the critical value, we find the configurational entropy has a global minimum, which is related to the predominant tachyonic states.

I. INTRODUCTION

The investigation of a D-brane with tachyon condensation \[1\] has opened up a new window to explore off-shell structure of string theory. It was found that rolling tachyon solutions through boundary conformal field theory and effective field theory describe the motion of the tachyon on unstable D-branes \[2,14\]. It was extensively studied for the inhomogeneous tachyon condensation \[5-8\], the emission of closed string radiation \[9,10\] and for instability of codimension-one D-branes \[9,11,13\]. In particular, the configurational entropy was investigated for dynamical tachyonic systems \[15-22\] and for instability of a variety of physical states \[14,15\]. It was found that the configurational entropy represents the informational content in physical systems with localized energy density configurations through measure of their ordering in field configuration space \[14,15\]. It was extensively studied for AdS/QCD holographic models \[16,22\] and for instability of a variety of physical systems \[15-17,23-29\]. In particular, the configurational entropy was investigated for dynamical tachyonic AdS/QCD holographic model. The authors showed that the corrections to dual mesonic states in the boundary QCD due to tachyonic fields become more dominant their states \[19\]. Since decay of unstable D-branes has rich tachyon kink solutions, it is intriguing to calculate the configurational entropy of tachyon kink solutions.

The paper is organized as follows: in the next section we will investigate configurational entropy in tachyon effective theory with Born-Infeld electromagnetic fields. Firstly, we will calculate the configurational entropy in the pure tachyon case. Next, we will turn on electric/electromagnetic field. Then the configurational entropy will be computed. We will also discuss their configurational entropy by varying the electromagnetic field. In the last section we will give our conclusion.

II. CONFIGURATIONAL ENTROPY OF TACHYON KINKS

The Boltzmann-Gibbs entropy \( S_{BG} \) is defined as

\[
S_{BG} = -k_B \sum p_i \ln p_i, \tag{2.1}
\]

with \( \sum p_i = 1 \). In fact, it is given as the most general formula between the entropy and the set of probabilities of their microscopic states in statistical thermodynamics. Here, \( k_B \) is the Boltzmann constant, and \( p_i \) the probability of a microstate, respectively. In particular, when each microstate has equal probability as the following

\[
p_i = \frac{1}{W}, \tag{2.2}
\]

with the number of microstates \( W \), \( S_{BG} \) \[2.1\] reduces to the configurational entropy \( S_C \) in the microcanonical ensemble

\[
S_C = k_B \ln W, \tag{2.3}
\]

since \( W \) can be treated as the number of possible configurations at a given energy. For example, there are two different molecules with the total number of molecules \( N_0 \), then the number of one type of molecule is \( N_1 \) and the number of another type of molecule \( N_2 \). One obtains the configurational entropy \( S_C \)

\[
S_C = k_B \ln W = k_B \ln \left( \frac{N_0^!}{N_1^! N_2^!} \right), \tag{2.4}
\]

and after employing Sterling’s approximation \( \ln N! \approx N \ln N \), one has

\[
S_C = k_B (N_0 \ln N_0 - N_1 \ln N_1 - N_2 \ln N_2). \tag{2.5}
\]

As another example, there is the system with spatially localized energy in \( d \)-dimensional space. When its energy density is given as a function of the position \( \rho = \rho(x) \), the energy density is written as

\[
\rho(k) = \left( \frac{1}{\sqrt{2\pi}} \right)^d \int \rho(x) e^{-ik\cdot x} d^d x \tag{2.6}
\]

through the Fourier transform and the modal fraction reads

\[
f(k) = \frac{|F(k)|^2}{\int |F(k)|^2 d^d k}, \tag{2.7}
\]

which measures the relative weight of a given mode \( k \). One defines the configurational entropy \( S_C[f] \) as

\[
S_C[f] = -\sum_{l=1}^n f_l \ln(f_l). \tag{2.8}
\]
and in the limit of \( n \to \infty \), one has \(^{23}^{24}\)

\[
S_C[f] = - \int_{-\infty}^{\infty} g(k) \ln[g(k)] dk,
\]

where \( g(k) = f(k)/f(k)_{\text{max}} \) and the maximum modal fraction \( f(k)_{\text{max}} \).

One introduces a runaway tachyon potential\(^1\)

\[
V(T) = \frac{1}{\cosh(T/T_0)},
\]

with \( T_0 = \sqrt{2} \) for the non-BPS D-brane in the superstring and \( T_0 = 2 \) for the bosonic string. The effective tachyon action for the unstable D3-brane system with the tension of the D3-brane \( T_3 \) is given by \(^{11}^{13}\)

\[
S = -T_3 \int d^3xV(T)\sqrt{-X},
\]

with

\[
X = \det(\eta_{\mu\nu} + \partial_\mu T\partial_\nu T + F_{\mu\nu})
\]

which leads to equations of motion for the gauge field \( A_\mu \) and for the tachyon \( T \)

\[
\partial_\mu \left( \frac{V(T)}{\sqrt{-X}} C^{\mu\nu}_A \right) = 0,
\]

\[
\partial_\mu \left( \frac{V(T)}{\sqrt{-X}} C^{\mu\nu}_S \partial_\nu T \right) + \sqrt{-X} \frac{dV(T)}{dT} = 0.
\]

Here \( C^{\mu\nu}_A \) and \( C^{\mu\nu}_S \) are asymmetric and symmetric part of the cofactor,

\[
C^{\mu\nu}_A = \bar{\eta}(\bar{F}^{\mu\nu} + \bar{\eta}^{\mu\alpha} \bar{\eta}^{\beta\gamma} \bar{F}^{*}_{\alpha\beta} \bar{F}^{*}_{\gamma\delta} \bar{F}^{\delta\nu}),
\]

\[
C^{\mu\nu}_S = \bar{\eta}(\bar{F}^{\mu\nu} + \bar{\eta}^{\mu\alpha} \bar{\eta}^{\beta\gamma} \bar{\eta}^{\delta\nu} \bar{F}^{*}_{\alpha\beta} \bar{F}^{*}_{\gamma\delta}),
\]

with determinant of barred metric \( \bar{\eta} \)

\[
\bar{\eta} = -(1 + \partial_\mu T\partial^\mu T),
\]

and inverse metric \( \bar{\eta}^{\mu\nu} \)

\[
\bar{\eta}^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^{\mu} T \partial^{\nu} T}{1 + \partial_\mu T\partial^\mu T}.
\]

Here, contravariant barred field strength tensor \( \bar{F}^{\mu\nu} \) denotes

\[
\bar{F}^{\mu\nu} = \bar{\eta}^{\mu\alpha} \bar{\eta}^{\beta\nu} \bar{F}_{\alpha\beta},
\]

and barred field strength tensor \( \bar{F}_{\mu\nu} \)

\[
\bar{F}_{\mu\nu} = F_{\mu\nu},
\]

and its dual field strength \( \bar{F}^{*}_{\mu\nu} \)

\[
\bar{F}^{*}_{\mu\nu} = \frac{1}{2} \bar{\epsilon}_{\mu\nu\alpha\beta} \bar{F}^{\alpha\gamma} \bar{F}^{\beta\delta} \bar{F}_{\gamma\delta}.
\]

Energy-momentum tensor \( T_{\mu\nu} \) is obtained as \(^{11}^{12}\)

\[
T^{\mu\nu} = T_3 V(T) \sqrt{-X} C^{\mu\nu}_S.
\]

One considers the tachyon field \( T(x) \) as the function of the spatial coordinate \( x \), electric field \( E(x) \) and the magnetic field \( B(x) \). Then through solving Eq. \(^{2.13}\), one obtains that

\[
\tau \equiv T_3 V(T) \sqrt{-X},
\]

with \( \tau = \text{constant} \), which leads to a single first-order equation

\[
\mathcal{E} = \frac{1}{2} T'^2 + U(T),
\]

with

\[
\mathcal{E} = -\kappa/(2\lambda)
\]

and

\[
U(T) = -(T_3 V(T))^2/(2\lambda\tau^2).
\]

Here \( \kappa \) and \( \lambda \) are

\[
\kappa = 1 - \frac{E^2 + B^2 - (E \cdot B)^2}{2},
\]

\[
\lambda = 1 + B_1^2 - E_2^2.
\]

A. Tachyon kink case

When the tachyon field and the abelian gauge field are given as the function of the position \( T(x) \) and \( F_{\mu\nu}(x) \), profiles of tachyon field \( T(x) \) on general unstable Dp-brane are the same to that on unstable D2-brane \(^{12}\). For simplicity, we consider from now on tachyon effective theory with Born-Infeld electromagnetic fields in three-dimensional spacetime and investigate the configurational entropy of the various tachyon kink solutions.

In the case of pure tachyon model, the Born-Infeld type effective action \(^{9,11}\) reduces to the following action\(^2\)

\[
S = -T_2 \int d^3x V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T\partial_\nu T)}, (2.29)
\]

\[\text{where } \Lambda_5 \text{ is the five-dimensional bulk cosmological constant and } \kappa_5 \text{ is the five-dimensional gravitational coupling constant.}\]
which leads to the energy density $\rho$ \cite{11,12}:

$$\rho \equiv T_{00} = \frac{-T_2^2/p_1}{1 + [(T_2/p_1)^2 - 1] \sin^2(x/T_0)}, \quad (2.30)$$

and the negative pressure $p_1$

$$p_1 \equiv T_{11} = -T_2^2 \frac{V(T)}{\sqrt{1 + T^2}} < 0, \quad (2.31)$$

The mechanical energy $E$ \cite{22.25} is given as $E = -1/2$ and the potential $U(T)$ \cite{22.26} is $U(T) = -(T_2 V(T))^2/2$. Then, solutions are classified by $-p_1/T_2$ and there are four possible cases: (i) When $-p_1/T_2 > 1$, there is no motion. (ii) When $-p_1/T_2 = 1$, there is hypothetical motion but the motion eternally stops at $U(0)$. (iii) When $-p_1/T_2 < 1$, there is oscillatory motion. (iv) When $-p_1/T_2 \rightarrow 0^+$, there is oscillatory motion with infinity period. Thus, we now calculate the configurational entropy of pure tachyonic field under $0 < -p_1/T_2 < 1$.

The potential $U(T)$ \cite{22.26} is explicitly obtained as

$$U(T) = -\frac{T_2^2}{2p_1} \text{sech}^2 \left( \frac{T}{T_0} \right). \quad (2.32)$$

After taking $T_2 = 1$, shapes of potential $U(T)$ for various values of the pressure $p_1$ are depicted in Fig. 1. The smaller the pressure $-p_1$ becomes, the more convex function in potential $U(T)$. Then, profiles of tachyon filed $T(x)$ for various the pressure $-p_1$ and profiles of energy density are depicted in Fig. 2 and Fig. 3.

which is adapted to a warped five-dimensional line element with an induced three-dimensional brane in a spatially flat cosmological background

$$ds^2 = e^{2f(\xi)}[-dt^2 + e^{2H_1}(dx^2 + dy^2 + dz^2) + d\xi^2],$$

with the warp factor $f(\xi)$ and the scale factor of the brane $e^{H_1}$, where $H_1$ is integration constant, and one can get the following field equation \cite{30}:

$$f'^2 + \frac{\kappa_5^2 A_5}{6} e^{2f} - H^2 = -\frac{\kappa_5^2}{6} \frac{e^{2f} V(T)}{\sqrt{1 + e^{-2f} T^2}}.$$ 

The five-dimensional gravitational coupling constant $\kappa_5$ is set to $\sqrt{6}$ and the above field equation in the absence of the negative bulk cosmological constant $\Lambda_5$ and the warp factor $f(\xi)$ reduces to the negative pressure \cite{22.31} $p_1 = -\frac{V(T)}{\sqrt{1 + T^2}}$ with taking the tension of the D2-brane $\gamma_2 = 1$. Then the more general potential discussed in footnote 1, reduces exactly to the runaway tachyon potential \cite{2.10} $V(T) = \frac{1}{\cos(T/T_0)^2}$.

The configurational entropy of tachyon filed $S_{C,T}$ is numerically calculated by Eqs. (2.6), (2.7), and (2.9), and is depicted in Fig. 4. As the negative pressure $-p_1$
grows up, $S_{C,T}$ starts at the minimum value ($S_{C,T} = 0.5441$) and saturates to the maximum value ($S_{C,T} = 1.7373$).

The momentum space plane waves with equally distributed modal fractions are sharply localized in position space while the momentum space singular modes broadly spread out. Position space localized distributions have the maximum configurational entropy due to large amount of momentum modes while position space widespread distributions have the minimum configurational entropy due to small amount of momentum modes. Furthermore, the larger configurational entropy of physical system becomes, the larger its amount of energy to be generated. Thus in position space, the energy density with more sharply localized shape needs a larger amount of energy to be generated. On the contrary, the more energy density $\rho(x)$ in the presence of pure tachyonic fields sharply localized, the smaller the pressure $-p_1$ becomes while the more energy density $\rho(x)$ broadly spreads out, the bigger the pressure $-p_1$ becomes as shown in Fig. 3. As shown in Fig. 4, however, the above behaviour of configurational entropy by increasing the pressure $-p_1$ seems to be natural physically since the tachyon energy density with more widespread shape needs a larger amount of energy to be generated. In other words, this result is consistent with the fact that the configurational entropy grows up as the energy increases.

![Fig. 4. Plot of tachyon configurational entropy $S_{C,T}$ as a function of the pressure $-p_1$.](image)

\[ \Pi = \frac{T_2}{2\pi^2} \frac{V(T)}{\sqrt{-X}} E, \quad (2.34) \]

where $E$ denotes the electric field. The mechanical energy \((2.25)\) is given as

\[ \mathcal{E}_E = \frac{1}{2} (1 - E^2), \quad (2.35) \]

and the potential \((2.20)\) is

\[ U_E(T) = -\frac{T_2}{2\pi^2} \frac{E^2}{V(T)^2} \]
\[ = -\frac{T_2 E^2}{2\pi^2} \frac{1}{\cosh^2(T/T_0)}. \quad (2.36) \]

Then, solutions are classified as $E$ or $E$. (i) When $\mathcal{E}_E < U_E(0)$ ($E^2 > 1/[1 + (T_2/\Pi)^2]$), there is no motion. (ii) When $\mathcal{E}_E = U_E(0)$ ($E^2 = 1/[1 + (T_2/\Pi)^2]$), there is hypothetical motion but the motion eternally stops at $U(0)$. (iii) $U_E(0) < \mathcal{E}_E < 0$ ($1/[1 + (T_2/\Pi)^2] < E^2 < 1$), there is oscillatory motion. Thus, we will calculate the configurational entropy $S_{C,E}$ in the range $0 < E < 1$.

After employing $T_2 = 1$, shapes of potential $U_E(T)$ for various values of electric field $E$ are depicted in Fig. 5. The smaller electric field $E$ becomes, the more convex function in potential $U_E(T)$. Then, profiles of tachyon filed $T(x)$ for various electric field $E$ and profiles of energy density are depicted in Fig. 6 and Fig. 7.

**B. Kink with electric field case**

As discussed in the previous section we will apply a similar analysis to the kink with electric field case.

The energy density $\rho$ is given as \[11, 12\]

\[ \rho \equiv T_{00} = \Pi E \]
\[ + \frac{T_2^2 E^2}{\left( \frac{T_2^2 E^2}{\Pi^2(1 - E^2)} - 1 \right) \sin^2(x\sqrt{1 - E^2}/T_0)}, \]
\[ (2.33) \]

and conjugate momenta of the gauge field $\Pi$

\[ \Pi = \frac{T_2}{2\pi^2} \frac{V(T)}{\sqrt{-X}} E, \quad (2.34) \]

after numerically evaluating Eqs. \((2.6), (2.7), \) and \((2.9)\), the configurational entropy $S_{C,E}$ is depicted in Fig. 8. Especially, as electric field $E$ grows up, the configurational entropy reaches the minimum value ($S_{C,E} = 0.3485$) at a critical point ($E = 0.63$). In fact, the smaller configurational entropy of physical system becomes, the more dominant such physical system states since the larger its configurational entropy becomes, the larger its amount of energy to be generated. Thus, it is expected that the predominant tachyonic states happens at the minimum configurational entropy.
C. Kink with electromagnetic field case

In the case of the kink with electromagnetic field, the energy density $\rho$ is given as

$$\rho \equiv T_{00} = T_{2} \frac{V(T)}{\sqrt{-\lambda}}(1 + T^2 + B^2)$$

$$= \frac{\Pi_1(E_1^2 - B^2E_2^2)}{E_1(1 - E_2^2)} + \frac{T_2^2E_1}{\Pi_1(1 - E_2^2)} V(T)^2.$$  (2.37)

The mechanical energy (2.25) is given as

$$\mathcal{E}_{EM} = -\frac{1 - E_1^2 + B^2}{2(1 - E_2^2)},$$  (2.38)

where electric field $E$ is given as

$$E = E_1(x)i + E_2(x)j.$$  (2.39)

Three-dimensional analogue of Faraday’s law is

$$\frac{\partial B}{\partial t} = -\epsilon_{0ij}\partial_i E,$$  (2.40)

which implies magnetic field $B$

$$B = B(x)$$  (2.41)

where $E_i = F_{i0}$, and $B = \epsilon_{0ij}F_{ij}/2$.

The potential (2.26) is

$$U_{EM}(T) = -\frac{T_2E_1^2}{2\Pi_1^2(1 - E_2^2)} V(T)^2$$

$$= -\frac{T_2^2E_1^2}{2\Pi_1^2(1 - E_2^2) \cosh^2(T/T_0)}.$$  (2.42)

When $\lambda > 0$ ($E_2 < 1$), tachyon configurations are exactly same to that in kink with electric field case. However, when $\lambda < 0$ ($E_2 > 1$), the potential $U_{EM}$ is flipped, the property of solutions changes. Thus, we will treat the case in which $\lambda$ is negative.

![Plot of tachyon configurational entropy with electric field $S_{CE}$ as the function of electric field $E$.](image1)

![Plot of potential $U(T)$ as the function of the tachyon field $T$ (red dotted-dashed curve for electric field $E_2 = 1.37$, orange solid curve for electric field $E_2 = 1.52$, blue dashed curve for electric field $E_2 = 1.96$, respectively).](image2)
(i) When $0 < E_{\text{EM}} < U_{\text{EM}}(0)$ ($0 < E^2 + B^2 < (T_2 E_1 / \Pi_1)^2$), there is a turning point at the origin ($x = 0$) as shown in Fig. 10. Then, the charge density is explicitly written as

$$\rho = \frac{\Pi_1 (B^2 E_2^2 - E_1^2)}{E_1 (E_2^2 - 1)} - \frac{T_2^2 E_1}{\Pi_1 (E_2^2 - 1)} \frac{1}{1 + (\chi^2 - 1) \cosh^2(x/\sigma)},$$

with

$$\chi = \frac{T_2^2 E_1^2}{\Pi_1^2 (1 - E^2 + B^2)},$$

and

$$\sigma = T_0 \sqrt{\left| \frac{1 - E_2^2}{1 - E^2 + B^2} \right|}.$$ (2.45)

After taking $T_2 = 1$, shapes of potential $U_{\text{EM}}(T)$ for various values of electric field $E_2$ are depicted in Fig. 9. The smaller electric field $E_2$ becomes, the more concave function in potential $U_{\text{EM}}(T)$. Then, profiles of tachyonic field $T(x)$ for various electric field $E_2$ and energy density are depicted in Fig. 10 and Fig. 11. Then, the configurational entropy $S_{\text{C,EM}}$ is depicted in Fig. 12. As electric field $E_2$ grows up, $S_{\text{C,EM}}$ starts at the minimum value ($S_{\text{C,EM}} = 3.8446$) and saturates to the maximum value ($S_{\text{C,EM}} = 3.8963$).

(ii) When $E_{\text{EM}} = U_{\text{EM}}(0)$ ($1 - E^2 + B^2 = (T_2 E_1 / \Pi_1)^2$, as shown in Fig 13.), there are the trivial ontop solution ($T(x) = 0$) and nontrivial tachyon half-kink solutions as shown in Fig 14. Then, the charge density is given as

$$\rho = \frac{\Pi_1 (B^2 E_2^2 - E_1^2)}{E_1 (E_2^2 - 1)} - \frac{T_2^2 E_1}{\Pi_1 (E_2^2 - 1)} \frac{1}{1 + \exp(2x/\sigma)},$$

After taking $T_2 = 1$, shapes of potential $U_{\text{EM}}(T)$ for various values of electric field $E_2$ are depicted in Fig. 13. The smaller electric field $E_2$ becomes, the more concave function in potential $U_{\text{EM}}(T)$. Then, profiles of tachyon
filed $T(x)$ for various electric field $E_2$ and energy density are depicted in Fig. 14 and Fig. 15. Then, the configurational entropy $S_{C, EM}$ is depicted in Fig. 16. As electric field $E_2$ grows up, $S_{C, EM}$ starts at the minimum value ($S_{C, EM} = 0.0002618$) and saturates to the maximum value ($S_{C, EM} = 3.7719$).

![Plot of the tachyon field $T(x)$ as the function of the position $x$.](image)

$E_2 = 1.04$
$E_2 = 1.19$
$E_2 = 1.49$

**FIG. 14.** Plot of the tachyon field $T(x)$ as the function of the position $x$ (red dotted-dashed curve for electric field $E_2 = 1.04$, orange solid curve for electric field $E_2 = 1.19$, blue dashed curve for electric field $E_2 = 1.49$, respectively).

![Plot of energy density $\rho(x)$ as the function of the position $x$.](image)

$E_2 = 1.04$
$E_2 = 1.19$
$E_2 = 1.49$

**FIG. 15.** Plot of energy density $\rho(x)$ as the function of the position $x$ (red dotted-dashed curve for electric field $E_2 = 1.04$, orange solid curve for electric field $E_2 = 1.19$, blue dashed curve for electric field $E_2 = 1.49$, respectively).

(iii) When $\mathcal{E}_{EM} > U_{EM}(0) (1 - E^2 + B^2 > (T_2 E_1/\Pi_1)^2$, as shown in Fig 17.), there is hybrid of two half-kink solutions joined at the origin, as shown in Fig 18. Then, the charge density is given as

$$\rho = \frac{\Pi_1(B^2 E_2^2 - E_2^2)}{E_1(E_2^2 - 1)} - \frac{T_2 E_1}{\Pi_1(E_2^2 - 1)} \frac{1}{1 + (1 - \chi^2) \sinh^2(x/\sigma)} \cdot (2.47)$$

After taking $T_2 = 1$, shapes of potential $U_{EM}(T)$ for various values of electric field $E_2$ are depicted in Fig. 13. The smaller electric field $E_2$ becomes, the more concave function in potential $U_{EM}(T)$. Then, profiles of tachyon filed $T(x)$ for various electric field $E_2$ and energy density are depicted in Fig. 18 and Fig. 19. Then, the configurational entropy $S_{C, EM}$ is depicted in Fig. 20. As electric field $E_2$ grows up, $S_{C, EM}$ starts at the minimum
value ($S_{C,EM} = 3.8459$) and saturates to the maximum value ($S_{C,EM} = 3.8963$).

III. CONCLUSION

We considered the tachyonic system coupled to Born-Infeld electromagnetism and investigated its configurational entropy. It was found that the configurational entropy saturates to the maximum value as the pressure of pure tachyonic field $-p_1$ grows up. We also showed that the configurational entropy in the presence of electromagnetic fields saturates to the maximum value as electric field $E_2$ increases. Interestingly, when the magnetic field is turned off, the magnitude of electric field $E$ reaches the critical value. Then, the configurational entropy has the global minimum, which is related to the predominant tachyonic states.

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