Isotropization from Color Field Condensate in heavy ion collisions

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based on:
S. Floerchinger and C. Wetterich, *Isotropization from Color Field Condensate in heavy ion collisions*, [JHEP 03 (2014) 121].
ions are strongly Lorentz-contracted

*some* medium is produced after collision

medium expands in longitudinal direction and gets diluted
Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...

- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down

- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more

- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more
The puzzle of thermalization / isotropization

- Hydrodynamic description works well when started at $\tau_0 \approx 0.5$ fm/c.
- Perturbative time-scale for thermalization is much longer [Baier, Mueller, Schiff, Son (2001)].
- Effective hydrodynamic description for some quantities may also be possible without local equilibrium and detailed balance.
- Some quantities e.g. pressure may thermalize faster than others: “Prethermalization” [Berges, Borsanyi, Wetterich (2004)].
- In praxis hydro description does assume early local equilibrium and it works rather well with that.
- There must be some nontrivial mechanism of thermalization / isotropization to be understood.
- Another puzzle is: How does entropy and particle production work?
Could macroscopic / classical fields be the solution?

- Field expectation value or “classical field” has influence on quasi-particle excitations and leads to
  - modified vertices
  - modified dispersion relations / self energies
- That could lead to higher scattering rates and faster thermalization.
- Dynamical evolution of classical fields itself might also contribute to isotropization.
- Classical fields can also induce instabilities / particle production.
Large occupation numbers versus condensate

- In thermodynamic limit (stationary, infinite volume) a classical field corresponds to large occupation number of zero-mode: a condensate.

- For realistic heavy-ion collision one may have
  - non-equilibrium situation
  - finite size
  - finite number of gluons.

- Distinction between condensate and large occupation numbers for a few modes is not so clear.

- Nevertheless, condensate picture may be easiest way to capture important features of situation with large occupation numbers.

- Gluon condensate were also discussed in kinetic theory framework.
  [Blaizot, Gelis, Liao, McLerran, Venugopalan, Epelbaum, Berges, Schlichting, Sexty, Kurkela, Moore,...]
Is a homogeneous and isotropic color field possible?

- Expectation value for vector field \( \langle A_\mu \rangle \) breaks rotation invariance except for \( \mu = 0 \) component.
- But \( A_0 \) is gauge degree of freedom.
- One can choose Weyl or temporal gauge, \( A_0 = 0 \).
- Seems to suggest that homogeneous and isotropic color field is *not* possible.
Modified rotation symmetry

- One can combine rotations with gauge transformations into a modified rotation transformation [Reuter, Wetterich (1994)].
- Group theoretic: embed $SU(2) \in SU(3)$.
- Gauge singlets rotate in the normal way.
- There are two inequivalent embeddings of this type. For one of them Lie algebra of $SU(2)$ spanned by Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$.
- Contains a singlet

$$ (A_j)_{mn} = \sigma \, \epsilon_{jmn} $$

- More general, temporal part $A_0$ transforms like

$$ 8 = 5 + 3, $$

and spatial part $A_j$ like

$$ 24 = 7 + 2 \times 5 + 2 \times 3 + 1. $$
Field configurations with cylindrical symmetry

- There is only one candidate for isotropic condensate $\sigma$, i.e. a singlet under three-dimensional rotations.
- For cylindrical symmetry, i.e. reduced symmetry under
  - rotations in the transverse plane of $x_1, x_2$,
  - rotations of $180^\circ$ around $x_1$ or $x_2$ axis,
  one has two more condensate candidates, $\tilde{\gamma}^A$ and $\tilde{\gamma}^B$.
- For space parity transformations $P(A_0, A_j) = (A_0, -A_j)$ one has
  \[
P \sigma = -\sigma, \quad P \tilde{\gamma}^A = -\tilde{\gamma}^A, \quad P \tilde{\gamma}^B = -\tilde{\gamma}^B.
  \]
- For color charge conjugation $C A_\mu = -A_\mu^*$ one has
  \[
  C \sigma = \sigma, \quad C \tilde{\gamma}^A = -\tilde{\gamma}^A, \quad C \tilde{\gamma}^B = \tilde{\gamma}^B.
  \]
  and accordingly for CP
  \[
  CP \sigma = -\sigma, \quad CP \tilde{\gamma}^A = \tilde{\gamma}^A, \quad CP \tilde{\gamma}^B = -\tilde{\gamma}^B.
  \]
Time evolution of condensates 1

- Time evolution of condensate in general quite complicated due to quantum effects.
- Qualitative guiding from classical Yang-Mills equations.
- For isotropic and homogeneous condensate $\sigma$

$$\partial_t^2 \sigma = -2g^2 \sigma^3.$$ 

Anharmonic oscillator, solution in terms of Jacobi elliptic functions.
Isotropic and cylindric condensates have coupled evolution equations. 
Can be easily solved numerically. 
Isotropic condensate $\sigma$ can be generated from $\tilde{\gamma}^A$, $\tilde{\gamma}^B$. For example:
Energy-momentum tensor due to condensates

- Energy-momentum tensor due to condensates

\[ T^{\mu \nu} = 2 \text{tr} F^{\rho \mu} F_{\rho \nu} - \frac{1}{2} g^{\mu \nu} \text{tr} F^{\alpha \beta} F_{\alpha \beta}. \]

- Assume that energy and momentum are dominated by this.

\[ T^{\mu \nu} = \text{diag}(\epsilon, p_{tr}, p_{tr}, p_l). \]

For same example as above:

- Condensates can contribute to quick isotropization!
CP-even cylindrical condensate

- Initial condition with only $\tilde{\gamma}^A$ is CP symmetric.
- CP-breaking isotropic condensate $\sigma$ not generated.
- Initial energy momentum tensor of the form

$$T^\mu\nu = \text{diag}(\epsilon, p_{tr}, p_{tr}, p_l) = \text{diag}(\epsilon, \epsilon, \epsilon, -\epsilon).$$

- Leads to oscillations between $p_{tr}$ and $p_l$
Longitudinal expansion

- In realistic heavy ion collision the time evolution is modified by several effects, in particular by longitudinal expansion.
- Condensates will be diluted.
- That will probably hinder oscillations.
- Compare here only different scenarios for time evolution to $1/\tau^{1/3}$ dilution.
Consider now excitations of other field modes in the presence of isotropic condensate $\sigma$.

Classify them according to the transformation behavior under modified rotations.

Investigate in particular dispersion relations for excitations in the presence of isotropic condensate $\sigma$. 
Decomposition of gauge field 1

Write spatial and temporal parts of gauge field

\[(A_j)_{mn} = \kappa_{jmn} + \gamma^A_{mk}\epsilon_{kjn} + \gamma^A_{nk}\epsilon_{kmj} + i\gamma^B_{jk}\epsilon_{kmn} + (\beta^A_m + i\beta^B_m)\delta_{jn} + (\beta^A_n - i\beta^B_n)\delta_{jm} - \frac{2}{3}\beta^A_j\delta_{mn} + i\sigma\epsilon_{jmn}\]

\[(A_0)_{mn} = \gamma^C_{mn} + i\beta^C_l\epsilon_{lmn}\]

with

- \(\kappa_{jmn}\) is real, completely symmetric, three-dimensional tensor of rank three, traceless with respect to all contractions,
- \(\gamma^A_{jk}, \gamma^B_{jk}\) and \(\gamma^C_{jk}\) are real, symmetric and traceless three-dimensional tensors,
- \(\beta^A_m, \beta^B_m\) and \(\beta^C_m\) are real, three-dimensional vectors.

In summary

\[24 = 7 + 2 \times 5 + 2 \times 3 + 1,\]
\[8 = 5 + 3.\]
Decomposition of gauge field 2

To analyze dispersion relations it is useful to decompose further

- vectors

\[ \beta_m = \partial_m \beta + \hat{\beta}_m \]

- \( \beta \) is a real scalar,
- \( \hat{\beta}_m \) is a real, divergence-less vector.

- tensors of rank two

\[ \gamma_{mn} = \hat{\gamma}_{mn} + \partial_m \hat{\gamma}_n + \partial_n \hat{\gamma}_m + (\partial_m \partial_n - \frac{1}{3} \delta_{mn} \partial_j^2) \gamma \]

- \( \hat{\gamma}_{mn} \) is real, traceless and divergence-less tensor
- \( \hat{\gamma}_m \) is real, divergence-less vector
- \( \gamma \) is a real scalar.

- and tensors of rank three

\[ \kappa_{jmn} = \hat{\kappa}_{jmn} + \partial_j \hat{\kappa}_{mn} + \partial_m \hat{\kappa}_{jn} + \partial_n \hat{\kappa}_{jm} + \ldots \]
Decomposition of gauge field 3

- Discrete symmetries C and P classify fields further.
- Fields in different representations do not mix on linear level.
- Gauge fixing to Weyl gauge implies $A_0 = 0$ or $\gamma^C_{mn} = \beta^C_m = 0$.
- At this point we are left with
  - C-even scalars $\sigma, \beta^B, \gamma^B$
  - C-odd scalars $\beta^A, \gamma^A, \kappa$
  - C-even vectors $\hat{\beta}^B_m, \hat{\gamma}^B_m$
  - C-odd vectors $\hat{\beta}^A_m, \hat{\gamma}^A_m, \hat{\kappa}_m$
  - C-even rank-two tensors $\hat{\gamma}^B_{mn}$
  - C-odd rank-two tensors $\hat{\gamma}^A_{mn}, \hat{\kappa}_{mn}$
  - C-odd rank-three tensor $\hat{\gamma}_{jmn}$

  which makes 24 real degrees of freedom.
- To reduce to 16 d.o.f. one needs the Gauss constraint.
Variation of action with respect to $A_0$ yields the Gauss constraint

$$\partial_j (E_j)_{mn} - ig(A_j)_{mk}(E_j)_{kn} + ig(E_j)_{mk}(A_j)_{kn} = D_j(E_j)_{mn} = 0.$$ 

Linearize this around constant background $\sigma$ and decomposed further

- tensor constraint

$$\partial_0 \left[ \partial_j \kappa_{jmn} + \epsilon_{kjn} (\partial_j \gamma^A_{mk}) + \epsilon_{kjm} (\partial_j \gamma^A_{nk}) 
+ \partial_m \beta^A_n + \partial_n \beta^A_m - \frac{2}{3} \partial_j \beta^A_j \delta_{mn} - 6 g \sigma \gamma^A_{mn} \right] = 0,$$

- vector constraint

$$\partial_0 \left[ \partial_k \gamma^B_{jk} + \epsilon_{jmn} \partial_n \beta^B_m + \partial_j \delta \sigma - 2 g \sigma \beta^B_j \right] = 0.$$
Instabilities and particle production

One can now determine the dispersion relations for independent excitation modes. For example, for symmetric tensor of rank three $\hat{\kappa}_{jmn}$

$$p_0^2 = \vec{p}^2 + 2g^2\sigma^2 \pm 4pg\sigma,$$

- One mode gapped with $\Delta = \sqrt{2g^2\sigma^2}$.
- Other mode has Nielsen-Olesen instability for intermediate momenta.
- Particles will be produced in that momentum regime.
- Time scale for particle production $\tau_{pp} \approx 1/\sqrt{g^2\sigma^2} \approx 1\text{ fm/c}$. 

![Graph showing $p_0^2/(g^2\sigma^2)$ vs $p/(g\sigma)$]
Dispersion relations 1

C-odd tensors of rank two

\[ p_0^2/(g^2 \sigma^2) \]

\[ p/(g \sigma) \]
Dispersion relations 2

C-even tensors of rank two

\[ p_0^2/(g^2 \sigma^2) \]

\[ p/(g\sigma) \]
Dispersion relations 3

C-odd scalars

\[ p_0^2 / (g^2 \sigma^2) \]
Dispersion relations 4

C-even scalars

\[ p_0^2 / (g^2 \sigma^2) \]

\[ \text{p}/(g\sigma) \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ 0 \quad 10 \quad 20 \quad 30 \quad 40 \]
Conclusions

- Color field condensate may be simple qualitative description for state with high gluon occupation numbers.
- Modified rotation symmetry (involving a gauge transformation) provides powerful ordering principle.
- Collective dynamics provides efficient mechanism for approximate isotropization.
- Nielsen-Olesen type instabilities can trigger decay of color field condensate into quasi-particle excitations.
- Particle production from decay of isotropic condensate can be approximately isotropic, as well.
BACKUP
Alternative embedding of $SU(2) \in SU(3)$

- Lie algebra of $SU(2)$ spanned by Gell-Mann matrices $\lambda_1, \lambda_2, \lambda_3$.
- Contains singlets
  - in spatial part
    \[
    (A_j)_{mn} = \sigma(\lambda_j)_{mn}.
    \]
  - in temporal part
    \[
    (A_0)_{mn} = \sigma'(\lambda_8)_{mn}.
    \]
- More general decomposition of gauge field according to
  - temporal part $8 = 3 + 2 \times 2 + 1$,
  - spatial part $24 = 5 + 2 \times 4 + 2 \times 3 + 2 \times 2 + 1$. 
Parametric resonances

- Here we considered excitations around constant background $\sigma$.
- For oscillating condensate one has additional parametric resonance phenomenon leading to an additional instability band [Berges et al, PRD 85, 034507 (2012)].
- Parametric resonance instability subleading compared to Nielsen-Olesen instability.