Research Article

Excess Pore Water Pressure and Ground Consolidation Settlement Caused by Grouting of Shield Tunnelling

Qiongfang Zhang,1,2 Qihui Zhou,1,2 Fengchun Ye,1,2 Jiajia Yan,1,2 Nianwu Liu,3 and Yadong Lou4

1POWER CHINA Huadong Engineering Corporation, Zhejiang, Hangzhou 310014, China
2Engineering Research Center of Smart Rail Transportation of Zhejiang Province, Zhejiang, Hangzhou 310014, China
3Research Center of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China
4Hangzhou Metro Group Co., Ltd., Zhejiang, Hangzhou 310003, China

Correspondence should be addressed to Qiongfang Zhang; zhang_qf2@hdec.com

Received 24 January 2022; Revised 21 March 2022; Accepted 6 April 2022; Published 17 May 2022

Academic Editor: Jose Renato de Sousa

Copyright © 2022 Qiongfang Zhang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This study investigates the effect of shield tunnel grouting on excess pore pressure distributions around tunnel and ground consolidation settlement. First, it presents half-plane excess pore pressure around an unlined tunnel caused by grouting pressure based on the complex variable theory and the conformal mapping method. Second, the analytical solution for the soil consolidation is derived using the conformal transformation method and the consolidation theory of Terzaghi–Rendulic. The drainage conditions of the tunnel lining are idealized into three cases: fully drained, partially drained, and impermeable along the tunnel circumference. The impacts of the burial depth of the tunnel, the permeability coefficient of the tunnel lining and the surrounding soil, the Poisson’s ratio of the soil, the lateral Earth pressure coefficient of the soil, and the shear modulus are investigated.

1. Introduction

Many field and theoretical works [1–5] have studied the long-term ground settlement caused by shield tunnelling. When the shield tunnel boring machine moves forward, the soil loss produces negative excess pore pressure, while the positive thrust and the grouting pressure generate positive excess pore pressure. It is reported that the influence of initial excess pore water pressures associated with tunnelling significantly influences the ground consolidation settlement [6–10]. Terzaghi [6] conducted field observation and proposed that excess pore water pressure dissipation and soil consolidation caused by tunnelling led to long-term ground settlement. Mair and Taylor [7] pointed out that the long-term settlement of tunnels was mainly affected by the following four factors: the value and distribution of excess pore water pressure in the surrounding soil, the mechanical properties and permeability of the soil, the permeability of the tunnel lining, and the variation in the groundwater level.

Moreover, Shirlaw [8] summarized a large amount of measured data on the long-term settlement of tunnels and found that the long-term settlement of tunnels accounted for 30%–90% of their total settlement. The measured data showed that the excess pore water pressure reached the peak value and then dissipated when the shield tail passed through, resulting in consolidation settlement.

Complex variable functions and the conformal mapping technique have many applications in solving the consolidation settlement of the soil around tunnels [11, 12]. For example, Zhan et al. [13] transformed a circular hole in an infinite plane into an annulus domain by a complex variable function, assuming that the excess pore pressure caused by shield tunnelling was evenly distributed along the tunnel circumference, and considered the permeability of the tunnel lining.

Wang et al. [14] also determined the pore water pressure and the surface consolidation settlement under the assumption of impermeable lining based on the two-
dimensional consolidation theory of Terzaghi–Rendulic. The predicted settlement was significantly smaller than the measured one since the drainage effect of the lining was not considered. In another work, Cao et al. [15] assumed that the pore pressure on the inner circumference of the lining was constant, as proposed by Zhan et al. [13]. They presented the analytical solutions for the nonlinear consolidation of soft soil around a shield tunnel with idealized sealing linings. Their results demonstrated that the different tunnel circumference conditions affected the final settlement of the ground. Wei et al. [16] proposed the formula for the initial excess pore water pressure of soil around tunnel lining and the formula for the initial excess pore water pressure of soil within the region of its distribution at any point using the stress relief and stress transfer theories. Li et al. [17] also derived the dissipation expression of excess pore water pressure around a tunnel with partially sealed shell lining in viscoelastic media employing the complex function method. Further, Liang et al. [18] predicted the tunnel-induced initial excess pore water pressure based on the theory of elasticity and Skempton’s [19] excess pore water pressure theory. These studies did not fully consider the permeability of the tunnel using analytical method, and some studies do not further consider the consolidation settlement of the ground.

Current studies seldom take account of the ground consolidation settlement caused by the dissipation of excess pore water pressure, which is potentially a risk to the building facilities on the ground. Moreover, little work is conducted on the analysis of tunnel permeability and soil characteristics. In this paper, complex variable functions deduce the formula for calculating the additional stress on the soil under shield grouting, and Skempton’s formula determines the distribution of the initial excess pore pressure of the soil. The two-dimensional consolidation theory of Terzaghi–Rendulic is employed to consider the consolidation settlement under three conditions of tunnel circumference: fully drained, partially drained, and impermeable. The effects of the thickness of the water layer, the burial depth of the tunnel, the permeability coefficient of the lining and the surrounding soil, the Poisson’s ratio, the static soil pressure, and the shear modulus are discussed in detail.

2. Initial Excess Pore Water Pressure due to Grouting

2.1. Basic Assumptions. This paper assumes that synchronous grouting fills the soil void instantaneously when the shield tunnel boring machine drives, and the Earth pressure in front of the shield tunnel boring machine balances its thrust, which does not produce additional stress. Due to the existence of the grouting pressure, the liner must be expanded radially, while the excess pore water pressure of soil around the tunnel forms simultaneously, as shown in Figure 1.

As shown in Figure 1, a circular tunnel with a radius of \( r \) is buried in a half plane. Here, the depth of burial of the tunnel \( (H) \) is defined as the distance from the center of the tunnel to the surface. Additionally, this paper assumes that:

1. There is no relative displacement between the liner and the ground.
2. The shield tunnel boring machine is balanced with the Earth pressure in front of it, and the front thrust does not produce additional stress.
3. The soil loss is compensated by the synchronous grouting instantaneously, and the grouting pressure only produces the excess pore pressure.
4. The grouting pressure \( p \) is constant, and the stress vectors are directed toward the center of the tunnel.

This paper is also simplified to solve the initial excess pore pressure of the soil along the tunnel circumference under the grouting pressure.

2.2. Soil Stress Functions under Grouting Pressure. Figure 1 depicts a schematic of the uniform pressure on the boundary of a circular tunnel in a half plane. The solutions describe excess pore pressure assuming the radial grouting stress and plane strain condition.

Figure 2 shows that the original plane is mapped into an annulus domain. The \( z \)-plane is the original plane, and the coordinate of the origin is located on the surface directly above the tunnel; \( r \) denotes the radius of the shield tunnel; and \( H \) represents the burial depth of the tunnel. The original plane surface is mapped into the external diameter of the annulus, and the tunnel circumference in the \( z \)-plane is the inner diameter of the annulus. (1) and (2) express the
mapping relationship between the two planes given by Verruijt [20]:

\[ z = w(\zeta) = -ih \frac{1 - R^2}{1 + R} \frac{1 + \zeta}{1 - \zeta} \Rightarrow (z) \]

\[ = \frac{iz(1 + R^2) - H(1 - R^2)}{iz(1 + R^2) + H(1 - R^2)}, \tag{1} \]

where

\[ R = \frac{H}{r} - \sqrt{\left(\frac{H}{r}\right)^2 - 1}. \tag{2} \]

With the assumption of \( \alpha = 1 - R^2/1 + R^2H \), (1) can be simplified to:

\[ z = w(\zeta) = -i\alpha \frac{1 + \zeta}{1 - \zeta} \Rightarrow (z) \]

\[ = \frac{z + i\alpha}{z - i\alpha}. \tag{3} \]

(4) can define the analytical functions of the \( z \)-plane using the conformal transformation formula:

\[ \phi(z) = iP \left[ -2(1 + R^2) + \frac{2(z - \alpha)}{z + \alpha} + \frac{2R^2(z + \alpha)}{z - \alpha} \right], \]

\[ \psi(z) = iP \left[ -3(1 + R^2) + \frac{2R^2(z - \alpha)}{z + \alpha} + \frac{2(z + \alpha)}{z - \alpha} + \frac{R^2(z + \alpha)^3}{(z - \alpha)^3} \right]. \tag{4} \]

where \( \phi(z) \) and \( \psi(z) \) are two analytic functions in the complex plane; \( P = -R^2 \rho H/(1 - R^2)(1 - R^4); R = H - \sqrt{H^2 - r^2}/r; \alpha = -iH1 - R^2/1 + R^2; \) \( p \) (MPa) denotes the grouting pressure and can be expressed by

\[ P = 4n\rho v^2 \left[ \frac{nP_0 r^2}{4(r^2 - r_0^2)} \right]. \tag{5} \]

Figure 2: The original \( z \)-plane and the transformed \( \zeta \)-plane.

where \( P_0 \) (MPa) is the working pressure of the grouting press, \( d \) (m) represents the diameter of the grouting pipe, \( r \\) (m) stands for the radius of the pipe, and \( n \) is the ratio of the grouting volume to the void volume due to the tunnelling.

Using the Muskhelishvili complex variable method, one may obtain [21]:

\[ 2G(u_x + iu_y) = \kappa \phi(z) - z \frac{d\phi(z)}{dz} - \psi(z). \tag{6} \]

\[ \sigma_x = 2\text{Re}[\phi'(z)] - \text{Re}[\overline{\phi''}(z) + \psi'(z)], \]

\[ \sigma_y = 2\text{Re}[\phi'(z)] + \text{Re}[\overline{\phi''}(z) + \psi'(z)], \tag{7} \]

\[ \tau_{xy} = \text{Im}[\overline{\phi''}(z) + \psi'(z)]. \]

where for the plane strain conditions, \( \kappa = 3 - 4\nu \), and for the plane stress conditions, \( \kappa = 3 - \nu/1 + \nu \).

Introducing (4) into (6) and (7) can define the displacement of and the stress on the soil in the half plane:

Based on the theory of elasticity [22], the expression of the principal stress is given by

\[ \begin{align*}
\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 &= 0, \\
I_1 &= \sigma_x + \sigma_y + \sigma_z, \\
I_2 &= \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_y - \tau_{xz}^2, \\
I_3 &= \sigma_x\sigma_y\sigma_z - \sigma_z\tau_{xz}^2.
\end{align*} \tag{8} \]

where \( \sigma \) is the additional principal stress, including three principal stresses \( \sigma_1, \sigma_2, \) and \( \sigma_3, \) and \( \sigma_1 > \sigma_2 > \sigma_3, \) \( I_1, I_2, \) and \( I_3 \) are the first, second, and third principal stress invariants, respectively.

In the case of a plane strain condition:

\[ \sigma_z = \sqrt{(\sigma_x + \sigma_y)}. \tag{9} \]

Based on the soil triaxial tests, Skempton [19] proposed a formula for calculating the excess pore pressure of the soil as follows:
\[
\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)],
\]

where \( \Delta u \) is the excess pore pressure; \( B \) represents the coefficient of the pore water pressure under the combined action of the isotropic stress, and the deviating stress is related to the saturation of the soil; \( B \) is equal to one for saturated soil; \( \Delta \sigma_3 \) indicates the isotropic stress; \( A \) denotes the coefficient of the pore water pressure under the deviating stress and is related to the stress history, the stress level, the initial stress state, and the strain value of the soil; and \( \Delta \sigma_1 \) stands for the maximum principal stress. Table 1 lists the values of \( A \) for different soils.

From the above equations, it can be inferred that the excess pore pressure of the soil is related to the burial depth of the tunnel (\( H \)), the tunnel radius (\( r \)), and coefficients \( A \) and \( B \) in Skempton’s formula.

2.3. Distribution of Excess Pore Pressure around Tunnel.

(4) demonstrates that the grouting pressure is proportional to the excess pore pressure of the soil. The value of the excess pore pressure of the soil is normalized by \( p \).

Figure 3 illustrates the change in the initial excess pore pressure of the soil under grouting. The initial excess pore pressure isoline near the tunnel circumference is approximately circular. Moreover, the initial excess pore pressure of the soil is positive and decreases along the radial direction; the initial excess pore pressure also declines with the distance from the tunnel.

Figures 4 and 5 depict the excess pore pressure of the soil when the burial depth of the tunnel is equal to 0.8 and 0.5, respectively. When the tunnel is buried at a shallow depth, the excess pore pressure on the surface increases, and the burial depth of the tunnel affects the distribution of the excess pore pressure around the upper part of the tunnel more but the distribution of the excess pore pressure around the lower part of the tunnel less.

Figure 6 displays the contours of the excess pore pressure at an \( A \) value of 1.25 in Skempton’s formula. Compared with Figure 3, the excess pore pressure rises with coefficient \( A \).

Figures 7 and 8 delineate the contours of the excess pore pressure of the soil at an \( r/H \) value of 0.2 and 0.4, respectively. Comparing Figure 7 with Figure 8 demonstrates that when the diameter of the tunnel enlarges, the excess pore pressure of the soil around the tunnel increases, and the distribution of the excess pore pressure varies.

2.4. Consolidation Settlement of Soil Caused by Dissipation of Excess Pore Pressure.

Figure 9 shows the distribution of the excess pore pressure of the soil above the tunnel under the grouting. When analyzing the consolidation settlement of a tunnel, only the excess pore pressure of the soil above the bottom of the tunnel is taken into account. At a shorter distance from the surface, the excess pore pressure of the soil is smaller, and the excess pore pressure decreases more slowly.

2.5. Basic Assumptions. In order to find the analytical solution to the consolidation function of the soil around the shield tunnels, the problem is simplified to a semi-infinite

**Table 1: The empirical values of \( A \) (Skempton [19]).**

| Types of clay                      | \( A \)       |
|-----------------------------------|--------------|
| Highly sensitive cohesive soil     | 0.75–1.5     |
| Normally consolidated clay         | 0.5–1        |
| Compacted sandy clay               | 0.25–0.75    |
| Slightly overconsolidated clay     | 0.2–0.5      |
| General overconsolidated clay      | 0–0.2        |
| Highly overconsolidated clay       | –0.5 to 0    |

**Figure 3:** The distribution of the initial excess pore pressure under the grouting pressure.

**Figure 4:** The contours of the excess pore pressure at a burial depth of the tunnel equal to 0.8 \((H = 0.8)\).
plane problem with circular holes by assuming the following conditions:

(1) The length of the tunnel is long enough to assume a plane strain problem. Moreover, the ratio of the burial depth of the tunnel to the radius of the tunnel is higher than five ($H/r > 5$).

(2) The permeability of the tunnel at the interface depends on the relative permeabilities of the ground and tunnel lining. The tunnel lining is fully drained, partially drained, or impermeable.

(3) The soil around the tunnel is an isotropic, saturated, and linear elastic medium, and the permeability and compressibility of the soil remain unchanged during the consolidation.

(4) The soil particles and pore water are incompressible.

(5) The shield tunnelling only changes the pore water pressure in the soil, and the total vertical stress on the soil remains unchanged.

2.6. Basic Equations. The sum of the total normal stresses on the soil remains unchanged as follows:
\[ \Theta = \sigma_x + \sigma_y + \sigma_z, \]

where \( \sigma_x, \sigma_y, \) and \( \sigma_z \) represent the stress in the \( x-, y-, \) and \( z- \) directions of the soil, respectively, and \( \Theta \) is the sum of the total normal stresses on the soil.

If the total stress at each point of the soil remains unchanged, one may obtain:

\[ \Theta = \Theta \big|_{t=0} = 3\Delta u, \]

where \( \Delta u \) is the initial excess pore pressure.

If the soil is an ideal elastic medium, the stress–strain relationship is defined as

\[ \varepsilon_x = \frac{1}{E'} \left[ \sigma_x' - v' \left( \sigma_y' + \sigma_z' \right) \right], \]
\[ \varepsilon_y = \frac{1}{E'} \left[ \sigma_y' - v' \left( \sigma_x' + \sigma_z' \right) \right], \]
\[ \varepsilon_z = \frac{1}{E'} \left[ \sigma_z' - v' \left( \sigma_x' + \sigma_y' \right) \right], \]

where \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z \) are the strains of the soil in the \( x-, y-, \) and \( z- \) directions, respectively; \( E' \) represents the effective elastic modulus of the soil; \( v' \) stands for the effective Poisson’s ratio of the soil; \( \sigma_x', \sigma_y', \) and \( \sigma_z' \) indicate the effective stress on the soil in the \( x-, y-, \) and \( z- \) directions, respectively.

The volumetric strain of the soil can be expressed by

\[ \varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = (1 - 2v') \frac{\Theta - 3u}{E'} \]

where \( \varepsilon_v \) is the volumetric strain of the soil and \( u \) represents the excess pore pressure of the soil.

In the case of a plane strain condition:

\[ \sigma_z' = v' \left( \sigma_x' + \sigma_y' \right) \]

Substituting (15) into (8) yields:

\[ \varepsilon_x = \frac{1 + v'}{E'} \left[ (1 - v) \sigma_x' - v \sigma_y' \right], \]

\[ \varepsilon_y = \frac{1 + v'}{E'} \left[ (1 - v) \sigma_y' - v \sigma_x' \right]. \]

\( H/r > 5 \) expresses a deeply buried tunnel and (17) defines the relation between \( \sigma_x' \) and \( \sigma_y' \) as

\[ \sigma_z' = K_0 \sigma_y' \]

where \( K_0 \) is the lateral Earth pressure coefficient of the soil.

Combining the above formulas defines the relationship between the vertical strain \( (\varepsilon_v) \) and the volumetric strain \( (\varepsilon_v') \) of the soil as

\[ \varepsilon_v = \frac{-v' K_0 + (1 - v')}{{(1 - 2v')(K_0 + 1)}} \varepsilon_v'. \]

By integrating the above equations, (19) expresses the consolidation settlement of the surface by

\[ s(t) = \int_{-h}^{0} \frac{(1 - v') - K_0 v'}{(1 + K_0)(1 - 2v')} \left( 1 - 2v' \right)^3 \frac{3u_0 - 3u}{E'} \ dy. \]

With the assumption that the pore water and the soil particles are incompressible, and the total stress remains unchanged, (20) expresses the variation in the excess pore water pressure of soil conforming to the consolidation theory of Terzaghi–Rendulic in:

\[ C_v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = \frac{\partial u}{\partial t}, \]

where

\[ C_v = \frac{kE'}{\gamma_w (1 + v')(1 - 2v')} \]

where \( C_v \) is the consolidation coefficient, \( u \) represents the excess pore water pressure, \( k \) denotes the permeability coefficient, and \( \gamma_w \) indicates the unit weight of water; \( E' \) and \( v' \) stand for the effective elastic modulus of the soil and the effective Poisson’s ratio of the soil, respectively.

(22) can express the effective shear modulus of the soil \( (G') \) through:

\[ G' = \frac{E'}{2(1 + v')} \]

### 3. Conformal Transformation

The problem of the half-plane circular holes is transformed into the annulus domain, and the \( \zeta \)-plane is mapped to the \( \xi \)-plane using the conformal transformation formula:

\[ C_v \left( \frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} \right) = \frac{4a^2}{\left( (1 - \xi)^2 + (1 + \eta)^2 \right)^2} \frac{\partial w}{\partial t}. \]
Transforming the above equation into the polar coordinates yields:

\[
C_r \left(1 - \rho^2 - 2\rho \cos \theta \right)^2 \left(\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u}{\partial \rho} - \frac{\lambda^2}{\rho^2} \frac{\partial^2 u}{\partial \theta^2} \right) = \frac{\partial u}{\partial t} \tag{24}
\]

Then, the method of separating variables solves the above equation as follows:

\[
u(\rho, \theta, t) = W(\rho, \theta)T(t). \tag{25}
\]

Substituting (25) into (24) yields:

\[
\frac{\partial^2 W}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial W}{\partial \rho} + \frac{\lambda^2}{\rho^2} \frac{\partial^2 W}{\partial \theta^2} + \left(1 + \rho^2 - 2\rho \cos \theta \right) W = 0, \tag{26}
\]

\[
\frac{\partial T}{\partial t} + \frac{C_r \lambda^2}{4a^2} T = 0,
\]

where \( \lambda \) is the eigenvalue of the equation.

Substituting \( \chi = 1/\sqrt{1 + \rho^2 - 2\rho \cos \theta} \) into (24) results in the following equation:

\[
\frac{\partial^2 W}{\partial \chi^2} + \frac{1}{\chi} \frac{\partial W}{\partial \chi} + \lambda^2 W = 0. \tag{27}
\]

The above function is the Bessel equation of order zero, and the general solution to it is given by

\[
W_0(\chi) = A_1 J_0(\lambda_1 \chi) + B_1 N_0(\lambda_1 \chi), \tag{28}
\]

where \( J_0 \) and \( N_0 \) are the Bessel function and Neumann function of the first kind of order zero, respectively. \( A_1 \) and \( B_1 \) indicate the coefficients determined by the \( i \)th ideal boundary condition.

4. Boundary Conditions

The dissipation of the pore water pressure is the reduction of the excess pore pressure. Considering the various boundary conditions of the tunnel lining, we deduce the analytical solution for the excess pore pressure of the soil around the shield tunnels according to the consolidation theory of Terzaghi–Rendulic and determine the variation of the ground consolidation settlement with time.

The permeability of the tunnel lining is idealized into three types to study the influence of the permeability of the tunnel lining on the surface settlement during the soil consolidation:

(1) The first type of the idealized boundary: FD, fully drained;
(2) The second type of the idealized boundary: UD, impermeable;
(3) The third type of the idealized boundary: PD, partially drained.

Taking the relative permeability of the tunnel lining and the soil into account, Li [23] defined a dimensionless parameter reflecting the permeability of the tunnel lining \( (k_0) \) based on Darcy’s law as follows:

\[
k_0 = \frac{k_1}{k_2} r_2 \ln \left( \frac{r_2}{r_1} \right) \tag{29}
\]

where \( k_1 \) and \( k_2 \) are the permeability coefficients of the tunnel lining and the soil, respectively; \( r_1 \) and \( r_2 \) represent the internal and external radii of the tunnel lining, respectively. From the above formula, it can be inferred that \( k_0 \) is related to the permeability coefficient of the tunnel lining, the permeability coefficient of the soil, and the geometric size of the tunnel. The boundary of the tunnel is considered impermeable when \( k1 \ll k2 \) and \( k0 \to 0 \) and fully drained when \( k1 \approx k2 \) and \( k0 \to \infty \). The influence of the seepage on the pore water pressure is not taken into account, that is, the static pore water pressure at the tunnel boundary remains unchanged.

When the tunnel is entirely permeable, the pore water pressure at the lining boundary is zero. When the tunnel reaches a new seepage equilibrium state, the reduction of the static pore water pressure of the soil is equal to the dissipation value of the excess pore pressure caused by the tunnel construction. The first type of the idealized boundary and the initial conditions are as follows:

\[
\Delta u|_{t=0} = u_0, \\
\Delta u|_{t=0} = 0, \\
\Delta u|_{r=0} = 0, \tag{30}
\]

where \( u_0 \) is the initial excess pore pressure of the soil.

The first type of the idealized boundary conditions can be transformed into:

\[
W|_{r=1} = 0, \\
W|_{r=R} = 0. \tag{31}
\]

Substituting (31) into (28) yields:

\[
\begin{cases}
A_1 J_0(\lambda_{1n}/\sqrt{2 - 2\cos \theta}) + B_1 N_0(\lambda_{1n}/\sqrt{2 - 2\cos \theta}) = 0,
A_1 J_0(\lambda_{1n}/\sqrt{1 + R^2 - 2R \cos \theta}) + B_1 N_0(\lambda_{1n}/\sqrt{1 + R^2 - 2R \cos \theta}) = 0.
\end{cases} \tag{32}
\]
To find the solutions for the undetermined coefficients $A_1$ and $B_1$, the characteristic equation must satisfy:

$$
\begin{align*}
&f(\lambda_{1n}) = J_0 \left( \frac{\lambda_{1n}}{\sqrt{2 - 2 \cos \theta}} \right) N_0 \left( \frac{\lambda_{3n}}{\sqrt{2 - 2 \cos \theta}} \right) = 0, \\
&J_0 \left( \frac{\lambda_{1n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) N_0 \left( \frac{\lambda_{3n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) = 0,
\end{align*}
$$

(33)

where $W_{1n}(\rho, \theta)$ can be expressed by

$$
W_{1n}(\rho, \theta) = A_{1n} \left\{ J_0 \left( \frac{\lambda_{1n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - J_0 \left( \frac{\lambda_{1n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) N_0 \left( \frac{\lambda_{1n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) \right\}
$$

(34)

When the tunnel is entirely impermeable, the derivative of the excess pore pressure with respect to the radial coordinates is zero. The second type of the idealized boundary and the initial conditions are thus given by

$$
\begin{align*}
\Delta u |_{t=0} &= u_0 \\
\Delta u |_{y=0} &= 0 \\
\frac{\partial u}{\partial n} |_{\rho^2 + (y+h)^2 = r^2} &= 0 \\
\Delta u |_{r=\infty} &= 0
\end{align*}
$$

(35)

Substituting (36) into (28) yields:

$$
\begin{align*}
A_2 J_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) + B_2 N_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) &= 0, \\
A_2 J_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) + B_2 N_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) &= 0,
\end{align*}
$$

(37)

To find the solutions for the undetermined coefficients $A_2$ and $B_2$, the characteristic equation must satisfy:

$$
\begin{align*}
&f(\lambda_{2n}) = \left| J_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) N_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) \right| = 0. \\
&J_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) N_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) \right| = 0.
\end{align*}
$$

(38)
where \( W_{2n}(\rho, \theta) \) can be expressed by

\[
W_{2n}(\rho, \theta) = A_{2n} \left\{ J_0 \left( \frac{\lambda_{2n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - J_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) \right\}
\]

When the tunnel is partially drained, the third type of the idealized boundary and the initial conditions are given by

\[
\Delta u_{|\rho=0} = u_0, \\
\Delta u_{|\rho=0} = 0, \\
\frac{\partial u}{\partial n}_{|x+x_1=\lambda^2} = \frac{k_1}{k_2} \frac{1}{r_2^2 1n(r_2/r_1)} = k_0 u, \\
\Delta u_{|z=\infty} = 0.
\]

The third type of the idealized boundary conditions can be transformed into:

\[
W_{|\rho=1} = 0, \\
\frac{\partial W}{\partial \rho}_{|\rho=R} = \kappa W_{|\rho=r}.
\]

Substituting (41) into (28) yields:

\[
\left\{ A_3 J_0 \left( \frac{\lambda_{3n}}{\sqrt{2 - 2 \cos \theta}} \right) + B_3 N_0 \left( \frac{\lambda_{3n}}{\sqrt{2 - 2 \cos \theta}} \right) = 0, \\
A_3 \cdot C(\lambda) + B_3 \cdot D(\lambda) = 0.
\]

The characteristic equation must satisfy the conditions given in (43) to find the solutions for the undetermined coefficients \( A_3 \) and \( B_3 \):

\[
f(\lambda_{2n}) = \left| \begin{array}{cc} J_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) & N_0 \left( \frac{\lambda_{2n}}{\sqrt{2 - 2 \cos \theta}} \right) \\
J_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) & N_1 \left( \frac{\lambda_{2n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) \end{array} \right| = 0.
\]

where

\[
f(\lambda_{3n}) = \left| \begin{array}{cc} J_0 \left( \frac{\lambda_{3n}}{\sqrt{2 - 2 \cos \theta}} \right) & N_0 \left( \frac{\lambda_{3n}}{\sqrt{2 - 2 \cos \theta}} \right) \\
C(\lambda_{3n}) & D(\lambda_{3n}) = 0 \end{array} \right|
\]

\[
C(\lambda) = \frac{\lambda_{3n}(R - \cos \theta)}{(1 + R^2 - 2R \cos \theta)^{\frac{3}{2}}} J_1 \left( \frac{\lambda_{3n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) - \kappa J_0 \left( \frac{\lambda_{3n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right),
\]

\[
D(\lambda) = \frac{\lambda_{3n}(R - \cos \theta)}{(1 + R^2 - 2R \cos \theta)^{\frac{3}{2}}} N_1 \left( \frac{\lambda_{3n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right) - \kappa N_0 \left( \frac{\lambda_{3n}}{\sqrt{1 + R^2 - 2R \cos \theta}} \right).
\]
$W_{3n}(\rho, \theta)$ can be expressed by

$$W_{3n}(\rho, \theta) = A_{3n} \left\{ J_0 \left( \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - J_0 \left( \frac{\lambda_{3n}/\sqrt{2 - 2 \cos \theta}}{N_0 \left( \lambda_{3n}/\sqrt{2 - 2 \cos \theta} \right)} \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) \right\}. \quad (46)$$

The characteristic equations, namely (36) and (41), and (46), are the even functions of $\lambda$. For each certain $\theta$, countless positive roots can be obtained from the equation. The sequence $\{\lambda_{1}, \lambda_{2}, \ldots \lambda_{ij}\}$ is the positive roots arranged from small to large roots, that is, the eigenvalues of the above equation. Usually, a finite term can fulfill the requirements of engineering calculations.

Introducing the eigenvalues $\lambda_{n}$ into (27) yields:

$$T_n(t) = Ce^{-C_n \chi^2 / 4 \pi^2 t}. \quad (47)$$

According to $T(0) = 1$, (48) expresses the initial excess pore pressure by

$$u_0 = \sum_{n=1}^{\infty} W_n(\chi) \quad (48)$$

$$\begin{align*}
\Omega_{1n}(\rho, \theta) &= J_0 \left( \frac{\lambda_{1n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - \frac{J_0 \left( \frac{\lambda_{1n}/\sqrt{1 + R^2 - 2R \cos \theta}}{N_0 \left( \lambda_{1n}/\sqrt{1 + R^2 - 2R \cos \theta} \right)} \frac{\lambda_{1n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right)}{N_0 \left( \lambda_{1n}/\sqrt{1 + R^2 - 2R \cos \theta} \right)} \frac{\lambda_{1n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}}, \\
\Omega_{2n}(\rho, \theta) &= J_0 \left( \frac{\lambda_{2n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - \frac{J_1 \left( \frac{\lambda_{2n}/\sqrt{1 + R^2 - 2R \cos \theta}}{N_1 \left( \lambda_{2n}/\sqrt{1 + R^2 - 2R \cos \theta} \right)} \frac{\lambda_{2n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right)}{N_1 \left( \lambda_{2n}/\sqrt{1 + R^2 - 2R \cos \theta} \right)} \frac{\lambda_{2n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}}, \\
\Omega_{3n}(\rho, \theta) &= J_0 \left( \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right) - \frac{J_0 \left( \frac{\lambda_{3n}/\sqrt{2 - 2 \cos \theta}}{N_0 \left( \lambda_{3n}/\sqrt{2 - 2 \cos \theta} \right)} \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} \right)}{N_0 \left( \lambda_{3n}/\sqrt{2 - 2 \cos \theta} \right)} \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}}, \quad (51)
\end{align*}$$

Then, (52) defines the coefficients $A_{3n}$ of the $i$th boundary conditions as

$$A_{3n} = \frac{1}{i \int_{R}^{1} \Omega_{1n}(\rho, \theta)/\sqrt{1 + \rho^2 - 2\rho \cos \theta} d\rho } \left[ \frac{1}{\int_{R}^{1} \Omega_{2n}(\rho, \theta)/\sqrt{1 + \rho^2 - 2\rho \cos \theta} d\rho } \right] \frac{\lambda_{3n}}{\sqrt{1 + \rho^2 - 2\rho \cos \theta}} i = 1, 2, 3. \quad (52)$$
According to the boundary conditions $T_n(0) = 1$, the expressions of the pore pressure under the three idealized tunnel lining conditions can be defined as

$$u_1(\rho, \theta, t) = \sum_{n=1}^{\infty} W_1n(\rho, \theta)T_{1n}(t),$$

$$u_2(\rho, \theta, t) = \sum_{n=1}^{\infty} W_2n(\rho, \theta)T_{2n}(t),$$

$$u_3(\rho, \theta, t) = \sum_{n=1}^{\infty} W_3n(\rho, \theta)T_{3n}(t).$$

Finally, (19) can express the solution for the ground consolidation settlement.

### 4.1. Analytical Solution Result and Parametric Analysis

The influences of the burial depth of the tunnel, the permeability coefficient of the soil, the Poisson’s ratio of the soil, the lateral Earth pressure coefficient of the soil, the shear modulus of the soil, and the permeability of the tunnel lining on the surface consolidation settlement are analyzed by assuming the following parameters in Table 2 to study the effect of the shield grouting pressure on the soil.

### 4.2. Effects of Shear Modulus, Poisson’s Ratio, And Lateral Earth Pressure Coefficient of Soil

Figures 10–12 depict the variation in the ground consolidation settlement with the shear modulus, the Poisson’s ratio, and the static lateral pressure coefficient of soil. The tangent of the consolidation curve is the consolidation rate. At the initial stage of consolidation, the surface consolidation settlement speed is fast, and the settlement speed gradually slows down with the passage of time. When the tunnel circumference is undrained (UD), the rate of surface consolidation settlement speed is the lowest, and the soil consolidation needs more time. When the tunnel circumference is fully drained (FD), the rate of surface consolidation settlement speed is the fastest, resulting in the final consolidation settlement in a short time. In conclusion, the higher the permeability values of the tunnel lining are, the quicker the surface consolidation settles. The tunnel lining permeability does not affect the final surface consolidation settlement.

The predicted consolidation settlement is inversely proportional to the shear modulus, the Poisson’s ratio, and the lateral Earth pressure of the soil, which can be obtained from the calculation formula of the settlement. These results demonstrate that when the tunnel boundary acts as fully drained, it accelerates soil consolidation significantly, as reported by Cao et al. [15]. When the Poisson’s ratio of the soil is equal to 0.499, the effect of the tunnel boundary can be neglected, the soil consolidation is complete in a short time, and the final surface consolidation settlement is small.

### 4.3. Effect of Burial Depth of Tunnel

Figure 13 delineates the impact of the burial depth of the tunnel on the surface consolidation settlement. The shallower the tunnel is buried, the shorter the time required for the consolidation.
settlement of the ground surface becomes, and the smaller the consolidation settlement of the ground surface is.

4.4. Effects of Permeability Coefficients of Soil and Tunnel Boundary. Figures 14 and 15 show the effect of the soil permeability \( k = 8 \times 10^{-9} \text{m/s} \) for silty clay and \( k = 8 \times 10^{-8} \text{m/S} \) for silt) and tunnel permeability on the surface consolidation settlement. The larger the permeability coefficient of the soil is, the quicker the ground settles. When the permeability of the soil is equal to \( 8 \times 10^{-9} \text{m/s} \), the tunnel circumference is nearly impermeable, the rate of surface consolidation settlement speed is the lowest, and the soil consolidation needs more than 900 days. When the tunnel circumference is partially drained and the permeability coefficient of the tunnel lining is equal to one \( (k_0 = 1) \), the rate of surface consolidation settlement speed is fast, leading to the final consolidation settlement in 200 days. When the tunnel lining is fully drained, the rate of surface consolidation settlement speed is the highest, resulting in the final consolidation settlement in 50 days. In conclusion, the higher the permeability values of the tunnel lining and the soil are, the quicker the surface consolidation settles. The soil and tunnel lining permeability does not affect the final surface consolidation settlement.
5. Conclusions

The complex variable functions and conformal mapping technique are used to estimate the excess pore pressure of the soil due to the shield grouting pressure. The analytical solution for the ground surface consolidation settlement is derived by calculating the variation of excess pore water pressure with time using idealized tunnel boundary conditions and the Terzaghi–Rendulic consolidation theory.

From the above calculations and comparative analyses, the following conclusions can be drawn:

(i) When uniform radial grouting pressure is applied to the tunnel circumference, the excess pore pressure of the soil is positive, and the contours have an approximately circular distribution symmetrical about the y-axis. The closer to the tunnel boundary the contours are, the denser they become, and the rapid the excess pore pressure of the soil around the tunnel decreases along the radial direction.

(ii) The burial depth of the tunnel has little influence on the excess pore pressure of the soil at the bottom of the tunnel, but it significantly impacts the distribution of the excess pore water pressure of the soil at the top of the tunnel. The shallower the tunnel is buried, the higher the excess pore pressure of the surface becomes. Moreover, the excess pore pressure of the soil around the tunnel is higher at a larger tunnel radius.

(iii) The increase in the surface consolidation settlement is directly proportional to the dissipation of the initial excess pore pressure. A higher elastic modulus of the soil, a larger lateral Earth pressure coefficient, and a higher Passion’s ratio of the soil result in a smaller final surface consolidation settlement. When the tunnel is buried deeply, a long time is required for the surface consolidation settlement, and the final surface consolidation settlement is significant.

(iv) Higher permeability coefficients of the soil and the tunnel lining give rise to the rapid surface consolidation settlement. However, the permeability coefficients of the soil and the tunnel lining do not impact the final surface consolidation settlement. Using similar calculation parameters, when the relative permeability coefficients of the tunnel lining and the soil are higher than a specific value of 100, the leakage of the tunnel as a drainage boundary is considered to be fully drained. Nevertheless, when the relative permeability coefficients of the tunnel lining and the soil are smaller than a specific value of 0.1, the tunnel boundary is considered to be impermeable.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was financially supported by the Key Research Plan Foundation of Zhejiang Province (grant no. 2020C01102).

References

[1] M. P. O’Reilly, R. J. Mair, and G. H. Alderman, "Long-term settlements over tunnels: an eleven-year study at Grimsby," International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, vol. 29, no. 2, p. A96, 1992.
[2] K. H. Bowers, D. M. Hiller, and B. M. New, "Ground movement over three years at the Heathrow express rail tunnel," in Proceedings of the Geotechnical Aspects of Underground Construction in Soft Ground-Preprint Volume of Proceedings from an International Symposium Held at City University, pp. 15–17, London, UK, April 1996.
[3] R. N. Hwang, C. B. Fan, and G. R. Yang, "Consolidation settlements due to tunnelling," Proc South East Asian Symposium on Tunnelling and Underground Space Development, Bangkok: Japan Tunnelling Association, pp. 79–86, 1995.
[4] J. J. R. Chen, T. Y. Ho, F. S. Chen, and C. S. Chao, "Consolidation settlements caused by a shield tunnel for the taipei mass rapid transit system," IAHS Publication, vol. 8, pp. 135–140, 1995.
[5] Y. S. Fang, S. J. Lin, and J. S. Lin, "Time and settlement in EPB shield tunnelling," Tunnels and Tunnelling, vol. 25, no. 11, pp. 27–28, 1993.
[6] K. Terzaghi, Theoretical Soil Mechanics, p. 300, J. Wiley and Sons, Hoboken, New Jersey, U.S, 1942.
[7] R. J. Mair and R. N. Taylor in Proceedings of the 14th ICSMFE, Hamburg, pp. 2343–2385, Germany, 1997.
[8] J. N. Shirlaw, "Observed and calculated pore pressures and deformations induced by an earth balance shield: Discussion," Canadian Geotechnical Journal, vol. 32, no. 1, pp. 181–189, 1995.
[9] J. Wongsaroj, K. Soga, and R. J. Mair, "Modelling of long-term ground response to tunnelling under St James’s Park, London," Géotechnique, vol. 57, no. 1, pp. 75–90, 2007.
[10] J. Wongsaroj, K. Soga, and R. J. Mair, "Tunnelling-induced consolidation settlements in london clay," Géotechnique, vol. 63, no. 13, pp. 1103–1115, 2013.
[11] Z. Zhang, Y. Pan, M. Zhang, X. Lv, K. Jiang, and S. Li, "Complex variable analytical prediction for ground deformation and lining responses due to shield tunneling..."
considering groundwater level variation in clays," *Computers and Geotechnics*, vol. 120, Article ID 103443, 2020.

[12] Q. Zhang, K. Cheng, C. Lin, Y. Lou, P. Guo, and X. Gan, "Ground displacements due to the deformations of shallow tunnels with arbitrary cross sections in soft ground," *Advances in Civil Engineering*, vol. 2021, no. 6, pp. 1–16, 2021.

[13] M. L. Zhan, J. H. Qian, and X. L. Chen, "Analytical solution of soil consolidation around tunnel in viscoelastic foundation," *Journal of Hehai University (Natural Science Edition)*, vol. 2, pp. 54–60, 1993.

[14] Z. L. Wang, M. Liu, and J. B. Xie, "Research on consolidation settlement of ground surface caused by shield tunnelling," *Rock and Soil Mechanics*, vol. 34, pp. 127–133, 2013.

[15] Y. Cao, J. Jiang, K.-H. Xie, and W.-M. Huang, "Analytical solutions for nonlinear consolidation of soft soil around a shield tunnel with idealized sealing linings," *Computers and Geotechnics*, vol. 61, pp. 144–152, 2014.

[16] X. J. Wei, W. J. Chen, and G. Wei, "Research on distribution of initial excess pore water pressure due to shield tunnelling," *Rock and Soil Mechanics*, vol. 33, no. 7, pp. 2103–2109, 2012, in Chinese.

[17] X. Y. Li, G. B. Liu, M. Liu, and Q. G. Feng, "Analytical study on dissipation of excess pore water pressure around tunnels with semi-permeable boundary conditions," *Chinese Journal of Geotechnical Engineering*, vol. 36, no. 1, pp. 75–82, 2014.

[18] R. Z. Liang, T. D. Xia, C. G. Lin, F. Yu, and S. M. Wu, "Initial excess pore water pressures induced by tunnelling in soft ground," *Journal of Central South University*, vol. 22, no. 11, 2015.

[19] A. W. Skempton, "The pore-pressure coefficients A and B," *Geotechnique*, vol. 4, no. 4, pp. 143–147, 1954.

[20] A. Verruijt, "A complex variable solution for a deforming circular tunnel in an elastic half-plane," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 21, no. 2, pp. 77–89, 1997.

[21] N. I. Muskhelishvili, *Some Basic Problems of Mathematical Theory of Elasticity*, P. Noordhoff, Ed., Groningen, Netherlands, 4th Edition, 1963.

[22] S. P. Timoshenko, *Theory of Elasticity*, McGraw-Hill, New York, NY, USA, Third Edition, 1970.

[23] X. Li, "Stress and displacement fields around a deep circular tunnel with partial sealing," *Computers and Geotechnics*, vol. 24, no. 2, pp. 125–140, 1999.

[24] A. Bobet and S. W. Nam, "Stresses around pressure tunnels with semi-permeable liners," *Rock Mechanics and Rock Engineering*, vol. 40, no. 3, pp. 287–315, 2007.

[25] M. B. Wang and G. Wang, "A modified stress-displacement solution for a pressure tunnel with a permeable liner in an elastic porous medium based on a new model," *Rock Mechanics and Rock Engineering*, vol. 46, no. 2, pp. 259–268, 2013.

[26] P. Arjnoi, J.-H. Jeong, C.-Y. Kim, and K.-H. Park, "Effect of drainage conditions on porewater pressure distributions and lining stresses in drained tunnels," *Tunnelling and Underground Space Technology*, vol. 24, no. 4, pp. 376–389, 2009.

[27] "Lining leakage and consolidation around soft-ground tunnels," *Master of Science Thesis*, Cornell University, 1980.