An improved algorithm for target trajectory prediction in the active protection system based on converted measurement Kalman filter

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Abstract. Fast and accurate prediction of the trajectory is the prerequisite for the successful interception of the incoming target by the active protection system (APS) of armoured vehicles. The traditional least-squares algorithm is susceptible to large errors in the initial stage of the tracking process, and the prediction of interception points converges slowly. This paper proposes an interception point prediction algorithm that combines the Kalman filter and least-squares estimation. By dynamically determining the convergence of the least-squares prediction, a converted measurement Kalman filter is used in the initial tracking stage to obtain higher prediction accuracy of the interception point. Simulations are performed in the scenario of the interception process of the APS. The results prove that the algorithm can effectively accelerate the convergence of the interception point prediction process.

1. Introduction

Active protection system (APS) is a self-defence system used by armoured vehicles to intercept and destroy anti-tank missiles or projectiles. The system consists of the following three parts: the first part is the threat detection system, which detects threats to the vehicle on the battlefield; the second part is the signal processing and decision-making system, which is responsible for processing, analysing and making countermeasure decisions; the third part is the confrontation system, which uses a close-range anti-missile defence system to intercept and destroy threats before they hit the vehicle [1].

To ensure that the intercepting ammunition accurately meets the incoming target and then causes damage to it, it is necessary to track and predict its trajectory based on the range, speed and orientation measured by the threat detection system. Targets of the APS are high-speed moving ammunitions. In the radar detection range, the gravity has little influence on the targets and their trajectories can be approximated as straight lines. Therefore, the trajectories can be predicted by a linear fit of the radar tracks using the least-squares method. However, because the least-squares method only relies on the spatial position of the measured data points for state estimation, the accuracy of its prediction is very unsatisfactory in the initial tracking stage with less measured data and relatively larger errors. This leads to a slower convergence process of the prediction of the final interception point, thereby restricting the performance of the entire system. First, the inability to accurately predict the intersection of the incoming target's trajectory and the protective surface in a short time will affect the system's identification and judgment of effective threats. Second, a longer prediction convergence time means a shorter time to perform the confrontation. Then the performance requirements for intercepting
launchers and ammunition will be more stringent. Finally, for incoming targets with faster flight speed, such as high-explosive anti-tank warheads, the system's response time will be further reduced, so the prediction process may fail to converge.

In the initial tracking stage, the Kalman filter method can be introduced to improve the prediction accuracy by establishing the target's motion model. Kalman filter is one of the commonly used algorithms in radar target tracking. Many studies have applied the Kalman filter and its various improved algorithms to related fields [2-7], but research on the APS is still relatively rare. This paper proposes an interception point prediction algorithm that combines the Kalman filter and least-squares estimation. By dynamically determining the convergence of the least-squares estimation, a converted measurement Kalman filter is used in the initial tracking stage to reduce the prediction error of the interception point; the least-squares algorithm is used to predict the interception point when it converges within the ideal range so that when the number of radar data points is sufficient, the measured data is fully utilized to obtain higher prediction accuracy. Simulations verified that this algorithm has an ideal balance of the convergence speed and accuracy of interception point prediction.

2. Problem Description

2.1. The motion model of targets

In the detection range of the APS, the flying time of the incoming target is extremely short, and no large manoeuvre will occur. Therefore, its movement can be simplified to uniform linear motion. Also, because the speed of the incoming target is much faster than the vehicle travelling speed, it can be assumed that the vehicle is stationary relative to the ground during the entire interception process [8]. Based on the above two assumptions, we can use the model shown in equation (1) to describe the motion of the target.

\[
X_{k+1} = \Phi_k X_k + W_k
\]

\[
\Phi_k = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 1 & 0 & 0 & T \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
X_k = [x_k, y_k, z_k, \dot{x}_k, \dot{y}_k, \dot{z}_k]^T
\]

is the motion state vector whose components are the position and velocity of the target in the three directions of x, y and z. \( \Phi_k \) is the state transition matrix. For uniform linear motion, its form can be easily obtained as shown in equation (2). \( T \) is the radar detection period. \( W_k \) is the process noise which is a white noise sequence with a mean value of zero and a covariance matrix of \( Q \).

2.2. Measurement Conversion

The radars in the APS use a spherical coordinate system for measurement. The position measurements are the radial distance \( r_k^m \), the azimuth angle \( \theta_k^m \) and the elevation angle \( \phi_k^m \). The corresponding measurement error can be expressed by equation (3).
\[
\begin{align*}
\dot{r}_k &= r_k + \tilde{r}_k \\
\dot{\theta}_k &= \theta_k + \tilde{\theta}_k \\
\dot{\phi}_k &= \phi_k + \tilde{\phi}_k
\end{align*}
\] (3)

\(\tilde{r}_k\), \(\tilde{\theta}_k\) and \(\tilde{\phi}_k\) are the corresponding measurement errors, all of which are Gaussian white noise with zero mean and independent of each other, and the variances are \(\sigma_r^2\), \(\sigma_\theta^2\) and \(\sigma_\phi^2\), respectively. Then we can use equation (4) to convert the measurements from the spherical coordinate system to the Cartesian coordinate system.

\[
\begin{align*}
x_k &= r_k \cos \phi_k \cos \theta_k = x_k + \tilde{x}_k \\
y_k &= r_k \cos \phi_k \sin \theta_k = y_k + \tilde{y}_k \\
z_k &= r_k \sin \phi_k = z_k + \tilde{z}_k
\end{align*}
\] (4)

In addition to position measurements, the radar used in the APS can also obtain Doppler measurement information which can be used to improve the accuracy of the target state estimation. The radial velocity measurement and the corresponding error are shown in equation (5).

\[
\tilde{r}_k = \tilde{r}_k + \tilde{\tilde{r}}_k
\] (5)

\(\tilde{\tilde{r}}_k\) is Gaussian white noise with zero mean and variance \(\sigma_r^2\). Considering the possible correlation between the Doppler measurement and range measurement of radars [9], let the correlation coefficients of \(\tilde{r}_k\) and \(\tilde{r}_k\) be \(\rho\). To reduce the strong non-linearity between the radial velocity measurement and the target motion state, a pseudo-measurement can be constructed as shown in equation (6) [10].

\[
\eta_k = \eta_k R_k = x_k \tilde{x}_k + y_k \tilde{y}_k + z_k \tilde{z}_k + \tilde{\eta}_k
\] (6)

By combining the position measurements with the Doppler pseudo-measurement, the observation equation of the motion model after measurement conversion can be given as shown in equation (7):

\[
Z_k = h(X_k) + V_k
\] (7)

where

\[
Z_k = [x_k, y_k, z_k, \eta_k]^T \\
h(X_k) = [x_k, y_k, z_k, x_k \tilde{x}_k + y_k \tilde{y}_k + z_k \tilde{z}_k]^T \\
V_k = [\tilde{x}_k, \tilde{y}_k, \tilde{z}_k, \tilde{\eta}_k]^T
\]

2.3. Statistical characteristics of converted measurement errors

The true error and covariance of the converted measurements can be calculated from equations (3) to (6), which are given by equations (8) and (9), respectively.

\[
\mu'_k = E(V_k | r_k, \theta_k, \phi_k, \tilde{r}_k)
\] (8)

\[
R'_k = \text{cov}(V_k | r_k, \theta_k, \phi_k, \tilde{r}_k)
\] (9)
When the real position is unknown, the mathematical expectation can be obtained for equations (8) and (9) under the conditions of known measurement values, as shown in equations (10) and (11) [11].

\[
\mu^a_k = E(\mu^r_k | r^m_k, \theta^m_k, \phi^m_k, \dot{r}^m_k)
\]  
(10)

\[
R^a_k = E(R^r_k | r^m_k, \theta^m_k, \phi^m_k, \dot{r}^m_k)
\]  
(11)

3. Prediction Algorithms

3.1. The recursive least-squares algorithm

For a linear system with input measurements \( \Psi \), output measurements \( A \) and measurement errors \( \epsilon \), the observation model can be expressed by equation (12).

\[ A = \Psi \Theta + \epsilon \]  
(12)

From equation (12), the best estimate of the parameter \( \Theta \) which minimizes the mean square error can be obtained using the least-squares method, as shown in equation (13).

\[ \hat{\Theta} = (\Psi^T \Psi)^{-1} \Psi^T A \]  
(13)

Through mathematical derivation, the recursive form of the least-squares algorithm above can be obtained, as shown in equation (14):

\[
\begin{align*}
L_{k+1} &= B_k \psi_k + (1 + \psi_k^T B_k \psi_k)^{-1} \\
B_{k+1} &= (I - L_{k+1} \psi_k^T B_k) B_k \\
\hat{\Theta}_{k+1} &= \hat{\Theta}_k + L_{k+1} (\lambda_k - \psi_k^T \hat{\Theta}_k)
\end{align*}
\]  
(14)

where \( \psi_k \), \( \lambda_k \) are single input and output measurements respectively and \( L_k, B_k \) are intermediate variables in the recursive process.

For the approximate motion model of the APS target used in this paper, the radar ranging value is used as the system input, and the x, y and z coordinates are fitted. That is, let the input \( \psi_k = [r^m_k 1]^T \), and the output be \( x^m_k, y^m_k \) and \( z^m_k \), and perform recursive calculation on the estimated parameter \( \hat{\Theta}_k \). Let the radius of the protective surface be \( r_p \), then after each update period of \( \hat{\Theta}_k \), let the input be \( \psi_p = [r_p 1]^T \) to get the 3D coordinates of the predicted interception point.

To determine the convergence of the least-squares prediction, the sliding time window method is used to investigate the change of the predicted interception point. In the recursive process, a buffer stores the spatial variations of the predicted interception points in the last \( T_w \) cycles; if the variations are always less than a certain threshold value \( \delta_w \), the least-squares prediction process is considered convergence. In this judgment criterion, the sliding time window width \( T_w \) and the spatial variation threshold \( \delta_w \) are the key parameters, which need to be reasonably selected according to the actual situation.

3.2. The extended Kalman filter

When the least-squares prediction does not converge, an extended Kalman filter algorithm is used to track and predict the target trajectory. From equation (7), we know that the converted measurement is a quadratic function of the target's motion state, so the second-order EKF is used for nonlinear tracking filtering, as shown in equation (15) [10].
\[
\begin{aligned}
\hat{X}_{n+1|k} &= \Phi \hat{X}_{n|k} \\
\hat{P}_{n+1|k} &= \Phi \hat{P}_{n|k} \Phi^T + Q \\
K_{n+1} &= P_{n+1|k} H_{n+1}^T (H_{n+1} P_{n+1|k} H_{n+1}^T + R_{n+1} + A_{n+1})^{-1} \\
\hat{X}_{n+1|n+1} &= \hat{X}_{n+1|k} + K_{n+1} [Z_{n+1} - \mu_{n+1} - h(\hat{X}_{n+1|k}) - \delta_{n+1}^2 / 2] \\
P_{n+1|n+1} &= \left(I - K_{n+1} H_{n+1}\right) P_{n+1|k}
\end{aligned}
\]

\(H_{n+1}\) and \(\delta_{n+1}^2\) are the Jacobian matrix and Hessian matrix of \(h(X_k)\) at \(\hat{X}_{n+1|k}\). \(A_{n+1}\) is an incidental term of the covariance when the second-order extended Kalman filter is applied, and its specific form is given in [10]. After the tracking process is completed every cycle, the position of the predicted interception point can be calculated based on the target's motion state vector and the motion model.

4. Simulation

4.1. Simulation environment settings
Simulations are performed in the scenario of the interception process of the APS. The tracking and interception point prediction process of the proposed algorithm is compared with the traditional least-squares algorithm.

Suppose the target trajectory starts with the range of 100m, the azimuth of 35°, the elevation of -5°, and the speed of -300m/s; the standard deviation of the azimuth and elevation measurements are both 0.5°; the standard deviation of the radar range and radial velocity measurements are 0.5m and 1m/s, respectively; the correlation coefficient of range and radial velocity measurements \(\rho = 0.5\); the radar detection period is 3ms; the sliding time window width \(T_w = 6\) and the spatial variation threshold \(\delta_{vp} = 0.5m\); The radius of the protective surface is 10m, and it is assumed that at the current target speed, to set aside time for the flight of the intercepting ammunition, the tracking and prediction process cut-off distance is 40m.

4.2. Simulation results
The EKF algorithm and the least-squares algorithm were used to track the simulated radar data, and the results are shown in figure 1. In figure 1, the least-squares trajectory is a steady-state after recursing to the cut-off distance. It can be seen that the least-squares algorithm can obtain higher steady-state accuracy when there are enough radar data points; the EKF algorithm has faster convergence speed, and its tracking process can converge in about 10 detection cycles.

![Figure 1. The target trajectory.](image)
The Monte Carlo simulation of the APS interception process was performed 100 times. For each radar detection cycle, the root-mean-square value $D_{wp}$ of the spatial distances between the predicted and actual interception points was used as an indicator to investigate the convergence process of different prediction algorithms. The results are shown in figure 2.

Figure 2. The root-mean-square error of interception point prediction.

It can be seen from figure 2 that the least-squares algorithm has a larger prediction error in the initial stage of the trajectory, which results in a slower convergence process. It takes about 30 cycles to keep the prediction error below 2m. While the prediction error of the EKF algorithm can stay at the same level from the beginning, which can significantly reduce the judgment time of effective incoming targets. When the number of radar data points is enough, the steady-state error predicted by the least-squares algorithm is slightly smaller than the EKF algorithm. The convergence judgment criterion proposed can effectively evaluate the prediction accuracy of the least-squares algorithm, thereby achieving a balance between the convergence speed in the initial stage and the steady-state accuracy.

5. Conclusion
Fast and accurate prediction of the trajectory is the prerequisite for the successful interception of the incoming target by the active protection system of armoured vehicles. To make up for the large error and slow convergence of the least-squares prediction algorithm in the initial tracking stage, this paper proposes an interception point prediction algorithm that combines the Kalman filter and least-squares estimation. By dynamically determining the convergence of the least-squares prediction, a converted measurement Kalman filter is used in the initial tracking stage. Simulations show that the algorithm can effectively improve the interception point prediction accuracy at the initial tracking stage, while retaining the high steady-state accuracy of the least-squares algorithm when there are enough measured data. It can help improve the target tracking performance of the APS and is suitable for engineering applications.

References
[1] Li C, Song H and Xin J 2015 Active protection system (APS) theory and design (Beijing: Ordnance Industry Press)
[2] Kwon J, Kwak N, Yang E and Kim K 2018 22nd Int. Microwave and Radar Conf. (Poznan: IEEE) p 395-9
[3] Raj K D and Krishna I M 2015 2nd Int. Conf. on Electronics and Communication Systems (Coimbatore: IEEE) p 878-83

[4] Shi Y, Yang Z, Zhang T, Lin N, Zhao Y and Zhao Y 2018 IEEE Int. Conf. on Smart Internet of Things (Xi’an: IEEE) p 250-4

[5] Bordonaro S V, Willett P, Bar-Shalom Y and Luginbuhl T 2018 IEEE Trans. Aerosp. Electron. Syst. 55 147-59

[6] Rohal P and Ochodnicky J 2017 Radar target tracking by Kalman and particle filter 2017 Communication and Information Technologies (Vysoke Tatry: IEEE) p 1-4

[7] Zhang Y 2017 Radar target tracking algorithm research under the nonlinear measurement (Chengdu: University of Electronic Science and Technology of China)

[8] Sun J, Chen X, Du Z, Lu Q and Xu G 2018 J. Test and Measurement Technology 32 468-74

[9] Bar-Shalom Y 2001 IEEE Trans. Aerosp. Electron. Syst. 37 1117-20

[10] Duan Z, Han C and Tao T 2005 Signal Processing 21 355-8

[11] Suchomski P 1999 IEEE Trans. Aerosp. Electron. Syst. 35 368-70

[12] Liu X 2006 Modern identification engineering (Beijing: National Defence Industry Press)