Constraining the universal lepton asymmetry

Vimal Simha\textsuperscript{1} and Gary Steigman\textsuperscript{2,3}

\textsuperscript{1} Department of Astronomy, Ohio State University, 140 West 18th Avenue, Columbus, OH 43210, USA
\textsuperscript{2} Department of Physics and Astronomy, Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
\textsuperscript{3} Center for Cosmology and Astro-Particle Physics, Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
E-mail: vsimha@astronomy.ohio-state.edu and steigman@pacific.mps.ohio-state.edu

Received 30 June 2008
Accepted 18 July 2008
Published 7 August 2008

Online at stacks.iop.org/JCAP/2008/i=08/a=011
doi:10.1088/1475-7516/2008/08/011

Abstract. The relic cosmic background neutrinos accompanying the cosmic microwave background (CMB) photons may hide a universal lepton asymmetry orders of magnitude larger than the universal baryon asymmetry. At present, the only direct way to probe such an asymmetry is through its effect on the abundances of the light elements produced during primordial nucleosynthesis. The relic light element abundances also depend on the baryon asymmetry, parameterized by the baryon density parameter ($\eta_B \equiv n_B/n_\gamma$) and on the early-universe expansion rate, parameterized by the expansion rate factor ($S \equiv H'/H$) or, equivalently, by the effective number of neutrinos ($N_\nu \equiv 3 + 43(S^2 - 1)/7$).

We use data from the CMB (and large scale structure: LSS) along with the observationally inferred relic abundances of deuterium and helium-4 to provide new bounds on the universal lepton asymmetry, finding for $\eta_L$, the analog of $\eta_B$, $0.072 \pm 0.053$ if it is assumed that $N_\nu = 3$ and, $0.115 \pm 0.095$ along with $N_\nu = 3.3^{+0.7}_{-0.6}$, if $N_\nu$ is free to vary.

Keywords: cosmological neutrinos, neutrino properties, big bang nucleosynthesis

ArXiv ePrint: 0806.0179
Constraining the universal lepton asymmetry

Contents

1. Introduction 2
2. Neutrino degeneracy and BBN 3
3. Adopted primordial abundances 5
4. Neutrino degeneracy ($\xi \neq 0$): standard expansion rate ($N_\nu = 3$) 6
5. Neutrino degeneracy ($\xi \neq 0$): non-standard expansion rate ($N_\nu \neq 3$) 7
6. Discussion and conclusions 8
Acknowledgments 11
References 11

1. Introduction

The standard models of particle physics and cosmology assume that, in the early, radiation-dominated universe only the known, massless or light ($mc^2 \ll kT$) particles, including three flavors of light, active neutrinos ($N_\nu = 3$), contribute to the energy density driving the universal expansion ($H^2 = 8\pi G \rho / 3$; $\rho = \rho_R$). It is also generally assumed that any universal lepton asymmetry is very small, of the order of the baryon asymmetry$^4$. However, as emphasized by Harvey and Kolb [19] and Langacker et al [26], this need not be the case. For example, Casas et al [11] have used the Affleck–Dine scenario [3] to produce a large lepton asymmetry which, if large enough, suppresses the B- and L-violating sphaleron transitions which tend to equilibrate the baryon and lepton numbers. We favor the argument of Serpico and Raffelt [41] that, since there is no experimental evidence for or against sphaleron effects, BBN and the CMB may provide a path to testing them.

In analogy with the measure of the baryon asymmetry provided by $\eta_B$, an asymmetry between neutrinos and antineutrinos of flavor $\alpha$ ($\alpha = e, \mu, \tau$) can be described in terms of the neutrino chemical potential $\mu_\alpha$ or, in terms of the dimensionless degeneracy parameter $\xi_\alpha \equiv \mu_\alpha / kT$, by

$$\eta_L \equiv \sum_\alpha n_{\nu_\alpha} - n_{\bar{\nu}_\alpha} = \frac{\pi^3}{12 \zeta(3)} \sum_\alpha \left[ \frac{\xi_\alpha}{\pi} \right] + \left[ \frac{\xi_\alpha}{\pi} \right]^3. \tag{1}$$

For big bang nucleosynthesis (BBN) the electron neutrinos play a key role through their charged-current weak interactions, which help to regulate the neutron-to-proton ratio ($p + e^- \leftrightarrow n + \nu_e$, $n + e^+ \leftrightarrow p + \bar{\nu}_e$, $n \leftrightarrow p + e^- + \bar{\nu}_e$). Since the BBN-predicted $^4$He abundance is, to a very good approximation, determined by the neutron-to-proton ratio at BBN, changes from the standard model value of this ratio will be reflected in

$^4$ Charge conservation ensures that the very small proton–antiproton asymmetry, of order $\eta_B$, is balanced by a correspondingly small asymmetry between electrons and positrons but there are no such constraints on the size of any asymmetry between neutrinos and antineutrinos.
its primordial abundance and, to a lesser extent, in the abundances of the other light elements produced during BBN. These relic abundances therefore provide probes of a universal lepton asymmetry. For further discussion and previous analyses, see, e.g., [4], [6]–[8], [12, 13, 16, 17, 20, 21, 23, 24], [35]–[37], [39]–[41], [44]–[46], [48].

There is another way in which a significant neutrino degeneracy may influence the early evolution of the universe. In the standard models of particle physics and cosmology, the energy density at, or just prior to, BBN is contributed by the cosmic background radiation photons, $e^\pm$ pairs and three flavors of extremely relativistic neutrinos. In the standard cosmology, the neutrinos constitute 40% of the total energy density. Any modification of the early-universe energy density (or expansion rate; see, e.g., [42]) can be parameterized in terms of the ‘effective’ number of neutrinos by

$$\rho \rightarrow \rho^\prime \equiv \rho + \Delta N_\nu \rho_\nu.$$  

For the standard models, $N_\nu = 3$ at BBN, while in the post-$e^\pm$ annihilation epoch probed by the CMB, $N_\nu = 3.046$ [28]. A secondary effect of neutrino degeneracy is to increase the energy density in relativistic neutrinos predicted by the standard model, so that $N_\nu \rightarrow N_\nu + \Sigma_\alpha \Delta N_\nu(\xi_\alpha)$, where

$$\Delta N_\nu(\xi_\alpha) \equiv \frac{30}{7} \left( \frac{\xi_\alpha}{\pi} \right)^2 + 15 \left( \frac{\xi_\alpha}{\pi} \right)^4.$$  

In general, if the three active neutrino flavors mix only with each other, neutrino oscillations ensure that their degeneracies equilibrate prior to BBN [1, 15, 27, 47]. In the following we will assume this is the case and use $\xi_e = \xi_\mu = \xi_\tau$. As a result

$$\eta_L = \frac{\pi^3}{4 \zeta(3)} \left[ \left( \frac{\xi_e}{\pi} \right) + \left( \frac{\xi_\mu}{\pi} \right)^3 \right],$$  

and

$$\Sigma_\alpha \Delta N_\nu(\xi_\alpha) = \frac{90}{7} \left( \frac{\xi_e}{\pi} \right)^2 + \frac{45}{7} \left( \frac{\xi_\mu}{\pi} \right)^4.$$  

Notice that, if $\xi_e = \xi_\mu = \xi_\tau$, then for $|\xi_e| \lesssim 0.2$, $\Sigma_\alpha \Delta N_\nu(\xi_\alpha) \lesssim 0.05$. Later, in section 6, we will relax this assumption so that $\xi_e \neq \xi_\mu$ and/or $\xi_\tau$. The only effect of non-zero values of $\xi_\mu$ and/or $\xi_\tau$ is to change the effective value of $N_\nu$, while non-zero values of $\xi_e$ also contribute to $N_\nu$ and, more importantly, they modify the neutron-to-proton ratio at BBN.

### 2. Neutrino degeneracy and BBN

The stage is being set for BBN when the universe is $\lesssim 0.1$ s old and the temperature is $\gtrsim$ a few MeV, at which time the neutral-current weak interactions ($e^+ + e^- \leftrightarrow \nu_\alpha + \bar{\nu}_\alpha$; $\alpha \equiv e, \mu, \tau$) are sufficiently rapid to maintain the neutrinos in equilibrium with the photon–$e^\pm$ plasma. When the temperature drops below $\sim 2$ MeV, the neutrinos begin to decouple from the photon–$e^\pm$ plasma. However, the electron neutrinos continue to interact with the neutrons and protons through their charged-current weak interactions ($n + \nu_e \leftrightarrow p + e^-$, $p + \bar{\nu}_e \leftrightarrow n + e^+$, $n \leftrightarrow p + e^- + \bar{\nu}_e$), enabling the neutron-to-proton ratio to track its equilibrium value of $n/p = \exp(-\Delta m/kT)$, where $\Delta m$ is the neutron–proton mass difference. In the presence of a neutrino–antineutrino asymmetry for the electron...
neutrinos, where $\xi_e \equiv \xi(\nu_e) = -\xi(\bar{\nu}_e)$, the neutron-to-proton equilibrium ratio is shifted to $n/p = \exp(-\Delta m/kT - \xi_e)$ which, for $\xi_e > 0$, results in fewer neutrons and less $^4\text{He}$.

While the BBN-predicted abundances of D, $^3\text{He}$ and $^7\text{Li}$ are most sensitive to the baryon density, that of $^4\text{He}$ is very sensitive to the neutron abundance when BBN begins and, therefore, to any electron neutrino degeneracy. The primordial abundances of D, $^3\text{He}$ or $^7\text{Li}$ are baryometers, constraining $\eta_B$, while the $^4\text{He}$ primordial mass fraction ($Y_P$) is a leptometer, constraining $\xi_e$. In the standard model of particle physics and cosmology with three species of neutrinos and their respective antineutrinos, the primordial element abundances depend on only one free parameter, the baryon density parameter, the post-$e^{\pm}$ annihilation ratio (by number) of baryons to photons, $\eta_B = n_B/n_\gamma$. This parameter is related to baryon mass density parameter, $\Omega_B$, the present-universe ratio of the baryon mass density to the critical mass–energy density (see [43]) by

$$\eta_{10} = 10^{10}n_B/n_\gamma = 273.9 \Omega_B h^2. \tag{5}$$

The abundance of $^4\text{He}$ is sensitive to $\xi_e$ and to the early-universe expansion rate $S \equiv H'/H = (\rho'/\rho)^{1/2} = (1 + 7\Delta N_\nu/43)^{1/2}$ [44]. In contrast, the abundance of $^4\text{He}$ is relatively insensitive to the baryon density since, to first order, all neutrons available when BBN begins are rapidly converted to $^4\text{He}$. To a very good approximation [23,44]

$$Y_P = (0.2482 \pm 0.0006) + 0.0016(\eta_{\text{He}} - 6), \tag{6}$$

where

$$\eta_{\text{He}} \equiv \eta_{10} + 100(S - 1) - 574\xi_e/4. \tag{7}$$

In equation (6), the effect of incomplete neutrino decoupling on the $^4\text{He}$ mass fraction is accounted for according to the results of [28].

In contrast to $^4\text{He}$, since the primordial abundances of D, $^3\text{He}$ and $^7\text{Li}$ are set by the competition between two body production and destruction rates, they are more sensitive to the baryon density than to a non-zero lepton asymmetry or to a non-standard expansion rate. For a primordial ratio of D to H by number, $y_D \equiv 10^5(D/H)_P$, in the range, $2 \lesssim y_D \lesssim 4$, to a very good approximation [23,44]:

$$y_D = 2.64(1 \pm 0.03)(6/y_D)^{1.6}, \tag{8}$$

where

$$\eta_D \equiv \eta_{10} - 6(S - 1) + 5\xi_e/4. \tag{9}$$

The effect of incomplete neutrino decoupling on this prediction is at the $\sim 0.3\%$ level [28], about ten times smaller than the overall error estimate.

A good fit to the primordial ratio of $^7\text{Li}$ to H by number, $y_{\text{Li}} \equiv 10^{10}(\text{Li/H})_P$, is provided by [23,44]

$$y_{\text{Li}} = 4.24(1 \pm 0.1)(\eta_{\text{Li}}/6)^2, \tag{10}$$

where

$$\eta_{\text{Li}} \equiv \eta_{10} - 3(S - 1) - 7\xi_e/4. \tag{11}$$

In our previous paper [42] we assumed that $\xi_e = 0$ and concentrated on the constraints on $\eta_{10}$ and $N_e$ which follow from BBN and the CMB (supplemented by large scale structure.
(LSS) data). Here we will first set $N_\nu = 3$ ($S = 1$) and repeat our analysis for $\eta_{10}$ and $\xi_e$. For consistency, in this case we will need to confirm that $|\xi_e|$ is bounded to be sufficiently small to justify the assumption that $N_\nu = 3 + \Sigma_\alpha \Delta N_\nu (\xi_\alpha) \approx 3$. Then, we will relax the assumption that $N_\nu = 3$ and use a combined BBN/CMB/LSS analysis to constrain all three parameters. In this case, any contribution to $\Sigma_\alpha \Delta N_\nu (\xi_\alpha)$ is automatically accounted for in our self-consistent determination of $N_\nu$.

Before discussing our results, the primordial abundances adopted for our analysis are outlined.

3. Adopted primordial abundances

Since deuterium is destroyed as gas is cycled through stars, the deuterium abundance inferred from high redshift (i.e. young), low metallicity (i.e. little stellar processing) QSO absorption line systems should provide the best estimate of its primordial abundance. The weighted mean of the seven, high redshift, low metallicity D/H ratios from [9, 10, 22, 30, 31, 33] and, most recently, from [34] is

$$y_D = 2.70^{+0.22}_{-0.20}. \quad (12)$$

For this relic abundance

$$\eta_D = 5.92^{+0.30}_{-0.33}. \quad (13)$$

Since the post-BBN evolution of $^4$He is also monotonic, with its mass fraction, $Y$, increasing along with increasing metallicity, at low metallicity, the $^4$He abundance should approach its primordial value $Y_P$. Observations of helium and hydrogen recombination lines from low metallicity, extragalactic H II regions are of most value in determining $Y_P$. At present, corrections for systematic uncertainties dominate the estimates of the observationally inferred $^4$He primordial mass fraction and, especially, its error. Following [42, 44], we adopt for our estimate here

$$Y_P = 0.240 \pm 0.006. \quad (14)$$

In this case

$$\eta_{He} = 0.88 \pm 3.75. \quad (15)$$

While the central value of $Y_P$ adopted here is low, the conservatively estimated uncertainty is relatively large (some ten times larger than the uncertainty in the BBN-predicted abundance for a fixed baryon density). In this context, it should be noted that, although very careful studies of the systematic errors in very limited samples of H II regions provide poor estimators of $Y_P$ as a result of their uncertain and/or model-dependent extrapolation to zero metallicity, they are of value in providing a robust upper bound to $Y_P$. Using the results of [18, 29, 32], we follow [42, 44] in adopting a $\sim 2\sigma$ upper bound of $Y_P \leq 0.255$. This upper bound to $Y_P$ corresponds to an upper bound to $\eta_{He} \leq 10.25$.

Although the BBN-predicted relic abundance of lithium provides a potentially sensitive baryometer ($[Li/H] \propto \eta_{10}^2$, for $\eta_{10} > 4$), its post-BBN evolution is complex and model-dependent. In addition, lithium is only observed in the Galaxy, in its oldest, most metal-poor stars in galactic globular clusters and in the halo. However, these oldest galactic stars have had the most time to modify their surface lithium abundances,
leading to the possibility that the observationally inferred abundances may require large, uncertain, model-dependent corrections in order to use them to infer the primordial abundance of $^7\text{Li}$.

In the absence of corrections for depletion, dilution or gravitational settling, the data of [2,38] suggest

$$[\text{Li}]_P \equiv 12 + \log(\text{Li}/\text{H}) = 2.1 \pm 0.1.$$  

(16)

In contrast, in an attempt to correct for evolution of the surface lithium abundances, Korn et al [25] use their observations of a small, selected sample of stars in the globular cluster NGC6397, along with stellar evolution models which include the effect of gravitational settling, to infer

$$[\text{Li}]_P = 2.5 \pm 0.1.$$  

(17)

In the following analysis the primordial abundances of D and $^4\text{He}$ adopted here are used, along with CMB/LSS data, to estimate $\eta_{10}$, $\xi_e$ and $\Delta N_\nu$. Then, using the inferred best-fit values and uncertainties in these three parameters, the corresponding BBN-predicted abundance of $^7\text{Li}$ is derived and compared to its observationally inferred value.

4. Neutrino degeneracy ($\xi \neq 0$): standard expansion rate ($N_\nu = 3$)

If attention is restricted to BBN alone, then there are two parameters inferred from observables, \{\eta_D, \eta_{\text{He}}\}, and two unknowns, \{\eta_{10}, \xi_e\}. These are related by

$$\xi_e = 4(\eta_D - \eta_{\text{He}})/579$$  

(18)

and

$$\eta_{10} = (574\eta_D + 5\eta_{\text{He}})/579.$$  

(19)

In the left-hand panel of figure 1 are shown the 68% and 95% contours in the $\xi_e$–$\eta_{10}$ plane derived from the adopted BBN abundances. Notice that $\xi_e$ and $\eta_{10}$ are virtually uncorrelated. From this approach it is found that $\xi_e = 0.035 \pm 0.026$ and that $\eta_{10} = 5.88^{+0.30}_{-0.33}$. The CMB/LSS provide an independent constraint on the baryon density,
\( \eta_{10} = 6.14^{+0.16}_{-0.11} \) \cite{42}. Since these two estimates for \( \eta_{10} \) are in agreement, we may use the CMB/LSS data to further restrict the allowed parameter space in the \( \xi_e - \eta_{10} \) plane, as shown in the right-hand panel of figure 1. In this case, the value of \( \xi_e \) is unchanged and \( \eta_{10} = 6.11^{+0.12}_{-0.11} \).

Using these results for \( \{ \eta_{10}, \xi_e \} \), the primordial abundance of \(^7\text{Li} \) can be predicted using equations (10) and (11). In this case we find that \( \eta_{\text{Li}} = 6.05^{+0.13}_{-0.12} \) and \([\text{Li}] = 2.63^{+0.04}_{-0.05} \). The non-zero value of \( \xi_e \) which is consistent with BBN and the CMB is incapable of reconciling the BBN-predicted and observationally inferred primordial abundances of \(^7\text{Li} \).

An alternate approach which mixes BBN and the CMB provides another way to constrain the lepton asymmetry. Since the \(^4\text{He} \) abundance is much more sensitive to \( \xi_e \) than is the deuterium abundance, equation (7) can be used along with the CMB value of \( \eta_{10} \) to constrain \( \xi_e \). This results in a virtually identical bound to that found from BBN alone, \( \xi_e = 0.037 \pm 0.026 \). In contrast, combining the deuterium abundance with the CMB value of \( \eta_{10} \) using equation (9) gives \( \xi_e = -0.176 \pm 0.272 \), a range that is too broad to be of much value.

From these results we see that, at 95% confidence, \( |\xi_e| < 0.09 \), so that \( \Sigma_\alpha \Delta N_\alpha(\xi_\alpha) < 0.01 \). Such a small neutrino degeneracy has a negligible effect on the universal expansion rate during radiation domination, confirming the validity of the assumption that \( S = 1(N_\nu = 3) \). The corresponding lepton asymmetry \( \eta_e = 0.072 \pm 0.053 \), while ‘small’, is orders of magnitude larger than the universal baryon asymmetry \( (\eta_B = 10^{-10} \eta_{10} = 6 \times 10^{-10}) \).

5. Neutrino degeneracy \( (\xi \neq 0) \): non-standard expansion rate \( (N_\nu \neq 3) \)

In this case, with \( N_\nu \) and \( \xi_e \) free, there are three parameters to be determined: \( \{ \eta_{10}, N_\nu, \xi_e \} \), but only two \textit{useful} relic abundances \( \{ y_D, Y_e \} \). To constrain these three parameters it is necessary to employ the CMB/LSS data along with that from BBN. To obtain the best constraints a bit of care is required. While the CMB/LSS provide a very good constraint on the baryon density parameter, they are less successful, at present, in constraining \( N_\nu \). For example, Simha and Steigman \cite{42} find from the CMB/LSS data that, while \( \eta_{10} = 6.14^{+0.16}_{-0.11} \) and \( N_\nu = 2.9^{+1.0}_{-0.8} \) (or \( S - 1 = -0.008^{+0.070}_{-0.068} \)). Such a large range in \( N_\nu \) will dilute the constraint on \( \xi_e \), resulting in a less than optimal constraint on \( \xi_e \) given the available data. However, as Simha and Steigman \cite{42} have noted, from the CMB/LSS the constraints on \( \eta_{10} \) and \( N_\nu \) are virtually uncorrelated. Therefore, in the following we adopt for our observational input \( \{ \eta_{10}, y_D, \eta_{\text{He}} \} \) and derive \( N_\nu \) and \( \xi_e \) from them. Since \( \xi_e \) is invisible to the CMB (except due to its contribution to the radiation energy density, which is accounted for in \( N_\nu \)), we may combine our constraints on the \( \{ N_\nu, \xi_e \} \) combination with the independent constraint on \( N_\nu \) from the CMB, to further constrain this combination.

With \( S(N_\nu) \) and \( \xi_e \) to be constrained from \( \eta_{10}, \eta_D, \) and \( \eta_{\text{He}} \), equations (7) and (9) may be recast as

\[
S - 1 = (579\eta_{10} - 574\eta_D - 5\eta_{\text{He}})/2944
\]

and

\[
\xi_e = (106\eta_{10} - 100\eta_D - 6\eta_{\text{He}})/736.
\]
Figure 2. In the left-hand panel are the 68% and 95% contours in the $\xi_e-N_\nu$ plane inferred from BBN and the adopted relic abundances of D and $^4$He, along with the CMB constraint on $\eta_{10}$. In the right-hand panel are the BBN constraints convolved with the CMB constraints on $N_\nu$ and $\eta_{10}$.

In the left-hand panel of figure 2 are shown the 68% and 95% contours in the $\xi_e-N_\nu$ plane derived from the adopted BBN abundances in combination with the constraint on $\eta_{10}$ from the CMB/LSS. From this approach we find $\xi_e = 0.073^{+0.056}_{-0.057}$ and $N_\nu = 3.7 \pm 0.9$.

The CMB/LSS provide an independent constraint on $N_\nu$, $N_\nu = 2.9^{+1.0}_{-0.8}$ [42]. Since these two estimates are in agreement, the CMB/LSS data may be used to further restrict the allowed parameter space in the $\xi_e-N_\nu$ plane, as shown in the right-hand panel of figure 2. In this case, we obtain $\xi_e = 0.056 \pm 0.046$ and $N_\nu = 3.3^{+0.7}_{-0.6}$.

Using these results for $\{\eta_{10}, N_\nu, \xi_e\}$, the primordial abundance of $^7$Li can be predicted using equations (10) and (11). In this case we find $[\text{Li}] = 2.62^{+0.06}_{-0.06}$. A non-zero value of $\xi_e$ along with a non-standard expansion rate, consistent with BBN and the CMB, are still incapable of reconciling the BBN-predicted and observationally inferred primordial abundances of $^7$Li.

An alternate approach, mixing BBN and the CMB, provides another way to constrain the lepton asymmetry. Since the $^4$He abundance is much more sensitive to $\xi_e$ than is the deuterium abundance, equation (7) can be used along with the CMB/LSS values of $\eta_{10}$ and $N_\nu$ to constrain $\xi_e$. This results in a virtually identical bound, $\xi_e = 0.053 \pm 0.046$. In contrast, combining the deuterium abundance with the CMB value of $\eta_{10}$ using equation (9) yields $\xi_e = -0.061^{+0.364}_{-0.376}$, whose range is too broad to be of much value.

From these results, the effect of the neutrino degeneracy on the universal expansion rate during radiation domination may be computed, yielding $\Sigma_\alpha \Delta N_\nu(\xi_\alpha) \leq 0.03$ at 95% confidence. The corresponding lepton asymmetry, $\eta_L = 0.115 \pm 0.095$, while constrained to be ‘small’, may nonetheless be orders of magnitude larger than the universal baryon asymmetry ($\eta_B = 10^{-10} \eta_{10} = 6 \times 10^{-10}$).

6. Discussion and conclusions

The left-hand panel of figure 3 shows the probability distribution of $\xi_e$ for the standard expansion rate $S = 0$ ($N_\nu = 3$), derived after marginalizing over $\eta_{10}$, from BBN and the adopted primordial abundances of D and $^4$He. The right-hand panel of figure 3 shows the
Constraining the universal lepton asymmetry

**Figure 3.** The left-hand panel shows the probability distribution for $\xi_e$, inferred from BBN and the adopted relic abundances of D and $^4$He, for the case where $N_\nu = 3$. In the right-hand panel, the dashed curve is the BBN constraint convolved with the CMB/LSS constraint on $\eta_{10}$ alone and the solid curve is the BBN constraint convolved with the CMB/LSS constraint on $\eta_{10}$ and $N_\nu$.

**Figure 4.** The dashed curve shows the probability distribution for $N_\nu$ for $\xi_e = 0$ inferred from BBN and the relic abundances of D and $^4$He convolved with the CMB/LSS constraints on $\eta_{10}$ and $N_\nu$. The solid curve shows the same for $\xi_e \neq 0$.

The probability distribution of $\xi_e$ for the more general case where a non-standard expansion rate $S \neq 0 (N_\nu \neq 3)$ is allowed. The constraints are based on combining BBN and the adopted primordial abundances of D and $^4$He with the independent constraints from the CMB/LSS. Additional information from the CMB/LSS constraints on $\eta_{10}$ are used before marginalizing over $\eta_{10}$ and $N_\nu$ to produce the dotted curve, while additional information from the CMB/LSS constraints on both $\eta_{10}$ and $N_\nu$ are used before marginalizing over $\eta_{10}$ and $N_\nu$ to produce the solid curve.

The dashed curve in figure 4 shows the probability distribution of $N_\nu$ for $\xi_e = 0$, derived from BBN and the adopted primordial abundances of D and $^4$He with the independent constraints from the CMB/LSS after marginalizing over $\eta_{10}$. The solid curve in figure 4 shows the probability distribution of $N_\nu$ for the more general case where a
Constraining the universal lepton asymmetry

non-zero neutrino degeneracy $\xi_e \neq 0$ is allowed. The constraints are based on combining BBN and the adopted primordial abundances of D and $^4$He with the independent constraints from the CMB/LSS. Additional information from the CMB/LSS constraints on $\eta_{10}$ and $N_\nu$ are used before marginalizing over $\eta_{10}$ and $\xi_e$. Both of these constraints on $N_\nu$ are consistent with each other and with the standard model prediction of $N_\nu = 3$.

Our results can be used to constrain any deviation in the universal expansion rate from its standard model value due to the increase in radiation energy density from neutrino degeneracy. Our constraint on the neutrino degeneracy leads to $\Sigma_\nu \Delta N_\nu(\xi_e) \lesssim 0.03$ at 95% confidence. In addition, the constraint on the neutrino degeneracy yields a corresponding constraint on any lepton asymmetry, $\eta_L$: $\eta_L = 0.072 \pm 0.053$ for $N_\nu = 3$ and $\eta_L = 0.115 \pm 0.095$ when $N_\nu \neq 3$.

Using the constraints on $\eta_{10}$, $N_\nu$ and $\xi_e$, the BBN-predicted primordial abundance of $^7$Li may be inferred. For $N_\nu = 3$, $[\text{Li}] = 2.63^{+0.04}_{-0.05}$ and for $N_\nu \neq 3$, $[\text{Li}] = 2.62^{+0.05}_{-0.06}$. Both of these are considerably higher than the value ($[\text{Li}]_P = 2.1 \pm 0.1$) determined from observations of metal-poor halo stars [2,38] without any correction for depletion, destruction or gravitational settling. If, however, the correction proposed by Korn et al [25] is applied, the predicted and observed $^7$Li abundances may, perhaps, be reconciled. It remains an open question whether this lithium problem is best resolved by a better understanding of stellar physics or if it is providing a hint of new physics beyond the standard model.

In our analysis we have assumed that $\xi_e = \xi_\mu = \xi_\tau$. Suppose instead that $\xi_\mu = \xi_\tau \neq \xi_e$ or that $\xi_e = \xi_\mu \neq \xi_\tau$ [14]. Since our constraints on $\xi_e$ come from BBN, and they constrain $\xi_e$ to be sufficiently small that the allowed degeneracy has minimal effect on the universal expansion rate ($\Delta N_\nu(\xi_e) \lesssim 0.01$), the only way to probe non-zero values of $\xi_\alpha \equiv \xi_\mu \equiv \xi_\tau$ or $\xi_\alpha = \xi_\tau$ is through their effect on the expansion rate ($\Sigma_\nu \Delta N_\nu$). In these cases, it is possible that $\xi_\alpha$ may be $\gg \xi_e$. For $N_\nu = 3.3^{+0.7}_{-0.6}$, $\Sigma_\nu \Delta N_\nu(\xi_\alpha) \lesssim 1.7$ at $\sim 2\sigma$. If it is assumed that $\xi_\alpha = \xi_\mu = \xi_\tau$, then $|\xi_\alpha| \lesssim 2.34$ and $|\eta_L| \lesssim 5.0$. If, instead, it is assumed that $\xi_\mu = \xi_e \ll \xi_\tau$ (or vice versa for $\xi_\mu$), then $|\xi_\tau| \lesssim 4.12$ and $|\eta_L| \lesssim 7.6$.

Of course, our results are sensitive to the relic abundances we have adopted. The simple but accurate fitting formulae [23] we have used make it easy to reevaluate our constraints for any adopted abundances. The constraint on $\xi_e$ is sensitive to the $^4$He abundance and is relatively insensitive to small changes in the D abundance. For example, we repeated our analysis for a different primordial $^4$He mass fraction, $Y_P = 0.247 \pm 0.004$. This alternative abundance, in combination with the D abundance used in this paper, and the independent constraints on $\eta_{10}$ and $N_\nu$ from the CMB/LSS from Simha and Steigman [42] yields $\xi_e = 0.023 \pm 0.041$.

Our results here are similar to, but considerably more restrictive than, those of [4] and of [41], due to the improved constraints on $N_\nu$ and $\eta_{10}$ from the WMAP five-year and other CMB and LSS data. The analysis described here seems to be related to that in recent papers by Popa and Vasile [35,36]. However, we fail to understand how they derive their constraints and why they find so much tighter bounds on $N_\nu$ and so much weaker bounds on $\xi_e$.

Except from its contribution to the radiation energy density and the early-universe expansion rate, a lepton asymmetry in the neutrino sector is invisible to the CMB. Future CMB experiments will improve the constraint on $N_\nu$ by measuring the neutrino anisotropic stress more accurately. According to Bashinsky and Seljak [5], Planck should determine $N_\nu$.
to an accuracy of $\sigma(N_{\nu}) \sim 0.24$ and CMBPOL, a satellite-based polarization experiment, might improve it further to $\sigma(N_{\nu}) \sim 0.09$, independent of the BBN constraints. Although still too large to provide a measure of the neutrino degeneracy, the tighter constraint on $N_{\nu}$ can be used to further narrow the allowed ranges of $\xi_e$ and $N_{\nu}$ shown in figure 2.

Acknowledgments

This research is supported at Ohio State University by a grant (DE-FG02-91ER40690) from the US Department of Energy. We thank J Beacom and G Gelmini for useful discussions.

References

[1] Abazajian K N, Beacom J F and Bell N F, 2002 Phys. Rev. D 66 013008 [SPIRES]
[2] Asplund M et al., 2006 Astrophys. J. 644 229 [SPIRES]
[3] Affleck I and Dine M, 1985 Nucl. Phys. B 249 361 [SPIRES]
[4] Barger V, Kneller J P, Langacker P, Marfatia D and Steigman G, 2003 Phys. Lett. B 569 123 [SPIRES]
[5] Bashinsky S and Seljak U, 2004 Phys. Rev. D 69 083002 [SPIRES]
[6] Beaudet G and Goret P, 1976 Astron. Astrophys. 49 415 [SPIRES]
[7] Beaudet G and Yahil A, 1977 Astrophys. J. 218 253 [SPIRES]
[8] Boesgaard A M and Steigman G, 1985 Ann. Rev. Astron. Astrophys. 23 319 [SPIRES]
[9] Burles S and Tytler D, 1998 Astrophys. J. 499 699 [SPIRES]
[10] Burles S and Tytler D, 1998 Astrophys. J. 507 732 [SPIRES]
[11] Casas A, Cheng W Y and Gelmini G, 1999 Nucl. Phys. B 538 297 [SPIRES]
[12] Cuoco A, Iocco F, Mangano G, Miele G, Pisanti O and Serpico P D, 2004 Int. J. Mod. Phys. A 19 4431 [SPIRES]
[13] David Y and Reeves H, 1980 Phil. Trans. R. Soc. A 296 415
[14] Dolgov A D and Takahashi F, 2004 Nucl. Phys. B 688 189 [SPIRES]
[15] Dolgov A D et al., 2002 Nucl. Phys. B 632 363 [SPIRES]
[16] Esposito S, Miele G, Pastor S, Peimbert M and Pisanti O, 2000 Nucl. Phys. B 590 539 [SPIRES]
[17] Freese K, Kolb E W and Turner M S, 1983 Phys. Rev. D 27 1689 [SPIRES]
[18] Fukugita M and Kawasaki M, 2006 Astrophys. J. 646 691 [SPIRES]
[19] Harvey J A and Kolb E W, 1981 Phys. Rev. D 24 2990 [SPIRES]
[20] Ichikawa K and Kawasaki M, 2003 Phys. Rev. D 67 063510 [SPIRES]
[21] Kang H-S and Steigman G, 1992 Nucl. Phys. B 372 494 [SPIRES]
[22] Kirkman D et al., 2003 Astrophys. J. Suppl. 149 1
[23] Kneller J P and Steigman G, 2004 New J. Phys. 6 117
[24] Kohri K, Kawasaki M and Sato K, 1997 Astrophys. J. 490 72 [SPIRES]
[25] Korn A J et al., 2006 Nature 442 657 [SPIRES]
[26] Langacker P, Segrè G and Soni S, 1982 Phys. Rev. D 26 3425 [SPIRES]
[27] Lunardini C and Smirnov A Y, 2001 Phys. Rev. D 64 073006 [SPIRES]
[28] Mangano G et al., 2005 Nucl. Phys. B 729 221 [SPIRES]
[29] Olive K A and Skillman E D, 2004 Astrophys. J. 617 29 [SPIRES]
[30] O’Meara J M et al., 2001 Astrophys. J. 552 718 [SPIRES]
[31] O’Meara J M et al., 2006 Astrophys. J. Lett. 649 L61
[32] Peimbert M, Luridiana V and Peimbert A, 2007 Astrophys. J. 666 636 [SPIRES]
[33] Pettini M and Bowen D V, 2001 Astrophys. J. 560 41 [SPIRES]
[34] Pettini M et al., 2008 Preprint 0805.0594
[35] Popa L A and Vasile A, 2007 J. Cosmol. Astropart. Phys. JCAP10(2007)017 [SPIRES]
[36] Popa L A and Vasile A, 2008 Preprint astro-ph/0804.2971
[37] Reeves H, 1972 Phys. Rev. D 6 3363 [SPIRES]
[38] Ryan S G et al., 2000 Astrophys. J. 530 L57 [SPIRES]
[39] Scherrer R J, 1983 Mon. Not. R. Astron. Soc. 205 683
[40] Schramm D and Steigman G, 1979 Phys. Lett. B 87 141 [SPIRES]
[41] Serpico P D and Raffelt G G, 2005 Phys. Rev. D 71 127301 [SPIRES]
[42] Simha V and Steigman G, 2008 J. Cosmol. Astropart. Phys. JCAP06(2008)016 [SPIRES]
Constraining the universal lepton asymmetry

[43] Steigman G, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)016 [SPIRES]
[44] Steigman G, 2007 Ann. Rev. Nucl. Part. Sci. 57 463 [SPIRES]
[45] Terasawa N and Sato K, 1985 Astrophys. J. 294 9 [SPIRES]
[46] Wagoner R V, Fowler W A and Hoyle F, 1967 Astrophys. J. 148 3 [SPIRES]
[47] Wong Y Y, 2002 Phys. Rev. D 66 025015 [SPIRES]
[48] Yahil A and Beaudet G, 1976 Astrophys. J. 206 26 [SPIRES]