Price fluctuations of commodities like cotton and wheat are thought to display probability distributions of returns that follow a Lévy stable distribution. Recent analysis of stocks and foreign exchange markets show that the probability distributions are not Lévy stable, a plausible result since commodity markets have quite different features than stock markets. We analyze daily returns of 29 commodities over typically 20 years and find that the distributions of returns decay as power laws with exponents \( \alpha \) which have values \( \alpha > 2 \), outside the Lévy-stable domain. We also find that the amplitudes of the returns display long-range time correlations, like stocks, while the returns themselves are uncorrelated for time lags \( \approx 2 \) days, much larger than for stocks ( \( \approx 4 \) min).

\[ g_i(t) = \frac{\ln S_i(t + \Delta t) - \ln S_i(t)}{\sigma_i}, \]

where \( \Delta t = 1 \) day, \( i = 1, 2, \ldots, 29 \) indexes the commodities, \( S_i(t) \) is the price and \( \sigma_i \) is the standard deviation. Figure 1 displays the price and corresponding returns of a typical commodity, high-sulphur fuel oil [Table I]. Figure 3 displays Hill estimates [16] of \( \alpha \) which have values \( \alpha > 2 \). Here we address the question whether the scaling of commodities is statistically distinguishable from that of stocks. We study the statistical properties of the price fluctuations for 29 commodities [3] and compare the results with the statistical properties of daily returns in stock markets.

We define the normalized price fluctuation (“return”) as:

\[ P(g_i > x) \sim \frac{1}{x^{\alpha_i}}, \]

where \( \alpha_i \) is outside the Lévy stable domain \( 0 < \alpha_i < 2 \). Figure 2 displays Hill estimates [16] of \( \alpha_i \) for all 29 commodities, calculated for \( x \) above a cutoff value \( x_{\text{cutoff}} \), which is estimated by evaluating \( \alpha_i \) starting from different trial values of \( x_{\text{cutoff}} \), and choosing \( x_{\text{cutoff}} \) as the minimum \( x \) value above which \( \alpha_i \) does not change.
significantly. Based on our analysis we choose for all 29 commodities the same value $x_{\text{cutoff}} = 2$. Note that for daily returns the number of data points beyond $x_{\text{cutoff}} = 2$ is typically 50-200. The average exponent is $\alpha = (\sum_{i=1}^{29} \alpha_i)/29 = 2.9 \pm 0.08$ for the positive tail and $\alpha = 2.7 \pm 0.07$ for the negative tail.

Next we compare our calculations of $\alpha_i$ with exponents $\alpha_i$ of daily returns evaluated for 7128 stocks from the CRSP database \cite{18}. We choose stocks in the same time period as the 29 commodities analyzed, and compute tail exponents of $P(x)$ by the same procedure. Figures 3(a) and 3(b) compare the probability density functions of tail exponents for both commodities and stocks. The pdf for stocks is same as is reported in \cite{1}, and the exponents are outside the Lévy stable region for both stocks and commodities.

We next discuss time correlations of returns. Figure 4(a) displays the autocorrelation function $\langle g_i(t)g_i(t+\tau) \rangle$ averaged over the 29 commodities. We observe that this averaged autocorrelation function $C(\tau)$ ceases to be statistically different from zero for time lags $\tau$ of 3 days or more. To further quantify time correlations, we use the detrended fluctuation analysis (DFA) method \cite{19}. The DFA method calculates fluctuations $F(n)$ in a time window of size $n$, and then plots $F(n)$ versus $n$. The slope $\hat{\alpha}$ in a log-log plot gives information about the correlations present. If $C(\tau) \sim \tau^{-\gamma}$ then $\hat{\alpha} = (2 - \gamma)/2$, while if $C(\tau) \sim e^{-\tau/\tau_c}$ then $\hat{\alpha} = 1/2$. We find in Fig. 4(b) that $\hat{\alpha} = 0.55 \pm 0.05$, consistent with the exponential decay of Fig. 3(a). We also observe that $|g_i|$, the absolute value of returns (one measure of volatility), are power law correlated with $\hat{\alpha} = 0.73 \pm 0.05$, which implies a power law decay of the autocorrelation of the absolute value of returns with $\gamma = 0.54 \pm 0.1$.

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\bibitem{17} If we assume that all 29 commodities follow the same distribution then we can aggregate the data, in this way we find $\alpha = 2.9 \pm 0.6$ for the positive and $\alpha = 2.7 \pm 0.05$ for the negative tail.
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\end{thebibliography}
| Index | Symbol | Description                                                                 | Period    | # points |
|-------|--------|------------------------------------------------------------------------------|-----------|----------|
| 1     | Brent  | Crude oil (UK Standard cf WTI in the US)                                     | 1/88–8/98 | 2770     |
| 2     | BUTANE | Butane chemical used as fuel                                                 | 2/93–8/98 | 1433     |
| 3     | Gasoil | Gas oil                                                                      | 1/88–7/93 | 1433     |
| 4     | HFO    | Heavy fuel oil                                                               | 1/88–8/98 | 2770     |
| 5     | HSFO arg | High-sulfur fuel oil from Arabian Gulf                                         | 1/88–8/98 | 2770     |
| 6     | HSFO New | High-sulfur fuel oil transported by New York barge                           | 1/88–8/98 | 2770     |
| 7     | Kero New | Kerosene transported by New York barge                                       | 1/88–8/98 | 2770     |
| 8     | LSFO New | Low-sulfur fuel oil transported by New York barge                           | 1/88–8/98 | 2770     |
| 9     | LSFO NYH | Low-sulfur fuel oil traded at New York Harbour                                | 1/88–8/98 | 2770     |
| 10    | Nap Med | Naphtha from Mediterranean (used for feed-stock)                             | 1/88–8/98 | 2770     |
| 11    | Nap New | Naphtha transported by New York barge                                         | 1/88–4/95 | 1897     |
| 12    | Prem unl | Premium unleaded automobile gasoline                                       | 6/92–8/98 | 1619     |
| 13    | CL     | Crude Oil                                                                    | 1/83–8/99 | 4164     |
| 14    | HO     | Heating Oil                                                                  | 9/79–8/99 | 4999     |
| 15    | C      | Corn                                                                         | 6/69–8/99 | 7593     |
| 16    | CT     | Cotton                                                                       | 6/69–8/99 | 7593     |
| 17    | FC     | Feeder Cattle                                                                | 7/79–8/99 | 5051     |
| 18    | KC     | Coffee                                                                       | 6/69–8/99 | 7593     |
| 19    | O      | Oats                                                                          | 6/69–8/99 | 7593     |
| 20    | S      | Soybeans                                                                      | 6/69–8/99 | 7593     |
| 21    | SB     | Sugar                                                                         | 1/80–8/99 | 4909     |
| 22    | W      | Wheat                                                                        | 1/75–8/99 | 6186     |
| 23    | LH     | Live Hogs                                                                     | 6/69–8/99 | 7593     |
| 24    | PB     | Pork Bellies                                                                  | 6/69–8/99 | 7593     |
| 25    | GC     | Gold                                                                          | 6/69–8/99 | 7593     |
| 26    | HG     | Copper High Grade                                                            | 1/71–8/99 | 7195     |
| 27    | PA     | Palladium                                                                     | 6/87–8/99 | 3039     |
| 28    | PL     | Platinum                                                                      | 6/87–8/99 | 3039     |
| 29    | SI     | Silver                                                                        | 6/69–8/99 | 7593     |
FIG. 1. (a) Prices $S_i(t)$ for a typical commodity, high-sulphur fuel oil HSFO arg [Table I]. (b) Normalized returns $g_i(t)$ defined by Eq. (1). Note the large fluctuations of up to $20\sigma$, which would have probability $\approx e^{-200}$ for a Gaussian distribution.

FIG. 2. Cumulative distributions of positive and negative returns for high-sulphur fuel oil (HSFO arg) [Table I]. The tails of the distributions shown have power law decays with an exponent $\alpha_5 = 2.6 \pm 0.3$ for the negative tail and $\alpha_5 = 2.9 \pm 0.06$ for the positive tail, where the exponents and the error bars are estimated by Hill’s method [16].

FIG. 3. (a) Exponents $\alpha_i$ of the negative tail and (b) the positive tail, where $i = 1, 2, \ldots, 29$ indexes the 29 commodities analyzed [Table I]. We employ Hill’s method [16] to estimate the exponent $\alpha_i$ of each probability distribution in the range $x \geq x_{\text{cutoff}}$ where we choose $x_{\text{cutoff}} = 2$. The dashed lines show the average values $\alpha = (\sum_{i=1}^{29} \alpha_i)/29$. Shaded regions indicate the range of Lévy stable exponents, $\alpha < 2$.

FIG. 4. Probability density function of (a) the negative tail exponents and (b) the positive tail exponents evaluated for the 7128 stocks and the 29 commodities analyzed. Observe that for both stocks and commodities the mean exponent is around 3.0, outside the Lévy stable region $0 < \alpha < 2$.

FIG. 5. (a) Autocorrelation function averaged over all 29 commodities. The autocorrelation function displays an exponential decay with a decay constant of 2.3 days (see inset). (b) The fluctuations $F(n)$ of returns $g_i$ and absolute value of returns $|g_i|$ averaged over all 29 commodities. The plot for $g_i$ is consistent with the exponential decay of Fig. 5(a). The exponent of $\tilde{\alpha} = 0.73 \pm 0.05$ for $|g_i|$ implies long-range power law correlation, a feature also seen for stock markets where $\tilde{\alpha} \approx 0.8$ [1].