Thermoacoustic analysis of a pulse tube refrigerator

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Abstract. Thermoacoustic devices use acoustic gas oscillations in place of pistons. They execute mutual energy conversion between work flow and heat flow through the heat exchange between the gas and the channel walls. Understanding of the acoustic field is necessary to control the resulting energy flows in thermoacoustic devices. We will present from experimental point of view the physical mechanism of a pulse tube refrigerator that is one of the travelling wave thermoacoustic heat engines.

1. Introduction
Simplifying heat engine’s hardware design while maintaining the highest possible efficiency has been a longstanding desire since the 19th century. One way to achieve this is to eliminate solid pistons from heat engines. A thermoacoustic device uses acoustic gas oscillations in place of pistons and hence it operates essentially with no moving parts.

Wheatley [1], Swift [2] and Tominaga [3] proposed work flow $I$ and heat flow $Q$ from a viewpoint of thermodynamics, and successfully gave a framework for understanding the thermoacoustic device working as a prime mover or a cooler. The work flow $I$ is equivalent to the acoustic intensity used in acoustics, and the heat flow $Q$ is the energy flow associated with the hydrodynamic transport of entropy. Thermoacoustic effects such as production and strong damping of $I$ by the temperature gradients [4], and occurrence of $Q$ against the temperature gradient take place in narrow channels where sound wave propagates. Such effects are governed by two factors; one is a nondimensional parameter $\omega \tau$ [3], a product of an angular frequency $\omega$ of the sound wave and a thermal relaxation time $\tau$ for a gas to equilibrate with the channel walls, and the other is the acoustic field expressed by the acoustic pressure and the axial acoustic particle velocity.

We will present the propagation of acoustic waves in narrow channels [5] and show the physical mechanism of a pulse tube refrigerator that is one of the thermoacoustic devices. Measurements of pressure and velocity oscillations show that pulse tube refrigerator controls the phase difference between them to enhance heat flow through the regenerator [6].

2. Sound waves in narrow and wide tubes
It was 1816 when Laplace gave a reasonable explanation on the speed of sound in air by assuming adiabatically reversible thermal processes. However, the sound waves in flow channels differ from the adiabatic one because of the thermal contact of the gas with the channel wall [7].
Figure 1 shows the propagation constant $\Gamma$ measured for sound waves running through cylindrical ducts with uniform temperature \cite{5}. Note that the acoustic plane wave can be expressed as $p = p_a \exp(i \omega t - k_0 \Gamma x)$, where $p_a$ is the pressure constant, $\omega$ is the angular frequency, and $k_0$ is the free space wave number given by $k_0 = \omega / c$ ($c$ is the adiabatic speed of sound). The phase velocity and attenuation constant are given by $v = c / \text{Im} \Gamma$, and $\beta = k_0 \text{Re} \Gamma$, respectively. Figure 1 presents the non-dimensional phase velocity $v/c$ and attenuation constant $\beta/k_0$ as a function of the parameter $\omega \tau$. Here, $\tau$ is the relaxation time for the gas to thermally equilibrate with the wall, which is given as $\tau = R^2 / (2 \alpha)$ by using the tube radius $R$ and the thermal diffusivity $\alpha$ of the gas.

Figure 1. Experimental propagation constants.

In the wide tube region with $\omega \tau \gg 2$, the phase velocity asymptotically reaches $c$, meaning that the entropy oscillation of the gas satisfies $S = 0$. Hence, the damping of the sound wave is not so strong because viscous and thermal interactions of the gas with the tube walls are weak. As a result, the thermoacoustic effects are negligibly small because $Q = 0$. On the other hand, in the narrow tube region with $\omega \tau \ll 2$, the sound wave shows anomalous dispersion, indicating that a high frequency wave travels faster than a low frequency wave. In this region, the gas motion reaches the isothermal process, and hence the entropy oscillation of the gas can be written as $S = (\partial S / \partial P)_T P$. However, very strong viscous damping, as indicated by a rapid increase of $\beta$ with decreasing $\omega \tau$, would mask any thermoacoustic effects in this region.

In the intermediate region with $\omega \tau \sim 2$, the gas can maintain the thermal contact with tube walls without suffering severe viscous damping. Thus the thermoacoustic devices adopt the porous material with this $\omega \tau$ region to promote the thermoacoustic effects. The porous materials roughly with $\omega \tau > 1$ are called as a stack, and those with $\omega \tau < 1$ as a regenerator. Figure 2 shows the results of the gain of acoustic power amplification \cite{4} when the traveling wave passed through the porous material having many square pores, where the gain difference $\Delta G$ between the gains with and without the temperature gradients along the pores are shown. A transition from the adiabatic sound wave to the isothermal one is visible near $\omega \tau \sim 2$, where $\Delta G$ changes rapidly with $\omega \tau$. Because the regenerator ($\omega \tau < 1$) can attain the thermal process close to the isothermal one, $\Delta G$ is larger than that for the stack ($\omega \tau > 1$),
meaning the regenerator-based thermoacoustic device can have a better thermal efficiency than that of the stack-based devices.

$$\Delta G$$ as a function of $$\omega \tau$$. Temperature ratio $$T_{\text{out}}/T_{\text{in}}$$ at ends of the regenerator was varied from 0.43 to 2.3.

3. Acoustic heat flow

The time averaged energy flux of the small-amplitude gas oscillation can be decomposed into the sum of heat flow $$Q$$ and work flow $$I$$ [1-3], where

$$I = \langle P \cdot V \rangle = \frac{1}{2} \rho |v| \cos \Phi$$  \hspace{1cm} (1)$$

and

$$Q = \rho_m T_m \langle S \cdot U \rangle.$$  \hspace{1cm} (2)$$

Here, $$P = \rho |v| e^{i\omega t}$$ is the acoustic pressure, $$U$$ is the axial acoustic particle velocity, and $$S$$ is the entropy oscillation of the gas; $$\rho_m$$ and $$T_m$$ are the time-averaged density and temperature, angular brackets mean the time average and over bar denotes the cross-sectional average; the cross-sectional averaged velocity $$V = \overline{U}$$ is expressed as $$V = |v| e^{i(\omega t + \Phi)}$$. If the isothermal process is achieved and viscous effects are sufficiently small, the heat flow can be expressed by inserting isothermal entropy oscillation $$S = (\partial S / \partial P)$$, $$P$$ into equation (2) as

$$Q = \rho_m T_m (\partial S / \partial P) \omega \tau I.$$  \hspace{1cm} (3)$$

Since $$(\partial S / \partial P) < 0$$, the magnitude of $$Q$$ is proportional to that of $$I$$, although its flow direction is opposite to it. As shown in equation (1), the work flow $$I$$ is maximized for the traveling wave phasing ($$\Phi = 0$$), whereas it becomes zero for the standing wave phasing ($$\Phi = \pm 90^\circ$$). Therefore, in order to enhance $$Q$$ in the thermoacoustic coolers, the appropriate phasing between $$P$$ and $$V$$ is necessary. Besides, the inevitable viscous losses should be kept small. In other words, the complex acoustic impedance given as

$$Z = \frac{P}{AV} = \frac{1}{A} \left| \frac{P}{A} \right| e^{i\Phi}$$  \hspace{1cm} (4)$$

needs to have a larger $$|Z|$$ and a traveling wave phasing $$\Phi = 0$$ at the regenerator ($$\omega \tau < 1$$) of the thermoacoustic cooler.
4. Pulse tube refrigerator

A pulse tube refrigerator is one of the regenerator-based thermoacoustic devices. The first pulse tube refrigerator, what we now call a basic pulse tube refrigerator, was developed in the mid 1960s [8]. It consists of a closed empty tube called a pulse tube, a regenerator made of stacked screen meshes and a compressor unit, as presented in figure 3(a). Merely by introducing gas oscillations into the pulse tube through the regenerator, the temperature at the end of the pulse tube decreased to 199 K. In 1984, Mikulin et al developed an orifice pulse tube refrigerator (OPTR) [9], presented in figure 3(b), by installing an orifice and a buffer tank near the warm end of the pulse tube. They succeeded thereby in lowering the temperature to 105 K. The remarkable difference of the cooling performance is attributable to the difference in the phasing between \( P \) and \( V \).

![Figure 3. Pulse tube refrigerators: (a) basic, (b) orifice, (c) inertance, and (d) double inlet.](image)

Figure 4 shows the schematic illustration of the small prototype pulse tube cooler constructed to verify the phasing. The working fluid was air at atmospheric pressure. The operating frequency was 5.9 Hz. A needle valve was used as an adjustable orifice. Its opening is expressed by the number \( \varepsilon \) of turns; 0 (fully open) \( \leq \varepsilon \leq 8 \) (close). When the valve is closed, the prototype works as the basic pulse tube refrigerator and it operates as the OPTR when the valve is open. A buffer tank has internal volume of \( 0.5 \times 10^{-4} \text{ m}^3 \).

The acoustic pressure \( P \) was measured using a small pressure transducer at the lower end of the pulse tube (labeled as A in figure 4). At the same axial position, the axial acoustic particle velocity \( U \) was measured on the central axis of the tube using a laser Doppler velocimeter. The measured central velocity \( U \) was converted to the cross-sectional mean velocity \( V \) using the theoretical result of the laminar flow theory. The complex acoustic impedance was then determined based on the definition in equation (4).

![Figure 4. The prototype OPTR. A loudspeaker connected with dynamic bellows was used as a compressor to introduce periodic gas oscillations with the frequency \( f = 5.9 \text{ Hz} \). The regenerator was made of stacked stainless-steel screen meshes. The mesh number per inch is 250, the wire diameter is 0.04 mm. The regenerator length and its internal diameter were, respectively, 50 and 15 mm. A 60-mm-long glass cylinder with internal diameter of 13.4 mm was used as a pulse tube.](image)
Figure 5 presents the measured $Z$, which is shown as a phasor in the complex plane. When the orifice valve is closed, $\varepsilon = 0$, $Z$ is positioned very closely to the imaginary axis. Therefore, the phase $\Phi$ of pressure $P$ relative to velocity $V$ is close to a standing wave phase $\Phi = -90^\circ$ in the basic pulse tube refrigerator. As $\varepsilon$ increases, $Z$ traces a semicircle in a counterclockwise direction. The phase $\Phi$ approaches 0 with $\varepsilon \geq 6$. This result confirms that the OPTR gains the better cooling performance than the basic type owing to the improvement of the phasing.

Figure 5 also shows $Z$ of the inertance pulse tube refrigerator (IPTR, figure 3(c)). The IPTR was built by inserting an inertance tube (5 m in length and 4 mm in inner diameter) between the pulse tube and the tank of the present OPTR. Although the travelling wave phasing was not obtained in the OPTR, the IPTR realized it with the valve opening $\varepsilon = 8$.

![Figure 5. Complex acoustic impedance $Z$ of the OPTR and the IPTR. Numbers refer to the turn $\varepsilon$ of the orifice valve.](image)

In contrast to the series configuration of the OPTR and the IPTR, a double-inlet pulse tube cooler (DIPTR, shown in figure 3(d)) [11] uses a bypass tube that goes in parallel with the pulse tube and the regenerator. Figure 6 presents the experimental acoustic impedance $Z$ of the DIPTR measured at the cold end of the pulse tube. Here we used a tube of 300 mm in length and 4 mm in diameter as a bypass tube, and a bypass valve (its opening is expressed as $\varepsilon_b$) to modify our OPTR. The impedance $Z$ of the DIPTR showed the arc going into the first quadrant when $\varepsilon_b = 5$. It is noteworthy that the magnitude of $Z$ with $\varepsilon_b = 5$ is much larger than that of the IPTR with the same traveling wave phasing $\Phi = 0$. This result verifies that the DIPTR can achieve both a desired phasing and very high acoustic impedance, the latter of which contributes to the reduction of viscous losses. Indeed, the first DIPTR achieved the lowest temperature of 25 K [11]. Thus, understanding of the acoustic impedance is of great importance to improve the thermoacoustic device performance.
Figure 6. Complex acoustic impedance $Z$ of the DIPTR when $\varepsilon = 6$. Solid circles represent the data of the OPTR reproduced from Fig. 5.

5. Summary

We have shown the propagation constant of sound waves in a wide $\omega \tau$ region to present importance of thermal contacts of the gas with the tube wall and of the moderate viscous damping. For the thermoacoustic heat pump, realization of both a traveling wave phasing and a high acoustic impedance is essential to achieve a better cooling performance. Progress of the pulse tube refrigerators lends strong support to the validity of thermoacoustical point of view.

References

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