Study of neutrino oscillation parameters at INO-ICAL detector using event-by-event reconstruction

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Plan of the Talk

- Neutrino oscillations
- INO-ICAL
- Analysis Procedure
  - Data Generation
  - Applying oscillations
  - $\chi^2$ analysis and systematics
- Results
  - With fluctuations
  - Without fluctuations
- Conclusions
Neutrino oscillations

- Neutrinos are massive and undergo oscillations ($\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$).
- Three flavors: mixing is governed by $3 \times 3$ unitary matrix.
  - Dirac type: 3 magnitudes ($\theta_{12}, \theta_{13}$ & $\theta_{23}$) and 1 irreducible phase ($\delta$)
  - Majorana: 3 magnitudes ($\theta_{12}, \theta_{13}$ & $\theta_{23}$) and 3 phases ($\delta, \alpha_{21}$ & $\alpha_{31}$)
- Oscillations are governed by these parameters along with two mass-square terms ($\Delta m_{21}^2$ & $\Delta m_{32}^2$)

![Diagram of two possible hierarchies of neutrino mass eigenstates.](http://www.staff.uni-mainz.de/wurmm/juno.html)

- Absolute masses are yet to be determined.
- Only mass-squared differences are known.
- $\Delta m_{21}^2$ is found to be positive.
- Sign of $\Delta m_{32}^2$ is yet unknown.
- Hence two possible Mass hierarchies (MH):
  - Normal Hierarchy (NH): $m_1 < m_2 < m_3$
  - Inverted Hierarchy (IH): $m_3 < m_1 < m_2$
ICAL at INO is proposed to detect atmospheric neutrinos.

Main goals of INO: measure $M_H$, $\Delta m_{32}^2$ and $\sin^2 \theta_{23}$.

50 kton detector with modular structure ($48.5 \text{ m} \times 16 \text{ m} \times 14.5 \text{ m}$)

A sandwich of iron and glass Resistive Plate Chambers (RPCs)

RPC’s: Active detector elements.

Magnetized up to 1.5 T via current carrying copper coils.

Figure: Schematic of INO-ICAL detector.

Note

- $\nu_\mu$ or $\bar{\nu}_\mu$ will give $\mu^-$ and $\mu^+$ respectively via charge current interaction.
- ICAL can distinguish $\mu^-$ and $\mu^+$ ($\nu_\mu$ from $\bar{\nu}_\mu$) using magnetic field.
- Measure the MH by observing matter effects separately in $\nu_\mu$ and $\bar{\nu}_\mu$. 
Motivation

- Devise a method, which can be used on real ICAL data.
- Study the reach of ICAL for the run of 5 years.
- To account for the tails of resolution functions, which have been approximated by single Gaussians and Vavilov functions in the previous studies.

Analysis procedure

- Our analysis: 1000 years NUANCE [1] unoscillated CC $\nu_\mu$ events (0.4 GeV to 500 GeV).
- Separate data sets were generated for 5 and 995 years after simulating in GEANT4 [2].
  - 5 year data: the experimental data set.
  - 995 year data: the probability distribution function (PDF).
  - Hence data is: uncorrelated with fluctuations.
  - To see the effect of fluctuations: chose sixty different combinations of data and PDF.
- To reconstruct: INO-ICAL code ($E_\mu^{\text{rec}}$ and $\cos \theta_z$).

Figure: Analysis flowchart.
Event selection

Figure: Magnetic field map in the central plate \((z = 0)\) of the central module [3].

Figure: The cross-sectional view of ICAL (all three modules) in \(x - y\) plane, showing the division of regions on the basis of the magnetic field strength.

- Event selection is applied in different regions [3].
- Cut on the \(\chi^2\) of track reconstruction, \(\chi^2 / \text{ndf} \leq 10\)
- Removal of horizontal events, \(|\cos \theta_z| \geq 0.35\)
- \(N_{\text{hits}}\) (hits in muon track) selection:
  - \(N_{\text{hits}} > 15\) for partially contained events in side and peripheral regions.
  - \(N_{\text{hits}} > 0\) for all other events.
Figure: Comparison of muon charge identification efficiency (bottom panel), reconstruction efficiency (above bottom panel), energy resolution (below top panel) and zenith angle resolution (top panel), with (WS) and without (WOS) event selection.

- Observed improvement in relative charge ID (cid) efficiency, angular and energy resolution of muons.
- But lost many events in the process (40% of reconstructed events).
- Hence, we also study the effect of selection cuts in our analysis.
Applying oscillations and binning scheme

\[
\frac{d^2 N}{dE_{\nu} \ d \cos \theta_z} = T \times N_D \times \sigma_{\nu\mu} \times \left[ P_{\mu\mu} \frac{d^2 \Phi_{\nu\mu}}{dE_{\nu} \ d \cos \theta_z} + P_{e\mu} \frac{d^2 \Phi_{\nu e}}{dE_{\nu} \ d \cos \theta_z} \right], \quad (1)
\]

- The fraction of $\nu_e$ in the sample is very low, hence only $\nu_\mu$ flux is used in the analysis.

- The data is subjected to three flavor matter oscillations, assuming PREM density profile of Earth [4].

Binning:

- Two variables: the reconstructed muon zenith angle ($\cos \theta_z$) and the muon energies, tagged positive for $\mu^+$ and negative for $\mu^-$ ($Q_{\mu} E_{\mu}$).
- $\nu_\mu$ and $\bar{\nu}_\mu$ are binned in separate bins in $Q_{\mu} E_{\mu}$ according to charge ID.
\( \chi^2 \) Analysis : pull method

- The pull approach [5] is used in defining the Poisson \( \chi^2 \) incorporating systematic uncertainties.

\[
\chi^2 = \min_{\{\xi_k\}} \sum_{i=1}^{N_{\cos \theta_z}} \sum_{j=1}^{N_{E_{\mu}}} \left[ 2 \left( N_{ij}^{\text{pdf}} - N_{ij}^{\text{data}} \right) - 2N_{ij}^{\text{data}} \ln \left( \frac{N_{ij}^{\text{pdf}}}{N_{ij}^{\text{data}}} \right) \right] + \sum_{k=1}^{2} \xi_k^2, \quad (2)
\]

\[
N_{ij}^{\text{pdf}} = R \left[ fT_{ij}^{\bar{\nu}} + (1 - f) T_{ij}^{\nu} \right] \left[ 1 + \sum_{k=1}^{2} \pi_{ij}^{k} \xi_k \right]. \quad (3)
\]

- \( T_{ij}^{\bar{\nu}} \) and \( T_{ij}^{\nu} \) are \( \bar{\nu}_\mu \) and \( \nu_\mu \) PDFs respectively.
- \( f \) is a free parameter describing the fraction of \( \bar{\nu}_\mu \) in the sample.
- The systematic errors are parametrized in terms of variables \( \{\xi_k\} \) called pulls, and \( \pi^k \) corresponds to the resulting uncertainty.
- Considered two systematic uncertainties :
  - 5% uncertainty on the zenith angle dependence of the flux.
  - 5% on the energy dependent tilt error.
- The normalization uncertainty is excluded from systematics and instead the free parameter \( R \) fixes the overall normalization.
The significance of the fit:

\[
\text{significance} = \sqrt{\Delta \chi^2_{\text{input}} - \Delta \chi^2_{\text{min}}}
\]

- WOS: significance < 1\(\sigma\).
- WS: significance < 2\(\sigma\).
- Relatively worse precision, as we lose 40% of events after selection cuts.
- Hence choose not to apply any cuts.
- Conclusion: loose selection cut gives better results.

Adding prior constrains on \(\sin^2 \theta_{13}\) and \(\Delta m^2_{12}\), was observed not to make any difference in the fit.
Effect of fluctuations

The 5 year pseudo-data is scaled from PDF, to reduce fluctuations.

Figure: Comparison of precision reach obtained from the fit to five year pseudo-data in $\sin^2 \theta_{23} - \Delta m^2_{32}$ plane. The solid black line shows the coverage area with 99% CL for the fit without fluctuations, and compares it to 99% CL coverage of few other fits using fluctuated data sets. The input (true) point is given by the green dot.
Figure: (a) Significance of convergence obtained from sixty different data sets and (b) comparison of unfluctuated precision reach and the average coverage area calculated from 99% CL coverages of 50 different sets, in $\sin^2 \theta_{23} - \Delta m^2_{32}$ plane.

- The analysis was repeated for sixty different data sets.
- Roughly the fit converges:
  - 68% of times within 1σ
  - 95% of times within 2σ
Mass hierarchy determination

- Five year pseudo-data is fit to true and false hierarchy.
- The $\Delta \chi^2$ resolution: $\Delta \chi^2_{\text{MH}} = \Delta \chi^2_{\text{false}} - \Delta \chi^2_{\text{true}}$

![Graph](a) $\Delta \chi^2$ as a function of $\sin^2 \theta_{23}$ for true and false fit and (b) distribution of $\Delta \chi^2_{\text{MH}}$ obtained from fit to sixty fluctuated data sets.

- Fit to sixty different data sets: mean $\Delta \chi^2_{\text{MH}} = 2.9$ ($\approx 1.7 \sigma$).
- 15% probability of identifying wrong MH.
- 13 year run of ICAL will give a $3\sigma$ separation to differentiate the MH.
Event-by-event reconstruction is used for the first time.

The effect of event selection is studied for the first time.

For the first time we study the effect of low event statistics on the precision and MH measurements, by introducing fluctuations in the data.

We also find a mean resolution of $\Delta \chi^2_{MH} = 2.9$ from an ensemble of sixty experiments, which rules out the wrong hierarchy with a significance of $\approx 1.7\sigma$. 
References:

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Thank You
Current best-fit values

| Parameter   | best-fit value | 3σ range   |
|-------------|----------------|------------|
| $\sin^2 \theta_{23}$ | (NH) 0.437      | 0.379 – 0.616 |
|              | (IH) 0.569      | 0.383 – 0.637 |
| $\sin^2 \theta_{13}$ | (NH) 0.0214  | 0.0185 – 0.0246 |
|              | (IH) 0.0218  | 0.0186 – 0.0248 |
| $\sin^2 \theta_{12}$ | 0.297     | 0.250 – 0.354 |
| $\Delta m^2_{21} [10^{-5} \text{ eV}^2]$ | 7.37       | 6.93 – 7.97 |
| $\Delta m^2 [10^{-3} \text{ eV}^2]$ | (NH) 2.50 | 2.37 – 2.63 |
|              | (IH) 2.46       | 2.33 – 2.60 |
| $\delta$ [rad] | (NH) 1.35       | 0.92 – 1.99 (2σ) |
|              | (IH) 1.32       | 0.83 – 1.99 (2σ) |

Table: The current best-fit values of neutrino oscillation parameters and their 3σ allowed ranges assuming normal (NH) and inverted (IH) neutrino mass hierarchies. The values are taken from Ref. [6]. For CP phase, $\delta$, at 3σ no physical values are disfavored, hence the 2σ range is given. Here $\Delta m^2 \equiv m_3^2 - (m_2^2 + m_1^2)/2$. 
### Event selections

| Item | Criterion | Region | Events |
|------|-----------|--------|--------|
| CS   | $\chi^2/\text{ndf} < 10$ | all    | all    |
| HS   | $|\cos \theta_z| \geq 0.35$ | all    | all    |
| ZS   | $z_v < 6 \text{ m}$ | all    | up going |
|      | $z_v > -6 \text{ m}$ | all    | down going |
| NS   | $N_{\text{hits}} > 0$ | CR     | all    |
|      | $N_{\text{hits}} > 15$ | PR     | FC     |
|      |                   | SR     | FC     |
| OS   | $E_{\mu}^{\text{rec}} \geq 0.2 \text{ GeV}$ | all    | all    |
|      | $E_{\mu}^{\text{rec}} \leq 50 \text{ GeV}$ |       |        |
|      | $|\cos \theta_z^{\text{rec}}| < 0.9999$ |       |        |
|      | $|\phi_{\text{rec}}| \geq 0.07 \text{ rad}$ |       |        |

**Acronyms**

- CS - $\chi^2$ selection
- HS - Horizontal selection
- ZS - Z vertex selection
- NS - Nhits selection
- OS - Other selection
- CR - Central region
- PR - Peripheral region
- SR - Side region
- FC - Fully contained
- PC - Partially contained

**Position (m)**

| Region         | Position (m) | Magnetic field |
|----------------|--------------|----------------|
|                | $x$   | $y$ | Strength (T) | Variation coefficient |
| Central        | $x \leq 20$ | $y \leq 4$ | 1.5 | 12% |
| Peripheral     | unconstrained | $y > 4$ | 1 | 28% |
| Side           | $x > 20$ | $y \leq 4$ | 1.3 | 13% |

**Magnetic field**

- Central: $|x| \leq 20$, $|y| \leq 4$ with strength 1.5 and variation coefficient 12%
- Peripheral: unconstrained $|y| > 4$ with strength 1 and variation coefficient 28%
- Side: $|x| > 20$, $|y| \leq 4$ with strength 1.3 and variation coefficient 13%

**PC:**

- $|x| \geq 23 \text{ m}$ or
- $|y| \geq 7.5 \text{ m}$ or
- $|z| \geq 7 \text{ m}$

- Else FC.
Fraction of $\nu_e$

\[
\text{Fraction of } \nu_e = \frac{(\nu_e \rightarrow \nu_\mu)}{(\nu_e \rightarrow \nu_\mu + \nu_\mu \rightarrow \nu_\mu)}
\]

**Figure:** Fraction of (a) $\bar\nu_e$ in $\bar\nu_\mu$, and (b) $\nu_e$ in $\nu_\mu$ sample (50 kton $\times$ 100 years).

- **(a)**
  - Blue line: Survived $\bar\nu_\mu \rightarrow \bar\nu_\mu$
  - Dashed red line: Oscillated $\bar\nu_e \rightarrow \bar\nu_\mu$

- **(b)**
  - Blue line: Survived $\nu_\mu \rightarrow \nu_\mu$
  - Dashed red line: Oscillated $\nu_e \rightarrow \nu_\mu$

**Table:**

- Entries: 33,579
- Mean: 0.185
- RMS: 0.681
Figure: Comparison of precision reach obtained in $\sin^2 \theta_{23} - |\Delta m^{2}_{32}|$ plane with and without the electron flux ($\Phi_e$). The solid (orange) and the dashed (purple) line shows the coverage area with and without $\Phi_e$ respectively. The input (true) point is given by the green dot.
Oscillations

Down-going events
\[-1 \leq \cos \theta_z \leq 0\]

Up-going events
\[0 < \cos \theta_z \leq 1\]

Figure: Comparison of zenith angle (\(\cos \theta_z\)) distributions with and without oscillations as obtained with (WS) and without (WOS) selection cuts.
Figure: (a) Binning in $\cos \theta_z$, and (b) binning in $E_{\mu}$ of five year oscillated pseudo-data. The binning after (solid red) and before (dashed blue) selection cuts.
Fit results: one parameter

Figure: \( \Delta \chi^2 \) as a function of \( \sin^2 \theta_{23} \), for an input value of \( \sin^2 \theta_{23}(true) = 0.5 \) and \( \Delta m^2_{32} \), for an input value of \( \Delta m^2_{32}(true) = 2.32 \times 10^{-3} \text{ eV}^2 \). The solid (blue) and dashed (red) line shows the \( \Delta \chi^2 \) distribution without (WOS) and with (WS) event selection.
Marginalizing $\sin^2 \theta_{13}$ and $\Delta m_{12}^2$

Figure: Precision reach obtained in $\sin^2 \theta_{23} - \Delta m_{32}^2$ plane for the fit to five year pseudo-data (a) with fluctuations and (b) without fluctuations. The solid (black) and broken (red) line shows the coverage area with 99% CL without (NC) and with (WC) constraints respectively on the parameters $\sin^2 \theta_{13}$ and $\Delta m_{12}^2$. The input (true) point is given by the green dot. The plus and the star sign signifies the best-fit point obtained for NC and WC respectively.
Effect of fluctuations: 1D

Figure: Effect of fluctuations on $\Delta \chi^2$ as a function of (a) $\sin^2 \theta_{23}$, for an input value of $\sin^2 \theta_{23}(\text{true}) = 0.5$ and (b) $\Delta m^2_{32}$, for an input value of $\Delta m^2_{32}(\text{true}) = 2.32 \times 10^{-3} \, \text{eV}^2$. The solid (blue) and the broken (red) curves represents the fit to data with (WF) and without fluctuations (WOF) respectively.