Probing dynamics of boson stars by fast radio bursts and gravitational wave detection

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Abstract. Boson stars may consist of a new type of light singlet scalar particles with non-trivial self-interactions, and may compose a fraction of the dark matter in the Universe. In this work, we study the dynamics of boson stars with Liouville and logarithmic scalar self-interaction potentials as benchmarks. We perform a numerical analysis as well as a semi-analytic study on how the compactness and the total mass will deviate from that of the usual boson stars formed with a quartic repulsive self-interaction. We apply the recently suggested Swampland conjecture to examine whether boson stars with such benchmark potentials belong to the Landscape of a quantum gravity. Using the mass constraint on the macroscopic compact halo object (MACHO) and the cold dark matter (CDM) isocurvature mode constraint from the cosmic microwave background (CMB), we derive the allowed mass range of scalar particles which compose the boson star. We further analyze applications of the lensing of fast radio bursts (FRBs) and the gravitational wave (GW) detection to probe the presence of such boson stars and constrain the parameter space of their corresponding models. We discuss how the two types of boson star potentials can be discriminated by the FRB and GW measurements.

Keywords: dark matter theory, gravity, particle physics - cosmology connection

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1 Introduction

The Higgs boson discovery in 2012 has provided encouraging evidence that the spin-0 scalar particles are among the most fundamental ingredients in nature, and there may well exist new singlet scalar particles beyond the standard model (SM) which are weakly coupled to the SM and may play a wide range of important roles, including the candidates for dark matter, dynamical dark energy, inflaton, axion, and some other things.

If there exits a new scalar \( \Phi \) in a dark sector (serving as a SM singlet), it may couple to the SM sector very weakly via gravitational interactions. As the temperature of the Universe decreases in its history, it is probable that such bosonic particles become cold enough to sit in the ground state at a certain moment. This happens when the de Broglie wavelength of the bosons, \( \lambda_{\text{dB}} \sim m_{\phi}^{-1/2}T^{-1/2} \), becomes comparable to the interparticle distance between the particles. Then, as a self-gravitating system, the clustering of Bose-Einstein condensate (BEC) would tend to continue until gravitational attraction is balanced by the repulsive quantum pressure due to the Heisenberg uncertainty principle. In consequence, a stable compact object (called boson star) could form, and this may possibly provide a certain fraction of the dark matter in the Universe.

With this motivation, there have been several works studying the dynamics of boson stars, including the possible maximum mass and compactness of a boson star as determined
by distinctive forms of the scalar potential. Given the competition between gravitational attraction and repulsive quantum pressure, a possible scalar self-interaction new force will cause a different hydrostatic equilibrium point, and thereby the characteristic physical quantities describing the boson star will vary. As a simplest setup, the stable boson star with a free massive scalar was studied in [1], and was investigated further [2] by using field quantization of a real scalar field. This boson star (called mini-boson star) has a mass \( \sim m_P^2/m_4 \), still less than the Chandrasekhar mass limit \( \sim m_P^2/m_2 \) which is the maximum achievable mass for a fermion star. (Here \( m_P = G^{-1/2}_N \) is the Planck mass.) Then, the scalar model with a repulsive quartic self-interaction was examined [3] in the context of boson stars, and it was shown that the total mass comparable to the Chandrasekhar limit can be realized. In the strong coupling regime, the compactness of this type of boson stars was found to be as high as \( C_{\text{max}} \approx 0.16 \) [4, 5]. Based on these works, many different potentials were further considered in the literatures [6–9].

In this work, we will study the dynamics of boson stars with certain distinctive scalar self-interaction potentials. Regarding the new forces governing the hydrostatic equilibrium for the boson star other than the gravity and the quantum pressure, a nontrivial question is how the compactness and total mass would change, provided a infinite series of repulsive self-interaction forces are introduced. Namely, we wonder how much increase from \( C_{\text{max}} \approx 0.16 \) can be observed when the quantum pressure is put together with the repulsive force of an infinite series of scalar self-interactions in competing with the gravitational attraction. Would it be so significant that it approaches a value as high as the compactness of a black hole? If not, then how much does the infinite series of repulsive forces make the stable boson stars be different from the usual boson stars (including the mini-boson star or the boson star from a repulsive quartic self-interaction)? In addition, the same question can be asked for a scalar potential of which the expansion gives infinite series of paired attractive and repulsive forces by having alternating signs. Answering these questions is valuable for identifying the source of a compact object-related astrophysical signal that is characterized by its compactness and mass. For instance, such astrophysical signals include gravitational waves (GWs) caused by the merger of two binary compact objects or the fast radio bursts (FRBs) due to the presence of a compact object serving as the lensing source. We further derive the allowed mass range of the scalar particles (which compose the boson star) by using (i) the current constraints on the fraction of the dark matter contributed by MACHO and the mass of MACHO, and (ii) the current CMB constraint on CDM isocurvature modes. (Here MACHO stands for the macroscopic compact halo object, and CDM for the cold dark matter.) We study whether the FRB lensing and GW detection can be used to probe the currently allowed scalar mass range. For this purpose, we will examine equilibrium configurations of the boson star due to two benchmark potentials of scalar self-interactions — the Liouville and Logarithmic potentials.

This paper is organized as follows. In section 2, we introduce a formalism for studying the dynamics of boson stars and set up two benchmark scalar potentials. In section 3, we present the results of both numerical and analytic computation for the compactness and mass of boson stars. Then, we apply the Swampland conjecture to the results and check whether the boson star models under consideration belong to the Landscape of a quantum gravity theory. In section 4, we derive the allowed mass range of the scalar particles for forming a boson star, by using the current constraints on the MACHO mass and the fraction of the CDM from the MACHO, and the current CMB constraint on CDM isocurvature modes. Then, we apply the two astrophysical measurements to probe the presence of such boson stars and the parameter space of the benchmark scalar potentials. We study how the two
types of boson star potentials can be discriminated by the FRB and GW measurements. Throughout this paper, we will adopt the natural unit $c = \hbar = 1$. We denote the Planck mass by $m_P = G_N^{-1/2}$ and the reduced Planck mass by $M_P = (8\pi G_N)^{-1/2}$, where $G_N$ is the Newton gravitational constant. For the cosmological parameters, we use values based on Planck TT,TE,EE+lowE+lensing at the 68% confidence level in ref. [10].

2 Boson star modeling

In this section, we present the full set of equations which governs the boson star solutions. These equations include the Einstein equations for the spacetime geometry and the Klein-Gordon equation of the scalar field. The equations of such a coupled system are usually called Einstein-Klein-Gordon (EKG) equations (cf. refs. [9, 11] for reviews). Then, we proceed to introduce two self-interacting scalar potentials of different underlying physics. We will motivate the choice of the two potentials. Finally, we discuss the definitions of physical quantities characterizing a boson star, including the total mass and compactness.

2.1 Einstein-Klein Gordon equations

Boson star is a self-gravitating system comprised of particles which correspond to a complex scalar field obeying the EKG equations. We start by considering the following action,

$$ S = \int d^4x \sqrt{-g}\left(\frac{R}{16\pi G_N} + \mathcal{L}_M\right), \quad (2.1) $$

where $R$ is the Ricci curvature scalar, $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor, and $\mathcal{L}_M$ is the Lagrangian density of the scalar field. For a complex scalar field $\Phi(r,t)$, we write the corresponding Lagrangian

$$ \mathcal{L}_M = -g^{\mu\nu}\partial_\mu \Phi^* \partial_\nu \Phi - U(|\Phi|^2), \quad (2.2) $$

where we require the scalar potential $U(|\Phi|^2)$ to be a function of the modulus of the scalar field.\footnote{Here we have dropped the cosmological constant term since its effect is negligible for the current boson star study according to [12].} This feature of the potential is crucial for making the action invariant under a global $\text{U}(1)$ symmetry. Variations of the action (2.1) with respect to the metric tensor and the scalar field lead to the following EKG evolution equations,

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}, \quad (2.3a) $$

$$ \Box \Phi - \frac{dV}{d|\Phi|^2} \Phi = 0, \quad (2.3b) $$

where $\Box$ is the covariant D’Alembert operator, $R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the scalar energy-momentum tensor,

$$ T_{\mu\nu} = \left( \partial_\mu \Phi^* \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu \Phi^* \right) - g_{\mu\nu} \left[ \partial^\alpha \Phi^* \partial_\alpha \Phi + U(|\Phi|^2) \right]. \quad (2.4) $$

Equation (2.3a) can be further expressed as $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, with $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ being the Einstein tensor. The invariance of the action (2.1) under a phase transformation,
By substituting the ansatz (2.6) into eqs. (2.3b)–(2.4), we obtain the temporal and radial

\[ N = \int d^3x \sqrt{-g} J^0. \quad (2.5) \]

Stable general relativistic boson stars satisfy the relation \( M < Nm_\phi \) due to the existence of
a negative binding energy associated with the spacetime geometry.

The choice of a complex scalar field over a real scalar field has a reason. As Friedberg, Lee and Pang showed [8], a localized time-independent matter configuration does not exist for a real scalar field. Since the energy-momentum tensor (2.4) depends on the modulus of the field, the product of gradients of the field and its conjugate, it is convenient to consider a harmonic ansatz for the scalar field which ensures a time-independent gravitational field. Thus, we write

\[ \Phi(r, t) = \phi(r) e^{i\omega t}, \quad (2.6) \]

where \( \phi \in \mathbb{R} \) is the real radial profile of the scalar field and \( \omega \) a real angular frequency.

We impose a spherically symmetric ansatz to look for equilibrium configurations which correspond to minimum energy solutions. With this ansatz, the energy-momentum tensor becomes time-independent, leading to static metric functions. Thus, we can write

\[ ds^2 = -e^{\gamma(r)} dt^2 + e^\lambda(r) dr^2 + r^2[d\theta^2 + \sin^2(\theta)d\phi^2], \quad (2.7) \]

where \( \gamma = \gamma(r) \) and \( \lambda = \lambda(r) \) depend only on the Schwarzschild-type radial coordinate \( r \).

By substituting the ansatz (2.6) into eqs. (2.3b)–(2.4), we obtain the temporal and radial components of the energy-momentum tensor and the equation of motion of the scalar,

\[ T_t^t = -U(\phi^2) - \omega^2 e^{-\gamma} \phi^2 - e^{-\lambda} \phi'^2, \]
\[ T_r^r = -U(\phi^2) + \omega^2 e^{-\gamma} \phi^2 + e^{-\lambda} \phi'^2, \]
\[ \phi'' = -\left[ \frac{2}{r} + \frac{\gamma' - \lambda'}{2} \right] \phi' + e^\lambda \phi \left( \frac{dU(\phi^2)}{d\phi^2} - e^{-\gamma} \omega^2 \right), \]

where a prime means a derivative with respect to \( r \). After computing \( G_t^t \) and \( G_r^r \), the first two components of the Einstein equations are given by

\[ e^{-\lambda} \left( \frac{\lambda'}{r} + \frac{e^\lambda}{r^2} - \frac{1}{r^2} \right) = 8\pi G_N \left[ U(\phi^2) + \omega^2 e^{-\gamma} \phi^2 + e^{-\lambda} \phi'^2 \right], \quad (2.9a) \]
\[ e^{-\lambda} \left( \frac{\gamma'}{r} - \frac{e^\lambda}{r^2} + \frac{1}{r^2} \right) = 8\pi G_N \left[ -U(\phi^2) + \omega^2 e^{-\gamma} \phi^2 + e^{-\lambda} \phi'^2 \right]. \quad (2.9b) \]

For the later computational convenience, we introduce the following dimensionless variables, \( \tilde{r} = rm_\phi, \quad \tilde{\omega} = \omega/m_\phi, \) and \( \tilde{\phi}^2 = \frac{8\pi G_N}{\phi^2} \). These rescaled variables lead to the resultant coupled system of differential equations,

\[ \gamma'\tilde{r} = \frac{e^{\lambda\tilde{r}} - 1}{\tilde{r}} + \tilde{r} e^{\lambda\tilde{r}} \left[ -\tilde{U} + \tilde{\omega}^2 e^{-\gamma\tilde{r}} \tilde{\phi}^2 + \tilde{\phi}^2 e^{-\lambda\tilde{r}} \right], \quad (2.10a) \]
\[ \lambda'\tilde{r} = -\left[ \frac{e^{\lambda\tilde{r}} - 1}{\tilde{r}} \right] + \tilde{r} e^{\lambda\tilde{r}} \left[ \tilde{U} + \tilde{\omega}^2 e^{-\gamma\tilde{r}} \tilde{\phi}^2 + \tilde{\phi}^2 e^{-\lambda\tilde{r}} \right], \quad (2.10b) \]
\[ \tilde{\phi}''\tilde{r} = -\left[ \frac{2}{\tilde{r}} + \frac{\gamma'\tilde{r} - \lambda'\tilde{r}}{2} \right] \tilde{\phi}' + e^{\lambda\tilde{r}} \tilde{\phi} \left( \frac{d\tilde{U}}{d\phi^2} - e^{-\gamma\tilde{r}} \tilde{\omega}^2 \right). \quad (2.10c) \]
where the prime denotes a derivative with respect to $\tilde{r}$. In the above, we have defined dimensionless quantities for convenience,

$$U = \frac{8\pi G_N}{m_\Phi^2} U(\phi^2), \quad \phi^2 = \tilde{\phi}^2 / 8\pi G_N. \quad (2.10d)$$

### 2.2 Scalar self-interaction potentials

Different from the mini-boson star without any self-interaction [1], it was shown [3] that the maximum total mass of a fermion star (Chandrasekhar mass limit) could be mimicked by a boson star provided the scalar field is allowed to have repulsive quartic self-interaction with a suitably chosen particle mass and interaction strength. The underlying physics is that the repulsive force due to quartic self-interaction with positive coupling ($\lambda_\Phi > 0$) together with the quantum pressure of boson particles can better compete with the attractive gravitational force, leading to an equilibrium radius of a larger size.\(^2\) Hence, along this line of thinking, it is natural to wonder what else can achieve the higher compactness. In other words, we may ask whether compactness of a boson star increases significantly under the presence of additional sources of either of a repulsive or attractive force. As a first possibility, there may be a series of repulsive self-interaction terms. To avoid a divergent repulsive force, we demand that the added new repulsive higher order self-interaction terms becomes smaller as the expansion order increases. In this case, can we have a stable boson star? And if so, will there be a significant change in compactness as compared to the mini-boson star? To address these questions, we may consider the following modified U(1) Liouville potential [14],

$$U_{\text{Liouville}}(\phi^2) = f^2 m_\Phi^2 \left( e^{\phi^2/2f} - 1 \right), \quad (2.11)$$

where $f$ serves as a coupling strength parameter. (For the boson star studies in the literature, the scalar $\Phi$ is normally defined as a SM singlet and joins only the gravitational interactions in addition to its self-interactions. The above effective nonlinear scalar self-interactions may be induced by the quantum gravity effects around the Planck scale.) For $|\phi| < f$ and using eq. (2.6), we may expand the potential around $\phi = 0$ as follows,

$$U_{\text{Liouville}} = m_\Phi^2 \phi^2 + \frac{m_\Phi^2 \phi^4}{2f^2} + \frac{m_\Phi^2 \phi^6}{6f^4} + \frac{m_\Phi^2 \phi^8}{24f^6} + O(m_\Phi^2 \phi^{10}/f^8), \quad (2.12)$$

where the scalar mass term arises from the leading order of the expansion. We have modified the usual Liouville potential to respect the global U(1) symmetry which ensures the particle number conservation. Indeed, the above modified Liouville potential has the desired property of generating an infinite series of repulsive interaction terms. On the other hand, such a convex exponential potential of a scalar field has been applied to a variety of problems in cosmology, ranging from dark matter [15], inflation [16–19], to dark energy modeling [20, 21].

As a second option, we may consider a potential of which the higher order contributions largely cancel with each other pairwise due to different signs, but the net effect is not negligible and differs from a single quartic self-interaction. Note that this case can have two possibilities: the sign of the quartic coupling can be either of positive ($\lambda_\Phi > 0$) or negative ($\lambda_\Phi < 0$). For $\lambda_\Phi > 0$, the net force due to scalar self-interactions is repulsive and the resulting compactness turns out to exceed that of the mini-boson star ($\approx 0.08$) as shown in [13] with the cosine

\(^2\)In addition to the repulsive quartic self-interaction, the back reaction from curvature is another source of the repulsive force against gravitational attraction as discussed before [13].
potential (cf. its figure 10). For the current study, we will examine the other possibility with a negative quartic coupling, in which case the net force due to self-interactions should be attractive. A well-known example of this kind is the QCD axion potential (cf. refs. [22–26] for reviews of axion stars). For this purpose, we consider the following logarithmic scalar potential,

$$U_{\text{Log}}(|\Phi|^2) = f^2 m_\Phi^2 \log \left( \frac{|\Phi|^2}{f^2} + 1 \right),$$

(2.13)

For $|\Phi| < f$, we make Taylor-expansion of the scalar potential around $\phi = 0$ and use eq. (2.6) to derive the following form,

$$U_{\text{Log}} = m_\Phi^2 \phi^2 - m_\Phi^2 \phi^4 + \frac{m_\Phi^2 \phi^6}{3 f^4} - \frac{m_\Phi^2 \phi^8}{4 f^6} + O(m_\Phi^2 f^8/f^{10}),$$

(2.14)

where the mass term arises from the leading order expansion again. Note that both potentials are functions of the modulus of the scalar field as required by the global U(1) symmetry. This logarithmic type potential can appear as the effective potential for a flat direction of the scalar field in the gauge mediated supersymmetry breaking scenario [27]. Besides, thermal logarithmic potential can be induced due to interaction of the real scalar field with other fields in the primordial plasma after inflation for a supersymmetric theory [28, 29]. Motivated by these, the logarithmic potential of the same form as eq. (2.13) for a real scalar field was studied in the context of I-ball formation in [30, 31].

To make the computation simpler, we define a dimensionless parameter $\tilde{f}$ by $f^2 = 8\pi G_N m_\Phi^2$. We also define a dimensionless coupling strength $\Lambda = 1/\tilde{f}^2$. Note that, for the special case of a usual quartic self-interaction, $U_{\text{quartic}} = m_\Phi^2 \phi^2 + \frac{1}{2} \lambda_\Phi \phi^4$, we have $\Lambda = \lambda_\Phi M_P^2/m_\Phi^2$.

Then, in terms of the dimensionless variables defined above, we can convert the potentials into the following form,

$$U_{\text{Liouville}}(\phi^2) \times (8\pi G_N/m_\Phi^2) = \tilde{f}^2 \left( e^{\frac{\phi^2}{2\tilde{f}^2}} - 1 \right) \equiv \tilde{U}_{\text{Liouville}},$$

(2.15a)

$$U_{\text{Log}}(\phi^2) \times (8\pi G_N/m_\Phi^2) = \tilde{f}^2 \log \left( \frac{\phi^2}{\tilde{f}^2} + 1 \right) \equiv \tilde{U}_{\text{Log}}.$$

(2.15b)

In the above, the dimensionless potentials $\tilde{U}_{\text{Liouville}}$ and $\tilde{U}_{\text{Log}}$ are the same as what appeared in eqs. (2.10a)–(2.10c).

### 2.3 Mass and compactness of the boson star

For a boson star, the physical quantities of interest are its total mass and compactness. The total mass can be computed as, $M = m(r \to \infty)$, where $(dm/dr) = -4\pi r^2 T^t_t$. Using the dimensionless variables defined in the previous subsection, we derive the following

$$\tilde{m}(\tilde{r}) = \frac{1}{2} \int_0^{\tilde{r}} \tilde{U} + \omega^2 \gamma \tilde{\phi}^2 + e^{-\lambda} \left( \frac{d\tilde{\phi}}{d\tilde{r}'} \right)^2 \tilde{r}^2 d\tilde{r}',$$

(2.16)

For the case of the quartic self-interaction, this definition for $\Lambda$ agrees with the definition given in ref. [3] after a field and strength coupling redefinition. Our Lagrangian in eq. (2.2) becomes equal to the Lagrangian in that reference by making $\Phi \to \Phi/\sqrt{2}$ and $\lambda_\Phi \to 2\lambda_\Phi$. Thus, $\Lambda = \lambda_\Phi/(4\pi G_N m_\Phi^2)$ as eq. (2) of [3].
where the dimensionless mass parameter is \( \tilde{m}(\tilde{r}) = (m_{\Phi}/m_{\Phi}^2) m(\tilde{r}) \). Since we are working in an asymptotically flat spacetime, \( \tilde{M} = \tilde{m}(\tilde{r} \to \infty) \) in eq. (2.16) agrees with the ADM-mass, which is obtained from equalizing the radial component of the metric (2.7) with the radial component of the Schwarzschild metric,

\[
\tilde{m}(\tilde{r}) \bigg|_{\tilde{r} \to \infty} = \left( 1 - e^{-\lambda(\tilde{r})} \right) \frac{\tilde{r}}{2} \bigg|_{\tilde{r} \to \infty}.
\] (2.17)

The conserved total number of particles associated with a boson star is calculated from eq. (2.5). In terms of the dimensionless variables defined earlier, we have

\[
\tilde{N} = \int_0^\tilde{r} d\tilde{r} \tilde{r}^2 \tilde{\omega} \tilde{\phi}^2 e^{\frac{\lambda - \gamma}{2}},
\] (2.18)

where \( \tilde{N} = (m_{\Phi}^2/m_{\Phi}^2) N \) and \( \tilde{r} \to \infty \) are understood.

Since the radial profile of scalar field vanishes at the physical infinity, the scalar field has non-compact support. This means that boson stars do not have a hard surface and thus the definition of a radius \( R \) would be ambiguous. This ambiguity is also transferred to the definition of compactness. Following ref. [4], we define an effective compactness,

\[
C(\phi_0, f) = \frac{0.99 M(\phi_0, f) G_N}{R_{99}(\phi_0, f)},
\] (2.19)

where \( R_{99} \) refers to the radius at which the 99% of the boson star mass \( M \) is enclosed. We will see that the effective compactness is a function of the central scalar field value \( \phi_0 \) and the coupling strength parameter \( f \) of the theory.

3 Analysis and results

In this section, we first present a numerical approach to solve the Einstein-Klein-Gordon equations in section 3.1. Then, we apply the recently suggested Swampland criteria to examine whether boson stars with the two scalar potentials of section 3.2 belong to the Landscape of a quantum gravity theory. Finally, we present a semi-analytic approach in section 3.3 to estimate the maximum total mass, the minimal radius and maximum compactness of boson stars.

3.1 Numerical implementation and results

The equilibrium configurations associated with boson stars are found by solving numerically the coupled system of eqs. (2.10a)–(2.10c) under suitable boundary conditions: (i). \( \gamma(\tilde{r} \to \infty) = \lambda(\tilde{r} \to \infty) = \tilde{\phi}(\tilde{r} \to \infty) = 0 \) ensures asymptotic flatness; (ii). \( \lambda(\tilde{r} = 0) = \tilde{\phi}'(\tilde{r} = 0) = 0 \), \( \gamma(\tilde{r} = 0) = \gamma_0 \), and \( \tilde{\phi}(\tilde{r} = 0) = \tilde{\phi}_0 \) ensure regularity at the center. For a given initial central value of the scalar field, \( \tilde{\phi}_0 \), the whole problem is simply reduced to an eigenvalue problem for the angular frequency \( \tilde{\omega} \). The value \( \gamma_0 \) can be arbitrarily chosen before numerical computation. We can always rescale the time variable in eq. (2.7) to satisfy the asymptotic flatness \( \gamma(\tilde{r} \to \infty) = 0 \). For a radius greater than \( 2G_N M \), the metric becomes the Schwarzschild metric.

There exists a discrete spectrum of eigenfrequencies \( \tilde{\omega} \) for each value \( \tilde{\phi}_0 \). The lowest eigenfrequency \( \tilde{\omega}_0 \) corresponds to the ground state configuration, whose radial profile \( \phi(\tilde{r}) \) does not include nodes. The other eigenfrequencies correspond to excited state configurations with zeroes in their radial profile. As mentioned above, we are mainly interested in the ground
state scalar field configuration for boson stars. Since the excited states would decay to the ground state by emission of scalar and gravitational radiations [32], they are not so relevant for our purpose.

We execute a numerical integration of the coupled equations of eqs. (2.10a)–(2.10c) by using the Mathematica Software. The upper limit of the integration, \( \tilde{r}_{\text{max}} \), is chosen to be much greater than the characteristic radius of the profile of the scalar field. According to the presence or absence of zeroes in the radius profile \( \tilde{\phi}(\tilde{r}) \), we define a range of eigenfrequencies \([\tilde{\omega}_a, \tilde{\omega}_b]\) such that \( \tilde{\omega}_a \lesssim \tilde{\omega}_0 \lesssim \tilde{\omega}_b \). After that, we perform a search via the bisection method around the unknown \( \tilde{\omega}_0 \) based on the way in which \( \tilde{\phi}(\tilde{r}) \) diverges. Near \( \tilde{\omega}_0 \) (or any eigenfrequency), the numerical solution diverges upward (downward) if the value chosen for \( \omega \) is smaller (larger) than \( \tilde{\omega}_0 \). Once we obtain a profile which does not diverges in the spatial region \([0, \tilde{r}_{\text{max}}]\), we continue to apply the binary search until \( |\tilde{\phi}(\tilde{r}_{\text{max}})| < \epsilon \), where \( \epsilon \) is the desired tolerance. Finally, we impose asymptotic flatness on the temporal component of the metric by rescaling the value of \( \gamma_0 \) and \( \tilde{\omega}_0 \) as \( \gamma_0 \to \gamma_0 + \ln(a) \) and \( \tilde{\omega}_0 \to \tilde{\omega}_0 \sqrt{a} \), where

\[
\ln(a) \equiv -[\gamma(\tilde{r}_{\text{max}}) + \lambda(\tilde{r}_{\text{max}})].
\]

Depending on the scalar potential of the boson star model, the binary search of the lowest eigenfrequency could be very challenging. For instance, a high level of numeric precision is required in the case of ground state solutions for solitonic boson stars. These solutions turn out to be extremely sensitive to the tiny changes of the eigenfrequency value as a consequence of its steep radial profile [33]. For the boson star models of our current interest, the level of complexity of numerical calculations is similar to that for the mini-boson stars or massive stars with a quartic self-interaction. A detailed analysis of numerical computations for boson stars was given before [34].

Stability properties of boson stars were studied in the literature both analytically [13, 35–37] and numerically [38–41]. Mass and compactness of the ground state of a boson star depend on the central value of the scalar field \( \phi_0 \) and the scalar potential \( U(\Phi^2) \). For the case of the mini-boson stars with free scalar field, there exists a critical point for the central value of the scalar field, \( \phi_0^* \), beyond which the ground state is unstable under small radial perturbations. As \( \phi_0 \) increases, stable configurations have larger masses but smaller effective radius, leading to a greater compactness. This behavior continues until the central value of the scalar field reaches \( \phi_0^* \), where the total mass of the star encounters a turnaround. This turnaround implies a maximum allowed mass for a boson star in the ground state, which is found to be \( M_{\text{max}} = 0.633 m_p^2 / m_\Phi \) [1, 2]. More generic potentials which include one or more self-interaction terms added to the mass-term show the same stability features as the case of the free-field potential [3, 6, 7, 14].

Figure 1 shows the radial profile of the ground state of a boson star with a U(1) Liouville potential for different values of \( \phi_0 \) and \( f \). As we expect for any boson star ground state, these radial profiles of the scalar field do not contain any node and, starting from a central value \( \phi_0 \), go to zero as the radius increases. In particular, while the red curve in figure 1 shows the radial profile of the ground state solution for \( \phi_0 = 0.273 M_p \) and \( f = M_p \), the red curves in figure 2 show the radial and temporal components of the metric associated with this solution. From figure 2, we see that for the region outside the boson star, the metric components of eq. (2.7) overlaps the corresponding Schwarzschild metric components (shown by the blue curves).

In figure 3, we present the boson stars mass \( M \) as a function of the central value of the scalar field \( \phi_0 \) for the scalar potentials \( U_{\text{Liouville}} \) (red curve) and \( U_{\text{Log}} \) (blue curve). In both cases, we have set the coupling strength parameter to be \( f = M_p \). We compute the mass of the ground state of these boson stars by using eq. (2.16), which agrees with the ADM-mass...
Figure 1. Radial profile of the scalar field as a function of the radius for different values of $\phi_0$ and the coupling strength parameter, in the case of U(1) Liouville potential. The field and the radius are shown in units of $M_p$ and $1/m_\phi$, respectively. The red, blue, and green curves correspond to the ground state solutions for $(0.273, M_p)$, $(0.185, 1/\sqrt{20} M_p)$, and $(0.140, 1/\sqrt{40} M_p)$ values of $(\phi_0, f)$, respectively. In particular, the red curve is obtained under the suitable values $\omega = 0.84732123346818m_\phi$ and $\gamma_0 = -0.80267626206664$.

Figure 2. Radial and temporal components of the metric (2.7) (red curves) for the ground state solution of a boson star with the U(1) Liouville potential ($f = M_p$). This solution corresponds to the radial profile with $f = M_p$ in figure 1. The blue curves are the radial and temporal components of the corresponding Schwarzschild metric. The radius is expressed in units of $1/m_\phi$.

in eq. (2.17). The maximum mass for both potentials is obtained at the critical central value of the field $\phi_0^* \approx 0.27$. The total mass of the star shows a turnaround at this critical central value. All configurations on the left-hand (right-hand) side of the critical point $\phi_0^*$ are stable.
Figure 3. Boson star mass $M$ (in units of $m_p^2/m_\Phi$) of the ground state configurations as a function of the central value of the scalar field $\phi_0$ (in units of $M_p$) for the scalar potentials $U_{\text{Liouville}}$ (red curve) and $U_{\text{Log}}$ (blue curve). In both cases, the coupling strength parameter $f$ is set as $f = M_p$.

(unstable). For the two scalar potentials $U_{\text{Liouville}}$ and $U_{\text{Log}}$, we have $M_{\text{max}} = 0.666 m_p^2/m_\Phi$ and $M_{\text{max}} = 0.602 m_p^2/m_\Phi$, respectively. In comparison to the maximum mass for a free-field potential, the repulsive self-interaction terms present in the expansion of the $U(1)$ Liouville potential enhance the value of $M_{\text{max}}$. In contrast, the lower value of $M_{\text{max}}$ for the case of the $U(1)$ logarithmic potential is caused by the presence of the net effective attractive self-interactions from its expansion series. For the case of $U_{\text{Liouville}}$ potential (as shown by the blue curve in figure 3), the value of $M_{\text{max}}$ and the $M(\phi_0)$ curve are in full agreement with [14].

In figure 4, we further present the relation between the mass $M$ (in units of $m_p^2/m_\Phi$) and number of particles $N$ (in units of $m_p^2/m_\Phi^2$) of a boson star with the logarithmic potential $U_{\text{Log}}$ (left panel) and its respective bifurcation diagram (right panel). The bifurcation diagram show cusps which denote changes in the stability properties of the boson star configurations. The first branch is the only stable branch. As we mentioned earlier, a negative binding energy, $E_B = M - N m_\Phi < 0$, is a necessary (but not sufficient) condition for the stability.

In the left panel of figure 5, we present the negative binding energy of the boson star, $E_B = M - N m_\Phi$, as a function of the self-interaction coupling strength $1/f^2$. For the Liouville potential $U_{\text{Liouville}}$, we observe that increasing self-interaction coupling strength raises the magnitude of the negative binding energy $E_B$. But, for the logarithmic potential $U_{\text{Log}}$, the magnitude of the binding energy first decreases up to the coupling strength $\Lambda \sim 5$ and then increases for $\Lambda \geq 5$. We will examine the reason of the different behaviors of $E_B$ for the two scalar potentials in section 3.3.

As we will show in the next subsection, the compactness is one of the most relevant physical quantities for studying the dynamics of boson stars because of its connection to possible astrophysical signatures. The right panel of figure 5 presents the evolution of the maximum compactness reached for ground state configurations of boson stars with respect to
Figure 4. Analysis of the ground state configurations for boson stars with a logarithmic potential $U_{\log}$, where the coupling strength parameter is $f = M_p$. The mass $M$ and number of particles $N$ of the boson star are shown as functions of the central value of the scalar field $\phi_0$ (left panel) and the corresponding bifurcation diagram (right panel). $M$, $N$, and $\phi_0$ are shown in units of $m_p^2/m_\Phi$, $m_p^2/m_\Phi^2$, and $M_p$, respectively.

the coupling strength $\Lambda$. Here we have used the total mass $M$ of the boson star rather than $N m_\Phi$ for computing the maximum compactness $C_{\text{max}}$. For comparison, we also show the case of the usual repulsive quartic self-interaction potential $U_{\text{Quartic}}$, in addition to the potentials $U_{\text{Liouville}}$ and $U_{\log}$. As anticipated, we see that all three cases converge to the compactness of the mini-boson stars, $C_{\text{max}} \approx 0.08$ (shown as black dot), in the weak coupling limit. However, their behaviors deviate from each other as the coupling strength increases. The compactness for the potential $U_{\log}$ increases with the coupling strength, but is slightly smaller than that of the potential $U_{\text{Quartic}}$. For instance, for $U_{\log}$ and $f = 0.1 M_p$, we find that its compactness is only less than that of $U_{\text{Quartic}}$ by 0.3%. The combination of an attractive leading order self-interaction term with alternating repulsive and attractive higher order terms from expanding the U(1) logarithmic potential, produces a similar effect to the compactness of a potential with repulsive quartic self-interaction alone. However, the situation changes for the case of $U_{\text{Liouville}}$. The net effect of all repulsive self-interaction terms from its expansion series increases the compactness significantly above that of the conventional $U_{\text{Quartic}}$ potential. Note that for the $U_{\text{Liouville}}$ potential with $f \approx 1/\sqrt{40} M_p$, the compactness reaches the asymptotic value of that for the $U_{\text{Quartic}}$ potential, $C_{\text{Quartic}}(\Lambda \to \infty) \approx 0.158$ [4]. In the next subsections, we will use the compactness value for both potentials with $f = 0.1 M_p$ as representatives of a strong coupling regime, where $C_{\text{Liouville}} \approx 0.176$. In section 3.3, we will return to the analysis of compactness by using a semi-analytic approach.

3.2 Compactness in landscape of quantum gravity

Although we motivated the scalar potentials introduced in section 2.2, one may further wonder whether these consistent-looking scalar theories coupled to gravity could be UV-completed by a consistent quantum gravity theory. When the answer is positive, the effective field theories (EFT) are said to reside in the Landscape of a quantum gravity theory. In contrast, if the answer is no, they are said to form the Swampland according to [42, 43]. As a way to distinguish EFTs in the Landscape from those in the Swampland, certain criteria are suggested to be checked. Throughout these checks, we want to examine whether $C_{\text{max}}$...
Figure 5. Left Panel: binding energy of the boson star, $E_B = M - N m_\phi$, is shown as a function of the self-interaction coupling strength $\Lambda = 1/\tilde{f}^2$ for scalar potentials $U_{\text{Liouville}}$ (red curve) and $U_{\text{Log}}$ (blue curve). Right Panel: effective compactness $C_{\text{max}}$ is depicted as a function of the self-interaction coupling strength for scalar potentials $U_{\text{Liouville}}$ (red dashed curve) and $U_{\text{Log}}$ (blue dashed curve). For comparison, we show the result for the usual repulsive quartic self-interaction potential, $U_{\text{Quartic}}$ (green dashed curve), including the asymptotic value of its effective compactness when the coupling strength $\Lambda = \lambda_\Phi M_P^2/m_\Phi^2$ goes to infinity: $C_{\text{Quartic}}(\Lambda \to \infty) \simeq 0.158$ [4] (black dashed line). The three curves converge to an initial point in the weak coupling limit (black dot), corresponding to the compactness of the ground state of a mini-boson star.

Figure 6. Plot of $f|\partial_\phi V(|\Phi|)|/V$ as a function of $\phi/f$. The red dashed line is $\tilde{c} \sim O(1)$ appearing in the Criterion 2, and $\tilde{c} = 1$ is taken for our analysis. The potential $U_{\text{Liouville}}$ satisfies the Criterion 2 ($M_P|\partial_\phi V(|\Phi|)|/V > \tilde{c}$) for all $\phi/f > 0$, while $U_{\text{Log}}$ does so for $0 \leq \phi/f \leq 2.6555$.

obtained in section 3.1 is from the Landscape or the Swampland. The similar question was discussed in [44] for the cases of the mini-boson stars and of boson stars with a positive quartic self-interaction potential.
To address the question for our study, we apply the following two criteria [45]:

- **Criterion 1:** \( \Delta \Phi \lesssim \Delta \),
- **Criterion 2:** \( M_P |\nabla \Phi| / V \gtrsim \tilde{c} \),

where \( \Delta \Phi \) in Criterion 1 is the difference between the maximum and the minimum of the field value in unit of \( M_P \). Although the exact values of \( (\Delta, \tilde{c}) \) have not been determined yet, both are expected to be \( \mathcal{O}(1) \). Following [44], we take \( \Delta \) and \( \tilde{c} \) to be the unity for our analysis.

For the complex scalar field in our case, the Criterion 2 reads \( M_P |\partial\Phi|/|V| \gtrsim \tilde{c} \). Using the two potential forms (2.11) and (2.13), we can recast this condition as

\[
F(x) = \frac{2xe^{x^2}}{e^{x^2} - 1} \gtrsim \frac{\tilde{c} f}{M_P}, \quad G(x) = \frac{2x}{(1+x^2)\log(x^2+1)} \gtrsim \frac{\tilde{c} f}{M_P},
\]

where \( x = \phi/f \), and \( F(x) \) and \( G(x) \) equal \( f|\partial\Phi|/|V| \) for \( U_{\text{Liouville}} \) and \( U_{\text{Log}} \) potentials, respectively. The largest coupling strength parameter with which we apply this test is \( f = (1/\sqrt{10})M_P \approx 0.316M_P \). So it suffices to check the inequalities (3.1) only for \( f = (1/\sqrt{10})M_P \) as long as the Criterion 2 is concerned.\(^4\) For the scalar field configuration corresponding to \( C_{\text{max}} \), if \( F(x) \) and \( G(x) \) conform to the inequalities (3.1) with \( F(x), G(x) \gtrsim 0.316 \), then the Criterion 2 is satisfied. For the boson star study, we focus on the regular field configuration without a node which continues to monotonically decrease from the center of the boson star and approaches zero as moving outward. This implies \( \Delta \Phi = \Phi(r=0) \). For a fixed coupling strength parameter \( f \), we want to check whether the field configuration corresponding to \( C_{\text{max}} \) (as found in section 3.1) satisfies the above Criteria 1 and 2. If a violation occurs, then \( C_{\text{max}} \) found in section 3.1 is considered originated from an effective scalar theory belonging to the Swampland.

In figure 6, we plot \( f|\partial\Phi|/|V| \) as a function of \( \phi/f \). We see that the Criterion 2 is readily satisfied by the Liouville potential for any \( \phi/f \gtrsim 0 \). This means that we only need to check whether \( \phi(0) \lesssim 1 \) for \( \phi(0) \) corresponding to \( M_{\text{max}} \) \( (C_{\text{max}}) \) with a given \( f \). As for the logarithmic potential, we note that \( f|\partial\Phi|/|V| \) is a monotonically decreasing function of \( \phi/f \) and the Criterion 2 is satisfied for \( 0 \leq \phi/f \leq 2.6555 \). Hence, to avoid being in the Swampland, we require the intersection between \( \phi(0) \lesssim 1 \) and \( \phi(0) \leq 2.6555f \) for a given \( f \) to be satisfied by \( \phi(0) \) value corresponding to \( M_{\text{max}} \) \( (C_{\text{max}}) \). We show the values of \( \phi(0)/f \) associated with \( C_{\text{max}} \) for each fixed \( f \) in table 1, where the numbers in each parentheses correspond to the values of \( \phi(0) \). In this table, we show the values of \( \phi(r=0) \) up to three decimal points, which are enough for our purpose of checking two Swampland criteria. For the Liouville potential, the inequality \( \phi(0) < 1 \) holds for all \( f \) values under consideration. For the logarithmic potential, both Criteria 1 and 2 turn out to hold for all \( f \) values considered. Hence, \( C_{\text{max}}^{\text{Liouville}} \) and \( C_{\text{max}}^{\text{Log}} \) in figure 5 are all from the scalar theories in Landscape for \( \Lambda \gtrsim 10 \).

3.3 Semi-analytic approach

In this subsection, we try to analytically understand the results we obtained in section 3.1 by following the logic of [13, 22, 26, 46, 47]. The strategy is to estimate the Hamiltonian of the

\(^4\)One may apply a higher value of the coupling strength parameter \( f \) which corresponds to smaller self-interaction coupling regime \( \Lambda < 10 \) in the right panel of figure 5. Since we mainly want to check \( C_{\text{max}} \) produced from strong coupling regime, we restrict our analysis to \( f \lesssim (1/\sqrt{10})M_P \), which corresponds to \( \Lambda = 1/f^2 \gtrsim 10 \).


\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$V(\Phi)$ & $f=1/\sqrt{10}$ & $f=1/\sqrt{20}$ & $f=1/\sqrt{40}$ & $f=1/\sqrt{60}$ & $f=1/\sqrt{80}$ & $f=1/\sqrt{100}$ \\
\hline
Liouville & 1.866 (0.59) & 2.996 (0.67) & 4.427 (0.70) & 5.593 (0.72) & 6.494 (0.72) & 7.3 (0.73) \\
\hline
Logarithmic & 0.727 (0.23) & 0.827 (0.18) & 0.885 (0.14) & 0.891 (0.11) & 0.939 (0.10) & 0.900 (0.09) \\
\hline
\end{tabular}
\caption{The values of $\phi(r=0)/f$ corresponding to $M_{\text{max}}$ ($C_{\text{max}}$) are presented for each potential and coupling strength parameter $f$ (in the unit of $M_P$). In each parentheses the value of $\phi(0)$ is also shown. For the Liouville potential, all the $\phi(0)$ values are less than 1 which is the intersection of the two criteria (see the text). For the logarithmic potential, $\phi(r=0)/f < 2.6555$ can be checked for $f$ considered in this study, and so does $\phi(0) < 1$. This shows that all the $C_{\text{max}}$ values shown in figure 5 can be regarded as arising from effective scalar theories UV-completed by a consistent quantum gravity.}
\end{table}

system from eq. (2.8a) by using an ansatz for the radial profile of the field in eq. (2.6). This ansatz controls the shape of the profile by a single length scale $R$. Following a variational approach, we extremize the Hamiltonian with respect to $R$ and obtain an estimate of the maximum total mass $M_{\text{max}}$, the minimum radius $R_{\text{min}}$, and the maximum compactness $C_{\text{max}}$.

We first introduce an exponential ansatz of the scalar field wavefunction by specifying $\Phi(r,t)$ in eq. (2.6) as

$$\phi(r) = \sqrt{\frac{N}{\pi m R^3}} e^{-r/R}, \quad \omega^2 = m^2 \left(1 - \frac{G_N m N}{R}\right),$$

(3.2)

where we expect that the kinetic energy is proportional to the gravitational potential energy up to a factor, which may be parametrized as $\alpha$ and will be determined later. In the weak gravity limit, we may approximate the metric (2.7) as

$$e^{\gamma(r)} \simeq 1 + 2V(r), \quad e^{\lambda(r)} \simeq 1 - 2V(r),$$

(3.3)

where $V(r) = -G_N M(r)/r$ is a gravitational potential. Approximating the mass of the boson star system as $m_{\Phi}N$, we may use the following definition for the boson star mass,

$$M^A(r) \equiv \int_0^r d^3r 4\pi r^2 (m_{\Phi}^2 \Phi(r)^2).$$

(3.4)

Using this together with eq. (3.2), we obtain the gravitational potential

$$V(r) = -\frac{G_N m_{\Phi} N}{r} \left[1 - e^{-\frac{2r}{R}} \left(1 + \frac{2r}{R} + \frac{2r^2}{R^2}\right)\right].$$

(3.5)

The superscript $A$ in eq. (3.4) stands for the mass to be used in our analytic approach. We stress that this mass differs from the total mass (2.16) that we used for the numerical computations in section 3.1. From eqs. (2.8a) and (3.3), we derive in the weak gravity limit,

$$T_0^0(r) = -\omega^2 [1-2V(r)]\phi^2 - [1+2V(r)](\partial_r \phi)^2 - U(\phi^2),$$

(3.6)

which leads to the following Hamiltonian of the boson star when integrating over the proper volume\footnote{The integration measure should be $d^3r \sqrt{-g}$. For simplicity of the analysis, we take the weak gravity limit and approximate $\sqrt{-g} \simeq 1.$}

$$H_{\text{tot}} = \int d^3r T_0^0(r).$$

(3.7)
We may decompose $H_{\text{tot}}$ into three different contributions,

$$H_{\text{grav}} + H_{\text{mass}} = - \int_0^\infty d^3 r \left\{ \omega^2 [1 - 2 V(r)] \phi^2 + m_\Phi^2 \phi^2 \right\}, \quad (3.8a)$$

$$H_{\text{kin}} = - \int_0^\infty d^3 r \left( \partial_r \phi \right)^2 [1 + 2 V(r)], \quad (3.8b)$$

$$H_{\text{int}} = - \int_0^\infty d^3 r \left[ U(\phi^2) - m_\Phi^2 \phi^2 \right], \quad (3.8c)$$

By substituting the explicit form of the ansatz (3.2) into eq. (3.8a), we obtain

$$H_{\text{grav}} + H_{\text{mass}} = -2m_\Phi N + \frac{G N m_\Phi^2 N^2 (8\alpha - 5)}{8R}, \quad (3.9)$$

where we have dropped an additional term $5G_N^2 m_\Phi^3 N^3 \alpha / 8R^2$ due to its large suppression by the squared Newton constant $G_N^2$. We expect eq. (3.9) to reduce to $H_{\text{grav}}$ in the non-relativistic limit and this requirement fixes the value $\alpha = \frac{5}{6}$. Thus, we obtain

$$H_{\text{grav}} + H_{\text{mass}} = -2m_\Phi N + \frac{5G_N m_\Phi^2 N^2}{8R}. \quad (3.10)$$

Similarly, we derive the kinetic energy arising from the gradient of the field,

$$H_{\text{kin}} = - \frac{N}{m_\Phi R^2} + \frac{5G_N N^2}{8R^3}. \quad (3.11)$$

Finally, we compute $H_{\text{int}}$ due to the two different potentials introduced in section 2.2. For the Liouville potential (2.11), the exponential ansatz leads to

$$H_{\text{Liouville}}^{\text{log}} = - \int_0^\infty dr \ 4\pi r^2 \left[ U(\phi^2) - m_\Phi^2 \phi^2 \right]$$

$$= -f^2 m_\Phi^2 \int_0^\infty dr \ 4\pi r^2 \sum_{k=2}^{\infty} \phi^{2k}$$

$$= -4\pi f^2 m_\Phi^2 \sum_{k=2}^{\infty} \frac{1}{f^{2k}k!} \left( \frac{N}{\pi m_\Phi R^2} \right)^k \int_0^\infty dr \ r^2 e^{-2kr/R}$$

$$= -\pi f^2 m_\Phi^2 R^3 \sum_{k=2}^{\infty} \frac{1}{f^{2k}k^3} \left( \frac{N}{\pi m_\Phi R^2} \right)^k, \quad (3.12)$$

which provides a series of repulsive forces. For the logarithmic potential (2.13), the exponential ansatz leads to

$$H_{\text{int}}^{\text{log}} = - \int_0^\infty dr \ 4\pi r^2 \left[ U(\phi^2) - m_\Phi^2 \phi^2 \right]$$

$$= -f^2 m_\Phi^2 \int_0^\infty dr \ 4\pi r^2 \sum_{k=2}^{\infty} (-1)^{k-1} \frac{\phi^{2k}}{k f^{2k}}$$

$$= -4\pi f^2 m_\Phi^2 \sum_{k=2}^{\infty} \frac{(-1)^{k-1}}{f^{2k}k} \left( \frac{N}{\pi m_\Phi R^2} \right)^k \int_0^\infty dr \ r^2 e^{-2kr/R}$$

$$= -\pi f^2 m_\Phi^2 R^3 \sum_{k=2}^{\infty} \frac{(-1)^{k-1}}{k^4 f^{2k}} \left( \frac{N}{\pi m_\Phi R^2} \right)^k, \quad (3.13)$$

which yields a series of pairs of attractive and repulsive forces.

\(^6\text{We refer the non-relativistic-limit result } H_{\text{grav}} = -5G_N m_\Phi^3 N^2 / 16R \text{ to ref. [22].}\)
Figure 7. Solutions of $\partial \tilde{H} / \partial \tilde{R} = 0$ for $f^2 G_N = 1$ and for the Liouville potential with truncation at $\phi^6$ term. For a fixed value of $\tilde{N}$, there exists a pair of $\tilde{R}$. The larger $\tilde{R}$ makes $\partial^2 \tilde{H} / \partial \tilde{R}^2 > 0$ (red curve, stable branch) and the smaller one makes $\partial^2 \tilde{H} / \partial \tilde{R}^2 < 0$ (blue curve, unstable branch). For the $\tilde{N}$ value at which the two different branches meet together, the boson star realizes its highest possible compactness. For $(\tilde{N}_{\text{max}}, \tilde{R}_{\text{min}}) = (0.939007500937, 1.70293)$, the relevant boson star is characterized by $C_A^{\text{max}} = 0.129882$.

Given the above explicit expressions for the different self-interaction Hamiltonians, we can estimate minimum size of the scale radius $R_{\text{min}}$ and an associated $N_{\text{max}}$ by extremizing $H_{\text{tot}}$. As shown in [22], it is convenient to go through the extremizing procedure after rescaling $R$, $N$ and $H$ into the dimensionless quantities,

$$\tilde{R} \equiv m \phi \sqrt{G_N} R, \quad \tilde{N} \equiv \frac{m^2 \phi \sqrt{G_N}}{f} N, \quad \tilde{H} \equiv \frac{m \phi}{f^3 \sqrt{G_N}} H. \tag{3.14}$$

This results in

\begin{align}
\tilde{H}_{\text{Liouville}} &= -\frac{2 \tilde{N}^2}{f^2 G_N} + \frac{5 \tilde{N}^2}{8 \tilde{R}^2} - \frac{\tilde{N}}{\tilde{R}} + \frac{5 (f^2 G_N) \tilde{N}^2}{8 \tilde{R}^3} - \sum_{k=2}^{\infty} \frac{(f^2 G_N)^{k-2}}{k^3 k! \pi^{k-1}} \frac{\tilde{N}^k}{\tilde{R}^{3k-3}}, \tag{3.15a} \\
\tilde{H}_{\text{Log}} &= -\frac{2 \tilde{N}^2}{f^2 G_N} + \frac{5 \tilde{N}^2}{8 \tilde{R}^2} - \frac{\tilde{N}}{\tilde{R}} + \frac{5 (f^2 G_N) \tilde{N}^2}{8 \tilde{R}^3} - \sum_{k=2}^{\infty} \frac{(-1)^{k-1} (f^2 G_N)^{k-2}}{k^4 \pi^{k-1}} \frac{\tilde{N}^k}{\tilde{R}^{3k-3}}. \tag{3.15b}
\end{align}

Next, for a set of values of $f^2 G_N \simeq (f/m_p)^2$, we estimate a minimum radius $\tilde{R}_{\text{min}}$ and a maximum number of constituent bosonic particles $\tilde{N}_{\text{max}}$ which can be realized for a stable boson star. For this purpose, we take a fixed value of $\tilde{N}$ and we search for $\tilde{R}$ associated with a local minimum and maximum of $\tilde{H}$, which correspond to the stable and unstable boson star branches, respectively.

In figure 7, we present an example of how to find $(\tilde{R}_{\text{min}}, \tilde{N}_{\text{max}})$ from Taylor-expanded Liouville potential with a truncation at $\phi^6$ term and $f^2 G_N = 1$. For a fixed value of $\tilde{N}$, we find a pair of $\tilde{R}$’s satisfying $\partial \tilde{H} / \partial \tilde{R} = 0$. The $\tilde{R}$ solution obeying $\partial^2 \tilde{H} / \partial \tilde{R}^2 > 0$ corresponds to the stable boson star solution and vice versa. For the high enough $\tilde{N}$, two branches merge
Figure 8. Plot of $\tilde{H}$ as a function of $\tilde{R}$ for $f^2G_N = 1$ and for the Liouville potential truncated at $\phi^6$ term. The red solid curve is for $\tilde{N} = 0.939007500937$, the green dashed curve for $\tilde{N} = 0.90$, and the blue dash-dotted curve for $\tilde{N} = 0.87$. In each case, there exist two extremum and those get closer to each other as $\tilde{N}$ increases. Eventually for $\tilde{N} = 0.939007500937$, those two merge into a single one which is identified as $\tilde{R}_{\text{min}}$. 

into a single solution of $(\tilde{N}, \tilde{R})$ eventually, which yields $\tilde{N}_{\text{max}}$ and $\tilde{R}_{\text{min}}$. In figure 8, we illustrate how the two extremum of $\tilde{H}$ approach each other as $\tilde{N}$ increases.

Given $(\tilde{N}_{\text{max}}, \tilde{R}_{\text{min}})$ for a choice of $f^2G_N$ and using eq. (2.19), we compute the maximum compactness which the boson star can realize,

$$C_{\text{max}}^A = \frac{G_N 0.99 M^A}{R_{\text{min}}} = \frac{G_N 0.99 m_{\Phi} N_{\text{max}}}{4.203 R_{\text{min}}} = \frac{0.99}{4.203} \times \frac{f^2G_N \tilde{N}_{\text{max}}}{\tilde{R}_{\text{min}}},$$

where the superscript on $C_{\text{max}}^A$ means that it is computed with $M^A$ defined in eq. (3.4) and the factor of 4.203 arises from

$$0.99N = \int_0^{4.203 R_{\text{min}}} d\tau 4\pi \tau^2 [m_{\Phi} \Phi^2(r)].$$

For instance, given $(\tilde{N}_{\text{max}}, \tilde{R}_{\text{min}}) = (0.939007500937, 1.70293)$ as in figure 7, we deduce $C_{\text{max}} = 0.129882$.

Note that for an estimate of compactness by the semi-analytic approach, we approximate the boson star mass as $m_{\Phi} N$ for simplicity, though this does not take into account the binding energy of the boson star. In the following, we will compare $C_{\text{max}}^A$ to $C_{\text{max}}$ from the complete numerical computation in order to understand different binding energy behaviors of boson stars of different potential observed in section 3.1 for the small coupling regime.

With the semi-analytic approach described above, we try to understand the numerical results obtained in section 3.1. For the maximum compactness in figure 5, we observe that starting from the same value in free theory limit ($1/f^2 = 0$), $C_{\text{max}}^A$ due to $U_{\text{Liouville}}$ and $U_{\text{Log}}$ splits up with the increasing self-interaction strength and ends up with the hierarchy,

$$C_{\text{max}}^\text{Liouville} > C_{\text{max}}^\text{Quartic} > C_{\text{max}}^\text{Log},$$

(3.18)
Figure 9. Analytic computation of $C_{\text{max}}^\Lambda$ from the Taylor-expanded Liouville potential and logarithmic potential with truncation at $\Phi^4$ and $\Phi^6$ terms.

where $C_{\text{max}}^{\text{Quartic}}$ is due to the repulsive quartic self-interaction ($\sim \lambda_\Phi \Phi^4$ with $\lambda_\Phi > 0$). This shows that an infinite number of repulsive self-interactions from expansion of $U_{\text{Liouville}}(|\Phi|^2)$ renders the boson star system more compact than that with the usual repulsive quartic self-interaction. On the other hand, alternating attractive and repulsive self-interactions arising from expanding $U_{\text{Log}}(|\Phi|^2)$ tend to cancel each other and yield a small net attractive non-gravitational force. Hence, this small net effect makes the relevant boson star less compact than that with the repulsive self-interaction.

In figure 9, we plot $C_{\text{max}}^\Lambda$ as a function of $f^2 G_N$, where $C_{\text{max}}^\Lambda$ is defined in eq. (2.19) following the semi-analytic approach explained above. The left and right panels show $C_{\text{max}}^\Lambda$ based on the Taylor-expanded Liouville and logarithmic potentials truncated at $\Phi^4$ and $\Phi^6$ terms, respectively. For Liouville potential, both $\Phi^4$ and $\Phi^6$ terms have positive signs and thus generate repulsive forces. The $\Phi^6$ contribution to the repulsive force together with that of the $\Phi^4$ term helps for competing with the gravitational attraction, which leads to the higher compactness for a fixed $f^2 G_N$. This tendency becomes more prominent as self-interaction strength increases (as $f^2 G_N$ decreases). When there are more additional repulsive forces provided by the rest of the higher order terms with positive sign in the Taylor-expanded Liouville potential, we expect the boson star would become more compact as compared to the case of the usual repulsive $\Phi^4$ only. This is well reflected in figure 5 with the blue dashed curve ($C_{\text{max}}$ due to complete Liouville potential) and green dashed curve ($C_{\text{max}}$ due to repulsive quartic interaction). In contrast, for the logarithmic potential, $\Phi^4$ term produces attractive force whereas $\Phi^6$ term is repulsive. Accordingly, the repulsive $\Phi^6$ term plays a role of diluting the attractive force of the $\Phi^4$ term. The attractive $\Phi^4$ term tends to decrease $C_{\text{max}}^\Lambda$ as compared to a boson star with a quartic repulsive self-interaction in a free scalar theory, and this becomes more significant for the higher self-interaction strength (for the smaller $f^2 G_N$). But, the degree of decrease in $C_{\text{max}}^\Lambda$ becomes weaker when the repulsive force of $\Phi^6$ term dilutes the attractive force from $\Phi^4$. In the end, we expect the net attractive force due to pairing attractive and repulsive forces from terms in Taylor-expanded logarithmic potential to make $C_{\text{max}}^\Lambda$ lie between 0.095 and 0.105 at $f^2 G_N = 0.1$.\footnote{We have checked that $C_{\text{max}}^\Lambda$ from interaction terms up to $\Phi^{10}$ in expansion of logarithmic potential takes a value of $C_{\text{max}}^\Lambda = 0.103$.}
According to the definition of $C_{\text{max}}^A$, we may infer the behavior of the negative binding energy $E_B$ from $C_{\text{max}}^N - C_{\text{max}}^A \propto E_B$, where $C_{\text{max}}^N$ is the maximum compactness computed numerically in section 3.1. The value $f^2G_N = 0.1$ corresponds to the coupling strength $\Lambda \simeq 0.4$, so changing from $f^2G_N = 1$ to $f^2G_N = 0.1$ corresponds to varying the coupling strength from $\Lambda \simeq 0.04$ to $\Lambda \simeq 0.4$. For Liouville potential, we observe that as the coupling $\Lambda$ rises, both $C_{\text{max}}^N$ and $C_{\text{max}}^A$ increase, and the difference $|C_{\text{max}}^N - C_{\text{max}}^A| \propto |E_B|$ also rises in the small coupling regime. This trend is in agreement with the red curve in the left panel of figure 5. On the other hand, for the logarithmic potential, $C_{\text{max}}^A$ decreases and $C_{\text{max}}^N$ increases for rising $\Lambda$ in the small coupling regime where $C_{\text{max}}^A > C_{\text{max}}^N$, so the difference $|C_{\text{max}}^N - C_{\text{max}}^A| \propto |E_B|$ decreases for the small coupling regime. This explains the behavior of the blue curve in the small coupling regime as shown by the left panel of figure 5.

4 Astrophysical probe of boson stars

In this section, we study two ways to experimentally probe the presence of boson stars by constraining their model parameter space. The boson stars may be responsible for a small fraction of the dark matter in the Universe by serving as a kind of MACHO (macroscopic compact halo object), which we infer from [48–52]. For the present study, we focus on the scenarios where the fraction of the DM provided by the boson stars is either less than 10% or 1%.

In section 4.1, we will derive the allowed mass range of the scalar particles which compose the boson stars of our interest. In section 4.2, we consider that a small fraction of the dark matter is attributed to the boson stars, and study how we can probe a boson star by the lensing of fast radio burst (FRB). Then, in section 4.3, we analyze whether the gravitational wave (GW) signals caused by the merger of two boson stars can be detected by the laser interferometer gravitational-wave observatory (LIGO). For both astrophysical probes, we will discuss the different implications for the boson stars with the two benchmark scalar potentials as studied earlier.

4.1 Scalar particle mass range for boson stars

In section 3.1, we computed the maximum total mass $M_{\text{max}}$ of a boson star in unit of $m_P^2/m_\Phi$, for each scalar potential with a coupling strength $\Lambda$. Given a fraction of the DM contributed by the boson stars ($\xi_{\text{DM}}$), we can derive the relevant mass range of the scalar particle $\Phi$ by comparing $M_{\text{max}}$ to the current constraint on the MACHO mass.

The microlensing survey provides constraints on $\xi_{\text{DM}}$ and $M_{\text{max}}$ for MACHO mass range $10^{-11} < M/M_\odot < 30$ [48–50, 52], which can be applied to the primordial black holes (PBHs) and the exotic compact objects including boson stars. For the heavier mass range of $M \gtrsim 100M_\odot$, the cosmic microwave background (CMB) anisotropy excludes PBHs as the dominant component of DM [53]. On the other hand, the survival of a star cluster near the core of Eridanus II and of a sample of compact ultra-faint dwarfs places constraints on $\xi_{\text{DM}}$ and $M_{\text{max}}$ for MACHO mass range $M \gtrsim 5M_\odot$ [51]. For exemplary fractions $\xi_{\text{DM}} = 0.1$ and 0.01, we find the allowed MACHO mass ranges to be $1 \lesssim M/M_\odot \lesssim 100$ and $10^{-7} \lesssim M/M_\odot \lesssim 10^3$, respectively. In table 2, for $\Lambda = 10$ and $\Lambda = 100$, we derive the corresponding constraints on the scalar boson mass for the two benchmark potentials, which is obtained based on the analysis of $M_{\text{max}}$ in section 3.1.

In addition, we may wonder whether the scalar mass range as allowed by the MACHO constraints (table 2) can be consistent with the cold dark matter (CDM) isocurvature modes in the CMB power spectrum. For the present boson star study, we consider the scenario
Table 2. Allowed mass ranges of the scalar particle under two benchmark potentials, as inferred from the MACHO mass constraints [48–52] for the DM fraction $\xi_{\text{DM}} = 0.1, 0.01$ and the coupling strength $\Lambda = 10, 100$. Here the unit of the scalar particle mass is eV.

| $\xi_{\text{DM}}$ | Liouville Potential ($\Lambda = 10$) | Logarithmic Potential ($\Lambda = 10$) |
|-------------------|---------------------------------|---------------------------------|
| 0.1               | $1.3 \times 10^{-12} \lesssim m_\Phi \lesssim 1.3 \times 10^{-10}$ | $6.4 \times 10^{-13} \lesssim m_\Phi \lesssim 6.4 \times 10^{-11}$ |
| 0.01              | $1.3 \times 10^{-13} \lesssim m_\Phi \lesssim 1.3 \times 10^{-3}$ | $6.4 \times 10^{-14} \lesssim m_\Phi \lesssim 6.4 \times 10^{-4}$ |
| $\xi_{\text{DM}}$ | Liouville Potential ($\Lambda = 100$) | Logarithmic Potential ($\Lambda = 100$) |
| 0.1               | $3.6 \times 10^{-12} \lesssim m_\Phi \lesssim 3.6 \times 10^{-10}$ | $8.4 \times 10^{-13} \lesssim m_\Phi \lesssim 8.4 \times 10^{-11}$ |
| 0.01              | $3.6 \times 10^{-13} \lesssim m_\Phi \lesssim 3.6 \times 10^{-3}$ | $8.4 \times 10^{-14} \lesssim m_\Phi \lesssim 8.4 \times 10^{-4}$ |

where the scalar particle mass is comparable to the Hubble expansion rate during inflation ($H_{\text{inf}}$) [54]. During inflation the scalar mass $m_{\Phi, \text{inf}} \approx H_{\text{inf}}$ can be generated by the gravitationally induced coupling between the scalar boson $\Phi$ and inflaton $\chi$ [54],

$$\mathcal{L}_{\Phi \chi} = c_{\Phi \chi} \frac{V(\chi)}{M_P^2} \Phi^2,$$  \hspace{1cm} (4.1)

where $V(\chi)$ is the inflaton potential and $c_{\Phi \chi}$ is a dimensionless coupling parameter. Since the scalar mass of our interest obeys $m_{\Phi} \ll H_{\text{inf}}$, the $\Phi$ mass during inflation ($m_{\Phi, \text{inf}}$) becomes effectively $m_{\Phi, \text{inf}} \approx H_{\text{inf}}$. For the post-inflationary epoch, $m_{\Phi, \text{inf}}$ reduces to the original small mass ($m_{\Phi}$) which corresponds to $V(\chi) = 0$ as the global minimum of the inflaton potential.

For the two benchmark scalar potentials in section 2.2, their mass term dominates over the quartic interaction and non-renormalizable terms arising from expanding the potential with $|\Phi| < f$. Hence, after inflation ends, the homogeneous scalar field obeys the time evolution equation in the expanding background,

$$\ddot{\Phi} + 3H \dot{\Phi} + m_{\Phi}^2 \Phi = 0,$$  \hspace{1cm} (4.2)

where $H$ is the Hubble expansion rate and the dot denotes the derivative with respect to time $t$. For the scalar mass $m_{\Phi} < O(10^{-3})$eV relevant to the boson stars serving as a candidate of MACHO, the relation $H > m_{\Phi}$ holds after inflation and the field value remains as $\phi_{\text{inf}}$ (the field displacement from the global minimum of the potential) due to the overdamping from the end of the inflation to the time when $H \approx m_{\Phi}$ is reached. Accordingly, the energy density of the scalar particles remains as a constant, $\rho_{\Phi} = m_{\Phi}^2 \phi_{\text{inf}}^2/2$, until $H \approx m_{\Phi}$ is realized. Then, $\rho_{\Phi}$ scales as $a^{-3}$. When the oscillation starts, we have $H \approx m_{\Phi}$ and the temperature of the Universe is given by

$$T_{\text{osc}} = \left(\frac{90}{\pi^2}\right)^{1/4} g^1/4 \sqrt{M_P m_{\Phi}},$$  \hspace{1cm} (4.3)
where \( g_s \) is the effective number of relativistic degrees of freedom. In our scenario, we denote the fraction of DM contributed by the boson stars as \( \xi_{DM} \). With \( T_{osc} \) in eq. (4.3) for \( n_{ph}/s \), we derive the relation [54],

\[
\xi_{DM}(\Omega_{DM,0}) = \frac{\rho_{\phi,0}}{\rho_{c,0}} = \frac{n_{ph,osc} s_0}{m_{\phi} s_{osc}}, \quad \Rightarrow
\]

\[
m_{\phi} \simeq \xi_{DM}^2(3 \times 10^{-5}) \left( \frac{g_s}{106.75} \right)^{1/2} \left( \frac{\phi_{inf}}{10^{11} \text{GeV}} \right)^{-4} \text{eV},
\]

(4.4a)

where \( n_{ph} \) is the number density of the scalar particle, \( s \) is the entropy density, and \( 0 \) (osc) in the subscript indicates that the quantities are evaluated for today (\( t_{osc} \)). For our current analysis, we will require the DM fraction from boson stars to be 1%, \( \xi_{DM} \approx 0.01 \), which serves as a benchmark condition. As explained in appendix A, for \( m_{\phi,inf} \approx H_{inf} \), the CMB constraint on the isocurvature parameter \( \beta_{iso} < 0.038 \) at \( k_{mid} = 0.050 \text{Mpc}^{-1} \) leads to the condition \( \phi_{inf} \lesssim 0.25 H_{inf} \). Using \( H_{inf} \approx 6.3 \times 10^{13} \sqrt{r/0.064} \text{GeV} \) [54] and the CMB constraint on the tensor-to-scalar ratio \( r \lesssim 0.064 \) [55], we finally derive

\[
m_{\phi} \gtrsim \xi_{DM}^2 \times 4.875 \times 10^{-6} \times \left( \frac{g_s}{106.75} \right) \times \left( \frac{r}{0.064} \right)^2 \text{eV}.
\]

(4.5)

For \( 10.75 \leq g_s \leq 106.75 \),\(^8\) we may expect the allowed minimum scalar mass for the boson stars with \( \xi_{DM} = 10^{-2} \) (or \( \xi_{DM} = 10^{-1} \)) to be \( m_{\phi} \gtrsim \mathcal{O}(10^{-10}) \text{eV} \) (or \( m_{\phi} \gtrsim \mathcal{O}(10^{-8}) \text{eV} \)). In comparison with the MACHO constraints in table 2, we find that the fraction \( \xi_{DM} = 0.01 \) is consistent with the current cosmological and astrophysical data for the scalar mass-range \( \mathcal{O}(10^{-10}) \text{eV} < m_{\phi} < \mathcal{O}(10^{-8}) \text{eV} \), while the fraction \( \xi_{DM} = 0.1 \) is excluded because the MACHO and CMB constraints on \( m_{\phi} \) do not overlap. Hence, in the following analyses, we focus on the benchmark case where the boson stars are responsible for 1% of the DM population in the Universe with the mass range \( \mathcal{O}(10^{-10}) \text{eV} < m_{\phi} < \mathcal{O}(10^{-8}) \text{eV} \).

4.2 Probe by lensing of fast radio bursts

The origin of the fast radio bursts (FRBs) remains unknown, and we expect that the boson stars can cause some portion of the generated FRBs to be lensed by their own gravitational field. Probing the dark matter (DM) mass and the fraction of DM as occupied by the primordial black hole was studied before [56]. It was also extended to the applications for probing the parameter space of exotic compact objects including the mini-boson stars, the boson stars with quartic self-interaction, and the fermion stars [57]. In this subsection, following [56, 57], we study how to use the lensing of FRBs to probe the presence of boson stars as well as to constrain the parameter space \( (m_{\phi}, \Lambda) \) of our boson star models. The FRBs and their usage were discussed before [56–58]. For the purpose of the present study, we briefly explain the computation of the optical depth in appendix B.

In our analysis, we assume the total number of observable FRB signals in the near future to be \( N_{FRB} = 10^4 \) according to [59] where it was shown that CHIME could detect thousands of FRBs throughout its envisioned project lifetime (3 years). For a set of values of the DM fraction \( \xi_{DM} \) as contributed by the boson stars, the boson star mass \( M_{\star} \) as a lens, and the time delay \( \Delta T \) between the two images from the FRB gravitational lensing, we can estimate

\(^8\)To be consistent with the MACHO constraint on the scalar particle mass \( m_{\phi} \), we have \( m_{\phi} \gtrsim \mathcal{O}(10^{-13}) \text{eV} \). The effective number of relativistic degrees of freedom \( g_s \) would be \( g_s(T \approx 10 \text{MeV}) \sim 10.75 \) at the time when \( H \approx m_{\phi} \sim \mathcal{O}(10^{-13}) \text{eV} \) holds.
Figure 10. Parameter space of the boson star models which can be probed by detecting the lensed fast radio bursts (FRBs). We assume that the DM fraction contributed by boson stars is $\xi_{DM}=1\%$. In each plot, the regions above the red horizontal lines can be probed by the lensed FRBs with the corresponding time delays $\Delta t$. For each given coupling strength parameter, the blue and magenta lines show the relation between the scalar particle mass and the maximum mass of the boson star.

We present in figure 10 the parameter space of the two benchmark models of boson stars that can be probed by detecting the lensed FRB signals by the boson star. We consider four different reference times ($\Delta t = 0, 0.3, 1, 3$ ms) following [56, 57] and an exemplary DM fraction contributed by boson stars ($\xi_{DM}=0.01$). As explained in [57], FRB170827 shows the
temporal profile with three different components. The narrowest component is characterized by the width \( \sim 30 \mu s \), which justifies the choice of the smallest reference time, i.e., 0.1ms if one intends to detect the lensed image of mini-bursts (cf. figure 2 of [60]). The solid and dashed blue lines in figure 10 represent the relation between the scalar particle mass \( m_\Phi \) and the maximum achievable boson star mass \( M_{\text{max}} \) for the dimensionless coupling strength \( \Lambda = 10 \) and \( \Lambda = 100 \), respectively. The relation line with the coupling strength between \( \Lambda = 10 \) and \( \Lambda = 100 \) is located between the solid and dashed blue lines. The relation lines are plotted based on our results in section 3.1. Besides, the magenta line is plotted with the asymptote of the maximum boson star mass when \( \Lambda \) approaches infinity, which we inferred by fitting and extrapolating \( M_{\text{max}} \) for \( \Lambda \leq 100 \) as numerically obtained in section 3.1. The horizontal red lines denote the minimum boson star mass above which detecting the FRBs lensed by the boson star with the corresponding time delays \( \Delta t \) becomes possible. These red lines are derived by requiring the probability \( \tau \geq 10^{-4} \) for the benchmark DM fraction \( \xi_{\text{DM}} = 0.01 \).

The optical depth computation reveals that the smaller time delay between the two images can better probe the presence of the boson star with smaller mass. As discussed in our previous section 4.1, the allowed mass range of the scalar particles which compose the boson star reads \( \mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV} \). Hence, what matters is whether \( M_{\text{max}} \) resulting from the scalar mass range of \( m_\Phi \) for a given potential can be large enough to be probed by detecting the lensed FRB signals. For the Liouville potential and in the scalar mass range \( \mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV} \), we find that \( M_{\text{max}} \) with \( \Lambda \geq 100 \) becomes large enough to be probed by FRB lensing with \( \Delta t = 0.1\text{ms} \). In contrast, the logarithmic potential is unable to produce a large enough \( M_{\text{max}} \) to be probed by FRB lensing in the same scalar mass range \( \mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV} \). For a given scalar particle mass \( m_\Phi \), the logarithmic potential tends to produce a boson star with lighter total mass than that of the Liouville potential. Hence, it is more challenging to probe the boson stars with logarithmic potential by FRB lensing because it requires a shorter time delay between the two images.

### 4.3 Probe by gravitational wave detection

If the boson stars described by our scalar potential models appear in the Universe, we may expect the two neighboring boson stars to form a binary system, go through inspiraling motion, and then merge to produce gravitational waves (GWs). The frequency of the GW depends on the mass and compactness of the boson stars. Hence they could be invoked to infer the information about the scalar particle mass \( m_\Phi \) and the coupling strength \( \Lambda \) of our potential models introduced in section 2.2. Using the LIGO GW measurement to probe the mini-boson star or boson stars with a quartic potential was discussed before [61]. In this subsection, we apply our two benchmark potential models (section 2.2) to study how the scalar particle mass \( m_\Phi \) and coupling strength \( \Lambda \) can be possibly probed by the LIGO GW detection. In appendix C, we will explain further about obtaining the sub-region in the plane of \( (C, M_{\text{max}}) \) which corresponds to the high enough signal to noise ratio (SNR) for the LIGO GW detector. Here we assume a boson star accomplishes its allowed maximum mass \( M_{\text{max}} \) and compactness \( C_{\text{max}} \) for stability. For the simplicity of analysis, we consider that the two merging boson stars share the common mass and compactness. We shall address the

\[8\] We note that a smaller DM fraction \( \xi_{\text{DM}} < 0.01 \) can relax the lower bound on \( m_\Phi \) according to eq. (4.5). Then, our figure 10(b) shows that for a low enough scalar mass \( m_\Phi \) to produce the total boson star mass \( M \geq 5M_\odot \), this is within \( m_\Phi \) range as allowed by the current \( \beta_{\text{iso}} \) constraint. So, if the planned future CHIME experiment allows the use of FRB lensing with \( \Delta t = 0.1\text{ms} \), then the lensed FRB signals can be detected by CHIME even for the Logarithmic potential.
following questions: for what values of \((m_{\Phi}, \Lambda)\), can the merger of two boson stars produce GW signals whose frequency falls into the sensitive range of the LIGO detector? Could such \(m_{\Phi}\) values be consistent with the mass range as we discussed in section 4.1?

We present the findings from our analysis in figure 11. For a set of values of \((C_{\text{max}}, M_{\text{max}})\) within the sub-region with cyan color, the merger of the binary boson stars will generate GW signals whose frequency lies in the range \((50\text{Hz} \leq \nu \leq 1000\text{Hz})\) where the noise spectral density \(S_n(\nu)\) is low enough for the LIGO sensitivity. In addition, the GW
signals from the boson stars within the sub-region of yellow color satisfy the high enough SNR with $\rho \geq 8$, where $\rho$ is defined in eq. (C.3). The (red, green and blue) dots correspond to the ($C_{\text{max}}$, $M_{\text{max}}$) values of the boson stars formed by the scalar particles of mass $m_\Phi$ and self-interaction coupling strength $\Lambda = 10 - 100$. For each color, we also show the asymptote values of ($C_{\text{max}}$, $M_{\text{max}}$) for a large $\Lambda$ which are obtained by fitting and extrapolating the results in section 3.1. The higher coupling strength $\Lambda$ leads to the higher $C_{\text{max}}$ and $M_{\text{max}}$.

For the case of Liouville potential, we find that boson stars with scalar particle mass $m_\Phi = 10^{-10} \text{eV}$ (for any $\Lambda$) or $m_\Phi = 10^{-9} \text{eV}$ (for $\Lambda > 100$) are lying in the overlapping region between the cyan and yellow areas, so their merger will produce GW signals whose frequency can probed by the LIGO detector. For $m_\Phi = 10^{-10} \text{eV}$, if the self-interaction strength $\Lambda \leq 20$, the boson star merger at the luminosity distance $D_L < 250 \text{Mpc}$ may produce detectable GWs by LIGO. For $m_\Phi = 10^{-9} \text{eV}$, if $\Lambda > 100$, then those mergers that occurred at the luminosity distance $D_L < 100 \text{Mpc}$ may produce detectable GWs. For boson stars with logarithmic potential, the scalar particle mass also needs to be $m_\Phi = 10^{-10} \text{eV}$ so that it can lie in the overlapping region of the ($C_{\text{max}}$, $M_{\text{max}}$) plane. We find that even the case with strong self-interaction strength $\Lambda > 100$ requires the merger of boson stars to occur at $D_L < 250 \text{Mpc}$ for the LIGO detection. In comparison, for the same scalar particle mass $m_\Phi$ and coupling strength $\Lambda$, the Liouville potential achieves a higher compactness $C_{\text{max}}$ and boson star mass $M_{\text{max}}$. Hence, the farther merging event can be probed for boson stars with Liouville potential rather than the logarithmic potential. In addition, the mass $m_\Phi = 10^{-10} \text{eV}$ which can be probed by the GW detection lies in the range $\mathcal{O}(10^{-10}) \text{eV} \lesssim m_\Phi \lesssim \mathcal{O}(10^{-3}) \text{eV}$ for both potentials. This implies that GW detection can compensate the FRB lensing probe for searching boson stars with the logarithmic potential.

5 Conclusions

In this work, we studied the dynamics of boson stars that consist of a new type of light singlet scalar particles with nontrivial self-interactions. Such boson stars may compose a small fraction of the dark matter in the Universe. We considered such light scalars with two distinctive self-interaction potentials as the benchmarks — the Liouville potential (which generates an infinite series of repulsive forces) and the logarithmic potential (which generates an infinite series of pairs of attractive and repulsive forces with a net attractive force).

In section 2, we introduced the Einstein-Klein-Gordon equations with which we can study properties of the scalar field $\phi(r)$ in the curved spacetime background. We discussed the underlying physics to motivate the two benchmark scalar potentials. We further defined the total mass and compactness of the boson star.

In section 3, we presented a numerical approach to solve the Einstein-Klein-Gordon equations and then applied a semi-analytic approach. We focused on the scalar field configuration for the ground state of the boson star which has the global maximum at the origin and approaches zero at the spatial infinity without a node. We found that the total mass of the stable boson star continues to increase with the rise of $\phi_0$ value until $\phi_0$ reaches a critical value $\phi_0^\ast$ that corresponds to the maximum total mass, where $\phi_0 = \phi(r)|_{r=0}$. Beyond $\phi_0^\ast$, the boson star becomes unstable. Furthermore, the stable boson star with either scalar potential was found to have negative binding energies. Intriguingly, as self-interaction strength increases, the negative binding energy for a boson star with the logarithmic potential decreases in magnitude up to the coupling strength $\Lambda \sim 5$ and then increases beyond this coupling value; while for the case of Liouville potential it monotonically increases in
magnitude. In addition, we studied the maximum compactness as a function of the coupling strength $\Lambda$, and obtained important insights. For the boson star with Liouville potential, the maximum compactness can reach as high as $C_{\text{max}} \sim 0.18$ for large coupling, and is larger than the case with a repulsive quartic interaction or a logarithmic interaction. On the other hand, the case of the logarithmic potential showed a slight deficit as compared to the case of a repulsive quartic potential. In the last part of this section, we applied the Swampland conjecture and found that the maximum compactness $C_{\text{max}}$ obtained by the full numerical computation for both potentials arises from low energy effective scalar field theories which could be UV-completed by a consistent quantum gravity theory.

In section 4, we studied the lensing of FRBs and the GW detection of LIGO as two astrophysical methods to probe the presence of boson stars and the parameter space of their corresponding scalar potentials. Given the future planned CHIME FRB detection experiment, we expect that boson stars with a total mass as light as $M = \mathcal{O}(1) M_\odot$ can be probed. Furthermore, LIGO will be sensitive to the GW signals from merging boson stars with a total mass $M = \mathcal{O}(1) M_\odot$. As for the individual scalar particle mass, LIGO is most sensitive to $m_\Phi \approx 10^{-10} \text{eV}$ for both potentials, whereas the FRB lensing detection can probe a sizable mass range $m_\Phi \approx \mathcal{O}(10^{-12} - 10^{-10}) \text{eV}$, depending on the time delay between two lensed FRB images by boson stars. We demonstrated that the current constraints on the DM fraction as contributed by MACHO and the CMB constraint on the cold DM isocurvature modes give the allowed scalar mass range $\mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV}$, where the scalar particles ($\Phi$) compose the boson stars. To suppress non-thermal production of the scalar particles in the early Universe, we considered a scenario where the scalar field has gravitationally induced coupling to the inflaton during the inflation. Because of this coupling, the effective mass of the scalar particle during inflation is comparable to the Hubble expansion rate at that time. After inflation ends, the scalar mass reduces to its original range $\mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV}$. Applying this mass range, we found that searching the boson stars with Liouville potential could be probed by both FRB lensing and GW detection. For boson stars with logarithmic potential, we find that the LIGO GW detection can probe the presence of the boson stars and the parameter space of the corresponding scalar theory. But, the FRB lensing can hardly probe boson stars with logarithmic potential since the mass range $\mathcal{O}(10^{-10}) \text{eV} < m_\Phi < \mathcal{O}(10^{-3}) \text{eV}$ does not produce a large enough total mass of boson star as needed by a valid FRB lensing. We anticipate the synergy between the FRB lensing and GW detection can help probing the boson stars, especially for the scalar particle mass around $m_\Phi \approx \mathcal{O}(10^{-10}) \text{eV}$.

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\section{Initial scalar displacement after inflation}

In this appendix, we follow the logic of \cite{54, 62} to obtain the expression of the scalar field displacement from the global minimum of the potential at the end of the inflation \( (|\Phi_{\inf}| = \phi_{\inf}) \) in terms of the tensor-to-scalar perturbation ratio \( r \). For a massive scalar field with \( m_{\Phi} < 1.5 H_{\inf} \), the Fourier mode of the field fluctuation on the super-horizon scale is given by \cite{62},

\[
|\delta \phi_k| \simeq \frac{H_{\inf}}{\sqrt{2k^3}} \left( \frac{k}{aH_{\inf}} \right)^{\frac{3}{2} - \nu_{\phi}},
\]

(A.1)

where \( \nu_{\phi} = \sqrt{\frac{9}{4} - \left( \frac{m_{\Phi}}{H_{\inf}} \right)^2} \). Integrating over all the super-horizon modes, one obtains the variance of the field value \cite{54},

\[
\langle \phi^2 \rangle \simeq \frac{1}{3 - 2\nu_{\phi}} \left( \frac{H_{\inf}}{2\pi} \right)^2.
\]

(A.2)

This result can be used for \( \phi_{\inf} \) value by approximating \( \phi_{\inf} \simeq \sqrt{\langle \phi^2 \rangle} \). Note that \( \phi_{\inf} \) will diverge for \( m_{\Phi} \ll H_{\inf} \), so it would generate too much scalar dark matter abundance. Hence, it is essential for our study to have \( m_{\Phi} \simeq H_{\inf} \) during inflation which could be realized by gravitationally induced coupling between the scalar and inflaton \cite{54}. From eqs. (A.1)–(A.2), we see that the isocurvature power spectrum is obtained as

\[
\Delta^2 I = \left( \frac{2}{\phi_{\inf}} \right)^2 P_{\delta \phi_k} = (3 - 2\nu_{\phi}) \left( \frac{k}{aH_{\inf}} \right)^{3 - 2\nu_{\phi}},
\]

(A.3)

where \( P_{\delta \phi_k} = (k^3/2\pi^2) |\delta \phi_k|^2 \) \cite{62}. The measurement on the isocurvature mode is written in terms of this isocurvature power spectrum and the amplitude of the adiabatic curvature power spectrum \( (\Delta^2_R(k)) \),

\[
\beta_{\text{iso}}(k) = \frac{\Delta^2_I(k)}{\Delta^2_R(k) + \Delta^2_I(k)}.
\]

(A.4)

Since \( \delta \phi \) and \( \delta \chi \) are independent, the CDM isocurvature modes are uncorrelated with the adiabatic modes \cite{54}. Hence, combined with \( \Delta^2_R(k_{\text{mid}}) \simeq 2.1 \times 10^{-9} \) \cite{55}, we find that the constraint on \( \beta_{\text{iso}} < 0.038 \) at \( k_{\text{mid}} = 0.050 \text{Mpc}^{-1} \) \cite{55} leads to \( \nu_{\phi} \lesssim 1.297 \) for 55 e-folds of inflation, and thus \( \phi_{\inf} \lesssim 0.25 H_{\inf} \) via eq. (A.2). We have used this result to derive the lower bound on the scalar particle mass \( m_{\Phi} \) in eq. (4.5).

\section{Computation of the optical depth}

For presenting our analysis in section 4.1, following \cite{56, 57}, we explain the procedure of computing the integrated optical depth \( \bar{\tau}(M_L) \) in this appendix.

The observable \( \bar{\tau}(M_L) \) is interpreted as the probability for a FRB lensed by a compact object with mass \( M_L \). The time delay between the two images resulting from the lensing of FRB by a compact object of mass \( M_L \) and location \( z_L \) is given as follows,

\[
\Delta t = \frac{4G_N M_L}{c^3} (1 + z_L) \left[ \frac{y}{2} \sqrt{y^2 + 4} + \log \left( \frac{\sqrt{y^2 + 4} + y}{\sqrt{y^2 + 4} - y} \right) \right],
\]

(B.1)
where \( y = \beta/\theta_E \). The parameter \( \beta \) is the angular impact parameter, and the angle \( \theta_E \) is an angular Einstein radius determined by the angular diameter distances to the source of FRBs \( (D_s) \) to a lens \( (D_L) \), and between the two \( (D_{LS}) \), as given by

\[
\theta_E = 2 \sqrt{\frac{G_N M_L D_{LS}}{c^2 D_S D_L}}. \tag{B.2}
\]

The following conditions to insure the strong enough lensing of a FRB give us \((y_{\text{min}}, y_{\text{max}})\). The parameter \( R_f \) is defined as the ratio of the size of the larger image to that of the smaller image. Requiring \( R_f \) to be smaller than the critical value \( \overline{R}_f = 5 \) provides the following maximal allowed value of \( y \),

\[
R_f = \frac{y^2 + 2 + y\sqrt{y^2 + 4}}{y^2 + 2 - y\sqrt{y^2 + 4}} \leq \overline{R}_f = 5, \quad \Rightarrow \quad y_{\text{max}} = \left( \frac{1 + \overline{R}_f}{\sqrt{\overline{R}_f}} - 2 \right)^{\frac{1}{2}} \simeq 0.8266. \tag{B.3}
\]

Demanding that the time delay \( (B.1) \) be greater than a reference time \( \Delta t \) gives the minimum value \( y_{\text{min}} \).

For a given set of \((y_{\text{max}}, y_{\text{min}})\), the optical depth \( \tau \) as a function of the mass of the lens \( M_L \) and the source position \( z_S \) can be computed from

\[
\tau(M_L, z_S) = \frac{3}{2} \xi_{\text{DM}} \Omega_c \int_0^{z_S} dz_L \frac{H_0^2}{c H(z_L)} \frac{D_L D_{LS}}{D_S} (1 + z_L)^2 \left[y_{\text{max}}^2 - y_{\text{min}}^2(M_L, z_S)\right], \tag{B.5}
\]

where \( H \) is the Hubble expansion rate and \( \xi_{\text{DM}} \) is the fraction of DM provided by the compact objects which cause the lensing of FRBs. We choose the values of cosmological parameters for computing \( \tau \) from ref. [10]. Different choices of \( \Delta t \) and \( M_L \) would lead to different \( y_{\text{min}} \) and thus different \( \tau \). For the present study, we assume the constant density redshift distribution function for sources [63],

\[
N_{\text{const}}(z) = N_{\text{const}} \frac{\chi^2(z)}{H(z)(1+z)} e^{-d_L^2(z)/[2d_L^2(z_{\text{cut}})]}, \tag{B.6}
\]

where \( d_L(z) \) is the luminosity distance, \( N_{\text{const}} \) is the normalization factor, and \( \chi(z) \) is the comoving distance. We choose \( z_{\text{cut}} = 0.5 \). Finally, convolving \( \tau \) in eq. (B.5) with the redshift distribution (B.6) removes \( z_S \) dependence and leads to the following integrated-optical depth,

\[
\bar{\tau}(M_L) = \int dz \tau(z, M_L) N(z). \tag{B.7}
\]

This quantity is interpreted as the probability for a single burst to be lensed.

### C Sensitive parameter space of boson stars to LIGO

In this appendix, following [61], we explain the procedure of determining the parameter space of the physical quantities \((C, M_{\text{max}})\) of the boson star which can be probed by the

\[
\Delta t \simeq \frac{8G_N M_L}{c^3} (1 + z_L) y. \tag{B.4}
\]

\[<\text{Label here}>\]
LIGO GW detector. For the simplicity of illustration, we consider the situation where the two merging boson stars have the same mass and compactness as described by the same the scalar potential. The GW emissions from the merger of the binary boson stars are characterized by the frequency,

$$\nu_{BS} = \frac{(C/3)^{3/2}}{2\pi M},$$  \hspace{1cm} (C.1)

where the parameters \((C, M)\) are the common (compactness, mass) of the binary boson stars. Requiring \(\nu_{BS}\) to lie within the GW frequency range \((50 - 1000)\text{Hz}\) to which LIGO detection is sensitive, one obtains the following relation

$$C^{3/2} \times 6.149 M_\odot \leq M \leq C^{3/2} \times 124.451 M_\odot,$$  \hspace{1cm} (C.2)

which must be satisfied by \((C, M_{\text{max}})\) of the binary boson stars to be probed with the low level noise. We draw the corresponding region of the parameter space by the cyan color as in figure 11. The signal to noise ratio (SNR) of the GW signals with strain \(h(t)\) reads

$$\rho^2 = \int_0^{\nu_{BS}} d\nu \frac{4|h(\nu)|^2}{S_n(\nu)},$$  \hspace{1cm} (C.3)

where \(h(\nu)\) is the Fourier transform of the strain and \(S_n(\nu)\) is the noise power spectral density (PSD). Note that the upper limit of the integral depends on both \(C\) and \(M\). We take \(S_n(\nu)\) from [64]. In the quadrupole approximation [65], the strain Fourier transform reads

$$\tilde{h}(\nu) \approx \frac{\sqrt{5/24}}{\pi^{2/3} D_L} M_c^{5/6}(\nu^{BS})^{-7/6},$$  \hspace{1cm} (C.4)

where \(D_L\) is the luminosity distance for the location at which the merger of the binary boson stars takes place, and \(M_c\) is the chirp mass defined as

$$M_c = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}},$$  \hspace{1cm} (C.5)

with \(M_1 = M_2 = M\) in our case. We will consider \(D_L < 450, 250, 100\) Mpc cases (with the corresponding redshifts \(z \ll 1\)), so the redshift effect is negligible as in [61]. Requiring \(\rho \geq 8\) for ensuring a large enough SNR, we identify the yellow colored region in the parameter space of \((C, M_{\text{max}})\) in figure 11.

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