Resummation for single-spin asymmetries in 
W boson production

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Abstract. To measure spin-dependent parton distribution functions in the production of W± bosons at the Relativistic Heavy Ion Collider, an accurate model for distributions of charged leptons from the W boson decay is needed. We present single-spin lepton-level cross sections of order $O(\alpha_S)$ for this process, as well as resummed cross sections, which include multiple parton radiation effects. We also present a program RHICBOS for the numerical analysis of single- and double-spin cross sections in $g^*, W^\pm$, and $Z^0$ boson production.

The measurement of longitudinal spin asymmetries in the production of W± bosons at the Relativistic Heavy Ion Collider (RHIC) will provide an essential probe of spin-dependent quark distributions at high scales $Q^2$ [1, 2]. At $pp$ center-of-mass energy $\sqrt{s} = 500$ GeV, about $1.3 \times 10^6 W^+$ and $W^-$ bosons will be produced by the time the integrated luminosity reaches 800 pb$^{-1}$. Due to the parity violation in the $Wq\bar{q}$ coupling, this process permits non-vanishing single-spin asymmetries $A_L(x)$, defined here as

$$A_L(x) \equiv \frac{d\sigma(p^+p\rightarrow WX) - d\sigma(p^-p\rightarrow WX)}{d\sigma(p^+p\rightarrow WX) + d\sigma(p^-p\rightarrow WX)}, \quad \text{where} \quad x = y_W, y_l, p_{Tl}, \ldots$$

(1)

Here $y_W$ is the rapidity of the W boson; $y_l$ and $p_{Tl}$ are the rapidity and transverse momentum of the charged lepton from the W boson decay in the lab frame, respectively. The lowest-order expression for the asymmetry $A_L(y_W)$ with respect to the rapidity $y_W$ of the W boson is particularly simple if the absolute value of $y_W$ is large. In that case, $A_L(y_W)$ reduces to the ratio $\Delta q(x)/q(x)$ of the polarized and unpolarized parton distribution functions [3]. Furthermore, $A_L(\xi)$ tests the flavor dependence of quark polarizations.

The original method for the measurement of W boson production at RHIC is based on the direct reconstruction of the asymmetry $A_L(y_W)$ [1]. Unfortunately, such reconstruction is obstructed by specifics of the detection of W± bosons at RHIC. First, RHIC detectors do not monitor energy balance in particle reactions. Due to the lack of information about the missing momentum carried by the neutrino, the determination of $y_W$ is in general ambiguous and depends on assumptions about the dynamics of the process. Second, due to the correlation between the spins of the initial-state quarks and final-state leptons, the measured value of $A_L(y_W)$ is strongly sensitive to experimental cuts imposed on the observed charged lepton. This feature is illustrated in Fig. 1, which shows...
Figure 1. Dependence of the asymmetry $A_L(y_W)$ on the cuts imposed on the momentum of the observed antilepton in the $W^+$ boson production $p^+p \rightarrow (W^+ \rightarrow l\nu_l)X$. The asymmetry is calculated using the resummation method described in the paper. The GRSV standard set [4] of the polarized PDF's was used. The error bars are calculated according to Eq. (13) in Ref. [1] assuming the integrated luminosity $L = 800 \text{ pb}^{-1}$ and beam polarization 70%.

The asymmetry in the $W^+$ boson production calculated without constraints on $y_l$ and $p_{Tl}$ (solid line), and with constraints $1.2 < |y_l| < 2.4, p_{Tl} > 20 \text{ GeV}$ (circles) and $|y_l| < 1, p_{Tl} > 20 \text{ GeV}$ (boxes). According to the Figure, there is a substantial difference between the asymmetries calculated with and without selection cuts. This difference arises due to the different dependence of the unpolarized and polarized cross sections on angular distributions of the leptons in the $W$ boson decay. For instance, at the lowest order in $W^+$ production

$$A_L(y_W, y_l) = \frac{-\Delta u(x_a)d(x_b)(1 - \cos \theta^*)^2 + \Delta d(x_a)u(x_b)(1 + \cos \theta^*)^2}{u(x_a)d(x_b)(1 - \cos \theta^*)^2 + d(x_a)u(x_b)(1 + \cos \theta^*)^2},$$

(2)

where $x_{a,b} \equiv (Q/\sqrt{s})e^{\pm y_W}$, and $\theta^*$ is the polar angle of the antilepton in the rest frame of the $W$ boson. Since $y_l$ is related to $\cos \theta^*$, as

$$y_l = y_W + \frac{1}{2} \ln \frac{1 + \cos \theta^*}{1 - \cos \theta^*},$$

it is clear that restrictions on the range of integration of $y_l$ strongly affect $A_L(y_W)$.

Since the only straightforward signature of the $W$ bosons at RHIC is the observation of secondary charged leptons, it is important to understand differential cross sections of spin-dependent $W$ boson production at the lepton level. Given that the radiative corrections are sizeable ($\sim 30\%$) both in the numerator and denominator of Eq. (1), and that
the measurement results will be used in the next-to-leading order (NLO) PDF analysis, it is necessary to derive these cross sections at NLO accuracy (i.e., at order $\mathcal{O}(\alpha_s)$).

Furthermore, most of the $W$ bosons are produced with small, but non-zero transverse momenta. Such non-zero $q_T$ is acquired through radiation of soft and collinear partons, which cannot be approximated by finite-order perturbative calculations. In order to obtain reliable predictions for differential cross sections, dominant logarithmic terms $\alpha_s^n \ln^m \left( \frac{q_T^2}{Q^2} \right)$ (where $0 \leq m \leq 2n - 1$) associated with such radiation should be summed through all orders of the perturbative series. In our work [5], we performed a complete lepton-level study for the production of $W^{\pm}$, $\gamma^*$, and $Z^0$ bosons for arbitrary longitudinal polarizations of incident protons. This study combined the $\mathcal{O}(\alpha_s)$ contributions with the all-order sum of small-$q_T$ logarithmic corrections. The resummation of the logarithms $\ln^m \left( \frac{q_T^2}{Q^2} \right)$ was performed with the help of the impact parameter space ($b$-space) resummation technique [6]. It extended the methodology developed for the unpolarized vector boson production [7] to the spin-dependent case.

Our study goes beyond those in the previous publications [8, 9, 10] in several aspects. It presents the fully differential NLO cross section at the lepton level, which was not available before. The resummed single- and double-spin cross sections for the production of on-shell vector bosons were presented earlier in Ref. [10]. We have derived a more complete resummed cross section, which also accounts for the decays of vector bosons, i.e., for spin correlations in the final state. The lepton-level cross section includes several additional angular structure functions, which do not contribute at the level of on-shell vector bosons. Moreover, resummation is needed not only for the parity-conserving angular function $1 + \cos^2 \theta^*$, which contributes to the on-shell cross section, but also for the parity-violating angular function $2\cos \theta^*$, which affects angular distributions of the decay products. The estimate of parity-violating contributions is more complicated since it involves $\gamma_5$-matrices and Levi-Civita tensors both from the electroweak Lagrangian and spin-projection operators. As a result, special care is needed to treat finite terms that arise in the factorization of collinear poles in $d \neq 4$ dimensions. We perform the calculation using the dimensional reduction and find that the resummed cross sections satisfy helicity conservation conditions for the incoming quarks. In addition, the coefficient functions in Ref. [10] were obtained in a non-conventional factorization scheme and cannot be used with the existing PDFs. In contrast, our results are fully consistent with the $\overline{\text{MS}}$ factorization scheme.

The resummed cross sections are incorporated in a numerical program RHICBOS for Monte-Carlo integration of the differential cross sections [11]. We are not able to discuss all aspects of our numerical study in this short report. However, as an example we show lepton-level asymmetries $A_L(y_l)$ for various cuts on $p_{Tl}$ (Fig. 2). We find that these asymmetries can be accurately measured for both $W^+$ and $W^-$ bosons. These directly observed asymmetries can efficiently discriminate between different PDF sets; hence, they provide a viable alternative to the less accessible asymmetry $A_L(y_W)$.

It is also useful to study distributions with respect to the transverse momentum $p_{Tl}$ of the charged lepton, not only because they are sensitive to the PDF’s, but also because they probe in detail dynamics of the QCD radiation. As was discussed above, the transverse momentum distributions for vector bosons are affected by the multiple parton radiation, which can be described only by means of all-order resummation. In
addition, the distributions at very small $q_T$ are sensitive to nonperturbative contributions characterized by large impact parameters $b > 1$ GeV$^{-1}$. As a result, the shape of the lepton-level distribution $d\sigma/dp_T^l$ around its peak at about $p_T^l = M_W/2$ is affected by both perturbative and nonperturbative QCD radiation. Remarkably, the shape of the Jacobian peak can be predicted by the theory, even though it cannot be calculated at any finite order of $\alpha_S$. The prediction is possible because the perturbative soft and collinear contributions are systematically approximated in the resummation formalism. The nonperturbative contributions currently cannot be derived in a systematical way; but there is substantial indirect evidence (spin independence of the perturbative soft radiation, quark helicity conservation) that such contributions may be practically independent of the proton spin and type of the vector boson. The impact of the nonperturbative contributions is illustrated in Fig. 3, which shows the number of events for the difference $d\sigma(p^+p)/dp_T^l - d\sigma(p^-p)/dp_T^l$ of single-spin cross sections at $L = 800$ pb$^{-1}$. This rate was calculated using two parameterizations [13, 14] of the nonperturbative part, which were found in the unpolarized vector boson production. It can be seen that the sensitivity to the nonperturbative input is small, but, nonetheless, visible near the Jacobian peak. For comparison, we also included the $\mathcal{O}(\alpha_S)$ finite-order cross section calculated using the phase space slicing method. The finite-order curve substantially deviates from the resummed curves, and, moreover, its shape can be drastically modified by varying the phase space slicing parameter $q_{sep}^T$. In contrast, the resummed curve is determined unambiguously once a parameterization of the nonperturbative input is obtained from the double-spin $\gamma'$ production, single-spin or double-spin $Z^0$ production, or even unpolarized $W$ production. Needless to say, the resummation predictions for the shape of the Jacobian peak, which directly follow from fundamental principles of QCD, must be tested at RHIC.
\[ \Delta p_T \to (W^+ \to l^+ \nu l^-) X \\]

\[ \sqrt{s} = 500 \text{ GeV}, L = 800 \text{ pb}^{-1} \]

\[ -1 \leq \chi \leq 1, \text{ GS-A} \]

Figure 3. The single-spin charged lepton transverse momentum distribution for $W^+$ boson production discussed in the main text. The nonperturbative parts of the resummed cross sections were calculated using the Ladinsky-Yuan parameterization \cite{13} (solid) and the most recent Brock-Landry Gaussian parameterization \cite{14} (dashed). The $\mathcal{O}(\alpha_s)$ finite-order cross section is shown as a dotted line.

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