1. Introduction

At the close of the 20th and beginning of the 21st century, a new technological process has emerged in sheet rolling production which has made it possible to obtain hot-rolled sheets with thickness less than 1.0 mm. This process consists of the use of a system of accelerated strip cooling between rough and finish stand groups and after the finish stand group of the sheet rolling mill.

It seems promising to use the existing rolling equipment in the new technology introduced at metallurgical enterprises. However, under certain conditions, accelerated strip cooling introduced on existing non-specialized equipment can cause the breakdown of expensive rolls and other emer-
gencies on the mill if reliable information about the thermal state of the metal is unavailable for each point of the deformation zone. Therefore, rational strip and roll cooling based on the mathematical model of this technological process is an important and urgent task provided that a reliable and simple method is created for temperature measurement in each point of the deformation zone.

2. Literature review and problem statement

The existing modes of roll and strip cooling were developed based on experimental studies of the operation of the existing rolling equipment. An attempt to experimentally elaborate elements of the new technological process on existing rolling mills has brought about significant losses because of equipment breakdowns. This is explained by the lack of a reliable procedure for taking into account temperature and speed parameters when designing draughting schedules and calculating rolling forces. When studying and developing the hot sheet rolling processes using accelerated cooling elements, it is necessary to have a procedure of determining the temperature of contacting strip and the roll sections in various points of the deformation zone.

The multitude of factors affecting the processes of accelerated strip cooling was studied and analyzed in [1] including the strip speed and temperature, water temperature, water spraying rate, etc. The effect of various cooling parameters on heat transfer coefficients was established. The values of heat transfer coefficients for various cooling parameters were obtained by a numerical calculation method. However, it was not indicated which of the numerical calculation methods was used and no complete procedure of determining the thermal state of the strip and rolls in the deformation zone was given.

A technology of cooling complex profiles with water jets was studied and tested in laboratory conditions [2]. According to the authors, it enables the achievement of an optimum self-regulation temperature by adjusting the cooling time. However, the article does not provide any thermal calculations and, what is more, its objective consists in the use of accelerated cooling for a somewhat different task: thermal hardening with the replacement of the conventional oil quenching technology.

Studies on special issues related to accelerated strip cooling are described in [3–5]. In particular, the results of studies of accelerated cooling by spraying saline water solutions (NaCl and MgSO₄) are presented in [3, 4]. It was shown that cooling with MgSO₄ solution gives 1.5 times faster cooling rate than cooling with water alone. Cooling with a NaCl solution provides merely a 1.2 time increase in the cooling rate. A method was proposed in [5] for estimating the operating temperature only for the rolls of metallurgical machines taking into account scale accumulation on the cooling channel surface. None of the mentioned studies offers an acceptable procedure for calculating the thermal state of a strip-roll contact pair.

Results of experimental studies of temperature distribution across thickness and width of the sheet hardened on a roll-type hardening machine at Severstal PJSC (Russia) are described in [6]. However, the thermal state of rolls is not considered in these studies and their results make it possible to make just a preliminary assessment of the effectiveness of the first water collecting header with a slot nozzle and subsequent sections with flat spraying nozzles.

Mathematical modeling using analog and digital computation means is one of the promising ways to solving practical problems of developing new technologies and equipment. However, most numerical methods of temperature modeling are based on the application of the finite-difference method as it is more efficient and simpler in software implementation. A mathematical model was created in [7] for calculating temperature during hot rolling at the 2000 mill of Severstal PJSC. At the same time, the created mathematical model of the process requires refinement and development since the strip speed during rolling is not taken into account in the heat equation used in this model. Besides, when solving the problems of modeling the thermal state of the metal, meshes with a uniform depth step were used which do not quite accurately take into account temperature distribution from the contact surface to the strip and roll bodies. No concrete procedure for calculating the thermal state of the strip-roll system during rolling was also given in the study.

Using the finite difference method, mathematical description of the heat transfer processes occurring in a strip-mill-scale-roll system including differential equations with corresponding boundary and initial conditions in characteristic points of the deformation zone was given in [8]. However, no specific procedure for determining the thermal state of the strip-roll system was given. It was only noted that the developed mathematical model is the basis for further theoretical studies and determining parameters of the new hot strip rolling technology. To elaborate on a reliable procedure for designing a section of a new technological process of ultrafast strip cooling rate during sheet rolling, test problems were solved in [9]. A possibility of mathematical modeling of elements of the new rolling processes was shown, reliability of the information on ultrafast cooling rate was given in the technical literature and the possibility of its use in practice was substantiated. The issues of determining temperature distribution in the system contact zone were not considered.

Study [10] addresses special issues of mathematical modeling of the thermal state of metal during sheet rolling. In this case, a mathematical model of the thermal state of metal on an output roller table of a hot wide-strip rolling mill was considered. The model takes into account heat generation due to the polymorphic transformation of the supercooled state of the austenitic phase and the influence of the steel chemical composition on the physical properties of the strip metal. However, it does not consider the thermal state of metal in the strip-roll system.

A mathematical model of hot rolling temperature parameters is presented in [11]. This model takes into account temperature losses caused by thermal radiation and convective heat transfer from and into the environment occurring in the interstand areas and on the discharge roller table of the hot wide strip rolling mill. Therefore, the presented model does not take into account the actual temperature distribution in the strip-roll contact area.

Taking into account the variety of rolling mill types, position of the strip in the stands (horizontal or vertical), the need for the reversal of the strip movement direction during the rolling process, it is practically impossible to link the coordinate axes of the strip and the roll [12]. Therefore, when designing specific systems, these features should be specially considered.
It is known that the roll durability largely depends on its surface hardness. The higher the roll surface hardness the higher the roll durability. The roll hardness can be ensured by using hypereutectoid steels with a carbon content of 1.6–1.8 %. The use of solid rolls of hypereutectoid steel is problematic at the loads occurring in the sheet mills because there is a risk of roll breakage. Consideration of a two-layer roll design is associated with the need to describe the thermal state of current rolling tools. Therefore, the design of a modern two-layer roll is considered in theoretical studies.

The mentioned models describe all main options of the sheet rolling technology using two-layer and solid roll designs. The developed models take into account heat exchange between the rolls, hot strip and water used to cool the rolls, geometric dimensions of the deformation and forced cooling zones as well as their relative position. The developed models serve as a basis for carrying out theoretical studies and determining parameters of the new technology for hot strip rolling at all steps of the technological process.

To obtain further results in a numerical form, switch from differential equations with partial derivatives to corresponding finite difference equations is usually applied. Finite difference equations are obtained by replacing derivatives with their differences.

Examples of transition from partial differential equations to finite difference equations are given in [13–15]. These equations have almost the same form and differ just in writing depending on the coordinate system (rectangular or cylindrical) and type (linear or nonlinear).

To determine temperature distribution in a two-dimensional or three-dimensional body using digital computation methods, the matrix inversion method can be used. However, an attempt to use texts of the matrix inversion programs (MATINV and EXCH) has shown that the texts were presented in a distorted form, so the use of these programs is inappropriate. Thus, in addition to the system of equations of energy balance for all nodes, independent development of programs for inverting large-sized matrices is also required. The way is known but it requires a significant timetable.

In the case of using the explicit finite-difference method, the problem is simplified: it is only necessary to independently compose finite-difference equations of energy balance for each node taking into account the geometry of the study objects. Thus, the analysis of scientific and technical literature has shown that when elaborating on the technological modes of thin sheet rolling processes with the application of accelerated cooling, it is necessary to have a reliable theoretical model for calculating the thermal state of the strip-roll system. Studies of the influence of various factors on the processes of accelerated strip cooling are presented in [1–15]. However, they either do not provide procedures giving just results of calculation of the thermal state of the strip-roll system or study the thermal state of one element of this contact pair. A complete procedure for calculating temperature distribution during rolling with accelerated cooling is not given.

It was also established that mathematical modeling using the finite difference method is the most acceptable method of theoretical study and solving on their basis practical problems of developing new technologies and equipment for rolling with accelerated cooling. However, the well-known examples of calculating the thermal state of a strip-roll system show that the adopted procedures need further refinement and improvement.

3. The aim and objectives of the study

The study objective is to develop a simplified procedure that reliably establishes temperature distribution in characteristic sections of the rolled strip and the rolls. The application of this procedure will improve the accuracy of solving non-linear problems described by the equations of non-stationary heat conductivity of the strip-roll system when rolling thin sheets.

To achieve the objective, the following tasks were set:
- to determine the intensity of heat release per unit volume in the deformation zone;
- to construct finite difference equations of energy balance for characteristic nodes of the conditional mesh applied on the slabs with a non-uniform thickness distribution of layers of the conditional mesh taking into account non-uniformity of the slab cross-section temperature distribution;
- to construct finite difference equations of energy balance for characteristic nodes of the conditional mesh applied on the rolls with a non-uniform thickness distribution of layers of the conditional mesh taking into account non-uniformity of the roll cross-section temperature distribution.

4. Materials used in studying the intensity of heat release in the deformation zone

To determine the intensity of heat release per unit volume in the deformation zone, it is necessary to know the angle of metal gripping by the rolls. It is known [16] that the gripping angle is found from the equation

\[ \cos \alpha = 1 - \frac{\Delta h}{2R} \]

where \( \Delta h = h_0 - h_1 \); \( h_0 \) is the thickness of metal at the entrance to the deformation zone; \( h_1 \) is the thickness of metal at the outlet of the deformation zone; \( R \) is the roll radius.

![Fig. 1. Scheme of division of the deformation zone into sections:](image)

- \( R \) (roll); \( M \) (rolled metal)
The deformation time in each section is 0.1Δτ. Equation (1) can be written as

\[ \frac{\Delta h_i}{2} = R (1 - \cos \alpha). \]  

(2)

Displacement of metal layers in the sections can be found from the equation

\[ \frac{\Delta h_i}{2} - \frac{\Delta h_{i-1}}{2} = R (1 - \cos \alpha_i). \]  

(3)

where \(i\) is the number of sections (\(i=10, 9, 8...3, 2, 1\)); \(\alpha\) is the gripping angle in the \(i\)-th section (\(\alpha=0.9\alpha\) for section 9; \(\alpha=0.8\alpha\) for section 8).

Having displacement of metal layers in each section and knowing time (0.1Δτ), one can find the speed of layer displacement to the middle of the slab where the temperature is higher and calculate temperature growth.

Temperature growth caused by plastic metal deformation takes into account the rate of heat release per unit volume (\(q_v\)). In [17], 10 dependences for determining \(q_v\) were proposed by different authors. Discarding six of these dependences which include empirical coefficients, remaining dependencies can be analyzed.

Identical dependences having the following form are proposed in [18, 19]

\[ \Delta T_i = \frac{A}{G \times c}, \]  

(4)

where \(\Delta T\) is the temperature increase due to deformation; \(A\) is the energy necessary for metal deformation; \(G\) is the rolled strip weight; \(c\) is the average specific heat of the metal.

To determine a temperature increase during rolling, a dependence is proposed in [20]:

\[ \Delta T_i = \frac{\sigma_s \times \ln \frac{h_i}{h_k}}{427 \times \gamma \times c}. \]  

(5)

where \(\sigma_s\) is the metal deformation resistance; 427 is the mechanical equivalent of the caloric unit.

Growth of metal temperature due to plastic metal deformation is presented in the form:

\[ \Delta T_i = \frac{q_v \times \Delta \tau}{\rho}, \]  

(6)

where \(\rho\) is the specific gravity of metal.

From equations (5) and (6), \(q_v\) can be found assuming that \(\gamma = \rho\)

\[ \sigma_s \times \ln \frac{h_i}{h_k} \times c \times \rho \times \Delta \tau \]

\[ q_v = \frac{\sigma_s \times \ln \frac{h_i}{h_k} \times c \times \rho}{427 \times \gamma \times c \times \Delta \tau}. \]  

(7)

where \(\Delta \tau\) is the time of transit of one metal point through the deformation zone, sec; \(q_v\) is the intensity of heat release per unit volume.

Taking into account the conversion coefficient (4.1868), the following is obtained for the mechanical equivalent of the caloric unit:

\[ q_v = -\frac{\sigma_s \times \ln \frac{h_i}{h_k}}{427 \times \Delta \tau} \times 10^6, \]  

(8)

where \(10^6\) is a coefficient enabling substitution of \(\sigma_s\) values, kg/mm².

5. Equations of energy balance and determination of temperature in characteristic nodes of a non-uniform mesh applied on the slabs

It is known that when a body is divided into 8 to 10 elementary layers, the volume discretization error approaches zero when the temperature is determined.

Test problems of modeling the metal thermal state were solved. First, uniform meshes were used with the division of the slab thickness by 10Δτ and 100Δτ. The results obtained (when dividing by 100Δτ) were used as the basis for assessing deviations of the solution in the nodes of the non-uniform mesh which coincide in the slab body with nodes of the uniform mesh. During rolling, the real contact time of a point on a metal surface with the rolls is approximately 0.08 sec. With such short-term contact, it is difficult to expect uniform penetration of the heat wave into the metal by 15 mm in each layer for a total slab thickness of 150 mm conditionally divided into 10 layers.

Therefore, the standard approach consisting of a uniform division of the slab into layers of the same thickness can lead to erroneous results. In view of the above, when solving heat transfer problems of the thin sheet rolling process, it is necessary to use the division of the slab with non-uniform steps in volume. Such an approach is described in detail in [21].

To determine the metal thermal state, a simplified calculation procedure will be used. It consists of the fact that according to the results of temperature field calculation, the general thermal state of the entire study object will be built based on individual dependences for the two-dimensional problem. That is, solution of the three-dimensional problem of determining \(T(x, y, z, \tau)\) shall be reduced to the solution of a two-dimensional problem \(T(x, z, \tau)\) for a plate or roll in which one linear mesh size (width) is larger than the other two.

Thus, we have a specified initial distribution \(T_0=\text{const}\) and time-varying limiting conditions of the third kind at the metal boundaries which reflect the change in heat exchange zones as they move along the metal surface. In this case, temperature field \(T(x, y, z, \tau)\) is approximated by two-dimensional fields \(T(x, z, \tau)\) obtained at discrete time instants displaced from each other by time Δτ. Each of them has its own limiting conditions and differs only in time position, that is, it has its own start of accounting displaced by an integer number of steps in time.

To solve \(T(x, z, \tau)\), it is necessary to sequentially solve problems in the \(i\)-th heat exchange zone in accordance with the set order of passage of heat exchange zones. The final temperature distribution in the \(i\)-th zone is the initial distribution for calculating temperatures in zone \(i+1\).

Since thermal influences differ only in their position in time, the distribution of \(T(x, y, z, \tau)\) for an arbitrary value of \(\tau\) can be obtained based on the results of the solution in the first section.

When calculating the metal thermal state, it is necessary to take into account the following changes in:

- metal thickness during the passage of the deformation zone;
- dimensions of the deformation zone;
– speed of movement of the metal layers;
– the intensity of heat release in the metal volume during its deformation.

Equation (9) describes the metal thermal state:

\[
c_\text{m} p_\text{m} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_\text{m} \frac{\partial T}{\partial x} \right) + p_\text{u} \frac{\partial}{\partial x} (c_\text{m} T) + q_x,\tag{9}
\]

where \( T_\text{m} \) is the strip temperature; \( u_\text{m} \) is the strip speed; \( q_x \) is the heat release due to plastic deformation of the strip; \( c_\text{m}, p_\text{m}, \lambda_\text{m} \) are the thermal characteristics of the strip.

Two possible approaches to determination of the thermal state of metal and rolls are described in [8, 9]. In the first case, metal and rolls are movable relative to the stationary surfaces. In the second case, metal and rolls are motionless while the sources of influence move along their trajectories. In the first case, metal and rolls are described in [8, 9]. In the second case, metal and rolls are movable relative to the stationary surfaces.

A diagram of dividing the slab by a mesh in a longitudinal direction is shown in Fig. 2.

Note that the mesh has a smaller step in subsurface layers in the direction of the \( x \) axis and a larger step closer to the middle of the slab.

The mesh is divided into steps \( \Delta z \) along the \( z \) axis. Let us assume that \( \Delta z = l_\text{g} \) is the length of the gripping arc in the metal deformation zone. The assumption that \( \Delta z = l_\text{g} \) is determined by the fact that the most intense heat sink with heat transfer coefficients 1–2 orders of magnitude higher than in other parts of the technological process is expected in the zone of metal deformation. If it becomes necessary to round the number of steps \( \Delta z \) in any part of the technological process to an integer value, this will not introduce a significant error in the calculation of the metal thermal state as a result of small values of heat transfer coefficients in these sections.

The procedure of obtaining algebraic equations for determining the temperature in the mesh nodes is similar to that described in [22]. For example, only coefficients will change for the internal node 24 (Fig. 2) because the node width is now equal to \( \Delta z \), and not \( 2\Delta y \).

Equation for node 24 (Fig. 2) will take the form:

\[
T_{24,1+2x} = 0.1666 F_0 \left( 2 T_{23,1+2x} + T_{25,1} \right) +
+ (1 - 0.5 F_0) T_{23,1+2x} + \frac{F_0}{B} (T_{24,1+2x} + T_{25,1} - 2 T_{23,1+2x}) +
+ \frac{q_x \Delta t}{\rho \Delta z} + \frac{U \Delta t}{3 \Delta x} (2 T_{23,1+2x} + T_{25,1} - 3 T_{24,1+2x}),
\]

where \( B \) is the dimensionless coefficient \( (B = \Delta z / \Delta x) \).

When \( \Delta z = 75 \text{ mm} \) and \( \Delta x = 1.5 \text{ mm} \), equation (10) will take the following form:

\[
T_{24,1+2x} = 0.1666 F_0 \left( 2 T_{23,1+2x} + T_{25,1} \right) +
+ (1 - 0.5 F_0) T_{23,1+2x} + 0.0004 F_0 \times
\times (T_{24,1+2x} + T_{25,1} - 2 T_{23,1+2x}) + \frac{q_x \Delta t}{\rho \Delta z} +
+ \frac{U \Delta t}{3 \Delta x} (2 T_{23,1+2x} + T_{25,1} - 3 T_{24,1+2x}),
\]

Equation (11) practically repeats equation (12) of [22] apart from coefficient (0.0004) at \( F_0 \) in the third term. Due to its smallness, the third component can be neglected and again, like in [22], the equation of the following form is obtained:

\[
T_{24,1+2x} = 0.1666 F_0 \left( 2 T_{23,1+2x} + T_{25,1} \right) +
+ (1 - 0.5 F_0) T_{23,1+2x} + \frac{q_x \Delta t}{\rho \Delta z} +
+ \frac{U \Delta t}{3 \Delta x} (2 T_{23,1+2x} + T_{25,1} - 3 T_{24,1+2x}),
\]

that is, a one-dimensional problem will be solved.

6. Equations of energy balance and determination of the temperature at characteristic nodes of a non-uniform mesh applied on the rolls

To determine the thermal state of the rolls for hot rolling, use the well-known statement [23–25] that there are two zones in the rolls during rolling that differ significantly from each other. The first of them is a thin surface layer in which cyclic temperature changes occur during one revolution: the largest on the surface and insignificant at a certain distance from it. In the second zone extending along the roll axis, the roll cross-section temperature fluctuations in layers are negligible.

The standard approach consists of a uniform division of an object into layers of the same size. With a roll diameter of 800 mm, division into 10 layers along the radius gives a layer thickness \( \Delta r = 400/10 = 40 \text{ mm} \). When averaging temperature over a 40 mm thick layer, erroneous results are most likely. In this regard, the well-known standard approach is unsuitable.
To obtain more reliable information when calculating the thermal state of the roll, it is desirable to have the smallest possible layer thickness. If we take \( \Delta r = 2 \) mm, then a more successful model of calculating the roll thermal state can be created. If the roll is evenly divided into layers, 200 layers are obtained in one direction (in the radial direction, Fig. 3). The problem cannot be solved even with the most modern computational equipment available. It is necessary to use a mesh with non-uniform steps in space; thicker where significant temperature changes are expected. In those places where temperature changes in neighboring nodes do not differ much in time, it makes sense to use a less dense mesh.

When analyzing the sizes of the steps between adjacent layers according to the fields of the studied area, it is easy to notice that the steps along the roll radius \( \Delta r \) are the smallest. When dividing the angle of the roll contact with the strip by 10 steps in time, the roll thermal state can be calculated for locations where temperature changes most sharply, i.e. in the deformation zone with slight changes in time \( \Delta t \). This makes it possible to move from solving a nonlinear heat conduction problem to solving a linearized problem.

Let us assume that \( \lambda, c, \rho \) of the roll are constant in an interval \( \Delta r \). It should be pointed out that due to the fact that most technical objects have a spatial length, their thermal state cannot be characterized by the problem of change of the object coordinates only in time [26].

When determining the thermal state of the objects with distributed parameters, the strip and the roll are assumed to be motionless and the sources of mobile influence move along their surfaces. During the time of passage of the sources of mobile influence of the deformation zone (0.08 s in the roughing stand and 0.001 s in the finishing stand of the rolling mill) with a decrease in \( \Delta t \) by a factor of 10, no significant changes occur in thermal characteristics of the roll material.

Let us consider the components of energy balance for the internal node 33 of the roll (Fig. 4).

Write the energy balance for node 33 in this form:

\[
q_{33\rightarrow 34} + q_{34\rightarrow 33} + q_{33\rightarrow 32} + q_{33\rightarrow 33} + q_{33\rightarrow 33}' = \frac{\partial U_{33}}{\partial \tau}.
\]

(13)

\[
\lambda 2 \Delta y \frac{2\pi (r_{33} - \Delta r)}{360} \Delta \varphi \times \frac{T_{33\rightarrow 32} - T_{33\rightarrow 34}}{\Delta \varphi} + \lambda 2 \Delta y \frac{2\pi (r_{33} - \Delta r)}{360} \Delta \varphi \times \frac{T_{33\rightarrow 34} - T_{33\rightarrow 33}}{\Delta \varphi} = \lambda 2 \Delta y \frac{2\pi (r_{33} - \Delta r)}{360} \Delta \varphi \times \frac{T_{33\rightarrow 34} - T_{33\rightarrow 33}}{\Delta \varphi}.
\]

(14)

where 32 is the nodal point above point 33, point 34 is under point 33 in radial direction; point 23 is the point in front and point 43 is behind the design point; point 33' is to the left and point 33'' to the right of the design point.
\[ T_{33,1} \text{ at point } 33 \text{ at the time instant } \tau; T_{33,1} \text{ is temperature at point } 33 \text{ at the time instant } \tau + \Delta \tau; r_{33} \text{ is the radius on which points } 33', 33, 33'' \text{ as well as points } 23 \text{ and } 43 \text{ are located.} \]

Following the obvious simplifications and taking into account that \( \Delta Y = 80 \Delta r \) and \( r_{33} = 197.5 \Delta r \), equation (6) will take the following form:

\[
T_{33,1} = \frac{F_0}{3} \left( 1 + \frac{\Delta \tau}{r_{33}} \right) T_{32,1} + \frac{F_0}{6} \left( 1 - \frac{\Delta \tau}{r_{33}} \right) T_{34,1} + \\
\left( 1 - 0.5F_0 \frac{r_{33}}{r_{33}} \right) T_{33,2} - \frac{0.08425 F_0}{\Delta \phi} \times \\
\left( T_{33,1} + T_{33,2} - 2T_{33,3} \right) + \\
+ 0.000039 F_0 \left( T_{33,1} - T_{33,1} \right) + \\
+ 0.0000156 F_0 \left( T_{33,1} - T_{33,1} \right). \tag{15}
\]

When analyzing equation (15), it can be asserted that the last two terms that take into account the influence of the nodes located along the roll axis from the calculated one can be discarded. With the gripping angle \( 10^\circ \) or more, the fourth term of the equation can also be discarded because its multiplier is quite small.

![Fig. 4. Scheme of typical internal and neighboring nodes in the middle of the roll](image-url)

In fact, we again come to the one-dimensional equation (in a cylindrical coordinate system). Besides, when calculating the thermal state of the active zone in which cyclic temperature changes occur during one revolution, we have an opportunity to switch from solving the problem in a cylindrical coordinate system to solving in a rectangular coordinate system. The error for the nodal point 33 that may occur during such a transition is:

\[ \frac{\Delta r}{r_{33}} = \frac{\Delta \tau}{197.5 \Delta r} = 0.005. \]

The transition to solving a one-dimensional strip-roll system greatly simplifies the calculation. An opportunity arises to determine the value of the heat transfer coefficient in the deformation zone by solving the boundary-value problem III for the roll and comparing the results obtained with the solutions for the strip-roll system in a theoretical way.

7. Discussion of results obtained in the study of the intensity of heat release and temperature distribution in the deformation zone during sheet rolling

The performed studies have made it possible to obtain a procedure for determining the temperature at any point of a steady deformation zone on the strip being deformed and temperature distribution from the surface into the depth of the roll body.

When determining the intensity of heat release per unit volume in the deformed section, the gripping angle in the deformation zone was divided into 10 sections in order to increase the accuracy of the results obtained (Fig. 1). This has made it possible to take into account heat release \( q \), in each of the selected layers and calculate the temperature rise in the direction from the strip surface to its core and therefore increase the accuracy of establishing the thermal state of the strip metal during rolling.

Unlike the known ones, the mathematical model of rolling a smooth strip in rolls has been refined in the part of the equation describing the thermal state of metal to additionally take into account speed of movement of the strip itself in the mill production stream in order to increase accuracy of the calculations carried out on its basis. Moreover, when determining the thermal state of the strip-roll system as an object with distributed parameters, the strip and the roll themselves are considered motionless while sources of mobile influence move along their surfaces with a speed \( v_m \).

To obtain more accurate information about the thermal state of the strip, the developed model provides for the creation of a non-uniform conditional mesh for dividing the studied areas of the strip (Fig. 2). The third task set in the work was solved in a similar way, i.e. using a non-uniform distribution of the nodes of the conditional mesh over the cross-section of the roll (Fig. 3). Finite difference equations of energy balance were compiled for the characteristic nodes of the conditional non-uniform mesh applied on the rolls. Unlike the conventional approach in which the objects are evenly divided into layers of the same size, this approach enables obtaining detailed data on the thermal state of the strip metal and rolls in their near-contact zones where the most intense heat transfer occurs. Reducing the number of design mesh nodes in areas where heat transfer is not as intense as in the contact area (in the region of central strip and roll sections) can significantly reduce the calculation volumes.

The thickness of the rolled strip is the parameter that limits the application of the proposed method of determining the thermal state of the strip-roll system. With a thickness of less than 1 mm, there is no significant temperature difference on the surface and in the core zones and there is no need to use meshes with a non-uniform step.

Further use of the study results is expedient to determine temperature and speed mode of thin strip cooling during rolling as well as to set the tasks of designing special equipment for accelerated cooling in a production stream of rolling mills.
6. Conclusions

1. To determine the intensity of heat release per unit volume taking into account temperature increase caused by plastic deformation of metal, the deformation zone in the gripping angle was divided into 10 sections. This has made it possible to take into account the non-uniform deformation rate during rolling and increase the accuracy of the results obtained.

2. The slabs hot rolled into strips were spatially divided into a rectangular coordinate system with a conditional mesh having a non-uniform step. The standard approach consisting of a uniform dividing the slabs into layers of the same thickness can lead to erroneous results when solving heat exchange problems of the rolling process. Finite difference equations of energy balance were compiled for characteristic mesh nodes which after approximation and simplifications made it possible to obtain an equation for determining the temperature at all nodes of the conditional mesh.

3. Finite difference equations of energy balance were obtained for characteristic nodes of the conditional mesh applied on the rolls with a non-uniform distribution of layers of the mesh in thickness taking into account non-uniformity of temperature distribution in the roll cross-section.

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