Understanding Exhaustive Pattern Learning

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Abstract

Pattern learning is an important problem in Natural Language Processing (NLP). Some exhaustive pattern learning (EPL) methods Bod (1992) were proved to be flawed Johnson (2002), while similar algorithms Och and Ney (2004) showed great advantages on other tasks, such as machine translation. In this article, we first formalize EPL, and then show that the probability given by an EPL model is constant-factor approximation of the probability given by an ensemble method that integrates exponential number of models obtained with various segmentations of the training data. This work for the first time provides theoretical justification for the widely used EPL algorithm in NLP, which was previously viewed as a flawed heuristic method. Better understanding of EPL may lead to improved pattern learning algorithms in future.

1. Introduction

Pattern learning is the crux of many natural language processing (NLP) problems. It is usually solved as grammar induction for these problems. For parsing, we learn a statistical grammar with respect to certain linguistic formalism, such as Context Free Grammar (CFG), Dependency Grammar (DG), Tree Substitution Grammar (TSG), Tree Adjoining Grammar (TAG), and Combinatory Categorial Grammar (CCG) etc. For machine translation (MT), we learn a bilingual grammar that transfer a string or tree structure in a source language into a corresponding string or tree structure in a target language.

What is embarrassing is that many of the grammar induction algorithms that provide state-of-the-art performance are usually regarded as less principled in the aspect of statistical modeling. Johnson (2002); Prescher et al. (2004) showed the Bod (1992)’s data oriented parsing (DOP) algorithm is biased and inconsistent. In the MT field, almost all the statistical MT models proposed in recent years rely on similar heuristic methods to extract translation grammars, such as Koehn et al. (2003); Och and Ney (2004); Chiang (2005); Quirk et al. (2005); Galley et al. (2006); Shen et al. (2008); Carreras and Collins (2009), to name a few of them. Similar heuristic methods have also been used in many other pattern learning tasks, for example, like semantic parsing as in Zettlemoyer and Collins (2005) and chunking as in Daumé III and Marcu (2005) in an implicit way.

In all these heuristic algorithms, one needs to extract overlapping structures from training data in an exhaustive way. Therefore, in the article, we call them exhaustive pattern learning (EPL) methods. The use of EPL methods is intended to cope with the uncertainty of building blocks used in statistical models. As far as MT is concerned, Koehn et al. (2003) found that it was better to define a translation model on phrases than on words, but there was no obvious way to define what phrases...
were. DeNero et al. (2006) observed that exhaustive pattern learning outperforms generative models with fixed building blocks.

In EPL algorithms, one needs to collect statistics of overlapping structures from training data, so that they are not valid generative models. Thus, the EPL algorithms for grammar induction were viewed as heuristic methods DeNero et al. (2006); Daumé III (2008). Recently, DeNero et al. (2008); Blunsom et al. (2009); Cohn and Blunsom (2009); Cohn et al. (2009); Post and Gildea (2009) investigated various sampling methods for grammar induction, which were believed to be more principled than EPL. However, there was no convincing empirical evidence showing that these new methods provided better performance on large-scale data sets.

In this article, we will show that there exists a mathematically sound explanation for the EPL approach. We will first introduce a likelihood function based on ensemble learning, which marginalizes all possible building block segmentations on the training data. Then, we will show that the probability given by an EPL grammar is constant-factor approximation of an ensemble method that integrates exponential number of models. Therefore, with an EPL grammar induction algorithm, we learn a model with much more diversity from the training data. This may explain why EPL methods provide state-of-the-art performance in many NLP pattern learning problems.

The rest of the article is organized as follows. We will first formalize EPL in Section 2. In Section 3, we introduce the ensemble method, and then show the approximation theorem and its corollaries. We discuss a few important problems in Section 4 and conclude our work in Section 5.

### 2. Formalizing Exhaustive Pattern Learning

For the purpose of formalizing the core idea of EPL, we hereby introduce a task called *monotonic translation*. Analysis on this task can be extended to other pattern learning problems. Then, we will define segmentation on training data, and introduce the EPL grammar, which will later be used in Section 3 theoretical justification of EPL.

#### 2.1 Monotonic Translation

**Monotonic translation** is defined as follows. The input $x \in X$ is a string of words $x_1x_2\ldots x_i$ in the source language. The monotonic translation of $x$ is $y \in Y$, a string of words, $y_1y_2\ldots y_i$, of the same length in the target language, where $y_j$ is the translation of $x_j$, $1 \leq j \leq i$.

In short, monotonic translation is a simplified version of machine translation. There is no word reordering, insertion or deletion. In this way, we ignore the impact of word level alignment, so as to focus our effort on the study of building blocks. We leave the incorporation of alignments for future work. In fact, we can simply view alignments as constraints on building blocks. Monotonic translation is already general enough to model many NLP tasks such as labelling and chunking.

#### 2.2 Training Data Segmentation and MLE Grammars

Without losing generality, we assume that the training data $D$ contains a single pair of word strings, $x_D$ and $y_D$, which could be very long. Let $x_D = x_1x_2\ldots x_n$, and $y_D = y_1y_2\ldots y_n$. Source word $x_i$ is aligned to target word $y_j$. Let the length of the word strings be $|D| = n$. Figure 1 shows a simple example of training data. Here $|D| = 4$.

We assume that there exists a hidden segmentation on the training data, which segments $x_D$ and $y_D$ into tokens. A *token* consists of a string of words, either on source or target, and it contains
at least one word. As for monotonic translation, the source side and the target side share the same topology of segmentation. Tokens are the building blocks of the statistical model to be presented, which means that the parameters for the model are defined on tokens instead of words.

A segmentation \( s_D \) of \( D \), or \( s \) for short, is represented as a vector of \( n - 1 \) Boolean values, \( s_1 s_2 ... s_{n-1} \). \( s_i = 0 \) if and only if \( x_i \) and \( x_{i+1} \) belong to the same token. \( s \) applies onto both the source and the target in the same way, which means \( x_i \) and \( x_{i+1} \) belong to the same token if and only if \( y_i \) and \( y_{i+1} \) belong to the same token.

If we segment \( D \) with \( s \), we obtain a tokenized training set \( D_s \). \( D_s \) contains a pair of token strings \( \langle u_s, v_s \rangle \). \( u_s = u_1 u_2 ... u_{|D_s|} \), and \( v_s = v_1 v_2 ... v_{|D_s|} \), where \( |D_s| \) is the total number of tokens in \( u_s \) or \( v_s \). Figure 2 shows an example of segmentation on training data. Here, \( s_2 = 0 \), so that we have a token pair that spans two words, \( (u_2, v_2) = (\text{LEFT FOR}, \text{went to}) \).

Given training data \( D \) and a segmentation \( s \) on \( D \), there is a unique joint probabilistic model obtained by the MLE on \( D_s \). Each parameter of this model contains a source token and target token. Since each token represents a string of words, we call this model a string-to-string grammar \( G_{D_s} \). Specifically, for any pair of tokens \( (u, v) \), we have

\[
Pr(u, v | G_{D_s}) = \frac{\#_s(u, v)}{|D_s|},
\]

where \( \#_s(u, v) \) is the number of times that this token pair appears in the segmented data \( D_s \).

As for the example segmentation \( s \) in Figure 2, its MLE grammar is simply as follows.

\[
Pr(\text{SOPHIE, sophia} | G_{D_s}) = \frac{1}{3} \\
Pr(\text{LEFT FOR, went to} | G_{D_s}) = \frac{1}{3} \\
Pr(\text{PHILLY, philadelphia} | G_{D_s}) = \frac{1}{3}
\]

However, for any given training data, its segmentation is unknown to us. One way to cope with this problem is to consider all possible segmentations. String distribution on the training data will lead us to a good estimation of the hidden segmentation and tokens. In Section 3, we will introduce an ensemble method to incorporate MLE grammars obtained from all possible segmentations. Segmentations are generated with certain prior distribution.
2.3 Exhaustive Pattern Learning for Monotonic Translation

Now we present an EPL solution. We follow the widely-used heuristic method to generate a grammar by applying various segmentations at the same time. We build a heuristic grammar $G_{D,d}$ out of the training data $D$ by counting all possible token pairs $(u, v)$ with at most $d$ words on each side, where $d \ll |D|$ is a given parameter.

$$P_r(u,v|G_{D,d}) = \frac{\#(u,v)}{Z_d},$$

where $\#(u,v)$ is the number of times that the string pair encoded in $(u,v)$ appears in $D$ and $Z_d = \sum_{(u',v')} \#(u',v') = \sum_{i=1}^{d} (|D| - i + 1) = (1 - \frac{d-1}{2|D|})d|D|$.

Therefore,

$$P_r(u,v|G_{D,d}) = \frac{\#(u,v)}{(1 - \frac{d-1}{2|D|})d|D|} \quad (2)$$

For example, the heuristic grammar for the training data in Figure 1 is as follows if we set $d = 2$.

- $P_r($SOPHIE, sophia $|G_{D,2}) = 1/7$
- $P_r($LEFT, went $|G_{D,2}) = 1/7$
- $P_r($FOR, to $|G_{D,2}) = 1/7$
- $P_r($PHILLY, philadelphia $|G_{D,2}) = 1/7$
- $P_r($SOPHIE LEFT, sophia went $|G_{D,2}) = 1/7$
- $P_r($LEFT FOR, went to $|G_{D,2}) = 1/7$
- $P_r($FOR PHILLY, to philadelphia $|G_{D,2}) = 1/7$

A desirable translation rule ‘LEFT FOR ⇒ went to’ is in this heuristic grammar, although its weight is diluted by noise. The hope is that, good translation rules will appear more often in the training data, so that they can be distinguished from noisy rules.

In the decoding phase, we use grammar $G_{D,d}$ as if it is a regular MLE grammar. Let $x = x_1x_2...x_i$ be an input source string. For any segmentation $a$ on the test sentence $x$, let $u_a = u_1u_2...u_k$ be the resultant string of source tokens. The length of the string is $|x| = i$, and the length of the token string is $|u_a| = k$. The translation that we are looking for is given by the target token vector $v$, such that

$$\langle \hat{v}, \hat{a} \rangle = \arg\max_{\langle v,a \rangle} P_r(u_a,v|G_{D,d}),$$

$$P_r(u_a,v|G_{D,d}) = \prod_{j=1,...,|u_a|} P_r(u_j,v_j|G_{D,d}) \quad (3)$$

$$= \prod_{j=1,...,|u_a|} \frac{m_j}{(1 - \frac{d-1}{2|D|})d|D|}$$

1. For the sake of convenience, in the rest of this article, we no longer distinguish a token and the string contained in this token unless necessary. We use symbols $u$ and $v$ to represent both. The meaning is clear in context.
where \( m_j = \#(u_j, v_j) \). As in previous work of structure-based MT, we do not calculate the marginal probability that sums up all possible target tokens generating the same word string, due to the concern of computational complexity.

Obviously, with \( G_{D,d} \), we can take advantage of larger context of up to \( d \) words. However, a common criticism against the EPL approach is that a grammarlike \( G_{D,d} \) is not mathematically sound. The probabilities are simply heuristics, and there is no clear statistical explanation. In the next section, we will show that \( G_{D,d} \) is mathematically sound.

3. Theoretical Justification of Exhaustive Pattern Learning

In this section, we will first introduce an ensemble model and a prior distribution of segmentation. Then we will show the theorem of approximation, and present corollaries on conditional probabilities and tree structures.

3.1 An Ensemble Model

Let \( D \) be the training data of \(|D|\) words. Let \( s \) be an arbitrary token segmentation on \( D \), where \( s \) is unknown to us. Given \( D \) and \( s \), we can obtain a model/grammar \( G_{D_s} \) with maximum likelihood estimation. Thus, we can calculate joint probability of \((u_j, v_j)\) given grammar \( G_{D_s} \), \( P_r(u_j, v_j|G_{D_s}) \).

There are potentially exponential number of distinct segmentations for \( D \). Here, we use an ensemble method to sum over all possible segmentations. This method would provide desirable coverage and diversity of translation rules to be learned from the training data. For each segmentation \( s \), we have a fixed prior probability \( P_r(s) \) which we will shown in Section 3.2. Thus, we define the ensemble probability \( L(u_j, v_j) \) as follows.

\[
L(u_j, v_j) = \sum_s P_r(u_j, v_j|G_{D_s})P_r(s). \tag{4}
\]

Prior segmentation probabilities \( P_r(s) \) serve as model probabilities in (4). Having the model probabilities fixed in this way could avoid over-fitting of the training data DeNero et al. (2006).

In decoding, we search for the best hypothesis \( \hat{v} \) given training data \( D \) and input \( x \) as follows.

\[
\langle \hat{v}, \hat{a} \rangle = \arg\max_{\langle v, a \rangle} L(u_a, v), \text{ where}
\]

\[
L(u_a, v) = \prod_{j=1\ldots|u_a|} L(u_j, v_j)
\]

What is interesting is that there turns out to be a prior distribution for \( s \), such that, under certain conditions, the limit of \( L(u_a, v)/P_r(u_a, v|G_{D,d}) \) as \(|D| \to \infty\) is a value that depends only on \(|x|\) and a parameter of the prior distribution \( P_r(s) \), to be shown in Theorem 3 \(|x|\) is a constant for all hypotheses for the same input. Therefore, \( P_r(u_a, v|G_{D,d}) \) is constant-factor approximation of \( L(u_a, v) \). Using \( G_{D,d} \) is, to some extent, equivalent to using all possible MLE grammars at the same time via an ensemble method.

3.2 Prior Distribution of Segmentation

Now we define a probabilistic model to generate segmentation. \( s = \langle s_1, s_2, \ldots, s_{|D|-1} \rangle \) is a vector of \(|D| - 1\) independent Bernoulli variables. \( s_i \) represents whether \( x_i \) and \( x_{i+1} \) belong to separated
tokens. 1 means yes and 0 means no. All the individual separating variables have the same distribution, \( P_q(s_i = 0) = q \) and \( P_q(s_i = 1) = 1 - q \), for a given real value \( q, 0 \leq q \leq 1 \). Since \( L(u_a, v) \) depends on \( q \) now, we rewrite it as \( L_q(u_a, v) \).

Based on the definition, Lemma 1 immediately follows, which will be used later.

**Lemma 1** For each string pair \((u, v)\), the probability that an appearance of \((u, v)\) in \(D\) is exactly tokenized as \(u\) and \(v\) by \(s\) is \(q^{\left| u \right| - 1}(1 - q)^2\).

### 3.3 Theorem of Approximation

Let \(x = x_1x_2...x_i\) be an input source string. Let \(a\) be a segmentation on \(x\), and the resultant token string be \(u_a = u_1u_2...u_k\). Let \(v = v_1v_2...v_k\) be a hypothesis translation of \(u_a\). Let \(m_j = \#(u_j, v_j)\), the number of times that string pair \((u_j, v_j)\) appears in the training data, \(1 \leq j \leq k\). Let \(m_{j,s} = \#_s(u_j, v_j)\), the number of times that this token pair appears in the segmented data \(D_s\). In order to prove Theorem 2 we assume that the following two assumptions are true for any pair of tokens \((u_j, v_j)\).

**Assumption 1** Any two of the \(m_j\) appearances in \(D\) are neither overlapping nor consecutive.

This assumption is necessary for the calculation of \(E[m_{j,s}]\), \(1 \leq j \leq k\). Based on Lemma 1, the number of times that \((u_j, v_j)\) is exactly tokenized as in this way with segmentation \(s\) is in a binomial distribution \(B(m_j, q^{\left| u_j \right| - 1}(1 - q)^2)\), so that

\[
E[m_{j,s}] = m_j q^{\left| u_j \right| - 1}(1 - q)^2,
\]

where \(\left| u_j \right|\) is the number of words in \(u_j\). In addition, since there is no overlap, these appearances cover a total of \(\left| u_j \right| m_j\) source words.

**Assumption 2** Let \(\eta_j = \frac{(\left| u_j \right| + 1)m_j}{|D|}\). We have \(\lim_{|D| \to \infty} \eta_j = 0\).

In fact, as we will see it in Section 4.1 we do not have to rely on Assumption 2 to bound the ratio of \(Pr(u_a, v|G_{D,d})\) and \(L_q(u_a, v)\). We know that \(\eta_j\) is a very small positive number, and we can build the upper and lower bounds of the ratio based on \(\eta_j\). However, with this assumption, it will be much easier to see the big picture, so we assume that it is true in the rest of this section.

**Theorem 2** Suppose Assumptions 1 and 2 hold for a given pair of tokens \((u_j, v_j)\), then we have

\[
\lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{Pr(u_j, v_j|G_{D,d})} = q^{\left| u_j \right|},
\]

where \(q = d/(d + 1)\).

Later in the section, we will show Theorem 2 with Lemmas 4 and 5. Theorem 3 immediately follows Theorem 2.

**Theorem 3** Suppose Assumptions 1 and 2 hold for any \(j\), then we have

\[
\lim_{|D| \to \infty} \frac{L_q(u_a, v)}{Pr(u_a, v|G_{D,d})} = q^{\left| x \right|},
\]

where \(q = d/(d + 1)\).
Here, $|x|$ is a constant for hypotheses of the same input. An interesting observation is that the prior segmentation model to fit into this theorem tends to generate longer tokens, if we have a larger value for $d$.

We will show Theorem 2 by bounding it from above and below via Lemmas 4 and 5 respectively. Now, we introduce the notations to be used in the proofs of Lemmas 4 and 5 in Appendixes A and B respectively.

First, we combine (1) and (4), and obtain

$$L_q(u_j, v_j) = E_s\left(\frac{m_j \cdot s}{|D_s|}\right). \quad (5)$$

With Assumption 1, we know the value of $E[m_{j,s}]$. However, $|D_s|$ depends on $m_{j,s}$, and this prevents us from computing the expected value on each individual item.

We solve it by bounding $|D_s|$ with values independent of $m_{j,s}$, or the separating variables related to the $m_j$ appearances in $D$. We divide $D$ into two parts, $H$ and $I$, based on the $m_j$ appearances of $(u_j, v_j)$ pairs. $H$ is the part that contains and only contains all $m_j$ appearances, and $I$ is the rest of $D$, so that the internal separating variables of $I$ are independent of $m_{j,s}$. An example is shown in Figure 3. Black boxes represent the $m_j$ appearances.

We concatenate fragments in $I$ and keep the $I$-internal separating variables as in $s$. There are two variants of the segmentation for $I$, depending on how we define the separating variables between the fragments. So we have the following two segmented sub-sets.

- $I_{s_0(I)}$: inter-fragment separating variable = 0.
- $I_{s_1(I)}$: inter-fragment separating variable = 1.

Here, $s_0(I)$ and $s_1(I)$ represent the two segmentation vectors on $I$ respectively, each of which has $|I| - m_j - 1$ changeable separating variables, where $|I|$ is the number words contained in $I$. The number of changeable variables that set to 1 follows a binomial distribution $B(|I| - m_j - 1, 1 - q)$. In Figure 3 fixed inter-fragment separating variables are represented in the bold italic font.

If there are $s$ changeable variables set to 1 in $I_{s_0(I)}$, the number of tokens in $I_{s_0(I)}$ is $|I_{s_0(I)}| = s + 1$. Similarly, if there are $s$ changeable variables set to 1 in $I_{s_1(I)}$, the number of tokens in $I_{s_1(I)}$ is $|I_{s_1(I)}| = s + m_j + 1$. 

Figure 3: An example of training data splitting.
In addition, it is easy to verify that
\[
|I_{s_0(t)}| \leq |D_s| \quad (6)
\]
\[
|D_s| \leq |I_{s_1(t)}| + |u_j|m_j \quad (7)
\]
Combining (5), (6) and the two assumptions, we have the upper bound in Lemma 4.

**Lemma 4** If Assumptions 1 and 2 hold,
\[
\lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{P_r(u_j, v_j|G_{D,d})} \leq q|u_j|,
\]
where \(q = d/(d + 1)\).

Similarly, combining (5), (7) and the two assumptions, we obtain the lower bound in Lemma 5.

**Lemma 5** If Assumptions 1 and 2 hold,
\[
\lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{P_r(u_j, v_j|G_{D,d})} \geq q|u_j|,
\]
where \(q = d/(d + 1)\).

The proofs of Lemmas 4 and 5 are given in the Appendixes A and B respectively. The proof of Lemma 4 also depends on Lemma 8 and its proof are given in Appendix C.

Therefore, Theorem 2 holds.

### 3.4 Corollaries on Conditional Probabilities

Theorem 2 is for joint distribution of token pairs. In previous work of using EPL, conditional probabilities were often used, for example, like \(P(u|v)\) and \(P(v|u)\). Starting from Theorem 2, we can easily obtain the following corollaries for conditional probabilities.

**Corollary 6** Suppose Assumptions 1 and 2 hold for a given pair of tokens \((u_j, v_j)\), then we have
\[
\lim_{|D| \to \infty} \frac{P_r(u_j, v_j|G_{D,d})}{\sum_u L_q(u, v_j)} = 1,
\]
where \(q = d/(d + 1)\).

**Proof** According to the definition, \(P_r(u, v_j|G_{D,d}) = 0\) and \(L_q(u, v_j) = 0\), if \(|u| \neq |v_j|\). Therefore, we only need to consider all pairs of \((u, v_j)\), such that \(|u| = |v_j| = |u_j|\). The number of distinct \(u\) is a finite number, since source vocabulary is a finite set. Therefore, according to Theorem 2, for any small positive number \(\epsilon\), there exists a positive number \(n\), such that, if \(|D| > n\), we have
\[
(1 - \epsilon)\frac{L_q(u, v_j)}{q^{|u_j|}} \leq P_r(u, v_j|G_{D,d}) \leq (1 + \epsilon)\frac{L_q(u, v_j)}{q^{|u_j|}} \quad (8)
\]
Therefore, we have

\[
Pr(u_j|v_j, G_{D,d}) = \frac{Pr(u_j, v_j|G_{D,d})}{\sum_{u:|u|=|u_j|} Pr(u, v_j|G_{D,d})} \\
\leq \frac{(1 + \epsilon)L_q(u_j, v_j)/q^{|u_j|}}{\sum_{u:|u|=|u_j|} (1 - \epsilon)L_q(u, v_j)/q^{|u_j|}} \quad \{\text{Eqn. (8)}\} \\
= \frac{1 + \epsilon}{1 - \epsilon} \frac{L_q(u_j, v_j)}{\sum_u L_q(u, v_j)}
\]

Thus,

\[
Pr(u_j|v_j, G_{D,d})/ L_q(u_j, v_j) \leq \frac{1 + \epsilon}{1 - \epsilon} \frac{L_q(u_j, v_j)}{\sum_u L_q(u, v_j)}
\]

Similarly, we have

\[
Pr(u_j|v_j, G_{D,d})/ L_q(u_j, v_j) \geq \frac{1 - \epsilon}{1 + \epsilon} \frac{L_q(u_j, v_j)}{\sum_u L_q(u, v_j)}
\]

Therefore,

\[
\lim_{|D| \to \infty} Pr(u_j|v_j, G_{D,d})/ \sum_u L_q(u, v_j) = 1
\]

Corollary 7 Suppose Assumptions 1 and 2 hold for a given pair of tokens \((u_j, v_j)\), then we have

\[
\lim_{|D| \to \infty} Pr(v_j|u_j, G_{D,d})/ \sum_v L_q(u_j, v) = 1,
\]

where \(q = d/(d + 1)\).

The proof of Corollary 7 is similar to that of Corollary 6 Therefore, conditional probabilities in EPL are reasonable approximation of the conditional ensemble probability functions.

The proofs for the conditional probabilities depend on a special property of monotonic translation; the length of \(u_j\) is the same as the length of \(v_j\). However, this is not true in real application of phrase-based translation. The source and the target sides may have different segmentations. We leave the modeling of real phrase-based translation for future work.

3.5 Extension to Tree Structures

Now we try to extend Theorem 2 to the string-to-tree grammar. First, we define a prior distribution on tree segmentation. We assign a Bernoulli variable to each tree node, representing the probabilities that we separate the tree at this node, i.e, with probabilities of \(1 - q\), we choose to separate each node.

Let \((u_j, v_j)\) be a string–tree pair, where \(u_j\) is a source string and \(v_j\) is a target tree. Let \(t_j\) be the number of words in \(u_j\), and let \(n_j\) be the number of non-terminals in \(u_j\), where \(t_j + n_j \leq d\), and \(\sum t_j\) is the length of the input sentence, \(|x|\). Thus, the probability that an appearance of \((u_j, v_j)\) in \(D\) is exactly tokenized as in this way is \(q^{t_j-1}(1 - q)^{n_j+1}\).
With similar methods used in the proofs for string structures, we can show that, if Assumptions 1 and 2 hold,

\[
\lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{Pr(u_j, v_j | G_{D,d})} = c q^{t_j - 1}(1-q)^{n_j},
\]

where \( c = \sum_{i^t_i + n_i \leq d} \#(u_i, v_i)/|D| \) is a constant, and \( q \) is a free parameter. We skip the proof here to avoid duplication of similar procedure. We define

\[
P_{q,d}(u_j, v_j) = c q^{t_j - 1}(1-q)^{n_j} Pr(u_j, v_j | G_{D,d}).
\]

Thus, \( P_{q,d}(u_j, v_j) \) approximates \( L_q(u_j, v_j) \), where

\[
\lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{P_{q,d}(u_j, v_j)} = 1.
\]

This result shows a theoretically better way of using heuristic grammar in string-to-tree models.

4. Discussion

In this section, we will focus on three facts that need more explanation.

4.1 On the Use of Assumption 2

In the proofs of Lemmas 4 and 5, Assumption 2 is only used in the very last steps. Therefore, we could build the upper and lowers bounds of the ratio without Assumption 2 by connecting Inequalities (12) and (13) in Appendixes A and B respectively.

4.2 On the Ensemble Probability

The ensemble probability in (4) can be viewed as simplification of a Bayesian model in (9).

\[
L(u_j, v_j) = Pr(u_j, v_j | D) = \sum_{G \in G(D)} Pr(u_j, v_j | G) Pr(G | D)
\]

In (9), we marginalize all possible token-based grammars \( G \) from \( D, G(D) \). Furthermore,

\[
Pr(G | D) = \sum_s Pr(G | D, s) Pr(s | D)
\]

Then, we approximate the posterior probability of \( G \) given \( D \) and \( s \) with point estimation. Thus, \( Pr(G | D, s) = 1 \) if and only if \( G \) is the MLE grammar of \( D_s \), which means all the distribution mass is assigned to \( G_{D,s} \), the MLE grammar for \( D_s \). We also assume that \( s \) is independent of \( D \). Thus,

\[
Pr(G | D) = \sum_s \mathbf{1}(G = G_{D,s}) Pr(s),
\]

4
where $Pr(s)$ is a prior distribution of segmentation for any string of $|D|$ words. With (10), we can rewrite (9) as follows.

$$L(u_j, v_j) = \sum_{G \in G(D)} Pr(u_j, v_j|G) \sum_s 1(G = G_{D_s}) Pr(s)$$

$$= \sum_s \sum_{G \in G(D)} Pr(u_j, v_j|G) 1(G = G_{D_s}) Pr(s)$$

$$= \sum_s Pr(u_j, v_j|G_{D_s}) Pr(s)$$

Equation (11) is exactly the ensemble probability in Equation (4).

4.3 On the DOP Model

The EPL method investigated in this article may date back to Data Oriented Parsing (DOP) by Bod (1992). What is special with DOP is that the DOP model uses overlapping treelets of various sizes in an exhaustive way as building blocks of a statistical tree grammar.

In our framework, for each pair $(u_j, v_j)$, we can use $u_j$ to represent the input text, and $v_j$ to represent its tree structure. Thus, it would be similar to the string-to-tree model in Section 3.5. Joint probability of $(u_j, v_j)$ stands for unigram probability $Pr(treelet)$.

However, the original DOP estimator (DOP1) is quite different from our monotonic translation model. The conditional probability in DOP1 is defined as $Pr(treelet|subroot-label)$, so that there is no obvious way to model DOP1 with monotonic translation. Therefore, theoretical justification of DOP1 is still an open problem.

5. Conclusion

In this article, we first formalized exhaustive pattern learning (EPL), which is widely used in grammar induction in NLP. We showed that using an EPL heuristic grammar is equivalent to using an ensemble method to cope with the uncertainty of building blocks of statistical models.

Better understanding of EPL may lead to improved pattern learning algorithms in future. This work will affect the research in various fields of natural language processing, including machine translation, parsing, sequence classification etc. EPL can also been applied to other research fields outside NLP.

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Appendix A. Proof for Lemma 4

\[ L_q(u_j, v_j) = E_s \left[ \frac{m_{j,s}}{|D_s|} \right] \{\text{Eqn. (5)}\} \]

\[ \leq E_s \left[ \frac{m_{j,s}}{|I_{s0(I)}|} \right] \{\text{Eqn. (6)}\} \]

\[ = E[m_{j,s}]E\left[ \frac{1}{|I_{s0(I)}|} \right] \]

\{Independence of \( m_{j,s} \) and \( I_{s0(I)} \)\}

\[ = E[m_{j,s}]E_{s0(I)}\left[ \frac{1}{|I_{s0(I)}|} \right] \]

\[ \leq \frac{E[m_{j,s}]}{(1-q)(|I|-m_j)}(1 - q|I|-m_j) \{\text{Lemma 8}\} \]

\[ \leq \frac{m_j q^{|u_j|-1}(1-q)^2}{(1-q)(|I|-m_j)} \{\text{Binomial Dist., Assumption 1}\} \]

\[ = \frac{m_j q^{|u_j|-1}(1-q)^2}{(1-q)(|D|-|u_j|m_j - m_j)} \]

\[ = \frac{m_j q^{|u_j|-1}(1-q)^2}{(1-q)(1-\eta_j)} \]

\[ = q^{|u_j|-1}(1-q)^2(1 - \frac{d-1}{2|D|})d \]

\[ \leq \frac{q^{|u_j|-1}(1-q)^2(1 - \frac{d-1}{2|D|})d}{(1-q)(1-\eta_j)} \]

\[ = \frac{Pr(u_j, v_j|G_{D,d})q^{|u_j|-1}(1-q)^2(1 - \frac{d-1}{2|D|})d}{(1-q)(1-\eta_j)} \]

\[ = Pr(u_j, v_j|G_{D,d})q^{|u_j|-1}(1-q)(1 - \frac{d-1}{2|D|})d \]

\[ \lim_{|D|\to\infty} \frac{L_q(u_j, v_j)}{Pr(u_j, v_j|G_{D,d})} \leq q^{|u_j|}(1-q)d \{\text{Assumption 2}\} \]

\[ = q^{|u_j|}. \]
Appendix B. Proof for Lemma 5

\[ L_q(u_j, v_j) = E_s \left[ \frac{m_{j,s}}{|D_s|} \right] \{ \text{Eqn. (5)} \} \]
\[ \geq E_s \left[ \frac{m_{j,s}}{|I_{s_1(t)}| + |u_j|m_j} \right] \{ \text{Eqn. (7)} \} \]
\[ = E[m_{j,s}]E_s \left[ \frac{1}{|I_{s_1(t)}| + |u_j|m_j} \right] \{ \text{Independence of } m_{j,s} \text{ and } I_{s_1(t)} \} \]
\[ \geq \frac{E[m_{j,s}]}{E[|I_{s_1(t)}| + |u_j|m_j]} \{ \text{Jensen’s inequality} \} \]
\[ = \frac{E[m_{j,s}]}{E[|I_{s_1(t)}|] + |u_j|m_j} \]
\[ = \frac{m_{j,q}u_j|u_j|^{-1}(1 - q)^2}{(1 - q)(|D| - |u_j|m_j) + q(1 + m_j) + |u_j|m_j} \{ \text{Binomial Dist., Assumption 1} \} \]
\[ = \frac{m_{j,q}u_j|u_j|^{-1}(1 - q)^2}{(1 - q)|D| + q(|u_j|m_j + m_j) + q} \]
\[ = \frac{m_{j,q}u_j|u_j|^{-1}(1 - q)^2}{(1 - q + qn_j + \frac{q}{|D|})|D|} \]
\[ = \frac{m_{j,q}u_j|u_j|^{-1}(1 - q)^2(1 - \frac{d-1}{2|D|})d}{1 - q + qn_j + \frac{q}{|D|}} \]
\[ = Pr(u_j, v_j|G_{D,d}) \frac{q^{|u_j|}(1 - q)^2(1 - \frac{d-1}{2|D|})d}{1 - q + qn_j + \frac{q}{|D|}q} \]
(13)

\[ \lim_{|D| \to \infty} \frac{L_q(u_j, v_j)}{Pr(u_j, v_j|G_{D,d})} \]
\[ \geq q^{|u_j|}(1 - q) \frac{d}{q} \{ \text{Assumption 2} \} \]
\[ = q^{|u_j|}. \]
Appendix C. Lemma and its Proof

Lemma 8  Let $X$ be a random variable of Binomial distribution $B(n, 1 - q)$, then

$$E\left[\frac{1}{X + 1}\right] = \frac{1 - q^{n+1}}{(1 - q)(n + 1)}$$

$$E\left[\frac{1}{X + 1}\right] = \sum_{k=0}^{n} \frac{1}{k + 1} \frac{n!}{k! (n - k)!} q^{n-k} (1 - q)^k$$

$$= \frac{1}{(1 - q)(n + 1)} \sum_{k=0}^{n} \frac{(n + 1)!}{(k + 1)! (n - k)!} q^{n-k} (1 - q)^{k+1}$$

$$= \frac{1}{(1 - q)(n + 1)} ((q + 1 - q)^{n+1} - q^{n+1})$$

$$= \frac{1 - q^{n+1}}{(1 - q)(n + 1)}$$

References

Phil Blunsom, Trevor Cohn, Chris Dyer, and Miles Osborne. A gibbs sampler for phrasal synchronous grammar induction. In Proceedings of the 47th Annual Meeting of the Association for Computational Linguistics (ACL), pages 782–790, Suntec, Singapore, August 2009.

Rens Bod. A computational model of language performance: Data Oriented Parsing. In Proc. of COLING92, 1992.

Xavier Carreras and Michael Collins. Non-projective parsing for statistical machine translation. In Proceedings of the 2009 Conference of Empirical Methods in Natural Language Processing, pages 200–209, Singapore, 2009.

David Chiang. A hierarchical phrase-based model for statistical machine translation. In Proceedings of the 43th Annual Meeting of the Association for Computational Linguistics (ACL), pages 263–270, Ann Arbor, MI, 2005.

Trevor Cohn and Phil Blunsom. A Bayesian model of syntax-directed tree to string grammar induction. In Proceedings of the 2009 Conference of Empirical Methods in Natural Language Processing, pages 352–361, Singapore, August 2009.

Trevor Cohn, Sharon Goldwater, and Phil Blunsom. Inducing compact but accurate tree-substitution grammars. In Proceedings of the 2009 Human Language Technology Conference of the North American Chapter of the Association for Computational Linguistics, pages 548–556, Boulder, Colorado, June 2009.

Hal Daumé III. Natural language processing blog: Teaching machine translation. http://nlpers.blogspot.com/2008/05/teaching-machine-translation.html, 2008.
Hal Daumé III and Daniel Marcu. Learning as search optimization: Approximate large margin methods for structured prediction. In *Proceedings of the 22nd International Conference on Machine Learning*, 2005.

John DeNero, Dan Gillick, James Zhang, and Dan Klein. Why generative phrase models underperform surface heuristics. In *Proceedings of the Workshop on Statistical Machine Translation*, 2006.

John DeNero, Alexandre Bouchard-Côté, and Dan Klein. Sampling alignment structure under a Bayesian translation model. In *Proceedings of the 2008 Conference of Empirical Methods in Natural Language Processing*, 2008.

Michel Galley, Jonathan Graehl, Kevin Knight, Daniel Marcu, Steve DeNeefe, Wei Wang, and Ignacio Thayer. Scalable inference and training of context-rich syntactic models. In *COLING-ACL ’06: Proceedings of 44th Annual Meeting of the Association for Computational Linguistics and 21st Int. Conf. on Computational Linguistics*, pages 961–968, Sydney, Australia, 2006.

Mark Johnson. The DOP estimation method is biased and inconsistent. *Computational Linguistics*, 28(1), 2002.

Philipp Koehn, Franz J. Och, and Daniel Marcu. Statistical phrase based translation. In *Proceedings of the 2003 Human Language Technology Conference of the North American Chapter of the Association for Computational Linguistics*, pages 48–54, Edmonton, Canada, 2003.

Franz J. Och and Hermann Ney. The alignment template approach to statistical machine translation. *Computational Linguistics*, 30(4), 2004.

Matt Post and Daniel Gildea. Bayesian learning of a tree substitution grammar. In *Proceedings of the 47th Annual Meeting of the Association for Computational Linguistics (ACL)*, pages 45–48, Suntec, Singapore, August 2009.

Detlef Prescher, Remko Scha, Khalil Sima’an, and Andreas Zollmann. On the statistical consistency of dop estimators. In *Proceedings of the 14th Meeting of Computational Linguistics in the Netherlands*, 2004.

Chris Quirk, Arul Menezes, and Colin Cherry. Dependency treelet translation: Syntactically informed phrasal SMT. In *Proceedings of the 43th Annual Meeting of the Association for Computational Linguistics (ACL)*, pages 271–279, Ann Arbor, MI, 2005.

Libin Shen, Jinx Xu, and Ralph Weischedel. A new string-to-dependency machine translation algorithm with a target dependency language model. In *Proceedings of the 46th Annual Meeting of the Association for Computational Linguistics (ACL)*, 2008.

Luke Zettlemoyer and Michael Collins. Learning to map sentences to logical form: Structured classification with probabilistic categorial grammars. 2005.