Oriented collisions for cold synthesis of superheavy nuclei

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The conditions of optimum orientations (lowest barrier and largest interaction radius) for deformed colliding nuclei are introduced in "cold" fusion of superheavy nuclei. Also, the role of (octupole and) hexadecupole deformations is studied. We have used the proximity potential and applied our method to Ca-induced reactions.

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Collisions between the deformed, oriented nuclei have been of much interest from time to time. In early 1980’s it got trigged off, beginning with a suggestion by Greiner [1] that oriented $^{238}$U+$^{238}$U collisions could lead to a very long lived (life-time $\sim 10^{-20}$ sec) giant molecule. A number of calculations were made [2–6] which all resulted in showing that the barrier is lowered due to deformations and orientations of colliding nuclei and that it is lowest for the $0^+\sim$180$^+$ orientations of two $^{238}$U nuclei, known as pole-to-pole (p-p) or nose-to-nose configuration. Note that $^{238}$U is a prolate deformed nucleus, and hence the above result is only for prolate-prolate collisions, though quoted in the literature loosely for all oriented collisions. In fact, we show in the present paper that colliding nuclei with different signs of their quadrupole deformations result in lowest barrier for different orientations (see Table 1). For example, for prolate-oblate collisions, the barrier is lowest for $0^+\sim$90$^+$ (equator-equator crossed, in short, e-c) configuration, as is envisaged very recently by Nörenberg [7]. Also, till recently [8], the role of multipoles higher than the quadrupole had not been investigated. We find that the inclusion of higher multipole deformations is favorable for fusion in some cases only i.e. the barriers are lowered only for some orientations. Some of these results require immediate attention and verification by alternative methods.

We use here the quantum mechanical fragmentation theory (QMFT), extended to include the higher multipole deformations and orientations degrees of freedom. In QMFT, cold synthesis of new and superheavy nuclei was first proposed by one of us and collaborators [9–11], where a method was given for selecting out an optimum "cold" target-projectile (T-P) combination. Cold compound systems were considered to be formed for all those T-P combinations that lie at the bottom of the minima in the potential energy surface of a given compound nucleus, calculated for all possible T-P combinations, referred to as "cold reaction valleys" or reaction partners leading to "cold fusion" [10–14]. This information on "cold fusion valleys" was further optimized [11] by the requirements of smallest interaction barrier, largest interaction radius and non-necked (no saddle) nuclear shapes, identifying the cases of "cold", "warm/ tepid" and "hot" fusion reactions. The key result behind the cold fusion reaction valleys is the shell closure effects of one or both the reaction partners. The QMFT was advanced as a unified approach both for heavy ion collisions, leading to fusion, and fission of nuclei including the cluster radioactivity (see e.g. the reviews in [15] and the references therein).

We choose to apply our method to the recent experiments of highly neutron-rich $^{48}$Ca beam bombarded on neutron-rich actinides $^{232}$Th, $^{238}$U, $^{242,244}$Pu and $^{248}$Cm, forming the compound systems $^{280}$110$^*$, $^{286}$112$^*$, $^{290,292}$114$^*$ and $^{296}$116$^*$ [16]. In these reactions, for near the Coulomb barrier energies, the compound nucleus excitation energy $E^*$ $\sim$30–35 MeV, in between the one for cold (10-20 MeV) and hot (40-50 MeV) fusion reactions. The use of neutron-rich (radioactive) nuclei is essential for overshooting the centre of island of superheavy nuclei (the next doubly magic nucleus) and their deformations and orientations could provide an added advantage since the fusion barrier gets lowered, or, in other words, the excitation energy of compound system gets reduced. This means a possibility that the "warm" and/or "hot" fusion reactions could also be reached in "cold" fusion, as is found to be the case here in the following calculations.

The QMFT is worked out in terms of the mass (and charge) asymmetry $\eta=(A_1-A_2)/(A_1+A_2)$ (and $\eta_Z=(Z_1-Z_2)/(Z_1+Z_2)$), the relative separation $R$, the deformations $\beta$, (so far $\lambda=2$ only, the quadrupole deformations) of two nuclei (1=2) or, in general, the two fragments, and the neck parameter $\epsilon$ [17–20]. We introduce here the higher multipole deformations $\lambda$=3 and 4, i.e. the octupole and hexadecupole deformations, as additional new parameters. Also, two orientation angles $\theta_i$ are included, as in [6] (see Fig. 1, illustrated for quadrupole deformations). So far, the time-dependent Schrödinger equation in $\eta$ is solved for non-oriented collisions and for weakly coupled $\eta$ and $\eta_Z$ motions:

$$H\Psi(\eta,t) = i\hbar \frac{\partial}{\partial t}\Psi(\eta,t),$$  

with $R(t)$ treated classically, and $\beta$, and $\epsilon$ fixed by minimizing the collective potential $V(R,\eta,\eta_Z,\beta,\epsilon)$. Eq. (1), solved for a number of heavy systems [19,20], shows that a few nucleon to a large mass transfer occurs for T-P combinations coming from outside the potential energy minima, whereas the same is zero for a T-P referring to potential energy minima. This means that for cold reaction partners, the two nuclei stick together and form a deformed compound system. A few nucleon transfer may, however, occur if a "conditional" saddle exists [21].
solution of Eq. (1) is very much computer-time consuming, and hence the following (next paragraph) simplifications are exercised on the basis of calculated quantities.

The potentials $V(R, \eta)$ and $V(R, \eta_Z)$ for non-oriented nuclei, calculated within the Strutinsky method by using asymmetric two-center shell model (ATCSM), show that the motions in both $\eta$ and $\eta_Z$ are much faster than the R-motion. This means that these potentials are nearly independent of the R-coordinate and hence R could be taken as a time-independent parameter. This reduces Eq. (1) to the stationary Schrödinger equation in $\eta$,

$$\left\{-\frac{\hbar^2}{2B_{\eta\eta}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V(\eta)\right\} \Psi^0(\eta) = E^0 \Psi^0(\eta). \tag{2}$$

Here R is fixed at the post saddle point, a choice justified by many calculations [15], and by an explicit, analytical solution of time-dependent Schrödinger equation in $\eta_Z$ coordinate [22]. An interesting result of these calculations is that the yields ($\propto |\Psi(\eta)|^2$ or $|\Psi(\eta_Z)|^2$, respectively, for mass or charge distributions) are nearly insensitive to the detailed structure of the kinetic energy term in the Hamiltonian which consisted of Cranking masses $B_{\eta\eta}$ consistently calculated by using ATCSM. In other words, the static potential $V(\eta)$ or $V(\eta_Z)$ contains all the important information of a colliding or fissioning system.

Since the potential $V(\eta, R)$ is nearly independent of the choice of R-value, for oriented nuclei, we define it as sum of two binding energies, and the deformation and orientation dependent Coulomb and proximity potentials:

$$V(\eta, R) = -\sum_{i=1}^{2} B_i(A_i, Z_i, \beta_{\lambda i}) + E_c(Z_i, \beta_{\lambda i}, \theta_i) + V_P(A_i, \beta_{\lambda i}, \theta_i). \tag{3}$$

Here, the binding energies $B_i$ are taken from Möller et al. [23] for $Z \geq 8$, and from experiments [24] for $Z \leq 7$. The Coulomb and proximity potentials, with higher multipole deformations included, are obtained by following the works of [25] and [6], respectively. The Coulomb potential

$$E_c = \frac{Z_1 Z_2 e^2}{R} + 3Z_i Z_2 e^2 \sum_{\lambda, i=1,2} \frac{R_{0i}^{\lambda}}{2 + 1} Y_{\lambda,0}^{(0)}(\alpha_i) \cdot \left[ \beta_{\lambda i} + \frac{4}{7} \beta_{\lambda i}^2 \cdot Y_{\lambda,0}^{(0)}(\alpha_i) \delta_{\lambda,2} \right], \tag{4}$$

with

$$R_i(\alpha_i) = R_{0i} \left[ 1 + \sum_{\lambda} \beta_{\lambda i} Y_{\lambda,0}^{(0)}(\alpha_i) \right], \tag{5}$$

where $R_{0i} = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$. A similar expression is obtained by Rumin et al. [26] which differs in the quadrupole interaction term proportional to $\beta_{\lambda i}^2$. The nuclear proximity potential

$$V_P = 4\pi \tilde{R}_\gamma b \Phi(s_0), \tag{6}$$

where the specific nuclear surface tension coefficient $\gamma = 0.9517 \left(1 - 1.7826 \left(\frac{Z}{A}\right)^2\right)$ MeV fm$^{-2}$; the surface thickness $b = 0.99$ fm; and the universal function, independent of the geometry of nuclear system, is

$$\Phi(s_0) = \left\{ -\frac{1}{2}(s_0 - 2.54)^2 - 0.0852(s_0 - 2.54)^3 \right\} \tag{7}$$

respectively, for $s_0 \leq 1.2511$ and $\geq 1.2511$, with

$$s_0 = [R - R_1(\alpha_1)cos\psi_1 - R_2(\alpha_2)cos\psi_2]/b, \tag{8}$$

the separation distance between the colliding surfaces, parallel to R taken along the collision axis, in units of b. For $s_0$ to be minimum, i.e. $\partial s_0/\partial \alpha_i = 0$, it follows from Fig. 1 that $\psi_1 = \theta_1 - \alpha_1$, $\psi_2 = 180 - \theta_2 - \alpha_2$ and $tan\psi_i = -R_i'/(R_i)$. Finally, $\tilde{R}$, the mean curvature radius characterizing the gap, for nuclei lying in the same plane, is $1/\tilde{R}^2 = 1/R_1 R_2 + 1/R_1 R_2 + 1/R_1 R_2 + 1/R_1 R_2$, where the four principal radii of curvature $R_1$ and $R_2$ at the points D (denoted 1) and E (denoted 2) of minimum $s_0$, are given by Eq. (15) in Ref. [6]. For further details, see [6]. Recently, Misure and Greiner [8] have also derived the heavy ion interaction potential by using a multipole expansion of the densities in a double folding procedure. Such a procedure is shown [27] to depend strongly on the number of terms included in the expansion. Three terms are found to be sufficient for the internal region of the nuclear potential, whereas up to five terms are shown necessary for the physically more relevant surface and tail region for heavy ion collisions.

For the fixed orientations, the charges $Z_i$ in (3) are fixed by minimizing the potential $V(R, \eta, \eta_Z, \beta_{\lambda i}, \theta_i)$ in $\eta_z$ coordinate (which fixes the deformation coordinates $\beta_{\lambda i}$ also). In Eq. (8), for fixed $R$, $s_0$ is different for different orientations, and for fixed $s_0$, $R$ is different for different orientations, which is used here in the following.

Table 1 gives the orientations of nuclei for the lowest barrier, for all possible combinations of different signs of their quadrupole deformations (prolate, oblate or spherical). These barriers also lie at the largest interaction radii. Fig. 2 illustrates our result for prolate-oblate $^{238}$Pu+$^{48}$Ar$\rightarrow^{286}$Ni122 reaction (see solid lines, where deformations are included to all orders, $\lambda=2,3,4$). According to the QMFT [11], as already stated above, the above conditions are for an optimum cold fusion reaction. In other words, the orientations in Table 1 are the optimum orientations for cold fusion reactions using deformed nuclei. We further notice from Fig. 2, that the inclusion of higher multipole deformations is not always favorable for fusion (compare solid lines, with dotted ones for $\lambda=2$ alone): the addition of $\beta_{\lambda i}$ term ($\beta_{\lambda i}=0$) lowers the barriers for some sets of orientations whereas it raises them for the other sets of orientations (illustrated in Fig. 2 for two cases each). Thus, the choice of nuclei having octupole and hexadecupole deformations for (cold) fusion reactions must be made judiciously, depending on not only the signs of their quadrupole deformations but also the orientation angles.
Fig. 3 shows the fragmentation potentials for optimum orientations of the different T-P combinations, at a fixed separation $s_0=1.5 \text{ fm}$, forming the same compound nucleus $^{286}_{112}$. The case of spherical nuclei [29] is also plotted for comparisons. Apparently, due to deformation and orientation degrees of freedom, all the potential energy minima are lowered, some new minima have appeared and some old ones have disappeared. Specifically, new deep minima occur at $^{56}_{26}\text{Cr} + ^{230}_{90}\text{Ra}$ and $^{106}_{42}\text{Mo} + ^{180}_{70}\text{Yb}$, which are in addition to the ones referring to the region of cluster radioactivity and/or ”hot” fusion (involving light nuclei of masses <30). The minima that have disappeared refer to well known cases of Ca and Pb (or neighbouring) nuclei; here $^{48,50}_{20}\text{Ca} + ^{238,236}_{92}\text{U}$ and $^{80}_{36}\text{Ge} + ^{206}_{82}\text{Hg}$ combinations. Similar results are obtained for the compound systems $^{290}_{114}$ and $^{296}_{116}$. For $^{280}_{110}$, however, Ca and Hg minima are still deep and could be used as cold fusion reactions (details to be published elsewhere). We further notice from Fig. 2 that, w.r.t. the g.s. energy, all minima now refer to much smaller excitation energies, some of them lying even below it. For example, for Ca minima it is reduced from ~35 MeV for spherical nuclei to <20 MeV for deformed and oriented collisions. This means that, as compared to spherical nuclei, oriented collisions result in cooler fusion reactions. Furthermore, all the optimum cold oriented collisions involve radioactive nuclei.

The above results are seen better in the calculated mass distribution yields $Y(A_i) = |\Psi(\eta(A_i))|^2 \sqrt{B_{\eta\eta}}$, for $\nu = 0$. Here, $\Psi(\nu)$ are the solutions of Eq. (3) and $B_{\eta\eta}$ are the classical hydrodynamical masses [30]. We take the view that, since fragments related to the minimum in $V(\eta)$ are more probable, the yields must give the intermediate (two) fragment formation yields or, in short the formation yields for a cool compound nucleus [29], where the contribution of barrier penetration is not included. Evidently, for oriented collisions, the yields for new T-P pairs $^{56}_{26}\text{Cr}+^{230}_{90}\text{Ra}$ and $^{106}_{42}\text{Mo}+^{180}_{70}\text{Yb}$ are larger than for their neighbouring Ca and Hg induced reactions. The (near) symmetric combination has the largest yield, but they are known to form necked-in shapes, signifying preformation of fission fragments [11,15,29].

Summarizing, we have extended the QMFT for use of oriented collisions and inclusion of higher multipole deformations, which result in the reduction of excitation energies of the compound system formed due to different T-P combinations. This means that both the ”warm” and ”hot” fusion reactions could now be reached in ”cold fusion” also. The idea of optimum orientations for cold fusion reactions is introduced for the first time, which leads to new cold fusion reaction partners. The choice of nuclei with hexadecupole deformations is shown to depend strongly on both the signs of their quadrupole deformations and orientation angles.

**Table 1:** The optimum orientations for ”cold” fusion of nuclei with all possible combinations of deformations.

| Nuclear deformations | Optimum orientations | Nuclear deformations | Optimum orientations |
|----------------------|----------------------|----------------------|----------------------|
| Prolate-Prolate      | $90^\circ - 180^\circ$| Prolate-Spherical    | $0^\circ - \dagger$  |
| Oblate-Oblate        | $90^\circ - 0^\circ$ | Oblate-Spherical     | $0^\circ - \dagger$  |
| Prolate-Oblate       | $0^\circ - 90^\circ$ | Spherical-Prolate    | $\dagger - 180^\circ$|
| Oblate-Prolate       | $90^\circ - 180^\circ$| Spherical-Oblate     | $\dagger - 90^\circ$ |

Here the spherical nuclei (denoted by $\dagger$) are considered to have zero octupole and hexadecupole deformations.
[1] W. Greiner, *International Advanced Course on Quantum Electrodynamics of Strong Fields*, Lahnstein 1981; M. Seiwert, et al., GSI Annual Report 1981.

[2] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[3] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[4] M.J. Rhodea-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[5] M. Seiwert, et al., Phys. Rev. C 29, 477 (1984).

[6] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[7] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[8] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[9] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[10] M. Seiwert, et al., Phys. Rev. C 29, 477 (1984).

[11] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[12] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[13] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[14] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[15] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[16] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[17] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[18] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[19] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[20] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[21] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[22] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[23] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[24] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[25] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[26] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[27] M.J. Rhoades-Brown, V.E. Oberacker, M. Seiwert, and W. Greiner, Z. Phys. A 310, 287 (1983).

[28] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

[29] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. A 388, 381 (1982).

[30] A.J. Baltz and B.F. Bayman, Phys. Rev. C 26, 1969 (1982).

Figure Captions

Fig. 1 Schematic configuration of two axially symmetric deformed, oriented nuclei, lying in the same plane ($\phi = 0^\circ$).

Fig. 2 Scattering potentials for the prolate-oblate $^{238}\text{Pu}^{+48}\text{Ar} \rightarrow ^{286}112^\ast$, at different orientations. The R-values at the top of the three lowest lying barriers are also shown.

Fig. 3 Fragmentation potentials of $^{286}112^\ast$ for the optimum orientations of different T-P combinations with $\lambda=2,3,4$ (solid line with symbols) and for spherical nuclei (solid line). For $Z\leq7$, the $\beta_{2i}$ are from relativistic mean field calculations using TM2 force [28], and for $Z>7$ from [23]. The $\beta_{3i} = \beta_{4i} = 0$. For the spherical case, $\beta_{2i} = \beta_{3i} = \beta_{4i} = 0$. The g.s. is the ground state energy.

Fig. 4 Calculated yields of $^{286}112$ for optimum orientations of different T-P combinations, with $\lambda=2,3,4$ (solid line with symbols) and spherical nuclei (solid line).
\[2^{^{238}}\text{Pu} + ^{^{48}}\text{Ar} \rightarrow 2^{^{286}}^{^{1}}\text{I}^{^{12}}*\]

\[\beta_{2i} = 0.215, -0.207\]
\[\beta_{4i} = 0.102, -0.067\]

\[\beta_{2i} + \beta_{3i} + \beta_{4i}\]
\[\beta_{3i} = \beta_{4i} = 0\]

1: \(0^0, 90^0\)
2: \(0^0, 135^0\)
3: \(0^0, 180^0\)
4: \(45^0, 90^0\)
5: \(45^0, 135^0\)
6: \(90^0, 90^0\)
7: \(90^0, 180^0\)
8: Spherical
\[ (258 \text{ Fm}) \]
\[ 28 \text{ Mg} \]
