Landscape of Supersymmetry Breaking Vacua in Geometrically Realized Gauge Theories

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We study vacuum structure of $\mathcal{N} = 1$ supersymmetric quiver gauge theories which can be realized geometrically by D brane probes wrapping cycles of local Calabi-Yau threefolds. In particular, we show that the $A_2$ quiver theory with gauge group $U(N_1) \times U(N_2)$ with $\frac{1}{2}N_1 < N_2 < \frac{2}{3}N_1$ has a regime with an infrared free description that is partially magnetic and partially electric. Using this dual description, we show that the model has a landscape of inequivalent meta-stable vacua where supersymmetry is dynamically broken and all the moduli are stabilized. Each vacuum has distinct unbroken gauge symmetry. The gaugino masses are generated by radiative corrections, and we are left with the bosonic pure Yang-Mills theory in the infrared. We also identify the supersymmetric vacua in this model using their infrared free descriptions and show that the decay rates of the supersymmetry breaking vacua can be made parametrically small.

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1. Introduction

The discovery \cite{1} that simple supersymmetric gauge theories such as the $\mathcal{N} = 1$ supersymmetric QCD with massive flavors have meta-stable vacua with broken supersymmetry may lead to a new paradigm in phenomenological model building of supersymmetric extensions of the Standard Model. Moreover, techniques developed in \cite{1} allow us to study non-perturbative aspects of these models that are not protected by supersymmetry. In this paper, we will try to understand how generic such phenomena are by applying their method to a class of quiver gauge theories. Not all models exhibit supersymmetry breaking vacua in regions that are accessible with our current technology. It is therefore interesting to note that these phenomena naturally happen in models that arise as low energy limits on D branes wrapping cycles in local Calabi-Yau manifolds.\footnote{See \cite{2} for another class of string inspired models with meta-stable supersymmetry breaking vacua.}

In the models studied in this paper, field content and gauge symmetry emerging from string theory conspire to ensure that the supersymmetry breaking configurations are locally stable in all directions.

One of the models we study in this paper is the $\mathcal{N} = 1$ supersymmetric quiver gauge theory associated to the $A_2$ Dynkin diagram with the gauge group $U(N_1) \times U(N_2)$. This model can be viewed as the $\mathcal{N} = 2$ supersymmetric $A_2$ quiver gauge theory with the supersymmetry broken to $\mathcal{N} = 1$ by superpotentials for the adjoint scalar fields. This model arises naturally from type IIB string theory as the low energy limit of D5 brane probes wrapping 2-spheres in the local Calabi-Yau three-fold which is an $A_2$ fibration on a plane \cite{3}. The superpotentials encode the geometric data on the fibration. We assume that the model is in the asymptotic free regime in the electric description.

When $N_1 < N_2$, it was shown in \cite{4} that this model has a Seiberg-like dual \cite{5,6} which is the same $A_2$ quiver theory but with the different gauge group $U(N_2 - N_1) \times U(N_2)$. In the type IIB string language, this duality is the Weyl reflection symmetry of the $A_2$ Dynkin diagram. This duality also has the following field theoretical interpretation. Let us assume that the strong coupling scales $\Lambda_1$ and $\Lambda_2$ for the $U(N_1)$ and $U(N_2)$ obey $\Lambda_1 \gg \Lambda_2$. (Geometrically, this means that we choose one of the 2-spheres to be much smaller than the other.) We can then performs the Kutasov-type duality \cite{7,8,9,10,11} on the $U(N_1)$ to land on a theory with the gauge group $U((k - 1)N_2 - N_1) \times U(N_2)$, where $k$ is the highest power in the superpotential $W_1(X_1)$ for the adjoint scalar $X_1$ for the electric gauge group.
$U(N_1)$. The $F$-term constraints force the gauge symmetry to be spontaneously broken to $U(N_2 - N_1) \times U(N_2)$, which is the dual gauge group identified in [3].

In this paper, we will consider the same model but in a different regime. When the superpotential $W_1(X_1)$ is cubic, the Kutasov-type dual becomes infrared free for both gauge groups when $\frac{1}{2} N_1 < N_2 < \frac{2}{3} N_1$.\(^2\) We find that, in this case, the $D$ and $F$-term constraints have no solutions near the origin of the field space. On the other hand, there are local minima of the classical $D$ and $F$-term potentials. Each of these minima has massless chiral multiplets at the tree level, but they all become massive by the one-loop effective potential. Thus, there are no flat directions at these meta-stable vacua. Higher order corrections to the potential can be made parametrically small because of the infrared freedom.

Each of the isolated meta-stable vacua has distinct unbroken gauge symmetry of the form $U(r_1) \times U(r_2) \times U(N_1 - N_2)$ with $r_1 + r_2 = 2N_2 - N_1$. At the supersymmetry breaking scale, all chiral multiplets become massive. Unless both $N_1$ and $N_2$ are even, the chiral anomaly combined with the superpotential breaks the R symmetry completely. Thus, the supersymmetry breaking also generates gaugino masses by radiative corrections, and we are left with the bosonic Yang-Mills theory for the unbroken gauge symmetry. In particular, the broken supersymmetry is not restored in the infrared. This gives the landscape of supersymmetry breaking vacua, each of which is characterized by the gauge symmetry breaking pattern.

We find that there are supersymmetric vacua away from the origin of the field space in the dual description. This is consistent with the electric description of the model and establishes the connection of the two descriptions. The identification of the supersymmetric vacua in the dual description also allows us to estimate the decay rates of the supersymmetry breaking vacua into the supersymmetric vacua. We find that the decay rates can be made parametrically small. Since the supersymmetry breaking vacua all have the same energy, the transition probabilities among them are equal to zero.

We also study the $A_2$ quiver theory with $SU(N_1) \times SU(N_2)$ gauge group, which has an analogous infrared free dual description. In the $U(N_1) \times U(N_2)$ model, the energies of

\(^2\) For a more general potential $W_1(X_1) \sim X_1^{k+1} + \cdots$, the corresponding condition is $\frac{1}{k+1} N_1 < N_2 < \frac{2k}{2k+3} N_1$. However, this condition is not compatible with the asymptotic freedom of the electric description unless $k \leq 3$. We restrict our attention to the case when $W_1(X_1)$ is cubic so that the original electrical description is ultraviolet complete.
the supersymmetry breaking vacua are degenerate. We find that the degeneracy is lifted in the $SU(N_1) \times SU(N_2)$ model.

Generalization of our results to gauge theories associated to more general quiver diagrams is currently under investigation \cite{12}. It would be interesting to find out whether the existence of meta-stable vacua with broken supersymmetry is a generic phenomenon for this class of gauge theories realized as low energy limits of string theory.

Study of meta-stable vacua with broken supersymmetry may provide a new insight into the flux compactification of string theory of the type pioneered in \cite{13,14}. In this connection, it would also be interesting to study properties of the meta-stable supersymmetry breaking vacua of the gauge theories from the geometric point of view of string theory. This has been attempted earlier for models with stable supersymmetry breaking vacua, for example in \cite{15} using the M theory fivebrane description of $\mathcal{N} = 1$ supersymmetric gauge theories \cite{16,17,18}. It would be worth revisiting this issue from the new perspective that is emerging.

2. $U(N_1) \times U(N_2)$ Gauge Theories

In this section, we consider the $U(N_1) \times U(N_2)$ quiver gauge theory as described in the introduction. The chiral multiplets of the theory are $X_1$ and $X_2$ which are adjoint in $U(N_1)$ and $U(N_2)$ respectively and $Q_{12}$ and $Q_{21}$ which are bi-fundamental in $U(N_1) \times U(N_2)$. In the $\mathcal{N} = 1$ language, the total superpotential is given by

$$W = W_1(X_1) + W_2(X_2) + \text{tr} \ Q_{21} X_1 Q_{12} + \text{tr} \ Q_{12} X_2 Q_{21}. \quad (2.1)$$

The last two terms are inherited from the $\mathcal{N} = 2$ quiver theory. The superpotentials $W_1(X_1)$ and $W_2(X_2)$ reduce the supersymmetry to $\mathcal{N} = 1$. We assume that $2N_1 - N_2 > 0$ and $2N_2 - N_1 > 0$ so that the model is asymptotically free. The strong coupling scales for $U(N_1)$ and $U(N_2)$ are denoted by $\Lambda_1$ and $\Lambda_2$ respectively.

2.1. Magnetic dual

Suppose $\Lambda_1 \gg \Lambda_2$. When $N_1 < N_2$, it is well-known that the model has a magnetic dual with the gauge group $U(N_2 - N_1) \times U(N_2)$. Geometrically, this duality is the Weyl reflection symmetry of the $A_2$ Dynkin diagram relating inequivalent blow-ups of the $A_2$ singularity \cite{4}.

\footnote{This is closely related to the duality cascade of \cite{13}, which corresponds to the affine $\hat{A}_1$ case.}
Here, we will consider another region $N_2 < \frac{2}{3} N_1$. We choose the superpotential terms to be

$$W_1(X) = t_0 \, \text{tr} \left( \frac{1}{3} X^3 - t_1^2 X \right), \quad W_2(X) = 0,$$

where $t_0$ and $t_1$ are constant parameters. As we mentioned in Footnote 2 in the introduction, we choose the highest power in $W_1(X_1)$ to be cubic. For simplicity, we set $W_2(X_2) = 0$, but it is straightforward to consider the case when $W_2$ is a general polynomial since we assume the $U(N_2)$ gauge sector is weakly coupled. A quadratic term is not included in $W_1(X_1)$ since it can be removed by a combined shift of $X_1 \to X_1 + c \, \mathbb{1}$ and $X_2 \to X_2 - c \, \mathbb{1}$ for some constant $c$. (We need to shift $X_2$ simultaneously in order to keep the last two terms in (2.1) unchanged.)

To identify the dual description, we first look at the theory at the scale $E$ where $\Lambda_2 \ll E$. There, the $U(N_2)$ gauge sector of theory is weakly coupled and can be treated as a spectator. On the other hand, the $U(N_1)$ gauge sector can be strongly coupled. This sector consists of the $U(N_1)$ gauge field coupled to $N_2$ fundamental fields $(Q_{12}, Q_{21})$ and the adjoint field $X_1$ with the superpotential $W_1(X_1)$. The weakly coupled magnetic dual of this model has been identified in [7,8,9,10] and consists of the gauge field for $U(2N_2 - N_1)$, $N_2$ fundamental fields $(q_{12}, q_{21})$, one adjoint field $Y$, and two neutral fields $M, M'$ which are in the adjoint representation of $U(N_2)$. The superpotential for the dual theory is

$$\tilde{W} = -t_0 \, \text{tr} \left( \frac{1}{3} Y^3 - t_1^2 Y \right) + \frac{t_0}{\mu_0} \, \text{tr} (M q_{21} q_{12} + M' q_{21} Y q_{12}) + \text{tr} (M + X_2 M'),$$

where $\mu_0$ relates the strong coupling scale $\Lambda_1$ of the electric gauge group $U(N_1)$ to the scale $\Lambda_1$ of its magnetic dual $U(2N_2 - N_1)$ as

$$\Lambda_1^{2N_1 - N_2} \Lambda_1^{3N_2 - 2N_1} = \left( \frac{\mu_0}{t_0} \right)^{2N_2}.$$

Since the dual theory has one adjoint field $Y$ and since $(q_{12}, q_{21})$ can be regarded as $N_2$ fundamental fields with respect to the $U(2N_2 - N_1)$, the coefficient of the beta-function is given by

$$b_{U(2N_2 - N_1)} = 3(2N_2 - N_1) - N_2 - (2N_2 - N_1) = 3N_2 - 2N_1,$$

\hspace{1cm} \text{Footnote 4: In particular, the matter content of the dual theory is independent of } W_2(X_2) \text{ but depends on } W_1(X_1).$
and it becomes negative when \( N_2 < \frac{2}{3}N_1 \). At the same time, since there are three adjoint fields \( M, M', X_2 \) and since \((q_{12}, q_{21})\) count as \((2N_2 - N_1)\) fundamental fields for \( U(N_2) \), we have

\[
b_{U(N_2)} = 3N_2 - (2N_2 - N_1) - 3N_2 = N_1 - 2N_2.\]

Thus, both \( U(2N_2 - N_1) \) and \( U(N_2) \) gauge couplings are infrared free when

\[
\frac{1}{2}N_1 < N_2 < \frac{2}{3}N_1.
\]

Moreover, in the magnetic dual, we land on the weak coupling regime of the \( U(N_2) \) gauge group. To see this, we note that the scale \( \tilde{\Lambda}_2 \) of this gauge group is related to its original scale \( \Lambda_2 \) in the electric description by the matching condition,

\[
\left( \frac{\Lambda_2}{E} \right)^{2N_2-N_1} = \left( \frac{\tilde{\Lambda}_2}{E} \right)^{-(2N_2-N_1)},
\]

namely,

\[
E = \sqrt{\Lambda_2 \tilde{\Lambda}_2}.
\]

Thus, when we start with the weak coupling regime \( E \gg \Lambda_2 \) in the asymptotic free electric description, we land on the the weak coupling regime \( E \ll \tilde{\Lambda}_2 \), well below the Landau pole in the infrared free magnetic description.

One may consider generalizing this construction by allowing higher powers of \( X_1 \) in the superpotential as

\[
W_1(X_1) = t_0 \text{ tr} X_1^k + \cdots.
\]

The Kutasov-type dual has the gauge group \( U((k-1)N_2-N_1) \times U(N_2) \), and the coefficients of the beta functions are

\[
b_{U((k-1)N_2-N_1)} = (2k-3)N_2 - 2N_1,
\]

\[
b_{U(N_2)} = N_1 - (k-1)N_2.
\]

If we require that the electric description is asymptotic free, in particular \( 2N_2 > N_1 \), we have \( b_{U((k-1)N_2-N_1)} > 0 \) for \( k > 3 \). Thus, the case with \( k = 3 \) is special in the sense that the electric description is ultraviolet complete and the magnetic description is infrared free.

Since the dual theory is in the infrared free range for \( k = 3 \), the Kähler potential is regular around the origin of the field space and can be expanded as

\[
K = \left( \frac{1}{\alpha \Lambda_1^2} \right)^2 M^\dagger M + \left( \frac{1}{\alpha' \Lambda_1} \right)^2 M'^\dagger M' + \frac{1}{\beta^2} (q^\dagger q + \bar{q}^\dagger \bar{q}) + \frac{1}{\gamma^2} Y^\dagger Y + \frac{1}{\gamma' r^2} X_2^\dagger X_2,
\]
for some unknown coefficients $\alpha, \alpha', \beta, \gamma, \gamma'$. It is reasonable to assume that these coefficients are regular for some range of the parameters, $t_0, t_1, \mu_0, \text{etc.}$ Trace symbols are implicit in the above. Since $M$ and $M'$ are identified with $Q_{12}X_1Q_{21}$ and $Q_{21}Q_{12}$ in the electric description, their dimensions are not equal to 1, which is why their kinetic terms are divided by the electric scale $\Lambda_1$. We will find it useful to rescale the fields, e.g. as $M \rightarrow \alpha \Lambda_1^2 M$, so that their coefficients in the Kähler potential are normalized to be 1. The superpotential (2.3) after this rescaling becomes

$$\tilde{W} = -t_0 \text{tr} \left( \frac{1}{3} \gamma^3 Y^3 - t_1^2 \gamma Y \right) + h \text{ tr} \left[ M (q_{21} q_{12} - \mu^2 1) \right] + h' \text{ tr} \left[ M' (q_{21} \gamma Y q_{12} - \mu'^2 X_2) \right],$$

(2.4)

where

$$h = \alpha \beta^2 t_0 \frac{\Lambda_1^2}{\mu_0^2}, \quad h' = \alpha' \beta^2 t_0 \frac{\Lambda_1}{\mu_0^2}, \quad \mu^2 = -\frac{1}{\beta^2} \frac{\mu_0^2}{t_0}, \quad \mu'^2 = -\frac{\gamma'}{\beta^2} \frac{\mu_0^2}{t_0}.$$

In the following, we will analyze the vacuum structure of the gauge theory using this dual description.

It is instructive to compare the above construction with that for $N_2 > N_1$. In this case, the dual $U(2N_2 - N_1)$ gauge sector is asymptotically free. Moreover, since $2N_2 - N_1 > N_2$, the $F$-term constraints require partial Higgsing of the gauge symmetry, and we end up with a magnetic dual with the $\mathcal{N} = 1$ quiver theory with the gauge group $U(N_2 - N_1) \times U(N_2)$, resulting in the cascade structure as shown in [4]. In contrast, in our case when $N_2 < \frac{2}{3}N_1$, the rank condition discussed below makes it impossible to solve the $F$-term constraints, leading to supersymmetry breaking.

2.2. Tree-level potential

The $F$-term conditions for the dual theory are

$$q_{21} q_{12} - \mu^2 1_{N_2} = 0,$$

$$q_{21} \gamma Y q_{12} - \mu'^2 X_2 = 0,$$

$$q_{12} M = M q_{21} = 0,$$

$$\gamma^2 Y^2 - t_1^2 1_{2N_2 - N_1} = 0,$$

$$M' = 0.$$

(2.5)
Note that \( q_{12} \) is a matrix of size \((2N_2 - N_1) \times N_2\). Since \(2N_2 - N_1 < N_2\), the rank of \( q_{12} \) is \((2N_1 - N_2)\) at most, and the first equation can never be satisfied. This is the rank condition mechanism of [1]. Thus, the supersymmetry is broken at the tree level in this description. What we need to check is whether there are field configurations that are locally stable.

The \( F \)-term potential contains a term proportional to \( \text{tr} \left| q_{21} q_{12} - \mu^2 \mathbf{1}_{N_2} \right|^2 \). This can be minimized by setting

\[
q_{21} = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad q_{12} = \begin{pmatrix} \tilde{\varphi}_0 & 0 \end{pmatrix}
\]

where \( \varphi_0 \) and \( \tilde{\varphi}_0 \) are \((2N_2 - N_1) \times (2N_2 - N_1)\) matrices satisfying

\[
\tilde{\varphi}_0 \varphi_0 = \mu^2 \mathbf{1}_{2N_2-N_1}.
\]

For this configuration,

\[
\text{tr} \left| \frac{\partial W}{\partial M} \right|^2 = |h|^2 \text{tr} \left| q_{21} q_{12} - \mu^2 \mathbf{1}_{N_2} \right|^2 = (N_1 - N_2) |h\mu|^2.
\]

As we will see below, the remaining \( F \)-term conditions and all the \( D \)-term conditions can be solved for this choice of \((q_{12}, q_{21})\). Moreover, the minimum value of \( \text{tr} \left| \partial W / \partial M \right|^2 \) depends only on the parameters of the model and not on the field variables, and cannot be minimized further. Thus, this gives the minimum value of the tree-level potential,

\[
V_{\text{min}}^{(\text{tree})} = (N_1 - N_2) |h\mu|^2 = (N_1 - N_2) |\alpha \Lambda_1^2|^2.
\]

For this choice of \((q_{12}, q_{21})\), the rest of the \( F \)-term conditions in (2.5) can be solved by setting

\[
M = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad M' = 0, \quad X_2 = \frac{1}{\mu^2} \tilde{\varphi}_0 \gamma Y \varphi_0, \quad \gamma^2 Y^2 = t_1^2 \mathbf{1}_{2N_2-N_1}
\]

where \( \Phi_0 \) is an arbitrary \((N_1 - N_2) \times (N_1 - N_2)\) matrix.

Let us turn to the \( D \)-term conditions. It is well-known that, for a supersymmetric theory with gauge group \( G \), any field configuration solving the \( F \)-term conditions can be mapped by a complexified gauge transformation \( G_c \) to a unique solution to the \( D \)-term constraints modulo the \( G \) gauge transformation [20]. In our case, since \( Y \) satisfies the last equation in (2.9), we can use \( GL(2N_2 - N_1, C) \) transformation to diagonalize it in the form,

\[
\gamma Y = \text{diag}(t_1, \cdots, t_1, -t_1, \cdots, -t_1).
\]
This configuration breaks $U(2N_2 - N_1)$ into $U(r_1) \times U(r_2)$ where $r_1 + r_2 = 2N_2 - N_1$ and $r_2$ is the number of minus signs in the above. We can also use $GL(2N_2 - N_1, C) \subset GL(N_2, C)$ to transform a solution to (2.7) into

$$\phi_0 = \tilde{\phi}_0 = \mu \mathbf{1}_{2N_2 - N_1}. \quad (2.11)$$

Finally, $\Phi_0$ in (2.9) can be diagonalized by the remaining gauge symmetry $GL(N_1 - N_2, C) \subset GL(N_2, C)$. It is straightforward to verify that the resulting field configuration solves the $D$-term constraints.

To summarize, we found that the tree-level $D$ and $F$-term potential is minimized by (2.6) and (2.9) where $Y$ and $\phi_0, \tilde{\phi}_0$ are fixed as in (2.10) and (2.11) and $\Phi_0$ is also diagonalized. The only flat directions that are not fixed by the tree-level potential are the eigenvalues of $\Phi_0$. In the following, we will compute a one-loop effective potential for $\Phi_0$ to show that all its eigenvalues are stabilized at one-loop.

2.3. One-loop effective potential

We are going to compute the one-loop effective potential for $\Phi_0$, which is the only massless chiral multiplet at the tree-level. Let us parametrize the fluctuations around one of the tree-level vacua as,

$$q_{12} = \mu \begin{pmatrix} 1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad q_{21} = \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \sigma_2 \\ \phi_2 \end{pmatrix},$$

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix} + \begin{pmatrix} \sigma_3 & \phi_4 \\ \phi_3 & \sigma_4 \end{pmatrix}, \quad M' = \begin{pmatrix} \sigma_5 & \phi_6 \\ \phi_5 & \sigma_6 \end{pmatrix}, \quad X_2 = \frac{\mu^2}{\mu'} \begin{pmatrix} \gamma Y_0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \sigma_7 & \phi_8 \\ \phi_7 & \sigma_8 \end{pmatrix}, \quad Y = Y_0 + \sigma_9,$$  

and expand the action to the quadratic order in $\phi$’s and $\sigma$’s. Here $Y_0$ is the vacuum value of $Y$ given by (2.10). We then perform the Gaussian integral for $\phi$’s and $\sigma$’s to compute the one-loop effective potential for $\Phi_0$. Since the vacuum configuration satisfies the $D$-term condition, one-loop contributions from the vector multiplets are canceled, even though some of their masses depend on $\Phi_0$. Since one-loop diagrams are planar, the effective potential $V_{\text{eff}}^{(1)}(\Phi_0)$ for $\Phi_0$ should be expressed in terms of a single trace. The quadratic term in the expansion of $V_{\text{eff}}^{(1)}$ around $\Phi_0 = 0$ should then be of the form $V_{\text{eff}}^{(1)} \sim \text{tr} \Phi_0^\dagger \Phi_0$ by the $U(N_2)$ symmetry. This means that all the eigenvalues of $\Phi_0$ get the same mass at one-loop. Without a loss of generality, we can set

$$\Phi_0 = X \mathbf{1}_{N_1 - N_2}.$$

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We can then expand $V_{\text{eff}}^{(1)}(X)$ in powers of $X$ and look at the quadratic term to find the mass for $\Phi_0$.

The one-loop effective potential for $X$ is then given by

$$V_{\text{eff}}^{(1)}(X) = \frac{1}{64\pi^2}S\text{Tr}M^4 \log \frac{M^2}{\Lambda^2} \equiv \frac{1}{64\pi^2} \sum \left( m_B^4 \log \frac{m_B^2}{\Lambda^2} - m_F^4 \log \frac{m_F^2}{\Lambda^2} \right),$$

where $\Lambda$ is the ultraviolet cutoff parameter, and $m_B$ and $m_F$ are the masses for the bosons and fermions that are given by expanding the $D$ and $F$-term potentials in the quadratic order in $\phi$’s and $\sigma$’s. The effective potential is a function of $X$ because the masses depend on it. The $D$-term potential for the $U(2N_2 - N_1)$ gauge symmetry does not contain fields that are directly coupled to the supersymmetry breaking sector, and therefore it does not contribute to the effective potential. The $D$-term potential for the $U(N_2)$ gauge symmetry contains such fields, but their effects are suppressed since the gauge coupling for the $U(N_2)$ is weak in this energy scale. Thus, we only consider the $F$-term potential to compute the one-loop effective potential.

The mass matrices we use to evaluate (2.13) are therefore given in terms of the superpotential $W$ as

$$m_B^2 = \begin{pmatrix} W_{\bar{a}c}^{\bar{a}b}W_{\bar{b}c}^{\bar{a}c} & W_{\bar{a}c}^{\bar{a}b}W_{\bar{b}c}^{\bar{c}b} \\ W_{\bar{a}c}W_{\bar{b}c}^{\bar{a}c} & W_{\bar{a}c}W_{\bar{b}c}^{\bar{c}b} \end{pmatrix}, \quad m_F^2 = \begin{pmatrix} W_{\bar{a}c}^{\bar{a}b}W_{\bar{b}c}^{\bar{a}c} & 0 \\ 0 & W_{\bar{a}c}W_{\bar{b}c}^{\bar{a}c} \end{pmatrix},$$

where $a, b, c, \ldots$ represent the fluctuations $\phi$’s and $\sigma$’s in (2.12), and the derivatives of $W$ are evaluated at the tree-level vacuum configuration. If $W_{\bar{a}c}W_{\bar{b}c}^{\bar{a}c} = 0$, the supertrace in (2.13) vanishes by cancellation between bosons and fermions contributions. Since the only field $c$ with $W_c \neq 0$ is $c = \sigma_4$ and since $\sigma$’s do not have non-zero $W_{\bar{a}c}$ with $\sigma_4$, only the $\phi$ fluctuations contribute to the one-loop effective potential.

Each $\phi$ in the above is an $(2N_2 - N_1) \times (N_1 - N_2)$ matrix. From this, it follows that the classical action for the quadratic fluctuations is a sum of $(2N_2 - N_1)(N_1 - N_2)$ copies of the O’Raifeartigh-type model with the superpotential

$$W = hX (\phi_1 \phi_2 - \mu^2) + h\mu (\phi_1 \phi_3 + \phi_2 \phi_4) + h'\mu \gamma Y (\phi_1 \phi_6 + \phi_2 \phi_5) - h'\mu' \gamma Y (\phi_6 \phi_7 + \phi_5 \phi_8).$$

Here $\gamma Y = +t_1$ for $r_1(N_1 - N_2)$ of them, and $\gamma Y = -t_1$ for the rest.

We can now evaluate the one-loop effective potential (2.13) and expand it in powers of $X$. It turns out that the potential is independent of $(r_1, r_2)$. Thus, both the vacuum
energy and the mass for $X$ are the same for all the vacua. The constant term in $V^{(1)}_{\text{eff}}$ gives the one-loop correction to the vacuum energy. Combining it the tree level result (2.8), we find that the vacuum energy is given by

$$V_{\text{vac}} = (N_1 - N_2) |h^2 \mu^4| + (N_1 - N_2)(2N_2 - N_1) \frac{|h^4 \mu^4|}{32\pi^2} \left[ 2 \log \left( |h^2 \mu^2| \Lambda^{-2} \right) + (2u + 2v)^2 \log(2u + 2v) + F_+(u, v)^2 \log F_+(u, v) + F_-(u, v)^2 \log F_-(u, v) - 2F_+(2u, 2v)^2 \log F_+(2u, 2v) - 2F_-(2u, 2v)^2 \log F_-(2u, 2v) \right],$$

where we set

$$u = \left| \frac{h^2 t_1^2}{2h^2} \right| = \frac{1}{2} \frac{\alpha'^2}{\alpha^2} \left| \frac{t_1^2}{\Lambda_1^2} \right|, \quad v = \left| \frac{h^2 \mu^4}{2h^2 \mu^2} \right| = \frac{1}{2} \frac{\alpha'^2 \gamma'^2}{\alpha^2 \beta^2} \left| \frac{\mu_0^2}{t_0 \Lambda_1^2} \right|,$$

and the function $F_\pm$ of $(u, v)$ are given by

$$F_\pm(u, v) = 1 + u + v \pm \sqrt{(1 + u + v)^2 - 4v}.$$

We note that the one-loop correction to the vacuum energy is independent of the gauge symmetry breaking pattern parametrized by $(r_1, r_2)$.

The mass squared, $m_X^2$, can be expressed analytically as

$$m_X^2 = (N_1 - N_2)(2N_2 - N_1) \frac{|h^4 \mu^2|}{16\pi^2} G(u, v),$$

for some function $G(u, v)$, where $(u, v)$ are defined in (2.17). In particular, the log $\Lambda$ terms in (2.13) are canceled out in $m_X^2$. The expression for $G$ is too lengthy to reproduce here. Its behavior for $0 \leq u, v \leq 1$ is displayed in Figures 1 and 2. Clearly in this range the function $G(u, v)$ stays positive, and the $X$ direction is stabilized at one-loop. We checked numerically that $G(u, v) > 0$ for a much larger range of $u$ and $v$. In the limit $u \to \infty$, $G(u, v) \to 0$. Thus, the one-loop effective potential for $X$ becomes asymptotically shallow for large $t_1$.

We observe that the dependence of $m_X^2$ on $v$ is relatively mild. In particular, its behavior for $u \to 0$ is independent of $v$ as

$$G(u, v) = 4(\log 4 - 1) + 16(\log 2 - 1)u + O(u^2).$$
We can also see this graphically in Figure 2. The leading behavior of $m_X^2$ for $u \to 0$ coincides with that of the supersymmetric QCD with flavors evaluated in [1].

![Graph of G(u,v) for 0 ≤ u, v ≤ 1](image1)

**Fig.1** $G(u,v)$ for $0 \leq u, v \leq 1$

![Graph of G(u,v) as a function of u for v = 0.001, 0.5 and 1](image2)

**Fig.2** $G(u,v)$ shown as a function of $u$ for $v = 0.001, 0.5$ and $1$.

To conclude, we have shown that the remaining flat directions parametrized by the eigenvalues of $\Phi_0$ are all lifted by the one-loop effective potential. It is worth mentioning that not all supersymmetric gauge theories have meta-stable supersymmetry breaking vacua of this type. Suppose, for example, that the $U(N_2)$ symmetry were not gauged. Then, we would not have the $D$-term constraint for this gauge symmetry, which is needed to make some of the fields massive at the tree level. One can see that, in such a model, extra flat directions emerge at the origin of the $\Phi_0$ space, and they may cause the runaway behavior. A similar problem happens to the stability of the vacua if we do not have the adjoint field $X_2$ for this group in the electric description even if the $U(N_2)$ is gauged. It is interesting that the geometric construction by string theory produces exactly the right
combination of field content and gauge symmetry so that the supersymmetry breaking configurations are locally stable in all directions.

2.4. Gaugino masses and the low energy limit

We found that the chiral multiplet $\Phi_0$ gains mass $m_X$ at the one-loop level. Thus, all the moduli around the meta-stable vacua are stabilized. However, this is not the end of the story; there is unbroken gauge symmetry $G = U(r_1) \times U(r_2) \times U(N_1 - N_2)$ with $r_1 + r_2 = 2N_2 - N_1$ at each of the vacua. We then need to find out the fate of the vector multiplet for the unbroken gauge symmetry.

Because of the term $h \text{tr} M(q_{21}q_{12} - \mu^2 1)$ in the superpotential $\widetilde{W}$, the $F$-term for the superfield $M$ is non-vanishing at these vacua as

$$F_M = \frac{\partial W}{\partial M} = h(q_{21}q_{12} - \mu^2 1_{N_2}) = -h\mu^2 \begin{pmatrix} 0 & 0 \\ 0 & 1_{N_1-N_2} \end{pmatrix},$$

where we used the vacuum values of $q_{12}$ and $q_{21}$ given in (2.6) and (2.7). Thus, this superpotential term gives rise to the so-called soft-breaking $B$-term for the bosonic components of $(q_{12}, q_{21})$ of the form,

$$\mathcal{L}_{B-term} = h\mu^2 \text{tr} \left[ \begin{pmatrix} 0 & 0 \\ 0 & 1_{N_1-N_2} \end{pmatrix} q_{21}q_{12} \right] + \text{c.c.}, \quad (2.18)$$

where $q_{12}, q_{21}$ refer to the bosonic components of the superfield. This is a part of the off-diagonal element in the mass matrix $m^2_B$ in (2.14). Moreover, unless both $N_1$ and $N_2$ are even, the $R$ symmetry becomes trivial. Thus, we expect that gaugino masses are generated by radiative corrections [21, 22, 23] for the entire gauge group $G$ except for the diagonal $U(1)$, for which $q_{12}, q_{21}$ are neutral.

The massive gauginos decouple in the infrared, and we are then left with the bosonic pure Yang-Mills theory for the gauge group $G' = G/U(1)$ and the free $U(1)$ supersymmetric Yang-Mills theory. Since the original electrical description of the theory contains no field charged with respect to the diagonal $U(1)$, this abelian Yang-Mills theory is decoupled from the beginning of the story and we can ignore it. Thus, the low energy limit is the

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5 To see this, we note that the superpotential given by (2.1) and (2.2) breaks the $U(1)$ $R$ symmetry to $Z_4$. On the other hand, the chiral anomaly breaks it to $Z_{2n}$ where $n$ is the largest common divisor of $(2N_1 - N_2)$ and $(2N_2 - N1)$. In particular, $n$ is odd unless both $N_1$ and $N_2$ are even. In this case, the intersection of $Z_4$ and $Z_{2n}$ is $Z_2$, which is the fermion number parity.
bosonic pure Yang-Mills theory for gauge group $G'$, and the supersymmetry is broken. The strong coupling scale of the bosonic Yang-Mills theory is determined by the matching condition at the supersymmetry breaking scale. This sets the lowest energy scale of the model, and the lowest excitations of the model around the meta-stable vacua are glueballs of the Yang-Mills theory.

2.5. Supersymmetric vacua and decay rates of meta-stable vacua

In addition to the meta-stable vacua with broken supersymmetry that we have looked at so far, the model also has supersymmetric vacua as one can see in the original electric description [3]. Here, we will show that these supersymmetric vacua can also be found in the dual description within the region of its validity, establishing the connection of the two description. We will use this result to estimate the decay rates of the supersymmetry breaking vacua into the supersymmetric vacua.

Following [1], we look for the supersymmetric vacua where $M$ is large. There $(q_{12}, q_{21})$ are heavy and can be integrated out. Thus, we are left with the superpotential,

$$
\widetilde{W} = t_0 \text{tr} \left( -\frac{1}{3} \gamma^3 Y^3 + t_1^2 \gamma Y \right) - h \mu^2 \text{tr} M - h' \mu'^2 \text{tr} M' X_2.
$$

The new scale $\Lambda_{\text{susy}}$ for the $U(2N_2 - N_1)$ gauge group after decoupling $(q_{12}, q_{21})$ is given by the matching condition at the mass scale $hM$ as

$$
(hM)^{N_2} \tilde{\Lambda}_1^{3N_2 - 2N_1} = \Lambda_{\text{susy}}^{2(2N_2 - N_1)}.
$$

On the other hand, the coupling constant $g_2$ for $U(N_2)$ stops running at the energy $hM$ since those charged with respect to this gauge group are $M, M'$ and $X_3$ and they make the same matter content as that for the $\mathcal{N} = 4$ supersymmetric gauge theory albeit with the different superpotential. The coupling constant $g_2$ is small since we assume $hM \ll \tilde{\Lambda}_2$ so that we stay well below the Landau pole. Thus, we can integrate out $M'$ and $X_2$ as they are free in the limit of small $g_2$.

The vacuum expectation value (2.10) of $Y$ breaks the gauge symmetry as

$$
U(2N_2 - N_1) \rightarrow U(r_1) \times U(r_2)
$$
with $r_1 + r_2 = 2N_2 - N_1$. The strong coupling scales $\Lambda_{U(r_a)}$ for $U(r_a)$ ($a = 1, 2$) are determined by the matching condition at the threshold as

$$\Lambda^3_{U(r_1)} = \gamma^3 t_0 \left( \frac{t_1}{\gamma} \right)^{r_1} \Lambda_{\text{susy}}^{\frac{r_2 - 2r_1}{r_1}} \Lambda^2_{U(r_1)} = \gamma^3 t_0 \left( \frac{t_1}{\gamma} \right)^{r_1} \tilde{\Lambda}_1^{\frac{3N_2 - 2N_1}{r_1}} (hM)^{\frac{N_2}{r_1}},$$

$$\Lambda^3_{U(r_2)} = \gamma^3 t_0 \left( \frac{t_1}{\gamma} \right)^{r_2} \Lambda_{\text{susy}}^{\frac{r_2 - 2r_1}{r_2}} = \gamma^3 t_0 \left( \frac{t_1}{\gamma} \right)^{r_2} \tilde{\Lambda}_1^{\frac{3N_2 - 2N_1}{r_2}} (hM)^{\frac{N_2}{r_2}}.$$

The effective superpotential $W_{\text{eff}}$ for $M$ would then be a function of these scales of the form:

$$W_{\text{eff}} = -h \mu^2 M + W_{\text{nonpert}}(\Lambda_{U(r_1)}, \Lambda_{U(r_2)}).$$

For fixed $t_0, t_1$ and $\tilde{\Lambda}_1$, the second term in the above is a non-trivial function of $M$. If it is sufficiently generic, there are solutions to the $F$-term condition,

$$\frac{\partial W_{\text{eff}}}{\partial M} = -h \mu^2 + \frac{\partial W_{\text{nonpert}}}{\partial M} = 0. \tag{2.21}$$

Thus, the non-perturbative $U(2N_2 - N_1)$ gauge dynamics can restore the supersymmetry for large $M$.

To make quantitative estimate of $W_{\text{nonpert}}$, let us assume that

$$\Lambda_{\text{susy}} \ll \frac{t_1}{\gamma}. \tag{2.22}$$

In this limit, fields that are bifundamental in $U(r_1) \otimes U(r_2)$ become heavier than the gauge theory scale and decouple. Thus, we can treat the two gauge group factors separately and estimate the effective superpotential as

$$W_{\text{nonpert}} \approx \sum_{a=1,2} r_a \Lambda^3_{U(r_a)} + O \left( \Lambda^6_{U(r_a)} \frac{\gamma^3}{t_1^3} \right). \tag{2.23}$$

Without loss of generality, we can assume $r_1 \leq r_2$. If $r_1 < r_2$, we have

$$\frac{\Lambda^3_{U(r_1)}}{\Lambda^3_{U(r_2)}} = \left( \frac{t_1}{\gamma \Lambda_{\text{susy}}} \right)^{\frac{2(r_2^2 - r_1^2)}{r_2^3}} \ll 1,$$

where we used (2.22). In this case, we can ignore $\Lambda_{U(r_1)}$ in the effective superpotential (2.23). On the other hand, $\Lambda_{U(r_1)} = \Lambda_{U(r_2)}$ for $r_1 = r_2$. Thus, in either case,

$$W_{\text{nonpert}} \approx r_2 \Lambda^3_{U(r_2)} = r_2 \gamma^3 t_0 \left( \frac{t_1}{\gamma} \right)^{\frac{r_2 - 2r_1}{r_2}} \Lambda_{\text{susy}}^{\frac{3N_2 - 2N_1}{r_2}} \tilde{\Lambda}_1^{\frac{3N_2 - 2N_1}{r_2}} (hM)^{\frac{N_2}{r_2}}.$$
With this non-perturbative term, the supersymmetry condition indeed has solutions of the form
\[ hM = \zeta^{2r_2} \eta^{2r_1 - r_2} \tilde{\Lambda}_1 1_{N_2}, \]
where
\[ \zeta^2 = \frac{1}{N_2 \gamma^3 t_0} \left( \frac{\mu}{\tilde{\Lambda}_1} \right)^2, \quad \eta = \frac{t_1}{\gamma \tilde{\Lambda}_1}. \]

Let us discuss the region of validity of our estimate. In order for our analysis to be consistent, we require the relevant particle masses, \( hM \) and \( t_1/\gamma \), are below the Landau pole at \( \tilde{\Lambda}_1 \). This means \( \eta \ll 1 \) and
\[ \zeta^{2r_2} \eta^{2r_1 - r_2} \ll 1. \tag{2.24} \]
If we require that this is satisfied for all the gauge symmetry breaking pattern, the strongest constraint from (2.24) comes from the case when \( r_1 = 0, r_2 = 2N_2 - N_1 \) (to be precise, this is allowed when \( N_2 \) is even). Thus, we have,
\[ \zeta \ll \eta^{\frac{1}{2}} \ll 1. \]

In addition, we require (2.22) so that we can use the explicit expression (2.23) for the non-perturbative effective potential. This condition can be expressed in terms of \( \zeta \) and \( \eta \) as,
\[ \zeta \ll \eta^{\frac{N_1}{N_2} - \frac{1}{2}}. \tag{2.25} \]
Since \( N_1 > N_2 \), this condition is stronger than \( \zeta \ll \eta^{\frac{1}{2}} \). Thus, all the inequalities are satisfied when
\[ \zeta \ll \eta^{\frac{N_1}{N_2} - \frac{1}{2}} \ll 1. \tag{2.26} \]

We can now estimate the decay rate of each meta-stable vacuum following \[1\]. At the semi-classical level, the decay probability is proportional to \( e^{-S} \) where \( S \) is the Euclidean action for the decay process. Using the formula in \[24\], we find
\[ S \sim \frac{(\Delta \Phi)^4}{V_+} = \frac{(\text{tr} hM)^4}{(N_1 - N_2) h^2 \mu^4} = \frac{N_2^2}{(N_1 - N_2) h^2 t_0^2 \gamma^6} \left( \frac{\eta^{2r_1 - r_2}}{\zeta^{N_2 - 3r_2}} \right)^4. \tag{2.27} \]
where $\Delta \Phi$ is the order of the difference of the vacuum expectations values at the meta-stable vacua and at the supersymmetric vacua, and $V_+ = (N_1 - N_2)|h^2 \mu^4|$. The meta-stable vacua are long-lived if $S \gg 1$, which means

$$\eta^{2r_1-r_2} \gg \zeta^{N_2-3r_2}. \quad (2.28)$$

Requiring this for all $r_1 \leq r_2$ with $r_1 + r_2 = 2N_1 - N_2$ and combining it with (2.26), we find,

$$\eta^{\frac{2N_2-N_1}{N_2}} \ll \zeta \ll \eta^{\frac{N_1-1}{2}} \ll 1 \quad \text{(if } N_2 > \frac{3}{5} N_1)$$

$$\zeta \ll \eta^{\frac{N_1}{N_2} - \frac{1}{2}} \ll 1 \quad \text{(if } N_2 \leq \frac{3}{5} N_1).$$

These conditions also allow us to ignore higher order correction to the Kähler potential even though the non-perturbative corrections to the superpotential are included.

One may be concerned about transitions among supersymmetry breaking vacua. Since they all have the same energy, their transition probabilities are all zero.

3. $SU(N_1) \times SU(N_2)$ Gauge Theory

Let us briefly describe our result for the quiver gauge theory with the gauge group $SU(N_1) \times SU(N_2)$. In this case, we can add a quadratic term to $W_1(X_1)$ as an independent superpotential term. It cannot be removed by constant shift of $X_1$ and $X_2$ because of the tracelessness condition on them. Let us parametrize $W_1(X_1)$ as

$$W_1(X_1) = t_0 \operatorname{tr} \left( \frac{1}{3} X_1^3 + \frac{t_2}{2} X_1^2 \right),$$

with the condition $\operatorname{tr} X_1 = 0$. Assuming $\Lambda_2 \ll \Lambda_1$, we can use the magnetic dual with respect to $SU(N_1)$ identified in [25]. Repeating the analysis in section 2.2 and using the same notation, we find that the tree-level vacua in the dual description break the gauge symmetry to $SU(r_1) \times SU(r_2) \times SU(N_1 - N_2) \times U(1)^2$ with $r_1 + r_2 = 2N_2 - N_1$ by the vacuum expectation value of $Y$ as

$$\gamma Y = \text{diag}(\lambda, \cdots, \lambda, -\lambda, \cdots, -\lambda), \quad (3.1)$$

where

$$\lambda = \frac{N_1 t_2}{2(r_2 - r_1)}, \quad (3.2)$$
and \( r_2 \) is the number of minus signs in (3.1). Without a loss of generality, we assume \( r_1 < r_2 \). Note that the eigenvalues of \( \gamma Y \) depend on \((r_1, r_2)\), namely on the choice of vacuum. This is in contract to the \( U(N_1) \times U(N_2) \), where the eigenvalues are given by \( \pm t_1 \).

Another difference is that there is an extra flat direction in addition to the \( \Phi_0 \) as in (2.9). By using the complexified gauge group \( SL(2N_2 - N_1, C) \), we can diagonalize \( \varphi_0 \) and \( \tilde{\varphi}_0 \) obeying \( \tilde{\varphi}_0 \varphi_0 = \mu^2 \mathbf{1}_{2N_2 - N_1} \). However, unlike the previous case, we cannot set \( \varphi_0 = \tilde{\varphi}_0 \) since \( SL(2N_2 - N_1, C) \) does not contain the overall scaling. The best we can do with this gauge group is

\[
\varphi_0 = \mu e^{\theta} \mathbf{1}, \quad \tilde{\varphi}_0 = \mu e^{-\theta} \mathbf{1}.
\]

This \( \theta \) is the additional flat direction for the tree-level potential.

The one-loop effective potential for \( \Phi_0 \) and \( \theta \) can be evaluated in the same way as in the \( U(N_1) \times U(N_2) \) model, and we have checked that both \( \Phi_0 \) and \( \theta \) are stabilized. Thus, all the flat directions are lifted at one-loop. A new feature of this model is that the one-loop effective action depends on the choice of vacuum. This follows from the fact that the vacuum expectation value of \( \gamma Y \) depends on \((r_2 - r_1)\) as in (3.2). In particular, the meta-stable vacua have different one-loop vacuum energies.

We have also estimated the lifetimes of these supersymmetry breaking meta-stable vacua. The analysis is the same as \( U(N_1) \times U(N_2) \) case with the substitution \( t_1 \rightarrow \lambda \) since the \( U(1) \) factors in \( U(N_1) \) does not affect the non-perturbative superpotential. Thus we conclude that the decay rates of the meta-stable vacua into the supersymmetric vacua can be made parametrically small in this model also. In this case, there may be non-zero transition probabilities between supersymmetry breaking vacua.

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