Two-Particle Separation Energies in the Superdeformed Well

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Abstract. The location of nuclear closed shells, as evidenced through discontinuities in binding energy and one- and two-particle separation energy systematics, remains one of the simplest tests of global nuclear models. How shell gaps evolve, whether with increasing mass, increasing neutron:proton ratio or increasing deformation, is still uncertain, and it has recently been suggested that one must go beyond a static meanfield picture to include the effects of dynamic fluctuations in the nuclear shape even in the ground state. The identification of key properties which may distinguish between competing approaches is thus vital. Comparison of the binding energies of superdeformed nuclei in the $A \approx 190$ region shows that two-proton separation energies are higher in the superdeformed state than in the normal state, despite the probably lower Coulomb barrier and lower total binding energy. Possible reasons for this difference are discussed. This somewhat counterintuitive result provides a critical test for global nuclear models.

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1. Introduction

The experimental identification of magic nucleon numbers has played an extremely important role in the development of nuclear models. One of the clearest signatures of a closed shell is a discontinuity in the particle separation energy, which decreases smoothly with increasing \( N(Z) \) along a particular isotope chain until a shell closure is reached, where a more abrupt decrease occurs. This is illustrated in Fig. 1, which shows the effect of moving beyond the \( N = 126 \) shell closure on the two-neutron separation energies in isotopes of Hg, Pb and Po [1] and the effect of moving beyond the \( Z = 82 \) shell closure in isotones with \( N = 124, 126, 128 \) on the two-proton separation energies.

![Figure 1. Two-neutron separation energies in isotopes of Hg, Pb and Po [1]](image)

It was the need to reproduce the observed shell closures in medium-mass and heavy nuclei which led to the introduction of the strong, inverted spin-orbit interaction in 1949 [2, 3]. More recently, there has been a great deal of interest in the evolution of shell structure with changing neutron:proton ratio (see, for example, [4, 5, 6]), with emerging interest in the roles of particular components of the nuclear force such as the tensor force [7] and spin-orbit interaction [8]. It has also recently been suggested that accurate reproduction of nuclear masses requires the inclusion of collective components in the ground state [9], particularly in nuclei away from stability.

A parallel emphasis has been placed on the superheavy island of stability, expected to arise due to spherical shell closures around \( Z = 114 \) or \( Z = 120 \) and \( N = 172 \) or \( N = 184 \) but with the precise location off the shell gaps highly sensitive to the properties of the nuclear mean-field (see, for example, [10]).

A third degree of freedom which results in the modification of shell structure is axial deformation exemplified by the phenomenon of superdeformation. Since the first observations of superdeformed nuclei [11], the existence of these exotic states as been explained as being due to the presence of shell closures at prolate deformations with major:minor axis ratios between 3:2 and 2:1 produced by the intrusion of orbitals originating from shells with \( N + 2 \) among orbitals with principal quantum number \( N \).
2. Location of superdeformed magic numbers

Modified harmonic oscillator calculations initially predicted superdeformed shell gaps at \( N, Z = 10, 16, 28, 40, 64, 86, 116 \) with sub-shell closures at \( N = 32, 44 \) [12]. Subsequent calculations indicated the possibility of additional deformed shell gaps including ones at \( Z = 80 \) and \( N = 110, 112 \) [13]. Experimental evidence for superdeformation has now been found in regions corresponding to almost all of these proposed magic numbers [14], including the \( A \approx 190 \) region of superdeformed nuclei with \( Z \approx 80 \) and \( N \approx 110 \).

Unfortunately, there has been little or no direct experimental evidence indicating the location and size of the superdeformed shell gaps in this or other regions. This is because the clear signatures obtained from binding energy systematics such as the two-particle separation energies shown in Fig. 1 require precise measurement of the masses of superdeformed states, which in turn rely on precise measurements of the excitation energies of those states relative to the ground state. Such measurements have proved very difficult, particularly in the “classic” regions of superdeformation centred around the supposed doubly-magic nuclei \(^{152}\text{Dy}\) and \(^{192}\text{Hg}\).

The key to a precise measurement of the excitation energies of superdeformed states is the unambiguous determination of decay sequences connecting them to known states in the normal minimum. Unfortunately, the relatively high excitation energy of the superdeformed states compared to the ground state means that the decay is distributed among a large number of pathways, and the spectrum of \( \gamma \) rays connecting the superdeformed and normal states is dominated by unresolved transitions. Only occasionally is sufficient strength concentrated in a transition directly linking a superdeformed and an yrast normal deformed state that it can be resolved.

3. Excitation energies and two-particle separation energies in the superdeformed well

Despite the experimental difficulties, a series of recent measurements has led to the observation of such “single-step” linking transitions in three even-even isotopes of Pb (\(^{192}\text{Pb}\) [15], \(^{194}\text{Pb}\) [16, 17] and \(^{196}\text{Pb}\) [18]), and two even-even isotopes of Hg (\(^{190}\text{Hg}\) [19] and \(^{194}\text{Hg}\) [20, 21]). These measurements have been possible because of the high efficiency and high granularity of multi-detector Ge arrays such as Gammasphere and Euroball, which allowed the examination of structures at the 0.01% level and the use of multiple gating requirements to isolate structures of interest. In addition, the presence of isomers in the normal deformed systems has been exploited to clarify decay paths in the Pb isotopes [15, 18].

The observation of single-step links establishes the excitation energy of the lowest observed levels in the superdeformed bands to within a few keV. Moment-of-inertia based extrapolations to lower spins provide estimates of the excitation energy of the superdeformed quasi-vacuum states to with a few tens of keV. A careful analysis of the quasicontinuum component of the decay of superdeformed \(^{192}\text{Hg}\) using a method validated on the known energy of superdeformed \(^{194}\text{Hg}\) has provided a less precise energy measurement in that nucleus [22], leading to an estimate of the quasi-vacuum excitation energy with an uncertainty of \( \pm 500 \) keV. Together, these data provide an opportunity to examine the binding energy systematics of superdeformed nuclei in the \( A \approx 190 \) region with the aim of identifying and characterising the expected superdeformed shell gaps.

Figures 2(a) and (b) compare the energies of the SD quasi-vacuum states in even-even nuclei in this region for which measurements have been made [15, 16, 17, 20, 21, 18, 22, 19]. The results of three theoretical approaches are also shown: Hartree-Fock-Bogolyubov (HFB) meanfield calculations employing state-of-the-art parameterizations of the competing Skyrme (SLy4) and Gogny (D1S) interactions [23, 24], and generator coordinate method (GCM) calculations [24] (also using the Gogny D1S interaction). The main difference between the HFB calculations and the GCM approach is that the latter takes into account long range correlations and thus allows
Figure 2. Known energies of SD quasi-vacuum states in even-even (a) Hg and (b) Pb nuclei compared with the results of HFB calculations using the SLy4 Skyrme [23] and DIS Gogny forces and GCM [24] calculations.

for the effects of quadrupole vibrational modes. Although the static meanfield calculations employing the Skyrme (Gogny) interaction reproduce the Hg (Pb) energies well, it is only the calculations including dynamic shape fluctuations that simultaneously reproduce the excitation energies in both isotope chains to within 0.5 MeV. Thus it appears that the need to include collectivity already noted for neutron-rich nuclei [9] extends to systems with large deformation.

The superdeformed bandhead energies shown in Fig. 2 depend on the properties of both true and quasi-vacuum states. These can be disentangled by examination of the two-particle separation energies for the superdeformed systems, which depend on the superdeformed quasivacuum properties alone. Figure 3 compares $S_{2n}$ and $S_{2p}$ in the normal [1] and SD wells. While the uncertainties on $S_{2n,SD}$ in the Hg isotopes remains somewhat high, the measurements obtained for the Pb isotopes are now known with sufficient precision to show that the decrease between $N = 110$ and $N = 112$ is larger than at normal deformations. This may be consistent with a superdeformed shell gap at $N = 110$; however measurements of the excitation energies of the superdeformed yrast states in either $^{190}$Pb or $^{198}$Pb would help to confirm this by showing whether the slope changes as observed in Fig. 1. As noted in [19], the HFB calculations employing the D1S interaction predict a pronounced shell closure at $N = 110$, larger than indicated by the difference in separation energies observed here. However, the inclusion of collective shape fluctuations via the GCM approach quench the predicted shell gap, resulting in an even less pronounced gap than indicated by the data.

The data shown in Fig. 3 raise another interesting question. Naively, one might expect a reduction in both $S_{2n}$ and $S_{2p}$ at superdeformation, since the binding energies per nucleon are lower and the average Coulomb barrier in the deformed system is likely to be similar to or a few percent lower than the spherical Coulomb barrier. The values of $S_{2n,SD}$ are consistently lower than $S_{2n,ND}$. In contrast, the values of $S_{2p,SD}$ are consistently larger than $S_{2p,ND}$. This behaviour presents a particular challenge to global nuclear models and also a question as to the physical origin of the high proton binding separation energies.
Figure 3. Two-neutron ($S_{2n}$) and two-proton ($S_{2p}$) separation energies as a function of neutron number. Open (filled) symbols indicate ground-state (SD) separation energies.

The two-particle separation energies depend on a variety of factors, including the strength of the pairing interaction, the overlap between the valence nucleon wavefunctions and the core, the variation in Coulomb barrier height over the nuclear surface and the neutron-proton interaction between nucleons close to the Fermi surface. Many of these factors are influenced by the character of the orbitals occupied by the least-bound nucleons.

The orbitals close to the Fermi surface at normal deformations are well-established, with the least-bound neutrons occupying orbitals of predominantly $i_{13/2}$ character, the configuration responsible for the $12^+$ isomers characteristic of moderately neutron-deficient Hg and Pb isotopes. The least-bound protons occupy orbitals of predominantly $s_{1/2}$ character in the Pb isotopes and $d_{5/2}$ character in the Hg isotopes.

Identification of occupied orbitals in the superdeformed systems is somewhat harder, relying on alignment properties and the extraction of $g$-factors from cross-talk between signature partners where possible. It has been suggested that the odd nucleon in the lowest energy superdeformed state in the $N = 111$ nucleus $^{191}$Hg occupies a $j_{15/2}$ orbital [25]. Magnetic moment measurements have established that the valence neutron in the $N = 113$ nucleus $^{193}$Hg occupies the $[512]5/2$ orbital of $h_{9/2}$ origin [26], while the valence neutron in the isotope $^{195}$Pb appears to occupy a $j_{15/2}$ orbital [27]. The behaviour of the yrast superdeformed bands in the $N = 115$ nuclei $^{195}$Hg and $^{197}$Pb both suggest occupation of a $j_{15/2}$ orbital [28, 29]. These results strongly suggest that the least bound pair of neutrons in superdeformed nuclei in this mass region occupies either $j_{15/2}$ or $h_{9/2}$ states.

The properties of the superdeformed states in the $Z = 79$ nucleus $^{191}$Au suggest that the wavefunction of the odd proton is dominated by $[532]5/2$ and $[530]1/2$ components, originating from $f_{7/2}$ and $h_{9/2}$ shells respectively [30]. Magnetic moment estimates in the Tl nuclei suggest the odd proton in these cases occupies the $[642]5/2$ orbital of $i_{13/2}$ origin [31, 32]. Finally, the properties of superdeformed bands in the Bi isotopes are consistent with the occupation of the $[651]1/2$ orbital of $g_{9/2}$ origin by the odd proton [33]. These results suggest that the least bound pair of protons in superdeformed Hg isotopes are likely to occupy states of mixed $h_{9/2}f_{7/2}$ character while the least bound protons in superdeformed Pb isotopes are likely to occupy states of relatively pure $i_{13/2}$ character.
The overlap of the least-bound proton wavefunctions with the core is large for both superdeformed and normal systems, suggesting this is unlikely to be responsible for the observed difference in $S_{2p}$. In addition, the spatial probability density of the high-$j$, low-$Ω$ intruder orbitals preferentially experiences the reduced Coulomb barrier at the tips of the prolate shape, which might be expected to lead to a reduction in separation energies for both neutrons and protons at superdeformation. One key difference is that the least-bound protons in the superdeformed (normal) systems occupy high-$j$ orbitals, while the least-bound neutrons occupy relatively high-$j$ orbitals in both cases. It is possible that the pairing interaction results in a more depressed $J=0$ coupling for the $i_{13/2}$ orbitals compared to the $s_{1/2}$, $d_{5/2}$ orbitals. However, the fact that the experimental observations are best reproduced with the inclusion of collective quadrupole vibrations suggests that these may play a crucial role in the structure of the superdeformed ground states.

This work has been supported by the German BMBF through grant no. 06BN907 and the EU under contracts ERBFMECT98-0145 and ERBFMRXCT97-0123, the OTKA through grant no. K72566 and the U.S. Department of Energy, Office of Nuclear Physics, under Contract No. DE–AC02–06CH11357.

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