The Propagation of Cosmic Rays from the Galactic Wind Termination Shock: Back to the Galaxy?

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Where are CRs in the shin region from?

- Cosmic ray can be accelerated at the GTS (Bustard+ 2016/1017)
- Can they diffuse back into the Galaxy?
  - What are the time scales?
  - How are their properties changed?
- What about secondaries?
SOFTWARE/
METHOD
CRPropa 3.2 – a modular structure

**SourceModel**
Spectrum
Evolution
Direction
Composition
...

**Module List**

**Candidate**
position, type, ...
isActive?

**Boundary**

**Interaction**

**Deflection**

**Observer**

**Output**

**Check isActive?**

**Tabulated data**
Infrared background
Radio background
...

**External libraries**
SOPHIA
DINT
...

**Propagation concepts**
Propagation CK
- single particle approach
Diffusion SDE
- multi particle approach

**Galactic lensing**

**Cosmology correction**

**Magnetic field**
Uniform
Grid
...

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CRPropa 3.2 – a rich toolbox

- SimplePropagation
- PropagationCK
- DiffusionSDE

- ElectronPairProduction
- PhotoPionProduction
- PhotoDisintegration
- NuclearDecay

- EM(Double/Triple)-PairProduction
- EMInverseCompton-Scattering

- Redshift
- SynchrotronRadiation
- AdiabaticCooling

Deflection

Nucleon-Interacion

EM-Interactions

General Interactions

- MaximumTrajectory-Length
- MinimumEnergy
- CubicBoundary
- SphericalBoundary
- ...

Boundaries/Thresholds

ObserverSmallSphere
- ObserverTracking
- ObserverPoint
- ObserverDetectAll
- ObserverTimeEvolution
- ...

Observer

ShellOutput
- TextOutput
- HDF5Output
- ParticleCollector

Output

- PerformanceModule

Others

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- Output
- Others

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Galactic Cosmic Rays

- Propagation of Cosmic Rays using the Transport equation
- Taking advantage of the collective behavior of the CRs

\[
\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \nabla \cdot (\hat{k} \nabla n) + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \kappa_{pp} \frac{\partial n}{\partial p} \right) + \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S
\]
Galactic Cosmic Rays

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\frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = \nabla \cdot (\hat{\kappa} \nabla n) + \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 \kappa_{pp} \frac{\partial n}{\partial p} \right) + \frac{p}{3} \nabla \cdot \mathbf{u} \frac{\partial n}{\partial p} + S
\]

Anisotropic diffusion in homogeneous background including advection
SIMULATION SET UP
The simulation model

Symmetry
- Radial / Archimedean spiral

Diffusion
- $D \propto E^{\delta}$
- $D_0 = 10^{28} \frac{cm^2}{s}$

Advection
- $v_0 = 600 \frac{km}{s}$
- $r_0 = 250 \text{kpc}$
The wind model – radial component
RESULTS
Adiabatic energy change

\[ \frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S \]
Adiabatic energy change

\[ \frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S \]

Cooling ($\nabla \cdot \vec{u} > 0$)
- Expansion of the plasma $\rightarrow$ energy loss

Heating ($\nabla \cdot \vec{u} < 0$)
- Compression of the plasma $\rightarrow$ energy gain
Adiabatic energy change

\[ \frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \frac{p}{3} \nabla \cdot \vec{u} \frac{\partial n}{\partial p} + S \]

Cooling (\( \nabla \cdot \vec{u} > 0 \))
- Expansion of the plasma \( \rightarrow \) energy loss

Heating (\( \nabla \cdot \vec{u} < 0 \))
- Compression of the plasma \( \rightarrow \) energy gain

Spherical

Archimedean
Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:

![Graph showing energy spectrum with spherical δ = 0.5 including wind.](image)
Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling

Spherical: $\delta = 0.5$ including wind

$t_{\text{obs}} = 50$ Myr
$t_{\text{obs}} = 100$ Myr
$t_{\text{obs}} = 150$ Myr
$t_{\text{obs}} = 200$ Myr
$t_{\text{obs}} = 250$ Myr
$t_{\text{obs}} = 300$ Myr
Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling, 2) diffusion

Spherical: $\delta = 0.5$ including wind

$\frac{dN}{dE^2} [\text{TeV}/(\text{s} \cdot \text{sr} \cdot \text{m}^2)]$

- $t_{\text{obs}} = 50$ Myr
- $t_{\text{obs}} = 100$ Myr
- $t_{\text{obs}} = 150$ Myr
- $t_{\text{obs}} = 200$ Myr
- $t_{\text{obs}} = 250$ Myr
- $t_{\text{obs}} = 300$ Myr

$E [\text{TeV}]$

- $1.0 \times 10^{-12}$
- $3.2 \times 10^{-02}$
- $1.0 \times 10^{03}$
- $3.2 \times 10^{03}$
- $1.0 \times 10^{04}$

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Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling, 2) diffusion, and 3) heating

Spherical: $\delta = 0.5$ including wind

![Energy Spectrum Graph]

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Energy Spectrum

- A significant change in the spectral shape is visible
- Three regions can be identified, dominated by:
  - 1) cooling, 2) diffusion, and 3) heating

Archimedean spiral: $\delta = 0.5, \kappa = 0.1$, including wind
Total proton flux

Spherical symmetric, no wind

\[
\begin{align*}
\delta &= 0.3 & \text{H: } 10^3 \text{ - } 10^4 \text{ TeV} \\
\delta &= 0.5 & \\
\delta &= 0.4 & \text{HGp: } 10 \text{ - } 10^5 \text{ TeV} \\
\delta &= 0.6 &
\end{align*}
\]
Total proton flux

Spherical

Archimedean

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Arrival Direction – Archimedean spiral

- Different field line length → double ring structure
- Maxima shift from the poles to the equator

Skymap in Galactic coordinates
Arrival Direction – Archimedean spiral

- Different field line length $\rightarrow$ double ring structure
- Maxima shift from the poles to the equator
- Perpendicular diffusion ($\kappa_\perp = 0.1\kappa_\parallel$) washes the structure out

Skymap in Galactic coordinates

Galactic latitude ($\Theta$ distribution)
Neutrino Flux

- Hadronic interaction: $p_{CR} + p_{target} \rightarrow \pi^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_\mu + \nu_e$
- $n_{target} \propto \frac{1}{r^2} \rightarrow$ Use accumulated column density to calculate flux
Neutrino Flux

- Hadronic interaction: $p_{\text{CR}} + p_{\text{target}} \rightarrow \pi^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_\mu + \nu_\tau$
- $n_{\text{target}} \propto \frac{1}{r^2}$ → Use accumulated column density to calculate flux
- Assumption: All neutrinos are produced at $r_{\text{obs}} = 10$ kpc

Column density

Flux for $\delta = 0.5$
SUMMARY / OUTLOOK
Summary

The GTS cannot be the sole source of CR in the shin region.

The expected spectra depend strongly on the model.

However, the neutrino flux is pretty stable.
Outlook

- Further propagation of the CRs to Earth
- Include other elements into the simulation
- Simulate the neutrino production
- Look at CRs leaving the wind termination shock → Starburst Galaxies
BACKUP
Green’s method

\[ S_{\text{burst}} = S_0 \delta(r_0) \delta(t - t_0) \]

\[ S_{\text{finite}} = \tilde{S}_0 \Theta(t - t_0) \Theta(t_1 - t_0); \text{Example: } t_{\text{obs}} = 100 \ \text{Myr}; \Delta t = t_1 - t_0 = 50 \ \text{Myr} \]
Model Assumption

Symmetry
- Radial / Archimedean spiral

Diffusion
- $D \propto E^\delta$
- $D_0 = 10^{28} \frac{cm^2}{s}$

Advection
- $v_0 = 600 \frac{km}{s}$
- $r_0 = 250 \text{ kpc}$

Boundaries
- $r_{\text{obs}} = 10 \text{ kpc}$
- $r_{\text{loss}} = 350 \text{ kpc}$

Simulation details
- $N = 10^7 - 8.5 \cdot 10^8$
- $\tau_{\text{CPU}} = O(10)h - O(1000)h$
OBSERVATION
Observation – Energy Spectrum

Fig 2. The energy spectrum of Cosmic rays.
Observation - Composition

Fig 3. Boron to carbon ratio as a measure for the column depth.

NASA. Imagine the Universe. access: (23.02.2015)
Observation – B/C ratio

Fig 3. Boron to carbon ratio as a measure for the column depth.

Oliva, A. for AMS-02 (2015)
Observation – Arrival Directions

Fig 4. Cosmic ray arrival anisotropy at a median energy $E=20$ TeV after the subtraction of the best fit dipole and quadrupole.
SDE - MATH
2 Propagation Models

Singleparticle picture

- Solving the equation of motion
- Backtracking and Reweighting of results is possible

Multiparticle picture

- Solving the transport equation
- Collective behavior, e.g. advection and diffusion is easy to implement

Numerics

Physics

Stochastic Differential Equations
Stochastic Differential Equation

Langevin Equation
\[
\frac{dx}{dt} = a(x, t) + b(x, t)\xi(t)
\]

Stochastic Integral Equation
\[
x(t) = x(0) + \int_{t_0}^{t} a[x(s), s]\, ds + \int_{t_0}^{t} b[x(s), s]\, dW(s)
\]

- These equations can be treated mathematically consistently.
- Numerical algorithm to solve them are available.
From Fokker-Planck Equ. to SDE

General Fokker-Planck Equation

$$\frac{\partial n(x, t; y, t')}{\partial t} = - \sum_i \frac{\partial}{\partial x_i} [A_i(x, t)n(x, t; y, t')] + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} [B_{ij}(x, t)n(x, t; y, t')]$$

Corresponding Stochastic Differential Equation

$$dr_{\nu} = A_{\nu} \, dt + D_{\nu \mu} \, d\omega^\mu$$

Calculation of stochastic tensor $D$

$$(\kappa + \kappa^t) = DD^\dagger$$

In the case of decoupled momentum and spatial operators
And diagonal diffusion tensor

$$D_{ij} = \delta_{ij} \sqrt{2\kappa_{ij}}, \quad D_{qq} = \sqrt{2\kappa_{qq}}$$
Integration scheme

\[ \vec{x}_{n+1} = \vec{x}_n + D_r \Delta \omega_r \]
\[ = \vec{x}_n + \left( \sqrt{2\kappa_\parallel} \eta_\parallel \vec{e}_t + \sqrt{2\kappa_{\perp,1}} \eta_{\perp,1} \vec{e}_n + \sqrt{2\kappa_{\perp,2}} \eta_{\perp,2} \vec{e}_b \right) \sqrt{h} \]

- Magnetic field line implementation (e.g. JF12 field) is not trivial.
- Calculation of the local trihedron is the crucial part.
- Use adaptive field line integration \( \rightarrow \) e.g. Cash-Karp algorithm.

\[ \vec{r}_{\text{end}} = \vec{r}_{\text{start}} + \int_0^L \vec{B}/B \, ds \quad \vec{r}_{\text{end}} = \vec{r}_0 + \sum_{j=0}^{2^n-1} \int_{2^{-n}Lj}^{2^{-n}L(j+1)} \vec{v}(s) \, ds \]
SDE and FPE

Ito’s lemma leads to equivalence between stochastic differential equation (SDE) and Fokker-Planck transport equation (FPE)

\[
\frac{\partial \Psi(r, t)}{\partial t} = \nabla \cdot (D \cdot \nabla \Psi(r, t)) + S(r, t)
\]

\[
dr = A \, dt + B \, d\omega \quad \text{with: } d\omega = \eta \sqrt{dt}
\]

\[
B = \sqrt{2 \cdot D}, \quad A = 0
\]

Diffusion tensor is diagonal in frame with \( \vec{B} = B_0 \cdot \vec{e}_z \)

\[
D = \begin{bmatrix}
\kappa_{\text{perp},1} & 0 & 0 \\
0 & \kappa_{\text{perp},2} & 0 \\
0 & 0 & \kappa_{\text{par}}
\end{bmatrix}
\]

with: \( \kappa_{\text{perp},1} = \kappa_{\text{perp},2} \)
CRPropa
# CRPropa: Interactions

## Nucleons
- ElectronPairProduction (Bethe Heitler)
- PhotoPionProduction
- PhotoDisintegration
- Nuclear Decay

## Electro-Magnetic
- EMPairProduction
- EMDoublePairProduction
- EMTripletPairProduction
- InverseComptonScattering

## General
- Redshift (accounts for adiabatic energy loss)
- SynchrotronRadiation
PERFORMANCE
Performance

Fig 15: Comparison of computation times. Conventional CRPropa3 (left) and propagation with the DiffusionModule (right)
PROBLEMS WITH THE JF12-FIELD
Problem: Fixed step length

Fig 19: Magnetic field line (blue) and end position after diffusion process (red dots).
Deviation vectors

Fig 20: Ninty smallest deviation vectors for E=10TeV
Deviation from field line II

**Fig 21:** Deviation from ideal trajectory is uniformly distributed in $\Phi$.

**Fig 22:** Deviation from ideal trajectory peaks around the plane perpendicular to magnetic field line.
VALIDATION
Validation I

First test of the diffusion in a homogeneous magnetic background field. A simple anisotropic diffusion tensor is implemented.

Fig 7. The algorithm reproduces the expected analytic results (simulation-barplot, theory-solid lines).
Stationary Test I

Stationary equation of anisotropic diffusion

\[-\nabla \cdot (\hat{\kappa} \nabla n(\vec{r})) = s(\vec{r})\]

Source term

\[s(\vec{r}) = \frac{4}{\pi^2} \left( \frac{\kappa_{zz}}{4R^2} + \frac{\kappa_{zz}}{2H^2} \right) \cdot \cos \left( \frac{x\pi}{2R} \right) \cos \left( \frac{y\pi}{2R} \right) \cos \left( \frac{z\pi}{2H} \right)\]

Not possible?

Indirect solution

\[\frac{\partial n}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla n) + s(\vec{r})\delta(t - t_i)\]

\[n_{sim}(\vec{r}) = \sum_i n(t_i) h_i w\]
Stationary Test II

Fig 8. Total number density depending on maximum integration time for different numbers of snapshots.
Stationary Test III

Fig 25. Total number density depending on maximum integration time for different integration time steps.
Validation IIa

- We test the accuracy of the algorithm in an artificial situation.
- A spiral with varying radius is used as the magnetic field line.
- The distance to the field line after the diffusion is taken as a measure for the algorithm accuracy.

Fig 9: Example of a spiral field line and a sample of end positions.
Validation IIb

Fig 10: Results for the accuracy test. The algorithm allows a user chosen precision for a pure parallel diffusion.
Adiabatic Cooling

\[ \frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n = \nabla \cdot (\hat{k} \nabla n) + \frac{1}{3} (\nabla \cdot \vec{u}) \frac{\partial n}{\partial \ln p} + S(\vec{x}, p, t) \]

**Fig 4.** Particle and energy density for advective test case.
First applications – Rigidity

Fig 14: Time Evolution of the total particle number.
Continuous source

Fig 24: Cosmic ray density for continuous uniform emission inside the Galactic disc.
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.5$
Time Evolution $\Delta t = 100$ Myr; $\delta = 0.3$
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.4$
4 Time Evolution $\Delta t = 100$ Myr; $\delta = 0.5$
Time Evolution $\Delta t = 100$ Myr; $\delta = 0.6$
4 Arrival $\delta = 0.6, \epsilon = 0.$; Wind
4 Arrival $\delta = 0.6, \epsilon = 0.1$; Wind
EXTENSION
Outlook

Use the local turbulence ratio $\eta$ with: 
\[ \eta = \frac{b_0^2}{b_0^2 + B_0^2} \]
to calculate diffusion tensor.

Fig 23: The turbulence ratio of the JF12 field.
COMPETITORS
## Comparison of Tools

**Tab. 1: Popular Propagation Programs**

| Name               | Propagation | Diffusion       | Integration            | Interaction | Remarks                        | Cite                      |
|--------------------|-------------|-----------------|------------------------|-------------|--------------------------------|---------------------------|
| GALPROP            | Trans. Equ. | Scalar          | Grid (Crank Nicolson)  | Yes         | Quasi stand.                   | Strong et al. (2011)      |
| DRAGON 2           | Trans. Equ. | 3dim anisotr.   | Grid                   | Yes         | Evoli et al. (2016)            |
| PICARD             | Trans. Equ. | 3dim anisotr.   | Grid                   | Yes         | Dedicated stat. Solver         | Kissmann et al. (2014)    |
| CRPropa 3 (PropagCK)| Equ. of Motion | No              | Cash Karp              | Partly      | UHECR                          | Batista et al. (2016)     |
| CRPropa 3.1 (DiffusionSDE)| Trans. Equ. | 3dim const. Eigenvalues | SDE adaptive | Partly | Arbitrary magn. field | Merten et al. (t.b.s.) |
|                    | Trans. Equ. | Fully anisotropic | SDE Euler-Mayurama    | No          |                                | Kopp et al. (2011)        |