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Incorporation of Material Variability in the Johnson Cook Model

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Abstract

The Johnson Cook (JC) material model is often used for modeling hypervelocity impacts (HVI) because it is capable of capturing high rate deformation and temperature effects. Within the JC model, material damage leading to fracture is aggregated using a path dependent damage parameter. Contributions to the damage parameter are calculated at each cycle as the quotient of incremental effective plastic strain over the effective failure strain. The effective failure strain is a function of the stress state, strain rate, and temperature which changes locally throughout the simulation. Solid bodies often demonstrate variability in resilience resulting from material inclusions and defects. Therefore, in addition to the deterministic, state-based variability in the effective failure strain inherent to the JC model, efforts are often made to capture the effect of general material non-uniformity. Although other approaches may be available, the Weibull probability distribution is often employed within failure analyses to allow simulations to diverge from a completely uniform solution.

In this paper, we investigate several methods of augmenting the JC damage model with Weibull variability. The implementation of each method provides a means for measured material uncertainty to enter the calculation. We focus on a standard three-parameter Weibull probability density function (pdf) although the methods proposed can be used with any probability distribution. Characteristics of the pdf are preserved within a solid body at its initial state but differ in the effect the JC effective failure strain has on these characteristics as the material state changes. The Weibull pdf under various loading conditions is discussed, and simulation results incorporating these methods are compared with those employing only the JC damage model or Weibull variability.

Keywords: Johnson Cook, Weibull, Material Variability, Uncertainty

1. Model formulations

Capturing variability of material behavior is an important component of modeling HVI events. Most materials will have some inhomogeneity due to imperfections within the crystalline structure, such as dislocations, grain boundaries, or other defects [1]. These non-uniformities introduce randomness in the material response, especially with regard to material failure and fracture at high strain rates. An approach is needed to model this variability in continuum mechanics based tools.

The best method for incorporating material variability in the Johnson-Cook (JC) damage model has not yet been established, yet with some assumptions data describing both can be gleaned from the same test [2]. The unknown factor allowing multiple approaches is the effect pressure, temperature and strain rate have on the distribution of failure strain values. In this section, the JC model and the three-parameter Weibull probability distribution function (pdf) are described and methods for combining the two are introduced. These methods primarily differ in the way in which the characteristics of the pdf change with the state of the material, which will be shown to have a significant effect on simulation results in the following sections.

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Since both the JC model and Weibull probability distributions can be used to provide the same information, some information must be discarded when developing each of the methods described below.

1.1. Johnson-Cook damage model

The Johnson-Cook (JC) constitutive relationship is intended to characterize material response under various loading conditions using one simple material model [3]. In this model, von Mises flow stress is calculated as a function of strain, strain rate, and temperature in the form

\[ \sigma_v = \left[ A + B\varepsilon^\eta \right] \left[ 1 + C \ln \dot{\varepsilon}^\ast \right] \left[ 1 - T^m \right] \]

where \( \varepsilon \) is the equivalent plastic strain, \( \dot{\varepsilon}^\ast \) is a dimensionless plastic strain rate, \( T^\ast \) is the homologous temperature, and the constants A, B, C, m, and n are material specific [3]. A path dependent fracture model that can be used in conjunction with the stress model has also been developed to characterize material failure, taking into account the effects of equivalent stress, pressure, strain rate, and temperature [4]. Material damage is accumulated locally using a damage parameter, calculated as follows:

\[ D = \sum_{i=0}^{\Delta \varepsilon} \varepsilon_{JC}^f \]

where \( \Delta \varepsilon \) is the incremental strain and \( \varepsilon_{JC}^f \) is the equivalent failure strain. The value of \( \varepsilon_{JC}^f \) is recalculated using Equation (3) each time step giving the damage parameter its path dependency.

\[ \varepsilon_{JC}^f = \left[ D_1 + D_2 \exp D_3 \sigma^\ast \right] \left[ 1 + D_4 \ln \dot{\varepsilon}^\ast \right] \left[ 1 + D_5 T^\ast \right] \]

Here, \( \sigma^\ast = P/\sigma \), where P is the pressure and \( \sigma \) is the von Mises equivalent stress. The material constants are \( D_1, D_2, D_3, D_4, \) and \( D_5 \).

This work focuses on incorporating the effects of material variability in the fracture model only. We will assume that the material behaves uniformly with regard to the stress-strain response and the flow stress is calculated using Equation (1). For simplicity, this effort does not consider the effect of strain rate on the failure strain. Since strain rates less than one could produce negative failure strains, and a strain rate of zero would cause numerical issues due to the natural logarithm, the distribution of failure strains at the initial state would depend on the minimum strain rate threshold in the model. Therefore, we will use \( D_1 = 0 \) to reduce complexity. We are left with

\[ \varepsilon_{JC}^f = \left[ D_1 + D_2 \exp D_3 \sigma^\ast \right] \left[ 1 + D_5 T^\ast \right] \]

In our implementation of the model, the homologous temperature, \( T^\ast \), is offset such that it is equal to zero when the material is at ambient temperature. Also, note that \( \sigma^\ast \) equals zero if no hydrostatic pressure exists, so Equation (4) becomes simply, \( \varepsilon_{JC}^f = D_1 + D_2 \) for the initial state.

1.2. Weibull distribution

The description of material damage described in the previous section does not include the effects of material inhomogeneity arising from defects, inclusions, or abnormalities. This type of variability can be described using a pseudorandom number generator and a probability density function (pdf) to vary material parameters spatially. The work presented here focuses on the Weibull pdf as a means of including material variability.

The three-parameter Weibull distribution is widely used in the field to describe variation of failure characteristics due to material inhomogeneity [5], and is expressed as follows:

\[ f(\varepsilon_{wc}^f) = \left( \frac{\beta}{\eta} \right) \left( \frac{\varepsilon_{wc}^f - \varepsilon_0^f}{\eta} \right)^{\beta - 1} \exp \left[ -\left( \frac{\varepsilon_{wc}^f - \varepsilon_0^f}{\eta} \right)^\beta \right], \quad \varepsilon_{wc}^f > \varepsilon_0^f \]
where $\beta$ and $\eta$ are the shape and scale parameters and $\varepsilon_0^f$ is a cut-off failure strain, providing a lower bound for the failure strain. Values for the failure strain that follow the probability distribution described by Equation (5) can be generated by Equation (6), which has been derived using the Inversion Method [6].

$$\varepsilon_W^f = \eta \left[ - \ln(1 - Y) \right]^{1/\beta} + \varepsilon_0^f$$  \hspace{1cm} (6)

Here, $Y$ is a random number with a uniform distribution on $[0,1]$. Fig. 1 shows an example of a set of $1E5$ failure strain values. The parameters used to generate this data set are $\beta = 2$, $\eta = 0.103$, and $\varepsilon_0 = 0.3$, which were obtained for 1018 steel from a pipe bomb experiment[2]. Note that no strains fall below the cutoff value ($\varepsilon_0^f$).

The expected value of the Weibull pdf, $E(\varepsilon_W^f)$, can be calculated as follows:

$$E(\varepsilon_W^f) = \int_0^\infty \varepsilon_W^f f(\varepsilon_W^f) d\varepsilon$$  \hspace{1cm} (7)

$$= \eta \Gamma\left( \frac{1}{\beta} + 1 \right) + \varepsilon_0^f$$  \hspace{1cm} (8)

where $\Gamma(x)$ is the gamma function.

Note that the Weibull failure strain values, $\varepsilon_W^f$, remain constant throughout the analysis, unlike $\varepsilon_{JC}^f$, which changes with material state.

![Figure 1: Sample failure strain values generated using a random number generator and the 3-term Weibull probability function. Parameters $\beta = 2$, $\eta = 0.103$, and $\varepsilon_0 = 0.3$ were used.](image)

1.3. Offset method

The first method of adding variability to the JC damage model works by shifting the Weibull pdf by an offset value that is calculated using the JC damage parameters. This method ignores the value of $D_1$ and shifts the original Weibull distribution using the values of $D_2$, $D_3$, and $D_5$. This is accomplished by replacing $D_1$ with $\varepsilon_W^f$ and removing the effect of $D_2$ from the initial distribution as follows:

$$D_1 = \varepsilon_W^f$$  \hspace{1cm} (9)

$$\varepsilon_{Offset}^f = [D_1 + D_2 (\exp D_5 \sigma^* - 1)][1 + D_3 \sigma^*]$$  \hspace{1cm} (10)

This formulation will produce the specified Weibull distribution at the initial state with $E(\varepsilon_W^f) = E(\varepsilon_{Offset}^f)$, and the cutoff value, $\varepsilon_0^f$. The shape of original distribution is preserved throughout the analysis because $D_1$ is constant, and the expected value and cutoff value are offset by an amount dictated by the state of the material.
1.4. $D_1$ reset method

The $D_1$ reset method also produces the specified Weibull pdf at the initial state and shifts it depending on the state of the material, but in this case, the shape of the distribution also changes with the material state. This is accomplished by using a multiplier to scale the JC damage equation and replacing the value of the $D_1$ parameter. The original Weibull pdf is modified using only the values of $D_2$, $D_3$, and $D_5$. Failure strain values for the $D_1$ reset method are calculated as shown below.

\[
D_1 = E(e_W^f) - D_2
\]

\[
E_{D_1\text{reset}}^f = \left[ \frac{e_W^f}{E(e_W^f)} \right] [D_1 + D_2 \exp D_3 \sigma^*][1 + D_5 T^*]
\]

As noted in Section 1.1, if the hydrostatic pressure is zero at the initial state, the second set of brackets contains only $D_1 + D_2$, which is equal to $E(e_W^f)$, and results in the original Weibull pdf for $e_{D_1\text{reset}}^f$. Therefore, at the initial state, $E(e_W^f) = E(e_{D_1\text{reset}}^f)$, and the cutoff value, $e_0^f$, is observed.

1.5. $D_2$ reset method

The $D_2$ reset method uses the same concept as the $D_1$ reset method, except that $D_2$ is modified rather than $D_1$.

\[
D_2 = E(e_W^f) - D_1
\]

\[
E_{D_2\text{reset}}^f = \left[ \frac{e_W^f}{E(e_W^f)} \right] [D_1 + D_2 \exp D_3 \sigma^*][1 + D_5 T^*]
\]

Once again, at $t = 0$, the expected value is that of the specified Weibull pdf, $E(e_W^f) = E(e_{D_2\text{reset}}^f)$, and the cutoff value is $e_0^f$.

1.6. Multiplier method

The multiplier method is similar to the reset methods, except that all the JC damage parameters are used, and the original Weibull distribution is not preserved at the initial state.

\[
e_{\text{Multiplier}}^f = \left[ \frac{e_W^f}{E(e_W^f)} \right] [D_1 + D_2 \exp D_3 \sigma^*][1 + D_5 T^*]
\]

For this case, in the initial state, $E(e_W^f) \neq E(e_{D_2\text{reset}}^f)$, and $e_0^f$ is not the cutoff value.

1.7. Example failure strain distributions for three material states

Fig. 2 shows the distribution of failure strains calculated using the methods described above for three different values of $\sigma^*$. The Weibull parameters used here are the same as those used to generate the histogram in Fig. 1. The effect that material state has on the distribution of failure strains is completely dependent on the values of the JC damage parameters. Damage parameters for 1018 steel derived from pipe bomb test results [2] were used in the example below because they were derived from the same data used to obtain the Weibull parameters above and because they are not subject to export restrictions.

| Table 1: Example Johnson Cook damage parameters |
|-----------------------------------------------|
| Sample JC Damage Parameters  | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|-----------------------------------------------|
| 0.24 | 0.54 | -1.50 | 0.00 | 0.00 |
Note that the scales vary for the different material states. The first column represents the initial state when no hydrostatic pressure is applied, the material in the second column is undergoing compression, and in the third column it is in tension. Distributions in the first row all follow the unmodified Weibull pdf. The same is true for the first column in every row except the last, because the multiplier method is the only method presented here that does not preserve the Weibull pdf in the initial state. The distributions in the second row all have the same profile, but are shifted by an offset value; whereas, the distribution profiles vary in the third, fourth, and fifth rows.

![Fig. 2: Probability distributions for a given set of JC parameters and Weibull coefficients for three different material states.](image)

The failure strain values for the offset method and the D1 reset method are quite low in the third column of the example shown in Fig. 2. In fact, with these parameters and a tensile state of $\sigma^* = 2/3$, a number of values within the distribution lie in the unphysical, negative region. This behavior is caused by the magnitude of D2 relative to the values of the Weibull effective failure strain. For these formulations, the effect of D1 is removed and a negative constant value of D2 is added. When values of $D_1 \sigma^*$ become negative, the exponential term diminishes and failure strain values can become negative.

Negative failure strains can also occur when the D2 reset method is used if the value of D1 is negative, or if the D1 parameter is greater than the expected value of the Weibull distribution. The latter case should be avoided entirely because if the value of D2 is negative, the effective plastic failure strain decreases with increasing pressure, and the formulation is unphysical.

To summarize, the following conditions can cause negative failure strains to occur:

- **Offset Method:** $D_2 > \varepsilon_0^f$

- **D1 Reset Method:** $D_2 > E(\varepsilon_W^f)$, $\sigma^* > \ln\left(\frac{(D_2 - E(\varepsilon_W^f))}{D_3}\right)$

- **D2 Reset Method:** $D_1 < 0$, $\sigma^* > \ln\left(\frac{-D_1}{(E(\varepsilon_W^f) - D_1)}\right)$, $D_1 > E(\varepsilon_W^f)$, *model not physical*

Furthermore, when the conditions discussed above for the D1 and D2 reset methods are met, the entire distribution is negative because a multiplier rather than an offset value is used, meaning that the effective failure strain for a material in that state is necessarily negative.

## 2. Simulation results

### 2.1. Test setup and summary

The effect that each of the methods described above has on a HVI event was investigated by simulating one of two spaced plates test events performed at the University of Alabama in Huntsville [7]. Images of the plates after the test event
are available for qualitative comparison with the simulation results and are shown in Fig. 9 in the next section. Fig. 3 shows a schematic of the test setup, in which a 354.2 gram, cylindrical projectile, consisting of a 7075-T6 aluminum core and a nylon outer layer, was fired at 4.8 km/s and struck the first plate with a yaw of 16° and a pitch of -11°.

In order to quantify the extent of the variability resulting from the methods described in this work, 60 trials were run for each. Pseudo-random numbers for each trial were generated using a different seed value, and each method was evaluated using the same set of 60 seed values. In these simulations, variability was only applied to the plates, using the Weibull pdf shown in Fig. 1. Damage parameters for the 6061-T6 aluminum JC model calculated by Johnson and Holmquist were used [8]. These are not tabulated here due to export restrictions. All simulations were carried out using Velodyne, Corvid’s in-house, massively parallel, hydro-structural code. Elements that fail during the analysis are converted to SPH particles to conserve mass and momentum.

2.2. Results

The figures in this section show histogram plots of the velocity, mass, and momentum of the second plate 5 ms after impact with the first plate. The results calculated using only the JC model with no variability are marked with a red line. The figures also include images depicting back and profile views of the rear plate for three representative calculations. The leftmost plate image had the lowest calculated momentum of all 60 trials, the rightmost had the highest momentum, and the middle plate was the result with the most average value of momentum. Some trends seemingly correlated with momentum are evident upon visual inspection of the extent to which the petals have spread open. For example, petals of the aluminum appear to have less curl in the plates with lower momentum.

Fig. 4: 2nd plate statistics and images of models deriving failure strains from the Weibull distribution only.

Fig. 5: 2nd plate statistics and images of models deriving failure strain using the offset method.
When comparing the images from a single method, it seems that the Weibull seed value had a larger effect on the distortion of the petals than on the extent of the cracks that formed them. Each example from a given method shows similar crack lengths, and the distribution ranges of velocity, mass, and momentum histograms for individual methods are narrow relative to the total range covered by all the methods.

Much can be gained from comparing the results of one method to another. Note the similarities between the offset method and the D₁ reset method. Results of both exhibit a large degree of cracking and break-up caused by finite element deletion. As more elements are removed to form cracks, the mass and total momentum of the plate decreases, and consequently, the mass, velocity, and momentum values are on the low side for these methods. The similarities noted between these results stem from the similarity in their formulations, as both methods replace the D₁ parameter and are largely affected by the D₂ parameter as described in Section 1.7.

3. Conclusion

Simulations using each of the methods described in this paper produce results that are quite different, qualitatively. The most notable difference in the results from this effort is the extent of material break-up produced by the offset and D₁ reset
methods as compared to the others. This can be explained by the low values of failure strain encountered during tensile states for these methods with the parameters applied in this work.

In this case, the characteristics of the simulated second plate seem most similar to photographs of the test when the D2 reset method was used. This is evident in Fig. 9, where the front and profile views of the rear plate are shown for both the test and simulated results for plates with the most average value of momentum. In the photograph, a limited number of petals have formed and have curled outward. With the offset and D1 reset methods, a greater number of long cracks have formed, allowing much of the center of the plates to bulge out rather than form isolated petals. Conversely, the Weibull and multiplier methods exhibit an overall lack of damage compared to the test. The shape and number of petals formed using the D2 reset method is most comparable with the test results.

![Image: Post-test photographs and simulation results](image)

Fig. 9: Post-test photographs and simulation results

The similarity between the D2 reset method simulation results and the test by no means infers that this formulation is the best, since much of the outcome of these simulations relies on the damage parameter values. A more realistic conclusion is that the method used to implement variability can have an effect on the results that is more significant than the effect of the variability itself, and care should be taken to ensure that the values of the Johnson Cook damage parameters and the Weibull failure strain distribution produce consistent, physical results. Since some information is lost in the implementation of any of these formulations, methods of adding variability should be chosen based on confidence in the parameters being used, and a means of handling negative failure strains should be in place if the method chosen makes them possible.

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