We present a unifying treatment for metric and scalar perturbations across different energy regimes in scalar-tensor theories of gravity. To do so, we introduce two connected symmetry-breaking patterns: one due to the acquisition of nontrivial vacuum expectation values by the fields and the other due to the distinction between background and perturbations that live on top of it. We show that the geometric optics approximation commonly used to enforce this separation is not self-consistent for high-frequency perturbations since gauge transformations mix different tensor and scalar sectors orders. We derive the equations of motions for the perturbations and describe the behavior of the solutions in the low and high-frequency limits. We conclude by describing this phenomenology in the context of two screening mechanisms, chameleon and symmetron, and show that scalar waves in every frequency range are screened, hence not detectable.
tation values (vevs), inspired by the averaging procedure discussed in [14]. Once developed, we aim to apply this formalism to two screening mechanisms, chameleon and symmetron, to understand whether the SW would be detectable.

**Spontaneous symmetry breaking.**— We consider a subset of generalized Brans-Dicke theories [23, 24] for which the action in Einstein frame (EF) can be written as the canonical action for a scalar field, \( \phi \), coupled to the matter fields, \( \Psi_i \), through a conformal transformation:

\[
S = \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right] + S_m(\Omega^2(\phi) g_{\mu\nu}, \Psi_i). \tag{1}
\]

In this expression, \( R \) is the Ricci scalar and \( V(\phi), \Omega(\phi) \) are two arbitrary functions modeling the field potential and the conformal coupling. We study the dynamics of perturbations in vacuum. This assumption is justified, for example, in cosmological settings when the scalar field dominates the energy content of the Universe and drives the expansion of the Universe, or in screening scenarios outside a localized matter field. Generalizing this assumption will be the topic of future works. We note that our results do not cover theories exhibiting the Vainshtein screening mechanism [37, 38], since they cannot be cast in the form of Eq. (1).

Breaking spontaneously the symmetries of the action (1), the fields acquire non trivial vevs

\[
\langle g_{\mu\nu} \rangle = \bar{g}_{\mu\nu}, \quad \langle \phi \rangle = \bar{\phi}. \tag{2}
\]

We require \( \bar{\phi} \) to be function of the spacetime coordinates in order to define the preferred vector field

\[
v_\mu \equiv \partial_\mu \bar{\phi} \neq 0 \tag{3}
\]

and the order parameter \( L \)

\[
L \equiv \sqrt{|v^\mu v_\mu|}. \tag{4}
\]

We assume that \( \{\bar{g}_{\mu\nu}, \bar{\phi}\} \) vary on the same length scale and, without loss of generality, we take \( |\bar{g}_{\mu\nu}|, |\bar{\phi}| \sim O(1) \), so that \( L \) measures the variation length scale of \( \bar{\phi} \).

Introducing an orthonormal tetrad such that \( v^a = e^a_\mu v^\mu \), under diffeomorphisms and local Lorentz transformations \( v^\mu \) transforms as

\[
\langle v_\mu \rangle' = \partial_\mu \langle \bar{\phi} - \xi^a \partial_a \bar{\phi} \rangle = v_\mu - \partial_\mu (v_\mu \langle \xi^a \rangle), \tag{5}
\]

\[
\langle v^a \rangle' = \Lambda^a_\mu e^b_\mu \partial^\mu \langle \bar{\phi} \rangle = \Lambda^a_\mu v^b, \tag{6}
\]

where \( \Lambda^a_\mu \) is a Lorentz matrix and \( \xi_\mu \) the generator of spacetime translations. The broken symmetry transformations are those such that \( \Lambda^a_\mu v^b \neq 0 \) and \( v_\mu \langle \xi^a \rangle \neq 0 \). Note that \( v^\mu \neq 0 \) is crucial to have an SSB since the case \( \bar{\phi} = \text{const} \) contains the maximally symmetric Minkowski, de Sitter and anti-de Sitter solutions.

**Definition of field perturbations.**— We study the dynamics of the field perturbations around their vevs

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \phi = \bar{\phi} + \delta \phi, \tag{7}
\]

defined via \( \langle h_{\mu\nu} \rangle = \langle \delta \phi \rangle = 0 \), compatibly with Eq. (2). We also assume that the amplitude of the perturbations is smaller than their background counterparts. This is quantified by the parameter \( \alpha \) such that \( |h_{\mu\nu}| \sim |\delta \phi| \sim \alpha \ll 1 \). To describe the behaviour of oscillatory perturbations, such as GWs and SWs, we also define

\[
\epsilon \equiv \lambda L, \tag{8}
\]

where \( \lambda \) is the order of magnitude of the derivatives of the perturbations: \( |\partial h_{\mu\nu}|, |\partial \delta \phi| \sim 1/\lambda \). This parameter allows us to introduce the ADM averaging scheme [15, 17] to formally evaluate the vevs \( \langle ... \rangle \): oscillatory perturbations average out to zero after integrating over volumes that are larger than \( \lambda \) but small enough to be independent of \( \epsilon \). In practice, \( \epsilon \) is used to separate between the so-called low-frequency modes, i.e., the background, and the high-frequency modes, i.e., the oscillatory perturbations. We note that the very existence of \( \{h_{\mu\nu}, \delta \phi\} \) requires \( \epsilon < 1 \), otherwise they would become part of the background as the perturbation’s wavelength \( \lambda \) approaches the background’s length-scale \( L \). The limit \( \epsilon \rightarrow 1 \) is particularly subtle since the volumes that need to be considered to make the ADM averages might become too big. Because the amplitudes of \( \{h_{\mu\nu}, \delta \phi\} \) can be made large or small via a gauge transformation and \( \{\bar{g}_{\mu\nu}, \bar{\phi}\} \) are unknown, the requirement \( \alpha \ll 1 \) is not enough to distinguish background from perturbation as in Eq. (7) [17]. This is the principal reason for introducing the parameter \( \epsilon \): perturbations and backgrounds are distinguished according to their different variation length scales via the averaging scheme used to take the vevs.

To discuss the role of diffeomorphisms, we decompose the vector field generating the gauge transformations as

\[
\xi^\mu = \xi^\mu + \delta \xi^\mu, \tag{9}
\]

where \( |\delta \xi^\mu| \lesssim \alpha \) and \( |\partial \delta \xi^\mu| \sim 1/\lambda \), such that \( \langle \delta \xi^\mu \rangle = 0 \). This way \( \xi^\mu \) generates the gauge transformations of \( \{\bar{g}_{\mu\nu}, \bar{\phi}\} \), while \( \delta \xi^\mu \), those of the HF perturbations \( \{h_{\mu\nu}, \delta \phi\} \). The field perturbations transform as

\[
h_{\mu\nu}' = h_{\mu\nu} - (\nabla_\mu \delta \xi^\nu + \nabla_\nu \delta \xi_\mu), \tag{10}
\]

\[
\delta \phi' = \delta \phi - v^\mu \delta \xi_\mu, \tag{11}
\]

where \( \nabla_\mu \) is the covariant derivative associated to \( g_{\mu\nu} \). To preserve the splitting of Eq. (7) we restrict the class of allowed HF diffeomorphisms requiring

\[
|\nabla_\mu \delta \xi^\nu|, |v^\mu \delta \xi_\mu| \lesssim \alpha, \tag{12}
\]
so that the amplitudes of the field perturbations after the gauge fixing are $\lesssim \alpha$. From Eq. (12) we then see that

$$|\partial_{\mu} \delta \xi_{\nu}| \sim |\delta \xi_{\nu}| / \lambda \sim |\delta \xi_{\nu}| \lesssim \alpha \rightarrow |\delta \xi_{\nu}| \lesssim \epsilon \alpha. \quad (13)$$

This is what we mean by second symmetry breaking: depending on the value of $\epsilon$, not every HF diffeomorphism is allowed [14, 15]. This requirement is not imposed in the EFT cosmological perturbation theory since the quantities $\{\tilde{g}_{\mu \nu}, \phi\}$ are assumed a priori and the perturbations are uniquely defined as $h_{\mu \nu} = g_{\mu \nu} - \tilde{g}_{\mu \nu}$ or $\delta \varphi = \phi - \tilde{\phi}$. Consequently, the EFT is able to describe perturbations varying on every length-scale, even those close to the background when $\epsilon \sim 1$, provided that they are below the energy cutoff. Conversely the traditional HF treatment does not assume a background, but works only in the $\epsilon \ll 1$ regime. Our vev-based definitions allow us to bridge the gap between the high-energy/HF and low-energy/EFT treatments by describing perturbations around unknown backgrounds in the entire $\epsilon < 1$ regime. This is why the formalism presented here acts as missing link between these two approaches.

**High-frequency expansion.** — We can use $\epsilon$ to set up the expansions

$$h_{\mu \nu} = h^{0}_{\mu \nu} + \epsilon h_{\mu \nu}, \quad b_{\mu \nu} = h^{1}_{\mu \nu} + \epsilon b^{2}_{\mu \nu} + \ldots, \quad (14)$$

$$\delta \varphi = \delta \varphi^{0} + \epsilon \psi, \quad \psi = \delta \varphi^{1} + \epsilon \delta \varphi^{2} + \ldots, \quad (15)$$

$$\delta \xi_{\mu} = \delta \xi^{0}_{\mu} + \epsilon \delta \xi^{1}_{\mu}, \quad \delta \xi^{0}_{\mu} = \delta \xi^{1}_{\mu} + \epsilon \delta \xi^{2}_{\mu} + \ldots, \quad (16)$$

where $|h^{0}_{\mu \nu}| \sim |\delta \varphi^{1}| \sim |\delta \xi^{0}_{\mu}| \sim \alpha$. If the fields perturbations satisfy a wave equation, one can assume the WKB ansatz where $\{h^{0}_{\mu \nu}, \delta \varphi^{0}\}$ coincide with the geometric optics (GO) order terms and $\{b_{\mu \nu}, \psi\}$ with the beyond geometric optics corrections [13, 14]. When $\epsilon \ll 1$ Eqs. (14)-16 are meaningful perturbative expansions and condition (13) leads to

$$\delta \xi^{0}_{\mu} = 0. \quad (17)$$

Moreover, when $\epsilon \ll 1$ the gauge transformations can be reorganized in powers of $\epsilon$.

$$h^{0}_{\mu \nu} = h^{0}_{\mu \nu} - \epsilon (\nabla_{\mu} \delta \xi_{\nu} + \nabla_{\nu} \delta \xi_{\mu}), \quad (18)$$

$$\{\delta \varphi^{0}\}' = \delta \varphi^{0}, \quad \psi' = \psi - \nu^{\mu} \delta \xi_{\mu}, \quad (19)$$

from which we see that $\delta \varphi^{0}$ is gauge invariant and that the leading order terms transform as

$$(h^{0}_{\mu \nu})' = (h^{0}_{\mu \nu}) - \epsilon (\partial_{\mu} \delta \xi_{\nu} + \partial_{\nu} \delta \xi_{\mu}), \quad (\delta \varphi^{0})' = \delta \varphi^{0}. \quad (21)$$

i.e., as if they lived on a flat background. This is not surprising, since covariant derivatives commute when acting on perturbations approximated at leading order in $\epsilon$ [13].

Eqs. (18) and (20) show that diffeomorphisms mix $h^{0}_{\mu \nu}$ and the second-order $\delta \varphi^{1}$ whenever $v_{\mu} \neq 0$, i.e. in the presence of an SSB. Therefore, fixing $h^{0}_{\mu \nu}$ generates $\delta \varphi^{1} \neq 0$, even if one started by neglecting it. Vice versa, keeping only the leading terms order of HF expansion may lead to inconsistencies because this implicitly assumes $\psi = 0$, using up one of the gauge freedom and leaving one less to fix $h_{\mu \nu}$. We conclude that keeping only the leading orders of the HF expansion, namely $\{h^{0}_{\mu \nu}, \delta \varphi^{0}\}$, is an inconsistent approximation scheme.

In contrast, $\delta \phi$ is gauged away at every order in the EFT formalism. In our framework, this is reproducible in the limit $\epsilon \lesssim 1$ where Eq. (14) becomes trivial. The difference between these two behaviours, namely at $\epsilon \ll 1$ versus $\epsilon \lesssim 1$, proves that some gauges choices are not suitable to describe perturbations across different energy scales. Known gauge-invariant quantities, such as the Bardeen’s potentials [39], fall in this category.

**Gauge Fixing and Equations of motion.** — Assuming $v^{\mu} v_{\mu} > 0$ we define the orthogonal projector

$$\Lambda_{\mu \nu} \equiv \tilde{g}_{\mu \nu} - n_{\mu} n_{\nu}, \quad n_{\mu} \equiv \frac{v_{\mu}}{L}, \quad (22)$$

and decompose the metric perturbation as

$$h_{\mu \nu} = n_{\mu} n_{\nu} A + (n_{\nu} B_{\mu} + n_{\nu} B_{\mu}) + C_{\mu \nu}, \quad (23)$$

with $A \equiv n^{\rho} n^{\sigma} \Lambda_{\rho \sigma}, \quad B_{\mu} \equiv n^{\rho} \Lambda_{\rho}^{\nu} h_{\nu \sigma}$ and $C_{\mu \nu} \equiv \Lambda_{\mu}^{\nu} \Lambda_{\nu}^{\rho} h_{\rho \sigma}$. We impose the conditions

$$A = 0, \quad B_{\mu} = 0. \quad (24)$$

Then, using the residual gauge freedom, we also fix:

$$C = 0, \quad \nabla^{\mu} C_{\mu \nu} = 0. \quad (25)$$

Note that we have exhausted the gauge freedom since Eqs. (23) and (24) amount to 4 conditions each. $B_{\mu}$ only has 3 independent components being orthogonal to $n^{\mu}$ and the condition $C = 0$ implies $n^{\mu} \nabla^{\nu} C_{\mu \nu} = 0$, in fact

$$n^{\mu} \nabla^{\nu} C_{\mu \nu} = -C_{\mu \nu} (K^{\mu \nu} - n^{\mu} a^{\nu}) = -C_{\mu \nu} K^{\mu \nu} \propto C, \quad (26)$$

where $K_{\mu \nu} \equiv \Lambda_{\mu}^{\rho} \nabla_{\rho} n_{\nu}$ and $a^{\mu} \equiv n^{\nu} \nabla_{\nu} n_{\mu}$. In the last step, we used the fact that scalars can be computed in any coordinate system and that choosing $\bar{\phi}$ as a coordinate implies $K_{\mu \nu} \propto \Lambda_{\mu \nu}$. Using the background equations of motion (EoMs),

$$\bar{R}_{\mu \nu} = \frac{1}{2} (v_{\mu} v_{\nu} + \tilde{g}_{\mu \nu} \bar{V}), \quad \Box \bar{\phi} = \bar{V}'. \quad (27)$$

1 Since it can be shown that $\alpha \ll \epsilon$ [17], we expand only to first order in $\alpha$.

2 This choice is suitable to investigate screening in a spherically symmetric and static spacetime. In cosmological settings where $v^{\mu} v_{\mu} < 0$ then $\Lambda_{\mu \nu} \equiv \tilde{g}_{\mu \nu} + n_{\mu} n_{\nu}$. 
where $\vec{V} = V(\phi)$ and $\vec{V}' = (\partial V/\partial \phi)|_{\phi}$, one can show that the combination

$$(\square \delta \varphi - \delta \varphi \vec{V}'') + \nabla^\mu [\epsilon^\mu h_{\mu
u}] - \frac{1}{2} v^\mu \nabla_\mu h = 0$$

(28)

is gauge invariant, where $\vec{V}'' = (\partial^2 V/\partial^2 \phi)|_{\phi}$, and $h$ is the trace of $h_{\mu\nu}$. The last two quantities in the equation above vanish in the chosen gauge, hence $\delta \varphi$ is invariant under the residual gauge freedom and different from zero. In the HF limit this result concerns $\psi$ since $\delta \varphi^0$ is already invariant.

We expand the action $\mathcal{I}$ to second order in the perturbations and find the EoMs

$$\square \gamma_{\mu\nu} + 2R_{\lambda\mu\nu\sigma} \gamma^{\lambda\sigma} = 0,$$

(29)

$$\square \delta \varphi - \delta \varphi \vec{V}'' = 0,$$

(30)

where we renamed the metric perturbation after the gauge fixings $\gamma_{\mu\nu}$. This system of equations is valid for every value of $\epsilon < 1$ and represents a spin 2 wave, $\gamma_{\mu\nu}$, and a spin 0 wave, $\delta \varphi$. Our gauge choice clarifies which are the dofs, especially highlighting that $\delta \varphi$ and $\psi$ are not independent. Eqs. (29), (30) do not display the damping term typical of non-minimally coupled ST theories [21] because they describe perturbations in EF. This factor can be recovered by going to Jordan Frame (JF) where matter is coupled to the JF metric $g_{\mu\nu} \equiv \Omega^2(\phi) g_{\mu\nu}$. Moreover, we do not find modifications in the propagation speed of the modes because this effect is not predicted in the ST theories considered here.

**High-frequency limit.**—We study the $\epsilon \ll 1$ limit of the EoMs above to study HF GWs and SWs. Following the considerations illustrated when discussing the gauge transformations, we keep the first non-null orders of the $\epsilon$ expansions [13, 14]. Since all of the dofs satisfy a wave equation, we make the following WKB ansatz

$$\gamma_{\mu\nu} = \Upsilon_{\mu\nu} e^{i\theta/\epsilon}, \quad \delta \varphi^0 = \Phi e^{i\theta/\epsilon}, \quad \psi = \Psi e^{i\theta/\epsilon},$$

(31)

where $\Upsilon_{\mu\nu}, \Phi, \Psi$ are complex and of order $\mathcal{O}(\epsilon^0)$, $\theta$ is real and they are all slow-varying functions of the spacetime coordinates. Because a derivative acting on the exponential brings down a $1/\epsilon$ factor, we can separate the EoMs into their $\epsilon^{-2}$, $\epsilon^{-1}$ and $\epsilon^0$ orders. The leading order gives

$$\ddot{g}^{\mu\nu} k_\nu k_\mu = 0,$$

(32)

where $k_\mu \equiv \partial_\mu \theta$ is the wave vector. Therefore $k_\mu$ is a null vector tangent to a null geodesic $k^\mu \nabla_\mu k_\nu = 0$ which are interpreted as the rays of the graviton and scalar bundles [13, 14]. At orders $\epsilon^{-1}$ and $\epsilon^0$ we find

$$2k^\mu \nabla_\mu \Upsilon_{\mu\nu} + \Upsilon_{\mu\nu} \nabla_\mu k^\nu = 0,$$

(33)

$$2k^\mu \nabla_\mu \Phi + \Phi \nabla_\mu k^\nu = 0,$$

(34)

$$2k^\mu \nabla_\mu \Psi + \Psi \nabla_\mu k^\nu = i(\square \Phi - \Phi \vec{V}'').$$

(35)

The equations above imply that the squared amplitudes of $(\Upsilon_{\mu\nu}, \Phi)$ scale with the inverse cross sectional area of the particle’s bundle, while $\Psi$ has an additional imaginary source/sink term.

**Observables.**—We can understand the effect of the gravitational and scalar waves on test particles by looking at the geodesic deviation equation in JF.

In the $\epsilon \ll 1$ limit, the JF metric perturbation is $\delta g_{\mu\nu} = H_{\mu\nu} e^{i\theta/\epsilon}$ with

$$\dot{H}_{\mu\nu} = \Omega^2(\phi) \left[ \Upsilon_{\mu\nu} + \frac{2}{\epsilon} \nabla_\rho \frac{\Omega'(\phi)}{\Omega(\phi)} (\Phi + c \Psi) \right]$$

(36)

and the perturbation of the JF Riemann tensor is

$$\delta R^{\rho\sigma\nu\mu} = -\frac{2}{\epsilon^2} k_{[\nu} k_{[\mu} \dot{H}_{\rho]\sigma]} e^{i\theta/\epsilon} +$$

$$\frac{2i}{\epsilon} [k_{[\nu} \nabla_{(\mu} \dot{H}_{\rho)\sigma]} + k_{[\nu} \dot{H}_{(\mu|\rho]} + \dot{H}_{[\sigma} \nabla_{\mu]} k_{\nu]}] e^{i\theta/\epsilon}.$$ 

(37)

where the square brackets stand for antisymmetrization and $\nabla_\mu$ is the covariant derivative associated to the background JF metric. We have verified that the JF Riemann tensor perturbation is invariant under both JF and EF gauge transformations up to order $\epsilon^{-1}$, as it should since it is related to observables. The acceleration between two nearby geodesics is given by the contraction of Eq. (37) with $\bar{u}^\nu \bar{u}^\rho$, the four-velocity of a JF observer. Being $\Upsilon_{\mu\nu}$ orthogonal to $\bar{v}_\mu$ and not $\bar{v}^\rho$, it could be that the polarization content seen by the observer is different than the standard case. Investigating this possibility will be the topic of further works.

Finally, we discuss the detectability of the SW in a screened region. In the low-frequency regime ($\epsilon^2 \gtrsim 1/\sqrt{\bar{V}''}$), it has been shown that Eq. (30) describes a damped wave [31]. Hence, waves in this energy range are screened. In the HF regime ($\epsilon \ll 1$), one has to use Eqs. (33), (34) which show that $\Phi$ is not affected by the background configuration of the scalar field, implying that a HF SW can pass through a screened region. However, the interaction with observers is regulated by the geodesic deviation equation. From Eq. (36) we see that the SW contribution to $\delta R^{\rho\sigma\nu\mu}$ is multiplied by $\Omega'(\phi)/\Omega(\phi)$, whose form depend on the type of screening mechanism. We consider two cases: chameleon and symmetron. Inside screened regions, the former requires $\Omega'/\Omega \sim 1/M$ where $M \sim 10^{-5}$ in units of Planck mass [11, 12], while in the latter requires $\Omega'/\Omega \sim 0$ [33]. Hence, we conclude that SWs would not be detectable in the high-energy limit because their interaction with matter is suppressed.

**Conclusions.**—The growth of matter perturbations and the propagation of GWs are two essential probes of the source of the late-time cosmic accelerated expansion, which must be used jointly. However, they span two distinct energy ranges, and the assumptions used to describe them are very different. The formalism introduced here,
based on the parameter \(\epsilon\), reproduces the results of the low-energy EFT regime (in the range \(\epsilon \lesssim 1\)), where \(\delta \phi\) can be entirely gauged away, and naturally includes the short-wavelength limit (\(\epsilon \ll 1\)) probed by GWs. We connected these two approaches to describe perturbations of an ST theory via two related symmetry-breaking patterns: the acquisition of nontrivial vevs by the fields and the separation of the high- and low-frequency modes. Working in the EF, we showed that the ST theory \([1]\) exhibits three propagating dofs (\(\gamma_{\mu\nu}, \delta \phi, \delta \chi\)) and discussed how the commonly used first-order GO approximation is not self-consistent in the presence of an SSB. This is because the next to leading order scalar field perturbation \(\psi\) mixes with the leading metric perturbation \(h^{0}_{\mu\nu}\) via the gauge transformations. We then derived the general perturbed EoMs. \((29), (30)\) and applied them to discuss the detectability of the additional dof through GW observations. We investigated the cases of chameleon and symmetron screenings, and concluded that the SWs present in these theories are not detectable on Earth no matter their wavelength. In the low-frequency limit, when \(\epsilon^2 \gg 1/V''\), the SW is damped by the nontrivial background profile. While this is not true when \(\epsilon \ll 1\), we showed that screening suppresses the interactions between the SW and matter via the multiplicative factor \(\Omega(\bar{\phi})/\Omega(\phi)\) in Eq \((37)\), making the SW undetectable also in this case. Therefore we conclude that a direct detection of a scalar wave inside a screened region would systematically rule out ST theories based on chameleon or symmetron screenings, and concluded that the SWs present in these theories are not detectable on Earth no matter their wavelength.

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1. Luca Amendola et al. Cosmology and fundamental physics with the Euclid satellite. *Living Rev. Rel.*, 21 (1):2, 2018. doi:10.1007/s41114-017-0010-3
2. Željko Ivezić et al. LSST: from Science Drivers to Reference Design and Anticipated Data Products. *Astrophys. J.*, 873(2):111, 2019. doi:10.3847/1538-4357/ab402f
3. Olivier Doré et al. Cosmology with the SPHEREX All-Sky Spectral Survey. 12 2014.
4. David J. Bacon et al. Cosmology with Phase 1 of the Square Kilometre Array: Red Book 2018: Technical specifications and performance forecasts. *Publ. Astron. Soc. Austral.*, 37:e007, 2020. doi:10.1017/pasa.2019.53
5. B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.*, 116 (6):061102, 2016. doi:10.1103/PhysRevLett.116.061102
6. Jolyon K. Bloomfield, Eanna E. Flanagan, Minjoon Park, and Scott Watson. Dark energy or modified gravity? An effective field theory approach. *JCAP*, 08:010, 2013. doi:10.1088/1475-7516/2013/08/010
7. Giulia Gubitosi, Federico Piazza, and Filippo Vernizzi. The Effective Field Theory of Dark Energy. *JCAP*, 02:032, 2013. doi:10.1088/1475-7516/2013/02/032
8. Jerome Gleyzes, David Langlois, Federico Piazza, and Filippo Vernizzi. Essential Building Blocks of Dark Energy. *JCAP*, 08:025, 2013. doi:10.1088/1475-7516/2013/08/025
9. Bin Hu, Marco Raveri, Noemi Frusciante, and Alessandra Silvestri. Effective Field Theory of Cosmic Acceleration: an implementation in CAMB. *Phys. Rev. D*, 89 (10):103530, 2014. doi:10.1103/PhysRevD.89.103530
10. Marco Raveri, Bin Hu, Noemi Frusciante, and Alessandra Silvestri. Effective Field Theory of Cosmic Acceleration: constraining dark energy with CMB data. *Phys. Rev. D*, 90(4):043513, 2014. doi:10.1103/PhysRevD.90.043513
11. Miguel Zumalacarregui, Emilio Bellini, Ignacy Sawicki, Julien Lesgourgues, and Pedro G. Ferreira. hi_class: Horndeski in the Cosmic Linear Anisotropy Solving System. *JCAP*, 08:019, 2017. doi:10.1088/1475-7516/2017/08/019
12. Tsutomu Kobayashi. Horndeski theory and beyond: a review. *Rept. Prog. Phys.*, 82(8):086901, 2019. doi:10.1088/1361-6633/ab2423
13. Richard A. Isaacs. Gravitational Radiation in the Limit of High Frequency. I. The Linear Approximation and Geometrical Optics. *Phys. Rev.*, 166:1263–1271, 1968. doi:10.1103/PhysRev.166.1263
14. Richard A. Isaacs. Gravitational Radiation in the Limit of High Frequency. II. Nonlinear Terms and the Effective Stress Tensor. *Phys. Rev.*, 166:1272–1279, 1968. doi:10.1103/PhysRev.166.1272
15. Alice Garoffolo, Gianmassimo Tasinato, Carmelita Carbone, Daniele Bertacca, and Sabino Matarrese. Gravitational waves and geometrical optics in scalar-tensor theories. *JCAP*, 11:040, 2020. doi:10.1088/1475-7516/2020/11/040
16. Charles Dalang, Pierre Fleury, and Lucas Lombriser. Scalar and tensor gravitational waves. *Phys. Rev. D*, 103 (6):064075, 2021. doi:10.1103/PhysRevD.103.064075
17. Michele Maggiore. *Gravitational Waves. Vol. 1: Theory and Experiments*. Oxford Master Series in Physics. Oxford University Press, 2007. ISBN 978-0-19-857074-5, 978-0-19-852074-0.
18. Robert Bluhm. Gravity with background fields and diffeomorphism breaking. In *14th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics, and Relativistic Field Theories*, 1 2016. doi:10.1142/97898132266090100
19. Shaoqi Hou, Yungui Gong, and Yunqi Liu. Polarizations of Gravitational Waves in Horndeski Theory. *Eur. Phys. J. C.*, 78(5):378, 2018. doi:10.1140/epjc/s10052-018-5869-y
20. D. M. Eardley, D. L. Lee, and A. P. Lightman. Gravitational-wave observations as a tool for testing relativistic gravity. *Phys. Rev. D*, 8:3308–3321, 1973. doi:10.1103/PhysRevD.8.3308
21. Enis Belgacem et al. Testing modified gravity at cosmological distances with LISA standard sirens. *JCAP*, 07:
2021. doi:10.1088/0034-4885/79/4/046902

[22] Enis Belgacem, Yves Dirian, Stefano Foffa, and Michele Maggiore. Modified gravitational-wave propagation and standard sirens. Phys. Rev. D, 98(2):023510, 2018. doi:10.1103/PhysRevD.98.023510

[23] Gianmassimo Tasinato, Alice Garoffolo, Daniele Bertacca, and Sabino Matarrese. Gravitational-wave cosmological distances in scalar-tensor theories of gravity. JCAP, 06:050, 2021. doi:10.1088/1475-7516/2021/06/050.

[24] Saeed Mirshekari, Nicolas Yunes, and Clifford M. Will. Constraining Generic Lorentz Violation and the Speed of the Graviton with Gravitational Waves. Phys. Rev. D, 85:024041, 2012. doi:10.1103/PhysRevD.85.024041.

[25] M. Punturo et al. The Einstein Telescope: A third-generation gravitational wave observatory. Class. Quant. Grav., 27:194002, 2010. doi:10.1088/0264-9381/27/19/194002.

[26] David Reitze et al. Cosmic Explorer: The U.S. Contribution to Gravitational-Wave Astronomy beyond LIGO. Bull. Am. Astron. Soc., 51(7):035, 2019.

[27] B. P. Abbott et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. Phys. Rev. Lett., 119(16):161101, 2017. doi:10.1103/PhysRevLett.119.161101.

[28] Claudia de Rham and Scott Melville. Gravitational Rainbows: LIGO and Dark Energy at its Cutoff. Phys. Rev. Lett., 121(22):221101, 2018. doi:10.1103/PhysRevLett.121.221101.

[29] K. Koyama. Cosmological Tests of Modified Gravity. Rept. Prog. Phys., 79(4):046902, 2016.