Improved Bayesian Network and Its Application in Autonomous Capability Evaluation

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Abstract: The Bayesian network (BN) is a new intelligent evaluation technology, which has become one of the effective methods to solve uncertainty problems and deal with asymmetric information. However, its application is limited in the case of complex indicator systems and interrelated variables. To solve these problems, an improved BN model based on game theory is proposed. First, an IHORAFA attribute reduction algorithm is used to optimize the index system. Then, a weighted BN evaluation model is proposed for the problem of correlation among indicators, which uses the improved combination weighting method of game theory to determine the optimal weight and improve the accuracy of weight calculation. Finally, the improved BN is applied to the autonomous capability evaluation of ground attack UAVs. The simulation results show that the improved BN model can be used for assessment and reasoning under uncertain conditions and variable correlation.

Keywords: Bayesian network; combination weighting method; game theory; IHORAFA; autonomous capability

1. Introduction

Comprehensive evaluation is a systematic and complex work, one of the most important means by which we know things, understand them, and influence them [1]. Generally, in it can be used qualitative methods and quantitative methods. Classic methods mainly include analytic hierarchy process (AHP) [2], principal component analysis [3], and grey relational analysis [4]. AHP is a multi-objective decision analysis method combining qualitative and quantitative analysis. The principal component analysis method can use the idea of dimension reduction to eliminate the correlation among indicators and find the main components containing the original indicator information. Based on the grey system theory, the grey relational analysis method can solve the uncertainty problems of less data and unclear internal relations. However, these classic methods have some disadvantages. AHP is mainly based on expert knowledge and has strong subjectivity. Principal component analysis requires precise data and the results are difficult to interpret. In the grey relational analysis method, it is difficult to determine the optimal sequence and its resolution is low.

Moreover, classic methods mainly consider the static characteristics of evaluation objects and lack the ability to perform dynamic evaluation and reasoning analysis. It cannot meet the needs of different evaluation stages.

(1) The forward problem.
We make a prediction on the expected benefits or risks of the evaluation event in the initial stage of the event to achieve the set goals and effects.

(2) The intermediate interference problem.
In the mid-stage, reasoning and analysis of unexpected situations to achieve effective monitoring of the evaluation process.
(3) The inverse problem.
Finding the key influencing factors at the end of the event to facilitate decision makers to adjust subsequent strategies and maximize benefits.

These problems lead to the fact that the values of evaluation object attributes vary from time to time and are in a dynamic change process, which shows the characteristics of asymmetry. Moreover, due to the existence of many uncertainties in the process of events, the construction of the evaluation model is difficult, and the reasoning ability of the evaluation model is required to be high. Therefore, we need to build an evaluation model with a strong reasoning ability to deal with these uncertainties.

The Bayesian network, as an effective intelligent algorithmic model with powerful uncertain knowledge representation and inference analysis capabilities, can achieve evaluation and decision making of uncertain information under complex conditions. It has been widely applied in risk assessment [5], medical diagnosis [6], and artificial intelligence [7]. Many scholars have confirmed the feasibility of applying BN in the field of evaluation. Huang et al. [8] constructed an urban flood hazard evaluation model based on BN and analyzed the key contributing factors. Zhang et al. [9] established a BN reliability evaluation model and formulated hierarchical management measures for ventilation systems based on the simulation results. Chen [10] realized the evaluation and inference of the operational effectiveness of unmanned aerial vehicle (UAV) formations under uncertain conditions based on BN. However, the classic BN still has limitations in handling uncertainty. It cannot solve the problem of strong correlation among variables [11]. Therefore, this paper improves the BN model based on existing research results. The innovation points of this paper are summarized as follows.

(i) To address the problem of cumbersome indicators of complex systems, an IHORAFA (improved heuristic optimal reduce algorithm based on frequencies of attributes) attribute reduction algorithm is proposed.
(ii) To address the problem of correlation among indicators, a weighted BN evaluation model is proposed, which weakens the conditional assumption of classic BN and enhances the applicability of the model. The optimal combination weights are determined using an improved game theory combination weighting method to improve the accuracy of the weight calculation.
(iii) An improved BN method based on the combination weighting method is proposed.

In this paper, an improved BN with decorrelation and attribute reduction ability is proposed to solve the problem of variables correlation and complex index system. A combination weighting method is used to determine the weights of weighted BN in an attempt to weaken the conditional assumptions in traditional BN and enhance the application of BN. An IHORAHA algorithm is proposed for attribute reduction of the index system. Finally, to illustrate the proposed methods, we apply them to UAV autonomous capability evaluation and use causal reasoning, influencing factor reasoning, and truncated analysis reasoning to analyze and reason for different task stages of UAV.

The chapters of this paper are organized as follows: Section 2 introduces the basic theory and evaluation process of BN. In Section 3, an improved BN and IHORAFA algorithm are proposed. A combination weighting method based on game theory is used for the optimal weights. In Section 4, to verify the rationality of the model, the proposed method is applied to the evaluation of the autonomous capability of UAV. The conclusions of this paper are given in Section 5.

2. Bayesian Network and Its Evaluation Process

BN is a good tool to describe and deal with uncertainty. It has a solid mathematical foundation and can model the uncertainty in the evaluation process. We first introduce its basic concepts and reasoning patterns, and then introduce the BN evaluation process and analyze the problems of classic BN.
2.1. Bayesian Network Theory

Bayesian network, also known as reliability network, is based on probability theory and graph theory [12]. It uses directed edges to represent the causal relationship between variables and a conditional probability table to represent the strength of the relationship between variables. In the case of uncertain knowledge and incomplete information, the powerful reasoning capability of BN can effectively fuse multi-source information for decision analysis, reasoning, and dynamic evaluation of uncertainty problems. We introduce three reasoning patterns of BN:

1) Causal reasoning: a forward reasoning process, i.e., the outcome is inferred from the causal information and the conditional probability distribution of the child nodes is inferred from the conditional probability of the parent nodes. Suppose that the set of evidence consisting of all parent nodes is $E_C$, then the probability that the state of the child node $M$ is $M_j$ defined by

$$P(M = M_j|E_C) = P(M = M_j|E_1 = e_1, E_2 = e_2, \ldots, E_n = e_n)$$

$$= \frac{P(M = M_j, E_1 = e_1, E_2 = e_2, \ldots, E_n = e_n)}{P(E_1 = e_1, E_2 = e_2, \ldots, E_n = e_n)}$$ (1)

where $E_i \in E_C, P(E_i = e_1, E_2 = e_2, \ldots, E_n = e_n)$ is the joint probability of nodes with known state, $n$ is the number of parent nodes with known status, $P(M = M_j|E_1 = e_1, E_2 = e_2, \ldots, E_n = e_n)$ is the conditional probability of forward conduction.

2) Influence factor reasoning: also known as diagnostic reasoning. It is reverse reasoning from result to cause. By changing the conditional probability of the child node, the probability of each state of the relevant parent node can be obtained, and then we can analyze the degree of influence of the child index on the upper index. Suppose that the state of the child node is known to be $M = M_j$, the posterior probability distribution of the parent node is

$$P(E_i = e_i|M = M_j) = \frac{P(E_i = e_i|M = M_j)}{P(M = M_j)}$$ (2)

3) Truncated analytical reasoning: a reasoning pattern that integrates forward reasoning ability and reverse diagnosis ability. When the values of intermediate index nodes are fixed, on the one hand, the state of evaluation index nodes can be obtained by forwarding reasoning, and on the other hand, the influence degree of sub-indexes can be obtained by reverse diagnosis.

2.2. Basic Process of BN Evaluation

Comprehensive evaluation with BN is a complex process, which mainly includes node selection, data processing, structure determination, parameter learning, reasoning, and simulation, and its basic process is shown in Figure 1. The BN model corresponds to the comprehensive evaluation process in each step, which proves the feasibility of its application in the field of comprehensive evaluation.

However, the classic BN still has certain defects, which greatly limits its application in practical problems [13]. The network structure and parameters of BN need to be adjusted in combination with the actual problems and the characteristics of the evaluation object. Moreover, the mathematical basis of BN is the Bayesian theorem, which is premised on the assumption of conditional independence, i.e., there is no correlation among any nodes or indexes. This assumption is difficult to hold in practical applications, which leads to the inability of classic BN to deal with the correlation among variables, which greatly affects the final reasoning accuracy. Therefore, our next step is to find a suitable method to improve the conditional independence assumption and carry out structural optimization and parameter learning in combination with practical problems.
3. An Improved BN Evaluation Method Based on Combined Weighting of Game Theory

Taking into account the complexity of the index system, we propose an IHORAFA attribute reduction algorithm to optimize the indicator structure. To improve the assumption of conditional independence and enhance the applicability of BN, the combination weighting method of game theory is used to determine the optimal weight.

3.1. An Improved HORAF Attribute Reduction Algorithm

In practical application, the one-sided pursuit of the comprehensiveness of the index system will lead to overly complex calculations, which do not highlight the main performance indexes. In addition, with the change of evaluation stage, some indexes may lose their practical significance, resulting in the whole index system not adapting to the new situation and causing delusion and confusion in the judgment of evaluators. At this point, it is necessary to optimize and adjust the index system to ensure the validity and real-time performance of the indexes.

Hu et al. [14] proposed a heuristic optimal reduction algorithm based on frequencies of attributes (HORAF). HORAF takes the frequency of attributes as the reduction basis, which avoids the calculation of complex concepts of rough sets and improves computational efficiency. The basic idea is that the information system is reduced by using the frequency of attributes in the discernibility matrix as heuristic information. The discernibility matrix is defined as

\[
m_{ij} = \begin{cases} \{a \in C : a(x_i) \neq a(x_j)\}, & x_i, x_j \in U \\
n & \text{else} \end{cases}
\]

where \( m_{ij} \) is the element of the discernibility matrix, \( a(x) \) is the value of record \( x \) on attribute \( a \), namely, \( a(x) = f(x, a) \). The discernibility matrix is a symmetric matrix, i.e., \( m_{ij} = m_{ji} \).

The more frequently an attribute appears in the discernibility matrix, the greater the ability of the attribute to differentiate the research object; the shorter the length of the element item in which the attribute is located, the more important the attribute is. On the basis of these two criteria, the attribute frequency function is defined as

\[
f(a) = f(a) + |A|/|c|
\]

where \(|A|\) is the total number of condition attributes in a discernibility matrix, \(|c|\) is the number of attributes contained in non-empty element items.
HORAFA is an incomplete algorithm, which may not find the optimal reduction of information systems in practical application. This paper makes the following improvements.

1. HORAFA is based on the discernibility matrix of Equation (3), which does not take into account the incompatibility of information systems and is not applicable to incompatible decision tables. To solve this problem, the YE discernibility matrix \[15\] is introduced, which is defined as

\[
m_{ij} = \begin{cases} 
\{a \mid \{a \in C : a(x_i) \neq a(x_j), D(x_i) \neq D(x_j)\} \text{ and } \min\{d(x_i), d(x_j)\} = 1\} & \text{if } \emptyset, \\
\text{else} & 
\end{cases}
\] (5)

where \(d(x_i)\) represents the cardinality of the attribute value set corresponding to all elements equivalent to \(x_i\) in \(U\).

2. To solve the problem that the HORAFA does not necessarily find the optimal reduction, the attribute frequency function is updated as follows.

\[
f(a) = f(a) + |A|/|c'|
\] (6)

where \(|c'|\) is the remaining number of attributes that are removed from the ones that join the core after reduction.

3. To address the incompleteness of HORAFA, the reverse elimination method is used to delete the attributes that can be deleted in the reduction. At the same time, this paper improves the reverse elimination algorithm. As the core attributes are necessary and cannot be deleted, it is not necessary to judge the core attributes. Only the reverse elimination of the noncore attributes in the reduction is required, thus, reducing the computation and improving the efficiency of reduction.

Based on the above improvements, an improved HORAF (IHORAF) algorithm is proposed. The specific process is shown in Algorithm 1.

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### Algorithm 1: IHORAF algorithm

**Input:** Decision table \(S = \{U, A, V, f\}, A = C \cup D\)  
**Output:** Reduced attribute set \(R\)

1. **initialization** \(R = \emptyset, Core = \emptyset, f(a_i) = 0 (i = 1, 2, \ldots, n)\);
2. **generate the difference matrix** \(M\) according to Equation (6);
3. **merge the same elements in** the matrix;
4. **add the attribute with only one element** to \(Core\), Namely, \(Core = Core \cup \{a_i\}\);
5. if \(|M| \neq 0\) then
   6. for \(i = 1 \to |C - R|\) do
      7. calculate \(f(a_i)\);
      8. end
   9. \(a = \max\{f(a_i)\}\);
   10. \(R = R \cup \{a\}\);
   11. delete the element with attribute \(a\) in \(M\);
   12. recalculate \(|M|\)
   13. end
   14. for \(i = 1 \to |C - R|\) do
      15. if \(a_i \in R - Core\) and \(POS_{R - \{a_i\}}(D) = POS_C(D)\) then
      16. \(R = R - \{a_i\}\)
      17. end
   18. end

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3.2. Weighted BN Based on Combination Weighting of Game Theory

BN is not applicable to the inference and evaluation of strongly correlated variables. In this section, we analyze the limitations of classic BN and the existing weighted BN and then
use the improved game theory combined weighting method to improve the conditional independence assumption.

3.2.1. Problem Analysis of Classic BN

The assumption of conditional independence limits the application of naive Bayesian networks. To address this problem, this paper determines the optimal weights of indexes based on game theory and constructs the weighted BN.

The premise of BN algorithm is the assumption of conditional independence, which means that each child node in the network is independent of other unrelated nodes under a given conditional node. Based on this assumption, when the prior probability and conditional probability table (CPT) are given, the posterior probability can be inferred by Equation (7).

\[
P(x|X_1, X_1, \ldots, X_n) = P(x) \prod_{i=1}^{n} P(X_i|x)
\]

where \(P(x|X_1, X_1, \ldots, X_n)\) is the posterior probability, \(P(x)\) is the a priori probability, \(P(X_i|x)\) is the conditional probability. The assumption of conditional independence improves the operational efficiency and probabilistic reasoning speed of BN, but there are too many factors and there is a certain correlation among the sub-indexes, so the assumption of conditional independence needs to be weakened or improved. An effective way to solve this problem is to assign different weights to each node to strengthen the connection among the nodes. The model obtained by this method is called a weighted Bayesian network and the formula is

\[
P(x|X_1, X_1, \ldots, X_n) = P(x) \prod_{i=1}^{n} P(X_i|x)^{w_i}
\]

Weight calculation is an important basis for weighted BN. Liu et al. [16] constructed weighted BN based on the entropy weight method to assess flood hazard risk effectively. Zhao et al. [17] applied weighted BN to the prediction of pancreatic cancer, which improves the accuracy of the prediction of pancreatic cancer. However, the existing research mainly uses a single weight calculation method, and the scientific nature of index assignment needs further study.

3.2.2. Combination Weighting Method Based on Game Theory

The subjective-objective combination weighting method has been a hot research topic in recent years. It can combine the advantages of the subjective weighting method and objective weighting method, considering the actual laws of objective data and reflecting the decision-making intention of the evaluator. Therefore, based on the improvement of the AHP and Entropy Weighting (EW) method, this paper introduces the idea of game theory to determine the optimal combination weights.

Subjective Weighting by IAHP

AHP is a combined qualitative and quantitative decision-making analysis method, but it is highly subjective and requires several consistency tests and correction of the judgment matrix, leading to cumbersome calculations [18]. The classic AHP is now improved to address this problem, and the steps of the improved AHP (IAHP) method are as follows.

Step 1: Construct the judgment matrix.

\[
A = (a_{ij})_{n \times n}
\]

where \(a_{ij}\) is the relative importance of factor \(i\) to \(j\), and satisfies \(a_{ii} = 1, a_{ij} > 0, a_{ij} = 1\).

Step 2: According to formula \(b_{ij} = \lg a_{ij}\), calculate the antisymmetric matrix \(B\) of the judgment matrix \(A\). Its characteristic is \(b_{ij} = -b_{ji}\).
Step 3: The optimal transfer matrix $C$ of the antisymmetric matrix $B$ is obtained according to Equation (10), so that $\sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} - b_{ij})^2$ is minimum.

$$c_{ij} = \frac{1}{n} \sum_{k=1}^{n} (b_{ik} - b_{jk})$$

(10)

Step 4: Construct the optimization matrix $A^*$, where $a_{ij}^* = 10^{c_{ij}}$.

Step 5: Calculate the weight vector.

1. Normalize the optimization matrix $A^*$.

$$a_{ij}^* = a_{ij}^* \frac{1}{\sum_{i=1}^{n} a_{ij}^*}$$

(11)

2. Add by row to get the sum vector.

$$W_i' = \sum_{j=1}^{n} a_{ij}^*$$

(12)

3. Normalize the sum vector to obtain the eigenvector of the optimization matrix, i.e., the weight vector.

$$W_i = \frac{W_i'}{\sum_{i=1}^{n} W_i'}$$

(13)

Objective Weighting by IEW

EW is a widely used objective weighting method, which is susceptible to extreme data. A small difference in information entropy can lead to an exponential change in entropy weight when the entropy value is in a specific interval [19]. Therefore, this paper introduces the standard processing method to eliminate the extreme value interference and improves the formula for calculating weights by information entropy. The steps to improve the entropy weight (IEW) method are as follows.

Step 1: Build raw data matrix $R'$.

$$R' = (x_{ij})_{n \times m} \ (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$$

(14)

where $m$ is the number of evaluation indexes, $n$ is the number of data samples, and $x_{ij}$ represents the value of the $j$th evaluation index of the $i$th sample.

Step 2: The standardized processing method is adopted to eliminate extreme-value interference, and the formula is

$$x_{ij}' = \frac{x_{ij} - \bar{x}_j}{s_j}$$

(15)

where $\bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij}$ is mean value of $j$th index, $s_j = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}$ is standard deviation of $j$th index.

Step 3: Non-negativity processing of indexes. EW requires index values to be positive, so the translation method makes the calculation of the information entropy meaningful. The formula is

$$x_{ij}^+ = x_{ij}' + l$$

(16)

where $x_{ij}^+$ is the value of index $j$ after non-negativity processing, $l$ is the translation distance, which needs to be selected according to the actual situation.
Step 4: Normalization processing.

\[ R = (y_{ij})_{n \times m} \quad (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \] (17)

\[ y_{ij} = \frac{x_{ij}^+}{\sum_{i=1}^{n} x_{ij}^+} \] (18)

Step 5: Calculate the information entropy of each index.

\[ H_j = -k \sum_{i=1}^{n} [y_{ij} \ln(y_{ij})] \] (19)

where \( k = 1/\ln n, i = 1, 2, \ldots, n \).

Step 6: Calculate index weights.

\[ w_j = \frac{1 - H_j}{\sum_{j=1}^{m} (1 - H_j)} \quad \sum_{j=1}^{m} w_j = 1 \] (20)

It should be noted that when the weights are calculated according to Equation (20), a small change of information entropy can lead to the abnormal phenomenon of large differences in the entropy weights when \( H_j \to 1 \). To solve this problem, the formula for calculating the entropy weight is improved, and the improved formula is

\[ w_j = \frac{1}{\sum_{j=1}^{m} (H_k + 1 - 2H_j)} \quad \sum_{j=1}^{m} w_j = 1 \] (21)

Combination Weighting

For the research on the combination of weighting, existing research mainly adopts the methods of addition synthesis and multiplication synthesis to synthesize different weights, but does not study how to coordinate the conflicts between different weighting methods to achieve the optimal or suboptimal combination of weights. The combination weighting method based on game theory uses the idea of game theory to resolve the conflicts between subjective and objective weighting methods, find the balance point between them, and maximize the common interests, so as to make the index weighting more scientific and reasonable [20]. The specific process is as follows.

The index weights obtained by IAHP and IEW are indicated as \( w_1 = (w_{11}, w_{12}, \ldots, w_{1n}) \) and \( w_2 = (w_{21}, w_{22}, \ldots, w_{2n}) \), respectively, and the combined weight can be expressed by a linear combination of \( w_1 \) and \( w_2 \).

\[ w = \alpha_1 w_1^T + \alpha_2 w_2^T \] (22)

where \( \alpha_1 \) and \( \alpha_2 \) represent the combination coefficients of subjective and objective weights, respectively.

We can find a balance between different weights and minimize the deviation between the combined weight and the subjective and objective weights. The optimization model is

\[ \min_{r=1, 2} \left\| \sum_{k=1}^{m} \alpha_k w_k^T - w_r^T \right\|, r = 1, 2 \] (23)

By solving the model, the optimal combination weight that takes into account both subjective factors and laws of objective data can be obtained. According to the differential
property of the matrix, to avoid negative combination coefficients, the optimal condition of the improved combination weighting method based on game theory is

$$\min f = \sum_{r=1}^{2} \left| \left( \sum_{k=1}^{2} a_k w_r w_k^T \right) - w_r w_r^T \right|$$  \hspace{1cm} (24)$$

where $a_k > 0$ and $\sum_{k=1}^{2} a_k^2 = 1$. The Lagrange function is established as

$$L(a_k, \lambda) = \sum_{r=1}^{2} \left| \left( \sum_{k=1}^{2} a_k w_r w_k^T \right) - w_r w_r^T \right| + \lambda \left( \sum_{k=1}^{2} a_k^2 - 1 \right)$$  \hspace{1cm} (25)$$

The solution is

$$a_k = \frac{\sum_{r=1}^{2} w_r w_k^T}{\sqrt{\sum_{k=1}^{2} \left( \sum_{r=1}^{2} w_r w_k^T \right)^2}}$$  \hspace{1cm} (26)$$

Based on IHORAFA and combination weighting, the improved BN can be applied to the evaluation under dynamic conditions and can better deal with the strong correlation among the indexes. The differences between improved BN and classic BN are shown in Table 1.

Table 1. Comparison between the improved BN and classic BN models.

| Model Conditions | Dynamic Condition and Strong Variable Relation | Model Solving Process | Structural Learning | Parameter Learning | Reasoning Mode |
|------------------|-----------------------------------------------|-----------------------|--------------------|--------------------|----------------|
| Classic BN       | Dynamic Condition and Strong Variable Relation | Manual construction based on knowledge | Expert score | Simple reasoning |
| Improved BN      | Dynamic Condition and Strong Variable Relation | IHORAFA | Membership weighting | Weighted reasoning |

3.2.3. The Basic Process of Improved BN Evaluation

According to the proposed method, the improved BN evaluation process is shown in Figure 2. First, to address the problem that the underlying data is difficult to deal with, the membership weighting method is used to determine the prior probability of the root node, and the main steps are shown in Algorithm 2. Secondly, we use the combination weighting method of game theory to determine the optimal weight and construct a weighted BN model. The IHORAFA algorithm is used to optimize the index system. Finally, the improved BN is applied to actual problems, and the simulation reasoning is carried out at different evaluation stages to solve the three uncertainty problems mentioned in the introduction.
Algorithm 2: A priori probability algorithm of root node based on membership weighting

| Input: Sub-index data \( \{X_1, X_2, \ldots, X_m\} \), where \( X_j = (x_{1j}, x_{2j}, \ldots, x_{nj})^T \); the attribute level of the root node \( V = \{V_1, V_2, \ldots, V_S\} \).
| Output: The prior probability of the root node \( \{P_1, P_2, \ldots, P_S\} \).

1. for \( j = 1 \) to \( m \) do

2. \[ \bar{x}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij} \]

3. \[ s_j^2 = \frac{1}{n} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2 \]

4. \[ \omega_j = s_j / \sum_{j=1}^{m} s_j \]

5. end

6. for \( t = 1 \) to \( S \) do

7. determine the membership degree \( \gamma_{tj} \) of the sub-index \( j \) belonging to grade \( V_t \)

8. calculate the prior probability of the root node at level \( V_t \): \( P_t = \sum_{j=1}^{m} \gamma_{tj} \omega_j \)

9. end

4. Model Application

In the intelligent war of the future, ground attack UAVs are one of the key combat forces in asymmetric warfare. An effective evaluation of the autonomous capability of ground attack UAVs in complex battlefield environments can accurately locate the comprehensive performance and technical deficiencies of UAVs, thus, guiding the development and improvement of subsequent models and laying a solid foundation for UAVs to adapt to complex battlefield environments and mission requirements [21]. In this section, we evaluate and reason about the autonomous capability of UAVs in different mission stages to verify the effectiveness of the method.
4.1. Indicator System Construction and Optimization

Autonomous capability is the combination of perception, analysis, communication, planning, decision-making, and behavior performed by unmanned systems, reflecting the degree of intelligence of unmanned systems [22]. Considering the connotation of autonomous capability [23] and the characteristics and operational processes of ground attack UAVs [24], the index system of the autonomous capability of ground attack UAVs is constructed from five aspects: sensing and detection, planning and decision making, operational execution, security management, and learning and evolution, as shown in Figure 3.

The index system is strongly relative in nature. It varies from time to time and from place to place, and the structure of the index system needs to be dynamically adjusted in relation to the requirements of the mission stage before evaluating autonomous capability. The index system shown in Figure 3 is based on the overall situation of combat tasks, but it is not necessarily optimal in each mission stage. Therefore, while taking into account the comprehensiveness and representativeness of the evaluation index, IHORAF is used to simplify the index system and obtain the reduced index shown in Table 2, taking into account the characteristics and actual situation of three typical combat mission stages, namely, the penetration stage, the reconnaissance and search stage, and attack stage (marked as S1, S2, and S3, respectively).
Table 2. Reduction index of each operation stage.

| Index | S1 | S2 | S3 |
|-------|----|----|----|
| B1    | ✓  | ✓  | ✓  |
| B2    | ✓  | ✓  | ✓  |
| B3    | ✓  | ✓  | ✓  |
| B4    | ✓  | ✓  | ✓  |
| B5    | ✓  | ✓  | ✓  |
| B6    | ✓  | ✓  | ✓  |
| B7    | ✓  | ✓  | ✓  |
| B8    | ✓  | ✓  | ✓  |
| B9    | ✓  | ✓  | ✓  |
| B10   | ✓  | ✓  | ✓  |
| B11   | ✓  | ✓  | ✓  |
| B12   | ✓  | ✓  | ✓  |
| B13   | ✓  | ✓  | ✓  |
| B14   | ✓  | ✓  | ✓  |
| B15   | ✓  | ✓  | ✓  |
| B16   | ✓  | ✓  | ✓  |
| B17   | ✓  | ✓  | ✓  |
| B18   | ✓  | ✓  | ✓  |
| B19   | ✓  | ✓  | ✓  |

4.2. Indicator System Construction and Optimization

According to the previous research [25], the autonomous capability level is divided into five levels (I, II, III, IV, and V), which are simple planning task, complex planning task, real-time planning task, multi-UAV cooperation, and clustering cooperation. The classification and specific connotation of autonomous capability level are shown in Table 3.

Table 3. Classification standard of autonomous capability.

| Level                      | Perceptual Detection Capability | Planning and Decision-Making Capability | Operational Execution Capability | Safety Management Capability | Learning Evolutionary Capability |
|----------------------------|---------------------------------|----------------------------------------|-----------------------------------|----------------------------|---------------------------------|
| I: Simple planning task   | Observe ground target specifically | Pre-programmed planning task | Single attack | Report status | Data processing |
| II: Complex planning task | Situation awareness | Adaptive planning | Single attack and damage assessment | Real-time fault diagnosis and isolation | Program automation |
| III: Real-time planning task | Complex environment perception | Route replanning | Timely threat avoidance | Simple troubleshooting | Computational intelligence |
| IV: Multi-UAV cooperation | Multi-machine information sharing | Tactical decision by leader | Multi-machine assisted attack | Fault prediction and fault tolerance control | Simple Thinking Wisdom |
| V: Clustering cooperation | Distributed/clustering situational awareness | Strategic decision | Clustering attack | Clustering diagnosis | Self-directed learning |

The priori probability of the root node can be obtained by expert scoring and historical data. Taking the node $B_1$ as an example, a group of experts in UAV development and use was invited to score the performance of the indexes under different tasks. A fuzzy
A classification method was used to establish a set of attribute levels. After discussion, it was specified that a score of 90 or more is good, 80–90 is average, and below 80 is poor. The scoring table is shown in Table 4.

Table 4. Scoring table of root node $B_1$.

| Expert Number | Task 1 | Task 2 | Task 3 | Task 4 |
|---------------|-------|-------|--------|--------|
| Expert 1      | 94    | 90    | 85     | 83     |
| Expert 2      | 81    | 95    | 88     | 78     |
| Expert 3      | 93    | 85    | 85     | 75     |
| Expert 4      | 82    | 86    | 90     | 80     |
| Expert 5      | 95    | 93    | 91     | 86     |
| Expert 6      | 97    | 96    | 86     | 75     |
| Expert 7      | 94    | 95    | 78     | 73     |
| Expert 8      | 80    | 82    | 77     | 75     |
| Expert 9      | 95    | 84    | 86     | 80     |
| Expert 10     | 96    | 81    | 78     | 82     |

According to the index data given in Table 4, the weight of the root node $B_1$ can be calculated by Step 1 of Algorithm 2.

$$\omega = \{0.3485, 0.2576, 0.2201, 0.1738\}$$  \hspace{1cm} (27)

Based on the index data in Table 4 and the specified attribute classification, the membership degree of the node $B_1$ can be obtained, as shown in Table 5.

Table 5. Index membership of the node.

| Task    | Good | General | Bad |
|---------|------|---------|-----|
| Task 1  | 0.7  | 0.3     | 0   |
| Task 2  | 0.5  | 0.5     | 0   |
| Task 3  | 0.2  | 0.5     | 0.3 |
| Task 4  | 0    | 0.5     | 0.5 |

According to Step 2 of Algorithm 2, the prior probability of node $B_1$ is calculated as $P = \{0.417, 0.430, 0.153\}$. The prior probabilities of other root nodes are shown in Table 6.

Table 6. A priori probability of root nodes.

| Node | Good   | General | Bad    |
|------|--------|---------|--------|
| $B_1$| 0.417  | 0.430   | 0.153  |
| $B_2$| 0.455  | 0.326   | 0.219  |
| $B_3$| 0.408  | 0.309   | 0.283  |
| $B_4$| 0.449  | 0.308   | 0.243  |
| $B_5$| 0.401  | 0.349   | 0.250  |
| $B_6$| 0.465  | 0.298   | 0.237  |
| $B_7$| 0.398  | 0.368   | 0.234  |
| $B_8$| 0.456  | 0.321   | 0.223  |
| $B_9$| 0.417  | 0.430   | 0.153  |
| $B_{10}$| 0.350 | 0.630   | 0.290  |
| $B_{11}$| 0.400 | 0.356   | 0.244  |
| $B_{12}$| 0.298 | 0.356   | 0.646  |
| $B_{13}$| 0.329 | 0.367   | 0.304  |
| $B_{14}$| 0.364 | 0.302   | 0.334  |
| $B_{15}$| 0.468 | 0.356   | 0.176  |
| $B_{16}$| 0.265 | 0.296   | 0.439  |
| $B_{17}$| 0.389 | 0.367   | 0.244  |
| $B_{18}$| 0.254 | 0.216   | 0.530  |
| $B_{19}$| 0.302 | 0.369   | 0.329  |
The improved combination weighting method is used to calculate the index weights according to the method described in Section 3, and the results are shown in Table 7. We can obtain the weighted CPT by integrating the weights of each index according to Equation (8), such as the CPT of node $A_1$ is shown in Table 8.

### Table 7. Weights of the evaluation index.

| Index | Subjective Weight | Objective Weight | Combined Weight |
|-------|-------------------|------------------|-----------------|
| $B_1$ | 0.0726            | 0.0948           | 0.0896          |
| $B_2$ | 0.0717            | 0.0227           | 0.0341          |
| $B_3$ | 0.0599            | 0.0564           | 0.0572          |
| $B_4$ | 0.0404            | 0.0412           | 0.0410          |
| $B_5$ | 0.0718            | 0.0734           | 0.0730          |
| $B_6$ | 0.0678            | 0.0398           | 0.0463          |
| $B_7$ | 0.0311            | 0.0315           | 0.0314          |
| $B_8$ | 0.0441            | 0.0153           | 0.0220          |
| $B_9$ | 0.0638            | 0.0288           | 0.0370          |
| $B_{10}$ | 0.0687          | 0.0754           | 0.0738          |
| $B_{11}$ | 0.0554           | 0.0865           | 0.0792          |
| $B_{12}$ | 0.0680           | 0.0231           | 0.0336          |
| $B_{13}$ | 0.0372           | 0.0154           | 0.0205          |
| $B_{14}$ | 0.0501           | 0.0697           | 0.0651          |
| $B_{15}$ | 0.0552           | 0.0543           | 0.0545          |
| $B_{16}$ | 0.0384           | 0.0715           | 0.0638          |
| $B_{17}$ | 0.0527           | 0.0886           | 0.0879          |
| $B_{18}$ | 0.0293           | 0.0671           | 0.0583          |
| $B_{19}$ | 0.0219           | 0.0345           | 0.0316          |

### Table 8. Weighted CPT of node $A_1$.

| $B_1$  | $B_2$  | $B_3$  | $B_4$  | $A_1$  |
|--------|--------|--------|--------|--------|
|        |        |        |        | High   | Medium | Low    |
| High   | High   | High   | High   | 0.97   | 0.03   | 0      |
| High   | High   | High   | Medium | 0.91   | 0.07   | 0.02   |
| High   | High   | High   | Low    | 0.85   | 0.11   | 0.04   |
| Low    | Low    | Low    | Low    | 0      | 0.01   | 0.99   |

4.3. Simulation Reasoning Based on Task Stage

The combat mission is the key factor driving the dynamic change of UAV autonomous capability. After establishing the BN model, the inference and analysis of UAV autonomous capability need to be combined with the combat phase to uncover deep-seated combat information. In this section, three reasoning patterns of BN are used to analyze the autonomy of the three processes: before, during, and after the combat mission, respectively.

4.3.1. Simulation Reasoning Based on Task Stage

The required autonomous capability for this mission is estimated before the operational mission to achieve overall control of the battlefield situation. We input the parameters of nodes and adopt causal reasoning with Equation (1). Moreover, to verify the scientificity and rationality of the method, the method in this paper is compared with other methods, as shown in Table 9.

From Table 9, we can get that the probability of UAV’s autonomous capability of level III is 51.93%, level II is 44.41%, and level I is 3.17%. According to the principle of maximum probability membership, the autonomous capability level of the ground attack UAV is level III, indicating that performing this task requires a high level of autonomous capability. Meanwhile, compared with other methods, the evaluation results of the improved BN are consistent. Because the improved BN considers the strong correlation among variables, the
difference among levels is more obvious, and it is easy to distinguish UAVs with similar autonomous capabilities, which reduces the ambiguity of the evaluation results.

Table 9. Reasoning results by different BN models.

|                      | I   | II  | III | IV  | V   | Evaluation Results |
|----------------------|-----|-----|-----|-----|-----|--------------------|
| Classic BN           | 0.0461 | 0.4607 | 0.4865 | 0.0036 | 0.0031 | III                |
| Weighted BN with entropy method | 0.0374 | 0.4498 | 0.5076 | 0.0029 | 0.0023 | III                |
| Improved BN          | 0.0317 | 0.4441 | 0.5193 | 0.0027 | 0.0022 | III                |

4.3.2. Battlefield Situation Deduction in Combat Mission

With the advancement of the mission process, UAVs will face uncertain events brought about by the complex and changing battlefield environment, so the autonomous capability is dynamic. How to adapt to the rapidly changing battlefield scenarios and effectively analyze, summarize and judge the multiple information has become the key to autonomous capability decision-making during the combat process.

After determining the dynamic structure, it is necessary to deduce the situation according to the battlefield situation and the mission situation at different stages. Taking the penetration stage as an example, we adjust the structure of BN according to the reduction indexes of S1 in Table 2, set the bad value to 100% and update the model. The result of S1 is shown in Figure 4. Similarly, the situational analysis can be performed for other mission stages. Due to the limited space of this paper, the reasoning simulation diagrams of other stages are not given. The trend of autonomous capability in each stage is shown in Figure 5, where the pre-mission is recorded as S0.

Figure 4. Reasoning diagram in S1.
From Figure 5, the autonomous capability of different mission stages is changing dynamically. The autonomous capability in S0 is the highest, followed by S1, and the autonomous capability in S3 is the lowest. S0 is mission global-oriented and requires a high autonomous capability. In S1, UAVs needs to face a complex battlefield environment and break through the enemy’s tight air defense system. In S2, UAVs need to use airborne detection equipment to complete multiple tasks such as search, identification, tracking, and locking. In S3, because it is the human who makes the final attack decision, the UAV only needs to execute the attack command. Therefore, the ranking result is in line with the reality of the task, which validates the scientificity and rationality of the evaluation method. On the other hand, in the process of UAV mission execution, it is necessary to timely adjust the UAV autonomy and human–computer interaction in combination with the actual mission requirements and battlefield situation so as to facilitate the smooth execution of the mission.

4.3.3. Analysis of Influencing Factors in Post Mission

After the combat mission, we need to find the key factors that affect autonomous capability. The probabilities of the five autonomous capability levels are set to 1, respectively, and the model is updated according to Equation (2). We can get the posterior probabilities of the nodes at each autonomy level, and the reasoning results of the indexes $A_1 \rightarrow A_5$ are shown in Figure 6.

The following conclusions can be drawn from Figure 6: (1) As the level of UAV autonomous capability rises, the performance of each index also improves. It indicates that each index has an impact on autonomous capability and the index system is reasonable and effective. (2) The main factors of the autonomous capability are changing under different autonomous capability levels. The main influencing factors of single aircraft autonomous capability (I to III) are perceptual detection, security management, and combat execution capability, respectively. At the level of multi-UAV cooperation/clustering cooperation (IV to V), learning evolution capability and planning and decision-making capability determine the upper limit of autonomous capability. The simulation results are in line with the development trend of UAV autonomous capability. (3) In the process of the development of UAV’s autonomous capability, the sensing and detection capability and planning and decision-making capability occupy a certain proportion, which needs to be paid attention to.
Figure 6. Results of influencing factor reasoning of index $A_1 \rightarrow A_5$ at each level: (a) level I; (b) level II; (c) level III; (d) level IV; (e) level V.

5. Conclusions

To address the uncertainty in the comprehensive evaluation, BN is applied to construct an evaluation model. To solve the problem of strong correlations among variables and meet the practical needs of different evaluation stages, we have done the following work: (1) The IHORFA algorithm is proposed to optimize the index system. (2) The model based on an improved BN is proposed, which relaxes the application conditions of BN and can better deal with the strong correlations among many factors. (3) The combination weighting method based on game theory is proposed. We combine the advantages of IAHP and IEW, which largely eliminates the limitations of a single method.

Finally, the improved BN is applied to the autonomous capability evaluation of ground-attack UAVs. The powerful analytical reasoning ability of BN is used to reason and analyze different mission stages, which better solves the forward problem, intermediate interference problem, and reverse problem in the evaluation process and verifies the feasibility of the proposed method. It should be pointed out that there are too many uncertainties and uncontrollable factors in the evaluation process. Therefore, the effectiveness of the evaluation model needs to be further tested with actual data, and the whole evaluation system needs to be continuously updated and optimized.

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