Digitization and axiomatics in modern metrology

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Abstract. The paper is addressing various viewpoints on theoretical metrology and components thereof. The necessity to develop a methodological basis for the theory of measurement is considered. The authors proposed to switch over to axiom-based constructing of metrology theoretical foundations; at that, axiom defining fundamental value of a priori knowledge, axiom describing measurement process procedure, basic postulate of metrology and axiom of measurement result assessment procedure — all of them take on the role of main ones. Authors scrutinized the postulate about existence of true value of measured quantity and gave attention to uncertainty and error estimates underlining that axiomatic fundamentals of metrology are still in nascent stage and, therefore, require focused attention of the researchers.

1. Introduction

Theoretical metrology as one of general metrology parts may be, in its turn, considered from various viewpoints [1]. Composition of general metrology is analyzed, on the one hand, with reference to main metrology sections, namely:
— basic concepts of metrology;
— general theory of measurements uniformity;
— general theory of measurements procedures;
— general theory of results exactness and measuring instruments accuracy [1].

On the other hand, it is required to develop methodological basis for measurement theory bearing in mind its integrating aspect which becomes particularly prominent for development of metrology as a single whole. In this regard, the following is being surveyed and proposed for use:
— gnoseological fundamentals of measurements theory [2];
— representative approach to measurements theory [3];
— algorithmic theory of measurements [4];
— gnoseological fundamentals of measurements theory [5, 6];

It is a well-known fact that measurement, as a procedure, refers to cognitive aspects this is why metrology has strong links with theory of knowledge — i.e. gnoseology. Substantiation of gnoseology fundamental principles with consideration of measurement process from ‘within’ [2], and ‘without’ [7] makes it possible to formulate basic metrological principles: (I) measurability, (II) unity of objective and subjective, (III) stipulation of measurement result, (IV) invariance of measurement result. However,
all these postulates claim that true value of measured quantity (which would ‘ideally reflect, in qualitative and quantitative respect, relevant property of an object’ as set forth by interstate standardization recommendations entitled ‘State measurement system. Metrology. Basic terms and definitions’)) exists. However, this comes to contradiction with inability to reflect physical essence of object itself and cannot, therefore, be used as a common basis of measurement theory [8, 9].

One more attempt to present formalized description of main concepts and provisions of metrology is caused by requirement of measurement theory establishment in capacity of independent scientific discipline. Besides, increasingly growing demands put forward by measurement practice and other scientific fields caused growth of ‘theory of scales’ — general theory of objects reflection with the use of number sets. This caused emerging of representative measurement theory (RMT) in which basic concept is represented by scale [3].

Modern RMT includes a number of main sections: systems with relations (relational systems), equivalence relations (by convention, reflexive, symmetric and transitive), theory of adequacy and some other [3]. Thus, the authors analyzed ordinal scales, interval scales, ratio scales and nominal scales; besides, absolute scale and some other scales with known mathematized RMT definitions were added. One of essential RMT drawbacks — it does not reflect the concept of measurement result error (uncertainty). Therefore, it constitutes the basis of scale theory, and its wide applicability is a part of measuring procedures analysis.

Wide spread and application of computing tools in measurement tasks, and, first of all, practical application of analog-to-digital converters (ADC), gave impetus to new scientific concept — the so-called algorithmic theory of measurements (ATM) which may be considered as one of approaches to formalized measurements description. This theory sets the task to plot comparison algorithms of observed physical quantities and to define numeric value of physical quantity measurement result using the said algorithm. Methodological basis of ATM is represented by constructive approach of mathematical logic. On the one hand, ATM is using measurement finiteness and potential realizability principle, on the other hand, principle of actual measurement procedures asymmetry (when comparison of physical quantities (observations) means that depending on results of previous step (comparison) the complexity at the next stage will be different) is used. To unexpected ATM results we may refer link between correlations obtained and classical combinatorial formulas [1].

Optimal step-wise measurement algorithm is generating binary, decimal numeration systems and even numerical series of Fibonacci coding system [1, 10].

In the majority of investigations and reviews dedicated to ATM namely quantity comparison axioms (including initial basic provisions of representative theory of measurements) are considered as a formal basis of mathematical theory of measurements. At that, even without any minimum acceptable reasoning, metrology is construed as a part of fundamental theory of measurements associated with provision of measurements uniformity and metrological support for applied problems of measurements.

It should be mentioned that while expounding the fundamentals of ATM considerable part of stages (relating to measuring procedures (for instance, specific features of primary measuring transformation of observed object properties) and to measuring experiments in real objects) are dropping off this theory coverage. Besides, it seems that this theory refers rather to theory of analog-digital conversion (numeric coding) of continuous quantities and, in this capacity, may be considered as one of sections of measuring transformation theory (aposteriori).

Any science including metrology as a measurement science [11] has its own fundamentals, postulates and axioms. Any science transition towards axiomatic basis adds harmony and completeness thereto. Attempts to build-up axiom-based metrology have been taken long ago [12–14]. Most popular among postulated axioms are:

— true value of measured quantity exists;
— is it impossible to define true value of measured quantity;
— measured quantity is constant.

In our opinion, axioms formulated by I. F. Shishkin [5] are most rational at the moment.
First axiom of metrology is defining the fundamental value of a priori knowledge. If someone knows nothing about the measured quantity it means it cannot be defined. From this it follows: ‘Measurement without a priori information is impossible’. This is the 1st metrology axiom. A priori information is represented in the previous investigations-measurements experience: in the form of object properties investigation during measurement task assignment, in the form specified quantity observations distribution law, in the form of its numerical characteristics (often by shaped object’s model), in the form of knowledge of influencing factors (non-informative parameters), sources and constituents of error (uncertainty). For instance, accuracy class of the measuring instrument serves as a generalized form of a priori information presentation.

Using the apriori information it is possible to solve an inverse problem of measurement theory — i.e. to proceed from random value measured on measuring instrument output to nonrandom value measured on its input.

The 2nd metrology axiom [5] is construed as measurement process description and reads as follows: ‘Measurement means dimensions comparison by experiment’. It is a well-known fact (L. Euler, M. F. Malikov) that the only way to obtain measurement information is to compare the uniform dimensions by experiment. This procedure forms the basis for using two ways of comparison in traditional metrology: ‘how many’ more/less (or equal) and ‘what fold’ more/less (or equal). This is reflected in the interval and ratio scales stipulated by regulatory documents, for instance, in the law entitled ‘On ensuring the uniformity of measurements’.

However, there is one more way to compare, i.e. the principle ‘more/less or equal’ which is represented (i.e. known in measurement practice) by ordinal scale. These scales are widely used in qualimetry when it is required to measure non-physical quantities, for organoleptic examinations and in many other spheres of scientific and practical knowledge. At that, such measurements are not supported by uniformity maintenance, and results thereof are considered illegitimate. This is why, the immediate objective of metrology and lawmakers is to approve ordinal scale-obtained measurement results as legitimate; this will allow obtaining practical utility in various spheres of socio-economic development [15].

One of the most important axioms identified as ‘main metrology postulate’ [5] and relating to measurement result reads as follows: ‘Result of measurement, if not rounded-off, is a random value’. It is naturally reflecting the real situation when measurement is undertaken in condition of the so-called ‘chaos’ [16] which is composed of influence exerted by numerous unpredictable factors on observation acquisition. This is why, the result of comparison between unknown dimension of definiendum \(X\) and known dimension \([X]\) represented by the size of unit of measure is a random digit

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q = X \cdot \left([X]\right)^{-1}, \tag{1}
\]
called reading.

This numerical value of measured quantity \(q\) is, for some reason, called ‘basic measuring equation’ with addition about evaluation of error of knowledge of \(q\) dimension as \(\pm \Delta_X\). Obviously, if you lower \(X\) — \([X]\) comparison accuracy in (1) of round-off the reading it means that \(q\), remaining unchanged [6], will stop being a random digit.
2. Discussion
This axiom explains numerous metrology provisions. Firstly, wide use of theory of probability and mathematical statistics which are used for measurement results evaluation. Secondly, this mathematical apparatus serves to identify intervals which, to various levels of probability, locate the measured quantity, i.e. knowledge of uncertainty (error) with which the result of measurement was obtained.

It seems that these three metrology axioms formulated by I. F. Shishkin [5] refer to situation of procedures before measurement, measurement procedures and measurement results, however, do not reflect the procedure of evaluation of that interval of uncertainty (error) which proceeds from definition of value of randomly measured quantity.

This is why; probably there is a demand for 4th metrology axiom that would read: ‘Quality (accuracy) of measurement results is the essence of its uncertainty (error)’.

3. Conclusion
Theory of measurement results and measuring instruments errors has always been considered by main sections of metrological investigations. Concept of measurement error is the most important element of measurements because namely error is characterizing the quality of measurements and without it neither result may be used in scientific or practical investigations [17, 18]. In this context, the postulate about existence of true value of measured quantity and, to the greater extent, about its constancy, has apparently no right of existence. As it is known [19, 20], considerable part of literature and legislative regulations is dedicated to evaluation of uncertainty and errors. Discussion thereof requires independent analysis.

Summing up, we can say that axiomatic fundamentals of metrology are in nascent stage and, therefore, shaping thereof will contribute to metrology completeness as a science.

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