Identification of strong and weak interacting two level systems in KBr:CN

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Tunneling two level systems (TLSs) are believed to be the source of phenomena such as the universal low temperature properties in disordered and amorphous solids, and $1/f$ noise. The existence of these phenomena in a large variety of dissimilar physical systems testifies for the universal nature of the TLSs, which however, is not yet known. Following a recent suggestion that attributes the low temperature TLSs to inversion pairs [M. Schechter and P. C. E. Stamp, arXiv:0910.1283, in its general form, and (b) positively identifies, for the first time, the relevant TLSs in a given disordered crystal. Our work (a) estimates parameters that support the theory in M. Schechter and P. C. E. Stamp, arXiv:0910.1283, and (b) re-enforces the prediction made in Ref.[12] that the $S$-TLSs are effectively noninteracting, and dictate the phonon attenuation. Thus, at $T < T_U$ the $S$-TLSs fulfil the assumptions of the STM. The small parameter of the theory is $g \equiv \gamma_w/\gamma_s \approx E_\phi/E_C \sim (1 - 3) \times 10^{-2}$, where $E_\phi, E_C$ are the typical elastic and Coulomb energies in the system. This small parameter gives the universality and smallness of the phonon attenuation. Defining $T_G$ as the ordering temperature of the $S$-TLSs, the emerging DOS of the $S$-TLSs at an energy $T_U \approx gT_G$, dictates $T_U$ as the energy scale below which universality is observed [12].

In this Letter we use DFT and ab-initio calculations to calculate the interaction of TLSs of types $\tau$ and $S$ with the phonon field $\gamma_w$ is small, as it results only from disorder induced local deviations from inversion symmetry. The $S$-TLSs are asymmetric with respect to local inversion, with a strong interaction with the phonon field $\gamma_s$. It was then shown [12] that the $S$-TLSs are gapped below $T_U$ by the $\tau$-TLSs through an Efros Shklovskii\textsuperscript{13} type mechanism, and that below $T_U$ the $\tau$-TLSs are effectively noninteracting, and dictate the phonon attenuation. Thus, at $T < T_U$ the $\tau$-TLSs fulfill the assumptions of the STM. The small parameter of the theory is $g \equiv \gamma_w/\gamma_s \approx E_\phi/E_C \sim (1 - 3) \times 10^{-2}$, where $E_\phi, E_C$ are the typical elastic and Coulomb energies in the system. This small parameter gives the universality and smallness of the phonon attenuation. Defining $T_G$ as the ordering temperature of the $S$-TLSs, the emerging DOS of the $S$-TLSs at an energy $T_U \approx gT_G$, dictates $T_U$ as the energy scale below which universality is observed [12].

In this Letter we use DFT and ab-initio calculations to calculate the interaction of TLSs of types $\tau$ and $S$ with the phonon field in the system KBr$_{1-x}$(CN)$_x$ (KBr:CN, Fig. 1). We find that $\gamma_w \approx 0.16eV$, and $\gamma_s \approx 3eV$. Our estimation of $\gamma_w$ compares well with the experimentally measured value for the relevant TLSs at low energies, of $\gamma \approx 0.12eV$ for impurity concentration $x= 0.25$ and $\gamma \approx 0.2eV$ for $x= 0.5$ [14, 15]. Our results also support the central arguments of the theory in Ref. [12] in (i) the categorization of the TLSs according to their symmetry under inversion, (ii) the ratio of the strengths of their interactions with the phonon field, constituting the small parameter of the theory, and (iii) the identification of the symmetric TLSs as the relevant TLSs dictating the low temperature universal properties in disordered solids. In addition, we re-enforce the prediction made in Ref. [12] for the existence, at higher energies, of a second type of TLSs (of $S$ type), with a much stronger coupling to the phonon field. Note, that although we focus here on the simplest single impurity excitations, our analysis does not exclude the possibility of symmetric and asymmetric

\textbf{Introduction} — Amorphous solids and many disordered lattices show peculiar universal characteristics at low temperatures\textsuperscript{[1–3]} [3]. Below $T_U \approx 3K$ systems which are otherwise very different have specific heat $C \propto T^\alpha$, with $\alpha \approx 1$, thermal conductivity $\kappa \propto T^\beta$ with $\beta \approx 2$, and internal friction $Q \approx 2\pi/\lambda \approx 10^3$, independent of $T$, $\lambda$, and with only a small variance between materials. Here $l$ is the phonon mean free path and $\lambda$ is the phonon wavelength. In an effort to explain this remarkable universality, Anderson Halperin and Varma\textsuperscript{[4]} and Philips\textsuperscript{[5]} suggested a phenomenological theory, where the existence of tunneling two level systems (TLSs) in these materials was postulated, and an ansatz for their density of states was given. This ”standard tunneling model” (STM) has been very successful in explaining the above mentioned phenomena. Still, the identity of the tunneling TLSs has remained unknown. Furthermore, the smallness and universality of the phonon attenuation, and the energy scale dictating $T_U$ are not accounted for by the STM.

Two level systems are also believed to be the cause of $1/f$ noise. Recently, it has been shown that $1/f$ noise is the main source for decoherence of superconducting qubits, and a major obstacle in their ability to perform quantum computation\textsuperscript{[6, 7, 9, 10]}. Also in these systems the nature of the TLSs is not known, yet assuming their existence and applying the STM has resulted in an explanation of the low frequency $1/f$ noise and high frequency linear in $f$ noise on the same footing\textsuperscript{[8]}. Extensive experimental investigations have revealed that the condition to observe universality is the presence of tunneling states and strong lattice strain\textsuperscript{[8, 9, 10]}, and that the phenomena in amorphous solids and disordered crystals are equivalent\textsuperscript{[10]}. Disordered crystals are advantageous for both experimental and theoretical investigation\textsuperscript{[11]}. Experimentally, they allow control of the nature and relative concentration of host material and impurities, and therefore a detailed study of different universal properties and their origin. The existence of lattice structure and the apparent candidates for tunneling states allows a favorable starting point for theoretical treatment as well.

Indeed, it was argued\textsuperscript{[12]} that, at least in disordered crystals, tunneling states can be categorized into two types of TLSs, denoted $\tau$ and $S$. The states of a $\tau$-TLS are related to each other by inversion. Consequently, the interaction of a $\tau$-TLS with the phonon field $\gamma_w$ is small, as it results only from disorder induced local deviations from inversion symmetry. The $S$-TLSs are asymmetric with respect to local inversion, with a strong interaction with the phonon field $\gamma_s$. It was then shown\textsuperscript{[12]} that the $S$-TLSs are gapped below $T_U$ by the $\tau$-TLSs through an Efros Shklovskii\textsuperscript{[13]} type mechanism, and that below $T_U$ the $\tau$-TLSs are effectively noninteracting, and dictate the phonon attenuation. Thus, at $T < T_U$ the $\tau$-TLSs fulfill the assumptions of the STM. The small parameter of the theory is $g \equiv \gamma_w/\gamma_s \approx E_\phi/E_C \sim (1 - 3) \times 10^{-2}$, where $E_\phi, E_C$ are the typical elastic and Coulomb energies in the system. This small parameter gives the universality and smallness of the phonon attenuation. Defining $T_G$ as the ordering temperature of the $S$-TLSs, the emerging DOS of the $S$-TLSs at an energy $T_U \approx gT_G$, dictates $T_U$ as the energy scale below which universality is observed\textsuperscript{[12]}.

In this Letter we use DFT and ab-initio calculations to calculate the interaction of TLSs of types $\tau$ and $S$ with the phonon field in the system KBr$_{1-x}$(CN)$_x$ (KBr:CN, Fig. 1). We find that $\gamma_w \approx 0.16eV$, and $\gamma_s \approx 3eV$. Our estimation of $\gamma_w$ compares well with the experimentally measured value for the relevant TLSs at low energies, of $\gamma \approx 0.12eV$ for impurity concentration $x= 0.25$ and $\gamma \approx 0.2eV$ for $x= 0.5$ [14, 15]. Our results also support the central arguments of the theory in Ref. [12] in (i) the categorization of the TLSs according to their symmetry under inversion, (ii) the ratio of the strengths of their interactions with the phonon field, constituting the small parameter of the theory, and (iii) the identification of the symmetric TLSs as the relevant TLSs dictating the low temperature universal properties in disordered solids. In addition, we re-enforce the prediction made in Ref. [12] for the existence, at higher energies, of a second type of TLSs (of $S$ type), with a much stronger coupling to the phonon field. Note, that although we focus here on the simplest single impurity excitations, our analysis does not exclude the possibility of symmetric and asymmetric...
multi-impurity excitations\textsuperscript{16–18}. Such excitations are expected to be significant especially for systems where single impurity excitations do not produce symmetric TLSs\textsuperscript{19, 20}.

For the specific KBr:CN system, early works have suggested, based on theories quite different from that of Ref.\textsuperscript{12}, that CN flips comprise the relevant low energy TLSs\textsuperscript{21, 22}. However, for long, advance in this direction was hindered because of experiments showing that the substitution of the symmetric N\textsubscript{2} molecules for the asymmetric CO molecules in N\textsubscript{2}/Ar/CO does not change its universal characteristics\textsuperscript{19, 20}. Our results here, in conjunction with the theory in Ref.\textsuperscript{12}, positively identify the 180° CN flips as the relevant TLS excitations dictating the low temperature characteristics in the KBr:CN system. Reconciliation of our results with the experiment in Ref.\textsuperscript{19, 20} stems from the fact that pairs of N\textsubscript{2} molecules do produce symmetric TLSs in the ArN\textsubscript{2} system\textsuperscript{23}.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Three 5x5 fragments of a KBr:CN lattice. The up state of the central impurity in fragment (b) is related by a τ excitation to a down state in fragment (a), and by an S excitation to a left state in fragment (c).}
\end{figure}

\textbf{Calculation — } KBr:CN is perhaps the most studied disordered lattice showing universal characteristics. The CN\textsuperscript{−} impurities have been found to orient either in the direction of the in-space diagonals, preferred for very low CN\textsuperscript{−} concentrations\textsuperscript{24} and for intermediate concentrations at high temperatures\textsuperscript{25}, or in the direction of the axes, preferred for intermediate CN\textsuperscript{−} concentrations at low temperatures\textsuperscript{25}. The six (eight) possible states of each impurity can be categorized into three (four) inversion pairs, each having two states related to each other by an 180° flip. Such flips constitute τ excitations, whereas rotations between different axis (diagonals) correspond to S excitations\textsuperscript{12, 26}.

The interaction of such a system with the lattice can be described by the Hamiltonian\textsuperscript{12, 26}

\begin{equation}
H_{\text{int}} = \sum_j \sum_{\alpha,\beta} \left[ \eta \delta_{\alpha,\beta} + \gamma_s^{\alpha\beta} S_j^{\gamma} + \gamma_w^{\alpha\beta} \tau_j^{\gamma} \right] u_{\alpha\beta}(r_j)
\end{equation}

where $\eta$ is an orientation-independent volume factor and $u_{\alpha\beta}(r_j)$ denotes the phonon field at point $r_j$. Whereas the central purpose of this Letter is the calculation of $\gamma_w$ and $\gamma_s$, we also calculate the parameter $\eta$ for both CN\textsuperscript{−} and Cl\textsuperscript{−} impurities. This parameter determines the strain, and thus the effective random field in the system\textsuperscript{26, 27}. Usually $\eta \lesssim \gamma_s$. In KBr:CN $\eta$ is significantly subdominant, as the Br\textsuperscript{−} and CN\textsuperscript{−} ions have similar volumes\textsuperscript{15}. In KBr:Cl this term is responsible for the strains allowing for the existence of universal properties upon minimal CN\textsuperscript{−} dilution\textsuperscript{8, 9}. This random field term is also central to the smearing of the glass transition and the peculiar disordering of dilute glasses\textsuperscript{27}.

Following the above definition of $\eta$, $\gamma_s$, and $\gamma_w$, we devise a series of numerical calculations to estimate them. In sum, we choose a number of lattice fragments and use DFT/ab initio methods to calculate the energy difference between the effects of phonon-like perturbations of the system with a central CN\textsuperscript{−} impurity in different states. For simplicity and without loss of generality, we restrict the excitations of the CN\textsuperscript{−} impurity under study to two dimensions, thus the possible states are $up$, $down$, $left$, and $right$. To those possible orientations, we apply vertical or horizontal phonons. Usually the symmetry is low enough to allow for several independent estimations for each parameter.

Our determination of $\gamma_w$ and $\gamma_s$ is performed as follows:

\begin{equation}
\gamma_{w,s} = \frac{1}{b} \left[ (E_{\text{ph}}^s(b) - E^i) - (E_{\text{ph}}^w(b) - E^j) \right]
\end{equation}

where $\{i, j\}$ are $\{up, down\}$ or $\{left, right\}$ for $\gamma_w$ and $\{up, left\}$ or $\{down, right\}$ for $\gamma_s$. $\text{ph}$ can stand for vertical or horizontal phonons, $E^i$ is the energy of an impurity $i$ surrounded by a lattice fragment in its equilibrium geometry and $E_{\text{ph}}^i$ is the energy of the same impurity after a lattice contraction by a fraction $b$ along a given crystallographic coordinate, mimicking the effect of a longitudinal phonon. For $\eta$, the same procedure is applied where $\{i,j\}$ means presence or absence of impurity, and for CN\textsuperscript{−} impurities all possible orientations are averaged.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Atomic coordinates of a lattice fragment in three key steps of a sample calculation. (a): all positions are optimized, without a central TLS, but in presence of an impurity (2,0,u). (b): a TLS (0,0,u) is substituted in the place of the central Br\textsuperscript{−}, its position is optimized, yielding $E'$. (c): the lattice is contracted by 2% vertically, yielding $E_{\text{vertical}}'(-2\%)$.}
\end{figure}

Fig. 2 illustrates the detailed procedure of a $\gamma$ calculation. We describe the position and orientation of the impurity as (2,0,u), where the numbers refer to the coordinates and the letter to the orientation, in this case the CN\textsuperscript{−} is lying on the abscise, at 2 interatomic spacings to the right, and the nitrogen is pointing up. We then proceed with the following steps: (i) for that particular system, the atomic positions are found which minimize the energy in the absence of the TLS under investigation, i.e., when the central position is occupied by a Br\textsuperscript{−}; that is our definition of the equilibrium geometry of the
lattice. (ii) a CN− (facing “up”, in this case) is substituted for the central Br−, and, freezing the lattice, only the position of the CN− is optimized, to obtain $E_{up}$. (iii) the atomic positions are contracted along the vertical axis to obtain $E_{vertical}^{up}(b)$. The calculation is done for $b \in \{-5\%, -2\%, -1\\%, -0.25\%, 0.25\%, 1\%, 2\%, 5\%\}$, and the limit of small $b$ is taken (see Fig.3). The repetition of this procedure for the vertical phonon and the “down”, “left” and “right” orientations of the central CN− allows for four non-independent estimations of both $\gamma_w$ and $\gamma_s$ (in 3D we also have the “front” and “back” orientations). In the case of $\eta$, the weighted average for all orientations yields one unique estimation. Note that in step (ii) we do not relax the whole lattice in the presence of the TLS. Our procedure is in line with both processes of lattice relaxation and phonon scattering by TLSs, which result from the out of equilibrium first order interaction of the TLS with the lattice.

We use the standard package Gaussian03 to perform quantum chemistry calculations on lattice fragments of different sizes and shapes and at different levels of sophistication. As we are dealing with a local phenomenon, and for cost reasons, most of the calculations are performed on small zero-dimensional squares or cubes, either 3x3, 3x3x3 or 5x5. The TLS under evaluation is always in the center, so that any deviation from a centrosymmetric situation felt by the TLS is due to the extra impurities and not to border effects. We mainly use the hybrid DFT/ab initio method B3LYP with small orbital sets, either 3-21G or 6-31G. The influence of a better description of the anions is tested by repeating the three calculations with a minimal basis set, with up to four impurities at positions (0,-2,r)(-2,2,r)(2,0,u),(2,2,d); these results should not be taken on equal footing with the rest of the table.

Results — Fig. 3 illustrates some tests of the range of linearity. One can see that the results are essentially the same for different fragments and levels of calculations: the first order approximation is very accurate at least for phonon amplitudes of one or two percent of the interatomic spacing. On a 3x3 fragment with a central CN− impurity, at MP2/6-31+G level, Fig. 3 shows the estimation of $\eta = 0.6\text{eV}$. The same conditions yield a comparable $\eta = 0.9\text{eV}$ for a Cl− impurity. In all cases a noticeable second-order correction of the order of 5eV can be fitted.

The central result of this Letter, reported in table I, is the calculation of $\gamma_s \simeq 3\text{eV}$ and $0 \leq \gamma_w \leq 0.15\text{eV}$. The finite size of our samples, the quality of our calculation methods, and differences between planar and cubic samples, all lead to some variance in the parameters. Yet, the strength of our results lies in the fact that our estimations are fairly consistent in their order of magnitude for very different lattice fragments and a variety of levels of calculation. This is true for additional calculations, e.g. for a non-central CN− impurity, not reported here.

| fragment | impurity | method      | $\gamma_s$ | $\gamma_w$ |
|----------|----------|-------------|------------|------------|
| 3x3      | none     | HF/6-311+G* | 3.02       | 0 (sym)    |
| 3x3      | none     | MP2/3-21G   | 1.91       | 0 (sym)    |
| 3x3      | none     | MP2/6-311+G*| 3.55       | 0 (sym)    |
| 3x3      | none     | B3LYP/3-21G | 3.28       | 0 (sym)    |
| 3x3      | none     | B3LYP/6-311G| 2.95       | 0 (sym)    |
| 3x3      | none     | B3LYP/3-21G | 4.20       | 0 (sym)    |
| 3x3      | (1,1,dr) | B3LYP/6-311G| 3.40       | 0.08       |
| 3x3      | (1,-1,dl)(1,1,ul) | B3LYP/3-21G | 2.86       | 0.03       |
| 3x3      | (1,-1,dl)(1,1,ul) | MP2/6-311G   | 1.80       | 0.003      |
| 3x3      | (0,-1,r)(1,-1,0,f) | HF/6-31+G   | 2.15       | 0.04       |
| 3x3      | (1,1,dl) | B3LYP/3-21G | n.a.       | 0.14       |
| 5x5      | (6,-2,r) | B3LYP/6-311G| 4.70       | 0.04       |
| 5x5      | (-1,-1,r) | B3LYP/3-21G | 2.10       | 0.04       |
| 5x5      | (2,0,d)  | B3LYP/3-21G | 2.40       | 0.11       |
| 7x7      | (see caption) | HFS/STO-3G   | 10 - 20    | 0.1 - 0.2  |

TABLE I: Some estimations, in eV, for $\gamma_s$ and $\gamma_w$. Absolute values are given, since the TLS orientation is arbitrary. Except for fragments (b-d), all possible TLSs were calculated, as shown for fragment (a) in table I and only the highest values are displayed here. In the absence of extra impurities, $\gamma_w = 0$ for symmetry reasons. For fragments (b-d), only one $\tau$-TLS and one $S$-TLS were chosen, and linearity of the energy response was tested as shown in Fig. 3. For these fragments the values for $\gamma_w$ should be considered as lower bounds. tr.ph. stands for ”transverse phonon”. Fragment (e) sums up three calculations with a minimal basis set, with up to four impurities at positions (0,-2,r)(-2,2,r)(2,0,u),(2,2,d); these results should not be taken on equal footing with the rest of the table.
particular can also be seen in the main panel of Fig. 3, where the displayed in Table II, for all TLSs in fragment (a). It with a variance which equals the typical value. This is all possible estimations of its values is peaked at zero, its values are dictated by the aforementioned deviation highest lar combination of fragment and calculation method, the very symmetric environment. Note that for each particu- tic deviations from symmetry, and are therefore a factor between different orientations is a result of the small elas-

The calculations using a minimal basis set serve to discard a correlation between $\gamma_w$ or $\gamma_s$ and the number of impurities.

One should point out that within a given sample and a given level of calculation, the variance in the values of $\gamma_s$ between different orientations is a result of the small elastic deviations from symmetry, and are therefore a factor of $g$ smaller than its typical value. With regard to $\gamma_w$, its values are dictated by the aforementioned deviation from local inversion symmetry. Thus, the distribution of all possible estimations of its values is peaked at zero, with a variance which equals the typical value. This is displayed in Table I for all TLSs in fragment (a). It can also be seen in the main panel of Fig. 3 where the particular $\tau$-TLS chosen for fragment (b) experiences a very symmetric environment. Note that for each particular combination of fragment and calculation method, the highest $\gamma_w$ values obtained among the four TLS-phonon combinations are denoted as our estimate values for $\gamma_w$ in Table I. These values are the most relevant for our purposes, as they are expected to be the best predictors for a real system with many impurities.

A noteworthy complication is presented by the in-plane diagonal orientations of the TLS, because of the small energy difference with the in-space diagonal states in the real system. Depending on the fragment, impurities and calculation method, the relative order of stability changes and the energy of one or more of the "axial" orientations rises above the most stable "diagonal" orientation. Some examples of these orientations, denoted as (dl, ul, dr, ur), are shown in Table I. As illustrated in Figs. 1 and 2 in the perimeter of the fragment, where the CN− suffer from intense border effects, we even find intermediate orientations. In the cases where the central TLS is affected by this problem -noted in Table I as "tilt"- the extraction of the parameters can be technically more difficult, but there is no fundamental physical difference between axial and diagonal orientations as in both cases there is a clear distinction between S-TLSs and $\tau$-TLSs.

**Summary** — Our numerical calculations confirm qualitatively and quantitatively the results of Ref. [12], the existence of weak and strong interacting TLSs in disordered solids, and the corresponding strength of their interaction with the phonon field. As TLSs in KBr:CN were experimentally measured to have a coupling constant of $\gamma \approx 0.12 – 0.18$eV with the phonon field, our calculations also verify that it is indeed the weak interacting $\tau$ TLSs which are the relevant TLSs at low temperatures, dictating the universal behaviour. Thus, we are able to clearly identify the relevant TLSs in this particular system. In Ref. [12] the plausibility that the same mechanism dictates universality in amorphous solids was argued for. The verification of this argument requires the identification of nearly inversion symmetric TLSs in amorphous solids, and the calculation of their coupling to the phonon field.

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| $\Delta \psi^{\text{ud}}$ | $\gamma_w$ | $\Delta \psi^{\text{lr}}$ | $\gamma_w$ |
|------------------------|-----------|-------------------------|----------|
| 2.84                   | 0.0036    | 2.82                    | 0.0070   |
| 2.86                   | 0.0001    | 2.84                    | 0.0273   |

TABLE II: Values in eV, summary of results for fragment (a) on Table I. Shorthand $\Delta \psi^{\text{ud}} = (E_{\text{ud}}^{\text{up}} - E_{\text{down}}^{\text{up}}) - (E_{\text{down}}^{\text{horizontal}} - E_{\text{down}}^{\text{down}})$ has been used for clarity.

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[1] R. C. Zeller and R. O. Pohl, Phys. Rev. B 4, 2029 (1971).
[2] S. Hunklinger, A. K. Raychaudhuri, Prog. Low. Temp. Phys. IX, 265-344 (1986).
[3] R. O. Pohl, X. Liu, and E. Thompson, Rev. Mod. Phys. 74, 991 (2002).
[4] P. W. Anderson, B. I. Halperin, and C. M. Varma, Phil. Mag. 25, 1 (1972).
[5] W. A. Phillips, J. Low Temp. Phys. 7, 351 (1972).
[6] R. W. Simmonds et al., Phys. Rev. Lett. 93, 077003 (2004).
[7] A. Shirman, G. Schon, I. Martin, and Y. Makhlin, Phys. Rev. Lett. 94, 127002 (2005).
[8] S. K. Watson, Phys. Rev. Lett. 75, 1965-1968 (1995).
[9] K. A. Topp, E. J. Thompson, and R. O. Pohl, Phys. Rev. B 60, 896 (1999).
[10] X. Liu et al., Phys. Rev. Lett. 81, 3171-3174 (1998).
[11] R. O. Pohl, X. Liu, and R. S. Crandall, Current Opin. Sol. St. Mat. Sci. 4, 281-287 (1999).
[12] M. Schechter and P. C. E. Stamp, arXiv:0910.1283.
[13] A. L. Efros and B. I. Shklovskii, J. Phys. C 8, L49-L51 (1975).
[14] J. F. Berret et al., Phys. Rev. Lett. 55, 2013 (1985).
[15] J. J. De Yoreo, W. Knaak, M. Meissner, and R. O. Pohl, Phys. Rev. B 34, 8828, (1986).
[16] E. R. Grannan, M. Randera, J. P. Sethna, Phys. Rev. B 41, 7784 (1990).
[17] A. L. Burin, J. Low Temp. Phys. 100, 309 (1995).
[18] A. L. Burin and Yu. Kagan, Phys. Lett. A 215, 191 (1996).
[19] C. I. Nicholls, L. N. Yadon, D. G. Haase, and M. S. Conrads, Phys. Rev. Lett. 59, 1317 (1987).
[20] L. N. Yadon, C. I. Nicholls, and D. G. Haase, Phys. Rev. B 40, 5215 (1989).
[21] J. P. Sethna and K. S. Chow, Phase Trans. 5, 317 (1985).
[22] M. P. Solf and M. W. Klein, Phys. Rev. B 49, 12703 (1994).
[23] A. Gaita-Ariño, V. F. González-Albúixech, and M. Schechter, in preparation.
[24] H. U. Beyeler, Phys. Rev. B 11, 3078 (1975).
[25] A. Loidl, K. Knorr, J. M. Rowe, and G. J. McIntyre, Phys. Rev. B 37, 389 (1988).
[26] M. Schechter and P. C. E. Stamp, J. Phys.: Condens. Matter 20, 244136 (2008).
[27] M. Schechter and P. C. E. Stamp, EPL 88 66002 (2009).
[28] Gaussian 03, Revision C.02, M. J. Frisch et. al., Gaussian, Inc., Wallingford CT, (2004).