Revisiting big bang nucleosynthesis with a new particle species: effect of co-annihilation with neutrons

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ABSTRACT: In big bang nucleosynthesis (BBN), the light matter abundance is dictated by the neutron-to-proton \((n/p)\) ratio which is controlled by the standard weak processes in the early universe. Here, we study the effect of an extra particle species \((\chi)\) which co-annihilates with neutron, thereby potentially changing the \((n/p)\) ratio in addition to the former processes. We find a novel interplay between the co-annihilation and the weak interaction in deciding the \((n/p)\) ratio and the yield of \(\chi\). At the initial stage of BBN for the large co-annihilation strength \((G_D)\) in comparison to the weak coupling \((G_F)\), more neutrons are removed from the thermal bath modifying the \((n/p)\) ratio from its standard evolution. We find that the standard BBN prediction is restored for \(G_D/G_F \lesssim 10^{-1}\), while the mass of \(\chi\) being much smaller than the neutron mass. When the mass of \(\chi\) is comparable to the neutron mass, we can allow large \(G_D/G_F\) values, as the thermal abundance of \(\chi\) becomes Boltzmann-suppressed. Therefore, the \((n/p)\) ratio is restored to its standard value via dominant weak processes in later epochs. We also discuss the viability of this new particle to be a dark matter candidate.
1 Introduction and Summary

Big bang nucleosynthesis (BBN) is one of the great achievements of both the standard model (SM) of particle physics and hot big bang cosmology. The observed primordial abundances of light matter in the universe agree with the theoretical prediction of BBN to a good approximation. One of the important quantities observed in this context is the primordial abundance of helium, i.e. the mass fraction of helium, $Y_p = 0.2449 \pm 0.0040$ [1], which is sensitive to the neutron-to-proton ($n/p$) ratio at the initial epoch of BBN. Neutrons and protons are in thermal equilibrium with the cosmic plasma after the QCD phase transition via SM weak processes. Weak processes become inefficient compared to the expansion of the universe at around temperature, $T = 1$ MeV. Subsequently the ($n/p$) ratio is frozen at a value around $(1/6 - 1/7)$[2]. This provides the initial condition for generating the light nuclei in correct abundance at later epochs. It is apparent that any alteration to this ratio due to some new physics phenomena would change the prediction of BBN to a great extent. The addition of a new particle species affects the ($n/p$) ratio broadly in two ways.

I. Additional particle species contributing significantly to the energy density of the universe during BBN, changes the expansion rate of the universe which in turn, delays or hastens the freeze-out of neutrons.

II. Chemical processes involving new particle species can alter the ($n/p$) ratio by removing or adding extra nucleons to the thermal bath.

In scenario I, the inclusion of an extra relativistic species during BBN is accounted as the effective number of extra neutrino species, defined as the following.

$$\Delta N_{\text{eff}} = \frac{8}{7} \frac{\rho_\chi}{\rho_\gamma}$$

(1.1)
Where, $\rho_\chi$ is the energy density of new particle species, $\chi$ and $\rho_\gamma$ is the photon counterpart. This parametrization works well for the species thermally decoupled from the SM plasma before BBN. In ref.\,[3, 4] the authors have shown that an additional particle species, strongly coupled either to photons or to neutrinos via elastic scatterings during BBN alters neutrino-to-photon temperature ratio ($T_\nu/T_\gamma$), thereby changing the standard weak interaction rates. As a result the freeze-out time of neutrons gets affected, in turn the final abundances of the light nuclei are altered.

In scenario II, the standard BBN (SBBN) prediction can potentially be altered due to the infusion of extra nucleons either from the decay [5] or from the annihilation [6, 7] of new particles. In this article we have introduced a new particle species $\chi$, which co-annihilates with neutrons. In the co-annihilation process [8] detailed in Sec.2, a neutron and a $\chi$ particle are removed from the thermal bath. Therefore, the $(n/p)$ ratio and the yield of $\chi$ can simultaneously be affected by the co-annihilation at the relevant epoch. The freeze-out value of the $(n/p)$ ratio and the abundance of $\chi$ are decided by the relative strength of the standard weak processes and the newly added co-annihilation, which we have discussed in great details in Sec.3. Additionally, we have considered the possibility of $\chi$ to be a viable dark matter (DM) candidate if it is stable over the cosmological time scale.

We now summarize our findings regarding the modification of the SBBN scenario due to newly added co-annihilation process. The effect of co-annihilation depends on the initial relative abundances of the neutron and $\chi$ in the cosmic soup. The number density of neutron is decided by the observed baryon asymmetry, whereas the number density of $\chi$ is assumed to be thermal. Hence, the number density is decided by the mass of $\chi$ ($m_\chi$) and the temperature of the cosmic soup. For $m_\chi < \mathcal{O}(m_n)$, $m_n$ being the neutron mass, $\chi$ freezes out relativistically, therefore the ambient number density becomes too large compared to the number density of neutrons. Consequently occasional co-annihilations are enough to eliminate neutrons even after the freeze-out, altering the $(n/p)$ ratio substantially from its SBBN value. This puts a constraint on the co-annihilation strength, i.e. $G_D/G_F \lesssim 10^{-1}$, $G_F$ being the weak interaction.

The constraint on the $G_D/G_F$ is significantly relaxed for $m_\chi \sim \mathcal{O}(m_n)$, unlike the previous case. This is due to an interesting interplay between the weak interaction and the co-annihilation in deciding the evolution of the $(n/p)$ ratio and the yield of $\chi$ over different epochs. Initially ($T >> 1$ MeV) there is a large number of $\chi$ and for large $G_D/G_F (\gtrsim 10^2)$ values the co-annihilation dominates over weak processes. Therefore, more neutrons are removed from the bath, thereby altering the $(n/p)$ ratio from its SBBN value. However for such a large co-annihilation strength $\chi$ remains longer in the thermal bath via the co-annihilation, consequently its number density becomes Boltzmann-suppressed. The co-annihilation rate per neutron becomes sub-dominant and the SBBN prediction of the $(n/p)$ ratio is eventually satisfied via weak processes in the later epoch. In this scenario, $\chi$ undergoes a non-relativistic freeze-out for which we find that the observed DM relic density is satisfied for $m_\chi \simeq 0.92$ GeV keeping SBBN predictions in tact. For $m_\chi \gtrsim 0.92$ GeV, $\chi$ contributes to the small fraction of the DM relic density. The universe becomes over-abundant with $\chi$ for $m_\chi \lesssim 0.92$ GeV. In that case, there must be some additional number-changing processes of $\chi$ to alleviate the situation.
2 Coannihilation with neutron

In the SBBN, the initial number densities of neutron and proton are controlled by the following electro-weak processes in which scattering processes freeze out at $T \sim 1 \text{ MeV}$ and the $(n/p)$ ratio becomes fixed. The frozen-out value of the $(n/p)$ ratio changes slightly due to occasional decays of neutron until the helium atom is produced, in which most of the neutrons are trapped.

A. $n + \nu_e \leftrightarrow p + e^-$, B. $n + e^+ \leftrightarrow p + \bar{\nu}_e$, C. $n \leftrightarrow p + e^- + \bar{\nu}_e$

We alter the standard scenario by incorporating a co-annihilation process, i.e. $n + \chi \leftrightarrow p + e^-$, where $\chi$ is considered as a charge-neutral Dirac fermion. Now, we can assign a lepton number, equal to that of electron to avoid the lepton number violation in the co-annihilation process. Such a process can be motivated by the lepto-quark mediator models [9] in solving B-physics anomalies as well as solving the neutron decay anomaly [10]. Moreover, the process considered here can be instrumental for detecting sub-GeV particles (DM or exotic neutrinos) on the beta-decaying nuclear targets [11–13]. Apart from being a fundamental particle, $\chi$ can well be a composite particle like neutron, which have been studied in Refs.[14, 15] and references therein. Here, we want to investigate the cosmological implication of the process being agnostic about the exact ultra-violet completion of the low energy effective theory, taken as

$$\mathcal{L} \supset \frac{G_D}{\sqrt{2}} (\bar{\psi}_p \Gamma^a \psi_n) (\bar{\psi}_e \Gamma^a \psi_\chi)$$

(2.1)

where $\Gamma^a$ in general, represents all possible independent combinations of Dirac matrices As a proto-type example of our scenario, We assume the standard $V - A$ structure for $\Gamma^a$, which is reminiscent of SM weak interaction. $\psi_i$ is the fermion field for the $i^{th}$ particle. We want to study only the effect of the co-annihilation on the SBBN case, therefore we take $m_\chi < m_n + m_p + m_e \sim 1.88 \text{ GeV}$ to prevent the decay, i.e. $\chi \to \bar{n} + p + e^-$ on the kinematic ground. In addition, to exclude the scenario I discussed in Sec.1, $\chi$ should not contribute significantly to the energy density of the universe during BBN. If $\chi$ is a decoupled but internally thermalized relativistic species, the measurement of $N_{\text{eff}}$ [16] sets an upper bound on the temperature $(T_\chi)$ of $\chi$ as,

$$(T_\chi/T_\gamma) \lesssim 0.6 \quad \text{ at 68\% C.L.}$$

(2.2)

If $\chi$ is thermalised with the photon bath during BBN, the lower bound on $m_\chi$ comes to be approximately 1.68 MeV. For more detailed analysis on this frontier see Ref.[17], and references therein. In the subsequent analysis, we take the number density of $\chi$ to be thermal to start with. Thus we get a very general mass window for a stable thermalised extra fermion species as,

$$1.68 \text{ MeV} \lesssim m_\chi \leq 1.88 \text{ GeV}$$

(2.3)

Now, the co-annihilation process is active once neutrons and protons are available in the bath after the QCD phase transition, $T \sim 150 \text{ MeV}$. To check the thermalization
condition for $\chi$ via the co-annihilation with neutrons in the expanding universe, we need $\Gamma(T) > H(T)$. Here, $\Gamma(T)$ is the co-annihilation rate per $\chi$ particle and $H(T)$ is the Hubble constant at temperature $T$. The interaction rate depends on the co-annihilation cross-section and the number density of neutrons at a certain epoch. The number density of neutrons is estimated from the observed baryon asymmetry, i.e.

$$\Delta_B = \frac{n_B - n_B^0}{s} = \frac{n_n + n_p}{s} = Y_n + Y_p \simeq 10^{-10}$$

(2.4)

Where, $Y_i = n_i/s$, $n_i$ is the number density of $i^{th}$ particle and $s$ is the entropy density of the universe. For $T \gg 1$ MeV the co-moving number density of neutron can be well be taken as $Y_n = Y_p = \Delta_B/2$. The s-wave contribution to the thermally averaged co-annihilation cross-section is given by,

$$\langle \sigma v \rangle \simeq \frac{G_D^2}{2\pi} \left( \frac{m_\chi^2}{m_n^2 + 2m_\chi m_n} \right)^{1/2} \left( \frac{0.1 \text{GeV}}{T} \right)^{1/2} \left( \frac{10}{g_*} \right)^{1/2}$$

(2.5)

in which we have assumed the electron to be massless and neutron and proton to have identical masses. Assuming usual radiation-dominating universe for the relevant epoch, we arrive at an approximate condition for thermalization of $\chi$ in the non-relativistic regime to be,

$$G_D \gtrsim 4 \times 10^{-4} \text{GeV}^{-2} \left( \frac{1 \text{GeV}^2}{m_\chi^2 + 2m_\chi m_n} \right)^{1/2} \left( \frac{0.1 \text{GeV}}{T} \right)^{1/2} \left( \frac{10}{g_*} \right)^{1/2}$$

(2.6)

Thus, the thermal number density of $\chi$ is ensured around $T \sim 100$ MeV($\sim m_n/10$), as shown in Figs.1, 2. The initial production of $\chi$ should happen anytime before $T \sim 100$ MeV either from the SM thermal bath or via some other mechanisms. Now, we assume that $\chi$ has been produced after $T \sim 100$ GeV to avoid baryogensis via leptogenesis [18], because $\chi$ has a lepton number and for $T > 100$ GeV leptons can be converted into baryons via the sphaleron effect [19].

To study the neutron freeze-out in this modified scenario we need to write Boltzmann equations for neutrons, protons and $\chi$. Due to high entropy density per baryon the light nuclei are not synthesized immediately after the neutron freeze-out, e.g. the synthesis of $^4\text{He}$ takes place around $T = 0.1$ MeV. Hence, for $T \gtrsim 0.1$ MeV the relevant processes are only the inter-conversions between neutrons and protons via three weak processes ($A,B,C$) and newly added co-annihilation. Hence, the total co-moving number density of baryons is conserved, i.e.

$$\frac{dY_p}{dx} + \frac{dY_n}{dx} = 0 \quad \text{for } T \gtrsim 0.1 \text{ MeV}$$

(2.7)

where $x = m_n/T$. Now, defining the ($n/p$) ratio as $R = Y_n/Y_p$ and $Y_\chi$ as the co-moving number density of $\chi$, we write Boltzmann equations using Eq.2.7 as the following.

$$\frac{dR}{dx} = \frac{1 + R}{Hx} \left[ \lambda(p \to n) - \lambda(n \to p)R \right] - \frac{(1 + R)s}{Hx} \langle \sigma v \rangle \left[ \left( Y_\chi - \frac{R_0}{R} Y_\chi^0 \right) \right]$$

$$\frac{dY_\chi}{dx} = - \frac{s}{Hx} \langle \sigma v \rangle \Delta_B R \left[ \left( Y_\chi - \frac{R_0}{R} Y_\chi^0 \right) \right]$$

(2.8)

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where \( Y_\chi^0 \) is the equilibrium co-moving number density for \( \chi \), \( R_0 = Y_n^0/Y_p^0 \simeq e^{-Q/T} \) and \( Q = m_n - m_p = 1.293 \text{MeV} \). \( \lambda(n \to p) \) is the neutron to proton conversion rate via SM weak processes defined assuming zero chemical potential for neutrinos as \([20]\),

\[
\lambda(n \to p) = \frac{1}{\tau} \int_1^\infty \frac{\sqrt{e^2 - 1} \epsilon (q + \epsilon)^2}{(\exp\left(\frac{m_\epsilon}{T}\right) + 1)} + \frac{\sqrt{e^2 - 1} \epsilon (\epsilon - q)^2}{(\exp\left(-\frac{m_\epsilon}{T}\right) + 1)} \frac{dT}{T^2}
\]

(2.9)

where, \( \tau \) is the neutron lifetime and \( T_\nu \) is the neutrino temperature and \( q = (m_n - m_p)/m_\epsilon = 2.53 \). The proton to neutron conversion rate, \( \lambda(p \to n) \) is achieved replacing \( q \) by \(-q\) in the expression of \( \lambda(n \to p) \). In Eq.2.9 \( T_\nu \) can be written as a function of photon temperature, \( T_\gamma = T \) and \( T \) is replaced by the scaled temperature variable \( x \).

It is apparent from above equations that both the \((n/p)\) ratio and the number density of \( \chi \) are inter-related due to added co-annihilation. In particular, \( Y_\chi \) follows its equilibrium form \((Y_\chi^0)\) as long as \( R = R_0 \), i.e. the \((n/p)\) ratio maintains its equilibrium value.

3 Results

3.1 Study of the evolution equations

We shall now study the evolution of \( R \) and \( Y_\chi \) by solving Eq.2.8 numerically keeping \( G_D \) and \( m_\chi \) as free parameters. The evolution equations for our scenario are solved upto \( x = 5000 \), which corresponds to \( T \simeq 0.2 \text{MeV} \). As discussed earlier, around \( T = 0.1 \text{MeV} \) other nucleons start building up significantly, prompting to include all nuclear reactions into our Boltzmann equations. The temperature range of the current study is adequate to capture the general feature of co-annihilation in the SBBN scenario.

It is apparent that in a co-annihilation-type process, the reaction rate per particle is different for each of the colliding particles. Therefore, the co-annihilation process is very much sensitive to the initial relative abundances of two particle species, which is decided by \( m_\chi \). This is because the number density of \( \chi \) thermal and the neutron number density is set by the baryon asymmetry. To demonstrate the effect of new physics on BBN we take two benchmark points (BPs) for \( m_\chi \), i.e. \( m_\chi = 50 \text{MeV} \) and \( m_\chi = 1.0 \text{GeV} \).

\( \text{Figure 1:} \) Evolution of the \((n/p)\) ratio \((R)\) \((\text{Left panel})\) and the co-moving number density \((Y_\chi)\) of the additional species \((\text{Right panel})\) shown for \( m_\chi = 50 \text{MeV} \), varying the scaled co-annihilation strength, \( G_D/G_F \).
BP - I: Relativistic freeze-out of $\chi$

In Fig.1 we have shown the evolution of $R$ (left panel) and $Y_\chi$ (right panel) for $m_\chi = 50$ MeV for different values $G_D$, scaled by $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$. In the right panel, the black dashed line denotes the equilibrium number density ($Y_\chi^0$) of $\chi$ and other lines correspond to the yields with different $G_D/G_F$ values. We find that the additional species, $\chi$ freezes out approximately at its equilibrium value, almost independent of the variation of $G_D/G_F$. For instance, $G_D/G_F$ varying from $10^{-2}$ to $10^{3}$, the freeze-out temperature is approximately the same, $T_F \simeq 23$ MeV which in terms of the scaled temperature becomes $m_\chi/T_F \simeq 2.2$. This indicates the relativistic freeze-out of $\chi$ as $m_\chi/T_F \lesssim 3$ [2].

In the left panel, the black dashed line denotes the equilibrium value of the ($n/p$) ratio $(R_0)$ and the cyan dashed line corresponds to that of the SBBN scenario, i.e. $G_D/G_F = 0$. As long as the co-annihilation keeps $\chi$ in chemical equilibrium, the ($n/p$) ratio follows the equilibrium value $(R_0)$ because there is no extra loss of neutrons via the co-annihilation. After the freeze-out of $\chi$, the ($n/p$) ratio departs from its standard evolution for $G_D/G_F = \{10, 10^2, 10^3\}$, whereas for $G_D/G_F = 10^{-2}$ (red solid line in the left panel) it is unaltered from the SBBN case. This feature can be understood from a comparison between the weak interaction rate ($\Gamma_w$) and the co-annihilation rate ($\Gamma_c$) per neutron as the following.

$$\frac{\Gamma_c}{\Gamma_w} \simeq \left(\frac{G_D}{G_F}\right)^2$$  \hspace{1cm} (3.1)

From Eq.3.1 we note that for $G_D/G_F = 10^{-2}$ the weak interaction dominates over the co-annihilation throughout the evolution history of neutron and $\chi$. For $G_D/G_F = \{10, 10^2, 10^3\}$ neutrons disappear due to the dominant co-annihilation, which eliminates more neutrons than the SBBN scenario. The disappearance of neutrons takes place even after the freeze-out of $\chi$. This is due to a large number of $\chi$ particle present in the cosmic soup compared to neutrons. In fact, in this case $Y_\chi \simeq 10^{-2}$ and $Y_n \simeq 10^{-10}$, which imply that there are $10^8$ number of $\chi$ particle available in 1 neutron in the bath. Therefore, the occasional co-annihilation is sufficient to modify the number density of neutrons drastically in later epochs. To sum up, for the relativistic freeze-out case the co-annihilation with larger interaction strength than the weak interaction strength reduces the ($n/p$) ratio by two orders of magnitude from the SBBN value at temperature, $T \simeq 10$ MeV (see the left panel of Fig.1). This eventually jeopardizes the predictions of light nuclei abundance, albeit the weak interaction is still on.

BP - II: Non-relativistic freeze-out of $\chi$

Similar to the previous one, we now show the evolution of $R$ (left panel) and $Y_\chi$ (right panel) in Fig.2 for $m_\chi = 1.0$ GeV for different values of $G_D/G_F$. Unlike the previous case, the freeze-out of $\chi$ depends on the co-annihilation strength. For example, the freeze-out temperature of $\chi$ for $G_D/G_F = 10^2$ (red solid line) is $T_F \simeq 50$ MeV, whereas for $G_D/G_F = 200$ (magenta dashed line for the corresponding yield) the freeze-out happens bit late, i.e. at $T_F \simeq 20$ MeV. To note, for $m_\chi = 1$ GeV the freeze out of $\chi$ happens in the non-relativistic regime, as $m_\chi/T_F >> 3$ in our example scenarios. In particular, there is
a non-trivial relation between freeze-out temperature and the co-annihilation strength, i.e. $T_F \propto G_D^2$ evident from Eq.2.6. This should be contrasted with the freeze-out temperature in the standard WIMP scenario of which the dependence on the coupling strength is rather weak, i.e. a logarithmic dependence [21]. The difference manifests from the fact that the freeze-out condition is determined by the baryon asymmetry in our case, whereas in the WIMP scenario it is decided by the thermal densities with vanishing chemical potentials. For $G_D/G_F = \{500, 10^3\}$ (blue dotted and orange dot-dashed line respectively) the co-annihilation keeps $Y_\chi$ in its equilibrium form ($Y_\chi^0$, black dashed line) for the relevant epoch.

In the left panel, for $G_D/G_F = 10^2$ (red solid line), $R$ freezes out at slightly smaller value than the SBBN scenario, i.e. $G_D/G_F = 0$ (cyan dashed line). For $G_D/G_F (> 10^2)$ (see magenta dashed, blue dotted and orange dot-dashed lines) the freeze-out value of $R$ agrees with the SBBN prediction. This is a sheer contrast to the relativistic case where largish value of $G_D/G_F$ destroys the SBBN prediction by changing the $(n/p)$ ratio significantly. In addition, for each of the parameter values there is a dip seen in the $(n/p)$ ratio at the initial epoch. This is a non-trivial feature emerging from a novel interplay between the weak interaction and the co-annihilation throughout the evolution history of neutron and $\chi$. To understand the situation, let’s look at the relative strengths of the co-annihilation and the weak interaction rate per neutron at the initial epoch, which is given by the following expression, noticeably different from Eq.3.1.

$$\frac{\Gamma_c}{\Gamma_w} \simeq \left(\frac{G_D}{G_F}\right)^2 \left(\frac{m_\chi^2 + 2m_\chi m_n}{T^2}\right) \left(\frac{m_\chi}{T}\right)^{3/2} e^{-m_\chi/T}$$ (3.2)

At around $T \approx 90$ MeV, for $G_D/G_F = 10^2$, $\Gamma_c/\Gamma_w \simeq 10^3$, i.e. the co-annihilation rate per neutron is large by several orders of magnitude from the standard weak processes. Therefore, at this epoch more neutrons are removed than in the SBBN scenario from the thermal bath via the dominant co-annihilation. As a result the $(n/p)$ ratio deviates from its standard evolution. However, the removal of neutrons does not go incessantly because very soon the weak processes become dominant. For large $G_D/G_F (> 10^2)$ values $Y_\chi$ follows its equilibrium form for longer period of time, thereby the number density of $\chi$ becomes Boltzmann-suppressed. In other words, the available $\chi$ particle per neutron be-
comes scarce to facilitate co-annihilation any further in the later epoch, e.g. at $T \sim 40 \text{ MeV}$ for $G_D/G_F = 10^2$, $\Gamma_c/\Gamma_w \simeq 10^{-2}$. Now, the small $(n/p)$ ratio indicates that there are more protons than neutrons in the thermal bath. Therefore, the proton-to-neutron conversion rate is enhanced compared to the neutron-to-proton conversion rate. Subsequently once ‘deceased’ neutrons reincarnate themselves via fast weak processes restoring the standard evolution of the $(n/p)$ ratio. Note that, the occasional co-annihilation of neutrons after the freeze-out of $\chi$ can not change the $(n/p)$ ratio substantially unlike the relativistic case, because of negligible relative abundance of $\chi$ to neutrons ($Y_{\chi}/Y_n$) in the bath. However, for $G_D/G_F = 10^2$, at the freeze-out temperature $Y_{\chi}/Y_n \sim O(1)$, which enables few occasional collisions, thereby changing the freeze-out value of $R$ by a small fraction. In passing, we also note that during the removal and reincarnation of neutrons $Y_{\chi}$ is also slightly deviated from its equilibrium value due to correlated chemical phases of $\chi$ and neutrons via the co-annihilation, apparent from Eq.2.8.

We can now summarize our discussion regarding the interplay between the weak interaction and the co-annihilation for both the cases, i.e. the relativistic and the non-relativistic freeze-out of $\chi$ using an instructive diagram shown in Fig.3. The freeze-out value of the $(n/p)$ ratio is denoted by $R_F$, which is calculated at $m_n/T_F = 5000$ varying $G_D/G_F$ continuously over several orders of magnitude for different $m_\chi$ values. In SBBN scenario, $R_F(BBN) \approx 1/7$ [22, 23], denoted by the black dashed line in Fig.3 including the effect of the neutron decay. As suggested by the previous two example scenarios, the mass of $\chi$ controls two different features in determining $R_F$.

![Figure 3](image-url)

**Figure 3**: The freeze-out value of the $(n/p)$ ratio, $R_F$ shown as a function of the scaled co-annihilation strength, $G_D/G_F$ for different values of $m_\chi$.

I. For $m_\chi < O(m_n)$, there is a large number of $\chi$ compared to neutrons, which makes weak processes inefficient for most of the time. Thus $R_F$ deviates from its SBBN value for $G_D/G_F \gtrsim 10^{-1}$. This is illustrated for two masses, i.e. $m_\chi = 100 \text{ MeV}$ (magenta dot-dashed line) and $m_\chi = 50 \text{ MeV}$ (orange solid line) in Fig.3. We note, for $G_D/G_F \approx 10^2$, $R_F \approx 10^{-11}$, which completely ruins the SBBN predictions.

II. For $m_\chi \sim O(m_n)$, $R_F$ remains at its SBBN value for two regions, i.e. for $G_D/G_F \lesssim 10^{-1}$ and for $G_D/G_F \gtrsim 10^2$. The SBBN prediction is altered only for the intermediate region, i.e. $10^{-1} \lesssim G_D/G_F \lesssim 10^2$. The situation is depicted through two illustrative masses, i.e. $m_\chi = 1 \text{ GeV}$ (red dashed line) and $m_\chi = 1.5 \text{ GeV}$ (blue dotted line).
line). For $G_D/G_F \lesssim 10^{-1}$ the weak interaction dominates over the co-annihilation throughout the evolution history, therefore the SBBN scenario is trivially satisfied. For $G_D/G_F \gtrsim 10^2$ the relative abundance of $\chi$ to neutrons ($Y_\chi/Y_n$) becomes too small to modify $R_F(BBN)$ via the interplay between the weak and the co-annihilation processes discussed earlier. In the intermediate region, the co-annihilation can not keep $Y_\chi$ in its equilibrium as indicated in Eq.2.6. Therefore, $Y_\chi/Y_n$ becomes large enough (but not as large as in the case of $m_\chi < O(m_n)$) to ruin BBN predictions.

To sum up, when $m_\chi < O(m_n)$, BBN prediction allows only small values of $G_D/G_F$. The parameter space for $G_D/G_F$ is enhanced for $m_\chi \sim O(m_n)$, as both small and large values of $G_D/G_F$ can reproduce BBN predictions.

### 3.2 Relic density constraint

Now, we shall study the possibility of $\chi$ to be a DM candidate while being consistent with the BBN predictions. In Fig.4 we have shown contours of constant relic densities of $\chi$ in the $G_D/G_F - m_\chi$ plane, where the red solid line represents the central value of the observed DM relic density, i.e. $\Omega_\chi h^2 = 0.12$ [24]. The grey shaded region in Fig.4 is allowed from the BBN prediction, i.e. $R_F = R_F(BBN)$. The kinematic bound on $m_\chi$ indicated by the black dashed line in the left panel ensures the stability of $\chi$, already shown in Eq.2.3. We note that for $m_\chi \simeq 0.92$ GeV, the new particle species contributes to the whole of the observed DM relic density, simultaneously satisfying the BBN constraint. For $m_\chi \gtrsim 0.92$ GeV, the energy density of $\chi$ contributes to the small fraction of the total DM density, e.g. see the magenta dashed and blue dot-dashed lines representing $\Omega_\chi h^2 = 10^{-3}$ and $\Omega_\chi h^2 = 10^{-6}$ respectively. For $m_\chi \lesssim 0.92$ GeV, the universe would be over-abundant with $\chi$, as shown by the black and orange dotted lines (in the right panel) denoting $\Omega_\chi h^2 = 10$ and $\Omega_\chi h^2 = 10^2$ respectively.

![Figure 4](image-url)

**Figure 4:** Contours in the $G_D/G_F - m_\chi$ plane represent the relic densities of $\chi$ particle, where the central value of the observed DM relic density, $\Omega_\chi h^2 = 0.12$ is achieved for the parameter points on the red solid line. The right panel is the zoomed in version of the left panel focusing on the parameter region, $G_D/G_F \in [100, 250]$ and $m_\chi \in [0.91, 0.93]$ GeV.
Therefore, the additional particle species, co-annihilating with neutron having $m_\chi \lesssim 0.92$ GeV is problematic from the DM relic density constraint, which calls for some extra number-changing processes to further reduce its number density. In passing, we notice that the additional species becomes a DM candidate only when its mass is near the neutron mass. This is a kind of generic feature in determining the relic via co-annihilations in which the mass difference between two particle species should be comparable to the temperature at the relevant epoch [8]. In particular, the freeze-out temperature of $\chi$ in the non-relativistic scenario turns out to be $T_F \sim (m_n - m_\chi) \sim \mathcal{O}(10)$ MeV for $m_\chi = 0.92$ GeV. It is clear that this condition is not satisfied for the relativistic freeze-out scenario when the mass of $\chi$ is away from the neutron mass.

4 Direct detection constraint

The direct detection of $\chi$ particle within the present effective theory comes from a loop-induced elastic scattering of neutron and $\chi$. For details see Appendix A. The spin-independent neutron-DM cross-section is found to be $2.3 \times 10^{-40}$ cm$^2$ for a typical benchmark point $m_\chi = 0.92$ GeV and $G_D/G_F = 250$ for which $\chi$ saturates the total DM density of the universe. For light DM ($m_\chi \lesssim 3$ GeV) as in our case, the bound for the spin-independent DM-nucleon cross-section is rather weak for XENON1T experiments [25]. This type of scenario can be probed at CRESST-III experiments [26] (and via Migdal effect [27, 28]) of which the current bound on the nucleon-DM scattering cross-section is $\sigma_{\chi n} \lesssim 10^{-38}$ cm$^2$ for $m_\chi \lesssim 1$ GeV. Hence, the present scenario with the DM mass of the order of the neutron mass is allowed by the current experiments. However, this type of scenario can be constrained with improved sensitivities of these experiments in near future. In passing, we also note for $m_\chi > 0.92$ GeV, $\chi$ contributes to the fraction of DM density, thereby the direct detection bound weakens in our scenario substantially.

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A Loop-induced scattering of neutron and $\chi$

To start with, we had only operator that enables the co-annihilation of $\chi$ and neutron producing a proton and an electron in the final state. This operator itself induces an effective operator at one-loop level, responsible for the elastic scattering of neutron and $\chi$ as shown in Fiq.5, i.e. relevant for direct detection experiments.
The amplitude for elastic scattering, $n(p_1) + \chi(p_2) \rightarrow n(p_3) + \chi(p_4)$ calculated from Eq.2.1 is given by,

\[
iM = \left(-i\frac{G_D}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \left[ \bar{u}(3) \Gamma^\mu \frac{i(\slashed{k} + m_p)}{k^2 - m^2_p} \Gamma^\nu u(1) \right] \left[ \bar{u}(4) \Gamma^\mu \frac{i(\slashed{P} - \slashed{k} + m_e)}{(P - k)^2 - m_e^2} \Gamma^\nu u(2) \right]
\]

(A.1)

where $u(i) = u(p_i)$ and $P^2 = (p_1 + p_2)^2 = s$, while $p_i$ is the on-shell four momentum. For $\Gamma^a = \gamma^a(I - \gamma_5)$, the form used for all calculations, we get $\Gamma^a \Gamma^b = 0$. Then the amplitude takes rather simpler form, i.e.

\[
iM = \left(\frac{G_D}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu (P - k)^\nu}{(k^2 - m^2_p)((P - k)^2 - m_e^2)} \left[ \bar{u}(3) \Gamma^\mu \gamma^\rho \Gamma^\nu u(2) \right] \left[ \bar{u}(4) \Gamma^\mu \gamma^\sigma \Gamma^\nu u(1) \right]
\]

(A.2)

The above integral is divergent in general, which is expected in an effective theory with coupling of negative mass dimension. Therefore, we put a cut-off scale, $\Lambda$ and calculate the amplitude as

\[
iM = \frac{iG_D^2}{32\pi^2} \left( A P^\rho P_\sigma + B \frac{A^2}{4} \eta_{\rho\sigma} \right) \left[ \bar{u}(3) \Gamma^\mu \gamma^\rho \Gamma^\nu u(1) \right] \left[ \bar{u}(4) \Gamma^\mu \gamma^\sigma \Gamma^\nu u(2) \right]
\]

(A.3)

where $A$ and $B$ given by Eq.A.6 are complex numbers in general, to be calculated numerically.

\[
A = \int_0^1 dx \; x(1 - x)F_1(x), \quad B = \int_0^1 dx \; F_2(x)
\]

(A.4)

\[
F_1(x) = \log \left( \frac{\Lambda^2 + \Delta}{\Delta} \right) - \frac{\Lambda^2}{\Lambda^2 + \Delta}
\]

\[
F_2(x) = 1 + \frac{\Delta}{\Lambda^2 + \Delta} - \frac{2\Delta}{\Lambda^2} \log \left( \frac{\Lambda^2 + \Delta}{\Delta} \right)
\]

\[
\Delta = m_e^2(1 - x) + m_p^2 x - x(1 - x)P^2
\]

Now, this is the most general structure for neutron-DM scattering from our effective theory. In direct detection experiments, the stringent bounds comes from the spin-independent operators. Therefore, we extract the spin-independent part ($M_{SI}$) from the above expression as the following using the relation, $\Gamma^\mu \gamma^\rho \Gamma^\nu = 4\gamma^\mu \eta^\rho\sigma(I - \gamma_5) - 2\gamma^\mu \gamma^\rho \gamma^\sigma(I - \gamma_5)$.
\[ iM = \frac{iG_F^2}{32\pi^2} \left( A P_\rho P_\sigma + B A^2 \eta_{\rho\sigma} \right) \left[ \bar{u}(3) \left( 4\gamma^\mu \gamma^\rho (I - \gamma_5) - 2\gamma^\mu \gamma^\rho \gamma^\sigma (I - \gamma_5) \right) u(1) \right] \\
\]
\[ iM_{SI} = \frac{iG_F^2}{2\pi^2} \left( A P^2 + B \Lambda^2 \right) \left[ \bar{u}(3) \gamma^\mu u(1) \right] \left[ \bar{u}(4) \gamma_\mu u(2) \right] \]  

(A.5)

Now, we set the cut-off scale, \( \Lambda = (m_n + m_\chi) \), which is the center-of-mass energy of the elastic scattering in the non-relativistic regime. The neutron-DM scattering cross-section (\( \sigma_{\chi n} \)) is calculated [29, 30] for the direct detection experiments by putting \( P^2 = (m_n + m_\chi)^2 \) and numerically evaluating \( A \) and \( B \).

\[ \sigma_{\chi n} \approx \frac{G_F^4}{\pi^5} \left( m_\chi m_n (m_\chi + m_n) \right)^2 \]  

(A.6)

For benchmark points, \( m_\chi = 0.92 \) GeV and \( G_D/G_F = 250 \), we get \( \sigma_{\chi n} \approx 2.3 \times 10^{-40} \) cm\(^2\).

References

[1] E. Aver, K. A. Olive and E. D. Skillman, JCAP 07, 011 (2015) doi:10.1088/1475-7516/2015/07/011 [arXiv:1503.08146 [astro-ph.CO]].

[2] E. W. Kolb and M. S. Turner, “The Early Universe,” Front. Phys. 69, 1-547 (1990) doi:10.1201/9780429492860

[3] E. W. Kolb, M. S. Turner and T. P. Walker, “The Effect of Interacting Particles on Primordial Nucleosynthesis,” Phys. Rev. D 34, 2197 (1986) doi:10.1103/PhysRevD.34.2197

[4] C. Boehm, M. J. Dolan and C. McCabe, “A Lower Bound on the Mass of Cold Thermal Dark Matter from Planck,” JCAP 08, 041 (2013) doi:10.1088/1475-7516/2013/08/041 [arXiv:1303.6270 [hep-ph]].

[5] M. Kawasaki, K. Kohri, T. Moroi and Y. Takaesu, “Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles,” Phys. Rev. D 97, no.2, 023502 (2018) doi:10.1103/PhysRevD.97.023502 [arXiv:1709.01211 [hep-ph]].

[6] E. W. Kolb and R. J. Scherrer, “Massive Neutrinos and Primordial Nucleosynthesis,” Phys. Rev. D 25, 1481 (1982) doi:10.1103/PhysRevD.25.1481

[7] M. Kawasaki, K. Kohri, T. Moroi and Y. Takaesu, “Revisiting Big-Bang Nucleosynthesis Constraints on Dark-Matter Annihilation,” Phys. Lett. B 751, 246-250 (2015) doi:10.1016/j.physletb.2015.10.048 [arXiv:1509.03665 [hep-ph]].

[8] K. Griest and D. Seckel, Dolan:2017xbu “Three exceptions in the calculation of relic abundances,” Phys. Rev. D 43, 3191-3203 (1991) doi:10.1103/PhysRevD.43.3191

[9] B. Belfatto, D. Buttazzo, C. Gross, P. Panci, A. Strumia, N. Vignaroli, L. Vittorio and R. Watanabe, “Dark Matter abundance via thermal decays and leptoquark mediators,” [arXiv:2111.14808 [hep-ph]].

[10] A. Strumia, “Dark Matter interpretation of the neutron decay anomaly,” JHEP 02, 067 (2022) doi:10.1007/JHEP02(2022)067 [arXiv:2112.09111 [hep-ph]].
[11] J. A. Dror, G. Elor and R. Mcgehee, “Absorption of Fermionic Dark Matter by Nuclear Targets,” JHEP 02, 134 (2020) doi:10.1007/JHEP02(2020)134 [arXiv:1908.10861 [hep-ph]].

[12] A. J. Long, C. Lunardini and E. Sabancilar, “Detecting non-relativistic cosmic neutrinos by capture on tritium: phenomenology and physics potential,” JCAP 08, 038 (2014) doi:10.1088/1475-7516/2014/08/038 [arXiv:1405.7654 [hep-ph]].

[13] A. G. Cocco, G. Mangano and M. Messina, “Probing low energy neutrino backgrounds with neutrino capture on beta decaying nuclei,” JCAP 06, 015 (2007) doi:10.1088/1475-7516/2007/06/015 [arXiv:hep-ph/0703075 [hep-ph]].

[14] J. M. Cline, “Dark atoms and composite dark matter,” doi:10.21468/SciPostPhysLectNotes.52 [arXiv:2108.10314 [hep-ph]].

[15] D. McKeen, M. Pospelov and N. Raj, “Cosmological and astrophysical probes of dark baryons,” Phys. Rev. D 103, no.11, 115002 (2021) doi:10.1103/PhysRevD.103.115002 [arXiv:2012.09865 [hep-ph]].

[16] B. D. Fields, K. A. Olive, T. H. Yeh and C. Young, “Big-Bang Nucleosynthesis after Planck,” JCAP 03, 010 (2020) [erratum: JCAP 11, E02 (2020)] doi:10.1088/1475-7516/2020/03/010 [arXiv:1912.01132 [astro-ph.CO]].

[17] R. An, V. Gluscevic, E. Calabrese and J. C. Hill, “What does cosmology tell us about the mass of thermal-relic dark matter?,” [arXiv:2202.03515 [astro-ph.CO]].

[18] M. Fukugita and T. Yanagida, “Baryogenesis Without Grand Unification,” Phys. Lett. B 174, 45-47 (1986) doi:10.1016/0370-2693(86)91126-3

[19] V. A. Rubakov and M. E. Shaposhnikov, “Electroweak baryon number nonconservation in the early universe and in high-energy collisions,” Usp. Fiz. Nauk 166, 493-537 (1996) doi:10.1070/PU1996v039n05ABEH000145 [arXiv:hep-ph/9603208 [hep-ph]].

[20] R. V. Wagoner, W. A. Fowler and F. Hoyle, “On the Synthesis of elements at very high temperatures,” Astrophys. J. 148, 3-49 (1967) doi:10.1086/149126

[21] V. A. Rubakov and D. S. Gorbunov, “Introduction to the Theory of the Early Universe: Hot big bang theory,” doi:10.1103/10447

[22] J. Bernstein, L. S. Brown and G. Feinberg, “COSMOLOGICAL HELIUM PRODUCTION SIMPLIFIED,” Rev. Mod. Phys. 61, 25 (1989) doi:10.1103/RevModPhys.61.25

[23] V. F. Mukhanov, “Nucleosynthesis without a computer,” Int. J. Theor. Phys. 43, 669-693 (2004) doi:10.1023/B:IJTP.0000048169.69609.77 [arXiv:astro-ph/0303073 [astro-ph]].

[24] N. Aghanim et al. [Planck], “Planck 2018 results. VI. Cosmological parameters,” Astron. Astrophys. 641, A6 (2020) [erratum: Astron. Astrophys. 652, C4 (2021)] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].

[25] E. Aprile et al. [XENON], Phys. Rev. Lett. 123, no.25, 251801 (2019) doi:10.1103/PhysRevLett.123.251801 [arXiv:1907.11485 [hep-ex]].

[26] A. H. Abdelhameed et al. [CRESST], “First results from the CRESST-III low-mass dark matter program,” Phys. Rev. D 100, no.10, 102002 (2019) doi:10.1103/PhysRevD.100.102002 [arXiv:1904.00498 [astro-ph.CO]].

[27] N. F. Bell, J. B. Dent, J. L. Newstead, S. Sabharwal and T. J. Weiler, “Migdal effect and photon bremsstrahlung in effective field theories of dark matter direct detection and coherent
elastic neutrino-nucleus scattering,” Phys. Rev. D 101, no.1, 015012 (2020) doi:10.1103/PhysRevD.101.015012 [arXiv:1905.00046 [hep-ph]].

[28] M. J. Dolan, F. Kahlhoefer and C. McCabe, “Directly detecting sub-GeV dark matter with electrons from nuclear scattering,” Phys. Rev. Lett. 121, no.10, 101801 (2018) doi:10.1103/PhysRevLett.121.101801 [arXiv:1711.09906 [hep-ph]].

[29] T. Lin, “Dark matter models and direct detection,” PoS 333, 009 (2019) doi:10.22323/1.333.0009 [arXiv:1904.07915 [hep-ph]].

[30] M. Lisanti, “Lectures on Dark Matter Physics,” doi:10.1142/9789813149441_0007 [arXiv:1603.03797 [hep-ph]].