Additive manufacturing for high precision structural properties via feedback control

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Abstract—This paper introduces the problem of printing a cantilever beam with precise structural requirements using Additive Manufacturing and feedback control. The manufactured cantilever beam is made of 500 horizontal layers with different widths, allowing for various final shapes of the beam. The whole printing process is therefore formalized as a finite-horizon discrete control problem, with a target stiffness to achieve and a cost function to minimize, and where the controls are the widths of the successive layers. By using a model linking the widths of the different layers of the printed part to its final stiffness, the shape is optimized by solving the related control problem. The dynamics describing the printing process are based on a simple structural mechanics model depending on four real parameters that are determined by printing several specimens parts prior to starting the manufacturing of the cantilever beam. During the printing process, stiffness measurements are performed on the partially built part in order to adjust the widths of the subsequent layers. This type of feedback control, consisting in repeatedly solving an optimization problem starting from the current state at each step, produces parts with more precise structural properties than the equivalent open-loop control problem that only solves the optimization problem once at the initial state. By using closed-loop control, we show that two sources of structural properties uncertainty in AM can be alleviated: the difficulty to correctly model printing processes and their variability.

I. INTRODUCTION

Additive Manufacturing (AM), often referred to as 3D printing, is becoming increasingly popular for manufacturing during the last years, with a growing market and a bigger demand for AM systems [32]. First considered a tool for rapid prototyping, it is now viewed as a technology that can be used to manufacture finished products, when there is a need for highly customized products, small batch production, or when complex geometries cannot be handled by traditional manufacturing technologies. AM has even made its way into industries that are characterized by strict certification processes, such as the aerospace or the biomedical industry [27]. Metal AM, especially, is now being utilized to build engine parts in commercial jets, such as in the CFM International LEAP engine, which is certified by the FAA and EASA [19]. In late 2017, the FAA has submitted a draft Additive Manufacturing Strategic Roadmap spanning eight years for review [31].

Despite the design possibilities offered by AM, its wide adoption has been slowed down by the difficulty in evaluating and guaranteeing the quality of the prints. Indeed, the lack of understanding of the physics of the different AM processes is an obstacle that often prevents manufacturers from having guarantees on the mechanical properties of the printed builds [6] [12] [18]. AM systems, from the low-cost plastic kit printers to the expensive metal printers used by industry, often have dozens or hundreds of control parameters. In the case of plastic AM, such parameters can include extrusion temperature, extrusion rate, print bed temperature, and nozzle diameter. Tuning these parameters is often a manual job, based on trials and errors and empirical considerations. Indeed, the relationships between printing parameters, external environment conditions, and the final properties of the builds are intricate and mostly unknown. Determining an accurate physics-based model for an AM process is extremely hard, time-consuming, and unlikely to generalize to other similar processes, printers, or even manufactured parts if their geometries differ too much.

A well-known engineering concept to palliate the negative outcomes of erroneous models is feedback (or closed-loop) control. In feedback control systems, the controls applied at each time step are influenced by the measurements performed on the system, whereas in open-loop control systems, a sequence of control inputs is pre-computed and implemented, and changes of the state of the system with time are not used to refine the subsequent controls at each stage of the printing process. AM systems mostly rely on open-loop controllers, or utilize closed-loop controllers based on low-level features of the system, such as extruding nozzle speed: work to date has essentially focused on local properties of the printed objects [17] [33].

This paper considers the problem of printing an object that must eventually satisfy a specific global property. A proof of concept is demonstrated as follows: The printed part is a vertical cantilever beam that must meet a prescribed stiffness specification. Several specimens are printed, both in an open-loop and a closed-loop fashion. The open-loop controls are based on a simple parameterized model made to fit the stiffness properties of previously printed test specimens. The closed-loop control architecture is based on the same model and relies on concurrent optimization techniques reminiscent of Model Predictive Control (MPC) [7].

The remainder of this paper is structured as follows. Section II describes previous work related to the topic of this paper, including modeling of AM processes, feedback control in AM, and a prior experiment that shows the relevance of using global properties for feedback control in AM is presented as a simple introductory experiment. In Section III the main experiment that constitutes the original contribution of this paper is described, along with the simple physics model that is considered to derive an optimal control law. The derivation of the closed-loop control architecture is given in Section IV. Last, we present the results of the printing experiments in Section V before concluding in Section VI.
II. RELATED WORK

There are two main fields of research for AM that are related to this paper. First, a very active topic of research deals with the problem of modeling the behavior of AM systems, mostly by using physical models to predict the structural properties of printed parts. There is a predominance in the literature about the modeling of metal-based printing processes. References [23, 11, 34, 28] study the physics of specific metal processes, while in [5, 14, 21] the structural properties of metallic 3D printed parts are evaluated. Among metal alloys, titanium-based ones are popular because their mechanical properties are the subject of many research works when used in AM [20, 30, 24, 4, 2, 8]. Although the amount of work regarding plastic materials is less significant, there have been some studies about the mechanical properties of parts build with Fused Deposition Modeling (FDM) [1, 3]. Besides the modeling of AM processes, a second field of research related with this paper is concerned with the optimization of the printing parameters and the feedback control of the printing processes. References [17] and [33] use closed-loop controllers to control the width of the printed layers in laser-based additive manufacturing. In [26], the build path is changed during the printing of a part to verify some temperature-based constraints. Reference [29] discusses the optimization of the process parameters of FDM, while [22] presents an evaluation of the measurements needs for closed-loop control of powder bed fusion processes. In [13], a leaf is manufactured using AM, validating the hypothesis that by introducing a high level feedback control loop during the 3D printing of an object, its mechanical properties can be improved. Leaf springs are springs made of several layers superposed on top of each other, forming a stack of unstuck layers. By changing the infill density of each leaf, the final springs have a stiffness whose error with the target is significantly reduced compared with the equivalent open-loop controller, from 11.63% to 1.43% and from 7.28% to 2.23%, with three and four leaves, respectively. Despite the encouraging result, the experiment presented in [13] relied on measurements taken outside of the printer, whereas the measurements taken in the experiment presented here are made in situ.

III. MANUFACTURING A CANTILEVERED BEAM TO MEET SPECIFIC STIFFNESS SPECIFICATIONS

A. Description of the experiment

The main experiment presented in this paper consists of the manufacturing of a vertical cantilever beam with a rudimentary 3D printer, using feedback control for the lateral stiffness of the beam to eventually reach a specific value. The stiffness is being defined here as the ratio of the load to the deflection of a beam (which is assumed to be constant for small deflections) under the cantilever beam deflection test, described below (see also Fig. 2). Unlike the foregoing leaf spring, the printed cantilever beam is made of a single part and the intermediate measurements are taken directly in situ. Each time a measurement is made, the manufacturing is paused. The other features of the cantilever beam such as height, thickness, printing pattern, and infill density are fixed.

B. Modeling of the printing process as a stochastic discrete dynamic problem

We begin the design of the feedback control architecture by designing a model binding the controls variables and the parameters of interest (here the final stiffness of the beam). The printing process via Fused Deposition Modeling (FDM) is hard to model because of the combination of different effects at different scales involving phase changes of a polymeric material and a high number of control and environmental variables [25]. The precise modeling of the process is not within the scope of this paper, and while this research direction could be beneficial to improve the results presented here, it is neither a requirement nor an implicit assumption. Instead, a macroscopic probabilistic model is used and adjusted by making some preliminary measurements on test specimens.

The controlled printing process is discretized in several stages, each one corresponding to the printing of one layer. A layer is a pass in a horizontal plane of the printer head throughout the object, since 3D printed objects are usually printed layer by layer after their 3D model has been sliced in multiple layers by a software called slicer. The state of the system at a given step is the vector of the reciprocals of the widths of the printed stacks (we will see why later). All non-printed stacks are attributed a value of 0. Such a representation of the state is justified by the equation of the final stiffness of the cantilever beam discussed thereafter. See Fig. 1 for a picture of a full cantilever beam.

1) Structural mechanics of the printed beam: The cantilever beam is considered a porous Euler-Bernoulli beam made of a single material, which is assumed isotropic for simplification purposes. Objects made with FDM are known for their anisotropic properties [1], however, this approximation is sufficient for small unidirectional deflections in the context of this work. Indeed, our work is not concerned with the accurate modeling of the physics of the printing process, but with feedback control strategies that do not require precise models for design. The fact that feedback control strategies do not require precise models for their design is their main advantage and confer strong robustness properties to them. Much of the role of feedback is, indeed, to improve the quality of the final product, despite the presence of components with significant lack of knowledge [10, 9]. The relation between load and deflection in the cantilever beam deflection test is derived from the Euler-Bernoulli equation:

$$\frac{d^2}{dz^2} \left( EI(z) \frac{d^2 w}{dz^2} \right) = 0.$$ 

Here, $z$ is the vertical axis, $w(z)$ is the deflection along the $x$ axis at a specific height (Fig. 2), $E$ is the Young’s modulus of the material, and $I(z)$ is the second moment of area of the cross-section of the beam at height $z$ along the $x$ axis. For more details about the Euler-Bernoulli equation and the definition of the second moment of area, see [15].

Because of the varying width of the printed beam, $I(z)$ is not constant. However, the beam can be divided in layers of equal height, with each single layer having a constant width. $I(z)$ is then a piecewise constant function.
For a cantilever beam made of \( n \) layers with same height \( h \), let \( I_k \) denote the second moment of area of the \( k \)-th layer (\( 1 \leq k \leq n \)).

Let \( F \) be the applied load on top of the beam, then

\[
 w(nh) = \frac{h^3 F}{E} \sum_{k=1}^{n} c_{n,k} \frac{1}{I_k} 
\]

(1)

with

\[
 c_{n,k} = \frac{3(2k-1)(n-k) + 3k^2 - 1}{6}. 
\]

(2)

A derivation of Eqs. (1) and (2) is provided in Appendix A.

From here onwards, we make the approximation that \( I_k \) is affine in the width \( v_k \) of the layer \( k \) (the width is the length of the layer along the \( y \) axis), which makes us write,

\[
 \frac{E}{h^3 F} I_k = \frac{1}{\alpha}(v_k + \gamma). 
\]

(3)

\( \alpha \) and \( \gamma \) are parameters of the model that depend on the geometrical and material properties of the beam. This assumption might seem unusual, and assuming a linear relationship could seem more natural. Indeed, when considering a homogeneous isotropic material, \( I_k \) scales linearly with \( w_k \). However, such an assumption could not explain the experimentally measured stiffnesses of the test specimens with a reasonable precision (See Section III-C). A possible explanation comes from the toolpath produced by the slicer software used in this experiment. The toolpath creates a uniform porous pattern with regularly spaced thin and dense support structures in the interior of the print. The constant term in the affine function illustrates the contribution of these support structures. It is important to repeat here that the goal of this work is not to accurately describe the physics behind the specific printing process used in this experiment. Instead, we want to show that, by using a basic model derived from empirical or physical considerations to fit a limited amount of data, one can design a good feedback control policy that will considerably improve the conformance of the final print to a given stiffness specification.

Under the assumption that the second moment of area \( I_k \) is affine in the width \( v_k \), the inverse of the stiffness of the vertical beam, or compliance, can be written

\[
 C = \frac{w(nh)}{F} = \alpha \frac{n}{\sum_{k=1}^{n} v_k + \gamma}. 
\]

(4)

Therefore, once these two parameters are estimated, knowing the reciprocals of the widths of each layers is enough to retrieve an analytical expression for the total stiffness of the beam, defined as

\[
 K = \frac{1}{C}. 
\]

2) Definition of the discrete dynamic system: For the printing of \( N \) layers, we consider a process of \( N \) stages. At each stage \( n \leq N \), the state \( s_n \) of the system is a vector of \( N \) real values, with the \( n \) first entries being the reciprocals of the layer widths, and the others being set to 0. The initial state is set to 0.

The dynamics across the stages are driven by the input \( u_k + \epsilon_k \), which is the width of the next layer. \( u_k \) is the control input while \( \epsilon_k \) is a stochastic term. This terms illustrates the
variability and external disturbances in the deposition process of the material that influence the actual layer width. Since at stage \( n \geq 1 \), the system is only changed by the addition of a new layer, only the \( n \)-th entry of the state vector, originally set zero, is modified. The dynamics of the system at step \( n \) are therefore given by

\[
s_n = s_{n-1} + \frac{1}{u_n + \gamma + \epsilon_n} e_n,
\]

where \( e_n \) is the \( n \)-th vector of the canonical basis, \( u_n + \gamma + \epsilon_n \) is the real width of layer \( n \), and the noise \( \epsilon_n \) has a normal distribution of mean 0 and standard deviation \( \sigma_n \). Under the assumption \( \epsilon_n \ll u_n + \gamma \), we make the first-order approximation

\[
s_n = s_{n-1} + \frac{1}{u_n + \gamma} \left( 1 - \frac{\epsilon_n}{u_n + \gamma} \right) e_n.
\]

Because \( \epsilon_n \) has a symmetric probability distribution function, we can reverse its sign and write

\[
s_n = s_{n-1} + \frac{1}{u_n + \gamma} \left( 1 + \frac{\epsilon_n}{u_n + \gamma} \right) e_n.
\]  

(5)

3) Observation of the state of the system: The system is observed with intermediate in situ measurements of the stiffness of the partially printed part, as described on Fig. 2. Based on equation (4), the inverse of the stiffness at step \( n \) is equal to the compliance

\[
C_n = \alpha C_n^T s_n
\]

where \( C_n = (c_{n,1}, \ldots, c_{n,n})^T \). In this experiment, the quantity directly measured is the stiffness of the cantilever beam. Consider the observation \( c_n^k = \frac{1}{c_n} + \nu_n \), where \( \nu_n \) is a random variable with normal distribution with zero mean and standard deviation \( \sigma_n \). The observed compliance at step \( n \) is given by

\[
o_n = \frac{1}{\alpha C_n^T s_n} + \nu_n \approx \alpha C_n^T s_n \left( 1 - \nu_n \alpha C_n^T s_n \right)
\]

that we rewrite

\[
o_n = \alpha C_n^T s_n \left( 1 + \nu_n \alpha C_n^T s_n \right)
\]

(6)

since the distribution of \( \nu \) is symmetric about zero.

C. Identification of the model parameters

Under the model of the state and its dynamics given by (5), and of the observation procedure (6), the unknown parameters are \( \alpha \), \( \gamma \), \( \sigma_n \), and \( \sigma_o \). They are estimated by running several stiffness measurements on test specimens built for that purpose.

1) Test specimens: Test specimens with fixed heights and thickness and with different constant widths were printed prior to attempting any closed-loop control experiment (See Fig. 3). The chosen specimen height is 50 mm, the thickness is 3 mm, while the different input widths are 5, 10, 15 and 20 mm. It is worth mentioning that the height of the test specimens is half the height of the full specimens printed in Section V below. Doing so is a way to validate the model over a broader range of specimen shapes than those with which it was calibrated. Three specimens per width were printed for a total of twelve specimens. The infill density was constant (50%).

2) Stiffness measurement procedure: On each test specimen, a stiffness measurement was performed by measuring the force applied to a load cell while deflecting the cantilever beam as shown in Fig. 4. The load cell was mounted on the 3D printer moving head support, and the deflection was controlled by directly moving the printer platform with a custom sequence of G-code instructions. A sequence of pairs of deflection and load measurements was obtained, and the measured stiffness of the specimen was determined by performing an affine interpolation of these values and by keeping the coefficient of the first-order term. We therefore assume that for a test specimen of stiffness \( K \), the different load and deflection pairs \((f_i, w_i)\) are related by the formula

\[
f_i = Kw_i + \delta,
\]

where \( \delta \) represents calibration errors of the measuring instruments.

3) Estimation of the parameters: Stiffness measurements were taken in order to determine the parameters \( \alpha \), \( \gamma \), \( \sigma_n \), and \( \sigma_o \) introduced by Eqs. (3), (5), and (6). In order to reduce sensor noise effects, five measurements per specimen were made. Let \( m \) be the number of different widths that were printed, \( p \) the number of printed specimens per width, and \( q \) the number of stiffness measurements performed per specimen. For a specific specimen of constant input width \( u_i \), made of \( n \) layers, defined uniquely by the pair \((i,j)\) with \( i \leq m \) and \( j \leq p \), we consider a stiffness measurement \( l \) with \( l \leq q \). Combining equation (5) and (6) gives the following measured compliances \( \hat{C}_{i,j,l} \)

\[
\hat{C}_{i,j,l} = \frac{\alpha}{u_i + \gamma} \left( \sum_{k=1}^{n} c_{n,k} + \frac{1}{u_i + \gamma} \sum_{k=1}^{n} c_{n,k} \epsilon_{i,j,k} \right)
\]

\[
\times \left( 1 + \frac{\nu_{i,j,k}}{u_i + \gamma} \sum_{k=1}^{n} c_{n,k} + \frac{1}{u_i + \gamma} \sum_{k=1}^{n} c_{n,k} \epsilon_{i,j,k} \right)
\]

where \( \epsilon_{i,j,k} \) is the process noise of the layer \( k \) of the specimen \((i,j)\) and \( \nu_{i,j,k} \) the observation noise of the measurement \( l \) taken on the same specimen. Then, we use \( \hat{C}_i = \frac{1}{pq} \sum_{j=1}^{p} \sum_{l=1}^{q} \hat{C}_{i,j,l} \) as an unbiased estimator of the expectation of the compliance of a test specimen of input width \( u_i \), that is

\[
\mathbb{E}(\hat{C}_i) = \frac{\alpha}{u_i + \gamma} \sum_{k=1}^{n} c_{n,k}.
\]
Then, we consider \( \hat{C}_{i,j} = \frac{1}{q} \sum_{l=1}^{q} C_{i,j,l} \), which is an unbiased estimator of the compliance of the specimen \((i,j)\), is equal to

\[
\alpha \frac{u_i + \gamma}{u_i} + \frac{1}{u_i + \gamma} \sum_{k=1}^{n} c_{n,k} \epsilon_{i,j,k}.
\]  

(7)

We obtain an estimator \( \hat{\sigma}_o^2 \) of \( \sigma_o^2 \) by considering the squares of

\[
\frac{C_{i,j,l} - \hat{C}_{i,j}}{C_{i,j,l}^2} \frac{u_i + \gamma}{\alpha}
\]

for each tuple \((i,j,l)\), and by taking their mean, which gives

\[
\hat{\sigma}_o^2 = \frac{1}{mpq} \sum_{i=1}^{m} \sum_{j=1}^{p} \sum_{l=1}^{q} \left( \frac{C_{i,j,l} - \hat{C}_{i,j}}{C_{i,j,l}^2} \frac{u_i + \gamma}{\alpha} \right)^2.
\]

From the estimator (7), we can also isolate the sum \( \sum_{k=1}^{n} c_{n,k} \epsilon_{i,j,k} \), which is a random variable with a normal distribution of mean 0 and variance \( \sigma_p^2 \sum_{k=1}^{n} c_{n,k}^2 \). From these sums we can finally estimate the value of \( \sigma_p^2 \) by taking

\[
\hat{\sigma}_p^2 = \frac{(\sum_{k=1}^{n} c_{n,k} \epsilon_{i,j,k})^2}{\sum_{k=1}^{n} c_{n,k}^2}.
\]

IV. FEEDBACK CONTROL LAW

A. State estimation

During the printing experiment, the state of the system \( s_n \) at step \( n \) is estimated with a linear Kalman filter. \( \mu_n \) is the mean of the estimated state while \( \Sigma_n \) is its covariance. Only the first \( n \) entries of the state are considered (the other ones, which are 0, are ignored). The update of the Kalman filter consists of two steps. The first one is the update after a control \( u_n \) is applied and before an observation is made (dynamics propagation). The second one is the update after a measurement of the compliance that gives a value \( o_n \) is made, it corresponds to the application of a Bayesian filtering step. Here, we write \( \alpha_n = \frac{1}{u_i + \gamma} \) to simplify the notation of the Kalman filter update steps. In the experiments of section V, several control steps are performed (and equivalently, a same number of layers are
The first component $L_1(u)$ is the quantity of used material, a natural quantity to minimize. The second one, $L_2(u)$, is the sum of the squares of the difference of successive controls. It prevents the series of control inputs to change too abruptly and favors a smooth transition from the initial width of the beam to a final value close to the minimum possible width. Finally, the third component $L_3(u)$ is the variance of the final compliance of the system after applying the controls $u$ without measurements. It drives the shape of the final beam to regions of the state space where the certainty on the final stiffness is higher. The idea of using such a cost function that encourages a specific form for the covariance of the final state has similarities with covariance control for linear systems [16]. $L_1(u)$, $L_2(u)$ and $L_3(u)$ can be written

$$L_1(u) = \sum_{k=n}^{N} u_k$$

$$L_2(u) = \sum_{k=n-1}^{N-1} \left( u_{k+1} - u_k \right)^2 + \left( u_N - u_{\min} \right)^2$$

$$L_3(u) = \alpha^2 \sigma_p^2 \sum_{k=n}^{N} \frac{C_{N,k}^2}{(u_k + \gamma)^2}$$

Note that the cost functions are not exactly the same at each stage, since the size of the vector they take as argument depends on the current stage. However, we use the same notation at each stage for better readability. At stage 1, in the cost function $L_2$, $u_0$ is replaced by $u_{\max}$, which is the width of the base on top of which the closed-loop control is applied.

### C. Application of the optimal control

From the cost function (12) and the deterministic dynamics given by the update of the Kalman filter (9), an observer-based optimal control strategy is derived by solving the related optimization problem. At step $n$, it therefore writes:

Minimize $u_n, \ldots, u_N$

subject to: $u_{\max} \geq u_1 \geq u_2 \geq \cdots \geq u_N \geq u_{\min}$,

$$\alpha \left( \sum_{k=1}^{n-1} c_{n,k} \mu_k + \sum_{k=n}^{N} c_{n,k} \right) = C^*$$

where $C^*$ is the target compliance and $u = (u_k)_{n \leq k \leq N}$ is the series of control inputs applied from the present state $\mu_n$ at step $n$ up to the end of the printing process.

To derive the control utilized in the experiment of this section, the three-component cost function

$$L(u) = \alpha_1 L_1(u) + \alpha_2 L_2(u) + \alpha_3 L_3(u)$$

is considered. The real coefficients $\alpha_1$, $\alpha_2$ and $\alpha_3$ weight and scale the contribution of each cost function in the total cost.

### V. Experiments

The goal of the performed experiments is to show that applying feedback control to the printing process can improve the global properties of the printed object by compensating for

...
two different sources of inaccuracy. The first one is the model error, which is the gap between the predicted properties of a part and its actual properties. The second one is the variability of the printing process, which may produce parts with different properties for the same control inputs. In order to evaluate the model error and the variance of the printing process, several specimens are printed open-loop, by directly applying the controls minimizing the cost function of equation (12) subject to the constraints (10) and (11). Each printed specimen has its stiffness measured by the procedure described in Section III-C2. The printer used for the manufacturing of the specimens is a Monoprice MP Select Mini 3D Printer V2.

The values chosen in this experiment are 10 cm for the height of the specimens, 3 mm for the thickness, and a width varying between 5 and 20 mm. Since the height of a single layer is 0.2 mm, the total number of layers used for the manufacturing of the cantilever beam is 500. A base of 250 layers (5 cm) of fixed width of 20 mm is first printed. In the case of an open-loop control, the remaining 250 layers with varying width are then directly printed. Pictures of the base and of a full specimen printed open-loop are shown in Figures 4 and 8, respectively. When closed-loop control is used, the rest of the printing process is divided in 10 stages. At each stage, the stiffness of the unfinished beam is measured with five repeated measurements, and 25 layers are printed before repeating the process until the cantilever beam is complete. In the Kalman filter update, the variance of the observation noise is scaled by a factor of $\frac{1}{5}$ since five measurements (which we assume independent) are taken.

### A. Open-loop control

First, we describe the results obtained from the open-loop printing experiments. The two main important values that are derived from these experiments are the average and standard deviation of the stiffness obtained on several specimens. The average stiffness characterizes the model error, whereas the standard deviation is an indicator of the global imprecision of the printing process. In this experiment, the target compliance is $0.12 \text{ mm.g}^{-1}$, which is equivalent to a stiffness of approximately $8.333 \text{ g.mm}^{-1}$. Five specimens are printed open-loop and the final stiffness is measured five consecutive times. The results of the stiffness measurements taken on the open-loop specimens are given in Table I.

| Specimen | Stiffness (g.mm$^{-1}$) | Error         |
|----------|------------------------|---------------|
| Specimen 1 | 10.3                  | 1.962 (23.54%) |
| Specimen 2 | 10.66                 | 2.326 (27.91%) |
| Specimen 3 | 13.34                 | 5.008 (60.09%) |
| Specimen 4 | 10.84                 | 2.503 (30.03%) |
| Specimen 5 | 10.3                  | 1.965 (23.58%) |
| Average   | 11.09                 | 2.753 (33.03%) |
| Standard deviation | 1.282                | —             |

### B. Closed-loop control

Three specimens were then printed using closed-loop control and the same target compliance of $0.12 \text{ mm.g}^{-1}$. The results of the final stiffness measurements are reported in Table I. The measured stiffness at each stage of the process is plotted in Fig. 11. For comparison, we also provide stiffness measurements at the height corresponding to every stage (from 5 to 10 cm) for the three last specimens printed open-loop. These measurements are taken after the specimens are completed and are therefore not used during the printing. They are also plotted in Fig. 11.

### C. Correction of model errors with closed-loop control

From the final results of the experiments performed with an open-loop control, we can observe that the final stiffness of each specimen is systematically higher than the target (Fig. 9). This cannot be totally explained by the variability of the printing process, since the average final stiffness value
error (2.753 g/mm) is significant compared to the standard deviation of the final stiffness values (1.282 g/mm). A more plausible explanation is the inaccuracy of the simple model that is used to derive the optimal controls, and a tendency for this model to underestimate the predicted final stiffness of the printed object. Nevertheless, the closed-loop controller, despite being based on the same inaccurate model, is able to lower the average final stiffness value error to 0.1658 g/mm, a considerable improvement of almost 94%. This result shows the relevance of closed-loop control for handling model inaccuracies. However, a legitimate question is to ask if closed-loop control can still be of any use when the model is good enough to predict the correct object properties, and to compute an open-loop control leading to the desired

| Specimen 1 | 8.683 | 0.3497 (4.197%) |
| Specimen 2 | 8.352 | 0.0189 (0.227%) |
| Specimen 3 | 8.205 | 0.1287 (1.545%) |
| Average    | 8.413 | 0.1658 (1.989%) |
| Standard deviation | 0.245 | — |

**Fig. 9:** Measured final stiffness of the five open-loop control specimens based on five successive measurements per specimen. The shaded area represents the standard deviation of the stiffnesses of all specimens around their average stiffness. The mean of the measured stiffness per specimen is represented by □, while the standard deviation is shown with △ and ∇.

**Fig. 10:** Measured final stiffness of the three closed-loop control specimens based on five successive measurements per specimen. The shaded area represents the standard deviation of the stiffnesses of all specimens around their average stiffness. The mean of the measured stiffness per specimen is represented by □, while the standard deviation is shown with △ and ∇.

**Fig. 11:** Measured stiffness of three closed-loop control specimens versus three open-loop control specimens based on five successive measurements per step. The shaded areas represent the standard deviations of the measurements per step. The y-axis is in logarithmic scale.
target properties, at least in expectation. We provide below what constitutes the beginning of an answer to that question.

D. Correction of the printing process variability with closed-loop control

By comparing the standard deviations of the final stiffness values in the case of the open-loop and of the closed-loop controls, we can observe that the closed-loop control can efficiently curb the imprecision of the final stiffness stemming from the inherent variability of the process. Indeed, the standard deviation of the final stiffness is decreased from 1.282 g/m/m to 0.245 g/m/m, that is a reduction of more than 80%. It is worth noticing, though, that the compared standard deviations do not correspond to the same average final stiffness values. However, the percentage of difference in the average final stiffness values (24%) is small compared to the standard deviation reduction (80%). Therefore, it seems reasonable to expect such an improvement, even if the open-loop control is modified to lead to the target stiffness in expectation. Besides compensating for model error, a consequence of the closed-loop control is then also to decrease the variability in the printed object properties, making the whole process more reliable overall. To support the validity of our statements, given the low number of experiments, a two-sample Kolmogorov-Smirnov test is performed on the set of our statements, given the low number of experiments, a two-sample Kolmogorov-Smirnov test is performed on the set of measurements made on the open-loop control specimens compared to the measurements made on the closed-loop control specimens. The resulting p-value is $5.6514 \times 10^{-7}$, which rejects the hypothesis that the two samples come from the same distribution at a very low significance level.

VI. Conclusion

This paper presents an experiment proving the feasibility and relevance of using feedback control for 3D printed parts to meet precise global mechanical properties. A simple dynamical model of the manufacturing process of a cantilever beam was calibrated from printed test specimens, and used to derive an optimal control law, with respect to a cost function and a stiffness target, for the manufacturing of new specimens. The control scheme was based on Model Predictive Control and a linear Kalman filter was used for the state estimation. All measurements were performed in situ by pausing the printing process and using the printer itself to measure the compliance of the cantilever beam during printing. The manufacturing with a open-loop control and a closed-loop control were compared by measuring the error relative to the final stiffness target. The specimens printed with open-loop control had an average error on their final stiffness of 33.03%, while the error of the specimens printed with closed-loop control had an error averaging 1.989%. This significant difference is explained by the ability of the closed-loop control system to counteract two negative side-effects of open-loop printing: the error of the model on which the control is based, and the variability and sensibility to disturbances of the printing process. We showed by analyzing the final stiffness measurements that these two effects were mitigated by the closed-loop control. Since both phenomena are general and apply to all AM processes, we expect the systematic utilization of feedback control in AM to bring a significant enhancement of the quality of the printed parts. Our approach, which consists of using feedback control to meet final, macroscopic mechanical properties of the 3D printed object, is totally compatible with approaches that use feedback control in AM at a lower scale. We expect a hierarchy of closed-loop controllers at different levels, to provide a comprehensive solution to improving AM technologies. Generalizing the sensor capabilities of AM systems for different processes, targeted mechanical properties, and materials, is a necessary step to broaden the scope of closed-loop control in AM. Finally, we strongly advocate the creation of a standardized framework to describe the relevant properties of AM built parts, and the relation of printing control variables to these properties, in order to systematize the application of feedback control algorithms in AM.

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The deflection along the vertical axis $z$ is defined by $w(z)$. We consider $n$ layers of height $h$ and Young’s modulus $E$. $F$ is a horizontal load applied at the top of the beam along the $x$ axis.

The Euler-Bernoulli equation reads

$$\frac{d^2}{dz^2} (EI(z) \frac{d^2w}{dz^2}) = 0.$$

The boundary conditions are given by:

$$w|_{z=0} = 0,$$
$$\frac{dw}{dz} \bigg|_{z=0} = 0,$$
$$\frac{d^2w}{dz^2} \bigg|_{z=L} = 0,$$
$$-\frac{d}{dx} \left( EI(z) \frac{d^2w}{dz^2} \right) \bigg|_{z=L} = F.$$

With $L = nh$, $n$ being a positive integer, we assume that:

$$\forall z \in [0, L], (i-1)h \leq z < ih \implies I(z) = I_i.$$

In the following derivation, $\epsilon$ belongs in $[0, h]$, and the Euler-Bernoulli equation is integrated four times.

1) **First integration of the Euler-Bernoulli equation:**

$$\frac{d}{dx} \left( EI(x) \frac{d^2w}{dx^2}(x) \right) = -F.$$

2) **Second integration of the Euler-Bernoulli equation:**

$$\frac{d^2w}{dx^2}(jh) - \frac{d^2w}{dx^2}(jh) = -\frac{F}{EI_{j+1}} \epsilon.$$

4) **Fourth integration of the Euler-Bernoulli equation:**

For $1 \leq j \leq n - 1$:

$$w(jh + \epsilon) - w(jh) = \frac{F}{EI_{j+1}} \epsilon + \frac{h^2}{2} \sum_{k=1}^{j} \frac{2k-1}{2I_k} + \frac{h^2 \epsilon_j}{6} \sum_{k=j+1}^{n} \frac{1}{I_k} + h^2 \epsilon^3 \sum_{k=j+1}^{n} \frac{1}{6I_{j+1}}.$$

We set $\epsilon = h$ and sum over $j \in \{i, \ldots, n - 1\}$:

$$\frac{d^2w}{dx^2}(ih) = \frac{hF}{E} \sum_{j=i+1}^{n} \frac{1}{I_j},$$
$$\frac{d^2w}{dx^2}(ih + \epsilon) = \frac{F}{E} \left[ h \sum_{j=i+1}^{n} \frac{1}{I_j} - \frac{1}{I_{i+1}} \right].$$
We set $\epsilon = h$ and sum over $j \in \{1, \ldots, i - 1\}$:

$$w(ih) = \frac{h^3 F}{E} \sum_{j=1}^{i} \left[ \sum_{k=1}^{j-1} \frac{2k - 1}{2I_k} + \frac{j + 1}{2} \sum_{k=j}^{n} \frac{1}{I_k} - \frac{1}{6I_j} \right]$$

$$= \frac{h^3 F}{E} \left[ \sum_{j=1}^{i} \frac{3(2j - 1)(i - j) - 1}{6I_j} \right. \right.$$
$$+ \sum_{j=1}^{i} \frac{j^2}{2I_j} + i^2 \sum_{j=i+1}^{n} \frac{1}{2I_j} \right. \left. \right]$$

$$= \frac{h^3 F}{E} \left[ \sum_{j=1}^{i} \frac{3(2j - 1)(i - j) + 3j^2 - 1}{6I_j} + i^2 \sum_{j=i+1}^{n} \frac{1}{2I_j} \right].$$

Finally,

$$w(nh) = \frac{h^3 F}{E} \left[ \sum_{j=1}^{n} \frac{3(2j - 1)(n - j) + 3j^2 - 1}{6I_j} \right].$$