Low energy dynamics of the one dimensional multichannel Kondo-Heisenberg Lattice

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We determine exactly the fixed point Hamiltonian of the one dimensional multichannel Kondo-Heisenberg lattice model for any number of channels $N \geq 2$. An anomalous singlet with non trivial internal dynamics is generated. We compute the correlation functions of the various conventional and unconventional order parameters of the system and find that for $N \leq 4$ the composite order parameter induce the dominant instabilities.

The Kondo lattice is one of the most challenging problems in contemporary theoretical condensed matter physics. If the single impurity Kondo problem, a local moment antiferromagnetically coupled to conduction electrons, is by now well understood thanks to a variety of theoretical techniques, the problem of a regular three-dimensional array of local moments in a metal (the Kondo lattice) still defies theoretical analysis. The difficulty stems from the fact that there are two competing Kondo effects, the tendency of the local moments to form singlets with the conduction electrons (or more complicated states if more than one band is available) and the tendency of the moments to order due to the RKKY interaction mediated by the conduction electrons. Besides its intrinsic theoretical interest for the general theory of strongly correlated fermions, the Kondo lattice model is believed to capture the non-Fermi liquid physics of a class of rare-earth or actinide compounds in which f-shell localized spins and their screening mechanisms gives rise to the so-called Kondo lattice antiferromagnetism. Adding an antiferromagnetic Heisenberg interaction between the local moments (Kondo-Heisenberg lattice) still defies theoretical analysis. The difficulty stems from the fact that there are two competing Kondo lattice interactions and dominant instabilities of the multichannel one dimensional Kondo-Heisenberg lattice model for incommensurate fillings and zero temperature. We find that screening of the local moments occurs via the "chiral stabilization" mechanism resulting in a ground state that is a chiral non-Fermi liquid we call coset singlet.

The Hamiltonian of the one dimensional multichannel Kondo-Heisenberg lattice is:

$$H = -t \sum_{i,n,\sigma} \left(c_{i,n,\sigma}^\dagger c_{i+1,n,\sigma} + c_{i+1,n,\sigma}^\dagger c_{i,n,\sigma}\right) + \lambda_R \sum_{i,n} \vec{S}_i \vec{S}_{i+1} + \lambda_K \sum_{i,n} \vec{S}_i \cdot \vec{S}_{i+1} \cdot \vec{S}_{i+2} \cdot \vec{S}_{i+3} \cdot \vec{S}_{i+4},$$

where $\sigma = \uparrow, \downarrow$ is the spin of an electron, $n = 1, \ldots, N$ is the channel index and $\vec{S}_i$ is a localized spin $1/2$. $t$ is the bandwidth, $\lambda_H > 0$ the Heisenberg coupling of the localized spins and $\lambda_K > 0$ the Kondo coupling.

To study the low energy physics of the model we follow the standard strategy: keep only linear electron modes around $\pm k_F$ captured in terms of the left and right moving fields $\psi_{L,R,n}(x)$, as well as the low-lying spin-lattice modes around $\vec{S}_i$. The low energy physics is then described by a continuum Hamiltonian expressed using non-abelian bosonization as a quadratic form of various currents: the $SU(2)_1$ (left and right) spin currents $\vec{S}_{L,R}$ of the local moments, the $SU(2)_N$ (left and right) spin current $\vec{S}_{L,R}(x)$ of the electrons and the electron charge and channel currents which decouple from the spin currents in the continuum. The notation $SU(M)_k$ indicates that the currents (generically denoted $J_k^{(L,R)}$) satisfy a Kac-Moody algebra: $[J_k^{(L)}(x), J_k^{(R)}(x')] = \delta(x-x') f^{abc} J_k^{(L)}(x) + \frac{k}{2\pi} \delta'(x-x')$, where $f^{abc}$ are the structure constants of the $SU(M)$ Lie algebra. A similar relation holds for the currents $J_R$, while the $J_R$ and $J_L$ currents commute. The KM algebra generates a conformal field theory (CFT) known in Lagrangian form as the Wess-Zumino-Novikov-Witten (WZNW) model. All the physical operators of the original lattice theory can be expressed in terms of the primary or descendant fields of this CFT allowing the calculation of the asym-
totic behavior of their correlators. This is at the heart of all applications of non-abelian bosonization to condensed matter physics.

Consider the Hamiltonian with $\lambda_K = 0$. The electrons are free, and in the basis described above charge, spin and channel excitations propagate independently of each other, and decouple from the local-moment excitations. Turning on a small antiferromagnetic Kondo coupling $0 < \lambda_K \ll t, \lambda_H$ at incommensurate filling, the charge and flavor excitations remain decoupled from the spin excitations. We can thus restrict ourselves to the latter, described by the Hamiltonian:

$$H = \int \left[ \frac{2\pi v}{N+2} (\vec{S}_L \cdot \vec{S}_L + \vec{S}_R \cdot \vec{S}_R) + \frac{2\pi v_s}{3} (\vec{\sigma}_L \cdot \vec{\sigma}_L + \vec{\sigma}_R \cdot \vec{\sigma}_R) + \lambda'_K (\vec{S}_L \cdot \vec{\sigma}_L + \vec{S}_R \cdot \vec{\sigma}_R) + \lambda''_K (\vec{S}_L \cdot \vec{\sigma}_R + \vec{S}_R \cdot \vec{\sigma}_L) \right]$$  \hspace{1cm} (2)

where we dropped irrelevant terms such as $\vec{\sigma}_L \cdot \vec{\sigma}_R$. The coupling $\lambda'_K$ describes the forward scattering, $\lambda''_K$ the backward scattering and $\lambda''_K = \lambda_K = \lambda_K$ to begin with. Under RG transformations $\lambda'_K$ does not flow near the weak coupling fixed point and merely renormalizes the velocity. On the other hand $\lambda''_K$ is relevant and drives the system to a new strong coupling fixed point. We will thus take $\lambda''_K = 0$ and $v_s = v$ in (1) and determine the fixed point. We will later prove that the neglected terms are irrelevant near the strong coupling fixed point.

Under these circumstances the spin sector is described by $H = H_1 + H_2$ where:

$$H_1 = \int dx \left[ \frac{2\pi v}{N+2} \vec{S}_R \cdot \vec{S}_R + \frac{2\pi v}{3} \vec{\sigma}_L \cdot \vec{\sigma}_L + \lambda_K \vec{S}_R \cdot \vec{\sigma}_R \right]$$

$$H_2 = \int dx \left[ \frac{2\pi v}{N+2} \vec{S}_L \cdot \vec{S}_L + \frac{2\pi v}{3} \vec{\sigma}_R \cdot \vec{\sigma}_R + \lambda_K \vec{S}_L \cdot \vec{\sigma}_L \right] .$$

In $H_1$ (resp. $H_2$), the left (resp. right) branch of the $SU(2)_N$ WZNW model is coupled to the right (resp. left) branch of the $SU(2)_1$ WZNW model, leading to a chiral asymmetry of $H_1$ (resp. $H_2$). We readily identify the strong coupling fixed point of (1) exactly via “chiral stabilization” [12]. The chiral asymmetry in $H_1$ or $H_2$ is invariant under the RG flow and characterizes the fixed point. We find this way that the theory flows under RG to a fixed point theory that is the product of a coset theory [13] by a WZNW theory:

$$H^* = SU(2)_1 \times SU(2)_N^{-1} \otimes SU(2)_1 \otimes SU(2)_N^{-1} \hspace{1cm} (3)$$

where the coset theory, $SU(2)_1 \times SU(2)_N^{-1}$, describes the local moment spin sector, and the $SU(2)_N^{-1}$ WZNW theory describes the electron spin sector. Note that at the fixed point the left and right components is $H_1$ and $H_2$ are recombined and chiral symmetry is globally preserved.

What is the physics around the fixed point? The coset theory describes a spin singlet which the local moments form with the electrons. It is a new type of a singlet, a \textit{coset singlet}: a fraction $\frac{6}{(N+1)(N+2)}$ of the local moment “modes” are paired with the same number of electron spin “modes”. Thus the system loses twice this amount of degrees of freedom as seen in the total specific heat (including channel and charge degrees of freedom):

$$C_{total} = \frac{\pi}{6} \left( 2N + 1 - \frac{12}{(N+1)(N+2)} \right) T .$$

The susceptibility is given by,

$$\chi = \frac{1}{2\pi v} (N - 1)$$

and the Wilson Ratio: $R_w = \frac{2N + 1}{N-1}$. The coset singlet still retains degrees of freedom whose number is given by the central charge of the theory $c = 1 - \frac{6}{(N+1)(N+2)}$. This fraction decreases with the number of channels $N$ since it is “easier” to form the singlet when $N$ increases. This also shows up in the effective coupling of the electrons to the local moments which decreases with with the number of channels, $\lambda''_K \sim 1/N$.

Consider the two-channel case, [14]. For $N = 2$, local moments are described by a $SU(2)_1 \otimes SU(2)_2 / SU(2)_2$ = Ising theory, or equivalently by a Majorana fermion. Such a Majorana fermion picture is very appealing since it is well known [15] that the the single impurity two channel Kondo model be described at the fixed point by a local Majorana fermion degree of freedom. These Majorana fermions form a band when coupled with each other, thus suppressing the single impurity residual entropy at $T = 0$.

Having obtained the low energy theory [13] describing the spin sector, we proceed to express the original operators in terms of the operators of the fixed point theory. This will enable us to check that the operators we discarded in [13] are indeed irrelevant at the strong coupling fixed point as well as determine the dominant instability of the Multichannel Kondo-Heisenberg Lattice. The needed identification of operators as well as the calculation of the scaling dimensions was done in Ref. [12]. Let us summarize briefly the method. To obtain the conformal weight of a given operator, we first decompose it into a product of operators belonging to each of the two decoupled chiral theories. Then, for each chiral theory, we decompose operators of $SU(2)_N \otimes SU(2)_1$ on operators of $SU(2)_N^{-1}$ in an expansion formally similar to the Clebsch-Gordan expansion, the role of the Clebsch-Gordan coefficients being played by operators of the coset theory [12]. The operator with the lowest scaling dimension in this expansion is then retained as the fixed point form of the original operator.

The results are summarized in the table [1] for the theory described by $H_1$. The conformal weights of the theory
described by $H_2$ are obtained by interchanging $L$ and $R$. These conformal weights are such that the operators $\tilde{S}_L(x),\tilde{S}_R(x), \tilde{\sigma}_L(x)$ and $(\nu_s - \nu)(\tilde{\sigma}_R\tilde{\sigma}_R + \tilde{\sigma}_L\tilde{\sigma}_L)$ that we have previously discarded are indeed irrelevant (marginally irrelevant for $N = 2$). This proves the self-consistency of our treatment.

The fixed point is a non-Fermi liquid. The Green's functions of the right moving fermions is given by:

$$\langle T\psi_R(x,t)\psi_R^\dagger(0,0) \rangle \sim \frac{1}{(x-vt)^{1+\frac{3}{2}\delta_N(3x+vt)^{\frac{1}{2}\delta_N}}},$$

where $\delta_N = \frac{3}{(N+1)(N+2)}$. Here we have taken, for simplicity, charge, spin, and channel velocities to be equal, and combined exponents from all sectors, neglecting the non-universal Luttinger interaction in the charge sector (it can be easily taken into account [10]). Further, if a contribution in the channel sector is generated it is ferromagnetic and flows to zero. $G_L(x,t)$ is obtained by replacing $x \pm vt$ by $x \mp vt$. We note that a weak singularity appears at the Fermi level $k_F$, and there is no large Fermi surface. Also note that all dimensions tend to their Fermi liquid values in the limit of large number of channels.

We now examine the possible order parameters of the system. Beginning with the localized moments, we have $\tilde{S}_i = \tilde{\sigma}_L(x) + \tilde{\sigma}_R(x) + e^{i\pi/\alpha}\tilde{u}(x)$, where $\tilde{u}(x)$, the staggered magnetization, is given by $\tilde{u}(x) = \frac{1}{2} \sum_{\alpha,\beta} \tilde{g}_{R,\alpha}\tilde{\sigma}_{\alpha,\beta}\tilde{g}_{L,\beta}$, where $\tilde{g}_{R,\alpha}$, $\tilde{g}_{L,\alpha}$ are the right (left) WZNW fields. We find:

$$\langle \tilde{u}(x)\tilde{u}^\dagger(x') \rangle \sim \frac{1}{|x-x'|^{1+\frac{3}{2}\delta_N}}.$$  \hspace{1cm} (6)

More order parameters are available in the electron sector. The order parameters for charge density wave (CDW), spin density wave (SDW), singlet (SS) and triplet (TS) superconducting are defined in the Table and their dimension is given, from which it follows that they are degenerate and fall with a power $2 + 2\delta_N$. As the fluctuations of these order parameters are weaker than in the one dimensional metal we are lead to investigate the possibility of dominant fluctuations associated with a non-conventional order parameter odd-frequency singlet pairing $\tilde{c}^\dagger$ (c-SP) and composite charge-density wave order (c-CDW): $O_{c-SP} = \tilde{u}\tilde{\sigma}_{TS}$ and $O_{c-CDW} = \tilde{u}\tilde{\sigma}_{CDW}$. The composite operators have momentum $\pm \frac{2}{\alpha}$ and $2k_F\pm \frac{2}{\alpha}$ respectively, and are associated with the gapless excitations predicted by Yamanaka et al. [3]; their correlations are expected display quasi-long range order. Indeed, they decay with the power $3 - \frac{6}{N+2}$. We observe that for $N \leq 5$, the composite order parameters have the most divergent correlations. In this case a large enough fraction of electron spin degrees of freedom is bound the the local moments to suppress the conventional order parameters and enhance the compos-
ations. A last question is whether the picture we have obtained persists for $\lambda_K \gg t$, $\lambda_H$ or very low density and if not what is the strong $\lambda_K$ regime. Presumably, such regime would be a ferromagnetic Nagaoka–like state as in the single channel case. Besides the various generalizations of the one dimensional Kondo-Heisenberg lattice problem, it would be interesting to determine whether the physics of the Kondo-Heisenberg problem persists in the Kondo limit $\lambda_H = 0$. It is known that this is not so in the one-channel case [8]. However, since in the multichannel case there is only a partial screening of the electrons by the spins, one may expect that a RKKY interaction could be generated even in the pure Kondo problem putting it in the universality class of the Kondo-Heisenberg problem.

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[1] P. W. Anderson, J. Phys. C 3, 2346 (1970); P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B 1, 4464 (1970); K. G. Wilson, Rev. Mod. Phys. 47, 773 (1975); N. Andrei, K. Furuya, and J. H. Lowenstein, Rev. Mod. Phys. 55, 331 (1983); A. M. Tsvelik and P. B. Wiegmann, Adv. Phys. 32, 453 (1983); I. Affleck, Acta Phys. Polon. B 26, 1869 (1995).
[2] P. Nozières and A. Blandin, J. Phys. (Paris) 41, 193 (1980).
[3] M. B. Maple, J. Low Temp. Phys 99, 223 (1995).
[4] D. L. Cox and A. Zawadowski, Ann. Phys. 47, 599 (1998).
[5] H. J. Schulz, in Mesoscopic quantum physics, Les Houches LXI, edited by E. Akkermans, G. Montambaux, J. L. Pichard, and J. Zinn-Justin (Elsevier, Amsterdam, 1995).
[6] S. R. White, Phys. Rev. Lett. 69, 2863 (1992).
[7] H. Tsunetsugu, M. Sigrist, and K. Ueda, Rev. Mod. Phys. 69, 809 (1997); N. Shibata and K. Ueda, J. Phys. Condens. Matter 11, R1 (1999).
[8] A. E. Sikkema, I. Affleck, and S. R. White, Phys. Rev. Lett. 79, 929 (1997).
[9] P. Coleman, A. Georges, and A. Tsvelik, J. Phys. Condens. Matter 79, 345 (1997); O. Zachar and A. M. Tsvelik, cond-mat/9909296 (unpublished).
[10] E. Witten, Commun. Math. Phys. 92, 455 (1984).
[11] I. Affleck, in Fields, Strings and Critical Phenomena, edited by E. Brezin and J. Zinn-Justin (Elsevier Science Publishers, Amsterdam, 1988); A. Tsvelik, Quantum Field Theory in Condensed Matter Physics (Cambridge University Press, Cambridge, 1995).
[12] N. Andrei, M. R. Douglas, and A. Jerez, Phys. Rev. B 58, 7619 (1998).
[13] P. Goddard, A. Kent, and D. Olive, Commun. Math. Phys. 103, 105 (1986).
[14] An interesting realization of the same fixed point Hamiltonian in a frustrated three-chain ladder was recently given by P. Azaria, P. Lecheminant, and A. A. Nersesyan, Phys. Rev. B 58, R8881 (1998). The authors also noticed the relation of their fixed point with the spin sector of the two channel Kondo Heisenberg lattice.
[15] V. J. Emery and S. A. Kivelson, Phys. Rev. B 46, 10812 (1992); A. M. Sengupta and A. Georges, Phys. Rev. B 49, 10020 (1994); P. Coleman, L. B. Ioffe, and A. M. Tsvelik, Phys. Rev. B 52, 6611 (1995); A. J. Schofield, Phys. Rev. B 55, 5627 (1997).
[16] V. L. Berzinsinskii, JETP Lett. 20, 287 (1974); A. Balatsky and E. Abrahams, Phys. Rev. B 45, 13125 (1992).
[17] M. Yamanaka, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. 79, 1110 (1997).
[18] A. B. Zamolodchikov, JETP Lett. 43, 730 (1986).
[19] E. Orignac and N. Andrei, 2000, article in preparation.
[20] A. M. Tsvelik, Phys. Rev. Lett. 72, 1048 (1994).
[21] P. Nozières, Ann. Phys. (Paris) 10, 19 (1985); P. Nozières, Eur. Phys. J. B 6, 447 (1998).

**TABLE I.** The conformal weights of the operators in the theory described by (1). Here, $\delta N = \frac{3}{(N+1)(N+2)}$. For $N = 2$, the conformal weights of $\bar{\sigma}_L$ and $\bar{\sigma}_R$ are respectively $(0, 1)$ and $(1, 0)$.

| Operator                        | Conformal weights at the fixed point |
|--------------------------------|-------------------------------------|
| $\psi_{R,n,\sigma}$           | $\left(\frac{1}{2} \pm \frac{3}{4} \delta_N, \frac{1}{2} \pm \frac{1}{4} \delta_N\right)$ |
| $\psi_{L,n,\sigma}$           | $\left(\frac{1}{2} \pm \frac{3}{4} \delta_N, \frac{1}{2} \pm \frac{1}{4} \delta_N\right)$ |
| $\tilde{g}_{L,\beta}$         | $\left(\frac{1}{2}, \frac{1}{4} \pm \frac{1}{4} \delta_N\right)$ |
| $\tilde{g}_{R,\beta}$         | $\left(\frac{1}{2}, \frac{1}{4} \pm \frac{1}{4} \delta_N\right)$ |
| $\bar{\sigma}_L(x)$           | $\left(\frac{1}{2}, 1 \pm \frac{1}{4} \delta_N\right), N \geq 3$ |
| $\bar{\sigma}_R(x)$           | $\left(1 + \frac{2}{N+1}, \frac{3}{N+1}\right), N \geq 3$ |
| $S_{\bar{L}}(x)$               | $(1, 0)$                             |
| $S_{\bar{R}}(x)$               | $(0, 1)$                             |
| $O_{CDW} = \psi_{L,n,\sigma}^{1} \bar{\psi}_{R,n,\sigma}$ | $\left(1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \delta_N\right)$ |
| $O_{SDW} = \psi_{L,n,\sigma}^{1} \bar{\sigma}_{R,n,\sigma}$ | $\left(1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \delta_N\right)$ |
| $O_{SS} = -\psi_{L,n,\sigma}^{1} \bar{\sigma}_{R,n,\sigma}$ | $\left(1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \delta_N\right)$ |
| $O_{TS} = -\psi_{L,n,\sigma}^{1} \bar{\sigma}_{R,n,\sigma}$ | $\left(1, \frac{1}{2}, \frac{3}{2}, \frac{1}{2} \delta_N\right)$ |
| $O_{LS} = \bar{\psi}_{R,n,\sigma} \bar{\sigma}_{R,n,\sigma}$ | $\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2} \delta_N\right)$ |
| $O_{C-CDW} = \bar{\psi}_{R,n,\sigma} \bar{\sigma}_{R,n,\sigma}$ | $\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2} \delta_N\right)$ |