Solution to the $B \to \pi K$ Puzzle in a Flavor-Changing $Z'$ Model

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Recent experiments suggest that certain $B \to \pi K$ branching ratios are inconsistent with the standard model expectations. We show that a flavor-changing $Z'$ provides a solution to the problem. Electroweak penguin amplitudes are enhanced by the $Z'$ boson for select parameters. We discuss implications for the $Z'$ mass and its couplings to the standard model fermions. We also show that the solution is consistent with constraints from the CP asymmetries of the $B \to \phi K_S$ decay.

I. INTRODUCTION

Recent experimental data show that some hadronic $B$ decays to two pseudoscalar mesons deviate from the standard model (SM) expectations. In $B \to \pi\pi$ decays, for example, the $\pi^0\pi^0$ mode is found to have a larger branching ratio than expected [1, 2] and the $\pi^+\pi^-$ mode has a large direct CP asymmetry [3, 4]. It was subsequently shown that agreement between theory and experiment on the above observables can be reconciled with a large color-suppressed tree amplitude ($C$), that has a nontrivial strong phase relative to the color-allowed tree amplitude ($T$), along with a gluonic penguin amplitude ($P$) that also has a sizeable strong phase relative to $T$. Such large final-state rescattering effects cannot be easily accounted for in perturbative approaches [5, 6]. A recent soft-collinear effective theory analysis [7, 8, 9], however, finds one solution that can explain the $\pi\pi$ data without a large strong phase between $T$ and $C$.

Using the flavor $SU(3)$ symmetry, the contributing amplitudes extracted from the strangeness conserving ($\Delta S = 0$) $\pi\pi$ decays can be related to those in the strangeness changing ($\Delta S = 1$) $\pi K$ decays, allowing predictions to be made for the $\pi K$ modes within the $SU(3)$ framework. Most of the CP asymmetries are consistent with the SM predictions, except for the direct $CP$ asymmetry of the $\pi^0 K^0$ mode, which has recently been measured but still has a sizeable uncertainty [10]. A perplexing pattern, however, is observed in the measured $\pi K$ branching ratios [4, 8, 12, 13, 14]. The $\pi K$ puzzle can be stated in terms of the ratios [12]

$$R_c = \frac{2 \left[ \frac{BR(B^+ \to \pi^0 K^+)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)} \right]}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)} = 1.15 \pm 0.12 \, ,$$

(1)

$$R_\alpha = \frac{1}{2} \left[ \frac{BR(B^0 \to \pi^- K^+)}{BR(B^0 \to \pi^0 K^0) + BR(B^0 \to \pi^0 K^0)} \right] = 0.78 \pm 0.10 \, ,$$

(2)

where the experimental values and those given below for the $CP$ asymmetries are taken from Ref. [10]. In the SM, these two ratios should be approximately equal, whereas values in Eq. (1) and (2) indicate a $2.4\sigma$ difference. This deviation may be due to an underestimation of the $\pi^0$ detection efficiency [14]. However, if the deviation is real, a sizeable new physics amplitude with a distinct weak phase is required for the $\pi^0 K^0$ and $\pi^0 K^\pm$ decays, which have a significant dependence on the color-allowed electroweak (EW) penguins [3, 12]. A related ratio that is not sensitive to the color-allowed EW penguins is [15]

$$R = \frac{BR(\bar{B}^0 \to \pi^- K^+)}{BR(B^0 \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)} \frac{\tau_{B^+}}{\tau_{\bar{B}^0}} = 0.91 \pm 0.07 \, .$$

(3)

where $\tau_{B^+}/\tau_{\bar{B}^0} = 1.086 \pm 0.017$ was used. The experimental value of $R$ is consistent with the SM prediction.

In a recent paper [16], we showed that in a model with an extra $U(1)'$ gauge boson it is possible to explain the current discrepancy between the measured mixing $CP$ asymmetry, $S_{\phi K_S}$, and its SM prediction based upon the $S_{J/\psi K_S}$ measurements if the $Z'$ has family non-universal couplings with the SM fermions. A large flavor-changing $Z'$ coupling between the bottom and strange quarks is experimentally allowed. Its implications in the $B_s$ system have been studied in Ref. [17]. Such models can naturally arise in certain string constructions [18], $E_6$-motivated models [19] and dynamical symmetry breaking models [20].

A common feature of the above-mentioned $\pi K$ modes and the $\phi K_S$ modes is that they are all dominated by $b \to s$ penguin-loop processes. In this letter, we demonstrate that the $Z'$ model can also provide the indicated new physics amplitude contributing to the EW penguin amplitudes in the $\pi K$ decays [21], while at the same time explaining the $\phi K_S$ $CP$ asymmetry. In Section II we specify our flavor-changing $Z'$ model, the relevant parameters involved in the analysis, and how the effective Hamiltonian responsible for hadronic $B$ decays is modified. We parametrize in Section III the EW penguin amplitudes where the new physics contributions would be manifest. In Section IV the master formulae used in the analysis of $\pi K$ decays are given along with the related hadronic parameters. Section V gives our numerical solutions to the $B \to \pi K$ puzzle. In Section VI we compare with the constraints from current $B \to \phi K_S$
data. We find that viable solutions to both problems exist.

II. FLAVOR CHANGING Z' MODEL

The basic formalism of the family non-universal Z' model with flavor-changing neutral currents (FCNCs) can be found in Ref. [22]. In this letter, we adopt the formalism and assumptions that we have introduced in Ref. [16] for explaining the observed anomalous indirect CP asymmetry in $B \to \phi K_S$. For simplicity, we consider the example in which flavor-changing effects are sizeable only in the left-handed couplings, leaving the right-handed couplings flavor-diagonal.

We write the $Z'$ part of the neutral-current Lagrangian in the gauge basis as

$$\mathcal{L}^{Z'} = -g_2 J'_\mu Z'^\mu,$$  

(4)

where $g_2$ is the gauge coupling constant of the $U(1)'$ group at the $M_W$ scale. We neglect its renormalization group (RG) running between the $M_W$ and $M_Z'$ scales in view of uncertainties in the parameters. The $Z'$ chiral current is

$$J'_\mu = \sum_{i,j} \bar{\psi}_i \gamma_\mu ((\epsilon_{\psi_L})_{ij} P_L + (\epsilon_{\psi_R})_{ij} P_R) \psi_j^T,$$  

(5)

where the sum extends over all flavors of the SM fermion fields, the chirality projection operators are $P_{L,R} \equiv (1 \mp \gamma_5)/2$, the superscript $I$ refers to the gauge interaction eigenstates, and $\epsilon_{\psi_L}$ $(\epsilon_{\psi_R})$ denotes the left-handed (right-handed) chiral coupling matrix. $\epsilon_{\psi_L}$ and $\epsilon_{\psi_R}$ are hermitian by the requirement of a real Lagrangian. The mass eigenstates of the chiral fields are $\psi_{L,R} = V_{\psi_{L,R}} \psi_{L,R}$ and the usual CKM matrix is given by $V_{\text{CKM}} = V_{us} V_{d\bar{s}}$. The chiral $Z'$ coupling matrices in the physical basis of up-type and down-type quarks are, respectively,

$$B_u^X \equiv V_{ux}^T v_{ux}, \quad B_d^X \equiv v_{dx}^T v_{dx}^T, \quad (X = L,R)$$

(6)

where $B_u^X (d)$ are hermitian. As long as the $\epsilon$ matrices are not proportional to the identity, the $B$ matrices will have non-zero off-diagonal elements that induce FCNC interactions at tree level. Our assumption of flavor-diagonal right-handed couplings demands $B_u^R \propto I$. However, the flavor-changing left-handed couplings will give new contributions to the SM operators. The effective Hamiltonian of the $\bar{b} \to \bar{s}q\bar{q}$ transitions mediated by the $Z'$ is

$$\mathcal{H}^{Z'}_{\text{eff}} = \frac{2 G_F}{\sqrt{2}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 \sum_q \left(B_{Lq}^b (\bar{b} \bar{q} \bar{q})_{V-A} \right)^2 + B_{Rq}^b (\bar{b} \bar{q} \bar{q})_{V+A} + h.c.,$$

(7)

where $g_1 \equiv \epsilon/(\sin \theta_W \cos \theta_W)$, and $B_{Lq}^b$ and $B_{Rq}^b$ refer to the complex conjugates of left- and right-handed effective $Z'$ couplings to the quarks $i$ and $j$ at the weak scale, respectively. Here we have suppressed the color indices as they match within the parentheses. A complete list of operators and their associated leading-order (LO) Wilson coefficients [22] evaluated at the $m_t$ scale can be found in Table 1. For various input parameters, we take $\alpha_s(M_Z) = 0.118$, $\alpha_{EM} = 1/128$, $\sin^2 \theta_W = 0.23$, $M_W = 80.42$ GeV along with the running quark masses $m_t = 170$ GeV, $m_b = 4.4$ GeV.

The above operators of the forms $(\bar{b} \bar{s})_{V-A} (\bar{q} \bar{q})_{V-A}$ and $(\bar{b} \bar{s})_{V-A} (\bar{q} \bar{q})_{V+A}$ already exist in the SM, and we can represent the $Z'$ effect as a modification to the Wilson coefficients of the corresponding operators. Thus,

$$\mathcal{H}^{Z'}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts} \sum_q \left( \Delta c_3 O_{3} + \Delta c_5 O_{5} \right) + \Delta c_7 O_{7} + \Delta c_9 O_{9} + h.c.$$  

(8)

The additional contributions to the SM Wilson coefficients at the $M_W$ scale in terms of $Z'$ parameters from Eq. (7) and (8) are determined to be

$$\Delta c_{3(5)} = -\frac{2}{3 V_{tb} V_{ts}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 B_{Lq}^b (B_{Lq}^L + 2 B_{Lq}^R)$$

(9)

$$\Delta c_{9(7)} = -\frac{4}{3 V_{tb} V_{ts}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 B_{Lq}^b (B_{Lq}^L - B_{Lq}^R).$$

(10)

While in general we can have a $Z'$ contribution to the QCD penguins $\Delta c_{3(5)}$ as well as the EW penguins $\Delta c_{9(7)}$, in view of the results found by Buras et al. [16] we assume $B_{Lq}^L \simeq -2 B_{Lq}^R$ so that new physics is primarily manifest in the EW penguins. (The same assumption has been used in Ref. [16].) This can be realized by $Z-Z'$
mixing with a small mixing angle $\theta$ at $O(10^{-3})$, as constrained by data. As a result, we have the $q\bar{q}$ current coupling in part to the $Z'$ with a flavor-universal coupling and in part to the Z with the SM couplings. In this case, the relation $B_{d^L} \approx -2B_{d^L}$ can be satisfied when $\epsilon_{\phi_L} \approx (\theta g_1 M_Z^2)/(6 g_2 M_Z^2)$ for the quarks of the first two generations. The requirement for the corresponding relation of the right-handed quarks is more relaxed because the $Z'$ couplings to quarks can be flavor-dependent.

The resulting $Z'$ contributions to the Wilson coefficients at the weak scale are

$$\Delta c_{3(5)} \simeq 0 \ ,$$

$$\Delta c_{9(7)} = 4 \left| \frac{V_{tb} V_{ts}^*}{V_{tb} V_{ts}} \right|^2 \xi^{LL}(R) e^{-i \phi_L} \ ,$$

where

$$\xi^{LL} = \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{d^L}^L B_{d^L}^X}{V_{tb} V_{ts}} ,$$

$$\phi_L = \text{Arg}[B_{d^L}^L] \ .$$

The diagonal elements of the effective coupling matrix $B_{d^L}^X$ are real due to the hermiticity of the effective Hamiltonian, but the off-diagonal elements, such as $B_{d^L}^X$, generally contain new weak phases.

Thus, there are 3 independent real parameters \{\xi^{LL}, \xi^{LR}, \phi_L\} in the model considered here. (Note that a possible negative sign of $B_{d^L}^X$ can be accounted for by shifting $\phi_L$ by $\pi$.) The relations $B_{d^L}^{L(R)} \simeq B_{d^L}^{L(R)}$ and $B_{d^L}^{d(L)} \simeq B_{d^L}^{d(L)}$ follow from the assumptions of universality for the first two families, as required by $K$ and $\mu$ decay constraints. We assume that the $Z'$ couplings to lepton pairs $(B_{d^L}^{9})$ are sufficiently small to satisfy experimental constraints from the related leptonic decays (i.e., decays to $\ell \ell$ instead of $q\bar{q}$).

The resulting effective Hamiltonian at the $M_W$ scale is then

$$\mathcal{H}_{\text{eff}}^{Z'} = -\frac{4 G_F}{\sqrt{2}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} \right)^2 B_{d^L}^L \sum_q \left( B_{d^L}^{q(q)} O_{d^L}^{(q)} + B_{d^L}^{R} O_{d^L}^{(q)} \right) + \text{h.c.}$$

Since heavy degrees of freedom (including the $Z'$) in the theory are considered to have been integrated out at the scale of $M_W$, the RG evolution of the Wilson coefficients down to low energies after including the new contributions from $Z'$ is exactly the same as in the SM.

### III. PARAMETRIZATION OF EW PENGUIN CONTRIBUTION

The ratio between the EW penguin amplitude $(P_{EW})$ and the tree contributions $(T' + C')$ in $|\Delta S| = 1$ decays can be given by the ratio of the corresponding Wilson coefficients

| Operator | $c_i^{SM}(m_b)$ | $\Delta c_i(m_b)$ |
|----------|-----------------|----------------|
| $O_{1}^{(q)}$ | $\bar{b} a q_s$ | $\bar{q} \bar{b} s_{\beta}$ | $V_{V-A}$ | 1.138 | 0.0 |
| $O_{2}^{(q)}$ | $\bar{b} a q_s$ | $\bar{q} \bar{b} s_{\beta}$ | $V_{V-A}$ | -0.296 | 0.0 |
| $O_{3}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | 0.014 | 0.0 |
| $O_{4}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | -0.029 | -0.1X' |
| $O_{5}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | 0.008 | 0.0 |
| $O_{6}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | -0.036 | -0.2X' |
| $O_{7}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | 0.000 | -3.6Y' |
| $O_{8}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | -0.000 | -1.5Y' |
| $O_{9}^{(q)}$ | $\bar{b} a s_{\beta}$ | $\bar{q} \bar{b} s_{\bar{\beta}}$ | $V_{V-A}$ | 0.000 | -4.5X' |

TABLE I: The Wilson coefficients for the SM and additions from the $Z'$ at the $m_b$ scale. The notation is $X \equiv \xi^{LL} e^{i \phi_L}$, $Y \equiv \xi^{LR} e^{i \phi_L}$. The numerical values here are rounded-off from the values used in the analysis.
minus sign comes from the sign flip of the axial current in the $\bar{q}q$ pairs in the $O_{7,8}$ operators.

One should note that although $c_{7,8}$ play a less important role compared with $c_{9,10}$ in the SM, they can receive contributions from the $Z'$ such that we cannot neglect them. In Ref. [3], it was implicitly assumed that new physics contributes dominantly to the $(V - A) \otimes (V - A)$ EW penguins. As one of their conclusions under this assumption, the current $S_{\phi K_S}$ data cannot be accommodated if one wants to explain the $\pi K$ anomaly. We will show in Section VI that both problems can be solved simultaneously once the right-handed currents are also taken into account.

We summarize here our simplifications to a general $Z'$ model: we assume (i) no right-handed flavor-changing couplings ($B_{ij}^R = 0$ for $i \neq j$), (ii) no significant RG running effect between $M_{Z'}$ and $M_W$ scales, (iii) negligible $Z'$ effect on the QCD penguin ($\Delta c_{3,5} = 0$) so that the new physics is manifestly isospin violating. With these simplifications, we have 3 parameters left in the model. This approach provides a minimal way to introduce the $Z'$ effect in the $B \to \pi K$ puzzle. Of course, more general $Z'$ models are possible.

As shown in Table II, the Wilson coefficients at the $m_b$ scale, $c_i(m_b)$, are all real in the SM. Possible new weak phases can be obtained from new physics couplings in the SM ($X$ and $Y$) defined by

$$X \equiv \xi^{LL} e^{i\phi_L}, \quad Y \equiv \xi^{LR} e^{i\phi_R}. \quad (21)$$

Because of the operator mixing from the RG running, the QCD penguins also receive contributions from the $Z'$ even though they are assumed to be unaffected at the weak scale. Their sizes should not be comparable to their SM counterparts to guarantee our assumption that new physics is manifest only in the EW penguin sector. We numerically checked that they change only up to several percent from their SM values for $|X| \leq 0.02$ and/or $|Y| \leq 0.02$, the natural sizes of the TeV-scale $Z'$ model [16]. The current-current terms, however, are virtually unaffected.

Following Ref. [3], we define $q$ and $\varphi$ as the magnitude and the weak phase of the ratio in Eq. (16) and ignore a small strong phase. From Table II, the Wolfenstein parameter $\lambda = 0.2240$ and $|V_{ub}/V_{cb}| = 0.086$ [28], we obtain

$$qe^{i\varphi} \equiv \frac{P_{EW}}{T^{L} + C^{L}} \approx 0.75 [1 + 410X^* + 450Y^*\eta + 180Y^*\bar{\eta}] \quad . (22)$$

Because of the sign difference between $c_{7,8}$ and $c_{9,10}$ terms (coming from the relative signs between $\eta^{(t)}$ and $\xi$), where dominant $Y$ and $X$ contributions respectively enter, $qe^{i\varphi}$ has contributions of opposite signs from $X$ and $Y$. In the SM ($X = Y = 0$), $q = 0.75$ and $\varphi = 0^\circ$.

**IV. $B \to \pi K$ DECAYS**

In terms of the hadronic parameters defined in Ref. [3], the $B \to \pi K$ amplitudes are

$$A(B^+ \to \pi^+ K^0) = -P', \quad (23)$$

$$\sqrt{2}A(B^+ \to \pi^0 K^0) = P' [1 - (e^{i\gamma} - qe^{i\varphi}) ] , \quad (24)$$

$$A(B^0_d \to \pi^- K^+) = P' [1 - re^{i\delta} e^{i\gamma} ], \quad (25)$$

$$\sqrt{2}A(B^0_d \to \pi^0 K^0) = -P' [1 + \rho_n e^{i\theta_n} e^{i\gamma} - qe^{i\varphi} re^{i\delta} ] , \quad (26)$$

where color-suppressed EW penguin and annihilation amplitudes have been neglected. As one immediately sees, only the $\pi^0 K^+$ and $\pi^0 K^0$ modes are sensitive to the color-allowed EW penguins (parametrized by $q$ and $\varphi$). The numerical values of the hadronic parameters extracted from the $\pi\pi$ modes are

$$r = 0.11^{+0.07}_{-0.09}, \quad \delta = +(42^{+23}_{-19})^\circ, \quad \rho_n = 0.13^{+0.07}_{-0.09}, \quad \theta_n = -(29^{+21}_{-26})^\circ, \quad (27)$$

$$r_c = 0.20^{+0.09}_{-0.07}, \quad \delta_c = +(4^{+23}_{-18})^\circ. \quad (28)$$

We will take the following CKM weak phase values [28]

$$\gamma = (65 \pm 7)^\circ, \quad \phi_d = 2 \beta = (47 \pm 4)^\circ. \quad (29)$$

To focus on the discussions of $Z'$ effects, we will use exclusively the central values of all the above parameters in Eqs. (27) and (28) in the following analysis, with the understanding that the allowed parameter space and the errors on predicted quantities only reflect those of Eqs. (27) and (28) that are predicted to be roughly equal within the SM. Written in terms of the above-mentioned hadronic parameters, one obtains

$$R_c = 1 - 2r_c \cos \delta_c \cos \gamma + r_c^2$$

$$+ qr_c \{ 2 \cos \delta_c \cos \varphi - r_c \cos (\gamma - \varphi) \} + qr_c \} , \quad (29)$$

$$R_n = \frac{1}{b} [ 1 - 2r \cos \delta \cos \gamma + r^2 \} , \quad (30)$$

with

$$b \equiv 1 - 2q r_c \cos \delta_c \cos \varphi + q^2 r_c^2 + \rho_n^2$$

$$+ 2\rho_n [ \cos \theta_n \cos \gamma - qr_c \cos (\theta_n - \delta_c) \cos (\gamma - \varphi) ] . \quad (31)$$

Both of these observables are sensitive to the color-allowed EW penguin amplitudes, as shown by the $q$ and $\varphi$ dependence. We also set a small strong phase $\omega = 0$ here and below as found in Ref. [3].
FIG. 2: (a) Two solutions of \( q, \varphi \) shown on the \( R_c-R_p \) plane. Solution (A) \( \{ q, \varphi \} = \{1.61, -84^\circ\} \) and solution (B) \( \{ q, \varphi \} = \{3.04, 83^\circ\} \) are respectively on the black (dark) curve and the red (light) curve traced out by varying \( \varphi \) from 0° to 360°. The 1σ experimental bounds and the SM prediction for \( \{ q, \varphi \} = \{0.75, 0^\circ\} \) are also indicated. (b) The contours of solutions (A) and (B) on the \( q-\varphi \) plane. Only the central values of the hadronic parameters in Eq. (27) are used in making these plots. Note that Solution (A) has been given by Ref. 3.

Other EW-penguin-sensitive observables are the \( CP \) asymmetry of \( B^+ \rightarrow \pi^0 K^\pm \):

\[
A_{\text{CP}}(\pi^0 K^\pm) = -\frac{2}{R_c} |r_c \sin(\delta_c) \sin(\gamma - qr_c \sin(\delta_c) \sin(\varphi))| \tag{32}
\]

and the time-dependent \( CP \) asymmetry of \( B \rightarrow \pi K_S \):

\[
A_{\text{CP}}(t) = A_{\pi K_S} \cos(\Delta M_{B_S} t) + S_{\pi K_S} \sin(\Delta M_{B_S} t). \tag{33}
\]

Both the direct and the indirect \( CP \) asymmetries have recently been measured by the BaBar collaboration 11. In terms of the hadronic parameters, the asymmetries are given by:

\[
A_{\pi K_S} = -\frac{2}{b} \left[ q r_c \sin(\delta_c) \sin(\varphi - \rho_n \sin(\delta_n \sin(\gamma - q r_c \sin(\delta_c) \sin(\varphi)))] \right. \tag{34}
\]

\[
S_{\pi K_S} = \frac{1}{b} \left[ \sin(\phi_d - 2 q r_c \cos(\delta_c) \sin(\phi_d + \varphi) + q^2 r_c^2 \sin(\phi_d + 2 \varphi) + 2 \rho_n \cos(\delta_n \sin(\phi_d + \gamma) - q r_c \cos(\delta_n \sin(\delta_c \sin(\phi_d + \gamma + \varphi)))] + \rho_n^2 \sin(\phi_d + 2 \gamma) \right]. \tag{35}
\]

Using the central values of the parameters given in Eqs. 24 and 25 along with \( \{ q, \varphi \} |_{\text{SM}} = \{0.75, 0^\circ\} \), the SM predictions are: \( R_c = 1.17 \), \( R_p = 1.15 \), \( A_{\text{CP}}(\pi^0 K^\pm) = -0.01 \), \( A_{\pi K_S} = -0.12 \) and \( S_{\pi K_S} = 0.86 \), which confirm what have been found in Ref. 3.

There are other \( B \rightarrow \pi K \) quantities, including \( R \) of Eq. (26), that are not sensitive to the EW penguin amplitudes (or the parameters \( q \) and \( \varphi \)):

\[
R = 1 - 2 r \cos(\delta \sin(\gamma + r^2) \tag{36}
\]

\[
A_{\text{CP}}(\pi^\pm K^0) = -\frac{2 r \sin(\delta \sin(\gamma + r^2)}{1 - 2 r \cos(\delta \sin(\gamma + r^2)} \tag{37}
\]

\[
A_{\text{CP}}(\pi^\pm K^0) = 0. \tag{38}
\]

Within our assumption of ignoring annihilation amplitudes, \( A_{\text{CP}}(\pi^\pm K^0) \) vanishes identically. Our SM predictions for the other observables are \( R = 0.94 \) and \( A_{\text{CP}}(\pi^\pm K^\pm) = -0.14 \), and these values do not change with the enhancement in the EW penguins by new physics.

V. FLAVOR VIOLATING \( Z' \) SOLUTION

To illustrate that the flavor-changing \( Z' \) model can provide the solution to the \( B \rightarrow \pi K \) puzzle through the enhanced EW penguin contribution, we take the central values of the \( B \rightarrow \pi K \) hadronic parameters in Eq. (26) and the SM weak phases in Eq. (25). From the two observables \( \{ R_c, R_p \} \) given in Eqs. 11 and 24, we find two solutions of the EW penguin parameters:

\[
\begin{align*}
\text{(A)} & \quad \{ q, \varphi \} = \{1.61, -84^\circ\}, \quad \text{(39)} \\
\text{(B)} & \quad \{ q, \varphi \} = \{3.04, 83^\circ\}. \tag{40}
\end{align*}
\]
with only left-handed currents), we find the solutions assume it is of the same order as $Z$.

Solution (A) $\{q, \varphi\} = \{1.61, -84^\circ\}$ is equivalent to the solution given in Ref. [2]. Solution (B) has a larger $q$ and a $\varphi$ of the opposite sign. This solution is not ruled out by other constraints such as rare kaon decays because of different $Z'$ couplings in our scenario. Fig. 2 shows the two-fold solutions on the $R_\pi$-$R_c$ plane and the $q$-$\varphi$ plane.

The solutions for the $Z'$ parameters $\xi^{LL}$, $\xi^{LR}$ and $\phi_L$ in terms of the general EW penguin parameter $q$ and $\varphi$ can be obtained by finding the solutions of Eq. (22).

Restricting ourselves to the case $\xi^{LR} = 0$ (in the limit with only left-handed currents), we find the solutions

$$\begin{align*}
(A_L) & \{\xi^{LL}, \phi_L\} = \{0.0055, 110^\circ\} , \\
(B_L) & \{\xi^{LL}, \phi_L\} = \{0.0098, -97^\circ\} .
\end{align*}$$

As Eq. (43) shows, the $\xi^{LL}$ values found above have implications on the coupling constant and the mass of $Z'$. In $E_6$ motivated models, $g_2$ has a value of the same order as $g_1$. The value of $|B_{sL}^{LL}B_{dL}^{LL}|$ is unknown, but if we assume it is of the same order as $|V_{ts}V_{tb}^*| \approx 0.04$, a size of $\xi^{LL} \sim O(0.01)$ would give $M_{Z'} \sim O(1 \text{TeV})$, consistent with our model of TeV-scale $Z'$.

In the special case where $\xi^{LR} = \xi^{LL}$ and $\tilde{\eta} = \tilde{\eta}' = -1$, we find the solutions

$$\begin{align*}
(A_{LR}) & \{\xi^{LL}, \xi^{LR}, \phi_L\} = \{0.0104, -70^\circ\} , \\
(B_{LR}) & \{\xi^{LL}, \xi^{LR}, \phi_L\} = \{0.0186, 83^\circ\} .
\end{align*}$$

The $180^\circ$ phase change of the solutions $(A_{LR})$ and $(B_{LR})$ from $(A_L)$ and $(B_L)$ can be understood as a result of the sign change in the dominant new physics term in Eq. (10).

These results are irrespective of whether right-handed currents are included. The predictions given by Solution (A) are consistent with Ref. [2]. The current experimental values are $A_{CP}(\pi^0K^\pm) = 0.00 \pm 0.12$ ($S = 1.79$), $A_{\pi K_S} = -0.40 \pm 0.29$, and $S_{\pi K_S} = 0.48 \pm 0.42$ [6]. Their SM values are already given in Section IV. The observables that are not sensitive to the EW penguin (or parameters $q, \varphi$) do not change from their SM expectations.

VI. RELATION TO $B \to \phi K_S$ MODE

The recent measurement of $S_{\phi K_S} = -0.96 \pm 0.50^{+0.11}_{-0.09}$ at Belle showing a $3.5\sigma$ deviation [9] from the SM expectation $0.736 \pm 0.049$ [60] has aroused great interest in new physics explanations. (The BaBar value [31], $0.47 \pm 0.34^{+0.08}_{-0.06}$, is consistent with the SM.) The $Z'$ solution to the $S_{\phi K_S}$ anomaly has been discussed in our
In this section, we compare our constrained parameter values extracted in the above sections to those in the $B \to \phi K_S$ process. The $\phi K_S$ CP asymmetries can be expressed as

$$A_{\phi K_S} = \frac{2v_0 \sin \Delta_0 \sin \phi'}{1 + 2v_0 \cos \Delta_0 \cos \phi' + v_0^2},$$

(45)

$$S_{\phi K_S} = \frac{1}{1 + 2v_0 \cos \Delta_0 \cos \phi' + v_0^2} \left[ \sin \phi_d + 2v_0 \cos \Delta_0 \sin(\phi_d + \phi') + v_0^2 \sin(\phi_d + 2\phi') \right],$$

(46)

where $v_0 \simeq 0.2$, the weak phase $\phi' = 0$ and the strong phase $\Delta_0 \simeq \pi$ in the SM. Assuming factorization, we have

$$v_0 e^{i(\Delta_0 + \phi')} \simeq \frac{3c_7(m_b) + c_8(m_b) + 4(c_9(m_b) + c_{10}(m_b))}{8(c_9(m_b) + c_4(m_b) + 6c_5(m_b) + 2c_6(m_b))}.$$  

(47)

From this equation we can see the $S_{\phi K_S}$ receives a roughly symmetric contribution of $c_7$ and $c_9$, and thus of $X$ and $Y$.

Since the measured $A_{\phi K_S} = 0.05 \pm 0.26$ is consistent with 0, we take $\Delta_0 \simeq \pi$ as in the SM. We can then obtain $v_0$ and $\phi'$ for each solution of the $B \to \pi K$ puzzle from the previous section and calculate $S_{\phi K_S}$. For $B \to \pi K$ solutions in the $C_L$ case, $S_{\phi K_S} = 0.99$ for solution ($A_L$) of Eq. (11) and $S_{\phi K_S} = -0.69$ for solution ($B_L$) of Eq. (12). Only the latter, with the larger $q$, satisfies the world-averaged experimental 1$\sigma$ range, $S_{\phi K_S} = -0.147 \pm 0.697 (S = 2.11)$. On the other hand, solutions in the $C_L = \pi C_L$ case give $S_{\phi K_S} = -0.83$ for solution ($A_{LR}$) of Eq. (13) and $S_{\phi K_S} = -0.65$ for solution ($B_{LR}$) of Eq. (13). Both solutions (A) and (B) satisfy the averaged experimental 1$\sigma$ range. As mentioned above, the reason we are able to find solutions to account for both the $\pi K$ and $S_{\phi K_S}$ anomalies lies in the fact that the contributions from the $O_{7,8}$ and $O_{9,10}$ operators interfere differently.

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