Break-up fragment topology in statistical multifragmentation models

Ad. R. Raduta

1 NIPNE, Bucharest-Magurele, POB-MG6, Romania
2 Institut de Physique Nucleaire, Universite Paris-Sud 11, CNRS/IN2P3, F-91406 Orsay cedex, France

Break-up fragmentation patterns together with kinetic and configurational energy fluctuations are investigated in the framework of a microcanonical model with fragment degrees of freedom over a broad excitation energy range. As far as fragment partitioning is approximately preserved, energy fluctuations are found to be rather insensitive to both the way in which the freeze-out volume is constrained and the trajectory followed by the system in the excitation energy - freeze-out volume space. Due to hard-core repulsion, the freeze-out volume is found to be populated un-uniformly, its highly depleted core giving the source a bubble-like structure. The most probable localization of the largest fragments in the freeze-out volume may be inferred experimentally from their kinematic properties, largely dictated by Coulomb repulsion.

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I. INTRODUCTION

For more than two decades, nuclear multifragmentation benefits from a constant scientific interest whose main motivation is the observation of a (liquid-gas-like) phase transition at sub-atomic scale [1, 2].

Relying on the presumptive existence of an equilibrated break-up stage in the simultaneous multi-particle decay of excited nuclei, statistical models with cluster degrees of freedom [3, 4, 5, 6, 7] represent particularly useful tools for the characterization of the equilibrated state of the source and, not less important, the study of the associated thermodynamics. The remarkable advantage of realistically incorporating most properties of bound and continuum states via empirical parameterizations of cluster energies or level densities explains their ability to well describe a wealth of experimental data produced over a broad energetic domain.

It was demonstrated that experimental data corresponding to a well-defined equilibrated source may be described by a unique solution of such a statistical model [8, 9]. It is nevertheless not true that the different statistical models converge to the same equilibrated source if the analysis is done by exclusively considering experimental (after-burner) information [10]. This is partly due to the different thermodynamical constraints imposed to the employed statistical ensembles or mathematical tricks designed in order to simplify the partition function or speed-up the simulation and, to a much larger extend, to the differences in the break-up fragment definition.

The aim of the present work is to contribute to a deeper understanding of the break-up stage of the multifragmentation decay as ruled by statistical laws. For this reason, contributions from dynamics (as radial collective flow) and sequential particle evaporations from primary fragments will be referred to only tangentially, despite that over an important region of the considered energy domain they play an important role. For the same reason we will ignore also eventual fragments recombination subsequent to the break-up, thoroughly considered by some authors [11, 12]. More precisely, we want to see

- whether fluctuations of different energetic degrees of freedom are mainly dictated by the localization of the decay event into the phase diagram or, conversely, by the dominant fragmentation modes,
- whether break-up nuclear matter distribution is uniform and, if not,
- whether is it possible to trace the un-homogeneities from experimentally accessible data.

The paper is organized as follows: Sec. II offers a brief review on the statistical models of multifragmentation with a special focus on the microcanonical ones, employed here; Sec. III investigates the sharing of system’s available energy among different degrees of freedom and the sensitivity of the energy fluctuations to the system phase properties and fragment partition; Sec. IV focuses on break-up patterns and the extend in which these may be inferred from kinetic energy distributions. Modifications of fragment charge distributions brought by considering that, at variance with the standard break-up picture, primary fragments interact through nuclear forces are also addressed in Section IV. Conclusions are drawn in Sec. V.

II. STATISTICAL TREATMENT OF MULTIFRAGMENTATION

Under the equilibrium hypothesis, statistical models reduce the physical problem under study to the estimation of the number of microscopic states compatible with the thermodynamical macroscopic constraints. This implies that assuming that it is possible to write down the mathematical expression of the statistical weight of a configuration $W_C$ in the appropriate statistical ensemble, all the thermodynamic quantities may be calculated
out of the characteristic partition sum,

$$Z = \sum_C W_C; \quad (1)$$

while any ensemble-averaged observable \(X\) may be expressed as,

$$\langle X_C \rangle = \frac{\sum_C W_C X_C}{\sum_C W_C}. \quad (2)$$

While for relatively large extensive systems, thermodynamical properties are not sensitive to the way in which the statistical ensemble is defined, when dealing with small systems, as the nuclear ones, it is important to choose the most appropriate replica of the physical phenomenon. The lack of any thermal or chemical potential reservoirs in the case of isolated multifragmenting nuclei, recommend the microcanonical ensemble as the most reasonable choice [8–11]. In this case, it is obvious that the conserved quantities are the total protons (\(Z\)) and neutrons (\(A - Z\)) numbers, the total energy (\(E\)), total momentum (\(\mathbf{P}\)) and, eventually, total angular momentum (\(\mathbf{L}\)). The freeze-out volume (\(V\)) may be considered either as fixed, either as fluctuating.

Defining a generic break-up configuration by the isotopic, internal and translational properties of each fragment, \(C = \{A_1, Z_1, \epsilon_1, r_1, \ldots, A_{N_C}, Z_{N_C}, \epsilon_{N_C}, r_{N_C}\}\), one gets for the statistical weight of the constant volume ensemble the equation [6],

$$W_C(A, Z, E, V) \propto \frac{1}{N_C!} \Omega \prod_{n=1}^{N_C} \left( \frac{\rho_n(\epsilon_n)}{h^3} (m A_n)^{3/2} \right)^{2\pi} \frac{1}{\Gamma(3/2(N_C - 2))} \frac{1}{\sqrt{\det(I)}} \frac{(2\pi K)^{3/2N_C - 4}}{(m A)^{3/2}}, \quad (3)$$

where \(I\) is the moment of inertia, \(K\) is the thermal kinetic energy and \(\Omega = \chi V^{N_C}\) stays for the free volume or, equivalently, accounts for inter-fragment interaction in the hard-core idealization. From Eq. (3) it is straightforward to calculate the statistical weight of a microcanonical ensemble with fluctuating volume as,

$$W_c(A, Z, E, \lambda) = \int W_C(A, Z, E, V) \exp(-\lambda V) dV. \quad (4)$$

It is worthwhile to mention at this point that working under a fixed total energy constraint, it results that the thermal kinetic energy, a key thermodynamic quantity related to the temperature through \(T^{-1} = (\partial S/\partial E) = 1/W(A, Z, E, V) \partial W(A, Z, E, V)/\partial E\), is determined by the amount of the energy available after extracting from the source excitation the costs of fragment formation \(\sum_i B_i\), fragment internal excitation \(\sum_i \epsilon_i\) and mutual fragment interaction \(\sum_{i<j} V_{ij}\),

$$K = E_{ex} - Q - \sum_i \epsilon_i - \sum_{i<j} V_{ij}. \quad (5)$$

This implies that also the fluctuations of \(K\) are strongly dependent on the fluctuations of the other three energetic degrees of freedom, as we shall see later on.

The results discussed hereafter have been obtained in the framework of the Microcanonical Model of Multifragmentation (MMM) [6] in the case of the medium size nucleus (130,60) within the commonly accepted scenario according to which the break-up fragments do not interact otherwise than via Coulomb forces. The consequences of considering in the spirit of Refs. [11, 12, 13] that break-up fragments feel also the nuclear proximity potential are discussed only with respect to fragment charge distributions, for the sake of completeness. Despite the particular choices regarding the model and the nucleus, the results are considered generic for the statistical break-up of multifragmenting nuclei.

Two arbitrary paths in the phase diagram have been considered, a constant volume path \(V = 6V_0\) (full symbols) and one along which the average volume increases with excitation energy (open symbols). The motivation of choosing a constant volume path is twofold. On one hand, it reproduces the fixed freeze-out volume statistical constrained which was used for treating multifragmentation over almost two decades and, on the other hand, it accounts for the belief that the freeze-out volume (average) value does not change significantly while increasing source excitation energy. Twofold is also the motivation of choosing the second path. First, it cancels the statistical constraint of constant volume and, secondly, it accounts for a freeze-out volume whose (average) value may increase with energy, as recent analyses of experimental data indicate [10, 17]. In this last case, the average freeze-out volume increases from \(3.5V_0\) at 2 MeV/nucleon...
to about 10.4V_0 at 14 MeV/nucleon, as indicated in the inset of Fig. 1. Even more importantly, the two paths differ by the regions of the system phase diagram they explore. Thus, following the evolution of the heat capacity,

\[ C^{-1} = -T^2 \left( \frac{\partial^2 S}{\partial E^2} \right) \]

\[ = 1 - T^2 \frac{1}{W(A, Z, E, V)} \frac{\partial^2 W(A, Z, E, V)}{\partial E^2} \]

\[ = 1 - T^2 \left( \frac{\frac{3}{2}N - 4}{K^2} \right), \quad (6) \]

plotted in Fig. 1 as a function of excitation energy, one may notice that the constant volume path is supercritical, while the increasing average-volume path crosses the phase coexistence region. Phase coexistence is signaled by negative values of the heat capacity.

### III. ENERGY SHARING AT BREAK-UP: AVERAGE AND RMS VALUES

Fig. 2 presents the average values of total fragment binding energy (upper panel), internal excitation (second upper panel), Coulomb interaction (third upper panel) and thermal kinetic energy (bottom panel) (left column) together with their RMS values (right column) corresponding to the break-up stage of a (130, 60) nucleus whose excitation energy ranges from 2 to 14 MeV/nucleon along the two considered trajectories. Distributions of the mean charge of the largest fragment and its RMS are superimposed on the third upper panels with full and open stars. Dashed lines are used in the bottom panels to indicate how total fragment kinetic energy, a quantity experimentally accessible, behaves with respect to source excitation.

One can see that, irrespectively the considered path, the more and more advanced fragmentation allowed by an increasing source energy, suggested by a rapidly decreasing \( Z_{\text{max}} \), leads to a monotonic diminish of the total binding energy and a monotonic increase of the total Coulomb interaction energy. The total binding energy decrease is due to the increasing fragment surfaces while the increase of the total Coulomb interaction energy is explained by an increasingly uniform occupation of the volume. While the curves corresponding to the two considered paths diverge with excitation, they are still far from one another, as for a given \( E_{\text{ex}} \), their values differ by at most 20% in the considered energy domain. In contrast with this, the amount of energy dissipated in fragment internal excitation has a more complex evolution and the relative difference among the values obtained along the two paths reaches 50% at \( E_{\text{ex}} = 14 \) MeV/nucleon. Nevertheless, the evolution and relative magnitude of the above quantities are such that the kinetic energy increases monotonically, as one would expect (see left bottom panel).

The right panels of Fig. 2 present the energy fluctuations and indicate that, as more and more fragment partitions are possible with the increasing energy, \( \sigma(B) \) and \( \sigma(E_{\text{int}}) \) rise as well. Very interestingly, \( \sigma(V_C) \) augments up to 4 MeV/nucleon and then decreases. The positive slope region corresponds to the energy domain where configurations containing one heavy residue are dominant. The negative slope interval corresponds to a regime of rather advanced fragmentation which allows for a more uniform population of the freeze-out volume. As one may notice, the peak of \( \sigma(Z_{\text{max}}) \) (full and open stars in the third right panel) and indicates that the largest fragment \( Z_{\text{max}} \)
dictates the geometrical arrangement of fragments and, finally, the Coulomb energy.

Another remark is that, because of the fact that the reduction of Coulomb energy fluctuation is less significant than the increase of internal excitation and binding energy fluctuations, $\sigma(K_{th})$ increases monotonically. Nevertheless, analysing the experimentally accessible fragment total kinetic energy distribution, one would note a peak at 3 MeV/nucleon, as the consequence of summing up the peaked $\sigma(V_C)$ with the monotonically increasing $\sigma(K_{th})$ (right bottom panel).

But, the first important result is that fluctuations of kinetic and configurational energetic channels prove rather insensitive to both freeze-out volume constraints and the trajectory followed by the system into the excitation energy - freeze-out volume plane, provided that the fragmentation pattern is preserved. The result is striking the more as the two considered trajectories explore different regions of the phase diagram.

**IV. Fragmentation Patterns and Nuclear Matter Radial Distributions**

The break-up fragmentation pattern corresponding to 4 MeV/nucleon excitation energy, where the largest fluctuations in $\sigma(Z_{max})$ and $\sigma(V_C)$ manifest themselves, is illustrated in the upper panel of Fig. 3 while the upper panel of Fig. 4 presents the fragmentation pattern obtained at a slightly higher source excitation, 6 MeV/nucleon. As no sensitivity was found to the way in which the freeze-out volume in constrained, from here on we shall consider only the case corresponding to $V = 6V_0$.

One can see that at $E_{ex} = 4$ MeV/nucleon the dominant fragmentation mode is characterized by a residue representing 80% of total system but multifragmentation configurations are already possible. For instance, configurations characterized by two intermediate size fragments ($Z_{max} \approx 30$ and $Z_{max2} \approx 20$), though five times less probable than the most probable fragmentation mode, are nevertheless frequent enough to induce a quite flat $Y(Z_{max})$. The diversity of fragmentation modes translated in broad $Z_{max}$ and $Z_{max2}$ distributions persists at 6 MeV/nucleon, but there is no more possible to identify a close competition among different fragmentation patterns. This means that there are no more distinct manners of filling up the available volume, whose co-existence leads to large fluctuation of the Coulomb energy.

We remind at this point that fragmentation patterns are nevertheless very sensitive to break-up fragments definition or modelisation of the break-up stage itself. If, for instance, one sticks to the non-interacting break-up fragments scenario but considers that, in agreement with Thomas-Fermi calculations, excited nuclei at freeze-out are diluted, the fragment charge distribution will be settled by the competition between the reduced free volume and the augmented thermal kinetic energy. The same qualitative situation is reached if, not the fragments density but, their internal excitation is modified. If, for example, one adopts for the nuclear level density an expression which leads to lower fragment internal excitation, in view of Eq. 4 $K_{th}$ will increase, favoring an increased reaction products multiplicity. This last quantity, in its turn, by making possible a more uniform population of the freeze-out volume characterized by a larger $V_C$, will tend to diminish $K_{th}$.

Much dramatic modifications are expected if one con-
FIG. 4: (Color online) The same as in Fig. 3 for the multifragmentation of the (130,60) nucleus with $V = 6V_0$ and $E_{ex} = 6$ MeV/nucleon. 

siders that break-up fragments interact not only via repulsive Coulomb, but also via attractive nuclear proximity potentials. This conceptually different approach is mainly justified by the fact that for break-up volumes of the order of few $V_0$ the distances between fragment surfaces may be inferior than $\sim 1$ fm. This situation has been discussed recurrently by Das, De, Samaddar, Satpathy, Bonasera and collab. \cite{11, 12, 15} together with break-up fragment subsequent recombination and shown to lead to an increased productivity of light and heavy fragments at the cost of the intermediate ones. As recombination, which occurs if two fragments approach each other during the Coulomb propagation, acts in the sense of washing-out the statistical properties of break-up fragment formation, here we shall restrict ourselves to comment exclusively on the consequences of modifying fragments energetics.

It is relatively easy to anticipate from Eq. (5) that, by considering an extra attractive potential, one will get an increase of thermal kinetic energy and reaction products multiplicity. The confirmation is given by the solid curves on the top panels of Figs. 3 and 4 obtained in the case in which the nuclear interactions are implemented as in Ref. \cite{11}. In both situations one may notice a dramatic enhancement of the light cluster multiplicity and the total suppression of fragments with $Z \geq 10$. These steep $Y(Z)$ distributions and the evolution of their slopes with source excitation may be reconciled with experimental data if and only if one assumes that final fragment formation is dominated by post-break-up dynamics (including collective flow) and multiparticle correlation \cite{12}. If this were the case, the freeze-out would occur much later than the break-up. The complete modelisation of this process is nevertheless a challenging task which goes beyond the goal of the present paper.

In addition to charge distributions, fragment average kinetic energy distributions represent robust and directly accessible experimental information and make up a key ingredient in the standard procedure of identifying the statistically equilibrated source by confrontation with predictions of statistical models \cite{8, 9}. The middle and bottom panels of Figs. 3 and 4 depict the average kinetic energy distributions of primary and, respectively, cold fragments corresponding to the same source (130,60) with $V = 6V_0$ and $E_{ex} = 4$ and 6 MeV/nucleon. The inclusive distributions are plotted with full circles, while distributions corresponding to the largest, second largest and third largest fragment are plotted with open circles, squares and triangles. Collective radial flow is set to zero to keep fragments statistical properties unaffected.

Fragment average kinetic energy distributions are qualitatively similar for the two source excitations. As one may notice, the maximum value of 49 (42) MeV reached by the primary (asymptotic) $< K(Z) >$ with $V = 6V_0$ and $E_{ex} = 4$ and 6 MeV/nucleon distribution exceeds by 25% (30 %) the maximum value obtained by the corresponding $< K(Z) >$ distribution. This result, in apparent contrast with what one would expect given the increase with 57% of the total Coulomb energy over the considered energy domain, may be understood taking into account the much stronger increase in the total number of reaction products \cite{18}.

A common and interesting feature is present in the charge domain where the $< K(Z) >$ distributions reach their maximum. Thus, the break-up and asymptotic average kinetic energies of the largest fragment are systematically smaller than the average kinetic energies of the second largest fragment which are, in their turn, smaller than the ones corresponding to the third largest fragment. This result has been already pointed out by the Indra collaboration in the case of Xe+Sn at 32 MeV/nucleon and Gd+U at 36 MeV/nucleon reactions \cite{19} and shown to diminish with the source excitation energy, in perfect agreement with the present results. Taking into account that fragment kinetic energies are to a large extent dictated by Coulomb, it becomes obvious that analyzing them one may get information on
FIG. 5: (Color online) Radial probability distributions of different size ($Z=1, 5, 10, 20, 30, 40$ and $50$) primary fragments at break-up. The statistically equilibrated source (130, 60) is characterized by an excitation energy of 4 MeV/nucleon and a freeze-out volume $V = 6V_0$.

the most probable fragment position at break-up. Having the same dependence as Coulomb on fragment mass and distance from the freeze-out volume center, radial collective flow, if present, would enhance this shift. If this reasoning is correct, it means that larger a fragment is, closer to the freeze-out volume center it is produced.

The answer to this issue is offered by Fig. 5 where radial probability distributions of different size fragments corresponding to the break-up stage of the (130, 60) source with $V = 6V_0$ and $E_{ex} = 4$ MeV/nucleon are plotted. As a first general remark, one may say that the probability to create a fragment inside the freeze-out volume, whatever its size, is highly un-uniform and strongly diminishes in the core region. Moreover, heavy fragments are localized preferentially towards the inner parts, while relatively light nuclei may be created over wider regions. This means that lighter is a fragment, stronger will be the Coulomb repulsion the charged core will exercise over it and, consequently, higher its final kinetic energy. This explains the observed systematic shift between the maximum values of kinetic energy corresponding to the three largest fragments. The systematic reduction of the volume accessible to a fragment as its mass increases is the consequence of the employed non-overlapping condition between a fragment and the wall of the container which mimics the freeze-out volume. In the case of the heaviest fragments ($Z=50$), this geometric condition is responsible for fragment concentration in a region which represents only 15% of the total freeze-out volume. We remind that the classification of multifragmentation events with respect to fragments' spatial arrangement and its influence on fragment-fragment correlation functions was discussed for the first time in Ref. [20] in the framework of MMMC model [3], where the authors have identified 'sun'- and 'soup'-like events.

As the source excitation energy increases and fragmentation becomes more advanced, a more uniform population of the freeze-out volume is expected, such that the largest fragments kinetic energy shifts become negligible. The evolution of the total charge radial distribution with source excitation is illustrated in Fig. 6 for the same multifragmenting nucleus, (130, 60) with $V = 6V_0$. Indeed, at 8 MeV/nucleon the matter in the inner regions of the freeze-out volume is 10 times denser that the one produced at 6 MeV/nucleon, but the overall distribution remains strongly outwards peaked, giving the source a bubble-like structure. Bubble-like structures of the nuclear matter at break-up have been obtained also in the framework of stochastic mean-field approaches [21] which explain fragmentation on behalf of growing volume and surface instabilities encountered during the expansion phase of the excited system, as recently reported in Refs. [22, 23]. This agreement between results of statistical models with cluster degrees of freedom and dynamical models with nucleonic degrees of freedom is far from being trivial taking into account the conceptually different scenarios the two categories of models advance for explaining multifragmentation and the almost complementary treatment of the physical process.

V. CONCLUSIONS

To conclude, using a microcanonical multifragmentation model with cluster degrees of freedom we have analyzed the break-up fragmentation patterns of a medium size equilibrated source who follows different paths through the excitation energy - freeze-out volume space. The constraints imposed on the freeze-out volume are
found to not affect significantly the magnitude of different energy fluctuations. Moreover, kinetic and configurational energy fluctuations are insensitive to the system phase properties as far as the considered fragment partitions are similar. Over the whole domain of excitation energy, spatial matter distribution at break-up is highly un-uniform, its outward peaked shape giving the source a bubble-like structure. The most probable localization of nuclear fragments at break-up depends on fragment mass and, because of Coulomb acceleration, it is possible to infer it from the experimentally accessible fragment average kinetic energy distributions, especially at intermediate values of source excitation. Thus, heavy fragments are found to be produced in the inner regions of the freeze-out volume, while the lighter ones are produced in a larger region of the freeze-out volume. Considering that break-up fragments interact not only through repulsive hard-core and Coulomb potentials but also via proximity potentials, one obtains dramatic modifications of the break-up fragmentation patterns which suggest that final fragment formation is strongly influenced by post-break-up dynamics and multiparticle correlations.

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