A feasibility study of multi-electrode high-purity germanium detector for $^{76}\text{Ge}$ neutrinoless double beta decay searching

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ABSTRACT: Experiments to search for neutrinoless double-beta ($0\nu\beta\beta$) decay of $^{76}\text{Ge}$ using a high-purity germanium (HPGe) detector rely heavily on background suppression technologies to enhance their sensitivities. In this work, we proposed a pulse-shape analysis method based on a neural network (NN) and a light gradient boosting machine (lightGBM; LGB) to discriminate single-electron (background) and double-electrons ($0\nu\beta\beta$ signal) events in a multi-electrode HPGe detector. In this paper, we describe a multi-electrode HPGe detector system, a data-processing system, and pulse-shape simulation procedures. We built a fully connected (FC) neural network and an LGB model to classify the single- and double-electron events. The FC network is trained with simulated single- and double-electron-induced pulses and tested in an independent dataset generated by the pulse-shape simulation. The discrimination efficiency of the FC neural network in the test set for the $0\nu\beta\beta$ double-electron events signal was 77.4%, the precision was 57.7%, and the training time was 430 min. The discrimination efficiency of LGB model was 73.1%, the precision was 64.0%, and the training time was 1.5 min. This study demonstrated that it is feasible to realize single- and double-electron discrimination on multi-electrode HPGe detectors using an FC neural network and LGB model. These results can be used as a reference for future $^{76}\text{Ge} \, 0\nu\beta\beta$ experiments.

KEYWORDS: Neutrinoless double beta decay; Background suppression; Multi-electrode high-purity germanium detector; Neural network model; lightGBM

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1 Introduction

In the past two decades, atmospheric and solar neutrino oscillation experiments have provided evidence for the existence of nonzero mass neutrinos [1]. If the neutrino is its antiparticle, it can obtain its mass through the Majorana mass term, which will significantly advance the understanding of the nature of the neutrino [2]. The Majorana nature of neutrinos leads to lepton number violation and emerges beyond standard model theories. Searching for neutrinoless double-beta ($0\nu\beta\beta$) decay is considered to be a promising method for proving the Majorana properties of neutrinos [3]. Under the ‘light Majorana neutrino’ mechanism, the effective neutrino mass is inversely proportional to the $0\nu\beta\beta$ half-lives, therefore can be determined by the detection of the $0\nu\beta\beta$ decay [4]. To investigate the $0\nu\beta\beta$ decay process, a variety of $0\nu\beta\beta$ experimental schemes have been proposed. Different $0\nu\beta\beta$ decay target isotopes have been proposed and used in various experiments. $^{76}$Ge radionuclide is one of the most important target nuclides. Several experiments have searched for $0\nu\beta\beta$ decay in $^{76}$Ge such as GERDA [5], MAJORANA collaboration [6], and CDEX [7] experiments. The GERDA and the Majorana collaborations are now merged into the LEGEND collaboration and are proposing a 200 kg-scale $0\nu\beta\beta$ experiment (LEGEND-200) aiming at setting the $0\nu\beta\beta$ decay half-life limit of $^{76}$Ge at $10^{27}$ yr [8].

The $0\nu\beta\beta$ experiment benefits from an extremely low background in the search for a rare $0\nu\beta\beta$ decay signal. One of the critical technologies of active background suppression is to analyze the
characteristics of the waveform of the signal and discriminate the signal from the background. In $0\nu\beta\beta$ experiments, the single-electron background events can be caused by the beta decay of cosmogenic nuclides inside the detector or by the Compton scattering of external gamma. The single- and double-electron signals are quite similar, and the track morphology is complex, so it is challenging to distinguish them [9–11]. Therefore, developing single- and double-electron event discrimination methods based on signal characteristics has always been a popular research topic.

High-purity germanium (HPGe) is ideal for detecting $0\nu\beta\beta$ decay because of its high energy resolution, low background, and high detection efficiency. The discrimination of single and double electrons has been seldomly investigated using a point-contact-style HPGe detector (such as BEGe detector) because their position sensitivity is limited to the millimeter-scale. The GERDA collaboration [12] and the MAJORANA collaboration [13] developed waveform discrimination methods such as $A/E$. $A/E$ is the ratio of the current waveform’s maximum amplitude to the event’s energy deposition. For single-site events, the peak value of the current waveform is approximately proportional to the total charge carried by the carrier. It is also proportional to the total energy deposition. For multi-site events, however, the energy deposition is distributed at different interaction points so the peak value of the current waveform is relatively smaller, and the $A/E$ characteristics of multi-site events are smaller than those of single-site events. This method can be used to discriminate single-site and multi-site events on the millimeter scale. However, for multi-site events, there are usually several types of Compton scattering, one photoelectric effect, and essentially several single electron energy deposition events. The signal event is a double electron energy deposition event for $0\nu\beta\beta$ detection. Because the energy deposition scale of single and double electron events is relatively small and the waveform difference is relatively unclear, it is not easy to distinguish them directly. However, due to the limit of the detector type (single electrode, signal readout), methods such as $A/E$ have not been demonstrated to discriminate single and double electron events. In the HPGe detector system, the single electron background comes from the beta decay of Ge’s cosmogenic nuclides such as $^{68}$Ge and $^{60}$Co. Alternatively, photons with energy more than 2039 keV are produced by background nuclides such as $^{238}$U and $^{232}$Th from materials surrounding the crystals. Photoelectric effect or secondary electrons are produced by single Compton scattering. After sufficient background suppression, these types of single electron background events will become important background sources [8, 14, 15].

In recent years, multi-electrode HPGe detectors have been widely used in many fields because of their position sensitivity and good energy resolution. The two main types of multi-electrode HPGe detectors by electrode structure are segmented multi-electrode HPGe and strip multi-electrode HPGe. The AGATA experiment [16] used a segmented multi-electrode HPGe detector to perform track reconstruction research to achieve measurement of the nuclear structure. The COSI experiment [17] used a strip multi-electrode HPGe detector for astronomical observations. No relevant research has been conducted, however, on the single- or double-electron event discrimination of $^{76}$Ge $0\nu\beta\beta$.

In this study, we built experimental and simulation platforms for a strip multi-electrode HPGe detector. We examined the generation mechanism of single- and double-electron event waveforms in a strip multi-electrode detector. We also studied single- and double-electron event waveform discrimination methods based on track features, a fully connected (FC) neural network model, and a light gradient boosting machine (lightGBM; the LGB model). All methods were applied to events with 2039 keV energies, the Q-value of $0\nu\beta\beta$ decay in $^{76}$Ge.
2 Method

2.1 Multi-electrode high-purity germanium detector experimental system

In this study, we used a multi-electrode HPGe detector developed by Tsinghua University [18]. We read the waveform signals of different electrodes created by the evaporation of amorphous germanium on the surface of the HPGe crystal. Figure 1 shows the structure diagram of the crystal of the detector. The germanium crystal had a diameter of 30.0 mm and a height of 10.0 mm. Seven strips of p+ Al electrodes were located on the upper surface of the germanium crystal as signal readout electrodes with a width of 2.5 mm and a center distance of 0.5 mm. To reduce the influence of surface leakage current, we installed a 2-mm-wide protective ring outside the readout electrode. The bottom lithium electrode biased the detector, and the depletion voltage of the detector was +290 V. Considering the influence of surface leakage current, the recommended operating voltage was +300 V.

As shown in figure 1, seven readout electrodes, a guard ring electrode (GR), and lithium electrodes were connected to a self-made nine-channel charge-sensitive preamplifier (CSP), and the output of each channel (except the GR channel) was connected to a reverse CR amplifier with zero-pole cancellation function. The CR output signal was then recorded and collected by a CAEN V1724 (14-bit) analog-to-digital converter (ADC) with a 100 MHz sampling rate. In the experiment, we collected 4000 samples (40 μs) for each waveform. This paper obtains the relevant physical parameters as the input of the subsequent simulation study through the established multi-electrode high-purity germanium detector experimental system (table 1).

Table 1. Parameters of the established multi-electrode high-purity germanium detector experimental system.

| Quantity               | Value | Unit |
|------------------------|-------|------|
| Geometry               |       |      |
| Germanium crystal diameter | 30    | mm   |
| Crystal height         | 10    | mm   |
| Electrode width        | 2.5   | mm   |
| Electrode gap          | 0.5   | mm   |
| Protective ring width  | 2     | mm   |
| Bias voltage           | 300   | V    |
| Sampling rate          | 100   | MHz  |
| Noise RMS              | 3.25  | keV  |

2.2 Multi-electrode high-purity germanium detector waveform simulation platform

In this study, we established a multi-electrode waveform simulation method for the self-developed multi-electrode HPGe detector, which provided support for detector performance optimization and subsequent waveform discrimination algorithm. We divided the pulse waveform simulation process of the HPGe detector into two parts: (1) the electric field calculation inside the germanium crystal and (2) the generation of the induced waveform caused by the drift of charge carriers. The specific process follows:
Figure 1. (Top image) Electrode distribution structure of the multi-electrode HPGe crystal. The inner diameter of the protective ring is 26 mm, and seven strip-shaped induction electrodes are evenly distributed in a circle with a diameter of 24 mm. (Bottom image) Diagram of the data acquisition system.

1. The electric field was calculated within the open-source SolidStateDector.jl software [19] which was capable of computing the electric and weight potential field for the multi-electrode HPGe detector.

2. The energy depositions of single and double electrons in the Ge crystal were calculated by Monte Carlo simulations using a GEANT4 [20] based simulation toolkit SAGE [21]. The energy deposition value and position of each step were clustered according to their energy deposition positions: steps with energy deposition positions within 0.1 mm were clustered into an interaction point.

3. For each interaction point obtained in step 1, the drift path of induced charge carriers was calculated in a 4 ns time step. The electric field-dependent charge carrier drifts velocity was calculated using a mobility model [22]: a linear relation between velocity and electric field at low field and a saturated velocity at high field.
4. The induced charge signal of each interaction point on one electrode was calculated according to the Shockley-Ramo theory [23]. The charge signal of the interaction point \( Q_i \) is given by formula (2.1), where \( q \) is the charge of the interaction point, \( \Phi_i \) is the weight potential field, \( r_e(t) \) and \( r_h(t) \) are the position of charge carrier electron and hold at time \( t \) respectively.

\[
Q_i(t) = q[\Phi_i(r_e(t)) - \Phi_i(r_h(t))] 
\tag{2.1}
\]

5. The output signal of an event was the sum of the signal of all interaction points within the event. Figure 2 shows the flow chart of the waveform simulation.

![Figure 2. Flow chart of waveform simulation.](image)

### 2.3 Digital waveform processing system

We established a digital waveform processing system (DPP) to extract the relevant physical characteristics from the simulated and measured waveforms. Figure 3 shows the flow chart of the DPP module used to process the original waveform, which had four steps:

1. **Data quality check.** We rejected noise events using a method similar to the GERDA experiment [24]. First, waveform parameters were extracted, including baseline mean, baseline slope, baseline root mean square (RMS), and trigger position. The waveform was preliminarily discriminated according to the baseline mean and baseline slope distribution. We then removed events where the same waveform contains multiple physical events by identifying and rejecting pile-up events. Finally, we recovered the waveform baseline. We extracted the baseline parameters using data points within the first 5 \( \mu s \) of the waveform, the mean value of the first 5 \( \mu s \) of the waveform as the baseline mean, and the linear fitting slope of the first 5 \( \mu s \) of the waveform as the baseline slope. A Gaussian function fit the distributions of baseline mean and baseline slope. We excluded the waveforms with a baseline mean and slope fall out of the \( \pm 3\sigma \) range of Gaussian distribution.

2. **Generation of the current waveform.** We obtained current waveform by smoothing and differential processing of the charge waveform.
3. Energy reconstruction. We use the trapezoidal filter \[25\] to convert the waveform output by the charge-sensitive preamplifier into an isosceles trapezoid and then extracted the amplitude of the trapezoid for energy reconstruction. The decay constant of the trapezoidal filter is $13 \, \mu s$ (measured by the charge-sensitive preamplifier output waveform). The rise time and the flat top width are set to $2 \, \mu s$, and the pole-zero cancellation is performed during the filtering process to eliminate the signal overshoot.

![Figure 3. Digital waveform processing method diagram.](image)

**Figure 3.** Digital waveform processing method diagram.

### 2.4 $X$-direction position resolution of a multi-electrode high-purity germanium detector

Figure 1 shows the coordinate system for the strip multi-electrode HPGe detector used in here. In the following simulations, the origin of the coordinate system was located at the bottom center. The position resolution of the detector for a single-site event was calculated via the signal amplitude induced by the charge collection electrode and its adjacent electrode. The position of the event in the $X$ direction for any electrode $A$ can be calculated by formula (2.2) \[26\]. For a given electrode structure, the spatial resolution at different positions can be calculated by the error transfer formula (2.3).

$$X = K \frac{S_R - S_L}{S_R + S_L} + X_A$$  \hspace{1cm} (2.2)$$

$$\sigma_X^2 = K^2 \cdot \sigma_S^2 \cdot \frac{4S_R^2 + 4S_L^2}{(S_R + S_L)^3} + \sigma_K^2 \cdot \left[ \frac{S_R - S_L}{S_R + S_L} \right]^2$$ \hspace{1cm} (2.3)$$

Here, $S_L$ and $S_R$ are the maximum amplitudes of the induced signals of adjacent electrodes on the left and right sides of electrode $A$, respectively; $X_A$ is the center position of electrode $A$; and $K$ is the slope constant obtained by fitting the linear relationship between the $X$ position and $(S_R - S_L)/(S_R + S_L)$. The $\sigma_S$ value is the waveform noise, and the $\sigma_K$ value is the uncertainty in the fit to $K$.

Here, $2 \, \text{mV}$ of noise was added when calculating the position resolution of a multi-electrode high-purity germanium detector. The noise was obtained according to the RMS value of the
noise measured in the experiment, and the corresponding Gaussian noise was superimposed on the waveform during the waveform simulation. In addition, due to the detector’s geometry, the detector has a poor position discrimination power in the $Y$-direction and $Z$-direction.

2.5 Fully connected neural network structure and training process

The neural networks currently used in particle physics are trained primarily in supervised learning [27, 28]. Many of the neural networks used in particle physics are trained by simulated data, which is the basic idea of the neural network-based single- and double-electron discrimination method in this study. Here, the single/double electron event waveforms were used to train a fully connected neural network model [29], and the waveforms of the test-set were then finally classified. The network training used the simulated waveform signal of the multi-electrode HPGe detector as the input. In the simulation, we used the $^{76}$Ge $0\nu\beta\beta$ double-electron event as the signal and used the 2039 keV single-electron event as the background. This paper uses SAGE for simulation with a cutoff value of 1 $\mu$m.

The waveform generated by the signal and the background event was directly output in the simulation. The waveform sampling rate was selected as 250 MHz, which was the same as that used in the experiment. We extracted the input pulse of the neural network from the raw charge pulse: a ±25 point window centered at the pulse central point M (defined as the point charge pulse rises to 50% of its amplitude) was used to select the input charge pulse. As shown in figure 4, these 50 points included the entire waveform rising edge area, and its amplitude would be differentiated and used as the input feature of the network. Seven different readout electrodes analog waveforms of each physical event were taken as a group. All groups of waveforms were labeled 1 for the $0\nu\beta\beta$ double-electron signal and 0 for the background. Finally, the network’s output was the label prediction results that corresponded to the waveform.

![Figure 4. Sample of analog signal waveform of multi-electrode HPGe detector for training.](image)

We simulated 90,000 $^{76}$Ge $0\nu\beta\beta$ signal events and 90,000 2039 keV background events for a total of 180,000 waveform data sets. We randomly divided the data into training, validation, and test sets according to the 70%, 20%, and 10% ratios. We used the training set for neural network training and the validation set to select the model and serve as a criterion for hyperparameter adjustment. We used the test set to evaluate network performance after training.
In this study, we implemented the neural network framework by Pytorch [30]. The structure diagram of the neural network is shown in figure 5. The network’s input was the multi-electrode HPGe detector seven-electrode waveform rising edge, which was evenly distributed on the 50 points $x_1$–$x_{50}$. The network extracted the waveforms of the seven electrodes separately, with three layers. The number of nodes in each layer was 32, 16, and 4. The excitation function for each layer was ReLU, and Dropout was set to retain 80% of the network. After feature extraction of seven electrode waveforms, we connected the output to a three-layer fully connected network with dimensions 16, 4, and 2. The excitation function was ReLU, and the Dropout was 50%. Finally, we conducted the two-class prediction of single- and double-electron events.

Figure 5. Structure diagram of fully connected neural networks for single- and double-electron event detection of $^{76}$Ge.

The initial learning rate of single- and double-electron discrimination network training was set to 0.001. The learning rate gradually decreased with an increase in the number of iterations in the training process. The optimizer selected the Adam model [31], which usually performed better training, and the loss function determined the cross-entropy loss function commonly used in classification problems. After multiple debugging, the training epoch was 150. Each batch size was set to 128.

2.6 LGB structure and training process

In addition to the FC neural network method, the decision tree is another common method used for binary classification in machine learning. Gradient boosting decision tree (GBDT) is an enduring model in machine learning. It uses the weak classifier (decision tree) iterative training to obtain the optimal model. The LGB framework is used to implement the GBDT algorithm, which supports efficient parallel training. It offers the advantages of faster training speed, lower memory consumption, better accuracy, and distributed processing of massive data [32].

– 8 –
In this study, we used the LGB model to classify and predict the single- and double-electron waveform discrimination of a multi-electrode HPGe detector $^{76}$Ge. The LGB framework was implemented by sklearn [33]. The input of the decision tree was 50 points $x_1$ to $x_{50}$ that were distributed evenly along the rising edge of the waveform of the seven electrodes of the multi-electrode HPGe detector. We used parameter grid search through GridSearchCV [34] to determine the decision tree parameters. The final use learning type was GBDT, the learning rate was 0.008, the tree depth was 30, the number of leaves was 97, and the proportion of random sampling of features when building a weak learner was 1. The minimum number of leaf node samples was 32, and the maximum number of iterations was 2000.

2.7 Evaluation index

In machine learning, loss function loss and accuracy are two important indicators of model training. However, if the number of categories in the data set is unevenly distributed — especially in the presence of extremely biased data — the accuracy rate cannot objectively evaluate the classification effect of the algorithm. However, it is still possible to judge the training of the model by observing the convergence of the accuracy curve and loss curve of the training set and the verification set at the same time. For the $0\nu\beta\beta$ discrimination of $^{76}$Ge, we selected common parameters to evaluate the performance of binary classifiers: accuracy, precision, recall, and area under the receiver operating characteristic (ROC) curve (AUC). For the binary classification model, we identified four combinations of predicted and actual situations (Table 2).

| Positive prediction | Negative prediction |
|---------------------|---------------------|
| Positive truth      | TP: True Positive   | FN: False Negative |
| Negative truth      | FP: False Positive  | TN: True Negative  |

Accuracy: Accuracy $= (TP + TN)/(TP + FN + FP + TN)$. Accuracy is the percentage of the predicted correct results in the total sample. If the sample was not balanced, the accuracy would fail. Therefore, when the proportion of different categories of samples was very uneven, the category with a large proportion often became the most important factor affecting the accuracy. For the binary classification process of single- and double-electron discrimination, accuracy was used only as a reference for the network training effect.

Recall: Recall $= TP/(TP + FN)$. Recall is the number of correctly predicted positive events divided by the total actual positive events. In the actual positive record, the predicted number was positive. We used the proportion correctly predicted in all positive examples to evaluate the detection coverage of the detector for all targets to be detected. The recall represented the proportion of the useful part of the entire test result in the useful part of the data set. Recall was the discrimination efficiency for the binary classification process of single- and double-electron discrimination.

Precision: Precision $= TP/(TP + FP)$. Precision is the number of correctly predicted positive events divided by the total number of predicted positive events. In the predicted positive record, the actual number was positive. We used the proportion of true positives in the predicted results to evaluate the accuracy of the detector based on successful detection. For the binary classification process of single- and double-electron discrimination, precision represented the proportion of the useful part in the whole detection result.
Area under the ROC curve (AUC): when drawing the ROC curve, the horizontal and vertical axes are 0 to 1, forming a square. AUC is the area enclosed by the ROC curve in this square. For the binary classification process of single- and double-electron discrimination, $AUC = 0.5$ represented a random classifier; $AUC < 0.5$ indicated that it was worse than the random classifier and had no modeling value; and $0.5 < AUC < 1$ was better than the random classifier.

3 Result and discussion

We verified the pulse-shape simulation by comparing the rise time of simulated and experimental waveforms. The 10–90% rise time distribution of the $^{57}$Co source (30 cm above the germanium crystal) measured in the experiment is shown in figure 6. The response functions of the CSP and CR circuits were considered in the simulation process. In practice, however, multiple parts may contribute noise, which makes it not wholly Gaussian. Thus, there is a difference between simulated and experimental waveforms. In addition, there may be electric field distortion due to the electrode processing errors and the inability to effectively measure the impurity concentration distribution. The simulated and actual waveforms will then be slightly different. In general, although the waveforms of the simulation and experiment are slightly different, the overall agreement is good. It is reasonable to study the related physical processes of multi-electrode high-purity germanium detectors via a simulation waveform method.

![Figure 6](image.png)

**Figure 6.** Simulation and experimental waveform rise time distribution.

3.1 $0\nu\beta\beta$ discrimination method based on track feature

Section 2.4 shows the position resolution of a multi-electrode high-purity germanium detector in the $X$ direction can be calculated. The root mean square of the noise is two mV. The simulation used a 122 keV collimated gamma point source, and the $Y$ and $Z$ directions were fixed at the center. The simulation results of spatial resolution are shown in figure 7. The values of $(S_R - S_L)/(S_R + S_L)$
at different $X$ directions were linearly fitted, and the $K$ value was 1.78 with a fitting error of 0.026. Therefore, we ignored the error term of the $K$ value. The resolution of the position directly below the electrode was about 0.3 mm, and the position at the edge of the electrode was about 0.15 mm. The results show that this structure’s multi-electrode HPGe detector had good position sensitivity in the $X$ direction. We evaluated the effect of self-expulsion and diffusion modeling on the $X$-direction position resolution of multi-electrode high-purity germanium detectors. The results show that self-repulsion and diffusion modeling will affect the $X$-direction position resolution of the multi-electrode HPGe detector. However, the difference is not significant. The position resolution results are within the acceptable range, and the training process of single/double electron discrimination in this paper does not consider the charge cloud effect due to simulations including group effects, which in turn result in significantly longer simulation times.

![Figure 7. $X$-direction spatial resolution comparison.](image)

To simulate the behavior of single electron events and $0\nu\beta\beta$ events, we use the SAGe package for simulation. The angular distribution is considered in calculating the $0\nu\beta\beta$ events in SAGe and is based on the G4SingleParticleSource. Two electrons are attached to the primary vertex to generate two electrons simultaneously by an embedded algorithm [21]. According to the simulation, the deposition of electrons with an energy of 2039 keV in Ge is mostly within 2 mm. In HPGe, the element of interest is $^{76}$Ge with a maximum $Q$ value of 2039 keV. Single- and double-electron events were uniformly sampled in the HPGe crystal, and emitted electrons were generated, discriminating for events that retained all energy deposition in the HPGe. In the $0\nu\beta\beta$ decay of $^{76}$Ge, two electrons were simultaneously emitted with a total energy of 2039 keV. We simultaneously simulated single-electron events to identify their characteristic differences from $0\nu\beta\beta$ events. The initial energy of the single-electron was 2039 keV, which corresponded to the $Q$ value of $^{76}$Ge. In the HPGe detector system, the single-electron background may have come from the $\beta$ decay of the Ge cosmogenic nuclides $^{68}$Ga and $^{60}$Co, from the secondary electrons generated by the photoelectric effect, or from the single Compton scattering of photons with an energy greater than 2039 keV in the crystal.
For the single-electron events and the $0\nu\beta\beta$ events generated by GEANT4, we recorded the energy deposition and $x$, $y$, and $z$ coordinates of each step level in the data set. Here, the energy deposition process of the 2039 keV single electron event and $0\nu\beta\beta$ double electron event in germanium are simulated; the step level information is counted. We then identified the two points with the largest energy deposition in each event and calculated the distance between the two points. The distance distribution of the maximum energy deposition point for single and double electrons is shown in figure 8. The multi-electrode high-purity germanium detector structure we studied here has a position resolution only in the $X$ direction. Figure 8 shows the track distribution of single/double electron events in the $X$ direction. It is not easy to distinguish single/double electron events effectively and directly. Therefore, developing a machine learning-based method for the multi-electrode high-purity germanium detector structure is necessary. In addition, as shown in figure 8 left panel, it is possible to perform event-by-event discrimination directly using track characteristics for future multi-electrode high-purity germanium detectors with three-dimensional position resolution.

![Figure 8](image.png)

**Figure 8.** (Left) Distance distribution of the maximum energy deposition point; (Middle) The $X$-direction distance distribution of the maximum energy deposition point; (Right) rise time distribution of single- and double-electron events.

### 3.2 $0\nu\beta\beta$ discrimination method based on FC neural network model

We used the Tesla V100 graphics card to train the network convergence, which took about 430 minutes. Figure 9 shows the relationship between the accuracy and loss function of the training set and the validation set with the number of training rounds after the FC network was trained. The correct rate of the training set and the validation set was about 50% at first and eventually approached 57.5% with the training. The loss function values of both were lower than 0.68. Figure 10 shows the ROC curves to classify test sets (15,000 sets of waveforms) using the FC neural network model trained in this study. The AUC of the network was about 0.61. According to the calculation, recall was close to 77.4%, and the precision was 57.7%. The results showed that using multi-electrode high-purity detectors, the FC neural network could serve as a single- and double-electron discriminator in the search for $^{76}\text{Ge}$ $0\nu\beta\beta$ decay.

### 3.3 $0\nu\beta\beta$ discrimination method based on LGB model

The dataset of LGB learning was the same as dataset of the FC neural network. The training of the LGB model took only 1.5 minutes. Figure 11 shows the relationship between the accuracy and
Figure 9. (Left) Accuracy of training set and validation set of FC neural network model; and (Right) loss of training and validation set of FC neural network model.

Figure 10. ROC curve of FC neural network model.

Figure 11. (Left) Accuracy of training set and validation set of LGB model; and (Right) loss of training and validation set of LGB model.
loss function of the training set and the validation set with the number of training rounds during the training process. The correct rate of the training set and the validation set was about 50% at first and eventually approached 64.3% with the training. The loss function values of both were lower than 0.64. Figure 12 shows the ROC curves to classify the test sets (15,000 sets of waveforms) using the LGB model trained in this study. The AUC of the network was about 0.69. According to the calculated, recall was close to 73.1%, and the precision was 64.0%. The results showed that the LGB was an effective method for searching $0\nu\beta\beta$ events of $^{76}$Ge for multi-electrode HPGe detector.

![ROC curve of LGB model.](image)

### 3.4 Effect comparison of discrimination methods

We studied the feasibility of waveform discrimination of $^{76}$Ge $0\nu\beta\beta$ single and double electrons in a multi-electrode HPGe detector according to three methods. We found that among the three methods, the track feature method was very dependent on the inherent position resolution of the detector. The detector structure in this paper could not directly and effectively utilize the characteristics of the waveform, so it was impossible to discriminate single and double electrons based on the track feature method. The FC neural network and LGB method based on machine learning could realize the extraction and learning of the waveform features, which provided a new method for multi-electrode HPGe detector for $0\nu\beta\beta$ single- and double-electron recognition of $^{76}$Ge. The AUC values of the single- and double-electron methods were 0.61 and 0.69, respectively, both greater than 0.5. The results indicated that both methods effectively discriminated the single and double electrons of $^{76}$Ge $0\nu\beta\beta$. The metrics parameters showed that the FC-based discrimination method recall was close to 77.4%, but the precision was only 57.7%. This method took a long time, and many places still could be improved for feature extraction of FC neural networks in the follow-up work. The LGB-based discrimination method achieved 73.1% recall and 64.0% precision, and the computation time was significantly reduced.
Table 3. Effect of Three Methods on Single- and Double-Electron Event Discrimination.

|                      | Track feature | FC  | LGB |
|----------------------|---------------|-----|-----|
| Feasibility          | FALSE         | TRUE| TRUE|
| ACC                  | \
| Recall               | \
| Precision            | \
| AUC                  | \
| Training Time        | \

4 Conclusion

In this study, we built a multi-electrode HPGe detector experimental system and waveform simulation research platform. We studied the feasibility of three methods for $0\nu\beta\beta$ single- and double-electron discrimination of $^{76}$Ge in multi-electrode HPGe detector using pulse-shape simulation. These simulations showed that the resolution of the position below the electrode is about 0.3 mm in the $X$ direction. The results showed that the multi-electrode HPGe detector with this structure had good position sensitivity in the vertical electrode distribution direction. Among the three methods, the track feature method was very dependent on the inherent position resolution of the detector. The detector structure in this study could not directly and effectively utilize the characteristics of the waveform, so it was impossible to discriminate single and double electrons based on the track feature method. We built an FC neural network and LGB model of the fully connected structure to train the simulation waveforms collected by each electrode of the multi-electrode HPGe detector. The discrimination between $0\nu\beta\beta$ double-electron events and 2039 keV single-electron events was feasible. In the absence of noise interference, the discrimination efficiency of the FC neural network was 77.4%, the precision was 57.7%, and the training time was 430 min. The discrimination efficiency of the LGB model was 73.1%, the precision was 64.0%, and the training time was 1.5 min. The results showed that it was feasible to realize single- and double-electron discrimination on multi-electrode HPGe detectors using the FC neural network and LGB model. These results can be used as a reference for future $^{76}$Ge $0\nu\beta\beta$ experiments.

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References

[1] Super-Kamiokande collaboration, Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. 81 (1998) 1562 [hep-ex/9807003].

[2] E. Majorana, Teoria simmetrica dell'elettrone e del positrone, Nuovo Cim. 14 (1937) 171.

[3] J.D. Vergados, H. Ejiri and F. Simkovic, Theory of Neutrinoless Double Beta Decay, Rept. Prog. Phys. 75 (2012) 106301 [arXiv:1205.0649].
[4] J.J. Gómez-Cadenas and J. Martín-Albo, Phenomenology of neutrinoless double beta decay, *PoS GSSI14* (2015) 004 [arXiv:1502.00581].

[5] M. Agostini et al., Search of Neutrinoless Double Beta Decay with the GERDA Experiment, *Nucl. Part. Phys. Proc.* **273-275** (2016) 1876.

[6] MAJORANA collaboration, The Majorana Demonstrator Neutrinoless Double-Beta Decay Experiment, *Adv. High Energy Phys.* **2014** (2014) 365432 [arXiv:1308.1633].

[7] CDEX collaboration, Search for neutrinoless double-beta decay of Ge76 with a natural broad energy germanium detector, *Phys. Rev. D* **106** (2022) 032012 [arXiv:2205.10718].

[8] LEGEND collaboration, The Large Enriched Germanium Experiment for Neutrinoless Double Beta Decay (LEGEND), *AIP Conf. Proc.* **1894** (2017) 020027 [arXiv:1709.01980].

[9] S. Cebrián, T. Dafni, H. Gómez, D.C. Herrera, F.J. Iguaz, I.G. Irastorza et al., The pattern recognition of $^{136}$Xe double beta decay events and background discrimination in a high pressure xenon TPC, *J. Phys. G Nucl. Phys.* **40** (2013) 125203 [arXiv:1306.3067].

[10] T.R. Bloxham and M. Freer, Evaluation of pixellated CZT detectors for neutrinoless double beta-decay measurements, *Nucl. Instrum. Meth. A* **572** (2007) 722.

[11] M. Zeng et al., 3-D topological signatures and a new discrimination method for single-electron events and 0νββ events in CdZnTe: A Monte Carlo study, *Nucl. Instrum. Meth. A* **858** (2017) 44 [arXiv:1601.06300].

[12] D. Budjas, M. Barnabe Heider, O. Chkvorets, N. Khanbekov and S. Schonert, Pulse shape discrimination studies with a Broad-Energy Germanium detector for signal identification and background suppression in the GERDA double beta decay experiment, 2009 *JINST* **4** P10007 [arXiv:0909.4044].

[13] MAJORANA collaboration, Multisite event discrimination for the majorana demonstrator, *Phys. Rev. C* **99** (2019) 065501 [arXiv:1901.05388].

[14] GERDA collaboration, Modeling of GERDA Phase II data, *JHEP* **03** (2020) 139 [arXiv:1909.02522].

[15] C.R. Haufe et al., Modeling Backgrounds for the MAJORANA DEMONSTRATOR, in proceedings of the 8th Topical Workshop on Low Radioactivity Techniques, 2022 [arXiv:2209.10592].

[16] AGATA collaboration, Physics opportunities with the Advanced Gamma Tracking Array: AGATA, *Eur. Phys. J. A* **56** (2020) 137.

[17] T. Siegert et al., Imaging the 511 keV positron annihilation sky with COSI, *Astrophys. J.* **897** (2020) 45 [arXiv:2005.10950].

[18] M. Yang, Y. Li, Z. Zeng, Y. Tian, T. Xue, M. Zeng et al., A prototype segmented planar high purity germanium detector using wraparound lithium diffusion electrode and amorphous germanium blocking contact, *Radiat. Detect. Technol. Meth.* **6** (2022) 433.

[19] I. Abt, F. Fischer, F. Hagemann, L. Hauertmann, O. Schulz, M. Schuster et al., Simulation of semiconductor detectors in 3D with SolidStateDetectors.jl, 2021 *JINST* **16** P08007 [arXiv:2104.00109].

[20] GEANT4 collaboration, GEANT4—a simulation toolkit, *Nucl. Instrum. Meth. A* **506** (2003) 250.

[21] Z. She et al., SAGE: a Monte Carlo simulation framework for experiments with germanium detectors, 2021 *JINST* **16** T09005 [arXiv:2104.13597].
[22] R. Quay, C. Moglestue, V. Palankovski and S. Selberherr, *A temperature dependent model for the saturation velocity in semiconductor materials*, Mater. Sci. Semicond. Process. 3 (2000) 149.

[23] Z. He, *Review of the Shockley-Ramo theorem and its application in semiconductor gamma-ray detectors*, Nucl. Instrum. Meth. A 463 (2001) 250.

[24] M. Agostini, L. Pandola and P. Zavarise, *Off-line data processing and analysis for the GERDA experiment*, J. Phys. Conf. Ser. 368 (2012) 012047 [arXiv:1111.3582].

[25] V.T. Jordanov, G.F. Knoll, A.C. Huber and J.A. Pantazis, *Digital techniques for real-time pulse shaping in radiation measurements*, Nucl. Instrum. Meth. A 353 (1994) 261.

[26] K. Vetter, M. Burks and L. Mihaiescu, *Gamma-ray imaging with position-sensitive HPGe detectors*, Nucl. Instrum. Meth. A 525 (2004) 322.

[27] A. Radovic, M. Williams, D. Rousseau, M. Kagan, D. Bonacorsi, A. Himmel et al., *Machine learning at the energy and intensity frontiers of particle physics*, Nature 560 (2018) 41.

[28] G. Vecchio, S. Amaducci, L. Cosentino and P. Finocchiaro, *Pulse identification and shape analysis by derivative-based peak detection using a Convolutional Neural Network*, 2022 JINST 17 P09040.

[29] R. Kruse, S. Mostaghim, C. Borgelt, C. Braune and M. Steinbrecher, *Multi-layer perceptrons*, in Texts in Computer Science, Springer International Publishing (2022), pp. 53–124.

[30] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan et al., *PyTorch: An Imperative Style, High-Performance Deep Learning Library*, in Advances in Neural Information Processing Systems, vol. 32, 2019 [arXiv:1912.01703].

[31] Z. Zhang, *Improved adam optimizer for deep neural networks*, in proceedings of the IEEE/ACM 26th International Symposium on Quality of Service, Banff, AB, Canada, 4–6 June 2018, pp. 1–2.

[32] G. Ke, Q. Meng, T. Finley, T. Wang, W. Chen, W. Ma et al., *Lightgbm: A highly efficient gradient boosting decision tree*, in Advances in Neural Information Processing Systems, vol. 30, 2017, https://proceedings.neurips.cc/paper_files/paper/2017/file/6449f44a102fde848669bddd9eb6b76fa-Paper.pdf.

[33] F. Pedregosa et al., *Scikit-learn: Machine Learning in Python*, J. Machine Learning Res. 12 (2011) 2825 [arXiv:1201.0490].

[34] T. Agrawal, *Hyperparameter optimization using scikit-learn*, in Hyperparameter Optimization in Machine Learning, Apress (2020), pp. 31–51.