Numerical study on pollutant transport in Dalian bay based on hydrodynamic model

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Abstract: Based on the depth-averaged two-dimensional shallow water and pollutant transport equation, the coupling model of water flow and water quality with explicit scheme is developed in this study. The unstructured triangular grid is adopted to locally refine the mesh around sewage outlet or in high-gradient regions of terrain change for the coupling model. The finite volume method is applied to ensure the conservation of mass for each element. This hydrodynamic model applies the Roe solver approximate Riemann solution with second-order accuracy to compute the water momentum flux on the grid interface. Taking Dalian Bay as the research object, the numerical model established is used to simulate the hydrodynamic characteristics and pollutant transport process. The computed results of the tide level, flow current and flow direction agree well with the measured data in Dalian Bay. The spatial and temporal distribution of pollutant in water are analyzed and discussed in this study. Simulated results show that the two-dimensional hydrodynamic and pollutant transport model can accurately simulate the mass transport in coastal waters, and it can provide a scientific basis on coastal water environment protection for the research water.

1. Introduction

Industrial and agricultural waste water and urban sewage discharged into the nearby water body will lead to the pollution of the water body. For the purpose of managing and protecting the ecological environment of water, it is essential to control transport and distribution of pollutants in the water body. Obviously, the process of pollutant transport is determined by the characteristics of the coastal circulation and the properties of pollutant over the entire ocean domain. Under the action of hydrodynamic factors, the numerical simulation method is used to calculate and analyze the movement of pollutants, which has the advantages of economy, flexible and high efficiency. Therefore, more and more scholars dedicate to the study of working on pollutant diffusion - transportation mathematical models. Kong et al (2013) presented a high-resolution numerical method for solving mass transport problems involving advection and anisotropic diffusion in shallow water based on unstructured mesh [1]. Zhang et al (2015) introduced a computationally efficient model based on a simplified adaptive grid system and the numerical scheme was able to provide well-balanced solutions of mass and solute concentration for applications involving wetting and drying over complex domain topographies [2]. Liang et al (2010) developed a numerical model for predicting solute transport in shallow waters, the alternating operator-splitting technique is used to separate the spatial discretization of the convection-diffusion terms in each step of the time marching procedure [3].

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About Dalian bay, there is little research on the application of the shallow water model to deal with mass transport problems. The present work focuses on developing a depth-averaged 2D numerical model to simulate the hydrodynamic characteristics and pollutant transport in Dalian bay. The purpose of this application is to verify the capacity of the model in simulating hydrodynamic and mass transport processes in natural shallow water bodies.

2. Governing Equations

The Shallow Water Equation (SWEs) written in conservation and vector form are

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = \frac{\partial F_d}{\partial x} + \frac{\partial G_d}{\partial y} + S$$

(1)

In which, \( t \) is the time, \( x \) and \( y \) are the horizontal coordinates, \( U \) is the vector of conserved variables, \( F \), \( G \), \( F_d \), and \( G_d \) are convection fluxes and diffusion fluxes in the \( x \) and \( y \) directions, respectively, and \( S \) is source terms which can be defined respectively as follows:

$$U = \begin{bmatrix} h \\ uh \\ vh \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ hu \\ hu^2 \\ huv \end{bmatrix}, \quad G = \begin{bmatrix} hv \\ hvu \\ hv^2 \end{bmatrix}$$

$$F_d = \begin{bmatrix} v_r \frac{\partial uh}{\partial x} \\ v_r \frac{\partial vh}{\partial y} \end{bmatrix}, \quad G_d = \begin{bmatrix} v_r \frac{\partial uh}{\partial y} \\ v_r \frac{\partial vh}{\partial x} \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ -gh \frac{\partial \eta}{\partial x} - \tau_{hx} \\ -gh \frac{\partial \eta}{\partial y} - \tau_{by} \end{bmatrix}$$

(2)

Where \( h \) is the flow depth, \( u \) and \( v \) are the depth-averaged velocity in the \( x \) and \( y \) directions, respectively, \( \eta \) is the water level, \( \tau_{hx} \) and \( \tau_{by} \) are the friction slopes in the \( x \) and \( y \) directions, respectively.

Mass transport equation:

$$\frac{\partial (hC)}{\partial t} + \frac{\partial (uhC)}{\partial x} + \frac{\partial (vhC)}{\partial y} = \frac{\partial (v_r h \frac{\partial C}{\partial x})}{\partial x} + \frac{\partial (v_r h \frac{\partial C}{\partial y})}{\partial y} + S_C$$

(3)

3. Finite volume method

Usually, the numerical methods may be classified into three groups: Finite-difference methods (FDM), finite-element methods (FEM) and finite volume methods (FVM). In literature, some 2D hydrodynamic models based on the shallow water equations have been implemented to calculate the flow pattern of ocean using the FEM, FDM or FVM (e.g. Sankaranarayanan et al 2003 [4]; Blain, 2005 [5]; Brière, 2007 [6]; Akbar et al, 2013 [7]; Kuang et al, 2011 [8]; Falcão et al, 2013 [9]).

In this paper, a cell-centered finite volume method is adopted to solve the SWEs. The computational area is divided into a set of triangular meshes. Equation (1) can be integrated over a control volume \( V \) as

$$\int_{V_r} \frac{\partial U}{\partial t} \, dv + \int_{V_r} \nabla \cdot E \, dv = \int_{V_r} \frac{\partial F}{\partial x} \, dv + \int_{V_r} \frac{\partial G}{\partial y} \, dv + \int_{V_r} S \, dv$$

(4)

Integrating equation (4) over the area of each control volume, applying Green’s theorem to equation (4), a line integral equation can be obtained:
\[ \frac{\Delta U_i}{\Delta t} A_i = -\int_{L_i} E^* \cdot n_i dl + \int_{L_i} (F_d \cdot n_x + G_d \cdot n_y) dl + \int_{V_i} S dl \cdot v \]  \hspace{0.5cm} (5)

where \( L_i \) is the boundary of cell \( V_i \), \( n_i \) is the unit outward normal vectors on the cell edges, is angle between outside the normal vector and the positive direction of the x-axis, \( A_i \) is the area of the cell \( i \).

After discretizing and arranging the line integral equation (5):

\[ \Delta U_i = -\frac{\Delta t}{A_i} \sum_{j=1}^{m} (E_{ij}^* \cdot n_{ij}) l_{ij} + \frac{\Delta t}{A_i} \sum_{j=1}^{m} (F_{d} \cdot n_{x} + G_{d} \cdot n_{y}) l_{ij} + \Delta t \cdot S \]  \hspace{0.5cm} (6)

where \( l_{ij} \) is the length of unit edge; \( E_{ij}^* \cdot n_{ij} \) is the normal vector numerical flux through the \( j \)th edge of the \( i \)th cell.

Mass transport equation can be also discretized over a triangular control volume, integrating equation (3) over the area of the \( i_{th} \) control volume, one can obtain

\[ \int_{V_i} \left[ \frac{\partial (ohC)}{\partial t} + \frac{\partial (uhC)}{\partial x} + \frac{\partial (vhC)}{\partial y} \right] dV = \int_{V_i} \left[ \frac{\partial}{\partial x} (v_i h \frac{\partial C}{\partial x}) + \frac{\partial}{\partial y} (v_i h \frac{\partial C}{\partial y}) + S_c \right] dV \]  \hspace{0.5cm} (7)

Applying the Green’s theorem to equation (7), discretizing the equation (7) over a time interval, one obtains

\[ \Delta C_i = -\frac{\Delta t}{A_i} \sum_{j=1}^{m} (Q_{ij} \cdot C_{ij} \cdot n_{ij}) + \frac{\Delta t}{A_i} \sum_{j=1}^{m} \left( v_i h \frac{\partial C}{\partial n} \right)_{ij} l_{ij} + \Delta t \cdot S_c \]  \hspace{0.5cm} (8)

In which, \( Q_{ij} \) is the flux across the element side; \( \left( \frac{\partial C}{\partial n} \right)_{ij} \) is the transport variable gradient of the outward normal direction.

4. Numerical validation and analysis

4.1. Introduction of Dalian Bay

The present model is applied to simulate the current situation of Dalian Bay in the northern Yellow Sea. The computational area is from 121°34′ to 122°10′ in east longitude and from 38°39′ to 39°09′ in north latitude, include Laohutan and Dalian Port, and there are two islands (Dasanshan island and Xiaosanshan island) in computed domain (see Figure 1). The water depth is smaller in the bay, about more than ten meters. While it is larger outside the bay, about forty meters. The water exchange capability in inner bay is poor since the flow velocity of it is lower than that of the bay mouth. Dalian bay is a typical semi-enclosed bay. Along the coast, there are a large number of municipal industrial sewage outfalls and kinds of ship terminals, which carries most of Dalian’s domestic sewage and industrial waste water. Considering the poor exchange of water discharge, the marine environment of the Dalian Bay get worse.

4.2. Model application

In this case, the computation domain of Dalian Bay is discretized into 15260 grid cells, because the domain is complex, a fine grid is used near the coastal line, island area and a coarse grid is used in open sea. The water elevations are given by the Tidal Model Driver (TMD). The tidal process covers 216 hours from 13th July, 2010 to 21th July, 2010. The Manning’s \( n \) is set as 0.03. The computational time step is 0.2 s. Figure 2 shows the comparison of simulated and the measured water surface elevation in Dalian port and Laohutan tidal stations. The simulated surface elevations are in good agreement with the observed data. Figure 3 shows the comparison of flow velocities in V1 and V2 monitoring stations. The simulated tide velocities have a slight deviation compared with the measured data, the agreement between them is reasonably good. The flow fields in flood and ebb tides are
reasonably well predicted, as shown in figure 4. Around the islands, the tide is in a spiral type, and the rest is basically reciprocating flow. The flood current is in the northeast into the gulf, the southwest direction is ebb current. Whether it is flood or ebb current, the velocity at the mouth of the bay is larger. In total, one can see that this model can applied to simulate the flow phenomena of coastal water, and it has practical application value.

Based on hydrodynamic simulation, the presently developed transport models applied to simulate the solute transport in the Dalian bay mouth. The parameters such as the grid, the open boundary and the time step of the simulation are similar to that of hydrodynamic simulation. The pollutant emission scheme is an imaginary scheme, the release point is shown in figure 1 and the emission rate is 30g/s, lasted for six hours. Figure 5 illustrates the movement of the pollutant cloud and the simulated flow fields after 6 hours of the release. From the figure we can see that the pollutant concentration field varies with the trend oscillation. As shown in figure 5, the solute oscillates with the tidal flow. As time progresses, the pollutant has gradually increased its distribution range and becomes less concentrated, which can be seen by comparing the concentration fields of figure 5 (a, b, c, and d). Although there is no measured data to verify this simulation, based on the characteristics of the hydrodynamic and pollutant transport and combined with the simulation results, one can see the concentration distribution is rational and the present solute transport model has the capacity to simulate mass transport processes in natural shallow water bodies.

Figure 1. Mesh and measurement stations and pollutant release point at computational area

Figure 2. Comparison of simulation and measurement water surface elevation at selected tidal stations
Figure 3. Comparison of flow velocities in V1 and V2 monitoring stations

Figure 4. Flood and ebb tide in Dalian bay

Figure 5. Development of the pollutant concentration field and flow field for the Dalian Bay mouth coastal water
5. Conclusions
Based on the two-dimensional shallow water equations and water quality equation, and the finite
volume method with an unstructured triangular grid are adopted, the coupling model of water flow and
water quality with explicit scheme is set up in this study. This model applies the Roe solver
approximate Riemann solution with second-order accuracy to compute the water momentum flux on
the grid interface. And then, the model is applied to calculate the tidal flow and simulate the transport
of a fictitious conservative solute in Dalian bay to verify its practicability. Through the example of the
pollutant migration and diffusion in the Dalian Bay, the calculation ability and the accuracy of the
model are verified in the practical application, which shows that the model has certain engineering
application value.

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