Non-local form factors for curved-space antisymmetric fields

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In the recent paper Buchbinder, Kirillova and Pletnev presented formal arguments concerning quantum equivalence of free massive antisymmetric tensor fields of second and third rank to the free Proca theory and massive scalar field with minimal coupling to gravity, respectively. We confirm this result using explicit covariant calculations of non-local form factors based on the heat-kernel technique, and discuss the discontinuity of quantum contributions in the massless limit.

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I. INTRODUCTION

The antisymmetric fields in four dimensions are interesting from various viewpoints. The most attractive part is that they emerge naturally as effective fields after compactification of the (super)string effective action. Therefore, the detection of these fields or their low-energy quantum effects may be regarded as indirect detection of (super)string theory. Naturally, the standard situation is that such fields emerge as parts of the corresponding supermultiplets. At the same time, at low energies the supersymmetry is supposed to be broken and the mainstream approach is the soft symmetry breaking related to the introduction of masses. Therefore, one can expect that the antisymmetric fields can be massive. At the same time, due to compactification of extra dimensions such a mass can be quite small, and hence it is interesting to see what happens in the massless limit, especially at the quantum level. Let us note that the antisymmetric tensor fields have also interesting applications to the construction of non-minimal Grand Unification models, where the interface between massless and broken-symmetry massive versions is one of the main issues.

The quantum aspects of massless antisymmetric fields have been explored in Refs. (13-14) (also, both massless and massive cases were explored within the worldline approach (14-15)). In particular, there was found a quantum equivalence with vector and scalar fields (classical equivalence was established before in (16)), and was shown that the massless third-rank field has no physical degrees of freedom (12, 13 - 19). Indeed, the first work where the equivalence between Proca model and antisymmetric second-rank field can be seen was published long before in 1960 (20). Taking these results into account, one of the interesting questions is about the possible discontinuity of the quantum effects of antisymmetric fields in the massless limit. Recently, quantum theory of massive antisymmetric fields was considered in Ref. (21). In particular, it was shown that the mentioned models are equivalent to vector and scalar massive fields, correspondingly. The equivalence holds in curved space-time, and not only at the classical level, but also in a semiclassical theory, that means the contributions of the corresponding fields to the effective action of vacuum are identical to the ones of vector and minimal scalar theories. The proof of (21) is very general and is based on $\zeta$-regularization. However, this type of proofs is always interesting to check by direct calculation, similar to what was done for the Proca model (22). In this case one can detect the discontinuity of the massless limit not only in the local divergent terms, but also in the complicated non-local contributions, which are typical for the massive field. Another interesting aspect is that the proofs of equivalence involve operations which are potentially dangerous with respect to the non-local multiplicative anomaly, which was previously detected only for fermionic determinants (23). It looks reasonable to to see whether a similar situation takes place in the case of antisymmetric fields and their quantum equivalence with massive vectors and minimal massive scalars.

In the present work we will derive the one-loop form factors using the heat-kernel technique based on the exact solution by Barvinsky and Vilkovisky and Avramidi (24, 25). Such a calculation was previously performed for various models, including scalar field (26), fermions and massive vectors (27). The equivalence with the derivation by means of Feynman diagrams has been shown in (26) and more recently in (28). Indeed, the heat-kernel based method is much more economic and since the application to antisymmetric fields is technically complicated, we chose this approach.

The paper is organized as follows. In Sect. III we briefly review the heat-kernel derivation of form factors according to (26) and present the final results for massive scalars and vectors. The second-rank antisymmetric tensor theory is worked out in Sect. IV and Sect. V deals with the third-rank case. Most of these two sections repeats

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the contents of [21] and other references. The reason to include this is the intention to make the work self-consistent. Therefore, this part is made as brief as possible, but at the same time we give sufficient details. Finally, the Conclusions are drawn in Sect. V.

II. DERIVATION OF THE ONE-LOOP FORM FACTORS

Let us review the derivation of the one-loop effective action up to the terms of the second order in curvatures. More details can be found in Refs. [26, 27]. The one-loop Euclidian effective action is given by the formula

\[ \tilde{\Gamma}^{(1)} = \frac{1}{2} \text{Tr} \ln \hat{H}, \]

where we assume the minimal form of the bilinear form of the action

\[ \hat{H} = \hat{\Box} - \hat{1} m^2 + \hat{P} - \frac{1}{6} R, \]

where \( \hat{1} \) is an identity matrix in the space of the fields of interest and \( \hat{P} \) operator depends on the metric and possibly other background fields. The commutator of the two covariant derivatives acting on the corresponding fields is \( \hat{S}_{\mu\nu} = [\hat{\nabla}_\mu, \hat{\nabla}_\nu] \).

The effective action \( \Gamma^{(1)} \) can be presented as an integral in the proper time \( s \), involving the heat kernel \( K(s) \),

\[ \Gamma^{(1)} = -\frac{1}{2} \int_0^\infty ds \frac{d}{s} \text{Tr} K(s). \]

The heat kernel can be expanded up to the second order in the curvatures, namely Ricci tensor \( R_{\mu\nu} \) and scalar \( R \), \( \hat{S}_{\mu\nu} \) and \( \hat{P} \). The second-order solution has the form [24]

\[ \text{Tr} K(s) = \frac{\mu^{2(2-\omega)}}{(4\pi s)^{\omega}} \int d^2x \sqrt{g} e^{-m^2 s} \text{tr} \left\{ \hat{1} + s\hat{P} 
+ s^2 \left( R_{\mu\nu} f_1(s\Box) R^{\mu\nu} + R f_2(s\Box) R
+ \hat{P} f_3(s\Box) R + \hat{P} f_4(s\Box) \hat{P}
+ \hat{S}_{\mu\nu} f_5(s\Box) \hat{S}^{\mu\nu} \right) \right\}, \]

where \( \omega \) is the parameter of dimensional regularization, \( \mu \) is renormalization parameter and the functions \( f_{1,2,\ldots,5} \) are given by the following expressions:

\[ f_1(\tau) = \frac{f(\tau) - 1 + \tau/6}{\tau^2}, \]
\[ f_2(\tau) = \frac{f(\tau) + f(\tau) - 1}{24\tau} - \frac{f(\tau) - 1 + \tau/6}{8\tau^2}, \]
\[ f_3(\tau) = \frac{f(\tau) + f(\tau) - 1}{12}, \quad f_4(\tau) = \frac{f(\tau)}{2}, \]
\[ f_5(\tau) = \frac{1 - f(\tau)}{2\tau}, \]

where

\[ f(\tau) = \int_0^\tau d\alpha e^{\alpha(1-\alpha)} \tau = -s\Box. \]

The integrals were taken in Ref. [26, 27] and we present only the final result,

\[ \tilde{\Gamma}^{(1)} = -\frac{1}{2(4\pi)^2} \int d^2x \sqrt{g} \left\{ l_{CC} L_0 + l_R L_1 R 
+ \sum_{i=1}^5 l_i^* R_{\mu\nu} M_i R^{\mu\nu} + \sum_{i=1}^5 l_i R M_i R \right\}, \]

where the coefficients \( l_{1,2,\ldots,5} \) and \( l_{CC,R,1,2,\ldots,5} \) are model-dependent and the integrals are universal. Within the dimensional regularization \( \omega \to 2 \) they have the form

\[ L_0 = \frac{m^4}{2} \left( \frac{1}{\epsilon} + 3 \frac{2}{\epsilon} \right), \]
\[ L_1 = -m^2 \left( \frac{1}{\epsilon} + 1 \right), \]
\[ M_1 = \frac{1}{\epsilon} + 2 Y, \]
\[ M_2 = \left( \frac{1}{\epsilon} + 1 \right) \left( \frac{1}{12} - \frac{1}{a^2} \right) - \frac{4Y}{3a^2} + \frac{1}{188}, \]
\[ M_3 = \left( \frac{1}{\epsilon} + 3 \frac{2}{\epsilon} \right) \left( \frac{1}{2a^4} - \frac{1}{12a^2} + \frac{1}{160} \right) \]
\[ + \frac{8Y}{180a^2} + \frac{1}{400}, \]
\[ M_4 = \left( \frac{1}{\epsilon} + 1 \right) \left( \frac{1}{4} - \frac{1}{a^2} \right), \]
\[ M_5 = \left( \frac{1}{\epsilon} + 3 \frac{2}{\epsilon} \right) \left( \frac{1}{2a^4} - \frac{1}{4a^2} + \frac{1}{32} \right), \]

where we used condensed notation

\[ \frac{1}{\epsilon} = \frac{1}{2 - \omega} - \gamma + \ln \left( \frac{4\pi \mu^2}{m^2} \right) \]

and \( \gamma \) is the Euler number (it was absorbed into \( \mu \)-dependence in [26, 27]). In the expressions \([8]-[14]\) we disregarded the terms with are \( O(2 - \omega) \). We also used the definitions

\[ Y = 1 - \frac{1}{a} \ln \left( \frac{2 + a}{2 - a} \right) \]

and

\[ a^2 = \frac{4\Box}{\Box - 4m^2}. \]

In order to arrive at the final form of the one-loop effective action, it is useful to introduce the basis which consists from the square of the Weyl tensor and of scalar curvature. For this end one can assume that for functions \( F = F(\Box) \) of our interest there is an expansion into power series in \( \Box \), and use the reduction formula (see, e.g., [29])

\[ R_{\mu\nu\alpha\beta} FR^{\mu\nu\alpha\beta} = 4R_{\mu\nu} FR^{\mu\nu} - RFR + O(R^3). \]
For the scalar field with non-minimal interaction to external gravity,
\[ S_0 = \frac{1}{2} \int d^4x \sqrt{g} \left\{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 + \xi R \phi^2 \right\} \]  \quad (19)
the \( \mathcal{O}(R^2) \) result is [21, 23]
\[ \Gamma_0^{(1)} = \frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ m^4 \left( \frac{1}{\epsilon} + \frac{3}{2} \right) \right. \]
\[ + \xi \left( \frac{1}{\epsilon} + 1 \right) m^2 R + R \left( \frac{1}{2\epsilon^2} + k_R^0 \right) R \]
\[ \left. + \frac{1}{2} C_{\mu\nu\alpha\beta} \left( \frac{1}{60\epsilon} + k_W^0 \right) C^{\mu\nu\alpha\beta} \right\}, \]  \quad (20)
where \( \xi = \xi - 1/6 \) and the non-local form factors have the form
\[ k_W^0 = k_W^0(a) = \frac{1}{150} + \frac{2}{45a^2} + \frac{8Y}{15a^4} \]  \quad (21)
and
\[ k_R^0 = k_R^0(a) = \frac{1}{108} \left( \frac{1}{a^2} - \frac{7}{20} \right) + \frac{Y}{144} \left( 1 - \frac{4}{a^2} \right)^2 \]
\[ + \left( \frac{1}{18} - \frac{Y}{6} + \frac{2Y}{3a^2} \right) \xi + Y \xi^2. \]  \quad (22)
In the next sections we will consider the form factors for a minimal (means \( \xi = 0 \)) scalar and also the ones for the Proca model in curved space,
\[ S_1 = \int d^4x \sqrt{g} \left\{ - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_{\mu}^2 \right\}, \]  \quad (23)
where \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \). The standard Stueckelberg procedure can be easily adapted to the curved space \[22\], yielding an equivalent action with an extra scalar field \( \phi \),
\[ S_1' = - \int d^4x \sqrt{g} \left\{ \frac{1}{4} F_{\mu\nu}^2 + \frac{m^2}{2} \left( A_{\mu} - \frac{1}{m} \partial_\mu \phi \right)^2 \right\}. \]  \quad (24)
The new action \[24\] is gauge invariant under the gauge transformations
\[ A_\mu \rightarrow A'_\mu = A_\mu + \nabla_\mu f, \quad \phi \rightarrow \phi' = \phi + mf. \]  \quad (25)
The original theory \[24\] is recovered in the special gauge \( \phi = 0 \). Since the gauge fixing dependence is irrelevant for the derivation of vacuum effective action, the practical calculation can be performed in a more useful gauge. The reader can consult Ref. \[22\] for the details, let us just present the final result
\[ \Gamma^{(1)} = - \frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ \frac{3m^4}{2} \left( \frac{1}{\epsilon} + \frac{3}{2} \right) \right. \]
\[ + \frac{m^2}{2} \left( \frac{1}{\epsilon} + 1 \right) R + R \left( \frac{1}{72\epsilon^2} + k_R^0 \right) R \]
\[ \left. + \frac{1}{2} C_{\mu\nu\alpha\beta} \left( \frac{13}{60\epsilon} + k_W^0 \right) C^{\mu\nu\alpha\beta} \right\}, \]  \quad (26)
where
\[ k_W = k_W^0(a) = -\frac{91}{450} + \frac{2}{15a^2} + Y + \frac{8Y}{5a^4} - \frac{8Y}{3a^2}, \]  \quad (27)
\[ k_R = k_R^0(a) = -\frac{1}{2160} + \frac{1}{36a^2} + Y \frac{48}{450} - \frac{8Y}{18a^2}. \]
As it was already discussed in \[22\], the massless limit in the expression \[24\] does not yield the effective action for a massless field, due to the discontinuity in the quantum corrections. In the next sections we shall meet two other examples of a similar discontinuity in the massless limit.

### III. MASSIVE ANTISYMMETRIC RANK-2 TENSOR

In this section we first present the well-known general considerations and then proceed to the derivation of non-local form factors.

#### A. General considerations

The model of massive antisymmetric second-rank tensor \( B_{\mu\nu} = -B_{\nu\mu} \) field is described by the action
\[ S_2 = \int d^4x \sqrt{g} \left\{ - \frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} \right\}, \]  \quad (28)
where
\[ F_{\mu\nu\lambda} = \nabla_\mu B_{\nu\lambda} + \nabla_\nu B_{\lambda\mu} + \nabla_\lambda B_{\mu\nu}. \]  \quad (29)
In four dimensions the theory \[28\] is classically equivalent to a massive axial vector field \( A^\mu \). The equivalence can be found through detailed analysis of the equations of motion \[4\]. The duality between the two theories is given by
\[ B_{\mu\nu} \propto \frac{1}{m} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}, \]  \quad (30)
where \( F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \).

The model \[28\] is an example of a theory with the softly broken gauge symmetry. The kinetic part of the action \[28\] is gauge invariant under the transformation
\[ B_{\mu\nu} \rightarrow B'_{\mu\nu} = B_{\mu\nu} + \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu, \]  \quad (31)
where the vector gauge parameters \( \xi_\mu \) in \[31\] are not unique. They can be transformed according to
\[ \xi_\mu \rightarrow \xi'_\mu = \xi_\mu + \nabla_\mu \phi, \]  \quad (32)

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1 Here we follow \[24\] and use an opposite sign for the mass term. This is reasonable taking into account possible applications to spontaneous symmetry breaking.

2 Let us note that the massive axial vector describes an antisymmetric torsion field. This identification comes from the requirement of quantum consistency \[34\]. The theory was eventually shown to violate unitarity when coupled to fermions \[53\].
with \( \varphi = \varphi(x) \) being an arbitrary scalar field. Equation (22) means that the gauge generators are linearly dependent. Using the background field method we can observe that the massive term in (31) violates gauge symmetry, but does not remove the degeneracy in the bilinear form in quantum fields

\[
\hat{H}_2 = \frac{1}{2} \frac{\delta^2 S_2}{\delta B_{\mu \nu}(x) \delta B_{\alpha \beta}(x')}.
\]  

(33)

Similar to the Proca model, the simplest approach for the Lagrangian quantization of the theory (23) requires the Stueckelberg procedure. Following [21], we introduce an extra vector field \( A_{\mu} \) and consider, instead of Eq. (31), the action

\[
S'_2 = \int d^4x \sqrt{g} \left\{ -\frac{1}{12} F_{\mu \nu \lambda} F^{\mu \nu \lambda} - \frac{\Lambda}{4} m^2 \left( B_{\mu \nu} - \frac{1}{m} F_{\mu \nu} \right)^2 \right\}.
\]  

(34)

The previous action (28) can be obtained from (34) in the specific gauge \( A_{\mu} = 0 \).

The new action (33) is gauge invariant under the simultaneous transformation

\begin{align*}
B_{\mu \nu} &\to B'_{\mu \nu} = B_{\mu \nu} + \nabla_{\nu} \xi_{\mu} - \nabla_{\mu} \xi_{\nu}, \\
A_{\mu} &\to A'_{\mu} = A_{\mu} + m \xi_{\mu},
\end{align*}

(35)

and it is also invariant under gauge transformation of the Stueckelberg field

\[
A_{\mu} \to A'_{\mu} = A_{\mu} + \nabla_{\mu} \Lambda,
\]  

(36)

with a scalar parameter \( \Lambda(x) \). Furthermore, we can consider a new scalar field \( \varphi(x) \) and note that the fields \( B_{\mu \nu} \) and \( A_{\mu} \) do not change if their gauge parameters transform as

\begin{align*}
\xi &\to \xi' = \xi + \nabla_{\nu} \varphi, \\
\Lambda &\to \Lambda' = \Lambda + m \varphi.
\end{align*}

(37)

Once again, the equations (38) and (39) reflect the fact that the gauge generators of the theory are linearly dependent.

The general formalism of Lagrangian quantization in theories with dependent generators is based on the Batalin-Vilkovisky method [33]. However, in the relatively simple theories such as (32), where the action is quadratic and the algebra of dependent gauge generators is Abelian, it is sufficient to make a successive multi-step application of the Faddeev-Popov method [12, 31, 33]. In the following we are going apply this approach to the theory (33).

According to the Faddeev-Popov method we replace the gauge group integral in the functional integral over gauge fields,

\[
\int DBDA e^{iS'_2[B,A]} ,
\]  

(40)

by the quantity

\[
\int DBDA e^{iS'_2[B,A]} \Delta \delta(\varphi^a[B,A] - l^a, \chi[A] - l),
\]  

(41)

where \( \Delta \) provides the identity

\[
1 = \Delta \int DBDA \delta(\varphi^a[B',A'] - l^a, \chi[A'] - l).
\]  

(42)

Here \( \varphi^a \) and \( \chi \) are the gauge fixing term which are related to the transformations (33), (35) and (37), respectively. For the theory (34) one can choose

\[
\chi_{\beta} = \nabla_{\alpha} B_{\alpha \beta} - m A_{\beta}, \quad \chi = \nabla_{\alpha} A_{\alpha}.
\]  

(43)

It is easy to verify the following constraint between the two gauge fixing conditions:

\[
\nabla_{\alpha} \chi_{\alpha} + m \chi = 0.
\]  

(44)

Due to the constraint (44), the delta-function \( \delta(\varphi^a[B,A], \chi[B]) \) in the definitions (41) and (42) is ill-defined, that represents the main difference with the standard Faddeev-Popov procedure. The same problem also affects the integral in (42), since the fields are invariant under transformations (35), (36) and (37) for the gauge parameters \( \xi_{\mu} \) and \( \Lambda \). To solve this issue, one can also apply the Faddeev-Popov trick second time to remove the integration along the gauge group orbits. Consider the Fourier representation for the delta-function

\[
\delta(\varphi^a, \chi) = \int D\zeta D\psi e^{i(\zeta_{\alpha} \varphi^a - \psi \chi)}.
\]  

(45)

After integration by parts and using (44) one can easily show that the integrand in (45) is invariant under the transformation

\begin{align*}
\zeta_{\alpha} &\to \zeta'_{\alpha} = \zeta_{\alpha} + \nabla_{\alpha} \phi, \\
\psi &\to \psi' = \psi + m \phi.
\end{align*}

(46)

(47)

In order to have well-defined definition one can extract from (45) the integral over gauge group orbit (40)–(47) by using the Faddeev-Popov trick, hence we arrive at

\[
\Delta \delta(\varphi^a[B,A] - l^a, \chi[A] - l) \times
\]  

\[
\times \delta(\nabla_{\alpha} \zeta^a - m \psi) \det \hat{H}_0^{min},
\]  

(48)

where \( H_0^{min} = \Box - m^2 \) is a minimal scalar operator. Let us remember that this operator depends on the external metric and its functional determinant is non-trivial.

For the integral in Eq. (22) one has to factorize the integrations over gauge group orbits (35)–(39). This means we replace \( D\xi D\Lambda \) by the product

\[
D\xi D\Lambda \delta(\nabla_{\alpha} \zeta^a - m \Lambda) \det \hat{H}_0^{min}
\]  

(49)

in the definition (12). Hence, the equation (12) for \( \Delta \) becomes

\[
\Delta^{-1} = \int D\zeta D\psi D\xi D\Lambda \delta(\nabla_{\alpha} \zeta^a - m \psi - s) \times
\]  

\[
\times \delta(\nabla_{\beta} \xi^a - m \Lambda - t) \times
\]  

\[
\times e^{i \{ \zeta_{\alpha} (\varphi^a[B'] - l^a) - \psi (\chi[A'] - l) \}} (\det \hat{H}_0^{min})^2.
\]  

(50)
For solving (50) one can use the fact that
\[
\chi^\alpha[B', A'] - l^\alpha = \chi^\alpha[B, A] - l^\alpha + (H_1)^\alpha_\beta \delta_\beta^\beta
\]
and \(\chi[A'] - l = \chi[A] - l + \Box \Lambda\), \(\chi = \mathcal{D}_\mu R^{-1}_\mu R^0_\mu - m^2 \delta_\mu^\mu\). \(\tag{52}\)

Because of the delta-function in (11), the fields satisfy the equations \(\chi^\alpha[B, A] - l^\alpha = 0\) and \(\chi^\alpha[A] - l = 0\). Therefore, introducing the identity factor in the form of the double integral \(\int Ds Dl e^{-iS_{1}} = 1\), we can take the integral over delta-functions, arriving at
\[
\Delta = \text{Det} \hat{H}_1^{-1} \cdot (\text{Det} \hat{H}_0^{\text{min}})^{-1}, \tag{53}\]
with \(\hat{H}_1\) is a minimal vector operator,
\[
\hat{H}_1 = (H_1)_\mu^\nu = \delta_\mu^\nu \Box - R^\mu_\nu - \delta_\mu^\nu m^2. \tag{54}\]

For the sake of completeness, let us remember the Stueckelberg procedure for the massive vector field \(22\),
\[
\hat{\Gamma} = \frac{1}{2} \text{Tr} \ln \hat{H}_1 = \frac{1}{2} \text{Tr} \ln \left(\delta_\mu^\mu - R^\mu_\mu - \delta_\mu^\mu m^2\right) - \frac{1}{2} \text{Tr} \ln \left(\Box - m^2\right) \tag{55}\]
Now, using the well-defined expressions (45) and (53) we can consider Eq. (11) and then the effective action. First one has to write the delta-function (45) in a more useful way. Using the Fourier representation
\[
\delta(\nabla \zeta - m \psi) = \int D\varphi e^{i(\nabla \zeta - m \psi) \varphi} = \int D\varphi e^{i(-\zeta \nabla \varphi - \psi m \varphi), \tag{56}\}
\]
we can make an integration over \(\zeta\) and \(\psi\) in (45) and find
\[
\delta (\chi^\alpha, \chi) = \int \text{Det} \hat{H}_0^{\text{min}}. \tag{57}\]

Hence, using Eqs. (57) and (53) we write the vacuum effective action in the form
\[
e^{i\Gamma_{[g_{\mu\nu}]}} = \int DB DA D\varphi e^{iS_{2}[B, A] + \delta (\chi^\alpha - \nabla_\alpha \varphi - l^\alpha) \times \delta (\chi - m \varphi - l) \text{Det} \hat{H}_1^{-1} \times \text{Det} \hat{H}_0^{\text{min}}. \tag{58}\]

As far as (68) does not depend of \(l^\alpha\) and \(l\) one can insert the identities in the form of \(\int Dl e^{-i \frac{1}{2} l^\alpha \Box l} = \int Dl e^{-i \frac{1}{2} l^2}\) to take the integrals. In this way we arrive at
\[
e^{i\Gamma_{[g_{\mu\nu}]}} = \int DB DA D\varphi e^{i\{S_{2}[B, A] + S_0[B, A] - S_0[\varphi]\} \times \text{Det} \hat{H}_1^{-1}, \tag{59}\]
where \(S_{gf} = -\frac{1}{2} \int d^4x \sqrt{g} (\chi_\alpha \chi^\alpha + \chi^2)\) and \(S_0[\varphi]\) is the action of scalar field (13). The action with the gauge-fixing term can be written in the form
\[
S_2^{\text{g.f.}} = \int d^4x \sqrt{g} \left\{ \frac{1}{4} B_{\alpha\beta} (H_1')_{\alpha\beta} B^\mu_\nu + \frac{1}{2} A_\mu (H_1')_{\mu\nu} A^\nu \right\}, \tag{60}\]
where
\[
\hat{H}_1' = (H_1')_{\mu\nu} = \delta_\mu^\nu \Box - R^\mu_\nu - \delta_\mu^\nu m^2. \tag{61}\]

is the identity matrix in the antisymmetric rank-2 tensor space and
\[
J_{\alpha\beta, \mu\nu} = \frac{1}{2} \left( g_{\alpha\mu} R_{\beta\nu} + g_{\beta\nu} R_{\alpha\mu} - g_{\alpha\nu} R_{\beta\mu} - g_{\beta\mu} R_{\alpha\nu} \right). \tag{62}\]

Eqs. (58) and (60) enable one to formulate the one-loop contribution to the vacuum Euclidean effective action in the form
\[
\Gamma^{(1)} = \frac{1}{2} \left( \text{Tr} \ln \hat{H}_2 + \text{Tr} \ln \hat{H}_1 \right) - \text{Tr} \ln \hat{H}_1^{\text{min}} \tag{63}\]
Using the relation for the Proca field contribution, Eq. (65), one can also write the one-loop effective action in the form
\[
\Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln \hat{H}_2' - \frac{1}{2} \text{Tr} \ln \hat{H}_1. \tag{64}\]

Finally, the effective action requires subtracting the contribution of the Stueckelberg massive vector from the one of the massive tensor operator, \(\text{Tr} \ln \hat{H}_2'\).

B. Derivation of form factors

The result (63) enables one to use the heat-kernel technique for deriving form factors. The first step is to identify the elements of the general expression (7),
\[
\hat{1} = \delta_\alpha^\beta_{\mu\nu}, \quad \hat{P}_2 = (P_2)_{\alpha\beta} = R_{\alpha\beta, \mu\nu} + \frac{1}{6} \delta_\alpha^\beta_{\mu\nu} R - J_{\alpha\beta, \mu\nu}. \tag{65}\]

The commutator of covariant derivatives on the antisymmetric tensor field \(B_{\mu\nu}\) is
\[
(\hat{S}_2)_{\mu\nu}^{[\alpha\beta]} = \left( (S_2)_{\mu\nu} \right)_{\alpha\beta}^{[\mu\nu]} = \frac{1}{2} (R_{\rho\mu\nu} \delta_\omega^\beta - R_{\rho\mu\nu} \delta_\omega^\alpha - R_{\omega\mu\nu} \delta_\rho^\beta + R_{\omega\mu\nu} \delta_\rho^\alpha). \tag{66}\]
Then, using the heat kernel representation, we arrive at the identification in the second order in curvatures,

\[
\frac{1}{2} \text{Tr} \ln H'_2 = -\frac{1}{2(4\pi)^2} \int d^2 x \sqrt{g} \left\{ 3m^4 \left( \frac{1}{\epsilon} + \frac{3}{2} \right) + m^2 \left( \frac{1}{\epsilon} + 1 \right) R + R \left[ \frac{1}{36\epsilon} + k_{2,R}^2(a) \right] R \right. \\
+ \left. \frac{1}{2} C_{\mu\nu\alpha\beta} \left[ \frac{13}{30\epsilon} + k_{2,W}^2(a) \right] C^{\mu\nu\alpha\beta} \right\},
\]

(67)

where \( l_{CC} \) and \( l_R \) are already inserted into (67), other coefficients are

\[
l_1 = -\frac{5}{16}, \quad l_2 = -\frac{5}{4}, \quad l_3 = -\frac{3}{4}, \quad l_4 = \frac{9}{8}, \quad l_5 = \frac{3}{4}, \\
l_1' = 1, \quad l_2' = 4, \quad l_3' = 6, \quad l_4' = -3, \quad l_5' = -6.
\]

Replacing these values and the integrals (8)-(14), we arrive at the expression

\[
\frac{1}{2} \text{Tr} \ln H'_2 = -\frac{1}{2(4\pi)^2} \int d^4 x \sqrt{g} \left\{ 3m^4 \left( \frac{1}{\epsilon} + \frac{3}{2} \right) \\
+ m^2 \left( \frac{1}{\epsilon} + 1 \right) R + R \left[ \frac{1}{36\epsilon} + k_{2,R}^2(a) \right] R \right. \\
+ \left. \frac{1}{2} C_{\mu\nu\alpha\beta} \left[ \frac{13}{30\epsilon} + k_{2,W}^2(a) \right] C^{\mu\nu\alpha\beta} \right\},
\]

(68)

where

\[
k_{2,W}^2(a) = -\frac{91}{225} + \frac{4}{15a^2} + 2Y + \frac{16Y}{5a^2} - \frac{16Y}{3a^2},
\]

(69)

\[
k_{2,R}^2(a) = -\frac{1}{1080} + \frac{1}{18a^2} + \frac{Y}{24} + \frac{2Y}{3a^4} - \frac{9a^2}{2}. 
\]

(70)

According to Eq. (63), we have to subtract from (68) the massive vector part, Eq. (26). Hence, we get

\[
\Gamma^{(1)} = -\frac{1}{2(4\pi)^2} \int d^4 x \sqrt{g} \left\{ 3m^4 \left( \frac{1}{\epsilon} + \frac{3}{2} \right) \\
+ m^2 \left( \frac{1}{\epsilon} + 1 \right) R + R \left[ \frac{1}{36\epsilon} + k_{2,R}^2(a) \right] R \right. \\
+ \left. \frac{1}{2} C_{\mu\nu\alpha\beta} \left[ \frac{13}{60\epsilon} + k_{2,W}^2(a) \right] C^{\mu\nu\alpha\beta} \right\},
\]

(71)

where the non-local form factors are

\[
k_{2,W}(a) = -\frac{91}{450} + \frac{2}{15a^2} + Y + \frac{8Y}{5a^2} - \frac{8Y}{3a^2},
\]

(72)

\[
k_{2,R}(a) = -\frac{1}{2160} + \frac{1}{36a^2} + \frac{Y}{48} + \frac{Y}{3a^4} - \frac{Y}{18a^2}.
\]

(73)

It is easy to see that the vacuum effective action for the massive rank-2 antisymmetric tensor, \( \Gamma^{(1)} \), is exactly the same as the vacuum effective action in the massive vector field case, given by Eq. (26). This confirms the conclusion of [21] that the massive rank-2 antisymmetric tensor is equivalent to the Proca theory at quantum level. Let us note that this conclusion has been achieved by the \( \zeta \)-regularization method, and we know that some of the relations of this kind can be violated by the non-local multiplicative anomaly [23]. Nothing of this sort occurs in the present case, as we have seen.

It is easy to check that in the massless limit the form factor \( k_{2,W}^2(a) \) reduce to the usual logarithmic expression \(-\frac{13}{60} \ln \left( \frac{-\Box}{4\pi^2} \right)\). On the other hand, we know that in the \( m = 0 \) case the rank-2 antisymmetric tensor is equivalent to a scalar field minimally coupled to gravity, where the duality looks like \( F_{\alpha\beta\omega} = \epsilon_{\alpha\beta\omega\gamma} \nabla^\gamma \varphi \). The form factor for the minimal massless scalar field is \(-\frac{1}{60} \ln \left( \frac{-\Box}{4\pi^2} \right)\). The difference between the two coefficients \( 1/5 = 13/60 - 1/60 \) demonstrates the discontinuity of quantum contributions in the massless limit for the rank-2 antisymmetric tensor theory. This difference is nothing else but the contribution of a massless vector field. To understand which vector is this, let us consider the effective action for a massless rank-2 antisymmetric tensor field \([21]\),

\[
\Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln H'_2 - \text{Tr} \ln H'_1 + \frac{3}{2} \text{Tr} \ln \hat{H}^\text{min}_0.
\]

(73)

The difference between (63) and (73) is

\[
-\frac{1}{2} \text{Tr} \ln \hat{H}'_1 + \text{Tr} \ln \hat{H}^\text{min}_0,
\]

(74)

which is the effective action for the free massless vector field. Another way to understand this is to recall that massive rank-2 antisymmetric tensor field is equivalent to a massive vector field model. According to \([22]\),

\[
\Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln (\delta_\mu^\nu \Box - R_\mu^\nu - \delta_\mu^\nu m^2) + \frac{1}{2} \text{Tr} \ln (\Box - m^2) \\
- \text{Tr} \ln (\Box - m^2)
\]

(75)

\[
= \frac{1}{2} \text{Tr} \ln (\delta_\mu^\nu \Box - R_\mu^\nu - \delta_\mu^\nu m^2) - \frac{1}{2} \text{Tr} \ln (\Box - m^2).
\]

Obviously, the difference in the contributions of a massive vector field and the one of the minimal scalar field is just the effective action of a massless vector field. In the massless limit this extra term does not disappear and this produce the quantum discontinuity.

**IV. MASSIVE ANTISYMMETRIC RANK-3 TENSOR**

As a second example, consider the model of massive totally antisymmetric rank-3 tensor field \( C_{\mu\nu\rho} = C_{[\mu\nu\rho]} \). The action is given by

\[
S_3 = \int d^4 x \sqrt{g} \left\{ -\frac{1}{48} F_{\mu\nu\rho\omega}^2 - \frac{1}{12} m^2 C_{\mu\nu\rho}^2 \right\},
\]

(76)

where

\[
F_{\mu\nu\rho\omega} = \nabla_\mu C_{\nu\rho\omega} - \nabla_\nu C_{\rho\mu\omega} + \nabla_\rho C_{\omega\mu\nu} - \nabla_\omega C_{\mu\nu\rho}.
\]

It is possible to prove that in four dimensional space the theory \([70]\) is classically equivalent to the theory of a real massive scalar field \( \varphi \) minimally coupled to gravity. The duality relation between the two theories is defined by the relation \( C_{\mu\nu\rho} \propto \frac{1}{m} \epsilon_{\mu\nu\rho\sigma} \nabla^\sigma \varphi \).
The kinetic term of the action (76) is invariant under the gauge transformations
\[ C_{\mu
u\rho} \rightarrow C'_{\mu
u\rho} = C_{\mu
u\rho} + \nabla_\mu \omega_{\nu\rho} + \nabla_\nu \omega_{\mu\rho} + \nabla_\rho \omega_{\mu\nu}, \]
with an antisymmetric tensor field parameter \( \omega_{\mu\nu} = -\omega_{\nu\mu} \). This parameter is defined up to the gauge transformation
\[ \omega_{\mu\nu} \rightarrow \omega'_{\mu\nu} = \omega_{\mu\nu} + \nabla_\mu \zeta_\nu - \nabla_\nu \zeta_\mu, \]
where \( \zeta_\mu \) is a vector gauge field parameter. Furthermore, \( \zeta_\mu \) is also defined up to the gauge transformation
\[ \zeta_\mu \rightarrow \zeta'_\mu = \zeta_\mu + \nabla_\mu \phi, \]
with the scalar field parameter \( \phi(x) \). Equations (78) and (79) mean that the gauge generators are linearly dependent. As in the previous case of the second-rank tensor, due to the gauge invariance of \( F^2_{\mu
u\rho\omega} \), we have to deal with a theory with softly broken gauge symmetry. Therefore, the quantization must be done with the Stueckelberg procedure.

One can restore the gauge symmetric by introducing an extra second-rank antisymmetric field \( B_{\mu\nu} \). Consider the following action:
\[ S'_3 = \int d^4x \sqrt{g} \left\{ -\frac{1}{48} F^2_{\mu\nu\rho\omega} - \frac{m^2}{12} \left( C_{\mu\nu\rho} - \frac{1}{m} F_{\mu\nu\rho} \right)^2 \right\}, \]
where \( F_{\mu\nu\rho} \) is defined in (29). The action (80) is gauge invariant under the simultaneous transformation (77) and
\[ B_{\mu\nu} \rightarrow B'_{\mu\nu} = B_{\mu\nu} + m \omega_{\mu\nu}. \]
It is also invariant under the gauge transformation of Stueckelberg procedure field
\[ B_{\mu\nu} \rightarrow B'_{\mu\nu} = B_{\mu\nu} + \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu, \]
where \( \xi_\mu \) is a vector gauge parameter, defined up to a gauge transformation \( \xi'_\mu = \xi_\mu + \nabla_\mu \phi \) with a scalar parameter \( \phi(x) \). Since the gauge generators of the theory are linearly dependent, the quantization of the theory differs from standard scheme and can be done in a way similar to the one described above for the second-rank field. The successive multi-step applications of Faddeev-Popov method for the antisymmetric rank-3 tensor is somewhat more tedious than for the antisymmetric rank-2 field, hence we are not going to bother the reader with the details and present only the final formula for the one-loop effective action,
\[ \Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln \hat{H}'_3 - \frac{1}{2} \text{Tr} \ln \hat{H}'_2 + \frac{1}{2} \text{Tr} \ln \hat{H}'_1 - \frac{1}{2} \text{Tr} \ln \hat{H}'_0^\text{min} = \frac{1}{2} \text{Tr} \ln \hat{H}'_3 - \frac{1}{2} \text{Tr} \ln \hat{H}'_2, \]
where
\[ \hat{H}'_3 = (H'^3)'_{\alpha\beta\omega} = \delta'_{\mu\nu\rho} (\Box - m^2) + K'_{\mu\nu\rho} - L'_{\mu\nu\rho}, \]
\[ \delta'_{\mu\nu\rho} = \frac{1}{6} \epsilon_{\alpha\beta\omega} \epsilon_{\mu\nu\rho}, \]
\[ K'_{\mu\nu\rho} = 3 \delta_{\gamma\delta} \delta_{\lambda\tau} R^\phi_{\gamma\delta} R^\tau_{\lambda\tau}, \]
and
\[ L'_{\mu\nu\rho} = \delta_{\gamma\delta} \delta_{\lambda\tau} (R^\phi_{\gamma\delta} R^\tau_{\lambda\tau} R_{\lambda\tau} + R^\phi_{\gamma\delta} R^\tau_{\lambda\tau} R_{\lambda\tau}). \]
Equation (85) defines the generalized Kronecker delta which serves as an identity matrix in the space of third-order totally antisymmetric tensors. It also has the property
\[ \delta'_{\mu\nu\rho} T_{\alpha\beta\omega} = T_{\mu\nu\rho}. \]
Due to the identity (85) one can write the expressions (80) and (87) in a compact way respecting their symmetries. From the technical side, by using the definition (85), calculation of divergences can be mainly reduced to contractions of the products of Levi-Civita symbols.

In accordance to the formula (80) we need to work with the third-rank tensor and subtract the second-rank contribution which is already known. Consider the field strengths for the first term,
\[ \tilde{P}_3 = (P^3)'_{\alpha\beta\omega} = K_{\alpha\beta\omega} + \frac{1}{6} \delta_{\mu\nu\rho} R - L_{\alpha\beta\omega}, \]
\[ (\tilde{S}_3)_{\mu\nu\rho} = [(S^3)'_{\mu\nu\rho}]_{\alpha\beta\omega} = \delta_{\eta\xi\zeta} (R^\phi_{\eta\xi} \delta_{\theta\gamma}) + R^\phi_{\eta\xi} \delta_{\theta\gamma} + R^\phi_{\eta\xi} \delta_{\theta\gamma}, \]
Then it is easy to obtain the expression
\[ \frac{1}{2} \text{Tr} \ln \hat{H}'_3 = \frac{1}{2(4\pi)^2} \int d^{2d} x \sqrt{g} \left\{ 4L_0 - \frac{1}{3} L_1 R + \sum_{i=0}^5 \sum_{l_i} l_i R M_i R + \sum_{l_i} l_i R M_i R \right\}, \]
where
\[ l_1 = -\frac{1}{8}, \quad l_2 = l_3 = -\frac{1}{2}, \quad l_4 = \frac{5}{12}, \quad l_5 = \frac{1}{2}; \]
\[ l'_1 = \frac{1}{2}, \quad l'_2 = 2, \quad l'_3 = 4, \quad l'_4 = -\frac{4}{3}, \quad l'_5 = -4. \]
By using the table of integrals (8)-(14) we find
\[ \frac{1}{2} \text{Tr} \ln \hat{H}'_3 = \frac{1}{2(4\pi)^2} \int d^4 x \sqrt{g} \left\{ 2m^2 \left( \frac{1}{\epsilon} + \frac{3}{2} \right) + \frac{m^2}{3} \left( \frac{1}{\epsilon} + 1 \right) R + R \left[ \frac{1}{30\epsilon} + k_{3,R}(a) \right] R + \frac{1}{2} C_{\mu\nu\rho} \left[ \frac{7}{30} + k_{3,W}(a) \right] C_{\mu\nu\rho} \right\}, \]
where
\[ k_{3,W}^3(a) = -\frac{44}{225} + \frac{8}{45a^2} + Y + \frac{32Y}{15a^4} - \frac{8Y}{3a^2}, \quad (93) \]
\[ k_{3,R}^3(a) = -\frac{7}{540} + \frac{1}{27a^2} + \frac{2Y}{12} + \frac{4Y}{9a^4} - \frac{2Y}{9a^2}. \quad (94) \]
Subtracting (71) from (92) we arrive at the final result
\[ \Gamma^{(1)} = -\frac{1}{2(4\pi)^2} \int d^4 x \sqrt{\mathcal{g}} \left\{ \frac{m^4}{2} \left( \frac{1}{\epsilon} + 3 \right) - \frac{m^2}{6} \left( \frac{1}{\epsilon} + 1 \right) R + R \left( \frac{1}{72\epsilon} + k^3_R \right) R + \frac{1}{2} \bar{\Gamma}^{\mu\nu\alpha\beta} \left( \frac{1}{60\epsilon} + k^3_W \right) C^{\mu\nu\alpha\beta} \right\}, \quad (95) \]
where
\[ k^3_W = k^3_{W}^3(a) = \frac{1}{150} + \frac{2}{45a^2} + \frac{8Y}{15a^4}, \quad (96) \]
\[ k^3_R = k^3_{R}^3(a) = -\frac{1}{80} + \frac{1}{108a^2} + \frac{Y}{16} + \frac{Y}{9a^4} - \frac{Y}{6a^2}. \]
In accordance to the general proof of [21], the effective action for the massive rank-3 antisymmetric tensor is exactly the same as the one for the massive scalar field minimally coupled to gravity, given by (20) with \( \xi = 0 \). There is no anomaly in the non-local part of effective action in this case.

Consider the massless limit for the form factor \( k^3_W^3(a) \) of the Weyl-squared term. Taking the \( m \to 0 \) limit in Eq. (96) we meet a non-zero contribution in the form \( \frac{1}{m^2} \ln \left( -\frac{m^2}{4\pi^2} \right) \). At the same time, for \( m = 0 \) the rank-3 antisymmetric tensor has no degrees of freedom and the result of the calculation is different. By using the methods explained in section [III] we arrive at the expression
\[ \tilde{\Gamma}^{(1)} = \frac{1}{2} \text{Tr} \ln \hat{H}_1' - \text{Tr} \ln \hat{H}_2' + \frac{3}{2} \text{Tr} \ln \hat{H}_1' - 2 \text{Tr} \ln \hat{H}_0^{\text{min}}. \quad (97) \]
Using previous results it is easy to check that the equation (97) in the massless case gives \( \tilde{\Gamma}^{(1)} = 0 \). The difference in the coefficients of the logarithmic form factors in \( k^3_W^3(a) \) of the massless limit in a massive theory and of the strictly massless case is 1/60. A similar situation holds for the factor \( k^3_R^3(a) \). In the \( m \to 0 \) limit of a massive theory the third-rank tensor the logarithmic coefficient in the form factor follows the divergent term and we find the quantum contribution \( -\frac{1}{36} \ln \left( -\frac{m^2}{4\pi^2} \right) \) for the \( R^2 \)-term. In the strictly massless case as the effective action \( \bar{\Gamma} \) vanishes and we meet no contribution. The difference between the two coefficients is 1/36 and represents the minimal scalar field contribution which do not disappear in the \( m \to 0 \) limit. This example once again demonstrates the quantum discontinuity for the massless limit.

Finally let us note that the conformal anomaly for vector and scalar massless fields can not be reproduced within the dual antisymmetric theories. The reason is that the duality takes place only in the massive theories and in the massless limits there is a discontinuity which makes reproduction of anomaly impossible. In the strictly massless theories we checked that the second-rank model does not possess the local conformal symmetry. This is a natural result due to the equivalence with a minimal (and hence non-conformal) scalar model.

V. CONCLUSIONS

By making direct calculations we have confirmed that the result of the paper [21] concerning the quantum equivalence of massive tensor fields of the second- and third-rank with vector and scalar models holds in the non-local form factors. Furthermore, one meets the discontinuity of the massless limits of quantum contributions in both cases. In fact, for the massless cases the mentioned equivalence does not hold. In particular, for the rank-3 tensor field in the massless case there is neither classical nor quantum dynamics and the theory is trivial. It would be interesting to formulate the same two types of fields on a more general backgrounds, e.g., including additional vector or axial vector fields. In these cases the proof based on \( \zeta \)-regularization may be more difficult, but there are apparently no obstacles in making explicit calculations.

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