Method and Research on Detecting Unknown Frequency Signal in Duffing Oscillator Detection Blind Zone

HUANG Ji-yao, CHEN Chang-xing, LING Yun-fei, LIN Xiang-yang, SUN Hao-xiang
(Air Force Engineering University, Basic Department, Xi’an Shaanxi 710043, CHN)

Abstract. In view of the problem of blind area in detecting weak iso-frequency signals of Duffing oscillator, an anova technique is proposed to detect the signal in blind area. By analyzing the condition of detecting blind area, the range of phase of the signal to be measured with the same frequency is obtained, and the variance calculation of the chaotic system and the signal to be measured is realized. The feasibility of the method is proved by experiments. It can also detect the existence of difference frequency signal. The results show that this method is a signal processing method with a lower threshold of SNR, which provides a basis for weak signal detection.

1. The Introduction
The measurement of weak periodic signals has a wide range of requirements in the fields of biomedicine, fault diagnosis, communication and transmission[1-3]. However, most of the traditional weak signal detection methods adopt the linear method, and there are inevitable defects in the detection of low SNR signals. Chaotic systems using nonlinear methods have strong advantages in weak signal detection and become a hot topic in weak signal detection. The chaotic class detection method is based on the nonlinear system, which uses the initial value sensitivity of chaotic system and the immunity to noise to complete the signal detection. Since it was proposed in the 1990s that chaotic vibration subsystem can be used to detect weak signals[4-6], many researchers have conducted in-depth research in this direction. Liu Haibo et al.[7] proposed a method of frequency measurement using the large period phase trajectory of the Duffing oscillator. Wu Yongfeng et al.[8] analyzed the detection effect of the two-way ring-coupled Duffing vibration subsystem. Lai Zhihui et al.[9] proposed a method for detecting the weak characteristic signal of a variable scale Duffing oscillator. Xie Tao et al.[10] studied the influence of noise on the disturbance of chaotic oscillator disposal. Qin Weiyang et al.[11] analyzed the synchronization of derivative system of Duffing equation to detect slight differences in excitation signal. Wang Yongsheng et al.[12] used chaotic phase transition to detect weak signals and established a simulation model. Li Yue et al.[13] analyzed the adaptability of chaotic systems to different types of noises. The above research results of detecting weak periodic signals based on Duffing oscillator involve theoretical analysis, simulation verification and practical application, which greatly promotes the application of this theory in signal detection under strong noise background. Chaotic vibration subclass detection method for weak signal is a nonlinear time domain signal processing technology with strong detection and anti-interference ability. Compared with the detection SNR threshold of only -10db achieved by traditional signal processing methods such as spectrum and high-order statistics[14], the detection SNR threshold of this kind of method can reach -90db[15], but there are still some problems to be solved at the present stage. Experimental simulation shows that in the critical state of chaos, the phase transition of the system may not be caused when the signal with the same
Common chaotic state determination methods are Lyapunov exponential method, Poincare cross section method, phase trajectory observation method, fractal dimension meter algorithm, spectral method, Kolmogorov entropy method, Melnikov method and so on. The calculation of Lyapunov index, fractal dimension and Kolmogorov entropy in these methods is complex. Poincare cross section method needs to select the appropriate cross section; observation method is more intuitive but the resolution is not enough; the resolution of spectral method is limited and it is difficult to distinguish chaos from long period solution. Melnikov method is an analytical method, more used in chaos threshold solution. Aimed at the problem of chaos system detection, this paper studies it from several aspects. Firstly, the theoretical basis of Duffing oscillator for detecting weak signals was analyzed and a model was established. On this basis, the relationship between the measured signal, the impulse angular frequency and the variance of the system output value is studied. The method of variance detection is proposed to detect the frequency of weak signal to be measured.

2. Weak signal detection principle and blind area of Duffing oscillator

Holmes Duffing equation is generally expressed as

\[ \ddot{x} + \mu \dot{x} - x + x^3 = F \cos(\omega t) + n(t) \]  

(1)

In this equation, \(\mu\) is the damping ratio, \(-x + x^3\) is the nonlinear restoring force, \(F \cos(\omega t)\) is the driving force term of the system, and \(n(t)\) is the noise in the conventional observation. With the \(F\) changes, the system will gradually experience chaotic state and large period state. Duffing oscillator phase track has the following features: Let’s fix the damping value \(\mu\) \(F\) gradually increased from zero to critical value, the phase diagram will continue to be chaotic state, when \(F\) gradually increase to more than critical value, the phase diagram by chaotic state into a big cycle state, the use of Duffing oscillator on the principle of weak signal detection is: \(F\) will be set to the critical value \(F_d\), when join the weak signal under test \(s(t)\), the system overall planning power will be bigger than the amplitude \(F_d\), which vary according to the state of the system to detect the signal under test, as shown in figure 1.
The figure above shows the phase space trajectories of the Duffing oscillator in a chaotic state and a large periodic state respectively. The bifurcation diagram of the Duffing oscillator can be obtained by using the fourth-order Runge-Kutta method according to the above parameters, as shown in figure 2.

The method of detecting the weak signal of the Duffing oscillator is as follows: first, the angular frequency $\omega$ in the policy dynamic term $F \cos(\omega t)$ of the Duffing oscillator is known, and it is set as $\omega_1$ with the same frequency as the signal to be measured. Then, the amplitude value $F$ was
adjusted to the critical value \( F_d \), and the signal to be tested was put into the outside world. Finally, the phase space trajectory of the system is detected whether the chaotic state enters the large periodic state or not. The method can detect signals of different frequencies by changing the frequency of the impulse term. After testing, it is more appropriate when the critical value \( F_d \) is 0.824\(^{[18]}\).

Set the signal to be measured 

\[
s(t) = A \cos(\omega t + \phi) = A \cos(\omega + \Delta \omega) t + \phi \quad (2)
\]

Where, \( \omega \), \( \omega + \Delta \omega \), when the influence of noise is not taken into account, the total driving force (synthetic driving force) of the system in equation (1) is

\[
F_d \cos(\omega t) + s(t) = F_d \cos(\omega t) + A \cos((\omega + \Delta \omega) t + \phi) = F(t) \cos(\omega t + \theta(t))
\]

Where, \( F(t) \) is the amplitude part of the synthetic driving force, \( \theta(t) \) is the initial phase part of the synthetic driving force, and the expressions are respectively

\[
F(t) = \sqrt{F_d^2 + 2F_d A \cos(\Delta \omega t + \phi) + A^2} \quad (4)
\]

\[
\theta(t) = \arctan \frac{A \sin(\Delta \omega t + \phi)}{F_d + A \cos(\Delta \omega t + \phi)} \quad (5)
\]

For weak signals, \( A \ll F_d \) is often used, so the term \( \theta(t) \) is approximately 0. Therefore, weak signal detection based on Duffing oscillator is to discuss the relationship between \( F_d \) and \( F_d \), so as to obtain the status of the system so as to realize the detection of the measured signal. Due to the existence of \( \Delta \omega \), signal detection can be divided into two cases of the same frequency or frequency difference.

When the signal to be measured is the same as the impulse frequency, i.e. \( \Delta \omega = 0 \), the impulse amplitude is

\[
F(t) = \sqrt{F_d^2 + 2F_d A \cos \varphi + A^2} \quad (6)
\]

\[
\varphi \in (0, \pi - \arccos(A/2F_d))
\]

When \( \bigcup(\pi + \arccos(A/2F_d), 2\pi) \), \( F(t) > F_d \), the phase diagram of the system will change from chaotic state to large period; However, when \( \varphi \in (\pi - \arccos(A/2F_d), \pi + \arccos(A/2F_d)) \) and \( F(t) \leq F_d \) are involved, the phase diagram of the system is still chaotic. Although the signal to be measured is added, the signal cannot be detected, that is, the signal to be measured falls into the detection blind area.

When the signal under test is different from the impulse frequency, i.e. \( \Delta \omega \neq 0 \), \( F_d - A \leq \sqrt{F_d^2 + 2F_d A \cos \varphi + A^2} \leq F_d + A \) at this time, \( F(t) \) will change within the interval \((F_d - A, F_d + A)\) and the system will present intermittent chaos. When \( \Delta \omega / \omega \leq 0.03 \) occurs, intermittent chaos can be distinguished\(^{[19]}\), so the presence of the signal to be tested can be detected.

3. A method for detecting the Duffing oscillator

The equation of state of Holmes Duffing equation is:
\[
\begin{align*}
\dot{x} &= y \\
y &= -x + x^3 - \mu x + F \cos(\omega t) + s(t) + n(t)
\end{align*}
\] (7)

Based on this equation, a method combining the phase trajectory observation method and the variance judgment method is proposed in this paper to realize the detection of weak signals under the condition of strong noise.

In this paper, weak signal detection was divided into two cases: the phase diagram of the Duffing vibration subsystem changed or not changed to study.

### 3.1. State Transition Occurs in Phase Trajectory Diagram

In chapter 2, we point out that when the signal frequency and Duffing oscillator equation under test strategy power of frequency at the same time, the state transition will happen phase diagram, the chaotic state in large cycle state, at this time to decide to enter Duffing subsystem of vibration signal frequency under test is the Duffing oscillator policy power frequency \( \omega = \omega_1 \), can achieve the purpose of weak signal detection.

### 3.2. No State Transition Occurs in The Phase Trajectory Diagram

In the existing research results, there are generally two reasons why the phase diagram of the Duffing oscillator does not change state: there is a signal to be measured that has the same frequency as the dynamic term of the Duffing oscillator, but the signal to be measured falls into a blind area and cannot be detected. It is found that when the output signal of the Duffing oscillator is in a chaotic and periodic state, the difference between the variance of the output value \( x \) and the difference between the impulse force and the frequency of the signal to be measured forms a regular peak distribution characteristic. When the frequencies of the two are consistent, the variance of the obtained \( x \) is the largest, as shown in figure 3.

![Fig. 3 Relationship between input signal Frequency and variance peak distribution](image)

In figure 3, the signal input to the Duffing oscillator under test subsystem, the output of the given in the variance and the relationship between the frequency of \( x \) is relatively obvious, laboratory set of Duffing oscillator power frequency \( \omega \) is 7 rad/s, \( \omega_1 \) signals to be measured can be get by looking for the distribution of the maximum value, get the frequency values of \( \omega_1 = 7 \) rad/s, this is consistent with the actual value, resulting in a detection blind area can be determined with the frequency signals cannot be the existence of Duffing oscillator detection signal under test.

### 4. Algorithm simulation and performance analysis

Variance of the presented method in MATLAB simulation analysis on the simulation platform, set system policy dynamic angular frequency \( \omega \) is 5 rad/s, the critical value \( F_d \) is 0.824, the external noise as gaussian white noise, signal under test for \( A \cos(\omega_1 t + \varphi) \), \( A \) is 1, \( \varphi \) take 20 rad/s, angular
frequency $\omega_1$ take 5 rad/s, $\mu$ take 0.5 fixed damping values, take 2000 sampling points, the method were used to detect signal frequency under test, as shown in figure 4. As can be seen from figure 5, the presence of the signal to be measured was successfully detected after entering the Duffing vibration subsystem through variance detection. The $x$ variance value reached the maximum value at the corresponding frequency point $\omega_1$, and was very obvious compared with other frequency values. It can be seen that under the condition of external white noise interference, the peak value of variance correctly reflects the relationship between variance value and frequency. When $\omega_1 = 5$ rad/s, $\Delta \omega = 0$, $\varphi \in (\pi - \arccos(A/2F_d), \pi + \arccos(A/2F_d))$, that is, $\varphi \in (-49.50, 55.78)$, the Duffing oscillator falls into the detection blind area. The technology proposed in this paper can detect the weak signal to be measured. It is confirmed that the presence of the signal can be detected when the Duffing oscillator falls into the detection blind area.

![Graph](image)

Fig. 4 Correlation diagram of $x$ prescription difference and frequency in blind area of same frequency detection
As shown in figure 5, the method proposed in this paper can detect not only the same frequency signal of the driving force when it falls into the detection blind area, but also the difference frequency signal.

5. Conclusion
In this paper, based on the principle of detecting weak signals of the Duffing oscillator, aimed at the problem of detecting blind spots in the same frequency signal detection of the Duffing oscillator, a technique of variance detection is proposed to detect the signals to be measured in the blind spots. When adding the signal to be measured, if the system remains in a chaotic state, the judgment of whether the signal exists or not will not be made at first, the signal to be measured and the chaotic system will be jointly tested for variance, and the presence of the signal with the same angular frequency as the policy force will prove the existence of the signal to be measured, so as not to cause misjudgment. At the same time, it can detect the existence of difference frequency signal. Simulation results show that this method has lower SNR threshold and provides a basis for weak signal detection.

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