Graph Interpolation Grammars: a rule-based approach to the incremental parsing of natural languages

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Abstract

Graph Interpolation Grammars are a declarative formalism with an operational semantics. Their goal is to emulate salient features of the human parser, and notably incrementality. The parsing process defined by GIGs incrementally builds a syntactic representation of a sentence as each successive lexeme is read. A GIG rule specifies a set of parse configurations that trigger its application and an operation to perform on a matching configuration. Rules are partly context-sensitive; furthermore, they are reversible, meaning that their operations can be undone, which allows the parsing process to be nondeterministic. These two factors confer enough expressive power to the formalism for parsing natural languages.

1 Introduction

1.1 Characteristics and rationale

A graph interpolation grammar is a grammar formalism with an operational semantics. A rule in a graph interpolation grammar specifies not only syntactic relations but also an elementary parsing operation.

Rules are lexicalized in the sense that each rule describes the combinatory properties of a lexical item. Parsing a sentence consists in matching each lexeme in the input string with a rule in the grammar and applying this rule to the current parse representation.

GIG-driven parsing was designed with incrementality and flexibility in mind, so as to emulate some features of the human parsing capability, in particular incremental processing, error tolerance, and handling of complex word orders.

In a complete model of discourse understanding, incremental parsing would be shown to work in tandem with some form of composition between partial semantic representations. It is to be thought that the collaboration between syntax and semantics in natural discourse understanding is fairly close, and that backtracking in the parser is frequently initiated by a clash found between semantic features. This report, however, will not attempt to give even the roughest idea of what semantic representations should look like and will therefore exclusively focus on syntactic phenomena, sometimes at the cost of simplifying the phenomena at hand.
1.2 Plan of the report

The first two sections give a fairly complete presentation of the concepts and processes involved in parsing with a Graph Interpolation Grammar. The syntactic structures generated when parsing with a GIG are described in Section 2, while the grammar rules and the parsing process are described in Section 3.

The next few sections apply the formalism to selected problems. Section 4 compares the handling of a simple expression language using a Context Free Grammar and a Graph Interpolation Grammar, and Section 5 analyzes Dutch Cross-Serial Dependencies.

Finally, the conclusion indicates topics for further research and related works.

2 Syntactic structures

2.1 Phrases

A phrase is made up of a head and its complements. It will be represented by a graph with a root and one labelled edge from the root to each nonroot node.

The root represents the head of the phrase, the other nodes represent its complements; and the edge labels represent grammatical functions.

2.1.1 Ordering relations

In addition, nodes in a phrase graph are connected through *ordering edges*. In the simplest case, the effect of ordering edges is to align graph nodes from left to right.

For example, the sentence

She gave me an apple.

contains a phrase the head of which is a ditransitive verb with three complements, the subject, the indirect object, and the direct object. This phrase can be represented by the following graph (in which *V* for *verb* and *NP* for *noun phrase*).

![A phrase graph](image)

**Figure 1:** A phrase graph

Since this is a case in which a total left-to-right alignment of nodes can be defined, ordering edges can be omitted and the ordering represented by the relative positions of the nodes in the diagram, as in Figure 2. Explicit orderings will not be used in this report, for a strict total ordering of phrase constituents exists in all examples given. At any rate, the possibility exists of specifying sophisticated order constraints as long as these constraints are local to a phrase.
2.1.2 Phrase head

The criteria for distinguishing a head in a phrase are both syntactic and semantic. From a syntactic point of view, the head determines the presence and functions of its complements. From a semantic point of view, the head acts as a predicate of its complements, adding semantic features to them, while the complements generally contribute to rooting these features into a specific situation.

On the basis of these criteria, an attributive adjective forms the head of a phrase made up of an adjective and a noun, such as *red leaf*. Indeed, the occurrence of the adjective implies the occurrence of a single noun, whereas the presence of the noun does not require or limit the number of attributive adjectives. On the other hand, though evidence for this type of criterion is not so easy to adduce, the adjective adds semantic features to the noun, as is shown in particular by its capacity to function as a predicate in a clause. Figure 3 illustrates a phrase headed by an attributive adjective. (*Adj* stands for *adjective.*)

![Figure 3: A modifying phrase](image)

Considering the adjective as the head of such a phrase can be counterintuitive insofar as the phrase in question functions as a noun. It is, however, necessary to distinguish the internal structure of a phrase—as described by its phrase graph—from its function with respect to other phrases. In order to bring out this distinction, a structure that interrelates phrase graphs must be defined. The next section defines a structure of this type, the *parse graph*.

2.2 Parse graphs

A parse graph is a structure in which phrase graphs are related through *parent-of* edges. Consider for example the phrase *good old days*. It contains the phrase *old days*, which has *old* as its head, but functions as a noun and is itself modified by the adjective *good* in the larger phrase *good old days*. The 'containment' relation just outlined can be rendered by drawing a *parent-of* edge between the noun node of the enclosing phrase and the head of the embedded phrase. Graphically, this can be represented as in Figure 4.

![Figure 4](image)

From now on, in parse graph representations, vertical edges will be assumed to be directed downward and represent *parent-of* edges.
On the other hand, a parent-of edge can also connect a phrase node to a lexical item. A complete parse graph for a phrase or sentence can thus be defined as a parse graph with a root node and such that every path leads to a lexical item. These facts give rise to the following principle and definition.

**Principle 1** A lexical node can occur in a parse graph as the destination of a parent-of edge.

**Definition 1** A complete parse graph is a parse graph with a root node, i.e., a node from which every node is reachable, and such that every path leads to a lexical node.

Figure 5 shows the complete parse graph for the phrase the good old days. (Det stands for determiner.)

Such a graph can be mapped to a tree in such a way that members of a phrase graph, including the head, are mapped to sibling nodes. Although the graph contains more information and is better adapted to incremental construction that its tree counterpart, it will often prove useful to apply tree terminology to parse graphs. For example, the terms descendant, ancestor, or frontier are to be understood with respect to the tree counterpart of a parse graph. Likewise, a dangling node is to be understood as a non-lexical node that occupies a terminal position in the tree counterpart of an incomplete parse graph. For further reference, here is the definition that justifies this terminology.
**Definition 2** The summary tree of a parse graph is a connected graph that contains the same set of nodes as the parse graph and such that each path of the parse graph consisting of either a parent-of edge or a parent-of edge followed by a functional edge is mapped to a parent-of edge of the summary tree.

![Figure 6: Summary tree for the parse graph on Figure](image)

### 2.3 Subtyping in node labels

The classification of lexical items can be made according to several possible criteria. For example, *the* can be categorized as an article and thus be distinguished from *this*. On the other hand, both of *the* and *this* are definite determiners and, as such, can be usefully grouped under one category. This suggests that lexical categories are usefully viewed as sets of syntactic properties, such as “determiner”, “article”, “definite”, etc. Furthermore, some of these properties are visible in ancestors of lexical items in a parse graph. For example, the presence of a definite determiner confers to a Noun Phrase the status of a definite description. A simple way of modeling this consists in labelling each node of a parse graph with a set of property names rather than an atomic name. Thus, the label of a node dominating *the* is a set containing the names “determiner”, “article”, and “definite”, and the label of an NP determined by *the*, *this*, a genitive NP, or any other definite determiner, is a set containing the name “definite”.

For simplicity, most of the graphs and rules given as examples have atomic node labels, even when some node types could be shown to subsume or share properties with other symbols. However, Section 4.2 shows how multi-valued node labels can contribute to the expressive power of a grammar.

### 3 Building parse graphs

Parse graphs, as just defined, can be incrementally built by scanning an input flow of lexemes and performing a building step as each lexeme is read. A building step is formally described by a *composition rule*, which adds to the graph under construction a subgraph associated with the current lexeme.
3.1 Form of a composition rule

Suppose that (i) the phrase being parsed is *a robin*, (ii) somehow (see Section 3.3), the parse graph on Figure 7 was built on encountering the determiner *a*, and (iii) the current lexeme is *robin*.

![Figure 7: An NP context](image)

The presence of the dangling —i.e. childless— noun node to the right of the previous lexeme, namely *a*, materializes the anticipation of a noun. Singular common nouns are generally inserted in just such a context, and so we can define a rule for the lexeme *robin* that stipulates (i) that the linear successor of the previous lexeme (i.e. *a*) in the parse graph should be a dangling noun node, (ii) that the new subgraph to integrate consists of a noun node dominating the lexeme *robin*, and (iii) that this new subgraph is to be substituted for the dangling noun node. (A graphic representation of this rule is given on Figure 8, in section 3.2.)

More generally, let the context be the parse graph obtained by parsing the input string up to the current lexeme, a composition rule consists of:

1. a pattern to match against the context, known as the context pattern, and
2. a subgraph to integrate into the context, known as the addendum.

Or, more formally:

**Definition 3** A composition rule is a pair made up of a context pattern and an addendum.

This definition in turn relies on the definitions of a context pattern and an addendum. The following definition of a context pattern is somewhat simplified and will be enriched later (Definition 11, Section 3.11).

**Definition 4** A context pattern is a parse graph with one distinguished node called the context anchor and an indication of whether the context anchor is an ancestor or immediate linear successor of the previous lexeme.

To proceed in depth-first order, here is the definition that explains the term linear successor.

**Definition 5** When all phrase constituents in a parse graph are strictly ordered from left to right, a linear order over the parse graph is given by the left-to-right ordering of nodes in the frontier of the summary tree (Definition 4, Section 2.3). It is therefore a partial ordering of parse graph nodes, under which only terminal nodes of the summary tree are comparable.
The context pattern indicates what the parse graph obtained so far must look like in order for the rule to be applicable. A context matches a context pattern if it is a supergraph of the pattern and it contains a node whose location and label match the anchor specifications.

Section 3.7.2 on free word order, examines the consequences of having several possible linearizations of the parse graph. This has computational implications but does not impact the above definitions in any essential way.

**Definition 6** An **addendum** is a parse graph with one or two distinguished nodes called addendum anchors and exactly one lexical node.

A rule is found applicable when the current lexeme matches the addendum lexeme and the context matches the context pattern. The conjunction of context and current lexeme forms a **parse configuration**.

**Definition 7** A **parse configuration** is a pair made up of a context and a current lexeme.

Beside a parse configuration, what a rule specifies is an operation to modify this configuration. This operation, called an **interpolation**, is specified by means of the context anchor and the addendum.

When the addendum contains a single anchor, such as the noun node in our example, the interpolation is performed by substituting the addendum anchor for the context anchor. This particular type of interpolation will sometimes be called an **insertion**, by opposition to a **proper interpolation**, which is characterized by the presence of two distinct anchors in the addendum.

When the addendum contains two anchors, they delimit a path that is substituted for the context anchor. Since a single anchor can be viewed as a path of length 0, this case subsumes insertion.

No formal definition of interpolation will be attempted at present. Let it simply be clear that interpolation involves in fact two operations:

1. the substitution of a path of the addendum for the context anchor,
2. the disjoint union of the addendum with the context.

These notions will be treated in more detail in Section 3.7.1.

### 3.2 Insertion example

An example of insertion is the example given informally above. The rule can be represented as on Figure 8.

In a graphic representation of a composition rule, the context pattern and the addendum appear on either side of an implies (⇒) sign, with the addendum on the right; and the anchoring specifications are materialized by placing a mark next to each anchor. This mark is an asterisk (*) next to an addendum anchor. Next to the context anchor, it is a double down arrow (⇓) or a left arrow (←) according as the context anchor is an ancestor or an immediate successor of the previous lexeme. The notion of **immediate successor** is defined with respect to the linear order indicated through
Applying the insertion rule on Figure 8 to the parse graph on Figure 7 results in the parse graph on Figure 9.

Figure 9: Result of applying the above insertion rule

3.3 Proper interpolation example

Figure 10 illustrates path-to-node anchoring. The composition rule interpolates a prepositional phrase into a noun phrase to produce a qualified noun phrase as in a bird on a tree. (PP stands for Prepositional Phrase and Prep for preposition.)

Figure 10: A proper interpolation rule

The sign next to the context anchor indicates that it is to be found above the previous lexeme rather than immediately after it. The diagram on Figure 11 shows the context to which this rule could apply and the resulting context after its application. An application diagram follows the following pattern.

context + addendum ⇒ new context
3.4 Anchor matching

The preceding examples have shown that the context anchor can be an ancestor or an immediate linear successor of the previous lexeme. The first case occurs typically with a proper interpolation, and the second with an insertion. This is, of course, just the typical case; Section 3.7.2 gives general criteria for locating the context anchor.

### 3.4.1 Matching an immediate successor anchor

No attempt to deal with free word order will be made until Section 3.7.2. Until then, we can rely on a strict left-to-right ordering of all phrase constituents. Therefore, the previous lexeme always has a unique immediate linear successor, which is its immediate successor in the frontier of the summary tree (Definition 2, Section 2.2).

### 3.4.2 Matching an ancestor anchor

Ancestor positions are also defined with respect to the summary tree of the parse graph (Definition 2, Section 2.2). On the other hand, for a node to match an ancestor anchor there is an additional condition beside location and label match. It can be stated as follows.

**Principle 2** A context anchor in ancestor position can only be matched by the root of a complete parse graph (see Definition 3, Section 2.2).

For example, an NP cannot be postmodified if it is incomplete, i.e. if the frontier of its summary tree contains nonlexical nodes.

### 3.4.3 Incidence of node subtyping on anchor matching

If node labels are sets of symbols, label matching requires that all symbols attached to the context anchor be present in the node to match. For example, a context pattern could contain the information that the anchor is a definite determiner, and any node that has at least the type atoms “definite” and “determiner” will satisfy label matching.

In other words, for a node to match a context anchor, its type must be a subtype of the type of the context anchor. On the other hand, this means that the formulation of a rule, and notably the addendum, must involve a special device to mention the
full type of the node that matched the anchor, so as not to lose information. (Indeed, a rule is allowed to change the type of the anchor, as explained in Section 3.5 and illustrated on Figure 13.) Accordingly, the special symbol $\ell$ can be used to assign to an addendum anchor the exact type of the node matching the context anchor.

An example of a grammar that uses subtyping is given in Section 4.2.

### 3.5 Parser initialization

To begin with, there is no context or, rather, the context consists of a single node with a generic label that subsumes all phrases with which an utterance is likely to start$^1$. No lexeme has been integrated to the context yet; but, for technical purposes, the initial node can be considered to “follow the previous lexeme”. In other words, a rule with an immediate-successor (←) mark next to the context anchor will match the initial context.

Suppose a sentence begins with the phrase *a robin*; then, the initial lexeme, *a*, can be integrated using the following insertion rule.

![Figure 12: Insertion rule for *a*](image)

This rule simply means that *a* can be considered as the beginning of a Noun Phrase. It does not say that *a* could be the beginning of a sentence. Yet, most utterances are sentences; so it would seem that this rule is generally inappropriate. It is not, however, if one considers that the definition of an interpolation that was (informally) given in Section 3.1 does not require the two ends of an interpolation path to bear the same label. And, indeed, should a phrase like *a robin* be followed with a verb, such as *flew*, the rule shown on Figure 13 can apply to make the root of the parse graph an $S$-node. ($Dir$ stands for “directional phrase”.)

![Figure 13: Interpolation rule for *flew*](image)

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$^1$Label subsumption is discussed in Section 2.3.
3.6 Backtracking

Matching a rule often involves making a prediction on the continuation of the utterance. For example, the rule on Figure 13 predicts that a directional phrase follows the verb *flew*. When several predictions are possible, as will often be the case, a trial-and-error approach is adopted.

A prediction conflict appears when several rules are available for a given context and current lexeme (i.e. a given parse configuration). It is supposed that the rules associated with each lexeme are totally ordered by priority, and each rule is tried in priority order until a successful parse is obtained.

For example, when the current lexeme is *flew* and an NP node dominates the previous lexeme, there could be two rules, with higher priority on the rule on Figure 13, and lower priority on the following rule, which makes *flew* an intransitive verb.

![Figure 14: An alternative rule for *flew*](image)

Given these two rules, the sentence *a robin flew swiftly* would first be mapped to the parse graph on Figure 15; then, on realizing that, by the end of the sentence, the parse graph is still incomplete, one would have to reconsider the choices made. Supposing there is a single analysis available for *swiftly*, the parser would then backtrack on *flew* and try the second rule, which would eventually lead to a successful parse.

![Figure 15: Graph built before backtracking](image)

Backtracking involves undoing some of the rules applied so far. This supposes that track is kept of the list of rules applied, and, more importantly, that each rule is reversible. As a matter of fact, rules, as defined, are reversible (owing to Principle 4).

If the selection process parallels a similar process in the human mind, a simple hypothesis would correlate rule priority with statistical efficiency; alternately, or complementarily, there could be structural criteria that correlated priority with simplicity or canonicity.
Section 3.7.1. In order to undo the effect of a rule, the addendum is to be removed and the interpolation path replaced back by the context anchor. If rules were not reversible, then, it would be necessary to keep track of at least the last few parse configurations.

Of course, backtracking does not always take place after reading the whole input string. It takes place whenever no further rule can be triggered and the parse graph is incomplete.

For another example, consider the sentence *he gave trouble to us* and suppose that the highest-priority rule for *gave* is the following. (The edge label *iObj* stands for “indirect object”.)

![Figure 16: Default rule for gave](image)

Then, on encountering *to*, no rule will match the context (in which an object NP is expected), thus leading the parser to backtrack on *trouble*, then *gave*, and select the following rule, which will eventually lead to a successful parse. (A *to*-Prep is a category that contains only the preposition *to* — and *unto* in archaic dialects.)

![Figure 17: Alternate rule for gave](image)

3.7 Interpolation and anchor matching

This section revisits concepts introduced in Section 3.1, giving more precise formulations and adding details that were initially overlooked.

3.7.1 Addendum incorporation

In all the examples given so far, an insertion consists in replacing a dangling node with a node with a child and a proper interpolation consists in interpolating a path whose first edge is a parent-of edge, so that the destination of the interpolation path dominates its source.
With interpolations of this type, merging an addendum into the context is a fairly straightforward process, which has not been explicated so far, but only illustrated through examples.

However, the concept of interpolation allows more flexibility than has been hitherto suggested, and, in order to consider further varieties of interpolation, it is important to bring out the graph invariants implicitly used and preserved during addendum incorporation.

**Invariant 1** *In a phrase graph, there cannot be distinct edges with the same label.*

**Invariant 2** *In a parse graph, any node except the root has exactly one incoming edge.*

**Invariant 3** *In a parse graph, there cannot be more than one parent-of edge from any node.*

According to invariant 2, an interpolation path can be defined as the unique oriented path that connects two addendum anchors. With respect to this path, there is a *source anchor* and a *destination anchor*. Invariant 2 also implies that the source anchor inherits the incoming edge of the context anchor as a result of an interpolation.

Furthermore, when one looks at the way the outgoing edges of the context anchor are distributed between the addendum anchors of an interpolation, invariant 3 can be seen to prevail in that the *parent-of* edge, in the examples seen so far, is transmitted to the destination anchor. The underlying principle can be stated as follows.

**Principle 3** *All edges of the context anchor are inherited by the source of the interpolation path except for edges present in the addendum, which are shifted to the destination of the interpolation path.*

Principle 3 itself presupposes that the addendum and the context form disjoint graphs. This principle is well worth spelling out, for it allows rules to be easily reversible in the sense of Section 3.6.

**Principle 4** *In a composition rule, the sets of nodes of the addendum and the context have an empty intersection.*

Indeed, if the graph union that is specified by a rule is a disjoint union, then, no information beyond the list of rules applied is necessary to undo the effect of these rules.

On the other hand, Principle 3 presupposes that, if the context anchor is a head, then, the addendum anchors must both be capable of inheriting functional edges, i.e. be phrase heads as well. Conversely, if the context anchor is not a head, substituting a head for it would create a second head within one phrase, which is ruled out by definition. These considerations can be summarized by the following principle.

**Principle 5** *The head status of the context anchor must be matched by the addendum anchors.*

This principle does not directly govern addendum incorporation, for it has to be built into the grammar rules. It is especially useful as a prerequisite of Principle 3.
Nonstandard interpolation  To see how Principle 3 applies to a nonstandard case of proper interpolation, i.e. a case in which the interpolation path does not start with a parent-of edge, consider the rule on Figure 18. (The box represents an empty category, or gap; the symbol that-Cnj informally denotes a subcategory whose main representative is the conjunction that.)

Nonstandard insertion  A standard insertion substitutes a node with an outgoing parent-of edge for a dangling node. But it is also conceivable to substitute a node with arbitrary nonredundant outgoing edges for a node with descendants.

A practical use of nonstandard interpolation could be the handling of optional complements. For example, supposing the rule that handles the by-complement of a

Figure 18: A nonstandard case of proper interpolation

Although the context anchor, which is a verb, is a dangling node, an empty object NP (i.e. an object gap) has been predicted, typically owing to the presence of a preceding object pronoun, i.e. a relative or interrogative pronoun, or a topicalized object NP. The occurrence of a verb that governs a sentence rather an object NP pushes the predicted object gap further. Since the object gap does not appear in the addendum (owing to Principle 4), its position in the graph after the rule has applied is not specified explicitly, but can be inferred from Principle 3. Indeed, since the object edge from the context anchor is redundant with the object edge that emanates from the source anchor of the addendum, it is inherited by the destination anchor. Therefore, after addendum incorporation, the context will contain the following subgraph.

Figure 19: Subgraph obtained after addendum incorporation
passive were the one represented on FigurepassiveAgent.fig, there would be no necessity to anticipate the presence of a by-complement on encountering a passive, and so no backtracking would be involved to handle its optionality.

3.7.2 Anchor matching

The principles assumed so far for matching the context anchor have to be stated explicitly and possibly extended; this is necessary, on the one hand, to predict the anchor position for any arbitrary interpolation, including a nonstandard one, and, on the other hand, to cater for variations in word order.

Section 3.4 specified two possible positions for the context anchor, as ancestor or immediate linear successor of the previous lexeme. But no principle to find out whether a given interpolation should take place at an ancestor or successor position has yet been stated. As a matter of fact, a simple principle underlies the examples given so far.

**Principle 6** If the addendum lexeme linearly follows either endpoint of the interpolation path, the context anchor is an ancestor of the previous lexeme. In all other cases, the context anchor is the immediate linear successor of the previous lexeme.

This principle is analogous to Principle 2 in that it prevents the occurrence of dangling nodes among the linear predecessors of the current lexeme. So it is a practical consequence of a more general principle.

**Principle 7** A valid interpolation cannot leave or create dangling nodes that linearly precede the lexeme it integrates.

Another consequence of Principle 7 on the form of rules is the following.

**Principle 8** An addendum cannot contain dangling nodes that linearly precede the anchor that inherits the parent-of edge of the context anchor.

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3The directional complement of the verb fly, used to illustrate backtracking in Section 3.6, could be handled in this way, unless there were good reasons to distinguish the two constructions from the point of view of lexical semantics.
Under strict word order, the form of rules can be controlled through Principles 8 and 6 to enforce Principle 7. When alternative orderings are allowed, the matching process itself may have to take this principle into account, for rules may have to admit addenda that *may* violate Principle 8 or 7 under conditions that can only be checked dynamically.

To see Principle 6 in application, compare the rule on Figure 21, which represents a premodification, to the rule on Figure 22, which represents a postmodification.

```
N -> N
  
  Adj -> N
    
    red
```

Figure 21: Premodification by the adjective *red*

```
V -> V
  
  Adv <- V
    
    swiftly
```

Figure 22: Postmodification by the adverb *swiftly*

It is now apparent that word order plays a key role in anchor choice during rule design and in anchor matching during rule application. So it is interesting to see what extensions to the principles just brought out are necessary in order to accommodate greater freedom in word order than has been presupposed up till now.

**Complex phrase ordering** Under a strict ordering of phrase nodes, a parse graph can be linearized by considering the frontier of the summary tree (Definition 2, Section 2.2). Then, in a given parse configuration, the previous lexeme has a unique immediate successor. However, when several possible orders are allowed, there may be more than one linear successor to the previous lexeme.

Consequently, several context nodes may be found to match the context anchor of an applicable rule. This introduces a degree of indeterminacy which is compounded with the indeterminacy resulting from rule conflicts (i.e. the availability of several matching rules for a given lexeme).

Not only does this type of indeterminacy arise in the presence of free word order, but it is also liable to arise in a fixed-order framework if several nodes along a parent-of path are allowed to bear the same label. Indeed, this situation could result in several nodes
matching an ancestor anchor; and a principle that ruled it out would be no more than an ad hoc feature that reduced the formalism’s expressive power without reducing its indeterminacy in any essential way. Therefore, indeterminacy due to anchor matching has to be handled somehow.

Once again, a trial-and-error approach seems to be the natural path to follow. The mechanism used to solve rule conflicts can be applied here too, provided competing anchor matches are somehow ordered by priority. As far as ancestor conflicts are concerned, proximity to the previous lexeme (i.e. depth) seems to correlate simply and naturally with priority. As far as successor conflict is concerned, however, it would seem that no natural solution is available unless, even when the placement of nodes in a phrase graph is relatively free, there is an underlying preferred order.

In addition to the possibility of finding multiple anchor matches, free word order also complicates the process of identifying context anchors, for context nodes do not stand in a simple topological relation with respect to the previous lexeme. From an algorithmic point of view, the solution seems to consist in updating the set of successors of the previous lexeme after each interpolation, as described in Procedure 1, Appendix A.

3.8 Coreference links

Incremental left-to-right parsing, as outlined, forms phrases exclusively from adjacent lexemes. However, immediate syntactic relations between distant constituents do exist in natural languages. Interrogative and relative clauses in English provide such examples. For example, in the following sentence, the pronoun who somehow functions as the subject of to win, although these two constituents occupy distant positions in the input stream.

Who do you want to win?

This difficulty will be solved through a new classic device in syntactic theory, namely coindexing of phrases. This consists in linking coreferential phrases together, typically by assigning to them equal referent indices. In the above example, the interrogative pronoun who and the empty subject of to win (whose presence is concretized by the unacceptability of Who d’you wanna win? as a spoken realization) are coindexed.

In fact, in order to reconcile the fact that the parsing model under discussion can only establish immediate syntactic relations between neighbouring constituents and the fact that various ordering constraints or rhetorical phenomena may separate phrase constituents, we will rely on coreference links. A coreference link is somewhat more general than NP coindexing is that it may apply to any part of speech rather than just noun phrases.

Nodes related by a coreference links have just one counterpart in the semantic interpretation to be derived from the parse graph. This means that all members of a coreference chain denote the same object in the universe of discourse. Syntactically, this entails that members of a coreference chain of NPs share gender and number characteristics, and members of a coreference chain of verbs share complements. (For example, one member will have a subject, another member no subject but a direct object, and a third an indirect object, thus forming three instances of a single verb with subject, object, and indirect object.)
Many phenomena in natural languages suggest that the human parser prohibits to a large extent the splitting of a phrase across distant lexemes but uses coindexing to handle the sharing of constituents between phrases and rhetorical disruptions to standard phrasal order.

Such disruptions are typically attenuated by the use of resumptive pronouns in spoken French, as in the following example.

Il est bizarre, ton chapeau.
It is weird, [is] your hat.

Here, the main verb finds its subject where the parser expects it, but this subject is only a pronoun coreferenced with the “real” subject, which, for intonational effect, is relegated to a secondary tone unit so as to allow full emphasis on the predicate.

When no overt pronoun shows the existence of a coreference chain, it is sometimes necessary to postulate the existence of empty lexemes, as in the interrogative clause cited above.

In Section 3, an analysis of Dutch cross serial dependencies will be attempted in terms of covert coreference chains.

The introduction of coreference links gives rise to the following definitions.

**Definition 8** An augmented parse graph is a parse graph augmented with coreference links.

**Definition 9** A coreference link is an undirected edge that relates two nodes standing in an equivalence relation. Links that can be inferred, such as the link from any node to itself, do not need to appear in the representation of an augmented parse graph. The equivalence relations represented in an augmented parse graph are coreferentiality relations.

**Definition 10** A coreferentiality is an equivalence relation over parse graph nodes bearing a specified label. Coreferentialities found useful are NP coreferentiality and verb coreferentiality.

This defines a graph hierarchy, with phrase graphs at the bottom, and augmented parse graphs at the top.

### 3.9 Multiple interpolation

Coreference links make it meaningful to define parallel interpolations operating simultaneously on several context anchors. Multiple interpolation seems useful only when the context anchors are located with respect to a coreference chain. Figure 23 shows an example of a double interpolation.

This example is purely formal, but concrete examples will be given in Section 3 on Dutch Cross-Serial Dependencies. Nonetheless, it illustrates an important feature of multiple interpolation: namely, the fact that the rule, albeit multiple, adds a single lexeme to the parse graph. Typically, it anticipates a Y-gap coreferential with this
lexeme. Gaps do not count as lexemes in so far as they are not physically present in the input string.

Furthermore, the formulation of multiple interpolation makes it necessary to define a third possible location for the context anchor, namely linear successor, denoted by a double left arrow \( (\Leftarrow) \), as opposed to immediate linear successor, denoted by a simple left arrow \( (\leftarrow) \).

3.10 Implicit reflexive-transitive closure in patterns

The definition of coreferentiality as an equivalence relation (Definition 10) yields somewhat unsuspected power to the occurrence of coreferential links in context patterns.

To see that, consider the example of adjectival definite descriptions in English, as found in the wealthy, the influential, etc. Leaving semantic features aside, one important syntactic requirement for forming such NPs is that the determiner be definite. This is captured by the rule on Figure 24. \( (\text{Def} \text{ stands for “definite determiner”}.) \)

Coreference links are created because the structure being built is a modification structure, not because of the presence of a noun gap\(^4\). The modification examples given so far did not contain coreference links simply because extended parse graphs had not been defined.

This rule requires that the noun node that is to be replaced by an adjective and an empty noun be immediately preceded by a definite determiner. Therefore, it will

\(4\)This “gap” does not play a pronominal role, but rather represents a semantically empty noun. A precise notation would include the name Pro in the label of “real” gaps, such as those used in Section \(3\).
not apply to a noun which is itself premodified by an adjective, as in *the idle rich*. On the other hand, one cannot devise a rule covering specifically a string of the form *Def Adj Adj*, for the premodification can be iterated, as in *the outrageous idle rich*. Consider therefore the rule on Figure 25.

```
Def        det
   N       N
⇒          *
        * N
     subj
Adj
wealthy
```

Figure 25: Comprehensive rule for adjectival definite descriptions

This rule allows the context anchor to be separated from the NP head by any number of premodification stages, including zero. For example, the last interpolation to be applied when parsing *the outrageous idle rich* can be represented by the diagram on 26

```
Def
the
NP
⇒
Def
the
NP
```

Figure 26: Last interpolation performed when parsing *the outrageous idle rich*

The existence of rules in which an anchor is related to the rest of the context by a coreference link leads to the following definition of a context pattern.

**Definition 11** A context pattern is a subgraph of an augmented parse graph with one or several distinguished nodes called anchors. Anchor location with respect to the previous lexeme takes one of the three values ancestor, immediate successor, and successor.

---

5This diagram represents a derivation step, as explained in Section 4.1.2.
4 A simple expression grammar

To show the specificities of Graph Interpolation Grammars, even in the absence of context-sensitivity in rules, this section applies them to the classical problem of a simple arithmetic expression language with two precedence levels and left associativity.

4.1 Deriving an GIG from a CFG

This section will derive as simply as possible a Graph Interpolation Grammar from the Context Free Grammar given in Figure 27. This kind of derivation could conceivably be performed automatically in order to benefit from the incrementality of GIG-driven parsing for existing context-free grammars. For example, in a language-based editor, the incremental building of parse representations as each token is read could be used to implement syntax-sensitive editing functions.

\[
\begin{align*}
E & \rightarrow E + T \\
E & \rightarrow T \\
T & \rightarrow T * F \\
T & \rightarrow F \\
F & \rightarrow ( E ) \\
F & \rightarrow 0 \\
F & \rightarrow 1
\end{align*}
\]

Figure 27: Context-Free Grammar for a simple expression language

4.1.1 Numbers

Moving from a CFG to a GIG involves lexicalization. This means that every rule is to be matched by a lexeme and contains a representation of the contexts in which this lexeme is likely to occur. As far as numbers are concerned, they can function either as factors, terms, or expressions. Therefore, each number will have three associated rules. Figure 28 shows the three rules associated with the number 0.

\[
\begin{align*}
\leftarrow E & \Rightarrow * E \\
\leftarrow T & \Rightarrow * T \\
\leftarrow F & \Rightarrow * F
\end{align*}
\]

Figure 28: GIG rules associated with the lexeme 0
4.1.2 Sum

A sum is identified on scanning the lexeme +. By representing a sum as an interpolation into an $E$, the fact that + is the lowest-priority operator is captured. Accordingly, the rule for + is as represented on Figure 29.

Since the addendum lexeme follows the destination of the interpolation path, i.e. the first operand of the sum, the context anchor is stipulated to be an ancestor of the previous lexeme (Principle 6).

The fact that the second operand is a term ($T$) rather than an expression ($E$) forces left associativity. This can be verified by unwinding the GIG derivation for the string $0 + 0 + 0$. This derivation, up to the insertion of the last zero, is represented on Figure 30.

A step in a GIG derivation consists in the marking of a context anchor in the result of the preceding step, and a representation of the parse graph as modified by the interpolation applied.

4.1.3 Product

The rule for a product is entirely analogous to the rule for a sum, except that it is allowed to apply to a term rather than an expression. This fact gives it precedence
over sum, for essentially the same reason as in the original context-free grammar.

Figure 31 shows the rule for the product operator, and Figure 32 shows the derivation for \( 1 + 1 \times 0 \) up to the insertion of the lexeme 0. It illustrates the relative priorities of sum and product.

\[
\text{Figure 31: The rule for } \times
\]

\[
\text{Figure 32: The derivation for } 1 + 1\times 4
\]

4.1.4 Parentheses

A rule for a left parenthesis is an insertion that announces an expression and a right parenthesis in a context where an expression, a term, or a factor could be expected. As a right parenthesis is not supposed to occur unless it has been announced by a left parenthesis, its rule is a simple lexical insertion. Accordingly, only the rules for the left parenthesis are shown on Figure 33.

Note that nonterminals \( \text{RPar} \) and \( \text{Rpar} \) are necessary according to principle 1, Section 2.2.

Applying these rules to the parsing of \((1 + 1) \times 0\), one obtains the derivation whose main steps are shown on Figure 34.
4.2 A more idiomatic approach

The existence of three rules to insert a number or a left parenthesis is the translation of a CFG idiom for handling precedence. The native GIG device for solving this type of problem relies on node subtyping.

For the expression language considered, we can have tree type atoms, $N$ (for number), $P$ (for product), and $S$ (for sum). They form two hierarchies that meet at the universal type $N,P,S$, as shown on Figure 33.

If we ignore preterminal labels (such as *Sum* or *LPar*), an $N,P,S$ matches any anchor; so it is an expression in a broad sense. It occurs typically as the initial parse graph and between parentheses. An $N,P$ is any expression except a sum, so it occurs as the right operand of a sum to enforce left association. Likewise, an $N$ is exclusively a number (or a parenthesized expression), such as occurs as the right operand of a product. As far as the right-hand branch of the hierarchy is concerned, a $P,S$ can occur as the left operand of a product or sum, while an $S$ can occur only as the left operand of a sum. These enforce the relative precedence of sum and product.

4.2.1 Numbers

There is a single rule for inserting a number. It requires the atom $N$ in the context anchor and does not modify its type. The symbol $\&$ that is used as the label of the addendum anchor indicates that this label is inherited as is from the node matching the context anchor. The rule for zero is represented on figure 36.

4.2.2 Sum

The rule for the operator $+$ is represented on Figure 37.

A sum can be interpolated only at a node whose type contains the type atom $S$. To guard against the interpolation of a product on top of a sum, the type $S$ is assigned to the root of the addendum. To guard against right association of additions, the type $N,P$ is assigned to the right operand. The left operand cannot be the object of further interpolations and simply inherits the type of the node matching the context anchor.

Figure 38 derives $0 + 0 + 0$ up to the last insertion to show that left associativity is enforced.
Figure 34: Main steps in the derivation for $(1 + 1) \times 0$
Figure 35: Type hierarchy for the expression grammar

Figure 36: The rule for zero

Figure 37: The rule for +

Figure 38: The derivation for 0 + 0+
When the second occurrence of + is encountered, only one of the ancestors of the previous lexeme has a type which contains S.

4.2.3 Product

The rule for the operator * exactly parallels the rule for the sum operator. It requires the anchor type to contain the atom P, assigns the type P,S to the addendum root, and assigns the type N to the right operand. It is represented on Figure 39.

```
\[ \psi_P \Rightarrow *_{P,S} \]
```

\[ \star \quad \text{Product} \quad \Rightarrow \quad N \]

Figure 39: The rule for the product operator

The fact that the root node of the addendum is a P,S rather than an S gives priority to multiplication over addition. Figure 40 outlines the derivations for 1 + 1 and 0 * 1.

```
\[ \leftarrow N,P,S \Rightarrow * \quad S \]
```

\[ \begin{array}{c}
N,P,S \\
\text{Sum} \\
1 + 1
\end{array} \quad \Rightarrow \quad N,P \\
\text{Product} \quad \Rightarrow \quad N \\
0 \quad \Rightarrow \quad 1
\]

Figure 40: Derivation outlines for 1 + 1 and 0 * 1

If the current lexeme after parsing 1 + 1 is *, only the closest ancestor of the previous lexeme will have a matching type. On the other hand, if the current lexeme after parsing 0 * 1 is a plus, only the root of the parse graph will have a matching type.

4.2.4 Parentheses

The rule for the left parenthesis is given on Figure 41.

Once it has applied, only an insertion at the linear successor of the left parenthesis is possible (due to Principle 2). This could be the insertion of a number or a left parenthesis. So, the supertype N as phrase head would also work, but no particular constraint is necessary here, which is why the most permissive type is indicated. The same remark holds for the initial context. In fact, the type N,P,S makes the grammar more open to future evolutions, but could be replaced by N without impairing its operation.
In Dutch, a few conjunctions, such as dat (*that*) or omdat (*because*), introduce a clause in which the verb comes after its object NP. Now, with perceptual and causative verbs, i.e. verbs which syntactically take both an NP object and an infinitive clause object, a double order prevails: all object NPs occur before the finite verb, in order of increasing nesting level, as in German for instance; and infinitive verbs occur after the finite verb in order of increasing nesting level, which reflects a right-branching construction for clausal objects, quite unlike German, in which the verbs occur in order of decreasing nesting level, thus inviting a recursive analysis.

In several papers on DCSDs, examples revolve around hippopotami. The example given below, taken from [Ren94], does not depart from this tradition. The indices relate each verb to its preceding NP object.

... dat ik Henk₁ haar₂ de nilparden₃ zag₁ helpen₂ voeren₃
... that I Henk₁ her₂ the hippopotamus₃ saw₁ help₂ feed₃
“that I saw Henk help her feed the hippopotamus”

The analysis that will be attempted is based on the principle that, when a verb in a dat-*clause* has an NP object and a clause object, the NP object is constrained to occur before the verb, and the clause object is constrained to occur immediately after it.

In itself, however, the existence of different positional requirements on the NP object and the clausal indirect object would not seem to require a special syntactic device when the ditransitive verb governs an ordinary transitive verb. Thus, in the light of the principle just outlined, one could expect the following incorrect construction to occur.

* ... dat ik haar zag de nilparden voeren.

In fact, there seems to be an additional constraint that amounts to forbidding any object in a dat-*clause* to occur before the finite verb. So the correct construction is instead as follows.

... dat ik haar₁ de nilparden₂ zag₁ voeren₂.

To recapitulate, the constraints at work to produce this construction are the following.

1. The NP object of any verb in a dat-*clause* or a clause nested therein occurs before it to form an ‘inverted’ verb phrase.
2. The clausal indirect object of a perceptual or causative verb, which is an infinitive clause with a subject gap, occurs after it.

3. In a dat-clause, no direct NP object, whatever the nesting level at which it occurs, occurs before the finite verb.

Now, the conjunction of the third constraint with the existence of a double order on the construction of perceptual and causative verbs in dat-clauses produces a rift in the construction, as a result of which a ditransitive verb may be separated from its preceding direct object by a theoretically arbitrary distance. In order to govern two complements across a rift, a verb, in the general framework adopted here, is in fact realized as two graph nodes, one of which is a verb gap. To see that, the representation below shows the phenomenon with one level of nesting. Verb gaps are represented by square boxes.

... dat ik haar \( \Box_1 \) de nijlpaarden \( \Box_2 \) zag_1 voeren_2

This representation postulates that haar and de nijlpaarden occur in inverted VPs headed by verb gaps and that the verb gaps are coindexed with lexical verbs occurring in a right-branching construction governed by the finite verb.

If extended one additional nesting level, the representation that is obtained is the following.

... dat ik Henk \( \Box_1 \) haar \( \Box_2 \) de nijlpaarden \( \Box_3 \) zag_1 helpen_2 voeren_3

Furthermore, each non-finite lexical verb heads a clause whose subject is a gap. Each such NP gap is coindexed with the NP object of the superordinate verb. The coreference pattern for NPs is the following.

... dat ik Henk_1 haar_2 de nijlpaarden zag \( \Box_1 \) helpen \( \Box_2 \) voeren

In order to capture these hypotheses as GIG rules, one will have to consider three cases: (i) a dat-clause headed by a perception or causative verb, (ii) a non-finite clause with an inverted VP whose head is a perception or causative verb, and (iii) a non-finite clause with an inverted VP whose head is a monotransitive verb.

5.1 Complex dat-clause

When the conjunction dat is encountered, one generally has no way of guessing that its verb will impose a cross-serial construction. So the rule on Figure 42 will be used only on backtracking after realizing that no rule can integrate the second object NP haar if the verb that is expected is an ordinary transitive verb.

The inverted verb phrase (split-inv-VP) that heads the sentence is split into an instance that governs the direct object (inv-VP) and an instance that governs the clausal indirect object (iObj-VP). Since neither of these half-instances qualifies as a head, an empty coordination conjunction is postulated to relate them. This empty conjunction represents the barrier between the left-branching construction for NPs and the right-branching construction for verbs.

Given our example, the lexemes whose occurrence is anticipated by this rule are (i) the subject of the split-inv-VP, namely ik, (ii) the direct object of its inv-VP
instance, namely Henk, (iii) the complex-V, namely zag, and (iv) the verb of its clausal object, namely helpen, which is labelled as nonfin-V.

The labels given to nodes here are more precise than required by the operation of the rules. But this precision hopefully helps legibility.

5.2 Complex nonfinite clause

Just as in the case of the previous rule, the decision to trigger the rule for an embedded complex verb cannot be taken the first time its object is encountered, but only after backtracking on the following NP.

The rule represented on Figure 42 is a double interpolation which acts simultaneously on both instances of a split VP. A relation of coordination is supposed between the inverted verb phrases on the left (top interpolation), for there is no clause hierarchy among them. The use of double interpolation on two graph areas connected by a coreference link allows these areas to lie at an arbitrary distance from each other.

Supposing the lexemes ik and Henk were inserted into the output of the rule represented on Figure 42, the effect of the last rule would be to produce the graph on Figure 43.

5.3 Simple nonfinite clause

With a simple transitive verb, the verb gap that is created is simply coreferenced to its counterpart in the context; so the bottom interpolation does not create any node. The presence of an article in the context specified on Figure 45 announces a transitive verb; and so no rollbacking need be involved to trigger the rule that is represented.
This rule adds an NP node to the left of the “syntactic rift”; this node is the object of an inv-VP containing an anticipatory verb gap coindexed with the second context anchor, which itself anticipates the occurrence of the verb voeren to the right of the rift. In short, the top interpolation adds a direct object, and the bottom interpolation connects it to the cascade of clausal objects after zag.

6 Conclusion

This report has defined a form of grammar, called Graph Interpolation Grammar, which produces parse graphs by simple derivation from an initial graph consisting of one node. Since derivation integrates input lexemes left-to-right, a GIG derivation is an incremental parse of an input sentence.

The basic operation, graph interpolation, which combines path substitution and disjoint graph union, has been described at some length. Two classic problems, one from the field of parsing (arithmetic expressions), and one from the field of linguistics (Dutch cross-serial dependencies), have been selected to illustrate problem-solving through GIG design.

The essential, hopefully, has been stated, but much remains to be done to improve the formalism and explore its potentialities. Areas for further research are presented in Section 6.2.

On the other hand, the GIG model shares intuitions with several other syntactic formalisms, which are briefly reviewed in Section 6.1.
Figure 44: The output of the rule on Figure 43.
6.1 Related works

Graph interpolation can be viewed as an extension of tree adjunction to parse graphs. And, indeed, TAGs \[\text{LT75}\], by introducing a 2-dimensional formalism into computational linguistics, have made a decisive step towards designing a syntactic theory that is both computationally tractable and linguistically realistic. In this respect, it is an obligatory reference for any syntactic theory intent on satisfying these criteria.

The basic intuition in GIGs, however, that of incrementally connecting lexemes by looking up their combinatory properties, can be found in link grammars \[\text{ST95}\]. Link grammars are mathematically interesting, but do not easily interface with a semantic component.

Categorial grammars \[\text{Ste86, Ste88}\] have a similar rationale. They use binary combinators on function types that represent grammatical categories. The parse structures generated, which are binary trees reflecting the order of combinator application, can be somewhat counterintuitive from a semantic point of view.

In Lexical Functional Grammars \[\text{Bre85}\], grammatical functions are loosely coupled with phrase structure, which seems to be just the opposite of what is done in a GIG, in which functional edges are part of the phrase structure. Nonetheless, these two approaches share the concern of bringing out a functional structure, even if much of what enters into an f-structure (i.e. a functional structure) in LFG is to be addressed by the semantic component — a topic for further research — in GIG.

6.2 Further research
6.2.1 Linguistic phenomena

Important categories of phenomena to analyze include ellipsis, notably under coordination, and word scrambling.

Furthermore, the discussion of free word order in this report has hovered at an uncomfortable level of abstraction. It is necessary to return to this issue with concrete examples.

On the other hand, the interface with a scanner and morphological analyzer has yet to be stated explicitly. Phenomena to account for include idioms, phrasal lexical items, agglutinative morphology, and scanning ambiguities.

6.2.2 Psycholinguistic considerations

The main natural feature embodied in Graph Interpolation Grammars is incremental left-to-right parsing, which is expected to provide a basis for incremental semantic evaluation of discourse, a feature in favour of which there seems to be conclusive experimental evidence.

Processing time The formalism would be particularly satisfactory as a model of human parsing if the computational complexity it predicts correlated with observed processing time in humans. What is mainly at stake here is the plausibility of the backtracking mechanism that has been described.

Garden-path sentences GIG parsing involves backtracking for sentences that humans do not identify as garden-path sentences. A hypothesis to test against further evidence is that the human analyzer is garden-pathed only when semantic interpretation has to be revised, not when purely structural adjustments are taking place. A thorough discussion of this question requires that a working model be proposed for the semantic component.

Error tolerance The capacity to pick up meaning from ill-formed utterances is a capability of the human language processor that does not seem beyond the reach of GIG-based modelling. The model could involve some form of partial match of context patterns to resort to when the parser is stymied.

Evolutionary aspect Languages undergo idiolectal variations and historical changes. A constant adjustment of rules takes place when language is used. If GIGs were endowed with the error recovery device delineated in the previous paragraph, a model of evolution could then be worked out to promote heavily activated near-matches to full matches, possibly at the detriment of competing weakly activated full matches, which could disappear in the process.

6.2.3 Mathematical properties

Expressive power GIGs are at least as expressive as TAGs, for graph interpolation can be used to express tree adjunction or tree substitution. This means that the formalism is at least mildly context-sensitive. Furthermore, multiple interpolation adds expressive power comparable to that found in Multi-Component TAGs [KJ87].
GIGs do seem to provide the “right” amount of context-sensitiveness, but this remains to be precisely quantified.

**Complexity** The backtracking mechanism induces a worst-case exponential complexity. On realistic grammars, however, both the base of the exponential —i.e. the number of lexemes past which it makes sense to backtrack— and the exponent —i.e. the number of competing rules for a given lexeme— seem to have very low upper bounds. What is a “realistic” grammar could be characterized precisely by determining such bounds by collecting linguistic facts. Beside quantified bounds, it is to be thought that some barrier effect makes it pointless to backtrack beyond certain syntactic barriers. On the other hand, the backtracking mechanism itself could be reconsidered to reduce its computational complexity, for example by allowing backtracking to skip back to a privileged place either through links built while parsing or using a form of pattern matching.

**Automatic generation of GIG rules** Section 4 suggests that a Graph Interpolation Grammar could be automatically derived from a Context Free Grammar, mostly for the purpose of using existing LR grammars.

The feasibility of converting automatically from other formalisms, and particularly Lexical Functional Grammars, seems worth exploring as well.

**Linearized syntax for rules** From a practical point of view, however, GIGs are not to be seen as a low-level formalism to be generated automatically from higher-level formalisms. But, for GIG writing to be practical, what could be needed is (i) a linearized syntax to type rules quickly rather than draw graphs, and (ii) a WYSIWYG tool that showed incrementally the graphic aspect of what is being typed. (This tool could of course be implemented on top of a GIG engine.)

A linearized syntax could involve a frame-like notation in which an augmented parse graph were a set of slots with a phrases slot that pointed (via slot values) to phrases with an optional pre-defined parent slot and user-defined functional slots.

For example, a linearized version of the addendum in Figure 25 could look like the frame expression on Figure 46. (Predefined slot names are boldfaced. Slot references are slash-separated paths starting at the parent of the slot being defined. A double dot (..) moves up one level in the slot hierarchy.)

On the other hand, many grammar designers would probably not feel frustrated by the lack of a linear syntax given an efficient graphic tool.

**References**

[Bre85] Joan Bresnan. *The mental representation of grammatical relations*. MIT Press series on cognitive theory and mental representation. MIT Press, 1985. Collected articles.

[JLT75] A. K. Joshi, L. S. Levy, and M. Takahashi. Tree adjunct grammars. *Journal of Computer and System Sciences*, 10:136–63, 1975.
addendum {
  phrases {
    0 { head N; }; 
    1 {
      parent ../0/head;
      head Adj;
      subj N;
    }; 
    2 {
      parent ../1/subj;
      head Gap;
    }; 
  }
  lexeme {
    parent ../phrases/1/head;
    spelling "wealthy";
  }
  anchors {
    source ../phrases/0;
    destination source;
  }
  coreferences {
    0 {
      0 ../../phrases/0/head;
      1 ../../phrases/1/subj;
    }; 
  }
}

Figure 46: A linearized addendum
A Anchor location procedures

The following procedure can be used to update the set of possible linear successors of the current lexeme after an interpolation. It is operational even when several linear orders are allowed.

Procedure 1

1. Based on ordering relations in the addendum, one identifies the set of possible linear successors of the current lexeme, which is here the lexeme just inserted.
2. The possible linear successors of the previous lexeme, as recorded when it was inserted, minus the node that matched the context anchor, are added to the set found in step 1.

Note that, since the addendum and the context are disjoint, the second step cannot add potential successors already added by the first step. This observation is important to determine whether rule application remains reversible in the presence of free word order. Indeed, undoing a rule with respect to the set of potential successors of the current lexeme can be done as follows.

Procedure 2

1. Based on ordering relations in the addendum, one identifies a subset of possible linear successors to the current lexeme, i.e. the lexeme on which to backtrack, and subtracts this subset from the set of possible successors.
2. Once the insertion itself has been undone, the context anchor for this insertion is added to the set obtained in the previous step, and this set becomes the set of potential successors of the previous lexeme.