Dynamics of a relativistic velocity collisionless ionization wave in an applied magnetic field

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Using simulations, we demonstrate that an applied 100 T-level magnetic field can restrict the expansion of a relativistic, high energy density plasma into a surrounding neutral gas. Without any applied magnetic field, an initial plasma filament launches a sustained, soliton-like collisionless ionization wave. This ionization wave traps hot electrons and can enable the rapid transport of a significant fraction of the original filament energy over hundreds of microns, in contrast with plasma expansion into vacuum. We find that the introduction of an applied magnetic field causes the ionization wave to lose energy, allowing the plasma expansion to be terminated by experimentally relevant magnetic field strengths. Using 1D particle-in-cell simulations, we demonstrate that the stopping of the ionization wave in an applied magnetic field is well-predicted by tracking the evolution of the ratio of thermal to magnetic pressure at the ionization wave front.

I. INTRODUCTION

The development of high peak power lasers 1 enables laser-produced plasma to access high energy density (HED) regimes with relativistic electron temperatures. Such plasmas are fundamental to application areas including inertial confinement fusion 2, laboratory astrophysics 3, and energetic particle 4, 5 and radiation 6, 7 sources. The addition or self-generation of strong magnetic fields in laser-produced HED plasma enables a host of novel magnetization-related phenomena, for example in the areas of magnetic reconnection 8, direct laser acceleration 9, 10, hot electron transport 11, 12, and ion acceleration 13, 14, 15, 16.

The expansion of a plasma filament with hot electrons and cold ions into vacuum is a fundamental and much-studied process 17, 18. Although this process is able to generate a population of high energy ions, the energy density carried by the expanding plasma quickly becomes small relative to the initial energy density of hot electrons.

In contrast, the expansion of an HED plasma filament into a neutral gas environment has recently been demonstrated to transport energy efficiently away from the filament surface via a propagating ionization wave 19. The ionization wave has been observed to form when the plasma filament has high energy density and relativistic electron temperature, under which conditions the sheath electric field created at the filament surface is sufficiently strong to ionize the neutral gas 19. As we will demonstrate in this work, this ionization wave can dramatically enhance the ability of hot electrons to transport energy away from the original plasma filament relative to the case of expansion into vacuum.

Magnetic field generation approaches capable of delivering static fields on the order of 100 T or more have recently been developed 20–27, motivated in part by the possibility of restricting energetic electron transport under high energy density conditions 28. While the effect of magnetic fields on the expansion of a plasma filament into a vacuum has been extensively studied 29–32, until now it has not been known how a magnetic field will affect plasma expansion into a neutral gas mediated by an ionization wave.

In this work, we evaluate the effect of a 100 T-level applied magnetic field on the transport of energy away from an initial plasma filament via an ionization wave. The addition of an applied magnetic field could affect not only the propagation and energy transport of the ionization wave but also the formation of the initial plasma filament and the initial launching of the ionization wave. A self-consistent, multidimensional investigation of the entire process of plasma generation, ionization wave formation, and ionization wave propagation is ultimately needed to characterize fully how an applied magnetic field can affect energy transport from a plasma filament in a neutral gas environment. However, in this manuscript, as a first step we consider the effect of an applied magnetic field on ionization wave propagation alone in one-dimensional planar geometry.

In order to isolate the effect of the magnetic field on the ionization wave propagation, we separate the simulation domain into non-magnetized and magnetized regions. The plasma filament is initialized in the non-magnetized region, which allows the ionization wave to form without interference from the applied magnetic field. The ionization wave then propagates into the magnetized region, where we investigate the effect of the magnetic field on its propagation.

We treat the initial spatially varying magnetic field profile as being formed by currents we do not directly
simulate, i.e. external field coils. We incorporate the role of these currents in creating this initially stationary magnetic field profile by allowing this ‘external magnetic field’ to act on particles but not to contribute to the time evolution of the electric field. In this way, we are able to isolate the effect of the magnetic field on the ionization wave propagation.

The outline of this paper is as follows. In Section II, we highlight important properties of the ionization wave propagation and energy transport in the absence of a magnetic field. In Section III, we evaluate ionization wave propagation in a magnetic field in terms of simple physical scales. In Section IV, we demonstrate that the instantaneous energy density carried by the ionization wave is crucial to its propagation and express succinctly the condition for an ionization wave to propagate in a magnetic field. In Section V, we summarize and discuss potential extensions to this work.

II. IONIZATION WAVE PROPAGATION IN NON-MAGNETIZED PLASMA

The expansion of a high energy density plasma filament into a surrounding neutral gas differs substantially from the expansion of a hot plasma into vacuum. While we will discuss the effect a magnetic field can have on restricting this expansion in Sections III and IV, in this Section we will first summarize the salient details of the expansion process in a non-magnetized case.

A sufficiently hot and dense plasma filament embedded in a neutral gas expands rapidly via ionization at the filament surface leading to the formation of a propagating ionization wave\textsuperscript{19}. We model the formation of the ionization wave and its propagation using Cartesian 1D simulations with the open source particle-in-cell (PIC) code EPOCH\textsuperscript{33}.

Ionization is modeled using the Monte Carlo ionization module in EPOCH\textsuperscript{33}. In each time step, the probabilistic ionization rate is calculated based on tunneling and barrier suppression ionization theory. The ionization module is discussed in more detail in Ref. 33.

The simulation setup is illustrated schematically in Fig. 1(a). We consider the expansion of an initial hydrogen plasma filament into a neutral hydrogen gas. The filament consists of hot electrons and cold ions extending over the region $|x| < 100 \mu\text{m}$. The plasma electrons are initialized with a water-bag momentum distribution with cutoffs at $p_x = \pm m_e c$. This choice of distribution and momentum cutoff roughly approximates the hot electron energy spectrum with temperature of a few hundred keV observed in experiments where ionization waves have been previously observed\textsuperscript{19}. The filament size is sufficiently large for the formation of the ionization wave not to depend on its size. The density was chosen to be representative of high energy density experiments where ionization waves have been observed in the past\textsuperscript{19}. Additional simulation parameters are given in Table I. In

![Graph](image-url)

FIG. 1: Ionization wave formation and propagation in non-magnetized case. (a) Schematic of simulation set up and soliton formation. (b) Electron phase space. The grey dash-dotted line denotes the average momentum of the original electrons inside the soliton, which agrees well with the speed of the ionization wave ($\approx 0.15c$). (c) Electric field in the soliton propagation ($x$) direction. (d) Electron density, normalized by the initial electron density. Figures (b)-(d) are snapshots at $t = 2$ ps.
**Plasma Parameters**

| Parameter                          | Value          |
|-----------------------------------|----------------|
| Plasma density \(n_0\)            | \(3.3 \times 10^{19} \text{ cm}^{-3}\) |
| Plasma location \(|x| < 100 \mu\text{m}\) |                |
| Electron momentum distribution \(|x| < 100 \mu\text{m}\) | Water-bag (cutoff: \(p_x = \pm m_e c\)) |

**Hydrogen gas Parameters**

| Parameter                          | Value          |
|-----------------------------------|----------------|
| Hydrogen density                  | \(3.3 \times 10^{19} \text{ cm}^{-3}\) |
| Hydrogen location \(|x| > 100 \mu\text{m}\) |                |

**Simulation Parameters**

| Parameter                          | Value          |
|-----------------------------------|----------------|
| Simulation domain                 | [-600 \mu\text{m}, 600 \mu\text{m}] |
| Cells/micron                      | 100/\mu\text{m} |
| Particles/cell                    | 200            |

| TABLE I: Parameters for 1D PIC simulation. The plasma ions are initialized cold. The plasma is treated as collisionless. |

all the scenarios we will consider, the plasma and any additional elements are symmetric about \(x = 0\). For convenience, we will discuss the evolution of the plasma filament in the half-space where \(x > 0\).

As the plasma expands, the sheath electric field at the edge of the expanding plasma filament ionizes the surrounding gas, creating a propagating ionization wave (Fig. 1). We henceforth call the electrons born by ionization ‘generated electrons’ in order to distinguish from the ‘original electrons’ initialized within the plasma filament.

As shown in Figs. 1(a),(c), the ionization wave forms a soliton-like structure consisting of a sheath electric field and a trapping electric field. This soliton structure travels with a relativistic speed (\(\approx 0.15 c\)) in excess of tens of picoseconds and propagates for hundreds of microns (Fig. 2(b)). Over the course of this propagation, the amplitude and spacing of the sheath field and the trapping field are maintained (Fig. 2(a)).

The underlying physics behind this long-lasting propagation can be explained by the combined effects of the sheath field and the trapping field\(^{19}\). The original electrons moving towards the soliton front (\(p_x \sim 0.1 m_e c\), Fig. 1(b)) create the strong sheath electric field which is responsible for the ionization. The generated electrons are born with negligible energy in this sheath field. As they are accelerated into the plasma by the sheath field, their density drops (Fig. 1(d)), creating an excess of ion charge behind the soliton ionization front. This creates a trapping field maintaining the original electron momentum distribution shown in Fig. 1(b). The trapping field can prevent the original electrons from leaving the soliton by keeping them bouncing between the trapping field and the sheath field. The two create a potential well which carries the original electrons along with the soliton, keeping the energy density inside the soliton unchanged. This in turn maintains the electric field structure at the same magnitude (Fig. 2(a)), enabling long-lasting propagation (Fig. 2(b)).

(1) Longitudinal electric field driving ionization wave propagation into neutral gas. (b) Kinetic energy density of the original electrons for plasma expansion into neutral gas. The purple dashed line represents the location of the plasma-neutral boundary. (c) Kinetic energy density of the original electrons for plasma expansion into vacuum. The black dashed lines are electron density contours (\(n_e/n_0\)). The red dashed line represents the location of the front of ions accelerated from the filament surface. The kinetic energy density in (b)-(c) is normalized by the initial kinetic energy density of the original electrons \(\epsilon_{k0}\).
Due to this field structure, plasma expansion in a neutral gas environment can carry substantial energy away with the propagating soliton. In contrast, plasma expansion in vacuum is much less efficient at transporting energy away from the initial plasma filament (Fig. 2(c)). In the case of plasma expansion in vacuum, it takes more than 10 ps for 30% of the initial energy within the plasma filament to be transported outside its initial volume, while in the case of the same plasma filament expanding in neutral gas, approximately the same amount of energy is carried away by the soliton in only 1.5 ps.

The enhanced energy transport and non-stopping ionization wave propagation is a potential concern in situations where plasma expansion is undesirable. We next investigate the effect of a magnetic field on the ionization wave propagation. The magnetic field can rotate the original electron momentum, which could create the possibility of reducing the sheath field and altering the wave propagation.

III. IONIZATION WAVE PROPAGATION IN A MAGNETIC FIELD

In this Section, we introduce a \( z \)-directed magnetic field to restrict the ionization wave propagation. Electromagnetic wave propagation in magnetized plasma features cutoffs defining regions of parameter space in which certain modes are able or unable to propagate. One possibility for ionization wave propagation is that it could follow similar criteria for propagation. There could be, for instance, a critical magnetic field above which the ionization wave cannot penetrate, analogous to the critical density for laser penetration in plasma. To investigate this possibility, we conduct a simulation with the same set up as in the previous Section except that we introduce a magnetic field beyond \( x = 200 \) \( \mu \)m. Using this approach, we study the effect of the magnetic field on the same ionization wave soliton as in Section II.

We consider two scale lengths that could set a threshold for the hypothetical critical magnetic field: the overall soliton size \( L \) and the width of the hot electron sheath \( l \) formed at the front of the soliton, as illustrated in Fig. 1(c). The magnetic field introduces an additional scale length associated with the gyroradius: \( \rho_e \equiv c p_{\perp} / |e|B \sim \varepsilon_e / |e|B \), where \( \varepsilon_e \) is the characteristic, relativistic kinetic energy of the original electrons in the soliton. If the gyroradius is smaller than the soliton size, then the original electrons are unable to make a full transit of the soliton structure. This could affect the ionization wave as original electrons at the back of the soliton would not be able to reach the front, making them unable to contribute to the sheath field. The corresponding condition of \( \rho_e \lesssim L \) requires a magnetic field of \( B_z \gtrsim 60 \) \( T \). Another possibility is that the gyroradius can become comparable to the sheath scale \( l \), which occurs at a much higher magnetic field. The sheath scale is related to the Debye length of the original electrons in the soliton.

![FIG. 3: Ionization wave propagation with external magnetic field \( B_{ext} = f(x) \), as given in Eq. (1).](image)

(a) Kinetic energy density of the original electrons \( \epsilon_k \) normalized by their initial kinetic energy density \( \epsilon_{k0} \).

(b) Total magnetic field \( B_z = B_{ext} + B_{plasma} \).

(c) Snapshots of the total magnetic field at the initial time \( t = 0 \), where \( B_z = B_{ext} \), and during soliton propagation at \( t = 10 \) ps, where \( B_z = B_{ext} + B_{plasma} \).
by \( l \sim \lambda_{De} \equiv \sqrt{\varepsilon_e/4\pi n_e e^2} \), where \( n_e \) denotes the density of electrons inside the soliton. If \( n_e \lesssim \lambda_{De} \), then the motion of electrons within the sheath may be restricted, which could also affect the formation of the sheath field. Estimation of the Debye length of the original electrons inside the soliton in the non-magnetized case shows that \( n_e \sim \lambda_{De} \) will require a magnetic field of \( B_z \gtrsim 600 \) T.

To determine whether one of these scales sets a critical magnetic field value for the wave to stop, we set an initial linearly increasing externally applied magnetic field profile \( B_{\text{ext}} \) beyond \( x = 200 \) \( \mu \)m.

\[
B_{\text{ext}} [T] = f(x) = \frac{2}{3}(x \, [\mu m] - 200) \quad (1)
\]

As we can see in Fig. 3(a), the ionization wave initially penetrates into the external magnetic field region but eventually comes to a stop. At the end of the propagation, the soliton structure is completely destroyed. The stopping point is \( x \sim 600 \) \( \mu \)m, which corresponds to \( B_{\text{ext}} \sim 300 \) T. This field value is well above the critical magnetic field value where the gyroradius is comparable to the overall soliton size as it enters the magnetized region (\( n_e \sim L \) at \( B_{\text{ext}} \sim 60 \) T), which suggests that this does not set the condition for the ionization wave to propagate in magnetized region. Additionally, as shown in Figs. 3(b)-(c), the external magnetic field is screened out at the ionization front and is excluded from both the soliton bulk and the region swept over by the soliton. However, the magnetic field at which the soliton stops is also less than the value where the gyroradius becomes comparable to the sheath size (\( n_e \sim l \) at \( B_{\text{ext}} \sim 600 \) T).

We next evaluate whether \( B_{\text{ext}} \approx 300 \) T represents a critical magnetic field beyond which the ionization wave cannot propagate. To this end, we consider a second case in which we alter the external magnetic field profile by increasing the magnetic field gradient by a factor of 6. That is, we set the external magnetic field as \( B_{\text{ext}} [T] = 4(x \, [\mu m] - 200) \quad \text{beyond} \quad x = 200 \) \( \mu \)m. Surprisingly, the ionization wave stops at \( B_{\text{ext}} \approx 360 \) T rather than \( B_{\text{ext}} \approx 300 \) T. We observe that as we increase the gradient of the magnetic field, the ionization wave actually stops at a higher magnetic field, which is contrary to the idea that there is one critical magnetic field value for the ionization wave to stop. This raises the question: what condition has to be satisfied for the ionization wave to stop?

IV. THE STOPPING OF AN IONIZATION WAVE IN A MAGNETIC FIELD

In the previous Section, we studied the ionization wave propagation in the magnetized region by increasing the gradient of the external magnetic field. The ionization wave stopped at a higher magnetic field in the higher-gradient case. However, even if we keep increasing the gradient until we get a sharp step-function-like magnetic field boundary, it seems unlikely that the ionization wave would be able to penetrate into an external magnetic field applied for \( x > 200 \) \( \mu \)m (orange dashed line in (a)-(b)).
field with any amplitude. We test this by setting a sharp rise to an external magnetic field of 600 T beyond \( x = 200 \mu m \). As shown in Fig. 4(a), the ionization wave cannot penetrate into the 600 T magnetic field region. The soliton structure is destroyed at the magnetic field boundary and cannot propagate further.

We find that the ionization wave is unable to penetrate into an external magnetic field greater than approximately 600 T. \( B_{ext} \) = 600 T corresponds to \( \rho_e / \lambda_{De} \approx 1 \). The ratio \( \rho_e / \lambda_{De} \) is linked to several important properties of the soliton sheath. First, when \( \rho_e / \lambda_{De} \lesssim 1 \), the original electrons are unable to transit the whole sheath size (\( l \approx \lambda_{De} \)) and are instead constrained by the gyroradius \( \rho_e \), which could affect the sheath formation. Second, \( \rho_e / \lambda_{De} \) at the ionization front is also linked to the ratio of thermal pressure to magnetic pressure, \( \beta_e \), by

\[
\frac{\rho_e}{\lambda_{De}} \sim \sqrt{\beta_e},
\]

where \( \beta_e \equiv 8\pi \epsilon_k / B^2 \), where \( \epsilon_k \) is the electron kinetic energy density.

We examine the ratio \( \rho_e / \lambda_{De} \) and the effect of the balance between thermal and magnetic pressure in simulations by the ‘effective beta’ \( \beta_{eff} \),

\[
\beta_{eff} = \frac{8\pi \langle \epsilon_k \rangle}{B_{ext}^2}.
\]

where \( \langle \epsilon_k \rangle \) denotes the average kinetic energy density of original electrons inside the soliton. For convenience, we evaluate \( \beta_{eff} \) using \( B_{ext} \). The plasma-generated magnetic field at the soliton front is relatively small compared to the external background field (\( B_{\text{plasma}} \ll B_{ext} \)).

We expect the magnetic field to significantly affect the ionization wave if the Larmor radius is smaller than the soliton sheath and the pressure exerted by the external magnetic field exceeds the thermal pressure of electrons in the soliton, i.e. if \( \beta_{eff} < 1 \). As a result, one might expect that we need an external magnetic field of at least 600 T to stop the ionization wave. However, this contradicts our earlier observation that the ionization wave stops at a weaker magnetic field in the cases discussed in Section III with the magnetic field gradient.

In order to reconcile these observations, we investigate the ionization wave propagation in a weaker uniform magnetic field. To ensure the ionization wave can fully penetrate into the magnetized region, we set an external magnetic field with a sharp rise to 200 T beyond \( x = 200 \mu m \), which initially results in \( \beta_{eff} \approx 9 \). The sharp magnetic field transition does not substantially disturb the soliton and the total energy in soliton is reduced by less than 10% during the transit into the magnetized region. Although the soliton initially propagates into the magnetized region, we find that the ionization wave is eventually stopped after propagating for 300 \( \mu m \) in the magnetized region (Fig. 4(b)). This indicates that even weak fields (\( \beta_{eff} > 1 \) initially) are capable of stopping the ionization wave. The question we need to ask is: how is the ionization wave stopped in a weak magnetic field?

When we previously considered magnetic fields below 600 T to have \( \beta_{eff} > 1 \), we made the assumption that the energy density of the electrons in the soliton does not change. This assumption is valid for a soliton traveling in a non-magnetized region. However, the energy density inside the soliton actually decreases during its propagation in a magnetized region.

The applied magnetic field can perturb the electrons inside the soliton by rotating the electron momentum from the \( x \)-direction to the \( y \)-direction, inducing currents in the \( y \)-direction. These currents introduce energy loss by converting electron kinetic energy to the emission of an \( x \)-propagating electromagnetic wave. Figures 3(c) and 4(c) show the \( +x \)-propagating electromagnetic wave which is launched from the front of the ionization wave. The electromagnetic wave emission begins when the soliton enters the magnetized region and continues until the ionization wave stops. As the ionization wave penetrates into the magnetic field of \( B_{ext} = 200 \) T, 60% of the original soliton energy is carried away by this electromagnetic wave emission.

The remainder of the soliton energy is carried away by the original electrons as they become less energetic through this process. These electrons are no longer energetic enough to keep up with the ionization wave and are left behind. The division of the original soliton energy into these two channels is shown in Fig. 5 and is discussed in more detail in Appendix A.

The energy density inside the soliton thereby decreases during its penetration in the magnetic field (Fig. 6(a)), which causes \( \beta_{eff} \) to drop (Fig. 6(b)). In addition to an external magnetic field of \( B_{ext} = 200 \) T, we consider the

\[
\text{FIG. 5: Time evolution of the total energy inside the soliton with an external magnetic field of 200 T.}
\]

\( t_0 = 2.7 \) ps is the time when soliton fully enters the \( B_{ext} \neq 0 \) region. The energy is normalized by \( \epsilon_0 \), the total energy in the soliton before it reaches the magnetic field boundary. The pink dashed line is the total magnetic field energy the soliton displaces from the magnetized region. This is the same case shown in Figs. 4(b)-(c). The details of how the energy is calculated are given in Appendix A.
evolution of $\beta_{\text{eff}}$ as the same ionization wave penetrates into external magnetic fields of 250 T, 300 T and 350 T beyond $x = 200 \, \mu m$. As shown in Fig. 6, although initially these four cases all have $\beta_{\text{eff}} > 1$ which allows the ionization wave to penetrate in the magnetized region, over the course of propagation, the plasma $\beta_{\text{eff}}$ drops to $\beta_{\text{eff}} \sim 1$, at which point the ionization wave stops (Fig. 6(b)). This indicates that the soliton can still be stopped even when initially $\beta_{\text{eff}} > 1$. Therefore, we clarify that $\beta_{\text{eff}} > 1$ is the instantaneous condition for the ionization wave to propagate in a magnetic field.

V. SUMMARY AND DISCUSSION

We demonstrated that the expansion of a high energy density plasma filament via ionization wave propagation into a surrounding neutral gas can be terminated by the application of a magnetic field. We found that the ability of the soliton-like ionization wave to propagate in the presence of a magnetic field depends on the instantaneous comparison of the scale lengths or pressures associated with the hot electrons trapped in the soliton structure and the applied magnetic field. The condition for the ionization wave to propagate can be conveniently summarized as $\beta_{\text{eff}} > 1$, where $\beta_{\text{eff}}$ is the ratio of the thermal pressure carried by the hot electrons in the soliton structure to the magnetic pressure of the applied magnetic field. We have additionally demonstrated that $\beta_{\text{eff}} > 1$ is an instantaneous condition and that the applied magnetic field introduces energy loss which can eventually reduce $\beta_{\text{eff}}$ and cause the ionization wave to stop. This reduction in $\beta_{\text{eff}}$ allows sub-500 T magnetic fields, such as are now experimentally available\textsuperscript{20–27}, to terminate plasma expansion.

In this work, we studied ionization wave propagation using an initially non-uniform externally applied magnetic field comprising two regions. We allowed the initial filament and formation of the ionization wave to occur in the non-magnetized region, followed by propagation of the ionization wave into the magnetized region. The initial magnetic field profile was assumed to be generated by currents not modeled in the simulation. While this configuration allowed us to determine the condition for the ionization wave to propagate, in the future, fully self-consistent modeling of the plasma filament formation and ionization wave launch should be performed to evaluate how the application of the magnetic field might impact these processes.

The impact of the magnetic field in higher dimensional geometry also remains to be seen. We conducted 1D Cartesian simulations in this work, which capture certain features of the ionization wave formation and propagation which are also seen in higher dimensional simulations. Ionization wave formation and propagation has previously been demonstrated in 2D geometry, which more closely matches the cylindrical filaments expected in experiments\textsuperscript{19}. In 2D, the expanding, initially cylindrical plasma filaments into a finger-like ionization wave structure. Each filament maintains a high electron energy density, enabling long-lasting propagation analogous to the 1D case. Although we expect the magnetic field to be screened out from the bulk of each filament, the rotation it introduces at the ionization front may introduce rotation of the filaments, which may affect the overall plasma expansion.

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Appendix A: Energy accounting of ionization wave propagation in magnetic field

As we illustrated in Section IV, the energy lost by the soliton is partly converted to electromagnetic wave emission and partly carried away by particle loss (See Fig. 5 in Section IV). In this Appendix, we demonstrate that the change in the soliton energy is entirely accounted for by these two energy loss mechanisms.

Figure. 7 is a schematic of the soliton propagation in the magnetized region. We consider a system comprising part of the simulation domain \((0 < x < x_b)\). The right boundary \(x_b = 550 \mu m\) is chosen to ensure that the soliton cannot leave the system. The front and the back of the soliton structure are respectively denoted by \(x_{\text{max}}(t)\) and \(x_{\text{min}}(t)\). For convenience, we divide the system into three parts. Region 1 \((0 < x < x_{\text{min}}(t))\) is the region behind the soliton. Region 2 \((x_{\text{min}}(t) < x < x_{\text{max}}(t))\) is the region inside the soliton. Region 3 \((x_{\text{max}}(t) < x < x_b)\) is the region in front of the soliton. We additionally consider energy transport out of the system at the boundary \(x = x_b\). The simulation is symmetric about \(x = 0\) and no energy is gained or lost through this boundary.

The energy per unit area inside the soliton \(\varepsilon_2\) is calculated as

\[
\varepsilon_2(t) = \sum_\alpha \int_{x_{\text{min}}(t)}^{x_{\text{max}}(t)} \epsilon_\alpha \, dx + \int_{x_{\text{min}}(t)}^{x_{\text{max}}(t)} \frac{E^2 + B^2}{8\pi} \, dx. \quad (A1)
\]

The first term is the kinetic energy per unit area of all particles inside the soliton, where \(\epsilon_\alpha\) stands for the kinetic energy density of particle species \(\alpha\). The second term is the field energy per unit area, where \(B = B_{\text{ext}} + B_{\text{plasma}}\) is the total magnetic field.

We similarly calculate the energy per unit area in regions 1 and 3,

\[
\varepsilon_1(t) = \sum_\alpha \int_0^{x_{\text{min}}(t)} \epsilon_\alpha \, dx + \int_0^{x_{\text{min}}(t)} \frac{E^2 + B^2}{8\pi} \, dx, \quad (A2)
\]

\[
\varepsilon_3(t) = \int_{x_{\text{max}}(t)}^{x_b} \frac{E^2 + B^2}{8\pi} \, dx. \quad (A3)
\]

The particle kinetic energy in front of the soliton (region 3) is negligible and is safely excluded from this calculation.

The energy density carried out of the boundary \(x = x_b\) by the Poynting flux is

\[
\varepsilon_{\text{out}}(t) = \int_{t_0}^t \frac{c}{4\pi} (E \times B) \cdot \hat{x} \, dt, \quad (A4)
\]

where \(t_0\) is the time at which the soliton fully enters the magnetized region, and \(\hat{x}\) is the unit vector normal to the simulation boundary.

The conservation of energy in our system written in terms of the energy density inside the soliton is

\[
\varepsilon_2(t) - \varepsilon_2(t_0) = -\left[\varepsilon_1(t) - \varepsilon_1(t_0)\right] - \left[\varepsilon_{\text{out}}(t) + \varepsilon_3(t) - \varepsilon_3(t_0)\right] = -\varepsilon_{\text{pl}} - \varepsilon_{\text{EM}}, \quad (A5)
\]

where we identify the sources of energy loss in the soliton (the terms in square brackets) with particle loss, \(\varepsilon_{\text{pl}}\), and electromagnetic wave emission, \(\varepsilon_{\text{EM}}\), at \(x = x_{\text{min}}\) and \(x = x_{\text{max}}\).

For comparison with Fig. 5, we normalize Eq. (A5) by \(\varepsilon_2(t_0)\), which gives

\[
\tilde{\varepsilon}_2(t) + \tilde{\varepsilon}_{\text{pl}}(t) + \tilde{\varepsilon}_{\text{EM}}(t) = 1, \quad (A6)
\]

where \(\tilde{\varepsilon}_2\) shows the evolution of normalized soliton energy. The left hand side of Eq. (A6) is shown in Fig. 8 and is indeed close to 1. Therefore, Eq. (A6) gives the energy balance during ionization wave propagation in the magnetic field.

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FIG. 8: Illustration of energy balance associated with ionization wave penetration into the magnetized region (see Eq. (A6)). The energy in the soliton ($\tilde{\epsilon}_2$) is well accounted for by the sum of energy carried out by particle loss ($\tilde{\epsilon}_{pl}$) and the electromagnetic wave emission ($\tilde{\epsilon}_{EM}$). The energy is normalized by initial energy inside the soliton $\epsilon_2(t_0)$.

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