Enhanced $K_L \to \pi^0\nu\bar{\nu}$ from Direct CP Violation in $B \to K\pi$ with Four Generations

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Recent CP violation results in $B$ decays suggest that $Z$ penguins may have large weak phase. This can be realized by the four generation (standard) model. Concurrently, $B \to X_s\ell^+\ell^-$ and $B_s$ mixing allow for sizable $V_{ts}^*V_{tb}$ only if it is nearly imaginary. Such large effects in $b \leftrightarrow s$ transitions would affect $s \leftrightarrow d$ transitions, as kaon constraints would demand $V_{td} \neq 0$. Using $\Gamma(Z \to bb)$ to bound $|V_{td}|$, we infer sizable $|V_{ts}| \leq |V_{td}| \leq |V_{us}|$. Imposing $\varepsilon K^0 \to \pi^0\nu\bar{\nu}$ and $\varepsilon'/\varepsilon$ constraints, we find $V_{ts}^*V_{tb} \sim \times 10^{-4}$ with large phase, enhancing $K_L \to \pi^0\nu\bar{\nu}$ to $5 \times 10^{-10}$ or even higher. Interestingly, $\Delta m_{B_d}$ and sin $2\Phi_{B_d}$ are not much affected, as $|V_{td}^*V_{tb}| \ll |V_{ts}V_{tb}| \sim 0.01$.

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Just 3 years after CP violation (CPV) in the B system was established, direct CP violation (DCPV) was also observed in $B^0 \to K^+\pi^-$ decay, $A_{K^+\pi^-} \sim -0.12$. A puzzle emerged, however, that the charged $B^0 \to K^+\pi^0$ mode gave no indication of DCPV, and is in fact a little positive, $A_{K^+\pi^0} \geq 0$. Currently, $A_{K^+\pi^-} - A_{K^+\pi^0} \sim 0.16$, and differs from zero with 3.8σ significance.

The amplitude $\mathcal{M}_{K^+\pi^-} \sim P + T$ is dominated by the strong penguin ($P$) and tree ($T$) contributions, while the main difference $\sqrt{2}\mathcal{M}_{K^+\pi^0} - \mathcal{M}_{K^+\pi^-} \approx \mathcal{P}_{EW} + C$ is from electroweak penguin (EWP, or $P_{EW}$) and color-suppressed tree ($C$) contributions which are subdominant. Thus, $A_{K^+\pi^+} \sim A_{K^+\pi^-}$ was anticipated by all models. As data indicated otherwise, it has been stressed that the C term could be much larger than previously thought, effectively cancelling against the CPV phase in $T$, leading to $A_{K^+\pi^0} \to 0$. While this may well be realized, a very large C (especially if $A_{K^+\pi^0} > 0$) would be a surprise in itself.

In a previous paper, we explored the possibility of New Physics (NP) effects in $P_{EW}$, in particular in the 4 generation standard model (SM4, with SM3 for 3 generations). A sequential $t'$ quark could affect $P_{EW}$ most naturally for two reasons. On one hand, the associated Cabibbo-Kobayashi-Maskawa (CKM) matrix element product $V_{ts}^*V_{tb}$ could be large and imaginary; on the other hand, it is well known that $m_{t'}^2$ in amplitude, and heavy $t'$ does not decouple.

Using the PQCD factorization approach at leading order, which successfully predicted $A_{K^+\pi^-} < -0.1$ (and C was not inordinately large), we showed that $A_{K^+\pi^0} \geq 0$ called for sizable $m_{t'} \geq 300$ GeV and large, nearly imaginary $V_{ts}^*V_{tb}$. As the $m_{t'}$ dependence is similar, we also showed that data on $B \to X_s\ell^+\ell^-$ and $B_s$ mixing concurred, in the sense that large $t'$ effect is allowed only if $V_{ts}^*V_{tb}$ is nearly imaginary. Applying the latter two constraints, however, $m_{t'}$ and $V_{ts}^*V_{tb}$ become highly constrained. In the following, we will take

$$m_{t'} \equiv 300 \text{ GeV}, V_{ts}^*V_{tb} \equiv r_{sb} e^{i\phi_{sb}} \sim 0.025 e^{i70^\circ}, \quad (1)$$

as exemplary values for realizing $A_{K^+\pi^0} - A_{K^+\pi^-} \geq 0.10$, without recourse to a large C contribution.

Comparing with $|V_{ts}V_{tb}| \approx 0.04, r_{sb} \sim 0.25$ is quite sizable. In our $b \leftrightarrow s$ study, we had assumed $V_{td} \to 0$ out of convenience, so as to decouple from $b \to d$ and $s \to d$ concerns. The main purpose of this note, however, is to show that, in view of the large $r_{sb}$ and $\phi_{sb}$ values given in Eq. (1), $V_{td} = 0$ is untenable, and one must explore $s \to d$ and $b \to d$ implications. The reasoning is as follows. Since a rather large impact on $V_{ts}V_{tb}$ is implied by Eq. (1), if one sets $V_{td} = 0$, then $V_{ts}V_{tb}$ would still be rather different from SM3 case. With our current knowledge of $m_{t'}$, the $\epsilon_K$ parameter would deviate from the well measured experimental value. Thus, a finite $V_{td}$ is needed to tune for $\epsilon_K$.

We find that the kaon constraints that are sensitive to $t'$ (i.e. $P_{EW}$-like), viz. $K^- \to \pi^-\nu\bar{\nu}, K_L \to \mu^+\mu^-$, $\epsilon_K$, and $\epsilon'/\epsilon$ can all be satisfied. Interestingly, once kaon constraints are satisfied, we find little impact is implied for $b \leftrightarrow d$ transitions, as $\Delta m_{B_d}$ and sin $2\Phi_{B_d}$.

That is, $V_{td} \to 0$ works approximately for $b \to d$ transitions, for current level of experimental sensitivity. The main outcome for $s \to d$ and $b \to d$ transitions is the enhancement of $K_L \to \pi^0\nu\bar{\nu}$ mode by an order of magnitude or more, to beyond $5 \times 10^{-10}$.

With four generations, adding $V_{ts}^*V_{tb}$ extends the familiar unitarity triangle relation into a quadrangle,

$$V_{us}V_{ub} + V_{cs}^*V_{cb} + V_{ts}V_{tb} + V_{ts}^*V_{tb} = 0. \quad (2)$$

FIG. 1: Unitarity quadrangles of (a) Eq. (1), with $|V_{us}V_{ub}|$ exaggerated; (b) Eq. (17), where actual scale is $\sim 1/4$ of (a). Adding $V_{ts}^*V_{tb}$ (dashed) according to Eq. (1), drastically changes the invariant phase and $V_{ts}^*V_{tb}$ from the SM3 triangle (solid), but from Eq. (1), the dashed lines for $V_{td}V_{tb}$ and $V_{td}V_{tb}$ can hardly be distinguished from SM3 case.
Using SM3 values for $V^*_{ub} V_{cb}$, $V^*_{cb} V_{tb}$ (validated later by our $b \to d$ decay), since they are probed in multiple ways already, and taking $V^*_{ts} V_{tb}$ as given in Eq. 11, we depict Eq. 2 in Fig. 1(a). The solid, rather squashed triangle is the usual $V^*_{us} V_{ub} + V^*_{cs} V_{cb} + V^*_{ts} V_{tb} = 0$ in SM3. Given the size and phase of $V^*_{ts} V_{tb}$, one sees that the invariant phase represented by the area of the quadrangle is rather large, and $V^*_{ts} V_{tb}$ picks up a large imaginary part, which is very different from SM3 case. Such large effect in $b \to s$ would likely spill over into $s \to d$ transitions, since taking $V^*_{tb}$ as real and of order 1, one immediately finds the strength and complexity of $V^*_{td} V_{ts}$ would be rather different from SM3, and one would need $V^*_{td} V_{ts} \neq 0$ to compensate for the well measured value for $\varepsilon_K$.

Note from Fig. 1(a) that the usual approximation of dropping $V^*_{us} V_{ub}$ in the loop remains a good one. To face $s \to d$ and $b \to d$ transitions, however, one should respect unitarity of the $4 \times 4$ CKM matrix $V_{CKM}$. We adopt the parametrization in Ref. 2 where the third column and fourth row is kept simple. This is suitable for $B$ physics, as well as for loop effects in kaon sector. With $V_{cb}, V_{tb}$ and $V^*_{ts}$ defined as real, one keeps the SM3 phase convention for $V_{ub}$, now defined as

$$\arg V^*_{ub} = \phi_{ub},$$

which is usually called $\phi_3$ or $\gamma$ in SM3. We take $\phi_{ub} = 60^\circ$ as our nominal value 8. This can in principle be measured through tree level processes such as the $B \to DK$ Dalitz method 6. The two additional phases are associated with $V^*_{ts}$ and $V^*_{td}$, and for the rotation angles we follow the PDG notation 8. To wit, we have

$$V^*_{td} = -c_{24} c_{34} s_{14} e^{-i\phi_{ub}},$$
$$V^*_{ts} = -c_{34} s_{24} e^{-i\phi_{ub}},$$
$$V^*_{tb} = -s_{34},$$

while $V^*_{ub} = c_{14} c_{24} c_{34}$, $V^*_{cb} = c_{13} c_{23} c_{43}$, $V_{ub} = c_{13} c_{23} s_{23}$ are all real. With this convention for rotation angles, from Eq. 6 we have $V^*_{ub} = c_{34} s_{13} e^{-i\phi_{ub}}$.

Analogous to Eq. 11, we also make the heuristic but redundant definition of

$$V^*_{td} V_{tb} \equiv r_{db} e^{i\phi_{db}}, V^*_{ts} V_{tb} \equiv r_{ds} e^{i\phi_{ds}},$$

as these combinations enter $b \to d$ and $s \to d$ transitions. Inspection of Eqs. 6, 11, 12 gives the relations

$$r_{db} V^*_{sh} = r_{ds} s_{34}, \phi_{ds} = \phi_{db} - \phi_{ub}.$$ (8)

As we shall see, $s \to d$ transitions are much more stringent than $b \to d$ transitions, hence we shall turn to constraining $r_{ds}$ and $\phi_{ds}$.

Before turning to the kaon sector, we need to infer what value to use for $s_{34} = |V^*_{ts}|$, as this can still affect the relevant physics through unitarity. Fortunately, we have some constraint on $s_{34}$ from $Z \to bb$ width, which receives special $t$ (and hence $t'$) contribution compared to other $Z \to q\bar{q}$, and is now suitably well measured.

Following Ref. 9 and using $m_{t'} = 300$ GeV, we find

$$|V^*_{tb}|^2 + 3.4 |V^*_{tb}|^2 < 1.14.$$ (9)

Since all $c_{ij}$ except perhaps $c_{34}$ would still likely be close to 1, we infer that $s_{34} \lesssim 0.25$. We take the liberty to nearly saturate this bound ($\Gamma(Z \to bb)$ is close to $1\sigma$ above SM3 expectation), by imposing

$$s_{34} \simeq 0.22.$$ (10)

to be close to the Cabibbo angle, $\lambda \equiv |V_{us}| \simeq 0.22$.

Note that Fig. 10 in Ref. 9 is somewhat below the expectation of “maximal mixing” of $s_{34}^2 \sim 1/2$ between third and fourth generations. Combining it with Eq. 11, one gets $|V^*_{ts}| \sim 0.11 \sim \lambda/2$. Its strength would grow if a lower value of $s_{34} \lesssim \lambda$ is chosen, which would make even greater impact on $s \to d$ transitions.

Using current values of $V_{cb}$ and $V_{ub}$ as input and respecting full unitarity, we now turn to the kaon constraints of $K^+ \to \pi^+\nu\bar{\nu}$, $\varepsilon_K$, $K_L \to \mu^+\mu^-$, and $\epsilon'/\epsilon$. The first two are short-distance (SD) dominated, while the last two suffer from long-distance (LD) effects.

Let us start with $K^+ \to \pi^+\nu\bar{\nu}$. The first observed event by E787 suggested a sizable rate hence hinted at NP. The fourth generation would be a good candidate, since the process is dominated by the $Z$ penguin. Continued running, including E949 data (unfortunately not greatly improving accumulated luminosity), has yielded overall 3 events, and the rate is now $B(K^+ \to \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ 11. This is still somewhat higher than the SM3 expectation of order $0.8 \times 10^{-10}$.

Defining $\lambda_{q}^d s_t^d \equiv |V_{q4} V^*_{qs}|$ and using the formula

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \frac{\lambda_{s_t}^d}{|V_{us}|} \frac{|P_{e}}{P_{\mu}} \frac{|\lambda_{q}^d}{|V_{us}|^2} \sum_{i} X_{t} (x_{t}^i)^2,$$ (11)

we plot in Fig. 2 the allowed range (valley shaped shaded region) of $r_{ds} - \phi_{ds}$ for the 90% confidence level (C.L.) bound of $B(K^+ \to \pi^+\nu\bar{\nu}) < 3.6 \times 10^{-10}$. We have used $\kappa_+ = (4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^8$ and $P_c = (0.39 \pm 0.07) \times (0.224/|V_{us}|)^4$. We take the QCD correction factors $\eta_{t}^0 \sim 1$, and $X_{t} (x_{t}^i)$ evaluated for $m_t = 166$ GeV and $m_{t'} = 300$ GeV. We see that $r_{ds}$ up to $7 \times 10^{-4}$ is possible, which is not smaller than the SM3 value of $4 \times 10^{-4}$ for $|V^*_{td} V_{ts}|$.

The SD contribution to $K_L \to \mu^+\mu^-$ is also of interest. The $K_L \to \mu^+\mu^-$ rate is saturated by the absorptive $K_L \to \gamma\gamma \to \mu^+\mu^-$, while the off-shell photon contribution makes the SD contribution hard to constrain. To be conservative, we use the experimental bound of $B(K_L \to \mu^+\mu^-)_{SD} < 3.7 \times 10^{-9}$ 13. It is then in general less stringent than $K^+ \to \pi^+\nu\bar{\nu}$, although the generic constraint on $r_{ds}$ drops slightly. We do not plot this constraint in Fig. 2.

The rather precisely measured CPV parameter $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$ 8 is predominantly SD. It maps
out rather thin slices of allowed regions on the $r_{ds}$-$\phi_{ds}$ plane, as illustrated by dots in Fig. 2, where we use the formula of Ref. 14 and follow the treatment. Note that $r_{ds}$ up to $7 \times 10^{-4}$ is still possible, for several range of values for $\phi_{ds}$. This is the aforementioned effect that extra CPV effects due to large $\phi_{ds}$ and $r_{ds}$ now have to be tuned by $t'$ effect to reach the correct $\varepsilon_K$ value. We have checked that $\Delta m_K$ makes no additional new constraint.

The DCPV parameter, $\text{Re}(\varepsilon'/\varepsilon)$, was first measured in 1999 13, with current value at $(1.67 \pm 0.26) \times 10^{-3}$ 8. It depends on a myriad of hadronic parameters, such as $m_s$, $\Omega_B$ (isospin breaking), and especially the non-perturbative parameters $R_6$ and $R_8$, which are related to the hadronic matrix elements of the dominant strong and electroweak penguin operators. With associated large uncertainties, we expect $\varepsilon'/\varepsilon$ to be rather accommodating, but for specific values of $R_6$ and $R_8$, some range for $r_{ds}$ and $\phi_{ds}$ is determined.

We use the formula

$$\text{Re}(\varepsilon'/\varepsilon) = \text{Im}(\lambda'^d_8)P_0 + \text{Im}(\lambda'^d_6)F(x_t) + \text{Im}(\lambda'^d_s)F(x_t),$$

(12)

where $F(x)$ is given by

$$F(x) = P_X X_0(x) + P_Y Y_0(x) + P_Z Z_0(x) + P_E E_0(x).$$

(13)

The SD functions $X_0$, $Y_0$, $Z_0$ and $E_0$ can be found, for example, in Ref. 15, and the coefficients $P_i$ are given in terms of $R_6$ and $R_8$ as

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8,$$

(14)

which depends on LD physics. We differ from Ref. 15 by placing $P_0$, multiplied by $\text{Im}(\lambda'^d_8)$, explicitly in Eq. 12. In SM4, one no longer has the relation $\text{Im}\lambda'^d_8 = -\text{Im}\lambda'^d_6$ that makes $\text{Re}(\varepsilon'/\varepsilon)$ proportional to $\text{Im}(\lambda'^d_6)$. We take the $r_i^{(j)}$ values from Ref. 15 for $\Lambda(4)_{\overline{MS}} = 310$ MeV, but reverse the sign of $r_i^{(j)}$ for above mentioned reason. Note that $\text{Re}(\varepsilon'/\varepsilon)$ depends linearly on $R_6$ and $R_8$. For fixed SD parameters $m_t'$ and $\lambda'^d_9 = V_{td}^* V_{ts}^*$, one may adjust for solutions to $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $\varepsilon_K$.

For the "standard" 13 parameter range of $R_6 = 1.23 \pm 0.16$ and $R_8 = 1.0 \pm 0.2$, we find $R_6 \sim 1.2$ and $R_8 \sim 1.0$–1.2 allows for solutions at $r_{ds} \sim (5-6) \times 10^{-4}$ with $\phi_{ds} \sim (35^\circ - 50^\circ)$, as illustrated by the elliptic rings on upper left part of Fig. 2. For $R_6 = 2.2 \pm 0.4$ found 10 in $1/N_C$ expansion at next-to-leading order (and chiral perturbation theory at leading order), within SM3 one has trouble giving the correct $\text{Re}(\varepsilon'/\varepsilon)$ value. However, for SM4, solutions exist for $R_6 \sim 2.2$ and $R_8 = 0.8$–1.1, for $r_{ds} \sim (3.5-5) \times 10^{-4}$ and $\phi_{ds} \sim (45^\circ - 60^\circ)$, as illustrated by the elliptic rings on upper right part of Fig. 2. We will take

$$r_{ds} \sim 5 \times 10^{-4}, \quad \phi_{ds} \sim -60^\circ \text{ or } +35^\circ,$$

as our two nominal cases that satisfy all kaon constraints. The corresponding values for $R_6$ and $R_8$ can be roughly read off from Fig. 2. We stress again that these values should be taken as exemplary.

To illustrate in a different way, we plot $\varepsilon_K$, $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $\text{Re}(\varepsilon'/\varepsilon)$ vs $\phi_{ds}$ in Figs. 3(a), (b) and (c), respectively, for $r_{ds} = 4$ and $6 \times 10^{-4}$. The current $1\sigma$ experimental range is also illustrated. In Fig. 3(c), we have illustrated with $R_6 = 1.1$, $R_8 = 1.2$ 15 and $R_6 = 2.2$, $R_8 = 1.1$ 10. For the former (latter) case, the variation is enhanced as $R_6$ ($R_8$) drops.

It is interesting to see what are the implications for the CPV decay $K_L \rightarrow \pi^0\nu\bar{\nu}$. The formula for $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ is analogous to Eq. 11, except 15 the change of $\kappa_{L_L}$ to $\kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$, and taking only the imaginary part for the various CKM products. Since $\phi_{ds} \sim -60^\circ$ or $+35^\circ$ have large imaginary part, while $r_{ds} \equiv |V_{td}^* V_{ts}^*| \sim 5 \times 10^{-4}$ is stronger than the SM3

![FIG. 2: Allowed region from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ (valley shaped shaded region), $\varepsilon_K$ (simulated dots) and $\varepsilon'/\varepsilon$ (elliptic rings) in $r_{ds}$ and $\phi_{ds}$ plane, as described in text, where $V_{td}^* V_{ts}^* \equiv r_{ds} e^{i\phi_{ds}}$. For $\varepsilon'/\varepsilon$, the rings on upper right correspond to $R_6 = 2.2$, and $R_8 = 0.8$, 1.1 (bottom to top), and on upper left, $R_6 = 1.0$, 1.2 (bottom to top), $R_8 = 1.2$.

![FIG. 3: (a) $\varepsilon_K$, (b) $B(K^+ \rightarrow \pi^+\nu\bar{\nu})$, (c) $\text{Re}(\varepsilon'/\varepsilon)$ and (d) $B(K_L \rightarrow \pi^0\nu\bar{\nu})$ vs $\phi_{ds}$, for $r_{ds} = 4$ and $6 \times 10^{-4}$ and $m_t = 300$ GeV. Larger $r_{ds}$ gives stronger variation, and horizontal bands are current $(1\sigma)$ experimental range $\mathcal{B}$ (the bound for (d) is outside the plot). For (c), solid (dashed) lines are for $R_6 = 2.2$, $R_8 = 1.1$ ($R_6 = 1.1$, $R_8 = 1.2$).]
the expectation of $\text{Im} V_{td}^* V_{ts} \sim 10^{-4}$, we expect the CP decay rate of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ to be much enhanced.

We plot $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs $\phi_{fs}$ in Fig. 3(d), for $r_{ds} = 4$ and $6 \times 10^{-4}$. Reading off from the figure, we see that the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ rate can reach above $10^{-9}$, almost two orders of magnitude above SM3 expectation of $0.3 \times 10^{-10}$. It is likely above $5 \times 10^{-10}$, and in general larger than $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Specifically, for our nominal value of $r_{ds} \sim 5 \times 10^{-4}$ and $\phi_{fs} \sim +35^\circ$, $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are 6.5 and $2 \times 10^{-10}$, respectively, while for the $\phi_{fs} \sim -60^\circ$ case, they are 12 and $3 \times 10^{-10}$, respectively. The latter case is closer to the Grossman-Nir bound $17$, i.e. $\text{nontrivial}$ since there is a large effect in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, but $B_{
u \bar{\nu}}$ is not yet updated. We see from Fig. 4(a) that $\Delta m_{B_d}$ does not rule out the parameter space around Eq. $19$ (equivalent to Eq. $15$). The overall dependence on $r_{ds}$ and $\phi_{db}$ is mild, and error on $f_{B_d} \sqrt{B_{B_d}}$ dominates. Seemingly, a lower value of $f_{B_d} \sqrt{B_{B_d}} \sim 215$ MeV is preferred. SM3 would give $\Delta m_{B_d} = 0.44 - 0.62 \text{ ps}^{-1}$ for $f_{B_d} \sqrt{B_{B_d}} = 208 \text{ MeV} - 246 \text{ MeV}$, so the problem is not with SM4.

We plot $\sin 2\Phi_{B_d}$ vs $\phi_{db}$ in Fig. 4(b), for $r_{db} = 8$ and $12 \times 10^{-4}$. One can see that $\sin 2\Phi_{B_d}$, which is not sensitive to hadronic parameters such as $f_{B_d} \sqrt{B_{B_d}}$, is well within experimental range of $\sin 2\Phi_{V_{ub}} = 0.73 \pm 0.04$ from PDG 2005 $8$ for the $\phi_{db} \sim 10^\circ$ case. However, for $\phi_{db} \sim 105^\circ$ case, which is much more imaginary, $\sin 2\Phi_{B_d}$ is on the high side $22$, and it seems that CPV in $B$ physics prefers $R_6 \sim 2.2$ over $R_6 \sim 1$. As another check, we find the semileptonic asymmetry $A_{SL} = -0.7 \times 10^{-3}$ ($-0.2 \times 10^{-3}$) for $\phi_{db} \sim 10^\circ$ (105$^\circ$), which is also well within range of $A_{SL} = (1 \pm 4 \text{ MeV}) / (2.4 \text{ MeV})$, respectively.

With Eqs. $19$, $10$ and $15$, together with standard (SM3) values for $V_{cb}$ and $V_{ub}$, we can get a glimpse of the typical $4 \times 4$ CKM matrix, which appears like

$$
\begin{pmatrix}
0.9745 & 0.2225 & 0.0038 e^{-i 60^\circ} & 0.0281 e^{i 61^\circ} \\
-0.2241 & 0.9667 & 0.0415 & 0.1164 e^{-i 66^\circ} \\
0.0073 e^{-i 25^\circ} & -0.0555 e^{-i 25^\circ} & 0.9746 & 0.2168 e^{-i 14^\circ} \\
-0.0044 e^{-i 10^\circ} & -0.1136 e^{-i 70^\circ} & -0.2200 & 0.9688
\end{pmatrix}
$$

(19)

for $\phi_{db} \sim 10^\circ$ case (Fig. $1$ and $V_{cs}$ pick up tiny imaginary parts, which are too small to show in angles). For the $\phi_{db} \sim 105^\circ$ case, the appearance is almost the same, except $V_{td} \simeq 0.0082 e^{-i 117^\circ}$ and $V_{ub} \simeq 0.029 e^{i 74^\circ}$. Note the “double Cabibbo” nature, i.e. the 12 and 34 diagonal submatrices appear almost the same. This is a consequence of our choice of Eq. $14$. To keep Eq. $14$ intact, however, weakening $s_{34}$ would result in even large $V_{ct}$, but it would still be close to imaginary. Since $V_{td}^\ast V_{td}$ is tiny compared to $V_{ud}^\ast V_{ud}$, we expect $M_{12}/|M_{12}|$, the unitarity quadrangle for $s \rightarrow d$ cannot be plotted as in Fig. 1.
However, note that $V_{ub}^* V_{ts}$ is almost real, and CPV in $s \rightarrow d$ comes mostly from $t'$. The entries for $V_{ub}$, $i = u, c, t$ are all sizable. $|V_{ub}| \sim 0.03$ satisfies the unitarity constraint $|V_{ub}| < 0.08$ from the first row, but it is almost as large as $V_{cb}$. However, the long standing puzzle of unitarity of the first row could be taken as a hint for finite $|V_{ub}| \sim 0.03$.

The element $V_{ub} \simeq -V_{ts}$ is even larger than $V_{tb}$ and close to imaginary. Together with finite $V_{ub}$, $V_{ub} V_{cb} \simeq 0.0033 e^{-i57^\circ}$ ($0.0034 e^{-i89^\circ}$) is not negligible, and one may worry about $D^0\bar{D}^0$ mixing. Fortunately the $D$ decay rate is fully Cabibbo allowed. Using $f_D \sqrt{B_D} = 200$ MeV, we find $\Delta m_{D^0} \approx 0.05$ ps$^{-1}$ for $m_D \approx 280$ GeV, for both nominal cases of Eq. (10). Thus, the current bound of $\Delta m_{D^0} < 0.07$ ps$^{-1}$ is satisfied, and the search for $D^0$ mixing is of great interest. This bound weakens by factor of 2 if one allows for strong phase between $D^0 \rightarrow K^-\pi^+$ and $K^+\pi^-$.

If $m_{b'} < m_s$, as slightly preferred by $D^0\bar{D}^0$ mixing constraint, the direct search for $b'$ just above 200 GeV at the Tevatron Run II could be rather interesting. Since $V_{ub}$ is not suppressed, the $b'$ quark would decay via charged current. Both $b'$ and $t'$, regardless of which one is lighter, with $m_{b'} \sim 300$ GeV and $|m_{s'} - m_{b'}| \lesssim 85$ GeV, can be easily discovered at the LHC.

The large and mainly imaginary element $V_{ts} \simeq -V_{ub}^*$ in Eq. (10), being larger than $V_{ts}$ and $V_{cb}$, may appear unnatural (likewise for $V_{ub}$ vs $V_{cb}$). However, it is allowed, since the main frontier that we are just starting to explore is in fact $b \rightarrow s$ transitions. The current situation that $A_{K^+\pi^-} \sim -0.12$ while $A_{K^+\pi^0} > 0$ in $B \rightarrow K\pi$ decays may actually be hinting at the need for such large $b \rightarrow s$ CPV effects. The limus test would be finding $\Delta m_B$, not far above current bound, but with sizable sin $2\Phi_{B_s} < 0$, which may even emerge at Tevatron Run II. Our results studied here are for illustration purpose, but the main result, that $K_L \rightarrow \pi^0 \nu\bar{\nu}$ may be rather enhanced, is a generic consequence of Eq. (10), which is a possible solution to the $B^+ \rightarrow K^+\pi^0$ DCPV puzzle.

In summary, the deviation of direct CPV measurements between neutral and charged $B$ decays, $A_{K^+\pi^0} - A_{K^+\pi^-}$, is 0.16 while $A_{K^+\pi^-} < -0.12$, is a puzzle that could be hinting at New Physics. A plausible solution is the existence of a 4th generation with $m_{t'} \sim 300$ GeV and $V_{ts} V_{tb} \sim 0.025 e^{i70^\circ}$. If so, we find special solution space is carved out by stringent kaon constraints, and the $4 \times 4$ CKM matrix is almost fully determined. $K^+ \rightarrow \pi^+\nu\bar{\nu}$ may well be of order $(1 - 2) \times 10^{-10}$, while $K_L \rightarrow \pi^0\nu\bar{\nu}$ $(4 - 12) \times 10^{-10}$ is greatly enhanced by the large phase in $V_{ts} V_{tb}$. With kaon constraints satisfied, $B_s$ mixing and $\sin 2\Phi_{B_s}$ are consistent with experiment. Our results are generic. If the effect weakens in $b \rightarrow s$ transitions, the effect on $K \rightarrow \pi\nu\bar{\nu}$ would also weaken. But a large CPV effect in electroweak $b \rightarrow s$ penguins would translate into an enhanced $K_L \rightarrow \pi^0\nu\bar{\nu}$ (and sin $2\Phi_{B_s} < 0$).

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Recent kaon decay results imply a more stringent bound of $1 - |V_{ud}|^2 - |V_{us}|^2 = 0.0004 \pm 0.0011$ ($|V_{ub}|^2$ is negligible), or $|V_{ub}'| < 0.047$ at 90% C.L., which is still satisfied by our value.