Inversion of elastic constants of composite materials by single test based on virtual fields method

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Abstract: In this paper, a method of obtaining four elastic constants of orthotropic composites by single tensile test is proposed. The principle of this method is to obtain the full field strain on the surface of the specimen through the tensile test of the specimen with a specific geometry, and to invert the measured in-plane strain field into the virtual fields method (VFM) to obtain the elastic constants. Based on the deformation field data generated by finite element numerical simulation, the effects of design parameters such as effective length, notch position, notch size and fillet radius on the identification results of elastic parameters of V-notch tensile specimen are studied. The results show that the inversion results of symmetrical V-notch specimen with 50 mm length, 12 mm notch size and 3.1 mm fillet radius are the best, and the sum of absolute relative errors of four elastic constants is less than 3%, which verifies the feasibility of the method.

1. Introduction
Fiber reinforced composites (FRCs) have the advantages of high specific modulus, high specific strength and strong designability of material properties which are widely used in aerospace, marine and vehicle engineering[1]. However, as an orthotropic material, it has four elastic constants, and at least three measurement tests need to be carried out by using the traditional measurement method. Among them, the tensile test along and perpendicular to the fiber direction is used to measure the elastic modulus and Poisson's ratio in two directions. For the measurement of shear modulus, there are two main methods: ±45° tensile test method and Iosipescu shear test. They are all direct measurement methods and have relatively mature standards, but need many tests which means high cost. As an alternative to the traditional method, the virtual fields method (VFM) is an effective method for solving mechanical inverse problems[2]. The theoretical basis of VFM is virtual work principle, and its basic idea is to map the elastic constants of materials through strain field. In order to characterize all elastic constants as accurately as possible, it is necessary to induce a complex strain field by changing the loading mode or the specimen shape. Therefore, test methods such as double V-notch short beam shear test[3], T-shaped specimen tensile test[4-5], shear test of short beams without openings[6], short beam three-point bending test[7] and elliptical hole specimen tensile test[8] have been proposed to inverse the orthotropic elastic constants of fiber-reinforced composites. However, the above methods, either the test program is complex, need special fixtures, or
the specimen processing is difficult, or the inversion accuracy is insufficient, which are difficult to be applied in engineering.

In this paper, the elastic properties of orthotropic materials identified by V-notch uniaxial tensile test are studied. Firstly, the VFM inversion formula of elastic constants of orthotropic materials is deduced theoretically, and then the tensile simulation of different V-notch specimens is carried out by using the finite element method. The simulated strain field is combined with VFM to invert the input parameters so as to select the optimal V-notch geometry. Finally, the influencing factors of identification results and improvement suggestions of method are discussed.

2. Virtual field method
The virtual work equation of deformed body under external force is

\[ \int_V \sigma_{ij} \varepsilon_{ij}^* dV = \int_V F_i^b u_i^* dV + \int_{S_f} F_i^s u_i^* dS + \sum F_i^c u_i^* \]  

(1)

Where, \( V \) represents the deformed body, \( S_f \) represents the surface of the deformed body, \( \sigma_{ij} \) is stress tensor, \( \varepsilon_{ij}^* \) and \( u_i^* \) are the virtual strain tensor and virtual displacement tensor respectively, and \( \varepsilon_{ij}^* = \frac{1}{2} (u_{ij}^* + u_{ji}^*) \), \( F_i^b \), \( F_i^s \) and \( F_i^c \) represent body force, surface traction and concentrated load respectively.

For the plane stress problem, when the gravity and concentrated load are ignored, the virtual work equation can be written as

\[ \int_S \sigma_{ij} \varepsilon_{ij}^* dS = \frac{1}{t} \int_{S_f} F_i^s u_i^* dS \]  

(2)

Where, \( t \) is the thickness of the specimen. The constitutive equation of orthotropic materials is

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_6
\end{bmatrix}
\]  

(3)

Substitute it into equation (2)

\[ Q_{11} \int_S \varepsilon_1^* \varepsilon_1^* dS + Q_{22} \int_S \varepsilon_2^* \varepsilon_2^* dS + Q_{12} \int_S (\varepsilon_1^* \varepsilon_2^* + \varepsilon_2^* \varepsilon_1^*) dS + Q_{66} \int_S \varepsilon_6^* \varepsilon_6^* dS = \frac{1}{t} \int_{S_f} F_i^s u_i^* dS \]  

(4)

where \( \varepsilon_i (i = 1, 2, 6) \) is the real strain field of the specimen, which can be measured by digital image correlation (DIC) in the test, and \( \varepsilon_{ij}^* (i=1,2,6) \) is the virtual strain field. It can be seen that each group of virtual fields corresponds to an equation about \( Q_{11}, Q_{22}, Q_{12} \) and \( Q_{66} \). As long as four groups of suitable virtual fields are set so that the coefficients of the four groups of equations are independent of each other, the four elastic constants can be obtained by solving a quaternion primary equation group.

However, there are some difficulties in the practical application of VFM. One is the selection of virtual field. According to equation (4), as the coefficient of the equation, the selection of virtual field directly affects the solution result. The virtual work principle requires that the virtual displacement should meet the boundary conditions. At the same time, in order to obtain accurate identification results, the loading mode and strain distribution of the specimen should be considered. At first, the virtual field is directly defined by specific mathematical functions. For example, the virtual field in the form of polynomial and trigonometric function is defined in the short beam shear test [6]. Although this method is fast in calculation, it can only be applied to specific specimens and loading methods. Therefore, Grédiac et al. proposed a method for automatically selecting the virtual field, which is called the special virtual field method[9-10], the problem of difficult selection of virtual field can be solved.
Another difficulty is to induce non-uniform strain fields in the specimens which fully reflect the inherent properties of the material. The short beam shear test and the tensile test of T-shaped sample need special fixture, the test program is complex, and the processing of tensile specimen with elliptical hole is difficult. These test schemes are difficult to be applied in engineering. Therefore, a V-notch specimen is designed in this paper.

3. Specimen design
In order to generate non-uniform strain field for specimens under uniaxial tensile loading, this paper carried out optimum design of V-notch specimens inspired by Meuwissen's work[11]. The notch is set as an isosceles right triangle and the width of the fixed specimen is 25mm, which meets the general tensile test standard. The four geometric parameters of specimen length, notch position, notch size and fillet radius at the notch are taken as the design variables. The geometry and design variables of the specimen are shown in Figure 1, and the value range of each parameter is shown in Table 1.

![V-notch specimen](image)

Figure 1. V-notch specimen.

The value range of each parameter is shown in Table 1.

| Parameter | Min / (mm) | Max / (mm) | Interval / (mm) | Quantity |
|-----------|------------|------------|----------------|----------|
| $x_1$     | 5          | 75         | 5              | 6        |
| $x_2$     | 5          | 12         | 1              | 8        |
| $x_3$     | 5          | 12         | 1              | 8        |
| $r$       | 1.5        | 3.1        | 0.2            | 9        |

Traverse each set of geometric parameters within the design range and use the commercial CAE software ABAQUS for modeling and simulation. The simulated strain field is substituted into the virtual field method for parameter inversion, and the inversion results are compared with the input parameters to calculate the relative error, so as to select the optimal geometry. The elastic constant of glass fiber-epoxy resin matrix composite[2] is used as the input parameter, that is, $Q_{11} = 41$, $Q_{22} = 10.3$, $Q_{12} = 3.1$, $Q_{66} = 4$, and the unit is Gpa. VFM selects the optimized special virtual field method[12] for more accurate results. The relative error formula $\text{Error}_i = \left( \frac{Q_{ij}^{VFM} - Q_{ij}^{input}}{Q_{ij}^{input}} \right)$ is selected to evaluate the quality of inversion results under each group of geometric parameters, where $Q_{ij}^{VFM}$ is the result of VFM inversion and $Q_{ij}^{input}$ is the finite element input parameter.
4. Results and discussion

4.1. Specimen geometry selection results
It is found that the best inversion result comes from the symmetrical V-notch specimen. The sum of the absolute values of the relative errors of the four elastic constants is 2.7246%, and the geometric parameters are $x_1 = 50$, $x_2 = 6$, $x_3 = 12$ and $r = 3.1$ respectively. The results are shown in Table 2.

| Table 2 The identification result with minimum relative error. |
|---------------------------------|----------|----------|----------|----------|
| Input value of FEM/ (GPa)       | $Q_{11}$ | $Q_{12}$ | $Q_{12}$ | $Q_{66}$ |
| Result of VFM/ (GPa)            | 41       | 10.3     | 3.1      | 4        |
| Relative error                  | -0.0097% | 0.4837%  | -0.5312% | -1.7000% |

The corresponding strain field is shown in Figure 2.

4.2. Discussion
For the four geometric parameters determined above, calculate the identification error of each elastic constant when fix the other three each time to observe the sensitivity of the identification results to each geometric parameter, as shown in Figure 3.
Figure 3. Sensitivity analysis of identification results to geometrical parameters of specimens, (a) length of the specimen, (b) position of V-notch, (c) size of V-notch, (d) corner radius of V-notch.

It can be seen from the figure that the most accurate elastic constant of the identification result is $Q_{11}$, because the deformation along the fiber direction is the most significant in the tensile test, and the characteristics of $Q_{11}$ are most obvious. The identification errors of $Q_{22}$ and $Q_{12}$ are very consistent, because $Q_{22}$ corresponds to the elastic modulus $E_2$ and $Q_{12}$ corresponds to the principal Poisson's ratio $\nu_{21}$. Their characteristics are reflected in the strain field $\varepsilon_2$ which is perpendicular to the fiber direction. It is not difficult to find that the identification error of $Q_{66}$ is large and negative in all geometries, because it corresponds to the shear modulus $G$, which is mainly identified by the shear strain field $\varepsilon_{12}$. In the tensile test along the fiber direction, the weight of shear strain is often lower than that of normal strain, resulting in the identified shear constant $Q_{66}$ lower than the actual value.

The geometric factor that has the greatest influence on the identification results is the notch position, because with the change of notch position, the principal strain distribution in the three planes of the specimen will change. When the notches are near the center of the specimen, the error of the identification results of each elastic constant is small. However, as the notch moves away from the center line of the specimen, the error of the identification result becomes large. The specimen length should not be too long, 50mm or 55mm is more appropriate. With the increase of specimen length, the proportion of high strain region in strain field $\varepsilon_2$ becomes smaller and smaller, and the identification error of $Q_{22}$ and $Q_{12}$ gradually increases and is negative. The identification error of $Q_{66}$ is the most sensitive to the notch size and gradually decreases with the increase of the notch, because the high strain area of shear strain is distributed near the bevel of the notch. As shown in Figure 3 (c), the larger the notch, the higher the proportion of shear strain value, and the better the identification effect of $Q_{66}$. Small notches can produce large high strain zone in $\varepsilon_2$, while the large notch can make the value of
high strain region in $\varepsilon_2$ larger, so when the notch is very small or very large, the identification error of $Q_{22}$ and $Q_{12}$ is the smallest. The most unstable geometric factor affecting the recognition results is the fillet radius $r$, but the overall trend is that the larger the radius, the smaller the recognition error, so the fillet radius should be as large as possible.

5. Conclusion
In this paper, the inversion of orthotropic elastic constants of fiber-reinforced composites by single tensile test based on virtual field method is studied. The characteristics of orthotropic materials are reflected by opening V-notches on both sides of the tensile specimen. The optimal geometry of the specimen is determined by finite element simulation. Finally, the sensitivity of the identification results to the geometric parameters of the specimen is analysed. The main conclusions are as follows:

1. According to the minimum relative error between the inversion results and the finite element input parameters, the optimal geometry of V-notch specimen with a length of 50 mm, notch located in the center line of the specimen, opening size of 12 mm and fillet radius of 3.1 mm is determined.

2. The relative errors of $Q_{11}$, $Q_{22}$, $Q_{12}$ and $Q_{66}$ recognition results are -0.0097%, 0.4837%, -0.5312% and -1.7000% respectively, and the sum of absolute values is no more than 3% which verified the effectiveness of this method.

3. The recognition result of $Q_{11}$ is the most accurate and stable, while the recognition error of $Q_{22}$, $Q_{12}$ and $Q_{66}$ decreases significantly with the increase of notch and fillet radius.

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