EPR before EPR: a 1930 Einstein–Bohr thought experiment revisited

Hrvoje Nikolić

Theoretical Physics Division, Rudjer Bošković Institute, PO Box 180, HR-10002 Zagreb, Croatia

E-mail: hrvoje@thphys.irb.hr

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Abstract
In 1930, Einstein argued against the consistency of the time–energy uncertainty relation by discussing a thought experiment involving a measurement of the mass of the box which emitted a photon. Bohr seemingly prevailed over Einstein by arguing that Einstein’s own general theory of relativity saves the consistency of quantum mechanics. We revisit this thought experiment from a modern point of view at a level suitable for an undergraduate readership and find that neither Einstein nor Bohr was correct. Instead, this thought experiment should be thought of as an early example of a system demonstrating nonlocal ‘EPR’ quantum correlations, five years before the famous Einstein–Podolsky–Rosen paper.

1. Introduction

The nonlocal nature of quantum correlations in systems with two or more particles is today a well-understood and widely known [1–8] property of quantum mechanics (QM). Although the nonlocal nature of quantum correlations was first clearly recognized by Bell in 1964 [9] and experimentally verified by Freedman and Clauser in 1972 [10], these nonlocal correlations are widely known as EPR correlations, after a 1935 paper by Einstein, Podolsky and Rosen [11]. Nevertheless, the nonlocality of QM was not even conjectured in the EPR paper. Instead, the paper was an attempt to prove that QM was incomplete, where one of the assumptions in the proof was that nature is local. Today we know that Einstein, Podolsky and Rosen were wrong, in the sense that this assumption of locality was incorrect. It does not imply that QM is complete (at the moment there is no consensus among physicists on that issue [7]), but today the EPR argument against completeness is no longer appropriate.

Although the nonlocality of QM was not discovered in the famous 1935 EPR paper, that paper played a pivotal historical role by influencing Bell to discover nonlocality in 1964 [9]. (To be more precise, Bell’s ‘discovery’ was a purely theoretical result, not an experimental one. He studied general properties of local theories describing measurement
outcomes in terms of objective properties possessed by the system. As a result he (i) derived an inequality involving correlations between measurement outcomes that any local theory of that kind should obey and (ii) found out that QM predicts violation of that inequality.) For that reason, it seems to be widely believed that the EPR paper is the first historical example of a quantum system which cannot be understood correctly without invoking quantum nonlocality.

In this paper, we point out that the EPR paper is not the first such example. Instead, a very similar example was provided by Einstein alone in 1930, i.e. five years before the EPR paper. In fact, this paper from 1930, together with Bohr’s reply, is well known and has been reviewed in many books [12–15]. In this example, Einstein presents a paradox in QM suggesting that QM is inconsistent, while Bohr attempts to save the consistency of QM by combining QM with Einstein’s general theory of relativity. Yet, despite the historical importance of that paradox, the similarity with the EPR paradox and the fact that this paradox is often presented in the literature, a correct resolution of it is rarely presented. Instead, many books like [13–15] still present Bohr’s resolution of the paradox in terms of general relativity as if such a resolution was correct.

Jammer in his book [12] is less superficial on that issue, discussing several criticisms of Bohr’s resolution, and yet none of these criticisms includes the correct resolution based on quantum nonlocality. At another place in [12] Jammer comes very close to the correct resolution.

Einstein’s own narrative shows in detail how the idea of a gravitating photon box gradually changed into that of a system of two interacting particles, as later used in the Einstein–Podolsky–Rosen paradox. […] The final elimination of the box itself and its replacement by a second ‘particle’ may have been prompted by the following development.

Some critical discussions of Bohr’s resolution of the paradox were also presented after Jammer’s book in the papers [16–21]. Most of these papers also do not present explicitly the correct resolution of the paradox in terms of quantum nonlocality, but some of them [17, 19] indicate such a resolution through some brief side remarks similar to those of Jammer above. The only paper we are aware of that explicitly presents the correct resolution in terms of quantum nonlocality is [21].

Unfortunately, these observations by Jammer, brief side remarks in [17, 19], and even the explicit claims in [21], remained widely unnoticed in the physics community and the correct resolution of Einstein’s 1930 paradox remained widely unrecognized. Thus, we believe that it is both important for historical reasons and instructive for pedagogic purposes to present in detail the correct resolution of that Einstein 1930 paradox. Hence, this is what we do in this paper.

The paper is organized as follows. We first review the Einstein 1930 paradox and Bohr’s resolution as presented in the existing literature [12–15], in section 2. After that, in section 3 we revisit this paradox from a modern point of view and find that the key to the correct resolution of the paradox is quantum nonlocality (and not general relativity as suggested by Bohr in 1930). Finally, the conclusions are drawn in section 4.

The paper is written at a level suitable for an undergraduate readership and contains material useful for physics education at university level. For example, it contains some simple qualitative explanations of the nonlocal features of QM, including the wavefunction collapse as well as a quantitative explanation of the time–energy uncertainty relation presented in a form rarely found in textbooks.
2. Einstein’s 1930 paradox and Bohr’s resolution

2.1. Einstein’s paradox

At the 6th Solvay conference in 1930 Einstein attempted to challenge the consistency of the time-energy uncertainty relation

\[ \Delta E \Delta t \gtrsim \hbar. \]  

(1)

For that purpose, he considered the following thought experiment. Consider a box filled with photons open during an arbitrarily short time \( \Delta t \). During this time a photon is emitted, so one knows the time of emission with an arbitrary precision \( \Delta t \). If (1) is correct, then a low time uncertainty \( \Delta t \) implies a high uncertainty of the photon energy

\[ \Delta E \gtrsim \frac{\hbar}{\Delta t}. \]  

(2)

However, despite the arbitrarily small \( \Delta t \), the energy of the photon can be measured with an arbitrary precision as well, in the following way. One can measure the mass \( m \) of the box before and after the photon left the box. (For concreteness, one can do that by reading the position of the box hanging on a spring scale in the gravitational field, which determines the weight of the box.) From the change of the mass and \( E = mc^2 \) one can determine the change of energy of the box with arbitrary precision. But the total energy must be conserved, so the energy decrease of the box must be equal to the energy carried by the photon. In this way, by measuring the mass of the box with arbitrary precision one also determines the energy of the photon by arbitrary precision, in contradiction with (2).

2.2. Bohr’s resolution

At first Bohr did not know how to reply to Einstein’s paradox, but eventually he constructed a reply which we now review. For the sake of clarity, we split his response into three steps.

The first step considers the precision of the measurement of energy. If energy of the box is measured with the precision \( \Delta E \), then \( E = mc^2 \) implies that mass is measured with the precision \( \Delta m = \Delta E/c^2 \). This implies that the gravitational force \( F = mg \) is determined by the precision

\[ \Delta F = \Delta m g = \Delta E \frac{g}{c^2}. \]  

(3)

The force acting during the time \( t \) determines the momentum transferred from the gravitational field to the box. This momentum is \( p = Ft \), so the transferred momentum is determined by the precision

\[ \Delta p = \Delta F t = \Delta E \frac{gt}{c^2}. \]  

(4)

Therefore, the energy of the box is measured by the precision

\[ \Delta E = \frac{\Delta p g}{ct}. \]  

(5)

This does not yet contradict Einstein’s conclusion that energy can be measured with arbitrary precision, but the second step considers another effect. Einstein argued that the time of the photon emission can be measured with arbitrary precision, but he did not take general relativity into account, which says that there is a relation between time and position. Namely, on Earth there is a weak gravitational potential \( \phi(x) \approx gx + \text{const} \), so general relativity for weak gravitational fields says that the lapse of time depends on the position \( x \) as

\[ t = \left( 1 + \frac{\phi(x)}{c^2} \right) t_0 \approx \left( 1 + \frac{gx + \text{const}}{c^2} \right) t_0. \]  

(6)
where $t_0$ is time lapsed at the position $x_0$ at which $\phi(x_0) = 0$. It is convenient to choose the additive constant in (6) so that $x_0$ is the position corresponding to the Earth’s surface. This implies that there is uncertainty in time of photon emission from the box due to the uncertainty of the box position

$$\Delta t \approx \frac{\xi \Delta x}{c^2} t_0.$$  

(7)

The third step is to consider the product $\Delta E \Delta t$. Equations (5) and (7) give

$$\Delta E \Delta t \approx \frac{t_0}{t} \Delta p \Delta x \approx \Delta p \Delta x,$$

(8)

where we have used $t_0/t \approx 1$. (The relation $t_0/t \approx 1$ means that the time lapsed at $x_0$ does not differ much from the corresponding time lapsed at $x$. This is because the effects of general relativity are small on Earth, i.e. $\phi(x)/c^2 \ll 1$ in (6).) So far the analysis is classical, but at this step it is legitimate to use the quantum bound $\Delta p \Delta x \gtrsim \hbar$, because Einstein in his argument has not refuted the momentum–position uncertainty relation. This finally gives

$$\Delta E \Delta t \gtrsim \hbar,$$

(9)

in contradiction with Einstein’s conclusion. The crucial step in the derivation of (9), which Einstein did not take into account, was general relativity, i.e. equation (6)).

In summary, to overcome Einstein, Bohr has used Einstein’s own weapon—general relativity. For that reason, it is often considered to be the greatest triumph of Bohr over Einstein. After that point, Einstein stopped trying to prove the inconsistency of QM and instead tried to prove that QM is incomplete.

3. The paradox revisited from a modern point of view

From a modern perspective, Bohr’s reply to Einstein does not look very rigorous. In particular, it does not involve any reference to a state in the Hilbert space. Besides, Bohr’s reply suggests that QM cannot be made self-consistent without incorporating general relativity into it, which is very far from a modern view of QM. (In fact, a consistent incorporation of general relativity into QM is one of the greatest unsolved problems in modern physics.) This motivates us to revisit the Einstein thought experiment from a different, more modern perspective. (By ‘modern’, we mean modern compared to the understanding of QM in 1930.) The discussion we present is quite elementary, but in the context of the Einstein–Bohr 1930 thought experiment it is widely unknown.

3.1. Energy and time uncertainties as properties of the quantum state

Let us first discuss what exactly it means that the photon’s energy is uncertain. In terms of states in the Hilbert space, it means that the quantum state is a superposition of different energies $E = \hbar \omega$, where $\omega$ is the photon’s frequency. For example, if the photon is in a pure state represented by a unit vector in the Hilbert space, then this state can be written as

$$|\psi\rangle = \int d\omega c(\omega) |\omega\rangle,$$  

(10)

where, for simplicity, the spin degrees of freedom of the photon are suppressed. The state is normalized such that $\langle \psi | \psi \rangle = 1$, which implies that

$$\int d\omega |c(\omega)|^2 = 1,$$  

(11)
and \( |c(\omega)|^2 \) is the probability density that the photon has the frequency \( \omega \). The uncertainty of frequency \( \Delta \omega \) is given by
\[
(\Delta \omega)^2 = \langle \omega^2 \rangle - \langle \omega \rangle^2, \tag{12}
\]
where
\[
\langle \omega \rangle = \int d\omega |c(\omega)|^2 \omega. \tag{13}
\]

One way to define the time uncertainty \( \Delta t \) in terms of Fourier transforms is as follows. One first introduces the Fourier transform
\[
\tilde{c}(t) \equiv \int \frac{d\omega}{\sqrt{2\pi}} c(\omega) e^{-i\omega t}, \tag{14}
\]
and then in analogy with (12) and (13) defines
\[
\langle t^n \rangle \equiv \int dt |\tilde{c}(t)|^2 t^n, \tag{15}
\]
\[
(\Delta t)^2 = \langle t^2 \rangle - \langle t \rangle^2. \tag{16}
\]

With such mathematical definitions, in the theory of Fourier transforms it can rigorously be proven that
\[
\Delta \omega \Delta t \geq \frac{1}{2}. \tag{17}
\]
Thus, the quantum relation \( E = \hbar \omega \) implies the uncertainty relation
\[
\Delta E \Delta t \geq \frac{\hbar}{2}. \tag{18}
\]

Another, more physical (but mathematically less rigorous) way to define \( \Delta t \) in QM is to study the behaviour of a physical clock. In this approach, one thinks of \( t \) as an abstract parameter appearing in our theoretical description of dynamics, while the quantity which we really observe is an observable \( Q \), the time-dependence \( Q(t) \) of which is described by the theory. Thus, the actual measurement of \( Q \) can be interpreted as a measurement of abstract time \( t \), so that \( Q \) can be interpreted as the clock observable. In particular, the uncertainty \( \Delta t \) can be expressed in terms of \( \Delta Q \) through the relation
\[
\Delta Q \approx \Delta t \left| \frac{d\langle Q \rangle}{dt} \right|. \tag{19}
\]
where \( \langle Q \rangle \equiv \langle \psi | \hat{Q} | \psi \rangle \) is the quantum-mechanical average of the observable \( Q \). The time derivative of any quantum operator \( \hat{Q} \) with only implicit dependence on time is given by
\[
\frac{d\hat{Q}}{dt} = i\hbar \{\hat{Q}, \hat{H}\}, \tag{20}
\]
where \( \hat{H} \) is the Hamiltonian. Therefore, (19) can be written as
\[
\Delta Q \approx \frac{\Delta t}{\hbar} \{\hat{Q}, \hat{H}\}. \tag{21}
\]
For any two operators \( \hat{Q} \) and \( \hat{H} \) it can rigorously be shown (see, e.g., [22]) that
\[
\Delta Q \Delta H \geq \frac{1}{2} |\{\hat{Q}, \hat{H}\}|, \tag{22}
\]
so (21) and (22) together give the time–energy uncertainty relation
\[
\Delta E \Delta t \gtrsim \frac{\hbar}{2}, \tag{23}
\]
where we have identified the Hamiltonian \( H \) with the energy \( E \).
3.2. Einstein’s thought experiment in terms of quantum states

Equation (10) would be an appropriate description of the state of the photon if the photon was independent of the state of box. However, this is not the case in Einstein’s thought experiment, because the photon is emitted by the box. The total Hamiltonian is an exactly conserved quantity in QM, so the sum of photon energy and box energy must be conserved exactly.

For simplicity, let as assume that the initial energy \( m_0c^2 \) of the box before the emission is not uncertain. Hence, if the photon energy after the emission is equal to \( \hbar \omega \), then exact energy conservation implies that the box after the emission must have energy \( m_0c^2 - \hbar \omega \).

Thus, instead of (10), the total joint quantum state of box and photon after the emission must have the form

\[
|\Psi\rangle = \int d\omega c(\omega) \left( \frac{m_0c^2}{\hbar} - \omega \right) |\omega\rangle,
\] (24)

where \( \left| \frac{m_0c^2}{\hbar} - \omega \right| \) are energy eigenstates of the box and \( |\omega\rangle \) are energy eigenstates of the photon. We see that energy of the photon in (24) is uncertain, just as in (10). Likewise, the energy of the box in (24) is as uncertain as the energy of the photon. And yet, the sum of energies of box and photon is not uncertain at all, because each product state \( |\frac{m_0c^2}{\hbar} - \omega\rangle |\omega\rangle \) has the same energy \( (m_0c^2 - \hbar \omega) + \hbar \omega = m_0c^2 \) not depending on the value of the integration parameter \( \omega \). In modern language, (24) describes the entanglement between box and photon, so that the photon energy is correlated with the box energy.

Now, let us consider the measurement of energy. Einstein was right that, in principle, the energy of the box can be measured with arbitrary precision and that such a measurement also determines the photon energy with arbitrary precision. But what happens when we perform such a measurement? The state (24) performs a transition to a new state

\[
|\Psi\rangle \rightarrow \left( \frac{m_0c^2}{\hbar} - \omega_{\text{meas}} \right) |\omega_{\text{meas}}\rangle,
\] (25)

where \( m_0c^2 - \hbar \omega_{\text{meas}} \) is the the energy of the box obtained by the measurement. The transition (25) is often referred to as collapse of the wavefunction. But this means that the measurement of the box modifies not only the state of the box, but also the state of the photon. It has two important consequences, which we discuss next.

3.3. Consistency of the time–energy uncertainty relation

The first important consequence of (25) is an explanation of how the time–energy uncertainty relation is saved. Before the measurement, the uncertainty of the photon energy \( \Delta E \) is given by the function \( c(\omega) \), through relations (12) and (13), and \( \Delta E = \hbar \Delta \omega \). Likewise, the time uncertainty before the measurement is also given by \( c(\omega) \), through relations (14)–(16).

Therefore, (18) is valid before the measurement. After the measurement the state collapses as in (25), which means that \( c(\omega) \) collapses into a new function

\[
c(\omega) \rightarrow \delta(\omega - \omega_{\text{meas}}).
\] (26)

But (18) is valid for any function \( c(\omega) \), including the distribution \( \delta(\omega - \omega_{\text{meas}}) \) on the right-hand side of (26). (There are some technical subtleties related to a proof that (17) is valid for the distribution \( \delta(\omega - \omega_{\text{meas}}) \), but the details are not important for our discussion. For our purpose, it is sufficient to say that \( \delta \)-distribution can be regularized by a finite-width Gaussian, which allows us to study the limit in which the Gaussian width approaches zero.) Therefore, both \( \Delta E \) and \( \Delta t \) change by the measurement, such that the property (18) remains intact.

Alternatively, if \( \Delta t \) is defined in terms of time measurement by a clock as in equations (19)–(23), then the time–energy uncertainty relation is saved in the following way. The uncertainty
\( \Delta t \) refers to a measurement during which the energy is measured, i.e. during which the collapse (25) takes place. Since \( \Delta E \) is zero (or very small in a more realistic case) due to (25), \( \Delta t \) in (23) is infinite (or very large in the more realistic case). Despite the uniform notation ‘\( \Delta t \)’, this large \( \Delta t \) in (23) is totally unrelated to the small \( \Delta t \) in (2). These two \( \Delta t \)'s refer to two different physical events; the small one is the duration of the box opening, while the large one, relevant in (23), is associated with the measurement of energy.

As we have seen in section 2, neither Einstein nor Bohr recognized in 1930 that the measurement of the box energy after the emission influences the uncertainty \( \Delta E \) of the photon. They both tacitly assumed in this thought experiment that \( \Delta E \) of photon before the box measurement must be the same as \( \Delta E \) of photon after the box measurement. Indeed, in 1930 it seemed absurd to both Einstein and Bohr that some operation performed on the box could have any influence on the decoupled photon travelling away from the box with the speed of light. Nevertheless, they were wrong, for the reason which brings us to the next subsection.

### 3.4. The role of nonlocality

The second important consequence of (25) is nonlocality. Indeed, similar to the argument in the 1935 EPR paper [11], in 1930 Einstein could argue that measurement of the box after the photon emission cannot influence \( \Delta E \) (or any other property) of the photon, because it would imply that some information from the box to the photon should travel faster than light. But today we know that Einstein was wrong [1–8]. The measured properties of one object may be strongly correlated with the measured properties of another object at a large distance from the first, such that no classical communication (involving only signals which do not travel faster than light) between them is possible. We see that such a nonlocal effect is exactly what happens in the Einstein–Bohr thought experiment. The measurement of the box induces the collapse (25), which nonlocally affects the state of the photon.

It is also instructive to compare it with the thought experiment in the EPR paper [11], which is usually considered to be the first example of a nonlocal effect in QM. For that purpose let us briefly present the main ideas of EPR, in a form somewhat simpler than in the original paper. Instead of time–energy uncertainty relation, the EPR paper deals with the momentum–position uncertainty relation

\[
\Delta p \Delta x \gtrsim \hbar.
\]

For that purpose, instead of (24) one deals with the state

\[
|\Psi\rangle = \frac{1}{\sqrt{2}}(|p\rangle - |p\rangle + |p\rangle).
\]

where \(|p\rangle\) and \(|-p\rangle\) are one-particle momentum eigenstates. The momentum of each particle can be either \(p\) or \(-p\), so the one-particle momentum is uncertain for each of the particles. Nevertheless, both terms in (28) have zero total momentum, i.e. \(p + (-p) = 0\) in the first term and \((-p) + p = 0\) in the second. Thus, the total momentum is not uncertain.

Now let us measure momentum of the first particle and position of the second particle. For definiteness, suppose that the momentum measurement of the first particle gives the value \(p\). This means that (28) suffers the collapse

\[
|\Psi\rangle \rightarrow |p\rangle - |p\rangle.
\]

But (29) determines also the momentum of second particle, equal to \(-p\). So we know both the momentum of the second particle (through the momentum measurement of the first particle) and the position of the second particle (through the position measurement of the second particle), in contradiction with (27). From that contradiction, EPR concluded that QM was incomplete.
As is well understood today, the correct resolution of the EPR contradiction is not incompleteness, but nonlocality. Namely, the momentum measurement of the first particle affects also the state of the second particle. Thus, the momentum measurement leading to (29) implies that the state of the second particle is $|\!\!p\!\!\rangle$, which has an infinite position uncertainty $\Delta x$. Consequently, contrary to the EPR argument above, one cannot determine both momentum and position for the second particle. The contradiction obtained by EPR was an artefact of the wrong assumption that measurement on one particle cannot influence the properties of the other.

The first experimental verification of quantum nonlocality was performed in 1972 by Freedman and Clauser [10]. They measured the linear polarization correlation of the photons emitted in an atomic cascade of calcium. Their results were in agreement with the predictions of QM and in contradiction with locality expressed as a variant of Bell inequality derived by Clauser et al [23]. Since then, many other improved experimental verifications of quantum nonlocality have also been performed (see, e.g., [24] for a review), the most famous being the experiment performed by Aspect [25].

In essence, we see that the 1935 EPR argument against the momentum–position uncertainty relation is very similar to the 1930 Einstein argument against the energy–time uncertainty relation. They both involve entanglement between two objects. (The entanglement describes energy correlations (24) in one case and momentum correlations (28) in the other.) They both involve an assumption that measurement on one object cannot influence the properties of the other. They both involve a collapse into a state in which each of the two objects has a definite value of the relevant observable (energy in (25) and momentum in (29)). And finally, to correctly reply to these two arguments against uncertainty relations, in both cases one needs to invoke quantum nonlocality.

Therefore, contrary to the widely held belief, the famous EPR paper was not the first serious challenge to QM that required quantum nonlocality for a correct resolution. Einstein had already presented such a challenge in 1930, five years before the EPR paper.

4. Conclusion

In 1930, Einstein presented a challenge to QM by considering a thought experiment involving a measurement of energy of the box which emitted a photon. Bohr attempted to resolve this challenge by appealing to Einstein’s general theory of relativity. Although it is often presented in the literature as a triumph of Bohr over Einstein, in this paper we have revisited this thought experiment from a modern point of view and found that Bohr’s resolution was not correct. Instead, the correct resolution involves quantum nonlocality originating from the entanglement between box and photon.

Since this 1930 challenge to QM from Einstein was presented five years before the EPR challenge, and since these two challenges are very similar in essence, and since the correct resolution of both involves quantum nonlocality as a crucial ingredient, we conclude that it is incorrect to think of the EPR paper as the first historical example of a quantum system that cannot be understood correctly without invoking quantum nonlocality.

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