Possible violation of spin-statistics connection in electron-electron scattering at low relativistic energies

R. N. Sen
Department of Mathematics
Ben-Gurion University of the Negev
Beer Sheva 84105, Israel
E-mail: rsen@cs.bgu.ac.il
May 1, 2020

Abstract

In 1954, Ashkin, Page and Woodward (hereafter APW) reported on the first counter experiments to measure the $e^--e^-$ and $e^--e^+$ scattering cross-sections at low relativistic energies (0.6–1.7 MeV). Their aim was to look for the spin and exchange or virtual annihilation effects predicted by the Møller and Bhabha formulae. Their experiments confirmed these effects, but the measured cross-sections at 0.61 MeV were significantly smaller than their predicted values. The authors remarked that these deviations were 'presumably due to multiple scattering'. However, careful reading of the unpublished theses of Page (1950) and Ashkin (1952), Page's letter (1951) and the APW paper reveals no credible evidence for multiple scattering at 0.61 MeV; if anything, the evidence rules against multiple scattering. If multiple scattering is ruled out, the observations may indicate a departure from quantum electrodynamics. This departure may be due to a non-Coulomb central force, a weakening of the spin-statistics connection, or both. Only experiment can tell which of these possibilities holds true, and therefore we suggest that new $e^--e^-$ scattering experiments be carried out at different energies (at 0.4–1.0 MeV) and different scattering angles, as well as specific tests for multiple scattering. We consider a non-Coulomb central force to be very unlikely, and advance the hypothesis that a fraction of the electron pairs scatter as spin-zero fermions (which would lower the observed cross-section). Numerical calculations show that this hypothesis may be tested quantitatively in a Page-type experiment, even with little improvement in the accuracy he achieved in 1950.

1 Introduction

The electron-electron ($e^--e^-$) scattering cross-section first attracted interest owing to its role in determining the penetrating power of fast electrons (from cosmic rays and radioactive substances) in their passage through matter [1]. Shortly after Møller published his celebrated formula in 1932 [2], Champion – who had been corresponding with Møller – published the results of his cloud-chamber experiments on the subject; his finding was that Møller's formula fitted the data better than five other candidates [3]. Champion did not have sufficient data to make a stronger assertion; the Møller formula contains terms which arise from spin and exchange, but their effects are most pronounced at large scattering angles which are found only in a very small fraction of scattering events. Therefore in the late 1940s Page and Ashkin, graduate students of Woodward at Cornell, carried out two separate experiments at energies between 0.6–1.7 MeV and 0.6–1.2 MeV respectively, using coincidence counters arranged to detect only large-angle
scattering events. Details of these experiments were presented in the Ph D theses of Page (1950) [4] and Ashkin (1952) [5], which remained unpublished, except for a brief letter by Page in 1951 [6]. A summary of their results on $e^-e^-$ and $e^-e^+$ scattering was published in 1954 in a joint paper by Ashkin, Page and Woodward (hereafter APW) [7].

The results left no doubt that, at large scattering angles, spin and exchange (or spin and virtual annihilation, for $e^-e^+$ scattering) modified the pure Coulomb scattering cross-section significantly, as had been suggested by Oppenheimer [8] and Mott [9] in the early years of quantum-mechanical scattering theory.

![Figure 1. $e-e$ scattering cross-section at $x = 0$ from collodion ($\bullet$) and beryllium ($\circ$) foils (after Ashkin, Page and Woodward [4])](image)

The results obtained with the ‘270°-apparatus’ used by Page (Fig. 5 of [7]) are reproduced in Fig. 4 (Ashkin used mylar foils for his experiments, but his results on $e^-e^-$-scattering agreed with those shown in Fig. 1; see Fig. 6 of [7].) Notice that the observed value at 0.6 MeV lies definitely below the Møller curve (solid line). This discrepancy will be the centre of our attention because, if it cannot be explained by experimental errors and/or multiple scattering, then it would imply a breakdown of QED at this energy. APW state that this discrepancy arose ‘presumably because of multiple scattering’. (The same discrepancy was observed at the same energy for $e^-e^+$ scattering by Ashkin; APW use exactly the same phrase to explain it.) In Sec. 2 we shall examine the evidence presented by APW in some detail by going back to the original sources [4, 5, 6], and shall conclude that the empirical data do not support the multiple scattering hypothesis. Therefore there is a very strong case for new experimental studies of the $e^-e^-$ cross-section at energies between 0.4 to 1.0 MeV.

The reader may wonder why a gap of such profound theoretical consequence has not been closed by experiment in the last 65 years. It would be presumptuous of the present author, who is not a historian of physics, to try to answer this question. He can only suggest a few references that capture some of the excitement, and record some of the concerns of physicists of the time: (i) the definitive history of QED by Schweber [10], (ii) the book by Bethe and de Hoffman for the state of ‘meson physics’ in 1954 [11], and finally (iii) Dyson’s account of how he put together his key paper ‘The radiation theories of Tomonaga, Schwinger and Feynman’ [12] in a Greyhound bus, given towards the end of chapter 6 of his memoirs [13]. Dyson’s paper predated the publications of Schwinger and Feynman!

The rest of this paper is organised as follows. In Sec. 2 we recall the scattering formulae with which we shall be concerned. They include the Møller formula, the formulae for the scattering cross-sections of two spin-zero fermions [sic!] and two spin-zero bosons of equal charge, and the Bhabha formula – suppressing some details – for $e^-e^+$ scattering [14]. We provide enough material to make the account more or less self-contained. In Sec. 3 we assemble some data from Page’s thesis and letter (pointing out a significant inconsistency between the two) and Ashkin’s thesis, with emphasis on the multiple scattering problem. We analyse the data in Sec. 4 and conclude that, contrary to the presumption of APW, the evidence for multiple scattering at 0.61 MeV, both in $e^-e^-$ and $e^-e^+$ scattering, is lacking. We also suggest a direct test for mul-

---

1Ashkin’s chief aim was to measure the ratio of $e^-e^-$ and $e^-e^+$ scattering cross-sections, which could be determined more accurately than either absolute cross-section.

2Photographic reproduction having proven unsatisfactory, we have redrawn the figure as best as we could.

3It would be convenient, for our purposes, to distinguish between multiple scattering and other sources of error.
tiple scattering. In Sec. 5 we try to devise an alternative explanation, and advance the hypothesis that with decreasing energy, an increasing fraction of the electron pairs scatter as spin-zero fermions (Hypothesis I, eq. (11)). In Sec. 4 we make numerical estimates of the experimental accuracy that would be required to put this hypothesis to test in off-foil scattering experiments, and conclude that it is indeed possible to do so even with little improvement on Page’s experiment of the late 1940s. (A further test for multiple scattering emerges in the process.) In section 7 we give some details about the colliding beam experiment of Williams et al (published 2014); an experiment with suitable beams may greatly reduce the possibility of errors due to multiple scattering, but we are unable to assess its feasibility at low relativistic energies. In the next section we comment on the far-reaching theoretical implications of hypothesis (11) being validated by experiment. In the last section we provide a brief sketch of a notion of ‘small’ violations of the Pauli principle in bound states and of the ongoing experiments to detect such violations, with references for the interested reader. An appendix gives the numerical tables on which the estimates of Sec. 6 are based.

2 Scattering formulae

When a particle of positive mass is scattered by another at relativistic energies, the differential cross-section for the process is most easily calculated in the centre-of-momentum system. One would expect that it has then to be transformed to the laboratory frame, in which the target particle is at rest, for use by the experimentalist. This transformation can be quite complicated in relativistic kinematics, even when both particles have the same mass. But Møller made the felicitous observation that it was not necessary to carry out this transformation; the scattering angle in the centre-of-momentum frame was very simply related to the fractional energy transfer – which could be observed directly – from the incident to the target particle, and he communicated his results by letter to Champion, who was performing the experiment (see [15]).

Champion wrote down Møller’s formula as follows (see [2], eq. 2); he noted with apparent surprise that Planck’s constant appears nowhere in it!

\[
\sigma'_M = \frac{d\sigma_M(x)}{dx} = 4\pi \left( \frac{r_0}{\beta v} \right)^2 \frac{2 + \gamma + 1}{ \gamma^2 } \left[ \frac{4}{(1 - x^2)^2} - \frac{3}{1 - x^2} \right] + \left( \frac{\gamma - 1}{4\gamma^2} \right) \left( 1 + \frac{4}{1 - x^2} \right)
\]

(1)

In the above, the independent variable \(x\) is the cosine of the scattering angle \((x = \cos \theta_{cm})\) in the centre-of-momentum system. The quantity \(\sigma'_M\) (M for Møller) is formally the \(x\)-derivative of the total cross-section \(\sigma_M\) in the laboratory system, \(r_0 = e^2/mc^2\) is the classical radius of the electron (in CGS units), \(v\) is the velocity of the incident electron, \(\beta = v/c\) and \(\gamma = (1 - \beta^2)^{-1/2}\). Finally, \(x\) is related to \(\theta\), the scattering angle in the laboratory system, by the complicated expression

\[
x = \cos \theta_{cm} = \frac{2 - (\gamma + 3) \sin^2 \theta}{2 + (\gamma + 1) \sin^2 \theta}
\]

(2)

where \(\theta_{cm}\) is the scattering angle in the centre-of-momentum system. What makes (1) usable by the experimentalist is Møller’s finding, mentioned above: the variable \(x\) may be expressed quite simply in terms of \(w\), the fraction of kinetic energy of the incident electron transferred to the target electron (assumed at rest)\(^4\)

\[
x = 1 - 2w
\]

(3)

If one calculates the same cross-section for a pair of spin-zero fermions \(\uparrow\uparrow\) – a physical impossibility if the spin-statistics theorem holds – one finds the same formula, but without the term

\[
\left( \frac{\gamma - 1}{4\gamma^2} \right) \left( 1 + \frac{4}{1 - x^2} \right)
\]

(4)

inside the square brackets in (1). This term “may thus be considered to be the contribution made by the spin” of the electron ([16], page 817). If the last two terms in the square brackets in (1) are dropped, what remains may be called the relativistic Rutherford formula with \(Z = Z^' = 1\) and \(m = m^'\); we shall

\(^4\)See Møller’s original article [2], page 569, the second unnumbered equation between the two equations each numbered (75').
denote this cross-section by $\sigma'_{b}$. Likewise, we shall denote the cross-section for the scattering of two charged spin-zero fermions by $\sigma'_{f}$.

Spin-zero bosons (szb) exist in nature. The electrodynamics of spin-zero particles (scalar electrodynamics) was constructed in 1950 (see [14] [15]). The lowest order scattering cross-section for two such particles of equal charge – in the laboratory system – turns out to differ from the Møller formula (2) only in the last two terms contained in the square brackets (see [19], page 286). The full formula is

$$\rho(E,x) = \frac{\frac{d\sigma_{M}}{d\Omega}}{\frac{d\sigma_{B}}{d\Omega}} = \frac{\sigma_{M}(x)}{\sigma_{B}(x)}$$

(8)

In the above, [Bhabha] consists of three separate terms which are functions of $x$ rather than $x^2$, as the two particles are not identical. We shall not write them down explicitly; the full formula may be found on p. 260 of [20]. Bhabha identified the first of these with the scattering of two spin-$\frac{1}{2}$ particles of opposite charge that are not antiparticles of each other. The second is the virtual annihilation term, and the third arises from interference between the two. All three are functions of $x$ and $\gamma$ (or $E$) only, and involve no physical constants. The ratio remains well-defined as $x \to 1$, unlike the individual cross-sections.

2.1 Nonrelativistic limits

All formulae for scattering cross-sections given above contain $r_0$ and $\beta$ in the combination $r_0/\beta^2 = e^2/mv^2$, independent of $c$. Therefore calculation of the nonrelativistic limit $c \to \infty$ is reduced to the substitution $\gamma = 1$. In this limit the Møller formula (1) loses the last term in the square brackets, and the resulting formula is identical with the (NR limit of) the scattering formula for two spin-zero fermions derived by Mott. Similarly, formula (5) for the scattering of two spin-zero bosons (of the same mass and charge) reduces to the Rutherford formula. At nonrelativistic energies, (i) an electron-electron scattering experiment cannot distinguish between spin-zero and spin-half fermions, and (ii) a scattering experiment cannot distinguish between two scalar bosons and two classical particles. The unwritten term [Bhabha] in (7) reduces, likewise, to

$$\frac{\sigma_{M}(x)}{\sigma_{B}(x)}_{NR} = \frac{1 + 3x^2}{(1 + x)^2}$$

(9)
3 The experiments of Page and Ashkin

In their paper, APW give a general overview of the experiments of Page and Ashkin. At the end of page 360 of [7], they say: ‘Care was taken to avoid errors due to multiple scattering in the scattering foil’, and describe tests to detect its presence. In this section we shall provide further details taken from the unpublished theses of Page [4] and Ashkin [5]. We shall also refer to the figure in Page’s brief communication [6]. The aim is to prepare the ground for a more detailed discussion, in Sec. 4, of the effect of multiple scattering on the observed cross-sections.

3.1 Page’s thesis and letter

Page’s letter [6] contradicts his thesis in a very significant manner. We shall discuss this after presenting some of his conclusions on the multiple scattering problem.

3.1.1 Page on the multiple scattering problem

Page measured the $e^- - e^-$ scattering cross-sections at 0.6, 0.8, 1.0 and 1.2 MeV with both beryllium and collodion foils, of densities 4.5 mg/cm$^2$ and 0.5 mg/cm$^2$ respectively. Since nothing except the foil was changed during the experiments, the ratio of the count rates $C_{\text{Be}}/C_{\text{coll}}$ at any energy should have depended – had there been no multiple scattering – only on the ratio of the densities of scatterers in the two foils, which was a constant. Page’s plot of $C_{\text{Be}}/C_{\text{coll}}$ against energy (his Fig. 4’) is shown as the upper graph in our Fig. 2. Had there been no multiple scattering, these points would have lain on the horizontal line marked in the figure. Page’s comment on it is as follows (p. 41 of his thesis, emphasis added):

‘In the absence of...precise formulas for multiple scattering (and clearly any side experiment aiming at such determination could easily dwarf the main experiment here)...one should simply accept the observed coincidence rate ratios (Fig. 4’) between thick and thin foils as the guide whereby certain of the data is rejected on nuclear scattering grounds.’

The evidence presented in Page’s Fig. 4’ clearly shows that, relative to the collodion foil, multiple scattering effects become progressively more important with decreasing energy in the beryllium foil. Use the word ‘nuclear’ rather than ‘multiple’ in the last sentence may require justification, but the point is not relevant to our discussion, because Page’s final data at the lower energies (0.6, 0.8 and 1.0 MeV) were obtained using the collodion foil.

[Image: Figure 2. Page’s Fig. 4 (lower figure) and Fig. 4’ (upper figure)]

Page’s Fig. 4 and Fig. 4’ are reproduced in our Fig. 2. (Note that Page expresses $E$ as multiples of $mc^2$, i.e., in units of 0.511 MeV.) All the observed points on his his Fig. 4 are close to the Möller curve. On page 42 of his thesis, he says:

‘...which implies that all the data of Fig. 4 is for all practical purposes free of multiple scattering loss.’
However, the last assertion was contradicted (implicitly) by his letter, and is discussed below.

3.1.2 The inconsistency

The raw data obtained by Page were used to draw three different energy vs cross-section graphs: (i) Fig. 4 in Page's thesis (lower graph of our Fig. 3), (ii) Fig. 2 in Page's letter [6] (which we have not reproduced) and (iii) APW's Fig. 5 (our Fig. 1). Even a quick glance shows that (i) and (iii) do not agree. In (i), the data points seem to be shifted upwards with respect to those in (iii), and the points at the two highest energies lie clearly above the Møller curve. The graphs in (ii) and (iii) do agree with each other. The remark from p. 42 of Page's thesis quoted above is not applicable to the graphs in (ii) and (iii), and is inconsistent with statements in APW. This suggests that at least one of the two formulae used for converting count rates to cross-sections given on pages 56 and 57 of Page's thesis was slightly modified to arrive at the graphs in (ii) and (iii). But no explanation is offered either in Page’s letter [6] or in APW; there is no reference to this inconsistency in either publication.

In view of this unexplained inconsistency, we shall use the graphs in (ii) and (iii) (our Fig. 1) as describing the results of Page’s experiments, rather than the one presented in his thesis, except when stated otherwise.

3.2 Ashkin’s thesis

Ashkin’s apparatus was similar to Page’s, but somewhat larger, to allow for lead shielding. (His positron source, Co$^{56}$, emitted three $\gamma$ rays for every positron.) Exigencies of shielding also restricted the trajectories of the scattered particles to $180^\circ$, as opposed to Page’s $270^\circ$. For $e^-e^+$ scattering, the magnetic field had to be reversed and one of the counters moved to the other side of the partition. (See Fig. 1 of Ashkin’s thesis or Fig. 3 of APW.) Thus the $e^-e^-$ scattering experiments of Page and Ashkin could be considered ‘essentially different’. Note that Ashkin used Mylar foils of densities 1.7 mg/cm$^2$ and 0.9 mg/cm$^2$ as scatterers (p. 14 of [5]).

Ashkin measured the $e^-e^-$ scattering cross-section at $E = 1.220$, 1.019, 0.818 and 0.611 MeV and $x = 0$. At the two higher energies, he used only the thicker foil, at 0.611 MeV, only the thinner foil, and both foils at 0.818 MeV. At this energy, the cross-section obtained with the thinner foil was 4.7% larger than that obtained with the thicker one, which he took as evidence of multiple scattering in the thicker foil. At 0.611 MeV, the value he obtained ($2.60 \times 10^{-24}$ cm$^2$) was about 3% smaller than the one obtained by Page ($2.69 \times 10^{-24}$ cm$^2$) with the 0.5 mg/cm$^2$ collodion foil and presented in his thesis (see Table II, p. 14 of [6] and Table III, p. 14 of [4]).

But, as we have noted earlier, the calculated cross-sections given in Page’s thesis and used in his Fig. 4 (our Fig. 2) do not agree with Fig. 5 of APW (our Fig. 1). Ashkin’s results are shown in graphical form in Fig. 6 in APW. This can be compared visually with Fig. 5 of APW, which is based on Page’s data. One sees that, at 0.6 MeV, the cross-section found by Ashkin is, if anything, *slightly larger* than the one shown in Fig. 5 of APW. As APW state in the penultimate paragraph on page 360, the results of Ashkin ‘agreed well’ with those of Page.

Ashkin also determined the ratio $\rho(E, x)$ of $e^-e^-$ and $e^-e^+$ cross-sections (8) at $x = 0$ and $E = 1.02$, 0.82 and 0.61 MeV directly from the coincidence counts, and compared the observed values with those calculated from Møller’s and Bhabha’s formulae. (Indeed, this was the main aim of his thesis.) He observed that most of the experimental errors, due to limitations of the apparatus, would cancel each other in the ratio. (We shall exploit this observation in Sec. 4.2.)

Even the 0.61 MeV point lies right on the theoretical curve. *This serves to substantiate the supposition that both absolute numbers for the $[e^-e^-$ and $e^-e^+]$ cross-sections at this energy were low due to multiple scattering* [our emphasis].

The same claim was repeated in APW. Neither Ashkin nor APW offered any justification for it. We shall examine it in Sec. 4.2.
4 The multiple scattering problem

The experiments of Page and Ashkin were based on the implicit assumption that single scattering dominates over other processes. Their results amply justify this assumption. Fig. 1 also suggests that the ratio of the observed $e^- e^-$ cross-section to the calculated Møller cross-section decreases with decreasing energy below 1.0 MeV. APW and Ashkin suggest that this effect (which turned out to be of little relevance to their main aim of verifying the existence of the exchange/virtual annihilation terms in the cross-sections for Møller/Bhabha scattering respectively) may be due to multiple scattering. The evidence they offered was of two kinds: (i) the effect of foil thickness, and (ii) the equality of observed and calculated ratios of the $e^- e^-$ and $e^- e^+$ cross-sections at certain energies. We shall consider them separately.

4.1 Effect of foil thickness

We shall now bring together the relevant data from the original sources, and set down the conclusions that may be drawn from them.

1. At 0.82 MeV and $x = 0$, the cross-section measured with a 1.7 mg/cm$^2$ Mylar foil is 4.7% smaller than that measured with a 0.9 mg/cm$^2$ Mylar foil (Ashkin [5]). This indicates that the influence of multiple scattering increases with increasing thickness of foils of the same material.

2. At 0.61 MeV and $x = 0$, the cross-section measured with a 0.9 mg/cm$^2$ Mylar foil by Ashkin [5] is 3% smaller than that measured with a 0.5 mg/cm$^2$ collodion foil by Page (as reported in his thesis [4], Table III, page 14). However, as pointed out earlier, the data of Page’s Table III are inconsistent with Fig. 5 of APW, and a visual comparison of Figs. 5 and 6 of APW suggests that the cross-section obtained by Ashkin with the 0.9 mg/cm$^2$ Mylar foil is, if anything, slightly larger than that obtained by Page with the 0.5 mg/cm$^2$ collodion foil.

Item 1 of the above is supportive of the idea that multiple scattering reduces the observed cross-section, at least in the thicker foil. Item 2 is more problematic; if we accept that APW overrides Page’s thesis, the near-equality of the cross-section measured with two different foils, one of which is nearly twice as dense as the other, would seem to indicate the absence of multiple scattering in both the experiments. Then the low value of the cross-section (at 0.61 MeV) remains to be explained. It should be emphasized that, in both cases, the data are too meager to draw hard conclusions.

4.1.1 Test for multiple scattering, I

The experiment, which can be a variant of Page’s experiment, would be carried out at fixed $E \approx 0.5$–0.6 MeV and $x = 0$, but with foils of different thicknesses made out of the same material, the thinnest having a lower density than Page’s collodion foil. All other parts of the apparatus will remain unchanged. The idea would be to obtain enough points in a plot of the cross section against (the decreasing) foil density to detect whether or not the cross section tends to level off at lower foil thicknesses. The hypothesis of dominance of single scattering would suggest that the cross section should level off with decreasing foil density. If it levels off at the Møller value, it would be strong evidence that multiple scattering alone suffices to explain the phenomenon. On the other hand, if it levels off at a value smaller than the Møller value, it would be strongly suggestive of a different mechanism, such as the one to be considered in Sec. 5, being active either alternatively or simultaneously.

The feasibility of the experiment would depend on the fabricability of suitable scattering foils. The range of thicknesses should be such that the levelling-off effect is observable. If such foils can be made, the experiment would be a decisive one.

4.2 Equality of observed and calculated ratios of $\sigma'_M/\sigma'_B$

Recall that Ashkin’s main aim was to determine the ratios $\rho(E, x)$ of the Møller and Bhabha cross-sections directly from the count ratios (using the same apparatus); many of the errors in the determination of the absolute cross-sections would cancel out in the ratio. If all errors cancelled out, the observed ratio would equal the calculated ratio. As pointed out in Sec. 4.2, the agreement was near-perfect at $E = 1.02$ and 0.61 MeV (at $x = 0$),
and very good at \( E = 0.82 \text{ MeV} \). We shall analyze Ashkin's 'supposition' that this near-equality explains the low observed values for the \( e^- e^- \) and \( e^- e^+ \) cross-sections at \( E = 0.61 \text{ MeV} \). Our analysis will be based on the observation that the 'dominance of single scattering' implies that corrections to single scattering will, in turn, be dominated by a single, second collision.

The cross-sections \( \sigma_{\text{M}}(E, x) \) and \( \sigma_{\text{B}}(E, x) \) diverge at \( \theta = 0 (x = 1) \) and then decrease very rapidly with increasing \( \theta \) (decreasing \( x \)). As a result, a particle that suffers a second collision in the foil will most probably be deviated only very slightly from its path, which may not be enough to prevent it from striking the counter. Only particles that suffer larger, less-probable deviations will avoid striking the counter. Owing to the divergence of the cross-section, the fraction of particles (electrons or positrons) that are prevented from reaching the counter due to a second collision in the foil cannot be estimated. However, the ratio \( \rho(E, x) \) of the \( e^- e^- \) and \( e^- e^+ \) cross-sections defined by (9) remains finite as \( x \to 0 \), and we can make a rough estimate of the effect of a second collision using this ratio – and the geometry of the apparatus – as follows.

We assume that an incoming electron or positron of energy \( E \) is scattered by an electron in the foil (considered at rest), imparting half its kinetic energy to the latter – i.e., \( x = 0 \) for the scattered particles. Next, one of the scattered particles suffers a second collision in the foil which changes its direction of flight by a small angle \( \theta_s \) which does not materially affect its time-of-flight to the counter. (The variables referring to the second collision will be distinguished by the subscript 's'.) If all second collisions scatter by the angle \( \theta_s \), their effect will be to multiply the Møller cross-section \( \sigma_{\text{M}}(E, x = 0) \) by a factor proportional to \( \sigma_{\text{M}}(E_s, x_s) \), where \( E_s = E/2 \), \( x_s = 1 - \delta \) and \( \delta \), only slightly greater than zero, is determined by the geometry of the apparatus. For positron scattering, the Bhabha cross-section \( \sigma_{\text{B}}(E, x = 0) \) will be modified by a factor proportional to \( \sigma_{\text{B}}(E_s, x_s) \). If, as in Ashkin’s experiments, the same apparatus is used for both experiments, the two proportionality factors should be the same. We may therefore assume that the ratio \( \rho(E, x = 0) \) will be multiplied by the factor

\[
F(E_s, x_s) = \frac{\sigma_{\text{M}}(E_s, x_s)}{\sigma_{\text{B}}(E_s, x_s)} \tag{10}
\]

where \( E_s = E/2 \) and \( x_s = 1 - \delta \).

**Figure 3.** Graphs of the ratio \( F(E_s, x_s) \) vs \( x_s \) for \( x_s \in [0, 1] \) and \( E_s = 0.0, 0.2, 0.3, 0.4 \) and 0.5 MeV [in grey scale, bottom to top on the \( x_s = 0 \) axis].

**Figure 4.** Graphs of the ratio \( F(E_s, x_s) \) vs \( x_s \) for \( x_s \in [0.95, 1] \) and \( E_s = 0.0, 0.2, 0.3, 0.4 \) and 0.5 MeV. Fig. 4 is a magnification of the above in the range \( x_s \in [0.95, 1] \). Except at \( x_s = 0 \), where they all meet, the graphs are totally disjoint, with those for higher \( E \) lying entirely above those for lower \( E \). The case \( E = 0 \) corresponds to the nonrelativistic limit, and its graph is that of the function (8). Except for \( E = 0 \), the graphs decrease monotonically from \( x_s = 0 \) to \( x_s = 1 \).
The scattering angle \( \theta_s \) (in the laboratory system) can be calculated for given \( x_s \) from (2). Table 1 shows the values of \( \theta_s \), in degrees, calculated using (2) for \( x_s = 0.995, 0.99, 0.98, 0.97 \) and 0.95 and \( E_x = 0.2, 0.3, 0.4 \) and 0.5 MeV. Recall that \( E_x = E/2 \), where \( E \) is the energy of the electron or positron incident upon the foil.

\[
\begin{array}{c|cccc}
\hline
 x_s & E_x, \text{ in MeV} \\
\hline
   & 0.2 & 0.3 & 0.4 & 0.5 \\
0.995 & 2.62 & 2.52 & 2.43 & 2.35 \\
0.990 & 3.71 & 3.57 & 3.44 & 3.32 \\
0.980 & 5.25 & 5.05 & 4.87 & 4.71 \\
0.970 & 6.44 & 6.19 & 5.97 & 5.55 \\
0.950 & 8.33 & 8.01 & 7.71 & 7.48 \\
\hline
\end{array}
\]

**Table 1.** The scattering angle \( \theta_s \) (in degrees) as a function of \( E_x \) for \( x_s = 0.995, 0.990, 0.980, 0.970 \) and 0.950

Finally, let us consider the geometry of the apparatus. The counters used by Ashkin were made of 0.8-inch (about 2 cm) square tubing ([7], p. 359). A very rough estimate based on Fig. 1 of Ashkin’s thesis shows that, for \( x = 0 \), the more energetic particles follow a trajectory about 30–40 cm long before reaching the counters. In the plane, a circular arc of length 2 cm subtends an angle of about 4° at 30 cm, and we shall not be far off in assuming that this remains true even for the helical paths that the particles traverse in the experiment. That is, the angular aperture of the detector at the point of scattering is about 4°. (For less energetic particles, the angular aperture will be smaller if the same counters are used.) Particles that are scattered *into* this angular aperture are counted; those that are scattered *out of* it are not.

We shall now try to put together the assumptions made and information obtained so far. The key assumption is that the effect of multiple scattering can be well approximated by the more precisely quantifiable idea of double scattering. The effect of a second collision that changes the direction of flight of a particle by an amount determined by \( x_s \) is to multiply the ratio \( \rho(E, x) \) defined by (2) by the factor \( F(E_x, x_s) \) defined by (10). To take all possible directions into account in the second scattering, we have to average \( F(E_x, x_s) \) over the interval \([0, x_s]\). This average is simply the area under the curve divided by the length of the interval. From Table 1 and the angular aperture of the counter estimated earlier, we see that for \( x_s \) around 0.99 or less, almost all the particles being scattered a second time would be reaching the counter. We now see from Figs. 3 and 4 that the contributions of the even larger interval \([x_s, 1]\) to the areas under the curves for \( E_x \geq 0.2 \) MeV is entirely negligible. Therefore, for each \( E_x \), the weighting factor becomes the area under the curve in the interval \([0, 1]\). These areas have to be computed numerically for each \( E_x \).

However, we do not have to carry out these numerical integrations. It is clear from Figs. 3 and 4 that \( F(E_x, x_s) \) is always greater than unity. Therefore the area under the curve will be significantly greater than unity, and *will increase significantly with increasing* \( E_x \).

\[
\begin{array}{c|c|c|c}
E, \text{ MeV} & \rho_{\text{obs}} & \rho_{\text{calc}} & \rho_{\text{obs}}/\rho_{\text{calc}} \\
\hline
1.02 & 3.74 \pm 0.49 & 3.76 & 1.0053 \\
0.82 & 3.29 \pm 0.19 & 3.42 & 1.0395 \\
0.61 & 3.09 \pm 0.22 & 3.08 & 0.9968 \\
\hline
\end{array}
\]

**Table 2.** Observed and calculated values of \( \rho \) (Ashkin) and their ratio

Table 2 gives the ratios \( \rho(E, x = 0) \) as observed and calculated by Ashkin, as well as the ratios \( \rho_{\text{obs}}/\rho_{\text{calc}} \). According to our estimates, this ratio, being the area under the curve of \( F(E_x, x_s) \), should be (i) significantly greater than unity, and (ii) an increasing function of \( E_x \). Ashkin’s data meet neither of these conditions. While three data points are not enough to reach a definitive conclusion, the data provided *certainly do not suggest* that the low values of the observed \( e^-e^- \) and \( e^-e^+ \) cross-sections are the results of multiple scattering. They may even suggest the contrary: that the near-equality of the observed and calculated ratios at \( E = 0.61 \) and 1.02 MeV are indicative of the insignificance of multiple scattering! But this putative conclusion is challenged by the data at \( E = 0.82 \) MeV.
4.3 Summary of this Section

We may sum up the discussion in this Section as follows:

1. Variation of the cross-sections with foil thickness – the foils being of the same material – does suggest the possibility of multiple scattering, while not ruling out other explanations. Experiments with foils of different thicknesses but the same material may be able to discriminate between multiple scattering and other possible causes.

2. The cross-section at 0.61 MeV, measured with a mylar foil of density 0.9 mg/cm$^2$, is about equal or slightly larger than the one measured with collodion foil of density 0.5 mg/cm$^2$. This suggests that multiple scattering is no longer effective at these densities. Then the fact that the measured cross-section is lower than its Møller value remains unexplained. The data are too meager to draw firm conclusions.

3. The data presented by Ashkin on the comparison of observed and calculated values of $\sigma'_M/\sigma'_B$ are not sufficient to establish a reason for the observed low values of $\sigma'_M$ and $\sigma'_B$ at $E = 0.61$ MeV and $x = 0$. The near-equality of the observed and calculated ratios at 0.61 and 1.02 MeV may even be taken to mean that multiple scattering is not significant at either energy, but the data are too meager to draw firm conclusions.

We need more experimental data to draw any conclusion.

5 The search for alternatives

If further experiments show that the low-energy large-angle scattering cross-sections are unquestionably smaller than their Møller values, what could be the possible explanations?

The electron-electron interaction contains three factors which combine seamlessly in QED: the Coulomb repulsion, the spin of the electron, and its statistics. However, the last two factors contribute separately to the Møller formula; the first term in the square brackets in (1) is the Rutherford (Coulomb) term, the second the exchange and the third the spin term. From a purely phenomenological point of view, one may therefore consider modifying the effects of these factors separately.

Modifying the Coulomb repulsion may be a step too radical; we consider it to be currently unwarranted. (See also Sec. 7.) Leaving the Coulomb interaction untouched would require modifying the effects of exchange, or spin, or both. Here the Møller formula itself offers a pointer: If the last term in the big square brackets in (1) is dropped, what remains is the formula for the scattering of two spin-zero fermions. If only the first term in the square brackets in (1) is kept, we obtain the (relativistic) Rutherford formula for equal charges and masses. Could it be that some (energy-dependent) fraction of the particles scatter according to the Møller formula, but the rest scatter as classical particles, as spin-zero bosons or even as spin-zero fermions?

![Graph 5](image)

Figure 5. From top to bottom: graphs of $R(E)$, $B(E)$, $M(E)$ and $F(E)$ for $x = 0$. The four graphs are pairwise disjoint.

Graphs of the terms in square brackets in the cross-sections for (i) Rutherford scattering $R(E)$, (ii) the scattering of two spin-zero bosons $B(E)$ (formula (5)), (iii) Møller scattering $M(E)$, and (iv) that of two spin-zero fermions $F(E)$, between $E = 0.3$ and 0.7 MeV at $x = 0$, are shown in Fig. 5. (The factor outside the square brackets is common to all four.) One sees immediately that the Rutherford and spin-zero boson cross-sections are always significantly larger than the Møller cross-section. Only the spin-zero fermions have a scattering cross-section which is smaller than the Møller! We therefore assume that

Hypothesis I. An observed cross-section $\sigma'_{\text{obs}}$
which is smaller than the Møller cross-section may be represented as

\[ \xi \sigma'_\text{obs} = (1 - \alpha) \sigma'_M + \alpha \sigma'_F \]  \hspace{1cm} (11)

where \( \sigma'_F \) is the cross-section for the scattering of two spin-zero fermions, \( \alpha = \alpha(E) \in (0, 1) \) is a parameter which depends of \( E \) but not on \( x \), and \( \xi = \xi(E) \) is a small correction factor \( (\xi(E) = 1 + \delta(E)) \) which accounts for the loss due to multiple scattering.

Note that in the nonrelativistic limit

\[ \sigma'_M = \sigma'_F \]

so that \( \sigma'_\text{obs}(E \to 0) \) is independent of \( \alpha \).

**Remarks II.** The right-hand side of (11) can be written as the right-hand side of the Møller formula (1), with the third term in square brackets being replaced by

\[ (1 - \alpha) \left( \frac{\gamma - 1}{4 \gamma^2} \right) \left( 1 + \frac{4}{1 - x^2} \right) \]  \hspace{1cm} (12)

where \( \alpha \) is the same as in (11). Alternatively, the extra factor \( (1 - \alpha) \) can be absorbed in \( \gamma \), as follows:

\[ \left( \frac{\gamma'}{4 \gamma'^2} \right) \left( 1 + \frac{4}{1 - x^2} \right) \]  \hspace{1cm} (13)

where \( \gamma'(< \gamma, \alpha) < \gamma \). Neither of these forms have as transparent an interpretation as (11); we shall return to them briefly in Sec. 8.

We shall now make some numerical estimates concerning the feasibility of subjecting the above hypothesis to experimental tests.

## 6 Feasibility estimates

The essential part of the model defined by (11) is that \( \xi \) and \( \alpha \) depend on \( E \) but not on \( x \). For given \( E \), measurement of \( \sigma'_\text{obs} \) for two different values of \( x \) will determine \( \xi(E) \) and \( \alpha(E) \) for that \( E \); measurements of \( \sigma'_\text{obs} \) for the same \( E \) but other values of \( x \) will then serve to test the hypothesis I; if the \( \xi \) and \( \alpha \) so determined fit the data for the same \( E \) but other \( x \), hypothesis I may be accepted provisionally; if not, it must be rejected.

To test (11) for given \( E \), one has to measure \( \sigma'_\text{obs}(x, E) \) for different values of \( x \). On the one hand, this should be done for as many values \( x \) as possible to accept or reject the hypothesis I with confidence; on the other hand, adjacent values of \( x \) for which \( \sigma'_\text{obs}(x, E) \) is measured should be separated enough to distinguish between the corresponding values of \( \sigma'_\text{obs}(x, E) \). These two requirements are in conflict, and the feasibility of the experiment will depend on whether they can be met simultaneously within the experimental error.

One more factor has to be taken into account. At small angles (those utilised for transmission electron microscopy, TEM; see [21, 22]) the \( e^-e^- \) scattering cross-sections are orders of magnitude greater than those at large angles (say for \( x = 0 \), as in the APW experiments). An experiment to detect possible departures from the Møller formula would have to be like the APW experiments – in which individual scattering events are detected and counted – and not like TEM runs in which images are formed by scattering events too numerous to count; the scattering angles at which the cross-sections are measured would have to be significantly larger than those used in TEM.

Numerical calculations show that for \( E \) from 0.3 to 0.7 MeV and \( x \) from 0.10 to 0.40, the scattering angle \( \theta \) lies between 36.6° and 25.0°. These angles should be far enough from TEM regimes for individual events to be detectable.

Let \( \sigma'_\text{obs}(x, E) \) be measured at \( x = x_0, \ldots, x_n \), with \( x_k < x_{k+1} \). To simplify the notation, write

\[ \sigma'_k(E) \text{ for } \sigma'_\text{obs}(x_k, E) \]

Then, to distinguish between \( \sigma'_k(E) \) and \( \sigma'_{k+1}(E) \), the error in the measurement of \( \sigma'_k(E) \) (or \( \sigma'_{k+1}(E) \)) must be discernibly less than

\[ \Delta_k(E) = 100 \left( \frac{\pi \sigma'_{k+1}(E) - \xi \sigma'_k(E)}{\xi \sigma'_{k+1}(E)} \right) \]  \hspace{1cm} (14)

The factors 100 in the above equation have been introduced to express the \( \Delta_k \) as percentages of \( \sigma'_{k+1} \). Note that \( \Delta \) is independent of \( \xi \).

For the energy values under consideration, \( \sigma'_k(E) \) is an increasing function of \( k \), so that the \( \Delta_k(E) \)
are positive numbers. The numerical value of \( \Delta x \) places a limit on how close \( x_k \) can be to \( x_{k+1} \) for \( \sigma'_{\text{obs}}(x_k, E) \) to be distinguishable from \( \sigma'_{\text{obs}}(x_{k+1}, E) \).

The errors in the APW experiments, carried out in the late 1940s and early 1950s, were reported as 7–8% (in their respective theses, both Page [4] and Ashkin [5] go into considerable detail about the various sources of error and their individual contributions to the total error.) However, in a Page-type experiment the ratios \( \Delta_k \) can be determined directly from the observed count rates (cf. Ashkin’s remarks about the greater accuracy of the ratio \( \rho(E, x) \) in Sec. 3.2), and should be much more accurate than the individual cross-sections. It will not be overly optimistic to assume that the \( \Delta_k \) would not exceed 4–5% even with the technology used by Page and Ashkin seventy years ago. We shall continue our analysis on the assumption that it will be possible to distinguish between \( \sigma'_{\text{obs}}(x_k, E) \) and \( \sigma'_{\text{obs}}(x_{k+1}, E) \) if \( \Delta_k > 5\% \).

We have calculated \( \Delta_k \) numerically for

1. \( E = 0.4, 0.5, 0.6 \) and 0.7.
2. For each \( E \), \( x_k = 0.10, 0.15, 0.20, 0.25, 0.30 \) and 0.35.
3. For each pair \((E, x)\), \( \alpha = 0.1, 0.2, 0.3, 0.4 \) and 0.5.

The results of the calculations are shown in tabular form in the Appendix, one table for each value of \( \alpha \). One sees from these tables that \( \Delta_k \) is less than 5.0 only for a few values of \( E \) and \( \alpha \) at \( x = 0.10 \). If the value \( x = 0.10 \) is altogether excluded from the measurements, then \( \Delta_k \geq 6.46 \), and one has five sets of data points available: \( x_k = 0.15, 0.20, 0.25, 0.30, 0.35 \). Any two of these will fix the values of \( \xi \) and \( \alpha \) in (11) for given \( E \); the other three can then be used to test the hypothesis itself. If one excludes the data point \( x_k = 0.15 \) as well, one will still have two data points for a rougher test of the hypothesis at any \( E \), with \( \Delta_k \geq (\text{a very accommodating}) 8.21 \).

We also see from the tables that, for larger \( x_k \), the \( \Delta_k \) are so large that more data points can be picked in the interval \( x \in [0.2, 0.35] \) than the three we have chosen to determine the feasibility of the experiment. Details are left to the experimentalist.

We may therefore conclude that hypothesis I is indeed susceptible to experimental test, even with a repetition of Page’s experiment.

### 6.1 Test for multiple scattering, II

Begin with the observation that for fixed \( E \) and \( x_1 \neq x_2 \), the ratio of cross-sections

\[
v(E; x_1, x_2) = \frac{\sigma'(E, x_1)}{\sigma'(E, x_2)} \tag{15}\]

can be determined more precisely than either cross-section if both measurements are carried out with the same apparatus and the same intensity of the incident beam. The ratio \( v \) will simply be the ratio of the observed count rates. This observation can be used to turn the experiment described above into a test for multiple scattering with very little extra effort. The extra effort would be to measure \( \sigma'(E, x) \) for each \( E \) at \( x = 0 \) as well, and then to compute the ratios

\[
\Upsilon_{\text{obs}}(E, x_k) = \frac{\sigma'(E, x_k)}{\sigma'(E, x = 0)} \tag{16}\]

for \( x_k = 0.10, 0.15, \ldots, 0.35 \) from the observed count rates. For any given \( E \), these points should be plotted together with the graph of the calculated ratio of \( \Upsilon_{\text{M}}(E, x) \) versus \( x \) for \( x \in [0, 0.4] \).

As \( x \) increases, the scattering angle will decrease, and so will the path traversed through the foil by the scattered particles. If multiple scattering is significant, then the \( \Upsilon_{\text{obs}} \) will depart more and more from the calculated graph with decreasing \( x \).

### 7 Free-free scattering; the experiment of Williams et al

In 2014, Williams et al published the results of an important experiment on the ‘Scattering of free electrons by free electrons’, which was probably the first of its kind [23]. (In the following, we shall call this free-free scattering.) In this experiment, the relative velocity \( v/c \) of the electrons varied between \( 10^{-2} \) and \( 10^{-1} \), i.e., \( \gamma \) varied from 1.00005 and 1.00504, small enough for the collisions to be considered nonrelativistic. As we have noted earlier, in the nonrelativistic limit the scattering cross-section does not distinguish between two electrons and two spin-zero fermions (of mass \( m = m_e \)), so that the abovementioned experiment does not provide the information that we are seeking. Page and Ashkin measured the cross-section at a single scattering angle (for given \( E \)); Williams et al measured
the cross-sections at several different scattering angles; their results agreed with the theory based on a strictly Coulomb central force.

Williams et al write that their experiment verified the Møller formula to within $\pm 4\%$. (One could say, with more drama but equal justice, that their experiment showed that the particles scatter like scalar fermions to within $\pm 4\%$.) Their runs yielded 48 data points, of which four were wildly off, and were disregarded. They added that “The additional time required for better precision was not pursued because the [energy] dependence was clear and consistent with Rutherford scattering’. By ‘Rutherford scattering’ they meant the factor $\beta^{-4}$ outside the square brackets in equation (1), which is common to all cross-sections under the Coulomb force. This factor also shows that, other things being equal, at higher energies (relative velocities), longer runs needed to achieve higher accuracies – or indeed to make any observations at all – will have to be very much longer: to get the same number of data points as at $E = 2.5$ keV, the run will have to be 2,340 times as long at 0.2 MeV and 7,940 times as long at 1.0 MeV!

The multiple scattering problem could exist in free-free scattering as well, at high intensities and large (beam) cross-sections. If it does, one may be able to minimize its effect by using beams that are almost two-dimensional. The present author is unable to assess the feasibility of this suggestion, or that of scaling up the experiment of Williams et al. (Williams et al write about extending the experiment to lower relative velocities.)

\section*{8 Implications for the theory}

Runs of experiments that are more extensive and precise than those of Page and Ashkin may have three possible outcomes.

1. The results agree with the Møller formula.
2. The results depart from the Møller formula, but do not validate hypothesis I.
3. The results depart from the Møller formula, and validate hypothesis I.

In the first case, we would conclude that there is no need to attempt a revision of QED; the experiments of Page and Ashkin had unexplained errors.

In the second case, we may have to consider a non-Coulomb central force, which would entail a study of the angular distribution (as opposed to Page and Ashkin, who studied only the scattering angle corresponding to $x = 0$ for each $E$). As stated earlier (and reinforced by the results of Williams et al), we consider this possibility to be very remote, and shall not say anything further about it. It is the third case – an abundance of observations conform to hypothesis I – that will be our main concern, but we first need to dispose of a somewhat different scenario that may be suggested.

Usual proofs of the spin-statistics theorem are based on relativistic invariance. Although there have been several attempts to establish the result in a nonrelativistic setting (Sudarshan \cite{24}, Balachandran et al \cite{25}, Berry and Robbins \cite{26}), quantum field theorists continue to believe that the theorem breaks down in nonrelativistic physics. Could it be that the spin-statistics connection breaks down because the dynamics becomes effectively nonrelativistic before the nonrelativistic limit is reached?

The answer is \textit{no}! At the energies we are considering, the \textit{kinematics} of a two-electron system will certainly be relativistic (even at $E = 0.2$ MeV, $\gamma = 1.3914$). Recall now that mass is a superselection rule in Galilei-invariant theories (Bargmann’s superselection rule \cite{27}). Therefore the \textit{in} and \textit{out} states in our experiment will belong to different superselection sectors. This will frustrate any attempt to combine relativistic kinematics with nonrelativistic dynamics.

As pointed out in Remark II, eq. (11), the defining equation of hypothesis I, can also be cast in two other forms: the Møller formula with a modified third term (in the square brackets), given by either (12) or (13). However, we have not been able to find any physical interpretation for these forms.

There remains one further possibility: modifying the second term

$$-rac{3}{1 - x^2}$$

in square brackets in the Møller formula (1). This term is negative, and multiplying it by a factor greater than one will lower the cross-section. It will also change the angular distribution. The effects will be most pronounced near the nonrelativistic limit. Williams et al did not find any such effect \cite{23}.

To sum up, verification of hypothesis I will create

13
serious problems for the spin-statistics connection for electrons that are not in bound states, and more generally for quantum electrodynamics and quantum field theories. It should also be noticed that the energy range in which the effect is observed is below the threshold of pair creation, where the notion of quantized fields is not called upon; indeed, Møller derived his formula within the Dirac theory, using his own relativistic generalization of a trick devised by Bethe [29] (see also [15], page 200). This may suggest that the whole notion of particles as field quanta – one of the most basic notions of theoretical physics today – has to be re-examined, the repercussions of which will be almost unimaginable. For example: is it really necessary to quantize the gravitational field? What questions does string theory answer? What are the objects of which quantum mechanics is a mechanics? Let alone a theory of everything, will we have a theory of anything at all?

9 Possible violation of Pauli’s principle in bound states

So far we have been considering the possible violation of exchange symmetry in states of free particles, albeit based on the assumption that the scatterer is also a free particle. The possible violation of the Pauli principle in bound states was first tested experimentally in 1990, and is the subject of a major ongoing collaboration. We shall describe it briefly in the following. For the background, we refer to the general discussion of exchange symmetries from the theorist’s point of view in Haag’s book [30] and the references quoted there. (The term parastatistics seems to have been introduced by Dell’Antonio, Greenberg and Sudarshan in [31].) We first describe an ansatz which produces a ‘small’ violation of the Pauli exclusion principle, one which has led to experiments.

In Fermi-Dirac statistics, the operator identity \((a_j^\dagger)^2 = 0\) (which follows from the anticommutation rules) ensures that the state \(j\) is not occupied more than once. If, instead, we had the operator identity \((a_j^\dagger)^3 = 0\), it would imply that the state \(j\) can be occupied twice, but no more. This, of course, would require trilinear commutation relations.

In 1987, Ignatiev and Kuzman published a model with trilinear commutation relations which contained a small parameter \(\beta\) [32]. Their model, which had only one level, was defined by

\[
a^2a^\dagger + \beta^2a^\dagger a^2 = \beta^2a \\
a^2a^\dagger + \beta^4a^\dagger a^2 = a^2a^\dagger a^\dagger a
\]

and their hermitian conjugates, together with

\[
a^3 = 0, \quad (a^\dagger)^3 = 0
\]

This model has proved difficult to generalize to systems with many degrees of freedom (for details, see [33] and the references cited there), but it has the advantage of being experimentally testable. For example, if one of the two \(1s\) states of an atom can be doubly occupied, albeit briefly, then its existence may be revealed by \(2p \rightarrow 1s\) transitions that would otherwise be forbidden when both \(1s\) states are occupied.

An experiment to test this possibility in an open system (to bypass the Messiah-Greenberg superselection rule [34]) was performed by Ramberg and Snow in 1990 [35]. They passed a large current through a copper trough and looked for \(2p \rightarrow 1s\) transitions. Normally the \(1s\) shell is already filled with two electrons, so that the transition, of energy \(\sim 8.05\) keV, is forbidden. A small violation of Pauli principle of the type (17) would, however, allow some such transitions. Owing to screening by existing electrons, these transitions would have an energy \(\sim 7.7\) keV, and the resulting X-rays can be detected. Ramberg and Snow found no such transitions during their run, which gave the bound \(\beta^2 \leq 1.7 \times 10^{-20}\).

The Ramberg-Snow experiment was greatly refined by a large international collaboration (the VIP collaboration) at the Laboratorio Nazionale di Frascati (LNF) of the INFN, Italy [36]. They obtained a much improved lower bound of \(\beta^2/2 \leq 4.5 \times 10^{-28}\). This experiment was subsequently moved to the Laboratorio Nazionale di Gran Sasso (LNGS) under the Gran Sasso mountain in the Abruzzi. In its new form (which is constantly being refined) it is known as the VIP 2 experiment [37]. The latest published bound from this experiment is \(\beta^2/2 \leq 3.4 \times 10^{-29}\) [38].

The project is continuing. For a status report from April 2019, see [39]. A good discussion of the theoretical background and derivation of the bound may be found in [40].
Acknowledgements

The author would like to thank Dr Luca De Paolis for bringing the VIP and VIP 2 experiments to his attention, and Dr Mayer Goldberg for help with the numerical calculations and the graphs.

References

[1] R. D. Birkhoff, The passage of fast electrons through matter, *Handbuch der Physik*, ed. S. Flügge, vol. 34, pp. 53–138 (1958), Springer-Verlag, Berlin-Göttingen-Heidelberg. Section B of this article gives a brief review of this subject up to 1958.

[2] C. Møller, Zur Theorie des Durchgangs schneller Elektronen durch Materie, *Ann Physik* 14, 786–795 (1932).

[3] F. C. Champion, The scattering of fast β-particles by electrons, *Proc. Roy. Soc. A* 137, 688–695 (1932).

[4] L. A. Page, A measurement of electron-electron scattering, *Ph D thesis* (unpublished), Cornell University, Ithaca, New York (1950). Available from Cornell University Library, Division of Rare Books and Manuscripts Collection.

[5] A. Ashkin, A measurement of positron-electron scattering and electron-electron scattering, *Ph D thesis* (unpublished), Cornell University, Ithaca, New York (1952). Available from Cornell University Library, Division of Rare Books and Manuscripts Collection.

[6] L. A. Page, Electron-electron scattering from 0.6 to 1.7 MeV, *Phys Rev* 81, 1062–1063 (1951).

[7] A. Ashkin, L. A. Page and W. M. Woodward, Electron-electron and positron-electron scattering experiments, *Phys Rev* 94, 357–362 (1954).

[8] J. R. Oppenheimer, On the quantum theory of electronic impacts, *Phys Rev* 32, 361–376 (1928).

[9] N. F. Mott, The collision between two electrons, *Proc Roy Soc* 126A, 259–267 (1930).

[10] S. S. Schweber, *QED and the Men who Made It*, Princeton University Press, Princeton, NJ, 1994.

[11] H. A. Bethe and F. de Hoffmann, *Mesons and Fields*, Vol. II, Mesons, Row, Peterson & Co., Evanston, Illinois, 1955.

[12] F. J. Dyson, The radiation theories of Tomonaga, Schwinger and Feynman, *Phys Rev*. 75,486–502.

[13] F. J. Dyson, *Disturbing the Universe*, Basic Books (1979).

[14] H. J. Bhabha, The scattering of positrons and electrons with exchange on Dirac’s theory of the positron, *Proc Roy Soc A* 154, 195–206 (1936).

[15] X. Roqué, Møller scattering: a neglected application of early quantum electrodynamics, *Archiv Hist Exact Sci* 44, 197–264 (1992).

[16] N. M. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, 3rd ed., Oxford: Clarendon Press (1965).

[17] P. T. Matthews, Spinless mesons in the electromagnetic field, *Phys Rev* 80, 292 (1950).

[18] F. Rohrlich, Quantum electrodynamics of charged particles without spin, *Phys Rev* 80, 666–687 (1950).

[19] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, International Edition, New York: McGraw-Hill Book Company (1985).

[20] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, Reading, MA, USA: Addison-Wesley (1955).

[21] R. F. Egerton, *Electron Energy-Loss Spectroscopy in the Electron Microscope*, 3rd ed., Springer US, New York (2011).

[22] R. F. Egerton, *Physical Principles of Electron Microscopy: An Introduction to TEM, SEM, and AEM*, 2nd ed., Springer International, Switzerland (2016).
[23] J. F. Williams, S. Samarin, O. Targhah, A. Hilton, K. Sudarshan, L. Pravica and A. Artamonov, Scattering of free electrons by free electrons, Phys Rev A 89, 062717 (2014).

[24] E. G. C. Sudarshan, The fundamental theorem on the relation between spin and statistics, Proc. Indian Acad. Sci. A67, 284–293 (1968).

[25] A. P. Balachandran, A. Daughton, Z.-C. Gu, R. D. Sorkin, G. Marmo and A. M. Srivastava, Spin-statistics theorems without relativity or field theory, Int. J. Mod. Phys. A 8, 2993–3044 (1993).

[26] M. V. Berry and J. M. Robbins, Indistinguishability for quantum particles: spin, statistics and the geometric phase, Proc. Roy. Soc. London A 453, 1771–1790 (1997).

[27] V. Bargmann, On unitary ray representations of continuous groups, Ann. Math. 59, 1–46 (1954).

[28] C. Møller Über den Stoß zweier Teilchen unter Berücksichtigung der Retardation der Kräfte, Zeits. Phys. 70, 786–795 (1931).

[29] H. Bethe, Zur Theorie des Durchgangs schneller Korpuskularstrahlen durch Materie, Ann. Phys. 5, 325–400 (1930).

[30] R. Haag, Local Quantum Theory, Springer-Verlag, Berlin-Heidelberg-New York, 1993.

[31] G.F. Dell’ Antonio, O.W. Greenberg and E.C.G. Sudarshan, Parastatistics: Axiomatic Formulation, Connection with Spin and Statistics and TCP Theorem for the General Field Theory, in Group Theoretical Concepts and Methods in Elementary Particle Physics, ed. F. Gürsey, Gordon and Breach, New York, 1964.

[32] A. Yu. Ignatiev and V. Kuzmin, Search for slight violations of the Pauli principle, JETP Lett 46, 529–432 (1987).

[33] A. Yu. Ignatiev, X-rays test the Pauli exclusion principle, arXiv:hep-ph/0509258v1 (2005).

[34] A. M. L. Messiah and O. W. Greenberg, Symmetrization postulate and its experimental foundation, Phys. Rev. 136, B248–B267 (1964).

[35] E. Ramberg and G. A. Snow, Experimental limit on a small violation of the Pauli principle, Phys Lett B 238, 438–441 (1990).

[36] S. Bartalucci et al (VIP collaboration), New experimental limit on the Pauli exclusion principle violation by electrons, Phys Lett B 641, 18–22 (available online at www.sciencedirect.com).

[37] A. Pichler et al, VIP 2: Experimental tests of the Pauli Exclusion Principle for electrons, https://arxiv.org/pdf/1602.00867.pdf (2016).

[38] H. Shi et al (the VIP-2 collaboration), Experimental search for the violation of Pauli exclusion principle, Eur J Phys C 78:319 (2018). https://doi.org/10.1140/epjc/s10052-018-5802-4.

[39] K. Piscicchia and C. Curceanu, https://agenda.infn.it/event/18795/contributions/87575/attachments/62043/74222/05_VIP-2_LNGS-Sci-Com.pdf (2019).

[40] E. Milotti et al, On the Importance of Electron Diffusion in a Bulk-Matter Test of the Pauli Exclusion Principle, Entropy 20, 515 (2018). doi:10.3390/e20070515.
Appendix: Tables of $\Delta_k(E, x; \alpha)$

Note: Values of $\Delta_k$ smaller than 5.0 are shown in boldface.

| $x$ | $E$, in MeV |  
|-----|-------------|
| 0.10 | 5.12 | **4.95** | **4.80** | **4.68** |
| 0.15 | 7.04 | 6.81 | 6.62 | 6.46 |
| 0.20 | 8.87 | 8.61 | 8.39 | 8.21 |
| 0.25 | 10.64 | 10.36 | 10.12 | 9.91 |
| 0.30 | 12.36 | 12.07 | 11.81 | 11.60 |
| 0.35 | 14.07 | 13.78 | 13.53 | 13.31 |

Table A1. $\Delta_k(E)$ for $\alpha = 0.1$

| $x$ | $E$, in MeV |  
|-----|-------------|
| 0.10 | 5.20 | 5.04 | **4.90** | **4.68** |
| 0.15 | 7.14 | 6.93 | 6.75 | 6.60 |
| 0.20 | 8.99 | 8.75 | 8.54 | 8.37 |
| 0.25 | 10.77 | 10.50 | 10.28 | 10.09 |
| 0.30 | 12.49 | 12.22 | 11.99 | 11.79 |
| 0.35 | 14.18 | 13.93 | 13.70 | 13.50 |

Table A2. $\Delta_k(E)$ for $\alpha = 0.2$

| $x$ | $E$, in MeV |  
|-----|-------------|
| 0.10 | 5.28 | 5.13 | 5.01 | **4.90** |
| 0.15 | 7.24 | 7.05 | 6.89 | 6.75 |
| 0.20 | 9.10 | 8.89 | 8.70 | 8.54 |
| 0.25 | 10.89 | 10.66 | 10.45 | 10.09 |
| 0.30 | 12.62 | 12.38 | 12.17 | 11.99 |
| 0.35 | 14.33 | 14.09 | 13.88 | 13.70 |

Table A3. $\Delta_k(E)$ for $\alpha = 0.3$

| $x$ | $E$, in MeV |  
|-----|-------------|
| 0.10 | 5.36 | 5.23 | 5.12 | 5.02 |
| 0.15 | 7.34 | 7.18 | 7.03 | 6.91 |
| 0.20 | 9.23 | 9.03 | 8.87 | 8.72 |
| 0.25 | 11.02 | 10.81 | 10.63 | 10.28 |
| 0.30 | 12.76 | 12.54 | 12.35 | 12.19 |
| 0.35 | 14.46 | 14.25 | 14.06 | 13.90 |

Table A4. $\Delta_k(E)$ for $\alpha = 0.4$

| $x$ | $E$, in MeV |  
|-----|-------------|
| 0.10 | 5.45 | 5.33 | 5.23 | 5.15 |
| 0.15 | 7.45 | 7.31 | 7.18 | 7.07 |
| 0.20 | 9.35 | 9.19 | 9.04 | 8.91 |
| 0.25 | 11.16 | 10.98 | 10.82 | 10.68 |
| 0.30 | 12.89 | 12.71 | 12.55 | 12.41 |
| 0.35 | 14.60 | 14.42 | 14.26 | 14.12 |

Table A5. $\Delta_k(E)$ for $\alpha = 0.5$