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Effect of material anisotropy on rolling contact fatigue life under dry and lubricated point contact conditions: a numerical study

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Rolling bearings have to operate reliably and efficiently under increasingly severe conditions. Previously ignored effects of material inhomogeneity with varying crystallographic orientation may significantly affect rolling contact fatigue life. In this paper, it is demonstrated that Multigrid techniques allow the computational analysis of such effects in a full 3D setting. Using grain configurations created by Voronoi tessellation, the dependence of the maximum von Mises stress, and of a predicted fatigue life stress field integral with load, friction, mean grain size and crystallographic orientation distribution is shown. Crystallographic orientation variations are shown to potentially significantly reduce predicted rolling contact fatigue life relative to homogeneous isotropic material. The results contribute to material optimization and to computational diagnostics of criticality of material crystallographic (sub)structures.

Keywords
Fatigue life; Polycrystalline anisotropic material; Dry and lubricated contact; Multigrid method

1. Introduction

The motion of the rolling element relative to the raceway in a rolling bearing results in cyclic loading and an alternating stress field distribution in the raceway subsurface. After millions of cycles, cracks are usually initiated in regions where the local stress exceeds the threshold of the fatigue limit of the material. These cracks gradually propagate to form a network, eventually leading to material spalling and bearing failure. This type of failure is known as rolling contact fatigue (RCF), and also occurs in other applications such as gears, wheel-rail contacts and camshaft-follower mechanisms.

Today’s high reliability of rolling bearings is built on a fundament of extensive research over the past decades dating back to the middle of the twentieth century. At that time, a wide spread in fatigue life was observed even in rolling bearing series from the same batch. The observed differences between the longest and the shortest life were as much as a factor twenty or even larger, which was obviously below acceptable engineering reliability [1]. Since 1939, the Weibull statistical theory has been used to describe the probability distribution of fatigue life [2]. In this model, the probability of survival is a function of the maximum shear stress, the number of stress cycles and the stressed volume. In 1947, Lundberg and Palmgren [3] established a fatigue life model (the L-P model) based on the Weibull distribution, in which the depth and the value of the maximum orthogonal shear stress were introduced. Based on the L-P model, the Chiu-Tallian-Mccool (C-T-M) model [4, 5, 6], the Ioannides-Harris (I-H) model [7], the Cheng-Cheng (C-C) model [8] and the Yu-Harris (Y-H) model [9] were proposed successively. In the C-T-M model [4, 5, 6], the surface initiated fatigue due to so-called type B defects (such as furrows, digs and pits) and type C defects (secondary micro pits formed during running) was investigated. In the I-H model [7], a fatigue limit is introduced below which the material will not fail due to classical RCF. The stress criterion used is not limited to the maximum orthogonal shear stress, it can also be the shear stress or the von Mises stress (VMS). Another difference in the I-H model compared to other models is that the overall risk of failure is based on the integral over the entire stressed volume rather than only on the value at a single location such as the
point of the maximum orthogonal stress in the L-P model. The C-C model [8] is similar to the I-H model, however, it is easier to use as no integrations are performed. In the Y-H model [9], the depth of the maximum orthogonal shear stress is eliminated and the three material constants \((e, c, h)\) are reduced into two. The differentiation aspect of Zaretsky model [10] is the inclusion of some external conditions affecting bearing life, for example contamination, bearing steel, etc. This is done by superposition of a series of operating-conditions independent factors, which are then multiplied to the fatigue life term. To obtain a model applicable to a wide range of test data, in 1996, Tallian proposed a data-fitted life prediction model for rolling bearings considering traction, lubricant film, surface roughness, bearing material, etc. [11, 12, 13]. The database contains 274 test groups from 25 sources ranging in date from 1950 to 1992. More recently, Morales-Espejel et al. [14] presented a rolling bearing life model, in which the surface initialized damage is explicitly formulated into the model instead of taken into account by means of a penalty factor as has been common practice so far.

With the developed models, the life of rolling bearings can be predicted more accurately. However, the trends in design and applications of downsizing, reduced material use, increased efficiency, leads to more severe operating conditions (e.g. higher temperature, larger loads, reduced lubricant supply and starvation) and often more complex material topology (e.g. coating [15], non-metallic inclusions [16], roughness and so on) push engineers and scientists ahead to develop models which e.g. can take into account the detailed topological and crystallographic effects of the material. In order to know more about the surface and subsurface originated failures, a numerical analysis of the stress field is essential. Sina et al. [17] studied the size, depth and stiffness of inclusions (Type A defect) on the butterfly wing formation, crack initiation and its propagation. It was found that stiffer inclusions are more detrimental to the RCF when they are closer to the maximum shear stress reversal point. Zhou [18] used the micro-macro contact model and real contact surface textures (surface profile with waviness and roughness, type C defect) to study the effect of surface topology on RCF with different lambda ratios. The results explained why the life of enhanced-finish bearings is longer than that of standard-finish bearings. Bearings with good surface finish and macroscopically homogeneous material can be achieved by modern manufacturing technology. However, for further improvement of predictions, more detailed insight in failure causes, and for computational diagnostics, research interest is directed to develop models and computational methods to take into account actual tomographic and crystallographic information of the local material and its anisotropy for prediction of fatigue life. Different from homogeneous isotropic material, the heterogeneous material topology resulting from the grains of anisotropic material with varying crystallographic orientation causes local stress concentrations around grain boundaries [19, 20, 21]. Besides, these different local mechanical properties also affect the pressure distribution at the surface under the dry contact and lubricated condition [22]. Due to the variation of properties on the scale of the grain size, the stress field calculation for heterogeneous anisotropic material requires a dense discretization (grid) which for 3D problems leads to systems of equations with many millions of unknowns. This requires an excessive computational effort unless a very efficient algorithm is used for solving the contact problem and the associated stress field. In earlier work [21][28], the authors have demonstrated that the multigrid method is well suited for the simultaneous solution of the (lubricated) contact problem and the 3D displacement and stress field in heterogeneous anisotropic material. In this paper, the developed algorithm is used in a fatigue life analysis considering the influence of the applied load, coefficient of friction, material grain size and orientation angles. The variable nature of the microstructural topology and crystallographic orientations results in scattering which depends on the grain size and orientation angle variations. The predicted fatigue life may significantly be reduced compared to the result for homogeneous isotropic material.
2. Theory

Figure 1 schematically shows the contact problem, a rigid ball loading against an elastic body. The contact pressure and the displacements on the top boundary can be solved with the Hertz-Signorini-Moreau condition:

\[
\begin{align*}
  h(x, y) > 0, p(x, y) = 0 & \quad \text{no contact} \\
  h(x, y) = 0, p(x, y) > 0 & \quad \text{contact}
\end{align*}
\]

(1)

Alternatively, for a lubricated contact the Reynolds equation should be used as in Ref. [23]. The side boundary condition of the 3D elastic body is a stress free or Neumann boundary condition. At the bottom surface, zero displacement (Dirichlet boundary condition) is imposed. At the top surface:

\[
\begin{align*}
  \sigma_{zx} &= \mu p \\
  \sigma_{zy} &= 0 \\
  \sigma_{zz} + p &= 0
\end{align*}
\]

(2)

with \( p \) determined by Eq. (1) for a dry contact or by the Reynolds equation for a lubricated contact. For a lubricated contact, pure rolling condition is considered in this paper. So, the forces in \( x \) and \( y \) directions can be assumed to be zero compared to the normal stress in the \( z \) direction. For a dry contact, both the rigid body and the elastic body keep still without relative motions. The effect of shear stress shown in Section 5.3 is obtained by applying shear force (\( \mu p \)) in the \( x \) direction. In other cases, the friction coefficient \( \mu \) equals to zero if the shear stress is not considered for a dry contact. The stress equilibrium equations for the material are:

\[
\begin{align*}
  \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\
  \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\
  \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0
\end{align*}
\]

(3)

where
\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{xz} \\
\sigma_{xy} \\
\end{bmatrix} = C \begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial w}{\partial z} \\
\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\
\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\end{bmatrix}, \quad (4)
\]

\[
(C_{i,j,k,l})_{\text{global}} = R_x(\alpha)R_y(\beta)R_z(\gamma)(C_{i,j,k,l})_{\text{local}} \left(R_x(\alpha)R_y(\beta)R_z(\gamma)\right)^T. \quad (5)
\]

\(\alpha, \beta\) and \(\gamma\) are the three Euler rotation angles, \(R\) is the rotation matrix, and \(C_{i,j,k,l}\) is the elastic stiffness matrix. For cubic anisotropic materials such as \(\alpha\)-Fe and \(\gamma\)-Fe, the local grain stiffness can be expressed in the following form [24]:

\[
(C_{i,j,k,l})_{\text{local}} = \begin{bmatrix}
c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{44} \\
\end{bmatrix} \quad (6)
\]

in which \(c_{11}, c_{12}\) and \(c_{44}\) are three material constants. The ratio

\[
A = \frac{2c_{44}}{c_{11} - c_{12}} \quad (7)
\]

is the anisotropy (Zener) ratio [24]. For isotropic materials \(A=1\), the stiffness matrix keeps its original sparsity under rotation. For anisotropic materials, the matrix in the global coordinate system may become full. Substitution of Eq. (4) (with Eq. (5) and Eq. (6)) in Eq. (3) gives the Navier-Cauchy equations in terms of the displacements \(u, v\) and \(w\), see Ref. [21]. From the solution of displacements, the associated stresses can be obtained from Eq. (4).

Subsequently, the stress integral over the whole calculation domain can be taken according to the I-H fatigue life equation [7] with the fatigue limit set to zero. The VMS is chosen as the stress criterion [26] though it must be noted that any other criteria can be included with equal ease. The integral equation reads:

\[
\ln \frac{1}{S} = A_s N^e \int \frac{\sigma^c}{z^\mu} d\nu', \quad (8)
\]

in which \(S\) is the probability of survival of \(N\) load cycles, with \(e=10/9, c=31/3, h_s=7/3\) [7]. The integral can be normalized using a reference case. This is taken as the same contact problem with a particular load for homogeneous isotropic material with the associated reference maximum Hertzian pressure:

\[
S_{r} = \frac{A_s N^e \int \frac{\sigma^c}{z^\mu} d\nu', (\text{varrying } p_h, \mu \text{ or material property})}{A_s N^e \int \frac{\sigma^c}{z^\mu} d\nu', (p_{h, \text{reference}}, \mu = 0, \text{isotropic material})} \quad (9)
\]
The variation of Sr with the material parameters and topology, e.g. grain topology, crystallographic
details such as variation of anisotropy orientation angle over the grains, is studied by means of
numerical calculations. All equations were made dimensionless using:

\[
X = \frac{x}{a}, \quad Y = \frac{y}{a}, \quad Z = \frac{z}{a}, \quad U = \frac{u}{a}, \quad V = \frac{v}{a}, \quad W = \frac{w}{a}
\]

\[
P = \frac{p}{p_h}, \quad \sigma_{ij} = \frac{\sigma_{ij}}{p_h}, \quad \tilde{c}_{ijkl} = \frac{c_{ijkl}}{p_h}, \quad H = \frac{h}{a}
\]

where \(a = \sqrt{\frac{3FR(1-\nu^2)}{4E}}\), \(p_h = \frac{3F}{2na^2}\) are the values of the Hertzian contact radius and maximum contact
pressure for the reference load condition with homogeneous isotropic material. For the contact and the
material (isotropic) parameters, see Table 1.

**Table 1** Contact and material parameters

| Parameter       | Value     |
|-----------------|-----------|
| \(F\)           | 40.34 N   |
| \(R\)           | 0.02 m    |
| \(p_h\)reference| 1.0 GPa   |
| \(E\)           | 206 GPa   |
| \(\alpha, \beta, \gamma\) | \(0 \sim \pi/2\) |
| \(X, Y\)        | -3 \sim 3 |
| \(Z\)           | 0 \sim 6  |
| \(\nu\)         | 0.3       |

3. Anisotropic Material Behaviour

Before presenting the details of the numerical solutions, we elaborate on the relation between the
anisotropy of the material and its compliance when loaded in a specific direction. Table 2 gives the
chosen parameters for a cubic anisotropic material.

**Table 2** Cubic anisotropic material parameters

| Parameter | \(A\) | \(A=2.4074^{[24]}\) | \(A=2.4167^{[20]}\) | \(A=3.77^{[22]}\) |
|-----------|-------|---------------------|---------------------|---------------------|
| \(c_{11}\) | 231.4 GPa | 237 GPa            | 204.6 GPa           |
| \(c_{12}\) | 134.7 GPa | 141 GPa            | 137.7 GPa           |
| \(c_{44}\) | 116.4 GPa | 116 GPa            | 126.2 GPa           |

Figure 2 (a) shows the elastic modulus in 3D according to Eq. (11) (Ref. [27]) for a cubic
anisotropic grain with anisotropic ratio \(A=2.4167\).

\[
\frac{1}{E(l, m, n)} = s_{11} - [2(s_{11} - s_{12}) - s_{44}](l^2m^2 + m^2n^2 + l^2n^2)
\]

in which \(s_{11}, s_{12}\) and \(s_{44}\) are elastic compliances of the material, \(E(l, m, n)\) is Young’s modulus along an
arbitrary loading direction \([l, m, n]\) for cubic crystals. The norm of the vector from the origin to the
point at the surface plane represents the elasticity. For isotropic material, the shape of the figure is a
sphere. For cubic anisotropic material, it has eight rounded protuberances in the corners and six
valleys at the centres of the faces. Its minimum value is \(1/s_{11}\) and the maximum value is \(1/(s_{11}-2(s_{11})^\sim\)
For an anisotropic grain with \(A=2.4167\), the Young’s modulus varies between 132.27 GPa to 283.34 GPa depending on the direction. When the grain is rotated, its stiffness tensor in the global coordinate system is changed according to Eq. (5).

As a result, the elastic moduli of anisotropic grains varies in the global coordinate system. Fig. 2 (b) compares the elasticity of anisotropic and isotropic material in the central \(XZ\) plane. With an increase of the anisotropy ratio, the ratio between the largest and the smallest value of the elastic modulus increases. This implies that with increasing anisotropy ratio, the elastic modulus varies more significantly over the grains for a stressed material with a heterogeneous topology of anisotropic grains.

![Fig. 2 Young’s modulus variation (GPa) on a single grain due to anisotropic elasticity](image)

### 4. Numerical Solution

The resulting system of partial differential equations in terms of the displacements was discretized on a uniform grid with second order accuracy. A Gauss-Seidel lexicographic relaxation in sequential order is used for solving the displacement equations where solving the contact equation as boundary condition is integrated in the relaxation algorithm. Multigrid techniques are used to accelerate convergence. For details regarding the algorithm and demonstration of the numerical accuracy of the solutions see Ref. [21][28]. From the solution of the displacement field, the stress field was computed exploiting a second order discretization of the first derivatives in Eq. (4) in the interior domain and boundaries. The stress integral (Eq. (8), Eq. (9)) was discretized assuming cubic elements of constant stress surrounding each grid point where the stress values are defined, which is also a second order discretization. The subsurface topology was generated by means of Voronoi tessellation with randomly distributed crystal nucleus positions and orientation angles, i.e. a statistically uniform distribution.

### 5. Results

The reference load condition is as given in Table 1. Table 2 gives the chosen parameters for a cubic anisotropic material. When not specifically indicated, otherwise the following parameters are used: (1) an average grain diameter of 15 \(\mu m\) for heterogeneous anisotropic material; (2) anisotropic material with a Zener ratio \(A=(2c_{44}/(c_{11}-c_{12})) = 2.4167\) [20]; (3) a contact load such that the maximum Hertzian contact pressure for homogeneous isotropic material would be \(p_c=1\) GPa. Note that the choice of a relatively large material grain size is made for convenience. It is not intended to represent actual bearing steels but rather to show the effect of material inhomogeneities and their orientation.
5.1 Comparison of dry contact and lubricated contact

First for a typical 3D material topology, the stress fields under dry contact and under lubricated contact are compared for the reference load condition which, under isotropic material, would give a maximum Hertzian pressure of 1 GPa. The additional parameters for the lubricated point contact are an entrainment velocity $u_m = 0.5 \text{ m/s}$, $n_0 = 0.08 \text{ Pa} \cdot \text{s}$, $\alpha_{oil} = 22 \text{ GPa}^{-1}$. In terms of the Moes dimensionless parameters for point contact $M = 211.7$, $L = 9.51$. The Dowson-Higginson density-pressure equation and the Roelands viscosity-pressure equation are used to describe the density and viscosity as a function of pressure respectively. The dimensionless equations used and the discretization method of the Reynolds equation, which, as mentioned above, is now a boundary condition, is the same as that in Ref. [23]. Further details of the EHL solver considering 3D heterogeneous anisotropic material can be found in Ref. [28].

![EHL pressure](image1)

![Dry contact pressure](image2)

![Dimensionless pressure difference $P_{EHL} - P_{Dry}$](image3)

![Sign distribution of pressure difference $P_{EHL} - P_{Dry}$](image4)

**Fig. 3** Pressure comparison between (a) lubricated and (b) dry contacts for heterogeneous anisotropic material

Figure 3 shows the pressure distribution under lubricated (Fig. 3(a)) and dry contact (Fig. 3(b)) conditions. Relative to the well-known smooth (near) Hertzian semi-elliptic pressure distribution for homogeneous isotropic material, clearly pressure fluctuations are observed. These results are due to the effect of the heterogeneous anisotropic crystallography of the material, see also Ref. [22]. For the dry contact condition, the grain boundaries are much clearer, which means the variation of pressure between grain boundaries is less smooth compared to the lubricated case in Fig. 3(a). Fig. 3(c) shows the difference between the (dimensionless) EHL pressure and the (dimensionless) dry contact pressure. The differences are localized around the edge of the Hertzian contact region. Figure 3 (d) further emphasizes the pressure difference by the sign function:
Obviously, the EHL pressure exceeds the dry contact pressure in the entire inlet region. In the outlet region, just after the outlet pressure spike it drops below the Hertzian pressure and decays subsequently to zero in this region. Inside the contact, the EHL pressure can either be smaller or larger. The transition points do not generally exactly coincide with the grain boundary as in the dry contact case, as the grain boundaries cannot be clearly observed in Fig. 3(d). For both cases, the VMS has been computed in the 3D volume below the surface. Fig. 4 shows the result in the central plane \( Y = 0 \).

\[
\text{sign}(P_{\text{EHL}} - P_{\text{Dry}}) = \begin{cases} 
1.0 & P_{\text{EHL}} > P_{\text{Dry}} \\
0 & P_{\text{EHL}} = P_{\text{Dry}} \\
-1.0 & P_{\text{EHL}} < P_{\text{Dry}} 
\end{cases}
\] (12)

The VMS stress fields (Fig. 4(a) and (b)) for the lubricated and dry contact are almost identical. Fig. 4(c) and 4(d) aim to emphasize the differences. As can be seen in Fig. 4(c), these differences are concentrated near the edge of the contact region close to the surface where in Fig. 3 (c) also the pressure differences were observed. In Fig. 4 (d), the sign of the difference between the stress fields is shown using Eq. (12) by replacing \( P \) with \( \bar{\sigma} \). In the inlet region and in the contact region near the top surface, the VMS of the lubricated case is larger than that of dry contact at most areas.

The relative stress integral values according to Eq. (9) for the lubricated and the dry contact are 3.918 and 3.556 respectively for the studied cases. As this is the reference load case, the value for the dry contact with isotropic material is unity so the heterogeneity and anisotropy at the grain size/scale significantly reduces the predicted fatigue life. The value for the lubricated condition is about 10% larger than for the dry contact condition. The major differences originate from the regions near the outlet pressure spike(s) where also the largest difference between the dry and lubricated pressure profile is observed. The relatively small difference between the value of the fatigue life integral for the lubricated contact and the dry contact is not unexpected. For homogeneous isotropic material this is
well known and justifies that most rolling contact fatigue life predictions at present are based on dry contact calculations. Note that for time varying solutions with roughness moving through the contact, this may be quite different. Nevertheless, under steady conditions also for the heterogeneous anisotropic material case it is justified to exploit dry contact calculations for fatigue life predictions provided the loads are sufficiently high. In view of the more severe operating conditions anticipated in the future, such as higher load, higher degree of starvation, higher temperature, the lubricant film will even become thinner and the EHL pressure will be even closer to that of dry contact. Besides, the time cost of dry contact (2h 8mins, Intel X5650 CPU at 2.66 GHz, 2 W cycles) is around 0.6 (depending on the number of grains and cycles [28]) of the time cost of an EHL case. Thus, here we restrict ourselves to the pressure and the stresses of a dry contact for the following analyses. Regarding the specific effects of the heterogeneous anisotropic material on the film thickness, the reader is referred to reference [22][28].

5.2 Effect of contact load

Figure 5 shows the VMS in the central XZ plane below the contact for a homogeneous anisotropic material aligned with the global coordinate system (Fig. 5(a)) and for a homogeneous isotropic material (Fig. 5(b)). The differences between the stress fields are visualized in the Figure 5(c) and 5(d). For the anisotropic material, the stress field extends over a wider region and decays more rapidly with depth. For the homogeneous isotropic material, the maximum dimensionless VMS is 0.62 and located in the centre at a dimensionless depth 0.47 below the surface. Quite differently, for the homogeneous anisotropic material aligned with the global coordinate system, the maximum VMS (0.5226) occurs at a circle below the surface near the Hertzian contact radius. This shows as two locations in the XZ plane ($X=0.4688$ and $X=-0.4688$) at a depth 0.4922 below the surface. This is related to the fact that for the aligned material the elastic modulus along the axis direction is the smallest and the material thus more compliant (see Fig.2).
Fig. 5 VMS in the central $XZ$ plane: (a) homogeneous anisotropic aligned. (b) homogeneous isotropic. (c) difference in VMS between anisotropic and isotropic. (d) sign of difference in VMS between homogeneous aligned anisotropic and homogeneous isotropic.

In Fig. 6, the value of the relative fatigue life stress integral $S_r$ (Eq. (9)) is given as a function of the contact load. Results are shown for several values of the Zener (anisotropy) ratio $A$ (Eq. (7)), including for the homogeneous isotropic material. Note that this latter curve has a value of unity for the reference load case with Hertzian pressure of 1 GPa. In the log-log plot, the relative integral value of both homogeneous anisotropic and isotropic material increases linearly with the increase of pressure. However, the values for the anisotropic material that is aligned with the global coordinate are lower than for the isotropic material. This is explained by the larger compliance in the load direction which is also reflected in the fact that the contact radius of for the anisotropic cases is 1.011–1.059 times larger than that of the isotropic case. When the homogeneous anisotropic material is not aligned with the global coordinate system ($0.5 \times \pi/2$ in each direction), its global stiffness increases and results in a larger relative value compared to homogeneous isotropic material.

Fig. 6 Effect of contact pressure on the relative stress integral $S_r$ for homogeneous aligned anisotropic and isotropic material.

Next the case is considered of a heterogeneous anisotropic material with the anisotropy direction randomly distributed over a granular structure defined by Voronoi tessellation. The average grain diameter is 15 $\mu$m which is 10 times smaller than the Hertzian contact radius. The VMS field in the central $XZ$ plane shown in Fig. 7 (a) is significantly different from that of homogeneous material in Fig. 5. The variation of the crystallographic alignment effectively changes the elastic modulus of each grain in the global coordinate directions leading to a higher or lower stress. This effect can clearly be seen in Fig. 7(b) which shows the difference between the present case and the values for the homogeneous isotropic case. Stress concentrations and larger values of the VMS occur in the boundary regions of grains across which the largest variation in crystallographic orientation occurs. The maximum value of the VMS is higher than for the homogeneous isotropic case. These variations are detrimental for the fatigue life as is indicated by the value of 3.43 for $S_r$. Note that for this case of a random distributed orientation and already quite small grains, the shape of the stressed region is quite similar to that for a homogeneous isotropic material. This is in line with the expectation that with decreasing grain size, one should eventually approximate the isotropic material. If these results were to be translated into a guideline for material crystallographic optimization to reduce the observed stress variations, one should consider: (1) reduce the grain size; (2) control the variation of rotation angles between adjacent grains. These effects will be discussed in more detail in section 5.4 and 5.5. This
would gradually reduce the red and blue regions in Fig. 7(b) indicating large stress difference to the uniform into green (small stress difference).

![Stress field of anisotropic material](image1.png)

![\sigma_{\text{Ani}} - \sigma_{\text{Iso}}](image2.png)

**Fig. 7** VMS of XZ plane comparison between heterogeneous anisotropic and isotropic material (Y=0)

Figure 8 (a) shows the influence of the contact load on the maximum VMS and the depth at which it occurs. Fig. 8 (b) shows the relative fatigue life integral value with the variation of contact load. The results for the homogeneous isotropic case match the theoretical values where the maximum VMS is $0.62 p_h$ at a depth $0.47a$, where $p_h$ and $a$ are the actual values of the Hertzian contact pressure and radius. As in the present case the results are scaled on the 1 GPa Hertzian pressure case one should see a linear increase of the maximum VMS value with slope 0.62 and a linear dependence of the depth at which it occurs with slope 0.47 for the homogeneous isotropic case. However, with the increase of load, the contact radius becomes larger and the side boundaries affect the stress field and thereby the stress integral value. As a result, the linear variation of the maximum VMS and its depth is affected. With an increasing large calculation domain, the effect of side boundaries decreases gradually and the linear variation with load could be observed. In Fig. 8(a) it can also be seen that with increasing load, the deviation of the maximum VMS from the value for the homogeneous isotropic case increases. The change of its depth with load is a stepwise function owing to the fact that with increasing load, the stressed region increases involving deeper laying grains. When a deeper grain whose strain and corresponding global stiffness can generate a larger stress is involved, this will become the location of the maximum VMS. With the increase of load, the relative integral value increases almost linearly on a log-log scale, see Fig. 8(b). For the grain structured material, the slope (on the log-log scale) is identical to the slope for the isotropic homogeneous material. However, due to the local stress concentrations around grain boundaries, the value of the relative stress integral is larger. The ratio varies between 2.76 and 4.65. Interpreting the observed results in terms of rolling contact design, it is obvious that crystallographic structure and orientation may considerably affect rolling contact fatigue life calculations and thus should be studied and understood in detail.
5.3 Effect of shear stress

So far friction was neglected. First for homogeneous anisotropic crystallography aligned with the main coordinate system and subsequently for a grain structured heterogeneous material with randomly varying crystallographic orientation, the influence of the coefficient of friction on the relative fatigue life stress integral $S_r$ is studied. Fig. 9 shows the results for the contact problem with friction for homogeneous anisotropic material aligned with the global grid, and for homogenous isotropic material as a function of the coefficient of friction for different values of the Zener ratio.

For small values ($\mu < 0.1$), the effect of the surface shear induced by friction on the integral value is small, although there is a clear difference between its value for the isotropic and the anisotropic case. For $\mu > 0.1$, the effect of the shear stress becomes more evident, and for $\mu > 0.2$, the effect of the value of the Zener ratio becomes visible. This is most likely related to the fact that for these values of the coefficient of friction, the stress distribution significantly differs from the reference case (without friction) as the maximum VMS now occurs at the surface. Although interesting from an academic point of view, these values of the friction coefficient are not representative for the lubricated roller-raceway contacts in rolling bearings as in these cases the friction is much lower.
Figure 10 shows the effect of the shear stress due to friction on the VMS field for the case of the heterogeneous grain structured material with stochastically distributed crystallographic orientation as considered above. The figure shows the difference between the VMS field with and without friction in the central XZ and the central YZ plane for the same case. In Fig. 10 (a) and (b), the difference in the VMS field for the frictionless case and for \( \mu = 0.1 \) is shown. It can be seen that the main change is a strong asymmetry in the XZ plane due to the friction with the maximum VMS moving closer to the surface on the inlet side. This trend adversely affects the RCF. Near surface stress concentrations of sufficient magnitude may induce surface cracks and pits close to the contact surface referred to as surface initiated RCF [29].

The variation of the maximum VMS and the depth at which it occurs with the coefficient of friction is shown in Fig. 11 (a). The value of \( S_r \) is shown in Fig. 11 (b). For homogeneous isotropic material, the depth of the maximum VMS depends on the normal load (stress) and the coefficient of friction. For the grain topology with varying orientation, the variation of the crystallographic orientation affects the grain stiffness and thereby the location of the maximum VMS. The overall trend of the dependence on the coefficient of friction is the same for both cases: the larger shear stress increases the value of the maximum VMS and moves its location from the inner material to the contact surface. The difference between the two cases is in the small shear stress region. As for the heterogeneous anisotropic material, the location is unaffected until a sufficiently high coefficient of friction to change it to the contact surface. The reason is that although the combined influence of normal stress and shear stress is the strongest, the variation of the stiffness from grain to grain (in the global coordinate system) still does not lead to a different overall maximum VMS value. Once occurring at the surface, the value of the maximum VMS strongly increases with friction. The ratio of the maximum VMS value between the two cases is 1.66~1.85 (on average 1.71±0.065). In Fig. 11 (b), the effect of the coefficient of friction on the stress integral value is shown. Similar to the effect of the load, the stress integral value for the grain topology case with varying crystallographic orientation is larger than that for homogeneous isotropic material. The ratio between the two cases is 3.02~3.95 (on average 3.44±0.309).
5.4 Effect of mean grain size

The material properties such as strength, stiffness, hardness, as well as the morphology of generated cracks in RCF are closely related to the crystallographic topology. For different values of the mean grain size in the Voronoi tessellation, two hundred microstructures were generated with randomly varying distribution of the orientation over the grains. The parameters for the Voronoi tessellation are given in Tab. 3. For each case, the contact problem was solved and the stress field evaluated.

Table 3 Configurations for the Voronoi tessellation

| mean grain size (µm) | relation with Hertzian contact width | total grain number | mean grain number in each direction |
|---------------------|-------------------------------------|--------------------|------------------------------------|
| 15                  | 0.108a                              | 171061             | 55.51                              |
| 20                  | 0.144a                              | 72166              | 41.63                              |
| 25                  | 0.180a                              | 36949              | 33.31                              |
| 30                  | 0.216a                              | 21383              | 27.76                              |
| 35                  | 0.252a                              | 13465              | 23.79                              |
| 40                  | 0.288a                              | 9021               | 20.82                              |
| 45                  | 0.324a                              | 6336               | 18.50                              |
| 50                  | 0.360a                              | 4619               | 16.65                              |
| 55                  | 0.396a                              | 3470               | 15.14                              |
| 60                  | 0.432a                              | 2673               | 13.88                              |

In Fig. 12(a), the variation of the contact area relative to the value for homogeneous isotropic material (Eq. (13)) is shown with the variation of the mean grain size. The spread of the value in the batch of two hundred generated cases for each grain size is also indicated.

\[
\text{relative contact area} = \frac{\sum \sum P(>0), \text{(varying grain size and crystallographic topology)}}{\sum \sum P(>0), \text{(}P_{\text{ref}, \mu = 0, \text{isotropic material})}}
\] (13)
With the decrease of grain size, the variation of the relative contact area in the batch decreases. From about 30 $\mu$m downwards, the decreasing average grain size seems to reduce the contact area a little which means that the apparent stiffness of the base material increases and agrees with the Hall-Petch’s Law [31]. In Fig. 12 (b), with finer mean grain size from 30 $\mu$m, it can be seen that the value of the relative fatigue life value decreases, i.e. the fatigue life increases with finer grain size. Also, the spread among the two hundred randomly generated cases reduces. This seems to align with the findings in the experimental study [30] that a fine grain size is beneficial to fatigue life and its dispersion would be smaller. The extension of fatigue life with finer grains can be explained as follows: The decrease of grain size introduces more grain boundaries and makes the stress field more continuous. The incidence of local stress concentration can be lowered. As a result, the integral value of stress field decreases (Fig. 12(b)) and the survival probability increases.

**Fig. 12** Effect of mean grain size on the relative contact area and the value of the relative fatigue life stress integral $S_r$ for grain structured material with random variation of the crystallographic orientation.

Figure 13 gives the influence of mean grain size on the maximum VMS and the depth at which it occurs. For the studied cases, the deviation of depth in Fig. 13(b) is larger compared to the maximum VMS value in Fig. 13(a). The tendency found in Fig. 12 does not apply to the variation of maximum VMS. It can be explained as follows: compared to isotropic and homogeneous anisotropic material, the value and the location of the maximum VMS for heterogeneous anisotropic material depends both on the microstructure and on the rotation angles. Even with the same microstructure, the stress field can be different when the rotation angles are different. One example is shown in Fig. 14. The stress fields are different even though their Voronoi tessellations are the same. This implies that the fatigue life integrals only based on a local maximum stress as the fatigue criterium may not be accurate when the effect of grains with varying crystallographic orientation needs to be predicted.
5.5 Effect of rotation angle

For the results shown before, the range of variation of the rotation angles $\alpha, \beta, \gamma$ is taken from 0 to $\pi/2$. In this part, the effect of restricting this range is investigated. It should be pointed out that the rotation angle is uniformly distributed at each rotation range. Fig. 15 shows the influence of the rotation range on the relative contact area and the relative stress integral value. For each case, twenty randomly generated microstructures are employed to check the influence of range of rotation angles. With the increase of range, the relative contact area decreases at first and then increases. It means the stiffness of the 3D body increases at first and then decreases. For the relative stress integral value, its trend is inverse. This implies that the fatigue life of bearing material can be optimized by control of the crystallography variations. At around $0.7 \times \pi/2$, the relative contact area and stress integral value reach the minimum and maximum value respectively. Different to the effect of the mean grain size shown in section 5.4, the stiffness and stress integral value show an opposite trend with the variation of range of rotation angle.

Figure 16 compares the stress fields with full rotation range ($0 \sim \pi/2$) and with restricted rotation ($0 \sim 0.7 \times \pi/2$) to that of isotropic material. The results are obtained for the same conditions and the same microstructure of grains. The stress integral value for the restricted rotation angle case (Fig. 16(b)
and (d)) is larger than that of full rotation (Fig. 16(a) and (c)). When comparing the stress fields in the $XZ$ and $YZ$ planes (Fig. 16(a) and (b), Fig. 16(c) and (d)), it is not easy to see which stress field would be more severe. A possible reason is that the total stiffness variation of adjacent grains has its maximum value when the rotation angle is restricted from $0$ to $0.7 \times \pi/2$.

![Fig. 16](image)

**Fig. 16** Effect of rotation angle on the stress field for heterogeneous anisotropic material

### 6. Conclusion

In this paper, a developed multigrid method for the numerical simulation of dry and lubricated contacts for the case of 3D elastic heterogeneous material with varying crystallographic orientation [21][28] was used to analyse the effects of various parameters on the predicted fatigue life using an Ioannides-Harris [7] based analysis. First uniform anisotropic cases were considered. Subsequently using Voronoi tessellation grain topologies where the crystallographic orientation was randomly varied with a uniform distribution. The effect of contact load (pressure), friction coefficient, mean grain size and effects of distribution of rotation angles on the stress field and the fatigue life stress integral value were investigated. It was shown that the developed numerical method is well suited to study the effects of crystallographic orientation induced anisotropy both for uniform as well as for granular topology with randomly varying orientation over the grains. The results show that mean grain size and crystallographic orientation variation significantly affect predicted fatigue life. Whereas the fatigue life integral value of homogeneous (aligned) anisotropic material under increasing pressure or tangential shear stress conditions is smaller than that of isotropic material, a granular topology with varying crystallographic orientation results in some local stress concentrations, and as a result, for the cases considered the fatigue life stress integral value is some 3-5 times larger than that of homogeneous isotropic material which could imply a significant reduction of the RCF. The study was conducted for grain sizes down to $15 \mu m$. The effects are strongest for the larger grains and with larger stress variation between neighbouring grains. With decreasing mean grain size, the detrimental effect is reduced. The results show that the developed method has excellent prospect to help assess criticality of heterogeneous material topology and crystallographic orientation variations down to a grain size that is about 10 times smaller than the Hertzian contact radius. It should be noted that this grain size is
most likely still too large to approach both the structure of bearing steels and the anticipated limiting behaviour of an homogenous isotropic material. This limit will anyhow numerically be very challenging to clearly demonstrate as a reduced grain size requires a finer grid to maintain comparable numerical accuracy. Parallelization of the solver to further decrease the grain sizes that can sufficiently accurately be computed as well as use of local grid refinement approaches where a particular structure is embedded in a homogeneous bulk are topics of current development. Nevertheless, the presented results already show an excellent capability to investigate the effects of varying crystallographic orientation in 3D. The developed method can be a very useful tool to aid microstructural criticality assessment, to perform computational diagnostics, and to help optimize EHL contact and bearing performance.

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Nomenclature

\( a \) half width of Hertzian contact (m)

\( A_i \) probability normalization factor, (-)

\( A \) anisotropy ratio, \( 2c_{44}/(c_{11}-c_{12}) \), (-)

\( c_{11}, c_{12}, c_{44} \) elastic constants for cubic material, (Pa)

\( e, \epsilon, h_i \) Lundberg-Palmgren exponents, (-)

\( C_{i,j,k,l} \) elastic stiffness matrix, (Pa)

\( \tilde{C}_{i,j,k,l} \) dimensionless of elastic stiffness matrix, \( C_{i,j,k,l}/p_h \)

\( E \) Young’s modulus, (Pa)

\( F \) applied load, (N)

\( h \) gap distance, (m)

\( H \) dimensionless gap distance, \( h/\alpha \)

\( l, m, n \) direction cosines, (-)

\( L, M \) Moes dimensionless parameters, (-)

\( N \) number of cycles, (-)

\( p \) pressure, (Pa)

\( P \) dimensionless pressure, \( p/p_h \)
\( p_h \) maximum Hertzian pressure, (Pa)
\( R \) spheres radius, (m)
RCF rolling contact fatigue
\( S \) survival probability
\( Sr \) relative value of fatigue life integral
\( s_{11}, s_{12}, s_{44} \) elastic compliances of material, (Pa\(^{-1}\))
\( u, v, w \) displacements in \( x, y, z \) directions, (m)
\( u_m \) entrainment velocity, (m/s)
\( v' \) volume, (m\(^3\))
\( U, V, W \) dimensionless displacements, \( u/a, v/a, w/a \)
VMS von Mises stress
\( x, y, z \) coordinate system, (m)
\( z' \) stress weighted average depth, (m)
\( X, Y, Z \) dimensionless coordinate system, \( x/a, y/a, z/a \)
\( \sigma_{ij} \) normal and shear stress, (Pa)
\( \bar{\sigma}_{ij} \) dimensionless normal and shear stress, \( \sigma_{ij}/p_h \)
\( \bar{\sigma}_{\text{VMS}} \) dimensionless Von Mises stress, \( \sigma_{\text{VMS}}/p_h \)
\( \alpha, \beta, \gamma \) Euler rotation angle, (rad)
\( \mu \) friction coefficient, (-)
\( \nu \) Poisson ratio, (-)
\( \eta_0 \) lubricant viscosity, (Pas)
\( a_{\text{oil}} \) pressure viscosity coefficient, Pa\(^{-1}\)

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Research highlights

1) In this paper, it is demonstrated that developed multigrid techniques allow the computational analysis of varying crystallographic orientation in a full 3D setting under dry and lubricated contact conditions.

2) Using grain structures created by Voronoi tessellation in this paper, we show the variation of the maximum von Mises stress, and of the value of a predictive fatigue life integral over the stress field with load, friction, mean grain size and crystallographic orientation distribution.

3) The results show that the developed method has excellent prospect to help assess criticality of heterogeneous material topology and crystallographic orientation variations down to a grain size that is about 10 times smaller than the Hertzian contact radius.
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: