From small to large $x$: toward a unified description of high energy collisions

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Abstract. Inclusive particle production at high $p_t$ (equivalently intermediate to large $x$) in high energy hadronic collisions is successfully described by perturbative QCD, using the collinear-factorization formalism. On the other hand at very high energies and not so high $p_t$ the Color Glass Condensate (CGC) formalism has been quite successful in describing particle production at small Bjorken $x$. Here we propose a way to unify the two approaches which enables one to apply it to particle production at all $x$ (all $p_t$).

1. Introduction

Perturbative QCD has been very successful in describing high $p_t$ particle production in high energy collisions. Its use depends on the concept of collinear-factorization where the particle production cross section can be written as a convolution of several independent quantities; parton distribution functions describing probabilities $f_1, f_2$ for finding partons of energy fractions $x_1$ and $x_2$ in the two incoming hadrons, the hard scattering of the two partons and a subsequent hadronization of the partons described by the parton-hadron fragmentation function $D_{p/h}$. This can be symbolically written as

$$d\sigma \sim f_1(x_1, Q^2) \otimes f_2(x_2, Q^2) \otimes d\sigma_{\text{parton}} \otimes D_{p/h}$$

where the parton distribution and fragmentation functions satisfy the DGLAP evolution equation [1] which describes further radiation of a parton. This formalism is ideal at high $p_t$ (equivalently at small distances) where parton density, number of partons per unit area, is small which justifies ignoring the fact that there are other partons in the colliding hadrons. While theoretically well-defined and on solid grounds at high $p_t$, collinear-factorization is expected to break down at small $x$ due to large parton densities.

It is an experimental fact that the number of partons, specially gluons, increases very fast as $x \to 0$. Due to this large number of partons occupying the same transverse area of a hadron the parton density becomes large (referred to as gluon saturation) which makes it very likely that more than one parton from each hadron will participate in the scattering which breaks the underlying approximations made in the collinear-factorization approach. This is specially significant for nuclei due to longitudinal coherence length of small $x$ partons which can be of the order of or larger than the nuclear size. If so this parton will see the effective parton density of the nucleus which is enhanced by $A^{1/3}$ compared to a proton. This indicates that parton density...
effects can be large for large nuclei and that one needs to go beyond collinear-factorization for processes dominated by small $x$ kinematics.

Due to large parton densities at small $x$ the concept of a quasi-free parton is not very useful anymore and it may be more appropriate to treat the system of partons collectively, i.e. as fields. In this approach, known as the Color Glass Condensate (CGC) formalism [2] the large $x$ degrees of freedom in the hadron act as sources of color charge which radiate a classical gluon field representing the collective small $x$ (gluon) degrees of freedom. One then includes quantum loop effects by re-summing the large logs of $x$ which become $O(1/\alpha_s)$ at small $x$ such that $\alpha_s \log 1/x \sim 1$. Due to the large parton densities at small $x$ multiple scattering (as compared to single scattering in pQCD) must be taken into account. This is efficiently done via the use of Wilson lines which re-sum multiple soft scatterings of a projectile from a dense target, represented by the strong classical color field. In the hybrid formulation [3] of particle production in CGC formalism the cross section is given in terms of correlations of products of Wilson lines [4] which satisfy the JIMWLK evolution equation [5]. For example, single inclusive hadron production in proton-nucleus collisions can be symbolically written as

$$d\sigma \sim f_1(x_1,Q^2) \otimes \sigma_{\text{dipole}} \otimes D_{p/h}$$

where $\sigma_{\text{dipole}}$ is the quark anti-quark dipole scattering cross section on the target which satisfies the JIMWLK equation. In this formalism the dipole cross section contains all the dependence on the target. When expanded in powers of density its first term contains the convolution of Leading Log gluon distribution function with the parton-gluon scattering cross section. In this sense, and very loosely speaking the dipole cross section contains the target parton distribution function and the parton-parton scattering cross section.

While each of the two formalisms are valid in their respective kinematic regions there is no single formalism which contains both in the appropriate limits. In addition to being theoretically unsatisfactory it also leads to practical problems which prevents one from unequivocally establishing the presence of gluon saturation at small $x$. For instance it is still not clear what region of $x$ contributes to single inclusive hadron production in the forward rapidity region of RHIC as the two competing formalisms are able to fit the data, and yet yield vastly different kinematics [6]. While some of these issues can be resolved experimentally in the proposed Electron-Ion Collider [7] it is essential to have a more general formalism that is valid in both kinematic regions.

2. Scattering from both large and small $x$ gluons in the target

In [8] we proposed a more general formalism which includes scattering of a projectile from both large and small $x$ degrees of freedom. As the parton density is large at small $x$ we include multiple scatterings of the projectile from the small $x$ gluons of the target. On the other hand the large $x$ parton density of the target is not large so that it is enough to consider a single scattering from the large $x$ partons of the target. In addition to the standard eikonal scattering employed in CGC formalism, there are three other classes of diagrams where 1) a projectile will multiply scatter from the soft gluons of the target both before and after a hard scattering from the large $x$ gluons of the target; 2) The large $x$ gluon itself will also interact with the soft gluon background, and finally, 3) both the scattered projectile and the large $x$ gluon will scatter from the soft background field. The amplitude is most easily evaluated using the spinor helicity formalism [9]. To show how spin asymmetries arise in our formalism, we just show our results for the Dirac numerators of the amplitude for scattering of a quark projectile on a target hadron or nucleus (ignoring the Wilson lines which encode the target information),

$$N_{1,a}^+ = p^+ \sqrt{q^+} \left\{ 2 A^a_a(x) - A^i_a(x) \left[ \frac{k_{1i} - i\epsilon_{ij} k^j_1}{q^+} + \frac{k_i + i\epsilon_{ij} k^j}{p^+} \right] \right\}$$
\[
\begin{align*}
N_{2,a}^+ &= \frac{p^+}{q_1^-} \sqrt{q^+ p^+} \left\{ \left(1 + \frac{q^+}{p^+} \right) q_\perp \cdot A_\perp^b (x) + i \left(1 - \frac{q^+}{p^+} \right) \epsilon^{ij} q_i A_j^a (x) \right\} \\
N_{3,a}^+ &= N_{2,a}^+ (q_i \to p_1 i) \\
&= \frac{p^+}{p_1^+} \sqrt{q^+ p^+} \left\{ \left(1 + \frac{q^+}{p^+} \right) p_1 \perp \cdot A_\perp^b (x) + i \left(1 - \frac{q^+}{p^+} \right) \epsilon^{ij} p_i A_j^a (x) \right\} 
\end{align*}
\]

and
\[
N_{1,2,3}^{-a} = [N_{1,2,3}^{+a}]^* 
\]

where \(p, q\) are the momenta of the incoming and outgoing quark respectively, \(k\) and \(k_1\) are the momenta of the internal quark line before and after the hard scattering which is integrated over and \(a\) is a color index. To get the cross sections for polarized scattering one needs to square the above amplitudes. Here we show one of the terms, specifically, the term \(|N_2^+|^2\) which is the simplest,

\[
|N_{2}^{+,b}_-|^2 = \frac{q^+}{p^+} \frac{1}{q_1^-} \left\{ \left[(4p^+ q^+) q_\perp \cdot A_\perp^b (x) q_\perp \cdot A_\perp^b (y) + (p^+ - q^+)^2 q_1^2 A_\perp^b (x) \cdot A_\perp^b (y) \right] \\
+ \epsilon^{ij} [(p^+)^2 - (q^+)^2] \left[ q_i A_j^b (x) q_\perp \cdot A_\perp^b (y) - q_i A_j^b (y) q_\perp \cdot A_\perp^b (x) \right] \right\} 
\]

Using the fact that \(N_{2}^{-b} = [N_{2}^{+,b}]^*\) we see that there is asymmetry in the polarized cross section, for example in the double longitudinal spin asymmetry \(A_{LL}\)

\[
A_{LL} = \frac{d\sigma^{++} - d\sigma^{--}}{d\sigma^{++} + d\sigma^{--}} 
\]

due to the presence of terms proportional to \(i\epsilon^{ij}\). We note that effects like this are already present [10] at small \(x\), the novel thing here is their appearance in our unified treatment in the intermediate \(x\).

Inclusion of scattering from the large \(x\) gluons of the target also leads to angular asymmetries in unpolarized scattering addition to the spin asymmetry. This is already seen in previous studies of sub-eikonal corrections to eikonal scattering [11, 12] at small \(x\). The difference between our results here and those in earlier studies is in the kinematic region where these corrections appear; in our formalism these corrections appear in the intermediate-large \(x\) region of the target which corresponds to intermediate to high \(p_t\) region of particle production.

Furthermore, this formalism leads to a rapidity shift of the projectile parton (commonly referred to as beam rapidity loss) which is absent in the eikonal approximation made in the standard CGC approach [13]. This is due to exchange of longitudinal momenta between the projectile and target which is not included in the standard CGC approach.

It will be interesting to apply this formalism to study cold matter energy loss. While the fully coherent energy loss is contained in parton radiation in the CGC formalism as part of higher order corrections (in \(\alpha_s\)) to inclusive production, the standard CGC approach does not contain partially coherent energy loss as in the small \(x\) kinematics a radiated parton can not resolve the longitudinal structure of the target nucleus. In our formalism this will not be the case, however, due to the fact that there is partially coherent radiation before and after hard scattering. To see this explicitly one will need to construct the gluon propagator [14] the same way as done for the projectile quark.

It will be easier to start with radiation of a real or virtual photon [15] first, as unlike gluons, photons will not interact with the target color fields. This should already exhibit the QED like
radiative energy loss pattern. Furthermore it will allow us to investigate two-particle correlations in this simpler process [16] before considering the more involved di-hadron or di-jet correlations.

Finally, and in the long term, it will be very fruitful to try to generalize this formalism by using an effective action approach [17]. This will allow one to use this formalism to study the early dynamics of high energy heavy ion collisions in a more robust way. Currently there is no formalism that can treat production of both the soft Quark-Gluon Plasma and the hard particles that probe its properties concurrently. Due to inclusion of large $x$ partons in the wave-function of a hadron or nucleus one will not only have the soft gluons forming the medium but also the high $p_t$ probes corresponding to the produced large $x$ partons.

In summary, we have introduced a new formalism which generalizes the Color Glass Condensate approach to particle production in high energy collisions by including large $x$ partons of a hadron or nucleus in the scattering process. We have shown that this leads to spin asymmetries in polarized scattering, as well as rapidity loss and angular asymmetries. It will be interesting to develop the formalism further and to apply it to phenomenology of both high and low $p_t$ particle production in high energy collisions such as those at RHIC and the LHC, as well as the proposed Electron-Ion Collider.

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