Dynamic mode of the mathematical model of an electric multipole with memresistive branches in conditions of interval uncertainty

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Abstract. This article discusses a method for obtaining a mathematical model of an electric multipole with memresistive branches in finite parameter increments for a large signal mode in time domain calculation when calculating a direct current based on a generalized model obtained earlier. A computational algorithm for calculating nanoelectronic circuits based on methods of interval arithmetic is also given.

1. Introduction

Information and computing technologies are used in almost all spheres of human activity. The modern development of computer technology has led to the emergence of a whole spectrum of new nanoelectronic components that greatly increase the computing power. However, the tasks of increasing productivity and reducing heat dissipation of modern computing devices without increasing cost remain urgent.

New nanoelectronic components, which have recently been used in the design of information and computing systems, require research aimed at improving the operational characteristics of computing technology.

One of the newest components of this kind is the memristor - a passive element in microelectronics capable of changing its resistance depending on the charge flowing through it. For a long time, the memristor was considered a theoretical model [1-7], which could not be realized in practice until the first sample of an element demonstrating the properties of a memristor was created in 2008 by a team of scientists led by R. S. Williams at the Hewlett-Packard research laboratory. The observed phenomenon of hysteresis in the memristor makes it possible to use it, among other things, as a memory cell [8-16].

Modern computers are built on the basis of the von Neumann architecture: both data and programs are stored in the memory of the machine in binary code, with the computational module separated from the storage devices, and the programs are executed sequentially, one after the other. The already studied properties of memristors allow us to say that on their basis it is possible to create computers of a fundamentally new architecture, significantly exceeding semiconductor ones in performance. The use of memristors in the near future will make, in principle, unnecessary individual hardware components of a computer - processors, video chips, memory and hard drives; the machine will be an
architecturally homogeneous device where all data will be simultaneously stored and all operations performed with them. For an upgrade, it will be enough to install additional memristor modules, and for repairs, replace those that have failed [5; 8; 16-20].

All the above makes us look at the electric multipole in a new way, including in its already known mathematical model a memristor, as one of the elements of the basic set. An additional element in the basic set also determines the appearance of new memresistive branches of the electric multipole. In accordance with the adopted classification [21-23], it is customary to distinguish the following typical operating modes of the investigated electronic devices, which is quite true for the elements of nanoelectronics:

- Quasi-linear mode of a small signal when calculating with an alternating current;
- Quasi-linear small signal mode when calculating in the time domain;
- Large signal mode when calculating for direct current (static mode);
- Large signal mode when calculating in the time domain (dynamic mode).

Depending on the selected mode, the initial model of the investigated device will be an electrical multipole of one of the following types: linear resistive, linear reactive, nonlinear resistive and nonlinear reactive. At the same time, the general methodological basis for studying all the listed types of models is their description in the basis of finite deviations of currents and voltages of the branches of the corresponding multipole, since the main goal of our study is to assess the ability of a device to maintain its characteristics within specified limits in the presence of disturbing influences with undefined properties. This article analyzes the large-signal mode in time-domain calculations (dynamic mode) for an electrical multipole with finite-increment memresistive branches.

2. Materials and methods

Mathematical models of an electric multipole with memresistive branches in the nominal form and using finite increments were built earlier [1-3] using the method of decomposition of branches of the directed graph of the circuit described in [19-22].

A full-size mathematical model in a full hybrid basis in finite increments of currents and voltages will look like this [20]:

\[
\begin{bmatrix}
0 & \frac{d}{dt} \begin{bmatrix} \Delta U_{C}^{P} \\ \Delta I_{C}^{X} \end{bmatrix} \\
-L C & 0
\end{bmatrix}
\begin{bmatrix}
\Delta U_{C}^{P} \\
\Delta I_{C}^{X}
\end{bmatrix}
= Q_1 \begin{bmatrix} \Delta U_{C}^{P} \\ \Delta I_{C}^{X} \end{bmatrix} + Q_2 \begin{bmatrix} \Delta U_{R}^{P} \\ \Delta I_{R}^{X} \end{bmatrix} + Q_3 \begin{bmatrix} E_{N}^{R} \\ E_{N}^{I} \end{bmatrix} + Q_4 \begin{bmatrix} \Delta U_{M}^{P} \\ \Delta I_{M}^{X} \end{bmatrix} - \begin{bmatrix} E_{E}^{P} \\ E_{E}^{I} \end{bmatrix} + H \cdot F\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta U_{R}^{P} \\
\Delta I_{R}^{X}
\end{bmatrix} = A_1 \begin{bmatrix} \Delta U_{C}^{P} \\ \Delta I_{C}^{X} \end{bmatrix} + A_2 \begin{bmatrix} \Delta U_{R}^{P} \\ \Delta I_{R}^{X} \end{bmatrix} + A_3 \begin{bmatrix} \Delta U_{M}^{P} \\ \Delta I_{M}^{X} \end{bmatrix} - H \cdot F\end{bmatrix}
+ G \begin{bmatrix} \Delta U_{C}^{P} \\ \Delta I_{C}^{X} \end{bmatrix} + G_2 \begin{bmatrix} \Delta U_{R}^{P} \\ \Delta I_{R}^{X} \end{bmatrix} + G_3 \begin{bmatrix} \Delta U_{M}^{P} \\ \Delta I_{M}^{X} \end{bmatrix} - W \begin{bmatrix} E_{N}^{R} \\ E_{N}^{I} \end{bmatrix} + K \begin{bmatrix} E_{M}^{P} \\ E_{M}^{I} \end{bmatrix}
\]

where \( \tilde{L} = \text{diag}(L_k + \Delta L_k) \) is the matrix of inductances \((k = 1, n_{L}^{X}, l = 1, n_{L}^{X}, n_{L}^{X})\), \( \tilde{C} = \text{diag}(C_k + \Delta C_k) \) is the capacity matrix \((k = 1, n_{C}^{X}, l = 1, n_{C}^{X}, n_{C}^{X})\), \( \Delta U_{C}^{P}, \Delta I_{C}^{X} \) - increments of voltages and currents on inductive chords; \( \Delta U_{R}^{P}, \Delta I_{R}^{X} \) - voltage and current increments on the capacitive edges; \( \Delta U_{M}^{P}, \Delta I_{M}^{X} \) - voltage and current increments on resistive edges and chords; \( \Delta U_{N}^{P}, \Delta U_{P}^{P}, \Delta I_{P}^{X} \) - voltage and current increments on nonlinear edges and chords.
increments on memresistive edges and chords; $Q_1 = \begin{bmatrix} -B_{II} & Z_E & 0 \\ V_E & -D_{II} & 0 \\ 0 & 0 & -D_{IV} \end{bmatrix}, \quad Q_2 = \begin{bmatrix} -B_{II} & 0 & 0 \\ 0 & -D_{II} & 0 \\ 0 & 0 & -D_{IV} \end{bmatrix}, \quad Q_3 = \begin{bmatrix} -B_{II} & 0 & 0 \\ 0 & 0 & -D_{II} \end{bmatrix} \cdot W$ and $Q_4 = \begin{bmatrix} -B_{II} & 0 & 0 \\ 0 & 0 & -D_{III} \end{bmatrix} + \begin{bmatrix} -B_{IV} & 0 & 0 \\ 0 & 0 & -D_{IV} \end{bmatrix} \cdot Z_E = \left[ \begin{bmatrix} \left[ 0 \right]_{n_Z \times n_Z}^{x} \right] \cdot \text{diag}\left( \left\{ Z_E^* + Z_E^T \left( \Delta i \right) \right\} \right)$ - matrix of equivalent resistances and $V_E = \left[ \begin{bmatrix} \left[ 0 \right]_{n_Z \times n_Z}^{x} \right] \cdot \text{diag}\left( \left\{ G_N^C + G_N^T \left( \Delta u \right) \right\} \right)$ - equivalent conductivities of nonlinear inductances and capacities; $D_i$ is the matrix of the main contours of the directed graph of the circuit; $B_i$ - matrix of the main sections of the directed graph of the circuit; $n$ is the number of branches and $k$ is the number of nodes; $E^l_E = \left[ \left[ \left[ E_N^l \right]_{n_k \times 1} \left[ E_N^T \left( \Delta i \right) \right]_{n_k \times 1} \right]^T \right]$ - equivalent vectors of current and voltage sources; 

$$A_1 = \begin{bmatrix} \Delta M^{-1} & Z_X (\Delta i^X) \\ G_p (\Delta U_H^p) & \Delta M \end{bmatrix}, \quad A_2 = \begin{bmatrix} \Delta M^{-1} & Z_X (\Delta i^X) \\ G_p (\Delta U_H^p) & \Delta M \end{bmatrix}, \quad A_3 = \begin{bmatrix} \Delta M^{-1} & Z_X (\Delta i^X) \\ G_p (\Delta U_H^p) & \Delta M \end{bmatrix}$$ 

and $\Delta M$ are diagonal matrices of equivalent inverse and forward memresistivities; $E$ is the identity matrix; $E_H^H, E_H^X, J^H, J^X, E_Z, E_N^H, E_N^X, J^H, J^X$ - equivalent sources of EMF and current on nonlinear chords and edges; $G_1 = \begin{bmatrix} B_{XII} & \hat{R} & Z_E \\ \hat{G} & D_{XII} & 0 \\ 0 & 0 & -D_{II} \end{bmatrix}, \quad G_2 = \begin{bmatrix} B_{XIV} & \hat{R} & Z_E \\ \hat{G} & D_{XIV} & 0 \\ 0 & 0 & -D_{IV} \end{bmatrix}, \quad G_3 = \begin{bmatrix} B_{XII} & \hat{R} & Z_E \\ \hat{G} & D_{XII} & 0 \\ 0 & 0 & -D_{II} \end{bmatrix}, \quad W = \begin{bmatrix} B_{XII} & \hat{R} & Z_E \\ \hat{G} & D_{XII} & 0 \\ 0 & 0 & -D_{II} \end{bmatrix}$ 

and $\hat{M} = M + \Delta M$ is the matrix of the branch memresistivities, $\hat{M}^{-1} = M^{-1} + \Delta M^{-1}$ is the matrix of inverse memresistivities of the branch; $E_N^M, J_N^M$ - equivalent vectors of independent voltage and current sources on memresistive branches.

Mathematical model of a multipole with memresistive branches in dynamic mode. Calculation of the dynamic characteristics of electronic circuits in the time domain consists in determining the type of transient processes in their structure, arising under the influence of sources of variable signals and pulse sequences. Because of such a calculation, the time intervals necessary to transfer the circuit from one static mode to another are found, or the time during which currents and voltages reach a given level. The form of the transient is often of interest. The development of electromagnetic processes in time is determined not only by the nature of the change in voltages on capacitors and currents in inductors, but also by the final increments of these parameters.

In cases where the levels of alternating signals in the circuit are significantly less than the levels of direct currents and voltages of the static mode, the analysis of transient processes is carried out in a
linearized circuit, where their small-signal models replace all nonlinear elements. In this case, the system of equations of state also becomes linear. To obtain such a model of the investigated circuit, it is necessary to form a hybrid basis containing state variables and exclude all other variables related to the resistive branches of the equivalent circuit, described in [19-20].

Using the conditions established in [20] for the existence of mathematical models of electrical multipoles, we construct such a model for the dynamic mode of an electronic circuit.

The reactive elements of the equivalent circuit - capacitance and inductance - are separated into special branches. As a result, the rest of the equivalent circuit will be a linear multipole with resistive and memresistive branches, which also includes dependent and independent sources. The capacities, the parametric description of which corresponds to the z-branches, must be referred to the edges, which can always be ensured if the assumption of the absence of main capacitive circuits is fulfilled. Likewise, inductances should be assigned to chords. To ensure the formulated requirements, the numbering of the branches should begin with capacitors and end with inductors. Based on the foregoing, the mathematical model of a multipole with memresistive branches (1) in the dynamic mode will take the form:

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \Delta U^P_C \\ \Delta I^X_C \\ \Delta U^P_R \\ \Delta I^X_R \\ \Delta U^P_M \\ \Delta I^X_M 
\end{bmatrix} &= Q_1 \begin{bmatrix} \Delta U^P_C \\ \Delta I^X_C 
\end{bmatrix} + Q_3 \begin{bmatrix} E^R \\ J^R_E \end{bmatrix} + Q_4 \begin{bmatrix} \Delta U^P_M \\ \Delta I^X_M 
\end{bmatrix} - \begin{bmatrix} E^L \\ J^L_E \end{bmatrix} \\
\begin{bmatrix} \Delta U^P_R \\ \Delta I^X_R 
\end{bmatrix} &= G_1 \begin{bmatrix} \Delta U^P_C \\ \Delta I^X_C 
\end{bmatrix} + G_3 \begin{bmatrix} \Delta U^P_M \\ \Delta I^X_M 
\end{bmatrix} - W \begin{bmatrix} E^R \\ J^R_E \end{bmatrix} \\
\begin{bmatrix} \Delta U^P_M \\ \Delta I^X_M 
\end{bmatrix} &= S_1 \begin{bmatrix} \Delta U^P_C \\ \Delta I^X_C 
\end{bmatrix} + S_3 \begin{bmatrix} \Delta U^P_R \\ \Delta I^X_R 
\end{bmatrix} - K \begin{bmatrix} E^M \\ J^M_E \end{bmatrix}
\end{align*}
\] (2)

Expression (2) belongs to the class of internal systems of linear algebraic equations, for the solution of which it is permissible to use Kahan's interval arithmetic and the Euler and Hook-Jeeves interval methods.

The essence of Kahan's interval arithmetic is that operations with intervals containing zero have the same result as in the case of other intervals and allows you to keep the interval expansion of functions unchanged, and also guarantees, under certain conditions, not only the distributiveness of operations, but also monotonicity with respect to inclusion [20; 23].

3. Results

Computational algorithm for external estimation of the solution sets of the mathematical model of a multipole with memresistive branches in a dynamic mode. Given the interval nature of the resulting model, this algorithm uses the Euler method for integrating differential equations and the configuration method (Hook-Jeeves method), which is a zero-order method and does not require the calculation of interval derivatives when solving a system of nonlinear algebraic equations [23]:

- We assume \( t_0 = 0 \) and \( l = 0 \);
- Calculate: \( \Delta U^P_C = [\Delta C^{-1}] [\Delta U^P_C - J^{HE3}_C - J^{MB}_C (\Delta U^P_C) - J^{HE3}_C (U^P_C)]t \) according to the first equation of the system (2), where \( \Delta C = \text{diag}\{\Delta C_1, \Delta C_2, \Delta C_3\} \) is the matrix of finite increments of capacities;
- Calculate: \( \Delta U^P_R = \frac{1}{\Delta} W \Delta U^P_C - \frac{1}{\Delta} D_1 V \Delta U^P_R \times \Delta \);  
- Calculate \( \Delta I^X_R = \frac{1}{\Delta} W \Delta U^P_C + \frac{1}{\Delta} V \Delta U^P_R \) with respect to the unknown vector \( \Delta I^X_R \) for the moment \( t_{i+1} = t_i + \Delta \) using the Newton-Raphson method for interval variables;
- We believe \( l = l + 1 \), \( t_i = t_i + \Delta \);
- Is the process over? If "yes", go to item next point; otherwise, go to item point 2;
- The end.
4. Discussion
The dynamic mode considered in this article for an electric multipole with memresistive branches confirms the applicability of the chosen approach to the analysis of nanoelectronic circuits of both memory elements and the architecture of information-measuring systems as a whole.

5. Conclusion
The form of presentation of the mathematical model for each mode of operation is quite consistent with the forms required for using the methods of interval analysis. Of course, it is necessary to check the adequacy of the presented model of an electric multipole with memresistive branches both in general form and in dynamic and static modes for real nanoelectronic circuits, which is the subject of further research.

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