Condensate Fraction and Pair Coherence Lengths of Two-Dimension Fermi Gases with Spin-Orbit Coupling

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Abstract

The effects of Rashba spin-orbit coupling on BCS-BEC crossover, the condensate fraction and pair coherence lengths for a two-component attractive Fermi gas in two dimension are studied. The results at $T = 0K$ indicate that (1) when the strength of SOC is beyond a critical value, BCS-BEC crossover does not happen in a conventional sense; (2) SOC enhances the condensate fraction, but suppresses pair coherence lengths.

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In a crystalline solid spin-orbit coupling (SOC), which occurs naturally in systems with broken inversion symmetry and makes the spin degree of freedom respond to its orbital motion, is responsible for many interesting phenomena, such as magnetoelectric effect [1,2,3], visionary Datta-Das spin transistor [4,5], topological insulator [6,7] and superconductivity [8,9]. Taking topological superconductivity for example, it has been predicted to occur in superconductors with a sizable spin-orbit coupling in the presence of an external magnetic field [10,11,12,13,14]. In these systems the transition to topological phases requires that critical magnetic field is much larger than the superconductivity gap above which an s-wave superconductor is expected to vanish in the absence of SOC. It is SOC that competes with a strong magnetic field to give rise to a topological superconducting phase.

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It is widely known that ultracold atom systems can be used to simulate many other systems owing to their many controllable advantages and operabilities [15, 16, 17]. Certainly the simulations to SOC, which are generally equivalent to produce non-abelian gauge potential with optical [18, 19, 20] or radio-frequency fields [21], are also possible and have been realized in a neutral atomic Bose-Einstein condensate (BEC) by dressing two atomic spin states with a pair of lasers [22]. Motivated by such a pioneer experiment and a practical proposal for generating SOC in $^{40}K$ atoms [23], BCS-BEC crossover in the two-component Fermi gases with SOC have been widely studied [24, 25, 26, 27, 28, 29, 30, 31]. On the one hand for balanced case SOC not only leads to an anisotropic superfluid [24], the signatures of which could be observed in the momentum distribution or the single-particle spectral function of atomic cloud, but also significantly enhances the superfluid transition temperature when scattering length $a_s < 0$, while suppresses it slightly when $a_s > 0$ [25]. In addition by adjusting the strength of SOC, one can engineer a BCS-BEC crossover even with a very weak attractive interaction that is unable to produce a two-body bound-state in free vacuum [26]. On the other hand for imbalanced case SOC and population imbalance are counteracting, and this competition tends to stabilize the uniform superfluid phase against the phase separation. However, SOC stabilizes (destabilizes) the uniform superfluid phase against the normal phase for low (high) population imbalances [30].

In this paper we consider BCS-BEC crossover of two-component Fermi gases with SOC positioned in a two-dimensional (2D) space, and are interested in the evolutions of condensate fraction and pair coherence lengths along the crossover. As is known to all that, without SOC, a spin-up fermion pairs with a spin-down fermion, i.e. fermion pairs happen in the singlet channel. This leads to that the condensate fraction comes from the contribution of singlet pairs [32, 33] and the coherence length of singlet pairs is defined [34, 35]. In the presence of SOC, SOC also induces triplet pairs in the system besides singlet pairs. Thus at this time both singlet and triplet pairs contribute to the condensate fraction, and we must define two coherence lengths related to singlet and triplet pairs.

The Hamiltonian of the system we consider is

$$H = \int d^2 \vec{r} \left\{ \sum_{\alpha,\gamma} \Psi_{\alpha}(\vec{r}) \left[ \frac{\vec{p}^2}{2m} + v_R(\sigma_x p_y - \sigma_y p_x) - \mu \right] \Psi_{\gamma}(\vec{r}) - U \Psi_{\uparrow}(\vec{r})\Psi_{\downarrow}(\vec{r}) \right\} ,$$

(1)

where a Fermi atom of mass $m$ for spin $\alpha$ is described by the field operator $\Psi_{\alpha}(\vec{r})$. $\sigma_x$ and $\sigma_y$ denote the Pauli matrices in the $x$ and $y$ directions. $-U (U > 0)$ corresponds to attractive contact interaction among fermions and $\mu$ is the chemical potential. Without loss of generality we assume SOC to be Rashba type and its strength to be $v_R$. Transforming the field operator $\Psi_{\alpha}(\vec{r})$ into momentum space

$$\Psi_{\alpha}(\vec{r}) = \frac{1}{\sqrt{V}} \sum_k \Psi_{\alpha}(\vec{k}) e^{i \vec{k} \cdot \vec{r}} ,$$

(2)

where $V$ is the volume of the system, and introducing the mean-field order parameter $\Delta = \frac{V}{V} \sum_k < \Psi_{\uparrow}(\vec{k})\Psi_{\downarrow}(\vec{k}) >$ the Hamiltonian (1) is written into

$$H_1 = \sum_k \xi_k [\Psi_{\uparrow}(\vec{k})\Psi_{\downarrow}(\vec{k}) + \Psi_{\downarrow}(\vec{k})\Psi_{\uparrow}(\vec{k})] + g_k [\Psi_{\uparrow}(\vec{k})\Psi_{\downarrow}(\vec{k}) + \Psi_{\downarrow}(\vec{k})\Psi_{\uparrow}(\vec{k})]$$

$$-\Delta \Psi_{\uparrow}(\vec{k})\Psi_{\downarrow}(\vec{k}) - \Delta \Psi_{\downarrow}(\vec{k})\Psi_{\uparrow}(\vec{k}) + \frac{V}{U} \Delta^2$$

(3)
with \( \xi_k = \frac{\hbar^2 k^2}{2m} - \mu \) and \( g_k = \hbar v_R (k_y + i k_x) \).

Explicitly the Hamiltonian \( \mathbf{H} \) is second order about field operators and can be solved exactly. To attack this goal we choose to use imaginary time Green function method \[36\] since in this frame some interesting physical quantities, such as atom number and order parameter, can be directly deduced from Green functions. Defining two normal Green functions \( G_{\uparrow \uparrow} (\tau) = -< T_\tau \Psi_\uparrow (k, \tau) \Psi_\uparrow (k, 0) >, G_{\downarrow \downarrow} (\tau) = -< T_\tau \Psi_\downarrow (k, \tau) \Psi_\downarrow (k, 0) > \) and two anomalous Green functions \( F_{\uparrow \uparrow} (\tau) = -< T_\tau \Psi_\uparrow (-k, \tau) \Psi_\uparrow (k, 0) >, F_{\downarrow \downarrow} (\tau) = -< T_\tau \Psi_\downarrow (-k, \tau) \Psi_\downarrow (k, 0) > \), and using the time evolution of an imaginary time operator \( O, -\hbar \frac{\partial}{\partial \tau} O = [O, H] \), we obtain the equation of motion

\[
M_{4 \times 4} (G_{\uparrow \uparrow}, G_{\downarrow \downarrow}, F_{\uparrow \uparrow}, F_{\downarrow \downarrow})^T = (1, 0, 0, 0)^T
\]

where \( G_{\uparrow \uparrow}, G_{\downarrow \downarrow}, F_{\uparrow \uparrow}, F_{\downarrow \downarrow} \) are Fourier transformation of \( G_{\uparrow \uparrow} (\tau), G_{\downarrow \downarrow} (\tau), F_{\uparrow \uparrow} (\tau), F_{\downarrow \downarrow} (\tau) \) respectively and

\[
M_{4 \times 4} = \begin{pmatrix}
(iw_n - \xi_k) & -g_k & \Delta & 0 \\
-g_k^* & (iw_n - \xi_k) & 0 & -\Delta \\
\Delta & 0 & (iw_n + \xi_k) & -g_k \\
0 & -\Delta & -g_k^* & (iw_n + \xi_k)
\end{pmatrix},
\]

with \( w_n = \frac{(2n+1)\pi}{\beta \hbar} \) representing the Matsubara frequency. From \[5\] the quasiparticle excitation spectrum is obtained by replacing \( iw_n \) with \( w \) and setting \( \text{det} M_{4 \times 4} = 0 \), leading to \( E_{k, \uparrow \uparrow}^2 = (\xi_k \mp |g_k|)^2 + \Delta^2 \). Naturally for \( v_R = 0 \), the excitation spectrum reduces to the standard BCS form \( E_k^2 = \xi_k^2 + \Delta^2 \). By using \( E_{k, \uparrow \uparrow} \), Green functions can also be figured out

\[
G_{\uparrow \uparrow} = \frac{(iw_n + \xi_k)[(iw_n)^2 - \xi_k^2 - \Delta^2] - (iw_n - \xi_k)\varrho_k^2}{(iw_n - E_{k, \downarrow})(iw_n + E_{k, \downarrow})(iw_n - E_{k, \uparrow})(iw_n + E_{k, \uparrow})},
\]

\[
F_{\uparrow \downarrow} = \frac{\Delta[\xi_k^2 + \Delta^2 + |\varrho_k|^2 - (iw_n)^2]}{(iw_n - E_{k, \downarrow})(iw_n + E_{k, \downarrow})(iw_n - E_{k, \uparrow})(iw_n + E_{k, \uparrow})},
\]

\[
F_{\downarrow \uparrow} = \frac{\Delta|\varrho_k|^2}{(iw_n - E_{k, \downarrow})(iw_n + E_{k, \downarrow})(iw_n - E_{k, \uparrow})(iw_n + E_{k, \uparrow})}.
\]

From \[6\] we know for a general \( k \), Green function \( F_{\downarrow \uparrow} \) is non-zero. This signifies that SOC induces triplet fermion pairs although the interaction is pure s-wave.

The evolution of the system is completely decided by the equations of order parameter \( \Delta \) and particle number \( N \)

\[
\frac{1}{U} = \frac{1}{V} \sum_k \left[ \frac{\tanh \frac{\beta}{2} E_{k, \downarrow}}{4 E_{k, \downarrow}} + \frac{\tanh \frac{\beta}{2} E_{k, \uparrow}}{4 E_{k, \uparrow}} \right]
\]

\[
N = \sum_k \left[ 1 - \frac{\xi_k - |\varrho_k|}{2 E_{k, \downarrow}} \tanh \frac{\beta}{2} E_{k, \downarrow} - \frac{\xi_k + |\varrho_k|}{2 E_{k, \uparrow}} \tanh \frac{\beta}{2} E_{k, \uparrow} \right]
\]

by using the relations \( \Delta = \frac{\varrho_k}{2 \hbar \beta} \sum_k \varrho_k \downarrow \uparrow \) and \( N = \frac{\varrho_k}{2 \hbar \beta} \sum_k \varrho_k \downarrow \uparrow \). It is easily found that the equation \[7\] is divergent due to the fact that we use contact interaction to simulate the
true two-body potential, whose Fourier transformation should fall off at large momentum. To regulate the divergence the strength of contact interaction $U$ should be substituted into

$$\frac{1}{U} = \frac{1}{V} \sum_k \frac{1}{\hbar^2 k^2/m + \epsilon_B},$$  \hspace{1cm} (9)$$

where $\epsilon_B$ is the 2D two-body binding energy $[37]$. We self-consistently solved the equations (7) and (8) for the different strengths $v_R$ of SOC at $T = 0K$. In Fig.1(a), the evolution of the order parameter $\Delta$ is shown and we can find that the larger $v_R$ is, the larger $\Delta$ is. From this perspective the existence of SOC enhances the superfluidity of the system. By comparison the evolution of $\mu$ is more interesting and plotted in Fig.1(b). It is easily found that with the increase of $v_R$, the chemical potential $\mu$ becomes negative even if the two-body binding energy $\epsilon_B$ is very small. Hence from a conventional viewpoint, that BCS-BEC crossover is exactly realized when the chemical potential $\mu$ crosses zero, this fact suggests that when $v_R$ is beyond a critical value $v^c_R$, BCS-BEC crossover does not happen at all and the system is into BEC. From numerical work $v^c_R \approx v_F / \sqrt{2}$, where $v_F$ is 2D Fermi velocity and has a relation with particle density $n v_F = \frac{\bar{\hbar} \sqrt{2 \pi n}}{m}$.

Now we analyze the condensate fraction of fermions in the BCS-BEC crossover with SOC. In terms of a superfluid Fermi system the condensate fraction $n_0$ is related to off-diagonal long-range order (ODLRO) $[38]$ and corresponds to the maximal eigenvalue $N_0$, divided by the whole particle number $N$, of two-particle density matrix

$$\rho_2(\vec{r}_1 \sigma_1, \vec{r}_2 \sigma_2 : \vec{r}_1' \sigma_1', \vec{r}_2' \sigma_2') = < \Psi^\dagger_{\sigma_1}(\vec{r}_1) \Psi^\dagger_{\sigma_2}(\vec{r}_2) \Psi_{\sigma_2'}(\vec{r}_2') \Psi_{\sigma_1'}(\vec{r}_1') >. \hspace{1cm} (10)$$

According to Leggett’s book $[39]$, $N_0$ can be decided as follows

$$N_0 = \sum_{\sigma_1, \sigma_2} \int d\vec{r}_1 \int d\vec{r}_2 |\Phi(\vec{r}_1 \sigma_1, \vec{r}_2 \sigma_2)|^2, \hspace{1cm} (11)$$

where $\Phi(\vec{r}_1 \sigma_1, \vec{r}_2 \sigma_2) = < \Psi_{\sigma_1}(\vec{r}_1) \Psi_{\sigma_2}(\vec{r}_2) >$ is an anomalous average which arises as a result of spontaneous breaking of the $U(1)$ gauge symmetry.

According to the above theory and a fact that SOC induces triplet pairs, the condensate fraction in the presence of SOC is

$$n_0 = \frac{2}{N} \sum_k \left[ |< \Psi^\dagger(\vec{k}) \Psi_{\downarrow}(-\vec{k}) >|^2 + |< \Psi^\dagger(\vec{k}) \Psi_{\downarrow}(-\vec{k}) >|^2 \right]. \hspace{1cm} (12)$$

The pre-factor 2 comes from the contributions of spin summation and time-reversal symmetry. In contrast to the case without SOC, there is an extra contribution to the condensate fraction from triplet pairs. Following the same procedure deriving the equations (7) and (8), we have

$$n_0 = \frac{\Delta^2}{4N} \sum_k \left[ \frac{\tanh^2 \frac{\beta}{2} E_{k,-}}{E^2_{k,-}} + \frac{\tanh^2 \frac{\beta}{2} E_{k,+}}{E^2_{k,+}} \right]. \hspace{1cm} (13)$$
When \( v_R = 0 \), \( E_{k,-} = E_{k,+} = E_k \) and \( n_0 = \frac{\Delta^2}{2N} \sum_k \tanh^2 \frac{\beta E_k}{2} \), same as the results in [33]. In Fig.2 we calculate the condensate fraction \( n_0 \) for different \( v_R \) at \( T = 0K \). The results are twofold. Firstly SOC also enhances the condensate fraction, which is consistent with Fig.1(a). In addition, with the increase of \( \epsilon_R \) \( n_0 \) rapidly increases to a large value for a large \( v_R \). Maybe this phenomenon can be illustrated from the results of Fig.1(b), that for a large \( v_R \) there is not BCS-BEC crossover and the system is situated in BEC, building on the fact that for a weakly interacting BEC, almost all atoms are into the condensate. A recent paper [40] also calculate the condensate fraction in the same system but do not think over the contribution from triplet pairs, so their result is qualitatively incorrect.

At last we determine the coherence lengths for singlet and triplet pairs. On general ground, information on pair coherence lengths can be extracted from the pair-distribution function

\[
g_{\sigma_1, \sigma_2}(r) = \frac{1}{n^2} < \Psi_{\sigma_1}^\dagger(\vec{r}) \Psi_{\sigma_2}^\dagger(0) \Psi_{\sigma_2}(0) \Psi_{\sigma_1}(\vec{r}) >.
\]

Following the same spirit that the Hartree-Fock term has been neglected in the Hamiltonian (3), at the mean-field level (14) becomes

\[
g_{\sigma_1, \sigma_2}(r) = \frac{1}{n^2} < \Psi_{\sigma_1}^\dagger(\vec{r}) \Psi_{\sigma_2}^\dagger(0) >^2,
\]

and pair coherence lengths \( \xi_{\text{pair}}^s \), \( \xi_{\text{pair}}^t \) for singlet and triplet pairs can be obtained as

\[
(\xi_{\text{pair}}^s)^2 = \frac{\int d\vec{r} \hat{g}^s_{\downarrow, \downarrow}(\vec{r})}{\int d\vec{r} \hat{g}^s_{\downarrow, \downarrow}(r)} = \sum_k \nabla_k \varphi_k^s \cdot \nabla_k \varphi_k^s
\]

(16)

and

\[
(\xi_{\text{pair}}^t)^2 = \frac{\int d\vec{r} \hat{g}^t_{\downarrow, \downarrow}(\vec{r})}{\int d\vec{r} \hat{g}^t_{\downarrow, \downarrow}(r)} = \sum_k \nabla_k \varphi_k^t \cdot \nabla_k \varphi_k^t
\]

(17)

with \( \varphi_k^s = \frac{\Delta}{4} \left[ \frac{\tanh \frac{\beta E_{k,-}}{2}}{E_{k,-}} + \frac{\tanh \frac{\beta E_{k,+}}{2}}{E_{k,+}} \right] \) and \( \varphi_k^t = \frac{\Delta}{4 \epsilon_k} \left[ \frac{\tanh \frac{\beta E_{k,-}}{2}}{E_{k,-}} - \frac{\tanh \frac{\beta E_{k,+}}{2}}{E_{k,+}} \right] \). Without SOC, \( \varphi_k^t = 0 \) but \( \varphi_k^s = \frac{\Delta}{2} \tanh \frac{\beta E_k}{2} \), consistent with the results in [34, 35]. Fig.3 and Fig.4 describe the behaviors of \( \xi_{\text{pair}}^s \) and \( \xi_{\text{pair}}^t \) in the process of evolution at \( T = 0K \), respectively. Very explicitly, SOC suppresses pair coherence lengths for both singlet and triplet pairs, and for a large \( v_R \), pair coherence lengths rapidly decrease to a small value. We think that this phenomenon can also be understood from Fig.1(b) in the light of the fact in BEC region pair coherence length is much smaller than in BCS region. Besides by comparing such two figures, triplet pair coherence length \( \xi_{\text{pair}}^t \) always is larger than singlet pair coherence length \( \xi_{\text{pair}}^s \). Physically this is the result from Pauli exclusion principle.

In summary we have discussed the evolution from BCS to BEC superfluids in the presence of Rashba SOC in two dimension and shown that when the strength of SOC is beyond a critical value, BCS-BEC crossover does not happen in a conventional sense. In addition, we studied the evolutions of the condensate fraction and pair coherence lengths. The results indicate that SOC enhances the condensate fraction, but suppresses pair coherence lengths. Furthermore we also give some physical interpretation for some phenomena.
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Figure 1: The evolutions of order parameter $\Delta$ (a) and chemical potential $\mu$ (b) as a function of two-body binding energy $\epsilon_B$, in units of the Fermi energy $\epsilon_F$, for different the strengths $v_R$ of SOC at $T = 0K$. In (a) from bottom to top and in (b) from top to bottom $v_R/v_F = 0, \sqrt{2}/2, 1, \sqrt{2}, 2$. 
Figure 2: The evolution of the condensate fraction $n_0$ as a function of two-body binding energy $\epsilon_B$, in units of the Fermi energy $\epsilon_F$, for different the strengths $v_R$ of SOC at $T = 0K$. From bottom to top $v_R/v_F = 0, \sqrt{2}/2, 1, \sqrt{2}, 2$. 
Figure 3: The evolution of singlet pairs coherence length $\xi_{\text{pair}}^s$ as a function of two-body binding energy $\epsilon_B$, in units of the inverse Fermi wavevector $k_F$ and Fermi energy $\epsilon_F$ respectively, for different the strengths $v_R$ of SOC at $T = 0K$. From top to bottom $v_R/v_F = 0, \sqrt{2}/2, 1, \sqrt{2}, 2$. 
Figure 4: The evolution of triplet pairs coherence length $\xi_{\text{pair}}^t$ as a function of two-body binding energy $\epsilon_B$, in units of the inverse Fermi wavevector $k_F$ and Fermi energy $\epsilon_F$ respectively, for different the strengths $v_R$ of SOC at $T = 0K$. From top to bottom $v_R/v_F = \sqrt{2}/2, 1, \sqrt{2}, 2$. 