An Incentive-Aware Job Offloading Control Framework for Mobile Edge Computing

Lingxiang Li Member, IEEE, Tony Q.S. Quek Fellow, IEEE, Ju Ren Member, IEEE
Howard H. Yang Member, IEEE, Zhi Chen Senior Member, IEEE, Yaoxue Zhang Senior Member, IEEE

Abstract—This paper considers a scenario in which an access point (AP) is equipped with a mobile edge server of finite computing power, and serves multiple resource-hungry mobile users by charging users a price. Pricing provides users with incentives in offloading. However, existing works on pricing are based on abstract concave utility functions (e.g., the logarithm function), giving no dependence on physical layer parameters. To that end, we first introduce a novel utility function, which measures the cost reduction by offloading as compared with executing jobs locally. Based on this utility function we then formulate two offloading games, with one maximizing individual’s interest and the other maximizing the overall system’s interest. We analyze the structural property of the games and admit in closed form the Nash Equilibrium and the Social Equilibrium, respectively. The proposed expressions are functions of the user parameters such as the weights of computational time and energy, the distance from the AP, thus constituting an advancement over prior economic works that have considered only abstract functions. Finally, we propose an optimal pricing-based scheme, with which we prove that the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point.

Index Terms—Mobile edge computing, multi-user offloading, game theory, wireless network economics

I. INTRODUCTION

With the development of internet services, a diverse variety of computing-intensive applications such as mobile shopping, face recognition, and augmented reality, are emerging and attracting great attention. The execution of those novel applications typically requires low latency and high power consumption [1]. Meanwhile, due to its physical size constraint, the mobile device usually has insufficient local computing power or limited battery capacities. As such, it may not provide satisfactory quality of experience to the users. An alternative way is to offload all or part of the jobs to the resource-rich cloud infrastructures such as Amazon EC2 via the access point (AP) associated with users. Such cloud-aided computing eases users’ burden in executing computing-intensive jobs in time. However, it also suffers from wide area network (WAN) delay between the cloud and the AP, as the cloud is often located far away from the users [2].

More recently, a new trend of moving the function of cloud computing to the network edge is happening. Along this line and in the cellular network, the network edge refers to the radio access network (RAN) side of the Internet, and the AP (e.g., the base station, the Wi-Fi) is equipped with one additional computing server, i.e., the mobile edge computing (MEC) server. There are two dominant advantages of applying MEC technology in cellular networks. First, instead of residing in a far-away cloud, the computing server is located at the AP, close to the mobile device users, thus avoiding the WAN delay between the AP and the cloud. Second, since the AP is in control of both the computing and the radio resource, a joint optimization of those resources could be performed, bringing about considerable improvement in the computing/radio resource efficiency.

Presently, researchers have been actively studying the joint optimization of computing and radio resources for different MEC based scenarios. Specifically, the works of [3]–[6] consider the single user case, jointly deciding the offloading decision, the CPU-cycle frequencies for mobile execution, and the transmit power for offloading. The works of [2]–[10] further examine the multi-user case, with users offloading jobs to the AP in a spatial division multiple access (SDMA), a time division multiple access (TMDA) or a frequency division multiple access (FDMA) mode. The case with multiple access points is studied in [11], wherein a mobile user allocates its jobs to multiple nearby access points. The D2D case is studied in [12], wherein mobile users can dynamically and beneficially share the computation and communication resources among each other. More complex energy harvesting driven networks are considered in [13], [14], where energy-constrained mobiles are considered and the energy harvesting technique is integrated into the mobile edge computing cellular networks. The above studies all formulate a centralized resource allocation problem, which involves solving a Mixed Integer Nonlinear Programming (MINP) problem, and most of them only give numerical results to illustrate the performance.

Due to the nature of sharing an MEC server, the expected delays of different users spent in the system required by the edge computing are coupled. Hence, it is usually very complicated to optimize the MINP problem centrally. Moreover, since users are strategic, making decisions to maximize individual welfare, it is not easy to control the decision variable directly. Therefore, another line of research targets resource allocation in a distributed way. In particular, the works of [15]–[19] reformulate the original centralized MINP problem into an interactive decision-making process among a
group of rational users. Based on game theory the equilibrium points of the interactive decision process are examined, and decentralized algorithms with low complexity are proposed. In [20], the original problem is decomposed into several easy-to-handle sub-problems and solved semi-distributively with convex optimization techniques.

A. Related works and main contributions of this paper

In this paper, we aim to do resource allocation in a decentralized way. Different from [15]–[20], we take queueing delays into account. Specifically, we consider an MEC assisted RAN serving multiple mobile device users as in [20], except that, unlike [20] which only considers a binary decision of offloading, we consider a more general case of controlling the flow rate of offloading. Our network comprises users that work in a FDMA mode to offload part/all of its jobs to the AP. Users may have different attributes, such as distance from the AP, battery capacity, and the application it runs. As such, different users have different utilities of offloading. In addition, since the computing power of the edge is limited, the users offloading jobs are competing for computing resource, and congestion may happen if too many jobs are offloaded to the edge. Therefore, waste would be reduced if the limited computing resources at the AP are reserved for the users that value them the most.

Pricing is a key tool to allocate resources to the users who value them the most, thus controlling congestion in resource competition cases [21], [22]. In addition, in economics there are some mechanisms such as the Vickrey-Clarke-Groves (VCG) mechanism, which is designed to provide incentives to the users to choose the socially optimal levels of demand and reveal their true utility functions; this mechanism is especially useful when the allocation requires users’ payoff functions and users may have motivations to cheat. However, existing works from the economics viewpoint such as [22], [23] are based on abstract utility functions (e.g., a simple logarithm function of the amount of product) and give no specific dependence of the utility function on the physical layer parameters. In this paper, we study the interactive decision-making process among users to optimize the physical layer parameters. Unlike the works of [15]–[19], which only study the Nash Equilibrium and no incentive-aware scheme is considered, in this paper we aim to integrate the ideas of economics into the optimization of MEC networks. Our goal is to provide users with the appropriate incentives to control their flows and improve the overall system performance in a distributed way. Our main contributions are summarized below.

1) We introduce a novel utility function (see eq. (13)), which subtracting the delay cost required by edge computing equals the profit a user achieves by offloading (14). This utility function measures the difference between the local computing cost and the offloading cost (including the offloading delay and the offloading energy consumption). We prove that the proposed utility function is concave and monotonously increasing with respect to users’ offloading flow rate, satisfying the basic nature of the utility function in economics, thus bridging the gap between the economics and physical layer parameter optimizations.

2) Based on the given utility function we propose an incentive-aware job offloading framework, in which the user decides the amount to offload by comparing its utility function with the delay cost required by edge computing. We determine in closed form the Nash Equilibrium point (see Theorem 1) as well as the Social Equilibrium point (see Theorem 3). Our analytical results fully describe the dependence of the Equilibria on the edge computing power as well as the user parameters such as the local computing power, the weights of the computational time and energy the distance from the AP, and apply to any number of users.

3) We derive in closed form the sufficient and necessary conditions for a positive unique Nash/Social Equilibrium to exist (see Corollary 1 and Corollary 2, respectively). We also propose an optimal pricing-based scheme (see Table Algorithm 1, which regulates users’ action by adding an appropriate price. With this pricing-based scheme, the individual objectives of self-interested users are aligned with that of the overall system, and the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point (see Theorem 4).

B. Organization of this paper

The rest of this paper is organized as follows. In Section II, we introduce the system model, including the computation model, the radio access model and the offloading policy. In Section III, we propose a business model for mobile edge computing, based on which we formulate two games, with one maximizing individual’s interest and the other maximizing the overall system interest. In Section IV, we examine the equilibrium points of the games, based on which we propose an optimal pricing-based scheme. We prove that with this pricing-based scheme, the individual objectives of self-interested users are aligned with that of the overall system. Numerical results are given in Section V and conclusions are drawn in Section VI.

II. System Model

We consider an MEC system (see Fig. 1(a)) consisting of an AP and multiple mobile devices. The wireless AP could be a small-cell base station, or a Wi-Fi AP. Aside from serving as a conventional AP to the core network, it is installed with an additional computing server and serves as an edge computing server. The mobile devices are running computation-intensive and delay-sensitive jobs, however, they may have insufficient computing power or limited battery energy to complete those jobs. As such, the devices that are short of computing or energy resources, should offload part of their jobs to the AP. In what follows, we will introduce the offloading policy, the wireless channel model, followed by the models for computing.
A. The offloading policy and the radio access model

Before proceeding to the offloading policy, we first introduce the job generation model. We assume that jobs arrive at the mobile device user following a Poisson process and at a rate of $\lambda_a$. On the other hand, in wireless communication systems time is divided into slots having a unit duration of $t_0 = 1$ ms. As such, we have a discrete version of the Poisson arrival process, i.e., a Bernoulli process with the parameter $p_a = \lambda_a t_0$. The inter-arrival times at the mobile device are thus independent and i.i.d. random variables, following a geometric distribution with the parameter $p_a$. Besides, the service times are assumed to be identically distributed (i.i.d.) and exponentially distributed with parameter $\mu_s$. In this paper, the job offloading is described by a tuple $(l_a, \mu_s)$, where $\mu_s$ represents the average CPU cycles required to accomplish a job, and the product $l_a \mu_s$ (e.g., the input parameters and the program codes) denotes the required offloading data size per job.

The mobile device users that are short of computing or energy resources, intend to offload all/part of their jobs to the AP. Specifically, when a job arrives, the mobile device will check its wireless channel condition to the AP. We consider flat-fading channels and assume that the channel coherence time is bigger than the time required to finish offloading a job. As such, we assume that once a decision to offload its current job is made, the mobile device user could finish offloading in a channel block. Hence, we consider the following offloading policy. When a job arrives and the channel is too bad to carry data transmission, the mobile device user shall choose local computing, instead of waiting for a favorable channel condition. On the other hand, if the channel power gain is higher than a threshold to support its expected transmission rate, denoted by $\beta_k$, the mobile device user would choose to offload its job to the AP.

In this paper, we consider the scenario with sufficient wireless frequency bands, and multiple mobile devices access to the AP in a FDMA mode. This is possible in the future five-generation (5G) wireless communication era, wherein an unprecedented spectrum and multi-Gigabit-per-second (Gbps) data rates are available to its users. In FDMA mode, each mobile device user is allocated a different frequency band, indicating that they suffer no multi-user interference from each other. Let $h_k$ denote the small-scale channel gain from the $k$-th mobile device to the AP, $k = 1, 2, \ldots, N$. The achievable uplink data rate can thus be computed by

$$R_k = \log(1 + d_k^{-\alpha}|h_k|^2 P_t / \sigma^2), \quad k = 1, 2, \ldots, N, \quad (1)$$

where $d_k$ denotes user $k$-th distance to the AP and $\alpha$ represents the path loss exponent. $P_t$ is the transmission power and $\sigma^2$ denotes the received noise power at the AP. Comparing the achievable data rate $R_k$, with the expected data rate $\beta_k$ and by Shannon theorem \[26\], one can see that when $R_k > \beta_k$ the mobile device could offload its job to the AP successfully. By some mathematical derivations and letting $P_k = d_k^{-\alpha} P_t / \sigma^2$ be the received SNR, we arrive at the following conditions of successful offloading,

$$|h_k|^2 > (e^{\beta_k} - 1) P_k^{-1}, \quad k = 1, 2, \ldots, N. \quad (2)$$

These, combined with the aforementioned offloading policy, indicate that the offloading frequencies (probabilities) are

$$x_k = \Pr(|h_k|^2 > (e^{\beta_k} - 1) P_k^{-1}), \quad k = 1, 2, \ldots, N. \quad (3)$$

According to the expressions in \[3\], it can be seen that the offloading frequency at the mobile device user is a decreasing function of the threshold value of $\beta_k$. As such, by adjusting $\beta_k$ the mobile device user is able to control the frequency it offloads jobs.

B. Computation model

According to the offloading policy, jobs that are offloaded to the AP will be pushed into a buffer of the AP, awaiting for edge computing. Otherwise, the jobs are pushed into a local buffer, awaiting for local computing. In what follows,
we discuss the total overhead/cost in terms of local computing and edge computing.

1) Local computing: In the local computing approach the mobile device user executes the job with its own computing power. Let \( f_m \) be the mobile device’s computing capability (CPU cycles per second), which indicates that the service rate of local computing as \( \mu_m t_0 = (f_m / \mu_a) t_0 \) (jobs per slot). Thus, for each job the expected time spent in the system (including both the computation execution time and the time awaiting in a local buffer) can be given as

\[
D^{LC}(x_k) = \frac{1}{\mu_m t_0 - p_a (1 - x_k) t_0} \overset{(a)}{=} \frac{1}{\mu_m - \lambda_a (1 - x_k)}, \quad k = 1, 2, \cdots, N, \tag{4}
\]

where (a) is due to \( p_a = \lambda_a t_0 \). The computational energy spent in local computing can be expressed as

\[
E^{LC}_k = \kappa_m f^2_m \mu_a, \quad k = 1, 2, \cdots, N, \tag{5}
\]

where \( \kappa_m f^2_m \) is the power consumption per CPU cycle, and \( \kappa_m \) is an energy consumption coefficient that depends on the chip architecture [27].

Combining (4) and (5) we can compute the total weighted cost by local computing as follows,

\[
Z^{LC}_k(x_k) = c^e_k E^{LC}_k + c^t_k D^{LC}(x_k), \quad k = 1, 2, \cdots, N, \tag{6}
\]

where \( 0 < c^e_k < 1 \) (in units 1/Joule) and \( 0 < c^t_k < 1 \) (in units 1/Second) are the weights of the computational energy and time. With different weights \( c^e_k \) and \( c^t_k \) we allow different users to put different emphasis in decision making. For example, when the mobile device is at a low battery state, it would tend to reduce the energy consumption in decision making. As such, the user will choose a higher value of \( c^e_k \). On the other hand, when the user has a more stringent quality of experience (QoE) requirement in delay, it would like to put more consideration to the delay cost and it will set a bigger value of \( c^t_k \).

2) Edge computing: In the edge computing approach the mobile device user will offload its job to the AP and resort to the computing power of the AP. First, it takes the mobile device some time to complete offloading, which can be calculated as follows,

\[
D^{EC}_{k,1}(x_k) = \frac{1}{\mu_B - \sum_{j=1}^{N} \lambda_j x_j}, \quad k = 1, 2, \cdots, N. \tag{7}
\]

This, combined with the the fact each mobile device transmits with power \( P_t \), indicates that the energy required by offloading is

\[
E^{EC}_k(x_k) = P_t \frac{1}{\mu_B - \lambda_k(x_k)} \overset{(a)}{=} P_t \frac{1}{\mu_B - \sum_{j=1}^{N} \lambda_j x_j}, \quad k = 1, 2, \cdots, N. \tag{8}
\]

On the other hand, it takes some time for the offloaded job to stay at the AP before it leaves after execution. Based on the aforementioned offloading policy, the mobile device will choose to offload its jobs with probability \( x_k \). This, combined with the splitting property of the queueing theory [25], indicates that the job offloading from a mobile device user also follows a Poisson process, with parameter \( \lambda_a x_k \). Therefore, the job arrival at the AP is a superposition of multiple Poisson processes from multiple mobile devices, which, according to the superposition property of queueing theory [25], is another Poisson process with the arrival rate as the sum arrival rate of the superposed processes, i.e., \( \sum_{k=1}^{N} \lambda_k x_k \). In addition, the service times are i.i.d. and exponentially distributed with parameter \( \mu_a \). Therefore, we get a M/M/1 queue for computing at the AP. Let \( f_B \) be the AP’s computing capability (CPU cycles per second). The service rate of the mobile device is then \( \mu_B = f_B / \mu_a \) (jobs per second). Hence, for each job the expected time spent at the AP (including both the computation execution time and the time spent awaiting in the edge buffer) can be expressed as,

\[
D^{EC}_{k,2}(x) = \frac{1}{\mu_B - \sum_{k=1}^{N} \lambda_k x_k}, \quad \forall k, \tag{9}
\]

where \( x = (x_1, x_2, \cdots, x_N) \).

Similar to many existing studies such as [13], [16], [18], we neglect the energy overhead of edge computing, due to the fact that normally the AP can access to wired chating and it has no lack-of-energy issues. As such, by combining (7) to (9) we could compute the total weighted cost by edge computing as follows,

\[
Z^{EC}_k(x) = c^e_k E^{EC}_k(x_k) + c^t_k (D^{EC}_{k,1}(x_k) + D^{EC}_{k,2}(x)), \quad k = 1, 2, \cdots, N. \tag{10}
\]

Note that the total cost by offloading as well as the end-to-end delay of computation \( D^{EC}_{k,2}(x) \) depend on both the local variable (i.e., \( x_k \) for the \( k \)-th user) and also the offloading frequency of other users. As we will see later, due to this coupled nature of delay we have coupled objective functions.

### III. Game Formulation

Due to the sharing nature of MEC server, the delay and hence the cost of offloading for users are coupled. As such, it is usually difficult to jointly optimize users’ offloading decisions centrally. On the other hand, the mobile devices are owned by different individuals who may have different QoE requirements on delay and pursue different interests. In that case, the economics theory is a useful tool in dealing with interactions among users, devising decentralized mechanisms with low complexity, and inducing users to self-organize into a mutually satisfactory solution.

In the following, we will formulate games from the perspective of economics. Towards that goal, we first introduce a business model for mobile edge computing, which includes physical layer parameters in the utility function and the cost function. Then, based on the proposed business model we formulate two games, with one maximizing the individual’s interest and the other maximizing the overall system’s interest.

#### A. Proposed economics model for mobile edge computing

By the aforementioned offloading policy, upon the arrival of a new job, the mobile device will choose to offload with probability \( x_k \), or push it into a local buffer for local
computing with probability $\bar{x}_k$ ($\bar{x}_k \triangleq 1 - x_k$). Therefore, the expected total cost can be written as follows,

$$Z_k(x) = \bar{x}_k Z_k^{EC}(x_k) + x_k Z_k^{LC}(x).$$

(11)

On the other hand, when there is no such edge server providing computing power, users have to complete every job by themselves and the average cost running a job locally is $Z_k^{LC}(0)$. This, combined with (11), indicates that the gross profit of offloading by the $k$-th user under a given offloading strategy $x$ is,

$$V_k(x) = Z_k^{LC}(0) - Z_k(x), \quad k = 1, \ldots, N.$$  

(12)

The key idea of the business model is to introduce the utility function and the cost function. Several observations are in order. Firstly, the profit each user obtains equals the cost savings from offloading, and it is a linear combination of the energy costs and the delay costs. Secondly, the coupled delay cost $D_{k,2}(x)$ reflects the harm/congestion each user causes to the other users, with a bigger value indicating that the AP provides worse service to the users. Thirdly, except for the expected time spent at the AP, i.e., $D_{k,2}(x)$, which depends on all users’ offloading decisions, the other items in the profit function only depend on each user’s own offloading frequency $x_k$. Motivated by these observations, we introduce a utility function $U_k(x_k)$ which includes the items in the profit function that only depend on the local variable $x_k$, i.e.,

$$U_k(x_k) = Z_k^{LC}(0) - \bar{x}_k Z_k^{LC}(x_k) - x_k (c_k E_k^{EC}(x_k) + c_k D_k^{EC}(x_k)).$$

(13)

This, combined with (12), indicates that

$$V_k(x) = U_k(x_k) - C_k(x), \quad k = 1, \ldots, N,$$  

(14)

where $C_k(x) \triangleq c_k x_k D_k^{EC}(x)$ can be regarded as the cost due to the sharing of an MEC server at the AP.

In economics, the utility function measures the welfare a consumer obtains, as a function of the consumption of real goods such as food, clothes, and so on. Here, we apply a utility function to capture the benefit a user can obtain by offloading. The offloading frequency $x_k$ quantifies the amount a user is willing to offload; a higher offloading frequency indicates that the user is more willing to offload. By comparing the utility function and the cost function in (14), each user could find its indifferent point of offloading frequency, i.e., the point at which the user achieves a maximum profit. It turns out that the utility function in (13) is strictly concave and strictly increases with respect to users’ offloading frequency, satisfying the law of diminishing marginal returns in economics; this is discussed in the following lemmas.

Lemma 1: Taking the derivative of $U_k(x_k)$ with respect to $x_k$, we arrive at the demand function of user $k$, i.e.,

$$g_k(x_k) \triangleq \frac{\partial U_k(x_k)}{\partial x_k}.$$  

For each mobile device user, it holds true that the demand function $g_k(x_k)$ is a monotonically decreasing function of the offloading frequency $x_k$. Moreover, there exists a unique solution of the equation $g_k(x_k) = 0$, denoted as $x_k^{up}$.

Proof: See Appendix A

Lemma 2: For each mobile device user, it holds true that $U_k(0) = 0$, and the utility function $U_k(x_k)$ is a monotonically increasing function of the offloading frequency $x_k \in [0, x_k^{up}]$.

Proof: By the definition of $U_k(x_k)$, it is easy to verify that $U_k(0) = 0$. On the other hand, according to Lemma 1, it is clear that $g_k(x_k^{up}) = 0$. In addition, since $g_k(x_k)$ is a monotonically decreasing function of $x_k$, it holds true that $g_k(x_k) > 0$ when $x_k < x_k^{up}$. This completes the proof.

Lemma 3: If the $i$-th mobile device user is closer to the AP than the $j$-th user, that is, $d_i < d_j$, then its utility function strictly dominates that of the $j$-th user, i.e., $U_i(x) > U_j(x)$.

Proof: The proof is omitted since it could be easily verified with the formula of $U_k(x_k)$ in (13).

![Fig. 2: The achievable utility for varying Mobile-AP distances.](image)

**Lemma 1 to 3** provide the properties of the utility curve. That curve is illustrated in Fig. 2 (as a function of the offloading frequency), for a system with $c_k^L = 0.9$, $c_k^C = 0.1$, $f_m = 0.1GHz$. The job arrives at a rate of $\lambda = 0.6$ jobs per second, with each job requiring an average of $\mu = 100M$ CPU-cycles to run and an average of $\tau = 100$ nats to offload. Different distances, i.e., $d = 10m$, $d = 50m$ and $d = 70m$, from the user to the AP are respectively considered. One can see that the utility function is strictly increasing but the increasing rate (i.e., the demand function of $g(x_k)$) decreases as the offloading frequency increases; this is consistent with the law of diminishing marginal returns in economics, i.e., the more the user consumes, the demand for additional unit of goods decreases. Moreover, the user closer to the AP has a greater demand to offload and can achieve a higher utility value with the same offloading amount. This is keeping with the intuition that the closer the user is to the AP, the better the wireless channel it experiences.

**B. Game formulations**

We first consider a decentralized computation offloading decision making problem, wherein for any given strategies of others, the mobile device user intends to maximize its own
profit $V_k(x)$ by optimizing the local offloading frequency $x_k$ via adjusting its threshold parameter $\beta_k$. Specifically, we have the following optimization problem that each mobile device user needs to solve for.

**Problem 1 (Selfish Problem):**

$$x^*_k \triangleq \arg \max_{x_k} V_k(x)$$
$$\text{s.t. } 0 \leq x_k \leq 1.$$  

**Definition 1:** A strategy profile $x^* = (x^*_1, \cdots, x^*_N)$ is a Nash Equilibrium (NE) of the offloading game if at the equilibrium $x^*$, no player can further increase its profit by unilaterally changing its strategy, which indicates that

$$V_k(x^*_k, x^*_{-k}) \geq V_k(x_k, x^*_{-k}), \forall x_k, k = 1, \cdots, N. \quad (16)$$

where $x^*_{-k} \triangleq (x^*_1, \cdots, x^*_{k-1}, x^*_{k+1}, \cdots, x^*_N)$.

With Problem 1, we formulate a game in which each mobile device user is selfish and responses to maximize its own benefit. The objective of this game is to find the NE point from which no mobile device user has incentives to deviate. However, the solution of Problem 1 is locally optimal; it may not coverage to a point where the sum profit of all the users is maximized.

On the other hand, we consider a social-welfare computation offloading decision making problem. Assume that the AP acts as a social planner, usually referred to as the hand of God in economics. It would like users to choose their offloading decisions such that the sum profit $\sum_{k=1}^N V_k(x)$ is maximized. Specifically, we have the following optimization problem that each mobile device user needs to solve for.

**Problem 2 (Social Problem):**

$$\bar{x}^*_k \triangleq \arg \max_{x_k} \sum_{k=1}^N V_k(x)$$
$$\text{s.t. } 0 \leq x_k \leq 1.$$  

**Definition 2:** A strategy profile $\bar{x}^* = (\bar{x}^*_1, \cdots, \bar{x}^*_N)$ is a Social Equilibrium (SE) of the offloading game if at the equilibrium $\bar{x}^*$, no player can further increase the sum profit of the whole system (social welfare) by unilaterally changing its strategy, which indicates that

$$\sum_{k=1}^N V_k(\bar{x}^*_k, \bar{x}^*_{-k}) \geq \sum_{k=1}^N V_k(x_k, \bar{x}^*_{-k}), \forall x_k. \quad (18)$$

where $\bar{x}^*_{-k} \triangleq (\bar{x}^*_1, \cdots, \bar{x}^*_{k-1}, \bar{x}^*_{k+1}, \cdots, \bar{x}^*_N)$.

Intuitively, in Problem 2 each user should also be concerned with the congestion it causes to other users and should keep its offloading under an appropriate amount for other users’ welfare; the difficulty lies in how to incentivise users to do so when users are selfish and will choose their offloading decisions such that its individual profit $V_k(x_k)$ is maximized.

Pricing is a useful tool in incentivising users to choose the socially optimal levels of demand. The key idea is to enforce users to pay for the congestion it causes to the other users. In the following Problem 3, we study the pricing-based scheme, which charges users an additional edge computing service fee to regulate users’ behavior.

**Problem 3 (Regulated Selfish Problem):**

$$\hat{x}^*_k \triangleq \arg \max_{x_k} V_k(x) - P x_k$$
$$\text{s.t. } 0 \leq x_k \leq 1,$$  

where $P$ denotes the unit price for offloading. In the following section, our goal is to derive an optimal price of $P$ such that the interactive decision-making process with regulated self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point. It is worth noting that in a practical system the optimal pricing could be learned based on the historical supply-demand relationship. Due to limitations of space, in this paper we only give discussions on the existence of such optimal price and show the advantages it brings in terms of the overall system performance.

**IV. Optimal Solutions of Offloading**

In this section, we aim to derive an optimal price to regulate users’ behavior, such that by this pricing-based scheme the individual objectives of self-interested users are aligned with that of the overall system. Towards that goal, we first analyze the structural property of the games and admit in closed form the Nash Equilibrium and the Social Equilibrium, respectively. Based on these results, we then propose the optimal price. Under this pricing-based scheme, the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point. In this way, we provide an algorithm which improves the overall resource efficiency and enhances the system performance in a distributed way.

For the purpose of getting more insights into the offloading problem, we first investigate the case in which users have the same time cost weight as well as the same energy cost weight, i.e., $c'_k = c'_0$, $c_k = c_0$, $\forall k$.

**A. Analysis of Nash Equilibrium**

The key idea for deriving the equilibrium is to look into the necessary conditions, i.e., at the equilibrium the derivative of the objective function should be zero.

We first look into Problem 1. At the equilibrium it holds that

$$\frac{\partial U_k(x_k)}{\partial x_k} - \frac{\partial x_k c'_k D^E_C(x)}{\partial x_k} = 0$$

$$\Leftrightarrow g_k(x_k) = c'_0 (\mu_B - \sum_{j=1}^N \lambda_n x_{j}) / (\mu_B - \sum_{j=1}^N \lambda_n x_{j})^2, \quad k = 1, \cdots, N. \quad (20)$$

Combining those $N$ equations in (20) yields the following Theorem.

**Theorem 1:** For the case with users of the same demand function, denoted by $g_k = g_0$, $\forall k$ (also referred to as the homogeneous user case in the following), at the Nash Equilibrium each mobile device user has the same value of offloading frequency $\bar{x}^*_0$, which can be achieved via setting a same parameter of $\beta^*_0$. Moreover, the offloading frequency satisfies the following equation,

$$N \lambda_n \bar{x}^*_0 + \frac{c'_0 (\mu_B - (N-1) \lambda_n \bar{x}^*_0)}{g_0(x^*_0)} = \mu_B.$$  

**(21)**
there exists a positive unique Nash Equilibrium. Otherwise, no mobile device would like to offload, which corresponds to a trivial Nash Equilibrium, i.e., $x_k^* = 0, \forall k$.

**Proof:** The proof is omitted since it is similar to that of Corollary 1.

**Corollary 3:** Comparing the Nash Equilibrium and the Social Equilibrium, the former converges to a Equilibrium point with a higher offloading frequency, i.e.,

$$x_0^* > x_k^*.$$

**Proof:** Clearly, it holds that $c_0^* (\mu_B - (N - 1) \lambda_a x_0^*) < c_0^* \mu_B$, which, combined with (21) and (24), indicates that $g_0(x_0^*) < g_0(x_k^*)$. Besides, according to Lemma 1 $g_0(x)$ is a monotonically decreasing function. As such, we have $x_0^* > x_k^*$. This completes the proof.

**Corollary 3** is consistent with the intuition that less jobs will be offloaded by the users who aim to solve for Problem 2, since they put additional consideration on the harm offloading causes to the other mobile device users.

### C. Optimal pricing-based scheme to achieve Social Equilibrium

According to Corollary 3, less jobs will be offloaded by users for the purpose of maximizing the sum profit of users. However, in the market every user is selfish and aims to maximize its own benefit.

In this subsection we study the regulated selfish problem, which regulates users’ action via charging them appropriate economic expenses for providing the edge computing service. The goal is to align the individual objectives of self-interested users with that of the overall system, such that the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point.

Recall Problem 3 where each mobile device user is selfish and aims to maximize its own benefit. The user is required to pay for offloading at a unit price of $\mu_B$. As such, we replace $V_k(x_k)$ in Problem 1 with $V_k(x_k) = V_k(x_k) - P x_k$. Taking the derivative of $\hat{V}_k(x_k)$ with respect to $x_k$ and letting it be zero, we arrive at similar formulas as in (20), i.e.,

$$g_k(x_k) - P = \frac{c_0^* (\mu_B - \sum_{j \neq k} \lambda_a x_j)}{g_0(x_0^*)} - \frac{c_0^* \sum_{j = 1}^{N} \lambda_a x_j}{g_0(x_0^*)^2}, \quad k = 1, \cdots, N. \quad (25)$$

Letting $\hat{g}_k(x_k) = g_k(x_k) - P$ and by some algebra derivations of the equations in (25), we obtain

$$\mu_B - \sum_{j \neq k} \lambda_a x_j = F_N(x_k), \quad k = 1, \cdots, N, \quad (26)$$

where $F_N(x_k) \triangleq \lambda_a x_k + \frac{c_0^*}{2 g_0(x_k)} + \sqrt{\left( \frac{c_0^*}{2 g_0(x_k)} \right)^2 + \frac{c_0^* \lambda_a x_k}{g_0(x_k)}}$.

The equations in (25) give the necessary conditions of the equilibrium of the game, indicating the best response of a user when the other users’ offloading decisions are given. Based on (26) we update the offloading frequency $x_k$ iteratively. The iteration stops until the offloading frequency difference of two

---

**Proof:** See Appendix B

**Corollary 1:** For homogeneous user case, if the following equation is satisfied,

$$c_0^* E_k^{LC} + \frac{c_0^* \mu_m}{(\mu_m - \lambda_a)^2} > \frac{c_0^*}{\mu_B}, \quad (22)$$

there exists a positive unique Nash Equilibrium. Otherwise, no mobile device would like to offload, which corresponds to a trivial Nash Equilibrium, i.e., $x_k^* = 0, \forall k$.

**Proof:** See Appendix C
specifically, substituting (25) into of users.

Theorem 4: Considering the homogeneous user case, for the Nash Equilibrium point to equal to the Social Equilibrium point, the price should be set as

\[ P = \frac{(N-1)\lambda_a}{\mu_B} \bar{x}_0 g_0(\bar{x}_0^+) \].

Proof: See Appendix E

V. NUMERICAL RESULTS

In this section, we conduct simulations to validate our theoretical findings and the proposed algorithm (see Algorithm 1) for both the homogeneous and the heterogeneous user cases. We consider a system model as illustrated in Fig. 1. For simplicity, we consider the symmetric case where are assumed to be known at the end user in the interactive decision-making process. In a practical system the latter is the path loss exponent. The small-scale channel gains are assumed to be identically distributed (i.i.d.) and exponentially distributed with parameter 1, i.e., \(|h_k|^2 \sim \exp(1)\). Substituting this distribution into yields

\[ x_k = \exp(-(e^{\beta_k} - 1)\rho_k^{-1}), \ k = 1, 2, \ldots, N. \]

The transmit power is \( P = 100\text{mW} \) and the noise power level is \( \sigma^2 = -40\text{dBm} \). Unless otherwise specified, we set \( d = 50\text{m} \). As such, the received SNR at the AP is \( \rho_k = d_k^{-\alpha} P_f / \sigma^2 = 0.89 \). The stop threshold \( \epsilon = 10^{-3} \).

A. The special homogeneous user case

In this subsection, we consider the homogeneous scenario in which users are uniformly distributed on a circle of radius \( R = d \) (unit: meters) and the AP locates at the center; in this case, all the users have the same utility functions.

![Fig. 3: Convergence of the proposed algorithm in the homogeneous user case.](image-url)

Fig. 3 illustrates the convergence of the proposed algorithm. The scheme with the price \( P = 0 \) and the proposed scheme (see Table Algorithm 1) with the optimal price (see Theorem 4) are considered respectively. The local information of \( F_N(x_k) \) and the edge delay-related information \( \mu_B - \sum_{j \neq k} \lambda_a x_j \) are assumed to be known at the end user in the interactive decision-making process. In a practical system the latter could be learned from the historical information on its edge.
computing delays. In the simulations we assume that such information could be perfectly estimated at the end users. It can be seen that the scheme with price equal to zero converges to the NE point given by Theorem 1. In contrast, the scheme with the optimal price converges to the SE point, where the SE point is given by Theorem 3. This accomplishes the goal that, via charging expenses for the edge computing service the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point. In this way, users’ behavior is regulated effectively and the edge computing resources go to the users who value them the most. Moreover, as compared with the latter scheme which needs around one hundred iterations to converge, one can see that the pricing-based scheme converges very fast, i.e., within only several iterations. The fast convergence property of the proposed pricing-based scheme is especially useful when facing an online user-behavior learning problem.

Fig. 4 illustrates the profit each user obtains by offloading a job to the edge. As expected, at the SE point a greater profit is achieved, and this improvement increases as the user moves closer to the AP. This is because the congestion becomes more serious as the offloading increases, while pricing helps to combat that. For the purpose of easy comparison, in Fig. 5 we further plot the ratio of the profit performance that we can get at the SE point and at the NE point, which indicates how the network performance benefits when applying the optimal pricing-based scheme. It can be seen that the ratio of SE/NE increases as the number of mobile device users connected to the AP increases. For the case of 200 users and with the proposed optimal pricing-based scheme, each user almost achieves a double profit as compared with applying the scheme charging no fees for the mobile edge computing service.

In Fig. 6 we plot the offloading frequency at the NE point and also at the SE point. It can be seen that more jobs will be offloaded when an user moves closer to the AP, since it experiences a smaller path loss and a better wireless channel. On the other hand, less jobs will be offloaded at the SE point as compared with that at the NE point. This is because, in the social-welfare game each user should pay for the congestion it causes to the other users. This suggests that by charging some appropriate edge computing service fee, users’ behaviors can be regulated effectively.

In Fig. 7 we check the average delays of local computing as well as edge computing. It can be seen that, the average delay of local computing increases a little bit even if it takes into account the congestion it causes to the other users. In contrast, the average delay of edge computing for the proposed pricing-based scheme decreases a lot. As such, at the SE point the user...
enjoys a smaller average delay. In addition, this improvement increases as the number of mobile device users increases. This should be expected, since in the social-welfare game the user offloads fewer jobs to the edge, which reduces the burden of the edge in computing especially when there are a large number of users.

B. The general heterogeneous user case

In the previous discussions, we have provided numerical results for the special case in which all the users have the same utility functions. In this subsection, we consider the heterogeneous user case where users are located at a random different distance to the AP. Specifically, we place mobile device users uniformly at random on a ring of radius $10 \leq d \leq 75$ (unit: meters) and center located at the AP; in this case and according to Fig. 8 one can see that the users have different utility functions. As such, it is difficult to derive closed-form results of the equilibrium as in the aforementioned homogeneous user case.

In Fig. 8 we compare the convergence of the pricing-based scheme and that of the optimal Social Equilibrium. The Social Equilibrium given by Theorem 3 for the homogeneous user case can not be applied here since each user may have a different utility function. Instead, by some algebra derivations of (23), we arrive at

$$\mu_B - \sum_{j \neq k}^N \lambda_{jk} x_j = F_S(x_k), \quad k = 1, \ldots, N, \quad (28)$$

where $F_S(x_k) \triangleq \lambda_{jk} x_k + \sqrt{c_0 \mu_B / g_k(x_k)}$. The equations in (28) give the necessary conditions of the equilibrium of the game, indicating the best response of a user when the other users’ offloading decisions are given. Based on (28) we update the offloading frequency $x_k$ one after another and iteratively; by Definition 2, if there exists an equilibrium point of this iteration, that equilibrium should be the Social Equilibrium.

From Fig. 8 it can be seen that this iterative process converges after around a hundred of rounds. On the other hand, the optimal price given by Theorem 4 also cannot be applied here. Instead, in the simulations we use the price of (27), where the offloading frequency of $x^*_k$ is given by the social problem. It can be seen that the interactive decision-making process converges to the same point as that by the social problem.

Fig. 8 plots the mean offloading frequency of users. In Fig. 8 we further respectively plot the profit each user obtains and its offloading frequency at the equilibrium. Interestingly, it can be seen that by the proposed pricing-based scheme and at the equilibrium, each user obtains the same profit as that at the Social Equilibrium. According to Theorem 2 that Social Equilibrium should be unique. Hence, the proposed pricing-based scheme is global optimal, under which the interactive decision-making process among users converges to a NE point equal to the SE point. In addition, as expected, the user closer to the AP offloads more frequently.

VI. CONCLUSION

In this paper, we have proposed an incentive-based offloading control framework for the mobile edge computing network, which consists of a radio access network (RAN) that is equipped with a mobile edge server (MEC) of finite computing power, and serves multiple resource-hungry mobile device users by charging users appropriate economic expenses for offloading computation jobs to the edge. Firstly, we have introduced a novel physical layer rooted utility function, which measures the reduction in cost by offloading as compared to executing jobs locally. We have proved this utility function satisfies some basic economic properties, thus bridging the gap between physical layer optimizations and economics. With this proposed utility function we then formulate two computation offloading decision making games, with one maximizing the individual’s interest and the other maximizing the overall system’s welfare. We have addressed analytically the structural property of the games and admit in closed form the Nash Equilibrium and the Social Equilibrium, respectively, based on which we then proposed an optimal pricing scheme, which regulates users’ action by charging expenses for providing the edge computing service. Numerical results support our theoretical results that under our proposed optimal pricing-based scheme, the interactive decision-making process with self-interested users converges to a Nash Equilibrium point equal to the Social Equilibrium point.
APPENDIX A
PROOF OF Lemma 1

By definition, the demand function is equal to the derivative of the utility function with respect to the offloading frequency $x_k$, which, combined with (13) indicates that

$$g_k(x_k) = c^d_k E_{k}^{LC} - \frac{\eta}{\beta_k(x_k)} + \frac{\eta \beta_k(x_k)}{\beta_k(x_k)} + \frac{c^d_k \mu_m}{(\mu_m - \lambda_a x_k)^2}$$

(29)

where $\eta := (c^c_k + c^e_k P_t) a \mu_a$.

Looking into the expressions in (29), the second term $-\frac{\eta}{\beta_k(x_k)}$ as well as the last term $\frac{c^d_k \mu_m}{(\mu_m - \lambda_a x_k)^2}$ decreases as $x_k$ increases, since $\beta_k(x_k)$ is a decreasing function with respect to $x_k$ (see (3)). In addition, it holds that $-x \beta'(x) > 0$. And in the sequel, we will further show that $-x \beta'_k(x_k)$ increases as $x_k$ increases, thus giving the proof that $g_k(x_k)$ decreases as $x_k$ increases.

Letting $\psi(x) = -x \beta'(x)$ and by the derivative of $\psi(x)$, we arrive at

$$\psi'(x) = \lim_{\Delta x \to 0} \frac{\psi(x + \Delta x) - \psi(x)}{\Delta x} = \lim_{\Delta x \to 0} -\frac{(x + \Delta x) \beta'(x + \Delta x) - x \beta'(x)}{\Delta x}.$$  

(30)

Besides this, by the definition of derivative it holds that

$$\beta'(x) = \lim_{\Delta x \to 0} \frac{\beta(x + \Delta x) - \beta(x)}{\Delta x},$$

(31a)

$$\beta'(x + \Delta x) = \lim_{\Delta x \to 0} \frac{\beta(x + \Delta x - \Delta x) - \beta(x + \Delta x)}{-\Delta x}.$$  

(31b)

Combining (31a) and (31b) yields

$$(x + \Delta x) \beta'(x + \Delta x) - x \beta'(x) = \beta(x + \Delta x) - \beta(x).$$

This, combined with (30), indicates that

$$\psi'(x) = -\beta'(x).$$

Since $\beta_k(x_k)$ is a decreasing function with respect to $x_k$, it holds true that $\psi'(x) > 0$. Hence, $-x \beta'(x)$ is a monotonically increasing function with respect to $x$. This completes the proof of the first part of Lemma 1.

According to the above argument, it shows that $g_k(x_k)$ is a monotonically decreasing function with respect to $x_k$. In addition, it can be verified that

$$\lim_{x_k \to 0^+} g_k(x_k) = c^d_k E_{k}^{LC} + \frac{c^d_k \mu_m}{(\mu_m - \lambda_a)^2} > 0,$$

(32)

$$\lim_{x_k \to 1^-} g_k(x_k) = -\infty.$$

Therefore, the solution of the equation $g_k(x_k) = 0$ exists and it is unique. This completes the proof.

APPENDIX B
PROOF OF Theorem 1

By definition, at the equilibrium the derivative of the objective function should be zero. Hence, according to (20) and at the equilibrium it holds that

$$(\mu_B - \sum_{j=1}^{N} \lambda_a x_j)^2 = \frac{\mu_B - \sum_{j=1}^{N} \lambda_a x_j}{g_0(x_k)}, \forall k.$$  

(32)

In what follows, we prove by contradiction and show that for any given offloading frequency vector $x = (x_1, \ldots, x_N)$ satisfying (32), $x_m = x_n$ holds true for all $m, n$. Substituting this result into (32), we arrive at the result of (21) in Theorem 1.

Assume that there exists a pair of $(x_m, x_n)$, with $x_m > x_n$. By Lemma 1, $g_0(x_k)$ is a monotonically decreasing function. As such, it holds that $g_0(x_m) < g_0(x_n)$. Besides, it holds that $\mu_B - \sum_{j=1}^{N} \lambda_a x_j > \mu_B - \sum_{j=1}^{N} \lambda_a x_n$, since $\mu_B - \sum_{j=1}^{N} \lambda_a x_j + \lambda_a x_n > (\mu_B - \sum_{j=1}^{N} \lambda_a x_n) + \lambda_a x_n$. Therefore, we have

$$\frac{\mu_B - \sum_{j=1}^{N} \lambda_a x_j}{g_0(x_m)} \geq \frac{\mu_B - \sum_{j=1}^{N} \lambda_a x_j}{g_0(x_n)}.$$  

(33)

This contradicts with (32). Similarly, we can prove that the assumption $x_m < x_n$ also contradicts with (32). In summary, it holds that $x_m = x_n, \forall m, n$. This completes the proof.

APPENDIX C
PROOF OF Corollary 1

For the ease of exposition, let the left side of Eq. (21) denote by

$$\phi(x_k) = N \lambda_a x_k + \sqrt{\frac{c^d_k (\mu_B - (N - 1) \lambda_a x_k)}{g_0(x_k)}}.$$  

(34)

It can be verified that $\phi(x_k)$ is a monotonically increasing function with respect to $x_k$. In addition, we will show later that there exists some $x_k$ such that $\phi(x_k)$ is greater than the right hand side of Eq. (21), i.e., $\mu_B$. As such, as long as there exists some $x_k$ such that $\phi(x_k)$ is less than $\mu_B$, we can infer that there exists a unique solution to Eq. (21).

Specifically, it can be verified that

$$\lim_{x_k \to x_k^{cp}} \phi(x_k) = +\infty > \mu_B, \lim_{x_k \to 0^-} \phi(x_k) = \frac{\sqrt{c^d_k (\mu_B)}}{g_0(0)}.$$  

Therefore, we arrive at a sufficient condition for the existence of solutions to Eq. (21) as follows,

$$\sqrt{\frac{c^d_k (\mu_B)}{g_0(0)}} \frac{\phi(x_k)}{\mu_B} \iff \frac{c^d_k}{\mu_B} < g_0(0) = c^d_k E_{k}^{LC} + \frac{c^d_k \mu_m}{(\mu_m - \lambda_a)^2}.$$  

(35)

And, that solution should be unique due to the monotonicity of $\phi(x_k)$ with respect to $x_k$. This completes the proof that if (35) holds true, there exists a positive unique Nash Equilibrium.

In the sequel, we prove that if $g_0(0) \leq \frac{c^d_k}{\mu_B}$, the profit function $V_k(x)$ is a non-increasing function with respect to $x_k$. In that case, the best response for the mobile device user is to stop offloading and compute all the jobs locally.
Some observations are in order. Firstly, it can be verified that
\[
\frac{c_0^\prime(\mu_B - \sum_{j=1}^N \lambda_a x_j)}{(\mu_B - \sum_{j=1}^N \lambda_a x_j)^2} \geq \frac{c_0^\prime}{(\mu_B - \sum_{j=1}^N \lambda_a x_j)^2} \geq \frac{c_0^\prime}{\mu_B}.
\]

Secondly, by assumption it holds that \(g_0(0) \leq \frac{c_0^\prime}{\mu_B}\). Combining the above observations yields
\[
\frac{c_0^\prime(\mu_B - \sum_{j=1}^N \lambda_a x_j)}{(\mu_B - \sum_{j=1}^N \lambda_a x_j)^2} \geq g_0(0) \geq g_0(x_k).
\]
This, combined with (33a)(33b), indicates that the derivative of the profit function \(V_k(x)\) is no more than zero. This completes the proof.

**APPENDIX D**

**PROOF OF Theorem 2**

As discussed in Sec. IV. B, the equations in (23) give the necessary conditions of the equilibrium of the game. Hence, for the purpose of studying the existence and the uniqueness of the Social Equilibrium point, it is sufficient to investigate the solution of the equations in (23).

The combination of the \(N\) equations in (23) indicates that at the equilibrium there exists some positive value of \(t\) such that \(g_k(x_k) = t, \forall k\). As such, the solutions in (23) can be reformulated as follows,
\[
\begin{align*}
\mu_B - \sum_{j=1}^N \lambda_a x_j &= \sqrt{\frac{c_0^\prime \mu_B}{t}}, \quad \text{(33a)} \\
g_k(x_k) &= t, \forall k. \\
\end{align*}
\]
In the sequel, we will first prove the existence of the solutions of (33a)(33b), followed by the proof of its uniqueness. In this way, we give the proof of Theorem 2.

There exists at least a solution to the equations in (33a)(33b). Some observations are in order.

1) Firstly, the nontrivial case where users cannot benefit from offloading is out of the scope of this paper. As such, only if a user’s demand at \(x_k = 0\) is greater than the edge computing cost, will it choose to offload its job to the edge i.e., it holds true that
\[
g_k(0) > \frac{c_0^\prime}{\mu_B} \iff \mu_B > \sqrt{\frac{c_0^\prime \mu_B}{g_k(0)}}.
\]
Therefore, at the offloading vector of \(x = 0\) it holds that
\[
\mu_B - \sum_{j=1}^N \lambda_a x_j > \sqrt{\frac{c_0^\prime \mu_B}{t}}. \quad \text{(34)}
\]
2) Secondly, it holds that \(\lim_{t \to 0^+} \sqrt{\frac{c_0^\prime \mu_B}{t}} = +\infty\). As such, there exists some offloading vector of \(x\) so as to
\[
\mu_B - \sum_{j=1}^N \lambda_a x_j < \sqrt{\frac{c_0^\prime \mu_B}{t}}. \quad \text{(35)}
\]
3) Thirdly, the left hand side of Eq. (33a) is monotonically decreasing while the right hand side is monotonically increasing.

Combining the above observations, we conclude that there exists at least a solution to the equations in (33a)(33b).

The solution of the equations in (33a)(33b) is unique: We give the proof by contradiction. Assume that there exist two equilibrium points \(x^1 = (x_1^1, \ldots, x_N^1)\) and \(x^2 = (x_1^2, \ldots, x_N^2)\), at which it satisfies that \(t_1 = g_k(x_k^1), \forall k,\) \(t_2 = g_k(x_k^2), \forall k,\) and \(t_1 \neq t_2\). Without loss of generality, let’s consider the case of \(t_1 > t_2\). As such, we have \(g_k(x_k^1) > g_k(x_k^2), \forall k\). In addition, by Lemma 1 the demand function \(g_k(x_k)\) is a monotonically decreasing function of the offloading frequency \(x_k\). Therefore, it holds that \(x_k^1 < x_k^2, \forall k\), which further indicates that
\[
\begin{align*}
\mu_B - \sum_{j=1}^N \lambda_a x_j^1 > \mu_B - \sum_{j=1}^N \lambda_a x_j^2, \\
\Leftrightarrow \sqrt{\frac{c_0^\prime \mu_B}{t_1}} > \sqrt{\frac{c_0^\prime \mu_B}{t_2}} \Leftrightarrow t_1 < t_2.
\end{align*}
\]
The inequality in (36) contradicts with the assumption of \(t_1 > t_2\). This completes the proof.

**APPENDIX E**

**PROOF OF Theorem 4**

Substituting (25) into Theorem 1 indicates that at the Nash Equilibrium it holds that
\[
N\lambda_1 \hat{x}_0^* + \sqrt{\frac{c_0^\prime(\mu_B - (N-1)\lambda_1 \hat{x}_0^*)}{g_0(\hat{x}_0^*) - P}} = \mu_B. \quad \text{(37)}
\]

For the purpose of having a Nash Equilibrium point equal to the Social Equilibrium point, we should have \(\hat{x}_0^* = \bar{x}_0^*\). As such, the solution \(\hat{x}_0^*\) should also satisfy Eq. (24). Combining Eq. (37) and Eq. (24) yields
\[
\begin{align*}
\frac{c_0^\prime(\mu_B - (N-1)\lambda_1 \bar{x}_0^*)}{g_0(\bar{x}_0^*) - P} &= \frac{c_0^\prime \mu_B}{g_0(\bar{x}_0^*)} \\
\Leftrightarrow P &= \frac{(N-1)\lambda_1 \bar{x}_0^*}{\mu_B}g_0(\bar{x}_0^*). \\
\end{align*}
\]
This completes the proof.

**REFERENCES**

[1] S. Liu and A. D. Striegel, “Exploring the potential in practice for opportunistic networks amongst smart mobile devices,” in 19th Annu. Int. Conf. Mobile Comput. Netw. (MOBICOM), Miami, Florida, 2013, pp. 315–326.

[2] M. Satyanarayanan, P. Bahl, R. Caceres, and N. Davies, “The case for VM-based cloudlets in mobile computing,” IEEE Pervasive Comput., vol. 8, no. 4, pp. 14–23, 2009.

[3] Y. Mao, J. Zhang, and K. B. Letaief, “Dynamic computation offloading for mobile-edge computing with energy harvesting devices,” IEEE J. Sel. Areas Commun., vol. 34, no. 12, pp. 3590–3605, 2016.

[4] C. You, K. Huang, and H. Chae, “Energy efficient mobile cloud computing powered by wireless energy transfer,” IEEE J. Sel. Areas Commun., vol. 34, no. 5, pp. 1757–1771, May 2016.

[5] J. Liu, Y. Mao, J. Zhang, and K. B. Letaief, “Delay-optimal computation task scheduling for mobile-edge computing systems,” in Proc. IEEE ISIT, Barcelona, Spain, 2016, pp. 1451–1455.

[6] Y. Tao, C. You, P. Zhang, and K. Huang, “Stochastic control of computation offloading to a dynamic helper,” in ICC Workshops, Kansas City, MO, USA, May 2018.

[7] Y. Mao, J. Zhang, S. H. Song, and K. B. Letaief, “Power-delay tradeoff in multi-user mobile-edge computing systems,” in GLOBECOM, Washington, DC, USA, 2016, pp. 1–6.
[8] ——, “Stochastic joint radio and computational resource management for multi-user mobile-edge computing systems,” IEEE Trans. Wireless Commun., vol. 16, no. 9, pp. 5994–6009, 2017.

[9] C. You, K. Huang, H. Chae, and B.-H. Kim, “Energy-efficient resource allocation for mobile-edge computation offloading,” IEEE Trans. Wireless Commun., vol. 16, no. 3, pp. 1397–1411, 2017.

[10] M.-H. Chen, B. Liang, and D. Ming, “Joint offloading and resource allocation for computation and communication in mobile cloud with computing access point,” in INFOCOM, Atlanta, GA, USA, 2017, pp. 1863–1871.

[11] T. Q. Dinh, J. Tang, Q. D. La, and T. Q. S. Quek, “Adaptive computation scaling and task offloading in mobile edge computing,” in WCNC, San Francisco, CA, USA, 2017, pp. 1–6.

[12] L. Pu, X. Chen, J. Xu, and X. Fu, “D2D fogging: An energy-efficient and incentive-aware task offloading framework via network-assisted D2D collaboration,” IEEE J. Sel. Areas Commun., vol. 34, no. 12, pp. 3887–3901, 2016.

[13] F. Wang and X. Zhang, “Dynamic interface-selection and resource allocation over heterogeneous mobile edge-computing wireless networks with energy harvesting,” in INFOCOM WKSHPS, Honolulu, HI, USA, 2018, pp. 190–195.

[14] F. Guo, L. Ma, H. Zhang, H. Ji, and X. Li, “Joint load management and resource allocation in the energy harvesting powered small cell networks with mobile edge computing,” in INFOCOM WKSHPS, Honolulu, HI, USA, 2018, pp. 754–759.

[15] X. Chen, “Decentralized computation offloading game for mobile cloud computing,” IEEE Trans. Parallel Distrib. Syst., vol. 26, no. 4, pp. 974–983, 2015.

[16] X. Chen, L. Jiao, W. Li, and X. Fu, “Efficient multi-user computation offloading for mobile-edge cloud computing,” IEEE/ACM Trans. Netw., vol. 24, no. 5, pp. 2795–2808, 2016.

[17] S. Guo, B. Xiao, Y. Yang, and Y. Yang, “Energy-efficient dynamic offloading and resource scheduling in mobile cloud computing,” in INFOCOM, San Francisco, CA, USA, 2016, pp. 1–9.

[18] J. Zheng, Y. Cai, Y. Wu, and X. S. Shen, “Dynamic computation offloading for mobile cloud computing: A stochastic game-theoretic approach,” IEEE Trans. Mobile Comput., vol. 99, no. 99, pp. 1–1, 2018.

[19] L. Tang and X. Chen, “An efficient social-aware computation offloading algorithm in cloudlet system,” in GLOBECOM, Washington, DC, USA, 2016, pp. 1–6.

[20] X. Lyu, H. Tian, C. Sengul, and P. Zhang, “Multiuser joint task offloading and resource optimization in proximate clouds,” IEEE Trans. Veh. Technol., vol. 66, no. 4, pp. 3435–3447, 2017.

[21] C. Courcoubetis and R. Weber, Pricing Communication Networks: Economics, Technology and Modelling. Hoboken, NJ, USA: Wiley, 2003.

[22] J. Huang and L. Gao, Wireless Network Pricing. San Rafael, CA, USA: Morgan & Claypool, 2013.

[23] W. Fang, X. Yao, X. Zhao, J. Yin, and N. Xiong, “A stochastic control approach to maximize profit on service provisioning for mobile cloudlet platforms,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 48, no. 4, pp. 522–534, 2018.

[24] A.-L. Jin, W. Song, P. Wang, D. Niyato, and P. Ju, “Auction mechanisms toward efficient resource sharing for cloudlets in mobile cloud computing,” IEEE Trans. Services Comput., vol. 9, no. 6, pp. 895–909, 2016.

[25] R. Hassin and M. Haviv, To Queue Or Not to Queue: Equilibrium Behavior in Queueing Systems. Kluwer Academic Publishers, 2003.

[26] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. Cambridge University Press, 2005.

[27] A. P. Miettinen and J. K. Nurminen, “Energy efficiency of mobile clients in cloud computing,” in HotCloud’10 Proceedings of the 2nd USENIX conference on Hot topics in cloud computing, Boston, MA, USA, 2010, pp. 1–7.

[28] H. Inaltekin, M. Chiang, H. V. Poor, and S. B. Wicker, “On unbounded path-loss models: Effects of singularity on wireless network performance,” IEEE J. Sel. Areas Commun., vol. 27, no. 7, pp. 1078–1092, Sep. 2009.