Holographic Model for Mesons at Finite Temperature

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Abstract

A holographic model of QCD at finite temperature is proposed and thermal properties for light mesons are examined in terms of this model. We find interesting temperature dependences of masses and decay constants for several mesons, and Nambu-Goldstone theorem for the chiral symmetry breaking and generalized Gell-Mann-Oakes-Renner relation are assured at finite temperature. Furthermore, we can see in the chiral limit that the speed of the pion in the thermal medium decreases to zero when the temperature approaches to the critical point of quark confinement and deconfinement transition, where the chiral symmetry is restored.

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1. Recently, from the gravity/gauge correspondence, probe brane approach to the gauge theory with flavor quarks has been largely developed in terms of the system of $D_p/D_{p+4}$ branes [1, 2]. In these, flavor quarks have been introduced by embedding $D_{p+4}$ branes as probe in the 10d background, which is deformed to the anti-de Sitter space (AdS) by the stacked $D_p$ branes. Then several important dynamical properties of QCD as chiral symmetry breaking and quark confinement have been explained, and furthermore, the flavored-meson masses have been calculated.

Inspired by these works, phenomenological, five-dimensional holographic models have been proposed to explain more quantitatively the physical properties of light mesons in a simpler setting [3, 4, 5, 6, 7]. Up to now, all these works of this framework are restricted to the case of zero temperature ($T = 0$). Meanwhile, it is interesting to extend this simple framework to the finite temperature scheme in order to compare the thermal properties of the quantities estimated in this framework and the recent experiments [8]. Up to now, such thermal properties of light mesons have been estimated by the lattice, chiral perturbations and sigma models. So, we extend here the model proposed at $T = 0$ in [3, 4] to the finite-temperature version in order to give the first holographic approach in this category.

In [3, 4], the gravity dual of the gauge theory with confinement at $T = 0$ is expressed by the 5d AdS with an infrared (IR) cutoff at an appropriate position of the fifth coordinate. This IR cutoff is essential to form a gravity dual of a gauge theory in the confinement phase, and the position of IR cutoff is very sensitive to the meson mass. In this sense, this point is fixed rigidly. The finite-temperature version is analogously obtained by replacing the AdS background with the AdS- Schwarzschild, and the horizon, which is related to the temperature of the gauge theory, in the AdS-Schwarzschild is set inside the IR cutoff at low temperature, then the horizon is hidden by the IR cutoff in the 5d bulk.

It is well-known that the 5d AdS-Schwarzschild background corresponds to the finite temperature version of CFT with deconfinement phase when the IR cutoff is removed. This situation is realized in the present case when the IR cutoff is put inside of the horizon. In this case, the IR cutoff is covered by the horizon, then the effect of the IR cutoff disappears. This setting corresponds to the high temperature phase of our model. On the other hand, at low temperature phase, the cutoff is set outside, the fields in the bulk end at this IR cutoff and appropriate boundary conditions for the fields are imposed at this end point. As a result, we find infinite series of discrete meson spectra. This situation is completely parallel to the AdS case, but the system is set at finite temperature in the present case.

The temperature is understood from the finiteness of the Euclidean time in the AdS-Schwarzschild configuration in order to evade the conical singularity which may arise at the horizon. This constraint for the Euclidean time should be preserved, even if the IR cutoff is introduced, in order to connect the manifold smoothly up to the critical temperature of quark confinement and deconfinement ($T_c$). So we can perform the analysis for the properties of the light mesons in the region of $0 < T < T_c$, by this configuration. As for the quark part, we use the effective 5d action ($S_{\text{meson}}$) introduced in [3, 4] for the sake of the simplicity.

2. The bulk background, which is dual to the finite temperature 4d QCD, is formed
by introducing the IR cutoff \([3, 6]\) \(z_m\) into the 5d AdS-Schwarzschild solution,

\[
ds_5^2 = \frac{1}{z^2} \left( -f^2(z) dt^2 + (dx^i)^2 + \frac{dz^2}{f^2(z)} \right), \quad f^2(z) = 1 - \left( \frac{z}{z_T} \right)^4. \tag{1}\]

where the radius of AdS\(_5\) is taken as unit. As is well-known, the imaginary time is restricted as 0 \(\leq it = \tau < \pi z_T\), to avoid the conical singularity. Then the temperature \(T\) is given as \(T = 1/(\pi z_T)\). As for the infrared cutoff \(z_m\), we consider the case of \(z_m < z_T\) in order to see the confinement phase at finite temperature. In this case, fields are restricted to the region \(0 \leq z \leq z_m\).

The condition \(0 \leq it = \tau < \pi z_T\) is not altered even if the cutoff is introduced. Then the temperature is retained as given above. When the cutoff has been introduced, however, the possible conical point is removed for \(z_m < z_T\), so it may be possible to change the temperature in order to find a smooth manifold for \(z < z_m\). But here we fix the temperature as \(T = 1/(\pi z_T)\) in order to extend smoothly the region of \(z_T\) up to \(z_T = z_m\). The confinement is thus assured since we have infinite series of discrete meson spectrum, quark and anti-quark bound states, due to the boundary at \(z_m\).

On the other hand, for the case of \(z_m > z_T\), the situation is the same with the finite temperature CFT. In this case, the cutoff has no meaning since \(z\) is restricted to \(z_T > z > 0\). Then we find the deconfinement phase with a few number of possible meson states, whose masses should be below the double quark-mass. This point is assured in \([2]\). This implies that the critical temperature \(T_c\) of confinement-deconfinement is set in the present case at \(T_c = 1/(\pi z_m)\) which is estimated here to be \(\sim 102\) MeV from meson spectra. This value is not so wrong since our model is not so strict.

3. For the sake of the simplicity, according to \([3, 6]\), we start with the following 5d meson-action under the background \([\Pi]\). This action is considered as the quark part represented by the probe brane in the string theory. And the fields on the brane represent meson states regarded as the bound states of quark and anti-quark. The fields considered here are the gauge fields, \(L_M\) and \(R_M\), and a scalar field \(\Phi\) whose vacuum expectation value (VEV) is connected to chiral symmetry breaking. Here, we consider \(N_f = 2\) flavors, and \(\Phi\) transforms as a \((2_L, 2_R)\). The action is

\[
S_{\text{meson}} = \int d^4xdz \sqrt{-g} \text{Tr} \left[ -\frac{1}{4g_5^2} (L_{MN}L^{MN} + R_{MN}R^{MN}) - |D_M\Phi|^2 - M_\Phi^2 |\Phi|^2 \right], \tag{2}\]

where the covariant derivative is defined as \(D_M\Phi = \partial_M\Phi + iL_M\Phi - i\Phi R_M\), \(g\) is the determinant of the metric, and \(L_M = L_M^a \tau^a\) for the Pauli matrices \(\tau^a\) and similarly for other fields. \(g_5\) is the 5d gauge coupling. We define \(\Phi = S e^{i\pi^a \tau^a}\) and \(\frac{1}{2}v(z) \equiv \langle S \rangle\), where \(S\) corresponds to a real scalar and \(\pi\) to a real pseudoscalar \((S \rightarrow S\) and \(\pi \rightarrow -\pi\) under \(L \leftrightarrow R\)). They transform as \(1 + 3\) under SU(2)\(_V\).

3-1. The 5d mass of the scalar is set as \(M_\Phi^2 = -3\) to consider the bulk field which corresponds in the gauge theory side to an operator with the conformal dimension \(\Delta = 3\). Then the equation of motion for \(v\) is given as,

\[
\left[ \partial_z^2 - \frac{4 - f^2}{zf^2} \partial_z + \frac{3}{z^2 f^2} \right] v(z) = 0. \tag{3}\]
And we obtain
\[ v(z) = z \left( m_q \frac{2F_1(\frac{1}{4}, \frac{1}{2}; \frac{z^4}{z_T})}{W} + c \frac{z^2}{2} \frac{2F_1(\frac{3}{4}, \frac{3}{2}; \frac{z^4}{z_T})}{W} \right). \] (4)

Here two integral constants, \( m_q \) and \( c \), are identified with the quark mass (explicit breaking of the chiral symmetry) and the chiral condensate (spontaneous breaking of chiral symmetry in the chiral limit), respectively. Actually, we get the approximate form, \( v(z) \sim m_qz + cz^3 \) near \( z = 0 \), which is consistent with the above statement.

We notice here that \( v(z) \) varies with temperature \( T \) or \( zT \), which is specific to our model, but the parameters \( m_q \) and \( c \) are independent of \( T \). In other words, the gauge theory considered here is characterized by these three parameters. However, the chiral condensate \( c \) should be determined dynamically and it should depend on \( T \). This point is the defect of the present model, and we must be careful about the results of our calculations for the quantities, which depend on this "parameter" \( c \), when we discuss the \( T \) dependence of them.

The fluctuation of \( S \), which is defined as \( S = v(z)/2 + \sigma \), can be observed as a singlet meson state (\( \sigma \)). Here and hereafter we consider the static mode, \( \partial_t \phi = 0 \), for any field \( \phi \) in order to derive the mass in a simple way. So the invariant mass, or pole mass, is defined here as \( -\partial^2 t \phi = m^2 \phi \). Then the equation for \( \sigma \) is given by adding this 4d mass term to Eq. (3) as
\[
\left[ \frac{m^2}{f^4} + \partial_z^2 - \frac{4 - f^2}{z f^2} \partial_z + \frac{3}{z^2 f^2} \right] \sigma = 0.
\] (5)

It should be noticed that this equation is independent of \( v(z) \). In this sense, the model given here is insufficient since all the fluctuations on the probe brane would be influenced by the shape of the embedded brane. But the improvement of this defect is postponed to the future.

The discrete mass-spectrum is obtained by solving this equation with the boundary conditions, \( \sigma(z)|_{z_0} = \partial_z \sigma(z)|_{z_m} = 0 \), where \( z_0 \) is the UV cutoff which is taken to zero after all. The mass depends on \( z_m \) and \( T \). By using the value of \( z_m \) determined by the \( \rho \) meson mass at \( T = 0 \) as \( 1/z_m = 0.323 \text{ GeV} \), the \( T \)-dependence of the \( \sigma \) meson mass is estimated. The numerical results are shown in the Fig. We can observe that the mass decreases with increasing \( T \) and approaches to zero for \( T \rightarrow T_c \). This behavior is easily understood from Eq. (4). The mass term, \( m^2/f^4 \), in the equation is largely enhanced near \( z = z_T \), where \( f \rightarrow 0 \), so \( m \) should be very small when \( z \) approaches to \( z_T \). This situation is realized for \( z_T \rightarrow z_m \) or \( T \rightarrow T_c \). This point is therefore the most prominent check point of this model. The experiments to assure this property are acquired.

3-2. The gauge bosons are separated to the vector and the axial vector bosons \( V_M \) and \( A_M \), and are defined as \( L_M \equiv V_M + A_M \) and \( R_M \equiv V_M - A_M \), respectively.

First of all, we consider the vector mesons. At finite temperature, it is impossible to write the equation of motion covariantly, so we consider the linearized equation for the spatial component \( V_i \) here. And it is (employing \( V_z = 0 \) gauge) given as
\[
\left[ \frac{m^2}{f^4} + \partial_z^2 - \frac{4 - 3f^2}{z f^2} \partial_z \right] V_i = 0.
\] (6)
This equation gives us the discrete eigenvalues \( m^2 = m_n^2 \). In order to see the decay constants, it is convenient to expand \( V_i(x,z) \) as \( V_i(x,z) = \sum_n V_i^{(n)}(x) h_n^V(z) \) for each mass eigen-state. The spatial components of the wave-function for each mode are normalized as

\[
\int_{z_0}^{z_m} dz \left( \frac{h_n^V(z)}{z} \right)^2 = 1, \tag{7}
\]

where we notice the factor \( 1/f^2(z) \) in this integrand. This is an important factor in determining the \( T \) dependence of the decay constant near \( T = T_c \). Actually, we find rapid decreasing of \( F_\rho \) near \( T = T_c \) due to this factor as shown in Fig.1.

The mass of vector meson is given by the boundary condition \( V_i(z_0) = \partial_z V_i(z_m) = 0 \) similarly to the case of the sigma meson. Note that Eq. (B) is also independent of \( v(z) \), then the masses of vector meson depend only on \( T \), and the \( T \)-dependence of the lowest one, which is identified with \( \rho \) meson, is shown in Fig.1.

\[
\begin{align*}
\text{Fig. 1: (A) The curves show the } T \text{ dependence of } m_\pi, m_\rho, m_\sigma \text{ and } m_{a_1} \text{ from the bottom. In (B), the solid curves denote the } T \text{ dependence of } F_\sigma, \sqrt{F_\rho} \text{ and } \sqrt{F_{a_1}} \text{ from the bottom. The dotted curve denoting } F_s \text{ is constant with high accuracy at } T < 100 \text{ MeV.}
\end{align*}
\]

We can see that the mass of \( \rho \) meson vanishes or becomes very small near \( T = T_c \). This is consistent with the expectation obtained from the equation (B) as in the case of the \( \sigma \)-meson.

3-3. The linearized equations of motion for the axial vector meson \( A_\mu \) and the pion \( \pi \) are obtained by decomposing \( A_\mu \) into the transverse and the longitudinal part, \( A_\mu = A_{\mu \perp} + \partial_\mu \phi \); for simplicity, flavor index is neglected. Using \( A_z = 0 \) gauge, one then one obtain

\[
\begin{align*}
\left[ \frac{m^2_A}{f^4} + \partial_z^2 - \frac{4 - 3f^2}{z f^2} \partial_z - g_5^2 \frac{v^2}{z f^2} \right] A_{\mu \perp} & = 0, \\
\left[ \partial_z^2 - \frac{1}{z} \partial_z - g_5^2 \frac{v^2}{z f^2} \right] A_{0 \perp} & = 0, \\
\left[ \partial_z^2 - \frac{1}{z} \partial_z \right] \phi - g_5^2 \frac{v^2}{z f^2} (\pi + \phi) & = 0, \\
m^2_\pi \partial_z \phi + g_5^2 \frac{v^2 f^2}{z f^2} \partial_z \pi & = 0,
\end{align*}
\tag{8-11}
\]
where the coupling constant $g_\pi$ is determined from the vector current two-point function at $T = 0$ [3]. Here $m_v$ and $m_\pi$ are the pole masses defined as $-\partial_i^2 \varphi = m_v^2 \varphi$ and $-\partial_i^2 \pi = m_\pi^2 \pi$. These equations are solved numerically under the boundary conditions, $A_{\mu \perp}(z_0) = \partial_z A_{\mu \perp}(z_m) = 0$ and $\varphi(z_0) = \partial_z \varphi(z_m) = \pi(z_0) = 0$.

The decay constants of the axial mesons and the pion are calculated from the wave functions as [3]

$$F_{av}^2 = \frac{1}{g_5} \left[ \frac{d^2 h_{\n}^A}{dz^2} \right]_{z_0}^2 (n \neq 0), \quad (F_{\pi \perp}^t)^2 = -\frac{1}{g_5} \left[ \frac{\partial_z A_{0,i \perp}}{z} \right]_{z_0}, \quad (12)$$

where $A_{i \perp}(x, z) = \sum_n \alpha_i^{(n)}(x) h_n^A(z)$ and $h_n^A(z)$ is normalized as $h_n^V(z)$ given in [7]. Furthermore, $A_{0,i \perp}^{(0)}$ (Eq. [8] with $m_a^2 = 0$), satisfying $A_{0,i \perp}^{(0)}(z_0) = 1$ and $\partial_z A_{0,i \perp}(z_m) = 0$, and $F_{\pi \perp}^{t,s}$ are the timelike and spatial components of the pion decay constant. Obviously, $F_{\pi \perp}^t$ and $F_{\pi \perp}^s$ are different from each other at finite $T$, while $F_{\pi \perp}^t = F_{\pi \perp}^s$ at $T = 0$.

The masses and the decay constants of axial-vector mesons and pion depend on four parameters $m_q$, $c$, $z_m$ and $T$ through $v(z)$ and $f(z)$ in Eqs. [8]-[11], while those of vector mesons do only on $z_m$ and $T$. For the consistency between the vector and axial-vector meson sectors, here we take the same $z_m$ as that determined in the vector meson sector; namely $1/z_m = 0.323$ GeV. Parameters, $m_q$ and $c$, are determined to reproduce the experimental values, $\bar{m}_\pi$ and $\bar{F}_\pi$, of $m_\pi$ and $F_\pi$ at $T = 0$; the resultant values are $m_q = 2.26$ MeV and $c = (0.333$ GeV$)^3$. This parameter set reproduces masses and decay constants of $\pi$, $\rho$, $a_1$ mesons at $T = 0$ within $\sim 10\%$ error [3, 2]. Furthermore, when $T = 0$, the present model satisfies the Gell-Mann-Oakes-Renner (GOR) relation $\bar{m}_\pi^2 \bar{F}_\pi^2 = 2m_v c$ [3, 6].

In principle, the chiral condensate $c$ depends on $T$, but it is known by lattice QCD [9] and the chiral perturbation theory to three loops [10] that $c$ little depends on $T$ in the region of $T < 100$ MeV. So we focus our discussion on the temperature region for a while and then neglect the weak $T$ dependence of $c$.

Figure [11] shows that the $T$ dependence of $m_\pi$ and $F_{\pi \perp}^t$ is almost constant in the region $T < 100$ MeV; more precisely, $m_\pi$ is slightly reduced and $F_{\pi \perp}^t$ is slightly enhanced there. This is consistent with the result of the chiral perturbation theory to two loops [11]. Furthermore, $m_\sigma$ is largely reduced as $T$ increases, as shown in Fig. [11] This is also consistent with the result of lattice OCD [12]; note that the pole mass is evaluated in the lattice calculation. Thus, the present model qualitatively simulates $T$ dependence of real QCD in the region $T < 100$ MeV.

In Fig. [11] the $T$ dependence of $m_\sigma$ is relatively weaker than that of $m_\rho$. The difference stems from a term $-(g_5 v)^2/(zf)^2$ which is present in Eq. [8] for the axial vector meson but not in the corresponding equation [6] for the vector meson. The term suppresses partially the enhancement of the mass term $m_\sigma^2/f^4$ near $T_c$, since the two terms have opposite signs to each other. In Fig. [11] we can also see a tendency for $m_\sigma$ to become small as $T$ reaches $T_c$. When $T \approx T_c$, the second term of Eq. [11] is enhanced by the factor $f^{-2}$ compared with the first term. This might indicate that $\varphi(z) \approx -\pi(z)$. Inserting this relation into Eq. [11], we can speculate that $m_\pi \propto v f$ near $T_c$. 

5
Fig. 2: The $m_q$ dependence of $m^2_\pi$ at $T = 100$ MeV. The solid line corresponds to $m^2_\pi(T)$ and the closed circles to $2m_qc/F^2_\pi(T)$.

3-4. In real QCD, a generalized GOR relation, $m^2_\pi(T)F^2_\pi(T) = 2m_qc$, is satisfied for finite $T$ \cite{13, 14}. Within the framework of the holographic QCD, a necessary condition for the GOR relation to be satisfied is that Eq. \eqref{eq:9} agrees with Eq. \eqref{eq:10} with $\pi(z) = 0$ \cite{3}. This condition is satisfied for any $T$ in the present model. Thus, the present model is expected to reproduce the generalized GOR relation properly.

Whether the generalized GOR relation is realized in the present model is tested through $m_\pi(T)$ and $F^\pi_\tau(T)$ calculated numerically. Figure 2 shows the $m_q$ dependence of $m^2_\pi(T)$ and $2m_qc/F^2_\pi(T)$ by the solid line and the closed circles, respectively. Here the case of $T = 100$ MeV is taken as an example. The $m^2_\pi(T)$ shown by the solid line tends to zero as $m_q$ decreases, as a reflection of the Nambu-Goldstone theorem at finite $T$. Comparing the solid line and the closed circles, one can see that the generalized GOR relation is well satisfied in the present model.

3-5. The pion velocity $v_\pi$ in the thermal medium can be estimated from $F^\pi_\tau(T)$ and $F^\pi_\omega(T)$ as $v_\pi^2 = F^\pi_\omega(T)/F^\pi_\tau(T)$ \cite{13, 14}. The pion velocity $v_\pi$ thus evaluated decreases quite slowly from 1 to 0.97 as $T$ increases from 0 to 100 MeV; here we assume $c$ does not depend on $T$. Now the value of $v_\pi$ is estimated in the limit of $T \to T_c$ by solving the Eqs. \eqref{eq:9} and \eqref{eq:10} at the chiral limit $m_q = 0$. At $T_c$, we expect the restoration of chiral symmetry, so we set as $v(z) = 0$ in the equations and the solutions are given as

\begin{equation}
A_i = a_1 + a_2 \frac{\tanh^{-1}(z/z_T)^2}{2z_T^2}, \quad A_0 = b_1 + b_2 z^2. \tag{13}
\end{equation}

The constants $a_{1,2}$ and $b_{1,2}$ are determined by the boundary conditions, $A_i,0|_{z=0} = 1$ and $\partial_z A_i,0|_{z=z_m} = \epsilon$, and we take the limit $\epsilon \to 0$ after obtaining $v_\pi$. Thus we get

\begin{equation}
v_\pi^2 = \sqrt{1 - \left(\frac{T}{T_c}\right)^4} \propto t^{1/2}, \tag{14}
\end{equation}

where $t = (T_c - T)/T_c$. This indicates $v_\pi(T_c) = 0$. This is consistent with the result obtained by Son-Stephanov \cite{14}. The corresponding critical exponent is $\nu = 1/2$ in our case.
4. The model we present is the first holographic model to describe QCD in the region of \( T < T_c \) nonperturbatively. In this model, we take the 5d AdS-Schwarzschild solution as the gravity background, and cut off the extra dimension at an appropriate infrared point \( z_m \) in order to introduce the confinement in a gauge theory dual to the gravity. The present model then describes the confinement phase in the region \( T < T_c \).

The present model makes it possible to evaluate \( T \) dependence of meson masses and decay constants nonperturbatively. In the region \( T < 100 \) MeV, the pion mass and the pion decay constant little depend on \( T \). This is consistent with the result of the chiral perturbation theory to two loops [11]. In contrast, the \( \sigma \) meson mass is largely reduced as \( T \) increases. This is consistent with the result of lattice calculation [12]. Furthermore, the Nambu-Goldstone theorem and the generalized GOR relation are assured at any finite \( T \). Thus, the present model reproduces the qualitative properties of QCD in the region \( T < T_c \). We then conclude that the present model is reliable enough to make qualitative predictions on QCD in the region \( T < T_c \).

Our prediction of the pion velocity is \( v_\pi(T_c) = 0 \). This agrees with the result of Son-Stephanov [14] based on scaling and universality. Thus, the present result also supports the statement that the measured pion velocity 0.65 [15], deduced from the pion spectra observed by STAR [8] at RHIC, would be a signal of QCD phase transition.

The present model has three parameters; quark mass \( m_q \), the chiral condensate \( c \) and the cutoff \( z_m \). In principle, \( c \) and \( z_m \) should be determined dynamically and be functions of \( T \). The \( T \) dependence of these quantities can not be obtained by the present model. The lattice calculation shows that the pion mass somewhat increases near \( T_c \). If it is true, this would imply that \( z_m \) becomes smaller there, since smaller \( z_m \) yields larger pion mass. Furthermore, it is well known that \( c \) has a strong \( T \) dependence near \( T_c \). This is essential for properties of light mesons near \( T_c \). Thus, it is quite important to improve the present model so that \( c \) and \( z_m \) can be determined dynamically. Furthermore, the vector meson mass and decay constant do not depend on \( c \) in the present model. This is also an important issue to be solved in future.

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