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Mirror instability in plasma with relativistic component

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Abstract. A kinetic model of the mirror instability in highly non-equilibrium anisotropic relativistic plasma is presented. The growth rate is derived for long-wavelength compressible magnetic perturbations in a magnetized collisionless plasma with an anisotropic pressure of the relativistic component. The mirror-type mode perturbations of the magnetic field are growing for the wave vectors preferentially directed transverse to the mean magnetic field. The mirror instability may occur in the vicinity of relativistic particle accelerators in the interstellar and intergalactic media.

1. Introduction

Instabilities in cosmic plasmas are responsible for relaxation of non-equilibrium states in various objects starting from the planetary magnetospheres to early cosmology. Instabilities of the relativistic component (RC) has long been recognized to play a role in the dynamics of the interstellar gas \cite{1}. The firehose and mirror instabilities occur in the case of anisotropic thermal background plasma pressure in the presence of magnetic fields \cite{2}. The mirror instability in the anisotropic magnetized interstellar and intergalactic plasma was studied in detail in \cite{3, 4, 5}. A pressure anisotropy of RC may result in the growth of a firehorse or mirror instability depending on the ratio of the partial pressures of relativistic particles along and transverse to the mean magnetic field. The firehose instability of the RC was studied recently in \cite{6, 7, 8}, while the mirror instability case requires a special kinetic treatment, which is outlined below.

2. Mirror instability of anisotropic RC distributions

The derivation given below treats kinetically the anisotropic RC which is the source of free energy for the mirror instability, while the background plasma is described with a magnetohydrodynamic (MHD) approximation as it was earlier done in \cite{8}.

The background plasma obey the continuity and the momentum conservation equations

\begin{equation}
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{u}) = 0,
\end{equation}

and

\begin{equation}
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} \right) = -\nabla p_g + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{1}{c} (j_{rc} - e n_{rc} \mathbf{u}) \times \mathbf{B} - \int p I[\phi_{rc}] d^3 \rho,
\end{equation}
where \( \mathbf{u}, \rho, \) and \( p_0 \) are the plasma bulk velocity, density, and the thermal gas pressure respectively. The electromagnetic fields \( \mathbf{E} \) and \( \mathbf{B} \) obey

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) ,
\]

and the kinetic equation for the RC is

\[
\frac{\partial \phi^{rc}}{\partial t} + \mathbf{v} \cdot \nabla \phi^{rc} + e\mathbf{E} \cdot \nabla \phi^{rc} - \frac{e c}{\mathcal{E}} \mathbf{B} \cdot \hat{\mathbf{O}} \phi^{rc} = I[\phi^{rc}] ,
\]

where \( \phi^{rc}, n_{rc}, j_{rc} \) are the RC distribution function, concentration, and electric current, \( \mathcal{E}, \mathbf{v}, \) and \( \mathbf{p} \) are the RC particle energy, velocity, and momentum, respectively, \( \hat{\mathbf{O}} = \mathbf{p} \times \frac{\partial}{\partial \mathbf{p}} \) is the momentum rotation operator, \( c \) is the speed of light, \( e \) is the particle charge, and \( \Omega = e B_0 c/\mathcal{E} \) is the RC particle gyro-frequency.

The collision operator \( I[\phi^{rc}] \) describes RC interactions with magnetic fluctuations carried by the thermal plasma. In the thermal background plasma frame

\[
I[\phi^{rc}] = -\nu (\phi^{rc} - \phi_{iso}^{rc}) ,
\]

where \( \phi_{iso}^{rc} \) is the isotropic part of the distribution function, \( \nu = \epsilon \Omega \) is the RC scattering frequency by magnetic fluctuations with \( \epsilon \leq 1 \), and \( \int \mathbf{p} I[\phi^{rc}] d^4 \mathbf{p} = -\epsilon B_0 j^{rc}/c \) in Eq. (2).

The small variations of the background plasma parameters in the plasma rest frame Eqs. (2)–(4) are \( \xi = \xi_0 + \delta \xi \), where \( \delta \xi \propto e^{i(kr - i \omega t)} \). Then in the case of the adiabatic process

\[
\nabla \delta \rho_g = c_s^2 \nabla \delta \rho ,
\]

where \( c_s = \sqrt{\gamma g p_0/\rho_0} \), and \( \gamma_g \) is the adiabatic index. The wavevector is \( \mathbf{k} = k_\parallel \mathbf{e}_z + k_\perp \mathbf{e}_x \), where the parallel direction is \( \mathbf{e}_z = B_0/B_0, \) while the transverse direction is \( \mathbf{e}_x \).

We will consider below instabilities initiated by the quadrupole anisotropy of the relativistic component. Such anisotropy of the relativistic particle distribution can be parameterized as

\[
\phi_{0}^{rc}(p, \mu) = \frac{n_{rc} F(p)}{4\pi} \left[ 1 + \frac{\zeta}{2} (3\mu^2 - 1) \right] ,
\]

where \( \int_0^\infty F(p) p^2 dp = 1 \) and \( \zeta \) is a quadrupole anisotropy parameter with \( |\zeta| < 1 \).

Keeping only linear responses to the perturbations in Eqs. (1)–(2), we obtain

\[
\left( \omega^2 - \omega^2 \left( c_a^2 + c_s^2 \right) k^2 + 2c_a^2 c_s^2 k^2 k_\parallel^2 \right) \delta B_z = \left( \omega^2 - c_s^2 k_\parallel^2 \right) i \frac{B_0^2 k_\parallel}{c \rho_0} (\delta j_y^{rc} - \epsilon \delta j_x^{rc})
\]

\[
-ic_a^2 k_\parallel^2 \frac{B_0^2 k_\perp}{c \rho_0} \epsilon \delta j_x^{rc} ,
\]

and

\[
\left( \omega^2 - c_s^2 k_\parallel^2 \right) \delta B_y = -i \frac{B_0^2 k_\parallel}{c \rho_0} (\delta j_x^{rc} + \epsilon \delta j_y^{rc}) ,
\]

where \( c_a = B_0/\sqrt{4\pi \rho_0} \) is the Alfvén velocity. The linearized Eq. (4) has the form

\[
[\nu + i \left( -\omega + k_\parallel v \cos \theta \right)] \delta \phi^{rc} + ik_\perp v \sin \theta \cos \phi \delta \phi^{rc} - \Omega \frac{\partial \delta \phi^{rc}}{\partial \varphi} = -e \mathbf{E} \cdot \frac{\partial \phi^{rc}}{\partial \mathbf{p}} + \frac{e c}{\mathcal{E}} \mathbf{B} \cdot \hat{\mathbf{O}} \phi^{rc} ,
\]

where \( \theta \) is the pitch angle and \( \varphi \) is the azimuthal angle between the field direction and the particle velocity.
In the weakly collisional case we have $\epsilon \ll 1$, and therefore one can omit the frequency $\nu$ compared to $\omega$ in Eq. (9). The response of relativistic particles’ current $\delta j^r e$ on the imposed magnetic perturbation $\delta B$ can be derived from Eq. (9) and has the form

$$
\delta j^r e_x = \frac{\pi e}{2B_0} \int dp \int dp' \int d\mu v S(\mu) \frac{\partial \phi^r_e (p, \mu)}{\partial \mu} \left\{ -\delta B_y \sum_{n=-\infty}^{\infty} \left[ J_n (\beta) \left( G_n^+ + G_n^- \right) + 2J_{n-2} (\beta) G_n^- \right] + 
+ i\delta B_x \sum_{n=-\infty}^{\infty} J_n (\beta) \left( G_n^+ - G_n^- \right) \right\},
$$

and

$$
\delta j^r e_y = \frac{\pi e}{2B_0} \int dp \int dp' \int d\mu v S(\mu) \frac{\partial \phi^r_e (p, \mu)}{\partial \mu} \left\{ \delta B_x \sum_{n=-\infty}^{\infty} \left[ J_n (\beta) \left( G_n^+ + G_n^- \right) - 2J_{n-2} (\beta) G_n^- \right] + 
+ i\delta B_y \sum_{n=-\infty}^{\infty} J_n (\beta) \left( G_n^+ - G_n^- \right) \right\},
$$

where

$$
G_n^\pm = \frac{J_n (\beta)}{a + i (n \pm 1)},
$$

$S(\mu) = 1 - \mu^2$, $J_n (\beta)$ is the Bessel function of integer $n$, $\beta = k_{\parallel} v \sin \theta / \Omega$, and $a = [\nu + i(k_{\parallel} v \cos \theta - \omega)] / \Omega$. The equations were obtained by neglecting the contributions proportional to $\delta E$ in (9), since they $\sim \frac{\omega}{c k}$.

The mirror instability appears mostly in the regime where $k_{\parallel} v / \Omega \ll 1$, $k_{\perp} v / \Omega \ll 1$, and $\omega / \Omega < 1$. In Eq. (13) we assumed $\epsilon \ll 1$ as well as $\beta \ll 1$ and $a \ll 1$. Therefore, in the long-wavelength regime from Eqs. (10) and (11) one can obtain the relativistic current response as

$$
\delta j^r e_x = \frac{\pi e}{2B_0} \int dp \int dp' \int d\mu v S(\mu) \left\{ 2\delta B_x - 2i \frac{k_{\parallel} v \mu}{\Omega} \delta B_y \right\} \frac{\partial \phi^r_e (p, \mu)}{\partial \mu},
$$

and

$$
\delta j^r e_y = \frac{\pi e}{2B_0} \int dp \int dp' \int d\mu v S(\mu) \left\{ 2\delta B_y + \delta B_x \left[ 2i \frac{k_{\parallel} v \mu}{\Omega} - i \frac{k_{\perp} v \mu}{\Omega} - \frac{\phi^r_e (p, \mu)}{\partial \mu} \right] \right\},
$$
The second term in the brace in Eq. (15) results in the firehorse instability which grows if the quadrupole anisotropy parameter $\zeta > 0$. On the contrary the mirror instability appears from the third term in the brace in Eq. (15) if $\zeta < 0$. Note that if the RC scattering rate by magnetic turbulence $\nu \sim \Omega$, which may occur if $\epsilon \sim 1$, then the mirror instability is suppressed.

Integrating Eq. (15) over $\mu$, with an account of Eq. (6), one obtains the asymptotic form of the third term in the brace in Eq. (15) for $\psi \ll 1$

$$\int_{-1}^{1} \frac{S(\mu)^2 \mu d\mu}{1 - \psi \mu} \to \frac{16}{105} \psi + O(\psi^3) ,$$

where $\psi = k || v / \omega$. For $\psi \gg 1$

$$\int_{-1}^{1} \frac{S(\mu)^2 \mu d\mu}{1 - \psi \mu} \to -\frac{16}{15\psi} + O\left(\frac{1}{\psi^2}\right) .$$

The response of the RC electric current in the last case is

$$\delta j_x^{rc} = -i \frac{\delta B_y \zeta n_{rc} k || c}{5B_0^2} \int_0^\infty vp^3 F(p) \, dp ,$$

and

$$\delta j_y^{rc} = i \frac{\delta B_x \zeta n_{rc} c}{5B_0^2} \left( k || - \frac{2k^2}{k ||} \right) \int_0^\infty vp^3 F(p) \, dp .$$

Let us define the unperturbed pressure

$$P_0 = \frac{n_{rc}}{3} \int_0^\infty vp^3 F(p) \, dp ,$$

and note that for the RC distribution function of Eq. (6)

$$P_\parallel = \int vp\mu^2 \phi_0^{rc}(p, \mu) \, d^3p = P_0 \left( 1 + \frac{2\zeta}{5} \right) ,$$

$$P_\perp = \int vpS(\mu) \cos^2 \varphi \phi_0^{rc}(p, \mu) \, d^3p = \int vpS(\mu) \sin^2 \varphi \phi_0^{rc}(p, \mu) \, d^3p$$

$$= P_0 \left( 1 - \frac{\zeta}{5} \right) ,$$

and

$$\delta P = P_\parallel - P_\perp = \frac{3\zeta}{5} P_0 .$$

Now, subsituting Eqs. (18) and (19) into (7) and (8) we obtain the dispersion relations

$$\omega^4 - \omega^2 \left( c_a^2 + c_s^2 \right) k^2 + c_a^2 c_s^2 k^2 k^2 || = - \left( \omega^2 - c_s^2 k^2 || \right) \frac{3\zeta P_0}{5\rho_0} \left( k^2 || - 2k^2_\perp \right) ,$$

and

$$\omega^2 - c_a^2 k^2_\parallel = - \frac{3\zeta P_0}{5\rho_0} k^2_\parallel .$$
The dispersion equation splits into two independent equations where the mirror instability is determined by Eq. (24) under conditions $2k^2_\perp > k^2_\parallel$ and $\zeta < 0$. For $k^2_\perp \gg k^2_\parallel$ Eq. (24) simplifies to
\[ \omega^2 - (c^2_a + c^2_s) k^2 = \frac{3\zeta P_0}{5\rho_0} 2k^2_\perp . \] (26)
Here the perturbation of the magnetic field is mostly $\delta B_z$ which is connected to the background plasma density variations by $\delta \rho \approx \rho_0 \delta B_z / B_0$.

Then from Eq. (23) with account of $k \approx k_\perp$, one finally obtains
\[ \omega^2 = \left( c^2_a + c^2_s + 2\frac{\delta P}{\rho_0} \right) k^2 . \] (27)
Eq. (27) shows that growing modes occur if $\delta P \propto \zeta < 0$ and $2\frac{\delta P}{\rho_0} > (c^2_a + c^2_s)$, i.e., the mirror instability is driven by the anisotropic RC pressure. Thus, the growth rate of the mirror instability rate from (27) is
\[ \Gamma_{\text{mir}} = k \sqrt{2\frac{\delta P}{\rho_0} - c^2_a - c^2_s} , \] (28)
for $\delta P \propto \zeta < 0$ and $2\frac{\delta P}{\rho_0} > (c^2_a + c^2_s)$.

If $k_\perp = 0$, $k_\parallel = k$ then Eq. (24) is reduced to the dispersion relation
\[ (\omega^2 - c^2_a k^2) \left( \omega^2 - c^2_s k^2 + \frac{3\zeta P_0}{5\rho_0} k^2 \right) = 0 . \] (29)
While the first bracket in Eq. (29) is the longitudinal acoustic mode, the second bracket and Eq. (25) gives
\[ \omega^2 - c^2_a k^2 + \frac{3\zeta P_0}{5\rho_0} k^2 = 0 , \] (30)
which may be unstable if $\delta P \propto \zeta > 0$ and $\frac{\delta P}{\rho_0} > c^2_a$. In this case the anisotropy of the RC pressure results in the growth of the long-wavelength firehose instability
\[ \Gamma_{\text{fh}} = k \sqrt{\frac{\delta P}{\rho_0} - c^2_a} , \] (31)
which was studied earlier in [8].

3. Discussion

Let us estimate the growth rate for the long-wavelength compressible mirror mode perturbation in the warm interstellar plasma with a typical number density about 0.1 cm$^{-3}$. The cosmic ray pressure in the interstellar medium is of the order of 1 eV cm$^{-3}$. Therefore, the anisotropy of the cosmic ray pressure of about $|\zeta| \sim 10^{-3}$ would provide the growth rate $\Gamma \sim 10^5 k$ s$^{-1}$ if the wavenumber $k$ is in cm$^{-1}$. Depending on the character of the pressure anisotropy which is determined by the sign of $\zeta$, either the mirror-type or the firehose mode perturbations of the magnetic field are growing on the time scale above some hundred years. The mirror instability should occur in the vicinity of relativistic particle accelerators. In the case of supernova shocks with super-diffusive particle transport the magnetic field amplification due to the mirror instability may be efficient [10].
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