Relativistic Effect of Gravitational Deflection of Light in Binary Pulsars

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Accepted 5 January 1995. Received November 1994.

ABSTRACT
An improved formula for the timing of binary pulsars that accounts for the relativistic deflection of light in the gravitational field of the pulsar’s companion is presented, and the measurability of this effect together with its variance estimates are discussed. The deflection of the pulsar’s beam trajectory in the gravitational field of its companion leads to variation in the pulsar’s rotational phase. This variation appears as the narrow sharp growth of the magnitude of the post-fit pulsar timing residuals in the vicinity of the moment of the superior conjunction of the pulsar with its companion. In contrast to the relativistic Shapiro effect the amplitude of the effect of gravitational deflection of the pulsar radio beam has two peaks with opposite signs, which become sharper as the inclination $i$ of the pulsar’s orbit approaches $90^\circ$. The effect under consideration influences the estimation of parameters of the relativistic Shapiro effect in the binary pulsars with nearly edgewise orbits. Its inclusion in the fitting procedure provide more careful measurement of sine of the orbital inclination $i$ as well as the masses of the pulsar $m_p$ and its companion $m_c$. This permits an improved testing of alternative theories of gravity in the strong field regime. Moreover, the measurement of this effect permits independent geometric constraints on the position of the pulsar rotational axis in space. The effect of the gravitational deflection of light has been numerically investigated for binary pulsars with nearly edgewise orbits. It is shown that the effect is observed in general only when $\cos i \leq 0.003$. This estimate becomes less restrictive as the pulsar’s spin axis approaches the line of the sight.

Key words: gravitation - pulsars - relativity - stars: neutron - stars: individual (PSRs B1855+09, B1913+16, B1534+12)

1 INTRODUCTION

High-precision timing observations of binary pulsars on relativistic orbits provide unique opportunities for testing of General Relativity (GR) through two mechanisms: the emission of gravitational radiation and the presence of strong-field effects. The effect of emission of gravitational waves have been confirmed using the PSR B1913+16 long term data with an accuracy better than 0.5% (Taylor & Weisberg 1989; Damour & Taylor 1991). This was accomplished by the measurements of three relativistic effects: 1) the rate of apsidal motion $\dot{\omega}$, 2) the Doppler and gravitational red shifts $\gamma$ of pulsar rotational frequency, and 3) the system orbital period decay $\dot{P}_b$, which is due to the effect of the gravitational waves emission, which has been rigorously justified from the mathematical point of view by Damour (1983a,1983b) and, independently, by Grishchuk & Kopeikin (1983,1986), as well as Schäfer (1985). However, timing of PSR B1913+16 is not sufficiently sensitive to the strong gravitational field effects. And, indeed, the theoretical investigations of Damour and Esposito-Farèse (1992) have shown that some alternative to the GR theories of gravity, which include the additional scalar fields, also fit the above mentioned $\dot{\omega} - \gamma - \dot{P}_b$ test, and, hence, can not be distinguished from the GR theory. Realizing this fact has led to the development of the parameterized post-Keplerian (PPK) formalism, which was designed to extract the maximum of possible information from the high-precision observational data for binary pulsars (Damour & Taylor 1992). The application of this methodology to the four pulsars B1913+16, B1855+09, B1534+12 and B1953+29 has already ruled out a number of the alternative gravity theories (Taylor et al. 1992).

The PPK formalism is especially effective in discriminating alternative theories of gravity in the strong field regime when there is a possibility of independently measur-
ing two post-Keplerian parameters characterizing the range \( r \equiv Gm/c^3 \) and the shape \( s \equiv \sin i \) of the famous Shapiro delay in propagation of radio signals of pulsar in the gravitational field of its companion. The most favorable case for the measurements of \( r \) and \( s \) is when the pulsar’s orbit is inclined to the plane of the sky at the angle \( i \) close to 90°. Timing observations of the "non-relativistic" binary pulsar B1855+09 by Ryba & Taylor (1991), and by Kaspi et al. (1993) have provided the very first such independent measurements of the parameters \( r \) and \( s \) that allowed the determination of the neutron star mass with a precision better than about 18%. It is worthwhile to stress that such mass determination does not require a knowledge of the rate of relativistic apsidal motion \( \dot{\omega} \) and the redshift parameter \( \gamma \) as it takes place in the case of the PSR B1913+16 system.

At the same time when a binary pulsar orbit is viewed nearly edge-on (as for B1855+09 and PSR B1534+12) the complementary effect in the propagation of the pulsar’s radio signal may be important. This is due to the well-known effect of the deflection of light rays in the gravitational field of a self-gravitating body (in the gravitational field of the pulsar’s companion in the event of the binary pulsar system). Due to the gravitational deviation of the pulsar’s beam from a straight line, the observer will see the pulsar’s pulse only when the pulsar is rotated by an angle compensating this deviation. In contrast to the relativistic Shapiro effect, the effect of gravitational deflection of light directly contributes to the rotational phase of the pulsar (as, for instance, the effect of aberration of pulsar’s beam does), but this is so small that it can be misinterpreted as an additional delay in the time-of-arrivals (TOA) of the pulsar pulses. Similar to the Shapiro delay, the deflection of the pulsar’s beam by the gravitational field of its companion manifests itself in the timing as the rapid, sharp growth of the magnitude of the post-fit residuals of TOA on a short time interval in the vicinity to the moment of superior conjunction of the pulsar and its companion. The effect in question is superimposed on the Shapiro effect, and, in principle, should be explicitly taken into account to avoid an incorrect determination of the parameters \( r \) and \( s \).

Another important feature of the effect of gravitational deflection of light in a binary pulsar is a strong dependence of the shape of TOA residuals on the spatial orientation of the pulsar’s rotation axis (see the eqs. (7), (9), (10) in the text below). Thus, the measurement of this effect would help to get further constrains on the position of the pulsar’s spin independently of the polarization observations. Let us emphasize that the effect of the relativistic aberration caused by the orbit motion of pulsar in binary systems (Smarr & Blandford 1976) also strongly depends on the spin’s orientation, but can not be observed directly due to the same time-dependence on the pulsar’s orbital phase as the classical Roemer effect (Damour & Deruelle 1986).

Epstein (1977) was, probably, the first who called attention to the phenomenon of deflection of the pulsar radio beam in the gravitational field of its companion when \( \cos i \leq 10^{-3} \). However, he did not make any calculations confirming this estimate. Narayan et al. (1991) also mentioned this effect as a useful tool for investigation of black hole - neutron star binary systems and verification of the existence of the black hole to an unparalleled degree of certainty. Schneider (1989, 1990) calculated the corresponding gravitational deflection time delay and lensing amplification factor of the pulse intensity in the edge-on binary pulsar, but, unfortunately, his presentation is not easily followed.

In the present paper we try to solve the problem more simply, giving both a complete theoretical treatment and a discussion of the measurability of the effect of the gravitational deflection of radio beam in binary pulsars which we call briefly the bending delay (BD) effect. It is shown how to include this effect in the fitting procedure of TOA for the evaluation of the pulsar parameters of binary pulsars whose orbital inclinations to the plane of the sky are extremely close to 90°.

In Sec. 2 of the paper we present an improved formula for the timing of binary pulsars with the BD effect taken into account. In Sec. 3 we investigate, using numerical simulations, the measurability of the masses and orbital inclination in binary systems when the BD effect is taken into account. A mathematical procedure for the direct measurement of parameters of the BD effect is outlined in Sec. 4 and applied to process the observational data for PSR B1855+09. In Sec. 5 we present a summary of our results and conclusions.

2 TIMING FORMULA

The self-consistent mathematical derivation of a timing formula for the calculation of pulsar rotational phase should be based upon the relativistic theory of astronomical reference systems together with the matching procedure developed previously by Kopeikin (1988), Brumberg & Kopeikin (1989a,b) and Brumberg (1991). The exact timing formula must include the kinematic effects of the galactic differential rotation, orbital and precessional motions of the pulsar, relativistic effect of aberration, quadratic Doppler and red shift effects, the Shapiro delay, as well as the deflection of the pulsar beam in the gravitational field of its companion. In the present paper we ignore the effects of differential galactic acceleration and precessional motion of the pulsar rotational axis suggesting for simplicity that vectors of the angular velocity \( \Omega \) and spin \( \dot{S} \) of the pulsar are coincident.

The timing formula obtained is a slightly modified version of that elaborated by Damour & Deruelle (1986). Some additional theoretical aspects of the derivation of the timing formula are discussed by Klioner & Kopeikin (1994) and Kopeikin (1994, 1995).

Let \( \eta \) and \( \lambda \) be the polar coordinates of the pulsar’s spin axis \( \dot{S} \), where \( \eta \) is the longitude of the pulsar’s spin axis in the plane of the sky measured from the ascending node of the pulsar’s orbital plane (\( 0° \leq \eta < 360° \)), and \( \lambda \) is the angle between the pulsar’s spin axis and the line of sight pointing from the terrestrial observer toward the pulsar (\( 0° \leq \lambda < 180° \)). The time evolution of the pulsar rotational phase \( \phi(T) \) is given by the equation:

\[
\phi(T)/2\pi \equiv N(T) = N_0 + \nu_T T + \frac{1}{2} \nu_T^2 T^2 + \frac{1}{6} \nu_T^3 T^3 ,
\]  

where \( N_0 \) is the initial phase; \( \nu_T, \nu_T', \nu_T'' \) denote the pulsar spin frequency and its time derivatives shifted by the Doppler factor, and \( T \) is the pulsar proper time related to the solar-system barycentric arrival time \( t \) according to the relationship:

\[
t - t_0 = T + \Delta_B(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T) + \Delta_B(T),
\]  

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Here $t_0$ is the initial barycentric epoch of observation; $\Delta R$, $\Delta E$ and $\Delta A$ correspond to the well-known Roemer, Einstein and aberration time delays, whose exact mathematical expressions can be found in the paper of Damour & Taylor (1992); $\Delta S$ is the Shapiro delay of radio pulses in the gravitational field of companion of the pulsar (Epstein 1977; Damour & Taylor 1992):

$$\Delta S = -2r \ln L,$$

(3)

$$L = 1 - e \cos u - s \{ \sin \omega (\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u \},$$

(4)

which depends on the just introduced post-Keplerian parameters $r$ and $s$ as well as the classical orbital parameters: eccentric anomaly $u$, eccentricity $e$, and longitude of the periastron $\omega$.

$\Delta B_{\nu}$ in the equation (2) is the additional bending delay caused by the deflection of the pulsar beam in the gravitational field of its companion. It has been calculated from the condition of emission of the pulsar’s pulse toward the observer (Kopeikin 1992). This condition is expressed in the pulsar’s proper reference frame by the equation:

$$\langle \vec{N} \cdot (\vec{\Omega} \wedge \vec{b}) \rangle = 0,$$

(5)

which defines a relation between the number of pulse, which has been emitted, and the moment of the emission reckoned in the pulsar proper time scale $T$. Here $\vec{N}$ denotes the spatial components of isotropic unit vector tangent to the trajectory of the emitted pulse, and directed toward the terrestrial observer; $\vec{b}$ is the unit vector in the fiduciary direction of maximal pulsar radio emission (it is not necessarily the direction of magnetic dipole); $\vec{\Omega}$ is the vector of the instantaneous angular velocity of the pulsar; the signs "∧" and "\wedge" denote the Euclidean scalar and cross products. We have assumed implicitly that the model of the conical rotating beacon for the description of the pulsar’s emission (Lyne & Manchester 1988) is true. However, we would like to point out that calculations being described in this paragraph are still valid in the case of a deviation of the polar beam geometry from the conical one as it is, for instance, in the model of Narayan & Vivekananda (1983).

Vector $\vec{N}$ does not keep a constant spatial orientation from the point of view of an external observer at rest but has small (caused by both classical and relativistic effects) spatial variations with respect to the constant unit vector $\vec{n}$ referred to the rest frame of the binary pulsar barycenter. To show this we use the simplified version of the relativistic relationship between the spatial components of the vectors $\vec{N}$ and $\vec{n}$ derived by Klioner & Kopeikin (1992) in the case of the relativistic $N$-body problem:

$$\vec{N} = \vec{n} - \frac{1}{R} (\vec{n} \wedge (\vec{r}_p \wedge \vec{n})) + \frac{1}{c} (\vec{n} \wedge (\vec{v}_p \wedge \vec{n}))$$

$$- \frac{2GM_n}{c^2} \left( \frac{\vec{n} \wedge (\vec{r}_p \wedge \vec{n})}{r_p + (\vec{n} \cdot \vec{r}_p)} \right),$$

(6)

to replace in the equation (2) the time dependent components $\vec{N}$ by the ones of the vector $\vec{n}$. The complete relation between $\vec{N}$ and $\vec{n}$ (Klioner & Kopeikin 1992) can be easily used for investigations of the second order aberration effects and physical parameters of a rotating black hole being considered as a possible companion of a pulsar (Narayan et al. 1991).

In formula (6), $c$ is the speed of light, $\vec{r}_p$ and $\vec{r}_R$ are the radius-vectors pointing respectively from the barycenter of the binary system and the center-of-mass of the pulsar’s companion to the pulsar (do not confuse these vectors with the notation $r$ for the parameter characterizing amplitude of the Shapiro delay), $\vec{v}_p = d\vec{r}_p/dT$ is the orbital velocity of the pulsar, and $R$ is the distance between the binary pulsar and the solar system barycenters. Let us note that it is possible to equate the vector $-\vec{n}$ to the unit vector $\vec{K}$ (that is $\vec{n} = -\vec{K}$) pointing from the Earth toward the pulsar system as it is usually done, though such procedure requires an appropriate justification from a theoretical point of view.

After substituting expression (6) into equation (5), expressing vectors $\vec{N}$, $\vec{\Omega}$, and $\vec{n}$ through the Euler variables, and solving the obtained trigonometric equation with respect to the fast Euler variable as described in (Doroshenko et al. 1995) we get the counting formula (1) with additional phase corrections due to the classical and relativistic terms in the relationship (6). To be specific, the second term in the right-hand side of this formula gives an additional but negligibly small classical parallax delay in the rotational phase of the pulsar (see, however, the paper of Kopeikin (1995) where the influence of the orbital parallax effect on the propagation time of pulsar radio signals is shown to be detectable); the third term in the right-hand side of (6) leads to the aberration delay $\Delta A(T)$ depending on two directly unobservable parameters $A$ and $D$ introduced by Damour & Deruelle (1986), and the very last term defines after straightforward calculations the gravitational bending delay being under consideration:

$$\Delta B_{\nu} = L^{-1} (C \sin(\omega + A_c) + D \cos(\omega + A_c)).$$

(7)

Here the true orbital anomaly $A_c$ is related to $u$ by the well-known equation:

$$A_c = 2 \arctan \left[ \left( 1 + \frac{e \cos u - c}{1 - e} \right)^{1/2} \tan \frac{u}{2} \right],$$

(8)

$a_R$ designates the semi-major axis of the relative orbit of the pulsar with respect to its companion, and the constants $C$ and $D$ have been introduced as the new post-Keplerian parameters:

$$C = \frac{cr(1 - s^2)^{1/2} \cos \eta}{\pi \nu \rho a_R \sin \lambda},$$

(9)

$$D = - \frac{cr \sin \eta}{\pi \nu \rho a_R \sin \lambda},$$

(10)

where $a_R = a_p(m_p + m_e)/m_e$, and $a_p$ is the major semi-axis of the pulsar orbit with respect to the barycenter of the binary system. It is interesting to note that the final expression for the BD effect, formula (9), does not include any dependence on the spatial orientation of vector $\vec{b}$. The same is true for the aberration delay $\Delta A$.

For a better understanding of influence of the BD effect on the TOA of the pulsar pulses let us compare the behavior of the functions $\Delta S$ and $\Delta B_{\nu}$ by inserting parameters of the system PSR B1855+09 into the expressions (3) and (6). The explicit dependence on time of the Shapiro (eq. (3)) and BD (eq. (6)) effects are shown in Fig.1, which has been drawn using observational parameters of the PSR B1855+09 reported by Kaspi et al. (1994).

For illustrative purposes we have chosen the artificially small numerical value for $\lambda = 0^\circ.005$, and two different val-
ues for the angle: \( \eta = -85^\circ \) (solid curve, \( D > 0, C > 0 \)), and \( \eta = 85^\circ \) (dotted curve, \( D < 0, C > 0 \)) to depict the BD effect in the most clear form. One can see from Fig.1 that the modulation of TOA due to the BD effect is completely different and quite sharper than that caused by the Shapiro delay. The total observational timing effect will be the sum of the Shapiro and BD delay curves. It is important to note that the asymmetry in the curve describing the BD effect does not depend on the value of the angle \( \lambda \) (since \( 0 \leq \sin \lambda \leq 1 \), when \( 0^\circ \leq \lambda \leq 180^\circ \)) and is sensitive to the numerical value of the angle \( \eta \) only. Therefore, we can say nothing from the investigation of the BD effect whether the pulsar spin axis \( \vec{S} \) is along the direction to observer or in the opposite one.

The considered example shows that there is in principal a possibility to detect the BD effect in the binary pulsars with nearly edgewise orbits. However, this effect was never included in existing pulsar timing algorithms. Therefore, it is important to check how strongly the BD effect perturbs measured parameters of such binary systems.

3 MEASURABILITY OF MASSES AND ORBITAL INCLINATION IN THE EDGE-ON BINARY SYSTEMS

To investigate the measurability of the BD effect we shall follow the linearization procedure for evaluation of the pulsar parameters, developed by Blandford & Teukolsky (1976), and improved by Damour & Deruelle (1986). The procedure is based upon, the so called, ”periodic approximation” of differential timing formula (see below). The idea is to introduce instead of the total set of orbital and spin parameters of the pulsar in the timing algorithm a convenient set of independent parameters, which absorb all the secular time dependencies in the form of polynomials of time \( t \). Coefficients of the differential timing formula are in such case only periodic functions of time.

Let the observed rotational phase of the pulsar be \( N_{\text{obs}} \). Then we can improve the initial estimates of any pulsar’s parameter \( e_i^{(0)} \) using the residuals of the pulsar’s rotational phase \( R(t_k) = N_{\text{obs}}(t_k) - N_{\text{cal}}(e_i^{(0)}, t_k) \), where the numerical values of \( N_{\text{cal}} \) are computed at the moments of observed arrival times \( t_k \) with the help of the equations (3) - (4) and (5) - (10). Assuming that the differences \( \delta e_i = e_i - e_i^{(0)} \) between the (yet unknown) true and initial numerical values of the parameters are small, one can represent an expression for the total derivative of the phase residuals (the differential timing formula) in the form:

\[
R(t) = \sum_{i=1}^{7} B_i \cdot \delta e_i,
\]

where the variations of the adjusted parameters in formula (11) are:

\[
\delta e_1 = -\frac{1}{v_p}(\delta N + t \cdot \delta v_p + \frac{e^2}{2} \cdot \delta \nu_p + \cdots)
\]

\[
\delta e_2 = \delta \alpha
\]

\[
\delta e_3 = \delta(\beta + \gamma)
\]

\[
\delta e_4 = -\delta \sigma - t \cdot \delta n
\]

\[
\delta e_5 = \delta e
\]

\[
\delta e_6 = \frac{\delta m_e}{m_e}
\]

\[
\delta e_7 = \frac{\delta \sin i}{\sin i},
\]

with \( \alpha = x \sin \omega \) and \( \beta = \sqrt{1 - e^2} x \cos \omega \); \( \sigma \) is the orbital reference phase; \( n \equiv 2\pi/P_o \), \( P_o \) is the orbital period; \( \gamma \) is the gravitational red shift plus transverse Doppler effect. Let us point out that to get relative accuracies in the determination of the mass \( m_e \) and \( \sin i \) we have used the substitutions for the parameters of the type \( m_e = m_e^{(0)} \cdot (1 + \mu) \) and \( \sin i = \sin i^{(0)} \cdot (1 + \xi) \), where \( m_e^{(0)} \) and \( \sin i^{(0)} \) are supposed to be fixed and equal to the current estimated values for these parameters.

The functions \( B_i = \partial N(t)/\partial e_i \) are the partial derivatives of the pulsar rotational phase \( N(t) \) with respect to the \( e_i \)-th adjusted parameter of the pulsar, and we neglect the relativistic post-Keplerian parameters \( \delta e \), \( \delta \sigma \), \( k \) and all hidden parameters (see for explanation of the physical meaning of the omitted parameters the papers of Damour & Deruelle (1986) and Kopeikin (1994)):

\[
B_1 = 1,
\]

\[
B_2 = \cos u - e,
\]

\[
B_3 = \sin u,
\]

\[
B_4 = \frac{\alpha \sin u - \beta \cos u}{1 - e \cos u},
\]

\[
B_5 = -\alpha - B_4 \sin u,
\]

\[
B_6 \equiv B_\mu = \Delta S + \Delta \beta,
\]

\[
B_7 \equiv B_\chi = L^{-1} \sin(\omega + A_e) \left[ (2r + \Delta B)(1 - e \cos u) - \frac{sc}{1 - s^2} \right] \times \sin(\omega + A_e).
\]

After the first step of the least-squares fit of the residuals \( R(t) \), the obtained corrections for the pulsar parameters \( \delta e_i^{(0)} \) are added to their initial values \( e_i^{(1)} = e_i^{(0)} + \delta e_i^{(0)} \) and are used again at the second step of the iteration procedure. The process is repeated until the limited values of the estimated parameters are obtained.
A similar model has been used by Ryba & Taylor (1991) and Kaspi et al. (1994) in the measurement of the parameters of the PSR B1855+09. However, these authors have relied upon the algorithm developed by Damour & Deruelle (1986). The main difference between our differential timing formula and the one of Damour & Deruelle (1986) is in the expressions for the partial derivatives $B_6$ and $B_7$ taking into account the BD effect in the explicit form. It is important to note that omitting the BD effect from the timing algorithm will result, in principle, in incorrect estimates for the relativistic post-Keplerian parameters $r$ and $s$. This fact must be taken into account for unambiguous testing of the alternative gravity theories in the strong field regime.

Using our improved expressions (18) and (19) for these functions one can examine how the BD effect will manifest itself in the fitted values of the pulsar parameters $\mu$ and $\xi$. To investigate this question we take numerical simulations using as an example imaginary binary systems having the same parameters as the pulsars B1855+09 and B1913+16 except for the inclination angle $i$ of the orbit. The inclination is varied in the range from 86.5° to 90°. Let us suppose that the standard deviation of the mock arrival times is $\epsilon$ (a constant measured in $\mu$s), and covariance matrix $C_{ij}$ for the fitting method is $\epsilon^2$ times the inverse of the $7 \times 7$ matrix:

$$ B_{ij} = \sum_{k=1}^{n} B_i(t_k) \cdot B_j(t_k). $$

Here $n$ is the total number of the TOA measurements and the functions $B_i (i = 1, 2, \ldots, 7)$ are given by equations (13) - (19). Then our variance estimates for the evaluated parameters $\epsilon_i$ are:

$$ \delta \epsilon_i = \sigma_{\epsilon_i} \frac{\epsilon(\mu s)}{\sqrt{n}}, $$

where the errors $\sigma_{\epsilon_i}$ of the parameters are computed by using the diagonal elements of the inverse matrix $b_{ij} = B_{ij}^{-1}$. We also suppose in our numerical simulations that the observational points are equally distributed over the pulsar’s orbital phase and their total number is equal to 1000. Two different timing models have been compared, namely, one of Damour & Deruelle (1986) without the BD effect, and that outlined above, which includes the BD effect.

First of all, we took calculations for binary systems having orbital parameters like the PSR B1855+09. Figs 2 and 3 illustrate the behavior of the fractional uncertainties $\sigma_{\mu} = \sigma_{m_c}/m_c$ and $\sigma_2 = \sigma_{\sin i}/\sin i$ plotted as functions of $\cos i$ for these two algorithms respectively, where we fixed the value $\eta = -85^\circ$, and supposed the inclination angle $i$ ranges from 86.5° to 90°. Our assumed values for $\lambda = 0^\circ, 1^\circ$, and $1^\circ$ were chosen for illustrative purposes. They correspond for the case of PSR B1855+09 to the values of the parameter $D = -18 ns$, and $-1.8 ns$, respectively. Note that the numerical values for the parameter $D$ depend on the inclination angle, and have been computed from the relation $C = -D \cot \eta \cos i$ following from equations (3) and (4). This relation between $C$ and $D$ has been used in equations (3), (4) for drawing the plots. In fact, the values of $C$ for the chosen set of $D$ and $\sin i$ lie in the range $0 \div 0.1 ns$. Replacing the positive sign of the parameter $D$ with the negative one with the same numerical values (that corresponds to the positive value of angle $\eta = 85^\circ$) does not change the Figures 2 and 3 perceptibly.

The same procedure has been applied to estimate the measurability of the ”range” and ”shape” parameters in the case of binary system PSR B1913+16. In such systems, with more massive pulsar companions, one can expect to measure the BD effect more easily. We took four numerical values for $\lambda = 0^\circ, 1^\circ$, $10^\circ$, and $80^\circ$, that corresponds to $D = -1572 ns, -1572 ns, -158 ns$, and $-28 ns$. The parameter $C$ lies in the range $0 \div 0.2 ns$. Results of the calculations are shown in the figures 4 and 5.

As one can see from the plots (2) and (3), the inclusion of the effect of the gravitational bending delay is of no practical consequence for reducing the relative errors in the mass and $\sin i$ determinations in such systems as the binary pulsars like PSR B1855+09 when $\cos i > 0.01$ and $\lambda > 1^\circ$. As for the binary pulsars like PSR B1913+16, possessing more
4 MEASURABILITY OF THE GRAVITATIONAL DEFLECTION OF LIGHT PARAMETERS IN THE PSR B1855+09

The numerical simulations made in the previous paragraph show that the amplitude of the BD effect is, probably, too small to be detected with confidence in the PSR B1855+09 since the real value of $\cos i = 0.04$. Nevertheless, in the same spirit of "looking for zero to be sure it is there", we have tried to find it using the 7-yr. observational data set for PSR B1855+09 obtained by J. Taylor and collaborators at the Arecibo observatory (Kaspi et al., 1994). To process the observational data we apply the timing program TIMAPR described in (Doroshenko & Kopeikin 1990). It has been generalized for timing of binary pulsars in accordance with the timing algorithm described in Sec. 3 with a secular drift of all relevant parameters. The celestial reference frame was based upon the DE200/LE200 ephemerides of the Jet Propulsion Laboratory (Standish 1982) and the relativistic time scales transformations in the Solar system developed by Fairhead & Bretagnon (1989) are used in our fitting procedure (see the paper of Brumberg & Kopeikin (1990) for more theoretical details concerning the origin and structure of these transformation). The polar motion corrections were introduced using the "raw values of X,Y, universal time for every 5 days" referred to the 1979 BIH system (BIH Annual Report) and the "normal values of the Earth orientation parameters at 5-day intervals" referred to the EOP (IERS) C02 system (IERS Annual Report).

The general concept of measurability of pulsar parameters is described in the papers of Taylor & Weisberg (1989) and Damour & Taylor (1992), and we follow the outlined procedure considering now $C$ and $D$ in the formula (5) as the new independent fitted parameters. It changes the formula (13) to:

$$R(t) = \sum_{i=1}^{g} \hat{B}_i \cdot \delta e_i$$

(22)

where parameters $e_1, ..., e_5$ are the same as in the equations (14), $\{e_i; 6 \leq i \leq 9\} = \{r, s, C, D\}$. Functions $\hat{B}_i = \hat{B}_i$ for $i = 1, 2, 3, 4, 5$ are taken from the equations (13)-(17) and:

$$\hat{B}_6 \equiv \hat{B}_7 = -2 \ln \Lambda,$$

(23)

$$\hat{B}_8 = L^{-1} (2r + \Delta B) (1 - \cos \nu) \sin (\omega + A_e),$$

(24)

$$\hat{B}_9 = \hat{B}_D \equiv L^{-1} \sin (\omega + A_e).$$

(25)

To determine parameters $r, s, C$ and $D$ we used the least-squares procedure of minimizing the statistic:

$$\chi^2 = \sum_{i=1}^{n} \left[ \frac{N(t_i) - N_i}{\sigma_i} \right]^2,$$

(27)

where $\sigma_i$ is the estimated uncertainty of the TOA $t_i$. The statistic $\chi^2$ measures the goodness of fit of the timing model. However, direct application of the least-squares method with all parameters considered to be varied freely fails to determine the parameters $C$ and $D$ because of their smallness. Moreover, the parameter $C$ includes the additional, small factor $\cos i = 0.04$, and, for this reason, has been assumed to be equal to zero identically. Then, to extract some value...
Figure 6. The smooth curve represents values of $\Delta \chi^2$ obtained in a data fit with the fixed values of the $r$ and $s$ relativistic post-Keplerian parameters obtained by Kaspi et al. (1994). The levels of confidence, corresponding to $1\sigma$, $2\sigma$ and $3\sigma$ determination of parameter $D$ are plotted with dotted lines.

able information, we investigate the structure of the function $\Delta \chi^2(e_i) = \chi^2(e_i) - \chi^2_0(e_{est})$ in the neighborhood of the global minimum $\chi^2_0$ in the multi-dimensional space of the fitted parameters (Taylor & Weisberg, 1989; Ryba and Taylor, 1991; Damour & Taylor, 1992).

We have calculated the functional dependence of $\chi^2$ statistic near the region of its global minimum with the minimization of the $\chi^2$ by the fit of the astrometric $\{N_0, \alpha, \delta, \mu_{\alpha}, \mu_{\delta}\}$, spin $\{\nu_p, \nu_{pd}, \nu_{pd}^b\}$, orbital Keplerian $\{x = a \sin i, e, T_0, \omega, P_b\}$, and post-Keplerian $\{P_{\mu}, \dot{e}, \dot{\omega}\}$ parameters of the pulsar with the parallax $\pi$, dispersion measure $DM$, and post-Keplerian parameters $r$ and $s$ held fixed at the unique values reported by Kaspi et al. (1994). In this procedure the parameter $D$ was held fixed at a number of different points from -40 ns to 40 ns. The results of the fitting procedure are shown in Fig. 6.

As one can see from this plot the corresponding curve has a parabolic shape, and the numerical value of the parameter $D = 12^{+10}_{-9}$ nanosecond is not zero with the 68 % confidence ($1\sigma$) limits. The values of other astrometric, spin, Keplerian, and post-Keplerian parameters for the PSR B1855+09 are in a perfect ($1\sigma$) agreement with the values reported by Kaspi et al. (1994). Although we can not say anything with the adopted ($3\sigma$) confidence about the real numerical value of the BD effect, the parabolic structure of the $\chi^2$ near region of its global minimum (with respect to the parameter $D$) indicates that the effect in question can be, perhaps, measured in the future.

5 CONCLUSION

We have derived the improved timing formula (1) for binary pulsars taking into account the relativistic effect of light deflection in the gravitational field of the pulsar’s companion. It has been shown the effect is perceptible only for the pulsars with nearly edge-on orbits. The measurement of the effect in question permits, in principle, the determination of the position angle $\eta$ of the angular velocity vector of the pulsar although it fails to determine direction of the rotation. The influence of the gravitational deflection of light on the measurability of the magnitude $r$ and shape $s$ parameters of the Shapiro time delay as well as the BD parameters $C$ and $D$ has been examined. Processing of the observational data for the binary pulsar B1855+09 was rendered in an effort to find some indication on the presence of effect of the gravitational deflection of light in this system.

Taking into account the effect of the gravitational deflection of light in the edge-on binary pulsars allows us to make more precise experimental tests of alternative theories of gravity in the strong gravitational field regime when the cosine of the orbital inclination to the plane of the sky is less than 0.003. This rather conservative estimate can be relaxed if the rotational axis of the pulsar is close enough to the line of sight and/or the total number of equally spaced observations exceeds 1000.

After this paper was submitted for publication, we were informed about a paper by Goicoechea et al., where the BD effect is discussed for the case of binary pulsars and X-ray binaries on circular orbits.

Acknowledgments

We are very grateful to Kaspi V.M., Taylor J.H. and Ryba M.F. for their kind permission to use their observational data on the PSR B1855+09 binary system before publication. We would also like to acknowledge K.Yokoyama (NAO of Japan, Mizusawa) who supplied us the polar motion corrections in machine readable form. We are thankful Yu.P.Ilyasov, K.Yokoyama, T.Fukushima, and H.Kinoshiba for participation in discussions. E.L.Presman made valuable remarks concerning statistical significance of the obtained results and we highly appreciate him. An expression of gratitude is due to Dr M.Holman for polishing the grammar of the manuscript and anonymous referee for helpful comments and suggestions.

S.M.Kopeikin is deeply indebted to the staff of the National Astronomical Observatory of Japan (Mitaka) for the support of this work. O.Doroshenko has been partially supported by the ISF grant M99000.

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Relativistic Effect of Gravitational Deflection of Light in Binary Pulsars

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ABSTRACT

An improved formula for the timing of binary pulsars that accounts for the relativistic deflection of light in the gravitational field of the pulsar's companion is presented, and the measurability of this effect together with its variance estimates are discussed. The deflection of the pulsar's beam trajectory in the gravitational field of its companion leads to variation in the pulsar's rotational phase. This variation appears as the narrow sharp growth of the magnitude of the post-fit pulsar timing residuals in the vicinity of the moment of the superior conjunction of the pulsar with its companion. In contrast to the relativistic Shapiro effect the amplitude of the effect of gravitational deflection of the pulsar radio beam has two peaks with opposite signs, which become sharper as the inclination \( i \) of the pulsar's orbit approaches 90°. The effect under consideration influences the estimation of parameters of the relativistic Shapiro effect in the binary pulsars with nearly edgewise orbits. Its inclusion in the fitting procedure provides a more careful measurement of some of the orbital inclination as well as the masses of the pulsar \( m_p \) and its companion \( m_c \). This permits an improved testing of alternative theories of gravity in the strong field regime. Moreover, the measurement of this effect allows independent geometric constraints on the position of the pulsar rotational axis in space. The effect of the gravitational deflection of light has been numerically investigated for binary pulsars with nearly edgewise orbits. It is shown that the effect is observed in general only when \( \cos i \leq 0.003 \). This estimate becomes less restrictive as the pulsar's spin axis approaches the line of the sight.

Key words: gravitation - pulsars - relativity - stars: neutron - stars: individual (PSRs B1855+09, B1913+16, B1534+12)

1 INTRODUCTION

High-precision timing observations of binary pulsars on relativistic orbits provide unique opportunities for testing of General Relativity (GR) through two mechanisms: the emission of gravitational radiation and the presence of strong-field effects. The effect of emission of gravitational waves have been confirmed using the PSR B1913+16 long term data with an accuracy better than 0.5% (Taylor & Weisberg 1989; Damour & Taylor 1991). This was accomplished by the measurements of three relativistic effects: 1) the rate of apsidal motion \( \dot{\omega} \), 2) the Doppler and gravitational red shifts \( \gamma \) of pulsar rotational frequency, and 3) the system orbital period decay \( \dot{P}_b \), which is due to the effect of the gravitational waves emission, which has been rigorously justified from the mathematical point of view by Damour (1983a,1983b) and, independently, by Grishchuk & Kopeikin (1983,1986), as well as Schäfer (1985). However, timing of PSR B1913+16 is not sufficiently sensitive to the strong gravitational field effects. And, indeed, the theoretical investigations of Damour and Esposito-Farèse (1992) have shown that some alternative to the GR theories of gravity, which include the additional scalar fields, also fit the above mentioned \( \dot{\omega} - \dot{P}_b \) test, and, hence, can not be distinguished from the GR theory. Realizing this fact has led to the development of the parameterized post-Keplerian (PPK) formalism, which was designed to extract the maximum of possible information from the high-precise observational data for binary pulsars (Damour & Taylor 1992). The application of this methodology to the four pulsars B1913+16, B1855+09, B1534+12 and B1953+29 has already ruled out a number of the alternative gravity theories (Taylor et al. 1992).

The PPK formalism is especially effective in discriminating alternative theories of gravity in the strong field regime when there is a possibility of independently measur-
ing two post-Keplerian parameters characterizing the range \( r \equiv Gm/c^2 \) and the shape \( s \equiv \sin i \) of the famous Shapiro delay in propagation of radio signals of pulsar in the gravitational field of its companion. The most favorable case for the measurements of \( r \) and \( s \) is when the pulsar's orbit is inclined to the plane of the sky at the angle \( i \) close to 90°. Timing observations of the "non-relativistic" binary pulsar B1855+09 by Ryba & Taylor (1991), and by Kaspi et al. (1993) have provided the very first such independent measurements of the parameters \( r \) and \( s \) that allowed the determination of the neutron star mass with a precision better than about 18%. It is worthwhile to stress that such mass determination does not require a knowledge of the rate of relativistic apsidal motion \( \omega \) and the red shift parameter \( \gamma \) as it takes place in the case of the PSR B1913+16 system.

At the same time when a binary pulsar orbit is viewed nearly edge-on (as for B1855+09 and PSR B1534+12) the complementary effect in the propagation of the pulsar's radio signal may be important. This is due to the well known effect of the deflection of light rays in the gravitational field of a self-gravitating body (in the gravitational field of the pulsar's companion in the event of the binary pulsar system). Due to the gravitational deviation of the pulsar's beam from a straight line, the observer will see the pulsar's pulse only when the pulsar is rotated by an angle compensating this deviation. In contrast to the relativistic Shapiro effect, the effect of gravitational deflection of light directly contributes to the rotational phase of the pulsar (as, for instance, the effect of aberration of pulsar's beam does), but this is so small that it can be misinterpreted as an additional delay in the time-of-arrivals (TOA) of the pulsar pulses. Similar to the Shapiro delay, the deflection of the pulsar's beam by the gravitational field of its companion manifests itself in the timing as the rapid, sharp growth of the magnitude of the post-fit residuals of TOA on a short time interval in the vicinity to the moment of superior conjunction of the pulsar and its companion. The effect in question is superimposed on the Shapiro effect, and, in principle, should be explicitly taken into account to avoid an incorrect determination of the parameters \( r \) and \( s \).

Another important feature of the effect of gravitational deflection of light in a binary pulsar is a strong dependence of the shape of TOA residuals on the spatial orientation of the pulsar's rotation axis (see the eqs. (7), (9), (10) in the text below). Thus, the measurement of this effect would help to get further constrains on the position of the pulsar's spin independently of the polarization observations. Let us emphasize that the effect of the relativistic aberration caused by the orbital motion of pulsar in binary systems (Smarr & Blandford 1976) also strongly depends on the spin's orientation, but can not be observed directly due to the same time-dependence on the pulsar's orbital phase as the classical Roemer effect (Damour & Deruelle 1986).

Epstein (1977) was, probably, the first who called attention to the phenomenon of deflection of the pulsar radio beam in the gravitational field of its companion when \( \cos i \leq 10^{-3} \). However, he did not make any calculations confirming this estimate. Narayan et al. (1991) also mentioned this effect as a useful tool for investigation of black hole - neutron star binary systems and verification of the existence of the black hole to an unparalleled degree of certainty. Schneider (1989, 1990) calculated the corresponding gravitational deflection time delay and lensing amplification factor of the pulse intensity in the edge-on binary pulsar, but, unfortunately, his presentation is not easily followed.

In the present paper we try to solve the problem more simply, giving both a complete theoretical treatment and a discussion of the measurability of the effect of the gravitational deflection of radio beam in binary pulsars which we call briefly the bending delay (BD) effect. It is shown how to include this effect in the fitting procedure of TOA for the evaluation of the pulsar parameters of binary pulsars whose orbital inclinations to the plane of the sky are extremely close to 90°.

In Sec. 2 of the paper we present an improved formula for the timing of binary pulsars with the BD effect taken into account. In Sec. 3 we investigate, using numerical simulations, the measurability of the masses and orbital inclination in binary systems when the BD effect is taken into account. A mathematical procedure for the direct measurement of parameters of the BD effect is outlined in Sec. 4 and applied to process the observational data for PSR B1855+09. In Sec. 5 we present a summary of our results and conclusions.

2 TIMING FORMULA

The self-consistent mathematical derivation of a timing formula for the calculation of pulsar rotational phase should be based upon the relativistic theory of astronomical reference systems together with the matching procedure developed previously by Kopeikin (1988), Brumberg & Kopeikin (1989a,b) and Brumberg (1991). The exact timing formula must include the kinematic effects of the galactic differential rotation, orbital and precessional motions of the pulsar, relativistic effect of aberration, quadratic Doppler and red shift effects, the Shapiro delay, as well as the deflection of the pulsar beam in the gravitational field of its companion. In the present paper we ignore the effects of differential galactic acceleration and precessional motion of the pulsar rotational axis suggesting for simplicity that vectors of the angular velocity \( \Omega \) and spin \( \vec{S} \) of the pulsar are coincident. The timing formula obtained is a slightly modified version of that elaborated by Damour & Deruelle (1986). Some additional theoretical aspects of the derivation of the timing formula are discussed by Klioner & Kopeikin (1994) and Kopeikin (1994, 1995).

Let \( \eta \) and \( \lambda \) be the polar coordinates of the pulsar's spin axis \( \vec{S} \), where \( \eta \) is the longitude of the pulsar's spin axis in the plane of the sky measured from the ascending node of the pulsar's orbital plane (\( 0° \leq \eta < 360° \)), and \( \lambda \) is the angle between the pulsar's spin axis and the line of sight pointing from the terrestrial observer toward the pulsar (\( 0° \leq \lambda < 180° \)). The time evolution of the pulsar rotational phase \( \phi(T) \) is given by the equation:

\[
\phi(T)/2\pi = N(T) = N_0 + \nu_p T + \frac{1}{2}\dot{\nu}_p T^2 + \frac{1}{6}\ddot{\nu}_p T^3,
\]

where \( N_0 \) is the initial phase; \( \nu_p, \dot{\nu}_p, \ddot{\nu}_p \) denote the pulsar spin frequency and its time derivatives shifted by the Doppler factor, and \( T \) is the pulsar proper time related to the solar-system barycentric arrival time \( t \) according to the relationship:

\[
t - t_0 = T + \Delta N(T) + \Delta \dot{N}(T) + \Delta \ddot{N}(T) + \Delta \phi(T) + \Delta \phi_p(T),
\]
Here $t_0$ is the initial barycentric epoch of observation; $\Delta R$, $\Delta \eta$ and $\Delta A$ correspond to the well-known Roemer, Einstein and aberration time delays, whose exact mathematical expressions can be found in the paper of Damour & Taylor (1992); $\Delta \eta$ is the Shapiro delay of radio pulses in the gravitational field of companion of the pulsar (Epstein 1977; Damour & Taylor 1992):

$$\Delta \eta = -2\eta \ln L,$$

$$L = 1 - e \cos u - e \{ \sin \omega (\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u \}.$$ (4)

which depends on the just introduced post-Keplerian parameters $r$ and $s$ as well as the classical orbital parameters: eccentric anomaly $u$, eccentricity $e$, and longitude of the periastron $\omega$.

$\Delta_B$ in the equation (2) is the additional bending delay caused by the deflection of the pulse beam in the gravitational field of its companion. It has been calculated from the condition of emission of the pulsar’s pulse toward the observer (Kopeikin 1992). This condition is expressed in the pulsar’s proper reference frame by the equation:

$$(\vec{N} \cdot (\vec{\Omega} \wedge \vec{b})) = 0,$$ (5)

which defines a relation between the number of pulse, which has been emitted, and the moment of the emission reckoned in the pulsar proper time scale $T$. Here $\vec{N}$ denotes the spatial components of isotropic unit vector tangent to the trajectory of the emitted pulse, and directed toward the terrestrial observer; $\vec{b}$ is the unit vector in the fiducial direction of maximal pulsar radio emission (it is not necessarily the direction of magnetic dipole); $\vec{\Omega}$ is the vector of the instantaneous angular velocity of the pulsar; the signs "\wedge" and "\Lambda" denote the Euclidean scalar and cross products. We have assumed implicitly that the model of the conical rotating beacon for the description of the pulsar’s emission (Lyne & Manchester 1988) is true. However, we would like to point out that calculations being described in this paragraph are still valid in the case of a deviation of the polar beam geometry from the conical one as it is, for instance, in the model of Narayan & Vivekanand (1983).

Vector $\vec{N}$ does not keep a constant spatial orientation from the point of view of an external observer at rest but has small (caused by both classical and relativistic effects) spatial variations with respect to the constant unit vector $\vec{n}$ referred to the rest frame of the binary pulsar barycenter. To show this we use the simplified version of the relativistic relationship between the spatial components of the vectors $\vec{N}$ and $\vec{n}$ derived by Kliore & Kopeikin (1992) in the case of the relativistic N-body problem:

$$\vec{N} = \vec{n} - 1/R (\vec{n} \wedge (\vec{r}_p \wedge \vec{n})) + 1/c (\vec{n} \wedge (\vec{v}_p \wedge \vec{n}))$$

$$- 2Gm_c (\vec{n} \wedge (\vec{r}_R \wedge \vec{n})) / c^2 \tau_R (\vec{n} \wedge (\vec{r}_R \wedge \vec{n}))$$

$$- 2Gm_c (\vec{n} \wedge (\vec{r}_p \wedge \vec{n})) / c^2 \tau_p (\vec{n} \wedge (\vec{r}_p \wedge \vec{n})).$$ (6)

to replace in the equation (5) the time dependent components $\vec{N}$ by the ones of the vector $\vec{n}$. The complete relation between $\vec{N}$ and $\vec{n}$ (Kliore & Kopeikin 1992) can be easily used for investigations of the second order aberration effects and physical parameters of a rotating black hole being considered as a possible companion of a pulsar (Narayan et al. 1991).

In formula (6), $c$ is the speed of light, $\vec{r}_p$ and $\vec{r}_R$ are the radius-vectors pointing respectively from the barycenter of the binary system and the center-of-mass of the pulsar’s companion to the pulsar (do not confuse these vectors with the notation $r$ for the parameter characterizing amplitude of the Shapiro delay), $\vec{v}_p = d\vec{r}_p / d\tau$ is the orbital velocity of the pulsar, and $R$ is the distance between the binary pulsar and the solar system barycenters. Let us note that it is possible to equate the vector $-\vec{n}$ to the unit vector $\vec{K}$ (that is $\vec{n} = -\vec{K}$) pointing from the Earth toward the pulsar system as it is usually done, though such procedure requires an appropriate justification from a theoretical point of view.

After substituting expression (6) into equation (5), expressing vectors $\vec{N}$, $\vec{n}$, and $\vec{K}$ through the Euler variables, and solving the obtained trigonometrical equation with respect to the fast Euler variable as described in (Doroshenko et al. 1995 we get the counting formula (1) with additional phase corrections due to the classical and relativistic terms in the relationship (6). To be specific, the second term in the right-hand side of this formula gives an additional but negligibly small classical parallax delay in the rotational phase of the pulsar (see, however, the paper of Kopeikin (1995) where the influence of the orbital parallax effect on the propagation time of pulsar radio signals is shown to be detectable); the third term in the right-hand side of (6) leads to the aberration delay $\Delta_A(T)$ depending on two directly unobservable parameters $A$ and $B$ introduced by Damour & Deruelle (1986), and the very last term defines after straightforward calculations the gravitational bending delay being under consideration:

$$\Delta_B = L^{-1} \left( C \sin (\omega + A) + D \cos (\omega + A) \right).$$ (7)

Here the true orbital anomaly $A$ is related to $u$ by the well-known equation:

$$A = 2 \arctan \left[ \left( \frac{1 + e}{1 - e} \right)^{1/2} \tan \frac{u}{2} \right],$$ (8)

where $A_R$ designates the semi-major axis of the pulsar with respect to its companion, and the constants $C$ and $D$ have been introduced as the new post-Keplerian parameters:

$$C = \frac{cr(1 - s^2)^{1/2} \cos \eta}{\pi \nu_p a_R \sin \lambda},$$ (9)

$$D = - \frac{cr}{\pi \nu_p a_R \sin \lambda},$$ (10)

where $a_R = a_p (m_p + M_c)/M_c$, and $a_p$ is the major semi-axis of the pulsar orbit with respect to the barycenter of the binary system. It is interesting to note that the final expression for the BD effect, formula (7), does not include any dependence on the spatial orientation of vector $\vec{b}$. The same is true for the aberration delay $\Delta_A$.

For a better understanding of influence of the BD effect on the TOA of the pulsar pulses let us compare the behavior of the functions $\Delta_A$ and $\Delta_B$ by inserting parameters of the system PSR B1855+09 into the expressions (3) and (7). The explicit dependence on time of the Shapiro (eq. (8)) and BD (eq. (7)) effects are shown in Fig.1, which has been drawn using observational parameters of the PSR B1855+09 reported by Kaspi et al. (1994).

For illustrative purposes we have chosen the artificially small numerical value for $\lambda = 0^\circ.006$, and two different val-
Periodic functions of time depend on the form of polynomials of time independent parameters, which absorb all the secular time measured parameters of such binary systems. For more explanations see the text.

The illustrative example of the TOA residuals for PSR B1855+09 caused by the Shapiro delay and deflection of the pulsar beam in the gravitational field of its companion - smooth or dotted curves. For more explanations see the text.

Figure 1. The illustrative example of the TOA residuals for PSR B1855+09 caused by the Shapiro delay and deflection of the pulsar beam in the gravitational field of its companion - smooth or dotted curves. For more explanations see the text.

3 MEASURABILITY OF MASSES AND ORBITAL INCLINATION IN THE EDGE-ON BINARY SYSTEMS

To investigate the measurability of the BD effect we shall follow the linearization procedure for evaluation of the pulsar parameters, developed by Blandford & Teukolsky (1976), and improved by Damour & Deruelle (1986). The procedure is based upon the so-called, "periodic approximation" of differential timing formula (see below). The idea is to introduce instead of the total set of orbital and spin parameters of the pulsar in the timing algorithm a convenient set of independent parameters, which absorb all the secular time dependencies in the form of polynomials of time \( t \). Coefficients of the differential timing formula are in such case only periodic functions of time.

Let the observed rotational phase of the pulsar be \( N_{\text{obs}} \). Then we can improve the initial estimates of any pulsar's parameter \( e_i^{(0)} \) using the residuals of the pulsar's rotational phase \( R(t_k) = N_{\text{obs}}(t_k) - N_{\text{cal}}(e_i^{(0)}, t_k) \), where the numerical values of \( N_{\text{cal}} \) are computed at the moments of observed arrival times \( t_k \) with the help of the equations (1)-(4) and (7)-(10). Assuming that the differences \( \delta e_i = e_i - e_i^{(0)} \) between the (yet unknown) true and initial numerical values of the parameters are small, one can represent an expression for the total derivative of the phase residuals (the differential timing formula) in the form:

\[
R(t) = \sum_{i=1}^{7} B_i \cdot \delta e_i, \quad (11)
\]

where the variations of the adjusted parameters in formula (11) are:

\[
\begin{align*}
\delta e_1 &= -\frac{1}{\nu_p} (\delta N + t \cdot \delta \nu_p + \frac{t^2}{2} \cdot \delta \nu_p + \cdots) \\
\delta e_2 &= \delta \alpha \\
\delta e_3 &= \delta (\beta + \gamma) \\
\delta e_4 &= -\delta \sigma - t \cdot \delta n \\
\delta e_5 &= \delta e \\
\delta e_6 &= \frac{\delta m_2}{m_2} \\
\delta e_7 &= \frac{\delta \sin i}{\sin i}
\end{align*}
\]

with \( \alpha = \sin \omega \) and \( \beta = \sqrt{1 - \sin^2 \omega} \cos \omega \); \( \sigma \) is the orbital reference phase; \( n \equiv 2\pi P_b \); \( P_b \) is the orbital period; \( \gamma \) is the gravitational red shift plus transverse Doppler effect. Let us point out that to get relative accuracies in the determination of the mass \( m_2 \) and \( \sin i \) we have used the substitutions for the parameters of the type \( m_2 = m_2^{(0)} (1 + \mu) \) and \( \sin i = \sin^{(0)} (1 + \xi) \), where \( m_2^{(0)} \) and \( \sin^{(0)} \) are supposed to be fixed and equal to the current estimated values for these parameters.

The functions \( B_i = \partial N(t)/\partial e_i \) are the partial derivatives of the pulsar rotational phase \( N(t) \) with respect to the \( e_i \)-th adjusted parameter of the pulsar, and we neglect the relativistic post-Keplerian parameters \( \delta \epsilon, \delta \theta, \delta k \) and all hidden parameters (see for explanation of the physical meaning of the omitted parameters the papers of Damour & Deruelle (1986) and Kopeikin (1994)).

\[
\begin{align*}
B_1 &= 1, \\
B_2 &= \cos u - e, \\
B_3 &= \sin u, \\
B_4 &= \frac{\alpha \sin u - \beta \cos u}{1 - e \cos u}, \\
B_5 &= \alpha - B_4 \sin u, \\
B_6 &= B_6 = \Delta S + \Delta B, \\
B_7 &= B_6 = L^{-1} \left[ \left( 2r + \Delta B \right) (1 - e \cos u) - \frac{\kappa C}{1 - e^2} \right] \times \sin(\omega + A_e). 
\end{align*}
\]

After the first step of the least-squares fit of the residuals \( R(t) \), the obtained corrections for the pulsar parameters \( \delta e_i^{(0)} \) are added to their initial values \( e_i^{(1)} = e_i^{(0)} + \delta e_i^{(0)} \) and are used again at the second step of the iteration procedure. The process is repeated until the limited values of the estimated parameters are obtained.
A similar model has been used by Ryba & Taylor (1991) and Kaspi et al. (1994) in the measurement of the parameters of the PSR B1855+09. However, these authors have relied upon the algorithm developed by Damour & Deruelle (1986). The main difference between our differential timing formula and the one of Damour & Deruelle (1986) is in the expressions for the partial derivatives $B_k$ and $B_l$ taking into account the BD effect in the explicit form. It is important to note that omitting the BD effect from the timing algorithm will result, in principle, in incorrect estimates for the relativistic post-Keplerian parameters $r$ and $s$. This fact must be taken into account for unambiguous testing of the alternative gravity theories in the strong field regime.

Using our improved expressions (18) and (19) for these functions one can examine how the BD effect will manifest itself in the fitted values of the pulsar parameters $\mu$ and $\xi$. To investigate this question we take numerical simulations using as an example imaginary binary systems having the same parameters as the pulsars B1855+09 and B1913+16 except for the inclination angle $i$ of the orbit. The inclination is varied in the range from 86.5° to 90°. Let us suppose that the standard deviation of the mock arrival times is $\epsilon$ (a constant measured in $\mu$sec), and covariance matrix $C_{ij}$ for the fitting method is $\epsilon^2$ times the inverse of the 7 x 7 matrix:

$$B_{ij} = \sum_{k=1}^{n} B_i(t_k) \cdot B_j(t_k).$$

(20)

Here $n$ is the total number of the TOA measurements and the functions $B_i$ ($i = 1, 2, \ldots, 7$) are given by equations (13) - (19). Then our variance estimates for the evaluated parameters $\epsilon_i$ are:

$$\delta\epsilon_i = \sigma_{\epsilon_i} \cdot \epsilon/\sqrt{n},$$

(21)

where the errors $\sigma_{\epsilon_i} = \sqrt{\mathbf{b}_{ii}}$ of the parameters are computed by using the diagonal elements of the inverse matrix $\mathbf{b}_{ij} = B_{ij}^{-1}$. We also suppose in our numerical simulations that the observational points are equally distributed over the pulse's orbital phase and their total number is equal to 1000. Two different timing models have been compared, namely, one of Damour & Deruelle (1986) without the BD effect, and that outlined above, which includes the BD effect.

First of all, we took calculations for binary systems having orbital parameters like the PSR B1855+09. Figs 2 and 3 illustrate the behavior of the fractational uncertainties $\sigma_\mu = \sigma_{m_\mu}/m_\mu$ and $\sigma_\xi = \sigma_{m_\xi}/\sin i$ plotted as functions of $\cos i$ for these two algorithms respectively, where we fixed the value $\eta = -85^\circ$, and suppose the inclination angle $i$ ranges from 86.5° to 90°. Our assumed values for $\lambda = 0^\circ, 1^\circ$, and $1^\circ$ were chosen for illustrative purposes. They correspond for the case of the PSR B1855+09 system to the values of the parameter $D = -18\,ns$, and $-1.8\,ns$, respectively. Note that the numerical values for the parameter $C$ depend on the inclination angle, and have been computed from the relation $C = -D \cos i \cos i Following from equations (9) and (10). This relation between $C$ and $D$ has been used in equations (18), (19) for drawing the plots. In fact, the values of $C$ for the chosen set of $D$ and $\sin i$ lie in the range $0 \leq 0.1\,ns$. Replacing the positive sign of the parameter $D$ with the negative one with the same numerical values (that corresponds to the positive value of angle $\eta = 85^\circ$) does not change the Figures 2 and 3 perceptibly.

The same procedure has been applied to estimate the measurability of the "range" and "shape" parameters in the case of binary system PSR B1913+16. In such systems, with more massive pulsar companions, one can expect to measure the BD effect more easily. We took four numerical values for $\lambda = 0^\circ, 1^\circ$, and $1^\circ$, that corresponds to $D = -16721\,ns$, $-16721\,ns$, $-158\,ns$, and $-28\,ns$. The parameter $C$ lies in the range $0 \div 0.2\,ns$. Results of the calculations are shown in the figures 4 and 5.

As one can see from the plots (2) and (3), the inclusion of the effect of the gravitational bending delay is of no practical consequence for reducing the relative errors in the mass and $\sin i$ determinations in such systems as the binary pulsars like PSR B1855+09 when $\cos i > 0.01$ and $\lambda > 1^\circ$. As for the binary pulsars like PSR B1913+16, possessing more

![Figure 2](image2.png)

**Figure 2.** The values of $\log_{10} \left( \sigma_\mu / \sigma_{m_\mu} \right)$ vs. $\cos i$ for measurement of the pulsar's companion mass in the two timing models for binary pulsars like PSR B1855+09. The dotted curve (S) represents the values $\sigma_{m_\mu}$ being expected from the data fit according to the Damour & Deruelle (1986) timing algorithm, and the two solid curves represent the values $\sigma_{m_\mu}$ being expected from the data fit with the gravitational bending delay taken into account. The assumed values for the angles $\eta = -85^\circ; \lambda = 0^\circ, 1^\circ$, and $1^\circ$ correspond to the curves labeled as (0), (1), and (1) respectively.

![Figure 3](image3.png)

**Figure 3.** The same as Fig.2, except for only that the plotted values are $\log_{10} \left( \sigma_\xi / \sigma_{m_\xi} \right)$ vs. $\cos i$.
massive pulsar's companion, the BD effect has no influence when $\cos i > 0.01$ and $\lambda > 80^\circ$. From an evolutionary point of view it is expected that for the pulsars under consideration $\lambda \sim i \approx 90^\circ$ and the only real hope to measure the influence of the BD effect is in this case when $\cos i < 0.003$. This value is close enough to that anticipated by Epstein (1977). It should be noted, however, that these estimates are true only for the total number of "observational points" being equal to 1000. Taking into account many more points or concentrating observations near the moment of superior conjunction makes the obtained estimates less conservative.

4 MEASURABILITY OF THE GRAVITATIONAL DEFLECTION OF LIGHT PARAMETERS IN THE PSR B1855+09

The numerical simulations made in the previous paragraph show that the amplitude of the BD effect is, probably, too small to be detected with confidence in the PSR B1855+09 since the real value of $\cos i = 0.04$. Nevertheless, in the same spirit of "looking for zero to be sure it is there", we have tried to find it using the 7-yr's observational data set for PSR B1855+09 obtained by J. Taylor and collaborators at the Arecibo observatory (Kaspi et al., 1994). To process the observational data we apply the timing program TIMAPR described in (Doroshenko & Kopeikin 1990). It has been generalized for timing of binary pulsars in accordance with the timing algorithm described in Sec. 3 with a secular drift of all relevant parameters. The celestial reference frame was based upon the DE200/LE200 ephemerides of the Jet Propulsion Laboratory (Standish 1982) and the relativistic time scales transformations in the Solar system developed by Fairhead & Brejag (1989) are used in our fitting procedure (see the paper of Brumberg & Kopeikin (1990) for more theoretical details concerning the origin and structure of these transformation). The polar motion corrections were introduced using the "raw values of X,Y, universal time for every 5 days" referred to the 1979 BIH system (BIH Annual Report) and the "normal values of the Earth orientation parameters at 5-day intervals" referred to the EOP (IERS) C02 system (IERS Annual Report).

The general concept of measurability of pulsar parameters is described in the papers of Taylor & Weisberg (1989) and Damour & Taylor (1992), and we follow the outlined procedure considering now $C$ and $D$ in the formula (7) as the new independent fitted parameters. It changes the formula (11) to:

$$ R(t) = \sum_{i=1}^{9} \hat{B}_i \cdot \delta e_i, $$

(22)

where parameters $e_1, \ldots, e_9$ are the same as in the equations (14), $\{e_i; 6 \leq i \leq 9\} = \{r, s, C, D\}$. Functions $\hat{B}_i = B_i$ for $i = 1, 2, 3, 4, 5$ are taken from the equations (13)-(17) and:

$$ B_6 \equiv \hat{B}_6 = -2 \ln L, $$

$$ B_7 \equiv \hat{B}_7 = L^{-1} (2r + \Delta B) (1 - e \cos \nu) \sin (\omega + A_c), $$

$$ B_8 \equiv \hat{B}_8 = L^{-1} \sin (\omega + A_c), $$

$$ B_9 \equiv \hat{B}_9 = L^{-1} \cos (\omega + A_c). $$

To determine parameters $r, s, C$ and $D$ we used the least-squares procedure of minimizing the statistic:

$$ \chi^2 = \sum_{i=1}^{n} \left( \frac{N(t_i) - N(t_i)}{\sigma v_p} \right)^2, $$

(27)

where $\sigma v_p$ is the estimated uncertainty of the TOA $t_i$. The statistic $\chi^2$ measures the goodness of fit of the timing model. However, direct application of the least-squares method with all parameters considered to be varied freely fails to determine the parameters $C$ and $D$ because of their smallness. Moreover, the parameter $C$ includes the additional small factor $\cos i = 0.04$, and, for this reason, has been assumed to be equal to zero identically. Then, to extract some valu-
Figure 6. The smooth curve represents values of $\Delta \chi^2$ obtained in data fit with the fixed values of the $\tau$ and $s$ relativistic post-Keplerian parameters obtained by Kaspi et al. (1994). The levels of confidence, corresponding to $1\sigma$, $2\sigma$ and $3\sigma$ determination of parameter $D$ are plotted with dotted lines.

5 CONCLUSION

We have derived the improved timing formula (2) for binary pulsars taking into account the relativistic effect of light deflection in the gravitational field of the pulsar's companion. It has been shown the effect is perceptible only for the pulsars with nearly edge-on orbits. The measurement of the effect in question permits, in principle, the determination of the position angle $\eta$ of the angular velocity vector of the pulsar although it fails to determine direction of the rotation. The influence of the gravitational deflection of light on the measurable information, we investigate the structure of the function $\Delta \chi^2 (e_1) = \chi^2 (e_1) - \chi^2 (e_1^{0 (e)})$ in the neighborhood of the global minimum $\chi^2$ in the multi-dimensional space of the fitted parameters (Taylor & Weisberg, 1989; Ryba and Taylor, 1991; Damour & Taylor, 1992).

We have calculated the functional dependence of $\chi^2$ statistic near the region of its global minimum with the minimization of the $\chi^2$ by the fit of the astrometric $\{N_0, \mu, \delta, \mu_0, \mu_0^\prime\},$ spin $\{\nu_p, \nu_y, \nu_z\}$, orbital Keplerian $\{z = a \sin i, e, T_0, \omega, P_b\}$, and post-Keplerian $\{P_b, \dot{\omega}, \dot{e}\}$ parameters of the pulsar with the parallax $\pi$, dispersion measure $DM$, and post-Keplerian parameters $\tau$ and $s$ held fixed at the unique values reported by Kaspi et al. (1994). In this procedure the parameter $D$ was held fixed at a number of different points from -40 ns to 40 ns. The results of the fitting procedure are shown in Fig. 6.

As one can see from this plot the corresponding curve has a parabolic shape, and the numerical value of the parameter $D = 12^{+10}_{-8}$ nanosecond is not zero with the 68 % confidence ($1\sigma$) limits. The values of other astrometric, spin, Keplerian, and post-Keplerian parameters for the PSR B1855+09 are in a perfect ($1\sigma$) agreement with the values reported by Kaspi et al. (1994). Although we can not say anything with the adopted ($3\sigma$) confidence about the real numerical value of the BD effect, the parabolic structure of the $\chi^2$ near region of its global minimum (with respect to the parameter $D$) indicates that the effect in question can be, perhaps, measured in the future.

ACKNOWLEDGMENTS

We are very grateful to Kaspi V.M., Taylor J.H. and Ryba M.F. for their kind permission to use their observational data on the PSR B1855+09 binary system before publication. We would also like to acknowledge K.Yokoyama (NAO of Japan, Miusawa) who supplied us the polar motion corrections in machine readable form. We are thankful Yu.P.Ilyasov, K.Yokoyama, T.Fukushima, and H.Kinosita for participation in discussions. E.L.Presman made valuable remarks concerning statistical significance of the obtained results and we highly appreciate him. An expression of gratitude is due to Dr M.Holman for polishing the grammar of the manuscript and anonymous referee for helpful comments and suggestions.

S.M.Kopeikin is deeply indebted to the staff of the National Astronomical Observatory of Japan (Mitaka) for the support of this work. O.Doroshenko has been partially supported by the ISF grant M99000.

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