Abstract

The interference of charge changing interactions, weaker than the $V-A$ Standard Model (SM) interaction and having a different Lorentz structure, with that SM interaction, can, in principle, produce effects near the end point of the Tritium beta decay spectrum which are of a different character from those produced by the purely kinematic effect of neutrino mass expected in the simplest extension of the SM. We show that the existence of more than one mass eigenstate can lead to interference effects at the end point that are stronger than those occurring over the entire spectrum. We discuss these effects both for the special case of Dirac neutrinos and the more general case of Majorana neutrinos and show that, for the present precision of the experiments, one formula should suffice to express the interference effects in all cases. Implications for ”sterile” neutrinos are noted.
1 Introduction

One of the outstanding problems of subatomic physics is that of neutrino mass. While this problem is being attacked on many fronts, the most direct approach would seem to be the use of Tritium beta decay to make a direct measurement of the mass of the electron anti-neutrino \[1, 2, 3, 4, 5, 6, 7, 8, 9\]. However, as was known \[10\] long before the advent of the Standard Model (SM), the exact form of the electron spectrum depends on the Lorentz structure of the weak current involved in beta decay. This observation was recently revisited \[11\], wherein it was shown that possible interference between the usual SM \(SU(2)_W\) current and a (weaker) non-SM current having a different Lorentz structure\(^1\) can be exacerbated by the presence of a CKM-like mixing \[13\] in the lepton sector. As we shall show in this paper, such effects could produce a signal that might be interpreted as a negative value of the square of the neutrino mass in an analysis which did not take interference into account. Whatever the eventual resolution of the current experimental situation, our discussion makes clear the fact that experimental analyses should not prejudice the result by assuming solely a SM structure.

The concept of using a detailed measurement of the high energy portion of the electron spectrum from nuclear beta decay to determine the mass of the neutrino was introduced in Fermi’s \[14\] original paper. In that work, he assumed that the interaction was due to a vector current. This gives rise (as we discuss below) to a different dependence on neutrino mass than the \(V - A\) current of the SM, as evidenced qualitatively by the curves shown in his paper. As the possibility of other Lorentz current structures was considered, Koefed-Hansen \[10\] pointed out the need to know the exact nature of the currents to properly extract a neutrino mass. Following the establishment of the dominance of \(V - A\), Enz \[15\] recast the problem in terms of possible interference between that current and other (weaker) Lorentz currents and Jackson, Treiman and Wyld \[16\] extended the discussion to include possible time reversal invariance violation.

These earlier discussions were based on only one neutrino. From LEP data we now know that there is a net of three light neutrinos with full SM coupling

\(^1\)Note that, here and throughout this paper, we are looking at extensions of the SM which are distinguishable at low energies. This does not include additional Left-chiral interactions \[13\].
to the $Z^0$. As we show below, both because the possible interferences depend on the neutrino mass and because Tritium beta decay experiments do not measure the neutrinos, it is appropriate to frame the discussion in terms of mass eigenstates. As this is more easily visualized with Dirac neutrinos, we begin our discussion with them, expanding to the more general and more widely accepted case of Majorana neutrinos in a following section. We then examine the effect that any such interference will have on the extraction of the mass parameters.

In a subsequent section, we present the effects of a possible background scalar field, interacting only with neutrinos \[17, 18\], on the Tritium spectrum. We also examine the possibility that the interference effects could manifest themselves at the upper end of the neutrino spectrum from other nuclear beta decays \[19\]. Following this, we examine the implications of the new interaction currents for neutrino neutral current scattering, which devolve from the necessity of “sterile” neutrino interactions. In the final section we reiterate our conclusions and that experimentalists should use our functional form for data analysis. We emphasize that this form must be used to avoid introducing unwarranted theoretical prejudice into the interpretation of the experimental results.

2 Dirac neutrinos

To present the general concepts that underlie our calculation in the most accessible form, we first do the analysis for Dirac neutrinos. In later sections we shall deal with the more general case of Majorana neutrinos. By dealing with Dirac neutrinos, all of the manipulations to calculate total rates from Feynman diagrams are standard and can be found in any text on relativistic quantum mechanics.

2.1 Mass eigenstates

For the purpose of the discussion in this section, we assume the existence of three massive Dirac neutrinos, $\nu_i^D$, with Dirac masses $m_i^D$. All three masses are less (really much less) than half the mass of the $Z^0$. We allow for the possibility that, as in the quark sector, current eigenstates and mass
eigenstates are not necessarily identical. In particular, we shall refer to the \( \nu_f, f = (e, \mu, \tau) \) as those linear combinations of mass eigenstates which are coupled to the corresponding charged leptons by the \( SU(2)_L \) currents of the SM. These current eigenstates are expressed in terms of the mass eigenstates as

\[
\nu_f = \sum_i \cos \theta_f^i \nu_i
\]

where the \( \cos \theta_f^i \) are the direction cosines in the coordinate system spanned by the mass eigenstates. (If two or more masses are degenerate, we choose an orthogonal set.) Under these assumptions, the current eigenstates also form an orthogonal set. This is depicted in Fig. 1.

If there are additional objects, besides the known massive weak vector bosons, that produce charge changing currents which couple to both the quarks and the leptons, we may define neutral leptons which are current eigenstates of this new interaction in exactly the same way, denoted by \( \hat{\nu}_f, f = (e, \mu, \tau) \), where now

\[
\hat{\nu}_f = \sum_i \cos \hat{\theta}_f^i \nu_i
\]

and one would naturally expect that, in general,

\[
\hat{\theta}_f^i \neq \theta_f^i.
\]

### 2.2 Interaction Hamiltonian

Interactions beyond the SM must be weaker, at low energies, than the usual Left-chiral \( SU(2) \) to avoid serious conflict with existing data. Presumably this is due to the boson mediating the interaction being much heavier than the known W’s and Z, and/or the coupling constant being smaller than that for the SM. Absent a particular Grand Unified Theory in which one wishes to embed the SM and the new interactions, that is all one can say.

For low energy physics, like nuclear \( \beta \)-decay in general and Tritium \( \beta \)-decay in particular, such new interactions can only appear as effective currents in the four fermion formulation of the theory with the usual space time structures of \( S, P, T, V \) or \( A \). Given the dominance of the SM, it is reasonable to recast this as \( S_L, S_R, T, R \) or \( L \), where \( R \sim (V + A) \) and \( L \sim (V - A) \), with a similar construction for \( S_L \) and \( S_R \) from \( S \) and \( P \). The effect on the
spectrum displayed below only occurs if there are additional currents in the
lepton sector that are different from \( L \). Corresponding changes in the hadron
currents affect only the scale of each new contribution. To emphasize this
fact, we employ an unconventional notation; more conventional descriptions
are given, for example, by Enz [15].

The effective low energy Hamiltonian for semi-leptonic decays is

\[
H_I = \sum_{\alpha,\beta=S,L,R,L,T} G^{\alpha\beta} \sum_f (J^\dagger_{h\alpha} \cdot J_{f\beta} + h.c.)
\]  

where, for example,

\[
J_{f\alpha} = \bar{\psi}_f \Gamma_{\alpha} \psi_{\nu_f}
\]  

with \( \psi_f \) representing a charged lepton of a given flavor, \( \psi_{\nu_f} \) a neutral lepton
associated with that charged lepton through the particular interaction (see
the discussion below for more detail). A similar construction can be made
on the hadron side. Note that, in this convention, the first Greek index in
Eq.(1) refers to the hadron current. Explicitly,

\[
\begin{align*}
\Gamma_S &= (1 - \gamma^5) \\
\Gamma_L &= \gamma^\mu (1 + \gamma^5) \\
\Gamma_R &= \gamma^\mu (1 - \gamma^5) \\
\Gamma_T &= [\gamma^\mu, \gamma^\nu] / 2.
\end{align*}
\]  

Most off diagonal combinations \((\alpha \neq \beta)\) in the sum vanish, the exceptions
being for \((S_R, S_L)\) and \((R, L)\). For nuclear beta decay in the SM, only \( \beta = L \)
and \( \alpha = L, R \) survive. The relation to the basic parameters of the SM is
given by [20]

\[
G^{RL} + G^{LL} = V_{ud} \frac{\pi \alpha_W}{\sqrt{2} M_W^2},
\]  

where \( V_{ud} \) is the appropriate element of the hadronic CKM matrix [13], \( \alpha_W \)
is the fine structure constant for the \( SU(2)_W \) of the SM, \( \alpha_W = \frac{g^2_W}{4\pi} \), where \( g_W \)
is the coupling constant, and \( M_W \) is the mass of the usual \( W^\pm \). To complete
the connection with conventional notation, note that

\[
(G^{RL} - G^{LL}) = G_A^L (G^{RL} + G^{LL}).
\]
The reason that the SM $SU(2)_L$ induces an hadronic Right-chiral coupling is that the quarks are confined and their wave functions cause $G_A$ to be renormalized with respect to $G_V$. If there is an $SU(2)_R$, mediated by a heavier vector boson or having weaker coupling constant, or both, one still expects the hadronic current to be modified. However, in parallel to the case of the SM interaction, the leptonic currents would be expected to be pure $V + A$. In fact, one also expects hadronic renormalization effects for $S$, $P$ and $T$. In addition, there will be, in general, a separate “CKM” matrix in the quark sector for each interaction, mirroring the discussion of mass and current eigenstates for neutrinos given above.

The various currents in the effective interaction Hamiltonian may be generated by several different assumptions about physics beyond the SM. The most prominent are the exchange of a charge-changing scalar, arising, for example, in supersymmetric models [21], or the existence of a vector boson coupled to Right-chiral fermions [22] which may or may not mix with the $W$ bosons of the SM. These are shown in Fig. 2, where we have included those cases which may impact Tritium beta decay. Although antisymmetric tensor interactions have been proposed in some (mostly experimental) contexts [23], they would normally be expected to arise in higher-loop order or by Fierz transformation from scalar leptoquark interactions [24] and so be exceptionally weak. In particular, Voloshin [25] has obtained a very stringent bound on the strength of such a tensor interaction ($10^{-4} G_F$), which applies unless a precise orthogonality holds for electron neutrino states defined by the different interactions.

As we will show below, it will be difficult to distinguish amongst the effective currents. Given an effect in Tritium from some such current, it will be even harder to trace that effect to a particular diagram in Fig. 2. However, this does mean that it makes sense to pursue such effects in Tritium beta decay whatever one’s prejudice about a particular source of “new physics”.

In terms of the diagrams of Fig. 2, we may define a new fine structure constant, $\hat{\alpha}$ (to go with $b$) or the first diagram of $c$) as $\hat{\alpha} = \frac{\hat{g}^2}{4\pi}$. Then the ratio of the effective strength of this interaction at low energy to that of the SM $SU(2)_L$ is given by $\hat{\rho}_X = \frac{\hat{g}^2 M_X^2}{\hat{\alpha}^2 M_X^2}$, where $M_X$ is the mass of the non-SM boson being exchanged.
2.3 Interference terms

If there are additional bosons that couple to the Left-chiral Vector fermion current $\mathcal{L}$, they can only renormalize $L$ and cannot lead to any new structure near the end point. For this study of Tritium beta decay all hadronic currents are evaluated in the approximation of no nuclear recoil, which eliminates a pure Pseudo-scalar current.

Nonetheless, we allow for an arbitrary mixture of Scalar and Pseudo-scalar, expressed as Left- and Right-chiral Scalar currents. Both couple hadronically through $S$ and couple independently to the lepton current. Consequently, we need to evaluate the effect on the Tritium beta spectrum of possible interferences between the SM Left-chiral Vector current and a Right-chiral Vector current, a Right-chiral Scalar current and a Left-chiral Scalar current. In this work, a Left-chiral Scalar current is defined as that Scalar current for which the Left-chiral projector acts on the neutrino field for the current defined as in Eq.(3), and a corresponding definition for the Right-chiral case.

Since the hadronic currents must be renormalized and the fitting procedure for any Tritium beta decay experiment fits the overall rate, we only quote here the relative factors from the lepton traces. In doing so, we make use of the fact that, when contracted with the hadron traces and integrated over the outgoing neutrino directions, the only terms that survive are those proportional to factors of the hadron mass or energy, which differ negligibly due to the small size of the momentum transferred to the final state hadrons ("no recoil" approximation):

$$
\begin{align*}
LL(SM) &= E_\nu E_\beta \\
RR &= E_\nu E_\beta \\
LR + RL &= -2m_\nu m_e \\
S_L S_L &= E_\nu E_\beta \\
S_R S_R &= E_\nu E_\beta \\
S_L S_R + S_R S_L &= -2m_\nu m_e \\
TT &= [E_\nu E_\beta - m_\nu m_e] \\
LS_R + S_R L &= -2m_\nu E_\beta \\
LS_L + S_L L &= 2E_\nu m_e \\
LT + TL &= 2[E_\nu m_e - m_\nu E_\beta]
\end{align*}
$$

The negative sign of the terms with a factor of $m_\nu$ arises from the fact that the neutral lepton in Tritium beta decay is an anti-neutrino.
For the interactions involving L (and by symmetry, R) currents, the Lorentz inner product of the hadron trace with the lepton trace produces an overall factor of \((G^2_V + 3G^2_A)\), and this same factor occurs for the interference terms between the \(W_L\) and \(W_R\) exchanges.

The case of \(W_L - W_R\) mixing is more complicated. It must include a factor like this for the interference term with the SM in which the \(W_R\) couples directly to both leptons and hadrons as above, but is more complicated and also involves other combinations, such as \((G^2_V - 3G^2_A)\) for the term involving the interference between the SM and the amplitude in which the mixed propagator couples via \(W_L\) to the hadrons and \(W_R\) to the leptons.

The corresponding result for the interference terms of a scalar current with the SM currents is proportional to \(G^2_V\) alone. This means that the scalar interactions are somewhat less efficient at producing new effects for the same relative strength as Right-chiral interactions.

Note that all interference effects of tensor interactions with the SM (which effects are proportional to factors of \(3G^2_A\)) are encompassed by the scalar terms and so we will not explicitly discuss tensor contributions further in this paper.

For our main purpose here, we need only consider the possibilities of interference between the SM current and either a Right-chiral Vector current (for Dirac neutrinos) or a Right-chiral Scalar current for either Dirac or Majorana neutrinos. (The effect of interference with the Left-chiral Scalar current appears in Sec.6 and an additional tensor interaction is just a linear combination of the Left-chiral Scalar and Right-chiral Scalar terms.)

### 2.4 Pre-existing limits

The best limits on the relative strengths of additional currents often make use of interference effects \([20]\), and are deduced assuming that the same neutrino is produced with the electron in all cases. As we discussed above, this is not a necessarily valid assumption. In fact, the interference effects need to be examined for each mass eigenstate and, depending upon the details of the interaction, could vanish over most of the spectrum even if the effective coupling does not.

As an example, consider the possible effect on the spectrum due to the inter-
ference between a Left-chiral Scalar and the SM Left-chiral interaction. As shown in the previous section, for each mass eigenstate $\nu_i$, this interference is proportional to $\hat{\rho}_{SL} E_\nu m_e \cos \theta_e^i \cos \theta^i e$. For most applications, the neutrino masses are negligible compared to $E_\nu$, so $q_\nu \equiv E_\nu$ and thresholds may be ignored. This means that, when one sums over the mass eigenstates, the result is proportional to $\hat{\rho}_{SL} \times \cos(x)$ where $x$ is the angle between $\bar{\nu}_e$ and $\nu_e$, not simply to $\hat{\rho}_{SL}$.

Recent limits [26] are given as
\[
\hat{\rho}_R \leq .07 \\
\hat{\rho}_{SR} \leq .1 \\
\hat{\rho}_{SL} \leq .01
\]

where the analysis has implicitly assumed unit value for $\cos(x)$.

3 Majorana neutrinos

In this section we discuss the more general possibility, arising from the lack of any conserved charge for the neutrinos (at least at the low energy scales below one-half the mass of the $Z_0$) that the mass eigenstates correspond to Majorana neutrinos.

3.1 Massive Weyl neutrinos

We follow the development presented by Ramond [27]. The basic object is a two complex component Weyl spinor which can be represented under the Lorentz group in either of two inequivalent irreducible representations, labelled conventionally as $(1/2, 0)$ and $(0, 1/2)$. These two representations of the same Weyl field are isomorphic; the isomorphism connecting them is the operation of Charge conjugation. The usual Dirac spinor in the Weyl (or chiral) representation [28] is constructed from one Weyl spinor in the $(1/2, 0)$ representation and an independent Weyl spinor in the $(0, 1/2)$ representation having the correct charges under appropriate interactions. A (Dirac) mass term coupling such representations allows for a common global or local phase and, in particular, allows for a Noether current that guarantees the conservation of particle number.
In this same representation, the four component representation of a Majorana spinor is constructed by taking, for the Weyl spinor in the $(0, 1/2)$ representation, plus or minus times the Charge conjugate of the Weyl spinor in the $(1/2, 0)$ representation. This produces even or odd Charge conjugation eigenstates, each of which contains exactly the information contained in the original Weyl spinor. The same procedure as used for a Dirac mass term now produces a Majorana mass term, only now, by construction, the two pieces of the spinor have conjugate behavior under a global or a local phase and one cannot construct a Noether current related to fermion number conservation. Also, the interaction charges are opposite in the two parts of the spinor, so this construction is not appropriate for any fermion with a conserved charge. Since both Left chiral $(1/2, 0)$ and Right chiral $(0, 1/2)$ representations are present, this Majorana spinor is adequate to describe all of the SM weak interactions and, if the mass is non-zero, can mediate neutrinoless double beta decay. \[29\]

### 3.2 Pure Weyl or “see-saw” neutrinos

Appealing only to the SM without extensions, one need only consider three Weyl neutrino fields associated with three (Majorana) mass eigenstates, possibly rotated from the current eigenstates. By convention, these masses are labelled $m_L$. With respect to Tritium beta decay, there can be no interference with Right-chiral Vector currents or with Right-chiral Scalar currents. On the other hand, interference with the Left-chiral Scalar current is possible, but, since that current is the most constrained by other data, the possibility of describing the data is less viable.

There is another nagging problem with pure, massive Weyl neutrinos. This has to do with the possibility, inherent in the SM, that the $SU(2)_L$ symmetry could be restored at high temperature. In this scenario, the SM Higgs loses its vacuum expectation value ($vev$), all four weak bosons become massless and the weak charge is conserved. The last statement is contradicted by the existence of the Majorana mass, which is certainly not connected to the standard Higgs.

To achieve such a mass, one generally introduces a new scalar field, the Majoron \[30\], the sole purpose of which is generate a Majorana neutrino mass via the $vev$ of the Majoron field. As this $vev$ must be small to avoid
distortion of the ratio of $W$ and $Z$ masses beyond experimental constraints, it may also be expected to evaporate as well when the $SU(2)_L$ symmetry is restored, although this is not guaranteed.

At some level, this is addressed by the usual see-saw \cite{31}. In that case one builds two distinct Majorana neutrinos, one from the Weyl field described above and one from a Weyl field that could be used with the first to build Dirac neutrinos. One then assumes that the usual Higgs produces some Dirac masses ($m_D$) (presumably of the order of charged lepton masses, although that is not critical) and that the new Weyl field, which is sterile to all known interactions, develops a Majorana mass through some means that does not affect the SM. That mass is conventionally termed $M_R$, and is actually a $3 \times 3$ (or larger, depending on the full model) matrix in flavor space.

This has the well known pleasant consequence that the masses of the light (active) neutrinos are of the order of $\frac{m_D^2}{M_R}$, leading to very small neutrino masses. However, it should clearly be noted that there is no principle leading to this construction. There is also no guarantee that the rank of $M_R$ is three, so there could be vastly different patterns \cite{32} in different generations. In this description (assuming rank three), the three light neutrinos are essentially indistinguishable from the pure Weyl case insofar as Tritium beta decay is concerned. However, since $m_D$ is assumed to be proportional to the Higgs vacuum expectation value, the problem with symmetry restoration is avoided. We do note, however, that $L-R$ symmetric models with a (relatively) low lying Right-chiral scale face the same problem of symmetry restoration.

### 3.3 Pseudo-Dirac neutrinos

For many years there has been discussion of the possibilities associated with a very small Majorana mass, either compared with Dirac masses or masses only off-diagonal in flavor space, the so-called “Pseudo-Dirac” case \cite{33,34,35}. In this case, which has recently been revisited in the literature \cite{36,37}, for Tritium beta decay there is really no distinction from the pure Dirac case. Any time evolution will be so slow that the implications for the spectra considered in this paper are negligible.
4 Implications for Tritium beta decay

We now examine the effect of the possibilities we have discussed on the analysis of the beta spectra in Tritium beta decay. We recognize that no attempt should be made to extract any parameters from the published literature, as proper analysis requires inclusion of specific experimental details. What we shall show is that, for parameter ranges which are not in conflict with other experimental results, the combination of non-vanishing neutrino masses and an interference at very low neutrino energy can cause an analysis, which assumes no interference, to produce negative values of $m^2_\nu$. Furthermore, there is a dependence of the extracted value on the range of $\beta$ energies used for the fit (for differential spectra) or the $\beta$ bias energy (for integral spectra) which dependence matches that reported by some experiments [2, 7, 8, 9].

4.1 Differential spectra

Our discussion in this paper is directed at those experiments using molecular Tritium as a source. Other sources require well known modifications to the discussion which are no different for our case than for the SM.

For the SM only, the differential spectrum, $\frac{dN}{dE_\beta}$, is already a sum of several individual spectra for each possible end point corresponding to a particular final energy of the $^3He-T$ molecular ionic system. Strictly speaking, the sum should include an integral over the continuum of ionic breakup states, although the analysis may find that it is adequate to represent that continuum with a finite sum. This may be expressed as

$$\left(\frac{dN}{dE_\beta}\right)_{SM} = \sum_i P_i \left(\frac{dN}{dE_\beta}(E_{i0})\right)_{SM}$$ (6)

where $P_i$ is the probability of leaving the ionic system in the $i^{th}$ final state and $E_{i0}$ is the maximum energy available to the $\beta$ for that final state, assuming that $m_\nu = 0$. At present, the $P_i$ are calculated [8] and current experiments estimate that the theoretical uncertainty which this introduces constitutes a small part of the error budget. The end point of the spectrum (for $m_\nu = 0$) is then $E_{i0}$ where $i = 0$ denotes the ground state of the molecular ion. Of course, the expression in Eq.(6) is theoretical only. In fitting to a particular experimental spectrum, any energy dependent experimental corrections must
either be included explicitly or allowed to vary, within reason, in the fitting procedure.

In this paper we shall illustrate the effects of possible interferences on the theoretical spectrum appropriate to $E_0^0$. The extension to the full spectrum follows Eq. (6). For zero mass neutrinos, where the SM weak current eigenstate may be taken as the only neutrino produced, we have

$$\left( \frac{dN}{dE_\beta}(E_0^0) \right)_{SM} = KF(E_\beta)q_\beta q_\nu E_\beta E_\nu$$

$$= KF(E_\beta)q_\beta E_\beta (E_0^0 - E_\beta)^2,$$  \hspace{1cm} (7)

where $K$ includes all those quantities, such as the nuclear matrix element, the source strength and the overall experimental efficiency, subsumed by the measured normalization, $F(E_\beta)$ is the Fermi function, $q_\beta$ and $E_\beta$ are the momentum and relativistic energy of the $\beta$ and $q_\nu$ and $E_\nu$ are the momentum and relativistic energy of the antineutrino. The second line reflects the assumptions that $E_\nu = E_0^0 - E_\beta$ and that $q_\nu = E_\nu$ for a massless neutrino.

The simplest extension to the SM consists of assuming that there are no additional interactions, that there is still only one relevant neutrino (strictly, $\bar{\nu}_e$), but that this neutrino may have a mass. In this case we make the replacement $q_\nu = \sqrt{E_\nu^2 - m_\nu^2}$ and fit

$$\left( \frac{dN}{dE_\beta} \right)_1 = \sum_i P_i \left( \frac{dN}{dE_\beta}(E_0^i) \right)_1$$

where, for example,

$$\left( \frac{dN}{dE_\beta}(E_0^0) \right)_1 = KF(E_\beta)q_\beta E_\beta E_\nu \sqrt{E_\nu^2 - m_\nu^2} \Theta(E_0^0 - E_\beta - m_\nu)$$

$$= KF(E_\beta)q_\beta E_\beta (E_0^0 - E_\beta)^2 \left[ 1 - \frac{m_\nu^2}{(E_0^0 - E_\beta)^2} \right]^{1/2} \Theta(E_0^0 - E_\beta - m_\nu),$$  \hspace{1cm} (9)

treating $m_\nu^2$ as an additional parameter. $\Theta(z)$ is the Heavyside function. As is well known, the reported values of $m_\nu^2 < 0$ are an indication of an excess of counts near the endpoint over the expectation for zero mass, with all other parameters determined from the robust spectrum far below the end point.
The first obvious extension is to include the possibility of different mass eigenstates, with the SM current eigenstates being mixtures of those mass eigenstates denoted by $\nu_k$,

$$\bar{\nu}_e = \sum_k \cos \theta^k \bar{\nu}_k$$

(10)

giving

$$\left( \frac{dN}{dE_\beta(E^0_0)} \right)_2 = K F(E_\beta) q_3 E_\beta (E^0_0 - E_\beta)^2 \sum_k \cos^2 \theta^k \sqrt{1 - \frac{m_k^2}{(E^0_0 - E_\beta)^2}} \times \Theta(E^0_0 - E_\beta - m_k),$$

(11)

which does not improve the fit near the endpoint [3].

The next extension, which is the point of this paper, is to include the possibility of interference with additional interactions which appear, at the low energies associated with nuclear beta decay, as currents with a different character under Lorentz transformations. As stated in Sec.2.3, we confine ourselves to Scalar currents, which can play a role for any neutrinos, and Right-chiral Vector currents which affect only Dirac and pseudo-Dirac neutrinos.

Such currents will produce direct effects on the rate, proportional to $\hat{\rho}_X^2$ as well as interference terms proportional to $\hat{\rho}_X$. The former will be hard to observe and, for Tritium beta decay, will be absorbed into normalization factors, but are included here for completeness.

In subsections 2.1 and 2.2 and in Eq.(10) above, we used the notation $\cos \hat{\theta}^i_f$ to refer to the direction cosines for new current eigenstates coupled to the charged lepton $f$ relative to the mass eigenstate labelled by $i$. In this case, the only flavor eigenstate considered is $e$ and we want to consider different possible currents, so we change the notation to read $\cos \theta^i_{iX}$ where $i$ refers to the mass eigenstate and $X$ refers to the current $R$, $S_R$ or $S_L$ (and is omitted for the SM current). We further define $\rho_X = \hat{\rho}_X (ME)_X$ where $(ME)_X$ accounts for the ratio of the hadronic matrix element of the designated current relative to that of the SM in the particular nuclear system, here $A = 3$.

This includes the (quark) CKM matrix elements appropriate to that non-SM interaction.

At the risk of being overly pedantic, we present the discussion for both of the cases under consideration first separately and then combine them. For a
Right-chiral Vector current (without L-R mixing), we make the substitution
\[
\cos^2 \theta_i E_\beta E_\nu \rightarrow \cos^2 \theta_i E_\beta E_\nu + \cos^2 \theta_i R \rho_R^2 E_\beta E_\nu - 2 \cos \theta_i \cos \theta_i R \rho_R m_\nu m_i.
\]
Defining
\[
\epsilon_{iR} = \rho_R \frac{\cos \theta_i R}{\cos \theta_i}
\]
we may recombine this as
\[
\cos^2 \theta_i E_\beta E_\nu \left( 1 - 2 \epsilon_{iR} \frac{m_\nu m_i}{E_\beta E_\nu} + \epsilon_{iR}^2 \right).
\]
We remind the reader that the sign of the middle term arises from the fact that the neutral lepton in Tritium beta decay, in terms of Dirac fields, is an anti-neutrino. Since \(\epsilon_{iR}\) contains the ratio of direction cosines, it also may have either sign. This allows for positive interference near the end point. Similar remarks apply throughout this section.

Turning now to the other case of interest to us here, of a Right-chiral Scalar interaction (as defined in Secs. 2.2, 2.3), we make the substitution
\[
\cos \theta_i^2 E_\beta E_\nu \rightarrow \cos \theta_i^2 E_\beta E_\nu + \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right)
\times \left[ \cos^2 \theta_i S_R \rho_{S_R}^2 E_\beta E_\nu - 2 \cos \theta_i \cos \theta_i S_R \rho_{S_R} E_\beta m_i \right]
\]
And defining
\[
\epsilon_{iS_R} = \rho_{S_R} \frac{\cos \theta_i S_R}{\cos \theta_i}
\]
we obtain the expression
\[
\cos^2 \theta_i E_\beta E_\nu \left[ 1 + \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right) \left( -2 \epsilon_{iS_R} \frac{m_i}{E_\nu} + \epsilon_{iS_R}^2 \right) \right].
\]
Finally, combining both of these possibilities (and keeping in mind that any tensor interaction contributions are encompassed by scalar terms), we obtain
\[
\left( \frac{dN}{dE_\beta} \right) = \sum_i P_i \left( \frac{dN}{dE_\beta} (E_\beta^i) \right)
\]
with
\[
\left(\frac{dN}{dE_\beta}(E_0^i)\right) = KF(E_\beta)q_\beta(E_0^i - E_\beta) \sum_k \Theta(E_0^i - E_\beta - m_k)
\]
\[
\times E_\beta(E_0^i - E_\beta) \sqrt{1 - \frac{m_k^2}{(E_0^i - E_\beta)^2}}
\]
\[
\times \left[ \cos^2 \theta_k + \cos^2 \theta_k \rho^2_R + \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right) \cos^2 \theta_k \rho^2_S \right]
\]
\[
-2m_e m_k \left[ \cos \theta_k \cos \theta_k \rho_R \right]
\]
\[
-2m_k E_\beta \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right) \left[ \cos \theta_k \cos \theta_k \rho_S \right].
\] (13)

While this expression has everything in it, it is not particularly useful for fitting experimental data. We can cast it into a more useful form by noticing that, near the end point, the dependence on $E_\beta$ is very gentle and one is really interested in the dependence on $E_\nu = (E_0^i - E_\beta)$. Furthermore, the product $F(E_\beta)q_\beta$ is nearly constant for $\beta$ energies near the end point. Note also that the ratio $\frac{m_e}{E_\beta}$ varies by less than 2 parts in $10^3$ as the kinetic energy of the $\beta$ varies from 17.5 keV to 18.5 keV, a much wider range than is used in modern fits \[3,4\]. These observations suggest the following approximations.

First, absorb $F(E_\beta)q_\beta E_\beta$ into a new “constant”

\[ K' = KF(E_\beta)q_\beta E_\beta. \]

Second, for some average value of $E_\beta$, take $\frac{m_e}{E_\beta} = \frac{m_e}{<E_\beta>}$ as constant\[4\] and define

\[ \varepsilon_k = \epsilon_{kR}^2 + \epsilon_{kS}^2 \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right) \]

and

\[ \phi_k = -2m_k \left( \frac{m_e}{1 + \varepsilon_k} \right) \left[ \frac{m_e}{<E_\beta>} \epsilon_{kR} + \epsilon_{kS} \left( \frac{G_V^2}{G_V^2 + 3G_A^2} \right) \right]. \]

\[ ^2\text{Since it is buried in a parameter to be fit, the actual value is not important.} \]
Then
\[
\frac{dN}{dE_\beta}(E_0^i) \approx K' \sum_k \cos^2 \theta_k (E_0^i - E_\beta)^2 (1 + \varepsilon_k) \left[ 1 + \frac{\phi_k}{(E_0^i - E_\beta)} \right] \\
\times \left[ 1 - \frac{m_k^2}{(E_0^i - E_\beta)^2} \Theta(E_0^i - E_\beta - m_k) \right] (1 + \varepsilon_k) \left[ 1 + \frac{\phi_k}{(E_0^i - E_\beta)} \right] .
\]
(14)

4.2 Integral spectra

While many of the experiments performed over the last few decades are differential measurements [1, 2, 3], the two ongoing experiments are integral measurements which accept all \(\beta\)s with energy above some cutoff energy [4, 5, 6, 7, 8, 9]. To obtain the theoretical form for the expected count rate for a given set of parameters, we should integrate Eq.(13) over \(E_\beta\) from \(E_C^\beta\) to \(\infty\). This daunting prospect requires numerical treatments. However, the form given in Eq.(14) is both a very good approximation and amenable to analytic integration. The integral needs to be evaluated for each endpoint energy \(E_0^i\) and for each mass eigenvalue \(m_k\). Let us explicate the procedure for \(E_0^i\) and one mass, \(m_k\).

We need to evaluate
\[
K' \cos^2 \theta_k \int_{E_C^\nu}^{\infty} dE_\beta (E_0^i - E_\beta)^2 \left[ 1 - \frac{m_k^2}{(E_0^i - E_\beta)^2} \Theta(E_0^i - E_\beta - m_k) \right] \\
\times (1 + \varepsilon_k) \left[ 1 + \frac{\phi_k}{(E_0^i - E_\beta)} \right] .
\]

Changing variables to \(E_\nu = (E_0^i - E_\beta)\) and defining \(E_C^\nu = (E_0^i - E_C^\beta)\), we get
\[
N(E_C^\nu) = K' \cos^2 \theta_k \int_{m_k}^{E_C^\nu} dE_\nu \sqrt{\frac{m_k^2}{E_\nu^2} E_\nu^2 (1 + \varepsilon_k)} \left( 1 + \frac{\phi_k}{E_\nu} \right) \\
= K' \cos^2 \theta_k (1 + \varepsilon_k) \left[ \frac{1}{3} ((E_C^\nu)^2 - m_k^2)^{3/2} \right. \\
\left. + \frac{\phi_k}{2} \left( E_C^\nu \sqrt{(E_C^\nu)^2 - m_k^2} - m_k^2 \ln \left( \frac{E_C^\nu + \sqrt{(E_C^\nu)^2 - m_k^2}}{m_k} \right) \right) \right] (15)
\]
4.3 Fitting fitted differential spectra

We emphasize, in this subsection and the next, that we attempt neither to fit data nor to extract reliable values of parameters, as that must be done with full knowledge of all experimental details. What we shall do is treat the published values of $< m^2 >_{\text{fit}}$ as a representation of the data. We then ask what parameters will produce similar values of $< m^2 >_{\text{fit}}$ if a fit were to be made, using only the SM expression, to our theoretical spectrum which includes interference effects. The purpose of doing that, in this paper, is to demonstrate that the interference terms apparently do allow a representation of the experimental data.

Furthermore, to simplify both the discussion and our task, we assume that the various experimental groups have correctly performed the weighted sums over the final states of the molecular ionic system. Therefore, we need only study the effects of interference compared with the SM analysis on one branch, which we shall take to be the ground state branch. In fact, the entire analysis of these next two subsections goes through unchanged for other branches, but the equations become needlessly cumbersome for simply the demonstration of our point.

Let us now further assume for simplicity that only one mass eigenstate is important for the fit near the end point. Let that mass eigenvalue be denoted as $m_1$.

Since, for Tritium beta decay, the range of $E_\beta$ is

$$m_e \leq E_\beta \leq 1.035m_e$$

one will not be able to distinguish between interference terms involving $E_\beta$ and $m_e$. On the other hand, since we are interested in effects where $E_\nu$ varies significantly compared to $m_\nu$, those terms will have very different effects on the spectrum.

Now scale $E_\nu$ and the $\phi_i$ in units of $m_1$,

$$x = E_\nu/m_1$$
$$f_i = \phi_i/m_1$$

(16)

In this case, defining $Y_1 = \cos^2 \theta_1 (1 + \varepsilon_1)$, the differential spectrum becomes

$$\frac{dN}{dE_\beta} = K'Y_1 x^2 (1 + \frac{f_1}{x}) \sqrt{1 - \frac{1}{x^2}} \Theta(x - 1).$$

(17)
If we were to fit this with a formula for the spectrum which is derived under the assumption that there is only one neutrino and that it has a mass extracted from the spectrum as \( < m^2 >_{\text{fit}} \), we would fit Eq. (17) with the function

\[
\frac{dN}{dE_\beta} = K' Y_T x^2 \sqrt{1 - \frac{< r^2 >_{\text{fit}}}{x^2}}
\]

(18)

where \( < m^2 >_{\text{fit}} = m_1^2 \) is the extracted (apparent) value of the neutrino mass-squared, and \( Y_T = \sum_k \cos^2 \theta_k (1 + \varepsilon_k) \). The reason that \( Y_T \) appears is that we assume that the experimental normalization of the data is taken from a region of lower electron energies where neutrino mass effects are presumed to be negligible and all mass eigenstates (and currents) contribute.

Setting Eqs. (17) and (18) equal, at some particular value of \( x \), gives an equation for \( < r^2 >_{\text{fit}} \),

\[
< r^2 >_{\text{fit}} = x^2 [1 - \zeta^4] - 2x f_1 \zeta^4 + [1 - f_1^2] \zeta^4 + 2x^{-1} f_1 \zeta^4 + x^{-2} f_1^2 \zeta^4 \]

(19)

where we have defined \( \zeta^2 = Y_1/Y_T \) for convenience. At \( x = 1 \), precisely at the end point of the physical spectrum, Eq. (19) gives the result \( < r^2 >_{\text{fit}} = 1 \).

It is instructive to consider the case in which \( \zeta^2 = 1 \) and the first term in Eq. (19) vanishes. (This can occur, for example, if \( \bar{\nu}_e \) is a mass eigenstate so that \( \cos \theta_1 = 1 \) and \( \cos \theta_2 = \cos \theta_3 = 0 \).) In this case, no matter how small the interference is, the fit value of \( m^2 \) will eventually become negative far enough away from the end point, assuming all other quantities were perfectly assigned. In fact, this could mean that a value of the mass much less than an eV could affect the fit to the spectrum several eV below the end point.

As we remarked above when discussing pseudo-Dirac neutrinos, a very small splitting, as suggested by the solar neutrino problem, would not appear in this discussion. Other possible flavor mixings would destroy this special condition. Existing reports from accelerator measurements \([39]\) and reactor experiments \([40]\) limit the size of such mixing, but allow it to be non-zero.

If \( \bar{\nu}_e \) is nearly a mass eigenstate, then the second term in Eq. (19) can still dominate, leading to \( < r^2 >_{\text{fit}} < 0 \) for some values of \( x \). The value of \( x \) at which \( < r^2 >_{\text{fit}} \) again becomes positive is a sensitive function of \( f_1 \) and \( \zeta^4 \). For example, taking \( \zeta = 0.9996 \), which corresponds to a minimum value of \( \sin^2(2\theta_1) \simeq 0.003 \) (obtained by setting \( \zeta = \cos \theta_1 \), i.e., ignoring possible corrections due to the small, but generally non-negligible values of the \( \varepsilon_k \).
in a two flavor mixing scenario, and \( f_1 = 0.075 \), which is within the limit on \( X_{SR} \) \cite{20}, \(< r^2 >_{f_{fit}} \) reaches -2.5 at \( x = 45 \) and turns positive a bit beyond \( x = 85 \). In such a case, the observed structure at a given point in the spectrum reflects a mass on a much smaller scale.

On the other hand, if the quantity \( \zeta^4 \) is small compared to 1, \(< r^2 >_{f_{fit}} \) will grow as \( x \) increases unless \( f_1 \) is so large that \((2f_1 + f_2^2)\zeta^4 > (1 - \zeta^4)\), so that the derivative with respect to \( x \) is negative at \( x = 1 \). (In fact, for \( f_1 = 1 \), analysis shows that \( \zeta^4 \) must be greater than \((16/27)\) to get a negative value of \(< r^2 >_{f_{fit}} \) for any value of \( x \).) For this case to be interesting, a second mass eigenstate must be involved with the interference term being destructive, so that, far from the end point, the interference cancels. Note that, for the cases at hand, the interference is proportional to the mass so that the more massive eigenstate can have a smaller admixture.

Since the modern integral experiments \cite{4, 5, 6, 7, 8, 9} provide the best data available, we defer numerical examples to the next subsection.

4.4 Fitting fitted integral spectra

To obtain sufficient statistics, experiments that measure differential spectra make a global fit to data over some range from the endpoint up to some value of \( E_\nu \), which translates, in practice, to fitting over a range of \( E_\beta \) down to some cut-off value. This is done automatically in those experiments that measure an integral spectrum.

For fitting to sums of differential spectra, the fitting procedure, viewed in terms of theoretical constructs only and ignoring essential experimental details, corresponds to finding the value of \(< r^2 >_{f_{fit}} \) that minimizes the integral

\[
I = \int_1^{x_c} dxx^4 \left[ Y_1(1 + \frac{f_1}{x})\sqrt{1 - \frac{1}{x^2}} - Y_T\sqrt{1 - \frac{< r^2 >_{f_{fit}}}{x^2}} \right]^2 \tag{20}
\]

where, as in the previous subsection, we have assumed, for simplicity of presentation, that one mass eigenstate with mass \( m_1 \) is important, \( x, < r^2 >_{f_{fit}}, \) and \( f_1 \) are as defined previously and \( x_c = E_\nu^C/m_1 \).

This was the form presented in Ref.\cite{11} and was appropriate for the earlier differential experiments \cite{4, 5, 6}. However, for the analysis of the integral
experiments, one wants to use Eq. (15). The appropriately scaled version is

\[ N(x_c) = K'Y_1 \left( \frac{1}{3}(x_c^2 - 1)^{3/2} + \frac{f_1}{2} \left[ x_c \sqrt{x_c^2 - 1} - \ln(x_c + \sqrt{x_c^2 - 1}) \right] \right). \]  

(21)

Fitting this with the usual SM formula, where \( K'Y_T \) is taken from the fit far from the end point (experimental normalization) and only one mass eigenstate is included, corresponds to equating

\[ N(x_c) = K'Y_T \int_{x_c}^{x_c} dxx^2 \sqrt{1 - \frac{<r^2>_{fit}}{x^2}} = K'Y_T \frac{1}{3} \left[ ((x_c^2 - <r^2>_{fit})^{3/2} - (1 - <r^2>_{fit})^{3/2} \right] \]  

(22)

to the same expression in Eq. (21). This procedure gives an implicit equation for \(<r^2>_{fit}\). In fact the lower limit in Eq. (22) makes a very small contribution for most values of \( x_c \) of interest, so that, to a good approximation, one may drop that term and obtain an explicit equation for \(<r^2>_{fit}\).

We used that equation to search through parameter space to find sets that gave a reasonable representation of the latest Mainz data [9]. We emphasize again that we have not constructed a fit, even to the representation in terms of \(<m^2>_{fit}\), but rather have selected a set of parameters that give a reasonable representation of the published results. We have done this to demonstrate that the inclusion of possible interference effects of non-SM currents shows no obvious evidence of being at odds with the data.

To that end we offer a comparison in Figs. 3-6. The data points were read from a reproduction of Fig. 3 of a preprint of Ref. ([9]) and should not be taken as a precisely accurate representation. In our Fig. 3, open circles correspond to the Q2 data set of their Fig. 3; in our Fig. 4, open squares correspond to their Q3 data set; in our Fig. 5, open diamonds correspond to their Q4 data set and in our Fig. 6, asterisks correspond to their Q5 data set. No attempt was made to generate fits; rather the parameter sets were chosen to give a maximum negative \(<m^2>_{fit}\) about 30eV below the endpoint, \(E_0\), and to turn \(<m^2>_{fit}\) positive at \(E_\beta \approx 18350 eV\). In the figures, the various calculated curves are labelled by the value of \(f_1\). In Table I, we list

3The experimentalists report substantial consistency of their results [6, 9]; we have simply found it more convenient to make use of the Mainz presentation of their data.
the full parameter set for each curve, where the parameters are $f_1$, $m_1$ and $\zeta^2$. The final row, labelled $\sin^2 2\theta$, is the minimum value one would deduce for that quantity in a two component oscillation formula, obtained by taking the value of $\zeta^2$ to equal $\cos^2 \theta_1$, i.e., treating the $\varepsilon_k$ as negligibly small.

Table I. Interference parameters for theoretical curves plotted in Figs. 3-6.

| $f_1$ | .05 | .07 | .09 | .11 | .13 |
|------|-----|-----|-----|-----|-----|
| $m_1$(eV) | 1.603 | 2.228 | 2.888 | 3.431 | 4.334 |
| $\zeta^2$ | .99954 | .99910 | .99850 | .99783 | .99675 |
| $\sin^2 2\theta$ | .00185 | .00360 | .00600 | .00870 | .01300 |

Since the authors of [9] chose not to combine these data sets, for reasons they describe, we also do not attempt to combine them. In fact, as we discuss in the next section, there are theoretical reasons to avoid combining runs taken at different times. In spite of that restriction, we believe that Figs. 3-6 make the point that, for values of the parameters which are within reason, the possibility of interference between the SM current and a weaker, non-SM current can give the observed negative values of $<m^2>_{fit}$ when the data is analyzed under the assumption that only the SM current is present.

4.5 Discussion

As we showed in Sec.(4.4), it is possible to generate values of $<m^2>_{fit}$ that resemble the published data for values of $f_1$ between .05 and .13. The lower end of this range is easily accommodated by existing limits, even without invoking the possibility, discussed in Sec.(2.4) that these limits are only appropriate for a sum over the mass eigenstates. The upper end of the range may be accommodated by invoking this last point or by noting that more than one non-SM current may be affecting the Tritium data in combination.

While we have not carried out an exhaustive analysis of the allowed parameter space, it is worth noting that, by studying Eq.(19), it is possible to get some sense for the allowed parameter ranges. For example, requiring $<m^2>_{fit} = -10eV^2$ at $E_\nu = 100eV$ relates $m_1$ to $f_1$. In particular, if $f_1 < 0.11$, then $1eV < m_1 < 10eV$. The lower limit comes from the obvious effect that the strength is $m_1 f_1$ and the upper limit from the fact that the kinematic effect
of \( q_\nu \) begins to close the phase space. We do not mean to emphasize \( f_1 = 0.11 \) particularly; our point is that the present results strongly suggest that fitting to data using our functional form will give \( m_1 \) near this approximate range.

The required mass eigenvalues, on the other hand, would appear to be in conflict with recent limits from double beta decay \[11\]. However, those limits are on the expectation of the Majorana mass and, as Wolfenstein pointed out \[12\], for a pseudo-Dirac neutrino it is only the difference between the mass eigenstates that is limited by bounds from neutrinoless double beta decay experiments. Taken together with the tiny \( \Delta m^2 \) inferred from solar neutrino studies \[13\], the values of \( m_1 \) (in the few \( eV \) range) that appear in this analysis strongly imply that the electron neutrino is either a Dirac particle or a pseudo-Dirac particle. \[36, 37\] The fact that \( \theta_1 \) is very small (inferred from the small value of \( 1 - \zeta^2 \)) in all of the preferred parameter sets is consistent with a small amount of flavor mixing, but the fact that it cannot be zero also indicates that some flavor mixing is required.

We note here that, if this scenario obtains, then there do exist additional neutrino degrees of freedom beyond those active in SM interactions. Such degrees of freedom are usually termed “sterile”; however, our analysis of Tritium beta decay clearly implies that these “sterile” neutrinos necessarily participate in new interactions beyond the SM. In Sec (7) we discuss other implications of this observation.

## 5 Environmental Effects

We \[17\] have previously studied the consequences that may arise if there is a very light scalar particle coupled only to neutrinos.\[4\] The primary effect is that, wherever in space the classical scalar field arising from such interactions is non-zero, the effective mass of the neutrino is altered from its vacuum mass (the mass a neutrino would have in a region of space devoid of other neutrinos, but including all field theoretic contributions). It is this effective mass at the site of a Tritium beta decay event which will govern the interference effects discussed above.

A second result reported in Ref.\((17)\) is that, for a wide range of parameters

\[4\] X.-G. He et. al. \[44\] have shown that such a scenario is possible in extended gauge theories.
which are not in conflict with known data, neutrinos will form clouds during an early stage in the development of the Universe. The extent and density of such clouds will depend on the details of the neutrino vacuum masses, the mass of the scalar and the size of the coupling between the scalar and neutrinos. However, there is no known impediment to considering a length scale between a solar radius and the size of a solar system, with the range of the scalar field being somewhat smaller. If the scale of the particular cloud in which the Solar system developed is on the order of the Earth’s orbit, it would be possible that the strength of the scalar field sampled at the Earth would vary with the Earth’s position along its orbit, leading to a time dependence of the effective mass.

Alternatively, the cloud itself could be undergoing various forms of collective motion (rotations of an ellipsoidal shape or vibrations in various modes), which will be reflected in variations of the strength of the scalar field.

The exact form of the variation of the effective mass as observed on Earth would further depend on unknown details. For example, if the electron antineutrino is mostly aligned with the heaviest vacuum mass eigenstate, then the stronger the scalar field, the smaller the effective mass. If, however, as we showed in Ref.([18]), the electron antineutrino is mostly aligned with one of the lighter vacuum mass eigenstates, as the scalar field gets larger, the magnitude of the effective mass increases. What is uniformly true is that, as the scalar field varies, the effective mass varies. Further, if the effective mass is a small fraction of the vacuum mass, small percentage variations of the scalar field can lead to larger percentage variations of the effective neutrino mass.

Another consequence that follows from this conjecture, as discussed in detail in [17], is that neutrinos could not constitute a hot dark matter component of the Universe during the formation of large scale structures, since they would be “locked up” within massive clouds with non-relativistic net kinetic energies. Thus a number of the reported limits on neutrino mass from cosmology would need to be revisited.

While there is, of course, no proof that such a background scalar field exists in the region of the Earth (or at all), the fact that it cannot be ruled out suggests that there is a value to independently analyzing experiments done at different times. If there is a time dependence in the effective mass matrix, this may imply a time dependence to the resulting direction cosines. In the
formulation presented here, that would affect both $\cos \theta_1$ and the value of $f_1$ through this and its dependence on $\cos \hat{\theta}_1$. Hence all the parameters of the fit may demonstrate a time dependence, and the correlations are very hard to predict \textit{a priori}. As bizarre as this possibility may seem, it is clearly prudent to await further experimental information before discarding it.

6 \ Neutrino endpoint effects

The interference terms discussed above display a general symmetry between the $\beta$ and the neutrino. M. Goldhaber [19] has raised the question of possible observable effects at the opposite end of the spectrum, where the neutrino carries away all of the available energy. In fact, the only terms that would be observable are complimentary to those discussed above, since any terms proportional to the neutrino mass eigenvalues will be completely dominated by $E_{\nu}$. This leaves the term proportional to $\rho_{S_R} \frac{m_\nu}{E_\beta}$. Since the scale of variation with energy is set by $m_e$, we would expect the variation to occur over several $MeV$. Consequently, Tritium is not the place to look for such effects.

As the Fermi function varies rapidly at small $E_\beta$, the analysis must make use of the full form given above. Furthermore, given the enhancement of the very low energy electron spectrum, compared with the suppression for a positron spectrum, electron emitters may be preferred. This particular current is the most severely constrained, and this part of the spectrum corresponds to large $E_\nu$ so that threshold effects are not relevant. Nonetheless, the fact that the scale of the variation is set by $m_e$ may make it possible to see such an effect in neutrino spectra of future experiments.

Although the observation of interference effects in such a transition would not directly impact the interpretation of Tritium beta decay, it would demonstrate the existence of non-SM currents and would be a very interesting result in its own right.

7 \ Implications for neutral currents

As we discussed in section (4.5), the combination of our analysis of Tritium beta decay and other experimental results strongly suggests that the elec-
tron neutrino is a pseudo-Dirac object with new interaction(s) involving the components which are sterile in the SM. If these new interactions devolve from Higgs mediated scalar currents, as in supersymmetric extensions of the SM [21], or from Right-chiral Vector currents, as in Left-Right symmetric models [22], there will be associated new neutral current interactions for the “sterile” components of the neutrino fields. In fact, such neutral currents occur in most proposed extensions of the SM.

These new interactions could affect the interpretation of any experiment sensitive to neutral currents. For example, in the case of the Sudbury Neutrino Observatory experiment (SNO) [45], oscillation of solar neutrinos into a “sterile” component could produce a neutral current signal intermediate in strength between that expected for oscillations among active neutrino components only and the reduction of signal observed for the charged current interactions. Absent any special quantum coherence effects, the bounds on the strength of any new charged current interactions suggest that the additional neutral current signal provided by this mechanism is not likely to be sufficiently large to confuse an oscillation into “sterile” components with those among active neutrinos of different flavors. Nonetheless, the SNO experimental group must specify the bounds on “sterile” neutral current strengths assumed in the analysis of their data.

Similarly, if, as one expects, these considerations apply to other flavors as well, experimentalists such as those involved in the SuperKamiokande experiment [46] need to consider the possible strength of neutral current interactions of “sterile” neutrinos. That particular case is exacerbated by the fact that the effect of interest is proportional to forward scattering, hence depends on amplitudes rather than rates.

8 Conclusions

In this paper we have examined the possibility that interference between the SM Left-chiral current and a weaker, non-SM current with a different Lorentz character may be the origin of the “anomaly” in the Tritium beta decay spectrum near the end point. On general theoretical grounds, it is expected that such currents must exist, the only uncertainty being their strength. To avoid the unwarranted prejudice that only the SM interaction
contributes to the decay, experimentalists should include these interference terms when fitting data.

Given the small variation in the electron energy, $E_\beta$, we have presented formulas for differential spectra, Eq.(14), and for integral spectra, Eq.(15), which are good approximations appropriate for fitting to data. At a minimum, these should be employed to obtain reasonable parameter values from which to initiate searches with more complete expressions.

Explicitly, we recommend the use of Eq.(15), with $k = 1$ and $\phi_1 = f_1 m_1$, for the analysis of ongoing integral experiments [6, 9], with all of sums over molecular end points implied by Eqs.(12 and 13) and experimental details included. This analysis should be carried out independently of other experimentally derived constraints as those generally reflect differing parameter combinations. In particular, this experiment is uniquely affected by neutrino mass thresholds.

Using a characterization of the data in terms of $\langle m^2 \rangle_{fit}$, we have shown that our interpretation of the negative values reported to date is possible for parameter values which are not, in fact, in conflict with other experimental constraints. For consistency with experimental results on neutrino oscillations and neutrinoless double beta decay, the electron neutrino must be either a Dirac or a pseudo-Dirac object with a small CKM-like mixture to other flavor eigenstates.

Finally, we examined three other related considerations: One is the possibility that neutrino endpoint effects might appear to be time dependent due to environmental factors. The second is the complementary effect of new interactions, as discussed here, on the high neutrino energy (low electron energy) end of beta spectra [19]. Lastly, and perhaps most dramatically, we noted that our analysis, when combined with other results, strongly implies the existence of new interactions involving “sterile” neutrinos. This last, in turn, has important implications for the interpretation of experiments studying neutral current interactions.

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10 Figure captions

Figure 1. Current eigenstate neutrinos displayed in the space spanned by the three mass eigenstates. The vectors denoted by $\nu_f$ are current eigenstates for the SM $SU(2)_L$ current, those denoted by $\hat{\nu}_f$ are current eigenstates for whatever other charged current one is considering.

Figure 2. Diagrams contributing to nuclear beta decay under various scenarios. a) SM interaction. Hadronic renormalization produces both L and R hadronic currents in the effective Hamiltonian. b) Scalar exchange. Recoil effects suppress the hadronic Pseudo-scalar coupling. In principle the two leptonic couplings, $S_L$ and $S_R$ can be different. c) Direct Right-chiral Vector couplings. d) Possible mixing between the $X^-_R$ and $W^-$. The analogous diagram in which the $W^-$ couples to the lepton current gives no observable change in the Tritium spectrum. Again, hadronic renormalization will lead to both L and R effective hadronic currents in both diagrams c) and d).

Figure 3. Neutrino mass-squared extracted from Mainz data set Q2 vs. integral cutoff on electron energy and corresponding results from the SM model analysis of a spectrum including interference effects, for various parameter values. (Details in text.)

Figure 4. The same as Fig.3 for Mainz data set Q3.

Figure 5. The same as Fig.3 for Mainz data set Q4.

Figure 6. The same as Fig.3 for Mainz data set Q5.
Fig. 2

(a) $n \rightarrow L,R \rightarrow W^- \rightarrow e^- \rightarrow \bar{\nu}_e$

(b) $n \rightarrow W^- \rightarrow \bar{\nu}_e$

(c) $n \rightarrow X^- \rightarrow e^- \rightarrow \bar{\nu}_e$

(d) $n \rightarrow W^- \rightarrow X \rightarrow X^- \rightarrow e^- \rightarrow \bar{\nu}_e$
Neutrino Mass–Squared Extracted
vs. Energy Cutoff for Mainz Data Set Q2

Extracted $m^2_{\nu,\ell}$ vs. Energy Cutoff for Mainz Data Set Q2

$\ell_1 = 0.05$
$\ell_2 = 0.07$
$\ell_3 = 0.09$
$\ell_4 = 0.11$
$\ell_5 = 0.13$

Fig. 3
Neutrino Mass–Squared Extracted

vs. Energy Cutoff for Mainz Data Set Q3

- $f_1 = 0.05$
- $= 0.07$
- $= 0.09$
- $= 0.11$
- $= 0.13$

Extracted $m^2_{\nu^2}$ vs. Energy Cutoff for Mainz Data Set Q3
Neutrino Mass–Squared Extracted

vs. Energy Cutoff for Mainz Data Set Q4

- $f_1 = 0.05$
- $f_2 = 0.07$
- $f_3 = 0.09$
- $f_4 = 0.11$
- $f_5 = 0.13$

Fig. 5
Neutrino Mass–Squared Extracted vs. Energy Cutoff for Mainz Data Set Q5

Fig. 6