The cutting vibration and surface information in whirlwind milling a large screw

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Abstract
The whirlwind milling (WM) technology is a competitive machining method, especially for large screws. A rotating large screw subjected to a rotating and moving WM cutting force has complex dynamics and formation. In the study, the WM cutting forces in radial, tangential, and axial directions were firstly acquired using a self-developed testing system. Then, the cutting vibration was modeled in consideration of the WM unique constraint, and the deflections in different directions were analyzed in detail. Lastly, the surface topography and roughness under cutting were geometrically modeled by superimposing the radial deflection on the static forming surface. Compared to single-factor experiments, the results demonstrated the consistency between surface roughness and experimental data. Therefore, the established vibration and surface roughness models are reasonable and effective for predicting, and thus favorable and beneficial for the optimization in whirlwind milling a large screw.

Keywords
Surface topography, cutting vibration, cutting force, experiments validation

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Introduction
The demand for large screws has become increasingly urgent in the development of large equipment manufacturing.1 Compared with turning and grinding, whirlwind milling (WM) is considered as one of the most promising technology for large-screw processing because of its great flexibility, high material removal rates and environmentally favored requirements. For improving the surface integrity, the cutting vibration, and surface roughness in WM are requisites in further studies.

Several theories and dynamic models have been developed to study the vibration. Chen2 proposed a dynamic model of a helical gear pair system, and applied Lagrange’s equation and Runge-Kutta method. Then Wang et al.3 developed a non-contact electromagnetic loading device, which measures the transient loading force of a high speed motorized spindle. Park and Hong4 developed an energy model for Euler–Bernoulli, Rayleigh, and Timoshenko beams. Bazehhour et al.5 used an analytical approach to present the free lateral vibration of a rotating Timoshenko shaft with various boundary conditions. Metsebo et al.6 applied the fast Fourier transform and phase trajectory to focus on the dynamics of a rotor–ball bearing system. Wang et al.7 explored a point vector method to express the shaft dynamic response using homogeneous
boundary conditions. Zhang et al.\(^8\) developed a new formulation with an equivalent internal damping when a slender beam was subjected to large deformations. While for WM, most studies were focused on the tool tip trajectory. Ni\(^9\) established the transforming relationship between tool profile and thread raceway profile. Li et al.\(^10\) obtained an accurate vibration model of AMB-rotor system, and verified on a test rig.

More studies have also focused on the formation of the machined surface. Duc et al.\(^11\) evaluated the variance of surface roughness in hard drilling. Strano et al.\(^12\) and Boschetto et al.\(^13\) developed a mathematical model to predict the surface roughness in selective laser melting, respectively. Considering with the system vibration, Shiau et al.\(^14\) predicted the surface roughness in grinding a ball screw under a moving force. Wojciechowski et al.\(^15\) found that the high surface finish in precise milling was strictly dependent on cutting force values. Twardowski et al.\(^16\) modeled the surface roughness through a kinematic–geometric projection of cutting edge in high-speed milling. Asiltürk and Čunkaş\(^17\) predicted the surface roughness combining multiple regression with the ANN method. Shalaby et al.\(^18\) proposed a mechanistic model based on Merchant’s analysis. Zein and Irfan\(^19\) developed an analytical model for surface roughness and investigated the effect of cutting parameters in orthogonal experiments. Marques et al.\(^20\) revealed experimentally that the surface roughness is in reduction trend with increasing cutting speed. Ghosh et al.\(^21\) utilized RSM to study the relationship between the input cutting parameters and output surface roughness in milling, and as Guo et al.\(^22\) did in WM. Guo et al.\(^23\) developed a static surface roughness model in WM.

In WM, although some studies on the tool tip trajectory and static surface formation have been examined, the cutting vibration caused by WM forces was ignored, and the resulted forming information of screw was still lack. Thus, the present study aims to develop effective models of cutting vibration and surface information based on tested cutting forces.

**Materials and methods**

Figure 1(a) showed a 10-m WM machine for experiments. The same tools in Figure 1(b) are symmetrically mounted on the tool holder. Tools rotate homodromously at high speed as screw at low speed. Figure 1(c) illustrates the partial enlarged detail of WM. The material of workpiece is GCr15 (similar to AISI 52100). It
was treated by surface low-frequency quenching, and the hardness is HRC 62. The tool employed in the cutting tests were polycrystalline cubic boron nitride with a negative 8° rake angle. The wedding agent was titanium nitride, and the granularity 2 mm. The high CBN content (85%) was used as recommended. In testing, the WM cutting forces in radial, tangential and axial directions were acquired by a team self-developed testing system.24

In Figure 1(d), the tool is in split type. It contains cutter body like toolbar, cutter blade and blade bearing which connected by screw. A 3D piezoelectric force sensor was used in the cutting force measurement, which was fastened between the tool apron and cutter holder. Passing by a multi-channel slip ring and filtered by a data acquisition system (P8020), the force signals were detected through a force sensor with built-in charge amplifier, and a cutting time of 1.5 s for force signal acquisition. After WM, the surface topography and roughness can be obtained by white light interferometer. Amplifier, and a cutting time of 1.5 s for force signal acquisition. After WM, the surface topography and roughness can be obtained by white light interferometer in Figure 1(e). The values of cutting force and surface roughness were all tested three times and averaged.

**Modeling cutting vibration and surface topography**

**Modeling cutting vibration**

In WM, a large screw is mounted and driven at a rotating speed \( n_w \). Both ends of the screw are configured with a clamped–hinged (left–right) boundary, whereas the remaining part is supported by three floating grippers and two clamping devices. In Figure 2, the two frames, a fixed reference frame \( X–Y–Z \) and a rotating reference frame \( x–y–z \), are utilized to describe the system motion. The \( Z \) and \( z \) axes are col-linear, and the two reference frames have a rotating angle \( \Omega \) difference at the \( Z \)-axis. The \( F_r \), \( F_t \), and \( F_u \) represent the three cutting force components acting on the screw in radial, tangential and axial directions, respectively. The deflections by the cutting force are classified as three translational (\( U, V, W \)) and two rotational displacements (\( B \) and \( S \)). Whilst cutting, three floating grippers (marked “I,” “II,” and “III”) which were simplified with axial \( k_x \) and lateral stiffness \( k_y \) emerge and proceed to support after the tools have passed some locations. Specified as concentrated mass “\( m \),” two clamping devices are symmetrically located on both sides of the cutting position and move with the tools.

In any cross section of the screw, the instantaneous axial displacement \( U \), radial \( V \), tangential \( W \), tangential rotational \( B \), and radial rotational \( S \) are expressed as the functions of location \( z \) and cutting time \( t \) shown in Figure 3(a), if assumed shape functions are selected (Figure 3(b)).

For the motion system, the two energies, potential energy \( U_s \) and kinetic energy \( T_s \), are expressed in Figure 3(c). The potential energy \( U_s \) includes the shear deformation, pure bending and axial deformation. The kinetic energy \( T_s \) includes the axial, translational, rotational inertia and gyroscopic effect. By the Lagrangian approach, the motion equations can be obtained (Figure 3(d)) and simplified (Figure 3(e)).

All motions should meet the constraints on the screw. The motion equation under constraints is shown in Figure 3(f). Thus, the deflection \( U, V, W \) caused by the WM cutting forces can be calculated.

**Modeling surface topography**

Most of surface topography models are examined in conventional cutting, such as turning and milling, and great progress has been achieved over the past years. However, reports on surface topography under cutting vibration in WM was little reported.

For modeling the cutting trajectory (Figures 1 and 4(a)), six coordinate systems shown in Figures 2 and 4(a) were utilized. In \( O_{1l} (x_{1l}, y_{1l}, z_{1l}) \), \( O_{tr} (x_{tr}, y_{tr}, z_{tr}) \), and \( O_{lc} (x_{lc}, y_{lc}, z_{lc}) \) local coordinate systems, the formula of points on different positions of the tool are expressed in Figure 4(b). By matrix \( M_{wdf} \), \( M_{wdc} \), and \( M_{wdrf} \), points on three tool’s local coordinate systems can be converted into the \( O_{wd} (x_{wd}, y_{wd}, z_{wd}) \) frame in Figure 4(c). Firstly in Figures 1 and 4(d), the transformation \( O_{df} (x_{df}, y_{df}, z_{df}) \rightarrow O_{wf} (x_{wf}, y_{wf}, z_{wf}) \) can be achieved by matrix \( M_{wdf} \). Secondly in Figures 2 and 4(d), the transformation \( O_{df} (x_{df}, y_{df}, z_{df}) \rightarrow O_{wd} (x_{wd}, y_{wd}, z_{wd}) \) can be expressed by matrix \( M_{wdf} \). Then in Figures 3 and 4(d), the transformation \( O_{wf} (x_{wf}, y_{wf}, z_{wf}) \rightarrow O_{wd} (x_{wd}, y_{wd}, z_{wd}) \) can be attained by matrix \( M_{wfwd} \). If the tool had a negative rake angle \( \gamma \), the transformation matrix can be expressed in Figure 4(d-4).
When considering cutting vibration in WM, the matrix $M_{wdh-vib}$ (Figure 4(e)) integrated the deflections by cutting vibration into the forming of surface topography. Finally, the finishing tool points under $O_{wf}$ ($X_{wf}$, $Y_{wf}$, $Z_{wf}$) can be modeled by substituting the above transformation matrices into expressions (Figure 4(f)). Thus combining with points on the tool in Figure 4(b),
all points on the workpiece under cutting vibration can be expressed by Figure 4(h).

For describing points in the cutting area, the workpiece is discretized in circumferential and radial directions (Figures 1 and 4(g)). The workpiece in the radial and circumferential direction are separately discrete as $R$ and $S$ equal parts. Accordingly, the cutting track on the workpiece is represented by a series of points.
of discrete points \( R_r \) \((r = 1, 2, ..., R; s = 1, 2, ..., S)\). All discrete points on the workpiece can be calculated in the Figure 4(g-2).

Through the superimposition of the tool contour on the workpiece surface, the lower contour between two adjacent cuts namely the \((i)\)th and \((i+1)\)th cutting is defined to generate a whirlwind-milled surface. In Figure 4(i), the radial lengths mapped on the workpiece are respectively marked \( r_{vib,i} \) \( r_{vib,i+1} \) in the \((i)\)th cutting and the \((i+1)\)th cutting. By comparing \( r_v(r, s) \) with \( r_{lim} \), the scallop height \( h_i(r, s) \) can be expressed in the Figure 4(j). The least scallop height \( H_{i+1} \) in two adjacent cutting is chosen as the final machined scallop height, shown in Figure 4(k).

### Deflection simulations and discussion

The governed \( V(I_{vib}) \) formula in Figure 3(f) was numerically solved by the Runge–Kutta method, and the first six shape functions were used to obtain the deflections by the WM cutting force. Table 1 lists the parameters. In this section, the enclasping device in WM was examined in terms of the enclasping torque \( M_f \).

#### Table 1. Parameter values in the simulation.

| Workpiece | \( \rho = 7812 \text{kgm}^{-3}, \kappa = 0.9, E = 217 \text{GPa}, G = 80 \text{GPa} \) |
|-----------|--------------------------------------------------|
| \( L = 6 \text{m}, d = 0.08 \text{m}, g = 9.8 \text{ms}^{-2} \) |
| Support | Enclasping device: \( m = 50 \text{kg} \) |
| Parameters | Floating gripper: \( k_v = 5.65 \times 10^6 \text{Nm}^{-1}, k_w = 4.34 \times 10^6 \text{Nm}^{-1} \) |
| | Enclasping torque: \( M_f = 1.6, 2.4, 4.8, 9 \text{Nm} \) |
| | Tool chamfer parameters: \( 0.15 \text{mm} \times 25^\circ \) |
| | \( \nu_c = 120, 160, 240, 320 \text{mm}^{-1} \) |
| | Cutting parameters: \( a_p = 0.06, 0.08, 0.12, 0.16 \text{mm} \) |
| | \( v_t = 0.04, 0.05, 0.08, 0.11 \text{mm}^{-1} \) |

#### Figure 5. Deflection \( V \) under different: (a) enclasping torques, (b) cutting speeds, (c) cutting depths, and (d) feeding speeds.
On maximum radial deflection $V_{max}$, the value in Figure 5(a) decreases with increasing $M_f$. The $M_f = 9.6$ N-m curve has gentle ups and downs, indicating that it can construct a relatively stable cutting condition and thus is selected in the subsequent analysis. Comparing with other cutting parameters at $z = 1.0$ m, the remarkable crest difference is caused by different enclasping torques. This phenomenon

Figure 6. Predicted $R_{a-pre}$ and measured $R_{a-mes}$ surface topography under various enclasping torques: (a) $M_f = 1.6$ N-m, (b) $M_f = 2.4$ N-m, (c) $M_f = 4.8$ N-m and (d) $M_f = 12.8$ N-m.
Figure 7. Predicted $R_{a-pre}$ and measured $R_{a-mes}$ surface topography under various cutting speeds: (a) $v_t = 120 \text{ m min}^{-1}$, (b) $v_t = 160 \text{ m min}^{-1}$, (c) $v_t = 240 \text{ m min}^{-1}$, and (d) $v_t = 320 \text{ m min}^{-1}$. 
indicates that the effect of the enclasping torque is crucial and discriminated from other processing systems. For cutting parameter (Figure 5(b)–(d)), the $V_{\text{max}}$ under $v_f = 240 \text{ m min}^{-1}$ is the minimum amongst other cutting speeds, and its stability is preferred in the full-length screw processing. As cutting depth $a_p$ increases, the $V_{\text{max}}$ decreases (Figure 5(c)). Meanwhile with increasing feeding speeds $v_f$ in Figure 5(d), the $V_{\text{max}}$ initially decreases and then increases. Eventually, the preferred value was attained under $v_f = 0.05 \text{ m min}^{-1}$. However, the influence of cutting parameter on $V_{\text{max}}$ is less significant than that of the enclasping torque.

**Simulation and experiment validation**

Based on the analysis of $V_{\text{max}}$, the surface topography under various cutting parameters were obtained by following Figure 4. If fixing parameters ($v_f = 200 \text{ m min}^{-1}$, $a_p = 0.06 \text{ mm}$ and $v_f = 0.07 \text{ m min}^{-1}$), the simulated surface roughness ($R_a$) decreases as the enclasping torque increases in Figure 6. The $Ra\text{-pre}_z$ value is the tool’s axial moving distance, and five cutting periods were chosen. If the axial moving speed $v_f$ and workpiece’s rotating speed $n_w$ were given, then the $Ra\text{-pre}_z$ value can be calculated as $z = v_f \times t = v_f \times 5T = 5v_f/n_w$ under different cutting conditions. Compared to experiments, the trend of the estimated data is similar to that of the test one.

As $M_f = 1.6 \text{ N-m}$, the workpiece undergoes significant fluctuation under the WM force. The machined surface exhibits a strong vibration in Figure 5(a), and the tested surface roughness is $R_a = 0.42 \mu\text{m}$. When it increases to $3.2 \text{ N-m}$, the waviness and roughness of the workpiece are all reduced. When the torque continuously increases to $12.8 \text{ N-m}$, the surface roughness tends to stabilize.

In Figure 7, the tested surface was obtained at a lower cutting speed of $160 \text{ m min}^{-1}$, which is high and uneven. By contrast, the tested surface at $240 \text{ m min}^{-1}$ is relatively uniform. As the cutting speed increased, the tested $R_a$ is reduced from 0.33 to 0.19 $\mu\text{m}$. Thus, the $R_a$ shows a maximum under $v_f = 160 \text{ m min}^{-1}$ and a minimum under $v_f = 240 \text{ m min}^{-1}$. After data comparison, the errors of the estimated and measured values of the surface roughness are within the 10% range.

**Conclusion**

In this study, the surface topography and dynamic characteristics of a rotating large screw were modeled and investigated when the screw is subjected to a moving and rotating WM cutting force. The results can be summarized as follows:

1. The established cutting vibration model considered the complicated constraints including floating support and enclasping device. The radial response in the case of the enclasping torque is evidently different from that of cutting parameters. The maximum radial deflection decreases with the increase in enclasping torque.

2. Based on the analysis of radial deflection, the surface topography was modeled by superimposing the radial deflection on the machined surface. The surface topography and roughness were simulated under various parameters. After data comparison, the errors of the estimated and measured values of the surface roughness were within the 10% range.

3. Research indicated that the condition under clasping torque of $12.8 \text{ N-m}$, cutting speed of $240 \text{ m min}^{-1}$, cutting depth of $0.12 \text{ mm}$ and feeding speed of $0.05 \text{ m min}^{-1}$ is preferred both in the simulation and experiment. Thus, the cutting vibration model and surface topography model are verified to be reasonable and effective for prediction and optimization.

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Appendix

Notations

| WM | Whirlwind milling |
|-----|------------------|
| n_w | Rotating speed of the workpiece |
| v_f | Axial moving speed of the tool |
| F_w, F_r, F_a | Cutting force components in radial, tangential, and axial directions |
| k_r, k_a | Axial and lateral stiffness of floating gripper |
| m | Specified concentrated mass of clasping device |
| U, V, W | Instantaneous axial, radial and tangential displacement |
| B, S | Tangential rotational and radial rotational displacement |
| n | Total number of assumed modes |
| u_i, v_i, w_i, p_i, q_i | Time-dependent generalized coordinates |
| \phi_i | Assumed shape functions |
| L | Full length of the screw |
| U_s, T_s | Potential energy, kinetic energy |
z, t  
Cutting location, time

A, I  
Cross-sectional area, the area moment of inertia of the screw

E, k, G  
Young’s modulus, shear form factor, shear modulus of the screw

$\rho, \Omega$  
Mass density, rotating speed of the screw,

$b_l, b_r$  
Distance from the left, right enclasping devices to the cutting point.

M, C, K, F  
System mass, damping, stiffness matrices, nonlinear force vector

$z_l, z_r$  
The left (clamped) and right (hinged) ends of the screw

$O_{tl}(X_{tl}, Y_{tl}, Z_{tl})$, $O_{tr}(X_{tr}, Y_{tr}, Z_{tr})$, $O_{tc}(X_{tc}, Y_{tc}, Z_{tc})$  
The coordinate systems of tool’s left, right and center arc.

$O_{wd}(X_{wd}, Y_{wd}, Z_{wd})$, $O_{tf}(X_{tf}, Y_{tf}, Z_{tf})$  
Dynamic, fixed coordinate systems of the workpiece

$M_{wdl}, M_{wdr}, M_{wdc}$  
Transformation from $O_{tl}(X_{tl}, Y_{tl}, Z_{tl})$, $O_{tr}(X_{tr}, Y_{tr}, Z_{tr})$, $O_{tc}(X_{tc}, Y_{tc}, Z_{tc})$ to $O_{wd}(X_{wd}, Y_{wd}, Z_{wd})$

$M_{wff}$  
Transformation from $O_{tl}(X_{tl}, Y_{tl}, Z_{tl})$ to $O_{wf}(X_{wf}, Y_{wf}, Z_{wf})$

$M_{wfd}$  
Transformation from $O_{tl}(X_{tl}, Y_{tl}, Z_{tl})$ to $O_{wd}(X_{wd}, Y_{wd}, Z_{wd})$

$M_{wdf}$  
Transformation from $O_{wf}(X_{wf}, Y_{wf}, Z_{wf})$ to $O_{wd}(X_{wd}, Y_{wd}, Z_{wd})$

$M_{wdb-vib}$  
Transformation integrating the deflections by cutting vibration

$R_{rs}$  
Series of discrete points in radial and circumferential direction

$r_s(r,s)$  
Instantaneous radius of the workpiece

$\tau_{limit}$  
Instantaneous radius of the workpiece surface corresponding to the arc transition point of the tool tip

$l_{vib}(r, s), l_{vib,i+1}(r, s)$  
Radial lengths mapped on the workpiece in $(i)$ th, $(i + 1)$th cutting

$h(r,s)$  
Scallop height

$H_i+1(r, s)$  
Least scallop height $H_i + 1(r, s)$ in two adjacent $(i)$th, $(i + 1)$th cutting

$M_f$  
Enclasping torque

$a_p, v_t$  
Cutting depth, cutting speed

$V_{max}$  
Maximum radial deflection

$R_{a-pre}, R_{a-mes}$  
Predicted, measured surface roughness