Fractionalized wave packets from an artificial Tomonaga–Luttinger liquid

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1. Derivation of equations (1) and (2) in the main manuscript

Wave equation for charge density wave in the form of an edge magnetoplasmon mode can be found in many articles [1–3]. Here, we use a simplified yet general form with a channel capacitance $C_{ch}$ defined per unit length, which relates the potential $V(x, t)$ and excess charge density $\rho(x, t)$ at position $x$ along the channel and time $t$ as $\rho(x, t) = C_{ch}V(x, t)$. Associated ‘Hall’ current $I(x, t) = \sigma_{xy}V(x, t)$ along the channel changes the charge distribution through the continuity equation $\partial \rho(x, t)/\partial t = -\partial I(x, t)/\partial x$. Considering the conductance of the channel $I(x, t)/V(x, t) = \sigma_{xy}$, the unidirectional wave equation reads

$$\frac{\partial}{\partial t} \rho(x, t) = -\frac{\sigma_{xy}}{C_{ch}} \frac{\partial}{\partial x} \rho(x, t),$$

where $\sigma_{xy}/C_{ch}$ gives the charge velocity in uncoupled channels.

Coupled charge density wave in our artificial TLL can be considered in a pair of two counter-propagating edge channels $i$ ($i = 1$ for propagating to the positive direction; $i = 2$ for negative direction). They are coupled with the cross capacitance $C_X$ per unit length. The relation between the charge density $\rho_i(x, t)$ and the potential $V_i(x, t)$ can be described in a matrix form as

$$ \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} C_{ch} + C_X & -C_X \\ -C_X & C_{ch} + C_X \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}. $$

(S2)

Using the continuity equation, the coupled wave equation for $\rho_i(x, t)$ is given by

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = -\frac{\partial}{\partial x} \begin{pmatrix} U & V \\ -V & -U \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix},$$

(S3)

where $U$ and $V$ are intra- and inter-channel interaction strengths, respectively. They are given by

$$U = \frac{\sigma_{xy}}{C_{ch}} \frac{C_{ch} + C_X}{C_{ch} + 2C_X}, \quad (S4)$$

$$V = \frac{\sigma_{xy}}{C_{ch}} \frac{C_X}{C_{ch} + 2C_X}. \quad (S5)$$

The wave equation (S3) exhibits transport eigenmode $-\rho_2/\rho_1$ or $-\rho_1/\rho_2 = V/(U + \sqrt{U^2 + V^2})$, which is defined as the coupling factor $r$, and the velocity $v^* = \sqrt{U^2 - V^2}$. Inserting the channel capacitance $C_{ch} = C_g + C_b$ for the model defined in the main manuscript gives equations (1) and (2).
2. Reflection coefficient

Figure S1. **Reflection coefficient between two TLL regions.** Schematic illustration of (i) the incident right moving mode $a_{\text{in}} u_{R1}$ in TLL$_1$, (ii) the reflected left moving mode $a_{\text{ref}} u_{L1}$ in TLL$_1$ and the transmitted right moving mode $a_{\text{tr}} u_{R2}$ in TLL$_2$.

Consider two TLL regions TLL$_1$ and TLL$_2$ with the coupling coefficient $r_1$ and $r_2$, respectively, connected with each other as shown in Fig. S1. We shall describe the eigenmodes in TLL$_i$ as $u_{Ri} = (1, -r_i)$ for right moving mode and $u_{Li} = (-r_i, 1)$ for left moving mode. Incident wave of amplitude $a_{\text{in}}$ from TLL$_1$ is partially transmitted into the TLL$_2$ with the amplitude $a_{\text{tr}}$ and reflected back to TLL$_1$ with the amplitude $a_{\text{ref}}$. Charge conservation law determines a relation

$$a_{\text{in}} \begin{pmatrix} 1 \\ -r_1 \end{pmatrix} = a_{\text{ref}} \begin{pmatrix} -r_1 \\ 1 \end{pmatrix} + a_{\text{tr}} \begin{pmatrix} 1 \\ -r_2 \end{pmatrix},$$

from which we obtain the transmission coefficient $T \equiv a_{\text{tr}} / a_{\text{in}} = (1 - r_2^2) / (1 - r_1 r_2)$ and the reflection coefficient $R \equiv a_{\text{ref}} / a_{\text{in}} = (r_2 - r_1) / (1 - r_1 r_2)$. For the abrupt junction between non-interacting channel ($r_1 = 0$) and TLL region ($r_2$) defined in Fig. 1a of the main manuscript, we used $T = 1$ and $R = r_2$ to evaluate the reflected charge. The above formula indicates that the reflection is associated with the difference of the coupling coefficients, and can be used to develop a time-domain reflectometry.
3. \( V_{G1} \) and \( \nu \) dependence of charge waveforms

Figure S2. \( V_{G1} \) and \( \nu \) dependence of charge waveforms. a, \( V_{G1} \) dependence of charge waveforms for the Type-I TLL region observed when \( \nu = 1.5 \). Traces are vertically offset for clarity. b, \( \nu \) dependence of charge waveforms observed when (i) no TLL regions are activated, (ii) Type-I TLL region is activated with the gate voltage \( V_{G1} = -0.20 \) V. Traces are vertically offset for clarity.

4. Charge waveforms

It is important to distinguish real signals from spurious ones in Fig. 2a in the main manuscript. Indeed, the additional small ripples seen at \( t \sim t_1 + 3.5 \) ns and \( t \sim t_2' + 3.5 \) ns in the traces (i) and (iii) are due to ripples in our voltage pulse waveform and irrelevant to the fractionalization processes. Electrostatic crosstalk between the leads also induces spurious signals in this type of experiments [4]. However, such crosstalk can be ruled out, as it should appear as a derivative of the incident wave (shown as a dashed line in the inset),
which cannot explain our observations. It is also known that plasmon mode exists in the bulk compressible region. However, such bulk plasmon mode is dissipative and exhibits a lower velocity [5]. Therefore, we do not consider the effects from the bulk plasmon mode in the analysis carried out in the main manuscript.

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