The Method of Calculating the Shock Effect of Falling Rock Research

Kexuan Guo1*, Hongkai Chen1,2 and Tao Chen1

1College of Hydraulic & environmental Engineering, China Three Gorges University, Yichang,Hubei,443002,China
2Institute of geotechnical engineering,Chongqing Jiaotong University,400074,China
*Email:374550280@qq.com

Abstract: The paper study on the process of rockfall falling, consider the air be low the rockfall will be compressed, calculate the force of the compressed air to the rockfall; Set up theory mode and divide the process into n parts, using the theory of Aerodynamics, Conservation of energy theorem and Air moving theory to derive the method of calculate the rockfall impacts; The results have certain reference, it can be used in the theory study of disaster reduction and technical of rockfall.

1. Introduction

China is a mountainous country with many high and steep slopes, dangerous rocks and rockfall are common forms of geological disasters rockfall. Disaster is characterized by randomness, sudden and unpredictable. Once it happens, it will cause serious casualties and economic losses. Such as August 1983,Xiangyu line Bashan a state highway section of the road slope affected by heavy rain caused rockfall disaster, a volume of about 12.6 rock fall from the height of about 80 meters, and a bridge was destroyed causing traffic interruption about 23 hours, the direct economic losses up to 22.1 million. So far, there are a large number of domestic and foreign scholars have carried out extensive and in-depth research of rockfall disaster [1], but they were committing on the research of the formation mechanism evolution [2] and fall to the impact of failure mechanism [3], neglecting the research of rockfall movement process and the impact factors for rockfall force was not considered perfect, the compressed air resistance when the dangerous rock fall. Based on the research status [4], this paper deduces the calculation method of impact velocity of dangerous rockfall by combined the viewpoint of energy with aerodynamics. The research results to improve the calculation accuracy of rockfall impact force [5], Further prediction of rockfall disaster, and to carry out disaster prevention work has a positive guiding significance.

2. Derivation the Method of Calculate the Speed of Rockfall

Dangerous rockfall has many characteristics, which are high speed and the huge destructive. Scholars at home or abroad often ignore the air impact but pay too much attention to free-faller when they research this type disasters [9]. Making inevitable error when we apply these conclusion to the prevention and repair on dangerous rockfall. In this paper, establishing a rockfall theory model, using aerodynamic theory, principle of conservation of energy and air dynamic theory aims to analyse the compression process, combined with the air leakage coefficient, to derived the calculation methods of rockfall fall speed.
2.1. Establish Model
The theory model of air compressed by dangerous rockfall [6]. Shown in figure 1.

Figure 1. The mechanical model of air be compressed by rockfall

Added a rockfall fall height of $H$, divided the $H$ into n parts, Constant n is large enough, each part $dh = H / n$. The process of rockfall fall is continuous. After a part of $dh$ relies on a part. The whole process is shown in figure 2. The first wave of $A_1 - A_1$, propagation speed is $C_0$, same as the sound and shown in figure 2-b. After the first $dh$, rockfall to continue the second $dh$, a new compression wave was create. As shown in figure 2-c. Propagation speed affected by transmission medium density and pressure, the wave of $A_2 - A_2$ propagate in compressed air by the wave of $A_1 - A_1$, propagation speed $C_1 > C_0$. In a similar way, and the third (figure 2-d, 2-e) even countless (figure 2-f) wave will be produced to compress air. Out of rockfall fall in the ith ($i \in \{1.2, \ldots, n\}$) part to analyze [7].
2.2. Derivation the Method of Calculate the Speed of Rockfall

Analysis the first $dh$ of rockfall falling.

First law of thermodynamics: Change of internal energy = The energy of action + heat transfer

The gas state equation form air dynamic theory

$$ E = \frac{\gamma}{2} kNT = \frac{\gamma}{2} k \frac{M}{m} T $$  \hspace{1cm} (2.1)

$\gamma$ is degree of freedom, air generally take 5. $k$ is ideal gas constant, generally take $8.314 \ J / mol$; $M / m$ can calculate the amount of gas materia.

Ignore the leaked air internal energy change. Combined with the energy conservation can get the formula

$$ \frac{1}{2} M v_{0}^{2} + \frac{1}{2} M_{0} C_{0}^{2} + Mgdh + M_{0} \frac{dL}{2} = \frac{1}{2} M v_{1}^{2} + \frac{1}{2} M_{1} C_{1}^{2} + \frac{\gamma}{2} k \frac{M}{m} (\Delta T) $$ \hspace{1cm} (2.2)

$v_{0} = 0 m / s$ said the rockfall original speed; $C_{0} = 340 m / s$ is wave propagation velocity in the air, $\Delta T$ is changed temperature of the compressed air

$$ M_{0} = \pi R^{2} \rho_{air} C_{0} t_{1} $$ \hspace{1cm} (2.3)

$M_{0}$ is the quality of air below the rockfall in initial stage; $R$ is the equivalent radius of rockfall

$$ dh = v_{0} t_{1} + \frac{1}{2} a_{1} t_{1}^{2} = \frac{1}{2} g t_{1}^{2} $$ \hspace{1cm} (2.4)

$a_{1}$ is the acceleration of the rockfall falling in first $dh$, take the free acceleration

$$ t_{1} = \sqrt{\frac{2dh}{g}} $$ \hspace{1cm} (2.5)

$t_{1}$ is the time of the rockfall falling in first $dh$

$$ dL_{1} = C_{0} t_{1} $$ \hspace{1cm} (2.6)

$dL_{1}$ is the length of the compression wave forward in first $dh$, $\frac{dL}{2}$ is the height of compressed air gravity drop

$$ v_{1} = v_{0} + gt_{1} $$ \hspace{1cm} (2.7)
$v_i$ is the the velocity of the rockfall when finished first $dh$

$$M_i = (1 - \alpha_i) M_0$$

(2.8)

$M_i$ is the quality of air below the rockfall when finished first $dh$; $\alpha_i$ is the leakage coefficient of air when finished first $dh$; Air leakage coefficient can be obtained through experiment.

Take formula form (2.3) to (2.8) into (2.2)

$$\frac{1}{2} \pi R^2 \rho_{aw} C_0^3 \sqrt{\frac{2dh}{g}} + \pi R^2 \rho_{aw} C_0^3 \frac{dh}{g} = \frac{1}{2} (1 - \alpha_i) M_0 C_i^3 + \frac{\gamma}{2} \frac{k (1 - \alpha_i) M_0}{m} (\Delta T)$$

(2.9)

By the gas state equation

$$PV = \frac{M}{m} kT$$

(2.10)

$$P_i = \frac{M}{mV_i} kT_i$$

(2.11)

$$T_i = T_0 + \Delta T$$

(2.12)

$$\rho_i = \frac{M}{V_i} = \frac{(1 - \alpha_i) M_0}{\pi R^2 (C_i t_i - dh)}$$

(2.13)

According to the wave propagation velocity formula which is included in aerodynamics

$$C = \sqrt{\frac{r p}{\rho}}$$

(2.14)

$r$ is the adiabatic coefficient, air usually take $1.402$; $p$ is the pressure of the propagation medium, $\rho$ is the density of the propagation medium; $C$ associated with the density and the pressure of the air.

The rockfall finished first $dh$ is shown in figure 2-d. Simultaneous equation can get the air temperature changes, then to derived the next movement $dh$;

That $i$ ($0 \leq i \leq n$) is the serial number of the current $dh$ in all $n$ parts $dh$; $M_i$ is the quality of air below the rockfall when finished ith $dh$; $V_i$ is the volume of air below the rockfall when finished ith $dh$; $a_i$ is the acceleration of the rockfall falling in ith $dh$; $P_i$ is the intensity of pressure of air to rockfall when finished ith $dh$; $v_i$ is the velocity of the rockfall when finished ith $dh$; $t_i$ is the time of the rockfall falling in ith $dh$; $C_i$ is the wave propagate velocity produce in the period of ith $dh$ which dpropagate in compressed air by i-1th wave, $\rho_i$ is the density of the propagation medium when finished ith $dh$, $d_{Li}$ is the length of the wave propagate in ith $dh$. $R$ is the rockfall equivalent radius. Analysis the ith $dh$ of rockfall falling. Strong energy balance equation and the change of each parameter conditions are as follows

$$\frac{1}{2} M v_{i+1}^2 + \frac{1}{2} M_i C_{i+1}^2 + Mg dh + M_{i+1} \frac{d_{i+1}}{2} = \frac{1}{2} M v_i^2 + \frac{1}{2} M_i C_i^2 + \frac{\gamma}{2} \frac{k M_i}{m} (\Delta T)$$

(2.15)

$$a_i = g - \frac{F_i}{M} = g - \frac{sp_{i+1}}{M} = g - \frac{\pi R^2 p_{i+1}}{M}$$

(2.16)

$$dh = v_{i+1} t_i + \frac{1}{2} a_i t_i^2$$

(2.17)
\[ t_i = \frac{v_{i-1}}{a_j} + \sqrt{\frac{v_{i-1}^2}{a_j} + \frac{2dh}{a_j}} \]  \tag{2.18}

The speed of rockfall falling
\[ v_i = v_{i-1} + a_j t_j = v_{i-1} + \left( g - \frac{\pi R^2 p_{i-1}}{M} \right) \left[ \left( \frac{-v_{i-1} \cdot Mg}{Mg - \pi R^2 p_{i-1}} \right) + \frac{v_{i-1} \cdot Mg + 2dhMg}{Mg - \pi R^2 p_{i-1}} \right] \]  \tag{2.19}

The length of the wave propagate
\[ d_{Li} = C_{i} \cdot t_j \]  \tag{2.20}

The volume of air below the rockfall when finished i-1th \( dh \)
\[ V_{i-1} = \pi R^2 d_{Li} = \pi R^2 C_{i} \cdot t_j \]  \tag{2.21}

The volume of air below the rockfall when finished ith \( dh \)
\[ V_i = \pi R^2 \left( d_{Li} - dh \right) = \pi R^2 \left( C_{i} \cdot t_j - dh \right) \]  \tag{2.22}

The process of rockfall fall is the process of air compressed. There is some air escaping in the compression process, so the \( M \) of air will changed before and after the compression.

The quality of the air before it be compressed is \( M_{i-1} \)
\[ M_{i-1} = \rho_{i-1} V_{i-1} = \pi R^2 \rho_{i-1} C_{i} \cdot t_j \]  \tag{2.23}

The quality of the compressed air is \( M_i \)
\[ M_i = \left( 1 - \alpha_i \right) M_0 \]  \tag{2.24}

The density of the compressed air is \( \rho_i \)
\[ \rho_i = \frac{M_i}{V_i} = \frac{\left( 1 - \alpha_i \right) M_0}{\pi R^2 \left( C_{i} \cdot t_j - dh \right)} \]  \tag{2.25}

\[ T_i = T_{i-1} + \Delta T_i \]  \tag{2.26}

\[ P_i = \frac{M_i}{mV_i} kT_i \]  \tag{2.27}

In the same way, take various parameters into type (2-16) to type (2-28). Simultaneous solution is obtained \( \Delta T_i \). When the air is continue compressed by the back wave, the propagation speed will increase. In the end, the wave before and after will be added together to promote with a common velocity. Finally rockfall and air motion with the same speed. Assumption after the \( s \)th \( dh \) \( (s < n, s \in i) \). Rockfall and air motion with the same velocity. That is:
\[ \frac{1}{2} M v_s^2 + \frac{1}{2} M_s C_s^2 + Mgdh + M_s \frac{C_s^{i+1}}{2} = \frac{1}{2} \left( M + M_{s+1} \right) v_{s+1}^2 + \frac{Z}{2} k \frac{M_{s+1}}{m} \left( \Delta T \right) \]  \tag{2.28}

Compression wave will contact with the ground with dangerous rockfall at the same time. Assumes that the compression wave all absorbed by the ground in the final.
\[ dL_i = C_{i} \cdot t_j > H - idh = \left( n - i \right) dh \]  \( (s \leq i \leq n) \)  \tag{2.29}
Said the rookfall motion a $dh$ length and compression wave also spread to the ground. Since then the pressure change is not a curve, but there is a saltation (figure 2-f), and each parameter changes

$$dl_i = (n-i)dh$$  \hspace{1cm} (2.30)

Among them $(s \leq i \leq n)$, the formulae of below all in the same way.

The volume of the air before it be compressed

$$V_{i-1} = \pi R^2 [(n-i)dh] = \pi R^2 (n-i+1)dh$$  \hspace{1cm} (2.31)

The volume of the compressed air

$$V_i = \pi R^2 [(n-i)dh] = \pi R^2 (n-i)dh$$  \hspace{1cm} (2.32)

The quality of the air before it be compressed

$$M_{i-1} = \rho_{i-1} \pi R^2 (n-i+1)dh$$  \hspace{1cm} (2.33)

The quality of the compressed air

$$M_i = (1 - \alpha_i) M_0$$  \hspace{1cm} (2.34)

The pressure change of air be compressed when finished ith $dh$

$$P_i = \frac{M_i \cdot kT_i}{m v_i}$$  \hspace{1cm} (2.35)

The calculation method is same as above. The air below the rockfall pressure gradually increase when rockfall fall, and the acceleration of rockfall will decrease. Until the adverse effect of compressed air for rockfall and the gravity of the rockfall are the same, and reach the limit state, Acceleration is reduced to 0. After, rockfall will fall with a constant velocity.

Assumption when $i = m$

$$P_m = \frac{M_g}{\pi R^2} = \frac{M_i \cdot kT_i}{m v_i}$$  \hspace{1cm} (2.36)

when $i > m$ accelerated velocity $a_i = 0$

$$\frac{1}{2} M v^2_{i-1} + \frac{1}{2} M_i C^2_{i-1} + M g dh + M_{i-1} \frac{C_{i-1}^2}{2} = \frac{1}{2} (M + M_i) v^2_i + \frac{\gamma}{2} k \frac{M_i}{m} (\Delta T_i)$$  \hspace{1cm} (2.37)

$$(2.37) \frac{1}{2} (M + M_{i-1}) v^2_{i-1} + M g dh + M_{i-1} \frac{dh}{2} = \frac{1}{2} (M + M_i) v^2_i + \frac{\gamma}{2} k \frac{M_i}{m} (\Delta T_i)$$  \hspace{1cm} (2.38)

The rookfall falling speed is

$$v_i = v_m$$  \hspace{1cm} (2.39)

The above content is the calculation method of the rookfall falling speed be effected by the air compression. The whole calculation process is relatively complicated. The calculation process can use the VB program.

3. The Method to Calculate the Impact Force of Rockfall

The scholar Ye Siqiaoproposed a new calculation method which is based on a variety of classical methods [8].
\[
F_{\text{max}} = k \left[ \frac{Mv_n (1 + e_n)}{\Delta t} + M g \cos \alpha \right] 
\]

\[\Delta t = \frac{1}{100} \left( 0.097 M g + 2.21 h + \frac{0.045}{H} + 1.2 \right)\]  

(3.1)

(3.2)

\(e_n\) is the normal recovery coefficient \(e_n = v_n / v_{an}\). \(v_n\) is the normal component of the velocity along the surface of the slope before the rockfall impact the slope surface \((m/s)\), \(v_{an}\) is the normal component of the velocity along the surface of the slope after the rockfall impact the slope surface \((m/s)\).

Combine this calculate method of the rockfall impact force with calculate method of the rockfall impact speed drove in this paper, and apply to engineering example. The results show that the affect of compressive effect is not negligible. Compared with other papers, the results were different because the starting point and the theory used of the study is different, but the difference is not huge. Further research is needed on the accuracy of the two, but there is no doubt that air compressibility has a certain effect on the impact of rockfall.

4. Conclusion

(1) In the process of rockfall falling the rock will compress the lower the air. During the process of compression, the part of the air leak and the other part density and pressure of the air are increase, the air pressure increase will form a certain air resistance to the rockfall mass. The air resistance directly affect the impact velocity of rockfall.

(2) The gas-shrinking process is equivalent to the propagation process of compressive wave, and the ideal hypothesis of rockfall fall height is divided into \(n\) parts. Combining aerodynamic theory, energy conservation theorem and aerodynamic theory (gas equation of state), the method of calculating the impact speed of rockfall is carried out by microanalysis.

5. References

[1] Chen H K, Dong P and Tang H M. Dangerous rock collapse disaster research present situation and the trend [J]. Journal of chongqing normal university (Natural science edition), 2015, 32(6): 53-60.

[2] Buzzi O, Giacomini A and Spadari M. Laboratory investigation on high values of restitution coefficients [J]. Rock mechanics and rock engineering, 2012, 45(1): 35-43.

[3] Mitchell W A, McSaveney M J, Zondervan A, Kim K, Dunning S A and Taylor P J. The Key long Serial rock avalanche, NW Indian Himalayas geomorphology and palaeoseismic implications [J]. Landslides, 2007,(4):245254

[4] Hu C. High and steep slope instability mechanism and the law of collapse rockfall motion [D]. Shandong University, 2014.

[5] Binal A and Ercanoglu M. Assessment of rockfall potential in the Kula (Manisa, Turkey) geopark region [J]. Environmental Earth Sciences, 2010, 61(7): 1361-1373.

[6] Alfredsson K S and Stigh U. Stability of beam-like fracture mechanics specimens [J]. Engineering Fracture Mechanics, 2012, 89: 98-113.

[7] Wang L, Li D F, Ye J, Xu H X and Yu X L. Compressed air brake vehicle engines cycle characteristics [J]. Journal of Zhejiang University (Engineering science), 2014, 48(1): 56-62.

[8] Ye S Q, Chen H K and Tang H M. Rockfall impact force calculation method [J]. China Railway Science, 2010, 31(6): 56-62.