Phantom-Like Behavior in $f(R)$-Gravity

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Abstract

We investigate possible realization of the phantom-like behavior in the framework of $f(R)$-gravity models where there are no phantom fields in the matter sector of the theory. By adopting some observationally reliable ansatz for $f(R)$, we show that it is possible to realize phantom-like behavior in $f(R)$-gravity without introduction of phantom fields that suffer from instabilities and violation of the null energy condition. Depending on the choice of $f(R)$, the null energy condition is fulfilled in some subspaces of each model parameter space.

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1 Introduction

One of the most astonishing discoveries of the last two decades is the observation of a positively accelerated phase of cosmic expansion. This amazing result comes from several sources of observational data such as: measurements of luminosity-distances of supernovae type Ia (SNIa) [1], the cosmic microwave background (CMB) temperature anisotropies with the Wilkinson Microwave Anisotropy probe (WMAP) satellite [2], large scale structure [3], the integrated Sachs-Wolfe effect [4], and the weak lensing [5]. Indeed, general relativity with ordinary matter content of the universe leads to a decelerating universe and therefore it cannot describe this accelerating expansion which has been confirmed by a huge amounts of observational data. In order to realize this late-time acceleration theoretically, several approaches have been proposed. One possibility is to consider an extra source of energy-momentum with a negative pressure in the matter sector of the Einstein field equations. However, the nature of this extra component (the so called dark energy) is yet unknown for cosmologists. A very simple and popular candidate for dark energy proposal is the cosmological constant [6], but this scenario suffers from some serious problems such as huge amount of fine-tuning and coincidence problems. Beside these problems, this scenario has not a dynamical behavior because of a constant equation of state parameter ($\omega_\Lambda = -1$). Another suggestion for dark energy is the dynamical models that include various scalar fields such as quintessence, k-essence, chaplygin gas, phantom fields, quintom fields and so on [7]. On the other hand, one of the most important results of the observational data comes from WMAP5 that the equation of state parameter of dark energy can be less than $-1$ and even can have a transient behavior [8]. While general relativity with one scalar field cannot realize such a crossing behavior, non-minimal coupling of scalar field and gravity leads to this crossing phenomenon [9].

There is another approach to realize the cosmic speedup: modifying geometric part of the gravitational theory. This proposal can be realized in braneworld scenario (DGP model and its extensions [10]), string inspired scenarios (Gauss-Bonnet terms in the action [11]) and so on. A very popular modified gravity model is the so called $f(R)$-gravity [12] where $f(R)$ is an arbitrary function of the scalar curvature $R$. This scenario has the interesting feature that choosing an observationally reliable $f(R)$, it is possible to describe the early inflation as well as the late time acceleration of the universe in a fascinating manner [12]. Recently it has been shown that one can realize the phantom-like effect (increasing of the effective dark energy density with cosmic time and an equation of state parameter less than $-1$) in the normal branch of the DGP cosmological solution without introducing any phantom fields that violate the null energy condition (NEC) [13,14]. This type of the analysis then has been extended by several authors [15]. The main goal in these studies is the realization of the phantom-like behavior without introducing any phantom fields in the matter sector of the theory. In fact, since phantom fields suffer from instabilities and violate the null energy condition, it is desirable to realize this behavior without introduction of phantom fields. With this motivation, in this paper we introduce another alternative to realize phantom like effect: We study possible realization of this behavior in the framework of $f(R)$-gravity models. We consider some observationally reliable versions of $f(R)$ gravity and investigate the phantom-like behavior of each model without introducing any phantom field that violates the null energy condition. Some of these model such as Hu and Sawicki (HS) model have passed the solar system tests in a very good manner as well as the perturbation theory [16]. We show that all of these models in some subspaces of the model parameter space realize a phantom-like behavior without introducing any phantom fields. We study the conditions
that are required in each case to fulfill the null energy condition.

2 \( f(R) \)-gravity

In this section we consider the metric formalism of \( f(R) \)-gravity and we summarize the field equations of the scenario. The action of a general \( f(R) \)-gravity theory is given by [12,17,18,19]

\[
S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \{ f(R) + \mathcal{L}_M \},
\]

(1)

where \( R \) is the scalar curvature, \( f(R) \) is an arbitrary function of \( R \) and \( \kappa = 8\pi G \) is the gravitational constant. The term \( \mathcal{L}_M \) accounts for the matter content of the universe. Using the metric approach, variation of this action with respect to \( g_{\mu\nu} \) provides the field equation

\[
f'(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f(R) + g_{\mu\nu} \Box f'(R) = \kappa T^{(M)}_{\mu\nu},
\]

(2)

where the prime denotes derivative with respect to \( R \) and the matter stress-energy density is defined as

\[
T^{(M)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_M)}{\delta(g^{\mu\nu})}.
\]

(3)

By assuming a spatially flat FRW metric, the Friedmann equation can be written as

\[
H^2 = \frac{8\pi G}{3} \left[ \frac{\rho_M}{f'(R)} + \rho_{\text{curv}} \right],
\]

(4)

where \( \rho_M \) is the energy density of the ordinary matter and \( \rho_{\text{curv}} \) is the energy density of the curvature fluid defined as

\[
\rho_{\text{curv}} = \frac{1}{f'(R)} \left\{ \frac{1}{2} [f(R) - R f'(R)] - 3H \dot{R} f''(R) \right\}.
\]

(5)

Throughout this paper we consider the Jordan frame, thus the continuity equation of the matter sector can be read as usual

\[
\dot{\rho}_M = \rho_M (t = t_0) = 3H_0^2 \Omega_M (1 + z)^3.
\]

(6)

Where \( \Omega_M \) is the present day matter density parameter. The continuity equation for the curvature fluid is given in the following form [17]

\[
\dot{\rho}_{\text{curv}} + 3H (1 + \omega_{\text{curv}} \rho_{\text{curv}}) = \frac{3H_0^2 \Omega_M \dot{R} f''(R) (1 + z)^3}{[f'(R)]^2}.
\]

(7)

By definition, the pressure of the curvature fluid is given by

\[
P_{\text{curv}} = \frac{1}{f'(R)} \left\{ 2H \dot{R} f''(R) + \ddot{R} f'(R) + \dot{R}^2 f''(R) - \frac{1}{2} [f(R) - R f'(R)] \right\}.
\]

(8)
The equation of state parameter corresponding to the curvature sector of the theory can be read as follows

$$\omega_{\text{curv}} = -1 + \frac{\dddot{R} f''(R) + \ddot{R} [\dot{R} f'''(R) - H f''(R)]}{\frac{1}{2} [f(R) - R f'(R)] - 3H \dot{R} f''(R)}.$$  \hspace{1cm} (9)

From the continuity equations (7) and field equation (4), the Hubble rate can be expressed as follows

$$\dot{H} = -\frac{1}{2f'(R)} \left\{ 3H_0^2 \Omega_M (1 + z)^3 + \dddot{R} f''(R) + \ddot{R} [\dot{R} f'''(R) - H f''(R)] \right\},$$  \hspace{1cm} (10)

where \( R = 6(\dot{H} + 2H^2) \). This equation is a very complicated equation and it is very difficult to solve it even with the simplest forms of the \( f(R) \)-gravity.

### 3 Phantom-like behavior of \( f(R) \)-gravity

With phantom-like behavior, we mean an effective energy density which is positive and grows with time and its equation of state parameter stays less than \(-1\). In this section, by adopting some cosmologically viable ansatz, we show that the modified gravity can lead to the effective phantom dark energy and phantom-like behavior without need to introduce any kind of the phantom (negative energy density) scalar fields that violate the null energy condition (see also [18], [19] for some earlier attempts in this regard). To do this end, the modified Friedmann equation (4) can be expressed in a familiar form

$$H^2 = \frac{8\pi G_{\text{eff}}}{3} \left[ \rho_M + f'(R) \rho_{\text{curv}} \right].$$  \hspace{1cm} (11)

This relation shows that in \( f(R) \) gravity the gravitational constant \( G \) can be replaced by an effective gravitational constant \( G_{\text{eff}} = \frac{G}{f(R)} \). The equation (11) can be recast in the following form

$$H^2 = \frac{8\pi G_{\text{eff}}}{3} \left[ \rho_M + \rho_{\text{eff}} \right],$$  \hspace{1cm} (12)

where \( \rho_{\text{eff}} = f'(R) \rho_{\text{curv}} \) with \( \rho_{\text{curv}} \) defined as (5) and therefore effective equation of state parameter is given by

$$\omega_{\text{eff}} = -\frac{1}{f'(R)} + \frac{\dddot{R} f''(R) + \ddot{R} [\dot{R} f'''(R) - H f''(R)]}{f'(R) \left( \frac{1}{2} [f(R) - R f'(R)] - 3H \dot{R} f''(R) \right)}. \hspace{1cm} (13)$$

Now we have all necessary ingredients to discuss phantom-like behavior of \( f(R) \)-gravity. To do this end, we consider some observationally reliable ansatz for \( f(R) \).

#### 3.1 Phantom-like effect with \( f(R) = R + f_0 R^\alpha \)

Phantom-like behavior is the growth of the effective energy density with cosmic time and in the same time, the effective equation of state parameter should stay always less than \(-1\). We mean as a first illustrative example, we consider the following ansatz [19]

$$f(R) = R + f_0 R^\alpha.$$  \hspace{1cm} (14)
with constant $f_0$ and $\alpha$. If $\alpha < 1$, in the small curvature limit the second term dominates. Note that in this class of models, a negative $\alpha$ implies the presence of a term inversely proportional to $R$ in the action that can lead to the present cosmic speed-up [20]. For $\alpha = 0$ the curvature freezes into a fixed value lead to producing a class of models that accelerate in a manner similar to the cosmological constant included models ($\Lambda CDM$). As has been stated, for $\alpha = -1$ this model can describe the acceleration of the universe, but the model evolves in the future into an unstable regime where $1 + f'(R) < 0$ and it does not contain $\Lambda CDM$ as a limiting case of parameter space [21]. Now the expression for effective quantities in this model take the following form

$$\rho_{\text{eff}} = (1 - \alpha)f_0R^\alpha \left[ \frac{1}{2} + \frac{3\alpha H \dot{R}}{R^2} \right]$$ \hspace{1cm} (15)

and

$$\omega_{\text{eff}} = \frac{-1}{1 + \alpha f_0 R^{\alpha - 1}} \left( 1 + \frac{\ddot{R}}{R^2} + \frac{\ddot{R}}{R} (\alpha - 2) - \frac{\ddot{R}}{2R} + 3H \dot{R} \right).$$ \hspace{1cm} (16)

To have an intuition of phantom-like behavior in this case, we adopt the ansatz $a(t) = a_0 t^\nu$. It is important to note that this is a solution of the Friedmann equation in our case. Especially, for $\nu > 1$ it gives an accelerating universe which is essentially realizable in $f(R)$-gravity [12]. In table 1 we have shown the acceptable ranges of $\alpha$ to realize phantom-like behavior in some subsets of the model parameter space. As mentioned before, theoretically negative values of $\alpha$ can account for cosmic acceleration, but they evolve in an unstable regime in the future. Especially, it is clear that for $\alpha = -1$ the null energy condition is violated. While for negative values of $\alpha$, $\rho_{\text{eff}}$ grows with decreasing $z$ but its values always remain negative and the effective equation of state parameter is quintessence-like. The case $\alpha = 0$ is corresponding to an effective cosmological constant with equation of state parameter $\omega_{\text{eff}} = -1$. Our numerical analysis shows that in this case phantom-like behavior can be realized if $0.5 \leq \alpha < 1$. The case $\alpha = 1$ with a redefinition of the Newtonian gravitational constant is corresponding to general relativity. For $\alpha > 1$, the effective equation of state parameter lies in the non-phantom region of parameter space and therefore it cannot account for phantom-like behavior. In figure 1 we plot the effective energy density and equation of state parameter of the model versus the redshift $z$ for $\alpha = 0.5$ (note that this choice is corresponding to $f(R) = R + f_0 \sqrt{R}$ model). As figure shows, in this case the effective energy density increases with decreasing $z$ and the effective equation of state parameter is less than $-1$ a typical realization of the phantom-like effect. From figure 2 we can derive the acceptable ranges of $\alpha$ to fulfill the null energy condition. As this figure shows, null energy condition is respected in some subspaces of the model parameter space and not in the entire parameter space. we note that for $\alpha < 0$ the phantom-like prescription breaks down since in this case $\rho_{\text{eff}}$ is negative.

### 3.2 Phantom-like effect with ln $R$ term

In this subsection we consider a modified gravity scenario with ln $R$ term in the form [19]

$$f(R) = R + \beta \ln \frac{R}{\mu^2} + \gamma R^m.$$ \hspace{1cm} (17)
Table 1: Acceptable range of $\alpha$ to have a phantom-like behavior with $f(R) = R + f_0 R^\alpha$.

| $\alpha$ | $\alpha < -2$ | $-2 < \alpha < -1.5$ | $-1.5 < \alpha < -1$ | $-1 < \alpha < -0.5$ | $-0.5 < \alpha < 0$ | $\alpha = 0$ | $0 < \alpha < 0.5$ | $0.5 < \alpha < 1$ | $\alpha > 1$ |
|----------|----------------|----------------------|-----------------------|----------------------|-------------------|--------------|------------------|----------------|------------|
| $\rho_{\text{eff}}$ | negative      | negative             | negative              | negative             | positive          | positive     | positive         | positive         | positive   |
| $\omega_{\text{eff}}$ | $\omega_{\text{eff}} > -1$ | $\omega_{\text{eff}} > -1$ | $\omega_{\text{eff}} > -1$ | $\omega_{\text{eff}} > -1$ | $\omega_{\text{eff}} = -1$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} > -1$ | $\omega_{\text{eff}} > -1$ |
| NEC      | not respected | respected           | not respected         | respected           | respected         | not respected | respected         | respected         | respected   |

Figure 1: Variation of the effective dark energy density versus the redshift (left hand side). The effective dark energy density increases with decreasing $z$ and therefore shows a phantom-like behavior. The effective equation of state parameter versus redshift (right hand side) which has entered in the phantom phase in the past.

Figure 2: $\rho_{\text{eff}} + p_{\text{eff}}$ versus $\alpha$ with $z = \pm 1, 0$. The null energy condition is violated for some values of $\alpha$, but there are subspaces of the model parameter space that respect this condition.
The second term in this ansatz containing \( \ln R \), is growing at small curvature. Basically this term is induced by quantum effects in curved spacetime. It has been shown that this model has a well defined Newtonian limit and is able to provide the late time acceleration without need to introduce any dark energy component [19]. Choosing \( m = 2 \), this model leads to a very interesting result: unification of the early time inflation and the late time acceleration. On the other hand, considering \( R^2 \) term can suppress the instabilities arises in the perturbation theory of the model as well as improving the solar system bounds, consequently the theory can be viable [19].

Now, the effective quantities in this model attain the following forms

\[
\rho_{\text{eff}} = -\frac{\beta}{2\mu^2} + \frac{\beta}{2} \ln \frac{R}{\mu^2} + (1 - m)\gamma R^2 \left( \frac{1}{2} + \frac{3mH \dot{R}}{R^2} \right) + \frac{3\beta H \dot{R}}{\mu^2 R^2} \quad (18)
\]

and

\[
\omega_{\text{eff}} = \frac{-1}{1 + \frac{\beta}{\mu^2 R} + m\gamma R^{m-1}} + \frac{(\ddot{R} - H\dot{R}) \left[ -\frac{\beta}{2\mu^2} + \frac{\beta}{2} \ln \frac{R}{\mu^2} + (1 - m)\gamma R^2 \left( \frac{1}{2} + \frac{3mH \dot{R}}{R^2} \right) + \frac{3\beta H \dot{R}}{\mu^2 R^2} \right]}{(1 + \frac{\beta}{\mu^2 R} + m\gamma R^{m-1}) \left[ -\frac{\beta}{2\mu^2} + \frac{\beta}{2} \ln \frac{R}{\mu^2} + (1 - m)\gamma R^2 \left( \frac{1}{2} + \frac{3mH \dot{R}}{R^2} \right) + \frac{3\beta H \dot{R}}{\mu^2 R^2} \right]} \quad (19)
\]

In figure 3 we plot the effective energy density and equation of state parameter versus the redshift \( z \) for \( m = 2 \). As this figure shows, in this case the effective energy density has a growing behavior with decreasing \( z \), so it displays the phantom-like behavior without introducing any phantom field. The effective equation of state parameter in the late times lies in the phantom region with no crossing behavior. To realize phantom divide line crossing we can introduce for instance a canonical scalar field in the matter sector of the theory. In figure 4 we have investigated the acceptable ranges of \( m \) to satisfying the null energy condition for \( z = \pm 0.2, 0 \). This figure shows that this model respects the null energy condition for \( m \geq 1.3 \). As has been pointed out in [19], the presence of higher derivative terms like \( R^2 \) (which may be responsible for early time inflation) in this model helps one to pass the existing arguments such as instabilities and solar system tests against such modification of the Einstein gravity.
Figure 3: Variation of the effective dark energy density versus the redshift (left hand side). The effective dark energy density increases with decreasing $z$, so it shows a phantom-like behavior. The effective equation of state parameter is less than $-1$ in the small redshifts (right hand side).

Figure 4: The null energy condition is fulfilled for $m \geq 1.3$. This figure is plotted for the redshifts $z = \pm 0.2, 0$. 
3.3 The Hu-Sawicki model

One of the most interesting modified gravity model has been proposed by Hu and Sawicki (HS model [22]) that can escape the severe constraint imposed by the solar system tests. The form of \( f(R) \) in this model is written as follows

\[
f(R) = R - M^2 \frac{c_1 (\frac{R}{M^2})^n}{c_2 (\frac{R}{M^2})^n + 1},
\]

where \( c_1 \) and \( c_2 \) are arbitrary dimensionless constants while \( M \) has the dimension of mass. This model yields an effective cosmological constant which generates the late-time accelerated expansion [13]. For \( R \gg M^2 \), equation (20) can be expanded to find

\[
f(R) \approx R - M^2 \frac{c_1}{c_2} + M^2 \frac{c_1}{c_2} \left( \frac{M^2}{R} \right)^n.
\]

In the limit of \( c_1 \rightarrow 0 \) at fixed \( c_2 \), this can be realized as an effective cosmological constant \( \Lambda_{\text{eff}} = M^2 \frac{c_1}{c_2} \) which produces the late time acceleration of the universe. The effective energy density in this model is given as follows

\[
\rho_{\text{eff}} = \frac{A}{2} + \frac{A}{2c_2 (\frac{R}{M^2})^n + 1} + \frac{2Ac_2 (\frac{R}{M^2})^n}{[c_2 (\frac{R}{M^2})^n + 1]^2} \left( 1 - \frac{6H \dot{R}}{R^2} \right) - \frac{3H \dot{\rho} A c_2 n^2 (\frac{R}{M^2})^n}{[c_2 (\frac{R}{M^2})^n + 1]^3 R^2} \left[ c^2 (\frac{R}{M^2})^n - 1 \right],
\]

where \( A \equiv M^2 \frac{c_1}{c_2} \). The effective equation of state parameter is a lengthy expression and we do not write it here explicitly. Figure 5 (left hand side) shows the behavior of the effective energy density versus the redshift for \( n = 4 \). Similar to previous cases, the effective energy density increases with decreasing \( z \). The effective equation of state parameter is in the phantom phase too, but it never crosses the phantom divide line (figure 5, right hand side). Figure 6 shows the acceptable ranges of \( n \) to fulfill the null energy condition. Table 2 shows the appropriate subspaces of the model parameter space to have phantom-like behavior and fulfilling the null energy condition in the HS model.
Table 2: Acceptable range of $n$ to have a phantom like behavior.

| value of $n$ | $n < 1.1$ | $1.1 < n < 1.7$ | $1.7 < n < 2.2$ | $2.2 < n < 3.4$ | $n > 3.4$ |
|--------------|-----------|-----------------|-----------------|-----------------|-----------|
| $\rho_{\text{eff}}$ | growing but negative | growing but negative | decreasing | decreasing | growing but negative | growing and positive |
| $\omega_{\text{eff}}$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} < -1$ | $\omega_{\text{eff}} < -1$ |
| null energy condition | not respected | respected | not respected | respected | not respected | respected |

Figure 5: The effective dark energy density versus the redshift (left hand side). The effective dark energy density increases with decreasing $z$ and therefore shows a phantom-like behavior. The effective equation of state parameter remains in the phantom region for small values of redshift (present day and future times evolution of the universe)(right hand side).
4 Summary and Conclusion

In this paper we have studied possible realization of the phantom-like behavior in some viable $f(R)$ gravity models. By phantom-like behavior, we mean increasing of the effective energy density with cosmic time while the effective equation of state parameter is less than $-1$. We have shown that some models of $f(R)$ gravity can display a phantom-like behavior without violating the null energy condition in some subspaces of their model parameter space. To do this end, first we have considered a modified gravity model with $f(R) = R + f_0 R^\alpha$ and we found that the phantom-like behavior can be obtained in the region of parameter space with $0.5 \leq \alpha < 1$ and in this domain null energy condition is respected. Although for negative values of $\alpha$ the effective energy density has an increasing behavior with cosmic time, but the null energy condition is violated. In the second stage, we have considered a model of modified gravity with a $\ln R$ and an additional power law term. With a suitable choice of the model parameters, this model which has potential to describe the early time inflation and late time acceleration of the universe, accounts for realization of the phantom-like behavior too. There are appropriate subspaces of the model parameter space that null energy condition is respected for this choice of $f(R)$. Finally we have considered the Hu-Sawicki which has a very good phenomenology and has been successful to pass the sever constraints imposed by solar system tests and the perturbation theory. We showed that this model is also capable to account for phantom like behavior without introducing any phantom fields that violate the null energy condition in the spirit of modified gravity.

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