An elastic-plastic analysis of interference fit connection

J F Jiang\textsuperscript{1,*} and Y B Bi\textsuperscript{2}

\textsuperscript{1}Qianjiang College, Hangzhou Normal University, Hangzhou, 310036, China
\textsuperscript{2}Department of Mechanical Engineering, Zhejiang University, Hangzhou, 310027, China

Corresponding author and e-mail: J F Jiang, jiang19jie81@zju.edu.cn

Abstract. Interference fit connection technique can effectively improve the fatigue life of aircraft structure. Considering the engineering practice, in this paper, the pin treated as an elastic deformed body is introduced in the analysis of interference fit connection. In the theoretical derivation, we obtain the contact stress between the hole and pin, the interference fit size of the elastic limit, and also the relationship between the plastic zone radius and interference fit size. At the same time, the finite element model is established, and the numerical simulation is conducted. By comparing theoretical and finite element results, the theoretical formulas are verified, which can be used furtherly in quick engineering calculation.

1. Introduction

The interference fit is a common connection technology, which has been very widely used in the field of mechanical connection. Interference fit connection is that there exists interference between the pin (or bolt) and hole, as a technique for improving the structure fatigue life, which has been applied in aircraft assembly [1-5]. In the field of the interference fit, a large number of scholars at home and abroad did lots of theoretical studies, to reveal the essence of interference fit connection.

Ярковец deduced the plate’s hole vicinity stress equation as interference fit size is less than 0.4% [6]. Right now the material is still in the elastic range, so the equation is only suitable for the structure sustained the low-level stress. In the elastic-plastic scope, the interference fit calculation is quite complicated. Wu simplified the interference fit connection as plane strain problem for stress analysis, but the fastener was treated as a rigid body, without considering the fastener elastic deformation [7].

In modern times, the finite element technology gradually develops maturely, which can be used to deal with interference fit problems and to compare with the theoretical solution. Pedersen, Paredes studied the interference fit connection in the elastic scope, Lame formula was used to analyze the average contact stress at the hole wall, and the result was close to the finite element results [8,9]. Jiang et al. have developed a two-dimensional axis-symmetric finite element model to simulate the interference fit bolt insertion process, and then analyzed the deformation and residual stress distribution around the hole [10,11].

In this paper, the pin is treated as a elastic body (because the fastener’s plastic limit is generally far more than the hole plate’s, when the plate occurs plastic deformation, the fastener is still in the elastic scope). By using the thick wall cylinder theory, interference fit connected problem is analyzed theoretically. Elastic and elastic-plastic analysis is considered respectively. Finite element model of
interference pin insertion is established, and numerical simulation is conducted. By an example, the comparative analysis is conducted between theoretical solutions and finite element results.

2. Theoretical analysis of elastic and elastic-plastic deformation

By using the thick-wall cylinder theory in the elastic-plastic mechanics, an elastic-plastic model of the interference fit is built up when the pin with larger diameter \( b_1 \) is inserted into the hole \( a_2 \), as shown in figure 1. Interference fit size \( I \) is the most important parameter in interference fit connection, and which often is used in the form of relative interference, here it can be defined as

\[
I = \frac{\delta}{a_2} = \frac{b_1 - a_2}{a_2} \times 100\%
\]  

(1)

Where \( \delta \) denotes actual interference, \( b_1 \) is the pin radius, \( a_2 \) is the hole radius. (In the paper, the parameters of pin are labeled with subscript 1, the parameters of plate holes are marked with subscript 2.)

![Figure 1. Elastic and plastic model of the interference fit.](image)

2.1. Elastic deformation

In order to simplify the problem, the interference fit connection is defined as the axis-symmetric plane strain problem. When the interference fit size is small, the pin and hole occurs elastic deformation, as shown in figure 1 (b). And \( p \) is the contact stress of pin and hole, \( r \) is the radius of one point around plate hole.

Stress and displacement of the pin are given by

\[
\sigma_{r1} = \sigma_{\theta 1} = -p \ , \ u_{r1} = -\frac{p(1+\nu_1)}{E_1}(1-2\nu_1)r
\]  

(2)

Where \( \sigma_{r1}, \sigma_{\theta 1}, u_{r1} \) denote the radial stress component, tangential stress component and radial displacement of the pin.

The hole vicinity stress distribution and displacement are given by

\[
\sigma_{r2} = \frac{a_2^2}{b_2^2 - a_2^2} p(1 - \frac{b_2^2}{r^2}) \ , \ \sigma_{\theta 2} = \frac{a_2^2}{b_2^2 - a_2^2} p(1 + \frac{b_2^2}{r^2}) \ , \ u_{r2} = \frac{(1+\nu_2)a_2^2b_2^2}{E_2(b_2^2 - a_2^2)} \left( \frac{1-2\nu_2}{b_2^2} \right) \left( \frac{1}{r} \right) + \frac{1}{r}
\]  

(3)

Where \( \sigma_{r2}, \sigma_{\theta 2}, u_{r2} \) are the radial stress component, tangential stress component and radial displacement around the hole.

Because \( u_{r1} \) and \( u_{r2} \) are the smaller values, at contact surface between pin and hole wall, \( r = a_2 = b_1 \) is supposed, so the total deformation (or displacement) is given as
\[ \delta = u_1 + u_2 = -\frac{p(1+v_2)}{E_1}(1-2v_1)a_2 + \left(1 + v_2\right) \frac{b_2^2 p}{E_2(b_2^2 - a_2^2)} \left(\frac{1-2v_2}{b_2^2 - a_2^2} a_2 + \frac{1}{a_2}\right) \]  
\hspace{1cm} (4)

The contact stress of the hole wall and pin can be derived as
\[ p = \delta \left[ \frac{(1+v_1)a_2 b_2^2}{E_2(b_2^2 - a_2^2)} \left(\frac{1-2v_2}{b_2^2 - a_2^2} a_2 + \frac{1}{a_2}\right) - \frac{(1+v_1)(1-2v_1)a_2}{E_1} \right] \]  
\hspace{1cm} (5)

When the hole happens to yield, existing Tresca condition \( \sigma_{\theta 2} - \sigma_{\theta 2}' = \sigma_s \), the limit of yield (\( \sigma_s \)) is given as
\[ \sigma_s = \sigma_{\theta 2} - \sigma_{\theta 2}' = \frac{a_2 b_2^2}{b_2^2 - a_2^2} p \frac{2}{r^2} = \frac{2b_2^2}{b_2^2 - a_2^2} p \]  
\hspace{1cm} (6)

When the plate is infinite, \( b_2 \to \infty \), and Eq. (5) is fed in Eq. (6), which can be rewritten as
\[ \sigma_s = \frac{2\delta}{(1+v_1)a_2} - \frac{(1+v_1)(1-2v_1)a_2}{E_2} \]  
\hspace{1cm} (7)

And then the interference fit size of elastic limit can be derived as
\[ l = \frac{\delta}{a_2} = \frac{1+v_1}{E_2} - \frac{(1+v_1)(1-2v_1)}{E_1} \]  
\hspace{1cm} (8)

The above equation is the maximum interference fit size for hole elastic deformation.

2.2. Elastic-plastic deformation

Elastic-plastic deformation occurs in the hole, as shown in figure 1 (c), and \( p_p \) is the contact stress of pin and hole wall, \( r_p \) is the radius of the plastic zone, \( q \) is the stress at the junction of the elastic zone and the plastic zone. Plastic deformation parameters that are same or similar to elastic deformation, which are added with superscript ' to distinguish.

In the plastic zone, stress components should satisfy equilibrium equation and yield conditions:
\[ \frac{d\sigma_{\theta 2}'}{d\sigma} + \frac{\sigma_{\theta 2}' - \sigma_{\theta 2}}{r} = 0 \]  
\hspace{1cm} (9)

\[ \sigma_{\theta 2}' - \sigma_{\theta 2} = \sigma_s \]  
\hspace{1cm} (10)

Eq. (10) is fed in Eq. (9), it can be rewritten as
\[ \frac{d\sigma_{\theta 2}'}{d\sigma} - \frac{\sigma_s}{r} = 0 \]  
\hspace{1cm} (11)

Eq. (11) is integrated as \( \sigma_{\theta 2}' = \sigma_s \ln r + C \).

On the hole wall \( (r=a_2) \), using boundary condition \( \sigma_{\theta 2}' = -p_p \), the constant \( C \) can be obtained as \( C = -p_p - \sigma_s \ln a_2 \).

Then stress components of plastic zone are given as
\[ \sigma_{\theta 2}' = \sigma_s \frac{\ln r}{a_2} - p_p, \sigma_{\theta 2} = \sigma_s \left(1 + \ln r\right) - p_p \]  
\hspace{1cm} (12)
And on the plastic zone boundary \((r=r_p)\): \(\sigma_{r_2} = -q\), so \(q = p_p - \sigma_s \ln \frac{r_p}{a_2}\).

In the elastic zone, radial stress and tangential stress components are given as

\[
\sigma_{r_2} = -\frac{r_p^2}{b_2^2 - r_p^2} q (1 - \frac{b_2^2}{r}) , \quad \sigma_{\theta_2} = -\frac{r_p^2}{b_2^2 - r_p^2} q (1 + \frac{b_2^2}{r})
\]

(13)

And on the elastic zone boundary \((r=r_p)\): \(\sigma_{\theta_2} - \sigma_{r_2} = \sigma_s\), then \(q = \frac{\sigma_s}{2} (1 - \frac{a_2^2}{b_2^2})\).

The stress \(q\) on plastic zone and elastic zone boundary are equal, and then the contact stress is obtained:

\[
p_p = \sigma_s \left( \ln \frac{r_p}{a_2} + \frac{1}{2} - \frac{r_p^2}{2b_2^2} \right)
\]

(14)

In the plastic zone, according to plane strain condition \((\varepsilon_z = 0)\) and volume incompressible condition, the equation \(\varepsilon_r + \varepsilon_\theta = 0\) exists. By using geometric equation, the equation can be rewritten as

\[
\frac{d u_2'}{dr} + \frac{u_2'}{r} = 0
\]

(15)

Eq. (15) is integrated as \(u_2' = \frac{C_1}{r}\).

On the elastic boundary \((r=r_p)\), the displacement is given as

\[
\left. u_2' \right|_r = \left(1 + \nu_2 \right) \sigma_s \frac{r_p}{E_2 (b_2^2 - r_p^2)} [ (1 + 2\nu_2) r_p + \frac{b_2^2}{r} ]
\]

(16)

The stress on boundary \(q = \frac{\sigma_2}{2} (1 - \frac{a_2^2}{b_2^2})\) is fed in Eq. (16). And on the junction of the elastic and the plastic zone, according to the continuity of displacement, the following equation exists:

\[
\left. u_2' \right|_r = \left. u_2' \right|_s = \frac{1 + \nu_2 \sigma_s r_p^2}{2E_2 b_2^2 r_p} [ (1 + 2\nu_2) r_p^2 + b_2^2 ] - \frac{C_1}{r_p}
\]

(17)

From Eq. (17), the constant of \(C_1\) can be obtained, and the displacement of the hole elastic-plastic deformation has been built up:

\[
\left. u_2' \right|_s = \frac{(1 + \nu_2 \sigma_s r_p^2}{2E_2 b_2^2 r} [ (1 + 2\nu_2) r_p^2 + b_2^2 ]
\]

(18)

The displacement of the pin elastic deformation is given as

\[
u_1' = -\frac{p_p (1 + \nu_2 \sigma_s r_p^2}{E_1} (1 - 2\nu_1) b_1
\]

(19)

Adding Eq. (18) to Eq. (19) and considering \(r=a_2=b_1\), the total displacement is derived as

\[
\delta' = u_1' + u_2' = -\frac{p_p (1 + \nu_2 \sigma_s r_p^2}{E_1} (1 - 2\nu_1) a_2 + \frac{(1 + \nu_2 \sigma_s r_p^2}{2E_2 b_2^2 r} [ (1 + 2\nu_2) r_p^2 + b_2^2 ]
\]

(20)
Eq. (14) is fed in the Eq. (20), and the interference fit size is given as

$$I = \frac{\delta}{a_z} = \frac{(1+\nu_1)(1-2\nu_1)}{E_1} \sigma_p \left( \ln \frac{r_p}{a_z} + 2 \frac{r_p^2}{2b_s^2} \right) + \frac{(1+\nu_2)\sigma_{fs}}{2E_2} \left[ (1-2\nu_2)r_p^2 + b_s^2 \right]$$

Equation (21)

When the elastic-plastic deformation of the hole occurs, Eq. (21) shows the relation of interference fit size and plastic zone radius.

3. Finite element model of interference fit pin insertion

To improve the calculated efficiency of the interference fit problem, the two-dimensional axisymmetric finite element model is employed in ABAQUS [12]. To facilitate the finite element calculation, the simplifying model is necessary. The import depart of pin front has a chamfer of 10° angle, which is advantageous to insert the pin into the hole and to reduce the insertion force. The plate’s thickness is 5 mm, the outer radius is 30 mm. If the pin or hole’s radius is 3 mm, the outer radius is equal to 10 times of the pin radius, so the big plate can approximately present the infinite plate. At the plate hole entry, there is an appropriate chamfering angle for convenient assembly. The finite element model of pin and plate’s initial assembly is shown in figure 2. Enhanced hourglass control unit CAX4R is chosen to mesh the model.

In the interference fit pin insertion, there exists obvious contact problem. In order to prevent a kind of the material embedded into another kind, we must define the contact pairs including the positive and slave surfaces. In this example, the pin shank and the imported surface is defined as the positive surface, the hole wall as the slave surface. Under ideal conditions, only the extrusion effect of the import depart acting on hole wall is considered, and the friction is ignored, so friction coefficient of 0 is defined between contact surfaces.

The boundary conditions are shown in figure 2. On the plate bottom surface, the boundary condition $U_Z=0$ is applied; which present there is a support. At the outer boundary of the plate ($R=30$mm, it means the plate is large enough), $U_R=0$ is applied. Because the pin is an axis-symmetric model, so there exists $U_R=0$, $U_Z=0$ on the central line of the pin ($R=0$). The pin is pressed into the hole using uniform velocity, the distance constraint $U_Z=-7$ mm is imposed on the pin top, which can ensure that the pin is filled in the hole completely.

A static analysis step is chosen, using the default time of 1s. In the same boundary conditions and constraint, by modifying the hole radius in the model, the pin insertion simulations with different interference fit size are carried out.

Figure 2. Finite element model.
4. Example and comparison

In the example, M6 Ti-6Al-4V titanium alloy bolt (pin) and 7050-T7451 aluminum alloy plate is chosen as the research objects, and the theoretical solution and the finite element result are compared. The performance of the two materials is shown in Table 1.

Table 1. Mechanical property of two materials.

| Material         | Density $\rho$ (g/mm$^3$) | Elastic modulus $E$ (MPa) | Poisson’s Ratio $\nu$ | Yield limit $\sigma_y$ (MPa) | Tensile limit $\sigma_b$ (MPa) |
|------------------|-----------------------------|---------------------------|-----------------------|-----------------------------|---------------------------------|
| Titanium alloy   | 4.8×10$^{-3}$               | 110,000                   | 0.3                   | 1,068                       | 1,137 ($\varepsilon_{pi}^{pl}$=0.18) |
| (Ti-6Al-4V)      |                             |                           |                       |                             |                                 |
| Aluminum alloy   | 2.8×10$^{-3}$               | 72,000                    | 0.33                  | 470                         | 525 ($\varepsilon_{pi}^{pl}$=0.15) |
| (7050-T7451)     |                             |                           |                       |                             |                                 |

4.1. Elastic limit interference fit size

As can be seen from Eq. (8), the maximum elastic interference fit size is not related to the hole radius ($a_2$). Elastic modulus and Poisson’s ratio in the table are substituted into Eq. (8), and interference fit size of the elastic limit is gotten as $I=0.33\%$.

In finite element model, the hole diameter is 6 mm ($a_2=3$ mm), the pin insertion is simulated. The simulation results show that when the interference fit size is 0.5%, the stress at the hole entry is just over 470 MPa (begin to yield). So the interference fit size of elastic limit has been gotten, $I=0.5\%$. Finite element result is greater than the theoretical solution.

4.2. Plastic zone radius

Eq. (21) shows that when the hole elastic-plastic deformation occurs, the plastic zone radius is related to the hole radius ($a_2$). The Elastic modulus, Poisson’s ratio, Yield limit and other parameters of two kinds of material are substituted into Eq. (21). When $a_2=3$ mm (bolt or pin’s diameter is 6 mm) and $b_2=30$ mm, the Eq. (21) is given by

$$I = 4.727 \times 10^{-6} \cdot 470 \cdot (\ln \frac{r_p^2}{3} + 0.5 - \frac{r_p^2}{2 \cdot 30^2}) + 1.140 \times 10^{-9} \cdot 470 \cdot (0.34 r_p^2 + 30^2)$$

The above formula is a nonlinear equation, the theoretical result can be gotten with the aid of Matlab software. If the interference fit size is given, the plastic zone radius can be calculated. In the finite element simulation result, we can measure the plastic zone radius when the interference fit size is a certain value.

Considering the beneficial range (0-3%) in the interference riveting, in this example, a series of interference fit size (from interference fit size of elastic limit to 3%) are chosen. When $a_2=3$ mm, the relation curves of plastic zone radius and interference fit size are shown in figure 3 from theoretical calculation and finite element results.
As can be seen from the figure 3, when the interference fit size is the same, the plastic zone radius from the finite element results is less than the theoretical solution. And the biggest relative gap is 12%. When interference fit sizes are 1% and 1.5%, the difference between finite element results and theoretical solutions are larger. And the interference fit sizes are 2% and 2.5%, the difference is smallest. As the interference fit size reaches to interference fit size of 3%, the gap increases again.

From the above analysis of the example, there has a certain gap between theoretical solution and finite element results. In the range of the hole elastic deformation, for interference fit size of elastic limit, the finite element result is 0.17% bigger than theoretical result. In the range of elastic-plastic deformation, for plastic zone of the same interference fit size, the theoretical solution is bigger than finite element result, and the biggest relative gap is 12%. The reason why relative difference occurs maybe is that the simplified conditions from theoretical derivation and finite element model are not exactly same, and the location of measuring plastic zone radius from the finite element, etc. The relative difference is in the acceptable range.

Since the theoretical formulas are verified by comparing theoretical and finite element results, which can be used for quick calculating the contact stress, interference fit size of elastic limit, the plastic zone radius in engineering practice. Also, according the stress requirements, the formulas are used to estimate concrete interference fit size.

5. Conclusions
By using the theory of thick-wall cylinder, the problem of interference fit connection is analyzed theoretically. In the range of the elastic deformation, interference fit size of the elastic limit is derived. Within the scope of the hole elastic-plastic deformation, the relation between the plastic zone radius and interference fit size is established.

Finite element model is established, and the interference fit pin insertion was simulated. By comparison of the theoretical solution and the finite element results in the example, the relative difference between theoretical and finite element results is in the acceptable range. So the theoretical formulas about the contact stress, the interference fit size and the plastic zone radius are available for quick calculation or estimation in the engineering practice.

Acknowledgement
This study is supported by Natural Science Foundation of Zhejiang Province (LGG18E050018) and General research project of Zhejiang Education Department (Y201737162) in China.
References

[1] Abazadeh B, Chakerlou T N, Farrahi G H and Akkerliesten R C. 2013 Fatigue life estimation of bolt clamped and interference fitted-bolt clamped double shear lap joints using multi-axial fatigue criteria. *Materials and Design* **33**: 327-336

[2] Kim S Y, Hennigan D J, Kim D and Seok C S 2012 Fatigue enhancement by interference-fit in a pin-loaded glass fiber-reinforced plastics laminate. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* **226** (6): 1437-1446

[3] Jiang J F, Bi Y B, Dong H Y, Ke Y L, Fan X T and Du K P 2014 Influence of interference fit size on hole deformation and residual stress in hi-lock bolt insertion. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* **228** (18): 3296-3305

[4] Bi Y B, Jiang J F and Ke Y L 2015 Effect of interference fit size on local stress in single lap bolted joints. *Advances in Mechanical Engineering* **7** (6): 1-12

[5] Jiang J F and Bi Y B 2016 Effect of parameters on local stress field in single-lap bolted joints with the interference fit. *Advances in Mechanical Engineering* **8** (5): 1-12

[6] Ярковец А И 1991 *The plane long life bolt connection and riveting technology*, translation by Zhang G L, Beijing: Aviation industry press

[7] Wu S 1990 Interference fit fastener holes of elastic-plastic engineering analysis. *Journal of Nanjing University of Aeronautics and Astronautics* **22** (4): 17-24

[8] Pedersen P 2006 On Shrink Fit Analysis and Design. *Computational Mechanics* **37**: 121-130

[9] Paredes M, Nefissi N and Sartor M 2012 Study of an interference fit fastener assembly by finite element modeling, analysis and experiment. *International Journal of Interactive Design and Manufacture* **6**: 171-177

[10] Jiang J F, Dong H Y and Ke Y L. 2013 Maximum interference fit size of hi-lock bolted joints. *Chinese Journal of Mechanical Engineering* **49** (3): 145-152

[11] Jiang J F, Dong H Y and Bi Y B 2013 Analysis of process parameters influencing protuberance during interference fit installation of hi-lock bolt. *Acta Aeronautica et Astronautica Sinica* **34** (4): 936-945

[12] ABAQUS 6.10-1 Software 2010 *ABAQUS analysis user’s manual*, Rhode Island: Dassault Systems Simulia Corporation