Comment on “Can gravity distinguish between Dirac and Majorana neutrinos?”

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In a recent letter\(^1\), Singh, Mobed and Papini (hereafter referred to as (I)) claim to show that the gravitational field associated with the Lense-Thirring metric can distinguish between Dirac and Majorana neutrinos. According to these authors, the matrix element of the interaction Hamiltonian is significantly different, depending on the Dirac or Majorana nature of the neutrino.

Here we point out that the treatment of Majorana neutrinos in (I) is not valid, due to an incorrect definition and construction of the spinor that corresponds to a Majorana particle. As a consequence, the results of this paper concerning Majorana neutrinos are not valid, and in particular the major claim stated above does not follow.

The Majorana condition is a requirement on the fermion field operator \(\psi\), viz., that it should be self-conjugate up to a phase factor \(\xi\),
\[
\psi^c(x) = \xi \psi(x), \tag{1}
\]
where the superscript \(c\) indicates the operation of the Lorentz-invariant complex conjugation. Instead of Eq. (1), the authors of (I) imposed the self-conjugacy condition on the spinors to construct a spinor \(W\) for the Majorana case. This is incorrect, and in particular the spinor \(W\) is not a solution of the free particle Dirac equation.

To be more specific, let us consider their Eqs. 10 and 11. In a more conventional notation, the authors write the spinor for the Dirac case in the form
\[
u(k) = u_L + u_R, \tag{2}
\]
where \(u_{L,R}\) are the left and right chiral projections of \(\nu(k)\). These two components satisfy
\[
\not{k} u_L = m u_R, \quad \not{k} u_R = m u_L, \tag{3}
\]
so that \(u(k)\) satisfies the Dirac equation \(\not{k} u = m u\). The spinors chosen in (I) for the Majorana case are given in their Eq. 11, which in our notation is
\[
W_{1,2} = u_L \pm (u_L)^c. \tag{4}
\]
This construction is meaningless. In fact, using Eq. (4) it can be verified that the spinors \(W_{1,2}\) are not solutions of the free particle equation \(\not{k} W_{1,2} = m W_{1,2}\), as they should be.

A Majorana free particle spinor satisfies the same equation as the Dirac free particle spinor but, in accord with Eq. (4), the four linearly independent solutions appear in the plane wave expansion of the Majorana field in the form
\[
\psi(x) = \int \frac{d^3p}{N_p} \left( a_{\lambda}(p) u_{\lambda}(p) e^{-ip \cdot x} + \xi^* a_{\lambda}^*(p) v_{\lambda}(p) e^{ip \cdot x} \right), \tag{5}
\]
with an implied sum over helicities \(\lambda\), \(v = u^c\), and the factor \(N_p\) that depends on the normalization of the spinors. For a Dirac field, the creation operator in Eq. (5) would be \(b_{\lambda}^d\), which is not the hermitian conjugate of \(a_{\lambda}\), and Eq. (4) would not hold. For a given operator bilinear in the Majorana field, its matrix element between two states of momenta \(p\) and \(p'\) is of the form
\[
\langle p' | \bar{\psi} O \psi | p \rangle \propto \bar{u}(p') O u(p) - \bar{v}(p) O v(p'). \tag{6}
\]
Two terms are obtained since both \(\psi\) and \(\bar{\psi}\) can either create or annihilate a particle. This is quite a different thing than calculating \(\bar{W} O W\) with the spinors given in Eq. (4).

Leaving behind the fact that the calculation for the Majorana case is not correctly done in (I), there remains the question of whether or not the relevant matrix element of the gravitational interaction term considered in (I) is different depending on whether the neutrino is a Dirac or a Majorana particle. Similar questions have been posed in the existing neutrino literature numerous times. As is well known, careful consideration of all the issues involved, such as the fact that only the matrix elements between the left-handed projections are physically relevant, shows in all cases that any differences are proportional to factors of \(m^2 / \langle E \rangle\), so that they vanish in the zero-mass limit.\(^2\) The situation in the present context is just the same in this respect. In particular, with the mean energy of the neutrino taken to be 1 MeV in (I), any difference between the Dirac and Majorana cases are unobservable in that context.

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\(^1\) Dinesh Singh, Nader Mobed and Giorgio Papini, Phys. Rev. Lett. 97, 041101 (2006).
\(^2\) See, e.g., Sec. 12.4 of “Massive neutrinos in Physics and Astrophysics”, by R. N. Mohapatra and P. B. Pal, (World Scientific, 3rd edition, 2004), and references therein.