Performance Analysis of Absolute Acceleration Shock Response Spectrum

Wenjuan Sun¹,a, Qiaoping Su²,b, Hongli Yuan¹,c and Song Chen¹,d,*
¹Institute of Information Engineering, Anhui Xinhua University, Hefei, China
²Electronic communication engineering college, Anhui Xinhua University, Hefei, China
a,bsunwenjuan@axhu.edu.cn, cyuanhongli@axhu.edu.cn,
* Corresponding Author: dchensong@axhu.edu.cn

Abstract. The Absolute Acceleration Shock Response Spectrum (AASRS), which is most commonly used in engineering, is analyzed in detail, and the effects of shock duration, shock amplitude, and shock waveform on AASRS are discussed respectively. Shock duration has the greatest influence on the shape and change trend of AASRS, followed by shock amplitude. However, shock amplitude only affects the amplitude of AASRS and does not change the overall change trend. Shock waveform has the smallest influence on AASRS and mainly affects the frequency point of peak acceleration.

1. Introduction
Shock response spectrum (SRS) has proven to be an effective standard tool for engineering analysis and quantification of shock environment [1-3], which describes the relationship between maximum response and natural frequency of a series of single degree-of-freedom (SDOF) oscillators under given base excitation. SRS transforms time domain shock excitation into frequency domain representation, which can be defined in the form of displacement, velocity or acceleration. Amongst different types of SRS, Absolute Acceleration SRS (AASRS) has been commonly used for shock test specifications and environment adaptability design in aerospace, electronics and other industries [1, 3-4]. AASRS is a calculation function based on acceleration time history that reflects the mapping between acceleration shock excitation in time domain and the maximum absolute acceleration response in frequency domain.

AASRS is a standard tool for structural dynamics analysis and given test specifications in aerospace. The most commonly used AASRS in engineering is analyzed in detail, and some enlightening conclusions are obtained.

2. Definition and Calculation Method of AASRS
Definition of AASRS. Shock response spectrum is a representation of transient shock acceleration signal in frequency domain. Biot [5] introduced the concept of SRS for the first time when studying the influence of earthquake on buildings. SRS allows the shock effect to be characterized on a dynamic standardization system to evaluate its severity or destructive potential. AASRS is defined as the relationship between the peak absolute acceleration response and the natural frequency of a series of single-degree-of-freedom (SDOF) oscillators with different frequencies excited by the same shock transient load. Therefore, the time domain impulse excitation is converted into AASRS representation in
the frequency domain, and the basic transformation relation is shown in Fig. 1 [6]. Different AASRS will be obtained for different shock excitation. Its value has no direct relation with the structural system to be studied, but only the shock excitation will be described by standard response tool (SDOF oscillator). The essence of AASRS still represents external excitation, but it is described by the results of standard response tool.

AASRS, as an effective tool to describe shock environment, has been widely used in the fields of national defense, aviation and earthquake.

Calculation Method of AASRS. To simplify the discussion, we consider a single SDOF. Fig. 2 shows a SDOF, where m, c and k are the mass, damping coefficient and stiffness of the SDOF respectively, \( f_n \) is its natural frequency, \( \ddot{y} \) is the input shock excitation acceleration signal to the SDOF, and \( \ddot{x} \) is the response under the shock signal.

According to Newton's law, the following governing differential equations can be obtained:

\[
m\ddot{x} + c\dot{x} + kx = cy + ky
\]

Defining the relative displacement \( z=x-y \) of the mass relative to the base, thus,

\[
m\ddot{z} + c\dot{z} + kz = -mx
\]

Undamped natural frequency:

\[
\omega_n = \sqrt{\frac{k}{m}}
\]

Damping ratio of system:
\[ \xi = \frac{c}{2m\omega_n} \]  

It should be noted that \( \xi \) is usually expressed by an amplification factor \( Q \), which is called the quality factor. Substituting equations (3) and (4) into equation (2) gives the motion equation of relative response:

\[ \ddot{z} + 2\xi\omega_n\dot{z} + \omega_n^2 z = -\ddot{y} \]  

The differential equation of Eq. 5 can be solved to obtain the relationship between the response peak value \( \max(\ddot{x}) \) and the natural frequency, thus converting the time domain acceleration into the response spectrum expression in the frequency domain.

The acceleration time domain excitation is applied to a series of SDOF oscillators with different natural frequencies. The improved slope-invariant digital filter recursive algorithm proposed by Smallwood [7] has become a commonly used numerical algorithm and engineering standard for AASRS [8] because of its clear physical meaning, concise algorithm, fast calculation speed and high computational accuracy.

The AASRS most commonly used in engineering is analyzed in detail below.

Calculation Steps of AASRS. AASRS calculation is carried out according to the following steps:

1. Input: the shock load on the structure is usually given in the form of acceleration-time history.
2. System: a series of single-degree-of-freedom mass-spring-damping systems. The selection of natural frequency of the system should include the frequency range required for calculation. In Fig. 3, the number of mass-spring-damping systems is numbered 1 to \( i \), where \( i \) is the total number of systems.

3. Response: record the acceleration and time response of each SDOF system to determine the peak value of each response.
4. Output: the peak acceleration response of each mass-spring-damping system is plotted as a function of the natural frequency of the system.

3. Performance Analysis of AASRS

Effect Analysis of Shock Duration Time on AASRS. In order to analyze the effect of shock duration time on AASRS, a typical half sine shock waveform is selected as the input load. Its acceleration peak is 1000 g, and the shock duration time \( t \) is taken as 2 ms, 4 ms, 6 ms, 8 ms, 10 ms and 14 ms respectively, thus obtaining six input excitation as shown in Fig. 4. Six kinds of half sine pulses are used as input excitation in turn, and the six AASRSs are obtained as shown in Fig. 5.
The results in Fig. 5 show that the six AASRS curves have the same maximum amplitude, while the frequency points of the maximum amplitude are different. From the overall trend, the shock duration has a greater effect on the low-frequency AASRS, showing a trend that the longer the duration, the higher the amplitude. This is because the shock attenuation is very fast for short duration, so the change of AASRS at the beginning is more severe than other curves. However, AASRS at high frequency is not sensitive to shock duration, especially at frequencies above 1000Hz, the amplitude is basically the same.

In addition, the duration of shock excitation also plays an important role in the energy transferred to the excited system. The energy contained in the shock pulse is proportional to the area enclosed by the acceleration-time curve. Therefore, a shock pulse with a longer duration gives the system more energy than a shorter duration pulse with the same amplitude.

Effect Analysis of Shock Amplitude on AASRS. The effect of acceleration amplitude on AASRS for the same duration is investigated. In the following, a half sine pulse with a shock duration of 8 ms is selected, and its acceleration amplitudes are 100g, 500g, 1000g, 1500g, 2000g and 3000g respectively, so as to obtain five half sine pulse input signals with different peaks and the same duration as shown in Fig. 6. The calculated AASRS curves are shown in Fig. 7.
The results in Fig. 7 show that for acceleration pulse signals with the same duration and different amplitudes, the change trend of AASRS is basically the same, but the amplitudes are different. The whole curves show that the amplitude gradually increases at low frequency after reaching the highest point, then gradually decrease, and after reaching 300Hz, the trend to be straight.

Effect Analysis of Shock Waveform on AASRS. The above discussion is based on half sine pulse, and this section will discuss the influence of different waveforms on AASRS. The ideal simple shock load can be expressed in a simple mathematical way, such as half sine, triangle, versed sine, and terminal peak sawtooth [9]. Four typical waveforms are selected, with acceleration amplitude of 1000 g and shock duration of 8 ms, the four waveforms are shown in Fig. 8. And the calculated AASRS curves are shown in Fig. 9.
Fig. 9 Comparison results of AASRS

The results in Fig. 9 show that the four AASRS curves corresponding to four different pulse shapes have the same change trend, with the increase of frequency, the AASRS firstly increase to reach the acceleration peak value, then gradually decrease, and then tend to be stable.

But they also have some differences.

1. Half sine, triangle and versed sine have similar AASRS shapes. Especially for triangle and versed sine, the AASRS acceleration amplitudes at all frequency points are almost identical except for the different peak acceleration.

The main reason is that the area enclosed by this two waveforms and the acceleration-time curve is basically equal, and the area is proportional to the energy contained in the shock pulse, which further indicates that AASRS is an effective quantification tool for shock energy, and different acceleration waveforms may have the same AASRS.

2. The AASRS of terminal peak sawtooth is basically the same as triangle and versed sine except that it is lower than the other three waveforms around 100Hz.

On one hand, the waveform change of terminal peak sawtooth and the time point of peak acceleration in time domain are different from the other three waveforms, resulting in different frequency points of AASRS peak; on the other hand, since the area enclosed by the acceleration-time curve is basically the same as that of triangle and versed sine, the three waveforms have the same AASRS variation curve and amplitude at low-high frequencies.

3. The AASRS amplitude of half sine pulse is higher than the other three kinds of waveforms in the whole frequency band, mainly because the area enclosed by the half sine AASRS and the acceleration-time curve is larger than the other three kinds of pulses.

4. The amplitude and shape of AASRS of the above four kinds of waveforms are completely consistent in high frequency band, and all tend to be flat, mainly because the input of shock energy is limited and decreases at high frequencies.

4. Summary

The effects of shock duration time, shock amplitude and shock waveform on AASRS are discussed respectively, and the following conclusions are obtained:

1. The shock duration has the greatest influence on the shape and change trend of AASRS, especially on low frequencies. With the increase of shock duration time, the amplitude of AASRS increases gradually.

2. Shock amplitude does not change the overall change trend of AASRS, but only affects the amplitude of AASRS, and its amplitude is proportional to the amplitude of shock acceleration input signal.

3. The shock waveform has little effect on AASRS, mainly affecting the frequency point where the maximum amplitude is located, while the overall envelope changes little.
Acknowledgments
The corresponding author is Song Chen. This research was financially supported by backbone teacher training program of Anhui Xinhua University (2015xgg07), Education and Training Plan for Outstanding Engineers in Computer Science and Technology (2018zygc063), and Excellent course of Anhui Xinhua University (2018jpkcx03).

References
[1] Jung-Ryul Lee, Chen Ciang Chia, Churl-Won Kong. Review of pyroshock wave measurement and simulation for space systems. Measurement. Vol. 45(4) (2012), p. 631-642
[2] Himelblau, H., AG Piersol, JE Manning, and S. Rubin. NASA-HDBK-7005. NASA Technical Handbook. (2001)
[3] ECSS-E-HB-32-25A. Space engineering. Mechanical shock design and verification handbook. (2015)
[4] B.W.Li, Q.M.Li. Damage boundary of structural components under shock environment. International Journal of Impact Engineering. Vol.118 (2018), p. 67-77
[5] BIOT M A. Transient oscillations in elastic systems. California Institute of Technology. (1932)
[6] ALEXANDER J E. Shock response spectrum-a primer. Sound & vibration. Vol. 43(6) (2009), p. 6-15
[7] David O. Smallwood. An improved recursive formula for calculating shock response spectra. Shock and Vibration Bulletin. Vol. 51 (1981), p. 211-217
[8] ISO/WD-18431-4. Mechanical vibration and shock Signal processing,Part4:Shock response spectrum analysis. American National Standards Institute. (2007)
[9] LALANNE C, CETINKAYA C. Mechanical shock. mechanical vibration and shock series. Applied Mechanics Reviews. Vol. 56(5) (2003), p. B67