Limits on the Stochastic Gravitational Wave Background and Prospects for Single Source Detection with GRACE Follow-On

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With a reinterpretation of recent results, the GRACE Follow-On mission can be applied to gravitational wave astronomy. Existing GRACE-FO data constrain the stochastic gravitational wave background to \( \Omega_{\text{GW}} < 3.3 \times 10^{-7} \) at 100 mHz. With a dedicated analysis of existing and future measurements, GRACE Follow-On may be able to detect the inspiral of local neutron star binaries, inspiral of subgalactic stellar-mass black hole binaries, or mergers of intermediate-mass black hole binaries within the Milky Way.

I. INTRODUCTION

GRACE Follow-On (GRACE-FO) is a satellite mission currently in Earth orbit studying the terrestrial gravitational field. We show that by interpreting recent results from this mission as strain, one can (1) constrain the gravitational wave background and (2) provide an opportunity for single-source gravitational wave (GW) searches.

Incoherent GW must be present throughout the universe at some amplitude. A broadband stochastic background is one of the few observable consequences of cosmic inflation [2] and a signature of proposed exotic phenomena such as cosmic strings [3]. Measurements of, and constraints on, such a background would yield insight into a wide range of phenomena, from inflationary models and predictions of string theory to populations of black holes and neutron stars. Existing limits on a stochastic background come from pulsar timing [4], satellite ranging [5, 6], torsion balances [7, 8], terrestrial interferometers [9], and geophysical observations [10–12].

The most prominent single-source GW are emitted either by binary systems or spinning asymmetric stars. While asymmetric stars emit GW with a quasistationary frequency, inspiraling binary systems radiate semi-monotone GW before a rapid merger and ring-down phase. The only direct observations of GW have been from the merger of black hole binaries and a neutron star binary [13] by the LIGO [14] and Virgo [15] observatories.

II. STRAIN MEASUREMENTS

As a GW passes two inertial masses nominally separated by a distance \( x_0 \) their relative displacement, \( x \), is [10]:

\[
x(t) = x_0 h_{xx} \cos(2\pi f t + \psi)
\]

\[
h_{xx} = h_+ (\cos^2 \theta \cos^2 \phi - \sin^2 \phi) + 2h_\times \cos \theta \sin \phi \cos \phi
\]

where \( h_+, h_\times \) are the plus and cross-polarization components of the gravitational wave, \( \theta \) and \( \phi \) are the polar and azimuthal angles of the source, \( f \) is the wave frequency, and \( \psi \) is an arbitrary phase. Here, the plus and cross polarizations are defined in the source frame.

The GRACE-FO satellite mission has produced sensitive relative-displacement measurements of an inertial mass pair formed by its two satellites separated by \( x_0 = 220 \) km. The published displacement spectrum [1] can be converted to strain via \( \tilde{h}_{xx}(f) = \tilde{x}(f)/x_0 \). The inferred strain-noise spectral density is shown in Figure 1.

As noted in Ref. [1], the noise below 30 mHz may be significantly reduced with subtraction of models of the terrestrial gravitational field.

![Strain amplitude spectral density for the GRACE-FO mission. The increase in apparent noise below 30 mHz is due to terrestrial gravitational signals and not instrumental noise.](image)

III. BACKGROUND CONSTRAINTS

Assuming an isotropic, unpolarized, stochastic gravitational wave background, this strain noise can be interpreted as limits on the cosmological energy density of gravitational waves, \( \Omega_{\text{GW}} \) [17], by integrating Equation

\[
\Omega_{\text{GW}}(f) = \frac{4\pi f^2}{c^4} h_{xx}(f)
\]

The resulting limits on \( \Omega_{\text{GW}} \) are shown in Figure 2. The black hole and neutron star binary inspirals are calculated using现有限的观测数据.

![Background constraints](image)
\[ \Omega_{GW}(f) = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{GW}}{df} = \frac{32\pi^3}{3H_0^2} f^3 \frac{15}{2} \tilde{h}\text{meas}(f)^2 \]  \hfill (3)

where \(\rho_{\text{crit}}\) is the critical energy density of the universe, \(\rho_{GW}\) is the energy density of gravitational waves, \(H_0\) is Hubble’s constant, and \(h_{\text{meas}}\) is the measured displacement interpreted as strain. A Hubble’s constant of \(H_0 = 70.3 \text{ km s}^{-1} \text{ Mpc}^{-1}\) was used \[13\]. Derived in Appendix A, the factor of \(15/2\) corrects for the polarization and angular sensitivity of the instruments in question.

Confidence intervals were extracted from the GRACE-FO strain spectrum by separating the data into frequency bins which encompassed 20 data points. Limits for each bin were set at the 95th percentile to yield the results shown in Figure 2. We anticipate that a dedicated analysis would improve these constraints further.

IV. SINGLE-SOURCES

In addition to setting stochastic GW limits, the GRACE-FO mission may allow for gravitational wave searches at frequencies between 30 mHz and 5Hz. Both stellar-mass black hole binaries and neutron star binaries emit GW in the band of interest during their inspiral phase, while intermediate-mass black hole binaries would merge within the band.

Due to the orbit of the satellites, a GW seen by GRACE-FO would be modulated as the antenna pattern sweeps across the sky. This modulation can be approximated by rotating \(\phi\) at the orbit frequency, \(f_{\text{orb}} = 1/94.5 \text{ min} \[4\].

\[ \phi(t) = 2\pi f_{\text{orb}} t + \phi_0 \]  \hfill (4)

Simulated waveforms from an equal-mass 5000 \(M_\odot\) binary as seen by GRACE-FO at a collection of \(\phi_0\) and \(\theta\) values are shown in Figure 3. Of particular interest, at select azimuthal angles and initial polar angles the peak emission is not visible.

Detailed analysis of the orbit of the satellites and backgrounds due to the terrestrial gravitational field would be required for a definitive search. However, an estimate of the characteristic strain \[19\] sensitivity can be calculated from the strain spectral density. This is shown in Figure 4 along with a collection of known and speculative sources. The best characteristic strain sensitivity is \(2 \times 10^{-15}\) at 0.5 Hz.

Intermediate-mass black hole binaries in the mass range of 440 - 22000 \(M_\odot\) would merge within GRACE-FO’s sensitive band. On the other hand, non-detection of sources by a thorough analysis would constrain the galactic and subgalactic populations of intermediate-mass black holes and stellar-mass black holes, respectively.
FIG. 3. Simulated waveforms of a face-on 5000 $M_\odot$ binary at 50 kpc as seen by GRACE-FO. Here $\theta$ is the polar angle of the source and $\phi_0$ is the azimuthal angle at the merger time. The initial, unmodulated waveform was generated by pycbc [20] with $t = 0$ defined as the merger time. Note that at particular $\theta$ and $\phi_0$ values the merger is not visible to GRACE-FO.
FIG. 4. Estimates of the characteristic strain sensitivity of GRACE-FO along with the expected strain from known white dwarf binaries [21] and a sensitivity curve for LIGO’s second observing run [22]. Also shown are lines that correspond to the predicted characteristic strain of an equal mass 1.4 $M_\odot$ binary at 150 pc, a 60 $M_\odot$ at 2 kpc, and a 5000 $M_\odot$ at 50 kpc [19]. The detection volume corresponding to a 50 kpc distance encompasses the entire Milky Way. The points at the end of each line note the merger frequency for the corresponding system. The approximations used here are valid only for the inspiral phase of binary evolution.

V. CONCLUSION

Existing GRACE Follow-On data constrain the stochastic gravitational wave background to levels of $\Omega_{GW} < 3.3 \times 10^{-7}$ at 100 mHz. These constraints are many orders of magnitude from expected sources [2], yet provide an independent set of limits in a frequency range where existing limits are sparse.

With a dedicated analysis, these missions can be an avenue for single-source GW searches. Promising target systems include the inspiral of local neutron star binaries or subgalactic stellar-mass black hole binaries, and the mergers of intermediate-mass black hole binaries within the Milky Way.

Although the application of this mission to gravitational wave astronomy will be supplanted in the near future by the launch of LISA [23], it has the capability to yield opportunistic insight into the local universe.

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VI. APPENDIX A

The strain along a single axis caused by a GW is [16]:

\[ h_{xx} = h_+ (\cos^2 \theta \cos^2 \phi - \sin^2 \phi) \]
\[ + 2h_\times \cos \theta \sin \phi \cos \phi \]  
(5)

where \( h_{xx} \) is the strain along the x-axis, \( h_+ , h_\times \) are the strain amplitudes respectively in the plus and cross polarization, and \( \theta \) and \( \phi \) are the polar and azimuth angles, respectively.

Assuming isotropic emission, the fraction of incident power that is captured by a single-axis instrument can be found by:

\[ \tilde{h}^2_{\text{meas}}(f) = \frac{1}{4\pi} \int d\Omega \tilde{h}^2_{xx}(f) \]  
(6)

\[ = \frac{1}{4\pi} \int d\Omega \left[ \tilde{h}_+^2(f) (\cos^2 \theta \cos^2 \phi - \sin^2 \phi)^2 
+ 4\tilde{h}_\times^2(f) (\cos \theta \sin \phi \cos \phi)^2 
+ 2 \tilde{h}_+(f) \tilde{h}_\times(f) (\cos^2 \theta \cos^2 \phi - \sin^2 \phi) \times \cos \theta \sin \phi \cos \phi \right] \]  
(7)

\[ = \frac{1}{4\pi} \left( \tilde{h}_+^2(f) \frac{22\pi}{15} + 4\tilde{h}_\times^2(f) \frac{\pi}{6} \right) \]  
(8)

Further assuming an unpolarized background, the polarization can be averaged to yield:

\[ \tilde{h}_+^2(f) = \tilde{h}_\times^2(f) = \frac{1}{2} \tilde{h}^2(f) \]  
(9)

\[ \tilde{h}_{\text{meas}}^2(f) = \frac{1}{8\pi} \tilde{h}^2(f) \left( \frac{22\pi}{15} + \frac{2\pi}{3} \right) \]  
(10)

\[ \tilde{h}_{\text{meas}}^2(f) = \frac{4}{15} \tilde{h}^2(f) \]  
(11)

VII. APPENDIX B

The technique used with GRACE-FO can also be applied to LISA Pathfinder (LISA-PF) [24] which was flown to demonstrate the drag-free technology needed for the future LISA mission [23]. For LISA-PF the mass-pair is formed by the proof masses, separated by \( x_0 = 376 \) mm. LISA-PF has published an acceleration noise spectrum which can be converted to strain via \( h_{xx}(f) = -\frac{1}{2\pi x_0 \omega} \ddot{x}(f) \). The corresponding strain spectral density is shown in Figure 5. This measurement can constrain the stochastic gravitational wave background to \( \Omega_{GW} < 6.5 \times 10^{-6} \) at 10 mHz as seen in Figure 2.

![FIG. 5. Strain amplitude spectral density for LISA-PF.](image-url)