An Application of Linear Programming in the Estimation of Technical Efficiency of DMU

B. Venkateswarlu, B. Mahaboob, K.A. Azmath, C. Narayana, C. Muralidaran

Abstract: The main objective of this research article is to propose linear programming problems for estimating the technical efficiency of DMU. This research article deals with the Shepard’s [1] input distance function and its properties are also evaluated. In addition to these extreme efficiency, efficiency but not extreme, weak efficiency and inefficient of a DMU are specifically examined here. In DEA the nature of returns to scale can be inferred. But we cannot quantify the returns to scale. The computations for the classification of RTS of a DMU are also derived in this discourse. In 2009, Barbara A. Mark et al. [2] in their paper, depicted an innovative method which is non-parametric to estimate technical efficiency. In 2011 S. Nuti et al. [3] inquired into the interrelations among technical efficiency scores, weighted per capita cost and overall performance.

I. INTRODUCTION

Efficiency measurement dates back to Farrell [6] who in his path breaking article introduced technical, allocative and cost efficiencies and their pictorial representations. Adding mathematical regour Charness et al. [7] proposed multiplier problems, input and output oriented, which can be readily transformed into linear programming problems. Banker et al. [8] formulated linear programming problems constructed axiomatically whose dual problems coincide with CCR multiplier problems. By the principle of duality the extreme values of the primal and dual objective functions are equal provided that both the problems are feasible. The BCC [9] problems are called the ‘envelopment problems’. A decision making unit (DMU) under evaluation turns out to be efficient or inefficient. The efficient DMUs are of three types, extremely efficient, efficient but not extremely efficient and or inefficient. The efficient units are ‘peerless’. Since the envelop is piecewise linear convex set, an extremely efficient DMU represents one of its vertices. If an efficient unit is efficient but not extremely efficient then its efficiency rating is unity. The input and output representation of such a unit belongs to the envelop, but it cannot represent a vertex. Two or more extremely efficient decision making units are its peers. For such DMU efficiency rating emerges to be unity and all input and output slacks are found vanishing.

II. INPUT DISTANCE FUNCTION

In the contest of multiple inputs and multiple output scenarios, Shephard [1] introduced the concept of IDF which is inversely related to Farrell’s input technical efficiency. The IDF is related to input level sets.

\[ L(u) = \{ x: x \text{ produces } u \} \]

where \( x \in \mathbb{R}^m_+ \) and \( u \in \mathbb{R}^s_+ \)

The structure imposed on \( L(u) \) forces \( L(u) \) to satisfy the following conditions.

1. \( L(0) = \mathbb{R}^m_+ \), every input vector produces null output vector due to inefficiency.
2. \( u_1 \geq u_2 \Rightarrow L(u_1) \subseteq L(u_2) \).
3. \( L(u) \) is closed set,
4. \( \lim_{u \to \infty} L(u) = \varnothing \). No input vector can produce infinite output vector
5. \( L(u) \) is convex set of inputs, if returns to scale are constant, \( L(\lambda \cdot u) = \lambda \cdot L(u), \lambda \geq 1 \)
6. \( L(u) \) satisfies the strong disposability of inputs. No cost is involved in disposing additional inputs due to inefficiency.

Shepard’s IDF is defined as follows:

\[ D(u_0, \lambda, \lambda_x) = \left[ \min \{ \lambda \cdot \lambda_x \in L(u_0) \} \right]^{-1} = F(u_0, x_0) \]

(i) The IDF is inversely related to the Farrell’s input technical efficiency measure.
(ii) \( D(u_0, \lambda, \lambda_x) \geq 1, D(u_0, \lambda, \lambda_x) = \lambda \cdot D(u_0, x_0) \)

\( u_0 \geq u_2 \Rightarrow D(u_0, x_0) \leq D(u_2, x_0) \)

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Let \( u_0 \geq u_1, x_0 \in L(u_0) \Rightarrow x \) produces \( u_0 \) 
Where \( x_0 \) can also produce every output vector smaller than \( u_0 \) \( \Rightarrow x \) produces \( u_1 \) 
\[ x \in L(u_0) \Rightarrow x \in L(u_1), \quad L(u_0) \subseteq L(u_1) \] 
Min \( \{ \lambda : \lambda x_0 \in L(u_1) \} \geq \min \{ \lambda : \lambda x_0 \in L(u_1) \} \] 
\[ D(u_0, x_0) \geq D(u_0, x_1) \] 

(iii) A DMU \( D \) is extremely efficient, if \( \hat{\lambda} = \min \lambda = 1 \), not all the slacks vanish.

(iii) A DMU \( D \) is inefficient, if \( \hat{\lambda} = \min \lambda < 1 \), if I denote the index set of the DMUs, then I = E U E^* U F U N

Where E: Index set of extremely efficient DMUs
E^*: Index set of efficient but not extremely efficient DMUs
F: Index set of weakly efficient DMUs and N: Index set of inefficient DMUs.

III. THE EFFICIENT OF A DMU

The convexity constraint \( \sum_{j=1}^{n} \lambda_j = 1 \) models variable returns to scale. BCC formulated a DEA model, which is input oriented under the axioms of inefficiency, ray unboundedness and minimum extrapolation.

The LP model admits constant return to scale.

\[ \hat{\lambda} = \min \lambda \] 

Subject to \( \hat{\lambda} x_1 + \hat{\lambda}_2 x_2 + \ldots + \hat{\lambda}_n x_n \leq \hat{\lambda} x_0 \) 
\[ \hat{\lambda}_1 y_{11} + \hat{\lambda}_2 y_{22} + \ldots + \hat{\lambda}_n y_{nn} \leq \hat{\lambda} y_{r0} \] 

Where \( \hat{\lambda}_j \) are non-negative.

The final solution of (3.1) leads to an optimal solution for which,

(i) \( \sum_{j=1}^{n} \hat{\lambda}_j = 1 \Rightarrow \) Constant returns to scale

(ii) \( \sum_{j=1}^{n} \hat{\lambda}_j \geq 1 \Rightarrow \) Non-increasing returns to scale

(iii) \( \sum_{j=1}^{n} \hat{\lambda}_j \leq 1 \Rightarrow \) Non-decreasing returns to scale.
The decision making units A and E are with week efficiency. G is a decision making unit without efficiency. Identifying weak efficient as inefficiency, (2.1) can be reformulated as,

\[ \lambda' = \min \{ \lambda : \lambda X_0 \in L_k(u_0) \} \]

subject to

\[ \sum_{j=1}^{n} \lambda_j x_j + \epsilon_j = \lambda x_m, \quad i = 1, 2, \ldots, m \]

\[ \sum_{j=1}^{s} \lambda_j y_j - \epsilon_j = y_r, \quad r = 1, 2, \ldots, s \]

\[ \sum_{j=1}^{k} \lambda_j = 1, \lambda_j \geq 0 \]

(3.2)

In the case of weak efficiency, we have, 1 - \left( s_1^i + s_2^i + \ldots + s_m^i \right) \left( s_1^v + s_2^v + \ldots + s_s^v \right) is less than unity. Since in the case of weak efficiency at least one slack emerges with a non-zero value, the input level sets \( L^l(u_0) \) and \( L^r(u_0) \) admit respectively constant, variable returns to scale.

\( L^l(u_0) \leq L^r(u_0) \)

To achieve pure technical efficiency, the producer should operate at Q; its input pure technical efficiency is,

\[ D^p(u_0, x_0) = \frac{OP}{OQ} = [\min \{ \lambda : \lambda x_0 \in L^k(u_0) \}]^{-1} \]

To achieve scale efficiency as well as technical efficiency the DMU shall operates at R.

\[ D^s(u_r, x_r) = \frac{OP}{OQ} = [\min \{ \lambda : \lambda x_r \in L^k(u_r) \}] \]

\[ D^s(u_r, x_r) \geq D^p(u_r, x_r) \]

\[ [D^s(u_r, x_r)]^{-1} \leq [D^p(u_r, x_r)]^{-1} \]

In data envelopment analysis the nature of returns to scale can be inferred. But we cannot quantify the returns to scale.

For DMU E returns to scale are constant

\[ D^k(x_0, u_0) = D^{NI}(x_0, u_0) = D^I(x_0, u_0) \]

To classify returns to scale of a DMU, we compute,

\[ D^I(u_0, x_0) \quad D^{NI}(u_0, x_0) \quad \text{and} \quad D^k(u_0, x_0) \]

If (i) \( D^I(x_0, u_0) > D^{NI}(x_0, u_0) \), RTS are increasing

(ii) \( D^I(x_0, u_0) \geq D^k(x_0, u_0) \), RTS are decreasing

(iii) \( D^I(x_0, u_0) < D^k(x_0, u_0) \)

\[ D^{NI}(u_0, x_0) = D^I(u_0, x_0) \]

Fig (3.5)
An Application of Linear Programming in the Estimation of Technical Efficiency of DMU

\[ D^k(u_0, x_0) \] 

RTS are constant

IV. CONCLUSION

In the above research paper LPPs are formulated to estimate the technical efficiency of a DMU and the pure technical and scale efficiency are achieved. The expressions by which the returns to scale of a DMU are classified are proposed. In addition to these the concept of input distance function is defined and its properties are discussed.

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