Enhanced yield ratio of light nuclei in heavy ion collisions with a first-order chiral phase transition

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Abstract Using a transport model that includes a first-order chiral phase transition between the partonic and the hadronic matter, we study the development of density fluctuations in the matter produced in heavy ion collisions as it undergoes the phase transition, and their time evolution in later hadronic stage of the collisions. With the production of deuterons and tritons described by the coalescence model from nucleons at kinetic freeze out, we find that the yield ratio $N_tN_d/N_p^2$, where $N_p$, $N_d$, and $N_t$ are, respectively, the proton, deuteron, and triton numbers, is enhanced if the evolution trajectory of the produced matter in the QCD phase diagram passes through the spinodal region of a first-order chiral phase transition.

1 Introduction

It is generally agreed that the quark-gluon plasma (QGP), which is believed to have existed at the very beginning of our universe, has been created in collisions of two heavy nuclei at extremely high energies [1–3]. In these collisions, the produced QGP undergoes a rapid expansion, leading to a decrease of its temperature until it is converted to a hadronic matter (HM) as a result of color confinement. The hadronic matter further expands and cools due to scatterings among hadrons until their kinetic freeze out when they suffer their last scatterings. According to calculations based on the lattice quantum chromodynamics (LQCD) at zero and small baryon chemical potentials [4], the phase transition from the QGP to HM is a smooth crossover. At finite baryon chemical potential ($\mu_B$) or net baryon density ($\rho_B$), many effective theories have suggested, however, that the quark-hadron phase transition is a first-order one [5–7], indicating the existence of a critical endpoint (CEP) on the first-order phase transition line in the $\mu_B - T$ plane. By changing the beam energy as well as selecting the size of the collision system and the rapidity range of produced particles, heavy ion collisions provide the possibility of exploring different regions in the $\mu_B - T$ or $\rho_B - T$ plane of the QCD phase diagram. To search for the CEP and locate the phase boundary in the QCD phase diagram are the main motivations for the experiments being carried out in the Beam Energy Scan (BES) program at RHIC, FAIR, NICA, and NA61/SHINE.

In heavy ion collisions, the created matter is expected to develop large baryon density fluctuations/correlations as a result of the spinodal instability [8–15] or the long-range correlations [16,17] if its evolution trajectory in the QCD phase diagram passes across the first-order phase transition line or the critical region of CEP. Although studying density fluctuations in heavy ion collisions offers the opportunity to locate the CEP in the QCD phase diagram, it is a challenge to identify observables that are sensitive to these density fluctuations or correlations and thus allow for the extraction of information on the phase structure of QCD from experimental measurements [3,18–20]. For the recently developed technique based on the machine learning with deep neural network, it provides a plausible way to discriminate results from different equations of state with no pre-defined physical observables [21,22] but has not yet been successfully applied to real experimental data. On the other hand, a well-known observ-
able is the fourth-order fluctuation $\kappa \sigma^2$ in the net-proton multiplicity distribution, given by the product of its kurtosis $\kappa$ and the variance $\sigma^2$, which is expected to show a non-monotonic behavior in its collision energy dependence [23, 24]. Another promising observable is the production of light nuclei, such as deuteron ($d$), triton ($t$), helium-3 ($^3\text{He}$) etc., due to their composite structures [11, 25–29]. In particular, it was shown in Refs. [26, 27] that the ratio $O_{p-d-t} = N_t/N_d^2$ of the proton, deuteron, and triton yields would be enhanced, and a peak structure is expected in its collision energy dependence if large spatial density fluctuations are developed during the QGP to HM phase transition and can survive later hadronic scatterings. Recently, this ratio has been measured in heavy ion collisions at both SPS energies [26, 30] and RHIC BES energies [31], and preliminary data indeed shows possible non-monotonic peak structures in its collision energy dependence. Although it has been shown in Ref. [13] that large spatial density fluctuations can develop during the QGP to HM phase transition, it is not known if they can survive later hadronic scatterings. To answer the question of whether the non-monotonic behavior of $N_t/N_d^2$ seen in experimental data is due to a first-order or second-order QGP to HM phase transition thus requires detailed dynamical modelling of the time evolution of density fluctuations in heavy ion collisions [32, 33].

Various hydrodynamic approaches are being developed to study the effect of density fluctuations in heavy ion collisions, and they include the stochastic fluid dynamics [34–36], the hydrokinetic approach [37–40], and the nonequilibrium chiral fluid dynamics [41]. Also, a partonic transport model based on the Nambu-Jona-Lasino (NJL) model [42, 43] in the mean-field approximation has been developed in Ref. [13] to study density fluctuations due to the first-order chiral phase transition of the quark matter. In the present study, we extend this study to include the hadronization of quarks to hadrons and the effect of hadronic scatterings on baryon density fluctuations as well as the effect of a first-order phase transition on light nuclei production in relativistic heavy ion collisions.

The NJL model used in the partonic transport model of Refs. [13, 44, 45] includes both the scalar and vector interactions between light quarks. Without the vector interaction, the chiral phase transition in the NJL model is a smooth crossover at small baryon chemical potentials, but changes to a first-order transition as the baryon chemical potential becomes large. Because of the repulsive nature of the quark vector interaction in baryon-rich quark matter, increasing its strength can change the location of the critical endpoint in the phase diagram of quark matter and even make it disappear. With proper initializations of the baryon-rich quark matter in relativistic heavy ion collisions, its evolution trajectory for the case of a first-order phase transition in its equation of state could enter the spinodal region of the phase diagram and develops large density fluctuations [13]. Converting these quarks to hadrons via the spatial coalescence model in a multiphase transport (AMPT) model [46], the quark density fluctuations are transferred to the density fluctuations in the produced hadronic matter. The effects of subsequent hadronic scatterings can then be modelled by a relativistic transport (ART) model [47] in the AMPT to study if the baryon density fluctuations can persist during the expansion of the hadronic matter.

Based on the nucleon coalescence model using the kinetically freeze-out nucleons from the above described approach for light nuclei production, we study in this work the effects of baryon density fluctuations induced by the first-order chiral phase transition of quark matter on their production in heavy ion collisions. We find that these density fluctuations can survive a long time and eventually leads to a yield ratio of $N_t/N_d^2$ of around 0.5, that is larger than those predicted in a recent study [48] based on the AMPT model with a smooth transition from the partonic phase to the hadronic phase, which is around 0.4 regardless of the collision energy. Our results thus suggest that the observed non-monotonic behavior of $N_t/N_d^2$, particularly its enhancement seen in experimental measurements, could be due to the occurrence of a first-order phase transition between QGP and hadronic matter.

Although the NJL model does not include explicitly the gluon degrees of freedom, the chiral phase transition in this model allows one to study how a first-order phase transition affects the baryon density fluctuation in quark matter. For a more realistic study of this effect, the present study can be extended by using the PNJL model [49, 50], which also includes the deconfinement transition via the Polyakov loop. Such a study requires, however, a consistent treatment of the scattering term in the transport model, which is currently being developed, as discussed in Ref. [88].

This paper is organized as follows. In Sect. 2, we give a detailed description of the model setup for the present study, which includes the 3-flavor NJL model, the resulting partonic transport model, the hadronization of quarks to hadrons, the hadronic evolution, and the nucleon coalescence model for producing deuterons and tritons from kinetically freeze-out nucleons. Results on the time evolution of the net-baryon density fluctuations and the yield ratio $N_t/N_d^2$ are then given in Sect. 3 for both cases with and without a first-order phase transition in the equation of state of the quark matter. We finally present the conclusions and outlook in Sect. 4. An appendix is included to show that the effect of density fluctuations on the yield ratio $N_t/N_d^2$ in the thermal model is similar to that in the coalescence model.
2 Model setup

In this section, besides a brief discussion on the models used for converting quarks to hadrons and treating the hadronic scattering, we describe in detail the transport model used in the present work for studying the dynamics of chiral phase transition in relativistic heavy ion collisions, and the nucleon coalescence model for light nuclei production from kinetically freeze-out nucleons. Details on these models can be found in Ref. [46], Ref. [47], Refs. [13,44,45] and Ref. [27], respectively.

2.1 Partonic interactions

To describe the partonic matter produced in relativistic heavy ion collisions, we follow Ref. [13] by adopting the 3-flavor NJL model [51] in terms of the following Lagrangian density,

\[ L = L_0 + L_S + L_V + L_{\text{det}}, \]

with

\[ L_0 = \bar{\psi} (i \gamma^\mu \partial_\mu - m_0) \psi, \]

\[ L_S = G_S \sum_{a=0}^{8} (\bar{\psi} \lambda^a \psi)^2 \]

\[ L_V = -g_V (\bar{\psi} \gamma^\mu \psi)^2, \]

\[ L_{\text{det}} = -K \left[ \det \bar{\psi} (1 + \gamma_5) \psi + \det \bar{\psi} (1 - \gamma_5) \psi \right], \]

where \( L_0 \) is the non-interacting Lagrangian density for free quarks, \( \psi = (u, d, s)^T \) represents the 3-flavor quark fields, and \( \bar{m} = \text{diag}(m_u, m_d, m_s) \) is the current quark mass matrix. In the above equation, \( L_S \) is the interaction Lagrangian density for the quark scalar interaction with \( G_S \) being the scalar coupling constant, \( \lambda^a(a=1,...,8) \) being the Gell-Mann matrices, and \( \lambda^0 = \sqrt{2/31} \). The term \( L_{\text{det}} \) is the Lagrangian density for the Kobayashi–Maskawa–t’Hooft (KMT) interaction [52] that breaks the \( U(1)_A \) symmetry with ‘det’ denoting the determinant in flavor space [53], i.e.,

\[ \det(\bar{\psi} \Gamma \psi) = \sum_{i,j,k} e_{ijk} (\bar{u} \Gamma q_i) (\bar{d} \Gamma q_j) (\bar{s} \Gamma q_k). \]

This term gives rise to the six-point interactions in three flavors. For the case of two flavors, the sum of scalar and pseudo-scalar interactions and the KMT interaction with \( K = -G_S \) reduces to the original NJL model. The term \( L_V \) represents the Lagrangian density for the flavor-independent vector interaction with \( g_V \) being the coupling strength [54].

As usually adopted in the application of the NJL model, we use the mean-field approximation [55] to linearize the interactions, and the Lagrangian density becomes

\[ L = \bar{u} (\gamma^\mu i D_\mu - M_u) u + \bar{d} (\gamma^\mu i D_\mu - M_d) d + \bar{s} (\gamma^\mu i D_\mu - M_s) s - 2G_S (\phi^a_u + \phi^a_d + \phi^a_s) + 4K \phi_a \phi_d \phi_s + g_V (j^u_\mu + j^d_\mu + j^s_\mu)(j^{u\mu} + j^{d\mu} + j^{s\mu}), \]

where

\[ M_u = m_u - 4G_S \phi_u + 2K \phi_d \phi_s, \]

\[ M_d = m_d - 4G_S \phi_d + 2K \phi_u \phi_s, \]

\[ M_s = m_s - 4G_S \phi_s + 2K \phi_u \phi_d \]

are the in-medium effective masses of \( u, d, \) and \( s \) quarks, respectively, with \( \phi_u = \langle \bar{u} u \rangle, \phi_d = \langle \bar{d} d \rangle, \) and \( \phi_s = \langle \bar{s} s \rangle \) being their respective condensates. In Eq. (4), \( j_{u\mu} = \langle \bar{u} u \gamma_\mu \rangle, j_{d\mu} = \langle \bar{d} d \gamma_\mu \rangle, \) and \( j_{s\mu} = \langle \bar{s} s \gamma_\mu \rangle \) denote, respectively, the u, d, and s net-quark vector currents, and \( i D_\mu = i \partial_\mu - A_\mu \) is the covariant derivative with

\[ A_\mu = 2g_V (j_{u\mu} + j_{d\mu} + j_{s\mu}) \]

being the effective vector potential.

2.2 The equation of state

The thermodynamic properties of a 3-flavor quark system are determined by the partition function \( Z \) with the path integral representation,

\[ Z = \text{Tr} e^{-\beta (\hat{H} - \mu \hat{N})} = \int D\psi D\bar{\psi} e^{i\beta \int L_{\text{det}} dx}, \]

where \( \beta = 1/T \) and \( \hat{H} \) are, respectively, the inverse of the temperature \( T \) and the Hamiltonian operator, and \( \mu \) and \( \hat{N} \) are the chemical potential and corresponding conserved charge number operator, respectively. The thermodynamic potential of the quark matter of volume \( V \) then reads

\[ \Omega = \frac{T}{V} \ln Z = \Omega_u + \Omega_d + \Omega_s + 2G_S (\phi^2_u + \phi^2_d + \phi^2_s) + 4K \phi_u \phi_d \phi_s - g_V (\rho_u + \rho_d + \rho_s)^2, \]

where \( \rho_i \) is the net-quark number density of quarks of flavor \( i = u, s, d, \) and

\[ \Omega_i = -2N_c \int \frac{d^3 p}{(2\pi)^3} \left[ E_i + T \ln \left( 1 + e^{\frac{E_i - \mu_i}{T}} \right) + T \ln \left( 1 + e^{\frac{E_i + \mu_i}{T}} \right) \right], \]
with $E_i = (M_i^2 + p^2)^{1/2}$ and $\mu_i^* = \mu_i - 2g_v(\rho_u + \rho_d + \rho_s)$ being the quark energy and effective chemical potential, respectively. We note that $p$ in the above is the kinetic momentum and is related to the canonical $P$ and the vector potential $A$ by $p = P \mp A$, with the minus and plus signs for quarks and antiquarks, respectively.

Minimizing the thermodynamical potential with respect to the quark in-medium masses and chemical potentials, i.e.,

$$\frac{\delta \Omega}{\delta M_i} = \frac{\delta \Omega}{\delta \mu_i^*} = 0,$$

leads to the following expressions for the quark condensates and the net-quark number densities,

$$\phi_i = 2N_c \int \frac{d^3p}{(2\pi)^3} \frac{M_i}{E_i} (n_i - \bar{n}_i - 1),$$

$$\rho_i = 2N_c \int \frac{d^3p}{(2\pi)^3} (n_i - \bar{n}_i),$$

where the occupation numbers are $n_i = \left(e^{E_i/T} + 1\right)^{-1}$ and $\bar{n}_i = \left(e^{E_i + \mu_i^*}/T + 1\right)^{-1}$ for quarks and antiquarks, respectively. The net-quark number density $\rho_i$ is the time component of the net-quark vector current,

$$j_{i\mu} = 2N_c \int \frac{d^3p}{(2\pi)^3} \frac{p_\mu}{E_i} (n_i - \bar{n}_i),$$

whose space components vanish in a static system, but is finite in an expanding quark matter. Because the NJL model is unrenormalizable, a momentum cutoff $\Lambda$ is usually introduced in the integrals in Eqs. (11)–(13).

From the thermodynamical potential $\Omega$, the pressure of a quark matter is simply given by $P = -\Omega + \Omega_0$ with $\Omega_0$ being the thermodynamical potential of the QCD vacuum. Its energy density $\varepsilon$ can also be calculated according to

$$\varepsilon = \frac{\partial \Omega}{\partial T} + \frac{\beta}{\partial \beta} \frac{\partial \Omega}{\partial \beta} + \sum_{i=u,d,s} \mu_i \rho_i - \varepsilon_0$$

$$= -2N_c \sum_{i=u,d,s} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_i (1 - n_i - \bar{n}_i)$$

$$+ 2G_S(\phi_u^2 + \phi_d^2 + \phi_s^2) - 4K \phi_u \phi_d \phi_s$$

$$+ g_v(\rho_u + \rho_d + \rho_s)^2 - \varepsilon_0,$$

where $\varepsilon_0$ is from the quark condensate contribution to the energy density in vacuum and is needed to ensure $\varepsilon = 0$ in vacuum. The entropy and free energy densities of the quark matter can then be obtained from the thermodynamical relations $s = (\varepsilon + p - \sum_{i=u,d,s} \mu_i \rho_i)/T$ and $f = \varepsilon - Ts$, respectively.

For the parameters in the NJL model, we use the values $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, $G_S \Lambda^2 = 1.835$, $K \Lambda^5 = 12.36$, and $\Lambda = 602.3$ MeV [51,56,57]. Figure 1 shows the quark matter phase diagram in the temperature and net-quark density plane from the three-flavor NJL model with $g_v = 0$ for equal $u$ and $d$ quark chemical potentials and zero strange quark chemical potential. The dash-dotted line denotes the coexistence line for the chiral symmetry restored and broken phases, while the solid line denotes the line on which the condition $\langle \partial P/\partial \rho_q \rangle_T = 0$ is satisfied, where $\rho_q = \rho_u + \rho_d + \rho_s$ is the net-quark number density. For quark matter in between these two lines, it is in the metastable phase. In the shaded region, where the quark matter has a convex anomaly in its pressure $\langle \partial P/\partial \rho_q \rangle_T < 0$ and is thus unstable against phase decompositions [9]. The critical temperature and net-quark density in this case are around 70 MeV and 0.85 fm$^{-3}$, respectively. We note that the critical temperature in the NJL model is lower than those obtained from approaches based on the Schwinger-Dyson equation [58,59] and the functional renormalization group method [60,61], but is close to that from the holographic black hole approach [62]. Including the repulsive vector interaction of coupling constant $g_v = G_S$, the critical point disappears in the phase diagram, and the transition from the chirally restored phase to the broken phase changes to a smooth crossover.

To quantify above discussions, we show in panel (a) of Fig. 2 the pressure in a quark matter at temperature $T = 40$ MeV as a function of its net-quark number density $\rho_q$. In the spinodal region and mixed phase from the case of $g_v = 0$, the quark matter pressure, depicted by the blue solid line and the red dashed line, respectively, shows a non-monotonic dependence on $\rho_q$. However, it changes to a monotonically
decreases, so the almost restored chiral symmetry at large increasing function of $\rho$ and $\mu$.

Fig. 2 a Pressure $P$, b in-medium quark mass $M_{q,d}$, c entropy density $s$, and d free energy per quark $f/\rho_q$ as functions of the net-quark number density $\rho_q$ in the three-flavor NJL model at $T = 40$ MeV with $\mu_s = \mu_d$ and $\mu_s = 0$.

increasing function of $\rho_q$ for $g_V = G_S$ as shown by the dash-dotted line. Shown in panel (b) of Fig. 2 is the in-medium quark mass as a function of $\rho_q$, which is the same for both cases of $g_V = 0$ and $g_V = G_S$. It is seen that the quark mass increases as $\rho_q$ decreases, so the almost restored chiral symmetry at large $\rho_q \sim 1.0$ fm$^{-3}$, when the quark mass is small, becomes strongly broken at small $\rho_q \sim 0.2$ fm$^{-3}$. In the spinodal unstable region for $g_V = 0$, where the quark mass is between the above two values, the system is unstable and large density inhomogeneity can be developed. The growth rate of the unstable modes during the first-order chiral phase transition in the NJL model has been studied in Ref. [50]. Although the transport model used in the present study neglects fluctuations due to multi-body correlations, it has been shown in Ref. [13] by two of present authors that the resulting density clumping due to the first-order phase transition is consistent with that from the linear response theory and can thus be described in the mean-field approximation.

Also shown in panel (c) of Fig. 2 is the net-quark density dependence of the entropy density $s$ of a quark matter at temperature $T = 40$ MeV. The results shown by the black solid line are obtained by assuming a uniform quark matter density. Taking into account the coexistence of the chirally restored and broken phases during the phase transition, corresponding to the case of $g_V = 0$, changes the dependence of the entropy density on the quark matter density to the one shown by the red dashed line. It is obtained from the black solid line via the Maxwell construction by using $s(\rho_q) = \alpha s(\rho_{q1}) + (1 - \alpha) s(\rho_{q2})$ with $\rho_q = \alpha \rho_{q1} + (1 - \alpha) \rho_{q2}$, where $\rho_{q1}$ and $\rho_{q2}$ are the net-quark densities at the coexistence lines in Fig. 1 for $T = 40$ MeV. From the corresponding free energy per quark shown in panel (d) of Fig. 2, it is seen that a quark matter in the non-uniform mixed phase has a smaller free energy, suggesting that the phase separation is a spontaneous process. Because of the small difference between the entropy densities of the uniform and non-uniform quark matter in Fig. 2c, the energy release due to the spinodal decomposition during the first-order chiral phase transition is, however, very small compared to the total quark energy.

2.3 Equations of motion in the partonic transport model

To construct a partonic transport model from the NJL model, one needs the equations of motion for quarks and antiquarks (partons) in their mean fields. With the flavor index of quarks omitted, the partonic equations of motion can be obtained from the vector mean-field potential $A^\mu = (A_0, \mathbf{A})$ in Eq. (6) and the in-medium quark mass $M$ of Eq. (5) through the single parton Hamiltonian,

$$H_\pm = \pm A_0 + \sqrt{M^2 + \mathbf{P}^2}, \quad (15)$$

where the upper (lower) sign is for quarks (antiquarks).

In terms of the electric and magnetic fields associated with the strong interaction, given by $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} + \nabla \times \mathbf{A}$ and $\mathbf{B} = \nabla \times \mathbf{A}$, respectively, and the in-medium energy of a parton, $E^* = \sqrt{M^2 + \mathbf{P}^2}$, the time evolution of the partonic matter produced in relativistic heavy ion collisions can be described by a transport equation for the parton phase-space distribution functions $f_\pm(r, \mathbf{p}, t)$, similar to that for the nucleonic matter based on the Walecka model [63,64], i.e.,

$$\frac{\partial f_\pm}{\partial t} + \mathbf{v} \cdot \nabla_r f_\pm + \left( -\frac{M}{E^*} \nabla_r M \pm \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_p f_\pm = \left( \frac{\partial f_\pm}{\partial t} \right)_{\text{coll}} \quad (16)$$

with $\mathbf{v} = \frac{\partial r}{\partial t} = \frac{\mathbf{p}}{E^*}$. In the above, $\left( \frac{\partial f_\pm}{\partial t} \right)_{\text{coll}}$ is the collision term for describing the change of the phase-space distribution function of partons due to their scatterings and is given by the integral
\[
\left( \frac{\partial f_{\pm}}{\partial t} \right)_{\text{coll}} = -\frac{1}{(2\pi)^3} \int \int dp_2 dp_3 \int d\Omega |v_{12}| \frac{d\sigma}{d\Omega} \\
\times \delta^{(3)}(p + p_2 - p_3 - p_4) \left\{ f_{\pm}(r, p, t) f(r, p_2, t) - f(r, p_3, t) f(r, p_4, t) \right\} \\
\times [2N_c - f(r, p_3, t)][2N_c - f(r, p_4, t)] - f(r, p_3, t) f(r, p_4, t) \\
\times [2N_c - f_{\pm}(r, p, t)][2N_c - f(r, p_2, t)].
\]

where \( f(r, p, t) \) denotes the phase-space distribution function of either quarks or antiquarks, \( v_{12} \) is the relative velocity between the two scattering partons, and \( d\sigma/d\Omega \) is their differential scattering cross section. In the present study, we take the parton scattering cross section to be isotropic and have a value of 3 mb. We further neglect the Pauli-blocking factors in the collision term of the transport equation as their effects are appreciable only in the very beginning of the time evolution of the system when the parton density is high and become unimportant as the system expands. We note that with an isotropic quark-quark cross section of 3 mb in the partonic phase, the AMPT model has been shown to describe very well many observables in relativistic heavy ion collisions. Also, a 3 mb cross section is comparable to that calculated from the NJL model based on the exchange of scalar and pseudoscalar mesons as shown in Ref. [65]. The equilibrated parton distributions from the transport model are \( f_+ = 2N_c n \) and \( f_- = 2N_c \bar{n} \) in terms of the quark and antiquark occupation numbers \( n \) and \( \bar{n} \) defined below Eq. (11) with the flavor index omitted.

Solving the transport equation with the test particle method [66] by using particles from an ensemble of physical events leads to the following equations of motion for a test parton between its scattering with another test parton in the same physical event:

\[
\frac{dr}{dt} = v, \\
\frac{dp}{dt} = \frac{M}{E^*}_r \mathbf{v} - \mathbf{E} \pm v \times \mathbf{B}.
\]

In the test particle method, the parton scalar and vector densities are determined by dividing the system into cells in space and including in each cell only test partons that have momenta in the rest frame of the cell less than the cutoff momentum \( \Lambda \). The in-medium quark masses and the scalar field are calculated in the cell rest frame, and the vector fields are calculated in the laboratory frame with corresponding coupling constants. For test partons with momentum above the cutoff, they are not affected by the mean fields and are thus treated as free particles. For the collision term in the transport equation in Eq. (16), only partons in the same physical event are allowed to scatter with each other and the geometrical method of Ref. [67] is used to treat their scatterings.

As in the study of nuclear gas-liquid phase transition in low energy nuclear collisions using transport models [9,68], both the energy of a quark during its propagation and also the energy of two quarks during their scattering are conserved to a good precision. The total energy of the quark matter in our transport model is thus conserved during its evolution, including the stage when it undergoes the chiral phase transition from the chirally restored phase of small quark masses to massive constituent quarks in the chirally broken phase. As a result, the energy released from the spinodal decomposition during the first-order chiral phase transition is dynamically included through the changes in quark masses, which is in contrast to the change of degrees of freedom during a first-order deconfinement transition.

In the present study, the transport equation is solved with 100 test particles. The cell size used in calculating the scalar and vector densities is taken to be \( 1 \times 1 \times 1 \text{ fm}^3 \). To suppress statistical fluctuations, both the scalar and vector densities are further averaged over neighboring \( 3 \times 3 \times 3 \) cells. For the time step used in solving the equations of motion of test particles, it is taken to be \( \Delta t = 0.2 \text{ fm}/c \). Unless stated explicitly, results shown in our study are from 100 ensembles of 100 physical events or test particles.

### 2.4 Hadronization and hadronic transport

Although the NJL model can be generalized to include the transition from quarks to hadrons [65,69], its implementation in the transport model has so far only for the quark to meson transition in ultrarelativistic heavy ion collisions where the produced hadronic matter is dominated by mesons [70,71]. Since the present study is about the first-order chiral transition of quark matter at finite baryon chemical potential, the resulting hadronic matter mainly consists of baryons. Instead of using the untested model of converting quarks to baryons dynamically via the introduction of diquarks and their interactions with quarks as suggested in Ref. [65], we adopt a simple hadronization scheme by converting quarks in each physical event to hadrons through a spatial quark coalescence model [46] at a global hadronization time when most partons have acquired around 70% of their constituent mass in vacuum. This approximate treatment of hadronization greatly simplifies the calculation and also allows a smooth transition of the baryon density distribution in the partonic phase to that in the hadronic phase as will be seen in Sect. 3. After hadronization, the subsequent scatterings and propagations of formed hadrons are then described by the ART model [47], which takes into consideration of various baryon-baryon, meson-baryon and meson-meson scatterings, including the familiar \( \pi + N \leftrightarrow \Delta \) reactions, that can affect the nucleon distribution.
2.5 Density fluctuations and light nuclei production

To quantify the density fluctuation, we consider the scaled density moments \( y_N = \rho^N / \rho^N \) [11] with

\[
\frac{\rho^N}{\rho} = \frac{\int d\rho (N+1) (x)}{\int d\rho (x)}.
\]  

(19)

It is easy to show that the first-order scaled density moment \( y_1 = 1 \) and that \( y_N = 1 \) if the density is a constant. In the present study, we are interested in the second-order scaled density moment, i.e.,

\[
y_2 = \left[ \int d\rho (x) \right] \frac{\int d\rho (x)}{\int d\rho^2 (x)}.
\]  

(20)

For small density fluctuations, \( \rho(x) = \rho_0 + \delta\rho(x) \) with \( \rho_0 \) being the average density, one can obtain the following relation

\[
y_2 \approx 1 + \frac{\int d\rho (\delta\rho(x))^2}{\int d\rho^2} \equiv 1 + \Delta\rho.
\]  

(21)

where \( \Delta\rho \) is defined as the relative density fluctuation averaged over space [26,27].

Although the density fluctuation in heavy ion collisions is not directly accessible in experimental measurements, it can affect the production of light nuclei as pointed out in Refs. [26,27]. In the nucleon coalescence model, the formation probabilities of deuteron and triton in the produced hadronic matter are given by their Wigner functions, which are taken to be

\[
W_d(x, p) = 8 \exp \left( -\frac{x^2}{\sigma_d^2} - \frac{\sigma_d^2 p^2}{\sigma_d^2} \right),
\]

\[
W_t(x, \lambda, p, p_c) = 8^2 \exp \left( -\frac{x^2}{\sigma_t^2} - \frac{\lambda^2}{\sigma_t^2} - \frac{\sigma_t^2 p^2}{\sigma_t^2} - \frac{\sigma_t^2 p_c^2}{\sigma_t^2} \right),
\]

(22)

by using a Gaussian or harmonic oscillator wave function for its internal wave function as usually assumed in the coalescence model for light nuclei production. The relative coordinates \( (x, \lambda, p, p_c) \) and the center-of-mass coordinates \( (X, P) \) are defined in Ref. [48]. The parameter \( \sigma_d \) in the Wigner function of the deuteron is related to its root-mean-square radius \( r_d \) through \( \sigma = \sqrt{2/3} \ r_d \approx 2.26 \text{ fm} \) [72–74], which is much smaller than the size of the nucleon emission source considered in the present study. Similarly, the parameter \( \sigma_t \) in the Wigner function of the triton is related to its root-mean-square radius \( r_t \) simply by \( \sigma_t = r_t = 1.59 \text{ fm} \) [72–74].

The yield ratio \( N_t N_p / N_d^2 \) in the nucleon coalescence model has been shown to be related to the density fluctuation by [26,27]

\[
\rho_{p-d-t} = \rho_{p-d-t} \approx \frac{N_t N_p / N_d^2}{1 + 2 C_{np} + \Delta\rho_n}.
\]  

(23)

where \( \Delta\rho_n \) is the neutron density fluctuation and \( C_{np} \) is the neutron and proton density fluctuation correlation averaged over space, similar to the \( \Delta\rho \) defined in Eq. (23).

In the case that the correlation between the neutron and proton density fluctuations \( C_{np} \) is small, one has in the leading-order approximation,

\[
\rho_{p-d-t} = \rho_{p-d-t} \approx \frac{N_t N_p / N_d^2}{1 + 2\Delta\rho_n} \approx \frac{N_t N_p / N_d^2}{2\sqrt{3}},
\]  

(24)

where the last step follows from the definition of the second-order scaled density moment in Eq. (20) for neutrons.

The above equation shows that the information on spatial density fluctuations is encoded in the yield ratio \( N_t N_p / N_d^2 \), and a large neutron density fluctuation would lead to an enhancement of its value. The relation between the neutron density fluctuation and the yield ratio \( N_t N_p / N_d^2 \) given by Eq. (24) is quite general and is not restricted to the coalescence model considered here. As shown in the Appendix, they can also be obtained in the thermal model that assumes local thermal and chemical equilibrium among nucleons, deuterons and tritons with a spatially varied chemical potential for nucleons. In the present study, we do not consider the medium effects on the binding energies and sizes of light nuclei [75], since their yields from the coalescence model calculations are obtained from nucleons at the kinetic freeze out of the hadronic matter when these effects are negligible.

3 Results

To illustrate the effect of a first-order quark to hadronic matter phase transition on light nuclei production in relativistic heavy ion collisions, we take the initial distribution of quarks and antiquarks in the coordinate space as in Ref. [13] by letting them to follow a spherical Woods-Saxon form:

\[
\rho(r) = \frac{\rho_0}{1 + \exp((r - R)/a)},
\]

(25)

with a radius \( R = 6 \text{ fm} \), a surface thickness parameter \( a = 0.6 \text{ fm} \), and a central net-quark density of \( \rho_0 = 1.5 \text{ fm}^{-3} \), similar to that expected from central Au+Au collisions. The momenta of these partons are taken to follow a relativistic Boltzmann distribution with the temperature \( T = 70 \text{ MeV} \) and the chemical potentials \( \mu_u = \mu_d \) and \( \mu_s = 0 \). For the equation of state of the partonic matter, we consider the two cases with and without a first-order chiral phase transition and a critical endpoint by setting the vector coupling constant in the NJL model to \( g_V = 0 \) and \( g_V = G_S \), respectively. We choose this relatively low initial temperature for the quark matter, which corresponds to that expected from heavy ion collisions at a nucleon–nucleon center-of-mass energy of \( \sqrt{s_{NN}} \approx 3 \text{ GeV} \), in order for the evolution...
Fig. 3 Evolution of the distribution of net-baryon number density in fm$^{-3}$ and the second-order scaled density moment $y_2$ (shown above individual windows) in the transverse plane $z = 0$ in the partonic phase (upper windows) and the hadronic phase (lower windows) for the case of $g_V = 0$, corresponding to a first-order quark to hadronic matter transition.

Fig. 4 Time dependence of the second-order scaled moment $y_2$ of net-baryon density for the two cases of $g_V = 0$ with a first-order chiral phase transition (solid line) and $g_V = G_S$ with a smooth crossover (dashed-dotted line). Red solid stars indicate the time at which hadronization takes place.

3.1 First-order chiral transition

We first consider the case of vanishing vector coupling constant $g_V = 0$, corresponding to a partonic matter that undergoes a first-order chiral phase transition. Shown in Fig. 3 is the distribution of net-baryon number density in fm$^{-3}$ in the transverse plane ($z = 0$) during the time evolution of both the partonic phase (upper windows) and the hadronic phase (lower windows) from an ensemble of 100 physical events. It is seen that the second-order scaled density moment $y_2$ during the partonic evolution, given in each window in Fig. 3 and also shown in Fig. 4 by the solid line, increases as the system expands.

The evolution trajectory of the partonic phase in this case is shown by red solid circles in Fig. 5. Each point on the trajectory is obtained from the average temperature and net-quark number density of the system over all cells weighted trajectory of the quark matter to go through the spinodal unstable region of the phase diagram for the case of $g_V = 0$. 

Fig. 5 Evolution trajectories of the partonic matter in the plane of temperature $T$ versus net quark number density $\rho_q$ for the two cases of $g_V = 0$ with a first-order chiral phase transition (red solid circles) and $g_V = G_S$ with a crossover (solid stars). The spinodal region in the case of $g_V = 0$ is denoted by the shaded region.
by the parton number in each cell, with the temperature in each cell determined from assuming the energy density and the net-quark number density in the rest frame of the cell being the same as those of a thermally equilibrated system. It is seen that the system enters the spinodal region of the phase diagram, shown by the shaded region, at around $t = 4$ fm/c when about 65% of the quarks have masses below 200 MeV with an average quark mass about 165 MeV as shown by the solid red lines in the panels (a) and (b) of Fig. 6, respectively. As given by Eq. (5), the mass of a quark is calculated from the quark condensates that are obtained self-consistently via Eq. (11). With decreasing temperature and density as the system expands, density fluctuations gradually develop as a result of the spinodal instability and reach a value of $y_2 \sim 2$ at $t = 15$ fm/c when the average quark mass increases to around 270 MeV, which is about 70 percent of its mass $M_0 = 367.7$ MeV in vacuum. At this time, about 80 percent of the partons have in-medium masses larger than 200 MeV as shown by the solid red lines in Fig. 6 and the corresponding average temperature of the quark matter is about 30 MeV. The continuous decrease of the average temperature with time shown in panel (c) of Fig. 6, instead of a constant temperature expected from a first-order phase transition, is due to the expansion of the quark matter and the conservation of its total energy. We note that the value of $y_2$ in the present study is larger than the value of $\sim 1.7$ in Ref. [13], and this is due to the slightly larger radius of initial quark matter and the smearing over neighbouring cells in calculating the density distribution $\rho(x)$ adopted in solving the transport equation. Also, for the initial conditions used in the present study, the scaled density moment $y_2$ of net-baryon number density has a similar value as that of the baryon number density. The large second-order scaled net-baryon density moment $y_2$ at hadronization remains substantial during the hadronic evolution as shown in Fig. 4 by the solid line starting from the red star when the hadronization occurs.

Although the chiral symmetry broken matter in the coexistent region inside the dash-dotted line in Fig. 5 should in principle be described in terms of hadronic degrees of freedom if one assumes that the chiral phase transition and the deconfinement transition coincide, we use the NJL model to describe its properties and dynamics in the present study for simplicity. In this respect, we adopt the condition for the hadronization of the partonic matter by choosing the in-medium quark mass to have an average value of around 270 MeV, which corresponds to $t_h = 15$ fm/c for the case of $g_V = 0$ mentioned in the above, and convert all partons in the NJL transport model to hadrons through the spatial quark coalescence model in the AMPT [46]. It is seen from Fig. 3 that the baryon density distribution at hadronization is entirely carried over from the partonic phase to the hadronic phase, suggesting the conservation of local baryon charge in the coalescence model used in the AMPT for hadronization. The small change in the baryon density distribution during hadronization is due to the spatial quark coalescence model used to convert nearby quarks to form mesons and baryons.

The above seemingly counter-intuitive result that the large density fluctuations developed at the phase transition can survive strong hadronic scatterings is due to the rapid expansion of the hadronic matter compared to the relative slow global equilibration of conserved charges through diffusion [76]. This phenomenon is analogous to the temperature fluctuations in the cosmic microwave background (CMB) at different angles, which is now considered as the remnant of quantum fluctuations in the primordial Universe during the rapid inflation epoch [77,78]. Also, according to Eq. (20), the second-order scaled density moment $y_2$ would remain a constant if the expansion of the hadronic matter is self-similar, i.e., $\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$, where $\lambda$ and $\alpha$ are any real functions of time $t$, similar to the expansion of the Universe.

Although there is a significant clustering in the quark matter during the first-order chiral phase transition as shown by the red solid line in Fig. 4, no reheating [79] of the system is seen in the corresponding phase trajectory in Fig. 5. The

![Fig. 6](image-url)
Fig. 7 Same as Fig. 3 for the case of $g_V = G_S$, corresponding to a smooth crossover from the quark to the hadronic matter

reason for the absence of visible reheating of the quark matter due to the spinodal decomposition during the chiral phase transition is because of the small energy release as discussed in the end of Sect. 2.2 and the closeness of the phase trajectory to the critical point, which makes the chirally restored and broken phases not well separated.

3.2 Smooth crossover

For the case of $g_V = G_S$ with a smooth crossover, results on the net-baryon number density distributions in the transverse plane during the evolution of the partonic and hadronic phases are shown in the upper and lower windows of Fig. 7, respectively. The corresponding $y_2$ shown in Fig. 4 by the dash-dotted line is much smaller than the case with a first-order chiral phase transition and only changes slightly throughout the partonic and hadronic evolutions of the system, as the density fluctuations in this case are largely due to statistical fluctuations and the finite size of the system. Also, the system in this case expands much faster than in the case of $g_V = 0$ with a first-order phase transition as shown by the solid stars in Fig. 5. Results on the time dependence of average in-medium quark mass and the fraction of quarks having mass smaller than 200 MeV in this case are shown by the dash-dotted lines in Fig. 6. A similar consideration based on the quark in-medium mass as in the case with a first-order chiral phase transition leads to a global hadronization at $t_h = 8$ fm/c when the average temperature of the quark matter is about 35 MeV, which is slightly higher than in the case of $g_V = 0$.

We note that a more realistic comparison of the quark matter expansion in the case of $g_V = G_S$ with that in case of $g_V = 0$ is to use a lower initial temperature to ensure that the total energy of the initial quark matter is the same for the two cases after including the potential energy due to quark interactions. Because of the reduced pressure with a lower temperature, the expansion of the quark matter becomes slower, which is expected to reduce the difference between the expansion times in the two cases considered here. Indeed, it was found in Refs. [11,15] that the two equations of state with and without spinodal instabilities give similar radial expansions for the produced matter in heavy ion collisions, although they lead to very different behaviors in the density moments. Therefore, the difference in the behavior of the scaled density moment $y_2$ in Fig. 4 between the two cases is due to whether there is a spinodal instability in the equation of state, not because of their different expansion rates.

3.3 Light nuclei production

Although spatial density fluctuations cannot be directly measured in experiments, it can lead to an enhancement [26,27] of the yield ratio $N_t N_p / N_d^2$ as seen from Eq. (24). Using the phase-space distribution of nucleons at the kinetic freeze out in AMPT, we can calculate the production probabilities of deuteron and triton according to Eq. (22). To achieve good statistics, test nucleons in each event are allowed to coalesce to deuterons and tritons. As shown in Fig. 8, the yield ratio $N_t N_p / N_d^2$ is about 0.49 ± 0.02 (solid star) for the case of a first-order chiral phase transition and is about 0.38 ± 0.02 (circles) for the case of a smooth crossover. Compared with the value of about 0.29, given by Eq. (24) for a uniform density distribution, shown by the dashed line in Fig. 8, the enhancements of the yield ratio $N_t N_p / N_d^2$ in these two cases are about 1.7 and 1.3, respectively. These values of enhancement are similar to those expected from the values of $y_2$ in Fig. 4, which, according to Eq. (24), would lead to the enhancement of about 2 and 1.2 for the two cases with and without a
first-order phase transition, respectively. The value of about 0.38 for \( N_1 N_p / N_d^2 \) in the case of \( g_V = G_S \) is consistent with that from a recent study [48] based on the AMPT model with a smooth partonic to hadronic phase transition, which also predicts a constant value of around 0.4 for \( N_1 N_p / N_d^2 \) in central Au+Au collisions from \( \sqrt{s_{NN}} = 7.7 \) GeV to 200 GeV. A similar collision energy independent constant value of \( N_1 N_p / N_d^2 \) is also seen in calculations from a pure hadronic JAM transport model [80] and a hybrid hydrodynamic and transport model [81] with light nuclei produced in both models via the coalescence model from kinetically freeze-out nucleons. Compared to these baseline results from models without a first-order phase transition in the equation of state, the enhancement in the yield ratio \( N_1 N_p / N_d^2 \) due to the first-order chiral phase transition in the present study is about 1.29, which is appreciable in comparison to the peak enhancement of 1.5 at \( \sqrt{s_{NN}} \approx 20 \) GeV in the preliminary data from the STAR Collaboration [31,82,83].

To see how the above result is affected by the value of \( g_V \), we increase it from \( g_V = G_S \) to \( g_V = 2G_S \), which also gives a crossover in the quark matter equation of state. The yield ratio \( N_1 N_p / N_d^2 \) in this case, also shown in Fig. 8, is 0.4 ± 0.02 and is consistent with that for \( g_V = G_S \). Our findings thus support the suggestion in Refs. [26,27] that the non-monotonic behavior in the collision energy dependence of the ratio \( N_1 N_p / N_d^2 \), especially its enhancement, could be a good probe to the first-order QCD phase transitions.

A possible enhanced production of light nuclei in the presence of a first-order phase transition has also been found in a previous study based on a fluid dynamic model using the equation of state from the Polyakov Quark Meson model that has a first-order phase transition [25]. In that study, the much heavier \( A = 5 \) and \( A = 8 \) nuclei were considered, which are, however, much harder to measure in high energy heavy ion collisions due to their very small numbers. Besides, the hadronic effects are not considered in Ref. [25].

### 4 Conclusions and outlook

To study how density fluctuations, which result from a first-order phase transition between the produced partonic matter to the hadronic matter in relativistic heavy ion collisions, evolve under subsequent hadronic scatterings, we have extended the study of Ref. [13] using a partonic transport model based on the 3-flavor NJL model to include the conversion of the partonic matter to the hadronic matter and its subsequent evolution. We have considered two equations of state in the NJL model for the partonic phase, with one having a first-order chiral transition and a critical point \( (g_V = 0) \) and the other having only a smooth crossover \( (g_V = G_S) \). With an initially thermalized partonic fireball, we have found that for the case with a first-order chiral phase transition, large density fluctuations can develop in the partonic phase as the evolution trajectory of the system passes through the spinodal region of the QCD phase diagram during its expansion. These density fluctuations are found to carry over entirely to the initial hadronic matter after hadronization, and largely survive after hadronic scatterings. Using the nucleon coalescence model for nuclei production from nucleons, which is known to allow the inclusion of the effect of nucleon density fluctuations, we have calculated the yields of deuterons and tritons and found an enhancement in the yield ratio of \( N_1 N_p / N_d^2 \) compared to that obtained from the case with a smooth crossover. The present study thus constitutes a first step towards the understanding of the measured non-monotonic behavior in the collision energy dependence of the ratio \( N_1 N_p / N_d^2 \) and the determination of the QCD phase structure through the production of light nuclei in heavy ion collisions.

Since the test particle method used in the present study suppresses the event-by-event fluctuations, which is expected to be significantly enhanced in the critical region, the present approach thus needs to be modified to address the critical phenomena associated with the critical point in the QCD phase diagram. As suggested in Refs. [29,84], the increase in the range of the nucleon-nucleon interaction in the vicinity of a critical point can further enhance the production of light nuclei in relativistic heavy ion collisions. It will be of great interest to extend the present study based on the mean-field approximation to include the long-range correlation between nucleons near the critical point. Also, to fully understand the collision energy dependence of the experimental results on light nuclei production from SPS [26,30] and RHIC [31,82,83], where the temperature of initially produced quark matter is expected to be much higher than 70

![Fig. 8 Yield ratio \( N_1 N_p / N_d^2 \) of proton (p), deuteron (d), and triton (t) for the case of \( g_V = 0 \) with a first-order phase transition (solid star) as well as for the cases of \( g_V = G_S \) and \( g_V = 2G_S \) with a smooth crossover (circles). The dashed line denotes the value of about 0.29 for this ratio when using a uniform density distribution in Eq. (24), and the dash-dotted line is for guiding the eye.](image-url)
MeV used in our study, more realistic equations of state and initial conditions for heavy ion collisions are needed. In this respect, the extended NJL model with a scalar-vector interaction introduced in Ref. [85] can be used because it allows a more flexible value for the critical temperature and baryon chemical potentials. To further take into account the effect of gluons, the present study can be extended by using the PNJL model [49,50] to also include the deconfinement transition of QGP via the Polyakov loop. Furthermore, a more realistic modeling of hadronization than that currently used in AMPT is also important. These improvements will make it possible to quantitatively understand the non-monotonic behavior of the yield ratio \(N_tN_p/N_d^2\) in the experimental data and to locate the position of the critical point in the QCD phase diagram.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Data from theoretical simulations are available upon request.]

Appendix A: Density fluctuations and light nuclei production in the thermal model

In the conventional thermal model for particle production in relativistic heavy ion collisions, the produced matter is assumed to be in global thermal and chemical equilibrium and to have a uniform density distribution. The effect of density fluctuations can be included in this model by assuming that the produced matter is in local thermal and chemical equilibrium with a space dependent temperature \(T(x)\) and chemical potential \(\mu(x)\). For the simplified case of a constant temperature, the nucleon number density is given by

\[
\rho_n(x) = \frac{2}{(2\pi)^3} 4\pi T^2 m^2 K_2 \left(\frac{m}{T}\right) e^{\frac{\mu_n(x)}{T}},
\]

where \(K_2\) is the modified Bessel function of second kind and \(\mu_n(x)\) denotes the space dependent chemical potentials for neutron and proton. Assuming that deuterons and tritons are in local thermal and chemical equilibriums with the nucleons, their number densities in this model are then

\[
\rho_d(x) = \frac{3}{(2\pi)^3} 4\pi T^2 (2m)^2 K_2 \left(\frac{2m}{T}\right) e^{\frac{\mu_d(x)+\mu_p(x)}{T}},
\]

\[
\rho_t(x) = \frac{2}{(2\pi)^3} 4\pi T (3m)^2 K_2 \left(\frac{3m}{T}\right) e^{\frac{2\mu_n(x)+\mu_p(x)}{T}}.
\]

The yield ratio \(O_{p-d-t}\) can be calculated in the non-relativistic approximation as

\[
O_{p-d-t} = \frac{K_2(\frac{m}{T}) K_2(\frac{2m}{T})}{4(K_2(\frac{2m}{T}))^2} \int d^3x \rho_n(x) \int d^3x \rho_p(x) d\rho_p(x)
\approx \frac{1}{2\sqrt{3}} \frac{1}{(1+C_n)^2}\left[\int d^3x \rho_n(x) \int d^3x \rho_p(x) d\rho_p(x)\right]^2
\approx \frac{1}{2\sqrt{3}} \frac{1}{(1+C_n)^2} + 2C_n + \Delta \rho_n.
\]

which is identical to Eq. (23) obtained from the nucleon coalescence model. Density fluctuations thus affect the yield ratio \(N_tN_p/N_d^2\) similarly in both the coalescence and the thermal model.

The above derivation is based on the assumption that the local chemical equilibrium among protons, deuterons, and tritons is maintained from chemical freeze out until kinetic freeze out as a result of the large production and dissociation cross sections of deuterons and tritons during the hadronic evolution [86]. If one assumes instead that the yields of deuterons and tritons produced at the chemical freeze out of identified hadrons remain unchanged during hadronic evolution as assumed in usual statistical hadronization model, there is an additional factor in Eq. (A3) from the contribution of resonance decays to protons [87].

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