Pupil engineering to create sheets, lines, and multiple spots at the focal region

A P Konijnenberg and S F Pereira

Optics Research Group, Delft University of Technology, Delft 2628 CH, Netherlands

E-mail: a.p.konijnenberg@student.tudelft.nl

Received 19 May 2015, revised 7 September 2015
Accepted for publication 17 September 2015
Published 5 November 2015

Abstract

In this paper we present several algorithms to find pupil functions which give focal fields with different desirable properties, such as a laterally elongated spot, a focal sheet, a spot with increased axial resolution, a lateral array of closely packed spots, and a lateral array of widely spaced diffraction-limited spots. All the algorithms work by writing the pupil function as a linear combination of appropriate basis functions, for which the coefficients are optimized. The focal field can be calculated repeatedly efficiently, since focal fields of each of the basis functions are precalculated. For each of the desired focal fields, the specific details of the algorithm are explained, simulation results are presented, and the results are compared to those in other publications.

Keywords: focal field shaping, pupil engineering, SLM

1. Introduction

The problem of finding Fourier transform pairs (i.e. a function and its Fourier transform) that meet certain constraints is a problem with a multitude of applications in optics, and has been investigated since the early 1970s. Gerchberg and Saxton in 1972 [1] developed an iterative algorithm that is relevant in the context of phase retrieval: given the intensity distributions in two different planes which are related to each other by a Fourier transform (such as an object and its far field), one can find the corresponding phase distributions by iteratively propagating the field back and forth (i.e. Fourier transforming it) and each time retaining the newly found phase distribution while replacing the intensity distributions by the ones given. Liu and Gallagher proposed in 1974 [2], and Fienup in 1980 [3], methods which try to achieve a similar goal, but now in the context of shaping a spectrum: rather than trying to reconstruct an object from measurement data as is done in phase retrieval, a function is looked for such that its Fourier transform has certain desirable properties. Realizing that a lens relates its pupil field to its focal plane by a Fourier transform as well, one can easily imagine how these methods find applications in focal field shaping, which is what we will discuss here. Since the Fourier transform of a pupil field gives the field in the focal plane only, it is not immediately obvious how the focal field can be shaped in the axial dimension, or more generally in a focal volume. In 2003 Shabtay noted [4] that by relating the pupil field to its focal volume by a 3D Fourier transform, one could also in this case apply a variant of the Gerchberg Saxton algorithm. A result was demonstrated experimentally in 2005 [5]. One should note however that performing many Fourier transforms (especially 3D Fourier transforms) can become computationally expensive if one wants to retain a high resolution. Therefore, instead of considering all physically allowed configurations like the iterative Fourier transform algorithms (or IFTAs) do, it may be more beneficial to look for pupil functions of a very specific form, appropriate for the type of focal field intensity distribution one wishes to shape. Especially if the desired focal field distributions are relatively small or simple, it would be overkill to shape the pupil by using IFTAs. For example, in [6] pupil functions that give axial or transverse superresolution are sought after, while requiring the pupil functions to be binary annular filters. In our earlier work [7] we looked for pupil functions that give an extended depth of focus, by assuming the pupil function is written as the linear combination of complex Zernike...
polynomials. In this work, we describe how the pupil function should be parametrized to find certain useful focal field configurations, such that the optimization algorithm is kept computationally inexpensive, as long as the desired intensity distributions are simple enough (e.g. straight lines, small planes, small arrays of spots, superresolved spots). Because the optimization problem is in this way simplified, it is then easier to fine tune a trade off between parameters (such as pupil transmission, height of sidelobes, and the goodness of fit with the desired intensity distribution), and the computational time may be reduced which can be useful if the fields need to be shaped as part of a feedback loop. Moreover, the same method can be extended to be applicable for pupil engineering in the vectorial case, whereas the currently known IFTAs only work in the scalar case. Especially since experiments performed with spatial light modulators (SLMs) have demonstrated their usefulness in optical systems [8], and the capabilities of shaping the phase and amplitude of wavefronts with either multiple SLMs [9–11] or just one SLM [12–15], it has become increasingly more interesting to see how structured illumination can further be utilized in optical systems.

We will look at algorithms that create a laterally elongated focal spot (a focal ‘line’, if you will), a focal spot which is elongated both laterally and axially (a focal ‘sheet’), a focal spot with increased axial resolution, and a lateral array of spots (as opposed to a continuous lateral focal line). After we explain how such focal fields are created and showing the simulation results, we compare (when possible) the properties of the obtained focal fields with other methods found in the literature.

Applications for the laterally elongated focal spot may be found in confocal microscopy [16, 17]. By scanning a sample with a focal line rather than a focal spot, the sample can be imaged at a higher frame rate.

A focal sheet may be useful in light sheet microscopy [18]. By illuminating a plane of the sample, a microscope objective perpendicular to the plane can observe the sample with a wide field of view, and increased axial resolution. The sheet is commonly generated by scanning an axially elongated spot laterally, but this will cause significant sidelobes. By creating focal spot in the shape of sheet, the spot does not have to be scanned continuously, which reduces the effect of sidelobes.

A focal spot with high axial resolution can be useful for 3D imaging and laser writing. Shaping the axial dimension of the focal spot by pupil engineering has been explored in [19–21].

A lateral array of widely spaced focal spots may be used for multi-spot confocal microscopy. This allows for imaging a sample more quickly than when it is scanned with a single spot, but it retains high lateral resolution as opposed to confocal line microscopy. Multiple scanning spots can be created with a Nipkow disk [22], but this requires mechanically moving parts, whereas pupil shaping can be performed with an SLM where no moving parts are present. A lateral array of closely spaced focal spots may be used when writing patterns close to each other. Research on arrays of spots have been performed for example in [23] where a set of diffraction orders is created with a phase grating designed so that the intensity of some orders are maximized. However, such an approach puts constraints on the possible distances between spots, and it cannot be trivially modified to work in the axial dimension.

2. The algorithms

2.1. Laterally elongated focal spots

A pupil of which the amplitude is modulated with a cosine will give two laterally displaced spots, since \( \cos(\text{mx}) = (e^{im\xi} + e^{-im\xi})/2 \). By increasing \( m \), the distance by which the two spots are separated increases. A pupil function given by an appropriate linear combination of cosines

\[
P(x_p, y_p) = \sum_{m=0}^{M} c_m \cos(mx_p)
\]

can give a focal spot which is elongated in the x-direction. Here \((x_p, y_p)\) defines a point in the pupil plane. Finding the coefficients \( c_m \) is done similarly as in [7]:

- A target function \( I_{\text{target}}(x_f, 0) = I_{[0,W/2]}(x_f, 0) \) is defined, where \( W \) indicates the width of the focal line we want to achieve, and \((x_f, y_f)\) denotes a point in the focal plane. The target function describes the intensity distribution in the focal plane we ideally wish to achieve. Because of symmetry, we only need to define the target function for \( x_f \geq 0 \).
- For each pupil function \( P_n(x_p, y_p) \) the cross-section of the focal field in the focal plane \( F_n(x_f, 0) \) is precalculated for \( x_f \in [0, W/2 + \Delta] \), where \( \Delta \) is some number sufficiently large to assure there will be no unnoticed sidelobes outside the domain of computation. One could compute the field in the focal field by performing a Fourier transform

\[
F_n(x_f, y_f) = \mathcal{F}\{P_n(x_p, y_p)\}.
\]

However, one may use any appropriate diffraction integral to compute the focal field: as long as the operation is linear, the method will work.

- With a least-squares algorithm coefficients \( c_m \) are calculated such that the normalized calculated intensity roughly matches the target function \( I_{\text{target}}(x_f) \). That is, we use a least-squares algorithm to solve the minimization problem

\[
\min_{c_m} \left[ \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \sum_{m=0}^{M} c_m F_m(x_f)} \right]^2 - I_{\text{target}}(x_f)^2.
\]

Whether or not the least-squares minimization finds a satisfactory solution may depend on the initial guess for
... so if no satisfactory solution is found, another random initial guess for \( c_m \) is tried.

- Using the \( c_m \) found with the least-squares minimization as the initial guess, we solve with a Nelder–Mead algorithm the maximization problem

\[
\max_{c_m} \min_{x_f \in [0,W/2]} \left[ \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \left[ \sum_{m=0}^{M} c_m F_m(x_f) \right]} \right]^2.
\]

By performing the maximization, \( c_m \) are found so that the target function \( I_{\text{target}}(x_f) \) is matched better, and so that we have a figure of merit

\[
I_{\text{min}} = \min_{x_f \in [0,W/2]} \left[ \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \left[ \sum_{m=0}^{M} c_m F_m(x_f) \right]} \right]^2
\]

which indicates how uniform the intensity along the focal line is (it is perfectly uniform if \( I_{\text{min}} = 1 \)).

- To make sure the pupil transmits an adequate amount of light, the pupil transmission is maximized under the constraint that \( I_{\text{min}} > t \), where \( t \) is a threshold value which can be chosen to be around 0.95 (it depends on how much uniformity of the intensity in the focal line one is willing to sacrifice for a higher pupil transmission). That is, with the \( c_m \) found by the Nelder–Mead maximization as the initial solution, we solve with a Nelder–Mead algorithm the constrained maximization problem

\[
\max_{c_m} \int_{x_f^2+y_f^2 \leq 1} \frac{\sum_{m=0}^{M} c_m F_m(x_f, y_f)}{\max_{(x_f,y_f)} \left[ \sum_{m=0}^{M} c_m F_m(x_f, y_f) \right]} \, dx_f \, dy_f
\]

under the constraint \( I_{\text{min}} > t \).

One should note that in case the functions \( P_m(x_f, y_f) \) depend on only one coordinate (such as \( x_f \) in this case, or the radial coordinate \( \rho \) in the case of a circularly symmetric pupil function), the integral

\[
\int_{x_f^2+y_f^2 \leq 1} \frac{\sum_{m=0}^{M} c_m P_m(x_f, y_f)}{\max_{(x_f,y_f)} \left[ \sum_{m=0}^{M} c_m P_m(x_f, y_f) \right]} \, dx_f \, dy_f
\]

can be reduced to a one-dimensional integral, which can be computed much more efficiently.

### 2.2. Laterally and axially elongated focal spots (focal sheets)

Now we need to find a set of pupil functions such that an appropriate linear combination could give a focal sheet (i.e. we need to find pupil functions which have focal fields which could be ‘stitched together’ to form a sheet). We have already found laterally elongated focal spots, so if we displace them axially and stitch them together, a focal sheet could be formed. Thus, if we define \( \tilde{P}_m(x_f, y_f) \) to be pupil functions that give laterally elongated spots, then

\[
\tilde{P}_m = \tilde{P}_m(x_f, y_f) Z_m^0(x_p, y_p)
\]

would give displaced focal lines (\( Z_m^0(x_p, y_p) \) is a radially symmetric complex Zernike polynomial [24]). So a focal sheet could be formed by a pupil function \( P(x_p, y_p) \) which is a linear combination

\[
P(x_p, y_p) = \sum_{m=0}^{N} \sum_{n=0}^{M} c_{nm} \tilde{P}_m(x_p, y_p).
\]

The coefficients \( c_{nm} \) are found in a similar way as the coefficients for a laterally elongated spot are found:

- A target function \( I_{\text{target}}(x_f, z_f) = I_{[0,W/2]}(x_f) I_{[0,W/2]}(z_f) \) is defined, where \( W_x \) and \( W_z \) indicate the width in the \( x_f \)-direction, and the depth in the \( z_f \)-direction of the focal sheet we want to achieve. The target function describes the intensity distribution in the focal plane we ideally wish to achieve. Because of symmetry, we only need to define the desired distribution for \( x_f \geq 0 \) and \( z_f \geq 0 \).

- For each pupil function \( P_m(x_p, y_p) \) the cross-section of the focal field in the \( (x_f, z_f) \)-plane \( F_m(x_f, z_f) \) is precalculated for \( x_f \in [0, W_x/2 + \Delta_x], z_f \in [0, W_z/2 + \Delta_z] \).

- With a least-squares algorithm, we solve the minimization problem

\[
\min_{c_{nm}} \left[ \frac{\sum_{m=0}^{M} c_{nm} F_m(x_f, z_f)}{\max_{(x_f,z_f)} \left[ \sum_{m=0}^{M} c_{nm} F_m(x_f, z_f) \right]} \right]^2 - I_{\text{target}}(x_f, z_f).
\]

- With a Nelder–Mead algorithm we find

\[
\max_{c_{nm}} I_{\text{min}},
\]

where we define

\[
\tilde{I}_{\text{min}} = \min_{x_f \in \left[-W_x/2, W_x/2\right], z_f \in \left[-W_z/2, W_z/2\right]} \left[ \frac{\sum_{m=0}^{M} c_{nm} F_m(x_f, z_f)}{\max_{(x_f,z_f)} \left[ \sum_{m=0}^{M} c_{nm} F_m(x_f, z_f) \right]} \right]^2.
\]

- With a Nelder–Mead algorithm we optimize the pupil transmission

\[
\max_{c_{nm}} \int_{x_f^2+y_f^2 \leq 1} \frac{\sum_{m=0}^{M} c_{nm} P_m(x_f, y_f)}{\max_{(x_f,y_f)} \left[ \sum_{m=0}^{M} c_{nm} P_m(x_f, y_f) \right]} \, dx_f \, dy_f
\]

under the constraint \( I_{\text{min}} > t \),

where \( t \) is a parameter indicating how uniform we want the intensity in the focal sheet to be at least.
2.3. Focal spot with increased axial resolution

Because we want to modify the properties of the focal spot in the axial direction, it makes sense to look for a pupil function which is the linear combination of axially displaced spots

\[
P(\rho_p) = \sum_{m=0}^{M} c_m \cos \left( m \rho_p^2 \right)
\]

Here, \( \rho_p \) is the radial pupil coordinate. However, to find the coefficients \( c_m \), it is not effective to pursue the usual method, i.e. to define a target function which is approximated using a least-squares method. Instead,

- For each pupil function \( P(\rho_p) = \cos(m \rho_p^2) \) we pre-calculate the focal fields \( F_m(z_f) \) along the axial dimension on a sufficiently large range \( z_f \in [0, \Delta] \). If we define \( z_0 \) to be the point where the first zero occurs for the Airy spot, then at least \( \Delta > z_0 \).
- With a Nelder–Mead algorithm, we maximize the derivative of the normalized intensity distribution on the interval \([0, z_0]\)

\[
\max_{\rho_p} \max_{z_f \in [0, z_0]} \left| \frac{d}{dz_f} \left( \sum_{m=0}^{M} c_m F_m(z_f) \right) \right|^2 \quad (14)
\]

- With a Nelder–Mead algorithm, we minimize the height of the axial sidelobes under the constraint that the

\[
\max_{z_f} \left( \sum_{m=0}^{M} c_m F_m(z_f) \right) \]
With a Nelder–Mead algorithm, we maximize the pupil transmission under the constraint that the derivative in the range \([0, z_0]\) stays above a certain value \(t_1\), and the height of the axial sidelobes remain under a certain value \(t_2\) of the axial sidelobes.

\[
\begin{align*}
\min_{c_m} & \quad \max_{z_f \in [z_0, \Delta]} \left| \left( \sum_{m=0}^{M} c_m F_m(z_f) \right)^2 \right| \\
\text{under the constraint} & \quad \max_{z_f \in [0, z_0]} \left| \frac{d}{dz_f} \left( \sum_{m=0}^{M} c_m F_m(z_f) \right) \right|^2 > t_1. \quad (15)
\end{align*}
\]

\[
\begin{align*}
\max_{z_f \in [0, z_0]} & \quad \left| \int_{x^2 + y^2 < 1} \frac{\sum_{m=0}^{M} c_m F_m(x_{y_f}, y_{y_f})}{\sum_{m=0}^{M} c_m F_m(x_{y_f}, y_{y_f})} \right| dx_{y_f} dy_{y_f} \\
\text{under the constraints} & \quad \max_{z_f \in [0, z_0]} \left| \frac{d}{dz_f} \left( \sum_{m=0}^{M} c_m F_m(z_f) \right) \right|^2 > t_1, \\
\max_{z_f \in [0, z_0]} & \quad \left| \sum_{m=0}^{M} c_m F_m(z_f) \right|^2 < t_2. \quad (16)
\end{align*}
\]
2.4. Lateral array of closely spaced spots

Since we try to modify the focal spot in the lateral direction, we again write the pupil function as a linear combination of cosines, as in equation (1) (although if we were to do this in the axial direction, it would also be possible by choosing different \( p_m(x_p, y_p) \)). To create a closely spaced array of spots, the integral of the absolute value of the derivative of the normalized intensity along the lateral axis is maximized with a Nelder–Mead algorithm

\[
\max_{\epsilon_{\infty}} \int \left| \frac{d}{dx_f} \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \sum_{m=0}^{M} |c_m F_m(x_f)|} \right|^2 \, dx_f. \tag{17}
\]

After that, as an attempt to make the intensity of each spot equal, the integral of the normalized intensity along the lateral axis is maximized under the constraint that the derivative remains high

\[
\max_{\epsilon_{\infty}} \int \left| \frac{d}{dx_f} \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \sum_{m=0}^{M} |c_m F_m(x_f)|} \right|^2 \, dx_f \quad \text{under the constraint}
\[
\int \left| \frac{d}{dx_f} \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \sum_{m=0}^{M} |c_m F_m(x_f)|} \right|^2 \, dx_f > \iota. \tag{18}
\]

Then, the pupil transmission is maximized under the constraint that other two parameters remain almost optimized

\[
\max_{\epsilon_{\infty}} \int_{x_f^2 + y_f^2 \leq 1} \frac{\sum_{m=0}^{M} c_m P_m(x_f, y_f)}{\max_{(x_f, y_f)} \sum_{m=0}^{M} |c_m P_m(x_f, y_f)|} \, dx_f \, dy_f
\]
Figure 4. Simulation of a spot with increased axial resolution. The pupil function is a linear combination of $M + 1 = 15$ cosines. The axial resolution is increased, but this gives high axial lobes. The Strehl ratio (i.e. the ratio of the peak intensities for the structured pupil and the aberration-free pupil) is found to be 0.19. The axial gain is almost the same as for the result in figure 6 of [19], although it must be noted those simulations were performed for NA = 1.4, while the ones presented here are for NA = 0.4.
Figure 5. Simulation of a spot with increased axial resolution. The pupil function is a linear combination of $M + 1 = 15$ cosines. The height of the axial lobes has not increased as much, but at the cost of the axial resolution. The Strehl ratio (i.e. the ratio of the peak intensities for the structured pupil and the aberration-free pupil) is found to be 0.31.
under the constraints
\[
\int \frac{d}{dx_f} \left[ \frac{\max_{x_f} \left( \sum_{m=0}^{M} c_m F_m(x_f) \right)^2}{\sum_{m=0}^{M} c_m F_m(x_f)} \right] \, dx_f > t_1,
\]
\[
\int \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \left( \sum_{m=0}^{M} c_m F_m(x_f) \right)^2} \, dx_f > t_2.
\]

2.5. Lateral array of widely spaced spots

Since we try to modify the focal spot in the lateral direction, we again write the pupil functions as a linear combination of cosines, as in equation (1) (although we could also do this in the axial dimension). To create a widely spaced array of spots, we define the points \( x_f^{(m)} \) along the lateral axis in the focal plane where we want the intensity \( I(x_f^{(m)}) \) to peak, and we define the points \( x_f^{(n)} \) along the lateral axis where we want the intensity \( I(x_f^{(n)}) \) to be minimal. With a Nelder–Mead algorithm we find

\[
\max_{c_m} \sum_{x_f^{(m)}} \int \left( I(x_f^{(m)}) - I(x_f^{(n)}) \right)^2 \, dx_f > t_2,
\]

where

\[
I(x_f) = \left[ \frac{\sum_{m=0}^{M} c_m F_m(x_f)}{\max_{x_f} \left( \sum_{m=0}^{M} c_m F_m(x_f) \right)^2} \right]^2.
\]

Then, the pupil transmission is maximized

\[
\max_{c_m} \int \sum_{x_f^{(m)} \in S} \frac{\sum_{m=0}^{M} c_m P_m(x_{f}, y_{p})}{\max_{(x_{f}, y_{p})} \left( \sum_{m=0}^{M} c_m P_m(x_{f}, y_{p}) \right)} \, dx_f \, dy_p
\]

under the constraint

\[
\sum_{x_f^{(m)} \in S} \int \left( I(x_f^{(m)}) - I(x_f^{(n)}) \right)^2 > t.
\]
3. Simulation results and discussion

3.1. Laterally elongated focal spots

Shown in figure 1 are simulation results for creating a laterally elongated spot. The number of cosines used to build the pupil functions is \( M + 1 = 50 \). The number of cosines that is used determines the maximum possible length of the focal line. If the line is chosen to be significantly shorter than the maximum possible length, the pupil transmission can be increased, but at the cost of introducing sidelobes which span the range of the maximum possible length. One may wonder whether a laterally elongated spot could just as well be achieved by modulating the pupil with a sinc function, since its Fourier Transform is a block function. However, as shown in figure 2, this introduces peaks at the ends of the focal line.

It has been proposed to use a focal line for confocal line scanning microscopy [16]. However, in this case a cylindrical lens is used, and the measured resolutions in \( z \) (depth), \( x \) (perpendicular to line focus), and \( y \) (direction of line focus) directions are 3.3 \( \mu m \), 0.7 \( \mu m \) and 0.9 \( \mu m \), respectively, with a 50\( \times \) objective lens. So the length of the spot in the \( y \)-direction is only a little longer than the length in the \( x \)-direction. In the simulations presented here the spot is easily several times longer than it is wide. In our case, no cylindrical lens is needed; the line is obtained with a normal microscope objective.

3.2. Laterally and axially elongated focal spots (focal sheets)

Shown in figure 3 is a simulation for a focal sheet. The pupil function is a linear combination of \( N + 1 = 4 \) laterally elongated spots multiplied with \( M + 1 = 4 \) Zernike polynomials. It is possible to increase the pupil transmission, but it will be at the cost of the uniformity of the sheet.

Focal sheets for light sheet microscopy have been created by scanning in the lateral direction a Bessel beam or an Airy beam. However, since the sidelobes of those beams are scanned as well, they contribute significantly to the light intensity around the focal sheet [18]. This decreases the axial resolution for the microscope objective perpendicular to the sheet (even though this can be countered by applying...
Figure 8. Simulation of a discretized version of figure 4. The Strehl ratio is found to be 0.29.
deconvolution). In the simulations presented here, the spot does not have to be scanned continuously since it has a large width, so effects of the sidelobes are reduced.

3.3. Focal spot with increased axial resolution

Shown in figures 4 and 5 are simulation results for spots with increased axial resolution. The pupil functions are linear.
combinations of $M + 1 = 15$ cosines. In figure 4 it is seen that the axial resolution is increased, but so is the height of the axial lobes, whereas in figure 5 the height of the lobes is decreased, but at the cost of a less improved axial resolution. Another method to find pupil masks which give axial superresolution, is given in [19]. The increase in axial resolution presented there is similar to the increase observed in these simulations. The advantages of their method is the fact that their pupil functions are only phase-modulated (binary, $0/\pi$) so that they have full transmission. However, a downside is that their method requires repeated forward and backward propagation of the field, which is computationally expensive and time-consuming. With the method presented here on the other hand, a solution is easily found in a matter of seconds. Moreover, by choosing the number of functions with which the pupil is constructed, it is easily controlled within what range there will be axial sidelobes. This control seems less obvious when creating a binary phase mask. Lastly, it is mentioned that the outcome of their algorithm is sensitive to their initial guess, which therefore has to be chosen carefully. The method presented here shows no such problems, since the solution depends in a straightforward manner on the constraints set to the focal field (i.e. the number of functions used to build the pupil, and the maximum height set for the axial sidelobes).

### 3.4. Lateral array of closely spaced spots

In figure 6 a simulation of a lateral array of spots is shown. The pupil function is a linear combination of $M + 1 = 50$ cosines.

### 3.5. Lateral array of widely spaced spots

In figure 7 a simulation of a lateral array of widely spaced spots is shown. The pupil function is a linear combination of $M + 1 = 60$ cosines.

### 4. Binary pupil functions

For experimental implementations it may be desirable if no continuous amplitude modulation is required, but instead the amplitude modulation is either 0 or 1. If the pupil transmission is high enough, one may make the pupil function binary while still mostly retaining the desired focal field intensity distribution, see figures 8 and 9 where the pupil functions of figures 4 and 5 have been made binary. The approach with which the pupil functions have been made binary is as follows:

- The pupil function amplitude is discretized into $r$ levels, i.e. instead of a continuously varying amplitude, the amplitude takes on one of the values $1/r$, $2/r$, ..., 1.
- We choose a circularly symmetric function (so that the discretized pupil function will consist of concentric circles) which rapidly oscillates between 0 and 1, for example $f(x_p, y_p) = 20 \sqrt{x_p^2 + y_p^2} \mod 1$.
- For each value $n/r$ the pupil amplitude can take, assign to each point $(x_p, y_p)$ for which the amplitude is $n/r$ the amplitude 1 if $f(x_p, y_p) \leq n/r$, and 0 otherwise.

### 5. Conclusion

We have explained several algorithms to find pupil functions which give focal fields with various intensity distributions in the focal volume. These algorithms all work by writing the pupil function as a linear combination of basis functions, and optimizing the coefficients. The algorithms only differ in the choice of basis functions and the parameter that is to be optimized. This straightforward yet effective approach has been shown to be able to find pupil functions that give line-shaped focal spots, sheet-shaped focal spots, spots with increased axial resolution, lateral arrays of closely spaced spots, and lateral arrays of widely spaced spots. In case other methods were found in the literature, the results presented here are comparable or an improvement to the results found using other methods.

Finally, we stress that these different intensity distributions in the focal region can be obtained by modifying the settings of a (or several) SLM(s). In this way, one could use these fields not only for imaging, but also in, for example, direct laser writing. By producing specific light distributions one could speed up the writing process. Also, lateral scanning of the focal field can be incorporated.

### Acknowledgments

We would like to thank Lei Wei for enlightening discussions.

### References

[1] Gerchberg R W and Saxton W O 1972 Optik 35 237–46
[2] Liu B and Gallagher N C 1974 Appl. Opt. 13 2470–1
[3] Fienup J R 1980 Opt. Eng. 19 297–305
[4] Shabtay G 2003 Opt. Commun. 226 33–7
[5] Whyte G and Courtial J 2005 New J. Phys. 7 117
[6] de Juana D M et al 2004 Opt. Commun. 229 71–7
[7] Konijnenberg A P et al 2014 Opt. Express 22 311–24
[8] Maurer C et al 2011 Laser Photonics Rev. 5 81–101
[9] Han W et al 2013 Opt. Express 21 20692–706
[10] Hsieh M et al 2007 Opt. Eng. 46 075001
[11] de Bougrenet de la Tocnaye J L and Dupont L 1997 Appl. Opt. 36 1730–40
[12] Davis J A et al 2003 Appl. Opt. 42 2003–8
[13] Yu Z et al 2015 Opt. Commun. 345 135–40
[14] Liu L Z et al 2014 Opt. Lett. 39 2137–40
[15] Tao S and Yu W 2015 Opt. Express 23 1052–62
[16] Im K et al 2003 Opt. Express 13 5131–6
[17] Mei E et al 2012 J. Microsc. 247 269–76
[18] Vettenburg T et al 2014 Nat. Methods 11 541–4
[19] Jabbour T G et al 2008 Opt. Commun. 281 2002–11
[20] Martinez-Corrall M et al 2002 Opt. Express 10 98–103
[21] Yun M et al 2005 J. Opt. Soc. Am. A 22 272–7
[22] Murphy D B and Davidson M W 2013 Fundamentals of Light Microscopy and Electronic Imaging 2nd edn (Hoboken, NJ: Wiley-Blackwell)
[23] Moreno I et al 2014 In Fringe 2013 (Berlin: Springer) pp 57–62
[24] Braat J J M, Janssen A J E M, Dirksen P and van Haver S 2008 Prog. Opt. 51 349–468