QUINTESSENCE AND THE ACCELERATING UNIVERSE

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Abstract. Observations seem to indicate that our universe is presently accelerating due to the presence of dark energy. Quintessence represents a possible way to model the dark energy. In these proceedings, we briefly review its main properties.

1 Introduction

There is now compelling evidence that dark energy is present in our universe and represents about 70% of the total energy density, i.e., a value of $\approx 10^{-47}\text{GeV}^4$ (Perlmuter 1998; Riess 1998). The challenge is therefore to understand the physical nature of dark energy. Concerning this question, a crucial ingredient is that, contrary to dark matter, dark energy must possess a negative pressure as required in order to have an accelerating universe. At first sight, the most natural candidate is a positive cosmological constant. A cosmological constant can be viewed as a fluid with a constant equation of state parameter (the ratio of its pressure to its energy density) equal to $\omega = -1$. Then, the equation expressing the conservation of the energy density, $\dot{\rho} + 3H(1 + \omega)\rho = 0$, implies that the corresponding energy density is constant. As it is well-known, explaining the dark energy with a cosmological constant runs into severe problems. Let us briefly mention only two of them linked to the constancy of the cosmological constant energy density (Weinberg 1989). The first problem is that we need to generate a large cosmological constant in the early universe (in order to ensure that a phase of inflation took place at this early epoch) and at the same time we also need a tiny value at the present time to explain the acceleration of the universe. This seems incompatible with the fact that the energy density of a $\Lambda$-term is constant. A second problem is the so-called coincidence problem. At high redshifts, say just after inflation $z \approx 10^{28}$, the energy density of the radiation, the dominant fluid at those redshifts, is of the order $\rho_R \approx 10^{61}\text{GeV}^4$, while of course we still have $\rho_\Lambda(z \approx 10^{28}) \approx 10^{-47}\text{GeV}^4$. This means that in order to have the correct amount of dark energy today, we need to fine-tune the initial

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conditions such that $\rho_h/\rho_\Lambda \simeq 10^{110}$, something which is a direct consequence of the fact that $\rho_\Lambda$ is a constant. This seems very unnatural.

The previous considerations lead us to the conclusion that the dark energy density must be time-dependent. As already mentioned above, the dark energy pressure must also be negative. As is well-known from inflationary model-building, a simple way of obtaining these features is to consider a minimally coupled scalar field $Q$ (named “quintessence”). The fact that the scalar field evolves with time causes the equation of state to be redshift-dependent. In addition, in a regime where the potential energy dominates the kinetic energy, one obtains a negative pressure. The question is now to discuss the shape of the potential $V(Q)$ such that the previous problems can be addressed.

2 A brief description of the prototypical model of Quintessence

Many potentials have been proposed in the literature. However, a simple one which allows us to discuss the main features of quintessence is the so-called Ratra-Peebles potential (Ratra & Peebles 1988)

$$V(Q) = \frac{M^{4+\alpha}}{Q^\alpha}. \quad (2.1)$$

This potential depends on two free parameters: the energy scale $M$ and the index $\alpha$ which is positive. The parameter $\alpha$ is a priori free whereas the scale $M$ is fixed by the requirement that, today, $\Omega_Q \simeq 0.7$. One can show that this implies the following link between $M$ and $\alpha$

$$\log_{10}[M(\text{GeV})] \simeq \frac{19\alpha - 47}{4\alpha + 4}. \quad (2.2)$$

This law is plotted in Fig. 1. One sees that for $\alpha > 3$, the scale $M$ is beyond the electro-weak scale. In this case, the inverse power-law shape of the Ratra-Peebles potential allows us to explain a very small scale in terms of a high-energy scale.

We now turn to the coincidence problem evoked in the introduction. In the case of quintessence, this problem is solved because there exists an attractor. This means that, whatever the initial conditions, the solution always converges towards the same solution. Therefore, contrary to the case of a cosmological constant, there is no need to fine-tune the initial conditions. This property is illustrated in Figs. 2 and 3. Starting from very different initial conditions (different by many orders of magnitude), one reaches the same solution at small redshifts. On the attractor, the quintessence energy density redshifts as $\rho_Q \propto a^{-3\alpha(1+\omega_B)/(2+\alpha)}$, where $\omega_B$ is the equation of state of the dominant hydrodynamical fluid before the quintessence era, i.e. either radiation ($\omega_B = 1/3$) or matter ($\omega_B = 0$). Since the quintessence energy density redshifts more slowly than the background, this explains why quintessence starts dominating at small redshifts while it is hidden during most of the cosmological evolution.

Finally, let us consider the problem of the equation of state. One can show that the present value of $\omega_Q$ is such that $-1 < \omega_Q < 0$. Due to the presence of the
Fig. 1. Evolution of the logarithm of the mass scale with the power-law index.

Fig. 2. Evolution of the energy density of radiation, matter and quintessence with the redshift. The dotted line represents radiation, the dashed line, matter, and the solid line, quintessence. The initial conditions corresponds to equipartition, i.e. initially $\Omega_Q \approx 10^{-4} \Omega_r$.

attractor, this value does not depend on the initial conditions. On the contrary, $\omega_Q$ depends on the power-law index $\alpha$. For $\alpha = 6$, one finds $\omega_Q \approx -0.4$ which may be a problem since the observations seems to indicate that $\omega_Q$ is quite close to $-1$. This can be easily improved. From a high-energy physics point a view, the
Fig. 3. Same as Fig. 2 but with different initial conditions such that initially $\rho_Q \ll \rho_r$.

SUGRA model (Brax & Martin 1999)

$$V(Q) = \frac{M^{4+\alpha}}{Q^\alpha} e^{\kappa Q^2/2},$$

with $\kappa = \frac{8\pi}{m_{\nu}^2}$, is better motivated. Due to the presence of the exponential factor (SUGRA corrections) the equation of state is pushed towards $-1$, more precisely, one has $\omega_Q \simeq -0.82$.

3 Conclusions

In this short letter, we have quickly reviewed the main properties of the quintessential scenario for dark energy. This scenario can improve the fine-tuning problem and solve the coincidence problem. Quintessence is a falsifiable hypothesis since it predicts a redshift-dependent equation of state which is different from $-1$ today. Measuring the dark energy equation of state is certainly one of the most important observational challenge for the future. Only this measure will able to tell us whether quintessence is an acceptable explanation of the dark energy of the universe.

References

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