Quantum information, an ingenious application of quantum mechanics within the field of information has attracted a lot of attentions recently. In particular almost all the branches of quantum communication have been developed quickly since the original protocol was proposed by Bennett and Brassard in 1984, such as quantum key distribution (QKD) [1,2,3,4,5,6,7,8,9], quantum secure direct communication (QSDC) [10,11,12,13,14], quantum teleportation [15,16,17], quantum secret sharing (QSS) [18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36], and so on. QKD provides a secure way for creating a private key between two remote parties. With a private key, most of the goals in classical secure communication can be accomplished. For example, a classical secret message can be transmitted securely by using classical one-time pad crypto-system with a private key. To date, QKD has progressed quickly and becomes one of the most mature applications of quantum information.

QSS is likely to play a key role in both transmitting of a classical information and protecting a secret quantum information, such as in secure operations of distributed quantum computation, sharing difficult-to-construct ancillary states and joint sharing of quantum money [24,25]. Suppose a banker, Alice, wants her two agents, Bob and Charlie who are at remote places to deal with her business according to her message \( M_A \). However Alice doubts that one of them may be dishonest and the business may be destroyed if the dishonest one manages it independently. Moreover Alice does not know who the dishonest one is, but she knows that the number of dishonest person is less than two. To prevent the dishonest man from destroying the business, classical cryptography provides the secret sharing scheme [37] in which Alice splits her message \( M_A \) into two sequences \( M_B \) and \( M_C \) and sends them to Bob and Charlie, respectively. They can read out the message \( M_A = M_B \oplus M_C \) if and only if they cooperate, otherwise they will get nothing.

QSS is the generalization of classical secret sharing [37] into quantum scenario and supplies a secure way for sharing not only a classical information [18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36], but also a quantum information [24,26,27,28,29,30,31,32,33,34,35,36]. In the latter, the sender, Alice will send an unknown quantum state to her \( m \) agents and one of them can recover it with the help of the others [32]. Compared with QKD, QSS can reduce the the resources necessary for the communication [23] for a classical secret information. Moreover, the sharing and the splitting of an unknown quantum state should resort to QSS with quantum entanglement, which is called quantum state sharing [36] (abbreviate it as QSTS for the difference from QSS of a classical information [32]) recently, and cannot be implemented in only a classical way. For example, Li et al. proposed a QSTS scheme for sharing an unknown single qubit with \( m \) agents. In their scheme, Alice shares \( m \) Einstein-Podolsky-Rosen (EPR) pairs with the \( m \) agents, and she performs an \( (m+1) \)-particle Greenberger-Horne-Zeilinger (GHZ) state measurement on the unknown quantum system and her \( m \) EPR particles. \( m-1 \) agents take a single-particle measurement on their EPR particles and the last agent can recover the original unknown state with the help of all the other agents.

Quantum teleportation [24,25] is a quantum technique with which the two remote parties, the sender Alice and the receiver Bob, exploit the nonlocal correlations of the quantum channel shared in advance, such as an Einstein-Podolsky-Rosen (EPR) pair, to teleport an unknown quantum state \( |\phi\rangle_u = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \). The task can be accomplished by means that Alice makes a Bell-basis measurement on her EPR particle and the unknown quantum system \( u \), and Bob reconstructs the state \( |\phi\rangle_u \) with a local unitary operation on his EPR particle according to the classical information published by Alice [24,25].

In this paper, we will propose a protocol for multiparty quantum secret splitting and quantum state sharing

**I. INTRODUCTION**

*Multi-party quantum secret splitting and quantum state sharing*

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A protocol for multi-party quantum secret splitting is proposed with an ordered \( N \) Einstein-Podolsky-Rosen (EPR) pairs and Bell-state measurements. It is secure and has the high intrinsic efficiency and source capacity as almost all the instances are useful and each EPR pair carries two bits of message securely. Moreover, we modify it for multiparty quantum state sharing of an arbitrary \( m \)-particle entangled state based on quantum teleportation with only Bell-state measurements and local unitary operations which make this protocol more convenient in a practical application than others.

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quantum secret splitting with EPR pairs following some ideas in Refs. [15, 17, 24]. It has the high intrinsic efficiency and source capacity as almost all the instances are useful and each EPR pair carries two bits of message securely. Moreover, we modify it for multiparty quantum state sharing (MQSTS) of an arbitrary $m$-particle entangled state based on quantum teleportation [35]. The parties exploit EPR pairs and Bell-state measurements to accomplish this task, which makes this protocol more convenient in a practical application than others.

II. MULTIPARTY QUANTUM SECRET SPLITTING WITH EPR PAIRS

In a QSS, its security is simplified to prevent the dishonest agent from eavesdropping freely as the parties can detected any eavesdropper if they can find out the dishonest one in the agents [14]. Our multiparty quantum secret splitting (MQSSP) protocol inherits this feature. For convenience, let us first describe a three-party quantum secret splitting protocol, and then generalize it to the case with $N$ agents, similar to Ref. [24].

An EPR pair is in one of the four Bell states shown as

$$|\psi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0 \rangle_A |1 \rangle_B \pm |1 \rangle_A |0 \rangle_B),$$

$$|\phi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0 \rangle_A |0 \rangle_B \pm |1 \rangle_A |1 \rangle_B).$$

where $|0 \rangle$ and $|1 \rangle$ are the eigenvectors of a two-level quantum system, and the subscripts $A$ and $B$ represent the two particles in an EPR pair. The four local unitary operations $U_i (i = 0, 1, 2, 3)$ can transform one of the Bell states into another.

$$U_0 \equiv I = |0 \rangle\langle 0| + |1 \rangle\langle 1|,$$

$$U_1 \equiv \sigma_z = |0 \rangle\langle 0| - |1 \rangle\langle 1|,$$

$$U_2 \equiv \sigma_x = |0 \rangle\langle 1| + |1 \rangle\langle 0|,$$

$$U_3 \equiv i\sigma_y = |0 \rangle\langle 1| - |1 \rangle\langle 0|,$$

where $I$ is the $2 \times 2$ identity matrix and $\sigma_i$ are the Pauli matrices.

Suppose the three parties in the MQSTS are the sender Alice and her two agents, Bob and Charlie. The basic idea of our MQSTS protocol is that the two photons in each EPR pairs prepared by the agent Bob are not transmitted simultaneously in the insecure quantum channel, and Alice codes her message with four local unitary operations after confirming the security of the channel, same as the two-step QSDC scheme [13]. We can describe our MQSTS protocol in detail as follows.

(1) Alice, Bob and Charlie agree on that the four local unitary operations $U_0, U_1, U_2$ and $U_3$ represent the two bits of information $00, 11, 01$ and $10$, respectively. They use the two measuring bases (MBs), $Z \equiv \{|0 \rangle, |1 \rangle\}$ and $X \equiv \{|+\rangle = \frac{1}{\sqrt{2}}(|0 \rangle + |1 \rangle), \{-\rangle = \frac{1}{\sqrt{2}}(|0 \rangle - |1 \rangle)\}$ (for instance, the polarizations of a photon along $z-$ and $x-$directions), to measure the sample photons randomly in the process of eavesdropping check.

(2) The agent Bob prepares an ordered $N$ EPR polarization photons pairs in the same quantum state, say $|\psi^-\rangle_{AC} = \frac{1}{\sqrt{2}}(|0 \rangle_A |1 \rangle_C - |1 \rangle_A |0 \rangle_C)$. We denote the $N$ ordered EPR pairs with $|(P_1(A), P_1(C)), (P_2(A), P_2(C)), (P_3(A), P_3(C)), \ldots, (P_N(A), P_N(C))\rangle$, same as Refs. [35, 43, 44].

(3) Bob takes one photon from each EPR pair to form an ordered EPR partner photon sequence, say $[P_1(A), P_2(A), P_3(A), \ldots, P_N(A)]$, called the $S_A$ sequence. The remaining EPR partner photons compose another EPR partner photon sequence $[P_1(C), P_2(C), P_3(C), \ldots, P_N(C)]$, which is called the $S_C$ sequence.

(4) Bob fist sends the $S_A$ sequence to Alice and keeps the $S_C$ sequence. Alice picks out a sufficiently large subset of photons from the sequence $S_A$ for the eavesdropping check of the transmission.

The check can be completed with the following procedures: (a) Alice tell Bob which photons he has chosen and Bob picks out the correlated photons in the sequence $S_C$. (b) Bob chooses randomly the measuring basis (MB) $Z$ or $X$ to measure the chosen photons. (c) Bob tells Alice which MB he has chosen for each photon and the outcomes of his measurements. (d) Alice uses the same MBs as Bob to measure the corresponding photons and checks the eavesdropping with the results of Bob’s. If no eavesdropping exists, their results should be completely opposite in an ideal condition, i.e., if Alice gets 0 (1), then Bob gets 1 (0). This is the first eavesdropping check.

After that, if the error rate $e_1$ is small, Alice and Bob can conclude that there are no eavesdroppers in the line. Alice and Bob continue to perform step 5; otherwise they have to discard their transmission and abort the communication.

(5) Bob chooses randomly one of the four local unitary operations $U_i (i = 0, 1, 2, 3)$ to encrypt each of the photons in the sequence $S_C$, say $U_B$, and then he sends the sequence $S_C$ to Alice.

(6) Alice analyzes the error rate $e_2$ of the transmission of $S_C$. This is the second eavesdropping check. For this end, Alice first picks up $k + j$ EPR photon pairs and then requires Bob to publish the operations $U_B$ with which he has operated on these pairs. Alice takes the single-photon measurements on the two photons of $k$ pairs by choosing randomly the MB $X$ or $Z$. For the other $j$ pairs, she only measures one photon in each pair with the MB $Z$ or $X$ chosen randomly. She uses the other photons in these pairs as the decoy photons for checking eavesdropping in the next step.

(7) Alice inserts randomly the decoy photons in the sequence $S_C$ and then she sends it to Charlie. They analyze the security of this transmission with the decoy photons. If they conclude that the quantum line is secure, they continue their communication to next step; otherwise, they discard the results and repeat the quantum communication from the beginning.
(8) Alice selects a subset of photons as the samples for eavesdropping check and chooses one of the four unitary operations randomly on each sample. For other photons in the $S_A$ sequence (except for those for eavesdropping check), Alice encodes her message $M_A$ on them with the four unitary operations, and then sends the sequence $S_A$ to Charlie.

(9) Charlie performs the Bell measurements on the EPR photon pairs and reads out the combination of the operations done by Alice and Bob, i.e., $U_C = U_A \otimes U_B$.

(10) Alice and Charlie finish the error rate of the samples selected by Alice, and determine whether there are eavesdroppers monitoring the quantum channel when the $S_A$ is transmitted from Alice to Charlie. For this end, Alice first requires Bob to publish the operations done on the correlated photons in the sequence $S_C$, and then she requires Charlie to tell her the results of the Bell measurements. If the transmission of the $S_A$ sequence is secure, Alice tells Bob and Charlie to collaborate for reading out the message $M_A$, otherwise they will abandon the results of the transmission.

This MQSSP protocol follows some ideas in the two-step QSDC scheme [15] and the Bennett-Brassard-Mermin 1992 (BBM92) QKD protocol [11]. The process for the transmission of the $S_A$ sequence from Bob to Alice is same as that in Ref. [12], and its security is same as the BBM92 QKD protocol [11], whose security is proven in both an ideal condition [11] and a practical condition [12]. So the process for the transmission of the $S_A$ sequence is secure for any eavesdropper including the agent Charlie. The operation done by Bob on each photon in the $S_C$ is equivalent to the encryption on the photon with a random key, which makes any other one have no ability for reading out the information on the photon, same as the quantum one time pad [7, 16]. That is, any eavesdropper except the agent Bob cannot eavesdrop the message $M_A$ even though he monitors the quantum channel in the subsequent processes.

The goal that the $S_C$ sequence is first transmitted to Alice before Alice encodes her message on the $S_A$ sequence is to prevent Bob from eavesdropping with a fake signal and cheat freely. The eavesdropping check of the transmission of the $S_C$ sequence is necessary for Alice and Charlie with some decoy photons. Also, the cheating of Bob’s will be found out by comparing the results of the measurements on the decoy photons. After the processes for the secure transmission of the $S_A$ sequence from Bob to Alice and that of the $S_C$ sequence from Alice to Charlie, any one cannot steal the message $M_A$ except that Bob and Charlie cooperate. Then the present MQSSP protocol is secure.

In fact, the security in each procedure of the transmission of the particle sequences is ensured by the eavesdropping check in this scheme, which is different to Ref. [24]. In the latter, the agent Bob does not sample his particles and measure them, which makes it insecure if the other agent Charlie wants to steal Bob’s information with multi-photon fake signal, similar to Ref. [25]. That is, Charlie intercepts the signal transmitted from Alice to Bob, and sends some photons in the EPR pairs with same Bell state in a time slot. Charlie can obtain almost all the information about the operations done by Bob with photon number splitters. Moreover, she can read out the information with a suitable delay Trojan horse attack [28]. This case does not happen in this scheme.

The generalization of this MQSSP protocol to the case with $N$ agents can be implemented in a simple way by modifying the processes in the case with two agents. We describe it after the step 4 discussed above.

(5') Bob chooses randomly one of the four local unitary operations $U_i$ ($i = 0, 1, 2, 3$) to encrypt each of the photons in the sequence $S_C$, say $U_B$, and then he sends the sequence $S_C$ to next agent, Charlie.

(6') Alice and Charlie analyze the error rate of this transmission by choosing randomly a subset of the EPR pairs and requires Bob to tell them his operations $U_B$ on these samples. They measure their sample photons with the correlated MBs, same as that between Alice and Bob. If the error rate is lower than the threshold value, Charlie operates the photons in the sequence $S_C$ by choosing randomly one of the four operations $U_i$ ($i = 0, 1, 2, 3$). Also, he picks up some samples from $S_C$ and performs a Hadamard operation on each sample. He sends the sequence to the next agent Dick.

(7') Alice and Dick complete the eavesdropping check of this transmission, same as that between Alice and Charlie. The difference is that Charlie should attend to check eavesdropping in this time. He publishes some positions of the the photons operated with Hadamard operations, and then Alice and Dick confirm whether there is an eavesdropper who has stolen the information about the operations done by Charlie. If the quantum communication is still secure, Dick repeats the procedure of Charlie’s and sends the sequence $S_C$ to the next agent.

(8') After repeating the step 7’ $N-3$ times, the $S_C$ sequence is received securely by Yang, the agent before the last one Zach. After the operations done by Yang, similar to Charlie, he sends the sequence $S_C$ to Alice (Not Zach!).

(9’) Alice analyzes the error rate of the transmission of $S_C$ between her and Yang. She first picks up $k + j$ EPR photon pairs and then requires all the agents except for the last one to publish their operations with which he has operated on these pairs. Alice takes the single-photon measurements on the two photons of $k$ pairs by choosing randomly the MB X or Z. For the other $j$ pairs, she only measures one photon in each pair with the MB Z or X chosen randomly. She uses the other photons in these pairs as the decoy photons for checking eavesdropping in the next step. Certainly, Yang should tell Alice the photons operated with Hadamard operations and they analyze their error rate after the other agents publish their operations on these photons.

(10’) Alice inserts randomly the decoy photons in the sequence $S_C$ and then sends it to the last agent Zach. They analyze the security of this transmission with the
decoy photons. If they conclude that the quantum line is secure, they continue their communication to next step; otherwise, they discard the results and repeat the quantum communication from the beginning.

(11’) Alice selects a subset of photons as the samples for eavesdropping check and chooses one of the four unitary operations randomly on each sample. For other photons in the $S_A$ sequence (except for those for eavesdropping check), Alice encodes her message $M_A$ on them with the four unitary operations, and then sends the sequence $S_A$ to Zach.

(12’) Zach performs the Bell-state measurements on the EPR photon pairs and reads out the combination of the operations done by Alice and Bob, i.e., $U_Z = U_A \otimes U_B \otimes U_C \otimes \cdots \otimes U_Y$.

(13’) Alice and Zach finish the error rate of the samples selected by Alice, and determine whether there are eavesdroppers monitoring the quantum channel when the $S_A$ is transmitted from Alice to Charlie. For this end, Alice first requires all the agents to publish the operations done on the correlated photons in the sequence $S_C$, and then she requires Zach to tell her the results of the Bell measurements. If the transmission of the $S_A$ sequence is secure, Alice tells all the agents to collaborate for reading out the message $M_A$, otherwise they will abandon the results of the transmission.

The security of this multiparty quantum secret splitting protocol is same as that with two agents as each process for the transmission of the photons is same as that in BBM92 QKD protocol [11]. That is, the process for secure transmission of photons in BBM92 QKD is repeated $N + 2$ times. So this multiparty quantum secret splitting is secure also.

III. MQSTS OF AN ARBITRARY M-PARTICLE STATE WITH EPR PAIRS

The state of an M-particle quantum system $x$ can be written as following:

$$|\psi_x\rangle = \sum_{i,j,\ldots,k \in \{0,1\}} C_{ij\ldots k}|ij\ldots k\rangle,$$

where

$$\sum_{i,j,\ldots,k \in \{0,1\}} |C_{ij\ldots k}|^2 = 1.$$  \hspace{1cm} (8)

The state $|\psi_x\rangle$ can be teleported with M EPR pairs completely [8]. That is, the protocol for quantum secret splitting of classical message discussed above can be modified to accomplish the task of multiparty quantum state sharing (MQSTS) with quantum teleportation. We depict it with three parties (Alice, Bob and Charlie) in brief as follows.

(II) Bob chooses randomly one of the four unitary operation $U_i$ ($i = 0, 1, 2, 3$) to encrypt each photon in the $S_C$ sequence, and then sends the sequence to Alice. She picks up $k + j$ EPR pairs and requires Bob to tell her the operations on these samples. She analyzes the security of the transmission for the sequence $S_C$ by measuring $K$ pairs with MB Z or X on the two photons in each pair. Then she performs a single-photon measurement on one photon in each of the other $j$ EPR pairs in the samples. She uses the photons collapsed as the decoy photons for checking eavesdropping in the next step by inserting them in the sequence $S_C$. She sends the sequence $S_C$ to Charlie.

(III) Alice and Charlie finish the eavesdropping check for the transmission of the $S_C$ sequence with the decoy photons. If the error rate is low, they continue to the next step, otherwise they abandon the results transmitted and repeat the quantum communication from the beginning.

(IV) Alice picks out $M$ EPR pairs from the N ordered EPR photon pairs ($N \geq M$), i.e., she select $M$ photon in the $S_A$ sequence in order. Then she performs a Bell-state measurement on each photon and a particle from the quantum system $x$, shown in Fig.1.

(V) Alice publishes the results of the Bell-state measurements in public, and Charlie can reconstructs the quantum system $x$ with the help of Bob’s.

In fact, this quantum state sharing protocol is just a process that Alice teleports the M-particle quantum system to Charlie who does not know the states of the quantum channel, i.e., those of the EPR pairs shared. Let us use a two-particle quantum system as an example to describe the principle in detail. The other case is the same as it with or without a little modification.

Assume that the unknown two-particle state $|\psi_x\rangle$ is $|\psi_x\rangle \equiv |\phi_{X_1, X_2}\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$, here $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Alice and Charlie first pick out two EPR pairs from the N ordered EPR pairs shared, say

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**FIG. 1:** The Bell measurements performed by Alice. The bold lines connect the two photons in an EPR pair or the M-particle quantum system $x$. The panes with dashed lines are the Bell-state measurements.
The relation between the local unitary operations and the known quantum state only when he collaborate with Bob. 

TABLE I: The relation between the local unitary operations and the arbitrary entangled state $|Φ\rangle_{X_1X_2}$. 

| $|φ⟩_{X_1A_1}$ | $|φ⟩_{X_2A_2}$ | $(U_{B_1}^{-1}⊗U_{B_2}^{-1})|Φ⟩_{C_1C_2}$ | $U_i⊗U_j$ |
|-----------------|-----------------|-------------------------------|-----------------|
| $φ^+$            | $φ^+$            | $(a|11⟩−b|10⟩+c|01⟩+d|00⟩)$ | $U_3⊗U_3$        |
| $φ^+$            | $φ^−$            | $(a|11⟩+b|10⟩−c|01⟩−d|00⟩)$ | $U_3⊗U_2$        |
| $φ^−$            | $ψ^+$            | $(a|10⟩−b|11⟩−c|00⟩+d|01⟩)$ | $U_3⊗U_1$        |
| $φ^−$            | $ψ^−$            | $(a|10⟩+b|11⟩−c|00⟩−d|01⟩)$ | $U_3⊗U_0$        |
| $ψ^+$            | $φ^+$            | $(a|01⟩−b|00⟩−c|11⟩+d|10⟩)$ | $U_2⊗U_3$        |
| $ψ^+$            | $φ^−$            | $(a|01⟩+b|00⟩−c|11⟩−d|10⟩)$ | $U_2⊗U_2$        |
| $ψ^−$            | $φ^+$            | $(a|00⟩−b|01⟩−c|10⟩+d|11⟩)$ | $U_2⊗U_1$        |
| $ψ^−$            | $φ^−$            | $(a|00⟩+b|01⟩−c|10⟩−d|11⟩)$ | $U_2⊗U_0$        |
| $ψ^+$            | $ψ^−$            | $(a|01⟩−b|00⟩+c|11⟩−d|10⟩)$ | $U_0⊗U_3$        |
| $ψ^−$            | $ψ^+$            | $(a|01⟩+b|00⟩+c|11⟩+d|10⟩)$ | $U_0⊗U_2$        |
| $ψ^+$            | $ψ^−$            | $(a|00⟩−b|01⟩+c|10⟩−d|11⟩)$ | $U_0⊗U_1$        |
| $ψ^−$            | $ψ^−$            | $(a|00⟩+b|01⟩+c|10⟩+d|11⟩)$ | $U_0⊗U_0$        |

The relation between the local unitary operations and the state of the two particles hold in the hand of Charlie after all the measurements are done by Alice; $U_i ⊗ U_j$ are the local unitary operations with which Charlie can reconstruct the unknown quantum state $|Φ⟩_{X_1X_2}$. 

IV. DISCUSSION AND SUMMARY

As discussed in Refs. [15, 16, 17, 18], the direct transmission of secret message should resort to the transmission of quantum data block. The parties should confirm the security of the quantum channel before the message is encoded on the states. In present multiparty quantum secret splitting and quantum state sharing protocols, the states are transmitted in quantum data block. The good feature is that the error correction and the quantum privacy amplification can be done on the states directly, which will ensure the security of those quantum communication protocols in a practical channel. In a loss channel, Alice and Charlie should prevent Bob from eavesdropping with quantum teleportation, same as the two step QSDC protocol [17]. That is, Charlie will determine which position in the $S\text{c}$ sequence has no photon and tell Alice not perform encoding the message on the correlated photon in the $S\text{c}$ sequence.

In present multiparty quantum secret splitting protocol, each EPR pair can carry two bits of information and its security is ensured by the eavesdropping checks with the single-photon measurements between the sender and the agents. The efficiency for qubits $q_t = \frac{2n}{q_a}$ approaches the maximal value 100% as almost all the EPR pairs are useful for carrying the message in principle, and here $q_a$ is the useful qubits and $q_t$ is the total qubits transmitted. The total efficiency $η_t$ in this protocol also approaches 100% as the classical information exchanged is not necessary except for the eavesdropping checks. $η_t$ is defined as

$$η_t = \frac{b_m}{q_t + b_t},$$

where $b_m$ is the number of bits in the message $M_A$, $q_t$ and $b_t$ are the qubits used and the classical bits exchanged between the parties in the quantum communication, respectively.

For sharing the unknown quantum state, the agent Bob plays a role for preparing the quantum source and encrypting a classical information on them also. The last agent will recover the unknown quantum state with the help of the other agents as he keeps the only quantum system for the reconstruction after all the Bell measurements are done by Alice. In this way, the present multiparty quantum state sharing protocol is asymmetrical, which is same as that in Ref. [27] and is different to the symmetrical protocols [21, 22] in which any one in the agents can act as the receiver with the help of others’. Certainly, any cheating done by the agents including the receiver can be detected if the sender and the agents accomplish a honesty check before the agents cooperate to recover the unknown states. That is, Alice inserts randomly some samples in the unknown quantum system $x$ and requires the agents to recover their states and measure them before they obtain the quantum information, same as that in Ref. [20]. The total efficiency $η_t$ of this multiparty quantum state sharing protocol is about 50% as Alice should publish two bits of classical information for two bits of quantum information.

In summary, we present a way for multiparty quantum secret splitting with an ordered $N$ EPR pairs following some ideas in Ref. [15, 17, 24]. It is secure and
each of the EPR pairs can carry two bits of message. Its efficiency for qubits and the total efficiency both approach the maximal value 100%. Moreover, we modify it for multiparty quantum state sharing (MQSTS) of an arbitrary $M$-particle entangled state based on quantum teleportation. For the latter, the task is completed with the quantum channel of two-particle entanglement and the two-particle Bell-state measurements, which make it more convenient in a practical application than others as it is difficult for producing and measuring a multipartite entangled states at present [44, 45, 46].

V. NOTE

This paper appeared in PLA. However, there is a security loophole in the original manuscript (also for other schemes with the similar principle). It is necessary for us to forbid the dishonest agent eavesdrop the quantum communication with a fake signal (a Bell state) and cheat. In this way, the boss Alice should have the capability of detecting cheat. In this revision, we added some procedures for detecting cheat and made this protocol be secure.

VI. ACKNOWLEDGEMENTS

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