Little String Theories in Heterotic Backgrounds\* 

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Abstract 

We study Little String Theories (LST) with \(\mathcal{N} = (1,0)\) supersymmetry arising, in a suitable double scaling limit, from 5-branes in heterotic string theory or in the heterotic-like type II/\((-)^F_L \times \text{shift}\). The limit in question, previously studied in the type II case, is such that the resulting holographically dual pairs, i.e. bulk string theory and LST are at a finite effective coupling. In particular, the internal (2,2) SCFT on the string theory side is non-singular and given by \(SL(2)/U(1) \times SU(2)/U(1)\) coset. In the type II orbifold case, we determine the orbifold action on the internal SCFT and construct the boundary states describing the non-BPS massive states of a completely broken \(SO\) gauge theory, in agreement with the dual picture of D5-branes in type II/\(\Omega \times \text{shift}\). We also describe a different orbifold action which gives rise to a \(Sp\) gauge theory with \((1,1)\) supersymmetry. In both the heterotic \(SO(32)\) and \(E_8 \times E_8\) cases, we determine the gauge bundles which correspond to the above SCFT and break down the gauge groups to \(SU(2) \times SO(28)\) and \(E_7 \times E_8\) respectively. The double scaling limit in this case involves taking small instanton together with small string coupling constant limit. We determine the spectrum of chiral gauge invariant operators with the corresponding global symmetry charges on the LST side and compare with the massless excitations on the string theory side, finding agreement for multiplicities and global charges. 

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1. Introduction

Little String Theories (LST) in type II or heterotic backgrounds received considerable attention in the last few years\[1, 2, 3, 4, 5, 6, 7\]. These 5+1 dimensional theories arise, as first observed in \[1\], by considering the world-volume theory of 5-branes in the decoupling limit where the string coupling constant \(g_s\) goes to zero, in such a way that the world-volume theory decouples from the bulk string theory but gives rise to a non-trivial, interacting 5+1 dimensional Quantum Field Theory, with \(\mathcal{N} = (2, 0)\) or \(\mathcal{N} = (1, 1)\) supersymmetry in type IIA or IIB, respectively. In both cases the theory contains string-like excitations which however do not give rise to gravity. In \[2\] it was argued that type II string theory in the linear dilaton background corresponding to a collection of coincident NS5-branes\[8\], has a holographic description in terms of the above 5+1 dimensional QFT. A class of observables in the LST, in short representations of the supersymmetry algebra, was found to match with a class of excitations in the bulk string theory. This analysis has been later extended in \[3\] to the heterotic string in the background of symmetric 5-branes, which also give rise to a SCFT involving a linear dilaton times an \(SU(2)\) WZW model. The problem with the linear dilaton is that, no matter how small is the asymptotic value \(g_s\) of the string coupling, it gives rise to a divergent string coupling near the NS5-branes, and therefore is not amenable for perturbative analysis. It was further observed in \[3, 4\] that, in the type II case, this problem can be overcomed by separating the NS5-branes: this has the effect of cutting-off the strong coupling region by adding a cosmological constant to the corresponding Liouville CFT, or, equivalently, replacing the Liouville theory with an \(SL(2, \mathbb{R})/U(1)\) coset CFT. The full internal CFT turns out to be an \(\mathcal{N} = (2, 2)\) SCFT, given by the tensor product of a \(\mathcal{N} = (2, 2)\) minimal model times an \(\mathcal{N} = (2, 2)\) Liouville theory or \(SL(2, \mathbb{R})/U(1)\) Kazama-Suzuki coset model \[^1\]. In this case one still takes the

\[^1\]Actually, in the internal, world-sheet SCFT, supersymmetry is enhanced to \(\mathcal{N} = (4, 4)\). Earlier work on (4,4) SCFT’s related to NS5-branes has appeared in \[3, 10, 11\].
decoupling limit \( g_s \to 0 \), but holds fixed the tension of the open D-string (in type IIB case) or open D2-branes (in type IIA case) connecting the NS5-branes. On the LST side this corresponds to moving off the origin of the Coulomb branch to a point where the original \( SU(2)_L \times SU(2)_R \) R-symmetry group is broken down to \( U(1) \times Z_N \), \( N \) being the number of NS5-branes. In [3, 4] various tests of the holographic correspondence in this background have been performed, including the matching of short multiplets on the two sides.

Purpose of this work is to extend the study of the holographic connection to theories which have \( 8 \) supercharges in the background of the 5-branes. In this case the LST has \( \mathcal{N} = (1,0) \) supersymmetry. Our main interest will be the heterotic string, but we will consider also a freely acting \( Z_2 \) orbifold of type II theory which removes the spacetime supercharges coming from, say, the left-moving sector. For this we require a compact \( S^1 \) direction and the \( Z_2 \) group element will then be \( (-)^{F_L \sigma_P} \), where \( F_L \) is the left-moving spacetime fermion number and \( \sigma_P \) is a shift of order 2 along \( S^1 \). The \( S^1 \) is considered to be part of the world-volume of the NS5-branes. In this case, in a dual description of the system in terms of D5-branes in type IIB/\( \Omega \sigma_P \), it has been shown in [26] that one can have both \( SO \) or \( Sp \) gauge groups. We will determine the appropriate action of the orbifold group on the internal SCFT and construct boundary states. We will find that the boundary states describe the massive gauge particles corresponding to the generators of a completely broken \( SO \) gauge group, in agreement with the dual D5-brane picture. As for the \( Sp \) case, we will find an orbifold action on the SCFT which results into boundary states agreeing with an \( Sp(N) \) group being broken down to \( Sp(1)^N \), with however unbroken \( \mathcal{N} = (1,1) \) supersymmetry. Thus the issue of reproducing a \( (1,0) \) \( Sp \) gauge theory is still open.

As for the heterotic string case, we recall that in the analysis of [8] the background corresponding to the linear dilaton (times \( SU(2) \) WZW model) SCFT involved a singular gauge bundle with a single fat \( SU(2) \) instanton superimposed to \( N - 1 \) small instantons \([29, 30]\) at the origin of \( R^4 \). We will show that the SCFT involving the \( (2,2) \) Liouville
theory with cosmological constant (or equivalently the \(SL(2, \mathbb{R})/U(1)\) coset) corresponds to a non-singular gauge bundle where the small instantons are given a (small) scale and are separated in \(R^4\). The double scaling limit in this case involves taking small instanton together with small string coupling limit. On the LST side this corresponds to moving in the Higgs branch to a point which has a global symmetry group containing the (double cover of) \(U(1) \times \mathbb{Z}_N\), as one expects from the world-sheet SCFT on the string theory side. We will compare, finding agreement, the spectrum of a class chiral gauge invariant operators from the LST, at the given point on the Higgs branch, with the corresponding on-shell states from bulk string theory.

The paper is organized as follows: in section 2 we review the main features of the (2,2) SCFT corresponding a configuration of 5-branes in type II string theory, together with the construction of boundary states describing the massive BPS states of the LST theory. In section 3 we consider type II/(\(-\)^{F_L} \times \sigma_P) in the above background together with the (non BPS) boundary states for the \(SO\) case. We will also describe the \(Z_2\) orbifold giving rise to \(Sp\) gauge group with (1,1) supersymmetry. In section 4 we will turn to the heterotic \(SO(32)\) and \(E_\text{8} \times E_\text{8}\) cases. First, we will describe the spectra of the two theories in the mentioned SCFT, which corresponds to a symmetric embedding of the gauge connection into the spin connection giving rise to unbroken gauge group \(SU(2) \times SO(28)\) and \(E_7 \times E_8\) in the two cases. We then analyze the ADHM constraints\([12]\) for \(SO(4)\) instantons relevant for the \(SO(32)\) theory, which are nothing but the F- and D-term constraints for a 4D, \(\mathcal{N} = 1\) gauge theory with \(Sp(N)\) gauge group and \(SO(4)\) flavour group \([29]\). We will identify the solution of the ADHM constraints which determines the gauge bundle corresponding to the SCFT under consideration and describe its global symmetries. With the resulting massless spectrum we construct gauge invariant chiral operators in the LST, compare with states one obtains from string theory and verify the matching of the multiplicities and global symmetry quantum numbers.
2. Review of the Type II case

In this section we will briefly review the CFT description of a system of $N$ separated NS5-branes in type II string theories. Let us remind that the CFT describing coincident NS5-branes has a bosonic part corresponding to the target space

$$R^{5,1} \times R_\varphi \times SU(2)_{N-2}. \quad (2.1)$$

The first factor describes the flat 5+1-dimensional Minkowski part, corresponding to the world volume coordinates $X^\mu$ of the NS5-branes. The second factor representing the radial direction transverse to the branes, is actually described by a Liouville field $\varphi$ with background charge $Q = \sqrt{2/N}$ (we set $\alpha' = 2$ here). Finally, the third factor is given by an $SU(2)$ WZW model at level $N-2$. Together with a total of 10 free fermions, the system has the critical central charge 15. In [2] the above CFT has been used to test the holographic correspondence between short multiplets in type IIA or IIB string theory on this background and those expected to arise in the corresponding 5+1-dimensional Little String Theories (LST).

However, string theory based on the above CFT is strongly coupled, due to the linear dependence of the dilaton on the coordinate $\varphi$, which manifests itself with the background charge term for $\varphi$. As a result, the string coupling constant diverges for $\varphi \to -\infty$.

In [3, 4] it has been argued that separating the NS5-branes, therefore moving in the Coulomb branch of the 5+1-dimensional world-volume theories, regularizes the strong coupling singularity.

The Coulomb branch in the cases IIA and IIB turns out to be $(R^4 \times S^1)^N/S_N$ and $(R^4)^N/S_N$ respectively, as dictated by the $\mathcal{N} = (2,0)$ respectively $\mathcal{N} = (1,1)$ nature of the 5+1 dimensional supersymmetry. In both cases there are 4 scalars $X^i$, $i = 6, 7, 8, 9$ in the adjoint of $U(N)$, parametrizing the transverse positions of the NS5-branes, whose Cartan components give rise to the above moduli spaces (in type IIA there is one more compact
scalar, giving rise to the additional $S^1$). Introducing the complex scalars $A = X^8 + iX^9$ and $B = X^6 + iX^7$, one particular point in the moduli space is characterized by the only non-trivial gauge invariant vacuum expectation value $<\text{tr}B^N> = \mu^N$, corresponding to the $N$ fivebranes symmetrically distributed on a circle of radius $\mu$ in the (6,7) plane. This arrangement breaks the original $SO(4)$ transverse rotational symmetry (which is an $R$-symmetry of the 6-dimensional theory in type IIB case) down to $SO(2) \times Z_N$. The double scaling limit is defined by taking $g_s \to 0, \mu \to 0$, with $M_W = \mu/g_s\alpha'$ being held fixed. This latter quantity is the “W-boson” mass, i.e. the mass of the D-strings stretched between the NS fivebranes in the type IIB case.

The regularization of the strong coupling singularity comes from the fact that switching on a non-trivial vev for $\text{tr}B^N$ has the effect of adding a cosmological constant to the worldsheet lagrangian for the Liouville field. To see this, is convenient to re write the background (2.1) as:

$$R^5 \times R_\phi \times (S^1 \times \frac{SU(2)}{U(1)})/Z_N.$$  

(2.2)

where $S^1$ has radius $\sqrt{2N}$ and its coordinate is denoted by $Y$. The Kazama-Suzuki coset $SU(2)/U(1)$ is an $\mathcal{N} = 2$ minimal model with central charge $3 - 6/N$, and the $Z_N$ orbifold is essentially the GSO projection. The field $\phi + iY$, together with its fermionic partners $\psi_L$, $\psi_R$ (and the conjugate $\bar{\psi}$) makes an $\mathcal{N} = 2$ superfield $\Phi$, giving rise to an $\mathcal{N} = 2$ version of the Liouville theory, with central charge $3 + 6/N$ [13]. The perturbation corresponding to the expectation value $<\text{tr}B^N>$ can be shown to be, in superfield notation

$$\delta L = \int d^2 \theta \exp \left( -\frac{1}{Q}\Phi \right) + c.c.$$  

(2.3)

The fact that the interaction in (2.3) grows exponentially for $\phi \to -\infty$ regularizes the strong coupling singularity. One can show that the effective string coupling is of order $1/M_W \sqrt{\alpha'}$.

A convenient dual (more precisely, mirror [14]) description of the perturbed $\mathcal{N} = 2$ Liou-
ville theory, which we will use in the following, is given by the non-compact \( SL(2, \mathbb{R})/U(1) \)
\( \mathcal{N} = 2 \) coset model \([15, 16, 17, 18]\). We are thus equivalently lead to consider type II string
theory on

\[
\mathbb{R}^{5,1} \times \left( \frac{SL(2, \mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)} \right)/\mathbb{Z}_N.
\] (2.4)

In this picture the strong coupling region \( \varphi \to -\infty \) of the cylinder with coordinates \((\varphi, Y)\) is cut off and the infinite cylinder is replaced by a semi-infinite cigar. The asymptotic
radius of the cigar \( Q \) is related by T-duality to that of the cylinder \( 2/Q = \sqrt{2N} \). The
\( SO(2) \times \mathbb{Z}_N \) symmetry of the problem is manifest in the perturbed Liouville CFT: the
perturbation in \( (2.3) \) carries momentum \( N \) and zero winding along the circle parametrized
by \( Y \), therefore winding is conserved (corresponding to the \( SO(2) \) symmetry), while mo-
mentum is conserved modulo \( N \) (corresponding to the \( \mathbb{Z}_N \) symmetry). Going to the dual
description given by \( \frac{SL(2, \mathbb{R})}{U(1)} \), momentum and winding are exchanged, in agreement with the
cigar geometry of the target space.

Finally, a useful correspondence comes from the fact that the CFT \( \left( \frac{SL(2, \mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)} \right)/\mathbb{Z}_N \)
is known \([17]\) to describe (for the case of \( A \)-type modular invariants) backgrounds given
by (deformed) ALE spaces of the type \( A_{N-1} \). The defining equation in \( \mathbb{C}^3 \) of these non-
compact Calab-Yau twofolds is \( x^N + y^2 + z^2 = \mu^N \), the complex deformation parameter
being \( \mu^N \). The precise statement is that type IIA(B) on ALE spaces of type \( A_{N-1} \) is
equivalent to type IIB(A) string theory on the background of \( N \) NS-fivebranes separated
\( \mathbb{Z}_N \) symmetrically on a circle \([17, 3, 4]\).

Since we can have arbitrarily small string coupling constant, we can analyze the per-
turbative spectrum on the background \([2.4]\) and compute scattering amplitudes among its
states. To do that we have to handle the non trivial, minimal model part of the CFT
\( \left( \frac{SL(2, \mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)} \right)/\mathbb{Z}_N \). Let us start from the coset \( SU(2)/U(1) \), representing an ordinary
\( \mathcal{N} = 2 \) minimal model at level \( N-2 \): the left-moving part of the primary fields is character-
ized by three integers \( \ell, m, s \), where \( \ell = 0, \ldots, N-2 \), \( m = -N, \ldots, N-1 \) and \( s = -1, 0, 1, 2, 0, 2 \) and \(-1, 1\) corresponding to the NS and R sector \( SO(2) \) conjugacy classes. There is a constraint \( \ell + m + s = \text{even} \) and a \( \mathbb{Z}_2 \) identification given by \( \ell \rightarrow N - 2 - \ell, m \rightarrow m + N, s \rightarrow s + 2 \). The corresponding conformal dimensions are given by

\[
\Delta = \frac{\ell(\ell + 2) - m^2}{4N} + \frac{s^2}{8}, \quad |m| \leq \ell, \quad (2.5)
\]

Similarly, for the right-moving part we have (for A-type modular invariants) the labels \( \bar{\ell} = \ell, \bar{m}, \bar{s} \). The non-compact part \( Sl(2, \mathbb{R})/U(1) \) is less understood, but we will restrict to those \( \mathcal{N} = 2 \) representations which correspond to the \( SL(2, \mathbb{R}) \) discrete series with \( \ell' = 0, \ldots, N - 2 \). We will have additional quantum numbers \( m', s', \bar{m}', \bar{s}' \). \( m' \) and \( \bar{m}' \) take values in \( \mathbb{Z} \) and are given in terms of momentum \( p \) and winding \( w \) as \( m' = p + wN, \bar{m}' = -p + wN \). The (left-moving) conformal dimensions are given by:

\[
\Delta' = \frac{m'^2 - \ell' (\ell' + 2)}{4N} + \frac{s'^2}{8}, \quad (2.6)
\]

and similarly for the right-moving ones.

An important fact for us is that the minimal model has a \( G = Z_N \times Z_2 \) chiral symmetry group which acts on the fields \( \phi^{\ell}_{m,s} \) as

\[
\phi^{\ell}_{m,s} \rightarrow e^{2\pi i \frac{m}{N}} \phi^{\ell}_{m,s}, \quad \phi^{\ell}_{m,s} \rightarrow (-)^s \phi^{\ell}_{m,s}. \quad (2.7)
\]

There is a similar action for the fields of the non-compact \( Sl(2, \mathbb{R})/U(1) \) part, with the formal flip of sign \( N \) to \(-N\), corresponding to a group \( G' \) again equal to \( Z_N \times Z_2 \). The same considerations apply to the right-moving sectors.

Physical states are constructed by tensoring states coming from the spacetime part, the \( Sl(2, \mathbb{R})/U(1) \) part and and \( SU(2)/U(1) \) part. As is familiar from Gepner’s construction

\footnote{Notice that here the windings are multiple of \( N \), however they become arbitrary integers as a result of the \( Z_N \) orbifold.}
involving $\mathcal{N} = 2$ minimal models, one has to impose the GSO projection condition on the total fermionic charge, which, in the present situation, amounts to the following constraint:

\[
\frac{m - m'}{N} - \frac{s + s'}{2} - \frac{\bar{\delta} \cdot \bar{w}}{4} \in 2\mathbf{Z} + 1,
\]  

(2.8)

where $\bar{\delta}$ is the spinor weight of $SO(4)$ and $\bar{w}$ is the $SO(4)$ weight of the state. In addition, there are conditions ensuring that all components are in the same sector (either R or NS):

\[
\bar{v} \cdot \bar{w} + \frac{s}{2} \in \mathbf{Z},
\]

\[
\bar{v} \cdot \bar{w} + \frac{s'}{2} \in \mathbf{Z},
\]

(2.9)

where $\bar{v}$ is the vector weight of $SO(4)$. Corresponding conditions hold for the right-moving sector.

Notice that the effect of the $Z_N$ orbifold is to trivialize on physical states the action of the diagonal $Z_N$ subgroup of $G \times G'$, and that the conditions (2.9) identify the actions of the two $Z_2$’s.

The mass shell condition, in the NS sector and for primaries with with $s = s' = 0$, is given by:

\[
\frac{k_{\mu}k^\mu}{2} + \frac{\ell(\ell + 2) - m^2}{4N} + \frac{m'^2 - \ell'(\ell' + 2)}{4N} = \frac{1}{2},
\]

\[
\frac{k_{\mu}k^\mu}{2} + \frac{\ell(\ell + 2) - \bar{m}^2}{4N} + \frac{\bar{m}'^2 - \ell'("ell' + 2)}{4N} = \frac{1}{2}.
\]

(2.10)

Massless states (scalars) are characterized by $m' = -\epsilon(l' + 2)$, $m = \epsilon l$ and $\bar{m}' = -\bar{\epsilon}(l' + 2)$, $m = \bar{\epsilon} l$ with $\ell + \ell' = N - 2$. The values $\epsilon, \bar{\epsilon} = \pm 1$ correspond, for a given $\ell$ ($\ell'$), to the four $(c,c)$, $(c,a)$, $(a,c)$, $(a,a)$ primary states of the internal $\mathcal{N} = (2,2)$ SCFT.

There are altogether $4(N - 1)$ scalars, matching with the number of Cartan generators of $SU(N)$. Also their $U(1)$ and $Z_N$ charges, given by $m' - \bar{m}'$ and $m' + \bar{m}'$ respectively, agree with those of the gauge invariant chiral operators $\text{tr}A^{(m'-\bar{m}')/2}$ and $\text{tr}B^{(m'+\bar{m}')/2}$ of the
5+1 world-volume theory. The gauge fields (in the type IIB case say) appear in the R-R sector and are obtained by spectral flow from the scalars in the NS-NS sector. Their quantum numbers are \( m = \ell + 1, m' = -\ell' + 1 \) and \( s = s' = 1 \), with similar values in the right-moving sector.

Charged, massive states ("W-bosons") should come as D-branes (D-strings in the type IIB case) stretched between NS fivebranes, and should have a boundary state representation within the CFT discussed above. They are BPS states, preserving 8 of the 16 supercharges.

The construction of boundary states for Gepner-like\(^3\) models has been first performed in \([20]\) and more recently analyzed by several authors \([21, 22]\). One starts from Ishibashi states, which are in one-to-one correspondence with (diagonal) primary fields. One then constructs Cardy states, which are linear combinations of Ishibashi states such that the corresponding cylinder amplitudes admit the interpretation of partition functions when rewritten in the open string channel. In our case, denoting Ishibashi states as \(|l, m, s, l', m', s', \vec{w}>\) and Cardy states with the corresponding capital letters, we have, up to an overall normalization:

\[
|L, M, S, L', M', S', \vec{W} > = \sum_{l,m,s,l',m',s',\vec{w}} S^L_{l'} S^{L'}_l e^{i\pi \frac{Mm-M'm'}{N}} e^{i\pi \frac{Ss+S's'}{2} + \frac{\vec{W} \cdot \vec{w}}{2}} |l, m, s, l', m', s', \vec{w} >, \tag{2.11}
\]

where

\[
S^L_l = \frac{\sin \frac{\pi (L+1)(l+1)}{N}}{\sqrt{\sin \frac{(l+1)}{N}}} \tag{2.12}
\]

(and similarly for \( S^{L'}_{l'} \)) is the S-modular transformation matrix for the \( SU(2)_{N-2} \) characters.

The sum in (2.11) is actually restricted to the states obeying the GSO condition plus the condition that all their components belong to the same sector, as given in (2.8) and (2.9). Notice that (2.8) implies that \( m - m' \) is a multiple of \( N \). These conditions can be imposed in (2.11) by starting with a free sum, introducing multipliers \( a, b, c = 0, 1, r = 0, \ldots, N-1 \)

\(^3\)Open strings in minimal models and Gepner models have been first discussed in \([19]\)
and then inserting in the sum the projection factors:

$$\frac{1}{N} \sum_{r=0}^{N-1} e^{2\pi i (m-m')/N} \frac{1}{2} \sum_{a=0}^{1} e^{\pi i (\ell + \ell')/4} a \frac{1}{2} \sum_{b=0}^{1} e^{2\pi i (\ell' - \ell)/2} b \frac{1}{2} \sum_{c=0}^{1} e^{\pi i (m-m')/N} \frac{1}{N} e^{-i\pi (l+1)/N} e^{i\pi \ell/2}$$

(2.13)

This has the effect of shifting the labels of the Cardy states, \(M, S, M', S', \vec{W}\), to

\[
\tilde{M} = M + 2r + c, \quad \tilde{M'} = M' + 2r + c, \quad \tilde{S} = S + 2a - c, \quad \tilde{S'} = S' + 2b - c,
\]

\[
\tilde{W} = \vec{W} + (a + b)\vec{v} + c\vec{\delta}/2.
\]

(2.14)

Therefore we can choose orbit representatives labelled by \(L, M, S\) and \(L'\). Furthermore we set \(L' = 0\), as states with \(L' > 0\) correspond to multi D-brane states.

The number of states obtained this way is \(N(N-1)\): using the \(Z_2\) identification mentioned before, we can choose \(0 \leq L \leq N-2\), \(0 \leq M < 2N\) and \(S = 0, 2\), with \(L + M \) even, giving the expected number of roots of \(SU(N)\). One can proceed further to prove that these states have the right charges, as in [23, 24, 13], by computing the intersection index, which gives the Cartan matrix of \(SU(N)\). Alternatively, one can evaluate directly the charge vectors by taking the overlap of the Cardy states with the R-R states corresponding to Cartan generators of \(SU(N)\), which amounts to project to Ishibashi states with \(m = \ell + 1, m' = -\ell' + 1, s = s' = 1\) etc., as discussed before. This gives for the charge vector \(q_l(L, M, S)\):

$$q_l(L, M, S) = \sin \pi \frac{(L + 1)(l + 1)}{N} e^{-i\pi \frac{M(l+1)}{N}} e^{i\pi \frac{S}{2}}.$$

(2.15)

By taking inner products of these vectors one can check that they span the root diagram of \(SU(N)\). Notice that the actions of \(Z_N\) and \(Z_2\) on Ishibashi states, as given in (2.7), induce on Cardy states the maps \(M \to M + 2\) and \(S \to S + 2\) respectively. The latter has the effect of flipping the sign of the charge vector \(q_l \to -q_l\), therefore it sends roots to their negatives, i.e. it reverses the orientation of the D-branes.

Finally, the tension of the D-branes is computed by taking the overlap of the Cardy states with the state corresponding to the identity operator \((l = m = s = 0)\). One gets a
factor proportional to
\[ \sin \pi \frac{(L + 1)}{N}, \]
which agrees nicely with the geometrical picture of D-branes stretched between \( N \) NS-fivebranes distributed \( Z_N \) symmetrically on a circle.

3. The case of Type II/\((-)^F_L \sigma_P\)

We now concentrate our discussion on the case of NS-fivebranes in a class of theories which, although considerably simpler, have many feature in common with the heterotic string. These theories are obtained by modding out type IIB string theory by \((-)^{F_L} \sigma_P\) where \( F_L \) is the left-moving spacetime fermion number and \( \sigma_P \) is a shift of half-period along a circle \( S^1 \), with \( P \) the corresponding momentum. In the untwisted sector this has the effect of giving a phase \((-)^P\) to the states with momentum \( P \). As a result, the left-moving spacetime supersymmetry is completely broken (twisted states are massive). S-duality relates this theory to the orientifold of IIB by \( \Omega \sigma_P \), \( \Omega \) being the world sheet parity operator. The presence of the shift avoids the introduction of D9-branes, allowing both \( SO \) or \( Sp \) projections for D5-branes with longitudinal shift [26]. Various non-perturbative aspects of these theories, including tests of S-duality have been discussed in [25, 26, 27], whereas boundary states have been analyzed in [28].

We will consider the case where the shift \( \sigma_P \) is longitudinal to the fivebranes, say along the direction 5. The background corresponding to \( N \) separated NS-fivebranes should be the same as the one in type IIB. So, string theory should be described by the CFT
\[ \mathbb{R}^{1,1} \times S^1 \times \left( \frac{SL(2,\mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)} \right)/Z_N, \]
which agrees nicely with the geometrical picture of D-branes stretched between \( N \) NS-fivebranes distributed \( Z_N \) symmetrically on a circle.
\[ \mathcal{N} = (1, 0) \] supersymmetry (if \( N \) is even we can have also \( Sp(N/2) \) gauge group) with vector multiplets in the adjoint of \( SO(N) \) and hypermultiplets in the second rank, symmetric tensor representation of \( SO(N) \). The separation of the D5-branes, in the symmetric configuration described in the previous section, is achieved by giving vev to the hypermultiplets, and as a result the gauge group is completely broken. The vector multiplets combine with the off-diagonal hypermultiplets to give massive non-BPS multiplets, leaving \( N - 1 \) massless scalars (excluding the center of mass degree of freedom). They correspond to the Cartan generators of \( SU(N) \), which, from the \( SO(N) \), \( \mathcal{N} = (1, 0) \) viewpoint, are the diagonal components of the hypermultiplets in the symmetric tensor representation. This is to be contrasted with the type IIB case, where we had \( \mathcal{N} = (1, 1) \) supersymmetry and the "W-bosons" were \( \frac{1}{2} \)-BPS states. In the \( \mathcal{N} = (1, 0) \) case the supersymmetry algebra does not admit a central extension and therefore there are no short massive multiplets. After compactifying to 4+1 dimensions, a central term appears and therefore there are short multiplets, which are associated to the Coulomb branch. We are however interested in the Higgs branch, since we give vev to the hypermultiplets, and this produces long (i.e. 16 dimensional) massive multiplets.

Let us go back to type IIB/(−)\( F_L \sigma_P \) on the background (3.1) and first discuss the perturbative spectrum we obtain: repeating the arguments of the previous section for the type II case, one sees that in the NS-NS sector there are \( N - 1 \) massless scalars, as expected from the previous arguments. In the notation introduced before they are characterized by the same values of \( m, \bar{m}, m', \bar{m}', \ell, \ell' \) given after equation (2.10). However, due to the projection by (−)\( F_L \sigma_P \), the R-R sector does not give massless gauge fields, since R-R states with \( P = 0 \) are projected out. This is in agreement with the dual D5-brane picture discussed in the previous paragraph, where we argued that the gauge group is completely broken.

As for the non-BPS "W-bosons", corresponding to the generators of \( SO(N) \), they should again appear as (possibly unstable) non-BPS D-branes, i.e. boundary states. The construc-
tion of Cardy states proceeds similarly to the type II case, the only modification arising from the introduction, in addition to (2.13), of projection factors taking into account the $(-)^F \sigma_P$ projection. Consider first the action of $\sigma_P$: denoting respectively by $|P>$ and $|x>$ the boundary states with given momentum $P$, respectively position $x$ along the circle $S^1$ transverse to the D-brane, we will have $|x> = \sum_P (-)^P e^{iP^R x} |P>$. Here $R$ is the radius of $S^1$. On the other hand, the operator $(-)^F \sigma_P$ can be realized on Ishibashi states by $e^{i\pi s}$: from the construction detailed in section 2, states with $s =$ odd belong to the left-moving $R$ sector. Therefore, we insert $\frac{1}{2} (1 + e^{i\pi s})$ in (2.11). This has the effect of shifting the quantum number $S$ of Cardy states to $S+2$, i.e. of sending a D-brane to an anti-D-brane. The boundary state invariant under the $(-)^F \sigma_P$ projection is therefore a superposition of a D-brane at $x$ and an anti-D-brane at $x + \pi R$. It is easy to see that the number of these states is $N(N-1)/2$, equal to the dimension of $SO(N)$ as expected.

By computing, for instance, the cylinder amplitude involving the above boundary state and then reading it in the open string channel, one can analyze the open string excitations of the D-brane /anti-D-brane system: the open string stretched between a D-brane at $x$ and an anti-D-brane at $x + \pi R$ (or vice versa) has a ground state whose mass is $R^2 - \frac{1}{2}$, and therefore it is tachyonic, i.e. unstable, for $R^2 < \frac{1}{2}$.

We have mentioned previously that for $N$ even (and with longitudinal shift) the dual system of $N$ D5-branes admits an $Sp(\frac{N}{2}) \times SU(2)$ gauge theory, with vector multiplets in the symmetric representation and hypermultiplets in the antisymmetric one. By giving vev to the latter ones we can realize a configuration in which $N/2$ pairs of D5-branes are distributed $Z_\frac{N}{2}$ symmetrically on a circle, thereby breaking the gauge group down to $SU(2)^{\frac{N}{2}-1}$, together with $\frac{N}{2} - 1$ massless scalars (we neglect here too the center of mass degrees of freedom). The question is how this configuration is realized in the present case of $N$ NS-fivebranes i.e. what is the corresponding CFT.

We have argued that the CFT $\frac{SL(2,\mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)}$, or the equivalent one involving the $\mathcal{N} = 2$
Liouville theory, describes the configuration of $N$ fivebranes distributed $Z_N$ symmetrically on $S^1$. As mentioned in section 2, in terms of the ALE space description this corresponds to the equation

$$\prod_{k=0}^{N-1} (x - \omega^k \mu) + y^2 + z^2 = x^N + y^2 + z^2 - \mu^N = 0,$$

(3.2)

with $\omega = e^{2\pi i N}$ the primitive $N$-th root of unity. The deformation $\mu^N$ corresponding to the cosmological term (2.3) in the $\mathcal{N} = 2$ Liouville theory. The configuration alluded to before, with $Z_{N/2}$ symmetry, should correspond to

$$\prod_{k=0}^{N/2-1} (x - \omega^{2k} \mu)^2 + y^2 + z^2 = 0.$$ 

(3.3)

Compared to the $Z_N$ case, (3.2), we see that there is an additional deformation, corresponding to the monomial $x^{N/2}$. It is easy to see that in turn this deformation corresponds, in the $\frac{SL(2,R)}{U(1)} \times \frac{SU(2)}{U(1)}$ CFT, to the scalar with quantum numbers $\ell = \pm m = \frac{N}{2}$, $m' = \pm (\ell' + 2)$, with $\ell + \ell' = N - 2$.

One can check, to first order, that indeed the above perturbation has the correct effect on the D-brane tension. The first order correction is basically given by the overlap of the boundary state with the state corresponding to the scalar field (plus its conjugate) and is proportional to

$$\cos \frac{\pi (L + 1)}{N} [e^{i\pi \frac{M}{N}} + e^{-i\pi \frac{M}{N}}].$$

(3.4)

This expression also agrees with what one would obtain geometrically in terms of branes wrapped on 2-cycles of the ALE manifold. In this case tensions are given by the (modulus of) the holomorphic volumes of the homology 2-cycles, i.e the integrals of the holomorphic 2-form over the homology 2-cycles. If one perturbs the original 2-form corresponding to (3.2) with $x^{N/2}$, then, to first order one gets (3.4).

It would be interesting to find the CFT to which the system is driven by the above field. The problem cannot be addressed perturbatively, since the strength of the perturbation is
tuned to the original cosmological term, as it is clear from (3.3). Moreover, the CFT in question is expected to be singular: the space (3.3) has vanishing cycles, therefore there are tensionless D-branes, responsible for the enhancement of the gauge symmetry to $SU(2)^{N-1}$. In any case, one may hope that the resulting CFT has a symmetry $Z_N \times (Z_2)^{\frac{N}{2}}$, and that modding out by a diagonal $Z_2$ produces the expected spectrum.

If $N$ is even, one may consider gauging a different $Z_2$, namely the diagonal subgroup of $Z_N \times Z_2$, whose action on the minimal model fields is given by:

$$
\phi^l_{m,s} \rightarrow e^{i\pi(m+s)}\phi^l_{m,s},
$$

(3.5)

together with $\sigma_P = (-)^P$ on the $S^1$ part of the tensor product fields. Notice that, since $l + m + s = \text{even}$, $e^{i\pi(m+s)} = (-)^l$. Supersymmetry is preserved by this orbifold projection, since the supercharge generators have $m = s = 1$, therefore we have $\mathcal{N} = (1, 1)$ supersymmetry in 5+1 dimensions. Out of the $N-1$ massless scalars found in section 2, those with $l$ even survive the projection, and this gives $N/2$ states. Accordingly, there are $N/2$ gauge fields from the RR sector. The construction of Cardy states proceeds following the general strategy indicated in section 2. The action (3.5) on Ishibashi states induces the action $S \rightarrow S + 2$, $M \rightarrow M + N$ on Cardy states. Therefore if we insert, in addition to (2.13), the projector $\frac{1}{2} \sum_{d=0,1} \exp(i\pi(m+s+P)d)$ in (2.11), we will have, in the notation of section 2, $\tilde{M} = M + 2r + c + dN$. We can therefore restrict $M$ to the range $0 \leq M \leq N-1$. Together with the allowed values $S = 0, 2$, $0 \leq L \leq \frac{N-2}{2}$, $L + M = \text{even}$, this gives a total of $N^2/2$ boundary states, which should correspond to the broken charged generators of the gauge group. We have therefore, including the $N/2$ Cartan generators, a group of rank $N/2$ and dimension $N(N + 1)/2$. To decide whether it is $Sp(N/2)$ or $SO(N+1)$ we compute the lengths of the root vectors: from (2.15) one can easily verify that the (squared) length of a root vector depends from $L$ as:

$$
|\vec{q}(L, M, S)|^2 = \sum_{l \text{ even}=0}^{N-2} \sin^2 \pi \frac{(L + 1)(l + 1)}{N} = \frac{N}{2} \left(1 + \delta_{L, \frac{N-2}{2}}\right), \quad L = 0, \ldots, \frac{N-2}{2}.
$$

(3.6)
From (3.6), taking into account the allowed values of $M$ and $S$, we see that there are $N$ long and $N^2/2 - N$ short roots, therefore we conclude that the gauge group is $Sp(N/2)$.

4. Heterotic $SO(32)$ and $E_8 \times E_8$ string

We will now discuss the little string theory for the standard heterotic string theory. The NS5 brane solution now involves, besides the metric, antisymmetric tensor and the dilaton, also the gauge fields. In the following we will restrict ourselves to the symmetric 5-branes, in which the spin connection is identified with the gauge connection. Since the spin connection sits in an $SU(2)$ subgroup of the transverse $SO(4)$ group, the gauge field will also be in an $SU(2)$ subgroup of $SO(32)$ or $E_8 \times E_8$. The unbroken gauge group is then $SO(28) \times SU(2)$ and $E_7 \times E_8$ respectively. These latter groups therefore will play the role of flavour symmetries in the 5-brane world volume theory.

CFT Analysis

Let us first discuss the $SO(32)$ case. The singular CFT for symmetric 5-brane background has been discussed in [5]. It is simply a left-right symmetric $SU(2)$ super-WZWN model at level $N$ together with the super Liouville field and the additional 28 free gauge fermions in the left moving bosonic sector. One of the important point that was made in this paper was that this CFT describes the situation where the $N$ 5-branes charge appears in the form of $N-1$ coincident small scale instantons and 1 large scale instanton. The world volume theory therefore only sees $Sp(N-1)$ gauge group. The number of operators (e.g. those charged under $SO(32)$) in this case will be proportional to $(N-1)$, which is indeed the number of primaries of the $SU(2)$ WZW model [4]. This idea of having one large

---

4A discrepancy pointed out in [5] between the string theory states and the gauge theory operators involved the left moving $SU(2)$ current algebra descendents of spin $(N-2)/2$ primary which gives states transforming under spin $N/2$ of the global part of this $SU(2)$. The corresponding gauge theory operator was missing. However the point is that such current algebra descendents also do not exist in the CFT as
scale instanton, is therefore the mechanism analogous to the factoring out of the center of mass $U(1)$ in the type II context.

What happens when we turn on the perturbation to go to the non-singular CFT given by the product of the $SL(2)/U(1)$ times $SU(2)/U(1)$ coset models. We will first obtain the results from the CFT and then later identify the heterotic background that this CFT corresponds to. As mentioned earlier we will restrict ourselves to the left-right symmetric case. What this means is that the internal theory is left-right symmetric $\mathcal{N} = 2$ minimal models based on $SU(2)/U(1) \times SL(2)/U(1)$. In fact in the present case $\mathcal{N} = 2$ is promoted to $\mathcal{N} = 4$ theory. Besides this of course we have the right moving $\mathcal{N} = 1$ free superconformal theory of $R^4$ (in the light cone gauge) and the left moving free bosonic theory of $R^4$ and 28 fermions. The Virasoro constraints now read:

$$\frac{k_{\mu}k^{\mu}}{2} + \frac{l(l+2) - m^2}{4N} + \frac{m'^2 - \ell'(\ell' + 2)}{4N} + \frac{s^2 + s'^2}{8} + \Delta = 1$$

$$\frac{k_{\mu}k^{\mu}}{2} + \frac{\ell(\ell + 2) - \bar{m}^2}{4N} + \frac{\bar{m}'^2 - \ell' (\ell' + 2)}{4N} + \frac{s^2 + \bar{s}'^2}{8} + \bar{\Delta} = \frac{1}{2}, \quad (4.1)$$

where $\bar{\Delta}$ is the contribution to $L_0$ from the descendants of the minimal models together with that of superconformal theory corresponding to $R^4$ (in the light cone gauge) and $\Delta$ is the contribution to $L_0$ from the descendants of the minimal models and from $R^4$ times 28 free fermion theory. The GSO condition on the right movers is the same as (2.8), while for the left movers it is:

$$\frac{m - m'}{N} - \frac{s + s'}{2} - \frac{\vec{\delta} \cdot \vec{w}}{4} \in 2\mathbb{Z}, \quad (4.2)$$

where now $\vec{\delta}$ is the spinor weight of $SO(28)$ and $\vec{w}$ is the $SO(28)$ weight that the state carries. In addition there is also the conditions analogous to (2.9) which ensure all the components are either in the same sector (either R or NS). This is the standard GSO projection that gives rise to $Spin(32)/\mathbb{Z}_2$ theory in the flat case.

Recall that $m'$ and $\bar{m}'$ are the left and right moving momenta of the free scalar that they are null states in the Verma module.
appears in the $SL(2)/U(1)$ coset. If these quantities are conserved then they would denote charges with respect to a $U(1)_L \times U(1)_R$ symmetry. However, as has been discussed in [3, 4], only the momenta $(m' + \bar{m}')/2$ are conserved while the windings $(\bar{m} - \bar{m}')/2$ are conserved only modulo $N$. This is due to the fact that this CFT is perturbed by an operator that carries $N$ units of windings. Besides this symmetry associated with the isometry group of the internal space, we have also the left moving $SO(28)$ Kac-Moody algebra coming from the 28 free fermions as well as the $SU(2)_f$ current algebra which is part of the left moving $\mathcal{N} = 4$ superconformal algebra. Note that, unlike the $SU(2)$ current algebra in the right moving supersymmetric sector which is broken by the picture changing operator, the $SU(2)_f$ is a good symmetry of the theory. The $U(1)_f$ subalgebra of the $SU(2)_f$ is part of the $\mathcal{N} = 2$ subalgebra of the $\mathcal{N} = 4$ superconformal algebra, which acts on the individual minimal models (the remaining generators mix the two minimal models). Important point to note is that $U(1)_L$ and $U(1)_f$ are not orthogonal to each other. Since the question of orthogonality involves only the $SL(2)/U(1)$ theory, it is sufficient to look at the $\mathcal{N} = 2$ superconformal algebra for this system. This system can be represented in terms of a free scalar $Y$, a Feigin-Fuchs field $\varphi$ and one complex fermion $\psi_\pm$ [13]. The algebra is given by the following generators:

\begin{align*}
T &= -\frac{1}{2}(\partial Y)^2 - \frac{1}{2}(\partial \varphi)^2 - Q\partial^2 \varphi - \frac{1}{2}(\psi_+ \varphi \psi_- - \varphi \psi_+ \psi_-) + T', \\
G_\pm &= (\partial \varphi \pm \partial Y)\psi_\pm + Q\psi_\pm + G'_\pm, \\
J &= \psi_+ \psi_- + iQ\partial Y + J'
\end{align*}

where $Q$ is the Feigin-Fuchs background charge and is equal to $\sqrt{2/N}$. $T'$, $G'_\pm$ and $J'$ are the contributions from the $SU(2)/U(1)$ theory. Now the vertex operator for a state that carries $m'$ quantum number, is $\exp im'QY/2$, therefore $J_L = \frac{iQ}{2}\partial Y$ measures the $U(1)_L$ charge $m'$. The singular part of the relevant OPE’s are

\begin{align*}
J_L(z)J(w) = \frac{2}{(z - w)^2}, \\
J(z)J(w) = \frac{2}{(z - w)^2}.
\end{align*}
where in the second equation we have used the fact that the $\mathcal{N} = 2$ algebra implies that the coefficient is given by the $\hat{c}$ of the full system (namely $SL(2)/U(1) \times SU(2)/U(1)$) which is 2. This implies that the current $\tilde{J}_L$ which is orthogonal to $U(1)_f$ current $J$ and therefore to the current algebra of $SU(2)_f$ is

$$\tilde{J}_L = J_L - J. \quad (4.5)$$

$\tilde{J}_L$ is the generator of $\tilde{U}(1)_L$ which is orthogonal to the $SU(2)_f$. This will be important for us in the following. For later purpose let us also recall that by bosonizing $U(1)_f$ current $J$, the raising and lowering generators $J_\pm$ of $SU(2)_f$ can be expressed as exponentials of the corresponding boson. In fact these are just the squares of the spin fields that take one from NS to R sector. In particular the states in a given representation of $SU(2)_f$ can be obtained from a particular state by spectral flow.

The states that give rise to poles in their two-point functions, and hence couple to the world volume theory, are the ones for which $|m' - s'| \geq \ell' + 2$ and $|\tilde{m}' - \tilde{s}'| \geq \ell' + 2$. We will restrict ourselves to such states. The massless bosonic states (i.e. coming from the right moving NS sector) among them which satisfy the GSO conditions are characterized by $\ell + \ell' = N - 2$, $m = \epsilon \ell$, $m' = -\epsilon (\ell' + 2)$, $\tilde{m} = \bar{\epsilon} \ell$ and $\tilde{m}' = -\bar{\epsilon} (\ell' + 2)$, where $\epsilon$ and $\bar{\epsilon}$ are $\pm 1$ and refer to the possible independent choices of chiral or anti-chiral primaries on the right and left sectors. Furthermore in the left-moving sector, in order for them to satisfy the Virasoro and GSO conditions, they must have $(s, s', \vec{w})$ equal to $(2, 0, 0)$ or $(0, 2, 0)$ or $(0, 0, \vec{v})$, where $\vec{v}$ are the weights of $SO(28)$ vector representation. For the states $(0, 0, \vec{v})$ with $m = \epsilon \ell$ and $m' = -\epsilon (\ell' + 2)$ we must also include the states obtained by spectral flow mentioned above in order to construct the full $SU(2)_f$ representation. Under this flow these states go to $(2, 2, \vec{v})$ with $m = \epsilon (\ell + 2)$ and $m' = -\epsilon \ell'$.

The $s = 2$ and $s' = 2$ states are the $G_\pm$ descendants of the $s = 0$ and $s' = 0$ states. Since the states above are chiral (anti-chiral) primaries, $G_+ (G_-)$ will annihilate them. While $m = +\ell$ and $m = -\ell$ are chiral and anti-chiral primaries respectively, the states $m' = \ell' + 2$
and \( m' = -(\ell' + 2) \) are chiral and anti-chiral respectively. This can be seen by using the fact that the Vertex operator for the primary \((\ell', m')\) is given by \( \exp(-((\ell' + 2)Q\phi + im'QY)) \) and applying \( G_\pm \) given in (4.3) on it. The \( U(1)_f \) quantum numbers of these states can then be simply computed by adding the quantum numbers of the primary and of the \( G_\pm \). We can now summarize various quantum numbers in table 1.

Note that in the first row \( \ell = 0 \) does not appear since for this case we have the ground state of the \( SU(2)/U(1) \) CFT which does not have a \( G'_\pm \) descendant. Starting from each of the above chiral (antichiral) states, by the spectral flow in the left moving supersymmetric sector we get all the states that fill out a hypermultiplet. Thus we have altogether \((2N - 3)\) \( SO(28) \times SU(2)_f \) singlet hypermultiplets and \((N - 1)\) hypermultiplets transforming as \((28, 2)\) under the flavour group.

As mentioned before \( U(1)_R \) and \( \tilde{U}(1)_L \) are not separately conserved due to the presence of the cosmological constant in the Liouville CFT. While half the difference (say \( p \)) is conserved, half of the sum (\( w \)) is conserved modulo \( N \). This would suggest that the symmetry group is \( U(1) \times Z_N \). From the above table, however we note that the the states in the third row which are \( SU(2)_f \) doublets, have half integer values of \( p \) and \( w \). Therefore the symmetry group is \( \tilde{U}(1) \times Z_{2N} \times SU(2)_f \times Spin(28)/(Z_2)^3 \), where \( \tilde{U}(1) \) is the double cover of \( U(1) \), and the modding by \((Z_2)^3\) keeps only 4 conjugacy classes (even, even, 1, \( sc \)), (even, even, 2, \( sp' \)), (odd, odd, 2, \( v \)) and (odd, odd, 1, \( sp \)) out of the total of 32 classes. We will see the appearance of double coverings of \( U(1) \) and \( Z_N \) in the following when we discuss the heterotic background corresponding to this CFT.

| \((s, s', \tilde{w})\) | \( \ell \) | \( U(1)_R \) | \( \tilde{U}(1)_L \) | \( SU(2)_f \) |
|-----------------|-------|-----------|---------------|---------|
| \((2, 0, 0)\)   | \( \ell = 1, \ldots, (N - 2) \) | \( \tilde{\epsilon}(N - \ell)/2 \) | \( \epsilon(N - \ell)/2 \) | 1       |
| \((0, 2, 0)\)   | \( \ell = 0, \ldots, (N - 2) \) | \( \tilde{\epsilon}(N - \ell)/2 \) | \( \epsilon(N - \ell)/2 \) | 1       |
| \((0, 0, \tilde{v})\) | \( \ell = 0, \ldots, (N - 2) \) | \( \tilde{\epsilon}(N - \ell)/2 \) | \( \epsilon(N - \ell - 1)/2 \) | 2       |

Table 1: Quantum Numbers of Massless states for \( SO(32) \) heterotic theory
Before proceeding further, let us discuss the case of $E_8 \times E_8$ heterotic string. The discussion is exactly as before with the exception that in the left moving GSO condition (4.2), $\vec{\delta}$ and $\vec{w}$ are the $SO(12)$ spinor weight and the weight carried by the state respectively. The remaining $SO(16)$ fermions have an independent spin structure sum which gives rise in the usual way to the unbroken $E_8$ group. The massless states are again given by the table 1, with $\vec{v}$ denoting now the $SO(12)$ vector. There are however, now additional massless states that come from the left moving R sector. It is easy to verify that these states are given by $(s, s', \vec{w}) = (1, 1, \vec{s}\vec{p})$ and $(m, m') = (\ell + 1, \ell' + 1)$ where $\vec{s}\vec{p}$ are the weights of the spinor representation of $SO(12)$. The $U(1)_f$ quantum number of such states is zero. The complete table for the massless states together with various quantum numbers is given in table 2.

The states in the last two columns transform under $SO(12) \times SU(2)_f$ as $(12, 2) + (32, 1)$ representations which together form the $(56)$- dimensional representation of $E_7$. This is as expected since we know that the left-right symmetric CFT (corresponding to identification of spin connection and the gauge connection) breaks one of the $E_8$’s to $E_7$. Exactly as in the $SO(32)$ case, here also the symmetry is $\tilde{U}(1) \times Z_{2N} \times E_7/(Z_2)^2$, where the modding by $(Z_2)^2$ keeps only 2 conjugacy classes (even, even, 1) and (odd, odd, 56) out of the total of 8 classes.

**Heterotic backgrounds corresponding to the heterotic CFT’s**

Now let us turn to the question of what heterotic backgrounds do these CFT’s describe.

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**Table 2: Quantum Numbers of Massless states for $E_8 \times E_8$ heterotic theory**

| $(s, s', \vec{w})$ | $\ell$ | $U(1)_R$ | $\tilde{U}(1)_L$ | $SU(2)_f$ |
|-----------------|-------|----------|-----------------|-----------|
| $(2, 0, 0)$     | $\ell = 1, \ldots, (N - 2)$ | $\tilde{\epsilon}(N - \ell)/2$ | $\epsilon(N - \ell)/2$ | $1$ |
| $(0, 2, 0)$     | $\ell = 0, \ldots, (N - 2)$ | $\tilde{\epsilon}(N - \ell)/2$ | $\epsilon(N - \ell)/2$ | $1$ |
| $(0, 0, \vec{v})$ | $\ell = 0, \ldots, (N - 2)$ | $\tilde{\epsilon}(N - \ell)/2$ | $\epsilon(N - \ell - 1)/2$ | $2$ |
| $(1, 1, \vec{s}\vec{p})$ | $\ell = 0, \ldots, (N - 2)$ | $\tilde{\epsilon}(N - \ell)/2$ | $\epsilon(N - \ell - 1)/2$ | $1$ |
We will first discuss the heterotic \( SO(32) \) theory since in this case the one can use the 5-brane world volume theory in the S-dual D5 branes of Type I theory. The brane world volume theory carries symplectic gauge group \( Sp(N) \) together with one hypermultiplet \( Y \) transforming in anti-symmetric representation of \( Sp(N) \) and 32 hypermultiplets \( q \) in fundamental representations which transform as vector representation of \( SO(32) \) flavour group. Under the transverse \( SO(4) = SU(2)_R \times SU(2)_L \), \( Y \) transforms in \( (2, 2) \) and \( q \) in \( (2, 1) \) representations. Furthermore \( q \)'s satisfy a reality condition

\[
q^* q = \epsilon_{\alpha\beta} \Omega_{IJ} q_I q_J
\]

where \( \alpha, \beta = 1, 2 \) denote the \( SU(2)_R \) indices, \( I, J = 1, \ldots, 2N \) denote \( Sp(N) \) indices and \( \Omega \) is the symplectic two-form. It is convenient to write the \( 2N \) indices \( I \) as pair of indices \( (\hat{\alpha}, i) \), where \( \hat{\alpha} = 1, 2 \) and \( i = 1, \ldots, N \). The symplectic form \( \Omega_{(\hat{\alpha}i)(\hat{\beta}j)} = \epsilon_{\hat{\alpha}\hat{\beta}} \delta_{ij} \) and the reality condition on \( q \) just says that it consists of \( N \) quaternions \( q_i \) with \( SU(2)_R \) acting on the right and \( i \)-th \( SU(2) \) subgroup in the decomposition \( Sp(N) \to (SU(2))^N \) acting on the left.

We will now break the symmetries in two steps. Firstly we give a large vev (of order \( \rho \)) to the fundamentals to break \( Sp(N) \) to \( Sp(N - 1) \). This will correspond to giving a large scale to one of the \( N \) instantons as in \cite{5}. The \( SO(32) \) symmetry is broken down to \( SO(28) \times SU(2)_f \). The states that become massive at this step can then be ignored, since in the limits we will be interested in they will be essentially infinitely massive. Furthermore massless states (and we will have some of them) that are neutral under the remaining \( Sp(N - 1) \) will not be localized on the remaining \( N - 1 \) branes. These will be the zero modes associated with the single instanton with large scale. Indeed in the large scale limit, the corresponding zero modes will be spread all over the transverse directions. We can therefore also ignore such states, since they would decouple from the world-volume physics in the double scaling limit.

In the second step, we will further break the \( Sp(N - 1) \) completely at a scale \( \lambda \ll \rho \), and take a double scaling limit \( \lambda \to 0 \) keeping \( \lambda/g_{st} \) finite. This will be done in such a way
that $SU(2)_R \times SU(2)_L$ global symmetry is broken to $U(1) \times Z_N$.

It is convenient to express the D-terms of $\mathcal{N} = 1$ in 6-dim. in the language of 4-dimensional $\mathcal{N} = 1$ where they appear as F- and D-terms. Denoting by $A$ and $B$ the two chiral fields contained in $Y$ (so that $A$ and $B$ have weights $(1/2, 1/2)$ and $(1/2, -1/2)$ with respect to $SU(2)_L \times SU(2)_R$ respectively), the F-terms are

$$F = [A, B] + \vec{q}\sigma^+\vec{q}^\dagger$$

and the D-terms are

$$D = [A, \bar{A}] - [B, \bar{B}] + \vec{q}\sigma_3\vec{q}^\dagger$$

where the vectorial notation refers to the $SO(32)$ vector and a dot product with respect to $SO(32)$ vector is understood. In fact we will be considering these vectors to lie in a four dimensional subspace corresponding to an $SO(4)$ subgroup of $SO(32)$. The most general solution (upto gauge equivalences) to the above equations, which has $U(1) \times Z_N$ symmetry referred to above, is

$$B = \lambda \text{diag}(1, \omega, \omega^2, ..., \omega^{N-1}) \ , \ A = 0$$

$$\vec{q}_i = \rho \bar{\vec{y}} \equiv \rho \vec{\tilde{\sigma}} \ , \ \vec{\tilde{\sigma}} = (1, i\sigma^1, i\sigma^2, i\sigma^3)$$

where $\omega = e^{2\pi i/N}$. The four $\vec{y}$’s appearing above are just unit quaternions that are orthogonal to each other. This was the solution given in [29]. In fact for the above solution the contribution of $q$, $A$ and $B$ to the F- and D-terms above separately vanish. Let us analyse the symmetries of this solution. For $\lambda = 0$, the gauge symmetry is broken to $Sp(N - 1)$ as can be readily seen by making a gauge transformation so that

$$\vec{q} \rightarrow \vec{q}', \ \vec{q}_i' = \delta_{iN} \sqrt{N} \rho \vec{y}$$

Such a gauge transformation can be done by an element $G$ of $Sp(N)$ which (expressed as $N \times N$ matrices with quaternion entries) is

$$G_{ij} = \frac{1}{\sqrt{N}} g^{(i-1)(j-1)} , \ g^N = 1, \ g \in SU(2)$$
On the other hand for $\rho = 0$, the $N$ branes are separated on a circle so that the gauge symmetry is broken down to $SU(2)^N$. When both $\rho$ and $\lambda$ are non-zero $Sp(N)$ is completely broken. As for the flavour symmetry, $SO(28)$ is clearly unbroken since $\vec{q}$ is only along 4-directions. The remaining $SO(4)$ acts on the 4 unit orthogonal quaternions $\vec{y}$ by rotating them. An $SU(2)$ subgroup of this $SO(4)$ (which we denote by $SU(2)_f$) can be however undone by the diagonal gauge $SU(2)$ subgroup of $SU(2)^N \in Sp(N)$ and the remaining $SU(2)$ by the $SU(2)_R$. Thus only the diagonal subgroup of the last two $SU(2)$'s survives and forms the new $SU(2)_R$ symmetry (in the absence of $\lambda$). For $\lambda \neq 0$, $SU(2)_L \times SU(2)_R$ is further broken down to $U(1) \times Z_N$ where the $U(1)$ acts on $A$ (i.e. it is $U(1)_{L+R}$) while the $Z_N$ rotation $B$ can be undone by a $Sp(N)$ transformations which $Z_N$ cyclic permutations of the eigenvalues of $B$ (note that this acts trivially on $q$). Thus the symmetry group is $U(1) \times Z_N \times SU(2)_f \times SO(28)$.

Since $\rho >> \lambda$, it is instructive to analyze the light spectrum by first taking $\lambda = 0$. As mentioned above the unbroken gauge group at this stage is $Sp(N - 1)$. The massless hypermultiplet spectrum can be easily obtained by going to the basis (4.10, 4.11) and expanding the fields around the vev in the F- and D-terms. The $Sp(N)$ anti-symmetric fields $Y$ remain massless and they decompose under the unbroken $Sp(N - 1)$ as an anti-symmetric field $Z$, a singlet and 2 fundamental fields $Q_{\alpha}$ which transform as a doublet of $SU(2)_f$. The $Q$'s satisfy a reality condition analogous to (4.6) with $SU(2)_R$ replaced by $SU(2)_f$. There are in fact 4 copies of $Z$ and $Q_{\alpha}$ transforming as $(2, 2)$ of $SU(2)_L \times SU(2)_R$. We can carry out a similar analysis for the fundamental $Sp(N)$ fields. The result is summarized in table 3. Here the symmetries are the new modified symmetries that leave the vev invariant. $Z'$ and $q'$ are the moduli (position, gauge orientation and the scale) of the large scale single instanton. These fields are clearly not localized in the world volume. $q''$ are $SO(28)$ vectors but are also not localized in the world volume. $q'''$.

\footnote{Note that in a symmetric comactification of heterotic theory, the total number of massless $SO(28)$ vectors is topological and does not change when we go to the limit of small scale instantons.}
Fields | $\text{Sp}(N - 1)$ | $SU(2)_R \times SU(2)_L$ | $SU(2)_f \times SO(28)$
--- | --- | --- | ---
$Z$ | $(N - 1)(2N - 3) - 1$ | $(2, 2)$ | $(1, 1)$
$Q$ | $2(N - 1)$ | $(2, 2)$ | $(2, 1)$
$q$ | $2(N - 1)$ | $(2, 1)$ | $(1, 28)$
$Z'$ | $1$ | $(2, 2)$ | $(1, 1)$
$q'$ | $1$ | $(3, 1) + (1, 1)$ | $(1, 1)$
$q''$ | $1$ | $(2, 1)$ | $(2, 28)$

Table 3: Quantum Numbers of Massless scalar fields after breaking $\text{Sp}(N)$ to $\text{Sp}(N - 1)$

$q'$ and $q''$ will decouple from the world volume physics. In the above table $Q$ transforms under $SU(2)_L \times SU(2)_f$ as $(2, 2)$ representation. This was because it originally was in the antisymmetric representation $Y$ of $\text{Sp}(N)$. However in $\rho \to \infty$ limit where we can restrict ourselves to $\text{Sp}(N - 1)$ theory there is an enhancement of symmetry. The action is invariant when $Q$ is transformed by $SU(2)' \times SU(2)_f$ where $SU(2)'$ is independent of $SU(2)_L$. Putting together $Q$ and $q$ therefore we get an $SO(32)$ vector representation. This is what was used in the analysis of $[5]$. Indeed in the corresponding CFT, the left moving sector had $SU(2)_L \times SO(32)$ symmetry.

The second step of symmetry breaking (i.e. $\lambda \neq 0$), corresponds to turning on $Z$ and $Q$. As mentioned above $\text{Sp}(N - 1)$ is completely broken and $SU(2)_L \times SU(2)_R$ is broken to $U(1) \times Z_N$ where the $U(1)$ acts on the $A$ direction and $Z_N$ on the $B$ direction. There are $(2N - 3)$ complex massless scalars coming from $Z$ and $Q$ with $U(1) \times Z_N$ charge $(1, 0)$ and 2 complex scalars each with $(0, m)$ with $m = 2, 3, \ldots, (N - 1)$ and one with charge $(0, 0)$. These are all $SU(2)_f \times SO(28)$ singlets. Finally from $q$ we have one complex scalar each with charge $(m/2, m/2)$ for $m = 1, \ldots, (N - 1)$ which transform as $(2, 28)$ under $SU(2)_f \times SO(32)$. The appearance of half integer charges implies extension of $U(1)$ and $Z_N$ to $\tilde{U}(1)$ and $Z_{2N}$ with certain $Z_2$ identifications as implied by the correlation of charges. Since this analysis only involves massless states, we do not see any $SO(28)$ spinors. However, the symmetry
The total number of massless scalars therefore agrees with the counting coming from CFT analysis given in table 1. However, as it happens in the type II case, the charge assignments are different. This is because the CFT states should couple to the gauge invariant composite operators in the world volume theory. We can construct \( Sp(N-1) \) invariant composite operators from the fields \( Z, Q \) and \( q \) exactly as in the type II case. Since \( Z \) and \( Q \) transform as \((2,2)\) under \( SU(2)_R \times SU(2)_L \), it is convenient to denote \( Z_A, Q_A \) along the \( A \) direction and \( Z_B, Q_B \) along the \( B \) direction. Gauge invariant chiral composite operators together with their quantum numbers are given in the table 4. Comparing this with table 1 (for \( \bar{c} = +1 \) which corresponds to the chiral primaries), we find complete agreement.

It is worth noting that although the above discussion made use of the effective world volume theory (namely \( Sp(N) \) gauge theory) in the S-dual D5 brane, we could have come to the same conclusions, directly, by working with the zero modes around the heterotic theory solution. The latter approach will have the advantage that it would not be tied to the details of the 5-brane world volume theory and therefore will be applicable also to the \( E_8 \times E_8 \) heterotic theory.

| Operators | \( m \) | \( U(1)_R \) | \( U(1)_L \) | \( SU(2)_f \times SO(28) \) |
|-----------|--------|-------------|-------------|-----------------|
| \( \text{tr} Z_A^m \) | \( m = 2, \ldots, N - 1 \) | \( m/2 \) | \( m/2 \) | (1,1) |
| \( \text{tr} Z_B^m \) | \( m = 2, \ldots, N - 1 \) | \( m/2 \) | \( -m/2 \) | (1,1) |
| \( Q_A^t Z_A^m Q_A \) | \( m = 0, \ldots, N - 2 \) | \( (m + 2)/2 \) | \( (m + 2)/2 \) | (1,1) |
| \( Q_B^t Z_B^m Q_B \) | \( m = 0, \ldots, N - 2 \) | \( (m + 2)/2 \) | \( -(m + 2)/2 \) | (1,1) |
| \( Q_A^t Z_A^m q \) | \( m = 0, \ldots, N - 2 \) | \( (m + 2)/2 \) | \( (m + 1)/2 \) | (2,28) |
| \( Q_B^t Z_B^m q \) | \( m = 0, \ldots, N - 2 \) | \( (m + 2)/2 \) | \( -(m + 1)/2 \) | (2,28) |

Table 4: Quantum Numbers of chiral composite operators of \( Sp(N - 1) \) theory.
The $SO(32)$ heterotic theory solution corresponds to $N$-instantons in an $SU(2)$ subgroup of the $SO(32)$ gauge theory in the target space. One of these $N$ instantons is of large size, while the others are approximately point like. The zero modes of interest are the ones localized near the point-like instantons, since they are the ones that will appear in the world volume physics. Imbedding $SU(2)$ in $SU(2) \times SU(2)_{f} \times SO(28)$ maximal subgroup of $SO(32)$, the zero modes can be described in terms of ADHM construction of the $SO(4)$ instantons. The $SO(32)$ gauge fields then transform as $(3, 1, 1) + (1, 3, 1) + (1, 1, \text{Ad}) + (2, 2, 28)$ under the maximal subgroup. The instanton involves non-trivial background only in the $(3, 1, 1)$ piece. Apart from the usual zero modes associated with the $SU(2)$ instantons, the zero modes will also come from the $(2, 2, 28)$ piece. The instanton solution and the zero modes can be conveniently described in terms of ADHM data. There is however a huge redundancy in this data which is given by the action of the ADHM symmetry group. It is therefore natural to describe the zero modes in terms of the ADHM group invariant quantities. The ADHM construction of the $SU(2)$ instantons can be done in different ways. For example, imbedding $SU(2)$ in $SO(4) = SU(2) \times SU(2)_{f}$, we can think of it as $SO(4)$ instanton. The ADHM group in this case is $Sp(N)$ and in fact the world volume analysis given above is precisely this case; the D terms are just the ADHM constraints. On the other hand we can also describe the $SU(2)$ instantons directly with the ADHM group being $SU(N)$ (or $SO(N)$ if we view $SU(2)$ as $Sp(1)$). It turns out however that $SU(N)$ description does not yield the zero modes associated with the gauge fields in $(2, 2, 28)$ representation since the latter satisfy a reality condition. $SU(N)$ is broken down to $SO(N)$ due to this reality condition. These different descriptions (i.e. $Sp(N)$ or $SO(N)$) must provide the same physical information of the zero modes if we restrict to the ADHM group invariant quantities. Let us therefore obtain the above results (which corresponded to the ADHM group being $Sp(N)$) by working instead with $SO(N)$ ADHM group.

We will now be general and consider some gauge group $G$ which has a maximal subgroup $Sp(1) \times G'$ such that adjoint representation of $G$ splits into adjoints of $Sp(1)$ and $G'$ plus
(2, R) where R is a pseudo-real representation of \( G' \). The (2, R) representation further satisfies a reality condition

\[
A_{\mu}^* = \sigma_2 A_\mu \Omega \tag{4.12}
\]

where \( A_\mu \) is a \((2 \times \dim(R))\) matrix and \( \Omega \) is a symplectic matrix associated with \( R \). For \( G = SO(32), G' = SU(2)_f \times SO(28) \) with \( R = (2,28) \) and for \( G = E_8, G' = E_7 \) with \( R = (56) \).

The ADHM data for \( N \) instantons in \( Sp(1) \) gauge theory is given in terms of a \((1+N) \times N\) matrix \( \Delta \) with quaternion entries

\[
\Delta_{\lambda,i} = a_{\lambda,i} + b_{\lambda,i} x; \quad x = x_\mu \sigma^\mu; \tag{4.13}
\]

where \( \lambda = 1, \ldots, 2 + 2N, i, j = 1, \ldots, N \) and \( a \) and \( b \) are constants. By using the symmetries one can choose \( b \) to be (writing \( \lambda = u + j\alpha \) with \( u = 1 \) being the \( Sp(1) \) gauge group index)

\[
b_{u,i} = 0; \quad b_{j,i} = \delta_{ji} 1 \tag{4.14}
\]

where \( 1 \) is the unit element of the quaternion. It is convenient to define

\[
a_{u,i} = w_i; \quad a_{j,i} = a'_{j,i} = a'_{j,i} \sigma^\mu \tag{4.15}
\]

\( a'^\mu \) transform in the symmetric tensor representation of the ADHM symmetry group \( SO(N) \), while \( w \) transforms as bi-fundamental of the gauge group \( Sp(1) \) and \( SO(N) \). The left and right \( SU(2) \) actions on the quaternion entries in \( a' \) are respectively the \( SU(2)_L \) and \( SU(2)_R \) actions \((SU(2)_L \times SU(2)_R \) is the Euclidean group acting on the 4-dim. space on which the instantons live and therefore is the transverse group to the 5-brane world volume). In other words the \( \mu \) index in \( a' \) transforms as a vector of the \( SO(4) = SU(2)_L \times SU(2)_R \). On the other hand the left and right actions on the quaternions in \( w \) are respectively the \( Sp(1) \) gauge group and \( SU(2)_R \) actions respectively. \( \Delta \) satisfies a quadratic constraint

\[
(\bar{\Delta}\Delta)_{ij} = f_{ij}^{-1} 1 \tag{4.16}
\]
where $f$ is a real symmetric non-singular $N \times N$ matrix and transforms in the symmetric tensor representation of $SO(N)$. This results in quadratic constraints on $a'$ and $w$ in the adjoint of $SU(2)_R \times SO(N)$ which are just the D-terms for the $SO(N)$ theory. The instanton solution for the $Sp(1)$ adjoint gauge field is

$$A_\mu = \bar{U} \partial_\mu U; \quad \bar{U} \Delta = \Delta \bar{U} = 0, \quad \bar{U} U = 1$$

(4.17)

where $U$ is a $(N + 1) \times 1$ matrix with quaternion entries. Solution (upto symmetries) to these constraints that breaks $SU(2)_L \times SU(2)_R$ to $U(1) \times Z_N$ is

$$w_i = \rho 1 \quad \text{for each } i$$

(4.18)

and $a'^\mu$ being diagonal matrices with $N$ different eigenvalues that are $Z_N$ symmetric in a plane. In other words, writing $a'^1 + ia'^2 = A$ and $a'^0 + ia'^3 = B$, this corresponds to choosing $A = 0$ and $B = \lambda \text{diag}(1, \omega, \omega^2, ..., \omega^{N-1})$ with $\omega = e^{2\pi i/N}$. As in the previous discussion we take $\rho$ to be large while $\lambda$ small. The ADHM group then is broken to an approximate $SO(N - 1)$ group. $A$ ($B$) splits into symmetric tensor $Z_A$ ($Z_B$) and a fundamental $Q_A$ ($Q_B$) of $SO(N - 1)$.

Finally we have to also consider the zero modes coming from the gauge fields in $(2, R)$ representation of $SU(2) \times G'$. These can be explicitly given as

$$A^{(2, R)}_\mu = \bar{U}_\lambda \partial_\mu \Delta_{\lambda,i} f_{ij} q_j$$

(4.19)

where $q_j$ are constant $2 \times \text{dim}(R)$ matrices satisfying a reality condition

$$q'^*_j = \sigma_2 q_j \Omega$$

(4.20)

$q$ therefore transforms as $(N, 2, R)$ under $SO(N) \times SU(2)_R \times G'$ together with the reality condition. Note that with this reality condition on $q$, the gauge fields $A^{(2, R)}_\mu$ satisfies the required reality condition \((1.12)\). In the limit of large $\rho$ only the $SO(N - 1)$ vector remains localized zero mode (the remaining one spreads over the entire transverse space and
therefore is irrelevant to the world-volume physics). We will, by a slight abuse of notation, in the following denote by \( q \) the localized zero modes in \( SO(N - 1) \) vector representation.

Note that for \( G' = SU(2)_f \times SO(28) \), \( Z, Q \) and \( q \) obtained here differ from the ones preceding the table 3, in that here \( Q \) is \( SU(2)_f \) singlet instead of doublet and \( q \) vice versa. This will, however, not matter when we construct the ADHM group invariant quantities. The chiral ones among these are \( trZ_A^m \) and \( trZ_B^m \) for \( m = 2, \ldots, N - 1 \) and \( Q_A^T Z_A^m Q_A \) and \( Q_B^T Z_B^m Q_B \) for \( m = 0, \ldots, N - 2 \) which result in the quantum numbers of the first 4 rows of operators in table 4. Besides this, we have also operators \( Q_A^T Z_A^m q^+ \) and \( Q_B^T Z_B^m q^+ \) for \( m = 0, \ldots, N - 2 \), with \( q^+ \) denoting the highest weight of \( SU(2)_R \). The quantum numbers of these operators reproduce the last two rows in table 4. Thus we see that the precise details of the world-volume theory, namely whether it is \( Sp(N) \) or \( SO(N) \) gauge theory was not necessary for obtaining the matching with the CFT. In fact it is determined by the zero mode structure around the instanton solution.

We can now apply this analysis to the case of \( E_8 \), since it does not require the details of the 5-brane world volume theory (which is not known at present). It is easy to see that for \( G = E_8 \) and \( G' = E_7 \) with the representation \( R = (56) \) the ADHM group invariant combinations given in the previous paragraph reproduce the table 2 coming from the CFT analysis. Alternatively one might have also worked with the \( SO(4) \) instanton giving rise to the ADHM group \( Sp(N) \), and go through the above steps to construct \( Sp(N) \) invariant quantities which also gives the result of Table 2. Thus although the matching with the chiral states of the CFT does not tell us much about the details of the 5-brane world-volume theory, it does suggest that it should have a local \( Sp(N) \) or \( SO(N) \) symmetry. How this symmetry is realized in a \((1,0)\) theory of tensor and hypermultiplets is an important open question.
5. Conclusions and Open Questions

In this paper, we have studied the physics of NS 5-branes in heterotic like theories with 16 supercharges. Non-singular CFT describing the string theory in these backgrounds is given by an $\mathcal{N} = 2$ minimal model together with and $SL(2)/U(1)$ Kazama Suzuki model. We have considered two classes of models; one which is obtained by projecting type II theories by $(-1)^F \sigma$ with $\sigma$ being a half shift on a circle longitudinal to the 5-brane. In this case the S-duality relation (for the IIB case), which maps the NS 5-brane to D5 brane in the S-dual model, is expected to hold due to adiabatic argument. We indeed find that the CFTs describe holographically the chiral gauge invariant operators in the dual theory. Furthermore we constructed the boundary states corresponding to massive charged states, which are non-BPS in the present case, and showed that their charges and tensions agree with the results expected from the brane world volume theory. It is interesting to note that although we are here dealing with non-BPS states their tensions are not renormalized.

The second type of models we considered are the standard $SO(32)$ and $E_8 \times E_8$ heterotic theories. Our discussion was restricted to the case of symmetric 5-branes, which gives rise to $\mathcal{N} = (2,2)$ CFT (actually this symmetry is extended to $(4,4)$). In this case the internal CFT turns out to be the same as above coset models (modulo the left moving gauge fermions). The $SU(2)_R \times SU(2)_L$ symmetry is broken down to $U(1) \times Z_N$, while the gauge symmetry is broken down to $SU(2) \times SO(28)$ and $E_7 \times E_8$ respectively. We obtained the chiral states which among others included states transforming as $(2,28)$ and $(56,1)$ under the above groups. For the $SO(32)$ case, by studying the D-terms of the $Sp(N)$ gauge theory living in the brane world volume, we identified the ground state which has the above symmetry and breaks completely the $Sp(N)$ group. We found that the gauge invariant chiral operators match with the CFT states. In the 10-dimensional heterotic theory this ground state corresponds to $N$ instantons in an $SU(2)$ subgroup of $SO(32)$, with one of the instantons having large scale while the remaining $N-1$ are separated and
have small scales. We studied the ADHM construction of such instanton configurations and showed that both $Sp(N)$ and $SO(N)$ ADHM groups give the same spectrum of chiral ADHM group invariant operators. This allowed us to verify the holographic correspondence also for the $E_8 \times E_8$ theory where we do not know the 5-brane world volume theory.

Although in this paper we have discussed 5-branes in the 10-dimensional supersymmetric heterotic theories, the discussion can be easily generalized to the non-supersymmetric heterotic theories. For example the tachyon free $SO(16) \times SO(16)$ model is obtained from $SO(32)$ theory by a $Z_2$ orbifolding generated by the element $(-1)^F \sigma_s$ where $\sigma_s$ is the shift by the weight $(0, sp)$ in the $SO(16) \times SO(16)$ decomposition of $SO(32)$. We can obtain the corresponding CFT by starting from the one given here and modding out by this $Z_2$ orbifold group.

There are several open questions. The most important one is an understanding of the massive charged states in the heterotic theory. In the first type of models we have analyzed, they appeared as D-strings (or D2 branes) stretched between the separated NS5 branes and we had a boundary state description for them. In the heterotic theory, on the hand there are no D-branes and boundary states. Therefore the description of these excitations remains an interesting open problem.

In the heterotic theory we considered only the symmetric 5-branes. There are moduli in the CFT which are charged under $SU(2) \times SO(28)$ or $E_7$ in the two theories. By turning on these moduli we can go to a non-symmetric situation coresponding to a $(4,0)$ CFT. This amounts to having the $N$ instantons not in an $SU(2)$ subgroup of $SO(32)$ or $E_8$. However in the $E_8 \times E_8$ case, these deformations still keep all the $N$ instantons in one $E_8$ factor. It will be interesting to construct CFTs which describe backgrounds with $N_1$ instantons in one $E_8$ and $N - N_1$ instantons in the second $E_8$. This will be also necessary in order to construct CFTs for CHL models[31].

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