Pseudo-Chemotactic Drifts of Artificial Microswimmers

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We numerically investigate the motion of active artificial microswimmers diffusing in a fuel concentration gradient. We observe that, in the steady state, their probability density accumulates in the low-concentration regions, whereas a tagged swimmer drifts with velocity depending in modulus and orientation on how the concentration gradient affects the self-propulsion mechanism. Under most experimentally accessible conditions, the particle drifts toward the high-concentration regions (pseudo-chemotactic drift). A correct interpretation of experimental data must account for such an “anti-Fickian” behavior.

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I. INTRODUCTION

Chemotaxis, defined as the movement of motile cells or organisms in response to a chemical gradient, is a well-studied phenomenon \(^1\). Bacteria and other single- or multi-cellular organisms propel themselves up or down the concentration gradient of a particular substance in their search for nutrients or to avoid antagonists. Inspired by chemotaxis in biology, researchers synthesized artificial microswimmers \(^2,3\) that can move in response to a chemical stimulus \(^4,5\). They showed that Janus particles (JP), in the form of two-faced Au-Pt colloidal rods that catalyze hydrogen peroxide redox, are attracted by a hydrogen peroxide source. Under such conditions, JP’s act as molecular “robots” and can thus be employed in practical applications, such as the design of new intelligent drugs \(^6\). More sophisticated chemical robots have been proposed that utilize artificial chemotaxis to navigate autonomously \(^7\).

The simplest and, possibly, best established model of self-propulsion is encoded by the Langevin equations \(^9–12\)

\[
\dot{x} = v_0 \cos \phi + \sqrt{D_0} \xi_x(t), \quad \dot{y} = v_0 \sin \phi + \sqrt{D_0} \xi_y(t),
\]

\[
\dot{\phi} = \frac{\sqrt{D_\phi}}{v_0} \xi_\phi(t),
\]

where \(x = (x, y)\) are the coordinates of the swimmer in the plane, \(v_0\) its self-propulsion speed, and \(D_\phi\) an orientational diffusion constant, whose reciprocal, \(\tau_\phi\), quantifies the time-persistence of the particle’s Brownian motion. Here, \(\xi_i(t)\), with \(i = x, y, \phi\), are zero-mean and delta-correlated Gaussian noises with \(\langle \xi_i(t) \xi_j(0) \rangle = 2 \delta_{ij} \delta(t)\). For long observation times \(t\), with \(t \gg \tau_\phi\), or lengths \(l\), with \(l \gg l_\phi \equiv v_0 \tau_\phi\), the effective diffusion constant, \(D_\text{eff}\), defined by the asymptotic law \(\lim_{t \to \infty} \langle r^2(t) \rangle = 4 D_\text{eff} t\) \(^13\), is \(D = D_0 + D_\phi\), where \(D_0\) is due to thermal fluctuations in the suspension, and \(D_\phi = v_0^2/2D_\phi\) is a (typically) much larger self-propulsion term, which depends on the chemical composition of the suspension itself.

Let us consider now a chemical reactor consisting of a narrow, straight channel of length \(L\) oriented along the \(x\)-axis, and a free JP moving in it (Fig. 1) \(^8\). A constant concentration gradient of the chemical that fuels the particle’s self-propulsion is maintained by connecting the channel to two reservoirs in thermal equilibrium with concentrations \(\rho_L < \rho_R\). The chemical concentration in the channel, \(\rho(x)\), will then grow linearly from left to right. At the channel ends, \(x = \pm L/2\), two porous membranes allow the chemical flow in and out, but prevent the JP from escaping into the reservoirs. We speculate, based on experimental observations \(^14–16\), that both \(v_0\) and \(D_\phi\) (and therefore \(D_\text{eff}\)) may depend on \(\rho(x)\) to some unspecified extent. We only assume that both \(v_0(x)\) and \(D_\phi(x)\) are non-decreasing functions of the channel coor-

![FIG. 1: (Color online) Chemical reactor with a stationary fuel concentration gradient, \(\rho(x)\) (see text). A Janus particle injected in the middle, (a), tends to drift to the right even if its probability density, \(P_0(x)\), peaks on the left (b). The data in (b) are for \(v_0(x) \propto \rho(x)\) with \(D_0 = 0.01\), \(\eta_v = 1\), \(\delta_v = 1\), \(\delta_\phi = 0\) and \(D_\phi = 0.1\).]
In conclusion, the injected JP is finally attracted toward the left (cold) exit, even if, immediately after injection, it may drift to either direction, depending on the $x$-dependence of the propulsion parameters $v_0$ and $\tau_0$. For the most common case when the $x$-dependence of $\tau_0$ is much weaker than $D_s$, the injected particle points decidedly to the right (hot) exit. Reconciling these seemingly conflicting mechanisms is of paramount importance to control the chemotaxis of artificial microswimmers as opposed to bacterial chemotaxis. To avoid misunderstandings we remark that the adjectives hot and cold refer here to the regions in the reactor where the effective swimmer diffusion due to the selfpropulsion, $D_s(x)$, is the highest and lowest, respectively. The thermal diffusion, $D_0$, is assumed to be $x$-independent, which means that thermal gradients do not enter our analysis. Accordingly, in the absence of thermal gradients and for low fuel concentrations, additional transport contributions due to hydrodynamic effects in the suspension fluid can be safely neglected.

This paper is organized as follows. In Sec. II we present numerical results for the “splitting probabilities” that a JP, injected at the center of the channel, exits it through the right or left end and the corresponding mean first-exit times. The particle clearly undergoes a transient drift toward the hot end of the channel, whereas its stationary distribution tends to accumulate at the opposite end. In Sec. III we interpret our data by means of a phenomenological 1D Langevin equation that describes the diffusion of a Brownian particle with the spatial dependent diffusion coefficient $D_s(x)$. The spatial dependence of $D_s(x)$ generates the drift term here detected as a transient drift. Finally in Sec. IV we discuss the implications of our findings in the interpretation of recent experiments on the diffusion of JP’s in concentration gradients.

II. NUMERICAL RESULTS

Our answers to questions (1) and (2) are based on the simulation data reported in Fig. 2. As a study case, we considered the $x$-dependent self-propulsion parameters,

$$v_0(x) = v_0(1 + \delta_v x/L)^{\eta_v}, \quad D_\phi(x) = D_\phi(1 + \delta_\phi x/L)^{\eta_\phi}$$

where $\delta_v = \Delta v_0/v_0$ and $\delta_\phi = \Delta D_\phi/D_\phi$ are both non-negative, and from now on, $v_0$ and $D_\phi$ are shorthands for $v_0(0)$ and $D_\phi(0)$ at the injection point. We also set $\eta_v = 1$ and $\eta_\phi = 2$, so that for $\delta_v = \delta_\phi$ the self-propulsion

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This might sound paradoxical, but we came to the conclusion that the injected JP is finally attracted toward the left (cold) exit, even if, immediately after injection, it may drift to either direction, depending on the $x$-dependence of the propulsion parameters $v_0$ and $\tau_0$. For the most common case when the $x$-dependence of $\tau_0$ is much weaker than $D_s$, the injected particle points decidedly to the right (hot) exit. Reconciling these seemingly conflicting mechanisms is of paramount importance to control the chemotaxis of artificial microswimmers as opposed to bacterial chemotaxis.

Fig. 2: (Color online) Janus particle in a concentration gradient, Eqs. (2), with $\eta_\delta = 1$, $\eta_\phi = 2$ and different $\delta_\delta$ and $\delta_\phi$: (a) $P_0(x)$ for $D_0 = 0.01$. Data for $D_0 = 0.01$ and different $\eta_\phi$ are plotted in the inset; (b) $N_R/N_L$ and $T_L/T_R$ (inset) vs $D_0$; and (c) $\epsilon$ vs $D_0$, Eq. (3). Other simulation parameters: $v_0 = 1$, $D_\phi = 1$, $L = 100l_\phi$, and channel width $y_L = 5$. The solid curves are the analytical predictions based on Eqs. (4)-(6) with $\alpha = 1/2$ and $\alpha = 1$, respectively, for $\delta_\phi = 0$ and $\delta_\phi = 0$.  

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III. PHENOMENOLOGICAL ANALYSIS

An analytical treatment of the model of Eqs. (1) is viable in two limiting cases, i.e., $\delta_0 = 0$, $\delta_v > 0$, and $\delta_v = 0$.

When one looks at the transient dynamics immediately following the particle injection, a surprising outcome appears. We injected the particle at $x = 0$ and clocked the time it takes to hit either the right or left containment membrane. We repeated this numerical experiment $N = 10^6$ times and determined the probability the particle first reached the right or left exit, $N_{R,L}/N$, and the corresponding mean-first-passage times (MFPT), $T_{R,L}$, from 0 to ±$L/2$. The ratios $N_{R}/N_{L}$ and $T_{L}/T_{R}$ are plotted, respectively, in the main panel and the inset of Fig. 2b. In the regime of low thermal noise, $D_0 \ll D_s$, we obtained distinct results, depending on which $x$-dependence is stronger, $v_0(x)$ or $D_\phi(x)$. [Note that we used the same $\eta_v$ and $\eta_\phi$ as in the main panel (a) for $P_0(x)$.] In the first case, the particle tries to leave the channel through the right exit and, accordingly, $T_L > T_R$. Vice versa, on suppressing the $x$-dependence of $v_0(x)$, while leaving $D_\phi(x)$ unchanged, the particle directs itself preferably toward to left exit and $T_L < T_R$.

This means that for $\delta_v \gg \delta_\phi$ the injected particle initially drifts up the $\rho(x)$ gradient (pseudo-chemotaxis), at odds with Ref. [17]. Only when the increase of $v_0(x)$ along the channel is accompanied by a suitably stronger increase of its orientational rate, $D_\phi$, the injected particle drifts immediately down the gradient, in agreement with Fick’s law for ordinary Brownian motion. Magnitude and orientation of the transient drift are characterized in the forthcoming section by means of the unique rectification factor $\epsilon$. This result is remarkable because $P_0(x)$ tends to accumulate in any case around the concentration minima. This behavior is reminiscent of the “drift without current” effect experimentally observed in Ref. [19, 20] and numerically investigated in Refs. [21] for thermal Brownian motion in confined geometries. However, the magnitude of the phenomenon reported here is significantly larger and more easily accessible to experimental demonstration.
\( \delta_0 > 0 \). For this purpose we implemented the approach of Ref. 22 to reduce the fully three-dimensional dynamics of Eq. 14 to the more tractable 1D phenomenological diffusion law,

\[
\dot{x} = \alpha D'_\alpha(x) + \sqrt{D_\alpha(x)} \xi(t),
\]

where the prime sign denotes an \( x \) derivation and (i) \( \alpha = 1/2 \) and \( D_{1/2}(x) = D_0 + v_0^2(x)/2D_\phi \), for \( \delta_0 = 0 \), and (ii) \( \alpha = 1 \) and \( D_1(x) = D_0 + v_0^2/2D_\phi \), for \( \delta_0 = 0 \). Here, the multiplicative noise term has to be handled according to It\'s prescription 13 and \( \xi(t) \) is defined like the noise of Eq. 11. Note that the Eq. 14 can be rewritten as \( \dot{x} = \sqrt{D_\alpha(x)} \circ \xi(t) \), with \( \circ \) denoting the Stratonovitch or anti-Ito prescription, respectively, in case (i) and (ii). The corresponding Fokker-Planck equation (FPE) is

\[
\frac{\partial}{\partial t} P(x,t) = \frac{\partial}{\partial x} \left[ -v_\alpha(x) + \frac{\partial}{\partial x} D_\alpha(x) \right] P(x,t) \quad (4)
\]

with \( v_\alpha(x) = \alpha D'(x) \) for the appropriate value of \( \alpha \) 17, 23. The stationary p.d.f. for zero net current, \( j_0 = \lim_{t \to \infty} \bar{j}(x,t) = 0 \), reads

\[
P_0(x) = \lim_{t \to \infty} P(x,t) = \mathcal{N}/D_\alpha(x)^{1-\alpha}, \quad (5)
\]

where \( \mathcal{N} \) is a normalization constant. In particular, for \( \alpha = 1 \), i.e., \( x \)-independent \( v_0 \), \( P_0(x) = 1/L \). The extension to cases with \( j_0 \neq 0 \) is straightforward.

Regarding the transient statistics of a particle injected at the center of the channel, \( x = 0 \), a simple “splitting probability” calculation 13 leads to

\[
\frac{N_R}{\mathcal{N}} = \frac{\int_{-L/2}^{0} [D_\alpha(x) P_0(x)]^{-1} dx}{\int_{-L/2}^{L/2} [D_\alpha(x) P_0(x)]^{-1} dx}, \quad (6)
\]

with \( N_R + N_L = \mathcal{N} \). Analogously, for the MFPT’s in a channel with absorbing endpoints, we have 13

\[
T_L(\delta_\alpha) = \langle \tilde{T}(x) \rangle_{0-L/2} - \langle \tilde{T}(x) \rangle_{-L/2-L/2} = T_R(-\delta_\alpha), \quad (7)
\]

where \( \delta_{1/2} = \delta_\nu \) and \( \delta_1 = \delta_\phi \),

\[
\tilde{T}(x) = \int_{-L/2}^{x} dz \psi_\alpha(z)/D_\alpha(z) \int_{z}^{L/2} dy/\psi_\alpha(y),
\]

and \( \langle \ldots \rangle_{(a,b)} = \int_{a}^{b} \ldots dx/\psi_\alpha(x) / \int_{a}^{b} dx/\psi_\alpha(x) \), with \( \psi_\alpha(x) = [D_\alpha(x)]^{\alpha} \). The second equality in Eq. 7 follows immediately from \( x \to -x \) symmetry considerations. The ratios \( N_R/N_L \) and \( T_L/T_R \) have been computed numerically. The results plotted for \( \alpha = 1/2 \) (Fig. 3) and for \( \alpha = 1 \) (Fig. 4) confirm the consistency of our phenomenological approach in both regimes.

Clearly, our approach hinges on the phenomenological Eq. 15 and the explicit expressions we used for \( v_\alpha \) and \( D_\alpha(x) \). We now justify our choice for both.

(i) \( \delta_\phi = 0, \delta_\nu > 0 \). In view of the third equation 11, we know that \( \cos \phi(t) \) behaves like a (non-Gaussian) colored noise with an asymptotic autocorrelation function \( \langle \cos \phi(t) \cos \phi(0) \rangle \approx (1/2)e^{-D_\phi |t|} \) for \( t \gg \tau_\phi \). The JP diffusion coefficient at \( x \) can thus be derived from Kubo’s formula 12, 23,

\[
D = D_0 + \lim_{t \to \infty} \int_{0}^{t} v_0'(x) \langle \cos \phi(t) \cos \phi(0) \rangle dt = D_{1/2}(x),
\]

as anticipated in Eq. 14.

The drift velocity, \( v_\alpha(x) \), of a JP with an \( x \)-independent self-propulsion time constant, \( \tau_\phi \), amounts to the average of \( v_0(x) \) over its persistence length \( l_\phi(x) = v_0(x) \tau_\phi \), i.e.,

\[
v_\alpha(x) = \frac{1}{2} \left[ v_0 \left( x + \frac{l_\phi}{2} \right) - v_0 \left( x - \frac{l_\phi}{2} \right) \right] \approx \frac{1}{2} v_0'(x)v_0(x) \tau_\phi,
\]

hence \( v_\alpha = \alpha D'_\alpha(x) \) as in Eq. 11 with \( \alpha = 1/2 \).

(ii) \( \delta_\phi > 0, \delta_\nu = 0 \). Calculating \( D(x) \) in this case is straightforward. The FPE corresponding to the first and third Langevin equations 11,

\[
\frac{\partial}{\partial t} \bar{P} = \left[ -v_0 \cos \phi \frac{\partial}{\partial x} + D_0 \frac{\partial^2}{\partial x^2} + D_\phi(x) \frac{\partial^2}{\partial \phi^2} \right] \bar{P},
\]

with \( \bar{P} = \bar{P}(x,\phi,t) \), admits a uniform p.d.f., as one can prove by substitution; hence, the \( P_0(x) \) of Eq. 14 with \( \alpha = 1 \). The diffusion coefficient will be calculated again through Kubo’s formula: since in the stationary regime \( x \) and \( t \) are statistically independent, \( D = D_1(x) \). Regarding the drift velocity, the condition \( j_0 = 0 \) in Eq. 14 requires that \( v_\alpha(x) = D_\alpha(x) [\ln[D_{\alpha}(x) P_0(x)]]' \), namely, for \( \alpha = 1 \), \( v_1(x) = D'_1(x) \), as expected.

Coming back to the plots of Figs. 3 and 4 we stress that:

(i) The insets of Figs. 3(b) and (c) illustrate the dependence of \( N_{R,L} \) and \( T_{L,R} \) on the channel length \( L \). \( T_{R,L} \) scale like \( L^2 \), whereas \( N_{R,L} \) grow insensitive to \( L \). Of course, both statements are valid as long as \( L \gg l_\phi \); (ii) Our expressions for \( T_{L,R}(\delta_\alpha) \), adapted from Ref. 13, correctly reproduce the limiting values \( T_{R,L}(\delta_\alpha) = L^2/8D_0 \) for \( D_0 \ll D_s \) (gradient effects are superseded by thermal noise), and \( T_{R,L}(0) = L^2/8D_s \) for \( D_0 = 0 \) and \( \delta_\alpha \to 0 \) (purely active Brownian motion); (iii) On comparing the currents for \( N_R/N_L \) in Figs. 3(b) and 4(b) and those for \( T_L/T_R \) in Figs. 3(c) and 4(c), the different dependence of the two ratios on \( D_\phi \) at low thermal noise becomes apparent. This can be easily explained by inspecting the corresponding analytical expressions in the limit \( D_\phi \to 0 \). For \( \delta_\phi = 0, \delta_\nu > 0 \), i.e., \( \alpha = 1/2 \), the two ratios are functions of \( \delta_\nu \) only and, therefore, independent of \( D_\phi \). For \( \delta_\phi = 0, \delta_\phi \neq 0 \), i.e., \( \alpha = 1 \), they grow insensitive to \( v_0 \), but do depend on \( \delta_\phi \) and, hence, \( D_\phi \). Accordingly, the limits \( D_0 \to 0 \) and \( v_0 \to 0 \) coincide, as confirmed, for instance, by the numerical data in Figs. 4(a) and (b).
the opposite pulls by rectification corresponds to pseudo-chemotactic drifts. The most effective right (left) the symmetry-breaking mechanism responsible for the general case (b) $N_R/N_L$ vs $D_0$ for $v_0 = 1.5$; and (c) $T_L/T_R$ vs $D_0$ for $v_0 = 1.5$. In (a)-(c) $D_0 = 0.1, L = 100\lambda_0$, and $y_L = 5$. The solid curves are the corresponding analytical predictions based on Eqs. (5)-(6) for $\alpha = 1$.

Finally, to fully answer our starting question (1), as the most likely exit end in the general case $\delta_v \neq 0$ and $\delta_\phi \neq 0$, we computed the rectification factor

$$\epsilon \equiv \langle v_\phi(0) \rangle / v_0 = \left( \frac{N_L T_L}{N_L T_R} - 1 \right) / \left( \frac{N_R T_L}{N_L T_R} + 1 \right),$$  

where the ratios $N_R/N_L$ and $T_L/T_R$ are combined together to quantify both the sign and magnitude of the symmetry-breaking mechanism responsible for the pseudo-chemotactic drifts. The most effective right (left) rectification corresponds to $\epsilon = \pm 1$, whereas for $\epsilon = 0$ the opposite pulls by $v_0(x)$ and $D_\phi(x)$ cancel each other.

Note that for $\eta_\nu = 1, \eta_\phi = 2$ and $\delta_v = \delta_\phi$, the transient rectification does not vanish [Fig. 2(c)], even if under the same conditions $T_L = T_R$ [Fig. 2(b), inset].

IV. CONCLUDING REMARKS

The phenomenon of “drift without current” has been explained in Refs. [19, 23], by noticing that the statistical ensemble governing the average drift (i.e., the rectification factor $\epsilon$ in our notation) is different from the one required to compute the average current, $j_0 = 0$. The former consists of the representative points exiting an infinitesimally narrow neighborhood with coordinate $x$, with equal $x$-dependent jump length in either direction, whereas the latter consists of all points crossing a channel cross-section with coordinate $x$ at a given time, no matter what their jumping length. The two ensembles differ as an effect of multiplicative noise [i.e., the $x$-dependence of $D_\alpha(x)$] and so do the currents thus calculated.

Self-propelling artificial microswimmers reproduce that very same situation as a combined effect of nonequilibrium and the higher dimensionality of their dynamics, Eq. (1). In contrast to bacterial chemotaxis [17], for an artificial microswimmer the self-propulsion parameters do not depend on the orientation. Here a dependence on the swimmer’s orientation might come into play due to, say, inertial or memory (i.e., nonlocal) effects, but surely not to some internal sensor-actuator pathways, like in bacteria [1]. The microswimmers considered here are characterized by very low Reynolds numbers and small dimensions compared to the $\rho(x)$ length scale; therefore, an orientation dependence of the swimmer’s self-propulsion mechanism is not an option.

For artificial microswimmers under the most common experimental conditions, $D_\phi$ is only weakly affected by the $\rho$ gradient, while $v_0(x)$ is reported to grow linearly with $\rho$ and then saturate at higher $\rho$. [3, 13, 14]. The onset of “anti-Fick” cold-to-hot (pseudo-chemotactic) currents can thus be easily demonstrated. For instance, a source steadily releasing fuel into a JP suspension, causes a concentration gradient around it; JP’s with $x$-independent rotational dynamics are driven away from the fuel source, whereas a tagged JP floating in such a depletion zone actually drifts toward the source. This prediction is in contrast with the experimental findings of Ref. [1], where Au-Pt micro-rods are reported to progressively cluster around an $H_2O_2$ fuel source. If we assume that the self-propulsion model of Eqs. (1) holds good for a free swimmer in the bulk (as established under the most diverse experimental conditions [3, 11, 24]), the only explanation for such a discrepancy is that, upon migrating toward the fuel source, the JP’s come into contact with one another and eventually aggregate, as suggested, e.g., in Ref. [27]. Drifts without current become observable at low swimmer concentration.
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[1] H. C. Berg, *E. coli in Motion* (Springer, New York, 2004).
[2] F. Schweitzer, *Brownian Agents and Active Particles* (Springer, Berlin, 2003).
[3] S. Jiang and S. Granick (Eds.), *Janus Particle Synthesis, Self-Assembly and Applications* (RSC Publishing, Cambridge, 2012); A Walther and A. H. E. Müller, Chem. Rev. **113**, 5194 (2013).
[4] Y. Hong, N. M. K. Blackman, N. D. Kopp, A. Sen, and D. Velegol, Phys. Rev. Lett. **99**, 178103 (2007).
[5] S. Sengupta, K. K. Dey, H. S. Muddana, T. Tabuillot, M. E. Ibele, P. J. Butler and A. Sen, JACS **135**, 1406 (2013).
[6] S. Sengupta, M. E. Ibele, and A. Sen, Angew. Chem. Int. Ed. **51**, 8434 (2012).
[7] I. Lagzi, Cent. Eur. J. Med. **8**, 377 (2013).
[8] see, e.g., P.S. Burada, P. Hänggi, F. Marchesoni, G. Schmid, and P. Talkner, ChemPhysChem **10**, 45 (2009).
[9] J.G. Gibbs and Y.-P. Zhao, Appl. Phys. Lett. **94**, 163104 (2009); J.R. Howse, R.A.L. Jones, A.J. Ryan, T. Gough, R. Vafabakhsh, R. Golestanian, Phys. Rev. Lett. **99**, 048102 (2007).
[10] S. van Teeffelen and H. Löwen, Phys. Rev. E **78**, 020101 (2008).
[11] G. Volpe, I. Buttinoni, D. Vogt, H.-J. Kümmerer, and C. Bechinger, Soft Matter **7**, 8810 (2013).
[12] for a minireview see X. Ao, P. K. Ghosh, Y. Li, G. Schmid, P. Hänggi, and F. Marchesoni, Eur. Phys. J. Special Topics **223**, 3227 (2014).
[13] C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1985).
[14] A. Sen, M. Ibele, Y. Hong, and D. Velegol, Faraday Discuss. **143**, 15 (2009).
[15] Y. Hong, D. Velegol, N. Chaturvedi, and A. Sen, Phys. Chem. Chem. Phys. **12**, 1823 (2010).
[16] F. Lugli, E. Brini, and F. Zerbetto, J. Phys. Chem. C **116**, 592 (2012).
[17] M. J. Schnitzer, Phys. Rev. E **48**, 2553 (1993).
[18] D. A. Clark and L. C. Grant, PNAS **102**, 9150 (2004).
[19] P. Lançon, G. Batrouni, L. Lobry, and N. Ostrowsky, EPL **54**, 28 (2001); Physica A. **304**, 65 (2001).
[20] G. Volpe, L. Helden, T. Brettschneider, J. Wehr, and C. Bechinger, Phys. Rev. Lett. **104**, 170602 (2010).
[21] P. F. Tupper and X. Yang, Proc. R. Soc. A **468**, 2012, 3864 (2012); R. Collins and T. Takemori, J. Phys.: Condens. Matter **1**, 3801 (1989).
[22] F. Marchesoni, Materials **6**, 3598 (2013) and references therein.
[23] C. Festa, L. Fronzoni, P. Grigolini and F. Marchesoni, Phys. Lett. A **102**, 95 (1984).
[24] P.K. Ghosh, V.R. Misko, F. Marchesoni, and F. Nori, Phys. Rev. Lett. **110**, 268301 (2013).
[25] M. Yang and M. Ripoll, J. Chem. Phys. **136**, 204508 (2012).
[26] D. Takagi, A. B. Braunschweig, J. Zhang, and M. J. Shelley, Phys. Rev. Lett. **110**, 038301 (2013).
[27] Y. Fily and M.C. Marchetti, Phys. Rev. Lett. **108**, 235702 (2012).