Color-singlet direct $J/\psi$ and $\psi'$ production at Tevatron in the $k_t$ factorization approach

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Abstract

Direct $J/\psi$ and $\psi'$ production rates at Tevatron are calculated in the $k_t$-factorization approach within the color-singlet model. In this approach, the production rates are enhanced by a factor of 20 compared to the naive collinear parton model. However, the theoretical predictions are still below the experimental data by at least one order of magnitude. This means that to explain charmonium productions at Tevatron, we still need to call for the contributions from color-octet channels or other production mechanisms.

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Studies of heavy quarkonium production in high energy collisions provide important information on both perturbative and nonperturbative QCD. In recent years, heavy quarkonium production has attracted much attention from both theory and experiment. To explain the \( J/\psi \) and \( \psi' \) surplus problem of large transverse momentum production at Tevatron [1], the color-octet production mechanism was introduced for the description of heavy quarkonium production [2,3] based on the NRQCD factorization framework [4]. During the last few years, extensive studies have been performed for the test of this color-octet production mechanism [5]. However, it was announced recently [6] that the color-octet contributions to charmonia productions at Tevatron are excluded by the experimental data under the \( k_t \)-factorization approach [7,8], where the authors calculated \( \chi_{cJ} \) production rates and found that the experimental data are successfully described only by the color-singlet contributions. Consequently, an urgent problem arises, which is whether the direct \( J/\psi \) and \( \psi' \) productions can also be described only by the color-singlet contributions in the \( k_t \) factorization approach. In this paper, we will try to tackle this problem. We will calculate the color-singlet direct \( J/\psi \) and \( \psi' \) production in the \( k_t \) factorization approach. We find the production rates are enhanced by a factor of about 20 compared to the conventional collinear parton model predictions after considering the \( k_t \) effects of the incident partons. However, the theoretical predictions are still below the experimental data by at least one order of magnitude. Therefore, to explain the Tevatron data on large transverse momentum charmonium productions, we still need to call for color-octet contributions or other production mechanisms.

The \( k_t \)-factorization approach differs greatly from the conventional collinear approximation because it takes the non-vanishing transverse momenta of the scattering partons into account. The conventional gluon densities are replaced by unintegrated gluon distributions which depend on the transverse momentum \( k_t \).

The lowest order graphs for heavy quark pair production in \( k_t \) factorization approach are plotted in Fig. 1, where we just give the typical graphs of [8]. The first two types of graphs (a) and (b) are conventional ones for the gluon-gluon fusion processes. The third type of graphs (c) are needed to preserve gauge invariance. To calculate heavy quarkonium production in this approach, we must project the \( Q\bar{Q} \) pair into a particular bound state. Because the \( Q\bar{Q} \) pairs in (b) and (c) are in color-octet, these two types of graphs will not contribute to heavy quarkonium production within the color-singlet model. In fact in [6], the productions of color-singlet \( P \)-wave charmonium states \( \chi_{cJ} \) are calculated, where there are only type (a) graphs contributing to their productions.

For the spin-triplet \( S \)-wave charmonium states \( J/\psi \) and \( \psi' \), in the lowest order, even type (a) graphs of Fig. 1 can not contribute to their productions because of \( C \) (charge) parity conservation. So, to produce the spin-triplet color-singlet \( S \)-wave heavy quarkonium states, one must go to higher order (in terms of \( \alpha_s \)) processes where an additional gluon must be emitted out. That is to say, in the lowest order, the spin-triplet \( S \)-wave quarkonia are only produced in the \( Q\bar{Q}g \) production processes. In Fig. 2, we give the typical graphs for \( Q\bar{Q}g \) production in the \( k_t \)-factorization approach, which include all contributing graphs in this order calculations in a gauge invariant formalism [8]. However, among these graphs only type (a) graphs of Fig. 2 contribute to \( S \)-wave quarkonium productions, because other graphs violate \( C \) parity or color conservations.

To calculate the production amplitude given by the type (a) graphs of Fig. 2, we make a Sudakov decomposition for every 4-momenta \( k_i \) as
\[ k_i = \alpha_i p_1 + \beta_i p_2 + \vec{k}_{iT}, \]  
where \( p_1 \) and \( p_2 \) are the momenta of the incoming hadrons. In the high energy limit, we have \( p_1^2 = 0, p_2^2 = 0, \) and \( 2p_1 \cdot p_2 = s, \) where \( s \) is the c.m. energy squared. \( \alpha_i \) and \( \beta_i \) are the momentum fractions of \( p_1 \) and \( p_2 \) respectively. \( k_{iT} \) is the transverse momentum, which satisfies

\[ k_{iT} \cdot p_1 = 0, \quad k_{iT} \cdot p_2 = 0. \]  
For the momenta of the incident gluons \( q_1 \) and \( q_2, \) we have the following decomposition \[8],

\[ q_1 = x_1 p_1 + q_{1T}, \quad q_2 = x_2 p_2 + q_{2T}. \]

That is to say, the longitudinal component of \( q_1 \) \((q_2)\) is only in the direction of light-like vector \( p_1 \) \((p_2)\).

Using the above defined Sudakov variables, we can express the cross section for \( J/\psi \) production from the type (a) graphs of Fig. 2 in the following form,

\[
\frac{d\sigma(p\bar{p} \to J/\psi X)}{d^4p} = \frac{1}{64 \times 16(2\pi)^4} \frac{d\alpha}{\alpha^2} \frac{d\alpha_3}{\alpha_3^2} d^2q_1d^2q_2 \frac{f(x_1; q_{1T}^2) f(x_2; q_{2T}^2)}{q_{1T}^2 q_{2T}^2} \frac{|A_0(q_{1T}, q_{2T})|^2}{S(q_{1T}, q_{2T})},
\]

where \( p_T \) is the transverse momentum of \( J/\psi, \) \( \alpha_\psi \) and \( \alpha_3 \) are the momenta fractions of \( p_1 \) carried by \( J/\psi \) and the outgoing gluon. The amplitude \( A_0 \) describes \( J/\psi \) production in the gluon-gluon fusion processes \( g + g \to J/\psi + g, \) with the off-shellness of the two incident gluons being \( q_1^2 = -q_{1T}^2 \) and \( q_2^2 = -q_{2T}^2 \) respectively. We have checked that the amplitude \( A_0 \) vanishes when \( q_{1T} \to 0 \) or \( q_{2T} \to 0, \) which is required by the gauge invariance of this approach.

\( f(x; q_T^2) \) is the unintegrated gluon distribution, which is related to the conventional gluon distribution by

\[ xg(x, \mu^2) = \int \frac{d\mu^2}{(2\pi)^4} \frac{dk_{iT}^2}{k_{iT}^2} f(x; k_{iT}^2). \]

The unintegrated gluon distribution \( f(x; q_T^2) \) includes the evolution in \( x \) and \( k_{iT}^2 \) by the BFKL and DGLAP equations. In the nonperturbative region of small \( k_{iT}^2 \) the unintegrated gluon distribution is not known, so in practice the above equation may be rewritten as \[8, 10\]

\[ xg(x, \mu^2) = xg(x, q_0^2) + \int_{q_0^2}^{\mu^2} \frac{d\mu^2}{(2\pi)^4} \frac{dk_{iT}^2}{k_{iT}^2} f(x; k_{iT}^2), \]

which introduces a priori unknown initial scale \( q_0^2 \) and the initial gluon distribution \( xg(x, q_0^2). \) According to this procedure, the amplitude has the following decomposition form \[8\],

\[
S(q_{1T}, q_{2T}) = \frac{|A_0(q_{1T}, q_{2T})|^2}{q_{1T}^2 q_{2T}^2} = S(0, 0)\theta(q_0^2 - q_{1T}^2)\theta(q_0^2 - q_{2T}^2) + S(q_{1T}, 0)\theta(q_{1T}^2 - q_0^2)\theta(q_{2T}^2 - q_0^2) + S(0, q_{2T})\theta(q_0^2 - q_{2T}^2)\theta(q_{2T}^2 - q_0^2) + S(q_{1T}, q_{2T})\theta(q_{1T}^2 - q_0^2)\theta(q_{2T}^2 - q_0^2).
\]
The function $S$ here is similar to the function $I$ of \[8\] called the impact factor, which is, in effect, an off-shell but gauge invariant cross section \[8\]. We note that $S(0, 0)$ represents the cross section for gluon-gluon fusion processes with no $k_t$ effects associated with the two incident gluons. Substituting this part of $S$ into Eq. (4), we can reproduce the conventional collinear parton model results for $S$-wave quarkonium production.

For numerical calculations, we choose the unintegrated gluon distribution of \[11\], where the authors determined it by using a combination of DGLAP and BFKL equations. With the initial conditions

$$ q_0^2 = 1 GeV^2, \quad xg(x, q_0^2) = 1.57(1 - x)^2.5, $$

they obtained an excellent fit to $F_2(x, Q^2)$ data over a wide range of $x$ and $Q^2$. As in Refs. \[12,6\], we set the scales $\mu^2$ for strong coupling constant $\alpha_s(\mu^2)$ in the amplitude squared $|A_0|^2$ to be $q_1^2 T$ for the interaction vertex associated with the incident gluon $q_1$, $q_2^2 T$ for the vertex associated with $q_2$, and $p_2^2 + m^2$ ($m = 2m_c$ is the quarkonium mass) for the vertex associated with the outgoing gluon attached to the $c\bar{c}$ line.

In the following we present the numerical results for color-singlet direct $S$-wave charmonia productions at Tevatron, compared with the experimental data \[1\]. Fig. 3 is for $J/\psi$, and Fig. 4 is for $\psi'$. In the calculations, we use charm quark mass as $m_c = 1.5 GeV$, and the color-singlet matrix elements as $\langle O^{\psi(3S_1)}_1 \rangle = 1.08 GeV^3$, and $\langle O^{\psi'(3S_1)}_1 \rangle = 0.67 GeV^3$ \[13\]. To compare with the experimental data we have imposed a cut on the rapidity of $J/\psi$ and $\psi'$: $|\eta| < 0.6$. From these two figures, we can see that the production rates are enhanced by a factor of about 20 compared to the conventional collinear parton model predictions after considering the $k_t$ effects of the incident gluons. However, the theoretical predictions are still below the experimental data by at least one order of magnitude. Therefore, to explain the Tevatron data on large transverse momentum charmonium productions, we still need to call for color-octet contributions or other production mechanisms.

In conclusion, we have calculated direct $J/\psi$ and $\psi'$ production at Tevatron in $k_t$-factorization approach within the color-singlet model. We found that the production rates are enhanced by a factor of 20 compared to the conventional collinear parton model. However, the theoretical predictions are still below the experimental data by at least one order of magnitude. This means that to explain charmonium productions at Tevatron, we still need to call for the contributions from color-octet channels or other production mechanisms.

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REFERENCES

[1] CDF collaboration, F. Abe et al., Phys. Rev. Lett. 69, 3704 (1992); Phys. Rev. Lett. 71, 2537 (1993); Phys. Rev. Lett. 79, 572 (1997); Phys. Rev. Lett. 79, 578 (1997)
[2] E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995); M. Cacciari, M. Greco, M.L. Mangano and A. Petrelli, Phys. Lett. B356 553 (1995).
[3] P. Cho and K. Leibovich, Phys. Rev. D53, 150 (1996); ibid, D53, 6203 (1996).
[4] G.T. Bodwin, L. Braaten, and G. P. Lepage, Phys. Rev. D51 1125 (1995); Err: ibid., D55, 5853 (1997).
[5] E. Braaten, S. Fleming, and T. C. Yuan, Annu. Rev. Nucl. Part. Sci. 46, 197 (1996); I. Rothstein, hep-ph/9911276.
[6] P. Högler, R. Kirschner, A. Schäfer, L. Szymanowski, and O.V. Teryaev, hep-ph/0004263.
[7] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B242, 97 (1990); Nucl. Phys. B366, 135 (1991).
[8] J.C. Collins and R.K. Ellis, Nucl. Phys. B360, 3 (1991).
[9] J. Kwiecinski, Z. Phys. C29, 561 (1985).
[10] M.G. Ryskin and Yu.M. Shabelski, Z. Phys. C61, 517 (1994); M.G. Ryskin, Yu.M. Shabelski, and A.G. Shuvaev, Z. Phys. C69, 269 (1994); hep-ph/9907507.
[11] J. Kwiecinski, A.D. Martin, and A.M. Stasto, Phys. Rev. D56, 3991 (1997).
[12] E.M. Levin, M.G. Ryskin, Yu.M. Shabelski, and A.G. Shuvaev, Sov. J. Nucl. Phys. 53, 657 (1991); ibid., 54, 867 (1992).
[13] G.T Bodwin, D.K. Sinclair, and S. Kim, Phys. Rev. Lett. 77, 2376 (1996).
Figure Captions

FIG. 1. The typical graphs for $Q\bar{Q}$ production in $k_t$ factorization approach in gauge invariant formalism.

FIG. 2. The typical graphs for $Q\bar{Q}g$ production in $k_t$ factorization approach in gauge invariant formalism.

FIG. 3. The color-singlet predictions for large $p_T$ direct $J/\psi$ production compared with the experimental data from Tevatron. The dashed line denotes the result in the conventional collinear parton model, and the solid line is the result in the $k_t$ factorization approach.

FIG. 4. The same as Fig. 3 but for $\psi'$. 
Fig. 1 The typical graphs for $Q\bar{Q}$ production in $\Lambda_c$ factorization approach in gauge invariant formalism.
Fig. 2 The typical graphs for $Q\bar{Q}g$ production in $k_t$ factorization approach in gauge invariant formalism.
direct $J/\psi$ production at Tevatron
color-singlet model prediction

Fig. 3
Fig. 4

direct $\psi'$ production at Tevatron
color-singlet model prediction

$B(\psi' \rightarrow \mu^+ \mu^-) \frac{d\sigma}{dP_T} (nb/GeV)$

$P_T$ (GeV)