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Observer analysis and synthesis for
Lipschitz nonlinear systems under discrete
time-varying measurements

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Abstract: Observer synthesis for nonlinear Lipschitz systems with time-varying sampling is
studied. To establish the exponential convergence of the observer, in this paper, we model
the impact of the sampling uncertainty by a reset integrator. First, generic conditions for
stability of a sampled data system are recalled. Second it is shown how to derive tractable
numerical conditions to analyze the robustness of a continuous-time Luenberger observer when
the sampling is discrete and time-varying. Then it is demonstrated that this emulation approach
can be passed over allowing for the direct computation of an observer gain. Simulations and
comparisons with related articles show the efficiency of the proposed methodology.

Keywords: Continuous-discrete observers, Lipschitz Nonlinear systems, Sampled-data systems,
LMIs

1. INTRODUCTION

In many control systems full state is not available for
measurements. The main reasons for this fact can either
be technical or economical and constitute the primary
motivation for observer synthesis. This topic has numerous
practical applications and has been the subject of an im-
mense body of work (among which [Luenberger (1971),
Be-
sancon (2007), Ellis (2002)]). The classical way to deal with
the observation when using digital device is to consider
periodic sampling and to analyze the resulting discrete
time system [Astrom (1997), Chen (1995)]. However,
in recent years a growing amount of attention has been
paid to networked control systems where information is
gathered discretely and communicated through a network
[Hespanha (2007)]. This network can introduce uncer-
tainty into the sampling, and the classical assumption of
periodic communication is no longer justified. This fact
has motivated a growing amount of research in the field of
sampled data systems [Antsaklis (2007), Hetel (2017)].

In this work we will consider observer synthesis for nonlin-
ear Lipschitz systems with discrete time-varying measurements. In the past few years, this problem has received
a lot of attention. Two main observation schemes are
considered. The first one uses a reset on the observer state and is usually described as a continuous-discrete
observer, where between sampling the model of the plant is
integrated and at sampling time an instantaneous reset of
the observer state occurs. In [Andrieu (2010), Chen
(2013), Dinh (2015), Etienne (2016), Mazenc (2015), Raff
(2007)] this structure is considered. In [Andrieu (2010),
Dinh (2015), Mazenc (2015)] a reachable set between sam-
plings is estimated, then a discrete time Lyapunov function
is derived that establishes the observer convergence. In
[Etienne (2016)] practical stability is considered, where
an event triggering mechanism is introduced in order to
reduce the communication burden. In [Raff (2007)] a set of
LMIs is given that allows to show stability of the sampled
system by using a time-varying periodic Lyapunov func-
tion. This approach was extended to the case of delayed
systems and time varying sampling in [Chen (2013)]. The
second observer structure the observer state trajectory is
piece-wise continuous and a reset occurs on the derivative
of the state.

This structure was considered in [Raff (2008)], where using
results for time-delay systems, a Luenberger-like observer is
synthesized. In [Farza (2014)] the gain of a Luenberger-like
observer exponentially decreases as the time from the
last sampled measurement increases. Here we will further
investigate this second observer structure. To establish our
results we will model the perturbation induced by sam-
pling as a reset integrator. This approach was considered
in [Mirkin (2007), Fujidaka (2009), Omran (2014), Omran
(2016)] and allows to use some structural properties of the
reset integrator to derive sufficient conditions on the max-
imal allowable sampling interval. In this case the resulting
approach is "emulation" based, i.e. it is assumed that a
continuous-time stabilizing controller/observer is already
known [Arcak (2004), Karafyllis (2009)].
The contribution of our paper is twofold. First, new conditions for analysis of an a priori known observer gain under discrete time-varying sampling will be provided. Then it will be shown how this emulation based analysis can be avoided to directly synthesize an observer for a nonlinear Lipschitz system with discrete time-varying measurements. With respect to other approaches the proposed method represents a trade-off between computational complexity and non conservatism of the computed intersampling time. Note that with respect to continuous-discrete observer we have smoother trajectories due to continuous corrective action.

The remainder of this paper is organized as follows: first some notations are introduced, then in Section 2 the model of the plant, the observer and the concept of solution under consideration are given. In Section 3 we recall some notions of the plant, the observer and the concept of solution under discrete time varying sampling will be provided. In what follows we consider a class of sampling sequences of the form:

\[ S_h = \{ (t_k)_{k \in \mathbb{N}} : t_0 = 0, 0 < t_{k+1} - t_k \leq h \} \]

with \( x(t) \in \mathbb{R}^n \) the state of the system at time \( t \) and \( u(t) \in \mathbb{R}^n \) the input applied to the system at time \( t \). Here \( A \in \mathbb{R}^{n \times m} \), \( B \in \mathbb{R}^{n \times l} \), \( C \in \mathbb{R}^{m \times n} \), \( G \in \mathbb{R}^{n \times m} \) and \( H \in \mathbb{R}^{m \times n} \), where the nonlinear term \( S_h \) satisfies the Lipschitz condition:

\[ |\phi(a) - \phi(b)| \leq \gamma|a - b|, \quad \forall (a, b) \in \mathbb{R}^m \times \mathbb{R}^m, \]

for some \( \gamma > 0 \). Furthermore it is assumed that \( u(t) \) is piece-wise continuous and bounded. As shown in Fig 1 the communication between the sensors and the observer is not continuous. We denote by \( (t_k)_{k \in \mathbb{N}} \) the sequence of sampling times, \( (t_k)_{k \in \mathbb{N}} \) is monotonously increasing. \( \nu_k \in \mathbb{R}^p \) is the sampled output available at time \( t_k \). In what follows we consider a class of sampling sequences of the form:

\[ S_h = \{ (t_k)_{k \in \mathbb{N}} : t_0 = 0, 0 < t_{k+1} - t_k \leq h \} \]

with \( \lim_{t \to \infty} t_k = \infty \). From (1) and the assumption in (2) for every initial condition \( x_0 \in \mathbb{R}^n \) and every time \( t \geq 0 \) the solution of (1) exist and is unique [Khalil (1996)].

The observer will be defined as follows:

\[ \dot{x}(t) = A \hat{x}(t) + Bu + G\phi(Hx(t)) \]

\[ + LCz(t_k), \quad \forall t \in [t_k, t_{k+1}) \]

where \( \forall t \in \mathbb{R}_+ \), \( \dot{x}(t) \in \mathbb{R}^n \) is the state of the observer at time \( t \), and \( L \in \mathbb{R}^{m \times q} \) is the observer gain.

Defining \( z(t) = x(t) - \hat{x}(t) \), the following observation error dynamics is obtained:

\[ \dot{z}(t) = Az(t) + G\left( \phi(Hx(t)) - \phi\left( H(x(t) - z(t)) \right) \right) \]

\[ + LCz(t), \quad \forall t \in [t_k, t_{k+1}) \]

**Definition 1.** The system (5) is globally exponentially stable with convergence rate \( \alpha > 0 \) if there exist \( M > 0 \) such that for any initial conditions \( x_0 \in \mathbb{R}^n \), any solution \( x(t) \) of (1) and any sampling sequence \( (t_k)_{k \in \mathbb{N}} \in S_h \) the solutions of (5) verify:

\[ |z(t)| \leq Me^{-\alpha t}|z_0|. \]

We are now able to formulate the problems that we will consider in this work:

**Analysis Problem (AP):** Consider system (5) with a given gain \( L \), a given set of admissible sampling sequence \( S_h \) and a convergence rate \( \alpha \). Given sufficient conditions to ensure that the system (5) is globally exponentially stable with convergence rate \( \alpha \) for any sampling sequence \( (t_k)_{k \in \mathbb{N}} \in S_h \).

**Synthesis Problem (SP):** Consider system (5) with a given set of admissible sampling sequence \( S_h \) and a convergence rate \( \alpha \). Find a gain \( L \) such that the system (5) is globally exponentially stable with convergence rate \( \alpha \) for any sampling sequence \( (t_k)_{k \in \mathbb{N}} \in S_h \).
3. PRELIMINARIES

In the case under consideration, system (5) can be described as an Linear Parameter Varying (LPV) system. This method is a classical one in the literature and can also be applied to some classes of non Lipschitz system. See for instance the work of [Zemouche (2008)]. In our setting it can be stated as follows:

Lemma 1. (Zemouche (2008)) Consider equations (1), (4), (5). Then, there exists a finite set of matrices \( R_i, i \in \mathcal{P} := \{1, 2, \ldots, p\} \), such that for any \((x, z) \in \mathbb{R}^n \times \mathbb{R}^n\)

\[
A_z + G \left[ \phi \left( H x \right) - \phi \left( H(x-z) \right) \right] \in \text{Cov} \{ R_i z \}_{i \in \mathcal{P}}.
\]

In practice, the set of matrices \( R_i, i \in \mathcal{P} \) can be easily computed numerically. See [Zemouche (2008)] for further details. Note that the proposed methodology obviously can also be applied to linear systems as a special case.

To find a suitable \( L \), in the case of continuous communication, numerous results are available. In particular, as stated previously, it is possible to recast (5) as an LPV system leading to a set of LMIs whose feasibility ensures asymptotic convergence of an observer (see [Zemouche (2008)])

\[
R_i P + PR_i + C_i Q + QC \leq 0, \forall i \in \mathcal{P}
\]  \hspace{1cm} (6)

with \( P = P^T > 0 \), and \( \mathcal{P} \) is obtained using the differential mean value theorem. If LMIs (6) are satisfied, then the resulting gain is given by \( L = P^{-1} Q \).

Using Lemma 1 and following the approach of [Fujikoa (2009), Omran (2016)], we model the discrepancy between \( z \) and \( z(t_k) \) in the following way:

\[
\dot{z} = (R(\lambda) + LC)z + LCw,
\]

\[
y = \dot{z}, \quad \forall t \in [t_k, t_{k+1}),
\]

with \( R(\lambda(t)) \in \overline{\text{Cov}}\{ R_i z \}_{i \in \mathcal{P}} \) and the reset integrator

\[
\Delta_{\text{sh}}: w(t) = z(t_k) - z(t) = -\int_{t_k}^{t} y(\rho)d\rho, \forall t \in [t_k, t_{k+1}).
\]  \hspace{1cm} (8)

Given the equations (5) and a sampling sequence \( (t_k)_{k \in \mathbb{N}} \in \mathcal{S}_k \), there is a unique solution that is defined recursively on each time interval of the form \([t_k, t_{k+1})\). Consider the sampled-data system (5) under the equivalent representation (7), (8). Assume that for every \((t_k)_{k \in \mathbb{N}} \in \mathcal{S}_k \):

H1. There exists a continuous function \( s(y, w) \) which satisfies the following integral property

\[
\int_{t_k}^{t} s(y(\rho), \omega(\rho))d\rho \leq 0, \forall t \in [t_k, t_{k+1}),
\]

along the trajectories of (7), (8).

H2. There exists a differentiable positive definite function \( V : \mathbb{R}^n \to \mathbb{R}_{\geq 0} \), and \( c_1, c_2, p > 0 \) such that \( c_1 |z|^p \leq V(z) \leq c_2 |z|^p \).

and

\[
\dot{V}(z(t)) + \alpha V(z(t)) \leq e^{-\alpha(t-t_\ell)} s(y(t), w(t)), \forall t \in [t_k, t_{k+1}), \forall z(t) \in \mathbb{R}^n.
\]  \hspace{1cm} (9)

We particularize as follows the result from [Omran (2016)] to our case

Theorem 1. (Omran (2016)). Consider a sampling sequence in \( \mathcal{S}_k \), and positive number \( \alpha, p > 0 \). Under the assumption H1-H2 the system (7) is globally exponentially stable with convergence rate \( \frac{\alpha}{p} \).

Theorem 1 does not provide a constructive way to obtain \( V \), or an observer gain to ensure convergence. Consider two technical Lemma’s.

Lemma 2. (Omran (2014)) Consider the system (7) and the reset integrator in (8). Then, for any \( y \in L_2^1 [t_k, t_{k+1}) \) and any \( 0 < \bar{Y} = Y' \in \mathbb{R}^{n \times n} \) we have the following inequality:

\[
\int_{t_k}^{t} w(\rho)' Xw(\rho) - \delta_0^2 y(\rho)' Xy(\rho)d\rho \leq 0
\]

for all \( t \in [t_k, t_{k+1}) \), where \( \delta_0 = \frac{2}{\pi} h \).

Lemma 3. (Omran (2014)) Consider the system (7) and the reset integrator in (8). Then, for any \( y \in L_2^1 [t_k, t_{k+1}) \) and any \( 0 < \bar{Y} = Y' \in \mathbb{R}^{n \times n} \) we have the following inequality:

\[
\int_{t_k}^{t} w(\rho)' Yy(\rho) + y(\rho)' Yw(\rho)d\rho \leq 0
\]

for all \( t \in [t_k, t_{k+1}) \), where \( \delta_0 = \frac{2}{\pi} h \).

4. TRACTABLE CONDITIONS TO SOLVE (AP) AND (SP)

4.1 Analysis of convergence with known observation gain.

We will first assume that a suitable observer gain for system (4) is known when communication is continuous (i.e. in (5), \( z(t_k) = z(t), \forall t \geq 0 \)) and give tractable sufficient conditions for analysis of a given observation gain and a given maximal inter sampling time \( h \). In other words, our approach gives sufficient conditions to test whether or not a couple \( (L, \bar{T}) \) (i.e. a given gain and a fixed maximal inter-sampling time) will guaranty a certain convergence rate.

In order to establish the convergence of the observer one defines the following LMIs:

\[
W(\alpha) := \begin{bmatrix}
H(\bar{C}_1 L C) + (R_1 + L C)C_2
+ P - G_1^T G_1
+ \alpha P
& H(\bar{C}_1 L C) + (R_1 + L C)C_3
& -H(\bar{G}_2)
& +\delta_0^2 Xl_2
& \bar{G}_2 L C
& -G_3 - V_1
& H(\bar{G}_2 L C)
& -Xl_2
\end{bmatrix} < 0,
\]  \hspace{1cm} (10)

\forall i \in \mathcal{P}, \forall j = 1, 2, \text{with } l_1 = 1, l_2 = e^{-\alpha h}, \alpha \geq 0, \delta_0 = \frac{2}{\pi} h,

\( G_1, G_2, G_3, P, X, Y \in \mathbb{R}^{n \times n} \), \( P = P^T > 0 \), \( X = X^T > 0 \), \( Y = Y^T > 0 \), then system (7) with observation gain \( L \) and a sampling period \( (t_k)_{k \in \mathbb{N}} \in \mathcal{S}_k \) is exponentially asymptotically stable with convergence rate \( \frac{\alpha}{2} \).

Proof 1. If the set of LMIs (10) is verified, then multiplying on the right by \( \xi := (z', y', w') \) and on the left by \( \xi^T \) leads to
2z′P(Riz + LCz + LCw) + ξ′He(Ｇ′Hi)ξ + αz′Pz ≤ −δ2||y||2X + ||w||2X + 2y′YW, ∀i ∈ P, (11)

and

\begin{align*}
2z′P(Riz + LCz + LCw) + ξ′He( Ｋ′Hi)ξ + αz′Pz ≤ e^{-αt}( -δ̂2||y||2X + ||w||2X + 2y′YW), & ∀i ∈ P,
\end{align*}

where \( H_i = [R_i, LC, -I, LC] \hat{G} = [G_1, G_2, G_3] \), and

Recalling that
\begin{align*}
\dot{z} = (R(λ(x, z)) + LC)z + LC(w)
\end{align*}

\( y = z, \quad ∀t \in [t_k, t_{k+1}) \),

Since \( R(λ(x, z)) = \sum_{i ∈ P} λ_i R_i \), for some \( λ ∈ \Delta_{Card(P)} \), one has by convexity,

\[ \sum_{i ∈ P} λ_i [ξ′(Ｇ′Hi + H_i′Ｇ)ξ] = 0. \]

Therefore by using (11) one obtains the following inequality:

\begin{align*}
2z′P(R(λ(t))z + LCz + LCw) + αz′Pz & ≤ −δ̂2||y||2X + ||w||2X + 2y′YW.
\end{align*}

and

\begin{align*}
2z′P(R(λ(t))z + LCz + LCw) + αz′Pz & ≤ e^{-αt}( -δ̂2||y||2X + ||w||2X + 2y′YW).
\end{align*}

Therefore \( \dot{V} + αV \leq e^{-α(t-t_i)}s(y(t), w(t)) \), where

\[ s(y(t), w(t)) = \left( \begin{array}{c} y \\ w \end{array} \right)' \left[ -δ̂2X \begin{array}{c} Y \\ Y \end{array} \right] \left( \begin{array}{c} y \\ w \end{array} \right). \]

By virtue of Lemma 2 and Lemma 3, \( s(y(t), w(t)) \) fulfils H1 of Theorem 1. Furthermore since

\[ λ_{min}(P)|z|^2 ≤ V(z) := \dot{z}'Pz ≤ λ_{max}(P)|z|^2. \]

One can apply Theorem 1:

\[ |z(t)| ≤ \sqrt{\frac{λ_{min}(P)}{λ_{max}(P)}} e^{-\frac{α}{2}}|z_0|. \]

This concludes the proof. □

The proposed method consists in two steps. First a gain \( L \) is computed based on continuous-time LMIs given by (6), then the robustness of the proposed controller with respect to sampled measurements is assessed using (10). However since the gain is directly computed using continuous-time equation some conservatism may be introduced.

**Remark 1.** Until now no observability hypothesis were made. However it can be seen that a necessary condition for (10) to be feasible is that there exist \( L, G_i, P > 0 \) such that for all \( i ∈ P \), \( G_i′(R_i + LC) + (R_i + LC)G_i + αP < 0, G_i + G′_i > 0 \). This inequality requires detectability of each linear system of the form \( \dot{x} = R_i x, y = Cx \).

### 4.2 Synthesis of an observation gain

In equation (10) the knowledge of the observation gain \( L \) is assumed, typically this \( L \) is computed using a continuous-time model. In order to find a \( L \) allowing (5) to converge for larger classes of sampling sequences, we show next how to circumvent the emulation based approach. Indeed modifying equation (10) one can deal simultaneously with both observer synthesis and convergence.

**Corollary 1.** If the following set of LMIs is feasible:

\[ Ω(α)_{ij} = \begin{bmatrix} αP + He(R_iG) + He(QC) & P + R_i′G + He(QC) & QC \\ * & -He(G) + δ̂0Xl & QC - Yl_j \\ * & * & -X_l_j \end{bmatrix} < 0, \]

\[ ∀i ∈ P, ∀j = 1, 2, with l_1 = 1, l_2 = e^{-αh}, α ≥ 0, δ̂0 = \frac{2h}{π}, \]

then system (7) with observation gain \( L = (Ｇ′)^{-1}Q \) is exponentially asymptotically stable with convergence rate \( \frac{2h}{π} \).

**Proof 2.** If the set of LMIs (17) is feasible then \( -G − G′ + δ̂0X < 0 \) with \( δ̂0X > 0 \). It follows that \( G \) is full rank and thus invertible.

Furthermore when (17) is feasible, (10) is feasible with a new gain \( L = G″^{-1}Q \) where \( G_1 = G_2 = G, G_3 = 0 \).

Therefore one can apply Theorem 2. This concludes the proof. □

Some generality is lost when setting new constraints on \( G_2 \) and \( G_3 \). However the resulting gain may allow to establish convergence of the observer for larger values of \( h \) than by using the emulation based approach. Moreover the gain obtained using Corollary 1 can be analysed using Theorem 2 to maximize the value of \( h \).

It has always been assumed that the value \( h \) was given a priori. However if unknown the maximal admissible \( h \) can be found by performing a line search.

If one is only interested in exponential convergence without known guaranteed convergence rate (10) and (17) can be simplified, leading to less computation.

**Corollary 2.** If the following set of LMIs is feasible

\[ \begin{bmatrix} +He(R_iG) + He(QC) & P + R_i′G + He(QC) & QC \\ * & -He(G) + δ̂0X & QC - Yl_j \\ * & * & -X_l_j \end{bmatrix} < 0, \]

\[ ∀i ∈ P, δ̂0 = \frac{2h}{π}, P, G ∈ R^{n×n}, P = P′ > 0, X = X′ > 0, Y = Y′ > 0 ∈ R^{n×n}, \]

Then (17) is feasible for \( α \) sufficiently small.

**Proof 3.** By assumption on LMIs (18)

\[ Ω(0)_{ij} < 0, ∀i ∈ P, j = {1, 2}. \]

Where \( Ω(0)_{ij} \) is defined in (17). Since \( ∀i ∈ P, j = {1, 2} \) \( Ω(α)_{ij} \) is a continuous matrix function of \( α \) it follows that there exists \( \tilde{α} \) sufficiently small, such that \( ∀α ≤ \tilde{α} \) (17) is verified. □
As shown in Table 2 it is possible to compare the maximal sampling time obtained with [Raff (2007), Raff (2008), Chen (2013)]. The class of sampling time considered in this articles is of the form $t_{k+1} - t_k \in [r, \bar{r}]$.

In the case where measurement are continuous , this same example has also been considered in [Zemouche (2008)] with $C = (1, 0, 0, 0)$. Considering the gain obtained in this case, it is possible to apply Theorem 2 and use an emulation approach. One finds out that the LMIs (10) is verified until $\bar{h} = 0.15$. On the same system using Corollary 1 along with LMIs (17) one finds a new gain $\bar{L}$ such that $\bar{h} = 0.38$. Using this new $\bar{L}$ with the LMIs (10) $\bar{h}$ can be slightly increased to $\bar{h} = 0.4$. In order to compare our approach with related work, we have tested the feasibility of different sets of LMIs using the Sedumi solver ([Sturm (1999)]) in MATLAB, the results are listed in Table 2.

6. CONCLUSION

In this work the problem of observer synthesis of nonlinear Lipschitz systems subject to time-varying sampling has been studied. In order to conduct the analysis one defines a reset integrator, whose purpose is to model the impact of the time-varying sampling on the system. New conditions are proposed to analyse the robustness of an a priori known gain for a continuous-time Luemberger observer when the sensors communicate information discretely and the sampling is time-varying. The proposed framework also allows to give new conditions for observer gain synthesis. The effectiveness of this approach is illustrated on a robot flexible link and a single-link direct-drive manipulator actuated by a permanent magnet DC brush motor. It appears that the proposed methodology is competitive with respect to related approaches. Further study will investigate robustness with respect to noise as well as decentralized sensor communications.

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| $h$  | $\alpha$ |
|------|----------|
| 0.38 | 0.2      |
| 0.31 | 0.6      |
| 0.2  | 1.0      |
| 0.18 | 1.5      |

Table 1. Sampling sequence and convergence rate
Table 2. Admissible sampling sequence for the flexible joint according to different approaches

| Raff (2007) | Raff (2008) | Chen (2013) | Chen (2013) | Th 10 |
|------------|-------------|-------------|-------------|-------|
| 0          | 0.1         | 0.3         | 0.45        | 0.1   | 0    |
| 0.1        | 0.3         | 0.45        | 0.34        | 0.41  |      |

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