Multi-antikaonic nuclei and in-medium kaon properties in dense matter

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Abstract. The effects of the \( \Lambda(1405) (\Lambda^\ast) \) and range terms (the second-order effects, SOE) on the structure of multi-antikaonic nuclei are studied in a relativistic mean-field theory coupled with the nonlinear effective chiral Lagrangian. It is shown that, due to the attractive interaction from the \( \Lambda^\ast \)-pole contribution, the \( K^- \) mesons and the protons are more attracted each other than the case without the SOE. On the other hand, the density distribution for neutron is pushed outward due to the repulsive effect from the range terms. The SOE is also discussed in connection with modification of kaon properties in a dense medium.

1. Introduction

Multi-strangeness system has recently attracted much interest as a new frontier of nuclear physics. In neutron stars, kaon condensation has been suggested to exist as a macroscopic appearance of strangeness, and its properties have been investigated extensively from viewpoints of nuclear physics and astrophysics\cite{1}. Hyperons may also be mixed in the ground state of neutron star matter\cite{2}. In terrestrial experiments, deeply bound kaonic nuclei have been proposed based on the strong \( \bar{K}-N \) attractive interaction\cite{3, 4}. Stimulated by studies of kaon condensation and kaonic nuclei, multi-antikaonic nuclei (MKN), where several numbers of antikaons (\( K^- \)) are embedded in nuclei, have been investigated theoretically\cite{5, 6}. Similarity and difference between the MKN and kaon condensation in neutron stars have been discussed.

We have considered properties of the MKN in a relativistic mean-field theory (RMF) coupled with the nonlinear effective chiral Lagrangian\cite{5}. It has been pointed out that repulsive \( \bar{K}-\bar{K} \) interaction becomes sizable in comparison with the attractive \( \bar{K}-N \) interaction as the number of embedded \( K^- \) mesons, \(|S|\), increases. Thereby the lowest \( K^- \) energy, \( \omega_{K^-} \), increases with \(|S|\), and it enters into the subthreshold resonance region of the \( \Lambda(1405) (\Lambda^\ast) \), where \( \omega_{K^-} \simeq m_{\Lambda^\ast} - m_N = 467 \) MeV. Much discussion has been made about the nature of the \( \Lambda^\ast \) including a two-pole structure for \( \Lambda^\ast \)\cite{7, 8}. In this paper, we take into account the \( \Lambda^\ast \) as a pole contribution of a point particle as well as range terms and clarify these effects on the structure of the MKN.

2. Formulation

The basic \( K-N \) and \( \bar{K}-\bar{K} \) interactions are described on the basis of the nonlinear effective chiral Lagrangian, where the \( s \)-wave \( K-N \) scalar interaction is simulated by the \( KN \) sigma term, \( \Sigma_{KN} \).
and the vector interaction (Tomozawa-Weinberg term) is incorporated model-independently. The $N - N$ interactions are given by the exchange of $\sigma$, $\omega$, and $\rho$ mesons in the RMF. The contact interactions between the nonlinear $K^-$ field and nucleons in the effective chiral Lagrangian are then replaced by the $\sigma$, $\omega$, and $\rho$ mesons-exchange within the RMF. It can be shown that the resulting $KN$ interactions in the meson-exchange picture formally equivalent to those prescribed by chiral symmetry through the relations, $g_{\sigma N}g_{\sigma K}/m_\sigma^2 = \Sigma_{KN}/(2m_K^2)$, $g_{\omega N}g_{\omega K}/m_\omega^2 = 3/(8f^2)$, and $g_{\rho N}g_{\rho K}/m_\rho^2 = 1/(8f^2)$, where $g_{\sigma K}$, $g_{\omega N}$, and $m_i$ ($i = \sigma, \omega, \rho$) are the coupling constants and meson masses, $m_K$ the free kaon mass, and $f$ (=93 MeV) is the meson decay constant[5].

Under the assumption that the MKN is spherically symmetric, the $K^-$ field is represented as $K^-(r) = f \theta(r)/\sqrt{2}$ with $\theta(r)$ being the chiral angle in the condensate approximation and $r$ the radial distance from the center of the MKN. The mass number $A$, the number of protons $Z$, and the number of embedded $K^-$ mesons $|S|$ are kept conserved, and the thermodynamic potential $\Omega$ for the MKN is derived under the Thomas-Fermi approximation for the nucleons[5].

After the second-order perturbation with respect to the axial-vector current of hadrons $\hat{A}_5^\mu$ with current algebra and PCAC, one obtains a correction to the energy density $\Delta \epsilon(r)$:

$$
\Delta \epsilon(r) = -i \int d^4z \langle x| T\bar{\psi}_K - \hat{A}_5^\mu(z)\bar{\psi}_K - \hat{A}_5^\mu(0)|x\rangle \times \left( -\frac{1}{2} \sin^2 \theta \right)_{\text{real part}}\right)
$$

$$
\Rightarrow -\frac{1}{2} f^2 \omega_K^2 \sin^2 \theta \left[ \rho_\sigma \left( d_p + \frac{g_\sigma^2}{2f^2} \frac{m_{\Lambda^*} - m_N - \omega_K - 2\gamma_{\Lambda^*}}{m_{\Lambda^*} - m_N - \omega_K - 2\gamma_{\Lambda^*}} \right) + \rho_\rho \right],
$$

where $\rho_\sigma^0(r)$ ($\rho_\rho^0(r)$) is the scalar density of the proton (neutron), and $\bar{\omega}_K(r) \equiv \omega_K - V_{\text{Coul}}(r)$ with $\omega_K$ being the ground-state energy of the $K^-$ and $V_{\text{Coul}}(r)$ the Coulomb potential. In Eq. (1), the pole term comes from the $\Lambda^*$ with $g_{\Lambda^*}$ being the coupling constant at the $K^-p\Lambda^*$ vertex, $\gamma_{\Lambda^*}$ the width, and the smooth parts $\propto d_p \rho_\sigma^0, d_n \rho_\rho^0$ are the range terms. These terms are absorbed into the effective nucleon masses. We call these contributions to the energy the second-order effects (SOE)[1]. The parameters, $d_p$, $d_n$, $g_{\Lambda^*}$, and $\gamma_{\Lambda^*}$ are determined so as to reproduce the on-shell s-wave $K-N$ scattering lengths[9]. One obtains $d_p = (0.35 - \Sigma_{KN}/m_K)/(f^2 m_K)$, $d_n = (0.23 - \Sigma_{KN}/m_K)/(f^2 m_K)$, $g_{\Lambda^*} = 0.58$, and $\gamma_{\Lambda^*} = 12.4$ MeV.

The classical $K^-$ field equation is given from $\delta \Omega/\delta \theta(r) = 0$ as

$$
\nabla^2 \theta(r) = \sin \theta(r) \left[ m_{\omega K}^2(r) - 2\bar{\omega}_K(r)X_0(r) - \bar{\omega}_K^2(r) \cos \theta(r) \left\{ 1 + d_n \rho_\rho^0(r) \right\} \right]
$$

$$
- \bar{\omega}_K^2(r) \cos \theta(r) \rho_\sigma^0(r) \left( d_p + \frac{g_\sigma^2}{2f^2} \frac{m_{\Lambda^*} - m_N - \omega_K - 2\gamma_{\Lambda^*}}{m_{\Lambda^*} - m_N - \omega_K - 2\gamma_{\Lambda^*}} \right),
$$

where $m_{\omega K}^2(r) (= m_{\omega K}^2 - 2g_{\omega K}m_K\sigma(r))$ is the square of the effective mass of the $K^-$, $X_0(r) (= g_{\omega K}\bar{\omega}_K(r) + g_{\rho K}R_0(r))$ represents the $K-N$ vector interaction. In these quantities, $\sigma(r)$, $\omega_K(r)$, and $R_0(r)$ are the mean fields of the $\sigma$ meson and the time components of the $\omega$ and $\rho$ mesons, respectively. Together with Eq. (2) one obtains the coupled equations of motion (EOM) for the other mesons $\sigma$, $\omega$, $\rho$, and the Poisson equation for the Coulomb potential $V_{\text{Coul}}(r)$:

$$
-\nabla^2 \sigma(r) + m_{\sigma}^2 \sigma(r) = \frac{dU}{d\sigma} + g_{\sigma N}(\rho_\sigma^0(r) + \rho_\rho^0(r)) + 2g_{\sigma K}m_Kf^2(1 - \cos \theta(r)),
$$

$$
-\nabla^2 \omega_K(r) + m_{\omega K}^2 \omega_K(r) = g_{\omega N}(\rho_\sigma^0(r) + \rho_\rho^0(r)) - 2g_{\omega K}\bar{\omega}_K(r)f^2(1 - \cos \theta(r)),
$$

$$
-\nabla^2 R_0(r) + m_{\rho K}^2 R_0(r) = g_{\rho N}(\rho_\sigma^0(r) - \rho_\rho^0(r)) - 2g_{\rho K}\bar{\omega}_K(r)f^2(1 - \cos \theta(r)),
$$

$$
\nabla^2 V_{\text{Coul}}(r) = 4\pi e^2(\rho_\rho^0(r) - \rho_K^0(r)),
$$

(3a, 3b, 3c, 3d)
where \( \rho_i(r) \ (i = p, n, K^-) \) are the number densities. The \( K^- \) number density is given by \( \rho_{K^-}(r) = \tilde{\omega}_{K^-}(r)f^2\sin^2\theta(r) + 2X_0(r)f^2(1 - \cos\theta(r)) \). The values of \( g_{\sigma N}, \ (i = \sigma, \omega, \rho) \) are determined so as to reproduce not only the properties of normal nuclear matter with saturation density \( \rho_0 = 0.153 \text{ fm}^{-3} \) but also the binding energy, proton-mixing ratio, and density distributions of proton and neutron for normal nuclei [5]. The coupling constants \( g_{\sigma K} \) and \( g_{\rho K} \) are chosen from the quark and isospin coupling rule as \( g_{\sigma K} = g_{\omega N}/3, \ g_{\rho K} = g_{\rho N} \). The unknown parameter \( g_{\sigma K} \) is related to the \( K^- \) optical potential \( U_K \ [\equiv -(g_{\sigma K} \sigma + g_{\omega K} \omega_0) \] in symmetric nuclear matter. The value of \( U_K \) is set to be \( -80 \) and \( -120 \text{ MeV} \), which corresponds to \( \Sigma_{KN} = 332 \text{ MeV} \) and 754 \text{ MeV} \), respectively.

3. Numerical results

3.1. Role of the \( \Lambda^* \) on the lowest \( K^- \) energy

Throughout this paper, a reference nucleus \( \{|S| = 0\} \) is taken to be the \(^{15}\text{O} \ (A=15, \ Z=8)\). The lowest energy \( \omega_{K^-} \) of the \( K^- \) mesons embedded in the MKN is shown as a function of \(|S|\) in Fig. 1. The dashed-dotted lines are the results with the SOE, and the solid lines are our previous results without the SOE [5]. For \( U_K = -80 \text{ MeV} \) (Fig. 1 (a)), the \( \omega_{K^-} \) lies well below the \( \Lambda^* \) resonance region as a result of the additional attraction from the \( \Lambda^* \) pole. The range terms \( \propto d_p\rho_p^2, d_n\rho_n^2 \) where \( d_p, d_n < 0 \) in Eq. (2) works repulsively, but they have a minor effect. On the other hand, in the case of \( U_K = -120 \text{ MeV} \) (Fig. 1 (b)), the repulsive effect from the range terms is large because of the large value of \( \Sigma_{KN} \) and overcomes the attraction from the \( \Lambda^* \) pole. As a result, the \( \omega_{K^-} \) with the SOE is slightly pushed up as compared with that without the SOE. The attraction from the \( \tilde{K} - N \) scalar interaction simulated by the \( \Sigma_{KN} \) and the repulsive effect from the range terms, both of which are of order \( O(m_{K}^2) \), are compensated with each other [Eq. (2)]. Therefore there is little dependence of \( \omega_{K^-} \) on the value of \( \Sigma_{KN} \) or \( U_K \), as seen from Figs. 1 (a) and (b).

With increase in \(|S|\), the repulsive \( \tilde{K} - \bar{K} \) interaction becomes as large as the attractive \( \tilde{K} - N \) interaction, and the lowest \( K^- \) energy \( \omega_{K^-} \) increases with \(|S|\) for \(|S| \geq 5\). For \(|S| \geq 12\) in the case of \( U_K = -80 \text{ MeV} \) [for \(|S| \geq 14\) in the case of \( U_K = -120 \text{ MeV} \), the \( K^- \) mesons become unbound, where \( \omega_{K^-} \geq m_{\Lambda^*} - m_N \) above the \( \Lambda^* \)-resonance region.
3.2. Density distributions

The density distributions of the protons, neutrons, and the distribution of the strangeness density \([-\rho_K(r)]\) are shown for \(|S|=1, 2, 4\) and \(8\) in the case of \(U_K = -80\) MeV in Figs. 2 (a)–(d). In each Figure, the dashed-dotted lines are the results with the SOE, and the solid lines are our previous results where the SOE is not taken into account. For reference, those of protons and neutrons \((\rho_p \sim \rho_n)\) for a reference nucleus \(^{15}\text{O}\) are shown in dotted lines in Fig. 2 (a). Without the SOE, we have seen that, even for \(|S|=1\) and 2, the protons and neutrons are attracted to the \(K^-\) mesons in the central region due to the \(\bar{K} - N\) attractive interaction, and the central baryon density \(\rho_B^{(0)}\) \(=\rho_p(r = 0) + \rho_n(r = 0)\) slightly increases as compared with the case \(|S|=0\). The protons are attracted more than neutrons since the \(\bar{K} - N\) attractive interaction is stronger for the isospin \(I = 0\) than for \(I = 1\). With the introduction of the SOE, the \(K^-\) mesons and the protons are attracted more to each other than the case without the SOE, since in the former the \(K^-\) lies below the resonance region of the \(\Lambda^*\) [see Fig. 1 (a)] and feels an additional attraction through coupling with the \(\Lambda^*\) pole. As a result, the central densities of the

![Diagram](image-url)

**Figure 2.** The density distributions of protons, neutrons, and the strangeness density \([-\rho_K(r)]\) for the MKN with \(A=15, Z=8\), and \(|S|=1\) (a), 2 (b), 4 (c), 8 (d) in the case of \(U_K = -80\) MeV.
protons and $K^-$ mesons become larger and the root-mean-square radii of the protons and $K^-$ mesons, $\sqrt{\langle r^2 \rangle}_p$ and $\sqrt{\langle r^2 \rangle}_{K^-}$, become smaller than those without the SOE. On the other hand, neutron distribution is shifted outward from the center of the MKN due to the repulsive effect from the range term ($\propto d_n \rho_n^*, d_n < 0$ in Eq. (2)), and $\sqrt{\langle r^2 \rangle}_n$ becomes larger than that without the SOE. Thus a “neutron skin” structure, with a thickness defined by $\delta_{np} \equiv \sqrt{\langle r^2 \rangle}_n - \sqrt{\langle r^2 \rangle}_p$, becomes remarkably remarkable by the SOE. These features are enhanced for large values of $|S|$ (see the cases of $|S|=4$ and 8). For instance, in the case of $|S|=8$, the central baryon density reaches $\rho_B^{(0)} \sim 3.5 \rho_0$, and the neutron skin thickness is estimated to be $\delta_{np}=0.90$ fm. In addition, for a larger $|S|$, the proton and $K^-$ density distributions tend to be more uniform near the center.

4. Structure of kaonic branches

We recapitulate qualitatively a sudden disappearance of a bound state of the $K^-$ mesons at a large value of $|S|$ with recourse to the structure change of $K^-$ and $\Lambda^* p^{-1}$ branches (the superscript “$-1$” denotes the hole state). For simplicity, we neglect finite-size effects of the MKN within a uniform matter approximation and omit the Coulomb potential $V_{\text{Coul}}(r)$. From the classical field equation for $\theta$ [Eq. (2) with neglect of $r$-dependence for all the relevant quantities], one obtains the lowest energy of $K^-$, $\omega_{K^-}(\rho_B, \theta)$ as functions of $\rho_B$ and $\theta$. In Fig. 3, the dependence of $\omega_{K^-}$ on $\theta$ is shown for $\rho_B = 1.0 \rho_0$ [(a)] and for $\rho_B = 3.5 \rho_0$ [(b)] in the case of $\Sigma_{KN}=300$ MeV. At a low density $\rho_B = 1.0 \rho_0$, in addition to the $K^-$ and $K^+$ branches corresponding to free $K^-$ and $K^+$ mesons in vacuum, respectively, there is a $\Lambda^* p^{-1}$ branch which disappears at $\theta \sim 1.0$ (rad). At a high density $\rho_B = 3.5 \rho_0$, which corresponds to the baryon density in the central region of the MKN with many $K^-$ mesons, these branches reveal complicated structure as a result of level-crossings between several branches.

For given $\rho_B$ and $|S|$, the strength of the $K^-$ field $\theta$ is determined through the relation, $\rho_{K^-}(\rho_B, \theta) = |S|/A \cdot \rho_B$. As $|S|$ increases, $\theta$ increases monotonically. Thus, together with the $\omega_{K^-}$-$\theta$ diagram (Fig. 3), one can obtain $\omega_{K^-}$ as a function of $|S|$ for a given $\rho_B$. For $\rho_B = 3.5 \rho_0$ [Fig. 3 (b)], the lowest energy state of the bound $K^-$ mesons is composed of the $\Lambda^* p^{-1}$ branch up to $|S| \lesssim 25$, while it transits to the $K^-$ branch discontinuously for $|S| > 25$. The energy $\omega_{K^-}$ for the $K^-$ branch for $|S| > 25$ is $\sim 500$ MeV (the free kaon mass), so that one cannot obtain a bound state of the $K^-$ mesons for such a large $|S|$.

Figure 3. The lowest energy of the $K^-$ for kaonic branches as functions of $\theta$ in matter with $\rho_p = Z/A \cdot \rho_B$ and $\rho_n = (A-Z)/A \cdot \rho_B$ in the case of $\Sigma_{KN}=300$ MeV. (a) is for $\rho_B = 1.0 \rho_0$ and (b) is for $\rho_B = 3.5 \rho_0$. For given $\rho_B$ and $|S|$, the strength of the $K^-$ field $\theta$ is determined through the relation, $\rho_{K^-}(\rho_B, \theta) = |S|/A \cdot \rho_B$. As $|S|$ increases, $\theta$ increases monotonically. Thus, together with the $\omega_{K^-}$-$\theta$ diagram (Fig. 3), one can obtain $\omega_{K^-}$ as a function of $|S|$ for a given $\rho_B$. For $\rho_B = 3.5 \rho_0$ [Fig. 3 (b)], the lowest energy state of the bound $K^-$ mesons is composed of the $\Lambda^* p^{-1}$ branch up to $|S| \lesssim 25$, while it transits to the $K^-$ branch discontinuously for $|S| > 25$. The energy $\omega_{K^-}$ for the $K^-$ branch for $|S| > 25$ is $\sim 500$ MeV (the free kaon mass), so that one cannot obtain a bound state of the $K^-$ mesons for such a large $|S|$.
5. Summary and concluding remarks
We have considered the structure of the MKN in the RMF coupled with the nonlinear effective chiral Lagrangian. A pole contribution from the $\Lambda^*$ and range effects (the second-order effects, SOE) have been taken into account so as to be consistent with the $s$-wave $K-N$ and $\bar{K}-N$ scatterings empirically. Due to the attractive interaction from the $\Lambda^*$-pole contribution, the $K^-$ mesons and the protons are more attracted each other than the case without the SOE. On the other hand, the density distribution for neutron is pushed outward due to the repulsive effect from the range terms. The gross structure of the MKN is then characterized as divided into two regions: (I) The central region, where baryon and $K^-$ meson densities may be as large as 3.5 times the normal saturation density, and one can extract information on $\bar{K}-N$ and $\bar{K}\bar{K}$ interactions at high baryon densities. (II) The outer region, where neutron skin structure becomes remarkable with the SOE, and one may extract information on neutron-rich matter at subnuclear densities. These features are hardly dependent upon the choice of the $KN\sigma$ term.

We also have discussed modification of in-medium properties of the $K^-$ mesons bound in the MKN. It has been suggested qualitatively within a simple model neglecting the finite-size effects that the ground state of the $K^-$ mesons trapped in highly dense MKN reveals a complicated structure, stemming from level-crossings between several kaonic branches, which may characterize the gross structure of the MKN and possible binding of the $K^-$ mesons in the MKN. The interplay of $K^-$ and $\Lambda^*p^{-1}$ branches should be clarified more realistically based on our present framework for the MKN. The work will be reported elsewhere.

Toward a more realistic consideration, an extension of the RMF to include hyperons, taking into account the $p$-wave coupling between $K^-$, nucleon, and hyperons, may be important. Inelastic channel-coupling effects associated with hyperons, $\bar{K}N \rightarrow \pi\Lambda, \pi\Sigma$ are responsible for not only the structure of the MKN but also their formation and decay processes. It has also been shown in a liquid-drop picture that hyperon-mixing may be crucial for deeper binding of the MKN[10]. Under chemical equilibrium conditions with respect to strong processes including hyperons, $K^-N = \Lambda, \Sigma$, and $K^-\Lambda = \Xi$, coexistence of antikaons and hyperons leads to highly dense self-bound objects, which may decay only through weak processes[10]. Further study is necessary before we can tell whether such deeply bound objects may exist or not[11]. For instance, finite-size effects, by which the lowest $K^-$ energy is pushed up to $\sim 400$ MeV as compared with that in the case of infinite matter $O(m_\pi)$, may affect hyperon-mixing ratio in the MKN through chemical equilibrium conditions[12]. Obtaining the energy levels beyond the local density approximation for baryons may also be important.

Acknowledgments
This work is supported in part by the Grant-in-Aid for Scientific Research (No. 20028009) and by the funds provided by Chiba Institute of Technology.

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