Level Excitation and Transition Probabilities
of Some Nuclei in The Lower $fp$-Shell

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Abstract

Unrestricted shell model calculations in the lower $fp$-shell region for the nuclei $^{46}$Ti, $^{46}$Cr and $^{46}$V have been performed for the isovector $T=1$ positive parity states using the shell model code OXBASH for Windows by employing the effective interactions GXPF1, FPD6 and KB3G. The level schemes and transition strengths $B(E2;\downarrow)$ are compared with the recently available experimental data. A very good agreement were obtained for all nuclei.

1 Introduction

The nuclear shell model has been very successful in our understanding of nuclear structure: once a suitable effective interaction is found, the shell model can predict various observables accurately and systematically. For light nuclei, there are several "standard" effective interactions such as the Cohen-Kurath [1] and the USD [2] interactions for the $p$ and $sd$ shells, respectively. On the other hand, in the next major shell, i.e., in the $fp$-shell, there were also "standard" interactions such as FPD6 [3] and GXPF1 [4].

The spectroscopy of nuclei, in the $fp$-shell region, has been well described within the shell model framework. Extensive shell model calculations have been performed in this mass region, using several model spaces and two-body interactions, the most remarkable work of Brown and co-workers [5, 6, 7, 8, 9, 10, 11]. Because of the quite importance of the $fp$-shell for variety of problems in nuclear structure, such as electron capture in supernova explosions. In this letter we report the shell model calculations in the lower $fp$-shell region for the nuclei $^{46}$Ti, $^{46}$Cr and $^{46}$V, to test the the ability of the present effective interactions in reproducing the experiment in this mass region.
Figure 1: Comparison of the experimental excitation energies taken from Ref. [14] with the present theoretical work using FPD6, GXPF1 and KB3G effective interactions.

2 Shell model calculations

2.1 Excitation energies

As mentioned in the earlier section, the main motivations for studying these nuclei lies in the lower $fp$-shell due to the importance of these in the recent applications in astrophysics and because of the spin-orbit splitting that gives rise to a sizable energy gap in the $pf$-shell between $f_{7/2}$ orbit and the other orbits $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$, producing the N or Z=28 magic number.

The calculations have been carried out using the code OXBASH for windows [12] in the FP model space which comprised of the $1p_{3/2}$, $1p_{1/2}$, $0f_{7/2}$ and $0f_{5/2}$ valence orbits outside the $^{40}$Ca. Three effective interactions were employed with FP model space for the calculations of level spectra and transition probabilities, these effective interactions are FPD6 [3], GXPF1 [4] and KB3G [13]. We should mention here that $^{46}$Ti and $^{46}$Cr have only isovector part T=1, while $^{46}$V have isovector part T=1 and isoscalar T=0, in our study we considered only the isovector T=1 for $^{46}$V.

Figure 1. presents the comparison of the experimental excitation energies of $^{46}$Ti with calculated values from FPD6, GXPF1 and KB3G effective interactions. The three effective interactions gives very good results in comparison with the experimental values up to $J^\pi=12^+$. From Fig1. we can notice that FPD6 are in excellent agreement with the experiment better than GXPF1 and KB3G.

In figure 2 and figure 3, same comparison were made using the three effective interactions for $^{46}$Cr and $^{46}$V respectively. From these figures same conclusion were drawn that FPD6 is the best for describing these nuclei lies in the lower part of the $fp$-shell.
Figure 2: Comparison of the experimental excitation energies taken from Ref. [14] with the present theoretical work using FPD6, GXPF1 and KB3G effective interactions.

Figure 3: Comparison of the experimental excitation energies taken from Ref. [15] with the present theoretical work using FPD6, GXPF1 and KB3G effective interactions.
2.2 Transition probabilities

Since the transition rates represent a sensitive test for the most modern effective interactions that have been developed to describe $fp$-shell nuclei. The transition strengths calculated in this work performed using the harmonic oscillator potential HO for each in-band transition by assuming pure $E2$ transition. Core polarization effect were included by choosing the effective charges for proton $e_p=0.7e$ and for neutron $e_n=0.5e$. Our results and the previous theoretical results using different models are listed in Table 1 for $^{46}$Ti.

In Th.1 and Th.2 [16], the effective charges for proton and neutron were taken as $1.38e$ and $0.83e$ respectively. The effective charges for protons and neutrons taken to be equal in value as $0.7e$ in Th.3 which is MONSTER [17] and $e_p=e_n=0.9e$ adopted in Th.4 "the $(f_{7/2})^6$ shell model [18]". As seen from Table 1, the $B(E2;\downarrow)$ values calculated in this work are in better agreement for the transitions $B(E2; 2^+_1 \rightarrow 0^+_1)$ and $B(E2; 4^+_1 \rightarrow 2^+_1)$ than the previous theoretical work, while the rest transitions, Th.1., Th.2, Th.3, Th.4 and Th.6 are in better agreement with the experimental data, except Th.5 “the rotational model [18]” do not follow the trend of experimental data.

Although FPD6 effective interaction is more successful in description of energy level spectra, but the calculation of the transition strengths prove that it not the standard effective interaction for this region and the results obtained by GXPF1 are in excellent agreement with experiment, also the result of KB3G are not so far from the experimental values.

For $^{46}$Cr the same comparison were made in Table 2, but the experimental data are not available, therefore we can not judge which effective interaction reproduce the experimental data better.

The effective charges for proton and neutron are taken to be $0.5e$ and $0.4e$ respectively, for the calculations of the transition strengths of $^{46}$V. Our theoretical results are in excellent agreement with the experimental values for the transitions $B(E2; 2^+_1 \rightarrow 0^+_1)$ and $B(E2; 4^+_1 \rightarrow 2^+_1)$ using GXPF1 effective interaction, also our theoretical predictions are in better agreement from the previous theoretical work Th.2 [15] and Th.3 [24] as summarized in Table 3.
Table 1: The $B(E2)$ values in the ground-state band of $^{46}\text{Ti}$. Their units are $e^2 \text{ fm}^4$. Exp. is the experiment [18, 19, 20, 21]; Th.1 is PPNC; Th.2 is the projected of the pure HF ground-state configuration [16]; Th.3 is MONSTER [17]; Th.4 is the $(f_{7/2})^6$ shell model [18]; Th.5 is the rotational model [18]; Th.6 is ANTOINE [21]. This work is assumed pure $E2$ transition limit.

| $J_i^\pi \rightarrow J_f^\pi$ | Exp. | Th.1 | Th.2 | Th.3 | Th.4 | Th.5 | Th.6 | Present work |
|-----------------------------|------|------|------|------|------|------|------|--------------|
| $2_1^+ \rightarrow 0_1^+$    |      | 132  | 134  | 138  | 116  | 215  | 116  | 183          | 195          | 233          |
| $4_1^+ \rightarrow 2_1^+$    |      | 186  | 184  | 186  | 127  | 304  | 154  | 233          | 256          | 328          |
| $6_1^+ \rightarrow 4_1^+$    |      | 196  | 188  | 189  | 110  | 342  | 154  | 213          | 241          | 309          |
| $8_1^+ \rightarrow 6_1^+$    |      | 183  | 175  | 172  | 122  | 325  | 140  | 211          | 233          | 291          |
| $10_1^+ \rightarrow 8_1^+$   |      | 143  | 157  | 119  | 69   | 362  | 101  | 160          | 174          | 222          |
| $12_1^+ \rightarrow 10_1^+$  | 29±3 | 56   | 124  | 51   | 41   | 372  | 41   | 65           | 75           | 84           |

$^a$Reference [19], $^b$Reference [20], $^c$Reference [18], $^d$Reference [21]

Table 2: The $B(E2)$ values in the ground-state band of $^{46}\text{Cr}$. Their units are $e^2 \text{ fm}^4$. Exp. is the experiment [22]. This work is assumed pure $E2$ transition limit.

| $J_i^\pi \rightarrow J_f^\pi$ | Exp. | Present work |
|-----------------------------|------|--------------|
|                            |      | GXP Fi | KB3G | FPD6 |
| $2_1^+ \rightarrow 0_1^+$   | 186±40 | 183     | 195  | 233  |
| $4_1^+ \rightarrow 2_1^+$   |      | 233     | 256  | 328  |
| $6_1^+ \rightarrow 4_1^+$   |      | 213     | 241  | 309  |
| $8_1^+ \rightarrow 6_1^+$   |      | 211     | 233  | 291  |
| $10_1^+ \rightarrow 8_1^+$  |      | 160     | 174  | 222  |
| $12_1^+ \rightarrow 10_1^+$ | 65    | 75      | 84   |
Table 3: The $B(E2)$ values in the ground-state band of $^{46}$V. Their units are $e^2$ fm$^4$. Exp. is the experiment [15, 23]. This work is assumed pure $E2$ transition limit.

| $J^z_i \rightarrow J^z_f$ | Exp. | Th.1 | Th.2 | Th.3 | Present work |
|--------------------------|------|------|------|------|--------------|
|                           |      |      |      |      | GXPF1 | KB3G | FPD6 |
| $2^+_i \rightarrow 0^+_1$ | $137\pm35^a$ | 537  | 142  | 142  | 137  | 145  | 175  |
|                           | $138\pm35^b$ |      |      |      |      |      |      |
| $4^+_i \rightarrow 2^+_1$ | $\geq 169^a$ | 676  | 187  | 187  | 173  | 191  | 245  |
| $6^+_i \rightarrow 4^+_1$ | 658  | 175  | 159  | 180  | 231  |
| $8^+_i \rightarrow 6^+_1$ | 601  | 167  | 156  | 173  | 217  |
| $10^+_i \rightarrow 8^+_1$ |      |      |      | 119  |      |      |      |
| $12^+_i \rightarrow 10^+_1$ | 54   | 48   | 56   | 63   |

$^a$Reference [23], $^b$Reference [15]

3 Summary

Full $fp$-space shell model calculations were performed using the code OXBASH for Windows. The FP model space were employed with the effective interactions GXPF1, FPD6 and KB3G to reproduce the level spectra and transition strengths $B(E2)$ for the nuclei $^{46}$Ti, $^{46}$Cr and $^{46}$V. Excellent agreement were obtained by comparing these calculations with the recently available experimental data for the level spectra using FPD6 effective interaction. Calculation of the transition strengths prove that GXPF1 is more consistent in reproducing the experiment than FPD6 for the lower $fp$-shell region.

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