OPEN STRINGS AND DUALITIES

Massimo BIANCHI

Dipartimento di Fisica, Università di Roma “Tor Vergata”
I.N.F.N. - Sezione di Roma “Tor Vergata”, Via della Ricerca Scientifica, 1
00133 Roma, ITALY

ABSTRACT

In the fruitful interplay between gauge fields and strings and in many conjectured M-theory dualities, open strings play a prominent role. We review the construction of open-string descendants (un-orientifolds) of closed-string theories admitting a generalized orientation reversal involution. We then specialize the construction to some classes of non-supersymmetric models in $D = 10$ that have been recently considered in the context of duality without supersymmetry. We also discuss the propagation of open and unoriented strings on the NS pentabranes (N5-brane). This background is a prototype of the configurations of branes and orientifold planes that represent a powerful alternative to the geometric engineering of Supersymmetric Yang-Mills Theories. The resulting description of D-branes in non-trivial backgrounds looks very different from the one naively expected. In particular the very distinction between different Dp-branes becomes ambiguous in the presence of strong curvature effects.
1 Introduction

1.1 In The Web of String Dualities

At the perturbative level, there are two rather distinct classes of superstring theories: those with only closed oriented strings (type IIA, type IIB, \( E(8) \times E(8) \) and \( SO(32) \) heterotic) and those with open and closed unoriented strings (type I) \([1]\). In the past, models with open strings have been studied to a lesser extent than models with only closed oriented strings. Though the initial proposal \([2]\) of identifying open-string theories as parameter-space orbifolds of left-right symmetric theories of closed oriented strings has been brought to a fully consistent systematization long time ago \([3]\), for almost ten years phenomenological considerations have oriented the interest of the string community towards perturbative vacua of the \( E(8) \times E(8) \) heterotic string preserving \( \mathcal{N} = 1 \) supersymmetry in \( D = 4 \) \([1]\). After the work of Seiberg and Witten on \( \mathcal{N} = 2 \) supersymmetric Yang-Mills theories \([4]\), these motivations seem to become much less compelling and a rather appealing scenario is taking shape according to which different string theories should be regarded as dual manifestations of a more fundamental M/F-theory \([5]\).

The most fashionable possibility is a theory of membranes (2-branes) and penta-branes (5-branes), known as M-theory, that almost by definition has 11D supergravity as its low-energy limit \([6]\). This theory has no analogue of the dilaton, whose (undetermined) vacuum expectation value plays the role of the string coupling constant, and does not allow a perturbative expansion prior to compactification. Upon dimensional reduction to \( D = 10 \) M-theory gives the type IIA superstring \([6]\). Upon compactification on a segment it is conjectured to give the \( E(8) \times E(8) \) heterotic string \([4]\) on the basis of symmetry between the two fixed “points”, surviving supersymmetry, anomaly cancellation and a consistent string interpretation. Relaxing the latter condition a theory such as \( \mathcal{N} = (1, 0) \) supergravity coupled to \( U(1)^{248} \times U(1)^{248} \) vector supermultiplets \([4]\) could not be excluded.

In the intricate web of conjectured dualities a prominent role is played by the type II solitons carrying Ramond-Ramond (R-R) charges \([8]\). These have been formerly identified with charged black p-brane solutions \([8]\) of the string equations of motion to lowest order in \( \alpha' \), the inverse string tension. A microscopic description \([8]\) in terms of hyperplanes where open strings can terminate with Dirichlet boundary conditions (D-branes) has opened the way to remarkable progress not only in the (string) duality realm but also in the context of black-hole thermodynamics \([10]\).

Much in the same way as the two type II superstrings are related by a non-geometrical \( Z_2 \)-orbifold procedure through the action of \((-)^{F_L}\) and similarly the two heterotic strings through \((-)^{F_{16}}\), the general construction of perturbative open-superstring vacuum configurations consists in a non-standard \( Z_2 \)-orbifold procedure that amounts to gauging the action of the world-sheet parity operator \( \Omega \) \([1]\). From the target-space viewpoint the \( \Omega \)-projection corresponds to the introduction of an orientifold 9-plane (O9-plane) that roughly speaking induces a doubling of the spacetime points.

\(^1\)Many authors refer to these as world-sheet orbifolds \([1]\) or as orientifolds \([8]\) though the name unorientifolds would be more suitable since the resulting string is unoriented and carries no conserved charge.
Open strings and D-branes play a crucial role in many recently conjectured string dualities [12]. The $SO(32)$ type I superstring may be considered as describing the excitations of a BPS configuration of 32 type IIB D9-branes needed to neutralize the R-R charge of an O9-plane. Similarly, the excitations of the type I D-string (D1-brane) exactly coincide with the (light-cone) degrees of freedom of the $SO(32)$ heterotic string [13]. This observation has strengthened the conjectured heterotic / type I strong-weak coupling duality [6, 14]. It is not the purpose of the present talk to review type I vacuum configurations in various dimensions and their heterotic duals that are analyzed in some detail in the talk of Carlo Angelantonj [15]. We simply remind the reader that many non-perturbative features of heterotic vacua allow for a quantitative description in the type I setting where open-string excitations of D-branes are included in the perturbative spectrum. In this respect, though all p-branes are believed to stand on an equal footing in the final non-perturbative formulation, open strings are representatives of a sort of open aristocracy [16] since they play a privileged role as far as perturbative description and explicit counting of BPS states are concerned.

The bridge between neutral vacuum configurations and charged configurations has been provided by the study of D-brane probes [17]. While the low-energy dynamics of a configuration of $N$ parallel Dp-branes is governed by the dimensional reduction of $\mathcal{N} = (1, 0)$ Supersymmetric Yang-Mills (SYM) theory with gauge group $U(N)$ from $D = 10$ to $D = p + 1$ [18] for other D-brane probes the dynamics is governed by lower-SYM theories possibly coupled to matter supermultiplets [17]. Notice that the Matrix Theory idea was born exactly in this context [19]. The non-polynomial Dirac-Born-Infeld action [8] has so far proven to be of very limited usage so far due to the difficulty intrinsic to a non-abelian generalization. It is remarkable that full Lorentz invariance is regained in the continuum limit $N \to \infty$ of M(atrix)-theory without any need of reference to the DBI formulation.

1.2 The Geometric Origin of Field-Theory dualities

Charged D-brane configurations and their open-string excitations have proven useful at least in two respects. On the one hand they allow for a consistent description of BPS configurations continuously connected with charged extremal black-holes that have allowed for a microscopic derivation of the Beckenstein-Hawking entropy formula [10]. On the other hand they have been used to explore the geometric origin of dualities in some SYM theories.

The prototype of such configurations has been introduced by Hanany and Witten [20]. It consists of a set of parallel D3-branes suspended between two parallel N5-branes [21] in such a way that the 4D world-volume of the D3-branes has 3 non-compact dimensions in common with the 6D world-volume of the N5-branes. This configurations break 1/4 of the original supersymmetries, i.e. preserve $\mathcal{N} = 4$ supersymmetry in $D = 3$. That D3-branes can end on N5-branes is U-dual to the fact that D3-branes, that are $SL(2, Z)$ singlets, can end on D5-branes, that transform into N5-branes under $SL(2, Z)$. A gas of transversal D3-branes can be added that does not break any further supersymmetry. Some remarkable properties of $\mathcal{N} = 4$ SYM in $D = 3$ can be derived as a consequence of allowed movements of these configurations and of a simple rule of anomalous creation of D-
Performing T-duality one can pass to configurations of N5-branes, D4-branes and D6-branes that preserve $\mathcal{N} = 2$ supersymmetry in $D = 4$ \cite{23}. Once again allowed movements and anomalous creation of D-branes give rise to a remarkable understanding of certain dualities in $\mathcal{N} = 2$ SYM theories. By rotating one of the two N5-branes \cite{24} one can even pass continuously from a configuration with $\mathcal{N} = 2$ supersymmetry to a configuration with $\mathcal{N} = 1$ supersymmetry \cite{22}. On the (3+1) non-compact directions of the $N_4$ D4-branes the effective low-energy dynamics is governed by $U(N_4)$ SQCD with $N_6$ flavours. In practice the D6-branes are much heavier and their dynamics is so slow that the $U(N_6)$ Chan-Paton (CP) group plays the role of a global symmetry. Separating the D6-branes has a field theory analogue in the introduction of superpotential mass-terms. A similar correspondence may be established for the $\mathcal{N} = 2$ configurations. Seiberg’s duality \cite{20} follows from transporting the D6-branes past one of the two N5-branes. In the movement a certain number of D4-branes is created. Adding O-planes one can analyze orthogonal and symplectic CP groups \cite{23}. More recently the dynamics of many configurations of branes and planes has been shown to be equivalent to the world-volume dynamics of a BPS configuration of a single M5-brane of complicated topology \cite{27}. In this M-theory approach the hyperelliptic curves that appear in the determination of the Wilsonian action for $\mathcal{N} = 2$ SYM theories are simply the surfaces around which the M5-brane world-volume is wrapped in order to give an effective 4D dynamics.

So far the dynamics of D-branes and O-planes has been studied mostly from a macroscopic point of view. In the second part of the talk we will address the issue of a microscopic description of the excitations of D-branes and O-planes in curved backgrounds. For the sake of a quantitative analysis we will restrict our attention to much simpler configurations such as the background of $k$ coincident N5-branes \cite{23, 28, 29}. More interesting configurations represent a challenge for future work in the field.

Some of the consistency requirements that are present in type I vacuum configurations, such as tadpole cancellation \cite{30, 31, 3}, may be relaxed when the R-R charge can leak out at infinity. The non-compact 4D background transverse to $k$ coincident N5-branes admits an exact $\mathcal{N} = (4, 4)$ superconformal field theory (SCFT) description in terms of an $SU(2)$ WZNW model at level $k$ and a Feigin-Fuchs (FF) boson with background charge $Q = \sqrt{2/(k + 2)}$ \cite{21}. Moreover, as a consequence of the linear dilaton background in the throat region accessible to CFT techniques, all closed-string states become massive for any finite $k$ and the very definition of a tadpole for an off-shell state is questionable in perturbative string theory \cite{32}. Nevertheless the factorization properties of the tube amplitudes and the relation between direct (open-string) channel and transverse (closed-string) channel are restrictive enough to fix the parametrization of the open-string spectrum almost completely. Not counting T-dualities along possible compact world-volume directions of the N5-branes, there are two different parent type IIA configurations that admit the introduction of D-branes and O-planes. We will discuss only the simplest configuration and refer the interested reader to \cite{29} for the other configuration. The former however already shows the main features of the problem. The distinction between different kinds of D-brane is ambiguous if not impossible for any finite $k$ due to the strong curvature of the background. From the underlying CFT viewpoint the ambiguity manifests itself in the non-abelian structure of the fusion algebra. Another amusing result follows the in-
troduction of a magnetic field in the above non-trivial background. It induces a twisting of the $SU(2)$ current algebra in the open-string sector with the consequent breaking of target-space supersymmetry [33].

For the sake of comparison, in the first part of the talk, we will present some $D = 10$ models where unphysical tadpole cancellation go hand by hand with anomaly cancellation [34, 3]. On the contrary, though tree-level vacuum stability tends to favour backgrounds with no dilaton tadpole [32, 3], tadpoles for physical massless states, such as the dilaton, may be disposed of in principle via the Fischler-Susskind (FS) mechanism [34] and have no relation whatsoever to anomalies.

The plan of the talk is as follows. In the next section we describe the general strategy for deriving open-string descendants of Rational Conformal Field Theories (RCFT) and discuss some non-supersymmetric models in $D = 10$. Their relevance to some recently proposed dualities that involve non-supersymmetric interpolating string models [35] will be streamlined. We then pass to charged configurations and apply the general strategy and some useful results for open-descendants of $SU(2)$ WZW models [36] to describe the propagation of open and unoriented strings on a class of BPS configurations of N5-branes, D-branes and O-planes. Finally we present lines for future investigation.

# 2 Open-string Descendants of RCFT

The introduction of D-branes and O-planes in a type II background requires the identification of the corresponding boundary (B) and crosscap (C) states. Consistency conditions for boundary states in curved backgrounds have been recently discussed in [37]. The procedure in fact may be reversed. Starting from a consistent type II background and more generally from a consistent CFT that admit an automorphism $\Omega$ exchanging left and right movers one can deduce the open-string descendant and then extract the information on the admissible B and C states from the resulting brane configurations. We briefly describe the general procedure$^2$ and then specialize to some non-supersymmetric models in $D = 10$.

## 2.1 General Strategy

The starting point of the construction is a left-right symmetric modular invariant torus partition function [2, 3]. The characters of the chiral algebra, that extends the (super)Virasoro algebra,

$$\chi_h(q) = Tr_{\mathcal{H}_h} q^{L_0 - \frac{c}{24}}$$  \hspace{1cm} (1)

encode the (chiral) closed-string spectrum of the RCFT and enter the torus amplitude in a modular invariant way

$$T = \sum N_{hh} \chi_h \bar{\chi}_h$$  \hspace{1cm} (2)

$^2$For a more detailed analysis see e.g. [38] or the original literature [4, 39, 36] on the subject.
The $\Omega$-projection introduces O-planes that are accounted for by the Klein-bottle amplitude

$$K = \frac{1}{2} \sum N_{hh} \sigma_h \chi_h \quad .$$

(3)

The signs $\sigma_h$ are restricted by the crosscap constraint [40] that e.g. requires $\sigma_i \sigma_j = \sigma_k$ if $[i] \times [j] = N_{ij}^k[k]$. The most general parametrization of the open-string spectrum involves the annulus partition function:

$$A = \frac{1}{2} \sum A_{ab}^h n^a n^b \chi_h \quad .$$

(4)

and the Möbius strip projection

$$M = \frac{1}{2} \sum M_{aa}^h n^a \hat{\chi}_h \quad .$$

(5)

where $\hat{\chi}_h$ form a proper basis of hatted characters [3]

$$\hat{\chi}_h(i\tau_2 + 1/2) = \exp(-i\pi(h - c/24)) \chi_h(i\tau_2 + 1/2) \quad .$$

(6)

Though the CP indices $a$ and the character indices $h$ vary in general over different sets, for the charge-conjugation modular invariant one is allowed to take them in the same range and let $A_{ij}^k = N_{ij}^k$ or an automorphism thereof [3]. Sewing of surfaces with holes and crosscaps implies some consistency conditions on the above parametrization. Most notably $A_{i}^a A_{j}^c = N_{ij}^k A_{k}^b$ [40]. After switching to the transverse closed-string channel via a modular S-transformation, $K$ yields the crosscap-to-crosscap amplitude

$$\tilde{K} = \sum |\Gamma_h|^2 \chi_h \quad .$$

(7)

and $A$ yields the boundary-to-boundary amplitude

$$\tilde{A} = \sum (B^h)^2 \chi_h = \sum (B_a^h n^a)^2 \chi_h \quad .$$

(8)

Given these two amplitudes, a consistency check arises from the boundary-to-crosscap amplitude that must be of the form

$$\tilde{M} = \sum \Gamma_h (B_a^h n^a) \hat{\chi}_h$$

(9)

where the $\Gamma$’s hide some sign ambiguity. The modular transformation between loop and tree channel of the Möbius strip is induced by $P = T^{1/2} ST^2 ST^{1/2}$ that acts on hatted characters and satisfies $P^2 = C$ [3]. The boundary reflection coefficients $B_a^k$ satisfy polynomial equations [40] that in general constitute an overcomplete set that determines the allowed boundary states. In its naivest form the equations read

$$B_i^{(a)} B_j^{(a)} = N_{ij}^k B_k^{(a)}$$

(10)

where the index $a$ labels the independent solutions i.e. the independent CP multiplicities. One thus has two alternatives. Either one starts from the direct channel and determines the signs $\sigma_h$ in $K$ and the coefficients $A_{ab}^h$ in $A$ imposing consistency of the transverse channel, or solves the polynomial equations for the reflection coefficients $B$’s, parametrizing the
transverse channel amplitudes, and then requires that the direct channel be compatible with the CP multiplicities $n$’s being integers.

In order for un-orientifolds of RCFT’s to describe consistent open-string vacuum configurations two more requirements are to be met. The first is the correct relation between spin and statistics \[11]. The second is the cancellation of the tadpoles of unphysical massless states \[30, 31]. In the following we will show that while in supersymmetric theories tadpoles of physical, e.g. the dilaton, and unphysical closed-string massless states are not independent, in non-supersymmetric theories the two are often independent and anomalies are related only to the unphysical massless tadpoles.

### 2.2 Non-supersymmetric Models in $D = 10$

The simplest rational closed-string theory that admits an open-string descendant is the type IIB theory in $D = 10$. The result is the type I superstring with gauge group $SO(32)$ \[2]. In $D = 10$ there are two more left-right symmetric theories that admit open-string descendants \[3]. The parent closed-string theories are tachyonic and are termed type 0A and type 0B \[35], in order to display the lack of supersymmetry. These theories are non-geometrical orbifolds of the type IIA and type IIB superstrings with respect to the $Z_2$-projection generated by $(-)^{F_L + F_R}$. Up to irrelevant factors, the closed-string spectrum is encoded in

$$T_A = |O|^2 + |V|^2 + S\bar{C} + C\bar{S}$$

$$T_B = |O|^2 + |V|^2 + |S|^2 + |C|^2$$

where \{O, V, S, C\} are the characters of the $SO(8)$ transverse Lorentz current algebra at level one \[38]. The conventional Klein-bottle projections

$$K_A = O + V$$
$$K_B = O + V - S - C$$

do not remove the closed-string tachyons from the spectrum. The massless bosonic spectrum of the A-model contains the graviton and the dilaton in the NS-NS sector together with a vector and a 3-form potential in the R-R sector. One may thus expect charged Dp-branes with $p = 0, 2, 4, 6$. The massless bosonic spectrum of the B-model contains the graviton and the dilaton in the NS-NS sector together with two 2-form potentials in the R-R sector. One may thus expect two independent sets of charged Dp-branes each with $p = 1, 5, 9$. From the tree-channel Klein bottle amplitudes that reads

$$\tilde{K}_A = 32 \times (O + V)$$
$$\tilde{K}_B = 2 \times 32 \times V$$

one deduces that the O9-planes neither carry R-R charge in the A-model nor in the B-model. A similar analysis of the transverse annulus amplitudes reveals that no R-R charge neutrality conditions emerge from unphysical tadpoles in the A-model, consistently with the absence of D9-branes. On the contrary in the B-model there are two unphysical D9-brane tadpoles to cancel. The resulting CP groups are $SO(N) \times SO(M)$ and $SO(N)^2 \times$
The doubling of the CP symmetry is clearly related to the doubling of D9-branes! In the A-model the open-string spectrum includes tachyons in the adjoint or in the symmetric tensor representation, depending on an unconstrained sign in the Möbius strip, and non-chiral spinors in the representation \((\mathbf{N}, \mathbf{M})\). In the B-model the open-string spectrum includes tachyons in the representations \((\mathbf{N}, \mathbf{N}, \mathbf{1}, \mathbf{1})\) and \((\mathbf{1}, \mathbf{1}, \mathbf{M}, \mathbf{M})\), left spinors in the representations \((\mathbf{N}, \mathbf{1}, \mathbf{M}, \mathbf{1})\) and \((\mathbf{1}, \mathbf{N}, \mathbf{1}, \mathbf{M})\) as well as right spinors in the representations \((\mathbf{N}, \mathbf{1}, \mathbf{1}, \mathbf{M})\) and \((\mathbf{1}, \mathbf{N}, \mathbf{1}, \mathbf{M})\). The irreducible anomaly cancels thanks to the mirror-like structure of the fermion representations. The reducible part does not factorize and requires the contribution of both R-R antisymmetric tensors \(\mathbf{[42]}\). Notice that for \(N + M = 32\) the dilaton tadpole cancels and the flat Minkowski vacuum is stable with respect to the lowest order (genus one half) quantum corrections. We stress again that the dilaton tadpole has no relation whatsoever to the anomaly and in principle it can be disposed of through the Fischler-Susskind mechanism \(\mathbf{[34]}\) since the dilaton is a physical state of the projected unoriented closed-string spectrum.

Keeping in mind that in the covariant extension of the operator content of these models it is \(V\) that plays the role of the identity in the fusion algebra, one easily finds that there are other unconventional Klein-bottle projections compatible with the crosscap constraint \(\mathbf{[40]}\):

\[
\begin{align*}
\mathcal{K}_{A1} & = V - O \quad (17) \\
\mathcal{K}_{B1} & = V + O + S + C \quad (18) \\
\mathcal{K}_{B2} & = V - O - S + C \quad (19) \\
\mathcal{K}_{B3} & = V - O + S - C \quad (20)
\end{align*}
\]

The last two are equivalent under 10D parity. In the models A1 and B2 the closed-string tachyon is projected out. In the R-R sector one has a vector and a 3-form in the A1-model, two scalar and a non-chiral (with no fixed duality properties) 4-form in the B1-model and a scalar, a 2-form and a self-dual 4-form in the B2-model. One thus expects the following charged D\(p\)-branes. In the A1-model \(p = 0, 2, 4, 6\). In the B1-model two sets with \(p = -1, 3, 7\). Finally in the B2-model \(p = -1, 1, 3, 5, 7, 9\). Notice that up to uninteresting factors, the transverse channel amplitudes read

\[
\begin{align*}
\mathcal{\tilde{K}}_{A1} & = -32 \times (S + C') \quad (21) \\
\mathcal{\tilde{K}}_{B1} & = +2 \times 32 \times O \quad (22) \\
\mathcal{\tilde{K}}_{B2} & = -2 \times 32 \times S \quad (23) \\
\mathcal{\tilde{K}}_{B3} & = -2 \times 32 \times C \quad (24)
\end{align*}
\]

The A1-model is manifestly inconsistent with the direct channel, \(i.e.\) with 10D Lorentz invariance and the absence of D9-branes. The B2(B3)-model require D9-branes while in the B1-model the O9-planes do not carry R-R charge. The open-string spectrum of the B1-model displays a \(U(\mathbf{N}) \times U(\mathbf{M})\) CP symmetry and includes tachyons in the representations \((\mathbf{N}(\mathbf{N} \pm 1)/2 + \text{c.c.}, \mathbf{1})\) and \((\mathbf{1}, \mathbf{M}(\mathbf{M} \mp 1)/2 + \text{c.c.})\), left-handed fermions in the representation \((\mathbf{N}, \mathbf{M}) + \text{c.c.}\) and right-handed fermions in the representation \((\mathbf{N}, \mathbf{M}^* ) + \text{c.c.}\). The non-abelian part of the irreducible anomaly cancels thanks to the mirror-like structure of the fermion representations. The abelian part is made innocuous by the analogue of the Dine-Seiberg-Witten (DSW) mechanism \(\mathbf{[43]}\) that allows the abelian open-string vector
to become massive by its coupling to the R-R scalar. After the decoupling of the massive photon there is no left-over reducible anomaly. The open-string spectrum of the B2-model displays a $U(N) \times U(M)$ CP symmetry and includes tachyons in the representation $(N, M^*) + \text{c.c.}$, left-handed fermions in the representations $(N(N-1)/2 + \text{c.c.}, 1)$ and $(1, M(M+1)/2 + \text{c.c.})$ and right-handed fermions in the representation $(N, M) + \text{c.c.}$. Tadpole cancellation require $N - M = 32$. The anomaly is partly cancelled by the GSS mechanism and partly by the DSW mechanism. Notice that there is no choice of $N$ and $M$ that allows the cancellation of the dilaton tadpole. In principle it has to be disposed of via the FS mechanism.

The interest in these models is twofold. On the one hand they neatly illustrate the procedure and the relation between D-branes, CP charges and tadpoles in flat spacetime. On the other hand they play a significant role in some recently proposed string dualities without supersymmetry \[35\]. Most dualities have so far relied heavily in the tight constraints supersymmetry imposes on the spectra and interactions and one has the right to wonder if string dualities are a property to be associated to the existence of non-pointlike constituents or are better an unescapable property of the low-energy supergravity, that however clearly knows about the presence of branes. Though based on the continuous connection in $D = 9$ between non-supersymmetric and supersymmetric models, the dualities proposed in \[35\] represent the first instances of a much wider context. In particular from the open-string perspective only a soliton interpolating between $SO(16) \times SO(16)$ and $SO(32)$ seems to play a role in the duality pattern so far proposed \[35\]. The other consistent open-string models described above should eventually find a raison d’être that hopefully might provide a string interpretation for the $\mathcal{N} = (1, 0)$ supergravities with gauge groups $U(1)^{496}$ or $E(8) \times U(1)^{248}$ in much the same way as the strong coupling limits of the type IIA and $E(8) \times E(8)$ heterotic strings provide string interpretations for the $\mathcal{N} = 1$ supergravity in $D = 11$ \[6, 7\].

Undoubtedly the continuous connection between supersymmetric and non-supersymmetric string models may prove very useful both in the study of black hole thermodynamics, where extremal or nearly extremal solutions have attracted so far most of the attention \[10\], and in the study of non-supersymmetric YM theories, where the problem of color confinement and chiral symmetry breaking are still poorly understood by analytic means.

3 From Spheres to Pentabranes

Having described the general strategy to derive open-string descendants of left-right symmetric closed-string models based on RCFT and discussed some unconventional open-string models in $D = 10$ to highlight the relation with the by-now standard D-brane technology \[8\] and string dualities \[35\], we would now like to turn to charged configurations of D-branes where the RR charges may leak out at infinity and the very distinction between different Dp-branes becomes rather fuzzy due to strong curvature effects. Following \[29\] very closely, we will discuss open string propagation in the background of $k$ coincident NS pentabranes (N5-branes). N5-branes are string solitons with NS-NS magnetic charge and may be visualized as extended objects with a 5 + 1-dimensional worldvolume \[21\]. After setting to zero all the R-R fields, the background of type II N5-branes is completely
determined by
\[ ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2\phi}(dr^2 + r^2 ds^2_3) \] (25)
\[ e^{-2\phi} = e^{-2\phi_0}(1 + \frac{k}{r^2}) \] (26)
\[ H = dB = *de^{-2\phi} = -kd\Omega_3, \] (27)

where the indices \( \mu, \nu = 0, 1, 2, 3, 4, 5 \) are tangent to the N5-brane, \( ds^2_3 \) and \( d\Omega_3 \) are the line and volume elements on \( S^3 \), respectively. The geometry of the space transverse to the N5-brane is that of a semi-wormhole with the size of the throat fixed by the axionic charge \( k \) (number of coincident N5-branes). In the throat region, \( r \to 0 \), where the dilaton diverges, the N5-brane background (27) admits an exact CFT description as the tensor product of the \( SU(2) \) WZNW model at level \( k \) times a FF boson \( X_4 \) with background charge \( Q = \sqrt{2/k + 2} \) [21]. Including the fermionic partners \( \{\psi^i, \psi^4\} \), the world-sheet theory enjoys an extended \( \mathcal{N} = (4, 4) \) superconformal symmetry which guarantees the absence of both perturbative and non-perturbative corrections in \( \alpha' \) [21]. The complete construction of the modular invariant spectrum of closed-string excitations around the semi-wormhole background for even values of \( k \) has been worked out in [44], where other classes of 4-d backgrounds with exact \( \mathcal{N} = (4, 4) \) superconformal symmetry have been constructed.

Many other exact 4D backgrounds (generalized hyper-kählerian manifolds) and their T-duals have been analyzed in [15] in relation to non-compact Calabi-Yau manifolds and axionic instantons. Stringy ALE and ALF instantons and their behavior under Buscher duality [40] were thoroughly analyzed in [47]. More recently, string dualities in \( D = 6 \) have been given support by the observation that type IIA (B) with \( k \) coincident N5-brane is equivalent (Buscher T-dual) to type IIB (A) around an ALE space \( R^4/\Gamma_k \) [48] at vanishing B-field [15]. The crucial observation is that the (non)-chiral type IIB(A) admit N5-branes with \( \mathcal{N} = (2, 0) \), respectively \( \mathcal{N} = (1, 1) \), supersymmetry [21]. For the type IIB N5-brane one expects an \( \mathcal{N} = (1, 1) \) vector multiplet whose four scalar components are the collective coordinates for the translation of the N5-brane in the transverse 4D space. This fits in with the conjectured \( SU(2, Z) \) U-duality of the type IIB superstring in \( D = 10 \) which relates the N5-brane to the D5-brane, the world-volume degrees of freedom of the latter being open-string massless excitations in an \( \mathcal{N} = (1, 1) \) vector multiplet. On the contrary, the type IIA N5-brane requires an \( \mathcal{N} = (2, 0) \) tensor multiplet with 5 scalars, that are very suggestive of an 11D interpretation in terms of M-theory. Indeed the type IIA N5-branes are conjectured to arise from M5-branes by simple dimensional reduction. On the contrary M5-branes wrapped around the eleventh dimension should give rise to D4-branes. As a matter of fact, we will show that an open descendant of the type IIA N5-brane can be consistently derived. A previous analysis, that we have not been able to reconcile with the non-abelian structure of the fusion algebra of the underlying SCFT at \( k \neq 1 \), has been performed in [28] and partly anticipated in [23].

After an anomalous chiral rotation the world-sheet fermions \( \Psi^i \) decouple from the \( SU(2) \) currents and the level gets shifted \( k \to k - 2 \). The problem of un-orientifolding the N5-brane in an a priori flat spacetime can be directly mapped to the already solved problem of un-orientifolding the \( SU(2) \) WZNW models [30]. Indeed, the contributions of the FF boson \( X^4 \), the flat bosonic coordinates \( X^\mu \) and the fermions \( \{\psi^\mu, \psi^i, \psi^4\} \) to the
torus partition function is rather trivial. Neglecting the discrete representations that are finite in number and thus do not contribute to the partition function but play the role of screening charges in the computation of scattering amplitudes, the FF boson and each non-compact coordinate give the standard contribution \( (\sqrt{\tau_2}|\eta|^2)^{-1} \).

For N5-branes in a priori flat spacetime the torus partition function factorizes
\[
T = (V - S)(\bar{V} - \bar{C}) \sum_{ab} I_{ab} \chi_a \bar{\chi}_b
\]
where \( \{O, V, S, C\} \) have been introduced in eq. (12) \( I_{ab} \) is one of the \( A - D - E \) modular invariant combination of \( SU(2) \) characters [31]. Since \( I_{ab} \) are all left-right symmetric one can always perform an \( \Omega \)-projection. Notice that \( \Omega \) combines world-sheet left-right interchange with the \( SU(2) \) involutions \( g \to \pm g^{-1} \) and a flip of the chirality of the spinors \( S_8 \to C_8 \) [23]. This is consistent with the following observation. Since the volume of the throat of the wormhole is quantized in units of \( k \). One can use overall factors of \( k \) to trace the scaling of the open and unoriented amplitudes with the volume and identify the configurations of branes and planes in the large volume limit. For finite \( k \) the distinction between different Dp-branes becomes less compelling due to strong curvature effects. Let us anticipate the results for the two descendants of the \( A \)-series. We will see that for the parametrization with real CP charges, keeping the lowest lying states with \( a < \sqrt{k} \) the amplitudes are independent of \( k \) for large \( k \) plus subleading terms. This we interpret as an indication that the unorientifold introduces D6-branes and O6-planes in this case. For complex CP charges, one finds amplitudes that up to subleading terms behave as \( k \) for large \( k \). This we interpret as an indication that the unorientifold introduces D8-branes and O8-planes in this case. Tadpoles for the type IIA R-R 9-form and 7-form need not be cancelled because the RR charge can flow to infinity and moreover all the states in the closed-string spectrum in the throat region get a mass from the linear dilaton background.

Since the only non-trivial contribution to string propagation in the throat of the N5-brane is given by the \( SU(2) \) part, let us briefly recall for completeness some known facts about \( SU(2) \) WZNW models and their open-string descendants. Open string propagation on \( S^3 \) has been considered in connection to 2-d charged black holes [11]. The problem was thoroughly addressed along the lines of [3] and completely solved in [36]. The central charge of the Virasoro algebra is \( c = 3k/(k+2) \) and the conformal weights of the integrable unitary representations are
\[
h_j^{(k)} = \frac{j(j+1)}{k+2}
\]
with isospin \( j \) in the range \( j = 0, ..k/2 \). The generalized character formula is given by
\[
\chi_j^{(k)}(\tau, z, u) = Tr \mathcal{H}_j^{(k)} q^{L_0} e^{2\pi i z J_0^{(3)}} =
\]
\[
e^{2\pi i k u} \frac{2 e^{2\pi i k u j}}{\sin(\pi j z)} \sum_n q^{(k+2)n^2+(2j+1)n} \sin[\pi z(2j+1+2n(k+2))] \prod_{n=1}^{\infty} (1 - q^n) (1 - e^{2\pi i q^n})(1 - e^{-2\pi i q^n})
\]
For later purposes it is convenient to label states and characters in terms of the dimension of the corresponding highest weight \( SU(2) \) representations \( a = 2j + 1 \). The modular transformations in the above basis are represented by the matrices
\[
S_{ab} = \sqrt{\frac{2}{k+2}} \sin \left( \frac{\pi ab}{k+2} \right),
\]
and

\[ T_{ab} = \delta_{ab} e^{i \pi (\frac{a^2}{2(k+2)} - \frac{1}{4})}. \quad (33) \]

The charge conjugation matrix is equal to the identity \( C = S^2 = (ST)^3 = 1. \) The modular transformation between loop and tree channel of the Möbius strip is induced by \( P = T^{1/2} ST^2 ST^{1/2}, \) for the \( SU(2) \) WZNW model \( P \) is represented by

\[ P_{ab} = \frac{2}{\sqrt{k+2}} \sin \left( \frac{\pi ab}{2(k+2)} \right) (E_k E_{a+b} + O_k O_{a+b}), \quad (34) \]

where \( E_n \) and \( O_n \) are projectors on even and odd \( n \) respectively. The fusion rule coefficients are given by the Verlinde formula \[ N_{ab}^c = \sum_{d=1}^{k+1} S_{ad}^c S_{bd} S_{cd}^d, \quad (35) \]

\( N_{ab}^c \) are non-zero, in fact equal to one, only for \( |a - b| + 1 \leq c \leq \min(k+1, a + b - 1). \) It turns out to be convenient to introduce also the integer (!) coefficients

\[ Y_{ab}^c = \sum_{d=1}^{k+1} S_{ad}^c P_{bd} P_{cd}^d. \quad (36) \]

For simplicity of description let us restrict our attention to the open descendants of the diagonal modular invariant, associated to \( A_k, \) that is available at any level \( k. \) The torus partition function reads

\[ T = \sum_{a=1}^{k+1} |\chi_a|^2. \quad (37) \]

Corresponding to the two geometrical involution on the \( SU(2) \) group manifold, i.e. \( g \rightarrow \pm g^{-1}, \) there are two different \( \Omega \)- projections (Klein bottle amplitudes) of the parent torus partition function. The two involutions have the same action on integer isospins, that trivialize the center of \( SU(2). \) On the half-integer isospins, the former (−) involution corresponds to keeping the antisymmetric part (\( e.g. \) the singlets) of the diagonal \( SU(2) \) subgroup of the parent \( SU(2)_L \times SU(2)_R \) symmetry. The latter (+) keeps the symmetric states and removes e.g. the singlets. We shall label the two choices by an index \( R \) and \( C \) in order to streamline their relation to real (orthogonal or symplectic) and complex (unitary) CP charge assignments. The diagonal \( A_k \) models allow for the introduction of \( k + 1 \) CP charges \( n^i \) or equivalently \( k + 1 \) independent boundary states \( B^{(i)} \) that are in one to one correspondence with the integrable \( SU(2) \) representations \[ [34]. \]

For the \( A \)-series with real CP charges the Klein-bottle \( (K_R), \) annulus \( (A_R) \) and Möbius strip \( (M_R) \) direct ("loop") channel amplitudes read

\[ K_R = \sum_{a=1}^{k+1} Y_{11}^a \chi_a = \sum_{a=1}^{k+1} (-1)^{a-1} \chi_a, \quad (38) \]

\[ A_R = \sum_{a,b,c=1}^{k+1} N_{ab}^c \chi_c n^a n^b, \quad (39) \]

\[ M_R = \pm \sum_{a,b=1}^{k+1} Y_{ab}^c \chi b n^a = \pm \sum_{a,b=1}^{k+1} (-1)^{a-1} (-1)^{b-1} N_{ab} \chi b n^a. \quad (40) \]
A modular transformation yields the transverse ("tree") channel amplitudes that are consistent with their interpretation as closed-string amplitudes between boundary and/or crosscap states.

For the $A$-series with complex CP charges the various contributions to the direct channel partition function read

$$K_C = \sum_{a=1}^{k+1} Y^{a}_{k+1,k+1} \chi_a = \sum_{a=1}^{k+1} \chi_a ,$$

$$A_C = \sum_{a,b,d=1}^{k+1} N_{ab}^d \chi_{k+2-d} n^a b^b ,$$

$$M_C = \pm \sum_{a,b=1}^{k+1} Y_{a,k+1}^b \hat{\chi}_b n^a = \pm \sum_{a,b=1}^{k+1} N_{ab}^a \hat{\chi}_{k+2-b} n^a .$$

The transverse channel amplitudes are manifestly compatible with the required factorization properties. Notice that positivity of the transverse channel requires the numerical identifications $n_{k+2-a} = \bar{n}_a = n_a$.

Since all closed-string states are massive it is not needed and in fact plagued by ambiguities imposing the vanishing of any tadpole. It is questionable even to interpret the CP symmetry as a gauge symmetry since for any finite $k$ the open-string vector multiplets are massive too. Indeed, the spectrum is non-chiral and the R-R charge, if any, can leak out at infinity with no bearing to anomaly consistency conditions. It is worth stressing once again that the proper identification of the various Dp-branes and Op-planes involved is possible only in the large $k$ limit, where the regulating mass goes to zero too. This completes our discussion of the simplest configuration of N5-branes, D-branes and O-planes. The open-descendants of the other configuration alluded to in the introduction has been analyzed in [29] where it was interpreted as arising from placing the N5-branes at D-type orbifold singularities. For lack of space and for the unavoidable technicalities involved in the discussion of this case we refer the reader to [29]. For completeness we simply mention that the resulting open-string spectrum allows the presence of both massless and tachyonic states. Choosing the CP factors in a proper way one can get rid of the latter while keeping the former that seem to play the role of collective coordinates of the bound state of D-branes and O-planes in the highly curved background.

### 3.1 Adding a Magnetic Field

As in toroidal and orbifold compactifications of open strings [39, 15], in the N5-brane background the introduction of a constant abelian magnetic field [33] can be fully taken into account. In the case at hand it corresponds to the insertion on the boundary of the operator

$$\mathcal{B}^i = J^i + \frac{i}{2} \epsilon^{ijk} \psi^j \psi^k .$$

This boundary deformation of the rational CFT is integrable and one can express the open-string spectrum in terms of the characters (31) with $z$ related to the magnetic field.
\( B \) and the \( U(1) \) charges of the open-string state through
\[
z = \frac{1}{\pi} (\arctg(q_1 B) + \arctg(q_2 B)) \tag{45}
\]

By \( SU(2) \) symmetry one can always choose a \( B \) pointing along the third direction. From the modular \( S \)-transformation
\[
\chi_a^{(k)}(-\frac{1}{\tau}, -\frac{z}{\tau}, u - \frac{z^2}{2\tau}) = \sum_b S_{ab} \chi_b^{(k)}(\tau, z, u) \tag{46}
\]
one immediately deduces the Casimir energy and the shift of the modes of the currents \( J^{(\pm)}_N \rightarrow J^{(\pm)}_{N\pm z} \). Notice that, since the modes of \( J^{(3)} \) are unaffected, the current algebra is preserved
\[
\begin{align*}
[J^{(+)}_{n+z}, J^{(-)}_{m-z}] &= 2J^{(3)}_{n+m} + k\delta_{n+m} \tag{47} \\
[J^{(3)}_n, J^{(\pm)}_{m\pm z}] &= \pm J^{(\pm)}_{n+m\pm z} \tag{48} \\
[J^{(3)}_n, J^{(3)}_m] &= \frac{k}{2}\delta_{n+m} \tag{49}
\end{align*}
\]
Indeed the introduction of the magnetic field simply amounts to a modulation of the boundary reflection coefficients. By world-sheet supersymmetry considerations an opposite shift of the modes is suffered by the fermions. However the total \( \mathcal{N} = 1 \) supercurrent, \( i.e. \) the one which couples to the worldsheet gravitino,
\[
G = J^i\psi_i + i\partial X^4\Psi_4 + \frac{i}{3!}\epsilon^{ijk}\Psi_i\Psi_j\Psi_k + Q\partial\Psi_4 \tag{50}
\]
seems to forbids a twist of \( \Psi_4 \) (and similarly of \( X^4 \)) due to the presence of the background charge. The twisting of only two currents and two fermions leads to an explicit breaking of the spacetime supersymmetry. Since the curvature of the spin connection with torsion is self-dual one may ask if there is any possibility of adding a self-dual field-strength, \( i.e. \) an instanton-like gauge field such as to make the background supersymmetric. A possibility of this kind is reminiscent of the standard embedding in the heterotic version of the N5-brane \([21, 47]\). This issue clearly deserves further study. It may also prove interesting to explore its generalization to compact curved background such as orbifolds, Gepner models and fermionic models \([15]\). The final goal would be to address the issue of consistency of magnetized D-branes inside Calabi-Yau (CY) manifolds \([52]\).

4 Final Comments

Real progress in understanding the dynamics of D-branes and O-planes in curved background may well benefit from exactly solvable CFT such as those described in \([13]\) or the string solitons described above. D-brane instanton corrections to superstring effective lagrangians clearly require a precise understanding of the weighting and counting of D-branes wrapped around non-trivial cycles in CY manifolds \([52]\). Explicit computations in this direction seem still rather difficult to perform.
Recently, D-branes and their open-string excitations have proven to be useful tools not only in the counting of microscopic degrees of freedom of supersymmetric black-holes [11] but also in the geometric engineering of SYM theories [18, 17, 23, 25]. In this context the dynamics of D-branes [20] and orientifold planes [23] in the background of symmetric penta-branes [21] leads to the anomalous creation of strings and branes [22] and provides a geometrical interpretation of some known dualities in SYM theories [26]. A unifying picture emerges from [27], where the relevant configurations of N5-branes and D-branes are interpreted as a single M5-brane wrapped around a Riemann surface.

Moreover, the very consistency of M-theory [4] and in particular of its Matrix-Theory interpretation [19] requires analyzing compactifications on curved backgrounds that break some or all the supersymmetries. In this respect the interpolating solitons studied in [35] provide a first step towards a complete understanding of the consistent non-supersymmetric strings [1, 3] that should play a role in the final non-perturbative scenario. The lesson that we are learning from the fruitful interplay between gauge fields, strings and branes is that any not manifestly inconsistent theory, such as the resurrected 11D supergravity or the still unexplored 10D supergravity with gauge group $U(1)^{496}$, deserves proper attention and is not to be prematurely cut by means of Ockam’s razor [53].

5 Acknowledgements

The second part of this talk is based on work done with Yassen Stanev that I would like to thank for an enjoyable collaboration. I would like to acknowledge useful conversations with C. Angelantonj, E. Kiritsis, G. Pradisi, S.-J. Rey, and A. Sagnotti and a fruitful discussion concerning Ockam’s razor with G. Preparata. I would also like to express my deep gratitude to the local organizers of the V Korean - Italian Meeting on Relativistic Astrophysics and particularly to Hyung-Won Lee for offering their kind hospitality and for creating a stimulating environment. Financial support has been provided by the KOSEF-CNR bilateral agreement.

References

[1] M. Green, J. Schwarz and E. Witten, _Superstring Theory_, Cambridge University Press, 1987.

[2] A. Sagnotti, in _Non-Perturbative Quantum Field Theory_, eds. G. Mack et al. (Pergamon Press, 1988), p. 521.

[3] M. Bianchi and A. Sagnotti, _Phys. Lett._ 247B (1990) 517; _Nucl. Phys._ B361 (1991) 519.

[4] N. Seiberg and E. Witten, _Nucl. Phys._ B426 (1994) 19; _ibid._ B431 (1994) 484.

[5] For a recent summary see _e.g._ S-J. Rey, in these Proceedings of the V Korean - Italian Meeting on Relativistic Astrophysics, Seoul - Suanbo, september 1997.
[6] E. Witten, *Nucl. Phys.* B**443** (1995) 85.

[7] P. Hořava and E. Witten, hep-th/9510209.

[8] J. Polchinski, hep-th/9510017; J. Polchinski, S. Chauduri and C. Johnson, *Notes on D-branes*, hep-th/9602052.

[9] M. Duff, R. Khuri, J. Lu, *Phys. Rep.* **259** (1995) 213.

[10] For a recent summary see J. Maldacena, hep-th/9705078 and references therein.

[11] P. Hořava, *Nucl. Phys.* B**327** (1989) 461, *Phys. Lett.* B**231** (1989) 251.

[12] For an elementary introduction see e.g. M. Bianchi, in the Proceedings of the IV Korean - Italian Meeting on Relativistic Astrophysics, Rome - Gran Sasso - Pescara, July 1995, *Nuovo Cim.* 112B (1997) 149.

[13] J. Polchinski and E. Witten, *Nucl. Phys.* B**460** (1996) 525.

[14] A. Dabholkar, *Phys. Lett.* B**357** (1995) 307; C. Hull, *Phys. Lett.* B**357** (1995) 545; A. Tseytlin, *Phys. Lett.* B**367** (1996) 84; *Nucl. Phys.* B**467** (1996) 383.

[15] C. Angelantonj, in these Proceedings of the V Korean - Italian Meeting on Relativistic Astrophysics, Seoul - Suanbo, September 1997.

[16] M. Bianchi, in the Proceedings of the XI SIGRAV Meeting on Gravitational Physics, Rome, September 1996.

[17] M. Douglas and M. Li, hep-th/9604041.

[18] E. Witten, hep-th/9510135, *Nucl. Phys.* B**460** (1996) 335.

[19] T. Banks, W. Fischler, S. Shenker and L. Susskind, hep-th/9610043.

[20] A. Hanany and E. Witten, hep-th/9611230.

[21] C. Callan, J. Harvey and A. Strominger, *Nucl. Phys.* B**359** (1991) 611; S.-J. Rey *Phys. Rev.* D**43** (1991) 526.

[22] C. Bachas, M. Douglas and M. Green, hep-th/9705074; O. Bergmann, M. Gaberdiel and G. Lyschyts hep-th/9705130; U. Danielsson, G. Ferretti and I. Klebanov, hep-th/9705084.

[23] N. Evans, C. Johnson and A. Shapere, hep-th/9703210; C. Johnson, hep-th/9705148; hep-th/9706135.

[24] J. Barbon, hep-th/9703051.

[25] S. Elitzur, A. Giveon and D. Kutasov, hep-th/9702014.

[26] N. Seiberg, hep-th/9411149; P. Argyres, R. Plesser and N. Seiberg, hep-th/9603042.

[27] E. Witten, hep-th/9703166.
[28] S. Forste, D. Ghoshal and S. Panda, hep-th/9706057.
[29] M. Bianchi and Ya. Stanev, hep-th/9711069.
[30] Y. Cai and J. Polchinski, Nucl. Phys. B376 (1991) 365.
[31] G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59.
[32] M. Bianchi and A. Sagnotti, Phys. Lett. B211 (1989) 411.
[33] A. Abouelsaood, C. Callan, C. Nappi, S. Yost, Nucl. Phys. B280 (1987) 599; C. Bachas and M. Porrati, Phys. Lett. B296 (1992) 77; C. Bachas, in String Theory, Quantum Gravity and the Unification of Fundamental Interactions, eds. M. Bianchi et al. (World Scientific, Singapore 1993) p.16; hep-th/9503030.
[34] W. Fischler and L. Susskind, Phys. Lett. B171 (1986) 383, ibid. B173 (1986) 262.
[35] J. Blum and K. Dienes, hep-th/9707148; hep-th/9707160.
[36] G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B354 (1995) 279; B356 (1995) 230.
[37] H. Ooguri, Y. Oz and Z. Yin, hep-th/9606112; K. Hori and Y. Oz, hep-th/9702173.
[38] G. Pradisi, in the Proceedings of the IV Korean - Italian Meeting on Relativistic Astrophysics, Rome - Gran Sasso - Pescara, july 1995, Nuovo Cim. 112B (1997) 467.
[39] M. Bianchi, G. Pradisi and A. Sagnotti, Nucl. Phys. B376 (1991) 365; Phys. Lett. B273 (1991) 389.
[40] D. Fioravanti, G. Pradisi and A. Sagnotti, Phys. Lett. B321 (1994) 349; G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B381 (1996) 97.
[41] M. Bianchi and A. Sagnotti, Phys. Lett. B231 (1990) 389.
[42] M. Green and J. Schwarz, Phys. Lett. B149 (1984) 117; A. Sagnotti, Phys. Lett. B294 (1992) 196.
[43] E. Witten, Phys. Lett. B149 (1984) 351; N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 589.
[44] I. Antoniadis, S. Ferrara, K. Kounnas, Nucl. Phys. B421 (1994) 343; E. Kiritsis and K. Kounnas, hep-th/9508078.
[45] E. Kiritsis, K. Kounnas and D. Lüst, Int. J. Mod. Phys. A9 (1994) 3007.
[46] T. Buscher, Phys. Lett. B201 (1988) 466.
[47] D. Anselmi, M. Billó, P. Fré, L. Giraradello and A. Zaffaroni, *Int. J. Mod. Phys.* **A8** (1993) 2351; in *String Theory, Quantum Gravity and the Unification of Fundamental Interactions*, eds. M. Bianchi et al. (World Scientific, Singapore 1993) p.28; M. Bianchi, F. Fucito, M. Martellini and G.C. Rossi, *Nucl. Phys.* **B440** (1995) 129; *ibid.* **B473** (1996) 367.

[48] H. Ooguri and C. Vafa, [hep-th/9511164](http://arxiv.org/abs/hep-th/9511164).

[49] P. Aspinwall, [hep-th/9508154](http://arxiv.org/abs/hep-th/9508154).

[50] A. Cappelli, J. C. Itzykson and J. B. Zuber, *Comm. Math. Phys.* **B113** (1987) 1.

[51] E. Verlinde, *Nucl. Phys.* **B300** (1988) 360; J. Cardy, *Nucl. Phys.* **B324** (1989) 581.

[52] K. Becker, M. Becker and A. Strominger, *Nucl. Phys.* **B456** (1995) 130.

[53] W. Ockam, *Summa Totius Logicae*. For the opposite viewpoint see G. Preparata, in these Proceedings of the V *Korean - Italian Meeting on Relativistic Astrophysics*, Seoul - Suanbo, September 1997, [hep-th/9711198](http://arxiv.org/abs/hep-th/9711198).