A multifrequency approach of the cosmological parameter estimation in the presence of extragalactic point sources

D. Paoletti,1,2* N. Aghanim,3* M. Douspis,3* F. Finelli,1,2* G. De Zotti,4,5* G. Lagache3* and A. Pénin3*

1INAF – IASF Bologna, Istituto di Astrofisica Spaziale e Fisica Cosmica di Bologna Istituto Nazionale di Astrofisica, via Gobetti 101, I-40129 Bologna, Italy
2INFN, Sezione di Bologna, Via Irnerio 46, I-40126 Bologna, Italy
3IAS Institut d’Astrophysique Spatiale Bât. 121 Université Paris Sud 11 & CNRS, 91405 Orsay Cedex, France
4INAF, Osservatorio Astronomico di Padova, Vicoletto Osservatorio 5, I-35122 Padova, Italy
5SISSA, Via Bonomea 265, I-34136 Trieste, Italy

Accepted 2012 July 16. Received 2012 June 18; in original form 2011 December 14

ABSTRACT

We present a multifrequency approach which optimizes the constraints on cosmological parameters with respect to extragalactic point source and secondary anisotropy contamination on small scales. We model with a minimal number of parameters the expected dominant contaminations in intensity, such as unresolved point sources and the thermal Sunyaev–Zel’dovich effect. The model for unresolved point sources, either Poisson distributed or clustered, uses data from Planck early results. The method presented is the first one where the models of the point-source contributions are based on the Planck early data. To reduce the number of parameters necessary to characterize the residuals, the method uses the knowledge of the frequency dependences of the residual signals coming from the data. The dependences are directly included in the parametrizations, allowing us to reduce the number of the residual parameters to the minimum of three. The overall three amplitudes of the residual contributions are included in a Markov chain Monte Carlo analysis for the estimate of cosmological parameters. We show that our method is robust: as long as the main contaminants are taken into account, the constraints on the cosmological parameters are unbiased regardless of the realistic uncertainties on the contaminants. Although general, the method is applied only to Planck.

Key words: cosmic background radiation – cosmological parameters.

1 INTRODUCTION

The cosmic microwave background (CMB) anisotropy data constitute a fundamental tool to test the standard cosmological model and its extensions. In particular, small-scale CMB anisotropies have a great importance in cosmology and are one of the current frontiers in CMB observations. Many experiments have been dedicated to the observations of small-scale anisotropies. The Planck satellite (Planck Collaboration 2006) covers all the scales up to \( \ell \sim 2500 \) but there are also ground-based experiments observing smaller regions of the sky but with higher angular resolutions, such as ACBAR (Reichardt et al. 2009), CBI (Redhead et al. 2004), QUaD (QUAD Collaboration 2009), SPT (Lueker et al. 2010) and ACT (Das et al. 2011). At these scales, the Silk damping suppresses the primordial CMB contribution with respect to extragalactic contamination and secondary anisotropies. These contaminations affect the constraints on cosmological parameters derived from small-scale data introducing biases and therefore it is necessary to properly account for them in the data analysis. We present an approach which has been developed to optimize, with respect to the contamination by point-source contributions, the constraints on cosmological parameters which could be obtained from high-resolution multifrequency data.

The problem of extragalactic sources and secondary anisotropies contaminating the angular power spectrum and consequently the cosmological parameters has already been addressed in several recent papers (Douspis, Aghanim & Langer 2006; Serra et al. 2008; Taburet et al. 2008; Taburet, Aghanim & Douspis 2010; Millea et al. 2011; Addison et al. 2012; Efstathiou & Migliaccio 2012), together with the specific tools developed by the data analysis groups of the most recent small-scale experiments (Dunkley et al. 2011; Keisler et al. 2011; Reichardt et al. 2012).

The approach we present in this paper consists of modelling the astrophysical contributions to the angular power spectrum with...
Table 1. Comparison between the resulting best fit and the input values for the cosmological and extragalactic parameters. The fiducial model for the mock data is reported in the fourth column. Second column: parameters recovered when taking into account the nominal contributions of the point sources and TSZ signals. Third column: parameters recovered when varying also the parameters of the contributions of the point sources and TSZ signals (note that the $A_{SZ}$ remains unconstrained). The CMB plus extragalactic contribution power spectrum obtained using the best-fitting values and the one generated using the fiducial model values do not differ by more than 0.2 per cent.

| Parameter | Best-fitting value I | Best-fitting value II | Input value |
|-----------|----------------------|-----------------------|-------------|
| $\Omega_b h^2$ | 0.02251 ± 0.0001 | 0.02254 ± 0.0001 | 0.0225 |
| $\Omega_{DM} h^2$ | 0.1121 ± 0.0011 | 0.1122 ± 0.0041 | 0.112 |
| $\tau$ | 0.0885 ± 0.0004 | 0.0874 ± 0.0004 | 0.088 |
| $n_s$ | 0.967 ± 0.0003 | 0.966 ± 0.004 | 0.967 |
| $\log [A_s 10^{10}]$ | 3.19 ± 0.0007 | 3.19 ± 0.0007 | 3.19 |
| $H_0$ | 69.96 ± 0.5 | 69.94 ± 0.54 | 70 |
| $A_{PS}$ | – | 0.99 ± 0.17 | 1 |
| $A_{CL}$ | – | 1.01 ± 0.22 | 1 |
| $A_{SZ}$ | – | – | 1 |

minimal parametrizations, where minimal relates to the number of parameters necessary to describe the signals, and considering the frequency dependences of the different contributions. This is the most important difference with respect to previous approaches (Dunkley et al. 2011; Keisler et al. 2011; Millea et al. 2011; Addison et al. 2012; Reichardt et al. 2012). These parametrizations are included as additional contributions to the primary CMB in the Markov chain Monte Carlo (MCMC) analysis to constrain cosmological parameters. The parameters characterizing the astrophysical contributions are included in the analysis together with the cosmological ones. As described in the following, we focus on the following frequencies: 70, 100, 143, 217 and 353 GHz. We use the available data from Planck early results (Planck Collaboration 2011a,b) to derive the parametrizations to describe the astrophysical signals. When no data are available, we rely on predictions from theoretical models and empirical simulations.

In the following, we use a fiducial cosmological model with the parameters from Komatsu et al. (2011) which are reported in the third column of Table 1. The polarization from point sources and Sunyaev–Zel’dovich (SZ) effect is negligible (Sazonov & Sunyaev 1999; Tucci et al. 2005; Niemack et al. 2010; Tucci & Toffolatti 2012); we thus do not take into account any residual contribution to the polarization autospectrum EE and temperature-polarization cross-spectrum TE in this analysis (the polarization autospectrum BB is always neglected here). We also do not account for residual signal, in intensity or in polarization, from our Galaxy.

The paper is organized as follows. In Section 2, we describe the astrophysical signals and we derive the parametrizations of their contributions to the angular power spectrum. In Section 3, we describe the analysis method and the frequency channel combinations. In Section 4, we present the results of the MCMC analysis for the cosmological and astrophysical parameters. In Section 5, we present the discussion and conclusions.

2 ASTROPHYSICAL CONTAMINATIONS

One of the major contributions of astrophysical contamination on small scales, and in the regions of the sky clean from the Galactic emission, are discrete sources and in particular galaxy clusters and extragalactic point sources. The former contribute through the SZ effect (Sunyaev & Zel’dovich 1970, 1972, 1980). We have considered only the contribution of the thermal SZ (TSZ) effect since the kinetic SZ effect from galaxy clusters has a much smaller amplitude at the angular scales considered in this work (da Silva et al. 2001). For the point-source contamination, we considered different contributions depending on the frequency. We aim in this section at parametrizing the different contributions by reducing to their minimum the number of parameters necessary to characterize the signals and including the frequency dependence in these parameters.

2.1 Thermal Sunyaev–Zel’dovich effect

The TSZ effect is a local spectral distortion of the CMB given by the interactions of CMB photons with the electrons of the hot gas in galaxy clusters (Sunyaev & Zel’dovich 1970, 1980) encountered during the propagation from the last scattering surface.

The TSZ contribution to the CMB signal strongly depends on the cosmological model and on the cluster distribution through the dark matter halo mass function. It also depends on the cluster properties through the projected gas pressure profile. The TSZ angular power spectrum can be computed using these two ingredients (e.g. Komatsu & Seljak 2002; Aghanim, Majumdar & Silk 2008). It was shown in Taburet et al. (2008) that the SZ contribution to the signal can be analysed in terms of a residual contribution to the total signal when the detected clusters are masked out from the maps, or in contrast the TSZ signal can be analysed together with the CMB one without taking out the detected clusters, providing the total SZ power spectrum is properly modelled (see Taburet et al. 2010). The latter approach is the one that is used in the CMB experiments such as WMAP (Komatsu et al. 2011), ACT (Das et al. 2011; Dunkley et al. 2011) and SPT (Lueker et al. 2010; Keisler et al. 2011; Reichardt et al. 2012).

We modelled the TSZ spectrum using the parametrization derived in Komatsu & Seljak (2002). It considers a template for the spectral shape, $\tilde{C}_\ell$, and a normalization which depends on the parameters $\sigma_8$ and $\Omega_c h^2$. The parametrization we chose to use is then

$$C_\ell^{TSZ}(\nu) = A_{SZ} g(\nu) \sigma_8^2 \Omega_c h^2 \tilde{C}_\ell,$$

where $A_{SZ}$ accounts for the uncertainty in the normalization. As discussed in Taburet et al. (2010), this parametrization does not bias cosmological parameters and at the same time is a physical representation of the signal. Thanks to the scaling with $\sigma_8$, the TSZ signal of this model depends on the cosmological model and gives a physical representation of the signal connected with the basic cosmological parameters. This connection may improve the sampling of the parameter space. The frequency dependence of the signal for the TSZ is given by the analytical function $g(\nu)$.

In the following, we used a fiducial TSZ power spectrum shape computed using the Tinker et al. (2008) mass function and a pressure profile provided in Arnaud et al. (2008).

2.2 Point-source contribution

An experiment such as Planck with its moderate angular resolution is not optimized for point-source detection. However, Planck, thanks to its full sky coverage and its nine frequencies, is detecting a large number of sources with the detection thresholds reported in Table 2 and taken from Planck Collaboration (2011b). For the CMB

\[ g(\nu) = \left( \frac{\nu}{5000.0} + 4 \right)^{\sigma_8/\Omega_c h^2} - 4 \]
Table 2. Poissonian contributions to the angular power spectrum for each frequency. First column: frequency channels; second column: the detection threshold for each channel; third column: values of the Poissonian angular power spectrum for radio sources obtained using the composite number counts; fourth column: values of the Poissonian angular power spectrum for IR sources as predicted in Planck Collaboration (2011b).

| Channels   | $S_{\text{max}}$ (Jy) | $C_{\text{Pois, Radio}}$ (μK$^2$) | $C_{\text{Pois, Original}}$ (μK$^2$) | $C_{\text{Pois, IR}}$ (μK$^2$) |
|------------|----------------------|-------------------------------|-------------------------------|-----------------------------|
| 70 GHz     | 0.57                 | 1.13 × 10^{-3}                | 1.08 × 10^{-3}                |                              |
| 100 GHz    | 0.41                 | 2.05 × 10^{-4}                | 2.25 × 10^{-4}                |                              |
| 143 GHz    | 0.25                 | 4.52 × 10^{-5}                | 4.86 × 10^{-5}                | 1.2 ± 0.2 × 10^{-5}          |
| 217 GHz    | 0.16                 | 1.60 × 10^{-5}                | 1.74 × 10^{-5}                | 6 ± 1 × 10^{-5}             |
| 353 GHz    | 0.33                 | 9.40 × 10^{-5}                | 1.03 × 10^{-4}                | 1.9 ± 0.3 × 10^{-3}         |

In the following, we describe the different contributions from point sources to the CMB anisotropy, we detail the proposed parametrizations and then provide the associated angular power spectrum.

2.3 Poissonian contribution

The Poissonian contribution of point sources is given by their random distribution in the sky. The coefficients of spherical harmonics of a Poissonian distribution of a population of sources in the sky with flux density $S$ are (Tegmark & Efstathiou 1996)

$$\langle a_{\ell m} \rangle = \begin{cases} \sqrt{4\pi} \bar{n} S & \text{for } \ell = 0, \\ 0 & \text{for } \ell \neq 0, \forall m, \end{cases}$$

where $\bar{n} = N/4\pi$ is the mean number of sources per steradian with flux density $S$. The angular power spectrum is given by $C_\ell = \langle |a_{\ell m}|^2 \rangle - |\langle a_{\ell m} \rangle|^2 = \pi S^2$, which can be generalized to $C_\ell = \Sigma \bar{n} S^2$ if we consider sources with different flux densities. For a continuum flux density up to the detection threshold, $S_{\text{max}}$, the angular power spectrum for a Poissonian distribution of sources in sky is given by a flat $C_\ell$:

$$C_{\ell,\text{Pois}} = \int_0^{S_{\text{max}}} \frac{dN(S)}{dS} S^2 dS,$$

where $dN(S)/dS$ are the source number counts. The flat spectral shape of the Poissonian contribution makes it easy to parametrize: it requires a simple amplitude term. One possible choice of parametrization is to use a different free parameter for the amplitude at each frequency (Efstathiou and Gratton, private communication). This would require in our case for the radio sources alone five parameters for the Poisson term. Since we aim at reducing the number of parameters to the minimum, we construct a single amplitude parameter for the total Poisson contribution and a fixed frequency dependence. This frequency dependence is complicated by the dependence on the detection threshold $S_{\text{max}}$ and thus no analytical expression can be constructed. We therefore use current data and galaxy evolution models to derive the fiducial amplitude and frequency dependence.

Both radio and IR galaxies contribute with a Poissonian term. We treated the former by fitting the number counts and then integrating on the flux density, whereas for the latter we used the available results from Bethermin et al. (2011) and Planck Collaboration (2011b). The two contributions are then summed in a single Poissonian term. The expected Poissonian contribution for each frequency can be computed using the number counts given a detection threshold. We used the predicted number counts for radio galaxies from the model of De Zotti et al. (2005, 2006). It is a good representation of the low flux density part of the observed number counts but the comparison with the Planck Early Release Compact Source Catalogue (ERCSC) showed a bias at high flux densities with the De Zotti et al. model overpredicting the counts (Planck Collaboration 2011a; Tucci et al. 2011). Since the ERCSC number counts are limited to bright flux densities, we constructed composite number counts combining the De Zotti model predicted counts, for lower flux densities roughly below 0.2–0.3 Jy (dots in Figs 1 and 2), whereas for flux densities greater than 1 Jy we used the actual Planck measurements reported in Planck Collaboration (2011a) (empty circles in Fig. 1).
Cosmological parameter estimation

Figure 1. Fits of the hybrid model/data number counts. The solid line is the empirical fit, whereas the dots represent the De Zotti et al. model number counts and the empty circles the Planck data for radio galaxies. The three curves under the fit are the three contributions (Low, Med and High, respectively, dotted, dashed and dot--dashed lines) which sum in the total fit. From top to bottom: 70, 217 and 353 GHz.

At 353 GHz, the radio sources are subdominant with respect to IR sources. In order to compute the associated Poisson term for the 353 GHz channel, we therefore used the predictions of the De Zotti et al. model that we rescaled to account for its likely overprediction, similar to that at 143 and 217 GHz. In practice, we have extrapolated the correction factors for the radio galaxy Poissonian contribution to the angular power spectrum of the 143 and 217 GHz channels.

The obtained number counts\(^1\) are represented for each frequency by the weighted sum of three different contributions: one at low \(F_\text{Low}^\nu\), one at intermediate \(F_\text{Med}^\nu\), and one at high \(F_\text{High}^\nu\) flux densities.

\[
S^{-2.5} \frac{dN(S)^{\text{tot}}}{dS_{\nu}} = \Sigma_{i=\text{Low,Med,High}} w_i^\nu F_i^\nu.
\]

Each term \(F_i^\nu\) writes as \(F_i^\nu = S^{-2.5} \frac{dN(S)^{i}}{dS_{\nu}}\), where the shape has been derived with empirical fits. For each term and each frequency, the exponents \(\alpha\) and \(\beta\) and the coefficients \(A\), \(B\) and \(C\), together with the weights of the sum \(w_i^\nu\), are all adjusted by comparing them to the composite number counts.

Table 3 summarizes the set of parameters used to construct the composite source counts and the fit to the hybrid model/data counts is shown as the thin line in Figs 1 and 2. We note the very good agreement between the analytical fit and the theoretical/measured points.

In Fig. 3, we compare our hybrid number counts to those of Tucci et al. (2011) based also on the ERCSC. Our fit overpredicts the number counts in the range 0.01–1 Jy with respect to the Tucci et al. model. This overprediction is due to the fact that we followed the De Zotti et al. model up to a flux density of 0.01 Jy, beyond which we modified the curve to fit the data at higher flux densities, whereas Tucci et al. number counts start to differ from the De Zotti et al. predictions at flux densities lower than 0.01 Jy. The number counts for radio galaxies at lower flux densities with respect to Planck have been measured also by recent experiments such as SPT. In particular, in Fig. 4 we show the comparison of our fits and theoretical models with the SPT results for 143 GHz (150 GHz for SPT) and 217 GHz (220 GHz for SPT; Vieira et al. 2010). We note how the empirical fits are in very good agreement with SPT number counts.

Using the obtained composite counts, we compute the Poissonian power spectra at the considered frequencies. They are given by

\[dN(S)^{i}/dS_{\nu} \propto S^{-2.5} \]
Table 3. The exponents $\alpha$, $\beta$ and the coefficients $A$, $B$, $C$ and $w_\nu$ for each term of the fits of the radio galaxies composite number counts.

| Channel (GHz) | 70 | 100 | 143 | 217 | 353 |
|--------------|----|-----|-----|-----|-----|
| $A^{\text{Low}}$ | 8.06 | 7.76 | 7.47 | 7.16 | 6.81 |
| $B^{\text{Low}}$ | $10^{14}$ | $10^{14}$ | $10^{14}$ | $10^{14}$ | $10^{14}$ |
| $C^{\text{Low}}$ | 1 | 1 | 1 | 1 | 1 |
| $\alpha^{\text{Low}}$ | 0.45 | 0.45 | 0.45 | 0.45 | 0.45 |
| $\beta^{\text{Low}}$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| $w_\nu^{\text{Low}}$ | 4.4 | 4.0 | 5.0 | 7.5 | 4.5 |
| $A^{\text{Med}}$ | 306.98 | 224.99 | 241.08 | 203.69 | 165.85 |
| $B^{\text{Med}}$ | 20408 | 40000 | 111111 | 308642 | 501187 |
| $C^{\text{Med}}$ | 1 | 1 | 1 | 1 | 1 |
| $\alpha^{\text{Med}}$ | 0.75 | 0.74 | 0.75 | 0.73 | 0.7 |
| $\beta^{\text{Med}}$ | 2.0 | 2.0 | 2.0 | 2 | 1.9 |
| $w_\nu^{\text{Med}}$ | 2.3 | 2.3 | 2.7 | 2.7 | 3.8 |
| $A^{\text{High}}$ | 58.41 | 22.66 | 70.89 | 23.12 | 51.17 |
| $B^{\text{High}}$ | 1.0 | 0.91 | 3.52 | 1.48 | 0.93 |
| $C^{\text{High}}$ | 1 | 0.25 | 1 | 0.29 | 1 |
| $\alpha^{\text{High}}$ | 0.75 | 0.85 | 0.8 | 0.85 | 0.75 |
| $\beta^{\text{High}}$ | 1.45 | 1.0 | 1.2 | 1.1 | 1.5 |
| $w_\nu^{\text{High}}$ | 1.0 | 1.0 | 1.04 | 1.0 | 1.03 |

following expressions:

$$C^{\text{RadioPois}}_\nu = \sum_{i=1,3} P_{i,\nu} S_{\text{max},i}(\nu) \times 2F1[a_i, \nu; b_i, \nu; c_i, \nu; d_i, \nu S_{\text{max},i}(\nu)],$$

(2.4)

where $2F1$ are the hypergeometric functions of second type (Abramowitz & Stegun 1965), the sum on $i$ is over the three contributions Low, Med and High, and the coefficients and exponents depend on the frequency and are reported in Table 4. The values of the Poissonian contribution to the angular power spectrum that we obtained from our fitted number counts are reported in the third column of Table 2. For comparison, in the fourth column we report the values computed from the original De Zotti model fits. As expected from the number counts’ comparison shown in Fig. 3, we have a higher Poissonian contribution than the one obtained with the Tucci et al. model, namely 30 per cent higher at 70 and 100 GHz, 40 and 60 per cent at 143 and 217 GHz, respectively (see Section 4.1). At 353 GHz we predict an amplitude twice larger.

Together with the term coming from the radio sources, it is necessary to consider the Poissonian contribution of IR galaxies which is relevant only for the higher frequency channels: the 143, 217 and 353 GHz. The values of the Poissonian IR contributions at these frequencies are computed from the Bethermin et al. (2011) model in Planck Collaboration (2011b) and reported in the fifth column of Table 2. We summed the two Poisson terms for radio and IR in the frequencies in a single Poissonian term:

$$C^{\text{PS, TOT}}_{\ell,\nu} = A_{\text{PS}} C^{\text{PS, Radio+IR}}_{\ell,\nu},$$

(2.5)

where $C^{\text{PS}}_{\ell,\nu}$ is the Poissonian contribution at the different frequencies, whereas $A_{\text{PS}}$, which incorporates deviations of real data from this model, remains the same for all the frequency considered.

2.4 Clustering term contribution

The cosmic IR background (CIB) is the integrated IR emission over all redshifts of unresolved IR star-forming galaxies (Lagache, Puget & Dole 2005). This contribution is mainly that of starburst galaxies, luminous IR galaxies ($10^{11} < L_{IR} < 10^{12} L_\odot$) and ultraluminous IR galaxies ($L_{IR} > 10^{12} L_\odot$) (with small contribution from active galactic nuclei; Lagache, Dole & Puget 2003; Lagache et al. 2005; Fernandez-Conde et al. 2008, 2010). One of the complexities related to the CIB is that different frequencies probe the contributions of IR sources from different redshift ranges (e.g. Fernandez-Conde et al. 2008, 2010).
Cosmological parameter estimation

Table 4. The coefficients and exponents of the radio galaxy Poissonian contribution to the angular power spectrum (equation 2.4).

| Channel (GHz) | 70 | 100 | 143 | 217 | 353 |
|-------------|----|-----|-----|-----|-----|
| $\lambda_i$ | 1.110^{-4} | 3.6 \times 10^{-5} | 1.1 \times 10^{-5} | 4.3 \times 10^{-6} | 1.8 \times 10^{-5} |
| $a_i$ | 0.95 | 0.95 | 0.95 | 0.95 | 0.95 |
| $b_i$ | 1 | 1 | 1 | 1 | 1 |
| $c_i$ | 1.334 | 1.34 | 1.34 | 1.34 | 1 |
| $d_i$ | -10^{14} | -10^{14} | -10^{14} | -10^{14} | -10^{14} |
| $e_i$ | 2.5 | 2.8 | 2.8 | 2.8 | 2.8 |

| Channel (GHz) | 70 | 100 | 143 | 217 | 353 |
|-------------|----|-----|-----|-----|-----|
| $\lambda_i$ | 1.28 | 1.24 | 1.2 | 1.15 | 1.2 |
| $a_i$ | 0.64 | 0.62 | 0.6 | 0.58 | 0.63 |
| $b_i$ | 1 | 1.0 | 1.0 | 1.0 | 1.0 |
| $c_i$ | 1.64 | 1.62 | 1.6 | 1.58 | 1.63 |
| $d_i$ | -20408.2 | -40000 | -1111110.0 | -3086420.0 | -501187 |
| $e_i$ | 2 | 2 | 2 | 2 | 1.9 |

| Channel (GHz) | 70 | 100 | 143 | 217 | 353 |
|-------------|----|-----|-----|-----|-----|
| $\lambda_i$ | 1.25 | 1.35 | 1.3 | 1.35 | 1.25 |
| $a_i$ | 0.86 | 1.0 | 1.0 | 1.0 | 0.83 |
| $b_i$ | 1.0 | 1.35 | 1.08 | 1.22727 | 1.0 |
| $c_i$ | 1.86 | 2.35 | 2.08 | 2.23 | 1.83 |
| $d_i$ | -1 | -3.64 | -3.52 | -5.10 | -0.93 |
| $e_i$ | 1.45 | 1.2 | 1.1 | 1.1 | 1.47 |

Fernandez-Conde et al. 2008, 2010). Finally, the bias, relating the emissivity fluctuations to the dark matter density fluctuations, also affects the clustering term and is poorly known. In view of the theoretical complexity of the clustering term, we rather used the observational clustering term extracted directly from Planck maps at 217 and 353 GHz by Planck Collaboration (2011b). In order to minimally parametrize this contribution, we have fitted the Planck best-fitting models. Since the Planck data are not available for the channels below 217 GHz, the values for the clustering term for 100 and 143 GHz channels were derived using the halo occupation distribution best-fitting parameters of the 217 GHz channels (table 7 of Planck Collaboration 2011b), combined with the emissivities at 143 and 100 GHz computed using the Bethermin et al. (2011) model.

The fit to the clustering term is given for all the frequencies by the following expression:

$$\ell(\ell + 1) C_{\text{clust}}(\ell, \nu) \propto \frac{\phi(\ell, \nu) + \beta(\nu) \log(\ell)^{\alpha(\nu)}}{(100 + \ell)^{\gamma(\nu)}} ,$$

where $\phi(\ell, \nu)$ is a fifth-order polynomial term $A_0 \ell^2 + A_1 \ell^6 + A_2 + A_3 \ell + A_4 \ell^5 + A_5 \ell^2 + A_6 \ell^4$ accounting for the steepening at high $\ell$, whereas the low-$\ell$ shape is dominated by a complex logarithmic form. The coefficients $A_{0,6}$ depend on the frequency. The first step to obtain a fit to the clustering term was to derive a spectral shape, namely the combination of a polynomial component with the logarithm form. Then we fitted the form given in equation (2.8) for each frequency, first fitting the logarithmic function and then the polynomial function. Finally, we have derived the frequency dependence of each coefficient $A_{0,6}$ and each exponent by fitting to the different frequencies. The resulting fit to the power spectrum is given by

$$\ell(\ell + 1) C_{\text{clust}}(\ell, \nu) = \sum_{j=0,6} A_j \ell^{(\nu-2)} + A_7(\nu) \log(\ell)^{\alpha(\nu)} (100 + \ell)^{\beta(\nu)},$$

where

$$A_j = \sum_{i=1,3} a_i^j \nu^j ,$$

Figure 4. Comparison of the fits of the hybrid model/data number counts with the SPT number counts. The solid line is the empirical fit, whereas the dots represent the De Zotti et al. model number counts and the empty circles the Planck data for radio galaxies. The Tucci et al. model is represented by empty triangles. The measured number counts for radio galaxies by SPT are represented by the red squares.

The clustering power spectrum is given by Knox et al. (2001), Fernández-Conde et al. (2008, 2010) and Penin et al. (2012):

$$C_{\ell, \text{clust}} = \int \frac{d^2 \mathbf{a}(z)}{dA_z} \frac{d \nu}{dA} \times j_i(z) P_g(k)|_{k=a/d_A(z)},$$

(2.6)

where $r$ is the proper distance, $d_A$ is the comoving angular diameter distance, $k = \ell d_A$ derives from the Limber approximation, the $\frac{d^2 \mathbf{a}(z)}{dA_z}$ term takes into account all geometrical effects and depends on the cosmological model adopted. $j_i(z)$ is the mean galaxy emissivity per unit of comoving volume and $P_g$ is the galaxy three-dimensional power spectrum. The emissivity of IR galaxies can be written as (Penin et al. 2012)

$$j_i(z) = \left( \frac{d \nu}{dA} \right)^{-1} \int S(L_{\nu}) \frac{dN}{dA} \frac{d \nu}{dA} \frac{dA}{dA(z)} ,$$

(2.7)

where $\frac{dA}{dA(z)}$ is the luminosity function, $a$ is the scale factor, $\chi(z)$ is the comoving angular diameter distance to redshift $z$ and $S$ is the flux. The clustering term as shown by equation (2.6) depends on the cosmological model through the geometrical factor, and through the galaxy power spectrum. Another complication is given by the indirect dependence of the emissivity on the assumptions on the galaxy spectral energy distribution (Knox et al. 2001;
Table 5. The coefficients and exponents \( (a_i^j) \) of the fits for the clustering term (equation 2.9).

| \( i \) | 0  | 1  | 2  | 3  |
|-------|----|----|----|----|
| 0     | 14541.6 | -298.31 | 1.84 | -3.26 \times 10^{-3} |
| 1     | -718.25 | 15.14 | -9.8 \times 10^{-2} | 1.9 \times 10^{-4} |
| 2     | 19.52 | -0.42 | 2.8 \times 10^{-3} | -5.99 \times 10^{-6} |
| 3     | -0.19 | 4.2 \times 10^{-3} | -2.8 \times 10^{-5} | 6.1 \times 10^{-8} |
| 4     | -2. \times 10^{-4} | 4.4 \times 10^{-6} | -3.03 \times 10^{-8} | 6.72 \times 10^{-11} |
| 5     | 1.83 \times 10^{-4} | -3.99 \times 10^{-10} | 2.78 \times 10^{-12} | 6.18 \times 10^{-15} |
| 6     | 4.58 \times 10^{-13} | -9.72 \times 10^{-15} | 6.50 \times 10^{-17} | -1.36 \times 10^{-19} |
| 7     | -108.25 | 2.40 | -1.67 \times 10^{-2} | 3.66 \times 10^{-5} |
| 8     | 3.81 | -6.03 \times 10^{-2} | 3.87 \times 10^{-4} | -6.53 \times 10^{-7} |
| 9     | 2.76 | -2.08 \times 10^{-3} | -1.13 \times 10^{-5} | 4.07 \times 10^{-8} |

where the first sum, on \( i \), is on the coefficients of the polynomial and the second, in \( j \), is on the polynomial dependence on the frequency. The coefficients \( a_i^j \) are reported in Table 5. Such complex formula ensures an accuracy of the fit at \( <1 \) per cent level over the whole \( \ell \) range, and at \(<0.1 \) per cent over about 90 per cent of the considered \( \ell \) range. The high accuracy allows the use of the fits not only for *Planck* data but also for data with higher accuracy on small scales which in turn will require higher accuracy in the residual models.

Finally, provided expression (2.9) giving the frequency-dependent CIB clustering power spectrum, the parametrization of the clustering term can be simply written as

\[
C_{\ell}^{\text{clust}}(\nu) = A_{\text{Cl}} C_{\ell}^{\text{clust}}(\nu),
\]

where the amplitude parameter \( A_{\text{Cl}} \) accounts for deviations of the real signal from our parametrization. Different models for the clustering contribution to the angular power spectrum of CMB anisotropies have been proposed in recent years and in particular the most recent ones are Serra et al. (2008), Dunkley et al. (2011), Millea et al. (2011), Keisler et al. (2011), Reichardt et al. (2012) and Xia et al. (2012). The main difference of our approach is that our parametrization is the only one based completely on the most recent data by Planck for the higher frequencies, whereas for the lower ones we rely on the most accurate theoretical models. A comparison with the templates used in the likelihood routines shows that our model is in agreement in the high multipole region with the ones proposed by the SPT and ACT teams (Dunkley et al. 2011; Keisler et al. 2011; Reichardt et al. 2012). Since our model together with the spectral shape and amplitudes fits also the frequency dependence of the signal, it requires a single parameter to describe the clustering contribution independently from the number of frequencies used.

3 METHODOLOGY

In the previous section, we have detailed how we construct frequency-dependent parametrizations for the point sources and TSZ contributions to the angular power spectrum. In Fig. 5, we display each contribution together with the primary CMB spectrum for the five frequencies of interest. As anticipated, we note that the higher frequency channels are dominated by the clustering term contribution, the 353 GHz channel being the most contaminated. The lower frequencies are in turn dominated by the Poissonian term contribution. We also note that at small scales the TSZ contribution remains subdominant with respect to that of the point sources. As seen in Fig. 5, displaying the comparison between the sum of all contributions with the primary CMB, the 353 GHz channel is the most contaminated due to the clustering contribution, whereas the cleanest channel is the 143 GHz due to the low contribution of both clustering and Poissonian terms. We show in Table 6 the relative importance of the two main contributions of clustering and Poissonian expressed in terms of the rms:

\[
\sigma = \sqrt{\Sigma (2l+1) b_l^2 C_{l}}/(4\pi).
\]

We note again how the high frequencies have a dominant clustering contribution, whereas for the 100 and 143 GHz the dominant contribution is the Poissonian term.
Since we are interested in small-scale anisotropies, it is possible to avoid the component separation process to remove diffuse emission from our galaxy and use instead single frequency angular power spectra derived by masking the Galaxy and the resolved point sources. The Galaxy may contribute even after masking with high latitude emissions and residuals which may remain depending on the aggressiveness of the mask used. In this work, we neglect the possible residuals, after masking, from Galactic contamination.

We combine the angular power spectra of the microwave sky at the considered frequency channels in a single power spectrum that we use to estimate the cosmological parameters. An optimal method to combine channels is the inverse noise variance weighting scheme (Efstathiou and Gratton, private communication). This weighting scheme indeed gives the larger weights to the channels with higher resolutions and lower noise levels. This scheme is particularly suitable for the case of high multipole analysis where the noise and the beam smearing start to dominate. We therefore use a weighted linear sum of single frequency power spectra:

$$C^\ell_{\text{tot}} = \sum_i w_i(\ell)C^i_{\ell},$$

where the summation in $i$ is over the frequencies.

For simplicity, the noise is modelled as an isotropic Gaussian white noise with variance $\sigma^2_N$ on the total integration time. We define the beam function as

$$b^2_\ell = e^{-(\ell+1)0.425 \text{FWHM}/60\pi/180^2},$$

(3.12)

where FWHM is the full width at half-maximum of the channel beams in arcmin. We illustrate our method on a Planck-like case taking the beams and noise characteristics from Planck HFI Core Team (2011) and Mennella et al. (2011). Moreover, the noise has been divided by 2 to account for the approved extension of the mission. The numbers are summarized in Table 7.

We can thus define the noise function as

$$N_\ell = \frac{(\sigma_N)^2\Omega_{\text{pix}}}{b^2_\ell},$$

(3.13)

where $\Omega_{\text{pix}}$ is the area corresponding to the pixel. Note that the total noise contribution to the angular power spectrum considers also the contribution of the observed sky fraction $\Delta C^i_{\ell}/C^i_{\ell} = N^i_{\ell}/\sqrt{2R(2\ell+1)}$. Assuming that the sky fraction in the different channels is the same, the contribution of this term cancels in the weights. With these definitions, the weights for the single frequency spectrum are given by

$$w_i(\ell) = \frac{1}{\sqrt{\sum_j b^2_j}} \frac{1}{N^j_{\ell}},$$

(3.14)

where $i$ stands for the specific channel, whereas $j$ runs over all channels.

We show the weights obtained using the inverse noise variance scheme in Fig. 6. We note how the dominant contribution among the various combinations is given by the 143 and 217 GHz channels which have higher resolution and lower noise level, whereas channels with lower angular resolution have very little weights, such as the 70 GHz whose contribution is strongly subdominant within this approach. In Fig. 7, we show the resulting angular power spectra for the different astrophysical components obtained weighting the frequencies with the inverse noise variance. In particular, we note how the dominance of the 217 GHz creates a decrease of the contribution by the TSZ in the weighted spectrum, whereas the major contribution comes from the clustering term which dominates the high multipole region of the 217 GHz.

![Weights of the different frequencies with the inverse noise variance scheme.](image)

Figure 6. Weights of the different frequencies with the inverse noise variance scheme: the dotted line is the 353 GHz, the short-dashed line is the 217 GHz, the dot–dashed line is the 143 GHz, the dotted line is the 100 GHz and the solid line is the 70 GHz.

![Weighted angular power spectra for the different astrophysical components.](image)

Figure 7. Weighted angular power spectra for the different astrophysical components: the solid line is the sum of astrophysical contributions, the dashed line is the Poissonian term, the dotted line is the clustering term and the dot–dashed line is the TSZ term.

### 4 ESTIMATING THE COSMOLOGICAL AND EXTRAGALACTIC PARAMETERS

We now show the results of our analysis where we have applied, to the mock data, our approach of frequency-dependent minimal parametrization of the extragalactic sources. We first show the capability of our approach to perfectly recover the input cosmological parameters in the presence of extragalactic contamination and exhibit the importance of the wide frequency range for the characterization of the extragalactic signals by comparing the results for different frequency combinations. Then we show the impact of the

---

**Table 7.** *Planck* channel performance characteristics (Mennella et al. 2011; Planck HFI Core Team 2011) for the nominal mission.

| Instrument | LFI | HFI |
|-----------|-----|-----|
| Centre frequency GHz | 70 | 100 | 143 | 217 | 353 |
| Mean FWHM (arcmin) | 13 | 9.88 | 7.18 | 4.71 | 4.5 |
| $\Delta T$ per pixel ($T$) | 24.2525 | 8.175 | 5.995 | 13.08 | 54.5 |
| $\Delta T$ per pixel ($Q$ and $U$) | 34.6075 | 13.08 | 11.1725 | 24.525 | 103.55 |

© 2012 The Authors, MNRAS 426, 496–509

Monthly Notices of the Royal Astronomical Society © 2012 RAS
astrophysical contributions to the power spectrum and to the cosmological parameters and their errors. We finally test the robustness of our approach by investigating the dependence of the results on the astrophysical models we used. We will show also the results in extended parameter space; in particular, we considered the case of a Λ cold dark matter (ΛCDM) with running spectral index and tensor modes.

We developed an extension of the publicly available CosmoMC code (Lewis & Bridle 2002), which, in addition to estimating the cosmological parameters, estimates at the same time the contributions of a Poissonian term, a clustering term and a TSZ term. This is done by deriving a minimal number of amplitude terms, namely $A_{SZ}$, $A_{PS}$ and $A_{CL}$, which represent the departures from the input point source and TSZ models.

In practice, we have modified the CAMB code (Lewis, Challinor & Lasenby 2000), version 2010 May, to compute for each given frequency channel the sum of point source and TSZ spectra as provided by our parametrization (Section 2) and the standard CMB angular power spectrum. The resulting total power spectra at the different frequencies are then combined in a single effective power spectrum, following equation (3.12), in the CosmoMC code. We have used mock data for $\ell < 2500$.

We first explore the frequency combination and its effect on the recovery of the cosmological and the astrophysical parameters. The latter are represented by the three amplitudes $A_{SZ}$, $A_{PS}$ and $A_{CL}$. We have considered the results using five combinations of frequencies: 143 and 217 GHz, 100–217 GHz, 100–353 GHz, 70–217 GHz and finally all the frequencies under consideration, 70–353 GHz, with a sky coverage of $f_{sky} = 0.85$. In order to establish which frequency combination recovers the cosmological and astrophysical parameters best, we have performed for each combination an MCMC analysis of mock data using a ΛCDM-based CMB plus the astrophysical contributions. For the latter we used the ‘nominal’ contributions, which means all the amplitude terms were set to 1 ($A_{SZ} = A_{PS} = A_{CL} = 1$). All the channel combinations perfectly recover the cosmological parameters. The major differences are in the recovery of the astrophysical parameters, $A_{SZ}$, $A_{PS}$ and $A_{CL}$ (see Fig. 8). More specifically, the simplest combination of the 143 and 217 GHz channels recovers the Poissonian term with the smallest errors but it gives large errors for the clustering term. In contrast, the combination of all considered channels, 70–353 GHz, gives the largest errors for the Poissonian term but significantly reduces the errors on the clustering term. The clustering term being the most uncertain with respect to the Poissonian one, we chose to use the combination of the five channels in order to improve the constraints on this major contribution to the microwave sky at small angular scales.

4.1 Standard ΛCDM model

We performed the analysis in the framework of a standard ΛCDM cosmological model. The input mock $C_{\ell}$ were produced with nominal values for the point source and TSZ contributions, namely $A_{SZ} = A_{PS} = A_{CL} = 1$. Two cases are compared: fixed values for the astrophysical parameters (fixed to 1), and free values for $A_{SZ}$, $A_{PS}$ and $A_{CL}$. The results are displayed in Fig. 9 and summarized in Table 1 (the second column is the case with astrophysical parameters fixed to 1, the third column is the case with varying astrophysical parameters), together with the input parameters (fourth column). We show that the input cosmological parameters are accurately recovered in both cases. When the astrophysical parameters $A_{SZ}$, $A_{PS}$ and $A_{CL}$ together with the cosmological parameters are let free, the errors bars are slightly enlarged. The astrophysical parameters, $A_{PS}$ and $A_{CL}$, which characterize the point sources (Poissonian and clustering terms) are recovered well but they exhibit a strong degeneracy. This degeneracy is expected; in fact, the Poissonian and the clustering terms have different shapes but they both contribute additively to increase the measured power at high multipoles. The results are the same when we use the Tucci et al. model rather than our hybrid model for the radio contribution. The amplitude of TSZ spectrum in turn remains unconstrained for the Planck-like mock data. This is due to its subdominance with respect to other contributions, but it is also affected by the weighting scheme. As shown in Figs 6 and 7, with the inverse noise variance scheme the major contribution at small scales is given by the 217 GHz, frequency at which the TSZ is null. Anyway, even if in the context of this work for the Planck-like mock data the TSZ remains unconstrained, we maintain it in our model because of its possible relevance for other experiments. We will not discuss it further.

We now explore the robustness of the approach, in terms of parameter recovery, with respect to the models we use for the point-source contributions. We do not vary the TSZ signal as it is subdominant and cannot be recovered. For the effects of TSZ on the cosmological parameters in absence of point sources, we refer the reader to Taburet et al. (2008, 2010). We generated mock data including the CMB, the nominal TSZ contribution and the point-source contributions (Poisson and clustering terms). The latter have been chosen to differ significantly from the nominal contributions.

We first focus on the Poissonian term and multiplied the nominal Poissonian term by 2 in the mock data for each frequency channel, leaving untouched the other contributions. In Fig. 10, we present the obtained cosmological and astrophysical parameters. It is worth noting that even with a much larger Poissonian contribution the
Figure 9. Cosmological (and astrophysical) parameters with and without varying the astrophysical contribution parameters. The five frequency channels are combined with the inverse noise variance weighting scheme. The black (solid) curve represents the analysis which in addition to cosmological parameters varies also the astrophysical ones, whereas the red (dashed) curve represents the analysis with astrophysical parameters fixed to the fiducial model values. In the lower panel, we show also the triangle plot which is useful to evidence possible degenerations. The curves are the 68 and 95 per cent confidence level.
cosmological and the astrophysical parameters, $A_{PS}$ and $A_{CL}$, are perfectly recovered.

If we now focus on the clustering term, many tests with respect to the nominal model can be envisaged to test the effect of the parametrization on the parameter recovery. We test the robustness of the results, or in other words the sensitivity to the clustering term, with respect to the frequency dependence of the clustering term by varying not only the amplitude but also the frequency dependence of the clustering term. Since the clustering term at 353 GHz is very well constrained by the present Planck measurements of the CIB, we choose not to vary it. The CIB measurements at 217 GHz by Planck, in turn, have larger errors (see Planck Collaboration 2011b) and the values adopted in the parametrization for the 100 and 143 GHz channels are empirical extrapolation based on the 217 GHz measure. Therefore, it is very likely that at these frequencies the clustering term may be significantly different from what we used in the model. For this reason, we chose to vary these three frequencies.

We first analyse a case where we multiply by 2 the clustering contribution for the 100 and 143 GHz channels. In Fig. 10, the results show that this modification does not affect significantly the cosmological or the astrophysical parameters. The reason is that at 100 and 143 GHz the clustering term is not the dominant astrophysical component and therefore in the frequency-combined spectrum the variation of the clustering term for these two frequencies does not modify the final result. In the inverse noise variance weighting combination scheme, the major contribution to the clustering term comes from the 217 GHz channel which has a strong clustering component and at the same time is the dominant channel at high multipoles in the frequency combination. Since the 217 GHz measurements of the clustering still allow a rather significant variation, we decided to test the effects of a variation of the 217 clustering contribution by varying it by a factor of 2. The obtained contribution differs by several $\sigma$ from the present constraints obtained by Planck. We see from Fig. 11 that the cosmological parameters are perfectly recovered when we either multiply or divide the 217 GHz clustering contribution by 2. Unsurprisingly, the recovered mean values of $A_{CL}$ are multiplied/divided by a factor of $\sim 2$, respectively. Due to the degeneracy between the Poissonian and clustering terms, the mean values of $A_{PS}$ are also shifted with respect to the input value. In Fig. 12, we show the comparison between best fit and fiducial model for the case of a clustering term for the 217 GHz doubled; we note how the angular power spectrum obtained with the recovered cosmological
and astrophysical parameters is in good agreement with the fiducial model used.

We now test the robustness of the cosmological parameter recovery with respect to the frequency dependence that we proposed in our parametrization of the clustering term. To do so, we multiply by 2 the clustering terms at the 100 and 143 GHz, whereas the clustering term at 217 GHz is multiplied and then divided by 2. Here again the cosmological parameters are perfectly recovered in both cases, whereas the amplitudes of the Poisson and clustering terms vary. The results obtained for the mean values of $A_{PS}$ and $A_{CL}$ are the same as those resulting from varying only the 217 GHz contribution. This comes from the dominance of the 217 GHz channel weight, in the channel combination, which in turn implies a dominance of the clustering term in the analysis. We also tested the case where the Poissonian contribution is completely different from our model. To do so, we analyse the data using our hybrid model, whereas we construct the mock data from the Tucci et al. model. By doing so, the Poissonian term varies from frequency to frequency, whereas the IR contribution from radio sources that we obtained with the analysis which considers the standard De Zotti et al. based Poisson fluctuations in the code (black curve).

We considered together with the six standard parameters both tensor perturbations and running spectral index, varying the tensor-to-scalar ratio, $r$, and the running, $n_{	ext{run}}$, together with the six standard parameters and the three astrophysical ones. We have generated the mock data with both extra parameters, $n_{	ext{run}}$, and $r$ set to zero. We fix the tensor spectral index to the second-order inflationary consistency condition $n_T = -(A_s/\Lambda^2)(2 - A_s/\Lambda - n_s)$. We included only the TE and EE mode in polarization. We do not include the B-mode polarization power spectra in the exact form of the CMB likelihood we exploit, since the main limitation in that respect is given by the ability of removing diffuse foregrounds rather than instrumental noise. In Fig. 14, we show the comparison of the analyses which include the variation of the tensor-to-scalar ratio and running spectral index, with fixed amplitudes of the point-source contributions (red dashed curve) and without fixing the amplitudes of the point-source contributions (black solid curves).

Similar to the standard $\Lambda$CDM model, leaving the astrophysical contributions free enlarges the errors bars including those of the running spectral index. The bidimensional plot of the scalar versus running spectral index in Fig. 15 shows, in addition to the enlargement of the errors, a degeneracy between the two cosmological parameters. The combined variation of the tensor-to-scalar ratio and running spectral index, without the inclusion of $B$-mode

**Figure 13.** Comparison of the cosmological and astrophysical parameters obtained with the analysis which considers the standard De Zotti et al. based radio Poisson fluctuations (red curve) and the one which considers the Tucci et al. based radio Poisson fluctuations in the mock data and the standard De Zotti et al. based Poisson fluctuations in the code (black curve).

**Figure 14.** Results of the analysis which considers together with cosmological and astrophysical parameters the variation of the tensor-to-scalar ratio and the running spectral index. Comparison between the results varying or not the astrophysical contributions: the black (solid) curves consider the astrophysical signals, whereas the red (dashed) curves represent the results with astrophysical contributions fixed to the fiducial value. Vertical bars are the input parameters.

### 4.2 Extended cosmological parameter space

We have shown the results of the cosmological and astrophysical parameter recovery for a standard $\Lambda$CDM six-parameter cosmological model. We now present some results obtained in the case of an extended cosmological parameter space.

We considered together with the six standard parameters both tensor perturbations and running spectral index, varying the tensor-to-scalar ratio, $r$, and the running, $n_{\text{run}}$, together with the six standard parameters and the three astrophysical ones. We have generated the mock data with both extra parameters, $n_{\text{run}}$, and $r$ set to zero. We fix the tensor spectral index to the second-order inflationary consistency condition $n_T = -(A_s/\Lambda^2)(2 - A_s/\Lambda - n_s)$. We included only the TE and EE mode in polarization. We do not include the B-mode polarization power spectra in the exact form of the CMB likelihood we exploit, since the main limitation in that respect is given by the ability of removing diffuse foregrounds rather than instrumental noise. In Fig. 14, we show the comparison of the analyses which include the variation of the tensor-to-scalar ratio and running spectral index, with fixed amplitudes of the point-source contributions (red dashed curve) and without fixing the amplitudes of the point-source contributions (black solid curves).

Similar to the standard $\Lambda$CDM model, leaving the astrophysical contributions free enlarges the errors bars including those of the running spectral index. The bidimensional plot of the scalar versus running spectral index in Fig. 15 shows, in addition to the enlargement of the errors, a degeneracy between the two cosmological parameters. The combined variation of the tensor-to-scalar ratio and running spectral index, without the inclusion of $B$-mode

© 2012 The Authors, MNRAS 426, 496–509

Monthly Notices of the Royal Astronomical Society © 2012 RAS
polarization, shifts the mean of the running spectral index towards negative values with respect to the input ones \( (n_{\text{run}} = 0) \).

We repeated also the main robustness analysis in this extended parameter space. In particular, we repeated the three main cases: the case with the Poissonian contribution multiplied by 2, the variation of both amplitude and frequency dependence of the clustering term with the multiplication by 2 of the 100 and 143, and multiplying and dividing by 2 the 217 term. In Fig. 16, we report the results of the analysis. We note how the recovery of the parameters is the same as in the standard astrophysical case, showing how our method is robust also for this extended parameter space.

5 DISCUSSION AND CONCLUSIONS

In the present study, we developed a multifrequency approach for the analysis of the CMB power spectrum aiming at obtaining unbiased cosmological parameters in the presence of unresolved extragalactic point sources and TSZ signal.

We have considered the three main extragalactic contributions to the CMB signal on small scales, namely radio sources, IR sources and SZ clusters. We have modelled these three signals in a minimal way using the observational constraints from Planck’s early results (Planck Collaboration 2011a,b). In particular, the radio and IR sources were modelled through a unique Poissonian term and a unique clustering term. In total, three amplitude terms suffice to account for and characterize the extragalactic contributions. This minimal number of parameters was obtained by considering the frequency dependences within our parametrizations of the TSZ signal, and IR and radio point sources.

The use of a data-based physical model including the frequency dependence of the astrophysical signals under consideration to build the parametrizations of the small-scale contribution to the CMB is one of the original points of the present work. Previous studies (Millea et al. 2011; Efstathiou and Gratton, private communication) chose parametrizations of the signals which consider more generic shapes and parametrize each frequency with a different set of parameters. Such an approach either leads to a rapid increase of the number of parameters necessary to describe the astrophysical signals, like in Millea et al. (2011), or requires the use of a limited number of frequencies. Other studies focused only on one single contribution using more detailed and physical based parametrizations which involve a higher number of parameters with respect to our approach (see e.g. Efstathiou & Migliaccio 2012, for the TSZ) or considered the frequency dependence on a much simpler level such as for the clustering contribution in Addison et al. (2012). A different approach, relying directly on the data, has been used by the SPT team (Lueker et al. 2010; Keisler et al. 2011; Reichardt et al. 2012). They took advantage of the high angular resolution to acquire data on scales where the CMB is suppressed. They used the data on smallest scales to measure the amplitudes of their templates for the astrophysical signals and consequently to remove the associated contamination from the CMB data.

We have applied our multifrequency minimal parametrization of the extragalactic contamination to estimate the cosmological parameters for a standard six-parameter \( \Lambda \)CDM model and for a model with tensor perturbations and a running spectral index, without including residuals from our Galaxy and without using the \( B \)-mode information. We show that the best combination of frequencies is the one which considers the frequencies from 70 to 353 GHz. It allows us to estimate the cosmological parameters and at the same time put the tightest constraints on the amplitude of the IR clustering term. We show, both in the standard \( \Lambda \)CDM model and in the ‘extended parameter’ model, that the input cosmological parameters together with the astrophysical ones (amplitudes of the TSZ, Poisson and clustering terms) are recovered as unbiased and with slightly enlarged error bars. We show that the Poisson and clustering terms are degenerate.
Our approach uses the whole available information on point sources and theoretical knowledge of the TSZ to have models of the astrophysical signals which are as close as possible to the expected signal. The use of the frequency dependence and the technique used to fit the data allowed a very restricted number of parameters to characterize the signals, in particular only a total of three parameters for all the signals.

We have investigated for both cosmological models, standard and extended, the dependence of our approach and of the derived parameters on the accuracy of our model of extragalactic contributions. In this way, we tested its robustness against our theoretical priors on frequency dependence. We tested different Poissonian and clustering contributions by varying the associated terms both in amplitude and in frequency dependence. We showed that for both cosmological models the cosmological parameters are recovered unbiased even in situations where the mock data differ by several $\sigma$ from the model. The astrophysical parameters, amplitudes $A_{PS}$ and $A_{CL}$, are recovered well in the case of a ‘real’ Poissonian term different from that of the parametrization. In the case of a different clustering term, the degeneracy with the Poissonian term induces a shift also in the Poissonian term.

We have provided a minimal parametrization to take into account the extragalactic contaminations on small scales. Our minimal analysis can be extended to include additional contributions such as residuals from the Galaxy contamination. However, it is ideal in the sense that systematic effects, beam uncertainties, etc. are not accounted for.

ACKNOWLEDGMENTS

We wish to thank Nicolas Taburet for useful discussions and for providing the TSZ power spectrum template, and Marco Tucci for providing the number counts of Tucci et al. (2011). This work is partially supported by ASI contract Planck-LFI activity of Phase E2, by LDAP. NA wishes to thank IASF Bologna for hosting, and DP thanks IAS for hosting. The simulations for this work have been carried on IASF Bologna cluster. We wish to thank the financial support by INFN IS PD51 for NA visit in Bologna. DP thanks the International Doctorate on AstroParticle Physics (IDAPP) by the Italian Ministry of University and Research (MIUR) for partial support. MD, NA and GL acknowledge support from CNES and PNCG. We wish to thank the reviewer for the useful comments and suggestions.

REFERENCES

Abramowitz M., Stegun I., 1965, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Table. Dover, New York
Addison G. E. et al., 2012, ApJ, 752, 120
Aghanim N., Majumdar S., Silk J., 2008, Rep. Progress Phys., 71, 066902
Amblard A. et al., 2011, Nat, 470, 510
Arnaud M., Pratt G. W., Piffaretti R., Boehringer H., Croston J. H., Pointecouteau E., 2010, A&A, 517, 92
Ashby M. L. N. et al., 2009, ApJ, 701, 428
Bethermin M., Dole H., Lagache G., Le Borgne D., Piau A., 2011, A&A, 529, 4
da Silva A. C., Barbosa D., Liddle A. R., Thomas P. A., 2001, MNRAS, 326, 155
Danese L., Franceschini A., Toffolatti L., de Zotti G., 1987, ApJ, 318, L15
Das S. et al., 2011, ApJ, 729, 62
De Zotti G., Ricci R., Mesa D., Silva L., Mazzotta P., Toffolatti L., González-Nuevo J., 2005, A&A, 431, 893

De Zotti G., Burigana C., Negrello M., Tinti S., Ricci R., Silva L., González-Nuevo J., Toffolatti L., 2006, in Del Toro Iniesta J. C., Alfaro E. J., Gorgas J. G., Salvador-Sole E., Butcher H., eds, in Proc. JENAM 2004 Astrophys. Rev., The Many Scales in the Universe, Springer, Dordrecht, p. 45
Douspis M., Aghanim N., Langer M., 2006, A&A, 456, 819
Dunkley J. et al., 2011, ApJ, 739, 52
Efstathiou G., Migliazzo M., 2012, MNRAS, 423, 2492
Fernandez-Conde N., Lagache G., Puget J.-L., Dole H., 2008, A&A, 481, 885
Fernandez-Conde N., Lagache G., Puget J.-L., Dole H., 2010, A&A, 515, 48
Ivison R. J. et al., 2010, MNRAS, 402, 245
Keisler R. et al., 2011, ApJ, 743, 28
Knox L., Cooray A., Eisenstein D., Haiman Z., 2001, ApJ, 550, 7
Komatsu E., Seljak U., 2002, MNRAS, 336, 1256
Komatsu E. et al., 2011, ApJS, 192, 18
Lagache G., Dole H., Puget J.-L., 2003, MNRAS, 338, 555
Lagache G., Puget J.-L., Dole H., 2005, ARA&A, 43, 727
Lagache G., Bavaudet N., Fernandez-Conde N., Ponthicou N., Rodez T., Dole H., Miville-Deschenes M. A., Puget J. L., 2007, ApJ, 665, L89
Lewis A., Bridle S., 2002, Phys. Rev. D, 66, 103511
Lewis A., Challinor A., Lasenby A., 2000, ApJ, 538, 473
Lueker M. et al., 2010, ApJ, 719, 1045
Marsden G. et al., 2009, ApJ, 707, 1729
Mennella A. et al., 2011, A&A, 536, A3
Millea M., Dor O., Dudley J., Holder G., Knox L., Shaw L., Song Y.-S., Zahn O., 2011, ApJ, 746, 4
Niemack M. D. et al., 2010, Proc. SPIE, 7741, 77411S
Penin A., Doge O., Lagache G., Bethermin M., 2012, A&A, 537A, 137
Planck Collaboration, 2006, preprint (astro-ph/0604069)
Planck Collaboration, 2011a, A&A, 536, A13
Planck Collaboration, 2011b, A&A, 536, A18
Planck HFI Core Team, 2011, A&A, 536, A6
QuaD Collaboration, 2009, ApJ, 700, L187
Redhead A. C. et al., 2004, ApJ, 609, 498
Reichardt C. L. et al., 2009, ApJ, 694, 1200
Reichardt C. L. et al., 2012, ApJ, 749, L9
Sazonov S. Y., Sunyaev R. A., 1999, MNRAS, 310, 765
Serra P., Cooray A., Amblard A., Pagano L., Melchiorri A., 2008, Phys. Rev. D, 78, 043004
Sunyaev R. A., Zel’dovich Y. B., 1970, Ap&SS, 7, 3
Sunyaev R. A., Zel’dovich Y. B., 1972, Comment Astrophys. Space Phys., 4, 173
Sunyaev R. A., Zel’dovich Y. B., 1980, ARA&A, 18, 537
Taburet N., Aghanim A., Langer M., 2006, A&A, 456, 819
Tinker J. L., Kravtsov A. V., Klypin A., Abazajian K., Warren M., Yepes G., Gottlber S., Holz D. E., 2008, ApJ, 688, 709
Toffolatti L., Argueso Gomez F., De Zotti G., Mazzini P., Franceschini A., Danese L., Burigana C., 1998, MNRAS, 297, 117
Toffolatti L., Negrello M., Gonzalez-Nuevo J., de Zotti G., Silva L., Granato G. L., Argueso F., 2005, A&A, 438, 475
Tucci M., Toffolatti L., 2011, preprint (arXiv:1204.0427)
Tucci M., Martínez-González E., Vielva P., Delabrouille J., 2005, MNRAS, 360, 935
Tucci M., Toffolatti L., De Zotti G., Martínez-González E., 2011, A&A, 533, A57
Vieira J. D. et al., 2010, ApJ, 719, 763
Xia J.-Q., Negrello M., Lapi A., de Zotti G., Danese L., Viel M., 2012, MNRAS, 422, 1324

This paper has been typeset from a LyX/LATEX file prepared by the author.