Test of quantum nonlocality for cavity fields

M. S. Kim\textsuperscript{1,2} and Jinhyoung Lee\textsuperscript{1}\ast
\textsuperscript{1} Department of Physics, Sogang University, CPO Box 1142, Seoul 100-611, Korea
\textsuperscript{2} Department of Applied Mathematics and Theoretical Physics, The Queen’s University of Belfast, Belfast BT7 1NN, United Kingdom
(November 1, 2018)

Abstract

There have been studies on formation of quantum-nonlocal states in spatially separate two cavities. We suggest a nonlocal test for the field prepared in the two cavities. We couple classical driving fields with the cavities where a nonlocal state is prepared. Two independent two-level atoms are then sent through respective cavities to interact off-resonantly with the cavity fields. The atomic states are measured after the interaction. Bell’s inequality can be tested by the joint probabilities of two-level atoms being in their excited or ground states. We find that quantum nonlocality can also be tested using a single atom sequentially interacting with the two cavities. Potential experimental errors are also considered. We show that with the present experimental condition of 5\% error in the atomic velocity distribution, the violation of Bell’s inequality can be measured.

PACS number(s); 03.65.Bz, 42.50Dv

\ast hyoung@quanta.sogang.ac.kr
I. INTRODUCTION

Entangled states have been at the focus of discussions in quantum information theory encompassing quantum teleportation, computing, and cryptography. Two-body entanglement allows diverse measurement schemes which can admit tests of quantum nonlocality. Using the atom-field interaction in a high-$Q$ cavity we can produce quantum entanglement between cavity fields, between atoms, and between a cavity and an atom. An entangled pair of atoms have been experimentally generated using the cavity QED (Quantum electrodynamics). The entanglement of atoms and fields in the cavity can be utilized towards a realization of the controlled-NOT gate for quantum computation.

A pair of atoms can be prepared in an entangled state using the atom-field interaction in a high-$Q$ cavity. The interaction of a single two-level atom with a cavity field brings about entanglement of the atom and the cavity field. If the atom does not decay into other internal states after it comes out from the cavity, the entanglement will survive for long and it can be transferred to a second atom interacting with the cavity field. The violation of Bell’s inequality can be tested by the joint measurement of atomic states.

There are proposals to entangle fields in two spatially separated cavities using the atom-field interaction. A two-level atom in its excited state passes sequentially through two resonant single-mode vacuum cavities and is found to be in its ground state after the second-cavity interaction. If the interaction with the first cavity is equivalent to a $\pi/2$ vacuum pulse and the second-cavity interaction is to a $\pi$ pulse then the atom could have deposited a photon either in the first cavity or in the second so that the final state $|\Psi_f\rangle$ of the two cavity field is

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|1, 0\rangle + e^{i\varphi}|0, 1\rangle)$$

where $|1, 0\rangle$ denotes one photon in the first cavity and none in the second, and $|0, 1\rangle$ vice versa. Using the entangled cavity field, an unknown atomic quantum state can also be teleported.

A three-level atom can act as a quantum switch to couple a vacuum cavity with the external classical coherent field. If the atom is in its intermediate state before it enters to the cavity, the AC Stark shift between the intermediate and upper-most states brings about the resonant coupling of the cavity with the external field so that the external field can be fed into the cavity. If the atom is initially in its lower-most state, the atom is unable to switch on the coupling between the cavity and the external field. Using the Ramsey interference and atomic quantum switch, Davidovich et al. suggested a coherent state entanglement $|\Psi_c\rangle$ between two separate cavities: $|\Psi_c\rangle = B_1|\alpha, 0\rangle + B_2|0, \alpha\rangle$, where $|\alpha, 0\rangle$ denotes the first cavity in the coherent state of the amplitude $\alpha$ and the second in the vacuum.

In this paper, we are interested in a test of nonlocality for the entangled field prepared in the spatially separated cavities. Despite the suggestions on the production of entangled cavity fields, the test of quantum entanglement, i.e., the measurement of the violation of Bell’s inequality for the entangled cavity fields has not been studied. To test quantum nonlocality, we first couple classical driving fields with the cavities where a nonlocal state is prepared. Two independent two-level atoms are then sent through respective cavities to interact off-resonantly with the cavity fields. The atomic states are measured after the
interaction. Bell’s inequality can be tested by the joint probabilities of two-level atoms being in their excited or ground states. We find that quantum nonlocality can also be tested using a single atom sequentially interacting with the two cavities. We also consider potential experimental errors. The atoms normally have the Gaussian velocity distribution with the normalized standard deviation less than 5%. We show that, even with the experimental errors caused by the velocity distribution, the test can be feasible.

II. BELL’S INEQUALITY BY PARITY MEASUREMENT

It is important to choose the type of measurement variables when testing nonlocality for a given state. Banaszek and Wódkiewicz [10] considered even and odd parities of the field state as the measurement variables, where a state is defined to be in the even (odd) parity if the state has even (odd) numbers of photons. The even and odd parity operators, $\hat{O}_E$ and $\hat{O}_O$, are the projection operators to measure the probabilities of the field having even and odd numbers of photons, respectively:

$$\hat{O}_E = \sum_{n=0}^{\infty} |2n\rangle\langle 2n| \quad ; \quad \hat{O}_O = \sum_{n=0}^{\infty} |2n+1\rangle\langle 2n+1|. \quad (2)$$

To test the quantum nonlocality for the field state of the modes $a$ and $b$, we define the quantum correlation operator based on the joint parity measurements:

$$\hat{\Pi}^{ab}(\alpha, \beta) = \hat{\Pi}^{a}_E(\alpha)\hat{\Pi}^{b}_E(\beta) - \hat{\Pi}^{a}_E(\alpha)\hat{\Pi}^{b}_O(\beta) - \hat{\Pi}^{a}_O(\alpha)\hat{\Pi}^{b}_E(\beta) + \hat{\Pi}^{a}_O(\alpha)\hat{\Pi}^{b}_O(\beta) \quad (3)$$

where the superscripts $a$ and $b$ denote the field modes and the displaced parity operator, $\hat{\Pi}_{E,O}(\alpha)$, is defined as

$$\hat{\Pi}_{E,O}(\alpha) = \hat{D}(\alpha)\hat{O}_{E,O}\hat{D}^\dagger(\alpha). \quad (4)$$

The displacement operator $\hat{D}(\alpha)$ displaces a state by $\alpha$ in phase space. The displaced parity operator acts like a rotated spin projection operator in the spin measurement [2]. We can easily derive that the local hidden variable theory imposes the following Bell’s inequality [10]

$$|B(\alpha, \beta)| \equiv |\langle \hat{\Pi}^{ab}(0,0) + \hat{\Pi}^{ab}(0,\beta) + \hat{\Pi}^{ab}(\alpha,0) - \hat{\Pi}^{ab}(\alpha,\beta) \rangle| \leq 2 \quad (5)$$

where we call $B(\alpha, \beta)$ as the Bell function.

III. PARITY MEASUREMENT IN CAVITY QED

Englert et al. [11] proposed an experiment to determine the parity of the field in a high-Q single-mode cavity. Let us consider a far-off resonant interaction of a two-level atom with a single-mode cavity field. If the detuning $\Delta = \omega_o - \omega$ of the atomic transition frequency $\omega_o$ from the cavity-field frequency $\omega$ is much larger than the Rabi frequency $\Omega$, there is no energy exchange between the atom and the field but the relative phase of the atomic states
changes due to the AC stark shift [12]. The change of the phase depends on the number of photons in the cavity and on the state of the atom:

\[
|e, \psi_f\rangle \rightarrow \exp[-i\Theta(\hat{n} + 1)]|e, \psi_f\rangle,
\]

\[
|g, \psi_f\rangle \rightarrow \exp[i\Theta(\hat{n})]|g, \psi_f\rangle
\]

(6)

where the atom-field state \(|e, \psi_f\rangle\) denotes the atom in its excited state and the cavity field in \(|\psi_f\rangle\). The phase \(\Theta(\hat{n})\) is a function of the number of photons in the cavity and the atom-field coupling time and strength.

If a \(\pi/2\) pulse is applied on a two-level atom before it enters the cavity, the atom initially in its excited state transits to a superposition state without changing the cavity field

\[
|e, \psi_f\rangle \rightarrow \frac{1}{\sqrt{2}}[|e, \psi_f\rangle + ie^{i\phi_1}|g, \psi_f\rangle],
\]

(7)

where \(\phi_1\) is determined by the phase of the pulse field. The atom, then, passes through a cavity and undergoes an off-resonant interaction with the cavity field where the atom-field coupling function is selected to be

\[\Theta(\hat{n}) = \frac{\pi}{2}\hat{n}.\]

(8)

After the atom comes out from the cavity, the atom is let interact with the second \(\pi/2\) pulse. If the phases of the first and the second pulses are chosen to satisfy the following relation

\[ie^{i(\phi_1 - \phi_2)} = 1,\]

(9)

the atom-field state after the second \(\pi/2\) pulse is

\[
\frac{1}{2}\left\{i[1 - (-1)^\hat{n}]|e, \psi_f\rangle + e^{i\phi_2}[1 + (-1)^\hat{n}]|g, \psi_f\rangle\right\}.
\]

(10)

If the atom is detected in its excited state, the field has the even parity. If the atom is detected in its ground state, the field has the odd parity. The atom tells us the parity of the field [11].

IV. QUANTUM NONLOCALITY OF CAVITY FIELDS

To test quantum nonlocality of the field, \(|\psi_{ab}\rangle\), prepared in spatially separated two single-mode cavities, we use two two-level atoms as shown in Fig. 1(a). In this paper we assume that the mode structures of the cavities are identical and the atoms are independent identical atoms. The atoms are labelled as \(a\) and \(b\) to interact, respectively, with the fields in the cavities \(a\) and \(b\). Each atom is initially prepared in its excited state and sequentially passes through interaction zones of the first \(\pi/2\) pulse, the cavity field, and the second \(\pi/2\) pulse. The atoms-field state then becomes

\[
|e_a, e_b\rangle|\psi_f^{ab}\rangle \rightarrow |\varphi(\hat{n}_a, \hat{n}_b)\rangle|\psi_f^{ab}\rangle
\]

(11)

where \(|\varphi(\hat{n}_a, \hat{n}_b)\rangle\) is the atomic state with the weights of the field operators (\(\hat{n}_a\) and \(\hat{n}_b\) are number operators of fields in the cavities \(a\) and \(b\)): 
\[ \varphi(n_a, n_b) = a(n_a)a(n_b)|e_a, e_b\rangle + a(n_a)b(n_b)|e_a, g_b\rangle + b(n_a)a(n_b)|g_a, e_b\rangle + b(n_a)b(n_b)|g_a, g_b\rangle \tag{12} \]

where \(d(n) = [e^{-i\Theta(n+1)} - e^{i(\phi_1 - \phi_2)}e^{i\Theta(n)}]/2\) and \(b(n) = ie^{i\phi_2}[e^{-i\Theta(n+1)} + e^{i(\phi_1 - \phi_2)}e^{i\Theta(n)}]/2\).

Choosing appropriate conditions for the atom-field couplings and pulse phases as shown in (8) and (9), the atoms-field state becomes

\[ \frac{1}{4}\{i[1 - (-1)^{n_a}]|e_a\rangle + e^{i\phi_2}[1 + (-1)^{n_a}]|g_a\rangle\} \times \{i[1 - (-1)^{n_b}]|e_b\rangle + e^{i\phi_2}[1 + (-1)^{n_b}]|g_b\rangle\} |\psi_f^{ab}\rangle. \tag{13} \]

If the atoms are jointly found in their excited states then we know that both the cavities are in the odd parity states. The joint probability \(P_{ee}\) of the atoms being in their excited states is related to the expectation value of the following joint parity operator:

\[ P_{ee} = \langle \hat{\Pi}_O^a(0)\hat{\Pi}_O^b(0) \rangle. \tag{14} \]

In Eqs. (3) and (5), it is seen that we need to know the joint parities of the displaced original fields to test quantum nonlocality. To displace the cavity field, external stable fields are coupled to the cavities as shown in Fig. 1(a) [13]. After a nonlocal field state is prepared in the cavities, we couple the cavities with the external fields to displace the original nonlocal field, then send two independent atoms through respective cavities. The \(\pi/2\) pulses shine atoms before and after the cavity interaction to provide Ramsey interference effects. The atomic states are detected after the second \(\pi/2\) pulses. \(P_{ee}(\alpha, \beta)\) denotes the joint probability of atoms being in their excited states when the original fields in the cavities \(a\) and \(b\) are displaced by \(\alpha\) and \(\beta\), respectively. The expectation value of the quantum correlation operator in (3) is obtained by the joint probabilities:

\[ \langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle = P_{ee}(\alpha, \beta) - P_{eg}(\alpha, \beta) - P_{ge}(\alpha, \beta) + P_{gg}(\alpha, \beta). \tag{15} \]

If there are any displacement factors \(\alpha\) and \(\beta\) which result in the violation of Bell’s inequality in Eq. (3), the field originally prepared in the cavities is quantum-mechanically nonlocal.

After a closer look at Eq. (3), we find that we do not need the individual parity of each cavity field to test the inequality (3). We need the parity of only the total field. Instead of sending two atoms to cavities, we now send a single two-level atom sequentially through cavities as shown in Fig. 1(b). The atom is initially prepared in its excited state and undergoes \(\pi/2\)-pulse interactions before and after the cavity interaction. The atom-field coupling strength is selected to satisfy Eq. (3) and the phases, \(\phi_a\) and \(\phi_b\), of the two \(\pi/2\) pulses are chosen as exp\([i(\phi_a - \phi_b)] = 1\) then the atom-field state becomes

\[ |e, \psi_f^{ab}\rangle \rightarrow -\frac{1}{2}[1 + (-1)^{n_a+n_b}]|e, \psi_f^{ab}\rangle + i\frac{1}{2}[1 - (-1)^{n_a+n_b}]|g, \psi_f^{ab}\rangle. \tag{16} \]

The external stable fields are taken to be coupled with the cavities to displace the cavity fields. The probability \(P_{e}(\alpha, \beta)\) of the atom being in its excited state after having passed displaced cavity fields and \(\pi/2\) pulses, is the expectation value of the parity operators:

\[ P_e(\alpha, \beta) = \langle \hat{\Pi}_E^a(\alpha)\hat{\Pi}_E^b(\beta) + \hat{\Pi}_E^a(\alpha)\hat{\Pi}_E^b(\beta) \rangle \tag{17} \]
where \( \alpha, \beta \) denote the displacements of the fields in the cavities \( a \) and \( b \). Similarly, the probability of the atom being in its ground state \( P_g(\alpha, \beta) \) is found to be related to the odd parity of the total fields. The expectation value of the quantum correlation function operator in Eq. (18) is simply

\[
\langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle = P_e(\alpha, \beta) - P_g(\alpha, \beta).
\] (18)

This does not tell us the parity of each mode but the parity of the total field which is enough to test the violation of Bell’s inequality.

V. REMARKS

We have suggested a simple way to test quantum nonlocality of cavity fields by measuring the states of atoms after their interaction with cavity fields. The test does not require a numerical process on the measured data. The difference in the probability of a single two-level atom being in its excited and ground states is directly related to the test of quantum nonlocality. In fact, this can also be used to reconstruct the two-mode Wigner function as the mean parity of the field is proportional to the two-mode Wigner function \[10,14,15]:

\[
W(\alpha, \beta) = \frac{2}{\pi} \langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle.
\] (19)

Experimental error can easily occur from the fluctuation in the atom-field coupling strength and time. The atom-field coupling function \( \Theta(\hat{n}) \) depends on the mode structure of the cavity field and on the duration of time for the atom to interact with the cavity field \[11\]. Because the atomic velocity has some fluctuations the interaction time is subject to the experimental error \[19\]. Another error source may be the \( \pi/2 \) pulse operation. We analyze the possibility to measure the violation of quantum nonlocality in the potential experimental situation.

The test of quantum nonlocality using the two-atom scheme in Fig. 1(a) is considered with potential experimental errors. The error in the atom-field coupling function is denoted by \( \Delta \Theta(\hat{n}) \), which is the departure of the experimental value \( \Theta(\hat{n}) \) from the required value \( \Theta_0(\hat{n}) \):

\[
\Delta \Theta(\hat{n}) = \Theta(\hat{n}) - \Theta_0(\hat{n}) = \delta \hat{n}
\] (20)

where \( \Theta_0(\hat{n}) = (\pi/2)\hat{n} \). The relative phases of atomic states given by the \( \pi/2 \)-pulse interactions are also subject to experimental errors. We take the phase error \( \Delta \phi \) as

\[
\Delta \phi = (\phi_2 - \phi_1) - \phi_0 \quad ; \quad i \exp(i\phi_0) = 1.
\] (21)

Note that the atomic state measurement is equivalent to the parity measurement as in Eq. (14) only when \( \Theta(\hat{n}) = \Theta_0(\hat{n}) \) and \( \phi_2 - \phi_1 = \phi_0 \).

The errors in the atom-field coupling and phases of the \( \pi/2 \) pulses bring about the departure \( \Delta \Pi^{ab}(\alpha, \beta) \) of the joint atomic state probabilities from the expectation value of the parity operators in Eq. (14). The mean error of \( \Delta \Pi^{ab}(\alpha, \beta) \) is calculated up to the second order of \( \delta \) and \( \Delta \phi \) as follows.
\[ \Delta \Pi^{ab}(\alpha, \beta) \equiv P_{ee}(\alpha, \beta) - P_{eg}(\alpha, \beta) - P_{ge}(\alpha, \beta) + P_{gg}(\alpha, \beta) - \langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle \]
\[ = -2 \langle \psi_f^{ab} | \hat{\Pi}^{ab}(\alpha, \beta) | (\Delta(\hat{n}_a(\alpha)) + \Delta(\hat{n}_b(\beta))) | \psi_f^{ab} \rangle \]  
\[ \text{(22)} \]

where
\[ \Delta(\hat{n}_{a,b}(\alpha)) = \frac{1}{4}[2\hat{n}_{a,b}(\alpha) + 1]^2 \delta^2 + \frac{1}{2}[2\hat{n}_{a,b}(\alpha) + 1]\delta \Delta \phi + \frac{1}{4}(\Delta \phi)^2 \]  
\[ \text{(23)} \]

and \[ \hat{n}_{a,b}(\alpha) = \hat{D}^\dagger(\alpha) \hat{n}_{a,b} \hat{D}(\alpha) \]  are the displaced number operators for the field modes in the cavities \( a \) and \( b \). The mean error \( \Delta B(\alpha, \beta) \) of the Bell function measurement in (3) is given by
\[ \Delta B(\alpha, \beta) = \Delta \Pi^{ab}(0,0) + \Delta \Pi^{ab}(\alpha,0) + \Delta \Pi^{ab}(0,\beta) - \Delta \Pi^{ab}(\alpha,\beta). \]  
\[ \text{(24)} \]

Consider an explicit example of a quantum nonlocal field (1) for an illustration of the experimental errors. For simplicity, we take the phase factor zero, \( i.e., \phi = 0 \). We know from an earlier work [10] that Bell’s inequality is maximally violated with \( B \sim -2.19 \) when \( \alpha = -\beta \) and \( |\alpha|^2 \sim 0.1 \). Substituting \( |\psi_f\rangle \) of Eq. (1) into \( |\psi_f^{ab}\rangle \) of Eq. (22), \( \Delta \Pi^{ab}(\alpha, \beta) \) is
\[ \Delta \Pi^{ab}(\alpha, \beta) \approx -2 \langle \psi_f^{ab} | \hat{\Pi}^{ab}(\alpha, \beta) | \psi_f^{ab} \rangle \langle \psi_f^{ab} | (\Delta(\hat{n}_a(\alpha)) + \Delta(\hat{n}_b(\beta))) | \psi_f^{ab} \rangle \]
\[ = -2 \langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle \{ c_1(\alpha, \beta) \delta^2 + c_2(\alpha, \beta) \delta \Delta \phi + (\Delta \phi)^2 \} \]  
\[ \text{(25)} \]

where the mean field approximation has been used [17]. The expectation value \( \langle \hat{\Pi}^{ab}(\alpha, \beta) \rangle = (2|\alpha - \beta|^2 - 1)e^{-2(|\alpha|^2 + |\beta|^2)} \), and the parameters \( c_1(\alpha, \beta) = 2(|\alpha|^4 + |\beta|^4) + \frac{13}{2}(|\alpha|^2 + |\beta|^2) + 5 \), and \( c_2(\alpha, \beta) = 2(|\alpha|^2 + |\beta|^2) + 4 \). The mean error \( \Delta B \) for \( \alpha = -\beta \approx \sqrt{0.1} \), is given by
\[ \Delta B \sim 10.2\delta^2 + 8.1\delta \Delta \phi + 2.0(\Delta \phi)^2. \]  
\[ \text{(26)} \]

The probing atoms normally have the Gaussian velocity distribution which causes the errors \( \delta \) and \( \Delta \phi \). When we consider the ensemble average over atoms, the second term vanishes in Eq. (24) and the first and third terms finally contribute to degrade the value of the Bell function.

For the test of nonlocality using single atoms as shown in Fig. 1(b), the mean error \( \Delta B \) is similarly obtained as
\[ \Delta B \sim 16.3\delta^2 + 8.2\delta \Delta \phi + 1.0(\Delta \phi)^2. \]  
\[ \text{(27)} \]

This is slightly larger than the error (24) for the two-atom scheme. The error enhancement in the single-atom scheme is due to the fact that the experimental errors are multiplied as the atom passes through two cavities. For the two-atom scheme, the error is a sum of errors occurred in each atom interaction with the cavity field and \( \pi/2 \) pulses.

We find that when the standard deviation of the atomic velocity distribution is 5\%, \( \Delta B \sim 0.06 \) for the two-atom scheme and \( \Delta B \sim 0.10 \) for the single-atom scheme, which still allows the observation of the violation of Bell’s inequality.

If the \( Q \) factor of a cavity is high, the cavity is very much closed. When an atom passes through the cavity walls the atom can lose the information on the phases of atomic states so that the scheme suggested in this paper cannot be used. However, recently, Nogues \textit{et al.} suggested a way to implement \( \pi/2 \) pulses and cavity fields interactions inside the cavity [18]. If this scheme is applied there will not be a problem of losing the \( Q \) value to keep the atomic phase information.
M.S.K. thanks Professor Walther for discussions and hospitality at the Max-Planck-Institut für Quantenoptik where a part of this work was carried out. This work was supported by the BK21 grant (D-0055) by the Korean Ministry of Education.
REFERENCES

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] J. S. Bell, Physics 1, 195 (1964).
[3] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).
[4] E. Hagley, X. Maitre, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 79, 1 (1997).
[5] M. P. Brune, P. Nussenzveig, F. Schmidt-Kaler, F. Bernardoe, A. Maali, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 72, 3339 (1994); Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. 75, 4710 (1995).
[6] L. Davidovich, N. Zagury, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A 50, 895 (1994).
[7] S. J. D. Phoenix and S. M. Barnett, J. Mod. Opt. 40, 979 (1993); I. K. Kudryavtsev and P. L. Knight, J. Mod. Opt. 39, 1411 (1992).
[8] P. Meystre, in Progress in Optics XXX, edited by E. Wolf (Elsevier, Amsterdam, 1992).
[9] L. Davidovich, A. Maali, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 71, 2360 (1993).
[10] K. Banaszek and K. Wódkiewicz, Phys. Rev. A 58, 4345 (1998); K. Banaszek and K Wódkiewicz, Phys. Rev. Lett. 82, 2009 (1999).
[11] B.-G. Englert, N. Sterpi, and H. Walther, Opt. Commun. 100, 526 (1993).
[12] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A 45, 5193 (1992).
[13] Coupling of the cavity with an external stable was experimentally demonstrated in [M. Brune et al., Phys. Rev. Lett. 76, 1800 (1996)].
[14] H. Moya-Cessa and P. L. Knight, Phys. Rev. A 48, 2479 (1993).
[15] For a single-mode cavity field, the reconstruction of the Wigner function is studied using the similar scheme as discussed in this paper by [L. G. Lutterbach and L. Davidovich, Phys. Rev. Lett. 78, 2547 (1997)].
[16] The interaction time is subject to thermal noise with a normalized standard deviation 1% [M. S. Kim, G. Antesberger, C. T. Bodendorf, and H. Walther, Phys. Rev. A 58, R65 (1998)].
[17] We checked the mean field approximation as we calculate the exact value of Eq. (22) for the nonlocal state (1). The result in Eq. (23) is within 0.1% of the exact algebraic calculation which implies that the correlation effect between $\hat{\Pi}$ and $\Delta(\hat{n})$ is negligible for fields of small amplitudes. We conjecture that the mean field approximation holds for the displaced nonlocal fields of the displacements $|\alpha| < 1$ and $|\beta| < 1$.
[18] G. Nogues, A. Rauschenbeutel, S. Osnaghi, M. Brune, J. M. Raimond, and S. Haroche, Nature 400, 239 (1999).
FIGURES

FIG. 1. Schematic diagram of the quantum nonlocality test for cavity fields. (a) Two two-level atoms pass through two cavities and joint measurements of atomic levels are performed after the cavity interaction. (b) A single two-level atom passes sequentially through two cavities and a measurement of atomic level is performed after the atom-field interaction in the cavities.
(a)

atomic state detector

$\alpha$

$\pi/2$ pulse

atomic state detector

$\beta$

$\pi/2$ pulse

$\pi/2$ pulse

$\pi/2$ pulse
(b)

\[ \pi/2 \text{ pulse} \rightarrow \alpha \rightarrow \beta \rightarrow \text{atomic state detector} \]

\[ \pi/2 \text{ pulse} \]