Review on Resistivity Inversion of Underground Abnormal Bodies

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Abstract: With the development of petroleum in China, most oilfields have entered the stage of high water cut development. Well ground resistivity method as a new type of electric prospecting method, which have influence on the formation of small, measuring low cost advantages, gradually become one of the key technology of remaining oil distribution in the detection of resistivity inversion method is through observation of the underground space apparent resistivity data reconstruction of the underground resistivity distribution, can realize the resistivity imaging of the underground space. In order to achieve the morphological characterization and spatial location of the abnormal area of underground resistivity, and then to carry out geological interpretation, the definition and properties of resistivity inversion are summarized, and the bottleneck problems encountered in practical engineering application are re-recognized and analyzed. On this basis, the theoretical method, numerical method and inversion method based on machine learning are introduced to solve the inverse resistivity problem of underground abnormal body. The inversion method based on deep learning is emphatically introduced, and its advantages and disadvantages and applicability are evaluated. It is pointed out that inversion is an ideal tool for data analysis. Then, it is pointed out that the development direction of resistivity inversion of underground abnormal body is to propose an optimized inversion network architecture based on deep learning.

Keywords: Underground Resistivity, Abnormal Body, Inversion Method, Deep Learning

1. Introduction

In the actual production process, some products need to be designed according to specific functions, and the above situations can be put forward as anti-problems. It can be seen that the emergence of the inverse problem is the result of the continuous progress of scientific exploration and the rapid improvement of production capacity. Meanwhile, the innovation of computing and processing technology also provides a strong thrust for its development. Now, the exploration of anti-problem has a profound influence on production practice. Although the specific forms of inverse problems are different in different disciplines, they have common characteristics in analysis methods and solving ideas [1]. This paper deals with the definition and characters of the inverse problem of induction and summary, this paper discusses the underground abnormal body resistivity inversion to solve inverse problems of theoretical method and numerical optimization algorithm and the inversion method based on the study, based on the above methods are reviewed and the comprehensive analysis, and the underground abnormal body resistivity inversion problem of commonly used methods are compared.

2. The Inverse Problem and Its Properties

2.1. Anti-problem Overview

It is called "inverse problem" in mathematical science and "inversion problem" in other fields. In scientific exploration, it plays a great role in finding laws of substances that cannot be
The method of solving the inverse problem is called "inversion method". The inverse problem is generally described mathematically [2] as follows: When data are expressed, if there are N data in the experiment, it can be expressed as formula (1) below.

\[ d = [d_1, d_2, ..., d_N]^T \]  

(1)

Then the set formed by D is called the data space. The model parameter K can be expressed in many ways due to its different models. For discrete cases, it is often expressed as a finite number of parameters. For example, for model M, it can be an M-dimensional column vector

\[ m = [m_1, m_2, ..., m_M]^T \]  

(2)

Let the vector function be \( f = [f_1, f_2, ..., f_n]^T \), then the general formula of the inversion problem is:

\[ f(d, m) = 0 \]  

(3)

Formula (1) is called the mathematical formula of the model. In general, it is written in the form (4) as follows:

\[ d = F(m) \]  

(4)

F is an N-dimensional vector, and form (4) is generally used as the basic theoretical equation of inversion.

### 2.2. The Antithesis Question Is Ill-Defined

In the process of solving the inversion problem, if the uniqueness of the solution, the existence of the solution, and the stability of the solution can be satisfied at the same time. Then, the inverse (inversion) problem can be said to be well-posed. However, the inverse problem of various engineering problems may often cause the unqualitability of their solutions due to the differences in the obtained fixed solution conditions, data space and observation data. In the process of solving the inverse problem, some relevant information can be added to overcome the inherent difficulties mentioned above. Therefore, the solution of the inverse problem is to recover the information of the original problem according to the relevant information provided, and then obtain the stable approximate solution of the inverse problem.

### 3. Abnormal Body Inversion of Underground Resistivity and Its Classification

Resistivity detection is a commonly used engineering detection technology at present. It has the advantages of low economic cost, small impact on underground and low efficiency, and has played an important role in transparent detection of urban underground space [3], and advance prediction of adverse geological conditions in tunnels and underground engineering. Resistivity detection against (inversion) problems by observing the apparent resistivity measurement underground space data, the data processing, and then reconstruct the underground resistivity distribution model, better realize the underground resistivity abnormal body shape features and spatial positioning, however, using the observation data for geological interpretation can not reveal the nature of the underground complex features, popular, it is to measure the electric field response, and then carry on the electric field response data processing, get a reasonable model, and then get the underground abnormal body spatial distribution. With the continuous development of underground space, geophysicists at home and abroad have been working on the reconstruction of more accurate and detailed inversion methods of underground attributes. According to the dimension of the model, it can be divided into one-dimensional, two-dimensional and three-dimensional inversion. According to the relationship between observed data and model parameters and the degree of linearization, it can be divided into linear inversion, iterative linear inversion and nonlinear inversion. It can be divided into traditional machine learning and deep learning inversion according to whether artificial construction model features are needed. In the development process, some scholars have studied and implemented multi-method and multi-data joint inversion method according to needs.

### 4. Inversion Method of Abnormal Body of Underground Resistivity

With the continuous development of underground space, many geophysicists have been committed to the inversion of underground anomalies with higher detection accuracy and resolution. Therefore, a variety of inversion methods have been proposed, which can be summed up as theoretical method, numerical method and learning based inversion method. It is worth noting that, in recent years, deep learning has demonstrated its outstanding ability in processing nonlinear function fitting in pathological inversion studies, such as ultra-high resolution imaging, medical image inversion, three-dimensional model reconstruction [4-6], etc., providing a new idea for underground resistivity inversion methods, such as: U-net [37], GAN [38], ResNet and FPN [39] and other optimized inversion network architectures based on deep learning.

#### 4.1. Theoretical Method

##### 4.1.1. Regularization

The process of solving the inverse resistivity problem is to search for the optimal resistivity spatial distribution model by using gradient and iteration methods under the condition that the model meets the acceptable fitting error range and other established constraints. Generally speaking, it is to find the minimum value of the total objective function:

\[ \varphi = \varphi_d + a\varphi_m \]  

(5)

Where, the total objective function is divided into two parts, observation objective function and roughness function, and is the regularization factor (Lagrange multiplier) of and.
OCCAM inversion method [2, 8] is one of the representative regularization inversion methods at present, which introduces smoothing factor and regularization ideas and greatly improves the multi-solution of inversion.

4.1.2. Constraint

Inversion method can usually by adding constraints to increase the stability of inversion, the various constraint conditions can build different model constraint object function, which is the typical model and square sum of the derivative and the parameters of the quadratic sum of squares is the smallest, in the detection process in order to improve the stability of inversion and, We can also impose constraints on the background and boundaries of the model. However, in practice, in order to reduce the fuzzy model, and avoid negative or unrealistic inversion values, some scholars in addition to conventional constraints, also impose inequality constraints, a fixed constraint of model parameter and time constraints, limit model parameters in a reasonable range, makes the theoretical model and the inversion model can fit well, Stable convergence of inversion [9, 10], better positioning and description [11], and can reduce the occurrence of false anomalies, better constrain underground dynamic targets, and track their changes with time [12].

4.2. Numerical Optimization Algorithm

4.2.1. Linear Inversion

Since the 1980s, linear methods have been continuously developed. The main method of linear method is to linearize nonlinear problems. In order to find the optimal solution of inversion, it iterates repeatedly on the gradient information of the objective function. Early successive α -center method [13-15], Zohdy method [16], Born approximation method [17] and other methods have made great progress in TWO-DIMENSIONAL and THREE-DIMENSIONAL resistivity inversion, but α -center method and Born approximation method have high requirements on the initial model contrast. Zohdy method is a direct fitting method. Although the iteration speed is fast, the overall inversion accuracy is not high. At present, the linear inversion method represented by the minimization of objective function constructed by regularized least squares [18-20] formula is more commonly used in research and application. Although linear method inversion provides a relatively stable inversion tool for resistivity detection, the inversion accuracy is insufficient and partial derivative matrix is difficult to solve. such as, in addition, the conditions are not fully known, initial model selection of discomfort, It will lead to local optimization or even wrong solution when the linear inversion method performs resistivity inversion imaging.

4.2.2. Iterative Linear Inversion

These methods can be classified according to the numerical optimization algorithm, as shown in the figure, where F(m) is the forward response function, CG is the conjugate gradient method, is the smooth factor, and H (Hessian matrix) is the second derivative of the objective function.

Quasi-linearization methods include Newton method and some inversion methods based on Newton method, including Gauss-Newton method, quasi-Newton method, OCCAM inversion method, gradient method, conjugate gradient method [21-23], improved gradient regularization method, etc.

Newton's method and quasi-Newton's method can solve the optimization problem with unconstrained conditions. Both of them are iterative algorithms. Quasi-newton's method simplifies the steps of Newton's method to obtain the Hessian matrix through positive definite matrix approximation, and gets better results.

OCCAM inversion [7, 8] is based on gauss-Newton method, which introduces the idea of smoothing factor and regularization. While finding the maximum fit between the model and the original data, it adjusts the Lagrange multiplier and calculates the roughness matrix. The model can be as smooth as possible, and the operation is basically stable and converging. Moreover, the smoothest inversion result is selected as the best inversion result, which greatly improves the multi-solution of inversion.

The conjugate gradient method [21] only uses the first derivative to solve the slow convergence problem of the fastest descent method and the complex calculation problem of Newton method. Its advantages are small storage volume, high stability, step convergence, and no need to add parameters outside the system.

4.2.3. Completely Nonlinear Inversion

Fully nonlinear inversion method, which can be called the fundamental method to solve nonlinear inverse problems, can realize the mapping from data space to model space, including
Monte Carlo method, exhaustive method, simulated annealing method [22-24], genetic algorithm [40], ant colony algorithm, etc. [26]. Monte Carlo method [25] blindly searches the solution space when solving inverse problems, while exhaustive method thoroughly searches the solution space, which is generally difficult to achieve in practical application due to the large amount of data calculation.

Simulated Annealing algorithm (SA) [22-24] does not require partial derivative matrix, requires little prior information, and has the advantages of strong continuous search ability and strong learning ability. However, in practical application, especially in multidimensional problems, resulting in large calculation time and cost. And in the application process will be limited by model space and search methods.

Genetic algorithm is a global optimal algorithm based on the survival of the fittest. In resistivity inversion application, some scholars put forward a kind of mutation direction control method based on joint algorithm [40], the improved genetic algorithm on the objective function and application of the smooth constraints and the inequality constraints, for the objective function of the non uniqueness and pathological change a certain degree of reduced, the method to optimize the mutation direction, eliminate dependence on initial model, Greatly improved efficiency.

Ant colony algorithm, with the global minimum, search capability advantages, have a kind of parallelism and positive feedback optimization algorithm, the method to simulate ants action model, based on the heuristic idea, use pheromones (Pheromone), gradually converge to find the global optimal solution, the ACO and other bionic optimization method for effective integration, formed the advantage with each other, To some extent, problems such as over-fitting, slow convergence rate and low accuracy [7, 30-32]. Based on this, domestic and foreign geophysicists have carried out in-depth research on this issue and put forward a series of improved methods, such as: Jiang et al. proposed COSFLA optimization based on wavelet packet denoising and ANFIS network [33] and resistivity imaging inversion based on kernel principal component wavelet neural network based on ISFLA training [34], which achieved good results by using kernel principal component wavelet neural network. Secondly, a resistivity inversion method based on CCSFLA-MSVR hybrid method was proposed [35]. Multi-output support vector regression of finite ERI learning samples was introduced into resistivity inversion research. Some scholars also proposed resistivity neural network inversion imaging based on IGA algorithm [36]. It combines BP neural network and immune genetic algorithm effectively, overcomes the above shortcomings of neural network, and improves the inversion accuracy and computational efficiency.

### 4.3.2. Based on Deep Learning Inversion Method

With the continuous exploration of deep learning, scholars have found that deep learning performs well in approximating very complex nonlinear mapping functions, and is even more superior to ill-posed inverse problems. Based on this, a series of inversion methods based on deep learning are proposed. Liu et al. (2020) [37] published Deep Learning Inversion of Electrical Resistivity Data introduces the deep learning method to solve the inverse Resistivity problem and proposes ERSInvNet, a Resistivity deep learning inversion network that can realize real-time reasoning based on U-NET structure (as shown in Figure 2). Convolutional neural network (CNNs) is used to directly establish the mapping relationship between the apparent resistivity data (input) and the resistivity model (output). Furthermore, depth weighting function and smoothing constraint are introduced into the loss function to construct the prior input structure of the layer feature image, which improves the accuracy of deep inversion and inhibits false anomalies.

![Figure 2. U-NET architecture.](image-url)
Due to general resistivity depth inversion using supervised learning method, has certain difficulty in the process of practical engineering exploration, according to the above problem, Shang Yuting et al. [38] is presented based on the generated against deep learning network architecture of resistivity inversion method, through antagonism training and confrontational loss mechanism, only need a small amount of tags in the process of training data, Achieving a semi-supervised learning training (as shown in figure 3 and a half to supervise net structure), effectively reduce the data set is hard work, and to improve the discrimination device, used to distinguish the data and model, the relationship between the inversion results overall approximation for supervised learning, and in terms of volume form, its precision is higher than supervised learning. Liu Yu et al. [39] proposed a convolution neural network combining ResNet (Deep residual network) and FPN (Multi-layer Scale feature Fusion Detection algorithm), which achieved good results in the application of THREE-DIMENSIONAL resistivity inversion, improved the accuracy of inversion interpretation, and could achieve small error of inversion results and prominent abnormal bodies.

5. Conclusion

With the development of resistivity inversion in recent years, various inversion methods have different application fields and solving advantages. Although the linearized inversion method provides relatively stable tools, it has disadvantages such as insufficient inversion accuracy and dependence on the initial model. The iterative linearized inversion method has the advantages of less calculation time, but it often depends on the selection of the initial value. The nonlinear inversion methods of global optimization have been developing continuously. Among them, simulated annealing method has the advantages of less dependence on the initial model and global minimum optimization, but it has some limitations in terms of operation time and cost.

In traditional machine learning and artificial neural network is one of the representative inversion method, which can be derived from the loss function of the gradient optimization, but exist in the practical application, such as slow convergence and low accuracy and over fitting phenomenon such as limitations, in the process of continuous development, for the main component of the wavelet neural network and the introduction of support vector regression, Good results were obtained. The inversion method based on deep learning directly establishes the mapping relationship between the data and the model, and proposes an effective deep learning optimization inversion network architecture, which effectively improves the resistivity inversion quality, improves the dependence of the initial model, and improves the local optimum.

6. Discussion

Resistivity of the inverse problem solving process is in the model to meet the acceptable data fitting error range and other established under constraint conditions, to seek the optimal process of underground abnormal body space distribution model, so the optimization inversion method in the problem solving of underground abnormal body resistivity inversion has been widely used, it has been committed to more accurate, more detailed reconstruction property.

Theoretical method and numerical optimization algorithm, has certain limitation in the process of inversion application, along with the deep learning method gradually introducing detection resistivity inversion domain, to the underground abnormal body resistivity inversion opened the new way of thinking and the depth of the main focus is to put forward effective learning optimization inversion network architecture, and then improve the inversion accuracy, but in the practical engineering. There are still some difficulties: 1) It is difficult to make a large number of data sets required by deep learning; 2) It is difficult to acquire a large number of real geological models corresponding to the observed data; 3) Specific data sets should be refined and enriched in different application fields; 4) In the face of the diversity of anomalous bodies, a scientific and reasonable deep learning optimization inversion network framework is proposed; 5) Four-dimensional resistivity inversion imaging cannot be realized to strengthen
the monitoring of abnormal bodies. These problems will become the research hotspot in the future and have great research value and significance.

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