Quantum nondemolition measurements and non–Newtonian gravity.

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Abstract

In the present work the detection, by means of a nondemolition measurement, of a Yukawa term, coexisting simultaneously with gravity, has been considered. In other words, a nondemolition variable for the case of a particle immersed in a gravitational field containing a Yukawa term is obtained. Afterwards the continuous monitoring of this nondemolition parameter is analyzed, the corresponding propagator is evaluated, and the probabilities associated with the possible measurement outputs are found. The relevance of these kind of proposals in connection with some unified theories of elementary particles has also been underlined.

1 Introduction

The equivalence principle (EP) is one of the fundamental cornerstones in modern physics, and comprises the underlying symmetry of general relativity (GR) [1]. At this point we must be more precise and state that EP has three different formulations, namely the weak, the medium strong, and finally, the very strong equivalence principle. In order to avoid misunderstandings, here we follow [1], namely weak equivalence principle (WEP) means the motion of any freely falling test particle is independent of its composition and structure, medium strong form (MSEP) means for every pointlike

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event of spacetime, there exists a sufficiently small neighborhood such that in every
local, freely falling frame in that neighborhood, all the nongravitational laws of physics
obey the laws of special relativity. Replacing all the nongravitational laws of physics
with all the laws of physics we have the very strong form of the equivalence principle
(VSEP).

The proposals that confront the predictions of GR with measurement outputs
include already a large amount of experiments, for instance, the gravitational time
dilation measurement [2], the gravitational deflection of electromagnetic waves [3],
the time delay of electromagnetic waves in the field of the sun [4], or the geodetic
effect [5]. The discovery of the first binary pulsar PSR1913+16 [6] allowed not only to
probe the propagation properties of the gravitational field [7], but it also offered the
possibility of testing the case of strong field gravity [8]. Of course, all these impressive
experiments are an indirect confirmation of the different EP.

Another important experimental direction comprises the attempts to test, directly,
WEP. Though these efforts are already more than a century old [9], the interest in
this area has not disappeared. Recently [10], WEP has been tested using a rotating 3
ton $^{238}$U attractor around a compact balance containing Cu and Pb test bodies. The
differential acceleration of these test bodies toward the attractor was measured, and
compared with the corresponding gravitational acceleration. Clearly, this proposal
is designed to test WEP at classical level, i.e., gravity acts upon a classical system.
At quantum realm the gravitational acceleration has been measured using light pulse
interferometers [11], and also by atom interferometry based on a fountain of laser–
cooled atoms [12]. Of course, the classical experiment by Colella, Overhauser, and
Werner (COW) [13], is also an experiment that explores the effects at quantum level
of gravity, and shows that at this level the effects of gravity are not purely geometric
[14].

The interest behind these experiments stems from the fact that various theoretical
attempts to construct a unified theory of elementary particles predict the existence
of new forces, and they are usually not described by an inverse–square law, and of
course, they violate one of the formulations of EP. By studying these violations one
could determine what interaction was producing these effects [15].

Among the models that in the direction of noninverse–square forces currently exist
we have Fujii’s proposal [16], in which a “fifth force”, coexisting simultaneously with
gravity, comprises a Yukawa term, $V(r) = -G_\infty \frac{mM}{r} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$, here $G_\infty$ describes
the interaction between $m$ and $M$ in the limit case $r \to \infty$, i.e., $G = G_\infty (1 + \alpha)$, where
$G$ is the Newtonian gravitational constant. This kind of deviation terms arise from
the exchange of a single new quantum of mass $m_5$, where the Compton wavelength
of the exchanged field is $\lambda = \frac{\hbar}{m_5c}$ [15], this field is usually denoted dilaton.
The experiments, already carried out, that intend to detect a Yukawa term have already imposed some limits on the parameters $\alpha$ and $\lambda$. For instance, if $10^{-4}m \leq \lambda \leq 10^{-3}m$, then $\alpha \sim 10^{22}$ [17], if $\lambda = 200\mu M$, then $\alpha \leq 8 \times 10^7$ [18] (for a more complete report see [15]).

To date, after more than a decade of experiments [19], there is no compelling evidence for any kind of deviations from the predictions of Newtonian gravity. But Gibbons and Whiting (GW) phenomenological analysis of gravity data [20] has proved that the very precise agreement between the predictions of Newtonian gravity and observation for planetary motion does not preclude the existence of large non-Newtonian effects over smaller distance scales, i.e., precise experiments over one scale do not necessarily constrain gravity over another scale. GW results conclude that the current experimental constraints over possible deviations did not severely test Newtonian gravity over the 10–1000m distance scale, usually denoted as the “geophysical window”.

The idea in this work is two-fold: firstly, the effects of a Yukawa term upon a quantum system (the one is continuously monitored) will be calculated; secondly, new theoretical predictions for one of the models in the context of quantum measurement theory will be found. In order to achieve these two goals we will obtain a non-demolition variable for the case of a particle subject to a gravitational field which contains a Yukawa term such that $\lambda$ has the same order of magnitude of the radius of the earth. At this point it is noteworthy to mention that the current experiments set constraints for $\lambda$ for ranges between 10km and 1000km [10], but the case in which $\lambda \sim$ Earth’s radius remains rather unexplored. Afterwards, we will consider, along the ideas of the so-called restricted path integral formalism (RPIF) [21], the continuous monitoring of this non-demolition parameter, and calculate, not only, the corresponding propagators, but also the probabilities associated with the different measurement outputs.

## 2 Yukawa term

Suppose that we have a spherical body with mass $M$ and radius $R$. Let us now consider the case of a Yukawa form of gravitational interaction [16], hence the gravitational potential of this body reads

$$V(r) = -G_{\infty} \frac{M}{r} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right).$$

Let us now write $r = R + z$, where $R$ is the body’s radius, and $z$ the height over its surface. If $R/\lambda \sim 1$ (which means that the range of this Yukawa term has the
same order of magnitude as the radius of our spherical body), and if \( z \ll R \), then we may approximate the Lagrangian of a particle of mass \( m \) as follows

\[
L = \frac{\vec{p}^2}{2m} + G_\infty \frac{mM}{R} \left[ 1 + \alpha \right] - \left[ \frac{1 + \alpha}{R} + \frac{\alpha}{2\lambda} \right] z + \left[ \frac{1 + \alpha}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha^2}{2\lambda^2} \right] z^2 .
\]  
(2)

### 3 Quantum Measurements

Nowadays one of the fundamental problems in modern physics comprises the so-called quantum measurement problem [22]. Though there are several attempts to solve this old conundrum (some of them are equivalent [23]), here we will resort to RPIF [21], because it allows us to calculate, in an easier manner, propagators and probabilities. RPIF explains a continuous quantum measurement with the introduction of a restriction on the integration domain of the corresponding path integral. This last condition can also be reformulated in terms of a weight functional that has to be considered in the path integral. Clearly, this weight functional contains all the information about the interaction between measuring device and measured system. This model has been employed in the analysis of the response of a gravitational antenna not only of Weber type [21], but also when the measuring process involves a laser-interferometer [24]. We may also find it in the quest for an explanation of the emergence of some classical properties, as time, in quantum cosmology [25].

Suppose now that our particle with mass \( m \) goes from point \( N \) to point \( W \). Hence its propagator reads

\[
U(W, \tau''; N, \tau') = \left( \frac{m}{2\pi i\hbar T} \right)^{\frac{1}{2}} \exp \left\{ \frac{im}{2\hbar T} \left[ (x_W - x_N)^2 + (y_W - y_N)^2 \right] \right\} \times \int_{z_N}^{z_W} d[p] d[z(t)] \exp \left\{ \frac{i}{\hbar} \int_{\tau'}^{\tau''} \left[ \frac{p^2}{2m} + (1 + \alpha) \frac{G_\infty mM}{R} + Ft + \frac{m}{2} \Omega^2 z^2 \right] dt \right\} .
\]  
(3)

Here we have introduced the following definitions

\[
F = -G_\infty \frac{mM}{R} \left[ \frac{1 + \alpha}{R} + \frac{\alpha}{2\lambda} \right] ;
\]  
(4)
\[ \omega^2 = -2G_\infty M \frac{1 + \alpha}{2R^2} \left( + \frac{\alpha}{2\lambda R} + \frac{\alpha}{2\lambda^2} \right). \quad (5) \]

\[ T = \tau'' - \tau', \quad \sqrt{(x_W - x_N)^2 + (y_W - y_N)^2} \] denotes the projection on the body’s surface of the distance between points \( W \) and \( N \). We also have that \( G = G_\infty[1 + \alpha] \), and \( G \) is the Newtonian gravitational constant \([15]\). In our case, \([\frac{1+\alpha}{2R^2} + \frac{\alpha}{2\lambda R} + \frac{\alpha}{2\lambda^2}] > 0 \), hence, \( \omega = i\Omega \), where \( \Omega \in \mathbb{R} \).

Suppose now that the variable \( A(t) \) is continuously monitored. Then we must consider, along the ideas of RPIF, a particular expression for our weight functional, i.e., for \( w_{[a(t)]}[A(t)] \). As was mentioned before, the weight functional \( w_{[a(t)]}[A(t)] \) contains the information concerning the measuring process.

At this point we face a problem, namely, the choice of our weight functional. In order to solve this difficulty, let us mention that the results coming from a Heaveside weight functional \([26]\) and those coming from a gaussian one \([27]\) coincide up to the order of magnitude. These last remarks allow us to consider a gaussian weight functional as an approximation of the correct expression. But a sounder justification of this choice stems from the fact that there are measuring processes in which the weight functional possesses a gaussian form \([28]\). In consequence we could think about a measuring device whose weight functional is very close to a gaussian behaviour.

Therefore we may now choose as our weight functional the following expression

\[ \omega_{[a(t)]}[A(t)] = \exp \left\{ -\frac{T}{2\Delta a^2} \int_{\tau'}^{\tau''} [A(t) - a(t)]^2 \, dt \right\}, \quad (6) \]

here \( \Delta a \) represents the error in our measurement, i.e., it is the resolution of the measuring apparatus.

4 Quantum nondemolition measurements

The basic idea around the concept of quantum nondemolition (QND) measurements is to carry out a sequence of measurements of an observable in such a way that the measuring process does not diminish the predictability of the results of subsequent
measurements of the same observable \[29\]. This concept stems from the work in the context of gravitational wave antennae. Indeed, the search for gravitational radiation demands measurements of very small displacements of macroscopic bodies \[30\]. Braginsky et al \[31\] showed that there is a quantum limit, the so called “standard quantum limit”, which is a consequence of Heisenberg uncertainty principle, the one limits the sensitivity of the corresponding measurement (the original work \[31\] involves the sensitivity of a gravitational antenna). This work allowed also the introduction of the idea of a QND measurement, in which a variable is measured in such a way that the unavoidable disturbance of the conjugate observable does not disturb the evolution of the chosen variable \[32\].

Let us now suppose that in our case \(A(t) = \rho p + \sigma z\), where \(\rho\) and \(\sigma\) are functions of time. In this particular case, the condition that determines when \(A(t)\) is a QND variable may be written as a differential equation \[21\]

\[
\frac{df}{dt} = \frac{f^2}{m} - m\Omega^2,
\]

(7)

where \(f(t) = \sigma/\rho\).

It is readily seen that a solution to (7) is

\[
f(t) = -m\Omega\tanh(\Omega t).
\]

(8)

Choosing \(\rho(t) = 1\), we find that in our case a possible QND variable is

\[
A(t) = p - m\Omega z\tanh(\Omega t).
\]

(9)

5 QND and non–Newtonian gravity: Propagators and probabilities

With our weight functional choice (expression (6)) the new propagator involves two gaussian integrals, and can be easily calculated \[33\]
\[
U_{[a(t)]}(W, \tau'', N, \tau') = \left(\frac{m}{2\pi \hbar T}\right) \exp\left\{\frac{im}{2\hbar T} \left[(x_W - x_N)^2 + (y_W - y_N)^2\right]\right\} 
\times \exp\left\{\frac{i}{\hbar}(1 + \alpha) G \frac{m M}{R} t\right\} \exp\left\{-T \Delta a^2 + i\frac{2m\hbar}{R} \int_{\tau'}^{\tau''} a^2(t)dt\right\} 
\times \exp\left\{-\frac{i\hbar}{2m\Omega^2} \int_{\tau'}^{\tau''} \left[ F + \frac{4m^2\hbar\Omega a}{4m^2\hbar^2 + T^2\Delta a^4} \tanh(\Omega t) + i\frac{2\hbar\Omega T \Delta a^2}{4m^2\hbar^2 + T^2\Delta a^4}\right]^2 \right\} 
\times \left[ \frac{4m^2\hbar^2 [1 + \tanh^2(\Omega t)] + T^2\Delta a^4 - i2m\hbar T \Delta a^2 \tanh^2(\Omega t)}{4m^2\hbar^2 [1 + \tanh^2(\Omega t)]^2 + T^2\Delta a^4}\right] dt}. \quad (10)
\]

The probability, \( P_{[a(t)]} \), of obtaining as measurement output \( a(t) \) is given by expression \( P_{[a(t)]} = |U_{[a(t)]}|^2 \) [21]. Hence, in this case

\[
P_{[a(t)]} = \exp\left\{-\frac{2T \Delta a^2}{4m^2\hbar^2 + T^2\Delta a^4} \int_{\tau'}^{\tau''} a^2(t)dt\right\} 
\times \exp\left\{\frac{\hbar}{m\Omega^2} \int_{\tau'}^{\tau''} \left[2I_1 I_2 I_3 + I_4 (I_2^2 - I_1^2)\right] dt\right\}. \quad (11)
\]

Here the following definitions have been introduced

\[
I_1 = \frac{F}{\hbar} + \frac{4m^2\hbar \Omega a}{4m^2\hbar^2 + T^2\Delta a^4} \tanh(\Omega t), \quad (12)
\]

\[
I_2 = \frac{2m\hbar \Omega T \Delta a^2}{4m^2\hbar^2 + T^2\Delta a^4} \tanh(\Omega t), \quad (13)
\]

\[
I_3 = \frac{4m^2\hbar^2 [1 + \tanh^2(\Omega t)] + T^2\Delta a^4}{4m^2\hbar^2 [1 + \tanh^2(\Omega t)]^2 + T^2\Delta a^4}, \quad (14)
\]

\[
I_4 = \frac{2m\hbar T \Delta a^2 \tanh^2(\Omega t)}{4m^2\hbar^2 [1 + \tanh^2(\Omega t)]^2 + T^2\Delta a^4}. \quad (15)
\]
6 Conclusions

In this work we have considered a Yukawa term coexisting with the usual Newtonian gravitational potential (expression (1)). Assuming that the range of this new interaction has the same order of magnitude than the Earth’s radius a QND variable was obtained for a particle with mass \( m \) (expression (9)), and its corresponding propagator was evaluated. Afterwards, it was assumed that this QND variable was continuously monitored, and, along the ideas of RPIF, the propagator and probability associated with the possible measurement outputs were also calculated, expressions (14) and (15), respectively.

Another interesting point around expressions (10) and (11) comprises the role that the mass parameter plays in them. It is readily seen that if we consider two particles with different mass, say \( m \) and \( \tilde{m} \), then they render different propagators and probabilities. This last fact means that “gravity” is, in this situation, not purely geometric. This is no surprise, the presence of a new interaction, coexisting with the usual Newtonian gravitational potential, could mean the breakdown of the geometrization of the joint interaction (Newtonian contribution plus Yukawa term). The detection of this interaction would mean the violation of WEP, but not necessarily of VSEP [15]. Nevertheless, the possibilities of using quantum measurement theory to analyze the possible limits of VSEP at quantum level do not finish here, indeed, the possible incompatibility between the different formulations of EP and measurement theory can also be studied along the ideas of this theory [42]. Of course, more work is needed around the validity at quantum level of the different formulations of EP. Indeed, as has already been proved [43], in the presence of a gravitational field, the generalization to the quantum level of even the simplest kinematical concepts, for instance the time of flight, has severe conceptual difficulties.

This proposal would also render new theoretical predictions that could be confronted (in the future) against the experiment, and therefore we would obtain a larger framework that could allow us to test the validity of RPIF [44]. As was mentioned before, this comprises our second goal in the present work.

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References

[1] I. Ciufolini and J. A. Wheeler, “Gravitation and Inertia”, Princeton University Press, Princeton, New Jersey, (1995).

[2] R. F. C. Vessot, L. W. Levine, Gen. Rel. and Grav. 10, 181 (1979); R. F. C. Vessot, et al., Phys. Rev. Lett. 45, 2081 (1980).

[3] R. N. Truehaft and S. T. Lowe, Phys. Rev. Lett. 62, 369 (1989).

[4] T. P. Krisher, J. D. Anderson, and A. H. Taylor, Astrophys. J. 373, 665 (1991).

[5] J. O. Dickey et al., Science 265, 482 (1994).

[6] R. A. Hulse and J. H. Taylor, Astrophys. J. Lett. 195, L51 (1975).

[7] J. H. Taylor, Class. Quant. Grav. 10, S167 (1993).

[8] T. Damour and J. Taylor, Phys. Rev. D45, 1840 (1992).

[9] R. v. Eötvös, “Über die Anziehung der Erde auf verschiedene Substanzen”, in Roland Eötvös Gesammelte Arbeiten, P. Selényi ed., Budapest, Akademiai Kiado (1953).

[10] G. L. Smith, C. D. Hoyle, J. H. Gundlach, E. G. Adelberger, B. R. Heckel, and H. E. Swanson, Phys. Rev. D61, 022001 (1999).

[11] M. Kasevich and S. Chu, Appl. Phys. B54, 321 (1992).

[12] A. Peters, K. Y. Chung, and S. Chu, Nature 400, 849 (1999).

[13] R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. 34, 1472 (1975); K. C. Littrell, B. E. Allman, and S. A. Werner, Phys. Rev. A56, 1767 (1997).

[14] J. J. Sakurai, “Modern Quantum Mechanics”, Addison-Wesley Publishing Company, Reading, Mass. (1995).

[15] E. Fishbach and C. L. Talmadge, “The Search for Non–Newtonian Gravity”, Springer–Verlag, New York (1999).

[16] F. Fujii, Nature 234, 5 (1971).

[17] G. Carugno, Z. Fontana, R. Onofio, and C. Rizzo, Phys. Rev. D55, 6591 (1997).
[18] R. Onofrio, Mod. Phys. Lett. A15, 1401 (1998).

[19] D. R. Long, Phys. Rev. D9, 850 (1974); F. D. Stacey, Rev. Mod. Phys. 59, 157 (1987).

[20] G. W. Gibbons and B. F. Whiting, Nature 291, 636 (1981).

[21] M. B. Mensky, “Continuous Quantum Measurements and Path Integrals”, IOP, Bristol and Philadelphia (1993).

[22] R. Omnes, “The interpretation of quantum mechanics”, Princeton University Press, Princeton 1994.

[23] C. Presilla, R. Onofrio, and U. Tambini, Ann. Phys. 248, 95 (1996).

[24] A. Camacho, Int. J. Mod. Phys. A14, 1997 (1999).

[25] M. B. Mensky, Class. Quan. Grav. 7, 2317 (1990); A. Camacho, “Decoherence and time emergence”, in Proceedings of the International Seminar: Current Topics in Mathematical Cosmology, M. Rainer and H.-J. Schin dt, eds., World Scientific Publishing Co., Singapore (1998); A. Camacho and A. Camacho–Galván, Nuov. Cim. B114, 923 (1999).

[26] M. B. Mensky, Phys. Rev. D20, 384 (1979).

[27] M. B. Mensky, Sov. Phys. JETP. 50, 667 (1979).

[28] M. B. Mensky, Physics–Uspekhi 41, 923 (1998).

[29] M. F. Bocko and R. Onofrio, Rev. Mod. Phys. 68, 755 (1996).

[30] K. S. Thorne, Rev. Mod. Phys. 52, 299 (1980).

[31] V. B. Braginsky, Yu. I. Vorontsov, and V. D. Krivchenkov, Sov. Phys. JETP 41, 28 (1975).

[32] V. B. Braginsky, Yu. I. Vorontsov, and F. Ya. Khalili, Pis’ma Zh. Eksp. Teor. Fiz. 73, 296 (1978).

[33] W. Dittrich and M. Reuter, “Classical and Quantum Dynamics”, Springer–Verlag, Berlin (1996).

[34] R. Thompson, “Quantum optics with trapped ions”, in Latin–American School of Physics XXXI ELAF, S. Hacyan, R. Jáuregui, and R. López–Peña, eds., American Institut of Physics, Woodbury, New York (1999).
[35] W. Paul, Rev. Mod. Phys. 62, 531 (1990); H. Dehmelt, Rev. Mod. Phys. 62, 525 (1990).

[36] M. F. Bocko and W. W. Johnson, Phys. Rev. Lett. 48, 1371 (1982).

[37] V. B. Braginsky and F. Ya. Khalili, Rev. Mod. Phys. 68, 1 (1996).

[38] J. F. Roch, G. Roger, P. Grangier, J.-M. Courty, and S. Reynaud, Appl. Phys. B55, 291 (1992).

[39] I. Marzoli and P. Tombesi, Europhys. Lett. 24, 515 (1993).

[40] A. Camacho, Phys. Lett. A256, 339 (1999); A. Camacho, Phys. Lett. A262, 110 (1999).

[41] A. Camacho, “Quantum–mechanical detection of non–Newtonian gravity”, Int. J. Mod. Phys. D, in press.

[42] A. Camacho, Mod. Phys. Lett. A14, 275 (1999); A. Camacho, Mod. Phys. Lett. A14, 2545–2556 (1999); A. Camacho, Mod. Phys. Lett. A15, 1461 (2000).

[43] L. Viola and R. Onofrio, Phys. Rev. D55, 455 (1997); R. Onofrio and L. Viola, Mod. Phys. Lett. A12, 1411 (1997).

[44] J. Audretsch, M. B. Mensky, and V. Namiot, Phys. Letts. A237, 1 (1997); A. Camacho and A. Camacho–Galván, Phys. Letts. A247, 373 (1998); A. Camacho, Phys. Lett. A277, 7 (2000).