A New Method of Reliability Evaluation Based on Wavelet Information Entropy for Equipment Condition Identification

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Abstract. Aiming at reliability evaluation of condition identification of mechanical equipment, it is necessary to analyze condition monitoring information. A new method of reliability evaluation based on wavelet information entropy extracted from vibration signals of mechanical equipment is proposed. The method is quite different from traditional reliability evaluation models that are dependent on probability statistics analysis of large number sample data. The vibration signals of mechanical equipment were analyzed by means of second generation wavelet package (SGWP). We take relative energy in each frequency band of decomposed signal that equals a percentage of the whole signal energy as probability. Normalized information entropy (IE) is obtained based on the relative energy to describe uncertainty of a system instead of probability. The reliability degree is transformed by the normalized wavelet information entropy. A successful application has been achieved to evaluate the assembled quality reliability for a kind of dismountable disk-drum aero-engine. The reliability degree indicates the assembled quality satisfactorily.

1. Introduction

The reliability evaluation of condition identification of mechanical equipment is very important for industrial enterprises. The reliability of mechanical equipment always has significant impact on operational safety and product competitiveness. To comply with these requirements, it is necessary to evaluate the operational reliability by means of condition monitoring information. Most of existing reliability methods focus on probability statistics analysis based on large number of sample data. However, these methods are usually difficult to evaluate equipment reliability with small number of sample data for lack of probability. Furthermore, these methods rarely utilize the operational condition information of mechanical equipment to evaluate the operational reliability and predict machine breakdowns or other serious risks occurrences. How to deal with the problems without probability statistics? How to obtain reliability degree (or level) that is very important evaluation indicator in reliability engineering? How to calculate the reliability degree according to information entropy instead of probability statistics?

Information entropy, which was first presented by Claude E. Shannon in 1948, is an effective indicator to measure a system uncertain degree. On the basis of the information entropy theory, the most uncertain probability distribution (such as the equal probability distribution) has the largest entropy, and the most certain probability distribution has the smallest entropy. From then on, the information entropy is widespread in engineering applications. Ren, et al. defined wavelet entropy,
relative wavelet entropy, and wavelet-time entropy to detect damage in structure [1]. El Safty, et al. adopted the wavelet information entropy with neural-fuzzy inference system to identify the transmission line fault and determine the phases involved in the fault of the power system [2]. Lin, J.L., et al. detected motor shaft misalignment using multiscale entropy with wavelet denoising [3]. Different types of the information entropy have been defined in accordance with their own usage, such as fuzzy entropy for adaptive bacterial foraging [4], sample entropy for tool state detection [5], hierarchical entropy for biological signal analysis [6], Renyi entropy for high-resolution scalar quantization [7], cross entropy for multi-target tracking [8], and so on. Since wavelet transform has its predominance in analyzing unsteady signals in both time domain and frequency domain. The wavelet information entropy is defined to represent dynamic characteristics of mechanical signals. Up to now, the wavelet information entropy has been applied in machinery condition monitoring and fault diagnosis. On the whole, the current researches are able to diagnose some mechanical faults, structural damage, fault classification [1-3] etc., but there are few studies of the reliability evaluation of mechanical equipment with the wavelet information entropy.

A new method of reliability evaluation based on the wavelet information entropy that is normalized is proposed, which is realized by extracting vibration signals from mechanical equipment and evaluating reliability using second generation wavelet package (SGWP) and the wavelet information entropy. The reliability degree is transformed by normalizing the wavelet information entropy that belongs to [0, 1]. In this instance, the reliability degree value is uniformly and monotonously mapped into [0, 1] that coincides with the traditional definition of the reliability degree. A successful application has been achieved to evaluate the assembled quality reliability for a kind of dismountable disk-drum aero-engine. The reliability degree indicates the assembled quality satisfactorily.

2. Wavelet information entropy

2.1 Second generation wavelet package (SGWP)

Analyzed signals often have a local correlation structure, that is, adjacent samples are much more correlated than those far from each other. Original signal $S = \{x(k), k \in Z\}$ can be divided into even series $s_e = \{s_e(k), k \in Z\}$ and odd series $s_o = \{s_o(k), k \in Z\}$.

\[
\begin{align*}
    s_e(k) &= x(2k) & k & \in Z \\
    s_o(k) &= x(2k + 1) & k & \in Z
\end{align*}
\]  

(2.1)

(2.2)

Since the odd and even subsets are highly correlated, several samples of even series can be used to predict a certain sample in the odd series, and the prediction difference is defined as detail signal. The even series can be updated using the obtained detail signal and modified even subset is defined as approximation signal.

\[
\begin{align*}
    s_{l1} &= s_{(l-1)10} - P(s_{(l-1)1e}) \\
    s_{l2} &= s_{(l-1)1e} + U(s_{l1}) \\
    \ldots \ldots \ldots
\end{align*}
\]

(2.3)

(2.4)

(2.5)

(2.6)

where $s_{l1}, s_{l2}, \ldots, s_{l2^l}$ are the decomposed signals respectively in each frequency band after $l^{th}$ decomposition; $s_{(l-1)10}, \ldots, s_{(l-1)2^{l-1}1_o}$ are odd series respectively after the $(l-1)^{th}$ decomposition;
\( s_{(l-1)e}, \ldots, s_{(l-2l-1)e} \) are even series respectively after the \((l-1)\)th decomposition; \( P( ) \) is defined as \( N \) point predictor whose prediction coefficients are \( p_1, p_2, \ldots, p_N \) and \( N \) is predictor order. \( U( ) \) is denoted as \( \tilde{N} \) point updater with update coefficients \( u_1, u_2, \ldots, u_{\tilde{N}} \) and \( \tilde{N} \) is updater order. The coefficients \( p_1, p_2, \ldots, p_N \) and \( u_1, u_2, \ldots, u_{\tilde{N}} \) can be got based on [9, 10].

We keep the signal in one frequency band to be reconstructed, and then set the others be zero. The signal reconstruction of SGWP for one appointed frequency band is as follows.

\[
\begin{align*}
    s_{(l-1)2^{l-1}e} &= s_{l2} - U(s_{l2^{l-1}}) \\
    s_{(l-1)2^{l-1}} &= s_{l2^{l-1}} + P(s_{(l-1)2^{l-1}e}) \\
    s_{(l-1)2^{l-1}e}(2k) &= s_{(l-1)2^{l-1}}(k) \quad k \in \mathbb{Z} \\
    s_{(l-1)2^{l-1}}(2k+1) &= s_{(l-1)2^{l-1}}(k) \quad k \in \mathbb{Z}
\end{align*}
\]

\[
\begin{align*}
    s_{(l-1)e} &= s_{l2} - U(s_{l1}) \\
    s_{(l-1)o} &= s_{l1} + P(s_{(l-1)e}) \\
    s_{(l-1)e}(2k) &= s_{(l-1)e}(k) \quad k \in \mathbb{Z} \\
    s_{(l-1)o}(2k+1) &= s_{(l-1)o}(k) \quad k \in \mathbb{Z}
\end{align*}
\]

where \( s_{(l-1)e}(2k), s_{(l-1)o}(2k+1), \ldots, s_{(l-2l-1)e}(2k), s_{(l-2l-1)o}(2k+1) \) are reconstructed signal of the appointed frequency band from the \((l-1)\)th reconstruction of SGWP.

2.2 Frequency band energy of SGWP

The SGWP decomposition obeys energy conservation principle because of its bi-orthogonal basis. After the \(l\)th decomposition, \(2^l\) frequency bands can be got that each frequency band has the same bandwidth and end to end. Let \( x_{i,j}(k) \) be the reconstructed signal at the \(l\)th decomposition in the \(i\)th frequency band, its energy \( E_{i,j} \) and relative energy \( \tilde{E}_{i,j} \) are respectively shown as follows.

\[
E_{i,j} = \frac{1}{n-1} \sum_{k=1}^{n} \left( x_{i,j}(k) \right)^2, \quad i = 1, 2, \ldots, 2^l, \quad k = 1, 2, \ldots, n, \quad n \in \mathbb{Z}
\]

\[
\tilde{E}_{i,j} = E_{i,j} \left( \sum_{i=1}^{2^l} E_{i,j} \right)^{-1}
\]

Obviously, \( \sum_{i=1}^{2^l} \tilde{E}_{i,j} = 1 \), the sum of total relative energy equals 1.

2.3 The normalized wavelet information entropy

The normalized wavelet information entropy \( \text{Ent} \) is defined as

\[
\text{Ent} = -\sum_{i=1}^{2^l} \tilde{E}_{i,j} \log_2 \tilde{E}_{i,j}
\]
In equation (2.17), the base of logarithm should be $2^l$. In this case the value $Ent$ belongs to $[0, 1]$. We can see that if $2^l$ frequency bands have the same value relative energy $(2^l)^{-1}$ (like equal probability distribution), the $Ent = 1$; if in $2^l$ frequency bands there is only one frequency band that concentrates whole energy, its relative energy is equal to 1 (like the most certain probability distribution), and the $Ent = 0$. So, we call $Ent$ the normalized wavelet information entropy.

2.4 The reliability degree based on normalized wavelet information entropy

When there are some faults occur in equipment, its conditions become more and uncertainty becomes larger. The normalized wavelet information entropy also becomes large in the nature of things, the equipment reliability becomes small. In the same way, the reliability degree also becomes small. Because the normalized wavelet information entropy $Ent$ belongs to $[0, 1]$, and the value of the reliability degree $R$ belongs to $[0, 1]$ too, we have a simple linear relation as follows

$$R = 1 - Ent, \quad R \in [0, 1]$$

(2.18)

Obviously, large $Ent$ means small $R$, contrarily small $Ent$ means large $R$.

3 Application in the assembled quality reliability evaluation for a kind of dismountable disk-drum aero-engine

3.1 Experimental system description

In order to validate the effectiveness of the proposed reliability evaluation method based on wavelet information entropy, it is applied in a kind of dismountable disk-drum aero-engine. The experiment system mainly contains a dismountable disk-drum aero-engine rotor, an exciter, a signal generator, sensors, and a data acquisition system. The sketch map of the experimental system is illustrated in Figure 1. The dismountable disk-drum aero-engine rotor is constituted of nine disks (serial number 1-9 in Figure 1), a labyrinth seal toothed disk (serial number 10 in Figure 1), and a shaft (serial number 11 in Figure 1). An exciter (serial number 12 in Figure 1) is installed under the shaft to excite the dismountable disk-drum aero-engine rotor. A signal generator is used to generate excitation signals for the exciter. Four accelerometers (serial number I-IV) are mounted on the sensor installation surface (A-A in Figure 1) which is close to the labyrinth seal toothed disk, so as to measure the dynamic response vibration signals of the dismountable disk-drum aero-engine rotor with bolts from looseness to tightness. Sony EX system is adopted as the data acquisition system to acquire and store the dynamic response vibration signals.

![Figure 1. Test bench of the dismountable disk-drum aero-engine. 1-the first disk; 2- the second disk; 3- the third disk; 4-the fourth disk; 5- the fifth disk; 6-the sixth disk; 7-the seventh disk; 8-the eighth disk; 9-the ninth disk; 10- labyrinth seal toothed disk; 11-shaft; 12-exciter; A-A- sensor installation surface; I -the first accelerometer; II -the second accelerometer; III-the third accelerometer; IV-the fourth accelerometer.](image-url)
3.2 Experimental results and analysis

The dynamic response signals are acquired from the experimental system from three assembled quality conditions: looseness condition 1 (tighten up the dismountable disk-drum aero-engine rotor with force moment $M_1$); looseness condition 2 (tighten up the dismountable disk-drum aero-engine rotor with force moment $M_2$ after condition 1); qualified condition 3 (tighten up the dismountable disk-drum aero-engine rotor with force moment $M_3$ after condition 2), $M_1 < M_2 < M_3$. For each condition, four signals are measured by the four accelerometers mentioned above. Therefore, twelve signals are attained from the three conditions.

The dynamic response signals under each assembled quality condition are damped in the time domain, shown in Figure 2. The sample frequency is 6400Hz. It is shown that the amplitude of the signal tested in looseness condition 1 (the dismountable disk-drum aero-engine rotor with force moment $M_1$) is smaller than both the amplitude of the signals tested in looseness condition 2 (the dismountable disk-drum aero-engine rotor with force moment $M_2$) and qualified condition 3 (the dismountable disk-drum aero-engine rotor with force moment $M_3$).

Since the SGWP is adopted to decompose signal of an assembled quality condition to the extent of level 3 by research experience, there are eight frequency bands attained. Each frequency band has the same bandwidth and end to end, which are shown in Figure 3. According to the equation (2.15), the frequency band energy of the SGWP from three assembled quality conditions are respectively computed, which are shown in Figure 4. It can be seen that the frequency band energy of the SGWP from three assembled quality conditions reflects several similarities and differences. The similarities are that the main frequency band energy of the SGWP from three assembled quality conditions lies in the fifth frequency band and the energy in other frequency band are much smaller. The differences are that the main frequency band (the fifth frequency band) energy of the SGWP increases and the sixth frequency band energy decreases with the assembled quality from looseness to tightness. The reason is that the structural stiffness is gradually increased with the assembled quality from looseness to tightness, so the rotor dynamic response takes the rotor natural frequency as the main information in qualified condition 3 (the dismountable disk-drum aero-engine rotor with force moment $M_3$) and some other signal information (such as the sixth frequency band) is incorporated in the rotor dynamic response of looseness condition 1 (the dismountable disk-drum aero-engine rotor with force moment $M_1$) and looseness condition 2 (the dismountable disk-drum aero-engine rotor with force moment $M_2$).

The normalized wavelet information entropy of the acquired four signals under each assembled quality condition is respectively computed according to the equation (2.17), which is shown in Table 1. Comparing with three assembled quality conditions, the normalized wavelet information entropy
decreases with the assembled quality from looseness (condition 1) to tightness (condition 3).

\[ X_{11}, X_{12}, X_{13}, X_{14} \]
\[ t/s \]

\[ X_{31}, X_{32}, X_{33}, X_{34} \]
\[ t/s \]

\[ X_{51}, X_{52}, X_{53}, X_{54} \]
\[ t/s \]

\[ X_{71}, X_{72}, X_{73}, X_{74} \]
\[ t/s \]

\[ X_{91}, X_{92}, X_{93}, X_{94} \]
\[ t/s \]

\[ X_{11}, X_{12}, X_{13}, X_{14} \]
\[ t/s \]

\[ X_{31}, X_{32}, X_{33}, X_{34} \]
\[ t/s \]

\[ X_{51}, X_{52}, X_{53}, X_{54} \]
\[ t/s \]

\[ X_{71}, X_{72}, X_{73}, X_{74} \]
\[ t/s \]

\[ X_{91}, X_{92}, X_{93}, X_{94} \]
\[ t/s \]

**Figure 3.** Signals analyzed by the SGWP from three assembled quality conditions. (a) condition 1; (b) condition 2; (c) condition 3

\[ X_{11}, X_{12}, X_{13}, X_{14} \]
\[ t/s \]

\[ X_{31}, X_{32}, X_{33}, X_{34} \]
\[ t/s \]

\[ X_{51}, X_{52}, X_{53}, X_{54} \]
\[ t/s \]

\[ X_{71}, X_{72}, X_{73}, X_{74} \]
\[ t/s \]

\[ X_{91}, X_{92}, X_{93}, X_{94} \]
\[ t/s \]

**Figure 4.** Frequency band energy of the SGWP from three assembled quality conditions.

**Table 1.** The normalized wavelet information entropy of three assembled quality conditions.

| Condition | Signal 1 | Signal 2 | Signal 3 | Signal 4 | Average |
|-----------|----------|----------|----------|----------|---------|
| Condition 1 | 0.3333 | 0.3261 | 0.2760 | 0.2570 | 0.2981 |
| Condition 2 | 0.1738 | 0.1454 | 0.1366 | 0.1382 | 0.1485 |
| Condition 3 | 0.1090 | 0.1062 | 0.0998 | 0.1039 | 0.1047 |
According to the equation (2.18), the reliability degree based on the normalized wavelet information entropy for each condition is respectively computed, which is shown in Table 2. Comparing with the three assembled quality conditions, it is found that the reliability degree gradually increases with the assembled quality from looseness (condition 1) to tightness (condition 3) and reaches the maximum in the qualified assembled quality (condition 3), which is in accordance with the reliability evaluation habit that the qualified assembled quality has higher reliability degree while the weak assembled quality has lower reliability degree.

| Table 2. The reliability degree of three assembled quality conditions. |
|---------------------------------------------------------------|
| Condition 1 | Signal 1 | Signal 2 | Signal 3 | Signal 4 | Average |
|------------|----------|----------|----------|----------|---------|
|            | 0.6667   | 0.6739   | 0.7240   | 0.7430   | 0.7019  |
| Condition 2| 0.8262   | 0.8546   | 0.8634   | 0.8618   | 0.8515  |
| Condition 3| 0.8910   | 0.8938   | 0.9002   | 0.8961   | 0.8953  |

Further, reliability evaluation for the dismountable disk-drum aero-engine rotor taking service for 460 hours is conducted. The acquired dynamic response signal is also damped in the time domain, which is shown in Figure 5(a). The eight frequency bands attained by SGWP are shown in Figure 5(b). The frequency band energy of the SGWP is computed according to the equation (2.15), which is shown in Figure 5(c). It is seen that the main frequency band energy of the SGWP is the fifth frequency band and the energy in the third frequency band is more than other frequency bands. The normalized wavelet information entropy of the acquired four dynamic response signals is computed according to the equation (2.17), which is shown in Table 3. It is found that the normalized wavelet information entropy of the acquired four dynamic response signals is in the range of [0.1618, 0.2196], and the average value is 0.1893. Meanwhile, the reliability degree based on the normalized wavelet information entropy is in the range of [0.7804, 0.8382], and the average value is 0.8107, which is shown in Table 4. Both of the normalized wavelet information entropy and the reliability degree indicate that the performance of the dismountable disk-drum aero-engine rotor has degraded during the flight mission of 460 hours. In addition, cracks are overhauled in the labyrinth seal toothed disk of the dismountable disk-drum aero-engine rotor.

Figure 5. The dynamic response signal of a dismountable disk-drum aero-engine rotor. (a) the original time signal; (b) the signal analyzed by the SGWP; (c) the frequency band energy of the SGWP.
Table 3. The normalized wavelet information entropy of a dismountable disk-drum aero-engine rotor.

| Signal 1 | Signal 2 | Signal 3 | Signal 4 | Average |
|----------|----------|----------|----------|---------|
| 0.2196   | 0.1618   | 0.1684   | 0.2075   | 0.1893  |

Table 4. The reliability degree of a dismountable disk-drum aero-engine rotor.

| Signal 1 | Signal 2 | Signal 3 | Signal 4 | Average |
|----------|----------|----------|----------|---------|
| 0.7804   | 0.8382   | 0.8316   | 0.7925   | 0.8107  |

4 Conclusions
In this study, a new method of reliability evaluation based on the normalized wavelet information entropy was proposed, which was realized by analyzing condition monitoring information of mechanical equipment. The vibration signals of mechanical equipment were analyzed by means of second generation wavelet package (SGWP). The relative energy in each frequency band of decomposed signal that equals a percentage of the whole signal energy was taken as probability. Normalized information entropy (IE) was obtained based on the relative energy to describe uncertainty of a system instead of probability. The reliability degree is transformed by normalizing the wavelet information entropy that belongs to [0, 1], which coincides with the traditional definition of the reliability degree. A successful application has been achieved to evaluate the assembled quality reliability for a kind of dismountable disk-drum aero-engine, which attains good results.

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6 Reference
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