A new T-circuit model of wind turbine generator for power system steady state studies

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ABSTRACT
Modeling of wind power plant (WPP) is a crucial issue in power system studies. In this paper, a new model of WPP for steady state (i.e. load flow) studies is proposed. Similar to the previous T-circuit based models, it is also developed based on equivalent T-circuit of the WPP induction generator. However, unlike in the previous models, the mathematical formulation of the new model is shorter and less complicated. Moreover, the derivation of the model in the present work is also much simpler. Only minimal mathematical operations are required in the process. Furthermore, the rotor voltage value of the WPP induction generator is readily available as an output of the proposed new model. This rotor voltage value can be used as a basis to calculate the induction generator slip. Validity of the new method is tested on a representative 9-bus electrical power system installed with WPP. Comparative studies between the proposed method (new model) and other method (previous model) are also presented.

1. INTRODUCTION
Modeling of WPP is a crucial issue in steady state and dynamic analyses of modern electric power system. Several interesting models have been investigated to enable incorporating the WPP into the studies. In the context of power system steady state (i.e. load flow) studies, some recent methods in the modeling can be found in [1-23]. The present paper investigates a new method for modeling WPP to be incorporated into modern power system load flow analysis.

Steady state model of WPP is developed where equivalent T-circuit of the WPP induction generator has been used as a basis in the model derivation [2, 16-18, 22, 23]. For each WPP, the developed model formulation consists of two nonlinear equations. Also, the unknown quantities (i.e. quantities to be determined or calculated) in the formulation are only the WPP active and reactive power output. However, in the method proposed in [2, 16-18, 22, 23], the WPP has been represented with relatively long and complex mathematical expressions.

In the present paper, a new model of WPP is proposed. Similar to the models discussed in [2, 16-18, 22, 23], it is also developed based on equivalent T-circuit of the WPP induction generator. However, the derivation process of the new model is much simpler than that proposed in [2, 16, 17, 18, 22, 23]. Only minimal mathematical operations are required in the process. As a result, simpler WPP steady state model can be obtained. Moreover, the rotor voltage value of the WPP induction generator is readily available as an output of the proposed model. This rotor voltage value can be used as a basis to calculate the induction...
generator slip. The validity of the proposed method is then tested on a representative 9-bus electrical power system installed with WPP.

To be systematic, the rest of the paper is arranged as follows: Section 2 gives the load flow problem formulation of a general power system without WPP. Section 3 addresses the proposed method of WPP modeling. Incorporation of the model into load flow analysis is also discussed in this section. Validity of the proposed method is investigated in section 4. Effects of shunt capacitor installation on WPP steady state performances (voltage profile, power output and slip) are also presented in this section. Finally, some important conclusions of the present work are given in section 5.

2. FORMULATION OF LOAD FLOW PROBLEM

The formulation of power flow problems can be obtained through equations that describe the performance of power system network in the forms of admittance. These equations are then combined with the formulations of bus power injection to obtain [24].

\[ S_{Gi} - S_{Li} - \sum_{j=1}^{n} Y_{ij}^* V_j^* = 0 \]  

where:

- \( i = 1, 2, \ldots, n \): bus number
- \( n \): total number of buses
- \( S_{Gi} = P_{Gi} + jQ_{Gi} \): power generation at bus \( i \)
- \( S_{Li} = P_{Li} + jQ_{Li} \): power load at bus \( i \)
- \( V = |V|e^{j\delta} \): voltage at bus \( i \)
- \( Y_{ij} = |Y_{ij}|e^{j\theta_{ij}} \): \( ij \)-th element of admittance matrix

To be able to find a valid solution to (1), in the power flow analysis, the following three types of system buses are usually defined: reference (slack), generator (PV) and load (PQ) buses as shown in Table 1. This definition is intended to make the number variables equal to the number of equations so that a correct and valid solution to (1) is obtainable.

| No | Bus Type | Known Variable                                                                 | Unknown Variable     |
|----|----------|-------------------------------------------------------------------------------|----------------------|
| 1  | Slack    | \([|Y|, q, P_c, Q_c, |V| and \(d=0\)]\)                                                        | \(P_c\) and \(Q_c\) |
| 2  | PQ       | \([|Y|, q, P_c, Q_c, \text{and } P_G=Q_G=0]\)                                | \(|V|\) and \(d\)    |
| 3  | PV       | \([|Y|, q, P_c, Q_c, P_G=Q_G=0]\)                                            | \(Q_G\) and \(d\)   |

3. PROPOSED MODEL OF WPP

In Figure 1(a), a WPP connected to bus \( k \) of an electric power system is shown. The main energy converter of the WPP is SCIG (squirrel cage induction generator). Separately, the SCIG is shown in Figure 1(b). Mechanical power input to the SCIG is \( P_m \), and the SCIG (i.e. WPP) electrical power output is \( S_G = P_G + jQ_G \). Figures 2 and 3 show the steady state equivalent circuits of SCIG. In Figure 3, \( Z_S, Z_R \) and \( Z_M \) represent the impedances of stator, rotor and magnetic core circuits, respectively. These impedances are given by:

\[ Z_S = R_S + jX_S \]  

\[ Z_R = R_R + jX_R \]  

\[ Z_M = \frac{jR_cX_m}{(R_c + jX_m)} \]

It is also to be noted that in Figure 2, \( s \) is the machine (i.e. SCIG) slip, and \( R_2(1-s)/s \) is a variable resistance where the turbine mechanical power \( P_m \) is assumed to be dissipated into. Based on Figures 2 and 3, the dissipated power can be computed using:
\[ P_m = -R_R \frac{1-s}{s} I_R I_R^* \] (3)

Figure 1. Wind power plant connected to power system

Figure 2. Steady state equivalent circuit of SCIG

Figure 3. Steady state equivalent circuit of SCIG in terms of impedances

Also, by looking at Figure 3, the formulas for WPP electrical power output \((S_g)\) and turbine mechanical power input \((S_m)\) can be written as:

\[ S_g = P_g + jQ_g = V_S I_S^* \] (4)

\[ S_m = P_m + j0 = V_R I_R^* \] (5)
Based on (4) and (5), the stator current ($I_s$) as a function of electric power output ($S_g$) and stator voltage ($V_s$), and the rotor current ($I_r$) as a function of mechanical power input ($P_m$) and rotor voltage ($V_r$) are formulated as:

$$I_s = \frac{S_g}{V_s}$$  \hspace{1cm} (6)

$$I_r = \frac{P_m}{V_r}$$  \hspace{1cm} (7)

Furthermore, by applying basic electric circuit theories (i.e. Kirchhoff’s and Ohm’s laws) to circuit in Figure 3, the following equations can also be obtained (derivation is given in the appendix):

$$V_s - V_r + Z_S I_s + Z_R I_R = 0$$  \hspace{1cm} (8)

$$V_s + (Z_S + Z_M) I_s - Z_M I_R = 0$$  \hspace{1cm} (9)

Substituting (6) and (7) into (8) and (9) will result in:

$$V_s - V_r + Z_S \frac{S_g}{V_s} + Z_R \frac{P_m}{V_r} = 0$$  \hspace{1cm} (10)

$$V_s + (Z_S + Z_M) \frac{S_g}{V_s} - Z_M \frac{P_m}{V_r} = 0$$  \hspace{1cm} (11)

In (10) and (11), are the proposed steady state model of WPP. It is to be noted that, in (10) and (11), $V_s$ is the stator voltage which is also the voltage at WPP terminal (i.e. $V_s=|V|e^{j\theta}$), and $V_r=|V|e^{j\alpha}$ is the rotor voltage. Moreover, the active and reactive power generations ($P_G$ and $Q_G$) at WPP bus, are also the WPP active and reactive power outputs ($P_g$ and $Q_g$). All of the equations to be solved and quantities to be determined in the complete load flow formulation are presented in Table 2. It is also worth mentioning here that there is an alternative expression to (11). This alternative formulation can be found in the appendix.

| No | Equation(s) | Bus Type | Known Variable | Unknown Variable |
|----|-------------|----------|----------------|-----------------|
| 1  | (1)        | Slack    | $|Y|$, 0, $P_c$, $Q_c$, $|V|$ and $\delta=0$ | $P_G$ and $Q_G$ |
| 2  | (1)        | PQ       | $|Y|$, 0, $P_c$, $Q_c$, and $P_G=Q_G=0$ | $|V|$ and $\delta$ |
| 3  | (1)        | PV       | $|Y|$, 0, $P_c$, $Q_c$, and $|V|$ | $Q_G$ and $\delta$ |
| 4  | (1), (10), (11) | WPP | $|Y|$, 0, $P_c$, and $Q_c$, $P_G=P_c=Q_G=Q_c$, $|V|$, $\delta$, $|V_B|$ and $\alpha$ | |

4. RESULT AND DISCUSSION

4.1. Test system

The new WPP model proposed in section 3 will be tested by using a power system shown in Figure 4. This power system is based on 9-bus system adopted from [25]. The test system is then modified by adding a WPP to bus 8 via a step-up transformer. Data for the test system (including the WPP) are shown in Tables 3-5 (all data are in pu on 100 MVA base).
Figure 4. Test system

Table 3. Branch (line) data

| Line | Bus p-q | Series Impedance (Z) | Shunt Admittance (Ysh/2) |
|------|---------|----------------------|--------------------------|
| 1    | 1–4     | j0.1184              | 0                        |
| 2    | 2–7     | j0.1823              | 0                        |
| 3    | 3–9     | j0.2399              | 0                        |
| 4    | 4–5     | 0.0100+j0.0850       | j0.0880                  |
| 5    | 4–6     | 0.0170+j0.0920       | j0.0790                  |
| 6    | 5–7     | 0.0320+j0.1610       | j0.1530                  |
| 7    | 6–9     | 0.0390+j0.1700       | j0.1790                  |
| 8    | 7–8     | 0.0085+j0.0720       | j0.0745                  |
| 9    | 8–9     | 0.0119+j0.1008       | j0.1045                  |

Table 4. Bus (node) data

| Bus | [V] | δ | Generation | Load     | Note    |
|-----|-----|---|------------|----------|---------|
| 1   | 1.04| 0 | -          | 0        | Slack   |
| 2   | 1.02| - | 1.50+j1.5  | 0        | PV      |
| 3   | 1.01| - | 1.00+j1.0  | 0        | PV      |
| 4   | -   | - | 0          | 0        | PQ      |
| 5   | -   | - | 1.25+j0.5  | 0        | PQ      |
| 6   | -   | - | 0.90+j0.3  | 0        | PQ      |
| 7   | -   | - | 0          | 0        | PQ      |
| 8   | -   | - | 0          | 1.00+j0.35| PQ      |
| 9   | -   | - | 0          | 0        | PQ      |

Table 5. Wind power plant data

| Component | Parameter                  | Value                  |
|-----------|----------------------------|------------------------|
| SCIG      | Stator: R_s=0.002, X_s=0.024|                        |
|           | Rotor: R_o=0.010, X_o=0.040 |                        |
|           | Magnetic core: X_m=8       |                        |
| Step-up transformer | Impedance: Z_t=0.002+j0.080 |                        |

4.2. Results and discussion

The load flow studies in the present work are carried out for various values of P_m (i.e. from 0.1 to 1.0 pu). The calculation results in terms of voltage magnitude and electric power output of the WPP are shown in Table 6. As a comparison, results of the calculation when the previous T-circuit model [23] is used are also shown in the table. It can be seen that both results are in exact agreement which indicates that the model proposed is also valid.
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Figure 8 shows the graphs of negative slip (-s) against turbine mechanical power input \( (P_m) \). It can be seen that for WPP without shunt capacitor, the values of slip vary in the range of \(-0.1080\%\) and \(-1.1732\%\). Whereas, for WPP with shunt capacitor, the range of variation is slightly smaller \((-0.1011\%\) and \(-1.0807\%\)). These results are expected since system with shunt capacitor has a better voltage profile and will lead to a higher (or less negative) value of slip.

![Figure 5. Variation of voltage magnitude](image1)
![Figure 6. Variation of active-power](image2)
![Figure 7. Variation of reactive-power](image3)
![Figure 8. Variation of slip](image4)

5. CONCLUSION
A new steady state model of WPP has been presented in this paper. Similar to the previous T-circuit based models, it is also developed based on equivalent T-circuit of the WPP induction generator. However, unlike in the previous models, the mathematical formulation of the new model is shorter and less complicated. Moreover, the derivation process of the new model is much simpler. Only minimal mathematical operations are required in the process. Also, the rotor voltage value of the WPP induction generator is readily available as an output of the proposed model. This rotor voltage value can be used as a basis to calculate the induction generator slip. The validity of the proposed method has been tested on a representative 9-bus electrical power system installed with WPP. Comparative studies between the proposed method (new model) and other method (previous model) have also been carried out. Results of the studies confirm the validity of the new model.

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APPENDIX

A.1. Derivation of (9)
Applying Kirchhoff voltage law to the first loop of circuit in Figure 3 gives:

\[ V_S - V_M + Z_S I_S = 0 \]  \hspace{1cm} (A.1)

Since:

\[ V_M = Z_M I_M = Z_M (I_R - I_S) \]  \hspace{1cm} (A.2)

Then, by substituting (A.2) into (A.1) and rearranging, (9) can be obtained.

\[ V_R - V_M - Z_R I_R = 0 \]  \hspace{1cm} (A.3)

A.2. Alternative expression to (11)
Applying Kirchhoff voltage law to the second loop of circuit in Figure 3 gives:

Substituting (A.2) into (A.3) will result in:

\[ V_R - (Z_R + Z_M) I_R + Z_M I_S = 0 \]  \hspace{1cm} (A.4)

On using (6) and (7) in (A.4) will give:

\[ V_R - (Z_R + Z_M) \frac{P_m}{V_R^*} + Z_M \frac{S_g^*}{V_S^*} = 0 \]  \hspace{1cm} (A.5)

Equation (A.5) is the alternative formulation to (11).

A.3. Determination of machine slip based on rotor voltage
Substituting (7) into (3) and rearrangement gives:

\[ s = R_R \left[ R_R - \frac{V_R V_R^*}{P_m} \right] \]  \hspace{1cm} (A.6)

Since:

\[ V_R V_R^* = |V_R|^2 \]  \hspace{1cm} (A.7)

Then, (A.6) can be rewritten as:

\[ s = R_R \left[ R_R - |V_R|^2 / P_m \right] \]  \hspace{1cm} (A.8)

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