A comparative study of critical failure surface determination in slope stability assessment

A Iravanian$^*$ and A Shlash$^1$

$^1$Near East University, Civil Engineering Department, Lefkosa, N. Cyprus, 99138, Mersin 10, Turkey

$^*$E-mail: anoosheh.iravanian@neu.edu.tr

Abstract. Slope stability analysis and design of manmade slopes has always been a challenging geotechnical problem. Different methods have been developed and created to examine two and three dimensional slope models based on various methods of assumption and assessment. Throughout this study two limit equilibrium, and finite element methods has been used to examine the stability of the slope and to determine the critical slip surface of failure by using PLAXIS, FLAC/Slope and SLOPE/W software programs respectively. Different values of shear strength parameters were chosen to investigate the efficiency of these methods. Various values of unit weight, cohesion, internal friction angle, and their impact on the safety factor were examined. Comparing the results for factor of safety indicated that finite element calculation used in PLAXIS gives more conservative results by 0.5% compared to SLOPE/W program. FLAC/Slope was observed as the most complicated software and finite difference method generally gave the lowest value for factor of safety in comparison with the other methods. Impact of internal friction angle, unit weight, and cohesion on length of failure arc was also evaluated, and no significant relation between length of failure arc (La) and safety factor was detected.

1. Introduction

There are many problems encountered by the engineer, student or executor, so it should be ready to set the groundwork for predicting such problems and devising appropriate solutions in case of occurrence. The goal of any natural or manmade slope's slope stability analysis is to determine the critical failure surface with the minimum value of safety factor. With the development in technology software packages use the finite difference and finite element methods have increased in popularity as they head to possess a wide range of characteristics [4]. At present, the problem of slope stability is one of the main issues facing workers and engineers in soil field; there is still a big distinction between the scientists about the best way to calculate the stability of the slopes [5]. The main objectives of this study are using various slope stability analysis software programs to evaluate the impacts of soil strength parameters (cohesion, unit weight, internal friction angle) and slope geometry parameters (Height of the slope, angle of the horizontal studied slope, Alpha, and the vertical angle of the slope, Beta) as shown in Figure 10 on factor of safety and critical slip surface, in addition to compare the results of the different analysis software programs of slope stability in this study. Over 100 types of soil with different strength parameters have been used in this study to be modeled and analyzed.
2. Factor of safety, FOS
Simply the ratio of resisting force of deriving force, $F = S (C)^{-1}$ [6], or ratio of shear strength to that required to keep the slope stable in case only stable slope (without any structure on it). [7] and [8] found that factor of safety which computed by using 3D analyzes will always be equal or higher than to factor of safety computed by using 2D analyzes. There are various methods for formulating FOS, usually each of the techniques of analysis has its own formulas for FOS, but the most popular formula assumes that the FOS is constant and can be divided into two types: Moment equilibrium and Force equilibrium [9].

3. Impact of geometry and soil strength parameters on factor of safety
To evaluate the influence of strength parameters of soil and geometry parameters on the safety factor, FOS versus these parameters was offered and drawn in the following figures. To study the influence of cohesion C, internal friction angle (Phi) $\phi$ and unit weight (UW) $\gamma$ on factor of safety FOS, the values of three parameters diverge from 15 to 32 kPa, degree, and kN/m$^3$ respectively were chosen while the other two parameters kept constant at 15.

![Figure 1. Cohesion effect on FOS.](image1.png)

![Figure 2. Internal friction angle impact on the FOS.](image2.png)

The cohesion effect on FOS has been shown in figure 1 while Figure 2 the relationship between Phi and FOS. Increasing the value of cohesion and Phi as a resistant forces, as expected, increased the value of FOS which is nearly linear with R$^2$ factor of 0.99.

![Figure 3. Effect of UW on FOS.](image3.png)

![Figure 4. The combined effect UW and cohesion the on FOS.](image4.png)

As can be shown from the Figure 3, the soil’s unit weight is inversely proportional to the safety factor because of the raise in unit weight which is the main reason of the derivative forces. In Figure 4 Cohesion and soil unit weight were raised together here, but the ratio stayed constant. The outputs dictate that the
potential surface of the slip is affected by the combination of c and φ, the function of which is defined as \( \lambda \) equivalent to

\[
\lambda = \frac{C}{(\gamma h \tan \phi)}
\]

(1)

As shown in Figure 5, decrease in the value of safety factor has achieved by raising phi value so \( \gamma \) value. This is due to the failure surface movement to the top, thus reducing the length of the arc of failure and so reducing the impact of resisting forces. Since the surface of potential failure is expected to be influenced by both of C and phi values, Figure 6 shows the relationship between the safety factor and C, \( \phi \) so because both of these parameters of shear strength are holding (resisting) forces, raising these two values results in an raise in the value of the safety factor. FOS is nearly linear with \( R^2 \) factor of 0.9999.

![Figure 5. The combined effect of Phi and UW on FOS.](image)

![Figure 6. The effect of Phi and C on FOS.](image)

Figure 7 (a) shows the impact of unit weight on failure slip surface (whereas Figure 7 (b) is zoomed variant of (a)). The tests follow a practical rule, by maximizing the soil’s unit weight, the failure surface is moved to the right, leading in a smaller volume of soil failure and thus decreasing the slip surface length. Less resistant force is activated due to the smaller surface for holding factors (friction and cohesion). For these reasons, a lower value for FOS is accomplished. Two angles of the slope \( \alpha \) and \( \beta \) used for this to test the influence of geometry on the safety factor. The results presented in the figures below.
Figure 8 represents that no noticeable variation is observed in the safety factor except the value of FOS between angles 4.6˚ and 6.8˚, it decreased unpretentiously after the angle 6.8˚ although it was increasing slightly before that value of Alpha angle. These decrease because it is possible by increase the Alpha angle as if adding an additional overhead load on the surface of the slope. While this rise in the surface of failure produces more resistant force, it simultaneously produces an raise in derivative force (weight of the surface of failure). The safety factor therefore remains constant, for angles more than 6.8˚, the rise in derivative force approach the value of the resisting force also from this angle value onwards, and the derivative force becomes greater than the resisting force, and therefore a decrease could be seen in safety factor’s value. Figure 9 represents the safety factor significantly increases by increasing Beta angle. The reason for this behavior is that only the failure arc length increase as resisting force and the weight of the failure shape as driving force stays constant by increasing the beta angle. For that by increasing the length of the arc result an increase in the resisting force and thus the safety factor.

4. Impact of geometry and soil strength parameters on the slip surface

Based on what was debated in the past part, it is easy to estimate that a correlation should exist between the strength parameters of soil and the geometry of the slope and the surface of failure; the following analyzed models will be studied in order to examine this condition. Many models were generated using SLOPE/W software in this step. The output in this aspect will become the safety factor and slip circle center coordinates as well as the circular failure surface radius. The circles have been drawn using AutoCAD software to determine failure arc’s length and locate entry and exit points in the slope area. Figure 10 reflects the general shape of the slope geometry that going to be used in the next models.
Figure 11 represents that the length of failure surface will rise by raising the value of cohesion. The cause is that in the instance of the constant failure surface’s location as the cohesion factor raise, the hold forces become greater in addition to the FOS. The deriving forces must raise that can be accomplished by increase the area of slope failure, in order Finding the smallest value of FOS (the initial objective for slope stability analyses). This results in a larger arc failure length and so a greater value of FOS. The impact of internal friction angle on failure surface length is shown in Figure 12. In the previous section, referring to the same explanation, it could be anticipated that the length of the arc must be in a direct correlation with \( \phi \), but as can be seen in Figure 12, \( L_a \) and \( \phi \) are related reversely. This reverse relationship is consistent with the paper [1] which noted that "when the distribution of slope geometry and unit weight in a homogeneous soil slope is given, the slip surface location for specific slice technique is only concerned with the ratio \( C \cdot \tan \phi \)\(^{-1} \) of the slope. This paper indicates that slip surface positioning and therefore \( L_a \) is in the inverse relation to the friction angle.

As shown in Fig 13 by raising the value of soil unit weight, the weight of the failed shape rises, resulting a small safety factor. And by considering lambda the surface of the slip shifts towards the slope's face whereas by reducing \( L_a \), the impacts of the friction angle and cohesion as the forces of resistance decrease, thus achieving a smaller safety factor. Constant unit weight over C ratio, which results in a constant \( \lambda \). As noted in the study [10], this means the same shape of failure and therefore a constant value of \( L_a \) as shown in Figure 14.
Figure 15. The effect of Phi and UW on La.

Figure 16. The effect of C and Phi on La.

The results shown in Figure 15, to show the variation impact for both of the strength factors. It can be seen that there will be a reduction in the length of the surface failure by increasing the value of $\gamma$ and $\tan \phi$. It is consists when talking about the $\lambda$ value, by maximizing this value, $\lambda$ declines; smaller value of $\lambda$ imply a surface of failure nearer to the surface of the slope and thus a smaller failure arc length. In Figure 16, it can be seen that $La$ will remain proportionally constant at value (11.2 m) when both of the factors cohesion and friction angle remain constant between the values (16-32). Since constant C and $\tan \phi$ results in a constant lambda, and constant lambda imply a constant surface of failure, $La$ also stays constant.

The length of the failure arc as a numerical value was estimated and drawn to observe the effect of the slope geometry on the surface of the failure as shown in Figures 17 and 18. Slope geometry has been shown to have a proportional relation with the stability of the slope also the properties of soil strength [11]. In the last models series, strength parameters of soil remained unchanged at the following values, whereas angles ranged from 0 ° to 63.4° for (Beta) angle while (Alpha) angle ranged from 0 ° to 19.8° in slope geometry (shown in Figure 4.4). Cohesion = 15 kPa, angle of friction = 15°, unit weight = 15 kN/m³.

Figure 17. Alpha angle impact on La.

Figure 18. Beta angle impact on La.

Results of analysis of the models indicate that the positioning of the failure surface is not significantly different by increasing the Alpha angle. The cause for increasing the length of the failure arc is only the slope surface motion and therefore the failure arc extension. By increasing Beta angle value and the remaining parameters in the formula (1) won’t change and therefore don’t affect the value of lambda), the failure surface depth doesn’t vary. But, for the other side, an raise in the value of Beta angle can shift the surface of the slope to the left and this is going to cause the arc to be protracted as shown in Figure 18.
5. Reanalyzing models by PLAXIS and comparison of results

The studied models were reanalyzed using PLAXIS software program to validate the outputs of the SLOPE/W program acquired in part 3. Table 1 shows the results of these analyzes.

Table 1. Models for the factor of safety analysis – PLAXIS.

| Model Number | Unit Weight [kN/m³] | Phi [°] | Cohesion [kPa] | FOS   |
|--------------|---------------------|---------|----------------|-------|
| 2            | 15                  | 15      | 16             | 1.051 |
| 6            | 15                  | 15      | 20             | 1.254 |
| 10           | 15                  | 15      | 24             | 1.391 |
| 16           | 15                  | 15      | 30             | 1.663 |
| 19           | 15                  | 16      | 15             | 1.047 |
| 23           | 15                  | 20      | 15             | 1.073 |
| 29           | 15                  | 26      | 15             | 1.237 |
| 33           | 15                  | 30      | 15             | 1.316 |
| 37           | 17                  | 15      | 15             | 0.867 |
| 41           | 21                  | 15      | 15             | 0.752 |
| 45           | 25                  | 15      | 15             | 0.689 |
| 48           | 31                  | 15      | 15             | 0.594 |
| 49           | 16                  | 15      | 16             | 1.024 |
| 52           | 21                  | 15      | 21             | 1.024 |
| 54           | 25                  | 15      | 25             | 1.024 |
| 57           | 31                  | 15      | 31             | 1.024 |
| 58           | 16                  | 16      | 15             | 0.954 |
| 61           | 21                  | 21      | 15             | 0.900 |
| 63           | 25                  | 25      | 15             | 0.838 |
| 66           | 31                  | 31      | 15             | 0.792 |
| 67           | 15                  | 16      | 16             | 1.048 |
| 70           | 15                  | 21      | 21             | 1.341 |
| 72           | 15                  | 25      | 25             | 1.617 |
| 75           | 15                  | 31      | 31             | 2.046 |
Table 2 summarizes the distinction between the FOS acquired from both programs (PLAXIS and SLOPE/W) and compares these results using the following formula.

\[
\text{Difference} = \frac{\text{FOS(SLOPEW)} - \text{FOS(PLAXIS)}}{\text{FOS (PLAXIS)}} \times 100
\]

| Model Number | SLOPE/W FOS | PLAXIS FOS | Difference% |
|--------------|-------------|------------|-------------|
| 2            | 1.081       | 1.051      | 0.029       |
| 6            | 1.272       | 1.254      | 0.014       |
| 10           | 1.483       | 1.391      | 0.066       |
| 16           | 1.787       | 1.663      | 0.075       |
| 19           | 1.033       | 1.047      | -0.013      |
| 23           | 1.089       | 1.073      | 0.015       |
| 29           | 1.215       | 1.237      | -0.018      |
| 33           | 1.284       | 1.316      | -0.02       |
| 37           | 0.927       | 0.867      | 0.030       |
| 41           | 0.786       | 0.752      | 0.017       |
| 45           | 0.701       | 0.689      | 0.006       |
| 48           | 0.613       | 0.594      | 0.0095      |
| 49           | 1.033       | 1.024      | 0.0045      |
| 52           | 1.033       | 1.024      | 0.0045      |
| 54           | 1.033       | 1.024      | 0.0045      |
| 57           | 1.033       | 1.024      | 0.0045      |
| 58           | 0.988       | 0.954      | 0.017       |
| 61           | 0.895       | 0.9        | -0.0025     |
| 63           | 0.83        | 0.838      | -0.004      |
| 66           | 0.804       | 0.792      | 0.01515     |
| 67           | 1.103       | 1.048      | 0.0275      |
| 70           | 1.456       | 1.341      | 0.0575      |
| 72           | 1.745       | 1.617      | 0.064       |
| 75           | 2.19        | 2.046      | 0.072       |

Table 3. Impact of slope’s geometry on the slope’s slip surface – PLAXIS.

| Model Number | α [°] | β [°] | Slope Height [m] | Factor of Safety |
|--------------|-------|-------|------------------|------------------|
| 1            | 19.8  | 0     | 20               | 1.013            |
| 2            | 17.75 | 0     | 19               | 1.029            |
| 4            | 13.5  | 0     | 17               | 1.061            |
| 6            | 9.1   | 0     | 15               | 1.064            |
| 8            | 4.6   | 0     | 13               | 1.057            |
| 10           | 0     | 0     | 11               | 1.048            |
| 11           | 0     | 11.3  | 11               | 1.216            |
| 12           | 0     | 21.8  | 11               | 1.365            |
| 14           | 0     | 38    | 11               | 1.609            |
| 16           | 0     | 50.2  | 11               | 1.877            |
| 20           | 0     | 63.4  | 11               | 2.274            |
Table 4. The difference of FOS between PLAXIS and SLOPE/W for slope geometry.

| Model No | SLOPE/W FOS | PLAXIS FOS | Difference (%) |
|----------|-------------|------------|----------------|
| 1        | 1.037       | 1.013      | 0.012          |
| 2        | 1.033       | 1.029      | 0.002          |
| 4        | 1.058       | 1.061      | -0.0015        |
| 6        | 1.048       | 1.064      | -0.008         |
| 8        | 1.079       | 1.057      | 0.011          |
| 10       | 1.053       | 1.048      | 0.0025         |
| 11       | 1.252       | 1.216      | 0.018          |
| 12       | 1.419       | 1.365      | 0.027          |
| 14       | 1.642       | 1.609      | 0.0165         |
| 16       | 1.928       | 1.877      | 0.0255         |
| 20       | 2.302       | 2.274      | 0.014          |

It can be seen in Table 3 and 4 that PLAXIS is a more conservative designing program. On average, PLAXIS gives less than 1% lower FOS (except few models) which gave FOS greater than SLOPE/W software that will make PLAXIS more conservative and therefore safer to design and analyze more significant slopes. By comparison, giving SLOPE/W a higher FOS makes it more beneficial to analyze and design less important slopes.

6. Reanalyzing the past models by using FLAC/Slope

To inspect the outputs from PLAXIS and SLOPE/W, a sample of the models were chosen up to their properties, and these models were reanalyzed by using FLAC/SLOPE program as shown in Tables 5 and 6. Considering that FLAC is not software with a complete LEM, the results may show a distinction between three software.

Table 5. Reanalyze models by using FLAC software (shear strength parameters models).

| Model Number | Unit Weight [kN/m³] | Phi [°] | Cohesion [kPa] | FOS |
|--------------|---------------------|---------|---------------|-----|
| 6            | 15                  | 15      | 20            | 1.029 |
| 11           | 15                  | 15      | 25            | 1.221 |
| 23           | 15                  | 20      | 15            | 0.607 |
| 28           | 15                  | 25      | 15            | 0.932 |
| 45           | 25                  | 15      | 15            | 1.049 |
| 54           | 25                  | 15      | 25            | 0.842 |
| 63           | 25                  | 25      | 15            | 0.764 |
| 72           | 15                  | 25      | 25            | 1.436 |

Table 6. Reanalyze models by using FLAC software (slope geometry models).

| Model No | α (°) | β (°) | Factor of Safety |
|----------|-------|-------|------------------|
| 6        | 9.1   | 0     | 0.869            |
| 15       | 0     | 45    | 1.827            |
Table 7. The difference between the software packages (shear strength parameters models).

| Model no | Factor of safety of FLAC | Factor of safety of SLOPE/W | Factor of safety of PLAXIS | Difference between FLAC and SLOPE/W (%) | Difference between FLAC and PLAXIS (%) |
|----------|--------------------------|----------------------------|---------------------------|----------------------------------------|---------------------------------------|
| 6        | 1.029                    | 1.272                      | 1.254                     | 0.236152                               | 0.218659                              |
| 23       | 0.607                    | 1.089                      | 1.073                     | 0.794069                               | 0.76771                               |
| 45       | 1.049                    | 0.701                      | 0.689                     | -0.33174                               | -0.34318                              |
| 54       | 0.842                    | 1.053                      | 1.024                     | 0.250594                               | 0.216152                              |
| 63       | 0.764                    | 0.83                       | 0.838                     | 0.086387                               | 0.096859                              |
| 72       | 1.436                    | 1.745                      | 1.617                     | 0.215181                               | 0.126045                              |

Table 8. The difference between the three software packages (slope geometry models).

| Model no | FACTOR OF SAFETY OF FLAC | FACTOR OF SAFETY OF SLOPE/W | FACTOR OF SAFETY OF PLAXIS | Difference between FLAC and SLOPE/W (%) | Difference between FLAC and PLAXIS (%) |
|----------|--------------------------|----------------------------|---------------------------|----------------------------------------|---------------------------------------|
| 6        | 0.869                    | 1.048                      | 1.064                     | 0.205984                               | 0.195                                 |
| 15       | 1.827                    | 1.774                      | 1.774                     | -0.02901                               | -0.053                                |

As shown in Tables 7 and 8, the average distinction between FLAC/SLOPE and other two programs is less than 1 percent, which is appropriate. In addition, it is noticeable that FOSs obtained from FLAC are lower than the other two programs in 75% of the models.

7. Conclusion and recommendations

- Cohesion (c) and friction angle (φ), as resistance forces are directly related to the safety factor while unit weight as deriving force is inversely related to the safety factor. An increase in the cohesion value results to higher value for length of failure arc (La). An increase in the friction angle value results to reduce the length of failure arc (La) value. Increasing the unit weight value of soil leads to increase in the of the length of failure arc (La) value. An increase in the value of Alpha angle to a specific value doesn’t have any noticeable influence on FOS. while, an increase in the value of the Beta angle affects directly on the factor of safety. The greater value of the Alpha angle, leads to a greater value of La. However, varying the value of Beta angle doesn’t affect noticeably on the value of La. PLAXIS is more conservative software for slope stability studies, and comparing to SLOPE/W which it gives about 0.5% lower value for factor of safety. FLAC/Slope is the most complicated software and usually gave out the lowest value for factor of safety in comparison with PLAXIS and SLOPE/W. There is no any significant relation between length of failure arc (La) and safety factor. At the end of this study I recommend analysis and modeling wider range in parameters of soil strength. Including the level of water content and consideration of unsaturated soils and the effect of water pressure and pore air. Including more slope geometry variables as overall slope’s angle and the slope’s length. Conduct a state study to verify the validation of the equations acquired to locate the critical surface of the failure.

1. Cohesion (c) and friction angle (φ), as resistance forces are directly related to the safety factor while unit weight as deriving force is inversely related to the safety factor.
2. An increase in the cohesion value results to higher value for length of failure arc (La).
3. An increase in the friction angle value results to reduce the length of failure arc (La) value.
4. Increasing the unit weight value of soil leads to increase in the length of failure arc (La) value.
5. An increase in the value of Alpha angle to a specific value doesn’t have any noticeable influence on FOS. While, an increase in the value of the Beta angle affects directly on the factor of safety.
6. The greater value of the Alpha angle, leads to a greater value of La. However, varying the value of Beta angle doesn’t affect noticeably on the value of La.
7. PLAXIS is more conservative software for slope stability studies, and comparing to SLOPE/W which it gives about 0.5% lower value for factor of safety.
8. FLAC/Slope is the most complicated software and usually gave out the lowest value for factor of safety in comparison with PLAXIS and SLOPE/W.
9. There is no significant relation between length of failure arc (La) and safety factor FOS.

8. Recommendations
The following recommendations analysis can be performed for other studies in connection with this thesis study:
1. Analysis and modeling wider range in parameters of soil strength.
2. Including the level of water content and consideration of unsaturated soils and the effect of water pressure.
3. Including more slope geometry variables as the angle of the slope itself and the length of the slope.
4. Conduct a state study to verify the validation of the equations acquired to locate the critical surface of the failure.

References

[1] Jiang JC, Yamagami T (2006) Charts for estimating strength parameters from slips in homogeneous slopes, Computers and Geotechnics, 33(6–7): 294-304, ISSN 0266-352X
[2] Shiguo X, Liping Y, Zhiqiang C (2011) A method combining numerical analysis and limit equilibrium theory to determine potential slip surfaces in soil slopes. Journal of Mountain Science, 8(5): 718–727
[3] Naderi MA (2013) Determination of the Critical Slip Surface in Slope Stability Analysis, Master Thesis Eastern Mediterranean University
[4] Hammouri, N. A., Husein Malkawi, A. I., & Yamin, M. M. A. (2008). Stability analysis of slopes using the finite element method and limiting equilibrium approach. Bulletin of Engineering Geology and the Environment.
[5] Oghli, Z. (2011). Analyzing slope stability by using shear strength reduction method. Aleppo University.
[6] Duncan, J. M., Wright, S. G., & Brandon, T. L. (2014). Soil strength and slope stability. Canada: John Wiley & Sons, Inc. Retrieved from.
[7] Mitchell, J. K., Chang, M., Seed, R. B., Mitchell, J. K., Chang, M., Cahill, E. G., & Cahill, J. R. (1993). The Kettleman Hills Landfill Failure: A Retrospective View of the Failure Investigations and Lessons Learned. International Conference on Case Histories in Geotechnical Engineering, 1–15.
[8] Duncan, J. M. (1992). State-of-the-art: Static stability and deformation analysis. In Proc. of Specialty Conf. Stability and Performance of Slopes and Embankments-II (Vol. 1, pp. 222-266). ASCE.
[9] Cheng, Y. M., Lansivaara, T., & Siu, J. (2008). Impact of convergence on slope stability analysis and design. Computers and Geotechnics, 35(1), 105–113. https://doi.org/10.1016/j.compgeo.2007.02.011.
[10] Lin, H., & Cao, P. (2011). Potential slip surfaces of slope with strength parameters. In Advanced Materials Research (Vol. 243, pp. 3315-3318). Trans Tech Publications.
[11] Namdar, A. (2011). Geometry in Slope Modeling and Design. e -Journal of Science & Technology (e-JST)(22), 9-21.