Analytic Compact Model of Short-channel Cylindrical ballistic GAA MOSFET Including SDT effect

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Abstract. We have proposed an analytic compact model describing the drain current characteristics valid in all operating regions, for ultra-short channel cylindrical gate-all-around metal-oxide-semiconductor field-effect transistors considering source-to-drain tunnelling effect. The drain-induced barrier lowering had been incorporated from one two-dimensional analysis in our previous compact model. In this study, to represent the energy level profile along the device channel direction into the Wentzel-Kramers-Brillouin approximation by substituting a parabolic function, we can analytically derive the expressions of the transmission coefficients for source-to-drain tunneling. In the subthreshold region, the source-to-drain tunneling current then can be evaluated using the Landauer formula. Finally, a fully analytic compact model is proposed for representing the drain current in all operating regions. The results compared with non-equilibrium Green’s function transport simulations can be obtained in a good agreement.

1. Introduction

Owing to the size of semiconductor devices keep shrinking down, gate-all-around metal-oxide-semiconductor field-effect transistors (GAA MOSFETs) with undoped Silicon channel have been attracting attentions as one promising candidate for the future electronics [1-2]. Thanks to the excellent controllability of their electrostatic potential over the channel [3-5], such devices are considered competent to overcome the short-channel effects (SCEs), such as drain-induced barrier lowering (DIBL) and threshold voltage roll-off [2,6]. Then, it has been well-studied that quantum transport needs to be taken into account of describing the properties in such MOSFETs whose gate lengths reach sub-7 nm regime [7, 8]. Particularly, to model the quantum tunneling and ballistic transport within GAA MOSFETs across source-to-drain potential barrier, several numerical simulators using non-equilibrium Green’s function (NEGF) formalism are reported [8-10]. However, such kinds of simulations on computing cannot be used in circuit level simulation, and the reduction of time consumption of the computations should be considered.

This paper studies source-to-drain tunnelling (SDT) effect to describe the ballistic current within the GAA MOSFET in the subthreshold region. We assume to use one unknown parameter to express the electron energy level profile along the device channel analytically in accordance with our previous study. We also assume that the distribution of the energy subband level can be approximately represented by...
using one parabolic function in order to calculate the tunnelling transmission coefficients analytically when considering the Wentzel-Kramers-Brillouin (WKB) approximation. Then, the model of cylindrical ballistic GAA-MOSFETs incorporating SDT effect can be obtained by one analytic formula for describing the drain current in all operating regions.

2. Calculation Theory

2.1. Model structure

As illustrated in figure 1, considering the intrinsic silicon channel of the cylindrical GAA MOSFET is surrounded by SiO2 gate oxide and metal gate. Here, the source and drain are assumed as ideal reservoirs of electrons with highly doped n-type silicon. Furthermore, we consider that there are no reflections back to the device channel when the electrons flow into the source and drain regions. The device channel length, radius, and gate oxide thickness are denoted as \( L_G \), \( R \) and \( T_{OX} \), respectively. In the following formulations, cylindrical coordinate with length, radius and angular component are denoted by \( z \), \( r \) and \( \phi \), respectively.

![Figure 1. Schematic of cylindrical GAA-MOSFET model structure.](image)

2.2. Energy level distribution

Figure 2(a) illustrates the profiles of the conduction band edge and higher subbands in \( z \)-axis. The electron total energy, Fermi energies the source and drain regions are denoted as \( E_{\text{total}} \), \( E_{\text{FS}} \) and \( E_{\text{FD}} \).

![Figure 2. (a) Conduction band edge and higher subband profiles along the \( z \)-axis at \( r=0 \) in the device. (b) Confinement potential energy distribution along \( r \)-axis at \( z_{\text{MAX}} \) in the cross section.](image)
respectively. The maximum subband energy level with the angular and radial quantum numbers of \( n_{\varphi} \) and \( n_r \), respectively, is denoted as \( E_{\text{MAX}, \varphi, r} \). Then, the confinement energy level with respect to \( E_{\text{FS}} \) is represented as \( E_{\text{CONF}, \varphi, r}(z_{\text{MAX}}) \) along the \( z \)-direction at the barrier top \( (z = z_{\text{MAX}}) \). Turning points \( z_{\text{MAX}} \), \( z_1 \) and \( z_2 \) have literal expressions. Difference between the source and drain in Fermi level, \( eV_{\text{DS}} \), corresponds to the voltage across the source and drain denoted as \( V_{\text{DS}} \). As shown in figure 2(b), the potential profile along \( r \)-axis at \( z_{\text{MAX}} \) in the cross section is illustrated. The electrostatic potential at the center and surface of the wire channel are defined as \( w_0 \) and \( w_S \), corresponding to the conduction band edges with respect to \( E_{\text{FS}} \) at the center and surface given by \( -eW_0 \) and \( -eW_S \), respectively. According to our previous work, an unknown parameter \( \Delta U_G \) describes the difference between \( -eW_0 \) and \( -eW_S \). Here, we use \( \Delta U_G(z) \) to describe the \( z \)-component dependence [11, 12]. Hence, the profile of electron energy subband, \( E_{\text{CONF}, \varphi, r}(z) \), is represented as a function of \( \Delta U_G(z) \) in following expression:

\[
E_{n_{\varphi}, n_r}(z) = E_{n_{\varphi}, n_r}^{(0)} - eV_{\text{GS}}^* + e \left( H_{n_{\varphi}, n_r} + \frac{4\pi \epsilon n_{\text{CH}}}{C_{\text{OX}}} \right) \Delta U_G(z),
\]

(1)

where \( V_{\text{GS}}^* = V_{\text{GS}} - \phi_{\text{GC}} + w_{\text{FB}}, \ V_{\text{GS}} \), \( \phi_{\text{GC}} \) and \( w_{\text{FB}} \) denote voltage between the gate-source, the work function difference between the gate and channel material, and the conduction band edge measured from \( E_{\text{FS}}(e) \) under the flat-band condition, respectively, \( E_{\text{CONF}, \varphi, r}^{(0)} \) represents the confinement energy level measured from \( -eW_S \) in an 1-D infinite square well potential, \( H_{n_{\varphi}, n_r} \) is the matrix element, \( C_{\text{OX}} \) and \( \epsilon_{\text{CH}} \) are the gate oxide capacitance per unit length and the dielectric constant of the channel [12]. In addition, \( \Delta U_G(z) \) is expressed as [11]:

\[
\Delta U_G(z) = A \exp(\gamma \cdot z) + B \exp(-\gamma \cdot z),
\]

(2)

where coefficients \( A \) and \( B \) are given by

\[
A = \frac{\left(V_{\text{bi}} - V_{\text{GS}}^*\right) \left[1 - \exp(-\gamma \cdot L_G)\right] + V_{\text{DS}} \cdot \left(-\frac{4}{\gamma^2 \cdot R^2}\right)}{\exp(\gamma \cdot L_G) - \exp(-\gamma \cdot L_G)},
\]

(3)

\[
B = \frac{\left(V_{\text{bi}} - V_{\text{GS}}^*\right) \left[\exp(\gamma \cdot L_G) - 1\right] - V_{\text{DS}} \cdot \left(-\frac{4}{\gamma^2 \cdot R^2}\right)}{\exp(\gamma \cdot L_G) - \exp(-\gamma \cdot L_G)},
\]

(4)

where \( \gamma \) is scaling parameter depending on device geometry described by \((2/R)(1+4\pi \epsilon_{\text{CH}}/C_{\text{OX}})^{1/2} \). Here, note that \( V_{\text{bi}} \) is the built-in potential at the junction between the source and channel. Moreover, by considering the derivative of (2) is zero, \( z_{\text{MAX}} \) is then read by

\[
z_{\text{MAX}} = \frac{1}{2\gamma} \ln \left(\frac{B}{A}\right).
\]

(5)

2.3. Transmission coefficient calculation

Since SDT current should be considered within the sub-7 nm channel length device in the subthreshold region [14], the WKB approximation is assumed to be used to compute the transmission coefficients of each electron subband \( T_{n_{\varphi}, n_r}(E_z) \) [13, 14]:

\[
T_{n_{\varphi}, n_r}(E_z) = \exp\left\{-\frac{2}{\hbar} \int_{z_1}^{z_2} \left[2m_r \left(E_{n_{\varphi}, n_r}(z) - E_z \right)\right]^{1/2} dz\right\},
\]

(6)

where \( z_1 \) and \( z_2 \) denote the classical turning points, \( m_r \) and \( E_z \) are the effective mass and the electron kinetic energy along \( z \)-direction, respectively. However, by assuming that the electron energy level distribution with respect to \( E_{n_{\varphi}, n_r}(0) \) can be approximately represented as a parabolic function in the following expression:
\[ E_{\alpha, \beta, B}(z) = a(z - z_{\text{MAX}})^2 + b, \]  
\[ a = -\frac{E_{\alpha, \beta}(z_{\text{MAX}}) - E_{\alpha, \beta}(0)}{z_{\text{MAX}}^2}, \]
\[ b = E_{\alpha, \beta}(z_{\text{MAX}}) - E_{\alpha, \beta}(0). \]
By substituting (7) into (6), the electron transmission coefficients can be analytically expressed by:
\[ T_{\alpha, \beta}(E) = \exp \left( -\frac{\pi}{h} \sqrt{\frac{2m}{a}} \left[ b - E_z \right] \right). \]  
2.4. Landauer formula
In general, we can use the Landauer formula to express the drain current [11-15]:
\[ I_{DS} = \frac{e}{\pi h} \sum_{\alpha} \sum_{\beta} \int_0^{\infty} dE_z T_{\alpha, \beta}(E_z) \left[ f(E_{F,S}, E_{\text{total}}) - f(E_{F,D}, E_{\text{total}}) \right], \]
\[ E_{\text{total}} = E_{\alpha, \beta}(z) + E_z, \]
where the Fermi-Dirac distribution function in the source and drain regions are denoted by \( f(E_{F,S}, E_{\text{total}}) \) and \( f(E_{F,D}, E_{\text{total}}) \). Furthermore, the SDT and ballistic currents consist of electrons in the channel, whose energies are \( E_{\text{total}} < E_{\text{MAX}, \alpha, \beta, \alpha} \) and \( E_{\text{total}} > E_{\text{MAX}, \alpha, \beta, \alpha} \), respectively. Therefore, the SDT current considering the WKB model in the subthreshold region, denoted by \( I_{DS, \text{sub}} \), can be developed by substituting (10) into (11). According to the ballistic model in our previous work, we can represent the drain current in the inversion region, denoted by \( I_{DS, \text{inv}} \), by using the Natori formula [12]. Then, we can implement a smoothing function to express the total drain current \( I_{DS, \text{tot}} \) in one analytic formula in all operating regions [11]:
\[ I_{DS, \text{tot}} = \beta I_{DS, \text{sub}} + I_{DS, \text{inv}}, \]
\[ \beta = \left( 1 + \exp \left[ \mu (\eta - \theta) \right] \right)^{-1}, \]
\[ \eta = \left( eV_{\text{GS}} - E_{\alpha, \beta} \right)/k_B T. \]
where \( \beta \) denotes one smoothing function, \( \mu \) and \( \theta \) are the fitting parameters without any physical meanings set as 0.5 and -7.5, respectively in this work.
3. Results and Discussion
In this paper, we assume the ballistic drain current can be represented by only taking the lowest the subband \((n_{\alpha, \beta} = 0, n_{\gamma} = 1)\) into consideration in all operating regions [12]. Comparisons of the lowest subband profile within the channel, between analytic compact models with and without parabolic approximation (solid and dashed curves) and NEGF simulation (open circle lines) [16], are shown in figure 3. Three lines correspond to the calculation results with \( V_{\text{GS}} = 0, 0.2 \) and 0.5 V, respectively. Interfaces between source/channel and drain/channel are illustrated by the dashed lines. Here, we expand the channel length into the source and drain regions by 0.8 and 1.45 nm, respectively, in this model to achieve good agreements. As demonstrated in this figure, the subband profiles with analytic compact model can successfully catch the shape of that with NEGF simulator in the subthreshold region. Figure 4 shows comparisons of compact model calculation with and without parabolic approximation for
dependencies of $E_z-T_{ne,nr}$ when implementing WKB approximation [17]. The solid and dashed lines relate to the calculation results with $V_{GS} = 0, 0.1, 0.2$ and 0.3 V. It should note that the transmission coefficients must be 1 under different gate bias correspondingly at the situation of $E_{\text{total}} > E_{0,1}(z_{\text{MAX}})$.

Figure 3. Comparison results between NEGF simulation and the compact models with and without parabolic approximation for the lowest subband profile calculation.

Figure 4. Transmission coefficient calculations with and without parabolic approximation.

Figure 5 demonstrates the subthreshold and inversion drain currents vs. gate voltage characteristics for different channel lengths set as 5 and 7 nm from the top. The solid and dashed lines and open circle show the results obtained by the ballistic and WKB models and NEGF simulation, respectively. It illustrates good agreement with the WKB and ballistic models in the subthreshold and inversion regions, respectively. Note that when the height of potential barrier in the channel becomes lower, the amount of electrons tunneling through that potential barrier decrease. It can be considered that the SDT current
reduces as $V_{GS}$ becomes large. Figure 6 illustrates a comparison results of $I_{DS}$-$V_{GS}$ characteristics between SILVACO NEGF numerical and analytic compact model calculations. The open and solid circle and solid lines represent calculation results of NEGF simulation and analytic model considering the SDT for the channel lengths of 5 and 7 nm from the top, respectively.

![Comparison results between the models considering the WKB approximation and ballistic transport and NEGF simulation, respectively, for drain current characteristics calculations in the subthreshold and inversion regions.](image1)

![Current characteristics from numerical SILVACO NEGF and analytic compact model calculation for all operating regions in $I_{DS}$-$V_{GS}$ characteristics, respectively.](image2)

**4. Conclusions**

We have proposed an analytic compact model of drain current for short-channel cylindrical ballistic GAA MOSFET including SDT effect. By implementing a parabolic function to approximate the subband profile over all the channel, it is practical that the WKB approximation can be used to evaluate the SDT transmission coefficients. From the Landauer formula, we can calculate SDT drain current in the subthreshold region by only taking the lowest subband into account. Finally, we represent an analytic compact model of GAA MOSFET incorporating both SDT and ballistic currents for all operating regions. This fully analytic compact model is tested against NEGF simulation, and showed a good agreement.

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