**Which way** interpretation of the dephasing of charge qubits in quantum dots

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**Abstract.** We show that phonon-induced dephasing of exciton states confined in a quantum dot may be interpreted in terms of *which way* information transferred from the carrier system to the lattice environment. Using distinguishability of quantum states as a measure for this information transfer we cast this interpretation in the form of a quantitative complementarity relation.

The ability of a quantum system to remain in a superposition of states is the key feature that distinguishes it from a classical one and provides the unusual power for quantum computing schemes. However, the phase coherence of such a quantum superposition is affected by interaction with the surrounding world. The resulting effects of decoherence or dephasing not only belong to the most fundamental aspects of the quantum theory [1] but also present serious limitations to quantum computing schemes, where maintaining system coherence over many control operations is of primary importance [2]. Thus, not only quantitative estimates but also qualitative understanding of such dephasing processes is of great interest.

Contrary to the classical situation, where system states are perturbed due to influence of external noise, dephasing of a quantum state may be interpreted as a result of perturbation exerted by the system on its environment [3]. A state-dependent trace imprinted on the environment may allow one to infer the system state by observing only the environment. Because of the historical relation to the interference of two spatial paths in the Young experiment destroyed by the knowledge of the system state, the latter is customarily referred to as *which way* (or “welcher Weg”) information. It should be stressed that dephasing is due to correlations between the two physical systems and is present no matter whether the *which way* information is really stored or extracted in a form useful to a human.

An exciton confined in a semiconductor quantum dot (QD) has been proposed as a natural implementation of an optically controlled quantum bit (qubit) with the two quantum logical values encoded as the presence (|1⟩) or absence (|0⟩) of the exciton in the QD [4]. Here, the dephasing effect is observed as a decay of the coherent polarization response from an optically excited QD (or an ensemble thereof) [5]. Another class of experiments where coherence properties of the system are manifested are time-domain interference experiments [6, 7, 8], where the final average number of excitons created in a QD with a phase-locked two-pulse sequence is measured as a function of the time delay between the pulses. This time delay is analogous to path length difference in the space-domain (Young) setup and produces a shift between the phase of the superposition state created by the first pulse and the phase of the second pulse, leading to an oscillation of the final average exciton number between 0 and 1. The visibility of
this exciton occupancy fringes is affected by the dephasing of the superposition state between the pulses.

In this paper we relate the dephasing of an exciton in a QD to the which way trace left in its lattice environment due to carrier–phonon interactions. By using a formal measure for information on the system state broadcast into the environment in the course of the dephasing process [9, 10] we will derive a quantitative relation between the degree of system coherence (as manifested by the amplitude of coherent radiation or visibility of exciton occupancy fringes) and the amount of which way information. In this way, we prove complementarity between the knowledge of the system state and the degree of coherence retained by the system.

The present discussion will be restricted to the ground state of an exciton confined in a QD, which is a very good approximation at low temperatures, under resonant excitation with an appropriately polarized laser beam. The observed decrease of coherence in such a system has been accounted for by a model [11] invoking interaction with acoustical phonons. The model reproduces experimental data very well [11, 12] and may serve as a reliable starting point to describe the evolution of the combined system of confined carriers and lattice modes. Under conditions assumed here, the most important effect is that of deformation potential coupling to longitudinal acoustic phonons.

The complete Hamiltonian of the system is

\[ H = \epsilon |1\rangle\langle 1| + H_{\text{ph}} + |1\rangle\langle 1| \sum_k \hbar \omega_k (g_k b_k + g_k^\dagger b_k^\dagger), \tag{1} \]

where the first term describes the energy of the confined exciton (\( \epsilon \) is the energy difference between the states without phonon corrections), \( H_{\text{ph}} = \sum_k \hbar \omega_k b_k^\dagger b_k \) is the Hamiltonian of the phonon subsystem (\( \omega_k \) is the frequency of a phonon with wave vector \( k \) and \( b_k, b_k^\dagger \) are the corresponding creation and annihilation operators) and the third term describes the interaction, with

\[ g_k = (\sigma_e - \sigma_h) \sqrt{\frac{1}{2\rho V_N^3 \hbar^2 k^3}} \int_{-\infty}^{\infty} d^3 r \psi^*(r)e^{-iE_t r} \psi(r), \tag{2} \]

where \( \sigma_{e,h} \) are deformation potential constants for electrons and holes, \( V_N \) is the normalization volume of the phonon system, \( \rho \) is the crystal density, \( c \) is the speed of sound, and \( \psi(r) \) are single-particle wave functions which are assumed identical for both types of carriers. In our calculations we use typical parameters for a self-assembled InAs/GaAs structure: \( \psi(r) \) modelled by Gaussians with 4 nm width in the \( xy \) plane and 1 nm along \( z \), \( \sigma_e - \sigma_h = 9.5 \) eV, \( \rho = 5300 \) kg/m\(^3\), \( c = 5150 \) m/s.

The Hamiltonian (1) is diagonalized by the unitary transformation \( W = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes w \), where \( w = \exp\left[\sum_k \left( g_k b_k^\dagger - g_k b_k \right)\right] \), \( I \) is the identity operator, and the tensor product refers to the carrier subsystem (first component) and its phonon environment (second component). This allows us to find the system evolution exactly. Let us assume that an ultrashort optical pulse at \( t = 0 \) performed a \( \pi/2 \) rotation, i.e., prepared the system state \( \rho_0 = (|\psi_0\rangle\langle \psi_0|) \otimes \rho_E \), where \( \rho_E \) is the density matrix of the phonon subsystem (environment) at thermal equilibrium and \( |\psi_0\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \). The system state at time \( t > 0 \) may be then written in the form [13]

\[ \rho(t) = \frac{1}{2} \left( e^{-iE_t \hbar/2} w^\dagger(t) \rho_E w(t) e^{iE_t/\hbar} \rho_E w^\dagger(t) w(t) \right), \tag{3} \]

where \( E = \epsilon - \sum_k \hbar \omega_k |g_k|^2 \), \( w(t) = e^{-iH_{\text{ph}} t/\hbar} e^{iH_{\text{ph}} t/\hbar} \).

The density matrix for the carrier subsystem is obtained by tracing out the phonon degrees of freedom,

\[ \rho_S(t) = \text{Tr}_E \rho(t) = \frac{1}{2} \left( e^{-iE_t \hbar/2} \frac{1}{w^\dagger(t) w(t)} e^{iE_t/\hbar} \frac{1}{w^\dagger(t) w(t)} \right). \tag{4} \]
The first of the two classes of experiments mentioned above consists in measuring the amplitude of the coherent optical polarization $P(t)$ which is proportional to the non-diagonal element of the reduced density matrix (4),

$$|P(t)|^2 = P_{0}^2 |\langle w^\dagger(t)w \rangle|^2. \quad (5)$$

In the other experiment, at time $t = \tau > 0$ one applies a second pulse that performs a $\pi/2$ rotation, $U_{\pi/2} = [I + i \sin \phi \sigma_x - i \cos \phi \sigma_y]/\sqrt{2}$, where $\phi = E\tau/\hbar$, and measures the average occupation of the state $|1\rangle$, i.e., the average exciton occupancy $N(\tau) = \langle 1|U_{\pi/2}\rho_S(\tau)U_{\pi/2}^\dagger|1\rangle = 1/2 \left[ 1 + |\langle w^\dagger(\tau)w \rangle| \cos(\phi + \theta) \right]$, where we wrote $\langle w^\dagger(\tau)w \rangle = |\langle w^\dagger(\tau)w \rangle| e^{i\theta}$. The value of $N(\tau)$ shows oscillations (interference fringes) in function of $\phi$, or of the time delay $\tau$, on the time scale $\hbar/E \sim 1$ fs, modulated by a much more slowly changing function $|\langle w^\dagger(\tau)w \rangle|$, related to dephasing. The visibility of the fringes is

$$V(\tau) = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}} = |\langle w^\dagger(\tau)w \rangle|. \quad (6)$$

It may be shown [13] that $|\langle w^\dagger(\tau)w \rangle| = \exp \left[ \sum_k |g_k|^2 (\cos \omega_k \tau - 1)(2n_k + 1) \right]$, where $n_k$ are bosonic equilibrium occupation numbers. For sufficiently regular coupling constants (as in the case of all carrier-phonon coupling mechanisms in QDs) the above quantity decreases form the initial value of 1 to a certain finite value that depends on the system parameters and temperature. As a result, the coherent polarization response is partly reduced over the first few picoseconds of the system evolution (Fig. 1a; cf. Ref. [14]). The visibility of time-domain interference fringes undergoes the same reduction as a function of the delay time between the pulses (Fig. 1b,c).

Let us now derive a quantitative complementarity relation between the degree of coherence in the quantum dot system and the amount of which way information transferred to the environment [9]. This relation is analogous to the visibility–distinguishability relation in the double-slit setup [9, 10].

A measure of information on the carrier system contained in its phonon environment is defined as follows. One performs a measurement on the environment and uses its result to predict if an exciton will be found in the QD in a subsequent measurement. The probability of a correct prediction ranges from 1/2 (guessing at random in absence of any correlations) to 1 (knowing for sure, when the systems are maximally entangled). Quantitatively, an intrinsic measure is provided by the distinguishability of states [9, 10], $D = 2(p - 1/2)$, where $p$ is the probability for a correct prediction for the exciton state maximized over all possible measurements on the

Figure 1. (a) Decay of the coherent radiation from a confined exciton at various temperatures, as shown. (b) Occupation interference fringes for different delay times $\tau = \tau_0 + \Delta\tau$ at 50K. The three panels show the dependence of the final exciton occupation on the delay time $\Delta\tau$ on femtosecond scale and correspond to three different time windows shifted by $\tau_0 = 0, 1$ and 2 picoseconds from the initial time (we assume $E/\hbar = 1.5$ fs$^{-1}$). (c) The envelope of the occupation fringes at three different temperatures as shown.
environment. In this way, guessing at random and knowing for sure correspond to $D = 0$ and $D = 1$, respectively.

For a general density matrix of the compound system $\rho = \sum_{i,j=0,1} |i\rangle\langle j| \otimes \rho_{ij}$, it may be shown [9, 13] that the distinguishability is given by $D = (1/2)\text{Tr}|\rho_{00} - \rho_{11}|$. Thus, for the carrier-phonon state of Eq. (3) one finds for the distinguishability of carrier states due to correlations with the phonon environment $D(t) = (1/2)\text{Tr}|\rho(t) - \rho(t)|$. One has, in general [15],

$$D^{2}(t) = 1 - \left[\text{Tr}\rho(t)\right]^{2} = 1 - \left[\text{Tr}\rho(t)\right]^{2}.$$

Since $\text{Tr}[A] \geq |\text{Tr}[A]|$ we may write

$$\text{Tr}[\rho(t)] \geq |\text{Tr}[\rho(t)]| = |\langle w(t)|w\rangle|.$$

Using Eqs. (5) and (6) and combining Eqs. (7) and (8) we find the inequalities

$$\left[\frac{P(t)}{P_0}\right]^{2} + D^{2}(t) \leq 1, \quad \gamma^{2}(\tau) + D^{2}(\tau) \leq 1,$$

which relate the degree of coherence to the which way knowledge of the system state.

Eqs. (9) show that extracting information on the system state must inevitably destroy coherence. They reflect the fundamental complementarity that lies at the foundation of quantum theory. While phase relations pertain to wave-like properties of the quantum system, an attempt to determine the system state refers to particle-like properties of the same system, like indivisibility of the relevant entity: when measured, an exciton in a QD may be either present or absent, with no intermediate states. Here we have shown that optical experiments with semiconductor quantum dots can contribute to these fundamental issues.

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