Large diamagnetic persistent currents

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\textbf{Abstract.} Magnetization of finite size systems in the form of mesoscopic rings has intrigued physicists for a long time. Theories to date predict paramagnetic behavior, but experiments consistently show diamagnetic behavior. We show that the evanescent modes that are always present in rings of finite thicknesses can carry very large diamagnetic currents that are also very sensitive to disorder. Their contribution has always been ignored so far. Their contribution has features similar to that observed in experiments.

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1. Introduction

When a magnetic field is applied through a metallic ring, it induces a current. In an extremely small and narrow ring, this current has peculiar features: it becomes persistent and shows periodic oscillations as a function of the magnitude of the magnetic flux through the ring [1]. These effects are caused by the coherence of the electron wavefunction, which in such small rings reaches throughout along the length of the ring. The magnetization caused by the persistent current is an example of orbital magnetization. While in infinite simple metals the orbital magnetization leads to the Landau diamagnetism, a simple theory for narrow quantum rings leads to paramagnetism, due to the different boundary conditions [2, 3]. Persistent currents also appear in open systems [4] and one knows that the closed system current for the $n$th level can be recovered in the limit that the coupling to the reservoirs tends to zero [4, 5].

Several experiments have been done that confirm the existence of persistent currents in quantum rings [6]–[13]. While earlier experiments [6]–[12] had some ambiguity, recent experiments [13] made with an ensemble of $10^5$ rings have carefully studied the sign and the periodicity of the persistent current. The results show a large persistent current that is diamagnetic in nature at low fields.

While in bulk samples standard principles tell us how to calculate ensemble averages, similar principles do not necessarily apply to mesoscopic systems. Several ways of ensemble averaging were employed [14]–[18], namely, disorder averaging at fixed $M$ ($M$ being the number of electrons in a ring), averaging over disorder as well as $M$, disorder averaging at fixed chemical potential $\mu$, disorder averaging at flux-dependent chemical potential $\mu$, etc. Different methods give slightly different results. All these works take the $n$th-level current to be proportional to $(n - 2\pi \phi/\phi_0)$ which at a small field is diamagnetic for even values of $n$ and paramagnetic for odd values of $n$. Here, $\phi$ is the flux through the ring and $\phi_0 = hc/e$. The ensemble-averaged current turns out to be paramagnetic [14]–[18]. At present, there is even no known mechanism that can account for such large diamagnetic persistent current as seen in experiments.

Magnetization or conductance in mesoscopic systems is very sensitive to disorder and will ensemble average to zero. But the second harmonic of $\phi_0/2$ (or $hc/2e$) periodicity does not obey this rule and gives a nonzero value [14, 15, 19, 20]. On this the periodicity theory and experiments agree. This is essentially because the second harmonic consists of time-reversed trajectories and the disorder configuration does not change the observed quantity randomly. This is very robust and manifests in a variety of phenomena briefly described hereinafter. Weak localization in disordered metallic or semi-conducting samples occurs because of this. Forward scattering probability beyond a certain length turns out to be negligibly small while the back scattering arising due to time-reversed trajectories always interferes constructively, irrespective of disorder configuration. As a consequence, the Aronov–Altshuler–Spivak weak localization correction to conductance has $\phi_0/2$ periodicity [19]. Also the response of a long cylinder to an applied magnetic field turns out to have $\phi_0/2$ periodicity [20]. $\phi_0/2$ periodicity of ensemble-averaged persistent current is due to the same reason that the first harmonic averages to zero, whereas the second does not [14, 15].

The first attempt to explain the discrepancy in sign and magnitude is based on repulsive interactions between electrons [15, 17]. This did not turn out to be the correct mechanism, because this yields a paramagnetic response at low fields, whereas a recent experiment has conclusively shown that the observed response is diamagnetic. For a recent analysis of the
effect of disorder and interactions, we suggest [21]. A more recent attempt to explain the experimental discrepancy is based on additional currents that may be generated in rings due to the rectification of high-frequency, non-equilibrium noise [22]. This mechanism can give a diamagnetic current in the absence of spin–orbit coupling and a paramagnetic response in presence of spin–orbit coupling. A recent experiment [13] has also ruled out this explanation as a paramagnetic response could not be observed in the presence of strong spin–orbit coupling. The origin of a high-frequency, non-equilibrium noise is also unclear.

The purpose of this work is to show a mechanism that can result in very large diamagnetic current. We show that the evanescent modes in a quasi-one-dimensional (Q1D) ring can carry very large diamagnetic currents. Evanescent modes are always present in realistic situations and they can occur for several reasons. An inspection of the Schrödinger equations shows that the evanescent modes are a natural solution that can be consistent with all boundary conditions. Some models account for them, whereas others do not. Surface scattering can create evanescent states that are more commonly called surface states. Surface states naturally occur in multiband, tight-binding models but not in single-band models. In continuous models, collisions (electron–electron, electron–phonon, or electron–impurity) can excite an electron to evanescent modes. It has been shown that in a one-dimensional (1D) ring, evanescent modes can carry a diamagnetic persistent current [23]. It has a very small magnitude compared to the persistent current in propagating modes and cannot exhibit $\phi_0/2$ periodicity as it is not sensitive to disorder. Rings used in the experiments have a finite thickness and are referred to as Q1D rings. In this work we show that in Q1D, evanescent modes can carry a very large diamagnetic persistent current that is comparable to that of propagating modes and is as sensitive to disorder as that due to the propagating modes. This contribution was ignored in [14]–[18]. We suggest that this mechanism is a natural explanation for the observed diamagnetic persistent currents.

All experiments have been done at finite temperatures. There are thermal effects wherein an electron can get energy from some collision and get excited to higher states. Such inelastic processes will not destroy the persistent currents. Persistent currents are actually observed in networks, where the total length is much greater than the inelastic mean free path [24]. Such inelastic processes are often accounted for with the help of a reservoir [4] or an infinite wire [5]. When a mechanism for excitation is present, evanescent modes can be excited.

2. Theoretical model

We consider the ring to be coupled to an infinite wire as schematically shown in figure 1. This basically constitutes an open system like that in [4, 5]. It is also known that it can simulate the effects of inelastic collisions and thermal effects [4]. This is essentially because it allows the energy to be fixed externally, as a parameter, without maintaining any phase correlation. This is exactly what inelastic collisions do, i.e. they give an arbitrary energy to the electron wavefunction and change its phase randomly. We shall see in our mathematical analysis how evanescent modes are excited in this system very naturally. Hence, this is a model that accounts for evanescent modes. The infinite wire in figure 1 could also be replaced with a reservoir, but we would not expect this to change the basic results. We consider two modes of propagation as the results can be generalized to any number of modes. There is a $\delta$-potential impurity present in the ring at any arbitrary position X (figure 1). We apply Aharonov–Bohm flux $\phi$ through the ring, perpendicular to the plane of the paper. The Schrödinger equation for a Q1D wire in the presence of a $\delta$-potential at $x = 0, y = y_i$ is (the third degree of freedom, i.e. $z$-direction, is
Figure 1. A ring connected to an infinite wire. A $\delta$-potential impurity is at position $X$. Distances $PQ$, $QX$, and $XQ$ are denoted by $l_1$, $l_2$ and $l_3$, respectively. The variables $\alpha$ and $\beta$ denote that phase shifts due to the flux at intervals $l_2$ and $l_3$ ($\alpha + \beta = 2\pi \phi/\pi_0$).

usually frozen by creating a strong quantization [25])

$$
-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V_c(y) + \gamma \delta(y - y_i) = E \psi(x, y),
$$

(1)

where the $x$-coordinate is along the wire (or ring) and the $y$-coordinate is perpendicular to the wire. We assume the ring to be so large that its curvature can be neglected. $V_c(y)$ is the confinement potential making up the quantum wires in figure 1. Solutions to the Schrödinger equation in the ring geometry can be obtained by applying Griffith’s boundary conditions (continuity of wavefunction and conservation of current at the junction Q). The magnetic field just appears as a phase of $\psi(x, y)$ that will be accounted for while applying boundary conditions. Away from the scattering region equation (1) can be separated as

$$
-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} = \frac{\hbar^2 k^2}{2m} \psi(x)
$$

(2)

and

$$
-\frac{\hbar^2}{2m} \frac{d^2 \chi_n(y)}{dy^2} + V_c(y) = E_n \chi_n(y).
$$

(3)

Here, we consider that electron is propagating along the $x$-direction. This means in regions I and II of figure 1, the $x$-direction is along the arrows. In region III, the $x$-direction is along the line joining P and Q. And in regions IV and V, the $x$-direction is along the perimeter of the ring. One can choose different axes in the different regions as the matrix equations for mode matching is independent of this choice as will be soon evident. The confinement potential $V_c(y)$ in different regions is then in the $y$ (transverse)-direction. We take $V_c(y)$ to be a square well potential of width $W$ that gives $\chi_n(y) = \sin[(n\pi/W)(y + (W/2))]$. In the first mode, the plane wave solution of equation (2) has the propagating wave vector $k_1 = \sqrt{(2mE/\hbar^2) - (\pi^2/W^2)}$, and in the second mode $k_2 = \sqrt{(2mE/\hbar^2) - (4\pi^2/W^2)}$, where $m$ is the electron mass, $E$ is the electron energy and $W$ is the width of the quantum wire. When electrons are incident along region I (in figure 1) in the first mode, the scattering problem can be solved exactly. The solutions to equation (2) in the different regions become

$$
\psi_1 = \frac{1}{\sqrt{k_1}} e^{ik_1x} + \frac{r'_{11}}{\sqrt{k_1}} e^{-ik_1x} + \frac{r'_{12}}{\sqrt{k_2}} e^{-ik_2x},
$$

(4)
where \( r_{11} ', r_{12} ', g_{11} ' \) and \( g_{12} ' \) are the scattering matrix elements and \( A, B, C, D, E, F, G, H, J, K, L \) and \( M \) are to be determined by mode matching.

Note that at \( P \) and \( Q \), we have a three-legged junction that is schematically shown in figure 2. A popularly used scattering matrix for a three-legged junction is given in [26]. This junction \( S \)-matrix does not include channel mixing and cannot account for any contribution from evanescent modes. In this work, we propose a three-legged junction scattering matrix \( S_J \) for a two-channel quantum wire that can be generalized to any number of channels. Just like in [26], this junction matrix is also based on the principle of unitarity. While in [26], all interchannel matrix elements are 0, in our \( S_J \) all matrix elements are nonzero. It is given by

\[
S_J = \begin{pmatrix}
    r_{11} & r_{12} & g_{11} & g_{12} & f_{11} & f_{12} \\
    r_{21} & r_{22} & g_{21} & g_{22} & f_{21} & f_{22} \\
    g_{11} & g_{12} & r_{11} & r_{12} & f_{11} & f_{12} \\
    g_{21} & g_{22} & r_{21} & r_{22} & f_{21} & f_{22} \\
    f_{11} & f_{12} & f_{11} & f_{12} & r_{11} & r_{12} \\
    f_{21} & f_{22} & f_{21} & f_{22} & r_{21} & r_{22}
\end{pmatrix},
\]

where

\[
\begin{align*}
    r_{11} &= \frac{3k_2 + k_1}{3k_1 + 3k_2} , \\
    g_{11} &= f_{11} = \frac{2k_1}{3k_1 + 3k_2} , \\
    r_{12} &= g_{12} = f_{12} = \sqrt{\frac{k_2}{k_1} \frac{2k_1}{3k_1 + 3k_2}} , \\
    r_{22} &= \frac{3k_1 + k_2}{3k_1 + 3k_2} , \\
    g_{22} &= f_{22} = \frac{2k_2}{3k_1 + 3k_2} , \\
    r_{21} &= g_{21} = f_{21} = \sqrt{\frac{k_1}{k_2} \frac{2k_2}{3k_1 + 3k_2}} .
\end{align*}
\]

One of the reasons for taking an infinite wire at \( P \) rather than a reservoir at \( P \) (for example, continuation of region III to infinity) is that this system has a \( 4 \times 4 \) \( S \)-matrix whose unitarity can be checked at any parameter region. This ensures the form of \( S_J \) in equation (9) is correct.
(Replacing the infinite wire by a reservoir at P would reduce the $S$-matrix to a number in some parameter regimes, with trivial unitarity.) Mode matching at the junction P (figure 1) gives

$$
\begin{pmatrix}
  r'_{11} \\
  r'_{12} \\
  g'_{11} \\
  g'_{12} \\
  A \\
  C
\end{pmatrix}
= S_J
\begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0 \\
  B \\
  D
\end{pmatrix}.
$$

(11)

Similarly, mode matching at the junction Q (figure 1) gives

$$
\begin{pmatrix}
  B e^{-ik_1 l_1} \\
  D e^{-ik_2 l_1} \\
  E \\
  G \\
  K e^{-ik_1 l_1} \\
  M e^{-ik_2 l_1}
\end{pmatrix}
= S_J
\begin{pmatrix}
  A e^{ik_1 l_1} \\
  C e^{ik_2 l_1} \\
  F e^{-i\alpha} \\
  H e^{-i\alpha} \\
  J e^{(k_1 l_3 + \beta)} \\
  L e^{(k_2 l_3 + \beta)}
\end{pmatrix}.
$$

(12)

Here $l_1$, $l_2$ and $l_3$ are shown in figure 1. In $\alpha + \beta = 2\pi \phi/\phi_0$, $\phi$ is the magnetic flux and $\phi_0 = hc/e$ is the flux quantum. Equations (11) and (12) apply Griffith’s boundary conditions to wavefunction at P and Q due to the unitarity of $S_J$. Mode matching at the impurity site X (figure 1) gives

$$
\begin{pmatrix}
  F e^{-ik_1 l_1} \\
  H e^{-ik_2 l_1} \\
  J \\
  L
\end{pmatrix}
= \left(\begin{array}{cccc}
  \tilde{r}_{11} & \tilde{r}_{12} & \tilde{r}_{11} & \tilde{r}_{12} \\
  \tilde{r}_{21} & \tilde{r}_{22} & \tilde{r}_{21} & \tilde{r}_{22} \\
  \tilde{t}_{11} & \tilde{t}_{12} & \tilde{r}_{11} & \tilde{r}_{12} \\
  \tilde{t}_{21} & \tilde{t}_{22} & \tilde{r}_{21} & \tilde{r}_{22}
\end{array}\right)
\times
\begin{pmatrix}
  E e^{(k_1 l_3 + \alpha)} \\
  G e^{(k_2 l_3 + \alpha)} \\
  K e^{-i\beta} \\
  M e^{-i\beta}
\end{pmatrix},
$$

(13)

where

$$
\tilde{r}_{pp'} = -i \left(\Gamma_{pp'}/2\sqrt{k_p k_{p'}}\right)
\left[1 + \sum_e (\Gamma_{ee'}/2\kappa_e) + i \sum_p (\Gamma_{pp'}/2\kappa_p)\right]
$$

(14)

in which $\sum_e$ represents the sum over all the evanescent modes and $\sum_p$ represents the sum over all the propagating modes. The variables $p$ or $p'$ can take values 1 and 2 as there are two propagating modes. $\kappa_e = \sqrt{(e^2\pi^2/W^2) - (2mE/\hbar^2)}$, where $e = 3, 4, \ldots$. The inter-mode
Figure 3. $I/I_0$ vs $8\pi^2mEW^2/\hbar^2$ with $\gamma = 4$, $l_1 = l_2 = l_3 = 1$, $\alpha = \beta = 0.3$ and $y_i = 0.1$.

(i.e. $p \neq p'$) transmission amplitudes are $\tilde{t}_{pp'} = \tilde{r}_{pp'}$ and intra-mode transmission amplitudes are $\tilde{t}_{pp} = 1 + \tilde{r}_{pp}$. $\Gamma_{pp'}$ is given as

$$\Gamma_{pp'} = \frac{2m\gamma}{\hbar^2} \sin \left( \frac{p\pi(y_i + W/2)}{W} \right) \sin \left( \frac{p'\pi(y_i + W/2)}{W} \right),$$

where $y_i$ is used for denoting the position coordinate of the $\delta$-potential impurity.

We calculate $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $J$, $K$, $L$ and $M$ numerically from equations (11), (12) and (13). Persistent current is defined by

$$I = \int_{-W/2}^{W/2} \frac{\hbar}{2im} (\Psi^\dagger \vec{\nabla} \Psi - \Psi \vec{\nabla} \Psi^\dagger) \, dy$$

which can be simplified to give

$$I = I^{(k_1)} + I^{(k_2)},$$

where

$$I^{(k)} = 2I_0(|E|^2 - |F|^2 + |G|^2 - |H|^2)^{(k)}$$

is the current when electron is incident along the left lead in $k_i$ channel ($i = 1$ or 2) and $I_0 = \hbar e/2mW^2$. This is the scattering problem defined by equations (11)–(13).

3. Results

First we take the energy range ($4\pi^2 \leq 2mEW^2/\hbar^2 \leq 9\pi^2$, i.e. $40 \leq 2mEW^2/\hbar^2 \leq 88$) in such a way that both the modes are propagating. The nature of current obtained from equation (17) is shown in figure 3. This gives the behavior that is captured in earlier works [14]–[18]. Earlier works take $n$th-level current to be $(n - 2\pi\phi/\phi_0)$. In figure 3, we are considering a two-channel ring, we can see that the first two peaks at $A$ and $B$ are diamagnetic, the next two peaks at $C$ and $D$ are paramagnetic and so on. As the coupling of the ring to the infinite wire is reduced, the peaks become narrower and in the limit of zero coupling, each becomes a $\delta$ function.
Figure 4. Reflection (solid line) and transmission (dashed line) coefficients of the system when second channel is evanescent. The system parameters are $l_1 = l_2 = l_3 = 1$, $\alpha = \beta = 0.3$, $y_1 = 0.1$ and $\gamma = 4$. 

peak carrying a current $(n - 2\pi \phi/\phi_0)$. This has been explicitly shown in a seminal paper by Büttiker [4] and put on firm theoretical grounds by Akkermans et al [5]. Figure 3 can also be obtained from the $S$-matrix following the method in [5]. The broadening of the peaks in figure 3 is due to inelastic effects modeled by the coupling to scattering states [4, 5]. Thus, single-ring current can be paramagnetic as well as diamagnetic as can be seen in figure 3. We shall show that when we make one of the modes evanescent, we will get a behavior that has not been discussed before. As soon as $2mEW^2/\hbar^2$ becomes less than $4\pi^2$, the second channel becomes evanescent. This is because $k_1$ is real, whereas $k_2$ is imaginary ($k_2 \to i\kappa_2$ in this regime). The rest of the paper will discuss this regime where one mode is propagating and the other is evanescent. We shall show that evanescent modes can only carry diamagnetic current, and hence the larger the number of evanescent modes considered, the larger the diamagnetic response will be.

When one of the modes is evanescent, the $S$-matrix becomes $2 \times 2$ and is given by

$$S = \begin{pmatrix} r'_{11} & g'_{11} \\ g'_{11} & r'_{11} \end{pmatrix}. \quad (19)$$

Although the $S$-matrix is $2 \times 2$, its calculation has to be done by using the $6 \times 6$ junction matrix $S_J$ defined in equation (9) and the $4 \times 4$ impurity $S$-matrix defined in equation (13). The scattering matrix elements $g'_{12}, r'_{12}$ etc are still nonzero; although they do not carry any current, they define the coupling to the evanescent mode. So equations (11), (12) and (13) still hold with $k_2 \to i\kappa_2$, where $\kappa_2 = \sqrt{(4\pi^2/W^2) - (2mE/\hbar^2)}$. Unitarity should imply $|r'_{11}|^2 + |g'_{11}|^2 = 1$ and indeed we get this from the junction matrix defined by $S_J$ in equation (9) and impurity $S$-matrix defined in equation (13). A plot of $|r'_{11}|^2$ and $|g'_{11}|^2$ versus $EW^2$ is shown in figure 4. This implies that $S_J$ is appropriate to account for realistic multichannel situations. An earlier junction matrix [26] does not take into account such effects and does not allow one to include coupling to evanescent modes. Current is expected to be continuous as the energy changes continuously from evanescent modes to propagating modes at $(2mEW^2/\hbar^2) = 4\pi^2$. This also comes out in our model.
No electron can be incident into an evanescent channel from infinity and so $I^{(k_2)}$ will not exist. But electrons incident in the $k_1$ channel can be excited into an evanescent channel in the ring implying that $G$ and $H$ in equation (18) are nonzero. A single impurity can excite an electron into the evanescent second channel (the more impurities, the better). Scattering at the junctions can also excite an electron into the evanescent second channel. An electron residing in an evanescent state will carry a current. This naturally arises in the scattering problem that is defined in equations (11)-(13). Evanescent mode current can be calculated by analytically continuing propagating-mode current below the barrier (that is $k_2 \rightarrow ik_2$). A plot of the total current $I^{(k_1)}$ in this regime is shown in figure 5, where the result is shown for the same parameters as in figure 3 ($l_1 = l_2 = l_3$) as well as for another set of parameters where the distances of $l_i$ have irrational ratios ($l_1 = 1, l_2 = \sqrt{2}, l_3 = \sqrt{3}$). It can be seen that the current is predominantly diamagnetic unlike that in figure 3. We have repeated the computations also for many other sets of parameters, and all the results show the diamagnetic tendency.

Note from equation (18) that the total current $I^{(k_1)} = I_1^{(k_1)} + I_2^{(k_1)}$, where $I_1^{(k_1)} = 2I_0(|E|^2 - |F|^2)^{(k_1)}$, $E$ and $F$ being the wavefunction amplitudes in the propagating channel and $I_2^{(k_1)} = 2I_0(|G|^2 - |H|^2)^{(k_1)}$, $G$ and $H$ being the wavefunction amplitudes in the evanescent channel. In figure 6, we have shown the plot of $I_1^{(k_1)}$ and $I_2^{(k_1)}$ versus $2mEW^2/h^2$. While $I_1^{(k_1)}$ can be positive (diamagnetic) as well as negative (paramagnetic), $I_2^{(k_1)}$ is seen to be only diamagnetic. $I_2^{(k_1)}$ is the current in an evanescent channel, and there is a fundamental difference with evanescent channel currents in 1D. In 1D evanescent channel current cannot oscillate with energy because evanescent wavefunction is not of wave nature [28] and cannot exhibit resonance. If we take a 1D ring of the same length as that considered in figure 6, then evanescent current is typically (independent of disorder) given by the dotted curve in figure 6. It is maximum at the highest energy, i.e. $2mEW^2/h^2 \approx 40$ and then monotonously decreases with Fermi energy. In 1D, we have to introduce an infinitesimal region of the ring where the electron can be propagating (evanescent in the rest of the ring), for the persistent current to be oscillating between paramagnetism and diamagnetism [29]. But in the present case, the second channel is evanescent throughout the length of the ring, its wavefunction is not of wave nature, and yet can oscillate with Fermi energy. The peaks in $I_1^{(k_1)}$ are resonance effects due to wave
nature of electron wavefunction wherein at these energies, the electrons can spend a long time in the propagating mode. The impurity also gets a long time to pump more electrons into the evanescent mode. So the evanescent-mode current $I_2^{(k_1)}$ also peaks at the same energies where $I_1^{(k_1)}$ peaks (see figure 6), although the evanescent mode wavefunction is not of wave nature. The difference between them being that while the peaks in $I_1^{(k_1)}$ can be in positive direction (diamagnetic) or in negative direction (paramagnetic), the peaks in $I_2^{(k_1)}$ are always in the positive direction. As impurity configuration changes, these peaks also change randomly. But the peaks in $I_2^{(k_1)}$ always follow the peaks in $I_1^{(k_1)}$. One can also see this mathematically. Although the evanescent mode wave-function is not of a wave nature, $G$ and $H$ are functions of $k_1$ and $\kappa_2$, due to the non-locality of quantum mechanics. $E$ and $F$ are also functions of $k_1$ and $\kappa_2$. Whereas $I_1^{(k_1)}$ will fluctuate around zero value, $I_2^{(k_1)}$ will fluctuate around a certain positive value as disorder configuration changes. Apart from this shift, $I_2^{(k_1)}$ will follow the same rules as $I_1^{(k_1)}$ as far as disorder averaging is concerned (as the peaks in $I_2^{k_1}$ follow the peaks in $I_1^{k_1}$, as disorder configuration is changed). Or more appropriately, $I_2^{(k_1)}$ will follow same averaging rules as conductance that fluctuate with disorder, remaining positive all the time [19]. It is much easier to take random values of $l_2$ and $l_3$ to show this for the average current (as demonstrated in figure 5).

4. Discussion and conclusions

Impurity scattering can localize an electron at the site of the impurity in the evanescent mode. This results in enhanced partial density of states\(^5\) (contribution to density of states coming from $G$ and $H$) in the evanescent channel. Such large density of states results in large current in the evanescent channel. In real space, the electron has a large probability to reside at the impurity site in the evanescent channel. An electron at Q can tunnel to this impurity site at X along the

\(^5\) The usual definition of partial density of states is different and can be seen in [30].

Figure 6. $I_1^{(k_1)}/I_0$ and $I_2^{(k_1)}/I_0$ versus $8\pi^2 mE W^2/h^2$. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_1 = 0.1$, $\alpha = \beta = 0.3$ and $\gamma = 4$. The dotted curve shows typical behavior of evanescent-mode current in 1D ring of the same length.

\[ I_1^{(k_1)}/I_0 \text{ and } I_2^{(k_1)}/I_0 \text{ versus } 8\pi^2 mE W^2/h^2. \]
region IV and then tunnel from the impurity site back to Q along the region V. As a result, the current in the ring is much larger than that if the impurity were absent. Because in the absence of an impurity, an electron at Q will have to tunnel all the way along the ring and come back to point Q in order to give rise to a current. This probability is very low.

We suggest that the large diamagnetic component, ignored so far, is a natural explanation for the discrepancy between the theory and experiment. One can further check the validity of our explanation by measuring how the response of the ensemble scales with the number of rings \( N \) in the ensemble. In practice, the total response of the ensemble is the arithmetic sum of the response of individual rings as there is no mutual inductance between the rings. Disorder configuration is different in different rings and one theoretically uses a model of ensemble averaging for estimating the sum of the response of all the rings. We have shown the principle for which periodicity of evanescent mode currents will be identical to that of propagating mode currents as far as the sum of the response of all the rings is concerned.

Magnetization of finite size systems in the form of mesoscopic rings has posed a serious challenge to our understanding for more than a decade. Theoretical estimates have always ignored the evanescent modes as their contribution was thought to be very small. We show that the solution of the Schrödinger equation in multichannel rings consists of evanescent modes that are naturally populated due to scattering. These evanescent modes carry only diamagnetic persistent currents. For a ring of given length, the contribution coming from an evanescent mode is comparable in magnitude to that of a propagating mode. The evanescent-mode current is also as sensitive to disorder as that of propagating modes. So the inclusion of the evanescent modes is not expected to change the flux periodicity of the total response. They provide an explanation for the discrepancy between the theory and experiments. Future experiments should try to isolate the role of the contributions coming from propagating and evanescent modes. In this work, we also propose a new form of junction matrix \( S_J \) that can help solve realistic multichannel problems.

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