A new possibility to monitor collisions of relativistic heavy ions at LHC and RHIC

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Abstract

We consider the radiation of particles of one bunch in the collective field of the oncoming bunch, called coherent bremsstrahlung (CBS). The main characteristics of CBS for LHC (in the Pb–Pb mode) and for RHIC are calculated. At LHC about $3.9 \cdot 10^8 dE_\gamma/E_\gamma$ photons per second are expected for photon energies $E_\gamma \lesssim E_c = 93$ eV. It seems that CBS can be a potential tool for optimizing collisions and for measuring beam parameters. The bunch length $\sigma_z$ can be found from the critical energy of the CBS spectrum $E_c \propto 1/\sigma_z$; the transverse bunch size $\sigma_\perp$ is related to the photon rate $dN_\gamma \propto 1/\sigma_\perp^2$. A specific dependence of $dN_\gamma$ on the impact parameter between the beams allows for a fast control over the beam displacement.

1. Introduction – In this paper a new type of radiation at colliders – the coherent bremsstrahlung (CBS)– is considered. The CBS is the radiation of one bunch particles in the short collective electromagnetic field of the oncoming bunch.

For definiteness, let us consider the photon emission by a single ion with a charge $Z_1e$ moving through a bunch of ions with the charge $Z_2e$. The ordinary (incoherent) bremsstrahlung of ions at colliders has a relatively small cross section. On the other hand, at small enough photon energies $E_\gamma$, the radiation due to interaction of the $Z_1$ ion with the second bunch becomes coherent resulting in large number of emitted photons. This increase arises from an extra factor $N_2$ where $N_2$ is the second bunch population. Therefore, the number of the emitted photons $dN_\gamma$ is already proportional to $N_2^2$

$$dN_\gamma \propto N_1 N_2^2 \frac{dE_\gamma}{E_\gamma}. \quad (1)$$

The CBS occurs when a coherence length $\sim 4\gamma_1^2 \hbar c/E_\gamma$ becomes comparable or larger to the length of the second bunch $\sigma_{2z}$ (here $\gamma_1 = E_1/(m_1c^2)$ is the Lorentz factor of
Z\textsubscript{1} ion). At photon energies
\[ E_{\gamma} \approx E_c = \frac{4\gamma^2 \hbar c}{\sigma_z} \] (2)
the radiation arises from the interaction of the Z\textsubscript{1} ion with the second bunch which looks like a “particle” with the huge charge Z\textsubscript{2}eN\textsubscript{2}.

A classical approach to CBS was given in [1], a quantum treatment of CBS and applications to some working and planned colliders can be found in [2]-[4]. Recently we have developed a new simple and transparent method to calculate this radiation [5]. For the application of CBS to relativistic heavy ion colliders we collect the needed formulae from [3] and [5].

The method of calculation is valid if the ion deflection angle \( \theta_d \) in the field of the oncoming bunch is small compared with the typical radiation angle \( \theta_r \sim 1/\gamma_1 \). The ratio of these angles is easily estimated knowing the electric and magnetic fields of the second bunch which are approximately equal in magnitude, \(|\mathbf{E}| \approx |\mathbf{B}| \sim 2Z_2eN_2/(\sigma_{2z}\sigma_{2\perp})\) (for LHC the effective field \(|\mathbf{E}| + |\mathbf{B}| \sim 0.1\text{T}\)

\[ \frac{\theta_d}{\theta_r} \sim \eta = \frac{Z_2 r_1 N_2}{Z_1 \sigma_{2\perp}} \] (3)

where \( \sigma_{2\perp} \) is the transverse size of the second bunch and \( r_1 = Z_1^2 e^2/(m_1 c^2) \) is the classical ion radius.

For LHC (in the mode Pb-Pb) and for RHIC the parameter \( \eta \) is of the order
\[ \eta \sim 10^{-4} - 10^{-3}, \] (4)
so the above mentioned method is valid.

It is worthwhile to notice that for the majority of the colliders \( \eta \) is either much smaller than one (all the pp, \( \bar{p}p \), some \( e^+e^- \) colliders and B-factories) or \( \eta \sim 1 \) (e.g., LEP, TRISTAN) and only the linear \( e^+e^- \) colliders have \( \eta \gg 1 \). Therefore, the CBS has a very wide region of applicability.

In calculating the CBS effect we have assumed that the densities of the bunches do not change during the collisions and that the bunches in the interacting region have Gaussian particle distributions with mean-squared transverse \( \sigma_{j\perp} \) and longitudinal \( \sigma_{jz} \) radii, \( j = 1, 2 \). All parameters which are necessary for the calculation are given in Table 1, for LHC they are taken from [6] and for RHIC from [7]. For collisions of identical beams we assume \( N_1 = N_2 = N \).

| Collider | \( N \)  | \( \gamma = E/(mc^2) \) | \( \sigma_z \) (cm) | \( \sigma_{\perp} \) (\( \mu \)m) |
|----------|--------|-------------------|-----------------|-----------------|
| \( Pb \) (LHC) | 0.9 \cdot 10^8 | 2980 | 7.5 | 15 |
| \( Au \) (RHIC) | 10^9 | 108 | 11.9 | 150 |
| \( p \) (RHIC) | 10^{11} | 268 | 7.2 | 110 |
2. Spectrum of CBS photons — The number of CBS photons for a single collision of the beams is equal to

\[ dN_\gamma = N_0 \Phi(E_\gamma/E_c) \frac{dE_\gamma}{E_\gamma}. \] (5)

Here for the round Gaussian bunches with \( \sigma_{1\perp} = \sigma_{1x} = \sigma_{1y}; \ \sigma_{2\perp} = \sigma_{2x} = \sigma_{2y} \),

\[ N_0 = \frac{4}{3} \frac{\alpha}{\pi} N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_{1\perp}} \right)^2 \ln \frac{(\sigma_{1\perp}^2 + \sigma_{2\perp}^2)^2}{2 \sigma_{1\perp}^2 \sigma_{2\perp}^2 + \sigma_{4\perp}^2}. \] (7)

For identical beams \( \sigma_{1\perp} = \sigma_{2\perp} = \sigma_{\perp} \),

\[ N_0 = \frac{4}{3} \ln \frac{4}{3} \frac{\alpha}{\pi} N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_{\perp}} \right)^2 = 8.91 \cdot 10^{-4} N_1 \left( \frac{Z_2 r_1 N_2}{\sigma_{\perp}} \right)^2. \] (9)

The function \( \Phi(x) \) is defined as follows

\[ \Phi(x) = \frac{3}{2} \int_0^\infty \frac{1 + z^2}{(1 + z)^4} \exp[-x^2(1 + z)^2] dz; \] (10)

\[ \Phi(x) = 1 \text{ at } x \ll 1; \ \Phi(x) = (0.75/x^2) \cdot e^{-x^2} \text{ at } x \gg 1 \] (11)

(some values of this function are: \( \Phi(x) = 0.80, 0.65, 0.36, 0.10, 0.0023 \) for \( x = 0.1, 0.2, 0.5, 1, 2 \)).

The calculated constants \( N_0 \), the critical energies \( E_c \) and wave lengths \( \lambda_c = 2\pi \hbar c/E_c \) are presented in Table 2.

| Parameter | Pb-Pb (LHC) | Au-Au (RHIC) | p-Au / Au-p (RHIC) | p-p (RHIC) |
|-----------|-------------|--------------|--------------------|------------|
| \( N_0 \) | 49          | 590          | 49 / 1800          | 170        |
| \( E_c \) (eV) | 93          | 0.077        | 0.48 / 0.13        | 0.79       |
| \( \lambda_c \) (\( \mu \)m) | 0.013        | 16           | 2.6 / 9.7          | 1.6        |

Table 2: Parameters of CBS

Note the following features of the CBS spectrum. The constant \( N_0 \) is proportional to \( 1/\sigma_{\perp}^2 \) and the shape of the spectrum strongly depends on the bunch length \( \sigma_{2x} \) via Eqs. (4) and (5). Therefore, measuring the photon rate and the shape of the spectrum one can obtain informations about the beam sizes.

It may be convenient for LHC to use the CBS photons in the range of visible light \( E_\gamma \sim 2 - 3 \text{ eV} \ll E_c = 93 \text{ eV} \). In this region the rate of photons is expected to be

\[ \frac{dN_\gamma}{\tau} \approx 3.9 \cdot 10^8 \frac{dE_\gamma}{E_\gamma} \text{ photons per second} \] (12)
(here $\tau = 0.125 \mu s$ is the time between bunch collisions at a given interaction point). Furthermore, for visible light the photon polarization should be easily measurable.

Let us discuss the question of a possible background due to synchrotron radiation (SR) on the external magnetic field of the collider $B_{\text{ext}}$. A complete treatment of this background can be done only knowing the details of the collider magnetic layout. Nevertheless, for all the discussed heavy ion colliders we expect this background to be small.

A rough estimation of the SR rate can be performed as follows. The first bunch moving through the uniform field $B_{\text{ext}}$ emits $dN_{\gamma}^{SR}$ photons in the time $\Delta t$

$$dN_{\gamma}^{SR} = N_1 \frac{dI}{E_\gamma} \Delta t = \frac{4}{9} \alpha N_1 F(E_\gamma/E_c^{SR}) \frac{dE_\gamma Z_1^3 eB_{\text{ext}}}{m_1 c} \Delta t.$$  \hspace{1cm} (13)

Here $dI$ is the classical synchrotron radiation intensity per unit time, $E_c^{SR}$ the critical SR energy

$$E_c^{SR} = \hbar \frac{3Z_1 eB_{\text{ext}} \gamma_1^2}{2m_1 c} = \hbar \frac{3\gamma_1^3 c}{2R_B}$$  \hspace{1cm} (14)

with the circular orbit of radius $R_B$. The normalized function $F(y)$ is defined as an integral over the modified Bessel function $K_{5/3}(x)$ \[1\]

$$F(y) = \frac{9}{8\pi} \sqrt{3} y \int_y^\infty K_{5/3}(x) \, dx.$$  \hspace{1cm} (15)

Since the CBS radiation is confined to angles $\lesssim 1/\gamma_1$. Only those SR photons should be considered which are emitted in the same angular interval. Using this condition one can estimate $\Delta t$

$$\Delta t \sim \frac{R_B}{\gamma_1 c} = \frac{m_1 c}{Z_1 eB_{\text{ext}}}$$  \hspace{1cm} (16)

which allows to write Eq. (13) in the form

$$dN_{\gamma}^{SR} \sim \frac{4}{9} \alpha N_1 Z_1^2 F(E_\gamma/E_c^{SR}) \frac{dE_\gamma}{E_\gamma}.$$  \hspace{1cm} (17)

In Fig. 1 we compare the rate for the CBS and SR photons using Eqs. (5), (17) and the parameters of LHC assuming an external field $B_{\text{ext}} = 1T$, for definiteness. The critical SR energy for this field strength is 0.35 eV (remember $E_c = 93$ eV for CBS). The ratio of the CBS rate to the SR rate changes drastically with the photon energy. It is equal to 1, 10, and 1000 for $E_\gamma = 6.5, 7.5,$ and 9.8 eV. So we conclude that for photon energies above 8eV the synchrotron radiation can be neglected. It may be useful to consider the case $E_\gamma = E_c \gg E_c^{SR}$ where the signal to background ratio can be estimated as follows

$$\frac{dN_\gamma}{dN_{\gamma}^{SR}} \sim 0.03 \left( \frac{Z_2 \gamma_1 N_2}{Z_1 \sigma_\perp} \right)^2 \frac{e^x}{\sqrt{x}}, \quad x = \frac{E_c}{E_c^{SR}}.$$  \hspace{1cm} (18)

Let us stress that the CBS spectrum is completely different to that of the synchrotron radiation in an uniform magnetic field. Both are classical radiations in the considered cases, but CBS is an emission in a short electromagnetic filed at the length scale of the bunch while SR corresponds to photon emission in a static magnetic filed with a much larger characteristic length scale. In particular, the critical energy $E_c$ of CBS (2) does not depend neither on the charge nor on the mass of the radiating ion contrary to $E_c^{SR}$. 

\[4\]
Figure 1: Number of photons emitted due to CBS and SR ($B_{\text{ext}} = 1 T$) at LHC (Pb-Pb) as function of the photon energy.

If the first bunch axis is transversely shifted by a distance $R$ from the second bunch axis, the luminosity $L(R)$ (as well as the number of events for the ordinary reactions) decreases very quickly

$$L(R) = L(0) \exp \left( - \frac{R^2}{2(\sigma_{1\perp}^2 + \sigma_{2\perp}^2)} \right).$$

On the contrary, for the discussed colliders the number of CBS photons decreases slowly and for small values of $R$ this number even increases. For the collision of identical round beams the maximum of the ratio $dN_{\gamma}(R)/dN_{\gamma}(0)$ is equal to 1.06 at $R \approx 1.3 \sigma_{\perp}$ (see Fig. 2) whereas the luminosity already decreases significantly. At large displacements $R \gg \sigma_{\perp}$ this ratio is

$$\frac{dN_{\gamma}(R)}{dN_{\gamma}(0)} = 6.95 \frac{\sigma_{\perp}^2}{R^2}.$$ 

Additionally, two cases of non-identical beams (p-Au and Au-p collisions at RHIC) are also shown in Fig. 2. The dependence of CBS on the beam axis displacement can be used for a fast control over the collision impact parameter between beams at the interaction point (especially during the beginning of every run) and over the transverse beam size. For the case of another type of radiation (beamstrahlung), such an experiment has already been successfully performed at SLC [9].

3. Azimuthal asymmetry and polarization — The angular distribution for central head-on collisions of round beams has no azimuthal asymmetry, it is given by the expression

$$dN_{\gamma} = \frac{3}{2} N_0 \frac{dE_{\gamma}}{E_{\gamma}} \frac{1 + z^2}{(1 + z)^4} \frac{dz}{dz} \exp \left[ - \left( \frac{(1 + z)E_{\gamma}}{E_c} \right)^2 \right].$$
Figure 2: Normalized number of CBS photons as function of the impact parameter $R$.

where $z = (\gamma_1 \theta)^2$, $\theta$ is the polar angle of the emitted photons. In Fig. 3 we present the normalized angular distribution for different photon energies as function of $\gamma_1 \theta$. For small photon energies ($E_\gamma \ll E_c$) it almost coincides with that of the ordinary bremsstrahlung. At photon energies of the order of $E_c$ the distribution shrinks considerably.

It may be useful to note that in the energy region

$$E_c^{SR} \ll E_\gamma \ll E_c$$

(22)

the angular distributions of CBS and SR photons are quite different. According to (21) the characteristic emission angle $\theta_{char}^{CBS}$ of CBS in this region is

$$\theta_{char}^{CBS} \approx \frac{1}{\gamma_1}.$$  

(23)

For photon energies (22) the distribution of SR photons over an angle $\vartheta$ (defined as the angle between the photon momentum and the orbit plane) can be obtained from [10] (see §74) as follows

$$\frac{dN_{SR}^{SR}}{dE_\vartheta dz^{SR}} \propto \frac{1 + 2z^{SR}}{\sqrt{1 + z^{SR}}} \exp \left( - \frac{E_\gamma}{E_{SR}} (1 + z^{SR})^{3/2} \right), \quad z^{SR} = (\gamma_1 \vartheta)^2.$$  

(24)

From this expression one can see that the SR photon rate is strongly suppressed for angles $\vartheta$ larger than

$$\vartheta_{char} \approx \frac{1}{\gamma_1 \sqrt{2E_{SR}^{SR} / 3E_\gamma}}.$$  

(25)
Figure 3: Angular distribution of the CBS photons for two typical energies, normalized at \( \theta = 0 \), \( r(\theta) = dN_\gamma(\theta, E_\gamma)/dN_\gamma(0, E_\gamma) \).

The characteristic angle of SR is considerably smaller than that of CBS. This fact can be used to suppress the SR background even stronger.

If the impact parameter \( R \) between beams is non-zero, the angular distribution over \( \theta \) (or \( z \)) is unchanged, but an azimuthal asymmetry of the CBS photons appears which can also be used for an operative beam control. For example, if \( R \) increases in the vertical direction, the first bunch is shifted to the region where the electric field of the second bunch is directed almost vertically. This moving field of the second bunch can be represented as a flux of equivalent photons [5]. These photons are linearly polarized in the vertical direction. The average degree of such a polarization \( l \) for the discussed colliders with identical and non-identical beams as function of \( R \) is shown in Fig. 4.

Let us define the azimuthal asymmetry of the emitted photons by the relation

\[
A = \frac{dN_\gamma(\varphi = 0) - dN_\gamma(\varphi = \pi/2)}{dN_\gamma(\varphi = 0) + dN_\gamma(\varphi = \pi/2)},
\]

where the azimuthal angle \( \varphi \) is measured with respect to the horizontal plane. This quantity does not depend on the photon energy and is equal to

\[
A = \frac{2(\gamma_1 \theta)^2}{1 + (\gamma_1 \theta)^4} l(R).
\]

Increasing \( R \) in the vertical direction (see Fig. 4), the fraction of photons emitted horizontally becomes larger than the fraction of photons emitted vertically.

Finally, let us briefly discuss the polarization of CBS photons. For linearly polarized equivalent photons the CBS photons also get a linear polarization in the same direction.
Figure 4: **Average degree of linear polarization as function of the impact parameter $R$.**

Denoting by $I^{CBS}$ the average degree of CBS photon polarization, the ratio $I^{CBS}/I$ varies as function of $E_\gamma$ in the interval from 0.5 to 1 [4] (see Table 3).

**Table 3**: Degree of the linear polarization of the CBS photons

| $E_\gamma/E_c$ | 0   | 0.2 | 0.4 | 0.6 | 0.8 | 1   | 1.5 | 2   |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| $I^{CBS}/I$    | 0.5 | 0.7 | 0.81| 0.86| 0.89| 0.94| 0.96| 0.97|

**Conclusions** — We have calculated the main characteristics of CBS photons for heavy ion colliders, the photon rate, the energy and angular distributions as well as the polarization. It seems that CBS can be a potential tool for optimizing collisions and for measuring beam parameters directly at the interaction point. Obtaining the critical energy $E_c$ from the CBS spectrum, the bunch length $\sigma_z$ can be found since $E_c$ is proportional to $1/\sigma_z$. The transverse bunch size $\sigma_\perp$ is related to the rate of photons $dN_\gamma \propto 1/\sigma_\perp^2$. The possible background due to synchrotron radiation on the external magnetic field of the collider is estimated. Furthermore, CBS may be very useful for a fast control over the impact parameter between the colliding bunch axes because the photon rate $dN_\gamma$ depends on this parameter in a very specific way.

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