Classical and Quantum Composite p-branes

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We discuss classical composite p-brane solutions and their quantization using the conjecture that their fluctuations may be described via degrees of freedom of Dirichlet strings ended on these p-branes. We work with Dirichlet (super)strings in framework of string field theory for open (super)strings. To elaborate in this scheme the eleventh dimension modes we take just a collection of Dirichlet strings which in their middle points have jumps in eleventh dimension. This theory can be seen as string field theory in infinite momentum frame of an eleven dimensional object.

1. Introduction

Recent remarkable developments in superstring theory led to the discovery that the five known superstring theories in ten dimensions are related by duality transformations and to the conjecture that $M$-theory underlying the superstring theories and 11 dimensional supergravity exists (see [1]-[3]).

D-branes [4–7] carrying Ramond-Ramond charges play crucial roles in non-perturbative understanding of string theories based on dualities. Duality requires the presence of D-p-branes in the superstring spectra. A derivation of the Bekenstein-Hawking formula for the entropy of certain extreme black holes was given by using the D-brane approach[7].

D-branes have an interpretation as solitons. p-brane classical solutions with Ramond-Ramond charges were found in supergravity theory which describes the low energy dynamics of superstring theory [8][9]. The classical p-brane solutions have the following specific features:

i) existence of multi-center solutions (multisoliton solutions);
ii) harmonic superposition rule;
iii) S-duality;
iv) T-duality.

As to quantum theory of these solitons it was proposed that D-branes internal degrees of freedom originate from the Dirichlet open strings. In particular, the degrees of freedom representing the collective coordinates for translation of D-branes come from a part of the massless gauge fields.

In this talk after a short review of known classical composite p-brane solutions we will try to push further this idea and to identify the quantum theory of D-branes with covariant bosonic string field theory (SFT) for Dirichlet (super)string.

Another approach to D-branes using SFT would consist in an identification of D-branes with classical solutions (solitons) in closed SFT and in using the quantization around D-branes a la background formalism in local field theories [1] (few comments about background formalism in SFT see [2]). However, this attempt meets a problem with a complicated form of covariant SFT for closed superstrings.

To elaborate the Witten idea that to get the strong coupling IIA theory [14] we have to take a collection of N D-0-branes we just take a matrix-valued SFT for Dirichlet string. As a low energy theory this theory in non-compact case reproduces M(artix) quantum mechanic of Banks-Fishler-Shenker-Susskind (BFSS) [15]. Taking SFT for D-0 branes on $R^1 \times T^k \times R^{10-k}$ one gets in low energy limit super Yang-Mills theory on $R^1 \times T^k$. Using an old idea of getting matrix

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2When this notes has been completing we got an interesting paper where another way of introducing D-branes into closed SFT was considered. The authors of [13] add to the SFT action a term describing the direct interaction between D-brane and closed string.
models from strings \[24\] we also argue that BFSS M(atrix) quantum mechanics for large \(N\) contains all string excitations. However this is not so for finite \(N\). To make contact with 11-th dimensional theory BFSS identify matrix indices with 11-th component of momentum in the infinite momentum frame (IMF). One can go further and identify ”colour” (or Chan-Parton) indices with the 11-th component of momentum for string fields. This interpretation gives a natural geometric picture of the interaction in IMF and proposes to identify a parameter of ”light-cone-like” covariant SFT \[13\] with 11-th component of momentum. Having in mind that dynamics of solitons in Sin-Gordon model is described by the Thirring model and that

\[
\text{Sin – Gordon} \iff \text{Thirring}
\]
duality \[\text{IIA SFT} \iff \text{D-SFT} \iff \text{IIB SFT}\] exists. A proposal how to get IIA SFT from zero-mode excitations of D-0 theory on \(R^1 \times T^1 \times R^8\) \[2\] has been recently proposed in \[22\].

The presentation of the material is the following. In Section 2 we sketch a general scheme of getting solutions of effective low energy field theory and in Section 3 we discuss the SFT for the Dirichlet superstrings.

2. Classical Composite p-brane Solutions

2.1. Algebraic method for finding solutions

To clarify the general picture of classical p-brane solutions it seems useful to have solutions in arbitrary spacetime dimension. In this section results obtained in recent works \[22\] (see also \[23\], the similar results was been obtained by T.Ortin \[2\]) will be presented. In these papers a systematic algebraic method of finding p-brane solutions in diverse dimensions was developed. A starting point is an ansatz for a metric on a product manifold. Using the Fock-De Donder harmonic gauge

\[
\int d^9 X \sqrt{-g} R - \frac{(\nabla \phi)^2}{2} - \sum_{I=1}^{k} \frac{e^{-a^I} \phi F_{dI}^2}{2(d_I + 1)!} \tag{1}
\]

The metric which solves equations of motion for the action (1) has the form

\[
ds^2 = \chi_{tot}(\sum_{K,L=0}^{D-s-3} \chi_{K,L} dy^K dy^L + \sum_\gamma dx^\gamma dx^\gamma)
\]

\[
\chi_{tot} = \prod_{I=1}^{k} (H_I^1 \cdots H_I^{l_I} E_{I}^1 \cdots E_{I}^{l_I})^{2l_I} \sigma^{l_I} \tag{2}
\]

The notations are the following

\[
u^I = \frac{d_I}{2(D-2)}, \quad \sigma^I = \frac{1}{d_I d_I + \frac{1}{4} d_I^2}.
\]

\[H_a^I, a = 1, \ldots, E_I \text{ and } U_b^I, b = 1, \ldots, M_I \text{ are harmonic functions depending on } x^\gamma \Delta_I^a = (\Delta^a_{KL}), a = 1, \ldots, E_I, \text{ and } \Lambda^a = (\Lambda_a^{KL}), b = 1, \ldots, M, L = 0, \ldots, D - 1 \text{ are electric and magnetic incidence matrices. Their rows correspond to independent branches of the electric (magnetic) gauge field and columns refer to the space-time indices. The entries of the incidence matrices are equal to 1 or 0. Incidence matrices for fixed } I \text{ have equal numbers of units in each row, and there are no rows which coincide. These matrices describe electric and magnetic configurations } A^a_c \text{ and } F^M_b.
\]

\[5\text{About duality in QFT see } [17,18]\]
(for more details see [23]) and form a brane incidence matrix
\[
\Upsilon_{PL} = \begin{pmatrix} \Delta L \\ \Lambda_L \end{pmatrix}, \quad \Delta L = \begin{pmatrix} \Delta^1_{gL} \\ \Delta_{2aL} \\ \vdots \\ \Delta^k_{aL} \end{pmatrix}, \quad (4)
\]
and the same for \( \Lambda \). To make a contact with our previous notations note that \( \Lambda^k_{bL} = 1 - \Lambda^k_{bL} \). Matrix \( \Upsilon \) has to satisfy the following characteristic equations
\[
\varsigma_{R R'} - \frac{\tilde{\alpha}_R \tilde{\alpha}_{R'}}{2} - \frac{d_R d_{R'}}{D - 2} = \sum_{L=0}^{D-1} \Upsilon_{RL} \Upsilon_{R'L} = 0; \quad R \neq R'.
\]
\( \varsigma_R = -1(+1) \) for the electric(magnetic)-branes.

2.2. S-duality
In order to demonstrate S-duality let us consider a new action, which is obtained from the action (1) by replacing an antisymmetric field \( F^I_{dI} \) by another field \( F^I_{dI} \), \( d_I = D - 2 - d_I \), and changing the signs of the corresponding dilaton coupling constants on the opposite ones: \( \tilde{\alpha}^I = -\tilde{\alpha}^I \).

S-duality transforms the solutions of the theory (1) into the solutions of the theory with a new action. The corresponding transformations of the incidence matrices are
\[
\Delta^I_{aL} \rightarrow \tilde{\Delta}^I_{aL} \rightarrow \tilde{\Delta}^I_{aL} = \Delta^I_{aL}. \quad (5)
\]
One can check that the new incidence matrices also satisfy the characteristic equations.

One can also perform S-duality transformation (6) only for some branches of the fields. In this case the dual theory may have more fields in comparison with the initial one.

2.3. Harmonic function rule
Our solution (3) has a very simple structure. This becomes obvious if one rewrites the metric in the following form:
\[
g_{KK} = \prod \prod g^{Ia}_{KK} \prod g^{Ib}_{KK}, \quad (6)
\]
where
\[
g^{Ia}_{KK} = (H^I_a)^{\tau I} (H^I_a(1-\Delta_{aK}))^{\rho I}, \quad (7)
g^{Ib}_{KK} = (U^I_b)^{-\tau I} (U^I_b(1-\Delta_{bK}))^{-\rho I}. \quad (8)
\]
The exponents are given by:
\[
\tau^I = -\frac{4(D-2-d_I)}{\Delta^I(D-2)}, \quad \rho^I = \frac{4d_I}{\Delta^I(D-2)}, \quad (9)
\]
\[
\Delta^I = \tilde{\alpha}^I + \frac{2d_I(D-2-d_I)}{D-2}. \quad (10)
\]

For given incidence matrices and values of \( \tau^I \) and \( \rho^I \) (4)-(9) gives the following rule for constructing a metric. For each space-time direction the coefficient in the metric is a product of functions \( H_a \) and \( U_b \) in an appropriate power. Namely, we put \( H_a^{\tau^I} (U_b^{-\rho I}) \) if the corresponding direction belongs to the \( n \)-th \((D-d-3)-\) electric \((D-d-3)-\) magnetic) brane, and we put \( H_a^{\tau^I} (U_b^{-\rho I}) \) if the corresponding direction is transverse to the \( n \)-th \((D-d-3)-\) electric \((D-d-3)-\) magnetic) brane. Note that \( \tau^I \) and \( \rho^I \) are the same as in the corresponding single brane (6).

2.4. T-duality
Let us consider generalized T-duality transformations. T-duality transforms solutions for the action (1) with one set of fields into solutions with another set of fields. We perform T-duality transformation along the direction corresponding to \( y_{n0} \) coordinate, \( q \leq i_0 \leq D - s - 3 \). T-duality acts on the brane incidence matrix \( \Upsilon_{RL} \) as follows. We select the \( i_0 \)-th column, change 1 into 0 and vice versa and obtain a new brane incidence matrix. This matrix satisfies the characteristic equation if we simultaneously change dilaton coupling constants. More precisely, new dilaton coupling constants \( \tilde{\beta}^I_R \) are connected with the old ones \( \tilde{\beta}^I_R \) in the following way
\[
\frac{\tilde{\beta}^I_R \tilde{\beta}^I_{R'}}{2} = \tilde{\alpha}^I_{R'} - 1 + \Upsilon_{Ri_0} + \Upsilon_{R'i_0} \quad (11)
\]
\[
(1 - 2\Upsilon_{Ri_0} + d_R)(1 - 2\Upsilon_{R'i_0}) + (1 - 2\Upsilon_{R'i_0})d_R
\]
\[
D - 2
\]
These relations give rather restrictive conditions on the initial theory parameters.

3. SFT for Dirichlet strings
We start with the bosonic part of the world-sheet action for a free (super)string in flat space-
time and in the conformal gauge
\[ \int_{\mathcal{M}} \frac{d\tau d\sigma}{4\pi\alpha'} (\partial_+ \eta_{\mu\nu} + c_+ \partial_- b_- + c_- \partial_+ b_+), \]
where \( \mathcal{M} \) is a surface with the boundary. To perform the covariant quantization we add ghost \( c(\tau, \sigma) \) and antighost \( b(\tau, \sigma) \) fields.

In the covariant string field theory (SFT) the string field \( A \) is a functional of the basic string variables. These in conformal gauge are the bosonic coordinates \( x^\mu(\sigma, \tau) \), \( \mu = 0, \ldots, 9 \), the ghost and antighost variables \( c(\sigma, \tau), b(\sigma, \tau) \) and fermionic variables \( \psi^\mu(\sigma, \tau), \bar{\psi}^\mu(\sigma, \tau) \) with corresponding ghosts \( \eta(\sigma, \tau) \) and \( \chi(\sigma, \tau) \) (see [12] for more details). Below for simplicity we will write only the formula for bosonic string. To deal with D-p-branes we suppose that the first \( p + 1 \) components of \( x \) satisfy the Neumann boundary conditions and the last components satisfy the Dirichlet boundary conditions. More precisely, let \( \sigma \in [0, \pi] \) be the normal coordinate and \( \tau \) be the tangential coordinate on an open string worldsheet and define \( z = e^{\tau + i\sigma} \). A "real" open string configurations \( x^\alpha(z, \bar{z}) \) have Neumann boundary conditions:
\[ \partial_+ x^\alpha = 0 \text{ at } \sigma = 0, \pi. \]

D-brane has Dirichlet boundary conditions \( x^i = 0 \) at \( \sigma = 0, \pi: \)
\[ x^i(z, \bar{z}) = -i \frac{\delta x^i}{2\pi} \ln \frac{z}{\bar{z}} + x^i_{osc}, \]
\[ x^i_{osc} = i \frac{\sqrt{\alpha'}}{2} \sum_{m \neq 0} \frac{\alpha^i_m}{m} (z^{-m} - \bar{z}^{-m}) \]
Neumann boundary conditions correspond to open strings with free ends (which move at the speed of light). Dirichlet strings are open strings attached to a fixed surface or point – the D-brane (in this case \( \delta x^0_0 \) is the displacement between two endpoints of an open string). We left unchanged the boundary conditions for ghosts fields
\[ c_\pm = \sum_{n=-\infty}^{\infty} c_n e^{\pm n\sigma}; \quad b_\pm = \sum_{n=-\infty}^{\infty} b_n e^{\pm n\sigma} \]

We will use a bosonized language for the ghosts where they are replaced by a scalar field \( \phi \) and one can treat it as an 11-th coordinate (\( \mu = 10 \)).

The basic starting point of the Witten SFT is a non-commutative differential calculus in string space. This calculus is given by a triplet: \( (\mathcal{H}, Q, *) \), where
i) \( \mathcal{H} \) is a Hilbert space with scalar product \( (\cdot, \cdot) \); this Hilbert space consists of string fields;
ii) \( Q \) is a nilpotent operator (an analog of a derivation);
iii) \( \wedge \) is the BRST operator. The scalar product \( (\cdot, \cdot) \) is given with the help of \( * \) and an integration \( \int, (A, B) = \int A * B \). The Witten integration \( \int \) reads
\[ \int dx^\mu e^{-i\frac{\pi}{4} x^{10}(\widetilde{x})} A[x^\mu] \prod_{0 \leq \sigma \leq \pi/2} \delta (x^\mu(\sigma) - x^\mu(\pi - \sigma)) \]
and it represents a folding of the string about its midpoint. The reason for this definition is that for achieving the following property of integration \( \int Q A = 0 \). The wedge product with two string functionals \( A \) and \( B \) associates the third one, \( C = A * B \)
\[ C[x^\mu_L, x^\mu_R] = \int dy_L^\mu d^\mu d^\mu e^{-i\frac{\pi}{4} y^{10} (\widetilde{x})} A[x^\mu_L, y^\mu_R], \]
\[ B[dy_L^\mu, x^\mu_R] \prod_{0 \leq \sigma \leq \pi/2} \delta (y^\mu_R(\sigma) - y^\mu(\sigma)) \]
Here \( x^\mu_L(\sigma) = x^\mu(\sigma), \quad 0 \leq \sigma \leq \pi/2; \quad x^\mu_R(\sigma) = x^\mu(\bar{\sigma} + \pi), \quad 0 \leq \sigma \leq \pi/2. \) The string field Lagrangian has the Chern-Simons form
\[ S = \int (A * QA + \frac{2}{3} A * A * A) \quad (12) \]
It is invariant under gauge transformations
\[ \delta A = QA + A * - A \quad (13) \]
This gauge invariance immediately follows from the properties obeying by the \( Q, * \) and \( \int \):
i) associativity \( A * (B * C) = (A * B) * C; \)
ii) the Leibniz rule \( Q(A * B) = (QA) * B + (-1)^{|A|} A * (QB) \);
iii) integration "by parts", \( \int (QA) * B = -\int A * (QB) \); and iv) \( \int A * B = (-1)^{|B|} \int B * A, \) with
Figure 1. A D-brane sandwich in 11 dimensions and corresponding string fields

\[ |A| = N_{ab}(A) - \frac{1}{2}, \quad N_{ab} \] is the ghost number. The string field has ghost number \(-\frac{1}{2}\).

In the case of N D-p-branes we deal with matrix valued string fields \( A_{ab}[x^M_L, x^M_R] \) with the following boundary conditions

\[
\begin{align*}
  x^\alpha_L(0) &= x^\alpha_R(\frac{\pi}{2}); & \alpha &= 0, \ldots, p, 10; \\
  x^i_L(0) &= x^i_R(\frac{\pi}{2}); & i &= p + 1, \ldots, 9;
\end{align*}
\]

For our purposes it is enough to consider \( \delta x^i = 0 \).

It is convenient for us to select an expect dependence of the center mass position in string fields

\[ A = (A_{ab}[x^M_L, x^M_R]) \quad a, b = 1, \ldots N, \] (15)

here \( x^M_L \) and \( x^M_R \) means only the oscillations of right and left parts of strings. We assume that these functionals may be considered as fields describing M-theory in 11 dimension in the infinite momentum frame (IMF). More precisely, we assume that in IMF only string configurations with

\[ x^{11}(\sigma) = C_1, \quad \frac{\pi}{2} < \sigma \leq \pi, \quad x^{11}(\sigma) = C_1, \quad \pi/2 < \sigma \leq \pi \] are important (see Fig.1) and

\[ A_{ab}[x^M] = A_{ab}[x^\mu_L, x^\mu_R, x^\sigma_0, x^{11}] = \] (16)

The interaction is presented on Fig.2.

A D-brane sandwich of open strings gives rise to the following massless states:

\[ A_{\mu ab}(x^0_\alpha)\alpha^{\mu}_{\alpha -1}|0> = \] (18)

\[ (A_{\beta ab}(x^\alpha_0)\alpha^\beta_{\alpha -1}|0>; \quad A_{i ab}(x^\alpha_0)\alpha^{-1}_{\alpha -1}|0>) \]

\( \mu = 0, 1, \ldots 9; \alpha, \beta = 0, 1, \ldots p; i = p + 1, \ldots 9. \)

\[ S = \int (A \ast QA) \] for the fields (18) gives the Maxwell action for \( A_{\beta ab}(x^0_\alpha) \) and a free scalar action for \( A_{i ab}(x^0_\alpha) \). \( A_{i ab} \equiv X_{1 ab} \) are considered as non-commutative coordinates of D-brane (compare with attempts of dealing with non-commutative structure of space-time in frameworks of quantum groups [26]). The interaction term [14] after integration over the ghost and auxiliary fields will reproduce a reduced version of ten-dimensional Yang-Mills action (N=1, d=10
super Yang-Mills action for NSR superstring). If one takes \( p = 0 \), D-particle, then the zero-modes theory is matrix N=8 super quantum mechanics.

Now we will show that all highest string excitations can also be encoded in zero-mode matrix fields. The trick is based on the following consideration [16]. Let us introduce a lattice \( \mathbb{Z}^D \) in \( \mathbb{R}^D \) with lattice spacing \( a \). String is a contour on the lattice and to specify the string configuration it is necessary to specify the initial point, say \( x \), as well as links along which the string lies. Let \( \Gamma_x(l) \) be the space of all strings on the lattice \( \mathbb{Z}^D \) of length \( l \) with starting point \( x \in \mathbb{Z}^D \), and we have a finite number, say \( N \), elements which can be enumerated in some way.

It is convenient for us to imagine the string of length \( 2l = 2ka \) to be composed of two "halves":

\[
\gamma^L_x = (x - \sum_i e_{\mu_i}, \mu_1, ..., \mu_k) \quad \text{(left-half)}
\]

\[
\gamma^R_x = (x, \mu_{k+1}, ..., \mu_{2k}) \quad \text{(right-half)}
\]

Here \( x \) is a middle point and \( \mu_i \) \( i = 1, ..., 2k \) are directions of string links and \( e_\mu \) is an unit vector along \( \mu \)-direction. For the Dirichlet string ending on the same plane we have a restriction \( \sum_{i=1}^{2k} e_{\mu_i} = 0 \). The space of right half \( \Gamma^R_x(2l) \) is isomorphic to \( \Gamma_x(l) \) and its elements can be enumerated in the same way as the elements of \( \Gamma_x(l) \).

The elements of the left halves \( \Gamma^L_x(2l) \) can be numerated as if they are the strings \( (x, -\mu_k, ..., -\mu_1) \). One can use the following notations: any string of length \( 2l \) with midpoint \( x \) will be denoted as \( (x, a, b) \), where \( a \) specifies the right half of string and \( b \) numerates the left half of the string. Therefore a scalar functional \( A[\gamma] \) on the string \( \gamma = (x, a, b) \) on the lattice is in fact a matrix function

\[
A[\gamma] = A_{ab}(x), \quad a, b = 1, ..., N, \quad x \in \mathbb{Z}^d
\]

and in continuous case one gets an isomorphism

\[
\text{strings} \iff \infty \times \infty \text{-matrices}
\]

Therefore we get a matrix realization of the Witten algebra where fields are matrix \( \Phi_{ab}(x) \) depending on parameter \( x \), the product * is the matrix product,

\[
(A * \Psi)_{ab}(x) = A_{ac}(x)\Psi_{cb}(x)
\]

and the integral is \( \int A = \sum_x \text{tr} A(x) \). It is clear that the condition i) is satisfied.

It is obvious that for Dirichlet strings ending on the same hyperplane we left with \( A_{ab}(x) \), \( x \in \mathbb{Z}^{p+1} \) and for the case of \( p = 0 \) we left with matrices depending only on one parameter.

In the case when there is one compact direction, say \( x^{p+1} \), the state \( |0> \) is specified by an extra integer parameter \( n \) (winding number), that gives
rize to the following fields (compare with 27)
\[
(A_{\alpha ab}(x_0^\alpha, n)\alpha_{-1}^\beta|0, n >; X_i ab(x_0^\alpha, n)\alpha_{-1}^\beta|0, n >),
\]

\[X_{p+1}(n) = X_{p+1}(0) + 2\pi RnI,\] that can be collected to fields on a dual torus

\[A_{\alpha ab}(x^\alpha, x^{p+1}) = \sum e^{inz^p_{p+1}} A_{\alpha ab}(x^\alpha, n),\]

\[A_{p+1 ab}(x^\alpha, x^{p+1}) = \sum e^{inz^p_{p+1}} X_{p+1 ab}(x^\alpha, n),\]

\[X_{i ab}(x^\alpha, x^{p+1}) = \sum e^{inz^p_{p+1}} X_{i ab}(x^\alpha, n),\]

\[\alpha = 0, \ldots, p; \ i = p + 2, \ldots, 9.\] In the case of \(p = 0\) this construction produces the \(d = 2\) \(N = 8\) super Yang-Mills on the cylinder.

In the case when we have from beginning two D-branes, D-p and D-p' branes, the Witten interacting vertex produces the states corresponding to an open string which is attached at one end to a D-brane while the other end is free. In this case one has the following mode expansion

\[x^{i'}(z, \bar{z}) = iV \sum_2^{p+1} \sum \frac{\alpha_m^{i'}}{m^2} \left( \frac{1}{z^m+1/2} - \frac{1}{\bar{z}^m+1/2} \right),\]

\[i' = p + 1, \ldots, p'.\] Note that this string has no zero modes and it does not produce the new massless fields. This gives an explanation of the harmonic superposition rule.

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