Cosmic Discordance: Are Planck CMB and CFHTLenS weak lensing measurements out of tune?

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ABSTRACT

We examine the level of agreement between low redshift weak lensing data and the CMB using measurements from the CFHTLenS and Planck+WMAP polarization. We perform an independent analysis of the CFHTLenS six bin tomography results of Heymans et al. (2013). We extend their systematics treatment and find the cosmological constraints to be relatively robust to the choice of non-linear modeling, extension to the intrinsic alignment model and inclusion of baryons. We find that the 90% confidence contours of CFHTLenS and Planck+WP do not overlap even in the full 6-dimensional parameter space of ΛCDM, so the two datasets are discrepant. Allowing a massive active neutrino or tensor modes does not significantly resolve the disagreement in the full n-dimensional parameter space. Our results differ from some in the literature because we use the full tomographic information in the weak lensing data and marginalize over systematics. We note that adding a sterile neutrino to ΛCDM does bring the 8-dimensional 64% contours to overlap, mainly due to the extra effective number of neutrino species, which we find to be 0.84 ± 0.35 (68%) greater than standard on combining the datasets. We discuss why this is not a completely satisfactory resolution, leaving open the possibility of other new physics or observational systematics as contributing factors. We provide updated cosmology fitting functions for the CFHTLenS constraints and discuss the differences from ones used in the literature.

Key words: cosmological parameter cosmology: observations gravitational lensing: weak cosmic background radiation dark matter dark energy

1 INTRODUCTION

The Cosmic Microwave Background (CMB) radiation has been the most powerful probe of cosmology for more than a decade. The Planck satellite (Planck Collaboration et al. 2013a) gives us an unprecedented view of the temperature fluctuations at recombination and the polarisation measurements are eagerly awaited. Meanwhile the Wilkinson Microwave Anisotropy Probe (WMAP, Bennett et al. 1997) still provides the most detailed maps of the polarisation fluctuations (Bennett et al. 2013). More recently the BICEP2 team (BICEP2 Collaboration 2014) have reported a ~ 5σ detection of the B-mode CMB polarisation signal expected from primordial gravity waves, although whether its origin is cosmological rather than galactic has been disputed (Mortonson & Seljak 2014; Flauger et al. 2014).

Planck and WMAP polarisation together provide a self-consistent constraint on the 6 parameter ΛCDM cosmological model i.e. a flat universe containing only cold dark matter and baryons, and a cosmological constant, Λ. The BICEP2 results have shaken-up this picture to include primordial gravity waves, which then creates tension with the previous CMB results unless some additional modification is made, for example a modified primordial power spectrum (BICEP2 Collaboration 2014).

At the same time, additional pressure is mounting on the ΛCDM model from tension between the CMB and low-redshift measurements of matter clumpiness. The primary CMB anisotropies place a constraint on the matter fluctuation amplitude at the time of recombination, which can be
extrapolated to the present day for a particular assumed cosmological model. The primary measures of the amplitude of matter fluctuation at low redshift are weak lensing, galaxy clustering and the abundance of galaxy clusters. Low-redshift observations seem to be finding a lower value for this fluctuation amplitude than expected in ΛCDM (Beutler et al. 2014a; Planck Collaboration et al. 2013b; Vikhlinin et al. 2009). This could be reconciled by new physics which reduces the rate of clustering between recombination and today (Planck Collaboration et al. 2013c; Hamann & Hasenkamp 2013; Battye & Moss 2014; Beutler et al. 2014a; Dvorkin et al. 2014; Leistedt et al. 2014; Archidiacono et al. 2014).

Gravitational lensing is the most direct method for measuring the distribution of matter in the low-redshift universe. The image distortion of distant galaxies in typical patches of sky was first detected in 2000 (Bacon et al. 2000; Kaiser et al. 2000; Van Waerbeke et al. 2000; Wittman et al. 2000) and last year the Canada-France-Hawaii Telescope Lensing Survey (CFHTLenS) provided the tightest constraints on cosmology yet from cosmic shear. This arguably provides one of the most robust and constraining low-redshift measures of cosmology, and thus this is the low-redshift dataset we focus on in this paper.

One way to reduce the matter clustering rate is for some of the matter to travel fast enough to leave the clumps and smear out the fluctuations (“free-streaming”). Active neutrinos are an obvious candidate for this hot dark matter, because we already know they have mass (Beringer et al. 2012) and particle physics experiments allow a mass range that would have a significant impact on cosmology (Lobashev et al. 1999; Weinheimer et al. 1999). They have been invoked at various times to reconcile CMB and low-redshift counts of galaxy clusters.

Even if the active neutrino has the smallest mass allowed by particle physics experiments, an alternative hot dark matter particle might be responsible for smearing out the fluctuations. A sterile neutrino is a promising candidate which would also affect the CMB anisotropies by introducing an additional relativistic species in the early universe.

In this paper we focus in detail on combining the CMB with the CFHTLenS low-redshift dataset to examine whether they alone warrant new physics. In contrast to the earlier papers, we use the full 6 tomographic redshift bins and marginalise over intrinsic alignments, as in Heymans et al. (2013) and described in Section 2. Earlier papers drew conclusions about agreement between datasets by comparing marginalised contours in one or two dimensions. In Section 3 we investigate whether these conclusions hold up in the full multi-dimensional parameter space, and extend the treatment of weak lensing systematics. In Section 4 we investigate the effect of cosmological extensions (massive active neutrinos, a massive sterile neutrino, tensors and running of the spectral index) to the base model. We compare with related work, and discuss other possible explanations for the tension in Section 5.

2 DATASETS AND METHODOLOGY

The Planck satellite (Planck Collaboration et al. 2013a) provided high resolution (∼10 arcminutes) temperature maps of the CMB at a range of frequencies between ∼25 and ∼1000 GHz. These observations allow the estimation of the CMB temperature power spectrum for 2 ≤ ℓ ≤ 2500 (Planck collaboration et al. 2013). We use the publicly available Planck likelihood codes which use this power spectrum, and the corresponding polarisation power spectrum from WMAP9 (Bennett et al. 2013). Throughout, we marginalise over the 14 nuisance parameters which account for astrophysical systematics in the Planck likelihood codes. We refer to this combination as Planck+WP.

The Canada France Hawaii Lensing Survey (Heymans et al. 2012), hereafter referred to as CFHTLenS, is a 154 square degree multi-filter survey which achieved an effective weighted number density of 11 galaxies per square arcminute with shape and photometric redshift estimates. Kilbinger et al. (2013) performed a 2d cosmic shear analysis of the CFHTLenS data, producing constraints on the ΛCDM model which they approximated by σ₈(Ω_m/0.27)^0.59 = 0.787 ± 0.032. Heymans et al. (2013) (HE13 henceforth) performed a tomographic cosmic shear analysis of the CFHTLenS data, dividing the galaxies into six tomographic redshift bins (with photo-z between 0.2 and 1.3) and taking into account the effect of galaxy intrinsic alignments using a free parameter for the overall intrinsic alignment amplitude. For each tomographic bin combination, they measured the real space shear-shear correlation functions ξ(θ) in 5 evenly log-spaced angular bins for 1 ≤ θ ≤ 40 arcmin. In this paper we use the full HE13 correlation functions and covariance matrices (which were obtained from N-body simulated mock surveys), and marginalise over the same model for intrinsic alignments as in HE13.

The analysis in this paper is performed with CosmoSIS, a new cosmological parameter estimation framework (Zuntz et al. 2014 in prep). A parameter estimation problem in CosmoSIS is represented as a sequence of independent modules each performing a specific part of the calculation and passing on their results to later modules. For this work the modules were: CAMB (Lewis et al. 2000), to calculate CMB and linear matter power spectra and expansion histories; HALOFIT for non-linear power; a module based on COSMOCALC³ to compute cosmic shear spectra and intrinsic alignments; a custom module to compute the 2-point shear correlation functions ξ(θ) from C_s; the commander, lowlike and CAMSpec Planck likelihood codes (Planck collaboration et al. 2013); and a custom CFHTLenS likelihood code. As a default we use the HALOFIT formulation as implemented by CAMB, which is Takahashi et al. (2012) with modified massive neutrino parameters (although we compare this nonlinear correction to others in section 3.2). We’ll refer to this implementation as TA12 from now on. Note that HE13 used the fitting formula of Eisenstein & Hu to get the linear matter power spectrum and the Smith et al. (2003) (SM03 henceforth) version of HALOFIT to perform the non-linear correction.

We use the following parameter definitions. Ω_b is the total matter density at redshift zero (as a fraction of the critical density at redshift zero). The present-day baryon density is given by Ω_b, σ₈ is the rms fluctuation in 8h⁻¹Mpc

³ https://bitbucket.org/beckermr/cosmocalc
spheres at the present day in linear theory. The spectral index of the scalar primordial power spectrum is given by \( n_s \). \( \tau \) is the optical depth due to reionization. The Hubble constant is written as \( h \), in units of \( 100(\text{km/s})/\text{Mpc} \). When we refer to ‘base ΛCDM’, we mean the same model as the Planck Collaboration et al. (2013d) baseline model - the normal 6 parameter ΛCDM model, assuming 1 massive active neutrino eigenstate, with \( m_\nu = 0.06\text{eV} \).

3 DISCORDANCE IN ΛCDM

We assess the level of agreement between CFHTLenS and Planck+WP in the 6 parameter base ΛCDM model, and provide an updated fitting function to the CFHTLenS data.

3.1 Quantifying the tension

Fig. 1 shows the Planck and CFHTLenS data superposed onto the present day matter power spectrum assuming the Planck best fit cosmology, using the method of Tegmark & Zaldarriaga (2002). Each coloured CFHTLenS point corresponds to an angular correlation function measurement. Cross correlations with tomographic bin 1 are magenta, with bin 2 (and not with bin 1) are red, with bin 3 (and not with bins 1 or 2) are yellow, bin 4 are green, bin 5 are cyan and bin 6 are blue. These have been averaged using the noise covariance matrix to make the black points.

This was also illustrated in Battye & Moss (2014) using the cosmic shear correlation function.

In Fig. 2 we show that the two-dimensional marginalised constraints from Planck+WP and CFHTLenS are discrepant in the \( \Omega_m-\sigma_8 \) plane: the 2σ contours do not touch. This is a significantly stronger conclusion than reached in other works, e.g. Leistedt et al. (2014), Beutler et al. (2014a). There are four main reasons for this: (i) We use an improved non-linear HALOFIT treatment (see Section 3.5 below), which was already shown to have a significant effect in Fig. 4 of Beutler et al. (2014a); (ii) we use exactly the same cosmological model (i.e. include an active neutrino with mass 0.06eV) for the CFHTLenS constraints as for the Planck+WP constraints. (iii) we use a full likelihood analysis rather than just a prior in the \( \Omega_m-\sigma_8 \) plane; (iv) we follow HE13 by using 6 bin tomographic results marginalised over intrinsic alignments (see Fig. 4 of HE13 for the effect of this).

However, the fact that the 2d marginalised CFHTLenS and Planck+WP contours do not overlap is not necessary or sufficient to prove that they are discrepant, since the ΛCDM model has 6 dimensions, so it is the amount of contour overlap in 6 dimensions that is important.

One way we can quantify the discrepancy between two datasets (e.g. Planck+WP and CFHTLenS) within a particular \( n \)-parameter cosmological model is by checking how much the \( n \)-dimensional posterior distributions overlap. We first calculate the positions of the 68% and 95% contours in the full \( n \)-dimensional parameter space for a given dataset and can then assess whether a given point lies within these confidence intervals. Or more generally we can identify the percentage contour a given point lies on for each dataset.

We find the multi-dimensional contour levels as follows. We perform fits of the model to the two measurements in-

\[ P(k) \]

\[ \begin{array}{cccc}
 10^4 & & & \\
 10^3 & & & \\
 10^2 & & & \\
 10^1 & & & \\
 \end{array} \]

\[ k [h/\text{Mpc}] \]

\[ 1.0 \]

\[ 0.7 \]

\[ 0.6 \]

\[ 0.10 \ 0.15 \ 0.20 \ 0.25 \ 0.30 \ 0.35 \ 0.40 \ 0.45 \]

\[ \Omega_m \]

\[ \sigma_8 \]

\[ 1.1 \]

\[ \text{ Figure 1. The Planck and CFHTLenS data superposed onto the present day matter power spectrum assuming the Planck best fit cosmology, using the method of Tegmark & Zaldarriaga (2002). Each coloured CFHTLenS point corresponds to an angular correlation function measurement. Cross correlations with tomographic bin 1 are magenta, with bin 2 (and not with bin 1) are red, with bin 3 (and not with bins 1 or 2) are yellow, bin 4 are green, bin 5 are cyan and bin 6 are blue. These have been averaged using the noise covariance matrix to make the black points.} \]

\[ \text{ Figure 2. Constraints in the clustering amplitude } \sigma_8 \text{ and dark matter density } \Omega_m \text{ plane from Planck+WP and CFHTLenS, assuming our base cosmological model. Dotted grey banana: 1 and 2σ CFHTLenS only constraints using all } \theta \text{ bins in } \xi_+/-(\theta). \text{ Filled green banana: 1 and 2σ CFHTLenS only constraints excluding small scales (see section 3.2 for cuts on } \theta). \text{ Small, purple contours: The constraints on the base model from Planck+WP. Discrepancy between the 2d marginalised Planck+WP and CFHTLenS contours is clear at } >95\%. \]
dividually, allowing us to obtain a histogram of probability values for each dataset. As in the 1d case, we define the 68% contour as the surface of equal likelihood which contains 68% of the probability distribution, or in the case of MCMC samples, 68% of the samples. This allows us to identify the probability value of the 68% contour for each dataset. More generally, we can use this histogram to read off the percentage contour for any point in parameter space, given its probability value.

As an example of a point of interest, we perform a joint fit, and define \( \sigma_i(\mathbf{p}_{\text{joint}}) \) as the percentile contour on which the joint-best fit point lies, for dataset \( i \) (where \( i \) is one of \( C \) and \( P \), denoting CFHTLenS and Planck+WP respectively). The values of \( \sigma_C(\mathbf{p}_{\text{joint}}) \) and \( \sigma_P(\mathbf{p}_{\text{joint}}) \) are given in Table 1. The best joint fit is a poor fit to CFHTLenS, lying outside the 99% contour. The fact that it is an acceptable fit to Planck+WP reflects the greater constraining power from Planck+WP, which pulls the best fit point close to the best fit to Planck+WP alone.

We also wish to know if there are regions of parameter space which are a good fit to both datasets (albeit a slightly worse fit to Planck+WP), i.e. the minimum percentile contours which overlap. For this we define \( \sigma_{\text{ov}} \), the minimum value of \( \sigma_C = \sigma_P \). Therefore \( \sigma_{\text{ov}} \) is the best percentile value for which equal percentile contours of Planck+WP and CFHTLenS touch. For base \( \Lambda \)CDM we find \( \sigma_{\text{ov}} = 96\% \), (or \( \sigma_{\text{ov}} = 93\% \) when cutting small scales from the CFHTLenS correlation functions, see section 3.2) so for example the 90% contours do not overlap. This means that the best points, or at least those where the tension is least, are still on at least the 96% (93%) contours of both probes. These \( \sigma \) values are collected for this and subsequent sections in Table 1.

### 3.2 Sensitivity to the choice of nonlinear matter power spectrum

The strength of weak lensing lies in its ability to constrain the matter power spectrum, \( P_s(k) \) however, it is most sensitive to scales where nonlinear effects on \( P_s(k) \) are significant. This is demonstrated by Fig. 3, the upper panel of which shows the weighting of \( k \) scales in \( \xi_+ \) for the autocorrelation of the highest redshift CFHTLenS redshift bin. We show \( W(\log(k), \theta) \), where

\[
\xi_+(\theta) = \int d\log(k) W(\log(k), \theta) P_s(k)
\]  

(1)

The 5 lines are the 5 angular bins (1 \( \lesssim \theta \lesssim 40 \) arcmin) used in the HE13 measurement, with larger angles peaking at lower \( k \). The lower panel shows the fractional difference between the two HALOFIT implementations, SM03 and TA12 and the prediction of the publicly available code Franken-Emu\(^2\), a matter power spectrum emulator based on the Coyote Universe simulations (Heitmann et al. 2014), which we’ll refer to as Coyote. Comparing the two panels, it is clear, particularly for smaller angular bins, that the choice of nonlinear correction is important for the \( k \)-scales being probed.

Beutler et al. (2014a) already noted a \( \approx 1 \) \( \sigma \) shift in the constraint on \( \sigma_8 \) from using the newer implementation of HALOFIT. This is not unexpected when we look at the fractional differences in \( P(k) \) for different nonlinear prescriptions, shown in Fig. 3. HE13 suggest the conservative approach of cutting some of the lower \( \theta \) bins in \( \xi_{+/−} \) as a way of reducing the importance of the nonlinear correction. They boost and decrease the SM03 non-linear correction by \( ±7\% \), and propose cutting all \( \theta \) bins where the predicted \( \xi \) changes by more than 10\%. For \( \xi_+ \), this corresponds to \( \theta \lesssim 3 \) arcmin for tomographic bin combinations including bins 1 and 2. For \( \xi_− \) (which is sensitive to higher \( k \) than \( \xi_+ \) for a given angular scale), this corresponds to \( \theta \lesssim 30 \) arcmin for tomographic bin combinations including bins 1, 2, 3, and 4, and \( \theta \lesssim 16 \) arcmin for tomographic bin combinations including bins 5 and 6.

We adopt this scheme for the rest of the paper, and perform the following simple test on the sensitivity to the choice of nonlinear correction: We fix all parameters except \( \sigma_8 \) to the best joint-fit Planck+WP and CFHTLenS cosmology, and obtain 1d CFHTLenS constraints on \( \sigma_8 \), for each of TA12, SM03 and Coyote. Fig. 4 shows the results of this test. Even after implementing the conservative \( \theta \) cut, there is still a small shift between SM03 and TA12, although the constraints from TA12 and Coyote are very similar for this slice of parameter space. Encouraged by this, we continue using TA12 for the rest of the paper, since it can be used consistently with non-zero neutrino mass.

### 3.3 Baryonic feedback

The matter power spectrum (and therefore the weak lensing convergence power spectrum) is also affected by baryonic feedback at \( k > 1 h^{-1} \text{Mpc} \), as pointed out by White (2004) and Zhan & Knox (2004) who, using simple models of the effect, reported up to several percent changes in the convergence power spectrum at \( l > 1000 \). Jing et al. (2006), Rudd et al. (2008), Guillet et al. (2010), Casarini et al.

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\(^2\) http://www.hep.anl.gov/cosmology/CosmicEmu/emu.html

**Figure 3.** Top panel - the weight functions, \( W(\log(k), \theta) \) for \( \xi_+(\theta) \), for the autocorrelation of the highest redshift CFHTLenS bin. The weight functions give the relative contribution to \( \xi_+(\theta) \) as a function of \( k \). The 5 lines are for the 5 \( \theta \) bins used (with bin centres at 1.65, 3.58, 7.76, 16.80, 36.18 arcmin), with lower \( \theta \) bins peaking at higher \( k \). Bottom panel - the nonlinear matter power spectrum at \( z=0.5 \) predicted by Coyote and SM03, as a fraction of the TA12 prediction.
Figure 4. CFHTLenS $\sigma_8$ constraints in an otherwise fixed fiducial cosmology, with 3 different nonlinear power spectrum treatments. Also shown is the constraint using TA12 and a prescription for AGN feedback described in Section 3.3.

Table 1. Goodness of fit of joint fit to individual datasets, for several extensions to $\Lambda$CDM. ‘C’ and ‘P’ denote CFHTLenS and Planck+WP respectively. ‘small scales cut’ refers to removing some of the $\xi(\theta)$ bins used in the CFHTLenS analysis, as described in section 3.2. $p_{\text{joint}}$ denotes the parameters of the best joint fit to the datasets. $\sigma_i$ values are defined at the end of Section 3.1.

| Model         | Datasets | small scales cut? | $\sigma_C(p_{\text{joint}})$ | $\sigma_P(p_{\text{joint}})$ | $\sigma_{eq}$ |
|---------------|----------|-------------------|------------------------------|------------------------------|--------------|
| $\Lambda$CDM  | P + C    | No                | 99%                          | 81%                          | 96%          |
| $\Lambda$CDM  | P + C    | Yes               | 98%                          | 60%                          | 93%          |
| $\Lambda$CDM + IA(z) | P + C    | Yes               | 99%                          | 43%                          | 92%          |
| $\Lambda$CDM + AGN | P + C    | Yes               | 99%                          | 25%                          | 92%          |
| $m_v\Delta N_{\text{eff}} \Lambda$CDM | P + C    | Yes               | 96%                          | 62%                          | 90%          |
| $\tau_{\text{AGN}} \Lambda$CDM | P + C    | Yes               | 90%                          | 5%                           | 64%          |
| $\tau_{\text{AGN}} \Lambda$CDM | P + C    | Yes               | 99%                          | 42%                          | 89%          |

We obtain a 1d constraint on $\sigma_8$ as before, shown by the dotted line labelled TA12 + AGN in Fig. 4. Largely due to the fact that we have cut the smallest scales from our analysis, the shift from introducing AGN feedback is smaller than the shift arising from the use of different Halofit implementations, which suggests that the effect of AGN feedback is probably not important here. Nevertheless, we also repeat the analysis of section 3.1, introducing a new parameter $\alpha_{\text{AGN}}$ given by

$$P_3 = \frac{P_{\text{AGN}}^{\delta}}{P_{\text{DMONLY}}^{\delta}} P_{\tau_{\text{TA12}}}^{\delta}. \quad (2)$$

Thus we allow the strength of the AGN feedback to vary by allowing $\alpha_{\text{AGN}}$ to vary. The top right panel of Fig. 5 shows the widening of the CFHTLenS alone contours when we allow $0 < \alpha_{\text{AGN}} < 2$. We repeat the analysis of section 3.1, but obtain only a small improvement in agreement between the two probes, with $\sigma_{eq} = 92\%$. In the joint fit, we allow $-3 < \alpha_{\text{AGN}} < 3$, and find $\alpha = 1.2^{+1.1}_{-1.2}$. The fact there is only weak preference for positive $\alpha_{\text{AGN}}$ is consistent with this parameter not being very helpful in resolving the tension.

This is the first combined CMB and lensing analysis to constrain baryonic feedback and cosmology simultaneously, although we note that this is a very simplistic prescription for AGN feedback, let alone baryonic feedback as a whole. Harnois-Déraps et al. (2014) use three of the OWLS to construct a fitting function for the effect of baryonic feedback on the power spectrum, and use CFHTLenS to constrain this model while fixing the cosmology. Unlike this analysis, they extend the CFHTLenS data to sub-arcminute scales, and find a strong preference for a universe with baryonic feedback. Eifler et al. (2014) show the importance of accounting for baryonic feedback for stage III and IV weak lensing experiments, and propose a PCA marginalisation approach that uses information from a range of hydrodynamical simulations, as a way of removing the bias with 3 or 4 nuisance parameters.
3.4 Sensitivity to the IA model

We follow HE13 by using the non-linear linear alignment model (henceforth NLA model, Bridle & King (2007)) to account for intrinsic alignments. The NLA model is based on the linear alignment model (Catelan et al. 2001; Hirata & Seljak 2004), which assumes that galaxies are aligned with their haloes which are in turn aligned with the local tidal gravitational field; for a given redshift the intrinsic galaxy ellipticity is taken to be proportional to the linear theory tidal field strength. In the linear alignment model, the intrinsic-intrinsic (II) and shear-intrinsic (GI) power spectra are given by

\[ P_{\text{II}}(k, z) = F_{\text{II}}^2(z) P_\delta(k, z), \quad P_{\text{GI}}(k, z) = F_{\text{GI}} P_\delta(k, z), \]  

(4)

where

\[ F(z) = -AC_1 \rho_{\text{crit}} \frac{\Omega_m}{D(z)}. \]  

(5)

\[ \rho_{\text{crit}} \] is the critical density at \( z = 0 \), \( C_1 = 5 \times 10^{-14} h^{-2} M_\odot^{-1} \text{Mpc}^3 \), and \( A \), the dimensionless amplitude, is the single free parameter.

In the NLA model, the linear matter power spectrum in equation 4 is replaced with the non-linear matter power spectrum. One of the main uncertainties of both of these alignment models is the redshift scaling - it may be that alignment was produced at high redshift during galaxy formation, but the strength of the signal is likely to have evolved over cosmic time. Hence we try a simple extension to the NLA model, by introducing a power law redshift scaling, \( \alpha_{\text{IA}} \), so that

\[ F(z) = -AC_1 (1 + z)^{\alpha_{\text{IA}}} \rho_{\text{crit}} \frac{\Omega_m}{D(z)}. \]  

(6)

We repeat the analysis of section 3.1, and obtain the marginalised constraints in the \( \Omega_m - \sigma_8 \) plane shown in the top left panel of Fig. 5. A small shift in the CFHTLenS contours is apparent when including the extra intrinsic alignment parameter, but by eye it does not appear significant in
resolving the tension with Planck+WP. We find \( \sigma_{\alpha A} = 92\% \) for this model (i.e. only the 92\% contours of the two probes’ posteriors touch), which supports this conclusion.

In agreement with HE13 we find that negative values of the intrinsic alignment amplitude parameter \( A \) are slightly preferred for \( \alpha_{IA} = 0 \). We allow a prior range of \(-5 < \alpha_{IA} < 5\) and find \( \alpha_{IA} \) to be unconstrained but preferred to be strongly negative for both CFHTLenS alone and for CFHTLenS + Planck+WP. This can be understood from the relatively large amount of power at low redshift in the CFHTLenS data - see HE13 Fig. 2. The negative power law index allows more intrinsic alignment contribution at low redshift. An even more negative intrinsic alignment amplitude \( A \) is preferred for both CFHTLenS alone and CFHTLenS + Planck+WP. This makes the contribution from the dominant intrinsic alignment term (GI) positive, to match the Planck+WP. This makes the contribution from the dominant intrinsic alignment term (GI) positive, to match the Planck+WP.

### 3.5 A New CFHTLenS Fitting Function

HE13 presented the constraint \( \sigma_s(\Omega_m/0.27)^{0.47} = 0.774 \pm 0.032 \) that has been used in combination with other datasets instead of running a full likelihood analysis. The HE13 analysis used the Smith et al. (2003) version of HALOFIT. Beutler et al. (2014a) showed a \( \sim 1\sigma \) reduction in the Kilbinger et al. (2013) CFHTLenS constraint on \( \sigma_s \) at \( \Omega_m = 0.3 \) when using a newer HALOFIT implementation, and we see a similar effect when using TA12. We agree with Beutler et al. (2014a) that the inclusion of a massive neutrino eigenstate of mass 0.06eV also makes a difference to the result. We find the following fitting function for the CFHTLenS constraints in the \( \Omega_m\sigma_s \) plane, for our base \( \Lambda \)CDM

\[
\sigma_s(\Omega_m/0.27)^{0.47} = 0.74 \pm 0.03, \quad (7)
\]

an \( \approx 1\sigma \) shift in the central value compared to the HE13 result, in agreement with Beutler et al. (2014a).

### 4 DISCORDANCE IN EXTENSIONS TO \( \Lambda \)CDM

In this section we try the following extensions to \( \Lambda \)CDM: massive active neutrinos, a massive sterile neutrino, and primordial tensor modes, and quantify how much they resolve the tension between Planck+WP and CFHTLenS.

#### 4.1 Discordance in \( m_\nu \Lambda \)CDM

We wish to know whether allowing a greater active neutrino mass alleviates the tension between Planck+WP and CFHTLenS. We allow the mass, \( m_\nu \) of the massive neutrino eigenstate in our base \( \Lambda \)CDM model to vary, above a lower bound of 0.06eV. We call this model \( m_\nu \Lambda \)CDM.

Line 5 of Table 1 summarises the consistency tests we performed for this model. \( \sigma_{\alpha A} \) is 86\% i.e. the 86\% 7d confidence regions only just touch, which is only a small improvement over \( \Lambda \)CDM. Some insight into why this happens can be gained from the bottom left panel of Fig. 5: the Planck+WP contours are extended along the same line of degeneracy as the CFHTLenS constraint. We explain this as follows: although increasing neutrino mass does reduce the growth of structure, hence driving the Planck+WP contours to lower \( \sigma_s \), massive neutrinos, which are relativistic before recombination, add to the radiation density, and hence encourage larger \( \Omega_m \) to preserve the time of matter-radiation equality. Therefore the datasets are still discrepant. We discuss other related analyses in Section 5.1.

Our principal component analysis gives a similar power law slope for this model, and we find the constraint

\[
\sigma_s(\Omega_m/0.27)^{0.48} = 0.72 \pm 0.03, \quad (8)
\]

grows for CFHTLenS alone in the \( m_\nu \Lambda \)CDM model. This power law can be used to approximate the CFHTLenS constraints when a varying active neutrino mass is allowed.

#### 4.2 A sterile neutrino: \( m_\nu \Delta N_{\text{eff}} \Lambda \)CDM

We add to our base model a sterile neutrino - an additional neutrino species with effective mass \( m_\nu \) and contribution to \( N_{\text{eff}} \) of \( \Delta N_{\text{eff}} = N_{\text{eff}} - 3.046 \), as proposed by Hamann & Hasenkamp (2013) and Battye & Moss (2014). The bottom right panel of Fig. 5 shows constraints in the \( \Omega_m - \sigma_s \) plane, and we see now there is better agreement between the two probes - the 95\% 2d marginalised contours do now touch. We find \( \sigma_{\alpha A} \approx 64\% \) for this model, a considerable improvement. We consider this agreement good enough to combine the measurements, and find \( m_\nu < 0.737 \)eV (95\%) and \( \Delta N_{\text{eff}} = 0.838 \pm 0.343 \). The 1d marginalised pdfs are shown in Fig. 6, and the cosmological constraints are shown in Table 2.
Bayesian evidence ratio for a model is favoured when the evidence ratio is less than one. The extended model has extra parameter(s) $p$ is given by

$$
P(d|M_0) = \frac{P(d|M_1, p = 0)}{\int dpP(d|M_1, p)Pr(p|M_1)}$$

where $P(d|M_1, p)$ is the likelihood of the data $d$ marginalised over all other parameters except $p$ and $Pr(p|M_1)$ is the normalised prior on $p$ (e.g. see Lewis & Bridle (2010), Trotta (2007) and references therein). The extended model $M_1$ is favoured if the ratio is less than one. If regions where the likelihood $P(d|M_1, p)$ is very small are allowed by a wide prior (i.e., a broad range) of $p$, the denominator can become very small, causing the extended model to be disfavoured. So the evidence ratio is very sensitive to the choice of prior.

To illustrate this, we compute the evidence ratio of $m_s \Delta N_{\text{eff}} \Lambda CD M$ compared to $\Lambda CD M$, as a function of the priors on $m_s$ and $\Delta N_{\text{eff}}$, shown in Fig. 7. Either model can be favoured, depending on the choice of prior. If the number of extra neutrino species is assumed to be less than 3.5 then the sterile neutrino model is favoured if we assume an upper limit on the sterile neutrino mass of 1 eV. More stringently, if the number of extra species is assumed to be less than 2 and the mass less than 0.3 eV then the sterile neutrino model is around a factor of five more probable than $\Lambda CD M$. Conversely, if the prior range on the mass and number of neutrino species is large then the sterile neutrino model is disfavoured. For example, if the mass is restricted to be less than 3 eV and the number of extra species less than 5, then $\Lambda CD M$ is three times as probable as $m_s \Delta N_{\text{eff}} \Lambda CD M$.

Again, we provide a power law representation of the evidence ratio of Planck+WP to Planck+WP

$$
\sigma_8(\Omega_m/0.27)^{0.48} = 0.72 \pm 0.03.
$$

This is the same result we found for the $m_s \Lambda CD M$, which is reasonable since low redshift probes like weak lensing are sensitive to the total neutrino mass, and not to $N_{\text{eff}}$ (i.e., they do not care whether the neutrino mass eigenstate is active or sterile).

| Model     | Base $\Lambda CD M$ | $m_s \Delta N_{\text{eff}} \Lambda CD M$ |
|-----------|----------------------|------------------------------------------|
| Data      | Planck+WP            | Planck+WP CFHTLenS                       |
| $\Omega_m$ | 0.315$^{+0.016}_{-0.018}$ | 0.271$^{+0.017}_{-0.027}$ |
| $\sigma_8$ | 0.829$^{+0.012}_{-0.012}$ | 0.777$^{+0.044}_{-0.029}$ |
| $h_0$     | 0.673$^{+0.027}_{-0.025}$ | 0.744$^{+0.020}_{-0.037}$ |
| $n_s$     | 0.966$^{+0.007}_{-0.007}$ | 1.000$^{+0.009}_{-0.015}$ |
| $\tau$    | 0.089$^{+0.012}_{-0.014}$ | 0.097$^{+0.013}_{-0.015}$ |
| $\Delta N_{\text{eff}}$ | - | 0.838$^{+0.353}_{-0.342}$ |
| $m_s$ [eV] | - | $< 0.737$ (95%) |

Table 2. Cosmological parameter constraints in the $m_s \Delta N_{\text{eff}} \Lambda CD M$ model. The values shown are means of the posterior distribution; errors are 68% confidence intervals unless specified. The Planck+WP base $\Lambda CD M$ are included for easy reference.

4.3 Primordial Gravity Waves

Inspired by the recent BICEP2 results (BICEP2 Collaboration et al. 2014) we investigate the effect of gravity waves on the tension between Planck+WP and CFHTLenS. Qualitatively we might expect the agreement between Planck+WP and CFHTLenS to improve due to gravity waves: the increase of power at low multipoles in the CMB from tensors will need to be compensated by a reduction in power from scalar modes; a reduction in scalar power would bring the Planck+WP contours closer to CFHTLenS. Meanwhile, the addition of tensor modes in the primordial power spectrum does not affect the matter power spectrum, which determines the shear-shear correlation function, and so has no effect on weak lensing. However, a detailed analysis is necessary to see how the values of the other cosmological parameters are affected by the change in shape of the CMB power spectrum due to the addition of tensors.

The BICEP2 measurement of $r = 0.20^{+0.07}_{-0.05}$ (BICEP2 Collaboration et al. 2014) is not compatible with the Planck+WP data unless an additional modification is made to $\Lambda CD M$, because Planck Collaboration et al. (2013d) showed that when only $r$ is added to the base $\Lambda CD M$ model, Planck+WP gives the constraint $r < 0.11$ (95% confidence). Therefore in this section we also consider the effect of adding a running spectral index of the primordial power spectrum, $\alpha_{\text{run}} = d\alpha_s/d\ln k$, which Planck Collaboration et al. (2013d) showed to relax the constraint on $r$ to $r < 0.26$ (95%), into agreement with BICEP2.

We repeat the investigation in $n$-dimensions described in 3.1, and find that the addition of gravity waves and running of the spectral index, in the $\sigma_{\text{run}} \Lambda CD M$ model, relaxes the tension between the datasets only slightly, with $\sigma_{\text{eq}} = 87\%$ i.e. the 87% contours from each dataset touch.

Therefore we conclude that gravity waves do not significantly resolve the tension between CFHTLenS and Planck+WP.
Cosmic Discordance: Are Planck CMB and CFHTLenS weak lensing measurements out of tune?

Figure 8. Comparison of constraints in the $\sigma_8$, $\Omega_m$ plane in $\Lambda$CDM from CFHTLenS (this work; green), Planck+WP (yellow, Planck Collaboration et al. (2013d)), Planck SZ cluster counts (orange, Planck Collaboration et al. (2013c)), X-ray clusters (red, Vikhlinin et al. (2009)) and CMASS $f_{\sigma_8}$ (blue, Beutler et al. (2014b)). In the left panel, the contours are obtained assuming $\Lambda$CDM, while in the right panel, the CFHTLenS and Planck+WP constraints allow a varying active neutrino mass. Of note is the improved consistency of the Planck+WP contours with the CMASS $f_{\sigma_8}$ and the Planck SZ contours when the neutrino mass is allowed to vary, driving the neutrino mass detections of Battye & Moss (2014) and Beutler et al. (2014a).

5 DISCUSSION

In this Section we compare our results with those from other analyses, and speculate on alternative potential explanations for the discrepancy. We will refer to Fig. 8, which shows a selection of other low-z probes of the growth of structure.

5.1 Comparison with other work

Several other authors have considered how to reconcile cosmology from the CMB and the amplitude of matter fluctuations measured by low-redshift probes. The most relevant to our work are by Planck Collaboration et al. (2013d), Battye & Moss (2014), Beutler et al. (2014a), Dvorkin et al. (2014), Leistedt et al. (2014), and Archidiacono et al. (2014). We discuss next the differences to our analysis.

The Planck Collaboration et al. (2013d) noted an approximately $2\sigma$ discrepancy between their Planck CMB analysis and the CFHTLenS analysis of Heymans et al. (2013) and noted that further work will be required to resolve the difference. They allow freedom in the effect of lensing on the primary anisotropies and find that a larger lensing amplitude is preferred when the Planck data is combined with smaller scale CMB measurements. Taken at face-value this suggests an increased $\sigma_8$ from low redshift data, unlike all the other low redshift data considered in the other papers we discuss below.

The tension between Planck Sunyaev–Zel’dovich (SZ) cluster counts and the primary anisotropies was discussed by the Planck Collaboration et al. (2013c). They discuss possible systematics in the SZ analysis and conclude that each is improbable, but that understanding the mass bias scaling relation is the key to further investigation. They find a 1.9$\sigma$ preference for a non-zero active neutrino mass by combining Planck+WP with the Planck SZ constraints marginalising over their preferred range in the mass bias.

Planck Collaboration et al. (2013e) used lensing of the CMB to measure the power spectrum of the gravitational potential at slightly higher redshift than that probed by CFHTLenS. This was combined with the constraints from the primary anisotropies and found to reduce the measured amplitude of fluctuations (Planck Collaboration et al. 2013d). One of the many extensions to $\Lambda$CDM investigated by the Planck team (Planck Collaboration et al. 2013d) was the mass of the active neutrino. When using the CMB lensing information, they found that this increased the upper limit on the neutrino mass relative to that from CMB primary anisotropies alone (the 95% upper limit increased from 0.66eV to 0.85eV), indicating some tension.

Battye & Moss (2014) found a preference for a non-zero active neutrino mass when combining CMB lensing, CFHTLenS and Planck SZ cluster counts (Planck Collaboration et al. 2013b) with the CMB. They use the correlation functions measured by Kilbinger et al. (2013), who performed a 2d cosmic shear analysis i.e. they did not use multiple redshift bins. They found similar but stronger pref-
ference for a non-zero sterile neutrino mass. They noted that both these joint fits come at the cost of a worse fit to the primary CMB data. Fig. 8 shows an orange band corresponding to the SZ cluster counts prior from Planck Collaboration et al. (2013b), $\sigma_8(\Omega_m/0.27)^{1/3} = 0.78 \pm 0.01 (68\%)$. The shallower degeneracy direction of this prior as compared to the lensing constraint allows more overlap with the Planck+WP confidence regions with an active neutrino, explaining the significant detection of neutrino mass Battye & Moss (2014) claimed when combining this prior with Planck+WP.

Beutler et al. (2014a) investigate the constraints on the active neutrino mass using Baryon Oscillation Spectroscopic Survey (BOSS), the CMB, and other low redshift measurements including a parameterised fit to the CFHTLenS cosmology constraints of Kilbinger et al. (2013). Their Figure 7 illustrates that the inclusion of a free active neutrino mass elongates the Planck contours in a parallel direction to the CFHTLenS constraints. They combine CMB constraints with those from the BOSS (Schlegel et al. 2009) CMASS DR11 galaxy clustering results of Beutler et al. (2014b), which come from BAO, Alcock-Paczinski and growth measurements. The push towards a positive neutrino mass comes mostly from the growth constraint (shown as blue contours in Fig. 8) since this is sensitive to the amplitude of clustering. They use various combinations of the data and find similar non-zero values for the neutrino mass to Battye & Moss (2014), and finally combine them all together to get a $\approx 3\sigma$ detection of the neutrino mass, with a similar result whether using WMAP or Planck temperature anisotropies.

Dvorkin et al. (2014) focus on the discrepancy between Planck and BICEP2, noting that extra relativistic species in the early universe can help alleviate the tension introduced into the Planck data by extra power from gravitational waves. They point out that the sterile neutrino can thus help alleviate the Planck-BICEP2 tension and additionally the CMB-low-z tension at the same time. They use local $H_0$, baryon acoustic oscillations and local X-ray cluster abundance measurements (Vikhlinin et al. 2009), shown as the red band in Fig. 8) for low-redshift information, and obtain a $\approx 3\sigma$ sigma detection of the sterile neutrino mass.

The cosmological constraints on sterile neutrinos are compared with those from short baseline neutrino oscillation experiments in Archidiacono et al. (2014). They use the CMB combined with low-redshift clustering measurements from the growth of structure obtained by Parkinson et al. (2012), Planck SZ and the parameterised fit to the CFHTLenS constraints of Kilbinger et al. (2013). They find a detection of non-zero sterile neutrino mass, and point out the significant inconsistency between its value and that found in the neutrino oscillation experiments.

However, the neutrino mass detections are disputed by Leistedt et al. (2014), who point out that they are driven by two highly constraining datasets from counting galaxy clusters (Planck Collaboration et al. 2013b; Vikhlinin et al. 2009). They describe some of the potential systematic effects in these two measurements. They use the same simple parameterised fit to the CFHTLenS constraints as Archidiacono et al. (2014) and Beutler et al. (2014a). Omitting the two most constraining datasets, they use each other low-redshift clumpiness measure one-at-a-time, and find only upper limits on the neutrino mass.

5.2 Other possible explanations

We have shown our results to be robust to uncertainties in the modelling of the nonlinear matter power spectrum (including AGN feedback) and intrinsic alignments. Two other weak-lensing systematics we have not considered in detail are photometric redshift errors and shape measurement errors. We can estimate how wrong these would have to be to account for the $\approx 20\%$ disparity (assuming the Planck+WP best-fit value of $\Omega_m$) between the CFIHTLenS and Planck+WP best fit values of $\sigma_8$. As a rule of thumb, equation 24 from Huterer et al. (2006) tells us that the lensing power spectrum (at $l = 1000$ and assuming all source galaxies are at $z_s = 1$) has dependence

$$P_s \propto \sigma_8^{2.9} \zeta_{p,\text{observed}}^{1.6}.$$  

Hence to observe the same signal (i.e. setting $P_s$ equal to a constant in 11), with a 20% higher $\sigma_8$, would require the redshifts to shift systematically by a fraction $\approx 1.2^{-1.8} = 0.72$ i.e. a systematic 30% error in photometric redshift, which would be very surprising in any one redshift bin, let alone all redshift bins with the same sign.

To estimate the effect of multiplicative bias on our results, we note that $\xi_{v/-} \propto (1 + m)^2$, where $m$ is the multiplicative bias (see e.g. Heymans et al. (2006) for an introduction to shape measurement biases). The information in $\xi_{v/-}$ comes from a mixture of linear and nonlinear scales. On linear scales we have $\xi_{v/-}^{\text{observed}} \propto (1 + m)^2 P_s \propto (1 + m)^2 \sigma_8^2$, whereas on nonlinear scales $\xi_{v/-} \propto (1 + m)^2 \sigma_8^2$. So an increase in $\sigma_8$ of 20% would require $(1 + m) = 1.2^{-1} = 0.83$ (assuming all information comes from linear scales) and $(1 + m) = 1.2^{-3/2} = 0.69$ (assuming all information comes from nonlinear scales). The multiplicative bias in the CFHTLenS shape measurements was calibrated using image simulations (Miller et al. 2013) and the average value of $(1 + m)$ was found to be 0.94. It’s clear then, that the value of the multiplicative bias estimated from simulations would have to be catastrophically wrong to produce such a significant shift in $\sigma_8$.

Spergel et al. (2013) reanalysed the Planck data, and claim that the 217GHz $\times 217$GHz detector set spectrum used in the Planck analysis is responsible for some of the tension with other cosmological measurements. The latest version of Planck Collaboration et al. (2013d) does discuss a residual systematic in the 217GHz $\times 217$GHz spectrum, but claim that this has an impact of less than half a standard deviation on cosmological parameters. The Planck 2014 releases will include a correction for this, but the Planck products used in this analysis do not.

Furthermore, the CMB constraints we use rely on the WMAP polarisation data primarily to constrain the optical depth to reionisation. The Planck satellite has measured the polarisation signal more precisely. A reduction in the optical depth to reionisation would be required to push the CMB contours towards those from CFHTLenS. This is because a lower optical depth increases the predicted CMB temperature anisotropy power spectrum on the majority of scales, and thus the underlying amplitude of scalar fluctuations must be reduced to retain a good fit to the observations. The Planck Collaboration et al. (2013d) noted that the Planck temperature anisotropies alone tightly constrain
the combination
\[ \sigma_{8 e^{-\tau}} = 0.753 \pm 0.011 \quad (12) \]
which would require \( \tau \) to be negligible to make a significant impact on the discrepancy.

As noted by Archidiacono et al. (2014), the cosmology constraints on the sterile neutrino are not compatible with those from short baseline neutrino experiments. However, the cosmology constraints are relatively generic for other relativistic particles in the early universe.

Finally we note that other extensions of \( \Lambda \)CDM can be explored, such as the variation of the dark energy equation of state or a deviation of the growth of structure from the GR prediction.

\section{6 CONCLUSIONS}
We have found that Planck+WP and CFHTLenS are inconsistent within the base \( \Lambda \)CDM cosmology. The inconsistency between the CMB and low redshift datasets have been examined by other authors, but this is the first time that such a strong conclusion has been drawn using just cosmic shear and the CMB.

We find that allowing massive active neutrinos does not in fact resolve the discrepancy, because the slope of the CFHTLenS contours runs parallel to the effect of adding active neutrinos to the CMB. Other works include other datasets with a shallower slope than CFHTLenS, which intersect the CMB contours at high active neutrino mass, thus leading to a detection of the neutrino mass. However, if taken at face value, the CFHTLenS and Planck+WP data already rule out this scenario. It was noted in Battye & Moss (2014) that the active neutrino mass detection comes at the cost of a decreased likelihood of the Planck+WP data. In this paper we have quantified the size of this decrease in terms of the full n-dimensional contours and used a more robust version of the cosmic shear data.

The addition of tensor modes, even with running of the spectral index also does not significantly affect the tension.

We have also added an extra, sterile species of neutrino, and find that the 64\% confidence contours in the 8 dimensional parameter space touch in this case. We find that the effective number of extra neutrino species (\( \Delta N_{\text{eff}} \)) is strongly favoured to be non-zero in the joint fit. Although the \( m_1 \Delta N_{\text{eff}} \Lambda \)CDM model does allow an acceptable joint fit, some tension remains between Planck+WP and CFHTLenS, since all points in the joint fit are on at least the 64\% contour of either the Planck+WP-only or CFHTLenS-only constraints.

Therefore we are not completely satisfied by the amount which the flexible sterile neutrino model reduces the discrepancy, and believe that investigating other new physics, and other sources of systematic error in either experiment, may lead to a better resolution of the tension.

If the \( \Lambda \)CDM discrepancy between CFHTLenS and Planck+WP is to be resolved it would require more than a 1\( \sigma \) shift in the CFHTLenS constrains upwards or similarly for the Planck+WP downwards. We look forward to future constraints from the CMB and other lensing analyses.

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