Squeezing of a nanomechanical oscillator

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We show that squeezing of a nanomechanical mirror can be generated by injecting broad band squeezed vacuum light and laser light into the cavity. We work in the resolved sideband regime. We find that in order to obtain the maximum momentum squeezing of the movable mirror, the squeezing parameter of the input light should be about 1. We can obtain more than 70% squeezing. Besides, for a fixed squeezing parameter, decreasing the temperature of the environment or increasing the laser power increases the momentum squeezing. We find very large squeezing with respect to thermal fluctuations, for instance at 1 mK, the momentum fluctuations go down by a factor more than one hundred.

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I. INTRODUCTION

The optomechanical system has attracted much attention because of its potential applications in high precision measurements and quantum information processing [1, 2, 3, 4, 5, 6, 7, 8, 9]. Meanwhile, it provides a means of probing quantum behavior of a macroscopic object if a nanomechanical oscillator can be cooled down to near its quantum ground state [10, 11]. Many of these applications are becoming possible due to advances in cooling the mirror [12, 13, 14, 15, 16, 17]. Further as pointed out in Refs. [19, 20, 21], the ground state cooling can be achieved in the resolved sideband regime where the frequency of the mechanical mirror is much larger than the cavity decay rate.

Squeezing of a nanomechanical oscillator plays a vital role in high-sensitive detection of position and force due to its less noise in one quadrature than the coherent state. A number of different methods have been developed to generate and enhance squeezing of a nanomechanical oscillator, such as coupling a nanomechanical oscillator to an atomic gas [22], a Cooper pair box [23], a SQUID device [24], using three-wave mixing [25] or Circuit QED [26], or by means of quantum measurement and feedback schemes [27, 28, 29, 30]. A recent paper [31] reports squeezed state of a mechanical mirror can be created by transfer of squeezing from a squeezed vacuum to a membrane within an optical cavity under the conditions of ground state cooling. We previously considered the possibility of using an OPA inside the cavity for changing the nature of the statistical fluctuations [32].

In this paper, we propose a scheme that is capable of generating squeezing of the movable mirror by feeding broad band squeezed vacuum light along with the laser light. The achieved squeezing of the mirror depends on the temperature of the mirror, the laser power, and degree of squeezing of the input light. One can obtain squeezing which could be more than 70%.

The paper is structured as follows. In Sec. II we describe the model, give the quantum Langevin equations, and obtain the steady-state mean values. In Sec. III we derive the stability conditions, calculate the mean square fluctuations in position and momentum of the movable mirror. In Sec. IV we analyze how the momentum squeezing of the movable mirror is affected by the squeezing parameter, the temperature of the environment, and the laser power. We also compare the momentum fluctuations of the movable mirror in the presence of the coupling to the cavity field with that in the absence of the coupling to cavity field. We find very large squeezing with respect to thermal fluctuations, for instance at 1 mK, the momentum fluctuations go down by a factor more than one hundred. Our predictions of squeezing are based on the parameters used in a recent experiment on normal mode splitting in a nanomechanical oscillator [33].

II. MODEL

The system to be considered, sketched in Fig. 1, is a Fabry-Perot cavity with one fixed partially transmitting mirror and one movable perfectly reflecting mirror in thermal equilibrium with its environment at a low temperature. The cavity with length $L$ is driven by a laser with frequency $\omega_L$, then the photons in the cavity will exert a radiation pressure force on the movable mirror due to momentum transfer. This force is proportional to the instantaneous photon number in the cavity. The mirror also undergoes thermal fluctuations due to environment. Under the effects of the two forces, the movable mirror makes oscillation around its equilibrium position. Here we treat the movable mirror as a quantum mechanical harmonic oscillator with effective mass $m$, frequency $\omega_m$ and momentum decay rate $\gamma_m$. We further assume that the cavity is fed with squeezed light at frequency $\omega_S$.

In the adiabatic limit, $\omega_m \ll \frac{\omega_S}{\pi}$ ( $c$ is the speed of light in vacuum), we ignore the scattering of photons to other cavity modes, thus only one cavity mode $\omega_c$ is considered [34]. In a frame rotating at the laser frequency,
and creation operators for the cavity field with $c, c^\dagger$ mirror to the cavity field via radiation pressure, the pa-

ture of the movable mirror to the cavity field. The

The equations of motion of the system can be written as

$$H = \hbar(\omega_c - \omega_L)n_c - \hbar g n_c c + \frac{\hbar \omega_m}{4}(Q^2 + P^2) + i\hbar \varepsilon (c^\dagger - c),$$  

(1)

we have used the normalized coordinates for the oscil-
lator defined by $Q = \sqrt{\frac{2m\omega_m}{\hbar}} q$ and $P = \sqrt{\frac{2m\hbar\omega_m}{\hbar}} p$ with $[Q, P] = 2i$. This normalization implies that in the ground state of the nanomechanical mirror $\langle Q^2 \rangle = \langle P^2 \rangle = 1$. Further in Eq. (1) the first term is the en-
ergy of the cavity field, $n_c = c^\dagger c$ is the number of the photons inside the cavity, $c$ and $c^\dagger$ are the annihi-
lation and creation operators for the cavity field with $[c, c^\dagger] = 1$. The second term comes from the coupling of the movable mirror to the cavity field via radiation pressure, the parameter $g = \frac{g}{\hbar} \sqrt{\frac{2m\omega_m}{\hbar}}$ is the optomechanical coupling constant between the cavity and the movable mirror. The third term corresponds the energy of the movable mirror. The fourth term describes the coupling between the input laser field and the cavity field, $\varepsilon$ is related to the input laser power $\varphi$ by $\varepsilon = \sqrt{\frac{2\omega_c}{\hbar \omega_L}}$ where $\kappa$ is the cavity decay rate associated with the transmission loss of the fixed mirror.

The equations of motion of the system can derived by the Heisenberg equations of motion and adding the corre-
ponding noise terms, this gives the quantum Langevin

equations

$$\dot{Q} = \omega_m P,$$

$$\dot{P} = 2gn_c - \omega_m Q - \gamma_m P + \xi,$$

$$\dot{c} = i(\omega_L - \omega_c + gQ)c + \varepsilon - \kappa c + \sqrt{2\kappa n_c},$$

$$\dot{c}^\dagger = -i(\omega_L - \omega_c + gQ)c^\dagger + \varepsilon - \kappa c^\dagger + \sqrt{2\kappa n_c}^\dagger.$$  

(2)

Here we have introduced the input squeezed vacuum noise operator $c_{in}$ with frequency $\omega_S = \omega_L + \omega_m$. It has zero mean value, and nonzero time-domain correlation functions

$$\langle \delta c_{in}(t)\delta c_{in}(t') \rangle = N\delta(t - t'),$$

$$\langle \delta c_{in}(t)\delta c_{in}^\dagger(t') \rangle = (N + 1)\delta(t - t'),$$

$$\langle \delta c_{in}(t)\delta c_{in}(t') \rangle = Me^{-i\omega_m(t+t')}\delta(t - t'),$$

$$\langle \delta c_{in}^\dagger(t)\delta c_{in}(t') \rangle = Me^{i\omega_m(t+t')}\delta(t - t'),$$

(3)

where $N = \sinh^2(r), M = \sinh(r) \cosh(r) e^{r^2}, r$ is the squeezing parameter of the squeezed vacuum light, and $\varphi$ is the phase of the squeezed vacuum light. For simplicity, we choose $\varphi = 0$. The force $\xi$ is the thermal Langevin force resulting from the coupling of the movable mirror to the environment, whose mean value is zero, and it has the following correlation function at temperature $T$ [36]:

$$\langle \xi(t)\xi(t') \rangle = \frac{\gamma_m}{\pi\omega_m} \int \int e^{-\omega(t-t')} \left[ 1 + \coth\left( \frac{\hbar \omega}{2k_B T} \right) \right] d\omega,$$  

(4)

where $k_B$ is the Boltzmann constant and $T$ is the tem-
perature of the environment. By using standard methods [37], setting all the time derivatives in Eq. (2) to zero, and solving it, we obtain the steady-state mean values

$$P_s = 0, Q_s = \frac{2g|c_s|^2}{\omega_m}, c_s = \frac{\varepsilon}{\kappa + i\Delta},$$  

(5)

where

$$\Delta = \omega_c - \omega_L - gQ_s = \Delta_0 - gQ_s = \Delta_0 - \frac{2g^2|c_s|^2}{\omega_m}$$  

(6)

is the effective cavity detuning, depending on $Q_s$. The $Q_s$ denotes the new equilibrium position of the movable mirror relative to that without the driving field. Further $c_s$ represents the steady-state amplitude of the cavity field. From Eq. (5) and Eq. (6), we can see $Q_s$ satisfies a third order equation. For a given detuning $\Delta_0$, $Q_s$ will at most have three real values. Therefore $Q_s$ and $c_s$ display an optical multistable behavior [38, 39, 40], which is a non-
linear effect induced by the radiation-pressure coupling of the movable mirror to the cavity field.

III. RADIATION PRESSURE AND QUANTUM FLUCTUATIONS

To study squeezing of the movable mirror, we need to calculate the fluctuations in the mirror’s amplitude. Assuming that the nonlinear coupling between the cavity field and the movable mirror is weak, the fluctuation of each operator is much smaller than the corresponding steady-state mean value, thus we can linearize the system around the steady state. Writing each operator of the form as the sum of its steady-state mean value and a small fluctuation with zero mean value,

$$Q = Q_s + \delta Q, \quad P = P_s + \delta P, \quad c = c_s + \delta c.$$  

(7)
Inserting Eq. (7) into Eq. (2), then assuming $|c_s| \gg 1$, the linearized quantum Langevin equations for the fluctuation operators can be expressed as follows,

$$
\delta \dot{Q} = \omega_m \delta P,
$$

$$
\delta \dot{P} = 2g(c_s^\dagger \delta c + c_s \delta c^\dagger) - \omega_m \delta Q - \gamma_m \delta P + \xi,
$$

$$
\delta \dot{c} = - (\kappa + i\Delta) \delta c + igc_s \delta Q + \sqrt{2\kappa} \delta c_m,
$$

$$
\delta \dot{c}^\dagger = - (\kappa - i\Delta) \delta c^\dagger - ig^* c_s \delta Q + \sqrt{2\kappa} \delta c^\dagger_m.
$$

Introducing the cavity field quadratures $\delta x = \delta c + \delta c^\dagger$ and $\delta y = i(\delta c^\dagger - \delta c)$, and the input noise quadratures $\delta x_{in} = \delta c_{in} + \delta c_{in}^\dagger$ and $\delta y_{in} = i(\delta c_{in}^\dagger - \delta c_{in})$, Eq. (8) can be rewritten in the matrix form

$$
\dot{f}(t) = Af(t) + \eta(t),
$$

in which $f(t)$ is the column vector of the fluctuations, $\eta(t)$ is the column vector of the noise sources. Their transposes are

$$
f(t)^T = (\delta Q, \delta P, \delta x, \delta y),
$$

$$
\eta(t)^T = (0, \xi, \sqrt{2\kappa} \delta x_{in}, \sqrt{2\kappa} \delta y_{in});
$$

and the matrix $A$ is given by

$$
A = \begin{pmatrix}
0 & \omega_m & 0 & 0 \\
-\omega_m & -\gamma_m & g(c_s + c_s^\dagger) & -ig(c_s - c_s^\dagger) \\
ig(c_s - c_s^\dagger) & 0 & -\kappa & \Delta \\
g(c_s + c_s^\dagger) & 0 & -\Delta & -\kappa
\end{pmatrix}.
$$

(11)

The system is stable only if the real parts of all the eigenvalues of the matrix $A$ are negative. The stability conditions for the system can be derived by applying the Routh-Hurwitz criterion. For example, we get

$$
\kappa \gamma_m \left[(\kappa^2 + \Delta^2)^2 + (2\kappa \gamma_m - 2\omega_m^2)(\kappa^2 + \Delta^2) + 4\omega_m^2 \left(4 \kappa^2 + \omega_m^2 + 2\kappa \gamma_m \right) \right] + 2 \omega_m \Delta g^2 \left|c_s\right|^2 > 0,
$$

(12)

$$
\omega_m \left[(\kappa^2 + \Delta^2) - 4 \Delta g^2 \left|c_s\right|^2 \right] > 0.
$$

All the external parameters chosen in this paper satisfy the stability conditions to ensure the system to be stable.

Fourier transforming each operator in Eq. (8) and solving it in the frequency domain, the position fluctuations of the movable mirror are given by

$$
\langle \delta Q(t)^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega d\Omega e^{-i(\omega + \Omega)t} \langle \delta Q(\omega) \delta Q(\Omega) \rangle,
$$

$$
\langle \delta P(t)^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega d\Omega e^{-i(\omega + \Omega)t} \langle \delta P(\omega) \delta P(\Omega) \rangle.
$$

(14)

To calculate the mean square fluctuations, we require the correlation functions of the noise sources in the frequency domain,

$$
\langle \delta c_{in}^\dagger(\omega) \delta c_{in}(\Omega) \rangle = 2\pi N \delta(\omega + \Omega),
$$

$$
\langle \delta c_{in}(\omega) \delta c_{in}^\dagger(\Omega) \rangle = 2\pi (N + 1) \delta(\omega + \Omega),
$$

$$
\langle \delta c_{in}(\omega) \delta c_{in}(\Omega) \rangle = 2\pi M \delta(\omega + \Omega - 2\omega_m),
$$

$$
\langle \delta c_{in}^\dagger(\omega) \delta c_{in}^\dagger(\Omega) \rangle = 2\pi M^* \delta(\omega + \Omega + 2\omega_m),
$$

$$
\langle \xi(\omega) \xi(\Omega) \rangle = 4\pi \gamma_m \omega_m \left[1 + \coth(\frac{\hbar \omega_m}{2kT})\right] \delta(\omega + \Omega).
$$

(15)

Combining Eqs. (13) - (15), after some calculations, the mean square fluctuations of Eq. (14) are written as

$$
\langle \delta Q(t)^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \omega_m^2 (A + Be^{-2\omega_m t} + Ce^{2\omega_m t}) d\omega,
$$

$$
\langle \delta P(t)^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \omega_m^2 A + \omega(\omega - 2\omega_m) Be^{-2\omega_m t}
$$

$$
+\omega(\omega + 2\omega_m) Ce^{2\omega_m t} d\omega.
$$

(16)

where

$$
A = \frac{8\kappa g^2 M}{d(\omega)(2\omega_m - \omega)} \left(N(1)[\kappa^2 + (\Delta + \omega)^2] + 2\gamma_m \omega_m^2 \Delta g^2 \left[\kappa^2 + (\Delta + \omega)^2\right] + 4\kappa^2 \omega_m^2 \left[1 + \coth(\frac{\hbar \omega_m}{2kT})\right]\right),
$$

$$
B = \frac{8\kappa g^2 \gamma_m M}{d(\omega)(2\omega_m - \omega)} \left[-i(\Delta + \omega)[\kappa - i(\Delta + 2\omega_m - \omega)]\right],
$$

$$
C = \frac{8\kappa g^2 \gamma_m M}{d(\omega)(2\omega_m - \omega)} \left[i(\Delta - \omega)[\kappa - i(\Delta + 2\omega_m + \omega)]\right].
$$

(17)

In Eqs. (16) and (17), the term independent of $g$ is from the thermal noise contribution; while those terms involving $g$ arise from the radiation pressure contribution, including the influence of the squeezed vacuum light. Moreover, either $\langle \delta Q(t)^2 \rangle$ or $\langle \delta P(t)^2 \rangle$ contains three terms, the first term is independent of time, but the second and third terms are time-dependent, which causes $\langle \delta Q(t)^2 \rangle$
and $\langle \delta P(t)^2 \rangle$ vary with time. The complex exponential in Eq. (16) can be removed by working in the interaction picture. Let’s define $b$ ($b^\dagger$) and $\tilde{b}$ ($\tilde{b}^\dagger$) be the annihilation (creation) operators for the oscillator in the Schrödinger and interaction picture with $[b, b^\dagger] = 1$ and $[\tilde{b}, \tilde{b}^\dagger] = 1$. The relations between them are $b = \tilde{b}e^{-i\omega_m t}$ and $b^\dagger = \tilde{b}^\dagger e^{i\omega_m t}$. Then using $Q = b + b^\dagger$, $P = i(b^\dagger - b)$, $\tilde{Q} = \tilde{b} + \tilde{b}^\dagger$, and $\tilde{P} = i(\tilde{b}^\dagger - \tilde{b})$, we get

$$\langle \delta \tilde{Q}^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \omega_m^2 (A + B + C) d\omega,$$

$$\langle \delta \tilde{P}^2 \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \omega^2 A + \omega(\omega - 2\omega_m)B + \omega(\omega + 2\omega_m)C d\omega.$$  \hspace{1cm} (18)

According to the Heisenberg uncertainty principle,

$$\langle \delta \tilde{Q}^2 \rangle \langle \delta \tilde{P}^2 \rangle \geq \frac{1}{2} |\langle \tilde{Q}, \tilde{P} \rangle|^2.$$  \hspace{1cm} (19)

If either $\langle \delta \tilde{Q}^2 \rangle < 1$ or $\langle \delta \tilde{P}^2 \rangle < 1$, the movable mirror is said to be squeezed.

From Eqs. (17) and (18), we find $\langle \delta \tilde{Q}^2 \rangle$ or $\langle \delta \tilde{P}^2 \rangle$ is determined by the detuning $\Delta_0$, the squeezing parameter $r$, the laser power $\varphi$, the cavity length $L$, the temperature of the environment $T$, and so on. Here we focus on the dependence of $\langle \delta \tilde{Q}^2 \rangle$ and $\langle \delta \tilde{P}^2 \rangle$ on the squeezing parameter, the temperature of the environment, and the laser power.

**IV. SQUEEZING OF THE MOVABLE MIRROR**

In this section, we numerically evaluate the mean square fluctuations in position and momentum of the movable mirror given by Eq. (18) to show squeezing of the movable mirror produced by feeding the squeezed vacuum light at the input mirror. We use the same parameters as those in the recent successful experiment on normal mode splitting in a nanomechanical oscillator: the wave length of the laser $\lambda = \frac{2\pi}{\omega_c} = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215 \times 10^3$ Hz, $\omega_m = 2\pi \times 947 \times 10^3$ Hz, the mechanical quality factor $Q' = \frac{\omega_c}{\gamma_m} = 6700$. In the case of $k_BT \gg \hbar\omega_m$, we may approximate coth($\hbar\omega/(2k_BT) \approx 2k_BT/\hbar\omega$). In the case of $T = 0$ K, if $\omega < 0$, coth($\hbar\omega/(2k_BT) \approx -1$, if $\omega > 0$, coth($\hbar\omega/(2k_BT) \approx 1$. Through numerical calculations, it is found that squeezing of $\langle \delta \tilde{Q}^2 \rangle$ doesn’t exist but squeezing of $\langle \delta \tilde{P}^2 \rangle$ exists. In the following we therefore concentrate on $\langle \delta \tilde{P}^2 \rangle$.

Note that in the absence of the coupling to the cavity field, the movable mirror is in free space, and is coupled to the environment. Then the fluctuations are given by

$$\langle \delta \tilde{Q}^2 \rangle = \langle \delta \tilde{P}^2 \rangle = 1 + \frac{2}{e^{h\omega_m/(k_BT)} - 1}$$

$$= \left\{ \begin{array}{ll}
1 & \text{for } T = 0 \text{ K}, \\
44 & \text{for } T = 1 \text{ mK}, \\
440 & \text{for } T = 10 \text{ mK}.
\end{array} \right.$$

As well known no squeezing of the movable mirror occurs.

Now we consider fluctuations in the presence of the coupling to the cavity field. If we choose $T = 1$ mK, and $\varphi = 6.9$ mW, the mean square fluctuations $\langle \delta \tilde{P}^2 \rangle$ are plotted as a function of the detuning $\Delta_0$ in the Fig. 2. Different graphs correspond to different values of the squeezing of the input light. In the case of no injection of the squeezed vacuum light ($r = 0$), which means that the squeezed vacuum light is replaced by an ordinary vacuum light, we find $\langle \delta \tilde{P}^2 \rangle$ is always larger than unity (the coherent level), the minimum value of $\langle \delta \tilde{P}^2 \rangle$ is 1.071, thus there is no momentum squeezing of the movable mirror. However, if we inject the squeezed vacuum light, it is seen that the momentum squeezing of the movable mirror occurs, and the maximum squeezing happens at about $r = 1$, the corresponding minimum value of $\langle \delta \tilde{P}^2 \rangle$ is 0.319, thus the maximum amount of squeezing is about 68%. So the injection of the squeezed vacuum light greatly reduces the fluctuations in momentum, because using the squeezed vacuum light increases the photon number in the cavity, which results in a stronger radiation pressure acting on the movable mirror. Note that the minimum value of $\langle \delta \tilde{P}^2 \rangle$ in the presence of the coupling to the cavity field is much less than that ($\langle \delta \tilde{P}^2 \rangle = 44$) in the absence of the coupling to the cavity field. So there is very large squeezing with respect to thermal fluctuations. The momentum fluctuations can be reduced by a factor more than one hundred.

![Fig. 2](image-url) (Color online) The mean square fluctuations $\langle \delta \tilde{P}^2 \rangle$ versus the detuning $\Delta_0$ ($10^6$ s$^{-1}$) for different values of the squeezing of the input field. $r = 0$ (red, big dashed line), $r = 0.5$ (green, small dashed line), $r = 1$ (black, solid curve), $r = 1.5$ (blue, dotdashed curve), $r = 2$ (brown, solid curve). The minimum values of $\langle \delta \tilde{P}^2 \rangle$ are 1.071 ($r = 0$), 0.467 ($r = 0.5$), 0.319 ($r = 1$), 0.468 ($r = 1.5$), 1.078 ($r = 2$). The flat dotted line represents the variance of the coherent light ($\langle \delta \tilde{P}^2 \rangle = 1$). Parameters: the temperature of the environment $T = 1$ mK, the laser power $\varphi = 6.9$ mW.

Then we fix the squeezing parameter $r = 1$, the mean square fluctuations $\langle \delta \tilde{P}^2 \rangle$ as a function of the detuning $\Delta_0$ for different temperature of the environment and laser power are shown in Figs. 3–5. For a given laser power, we find that the minimum value of $\langle \delta \tilde{P}^2 \rangle$ decreases with decrease of the temperature of the environment as ex-
FIG. 3: (Color online) The mean square fluctuations \(\langle \delta \hat{P}^2 \rangle\) versus the detuning \(\Delta_0\) \((10^6 \text{ s}^{-1})\), each curve corresponds to a different temperature of the environment. \(T=0\) K (blue, solid curve), 1 mK (red, small dashed curve), 5 mK (brown, big dashed curve), 10 mK (green, dotdashed curve). The minimum values of \(\langle \delta \hat{P}^2 \rangle\) are 0.252 \((T=0\) K\), 0.611 \((T=1\) mK\), 2.082 \((T=5\) mK\), 3.919 \((T=10\) mK\). The flat dotted line represents the variance of the coherent light \(\langle \delta \hat{P}^2 \rangle \) =1\). Parameters: the squeezing parameter \(r = 1\), the laser power \(\varphi = 0.6\) mW.

FIG. 4: (Color online) The mean square fluctuations \(\langle \delta \hat{P}^2 \rangle\) versus the detuning \(\Delta_0\) \((10^6 \text{ s}^{-1})\), each curve corresponds to a different temperature of the environment. \(T=0\) K (solid curve), 1 mK (dashed curve), 10 mK (dotdashed curve). The minimum values of \(\langle \delta \hat{P}^2 \rangle\) are 0.261 \((T=0\) K\), 0.330 \((T=1\) mK\), 0.611 \((T=1\) mK\), 0.731 \((T=10\) mK\). The flat dotted line represents the variance of the coherent light \(\langle \delta \hat{P}^2 \rangle \) =1\). Parameters: the squeezing parameter \(r = 1\), the laser power \(\varphi = 6.9\) mW.

FIG. 5: (Color online) The mean square fluctuations \(\langle \delta \hat{P}^2 \rangle\) versus the detuning \(\Delta_0\) \((10^6 \text{ s}^{-1})\), each curve corresponds to a different temperature of the environment. \(T=0\) K (solid curve), 1 mK (dashed curve), 10 mK (dotdashed curve). The minimum values of \(\langle \delta \hat{P}^2 \rangle\) are 0.275 \((T=0\) K\), 0.319 \((T=1\) mK\), 0.731 \((T=10\) mK\). The flat dotted line represents the variance of the coherent light \(\langle \delta \hat{P}^2 \rangle \) =1\). Parameters: the squeezing parameter \(r = 1\), the laser power \(\varphi = 0.6\) mW.

FIG. 4: (Color online) The mean square fluctuations \(\langle \delta \hat{P}^2 \rangle\) versus the detuning \(\Delta_0\) \((10^6 \text{ s}^{-1})\), each curve corresponds to a different temperature of the environment. \(T=0\) K (solid curve), 1 mK (dashed curve), 10 mK (dotdashed curve). The minimum values of \(\langle \delta \hat{P}^2 \rangle\) are 0.252 \((T=0\) K\), 0.330 \((T=1\) mK\), 0.611 \((T=5\) mK\), 3.919 \((T=10\) mK\). The flat dotted line represents the variance of the coherent light \(\langle \delta \hat{P}^2 \rangle \) =1\). Parameters: the squeezing parameter \(r = 1\), the laser power \(\varphi = 3.8\) mW.

V. CONCLUSIONS

In conclusion, we have found that squeezing of the movable mirror can be achieved by the injection of squeezed vacuum light and a laser. The result shows the maximum momentum squeezing of the movable mirror happens if squeezed vacuum light with \(r \approx 1\) is injected into the cavity. For a given squeezing parameter and laser power, decreasing the temperature of the environment can enhance the maximum momentum squeezing of the movable mirror. In addition, the momentum squeezing of the movable mirror may be achieved by increasing the input laser power. Generation of squeezing of the movable mirror provides a new way to detect a weak force. Further the “feeding” of squeezed light can be used to squeeze collective degrees of freedom for several mirrors inside the cavity.

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