Skyrmion Dynamics and Topological Sorting on Periodic Obstacle Arrays

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Abstract. We examine skyrmions under a dc drive interacting with a square array of obstacles for varied obstacle size and damping. When the drive is applied in a fixed direction, we find that the skyrmions are initially guided in the drive direction but also move transverse to the drive due to the Magnus force. The skyrmion Hall angle, which indicates the difference between the skyrmion direction of motion and the drive direction, increases with drive in a series of quantized steps as a result of the locking of the skyrmion motion to specific symmetry directions of the obstacle array. On these steps, the skyrmions collide with an integer number of obstacles to create a periodic motion. The transitions between the different locking steps are associated with jumps or dips in the velocity-force curves. In some regimes, the skyrmion Hall angle is actually higher than the intrinsic skyrmion Hall angle that would appear in the absence of obstacles. In the limit of zero damping, the skyrmion Hall angle is $90^\circ$, and we find that it decreases as the damping increases. For multiple interacting skyrmion species in the collective regime, we find jammed behavior at low drives where the different skyrmion species are strongly coupled and move in the same direction. As the drive increases, the species decouple and each can lock to a different symmetry direction of the obstacle lattice, making it possible to perform topological sorting in analogy to the particle sorting methods used to fractionate different species of colloidal particles moving over two-dimensional obstacle arrays.
1. Introduction

For a particle moving over a two dimensional periodic array of scattering sites, the type of motion that occurs depends on the direction of drive relative to the symmetry directions of the array. For certain drive angles, the particle can follow a straight trajectory without encountering any obstacles, while for other angles, the particle collides unavoidably with the obstacles. Such systems can exhibit a directional locking effect in which particles preferentially channel along certain directions. In a square lattice of obstacles, these angles include $\phi = 0^\circ$, $45^\circ$ and $90^\circ$, and they are more generally described by the relation $\phi = \arctan(n/m)$ where $n$ and $m$ are integers $[1, 2, 3, 4]$. Dynamical directional locking effects have been studied for vortices interacting with square and triangular pinning arrays $[1, 5, 6]$, electrons moving through antidot arrays $[2, 7]$ and colloidal assemblies moving on two dimensional periodic substrates $[3, 4, 8, 9, 10, 11, 12, 13, 14]$. In these systems, as the angle of the drive is varied with respect to the symmetry direction of the substrate, the velocity vector or velocity-force curves show a series of steps corresponding to drive angle intervals over which the direction of the motion of the particles remains locked to the substrate instead of following the drive direction. The steps are similar to the phase locking phenomenon studied in systems of particles moving in a fixed direction over a periodic substrate under combined ac and dc driving. When phase locking occurs, the frequency of the oscillations generated by the motion of the particle over the periodic substrate locks or comes into resonance with the ac drive frequency and its higher harmonics for a fixed range of drive intervals, producing steps in the velocity-force curves of the type found in Josephson junctions (which are known as Shapiro steps) $[15, 16]$, incommensurate sliding charge density waves $[17]$, driven Frenkel-Kontorova systems $[18]$, vortices in type-II superconductors moving over a periodic pinning substrate $[19, 20, 21]$, and colloidal particles driven over periodic substrates $[22, 23]$. In the case of the directional locking, there is no ac driving; however, two frequencies are still present, where one is associated with motion in the direction parallel to the drive and the other is associated with motion in the direction perpendicular to the drive. Directional locking can also arise in a system with a quasiperiodic substrate, where there are five or seven symmetry locking directions $[24, 25]$. The locking can be harnessed for applications such as the sorting of different species of particles which have different sizes, charges, or damping, where a spatial separation of the species is achieved over time when one species locks to one angle while the other species locks to a different angle $[26, 27, 28, 29, 30, 31, 32]$. Up to this point, directional locking effects have been studied solely in overdamped systems where the transitions between directionally locked states can only occur when the direction of the drive with respect to the substrate symmetry is changed.

Skyrmions in chiral magnets are another type of particle-like system with distinctive properties $[33, 34, 35, 36]$. Magnetic skyrmions have been found in a wide variety of materials with skyrmion sizes ranging from a micron down to 10 nm, and in a number of materials, the skyrmions are stable at room temperature $[37, 38]$. Skyrmions are readily set into motion via the application of an external current $[39, 40, 41, 42]$, and the resulting velocity-force relations show a pinned to sliding transition that can be observed in transport experiments by measuring changes in the topological Hall effect $[39, 43]$ or performing direct imaging of the skyrmion motion $[37, 40, 44]$. It is also possible to examine skyrmion dynamics using neutron scattering $[45]$, X-ray diffraction $[46]$, and changes in the noise fluctuations as a function of drive $[47, 48]$.
Due to their stability, size scale, and manipulability, skyrmions are very promising candidates for a variety of applications including memory, logic devices, and alternative computing architectures [49, 50]. The capability to precisely control the direction, traversal distance, and reversibility of skyrmion motion could open up new ways to create such devices, and there are already a number of proposals for controlling skyrmion motion using structured substrates such as race tracks [49, 51, 52], periodic modulations [53], or specially designed pinning structures [54, 55, 56, 57, 58]. One proposal for controlling skyrmion motion involves having the skyrmions interact with a two dimensional periodic substrate of the type that has already been realized for colloidal particles and vortices in type-II superconductors, and there are existing experimental realizations of skyrmions interacting with two-dimensional (2D) antidot arrays [59].

A key feature that distinguishes skyrmions from colloids and superconducting vortices is the strong non-dissipative Magnus component in the skyrmion dynamics caused by topology [34, 36, 60]. This affects both how the skyrmions move under a drive and how they interact with a substrate potential or pinning. In the absence of a substrate, the skyrmion moves at an angle with respect to the drive which is known as the intrinsic skyrmion Hall angle $\theta_{sk}^{int}$ [34, 36]. The magnitude of this angle increases as the ratio of the Magnus force to the damping term increases. In principle, skyrmion Hall angles of close to $\theta_{sk} = 90^\circ$ are possible in certain systems. Experimental measurements of $\theta_{sk}$ give values ranging from just a few degrees up to $\theta_{sk} = 55^\circ$ [61, 62, 63, 64, 65]. These experiments are generally performed on larger skyrmions that can be observed directly, but much larger skyrmion Hall angles could be present in systems with smaller skyrmions. In race track devices, the skyrmion Hall angle can limit the distance the skyrmion can move, so there is currently considerable interest in identifying ways to reduce or control the skyrmion Hall angle [49]. The skyrmion Hall angle is known to have a strong drive dependence in the presence of pinning. At low drives, the skyrmion Hall angle is small or zero, but $\theta_{sk}$ increases with increasing drive before saturating at a value close to the pin-free limiting value for high drives [61, 62, 63, 64, 65, 66, 67, 68, 69, 70]. This drive dependence can arise due to a side jump or swirling motion of the skyrmions that occurs when they interact with pinning sites, where faster moving skyrmions undergo a smaller side jump [71, 72]. Most work on skyrmions and pinning has focused on systems with randomly placed pinning; however, nanostructuring techniques provide a wide variety of ways to create periodic substrates that could act as attractive or repulsive scattering sites for skyrmions [56, 57]. Using a particle-based model for individual skyrmions in the presence of a 2D pinning array, Reichhardt et al. demonstrated that the skyrmion Hall angle is initially small and increases with increasing driving force, but that this increase takes the form of a series of jumps, giving a quantization of the skyrmion Hall angle that is absent for random pinning [72]. This is similar to the symmetry locking found in superconducting vortex [11, 16] and colloidal systems [8, 9, 10] under a rotating external drive; however, in the skyrmion system, the drive direction remains fixed and it is the velocity dependence of the skyrmion Hall angle which generates the change in the skyrmion flow direction. Individual skyrmions interacting with a periodic array of antidots have also been studied numerically using micromagnetic and Theile equation approaches, where it was shown that the skyrmion motion can be carefully controlled with a drive and can lock to certain directions, and that there are localized states in which the skyrmion undergoes circular motion in the interstitial regions between antidots or moves around the edge of an antidot [73].
In this work we examine skyrmions interacting with a two dimensional periodic array of obstacles or antidots while being driven with a force that is applied in a fixed direction. We find directional symmetry locking of the skyrmion motion similar to that reported in previous work [72]; however, here we also examine the effects of changing the obstacle size. In some regimes we find that the skyrmion motion locks parallel to the driving direction, and in all cases the locking phases are associated with both a velocity locking of the skyrmion and with a quantized skyrmion Hall angle. We also consider dc driving of a mixture of skyrmion species with different Magnus forces through the same periodic obstacle array. In this case, the flow is more complex, the steps in the velocity-force curves become less distinct, and we observe directional locking effects in which one species locks a symmetry direction while the other does not. This effect can be harnessed in order to achieve the topological sorting of different skyrmion species in analogy to the sorting of different species of particles or colloids moving over periodic arrays in overdamped systems [26, 27, 28, 29, 30, 31]. For increasing skyrmion density, the sorting effect is slightly reduced but remains robust up to very high skyrmion density, where a jammed phase appears in which both species move together either in the driving direction or along a symmetry direction of the substrate array.
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Figure 2. (a) $\langle V\parallel \rangle$ (black) and $\langle V\perp \rangle$ (red) vs $F^D$ for a system containing a single skyrmion where $a_0 = 0.65$ and $\alpha_m/\alpha_d = 0.45$. When $F^D < 1.0$, the motion is strictly along the x-direction, parallel to the drive. (b) The corresponding skyrmion Hall angle $\theta_{sk} = \arctan(R)$ vs $F^D$, where $R = \langle V\perp \rangle/\langle V\parallel \rangle$.

2. Simulation

In this work we consider a two-dimensional skyrmion system of size $L \times L$ with periodic boundary conditions in the $x$ and $y$ directions. We initially focus on a single skyrmion moving through a square obstacle array, and later we introduce multiple interacting skyrmions of different types. The total number of skyrmions is $N = N_a + N_b$, where $N_a$ ($N_b$) is the number of skyrmions of species $a$ ($b$). We set $N_a = N_b = N/2$ except when $N = 1$. The different skyrmion species represent skyrmions with different sizes that can coexist in a sample. The total skyrmion density is $n_s = N/L^2$.

The dynamics of skyrmion $i$ is obtained using a particle based skyrmion model [74], with the following equation of motion:

$$\alpha^\gamma_d v_i + \alpha^\gamma_m \hat{z} \times v_i = F^C_i = F_s^i + F_o^i + F^D. \quad (1)$$

The first term on the left is the damping term for species $\gamma$, where $\gamma = (a, b)$ and the damping constant is $\alpha^\gamma_d$. The damping originates from the spin precession and dissipation of electrons localized in the skyrmion core. The second term on the left represents the Magnus force, where $\alpha^\gamma_m$ is the Magnus term for species $\gamma$. The Magnus force is oriented perpendicular to the skyrmion velocity. The repulsive skyrmion-skyrmion interaction force is given by $F_s^i = \sum_{j}^{N} K_1 (r_{ij}/\xi) \hat{r}_{ij}$. Here, $\xi$ is the screening length which we take to be 1.0 in dimensionless units, $r_{ij} = |r_i - r_j|$ is the distance between skyrmions $i$ and $j$, and $\hat{r}_{ij} = (r_i - r_j)/r_{ij}$. In order to enhance computational efficiency, we cut off the skyrmion-skyrmion interaction beyond $r_{ij} = 6.0$ where its magnitude becomes negligible. We set the sample size to $L = 36\xi$. We model the potential energy of the skyrmion-obstacle interaction using the Gaussian form.
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Figure 3. (a-d) The obstacles (open circles) and the skyrmion trajectory (lines) for the system in Fig. 2 with $a_0 = 0.65$ and $\alpha_m/\alpha_d = 0.45$. (a) $F^D = 0.5$, where $R = 0$. (b) $F^D = 1.25$, where $R = -1/3$ and the system is on a locking step. (c) $F^D = 2.0$, where $R = -0.5$. (d) Obstacles and skyrmion trajectories for the system in Fig. 4 with $a_0 = 0.15$ and $\alpha_m/\alpha_d = 0.45$ at $F^D = 0.5$, where $R = -0.45$. (e-f) The same for the system in Fig. 4 with $a_0 = 0.2$ and $\alpha_m/\alpha_d = 0.45$. (e) $F^D = 0.5$, where $R = -0.5$. (f) $F^D = 2.0$, where $R = -0.45$.

$U_o = C_o e^{-(r_{io}/a_o)^2}$, where $C_o$ is the strength of the obstacle potential. The skyrmion-obstacle interaction is thus given by $F_i^o = -\nabla U_o = -F_o r_{io} e^{-(r_{io}/a_o)^2} \hat{r}_{io}$, where $F_o = 2U_o/a_o^2$. $r_{io}$ is the distance between skyrmion $i$ and obstacle $o$, and $a_o$ is the obstacle radius. We cut off this interaction beyond $r_{io} = 2.0$ where the force becomes negligible. We consider an obstacle density of $\rho_o = 0.093/\xi^2$. The skyrmion-external current interaction is given by $\mathbf{F}^D = F^D \hat{d}$, where $\hat{d}$ is the direction of the driving force, which is fixed in the $x$ direction, $\hat{d} = \hat{x}$.

In Fig. 1, we show a schematic of our system highlighting the obstacle array and a skyrmion moving through the array. We measure the skyrmion velocity parallel, $\langle V_\parallel \rangle$, ...
and perpendicular, \(\langle V_\perp \rangle\), to the drive. When the skyrmion is flowing in the absence of obstacles, in the overdamped limit of \(\alpha_m/\alpha_d = 0\) it moves only in the direction of the drive. If \(\alpha_m/\alpha_d\) is finite, the skyrmion instead moves with a Hall angle of \(\theta_{sk} = \arctan(\langle V_\perp \rangle/\langle V_\parallel \rangle)\). In order to quantify the direction of the skyrmion motion we measure \(R = \langle V_\perp \rangle/\langle V_\parallel \rangle\), where the skyrmion Hall angle is given by \(\theta_{sk} = \arctan(R)\). We increase the drive in small steps of \(\delta F = 0.001\) and we wait 10^5 simulation time steps between increments to ensure that the system has reached a steady state. Unless otherwise noted, we normalize the damping and Magnus coefficients such that \((\alpha_\gamma)^2_d + (\alpha_\gamma)^2_m = 1\).

3. Symmetry Locking and Quantized Hall Angle

3.1. Changing obstacle size

We first consider the case of a single skyrmion moving through an obstacle array for varied obstacle size and driving force. In Fig. 2(a) we plot \(\langle V_\parallel \rangle\) and \(\langle V_\perp \rangle\) versus \(F_D\) for a system with fixed \(a_0 = 0.65\) and \(\alpha_m/\alpha_d = 0.45\), while Figure 2(b) shows the corresponding skyrmion Hall angle \(\theta_{sk} = \arctan(R)\). If there are no obstacles, the skyrmions move at an intrinsic angle of \(\theta_{sk}^{int} = 24.23^\circ\). We find that when \(F_D < 1.0\), \(\langle V_\perp \rangle = 0\) and \(\langle V_\parallel \rangle \) increases with increasing \(F_D\), so that the skyrmion Hall angle is \(\theta_{sk} = 0\). As shown in Fig. 3(a), at \(F_D = 0.5\) the skyrmion exhibits an oscillatory motion but translates only in the \(x\)-direction. As \(F_D\) increases, the skyrmion starts to move in both the \(+x\) and \(−y\) directions. At the transition to finite \(\langle V_\perp \rangle\), there is a drop in \(\langle V_\parallel \rangle\), indicating that the particle is actually slowing down in the \(x\)-direction as a function of increasing \(F_D\), indicating the appearance of negative differential conductivity with \(d\langle V_\parallel \rangle/dF_D < 0.0\). For higher \(F_D\), several additional dips or cusps appear in both \(\langle V_\parallel \rangle\) and \(\langle V_\perp \rangle\). As shown in Fig. 2(b), the skyrmion Hall angle is quantized, with steps at \(R = −0.33\) (\(\theta_{sk} = −18.43^\circ\)) and \(R = −0.25\) (\(\theta_{sk} = −14.03^\circ\)).
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Figure 5. The locking step force interval \( \Delta F_D \) vs \( a_0 \) at \( \alpha_m/\alpha_d = 0.45 \) for the \( R = 0.0 \) (black squares), -0.25 (red circles), -0.33 (blue up triangles), and -0.5 (green down triangles) locking steps.

corresponding to orbits in which the skyrmion moves one lattice constant in the \(-y\)-direction and three or four lattice constants in the \(x\)-direction, respectively.

In Fig. 3(b), we illustrate the skyrmion orbit for the \( R = -0.33 \) step at \( F_D = 1.25 \). The orbit winds periodically around the system and the skyrmion undergoes multiple collisions with the obstacles during each complete orbit. As \( F_D \) increases, the magnitude of both \( \langle V_\parallel \rangle \) and \( \langle V_\perp \rangle \) jumps up and the system reaches the \( R = -0.5 \) step. The corresponding orbit appears in Fig. 3(c) for \( F_D = 2.0 \), where the skyrmion moves 2 lattice constants in the \(+x\) direction for every one lattice constant in the \(-y\) direction. Interestingly, this means that the skyrmion is moving at an angle of \( \theta_{sk} = -26.565^\circ \), which is larger than the intrinsic skyrmion Hall angle of \( \theta_{sk} = -24.23^\circ \). The strong locking on the \( R = -0.5 \) step pulls the skyrmion to this higher value of \( \theta_{sk} \) over an interval of \( F_D \) due to the symmetry of the square obstacle lattice. The skyrmion remains locked to the \( R = -0.5 \) state until \( F_D > 2.3 \), after which the skyrmion motion drops to a lower value of \( \theta_{sk} \), which rapidly approaches the intrinsic skyrmion Hall angle value.

In Fig. 4 we plot \( \theta_{sk} \) versus \( F_D \) for a system with \( \alpha_m/\alpha_d = 0.45 \) at varied \( a_0 = 0.15 \) to 1.0. For smaller \( a_0 \), the skyrmion initially moves at an angle close to the intrinsic skyrmion Hall angle of \( \theta_{sk}^{int} = -24.23^\circ \) since it experiences very few collisions with the obstacles. When \( a_0 = 0.2 \), the system starts off in the \( R = -0.5 \) \( (\theta_{sk} = -26.565^\circ) \) state and jumps to the \( R = -0.45 \) \( (\theta_{sk} = 24.23^\circ) \) state at higher drives. For \( a_0 \geq 0.65 \), the system initially locks to the \( R = 0.0 \) \( (\theta_{sk} = 0) \) state. At the largest value of \( a_0 = 1.0 \), we find a series of smaller steps at rational fractional ratios of \( R = -n/m \), and at large drives \( R \) gradually approaches the intrinsic value.

In Fig. 3(d) we plot the skyrmion trajectory for the system in Fig. 4 with \( a_0 = 0.15 \) at \( F_D = 0.5 \), where the motion is locked to the \( R = -0.45 \) state, while in Fig. 3(e) at \( F_D = 0.5 \) and \( a_0 = 0.20 \), the motion is now locked to the \( R = -0.5 \) state, which has a distinct trajectory. Figure 3(f) shows that at \( F_D = 2.0 \) and \( a_0 = 0.2 \), the higher drive \( R = -0.45 \) state has a trajectory that is closer to linear.
respectively, along with numerous higher order steps. When a to remains locked at $R = -1.5, -2.0$, and for higher drives it undergoes two transitions before reaching $R$ possible to observe a larger number of locking phases. In Fig. 6(a) we plot $\theta_{sk}$ locking step appears at the higher order values of a. The widths of the $a$ is large when $R$ of additional steps corresponding to a. For increased $\alpha_m/\alpha_d$, the skyrmion can move at larger Hall angles, making it possible to observe a larger number of locking phases. In Fig. 6(a) we plot $\theta_{sk}$ versus $F_D$ for samples with $\alpha_m/\alpha_d = 1.91$ at $a_0 = 0.15$ to 1.0. We find steps at $R = -1.0, -1.5, -2.0, -1.91$, corresponding to $\theta_{sk} = -45^\circ, -56.3^\circ, -63.43^\circ$, and $-62.37^\circ$, respectively, along with numerous higher order steps. When $a_0 = 0.15$, the skyrmion remains locked at $R = -1.91$, while when $a_0 = 0.25$, the system is initially locked to $R = -2.0$, corresponding to a Hall angle that is higher than the intrinsic $\theta_{int}$, and for higher drives it undergoes two transitions before reaching $R = -1.91$. When $a_0 > 0.64$, the $R = 0$ state appears at low drives, and for $a_0 = 0.85$, there are a number of additional steps corresponding to $R = -1.25, -1.33, -1.67$, which are equivalent to $\theta_{sk} = -51.34^\circ, 53.06^\circ$, and $-59.09^\circ$, respectively. In Fig. 7(a) we plot the skyrmion trajectories for the system in Fig. 6 with $a_0 = 0.65$ at $F_D = 0.04$ in the $R = 0$ state, while in Fig. 7(b) we show the trajectories in the same system for the $R = -1.0$ state at $F_D = 0.5$, where the skyrmion moves at an angle of $\theta_{sk} = -45^\circ$. At $F_D = 1.0$ in the $R = -1.5$ state, illustrated in Fig. 7(c), the skyrmion Hall angle is $\theta_{sk} = 56.31^\circ$ and the skyrmion moves $3a$ in the $y$-direction while translating a distance $2a$ in the $x$-direction. If the pinning site radius is reduced to $a_0 = 0.2$, then for $F_D = 1.0$ the system is in the $R = -1.85$ state, shown in Fig. 7(d). In Fig. 7(b), we plot $\Delta F_D$ vs $a_0$ for the same system highlighting the regions in which the $R = 0, -1, -1.5$, and $-2.0$ states appear. We find that the widths of the $R = -1.0$ and $R = -1.5$ states are maximized at a particular value of $a_0$. 

![Graph](image-url)

**Figure 6.** (a) $\theta_{sk} = \arctan(R)$ vs $F_D$ for varied $a_0$ of 0.15, 0.20, 0.25, 0.30, 0.35, 0.65, 0.85, and 1.0 in samples with $\alpha_m/\alpha_d = 1.91$. Locking steps appear at $R = 0, -1.0, -1.5, -1.91$, and $-2.0$. (b) $\Delta F_D$ vs $a_0$ for the system in (a) for the $R = 0$ (black squares), -1.0 (red circles), -1.5 (blue up triangles), and 2.0 (green down triangles) steps.
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Figure 7. The obstacles (open circles) and the skyrmion trajectory (lines) for the system in Fig. 6 with $\alpha_m/\alpha_d = 1.91$. (a) The $R = 0.0$ state for $a_0 = 0.65$ at $F^D = 0.04$. (b) The $R = -1.0$ state for $a_0 = 0.65$ at $F^D = -1.0$. (c) The $R = -1.5$ state for $a_0 = 0.65$ at $F^D = 1.0$. (d) The $R = -1.85$ state for $a_0 = 0.2$ at $F^D = 1.0$. 

Figure 8. (a) $\theta_{sk} = \arctan(R)$ vs $F^D$ for varied $a_0$ of 0.15, 0.20, 0.25, 0.30, 0.35, 0.65, 0.85, and 1.0 in samples with $\alpha_m/\alpha_d = 4.925$. (b) $\Delta F^D$ vs $a_0$ for the system in (a) for $R = 0.0$ to $R = -4.8$. As $a_0$ increases, a larger number of steps appear.

In Fig. 8(a) we plot $R$ versus $F^D$ for samples with $\alpha_m/\alpha_d = 4.925$ where $a_0$ ranges from $a_0 = 0.15$ to $a_0 = 1.0$. Here we find a larger number of possible locking steps ranging from $R = 0.0$ to $R = -4.8$, with prominent steps appearing at $R = 0.0$, -1, -2, -3, and -4 as well as numerous additional smaller steps such as those at $R = -2.5$ or $R = -3.5$. In Fig. 8(b) we show $\Delta F^D$ versus $a_0$ for the $R = 0$ to $R = -4.8$ steps for the system in Fig. 8, where the number of steps present increases as $a_0$ increases. In Fig. 8(b) we illustrate some of the skyrmion trajectories that appear at $a_0 = 1.0$. Figure 9(a) shows the $R = -1.0$ state where the skyrmion follows a sinusoidal path.
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Figure 9. The obstacles (open circles) and the skyrmion trajectory (lines) for the system in Fig. 8 with $\alpha_m/\alpha_d = 4.925$ at $a_0 = 1.0$. (a) $R = -1.0$. (b) $R = -2.0$. (c) $R = -3.0$. (d) $R = -4.0$.

between the obstacles. In Fig. 9(b) we plot the trajectories for the $R = -2.0$ state, while in Fig. 9(c) we show the $R = -3.0$ state. At $R = -4.0$ in Fig. 9(d), the skyrmion trajectory becomes increasingly tilted.

3.2. Changing Damping Effect

We next consider the effect of changing the damping by fixing $\alpha_m = 1.0$ and varying $\alpha_d$. In Fig. 10(a) we plot the perpendicular $\langle V_\perp \rangle$ and parallel $\langle V_\parallel \rangle$ velocity components versus $F_D$ for a system with $a_0 = 0.65$ at $\alpha_d = 0.0$, 0.2, and 0.4. We observe a series of dips in both velocity components that are associated with the directional locking effects. In Fig. 10(b), where $\alpha_d = 0.6$, 0.8, and 1.0, the $\alpha_d = 0.6$ curves exhibit the most prominent steps. The velocity force curves for $\alpha_d = 1.5$, 2.0, 5.0, and 10.0 in Fig. 11(b) indicate that $\langle V_\parallel \rangle$ becomes more dominant as $\alpha_d$ increases, and we find that at $\alpha_d = 5$ and above, the system remains locked in the $R = 0.0$ state over the entire range of $F_D$.

In Fig. 11(a) we plot $\theta_{sk}$ versus $F_D$ for the system in Fig. 10(a,b) with $\alpha_d = 0.2$, 0.4, 0.6, 0.7, and 1.0. As $\alpha_d$ increases, fewer steps appear in $\theta_{sk}$. The largest number of transitions occur for the lowest damping of $\alpha_d = 0.2$, and in all cases, we find a locking step corresponding to $R = -1.0$. Figure 11(b) shows the $\theta_{sk}$ versus $F_D$ curves for $\alpha_d = 1.5$, 2.0, 5.0, and 10, where steps occur at $R = 0.0$, 0.33, 0.5, and 0.66. For $\alpha_d = 5$ and $\alpha_d = 10$, the system remains locked in the $R = 0.0$ state over the entire range of drives we have investigated.

In Fig. 12 we plot the skyrmion trajectories for the system in Figs. 10 and 11 for $\alpha_d = 0.0$, 2.0, and 5.0 at $F_D = 1.0$. In the case of zero damping with $\alpha_d = 0.0$, the skyrmion moves at $\theta_{sk} = 90^\circ$ with respect the drive. For $\alpha_d = 5.0$ the skyrmion is
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Figure 10. (a) $\langle V_\parallel \rangle$ (curves above zero) and $\langle V_\perp \rangle$ (curves below zero) vs $F^D$ for a system with $\alpha_m = 1.0$, $a_0 = 0.65$, and varied $\alpha_d$. (a) $\alpha_d = 0.0$ (light blue, $\langle V_\parallel \rangle$; red, $\langle V_\perp \rangle$), $\alpha_d = 0.2$ (light green, $\langle V_\parallel \rangle$; dark blue, $\langle V_\perp \rangle$), and $\alpha_d = 0.4$ (dark green, $\langle V_\parallel \rangle$; magenta, $\langle V_\perp \rangle$). (b) $\alpha_d = 0.6$ (red, $\langle V_\parallel \rangle$; light blue, $\langle V_\perp \rangle$); $\alpha_d = 0.8$ (light green, $\langle V_\parallel \rangle$; dark blue, $\langle V_\perp \rangle$); and $\alpha_d = 1.0$ (dark green, $\langle V_\parallel \rangle$; magenta, $\langle V_\perp \rangle$). (c) $\alpha_d = 1.5$ (dark blue, $\langle V_\parallel \rangle$; magenta, $\langle V_\perp \rangle$); $\alpha_d = 2.0$ (dark green, $\langle V_\parallel \rangle$; light blue, $\langle V_\perp \rangle$); $\alpha_d = 5.0$ (cyan, $\langle V_\parallel \rangle$; red, $\langle V_\perp \rangle$); and $\alpha_d = 10.0$ (orange, $\langle V_\parallel \rangle$; light green, $\langle V_\perp \rangle$).

locked to $\theta_{sk} = 0.0^\circ$, while for $\alpha_d = 2.0$ the skyrmion moves at $R = -0.5$ corresponding to $\theta_{sk} = -26.56^\circ$, indicating that increasing the damping reduces the magnitude of the skyrmion Hall angle.

In Fig. 13 we show $\langle V_\parallel \rangle$ and $\langle V_\perp \rangle$ versus $F^D$ for a system with $a_0 = 1.3$ at $\alpha_d = 0.0$, 0.2, and 0.4. For $\alpha_d = 0.0$, $\langle V_\parallel \rangle = 0.0$ for all $F^D$. In this case we observe a finite depinning threshold at $F^D = 0.255$, where $\langle V_\perp \rangle$ first rises above zero. There is also a partial two step depinning process, with the second depinning transition occurring at $F^D = 0.976$. In Fig. 13(b,c) we illustrate the skyrmion trajectory in the system with $\alpha_d = 0.0$. At $F^D = 0.5$, shown in Fig. 13(b), the skyrmion is oscillating between the obstacles, while in Fig. 13(c) at $F^D = 1.0$, above the second step, the skyrmion runs along the edge of a column of obstacles. At $\alpha_d = 0.2$ in Fig. 13(a),
Figure 11. \( \theta_{sk} \) vs \( F_D \) for the system in Fig. 10(a,b) with \( \alpha_m = 1.0 \) and \( a_0 = 0.65 \) at different values of \( \alpha_d \). (a) \( \alpha_d = 0.2 \) (teal), 0.4 (red), 0.6 (blue), 0.8 (magenta), and 1.0 (green). (b) \( \alpha_d = 1.5 \) (black), 2.0 (red), and 5.0 (blue). For \( \alpha_d = 5.0 \) and higher the system remains locked to \( \theta_{sk} = 0^\circ \).

there is still a sharp depinning transition along with a series of jumps in both \( \langle V_{||} \rangle \) and \( \langle V_{\perp} \rangle \). We also find an extended interval of \( F_D \) over which \( \langle V_{||} \rangle \) decreases with increasing \( F_D \), but for \( F_D > 1.0 \), \( \langle V_{||} \rangle \) increases monotonically with increasing \( F_D \). At \( \alpha_d = 0.4 \), there are a larger number of jumps in the transport curves.

In Fig. 14(a) we plot \( \langle V_{||} \rangle \) and \( \langle V_{\perp} \rangle \) versus \( F_D \) for a system with \( \alpha_m = 1.0 \) and \( a_0 = 1.3 \) at \( \alpha_d = 0.6, 0.8, \) and 1.0. There is a finite depinning threshold for motion in both the parallel and perpendicular directions, and the width of the \( R = 0.0 \) step grows with increasing damping. We show \( \langle V_{||} \rangle \) and \( \langle V_{\perp} \rangle \) versus \( F_D \) for \( \alpha_d = 1.5, 2.0, 5.0, \) and 10 in Fig. 14(b), where we find that the depinning transition is still sharp and that the number of steps in the velocity-force curves decreases with increasing damping. In general, we find that for the drive at which \( \langle V_{\perp} \rangle \) becomes finite, the magnitude of \( \langle V_{||} \rangle \) drops. Figure 14(c) shows \( \theta_{sk} \) versus \( F_D \) for the samples in Figs. 14(a,b) where a series of locking steps occur. In the case of \( \alpha_d = 0.2 \), there are steps at \( R = -2.0, -3.0, \) and \(-4.0 \).

Our results indicate that the skyrmion Hall angle can be controlled precisely in systems with periodic substrates. This also implies that it may be possible to create new types of transistor-like devices by exploiting the sharp jumps between the different Hall angles. In this case, certain values of \( R \) or \( \theta_{sk} \) could be used to achieve a specific resistance value. It could also be possible to create periodic arrays with obstacles of different sizes that would allow skyrmions to have one value of \( \theta_{sk} \) in one part of the sample and a different value of \( \theta_{sk} \) in a different part of the sample.
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3.3. Attractive Pinning Sites

We next consider the effect of changing the substrate from obstacles to attractive pinning sites, which we achieve by changing the sign of $C_0$, the strength of the potential. A single skyrmion not subjected to a drive will come to rest inside such a pinning site, and there is a finite depinning threshold or driving force required to set the skyrmion in motion. In Fig. 15(a) we plot $\langle V_\parallel \rangle$ and $\langle V_\perp \rangle$ versus $F_D$ for the same system as in Fig. 2 containing a single skyrmion where $a_0 = 0.65$ and $\alpha_m/\alpha_d = 0.45$, but for attractive pinning sites. Figure 15(c) shows the corresponding skyrmion Hall angle. Unlike the system with obstacles, where the skyrmions can initially slide along the $x$ direction at low drives, for the attractive pins we find a pinned regime in which the skyrmion velocity is zero in both directions. There are still steps in the velocity-force curves and in $\theta_{sk}$ but they are strongly reduced in size compared to the system with obstacles. In Fig. 15(b) we plot $\langle V_\parallel \rangle$ and $\langle V_\perp \rangle$ for the same system but at $\alpha_m/\alpha_d = 1.91$ and $a_0 = 1.0$. Here the depinning threshold is lower, and although steps appear in the velocities and in the skyrmion Hall angle shown in Fig. 15(d), they are strongly reduced in size. This result indicates that the attractive pinning sites produce much weaker directional locking effects than the obstacles.

4. Collective Effects and Topological Sorting

4.1. Sorting

We next consider the effects of multiple interacting skyrmions moving in periodic obstacle arrays for varied species of skyrmions. In Fig. 16(a) we plot $\langle V_\parallel \rangle$ and $\langle V_\perp \rangle$
Figure 13. (a) $\langle V \parallel \rangle$ (curves above zero) and $\langle V \perp \rangle$ (curves below zero) vs $F^D$ for a system with $\alpha_m = 1.0$ at $\alpha_d = 0.0$ (black, $\langle V \parallel \rangle$); red, $\langle V \perp \rangle$); $\alpha_m = 0.2$ (light blue, $\langle V \parallel \rangle$; pink, $\langle V \perp \rangle$); and $\alpha_m = 0.4$ (green, $\langle V \parallel \rangle$; dark blue, $\langle V \perp \rangle$). (b,c) The obstacles (open circles) and the skyrmion trajectory for the $\alpha_d = 0.0$ system at (b) $F^D = 0.5$ and (c) $F^D = 1.0$.

for each species versus $F^D$ for a system with $a_0 = 0.65$ containing 44 skyrmions at a density of $n_s = 0.034$. Half of the skyrmions have $\alpha_m/\alpha_d = 0.45$, which we refer to as species $a$, and the other half have $\alpha_m/\alpha_d = 1.91$, which we refer to as species $b$. Figure 16(b) shows the corresponding $R$ versus $F^D$ curves for the two species. Here each species has a different value of $R$ at all drives. We find that $|R|$ is always lower than the intrinsic value for either species, indicating that the skyrmion-skyrmion interactions cause a reduction in the Hall angle for both species. The pronounced steps associated with locking effects are not present in this system. This is due to the dis ordering effect that occurs when the different skyrmion species each move at different angles, causing collisions among the skyrmions and resulting in the disordered trajectories illustrated in Fig. 17 for $F^D = 0.5$. Here, species $a$ is moving at $R = -1/3$, but when the species $a$ skyrmions collide with the species $b$ skyrmions, the trajectories begin to disorder. As $F^D$ is increased, species $a$ eventually jumps out of the $R = -1/3$ state and moves at a higher angle. For collective effects among monodisperse skyrmions, locking steps similar to those found for the single skyrmion
case occur. The fact that different skyrmion species can lock to different directions of motion indicates that it would be possible to perform skyrmion sorting in systems containing skyrmions with different sizes or different winding numbers, similar to the species fractionation that can be achieved for different species of colloidal particles moving over periodic substrates [10, 11, 13, 27]. The separation will be more difficult in cases where there are only small differences between the skyrmion species, but it should be possible to tune $F^D$ carefully to the edge of a locking step such that one species is in a locked state while the other species is not.
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Figure 15. (a) $\langle V_{||} \rangle$ (black) and $\langle V_{\perp} \rangle$ (red) vs $F^D$ for the system in Fig. 2 with $a_0 = 0.65$ and $\alpha_m/\alpha_d = 0.45$ but for attractive pinning sites. When $F^D < 1.25$, the system is in a pinned state. (b) $\langle V_{||} \rangle$ (black) and $\langle V_{\perp} \rangle$ (red) for an attractive pinning site system with $\alpha_m/\alpha_d = 1.91$ and $a_0 = 1.0$. (c) The skyrmion Hall angle $\theta_{sk}$ vs $F^D$ for the system in panel (a), where $R = \langle V_{\perp} \rangle / \langle V_{||} \rangle$. (d) $\theta_{sk}$ vs $F^D$ for the system in panel (b).

Figure 16. (a) $\langle V_{||} \rangle$ (curves above zero) and $\langle V_{\perp} \rangle$ (curves below zero) for each individual species versus $F^D$ for a system with two different species of skyrmions at a total $n_s = 0.034$. Species $a$ has $\alpha_m/\alpha_d = 0.45$ (black: $\langle V_{||} \rangle$; red: $\langle V_{\perp} \rangle$) and species $b$ has $\alpha_m/\alpha_d = 1.91$ (green: $\langle V_{||} \rangle$; blue: $\langle V_{\perp} \rangle$). (b) The corresponding $R$ vs $F^D$ curves for the two species, $a$ (black) and $b$ (red).
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Figure 17. The obstacles (open circles) and skyrmion trajectories for species $a$ (black lines) and species $b$ (blue lines) for the system in Fig. 16 at $F_D = 0.5$, where skyrmion species $a$ is locked to the $R = -1/3$ state and species $b$ is locked to the $R = -1$ state.

4.2. Jamming Effect

As the density of the two species system increases, the skyrmion-skyrmion interactions become stronger, and low drive regimes emerge in which the two species move while rigidly locked together. In Fig. 18(a) we show $\langle V_{||} \rangle$ and $\langle V_{\perp} \rangle$ for each species versus $F_D$ in a sample with $n_s = 0.085$, and in Fig. 18(b) we plot the corresponding $R$ versus $F_D$ curves. Species $a$ reaches a step with $R = -0.5$, which is lower than the expected intrinsic value of $R = -0.45$, indicating that there is dragging effect from species $b$ on species $a$. At higher drives, species $a$ settles onto a step with $|R| = 0.4$. At the lowest values of $F_D$, $R$ is close to zero when the system forms a jammed state where the skyrmions of both species start to form an elastically moving lattice due to the repulsive skyrmion-skyrmion interactions. In Fig. 19(a) we show the skyrmion trajectories for the system in Fig. 18 at $F_D = 0.01$, where the skyrmions move mostly in the $R = 0.0$ state with some small jumps in the $y$-direction, and where the two species remain locked together. Figure 19(b) shows that at $F_D = 0.5$, species $b$ is locked to $R = -1.0$ and species $a$ is partially locked to $R = -1/3$.

To quantify the efficiency of the separation of the two species, we measure the difference $A_{ab} = \langle V_a \rangle - \langle V_b \rangle$ between the velocity of the two species in both the parallel and perpendicular directions. In Fig. 20 we show that the efficiency $A_{ab}$ of the separation drops with increasing skyrmion density for the system from Figs. 17 and 18. For the perpendicular direction, there is a monotonic decrease in the efficiency with increasing $n_s$ due to the increasing drag effect between the two species that appears as the density increases, while for the parallel direction, the behavior of the efficiency is non-monotonic as a function of $n_s$ due to a partial jamming effect. The jamming effect appears at low drives and can also occur for motion in different directions. In Fig. 21 we plot the width $\Delta F_D^{jam}$ of the jammed phase as a function of $n_s$, where the jammed phase is defined to extend from the depinning threshold to the drive at which
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Figure 18. (a) $\langle V_{||} \rangle$ (curves above zero) and $\langle V_{\perp} \rangle$ (curves below zero) for species a (black: $\langle V_{||} \rangle$; red: $\langle V_{\perp} \rangle$) and b (green: $\langle V_{||} \rangle$; blue: $\langle V_{\perp} \rangle$) in a system with $n_s = 0.085$. (b) The corresponding $R$ vs $F^D$ for species a (black) and b (red).

Figure 19. The obstacles (open circles) and skyrmion trajectories for species a (black lines) and species b (blue lines) for the system in Fig. 18. (a) At $F^D = 0.01$, the system forms a jammed phase close to $R = 0.0$ where all of the skyrmions move together in elastic flow. (b) At $F^D = 0.5$, species b is locked to the $R = -1.0$ state and species a is partially locked to the $R = -0.33$ state.
Figure 20. The area $A_{ab}$ between the velocity curves vs skyrmion density $n_s$ for $\langle V_\parallel \rangle$ (black curve) and $\langle V_\perp \rangle$ (red curve), as a measure of the efficiency of the sorting of the two species. The efficiency drops monotonically with increasing $n_s$ in the perpendicular direction but is non-monotonic for the parallel direction.

Figure 21. $\Delta F_{\text{jam}}^D$, the width of the window in $F^D$ where jamming behavior occurs, vs skyrmion density $n_s$ at the lower drives, highlighting the efficiency of the jammed phases. The vertical lines indicate the direction in which the jammed phase is moving for the $R = 0$, -0.5, and -1.0 states.
the two skyrmion species first begin to move at different velocities. For \( n_s \leq 0.0594 \), there is no jamming since the skyrmion density is low enough that skyrmions can easy pass by each other with minimal interactions. For \( 0.0594 < n_s < 0.0933 \), the initial motion occurs in the \( R = 0.0 \) jammed state, while for \( 0.0933 < n_s < 0.1188 \), the skyrmions form a jammed state that moves in the \( R = -0.5 \) direction. Over the interval \( 0.1188 < n_s < 0.1867 \), the jammed state moves in the \( R = -1.0 \) direction, while for \( n_s > 0.1867 \), another jammed state appears that is locked in the \( R = 0.0 \) direction. For larger values of \( n_s \), the jammed states increase in extent but can move along different directions.

5. Summary

We have examined individual and multiple interacting skyrmions moving through a square array of obstacles. In the single skyrmion case, we observe a series of directional locking effects where the skyrmion Hall angle increases in both a quantized and a continuous manner. The transitions between the different locking steps are associated with dips or cusps in the velocity-force curves as well as with a quantized skyrmion Hall angle. For small obstacles, the skyrmion motion is oriented close to the intrinsic Hall angle, but directional locking to higher or lower Hall angles can occur. For larger obstacles, the number of directional locking steps is increased. The angle of skyrmion motion is \( 90^\circ \) with respect to the drive at zero damping, and it decreases with increasing damping until, for high damping, the skyrmion remains locked in the drive direction. When multiple species of interacting skyrmions are present, we show that it is possible to achieve a sorting effect in which one species of skyrmion locks to a symmetry direction of the obstacle lattice while the other species does not. The sharp steps in the velocity force curves disappear when there are multiple skyrmion species due to the disordering of the skyrmion trajectories that occurs when the different species move in different directions and collide. At lower drives, as the skyrmion density increases we observe a jammed state in which the two species form a rigid assembly and all move in the same direction, while at higher drives the motion of the two species decouples. For increasing skyrmion density, we observe a series of transitions among jammed phases that move at different angles with respect to the drive.

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