Charge Hall effect driven by spin-dependent chemical potential gradients and Onsager relations in mesoscopic systems

E. M. Hankiewicz,1 Jian Li,2 Tomas Jungwirth,3 Qian Niu,3 Shun-Qing Shen,2 and Jairo Sinova1

1Department of Physics, Texas A&M University, College Station, TX 77843-4242
2Department of Physics, The University of Hong-Kong, Pukfulam Road, Hong-Kong, China
3Department of Physics, University of Texas, Austin, TX 78712-0264

(Dated: October 11, 2018)

We study theoretically the spin-Hall effect as well as its reciprocal phenomenon (a transverse charge current driven by a spin-dependent chemical potential gradient) in electron and hole finite size mesoscopic systems. The Landauer-Buttiker-Keldysh formalism is used to model samples with mobilities and Rashba coupling strengths which are experimentally accessible and to demonstrate the appearance of measurable charge currents induced by the spin-dependent chemical potential gradient in the reciprocal spin-Hall effect. We also demonstrate that within the mesoscopic coherent transport regime the Onsager relations are fulfilled for the disorder averaged conductances for electron and hole mesoscopic systems.

Introduction. In the very active field of semiconductor-based spintronics the control of spin can be achieved by the manipulation of the strength of spin-orbit (SO) interactions in paramagnetic systems. Within this context, the newly proposed intrinsic spin-Hall effect (SHE) in p-doped semiconductors by Murakami et al.1 and in a two-dimensional electron system (2DES) by Sinova et al.2 offers new possibilities for spin current manipulation and generation in high mobility paramagnetic semiconductor systems. In contrast to the earlier proposed extrinsic spin-Hall effect3 4 5, which is associated with scattering from impurities, the intrinsic spin-Hall arises purely from host semiconductor band structure and represents a spin-current response generated perpendicular to the driving electric field.

Recently, the spin Hall effect was experimentally observed by Kato et al.6 in n-doped GaAs using the Kerr effect and by Wunderlich et al.6 in the p-n junction light-emitting diodes based on two dimensional hole gas (2DHG)(Al,Ga)As. Although, the experiment by Wunderlich et al. seems to be in the regime where the intrinsic effect in 2DHG is dominant, the main theoretical focus has been concentrated so far on 2DEG with Rashba SO interactions, where Rashba term is linear with k and reciprocal charge-Hall conductivity was established directly by detecting charge voltages induced by the spin-dependent chemical potential, was proposed in a context of exact symmetry, the pure cubic Rashba term was not considered in detail within the LB formalism so far.

The observation of spin-Hall effect through transport measurements is one of the urgent experimental challenges facing this spin-transport physics. Recently, an H-probe structure has been proposed to measure the effect where the spin-Hall effect could be measured indirectly by detecting charge voltages induced by the reciprocal spin-Hall effect (RSHE)10. This RSHE, where transverse charge current is driven by spin dependent chemical potential, was proposed in a context of extrinsic spin-Hall effect by Hirsch11 and formulated in a semiclassical approach by Zhang and Niu12. Also, the Onsager relation between the spin-Hall conductivity and reciprocal charge-Hall conductivity was established within a wave packet model through a redefinition of the spin-current including spin-torque terms in the bulk13. We show here that within the mesoscopic regime, and more specifically within the Landauer-Buttiker-Keldysh formalism, the Onsager relations are satisfied within the models studied for the disorder averaged conductances. Because the conductances are formulated with respect to the leads which have no spin-orbit coupling, it is not necessary nor consequential to introduce the spin current

momentum space show finite value of SHE in a weak scattering regime which goes to constant in the thermodynamic limit14.
In this paper we compare the magnitude of the SHE as well as the RSHE in finite electron and hole mesoscopic systems within the LB formalism. We show that the conductances associated with both effects are significantly larger in the hole systems. Furthermore, we analyze the possible experimental setup to measure the RSHE. We show that the charge current driven by a spin-dependent chemical gradient is on the order of hundred nano-amperes for typical voltages in hole systems and should be experimentally measurable.

**Model Hamiltonian for hole system and LB treatment of the spin-Hall effect and its reciprocal correspondent.**

The observation of the spin-Hall effect and its reciprocal phenomenon in transport is the next experimental challenge in the subfield of spintronics using spin-orbit interactions to manipulate the spin.

The continuum effective mass model for 2DHG in a narrow inversion asymmetrical well is given by [29]:

\[
\hat{H} = \sum_{j,\sigma} \epsilon_j c_{j,\sigma}^\dagger c_{j,\sigma} - t \sum_{j,\delta,\sigma} c_{j+\delta,\sigma}^\dagger c_{j,\sigma} \\
+ t_{SO-k^3} \left( \sum_j c_{j,\uparrow}^\dagger c_{j+2x,\downarrow} - c_{j,\uparrow}^\dagger c_{j-2x,\downarrow} \right) \\
+ \frac{i}{2} \sum_j c_{j,\uparrow}^\dagger c_{j+2y,\downarrow} - c_{j,\uparrow}^\dagger c_{j-2y,\downarrow} \\
+ 3(1-i) \sum_j c_{j,\uparrow}^\dagger c_{j-a_x+a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j+a_x-a_y,\downarrow} \\
+ 3(1+i) \sum_j c_{j,\uparrow}^\dagger c_{j-a_x-a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j+a_x+a_y,\downarrow} \\
+ 4i \sum_j c_{j,\uparrow}^\dagger c_{j+a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j-a_y,\downarrow} \\
+ 4 \sum_j c_{j,\uparrow}^\dagger c_{j+a_x,\downarrow} - c_{j,\uparrow}^\dagger c_{j-a_x,\downarrow} + h.c.,
\]

where \( t = \hbar^2/2m^* a_0^2 \) and \( t_{SO-k^3} = -\lambda/2a_0^3 \), \( a_0 \) is the mesh lattice spacing, \( \delta = \pm a_0 \hat{x}, \pm a_0 \hat{y} \). The first term represents a quenched disorder potential and disorder is introduced by randomly selecting the on-site energy \( \epsilon_j \) in the range \([-W/2, W/2]\). The continuum effective mass model for 2DES and its tight-binding correspondent can be found elsewhere [18, 19].

Within the leads the SO coupling is zero and therefore each lead should be considered as having two independent spin-channels. Moreover, in leads without the SO coupling, the spin current is measured in a medium where spin is conserved removing the ambiguity of spin-current definition. For SHE, leads constitute reservoirs of electrons at chemical potential \( \mu_1, \ldots, \mu_N \), where \( N \) is the number of leads that we consider to be four (see Fig. 1b). For RSHE, the chemical potential is spin dependent in leads 1 and 2 allowing the generation of spin-force in the x direction (see Fig. 1a). In the low temperature limit \( k_BT << E_F \) and for low bias-voltage, the particle current going through a particular channel is given within the LB formalism by [30]

\[
I_{p,\sigma} = \frac{e}{h} \sum_{q\sigma} T_{p,\sigma; q,\sigma} [V_p - V_q],
\]

where \( p \) labels the...
lead and \( T_{p,\sigma,q,\sigma'} \) is the transmission coefficient at the Fermi energy \( E_F \) between the \((p,\sigma)\) channel and the \((q,\sigma')\) channel. This transmission coefficient is obtained by \( T_{p,\sigma,q,\sigma'} = \text{Tr}[\Gamma_{p,\sigma}G_R\Gamma_{q,\sigma'}G_A] \) where \( \Gamma_{p,\sigma} \) is given by \( \Gamma_{p,\sigma}(i,j) = i[\Sigma_{p,\sigma}^{R,i,j} - \Sigma_{p,\sigma}^{A,i,j}] \). The retarded and advanced Green’s function of the sample \( G^{R/A}_{i,j} \) with the leads taken into account through the self energy \( \Sigma_{p,\sigma}^{R/A}(i,j) \) has a form \( G^{R/A}(i,j) = [E\delta_{i,j} - H_{i,j} - \sum_{p,\sigma} \Sigma_{p,\sigma}^{R/A}(i,j)]^{-1} \). Here the position representation of the matrices \( \Gamma_{p,\sigma}, G^R, H_{i,j} \), and \( \Sigma^R \) are in the subspace of the sample. Within the above formalism the spin current through each channel is given by \( I_{p,\sigma} = (e/4\pi) \sum_{q,\sigma'} T_{p,\sigma,q,\sigma'}[V_{p,\sigma} - V_{q,\sigma'}] \).

The spin force driven charge-Hall conductance, \( G_{xy}^{\text{SC}} \), is defined as the ratio of charge current in the y-direction induced by the spin-dependent chemical potential along the x-axis to this spin-dependent chemical potential difference (see Fig 1a).

\[
G_{xy}^{\text{CS}} = \frac{(I_{31} + I_{32})}{V_y - V_1}, \tag{2}
\]

where \( V_1 = \mu_1/e \). The spin-Hall conductance \( G_{xy}^{\text{SH}} \), is defined as the ratio of spin-current in the x-direction induced by charge voltage difference in y direction to this voltage difference

\[
G_{xy}^{\text{SC}} = \frac{(I_{11} - I_{12})}{V_y - V_3}, \tag{3}
\]

where \( V_3 = \mu_3/e \), and the labels are indicated in Fig. 1b. \( G_{xy}^{\text{CS}} \) and \( G_{xy}^{\text{SC}} \) are defined by analogy. We set the absolute value of voltage for spin and spin force driven charge-Hall effects as \( V = \mu/e = 2.5 \text{mV} \).

**Results and discussion.** In order to address the key issue of experimental observation of spin driven charge-Hall effect as well as to establish the Onsager relation between spin-Hall effect and it reciprocal correspondent we choose realistic parameters for our calculations which model currently attainable systems. We consider an effective mass of \( m^* = 0.05 m_e \) for electron systems and \( m^* = 0.5 m_e \) for hole ones. The disorder strength \( W = 0.09 \text{meV} \) corresponds to the mobility of \( 250,000 \text{cm}^2/\text{Vs} \), which is typical for a semiconductor like (In,Ga)As. We take the Rashba parameter \( \lambda \) in the range from 0 to 100 meV nm, easily obtained in experiments [31, 32], and we choose the electron concentration \( n_{2D} \) in a range between \( 3 \cdot 10^{11} \text{cm}^{-2} \) and \( 1.3 \cdot 10^{12} \text{cm}^{-2} \). The Fermi energy is obtained from the chosen carrier concentration assuming an infinite 2D gas. In the detection of spin force driven charge-Hall effect the first task is to generate a spin force which can be realized by spin dependent chemical potential in the leads. The ferromagnetic leads are not the good candidates because although magnetization exists in ferromagnetic leads, the chemical potential is the same for both spin directions. Here, we propose the optical method of spin-dependent chemical potential generation by shining the beam of circularly polarized light on the leads (see Fig. 2). The right-circularly polarized beam shines on the right lead (lead 2 in Fig. 1a) and the left circularly polarized beam on the left one (lead 1 in Fig. 1a). The sample as well as the transverse leads should be covered by mask, preventing the light absorption anywhere except the small part of longitudinal leads as shown in Fig. 2a. Choosing semiconductor leads for this setup e.g. GaAs, will cause the opposite spin polarizations in left and right leads through optical selection rules. Using the beam splitter should produce the same light intensity in each leads providing simultaneously the spin-dependent chemical potential between leads 1 and 2 and the total charge current across a sample equal zero. Having produced the spin-dependent chemi-

---

**FIG. 4:** The spin-force driven charge current as a function of hole concentration and Rashba coupling \( \lambda \), for mesoscopic square sample 100nm by 100nm and \( \mu = 250,000 \text{cm}^2/\text{Vs} \).
the charge current also oscillates with our calculations. For hole mesoscopic systems (see Fig. 4), the charge current for electron systems show oscillations with respect to the electron density and SO coupling. The period of current oscillations depends on system size, however its maximal value seems to be around 40 nA for systems sizes achievable in our calculations. For hole mesoscopic systems (see Fig. 4), the charge current also oscillates with λ and for system sizes on the order of 100 nanometers is on the order of hundred nano-amperes in a wide range of densities starting from $6 \times 10^{11}$ cm$^{-2}$. Hence the charge current is much larger in hole systems and shall be detectable in experiments. Fig. 5 presents the spin-Hall conductance $G_{SC}^{xy}$, as a function of $\lambda$ for electron and hole systems. One can see that spin-Hall conductance (Fig. 5) and spin-force driven charge conductances (see Figs. 3 and 4) behave similarly. Spin-Hall conductances for electron and hole systems oscillate with SO coupling. Moreover, $G_{SC}^{xy}$ is several times larger for hole systems in comparison with electron ones, which is associated with much larger effective mass in a case of hole systems. Let us emphasize that for electron and hole systems with the same effective masses the spin-Hall effect can be larger for electron systems. Moreover, in mesoscopic systems where Fermi energy as well as multichannel effects are important the straightforward renormalization of effective mass of electron and hole systems to compare the spin-Hall conductance suggested by [27] does not have to be correct. Our calculations in mesoscopic systems are in agreement with the linear response Kubo calculation for 2DEG and 2DHG which show that spin-conductance is much more larger for hole systems [2, 29]. The sign of spin-Hall and charge Hall conductances in mesoscopic systems depends on the Fermi energy.

![Diagram](image_url)

**FIG. 6:** Onsager relation between the spin-Hall conductance $G_{SC}^{xy}$ (open symbols) and spin-force driven charge-Hall conductance $G_{CS}^{xy}$ (close symbols) for different L by L hole systems.

The Onsager relations express the symmetry between the transport coefficients describing reciprocal processes in systems with a linear dependence of response to the driving forces. Within the models studied, considering disorder averaged conductances, the relations can be derived by utilizing the time reversal symmetry and the inversion symmetry on the x-y plane which relate $T_{\sigma m;\sigma m} = T_{\bar{\sigma}m;\bar{\sigma}m}$ and $T_{\bar{\sigma}n;\bar{\sigma}n} = T_{\sigma n;\sigma n}$ respectively, where $\sigma, \alpha$ and $n, m$ are the spin and lead labels and a bar represents the opposite direction. Within the SHE $V_1^\sigma = V_2^\sigma = 0$ and $V_3^\sigma = -V_4^\sigma = V_0^\sigma = V_0/2$ following the labels of Fig. 1. Within the RSHE $V_1^\sigma = V_2^\sigma = 0$ and $V_3^\sigma = V_2^\sigma = s(\sigma)V_0/2$ where $s(\uparrow) = +1$ and $s(\downarrow) = -1$. Within the LB formalism and these boundary conditions we obtain for the spin-current associated with the SHE $I_1^{spin} = I_1^{\uparrow} - I_1^{\downarrow} = \frac{1}{2} \sum_{\sigma, \alpha} s(\sigma)(T_{\sigma 1;\alpha 3} - T_{\sigma 1;\alpha 4})$ and for the charge current associated with the RSHE $I_1^c = I_1^{3} + I_1^{4} = \frac{1}{2} \sum_{\sigma, \alpha} s(\alpha)(T_{\sigma 3;\alpha 1} - T_{\sigma 3;\alpha 2})$. The above symmetries imply that $T_{\sigma 1;\alpha 4} = T_{\sigma 2;\alpha 3} = T_{\bar{\sigma} 3;\alpha 2}$. This then yields $I_1^{spin} = -I_1^{3}$ which implies the Onsager relation

$$G_{SC}^{xy} = -G_{CS}^{xy}$$

This is verified numerically in Fig. 6 which presents the disorder averaged spin-Hall conductance and charge Hall conductance for hole systems of different sizes. For a specific disorder realization this relation does not hold and is only approximate depending on the strength of...
the fluctuations induced by disorder. This relation between $G_{SC}$ and $G_{CS}$ is consistent with predictions of semiclassical wave-packet theory, where standard definition of spin-current was modified by a spin-torque term\cite{28,33}. However, as seen from the above derivation and noted in the introduction, our finding of an Onsager relation in mesoscopic coherent systems do not involve such spin-torque term since all spin-currents are defined in the non-spin-orbit coupled leads.

Summary. We have analyzed the spin Hall effect as well as its reciprocal effect in mesoscopic hole and electron systems. We have shown that the spin-Hall as well as the spin-dependent chemical potential gradient driven charge-Hall conductances are several times larger for hole systems. Further we have proposed the experimental setup to detect the transverse charge current driven by the spin-dependent chemical potential gradient through transport measurements. We have shown that this charge current is of the order of hundred nano-amperes in hole systems and should be detectable. Also, we have established a direct relation between the disorder average spin-Hall conductances and their reciprocal in mesoscopic systems.

Acknowledgments We thank A. H. MacDonald for stimulating discussions. The work was partly supported by the Research Grant Council of Hong Kong (SQS) and DOE grant DE-FG03-02ER45958.

\begin{thebibliography}{99}
\bibitem{1} S. Murakami, N. Nagaosa, and S.-C. Zhang, Science \textbf{301}, 1348 (2003).
\bibitem{2} J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. MacDonald, Phys. Rev. Lett. \textbf{92}, 126603 (2004).
\bibitem{3} M. I. Dyakonov and V. I. Perel, Zh. Eksp. Ter. Fiz. \textbf{13}, 657 (1971).
\bibitem{4} J. E. Hirsch, Phys. Rev. Lett. \textbf{83}, 1834 (1999).
\bibitem{5} S. Zhang, Phys. Rev. Lett. \textbf{85}, 393 (2000).
\bibitem{6} Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science \textbf{306}, 1910 (2004).
\bibitem{7} J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. \textbf{94}, 047204 (2005).
\bibitem{8} D. Culcer, J. Sinova, N. A. Sinitsyn, T. Jungwirth, A. H. MacDonald, and Q. Niu, Phys. Rev. Lett. \textbf{93}, 046602 (2004); N. A. Sinitsyn, E. M. Hankiewicz, W. Teizer, and J. Sinova, Phys. Rev. B \textbf{70}, 081312 (2004); B. A. Bernevig and S.-C. Zhang cond-mat/0408442, J. Hu, B. A. Bernevig, and C. Wu cond-mat/0310093; S. I. Erlingon and J. Schliemann and D. Loss, Phys. Rev. B \textbf{71}, 035319 (2005); S.-Q. Shen, Phys. Rev. B \textbf{70}, 081311 (2004); S.-Q. Shen, M. Ma, X.C. Xie, and F.C. Zhang, Phys. Rev. Lett. \textbf{92}, 256603 (2004); L. Hu, J. Gao, and S.-Q. Shen, Phys. Rev. B \textbf{69}, 165304 (2004); A. A. Burkov, A. S. Nunez, and A. H. MacDonald, Phys. Rev. B \textbf{70}, 155308 (2004); S. Zhang and Z. Yang, Phys. Rev. Lett. \textbf{94}, 066602 (2005).
\bibitem{9} E. I. Rashba, Phys. Rev. B \textbf{68}, 241315 (2003).
\bibitem{10} E. I. Rashba, Phys. Rev. B \textbf{70}, 161201 (2004).
\bibitem{11} O. V. Dimitrova cond-mat/0405339; cond-mat/0407612.
\bibitem{12} E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. \textbf{93}, 226602 (2004).
\bibitem{13} K. Nomura, J. Sinova, T. Jungwirth, Q. Niu, and A. H. MacDonald, Phys. Rev. B \textbf{71}, 041304 (2005).
\bibitem{14} J. Inoue, G. E. W. Bauer, and L. W. Molenkamp, Phys. Rev. B \textbf{70}, 041303 (2004).
\bibitem{15} O. Chalaev and D. Loss, cond-mat/0407342.
\bibitem{16} A. Khaetskii, cond-mat/0408136.
\bibitem{17} B. K. Nikolić, L.P. Zarbo and Z. Souma, cond-mat/0408693.
\bibitem{18} L. Sheng, D. Sheng, and C. Ting, Phys. Rev. Lett. \textbf{94}, 016602 (2005).
\bibitem{19} E. M. Hankiewicz, L. W. Molenkamp, T. Jungwirth, and J. Sinova, Phys. Rev. B \textbf{70}, 241301 (2004).
\bibitem{20} N. Sugimoto, S. Onoda, S. Murakami, and N. Nagaosa, cond-mat/0503475.
\bibitem{21} S. Murakami, cond-mat/0504353.
\bibitem{22} D. N. Sheng, L. Sheng, Z. Y. Weng, and F. D. M. Haldane, cond-mat/0504218.
\bibitem{23} K. Nomura, unpublished.
\bibitem{24} S. Murakami, Phys. Rev. B \textbf{69}, 241202 (2004).
\bibitem{25} B. A. Bernevig and S.-C. Zhang, cond-mat/0412550.
\bibitem{26} J. Li, L. Hu, and S.-Q. Shen, cond-mat/0502302.
\bibitem{27} M. W. Wu and J. Zhou, cond-mat/0503616.
\bibitem{28} P. Zhang and Q. Niu, cond-mat/0406436.
\bibitem{29} J. Schliemann and D. Loss, Phys. Rev. B \textbf{71}, 085308 (2005).
\bibitem{30} S. Datta, \textit{Electronic transport in mesoscopic systems} (Cambridge University Press, Cambridge, 1995).
\bibitem{31} T. K. J. Nitta, T. Akazai, and H. Takayanagi, Phys. Rev. Lett. \textbf{89}, 046801 (2002).
\bibitem{32} E.Johnston-Haleprin, D. Lofgreen, D.K.Young, L. Col-dren, A. Gossard, and D. Awschalom, Phys. Rev. B \textbf{65}, 041306 (2002).
\bibitem{33} P. Zhang, J. Shi, D. Xiao, and Q. Niu, cond-mat/0503505.
\end{thebibliography}