Light-Element Reaction flow and the Conditions for r-Process Nucleosynthesis

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\textbf{ABSTRACT}

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We deduce new constraints on the entropy per baryon \((s/k)\), dynamical timescale \((\tau_{\text{dyn}})\), and electron fraction \((Y_e)\) consistent with heavy element nucleosynthesis in the r-process. We show that the previously neglected reaction flow through the reaction sequence \(4\text{He}(t,\gamma)\text{Li}(n,\gamma)\text{Li}(\alpha,n)\text{B}\) significantly enhances the production of seed nuclei. We analyze the r-process nucleosynthesis in the context of a schematic exponential wind model. We show that fewer neutrons per seed nucleus implies that the entropy per baryon required for successful r-process nucleosynthesis must be more than a factor of two higher than previous estimates. This places new constraints on dynamical models for the r-process.

Subject headings: \(\alpha\)-capture, r-process nucleosynthesis, nuclear reactions, supernovae

1. Introduction

The astronomical site for r-process nucleosynthesis by rapid neutron capture has not yet been unambiguously determined. Observations of metal poor stars (e.g. Sneden et al. 1996) indicate an abundance pattern for the early Galactic r-process elements which is very similar to that of the Solar r-process abundance distribution. Hence, it is often argued that core-collapse supernovae (e.g. Type II SNe) are the most likely site for r-process nucleosynthesis. Such events are the first contributors to the abundances observed at lowest metallicity. The possibility remains, however, that the r-process could be associated with neutron star mergers (Freiburghaus et al. 1999) or gamma-ray burst environments (Inoue et al. 2003) in which the required neutron-rich conditions can also be realized.

Moreover, the environment suitable for the r-process is not yet fully understood numerically. Even in the presently popular paradigm of neutrino-driven winds from Type II SNe, physical conditions of the r-process environment are largely dependent on the details of the adopted numerical simulations (Meyer et al. 1992; Woosley et al. 1994; Witti et al. 1994; Takahashi et al. 1994; Qian & Woosley 1996; Cardall & Fuller 1997; Hoffman et al. 1997; Otsuki et al. 2003; Sumiyoshi et al. 2000; Wanajo et al. 2001; Otsuki et al. 2003).

As a useful guide for numerical studies of the r-process environment, Hoffman et al. (1997) determined empirical conditions required to produce the platinum peak in the r-process. They deduced a phenomenological constraint on the parameter space of \(s/k\), \(\tau_{\text{dyn}}\), and \(Y_e\), i.e. \((s/k \propto Y_e \tau_{\text{dyn}}^{1/3})\).

In the present work we reinvestigate these phenomenological constraints and deduce a new allowed parameter space for \(s/k-\tau_{\text{dyn}}-Y_e\). We deduce significantly greater restrictions
on the r-process environment. The difference of the present work with the previous study can be traced to the treatment of the α-process. In formulating the $s/k-\tau_{\text{dyn}}-Y_e$ relation, Hoffman et al. (1997) considered only the reaction flow through $^4\text{He}(\alpha, \gamma)^9\text{Be}(\alpha, n)^{12}\text{C}$ as an α-process path (see section 4 in Hoffman et al. (1997)). In the present work, however, we also include the reaction sequence $^4\text{He}(t, \gamma)^7\text{Li}(n, \gamma)^8\text{Li}(\alpha, n)^{11}\text{B}$.

Although the reaction sequence $^4\text{He}(\alpha, \gamma)^9\text{Be}(\alpha, n)^{12}\text{C}$ is usually the dominant flow in the α-process, alternative reaction sequences such as the $^4\text{He}(t, \gamma)^7\text{Li}(n, \gamma)^8\text{Li}(\alpha, n)^{11}\text{B}$ path can provide enhanced reaction flow toward seed nuclei (Terasawa et al. 2001). The production of heavy nuclei is, therefore, quite sensitive (Sasaqui et al. 2005a,b) to the rates for these reactions. Also, since the work of Hoffman et al. (1997), new measured rates (Hashimoto 2004) are available for the $^8\text{Li}(\alpha, n)^{11}\text{B}$ reaction.

The purpose of this short paper is therefore to reformulate the $s/k-\tau_{\text{dyn}}-Y_e$ constraints on the SN dynamics by incorporating the important effects from the reaction sequence $^4\text{He}(t, \gamma)^7\text{Li}(n, \gamma)^8\text{Li}(\alpha, n)^{11}\text{B}$.

2. Calculation

2.1. Exponential Model

For the present studies we utilize a schematic exponential model similar to that adopted by Meyer et al. (1992); Meyer & Brown (1997). This model provides an adequate approximation to the evolution of ejected material in a wide variety of the plausible conditions of the r-process such as may occur for example in both delayed and prompt SNe (Hillebrandt et al. 1984; Sumiyoshi et al. 2001; Wanajo et al. 2003), neutron-star mergers (Freiburghaus et al. 1999), or gamma-ray burst (GRB) environments.

In this model the dynamical expansion timescale, $\tau_{\text{dyn}}$, denotes how rapidly the temperature evolves,

$$\tau_{\text{dyn}}^{-1} = -\frac{1}{T - T_a} \frac{dT}{dt}, \quad \text{(1)}$$

where $T_a$ is the asymptotic temperature (Otsuki et al. 2003) of the material. This temperature determines the freeze-out of the neutron-capture flow. The model assumes adiabatic expansion. Hence, the entropy per baryon $s/k \propto T^3/\rho = \text{constant}$. The temperature and density thus evolve according to:

$$T_9(t) = T_9(0) \exp(-t/\tau_{\text{dyn}}) + T_a \quad \text{,} \quad \text{(2)}$$
\[ \rho(t) = \rho(0) \left( \frac{T_g(t)}{T_g(0)} \right)^3, \]  

where we adopt \( T_g(0) = 8.40 \), \( T_a = 0.62 \), and \( \rho(0) = 3.3 \times 10^8 \) g cm\(^{-3} \) from Otsuki et al. (2003) and Sasaqui et al. (2005a).

### 3. Nucleosynthesis Network

We employ the nucleosynthesis reaction network used in Otsuki et al. (2003), which was derived from the network code described in Meyer et al. (1992) and Woosley et al. (1994), and expanded in Terasawa et al. (2001). Several further important modifications have also been made in the present reaction network. The main features are the following.

The reaction \(^4\text{He}(\alpha,\gamma)^9\text{Be}\) is still important even in wind models with a short dynamical expansion timescale. The three-body reaction rate for \(^4\text{He}(\alpha,\gamma)^9\text{Be}\) is taken from the network estimate of Sumiyoshi et al. (2002) based on recent experimental data (Utsunomiya et al. 2001) for this reaction cross section which spans the low energy region of astrophysical interest. However, it now shares the main nuclear reaction chain with a new flow path \(^4\text{He}(t,\gamma)^7\text{Li}(n,\gamma)^8\text{Li}(\alpha,n)^{11}\text{B}\) (Terasawa et al. 2001). The \(^8\text{Li}(\alpha,n)^{11}\text{B}\) reaction in particular has been identified (Terasawa et al. 2002; Sasaqui et al. 2004) to be critical in the production of intermediate-to-heavy mass elements. Hashimoto (2004) have carried out very precise measurements of the exclusive (i.e. individual states) reaction cross section for \(^8\text{Li}(\alpha,n)^{11}\text{B}\). Their results confirm that transitions leading to several excited states of \(^{11}\text{B}\) make the predominant contribution to the total reaction cross section. This is in good agreement with the previous measurements of the inclusive (i.e. sum of excited states) reaction cross section (Boyd et al. 1992; Gu et al. 1995; Mizoi et al. 2000). Hence, we employ the newest cross section data from Hashimoto (2004).

We also note that we calculate the nucleosynthesis sequence from nuclear statistical equilibrium (NSE), the \(\alpha\)-process, \(\alpha\)-rich freeze-out, the r-process and subsequent beta-decay and alpha-decay in a single network code rather than to split the calculation into two parts as was done in Woosley et al. (1994). It is important to compute self-consistently the evolution of seed nuclei along with heavy element production in the r-process (Sasaqui et al. 2005a,b). Computed final r-process abundances for \(Y_e = 0.45\) and various values for the dynamical timescale and entropy are shown in Figure 1. As noted in Hoffman et al. (1997), this entire line of inquiry will only be relevant if the conditions important to seed production prior to the r-process occur in an environment with a neutron excess.
4. Analytic Treatment Of The $\alpha$-process

As in Hoffman et al. (1997), we analyze the $\alpha$-process in detail in order to provide new dynamical constraints on the $s/k-\tau_{dyn}-Y_e$ parameter space relevant to the r-process. The $\alpha$-process is particularly important as it is the means for producing seed nuclei for subsequent r-process neutron capture.

As the temperature drops below $T_9 \sim 5.0$ the reaction flow falls out of NSE and the $\alpha$-process operates until the temperature drops below $T_9 \sim 2.5$. During this process, $\alpha$ particles are consumed through the main bottleneck reaction sequence $^4\text{He}(\alpha,n)^9\text{Be}(\alpha,n)^{12}\text{C}$ and also the secondary reaction path $^4\text{He}(t,\gamma)^7\text{Li}(n,\gamma)^8\text{Li}(\alpha,n)^{11}\text{B}$. Seed nuclei for the r-process are subsequently produced by a sequence of $\alpha$-capture reactions starting with $^9\text{Be}(\alpha,n)^{12}\text{C}$ or $^8\text{Li}(\alpha,n)^{11}\text{B}$.

4.1. $^4\text{He}(\alpha,n)^9\text{Be}(\alpha,n)^{12}\text{C}$

The approximate time evolution of the abundances of $\alpha$ particles ($Y_\alpha$) and neutrons ($Y_n$) is expressed as in Hoffman et al. (1997),

$$\frac{dY_\alpha}{dt} \approx -\frac{\bar{Z}}{2} Y_\alpha Y_9 \rho N_A \langle \sigma v \rangle_{\alpha n}, \quad (4)$$

$$\frac{dY_n}{dt} \approx -(\bar{A} - 2\bar{Z}) Y_\alpha Y_9 \rho N_A \langle \sigma v \rangle_{\alpha n}, \quad (5)$$

where $Y_9$ is the abundance of $^9\text{Be}$ and $N_A \langle \sigma v \rangle_{\alpha n}$ is the $^9\text{Be}(\alpha,n)^{12}\text{C}$ reaction rate. The quantities $\bar{A}$ and $\bar{Z}$ are the mean mass number and mean proton number, respectively, of typical seed nuclei as defined in Hoffman et al. (1997).

Because of the low $Q$-value for the $^9\text{Be}(\alpha,n)^{12}\text{C}$ reaction rate, statistical equilibrium is realized between $^9\text{Be}$ and $^4\text{He}$ over the temperature range of interest. Hence, we can write (Hoffman et al. 1997)

$$Y_9 = Y(4, 9) \approx G(4, 9)[\zeta(3)^8 \pi^{-4/2}] 2^{11} 9^{3/2} \left(\frac{kT}{m_N c^2}\right)^{12} \phi^{-8} Y_\alpha^4 Y_n^5 \exp \left(\frac{B(4, 9)}{kT}\right),$$

$$Y_\alpha = Y(2, 4) \approx G(2, 4)[\zeta(3)^3 \pi^{-3/2}] 2^{7/2} \left(\frac{kT}{m_N c^2}\right)^{9/2} \phi^{-3} Y_\alpha^2 Y_n^2 \exp \left(\frac{B(2, 4)}{kT}\right).$$

Here, $B(4,9)=58.16$ MeV and $B(2,4)=28.29$ MeV. Therefore,

$$Y_9 \approx 8.66 \times 10^{-11} \rho_5^2 T_9^{-3} Y_\alpha^2 Y_n \exp \left(\frac{18.26}{T_9}\right).$$
Adopting the exponential dynamical model [i.e. Eqs. (2) and (3)], equations (4) and (5) become,

\[ \frac{dY_\alpha}{dT_9} \approx \frac{\bar{Z}}{2} Y_\alpha^3 Y_n f(T_9) \tau_{dyn}, \]

\[ \frac{dY_n}{dT_9} \approx (\bar{A} - 2\bar{Z}) Y_\alpha^3 Y_n f(T_9) \tau_{dyn}, \]

where \( f(T_9) \) is given by

\[ f(T_9) \approx 8.66 \times 10^{-6} \rho_5^3 T_9^{-4} \exp(18.31/T_9) N_A \langle \sigma v \rangle_{\alpha n} \text{ sec}^{-1}. \]

Now inserting \( \rho_5 \approx 3.33 T_9^3/(s/k) \), we have

\[ f(T_9) \approx 3.20 \times 10^{-4}(s/k)^{-3} T_9^5 \exp(18.31/T_9) N_A \langle \sigma v \rangle_{\alpha n} \text{ sec}^{-1}. \]

Now integrating Eq. (7) in the range between \( T_9 = 2.5 \) and \( T_9 = 5.0 \) where the \( \alpha \)-process is dominant, we obtain the final neutron abundance,

\[ Y_{n,f} \approx Y_{n,0} \exp \left[ - (\bar{A} - 2\bar{Z}) Y_\alpha^3 Y_n \tau_{dyn} \int_{T_9=2.5}^{5.0} \frac{f(T_9) dT_9}{\Delta T_9} \right], \]

where we have made use of the fact that \( Y_\alpha \approx Y_{\alpha,0} = X_{\alpha,0}/4 \approx 1/2 Y_e,i \) during the \( \alpha \)-process.

The integral can be approximated by \( \int_{T_9=2.5}^{5.0} f(T_9) dT_9 \approx 7.44 \times 10^8(s/k)^{-3} \). We introduce (Hoffman et al. 1997) the produced r-process nucleus of interest. This leads to a lower limit on the entropy per baryon required to produce an r-process nucleus of mass number \( A \),

\[ s/k \approx Y_{e,i} \left\{ \frac{9.3 \times 10^7 (\bar{A} - 2\bar{Z})}{\ln \left( (1 - 2\bar{Z}/A)/(1 - A/A) \right)} \right\}^{1/3} \tau_{dyn}. \]

The results of this analysis is expressed in the lower part of Figure 2. These expressions are essentially identical to those of Hoffman et al. (1997). The only differences stem from the use a different reaction rate in the \( ^4\text{He}(\alpha,\gamma)^9\text{Be}(\alpha,n)^{12}\text{C} \) sequence. The linear scaling of this result on the choice of \( Y_e \) is also apparent.

### 4.2. \( ^4\text{He}(t,\gamma)^7\text{Li}(n,\gamma)^8\text{Li}(\alpha,n)^{11}\text{B} \)

As described in Section 3, the reaction flow through \( ^4\text{He}(t,\gamma)^7\text{Li}(n,\gamma)^8\text{Li}(\alpha,n)^{11}\text{B} \) is also important. We make an analogous treatment of the \( ^8\text{Li}(\alpha,n)^{11}\text{B} \) reaction to that of the \( ^9\text{Be}(\alpha,n)^{12}\text{C} \) reaction sequence of Section 4.1. Hence, we write,

\[ \frac{dY_\alpha}{dt} = - \frac{\bar{Z}}{2} Y_\alpha Y_8 \rho N_A \langle \sigma v \rangle_{\alpha n}, \]
\[
d\frac{dY_n}{dt} = -(\bar{A} - 2\bar{Z})Y_\alpha Y_8 \rho N_A \langle \sigma v \rangle_{an},
\]  

(13)

where \(Y_8\) is the abundance of \(^8\)Li, and in this case \(N_A \langle \sigma v \rangle_{an}\) is the reaction rate of \(^8\)Li(\(\alpha\),n)\(^{11}\)B.

Because of the low \(Q\)-value for the \(^8\)Li(\(\alpha\),n)\(^{11}\)B reaction, statistic equilibrium is again realized (Hoffman et al. 1997) between \(^9\)Li and \(^4\)He. Hence, we write

\[
Y_8 = Y(3,8) = G(3,8)[\zeta(3)7^{-7/2}2^{19/2}]8^{3/2}(\frac{kT}{m_NC^2})^{21/2} \phi^{-7}Y^3 Y^5 \exp \left( \frac{B(3,8)}{kT} \right),
\]  

(14)

where, \(B(3,8) = 41.28\) MeV. Using \(Y_\alpha\) as defined in §4.1 then we deduce

\[
Y_8 \approx 7.96 \times 10^{-14} Y_n^2 Y_\alpha^3 \exp(-\frac{13.39}{T_9}) T_9^{-15/4} \rho_5^{5/2}.
\]  

(15)

Once again adopting an exponential model [Eqs. (2) and (3)], equation (13) becomes

\[
d\frac{dY_n}{dT_9} = (\bar{A} - 2\bar{Z}) Y_\alpha^{3/2} Y_n^2 h(T_9) \tau_{dyn},
\]  

(16)

where \(h(T_9)\) is given by

\[
h(T_9) \approx 7.6 \times 10^{-9}(s/k)^{-7/2} T_9^{23/4} \exp(-13.39/T_9) N_A \langle \sigma v \rangle_{an}.
\]  

(17)

Here we have made use of the fact that \(s \sim 3.33 T_9^3 / \rho_5\). Integrating Eq. (16) from \(T_9 = 2.5\) and \(T_9 = 5.0\) we have,

\[
-\frac{1}{Y_{n,f}} + \frac{1}{Y_{n,0}} = (\bar{A} - 2\bar{Z}) \tau_{dyn} Y_\alpha^{3/2} \int_{2.5}^{5.0} h(T_9) dT_9,
\]

where we again use the fact that \(Y_\alpha \approx Y_{\alpha,0} = X_{\alpha,0}/4 \approx 1/2Y_{e,i}\) during the \(\alpha\)-process and we invoke the approximation, \(\int_{2.5}^{5.0} h(T_9) dT_9 \approx 3.6 \times 10^3 s^{7/2}\) for \(N_A \langle \sigma v \rangle_{an}\) given by Hashimoto (2004).

As a result,

\[
s/k \approx \left\{ \frac{3.57 \times 10^3 Y_{e,i}^{3/2}(\bar{A} - 2\bar{Z})(1 - \bar{A}/A)(1 - 2Y_{e,i})}{2^{5/2} Z/A} \right\}^{2/7} \tau_{dyn}.
\]  

(18)

This result is also shown on the lower part of Figure 2. Here, the need that \(Y_e < 0.5\) is evident as is the scaling of these results with \(Y_e\).
4.3. Total Sequence

Combining these two reaction branches, the total change of neutron density with temperature now becomes:

\[
\left( \frac{dY_n}{dT_9} \right)_{tot} = (\bar{A} - 2\bar{Z}) \left[ Y_\alpha^3 Y_n f(T_9) + Y_\alpha^{3/2} Y_n^2 h(T_9) \right] \tau_{dyn},
\]

Integrating Eq. (19) from \(T_9 = 2.5\) to \(T_9 = 5.0\) we have,

\[
\frac{1}{Y_{n,f}} \approx \left( \frac{1}{Y_{n,0}} + \frac{\alpha_1}{\beta_0} (1 - e^{-2.5\beta_0}) Y_{n,f} h(5.0) \right) \exp \left[ \alpha_0 \int_{2.5}^{5.0} f(T_9) dT_9 \right].
\]

Here, \(\alpha_0 = (\bar{A} - 2\bar{Z})\tau_{dyn} Y_\alpha^3\), \(\alpha_1 = (\bar{A} - 2\bar{Z})\tau_{dyn} Y_\alpha^{3/2}\), and \(\beta_0 = \left( \frac{k'(T_i)}{h(T_i)} + \alpha_0 f(T_i) \right)\). Since the left side \(1/Y_{n,f}\) is fixed once the initial conditions are specified, it must be constant. Hence, the right hand side must be constant as well. By this requirement, we suppose that the both parts of the right hand side are constant. The term in parentheses on the right hand side yields \(s/k \propto \tau_{dyn}^{2/7}\) (similar to the result of §4.2, i.e. Eq. (11)). The later exponential term on the right introduces \(s/k \propto \tau_{dyn}^{1/3}\) (similarly to Eq. (18) of §4.1). As a result we find:

\[
s/k \propto \tau_{dyn}^{13/21}
\]

The exact solution cannot be expressed analytically like Eqs. (11) or (18). However, the details of the numerical calculation are also shown on Figure 2 and compared with the above analytic results. The analytic result is in good agreement with the numerical simulation.

5. Results and Summary

As noted in §4, the reaction sequence \(^4\text{He}(t,\gamma)^7\text{Li}(n,\gamma)^8\text{Li}(\alpha,n)^{11}\text{B}\) can be a competitor to the \(^4\text{He}(\alpha,\gamma)^9\text{Be}(\alpha,n)^{12}\text{C}\) sequence in the \(\alpha\)-process. We have analyzed this by adding
the contribution of the reaction $^8\text{Li}(\alpha,n)^{11}\text{B}$ to the previous analysis (Hoffman et al. 1997) of the entropy constraint

In Figure 2 we compare this analytic model for the entropy constraint as a function of dynamical timescale with the nuclear simulation results. We have selected a comparatively wide range model parameters, $28 < \bar{Z} < 36$, and $85 < \bar{A} < 105$ after Hoffman et al. (1997). In the present analysis we also consider the production of the actinide nuclei ($^{232}\text{Th}$, $^{235}\text{U}$, and $^{238}\text{U}$). We consider such an analysis to be worthwhile since ultimately the actinides must be produced in an r-process environment. Indeed, it is possible to produced the second and third r-process peaks without producing actinides (cf. Woosley et al. (1994)). Moreover, the actinides are particularly sensitive to the production of seed nuclei by light-element reactions (Sasaqui et al. 2005a,b) and are also important for cosmochronology.

Even so, we note that there are additional uncertainties associated with the formation of the actinide nuclei due for example to uncertainties in atomic mass extrapolations, fission barriers, beta-delayed fission, etc. Nevertheless, such an application is within the spirit of the schematic model analysis applied here and in Hoffman et al. (1997) and provides additional insight into the plausible conditions for a successful r-process.

We adopt the following values in calculating the r-process production: an initial electron fraction of $Y_{e,i}$, 0.45 and dynamical time-scales from 1-50 msec. In most successful simulations $Y_{e,i}$ remains fixed at near 0.45 by the ambient weak interaction rates. Hence, although these results will change for different values of $Y_e$, we adopt a fixed value for this figure. On the other hand, various values of $\tau_{dyn}$ have been proposed in the literature. We then search for entropy values for which the r-process abundance distribution is consistent with observation for each adopted dynamical timescale. Examples of consistent entropy values are summarized in Figure 1. The right most figures roughly correspond to expansion timescales studied in Hoffman et al. (1997). For $Y_e = 0.45$, they obtained minimum entropies of 140 (5 msec) and 300 (50 msec). That is about a factor of two, and factor of 6 respectively less than the values deduced in the present work. These results, however, will change for different values of $Y_e$ as is evident from Eqs. (11) or (18).

Figure 2 shows the relation between the analytic model (dotted lines) and numerical simulation (points). Shown are the lower limits on the entropy required to form A=232 (Th) nuclei consistent with observation. Both results are similar. Figure 2 also shows a comparison between the present lower limits and those of Hoffman et al. (1997). The new relation implies that the required entropy is typically a factor of two greater than the previous $s/k-\tau_{dyn}$ estimate.

In summary, we have shown that the $^8\text{Li}(\alpha,n)^{11}\text{B}$ reaction is an important competing
reaction flow channel for r-process nucleosynthesis. This reaction in particular implies a more efficient production of seed nuclei so that a larger neutron/seed ratio is required for a successful dynamical r-process model. For the schematic exponential models considered here, the implied lower limit to the entropy per baryon increases by about a factor of two from previous estimates. This places a serious constraint on models for the astrophysical site for the production of r-process nuclei.

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A.

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B. Appendix A

The following updated expressions were utilized to calculate the reaction rates $N_A\langle\sigma v\rangle$. Although these rates differ from those used in Hoffman et al. (1997), employing them does not substantially change the results.

For the reaction $^9$Be($\alpha$,n)$^{12}$C reaction we use;

$$N_A\langle\sigma v\rangle_{\text{NOW}} = 4.62 \times 10^{13}/T_9^{2/3} \exp (-23.870/T_9^{1/3} - (T_9/0.049)^2)$$
$$\times (1 + 0.017 \times T_9^{1/3} + 8.57 \times T_9^{2/3} + 1.05 \times T_9 + 74.51 \times T_9^{4/3} + 23.15 \times T_9^{5/3})$$
$$+ 7.34 \times 10^{-5}/T_9^{3/2} \exp (-1.184/T_9)$$
$$+ 0.227/T_9^{3/2} \exp (-1.834/T_9)$$
$$+ 1.26 \times 10^5/T_9^{3/2} \exp (-4.179/T_9)$$
$$+ 2.40 \times 10^8 \times \exp (-12.732/T_9).$$

On the other hand Hoffman et al. (1997) used Wrean et al. (1994);

$$N_A\langle\sigma v\rangle_{\text{WREAN}} = 6.476 \times 10^{13}/T_9^{2/3} \exp (-23.8702/T_9^{1/3}) \times (1.0 - 0.3270 \times T_9^{1/3})$$
$$+ 6.044 \times 10^{-3}/T_9^{3/2} \exp (-1.041/T_9)$$
$$+ 7.268/T_9^{3/2} \exp (-2.063/T_9)$$
$$+ 3.256 \times 10^4/T_9^{3/2} \exp (-3.873/T_9)$$
$$+ 1.946 \times 10^5/T_9^{3/2} \exp (-4.966/T_9)$$
$$+ 1.838 \times 10^9/T_9^{3/2} \exp (-15.39/T_9).$$

We use the newest reaction rate for $^8$Li($\alpha$,n)$^{11}$B from X.Gu et al. (1995) and Hashimoto et al. (2005);

$$N_A\langle\sigma v\rangle_{\text{Gu}} = 4.929 \times 10^6/T_9^{3/2} \exp (-4.410/T_9)$$
$$+ 5.657 \times 10^8/T_9^{3/2} \exp (-6.846/T_9)$$
$$+ 4.817 \times 10^9/T_9^{3/2} \exp (-11.836/T_9)$$
$$+ 1.0 \times 10^{12}/T_9^{2/3} \exp (-19.45/T_9^{1/3}) \times (10.03/T_9^{1/3} + 4.814).$$

Then we can get numerically these results;

$$\int_{2.5}^{5.0} f(T_9)dT_9 = \begin{cases} 6.4 \times 10^8 s^{-3} & \text{(for Wrean et al. (Hoffman et al. 97))} \\ 7.4 \times 10^8 s^{-3} & \text{(for our version)} \end{cases} \quad (B1)$$

$$\int_{2.5}^{5.0} h(T_9)dT_9 = \begin{cases} 1.5 \times 10^4 s^{-7/2} & \text{(for X.Gu et al. version)} \\ 3.6 \times 10^3 s^{-7/2} & \text{(for Hashimoto et al. version)} \end{cases} \quad (B2)$$
C. Appendix B

Eq. (19) is solved by the following mathematical method. There exists such a function \( y(t) \) that satisfies this differential equation,

\[
\frac{dy}{dt} = f(t)y + h(t)y^2 ,
\]

where \( f(t) \) and \( h(t) \) are any functions of \( t \).

First, we replace \( y^{-1} = z \). Then \( z' = -y^{-2}y' \). This leads to the homogeneous first order differential equation,

\[
\frac{dz}{dt} + zf(t) = -h(t) .
\]

Next, multiplying the both sides by \( e^{F(t)} \), here \( F(t) = \int f(t)dt \), we have,

\[
\frac{d}{dt} \left( z(t)e^{F(t)} \right) = -h(t)e^{F(t)} .
\]

Now integrating the above equation between \( t = [t_i : t_f] \) we have,

\[
z(t_f)e^{F(t_f)} - z(t_i)e^{F(t_i)} = -\int_{t_i}^{t_f} h(t)e^{F(t)}dt ,
\]

\[
z(t_f) = z(t_i)e^{F(t_i) - F(t_f)} - e^{-F(t_f)} \int_{t_i}^{t_f} h(t)e^{F(t)}dt ,
\]

where \( F(t_i) - F(t_f) = \int_{t_f}^{t_i} f(t)dt \).

Finally, transforming the valuable \( z(t) \) back to \( y(t) \), we can get the solution:

\[
\frac{1}{y_{n,f}} = \frac{1}{y_{n,0}} \exp \left[ \int_{t_f}^{t_i} f(t)dt \right] + \int_{t_f}^{t_i} h(t) \exp \left[ \int_{t_f}^{t_i} f'(t')dt' \right] dt .
\]

(C3)
D. Appendix C

Let us suppose that there exists such a function \( p(T) \) such that \( p(T) \) is very large for the \( T_i \) of interest. Then,

\[
p(T) \approx p(T_i) \exp\left\{ -\left[ \frac{\ln p(T)}{T_i} \right] (T_i - T) \right\}.
\]

Because \( p(T) = \exp[\ln p(T)] = \exp[p(T_i) + \left[ \frac{\ln p(T)}{T_i} \right] (T - T_i)] \),

we can deduce the following formula:

\[
p(T) \approx \left\{ \alpha_1 h(T_i) e^{\alpha_0 \int_{T_f}^{T_i} f(T')dT'} \right\} \times \exp\left[ -\left( \frac{h'(T_i)}{h(T_i)} + \alpha_0 f(T_i) \right) (T - T_i) \right], \tag{D1}
\]

where we have replaced \( \left( \frac{h'(T_i)}{h(T_i)} + \alpha_0 f(T_i) \right) \) with \( \beta_0 \).

\[
\int_{T_f}^{T_i} p(T)dT \approx \left\{ \alpha_1 h(T_i) e^{\alpha_0 \int_{T_f}^{T_i} f(T')dT'} \right\} \times \int_{T_f}^{T_i} e^{-\beta_0 (T_i - T)}dT
\]

\[
= \left\{ \alpha_1 h(T_i) e^{\alpha_0 \int_{T_f}^{T_i} f(T')dT'} \right\} \times \frac{1}{\beta_0} \left[ 1 - e^{-\beta_0 (T_i - T_f)} \right].
\]
Fig. 1.— Comparison of the final computed $r$-process abundances based upon our adopted network (Otsuki et al. 2003) for exponential models with $Y_e = 0.45$ and various values of $\tau_{dyn}$ and $s/k$ as labeled. Abundances are normalized to $Y/Y(Eu) = 1$ [for $Y(Eu) = Y(^{151}Eu) + Y(^{153}Eu)$]. For each figure $\tau_{dyn}$ is given in the upper left corner while values for $s/k$ are shown near the bottom. These models have been chosen so as to produce final abundances consistent with observation. Inserts in the upper right corner show an expanded view of the calculated and observed Th and U abundances as tabulated in Table 3 of Sasaqui et al. (2005b).
Fig. 2.— The minimum entropy required as a function of the dynamical timescale to produce the observed abundance of characteristic nuclei in the r-process based upon the numerical network calculation of Figure 1. The upper figure (points and dot-dashed lines) shows the entropy required to produce the 1st (lower curve), 2nd (middle curve) r-process peaks and (A=232) actinide nuclei (upper curve). The numerical result for the 2nd r-process peak is also shown on the lower figure where it is compared with the the analytic limits (dotted lines) deduced for the 2nd (lower line) r-process peak and (A=232) actinide nuclei (upper line). The solid lines show the entropy required when considering only the \(^4\)He\((\alpha,n)\)^9Be flow of Eq. (11) (i.e. Hoffman et al. (1997)) or the \(^4\)He\((t,\gamma)\)^7Li\((n,\gamma)\)^8Li\((\alpha,n)\)^11B flow alone as labeled.