Expansion of the investment portfolio performance assessment model based on value-at-risk using a time series approach

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Abstract. Portfolio performance assessment needs to be carried out before or after the investment decision is taken, in order to minimize the possibility of risk loss. This paper discusses the expansion of the investment portfolio performance appraisal model based on Value-at-Risk, where the analyzed stock returns on mean and volatility is non-constant. The aim is to increase the likelihood of achieving investment objectives by investors. In this paper the mean is estimated using autoregressive moving average models, while the non-constant volatility is estimated using generally autoregressive conditional heteroscedastic models. The estimator’s of mean and non-constant volatility are then used for the analysis of investment portfolio optimization. Portfolio optimization issues are followed based on the basic framework of the Mean-Value-at-Risk model. The solution to the investment portfolio optimization problem is done by using the Lagrange multiplier technique and the Kuhn-Tucker method. Assessment of investment portfolio performance is based on Reward to Value-at-Risk, which is then used to compare the two investment portfolios A and B are analyzed. The results of the analysis show that portfolio A has better performance than portfolio B. So it is recommended to investors to choose an investment portfolio A, to achieve a better level of success.

1. Introduction
In dealing with risky investments, investors must make a decision to choose an efficient portfolio that has better performance. To make a decision, an assessment of an efficient portfolio needs to be done. The performance assessment of an efficient portfolio can be carried out before or after the investment decision is taken [1][2][3]. Efficient portfolio performance evaluation is to increase the likelihood of achieving investor goals. In conditions of uncertainty, investors cannot choose investment opportunities only by considering the level of profit offered. Investors need to consider the element of risk [4][5]. Therefore, the assessment of investment performance will be based on the level of profit and risk [6][7].
Portfolio risk is the possibility of a level of profit deviating from what is expected. Therefore, certain dispersal measures are often used as a measure of risk [8][9]. Standard deviation or variance is often used as a measure of investment portfolio risk [10]. However, many loss risk events exceed the standard deviation or variance. Therefore, the idea arises to measure risk using a quantile, or more popularly called Value-at-Risk (VaR) [10][11]. The amount of VaR depends on the average parameter value and volatility of a stock return, as well as the probability of possible risk of loss. Stock returns often have a non-constant mean and volatility, and even have long memory effects [12][13].

This paper aims to analyze portfolio performance measurement based on VaR risk measure. Average and volatility as constituents in VaR will be analyzed using a time series approach. The average is estimated using autoregressive fractionally integrated moving average (ARFIMA) models. Non-constant volatility is estimated using the generally autoregressive conditional heteroscedastic (GARCH) models. For efficient portfolio fractionally integrated moving average (ARFIMA) models. While to measure portfolio performance is done using the Reward to Value-at-Risk (RVaR) approach. The application of this method is used to analyze ten stocks traded in the capital market in Indonesia. The aim is to compare and choose a portfolio that has better performance than other portfolios. This study is useful for investors to find out the performance of two investment portfolios, where risk is measured by Value-at-Risk, where data follows a time series pattern.

2. Methodology
Suppose \( P_{it} \) and \( \hat{r}_{it} \) successively stated prices and stock returns \( i (i = 1, \ldots, N \) and \( N \) the number of stocks analyzed), at the time \( t (t = 1, \ldots, T, \) \( T \) data observation period). Stock returns \( \hat{r}_{it} \) calculated using formula \( \hat{r}_{it} = \ln(P_{it} / P_{i(t-1)}) \) [5; 14]. The return data model estimation is then performed on mean and volatility as follows.

2.1 Modeling of mean and volatility

**Modeling of mean.** Identification of long memory effects on stock returns data \( \hat{r}_{it} \). Identification is done using the rescale (R / S) method or the Geweke and Porter-Hudak (GPH) methods. Estimation of fractional differentiation parameters \( d_{i} \) (\( i \in \{1, \ldots, N \} \) \( \) and \( N \) the number of stocks analyzed), carried out using the maximum likelihood method [11; 13]. Confidence interval \((1 - \alpha)100\%\) for \( d_{i} \) is \( \hat{d}_{i} - \frac{\sigma_{d}}{2} < d_{i} < \hat{d}_{i} + \frac{z_{1-\alpha}/\sigma_{d}}{2} \) with \( \hat{d}_{i} \) is estimator of \( d_{i} \), and \( z_{1-\alpha}/\sigma_{d} \) standard normal distribution percentile if given a level of significance \( \alpha \). Suppose \( d_{i} \) fractional differentiation to be tested by the hypothesis. Suppose also \( \sigma_{d} \) standard deviation of \( d_{i} \). Hypothesis testing is carried out against \( H_{0}: \hat{d}_{i} = 0 \) and \( H_{1}: \hat{d}_{i} \neq 0 \) use \( z_{d_{i}} = \hat{d}_{i} / \sigma_{d} \). Test criteria are reject \( H_{0} \) if value \( z_{d_{i}} < z_{1-\alpha}/2 \) atau \( z_{d_{i}} > z_{1-\alpha}/2 \) [12; 14].

Fractional differentiation processes are defined as:

\[
(1 - B)^{d_{i}} \hat{r}_{it} = a_{it}, \quad -0.5 < d_{i} < 0.5;
\]

with \( a_{it} \) is residual white noise series, and \( B \) stated the backshift operator. If fractional differentiation series \( (1 - B)^{d_{i}} \hat{r}_{it} \) follow the model of ARMA \((p, q)\), then \( \hat{r}_{it} \) called the autoregressive fractionally integrated moving average degree process \( p \) and \( q \) or ARFIMA \((p, d, q)\) [14]. Model equation of ARFIMA \((p, d, q)\) is:

\[
\hat{r}_{it} = \psi_{0} + \sum_{g=1}^{p} \psi_{ig} \hat{r}_{it-g} + a_{it} + \sum_{h=1}^{q} \theta_{ih} a_{it-h},
\]
with $\psi_{i0}$ constants, and $\psi_{ig} \ (g = 1, ..., p)$, and $\theta_{ih} \ (h = 1, ..., q)$ parameter coefficient of the mean stock return model $i \ (i = 1, ..., N$ and $N$ the number of stocks analyzed). Assumed $\{a_{it}\}$ residual white noise sequence with zero mean and variance $\sigma_{a}^2$ [14][15].

The stages of the average modeling process include: (i) Model identification, (ii) Estimation of parameters, (iii) Test diagnosis, and (iv) Prediction [14].

**Modeling of volatility.** Stock return volatility modeling is performed using generalized autoregressive conditional heteroscedastic (GARCH) models. Suppose $\mu_{it}$ and $\sigma_{it}^2$ successive mean and volatility of stock returns $i \ (i = 1, ..., N$ and $N$ the number of stocks analysed), at the time $t \ (t = 1, ..., T$ and $T$ data observation period). Residual $a_{it}$ the above has an equation

$$a_{it} = \sigma_{it} e_{it}, \quad \sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} a_{it-k}^2 + \sum_{l=1}^{n} \beta_{il} \sigma_{it-l}^2 + \varepsilon_{it}, \quad (3)$$

with $\alpha_{i0}$ constants, and $\alpha_{ik} \ (k = 1, ..., m)$ and $\beta_{il} \ (l = 1, ..., n)$ parameter coefficient of stock return volatility model $i \ (i = 1, ..., N$ and $N$ the number of stocks analyzed). Assumed $\{e_{it}\}$ sequence of random variables are mutually independent and have identical distributions (iid) with an average of 0 and variance $1 \alpha_{i0} > 0, \ alpha_{ik} \geq 0, \ beta_{il} \geq 0$, and $\sum_{k=1}^{m} (\alpha_{ik} + \beta_{ik}) < 1$ [14][16].

Volatility $\sigma_{it}^2$ will follow the GARCH model with degrees $m$ and $n$ or written as $GARCH(m,n)$, when:

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} a_{it-k}^2 + \sum_{l=1}^{n} \beta_{il} \sigma_{it-l}^2 + \varepsilon_{it}, \quad (3)$$

with $\alpha_{i0}$ constants, and $\alpha_{ik} \ (k = 1, ..., m)$ and $\beta_{il} \ (l = 1, ..., n)$ parameter coefficient of stock return volatility model $i \ (i = 1, ..., N$ and $N$ the number of stocks analyzed). Assumed $\{e_{it}\}$ sequence of random variables are mutually independent and have identical distributions (iid) with an average of 0 and variance $1 \alpha_{i0} > 0, \ alpha_{ik} \geq 0, \ beta_{il} \geq 0$, and $\sum_{k=1}^{m} (\alpha_{ik} + \beta_{ik}) < 1$ [14][16].

Stages of the volatility modeling process include: (i) Estimation of the average model, (ii) ARCH effect test, (iii) Model identification, (iv) Estimation of the volatility model, (v) Test diagnosis, and (vi) Prediction [14].

Using the mean model (2) and volatility (3), the prediction is carried out aimed at calculating the mean $\hat{\mu}_{it}$ and variance $\hat{\sigma}_{it}^2$, namely the prediction of the $l$-step forward after the time period to $T$ [14].

2.2 **Portfolio modal and Value-at-Risk**

In the formation of an investment portfolio $w$, will relate to the proportion of funds allocated to each of the shares analyzed. Suppose $w_i$ is the proportion of funds allocated to stocks $i$, where $\sum_{i=1}^{N} w_i = 1$, then portfolio return can be expressed as:

$$R_{wt} = \sum_{i=1}^{N} w_i R_{it}, \quad (4)$$

where $R_{wt}$ portfolio return $w$ at time $t$, and $N$ the number of stocks in the formation of a portfolio[10; 14].

Based on (5), the mean (expectation) portfolio is obtained with weights $w_i$ can be stated as:

$$\hat{\mu}_{wt} = \sum_{i=1}^{N} w_i \hat{\mu}_{it} \quad (5)$$

While portfolio variance can be expressed as:

$$\sigma_{wt}^2 = \sum_{i=1}^{N} w_i^2 \sigma_{it}^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \ ; \ i \neq j \quad (6)$$

where $\sigma_{ij} = \text{Cov}(r_{it}, r_{jt})$.

Suppose the amount of the initial investment is one unit, and the level of significance of the risk of loss is $\alpha$, then $VaR$ for portfolios with weights $w_i$ is:
\[ VaR_{wt} = z_\alpha \tilde{\sigma}_{wt} - \tilde{\mu}_{wt} \]  

where \( z_\alpha \) is the percentile value of a standard normal distribution with a level of significance \( \alpha \) \([10; 11; 14]\).

### 2.3 Mean-VaR Portfolio Optimization

Suppose that the vector values of expectations and covariance matrices are given successively by:
\[ \mu^T = (\mu_1, \ldots, \mu_N), \quad \Sigma = (\sigma_{ij})_{i,j=1,\ldots,N}, \quad \text{with } \sigma_{ij} = \text{Cov}(r_{it}, r_{jt}), \]
\( i, j = 1, \ldots, N \). Refer to the previous discussion, the weight of stock returns in a portfolio \( w^T = (w_1, \ldots, w_N) \), where \( \sum_{i=1}^{N} w_i = 1 \) or \( e^T w = 1 \) with \( e = (1, \ldots, 1)^T \) vector with one-on-one element. Referring to equation (5) can be rewritten as:
\[ \mu_{wt} = E[R_{wt}] = \mu^T w, \]  
and equation (6) is rewritten as:
\[ \sigma^2_{wt} = Var(R_{wt}) = \sigma^T \Sigma \sigma. \]  

Use the level of significance \( \alpha \), the percentile \( z_\alpha \) obtained from the standard normal distribution table. So the Value-at-Risk investment portfolio equation (13) can be rewritten as:
\[ VaR_{wt} = z_\alpha \sigma_{wt} - \mu_{wt} = z_\alpha (w^T \Sigma w)^{1/2} - \mu^T w. \]  

A portfolio \( w^* \) called \( \text{(Mean-VaR)} \) efficiently if there is no portfolio \( w \) with \( \mu_{wt} \geq \mu_{wt}^* \) and \( VaR_{wt} < VaR_{wt}^* \) \([3; 7; 9]\). To get an efficient portfolio, the objective function is determined maximally
\[ \text{max}(2x \mu^T w - z_\alpha (w^T \Sigma w)^{1/2} + \mu^T w) \]  

kendala \( e^T w = 1 \)

Because of the covariance matrix \( \Sigma \) semidefinite positive, the objective function is quadratic concave. Therefore, (12) is a concave quadratic optimization problem. The Lagrange function is given by:
\[ L(w, \lambda) = (2\tau + 1) \mu^T w - z_\alpha (w^T \Sigma w)^{1/2} + \lambda (e^T w - 1). \]

Using the Kuhn-Tucker theorem, the optimality requirement is:
\[ \nabla L / \nabla w = (2\tau + 1) \mu - z_\alpha \Sigma w / (w^T \Sigma w)^{1/2} + \lambda e = 0 \quad \text{dan} \quad \nabla L / \nabla \lambda = e^T w - 1 = 0. \]  

Based on algebraic calculations, if for example \( A = e^T \Sigma^{-1} e, \quad B = (2\tau + 1) (\mu^T \Sigma^{-1} e + e^T \Sigma^{-1} \mu) \) and \( C = (2\tau + 1)^2 (\mu^T \Sigma^{-1} \mu - z_\alpha^2) \), then the ABC formula is obtained:
\[ \lambda = [-B + (B^2 - 4AC)^{1/2}] / 2A \]  

For \( \tau \geq 0 \), solving equation (12) obtained a portfolio with weight vector \( w^* \)
\[ w^* = \frac{(2\tau + 1) \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{(2\tau + 1) e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}. \]  

If vector \( w^* \) substituted into equations (8) and (10), then the average portfolio return and optimum Value-at-Risk will be obtained \([2; 9]\).

### 2.4 Reward-to Value-at-Risk (RVaR)

Because in this case portfolio risk is measured based on Value-at-Risk (VaR), the size of the Reward to Volatility (RVaR) portfolio performance is expanded to RVaR. Meaning RVaR measures the
comparison between portfolio risk premiums and Value-at-Risk (VaR). The mathematical equation of RVaR is:

\[
RVaR = \frac{\mu_{wt} - \mu_f}{VaR_{wt}}.
\]

(15)

Where \(\mu_{wt}\) mean portfolio return at time \(t\), \(\mu_f\) mean of risk-free asset return, and \(VaR_{wt}\) Value-at-Risk portfolio \(w\) at time \(t\). Measures of good portfolio performance are determined based on the greatest value of RVaR [11].

3. Result and Analysis

The analyzed stock data is accessed through the website http://www.finance.go.id/\. The data consists of 10 (ten) selected shares, for the period January 2, 2015 up to June 4, 2018, which includes shares: INDF, DEWA, AALI, LSIP, ASII, TURB, HDMT, BMRI, UNTR, BBRI . Next, it is called sequentially \(S_1\) up to \(S_{10}\) . Share prices include the opening price, the highest price, the lowest price, and the closing price, but only the closing price is closed.

3.1 The results of mean and volatility modeling

In this section ten shares are analyzed \(S_1\) up to \(S_{10}\) . The analysis starts with calculating the return of each stock, then identifying the long memory effect, estimating the average model and the volatility model.

Identification of long memory effects. To identify long memory effects, it is done by estimating fractional differentiation parameters \(d_i\) \((i=1,\ldots,10)\) in equation (1). Estimates were made using the Gewek and Porter-Hudak methods, with the help of software R. The estimation results obtained fractional differentiation values \(\hat{d}_1=0.333742\), \(\hat{d}_7=0.016421\), and \(\hat{d}_{10}=-0.062398\). Based on the results of the hypothesis test shows that stocks \(S_1\), \(S_7\) and \(S_{10}\) there is a significant long memory effect, with fractional differentiation \(\hat{d}_1\), \(\hat{d}_7\) and \(\hat{d}_{10}\). Whereas other stocks do not have long memory effects.

Results of fractional differentiation estimates for stocks \(S_1\) up to \(S_{10}\) given in Table-1 column \(\hat{d}_i\).

Mean model estimates. In this section Eviews 9 software is used for estimating the mean model. The fractional stock return data will be estimated for the mean model. The first stage is the identification and estimation of the mean model. Identification was carried out through the fractional function (ACF) and partial autocorrelation function (PACF) samples. Based on the ACF and PACF patterns, tentative models are possible for the return data of each stock \(S_1\) up to \(S_{10}\) determined. Then the model estimation is done by referring to equation (2). From the model estimation and diagnostic test can be obtained a significant mean model of stocks \(S_1\) up to \(S_{10}\), the results are given in Table-1 below.

Estimation of volatility models. In this section Eviews 9 software is also used to estimate the volatility model. First, the detection of the existence of an element of autoregressive conditional heteroscedasticity (ARCH) against residuals \(\sigma_t\) \((i=1,\ldots,10)\) from the mean model. Detection is done using the ARCH-LM test method. Detection results indicate that the calculation value of \(\chi^2\) \((\text{obs} \ast R-Square)\) on each stock \(S_1\) up to \(S_{10}\) generate a probability of 0.0000 or 5% smaller, which means there are elements of ARCH.
Second, identification and estimation of volatility models are carried out. The volatility model used is the generalized autoregressive conditional heteroscedasticity (GARCH) model referring to equation (3). Based on correlogram residual squares $\hat{\sigma}^2_{it} (i = 1,...,10)$, set a tentative volatility model for each stock $S_1$ up to $S_{10}$. Estimation of the volatility model is carried out simultaneously with the average model. In the volatility modeling process it is also shown that based on the ARCH-LM test, residuals $\hat{e}_{it} (i = 1,...,10)$ from the volatility model of each stock $S_1$ up to $S_{10}$ is white noise. The estimation results of the average model and volatility for stocks $S_1$ up to $S_{10}$ given in Table-1 column of "Time Series Model". Furthermore, the mean and volatility equations are used to estimate values $\hat{\mu}_t = \hat{\mu}_t (1)$ and $\hat{\sigma}^2_t = \hat{\sigma}^2_t (1)$ $(i = 1,...,10)$; is a recursive 1-step forward prediction. The results for each stock $S_1$ up to $S_{10}$ given in Table-1 column of $\hat{\mu}_t$ and $\hat{\sigma}^2_t$.

### 3.2 The results of portfolio optimization A and B

In this section analyzes two portfolios, namely portfolios A and B. portfolio of stocks that were analyzed include stocks $S_1$ up to $S_{10}$, which is divided into two portfolios. Portfolio A consists of stocks $S_1$ up to $S_{5}$, while portfolio B consists of stocks $S_6$ up to $S_{10}$.

#### Portfolio Optimization A

A portfolio of five stocks, namely stock $S_1$ up to $S_{5}$. Estimator based on the mean stock return $S_1$ up to $S_{5}$, in Table-1 the average vector is arranged as follows:

$$\bar{\mu}^T = (0.015399 \ 0.039007 \ 0.003315 \ 0.008672 \ -0.000262)$$

Portfolio A consists of five stocks, so the unit vector $e^T = (1 \ 1 \ 1 \ 1 \ 1)$. Next, use a volatility estimator in Table-1 and a return covariance estimator between stocks $S_1$ up to $S_{5}$ covariance matrix is formed as:

| Stocks ($S_i$) | Time Series Model | Frac. Diff. ($\hat{\phi}_i$) | Mean ($\hat{\mu}_i$) | Variance ($\hat{\sigma}^2_i$) |
|---------------|-------------------|-------------------------------|----------------------|-------------------------|
| $S_1$         | ARFIMA(1, $\hat{\phi}_0$)-GARCH(1,1) | 0.333742                     | 0.015399             | 0.002643                |
| $S_2$         | ARFIMA(2, $\hat{\phi}_0$)-ARCH(1)-M | 0                             | 0.039007             | 0.002797                |
| $S_3$         | ARFIMA(0, $\hat{\phi}_1$)-GARCH(3,3) | 0                             | 0.003315             | 0.001331                |
| $S_4$         | ARFIMA(1, $\hat{\phi}_1$)-GARCH(1,1) | 0                             | 0.008672             | 0.001921                |
| $S_5$         | ARFIMA(0, $\hat{\phi}_1$)-GARCH(1,1) | 0                             | -0.000262            | 0.001873                |
| $S_6$         | ARFIMA(1, $\hat{\phi}_1$)-FIGARCH(1,1) | 0                             | 0.022085             | 0.001181                |
| $S_7$         | ARFIMA(1, $\hat{\phi}_1$)-GARCH(1,1) | 0.016421                      | 0.003564             | 0.001073                |
| $S_8$         | ARFIMA(0, $\hat{\phi}_1$)-EGARCH(1,1) | 0                             | 0.001594             | 0.001411                |
| $S_9$         | ARFIMA(2, $\hat{\phi}_2$)-TGARCH(1,1) | 0                             | 0.020709             | 0.013362                |
| $S_{10}$      | ARFIMA(0, $\hat{\phi}_1$)-GARCH(1,1) | -0.062398                     | -0.000865            | 0.001237                |
Based on the matrix $\Sigma_A$, then the inverse matrix can be calculated $\Sigma_A^{-1}$.

Optimization is carried out based on portfolio problems in equation (11). Next, vector $\mu^T$ and $\epsilon^T$ and matrix $\Sigma_A^{-1}$, used to calculate the optimum weight vector using equation (14). Where risk tolerance $\tau$ on condition $\tau \geq 0$ in portfolio optimization here is simulated by taking several values that meet the requirements $\epsilon^T w = 1$. Assuming short sale is not permitted, taking the risk tolerance value is only for value $0 \leq \tau \leq 0.486$. This is due to the risk tolerance value $\tau > 0.486$ produce a negative weight.

For each risk tolerance value $0 \leq \tau \leq 0.486$ generate portfolio mean return $\mu_A$ and the level of risk $VaR_A$ different. Curved lines between pairs $\mu_A$ and $VaR_A$ form an efficient surface. Where the mean minimum portfolio return is 0.015736 with a minimum VaR of 0.019629, and the mean highest portfolio return is 0.025191 with a maximum VaR of 0.025022.

Ratio between $\mu_A$ and $VaR_A$ the biggest is 0.90099912 or obtained when risk tolerance $\tau = 0.486$. Ratio between $\mu_A$ and $VaR_A$ continue to experience an increase in risk tolerance intervals $0 \leq \tau \leq 0.486$. Based on Mean-VaR portfolio optimization analysis, the optimal portfolio composition of stocks $S_1$ up to $S_5$ produce a weight vector $w^T = (0.21701 \ 0.51389 \ 0.09856 \ 0.17038 \ 0.00016)$. Where the composition of the optimal portfolio produces $\mu_A = 0.025191$ with $VaR_A = 0.025022$ which is also a maximum portfolio.

**Portfolio Optimization B**

Portfolio B consists of five stocks, namely stocks $S_6$ up to $S_{10}$. From the average estimator of stock returns $S_6$ up to $S_{10}$, in Table-1 the average vector is arranged as follows:

$$\mu_B = (0.022085 \ 0.003564 \ 0.001594 \ 0.020709 \ -0.000865)$$

Portfolio B consists of five stocks, so the unit vector $\epsilon^T = (1 \ 1 \ 1 \ 1 \ 1)$. Next, from the volatility estimator in Table-1 and the return covariance estimator between stocks $S_6$ up to $S_{10}$ covariance matrix are formed as follows:

$$\Sigma_B = \begin{pmatrix}
0.001181 & 2.05888 \times 10^{-7} & 2.63104 \times 10^{-7} & 1.60945 \times 10^{-7} & 1.60497 \times 10^{-7} \\
2.05888 \times 10^{-7} & 0.001073 & -2.65181 \times 10^{-8} & -4.58792 \times 10^{-8} & -2.38116 \times 10^{-8} \\
2.63104 \times 10^{-7} & -2.65181 \times 10^{-8} & 0.001411 & 7.18467 \times 10^{-7} & 7.62223 \times 10^{-7} \\
1.60945 \times 10^{-7} & -4.58792 \times 10^{-8} & 7.18467 \times 10^{-7} & 0.0013362 & 6.47969 \times 10^{-7} \\
1.60497 \times 10^{-7} & -2.38116 \times 10^{-8} & 7.62223 \times 10^{-7} & 6.47969 \times 10^{-7} & 0.001233
\end{pmatrix}$$

Based on the matrix $\Sigma_B$, then the inverse matrix can be calculated $\Sigma_B^{-1}$. 


The portfolio optimization problem is based on equation (11). Using vectors $\mu^T$ and $\epsilon^T$ and matrix $\Sigma_B^{-1}$, optimal weight vector is calculated using equation (14). Risk tolerance $\tau$ on condition $\tau \geq 0$ in portfolio optimization here is simulated by taking several values that meet the requirements $\epsilon^T \mathbf{w} = 1$. Assuming short sale is not permitted, taking the risk tolerance value is only for value $0 \leq \tau \leq 0.409$. Because for the risk tolerance value $\tau > 0.409$ produce a negative weight.

Each risk tolerance value $0 \leq \tau \leq 0.409$ produce $\hat{\mu}_B$ and $\text{VaR}_B$ different. Curved lines between pairs $\hat{\mu}_B$ and $\text{VaR}_B$ it forms an efficient surface. Where the mean minimum portfolio return is generated $\hat{\mu}_B = 0.013436$ with a minimum risk level $\text{VaR}_B = 0.014542$. The highest mean return portfolio is $0.018832$ with a maximum VaR of $0.017021$.

Ratio between $\hat{\mu}_B$ and $\text{VaR}_B$ the biggest is $0.9509312$ or obtained when risk tolerance $\tau = 0.409$. Ratio between $\hat{\mu}_B$ and $\text{VaR}_B$ continue to experience an increase in risk tolerance intervals $0 \leq \tau \leq 0.409$. Based on Mean-VaR portfolio optimization analysis, the optimal portfolio composition of stocks $S_6$ up to $S_{10}$ is a stock weight vector $\mathbf{w}^T = (0.46850 \ 0.09989 \ 0.04210 \ 0.38933 \ 0.00018)$. Where the composition of this optimal portfolio produces $\hat{\mu}_B = 0.018832$ dan $\text{VaR}_B = 0.017021$ which is also $\hat{\mu}_B$ and $\text{VaR}_B$ maximum.

### 3.3 Performance of portfolio A and B

In this section an assessment of the performance of portfolios A and B is carried out, the aim is to determine the performance of each portfolio at various risk tolerances that meet both portfolios. Performance appraisal is carried out referring to equation (15). Calculation $RVaR_A$ for portfolio A and $RVaR_B$ for portfolio B, the results is summarized in the following Table-2. Based on the RVaR values presented in Table 2, it appears that for the risk tolerance value $0 \leq \tau \leq 0.50$ the values $RVaR_A$ greater than values $RVaR_B$.

| No | $\tau$ | $RVaR_A$ | $RVaR_B$ |
|----|--------|----------|----------|
| 1  | 0.00   | 0.666721 | 0.387433 |
| 2  | 0.05   | 0.697884 | 0.405700 |
| 3  | 0.10   | 0.728374 | 0.423855 |
| 4  | 0.15   | 0.757918 | 0.441876 |
| 5  | 0.20   | 0.786218 | 0.459663 |
| 6  | 0.25   | 0.812914 | 0.477149 |
| 7  | 0.30   | 0.835589 | 0.494258 |
| 8  | 0.35   | 0.859494 | 0.510899 |
| 9  | 0.40   | 0.878353 | 0.526912 |
| 10 | 0.45   | 0.893241 | 0.542234 |
| 11 | 0.50   | 0.900935 | 0.556655 |
| 12 | 0.55   | -        | 0.570065 |
| 13 | 0.60   | -        | 0.582166 |
| 14 | 0.658  | -        | 0.594192 |

For more details, RVaR values in Table-2 can be presented in graphical form as given by Figure 1.
In Figure-1 it is also seen that for the risk tolerance value $0 \leq \tau \leq 0.50$ graph of $RVaR_A$ always above than the graph of $RVaR_B$. This situation shows that portfolio A performs better than portfolio B. Therefore, it is advisable for investors to choose portfolio A which consists of stocks $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$.

4. Conclusion

In this paper a discussion has been carried out on the expansion of the investment portfolio performance assessment model based on Value-at-Risk using a time series approach. Based on the identification of the long memory effect shows that stock returns $S_1$, $S_7$ and $S_{10}$ there is an element of long memory. Average modeling and non-constant volatility indicate that stocks return $S_1$, $S_7$ and $S_{10}$ following the ARFIMA-GARCH model; $S_2$ ARMA-ARCH-M model; $S_4$, $S_4$ and $S_5$ ARMA-GARCH model; $S_6$ ARMA-FIGARC model; $S_8$ ARMA-EGARCH model; and $S_9$ ARMA-TGARCH model. The average model and volatility are used to estimate the average values and variances of each stock. The portfolio A is composed of $S_1$ up to $S_5$, whereas portfolio B consists of $S_6$ up to $S_{10}$. Portfolio optimization is formed based on the Mean - Value-at-Risk model. Portfolio performance appraisal has been carried out based on the Reward to Value-at-Risk approach, and the results show that portfolio A has better performance than portfolio B. Based on the analysis of investment portfolios which include stocks $S_1$ up to $S_{10}$, investors are recommended to choose portfolio A. The limitation of this study is that the comparison of the performance of the investment portfolio is only valid if the risk is measured by Value-at-Risk, and the data follows a time series pattern.

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