The use of light polarization in weak-lensing inversions

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ABSTRACT

The measurement of the integrated optical polarization of weakly gravitationally lensed galaxies can provide considerable constraints on lens models. The method outlined depends on the fact that the orientation of the direction of optical polarization is not affected by weak gravitational lensing. The angle between the semi-major axis of the imaged galaxy and the direction of integrated optical polarization thus informs one of the distortion produced by the gravitational lensing. Although the method depends on the polarimetric measurement of faint galaxies, large telescopes and improved techniques should make such measurements possible in the near future.

Key words: Cosmology: theory – dark matter – gravitational lensing – polarization

1 INTRODUCTION

Gravitational lensing of distant galaxies by foreground galaxy clusters provides a powerful method of determining the distribution of matter on a large scale. Its strength lies in the fact that it is sensitive to all gravitating matter, and that it gives the matter distribution directly, without invoking other physics or other assumptions. In this way it is different from other methods based on such mechanisms as the Sunyaev-Zeldovich effect which depend on the hydrodynamical state of the cluster ([Wu & Fang 1996] and references therein).

There are of course weaknesses of the method which arise from the limited data available, and the difficulty with which these data are obtained. With the improvement of technique we can expect these problems to be less of an obstacle in the future.

Considerable effort has gone into inverting images produced by gravitational lensing by clusters (Kochanek 1990), Lupino & Kaiser 1997, Bonnet et al. 1993, Seitz et al. 1996) to find the properties of the gravitational field and mass of the lensing cluster. One distinguishes two regimes - that of strong lensing, where the mass density is near the critical mass density for the lensing system, which produces arcs, extreme distortions, and multiple images of galaxies, and weak lensing where only minor distortions are observed.

In this paper we concentrate entirely on the weak lensing regime which is appropriate for the outer part of galaxy clusters. In the weak lensing regime one is looking for distortions of background galaxies which are at high redshift and very faint. Redshifts are difficult to determine for most imaged galaxies, and the determination of their shape subject to considerable uncertainty. The effect of weak lensing on an elliptical source, as might be provided by an spiral galaxy inclined to the line of sight, is to change its apparent size, orientation and ellipticity. In principle measurement of this change can yield information about the gravitational field of the lens, and hence the matter distribution within the cluster. In practice this is difficult because one lacks information. Even if one assumes that both the distance of the lens and that of the source can be determined from redshift analyses, this still leaves an uncertainty in the intrinsic size, ellipticity and orientation of the source galaxy. These uncertainties have led researchers to treat the problem as a statistical one, where the background galaxies are treated as a statistical ensemble without any attempt to use information about their intrinsic properties. (Bonnet et al. 1993, Kaiser et al. 1995, Seitz et al. 1996)

If in some way one’s ignorance of the source parameters could be reduced the lens parameters could be better determined. Measurement of the apparent brightness and redshift are obvious ways to improve inferences, but are difficult to achieve given the faintness of the images. Another interesting property that has yet to be exploited is the optical polarization of the source galaxy. Measurement of galactic polarization of high redshift galaxies would of course be difficult, given the faintness of the objects, although this has been carried out for a number of radio galaxies in both radio (Gabuzda et al. 1992) and optical bands (Tadhunter et al. 1992). Potentially it could yield valuable information. (Indeed, in the case of radio observations these measurements have been carried out with considerable accuracy with a resolution of milliarcseconds.) The reason is the following. Light polarization is not affected, at least if the lensing object has small angular momentum (in relativistic

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units) (Scheinder et al. 1992), by gravitational lensing in the sense that the direction of polarization will not be altered. For a typical spiral galaxy containing free electrons and dust in the galactic disc the integrated optical polarization arising from scattered starlight could be as high as 2 percent and one would expect it to be in a direction perpendicular to the disc (Bianchi et al. 1996). Even if other mechanisms such as dichroic extinction are at play, the reasonable assumption of large scale axial symmetry requires that the integrated polarization should be either in the direction of the major axis of the ellipse or perpendicular to it. Thus the integrated polarization could in principle be used to fix the orientation of the source galaxy, modulo 90 degrees.

The degree of polarization will obviously depend on the angle of inclination of the source galaxy to the line of sight and the number of scattering particles in the galactic disc. If we assume Thomson or Rayleigh scattering and axial symmetry in the distribution of sources and scatterers, one can show (Simmons & Audit 1998) that the degree of polarization for optically thin galaxies is proportional to \( \sin^2 i \) and the optical depth along the galactic radius, where \( i \) is the angle of inclination of the axis of symmetry of the galaxy to the line of sight.

One signature of a lensed galaxy would be an observed deviation of the direction of polarization from the perpendicular to the semi-major axis of the image ellipse. In principle it is possible for the direction of integrated polarization to be in the direction of the semi-major axis of the ellipse, owing to other mechanisms as discussed by several authors (Draper 1995, Scarrott et al. 1990, Wood 1997, and Wood and Jones 1997). Small scale deviations from rotational symmetry produced by clumpiness in the scattering distribution, as discussed by Witt & Gordon 1999, or inhomogeneities in the galactic magnetic field in the case of dichroism, should not greatly affect the direction of integrated polarized flux although the degree of integrated polarization could be reduced.

In the ideal situation the degree of polarization should yield information about the inclination of the source galaxy to the line of sight. It would in fact, however, be difficult to accurately measure the degree of polarization, or to unambiguously infer from it the inclination, and hence the ellipticity on the sky of the spiral galaxy. (High inclination gives rise to high polarization.)

In this paper we investigate how measurement of galactic light polarization can be used to sharpen and improve the determination of the lens parameters. In section 2 we give a rough estimate of the degree of polarization produced in a spiral galaxy due to dust and electron scattering. Our main purpose in doing this is to obtain a ball-park value for the degree of polarization. More accurate modeling has been carried out for the optically thin case (Simmons & Audit 1998), and by Monte Carlo treatment of radiative transfer (Wood 1997, Wood and Jones 1997, and Bianchi et al. 1996) for the optically thick case, but with the view to obtaining a resolved polarization map rather than the integrated polarization. In section 3 we discuss how information on the inclination of the source galaxy together with measurements of the ellipticity and orientation of the images can be used to infer the parameters of the lens, and in section 4 we compare the results obtained when this polarimetric information is available with the case when it is not, in order to assess the usefulness of the method. The last section contains a short discussion of the feasibility of carrying out such a polarimetric measurement and our conclusions.

## 2 GALACTIC POLARIZATION

Measurement of optical polarization of spiral galaxies has been carried out for a few nearby galaxies (Scarrott et al. 1990, Neininger et al. 1990). Various mechanisms could produce this optical polarization, the most obvious being scattering by dust and electrons. Most recent studies however have largely been concerned with the detailed structure of the galaxies and the possible existence of appreciable magnetic fields. Accordingly they have aimed at providing a resolved map of polarization in optical and infrared, which could give some information not only about the distribution of scattering grains and electrons, but also about the presence of magnetic fields and of dust lanes near the galactic nucleus. Dust grains, assumed oriented by the magnetic field of the galaxy, preferentially absorb light along their longer axis, and hence could produce polarization perpendicular to this axis of the grain (dichroism). Such a model (Wood and Jones 1997) has been partly successful in explaining the observed pattern for several high inclination spiral galaxies where the extinction would be most pronounced. In such cases, near to the nucleus the polarization is parallel to the galactic plane. However, that is not necessarily the case for other spiral galaxies (Scarrott & Draper 1996).

In this paper we are interested only in the polarized flux integrated over the whole galaxy, which should in principle give an indication of the orientation of the galaxy. If the spiral galaxy has an axis of symmetry, so that its isophotes are ellipses, we should, on the grounds of symmetry, expect the linear polarization flux to be either perpendicular the major axis of the ellipse, or possibly, if such effects as dichroism were important, along the minor axis. Although the small scale deviations from axial symmetry have been observed for spiral galaxies at high inclinations, we should expect most of these effects to be largely cancelled out in the integrated flux, which we would expect to be either perpendicular to the major axis of the ellipse or along the major axis. The apparent rotation brought about by weak lensing would not usually be more than a few tens of degrees, and thus the determination of the direction of integrated polarization ‘modulo’ 90 degrees would be sufficient to provide pretty unambiguous information about the orientation of the source galaxy.

In the case of spiral galaxies, one would expect optical polarization to arise as a result of Thomson, Rayleigh and dust scattering of starlight primarily in the galactic disc of the spiral galaxy, with minor contributions from the halo. For Thomson and Rayleigh scattering, light scattered through 90 degrees will be entirely polarized in a direction perpendicular to the scattering plane. (For other scattering mechanisms we would expect the polarization to be either in the plane or perpendicular.) Thus if Thomson or Rayleigh scattering were dominant, a galaxy inclined to the line of sight should display...
polarization, and one would expect in the case of a rotationally symmetric disc that the direction of polarization to be along the minor axis of the ellipse (Bianchi et al. 1996), although in exceptional circumstances it could be along the major axis.

2.1 A simple model for the integrated polarization

One can show that under fairly general assumptions the degree of polarization produced by Thomson or Rayleigh scattering, in the single scattering regime depends on \( \sin^2 i \) and the total number of scattering particles (Simmons & Audit 1998). In this section we shall give a very simplified derivation of this result to obtain an order of magnitude for the galactic polarization.

Consider a completely flat galactic disc with the light source at the centre (see Fig. 1). Take the surface density of electrons in the disc to be \( \mathcal{N}_e \) and the luminosity of the galaxy to be \( L \). The axis of symmetry of the galaxy, which we have taken to be the \( z \)-axis, is at inclination \( i \) to the line of sight. We take the \( x \)-axis to be in the plane of symmetry.

The flux arriving at the scattering element at \( a \) is given by \( L/4\pi a^2 \). Thus the energy scattered per steradian per unit time into the line of sight by this element is simply

\[
dF = \frac{L \sigma}{4\pi a^2} \mathcal{N}_e(a) a \ d\theta \ da \ 3 \frac{3}{16\pi} (1 + \cos^2 \chi)
\]

where \( \theta \) is the angle between \( a \) and the \( x \)-axis, \( \chi \) is the scattering angle and \( \sigma \) the total scattering cross section. We obtain similar expressions for the polarized flux. In terms of the Stokes parameters referred to the scattering plane we have

\[
dF_Q = \frac{L \sigma}{4\pi a^2} \mathcal{N}_e(a) a \ d\theta \ da \ 3 \frac{3}{16\pi} \sin^2 \chi
\]

and

\[
dF_U = 0.
\]

To obtain the total scattered energy per steradian per unit time we integrate over the disc to obtain

\[
F = \int \int \frac{L \sigma}{4\pi a^2} \mathcal{N}_e(a) a \ 3 \frac{3}{16\pi} (1 + \cos^2 \chi) d\theta da.
\]

Using the properties of the Stokes parameters under rotations, total polarized energy \( F_Q \) expressed in the observers frame (with the polarimeter aligned with the axis of symmetry) is given by

\[
F_Q = \int \int \frac{L}{4\pi a^2} \mathcal{N}_e(a) a \ 3 \frac{3}{16\pi} \sin^2 \chi \cos 2\phi \ d\theta da
\]
and

\[ F_U = \int \int \frac{L \sigma}{4\pi a^2} N_e(a) a \frac{3}{16\pi} \sin^2 \chi \sin 2\phi d\theta da, \]

where \( \phi \) is the angle between the scattering plane and the plane defined by the line of sight and the axis of symmetry of the galaxy. The scattering angle \( \chi \) and the angle \( \phi \) may be expressed in terms of \( \theta \) and \( i \)

\[
\cos \chi = -\cos \theta \sin i
\]

and

\[
\cos \phi = \frac{\cos \theta \cos i}{\sin \chi}.
\]

Substitution of these into the integral expressions yields after a bit of reduction

\[ F_Q = \frac{L \sigma}{4\pi} \frac{3\pi}{8 \sin^2 i} \int \frac{N_e(a)}{a} da \]

and

\[ F_U = 0, \]

signifying that the polarization lies along the axis of symmetry (because \( F_Q \) is always greater or equal to zero). We shall ignore the unpolarized scattered power as this will be swamped by the direct light from the galaxy. The degree of polarization is simply \( F_Q/F_{\text{direct}} \) where \( F_{\text{direct}} \) is the direct power per steradian, \( L/4\pi \). Thus the degree of polarization is

\[ p = \frac{3\pi \sigma}{8 \sin^2 i} \int \frac{N_e(a)}{a} da. \]

The degree of polarization depends on \( \sin^2 i \). There are obvious problems in evaluating this integral for most density distributions because of the possible singularity at \( a = 0 \) which arises from the two dimensional treatment of the problem. To avoid such difficulties we take the surface density to be zero within a certain radius of the galactic centre. Again since we are really only interested in orders of magnitude here, we take the surface number density to be constant within an annulus with inner radius \( R_1 \) and outer radius \( R_2 \). We then obtain for the degree of polarization

\[ p = \frac{3\pi N_e \sigma}{8 \sin^2 i} \ln \frac{R_2}{R_1}. \]

If we denote the optical depth through the galactic disc by \( \tau \) and assume a geometric thickness for the galactic disc of \( \Delta \) then the degree of polarization may be written

\[ p = \frac{3\pi \sigma}{8 \sin^2 i} \frac{\Delta}{R_2 - R_1} \ln \frac{R_2}{R_1}. \]

Typically we can expect the ratio of the thickness of the disc to its radius to be 0.1, so that the degree of polarization is approximately \( p = 0.17 \sin^2 i \) where we take the value of \( \ln R_2/R_1 \) to be of the order of 1. Thus with an optical depth in the range 0.1 to 1 we obtain a maximum polarization of one to ten percent. The actual polarization depends of course also on the inclination of the galaxy through the \( \sin^2 i \) factor.

This simple model seems to give the right order of magnitude for the integrated polarization when compared to more sophisticated models (Bianchi et al. 1996).

### 3 THE LENS EQUATION AND WEAK LENSING

A simplified picture of the lensing situation is represented in figure 2 which serves to define the quantities entering into our equations. The lens equation relates the position, \( \eta \) of the source galaxy (in the source plane which is a distance \( D_s \) from the observer) to the position, \( \xi \), of the image galaxy in the lens plane, which is a distance \( D_d \) to the observer. (Distance here means the angular size distance). Elementary geometrical arguments yield

\[ \eta = \frac{D_s}{D_d} \xi - D_{ds} \alpha'. \]
where $\alpha'$ is the vector angular deflection produced by the lens. If we use angular coordinates instead this equation may be written

$$y = x - \alpha,$$

where $\alpha$ is defined to be $\alpha' D_{ds}/D_s$. The deflection angle is determined by the matter distribution in the lens plane. With the usual assumptions (see Schneider et al. 1992) we may write

$$\alpha = \nabla \Psi$$

where

$$\Psi(x) = \frac{1}{\pi} \int d^2 x' \kappa'(x') \ln |x - x'|$$

and $\kappa' = \Sigma/\Sigma_{crit}$, and $\Sigma_{crit} = c^2 D_s/4\pi G D_d D_{ds}$, where $\Sigma$ is the projected mass density. It follows that

$$\nabla^2 \Psi = 2 \kappa'.$$

The amplification matrix, $A$, is then defined as the inverse of the jacobian matrix $(\partial y/\partial x^i)$, and so yields

$$A^{-1} = \begin{pmatrix} 1 - \Psi_{11} & \Psi_{12} \\ \Psi_{12} & 1 - \Psi_{22} \end{pmatrix}.$$ (5)

It follows from a linear expansion of equation 2 that in the weak regime, an elliptical source (i.e. elliptical isophotes) will be transformed into an elliptical image (Kochanek 1990). The relation between the two depends on the properties of the lens, and is given below by equations (6-9) in terms of the characteristics of the source and image ellipses, and the lens. Following Kochanek, we characterise the source ellipse by the triplet $S = (\lambda_s, \Delta \lambda_s, \alpha_s)$, the image by $I = (\lambda_i, \Delta \lambda_i, \alpha_i)$ ($\alpha$ gives the orientation of the ellipse and $(\lambda + \Delta \lambda)^{1/2}$ (resp. $(\lambda - \Delta \lambda)^{1/2}$) its major (resp. minor) axis length) and the lens by $\mathcal{L} = (\kappa, \gamma, \theta)$. Effectively these provide

$$\Delta \lambda_s S_s = (\kappa^2 - \gamma^2) S_i \Delta \lambda_i$$

$$\lambda_s = \lambda_i (\kappa^2 + \gamma^2) + 2 \Delta \lambda_i \kappa C_i$$

$$\lambda_s \pm \Delta \lambda_s C_s = (\lambda_i \pm \Delta \lambda_i C_i) (\kappa \pm \gamma)^2$$

$$\Delta \lambda S = 2 \kappa \gamma \lambda_i + \Delta \lambda_i C_i (\kappa^2 + \gamma^2)$$

where we have defined: $C_x = \cos 2(\alpha_x - \theta)$ and $S_x = \sin 2(\alpha_s - \theta)$. There are 9 unknown parameters and three independent equations. Thus in principle any one of $S$, $I$, or $\mathcal{L}$ can be determined from a knowledge of the other two. In particular the source parameters, $S$, can be determined if one knows the image, $I$, and lens parameters, $\mathcal{L}$.

In what follows we shall also use the parameter $\epsilon$ describing the source and image ellipticity defined in terms of $\lambda$ and $\epsilon$.
\[ \Delta \lambda = \frac{1}{\sqrt{1 + \Delta \lambda / \lambda}}. \]

It is interesting to note, before making any attempt to solve these equations, that they have a number of scaling laws which should make their solution easier. Thus for a given lens, if \((\lambda_0, \Delta \lambda_0, \alpha_0, \lambda_0, \Delta \lambda_0, \alpha_0)\) is a solution, then so is \((x \lambda_0, x \Delta \lambda_0, \alpha_0, x \lambda_0, x \Delta \lambda_0, \alpha_0)\) where \(x\) is a constant. Moreover, for a given image, if \((\lambda_0, \Delta \lambda_0, \alpha_0, \kappa, \gamma, \theta)\) is a solution, then so also is \((y^2 \lambda_0, y^2 \Delta \lambda_0, \alpha_0, y \kappa, y \gamma, \theta)\). This means that one can arbitrarily take \(\lambda_0\) and \(\kappa\) to be 1. Dividing the equations \(\text{[1]}-\text{[13]}\) by \(\kappa^2 \lambda_0\) \((x = 1/\Delta \lambda_0\) and \(y = 1/\kappa)\) one obtains:

\[
\begin{align*}
\Delta I_s S_i & = (1 - \Gamma^2) S_i \Delta I_i \quad (10) \\
l_s & = (1 + \Gamma^2) + 2 \Delta I_i \Gamma C_i \quad (11) \\
l_s \pm \Delta I_s C_s & = (1 \pm \Delta I_i C_i)(1 \pm \Gamma)^2 \quad (12) \\
\Delta I_i C_s & = 2\Gamma + \Delta I_i C_i(1 + \Gamma^2) \quad (13)
\end{align*}
\]

where \(\Delta I_i = \Delta \lambda_i/\lambda_i\), \(l_s = \lambda_s/(\kappa^2 \lambda_i)\), \(\Delta I_s = \Delta \lambda_s/(\kappa^2 \lambda_i)\), and \(\Gamma = \gamma/\kappa\).

### 4 INFERRING THE LENS PARAMETERS

The determination of the lens parameters in the weak lensing regime is usually treated as a statistical problem. In the absence of additional information about the source galaxies, in particular about the orientation of their axis of symmetry (if they have one), most analyses to date have attempted to solve the problem of inferring the lens parameters by matching the distribution of source parameters obtained from the data by the lens model, with an assumed distribution of source galaxy parameters e.g. a uniform distribution of orientations of source galaxies. One can obtain the best fit model parameters by standard routines such as \(\chi^2\)-fitting. This sort of approach has been adopted by, for example, \(\text{[Bonnet et al. 1993]} \). Although very promising, the measurements are not good enough to provide accurate determination of the lens parameters. The techniques developed to reconstruct the cluster potential from the lensed galaxies are now extremely sophisticated and they treat with great care the observational and statistical uncertainties \(\text{[Kaiser et al. 1995], [Schneider 1996], [Kochanek 1990]}\)). We do not intend in this paper to go into such details. Our point is just to show that polarimetric measurements put strong constraints on the cluster potential reconstruction.

We have already argued that information about the source galaxies, and in particular about their orientation on the sky, which could be obtained from measurement of their optical polarization, would lead to more accurate determinations. Polarization measurements would undoubtedly be difficult for such faint galaxies. However, even the measurement of the polarization for a small subset of the galaxy sample can considerably improve the accuracy on the lens parameters determination. In the following section we illustrate this numerically for simple lens models.

To assess what can be obtained via polarization measurements, we have carried out a simple simulation. First we decide on a lens model, which fixes the values of \(\kappa(x, y), \gamma(x, y)\) and \(\theta(x, y)\) in the lens plane. We then randomly generate a sample of source galaxies from a given distribution with the following properties. We take the distribution of the symmetry axis to the line of sight to be isotropic, which implies a uniform distribution in orientation on the sky, and uniform distribution in ellipticity (if we assume the galactic discs to be circular). Since we are going to use the system \(\text{[10]-[13]}\) which contains only “reduced” variables, we do not need to know the brightness of the galaxies at this stage. These galaxies are then lensed by a given lens model to give a distribution of images. We then attempt to reestablish the lens parameters from the sample of images. We shall see below that the effectiveness of the retrieval depends crucially on the assumptions about the model, and whether or not we use potential information about the orientation of the source galaxies, which we argue, could be furnished by polarimetric data.

#### 4.1 A spherical lens model

To illustrate our argument we take an isothermal sphere with a core radius as our lens model \(\text{[Schneider et al. 1992]}\). The lens parameters are then given by

\[
\begin{align*}
\kappa & = \frac{1}{2} \Phi_0 \left( d^{-3/2} + d^{-1/2} \right) \quad (14) \\
\gamma & = \frac{1}{2} \Phi_0 \left( d^{-3/2} - d^{-1/2} \right) \quad (15)
\end{align*}
\]

where \(d = (1 + (r/r_0)^3)\) and \(r\) is the distance to the lens centre. In addition to the values of \(r_0\), and \(\Phi_0\), the coordinates of the centre of symmetry, introduce two further parameters. The centre of the lens can be inferred in various ways (see for example \(\text{[Kochanek 1990]}\)) and we shall assume in the rest of this article that it is already known. We have used a lens with \(r_0 = 1\) and \(\Phi_0 = 0.9\) (Fig.\(\text{[1]}\)).

If we assume values for the pair of parameters \((r_0, \Phi_0)\), and the image parameters are assumed to have been determined exactly, then it is possible to reconstruct the parameters for the source. If we assume also that the only information that
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Figure 3. Lens parameters for the isothermal sphere. $\kappa$ long-dashed line, $\gamma$ short-dashed line, $\Gamma = \gamma/\kappa$ full-line.

we have about the source is from the measurement of the polarization, which provides us with the orientation of the source galaxies with a standard error of $\epsilon$, then we can find the lens parameters that best fit the observed polarization direction. The natural way to do this is to carry out a chi-squared minimisation, i.e. we minimise the following expression over the lens parameters.

$$
\chi^2_1(r_0, \Phi_0) = \sum_{i=1}^{N} \frac{(\alpha_r^i(r_0, \Phi_0) - \alpha_r^m)^2}{\epsilon^2}.
$$

(16)

The summation in equation (16) is carried out over the set of galaxies whose polarization has been determined. $\alpha_r^i$ is the direction which has been reconstructed from the parameters of the lens model, and $\alpha_r^m$ is the measured direction. We have reconstructed lens parameters for the cases when the number of galaxies, $N$, for which the polarization has been measured is given by $N = 50, 10$ and $5$. For each of these cases we consider the error on the polarization direction, $\epsilon$, to be given by $2, 5,$ and $10\%$. In order to get an idea of the confidence one should have in the values of $r_0$ and $\Phi_0$ obtained in this way, we have for each pair of values $(N, \epsilon)$, generated 500 different samples of galaxies, and from these used the chi-square estimator of the values of $(r_0, \Phi_0)$. Figures 4 and 5 show ellipses in the $(r_0, \Phi_0)$ plane which contain 70% and 90% of the reconstructed values.

In order to quantify the information that the measurement of the polarization provides, we have also carried out a reconstruction of the lens (that is determined $r_0$ and $\Phi_0$) assuming no knowledge of the orientation of the source galaxies. We did this by comparing the distribution of source parameters inferred from an assumed pair of lens parameters and sample of image galaxies with a distribution of source galaxies that is assumed to be uniform in orientation and ellipticity. We compared the joint distributions in ellipticity and orientation by chi-squared fitting of the two distributions. First we divide the $\alpha_s, e_s$ plane ($\alpha_s \in [0, \pi]$ is the source orientation, and $e_s \in [0, 1]$ is its ellipticity) into $N_c$ bins. We then minimise

$$
\chi^2_2(r_0, \Phi_0) = \sum_{i=1}^{N_c} \frac{(n_i^s(r_0, \Phi_0) - n_i^s)^2}{n_i^s}
$$

(17)

with respect to $(r_0, \Phi_0)$. $n_i^s$ is the observed number of galaxies in bin $i$ and $n_i^s$ the predicted number for any assumed pair of lens parameters. We have binned the data in such a way that $n_i^s = 20$. This procedure was carried out for several hundred samples of 500 galaxies. The reconstructed $(r_0, \Phi_0)$ are plotted on figure 6. We can see that there is a great dispersion in the reconstructed data. This arises from the fact that the statistic of the reconstructed sources is fairly insensitive to the lens parameters. Therefore the above $\chi^2$ is small for a wide range of parameters over which the minimum can be found.

Although this method of finding confidence regions for the lens parameters in the absence of polarization information may not be optimal, it does however given some idea of the advantages of using polarimetric data (see Fig. 2).
4.2 An arbitrary spherical lens

As we have already mentioned, if one does not use the polarization to determine the orientation of the source galaxy, the only measurable quantities are the image parameters. We are thus left with five unknown parameters \((l_s, \Delta l_s, \alpha_s, \Gamma, \theta)\) but still have only 3 equations, and so it is impossible to solve the system.

If we assume only that the lens is spherically symmetric, and, as before, that the centre of the lens is known, so that the angle \(\theta\) is determined, and that the polarization has been measured, then the system of equations (10), (11) and (13) will
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Figure 6. Each point in this figure gives the couple \((r_0, \Phi_0)\) reconstructed from a sample of 500 galaxies without using the information given by the polarization. (There are no points outside this square because during the minimisation process, we limited the variation of \(r_0\) to \([0.5, 2.1]\) and that of \(\Phi_0\) to \([0.4, 1.5]\).)

Figure 7. The average error, expressed in percent, on the reconstructed value of \(\Gamma\) as a function of the error of the orientation, \(\alpha_s\), of the source galaxy, expressed in degrees. One can see that if \(\alpha_s\) is given with a better than 5° precision then the error on \(\Gamma\) is less than 30%.

contain three unknowns \((\iota_s, \Delta \iota_s, \Gamma)\). It is thus possible to deduce the remaining source parameters as well as \(\Gamma\). From equations (10) and (13) one can show that \(\Gamma\) is given by

\[
(C_i S_i + S_i C_i) \Delta \iota_i \Gamma^2 + 2 S_i \Gamma + (C_i S_i - S_i C_i) \Delta \iota_i = 0.
\] (18)

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-filled in triangles correspond to the values found for 10 randomly selected galaxies. The left (right) diagram corresponds to a population of galaxies with a $1 - \sigma$ dispersion of $2.5^\circ$ ($5^\circ$) on the measure of $\alpha_s$.

If there is no measurement error and if the lens is really spherical, this method would provide an exact determination of the lens parameters. Of course these assumptions are not very realistic, and it is interesting to investigate the case described by equation (18) when there is a measurement error $\alpha_s$. In order to carry out this analysis, we have reconstructed $\Gamma$ for 10000 randomly selected galaxies as before, and including a random error on the measurement of $\alpha_s$. Fig. 8 shows the mean error in percent on the value obtained for $\Gamma$ ($\Delta\Gamma_s = (\Gamma_r - \Gamma)/\Gamma$ where $\Gamma_r$ and $\Gamma$ are the inferred and the actual value) as a function of the error, measured in degrees, on $\alpha_s$ ($\Delta\alpha_s = |\alpha'_s - \alpha_s|$). Evidently, if one can measure $\alpha_s$ to within a few degrees, it is possible to locally determine $\Gamma$ to a precision of around 20% using just one galaxy.

We generated two samples of galaxies with a gaussian error distribution on $\alpha_s$ with a standard deviation of $2.5^\circ$ and $5^\circ$. The mean value and the one sigma error on the on the retrieved values $\Gamma$ are shown in Fig 8. This figure shows that even with only ten or so galaxies for which the polarization have been measured, it is possible to fairly accurately determine the function $\Gamma(r)$. It is also important to point out that if one uses two galaxies at similar radial distances from the centre, it is possible to test the assumption of spherical symmetry.

### 4.3 Arbitrary lenses

In this section we discuss the problem of determining the lens parameters when no assumptions are made about the lens. If we no longer assume that the lens is spherical, then the angle $\theta$ is not known a priori. The system of equations (10), (11) and (12) then consists of 3 equations for 4 unknowns ($l_i, \Delta l_i, \Gamma, \theta$), and so it is impossible to solve for the unknowns. However if we assume that there are two galaxies in the sample that can effectively be associated with the same values of $\Gamma$ and $\theta$, then the previous system written for both galaxies will contain 6 unknowns, $(l_1^i, l_2^i, \Delta l_1^i, \Delta l_2^i, \Gamma, \theta)$, and six equations, which means that it can be solved exactly for $\Gamma$ and $\theta$ and the source galaxies’ parameters. One should note, however, that if the orientation of the source galaxies is not known, adding more galaxies in this way cannot close the system, as there will always be more unknowns than independent equations: each new galaxy brings three unknowns and three equations, and so the equations are always underdetermined. There are a number of methods for finding galaxies associated with the same pairs of values of $(\Gamma, \theta)$. If we make no assumptions about the lens, then one has to take two galaxies that have an angular separation much smaller than the angular diameter of the lensing cluster. One can then make the approximation that the potential is the same for both galaxies. If on the other hand we assume that the potential is spherically symmetric, then one can take galaxies which are well separated but at the same distance from the centre, so they have the same $\Gamma$ and $\theta$. There could of course be other symmetries that allow one to do this.

The equation (18) for each galaxy gives the following system:

$$
\begin{align*}
(C^0_1 S^0_1 + S^0_1 C^0_1) \Delta l^0_1 \Gamma^2 + 2 S^0_1 \Gamma + (C^0_1 S^0_1 - S^0_1 C^0_1) \Delta l^0_1 &= 0 \\
(C^0_2 S^0_2 + S^0_2 C^0_2) \Delta l^0_2 \Gamma^2 + 2 S^0_2 \Gamma + (C^0_2 S^0_2 - S^0_2 C^0_2) \Delta l^0_2 &= 0
\end{align*}
$$

where $g_1$ (resp. $g_2$) denote quantities related to the first (resp. second) galaxy. The parameters $C^0_k$ and $S^0_k$ ($k = i, s$) depend only on $\theta$. This system can be solved for $\Gamma$ and $\theta$. As before, we tested the possibility of reconstructing lens parameters by generating a sample of 10000 galaxies. Two galaxies are selected which correspond to lens parameters which are only slightly different. Thus we have considered only pairs of galaxies separated by less than 0.2 in the lens plane. To simplify the presentation we shall use the following notation.

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Figure 9. Error on $\Gamma$ plotted against the difference between $\Gamma^{g1}$ and $\Gamma^{g2}$ (see text) for galaxies for which $\alpha_s$ is measured exactly (squares) and a standard deviation of 2.5°.

- $\Gamma^{gi}$ parameter corresponding to the $i^{th}$ galaxy.
- $\Gamma$ retrieved value
- $\Gamma_m = (\Gamma^{g1} + \Gamma^{g2})/2$, the average value.
- $\Delta\Gamma_{12} = |\Gamma^{g1} - \Gamma^{g2}|/\Gamma_m$, the initial difference in $\Gamma$ for the two galaxies.
- $\Delta\Gamma = |\Gamma - \Gamma_m|/\Gamma_m$: the error on the retrieved value
- $\Delta\alpha_s = (\Delta\alpha^{g1}_s + \Delta\alpha^{g2}_s)/2$. The quantities $\Delta\alpha^{gk}_s$ are defined as in the paragraph above.

In Fig. 9 we have plotted the average value of $\Delta\Gamma$ as a function of $\Delta\Gamma_{12}$. If there is no measurement error, the error on the retrieved $\Gamma$ is roughly equal to the difference between the values of $\Gamma^{g1}$ and $\Gamma^{g2}$. If on the other hand one plots the same curve but for a population of galaxies when a measurement error of 2.5° is made, then the error on $\Gamma$ is noticeably greater, and around 15 – 20%. In order to establish the precision on $\alpha_s$ necessary to establish $\Gamma$ up to a certain level of accuracy, we have plotted the mean value of $\Delta\Gamma$ against $\Delta\alpha_s$ for pairs of galaxies satisfying $\Delta\Gamma_{12} < 10\%$. Indeed one sees that just one pair of nearby galaxies whose polarization yields $\alpha_s$ with 5° accuracy allows $\Gamma$ to be determined with an accuracy of 25% without making any assumptions about the form of the lens. Evidently, if one has a large number of galaxy pairs, that it is possible to still further reduce this error.

One should also not that if one has several pairs of neighbouring galaxies, the number of equations in (19) is greater than the number of unknowns, and one should use statistical techniques for the determination of the lens parameters.

5 DETERMINATION OF $\kappa$ AND $\gamma$

Once $\Gamma$ has been determined, it is still necessary to determine the convergence $\kappa$ and the shear $\gamma$, which are the parameters directly linked to the mass density distribution in the lens plane. These parameters can be obtained from equation (7), which can be written in the form

$$\lambda_s = \kappa^2 \left( \lambda_s (1 + \Gamma^2) + 2 \Delta\lambda_s \Gamma C_s \right) = a\kappa^2.$$  \hspace{1cm} (20)

If we now average the above relation over a subset $S$ of galaxies having the same value of $\kappa$, either because they are sufficiently close together or because of some symmetry (all the galaxies can be used and not only those with a measured polarization) we obtain $\kappa^2 = \bar{\lambda}_s/\bar{a}$, where $\bar{a}$, the average of $a$ over the subset $S$, can be computed using the image and the reconstructed value of $\Gamma$ and $\theta$ (which are equal for all galaxies in $S$). The average of $\lambda_s$, $\bar{\lambda}_s$, cannot be computed since the intrinsic brightness of the sources are not known, but since the source galaxies (and therefore their properties) are uniformly distributed in the source plane, $\bar{\lambda}_s$ should be constant (independent of the set $S$ chosen) if the average is taken over a sufficiently large number of galaxies. Therefore we can arbitrarily set $\bar{\lambda}_s = 1$ for the whole subset of galaxies. This allow to compute $\bar{\kappa} = 1/\bar{a}$ and $\bar{\gamma} = \Gamma \bar{\kappa}$ which are proportional to $\kappa$ and $\gamma$ respectively, the proportionality factor being a constant.
throughout the lens plane. The reliability of this method will strongly depend on the number of galaxy contained in each subset and on the luminosity dispersion of the source galaxies. Each subset should contain enough galaxies in order for to assume the $\lambda_s$ is equal to the average luminosity of the source galaxies.

6 OBSERVATIONAL CONSIDERATIONS

In the previous section we saw how knowledge of the orientation of the source galaxy enabled us to improve the determination of the gravitational field of the cluster. In principle this orientation would be given by the direction of the polarization. Similarly measurement of the degree of polarization would yield information about the ellipticity of the source, although in view of the possible uncertainty concerning the mechanisms involved in the production of the polarization detailed modelling might be required. Potentially, determination of the intrinsic polarization of lensed galaxies could yield valuable information. Thus a question of crucial importance is the feasibility of such measurements and the reliability of their interpretation.

The main difficulty evidently arises from the faintness of the source galaxy, and the relatively low level of polarization. It should be borne in mind, however, that although we are looking at objects of magnitude 25 or fainter, it is the integrated polarization that we are interested in, and measurements could be taken over broad band. Such observations in the optical have been carried out for a number of high redshift radio galaxies (Tadhunter et al. 1992), with the galaxies showing high polarization, which appears to increase with higher redshift. Although our concern is mainly with spiral galaxies, it is possible that radio galaxies could also be used in the analysis, since these appear to display net polarization oriented in a definite direction to the radio axis. Our main point here however is that optical polarimetric measurements would be difficult, but not impossible, particularly on a large telescope. It is important to point out also that in our analysis it is only the direction of polarization that is important, and this is much easier to establish accurately than the degree of polarization (Treanor 1968).

Another problem would be the possible effect of Faraday rotation, yet in the visible we would expect this to be fairly weak. If we were to use polarimetric measurements in the radio, then this of course would need to be taken into account.

7 CONCLUSIONS

The polarimetric study of weakly lensed galaxies can in principle provide information about the lensing gravitational potential that otherwise would be extremely difficult or impossible to obtain. The method relies on the property that the plane of polarization of light is not affected by the gravitational potential. Thus the direction of polarization provides a marker for the orientation of the source galaxy, and hence information about the distortion in the image induced by the lens. This reduces the uncertainty in the estimation of the lens parameters, and can even, in certain situations when a few source galaxies can actually be measured polarimetrically, break the degeneracy of the equations that determine the lens parameters.

Although to a large extent we have assumed that Thomson or Rayleigh scattering is responsible for the polarization flux, the our conclusions are not greatly affected by the presence of such small scale mechanisms as dichroism provided that on
the large scale axial symmetry holds. Only in exceptional circumstances would one expect the direction of the polarization not to be perpendicular to the major axis of the elliptical isophotes. Since weak lensing would induce a rotation of much less than 90 degrees, even if the integrated polarization were aligned parallel to the major axis of the source, this should not lead to confusion and hence measurement of the direction of polarization would still provide rigorous constraints on the lensing parameters.

Measurement of the optical polarization of faint galaxies would require large telescopes and integration times, but it is important to realise that the main advantage of the proposed method relies on the use of the direction of polarization, and not the degree of polarization and the former is considerably easier to establish.

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