Small-signal modeling and stability analysis of grid-connected offshore wind power based on virtual synchronous generator control

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Abstract. With the increasing installed capacity of offshore wind power, the grid-connected of offshore wind power technology based on virtual synchronous generators (VSG) control can simulate the inertia and damping characteristics of synchronous generator, which is helpful to improve the networking ability and inertial supportability of grid-connected inverter. However, VSG control also inevitably causes system oscillation, endangers the safe and stable operation of the power grid. Hence, the main circuit and control circuit of offshore wind power grid connection are firstly modeled, and the small-signal model of offshore wind power grid-connected inverter based on VSG control is obtained. Then the correctness of the model is verified by Matlab/Simulink software. Finally, the root trajectory method is used to identify the effects of VSG control parameters and line parameters on the system stability. The results show that VSG control has a significant influence on the stability of the grid-connected system, and reasonable design parameters are needed to ensure the safe and stable operation of the power grid.

1. Introduction

With the continuous growth of energy demand and the increasingly prominent environmental problems caused by the development and utilization of fossil energy, offshore wind energy, as the most potential distributed energy, is developing rapidly, and its proportion in the energy structure is constantly increasing[1]. Grid-connected inverters, as an important link of distributed energy to the power grid, have the advantages of flexible control and rapid response. However, distributed energy based on power electronic inverters also has disadvantages such as low rotational inertia and low damping component, resulting in insufficient inertia and damping to participate in the power grid regulation when the system disturbance occurs. It endangers the safe and stable operation of the power grid and the reliability of power supply[2].

Given the above problems, some scholars proposed the concept of virtual synchronous generator (VSG)[3-5]. Specifically, the inertia and damping of the synchronous generator were introduced into the control algorithm of the power electronic converter. Provide inertia and damping support for the power grid to solve the lack of frequency and voltage regulation capacity of new energy power
generation equipment. At present, scholars have carried out a series of studies on VSG ontology modeling, control technology optimization, parameter design, and grid-connected stability. Reference[6-7], the small-signal model of VSG power frequency was established, the decoupling conditions of VSG active and reactive power loop were given from the perspective of frequency domain analysis, and the design of VSG control parameters was completed according to the requirements of phase margin. Reference[8], a high-order small-signal model with the robust droop control VSG was established as the research object. Although the accuracy of analysis was improved, the complexity of the modeling of multi-machine and the multi-inverter system was increased due to the high order of the model, and little influence was exerted on the damping analysis and controller parameter design. Reference[9] aiming at the problem of operating point shift caused by system oscillation and random fluctuation of PV VSG, a global adaptive control parameter design method suitable for photovoltaic VSG is proposed, and analyzed the small-signal stability and low-frequency oscillation characteristics of photovoltaic power system. Reference[10] proposed an active disturbance rejection control strategy based on retaining the basic framework of PID controller for the oscillation risk brought by VSG in a complex environment, and added compensation at the output end of the controller. However, the selection principle of parameters $K$, $\beta_1$, $\beta_2$, and $b$ in the active disturbance rejection control algorithm was not given. Reference[11] has put forward the adaptive control strategy of moment of inertia to makes up for the disadvantage that the moment of inertia of VSG control is fixed, and the tuning method of some parameters is given, but does not point out the applicability of the adaptive control algorithm to the infinite power grid. Reference[12] compares VSG control with droop control based on frequency change rate from the mathematical model, transient output characteristics, and small-signal stability, and on this basis identifies stability in different system scenarios.

To sum up, there have been many researches on VSG modeling and parameter adaptive optimization, and the stability researches on VSG also mainly take microgrid as the application scenario. As penetration of offshore wind power increases, there are relatively few researchs on the small-signal stability of VSG-based offshore wind power connected to the infinite power grid, and some papers fail to obtain all the modal information of the system due to the neglect of LC filter and the control loop of voltage current in the modeling[13]. Therefore, it is urgent to carry out the grid-connected stability analysis and research of offshore wind power connected to the infinite power grid based on VSG control.

Given the above problems, this paper first selects appropriate state variables and adopts the state space method to establish a small-signal state-space model for each link of the offshore wind power grid-connected system based on VSG control. Then, the inverter, filter circuit, voltage and current loops are combined to form a small-signal model of the overall VSG grid-connected system, and the correctness of the model is verified by Matlab/Simulink software. Finally, the influence of VSG control and line parameters on system stability is identified by using root locus method.

2. VSG basic principles

2.1. VSG system topology

Fig.1 is the topology of the offshore wind power grid-connected inverter based on VSG control. $v_{in}$ is the offshore wind power ideal DC power supply, $L_1$ and $C_1$ are the filter inductor and capacitor respectively, and $R_g$ and $L_g$ are the transmission line resistance and inductance. VSG controls the reference voltage of the output, and after passing through the voltage and current loop, sinusoidal pulse width modulation (SPWM) is used to generate signals to drive the action of power electronic devices[14].
2.2. VSG controller

The VSG controller block diagram is shown in Fig.2. In the active power-frequency control, \( J \) and \( D \) simulate the inertia and damping characteristics of synchronous generator respectively. Reactive-voltage control represents the voltage regulating characteristics of synchronous generators to fulfill the droop characteristics of reactive power and output voltage amplitude[15].

\[
\begin{align*}
\omega_n &= \omega, \\
\omega_n &= J_0 \frac{d(\omega - \omega_n)}{dt} + K_d (\omega_n - \omega) + P_{ref} - P_e - D(\omega - \omega_n) \\
\epsilon_d &= K_q (Q_{ref} - Q_e) + v_{ref}
\end{align*}
\]

Where, \( \omega \) is the virtual angular velocity, \( \omega_n \) is the rated angular velocity, \( P_{ref} \) and \( P_e \) are the given value and measured value of active power respectively, \( J \) is the virtual moment of inertia, \( D \) is virtual damping coefficient, \( K_d \) is the active power frequency modulation coefficient, \( v_{ref} \) is the voltage reference set value, \( K_q \) is the reactive voltage regulating coefficient, \( Q_{ref} \) and \( Q_e \) are the given value and measured value of reactive power respectively. \( \epsilon_d \) is the reference voltage of the output of the reactive power-frequency control link.

3. Small-signal modeling of VSG grid-connected system

3.1. Small-signal modeling for VSG power control

The power calculation can be obtained after the voltage at the point of common coupling (PCC) flows through the instantaneous power calculation module.

\[
\begin{align*}
P_e &= 1.5(v_{odq}i_{odq} + v_{idq}i_{dq}) \\
Q_e &= 1.5(v_{odq}i_{dq} - v_{idq}i_{odq})
\end{align*}
\]

Where, \( v_{odq} \) and \( i_{odq} \) respectively represent the \( dq \) components of voltage and current at PCC.

The small-signal model of power calculation can be obtained by linearizing Equation (2):

\[
\begin{align*}
\Delta P_e &= 1.5(I_{odq}\Delta v_{odq} + I_{idq}\Delta v_{idq} + V_{odq}\Delta i_{odq} + V_{idq}\Delta i_{idq}) \\
\Delta Q_e &= 1.5(-I_{odq}\Delta v_{odq} + I_{idq}\Delta v_{idq} + V_{odq}\Delta i_{odq} - V_{idq}\Delta i_{idq})
\end{align*}
\]

To facilitate the analysis, the coordinate system of VSG is used as the reference coordinate system. By linearizing Equation (1) can be obtained:
3.2. Small-signal modeling of inverter side filter circuit

As shown in Fig. 1, the output end of the inverter is mainly composed of a filter circuit, and the state equation corresponding to this link is:

$$\frac{d\Delta q}{dt} = \frac{1}{L_i} (-R_{iL} \Delta i_q + v_o - v_{oa})$$
$$\frac{d\Delta d}{dt} = \frac{1}{L_i} (-R_{iL} \Delta i_d + v_o - v_{oa})$$
$$\frac{d\Delta i_{oq}}{dt} = \frac{1}{C_i} (\omega C_i v_{oa} + i_q - I_{oa})$$
$$\frac{d\Delta i_{od}}{dt} = \frac{1}{C_i} (-\omega C_i v_{od} + i_d - I_{od})$$

(4)

Where, $v_{oq}$ and $i_{od}$ are respectively $dq$ components of the inverter output voltage and current.

By linearizing the above equation can be obtained:

$$\begin{align*}
\Delta i_{od} & = A_i \Delta i_{oq} + B_i \Delta v_{oq} + C_i \Delta i_{oa} + D_i \Delta \omega \\
\Delta v_{oq} & = A_v \Delta i_{oq} + B_v \Delta v_{oq} + C_v \Delta i_{oa} + D_v \Delta \omega
\end{align*}$$

(7)

3.3. Small-signal modeling of transmission line and network side

The network side is mainly made up of the infinite power grid and transmission line, and the corresponding state equation is shown in Equation (8):

$$\begin{align*}
\frac{d\Delta v_{od}}{dt} & = \frac{1}{L_q} (-\omega L_q i_q + v_{od} - v_{og}) \\
\frac{d\Delta v_{gq}}{dt} & = \frac{1}{L_q} (-\omega L_q i_q + v_{gq} - v_{gq})
\end{align*}$$

(8)

Where, $v_{og}$ and $v_{gq}$ are the infinite grid voltage in the coordinate system $dq$.

Under small interference, it can be considered that the amplitude of the power grid voltage $E_g$ and the frequency $\omega_g$ are constant values, and the phase difference between the grid and the output voltage of the reference VSG is $\alpha$. When the grid voltage is transformed to the common coordinate system [16]:

$$\begin{align*}
\Delta v_{od} & = E_g \sin \alpha \Delta \alpha \\
\Delta v_{gq} & = -E_g \cos \alpha \Delta \alpha
\end{align*}$$

(9)

By linearizing Equation (8), the small-signal model of transmission line and network side can be obtained:

$$\begin{align*}
\Delta i_{od} & = A_i \Delta i_{oq} + B_i \Delta v_{oq} + C_i \Delta i_{oa} + D_i \Delta \omega \\
\Delta v_{oq} & = A_v \Delta i_{oq} + B_v \Delta v_{oq} + C_v \Delta i_{oa} + D_v \Delta \omega
\end{align*}$$

(10)

3.4. Voltage and current loop control

![Fig. 3 Voltage current loop control block diagram](image-url)
Fig. 3 is the control block diagram of the voltage and current loop [17]. The corresponding small-signal model is shown in Equation (11)-(14):

\[
\begin{align*}
\Delta \phi_{dq} &= A_x \Delta \phi_{dq} + B_x \Delta \gamma_{dq} + C_x \Delta \delta_{dq} \Delta \nu_{odq} \\
\Delta \delta_{dq} &= A_x \Delta \phi_{dq} + B_x \Delta \gamma_{dq} + C_x \Delta \delta_{dq} \Delta \nu_{odq} \\
\Delta \delta_{dq} &= A_x \Delta \phi_{dq} + B_x \Delta \gamma_{dq} + C_x \Delta \delta_{dq} \Delta \nu_{odq} \\
\Delta \nu_{dq} &= A_x \Delta \phi_{dq} + B_x \Delta \gamma_{dq} + C_x \Delta \delta_{dq} \Delta \nu_{odq}
\end{align*}
\]

(11)  
(12)  
(13)  
(14)

3.5. Small-signal model of VSG grid-connected system

By combining equations (1)-(14), the small-signal model of VSG grid-connected in a common coordinate system can be obtained:

\[
\Delta x = A \Delta x
\]

Where, \( \Delta x = [\Delta \omega, \Delta \alpha, \Delta i_{dq}, \Delta \nu_{odq}, \Delta \delta_{dq}, \Delta \phi_{dq}]^T \), \( A \) is the system state matrix.

3.6. Small-signal model verification

To verify the correctness of the deduced small-signal model, the actual simulation model and small-signal model are built in Matlab/Simulink respectively. The parameters are shown in Table I. When the active power reference value \( P_{ref} \) takes a step of 10 kW, the curves of the active power \( P_e \), reactive power \( Q_e \) and virtual angular velocity \( \omega \) in the small-signal model are compared and basically coincide with the simulation curves of the actual model in Simulink, as shown in Fig. 4:

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( R_1/\Omega \) | 0.02  | \( P_{ref}/kW \) | 30    |
| \( L_1/mH \) | 3.2   | \( J \)    | 10    |
| \( C_1/\mu F \) | 100   | \( D \)   | 75    |
| \( R_g/\Omega \) | 0.1   | \( K_d \) | 314.2 |
| \( L_g/mH \) | 2     | \( K_q \) | 0.005 |

Fig.4 Comparison of models of Simulink and small-signal

3.7. Solution of system eigenvalues

All the eigenvalues of the system can be obtained by solving the eigenvalue of the state matrix \( A \). As shown in Table II, the system has 12 eigenvalues, among which \( \lambda_{7,8} \) and \( \lambda_{9,10} \) are closest to the imaginary axis and have the greatest influence on the system stability. The following will focus on analyzing these two pairs of characteristic roots.
TABLE II. SYSTEM EIGENVALUE

| Eigenvalue | Real Part | Imaginary Part | Oscillation Frequency/Hz | Damping Ration |
|------------|-----------|----------------|--------------------------|----------------|
| $\lambda_{1-2}$ | -879.76 | ±3992.61 | 635.76 | 0.22 |
| $\lambda_{3-4}$ | -86.53 | ±3245.87 | 516.86 | 0.03 |
| $\lambda_{5-6}$ | -197.26 | ±356.04 | 56.69 | 0.48 |
| $\lambda_{7-8}$ | -3.31 | ±2.59 | 0.41 | 0.79 |
| $\lambda_{9-10}$ | -1.43 | ±1.06 | 0.16 | 0.81 |
| $\lambda_{11-12}$ | -60 | ±319.94 | 50.94 | 0.18 |

4. Analysis of the influence of parameter variation on the stability of VSG grid-connected

4.1. Impact of virtual moment of inertia $J$
When $J$ changes from 7 to 70, while other parameters remain unchanged. The influence on the eigenvalue is shown in Fig.5. The arrow direction in the figure indicates the trend of the eigenvalue increasing with parameter $J$. Fig.5 shows that, with the increase of $J$, the eigenvalue $\lambda_{7-8}$ moves monotonically and rapidly to the right, and the damping of the corresponding oscillation mode decreases rapidly. When $J$ increases to a certain extent, the eigenvalue $\lambda_{7-8}$ will be close to the origin of the coordinate, and the system stability will be seriously threatened[18].

4.2. Impact of virtual damping $D$
When $D$ changes from 60 to 180, and its influence on the eigenvalue is shown in Fig.6. The arrow direction in the figure indicates the trend of the eigenvalue increasing with parameter $D$. As shown in Fig.6, with the increase of $D$, the eigenvalue $\lambda_{9-10}$ is insensitive to $D$, and the eigenvalue $\lambda_{7-8}$ moves towards the direction near the real axis, and the damping of the system increases. When the value of $D$ exceeds 90, the eigenvalue $\lambda_{7-8}$ moves along the real axis, but the system is still in a stable state.
4.3. Impact of Active power frequency modulation coefficient $K_d$
When analyzing the influence of the active power-frequency drooping coefficient $K_d$ on the eigenvalue, $K_d$ changes from 300 to 1000, and the influence on the eigenvalue is shown in Fig.7. The arrow direction in the figure represents the trend of the eigenvalue changing with the increase of $K_d$. As can be seen from Fig.7, when $K_d$ changes, the eigenvalue is basically unchanged. This is because the frequency difference of the system is small in the case of small disturbance, so $K_d$ has little influence on the stability of the system.

![Fig.7 Influence of $K_d$ on eigenvalue](image)

4.4. Impact of line parameters
The influence of line parameter changes on the system eigenvalue is shown in Fig.8. The arrow direction in the figure represents the trend of eigenvalue changes with the increase of $R_g$ and $L_g$. The resistance of the line gradually increases from 0.01Ω to 1Ω. Compared with the eigenvalue $\lambda_{3,4}$, the eigenvalue $\lambda_{5,6}$ and $\lambda_{11,12}$ change significantly, but the eigenvalue is far away from the imaginary axis, which is beneficial to the system stability. The inductance of the line increases gradually from 2mH to 50mH, and the eigenvalue $\lambda_{5,6}$ and $\lambda_{11,12}$ rapidly approach the imaginary axis, which is not good for system stability.

![Fig.8 Influence of line parameters on eigenvalue](image)

5. Conclusion
In this work, topology of offshore wind power grid connection based on VSG control are modeled, and the small-signal model of offshore wind power grid-connected inverter is obtained. Then, on this basis, the root trajectory method is used to identify the effects of VSG control parameters and line parameters on the system stability. The main conclusions are as follows:

(1) The introduction of VSG control has a significant impact on the stability of the grid-connected system, therefore, it is necessary to reasonable set the controller parameters to ensure the safe and stable operation of the system.

(2) In VSG control parameters, the $J$ and $D$ have a great influence on the low-frequency characteristic root of the system, and the active power frequency modulation coefficient $K_d$ has little
influence on the system stability because the frequency difference of the system is far less than the frequency change rate.

(3) Line parameters mainly affect the medium and high frequency characteristic roots, and the degree of influence on the medium frequency characteristic roots is greater than that on the high frequency characteristic roots.

Appendix
1. Filter circuit matrix parameters
\[
A = \begin{bmatrix}
-\omega & 0 & 0 \\
\omega & -1 & 0 \\
0 & 0 & -\omega
\end{bmatrix} \quad B = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \\
C = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \quad D = \begin{bmatrix}
I_q - I_d & V_{eq} & V_{od}
\end{bmatrix}^T
\]

2. Line and network side matrix parameters
\[
A_s = \begin{bmatrix}
\frac{R_s}{L_s} & \omega \\
-\omega & -\frac{R_s}{L_s}
\end{bmatrix} \quad B_s = \begin{bmatrix}
0 \\
0
\end{bmatrix} \quad C_s = \begin{bmatrix}
I_{eq} & I_{od}
\end{bmatrix}^T
\]

3. Voltage current loop matrix parameters
\[
A_i = A_i = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \quad B_i = B_i = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \quad C_i = \begin{bmatrix}
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1
\end{bmatrix} \quad A_{ii} = \begin{bmatrix}
K_{n1} & 0 \\
0 & K_{n1}
\end{bmatrix} \quad B_{ii} = \begin{bmatrix}
K_{p1} & 0 \\
0 & K_{p1}
\end{bmatrix}
\]

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References
[1] Lü Zhipeng, Sheng Wanxing, Zhong Qingchang, et al. “Virtual synchronous generator and its applications in micro-grid,” Proceedings of the CSEE, vol. 34, no. 16, pp. 2591-2603, Jun. 2014.
[2] Chai Jianyun, Zhao Yangyang, Sun Xudong, et al. “Application and prospect of virtual synchronous generator in wind power generation system,” Automation of Electric Power Systems, vol. 42, no. 9, pp. 17-25, May. 2018.
[3] Zhong Q C, Weiss G. “Synchronverters: inverters that mimic synchronous generators,” IEEE Transactions on Industrial Electronics, vol. 58, no. 4, pp. 1259-1267, 2011.
[4] Torres M, Lopes L. “Virtual synchronous generator: a control strategy to improve dynamic frequency control in autonomous power systems,” Energy & Power Engineering, vol. 5, no. 2, pp. 32-38, 2013.
[5] Yuko, Hirase, Kazuhiro, et al. “A grid connected inverter with virtual synchronous generator model of algebraic type,” IEEE Transactions on Power and Energy, vol. 132, no. 4, pp. 371-380, 2012.
[6] Wu Heng, Yang Dongsheng, Chen Xinran, et al. “Small-signal modeling and parameters design for virtual synchronous generators,” IEEE Transactions on Industrial Electronics, vol. 63, no. 7,
pp. 1-11, Feb. 2016.

[7] Chen Junru, O’Donnell Terence. “Parameter constraints for virtual synchronous generator considering stability,” IEEE Transactions on Power Systems, vol. 34, no. 3, pp. 1-3, May. 2019.

[8] Liu Ni, Zhang Changhua, Duan Xue, et al. “Comparison and applicability analysis of small-signal modeling methods for grid-connected inverter,” Automation of Electric Power Systems, vol. 42, no. 23, pp. 134-141, Dec. 2018.

[9] Deng Jun, Xia Nan, Yin Junghang, et al. “Small-signal modeling and parameter optimization design for photovoltaic virtual synchronous generator,” Energies, vol. 13, no. 2, pp. 398-412, Jan. 2020.

[10] Yu Yunjun, Hu Xianyu. “Active disturbance rejection control strategy for grid-connected photovoltaic inverter based on virtual synchronous generator,” IEEE Access, vol. 1, pp. 1-10, 2019.

[11] Song Qiong, Zhang Hui, Sun Kai, et al. “Adaptive control of inertia for virtual synchronous generators in islanding micro-grid with multiple distributed generation units,” Proceedings of the CSEE, vol. 37, no. 2, pp. 412-424, Jan. 2017.

[12] Sun Dawei, Liu Hui, Gao Shunnan, et al. “Comparison of different virtual inertia control methods for inverter-based generators,” Journal of Modern Power Systems and Clean Energy, vol. 8, no. 4, pp. 768-777, May. 2020.

[13] Ge Jun, Liu Hui, Jiang Hao, et al. “Analysis and investigation on grid-connected operation adaptability of virtual synchronous generators,” Automation of Electric Power Systems, vol. 42, no. 9, pp. 26-35, May. 2018.

[14] Ren Biying, Qiu Jiaojiao, Liu Huan, et al. “Optimization control strategy of self-adjusting parameter based on dual-parallel virtual synchronous generators,” Transactions of China Electrotechnical Society, vol. 34, no. 1, pp. 128-138, Jan. 2019.

[15] Zheng Tianwen, Chen Laijun, Chen Tianyi, et al. “Review and prospect of virtual synchronous generator technologies,” Automation of Electric Power Systems, vol. 39, no. 21, Nov. 2015.

[16] Zeng Deyin, Yao Jun, Zhang Tian, et al. “Research on frequency small-signal stability analysis of multi-parallel virtual synchronous generator-based system,” Proceedings of the CSEE, vol. 40, no. 7, pp. 2048-2061, Apr. 2020.

[17] Tu Chunming, Xie weijie, Xiao fan, et al. “Influence analysis of control parameters of parallel system with multiple virtual synchronous generators on stability,” Automation of Electric Power Systems, vol. 44, no. 15, pp. 77-86, Aug. 2020.

[18] Yang Yun, Mei Fei, Zhang Chenyu, et al. “Coordinated adaptive control strategy of rotational inertia and damping coefficient for virtual synchronous generator,” Electric Power Automation Equipment, vol. 39, no. 3, pp. 125-131, Mar. 2019.