Effective field theory of Bose–Einstein condensation of α clusters and Nambu–Goldstone–Higgs states in $^{12}$C

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An effective field theory of α cluster condensation is formulated as a spontaneously broken symmetry in quantum field theory to understand the raison d’être and nature of the Hoyle and α cluster states in $^{12}$C. The Nambu–Goldstone and Higgs mode operators in infinite systems are replaced with a pair of canonical operators whose Hamiltonian gives rise to discrete energy states in addition to the Bogoliubov–de Gennes excited states. The calculations reproduce well the experimental spectrum of the α cluster states. The existence of the Nambu–Goldstone–Higgs states is demonstrated.

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Alpha cluster condensation in nuclei has attracted much attention since the observation of Bose–Einstein condensation (BEC) of trapped cold atoms [1]. In $^{12}$C, the three-α structure was most thoroughly investigated by Uegaki et al. [2], who showed that the $0^+_1$ state at an excitation energy $E_x$ of 7.654 MeV, the Hoyle state, which is crucial for nuclear synthesis, the evolution of stars, and the emergence of life, has a dilute structure in a new “α-boson gas phase” and clarified the systematic existence of a “new phase” of three α clusters above the α threshold. The Hoyle state has been extensively studied theoretically [3–13] and experimentally [14–21], and has been considered widely as an α cluster condensate. It has a gas-like structure with a dilute matter distribution of three-α clusters, 70% of which are in the 0s state [6]. However, no firm evidence of BEC, such as superfluidity, has been found.

In $^{12}$C, all the excited states except the $2^+_1$ state at 4.44 MeV appear above the α particle threshold (7.367 MeV). Recently, α cluster states above the Hoyle state, which are also candidates for an α cluster condensate, that is, the $0^+_3$ state at 9.04 MeV, $0^+_4$ state at 10.56 MeV, $2^+_2$ state (∼9.75 MeV) [14–18], and $4^+_1$ state (∼13.3 MeV) [19, 20] have been observed. To date, studies using α cluster models [6–8] and ab initio calculations [9–13] explain the Hoyle state and the excited gas-like states as collective states of α clusters or nucleons in configuration space. Collective motions arise owing mostly to spontaneously broken symmetries (SBSs) in the configuration space, such as rotational and translational ones, or in the gauge space [22]. The BEC of α clusters is a manifestation of the SBS of the global phase. It would be difficult from the standpoint of traditional α cluster models or ab initio calculations to conclude that BEC is truly realized, because it is not clear then what type of symmetry is broken for the Hoyle state and the α condensate states above it.

In the study of α cluster condensation, it is important to treat the SBS of the global phase on the basis of quantum field theory because of its unifying view and underlying principle. SBS is ubiquitous [23]; when it occurs, a Nambu–Goldstone (NG) mode (phason) appears according to the NG theorem [24, 25], and a Higgs (amplitude) mode (amplitudon) usually accompanies it. For example, in infinite superconducting systems, the NG mode [26], which is eaten by the plasmon, and the Higgs mode [27, 28] have been observed. For systems with a finite particle number, both the NG and Higgs modes have been confirmed in superfluid nuclei as a pairing rotation and pairing vibration, respectively [29]. The observation of the Higgs boson in particle physics [30] has stimulated a search for Higgs modes in other phenomena, including a recent experiment on Higgs mode excitation in a superconductor using a terahertz pulse [31]. It is intriguing to reveal the emergence of the NG and Higgs modes theoretically in an α cluster condensate and to observe them experimentally. Because the system is finite in size and particle number, they would manifest themselves not as particle excitations but as resonant states with discrete energy levels. From this viewpoint, Ref.[32] discussed a possible emergence of such states for an α cluster condensate in $^{12}$C and $^{16}$O qualitatively.

The purpose of this paper is to show for the first time that the dilute excited α cluster states, the Hoyle state and those above it, can be understood as new discrete states that follow naturally in the formulation of quantum field theory [33] for BEC of α clusters in terms of the field equation, canonical commutation relations (CCRs), and global gauge invariance. First, we clarify from quantum field theory for the α cluster condensate that the canonical operators [33], which replace the NG and Higgs mode operators in infinite systems with SBS, emerge and that the spectrum of their quantum mechanical system is discrete.

We start with the following Hamiltonian for the α clus-
ter system described by the field operator $\hat{\psi}$:

$$\hat{H} = \int d^3 x \hat{\psi}^\dagger(x) \left( -\nabla^2/2m + V_{\text{ex}}(x) - \mu \right) \hat{\psi}(x) + \frac{1}{2} \int d^3 x \, d^3 x' \, \hat{\psi}^\dagger(x) \hat{\psi}^\dagger(x') U(|x - x'|) \hat{\psi}(x') \hat{\psi}(x), \quad (1)$$

where $m$ and $\mu$ denote the mass of the $\alpha$ particle and the chemical potential, respectively. The external isotropic confinement potential $V_{\text{ex}}(x)$ is introduced phenomenologically. The interaction potential $U(r)$ is the sum of the nuclear $\alpha-\alpha$ potential, $V_{\text{NNN}}^\text{a-a}(r)$, and the Coulomb potential, $V_{\text{Coa}}(r)$.

Assuming $\alpha$ condensation, namely, the condensation phase, we divide $\psi$ into a condensate c-number component $\xi$ and an excitation component $\hat{\phi}$ using the criterion $\langle 0 | \hat{\phi} | 0 \rangle = 0$. The order parameter $\xi$ is taken to be stationary, isotropic, and real, and is normalized to the condensed particle number as $\int d^3 x \, \xi^2(x) = N_0$, where we fix $N_0 = 3$ for $^{12}\text{C}$ below. The Hamiltonian (1) is rewritten in terms of $\phi$ as $\hat{H} = \hat{H}_2 + \hat{H}_{3,4}$, where

$$\hat{H}_2 = \frac{1}{2} \int d^3 x \, d^3 x' \left( \hat{\phi}^\dagger(x) - \hat{\phi}(x) \right) \times \left( \begin{array}{cc} \mathcal{L}(x, x') & \mathcal{M}(x, x') \\ -\mathcal{M}^*(x, x') & -\mathcal{L}(x, x') \end{array} \right) \left( \begin{array}{c} \hat{\phi}(x') \\ \hat{\phi}^\dagger(x') \end{array} \right), \quad (2)$$

$$\hat{H}_{3,4} = \frac{1}{2} \int d^3 x \, d^3 x' \, U(|x - x'|) \times \left\{ 2\xi(x') + \hat{\phi}^\dagger(x') \right\} \hat{\phi}(x) \hat{\phi}(x') + \text{h.c.}, \quad (3)$$

with

$$V_H(x) = \int d^3 x' \, U(|x - x'|) \xi^2(x'), \quad (4)$$

$$\mathcal{M}(x, x') = U(|x - x'|) \xi(x) \xi(x'), \quad (5)$$

$$\mathcal{L}(x, x') = \delta(x - x') (-\nabla^2/2m + V_{\text{ex}}(x)) - \mu + V_H(x) + \mathcal{M}(x, x'). \quad (6)$$

The requirement that the $\phi$-linear term in $\hat{H}$ must vanish leads to the Gross–Pitaevskii equation [34]

$$(-\nabla^2/2m + V_{\text{ex}}(x) - \mu + V_H(x)) \xi(x) = 0. \quad (7)$$

According to the method developed in cold atomic physics, $\hat{\phi}$ is expanded as [35, 36]

$$\hat{\phi}(x) = \hat{\phi}_{\text{ex}}(x) - i\hat{Q}(t) \xi(x) + \hat{\phi}(t) \eta(x). \quad (8)$$

The field $\hat{\phi}_{\text{ex}}(x)$ is expanded as $\hat{\phi}_{\text{ex}}(x) = \sum_n \hat{a}_n(t) u_n(x) + \hat{a}_n^\dagger(t) v_n^\dagger(x)$, where $u_n$ and $v_n$ are the elements of the Bogoliubov–de Gennes (BdG) eigenfunction [37, 38],

$$\int d^3 x' \left( \mathcal{L} \mathcal{M} \right) \left( \begin{array}{c} u_n \\ v_n \end{array} \right) = \omega_n \left( \begin{array}{c} u_n \\ v_n \end{array} \right), \quad (9)$$

with a normalization condition $\int d^3 x \left[ |u_n|^2 - |v_n|^2 \right] = 1$. The isotropic $\xi$ implies $\mathbf{n} = (n, \ell, m)$, a triad of the main, azimuthal, and magnetic quantum numbers. In Eq. (8), $\xi$ is the element of the BdG eigenfunction belonging to zero eigenvalue, and $\eta$ is its adjoint function, calculated as $\eta = \partial \xi / \partial N_0$, with a normalization condition $\int d^3 x \, [\xi^\dagger \eta + \eta \xi] = 1$. The CCR of $\hat{\phi}$ and $\hat{\phi}^\dagger$ yields $[\hat{a}_n, \hat{a}_n^\dagger] = \delta_{nn'}$, $[\hat{Q}, \hat{P}] = i\hbar$, $(\text{otherwise}) = 0$. The pair of canonical operators $\hat{Q}$ and $\hat{P}$, which are associated with the eigenfunctions with zero eigenvalue and stem from the SBS of the global phase, are counterparts of the NGH operators and NGH subspace, respectively. The excitation mode created by $\hat{a}_n^\dagger$ is referred to as the BdG mode. We note that the NGH operators exist in our finite model of superfluid type irrespective of the fact that the Higgs mode is absent in non-relativistic infinite models of this type [28].

Let us seek the vacuum $|0\rangle$, with which we identify the Hoyle state. A naive choice of the unperturbed Hamiltonian would be $\hat{H}_2$, because the system is a dilute, weakly interacting gas-like one, so the higher powers of $\hat{\phi}$, $\hat{H}_{3,4}$ could be ignored in the leading order. Substituting Eq. (8) into Eq. (2), we obtain $\hat{H}_2 = \hat{P}^2/2 + \sum_n \omega_n \hat{a}_n^\dagger \hat{a}_n$, with $\hat{I} = \partial \mu / \partial N_0$. The Hamiltonian of the NGH operators, which has the free-particle form and therefore a continuous spectrum, causes serious defects, that is, the non-existence of a stationary normalized vacuum and the diffusing phase of $\xi$ [35].

To avoid these defects, a modified unperturbed Hamiltonian [33], which retains the nonlinear terms of $\hat{Q}$ and $\hat{P}$ in $\hat{H}_{3,4}$, has been proposed, because it is unfounded to neglect them, unlike the higher powers of the BdG modes. Concretely, we replace the term $1\hat{P}^2/2$ above with

$$\hat{H}_{u}^\text{p} = - (\delta \mu + 2C_{2002} + 2C_{1111}) \hat{P} + \frac{I - 4C_{1102}}{2} \hat{P}^2$$

$$+ 2C_{2011} \hat{P} \hat{Q} + 2C_{1102} \hat{P}^3 + \frac{1}{2} C_{2020} \hat{Q}^4 - 2C_{2111} \hat{Q}^2$$

$$+ C_{2002} \hat{P}^2 \hat{Q} + \frac{1}{2} C_{0202} \hat{P}^4, \quad (10)$$

where $C_{ijij'} = \int d^3 x d^3 x' U(r) \xi^i(x) \xi'^j(x') \xi^i(x') \xi'^j(x')$ with $r = |x - x'|$, and $\delta \mu$ is to be determined self-consistently to satisfy the criterion $\langle 0 | \hat{\phi} | 0 \rangle = \xi$. The fact that the spectrum of $\hat{H}_{u}^\text{p}$ is discrete is especially significant. It is implicitly postulated in the introduction of Eq. (10) that the unperturbed state of the total system is factorized as $|\Psi\rangle = |\xi\rangle \otimes |\xi\rangle$, where $|\Psi\rangle$ and $|\xi\rangle$ are a wave function in the NGH subspace and a Fock state associated with $\hat{a}_n$, respectively. Accordingly, all the cross terms such as $\hat{a}_n \xi \hat{P}$ are included in the interac-
tion Hamiltonian and should be treated perturbatively. The unperturbed vacuum $|0\rangle$, which is identified with the Hoyle state, is now given by $|\Psi_0\rangle|0\rangle_{\text{ex}}$, where $|\Psi_0\rangle$ is the ground state in the eigenequation

$$\hat{H}_u^{QP}|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle \quad (\nu = 0, 1, \cdots).$$

(11)

The excitation in the NGH subspace is a new and original concept and our prediction, for which the adoption of the non-quadratic Hamiltonian in Eq. (10) is crucial [33]. Note that this excitation does not change the value of the angular momentum $J$. The states $|\Psi_\nu\rangle|0\rangle_{\text{ex}} (\nu = 1, 2, \cdots)$, which have gap energies from the Hoyle state $E_\nu - E_0$, are referred to as the NGH states below. The BdG excitation energy $\omega_n$ is measured from the energy of the Hoyle state, and the state $|\Psi_0\rangle(a_n^\dagger |0\rangle_{\text{ex}})$ is termed the BdG state. Its experimental $J$ is given by the azimuthal quantum number $\ell$ of $n$. Solving Eqs. (7), (9), and (11) simultaneously, we obtain theoretical predictions that can be compared with experimental data, as shown below.

In the calculations, we take a phenomenological Ali–Bodmer potential (the parameter set $d_0$) [39] for $V_{\alpha-\alpha}^{\text{Nuc}}(r)$, which was obtained by fitting the s-wave phase shifts of $\alpha-\alpha$ scattering and has been used in three-\(\alpha\) cluster structure studies of $^{12}\text{C}$ [4]. The Coulomb potential, $V_{\alpha-\alpha}^{\text{Coul}}(r)$, is taken as $(4e^2/r)\text{erf}(\sqrt{3r/2h})$, where the size parameter of the $\alpha$ particle $b$ is 1.44 fm. The external confinement potential is assumed to be harmonic: $V_{\text{ex}}(r) = m\Omega^2r^2/2$, with a parameter $\Omega$.

The rms radius and density profile of the Hoyle state, calculated from $\xi(\mathbf{x})$, are shown in Fig. 1. The strength of the repulsive core of the Ali–Bodmer potential decreases slightly from 500 MeV to 405 MeV, which is consistent with the finding in Ref. [40] that the $\alpha-\alpha$ interaction in the three-$\alpha$ system is more attractive than that determined in free $\alpha-\alpha$ scattering. The Hoyle state is found to be dilute for all the $\Omega$ values. The rms radius and the peaks of the radial density distribution, located around 4–5 fm, are not very sensitive to $\Omega$, which implies that the three-$\alpha$ clusters are essentially self-binding. We take $\Omega = 2.0 \text{ MeV}/h$. The calculated rms radius of the Hoyle state is 4.18 fm, which is comparable with the calculations in Refs. [2, 3, 6].

In Fig. 2, the calculated energy levels are compared with the observed $\alpha$ cluster states. The agreement between the calculated and experimental energy levels is good, and the order of the levels is correctly reproduced. Our calculation reproduces the two $0^+$ NGH states ($\nu = 1, 2$), which correspond well to the $0^+_4$ at 9.04 MeV and $0^+_5$ at 10.56 MeV, respectively. The existence of the NGH states is critical for the assignments, because there is no BdG state with $\ell = 0$ near the energies of $0^+_4$ and $0^+_5$. Then, quite naturally, the excitations $2^+_2$ and $4^+_4$ are identified as the BdG states with $\ell = 2, 4$. All the observed positive parity states are well reproduced as BEC states of $\alpha$ clusters. This shows that the present field theory is useful even for a few-body system.

In Figs. 2 and 3, the calculation shows two almost degenerate $0^+$ states around 12.5 MeV, where no corresponding excitation has been established experimentally yet. These are the NGH state with $\nu = 3$ and the BdG state $|\Psi_0\rangle(a_{1,0}^\dagger |0\rangle_{\text{ex}})$, denoted simply as $|\text{h}\rangle$ and $|\text{BdG}\rangle$, respectively. Because of the degeneracy and because the interaction Hamiltonian allows mutual transitions, we expect that they mix with each other considerably to make new two energy eigenstates. A rough estimation of diagonalizing $\hat{H}$ in the subspace of $|\text{h}\rangle$ and $|\text{BdG}\rangle$ gives the eigenstates, $0.73|\text{BdG}\rangle + 0.68|\text{h}\rangle$ with an energy of 12.45 MeV and $0.73|\text{h}\rangle - 0.68|\text{BdG}\rangle$ with 12.64 MeV. We also note that doubly excited states, e.g., $|\Psi_1\rangle(a_{0,2}^\dagger |0\rangle_{\text{ex}})$ are possible.

Our interpretation of the $\alpha$ cluster states as phase locking due to BEC is quite different from the traditional $\alpha$ cluster model, ab initio calculations, and other approaches that try to explain them as collective modes in configuration space, e.g., the rotational band or vibrational states caused by breakdown of rotational or
translational symmetries.

In the traditional models, there has been a long-standing question about which excited states are the rotational band members built on the Hoyle state [21]. In other words, which of the $0^+_3$ and $0^+_4$ states is the bandhead of the observed $2^+_2$ and $4^+_2$ states? The first and traditional $\alpha$-cluster model picture regards the Hoyle state as the bandhead state [20, 41, 42]. In the $\alpha$-condensate model [6], the $2^+_2$ state is interpreted as a state in which an $\alpha$-cluster is lifted from the Hoyle state to the $D$ state in configuration space, and both states have essentially the same weakly coupled $[^8\text{Be}(0^+) \times \alpha]_J$ cluster configuration revealed in Refs. [2, 8]. In these cluster model pictures, because the Hoyle and $2^+_2$ states have a gas-like spherical structure, it is difficult to consider logically that a rotational band is built. In ab initio lattice [13] and no-core shell model [12] calculations, the Hoyle, $2^+_2$, and $4^+_2$ states are understood to be rotational band states. The second interpretation is that the $0^+_3$ state is a bandhead state on which the rotational $2^+_3$ and $4^+_3$ states are built [41]. Ref. [8] suggests that the $0^+_3$ state is a higher nodal state with the $[^8\text{Be}(0^+) \times \alpha(L = 0)]_{J = 0}$ structure. A calculated large $B(E2)$ value of the $2^+_2 \rightarrow 0^+_3$ transition [9] is reported, although no experimental data are available.

The reason that these two different interpretations have been presented is entirely due to the appearance of the Hoyle and $0^+_3$ states so closely above the $\alpha$-threshold. If rotational invariance of the $0^+_2$ and $0^+_3$ states in configuration space is broken, a rotational band should appear individually on both the $0^+_2$ and $0^+_3$ states, in contradiction with the experimental data. It seems difficult to determine which interpretation is correct as long as these are considered as collective modes with $\alpha$-cluster structure in configuration space. In our picture above, the question does not arise in principle. Our calculations show that the $2^+_3$ and $4^+_3$ states are the BdG states and need not be rotational member states on either the Hoyle state or the $0^+_3$ state. In fact, the $J(J + 1)$ plot of the excitation energy of the observed states of the band based on the above two pictures deviates from a straight line.

Why and how does nature allow in principle the emergence of the $0^+_3$ state, which is interpreted as a linear chain-like $\alpha$ cluster state in Refs. [8–10, 43], so close to the $0^+_3$ and $0^+_2$ states? In our picture, the close $0^+_3$ and $0^+_4$ states emerge naturally and fundamentally as the NGH states, which is a logical consequence of BEC of the Hoyle state, and the three are closely interrelated.

To summarize, we have studied the $\alpha$ cluster structure above the $\alpha$-condensate Hoyle state in $^{12}$C by formulating an effective field theory of $\alpha$ cluster condensation that properly treats spontaneous symmetry breaking of the global phase. The observed well-developed $\alpha$-cluster states, i.e., the $0^+_3$ (9.04 MeV), $2^+_3$ (9.75 MeV), $0^+_4$ (10.56 MeV), and $4^+_3$ (13.3 MeV) states, are well reproduced. It is demonstrated for the first time that the NGH states emerge just above the Hoyle state. It would be intriguing to study the NGH states in other nuclei such as $^{16}$O and $^{20}$Ne.

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