Multi-criteria decision making process using complex cubic interval valued intuitionistic fuzzy set

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Abstract. This paper presents a new notion of complex cubic interval valued intuitionistic fuzzy set (CCIVIFS) which is an extension from the innovative concept of a cubic interval valued intuitionistic fuzzy set (CIVFS). The novelty of CCIVIFS is to achieve more range of values with the combination of interval-valued membership, interval-valued non-membership and fuzzy membership. We define some basic operations namely complement, union, intersection and notions of $\tilde{\alpha}$-internal, $\tilde{\beta}$-internal, $\tilde{\alpha}$-external and $\tilde{\beta}$-external complex cubic interval valued intuitionistic fuzzy set are introduced. Also P-union, P-intersection, R-union and R-intersection of $\tilde{\alpha}$-internal and $\tilde{\alpha}$-external complex cubic IVIF sets are discussed. Furthermore, a group decision-making method is discussed to illustrate the applicability and validity of the proposed approach.

1. Introduction
Zadeh[7] introduced the concept of fuzzy set and studied its properties. Since then many articles on fuzzy sets appeared highlighting the importance of the concept and its applications to logic, set theory, group theory, real analysis, measure theory, topology, etc. In 1975, Zadeh[15] introduced a new notion called interval-valued fuzzy subsets, where the values of the membership function are closed interval of numbers instead of a number. In 1999, Molodstov[4] introduced the concept of soft sets and discussed the fundamental results of the new theory. To deal with it, the theory of fuzzy set[7] or extended fuzzy sets such as intuitionistic fuzzy set [2], interval-valued intuitionistic fuzzy set [3] are the most successful ones, which characterize the parameter values in terms of membership degree. Under these environments, several researchers have paid great attention and successfully applied the concept of these theories to many practical areas. Jun[8] introduced the concept of cubic set which is a combination of interval-valued fuzzy set and fuzzy set and investigated several properties of cubic sets. In 2017, Chinnadurai and Barkavi[10-13] introduced a new concept of cubic soft matrix and studied its properties. In 2002, Ramot et. al.,[5] introduced the concept of complex fuzzy set whose range is expanded to a unit circle in a complex plane. Madad Khan et. al.,[14] introduced complex fuzzy soft matrices and discussed some of its operations. Alkouri and Saleh[1] extended the concept of complex fuzzy set to complex intuitionistic fuzzy set (CIFS) by adding the degree of non-membership and defined the basic operations such as union, intersection, complement etc. Then Rani and Gare[6] extended the concept of complex interval-valued intuitionistic fuzzy sets and operation on aggregation. Jun and Zun[9] introduced the concept of cubic interval-valued intuitionistic fuzzy sets and its applications.
In this paper, we define some operations on complex cubic interval-valued intuitionistic fuzzy sets. We discuss $\tilde{\alpha}$-internal, $\tilde{\beta}$-internal, $\bar{\alpha}$-external and $\bar{\beta}$-external complex cubic interval-valued intuitionistic fuzzy sets. Also P-union, P-intersection, R-union and R-intersection of $\tilde{\alpha}$-internal and $\bar{\alpha}$-external complex cubic intuitionistic fuzzy sets are introduced. Finally, a real life application is discussed to show the reliability of the tool.

2. Preliminaries

In this section we define some basic concepts and definition.

Definition 2.1[7]
A fuzzy set $A$ on a universe $U$ is a mapping $A : U \rightarrow [0,1]$ , for all $u \in U$. For any $u \in U$, $A(u)$ denotes the membership degree of $u$ in $A$. The class of fuzzy sets defined on $U$ is denoted by $\mathcal{F}(U)$.

Definition 2.2[2]
A generalization of the notions of intuitionistic fuzzy set (IFS) $A^I = \{(x, \mu_A(x), \nu_A(x)) \mid x \in E\}$, where the functions $\mu_A : E \rightarrow L$ and $\nu_A : E \rightarrow L$ define the degree of membership and the degree of nonmembership of the element $x \in E$ to $A \subseteq E$, respectively the function $\mu_A$ and $\nu_A$ should satisfy the condition:

$$\forall x \in E \ (0 \leq \mu_A(x) + \nu_A(x) \leq 1)$$

Definition 2.3[3]
Let a set $E$ be a fixed. An interval-valued intuitionistic fuzzy sets (IVIFS) $A$ over $E$ is an object having the form $A = \{\langle x, M_A(x), N_A(x) \rangle \mid x \in E\}$, where $M_A(x) \subseteq [0,1]$ and $N_A(x) \subseteq [0,1]$ are intervals and for every $x \in E$

$$\sup M_A(x) + \sup N_A(x) \leq 1$$

Definition 2.4[9]
Let $X$ be a nonempty set. By a cubic interval-valued intuitionistic fuzzy set in $X$ we mean a structure $A = \{\langle x, \mu_A(x), A(x) \rangle \mid x \in X\}$ in which $\mu(x)$ is a fuzzy set in $X$ and $A(x)$ is an interval-valued intuitionistic fuzzy set in $X$.

Definition 2.5[6]
Let $U$ be the universe of discourse. A complex interval-valued intuitionistic fuzzy sets defined on $U$ is a set given by $A = \{\langle x, [\mu_A(x), \mu_A^+(x)], [\nu_A(x), \nu_A^+(x)] \rangle \mid x \in U\}$ where $\mu_A(x), \mu_A^+(x)$ and $\nu_A(x), \nu_A^+(x)$ represent the degree of lower and upper bound of the membership and non-membership which are defined as,

$$\mu_A(x) = \mu_A^{-}(x) e^{i\omega_A^{-}(x)}; \nu_A(x) = \nu_A^{-}(x) e^{i\omega_A^{-}(x)}$$

and

$$\mu_A^+(x) = \mu_A^{+}(x) e^{i\omega_A^{+}(x)}; \nu_A^+(x) = \nu_A^{+}(x) e^{i\omega_A^{+}(x)}$$

while $\mu_A(x) \leq \mu_A^+(x)$ and $\nu_A(x) \leq \nu_A^+(x)$. The amplitude terms $r_A, k_A, k_A^+ \in [0,1]$ and $0 \leq r_A \leq k_A \leq 1$. The phase terms $\omega_A, \omega_A^+ \in [0,2\pi]$ and $0 \leq \omega_A^+ + \omega_A^+ \leq 2\pi$ for all $x \in U$.

3. Complex cubic interval valued intuitionistic fuzzy set (CCIVIFS)

In this section we define complex cubic interval-valued intuitionistic fuzzy sets and investigate $\tilde{\alpha}$-internal, $\tilde{\beta}$-internal, $\bar{\alpha}$-external and $\bar{\beta}$-external complex cubic interval-valued intuitionistic fuzzy sets.

Definition 3.1
Let $U$ be the universe of discourse. A Complex cubic interval valued intuitionistic fuzzy set (CCIVIFS) defined on $U$ is a set given by,

$$A^k = \{x, \left[p^{-}_k(x) e^{i\theta^{-}_k(x)}, p^+_k(x) e^{i\theta^+_k(x)}\right], \left[q^{-}_k(x) e^{i\theta^{-}_k(x)}, q^+_k(x) e^{i\theta^+_k(x)}\right], r^+_k(x) e^{i\theta^+_k(x)} \mid x \in U\}$$

where, $\left[p^{-}_k(x) e^{i\theta^{-}_k(x)}, p^+_k(x) e^{i\theta^+_k(x)}\right]$ and $\left[q^{-}_k(x) e^{i\theta^{-}_k(x)}, q^+_k(x) e^{i\theta^+_k(x)}\right]$ represent the degree of lower and upper bound of the membership and non-membership which are defined as, $\tilde{\alpha}_k(x) =$
Let $U$ be the universe set. If $\beta \in \mathbb{R}$ and satisfy the inequality $\bar{\alpha}_k \leq \alpha_k^+ \leq \beta_k$ and $\bar{\alpha}_k + \beta_k^+ \leq 1$ for all $x \in U$. The phase terms $\theta_{p_k}^-(x), \theta_{p_k}^+(x), \theta_{q_k}^-(x), \theta_{q_k}^+(x)$ are real valued which lie with in the interval $[0, 2\pi]$ and satisfy the inequality $\theta_{p_k}^-(x) \leq \theta_{p_k}^+(x), \theta_{q_k}^-(x) \leq \theta_{q_k}^+(x)$ and $\theta_{p_k}^+(x) + \theta_{q_k}^+(x) \leq 2\pi$ for all $x \in U$. Where $\mu_k^i(x) = |z_f| = r_k^i(x)e^{i\theta_k^i(x)}$. The fuzzy amplitude terms $r_k^i \in [0, 1]$ and the fuzzy phase term $\theta_k^i(x) \in [0, 2\pi]$ for all $x \in U$. Therefore, mathematically [CCIVIFS] $A^k$ defined on $U$ can be represented as,

$$A^k = \left\{ x, \left( [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)], \left[ \bar{\beta}_k^-(x), \bar{\beta}_k^+(x) \right], \mu_k^i(x) \right) : x \in U \right\}$$

**Definition: 3.2**

Let $U = u_1, u_2, \ldots, u_m$ be a finite universal set and $E = e_1, e_2, \ldots, e_n$ be a finite set of parameters. Let $A \subseteq E$. Then complex cubic interval valued intuitionistic fuzzy set $A^k$ can be expressed in matrix form as

$$A^k_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

such that $A^k = [a_{ij}] = \left( [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)], \left[ \bar{\beta}_k^-(x), \bar{\beta}_k^+(x) \right], \mu_k^i(x) \right)$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. $A^k$ is called an $m \times n$ complex cubic interval valued intuitionistic fuzzy matrix.

**Definition: 3.3**

Given a set $A^k = \left( [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)], \left[ \bar{\beta}_k^-(x), \bar{\beta}_k^+(x) \right], \mu_k^i(x) \right)$ of a CCIVIFS in $U,(k = 1, 2, 3, \ldots, n)$ for all $x \in U$. We define

$\bar{\alpha}$—internal:

Let $U$ be the universe set. If $\mu_k^i(x) \in [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)]$ such that the amplitude term $r_k^i(x) \in [p_k^-(x), p_k^+(x)]$ and the phase term $\theta_{p_k}^i(x) \in [\theta_{p_k}^-(x), \theta_{p_k}^+(x)]$ for all $x \in U$.

$\bar{\beta}$—internal:

Let $U$ be the universe set. If $\mu_k^i(x) \in [\bar{\beta}_k^-(x), \bar{\beta}_k^+(x)]$ such that the amplitude term $r_k^i(x) \in [q_k^-(x), q_k^+(x)]$ and the phase term $\theta_{q_k}^i(x) \in [\theta_{q_k}^-(x), \theta_{q_k}^+(x)]$ for all $x \in U$.

$\bar{\alpha}, \bar{\beta}$—internal:

Internal if it is both $\bar{\alpha}, \bar{\beta}$—internal we have, $\mu_k^i(x) \in \{ [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)], \left[ \bar{\beta}_k^-(x), \bar{\beta}_k^+(x) \right] \}$ such that amplitude term $r_k^i(x) \in \{ [p_k^-(x), p_k^+(x)], [q_k^-(x), q_k^+(x)] \}$ and the phase term $\theta_{p_k}^i(x) \in \{ [\theta_{p_k}^-(x), \theta_{q_k}^-(x)], [\theta_{q_k}^-(x), \theta_{q_k}^+(x)] \}$ for all $x \in U$.

$\bar{\alpha}$—external:

Let $U$ be the universe set. If $\mu_k^i(x) \notin [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)]$ such that the amplitude term $r_k^i(x) \notin [p_k^-(x), p_k^+(x)]$ and the phase term $\theta_{p_k}^i(x) \notin [\theta_{p_k}^-(x), \theta_{p_k}^+(x)]$ for all $x \in U$.

$\bar{\beta}$—external:

Let $U$ be the universe set. If $\mu_k^i(x) \notin [\bar{\beta}_k^-(x), \bar{\beta}_k^+(x)]$ such that the amplitude term $r_k^i(x) \notin [q_k^-(x), q_k^+(x)]$ and the phase term $\theta_{q_k}^i(x) \notin [\theta_{q_k}^-(x), \theta_{q_k}^+(x)]$ for all $x \in U$.

$\bar{\alpha}, \bar{\beta}$—external:

Internal if it is both $\bar{\alpha}, \bar{\beta}$—internal we have, $\mu_k^i(x) \notin \{ [\bar{\alpha}_k^-(x), \bar{\alpha}_k^+(x)], \left[ \bar{\beta}_k^-(x), \bar{\beta}_k^+(x) \right] \}$ such that amplitude term $r_k^i(x) \notin \{ [p_k^-(x), p_k^+(x)], [q_k^-(x), q_k^+(x)] \}$ and the phase term $\theta_{p_k}^i(x) \notin \{ [\theta_{p_k}^-(x), \theta_{p_k}^+(x)], [\theta_{q_k}^-(x), \theta_{q_k}^+(x)] \}$ for all $x \in U$. 

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Example:3.1
Let $A^k = \left( [0.2e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.55)}], [0.1e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.45)}] \right)$ and $\mu^k_k(x) = r^k_k(x)e^{i\theta^k_k(x)}$ for all $x \in U$.

$\tilde{\alpha}$ internal:
If the amplitude term $r^k_k(x) \in (0.3, 0.5]$ and the phase term $\theta^k_k(x) \in 2\pi(0.45, 0.55]$ which is not $\tilde{\beta}$ internal.

$\beta$ internal:
If the amplitude term $r^k_k(x) \in [0.1, 0.2]$ and the phase term $\theta^k_k(x) \in 2\pi[0.2, 0.4)$ which is not $\tilde{\alpha}$ internal.

$\tilde{\alpha}, \beta$ internal:
If the amplitude term $r^k_k(x) \in [0.2, 0.3]$ and the phase term $\theta^k_k(x) \in 2\pi[0.4, 0.45]$.

$\tilde{\alpha}$ external:
If the amplitude term $r^k_k(x) \geq 0.5$ or $r^k_k(x) \leq 0.1$ and the phase term $\theta^k_k(x) \leq 2\pi(0.2)$ (or) $\theta^k_k(x) \geq 2\pi(0.55)$.

$\beta$ external:
If the amplitude term $r^k_k(x) \leq 0.2$ or $r^k_k(x) \geq 0.5$ and the phase term $\theta^k_k(x) \leq 2\pi(0.4)$ (or) $\theta^k_k(x) \geq 2\pi(0.55)$ but may not be $\tilde{\beta}$ external.

$\tilde{\beta}$ external:
If the amplitude term $r^k_k(x) \leq 0.1$ or $r^k_k(x) \geq 0.3$ and the phase term $\theta^k_k(x) \leq 2\pi(0.2)$ (or) $\theta^k_k(x) \geq 2\pi(0.45)$ but may not be $\tilde{\alpha}$ external.

Proposition:3.1
Let, $A^k = \left( [\tilde{\alpha}^k_k(x), \tilde{\alpha}^+_k(x)], [\tilde{\beta}^-_k(x), \tilde{\beta}^+_k(x)], \mu^k_k(x) \right)$ be a CCIVIFS in non-empty set U in which, $\tilde{\beta}^-_k(x) \leq \tilde{\alpha}^+_k(x) < \frac{\tilde{\alpha}^+_k(x)}{\tilde{\beta}^-_k(x)}$ for all $x \in U$. If $A^k$ is $\tilde{\alpha}$ internal, then it is $\tilde{\beta}$ internal and so internal. Also if $A^k$ is $\beta$ external, then it is $\tilde{\alpha}$ external and so external.

Proof: Straight forward.

Theorem:3.1
Let, $A^k = \left( [\tilde{\alpha}^-_k(x), \tilde{\alpha}^+_k(x)], [\tilde{\beta}^-_k(x), \tilde{\beta}^+_k(x)], \mu^k_k(x) \right)$ be a CCIVIFS in non-empty set U in which the right end point of membership value (or non-membership value) is equal to the left end point of non-membership (or membership) for all $x \in U$ if we define $\mu^k_k(x)$ by $\mu^l_k(x) = \tilde{\alpha}^-_k(x) = \beta^+_k(x)$ (or) $\mu^l_k(x) = \tilde{\alpha}^-_k(x) = \beta^-_k(x)$ then $A^k$ is external.

Proof: Straight forward.

Theorem:3.2
Let $A^k = \left( [\tilde{\alpha}^-_k(x), \tilde{\alpha}^+_k(x)], [\tilde{\beta}^-_k(x), \tilde{\beta}^+_k(x)], \mu^k_k(x) \right)$ be a CCIVIFS in non-empty set U. If $A^k$ is both $\tilde{\alpha}$ internal and $\tilde{\alpha}$ external (or) $\beta$ internal and $\beta$ external, then $\mu^k_k(x) \in [\tilde{\alpha}^-_k(x) \cup \tilde{\alpha}^+_k(x)]$ (or) $\mu^k_k(x) \in [\tilde{\beta}^-_k(x) \cup \tilde{\beta}^+_k(x)]$ for all $x \in U$.

Proof: Assume that $A^k = \left( [\tilde{\alpha}^-_k(x), \tilde{\alpha}^+_k(x)], [\tilde{\beta}^-_k(x), \tilde{\beta}^+_k(x)], \mu^k_k(x) \right)$ is both $\tilde{\alpha}$ internal and $\tilde{\alpha}$ external. Then $\mu^k_k(x) \in [\tilde{\alpha}^-_k(x) \cup \tilde{\alpha}^+_k(x)]$ is $\tilde{\alpha}$ internal and $\mu^k_k(x) \notin [\tilde{\alpha}^-_k(x) \cup \tilde{\alpha}^+_k(x)]$ is $\tilde{\alpha}$ external, that is $\mu^k_k(x) \in \tilde{\alpha}^-_k(x)$ or $\mu^k_k(x) \in \tilde{\alpha}^+_k(x)$. Hence $\mu^k_k(x) \in [\tilde{\alpha}^-_k(x) \cup \tilde{\alpha}^+_k(x)]$ for all $x \in U$. Similarly if $A^k$ is both $\beta$ internal and $\beta$ external then $\mu^k_k(x) \in [\tilde{\beta}^-_k(x) \cup \tilde{\beta}^+_k(x)]$ for all $x \in U$.

Definition:3.4
Given an CCIVIFS sets $A^k$ in U, the complement of $A^k$ is denoted by $(A^k)^c$ and is defined as follows,

$$(\tilde{\alpha}^-_k(x))^c = 1 - p^+_k(x)e^{i(2\pi - \theta^+_k(x))}, (\tilde{\alpha}^+_k(x))^c = 1 - p^-_k(x)e^{i(2\pi - \theta^-_k(x))}$$

and
\[(\tilde{\beta}_k^-(x))^c = 1 - q_k^c(x)e^{i(2\pi - \theta_k^+(x))}, (\tilde{\beta}_k^+(x))^c = 1 - q_k^c(x)e^{i(2\pi - \theta_k^-(x))}\] for all \(x \in U\).

**Definition:** 3.5

For any CCIVIFS set \(A^1 = \langle [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)], \mu_1^i(x) \rangle \) and \(A^2 = \langle [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)], \mu_2^i(x) \rangle \) in \(U\) we define equality of \(A^1\) and \(A^2\), we have

(i) \(A^1 = A^2 \iff \langle [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)] \rangle = \langle [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)] \rangle \) and \(\mu_1^i(x) = \mu_2^i(x)\) for all \(x \in U\).

(ii) **P-order:** \(A^1 \otimes_P A^2 \iff \langle [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)] \rangle \subseteq \langle [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)] \rangle \) and \(\mu_1^i(x) \leq \mu_2^i(x)\) for all \(x \in U\).

Example: 3.2

(i) **P-order:** Let \(A^1, A^2\) be two CCIVIFSs. Then P-order of \(A^1 \otimes_P A^2\) is

\[A^1 = \langle [0.1e^{i2\pi(0.2)}, 0.5e^{i2\pi(0.4)}], [0.4e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.6)}], 0.3e^{i2\pi(0.4)} \rangle \]

and \(A^2 = \langle [0.2e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.5)}], [0.3e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.5)}], 0.5e^{i2\pi(0.6)} \rangle \) for all \(x \in U\).

(ii) **R-order:** Let \(A^1, A^2\) be two CCIVIFS. Then R-order of \(A^1 \otimes_R A^2\) is

\[A^1 = \langle [0.1e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)}], [0.4e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.6)}], 0.3e^{i2\pi(0.4)} \rangle \]

and \(A^2 = \langle [0.2e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.5)}], [0.3e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.5)}], 0.3e^{i2\pi(0.4)} \rangle \) for all \(x \in U\).

**Definition:** 3.6

Let \(A^1 = \langle [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)], \mu_1^i(x) \rangle \) and \(A^2 = \langle [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)], \mu_2^i(x) \rangle \) be two CCIVIFSs.

Then the union of \(A^1\) and \(A^2\) is,

\[A^1 \cup A^2 = \max \{ [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)] \}, \min \{ [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)] \} \]

for all \(x \in U\).

and intersection of \(A^1, A^2\) is,

\[A^1 \cap A^2 = \min \{ [\tilde{\alpha}_1^-(x), \tilde{\alpha}_1^+(x)], [\tilde{\alpha}_2^-(x), \tilde{\alpha}_2^+(x)] \}, \max \{ [\tilde{\beta}_1^-(x), \tilde{\beta}_1^+(x)], [\tilde{\beta}_2^-(x), \tilde{\beta}_2^+(x)] \} \]

for all \(x \in U\).

**Definition:** 3.7

Given a set \(A^k = \langle [\tilde{\alpha}_k^-(x), \tilde{\alpha}_k^+(x)], [\tilde{\beta}_k^-(x), \tilde{\beta}_k^+(x)], \mu_k^i(x) \rangle \) of a CCIVIFS in \(U,(k = 1, 2, 3, ..., n)\) we define for all \(x \in U\).

\[
\begin{align*}
(1) (P\text{-union}) & \quad \bigcup_{k \in N}^P A^k = \bigcup_{k \in N} \left( [\tilde{\alpha}_k^-(x), \tilde{\alpha}_k^+(x)], [\tilde{\beta}_k^-(x), \tilde{\beta}_k^+(x)] \right), \bigvee_{k \in N} \mu_k^i(x) \\
(2) (R\text{-union}) & \quad \bigcup_{k \in N}^R A^k = \bigcup_{k \in N} \left( [\tilde{\alpha}_k^-(x), \tilde{\alpha}_k^+(x)], [\tilde{\beta}_k^-(x), \tilde{\beta}_k^+(x)] \right), \bigwedge_{k \in N} \mu_k^i(x)
\end{align*}
\]
(3) (P-intersection) \[ \bigcap_{k \in N} A_k = \left( \bigcap_{k \in N} \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right] \right) \right) \bigcap_{k \in N} \mu_k^f(x) \]

(4) (R-intersection) \[ \bigcap_{k \in N} A_k = \left( \bigcap_{k \in N} \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right] \right) \right) \bigvee_{k \in N} \mu_k^f(x) \]

**Theorem 3.3**

Let \( A^k \) be a CCIVIFS in \( U \). If \( A^k = \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right], \mu_k^f(x) \right) \) is \( \tilde{\alpha} \)-internal (respectively \( \tilde{\alpha} \)-external), then so is the complement of \( A^k \).

**Proof:**

Assume that \( A^k = \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right], \mu_k^f(x) \right) \) is \( \tilde{\alpha} \)-internal. Then \( \mu_k^f(x) \in \left[ \tilde{\alpha}_k^{+}(x), \tilde{\alpha}_k^{-}(x) \right] \) that is, \( \tilde{\alpha}_k^{-}(x) \leq \mu_k^f(x) \leq \tilde{\alpha}_k^{+}(x) \) for all \( x \in U \). Thus \( 1 - \tilde{\alpha}_k^{-}(x) \leq 1 - \mu_k^f(x) \leq 1 - \tilde{\alpha}_k^{+}(x) \) for all \( x \in U \), that is \( \left( \mu_k^f(x) \right)^c \in \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right] \) for all \( x \in U \). Hence \( A^k \)^c is \( \tilde{\alpha} \)-internal. If \( A^k \) is \( \tilde{\alpha} \)-external, \( \mu_k^f(x) \notin \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right] \) for all \( x \in U \), and so \( \mu_k^f(x) \notin \left[ 0, \tilde{\alpha}_k^{-}(x) \right] \) (or \( \mu_k^f(x) \notin \left[ \tilde{\alpha}_k^{+}(x), 1 \right] \). It follows that \( 1 - \tilde{\alpha}_k^{-}(x) \leq 1 - \mu_k^f(x) \leq 1 \) or \( 1 - \tilde{\alpha}_k^{+}(x) \leq 1 - \mu_k^f(x) \leq 1 - \tilde{\alpha}_k^{-}(x) \), and so that \( \left( \mu_k^f(x) \right)^c = 1 - \mu_k^f(x) \notin [1 - \tilde{\alpha}_k^{-}(x), 1 - \tilde{\alpha}_k^{+}(x)] \). Therefore \( A^k \)^c is \( \tilde{\alpha} \)-external.

**Theorem 3.4**

Let \( A^k \) be a CCIVIFS in \( U \). If \( A^k = \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right], \mu_k^f(x) \right) \) is \( \tilde{\beta} \)-internal (respectively \( \tilde{\beta} \)-external), then so is the complement of \( A^k \).

**Proof:**

It is similar to the proof of theorem 3.3.

**Theorem 3.5**

If \( A^k = \left( \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right], \mu_k^f(x) \right) \) is a set of \( \tilde{\alpha} \)-internal CCIVIFS in \( U \). Then the P-union and the P-intersection of \( A^k = \left( \left[ \alpha_k^{-}(x), \alpha_k^{+}(x) \right], \left[ \tilde{\beta}_k^{-}(x), \tilde{\beta}_k^{+}(x) \right], \mu_k^f(x) \right) \)

\( (k = 1, 2, 3, ..., n) \) are also \( \tilde{\alpha} \)-internal CCIVIFS in \( U \).

**Proof:**

Since \( A^k \) is \( \tilde{\alpha} \)-internal, we have \( \mu_k^f(x) \in \left[ \tilde{\alpha}_k^{-}(x), \tilde{\alpha}_k^{+}(x) \right] \), that is \( \tilde{\alpha}_k^{-}(x) \leq \mu_k^f(x) \leq \tilde{\alpha}_k^{+}(x) \) for all \( x \in U (k = 1, 2, 3, ..., n) \). It follows that

\[ \bigcup_{k \in N} \tilde{\alpha}_k^{-}(x) \leq \bigvee_{k \in N} \mu_k^f(x) \leq \bigcup_{k \in N} \tilde{\alpha}_k^{+}(x) \]

\[ \bigcap_{k \in N} \tilde{\alpha}_k^{-}(x) \leq \bigwedge_{k \in N} \mu_k^f(x) \leq \bigcap_{k \in N} \tilde{\alpha}_k^{+}(x) \]

Therefore \( \bigcup_{k \in N} A_k \), \( \bigcap_{k \in N} A_k \) are \( \tilde{\alpha} \)-internal CCIVIFS in \( U \).

**Example 3.3**

Then the following example R-union and R-intersection of \( \tilde{\alpha} \)-internal CCIVIFSs need not be \( \tilde{\alpha} \)-internal. Let \( A^1 \), \( A^2 \) be two CCIVIFSs.

\[ A^1 = \left\{0.1e^{2\pi i (0.2)}, 0.3e^{2\pi i (0.4)} \right\}, \left\{0.2e^{2\pi i (0.3)}, 0.4e^{2\pi i (0.5)} \right\}, \left\{0.15e^{2\pi i (0.25)} \right\} \]

and \( A^2 = \left\{0.5e^{2\pi i (0.6)}, 0.6e^{2\pi i (0.7)} \right\}, \left\{0.1e^{2\pi i (0.2)}, 0.2e^{2\pi i (0.3)} \right\}, \left\{0.55e^{2\pi i (0.65)} \right\} \) for all \( x \in U \)

Then R-union is \( \left\{0.5e^{2\pi i (0.6)}, 0.6e^{2\pi i (0.7)} \right\}, \left\{0.1e^{2\pi i (0.2)}, 0.2e^{2\pi i (0.3)} \right\}, 0.15e^{2\pi i (0.25)} \)

Then R-intersection is \( \left\{0.1e^{2\pi i (0.2)}, 0.3e^{2\pi i (0.4)} \right\}, \left\{0.2e^{2\pi i (0.3)}, 0.4e^{2\pi i (0.5)} \right\}, 0.55e^{2\pi i (0.65)} \) which is not \( \tilde{\alpha} \)-internal. For all \( x \in U \)

We provide a condition for the R-union of two \( \tilde{\alpha} \)-internal CCIVIFS to be \( \tilde{\alpha} \)-internal.
Theorem 3.6
Let \( A^1 = \left\{ [\tilde{\alpha}^-_1(x), \tilde{\alpha}^+_1(x)], [\tilde{\beta}^-_1(x), \tilde{\beta}^+_1(x)], \mu^1_1(x) \right\} \) and
\( A^2 = \left\{ [\tilde{\alpha}^-_2(x), \tilde{\alpha}^+_2(x)], [\tilde{\beta}^-_2(x), \tilde{\beta}^+_2(x)], \mu^2_1(x) \right\} \) be CCIVIFSs in \( U \). Such that
\[
\max \{ \tilde{\alpha}^-_1(x), \tilde{\alpha}^-_2(x) \} \leq \left( \mu^1_1(x) \land \mu^2_1(x) \right).
\] (1)
If \( A^1 \) and \( A^2 \) are \( \tilde{\alpha}^- \)-internal, then so is the R-union of \( A^1 \) and \( A^2 \).

Proof: Assume that \( A^1 = \left\{ [\tilde{\alpha}^-_1(x), \tilde{\alpha}^+_1(x)], [\tilde{\beta}^-_1(x), \tilde{\beta}^+_1(x)], \mu^1_1(x) \right\} \) and
\( A^2 = \left\{ [\tilde{\alpha}^-_2(x), \tilde{\alpha}^+_2(x)], [\tilde{\beta}^-_2(x), \tilde{\beta}^+_2(x)], \mu^2_1(x) \right\} \) be CCIVIFSs in \( U \) that satisfies the condition (1). Then \( \tilde{\alpha}^-_1(x) \leq \mu^1_1(x) \leq \tilde{\alpha}^+_1(x) \) and \( \tilde{\alpha}^-_2(x) \leq \mu^2_1(x) \leq \tilde{\alpha}^+_2(x) \) for all \( x \in U \), which imply that
\[
\left( \mu^1_1(x) \lor \mu^2_1(x) \right) \leq \left[ \tilde{\alpha}^-_1(x) \cup \tilde{\alpha}^-_2(x) \right].
\] It follows from (1) that
\[
\left[ \tilde{\alpha}^-_1(x) \cup \tilde{\alpha}^-_2(x) \right] = \max \{ \tilde{\alpha}^-_1(x), \tilde{\alpha}^-_2(x) \} \leq \left( \mu^1_1(x) \lor \mu^2_1(x) \right) \leq \left[ \tilde{\alpha}^+_1(x) \cup \tilde{\alpha}^+_2(x) \right] \text{ for all } x \in U.
\] Therefore \( A^1 \cup^R A^2 \) is an \( \tilde{\alpha}^- \)-internal CCIVIFS in \( U \).

Theorem 3.7
Let \( A^1 = \left\{ [\tilde{\alpha}^-_1(x), \tilde{\alpha}^+_1(x)], [\tilde{\beta}^-_1(x), \tilde{\beta}^+_1(x)], \mu^1_1(x) \right\} \) and
\( A^2 = \left\{ [\tilde{\alpha}^-_2(x), \tilde{\alpha}^+_2(x)], [\tilde{\beta}^-_2(x), \tilde{\beta}^+_2(x)], \mu^2_1(x) \right\} \) be CCIVIFSs in \( U \) such that
\[
\min \{ \tilde{\alpha}^+_1(x), \tilde{\alpha}^+_2(x) \} \geq \left( \mu^1_1(x) \lor \mu^2_1(x) \right).
\] (2)
If \( A^1 \) and \( A^2 \) are \( \tilde{\alpha}^- \)-internal, then so is the R-intersection of \( A^1 \) and \( A^2 \).

Proof: Let \( A^1 = \left\{ [\tilde{\alpha}^-_1(x), \tilde{\alpha}^+_1(x)], [\tilde{\beta}^-_1(x), \tilde{\beta}^+_1(x)], \mu^1_1(x) \right\} \) and
\( A^2 = \left\{ [\tilde{\alpha}^-_2(x), \tilde{\alpha}^+_2(x)], [\tilde{\beta}^-_2(x), \tilde{\beta}^+_2(x)], \mu^2_1(x) \right\} \) be CCIVIFSs in \( U \) that satisfies the condition (2). Then \( \tilde{\alpha}^-_1(x) \leq \mu^1_1(x) \leq \tilde{\alpha}^+_1(x) \) and \( \tilde{\alpha}^-_2(x) \leq \mu^2_1(x) \leq \tilde{\alpha}^+_2(x) \) for all \( x \in U \), which imply that
\[
\left( \mu^1_1(x) \lor \mu^2_1(x) \right) \leq \left[ \tilde{\alpha}^-_1(x) \cap \tilde{\alpha}^-_2(x) \right].
\] It follows from (2) that
\[
\left[ \tilde{\alpha}^-_1(x) \cap \tilde{\alpha}^-_2(x) \right] \leq \left( \mu^1_1(x) \lor \mu^2_1(x) \right) \leq \min \{ \tilde{\alpha}^+_1(x), \tilde{\alpha}^+_2(x) \} = \left[ \tilde{\alpha}^+_1(x) \cap \tilde{\alpha}^+_2(x) \right] \text{ for all } x \in U.
\] Therefore \( A^1 \cap^R A^2 \) is an \( \tilde{\alpha}^- \)-internal CCIVIFS in \( U \).

The following example shows that the P-union and P-intersection of \( \tilde{\alpha}^- \)-external CCIVIFSs need not be \( \tilde{\alpha}^- \)-external.

Example 3.4
Let \( A^1 = \left\{ [0.1e^{2\pi(0.2)}, 0.3e^{2\pi(0.4)}], [0.2e^{2\pi(0.3)}, 0.4e^{2\pi(0.5)}], 0.55e^{2\pi(0.6)} \right\} \) \( \) and \( A^2 = \left\{ [0.5e^{2\pi(0.6)}, 0.6e^{2\pi(0.7)}], [0.1e^{2\pi(0.2)}, 0.2e^{2\pi(0.3)}], 0.25e^{2\pi(0.3)} \right\} \) be CCIVIFSs in \( U \). For all \( x \in U \), then \( A^1 \) and \( A^2 \) are \( \tilde{\alpha}^- \)-external. The P-union of \( A^1 \) and \( A^2 \) is
\( A^1 \cup^P A^2 = \left\{ [0.5e^{2\pi(0.6)}, 0.6e^{2\pi(0.7)}], [0.1e^{2\pi(0.2)}, 0.2e^{2\pi(0.3)}], 0.25e^{2\pi(0.3)} \right\} \) \( \) and it is not \( \tilde{\alpha}^- \)-external. The P-intersection of \( A^1 \) and \( A^2 \) is
\( A^1 \cap^P A^2 = \left\{ [0.1e^{2\pi(0.2)}, 0.3e^{2\pi(0.4)}], [0.2e^{2\pi(0.3)}, 0.4e^{2\pi(0.5)}], 0.25e^{2\pi(0.3)} \right\} \) \( \) which is not \( \tilde{\alpha}^- \)-external. For all \( x \in U \)
we consider conditions for the P-union and P-intersection of two \( \tilde{\alpha}^- \)-external CCIVIFSs to be \( \tilde{\alpha}^- \)-external.

Theorem 3.8
Let \( A^1 \) and \( A^1 \) be CCIVIFSs in \( U \) such that
\[
\{ x \in U | \mu^1_1(x) \leq \tilde{\alpha}^-_1(x), \mu^2_1(x) \geq \tilde{\alpha}^-_2(x) \}
\] (3)
\[ \{ x \in U | \mu_1^x (x) \geq \bar{\alpha}_1^+ (x), \mu_2^x (x) \leq \bar{\alpha}_2^x (x) \} \] (4).

If \( A^1 \) and \( A^2 \) are \( \bar{\alpha} \)-external, then so are the \( R \)-union and \( R \)-intersection of \( A^1 \) and \( A^2 \).

**Proof:** If \( A^1 \) and \( A^2 \) are \( \bar{\alpha} \)-external, then \( \mu_1^x (x) \notin \{ \bar{\alpha}_1^+ (x), \bar{\alpha}_1^- \} \) and \( \mu_2^x (x) \notin \{ \bar{\alpha}_2^+ (x), \bar{\alpha}_2^- \} \) that is, \( \mu_1^x (x) \leq \bar{\alpha}_1^+ (x) \) or \( \mu_1^x (x) \geq \bar{\alpha}_1^- (x) \) and \( \mu_2^x (x) \geq \bar{\alpha}_2^+ (x) \) or \( \mu_2^x (x) \leq \bar{\alpha}_2^- (x) \) for all \( x \in U \). It follows from conditions (3),(4) that

\[ \begin{align*}
\mu_1^x (x) & \leq \bar{\alpha}_1^+ (x), \mu_2^x (x) \leq \bar{\alpha}_2^- (x) \\
\mu_1^x (x) & \geq \bar{\alpha}_1^- (x), \mu_2^x (x) \geq \bar{\alpha}_2^+ (x)
\end{align*} \] (5)

The condition (5) and (6) induce \( \mu_1^x (x) \leq \max \{ \bar{\alpha}_1^+ (x), \bar{\alpha}_2^- (x) \} \) and \( \mu_1^x (x) \geq \min \{ \bar{\alpha}_1^- (x), \bar{\alpha}_2^+ (x) \} \) respectively, for all \( x \in U \).

Hence \( \mu_1^x (x) \leq \max \{ \bar{\alpha}_1^+ (x), \bar{\alpha}_2^- (x) \} \) and \( \mu_1^x (x) \geq \min \{ \bar{\alpha}_1^- (x), \bar{\alpha}_2^+ (x) \} \) respectively, for all \( x \in U \). Therefore \( z^1 \) and \( z^2 \) is an \( \bar{\alpha} \)-external CCIVIFSs in \( U \).

The following example shows that the \( R \)-union and \( R \)-intersection of \( \bar{\alpha} \)-external CCIVIFSs in \( U \) may not be \( \bar{\alpha} \)-external.

**Example: 3.5**

Let \( A^1 = \left\{ [0.2e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.5)}], [0.3e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.5)}] \right\}, \mu_1^x (x) \) and \( A^2 = \left\{ [0.3e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.7)}], [0.2e^{i2\pi(0.25)}, 0.4e^{i2\pi(0.3)}] \right\}, \mu_2^x (x) \) be CCIVIFSs in \( U \). For all \( x \in U \), then \( A^1 \) and \( A^2 \) are \( \bar{\alpha} \)-external.

(i) The \( R \)-union of \( A^1 \) and \( A^2 \) is
\[ A^1 \cup^R A^2 = \left\{ [0.3e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.7)}], [0.2e^{i2\pi(0.25)}, 0.4e^{i2\pi(0.3)}] \right\}, \mu_1^x (x) \]

If \( \mu_1^x (x) = 0.5e^{i2\pi(0.6)} \) and \( \mu_2^x (x) = 0.7e^{i2\pi(0.8)} \) for all \( x \in U \), then \( A^1 \) and \( A^2 \) are both \( \bar{\alpha} \)-external. The \( R \)-union of \( A^1 \) and \( A^2 \) is not \( \bar{\alpha} \)-external. for all \( x \in U \)

(ii) The \( R \)-intersection of \( A^1 \) and \( A^2 \) is
\[ A^1 \cap^R A^2 = \left\{ [0.2e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.5)}], [0.3e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.5)}] \right\}, \mu_1^x (x) \]

If \( \mu_1^x (x) = 0.1e^{i2\pi(0.15)} \) and \( \mu_2^x (x) = 0.3e^{i2\pi(0.35)} \) for all \( x \in U \), then \( A^1 \) and \( A^2 \) are both \( \bar{\alpha} \)-external. The \( R \)-intersection of \( A^1 \) and \( A^2 \) is not \( \bar{\alpha} \)-external. for all \( x \in U \)

4. Value extraction from CCIVIFS

In this section, we present a problem to select suitable region for well drilling using CCIVIFM. An algorithm is developed for the same. The working of the algorithm is illustrated with an example.

**Definition: 4.1**

For each \( v_{ij} \) the value function is defined as,
\[ v_{ij} = \left( \frac{p_k^- + q_k^+ + q_k^- + q_k^+}{2} - r_k^f \right) e^{i \left( \frac{\theta_{p_k}^+ + \theta_{q_k}^+ + \theta_{q_k}^-}{2} - \theta_k^f \right)} \]

**Definition: 4.2**

Let \( B = [v_{ij}] \) be the matrix of the value function of a CCIVIFM \( A \). The quantity \( T_i = \sum_{j=1}^{n} v_{ij} \) gives the total of the value function values for the \( i^{th} \) row of CCIVIFM. \( T_i \) represent the total
value for the element \( x \) with regards to all the characteristics under consideration.

4.1. Statement of the problem
Let \( U = \{a_1, a_2, .., a_m\} \) be the list of regions taken into consideration. Let \( E = \{e_1, e_2, .., e_n\} \) be the set of parameters based on which the well drilling region is to be finalized. Let \( F = \{d_1, d_2, .., d_k\} \) be the set of drilling engineering experts. The entire data is presented in the form of CCIVIFM by each expert. The respective CCIVIFM’s are denoted by \( Z^1, Z^2, .., Z^k \). The problem is to convert the CCIVIFM’s into significant matrices which provides the suitable region for well drilling.

4.2. The Method
Let \( Z^1, Z^2, .., Z^k \) be the CCIVIFM’s obtained from each expert. Using Definition 4.1, convert each \( |a_{ij}| \) into corresponding value function \( v_{ij} \). Then using the Definition 4.2, construct the value function matrices \( B^r = [v_{ij}]^r, r = 1, 2, .., k \). Calculate the total value \( T^r_i = \sum_{j=1}^{n} v_{ij}^r \) from each of the \( B^r \) matrices. Now the overall total score \( OT_i \) for each region is obtained as \( OT_i = \sum_{r=1}^{k} T^r_i \), \( i = 1, 2, .., m \). Arrange the \( OT_i \) values in increasing order. The region with the highest \( OT_i \) value is the most suitable region for well drilling from the given list.

4.3. Algorithm
Step:1 Identify the list of suitable region for well drilling and the list of parameters.

Step:2 Form the CCIVIFM \( Z^1, Z^2, .., Z^k \) for each expert.

Step:3 Calculate the value function \( B^r = [v_{ij}]^r \) by using Definition 4.1.

Step:4 Calculate the total value \( T^r_i \) using Definition 4.2 from each of the \( B^r \) matrices.

Step:5 Evaluate overall total score \( OT_i \) for each of the region.

Step:6 Order the \( OT_i \) values and the region with the highest \( OT_i \) value confirms the suitable region for well drilling.

4.4. Case Study:
In this section, we present a case study to illustrate the working of the algorithm. A group of drilling engineering experts is in the process of selecting suitable region for well drilling and independently based on the set of parameters.

1. Let \( U = \{a_1, a_2, a_3, a_4\} \) be the set of suitable region for well drilling and \( d_1, d_2 \) be the set of experts. Let \( E = \{e_1, e_2, e_3\} \) be the set of parameters for the expert \( d_1 \). Here \( e_1 = \text{Geology}, e_2 = \text{Drilling equipment}, e_3 = \text{Temperature} \). Let \( E = \{e_4, e_5, e_6\} \) be the set of parameters for the expert \( d_2 \). Here \( e_4 = \text{casing limitations}, e_5 = \text{hole sizing}, e_6 = \text{budget} \).

2. Let the experts inspect the selected region individually based on the parameter set and provide their observation details in CCIVIFM \( (Z^1, Z^2) \) by applying Definition 3.2.
4. By applying Definition 4.2, the total of the value function are calculated as,
\[
T_i^1 = \begin{bmatrix} 1.61 \\ 1.2 \\ 1.4 \\ 1.46 \end{bmatrix}, \quad T_i^2 = \begin{bmatrix} 1.17 \\ 1.1 \\ 1.61 \\ 1.16 \end{bmatrix}
\]

5. The total score for each region is calculated and presented as,
\[
OT_i = \begin{bmatrix} 2.78 \\ 3.01 \\ 3.10 \end{bmatrix}
\]

By arranging the total score value for each region the most suitable regions for well drilling can be determined.

**Table 1.** Tabular representation of region’s total score values.

| \(OT_i\) | a_3 | Score | Rank |
|----------|-----|-------|------|
| OT_1     | a_3 | 3.01  | 1    |
| OT_2     | a_1 | 2.78  | 2    |
| OT_3     | a_4 | 2.62  | 3    |
| OT_4     | a_2 | 2.30  | 4    |

We predict from Table 1 that the region \(a_3\) is the most suitable for well drilling.

5. Conclusion
In this paper, we have introduced the concept of complex cubic interval valued intuitionistic fuzzy set and matrix to deal with uncertainty. Also, a decision making method has been proposed to demonstrate the reliability and validity of the tool.

6. References
[1] Alkouri A and Salleh A 2012. *2nd International conference on fundamental and applied sciences 2012*. Complex intuitionistic fuzzy sets 1482, 464-470
[2] Atanassov KT 1986. *Fuzzy sets and systems*. Intuitionistic fuzzy sets 20, 87-96
[3] Atanassov K and Gargov G 1989. *Fuzzy sets and systems*. Complex intuitionistic fuzzy sets 31, 343-349
[4] Molodstov D.A 1999. *Computers and mathematics with applications*. Soft set theory -first results 37, 19-31
[5] Ramot D, Milo R, Friedman M and Kandel A 2012. *IEEE Trans. Fuzzy Syst*. Complex fuzzy sets 2, 171-186
[6] Harish Garg and Dimple Rani 2019. *Fundamenta Informaticae*. Complex interval-valued intuitionistic fuzzy sets and their aggregation operators 164, 61-101
[7] Zadeh L.A 1965. *Information and control*. Fuzzy sets 8, 338-353
[8] Young Bae Jun, Chang Su Kim and Ki Oong Yang 2012. *Annals of fuzzy mathematics and informatics*. Cubic sets 4, 83-98
[9] Young Bae Jun, Seok-Zun Song and Seon Jeong Kim 2018. *Axioms 2018*. Cubic interval-valued intuitionistic fuzzy sets and their application in BCK/BCI-Algebras 7
[10] Chinnadurai V and Barvkavi S 2017. *International journal for research in mathematics and statistics*. Cubic Soft Matrices (3)4
[11] Chinnadurai V and Barvkavi S 2017. *International journal of scientific research and modern education*. Operations on P-order of internal cubic soft matrices (2)1
[12] Chinnadurai V and Barvkavi S 2017. *International journal of scientific research and modern education*. Operations on P-order of external cubic soft matrices (2)1
[13] Chinnadurai V and Barvkavi S 2017. *International journal of scientific research and modern education*. Operations on internal and external cubic soft matrices cubic (2)1
[14] Madad Khan, Saima Anis, Seok-Zun Song and Young Bae Jun. *Hacettepe journal of mathematics and statistics*. Complex fuzzy soft matrices with application. 1-8
[15] Zadeh L.A 1975. *Information and sciences*. The concept of a linguistic variable and its application to approximate reasoning-I (8) 199-249