Study Of Numerical Approach To Solve Fluid Flow Equations

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Abstract
In this paper we numerically investigated the nonlinear boundary value problem. We took the stretching sheet with linear velocity. Choose the suitable similarity variable to convert the nonlinear partial differential equation into nonlinear ordinary differential equation and use suitable numerical approach like runge kutta with shooting method to solve the equations.

Keywords: Fluid flow, stretching sheet, Numerical approach.

1- INTRODUCTION
In engineering activity have a stretching area is main issues like rubber sheet electrolyte manufacture of plastic cooling of a large metallic plate in bath. Polymer sheet and filaments are manufactured in construction by regular extrusion of the polymer from a die to a windup roller. That is placed at a signified space from there polymer area. A climate fluid along the thin polymer surface compose a steady moving area accompanied a non-uniform velocity along an climate fluid [1]. A test appear that the velocity of stretching sheet proportional to the space from the orifice [2]. Crane [3] studied the steady two-dimensional incompressible boundary layer flow of a Newtonian fluid because the stretching of an elastic flat sheet which moves in its self-plane along with the space from a preset point due to the appliance of a uniform stress. Stretching membrane has different value of stretching ratio for positive and negative value in two dimensional for both numerical physical. The numerical technology does not useful for small cases. We take similarity solution if stretching force linearly decrease or with respect to time and magnetic force and unsteadying factor show the direct impression on well temperature then at the surface applied a constant heat flux [4-9]. The experiment show that unusually raise the conventional heat transfer capacity through suspending Nano particles in these base fluids. The result of experiment on the particle moments are needed to understand heat transfer and fluid flow actions of Nanofluid [10]
An innovative technology to improve heat transfer is by using Nano scale particles in the base fluid [11]. The thermal conductivity of the fluid is improving up to about two times with the adding of a little amount of Nano particles to standard heat transfer [12]. At the molecular level nanotechnology take the motive of manipulating the structure of the matter with the ambition for revelation in field of Biological Science, Physical Technology, Electronics cooling, National Security and the Environment. The

2- PROBLEM FORMULATION

We assumed that stretching sheet with linear velocity \( u_w = ax \), here \( a \) is constant. In the flow problem we can describe the continuity, momentum and energy equations as fellow:

\[
\frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U - u)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\theta_0}{\rho c_p} (T - T_\infty)
\]

Velocity field with boundary conditions

\( u = u_w = cx + L \frac{\partial u}{\partial Y}, \quad v = v_w = ax \) at \( y = 0 \)

\( u \rightarrow 0 \) as \( y \rightarrow \infty \)

Here respectively, in the x and y direction u and v are the velocity component. Temperature is denoted by \( T \), kinematic viscosity denoted by \( \nu \), thermal diffusivity is denoted by \( \alpha \) and \( L \) is the proportional constant of the velocity slip and magnetic parameter \( M = \frac{\sigma B_0^2}{\rho} \) and \( \lambda = \frac{\theta_0}{\rho c_p} \).

For the similarity solution we look for Aqsa. (1)-(3) with boundary condition by following transformation

\[ \eta = y \sqrt{\frac{a}{U_f}}, \quad u = ax f'(\eta) \) & \( v = -\sqrt{aU_f} f(\eta) \)

\[ \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty} \]
Where dimensionless stream function is denoted by $f$, and dimensionless temperature is denoted by $\theta$.

We get nonlinear ordinary differential equations:

$$f'''' + ff'' - (f')^2 + M(1 - f') = 0$$  \hspace{1cm} (7)

$$\frac{1}{Pr} \theta'' + f\theta' + \lambda \theta = 0$$  \hspace{1cm} (8)

The boundary condition for transformations can describe as:

$$f = 0, f' = 1, \theta = 1, \text{ at } \eta = 0$$  \hspace{1cm} (9)

$$f' \to 0, \theta \to 0, \text{ at } \eta \to \infty$$  \hspace{1cm} (10)

Where primes denoted differential with respect to $\eta$ (similarity variable).

The quantities of local skin friction coefficient $C_f$ are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2}$$  \hspace{1cm} (11)

Where $\tau_w$ is the surface shear stress and the $q_w$ is the surface heat flux are define as

$$\tau_w = \mu \frac{\partial u}{\partial y}, \quad q_w = -k \frac{\partial T}{\partial y}$$  \hspace{1cm} (12)

With respectively $\mu$ and $k$ is the dynamic viscosity and the thermal conductivity. We using similarity variable

$$\frac{1}{2} C_f Re_x^{-\frac{1}{2}} = f''(0), \quad Nu_x Re_x^{-\frac{1}{2}} = -\theta'(0)$$  \hspace{1cm} (13)

Where $Re$ is denoted Reynolds number.

### 3- RESULT AND DISCUSSION

We used similarity transform to changing these equation (1), (2) and (3) respectively equation of continuity equation of motion and equation of energy with boundary condition (4) and (5) in ordinary differential equation (7) and (8) with boundary condition (9) and (10).

Then we were using shooting method and runge kutta method for solving numerically of nonlinear ordinary differential equation and subject to the boundary condition. For the various value of intricate parameter numerical computation achieved like prandtl number, sink parameter $\lambda$, magnetic parameter $M$.

Figure 1-3 show the temperature and velocity profile which satisfy the far field boundary condition. We see that the prandtl number $pr$ don’t show any effect to the flow field and we can say that by equation 7-10.

The effectof the prandtl number $pr$ and temperature distribution on the field. We see from figure 1 that the prandtl is increasing the temperature is also increasing.
The effect of dimensionless velocity of different magnetic parameter on the field. We see from figure 3 that we increase magnetic parameter the velocity is decrease. We increase the magnetic parameter then the flow is slow.

4- CONCLUSION
In this paper numerical study of magnetic field on boundary layer flow with natural convection boundary condition is studied. We are used similarity transformations to reduce the partial differential equation into ordinary differential equation. We were presented graphically and discussed the effect of prandtl number $\text{Pr}$, magnetic parameter $M$ and slip parameter $\lambda$ on the fluid flow. We were using shooting method and runge kutta method for numerical solving problem of boundary layer flow over a stretching sheet with a convective surface boundary condition and slip effect. We were got that both magnetic field parameter and the prandtl number is increase at the surface as the temperature is also increase. Moreover the magnetic field parameter is increase but the velocity is decrease.

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