Theoretical Determination of the $\Delta N\gamma$ Electromagnetic Transition Amplitudes in the $\Delta(1232)$ Region

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Abstract

We utilize non-perturbative and fully relativistic methods to calculate the $\Delta N\gamma$ electromagnetic transition amplitudes $G_M^*(q^2)$ (related to the magnetic dipole moment $M_{1+}^{3/2}(q^2)$), $G_E^*(q^2)$ (related to the electric quadrupole moment $E_{1+}^{3/2}(q^2)$), the electromagnetic ratio $R_{EM}(q^2) \equiv -G_E^*(q^2)/G_M^*(q^2) = E_{1+}^{3/2}(q^2)/M_{1+}^{3/2}(q^2)$, and discuss their $q^2$ behavior in the $\Delta(1232)$ mass region. These are very important quantities which arise in all viable quark, QCD, or perturbative QCD models of pion electroproduction and photoproduction.

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I. INTRODUCTION: THE $\Delta N\gamma$ TRANSITION FORM FACTORS AND MULTIPOLES

The $\Delta N\gamma$ transition form factors \cite{1} $G^*_M(q^2)$, $G^*_E(q^2)$, and $G^*_C(q^2)$ and their multipole counterparts $M_{1+}$, $E_{1+}$, and $S_{1+}$ in nuclear and elementary particle physics are very important in nuclear and elementary particle physics because they provide a basis for testing theories of effective quark forces or production models.

- They are especially important in the understanding of perturbative QCD (PQCD) models \cite{2} involving gluon exchange mechanisms, tensor interactions, or possible hybrid baryonic states;

- They are important in enhanced quark models in which the transition form factors may be calculated as a function of $q^2$; in electroproduction and photoproduction processes; in symmetry schemes such as SU(6) and U(6,6), and Melosh transformations; in bag models; in dispersion relation and Bethe-Salpeter approaches; in current algebra baryon sum rules; and in nonperturbative methods such as lattice QCD, QCD sum rules, and algebraic formulations.

- The fundamental reason that the transition form factors are such good QCD probes lies in the fact that in many quark, symmetry, and potential models, $G^*_E(q^2)$ and/or $G^*_C(q^2)$ are identically zero thus giving rise to pure magnetic dipole $M_{1+}$ transitions

- In the naïve quark model, it can be shown that the quantity $E_{1+}/M_{1+} = -G^*_E/G^*_M \equiv R_{EM} = 0$. Experimentally, however, the $R_{EM}$ appears to be non-zero but small in magnitude and of the order of a few percent. Most analyses predict the $R_{EM}$ to be small and negative at small momentum transfer, however a recent analysis \cite{3} extracted the value $R_{EM}(q^2 = -3.2 (GeV/c)^2) \approx (+6 \pm 2 \pm 3)\%$. Subsequently, another even more recent analysis \cite{4} predicts that $R_{EM} = -(2.5 \pm 0.2 \pm 0.2)\%$ at the maximum of the $\Delta(1232)$ resonance and $R_{EM} = -3.5\%$ when background scattering amplitude contributions are taken into consideration.

- Clearly, the capability of any particular theoretical model (including ours) to predict accurately and precisely non-zero $R_{EM}$ values of the right sign and magnitude for particular values of $q^2$ in agreement with experiment is critical. As has been noted, the $R_{EM}$ ratio is especially effective for testing effective quark forces such as occur in QCD one-gluon exchange tensor interactions, various types of enhanced quark models, symmetry schemes such as SU(6), U(6,6), melosh transformations, dispersion relations and sum rules (where the $\Delta$ always plays an important role).

A. Importance of the $\Delta \to N + \gamma$ Transition Form Factors

- Provide a basis for testing theories of effective quark forces and production models

- QCD: One gluon exchange mechanisms, tensor interactions, and possible hybrid baryonic states.
• Enhanced Quark Models: They should be capable of predicting accurately the $\Delta-N$ transition form factors as a function of $q^2$.

• Bag models of hadrons

• Current Algebra approaches to hadron physics

• Non-perturbative approaches to hadron physics such as lattice QCD

• Electroproduction and Photoproduction: Important for correct theoretical description.

• Symmetry Schemes: The $\Delta$ always plays an important role in models involving $SU(6)$, $U(6,6)$, etc., and melosh transformations.

• Dispersion relations: The $\Delta$ always plays an important role.

• Baryon Sum Rules: The $\Delta$ always plays an important role.

B. The $\Delta \to N + \gamma$ Transition Form Factors and Transition Amplitudes

In general one may write for the $\Delta \to N + \gamma$ transition amplitude the following expression:

$$\langle p|j_\mu(0)|\Delta^+\rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{mm^*}{E_P E_{\Delta^+}}} \bar{u}(p, \lambda_P) \left[ \Gamma_{\mu\beta} \right] u(\Delta^+, \lambda_{\Delta^+})$$

where

$$\Gamma_{\mu\beta} = \frac{3(m^* + m)}{2m} (G_M^* - 3G_E^*) \Theta^{-1} m^* q_\beta \epsilon_{\mu}(qp\gamma)$$

$$- \frac{3(m^* + m)}{2m} (G_M^* + G_E^*) \Theta^{-1} [2\epsilon_{\beta\sigma}(p^* p) \epsilon_{\mu\nu}(p^* p) \gamma_5 - im^* q_\beta \epsilon_{\nu}(qp\gamma)]$$

$$+ \frac{3(m^* + m)}{m} G_C^* \Theta^{-1} q_\beta [p \cdot qq_{\mu} - q^2 p_{\mu}] \gamma_5.$$  (2)

In Eqs. (1) and (2), the electromagnetic current is denoted by $j_\mu$. $q \equiv p^* - p$, $p^*$ and $p$ are the four-momenta of the $\Delta^+$ and nucleon respectively. $\Theta^{-1} \equiv \left[ (m^* + m)^2 - q^2 \right] \left[ (m^* - m)^2 - q^2 \right]^{-1}$ is a kinematic factor which depends on $q^2$, $m^*$ (the $\Delta^+$ mass), and $m$ (the proton mass); $\lambda_P$ and $\lambda_{\Delta^+}$ are the helicities of the proton and $\Delta^+$ respectively. We note that the first, second, and third terms in Eq.(2) induce transverse $h_3$ ($h_3$), transverse $h_2$ ($h_2$), and longitudinal helicity transitions ($h_1$) respectively in the rest frame of the $\Delta^+$ isobar. $G_M^*$, $G_E^*$, and $G_C^*$ are related to the helicity form factors $h_1$, $h_2$, and $h_3$ by the relations:

$$h_3 = -\frac{3(m^* + m)}{2m} (G_M^* - 3G_E^*)$$

$$h_2 = -\frac{3(m^* + m)}{2m} (G_M^* + G_E^*)$$

$$h_1 = \frac{3(m^* + m)}{m} G_C^*.$$  (3)
For the transition amplitude governing the virtual process $p \rightarrow p + \gamma$, we have similarly

$$\langle p(\vec{s}, \lambda) | j_\mu(0) | p(\vec{t}, \lambda^*) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m^2}{E_p E_{p'}}} \bar{u}_p(\vec{s}, \lambda) [\Gamma_\mu] u_p(\vec{t}, \lambda^*)$$

(4)

where

$$\Gamma_\mu = [1 - \tilde{q}^2/(4m^2)] \left[ (i/(4m^2)) G_M(\tilde{q}^2) \epsilon_\mu \left( \tilde{P} \tilde{q} \gamma_5 \right) + (1/(2m)) G_E(\tilde{q}^2) \tilde{P}_\mu \right],$$

(5)

$$\tilde{P}_\mu \equiv \tilde{p} + \tilde{p}^*,$$

$$\tilde{q} = \tilde{p}^* - \tilde{p} = (\tilde{p}^0, \tilde{t})$$

and

$$\tilde{p} = (\tilde{p}^0, \tilde{s}).$$

$G_M(\tilde{q}^2)$ and $G_E(\tilde{q}^2)$ in Eq. (5) are the familiar nucleon Sachs form factors.

C. Relationship Between the $\Delta N\gamma$ Form Factors and Multipoles

The magnetic, electric, and coulombic multipole transition moments given by $M_{1+}(q^2)$, $E_{1+}(q^2)$, and $S_{1+}(q^2)$ can be written \([5]\) in terms of $G^*_M(q^2)$, $G^*_E(q^2)$, and $G^*_C(q^2)$. Indeed one has

$$M_{1+} = \alpha_1 \sqrt{Q} G^*_M$$

$$E_{1+} = \alpha_2 \sqrt{Q} G^*_E$$

$$S_{1+} = \alpha_3 Q^{-\sqrt{Q}} G^*_C$$

(6)

where $\alpha_1, \alpha_2 = -\alpha_1, \alpha_3$ are functions of parameters governing the process $\Gamma(\Delta \rightarrow \pi N)$ (and in particular are dependent on the $\Delta$ mass $m^*$) and where $Q^\pm \equiv \sqrt{(m^* \pm m)^2 - q^2}$.

D. $A_2$ and $A_2$ Photon Decay Helicity Amplitudes

The total $\Delta$ radiative width $\equiv \Gamma^T_\gamma$, for decay into $p + \gamma$ is given by:

$$\Gamma^T_\gamma = \frac{mq_e^2}{2m^* \pi} \sum_{\lambda=\frac{1}{2}, \frac{3}{2}} A^2_\lambda$$

(7)

where

$$A_1 = -e \left( \frac{\sqrt{3}}{8} \right) \left[ \frac{m^* - m^2}{m^3} \right]^{\frac{1}{2}} \left[ G^*_M(0) - 3G^*_E(0) \right],$$

(8)

$$q_e^2 = \text{CM momentum, and}$$

$$A_2 = -e \left( \frac{3}{8} \right) \left[ \frac{m^* - m^2}{m^3} \right]^{\frac{1}{2}} \left[ G^*_M(0) + G^*_E(0) \right]$$

(9)

Experimentally \([4]\),

$$A_1 \cong (-140 \pm 5) \times 10^{-3} GeV^{-1/2} \quad \text{and} \quad A_2 \cong (-258 \pm 6) \times 10^{-3} GeV^{-1/2}$$

(10)
II. CALCULATION

- Our treatment is non-perturbative and performed in a broken symmetry hadronic world [7]. We do not require the use of “mean” mass approximations;
- Physical masses are used at all times. Thus, \( G_E^* \) is not forced to equal zero as in the naïve quark model;
- Our treatment is completely relativistic. Current conservation is guaranteed. Additionally, the correct electromagnetic transition operator is used in all calculations;
- We use the infinite-momentum frame for calculations or equivalently—“infinite” Lorentz boosts that are not always in the z-direction, thus implicitly (and often explicitly) bringing into play Wigner rotations resulting in mixed helicity particle states.
- In order to proceed with the calculation of \( G_M^*(q^2) \) and \( G_E^*(q^2) \), we consider helicity states with \( \lambda = \pm 1/2 \) and \( \lambda_\Delta = 1/2 \) (i.e. spin flip and non-flip sum rules) and the non-strange \( (S = 0) \) \( L = 0 \) ground state baryons \( (J^{PC} = \frac{1}{2}^+ , \frac{3}{2}^+ ) \). We will ultimately find two independent constraint equations which will then allow one to calculate \( G_M^*(q^2) \) and \( G_E^*(q^2) \), and their multipole counterparts.

- It is well-known [4] that if one defines the axial-vector matrix elements:
  \[ \langle p, 1/2 | A_{\pi^+} | n, 1/2 \rangle \equiv f = g_A(0), \langle \Delta^{++}, 1/2 | A_{\pi^+} | \Delta^+, 1/2 \rangle \equiv -\sqrt{\frac{3}{2}} g, \]
  and
  \[ \langle \Delta^{++}, 1/2 | A_{\pi^+} | p, 1/2 \rangle \equiv -\sqrt{6} h, \]
  and applies asymptotic level realization to the chiral \( SU(2) \otimes SU(2) \) charge algebra \( [A_{\pi^+}, A_{\pi^-}] = 2V_3 \), then \( h^2 = (4/25)f^2 \) (the sign of \( h = +(2/5)f \), can be fixed by requiring that \( G_M^*(0) > 0 \) and \( g = (\sqrt{2}/5)f \).
- If one inserts the algebra \( [j_3^\mu(0), A_{\pi^+}] = A_{\pi^+}^\mu(0) \) (\( j_3^\mu \) is isovector part of \( j_3^\mu \), \( j_3^\mu \) and \( j_3^\mu \) is isoscalar) between the ground states \( (B(\alpha, \lambda = \pm 1/2, \vec{s}^\prime) | B'(\beta, \lambda = 1/2, \vec{t}^\prime) \rangle \) with \( |\vec{s}^\prime| \to \infty, |\vec{t}^\prime| \to \infty \), where \( \langle B(\alpha) \rangle \) and \( \langle B'(\beta) \rangle \) are the following \( SU_F(2) \) related combinations: \( \langle p, n \rangle, \langle p, \Delta^0 \rangle, \langle \Delta^{++}, p \rangle, \langle n, \Delta^- \rangle, \langle \Delta^{++}, \Delta^+ \rangle, \langle \Delta^+, \Delta^0 \rangle, \langle \Delta^0, \Delta^- \rangle \) and \( \langle \Delta^+ , n \rangle \), then one obtains (we use \( < \Delta | j_3^\mu \Delta > = 0 \) ) the following two independent sum rule constraints:

  \[
  \langle p, 1/2, \vec{s} | j_3^\mu(0) | \Delta^+, 1/2, \vec{t}^\prime > = \frac{5\sqrt{2}}{4} \left\{ \frac{\langle p, 1/2, \vec{s} | A_{\pi^+}^\mu(0) | n, 1/2, \vec{t}^\prime >}{2f} - \langle p, 1/2, \vec{s} | j_3^\mu(0) | p, 1/2, \vec{t}^\prime \rangle \right\}. \tag{11}
  \]

  and

  \[
  \text{Spin Flip Sum Rule}
  \]

  \[
  < p, -1/2, \vec{s} | j_3^\mu(0) | \Delta^+, 1/2, \vec{t}^\prime > = \frac{5}{8} \sqrt{2} \langle p, -1/2, \vec{s} | j_3^\mu(0) | p, 1/2, \vec{t}^\prime \rangle. \tag{12}
  \]
Now take the limit $|\vec{t}| \to \infty$ and $|\vec{s}| = \to \infty$ and evaluate directly each of the matrix elements in Eq. (11) and Eq. (12). We find respectively that:

$$G_M^*(q^2) + \left[ \frac{2m^*(4m^* - m^2) + m^2 - 2q^2}{2(m^* + m^2 - q^2)} \right] G_E^*(q^2) =$$

$$\frac{5\sqrt{3}}{3} \frac{2m^2}{(m^* + m)(m^* + m^2 - q^2)} \sqrt{\frac{(m^* + m)^2 - q^2}{4m^2 - q^2}} G_M^*(q^2)$$

and

$$G_M^*(q^2) - 3G_E^*(q^2) = \left[ \frac{5\sqrt{3}m\sqrt{-q^2}}{3(m^* + m) [(m^* - m^2 - q^2)^{1/2}]} \right] G_M^*(q^2),$$

where in Eq. (14), a collinear limit of $|\vec{t}|$ and $|\vec{s}|$ was taken in such a fashion that $|\vec{s}| = r |\vec{t}|$ ($\vec{s}$ and $\vec{t}$ are taken along the z-axis, $0 < r \leq m^2/m^2$) and $q^2$ and $\vec{q}$ are related by the equations $\vec{q}^2 = (1 - r) [q^2 - (1 - r)m^2]$ and $q^2 = \frac{(1 - r)}{r}(m^2 r - m^2)$.

Eq. (13) and Eq. (14) may be solved for $G_M^*(q^2)$ and $G_E^*(q^2)$. The numerical results (Figure 1.) are as follows:

### III. RESULTS AND CONCLUSIONS

- $G_M^*(q^2)$, $G_E^*(q^2)$, $R_{EM}(q^2)$, $A_{\frac{1}{2}}^*$, and $A_{\frac{3}{2}}^*$ are computed in good agreement with experiment (Figure 1).

1. At the $\Delta$ pole mass, we find that $G_M^*(0) = 3.18$, $G_E^*(0) = +0.07$, and $R_{EM}(0) = -2.19\%$. We also determine that $A_{\frac{1}{2}}^* \approx -134 \times 10^{-3} GeV^{-1/2}$, and $A_{\frac{3}{2}}^* \approx -254 \times 10^{-3} GeV^{-1/2}$.

2. When the $\Delta$ mass is taken to be $1.232 GeV/c$, $G_M^*(0) = 3.09$, $G_E^*(0) = +0.12$, and $R_{EM}(0) = -3.94\%$. We also determine that $A_{\frac{1}{2}}^* \approx -128 \times 10^{-3} GeV^{-1/2}$, and $A_{\frac{3}{2}}^* \approx -262 \times 10^{-3} GeV^{-1/2}$.

- We find that $R_{EM}(q^2)$ is negative for $0 \leq -q^2 \lesssim 5 GeV^2/c^2$, changes sign in the region $-q^2 \approx 6 - 7 GeV^2/c^2$, and very slowly approaches 1 as $-q^2 \to \infty$.

- The PQCD prediction that $R_{EM}(q^2) \to 1$ as $-q^2 \to \infty$ is verified but only as an asymptotic condition applicable only at very high momentum transfer $E$.

- The $R_{EM}$ is particularly sensitive to the $\Delta$ mass (i.e. mass parametrization used) when $0 \leq -q^2 \lesssim 1 GeV^2/c^2$ (i.e. photoproduction).
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