How general is Legett’s conjecture for a mesoscopic ring?

P. Singha Deo

Institute of Physics, Bhubaneswar 751005, India

Abstract

It has been shown[19] that in a loop of length u to which a single stub of length v is attached (fig. 1 in ref. 19), the parity effect is completely destroyed when v/u > 2. It was also shown that such minute topological defects (v/u<< 1) act as singular perturbations. However ref. 19 studies the effect of a single topological defect and says that for v/u<< 1 parity effect is not violated in the ring. In this paper we show that topological defects of the type v/u<< 1 can also violate the parity effect depending on the exact value of v/u and the parity effect is significantly destroyed if we have many such geometric scatterers. This paper brings out the physical reasons for the destruction of parity effect. We show that the generic feature of topological defects as this is that they can produce discontinuous phase change (with change in energy) of the electron wavefunction in the ring for special value of v/u and then Legett’s conjecture breaks down. So Legett’s conjecture which generalises parity effect in presence of any arbitrary 2 body scattering and any arbitrary 1 body scattering need not be true when the one body scattering can produce discontinuous phase change of the electron wave function in the ring. This may have implications on the experimental observations.

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A normal metal ring pierced by a magnetic field carries a persistent current and has a magnetic response. The ring shows strong parity effect in the sense that the nature of response of a ring (paramagnetic or diamagnetic) is extremely sensitive to the number of electrons present in the ring. For spinless electrons, a clean ring with odd number of electrons has a diamagnetic response and that with an even number of electrons has paramagnetic response[1]. This happens because when the number of electrons in the ring changes from odd to even, there is a statistical half flux quantum which shifts the energy flux dependence by exactly $\phi_0/2$[2]. It was conjectured by Legett that this fact follows just from symmetry property of the wavefunction (the electrons being fermions, the wave function must be antisymmetric) and is independent of electron electron interaction and impurity scattering[3]. Refs. [4,5,6] are devoted to proving rigorously the so called Legett’s conjecture. At high temperature and disorder the parity effect shows up as a shift by $\pi$ of the persistent current versus flux curve with the change of number of electrons from odd to even[1,2]. For electrons with spin we get double parity effect[2]. Only in case of electrons with spin as well as interactions the parity effect may be destroyed because of fractional Aharonov Bohm(AB) effect[7-12]. But such fractional Aharonov Bohm effect has not yet been observed experimentally. Parity effect is seen in multichannel simulations too[13].

The first experiment[14] was done with $N=10^7$ rings and due to parity effect one expects the ensemble averaged response to scale with $\sqrt{N}$. Contradictory to it the actual response measured in the experiment is quiet high along with a $\phi_0/2$ periodicity. There are many possible effects that can give the $\phi_0/2$ periodicity[15] but the magnitude is a puzzle[16] and so it is for a single ring (where alternate levels have opposite response) experiment[17]. In a recent single loop experiment[18] one observes fair agreement.

It has been shown[19] that in a loop of length $u$ to which a single stub of length $v$ is attached (fig. 1 in ref. 19), the parity effect is completely destroyed when $v/u > 2$. It was also shown that minute topological defects ($v/u << 1$) act as singular perturbations. However ref. 19 studies the effect of a single topological defect and says that for $v/u << 1$ parity effect is not violated in the ring. In this paper we show that topological defects of the
type $v/u << 1$ can also violate the parity effect depending on the exact value of $v/u$ and the 
parity effect is significantly destroyed if we have many such geometric scatterers and that 
can well affect the experimental observations. This paper brings out the physical reasons 
for the destruction of parity effect. We show that the generic feature of topological defects 
as this is that they can produce discontinuous phase change of the electron wavefunction 
in the ring for special value of $v/u$ and then Legett’s conjecture breaks down. So Legett’s 
conjecture which generalises parity effect in presence of any arbitrary 2 body scattering and 
any arbitrary 1 body scattering need not be true when the one body scattering can produce 
discontinuous phase change of the electron wave function in the ring.

The thickness of the experimental ring could not have been uniform. If there are some 
sharp variations in thickness then some resonant cavities may be formed at certain places 
(at random) in the ring. 1-D modelling of the ring helps to understand the basic physics and 
its result can be easily extended to the multichannel case. Resonant cavities can be taken 
as stubs[20] and width of the resonant cavities result only in lowering the energy[21]. 

The allowed modes in the system are given by the following condition[19]. 

$$\cos(\alpha) = \frac{\sin(ku)\cot(kv)}{2} + \cos(ku) \quad (1)$$

$k$ is the allowed wave vectors and $\alpha = 2\pi\phi/\phi_0$, is the AB phase, $\phi$ being the flux through the 
ring and $\phi_0$ is the flux quantum. The above condition is the simplified form of the following 
condition[19].

$$\cos(\alpha) = re(1/T) \quad (2)$$

where $T$ is the transmission amplitude across the ring when the ring is cut open. The above 
equation immediately suggests something. For the clean ring the bound state condition is 
$e^{i(ku+\alpha)} = 1$. Whereas eqn(2) is just the condition (It is worth mentioning that $\cos(\alpha) = 
\cos(2n\pi - \alpha)$)

$$e^{[\cos^{-1}(re\frac{1}{T})+\alpha]} = 1 \quad (3)$$
Büttiker et al[22] has shown that an electron in a ring with a random potential is effectively moving in a periodic system whose unit cell is the ring when cut open. It is also well known that $\cos^{-1}(re_T^1)$ is the Bloch phase (Ku where K is the Block momentum) acquired by the electron in traversing an unit cell of an infinite periodic system where T is the transmission amplitude across the unit cell of the periodic system[19]. Hence eqn(3) is just $e^{i(Ku+\alpha)}=1$ and is in perfect agreement with ref. [22]. It also suggests that inside the ring the electron moves with the momentum K and not with the free particle momentum $\pm k$. Hence it is not surprising that $\pm k$ states are degenerate even in the presence of magnetic field (because both $+k$ and $-k$ satisfy eqn (3) ). Inside the ring the electron is moving anticlockwise or clockwise with momentum K ($Ku=\cos^{-1}(re_T^1)$). One of them is a diamagnetic (anticlockwise moving) state and the other is a paramagnetic (clockwise moving) state. Initially as the magnetic field is increased then the two states move away from each other, however the diamagnetic state and the paramagnetic state are not degenerate for any value of $\phi$ for reasons explained later. In a clean ring $(Ku+\alpha)$ is the phase aquired by the electron in going round the ring once whereas in a ring with scatterers (potential or geometric) $Ku+\alpha$ is the phase aquired in moving round the ring once and eqn (3) is due to the single valuedness of the wavefunction. So eqn (2) suggests that a particular mode is allowed in the ring if the phase acquired by an electron wave fn. in that mode (apart from the AB phase) in going round the ring once,i.e. $\cos^{-1}(re_T^1)$, equals $\alpha$. We can alternately state it as - if the Block phase of an electron in travelling a unit cell of an infinite periodic system i.e., Ku equals $\alpha$ then single valuedness of wavefunction is obtained in the ring made of the unit cell and we get a bound state. So the boundstates can be determined by graphically solving $\text{re}(1/T)=\cos(\alpha)$. This occurs at certain k values and then $k^2$ is the energy of the electron in that particular state K. It has an analogy with a scattering problem where k is the momentum outside the potential where it vanishes, whereas the momentum K inside the potential can be quiet different. However the energy throughout is $k^2$ (we have set $\hbar = 1$ and $2m=1$).

So in fig(1) we show a simple plot where the solid curve is a plot of $y=\text{re}(1/T)$ with $ku$ for $v/u=.2$. Wherever this curve intersects the straight line $y=\cos(\alpha)$, the corresponding
k value is a bound state for the system. Let us start with $\alpha=0$ and then $y=\cos(0)$ curve is shown in fig. 1 by dotted lines. Two consecutive points where the curve $y=\text{re}(1/T)$ intersects the straight line $y=\cos(0)$ are denoted by A and B in the fig. The corresponding k values are denoted as $k_1$ and $k_2$ in the fig. If $\alpha$ is increased gradually then the straight curve $y=\cos(\alpha)$ shifts gradually downwards towards the dashed curve. As the curve $y=\cos(\alpha)$ gradually go downwards the allowed wave vectors $k_1$ and $k_2$ slowly drift rightwards and leftwards respectively along the k axis. As $k_1$ drifts rightwards with $\alpha$ i.e., towards higher energy, $k_1$ is a diamagnetic state. Similarly $k_2$ is a paramagnetic state. That $k_1$, $k_2$, etc. gradually increase or decrease with $\alpha$ gives rise to a dispersion with $\alpha$ (E vs $\alpha$) with close by alternate states going further away from each other with $\alpha$ upto $\alpha=\pi$. $y=\cos(\pi)$ is also shown in fig. 1 with dashed lines. If we increase $\alpha$ further then the straight curve $y=\cos(\alpha)$ start moving upwards and comes back to its original position at $\alpha=2\pi$. This ensures $\phi_0$ periodicity of the dispersion curves. Since $\cos(\alpha)$ can vary from -1 to +1 (dotted lines to dashed lines) the dispersion curve for any two consecutive states can never cross (see fig. 1). So the dispersion curve is exactly similar to that of a ring with a random potential. The cause of gaps in the dispersion curve in that case is the breakdown of rotational symmetry of the ring by the random potential and hence the removal of degeneracy of states that cross over for a clean ring. In our case the rotational symmetry is destroyed by the topological defect. Some gaps (the ones around $kv=n\pi$) are very large but most gaps are very small. In fact some special gaps may actually go to zero for reasons explained later. Hence from fig. 1 it is evident that alternate states carry persistent currents with opposite signs and have opposite magnetic properties up to infinite energy. This is exactly the same as in case of potential scattering.

But this effect is not observed when we plot the same curves for different values of $v/u=.21$ (fig. 2) (in fact $v/u=.2\pm\epsilon$ is sufficient). Consider the intersections between the graphs $y=\text{re}(1/T)$ and $y=\cos(0)$. The first few alternate states have opposite magnetic properties but the fifth and the sixth states (two consecutive states marked A and B) are both diamagnetic disobeying the parity effect. Slowly increase $\alpha$ to see that. Parity effect
is again violated for the 11th and the 12th states both of which are paramagnetic. After a regular spacing of five levels we always find two consecutive levels that violate the parity effect. Hence parity effect due to the antisymmetric property of the electron wave function is not generic to a ring with topological defects. We shall soon see why it does not happen for specific values of \( v/u \).

Note that for \( kv=n\pi \), the transmission across a stub is zero[23] due to the formation of a node at the junction between the ring and the stub. This mode always lie in a gap of the dispersion curve and is never an allowed mode.

Scattering by a topological defect like a stub is still a poorly understood phenomenon. Ref. [23] tries to explain scattering by a stub on the same footing as scattering by potentials. Here we intend to understand the problem by mapping it onto an effective delta potential. A special feature of the delta potential is that \( | T |^2= \text{re}(T) \). This feature is not seen for any other potential. However this feature is also observed in case of a stub. This makes it possible to map a single stub onto an effective delta potential \( V(x)=k \cot(kv)\delta(x) \). So the strength of the delta potentials depend on the fermi energy. That is why at certain energies \( V(x) \) becomes zero and then the gap vanishes. Now let us start with \( k=0 \) and then slowly increase \( k \) continuously. For \( k=0 \) \( V(x)=1/v \) which means it starts with a small positive value. Then it decreases and soon goes to zero. After this the strength of the potential monotonously increase on the negative side and finally becomes \(-\infty \) at \( kv=\pi \). After this \( V(x) \) undergo a discontinuous jump from \(-\infty \) to \(+\infty \). If the strength of the \( \delta \) potential at \( kv=\pi \) and \( kv=\pi+\epsilon \) are discontinuous the scattering phase shift and hence the Block phase will also undergo discontinuous jump. \( \text{re}(1/T) \) also make a discontinuous jump from \(-\infty \) to \( \infty \) and hence the Block phase jumps by \( \pi \) (see fig. 2) (Block phase of the infinite periodic system has to be defined to a modulo of \( 2\pi \) i.e., \(-1 < \text{re}(1/T) < 1 \)). The next allowed Block phase of the infinite periodic system of stubs after that at \( D \) is that at \( B \) and they differ by \( \pi \). This is markedly different from the next allowed Block phase at any other gap, e.g., the Block phase at \( C \) is same as that at \( A \). We have seen that if the Block phase of the periodic system of stubs equals the AB phase \( \alpha \) (for the time being we have taken \( \alpha=0 \)
then the single valuedness of the wave function gets satisfied in the ring and we get a bound state. This additional phase results in satisfying this condition and creating a state at B close to the value \( kv = \pi \) which otherwise would not have been there had the phase change across \( kv = \pi \) been continuous. If it so happens that this singularity in the Bloch phase due to a singularity in the effective potential \( V(x) \) is cancelled by another singularity then the phase difference between two consecutive Bloch phase would not have been \( \pi \) and then this state at B would not exist because total phase acquired in this state is not enough to satisfy the single valuedness condition in the ring. All other states would have been as usual and would have been qualitatively same as that of a ring with a random potential. This is what happens in case of fig. 1. For \( v/u = .2 \) at \( kv = n\pi \cot(kv) = \pm \infty \) but \( \sin(ku) = 0 \). And so there is no discontinuity in \( \text{re}(1/T) \). Hence the state at B of fig. 2 will not exist. See fig. 3 where we have superposed the two graphs of fig. 1 and fig. 2. The dotted curve is for \( v/u = .2 \) whereas the solid curve is for \( v/u = .21 \). All the states for both the cases are very close to each other except that the solid curve shows a state at A (which is the state at B in fig. 2) where the dashed curve does not show any state at all. Other states of the solid curve are very close but slightly at lower energies than those of the dashed curve because an increase in \( v/u \) means an increase in the phase space of the electrons. The state at A has no partner and locked between a diamagnetic state and a paramagnetic state it has to break the parity effect. It is easy to see from fig 2 and 3 that slope of \( \text{re}(1/T) \) is such that if it jumps from \(-\infty \) to \(+\infty \) then the broken parity state is diamagnetic and paramagnetic for the other case. Specific values of the parameter \( v/u \) at which these two singularities exactly cancel are negligibly few compared to the values where they do not and is hardly likely in a real situation.

We then study the spectrum of a ring with four small topological defects or stubs present in it. To find the spectrum we have to solve eqn (2) numerically using the transfer matrix mechanism to compute \( T^{[24]} \). A portion of plot is shown in figs. 4a, 4b (solid lines). Within a certain energy range (\( 3400 > Eu^2 > 150 \)) there are much more diamagnetic levels than paramagnetic. Higher above there are however more paramagnetic levels than diamagnetic.
The length of the stubs as well as the separation between the stubs has been chosen by random number generating subroutines (0.1 < v/u < 0.3). We have plotted for only one configuration because we do not intend to take an average over configurations and compare with the experimentally observed numbers, because our calculations are in 1-D where localisation effects are very strong. But the usual alternate paramagnetic and diamagnetic states apart from the parity breaking states are almost unaltered by the defects because they feel a very weak V(x). We have checked that the graphs qualitatively remain the same for other configurations and for each case there is a substantial breakdown of parity effect. In a 3D multichannel ring there are many subbands and very close by levels. Each subband exhibits the parity effect [3]. Topological defects will destroy the parity effect of each subband with the first one being diamagnetic for each. Even one appropriate topological defect in that case can give many broken parity states.

The purpose of this paper has been to show the breakdown of Legett’s conjecture and parity effect due to topological defects (v/u << 1). We have also shown that a discontinuity in the scattering phase is the physical reason for it because nature of a state depend on how its boundary conditions get tuned by the phases. The discontinuity in the phase shifts the origin of the total phase by π but does not determine wheather the total phase should increase or decrease with α and k. However we would like to say that any breakdown in parity effect is sure to enhance the persistent current in the single ring as well as in the many ring experiment specially when the usual states (not the parity breaking ones) are not affected much by the topological defects. Also initially upto a certain energy there is no breakdown of parity effect which means that the persistent current in a ring with very few propagating channels (only four in ref. 18) will hardly be affected by these defects. Also the abundant paramagnetic states much higher up may not be occupied and hence may be of no consequence. No theories proposed so far try to find a common explanation for the three experiments. Also all except one [GK in 16] completely overlook the fact that the rings can exhibit coexistence of large persistent currents and small conductances [17]. In our case as for every parity breaking state there is a state at which the conductance across the ring
becomes zero ($| T |^2=0$) the same reason that enhances the persistent current can decrease the conductance.

$v/u$ values taken in this paper for qualitative analysis are much higher than realistic values. Smaller is $v/u$ the higher up will be the diamagnetic states. Again more the width of the resonant cavities lower again will be the parity breaking states[22]. A 3D simulation is needed to study the real situation and it will be reported in the near future.

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* prosen@iopb.ernet.in

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FIGURE CAPTIONS

Fig. 1. To show graphical solutions for the allowed modes for $v/u=.2$.

Fig. 2. To show the graphical solutions for the allowed modes for $v/u=.21$.

Fig. 3. Superposition of fig. 1. and fig. 2.

Fig. 4a. $E$ versus $\phi$ dispersion curves in the range $1000 < Eu^2 < 2000$.

Fig. 4b. $E$ versus $\phi$ dispersion curves in the range $2000 < Eu^2 < 3400$. 