On Visibility in the Afshar Two-Slit Experiment

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ABSTRACT. A modified version of Young’s experiment by Shahriar Afshar indirectly reveals the presence of a fully articulated interference pattern prior to the post-selection of a particle in a “which-slit” basis. While this experiment does not constitute a violation of Bohr’s Complementarity Principle as claimed by Afshar, both he and many of his critics incorrectly assume that a commonly used relationship between visibility parameter V and “which-way” parameter K has crucial relevance to his experiment. It is argued here that this relationship does not apply to this experimental situation and that it is wrong to make any use of it in support of claims for or against the bearing of this experiment on Complementarity.
Figure 1. The setup for the Afshar experiment.

1. Background

The Afshar experiment (shown schematically in Figure 1) is a two-slit interference experiment with the addition of a lens which focuses the beams emerging from the two slits onto detection areas serving as “which-slit” detectors, at plane $\sigma_2$. At plane $\sigma_1$ in between the slits and the final detection, a thin wire grid is placed which intercepts areas in which an interference pattern would have minima; i.e., where the probability of particle interception at plane $\sigma_1$ is zero. Afshar uses the grid to indirectly reveal the presence of an interference pattern at $\sigma_1$, since if no interference existed there, the intensity of detection at the final screen would be diminished by a known amount, and this does not happen.

In typical “which-slit” or “which-way” experiments involving detectors of varying efficiency placed right behind the slits or in arms of an interferometer, two complementary parameters $V$ and $K$ have traditionally been used to describe the degree to which one can ascertain either which way the particle went (in which case $K = 1$) or one can see a fully articulated interference pattern showing the loss
of which-way information (in which case $V = 1$). $K$ and $V$ have been shown to obey the relationship $^1$

$$K^2 + V^2 \leq 1$$

(1)

The relationship (1) has often been taken as an expression of Bohr’s Complementarity Principle (CP), since it displays the fact that one cannot attribute both a precise “which-way” property and an interference (or “both ways”) property to a particle in such an experiment; the corresponding observables are complementary.

Since the Afshar experiment shows with high reliability that the intensity at $\sigma_2$ is not diminished when the wire grid is in place, there is indeed interference occurring at $\sigma_1$. Since interference is clearly demonstrated and the final detection measurement is precise (i.e., with one slit blocked the particle is always detected at the detector corresponding to the open slit), Afshar concludes that both $K$ and $V$ are equal to unity, in apparent violation of the relationship (1). Indeed he uses this apparent violation of (1) to argue that his experiment violates CP.

I have argued elsewhere (Kastner 2005) that this experiment does not show any violation of CP since it is completely analogous to a commonplace spin experiment in which a spin-$\frac{1}{2}$ particle is prepared in the state $|x = +1\rangle$, confirmed to be in that state at an intermediate time through a non-destructive measurement, and then subjected to a sharp (i.e., precise) measurement of spin along a different (noncommuting) direction, say $z$.

Other authors agree that Afshar’s experiment does not demonstrate a violation of Complementarity. However, there seems to be some disagreement in the literature as to exactly why the Afshar experiment does not violate CP. Some authors argue that it has something to do with Afshar’s specific claims about fringe visibility $V$ (e.g., Drezet, A. (2005) and Steuernagel (2005))^2

The aim of this paper is to show that arguments about $V$, both for and against Afshar’s claims, are irrelevant to what is going on in this experiment.

In the next section, we discuss the result (1) and consider how it should properly be interpreted.

2. Visibility versus Which-Way Information: several approaches
A very simple and straightforward treatment of the tradeoff between interference and which-way information is presented by Feynman in his (1965, pp. 3-5 and 3-6). Feynman considers a Young’s experiment setup for electrons in which a light source is positioned just downstream from the slits, so that a photon probe can be emitted whenever an electron goes through the slits. Two appropriately placed detectors serve to detect any scattered photons, indicating which slit the electron went through. If $\phi_i$ is the amplitude for the electron to go through slit $i$ and land at screen position $x$, $a$ is the amplitude for a photon to be scattered into the correct detector $i$, and $b$ is the amplitude for the photon to be scattered into the incorrect detector $j$, then the probability of the electrons’s detection at screen position $x$, given a photon detection at either detector, is:

$$|a\phi_1 + b\phi_2|^2 + |a\phi_2 + b\phi_1|^2$$

(2) clearly exhibits the relationship between precise which-way detection and fringe visibility, since the former corresponds to $b = 0$ (which wipes out the interference terms) and the latter corresponds to $a = b$.

A much more general approach to this problem is presented by Englert (1996) who considers an interferometer augmented with a which-way detection device upstream from the region of recombination of the two beams. If the which-way detector is initially in a pure state $|d\rangle$, its state following interaction with a particle is given by $U_\pm |d\rangle$ where the plus/minus denotes which path is taken. Englert finds that the fringe visibility and which-way parameters $V$ and $K$ are given by:

$$V = |\langle d|U_- U_+^\dagger |d\rangle|,$$

$$K = (1 - |\langle d|U_- U_+^\dagger |d\rangle|^2)^{1/4}$$

(3)

and thus

$$V^2 + K^2 = 1$$

(4)

Again, the precision of the which-way detection depends on the separation of the two states $U_\pm |d\rangle$, which is analogous to the smallness of the photon’s wavelength as it interacts with the passing electrons. A photon with a wavelength
that is large in comparison to the dimensions of the experiment will only perform a “weak” measurement of slit location. Similarly if the two states $U \pm |d\rangle$ overlap significantly, the which-way measurement will be weak.

Based on Feynman’s presentation, we can make the correspondence

$$V = |2ab|,$$
$$K = (1 - |2ab|^2)^{\frac{1}{2}},$$

whence it can be seen that if $a = 1, b = 0$, then $V = 0$, and if $a = b = \frac{1}{\sqrt{2}}$, then $V = 1$.

3. Analysis

The above results are straightforward implications of the quantum formalism for the experiments discussed. However, Afshar’s experiment seems to involve both a which-slit detection and fringe visibility, and he therefore claims that $V^2 + K^2 = 2$. Are Feynman and Englert both wrong, or has Afshar found an interesting loophole in their derivation? No. Their derivations only apply to the experimental situations considered by them, in which the which-way detection occurs upstream from the region of interference, and the relationship (1) also applies only to those situations. What this means is that Afshar is wrong in presenting his experiment as providing any sort of *interesting* violation of the relationship (1) between $V$ and $K$, since the derivation of (1) unambiguously applies to a different experiment.

Some critics, as noted earlier, try to refute his claim to have shown a violation of complementarity by arguing that $V \neq 1$, which tacitly accepts the notion that (1) is applicable and tries to argue against the claim that it is violated. Such arguments generally take the form of claiming that the only way to have $V = 1$ is to have a full detection of all particles involved in the interference pattern. But these arguments miss the point, for they don’t dispute that the grid successfully, if *indirectly*, reveals the presence of a fully articulated interference pattern between the slits and the final screen (since virtually no particles are blocked by the grid at the interference minima loci). For the argument asserting $V \neq 1$ to have any force as an upholding of eqn. (1) against Afshar’s claim that it is violated, it needs to show that the interference pattern is distorted to the extent that an accurate slit basis measurement takes place at the final screen. But obviously this isn’t
happening—the final detection does nothing to eradicate or distort the interference pattern. On the contrary, the undiminished intensity of the final detection serves as indirect proof that the interference pattern remains intact in the context of a sharp post-selection measurement of a noncommuting observable. Thus, what Afshar shows is that you can post-select for a particular value of the slit basis observable and not lose interference prior to that post-selection. This is a different experiment than the one considered by Feynman and Englert and their V and K analysis does not apply.

Interestingly, the essence of the Afshar experiment is presented by Srikanth in his (2001), “Physical Reality and the Complementarity Principle.” Srikanth considers a two-slit experiment in which unitarity is explicitly preserved, via an additional internal degree of freedom of the detector elements (which can be considered a “vibrational” component), as the amplitude contributions from each slit evolve toward final detection on a screen composed of those detector elements.

The evolution of the particle + detector state from the slits to the final screen with initial detector state \{\ket{0}\}, activated detector spatial basis states \{\ket{\phi_x}\} and vibrational basis states \{\ket{v_U}, \ket{v_L}\} is then given by (where amplitudes \(a_x\) and \(b_x\) depend on wave number, distance, and slit of origin, and \{\ket{x}\} are final particle basis states):

\[
\frac{1}{\sqrt{2}} (\ket{U} + \ket{L}) \otimes \ket{0} \rightarrow \sum_x \ket{x} \otimes [a_x \ket{\phi_x} \ket{v_U} + b_x \ket{\phi_x} \ket{v_L}]
\] (6)

Upon detection at a particular location \(x\), one term remains from the sum on the right-hand side of (6):

\[
\ket{x} \otimes [a_x \ket{\phi_x} \ket{v_U} + b_x \ket{\phi_x} \ket{v_L}]
\] (7)

which still, however, allows for a post-selection measurement of each detector’s vibrational component at each detection event to obtain either \(v_U\) or \(v_L\) and is therefore analogous to the “which-slit” measurement performed by Afshar via the lens setup. This is even more dramatic than the Afshar result because clearly \(V = 1\) since a fully articulated interference pattern has been irreversibly recorded—not just indicated indirectly—and yet a measurement can be performed after the fact that seems to reveal “which slit” the photon went through. However, the point is that the detector’s vibrational mode remains in a superposition until that
measurement is made, implying that each photon indeed went through both slits. As Srikanth puts it, “...the amplitude contributions from both paths to the observation at [detector element] \( x \) results in a superposition of vibrational modes. The initial superposition leaves behind a remnant superposition.” \(^5\) So, just because one can “post-select” by measuring the vibrational observable and end up with a particular corresponding slit eigenstate doesn’t mean the particle went through that slit alone; in a very concrete sense, it went through both slits.

4. Conclusion

The inverse relationship between \( V \) and \( K \) derived independently by both Feynman and Englert depends on an experimental situation in which a pre-existing superposition of slit states is “collapsed” to some degree by a measurement of an observable whose eigenstates are components of the superposition, before the interference due to the superposition can be recorded. This collapse, characterized by an increase in \( K \), is what causes the corresponding decrease in \( V \).

In the Srikanth thought experiment, the collapse takes place only after the interference indicating \( V = 1 \) has already been recorded. In the Afshar experiment, the collapse takes place after the fully articulated interference has been indirectly indicated to exist by the fact that the grid does not significantly diminish the intensity of the final detection. In cases like these, the inverse relationship between \( V \) and \( K \) does not apply; you can get a full interference pattern and than sharply post-select for a slit eigenstate. But the latter measurement doesn’t give any physically meaningful “which-slit” information since the particle already went through both slits. So thinking of \( K \) as a true “which-way” parameter in this kind of post-selection is misleading, and it is inappropriate to argue either that (1) Afshar’s claims about CP are wrong because his \( V \neq 1 \) or that (2) Afshar is right because \( V = 1 \) and \( K = 1 \).

\(^1\)Cf. Feynman, 1965.
\(^2\)Steuernagel presents a calculation that purports to show that the visibility \( V \) in Afshar’s experiment is quite low. However, this calculation seriously misrepresents both the maximum and minimum irradiances by defining them with such a low resolution that many photons counted as contributing to \( I_{\text{max}} \) will also be counted as contributing to \( I_{\text{min}} \). This double-counting necessarily results in little difference between \( I_{\text{max}} \) and \( I_{\text{min}} \) and leads to a value for \( V \) that bears little, if any, relation to the actual interference pattern.

\(^3\)Englert uses ‘\( D \)’ instead of ‘\( K \)’ for the which-way parameter.

\(^4\)This formulation makes the assignment \( |d\rangle = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \) and assumes that \( U_+ \) and \( U_\text{−} \) rotate \( |d\rangle \) to \( (a, b) \) and \( (b, a) \), respectively, where \( |a|^2 + |b|^2 = 1 \).

\(^5\)Srikanth (2001), p. 2
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