U(1) Goldberger-Treiman Relation
and Its Connection to the Proton Spin

Hai-Yang Cheng
Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

Abstract

The U(1) Goldberger-Treiman (GT) relation for the axial charge \( g_A \) is reexamined. It is stressed that the isosinglet GT relation in terms of the \( \eta_0 \) holds irrespective of the quark masses and the axial anomaly. We pointed out that the identification of the \( \eta_0 - N \) and \( \partial \cdot K - N \) coupling terms with the quark and gluon spin components respectively in a proton is possible but valid only in the chiral-invariant factorization scheme. In general, the two-component U(1) GT relation can be identified in a gauge-invariant way with connected and disconnected insertions. The observation that \( (\sqrt{3}f/2m_N)g_{\eta'NN} \) is related to the connected insertion i.e., the total valence quark contribution to the proton spin enables us to determine the physical coupling constants \( g_{\eta'NN} \) and \( g_{\eta NN} \) from the GT relations for \( g_A^0 \) and \( g_A^8 \). We found \( g_{\eta'NN} = 3.4 \) and \( g_{\eta NN} = 4.7 \).
One important thing we learn from the derivation of the isotriplet Goldberger-Treiman (GT) relation

\[ g_3^A(0) = \frac{\sqrt{2} f_\pi}{2m_N} g_{\pi N N}, \]  

where \( f_\pi = 132 \text{ MeV} \), is that this relation holds irrespective of the light quark masses. For \( m_\pi^2 \neq 0 \), it is derived through the use of PCAC; while in the chiral limit, \( g_3^A(q^2) \) is related to the form factor \( f_3^A(q^2)q^2 \), which receives a nonvanishing pion-pole contribution even in the \( q^2 \to 0 \) limit. By the same token, it is tempting to contemplate that the flavor-singlet GT relation

\[ g_0^A(0) = \frac{\sqrt{3} f_\pi}{2m_N} g_{\eta_0 NN} \]  

with \( g_{\eta_0 NN}^{(0)} \) being a bare direct coupling between \( \eta_0 \) and the nucleon, should be also valid irrespective of the meson masses and the axial anomaly. This is indeed the case: the U(1) GT relation remains totally unchanged no matter how one varies the anomaly and the quark masses. This salient feature was first explicitly shown in [1,2] (see also [3] for a general argument). It was also pointed out in [4] that this U(1) relation is independent of the interaction of the ghost field \( \partial^\mu K_\mu \) (\( K_\mu \) being the Chern-Simons current) with the nucleon.

Many discussions on the isosinglet GT relation around the period of 1989-1992 [1-7] were mainly motivated by the desire of trying to understand why the axial charge \( g_3^A \) inferred from the EMC experiment [8] is so small, \( g_3^A(0) = 0.12 \pm 0.18 \) at \( Q^2 = 10.7 \text{ GeV}^2 \) (pre-1993). At first sight, the U(1) GT relation seems not to be in the right ballpark as the naive SU(6) quark model prediction \( g_{\eta_0 NN}^{(0)} = (\sqrt{3}/5)g_{\pi NN} \) yields a too large value of \( g_3^A(0) = 0.80 \). Fortunately, in QCD the ghost field \( G \equiv \partial^\mu K_\mu \), which is necessary for solving the \( U_A(1) \) problem, is allowed to have a direct \( U_A(1) \)-invariant interaction with the nucleon. This together with the mixing of \( \partial^\mu K_\mu \) with the \( \eta_0 \) implies that the net “physical” \( \eta_0 - N \) coupling \( g_{\eta_0 NN} \) is composed of the bare coupling \( g_{\eta_0 NN}^{(0)} \) and the ghost coupling \( g_{GNN} \). As a consequence, a possible cancellation between \( g_{\eta_0 NN} \) and \( g_{GNN} \) terms will render \( g_3^A \) smaller. However, this two-component expression for the axial charge is not free of ambiguity. For example, \( g_{GNN} \) is sometimes assumed to be the coupling between the glueball and the nucleon in the literature.

Since the earlier parton-model analysis of polarized deep inelastic scattering seems to indicate a decomposition of \( g_3^A \) in terms of the quark and gluon spin components [9], this has motivated many authors to identify the term \( (\sqrt{3} f_\pi/2m_N)g_{\eta_0 NN} \) with the total quark spin \( \Delta \Sigma \) in a proton, and the other term with the anomalous gluon contribution. However, it is also known that the lack of a local and gauge-invariant operator definition for the quark and gluon spins in this two-component picture leads to a clash between the OPE approach and the parton model. In the former approach, \( g_3^A \) is identified with the total quark spin in a proton. This casts doubt on the usual two-component interpretation of the axial charge.

\footnote{The \( q^2 \) of the form factor should not be confused with the momentum transfer \( Q^2 \) occurred in deep inelastic scattering.}
The purpose of this Letter is two-fold. First, we would like to clarify and present a pertinent physical interpretation for the two-component isosinglet GT relation. We argue that the term \((\sqrt{3}f_\pi/2m_N)g_{\eta_{NN}}\) should be identified with the connected insertion i.e., the total valence quark spin in a proton. Second, with the valence quark spin inferred from data or from the quark model, we can employ the GT relations for \(g_A^8\) and \(g_A^0\) to determine the physical coupling constants \(g_{\eta_{NN}}\) and \(g_{\eta'_{NN}}\).

2. The easiest way of deriving the U(1) GT relation is to first work in the chiral limit. Defining the form factors in terms of \(\eta\) and \(\pi\) arises entirely from the axial anomaly, we are led to the isosinglet GT relation (2). When \(\theta_0\) mass \(m_{\eta_0}\) arises, we get

\[ 2m_N g_A^0(0) = \langle N|\partial^\mu J_{\mu}^5|N\rangle = 3\langle N|\partial\cdot K|N\rangle. \]

Assuming the \(\eta_0\) pole dominance for \(\partial\cdot K\), namely \(\partial\cdot K = \frac{1}{\sqrt{3}}m^2_{\eta_0}f_\pi\eta_0\), where the \(\eta_0\) mass \(m_{\eta_0}\) arises entirely from the axial anomaly, we are led to the isosinglet GT relation (2). When the quark masses are turned on, chiral symmetry is explicitly broken but the GT relation in terms of \(\eta_0\) remains intact, as shown in [1,2]. Nevertheless, the \(\eta_0\) is no longer a physical meson, and it is related to the mass eigenstates via

\[
\begin{pmatrix} \pi_3 \\ \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} 1 & \theta_1 \cos \theta_3 + \theta_2 \sin \theta_3 & \theta_1 \sin \theta_3 - \theta_2 \cos \theta_3 \\ -\theta_1 & \cos \theta_3 & \sin \theta_3 \\ \theta_2 & -\sin \theta_3 & \cos \theta_3 \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix},
\]

where \(\theta_1, \theta_2, \text{and } \theta_3\) are the mixing angles of \(\pi^0 - \eta, \pi^0 - \eta'\) and \(\eta - \eta'\) respectively, and their analytic expressions are given in [6] with the numerical values

\[ \theta_1 = -0.016, \quad \theta_2 = 0.0085, \quad \theta_3 = -18.5^\circ. \]

In Eq.(5) only terms linear in small angles \(\theta_1\) and \(\theta_2\) are retained. Consequently, the complete GT relations in terms of physical coupling constants read [2] \[2\]

\[
g_A^3(0) = \frac{\sqrt{2}f_\pi}{2m_N}g_{\pi_{3NN}} = \frac{\sqrt{2}f_\pi}{2m_N}(g_{\pi_{NN}} \pm g_{\eta'_{NN}}(\theta_1 \sin \theta_3 - \theta_2 \cos \theta_3) \\
\pm g_{\eta_{NN}}(\theta_1 \cos \theta_3 + \theta_2 \sin \theta_3)),
\]

\[
g_A^8(0) = \frac{\sqrt{6}f_\pi}{2m_N}g_{\eta_{8NN}} = \frac{\sqrt{6}f_\pi}{2m_N}(g_{\eta_{NN}} \cos \theta_3 + g_{\eta'_{NN}} \sin \theta_3 \pm g_{\pi_{NN}} \theta_1),
\]

\[
g_A^0(0) = \frac{\sqrt{3}f_\pi}{2m_N}g_{\eta_{0NN}}^{(0)} = \frac{\sqrt{3}f_\pi}{2m_N}(g_{\eta'_{NN}} \cos \theta_3 - g_{\eta_{NN}} \sin \theta_3 \pm g_{\pi_{NN}} \theta_2) + \cdots,\]

\(^2\)For the axial charge \(g_A^0\), the authors of [6] obtained a result something like (see Eq.(24) of the second reference of [6])

\[
\frac{\sqrt{3}f_\pi}{2m_N} \left( g_{\eta'_{NN}} \cos \theta_3 - \Delta m_{\eta'_{NN}} g_{Q_{NN}} \right) - \frac{1}{\sqrt{2}} g_A^3 \tan \theta_3 \pm \frac{3}{2} g_A^8 (\theta_2 - \theta_1 \tan \theta_3)
\]

and claimed that in the limit of \(\theta_1, \theta_2 \rightarrow 0\) but \(\theta_3 \neq 0\), it reproduces the result of Veneziano [4] only if the first order correction from \(\theta_3\) (i.e., the \(g_A^8 \theta_3\) term) is neglected. However, using Eqs.(8) and (12) one can show that (7) is nothing but \((\sqrt{3}f_\pi/2m_N)g_{\eta_{0NN}}^{(0)}\), as it should be.

3
where the first sign of ± or ∓ is for the proton and the second sign for the neutron, and the ellipsis in the GT relation for $g_A^0$ is related to the ghost coupling, as shown below. Since the mixing angles $\theta_1$ and $\theta_2$ are very small, it is evident that isospin violation in (8) is unobservably small.

As we have accentuated before, the isosinglet GT relation in terms of the $\eta_0$ remains unchanged no matter how one varies the quark masses and the axial anomaly. However, the $\eta_0$ field is subject to a different interpretation in each different case. For example, when the anomaly is turned off, the mass of $\eta_0$ is the same as the pion (for $f_{\eta_0} = f_\pi$). When both quark masses and anomaly are switched off, the $\eta_0$ becomes a Goldstone boson, and the axial charge at $q^2 = 0$ receives its contribution from the $\eta_0$ pole.

When the SU(6) quark model is applied to the coupling $g_{\eta_0 NN}^{(0)}$, it is evident that the predicted $g_A^0 = 0.80$ via the GT relation is too large. This difficulty could be resolved by the observation that a priori the ghost field $G \equiv \partial \cdot K$ is allowed in QCD to have a direct coupling with the nucleon

$$L = \cdots + \frac{g_{GNN}}{2m_N} \partial^\mu G \text{Tr}(\bar{N} \gamma_\mu \gamma_5 N) + \frac{\sqrt{3}}{f_\pi} (\partial \cdot K) \eta_0 + \cdots,$$

so that

$$\partial \cdot K = \frac{1}{\sqrt{3}} m_{\eta_0} f_\pi \eta_0 + \frac{1}{6} m_{\eta_0}^2 f_\pi \text{Tr} \partial^\mu \text{Tr}(\bar{N} \gamma_\mu \gamma_5 N).$$

However, the matrix element $\langle N | \partial \cdot K | N \rangle$ remains unchanged because of the presence of the $\partial \cdot K - \eta_0$ mixing, as schematically shown in Fig. 1:

$$\langle N | \partial \cdot K | N \rangle = \frac{1}{\sqrt{3}} f_\pi g_{\eta_0 NN}^{(0)} - \frac{1}{3} m_{\eta_0}^2 f_\pi g_{GNN} + \frac{1}{3} m_{\eta_0}^2 f_\pi g_{GNN} = \frac{1}{\sqrt{3}} f_\pi g_{\eta_0 NN}^{(0)}.$$ (11)

We see that although it is still the bare coupling $g_{\eta_0 NN}^{(0)}$ that relates to the axial charge $g_A^0$, the “physical” $\eta_0 - N$ coupling is modified to (see Fig. 1)

$$g_{\eta_0 NN} = g_{\eta_0 NN}^{(0)} + \frac{1}{\sqrt{3}} m_{\eta_0} f_\pi g_{GNN},$$

where the second term arises from the $\eta_0 - \partial \cdot K$ mixing. As a consequence, the quark model should be applied to $g_{\eta_0 NN}$ rather than to $g_{\eta_0 NN}^{(0)}$, and we are led to

$$g_A^0(0) = \frac{\sqrt{3} f_\pi}{2m_N}(g_{\eta_0 NN} - \frac{1}{\sqrt{3}} m_{\eta_0}^2 f_\pi g_{GNN}).$$

\(^3\)A smooth extrapolation of the strong coupling constant from on-shell $q^2$ to $q^2 = 0$ is understood.

\(^4\)A two-component expression for the U(1) GT relation was first put forward by Shore and Veneziano [5].
It has been proposed that the smallness of $g_A^0$ may be explained by considering the pole contributions to $\partial \cdot K$ from higher single particle states $X$ above the $\eta_0$, so that the isosinglet GT relation has the form (see e.g., Chao et al. [4], Ji [4], Bartelski and Tatur [10])

$$g_A^0(0) = \frac{\sqrt{3}}{2m_N}(f_{\eta_0} g_{\eta_0 NN} + \sum_X f_X g_{XNN}).$$

(14)

The state $X$ could be the radial excitation state of $\eta_0$ or a $0^{-+}$ glueball. (Note that the ghost field $\partial \cdot K$ is not a physical glueball as it can be eliminated via the equation of motion.) However, we will not pursue this possibility further for two reasons: (i) It is entirely unknown whether or not the $X$ states contribute destructively to $g_A^0$. (ii) As we shall see later, the contribution from a direct interaction of the ghost field with the nucleon corresponds to a disconnected insertion, which is shown to be negative according to recent lattice QCD calculations [11,12]. Therefore, the ghost-field effect is realistic, and if the contributions due to the states $X$ are taken into account, one should make the following replacement

$$g_{\eta_0 NN} \rightarrow g_{\eta_0 NN} - \frac{1}{\sqrt{3}} m_{\eta_0}^2 f_\pi g_{GNN}, \quad g_{XNN} \rightarrow g_{XNN} - \frac{1}{\lambda} m_X^2 g_{XNN}$$

(15)

in Eq.(14), where $\lambda$ is the $\partial \cdot K - X$ mixing.

3. It has been claimed in the parton-model study of polarized deep inelastic scattering that $g_A^0$ is related to the flavor-singlet quark spin and the anomalous gluon contribution [9]:

$$g_A^0(0) = \Delta u' + \Delta d' + \Delta s' - \frac{3\alpha_s}{2\pi} \Delta G \equiv \Delta \Sigma' - \Delta \Gamma,$$

(16)

where $\Delta q' = q^\uparrow + \bar{q}^\uparrow - q^\downarrow - \bar{q}^\downarrow$ is the net helicity of the quark flavor $q$ in a proton, and $\Delta G = G^\uparrow - G^\downarrow$. In Eq.(16) a superscript “prime” is used to denote a quark spin different from the one appearing in the OPE approach (see below). By comparing (16) with (13), it is tempting to identify the two components of the U(1) GT relation as

$$\Delta \Sigma' = \frac{\sqrt{3} f_\pi}{2m_N} g_{\eta_0 NN}, \quad \Delta \Gamma = \frac{m_{\eta_0}^2 f_\pi^2}{2m_N} g_{GNN}.\quad (17)$$

On the contrary, in the OPE approach only the quark operator contributes to the first moment of the proton structure function $g_1^p(x)$ at the twist-2 and spin-1 level [13], so that

$$g_A^0(0) = \Delta u + \Delta d + \Delta s \equiv \Delta \Sigma.$$

(18)

Therefore, one may wonder if the identification (17) is unique and sensible.

The above issue has to do with whether or not gluons contribute to $\Gamma_1^p$, the first moment of the polarized proton structure function $g_1^p(x)$ at the twist-2 and spin-1 level [13]. Since this issue has been addressed and resolved by Bodwin and Qiu [14], in the following we will simply outline the main arguments (see also [15]).

The gluonic contribution to $\Gamma_1^p$ is governed by the first moment of the differential polarized photon-gluon scattering cross section denoted by $\Delta \sigma(x)$. A direct calculation of the photon-gluon scattering box diagram shows that $\Delta \sigma(x)$ has collinear and infrared singularities at
\( m^2 = p^2 = 0 \), with \( m \) the quark mass and \( p \) the momentum of the gluon. With two different choices of the soft cutoff, one obtains

\[
\Delta \sigma_{CCM}(x) = (1 - 2x) \left( \ln \frac{Q^2}{-p^2} + \ln \frac{1}{x^2} - 2 \right),
\]

for \( m^2 = 0 \) and \( p^2 \neq 0 \) (Carlitz et al. [9]), and

\[
\Delta \sigma_{AR}(x) = (1 - 2x) \left( \ln \frac{Q^2}{m^2} + \ln \frac{1 - x}{x} - 1 \right) - 2(1 - x),
\]

for \( m^2 \neq 0 \) and \( p^2 = 0 \) (Altarelli and Ross [9]). At first sight, it appears that \( \int_0^1 \Delta \sigma_{hard}(x) dx = 1 \) in both regulator schemes because the \( 2(1 - x) \) term in \( \Delta \sigma_{AR}(x) \) arising from \( k^2_\perp \sim m^2 \) is a soft contribution and because the \( \ln(Q^2/p^2) \) and \( \ln(Q^2/m^2) \) terms, which depend logarithmically on the soft cutoff, make no contribution to the first moment due to chiral symmetry or helicity conservation, recalling the splitting function \( \Delta P_{qG}(x) = \frac{1}{2}(2x - 1) \). However, the cancellation of the soft contribution from different \( x \) regions is not reliable because chiral symmetry may be broken at some hadronic scale \( \Lambda \) through some nonperturbative effects. As a consequence, one has to introduce a factorization scale \( \mu_{fact} \) to subtract the unwanted soft contribution, i.e., the contribution arising from the distribution of quarks and antiquarks in a gluon:

\[
\Delta \sigma_{hard}(x, Q^2/\mu_{fact}^2) = \Delta \sigma(x, Q^2) - \Delta \sigma_{soft}(x, \mu_{fact}^2).
\]

In practice, one makes an approximate expression for the box diagram that is valid for \( k^2_\perp \ll Q^2 \) and then introduces an ultraviolet cutoff on the integration variable \( k_\perp \) to ensure that only the region \( k^2_\perp \ll \mu_{fact}^2 \) contributes to the soft part [14]. The choice of the regulator specifies the factorization convention. There are two sources contributing to the first moment of \( \Delta \sigma(x) \): one from \( k^2_\perp \sim Q^2 \) and the other from chiral symmetry breaking. When the ultraviolet cutoff is gauge invariant, it breaks chiral symmetry due to the presence of the axial anomaly and hence makes a contribution to \( \Delta \sigma_{soft} \). So we have

\[
\int_0^1 \Delta \sigma_{CCM}^{soft}(x) dx = 1, \quad \int_0^1 \Delta \sigma_{AR}^{soft}(x) dx = 0.
\]

In the mass-regulator scheme, the original soft contribution coming from \( k^2_\perp \sim m^2 \), where chiral symmetry is explicitly broken by the quark mass, is canceled by the contribution arising from chiral symmetry breaking induced by the ultraviolet cutoff. Therefore, in the gauge-invariant factorization scheme \( \int_0^1 \Delta \sigma_{hard}(x) dx = 0 \) and hence \( g^0_\mu(0) = \Delta \Sigma \). In this scheme, the quark spin has a gauge-invariant local operator definition: \( s_\mu \Delta q = \langle p|\bar{q}\gamma_\mu \gamma_5 q|p\rangle \). It is \( Q^2 \) dependent because of the nonvanishing anomalous dimension associated with the flavor-singlet quark operator. By contrast, it is also possible to choose a chiral-invariant but gauge-variant ultraviolet cutoff, so that

\[
\int_0^1 \Delta \sigma_{CCM}^{soft}(x) dx = 0, \quad \int_0^1 \Delta \sigma_{AR}^{soft}(x) dx = 1.
\]
This together with Eqs. (19) and (20) leads to \( \int_0^1 \Delta \tilde{\sigma}_{\text{hard}}(x) dx = 1 \). It is thus evident that gluons contribute to \( \Gamma_p \) and \( g^0_A(0) = \Delta \Sigma - \Delta \Gamma \) in the chiral-invariant factorization scheme. Contrary to the first scheme, \( \Delta q' \) here cannot be written as a matrix element of a gauge-invariant local operator; it is either gauge variant or involves a nonlocal operator. Moreover, \( \Delta q' \) is \( Q^2 \) independent as the gauge-variant ultraviolet cutoff in this scheme does not flip helicity. It is thus close to the naive intuition in the parton model that the quark helicity is not affected by gluon emissions.

It is clear that the issue of whether or not gluons contribute to \( \Gamma_p \) is purely a matter of the factorization scheme chosen in defining the quark spin density \(^5\) and the hard gluon-photon scattering cross section [14]. We thus conclude that the identification of the U(1) GT relation with the quark and gluon spin components in a proton as given in Eq. (17) is possible but valid only in the chiral-invariant factorization scheme. Next, one may ask what will be the physical interpretation for the gauge-invariant \( g_{\eta NN} \) and \( g_{GNN} \) terms in the two-component isosinglet GT relation (13) in the gauge-invariant factorization scheme in which \( g^0_A = \Delta \Sigma \)? We note that the evaluation of the hadronic flavor-singlet current involves a disconnected insertion in addition to the connected one (see Fig. 2). The connected and disconnected insertions are related to valence quark and vacuum polarization (i.e., sea quark) contributions respectively (Liu [4]) and are separately gauge invariant. A recent lattice calculation [10] shows a sea polarization in a polarized proton: \( \Delta u_s = \Delta d_s = \Delta s = -0.12 \pm 0.01 \) from the disconnected contribution. This empirical SU(3)-flavor symmetry for sea polarization, which is known to be not true for the unpolarized counterpart, implies that the disconnected insertion is dominated by the axial anomaly of the triangle diagram. Since the triangle contribution is proportional to \( \partial \cdot K \), the ghost field, it is thus quite natural to make the gauge invariant identification:

\[
\frac{\sqrt{3} f_\pi}{2m_N} g_{\eta_{NN}} = \text{connected insertion}, \quad - \frac{m^2_{\eta_{NN}} f_\pi^2}{2m_N} g_{GNN} = \text{disconnected insertion},
\]

which is valid in both factorization schemes. In the gauge-invariant factorization scheme, the disconnected insertion, which is responsible for the smallness of \( g^0_A \), should be interpreted as a screening effect for the axial charge owing to the negative sea polarization rather than an anomalous gluonic effect.

4. Having identified the two-component U(1) GT relation (13) with connected and disconnected insertions, we are now able to extract the physical coupling constants \( g_{q_{NN}} \) and \( g_{\bar{q}_{NN}} \). This is because the connected insertion (CI) corresponds to the total valence

---

\(^5\)In principle, the choice of \( \Delta q \) and \( \Delta \tilde{\sigma}_{\text{hard}}(x) \) or \( \Delta q' \) and \( \Delta \tilde{\sigma}_{\text{hard}}(x) \) is just a matter of convention. In practice, the gauge-invariant \( \Delta q \) is probably more useful than the \( Q^2 \)-independent \( \Delta q' \) since the former can be expressed as a nucleon matrix element of a local gauge-invariant operator and is thus calculable in lattice QCD. Moreover, the polarized Altarelli-Parisi equations cannot be applied to \( \Delta q' \) directly [16]. It has been advocated that \( \Delta q' \) and \( \Delta G \) have a simple partonic definition: the former (latter) can be identified in one-jet (two-jet) events in polarized deep inelastic scattering (Carlitz et al. [9]). However, as pointed out in [17], it is impossible to separate the jets when the target is at rest because the longitudinal momentum is of order \( Q^2/M \), whereas the transverse momentum \( k_\perp \) can only be of order \( Q \). Consequently, the \( q \) and \( \bar{q} \) jets are collinear even they may have large transverse momentum.
quark contribution to the proton spin, so it is related to the quark model expectation; that is,

\[ \sqrt{3} \frac{f_\pi}{2m_N} g_{g_\pi NN} = g_A^0 (\text{CI}) = \Delta u_v + \Delta d_v = 3F - D, \]  

(25)

where last identity follows from the fact that in the quark model \( g_A^8 = 3F - D = \Delta u_v + \Delta d_v \). Another way to see this is that \( g_\pi^8 = \Delta u_v + \Delta d_v - 2\Delta s \to \Delta u_v + \Delta d_v \) due to the aforementioned SU(3) symmetry for sea polarization. Unlike the previous identification (17), \( g_A^0 (\text{CI}) \) here is not identified with the total quark spin \( \Delta \Sigma \). In the nonrelativistic quark limit, \( F = \frac{2}{3} \), \( D = 1 \), and hence \( \Delta u_v + \Delta d_v = 1 \). With the inclusion of the relativistic effects, \( F \) and \( D \) are reduced to \( F = 0 \) and \( D = 0 \).

From Eqs. (8) and (25), the GT relations for \( g_A^8 \) and \( g_A^0 \) are recast to

\[ 3F - D = \sqrt{6} \frac{f_\pi}{2m_N} g_{g_\pi NN} = \sqrt{6} \frac{f_\pi}{2m_N} (g_{\eta NN} \cos \theta_3 + g_{\eta' NN} \sin \theta_3), \]

\[ 3F - D = \sqrt{3} \frac{f_\pi}{2m_N} g_{g_\pi NN} = \sqrt{3} \frac{f_\pi}{2m_N} (g_{\eta' NN} \cos \theta_3 - g_{\eta NN} \sin \theta_3), \]  

(26)

where the tiny isospin-violating effect has been neglected. Note that we have \( g_{\eta NN} \) instead of \( g_{\eta NN}^{(0)} \) on the second line of the above equation. Using \( \theta_3 = -18.5^\circ \) [see Eq. (6)], it follows from (26) that

\[ g_{\eta' NN} = 3.4, \quad g_{\eta NN} = 4.7, \]  

(27)

while

\[ g_{\eta NN} = 4.8, \quad g_{\eta NN} = 3.4. \]  

(28)

It is interesting to note that we have \( g_{\eta' NN} < g_{\eta NN} \), whereas \( g_{\eta NN} > g_{\eta NN} \). Phenomenologically, the determination of \( g_{\eta' NN} \) and \( g_{\eta NN} \) is rather difficult and subject to large uncertainties. The analysis of the NN potential yields \( g_{\eta' NN} = 7.3 \) and \( g_{\eta NN} = 6.8 \) [19], while the forward NN scattering analyzed using dispersion relations gives \( g_{\eta' NN} \), \( g_{\eta NN} < 3.5 \) [20]. But these analyses did not take into account the ghost pole contribution. An estimate of the \( \eta' \to 2\gamma \) decay rate through the baryon triangle contributions yields \( g_{\eta' NN} = 6.3 \pm 0.4 \) [21].

Finally, the ghost coupling is determined from the disconnected insertion (DI)

\[ - \frac{m_\pi^2 f_\pi^2}{2m_N} g_{GNN} = g_A^0 (\text{DI}) = \Delta u_s + \Delta d_s + \Delta s \to 3\Delta s. \]  

(29)

A combination of all EMC, SMC, E142 and E143 data for \( \Gamma_1^p \) at \( \langle Q^2 \rangle = 10 \text{GeV}^2 \) yields [22]

\[ \Delta u = 0.83 \pm 0.02, \quad \Delta d = -0.43 \pm 0.02, \quad \Delta s = -0.09 \pm 0.02, \]  

(30)

and hence

\[ g_A^0 (0) = \Delta \Sigma = 0.31 \pm 0.06. \]  

(31)
From (29) and (30) we obtain

$$g_{GNN} \approx 55 \text{ GeV}^{-3}.$$  \hspace{2cm} (32)

5. To summarize, we have emphasized that the U(1) GT relation in terms of the $\eta_0$ remains totally unchanged no matter how one varies the quark masses and the axial anomaly, and pointed out that the two-component expression of the isosinglet GT relation should be identified with the connected and disconnected insertions; the identification with the quark and gluon spin components in a proton is possible only in the chiral-invariant factorization scheme. Since $(\sqrt{3}f_\pi/2m_N)g_{\eta_0NN}$ is related to the total valence quark contribution to the proton spin, we have determined the physical coupling constants $g_{\eta'NN}$ and $g_{\eta NN}$ from the GT relations for $g^0_A$ and $g^A_A$ and found that $g_{\eta'NN} = 3.4$ and $g_{\eta NN} = 4.7$.

ACKNOWLEDGMENT

This work was supported in part by the National Science Council of ROC under Contract No. NSC84-2112-M-001-014.

REFERENCES

1. J. Schechter, V. Soni, A. Subbaraman, and H. Weigel, Phys. Rev. Lett. 65, 2955 (1990); Mod. Phys. Lett. A5, 2543 (1990); Mod. Phys. Lett. A7, 1 (1992).

2. J. Bartelski and S. Tatur, Phys. Lett. B265, 192 (1991).

3. G.M. Shore and G. Veneziano, Nucl. Phys. B381, 23 (1992).

4. G. Veneziano, Mod. Phys. Lett. A4, 1605 (1989); T.D. Cohen and M.K. Banerjee, Phys. Lett. B230, 129 (1989); T. Hatsuda, Nucl. Phys. B329, 376 (1990); X. Ji, Phys. Rev. Lett. 65, 408 (1990); M. Birse, Phys. Lett. B249, 291 (1990); K.T. Chao, J. Wen, and H. Zeng, Phys. Rev. D46, 5078 (1992); M. Wakamatsu, Phys. Lett. B280, 97 (1992); K.F. Liu, Phys. Lett. B281, 141 (1992).

5. G.M. Shore and G. Veneziano, Phys. Lett. B244, 75 (1990).

6. A.V. Efremov, J. Soffer, and N.A. Törnqvist, Phys. Rev. Lett. 64, 1495 (1990); Phys. Rev. D44, 1369 (1991).

7. T. Hatsuda, Nucl. Phys. (Proc. Suppl.) 23B, 108 (1991).

8. EMC Collaboration, J. Ashman et al., Nucl. Phys. B238, 1 (1990); Phys. Lett. B206, 364 (1988).
9. G. Altarelli and G.G. Ross, *Phys. Lett.* **B212**, 391 (1988); R.D. Carlitz, J.C. Collins, and A.H. Mueller, *Phys. Lett.* **B214**, 229 (1988); A.V. Efremov and O.V. Teryaev, in *Proceedings of the International Hadron Symposium*, Bechyne, Czechoslovakia, 1988, eds. Fischer et al. (Czechoslovakian Academy of Science, Prague, 1989), p.302.

10. J. Bartelski and S. Tatur, *Phys. Lett.* **B305**, 281 (1993).

11. S.J. Dong, J.-F. Lagaè, and K.F. Liu, *Phys. Rev. Lett.* **75**, 2096 (1995).

12. M. Fukugita, Y. Kuramashi, M. Okawa, and A. Ukawa, *Phys. Rev. Lett.* **75**, 2092 (1995).

13. R.L. Jaffe and A.V. Manohar, *Nucl. Phys.* **B337** 509 (1990).

14. G.T. Bodwin and J. Qiu, *Phys. Rev.* **D41**, 2755 (1990), and in *Proc. Polarized Collider Workshop*, University Park, PA, 1990, eds. J. Collins et al. (AIP, New York, 1991), p.285.

15. H.Y. Cheng, H.H. Liu, and C.Y. Wu, IP-ASTP-17-95 (1995).

16. S.D. Bass and A.W. Thomas, *J. Phys.* **G19**, 925 (1993); Cavendish preprint 93/4 (1993).

17. A.V. Manohar, *Phys. Lett.* **B255**, 579 (1991).

18. F. Close and R.G. Roberts, *Phys. Lett.* **B316**, 165 (1993).

19. O. Dumbrajs et al., *Nucl. Phys.* **B216**, 277 (1983).

20. W. Brein and P. Knoll, *Nucl. Phys.* **A338**, 332 (1980).

21. B. Bagchi and A. Lahiri, *J. Phys.* **G16**, L239 (1990).

22. C.Y. Prescott, SLAC-PUB-6620 (1994); J. Ellis and M. Karliner, *Phys. Lett.* **B341**, 397 (1995).

**FIGURE CAPTIONS**

**Fig. 1** Contributions to the matrix element $\langle N|\partial \cdot K|N\rangle$ from (1) the $\eta_0$ pole dominance, (2) a direct coupling of the ghost field with the nucleon, and (3) the $\partial \cdot K - \eta_0$ mixing.

**Fig. 2** Connected and disconnected insertions.
\[ \begin{align*}
\text{Figure 1}
\end{align*} \]
Figure 2