Spin Dynamics and Multiple Reflections in Ferromagnetic Film in Contact with Normal Metal Layers

E. Šimánek
Department of Physics, University of California, Riverside, CA 92521

Spin dynamics of a metallic ferromagnetic film imbedded between normal metal films (N) has received much attention recently. Ferromagnetic resonance (FMR) experiments on N/F/N structures show that imbedding produces an additional spin damping. A possible explanation of these observations is that the localized spins in the ferromagnet interact, via s-d exchange, with the conduction electrons in the normal metal. Due to this interaction, a transfer of angular momentum takes place from the precessing ferromagnetic moment to the conduction electrons in the adjacent N layers. The reactive torque generated by this transfer acts back to the conduction electrons in the adjacent N layers.

An interesting formulation of this mechanism has been given in recent work by Tserkovnyak et al. They used the Landau-Lifshitz-Gilbert (LLG) equation of the N/F/N structure. The modifications of the spin scattering approach of parametric charge pumping by these authors calculate the spin current pumped through the N/F and F/N contacts of the N/F/N structure. The FMR frequency, whereas the Gilbert damping is in an order of magnitude agreement with the damping observed by Urban et al. The results are compared with the linear response theory of Mills [Phys. Rev. B 68, 0144419 (2003)].

I. INTRODUCTION

Spin dynamics in multilayers consisting of a thin ferromagnetic film (F) imbedded between normal metal films (N) has been studied using the spin-pumping theory of Tserkovnyak et al. [Phys. Rev. Lett. 88, 117601 (2002)]. The scattering matrix for this structure is obtained using a spin-dependent potential with quantum well in the ferromagnetic region. Owing to multiple reflections in the well, the excess Gilbert damping and the gyromagnetic ratio exhibit quantum oscillations as a function of the thickness of the ferromagnetic film. The wavelength of the oscillations is given by the depth of the quantum well. For iron film imbedded between gold layers, the amplitude of the oscillations of the Gilbert damping is in an order of magnitude agreement with the damping observed by Urban et al. [Phys. Rev. Lett. 87, 217204 (2001)]. The results are compared with the linear response theory of Mills [Phys. Rev. B 68, 0144419 (2003)].

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\[\frac{1}{\gamma} = \frac{1}{\gamma_0} \left\{ 1 + \frac{g_L \mu_B}{4\pi M} [A_{L}^{(L)} + A_{R}^{(R)}] \right\}\]

\[\alpha = \frac{\gamma}{\gamma_0} \left\{ \alpha_0 + \frac{g_L \mu_B}{4\pi M} [A_{L}^{(L)} + A_{R}^{(R)}] \right\}\]

where parameters \(\alpha_0\) and \(\gamma_0\) are the bulk values of \(\alpha\) and \(\gamma\). \(g_L\) is the Landé factor, \(\mu_B\) is the Bohr magneton, and \(M\) is the total moment of the ferromagnetic film. The superscripts (L) and (R) correspond to the left (N/F) and right (F/N) interfaces of the N/F/N structure. The parameters \(A_{L}^{(L,R)}\) and \(A_{R}^{(L,R)}\) stem from the spin currents pumped into the (L,R) normal layers (leads). These parameters are expressed in terms of the scattering amplitudes as follows:

\[A_{L} = \frac{1}{2} \sum_{mn} \{r_{mn}^{\uparrow} - r_{mn}^{\downarrow} \}^2 + t_{mn}^{\uparrow} - t_{mn}^{\downarrow} \]  

\[A_{i} = \text{Im} \sum_{mn} \{r_{mn}^{\uparrow} (r_{mn}^{\downarrow})^* + t_{mn}^{\uparrow} (t_{mn}^{\downarrow})^* \}\]

where \(r_{mn}^{\uparrow,\downarrow}\) and \(t_{mn}^{\uparrow,\downarrow}\) are the reflection and transmission amplitudes for electrons with up and down spins. The expressions (4) and (5) are to be evaluated with transverse modes \((m,n)\) taken at the Fermi energy.

From Eqs. (2) and (3) we see, in conjunction with Eq. (1), that the parameter \(A_{L}\) is responsible for the shift of the FMR frequency, whereas \(A_{R}\) increases the FMR linewidth. For a symmetric N/F/N structure, we have \(A_{L}^{(L)} = A_{L}^{(R)} = A_{L}\), and \(A_{R}^{(L)} = A_{R}^{(R)} = A_{R}\). When the electrons scattered from one interface interfere incoherently with the other interface, the parameters \(A_{L}^{(L)}\) and \(A_{R}^{(L)}\) contribute independently of the parameters \(A_{L}^{(R)}\) and \(A_{R}^{(R)}\). As pointed out in Ref. 4, this applies...
for ferromagnetic films that are thick compared to the coherence length \( \lambda_c \sim (k_F^+-k_F^-)^{-1} \) where \( k_F^+ \) and \( k_F^- \) are the spin-up and spin-down Fermi wave vectors, respectively. In this case, the parameters \( A_1 \) and \( A_\alpha \) can be expressed in terms of the mixing conductances of the F/N and N/F contacts.\(^8\)

This approximation suppresses the effect of multiple reflections in ferromagnetic layers that are not much thicker than \( \lambda_c \). The physics is very similar to that of the Fabry-Perot interferometer in optics. One manifestation of such interferences is oscillation of interlayer exchange coupling in F/N/F multilayers as a function of ferromagnetic-layer thickness. This effect has been first predicted from numerical calculations by Baranov.\(^9\) Subsequently, Bruno\(^10\) offered an explanation based on quantum interference picture. Results of his work have been confirmed by Bloemen et al.\(^11\) who succeeded to see oscillations of the interlayer exchange coupling in Co/Cu/Co as the thickness of the Co layers is varied.

The subject of the present work is to apply the spin-pumping approach to a model which allows multiple reflections in the ferromagnetic layer to be manifested in the renormalization of the parameters \( \alpha \) and \( \gamma \). Primary motivation for this investigation comes from the recent work by Mills\(^12\).

The starting point of Mills work is an approach\(^13\) which generalizes the Ruderman-Kittel-Kasuya-Yosida (RKKY) theory to time-dependent source represented by the precessing vector \( \mathbf{m}(t) \). In this approach, the renormalization of the parameters \( \gamma \) and \( \alpha \) is found proportional to the frequency derivative of the real and imaginary parts of the transverse susceptibility, respectively.

Mills calculates the transverse susceptibility from a model which allows for multiple reflections in the ferromagnetic layer. Instead of representing the ferromagnetic layer by two independently scattering outside interfaces, (as done in Refs.\(^ [4] \) and \( [13] \)), the electrons in Ref.\(^ [12] \) are assumed to move in a spin-dependent potential of the trilayer in the form of a quantum well. This analysis yields an excess damping parameter \( \alpha' = \alpha - \alpha_0 \) which exhibits strong quantum oscillations with the thickness of the ferromagnetic film.

Ref.\(^ [13] \) and recent work of the present author\(^14\) show that, for a ferromagnetic monolayer, the dynamic RKKY approach and the spin pumping theory yield the same results for the parameter \( \alpha' \). Thus, we anticipate that application of the spin-pumping method to scattering by a quantum well potential will produce renormalizations of the parameters \( \alpha \) and \( \gamma \) which also exhibit oscillations with the thickness of the ferromagnetic film.

In the present investigation we confirm this anticipation. However, it turns out that our results are, in several aspects, different from those of Ref.\(^ {12} \). Primarily, we find that besides oscillations of \( \alpha' \) there are also oscillations of the \( g \)-shift with similar period and magnitude. Moreover, the periods of the oscillations show a dependence on the exchange splitting that is different from that of Ref.\(^ {12} \).

II. SCATTERING BY A SQUARE WELL

We consider a trilayer, N/F/N, consisting of a ferromagnetic layer of thickness \( D \) imbedded between normal metal layers, each of infinite thickness. The conduction electrons move in a spin-dependent potential \( U_\alpha(x) \) where \( x \) is the coordinate perpendicular to the layers, and \( \alpha \) specifies the orientation of the spin relative to the direction of the magnetization of the ferromagnet. We assume that the electrons in the F layer are described by a single-band Hubbard model.\(^15\) In the Hartree-Fock approximation, this model yields one-electron energies \( \varepsilon_{k,\alpha} = \varepsilon_k + I < n_{-\alpha} > \) where \( I \) is the on-site screened Coulomb interaction. To simplify the calculations, we shift the energies in the F-region by \(-I < n \uparrow \rangle \). This brings the zero of energy to the bottom of the minority band. In this case, the spin-up conduction electron sees an attractive square well potential of depth \( \Delta = I < n \uparrow \rangle - < n \downarrow \rangle \), a quantity defining the exchange splitting between the majority and minority spin bands. The down-spin electron moves in a potential that is constant and equal to zero for all \( x \).

Thus, \( U_\uparrow(x) = 0 \) for \(-\infty < x < \infty \), \( U_\uparrow(x) = -\Delta \) for \(-D/2 < x < D/2 \), \( U_\uparrow(x) = 0 \) for \( x < -D/2 \) and \( x > D/2 \).

If the zero of energy is taken at the bottom of the majority band, the up-spin electron sees a potential barrier of height \( \Delta \), while the down-spin electron moves in constant zero-value potential. This choice of potential has been previously used by Bruno\(^ {10,16} \). Mills\(^12\) assumes that the potential energy of the up-spin electron is \( V_0 \) above the zero of energy (set at the bottom of the conduction band) whereas the down-spin has potential energy \( V_0 + \Delta \). Thus, Bruno’s potential follows by setting \( V_0 = 0 \) and our choice corresponds to taking \( V_0 = -\Delta \). The fact that the results\(^ {12} \) for the \( D \)-dependence of \( \alpha' \) are not very sensitive to the choice of \( V_0 \) provides some justification for the present choice of the potential. Our main motivation for this choice is that the \( x \)-component of the electron wave vector for spin up stays real (see Eq.\(^ {11} \)). This simplifies the summation over the transverse modes in Eqs.\(^ {4} \) and \( 5 \).

The single-band Hubbard model implies that, owing to the high density of states, the effective mass of the itinerant electrons in the ferromagnetic layer is larger than the free electron mass. This complicates the scattering problem since, besides the potential well, there is also a "mass barrier" in the F-region. Similar to Refs.\(^ {10} \) and \( 16 \), we replace this position dependent mass by a constant average mass. It should be pointed out that an enhancement of the effective mass can also take place in normal layers due to electron-electron interactions. This effect is especially pronounced for Pd and Pt layers\(^17\).

We first consider the reflection amplitudes \( r_{m,n}^{(\alpha)} \). Since the transverse momentum is conserved, we have \( r_{m,n}^{(\alpha)} = r_{m}^{(\alpha)} \delta_{m,n} \) where \( r_{m}^{(\alpha)} = r_{0}^{(\alpha)} (k_{F}^{(\alpha)}) \) where \( k_{F}^{(\alpha)} \) is the component of the wave vector along axis \( x \). For our choice of
the potential, we have \( r_{0}^{r}(k_{\perp}^{\uparrow}) = 0 \), and

\[
r_{0}^{r}(k_{\perp}^{\uparrow}) = r_{\infty}^{r}(k_{\perp}^{\downarrow}) \frac{1 - \exp(2ik_{\perp}^{\uparrow}D)}{1 - |r_{\infty}^{r}(k_{\perp}^{\downarrow})|^2 \exp(2ik_{\perp}^{\downarrow}D)}
\]

where

\[
r_{\infty}^{r}(k_{\perp}^{\downarrow}) = \frac{k_{\perp} - k_{\perp}^{\downarrow}}{k_{\perp} + k_{\perp}^{\downarrow}}
\]

where \( k_{\perp} \) and \( k_{\perp}^{\downarrow} \) are the wave vectors in the N and F-regions, respectively. Since the expressions (4) and (5) are to be evaluated at the Fermi energy, we have

\[
k_{\perp} = (k_{F}^{2} - k_{\perp}^{2})^{\frac{1}{2}}
\]

and

\[
k_{\perp}^{\downarrow} = (k_{F}^{2} - k_{\perp}^{2} + \delta^{2})^{\frac{1}{2}}
\]

where \( k_{||} \) is the magnitude of the in-plane wave vector \( k_{||} \), and \( \delta^{2} = 2m\Delta/h^{2} \). Eq. (9) implies that \( k_{\perp}^{\downarrow} \) stays real over the whole range, \((0, k_{F})\), of the variable \( k_{||} \).

Next let us consider the transmission amplitudes. The reflection symmetry of the potential implies \( t_{m,n}^{(\alpha)} = t_{n,m}^{(\alpha)} \).

Due to the conservation of transverse momentum, we have \( t_{m,n}^{(\alpha)} = t_{0}^{(\alpha)}(k_{\perp}^{\downarrow})\delta_{m,n} \) where \( t_{0}^{(\alpha)}(k_{\perp}^{\downarrow}) = 1 \) and

\[
t_{0}^{r}(k_{\perp}^{\downarrow}) = \frac{1 - |r_{\infty}^{r}(k_{\perp}^{\downarrow})|^2}{1 - |r_{\infty}^{r}(k_{\perp}^{\downarrow})|^2 \exp(2ik_{\perp}^{\downarrow}D)}
\]

**III. PARAMETERS \( A_r, A_i \)**

We are now ready to evaluate the quantities \( A_r \) and \( A_i \). The transverse-mode sums in Eqs. (4) and (5) can be written as a sum over the in-plane wave vectors \( k_{\perp} \). Converting the sum to a two-dimensional integral and using Eq. (9), we have

\[
\sum_{m,n} = \frac{L^{2}}{2\pi} \int_{0}^{k_{F}} dk_{||}k_{||} = -\frac{L^{2}}{2\pi} \int_{k_{1}}^{k_{2}} dk_{\perp}^{\downarrow} k_{\perp}^{\downarrow}
\]

where \( L \) is the lateral dimension of the film. Owing to the presence of the term \( \exp(2ik_{\perp}^{\downarrow}D) \) in Eqs. (6) and (10), it is convenient to integrate over the variable \( k_{\perp}^{\downarrow} \). According to Eq. (9), the integration limits for this variable are

\[
k_{1} = (k_{F}^{2} + \delta^{2})^{\frac{1}{2}}
\]

\[
k_{2} = \delta
\]

Using the unitarity property of the scattering matrix, we have

\[
(r_{m,n}^{r})^{2} + (t_{m,n}^{r})^{2} = 1
\]

Using this relation and the fact that \( r_{m,n}^{\uparrow} = 0, t_{m,n}^{\uparrow} = 1 \), equation (4) can be simplified to the following form

\[
A_{r} = \sum_{m,n} (1 - \text{Re}t_{m,n}^{r})
\]

Performing the summation with use of the prescription (11) and using Eq. (10), we obtain from Eq. (15)

\[
A_{r} = \frac{L^{2}}{2\pi} \int_{k_{1}}^{k_{2}} dk_{\perp}^{\downarrow} k_{\perp}^{\downarrow}
\times \frac{[r_{m,n}^{r})^{2} + (r_{\infty}^{r})^{4}] [1 - \cos(2k_{\perp}^{\downarrow}D)]}{1 - 2(r_{\infty}^{r})^{2} \cos(2k_{\perp}^{\downarrow}D) + (r_{\infty}^{r})^{4}}
\]

where

\[
r_{\infty}^{r} = \frac{[(k_{\perp}^{\downarrow})^{2} - \delta^{2}]^{\frac{1}{2}} - k_{\perp}^{\downarrow}}{[(k_{\perp}^{\downarrow})^{2} - \delta^{2}]^{\frac{1}{2}} + k_{\perp}^{\downarrow}}
\]

Substituting \( r_{m,n}^{\uparrow} = 0, t_{m,n}^{\uparrow} = 1 \) into Eq. (5), we obtain

\[
A_{i} = \sum_{m,n} \text{Im}t_{m,n}^{r}
\]

This result can be expressed using Eqs. (10) and (11) as an integral over \( k_{\perp}^{\downarrow} \)

\[
A_{i} = \frac{L^{2}}{2\pi} \int_{k_{1}}^{k_{2}} dk_{\perp}^{\downarrow} k_{\perp}^{\downarrow}
\times \frac{[r_{m,n}^{r})^{2} - (r_{\infty}^{r})^{4}] \sin(2k_{\perp}^{\downarrow}D)}{1 - 2(r_{\infty}^{r})^{2} \cos(2k_{\perp}^{\downarrow}D) + (r_{\infty}^{r})^{4}}
\]

Making a substitution \( u = 2Dk_{\perp}^{\downarrow} \), we obtain from Eqs. (16) and (17)

\[
A_{r} = \frac{L^{2}\delta^{2}}{2\pi} I_{r}(u_{1}, u_{2})
\]

where

\[
I_{r}(u_{1}, u_{2}) = \int_{u_{2}}^{u_{1}} du \frac{(1 - \cos u)\{u_{2}^{2} + [(u_{2}^{2} - u_{2}^{2})^{\frac{1}{2}} - u]^{4}\}}{2u_{2}^{2}(1 - \cos u) + 16u_{2}^{4}(u^{2} - u_{2}^{2})^{2}}
\]

According to Eqs. (12) and (13), the dimensionless parameters \( u_{1} \) and \( u_{2} \) are given by

\[
u_{1} = 2Dk_{1} = 2D(k_{F}^{2} + \delta^{2})^{\frac{1}{2}}
\]

\[
u_{2} = 2Dk_{2} = 2D\delta
\]

With the same substitution, Eq. (19) yields

\[
A_{i} = \frac{L^{2}\delta^{2}}{2\pi} I_{i}(u_{1}, u_{2})
\]
where

\[
I_i(u_1, u_2) = \int_{u_2}^{u_1} du u \sin\left(\left|u^2 - u_0^2\right|^\frac{3}{2} - u^4 - u_0^4\right) - \frac{1}{2u_0^2(1 - \cos u) + 16u_0^2u^2(u^2 - u_0^2)}
\]  

(25)

Integrals (21) and (25) are evaluated numerically as a function of the parameter \(u_2 = 2D\delta\) by setting \(u_1/u_2 = 2.38\). This value results from Eqs. (22) and (23) by taking \(\varepsilon_F = 7\) eV and \(\Delta = 1.5\) eV as adopted by Bruno\(^\text{18}\) for the Co/Cu/Co system. We note that this set of parameters corresponds to replacing the \(x\)-dependent electron mass by the free-electron mass.

The results are plotted in Figs. 1 and 2. The data are given for the range \(0 < 2D\delta < 60\). Taking \(k_F \approx \pi/a\) where \(a\) is the lattice constant of the ferromagnetic film, the upper limit of this range corresponds to \(D_{\text{max}} \approx 2a\). From Fig. 1 we see that, for large values of \(D\), the integral \(I_{u}(D)\) tends to a constant equal to 0.08. As \(I_{u}(D)\) approaches this value, it exhibits large quantum oscillations with an initial amplitude of 0.12. The period of these oscillations is about \(\pi/\delta\). This result can be traced to the singular behavior of the integrands of Eqs. (21) and (25). In fact, a numerical plot of the coefficient of the factor \(1 - \cos u\) in the integrand of Eq. (21) shows a sharp peak at \(u_2 \approx 2D\delta\). Also the coefficient of the factor \(\sin u\) in the integrand of Eq. (25) shows a peak at this value of \(u_2\). This behavior of the integrands is also responsible for the fact that the integrals (21) and (25) are insensitive to changes of the ratio \(u_1/u_2\). For instance, on increasing this ratio from 2.38 to 10, the plots of the \(I_{u}\) and \(I_{v}\) functions shown in Figs. 1 and 2 sustain less than a 10% change.

The amplitude envelope of the oscillations of the function \(I_{u}\) decreases slowly with increasing \(D\) as a power law \(D^{-\gamma}\) where the exponent \(n \approx 0.6 \pm 0.1\).

The integral (25), displayed in Fig. 2 as a function of the parameter \(2D\delta\), exhibits oscillations of similar character taking place about \(I_{u}(D) = 0\). The position and magnitude of the first peak of the function \(I_{u}(D)\) roughly agree with those of the oscillatory part of the \(I_{u}(D)\) curve.

IV. GILBERT DAMPING

The excess damping constant \(\alpha'\) is given by the second term on the right hand side of Eq. (3). Assuming that the increment the gyromagnetic ratio is small, we set \(\gamma/\gamma_0 \approx 1\) and obtain with use of Eq. (20)

\[
\alpha' \approx \frac{g_L\mu_B A_r}{2\pi M_s L^2 D} = \frac{g_L\mu_B\delta^2 I_{u}(D)}{4\pi^2 M_s D}
\]  

(26)

where \(M_s\) is the saturation magnetization of the ferromagnetic layer.

Invoking Fig. 1, we see from Eq. (26) that, for \(D \gg \delta^{-1}\), \(\alpha'(D)\) involves a \(D^{-1}\) dependence similar to previous works\(^\text{1,4,13}\). Superimposed on this dependence are large slowly decaying short-wave oscillations with a period close to \(\pi/\delta\). In the limit of \(D \gg 1/\delta\), the N/F and F/N interfaces are decoupled, and presumably contribute to \(\alpha'\) independently\(^\text{4,13}\). In Ref. 13, the interfaces were represented by monolayers acting as delta function scatterers. The expression for \(\alpha'\) obtained in this approximation is, to leading order in \(J_{\text{sd}}\), proportional to \(f_{\Delta e}^2 \propto \Delta^2\). This contrasts with Eq. (26) showing a linear dependence on \(\Delta\). A comparison of this result with the dynamic RKKY theory of Mills\(^\text{12}\) is made in Sec. VI.

Let us now estimate the magnitude of \(\alpha'\) for the Cu/Co/Cu system. Consistent with the choice \(u_1/u_2 = 2.38\) made in the evaluation of the integrals (21) and (25), we set \(\delta^2 \approx 2m\Delta/h^2\) where \(\Delta = 1.5\) eV. Letting \(M_s \approx 1400\) G, \(D \approx 40\) A and \(I_{u}(D) \approx 0.1\), we obtain from Eq. (26) \(\alpha' \approx 3 \times 10^{-4}\). This is possibly an underestimate since a free-electron mass is assumed\(^\text{10}\). An enhancement of the average mass due to high density of states in the Co layer can increase the value of \(\delta^2\) in Eq. (26) yielding \(\alpha'\) of order \(10^{-3}\).

Heinrich et al.\(^\text{17}\) studied, using FMR, epitaxially stabilized ultrathin fcc films of Co deposited on bulk Cu(001). For a 10 monolayer (ML) Co film, they find \(\alpha \approx 10^{-2}\). Schreiber et al.\(^\text{18}\) measured Gilbert damping of single crystal Fe\(_2\),\(\text{Co}_{1-x}\) alloy films prepared by sputtering on MgO(001) substrates. For pure Co films 200 A thick, they measure \(\alpha \approx 10^{-2}\) for the hard and \(\alpha \approx 6 \times 10^{-3}\) for the easy direction, respectively. On the other hand, the pure Fe films show substantially lower damping (\(\alpha \approx 2 \times 10^{-3}\)) that is practically isotropic\(^\text{18}\). On comparing our predicted damping with the experimental results on Co films\(^\text{17,18}\), it is evident that the magnitude of the oscillatory part of \(\alpha\) is within the error margins of the measured damping.

Another way to estimate the Gilbert damping constant is to explore the current-induced switching of magnetic moments in Co/Cu/Co systems. Katine et al.\(^\text{19}\) used thin-film pillars containing a thin (25 A) Co film coupled to a thicker (100 A) Co film. During the rotation of the magnetization of the thinner film, the magnetization of the thicker film remains fixed. This enables the determination of the polarity of the current bias associated with the spin-transfer excitations in the thinner layer. Moreover, the thicker layer acts as a spin sink. In this way, problems with spin accumulation in the Cu layers can be circumvented\(^\text{19}\). The Gilbert damping constant is determined starting from the Landau-Lifshitz equation augmented by the current-induced spin transfer torque\(^\text{20}\). The stability of the given magnetic configuration of the pillar is determined by a competition between the spin-transfer torque and the Gilbert damping torque. From the way the critical switching current scales with the effective magnetic field, Katine et al.\(^\text{19}\) find that an agreement with their experiment is obtained by assuming \(\alpha \approx 7 \times 10^{-3}\). This value is in good agreement with the FMR data\(^\text{17,18}\). The current-induced switching has been also studied by injecting the current into the multilayer
through a point contact\textsuperscript{21}. Interestingly, the damping constant deduced by this method is 10-50 times larger than the value obtained in the pillar geometry\textsuperscript{19}. This could be related to the fact that, in the point-contact study, the spin-wave excitation is induced in a localized region exchange coupled to the unbounded ferromagnetic film.

Due to the relatively small FMR linewidth of Fe films\textsuperscript{18}, multilayers involving Fe layers seem more suitable for the observation of the quantum oscillations. Urban et al.\textsuperscript{2} studied FMR on two Fe layers separated by nonmagnetic Au spacer. The thickness of the thinner layer is varied from 8 to 31ML. The thicker, 40ML, layer exhibits precession of negligible amplitude and serves as a spin sink for the spin currents emitted from the precessing thin layer. The FMR linewidth follows a $D^{-1}$ dependence without superimposed oscillatory component. For the 16ML film, the measured excess Gilbert damping constant, $\alpha' \approx 2 \times 10^{-3}$, is comparable to the intrinsic damping in the single Fe film\textsuperscript{18}.

Using Eq. (26), we estimate the theoretical value $\alpha_{th}'$ for the 16ML Fe film imbedded between Au layers. We assume that the electronic parameters are similar to those of the Fe/Cu/Fe system discussed by Hood and Falicov\textsuperscript{22}. The conduction electron mass is taken, by these authors, independent of the material and the spin orientation and equal to $m^* \approx 4 \times$ free electron mass. The exchange splitting for Fe is $\Delta \approx 2.5$ eV. The resulting value of the parameter $\delta^2 = 2m^*\Delta/h^2$ is $2.6 \times 10^{16}$ cm$^{-2}$. Consequently, the period of the oscillations of the functions $I_r$ and $I_s$ is $\pi/\delta \approx 2 \times 10^{-8}$ cm. Since this period is comparable to the lattice constant of Fe, films differing in thickness by 1ML should be investigated to detect the oscillations. Using this value of $\delta$, and taking $M_s \approx 1700$ G, $D = 46$ A and $I_r \approx 0.1$, we obtain from Eq. (26) the excess damping constant $\alpha_{th}' \approx 1.7 \times 10^{-3}$. This is comparable with the experimental value\textsuperscript{2}.

V. GYROMAGNETIC RATIO

Next we consider the renormalization of the gyromagnetic ratio $\gamma$. Putting $\gamma = \gamma_0 + \Delta \gamma$ where $\Delta \gamma \ll \gamma_0$, we obtain from Eq. (2)

$$\Delta \gamma = \frac{\Delta g}{g_0} \approx -\frac{g_\text{L}\mu_B}{2\pi M} A_i$$  \hspace{1cm} (27)

where $\Delta g = g - g_0$ is the change of the $g$-factor due to the spin-pumping mechanism.

Using Eqs. (20), (24), (26) and (27), we evaluate the ratio

$$\frac{\Delta g}{g_0\alpha'} = \frac{\Delta \gamma}{\gamma_0} \approx \frac{A_{th}}{A_r} = \frac{I_r(u_1,u_0)}{I_r(u_1,u_2)}$$ \hspace{1cm} (28)

From Figs. 1 and 2 we see that the oscillatory part of $I_r$ is of similar form and magnitude as $I_r$. This implies, that an observation of quantum oscillations of $\alpha'$ should be accompanied by an oscillation, of similar magnitude and period, of the ratio $\Delta g/g_0$.

Fig. 2 implies that for thicker films, such that quantum coherence is suppressed, the shift of the $g$-factor should be negligible compared to the measured value of $\alpha'$. This prediction is in disagreement with the FMR data on Pd/Pt/Pd and Pt/Pt/Pt where Pt is a permalloy film\textsuperscript{3}. These data yield a damping parameter $\alpha'$ that is proportional to $1/D$ and does not exhibit any superimposed quantum oscillations. This suggests that the tri-layers are in a quantum-incoherent regime for which the present theory predicts $\Delta g/g_0 \approx 0$. In contrast, the experimental value of this quantity is comparable with the experimental damping constant $\alpha'$. For instance, for the Pd/Pt/Pd system with $D = 50$ A, $\alpha' \approx \Delta g/g_0 \approx 10^{-2}$. Thus we have

$$\left[ \frac{\Delta g}{g_0\alpha'} \right]_\text{expt} \approx 1 \hspace{1cm} (29)$$

It is interesting that an order of magnitude agreement with this result can be obtained by evaluating the ratio $\Delta g/(g_0\alpha')$ from the model of two monolayers represented by a delta function potential\textsuperscript{13,14}. For Pd and Pt, the Coulomb interactions between the electrons need to be taken into account. In Ref. 14 we have shown that the effect of these interactions is to enhance the delta function potential of the monolayer by the Stoner factor $S_E$.

As a result, the quantity $A_i$ for the monolayer is changed to

$$\bar{A}_i \approx \frac{1}{\pi} L^2 k_F \beta S_E$$ \hspace{1cm} (30)

where $\beta \approx k_F J_{sd}/(2\varepsilon_F)$. In what follows, we assume that $J_{sd}$ is negative\textsuperscript{23}. The origin of this antiferromagnetic s-d interaction is quantum-mechanical mixing between conduction and localized d-orbitals. In Sec.II we found that the single-band Hubbard model\textsuperscript{15} yields a ferromagnetic interaction between the itinerant electron and the magnetization. This results from the Hartree-Fock approximation which linearizes the on-site Coulomb interaction of two electrons with opposite spin. Thus the energy of the up-spin electron is lower relative to the down-spin one. There are no consequences of this dichotomy for the quantity $\alpha'$. However, the sign of $\Delta g$ changes as the interaction goes from the antiferromagnetic to the ferromagnetic one.

The quantity $A_r$ for the monolayer becomes\textsuperscript{14}

$$\bar{A}_r \approx \frac{1}{\pi} L^2 \beta^2 S_E \left[ \ln \left( \frac{k_F^2}{S_E^2 \beta^2} + 1 \right) - \frac{k_F^2}{k_F^2 + \beta^2 S_E^2} \right]$$ \hspace{1cm} (31)
Setting $J_{sd} \approx -\varepsilon_F/10$ and $S_E \approx 5$, Eqs. (30) and (31) yield

$$\frac{\Delta g}{g_0 \alpha} \approx -\frac{\tilde{A}_i}{\tilde{A}_r} \approx 0.8 \quad (32)$$

Applying this single-monolayer result to the model of two monolayers located at the N/F and F/N interfaces, the same result is obtained for the ratio $\Delta g/(g_0 \alpha)$. We see that the theoretical result (32) is in order of magnitude agreement with Eq. (29).

VI. COMPARISON TO DYNAMIC RKKY THEORY

We now compare the results of Sec. IV and V to the dynamic RKKY theory of Mills. Let us begin by considering the excess Gilbert damping given by Eq. (26). We note that the formalism of the spin-pumping approach is very different from the dynamic RKKY theory. Thus, we limit ourselves to comparing only the final results.

Using the relation $G' = \gamma M_0 \alpha$, we obtain from Eq. (26) the excess Gilbert damping constant

$$G' \approx \frac{\mu_B^2 \delta^2 I_r(D)}{\pi^2 \hbar D} \quad (33)$$

We wish to compare this equation with the excess damping constant obtained using the dynamic RKKY method (see Eq. (29) of Ref. 12)

$$\Delta G = \frac{\mu_B^2 k^4(\varepsilon_F) f(D)}{8\pi^2 \hbar D} \quad (34)$$

where $f(D)$ is a dimensionless function which, for $k^4(\varepsilon_F) D \gg 1$, approaches a constant not far from unity. This function exhibits quantum oscillations that consist of short-wave oscillations with period $2/[k^4(\varepsilon_F) + k^4(\varepsilon_F)]$ superposed on long-wave oscillations with period $2/[k^4(\varepsilon_F) - k^4(\varepsilon_F)]$. In contrast, the function $I_r(D)$ exhibits only short-wave oscillations with period $\pi/\delta$. It approaches, as $D$ increases, a constant close to 0.1.

As far as the magnitude of the damping is concerned, the main difference between Eqs. (33) and (34) is that the factor $k^4(\varepsilon_F) f(D)$ is replaced by $\delta^2$. This implies that the magnitude of $G'$, for large $D$, is reduced compared to $\Delta G$ by a factor of order $\Delta/\varepsilon_F$. A more serious discrepancy is that Eq. (34) yields a nonzero damping even when the splitting of the up and down-spin bands vanishes ($k^4(\varepsilon_F) = k^4(\varepsilon_F)$). According to Eq. (4), $A_r$ vanishes in this case since the reflection and transmission amplitudes for the up-spin cancel those for the down-spin electron. This feature of the spin-pumping theory has been emphasized by Tserkovnyak et al. when comparing their formula for excess damping with the result of Berger. The requirement of nonzero spin-splitting also follows from the general dynamic RKKY approach to Gilbert damping. We return to a possible resolution of this difficulty at the end of this section.

In what follows, we argue that the absence of long-wave oscillations in $L_r(D)$ is an upshot of the simplified potential ($U_r(x) = 0$). Similar argument has been raised by Bruno for the interlayer exchange coupling. When both the up and down-spin electrons are scattered, a long-wave modulation of the short-wave oscillation is expected for the $D$-dependence of this coupling.

To substantiate this idea, we derive $A_r$ for the case when both $U_r(x)$ and $U_r(x)$ are nonzero. Using Eqs. (6-10), we obtain from Eq. (4)

$$A_r \approx \frac{L^2}{2\pi} \int_{0}^{k_F} \frac{dk}{k||} (B||B^\perp)^{-1} \left\{ B||B^\perp \right\}$$

$$-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D)$$

$$\left\{ \begin{array}{l}
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D) \\
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D)
\end{array} \right\}$$

$$\left\{ \begin{array}{l}
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D) \\
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D)
\end{array} \right\}$$

$$\left\{ \begin{array}{l}
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D) \\
-\{r^\perp r^\perp (1-r^\perp r^\perp)\} \cos (2k^\perp D)
\end{array} \right\}$$

where $B^{(\alpha)} = 1 - 2(r^\alpha)^2 \cos (2k^\perp D) + (r^\alpha)^4$ (36)

Generalizing Eq. (9) to both spin components, we set

$$k^\perp = \left[ (k^\perp)^2 - k^\parallel\right]^{1/2} \quad (37)$$

where $k^\parallel = (k^2 + \delta^2)^{1/2}$ is to be identified with $k^\parallel(\varepsilon_F)$ of Ref. 12.

To determine the periods of the oscillations of $A_r(D)$, we consider the asymptotic estimate of the integral (35) for $D$ large. We note that the major contribution to this integral arises from the immediate vicinity of the end points of the integration interval $(0, k_F)$ and from the vicinity of the stationary points of the function $k^\perp$. From Eq. (37) we see that the derivative of this function vanishes at the stationary point $k_\parallel = 0$. However, the contribution of this stationary point to the integral (35) vanishes owing to the factor $k^\parallel$ which causes the coefficients of all cosine terms to vanish.

For the same reason, the partial integration method yields a nonzero contribution only from the end point $k_\parallel = k_F$. This conclusion holds also for the simplified potential used in Sec. III. In fact, setting $r^\perp = 0$, Eq.(35) reduces to the integral (16) which is dominated by vicinity of $k^\perp (k_\parallel = k_F) = \delta$.
Consequently, Eq. (35) yields three short-wave oscillatory terms with periods $\pi/\delta_\uparrow, \pi/\delta_\downarrow, \pi/(\delta_\uparrow + \delta_\downarrow)$ and a long-wave term with period $\pi/(\delta_\uparrow - \delta_\downarrow)$. Hence, once electrons of both spin orientations are allowed to scatter, the spin-pumping approach exhibits oscillations similar to those of Ref. 12. The main distinction is that the wave vectors $k^{(s)}(\varepsilon_F)$ determining the periods of the oscillations in the dynamic RKKY theory are replaced by the wave vectors $\delta_\alpha$.

We now return to the aforementioned problem concerning the product $k^{(s)}(\varepsilon_F)k^{(s)}(\varepsilon_F)$ in Eq. (34). First, we examine the integral (35) for the case when $\delta_\uparrow = \delta_\downarrow$ (implying $r^{\uparrow}_\infty = r^{\downarrow}_\infty$). In this case, it is possible to verify explicitly that $A_r(D) \to 0$ as $D \to \infty$. Specifically, the rapid oscillations tend to cancel the contribution of the three short-wave cosine terms in the integrand. The remaining terms, upon substituting $r^{\uparrow}_\infty = r^{\downarrow}_\infty$, exactly cancel so that the integral (35) vanishes. This is in accord with the more general formula (4). Moreover, it provides an independent check that the rather complicated integrand of Eq. (35) is correct. Based on this result and the fact that, upon letting $r^{\uparrow}_\infty = 0$, this equation goes over to Eq. (16), we propose the following generalization of Eq. (20)

$$A_r(D) \propto L^2(\delta_\uparrow - \delta_\downarrow)^2F_r(D)$$

(38)

where $F_r(D)$ tends to a constant for $D$ large and contains a superposition of slowly decaying oscillations with periods $\pi/\delta_\uparrow, \pi/\delta_\downarrow, \pi/(\delta_\uparrow + \delta_\downarrow)$ and $\pi/(\delta_\uparrow - \delta_\downarrow)$. Eq. (38) also suggests that replacing the product $k^{(s)}(\varepsilon_F)k^{(s)}(\varepsilon_F)$ by $[k^{(s)}(\varepsilon_F) - k^{(s)}(\varepsilon_F)]^2$ may endow Eq. (34) with a correct dependence on the splitting of the up and down-spin electron bands.

In the preceding section we came to the conclusion that quantum oscillations of the damping constant $\alpha'$ are accompanied by oscillations of similar magnitude and period of the relative shift, $\Delta g/g_0$, of the gyromagnetic ratio $g$. This spin-pumping result disagrees with that of the dynamic RKKY method. In this method, the renormalization of $g$ stems from the frequency derivative, called $\Lambda_1$, of the transverse dynamical susceptibility. Though Ref. 12 points out the general importance of this quantity in the spin dynamics, it reports rough estimates indicating that the renormalization effects due to $\Lambda_1$ are small.

VII. DISCUSSION

In the itinerant model of a bulk ferromagnet, the $g$-shift and the damping $\alpha'$ have been attributed to the combined effect of spin-orbit interaction and the electron-lattice interaction. Mizukami et al. assume that $\Delta g$ and $\alpha'$ observed on Pd/Py/Pd and Pt/Py/Pt trilayers are due to similar interactions taking place at the interfaces.

The results outlined in Eqs. (30)-(32) represent a strong departure from these ideas. In the framework of the spin-pumping theory, the enhancement of the gyromagnetic ratio comes from the angular momentum of the conduction electron spin carried away from the F-layer into the N-layers. It is the spin current that is in the phase with the precession that contributes to the $g$-factor. The out of the phase part gives rise to the excess damping $\alpha'$.

Now in order that this mechanism works, it is necessary that the conduction electron spins are at equilibrium with the normal metal reservoirs. Otherwise, spin accumulation takes place which blocks the flow of the spin current. The equilibrium is established by spin-lattice relaxation in the normal metal. This relaxation process involves the spin-orbit interaction combined with the electron-lattice interaction similar to the mechanism of Ref. 24. In the spin-pumping approach, the conduction electron spin is first transported into the normal metal where it relaxes. Thus we may view the relaxation of the ferromagnetic magnetization, in this approach, as a nonlocal process mediated via spin current. On the other hand, the itinerant electrons of the bulk ferromagnet relax via local spin-orbit interaction. Clearly, the spin-orbit interaction is an inevitable component for magnetization relaxation in both mechanisms.

It remains to address the reasons for the absence of quantum oscillations in the FMR experiments which study the Gilbert damping and the $g$-factor as a function of the thickness of the ferromagnetic film. It is, of course, possible that another mechanism intervenes and obliterates the predictions of Eqs. 26 and 27.

Another possibility is that the oscillations are washed out by interface roughness. Note that the wavelength of the short-wave oscillations, considered here, is about $5A$ for Cu/Co/Cu and only about $2A$ for Cu/Fe/Cu. It is conceivable that roughness introduces fluctuations of $D$ larger than these wavelength. Averaging over these fluctuations strongly suppresses the amplitude of the oscillations. Consequently, one essentially sees an average which is a constant of order 0.1 for Fig. 1, and zero for Fig. 2. This is in apparent disagreement with the monolayer model invoked in Sec. V to explain the large shift of the $g$-factor observed on Pd/Py/Pd systems. To clarify this point, we note that the monolayer model assumes that the quantum oscillations in the ferromagnetic film rapidly decay with the distance from the interface. In the extreme case of overdamping, the function $I_r(D)$ does not change sign upon increasing $D$ by one wavelength form the origin. Thus fluctuations of the thickness do not average the $g$-shift to zero in the monolayer model.

Though short-wave oscillations are expected to be washed out by surface roughness, there are also long-wave terms in the expressions for the Gilbert damping and the shift of the gyromagnetic ratio that may survive the averaging over $D$. As seen from Eq. (35), these terms appear in $A_r$, when electrons of both spin orientations are subject to scattering. It can be shown in a similar way
that these terms are also present in the expression for $A_i$. Since the period of these oscillations is $\pi/(\delta_\uparrow - \delta_\downarrow)$, multilayers with small exchange splitting should be used to increase the chance of observing the long-wave oscillations.

An alternative way of observing the quantum oscillations is suggested by the fact that $I_\uparrow$ and $I_\downarrow$ are functions of the product $2D\delta$. Thus, instead of changing the thickness of the ferromagnetic film, we may vary the parameter $\delta$ while keeping $D$ fixed. For the case when both $U_\uparrow$ and $U_\downarrow$ are nonzero, we have $\delta^2 = 2m\Delta_\alpha/\hbar^2$ where $\Delta_\alpha$ is the depth of the potential well for electron of spin $\alpha$. These potential wells can be varied by shifting the chemical potentials, $\mu_\alpha$, of the electrons in the N-region.

One possibility is to apply a potential difference across the N/F interface. This would change the parameters $\delta_\uparrow$ and $\delta_\downarrow$ by the same amount. Consequently, only short-wave oscillations could be tuned in this manner. It is worth noting that, for the N/F interfaces considered here, a voltage drop of order 1eV is needed to sweep through several periods of the short-wave oscillations. To minimize Joule heating associated with such a voltage drop, the interface resistance should be large. This could be achieved by a thin layer of oxide incorporated into the interface.

Tuning of long-wave oscillations requires that the difference $\Delta_\uparrow - \Delta_\downarrow$ is varied. It is tempting to consider the shift $\Delta \mu = (\mu_\uparrow - \mu_\downarrow)$ due to the spin polarized current normal to the interface. Using Eq. 12 of Ref. 26, we estimate that the current density required to generate a shift of 1eV is of order $10^{10}$A cm$^{-2}$ which rules out this idea.

It is of interest, both experimentally and theoretically, to examine other processes contributing to the decay of multiple reflections in ideal multilayers. Recent work of Stiles and Zangwill is of relevance in this context. These authors show that the transverse [i.e. perpendicular to $m$] component of the spin current that flows from the normal metal into the ferromagnet is absorbed within a distance of several lattice constants. In the spin-pumping approach, it is also the transverse spin current that determines the Gilbert damping and the gyromagnetic ratio. The parameters $A_r$ and $A_i$ give the magnitude of these currents. Now, the present calculation of these parameters shows slowly-decaying oscillations in disagreement with Ref. 27. Also the numerical results of Mills do not show fast decaying oscillations of Gilbert damping. We note that Stiles and Zangwill have identified three distinct processes contributing to the absorption of the transverse spin current: (1) spin dependent reflection and transmission; (2) rotation of reflected and transmitted spins; and (3) spatial precession of spins in the ferromagnet. Out of these processes, it is the first one that the present theory and that of Ref. 12 takes into account. It should be interesting to incorporate the other two processes into these theories to see if a fast decay of the quantum oscillations ensues.

It should be emphasized that these decay processes do not affect the longitudinal component of the spin current. Incidentally, it is also this component which determines the exchange coupling between ferromagnetic films separated by normal metal spacer. Thus, we expect that the D-dependence of the the coupling exhibits slowly decaying oscillations. This agrees with the theory of Bruno and its experimental verification.

In view of the predicted fast decay of the transverse spin current, the D-dependent oscillations may be decaying faster than calculated in the present simplified model, and therefore less easy to detect than the oscillations of exchange coupling. In this context, it is worth noting that the decay of the transverse spin current depends on the Fermi surface mismatch between the ferromagnetic and normal metal Fermi surfaces. A slower decay is predicted when the mismatch is small ($k_F^\uparrow/k_F \approx k^\downarrow/k_F$).

From this point of view, multilayers with small exchange splitting are preferable in order to mitigate the effect of the decay processes on quantum oscillations of the Gilbert damping and gyromagnetic ratio.

VIII. ACKNOWLEDGEMENT

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FIG. 1: Integral $I_r(u_1, u_2)$ defined in Eq. (21). The plot is obtained by numerical integration as a function of $u_2 = 2\delta D$ for $u_1/u_2 = 2.38$. $D$ is the thickness of the ferromagnetic layer, and $\delta = (2m\Delta/h^2)^{1/2}$ where $\Delta$ is the depth of the potential well.

FIG. 2: Plot of the quantity $I_i(u_1, u_2)$ evaluated from Eq. (25) as a function of $2\delta D$ for $u_1/u_2 = 2.38$. 

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