Bursts of cooperation triggered by external stimuli in violent and other uncertain situations

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Abstract

Individuals are reluctant to cooperate for public goods if others freeride on their efforts. Cooperation can be fostered by mechanisms that make benefits exceed costs, such as norms with sanctions, but these take time to develop and may be ineffective for impromptu challenges such as defense against or defeat of opponents. Nevertheless, cooperation can be triggered by situational stimuli, for example the sight of victims or provocations by opponents. An Ising spinglass model demonstrates that at a critical level of stimuli, few accidental cooperators entail a burst of cooperation among the rest. If the proportion of defectors is above the critical threshold, however, bursts do not occur, and only a few group members cooperate. The distinction between bursts and fizzles is strongly supported by video data of street violence. Ultimately, the model is generalized by incorporating the usual mechanisms, and can then explain cooperation without stimuli.

Models to explain participation in collective action are based on benefits and costs (evolutionary or rationally), which are codetermined by others’ actions \[23, 28, 45, 44, 48\]. Missing in these models is the immediate effect of stimuli in the situation where collective action is to take place. We examine video recorded street fights between groups of (mostly) young men, where the public good is the defeat of the opponent(s) or the defense against them. Street fighters’ stimuli are opponents’ provocations, which do not change the benefits or costs of violence and can therefore not be expressed in the usual models. When these stimuli reach a critical point, however, collective violence is triggered. Stimuli reaching a tipping point also explain protests against
repressive regimes, the support of another country invaded by a third country, and the rescue of victims of disasters. In these examples, the stimulus is the perception of human suffering. Individuals of other species also respond cooperatively to certain stimuli, for example, an imminent threat of lions that provokes a counter attack by buffalo herd bulls [18]. To explain cooperation for public goods due to stimuli, and in particular its temporal unfolding, we present an Ising spinglass model [12, 17, 57, 31, 22, 11, 68].

In the Ising model, the dilemma of collective action can be visualized as follows. Think of two individuals, each with the behavioral options to defect or cooperate (corresponding to magnetic spins in the original). They may both defect, which avoids exploitation but does not yield any public good, and leaves the participants somewhat dissatisfied. Alternatively one individual may cooperate while being exploited by the freeriding other, which yields only half of the public good and results in higher average dissatisfaction. Finally, they may both cooperate at a cost, which maximizes the public good and minimizes average dissatisfaction. Fig. 1 plots the dilemma as a hill that stands in the way of proceeding from full defection on the left to full cooperation on the right, with average dissatisfaction [22] (as an interpretation of energy) on the vertical axis. The dilemma is drawn for a group of two and a clique (fully connected network) of five. Due to prior discussions with group members or earlier experiences, people may have become sensitive to certain signals (corresponding to temperature in the original model). This sensitivity can be due to an awareness of potential consequences or opportunities, or a learned response. If individuals become agitated by these stimuli, some may accidentally cooperate, called “trembling hands” in game theory [16], which influences proximate others and may result in a cascade of cooperation.

An influential overview of field studies of violent confrontations [15] concluded that during acrimonious encounters of opposing groups, tension builds, followed by short bursts of violence committed by small subgroups (from a larger and more dispersed group) of individuals in close proximity and with face-to-face contact [14, 36]. Their targets are often stumbling, outnumbered, or otherwise vulnerable individuals [15, 40]. Other people present on the scene may form an audience or try to deescalate [34, 49, 67], which happens often [50]. We model the proximate face-to-face contacts as a network, and use the Ising model to explain the outbreak of cooperation. In our empirical study, stimuli for collective violence take the form of turmoil generated by opponents’ actions. Simulations of the Ising model demonstrate that if the proportion of deescalators and other defectors (with respect to violence) is below a critical threshold, cooperation breaks out in a burst. Above the threshold, there is only a fizzle of violence by a small minority who start fighting asynchronously. The model also explains why attacker
subgroups are small, whereas larger groups have a better chance to win at lower individual costs.

Stimuli can drive cooperation if there is only a network of visual contact, without other mechanisms mentioned in the literature: material or reputational rewards and punishments, called selective incentives [44], with norms about (in)appropriate behavior [19] in a given situation, as well as monitoring of the group members [51], and transmission of information (i.e., gossip) through the group’s network, which leads to reputations [41] that feed back through selective incentives, with or without leaders. This package of mechanisms is crucial for ongoing cooperation in the longer run but needs time to develop, which may not be available when problems are urgent. The more time people have, the better they can prepare themselves, which is especially important for high-risk situations such as violent conflict. Police, soldiers, firefighters, and combat medics receive professional training that enables them to cooperate effectively and respond to situational stimuli in predetermined manners rather than spontaneously. Ordinary citizens who face sudden disasters or violent opponents are less prepared or not prepared at all. For them, the uncertainties of outcomes, benefits, and costs are higher. Under uncertainty, people become more conformist [70, 39], which makes sense from an evolutionary perspective when payoffs are hard to predict [63]. At the same time, people become less cooperative [32], which increases our challenge of explaining collective action. Our empirical study focuses on inexperienced individuals under high uncertainty, for whom the usual package of mechanisms is ineffective or absent. Finally, we generalize the model by incorporating the package.

Cooperation is mostly studied in public goods experiments, where in the absence of strong norms, more than half of participants are conditional cooperators willing to contribute if others do [13], thus conforming to their (weighted) average neighbor in the network [7, 60]. However, not everyone conforms, and some tend to defect permanently. In violent situations, people may defect for several reasons. They may be too scared to fight [15], have empathy with their opponents, disagree with violence, i.e., value the public good differently, try to deescalate, or they may have fought but got wounded, were wrestled to the ground, or got exhausted at some point. The proportion of unconditional (i.e., temporally steady) defectors leads to the prediction of bursts versus fizzles that we empirically test.

An earlier Ising model of cooperation was a two-person public goods game [1, 52], which we generalize to groups of any size, and we use straightforward payoffs from evolutionary game theory [48]. Whereas spins are generally assigned the value 1 or -1 (and in rare cases 1 and 0), we use asymmetric values 1 and -1/2 for, cooperation and defection, respectively [9]. The reason is that
if the public good and the status quo have equal value (are at the same level on the vertical axis in Fig. 1), cooperation yields no more payoff than defection, whereas if defection equals zero, there is no dilemma (only a downward slope to the right). Without any further information, choosing defection’s value halfway between the trivial extremes seems to be a reasonable first approximation that can be generalized to variation across individuals—at the cost of many degrees of freedom, though.

There are earlier models of cooperation without the usual mechanisms of cooperation, namely thresholds [26], cascades [65], and critical mass [35]. These models draw on initiative takers or leaders to initiate cooperation. The spinglass model is more parsimonious because it has no such assumptions, and cooperation can be started by accidental cooperators rather than exceptionally zealous ones, as was already known for the prisoners’ dilemma [16]. If there are initiative takers or leaders [42], however, they can be accommodated, as well as exit as a third behavioral option. The spinglass model is also more parsimonious because it has no assumptions of rationality [35] or (nearly) perfect information about others’ behavior [26].

Model

Members of a (fledgling) group can defect, $D$, or contribute, $C$, to the public good, with $0 < D < C$. Behavioral variable $S_i$ can take the value $S_i = C$ or $S_i = -D$. Before a collective action, everyone defects. Perhaps posing as a member of a group that confronts opponents is already a dilemma of itself, but our focus is on the subsequent dilemma of participation in violent collective action. Network tie $A_{ij}$ means that $i$ is in close proximity and pays attention to group member $j$. We assume that attention is reciprocal (but not necessarily equal), except in simulations of tie disruption. Because people tend to respond to proportions of their social environment rather than absolute numbers [65, 21], ties are row-normalized [with $w_{ij} = A_{ij} / \sum_j A_{ij}$ such that $\sum_j w_{ij} = 1$].

We do not assume that individuals know their payoffs in advance, but they will heuristically and perhaps wrongly distinguish between valuable ($C > D$) and nonvaluable ($C < D$) public goods. Note that payoffs are not used in the model’s calculations but are defined for a meaningful interpretation. When $i$ chooses between $C$ and $D$ amidst $N_C$ cooperators, payoffs for cooperation, $P_C = \theta(N_C + 1)/n - 1$, and defection, $P_D = \theta N_C/n + Q$, with a synergy or enhancement factor $\theta \geq 1$, are the same as in evolutionary game theory [48] except for $Q$. This additional factor $Q$ assures that if $D$ approximates $C$, which means that the outcomes of defection and cooperation become equally
Figure 1: The dilemma of cooperation presented as a hill in between full defection (left) and full cooperation (right), with normalized cooperation ($N_C/n$) on the horizontal axis. Average dissatisfaction ($H/n$) is on the vertical axis, for a dyad and a clique of five individuals. The larger the group is, the more rounded the hill becomes.
valuable, \( P_D \) approximates \( P_C \) \([Q = (\theta/n - 1)(1 - R); R = (C - D)/(C + D); \theta = \theta_0 + R, \text{ with a base rate } \theta_0 \geq 1]\).

The dynamics are modeled by minimizing the following Hamiltonian equation\[5, 64]\,
\[H = -\sum_{i\neq j} w_{ij} S_i S_j.\] (1)

At the beginning, \( H \) is at a local minimum where everybody defects at the left-hand side of Fig. [1]. The influence of opponents is exerted at the aggregate level through stimuli, \( T \), in our case the turmoil of the acrimonious confrontation. At stepwise increasing levels of turmoil, collective (in)action of the focal group is modeled through large numbers of Monte Carlo steps. For a given level of \( T \), at each Monte Carlo step an individual \( i \) is randomly chosen. As a conditional cooperator, \( i \) uses the Metropolis algorithm\[5\] to decide how to react to social contacts \( j \). First, \( H_i = -\sum_j w_{ij} S_i S_j \) is calculated; then, \( i \)'s behavior is flipped to its opposite, e.g., from \( D \) to \( C \), and the calculation is performed again, resulting in \( H'_i \). The flip is implemented if \( H'_i < H_i \), or with a probability that increases with \( T \) [if for a random number \( 0 \leq c_r \leq 1, c_r < \exp(-(H'_i - H_i)/T)]\). An implemented flip changes the network neighborhood of everyone connected to \( i \). Individuals’ behavior thus depends on social influence \( \sum w_{ij} S_i S_j \), possibly wrongly expected benefits and costs (through \( C \) and \( D \)), an external stimulus \( T \), and a portion of randomness \( c_r \).

Beyond our empirical study, the payoffs in the asymmetric Ising model can be generalized by relating \( C \) and \( D \) to the symmetric model through a mapping \( \{C, -D\} \rightarrow \{S_0 + \Delta, S_0 - \Delta\} \), with a bias \( S_0 = (C - D)/2 \) with respect to 0, and the two behavioral options are symmetrical at each side of \( S_0 \) at an offset \( \Delta = (C + D)/2 \). It can be shown that the asymmetry in \( S \) is equivalent to the symmetric model with an external field \( 2S_0 \)\[9\]. The bias and offset are in the payoffs through \( R = S_0/\Delta \). If \( \Delta \) is set to a fixed value \((0.75 \text{ in our examples})\), decreasing \( S_0 \) makes cooperation less valuable and is equivalent to an increasing threshold of cooperating network neighbors, also in other binary decision models\[65, 26\]. Cooperation can be made more valuable by increasing \( S_0 \), which corresponds to a decreasing proportion of cooperating network neighbors. For parsimony, we set \( S = \{1, -1/2\} \) for all conditional cooperators, as in two earlier nonempirical papers\[9, 8\]. One could therefore say that the two values that determine the prediction of the critical threshold (below) were preregistered. For the steady defectors we set \( S_i = -D \), irrespective of their reasons.

The continuous black line in Fig. [2] shows the dynamics of a group of conditional cooperators \( (n = 5) \). Without turmoil, collective action does not
start, but at a critical level $T_c$, almost everybody bursts into cooperation, with a maximum at or near $T_c$ ($N_C/n \approx 1$). Cooperation ends when exhaustion sets in, a winner stands out, or others intervene. If there are initiative takers or leaders $i$ with higher $S_{0,i}$ values than the majority, simulations point out that they start cooperating at lower $T$ and thereby reduce $T_c$ for the entire group [8]. Locally stronger turmoil has the same effect. Anger, ideology, and concerns for reputation [11, 46, 25] may push individuals’ $S_{0,i}$ upward, whereas an intimidating majority of opponents will pull it downward.

If there are steady defectors (red line), $T_c$ increases, which is hardly visible in Fig. 2 but more pronounced in larger networks, and maximum cooperation decreases. If the proportion of steady defectors reaches a critical level, $p_c$, there is no burst but a fizzle of gradually increasing cooperation (dashed blue line in Fig. 2) to a lower maximum at a higher level of turmoil. In a mean field analysis, there is no (burst of) cooperation if $p_c > S_0/\Delta = R$, independent of network size and density. For our empirical study, we thus predict that $p_c = 1/3$. Simulations come very close to this value. If defectors are clustered together, however, they are less in the way of collective action (higher $p_c$) than if they are evenly spread out across the network (mean field).

The threshold of agitation ($T_c$) increases with network size at a decreasing rate [8], but it also increases with the proportion of steady defectors until $p_c$ is traversed and there is no $T_c$ anymore. At $T_c$, cooperation starts in the smallest clusters of conditional cooperators. This bottom up mounting of cooperation is similar to bottom up synchronization in the Kuramoto model [3]. The effect of turmoil is nonmonotonic (Fig. 2) and the level of cooperation decreases with $T$ beyond $T_c$, which means that very strong turmoil becomes more confusing than agitating.

**Street violence**

To study violence, lab experiments lack the turmoil, agitation, and emotional intensity of violent confrontations due to ethical restrictions. Field studies, in contrast, cannot be based on a random sample of participants or groups, yet they are invaluable for realism. We obtained 42 videos from websites such as YouTube, LiveLeak, and WorldStarHipHop using search terms with the English keywords “brawl,” “street fight,” and “assault.” This sample is random with respect to temporal unfolding and (sub)group size. Of these clips, 36 are from English-speaking countries (mainly the US and the UK, with one from Canada and one from India); five of the remaining clips are from the Netherlands, and one is from Colombia. We did not observe differences in relevant behavior related to the location of the recording. To
Figure 2: Level of cooperation ($N_C/n$) with increasing turmoil ($T$) in a clique of five individuals. The black line depicts the group without steady defectors ($p = 0$), the red line with one steady defector ($p < p_c$), and the dashed blue line with two steady defectors ($p > p_c$).
keep distracting factors away from our analysis, we excluded clips with professional fighters, long range weapons, protective clothing, a referee, ambush attacks, or youths in a school yard. Most of our videos are phone recorded by bystanders and are left-truncated. In all likelihood, there had already been some turmoil that motivated bystanders to start filming. The shortest lasted 30 sec. and the longest was nearly 5 minutes (mean 101 sec.; s.d. 59 sec.). Out of a potential 2 x 42 groups, 25 groups attacked a single individual rather than a group, who could not act collectively alone, which leaves 59 groups to examine. Most groups were small, 2 ≤ n < 10 (mean 3.6), but one had 14 members. The smaller ones were simulated as cliques wherein everyone could see one another unless there were obstacles or deescalators obstructing visual contact. Obstruction was simulated by randomly removing m ties.

The videos were coded using Noldus Observer XT 14 software. Clips were played at half speed many times over, and one of us discussed the coding of each with one or two assistants. The assistants were unaware of the theoretical expectations. Each of 406 individuals was coded for belonging to a focal, opponent, or third-party group. Their behavior was interpreted and represented on the timeline.

We coded violence when force was used against another’s body (punching, slapping, kicking, hitting, stomping) and/or when another person’s body was forcefully moved (by pushing, shoving, dragging, wrestling, holding, etc.). Collective violence implies at least two fighting focal group members. For a burst of violence, we required that at least half of a group participated (Fig. 2), or both individuals did in a dyad, and they started fighting less than 2 seconds after the first, with a 5% margin [1]. In the videos, it was not possible to distinguish leaders from initiative takers, but we noticed individuals who started violence on their own.

We subsumed the following behaviors of members of the opponent group under turmoil for the focal group: aggressing, including fighting gestures; pulling off clothing (jackets or vests); pulling up pants; pointing toward opponents; provocative gesturing with fingers or hands (as an invitation to engage); bending forward toward an opponent; encroaching (invading opponents’ personal space through using or damaging objects belonging to them); teasing, such as lightly hitting or ridiculing; and violence. We also included stumbling and falling because vulnerability tends to agitate opponents [15, 40, 66] as well as approaching the focal group in the context of confrontational tension, which under normal circumstances would not provoke. We calculated the total level of turmoil from the beginning of the

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1The requirement that $N_C \geq 0.5n$ is based on simulations of small networks just above $p_c$, but in large networks with $p \geq p_c$, $N_C < 0.5n$. 

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clip until a focal group’s maximum participation in violence by the duration of each instance of turmoil and multiplying it by the number of individuals involved.

*Deescalation* was coded as follows [34]: open-handed gestures in the direction of other individuals; waving arms to stop or dampen in the direction of others; touching or patting; guiding a person away; pulling people apart; and putting one’s body in between opponents. Other reasons and causes for not participating in violence could be distraction by deescalator(s), spatial constraints blocking the way, spatial distance, or harm inflicted by opponents. Everyone who, for whatever reason, did not participate in violence was considered a defector with respect to the collective goal of attack or defense.

A plausible alternative explanation of the onset of collective action is synchrony of motion [15, 37], which yields a feeling of oneness among group members [20] and a stronger willingness to fight and take risks for group mates [59]. To measure the degree of *synchronization*, we counted the number of synchronous pairs in a group with respect to simultaneous aggressing or moving toward or away from opponents, and divided the score by the maximum possible number of pairs.

**Ethics**

The use of videos for research purposes poses distinct ethical challenges, largely due to the non-anonymous content of the videos. However, ethical guidelines for digital spaces tend to be less restrictive [38], with the consent of the participants being less stringent for data acquired from the public domain, including the internet. While our video corpus is open to use and inspection by other researchers upon request, we require that they take the same measures to ensure the anonymity of the persons portrayed as we did.

**Results**

Of the 59 groups considered, there were 23 groups where violence started in a burst, 15 groups where violence was collective without a burst, and 21 cases of violence by a single group member. Turmoil preceded all collective violence with one exception, where two individuals suddenly assaulted a passive victim. The critical level of turmoil ($T_c$) for bursts is case-specific and depends on group size, both in absolute numbers and relative to the size of the opponent group, and on the proportion of steady defectors. Additionally, the use of weapons has an effect.
Simulations show that smaller subgroups burst into action at lower turmoil than larger groups [8], and lower levels are of course reached earlier. Bursts developed in 13 (37%) of the 35 smallest groups (dyads and triads, i.e., fully connected triples) and in 10 (42%) of the 24 larger groups. In bursts, the correlation between group size and turmoil is 0.53. The size turmoil pattern is slightly disturbed by dyads more often facing a larger opponent group and therefore less likely to fight collectively than triads, which in turn are more robust in general [54]. In fact, larger groups always defeated smaller opponent groups (made them flee or worked them to the ground) with only one exception.

The proportions of steady defectors in groups with burst (mean = 0.19; s.d. = 0.21) and groups without (mean = 0.49; s.d. = 0.27) are box-plotted in Fig. 3 (Welch test $t = 4.796; p = 6.411 \times 10^{-6};$ df = 54.76). Despite the simplicity of the model, the predicted critical threshold (vertical line in the figure) neatly separates the two boxes containing bursts and nonbursts. As expected, there was more turmoil preceding collective violence in the nonburst cases (16.6 versus 14.5 for bursts on average). The threshold yields no perfect partitioning, and Fig. 3 also plots the groups that are incorrectly predicted by the model.

We now focus on the 21 groups where violence was committed by a single
member; 13 of these groups were dyads. In 11 of these 21 groups, one or few deescalators in the focal group successfully prevented participants from using violence. In 2 groups, members of the opponent group were able to avoid collective violence by hampering focal group members from joining the fight. In 2 dyads, the participants were separated by their opponents and could not adjust their actions to each other any longer, leaving them to either fight on their own or flee. All these outcomes are consistent with the Ising model once interrupted ties and/or steady defectors are simulated. In 4 of the 6 remaining groups, opponents carried knives, a machete, a bat, or looked too intimidating even when unarmed, which apparently lowered $S_0$ in the focal groups. In the last 2 groups, members took turns attacking a fallen victim instead of using violence simultaneously, perhaps confident that they were in control (low uncertainty; high $S_0$). Because we took the same $S_0$ values for all conditional cooperators, variations of these values are below the resolution of our simple model, though.

An interesting alternative explanation is synchrony of motion [15, 20, 59]. Although in 21 out of 23 bursts, some degree of synchronization (10.8 on average) preceded collective violence, there were 18 cases in which synchronization (9.6 on average) was not followed by collective violence. In some of the latter cases, synchrony turned out to be a deceptive performance composed of blustering and aggrandizing [61] without commitment to fighting. This does not imply that synchronization is unimportant, just that it does not predict collective violence.

Discussion and conclusion

The simple Ising model is now a century old [33] and has been applied to a wide range of problems [17, 58], to which we add the dilemma of collective action. It explains cooperation parsimoniously, based on stimuli without recourse to rationality, initiative takers, reputations, norms, feedback through selective incentives, or reliable information passing through the network. The Ising model provides insights into the temporal pattern of cooperation under uncertainty by predicting a critical threshold of steady defectors that distinguishes a burst of collective action from a fizzle and is supported by the data. The model also explains why violent groups are often small or are small subgroups of larger groups despite greater risk. Small (sub)groups have a lower critical threshold of turmoil, and in a confrontation with opponents, lower levels are reached first. Whether the magnitude of turmoil predicts the intensity of violence remains a question for future studies. We also investigated whether synchronous action precedes violence, but we found that synchro-
nization precedes both collective and solitary violence, and cannot predict either of these outcomes. However, synchronization may still be important to increase solidarity.

This study has several limitations. Because we selected the videos for violence, we cannot be certain that turmoil is its cause. When investigating the videos, however, we observed time and again that people reacted violently to provocations, hence it seems quite plausible that turmoil triggers violence. Our measurements underestimated turmoil because the videos are left-truncated and depend on camera angle and vision width, thus we erred on the safe side. Another limitation is in the Ising model. Despite predicting the threshold of steady defectors well in general, it misclassifies some of the groups. In future studies, it is important to expand the number and diversity of cases, to code videos by different coders and to complement field studies by ethically responsible lab experiments. Perhaps one day, the time-consuming coding can be done accurately by AI.

Here, we applied the Ising model to violence, but it can also be applied to other situations. Its dynamics are entirely consistent with temporal patterns of protests [24], which break out more often if (rumors say that) a government or its police are weakened [55, 62], analogous to vulnerable individuals in our data. Interestingly, as most groups look for a justified reason to attack, some of our group members provoked opponents in order to be reciprocally provoked, such that the start of violence could be othered. Some groups in other studies generated their own turmoil, for example by a rapid increase in online messaging [30], whereas autocrats censor social media to keep turmoil below the critical level. The model also seems applicable to spontaneous lynchings [6], as well as to helping victims under uncertainty [50]. It might even be applicable to other species, for example, quorum sensing and cooperating microbes [69].

In our empirical study, people were agitated by turmoil, but in general, they can be won over to cooperate by a broader range of stimuli. A case in point is the urgency of the public good to be achieved or protected, for example the global climate. A group consensus or a leader’s imposition of the collective goal may lead to the establishment of prosocial norms (indexed $q$), which can be represented by additional terms $-h_q \sum \mu_{q,i} S_i$ in Eq. 1, with $h_q$ an increasing function of $N_q$ norm maintainers (who are unconditional cooperators, possibly paid to act that way) and $\mu_{q,i} \geq 0$ increasing with norm internalization by $i$. If the norm is sufficiently strong (internalized or $N_q$ large enough), cooperation can start by a weak stimulus ($T < T_c$) or none at all ($T = 0$), in a gradual manner instead of a burst, and foster ongoing cooperation. Norms also preclude a great deal of situational uncertainty (at low $T$), but they cannot prevent the decline of cooperation at high $T$ and
come at additional costs for the maintainers and punished defectors. If the maintainers act collectively, the cost per individual decreases [43]; however, the second-order dilemma is completely solved if sanctioning is centrally organized [53, 4, 27] and paid through taxation or fines. Punishment can have unintended consequences, however, for example, when severely punished individuals suffer great losses, whereafter they may decide to leave the group. In general, rewards and punishments as well as psychological factors and different valuations of the public good can be implemented in the model in terms of $S_{0,i}$ and payoffs, and kinship or friendship can be expressed as tie strength.

Furthermore, when individuals find themselves more often in similar situations, they will learn, which is easier in smaller groups where they have a larger influence on their payoffs [10]. Some will change their decision rule, or strategy, and turn into unconditional defectors [2] who try to exploit other group members and maximize their individual payoff instead of maximizing the group’s payoff. Strategy changes as well as network dynamics can be dealt with during subsequent Monte Carlo steps.

Along with situational uncertainty, or turmoil, there can be endogenous uncertainty in the information that participants $i$ have about $j$’s behavior, i.e., when reputation $r_{ij}$ is tainted with noise and bias [29]. At each Monte Carlo step when $j$ decides, gossip about $j$ spreads into the network with a chance $p_t$ at each transmission that it does not make it to the next individual and a chance $p_e$ that the gossip is incorrect. Group members $i$ then assess $j$’s reputation on the basis of received gossip by taking a (weighted) average of the gossips and, if they are connected to $j$, their personal observation of $j$ [56]. They decide by the Metropolis algorithm applied to $H_i = -\sum_{k=1}^n w_{ik} S_i r_{ik}$, with $j$ in the index $k$ of some $i$. If reputations are free of error ($r_{ij} = S_j$), for example, in small cliques, Eq. 1 is recovered. The important message is that among conditional cooperators, erroneous reputations get cooperation started more easily (in fewer Monte Carlo steps), but once collective action has been mounted, noise lowers the average level of cooperation.

These elaborations point out that the usual mechanisms of cooperation, as well as network dynamic and individual differences, can all be expressed in the Ising model, thereby making it suitable as a general approach to collective action. It goes beyond current models by incorporating situational stimuli and predicting how collective action will unfold. In this first empirical application, we showed that it can explain the dynamics of street violence, and in all likelihood, many more discoveries lay ahead.
Appendix

In line with the social sciences, cooperation is defined as $N_C/n$ in the main text. Here we stay close to the Ising model and define cooperation in terms of the order parameter, $M = 1/n \sum_{i=1}^{n} S_i$. Consequently, $N_C/n = (M + D)/(C + D)$. Now the mean field assumption can be stated simply as $S_i = \tilde{S}_i = M$.

We start out with the Hamiltonian, $H = -\sum_{i,j} w_{ij} S_i S_j$. We use the mapping $\{C, -D\} \rightarrow \{S_0 + \Delta, S_0 - \Delta\}$ with bias $S_0 = (C - D)/2$ and offset $\Delta = (C + D)/2$ to rewrite the Hamiltonian as

$$H = -\sum_{i,j} w_{ij} (S_0 + \hat{S}_i)(S_0 + \hat{S}_j), \quad (2)$$

with $\hat{S}_i$ and $\hat{S}_j \in \{-\Delta, \Delta\}$.

### Mean field without and with unconditional defectors

To calculate the Boltzmann probabilities of a single spin (or an individual’s probabilities to cooperate or defect), we define the pertaining Hamiltonian $H_i$, taking into account the row normalization of the adjacency matrix ($\sum_j w_{ij} = 1$).

$$H_i = -\sum_j w_{ij} (S_0 + \hat{S}_i)(S_0 + \hat{S}_j) \quad (3)$$

$$H_i = -\sum_j w_{ij} (S_0 + \hat{S}_i)M$$

$$= -(S_0 + \hat{S}_i)M$$

$$H_i^\pm = -S_0 M \mp \Delta M. \quad (4)$$

In the subsequent derivation, $\beta = 1/T$, without the Boltzmann constant. The average value of a spin, $\bar{S}_i$, according to the Boltzmann distribution, with $P(S_i^-)$ standing for the probability that $S_i$ is negative and $P(S_i^+)$ that
it is positive, is

\[ S_i = S^- P(S^-_i) + S^+ P(S^+_i) \]
\[ = S^- e^{-\beta H_-^i} + S^+ e^{-\beta H_+^i} \]
\[ = S^- e^{-\beta(-S_0M + \Delta M)} + S^+ e^{-\beta(-S_0M - \Delta M)} \]
\[ = S^- e^{-\beta \Delta M} + S^+ e^{\beta \Delta M} \]
\[ = (S_0 - \Delta) e^{-\beta \Delta M} + (S_0 + \Delta) e^{\beta \Delta M} \]
\[ = S_0 e^{-\beta \Delta M} + \Delta \frac{-e^{-\beta \Delta M} + e^{\beta \Delta M}}{e^{-\beta \Delta M} + e^{\beta \Delta M}} \]
\[ = S_0 + \Delta \tanh (\beta \Delta M). \]  

So far, we only dealt with conditional cooperators, but there are also unconditional defectors in proportion \( p \). Accordingly, we define \( M_{cc} \) as the average spin value of the conditional cooperators and \( M_{ud} \) as the average spin value of the unconditional defectors. Note that \( M_{ud} = S^- \). We assume that the unconditional defectors are homogeneously distributed across the network. Consequently, the mean field equation becomes

\[ S_i = M = pM_{ud} + (1 - p)M_{cc} \] 
\[ = pS^- + (1 - p)M_{cc}. \] 

The Hamiltonian for a single conditional cooperator becomes

\[ H^\pm_i = -(S_0 \pm \Delta)(pS^- + (1 - p)M_{cc}) \]
\[ = -S_0 pS^- \mp \Delta pS^- - S_0 (1 - p)M_{cc} \mp \Delta (1 - p)M_{cc}. \]

In the derivation of Eq. 6, all terms that did not contain \( \mp \Delta \) canceled each other out. For clarity, we remove these terms from Eq. 10 which results in

\[ H^\pm_i = \mp \Delta pS^- \mp \Delta (1 - p)M_{cc} \]
\[ = \mp \Delta (pS^- + (1 - p)M_{cc}). \]
The mean field analysis for conditional cooperator $i$ is

$$
\bar{S}_i = S^- P(S^-_i) + S^+ P(S^+_i)
$$

$$
= S^- e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + S^+ e^{\beta \Delta (pS^- + (1-p)M_{cc})}
$$

$$
= S_0 + \Delta e^{-\beta \Delta (pS^- + (1-p)M_{cc})} + e^{\beta \Delta (pS^- + (1-p)M_{cc})}
$$

$$
= S_0 + \Delta \tanh (\beta \Delta (pS^- + (1-p)M_{cc})) = M_{cc}.
$$

Using Eq. 8, we can express the self-consistency equation of $M_{cc}$ in $M$,

$$
M = pS^- + (1-p)(S_0 + \Delta \tanh (\beta \Delta (pS^- + (1-p)M_{cc})))
$$

$$
= pS^- + (1-p)(S_0 + \Delta \tanh (\beta \Delta M))
$$

$$
= p(S_0 - \Delta) + (1-p)(S_0 + \Delta \tanh (\beta \Delta M))
$$

$$
= S_0 - p\Delta + (1-p)\Delta \tanh (\beta \Delta M).
$$

**Critical proportion of unconditional defectors**

Depending on $\beta$, the self-consistency equation has two stable and one unstable ferromagnetic solutions, or one stable paramagnetic solution. At a critical $\beta$ (or $T$), the system transitions between these two states. When the system is paramagnetic,

$$
\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) < 1
$$

at the solution of $M$. When the system is ferromagnetic,

$$
\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) > 1
$$

at the unstable solution of $M$. We can identify a critical $\beta$ when

$$
\frac{\partial}{\partial M} (S_0 - p\Delta + \Delta \tanh (\beta \Delta M)) = 1
$$

$$
\frac{1}{\cosh^2 (\beta \Delta M)} \beta \Delta^2 (1-p) = 1.
$$

This equation can be rewritten as

$$
\cosh (\beta \Delta M) = \sqrt{\beta \Delta^2 (1-p)}
$$

$$
\beta \Delta M = \pm \text{arcosh} \left(\sqrt{\beta \Delta^2 (1-p)}\right)
$$

$$
M \Delta = \pm \text{arcosh} \left(\gamma \sqrt{(1-p)}\right)
$$

$$
\gamma^2 = \frac{\beta \Delta^2}{\beta \Delta^2 (1-p)},
$$
with $\gamma = \sqrt{\beta \Delta^2}$. We can substitute this expression \[20\] in the self-consistency equation \[16\]

\[
M = S_0 - p\Delta + (1 - p)\Delta \tanh (\beta \Delta M)
\]

\[
\Delta \frac{M}{\Delta} = S_0 - p\Delta + (1 - p)\Delta \tanh (\gamma^2 \frac{M}{\Delta})
\]

\[
\Delta \pm \frac{\text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2} = S_0 - p\Delta + (1 - p)\Delta \tanh (\pm \text{arcosh} (\gamma \sqrt{(1 - p)}))
\]

\[
\pm \frac{\text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2} = S_0 \Delta - p \pm (1 - p) \tanh (\text{arcosh} (\gamma \sqrt{(1 - p)}))
\]

\[
\pm \frac{\text{arcosh} (\gamma \sqrt{(1 - p)})}{\gamma^2} = S_0 \Delta - p \pm (1 - p) \sqrt{\frac{\gamma^2(1 - p) - 1}{\gamma^2(1 - p)}}.
\] (21)

The last substitution uses hyperbolic identities. Changing back to the original variables yields

\[
\pm \frac{\text{arcosh} (\sqrt{\beta \Delta^2(1 - p)})}{\beta \Delta^2} \mp (1 - p) \sqrt{1 - \frac{1}{\beta \Delta^2(1 - p)}} = \frac{S_0}{\Delta} - p.
\] (22)

We discard the equation that has no real numerical solutions and keep

\[
- \frac{\text{arcosh} (\sqrt{\beta \Delta^2(1 - p)})}{\beta \Delta^2} + (1 - p) \sqrt{1 - \frac{1}{\beta \Delta^2(1 - p)}} = \frac{S_0}{\Delta} - p.
\] (23)

Solutions of this equation become complex if $p > \frac{S_0}{\Delta}$, hence $p \leq \frac{S_0}{\Delta}$. The choice of $C = 1$ and $D = 1/2$ implies that there is no (burst of) cooperation if $p > 1/3$. Graphical illustrations are in Fig. 4.

**Author contributions**

JB made the asymmetric Ising model; wrote the software and the paper.

DW collected, interpreted, and analyzed the data. BM did the mean field analysis.

**Code and data availability**

The data files, the R script used to produce plots from coded video data, and a Fortran script for simulations of the Ising spinglass are available at https://osf.io/f25nq/
Figure 4: Given $S_0 = 0.2$, the critical level of stimuli, $T_c$, is plotted as a function of the proportion of steady defectors, $p$, for various levels of $\Delta$. The critical level, $p_c$, is reached at the right-hand end of the lines.

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