OBSERVATION OF PARAMETRIC DECAY CORRELATED WITH EDGE HEATING USING AN ION BERNSTEIN WAVE ANTENNA ON DIII-D

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ABSTRACT. Significant levels of parametric decay activity and correlated edge ion heating were observed during injection of high power ion Bernstein waves (IBWs) in DIII-D. Both minority hydrogen ions and majority deuterium ions showed the formation of a high energy perpendicular tail; no parallel heating was observed. The edge ion heating and the parametric decay activity were both strongest when an ion cyclotron harmonic was present at the plasma edge. Ion tail formation had a power threshold of several hundred kilowatts, above which the tail size increased with antenna power; a comparable power threshold for parametric decay instability (PDI) was observed. Both the PDI and the associated edge deuterium heating were found to be sensitive to the hydrogen-to-deuterium ratio.

1. INTRODUCTION

The application of externally launched ion Bernstein waves (IBW) to heat a tokamak plasma was proposed [1, 2] in 1979. In principle, IBW heating combines the advantages of lower hybrid heating (good wave coupling to relatively low edge densities, waveguide launching structures in high field devices) and fast wave heating in the ion cyclotron range of frequencies (ICRF) (good wave accessibility to high central densities, and the possibility of efficient, well localized ion heating). It is not unreasonable to expect that IBW heating would also suffer from some of the problems of the latter two RF heating schemes.

Indeed, the problem of RF enhanced impurity generation and concomitant radiated power losses, experienced in most fast wave heating experiments, has also presented difficulties in the reported high power IBW heating experiments. In some recent fast wave heating experiments, parametric decay instabilities (PDI) have been observed [3, 4]. Though the presence of parametric decay processes at some low level does not necessarily constitute a problem for an RF heating scheme, in at least one large tokamak, JT-60, the PDI has been found to be well correlated with enhanced impurity production [5, 6]. Parametric decay might be expected to play an important role in IBW heating experiments, for the following reasons. The mechanism by which an antenna at the edge of the plasma can couple to the IBW, known as 'mode transformation' [7], involves launching an electron plasma wave (EPW), which is the low frequency limit of the slow lower hybrid (LH) wave. In the linear (low RF power density) regime, the EPW propagates up to the density at which the wave frequency is comparable with the lower hybrid resonance frequency, where the EPW transforms.
into the IBW, given that the ion temperature in this region is greater than a few electron volts. In tokamak experiments where lower hybrid waves at frequencies of the order 1 GHz are launched from waveguide couplers, substantial parametric decay activity is often observed, particularly when the wave frequency is less than about half of the lower hybrid resonance frequency. In many cases, the PDI is correlated with strong RF–ion interaction (for an early example, see Ref. [8], while some recent work in this area is reported in Refs [9] and [10]). Therefore, parametric decay and associated ion interactions could occur in the edge region in IBW experiments, where the wave is actually a low frequency lower hybrid wave approaching the cold plasma LH resonance layer. However, the only reported direct observation of parametric decay to date has been in DIII-D [11], though some features of the Alcator C IBW results have been interpreted in terms of parametric decay processes [12, 13].

Non-linear effects with strong pondermotive forces have been invoked to explain the results of IBW antenna coupling measurements in DIII-D [14,15]. As will be discussed in Section 5 of the present paper, the same non-linear phenomena may raise the wave energy density in the edge region and significantly lower the threshold power for parametric decay instability. The PDI could then be an important mechanism for dissipating the non-linearly trapped wave power in the edge of the plasma. Thus, the observed lack of efficient central heating in the DIII-D IBW experiments [11, 14], the necessity of non-linear effects to explain the antenna coupling in these experiments, and the observed parametric decay are all part of the explanation of the DIII-D IBW results. In this paper, we focus on the detailed observations of parametric decay and associated edge heating and touch on the other aspects of the experiment only as they relate to PDI and edge heating.

Various parametric decay processes that might be important in IBW experiments have been discussed in the literature [16,17]. Some of these parametric decay instabilities are depicted in Fig. 1. The pump wave can decay into short wavelength daughter waves which may be IBWs at a lower frequency than the pump and into non-resonant (heavily damped) quasi-modes. Note that in these processes the pump wave itself is not necessarily an IBW (unless its frequency is just below an ion cyclotron harmonic) since by assumption $k_{\perp} \rho_i \ll 1$ (where $k_{\perp}$ is the perpendicular wavenumber of the pump and $\rho_i$ is the ion Larmor radius). The quasi-modes are characterized by $\omega \approx n \Omega_i$, where $\Omega_i$ is the ion cyclotron frequency, or $\omega \approx k_{\perp}v_{\text{te}}$, in which $v_{\text{te}}$ is the electron thermal velocity, and are absorbed by ion cyclotron and/or electron Landau damping. The processes shown in Fig. 1 include the following: (i) decay into an IBW and an ion cyclotron quasi-mode (ICQ), the latter having frequencies which correspond to the deuterium, and/or hydrogen ion cyclotron frequency; (ii) decay into an IBW and an electron Landau damped quasi-mode (ELDQ); (iii) resonant decay into two ion Bernstein waves.
All of these processes have been observed in the DIII-D IBW experiments.

The remainder of the paper is organized as follows. In the next section, the apparatus used in these studies is described. Section 3 is an account of the RF spectra observed during IBW injection. The decay modes are identified, and the dependences of the decay wave amplitudes on plasma parameters are shown. In Section 4, we demonstrate the good correlation between the observation of substantial edge heating and the observed parametric decay activity. This correlation has never before been reported from an IBW experiment. Scaling of the edge heating with plasma parameters is studied. The edge heating is shown possibly to account for a substantial fraction of the applied RF power.

Finally, in Section 5 we give a brief assessment of the relationship between theoretical predictions and the observations, and draw conclusions concerning the role of parametric decay activity in IBW heating experiments.

2. EXPERIMENTAL APPARATUS

The IBW experiments described here were performed on the DIII-D tokamak [18]. Many of the diagnostics employed for the present ICRF studies have been described recently [19]. The three tipped movable Langmuir probe described in Ref. [19] was not only used to measure electron density and temperature in the scrape-off layer (SOL), but one of the tips was also configured as an electrostatic RF probe for studying RF spectra in the edge plasma. Alternatively, a single turn magnetic loop probe, 5 cm in diameter, was connected to the spectrum analyser. The electrostatic probe was located in an outside midplane port approximately 2 m away from the IBW antenna, and was usually placed at the same major radius as the face of the Faraday shield of the antenna. The magnetic loop probe was located about 5 m away from the IBW antenna.

Neutral particles were detected by a scannable E || B charge exchange (CX) analyser [20]. This instrument employs a microchannel plate with 120 anodes arranged in two columns to measure the energy spectra of escaping hydrogen and deuterium neutrals as a function of time (1 ms resolution). The CX analyser is located in the horizontal midplane and rests on a cart that can be scanned shot-to-shot from a perpendicular orientation to a tangential orientation. For these experiments, no heating beams were employed so the source of neutrals for the charge exchange reactions was the plasma edge (passive charge exchange). At typical densities and energies, the mean free path of escaping neutrals was comparable with the minor radius (\( \lambda \lesssim a \)); however, during IBW the signal is thought to be dominated by neutrals that originate in the plasma edge (as discussed in Section 4), so no attenuation correction has been applied to the data. The raw data were corrected for various anode sensitivities and the appropriate charge exchange and ionization cross-sections to obtain the spectrum of neutral flux versus energy. When the analyser electric field is properly set, a few per cent of the incident deuterium neutrals spill over onto the hydrogen column (and vice versa) [20]. The mass discrimination was checked by extrapolating the spectra back to zero energy to obtain the ratio of hydrogen to deuterium; the charge exchange H:D ratio agreed with measurements based on H\textalpha and D\textalpha emission to within ~20%. The hydrogen and deuterium temperatures inferred from the data in Ohmic plasmas were also usually consistent. For some of the CX analyser data presented in this paper, however, a broken amplifier caused the analyser electric field to fluctuate erratically, resulting in poor mass discrimination between columns. For these data, only the high energy hydrogen data and the low energy deuterium data were valid.

Several details of how the DIII-D tokamak was operated during the IBW experiments are potentially important for understanding the observed phenomena. Forty per cent of the Inconel vacuum vessel inner surface was covered with graphite tiles; to obtain reproducible density control, several minutes of helium glow discharge conditioning...
were routinely performed before each tokamak discharge [21]. The graphite tiles and the glow conditioning result in two significant changes relevant to IBW coupling: (a) the SOL densities in diverted discharges were generally much lower than in previously reported work, such as Ref. [19], and (b) the helium glow introduced an unavoidable small concentration of helium ions into the subsequent tokamak discharge. Furthermore, most of the ongoing studies on DIII-D involve deuterium neutral beam injection into deuterium plasmas, so that on any particular day it proved difficult to produce a pure hydrogen plasma free of a substantial deuterium minority. The edge plasma in these experiments, therefore, was composed of a mixture of hydrogen, deuterium, singly and fully stripped $^4$He, and small amounts of various charge states of low Z impurities (C, O). Typical parameters during these experiments were: $B_T = 1.0-2.1$ T, $n_e = 7 \times 10^{12}-5 \times 10^{13}$ cm$^{-3}$, $T_e(0) \approx T_i(0)$ $\sim 1$ keV, $I_p = 0.7-1.4$ MA.

The IBW coupler used in these experiments consisted of a pair of cavity type, end fed loop antennas, oriented along the toroidal field, similar to the antennas used in almost all of the previous IBW heating experiments. A diagram of the antenna is given in Ref. [15]. Each current strap was 41 cm long, 16 cm wide and 1.3 cm thick. The loops were mounted colinearly in a single 1 m wide port at the outside midplane of DIII-D, and the entire structure was movable over a distance of 5 cm in the radial direction. The external impedance matching network included a phase shifter between the two antenna feeds, which permitted control of the launched $k_{\parallel}$ spectrum. Two different Faraday shields were used: the first shield consisted of a single row of molybdenum rods 9.5 mm in diameter, spaced on 19 mm centres, with 1.6 mm thick graphite tiles brazed onto the plasma facing side. Most of the experiments employed an optically opaque Faraday shield consisting of two offset rows of 9.5 mm molybdenum rods coated with a thin layer of TiC spaced on 11 mm centres. The antenna was operated at power levels up to 1 MW, for pulse lengths of several seconds, at frequencies of 32, 34, 38, 38 and 60 MHz. Most of the experiments reported here were performed at 36 or 38 MHz, so that $\omega/\Omega_H \simeq 1.2-2.1$ for the range of toroidal fields employed ($B_T = 1-2.1$ T, $B_{\text{edge}} \simeq 0.7 B_T$).

3. PARAMETRIC DECAY

Radiofrequency spectra showing evidence of parametric decay were obtained with loop probes and with the electrostatic probe during IBW injection, under a wide variety of discharge conditions. Most of the spectra presented in this paper were recorded using the electrostatic probe, for the following reasons: the loop probes were further from the IBW antenna than the electrostatic probe and no effort was made to minimize electrostatic pick-up on the electromagnetic loop probes. Furthermore, the spectra obtained with the electrostatic probe were somewhat better defined than those obtained with the loop probe. Unfortunately, only one electrostatic probe at an essentially fixed position relative to the IBW antenna was available, so no data on the spatial structure of the wave fields could be obtained. Without such spatial information, it is impossible to evaluate quantitatively the significance of the observed parametric decay in the global power balance solely on the basis of the probe data. In particular, observation of parametric decay activity at an amplitude 40 dB lower than that of the pump frequency does not necessarily imply that only a fraction of $10^{-4}$ of the pump power is converted to daughter waves. We may remark that PDI may occur some distance inside the plasma and the decay waves may not necessarily propagate to the RF probe. Since the decay wave peaks are often characterized by a much broader frequency bandwidth than the pump wave, a simple comparison of peak amplitudes is meaningless. Therefore, we shall plot frequency band integrated power, rather than peak amplitudes.

Figure 2 illustrates some of these points. It shows the spectrum which was observed under conditions where the edge ion heating and decay wave
amplitudes were maximized: a low line averaged density \( \overline{n_e} \approx 2 \times 10^{13} \text{ cm}^{-3} \) inside-wall limited discharge in deuterium with a small percentage of hydrogen, with the toroidal field satisfying the condition \( f_0 = 36 \text{ MHz} = 2f_H \) near the IBW antenna. Two spectra are shown; the only difference between the two discharges was the sensitivity of the spectrum analyzer. The 0 dB reference level corresponds to the marginally saturated amplitude for both spectra. In the first case, a peak at the generator frequency \( f_0 \) is observed with a bandwidth smaller than the resolution of the spectrum analyzer (300 kHz). The peak at \( 2f_0 \) (the so-called 'second harmonic') is lower in amplitude by 25 dB than the pump, though the second harmonic from the transmitter is between 30 dB and 50 dB below the fundamental. This harmonic generation may be due to sheath rectification either at the antenna [22-24] or at the receiving probe. The only other feature evident in the spectrum is a peak at 17 MHz; its peak amplitude is 30 dB lower than the pump. However, its bandwidth is significantly larger than that of the pump signal – the ratio of the band integrated power at the pump frequency to that at 17 MHz is 22 dB.

In the second case shown, the sensitivity of the spectrum analyzer is increased by 40 dB. The signal at the pump frequency is saturated. Of course, the peak amplitude of the second harmonic signal and of the signal at 17 MHz are higher by 40 dB than in the previous case. Peaks are now evident at frequencies of 9.3, 17.2, 26.6, 36, 45.3, 53.5, 62.5, 72, 81.5 and 89.6 MHz. The broadening of the peaks at 36 and 72 MHz is somewhat asymmetric towards the low frequency side; this feature is much more distinct in some other spectra (Fig. 6). This spectrum may be interpreted as resulting from parametric decay of the pump wave into IBWs and ion cyclotron quasi-modes – the fundamental ion cyclotron frequency for deuterons in the immediate vicinity of the IBW antenna is 9.3 MHz. In this case, the peak that would be expected at 18.6 MHz (hydrogen ICQ) is not well resolved due to the broad bandwidth of the IBW at 17.2 MHz. Again, in many other spectra, the hydrogen ICQ is clearly resolved, such as in Fig. 6. The usual frequency selection rule \( \omega_0 = \omega_1 + \omega_2 \) is satisfied in this as well as in all other cases observed in DIII-D in which both decay waves are detected. Mixing of the harmonic signals with the decay waves in the antenna or probe sheath could produce the additional peaks observed at \( f > f_0 \).

In general, the sensitivity of the spectrum analyzer was adjusted as in the second case in Fig. 2, where the pump is saturated, except in cases where the dependence of the pump amplitude on power was being studied. In the spectra exhibited in the remainder of this paper, the reference power level is chosen to be the saturation level, so that the amplitude of the peak at the pump frequency is always 0 dB. Therefore, no inferences regarding the amplitude of the pump relative to that of the decay waves are made.

The mode of parametric decay into ICQs and IBWs is identified by varying the value of the toroidal magnetic field and observing the variation of the frequencies of the decay waves. By ramping down the current in the toroidal field coil during...
the 2.0 s long RF pulse, spectra were obtained every 
0.12 s over a range of toroidal fields. The frequency 
at which decay waves were observed as a function 
of central toroidal field $B_T$ is plotted in Fig. 3.
The dashed lines are the deuteron ($\Omega_D/(2\pi)$) and 
proton ($\Omega_H/(2\pi)$) cyclotron frequencies evaluated 
at the outboard edge of the tokamak ($R = R_0 + a$), and the dotted lines represent the difference 
between the pump frequency (36 MHz) and the 
cyclotron frequencies. In this case, the hydrogen 
decay waves were observed only near the field at 
which $\omega = 2\Omega_H$ near the outside edge of the 
plasma (a central field of $B_T \approx 1.7$ T), and again 
at low field. This behaviour is characteristic of a 
very low hydrogen minority fraction, as discussed 
below; in this discharge $H_\alpha/D_\alpha$ measurements 
indicated a value of $n_H/(n_H + n_D) \approx 0.03$. If it 
is assumed that the location of the decay activity is 
where the frequency of the $\approx$10 MHz peak equals the 
local value of $\omega_D$, a value of $R_{\text{decay}} = 228\pm8$ cm is 
obtained. In this calculation, the local value of both 
the toroidal and poloidal components of the magnetic

field must be taken into account. The outer major 
radius of the last closed flux surface in this inside-
wall limited discharge is $R = 232$ cm, and the face 
of the IBW antenna Faraday shield is at $R = 238.5$ cm.

The power threshold for the decay activity 
was observed to depend on the plasma density 
in the neighbourhood of the antenna. In Fig. 4 
the band integrated power in two bands observed 
with the electrostatic probe is plotted against the 
net RF power coupled with the IBW antenna for 
two different conditions. The single null divertor

![Figure 3](image_url)  
**FIG. 3.** Frequency of peaks in the spectrum 
obtained in several discharges with toroidal field 
ramp down during the RF pulse. ($I_p = 0.9$ MA, 
$n_e = 1.9 \times 10^{18}$ cm$^{-3}$, $P_{RF} = 450$ kW.)

![Figure 4](image_url)  
**FIG. 4.** Band integrated power in two frequency 
bands as a function of net RF power obtained 
in two discharges with different edge densities: 
(a) single null divertor discharge (low edge 
density), (b) inside-wall limited discharge (high 
edge density). (Both discharges: $I_p = 1.4$ MA, 
$n_e = 4 \times 10^{18}$ cm$^{-3}$, $B_T = 1.7$ T.)
condition is characterized by a very low electron density in the outboard midplane region outside the separatrix: measurements with the midplane Langmuir probe indicate a value of \( n_e \leq 5 \times 10^9 \text{ cm}^{-3} \) at the major radius of the backup limiter \( (R = 237 \text{ cm}) \), with the exact value depending on the separatrix-backup limiter gap. Density values below about \( 1 \times 10^{10} \text{ cm}^{-3} \) could not be measured accurately by the probe; the value given is a rough upper estimate. By contrast, in the inside wall limited configuration the density in the outboard midplane region was found to be about \( 8 \times 10^{11} \text{ cm}^{-3} \).

For the 36 MHz launching frequency used in this comparison, the deuterium density at which \( \omega = \omega_{pi} \) is \( 6 \times 10^{10} \text{ cm}^{-3} \). In the high density case (\( \omega \ll \omega_{pi} \)), a power threshold of about 120 kW for observation of parametric decay activity is evident, while in the low density case (\( \omega \gg \omega_{pi} \)) the threshold, if any, is at a power level near or below 10 kW.

The dependence of decay amplitude on power level and edge density was measured at a fixed central toroidal field of 1.68 T, chosen so that \( \omega = 2 \Omega_H = 4 \Omega_D \) at the antenna, which is the optimal field to couple to the IBW. The decay wave amplitude was measured as a function of \( \omega/\Omega_H \) at a fixed power level in the same discharges that produced the frequency data shown in Fig. 3. The frequency integrated power in the 14 to 22 MHz band is plotted as a function of the central toroidal field in Fig. 5. This frequency band includes the hydrogen (or second harmonic deuterium) quasi-mode and the associated IBW. A remarkable peaking of the signal at 1.68 T is observed, just where the pump frequency of 36 MHz equals twice the proton (or four times the deuteron) gyrofrequency in the neighbourhood of the antenna. The peaking occurs not only for the hydrogen ICQ and for the associated IBW, but also for the deuterium ICQ and for the associated IBW. For either ICQ, the width of the peak is too large to determine with certainty whether the maximum occurs when the harmonic layer is at a slightly greater major radius than that of the antenna or at a slightly smaller major radius.

Another peak is evident when \( \omega = 5 \Omega_D \) near the antenna, at \( B_T = 1.33 \text{ T} \).

Strong peaking at \( \omega \approx 2 \Omega_H \) was observed only when the edge density was high (\( \omega \ll \omega_{pi} \)), in the inside-wall limited configuration. The nature of this resonant behaviour was confirmed by repeating the measurement with a pump frequency of 38 MHz; in the latter case, the peak amplitude was observed when \( B_T = 1.78 \text{ T} \), again where \( \omega = 2 \Omega_H \) near the antenna. On the basis of the RF probe data alone, it is difficult to rule out the possibility that the peak in the signal when \( \omega = 2 \Omega_H \) near the antenna might arise from a peak in the probe sensitivity when \( \omega = 2 \Omega_H \) at the probe, since the antenna and the probe were at nearly the same major radius. However, the strong peaking of edge ion heating under the same conditions, discussed in Section 4, makes this explanation unlikely. Another measurement with a pump frequency of 32 MHz showed a peak where \( \omega = 3 \Omega_D \) near the antenna (1.98 T); the toroidal field ramp-down did not extend to 1.50 T, at which the 2 \( \Omega_H \) peak would have been expected.

The amplitudes of the deuterium quasi-mode and the associated IBW were found to depend sensitively
on the hydrogen minority fraction. At hydrogen fractions greater than a few per cent, the deuterium quasi-mode and the associated IBW were almost unobservably weak. This phenomenon was clearest in an experiment in which hydrogen was puffed into a nearly pure deuterium discharge during the RF pulse. The decay spectrum before the hydrogen was introduced is shown in Fig. 6(a). This spectrum was typical of those obtained in inside-wall limited discharges with a low hydrogen fraction; in this case, the measured hydrogen fraction was less than 2%. A strong deuterium quasi-mode at 9 MHz with the associated IBW near 27 MHz is observed, along with the hydrogen (or second harmonic D) quasi-mode and the associated IBW near 18 MHz. A distinctly asymmetric broadening of the pump towards the low frequency side is observed, with a peak at 34 MHz. Upon injecting hydrogen, \( n_H/(n_H + n_D) \) rose to about 12%, and the peaks at 9 and 27 MHz decrease nearly to the noise level, as shown in Fig. 6(b). The amplitudes of the hydrogen quasi-mode and the associated IBW increase by about 10 dB, and become two well resolved peaks at 17.2 and 18.9 MHz. The amplitude of the peak at 34 MHz increases by about 7 dB. Simultaneously, an increase of the signal near 2 MHz is observed, although the instrumentation is not optimized for low frequencies. The low frequency mode corresponds to the electron Landau damped quasi-mode that results in the asymmetric pump broadening. A similarly asymmetric broadening of the signal around 72 MHz is also evident.

Though the effect of the hydrogen concentration on the amplitudes of the deuterium quasi-mode and associated IBW is most apparent in a shot with hydrogen puffing during the RF pulse, the effect is not dependent on active puffing (e.g. on the cooling of the edge associated with puffing). The same phenomenon was observed on a series of discharges with hydrogen concentrations varying from shot to shot only because of evolving wall conditions. Again, the deuterium quasi-mode and the corresponding IBW were observed only when the hydrogen fraction is on the order of a few per cent or less.

**FIG. 6.** (a) The RF spectrum obtained in a deuterium inside-wall limited discharge with \( H/(H+D) \leq 2\% \). \( (I_p = 0.84 \text{ MA}, n_e = 1.9 \times 10^{13} \text{ cm}^{-3}, p_{RF} = 450 \text{ kW}, B_T = 1.7 \text{ T}) \) (b) The RF spectrum obtained during hydrogen injection into the same discharge as shown in (a).

The observations of PDI with probes may be briefly summarized as follows. Parametric decay instabilities in which the pump wave decays into an IBW and an ICQ or into an IBW and a low frequency ELDQ were observed with either loop probes or with an electrostatic probe during high power IBW injection. The PDI exhibited a power threshold which was dependent on the plasma density in the SOL; at high edge density, the power threshold for decay into ICQ + IBW was on the order of 100 kW, while at lower edge density, the decay threshold was lower. Under high edge density conditions, a strong peaking of the decay wave amplitudes was observed when the condition \( \omega = 2\Omega_H \) was satisfied near the antenna. Deuterium quasi-modes were observed only when the hydrogen fraction was less than 2%.
concentration in a nominally deuterium discharge was less than a few per cent. In the next section, we shall discuss the observations of edge heating during IBW injection, and the correlations between the observation of edge heating and the presence of parametric decay instability.

4. EDGE HEATING

4.1. Ion heating

Strong heating of the ions in the edge of the plasma was observed with the neutral particle CX analyser during high power IBW injection. Both minority hydrogen ions and majority deuterium ions showed high energy perpendicular tail formation; no significant parallel heating was observed (determined from an angular scan of the CX analyser). The edge ion heating has been observed only in inside-wall limited discharges with low line averaged densities ($n_e \leq 3 \times 10^{13} \text{ cm}^{-3}$). No edge ion heating was seen in diverted discharges, in high line averaged density discharges, or in outside limited discharges. Typical SOL plasma parameters in the Ohmic heating phase of a discharge in which edge ion heating was observed were $n_e(\text{edge}) = 2 \times 10^{11} \text{ cm}^{-3}$ and $T_e \approx T_i \approx 10$ eV. Note that the SOL density was almost independent of the line averaged density in the range of discharges used for the IBW experiments, but was very dependent on whether the plasma is inside or outside limited (high edge density) or diverted (low edge density) [15].

Typical neutral particle energy spectra for hydrogen and deuterium ions before and during IBW injection are shown in Fig. 7. The IBW power for this case was 860 kW and the hydrogen fraction was $\approx 3\%$. The hydrogen and deuterium tail temperatures are 5.5 and 1.8 keV, respectively. The deuterium thermal temperature of 0.91 keV is in good agreement with the central ion temperature of 0.84 keV from the charge exchange recombination (CER) spectroscopy system. The perpendicular ion tail measured by the CX analyser increases with IBW power, as shown in Fig. 8 for hydrogen. For 240 kW of IBW power, no ion tail was observed, indicating a power threshold for ion tail formation.

The ion heating seen by the CX analyser has been identified as edge heating, as opposed to central heating, for the following reasons: (1) when the IBW power was abruptly turned off, the high energy counts on the CX analyser disappeared in less than 1 ms, which is the time resolution of the CX analyser;
no ion temperature increase in the core of the plasma was measured with the CER spectroscopy system; (3) no increase in the plasma stored energy (measured from either plasma diamagnetism or magnetic reconstruction of the equilibrium) was observed; (4) for cases in which a tail is observed for deuterium ions, no increase in the neutron count rate was observed; (5) no significant tail in the parallel direction was seen by the CX analyser. These points demonstrate that the confinement time of the fast ions is much smaller than their slowing down time [25], so that the fast ions must originate near the plasma edge.

The edge ion heating was strongly dependent upon the toroidal magnetic field. Figure 9(a) shows the ion tail densities, as measured by the CX analyser, for hydrogen and deuterium as a function of the toroidal field, where the ion tail density is defined as

\[
n_{\text{tail}} = \int_{E_1}^{E_2} \left( f_{\text{exp}}(E) - n_0 \sqrt{2m_i \over \pi T_i} e^{-E/T_i} \right) \times {dE} \over \sqrt{2m_i E} \tag{1}
\]

where \(E_2\) is the maximum particle energy the CX analyser detects and \(E_1\) is the particle energy where the ion tail first appears. The bulk ion temperature \(T_i\) was determined from a fit of the CX analyser data in the range \(0 < E < E_1\). The IBW power for Fig. 9(a) was \(\approx 500\) kW and the hydrogen fraction was \(\approx 2\%\). The deuterium tail density is seen to peak strongly near the magnetic field for which efficient coupling to the IBW was expected (1.68 T at 36 MHz, so that \(\omega = 2 \Omega_H\) near the antenna).
The hydrogen tail density is small for large magnetic fields and starts to increase as the magnetic field for efficient IBW coupling is approached. However, the hydrogen tail density can be seen to peak at a much lower magnetic field than the deuterium tail density.

Data were also taken by ramping down the toroidal field during the IBW pulse. This data for the hydrogen tail density is shown in Fig. 9(b). The peak in the tail density occurs at a central field of 1.65 T, when the hydrogen resonance is 15 cm in front of the IBW antenna. The RF induced impurity influx was greatly increased at this toroidal field, which in turn triggered locked modes which substantially degraded confinement. It is possible that the edge ion heating at \( B_T = 1.26 \) T, where the \( \omega = 5 \Omega_D \) layer is located in the edge plasma, is actually deuterium heating which is observed on the hydrogen channel due to imperfect mass discrimination. The RF power for the \( B_T \) ramp data is \( \approx 300 \) kW, somewhat smaller than the \( \approx 500 \) kW value for Fig. 9(a). This power difference probably accounts for the lack of a peak in the edge heating during the \( B_T \) ramp as the ion cyclotron resonance moves through the antenna location; the observed power threshold for tail formation is discussed below.

There is a striking similarity between Figs 5 and 9(a), which were taken under the same plasma conditions. Both the ICQ amplitude and the edge ion heating show a peaking near the toroidal magnetic field at which efficient coupling to the IBW was expected.

The deuterium tail density increases with the IBW power as shown in Fig. 10. The power threshold for tail formation is comparable with the power threshold for PDI in a plasma with high edge density, as was discussed in Section 3. A straight line fit to the data in Fig. 10 indicates a threshold power for tail formation of 200 kW. The tail size increases approximately linearly with IBW power above this threshold. For comparison, Fig. 11 shows the sum of the amplitudes of the deuterium quasi-mode and the associated IBW as a function of RF power for the same discharges as shown in Fig. 10. A straight line fit to this data results in a threshold power of 200 kW, the same as in Fig. 10. The PDI amplitudes also increase roughly linearly with IBW power above this threshold. The data shown in Figs 10 and 11 show the correlation between deuterium...
tail formation and deuterium quasi-mode decay; a similar correlation with a similar threshold power was observed for hydrogen tail formation and the corresponding quasi-mode decay.

Figures 10 and 11 suggest a strong correlation between edge ion heating and the PDI amplitude. The data from the previous two figures are replotted in Fig. 12 to show that the ion tail density for deuterium increases linearly with the associated PDI amplitudes. This suggests that the ion heating in the edge plasma is due to the cyclotron damping of the ICQs. This explanation accounts very well for the deuterium heating. For hydrogen, this explains the heating which was observed when \( \omega \approx 2 \Omega_H \) near the antenna (the optimal condition for direct IBW coupling); however, at lower toroidal fields strong hydrogen heating persists though relatively weak PDI is observed. Since for this condition the second harmonic resonance for hydrogen is in front of the IBW antenna, the edge hydrogen ions are perhaps being directly heated by the near fields of the antenna. Fourth harmonic heating of deuterium by this mechanism would be negligible.

Good correlation between edge ion heating and PDI activity was observed not only in the case of a small hydrogen minority in a deuterium plasma, but also in a case with \( n_H/(n_H + n_D) \approx 0.70 \). In the hydrogen majority case, with \( \omega \approx 2 \Omega_H \) at a slightly larger major radius than the antenna, a strong perpendicular tail was observed in the hydrogen CX neutral spectrum during RF injection. The tail temperature was well correlated with the amplitude of the hydrogen quasi-mode and the associated IBW, observed in this case with the magnetic loop probe located 5 m away from the IBW antenna.

Further experimental evidence of the link between edge ion heating and PDI amplitude is found in cases of hydrogen gas puffing in a deuterium plasma. As shown in Fig. 6, the deuterium ICQ amplitude was strongly affected by the hydrogen fraction: a strong deuterium cyclotron quasi-mode (and the corresponding IBW) was observed only in discharges with low hydrogen fractions. Similarly, deuterium edge ion heating was observed only when the hydrogen fraction is <3%. Figure 13 shows the neutral particle energy spectrum measured by the CX analyser during IBW injection for two discharges: one with a hydrogen fraction of 1.5% and the other with one of 4%. A strong deuterium tail is seen in the former but not in the latter condition. The PDI spectra shown in Fig. 6 represent a similar comparison. In the pair of RF spectra corresponding to Fig. 13, the frequency integrated deuterium ICQ amplitude (7–11 MHz) for the 4% hydrogen condition is 13 dB smaller than in the 1% hydrogen condition. Therefore, reducing the deuterium ICQ amplitude by increasing the hydrogen fraction simultaneously reduces the deuterium edge ion heating.

In the steady state, the power required to heat the edge ions is

\[
P_{\text{ions}} = \frac{n_i V \Delta k T_i}{\tau_{E_i}}
\]

where \( n_i \) is the density of the edge ions, \( V \) is the edge volume, \( \Delta k T_i \) is the temperature increase of the edge ions and \( \tau_{E_i} \) is the energy confinement time of the edge ions. Since it is difficult to determine many of
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These quantities accurately, especially $\tau_E$, Eq. (2) is useful only for an order-of-magnitude estimate. We now may calculate the power required to heat the edge ions for the case shown in Fig. 7. The electron density in the edge plasma is determined from Thomson scattering and double Langmuir probe measurements, giving $n_e \simeq 4.5 \times 10^{12} \text{ cm}^{-3}$. In order to determine the edge ion density, $Z_{\text{eff}}$ must be known. Using visible bremsstrahlung measurements along with electron density and temperature profiles, a $Z_{\text{eff}}$ of 2.0 is estimated (for a carbon impurity). For a 3% hydrogen fraction, the hydrogen and deuterium edge ion densities are therefore $1.1 \times 10^{11}$ and $3.5 \times 10^{12} \text{ cm}^{-3}$, respectively. The edge volume is estimated from the Doppler width of the edge ion cyclotron resonance to be about 2 cm, which gives a heating volume of 0.42 m$^3$ for DIII-D. Since the CX analyser which observed the edge ion heating was located on the opposite side of the tokamak from the IBW antenna, we may postulate that the edge volume is uniformly filled with energetic ions. Since the ion temperature in the edge region is $10 \lesssim T_i \lesssim 100$ eV, which is much less than the characteristic energy of the tail particles, $\Delta kT_i$ is approximately equal to the mean tail energy.

An upper bound of 1 ms can be placed on the tail confinement time from the observation that the high energy counts disappear less than 1 ms after the IBW power is turned off. Substituting these estimates into Eq. (2) gives a lower bound of the required heating power of 40 kW for hydrogen ions and 420 kW for deuterium ions, for a total heating power of at least 460 kW. The injected RF power for this case was 860 kW. Therefore, power loss to edge ion heating may represent a substantial fraction of the injected RF power for a regime in which central ion heating is desired.

For discharges with hydrogen minority fractions of greater than about 3%, in which no deuterium heating was observed, the power required to account for the observed edge hydrogen heating decreases to less than 10% of the coupled RF power. However, as the hydrogen fraction increases further the necessary edge ion heating power again increases. In a discharge with $n_H/(n_H + n_D) \simeq 0.70$, the edge hydrogen temperature increased 700 eV for $P_{RF} = 300$ kW. The edge hydrogen density calculated from edge Thomson scattering and $Z_{\text{eff}}$ measurements is $2.4 \times 10^{12} \text{ cm}^{-3}$. Using the same edge volume and energy confinement time of the previous examples, the estimated edge ion heating power is 110 kW, and therefore may again be a substantial fraction of the launched RF power.

4.2. Electron heating

Edge heating of electrons during IBW injection was observed in the SOL of DIII-D by a double Langmuir probe. The Langmuir probe was usually located at a major radius 1 mm larger than that of the outside limiter in the SOL of the plasma. The SOL electron temperature typically increased from 5 to 15 eV during IBW injection. The SOL electron density was usually unchanged. The edge electron heating was observed in both limiter and divertor configurations, unlike edge ion heating which was observed only in limiter configurations (high edge density conditions).
Electron heating could compete with or even dominate ion heating, as a result of electron Landau damping of the ion cyclotron quasi-modes or by excitation of electron Landau damped quasi-modes. Direct heating of edge electrons by the rippled (short wavelength) $E_{\parallel}$ near fields in the immediate vicinity of the Faraday shield elements [26] cannot be excluded with certainty. Even the linearly excited IBW at the intended relatively long parallel wavelength may be electron Landau damped in the edge, as a result of $k_{\parallel}$ upshifts caused by propagation of IBW with non-zero $k_{\parallel}$ in the sheared magnetic fields in the edge region [27].

As shown in Fig. 14 for the diverted configuration (albeit with a small enough gap between the last closed flux surface and the wall so that the density at the probe is high enough to yield good probe characteristics), the SOL electron temperature increased with RF power. There appears to be a rapid rise of SOL electron temperature with RF power up to 50 kW, above which the edge $T_e$ increases more slowly with power.

It is difficult to estimate the heating power flowing to electrons in the SOL layer of the plasma due to the limited diagnostics. The power necessary to heat the edge electrons can be estimated from

$$P_{\text{elec}} = \frac{3}{2} \frac{n_e V \Delta k T_e}{\tau_{\text{Te}}}$$  \hspace{1cm} (3)

The quantity with the greatest uncertainty in this equation is the electron energy confinement time, $\tau_{\text{Te}}$. Assuming that the electrons in the plasma edge are lost in a few toroidal transits gives $\tau_{\text{Te}} \approx 10^{-8}$ s. Using the edge electron density of $4 \times 10^{12}$ cm$^{-3}$ obtained from Thomson scattering and assuming the same edge volume as was assumed for edge ion heating, $\Delta k T_e \approx 10$ eV corresponds to an edge electron heating power of $\approx 400$ kW. Therefore, the magnitude of observed edge electron heating can account for the injected IBW power.

Reducing the IBW power flow to edge heating of deuterium by increasing the hydrogen fraction (see Fig. 13) apparently results in the excess power going to edge electron heating, as shown in Fig. 15.
In this case, already discussed in connection with Fig. 6, hydrogen gas was puffed during RF injection. The deuterium tail density was seen to decrease rapidly once the gas puffing begins, indicating that deuterium edge ions were no longer being heated. Simultaneously, the electron temperature in the SOL began to increase. The electron heating correlates with the observation of ELDQ, as was shown in Fig. 6. Thus, RF power not absorbed by edge ions appears to result in edge electron heating.

5. DISCUSSION AND CONCLUSIONS

In order to compare the experimental results discussed in this paper with the predictions of the theory of parametric decay [16], one first must have information on the magnitude and the spatial dependence of the pump wave fields. As a first step, then, the IBW antenna–plasma coupling should be understood. This is the topic of another paper [15], the results of which may be briefly summarized as follows. The character of the antenna loading is consistent only with coupling to the EPW. In particular, the lack of dependence of the loading on the value of $\omega/\Omega_H$ near the antenna indicates at most a very weak direct coupling to the IBW at the pump frequency. This result is obtained either with low edge density (so that $\omega > \omega_{LH}$ immediately in front of the antenna) or with high edge density ($\omega \ll \omega_{LH}$). Lack of dependence of the loading on $\omega/\Omega_H$ near the antenna would be expected in the low edge density case, but not in the high edge density case.

The apparent anomaly can be explained by invoking the ponderomotive force, which depresses the density in the near field region so that $\omega > \omega_{LH}$ (in which $\omega_{LH}$ denotes the lower hybrid resonance frequency) is always satisfied in front of the antenna, and the antenna consequently excites the EPW regardless of the initial density immediately adjacent to the antenna. When the ponderomotive depression of the density is taken into account in an antenna coupling model in an approximate way, the IBW antenna loading can be explained qualitatively [14], and roughly quantitatively [28]. Measurements of the reactive antenna loading have yielded direct evidence for the importance of ponderomotive effects [14, 15].

For sufficiently large values of the electric field, the ponderomotive force will depress the density not only in the antenna near field region, but also along the resonance cone trajectories of the EPW. Therefore, $\omega > \omega_{LH}$ may be satisfied at all points along the wave trajectory. We thus arrive at a picture of the pump wave fields wherein the non-linearly propagating EPW is trapped in the edge region until the wave energy is dissipated by non-linear effects such as PDI. Coupling of the non-linearly trapped EPW to the IBW, expected to occur in the linear (mode transformation) regime [29] near $\omega \approx \omega_{LH}$, may be prevented for sufficiently steep gradients at the edges of the non-linear resonance cones. It is important to note that the antenna loading showed none of the sensitivity to the H/D ratio and to $\omega/\Omega_H$ that the PDI exhibited, so that the wave energy dissipation associated with the PDI cannot have occurred in the antenna near field.

Spatially resolved, absolute measurements of the pump wave amplitude would be necessary to demonstrate directly the validity of this picture of the pump wave field structure. Unfortunately, no such measurements were possible in the DIII-D IBW experiments. Also, the parametric decay spectra were obtained with a single probe at an essentially fixed location 2 m away from the antenna, so that no data on the spatial extent and variation with location of PDI amplitudes and spectra were available. As a result of the very strong electric fields, the conventional theory of PDI [30] is not strictly valid in front of the antenna for RF power levels in excess of $P_{RF} \sim 50$ kW. Clearly, a quantitative comparison of the PDI results with the theory is difficult at best.

It is possible, however, that the theory may be applicable at the location of the probe and at the location of the CX analyser due to the lower electric fields that probably exist at these locations, though this depends critically on an accurate description of the non-linear pump wave propagation, which is not available. Some of the conclusions of a detailed
study of the application of parametric decay theory to the present situation [30] are summarized in the following.

The theory predicts decay of a long wavelength pump into IBW and associated hydrogen and deuterium ICQ, as well as electron Landau damped low frequency quasi-modes. In addition, decay into a pair of IBW is also predicted in some regimes. Qualitatively, each of these decay processes has been observed in the RF spectra in this experiment. Detailed numerical work undertaken recently [30] verifies earlier analytical estimates of the growth rates [17]. In general, the largest amplitude decay waves are experimentally observed near the optimal magnetic field for efficient coupling to the IBW at the pump frequency. Theoretically, growth rates are not maximized here; rather, the largest growth rates occur for pump wave frequencies near \((\pi \pm \frac{1}{2})\Omega_i\), with \(n\) an integer. On the other hand, convective losses are minimized for pump frequencies just below \(n\Omega_i\) with \(n \geq 2\) (i.e. the optimal situation for linear IBW coupling), because the daughter IBW is also just below an ion cyclotron harmonic and has a low group velocity \(v_g = \partial\omega/\partial k_\perp\) (see Fig. 1). Also, the pump wave amplitude itself may also peak near \(\omega \approx n\Omega_i\), at least in regions where the electric fields are low enough. In some cases the thermal fluctuation levels have been computed and convective thresholds for PDI were shown to be exceeded, resulting in significant pump wave depletion. The observation may be made that at high densities \((\omega < \omega_{LH})\), generally \(k_\perp\rho_i \gtrsim 1\) (where \(\rho_i\) is the ion Larmor radius) for the daughter waves, so that the damping of the ICQ is dominated by cyclotron damping, which results in ion tail formation. By contrast, at lower densities \((\omega > \omega_{LH})\), electron Landau damping dominates (since \(k_\perp\rho_i \ll 1\) and we expect electron heating, though the maximum growth rate is still at \(\omega \approx \Omega_i\). In addition, the absorption of the parametrically excited IBW will have to be assessed to determine whether the power flow is into the electron or ion channel. For example, strong shear near the plasma edge may result in absorption of the IBW by electrons [27].

Were the IBW excited at the pump frequency at high edge densities, a decay described by mode-mode coupling theory could occur. We find that the threshold for such a decay may also be exceeded [17]. Such decay processes were considered previously in the context of the Alcator C IBW experiments [12] which were performed at high magnetic fields and high pump wave frequencies \((f \sim 180\text{ MHz})\). In such a situation the ponderomotive force is significantly reduced (but still may be important). Nevertheless, even in the Alcator C experiments the heating results could be explained only by invoking nonlinear decay processes since the observed heating was not resonant as the magnetic field was varied [13]. It should be noted that the plasma minor radius in Alcator C was in the range 10–12 cm, which in DIII-D corresponds to the edge region. Thus, while in Alcator C PDI waves could produce core heating, in DIII-D a similar phenomenon would result in edge heating. The non-linearity of the situation in DIII-D is exacerbated by the low frequencies and the consequent strong ponderomotive forces necessitated by the relatively low magnetic field of the device.

At present, the disappearance of the deuterium ICQ (and the corresponding vanishing of the deuterium tail) as the hydrogen minority fraction is raised is not well understood. In general, the predicted growth rates for the ICQ in a Maxwellian plasma do not change significantly as \(n_H/(n_H + n_D)\) is increased from 0.02 to 0.10 or beyond. It is possible that the presence of the non-Maxwellian tail will have to be considered in the PDI theory. Alternate explanations may require consideration of the spatial evolution of the pump wave, including pump depletion. Theoretical exploration of this possibility is ongoing. As mentioned previously, it was not possible to obtain experimental data on the spatial evolution of either the pump or the daughter waves in this experiment.

In summary, high power external launching of the IBW in low to medium magnetic field tokamaks is necessarily in a strongly non-linear regime, characterized by strong ponderomotive forces and parametric decay instabilities. These phenomena
have been shown to be critically important in understanding the DIII-D IBW experimental results with respect to the antenna coupling [15] and the absorption mechanisms in the plasma edge, the latter area constituting the subject of the present paper. The non-linear effects have been shown to result in strong edge absorption which was consistent with the lack of observable core heating in these experiments [11, 14]. These results are not inconsistent with the earlier observations on the Alcator C tokamak, which implied strong non-linear effects in a regime of significantly lower ponderomotive forces than in DIII-D. It is recommended that a more thorough study of these phenomena, including detailed measurements of the non-linear spatial evolution of the pump and daughter wave fields, should be undertaken in smaller, low temperature devices [31] in which such measurements are practical.

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