The regularized BRST Jacobian of pure Yang-Mills theory

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The Jacobian for infinitesimal BRST transformations of path integrals for pure Yang-Mills theory, viewed as a matrix $1 + \Delta J$ in the space of Yang-Mills fields and (anti)ghosts, contains off-diagonal terms. Naively, the trace of $\Delta J$ vanishes, being proportional to the trace of the structure constants. However, the consistent regulator $\mathcal{R}$, constructed from a general method, also contains off-diagonal terms. An explicit computation demonstrates that the regularized Jacobian $\text{Tr} \ (\Delta J \exp - \mathcal{R}/M^2)$ for $M^2 \to \infty$ is the variation of a local counterterm, which we give. This is a direct proof at the level of path integrals that there is no BRST anomaly.

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The rigid BRST symmetry of the quantum action for gauge theories has become an essential tool in the path-integral approach. One uses the BRST symmetry to obtain Ward identities, which are then used to prove perturbative renormalizability and unitarity. In principle, the Ward identities can contain an anomaly which appears in the path-integral approach as a deviation from unity of the Jacobian for BRST transformations. It is often argued that there is no anomaly in the BRST Ward identities because $\Delta J$ is proportional to the trace of the structure constants, which vanishes for semi-simple gauge groups. However, this argument, though widely repeated and presented in various textbooks, is patently false. For example, the same line of reasoning would conclude that there never are chiral anomalies since $tr \gamma_5 = 0$. For chiral anomalies one knows that after regularization with a regulator $R$, the regularized trace $tr \gamma_5 \exp -R/M^2$ for $M \to \infty$ no longer vanishes in general and yields the chiral anomaly. For chiral symmetries, the choice of $R$ is not very important, as “[one] can show that under broad assumptions the one loop anomalies depend only on the quantum numbers of the elementary fields, and not on the specific Lagrangians chosen”. This is due to the topological nature of the chiral anomaly. In general, the choice of $R$ clearly matters. For example, the Jacobian for Weyl symmetry is field-independent, and if one were to choose a regulator which is also field-independent, one could never obtain anomalies proportional to the curvatures. Recently, a general method was developed for constructing a consistent regulator for the measure of any quantum field theory. This regulator is equivalent to Pauli-Villars regularization of the Feynman diagrams of the effective action, and yields anomalies which satisfy the consistency conditions. It is this regulator which we will use to compute the BRST anomaly. For BRST symmetry it is computationally not obvious that the regularized trace $Tr \Delta J \exp -R/M^2$ still vanishes in the limit $M \to \infty$, although one would expect so, given that indirect (cohomological) arguments say so. In this article we will demonstrate by a direct calculation that the BRST anomaly in the Ward identities indeed vanishes.

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1For local (gauged) BRST symmetry, see F.J. Ore and P. van Nieuwenhuizen, Nucl. Phys. B204 (1982) 317.

2For convenience we will use the word ‘anomaly’ for this regulated trace, even when it is the variation of a local counterterm and can be removed.
The derivation of the Ward identities is based on the Shakespeare theorem according to which one may rename the integration variables \( \phi^j \) everywhere (in the measure, in the action and in the coupling to external sources) by 
\[
\phi'{}^j = \phi^j + \delta \phi^j
\]

\[
\begin{align*}
Z(J) &= \int \mathcal{D}\phi^j \exp \left( \frac{i}{\hbar} \left[ S_{qu}(\phi) + \int J^j \phi^j d^4 x \right] \right) \\
&= \int \mathcal{D}\phi'^j \exp \left( \frac{i}{\hbar} \left[ S_{qu}(\phi') + \int J^j \phi'^j d^4 x \right] \right) .
\end{align*}
\] (1)

We take for \( \delta \phi^j \) an infinitesimal BRST transformation. The quantum action \( S_{qu}(\phi) \) consists of an \( \hbar = 0 \) part which is BRST invariant, and possibly local counterterms \( \hbar M_1 \), where \( M_1 \) is in general a power series in \( \hbar \). This leads to the formal Ward identity

\[
\left\langle Tr \Delta J^j + i \delta M_1 + \frac{i}{\hbar} \int J^j \delta \phi^j d^4 x \right\rangle = 0
\] (2)

where

\[
\Delta J^j_k = \frac{\partial \delta \phi^j}{\partial \phi^k}
\] (3)

(with right derivatives) is the deviation of the Jacobian matrix from unity.

We shall discuss the regularization of the trace \( Tr \Delta J \) in pure Yang-Mills theory, with \( \phi^j \) equal to the fields \( b \) (antighosts), \( Q_\mu \) (quantum Yang-Mills fields) and \( c \) (ghosts). However, for reasons to be explained, we shall use an action containing an extra, external, gauge field \( B_\mu \), which interpolates between ordinary quantum field theory and the background field method. This action \( S = S_{qu}(\hbar = 0) \) reads

\[
S = Tr \int d^4 x \left\{ -\frac{1}{4} F_{\mu\nu}(Q)^2 - \frac{1}{2} [D^\mu(B)(Q_\mu - B_\mu)]^2 - D_\mu(B)b \cdot D^\mu(Q)c \right\}
\] (4)

... \( \tilde{\phi} \) be some other name.

What's in a name? that which we call a rose,
By any other word would smell as sweete,
So Romeo would were he not Romeo cald,
Retaine that deare perfection...
where $D_\mu(X)Y = \partial_\mu Y + [X_\mu, Y]$ and the trace $Tr$ is over gauge indices. For $B_\mu = 0$, we obtain the Feynman gauge, while for $B_\mu \neq 0$ we recognize the action for the background field formalism (after shifting $Q_\mu$ to $Q_\mu + B_\mu$). This last action is invariant under two symmetries

1. local background symmetry, under which both the background field $B_\mu$ and the quantum field $Q_\mu$ transform as gauge fields, and $b$ and $c$ as vectors

\[
\begin{align*}
\delta Q_\mu &= D_\mu(Q)\lambda \\
\delta B_\mu &= D_\mu(B)\lambda \\
\delta b &= [b, \lambda] \\
\delta c &= [c, \lambda].
\end{align*}
\]

with $\lambda$ the local commuting Lie-algebra valued Yang-Mills parameter.

2. rigid BRST symmetry, under which the background field is inert

\[
\begin{align*}
\delta Q_\mu &= D_\mu(Q)c\Lambda \\
\delta B_\mu &= 0 \\
\delta b &= -D^\mu(B)(Q^\mu - B^\mu)\Lambda \\
\delta c &= \frac{1}{2}[c, c]\Lambda
\end{align*}
\]

with $\Lambda$ the constant, anticommuting BRST parameter.

In this article we shall discuss the Jacobians for these symmetries, as they appear in the path-integrals in (2).

If one (erroneously) neglects regularization, one would conclude that the trace of $\Delta J$ in (2) vanishes for both symmetries in (5) and (6), as the diagonal entries are proportional to $f_{a b c}$ in each case (where $f_{a b c}$ are the structure constants of a Lie algebra, which are traceless for the semi-simple groups which we consider). As we explained above this conclusion is incorrect.

In view of the importance of BRST symmetry, we think that a direct, explicit calculation of the anomaly (the trace of $\Delta J \exp -\mathcal{R}/M^2$) for both symmetries is useful. Of course we shall discuss how to determine $\mathcal{R}$. In addition, we shall determine by direct computation whether the anomaly
can be written as the variation of a local counterterm (and thus removed) and when it actually vanishes.

We work in the generalized gauge with $B_\mu$ present in order to avoid an accidental vanishing of the anomaly. To draw a comparison with string theory, we prefer to work in a ‘general gauge’ (like $g_{\alpha\beta} = G_{\alpha\beta}$ where $G_{\alpha\beta}$ is an arbitrary background field) rather than a special gauge. If one were to choose the gauge $g_{\alpha\beta} = \eta_{\alpha\beta}$, the anomaly $A = c\sqrt{\gamma}R(g) = 0$ would seem to vanish. Of course, this is not an allowed gauge since it cannot be reached using the gauge symmetries without anomalies. Similarly, in our case, we still have to prove that there are no Yang-Mills anomalies, so we should not use the Yang-Mills symmetry to choose a special gauge. In the Batalin-Vilkovisky field-antifield formalism \cite{10} there is a natural way to work with general gauges \cite{11}. Usually one eliminates antifields by putting $\phi^*_j = \frac{\partial \Psi}{\partial \phi_j}$, where $\Psi$ is a ‘gauge fermion’. This is equivalent to first making a canonical transformation with a ‘generating function’ $\Psi$, and then projecting onto the hypersurface $\phi^*_j = 0$. However, if one does not project onto this hypersurface and keeps the antifield dependence, then these play the role of arbitrary gauge parameters. For the string e.g., the choice $\Psi = b^{\alpha\beta}(g_{\alpha\beta} - \eta_{\alpha\beta})$ yields $g_{\alpha\beta} = \eta_{\alpha\beta} - b^*_{\alpha\beta}$, where the last term is the antifield of the antighost. Clearly, whether the anomaly is expressed as a function of a general background metric $G_{\alpha\beta}$, or as a function of the $b^*_{\alpha\beta}$-antifield is the same thing. We have done the calculations also with antifields (for $B_\mu = 0$, which is then sufficient), but for simplicity, we shall present here the calculations with (4) (and zero antifields), as this action is of practical interest and general enough for our purposes.

The most practical method for regularisation of Feynman diagrams is the dimensional regularisation method. It keeps gauge invariance at all stages when there are no $\gamma_5$ or similar dimension-dependent objects present. Therefore there are no anomalies in this regularization scheme in these cases. However, dimensional regularization can not be applied directly to the path integral measure. As shown by Fujikawa \cite{12}, in path integrals the anomalies come from the measure. However, in the original works, it was not known which regulators would give consistent \cite{1} anomalies. This problem was solved in \cite{6}, where a general recipe was obtained which yields a consistent regulator for any quantum field theory, once the quantum action is given.
Input for this method is a mass matrix $\phi^i T_{ij} \phi^j$, which must be non-singular. Output is a consistent regulator $\mathcal{R}$, given by

$$\mathcal{R}^i_j = (T^{-1})^{ik} S_{kj}; \quad S_{ij} = \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi^j} S.$$  

(7)

The regularized trace (really a supertrace due to the presence of the (anti)ghosts, hence denoted by $\str$) is then given by

$$\mathcal{A} = \lim_{M^2 \to \infty} \str \Delta J \exp(T^{-1} S/M^2)$$

$$= \lim_{M^2 \to \infty} \str \Delta J e^{T^{-1} S/M^2},$$

(8)

with

$$\Delta J_s \equiv \frac{1}{2} \left( \Delta J + T^{-1} \Delta J^T \right).$$

(9)

The transposition on $\Delta J$ in this equation refers to a supertransposition of the $\Delta J$ matrix, a transposition of the derivative operators which amounts to an extra minus sign after partial integration, and a transposition of the Lie-algebra representation matrices which amounts to another minus sign (in the adjoint representation). This rewriting of $\mathcal{A}$ makes use of the super-symmetry of $S_{ij}$ and $T_{ij}$, and will simplify calculations later on.

It is conjectured (and checked in many examples) that
- this method gives a consistent anomaly, i.e., the Wess-Zumino conditions are satisfied.
- the expression for $\mathcal{A}$ is gauge dependent, but this dependence can be absorbed in a counter-term.
- different choices of the mass term lead to different expressions of $\mathcal{A}$ which

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Footnote 1: All our matrices are supermatrices of bosonic type. Then the supertranspose for matrices with 2 lower indices, as $S$ and $T$, is defined by $(T^t)_{ij} = T_{ji}(-)^{i+j+ij}$, where $(-)^i$ is $+$ for entries related to $Q_\mu$ and $-$ for entries related to $b$ and $c$. This rule follows from the definition that $\phi^i (T^t)_{ij} \phi^j = \phi^i T_{ji} \phi^j$. For matrices with two upper indices, we have $(M^t)^{ij} = M^{ji}(-)^{ij}$, as can be derived for $T^{-1}$ if we impose $(T^{-1})^T T^t = 1$. Finally for matrices with mixed indices like $J^t j$ the supertransposition rule follows from the product of matrices of the previous types, imposing $(MT)^t = T^t M^t$. We have $(J^t)^{ij} = (-)^{i(j+1)} J^t_{ji}$. For these matrices the supertrace is $\str J = (-)^i J^t i$ which has the property that $\str J^t = \str J$ and $\str AB = \str BA$. These properties, with the knowledge that $S$ and $T$ are super-symmetric by their definition, lead to the equality in (8).
again differ by the variation of a local counterterm. This example will provide another check on the first two of these conjectures.

We choose a mass term which is invariant under rigid Yang-Mills transformations: \( tr \int d^4 x [Q_\mu Q^\mu + 2bc] \). We write the matrix-entries in order of decreasing ghost number, namely in the order \( b, Q_\mu, c \); this makes the triangular nature of the matrices to follow more manifest. Then

\[
T = \begin{pmatrix}
0 & 0 & 1 \\
0 & \eta_{\mu\nu} & 0 \\
-1 & 0 & 0
\end{pmatrix}; \quad T^{-1} = \begin{pmatrix}
0 & 0 & -1 \\
0 & \eta^{\mu\nu} & 0 \\
1 & 0 & 0
\end{pmatrix}. \tag{10}
\]

For the local background symmetry, \( \Delta J \) is diagonal, with entries \( f^{ab}_c \lambda^c \delta^\mu_\nu \) and \( f^{ab}_c \lambda^c \), respectively. One may verify that \( \Delta J_s \) vanishes in this case. Hence, the background symmetry is preserved at the quantum level in an almost obvious way.

For the rigid BRST symmetry, to which we devote the rest of this article, we find that \( \Delta J \) is off-diagonal and contains derivatives,

\[
\Delta J = \begin{pmatrix}
0 & -D_\nu(B) & 0 \\
0 & -c\delta^\mu_\nu & -D^\mu(Q) \\
0 & 0 & -c
\end{pmatrix} (x)\delta(x - y)\Lambda, \tag{11}
\]

where entries such as \( c \) stand for the matrices \( f^{a}_{cb}c^c \). The symmetrized \( \Delta J_s \) contains no derivatives but is purely algebraic:

\[
\Delta J_s = \frac{1}{2} \begin{pmatrix}
c & Q_\nu - B_\nu & 0 \\
0 & 0 & -Q^\mu + B^\mu \\
0 & 0 & -c
\end{pmatrix} (x)\delta(x - y)\Lambda. \tag{12}
\]

This fact will greatly simplify the evaluation of the supertrace in (8). Furthermore, the operator matrix \( T^{-1}S \) is given by

\[
T^{-1}S = \begin{pmatrix}
D_\alpha(Q)D^\alpha(B) & -(D_\nu(B)b) & 0 \\
cD^\mu(B) & R^\mu_\nu & (D^\mu(B)b) \\
0 & D_\nu(B)c & D_\alpha(B)D^\alpha(Q)
\end{pmatrix} (x)\delta(x - y), \tag{13}
\]

where

\[
R^\mu_\nu = D_\alpha(Q)D^\alpha(Q)\delta^\mu_\nu - D^\mu(Q)D_\nu(Q) + D^\mu(B)D_\nu(B) + 2F^\mu_\nu(Q) \tag{14}
\]
and the covariant derivatives act as far as \( \delta^4(x-y) \), unless put within explicit brackets. This expression can be cast into the form

\[
T^{-1}S = (\partial_\alpha \mathbf{1} + \mathcal{Y}_\alpha) \eta^{\alpha\beta} (\partial_\beta \mathbf{1} + \mathcal{Y}_\beta) + E
\]

where \( \mathbf{1} \) and \( \mathcal{Y}_\alpha \) are \( 6 \times 6 \) matrices with entries in the adjoint representation of the Yang-Mills Lie algebra, and \( \alpha, \beta \) are ordinary Minkowski indices. The derivatives in (15) are explicit, i.e., \( \mathcal{Y}_\alpha \) and \( E \) do not contain free derivative operators any more. We find the following expression for \( \mathcal{Y}_\alpha \) and \( E \) in \( d \) dimensions

\[
\mathcal{Y}_\alpha = Q_\alpha \mathbf{1} + \frac{1}{2} \begin{pmatrix}
-Q_\alpha' & 0 & -Q_\alpha' \eta_{\alpha\nu} - Q_\nu' \delta_\alpha & 0 \\
c\delta_\alpha & -Q_\mu \eta_{\mu\alpha} - Q_\nu' \delta_\mu & 0 \\
0 & -c\eta_{\mu\nu} & -Q_\alpha'
\end{pmatrix}
\]

\[
E = -\frac{1}{4} Q'^2 \mathbf{1} + \left( -\frac{1}{2} D_\alpha(B) Q'^\alpha - D_\nu(B) b \right)
\]

\[
\frac{E^{\mu\nu}}{4c^2} - \frac{D_\mu(B) b}{2c}
\]

where \( Q'^\nu = Q'^\nu - B'^\nu \) and\footnote{Antisymmetrization \([\mu\nu]\) is done with weight 1, i.e., \( \frac{1}{2}(\mu\nu - \nu\mu) \).

where \( Q'^\nu = Q'^\nu - B'^\nu \) and\footnote{Antisymmetrization \([\mu\nu]\) is done with weight 1, i.e., \( \frac{1}{2}(\mu\nu - \nu\mu) \).

The trace in (8) can be evaluated using the heat kernel. As long as \( \Delta J_s \) is algebraic, i.e., contains no derivative operator (which is the case for all applications we consider in this paper), only the value of this kernel at coincident points is needed. In the limit of large \( M^2 \) it can be calculated by a variety of methods. We read off the result from \[13\], generalized to the mixed bosonic and fermionic case. We are interested in the terms independent of \( M^2 \). In four dimensions they are usually denoted by \( a_2 \) and read

\[
a_2 = \frac{1}{(4\pi)^2} \left( \frac{1}{12} W_{\alpha\beta} W^{\alpha\beta} + \frac{1}{2} E^2 + \frac{1}{6} \Box E \right)
\]

\[
(18)
\]
where
\[ W_{\alpha\beta} = \partial_\alpha Y_\beta - \partial_\beta Y_\alpha + [Y_\alpha, Y_\beta] \]
\[ \Box E = \nabla_\alpha \nabla^\alpha E \]
\[ \nabla_\alpha X = \partial_\alpha X + [Y_\alpha, X] . \]
\[ (19) \]

In two dimensions they are denoted by \( a_1 \) and read
\[ a_1 = \frac{1}{4\pi} E . \]
\[ (20) \]

As a check on these results we compute the trace anomalies. They are obtained by taking the trace of the Weyl Jacobian \( J_W \) with the \( a_n \) coefficients. For two dimensions the Weyl weights of \( c, b \) and \( Q_\mu \) are all equal, see [14], hence the Weyl anomaly becomes proportional to the trace of \( E (d = 2) \). This vanishes, in agreement with the fact that the trace anomaly for spin 1 fields in \( d = 2 \) is zero. For \( d = 4 \) one has \( J_W = \text{diag}(0, \frac{1}{2}\sigma(x), \sigma(x)) \), see [14], yielding the correct result. Another check on the correctness of \( a_2 \) (except the \( \Box E \) term) is that after integration over space-time, they yield the one-loop counterterms, deduced by dimensional regularization and Feynman diagrams in [14].

As we already mentioned, in the Yang-Mills case \( \Delta J_s \) is also algebraic, due to the symmetrization in (8). The anomaly is obtained by computing the supertrace
\[ \mathcal{A} = \frac{1}{(4\pi)^2} \text{str} \Delta J_s \left( \frac{1}{12} W_{\alpha\beta} W^{\alpha\beta} + \frac{1}{2} E^2 + \frac{1}{6} \Box E \right) . \]
\[ (21) \]

The problem of obtaining the BRST anomaly for pure Yang-Mills theory is thus reduced to the evaluation of the supertrace in (21). The result reads
\[ \mathcal{A} = \frac{1}{(4\pi)^2} \frac{1}{12} \text{tr} \left[ D^\nu (B) c \right] \left[ 4Q_\mu' Q_\nu' Q'^\mu - 8Q'^\mu D_{[\mu} (B) Q_{\nu]} - 4Q_\nu' D_{\mu} (B) Q'^\mu + D_{\mu} (B) D^\mu (B) Q_\nu' - 3D_{\nu} (B) D_{\mu} (B) Q'^\mu \right] . \]
\[ (22) \]

If this is to be a consistent anomaly, its BRST variation should vanish. Indeed, it does. Expecting that there is no genuine BRST anomaly, this expression is expected to be the BRST variation of a local counterterm. Indeed, it
is: $\mathcal{A} = \delta M_1$, with

$$M_1 = \frac{1}{(4\pi)^2} \frac{1}{12} \text{tr} \left[ \frac{3}{2}(D_\mu(B)Q'^\mu)^2 - \frac{1}{2}(D_\mu(B)Q'_\nu)(D^\nu(B)Q'^\mu)ight.$$

$$-2Q'^\mu(D_\mu(B)Q'_\nu)Q'^\nu + \frac{3}{2}Q'_\mu Q'_\nu Q'^\mu Q'^\nu - \frac{1}{2}Q'^2 Q'^2 \left. \right] . \quad (23)$$

For computations, a suitable alternative form is given by

$$M_1 = \frac{1}{(4\pi)^2} \frac{1}{12} \text{tr} \left[ \frac{3}{2}(D_\mu(B)Q'^\mu)^2 - \frac{1}{2}(D_\mu(B)Q'_\nu)(D^\nu(B)Q'^\mu)ight.$$

$$-(D_\mu(B)Q'_\nu)(D^\nu(B)Q'^\mu) + Q'_\mu Q'_\nu Q'^\mu Q'^\nu + \frac{1}{4}F'_{\mu\nu} F'^{\mu\nu} \right] \quad (24)$$

where

$$F'_{\mu\nu} = D_\mu(B)Q'_\nu - D_\nu(B)Q'_\mu + [Q'_\mu, Q'_\nu] = F_{\mu\nu}(Q) - F_{\mu\nu}(B). \quad (25)$$

In the background field formalism $Q_\mu$ gets replaced by an external field $A_\mu$, and then one usually chooses the two external fields equal ($B_\mu = A_\mu$). In this case the anomaly vanishes without having to invoke a counterterm. In ordinary field theory one puts $B_\mu = 0$; in this case one needs a nontrivial counterterm $M_1$.

In conclusion, we have seen that within the framework of path-integrals, one can give a direct and complete derivation of the BRST Ward identities, on which perturbative unitarity and renormalizability are based. The precise form of the local counterterm $M_1$ which must be added to the quantum action in order that the BRST anomaly cancels, depends of course on the regularization scheme used to compute the effective action. If one uses Pauli-Villars regularization with the given mass term, $M_1$ is given in (24). If one uses dimensional regularization, one has $M_1 = 0$. However, the precise form of $M_1$ is not needed in general.

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