A Machian Version of Einstein’s Variable Speed of Light Theory

Alexander Unzicker
Pestalozzi-Gymnasium München, Germany
Jan Preuss
Technische Universität München, Germany
e-mail: aunzicker et web.de
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Abstract

It is a little known fact that while he was developing his theory of general relativity, Einstein’s initial idea was a variable speed of light theory [1]. Indeed space-time curvature can be mimicked by a speed of light $c(r)$ that depends on the distribution of masses. Einstein’s 1911 theory was considerably improved by Robert Dicke in 1957, but only recently has the equivalence of the variable speed of light approach to the conventional formalism been demonstrated [2]. Using Green’s functions, we show that Einstein’s 1911 idea can be expressed in an analytic form, similar to the Poisson equation. Using heuristic arguments, we derive a simple formula that directly relates curvature $w$ to the local speed of light, $w = -c^2 \Delta$. In contrast to the conventional formulation, this allows for a Machian interpretation of general relativity and the gravitational constant $G$. Gravity, though described by local equations, has its origin in all other masses in the universe.

1 Introduction: Einstein’s unrecognized masterstroke.

As a direct consequence of his equivalence principle of 1907, Einstein deduced that light rays both in accelerated and in gravitationally influenced frames must be curved. His immediate idea was that masses could influence the velocity of light in their vicinity, much as in optics a medium reduces the local speed of light. He assumed that:

‘... the velocity of light in the gravitational field is a function of the place, we may easily infer, by means of Huyghens’s principle, that light-rays propagated across a gravitational field undergo deflexion.’

Einstein realized that a variable speed of light must also influence the local units of time, and in a subsequent paper [3] in 1911 he stated:

Nothing forces us to assume [...] the clocks to be running at equal speed.’

This does not contradict the constancy of the speed of light with respect to Lorentz frames. With regard to special relativity, he explicitly stated [4] that

‘The principle of constancy of the speed of light can be upheld only when one restricts oneself to regions of constant gravitational potential.’

The gravitational potential will play an important role in the following considerations, being directly related to the speed of light. However, Einstein’s idea remained relatively unknown because of a series
of coincidences. First and foremost, Eddington’s spectacular eclipse observations of 1919 were seen as a confirmation of the 1915 form of general relativity. This overshadowed the fact that Einstein’s earlier attempts had a merit of their own. Then, because he had not considered length scales, Einstein derived a wrong value of 0.83 arc seconds with his variable speed of light theory. Coincidentally, another unsuccessful attempt, the ‘Entwurf theory’ developed in 1913 with Grossmann, predicted the same wrong value. Thus the variable speed of light attempt is easy to confound with that failed idea.

Einstein’s theory of 1911 was wrong in the sense that he did not consider a variation of length scales. This omission was not obvious prior to the 1919 eclipse results, which seemed to single out the geometric formulation of general relativity. It was not until 1957 that Einstein’s error was corrected in a paper by Robert Dicke, the Princeton astrophysicist who later became famed for his contribution to the CMB discovery. Dicke noted that when he included length scales, the variable speed of light formulation of general relativity was in agreement with the classical tests, giving an explicit formula for the dependence of $c$. Dicke therefore introduced a variable index of refraction \( \frac{c_0}{c} = 1 + \frac{2GM}{rc^2} = 1 + 2\alpha \) \( (1) \)

whereby \( c_0 \) is the velocity far from all masses. Given that the small right term depends on the Sun, Dicke discovered a relation with Mach’s principle: He suspected that the main part of the right term, 1, might have its origin in the remainder of the matter in the universe. The intriguing consequences for cosmology with respect to Dirac’s large number hypothesis, and Dicke’s further considerations are addressed in [6]. Although Dicke still had difficulties in reproducing the same results as he obtained for general relativity, the equivalence of both formulations was proven by [2]. It is worth mentioning that the notion of a variable speed of light is present in the conventional formalism of general relativity as well, usually denoted as ‘coordinate velocity.’ (For numerous references, see [2], ref.[70] herein.)

Since the considerable differences in the mathematical formalisms have obviously hindered the general acceptance of the variable speed of light formulation, we show in the following that Dicke’s idea can be constituted in a form much closer to Einstein’s classical equations. In 1912, Einstein [7] (eq. 5b) also suggested an analytic form resembling the Poisson equation, but in a quite different manner.

## 2 General relativity and variable speed of light

Before we modify Dicke’s idea, it is worth mentioning that other researchers have tried to incorporate Mach’s principle into a theory of gravitation. A couple of years earlier than Dicke, Sciama [8] derived a hypothesis on the gravitational constant,

\[
G = \frac{c^2}{\sum_i \frac{m_i}{r_i}}
\]

The coincidence was, however, taken as an approximation. How Sciama motivated his approach based on Mach’s principle is discussed in [6]. As early as 1925, Erwin Schrödinger made similar suggestions in a far-sighted paper [9].

### 2.1 Recovery of Newton’s law

We first show that that Dicke’s and Sciama’s idea are both embedded in the formula

\[
c^2 = \frac{c_0^2}{\sum_i \frac{m_i}{r_i}},
\]

\( c_0 \) being a constant velocity at an infinite distance. Since in \( \frac{c_0^2}{c^2} \approx 1 + 4\alpha \) holds. If one assumes the ‘1’ arising from a corresponding sum, Dicke’s suggestion is equivalent to a
Newtonian gravitational potential in the form

\[ \phi_{\text{Newton}} = \frac{1}{4}c^2. \]  

(4)

Newton’s law for the acceleration of a test mass at \( r = 0 \) can be recovered by

\[ \vec{a}(\vec{r}) = -\frac{1}{4} \nabla c(\vec{r})^2 = \frac{c_0^2}{4\Sigma} \sum_i \frac{\vec{r}_i}{r^2_i}, \]

(5)

where \( \Sigma \) is an abbreviation for \( \sum_i \frac{m_i}{r_i} \). When substituting \( c_0^2 \),

\[ a = \frac{c^2}{4\Sigma} \sum_i \frac{\vec{r}_i}{r^2_i} \]

follows, yielding the inverse-square law, while the gravitational constant is expressed by \( G = \frac{c^2}{4\Sigma} \).

Note that \( c_0 \) does not appear any more in (6), thus this is a form of Dicke’s idea where the Newtonian force is perceived in local, dynamic units.

### 2.2 The Einstein-Dicke variable speed of light theory in analytic form

We now present the Sciama-Dicke idea of a Machian law of gravity in a new analytic form. Making use of Einstein’s gravitational constant

\[ \kappa = \frac{8\pi G}{c^4}, \]

(7)

Sciama’s (modified) hypothesis

\[ G = \frac{c^4\kappa}{8\pi} = \frac{c^2}{4\Sigma} \sum_i \frac{m_i}{r_i}, \]

(8)

can be rewritten as

\[ \frac{1}{c^2} = \frac{\kappa}{2\pi} \sum_i \frac{m_i}{r_i}. \]

(9)

The corresponding integral form is

\[ \frac{1}{c^2} = \frac{\kappa}{2\pi} \int d\vec{r}' \frac{\rho}{|\vec{r}' - \vec{r}|}. \]

(10)

Taking the spatial Laplace operator on both sides yields

\[ \nabla^2 \frac{1}{c^2} = \frac{\kappa}{2\pi} \int d\vec{r}' \nabla^2 \frac{\rho}{|\vec{r}' - \vec{r}|}. \]

(11)

Using the fact that

\[ -\frac{1}{4\pi} \frac{\Delta}{|\vec{r}' - \vec{r}|} = \rho \delta^3(\vec{r}' - \vec{r}), \]

(12)

Equation (11) transforms to

\[ \Delta \frac{1}{c^2} = -2\kappa \rho. \]

(13)

It is interesting to compare this with Einstein’s 1912 proposal \[ \Delta c = \kappa \rho, \] considered ‘the simplest equation of this kind’ by Einstein. Evidently, he had seen the need to express his idea in a local form, though his solution is obviously not the only one. However, such a local formulation of the Sciama-Dicke hypothesis suggests further modifications that shed light on the Machian nature of the approach taken by Dicke.
2.3 The elimination of Newton’s constant $G$

Since neither Dicke’s nor Sciama’s approach has been built into a complete theory, (13) should not be considered as a rigorous consequence, but rather as a starting-point for what was behind Dicke’s and Sciama’s efforts, namely a Machian approach to gravity.

Einstein’s gravitational constant $\kappa = \frac{8\pi G c^4}{m}$ possesses an interesting property. When we look at its physical units $\frac{s^2}{mkg}$, there have been several theoretical approaches to redefine the unit of mass $kg$, which is, from a fundamental perspective, an unjustified quantity. Julian Barbour [10], for example, suggested that the unit of mass could as well be expressed as an inverse acceleration $\frac{s^2}{m}$, since it is the nature of an (inertial) mass to resist acceleration. If we do so, we imply the validity of Newton’s third law by definition. $\kappa$ therefore can be seen as a conversion factor that transforms the regular unit $kg$ into a Machian unit $\frac{s^2}{m}$.

This has further implications, such as the unit of energy becoming $\frac{m}{m^2 s^2}$ instead of $kg m^2 s^{-2}$.

Considering the preliminary form of (13), one may replace the mass density $\rho$ by an energy density $w$. Then, multiplying (13) by $c^2$, allows the form

$$-c^2 \Delta \frac{1}{c^2} = w,$$

(14)

$w$ being the energy density with units $\frac{1}{m^2}$, obviously related to the unit of the Riemannian curvature tensor. If we remember Dicke’s original idea, this yet more simple form is already an implementation of Mach’s principle. In (14), the gravitational constant $G$ has disappeared completely, as suggested by Dicke. So far we have considered the static case only. In general relativity, as in all other circumstances, the time derivative that corresponds to the Laplacian (and its extension, the Riemann tensor) must have an extra factor $\frac{1}{c^2}$. Thus without formal justification, one may assume that the relativistic generalization of (14) must have the form

$$\frac{\partial^2}{\partial t^2} \frac{1}{c^2} - c^2 \Delta \frac{1}{c^2} = w.$$

(15)

Despite being a nonlinear equation, the linear approximation of a locally constant $c$ is equivalent to a wave equation in empty space. A number of questions remain for further investigation. It will be interesting to see how (15) matches the gravitational wave predictions. Another issue is the role of the metric. In the conventional formalism, the speed of light $c$ enters the metric merely as a conversion factor, whereas here it seems that the metric itself is a function of $c$. If it can be shown that (15) is an equivalent description of the phenomenology of general relativity, it would be a strikingly simple form of Einstein’s theory.

3 Outlook

These purely heuristic considerations can only be seen as a first step toward incorporating Mach’s principle into a theory of gravity. Yet they constitute a novel way to link Dicke’s alternative theory of gravity of 1957 (not to be confused with the Brans-Dicke theory) with Einstein’s variable speed of light attempt in 1911. The content of (13) is certainly an unusual form of Ernst Mach’s idea of gravity having its origin in all other masses in the universe. The idea had always fascinated Einstein, though he never succeeded in incorporating it into general relativity. We have shown that his own 1911 idea, widely unknown in contemporary research, potentially offers an intriguing explanation of gravity.

The big epistomological difference between conventional and Machian gravity is that the latter allows for the elimination of a fundamental constant, Newton’s constant $G$. All revolutionary developments in physics have reduced the number of fundamental constants while expressing one constant in terms of others. Calculating $G$ by the mass distribution of the universe, as first suggested by Dicke, is therefore a radical idea that needs to be evaluated with care.

Not to be confused with the experimental efforts to redefine the kilogram.
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References

[1] A. Einstein. Über das Relativitätsprinzip und die daraus gezogenen Folgerungen. *Jahrbuch der Elektrizität und Elektronik*, 4, 1907.

[2] J. Broekaert. A Spatially-VSL Gravity Model with 1-PN Limit of GRT. *Foundations of Physics*, 38:409–435, May 2008.

[3] A. Einstein. Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. *Annalen der Physik*, 35(transl.: on the influence of gravitation on the propagation of light): 898–908, 1911.

[4] A. Einstein. Relativität und Gravitation. Erwiderung auf eine Bemerkung von M. Abraham. *Annalen der Physik*, 38:1062ff, 1912.

[5] R. H. Dicke. Gravitation without a principle of equivalence. *Review of Modern Physics*, 29(3):363–376, 1957.

[6] A. Unzicker. A Look at the Abandoned Contributions to Cosmology of Dirac, Sciama and Dicke. *Annalen der Physik*, 18:57.

[7] A. Einstein. Lichtgeschwindigkeit und Statik des Gravitationsfeldes. *Annalen der Physik*, 38:355ff, 1912.

[8] D. W. Sciama. On the origin of inertia. *Monthly Notices of the Royal Astronomical Society*, 113:34–42, 1953.

[9] E. Schrödinger. Die Erfüllbarkeit der Relativitätsforderung in der klassischen Mechanik. *Annalen der Physik*, 382:325–336, 1925.

[10] J. Barbour. *The End of Time*. Oxford University Press, 2000.