Higgs Production in Association with a Vector Boson Pair at Future $e^+e^-$ Colliders.

M. Baillargeon$^a$, F. Boudjema$^a$, F. Cuypers$^b$, E. Gabrielli$^c$ and B. Mele$^d$

$^a$ Laboratoire de Physique Théorique, ENSLAPP †
B.P.110, 74941 Annecy-Le-Vieux Cedex, France

$^b$ Sektion Physik der Universität München,
Theresienstraße 37, D-8000 Munich 2, Germany

$^c$ Theory Division, CERN, CH-1211 Geneva 23, Switzerland

$^d$ INFN, Sezione di Roma, Italy and
Dipartimento di Fisica, Università “La Sapienza”,
P.le Aldo Moro 2, I-00185 Rome, Italy

Abstract

We study Higgs boson production in association with a pair of electroweak vector bosons ($WW, ZZ, Z\gamma$) at future $e^+e^-$ colliders in the framework of the Standard Model. Total cross sections and distributions for the intermediate-mass Higgs are presented, with special emphasis on the Next Linear Collider (NLC) case operating at a centre-of-mass energy $\sqrt{s} \simeq 500$ GeV, where the cross sections turn out to be more favourable than in larger-$\sqrt{s}$ collisions. We find that with an integrated luminosity of 20 fb$^{-1}$ there is a sizeable event rate for the $HWW$ and $HZ\gamma$ (with a high $p_T^\gamma$) channels, while a larger integrated luminosity is needed to study the $HZZ$ production. We take into account various backgrounds, notably top-pair and triple vector-boson production, and show ways to significantly reduce their effects.

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* On leave from Laboratoire de Physique Nucléaire, Université de Montréal, C.P. 6128, Succ. A, Montréal, Québec, H3C 3J7, Canada.

† URA 14-36 du CNRS, associée à l’E.N.S. de Lyon et au LAPP d’Annecy-le-Vieux.
1 Introduction

The clarification of the mechanism for electroweak symmetry breaking is presently a basic issue for high energy physics. One way to attack this problem is to look for Higgs bosons that arise in the Standard Model, after spontaneously breaking the $SU(2) \times U(1)$ electroweak symmetry. Extensive studies have been carried out on the potential of present and future high energy colliders for discovering Higgs bosons predicted within and beyond the SM (for recent reviews see for instance [1]-[5]). In this respect, $e^+e^-$ colliders compared to hadron machines offer the advantage of producing Higgs bosons in a particularly clean environment. This is essential since the scalar Higgs, $H$, couples mainly to heavy particles, and consequently production cross sections are rather small. Furthermore, for $m_H \leq 135$ GeV, $H$ decays most of the time into the heaviest fermion pair allowed by phase-space ($H \to b\bar{b}$ for $m_H \gtrsim 10$ GeV). Hence, at hadron colliders, in this range of $m_H$, Higgs detection is made very difficult by the huge QCD backgrounds. Present limits on $m_H$ ($m_H \gtrsim 62.5$ GeV for a Standard Model Higgs [3]) derive from the lack of observation of a Higgs signal at LEP I. LEP II will be able to cover the range up to $m_H \lesssim M_Z$. A Higgs in the intermediate mass range $80$ GeV $\lesssim m_H \lesssim 140$ GeV could be observed at future hadron colliders (LHC/SSC) only through very dedicated (and costly) detectors [7]. Even in the optimistic case of detecting a signal at LHC/SSC, it would be impossible to study in detail the Higgs properties, such as its couplings, spin and parity characteristics.

On the other hand, an $e^+e^-$ linear collider with $\sqrt{s} \simeq (300-500)$ GeV and integrated luminosity $L \simeq 10-20$ fb$^{-1}$ (NLC) would be an ideal place to observe and study in detail an intermediate-mass or even a heavier Higgs [3]-[4]. At this machine, Higgs bosons would be produced mainly through the bremsstrahlung process $e^+e^- \to HZ$ and the $WW/ZZ$ fusion processes $e^+e^- \to H\nu\bar{\nu}$ and $e^+e^- \to He^+e^-$. By adding all these contributions, one gets a cross section larger than 100 fb for Higgs production in the intermediate-mass range in the Standard Model.

In this paper we consider another class of processes that are interesting for Higgs studies at future $e^+e^-$ colliders. We consider Higgs production in association with a pair of electroweak vector bosons

$$e^+e^- \to HWW$$

(1)

$$e^+e^- \to HZZ$$

(2)

$$e^+e^- \to HZ\gamma$$

(3)

The relevant tree-level Feynman diagrams are shown in Fig. 1. The $\gamma$ in the third channel
is a high-\(p_T\) observable photon. A Standard Model Higgs is assumed. Although of the same order in the electroweak coupling as the \(WW/ZZ\) fusion processes, the new channels are suppressed because of the narrower available phase space. Nevertheless, we will see that production rates for \(HWW\) and \(HZ\gamma\) can be non-negligible at the NLC.

Higgs production in association with two \(W\) or \(Z\) can provide further tests than those that may be probed in the fusion processes, on the \(HWW\) and \(HZZ\) couplings. Possible anomalous couplings in quadrilinear vertices, not present at tree-level in the SM such as \(HZWW\), \(H\gamma WW\), or even the C-violating \(HZZZ\) or \(H\gamma ZZ\) could be uniquely directly investigated in these processes as they would cause deviations from the predicted signal. Moreover, an accurate estimate of the channels (1)-(3) as well as some characteristic distributions is essential, as these processes could be potential backgrounds for possible new physics. For instance, it has been pointed out [8] that some \(ZZH\) events have the same signature as neutralino production when the latter decays into a Higgs or a vector boson.

The plan of this paper is the following. In section 2, we describe our procedures for computing total cross sections and distributions for the processes under study. We will present explicit and compact results for the \(HZ\gamma\) case and show how the matrix elements can be factorized in terms of the two-body reactions \(e^+e^- \rightarrow ZH\) times a photonic radiator or alternatively \(e^+e^- \rightarrow Z\gamma\) times a “Higgs radiator”. This Higgs radiator is also present in the other two reactions, and for \(ZZH\) production one can also write the result in a factorized form relating it directly to \(Z\) pair production. Section 3 is devoted to the results pertaining to total cross sections. We discuss the sensitivity of the cross sections to the Higgs mass and the variation with the centre-of-mass energy and compare with the other “standard” mechanisms of Higgs production. In section 4 we present and discuss some characteristic distributions. In section 5 we study in detail how these cross sections translate into numbers of events when we include the branching fractions and when we take into account various backgrounds. We show how the most dangerous backgrounds for \(WWH\) (and \(ZZH\)) production can be eliminated and how the production of three vector bosons \(WWZ, ZZZ, ZZ\gamma\) do not cause any serious problem. Some comments together with our conclusions are reported in section 6.

The processes in (1)-(3) have also been considered by Barger et al. in ref. [9]. We have checked that our cross sections agree with theirs. In contrast with the previous analysis, which concentrates on a Higgs not heavier than 50 GeV (already ruled out by LEP I), we cover here the case of the intermediate-mass Higgs and perform an extensive analysis of the backgrounds.
2 Description of the computation

In this section we describe the procedures adopted for evaluating the cross sections and distributions for the $HVV$ processes in (1)-(3).

The Feynman diagrams corresponding to $HVV$ processes at tree-level are shown in Fig. 1 in the unitary gauge (neglecting diagrams with direct Higgs-fermion couplings, which are suppressed by fermion masses). They can be obtained in a straightforward way by radiating a Higgs boson from every $W/Z$ external leg or propagator in the set of diagrams corresponding to the processes $e^+e^- \rightarrow WW, ZZ, Z\gamma$.

We have used two different and independent procedures for evaluating the corresponding matrix elements squared, both different from the one adopted in ref. [9]. In the first method we squared the amplitudes summing over initial and final polarization with the help of Schoonschip [10]. The output (which is a function of the five independent invariants of the particular process) was then integrated numerically in order to get various kinematical distributions and total cross sections. For this purpose, we used both a RGAUSS-like Fortran routine and a Vegas Monte Carlo [11] program to check the results.

The second way of obtaining the matrix elements squared relied exclusively on the computer program CompHep [12]. This software generates automatically the Feynman diagrams and then yields the matrix elements squared either in Reduce [13] or Fortran code. We then fed the output of CompHep into a Vegas-based Monte Carlo integration routine.

We do not show here the lengthy final expressions for the matrix elements squared $|M|^2$ for $HWW$ and $HZZ$. Instead, we make some comments on the explicit form of $|M|^2$ for $e^+e^- \rightarrow HZ\gamma$, which is rather compact and exhibits some interesting features.

The matrix element squared for the reaction $e^+e^- \rightarrow HZ\gamma$ can be cast into the form:

$$|M|^2 = G^2 \frac{1}{(q^2 - M_Z^2)^2} \left( \frac{A}{p_1 \cdot k} + \frac{B}{p_2 \cdot k} + \frac{C}{(p_1 \cdot k)(p_2 \cdot k)} \right),$$

(4)

where the momenta are defined as $e^-(p_1)e^+(p_2) \rightarrow H(h)Z(z)\gamma(k)$, with $q = h + z = p_1 + p_2 - k$ and $P = p_1 + p_2$ ($P^2 = 2p_1 \cdot p_2 = s$) and with

$$A = p_2 \cdot k + \frac{2}{M_Z^2} (p_2 \cdot z)(k \cdot z) \quad B = A(p_2 \rightarrow p_1) \quad C = s \left( \frac{s}{2} - k \cdot P \right) +$$
\[
\frac{2}{M_Z^2} \cdot \left( (p_1 \cdot z)(p_2 \cdot z)(s - k \cdot P) - \frac{s}{2} (P \cdot z)(k \cdot z) + (p_1 \cdot z)^2(p_2 \cdot k) + (p_2 \cdot z)^2(p_1 \cdot k) \right)
\]

The constant \( G \) in eq. (4) is defined as

\[
G^2 = 4\pi^3 \frac{M_Z^2}{s_W c_W^4} \alpha^3 (1 + 1 - 4s M_Z^2 \left( p_1 \cdot z \right) \left( p_2 \cdot z \right)) + \left( p_1 \cdot z \right)^2 \left( p_2 \cdot k \right) + \left( p_2 \cdot z \right)^2 \left( p_1 \cdot k \right).
\]

Equation (3) clearly displays the collinear and the soft-photon singularities. These are regularized by imposing a \( p_T \) cut on the photon such that \( p_T^\gamma > p_T^{cut} \), which leads to a description of the \( ZH\gamma \) production with an observable photon in the final state.

It is interesting to note that eq. (4) shows factorization in the limit of both soft photons and soft Higgses. The notion of a “soft Higgs” refers to a situation where the Higgs is massless and has a very small energy. Let us analyse in detail the two cases.

### 2.1 Factorization of the low-energy photons

We can readily recover the low-frequency collinear photons by only keeping the terms that are most singular in \( 1/|k_0| \) in eq. (4). These are obtained by letting \( k \to 0 \) in \( A, B \) and \( C \), which shows that only the cross-term \( C \) survives. Therefore, the contributing terms to the leading-log approximation are:

\[
|M|^2 \to G^2 \frac{1}{(q^2 - M_Z^2)^2} \frac{s^2}{2} \frac{1}{(p_1 \cdot k)(p_2 \cdot k)} \left( 1 + \frac{4}{s M_Z^2} (p_1 \cdot z)(p_2 \cdot z) \right),
\]

which agrees with the low-energy factorization:

\[
|M(e^+e^- \to HZ\gamma)|^2 \to |M(e^+e^- \to HZ)|^2 \left( -e^2 g_{\alpha\beta} \left( \frac{p_1^\alpha}{p_1 \cdot k} - \frac{p_2^\alpha}{p_2 \cdot k} \right) \left( \frac{p_1^\beta}{p_1 \cdot k} - \frac{p_2^\beta}{p_2 \cdot k} \right) \right)
\]

\[
= e^2 \frac{s}{(p_1 \cdot k)(p_2 \cdot k)} |M(e^+e^- \to HZ)|^2.
\]

Indeed we recognize the first term in the last expression to be the “photon radiator” and we check, by explicit calculation, that the two-body process \( e^+e^- \to HZ \) is given by

\[
|M(e^+e^- \to HZ)|^2 = \left( \frac{G}{e} \right)^2 \frac{1}{(q^2 - M_Z^2)^2} \frac{s}{2} \left( 1 + \frac{4}{s M_Z^2} (p_1 \cdot z)(p_2 \cdot z) \right).
\]

Note that, at the lowest order, the total integrated cross section for \( ZH \) production writes:
\[
\sigma(HZ) = \frac{\pi\alpha^2}{48s_W^4c_W^4} \left(1 + (1 - 4s_W^2)^2\right) \frac{M_Z^2}{(s - M_Z^2)^2} \beta \left(3 + \frac{s\beta^2}{4M_Z^2}\right),
\]  

(9)

where we have defined

\[
s\beta = \sqrt{(s - (M_Z + m_H)^2)(s - (M_Z - m_H)^2)}.
\]

(10)

For later reference, let us point out that for large momentum Higgs, the Z are produced with a predominantly longitudinal polarization and are hence essentially orthogonal to the beam. We recall that the angular distribution is written as

\[
\frac{d\sigma}{d\cos\theta} \propto (1 + \cos^2\theta) + \frac{s}{4M_Z^2} \left(1 + \frac{M_Z^2}{s} - \frac{m_H^2}{s}\right)^2 \sin^2\theta.
\]

(11)

The first term, \(1 + \cos^2\theta\), represents the transverse Z contribution, while the longitudinal one has an “enhanced” coupling: \(s/M_Z^2\).

### 2.2 Factorization in low-energy, low-momentum Higgses

One can also factorize out the “Higgs radiator” by going into the massless low-energy Higgs limit, although the present limit on the mass of the Higgs makes this a purely but interesting “academic exercise” (which can nonetheless be used as yet an additional check on our computations). The cross section in this case can be written as the product of the \(e^+e^- \to Z\gamma\) cross section times the “Higgs radiator”. One first verifies that the coefficients of \(1/M_Z^2\) in \(A\), \(B\) and \(C\) vanish in the limit \(h \to 0\), and that the remaining parts of \(A\), \(B\) and \(C\) add up to give the \(e^+e^- \to Z\gamma\) matrix element squared times a Higgs radiator, that is the propagator factor \(1/(q^2 - M_Z^2)^2\) modulo the \(HZZ\) coupling \(g_{HZZ}^2\):

\[
|M(e^+e^- \to HZ\gamma)|^2 \to \frac{g_{HZZ}^2}{(q^2 - M_Z^2)^2} |M(e^+e^- \to Z\gamma)|^2.
\]

(12)

In fact this factorization works also in the case of \(HZZ\) production. Denoting the external Z momenta by \(z_1\) and \(z_2\), the amplitudes may be written in the limit of a soft Higgs:

\[
\mathcal{M}(e^+e^- \to HZZ) \to \left(\frac{g_{HZZ}}{(z_1 + h)^2 - M_Z^2} + \frac{g_{HZZ}}{(z_2 + h)^2 - M_Z^2}\right) \mathcal{M}(e^+e^- \to ZZ).
\]

(13)
In the case of $W^+W^-H$ production, this factorization fails due to the emission by the “internal” $Z$ line. However we would like to point out, based on the analogy with the “backbone” reaction $e^+e^- \rightarrow W^+W^-$, that at threshold one expects a dominance of the neutrino-exchange diagrams. Indeed the $P$-wave nature of the $s$-channel $Z$ and $\gamma$ exchanges means that these are suppressed at threshold. We have checked this numerically. For instance, for a Higgs mass of 90 GeV, the approximate ($t$-channel) and total cross sections compare as follows

| $\sqrt{s}$ (GeV) | $t$-channel only (fb) | Total (fb) | Relative error |
|------------------|-----------------------|------------|----------------|
| 260              | 0.38                  | 0.37       | -3%            |
| 270              | 1.3                   | 1.2        | -8%            |
| 300              | 5.7                   | 4.2        | -26%           |

We see that within 10 GeV about the threshold, the agreement is better than 3%; however, already at 300 GeV the $s$-channel is badly required.

3 Total cross sections

In this section we study total cross sections for $HVV$ production as a function of the Higgs mass $m_H$ and the $e^+e^-$ centre-of-mass energy $\sqrt{s}$. We concentrate on an intermediate-mass Higgs and $0.3 \text{ TeV} \lesssim \sqrt{s} \lesssim 2 \text{ TeV}$. The cross sections are calculated with $M_Z = 91.18$ GeV, $M_W = 80.1$ GeV and we take an effective $\sin^2\theta_W = 0.232$. Moreover, apart from the case of $HZ\gamma$ where we take the electromagnetic coupling constant for the real photon at $q^2 = 0$ (i.e $\alpha(0) = 1/137$), we use $\alpha(M_Z^2) \simeq 1/128$.

Since, as we shall see below, at centre-of-mass energies around $\sqrt{s} \simeq 500$ GeV the production rates for $e^+e^- \rightarrow HWW, HZZ$ in the case of an intermediate-mass Higgs are the largest, we show in fig. 2 a comparison, at $\sqrt{s} = 500$ GeV, of our three processes with the main production channels of the Higgs, that is the bremsstrahlung process $e^+e^- \rightarrow HZ$ and the $WW/ZZ$ fusion processes $e^+e^- \rightarrow H\nu\nu$ and $e^+e^- \rightarrow He^+e^-$. The cuts $p_T^\gamma > 10$ GeV and $|y\gamma| < 2$ are imposed on the $HZ\gamma$ process. We can observe that for intermediate $m_H$ the rates for $HWW$ are comparable to those for $H$ production via $ZZ$ fusion. In particular, assuming an integrated luminosity of 20 fb$^{-1}$, one gets a sizeable (200-60) $HWW$ events (not including branching fractions) by varying $m_H$ in the range $(M_W - 2M_W)$. The $HZ\gamma$ channel is a bit lower than $HWW$ for intermediate $m_H$, but
exceeds it for larger Higgs masses. The $HZZ$ process has the lowest rate and will need higher integrated luminosity in order to be studied also in the intermediate-Higgs range. Although Higgs production through both the $WW$ fusion process and the Bjorken process is about an order of magnitude higher than through $HWW$ or $HZ\gamma$ production, the latter reactions are a welcome additional means of producing sizeable numbers of Higgs events. Even for Higgs masses up to $m_H \simeq (250-300)$ GeV one still expects a few $HWW$ raw events per year to be produced at the NLC. In the $HZZ$ case, a few events are still collected up to $m_H \simeq (150-200)$ GeV.

In Table I we give again total cross sections at $\sqrt{s} = 500$ GeV vs. $m_H$. Cross sections for the $HWW$ and $HZZ$ processes are also shown in figs. 3 and 4. In fig. 3 the $HWW$ cross section is plotted versus $\sqrt{s}$ for intermediate $m_H$. We can observe that the $HWW$ production is peaked for values of the centre-of-mass energy around $\sqrt{s} \sim 500$ GeV for $90 < m_H < 150$ GeV. In particular, $\sigma_{\text{MAX}} \simeq 9$ fb for $m_H = 90$ GeV, while at $\sqrt{s} \simeq 2$ TeV, production rates are about four times smaller for the same $m_H$. The same pattern holds for the $HZZ$ production (cf. fig. 4), but the $HZZ$ yield is about ten times smaller than that of $HWW$, for same $m_H$ and $\sqrt{s}$. This is mainly because the $Z$ has weaker couplings to the initial fermions than the $W$.

Cross sections for the $HZ\gamma$ channel are shown in figs. 5 and 6. In order to select high-$p_T$ observable photons we impose a cut of 10 GeV or more on the $\gamma$ transverse momentum. We also cut on the photon pseudorapidity imposing everywhere $|y\gamma| < 2$.

In fig. 5 we plot the total cross section versus $\sqrt{s}$ in the range $0.2 < \sqrt{s} < 3$ TeV, for $m_H = 90, 120, 150$ GeV and $p_T\gamma > 10$ GeV. Compared to $HWW$ and $HZZ$, cross sections for $HZ\gamma$ are peaked in a lower range of $\sqrt{s}$, due to the altogether lighter final state. For example, for $m_H = 90$ GeV $\sigma(HZ\gamma)$ has its peak at $\sqrt{s} \simeq 300$ GeV, where $\sigma(HZ\gamma) \simeq 10$ fb.

Figure 6 shows $\sigma(HZ\gamma)$ versus $m_H$ at $\sqrt{s} = 300$ GeV and 500 GeV. The effect of increasing the cut on $p_T\gamma$ is also shown in both cases. We can see that for $p_T\gamma > 10$ GeV, a lower-$\sqrt{s}$ machine can be better than the NLC with $\sqrt{s} \simeq 500$ GeV for studying $HZ\gamma$ with $m_H \simeq 150$ GeV. This is no longer true if one increases the cut on $p_T\gamma$ to about 40 GeV or more, since for small cuts on $p_T\gamma$, the behaviour of $\sigma(HZ\gamma)$ with $\sqrt{s}$ is closer to the one of $\sigma(HZ)$, which indeed decreases as $1/s$.

4 Characteristic distributions
| $m_H$ (GeV) | $\sigma(HWW)$ (fb) | $\sigma(HZZ)$ (fb) | $\sigma(HZ\gamma)$ (fb) |
|------------|---------------------|---------------------|------------------------|
| 60         | 15.2                | 1.43                | 5.08                   |
| 70         | 12.7                | 1.21                | 4.66                   |
| 80         | 10.7                | 1.03                | 4.33                   |
| 90         | 9.05                | 0.874               | 4.02                   |
| 100        | 7.71                | 0.749               | 3.74                   |
| 110        | 6.59                | 0.643               | 3.49                   |
| 120        | 5.63                | 0.551               | 3.25                   |
| 130        | 4.82                | 0.472               | 3.03                   |
| 140        | 4.12                | 0.403               | 2.82                   |
| 150        | 3.51                | 0.343               | 2.62                   |
| 160        | 2.98                | 0.291               | 2.42                   |
| 170        | 2.52                | 0.245               | 2.24                   |
| 180        | 2.12                | 0.204               | 2.07                   |
| 190        | 1.77                | 0.169               | 1.90                   |
| 200        | 1.47                | 0.138               | 1.74                   |
| 210        | 1.21                | 0.112               | 1.59                   |
| 220        | 0.978               | 0.0888              | 1.44                   |
| 230        | 0.783               | –                   | 1.30                   |
| 240        | 0.617               | –                   | 1.17                   |
| 250        | 0.476               | –                   | 1.05                   |
| 260        | 0.359               | –                   | 0.929                  |
| 270        | 0.262               | –                   | 0.818                  |
| 280        | 0.183               | –                   | 0.713                  |
| 290        | 0.121               | –                   | 0.616                  |
| 300        | 0.0737              | –                   | 0.525                  |

**Table I:** Total cross sections for $e^+e^- \rightarrow HWW, HZZ, HZ\gamma$ at $\sqrt{s} = 500$ GeV vs. $m_H$.

The cuts $p_T^{\gamma} > 10$ GeV and $|y^{\gamma}| < 2$ are applied to the final photon in the process $e^+e^- \rightarrow HZ\gamma$. 
4.1 \( e^+e^- \rightarrow HZ\gamma \)

The various distributions in this process can be easily understood when viewing the photon radiated from the electron/positron legs as a photon of a predominantly bremsstrahlung nature: the cross section is largest for the lowest-\( p_T \) photons independently of the mass of the Higgs. This is well rendered in fig. 7.1. This feature has an impact on the characteristics of the \( Z \) and \( H \) distributions, which are to be likened to those in the process \( e^+e^- \rightarrow ZH \). In fact the energies of both the \( Z \) and the \( H \) tend to peak around the beam energy, especially for small values of the \( p_T^\gamma \) cut as exemplified in figs. 7.2 and 7.4, where \( p_T^\gamma > 10 \) GeV. For higher values of this cut these spectra are slightly broader and away from \( \sqrt{s}/2 \) due to the reduction in the “effective” centre-of-mass energy of the process \( e^+e^- \rightarrow ZH \) as shown in figs. 7.3 and 7.5 for \( p_T^\gamma = 30 \) GeV. Anyhow, in all cases, the Higgs is emitted preferentially with a high \( p_T \) (see figs. 7.6 and 7.7). In fact the Higgs and the \( Z \) (since the photon has a low energy and prefers to be collinear to the beam most of the time) favour the central region, especially for low \( p_T^\gamma \) and smaller Higgs masses (figs. 7.8-7.11). This is again supported by the fact that the \( Z \) and the Higgs are almost back-to-back (see figs. 7.12 and 7.13). This feature of the Higgs takes its source from the process \( e^+e^- \rightarrow ZH \) and reflects the fact that the \( Z \), at high energies, is almost longitudinal. This particular distribution, as we will see below, is common to all three reactions that we studied. Probably more telling is the scatter plot, which confirms that the events cluster around \( E_H \approx E_{beam} \) while, at the same time, \( E_{\gamma} \) is essentially below 30 GeV (fig. 7.14). For \( m_H = 120, 150 \) GeV (\( p_T^\gamma > 10 \) GeV), the scatter plot is very similar to the one in fig. 7.14. These characteristics do not change significantly when we move to a higher centre-of-mass energy as depicted, for \( \sqrt{s} = 1 \) TeV in figs. 7.15 and 7.16.

4.2 \( e^+e^- \rightarrow HZZ \)

At 500 GeV the shape of the various distributions are not sensibly different for our three choices of the intermediate-Higgs mass, \( m_H = 90, 120, 150 \) GeV, apart from the distribution in the energy of the Higgs, \( E_H \). First, the transverse momentum of the Higgs (see fig. 8.1) is broadly peaked around 100 GeV, which is about half the value allowed by the kinematics. Once again the Higgs is produced centrally (fig. 8.2). It is preferentially within about 30° from the plane orthogonal to the beam direction. These two facts explain the shape of the distribution in the energy of the Higgs. Given \( p_T^H \), we essentially have \( E_H \approx \sqrt{(p_T^H)^2 + m_H^2} \). With respect to the most energetic \( Z \), the Higgs is emitted preferentially in the opposite direction, in fact the two particles are almost back to back.
as shown in fig. 8.3 where we see that this spectrum has a smooth hump around a value \(\sim 160^\circ\). On the other hand, with respect to the least energetic Z, the Higgs is rather orthogonal (fig. 8.4). A very useful and revealing distribution is exhibited as a scatter plot in fig. 8.5 (for \(m_H = 120, 150\) GeV, the plot is very similar to the one for \(m_H = 90\) GeV). This plot shows that the most favourable situation is when both the Higgs and the most energetic Z are orthogonal to the beam direction. These features are all to be compared with the previous reaction, \(e^+e^- \rightarrow HZ\gamma\). One can liken the rôle of the least energetic Z to that played by the photon, the Z mass providing in a sense a natural “energy” cut. Then, while we expect the least energetic Z to be emitted off the electron line, the most energetic Z is preferentially produced as a longitudinal Z together with the Higgs, i.e., it is radiated by the \(Z^*ZH\) vertex. Of course, since the polarization vector of a longitudinal energetic Z introduces the enhancement factor \(E_Z/M_Z\), and the Z current in \(Z^*ZH\) is not conserved (or rather not transverse), we can understand the fact that the \(ZZH\) cross section is largest when it is the most energetic Z that is emitted from the \(ZZH\) vertex. Since this is a situation akin to the Bjorken process, one can also understand that the energetic Z favours the central region.

### 4.3 \(e^+e^- \rightarrow HW^+W^-\)

A comparison among the shapes of the distributions in the transverse momentum, the angle with respect to the beam, and the energy of the Higgs in the reaction \(e^+e^- \rightarrow HW^+W^-\) and \(e^+e^- \rightarrow HZZ\), shows that these are almost identical for all three representative values of the Higgs mass \(m_H = 90, 120, 150\) GeV (figs. 9.1-9.3). This a reflection that, modulo the strengths of the couplings of the W and the Z to the electron and Higgs, these distributions are dictated by the behaviour of the \(t\)-channel. For \(WWH\) this is a behaviour similar to the one exhibited by the \(t\)-channel neutrino-exchange diagram in \(e^+e^- \rightarrow W^+W^-\). After all, \(e^+e^- \rightarrow W^+W^-H\) is directly related to \(WW\) by “grafting” a Higgs. This observation is further supported by the fact that \(W^-\) favours being emitted in the forward direction, the electron direction, as fig. 9.4 shows. In this case, as the scatter plot in fig. 9.5 shows, the \(W^-\) tends to take the maximum kinematically allowed energy.

Figure 9.6 shows that the Higgs prefers to be in the direction perpendicular to the beam, while at the same time the \(W^-\) is almost exclusively in the forward hemisphere,

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\(^1\)The shape of the angular distribution in \(W^+W^-\) production reflects essentially the \(t\)-channel exchange.
\[ \theta_{e^-W^-} < 90^\circ. \] The \( W^- \) distribution around the electron direction is a reflection of the fact that half of the time it is emitted longitudinally off the Higgs (hence it would rather be perpendicular to the beam) and half of the time (when the Higgs is emitted by the \( W^+ \)) the \( W^- \) is produced in the very forward region, as in \( e^+e^- \rightarrow W^+W^- \).

These features remain essentially unaltered for the three values of \( m_H \) that we considered.

## 5 Signatures and backgrounds

The intermediate-mass Higgs that we consider will decay predominantly into \( b\bar{b} \). All the processes we have studied will therefore consist of at least one pair of \( b \) quarks with a high transverse momentum as we saw in the previous section; \( b \) tagging with a good vertex detector will be very helpful. We note that the \( b \) pair should be reconstructed with an invariant mass around\( m_H \) the Higgs mass, which should be well measured in the fusion or/and the Bjorken process. The distribution of the \( b \) quark is isotropic in the rest frame of the Higgs, contrary, for instance, to \( b \) quarks emanating from a \( Z \), which are distributed mainly\(^3\) according to \( (1 + \cos^2 \theta^*) \), where \( \theta^* \) is the angle measured in the \( Z \) rest frame between the decaying \( b \) and the axis corresponding to the \( Z \) flight direction. Therefore, in principle, reconstruction of these distributions could help in Higgs detection. However, one needs a large enough sample of \( b \) in order to reconstruct these angular distributions, so that for the reactions we consider it will suffice to tag the \( b \). One type of potential background (especially for \( m_H \sim M_Z \)) to the reactions we study are precisely those where an \( H \) is replaced by a \( Z \), namely \( e^+e^- \rightarrow WWZ, ZZZ, ZZ\gamma \), with one of the \( Z \) decaying subsequently into \( b\bar{b} \). At centre of mass of 500 GeV, the cross sections for these three reactions are \(^{[14]} \)

\[
\begin{align*}
\sigma(e^+e^- \rightarrow WWZ) &= 39 \text{ fb with } m_H < 2M_W \\
\sigma(e^+e^- \rightarrow ZZZ) &= 1 \text{ fb with } m_H < 2M_W \\
\sigma(e^+e^- \rightarrow ZZ\gamma) &= 15 \text{ fb for } p_T^\gamma > 20 \text{ GeV , } |y^\gamma| < 2
\end{align*}
\]

As the \( Z \) branching ratio into \( b \)'s is \( \sim 15\% \), \( b \) tagging will help considerably since otherwise one has to consider a \( Z \) into jets branching ratio of about \( 70\% \). Hence, \( b \) tagging reduces

\(^2\)Allowing for the experimental resolution.

\(^3\)Standard \( Z \)'s at these energies are essentially transverse in processes not involving the Higgs.
these eventual $Z$-initiated backgrounds by more than a factor 4. By taking any of the $Z$ into $b\bar{b}$, while the other weak bosons can decay into anything, one gets

\[
\begin{align*}
\sigma(e^+e^- \rightarrow WWZ \rightarrow WWb\bar{b}) &= 5.85 \text{ fb with } m_{H} < 2M_{W} \\
\sigma(e^+e^- \rightarrow ZZZ \rightarrow ZZb\bar{b}) &= 0.45 \text{ fb with } m_{H} < 2M_{W} \\
\sigma(e^+e^- \rightarrow ZZ\gamma \rightarrow Z\gamma b\bar{b}) &= 4.5 \text{ fb for } p_t^\gamma > 20 \text{ GeV, } |y^\gamma| < 2
\end{align*}
\] (15)

These backgrounds are most serious for $m_{H} \sim M_{Z}$. However, in this case, the $H$ branching ratio into $b\bar{b}$ is about 85%. At $\sqrt{s} = 500 \text{ GeV}$, we find for $m_{H} = M_{Z}$, $\sigma(WWH \rightarrow WWb\bar{b}) \simeq 9 \times 85% \simeq 7.7 \text{ fb}$ and $\sigma(ZZH \rightarrow ZZb\bar{b}) \simeq 0.87 \times 85% \simeq 0.74 \text{ fb}$ which are sensibly above the three-vector-boson background. On the other hand, $\sigma(Z\gamma H \rightarrow Z\gamma b\bar{b}) \simeq 3.2 \times 85% \simeq 2.7 \text{ fb}$ for $p_t^\gamma > 20 \text{ GeV}$, which is below, but comparable, to the corresponding three-vector-boson background. Therefore the Higgs signal can easily be disentangled from triple vector-boson production. For larger Higgs masses, both the Higgs cross sections and $Br(H \rightarrow b\bar{b})$ drop. For instance, for $m_{H} = 120 \text{ GeV}$, one gets $\sigma(WWH \rightarrow WWb\bar{b}) \simeq 5.6 \times 70% \simeq 3.9 \text{ fb}$, $\sigma(ZZH \rightarrow ZZb\bar{b}) \simeq 0.55 \times 70% = 0.39 \text{ fb}$ and $\sigma(Z\gamma H \rightarrow Z\gamma b\bar{b}) \simeq 2.5 \times 70% = 1.7 \text{ fb}$. If $m_{H} = 150 \text{ GeV}$, one collects “only” $\sigma(WWH \rightarrow WWb\bar{b}) \simeq 3.5 \times 50% \simeq 1.75 \text{ fb}$, $\sigma(ZZH \rightarrow ZZb\bar{b}) \simeq 0.34 \times 50% = 0.17 \text{ fb}$ and $\sigma(Z\gamma H \rightarrow Z\gamma b\bar{b}) \simeq 1.9 \times 50% = 1 \text{ fb}$. However, although the signal for $m_{H} = 150 \text{ GeV}$ is about three to four times smaller than the corresponding three vector-boson background, the invariant mass of the $b\bar{b}$ system is such that it should not be mistaken as coming from the $Z$, hence almost eliminating this background. Therefore, the three vector-boson production does not seem to pose any serious problem for $V VH$ detection.

In fact a “huge” background to $WWH$ detection comes from top-pair production with the top decaying exclusively into $Wb$, leading to a topology $W^+W^-b\bar{b}$. The cross section for this process at $\sqrt{s} = 500 \text{ GeV}$ is $\sim 660 \text{ fb}$ for $m_t = 150 \text{ GeV}$. This is about two orders of magnitude above the signal. To study and show ways to reduce this background, we will take the representative value of $m_t = 150 \text{ GeV}$. The following discussion does not change much for other values of $m_t$ favoured by LEP I data. Even in this situation, $b$ tagging is crucial. In order of reducing the $t\bar{t}$ background, one can impose a cut on the invariant mass of the $b\bar{b}$ system, $m_{b\bar{b}}$, within 10 GeV of the Higgs mass. A Pythia-based simulation of $t\bar{t}$ events, with subsequent decays of top into $b\bar{b}$ reveals that the $m_{b\bar{b}}$ distribution shows a broad hump around values corresponding to an intermediate $m_{H}$. For both $M_{Z} - 10 \text{ GeV} < m_{b\bar{b}} < M_{Z} + 10 \text{ GeV}$ and $110 \text{ GeV} < m_{b\bar{b}} < 130 \text{ GeV}$ (relevant

\footnote{The full spin-correlations are kept. We are grateful to G. Azuelos for running Pythia for us.}
for $m_H = 120$ GeV) we find that there are still 10% of the $t\bar{t}$ events that pass this cut. This still amounts to a cross section of $\sim 62$ fb. We found that a much more efficient and simple selection criterion was to reject all the Higgs events in $WWH$ that simulate $t\bar{t}$ when the invariant mass of both the tri-jet (in our case $Wb$) system falls within $\pm 15$ GeV of the top mass. Once again, to reduce as much as possible the error in assigning the jet to its parent particle, $b$-tagging will be extremely useful. This is because, out of the six jets, the $W$ is experimentally reconstructed by only “pairing” the 4 non-$b$ jets, so that each pair recombines to give the $W$ mass. We would then recombine the $Wb$ system to give the top. As we want to exploit a good vertex detector for $b$-tagging without charge identification, and since by using the hadronic decays of both $W$ it would be extremely difficult to reconstruct their charges anyway, we tried in our program both combinations of $Wb$ to reconstruct the top. To perform this analysis, we included the $H$ decay into $b\bar{b}$ by first taking an isotropic distribution in the Higgs rest frame then boosting the events in the laboratory frame. We then demand that all $WWH \rightarrow WWb"b'"$ events passing the simultaneous cuts

$$m_t - 15 \text{ GeV} < M_{W+b} < m_t + 15 \text{ GeV} \quad \text{and} \quad m_t - 15 \text{ GeV} < M_{W-b'} < m_t + 15 \text{ GeV}$$

or

$$m_t - 15 \text{ GeV} < M_{W+b'} < m_t + 15 \text{ GeV} \quad \text{and} \quad m_t - 15 \text{ GeV} < M_{W-b} < m_t + 15 \text{ GeV}$$

(16)

not be counted as a $WWH$ signal. We find that the number of events is practically unaltered by this cut (the loss is about 3%):

| $m_H$(GeV) | $\sigma(WWH)$ (fb) before cut | $\sigma(WWH)$ (fb) after cut | Percentage loss |
|------------|-------------------------------|-------------------------------|----------------|
| 90         | 9.05                          | 8.77                          | 3.1%           |
| 120        | 5.63                          | 5.45                          | 3.2%           |
| 150        | 3.51                          | 3.37                          | 4.0%           |

We should add that this method should also work when one of the $W$ decays leptonically, as there are enough constraints to reconstruct the neutrinos and hence both invariant masses. Let us point out that a reconstruction of $t\bar{t}$ events away from threshold,

---

5This is because the charge analysis is necessarily done with a much reduced, $\sim 10\%$, sample of $b$ relying on the high-$p_T$ lepton from the semileptonic decay of the $b$ and would entail a considerable loss in our signal of Higgs.
based on the 3-jet clustering, was conducted in [15]. It shows that a tail remains. But this tail is mainly due to misassigned jets. This combinatorial error, as we noted above, will be much reduced if a preliminary identification of the two $b$ jets is done. In practice, this tail will also be further reduced by imposing our first cut on the invariant mass of the $b\bar{b}$ system, which cuts the $t\bar{t}$ by an order of magnitude. We conclude that $t\bar{t}$ is not a problem.

With at least one $W$ decaying into jets, and not taking into account decays into $\tau$'s, the useful combined branching fraction of the $WW$ is as large as 77%. After the cut on the $t\bar{t}$ “misidentification”, one gets a clean number of events of reconstructible $WWH$ at $\sqrt{s} = 500$ GeV. Assuming an integrated luminosity of $\mathcal{L} = 20$ fb$^{-1}$, one collects:

\begin{align}
N_H &\sim 115 \quad \text{for} \quad m_H = 90 \text{ GeV} \\
N_H &\sim 60 \quad \text{for} \quad m_H = 120 \text{ GeV} \\
N_H &\sim 26 \quad \text{for} \quad m_H = 150 \text{ GeV} 
\end{align}

(17)

We see that we have a healthy number of events and even if we take an overall efficiency of 50%, it is still observable for $m_H$ as large as 150 GeV.

For the $ZZH$ signal with $m_H = 90$ GeV one has $\sigma(ZZH \rightarrow ZZb\bar{b}) \simeq 0.74$ fb, which is above the corresponding $3Z$ background. Allowing both $Z$ to decay into non-$b$ jets, once again we will have a large background due to top-pair production, with both $W$’s decaying hadronically, as it will be difficult to disentangle a dijet invariant mass clustering around $M_W$ from one clustering around $M_Z$. In principle, we should apply the same 3-jet veto as for the $WWH$ production to cut the $t\bar{t}$ background. However, the signature (when the jet from the $Z$ is not “tagged” as a $b$ quark) is the same as for $WWH$, with the $W$ decaying into hadrons. Therefore we suggest that, for this particular signature, we should just add the $ZZH$ events to those from $WWH$ since these few $ZZH$ events represent about a tenth of the similar $WWH$ events. We do not attempt to find criteria to disentangle these $ZZH$ events from the $WWH$ ones because, for both processes, the distributions in the variables of the weak vector bosons and the Higgs are very similar (see section 4). Moreover, even for $m_H \sim 90$ GeV, taking only the hadronic decays not containing $b$ quarks for both $Z$ only amounts to about 5 events with $\mathcal{L} = 20$ fb$^{-1}$. Events that would not be mistaken as coming from $WWH$ arise from only one $Z$ decaying into jets while the other decays into leptons (in this case mostly neutrinos, i.e. large transverse missing energy). This corresponds to a combined branching fraction $B_{\text{comb.}} \sim B(H \rightarrow b\bar{b}) \times 0.36$
Alternatively one could demand that one Z decays into b’s while the other is allowed to decay into anything corresponding to $B_{\text{comb.}} \sim B(H \to b\bar{b}) \times 0.23$. In this situation we will have four b jets. Taking into account both signatures we end up with an almost background-free branching ratio of $B_{\text{comb.}} \approx B(H \to b\bar{b}) \times 0.59$. Assuming an integrated luminosity of 20 fb$^{-1}$ one would collect $\sim 10$ Higgs events, through $ZZH$ production for a Higgs mass of 90 GeV and about 6 for a mass of 120 GeV at $\sqrt{s} = 500$ GeV. At 1TeV taking a luminosity of 60 fb$^{-1}$ these rates are 14 and 11 events respectively.

The $HZ\gamma$ production does not suffer from the top pair production background. One should however still insist on b tagging. In fact, due to the large Z branching ratio into jets, the bulk of the $HZ\gamma$ events will consist of four jets and a photon. This is the same signature as the radiative $WW\gamma$ production process, i.e., $e^+e^- \to WW\gamma$ with both W decaying into jets. At $\sqrt{s} = 500$ GeV and with $p_T^\gamma > 20$ GeV and $|y| < 2$, this process has a cross section of 66 fb [14] (after folding with the branching ratios for $W \to jj$). This background is more important when $m_H \sim M_Z$, but again it should be under control after tagging the b-jets from H. The only potential background left when b-tagging is effective is due to $ZZ\gamma$ production, with one Z decaying into b quarks, especially when $m_H \sim M_Z$. Imposing the cuts $p_T^\gamma > 20$ GeV and $|y^\gamma| < 2$, and allowing the second Z from $ZZ\gamma$ to decay into anything (in the case of $\nu\bar{\nu}$, one will require a large missing $p_T$), one gets about 90 events at $\sqrt{s} = 500$ GeV with an integrated luminosity of 20 fb$^{-1}$. This is to be contrasted with the $HZ\gamma$ signal which, for a Z decaying into anything will produce about 53 events for $m_H = 90$ GeV, 34 for $m_H = 120$ GeV and 18 for $m_H = 150$ GeV. Hence, for $m_H \sim M_Z$, when this background is more dangerous, the signal clearly stands out. For this value of the Higgs mass, the ratio of signal over background is $S/B \approx 0.6$.

6 Conclusions

One of the primary motivations for the construction of a linear $e^+e^-$ collider with $\sqrt{s} \sim 500$ GeV is the production and the study of the properties of the Higgs with an intermediate mass ($M_W \approx m_H \approx 2M_W$). Such a Higgs will be difficult to detect at the planned $pp$ machines. In this paper we have investigated new mechanisms for the production of the Higgs in $e^+e^-$ collisions, namely the associated production of the Higgs with a pair of vector bosons, taking advantage of the large $WW$ and $Z\gamma$ cross sections to which we have “grafted” a Higgs.

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6 In all cases we have not included $Z \to \tau\tau$.

7 We have not double counted the $llb\bar{b}$, present in the previous sample.
We find that although Higgs production through $WWH$ and $ZH\gamma$ are about an order of magnitude smaller than for the main Higgs production mechanisms through $WW$ fusion or $ZH$ production, the number of events one collects with an integrated luminosity of $\mathcal{L} = 20 \, \text{fb}^{-1}$ at a centre-of-mass energy of 500 GeV is quite substantial. We have shown how some processes, which can at first be considered as serious backgrounds (like top pair production and triple vector-boson productions), can be efficiently eliminated, especially by requiring $b$-tagging. Leaving aside the issue of detection efficiencies and systematics, which can only be reliably estimated with a proper detector simulation, but taking into account the observable decays of the final particles, we find that one can have about 120 $WWH$ events for $m_H \simeq 90$ GeV and about 60 for a Higgs mass of 120 GeV, not including those events which may simulate some backgrounds. The $ZH\gamma$ process also provides another 50 ($m_H = 90$ GeV) or 30 ($m_H = 120$ GeV) events for $p_T^\gamma > 20$ GeV, or even more for lower cuts. The $ZZH$ process is unfortunately an order of magnitude smaller than the $WWH$.

These cross sections are largest at a centre-of-mass energy around 500 GeV for $WWH$ and $ZZH$, almost independently of $m_H$ in the intermediate region. Although they fall off with energy the decrease is not so drastic. For instance, at 1 TeV they are about two times smaller for $m_H \simeq 90$ GeV and are comparable for $m_H \simeq 150$ GeV. This will be largely compensated for by the increase in integrated luminosity, since we contemplate at this energy a value of about 60 fb$^{-1}$. This means that at $\sqrt{s} = 1$ TeV we will collect an even larger “healthy” number of Higgs events than at 500 GeV. For example, $HWW$ will provide a welcome 200 ($m_H = 90$ GeV) to 130 ($m_H = 120$ GeV) additional Higgs events to those produced in the conventional processes at 1 TeV.

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Figure captions

Fig. 1 Feynman diagrams contributing to the processes $e^+e^- \rightarrow HW^+W^- (1a)$, $HZZ (1b)$, $HZ\gamma (1c)$.

Fig. 2 Comparison among the different main modes of single Higgs production at a 500 GeV $e^+e^-$ machine, as a function of the Higgs mass. For $HZ\gamma$ we impose $p_T^\gamma > 10$ GeV and $|y^\gamma| < 2$.

Fig. 3 Cross section for $e^+e^- \rightarrow HW^+W^-$ as a function of the centre-of-mass energy for three values of $m_H = 90, 120, 150$ GeV (the largest cross sections correspond to the smallest mass).

Fig. 4 As in the previous figure, but for $e^+e^- \rightarrow HZZ$.

Fig. 5 Cross section for $e^+e^- \rightarrow HZ\gamma$ as a function of the centre-of-mass energy for the three values of $m_H = 90, 120, 150$ GeV including the cuts $p_T^\gamma > 10$ GeV and $|y^\gamma| < 2$ (the largest cross sections correspond to the smallest mass.)

Fig. 6 $e^+e^- \rightarrow HZ\gamma$ versus the Higgs mass at $\sqrt{s} = 300$ GeV and 500 GeV, with the effect of different $p_T^\gamma$ cuts, but keeping $|y^\gamma| < 2$.

Fig. 7.1 Distribution in the transverse momentum of the photon in the reaction $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV, for three values of the Higgs mass. The cut $|y^\gamma| < 2$ is applied.

Fig. 7.2 Distribution in the energy of the $Z$ for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV. The three curves correspond to three values of the Higgs mass as labelled in the previous graph.

Fig. 7.3 As in Fig. 7.2, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.4 Distribution in the energy of the Higgs for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV.

Fig. 7.5 As in Fig. 7.4, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.6 Distribution in the transverse momentum of the Higgs for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV.

Fig. 7.7 As in Fig. 7.6, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.8 Distribution in the angle between the beam and the direction of the Higgs for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV.

Fig. 7.9 As in Fig. 7.8, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.10 Distribution in the angle between the beam and the direction of the $Z$ for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV.

Fig. 7.11 As in Fig. 7.10, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.12 Distribution in the angle between the $Z$ and the Higgs for $p_T^\gamma > 10$ GeV in $e^+e^- \rightarrow HZ\gamma$ at $\sqrt{s} = 500$ GeV.
Fig. 7.13 As in Fig. 7.12, but with a cut $p_T^\gamma > 30$ GeV.

Fig. 7.14 Scatter plot in the energy of the Higgs $E_H$ versus the energy of the photon for $p_T^\gamma > 10$ GeV and $m_H = 90$ GeV at $\sqrt{s} = 500$ GeV.

Fig. 7.15 Scatter plot in the energy of the Higgs $E_H$ versus the energy of the photon for $p_T^\gamma > 10$ GeV and $m_H = 90$ GeV at $\sqrt{s} = 1$ TeV.

Fig. 7.16 As previously, but with $m_H = 150$ GeV and $p_T^\gamma > 30$ GeV.

Fig. 8.1 Distribution in the transverse momentum of the Higgs in the process $e^+e^- \rightarrow ZZH$ at $\sqrt{s} = 500$ GeV.

Fig. 8.2 Distribution in the angle between the beam and the Higgs in the reaction $e^+e^- \rightarrow ZZH$ at $\sqrt{s} = 500$ GeV.

Fig. 8.3 As in Fig. 8.2 for the distribution in the angle between the Higgs and the most energetic of the two $Z$.

Fig. 8.4 As in Fig. 8.3, but with respect to the least energetic $Z$.

Fig. 8.5 Scatter plot in the angle between the Higgs and the beam versus the angle between the most energetic $Z$ and the beam in $e^+e^- \rightarrow ZZH$ at $\sqrt{s} = 500$ GeV for $m_H = 90$ GeV.

Fig. 9.1 Distribution in the transverse momentum of the Higgs in the process $e^+e^- \rightarrow W^+W^-H$ at $\sqrt{s} = 500$ GeV.

Fig. 9.2 Distribution in the energy of the Higgs in the process $e^+e^- \rightarrow W^+W^-H$ at $\sqrt{s} = 500$ GeV.

Fig. 9.3 Distribution in the angle between the beam and the Higgs in the reaction $e^+e^- \rightarrow W^+W^-H$ at $\sqrt{s} = 500$ GeV.

Fig. 9.4 As in Fig. 9.3, but concerning the angle between the electron and $W^-$. 

Fig. 9.5 Scatter plot in the $W^-$ energy versus the angle between the electron and the $W^-$ for $m_H = 90$ GeV in $e^+e^- \rightarrow W^+W^-H$ at $\sqrt{s} = 500$ GeV.

Fig. 9.6 Scatter plot in the angle between the beam and the Higgs versus the angle between the electron and the $W^-$ for $m_H = 90$ GeV in $e^+e^- \rightarrow W^+W^-H$ at $\sqrt{s} = 500$ GeV.