Active Microrheology in Active Matter Systems: Mobility, Intermittency and Avalanches

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We examine the mobility and velocity fluctuations of a driven particle moving through an active matter bath of self-mobile disks for varied system densities and activities. The driven particle mobility is strongly non-monotonic and is correlated with distinct spatial-temporal structures that arise in the active media. We identify an activity-induced crystallization regime that is distinct from the higher activity-induced phase-separated cluster regime. The distributions of the velocity fluctuations of the probe particle exhibit specific features in the different dynamic regimes. In the cluster phase, we observe telegraph noise, while in the denser active jamming regimes, the probe particle moves in intermittent jumps or avalanches of sizes that are power-law distributed.

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I. INTRODUCTION

In active microrheology, the properties of a medium are probed by examining the mobility and velocity fluctuations of an externally driven probe particle that is on the order of the same size as the particles that comprise the medium. For example, the nonlinear mobility of a probe particle dragged through a colloidal system was shown to change across the glass transition. Other studies have examined anomalous diffusion properties, features of the velocity fluctuations of the driven particle, and threshold to motion phenomena. In ordered systems, a driven particle can create a localized melting region or shear thinning effects. Driven particles or intruders have also been used in granular media, where the particle mobility is strongly reduced, its motion becomes increasingly intermittent, and the velocity fluctuations become power-law distributed as the jamming transition is approached.

Another class of system to which active microrheology can be applied is active matter or assemblies of self-mobile particles, such as run-and-tumble bacteria or self-mobile colloidal particles. The most common models of active systems are self-mobile sterically interacting disks that undergo either active Brownian motion or run-and-tumble dynamics. These have been shown to exhibit a transition from a uniform liquid state at low activities and densities to a cluster or phase separated state at higher activities and densities, where close-packed clusters form surrounded by a low density gas. For monodisperse disks confined to two dimensions, the particles within the clusters have a triangular ordering, leading to the term “living crystals” since the crystallites move through the system, break up, and reform. A driven probe particle moving through an active matter system should show clear changes in mobility or velocity fluctuations depending on the spatio-temporal behavior exhibited by the active matter media, and thus could serve as a powerful tool for understanding a wide range of active systems. In numerical and theoretical studies on the driven dynamics of a probe particle moving through an active nematic system, anomalous viscosity effects such as a negative drag were predicted. Experiments on cargo transport through crowded living cells showed that motion occurs in intermittent bursts which exhibit scaling behavior similar to that found in critical jammed solids.

Here we examine the dynamics of a driven probe particle as it moves through a bath of run-and-tumble sterically interacting disks for varied activity or run length and density. We show that the probe mobility and velocity fluctuations are correlated with distinct dynamic spatial structures formed in the system. At low densities the probe mobility decreases with increasing run length, and the transition from a uniform active liquid to a living crystal state coincides with a pronounced mobility drop. At high densities the mobility becomes increasingly non-monotonic as a function of the activity. Initially the mobility decreases with activity due to activity-induced formation of a uniform triangular lattice; however, for a further increase in the activity, the system melts into a liquid state and the mobility increases. At large run lengths the system forms an active jammed state where the mobility is strongly reduced. These different states can also be identified by measuring the probe velocity fluctuation distributions, which have a two-state or telegraph noise character in the cluster phase. At higher densities and long run lengths where the system forms an active jammed state, the probe velocity often drops to zero and the probe moves only in sharp intermittent bursts or avalanches with a jump size distribution that can be fit to a power law. These results suggest that the activity can induce critical behavior of the type associated with jamming at densities significantly below the non-active jamming density.

II. SIMULATION AND SYSTEM

We consider a two-dimensional system of size $L \times L$ with periodic boundary conditions containing $N$ run-
and-tumble monodisperse disks of radius \( r_d \). The disks interact via a repulsive harmonic potential and have a density \( \phi = N \pi r_d^2 / L^2 \). In the absence of activity, a hexagonal solid forms when \( \phi > 0.9 \). The dynamics of a single disk or particle \( i \) is obtained by integrating the overdamped equation of motion

\[
\eta \frac{d \mathbf{R}_i}{dt} = \mathbf{F}_i^m + \mathbf{F}_i^s.
\]

Here \( \eta = 1.0 \) is the damping constant and \( \mathbf{F}_i^m \) is the motor force, which drives the particle in a fixed randomly chosen direction under a force \( F^m \) during a run time \( \tau_r \). After the run time has elapsed, the particle moves in a new randomly chosen direction for the next run time. A convenient measure is the run length \( R_l \equiv F_m \tau_r \), the distance the particle would move in the absence of other particles. The steric disk-disk interactions are given by

\[
\mathbf{F}_i = \sum_{i,j}^N (k/r_d) (R_{\text{eff}} - |\mathbf{r}_{ij}|) \theta (R_{\text{eff}} - |\mathbf{r}_{ij}|) \mathbf{r}_{ij},
\]

where \( \mathbf{r}_{ij} = \mathbf{R}_i - \mathbf{R}_j \), \( \mathbf{r}_{ij} = \mathbf{r}_{ij}/|\mathbf{r}_{ij}| \), \( r_d = 1.0 \), \( k = 20 \), \( R_{\text{eff}} = 2 r_d \), and \( \theta \) is the location of particle \( i(j) \). We add a single non-active probe particle to the system with the same radius and steric interactions as the active disks, but with which \( F^m = 0 \), and apply a constant force \( F_d \) in the \( x \) direction to only the probe particle, as shown in Fig. 1(a). We measure the time series of the probe velocity fluctuations \( V_x(t) = (d \mathbf{R}_p/dt) \cdot \mathbf{e}_x \), where \( \mathbf{R}_p \) is the location of the probe; in the absence of any other particles, \( F_d/(\langle V_x \rangle) = 1.0 \). In previous simulations of this same type of run-and-tumble system with no probe particle, a transition from a liquid state to a cluster state occurred for fixed \( \phi \) and increasing \( R_l \), or for fixed \( R_l \) and increasing \( \phi \).

Very similar results were obtained for active Brownian particles, where at a fixed density a transition to a cluster state occurs for increasing persistence length. It has been shown that run-and-tumble dynamics and active Brownian motion produce equivalent results when the mobility of the particles is density dependent, which occurs when particle-particle interactions are present, so we expect that our results will be general to either type of system.

### III. RESULTS

In Fig. 1 we show snapshots of the active particle positions and highlight the location of the probe for different dynamic regimes. Figure 1(a) illustrates a uniform liquid state at \( \phi = 0.1885 \) and \( R_l = 160 \), while in Fig. 1(b) at \( \phi = 0.5 \) and \( R_l = 160 \), a phase separated or cluster state occurs composed of a crystalline cluster with local density near \( \phi = 0.9 \) surrounded by a low density gas. In Fig. 1(c) at \( \phi = 0.5 \) and \( R_l = 1.0 \), the system forms a uniform liquid state. Figure 1(d) shows the Voronoi construction obtained from the particle positions in a system at \( \phi = 0.801 \) and \( R_l = 0.04 \), well below the non-active crystallization density of \( \phi = 0.9 \). Here we observe a uniform activity-stabilized triangular lattice. In Fig. 1(e), at \( \phi = 0.801 \) and \( R_l = 2 \) the system is now disordered forming a uniform liquid state, while Fig. 1(f) shows that at \( \phi = 0.801 \) and \( R_l = 160 \), there is a jammed state interspersed with fluctuating voids.

In Fig. 2(a) we plot the average mobility \( \langle V_x \rangle \) of the probe versus \( R_l \) for \( F_d = 0.5 \) at \( \phi = 0.1885, 0.3456, 0.5, 0.6273, 0.691, 0.754, 0.801 \), and 0.8482, with \( \phi \) increasing from top to bottom. To characterize the structure in the system, we measure the fraction of six-fold coordinated particles \( P_b = N^{-1} \sum_{i=1}^N \delta(z_i - 6) \), where \( z_i \) is...
were obtained.

The mobility again decreases with increasing $P$ for $\phi = 0.1885, 0.3456, 0.5, 0.6273, 0.691, 0.754, 0.801, \text{and} 0.8482$, from top to bottom. In Fig. 2(a) for $\langle V_x \rangle$ vs $R_l$ initially increases with increasing $R_l$ before reaching a minimum in the range $0.03 < R_l < 0.2$. For higher values of $R_l$, the mobility increases with increasing $R_l$ up to a broad maximum centered in the range $1.0 < R_l < 20$. For $R_l > 20$, the mobility again decreases with increasing $R_l$. These features in the mobility correlate with features in $P_b$, where $\phi = 0.84721, 0.701, \text{and} 0.754$, the $P_b$ curves in Fig. 2(b) initially increase with increasing $R_l$ until reaching a maximum near $0.03 < R_l < 0.2$ corresponding to the minimum in $\langle V_x \rangle$ in Fig. 2(a). As $R_l$ increases further, $P_b$ decreases, corresponding to an increase in $\langle V_x \rangle$, and for $R_l > 20$ clusters begin to form, leading to an increase in $P_b$ and a decrease in the mobility. Figure 1(d) illustrates the activity-induced crystalline phase at $\phi = 0.801$ at $R_l = 0.04$, where the probe has more difficulty moving through the solid phase. At $R_l = 2.0$ and $\phi = 0.801$ in Fig. 1(e), the system forms a disordered liquid state and the probe can move more easily, while for $\phi = 0.801$ and $R_l = 160$, Fig. 1(f) shows that the system forms an ordered jammed state and it is again more difficult for the probe to move.

These results indicate that activity can induce the formation of a crystalline state that is distinct from the cluster phase. Crystallization occurs with increasing $R_l$ at high $\phi$ since the longer run lengths cause the effective radii of the particles to increase, increasing the effective density $\phi'$ of the system to a value closer to the crystallization density of 0.9. For our parameters a run length $R_l = 0.03$ at $\phi = 0.801$ corresponds to an effective density of $\phi' = 0.9$, obtained from $\phi' = N \pi r_{\text{eff}}^2 / L^2$ with $r_{\text{eff}} = r_b + R_l$. The value of $R_l$ at which crystallization is predicted to occur is close to the value at which $P_b$ in Fig. 2(b) passes through a maximum. This effect can also be viewed as an example of the “freezing by heating” phenomenon in which driven fluctuations can induce crystallization in hard sphere systems. For $\phi < 0.754$ there is no longer any induced crystalline phase and $P_b$ in Fig. 2(b) decreases with increasing $R_l$, reaching a minimum in the range $1.0 < R_l < 2.0$ that is correlated with a small maximum in the corresponding $\langle V_x \rangle$ in Fig. 2(a).

At the onset of the clustering for $R_l > 20$, $P_b$ increases and the mobility decreases. Figure 1(c) shows the uniform liquid state at $\phi = 0.5$ and $R_l = 1.0$, where $P_b$ is low and the probe mobility is high, while Fig. 1(b) shows the phase-separated state at $\phi = 0.5$ and $R_l = 160$, where the mobility drops and a large fraction of the particles form triangular lattice ordering within the clusters. For $\rho = 0.1885$ in Fig. 2(b), cluster formation does not occur and the mobility does not show any significant changes as a function of $R_l$, as indicated in Fig. 2(a).

In Fig. 3(a) we plot the mobility versus $\phi$ for $R_l = 160$ and $R_l = 1.0$, while in Fig. 3(b) we show the corresponding $P_b$ values. For $0 < \phi < 0.3$, $\langle V_x \rangle$ is independent of $R_l$ and decreases linearly with increasing $\phi$. For $\phi > 0.3$, the $R_l = 160$ system undergoes a transition to the cluster state, correlated with a sharp increase in $P_b$ in Fig. 3(b) and a drop in the mobility in Fig. 3(a). As $\phi$ increases, the mobility of the $R_l = 1.0$ system continues to decrease linearly while the mobility of the $R_l = 160$ system

![FIG. 2: (a) Mobility $\langle V_x \rangle$ of the probe vs run length $R_l$ for $\phi = 0.1885, 0.3456, 0.5, 0.6273, 0.691, 0.754, 0.801, \text{and} 0.8482$, from top to bottom. (b) The fraction of six-fold coordinated particles $P_6$ vs $R_l$ for $\phi = 0.1885, 0.3456, 0.5, 0.6273, 0.691, 0.754, 0.801, \text{and} 0.8482$, from bottom to top. The letters $a - f$ indicate the points at which the panels in Fig. 1 were obtained.

![FIG. 3: (a) Mobility $\langle V_x \rangle$ vs $\phi$ for $R_l = 160$ (diamonds) and $R_l = 1.0$ (circles). Inset: corresponding $d\langle V_x \rangle/d\phi$. (b) $P_b$ vs $\phi$ for $R_l = 160$ (diamonds) and $R_l = 1.0$ (circles), highlighting the coordination of the onset of clustering with the drop in the mobility.](image_url)
broad distribution of jump sizes, as indicated by the non-averaged fluctuations in short bursts or avalanches with a skewed Gaussian shape with a maximum near $V_x = 0.4$. The mobility is a strongly nonmonotonic function of the running particle density. In the dense regime, the probe mobility is substantially reduced for $\phi > 0.75$. The mobility for both systems drops nearly to zero at $\phi = 0.9$, the non-active crystallization density. The inset of Fig. 3(a) shows the derivative of the mobility, $d(V_x)/d\phi$, indicating that the $R_l = 160$ system experiences a much earlier drop in mobility, so that near $\phi = 0.6$ the value of $\langle V_x \rangle$ in the $R_l = 160$ system is more than ten times smaller than in the $R_l = 1$ system. The mobility of the $R_l = 1$ system drops near $\phi = 0.8$, a little below the crystallization density of $\phi = 0.9$, and close to the crystallization density the two systems have nearly equal mobilities.

Figure 4(a) shows a representative portion of the velocity time series $V_x(t)$ of the probe in the liquid phase at $\phi = 0.1885$ and $R_l = 160$, while in Fig. 4(b) the corresponding probability distribution function $P(V_x)$ has a skewed Gaussian shape with a maximum near $V_x = 0.4$. Similar p.d.f.’s generally occur in the uniform liquid states. For $\phi = 0.5$ and $R_l = 160$, the system forms a cluster phase and the noise fluctuations exhibit a two-level or telegraph noise feature where $V_x$ jumps between $V_x = 0.0$ and $V_x = 0.5$, as illustrated in Fig. 4(c). This produces two clear peaks in $P(V_x)$ in Fig. 4(d). The two-level behavior arises because the probe velocity is nearly zero when the probe becomes trapped in a cluster, and nearly $P_d = 0.5$ when the probe moves through the low density gas surrounding the clusters where it undergoes very few collisions. We observe similar telegraph noise distributions in the phase separated regime for $R_l = 160$ in the range $0.377 < \phi < 0.75$.

For $\phi > 0.75$ and $R_l = 160$, the probe is mostly stationary and moves in short bursts or avalanches with a broad distribution of jump sizes, as indicated by $V_x(t)$ in Fig. 5(a) for $\phi = 0.817$. At this same density but for $R_l = 1.0$, the system forms a disordered liquid state and the probe motion is no longer intermittent, as shown in the plot of $V_x(t)$ in Fig. 5(b). Figure 5(c) illustrates the corresponding $P(V_x)$ curves on a log-linear scale for $R_l = 1.0$ and $R_l = 160$. For $R_l = 160$, $P(V_x)$ shows a pronounced peak near $V_x = 0.0$ and a broad tail for higher values of $V_x$, while for $R_l = 1.0$, $P(V_x)$ falls off more rapidly and the maximum is centered above $V_x = 0.0$. The inset of Fig. 5(c) shows a log-log plot of $P(V_x)$ for positive values of $V_x$ at $R_l = 160$. The solid line is a power law fit to the form $P(V_x) \propto V_x^{-\alpha}$. Similar fits can be made for $\phi > 0.75$ and $R_l = 160$. For non-active matter systems, a driven probe particle very near the jamming transition also exhibits avalanches with a size distribution that can be fit to a power law. The active matter avalanche behavior suggests that the active system exhibits a significantly extended range of densities at which critical jamming type behavior occurs compared to non-active systems. The exponent of $-2.0$ has been suggested to fall into the class of time-directed avalanche systems.

**IV. SUMMARY**

We have examined the mobility and fluctuations of an externally driven probe particle moving through a bath of active matter particles for varied activity and particle density. In the dense regime, the probe mobility is a strongly nonmonotonic function of the running length, and the activity initially decreases the mobility of the probe due to an activity-induced crystallization effect, while at higher activities the mobility increases...
again when the system becomes disordered. At very high activities where the system forms a phase separated cluster state, the mobility again decreases. The probe velocity fluctuations show distinct features for the different regimes, exhibiting biased Gaussian fluctuations in the uniform liquid state, two-level or telegraph noise fluctuations in the cluster state, and discrete jumps or avalanches of power-law distributed size for higher density and high activity. The distributions of the avalanches suggest that the dense active system may exhibit an extended range of the critical behavior associated with the jamming of non-active systems. Our results should be general to both run-and-tumble active systems as well as active Brownian particles.

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