\[ \Xi^- d \to n \Lambda \Lambda \] and the \( \Lambda \Lambda \) final state interaction

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Abstract

The reaction \( \Xi^- d \to n \Lambda \Lambda \) is studied within the framework of the Faddeev equations as a possible tool to gain insight into the final state \( \Lambda-\Lambda \) interaction. The neutron differential energy spectrum gives a final state interaction that is sensitive to both the \( \Lambda-\Lambda \) amplitude at threshold, and the coupling between the \( \Lambda-\Lambda \) and \( \Xi-N \) channels. The latter is a result of interference between two mechanisms for the production of the final state, which suggests that this reaction could give a measure of flavor \( SU(3) \) violation in the two-baryon system.

21.80.+a, 21.30.-x, 21.10.Dr, 21.45+v
I. INTRODUCTION

Interest in the reaction \( \Xi^- d \rightarrow n \Lambda \Lambda \) is twofold: (i) It can be used to examine the final state interaction between the two \( \Lambda \) hyperons and, therefore, to gain some insight into the baryon-baryon interaction. The fact that this reaction requires coupling between the \( \Xi^- N \) and \( \Lambda - \Lambda \) channels to proceed, suggests that one may gain further understanding of the importance of this coupling. This reaction might possibly set a constraint on its magnitude, which in turn could give a measure of \( SU(3) \) violation in this system as well as the general two baryon system. (ii) It can be used to test the hypothesis that there is an \( H \) dibaryon, a six quark state. The existence of such a dibaryon would be a signature that meson-baryon degrees of freedom are not sufficient to describe this reaction and possibly the baryon-baryon system in general. If the \( H \) has a mass significantly below the \( \Lambda - \Lambda \) threshold, then the present reaction would provide a clean signal (a mono energetic neutron) in the neutron differential energy spectrum (NDES). However, if the \( H \) has a mass comparable to the \( \Lambda - \Lambda \) threshold, then it would be difficult to distinguish, in the NDES, between the \( H \) dibaryon and the final state \( \Lambda - \Lambda \) interaction. In this case one needs to compare the experimental data with a theoretical calculation based upon meson-baryon dynamics. To that extent this investigation could be used as a guide in the search for \( H \) dibaryons.

Aerts and Dover [1] first examined the \( \Xi^- d \rightarrow n \Lambda \Lambda \) reaction within the context of estimating rates and spectra for the \( (\Xi^- d)_{atom} \rightarrow nH \) reaction. They approximated the decay rate for the \( (\Xi^- d)_{atom} \rightarrow n \Lambda \Lambda \) reaction by evaluating the lowest order diagram, including no multiple scattering in the \( \Xi NN \) intermediate state or between the \( \Lambda \Lambda \) pair in the final state. Taking into account such multiple scattering is essential if one is to model the enhancement in the neutron energy spectrum due to a strong \( \Lambda \Lambda \) interaction; transfer of the kinetic energy from the \( \Lambda \Lambda \) pair created by the \( \Xi^- N \rightarrow \Lambda \Lambda \) transition to the neutron requires a complex series of multiple scatterings in the final state if the spectator neutron is to carry off almost all the available energy and probe the \( \Lambda \Lambda \) zero-energy scattering length. This is the analog of the \( nd \rightarrow nnp \) breakup experiment (see, for example, Tornow et al. [2] for a recent review) in which measurement of the proton energy spectrum was proposed as a means to study the \( nn \) zero-energy scattering length. The Watson-Migdal approximation fails to properly describe the proton energy spectrum; multiple scattering calculations [3,4] which include fully three-body dynamics are essential.

To examine the reaction \( \Xi^- d \rightarrow n \Lambda \Lambda \) within the framework of three-body dynamics, we must derive a set of equations for this specific problem. The new feature, unique to this problem, is the fact that the initial \( \Xi NN \) system and the final \( \Lambda \Lambda N \) system have two (separate) identical spin \( \frac{1}{2} \) particles, which must be in antisymmetric states. In Sec. I we first derive the expression for the breakup amplitude with both the initial two nucleons and the final two \( \Lambda \) hyperons in antisymmetric states. We then proceed in Sec. II to adapt the Alt-Grassberger-Sandhas (AGS) three-body equations [5] to generate the antisymmetric elastic and rearrangement amplitudes required to determine the breakup amplitude. In this way we minimize the number of coupled integral equations needed to extract the final breakup amplitude. In Sec. III we write the breakup cross section as the coherent sum of three reaction mechanisms. To calculate the NDES for this reaction, we define the input two-body amplitudes in Sec. III A and demonstrate in Sec. III B that the final amplitudes have been accurately calculated to satisfy three-body unitarity. The fact that the final state
interaction between the two Λ hyperons is part of the coupled channel ΛΛ−ΞN problem implies that there are two ways of converting the Ξ to a Λ, the first is in the final state interaction [see diagram (a) of Fig. 1], the second in the rearrangement amplitude [as in diagram (b) of Fig. 1]. We find in Sec III C that these two amplitudes, each dominated by the final interaction in the ΛΛ−ΞN channel, are out of phase and almost cancel one another. It is only when the background term from diagram (c) of Fig. 1 is also included in the evaluation of the cross section that a relatively weak final state interaction peak is present in the NDES, and only if the ΛΛ−ΞN amplitude is dominated at low energies by a virtual bound state pole near the Λ−Λ threshold. The cancellation between diagrams (a) and (b) of Fig. 1 results in a final state interaction peak that is sensitive to the coupling between the Λ−Λ and the Ξ−N channels. Finally, in Sec. IV we present some concluding remarks about the implication of the results.

II. THEORY

The reaction Ξ−d → nΛΛ differs from the standard three-body problem, e.g. nd → nnp, in that the particles in the initial state ΞNN differ from those in the final state NΛΛ, and in that both initial and final states contain two different pairs of identical Fermions, for which we need to insure antisymmetric wave functions. In addition, the ΞN → ΛΛ conversion can take place on either nucleon in the initial state. All of these features can be included naturally if we work within the framework of SU(3) rather than the SU(2) of isospin. However, then we cannot include the mass splitting in the baryon octet, and we must carry out the necessary re-coupling within the framework of SU(3) algebra. To avoid this complication, and to take into consideration the mass splitting in the baryon octet, we have resorted to antisymmetrizing explicitly both initial and final states. In this section we first derive an expression for the antisymmetric breakup amplitude describing the reaction Ξ−d → nΛΛ in terms of antisymmetric off-shell elastic and rearrangement amplitudes. We then proceed, with the help of the AGS equations to derive coupled integral equations for the elastic and rearrangement amplitudes. By deriving coupled integral equations for the antisymmetric amplitude, we reduce the number of coupled integral equations we need to solve. Finally, we write the cross section for this reaction in terms of these antisymmetric amplitudes.

A. Antisymmetry

As a first step in determining the antisymmetric amplitude for Ξd → NΛΛ, we construct the initial and final states for this reaction. For the initial Ξd state, we designate the two nucleons as particles 1 and 2 and the Ξ as particle 3, while for the final state the nucleon can be either particle 1 or 2 depending on which nucleon the Ξ converted into a Λ. We now introduce the antisymmetrization operator $A_{ij}$ defined in terms of the permutation operator $P_{ij}$ that exchanges the coordinates of particles $i$ and $j$ as:

$$A_{ij} = \frac{1}{2} (1 - P_{ij}) , \quad (1)$$
where $A_{ij}A_{ij} = A_{ij}$. This allows us to define an initial state that is antisymmetric in the two nucleons that form the deuteron as

$$|\Xi^- d\rangle^{AS} = A_{12} |\Xi^- d\rangle,$$

with the antisymmetrized state being normalized; i.e.,

$$AS\langle d\Xi^- |\Xi^- d\rangle^{AS} = 1.$$

We note that, if the deuteron is in the $^3S_1$-$^3D_1$ channel, then $A_{12} |\Xi^- d\rangle = |\Xi^- d\rangle$.

For the final state we have the two Λ hyperons which must be in an antisymmetric state. However, now we need to recall that the final nucleon can be either nucleon 1 or nucleon 2, depending on which nucleon the Ξ converted to a Λ. We therefore have two possible configurations [7]:

$$|N_1\Lambda\Lambda\rangle^{AS} = \sqrt{2} A_{23} |N_1\Lambda_2\Lambda_3\rangle,$$

$$|\Lambda N_2\Lambda\rangle^{AS} = \sqrt{2} A_{13} |\Lambda_1 N_2\Lambda_3\rangle.$$

The factor of $\sqrt{2}$ was introduced to guarantee that the antisymmetric states are normalized to one. Since neither of these states is the physical state, we need to take the linear combination of these states such that the final state is antisymmetric with respect to $N_1$ and $N_2$, i.e.

$$|N\Lambda\Lambda\rangle^{AS} = \sqrt{2} A_{12} |N_1\Lambda\Lambda\rangle^{AS}$$

$$= A_{23} |N_1\Lambda_2\Lambda_3\rangle - A_{13} |\Lambda_1 N_2\Lambda_3\rangle.$$

In writing the above result, we have made use of the multiplication table of the permutation operators, and in particular the fact that

$$P_{12} P_{23} = P_{123} = P_{13} P_{12}.$$

We will see next that the antisymmetry in the initial state between the two nucleons propagates through a symmetric breakup operator to project only the antisymmetric combination considered in Eq. (5).

We are now in a position to define the physical amplitude for $\Xi d \rightarrow N\Lambda\Lambda$ as the matrix element of the breakup operator for distinguishable particles between antisymmetric and normalized initial and final states [8]: i.e.,

$$T_{N\Lambda\Lambda\rightarrow\Xi d} = AS\langle N\Lambda\Lambda| U_{[03]} |\Xi d\rangle^{AS}$$

$$= AS\langle N\Lambda\Lambda| \sum_{\alpha} T_{\alpha} G_{0} U_{\alpha 3} |\Xi d\rangle^{AS}.$$

We note here that $U_{03}$ commutes with the permutation operator $P_{12}$, and it is this feature of the breakup operator that required we write the antisymmetrized final state in the form given in Eq. (4). In Eq. (7), the operators $U_{\alpha 3}$ are the AGS [5] operators that are a solution of the AGS equations

$$U_{\alpha\beta}(E) = \tilde{\delta}_{\alpha\beta} G_{0}^{-1}(E) + \sum_{\gamma} \tilde{\delta}_{\alpha\gamma} T_{\gamma}(E) G_{0}(E) U_{\gamma\beta}(E).$$
Here $\delta_{\alpha\beta} = 1 - \delta_{\alpha\beta}$, while $G_0$ is the free three-body Green’s function, and $T_\alpha$ the amplitude for particles $\beta$ and $\gamma$.

We are now in a position to write the physical amplitude for $\Xi d \to N\Lambda\Lambda$ in terms of a linear combination of the AGS operators $U_{\alpha\beta}$ to maintain the antisymmetry in both initial and final states. We then can use the AGS equations to derive integral equations for these antisymmetric combinations of $U_{\alpha\beta}$. The advantage of this procedure is a reduction in the number of coupled integral equations we need to solve to construct the breakup amplitude.

Combining Eq. (5) with Eq. (7), we can write the physical amplitude as:

$$T_{N\Lambda\Lambda \leftarrow \Xi d} = \langle N_1 \Lambda_2 \Lambda_3 | A_{23} \sum_\alpha T_\alpha G_0 U_{\alpha 3} A_{12} | \Xi d \rangle - \langle \Lambda_1 N_2 \Lambda_3 | A_{13} \sum_\alpha T_\alpha G_0 U_{\alpha 3} A_{12} | \Xi d \rangle .$$

The permutation operator $P_{12}$ exchanges the coordinate of particles 1 and 2. In the final state, this interchanges the position of the particle in the ket and the label on the nucleon; i.e.,

$$\langle \Lambda_1 N_2 \Lambda_3 | = \langle N_1 \Lambda_2 \Lambda_3 | P_{12} .$$

With the help of the identities

$$P_{12} A_{13} = A_{23} P_{12} \quad P_{12} A_{12} = -A_{12} \tag{11}$$

that follow from the multiplication table of the permutation operators, we can write the breakup amplitude as

$$T_{N\Lambda\Lambda \leftarrow \Xi d} = 2 \langle N_1 \Lambda_2 \Lambda_3 | A_{23} A_{12} U_{03} A_{12} | \Xi d \rangle - 2 \langle N_1 \Lambda_2 \Lambda_3 | A_{23} A_{12} \sum_\alpha T_\alpha G_0 U_{\alpha 3} A_{12} | \Xi d \rangle . \tag{12}$$

In writing the above expression for the breakup amplitude, we have maintained the antisymmetry operators on both sides of the breakup amplitude $U_{03}$. To reduce the right hand side of the above expression for the breakup amplitude in terms of antisymmetric two-cluster final state amplitudes, we make use of the fact that

$$T_\alpha P_{\alpha\beta} = P_{\alpha\beta} T_\beta, \quad \text{and} \quad T_\gamma P_{\alpha\beta} = P_{\alpha\beta} T_\gamma \quad \text{for} \quad \alpha \neq \gamma \neq \beta, \tag{13}$$

and operate with $A_{12}$ in the final state on the breakup amplitude. This will assure us that the final state is antisymmetric in particles 1 and 2. The resultant breakup amplitude is the sum of two terms. The first has a final state interaction (FSI) between the two $\Lambda$ hyperons, while the second term has an $N\Lambda$ FSI; i.e.,

$$T_{N\Lambda\Lambda \leftarrow \Xi d} = \langle N_1 \Lambda_2 \Lambda_3 | A_{23} T_1 G_0 [ U_{13} - P_{12} U_{23} ] A_{12} | \Xi d \rangle + \langle N_1 \Lambda_2 \Lambda_3 | A_{23} T_2 G_0 [ (U_{23} - P_{23} U_{33}) - P_{12} (U_{13} - P_{13} U_{33}) ] A_{12} | \Xi d \rangle . \tag{14}$$

The linear combination of the amplitudes in Eq. (14) insures that the antisymmetry in the final state is preserved at the operator level.
The AGS equations take the form of a closed set of coupled integral equations for physical amplitudes only if the two-body interaction is assumed to be separable. Since we will be using separable interactions for calculating the cross section for this reaction, it is most convenient to introduce this approximation at this time as it will give a physical meaning to the terms resulting from Eq. (14). Production of the $N\Lambda\Lambda$ final state will require the conversion of $\Xi N$ into $\Lambda\Lambda$. This can take place either in the two-body amplitude $T_1$ in the FSI where the conversion takes place on nucleon 2, or in the three-body AGS amplitudes $[U_{13} - P_{12} U_{23}]$. To expose the mechanism for conversion, we need to write the amplitude $T_1$ in the form of $2 \times 2$ matrix. In three-body Hilbert space this takes the form

$$T_1(E) = \sum_{\alpha\beta} |g_{\alpha}^c; N_1\rangle \tau_{\alpha\beta}^c(E - \epsilon_1) \langle g_{\beta}^c; N_1|,$$  \hspace{1cm} (15)

where the sum $\alpha, \beta$ runs over the coupled channels $N\Xi, \Lambda\Lambda$, and the superscript $c$ indicates that this is a coupled channel partial wave. Since we have written the two-body amplitude $T_1(E)$ in three-body Hilbert space, $\epsilon_1$ is the energy of the spectator nucleon 1.

Because the $\Lambda\Lambda$ is an isospin zero system, this matrix structure for $T_1(E)$ is only present for this isospin channel and partial waves in which the space-spin wave function are antisymmetric. In all other partial waves the $\Xi N$ system does not couple to the $\Lambda\Lambda$ channel, and the corresponding amplitude is a single channel amplitude, and has the same form as the $N\Lambda$ interaction, which can be written as

$$T_i(E) = |g_{N\Lambda}; N_i\rangle \tau_{N\Lambda}(E - \epsilon_i) \langle g_{N\Lambda}; N_i| \ j \neq i = 2, 3. \hspace{1cm} (16)$$

In writing the separable representation for the $N\Lambda$ amplitude, we have excluded the particle label from the $\tau_{N\Lambda}(E)$ because this quantity is the same for the nucleon interacting with either $\Lambda$.

Making use of the above separable representation for the two-body amplitude, we can write the amplitude for $N\Lambda\Lambda \leftrightarrow \Xi d$ as

$$T_{N\Lambda\Lambda\leftrightarrow \Xi N} = \langle N_1\Lambda_2\Lambda_3| A_{23} |g_{\Lambda\Lambda}; N_1\rangle \left[ \tau_{\Lambda\Lambda;\Xi N}^c X_{N_1;\Xi}^c + \tau_{\Lambda\Lambda;\Lambda\Lambda}^c Y_{N;\Xi} \right] + 2 \langle N_1\Lambda_2\Lambda_3| A_{23} |g_{N\Lambda}; N_2\rangle \tau_{N\Lambda} Y_{\Lambda;\Xi} . \hspace{1cm} (17)$$

The first two terms on the right hand side correspond to the final interaction occuring in the $N\Xi-\Lambda\Lambda$ channel, while the last term has the interaction in the $N\Lambda$ channel. The antisymmetric AGS amplitudes $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ are defined as

$$X_{N;\Xi} \equiv \frac{1}{\sqrt{2}} \langle g_{\Xi N}; N_1| G_0 \left[ U_{13} - P_{12} U_{23} \right] G_0 |g_{NN}; \Xi \rangle$$

$$= \frac{1}{\sqrt{2}} \left[ X_{N_1;\Xi} - X_{N_2;\Xi} \right] \hspace{1cm} (18)$$

for the antisymmetric $(\Xi N)N \leftrightarrow \Xi d$, while the antisymmetric amplitude for $(\Lambda\Lambda)N \leftrightarrow \Xi d$ is defined as

$$Y_{N;\Xi} \equiv \frac{1}{\sqrt{2}} \langle g_{\Lambda\Lambda}; N_1| G_0 \left[ U_{13} - P_{12} U_{23} \right] G_0 |g_{NN}; \Xi \rangle$$

$$= \frac{1}{\sqrt{2}} \left[ Y_{N_1;\Xi} - Y_{N_2;\Xi} \right] . \hspace{1cm} (19)$$
In writing the above two definitions for the amplitudes, we have dropped the superscript as the definitions are valid for both coupled and uncoupled channels. Finally the antisymmetric amplitude for the reaction \( N\Lambda \leftarrow \Xi d \) is given by

\[
Y_{\Lambda;\Xi} \equiv \frac{1}{2} \left( g_{N1;\Lambda2} | G_0 \left[ (U_{23} - P_{23} U_{33}) - P_{12} (U_{13} - P_{13} U_{33}) \right] G_0 | g_{NN}; \Xi \right) = \frac{1}{2} \left[ Y_{\Lambda;\Xi}^{N1} - Y_{\Lambda;\Xi}^{N2} + Y_{\Lambda;\Xi}^{N3} \right].
\]  

(20)

In defining \( X_{\alpha\beta} \) and \( Y_{\alpha\beta} \), we have made use of the fact that in a separable approximation we can write the initial state \( |\Xi d\rangle = G_0 |g_d; \Xi \rangle \). In addition, it is assumed that the deuteron is in an antisymmetric state; i.e., \( A_{12} |g_d; \Xi \rangle = |g_d; \Xi \rangle \). The amplitude given in Eq. (17) can be represented diagrammatically as in Fig. 1.

**B. The AGS equations for \( \Xi d \rightarrow N\Lambda\Lambda \)**

We are now in a position to derive integral equations for the antisymmetric amplitudes \( X_{\alpha\beta} \) and \( Y_{\alpha\beta} \) defined in Eqs. (18)-(20), and required in Eq. (17) to construct the total amplitude for breakup. In addition to these amplitudes, we need the amplitude for \( \Xi d \) elastic scattering, which is basically a matrix element of \( U_{33} \). Thus for \( X_{N;\Xi} \) we have, after making use of the AGS equation,

\[
X_{N;\Xi}^c \equiv \frac{1}{\sqrt{2}} \langle g_{N2;\Xi} ; N1 | G_0 \left[ (1 - P_{12}) G_0 \right] | g_{NN}; \Xi \rangle
\]

(21)

Introducing the separable representation for the amplitudes \( T_2 \) and \( T_3 \) allows us to turn the above expression into an integral equation of the form

\[
X_{N;\Xi}^c = Z_{N;\Xi}^c + Z_{N;\Xi}^c \tau_{N;\Xi} X_{N;\Xi} + Z_{N;\Xi}^c \tau_{N;\Xi} X_{N;\Xi}^c + Z_{N;\Xi}^c \tau_{N;\Xi} Y_{N;\Xi} + Z_{N;\Xi}^c \tau_{N;\Xi} Y_{N;\Xi}^c
\]

(22)

where the antisymmetric elastic amplitude is given by

\[
X_{\Xi;\Xi} \equiv \langle g_{NN}; \Xi | G_0 U_{33} G_0 | g_{NN}; \Xi \rangle,
\]  

(23)

while the antisymmetric Born amplitudes \( Z_{\alpha\beta} \) needed in Eq. (22) are defined as

\[
Z_{N;\Xi} \equiv \frac{1}{\sqrt{2}} \langle g_{N2;N1} ; N1 | (1 - P_{12}) G_0 | g_{NN}; \Xi \rangle = \frac{1}{\sqrt{2}} \left[ Z_{N1;\Xi} - Z_{N2;\Xi} \right].
\]  

(24)
and

\[ Z_{N;N} \equiv - \langle g_{3;N_2}; N_1 | G_0 | g_{N;N_2} \rangle = - Z_{N_1;N_2} \, . \] (25)

In Eq. (22) we have the amplitude \( X_{N;\Xi}^c \) in terms of the amplitudes \( X_{N;\Xi}^c, Y_{N;\Xi}^c, X_{N;\Xi}, \) and \( X_{\Xi;\Xi} \). To close this set of integral equations, we generate an equation for each of these amplitudes following the same procedure we adopted for deriving Eq. (22). For the amplitude \( X_{N;\Xi}^c \), we have by definition the same expression as in Eq. (21) without the superscript \( c \).

This results in an equation for the \( X_{N;\Xi} \) amplitude which is identical to Eq. (22) without the left hand superscript \( c \).

For the amplitude \( Y_{N;\Xi}^c \), we have

\[ Y_{N;\Xi}^c \equiv \frac{1}{\sqrt{2}} \left\langle g_{\Lambda\Lambda}^c; N_1 | G_0 \left[ U_{13} - P_{12} U_{23} \right] G_0 | g_{NN}; \Xi \right\rangle = \frac{1}{\sqrt{2}} \left\langle g_{\Lambda\Lambda}^c; N_1 | G_0 T_2 G_0 \left[ U_{23} - P_{12} U_{13} \right] G_0 | g_{NN}; \Xi \right\rangle + \frac{1}{\sqrt{2}} \left\langle g_{\Lambda\Lambda}^c; N_1 | G_0 T_3 G_0 \left[ U_{33} - P_{12} U_{33} \right] G_0 | g_{NN}; \Xi \right\rangle . \] (26)

Making use of the fact that \( T_3 = P_{23} T_2 P_{23} \) and \( \langle g_{\Lambda\Lambda}^c; N_1 | P_{23} = - \langle g_{\Lambda\Lambda}^c; N_1 \rangle \), the integral equation for \( Y_{N;\Xi} \) reduces to

\[ Y_{N;\Xi}^c = Z_{N;\Lambda}^c \tau_{\Lambda} Y_{\Lambda;\Xi} \, . \] (27)

where

\[ Z_{N;\Lambda} \equiv \sqrt{2} \left\langle g_{\Lambda\Lambda}^c; N_1 | G_0 | g_{N\Lambda}; \Lambda \right\rangle = \sqrt{2} Z_{N_1;\Lambda_2} \, . \] (28)

In this same way we establish an equation for \( Y_{\Lambda;\Xi} \) to be

\[ Y_{\Lambda;\Xi} = Z_{\Lambda;N}^c \tau_{\Lambda;\Xi} X_{N;\Xi}^c + Z_{\Lambda;N}^c \tau_{\Lambda;\Xi} Y_{N;\Xi} \, . \] (29)

where \( Z_{\Lambda;N} \) is defined as

\[ Z_{\Lambda;N} \equiv \frac{1}{\sqrt{2}} \left\langle g_{N_1;\Lambda_2}; A_2 | (1 - P_{23}) G_0 | g_{\Lambda\Lambda}; N_1 \right\rangle = \sqrt{2} Z_{A_2;N_1} \, . \] (30)

and

\[ Z_{\Lambda;A} = - Z_{A_2;A_2} \, . \] (31)

Finally to close the set of coupled equations, we must write the equation for the elastic amplitude. Making use of the definition of \( X_{\Xi;\Xi} \), given in Eq. (23), we get
\[ X_{\Xi;\Xi} = Z_{\Xi;N} \tau_{\Xi N} X_{N;\Xi} + Z_{\Xi;N}^{c} \tau_{\Xi N;\Xi}^{c} X_{N;\Xi}^{c} \]
\[ + Z_{\Xi;N}^{c} \tau_{\Xi N;\Lambda}^{c} Y_{N;\Xi}^{c}, \]  
(32)

where
\[ Z_{\Xi;N} \equiv \frac{1}{\sqrt{2}} \langle g_{NN;\Xi} | (1 - P_{12}) G_{0} | g_{\Xi N;N} \rangle \]
\[ = \frac{1}{\sqrt{2}} \left[ Z_{\Xi_{1};N_{1}} - Z_{\Xi_{2};N_{2}} \right]. \]  
(33)

Equations (22), (27), (29), and (33) now form a closed set for the two-cluster to two-cluster antisymmetrized amplitudes required to construct the breakup amplitude and, therefore, the cross section. In the appendix we provide a summary of the antisymmetrized Born amplitudes in terms of the single particle exchanged amplitudes.

C. The cross section for \( \Xi d \rightarrow N\Lambda\Lambda \)

We now turn to the determination of the cross section, and in particular the neutron energy spectrum for the capture of \( \Xi^- \) on the deuteron in terms of the amplitudes defined in Sec. IA. Since our ultimate aim is to gain some insight into the low energy \( \Lambda\Lambda \) interaction in a reaction in which the \( \Xi^- \) is captured from an atomic orbit, we will restrict our analysis to \( S \)-wave two-body interactions. With this simplification, the only angular momentum in the problem is the spin of the three particles, and the total spin will be either \( \frac{1}{2} \) (doublet) or \( \frac{3}{2} \) (quartet). However, because the two \( \Lambda \) hyperons are in a \( ^1S_0 \) state, only the doublet channel contributes. In addition, by including isospin and taking into consideration the fact that the two \( \Lambda \) hyperons and the deuteron are in an isospin zero state, the total isospin is \( \frac{1}{2} \) with a projection \( -\frac{1}{2} \), corresponding to a \( \Xi^- \) in the initial state. With these restrictions, we can write the breakup amplitude as the sum of the three diagrams in Fig. 1 in the spin isospin \( \frac{1}{2} \) state as:

\[ T_{\Lambda\Lambda\Xi;\Xi-d} = \sum_{S_N} \sum_{M_S=\pm \frac{1}{2}} \left( \frac{1}{2} m_{s_2} \frac{1}{2} m_{s_3} |S_{N} M_{S_{N}} \rangle (S_{N} M_{S_{N}} \frac{1}{2} m_{s_1} |\frac{1}{2} M_S \rangle \right) \times \left[ A_{\ell_{N};\ell_{\Xi}} + B_{\ell_{N};\ell_{\Xi}} + C_{\ell_{N};\ell_{\Xi}} \right] \left( S_d M_{S_d} \frac{1}{2} m_{s_1} |\frac{1}{2} M_S \rangle \right), \]  
(34)

where \( \ell_{\alpha} = \{S_{\alpha}, t_{\alpha} \} \), with \( S_{\alpha} \) and \( t_{\alpha} \) the spin and isospin of the pair \( (\beta\gamma) \) respectively. This basically defines the channels in the two-body subsystems. The amplitude \( A_{\ell_{N};\ell_{\Xi}} \) corresponding to Fig. 1(a) is given in terms of the AGS amplitude \( X_{\alpha\beta} \) by

\[ A_{\ell_{N};\ell_{\Xi}}(p_1, k_1; k_{\Xi}^{0}) = \frac{\delta_{S_{N};0}}{\sqrt{2\pi}} g_{\ell_{N}}(p_1) \tau_{\Lambda\Lambda;\Xi N} [E - \epsilon(k_1)] X_{\ell_{N};\ell_{\Xi}}(k_1, k_{\Xi}^{0}), \]  
(35)

where \( k_{\Xi}^{0} \) and \( k_1 \) are the initial on-shell momentum of the \( \Xi^- \) and the final momentum of the neutron respectively. The momentum \( p_1 \) in Eq. (33) is the relative momentum of the two \( \Lambda \) hyperons in the final state. The AGS amplitude \( X_{\ell_{N};\ell_{\Xi}} \) is that defined in Eq. (18) for a state with total spin and isospin \( \frac{1}{2} \). Here we note that the \( \delta_{S_{N};0} \) restricts the contribution of Fig. 1(a) to the amplitude in which the \( \Lambda\Lambda \) interaction occurs in the \( ^1S_0 \) state. The
amplitude corresponding to diagram Fig. [I(b)], in which the conversion from ΞN to ΛΛ takes place in the multiple scattering before the FSI, is given by

\[ B_{\ell N;\ell \Xi}(p_1, k_1; k^0_\Xi) = \frac{\delta S_{N,0}}{\sqrt{2\pi}} g_{\ell N}(p_1) \tau_{\Lambda \Lambda;\Lambda \Lambda}[E - \epsilon(k_1)] Y_{\ell N;\ell \Xi}(k_1, k^0_\Xi), \]  

(36)

where the AGS amplitude \( Y_{\ell N;\ell \Xi} \) is defined in Eq. (13) and is non-zero in total spin, isospin \( \frac{1}{2} \) with the two Λ hyperons in the \( ^1S_0 \) state. If we compare the full breakup amplitude given in Eq. (17) and (14), we observe that the antisymmetrization operator \( A_{23} \) in the first two terms on the right hand side of Eq. (17) are not present in the expressions for \( A_{\ell N;\ell \Xi} \) and \( B_{\ell N;\ell \Xi} \). This is due to the fact that the ΛΛ interaction in the final state has been restricted to those channels that satisfy the Pauli principle for the Λ hyperons. This was simple to achieve by proper choice of partial waves. Unfortunately, that is not possible for the last term in Eq. (17), and in this case we need to include the antisymmetrization operator explicitly. In addition, we need to maintain the same spin coupling as the diagrams in Figs. [I(a) and [I(b)]. This will involve a re-coupling of the spin, with the resultant amplitude given by:

\[ C_{\ell N;\ell \Xi}(p_1, k_1; k^0_\Xi) = \sum_{S_{\Lambda}} (-1)^R \hat{S}_N \hat{S}_{\Lambda} \left\{ \frac{1}{2}, \frac{1}{2}, S_N \right\}
\times \left[ C_{\ell N;\ell \Xi}^{(2)}(p_2, k_2; k^0_\Xi) + (-1)^{R'} C_{\ell N;\ell \Xi}^{(3)}(p_3, k_3; k^0_\Xi) \right], \]  

(37)

where \( \hat{a} = \sqrt{2a + 1} \), \( R = S_{\Lambda} \) and \( R' = 1 - S_N \). The phase \((-1)^{R'}\) ensures that the two Λ hyperons are in states that satisfy the Pauli principle. The amplitude \( C_{\ell N;\ell \Xi}^{(i)} \), \( i = 2, 3 \) is given by

\[ C_{\ell N;\ell \Xi}^{(i)}(p_1, k_i; k^0_\Xi) = \frac{1}{\sqrt{4\pi}} g_{\ell N}(p_i) \tau_{\Lambda \Lambda}[E - \epsilon(k_i)] Y_{\ell N;\ell \Xi}(\bar{k}_i, k^0_\Xi), \]  

(38)

where the AGS amplitude \( Y_{\ell \Lambda;\ell \Xi} \) is defined in Eq. (20).

To calculate the neutron differential energy spectrum (NDES) we square the breakup amplitude, perform spin averaging over the incoming particles, and integrate over all the kinematical allowed final states for the two Λ hyperons. This gives the cross section as a function of the final neutron energy. In the three-body center of mass system, the NDES takes the form

\[ \frac{d^3\sigma}{dE_n d\Omega_n} = \frac{(2\pi)^4 \mu_{N,\Xi} k_1 m_N}{2k^0_\Xi} \int d^3 k_2 \int d^3 k_3 \delta(k_1 + k_2 + k_3) \delta(E - \frac{k_1^2}{2m_N} - \frac{k_2^2}{2m_\Lambda} - \frac{k_3^2}{2m_\Lambda}) |M(k_1, k_2, k_3; k^0_\Xi)|^2, \]  

(39)

where the \( \delta \)-functions maintain energy momentum conservation. The amplitude \( |M|^2 \) is given as the square of the sum of the diagrams in Fig. [I], i.e., one obtains

\[ |M|^2 = \frac{1}{3} \sum_{S_1} |A_{\ell N;\ell \Xi} + B_{\ell N;\ell \Xi} + C_{\ell \Lambda;\ell \Xi}|^2, \]  

(40)

with \( A_{\ell N;\ell \Xi} \), \( B_{\ell N;\ell \Xi} \), and \( C_{\ell \Lambda;\ell \Xi} \) given in Eqs. (35), (36), and (37). Because we are summing over the spin projections and integrating over the momenta of the two identical Λ hyperons,
we have included a factor of $1/2!$ in Eq. (39). This factor is physically crucial as it avoids double counting of states, that after all represent only interchanges of identical particles. The integral in Eq. (40) can be reduced to a two-dimensional one which needs to be evaluated with some degree of accuracy [3,11].

III. NUMERICAL RESULTS

Having established that the neutron differential energy spectrum (NDES) is basically the square of the coherent sum of the diagrams in Fig. 1, we are now in a position to determine whether the reaction $\Xi^-d \rightarrow n\Lambda\Lambda$ is, in fact, a means to investigate the final state $\Lambda\Lambda$ interaction. To achieve this we must: (i) Solve the coupled integral equations for the amplitudes $X_{N;\Xi}, Y_{N;\Xi},$ and $Y_{\Lambda;\Xi},$ i.e. Eqs. (22), (27), (29), and (32), and then construct the breakup amplitudes $A_{\ell;N;\Xi}, B_{\ell;N;\Xi},$ and $C_{\ell;N;\Xi}$ defined in Eqs. (35), (36), and (37). (ii) Establish the existence of an enhancement in the FSI region that is sensitive to the choice of the $\Lambda\Lambda$ interaction.

A. The input amplitudes

The proposed reaction is initiated by $\Xi^-$ capture from an atomic $s$ orbit; as a result we consider a low energy reaction in which the two-body interactions are dominated by the $S$-wave contribution. Inclusion of the Coulomb interaction in the initial state to get the $\Xi^-$ in an atomic orbital turns a relatively simple three-body problem into a much more difficult numerical calculation due to the complex analytic structure of the Coulomb amplitude. Because we are predominantly interested in the final $\Lambda–\Lambda$ interaction, we have assumed that the initial $\Xi^-$ is incident on the deuteron at an energy of one MeV, which is below the deuteron breakup threshold. This allows us to neglect the Coulomb potential, since we don’t have an initial bound state, and will enable us to check the numerical accuracy of our amplitudes with the help of three-body unitarity.

Since we are assuming an $S$-wave interaction and considering an initial state below the breakup threshold for $\Xi NN,$ we may assume that all orbital angular momenta in the problem can be set to zero. As a result, the partial wave expansion for the elastic and rearrangement amplitudes reduces to

$$X_{\ell;\alpha;\beta}(k_\alpha; k_\beta) = \frac{1}{4\pi} X_{\ell;\alpha;\beta}(k_\alpha; k_\beta).$$

(41)

With this limitation on the angular momentum, the coupled integral equations can be reduced to a set of one dimensional equations with the channel $\ell$ labeled by the total spin and isospin $(S_\alpha, t_\alpha)$ of the pair $(\beta, \gamma)$. In Table 1 we present the nine channels included in the solution of our coupled integral equations for total spins and isospin $\frac{1}{2}$. Because the kernel of these equations have the standard moving singularities encountered in any three-body breakup problem, we have followed the same procedure implemented in the $n–d$ breakup reaction using the rotation-of-contour method [3,11]. To test the accuracy of our numerical procedures in solving the coupled integral equations and the construction of the breakup amplitudes corresponding to the three diagrams in Fig. 1, we have made use of...
three-body unitarity to compare the imaginary part of the elastic \( \Xi - d \) amplitude to the total cross section. To calculate the total cross section for elastic \( \Xi d \) scattering, and in particular the imaginary part of the elastic amplitude, we also need to solve the integral equation for the spin quartet, or \( S = \frac{3}{2} \) case. In Table II we have the three-body channels for the \( S = \frac{3}{2} \) amplitudes.

To evaluate the NDES, we must define the input two-body separable potential. From Tables I and II it is clear that we need the two-body interactions in the following channels:

\[
\begin{align*}
NN : & \quad ^1S_0, \quad ^3S_1 \\
\Xi N : & \quad ^1S_0 \ t = 0, \quad ^3S_1 \ t = 0, 1 \\
\Xi N - \Lambda \Lambda : & \quad ^1S_0.
\end{align*}
\] (42)

For the \( NN \) potentials we have used the \( S \)-wave separable potentials of Afnan and Gibson \[9\], while the \( \Xi N \) and \( \Xi N - \Lambda \Lambda \) potentials are those constructed to give the same effective range parameters as the \( SU(3) \) rotated Nijmegen model D \[10\] with soft core \[11,12\]. Since we will examine the sensitivity of the FSI peak to the choice of the \( \Lambda \Lambda - \Xi N \) interaction, we present in Table III the effective range parameters and the \( \Lambda - \Lambda \) binding energy for the four potentials under consideration. Here we note that potentials SA and SB do not support a \( \Lambda - \Lambda \) bound state, while potential SC1 and SC2 do support a bound state. In particular, the potential SB has a \( \Lambda - \Lambda \) scattering length that is comparable to the \( ^1S_0 \) \( n - n \) scattering length; i.e., it has a virtual state with a pole in the amplitude on the second energy sheet near the \( \Lambda - \Lambda \) elastic threshold.

### B. Three-body unitarity

Three-body unitarity for this reaction gives a relation between the imaginary part of the elastic \( \Xi d \) amplitude and the total cross section at the corresponding energy; i.e., one has

\[
Im [X_{\Xi,\Xi}] = -\frac{1}{16\pi^3} \nu_{\text{rel}} \sigma_{\text{Tot}},
\] (43)

where the relative velocity \( \nu_{\text{rel}} = k_\Xi^0 / \mu_{\Xi d} \), and the total cross section is the sum of the elastic, breakup, and reaction cross sections, or

\[
\sigma_{\text{Tot}} = \sigma_{\text{el}} + \sigma_{\text{B-up}} + \sigma_{\text{reac}}.
\] (44)

We can now determine the total cross section \( \sigma_{\text{Tot}} \) from unitarity, while the elastic scattering cross section can be determined by integrating the elastic differential cross section. This gives us, in the absence of a \( \Lambda - \Lambda \) bound state, the total breakup cross section as

\[
\sigma_{\text{B-up}} = \sigma_{\text{Tot}} - \sigma_{\text{el}},
\] (45)

while in the presence of a \( \Lambda - \Lambda \) bound state

\[
\sigma_{\text{B-up}} = \sigma_{\text{Tot}} - \sigma_{\text{el}} - \sigma_{\text{reac}},
\] (46)

with \( \sigma_{\text{reac}} \) being calculated by integrating the rearrangement differential cross section for \( \Xi + d \rightarrow n + (\Lambda \Lambda) \).
As a test of the numerical accuracy in the solution of the integral equation, we compare the results for the total breakup cross section as calculated from unitarity and the elastic and reaction cross section (i.e. Eq. (45) or Eq. (46)), with the result of integrating the NDES over the neutron energy. In Table IV we compare the results for these two methods of calculating the total breakup cross section. We have included results for two of the four potentials we will consider. Potential SB has no Λ–Λ bound state, while potential SC1 has a bound state, and therefore a rearrangement cross section. The comparison is done at an energy of 25.7 MeV above the NΛΛ threshold which is just below the ΞNN threshold. The good agreement between the two methods used to construct the total breakup cross section is a clear indication that the numerical procedure for calculating NDES is satisfactory. This is particularly true when the unitarity approach for calculating σ_{B-up} involves taking the difference between the total and total elastic cross sections with both being an order of magnitude larger than the total breakup cross section.

C. The neutron differential energy spectrum

We are now in a position to examine the NDES for the different interactions. In Fig. 2 we present the neutron differential energy spectrum for the four potentials under consideration. Here we observe that for low energy neutrons all four potentials give basically the same shape for the cross section with the broad peak for low energy neutrons being independent of the choice of potential for the ΛΛ–ΞN 1S0 channel. On the other hand in the FSI region (i.e. for large neutron energies) the four potentials give different results with potential SB having a small FSI peak. Considering the similarity between the potential SB and the 1S0 n–n potential, it is surprising that we don’t see a large FSI peak similar to that observed in the n–d breakup reaction. In the n–d breakup, the enhancement in the cross section at the end of the proton spectrum is a result of the fact that the energy available for the final n–n interaction is small, and therefore the cross section is dominated by the pole in the two-body n–n amplitude near zero energy which is known as the 1S0 virtual state. The questions then are: (i) Why don’t we see a similar FSI peak in the present reaction? (ii) Why is the broad peak at the low energy part of the spectrum the same for the four potentials?

To answer the first question, we consider the cross section for potential SB in more detail. Since the FSI peak should be the result of a pole near zero energy in the ΛΛ–ΞN 1S0 amplitude, we expect a contribution to this FSI to come from diagrams (a) and (b) of Fig. 1. We therefore consider the NDES for the following combination of diagrams: (i) The diagrams (a) and (c) of Fig. 1, see Fig. 3. (ii) The diagrams (b) and (c) of Fig. 1, see Fig. 4. Here we observe that in both cases there is an enhancement in the FSI as we originally expected. However the magnitude of the FSI peak is considerably larger in Fig. 4 than in Fig. 3. This suggests that we have constructive interference between diagrams (b) and (c), and destructive interference between diagrams (a) and (c) of Fig. 1. In this way we establish the fact that diagrams (a) and (b) of Fig. 1 are of opposite sign leading to a cancellation between the two breakup amplitudes that provide the enhancement due to the final state Λ–Λ interaction.

To establish the relative contribution of diagrams (a) and (b) of Fig. 1, we present in Figs. 3 and 4 the NDES for diagrams (a) and (b) respectively. A comparison of the magnitude of the FSI peak in these two diagrams establishes the fact that the contribution
of these two diagrams to the total amplitude is the same. This is expected considering the fact that it is the same pole that gives the FSI peak in the two figures. From the above analysis we may conclude that diagrams (a) and (b) almost cancel each other and the only reason we have any FSI peak in the total amplitude is a result of the fact that diagram (c) enhances diagram (b) relative to diagram (a). The obvious difference between this reaction and that for the $n-d$ breakup is the fact that in this reaction we have a coupled channel problem for the FSI (i.e., $\Lambda\Lambda-\Xi N$), while in the $n-d$ case we have only one channel for the final $n-n$ interaction. Finally, we note that this cancellation does not depend on the sign of the coupling $C_{\Lambda;\Xi}$ between the $\Lambda\Lambda$ and $\Xi N$ in the potential, but requires that the same interaction be used for the FSI as is used in generating the inelastic and breakup amplitudes. To understand this we note that a change in the sign of $C_{\Lambda;\Xi}$ changes the sign of $\tau_{\Lambda\Lambda;\Xi N}$ but not the sign of $\tau_{\Lambda\Lambda;\Lambda\Lambda}$, and since each term in the multiple scattering series for each of the three amplitude has an odd number of $\tau_{\Lambda\Lambda;\Xi N}$, the sign of all three amplitudes changes under the substitution

$$C_{\Lambda;\Xi} \rightarrow - C_{\Lambda;\Xi}.$$ 

As a result the cross section does not give us any insight into the sign of the coupling between the $\Lambda\Lambda$ and $\Xi N$ channels.

We now turn to the broad peak in the NDES at the low end of the neutron spectrum. Here, we first observe from Fig. 6 that diagram (b) does not contribute to this peak. The fact that it is not present in the NDES resulting from diagrams (b) and (c), see Fig. 4, suggests that the main contribution to the broad peak at the low energy end of the neutron spectrum is due to diagram (a), which in lowest order is given by

$$g_N(p_1) \tau_{\Lambda\Lambda;\Xi N} X_{N;\Xi} \approx g_N(p_1) \tau_{\Lambda\Lambda;\Xi N} Z_{N;\Xi},$$

and corresponds to the $\Xi^-$ interacting with the proton in the deuteron to generate two $\Lambda$ hyperons. In this case the neutron spectrum is determined by the momentum distribution in the deuteron. To illustrate this we present in Fig. 7 the lowest order contribution from diagram (a) of Fig. 1 to the NDES in which the FSI is almost non-existent. However, as we add the higher order contributions in the (divergent) multiple scattering for $X_{N;\Xi}$ to the NDES, the low energy peak is reduced in magnitude and is converted from a sharp peak to the broad peak we found in the full calculation. From this we may conclude that the broad peak in the neutron spectrum is to a large extent determined by the momentum distribution in the deuteron, and since the same deuteron wave function was used to generate the results for the four $\Lambda\Lambda-\Xi N$ potentials in Fig. 2, the shape of the peak is basically the same. The difference in magnitude is due to the difference in the $\Lambda\Lambda-\Xi N$ potentials used.

The fact that the FSI peak is suppressed as a result of the cancellation between the diagrams (a) and (b) of Fig. 1, suggests that the height of the peak might be sensitive to the coupling between the $\Lambda-\Lambda$ and $\Xi-N$ channels, since the absence of such coupling in the FSI would have given us the result in Fig. 4 in which the NDES is dominated by the FSI peak. The potential SB is based on a separable approximation to a one boson exchange potential (OBE) in which the long range part was determined by the $SU(3)$ rotation of Nijmegen model D potential. The short range part of the interaction was adjusted in such a way that the $SU(3)$ rotated OBE potential was not altered outside of 0.8 fm [11,12]. Because the long range part of the coupling potential is determined by the $K$- and $K^*$-exchanges which
in turn is determined by $SU(3)$ except for the masses of the mesons and baryons, we expect the degree of cancellation between diagrams (a) and (b) may be determined by $SU(3)$. In an attempt to test this idea, we scaled the coupling between the $\Lambda-\Lambda$ and $\Xi-N$ channels; i.e., we assumed

$$C_{\Lambda\Xi} \rightarrow R C_{\Lambda\Xi},$$

where $C_{\Lambda\Xi}$ is the strength of the coupling between the $\Lambda-\Lambda$, and the $\Xi-N$ channels in the separable potential. Here, we chose the scale factor $R = 0.5, 0.75, 1.0, \text{ and } 1.25$. In this way we could investigate changes in the FSI peak with variation in the coupling between the $\Lambda-\Lambda$ channel and the $\Xi-N$ channel. To maintain the position of the virtual pole in the amplitudes fixed, we adjusted the other parameters of the potential to retain a scattering length $a_{\Lambda\Lambda} \approx 21 \text{ fm}$.

In Fig. 8 we present the NDES in the FSI region for different values of $R$. We note here that the $R = 1$ case gives the largest FSI peak, with the other values of $R$ invariably reducing the magnitude of the FSI peak. This suggests that a measurement of the NDES could give more than the $\Lambda-\Lambda$ scattering length in that the height of the peak could shed some light on the strength of the coupling between the $\Lambda-\Lambda$ and $\Xi-N$ channels and, in this way, could test the degree of violation of $SU(3)$ symmetry in the two baryon system. If the suggestion of Dover and Feshbach [14] that there is much less dynamic $SU(3)$ symmetry breaking in the $\Lambda\Lambda \rightarrow \Xi N$ than in $\Lambda\Lambda \rightarrow \Lambda\Lambda$, then the height of the FSI peak might be used to explore this question.

IV. CONCLUSIONS

The main motivation for examining the reaction $\Xi^-d \rightarrow n\Lambda\Lambda$ was to determine whether this reaction might provide some insight into the $\Lambda-\Lambda$ interaction at low energies and possibly give a measure of the scattering length. In this way we can establish whether the $\Lambda-\Lambda$ and the $n-n$ belong to the same flavor multiplet as predicted by $SU(3)$. At the same time if this reaction is to be used to establish the existence of the $H$ dibaryon, then we should establish some quantitative measure of what the cross section would be in the absence of the $H$. In particular, we need to establish which features of the cross section can be reproduced without the introduction of the $H$ particle.

From the results of the above calculation we have established that the neutron differential energy spectrum does, in fact, give a FSI peak in the event that the $\Lambda-\Lambda$ amplitude has a pole on the second Riemann sheet of the energy plane near the $\Lambda-\Lambda$ threshold. Thus, experimental observation of a FSI peak is a strong signature that the $\Lambda-\Lambda$ interaction is, in fact, similar to the $n-n$ interaction as suggested by flavor $SU(3)$. In contrast to the $n-n$ interaction, the $\Lambda-\Lambda$ interaction is part of the coupled channel $\Lambda\Lambda-\Xi N$, and we have established that in the case when the FSI is a coupled channel problem, then there are two distinct diagrams that contribute to the FSI peak. In this case the two diagrams (a) and (b) of Fig. 1 almost cancel each other resulting in a suppression of the FSI peak. In fact the background contribution from the diagram (c) in Fig. 1 effectively preserves the final state interaction by interfering constructively with diagram (b) and destructively with diagram (a). This cancellation is a unique feature of the coupled channel nature of the FSI.
The presence of the interference between diagrams (a) and (b) of Fig. 1 makes this reaction a possible tool to measure the strength of the coupling between the $\Lambda-\Lambda$ and $\Xi-N$ channels. Since this coupling is determined in the OBE model by the $K$- and $K^*$-exchanges whose coupling to the baryon is determined by $SU(3)$, we expect the magnitude of the FSI peak to be a candidate for the determination of $SU(3)$ violation in the two baryon system.

Finally, we observe that the low energy part of the NDES has a broad peak which is largely determined by the momentum distribution in the target deuteron. Although this is not the ideal reaction to examine the deuteron wave function, the fact that we have a major cancellation between the three diagrams that contribute to the FSI suggests that the FSI peak might be sensitive to the choice of deuteron wave function. In the present investigation we have used the simplest possible $^3S_1$ deuteron, and a more realistic wave function for the deuteron (e.g. one with short range repulsion) could give a different result for the magnitude of the FSI peak. The magnitude of the FSI peak could also be sensitive to the choice of the $\Xi-N$ and the $\Lambda\Lambda-\Xi N$ interactions. For the present investigation we have chosen to use simple separable potentials that give the same effective range parameters as the OBE potentials. The existence of experimental data would be required to justify the extension of the present results to include more realistic two-body input.

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APPENDIX A: THE KERNEL OF THE AGS EQUATIONS

To solve the coupled integral equations for the elastic and rearrangement amplitudes $X_{\ell_\alpha,\ell_\beta}$ and $Y_{\ell_\alpha,\ell_\beta}$, we need to define the kernel or the Born amplitudes $Z_{\ell_\alpha,\ell_\beta}$. Since the reaction under consideration involves two Hilbert spaces, i.e. the $\Xi NN$ and $\Lambda\Lambda N$, and the Born term does not couple these two spaces, we can define the Born term for each space separately. Thus, in the $\Xi NN$ Hilbert space we have the nucleon exchange amplitude $Z_{\ell N;\ell_\Xi}$ and the $\Xi$ exchange amplitude $Z_{\ell_\Xi;\ell N}$. To evaluate these amplitudes we need to relate them to the corresponding amplitudes in which the initial and final state are related by the cyclic permutation of the particle label for which we have a standard expression \[. Thus the $N$-exchange is given by:

$$Z_{\ell N;\ell_2} = [Z_{\ell_2;\ell N}]^\dagger \equiv \frac{1}{\sqrt{2}} \left[ Z_{N_1;\Xi_3} - Z_{N_2;\Xi_3} \right] = \sqrt{2} Z_{1;3} \equiv \sqrt{2} \langle (23)1|G_0|(12)3 \rangle,$$

(A1)

where particles 1 and 2 are the nucleon, while particle 3 is the $\Xi$. The $\Xi$ exchange Born amplitude can be written in a similar manner as:
\[ Z_{\ell_N: \ell_N} \equiv -Z_{N_1: N_2} = (-1)^{P+1} Z_{1:2} \]
\[ \equiv (-1)^{P+1} \langle (23)1|G_0|(31)2 \rangle . \]  

(A2)

Here the phase \( P \) results from the exchange of the coordinates of particles 1 and 3 in the coupling coefficient and is given by \( P = s^1 + s^3 - S + \tau_1 + \tau_3 - t_2 \), where \( s_i \) and \( \tau_i \) are the spin and isospin of particle \( i \), which in this case is \( \frac{1}{2} \), while \( S_\alpha \) and \( t_\alpha \) are the total spin and isospin of the pair \( (\beta \gamma) \).

On the other hand, in the \( \Lambda \Lambda N \) Hilbert space, we need the \( N \)-exchange Born amplitude \( Z_{\ell_N: \ell_\Lambda} \) and the \( \Lambda \) exchange amplitude \( Z_{\ell_\Lambda: \ell_\Lambda} \). These are given in terms of the cyclicly defined Born amplitudes as:

\[ Z_{\ell_N: \ell_\Lambda} = \sqrt{2} Z_{N_1: \Lambda_2} = \sqrt{2} Z_{1:2} \]
\[ \equiv \sqrt{2} \langle (23)1|G_0|(31)2 \rangle , \]  

(A3)

and

\[ Z_{\ell_\Lambda: \ell_\Lambda} \equiv - Z_{\Lambda_3: \Lambda_2} = (-1)^{P'+1} Z_{2:3} \]
\[ \equiv (-1)^{P'+1} \langle (31)2|G_0|(12)3 \rangle . \]  

(A4)

In this case the the phase \( P' \) results from the exchange of the labels on particles 1 and 2, and is given by \( P' = s^1 + s^2 - S + \tau_1 + \tau_3 - t_3 \). The cyclic order Born amplitudes can in general be partial wave expanded for any three particles with specific spin and isospin \( [9,8] \).
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TABLES

**TABLE I.** Three-body channels allowed for the $S = \frac{1}{2}$, $T = \frac{1}{2}$ configuration.

| Channel | $t_\alpha$ | $S_\alpha$ |
|---------|------------|------------|
| $\Xi(\Lambda\Lambda)$: | | |
| NN ($^3S_1$) | 0 | 1 |
| NN ($^1S_0$) | 1 | 0 |
| $N(\Xi N)$: | | |
| $\Xi N$ ($^1S_0$) | 0 | 0 |
| $\Xi N$ ($^1S_0$) | 1 | 0 |
| $\Xi N$ ($^3S_1$) | 0 | 1 |
| $\Xi N$ ($^3S_1$) | 1 | 1 |
| $N(\Lambda\Lambda)$: | | |
| $\Lambda\Lambda$ ($^1S_0$) | 0 | 0 |
| $\Lambda(\Lambda N)$: | | |
| $\Lambda N$ ($^1S_0$) | $\frac{1}{2}$ | 0 |
| $\Lambda N$ ($^3S_1$) | $\frac{1}{2}$ | 1 |

**TABLE II.** Three-body channels allowed for the $S = \frac{3}{2}$, $T = \frac{1}{2}$ configuration.

| Channel | $t_\alpha$ | $S_\alpha$ |
|---------|------------|------------|
| $\Xi(\Lambda\Lambda)$: | | |
| NN ($^3S_1$) | 0 | 1 |
| $N(\Xi N)$: | | |
| $\Xi N$ ($^3S_1$) | 0 | 1 |
| $\Xi N$ ($^3S_1$) | 1 | 1 |

**TABLE III.** The effective range parameters for the $\Lambda\Lambda$–$\Xi N$ coupled channel separable potentials in the $^1S_0$ partial wave.

| Pot. | $a_{\Lambda\Lambda}$ (fm) | $r_{\Lambda\Lambda}$ (fm) | $a_{\Xi N}$ (fm) | $r_{\Xi N}$ (fm) | B.E. (MeV) |
|------|-------------------------|-------------------------|-----------------|-----------------|-----------|
| SA   | -1.90                   | 3.33                    | -2.08-0.81i     | 3.44-0.22i      | UB        |
| SB   | -21.0                   | 2.54                    | -2.07-6.52i     | 2.62-0.15i      | UB        |
| SC1  | 7.84                    | 1.48                    | 3.05-5.28i      | 1.45+0.074i     | 0.71      |
| SC2  | 3.36                    | 1.0                     | 3.35-2.50i      | 1.83-0.10i      | 4.73      |
TABLE IV. The total breakup cross section as calculated via unitarity ($\sigma_{B-\uparrow}^u$) and by integrating the NDES ($\sigma_{B-\uparrow}^d$) at an energy of 25.7 MeV.

| $\Lambda\Lambda - \Xi N$ | $\sigma_{B-\uparrow}^u$ | $\sigma_{B-\uparrow}^d$ |
|--------------------------|------------------------|------------------------|
| SB                       | 83.32                  | 83.77                  |
| SC1                      | 111.69                 | 111.69                 |
FIG. 1. The three distinct contributions to the breakup amplitude, with (a) and (b) corresponding to the first term in Eq. (17) and (c) corresponding to the last two terms in Eq. (17).

FIG. 2. The NDES following the capture of $\Xi^-$ on the deuteron for the potentials SA, SB, SC1, and SC2.
FIG. 3. The NDES following the capture of $\Xi^-$ on the deuteron for the potentials SB including diagrams (a) and (c) of Fig. [1].

FIG. 4. The NDES following the capture of $\Xi^-$ on the deuteron for the potential SB including diagrams (b) and (c) of Fig. [1].
FIG. 5. The NDES following the capture of $\Xi^-$ on the deuteron for the potentials SB including diagram (a) of Fig. [1].

FIG. 6. The NDES following the capture of $\Xi^-$ on the deuteron for the potentials SB including diagram (b) of Fig. [1].
FIG. 7. The NDES following the capture of Ξ− on the deuteron for the potentials SB including diagram (a) of Fig. 1 to lowest order, i.e. $g_N \tau_{\Lambda\Lambda;\Xi N} Z_{N;\Xi}$.

FIG. 8. The NDES following the capture of Ξ− on the deuteron, over the FSI region, for a series of interactions in which the strength of the coupling between the Λ–Λ and Ξ–N is modified by multiplying by a factor $R$, i.e. $C_{\Lambda\Xi} \rightarrow R C_{\Lambda\Xi}$ with $R = 0.5, 0.75, 1.0, \text{ and } 1.25$. 