An Optimal Design Scheme of Missile Trajectory

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Abstract—The optimization of missile trajectory is an important part of missile design. Its difficulty lies in the complex force of the missile and the different laws of the trajectory parameters in each stage. The existing trajectory simulation models have problems such as complex models, low accuracy, simplifying the initial motion state of the missile, and failing to fully reflect the characteristics of the active and passive motions. In order to solve these problems, this paper extracts important model variables on the basis of analyzing the force of the missile at different stages, and establishes a ballistic model with time-varying parameters. At the same time, according to the engineering design method of the missile flight program, the change rule of the pitch angle of the active phase of the missile with time is drawn up. Finally, an intercontinental ballistic missile is used as the object to simulate and analyze the proposed model. The results show that the method proposed in this paper can design and optimize missile trajectory well.

1. Introduction
The optimization of missile trajectory is a classic problem in aircraft design and simulation. Based on the given parameters, designing the optimal missile trajectory (usually the trajectory with the farthest range) through computer simulation has important guiding significance for the actual engineering realization[1]. In order to describe the flight process of the missile, previous scholars have established various ballistic models. However, the existing ballistic models are often too complex and redundant in parameters, which are disadvantageous to the efficiency and accuracy of the solution[2]. In this regard, this paper proposes a simplified model for missile trajectory optimization, which greatly improves the calculation efficiency without affecting the accuracy, providing a new idea for the optimization of missile trajectory design.

2. Model establishment and simplification
2.1. mathematical model
During the movement of the missile, the instantaneous equilibrium assumption is used to ignore the movement of the missile around the center of mass and only consider the movement of the center of mass[3]. The factors that affect the acceleration of the center of mass are: thrust \( P \), aerodynamic force...
R, gravitational force $mg$, additional Coriolis force $F_e$ and centrifugal inertial force $F_k$. Consider in the launch coordinate system:

Thrust $P$:

$$
\begin{bmatrix}
P_x \\
P_y \\
P_z
\end{bmatrix} = G_B \begin{bmatrix}
P \\
0 \\
0
\end{bmatrix}
$$

(1)

Aerodynamic force $R$:

$$
\begin{bmatrix}
R_x \\
R_y \\
R_z
\end{bmatrix} = G_B \begin{bmatrix}
-C_x qS_m \\
C_y qS_m \alpha \\
C_z qS_m \beta
\end{bmatrix}
$$

(2)

Gravitational force $mg$:

$$
m\ddot{g} = mg_r \ddot{r}^0 + m g_\omega \dot{\omega}_e^0
$$

(3)

$$
g_r = -\frac{J^M}{r^2} \left[1 + J \left(\frac{a_e}{r}\right)^2 \left(1 - 5\sin^2 \phi\right)\right]
$$

(4)

$$
g_\omega = -2 \frac{J^M}{r^2} J \left(\frac{a_e}{r}\right)^2 \sin \phi
$$

(5)

Additional Coriolis force $F_e$:

$$
\vec{m}\ddot{\alpha}_e = -2m\dot{\omega}_e \times \vec{v}
$$

(6)

Centrifugal inertial force $F_k$:

$$
\vec{m}\ddot{a}_e = -m\ddot{\omega}_e \times (\dot{\omega}_e \times \vec{r})
$$

(7)

Therefore the acceleration is:

$$
\ddot{a} = \frac{\ddot{p}}{m} + \frac{\ddot{r}}{m} + \ddot{g} + \ddot{\omega}_e \times \vec{v} + \ddot{\alpha}_k
$$

(8)

The overall quality at a certain moment is:

$$
m = m_0 - dm \times t
$$

(9)

The above formulas are the equations of the center of mass motion of the missile during flight. Among them, the atmospheric density refers to the US1976 table.

2.2. Basic assumptions

2.2.1. Earth's gravitational field and shape model. The accurate calculation of the Earth's gravitational field is very complicated. In this model, in addition to the basic spherical gravity, only the J2 term is considered for non-spherical gravity. Assume that the shape of the earth is a standard ellipsoid.

2.2.2. Thrust model. The engine thrust and specific impulse are both at nominal values. After turning on, the thrust immediately reaches the rated value; after turning off, the thrust instantly drops to 0. The mass of the missile is calculated according to the consumption per second, and the mass of the missile is the mass of the warhead after shutting down.

2.2.3. Earth's atmosphere parameters. The Earth’s atmosphere model adopts the standard atmosphere table, and the aerodynamic force is only considered when the altitude is below 90km. The influence of aerodynamic force is not considered in the reentry section.
2.2.4. Pitch angle model. Assuming that the yaw angle and the roll angle are constant at 0, only the change in the pitch angle is considered. The pitch program angle of the active section is assumed to be in the form of a function shown in Figure 1.

When the missile takes off vertically, the starting program angle is $\phi_{p0} = 90^\circ$. After $t_1$ time of fixed program angle flight, it enters the turning phase. At this stage, the program angle decreases linearly. After $t_2 - t_1$ time, the fixed program angle flight is maintained again. After reaching the predetermined shutdown time $t_k$, the engine shuts down.

![Pitch program angle](image)

Figure 1. Pitch program angle

Among them, the flight time of the two fixed program angles are both set to 5s, that is, $t_3 = 5s$, $t_k - t_2 = 5s$. The intermediate linear descent process can be expressed by the following formula

$$\phi_p(t) = K \cdot t + \phi_{p0}$$

(10)

It can be seen from Figure 1 that $\phi_{pf} = \phi_p(t_2)$, and the terminal pitch program angle satisfies the constraint $10^\circ \leq \phi_{pf} \leq 70^\circ$.

3. Missile trajectory optimization

According to the initial time conditions, the acceleration can be calculated and iterated. The velocity vector can be found by performing the first-order integration on the acceleration; similarly, the acceleration vector can be obtained by performing the second-order integration on the acceleration. The acceleration at the next moment can be solved by using the fourth-order Runge-Kutta equations[4], so that the velocity and position at each moment can be calculated.

Given the pitch program angle law shown in Figure 1, the parameters that need to be designed are mainly the terminal pitch program angle $\phi_{pf}$. By adjusting this parameter, the missile's flight trajectory can be changed. When this parameter is determined, the missile's maximum range flight trajectory is also uniquely determined. With the aid of the simulated annealing algorithm[5], the terminal pitch program angle is optimized. Set the optimization interval to $[10^\circ, 70^\circ]$, and finally output the maximum range and the program angle corresponding to the maximum range.

4. Optimization results

Taking a certain type of missile as an example, its parameters are shown in the tables.
Table 1. Basic parameters of missile.

| Parameter                                         | Data  | Unit  |
|---------------------------------------------------|-------|-------|
| Reference area                                    | 2.2   | m²    |
| Maximum takeoff mass                              | 30000 | kg    |
| Warhead mass                                      | 1200  | kg    |
| Shell quality                                     | 1800  | kg    |
| Rated specific impulse of engine                  | 320   | s     |
| Rated engine thrust                               | 400   | kN    |
| Axial force coefficient                           | 0.30  | /     |
| Derivative of normal force coefficient to angle of attack | 0.05  | 1/deg |

Table 2. Launch mission parameters.

| Parameter              | Data  | Unit |
|------------------------|-------|------|
| longitude              | 112.6 | deg  |
| Geographic latitude    | 37.5  | deg  |
| Launch height          | 1500  | m    |
| Launch azimuth         | 270   | deg  |

Using computer for simulation, the best optimized missile trajectory parameters are shown in Table 3:

Table 3. Optimization Results.

| Maximum range angle | Maximum range  | The best terminal program angle | Simulation time |
|---------------------|----------------|---------------------------------|-----------------|
| 41.788°             | 4646.913 km    | 24.349°                         | 1267 s          |

Draw the change graph of each time-varying parameter, as shown in Figure 2.

(a) Pitch program angle.  
(b) Flight altitude.
5. Data analysis

Through computer simulation, it can be found that the optimal fuel loading is not intuitively full load (27000kg, 4568.882km), but slightly lower than full load (26962.23kg, 4646.913km). This is due to the impact of iteration accuracy. In the case of little difference in mass (20-40kg), the magnitude of the difference in shutdown time is only 0.1s. In order to ensure the optimization efficiency, the selected step size is 1s. This may cause the missile with more fuel to fail to perform effective calculations at the last shutdown point, making the range smaller than that of the missile with slightly less fuel. Plot the In order to study the relationship between step length and range, the Figure of range difference versus step length is drawn.
It can be seen that as the accuracy increases (the step size is reduced), the range difference gradually converges, but the calculation efficiency also decreases. Therefore, when the fuel mass is not much different, due to the limitation of calculation accuracy, there will be cases where the missile with less fuel mass will fly farther.

In addition, the theoretical landing point of a missile launched in the west direction should remain unchanged, but the actual landing point is southerly due to inertial centrifugal force. In the same way, if it is launched in the southern hemisphere, the landing site will be northerly. This is also consistent with the conclusion of theoretical mechanics[6].

6. Conclusion
In actual engineering applications, the missile trajectory is often optimized to make the corresponding performance of the missile meet the design requirements. The difficulty is that the optimization design of missile trajectory is tedious and complicated. The method proposed in this paper can effectively improve the optimization efficiency by simplifying the missile trajectory model within the allowable range of error. It has good application value in the initial stage of overall missile design. However, this algorithm cannot get rid of the limitation of accuracy on efficiency. But with the improvement of computer performance and the innovation of algorithms, this method will have a larger application space.

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