Investigation of multidimensional control systems in the state space and wavelet medium

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Abstract. The notions are introduced of “one-dimensional-point” and “multidimensional-point” automatic control systems. To demonstrate the joint use of approaches based on the concepts of state space and wavelet transforms, a method for optimal control in a state space medium represented in the form of time-frequency representations (maps), is considered. The computer-aided control system is formed on the basis of the similarity transformation method, which makes it possible to exclude the use of reduced state variable observers. 1D-material flow signals formed by primary transducers are converted by means of wavelet transformations into multidimensional concentrated-at-a-point variables in the form of time-frequency distributions of Cohen’s class. The algorithm for synthesizing a stationary controller for feeding processes is given here. The conclusion is made that the formation of an optimal control law with time-frequency distributions available contributes to the improvement of transient processes quality in feeding subsystems and the mixing unit. Confirming the efficiency of the method presented is illustrated by an example of the current registration of material flows in the multi-feeding unit. The first section in your paper.

1. Introduction

With bringing the concept of “wavelets” into the sphere of automatic control, a need arose to introduce and separate the notions “one-dimensional-point” and "multidimensional-point" concerning automatic control systems (ACS’s). In the first - at each physical point of the system a one-dimensional (1D-) signal is considered, in the ACS's of the second type - a multidimensional one (MD-signal). In this case, both these ACS’s can be called and considered as multidimensional systems, but putting different semantics into these concepts: 1) a one-dimensional-point ACS from the point of view of the state space method as the basic apparatus of the control theory within the modern paradigm is a multidimensional, or vector, system; the signal vectors (of control u(𝑡), state x(𝑡), and output y(𝑡)) are formed in such systems as sets of scalar (1D-) signals distributed over different points of the system; 2) a multidimensional-point system is multidimensional one in its genetics of the representations (i.e., MD-images proper) for 1D-signals, by which the latter are replaced in the system; the signal vectors are formed as vectors physically concentrated at a corresponding point, and they are interpreted as vector-concentrated signals.

2. Optimal systems - internal features

To demonstrate the joint use of approaches based on the concepts of state space and wavelet transforms, a method for optimal control of various technological objects in a state space medium...
represented in the form of time-frequency representations (maps), is considered. The technological object analyzed is a mixture-producing aggregate for the production of disperse mixtures.

The computer-aided control system is formed on the basis of the similarity transformation method, which makes it possible to exclude the use of reduced state variables observers.

In the system presented here, the so-called Cohen class distributions [1] are used – to indicate with their help the signals of current material flows. This feature makes the control system effective, and from the user’s point of view – more informative and semantically clear.

We consider the linear stationary system of multi-ingredient dosing in the pertinent block of the mixture-producing aggregate, the dynamics of which is described by the equations

\[
\begin{align*}
x(t) &= Ax(t) + Bu(t), \\
y(t) &= Cx(t) + Du(t),
\end{align*}
\]

The system is strictly causal [2], therefore the over-control output matrix \( D \) is equal to zero. Here, \( x(t) \in \mathbb{R}^n \) is the vector of state variables (VSV); \( u(t) \in \mathbb{R}^m \) – the control vector; \( y(t) \in \mathbb{R}^r \) – the output vector; \( x(0) \in \mathbb{R}^n \) – the VSV at time \( t = t_0 \), \( A, B, C \) are matrices of dimensions \((n \times n), (n \times m), (r \times n)\).

The problem is to transfer the multi-dosing system from the initial state \( x_0 \) to the lowest possible state in a temporal interval \([t_0, t_f]\), without the cost of unduly large control actions \( u(t) \) to achieve this goal. In this case, the problem of the optimal regulator (linear quadratic regulator) is formulated as the problem of determining the optimal control law \( u^*(t) \) on the interval \([t_0, t_f]\), within which the following optimal criterion \( J \) must be minimized:

\[
min J\{x(t_0), x(t_f)\} = \int_{t_0}^{t_f} \left[ x^T(t) Q_s x(t) + u^T(t) R u(t) \right] dt + x^T(t_f) Q_r x(t_f),
\]

where \( Q_s \in \mathbb{R}^{n \times n} \) and \( Q_r \in \mathbb{R}^{r \times r} \) are symmetric non-negatively definite matrices (respectively, the matrices of weighted integral quadratic quality estimates for the current \( Q_s \) and final \( Q_r \) state vectors of the dosing process, and \( R \in \mathbb{R}^{m \times m} \) is a symmetric positively definite matrix reflecting the contribution of each scalar component of the control vector \( u(t) \) to the optimal criterion. In other words, here the so-called Boltz problem [2] is being solved, that is, in fact, a solution is given to the problem of a purposeful change in an optimal control law in response to a detectable effect - during the ongoing monitoring - of changing the complex of characteristics in the material flow at the output of the \( j \)-th feeder, or at the output of the multi-dosing block. The inclusion of the unit for estimating the unevenness of the material flow in the mixture-producing aggregate into the structure of the optimal control system will enable to realize the optimal control of mixture-producing processes in the state-space medium using current monitoring in a wavelet [3] environment.

In this case, 1D-material flow signals formed by primary transducers (strain gage and piezoelectric sensors) are converted by means of wavelet transformations into multidimensional concentrated-at-a-point variables in the form of time-frequency distributions of Cohen’s class [1]. Thus, the control law for the total state vector written in the format of the mentioned distributions will look like this:

\[
u(t) = K \cdot Coh(t, \omega),\]

where \( dim \ Coh(t, \omega) = (n \times 1) \), \( t \) is lapsed time, \( \omega \) is the instantaneous angular frequency of the atoms on the time-frequency distribution (time-frequency map – TFM), \( K \) is the state feedback matrix. Consequently, the equations of state and output for causal systems are written accordingly as follows:

\[
\begin{align*}
\partial_c \{ Coh(t, \omega) \} / \partial t = & A \cdot Coh(t, \omega) + B \cdot K \cdot Coh(t, \omega) = (A + B \cdot K) \cdot Coh(t, \omega) = H \cdot Coh(t, \omega), \\
y(t) = & C \cdot Coh(t, \omega) + D \cdot K \cdot Coh(t, \omega) = (C + D \cdot K) \cdot Coh(t, \omega).
\end{align*}
\]
Taking into account that a number of state variables (SV’s) cannot be registered with sensors, it was decided to form alternative state variable vectors \( \bold{v} \) based on the similarity transformation method. In this case, the state, control, and over-state output matrices change according to the expressions:

\[
A_v = P^{-1}A \cdot P; \quad B_v = P^{-1}B; \\
C_v = C \cdot P; \quad D_v = D,
\]

where \( P \) is a transformation matrix. Note that a pertinent alternative vector \( Coh(t,\omega)^v \) is formed depending on the possibilities/impossibilities of their actual registering and also a number of regime/constructive parameters of the dosing unit. In particular, the alternative VSV reflecting the dosing process in a multi-ingredient unit including a discrete-type feeder and two continuous-type feeding devices, was formed, and its vector-matrix model analyzed. The scalar components of the alternative vector were generated in the form of linear combinations of the original vector, provided that only the actual signal components were included in the new VSV: the virtual SV’s of the original vector that occurred while representing a signal of the discrete-type feeder as a Fourier model, weren’t taken into account. Consequently, when using a new alternative VSV in our research and forming a control signal from the full feedback vector over the converted VSV, there was no need for the use of reduced observers. The latter greatly simplified the structure of the control system. For example, the process of feeding in a multi-component unit was described by a pertinent 17th-order vector-matrix model, and the procedure of inverting the transformation matrix \( P \) to turn an initial state vector into a relevant alternative one was realized in the MathCad package.

The optimization of the stationary feeding system in terms of quadratic integral quality estimates was carried out in accordance with the optimality criterion written as the following quadratic form

\[
Coh^T(t,\omega) \cdot K \cdot Coh(t,\omega) = J \quad \text{(Bellman function)},
\]

in which the symmetric matrix \( K \) is determined while solving a relevant Riccati matrix differential equation; in such a case, the quadratic form with matrix \( K \) can be written as the Boltz/Lagrange functional [2].

Thus, under the optimal criterion in the Lagrange form, the functional is as follows (the problem of Lagrange):

\[
J = Coh^T(t,\omega) \cdot K \cdot Coh(t,\omega) = \int_{t_0}^{t_f} \left[ Coh^T(t,\omega) \cdot Q \cdot Coh(t,\omega) + u^T(t) \cdot R \cdot u(t) \right] dt \quad \text{(1)}
\]

The integrand consists of two quadratic forms:

1) the quadratic form of the system state \( Coh^T(t,\omega) \cdot Q \cdot Coh(t,\omega) \) with positively definite symmetric matrix \( Q, \dim Q = (n \times n) \);

2) the quadratic control form \( u^T(t) \cdot R \cdot u(t) \) with a positively definite symmetric matrix \( R, \dim R = (m \times m) \). It is required to find the optimal control with a full state feedback, transferring the system from an arbitrary initial point \( Coh(t_0,\omega) = Coh(0) \) to a finite point \( Coh(\infty) = 0 \) and providing the functional minimum.

3. Algorithm of controller synthesis

The algorithm for synthesizing a stationary linear state controller for feeding processes looks like the next one below.

The following conditions must be specified in the source data:

- the system state equation (or directly matrix \( A \) and \( B \) for the multi-component feeding unit);
- the objective functional \( J \) (i.e., the optimal criterion) in the form of equation (1).

The functional \( J \) and the two integrands as the quadratic forms are scalar functions.

1. Determine the matrix \( Q \) from the quadratic form \( Coh^T(t,\omega) \cdot Q \cdot Coh(t,\omega) \) by performing the operations of multiplying the matrix \( Q \) with the Cohen’s distribution vectors and comparing the result obtained with the form of the given functional \( J \).
2. Similarly, we determine the weight matrix $R$ in the quadratic control form.

3. The next step is the calculation of the elements of the matrix $K$ that is part of the quadratic form, the Bellman function, $J = Coh^2(t, \omega)\cdot K\cdot Coh(t, \omega)$, from the algebraic Riccati equation [2]

$$- K\cdot A - A^T\cdot K + K\cdot B\cdot R^{-1}\cdot B^T\cdot K - Q = 0.$$ 

Performing the operations of matrix multiplication and, ultimately, addition operations, yields an equation of the form

$$\begin{bmatrix}
  z_1 & \ldots & z_n \\
  \vdots & \ddots & \vdots \\
  z_n & \ldots & z_m
\end{bmatrix} = \begin{bmatrix}
  0 & \ldots & 0 \\
  \vdots & \ddots & \vdots \\
  0 & \ldots & 0
\end{bmatrix},$$

$$\dim z = (n \times n), \dim 0 = (n \times n).$$

From the last matrix equation we write $n^2$ uniform equations, and solving them together, we determine the coefficients $k_{ij}$ of the matrix $K$.

4. We check the elements $k_{ij}$ of the matrix $K$ for positive definiteness by the Sylvester’s criterion [2], and, according to the criterion, determine the true values of the coefficients $k_{ij}$.

5. Substituting the found coefficients $k_{ij}$ into the expression

$$u^*(t) = R^{-1}\cdot B^T\cdot K\cdot Coh(t, \omega)$$

yields the optimal control law.

4. Cohen’s class distributions - applied aspects

To obtain a graphical representation of the optimal control law $u^*(t)$, it is necessary to determine all SV’s in the Cohen distribution format, included in the optimal control law. The SV’s are calculated from the scalar state equations with the SV’s in the form of time-frequency distributions, taking into account the matrices $A$ and $B$.

To minimize the functional $J$, we used (as mentioned above) multidimensional-point variables in the form of time-frequency distributions of Cohen’s class $Coh(t, \omega)$ (in our case, the Wigner-Ville [4] and Choi-Williams [1, 5] distributions).

When calculating the time-frequency distributions of complex multi-component signals, the latter are expressed in the form of discrete comb functions. In this case, in accordance with the sampling theorem [6], the Wigner-Ville distribution of a comb signal is defined as

$$W(t, \omega) = \frac{T}{\pi} \sum_{k=0, \ldots, 1/T} s^*(t - kT)s(t + kT)\exp(-j2\omega kT),$$

where $1/T$ is sampling frequency (sample rate); it is selected from the relation $T \leq \pi/2\omega_{\text{max}}$, where $\omega_{\text{max}}$ is the maximum frequency in the signal spectrum. As it can be seen from the expression (2), the discrete Wigner distribution is periodic with period $\pi$ (since the frequency of the imaginary exponential is equal to $2\pi$), and not $2\pi$ - as in the case of a continuous signal. Hence, the sampling rate should be twice the Nyquist frequency, that is:

$$\omega_s \geq 2\omega_N = 4\omega_{\text{max}}.$$

With the comb form of the material flow signal sampled in time

$$\hat{x}(t) = \sum_{n=0}^{\infty} x(nT) \delta(T - nT)$$
and a continuous frequency $\omega$, we obtain a discrete version of the Wigner distribution:

$$\hat{W}(n, \omega) = \sum_{k=0}^{\infty} s^*(n - 2k)T_s(kT)\exp[-j\omega(2k - n)T]$$

This expression corresponds to the condition of loss-free (in informational sense) restoration of a continuous one-dimensional material flow signal from its discrete distribution.

In cases where the process model is characterized by a transfer function that has the form of a rational function with zeros available, it is necessary to divide the transfer function model to pole- and zero-containing parts, while for the pole-containing part (for the differential-free fragment of the structural scheme) the vector-matrix model of the multi-feeding system can be described both in a Frobenius form and with the help of a similarity transformation using actual physical signals recorded by measuring converters. The need for such a procedure arises in the analysis of feeding processes, when one or more feeders operate in a batch (discrete) mode, provided that the description of material flow signals is performed on the basis of Fourier models.

It should be noted that with modal controllers regulators supplied with virtual sensors (state observers), used in systems for which vector-matrix models are formed on the basis of augmented (i.e., original) structural schemes, calculations of the feedback matrix over the vector of state variables and the observer matrix are carried out according to corresponding Ackerman’s formulas [2].

The data of current recording all of the material flow signals with synchronous visualization of them in the form of time-frequency distributions (time-frequency maps of controlled processes) make it possible to efficiently control and monitor – at the dispatch level – the processes occurring in different fragments of the mixture-producing aggregate, and the formation of an optimal control law with time-frequency distributions available (the latter registered in the wavelet medium and used instead of one-dimensional SV’s), contributes to the improvement of transient processes quality in feeding subsystems, inside the equipment for transferring ingredient substances within the production space, and in the mixing unit - when readjusting their modes of operation.

As an example of the current registration (carried out synchronously with the control function) of material flows in the multi-feeding unit of the mixing aggregate, figure 1 shows the representation of the steady-state feeding process in the form of Wigner distributions at the outlet of the multi-feeding unit. The latter consists of one batch feeder, two spiral ones running continuously (current frequencies are of 0.20Hz, 3.23Hz, and 4.02Hz), and a spiral feeder that functions discretely in time with a central frequency of 6.9 Hz in the spectrum band and a frequency of 0.077 Hz as a dose-forming one.

**Figure 1.** Display of the steady-state feeding process in the form of Wigner distributions at the outlet of the feeding unit.
Thus, realizing the procedure of synchronous feeding processes monitoring in the wavelet medium, combined with the controlling process, makes the automatic control system effective, and from the user’s point of view – more informative regarding the semantics of the processes occurring in the mixture-producing unit.

5. Conclusion

To demonstrate the joint use of approaches based on the concepts of state space and wavelet transforms, a method for optimal control of a technological object in a state space medium is considered. In this case, the state variables are represented here in the form of time-frequency representations (wavelet maps).

The difference from the traditional method of synthesizing an optimal control system is that instead of one-dimensional vectors of state variables and of control in a closed system, multidimensional distributions of Cohen’s class are used, in particular, time-frequency Wigner time-frequency representations.

The control plant is the feeding unit of the mixture-producing aggregate, and the optimal control system itself is strictly causal.

The computer-aided control system is formed on the basis of the similarity transformation method, which makes it possible to exclude the use of reduced state variable observers.

The Riccati equation is used to determine the state feedback matrix.

And finally, the results of monitoring the flow at the output of the multi-component feeding unit are presented. The example of this demonstrates the effectiveness of the computer-aided control system using multi-dimensional representations in the wavelet medium instead of 1D- state variables.

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References

[1] Cohen L 1995 Time-Frequency Analysis (Englewood Cliffs: Prentice Hall) p 299
[2] Kim D P 2004 Theory of Automatic Control (Moscow: Fizmatlit) vol 2 p 464
[3] Daubechies I 1992 Ten Lectures on Wavelets: Regional Conference Series in Applied Mathematics (Philadelphia PA: CBMS-NSF SIAM) p 357
[4] Fedosenkov B A, Nazimov A S and Sheboukov A V 2004 Automation and Modern Technologies: Automation of Scientific Research and Production Processes 8 7–13
[5] Choi H I and Williams W J 1989 IEEE Trans. Acoust., Speech, and Signal Proc. 37(6) 862–71
[6] Luenberger D G 2006 Information Science (Princeton NJ: Princeton University Press) p 448