An Equivalent Circuit Model for Electrostatic Energy Harvester utilized Energy Harvesting System

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Abstract. In this study, we report an equivalent circuit model of an electrostatic energy harvester for a SPICE circuit simulator. In order to simulate a harvesting system, the output power of the device is calculated in the simulator. The capacitance between the electrodes is obtained by FEM analysis by taking the fringing effect into account and the result is applied to a sub-circuit model for the simulator. Mechanical vibrations are converted into electricity by an equivalent circuit model of a mass-spring structure and an electrostatic energy harvester. The simulated output power and output waveform correspond with the measurement results of our electrostatic energy harvester. We also simulate the operation of a harvesting system connected with a power management IC.

1. Introduction
Recently, energy harvesters are attracted as a source of power for wireless sensor networks[1]. However, there are a few studies about harvesting system as electronic circuit components. We fabricated a electrostatic vibration energy harvester. Charge transfer on the counter electrode is caused by capacitive induction from an electret, which is a material with electric charge. The output power depends on the surface potential of the electret and the change in capacitance between counter electrodes and the base electrodes. It is possible to estimate the output power by applying those parameters to a SPICE circuit simulator. In a previous research, some circuit models of a PZT harvester[2] or an actuator[3] have been analyzed. We can also evaluate the harvesting system by combining a power management IC for vibration energy harvester.

In this study, the capacitance between the electrodes is obtained by FEM analysis and implemented in the equivalent circuit model of an electrostatic energy harvester. We also express a mass-spring system in a SPICE simulator. The simulation results are in good agreement with the measurement result. Then, the harvesting system is simulated by combining a power management IC.

2. Capacitance analysis
In our previous study, our electrostatic energy harvester consists of a Si counter electrode, an electret with base electrodes placed on a mass-spring structure, and a cover glass as shown in figure 1. The change in capacitance between the electrodes is caused by variation of the overlapping area due to mass vibration.
FEM analysis (ANSYS 14.0) is used to calculate the capacitance between the electrodes by taking the fringing effect into account. Figure 2 shows the FEM analysis model considered bipolar charged electret[5]. All dimensions are as same as our previous electrostatic energy harvester. A 2.5 μm thick CYTOP and 0.5 μm thick buried grid electrodes (BGE-N, BGE-P) are fabricated on a 0.5 μm thick SiO₂ isolation layer on a 500 μm thick Si substrate. The height of the slit-shaped Si counter electrode is 260 μm. The depth of all the elements is 10 mm. The change in capacitance for each displacement of the counter electrode is approximated as follows:

\[ C(x) = C_{\text{init}} + \Delta C \cos \left( \frac{1}{p} \pi x \right), \]

where \( C_{\text{init}} \), \( \Delta C \), \( p \), and \( x \) denote the initial capacitance, the change in capacitance, the pitch length of the base electrode, and the mass displacement, respectively. The change in capacitance between the counter electrodes and the BGE-N is anti-phase to that between the counter electrodes and the BGE-P.

![Figure 1. Electrostatic energy harvester[4].](image1)

![Figure 2. Analysis model and change in capacitance for displacement.](image2)

3. Equivalent circuit model

3.1. Variable Capacitance model

We developed an equivalent circuit model of the electrostatic energy harvester with a free SPICE circuit simulator LTSpice IV (Linear Technology Corp., USA). Figure 3 shows (a) the circuit symbol and (b) the internal circuit of a variable capacitance model for taking the time-function of capacitance \( C(t) \). It converts controlling voltage into capacitance at the ratio of 1 pF/V. This model calculates the output current by following equation:

\[ i = \frac{dQ(t)}{dt} = \frac{d(C \times V)}{dt} = \frac{d(V_{\text{con}} \times V_{\text{in}})}{dt}, \]

where \( V_{\text{con}} \) and \( V_{\text{in}} \) denote the capacitance controlling voltage and the voltage across the variable capacitance model, respectively.

![Figure 3. Variable capacitance model.](image3)
3.2. Equivalent circuit model of electrostatic energy harvester
The output transient voltage of the electrostatic energy harvester is calculated from the change in capacitance and the surface potential of the electret and time-function of the mass displacement as follows:

\[ I(t) = \frac{dQ(t)}{dt} = \frac{V_{\text{elect}}}{C(t)} \frac{dC(t)}{dt}, \]

where \( V_{\text{elect}} \) and \( R_L \) denote the surface potential of the electret and the load resistance, respectively.

Figure 4 shows the equivalent circuit model of the electrostatic energy harvester connected with a load resistance. \( V_p, V_n \) denote the positive, negative potential surface voltage of the electret, respectively. Two variable capacitance models \( (C_N, C_P) \) connected in series with the bipolar voltage due to our bipolar charged electret. \( C_N \) expresses the capacitance between the counter electrode and BGE-N, \( C_P \) of the other expresses that between the counter electrodes and BGE-P. The voltage sources for controlling the variable capacitance models \( (V_{\text{con}N}, V_{\text{con}P}) \) are used to express the approximation equation of (1) that is obtained by FEM analysis. We consider the parasitic capacitance \( (C_{\text{par}}) \) and it is connected in parallel with the load resistance.

3.3. Equivalent circuit model of mass-spring structure
It is possible to simulate the output power from human or machine motion by connecting harvester sub-circuit model with a mass-spring circuit model. A basic mechanical system consists of applying force \( (F_m) \), a proof mass \( (M) \), a damper \( (D) \), and a spring \( (K) \). The equation of motion is used to simulate the response of mechanical system and expressed as follows:

\[ M \frac{d^2x(t)}{dt^2} + D \frac{dx(t)}{dt} + Kx(t) = F_m(t). \]

The equation is solved by a RLC series circuit in a SPICE circuit simulator and the equation (4) is rewritten as follows:

\[ L \frac{d^2q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{c} q(t) = V(t). \]

Figure 5 shows the equation circuit model of a mass-spring structure with a limit displacement of the mass. We used a capacitor of 1 F as mathematic integrator to describe the mass displacement \( x \) (charge \( q \) in an electronic field) as voltage. There is a limit displacement of the mass \( x_{\text{lim}} \) depending on the device size. When the mass contacts the frame \( (x=x_{\text{lim}}) \), the kinetic energy of the mass is reduced. At the same time, the mass receive a repulsive force from the frame. We used a resistor and a capacitor as the frame to implement with the fundamental damper and spring as shown figure 5. Here, \( D_f \) and \( K_f \) denote the damping coefficient and the spring constant of the frame, respectively; these parameters are determined from experimental results. The voltage controlled switches (SW) changes two closed-circuit in case that the mass displacement achieves its limit \( (x=x_{\text{lim}}) \).
4. Simulation and comparison with measurement

Our electrostatic energy harvester was attached to the shaker (ET-126; Labworks Inc., USA) and vibrated at 352 Hz; the resonant frequency of the mass-spring structure. The acceleration of 7 G is sufficiently large to ignore the electrostatic force, which is caused by charged electret. We simulated under the same condition in the LTSpice. Table 1 summarizes the simulation parameters. Figure 6 shows the measurement and the simulated maximum output power versus load resistance. From the result, the maximum output power is 9.8 μW with the optimal load resistance of 0.9 MΩ and it corresponds with simulated result. Figure 7 shows the output waveform connected with the load resistance of 0.9 MΩ. The response of the electrostatic energy harvester is simulated on the equivalent circuit model with our device. The output frequency of our electrostatic energy harvester is larger than the applied acceleration frequency because the amplitude of the vibration is larger than the electrode pitch.

We evaluate the harvesting system by combining with a power management IC with our harvester sub-circuit. We use the LTC3588-1 as a power management IC. Since the LTSpice and LTC3588-1 are released from the same company, we can easy to combine whole harvesting system in the LTSpice as shown in figure 8(a). From the simulation result, we found that our previous electrostatic energy harvester has too large parasitic capacitance of 70 pF that wastes the generated power to charge the battery by LTC3588-1. If the parasitic capacitance can be reduced to 50 pF, an appropriate power managed voltage of 1.8 V will be obtained at intervals of approximate 150 s as shown in figure 8(b).

Table 1. Simulation parameters

| Parameter                     | Value  |
|-------------------------------|--------|
| Air gap: \(g\)                | 43 μm  |
| Electrode width: \(w\)        | 74 μm  |
| Electrode pitch: \(p\)        | 100 μm |
| Initial displacement: \(x_{\text{ini}}\) | 38 μm  |
| Initial capacitance: \(C_{\text{init}}\) | 7.57 pF |
| Change in capacitance: \(\Delta C\) | 1.24 pF |
| Surface potential of bipolar electret: \(V_p, V_n\) | ± 250 V |
| Parasitic capacitance: \(C_{\text{par}}\) | 70 pF  |
| Mass weight: \(m\)            | 120 mg |
| Resonant frequency: \(F_r\)   | 352 Hz |
| Q factor: \(Q\)               | 100    |
| Limit displacement of the mass: \(x_{\text{lim}}\) | 200 μm |
| Damper of the frame: \(D_f\)  | 5      |
| Spring of the frame: \(K_f\)  | 10 kN/m |
Figure 6. Output power versus load resistance.

Figure 7. Output waveform connected with the load resistance of 0.9 MΩ

(a) SPICE circuit connected with LTC3588-1

(b) Waveform connected with LTC3588-1

Figure 8. Harvesting system connected with LTC3588-1. The parasitic capacitance $C_{par}$ is 50 pF.

5. Conclusion
In this study, an electro-mechanical model in a SPICE circuit simulator is investigated. The relationship between the capacitance and the counter electrode is analyzed by FEM analysis by taking the fringing effect into account. Applying the result to the sub-circuit model, the output power and output waveform can be calculated in the LTSpice. We evaluate harvesting system by combining a power management IC.

Reference
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