Influence of Photovoltaic Generation Harmonic on Ferromagnetic Resonance of Distribution Network

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Abstract. When the Distribution network is disturbed, the Potential transformer(PT) saturates. It can cause distortion of voltage and current waves to generate harmonics, which further causes ferromagnetic resonance in the Distribution network. This article aims to analyze the effect of high-frequency and wide-frequency harmonics caused by photo-voltaic on the ferromagnetic resonance of the Distribution network. Firstly, the resonant circuit and resonance conditions of the underground neutral Distribution network are described. Secondly, based on resonant circuit, relationship between the PT flux linkage and harmonic voltage is established. The relationship between harmonics caused by photo-voltaic power generation and the influence of the harmonics on the ferromagnetic resonance of the Distribution network is analyzed. The conclusion is that this harmonic has an effect on the ferromagnetic resonance of the Distribution network. The effect of low-order harmonics is more obvious than that of higher harmonics. Finally, MATLAB/Simulink is used to verify the correctness of the theoretical results obtained.

1. Introduction

As a common electrical equipment in the power grid, the Potential transformer’s operating status is closely related to the safe and reliable operation of the power grid. In China, the Distribution network usually uses the operation mode that the neutral point is not directly grounded. When the Distribution network fails or performs some operation, there can be Potential transformer damages [1-2]. The research shows that it is mainly due to the occurrence of ferromagnetic resonance in the Distribution network [3-4].

In recent years, environmental pollution, energy shortages and other problems have become increasingly prominent. The large-scale photo-voltaic power generation has become the trend of energy development in the future [5]. However, when the large-scale photo-voltaic power generation is connected to the Distribution network, it will cause a series of adverse effects on the power grid [6]. The Distribution network would not have occurred in the ferromagnetic resonance. There is resonance due to the photo-voltaic power generation access. To find reasons, literature [7-8] analyzed and discussed various overvoltages in the Distribution network. It concluded that the main reason for the over-voltage is related to the type of distributed power supply. Literature [9-10] pointed out that the large-scale distributed photo-voltaic generation harmonics are related to the ferromagnetic resonance of the Distribution network. Therefore, this article is based on that the large-scale photo-voltaic power generation is connected to Distribution network. It analyzes the influence of photo-voltaic harmonics on the ferromagnetic resonance of the Distribution network.
2. Resonance Circuit and Resonance Condition of Ungrounded Distribution Network with Neutral Point

The equivalent circuit of the Distribution network is shown in Figure 1. In the figure, \( u_a, u_b \) and \( u_c \) are the three-phase equivalent power supplies for the Distribution network. \( C_0 \) is the capacitance of the transmission line to the ground in the Distribution network. \( L_A, L_B \) and \( L_C \) are the three-phase equivalent excitation inductances of the PT. When the Distribution network is operating normally, the PT is not saturated and its excitation inductance \( X_L = \omega L \) is greater than the transmission line to ground capacitance \( X_{C0} = \frac{1}{\omega C_0} \). When the power Distribution system is disturbed, the core of the PT is saturated and the inductive reactance \( X_L \) is reduced, so that the inductive reactance \( X_L \) is smaller than the capacitive reactance of the transmission line to the ground.

![Figure 1. Equivalent circuit of Distribution network.](image1)

When the Distribution network is disturbed, the core of the PT is saturated, which will cause harmonic distortion of the grid voltage and current waveform distortion and resonance in the LC loop. The equivalent circuit is shown in Figure 2 [11]. In the figure, \( C_0 \) is the capacitance of the transmission line to ground. \( L \) is the equivalent excitation inductance of the PT. \( u_G \) is the nonlinear harmonic equivalent voltage of the excitation inductance. Set the equivalent harmonic voltage as:

\[
U_G = \sum_{k=2,3\ldots} U_{G_k} \sin k\omega t
\]

Where \( U_{G_k} \) is the amplitude of the voltage corresponding to the k-th harmonic.

![Figure 2. Ferroresonance Equivalent Circuit.](image2)

If the equivalent harmonic voltage makes the parameters of the PT satisfy: \( X_{G0} / X_L > 0.01 \), which the Distribution network will have resonance, but when the parameters not satisfies, the resonance will not occur [12].

3. The influence of Output Harmonics of Photovoltaic Generation on Ferromagnetic Resonance of Distribution Network

When photo-voltaic power is connected to the Distribution network, it makes the Distribution network harmonics have the characteristics of broad-frequency domain and high frequency [13].

Let the voltage of the Distribution network as:
\[ u(t) = U_m \sin \omega t + \sum_{k=3,5,\ldots} U_{km} \sin k \omega t \] (1)

Where \( U_{km} \) is the Voltage amplitude corresponding to the \( k \)-th harmonic caused by photo-voltaic power generation.

Therefore, the ferromagnetic resonance equivalent circuit becomes Figure 3, where \( u_k \) is the harmonic voltage caused by photo-voltaic power generation.

**Figure 3. Ferroresonance Equivalent Circuit Containing Harmonic Of Distribution Network.**

For the circuit in the Figure 3, according to the KCL can be drawn:

\[ \frac{d\psi}{dt} + \frac{1}{C_0} \int i dt = u_G + u_k \] (2)

Where \( \psi \) is the flux linkage of PT.

Equation (2) can be simplified Equation (3):

\[ C_0 \frac{d^2\psi}{dt^2} + i = \alpha C_0 ( \sum_{k=2,3,\ldots} k U_{ck} \cos k \omega t + \sum_{k=3,5,\ldots} k U_{km} \cos k \omega t) \] (3)

Because the magnetizing inductance of PT is nonlinear, its magnetization characteristic can be represented by formula (4) [14].

\[ i = g(\psi) = a\psi + b\psi^3 \] (4)

In the formula: \( a \) and \( b \) are the coefficients of the fitting curves of magnetization, which are all greater than zero.

Substituting Equation (4) into (3):

\[ C_0 \frac{d^2\psi}{dt^2} + (a\psi + b\psi^3) = \alpha C_0 ( \sum_{k=2,3,\ldots} k U_{ck} \cos k \omega t + \sum_{k=3,5,\ldots} k U_{km} \cos k \omega t) \] (5)

It is not easy to solve the upper Equation because it is second-order nonlinear differential equations. the Equation (5) can be simplified to Equation (6):

\[ \frac{b}{20} \psi^5 + \frac{a}{6} \psi^3 + C_0 \psi = -C_0 ( \sum_{k=2,3,\ldots} \frac{U_{ck}}{k\omega} \cos k \omega t + \sum_{k=3,5,\ldots} \frac{U_{km}}{k\omega} \cos k \omega t) \] (6)

The above quintic equation about flux linkage can be solved by Tianheng formula. Through Using Tianheng formula, the multiple root discriminants are: \( A = -\frac{ab}{24} \), \( B = 0 \), \( C = \frac{a^2b^2}{576} - \frac{b^3C_0}{64} \) and \( D = \frac{5b^4C_0}{256} ( \sum_{k=2,3,\ldots} \frac{U_{ck}}{k\omega} \cos k \omega t + \sum_{k=3,5,\ldots} \frac{U_{km}}{k\omega} \cos k \omega t) \). The total root discriminant: \( \Delta = D^2 - 4A^3 > 0 \).
When solving with the Tianheng formula, the number of the Equations is related to $A$, $B$, $C$, $D$ and $\Delta$.

1) When $A = B = C = D = 0$, Equation (6) has 5 identical real roots. Considering that the parameters $a$, $b$, and $C_0$ of the actual Distribution network cannot be zero, therefore the condition of $A = B = C = D = 0$ does not hold and Equation (6) has no solution.

2) When $AD \neq 0$, $B = C = \Delta = 0$, Equation (6) has 5 identical real roots, there are two identical roots. Considering that the total root discriminant $\Delta$ cannot be equal to zero in an actual Distribution network, so the solution of Equation (6) is also not satisfied too.

3) When $B = C = 0$, $\Delta > 0$, Equation (6) has a real root and two pairs of unequal conjugate imaginary roots. When the parameters of the Distribution network satisfy $C = \frac{a^2b^2}{576} - \frac{b^2C_0}{64} = 0$ or $a^2 = 9bC_0$, Equation (6) has a satisfactory solution.

Therefore, when the parameters of the Distribution network are satisfied $a^2 = 9bC_0$, the solution that satisfies equation (6) is:

$$\psi = \frac{4(\sqrt[4]{M_1} + \sqrt[4]{M_2})}{b}$$  \hfill (7)

In the formula: $M_{1,2} = \frac{D \pm \sqrt{D^2 - 4A^2}}{2}$.

Considering that the magnetizing characteristic parameters $a$ and $b$ of the PT in the Distribution network are small and the multiple root discriminants $A$ can be approximated as $A = 0$. Then $M_1 = D$ and $M_2 = 0$.

According to Equation (7), the flux linkage of the PT is: $\psi = 4\sqrt{D} / b$.

The reciprocal of the equivalent magnetizing inductance $L$ of the PT is:

$$\frac{1}{L} = \frac{di}{d\psi} = a + 3b\psi^2$$  \hfill (8)

When the Distribution network is disturbed, the derivative of the magnetizing inductance of the PT is:

$$\frac{1}{L} = \frac{ab + 48\left(5b^4C_0\frac{1}{256} \sum_{k=2,3} U_{\omega \cos k\omega} \right)^2}{b}$$  \hfill (9)

Assuming that the parameters of the Distribution network satisfy: $X_{c_0} / X_L = 1 / \omega^2 L C_0 \leq 0.01$, no ferromagnetic resonance occurs in the Distribution network.

When photo-voltaic power is connected to the Distribution network, the Distribution network is subject to the same perturbation PT excitation inductance becomes:

$$\frac{1}{L} = \frac{ab + 48D^2}{b}$$  \hfill (10)

Compared Equations (9) with (10), due to the influence of photo-voltaic power generation, the reciprocal of the magnetizing inductance of the PT increases under the Distribution network is subject to the same disturbance, indicating that the magnetizing inductance of the PT is reduced.
According to the literature [12], the critical conditions for the occurrence of ferromagnetic resonance in Distribution networks is \( \frac{X_{C0}}{X_L} = 0.01 \). Therefore, when the Distribution network is disturbed, the critical condition for the occurrence of ferromagnetic resonance is:

\[
\frac{X_{C0}}{X_L} = \frac{1}{\omega^2 LC_0} = \frac{ab + 48D^2}{\omega^2 b C_0} = 0.01
\]

Solving the above Equation: \( D = \left[ \frac{\omega^2 b C_0 - 100ab}{4800} \right]^\frac{5}{2} \), Which can be rewritten as:

\[
\sum_{k=2,3,\ldots}^\infty \frac{U_{ok}}{k\omega} \cos k\omega t + \sum_{k=3,5,\ldots}^\infty \frac{U_{mk}}{k\omega} \cos k\omega t = \frac{256}{5b^2 C_0} \left[ \frac{\omega^2 b C_0 - 100ab}{4800} \right]^\frac{5}{2}
\]

The ferromagnetic resonance of the Distribution network that will not occur after the disturbance is caused by the access of photo-voltaic power generation. The critical value of the harmonic voltage caused by photo-voltaic power generation is:

\[
\sum_{k=3,5,\ldots}^\infty \frac{U_{mk}}{k\omega} \cos k\omega t = \frac{256}{5b^2 C_0} \left[ \frac{\omega^2 b C_0 - 100ab}{4800} \right]^\frac{5}{2} - \sum_{k=2,3,\ldots}^\infty \frac{U_{ok}}{k\omega} \cos k\omega t = E
\]  \( \text{(11)} \)

In the formula: \( E = \frac{256}{5b^2 C_0} \left[ \frac{\omega^2 b C_0 - 100ab}{4800} \right]^\frac{5}{2} \). When the Distribution network is disturbed, the parameters of the system may be satisfy \( \frac{X_{C0}}{X_L} > 0.01 \), and then ferromagnetic resonance of the Distribution network occurs.

As seen from the left side of Equation (11), the harmonic voltage caused by photo-voltaic power generation is inversely proportional to the harmonic frequency. In other words, under the same harmonic content, the higher harmonic frequency is, the smaller the voltage value is. The results show that lower-order harmonics are more easily satisfied with the above conditions than higher-order harmonics. Thus, low-order harmonic has relatively obvious effect on ferromagnetic resonance of Distribution network.

Based on the above analysis, when the parameters of the Distribution network satisfies \( a^2 = 9bC_0 \) and the harmonic voltage caused by photo-voltaic power generation satisfies \( \sum_{k=3,5,\ldots}^\infty \frac{U_{mk}}{k\omega} \cos k\omega t > E \), the Distribution network that produces no ferromagnetic resonance after the effect of disturbance may produce ferromagnetic resonance under the action of the harmonic voltage, and the low-order harmonic has significantly higher influence than high-order harmonic. when the harmonic voltage caused by photo-voltaic power generation satisfies \( \sum_{k=3,5,\ldots}^\infty \frac{U_{mk}}{k\omega} \cos k\omega t \leq E \), the harmonic has relatively slight effect on the ferromagnetic resonance of the Distribution network.

4 Simulation analysis

According to the analysis of Section 3, it can be seen that when the same disturbance occurs in the Distribution network, the ferromagnetic resonance may occur in the Distribution network due to the access of photo-voltaic power generation. In order to check the correctness of the theoretical analysis, a simulation model of the ferromagnetic resonance of the Distribution network is built by MATLAB/Simulink, in which the three-phase PT is simulated with a saturable transformer model, and
the high-side DC resistance is 250 Ω, the excitation resistance is 800 kΩ. The relationship between $\psi(i) - i$ of PT is shown in Table 1.

| i/mA  | 0.283 | 0.655 | 0.977 | 1.377 | 1.650 | 1.998 | 3.899 | 8.455 | 14.020 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| $\psi / \mu B$ | 18.34 | 36.50 | 55.90 | 74.85 | 92.04 | 115.46 | 139.62 | 169.43 | 173.45 |

According to the data in Table 1, the magnetization characteristics $i = 0.013\psi + 6.44 \times 10^{-6} \psi^3$ of the PT are obtained by the least squares method. The resistance of the transmission line is $R = 10 \Omega$, and the capacitance to the ground is $C_0 = 2.91 \mu F$. Take the fault earthing ground fault of phase A as an example to illustrate the influence of harmonics caused by photo-voltaic power generation on the ferromagnetic resonance of the Distribution network.

1) There is no photo-voltaic power generation in the Distribution network. There is a phase-to-phase earth fault occurs in phase A within 0.05-0.22s. The simulation results are shown in Figure 4.

![Figure 4](image-url)

From the simulation results in Figure 4, it can be seen that after the 0.22s fault disappears, although the voltage fluctuates, it recovers in line as the voltage before, which shows that no resonance occurs in the Distribution network at this time.

2) When photo-voltaic power generation is connected to the Distribution network, set the simulation parameters and time to be consistent with the above. If the Distribution network harmonic voltage is dominated by 17 and 19, the Distribution network voltage simulation result is shown as Figure 5.

![Figure 5](image-url)

From the simulation results in Figure 5, it can be seen that when the harmonics of the Distribution network are dominated by 17, 19, etc., the voltage simulation waveform does not change much compared with Figure 4, indicating that the higher harmonic voltage has less influence on it.
Under the same conditions, when photo-voltaic power generation has been accessed, made the harmonic waves are mainly low-order harmonic such as 3 or 5 orders, the grid voltage waveform has been shown as Figure 6.

![Figure 6. Distribution network voltage waveforms when the Distribution network is dominated by low harmonics such as 3 and 5.](image)

According to the simulation results shown in Figure 6, it can be seen that a single-phase ground fault occurs in the Distribution network within 0.05-0.22s, the voltage to earth of A is zero, and the voltages of phases B and C increase. After 0.22s, all the voltage earth of the Distribution network increases at the same time and distortion occurs, which satisfies the characteristics of this type of ferromagnetic resonance. At the same time, the voltage waveform of the neutral point of the Distribution network shown in Figure 7 shows that in the 0-0.05s Distribution network, the fluctuation of the neutral point voltage is small; the voltage fluctuation at the neutral point becomes larger when a single-phase grounding fault occurs in the Distribution network within 0.05-0.22s; after 0.22s, the frequency of the voltage becomes quicker than before, besides the neutral voltage is harmonic voltage.

![Figure 7. Distribution network neutral voltage waveform.](image)

5 Conclusion
Through theoretical analysis and simulation, results can be obtained that when the parameters of the Distribution network and the harmonic voltage caused by photo-voltaic power generation satisfy some certain conditions, high and wide frequency which are caused by photovoltaic power generation can provide harmonic energy for the ferromagnetic resonance of Distribution networks. Ferromagnetic resonance may happen in a Distribution network in which ferromagnetic resonance does not occur, and low harmonics have a greater influence than high harmonics.

References
[1] LIU Zengliang, LIU Tieling and LIU Guoting 2006 J. Electric Power Automation Equipment 26 21-24
[2] LI Li, HE Yue and WANG Junbing 2013 J. High Voltage Engineering 39 1114-19
[3] WANG Wei, JI Shengchang and LI Yanming 2009 J. TRANSFORMER 46 30-33
[4] LI Shunfu. 2003 J. Qinghai Electric Power 4 20-25
[5] DING Ming, WANG Weisheng and WANG Xiuli 2014 *J. Proceeding of the CSEE* 34 1-14
[6] QIN Rui, DONG Kaisong 2014 *J. ELECTRICAL AUTOMATION* 36 57-60
[7] ZAHNG Duxi, XU Xianghai and YANG Li 2007 *J. Automation of Electric Power Systems* 31 50-56
[8] HANG Zhifeng 2015 *J. TECHNOLOGICAL DEVELOPMENT OF ENTERPRISE* 34 102-104
[9] CHANG Fangyuan 2014 *D. China Electric Power Research Institute*
[10] ZENG Yuanjing 2016 *D. Southeast University*
[11] LI Shubing 2003 *J. Northeast Electric Power Technology* 5 14-18
[12] Andrei R G, Halley B R 1989 J. IEEE Transaction on Power Deliver 43 1773-78
[13] XIE Ning, LUO An and MA Fujun J. *Proceeding of the CSEE* 33 9-16
[14] CHEN Weixian 1996 *M. China Electric Power Press* 165-166