Response-Time Analysis and Optimization for Probabilistic Conditional Parallel DAG Tasks

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Abstract—Real-time systems increasingly use multicore processors in order to satisfy thermal, power, and computational requirements. To exploit the architectural parallelism offered by the multicore processors, parallel task models, scheduling algorithms and response-time analyses with respect to real-time constraints have to be provided. In this paper, we propose a reservation-based scheduling algorithm for sporadic constrained-deadline parallel conditional DAG tasks with probabilistic execution behaviour for applications that can tolerate bounded number of deadline misses and bounded tardiness. We devise design rules and analyses to guarantee bounded tardiness for a specified bounded probability for $k$-consecutive deadline misses without enforcing late jobs to be immediately aborted.

Index Terms—Real-Time Scheduling, Distributed Computing, Parallel Task Models

I. INTRODUCTION

A real-time system is a system where the missing of a deadline may lead to a catastrophe and thus warrants to formally verify the temporal behaviour of the system to ensure safety. In the last decade real-time systems have shifted from uniprocessor to multiprocessor systems in order to deal with the computational, thermal and energy constraints of modern complex applications. To that end, a lot of research has been conducted with regards to the challenge of how to make use of the parallelism provided by multiprocessors for task sets with inter- and intra-task parallelism whilst satisfying deadline constraints. Inter-task parallelism refers to the potential concurrent execution of distinct tasks that execute sequentially, whereas intra-task parallelism refers to tasks that allow for parallel execution. Fork/join models [18], synchronous parallel task models, real-time scheduling algorithms and response-time analyses thereof have been published, e.g., [29], and DAG (directed-acyclic graph) based task models [2], [6], [14], [15], [25]. These models enable tasks with higher execution demands and inherent parallelism such as computer vision, radar tracking or video applications to be scheduled with tighter deadlines.

Besides the different approaches and justifications to represent intra-task parallelism using the above models, parallel applications in the domain of autonomous driving and image processing are subject to multiple conditional branches and control flow instructions as stated by Melani et. al [25]. Moreover, the execution times of the subjobs of parallel algorithms in these domains are highly varying due to varying sensor inputs, e.g., images for object detection in autonomous vehicles. Beyond that, it was shown that the multicore architecture complicates the worst-case timing analysis. This is due to interference effects from contention on shared resources, e.g., caches, memory etc. The authors in [13] argue that the arbitration delay and state perturbation caused by resource sharing must be captured in the worst-case bounds. All these uncertainties eventually lead to pessimistic response-time analyses in real-time systems and thus lead to resource underutilization. These architectural impacts on the worst-case execution time analysis have been thoroughly researched by e.g., cache partitioning [11] or bandwidth sharing mechanisms for memory accesses [34].

Another approach to this problem is to accept the uncertain execution behaviour of the parallel tasks and to focus on the probabilistic response-time characteristics. For many applications, e.g., closed-loop feedback controllers, hard real-time system engineering (with a safe but very pessimistic upper bound) is not required due to the inherent controller robustness towards timing non-idealities like jitter and deadline misses. In fact, if only a limited number of deadlines of a control application are missed, the required quality of control can still be satisfied.

Recently, many research efforts have been focused on formalizing and analyzing relaxations of deadline constraints [28], e.g., weakly hard systems where $m$ out of $k$ task instances must meet the deadlines. Moreover, Maggio et al. [22] investigate the closed-loop control system stability under consecutive deadline-miss constraints, which further motivates the need for scheduling algorithms that can guarantee probabilistic bounds on consecutive deadline misses to the application.

In order to formally describe and verify quantitative guarantees of deadline misses, some quantifications are of importance for soft real-time systems: Probability of a deadline miss, probability for $k$ consecutive deadline misses, maximum tardiness of a job. Despite the guarantees are soft, the precise quantification of such deadline misses are hard and challenging even for the ordinary sequential real-time task models that are scheduled upon a uniprocessor system. A summary of the literature in this research direction is provided in Section II. They can only be derived under strict model assumptions, e.g., that a job is aborted whenever a job exceeds its deadline in the state-of-the-art analyses. The reason for this complexity is partly due to inter task interference, i.e., the preemption and
interference patterns of the task system due to higher-priority jobs, which results in a large number of system states that must be considered in a response-time analysis.

We aim to analyze, optimize and verify the schedulability of probabilistic conditional parallel DAG tasks on identical multiprocessors with respect to quantities such as deadline-miss probabilities, consecutive deadline-miss probabilities and tardiness constraints. When considering the scheduling and analysis of probabilistic parallel DAG tasks, not only inter-task, but also intra-task interference, and multiprocessor scheduling anomaly effects (the early completion of jobs may lead to longer response-times) must be considered, which complicate the analyses for the above mentioned quantities.

**Contributions:** We propose scheduling algorithms based on reservations, i.e., service provisioning, for the probabilistic analysis of parallel DAG tasks to avoid inter-task interference induced complexities and anomaly effects and are thus firstly able to solve the stated objective. More precisely, we make the following contributions:

- We propose a probabilistic version and formal description of the widely used conditional parallel DAG task model in Section II.
- We contribute scheduling algorithms and response-time analyses for probabilistic conditional parallel DAG tasks based on resource reservation. The reservations can be scheduled alongside real-time workloads using any existing scheduling paradigm. In addition, we provide design rules to devise reservations that guarantee probabilistic characteristics such as bounded tardiness, stability, and probabilistic upper-bounds for $k$-consecutive deadline misses. Our approach is anomaly-free because any early completions due to scheduling or dynamic DAG structures are handled by the adoption of resource reservation and the abstraction of the workload model.

To the best of our knowledge, this is the first paper that addresses the analysis and optimization for probabilistic conditional parallel DAG task sets with quantitive guarantees.

**II. RELATED WORK**

The scheduling of parallel real-time tasks with worst-case parameters, e.g., worst-case execution times, upon multiprocessor systems has been extensively studied for different parallel task models. An early classification of parallel tasks with real-time constraints into rigid, moldable or malleable has been described by Goosens et al. [16]. Early work concerning parallel task models focuses on synchronous parallel task models, e.g., [11], [23], [29]. Synchronous models are an extension of the fork-join model [12] in the sense that it allows different numbers of subtasks in each (synchronized) segment and that this number could be greater than the number of processors. Many of the proposed scheduling algorithms and analyses are based on decomposition, i.e., the decomposition of the parallel task into a set of sequential tasks and the scheduling thereof.

Recently, the directed-acyclic graph (DAG) task model has been proposed and been subject to scheduling algorithm design and analysis. The DAG task is a more general parallel structure where each task is described by a set of subtasks and their precedence constraints that are represented by a directed-acyclic graph. This parallel model has been shown to correspond to models in parallel computing APIs such as OpenMP by Melani et al. [31] or Sun et al. [32]. This model has been studied in the case of global scheduling in e.g., [6], [26] or partitioned scheduling algorithms [8], [13]. There has also been research regarding approaches of synchronous and general DAG tasks that are not decomposition based, e.g., federated scheduling as proposed by Li et al. [21] that avoids inter-task interference for parallel tasks. In federated scheduling, the set of DAG tasks are partitioned into tasks that can be executed sequentially on a single processor whilst meeting it’s deadline requirements and tasks that need to execute in-parallel in order to meet it’s deadline. The latter tasks are then assigned to execute on a set of processors exclusively.

Motivated by the conditional execution behaviour of modern parallel applications, e.g., autonomous driving or computer vision, the conditional DAG task model has been proposed. A plethora of research concerning the real-time schedulability of this model has been conducted by e.g., [3], [10], [25]. Most recently, the computational complexity of the scheduling of conditional DAG with real-time constraints has been investigated by Marchetti et al. [24]. However, due to the worst-case parameters and the worst-case conditional structure that has to be considered during real-time verification of the scheduling algorithms, resource over-provisioning is inevitable.

For soft real-time applications that can tolerate a bounded number of deadline-misses, probabilistic task models and response-time analyses for these kind of parallel tasks are of interest. Moreover, the worst-case parameter inference is increasingly complex and pessimistic for parallel architectures further bolstering the importance of probabilistic models and analyses. For sequential stochastic tasks a plethora of prior work concerning probabilistic analyses exists, e.g., [17], [30]. Recent work focused on the improvements of efficiency in convolution-based probabilistic deadline-miss analysis approaches. In Brüggen et al. [7], the authors propose efficient convolutions over multinomial distributions by exploiting several state space reduction techniques and approximations using Hoeffding’s and Bernstein’s inequality and unifying equivalence classes. Chen et al. [9] propose the efficient calculation of consecutive deadline-misses using Chebyshev’s inequality and moment-generating functions and optimizations thereof. There has also been efforts to use reservation servers to schedule probabilistic sequential tasks. For example, Palopoli et al. [27] have shown how to calculate the probability of a deadline miss for periodic real-time tasks scheduled using the constant bandwidth server (CBS). The authors have reduced the computation to the computation of a steady state probability of an infinite state discrete time markov chain with periodic structure. In the context of parallel DAG tasks Ueter et al. proposed a reservation scheme to schedule sporadic arbitrary-deadline DAG tasks [33] with real-time constraints.
Other approaches to tackle the probabilistic analysis of real-time tasks is real-time queuing theory by Lehoczky et al. [19], which is an extension of classical queuing theory to systems with deadlines. An initial work that analyzed the probabilistic response-times of parallel DAG tasks was proposed by Li [20]. Li extended prior work on federated scheduling [21] by facilitating queuing theory to devise federated scheduling parameters such that each task’s tardiness is bounded and soft real-time requirements are met. A more recent work on the probabilistic response-time analysis of parallel DAG tasks is by Ben-Amor et al. [41], [42]. The authors have studied the probabilistic response-time analysis of parallel DAG tasks upon multiprocessor systems using partitioned fixed-priority scheduling at the subtask level. In their model each subtask is described by a probabilistic worst-case execution time and static precedence constraints between them. Based on the above, the authors derive probabilities for subtask response-times using convolution-based approaches and compose an overall response-time.

III. Task and Problem Model

We consider a given set $T$ of probabilistic sporadic constrained-deadline conditional parallel directed-acyclic graph (DAG) tasks in a multiprocessor system that is comprised of $M$ identical (homogeneous) processors. Each task releases an infinite sequence of task instances, namely jobs. Each conditional parallel DAG task $\tau_i \in T$ is defined by a conditional DAG structure $G_i$ (to be defined later), a relative deadline $D_i$ and a minimal inter-arrival time $T_i$, which denotes the minimal distance between two job releases. In this paper we only consider constrained-deadline tasks, i.e., $D_i \leq T_i$ for every task $\tau_i$. An exemplary probabilistic conditional DAG is illustrated in Figure 1. A probabilistic conditional directed-acyclic graph is composed of nodes $V$ and edges $E$ that denote precedence and control flow constraints. Each node is either a subjob node with an associated execution time or a condition node that denotes probabilistic conditional branching to subjobs. In the illustrated example, two decision nodes with two possible branching options each are given. The given structure yields four different enumerable DAG realizations whose probability of realization is given by the probability of traversing a specific path of condition nodes. A conditional DAG is composed of finitely many DAGs, each of which consist of a tuple $(V, E)$, where $V$ denotes the finite set of subjobs and the relation $E \subseteq V \times V$ denotes the precedence constraints of these subjobs such that there are no directed circles in the underlying graph. For each of these DAGs the volume and length parameters are calculated as follows. We use $\text{pre}(v_i) := \{v_j \in V \mid (v_j, v_i) \in E\}$ and $v_j \prec v_i$ if $v_j \in \text{pre}(v_i)$ Conversely, we use $\text{succ}(v_i) := \{v_j \in V \mid (v_i, v_j) \in E\}$ and $v_j \succ v_i$ if $v_j \in \text{succ}(v_i)$.

Definition 1 (Path). A path $\pi$ in a directed-acyclic graph $G$ is any sequence of subjobs $v_{i_1} \prec v_{i_2} \prec \ldots \prec v_{i_k}$ for $v_i \in V$ such that $\text{pre}(v_{i_1}) = \emptyset$ and $\text{succ}(v_{i_k}) = \emptyset$.

Definition 2 (Length). Let a path $\pi$ be a sequence of subjobs such that each subjob in the sequence is an immediate successor of the previous subjob in terms of precedence constraints. Then the length of a path is given by

$$\text{len}(\pi) := \sum_{v_i \in \pi} \text{len}(v_i)$$

where the length of a subjob denotes its execution time. Subsequently, the length of DAG $G$ is given by

$$\text{len}(G) := \max\{\text{len}(\pi) \mid \pi \text{ is a path in } G\}.$$  

Definition 3 (Volume). The volume of DAG $G$ is given by the graph’s cumulative execution time, i.e.,

$$\text{vol}(G) := \sum_{v_i \in V} \text{len}(v_i).$$  

A. Probabilistic Parametric Description

Each probabilistic conditional DAG task is described by the tuple $\tau_i = (G_i, D_i, T_i)$ where $G_i$ denotes a probabilistic conditional DAG structure, $D_i$ denotes the relative deadline and $T_i$ denotes the minimal inter-arrival time between two job releases. For each task $\tau_i \in T$ a cumulative distribution function (CDF) is inferred from the conditional DAG structure, where $F_i(u, v)$ describes the probabilistic behaviour of the volume and length of a DAG instance. That is each task $\tau_i$ releases an infinite number of jobs $\tau_{i,\ell}$, $\ell = 0, 1, 2, \ldots$ and each job is associated with a DAG instance $G_{i,\ell}$ such that the parameters volume and length of $G_{i,\ell}$ are realizations according to the probabilistic characterization of the distribution function.

For instance the distribution function of the conditional DAG illustrated in Figure 1 is devised by the calculation of the probability for each of the DAG’s realizations and

| PROBABILITY | LENGTH | VOLUME |
|-------------|--------|--------|
| 0.42        | 12     | 13     |
| 0.18        | 13     | 14     |
| 0.28        | 9      | 10     |
| 0.12        | 11     | 11     |

TABLE I

Tabular representation of the probabilities of the parameters volume and length for the probabilistic conditional DAG task illustrated in Figure 1.
its respective parameter values. The instance illustrated in Figure 2 represents the graph where both upper edges are chosen for which the probability is 0.7 · 0.6 = 0.42. The associated length is 12 and the associated volume is 13. By similar reasoning, choosing the edges with probability 0.7·0.4, 0.3·0.6, and 0.3·0.4 yield 0.28, 0.18 or 0.12 realization probability of the associated DAG structures. Calculating the volume and length of each of these realizations yields the data listed in Table I. Consequently, we derive $F_i(u, v) = \mathbb{P}(\text{vol}(G_i) \leq u, \text{len}(G_i) \leq v)$ as follows:
\[
\mathbb{I}(u-13) \cdot \mathbb{I}(v-12) \cdot 0.42 + \mathbb{I}(u-14) \cdot \mathbb{I}(v-13) \cdot 0.18 \\
+ \mathbb{I}(u-10) \cdot \mathbb{I}(v-9) \cdot 0.28 + \mathbb{I}(u-11) \cdot \mathbb{I}(v-11) \cdot 0.12
\]
where $\mathbb{I}$ denotes the step function, i.e., $\mathbb{I}(x)$ is 1 if $x \geq 0$ and 0 otherwise. We note that for probabilistic conditional DAG tasks as presented, the CDF is a step function with finitely many steps. Moreover, we assume that the probabilities of DAG instances are independent.

### B. Tardiness

Every job that misses its deadline must be handled by the system, i.e., a mechanism must be devised that decides the actions taken upon such events. A common mechanism is the immediate abortion of every job which exceeds its deadline to avoid any interference of subsequent jobs. This approach is inefficient in the sense that all computation results are revoked for consistency reasons, which holds especially true if the amount of time that the deadline is exceeded is rather small. Informally speaking, the tardiness of a job measures the delay of job with respect to its deadline.

**Definition 4 (Tardiness).** Let $\delta_i(\ell)$ denote the tardiness of the $\ell$-th job of task $\tau_i$, i.e., the amount of time that the $\ell$-th job exceeds the task’s deadline under the consideration of possibly pending workload from prior jobs. The tardiness can be recursively stated as $\delta_i(\ell) = \max\{\delta_i(\ell - 1) + (R_{i,\ell} - D_i), 0\}$, where $R_{i,\ell}$ denotes the response time of the $\ell$-th job of task $\tau_i$. Furthermore $\delta_i(0) = 0$ by definition.

We note that due to this definition, the $\ell$-th job of task $\tau_i$ does meet its deadline if $\delta_i(\ell) = 0$, and it does miss its deadline if $\delta_i(\ell) > 0$. In pursuance of improving this problem we intend to bound the tardiness of each job of a task by a tardiness bound.

**Definition 5 (Tardiness Bound).** A task $\tau_i$ is said to have a tardiness bound $\rho_i > 0$ if any job of that task will be aborted if the job’s tardiness exceeds $\rho_i$, i.e., we have $0 \leq \delta_i(\ell) \leq \rho_i$ for all $\ell \geq 0$.

The tardiness bound is user-specified and refines the formal description of a probabilistic sporadic constrained-deadline parallel DAG task to the tuple $(F_i, D_i, T_i, \rho_i)$.

### C. Deadline Misses

We pursue to design reservation systems that provide sufficient service to each task $\tau_i$ in the task set $\mathbb{T} = \{\tau_1, \tau_2, \ldots, \tau_n\}$ such that the probability of $k$ consecutive deadline misses is bounded.

**Definition 6 (Consecutive Deadline Misses).** Any sequence of $k$ consecutive job releases $\tau_{i,\ell}, \tau_{i,\ell+1}, \ldots, \tau_{i,\ell+k-1}$ for $\ell \geq 0$ is subject to $k$-consecutive deadline misses if the following conditions hold:

- All jobs in the sequence miss their deadline
- Either $\ell = 0$ or the previous job $\tau_{i,\ell-1}$ does not miss its deadline.

For each task we define a function $\theta_i : \mathbb{N} \rightarrow [0, 1]$ to specify that we tolerate $k$ consecutive deadline misses for a given probability of at most $\theta_i(k)$.

**Definition 7 (k Consecutive Deadline Miss Constraint).** Let $\phi_i(j, k) := \mathbb{P}(\delta_i(j) > 0, \ldots, \delta_i(j+k-1) > 0 | j = 0 \text{ or } \delta_i(j-1) = 0)$ denote the probability that the sequence $\tau_{i,j}, \tau_{i,j+1}, \ldots, \tau_{i,j+k-1}$ suffers from $k$-consecutive deadline misses. Then a probabilistic conditional DAG task $\tau_i$ is said to satisfy the deadline constraint $\theta_i(k)$ if

\[\sup_{j \geq 0} \{\phi_i(j, k)\} = \phi_i(0, k) \leq \theta_i(k), \quad (1)\]

i.e., at each position $j$ the probability $\phi_i(j, k)$ does not exceed the threshold $\theta_i(k)$.

We note that the equality in Eq. (1) is due to the lack of pending workload prior to the release of job $\tau_{i,j}$.

### IV. SCHEDULING PROBLEM

We use a reservation system to handle the scheduling of the DAG tasks and use any partitioned scheduling algorithm to schedule the reservation system and other tasks in the system.

#### A. Reservations

In a reservation system service is reserved for each probabilistic parallel DAG task $\tau_i$ due to some regulation. At those reservations the task instances of $\tau_i$ can be processed.

The reservation system is $m_i$-in-parallel if there are at most $m_i \in \mathbb{N}$ reservations at the same time. In this work we consider a simplified version of in-parallel reservation system:

**Definition 8 (Our Reservation System).** A reservation system consists of $m_i$ reservation servers that provide $E_i$ amount of service each and that is replenished every $P_i > 0$ time units. More specifically, to provide the service, each $P_i$ time units there are activated a multisets of $m_i \in \mathbb{N}$ distinct reservations.
that each guarantee a service of $E_i$ time units over an interval of length $P_i$.

The instances of a task are assigned to the provided service in first-in-first-out (FIFO)-manner. Furthermore, we assume that at each time all assigned reservations only serve the subjobs of a single DAG job by the FIFO-policy. The reservation system is scheduled upon $M$ identical multiprocessors according to any scheduling paradigm and provides service to the DAG jobs whenever they are scheduled as follows.

**Definition 9 (List-Scheduling).** In a list schedule on $m_i$ in-parallel reservation servers a subjob of a given DAG job $G = (V,E)$ is executed on any reservation server that is idle and scheduled for execution and as soon as all preceding subjobs have executed until completion. More formally, the starting time $s_i$ for each subjob $v_i$ is given by $\min\{t \mid \text{some scheduled reservation idles at } t, t \geq \max\{f_j \mid v_j \in \text{pre}(v_i)\}\}$. $\GreyBox$.

For the remainder of this section, we assume the existence of a feasible schedule $S$ upon $M$ identical multiprocessors, meaning that all reservations will provide the promised service.

**Definition 10 (Work).** Let $\text{work}^S_i(t_1, t_2)$ denote the amount of workload from DAG jobs derived by task $\tau_i$ that was worked during the time interval $t_1 \leq t_2$ given the schedule $S$. $\GreyBox$.

Based on this definition, the worst-case response time of a job $\tau_{i,\ell}$ of a DAG task $\tau_i$ that was released at $t_{i,\ell}$ is given by the smallest $t' \geq t_{i,\ell}$ such that $\text{work}^S_i(t_{i,\ell}, t') \geq \text{vol}(G^\ell) + \text{backlog}^S_i(t_{i,\ell})$, where $\text{backlog}^S_i(t_{i,\ell})$ is the amount of unfinished work at time $t_{i,\ell}$ of jobs $\tau_i$ released before $t_{i,\ell}$. Note that $\text{backlog}^S_i(t_{i,\ell}) = 0$ if there are no previous deadline misses since we assume $D_i \leq P_i$ in our system model. In the following we express the processed work in provided service and develop a response-time bound as stated in Theorem [1].

For sake of argument, let $S$ denote a feasible schedule of a reservation system that works a job of a DAG task $\tau_i$ until completion. Furthermore let $\text{serv}^S_i(t_1, t_2)$ denote the service that is provided to the DAG job during the time interval from $t_1$ to $t_2$ in the schedule $S$.

**Definition 11 (Envelope).** Let $S$ be a concrete schedule of $T$. Consider a given DAG job instance $G$ of some task in $T$ with subjobs $V = \{v_1, \ldots, v_k\}$. Let each subjob $v_k$ have the starting time $s_k$ and finishing time $f_k$ in $S$. We define the envelope $s_{k_1}, f_{k_1}, s_{k_2}, f_{k_2}, \ldots, s_{k_p}, f_{k_p}$ of $G$, with $p \in \{1, \ldots, \ell\}$, recursively by the following properties:

1. $k_i \neq k_j$ for all $i \neq j$
2. $v_{k_p}$ is the subjob of $V$ with maximal finishing time
3. $v_{k_{i-1}}$ is the subjob in $\text{pre}(v_{k_i})$ with maximal finishing time, for all $i \in \{p, p-1, \ldots, 2\}$
4. $\text{pre}(v_{k_1}) = \emptyset$

We note that the definition of an envelope for a DAG job instance may be not unique if there are subjobs with equal finishing time. In this case we choose one among them arbitrarily. $\GreyBox$.

Based on the definition of an envelope, we are able to formally state the following lemma.

**Lemma 1.** Given a schedule $S$ of the DAG job $G = (T, E)$ with an $m_i$-in-parallel reservation system. Let $G_i,t \geq 0$ be one DAG job instance of $G_i$ with envelope $s_{k_1}, f_{k_1}, \ldots, s_{k_p}, f_{k_p}$. Then the amount of work that is finished during the interval from $f_{k_{q-1}}$ to $f_{k_q}$ for $q \in \{2, \ldots, p\}$ is lower bounded by

$$\text{work}^S_i(f_{k_{q-1}}, f_{k_q}) \geq \text{serv}^S_i(f_{k_{q-1}}, s_{k_1}) + \text{serv}^S_i(s_{k_1}, f_{k_q}) - (m_i - 1)\text{len}(v_{k_q})$$

where $v_{k_q}$ is the subjob of the envelope starting at time $s_{k_1}$ and finishing at $f_{k_q}$.

**Proof:** In the proof we split the work at time $s_{k_1}$ and estimate each summand of $\text{work}^S_i(f_{k_{q-1}}, f_{k_q}) = \text{work}^S_i(f_{k_{q-1}}, s_{k_1}) + \text{work}^S_i(s_{k_1}, f_{k_q})$ on its own. Combining both estimations yields the desired result.

In a first step we will prove that between finish and start of two consecutive subjobs in the envelope, the provided service is fully utilized by the DAG instance, i.e.,

$$\text{work}^S_i(f_{k_{q-1}}, s_{k_1}) = \text{serv}^S_i(f_{k_{q-1}}, f_{k_q})$$

holds for all $q \in \{2, \ldots, p\}$. Given the workload conserving properties of list-scheduling used to dispatch subjobs to the service, an eligible subjob is scheduled whenever service is available. Since by definition $s_{k_1}$ is the earliest time that $v_{k_1}$ is able to execute, all service during $f_{k_{q-1}}$ to $s_{k_1}$ must have been used to work on other (non envelope) subjobs.

Secondly, we show that the workload $\text{work}^S_i(s_{k_1}, f_{k_q})$ from start to finish of a subjob in the envelope can be estimated by

$$\max\{\text{serv}^S_i(s_{k_1}, f_{k_q}) - (m_i - 1)\text{len}(v_{k_q}), \text{len}(v_{k_q})\}.$$}

Clearly, during the starting time and finishing time of $v_{k_q}$ at least $\text{len}(v_{k_q})$ will be worked. Additionally, given the provided service $\text{serv}^S_i(s_{k_1}, f_{k_q})$ due to sequential execution of $v_{k_q}$ at most $m_i - 1$ reservations of duration $\text{len}(v_{k_q})$ may be unused. Therefore $\text{work}^S_i(s_{k_1}, f_{k_q}) \geq \max\{\text{serv}^S_i(s_{k_1}, f_{k_q}) - (m_i - 1)\text{len}(v_{k_q}), \text{len}(v_{k_q})\}. \GreyBox$

Based on this lemma, we can calculate the response-time of a DAG job. To do this we first extend the Lemma.

**Lemma 2.** Under the conditions of Lemma [1] we have that

$$\text{work}^S_i(r_G, r_G + t) \geq \text{serv}^S_i(r_G, r_G + t) - (m_i - 1)\text{len}(G)$$

holds, where $r_G$ is the release of job $G$ and $0 \leq t \leq f_{k_p}$.

**Proof:** The main part to prove this lemma is already done in Lemma [1]. We just have to be careful about the scenarios where $t$ is not a time instant of the envelope.

Similarly to the proof of Lemma [1] we can show that $\text{work}^S_i(f_{k_{q-1}}, t) = \text{serv}^S_i(f_{k_{q-1}}, t)$ for all $t \in [f_{k_{q-1}}, s_{k_1}]$ and that $\text{work}^S_i(s_{k_1}, t) \geq \text{serv}^S_i(s_{k_1}, t) - (m_i - 1)\text{len}(v_{k_1})$ for all $t \in [s_{k_1}, f_{k_1}]$. Furthermore, by the same reasoning $\text{work}^S_i(r_G, t) = \text{serv}^S_i(r_G, t)$ holds for all $t \in [r_G, s_{k_1}]$. 

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We conclude that work\((r_G, r_G + t)\) is again bounded by the envelope of combining both of the results, we obtain the lower bound for the response time, then proof by contraposition: If we assume that into parts already described above and estimating all of them at the same time. To formalize this, we define
\[
\mu := (r_G, s_{k_1}, f_{k_1}, \ldots, s_{k_p}, f_{k_p}).
\]
For \(q \in \{1, \ldots, 2p + 1\}\) we denote by \(\mu(q)\) the \(q\)-th entry of \(\mu\) and by \(\mu^t(q) := \min\{\mu(q), t\}\) the \(q\)-th entry bounded by \(t\).

By decomposing \(work_t^S(r_G, r_G + t)\), we obtain that it can be written as the sum of \(\sum_{q=1}^p work_t^S(\mu^t(2q) - 1, \mu^t(2q))\) and of \(\sum_{q=1}^p work_t^S(\mu^t(2q), \mu^t(2q + 1))\). The first summand is lower bounded by the sum of the corresponding service values \(\sum_{q=1}^p serv_t^S(\mu^t(2q) - 1, \mu^t(2q))\), and the second summand from above is lower bounded by \(\sum_{q=1}^p (\text{serv}_t^S(\mu^t(2q), \mu^t(2q + 1)) - (m - 1)\text{len}(v_{k_q}))\). By combining both of the results, we obtain the lower bound
\[
2p \sum_{q=1}^p serv_t^S(\mu^t(q), \mu^t(q + 1)) - (m - 1)\left(\sum_{q=1}^p \text{len}(v_{k_q})\right),
\]
which is again bounded by \(\text{serv}_t^S(r_G, r_G + t) - (m - 1)\text{len}(G)\). We conclude that \(work_t^S(r_G, r_G + t) \geq serv_t^S(r_G, r_G + t) - (m - 1)\text{len}(G)\).

**Definition 12 (Service Bound Function).** For a task \(\tau_i \in \mathbb{T}\) the minimal service that is provided by the reservation system during an interval of length \(t \geq 0\) is denoted by \(\text{sbf}_i(t)\). We call \(\text{sbfi}\) the service bound function of \(\tau_i\).

We use the service bound function to provide a lower bound \(\text{serv}_t^S(r_G, r_G + t) \geq \text{sbfi}(t)\) for all schedules \(S\). This leads us to the following theorem.

**Theorem 1 (Response-Time Bound).** We consider a task \(\tau_i \in \mathbb{T}\). Assume that the reservation system of \(\tau_i\) is \(m_i\)-in-parallel and its minimal service is described by \(\text{sbfi}\). Let \(G\) be the DAG which describes the task instance \(\tau_{i,j}\) of \(\tau_i\). Then the response time of \(G\) is upper-bounded by
\[
\min\{t > 0 \mid \text{sbfi}(t) \geq W_t^G\}. \tag{3}
\]

where \(W_t^G := \text{vol}(G) + (m_i - 1)\cdot \text{len}(G) + \text{backlog}^G_{\text{min}}(r_G)\) for notational brevity.

**Proof:** Let \(t' := \min\{t > 0 \mid \text{sbfi}(t) \geq W_t^G\}\). We do the proof by contraposition: If we assume that \(t'\) does not bound the response time, then \(t' < f_{k_p}\), where \(f_{k_p}\) is the last entry in the envelope of \(G\). In this case Lemma 2 yields:
\[
\text{work}_{t'}^G(r_G, r_G + t') \geq \text{serv}_{t'}^G(r_G, r_G + t') - (m_i - 1)\text{len}(G)
\geq \text{sbfi}(t') - (m_i - 1)\text{len}(G)
\]
By the definition of \(t'\) we have \(\text{sbfi}(t') \geq \text{vol}(G) + (m_i - 1)\cdot \text{len}(G) + \text{backlog}^G_{\text{min}}(r_G)\). Hence,
\[
\text{work}_{t'}^G(r_G, r_G + t') \geq \text{vol}(G) + \text{backlog}^G_{\text{min}}(r_G)
\]
the job \(G\) is finished at time \(t'\), i.e., \(t' \geq f_{k_p}\).

We emphasize that the reservation schemes and respective supply-bound function are not enforced to follow any specific kind of reservation scheme. The complexity of the calculation of the response-time depends only on the supply bound function. For instance, Figure 3 shows the supply-bound function of a our reservation system from Definition 8. As depicted, there may be no service provided to the task for up to \(2(P_i - E_i)\) time units in the worst case. We note that the first activation of reservations has to occur no later than at the release of the first job of \(\tau_i\). Otherwise, our analysis becomes invalid. However, the reservation system can stop assigning new reservation servers if there is no pending or unfinished job of \(\tau_i\), as long as it starts assigning new reservations if new jobs arise in the ready queue.

If we assume a reservation server as in Definition 8 then the response-time or service-time of a DAG job \(G\) is described by the following theorem.

**Theorem 2 (Service Time).** Let \(G = \tau_{i,j}\) be a task instance of \(\tau_i\). We assume that for \(\tau_i\) we have a reservation system as in Definition 8 with \(m_i\) equal sized in-parallel services \(E_i, \leq P_i\). We can give an upper bound \(R_G\) on the response time of \(G\) by
\[
R_G = \left(\frac{W_G^G}{m_i E_i}\right) + 1 \cdot (P_i - E_i) + \frac{W_G^G}{m_i} \tag{4}
\]
where \(W_G := \text{vol}(G) + (m_i - 1)\cdot \text{len}(G) + \text{backlog}^G_{\text{min}}(r_G)\) for notational brevity.

**Proof:** For the proof we assume that \(\text{vol}(G) > 0\) since otherwise no work has to be done and \(R_G = 0\) is already a trivial response-time bound. We aim to utilize Theorem 1. Therefore, we have to find the minimal \(t > 0\) such that \(\text{sbfi}(t) = W_G\). In the following we show one illustrative and one formal proof to justify that this minimal \(t\) is in fact \(R_G\) from Eq. (3).

We assume the worst-case service as depicted in Figure 3. We can see in the figure that every time when service is provided, it is done on \(m_i\) resources simultaneously. Hence, the total time which \(\tau_i\) has to be served, until \(G\) is finished,
is $\frac{W_G}{m_i}$. This happens during $\left\lceil \frac{W_G}{m_i} \right\rceil + 1$ service cycles. 

Therefore, we have to add this many times the amount of the service cycle, where $\tau_i$ is not served, i.e., $(P_i - E_i)$. In total, the response time is $\left\lceil \frac{W_G}{m_i} \right\rceil + 1 (P_i - E_i) + \frac{W_G}{m_i}$.

For the more formal proof, we also assume the worst-case service from Figure 5. For the function $g : \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$ with

$$g(t) := \left( \left\lfloor \frac{t}{m_i E_i} \right\rfloor + 1 \right) (P_i - E_i) + \frac{t}{m_i}$$

the composition $sbf \circ g$ is the identity and the function $g$ picks the minimal value of the inverse image of $sbf_i(t)$, i.e., $g(t) = \min(sbf_i^{-1}(t))$ holds. Hence, we obtain $g(W_G) = \min\{g(E_i)\}$.

In general, if we know an upper bound $b$ on the backlog of the previous job, we can state the response time bound from Eq. (4) independent from the previous schedule, by

$$R'_G(b) = \left( \left\lfloor \frac{V_G(b)}{m_i E_i} \right\rfloor + 1 \right) (P_i - E_i) + \frac{V_G(b)}{m_i}$$ (5)

where $V_G(b) := vol(G) + (m_i - 1)ecn(G) + b$. Based on Eq. (5), we bound the response time for the case that the preceding job has a deadline miss and for the case that the preceding job has no deadline miss.

**Corollary 1.** Under the assumptions of Theorem 2, $R'_G(\rho_i \cdot m_i)$ is an upper bound on the response time of $G$ if the preceding job has a deadline miss, and $R'_G(0)$ is an upper bound if the preceding job has no deadline miss.

**Proof:** This follows directly from Theorem 2 by using either $backlog^R(r_G) \leq \rho_i \cdot m_i$ (in case of a deadline miss) or $backlog^S(r_G) = 0$ (in case of no deadline miss).

V. RESERVATION ANALYSIS AND OPTIMIZATION

In this section we devise the analysis and optimization algorithm to generate reservation systems that provably respect the upper-bounds for $k$ consecutive deadline misses in a probabilistic sense. We emphasize that in order to co-design the $k$ consecutive deadline-miss constraints with the reservations configurations time-efficient algorithms are required to calculate the probabilities for $k$ consecutive deadline misses for any given reservation configuration.

A. Analysis of Reservation Systems

Based on the finite sample space of DAG structures $G$ of the probabilistic conditional DAG tasks $\tau_i$ we define the random variables $R^\ell_i := (G \rightarrow R'_G(\rho_i \cdot m_i))$ and $R^0_i := (G \rightarrow R'_G(0))$, which yield for each DAG job the response time bounds from Corollary 1 with and without a previous deadline miss. According to Definition 7 the constraint for $k$ consecutive deadline misses is fulfilled if

$$\phi_i(0, k) \leq \theta_i(k),$$ (6)

where $\phi_i(0, k)$ is the probability that the first $k$ jobs of $\tau_i$ miss their deadline, and $\theta_i(k)$ is some predefined value.

Since $\phi(0, k) = \mathbb{P}(\delta_i(k) > 0, \delta_i(k - 1) > 0, \ldots, \delta_i(1) > 0)$, we can use Bayes’ Theorem, to reformulate $\phi(0, k)$ as

$$\mathbb{P}(\delta_i(k) > 0 | \delta_i(k - 1) > 0, \ldots, \delta_i(1) > 0) \cdot \phi_i(k - 1).$$

The probability that $\tau_i$ does not meet its deadline does not decrease if the tardiness of the preceding job is increased. Therefore, if $\delta_i(k - 1) = \rho_i$, then the probability for a deadline miss of $\tau_i$ is maximal. In this case, the amount of tardiness of the other jobs $\delta_i(k - 2), \ldots, \delta_i(1)$ is irrelevant for the tardiness of $\tau_i$. More specifically,

$$\mathbb{P}(\delta_i(k) > 0 | \delta_i(k - 1) > 0, \ldots, \delta_i(1) > 0) \leq \mathbb{P}(\delta_i(k) > 0 | \delta_i(k - 1) = \rho_i)$$

holds and we can thus bound the probability for $k$ consecutive deadline misses by

$$\phi_i(0, k) \leq \mathbb{P}(\delta_i(k) > 0 | \delta_i(k - 1) = \rho_i) \cdot \phi_i(0, k - 1).$$ (7)

Then by Corollary 1 we know that

$$\mathbb{P}(\delta_i(k) > 0 | \delta_i(k - 1) = \rho_i) \leq \mathbb{P}(R^0_i > D_i)$$

and for the probability of the first job (without previous deadline miss)

$$\phi_i(0, 1) = \mathbb{P}(\delta_i(1) > 0) \leq \mathbb{P}(R^0_i > D_i).$$

Combining the results yields a bound on the probability of $k$ consecutive deadline misses:

$$\phi_i(0, k) \leq \mathbb{P}(R^1_i > D_i) \cdot \phi_i(0, k - 1)$$

$$\leq \ldots \leq \mathbb{P}(R^k_i > D_i) \cdot \phi_i(0, 1)$$

$$\leq \mathbb{P}(R^k_i > D_i) \cdot \mathbb{P}(R^0_i > D_i).$$

Since $\mathbb{P}(R^0_i > D_i) \leq \mathbb{P}(R^1_i > D_i)$, we also derive a simplified bound for the probability of $k$ consecutive deadline misses of task $\tau_i$ by

$$\phi_i(0, k) \leq \mathbb{P}(R^k_i > D_i).$$ (8)

As a prerequisite to derive upper-bounds on response-times for queuing systems it must be shown that the system is stable. Informally speaking this means that all backlog of the reservation system will have been worked at some point in time. We first give a formal definition of stability and then show that our devised reservation-based queuing system is stable by construction.

**Definition 13** (Stability). A reservation system $R_i$ is considered stable if for all $\ell \geq 0$ with $\delta_i(\ell) = 0$ it is almost certain that there exists $k > 0$ such that $\delta_i(k + \ell) = 0$. More formally,

$$\lim_{k \rightarrow \infty} \phi(0, k) = 0,$$ (9)

i.e., the probability for $k$ consecutive deadline misses approaches 0 for $k \rightarrow \infty$.

**Theorem 3** (Stability). A reservation system $R_i$ is stable if $\mathbb{P}(R^k_i > D_i) < 1$.

**Proof:** The probability for $k$ consecutive deadline misses is bounded by $\phi_i(0, k) \leq \mathbb{P}(R^k_i > D_i)$ according to Eq. (8).
If \((R_i^1 > D_i) < 1\), then \(\mathbb{P}(R_i^1 > D_i)^k \to 0\) for \(k \to \infty\). This concludes the theorem.

In consequence we do not have to especially consider stability concerns in the design of the reservation systems other than \(k\)-consecutive deadline constraints.

### B. Distribution Function Calculation

In this section, we show how to practically calculate the response-time upper bounds. First, we define the auxiliary random variable

\[ X_i := \frac{vol(G) + (m_i - 1) \cdot len(G) + \rho_i \cdot m_i}{m_i \cdot E_i} = \frac{V_i^G}{m_i E_i} \]

for which the distribution function \(\mathbb{P}(X_i \leq u)\) can be directly computed from the probabilistic DAG task model, i.e., by enumerating over all possible DAG job structures weighted by their realization probabilities as previously described. With reference to Corollary \(\square\) the distribution function of \(R_i^1\) can be written as follows:

\[ \mathbb{P}(R_i^1 \leq u) = \mathbb{P}\left( \left( \frac{Y_i}{E_i} \right) + 1 \cdot (P_i - E_i) + Y_i > D_i \right) \]

Let \(dom(X_i)\) denote all values that \(X_i\) can take, then we define the set of constant values \(I_i := \{ \ell \in \mathbb{N} \mid \inf(dom(X_i)) \leq \ell \leq \sup(dom(X_i)) \}\). Moreover given \(I_i\) the domain of \(\psi(X_i) = (P_i - E_i) \cdot (X_i + 1) + E_i \cdot X_i\) can be partitioned as follows:

\[ \bigcup_{\ell \in I_i} \left\{ (P_i - E_i) \cdot (\ell + 2) + E_i \cdot X_i \mid \ell < X_i \leq \ell + 1 \right\} \]

by the fact that \([X_i] \to \ell + 1\) for every \(X_i \in (\ell, \ell + 1]\). By the \(\sigma\)-additivity property of distribution functions and rearrangements yields

\[ \sum_{\ell \in I_i} \mathbb{P}(X_i \leq u - \frac{(P_i - E_i) \cdot (\ell + 2)}{E_i}) \mid \ell < X_i \leq \ell + 1 \]

### C. Optimization of Reservation Systems

In this section we present Algorithm \(\square\) to calculate reservation systems for the scheduling of probabilistic constrained-deadline conditional DAG tasks. Under the consideration of probabilities of upper-bounds for the maximal number of tolerable \(k_i\) consecutive deadline misses and given tardiness bounds the objective is to find minimal numbers of in-parallel reservations \(m_i\) and associated minimal amounts of service time \(E_i\). For each probabilistic constrained-deadline conditional DAG task the algorithm determines all feasible configurations \((m_i, E_i)\) by iterating through the number of in-parallel reservations \(m_i \in [1, \Omega_i]\) and search for the smallest required reservation service to still comply with the consecutive deadline-miss constraints.

**Theorem 4 (Monotonicity).** The functions

\[ \Phi_i^n : \mathbb{R}_{>0} \to \mathbb{R}_{>0}, \quad E_i \mapsto \mathbb{P}(R_i^1 > D_i | m_i = n) \]

that yield the probabilities of an upper-bound of a deadline-miss for a fixed number of in-parallel reservations with respect to the service time \(E_i\) are monotonically decreasing.

**Proof:** For easier readability let

\[ Y_i := \frac{vol(G_i) + (m_i - 1) \cdot len(G_i) + \rho_i \cdot m_i}{m_i} \]

for which the distribution function is independent of \(E_i\) for every fixed \(m_i\). According to the definition of \(\mathbb{P}(R_i^1 > D_i)\) in the beginning of this section, we have to prove that

\[ \mathbb{P}\left( \left( \frac{Y_i}{E_i} \right) + 1 \cdot (P_i - E_i) + Y_i > D_i \right) \geq \mathbb{P}\left( \left( \frac{Y_i}{E_i + \delta} \right) + 1 \cdot (P_i - (E_i + \delta)) + Y_i > D_i \right) \]

for any positive arbitrary increment \(\delta > 0\) and any realizations of \(Y_i \geq 0\). Let an arbitrary realization \(Y_i \geq 0\) satisfy

\[ \left( \frac{Y_i}{E_i + \delta} \right) + 1 \cdot (P_i - (E_i + \delta)) + Y_i < D_i \]

In this case \(Y_i\) satisfies

\[ \left( \frac{Y_i}{E_i} \right) + 1 \cdot (P_i - E_i) + Y_i > D_i \]

as well which yields the assumption by the property of distribution functions.

Due to the monotonicity of the functions \(\Phi_i^n\) as shown in Lemma \(\square\) it is possible to find the minimal amount of reservation service to guarantee compliance with the consecutive deadline-miss constraints by using binary search in the interval \((0, D_i]\). We emphasize that \(\Omega_i\) is an upper-bound specified by the user that can be set to an arbitrary fixed number that is larger than the number of available processors or determined as the point where an increase in the number of in-parallel reservations does not yield a significant decrease in the amount of required service to satisfy the deadline-miss probability constraints.

**VI. Conclusion and Future Work**

In this paper we proposed a probabilistic version and formal description of the widely used conditional parallel DAG task model and proposed a resource reservation system that allows for scheduling *anomaly free* scheduling whilst provably guaranteeing probabilistic quantities such as bounded tardiness.
stability, and probabilistic upper-bounds of $k$ consecutive deadline misses. In addition, we provided an algorithm to optimize the reservations systems with respect to the above quantities and showed that probabilistic conditional DAG tasks with a high degree of parallelism can improve a system’s resource usage if deadline misses are allowed. In the future we intend to improve the tightness of our proposed bounds and evaluate the effectiveness of the approach by implementing a prototype system.

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