Can Future Observation of the Living Partner Influence the Past Decorayed State in Entangled Neutral K-Mesons?

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Entangled neutral K-mesons allow the study of their correlated dynamics at interference and decoherence times not accessible in any other system. We find novel quantum phenomena associated to a correlation-in-time between the two partners: the past state of the first decayed kaon, when it was entangled before its decay, is influenced by the result and the time of the future observation of the second decay channel. This surprising “from future to past” effect is fully observable and leads to the unique experimental tag of the K\(_S\)-state, an unsolved problem since the discovery of CP violation.

Since long ago, several authors have stated the crucial role that the neutral kaon system has played for understanding the intricacies of the quantum world. In particular the words of R. Feynman [1], T.D. Lee [2] and L. B. Okun [3] are all emphasising the uniqueness of this system as a jewel donated to us by nature. They were referring to the peculiar properties of single neutral kaon states, which display several remarkable phenomena like the strangeness oscillation, the tiny mass splitting and the large difference in lifetimes of the physical states, the violation of the fundamental discrete symmetries Charge-Parity (CP) and Time-Reversal (T), the regeneration when traversing a slab of material.

The present research is related to another peculiar character of neutral kaons: the “strange entanglement”, i.e. the entanglement which is specific to two neutral kaon systems with all the interconnections with the above properties. It is worth reminding here that the entanglement is one of the most striking features of quantum mechanics, as stressed by E. Schrödinger [4], in reply to the famous argument by A. Einstein, B. Podolsky and N. Rosen [5] (EPR) based on local realism.

The experimental investigation of strange entanglement started more than two decades ago with the CPLEAR experiment [6] at CERN, and continued with the KLOE experiment and its successor KLOE-2 [7–9] at the DAΦNE -factory [10–12], yielding several precision results [13–20].

The characteristic behaviour of strange entanglement, with the peculiar properties of neutral kaons not found in any other system, makes possible the exploration of novel phenomena: the surviving correlation-in-time from the observation of the future decay of the living partner at a given time to the identification of the past kaon state leading to the first decay. This from the future to the past information in a system with non-trivial time evolution, entering into times in which the system was still entangled, could contribute to unveil the kind of reality to be associated to each part of the system.

The methodology that we follow consists in comparing the description of the double decay distribution at times \(t_1, t_2\) with \(\Delta t = t_2 - t_1 > 0\) using (i) the formalism of the two decay times state first introduced by Lee and Yang (LY) [21–24] with (ii) the time history (TH) of the entangled state from the coherent correlated neutral kaon system until its fate. The quantum consistency of the two approaches and the \(t_1, t_2\) symmetry of the first approach, with no special role of one of the two decay times, naturally demand the study of a novel problem: is it possible to infer the initial kaon state previous to the first decay at \(t_1\) from the observation of the second decay at time \(t_2 > t_1\), i.e. a correlation able to provide information from the future to the past? Contrary to the information from the past to the future, i.e., the prediction of the kaon state at time \(t_2\) from the observation of the first decay at time \(t_1\), the question formulated in this paper involves information on a part of the system at times in which the state was still fully entangled, i.e. before the first decay, when asking which is which is considered to be unspeakable in John Bell’s terminology [25].

In the following, first we analyze the correlation from past to future, i.e. which is the state before the second decay at \(t_2\), from the observation of the first decay channel at \(t_1\). Then we infer the correlation from future to past, i.e. which is the state before the first decay at \(t_1\), from the observation of the second decay channel at \(t_2\). We identify the decoherence region in \(\Delta t\) in which the surviving correlation tells us \(K_L\) at \(t_2\) and \(K_S\) at \(t_1\), providing the unique way to tag a \(K_S\) experimentally. We summarize the results presenting some final remarks and our conclusion.

We consider an entangled two body neutral kaon system, as actually realised at DAΦNE, with \(\phi \to K^0\bar{K}^0\) decays, the source of EPR coherent \(K^0\bar{K}^0\) pairs in the \(C = -1\) antisymmetric state: \(|i\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle)|. Under particle exchange we call particle-1 the first one...
to decay at time $t_1$, particle-2 the last to decay at time $t_2$. We remind the reader that Quantum Entanglement is associated to Non-Separability in two aspects: (i) we cannot identify which is which due to indistinguishability, and (ii) we cannot specify the two parts of the system which are not definite, showing that the parts have no local physical reality.

In fact, the antisymmetric state $|i\rangle$ is unique and therefore identically given in terms of any two generic linearly independent neutral kaon states, orthogonal or not. As a particular case, it can be written in terms of the $K_S, K_L$ states with definite time evolution \[ |i\rangle = \frac{N}{\sqrt{2}} (|K_S\rangle e^{i\lambda_S t}|K_L\rangle e^{-i\lambda_L t} - |K_L\rangle e^{i\lambda_L t}|K_S\rangle e^{-i\lambda_S t}) = e^{-i(\lambda_S + \lambda_L) t} |i\rangle. \tag{1} \]

If nothing is registered after the observation of the first decay at time $t_1$ (i.e. integrating over all subsequent decays at times $t_2$ of particle-2), the survival probability of the entangled state is necessarily characterised by the total width $\Gamma = \Gamma_S + \Gamma_L$ of the system \[ P(t_1) = e^{-\Gamma t_1}. \]

The two decay times formalism defines in the combined two terms of the entangled state \[ |i_{t_1,t_2}\rangle = \frac{N}{\sqrt{2}} (|K_S\rangle e^{-i\lambda_S t_1}|K_L\rangle e^{-i\lambda_L t_2} - |K_L\rangle e^{-i\lambda_L t_1}|K_S\rangle e^{-i\lambda_S t_2}). \tag{2} \]

measurement and its associated projection. Accordingly, the decay amplitude of the initial state $|i\rangle$ to channel $f_1$ at time $t_1$ for particle-1 and channel $f_2$ at time $t_2$ for particle-2, and the corresponding observable double differential decay rate $I(f_1,t_1;f_2,t_2)$ can be readily calculated \[ I(f_1,t_1;f_2,t_2)_{LY} = |\langle f_1(t_1)f_2(t_2)|T|i(t)\rangle|^2 = |\langle f_1f_2|T|i_{t_1,t_2}\rangle|^2 \]

with $\langle f_1|T|K_S\rangle$ and $\langle f_1|T|K_L\rangle$ the decay amplitudes to the $f_1$ channel of $K_S$ and $K_L$, $\eta_1 \equiv |\eta_1|^2 e^{i\phi_1} = \langle f_1|T|K_S\rangle / \langle f_1|T|K_L\rangle$, and $C_{12} = |\langle f_1|T|K_S\rangle \langle f_2|T|K_S\rangle|^2 / 2$.

As a corollary of the above approach one can notice that at an intermediate step of the calculation – after the first decay at time $t_1$ – the state of the surviving kaon (particle-2) immediately before its decay at time $t_2$ is expressed as:

\[ |K^{(2)}(t = t_2)\rangle = \frac{N}{\sqrt{2}} \langle f_1|T|i_{t_1,t_2}\rangle e^{-i(\lambda_S + \lambda_L) t_1} \left[ e^{-i\lambda_L t} |K_L\rangle - \eta_1 e^{-i\lambda_S t} |K_S\rangle \right]. \tag{4} \]
evolution from time $t_1$ to time $t_2$ of the pure state
\[ |K^{(2)}(t = t_1)⟩ = N_2 (|KL⟩ − η|KS⟩) , \tag{5} \]
with $N_2$ a suitable normalization factor. This is precisely the state of the living particle-2 which cannot decay to $f_1$, as a result of the projection by the decay of particle-1 at $t_1$ as a filtering measurement – see eqs. (7) and (8) below.

It is worth noting here that due to $ΔΓ = Γ_S − Γ_L ≠ 0$ two regimes can be identified in the time evolution of state (5): (i) the generic interference region and (ii) the decoherence region, with the relative weight of the $K_S$ component negligible when the following condition is satisfied:
\[ |η| e^{−ΔΓΔt/2} ≪ 1 \text{ [K}_L\text{-tag].} \tag{6} \]

At long enough $Δt$ – depending on what $f_1$ was – the living partner is always a $|K_L⟩$. This property is well understood and it has been used in the past in order to have $K_L$ beams “for all practical purposes” (FAPP) in Bell’s terminology \[25\].

**Time history (TH).** – It is worth to point out that the result \[5\] for the living partner is in agreement with the EPR instantaneous information due to the first decay when following the time history of strange entanglement, which we are now going to study in detail.

We first notice that in the case of decay processes, any initial state has some probability per unit time to decay to a given decay channel $f$ except that with zero probability. In particular the linear combination
\[ |K_{−f}⟩ = N_{−f} (|KL⟩ − η_f|KS⟩) , \tag{7} \]

having a vanishing decay amplitude $⟨f|T|K_{−f}⟩ = 0$, cannot decay to $f$. Thus the orthogonal state to (7), $|K_{−f}⟩$, is filtered by the decay. From the entangled system $|i⟩$, which can be rewritten for convenience in terms of these two orthogonal states as:
\[ |i⟩ = \frac{1}{\sqrt{2}} \{ |K_{−f_1}⟩|K_{−f}⟩ − |K_{−f}⟩|K_{−f_1}⟩\} , \tag{8} \]

the first decay to $f_1$ at time $t_1$ tags the state of the surviving partner as given by Eq. (7) with $f = f_1$ and filters at the same time – on an event-by-event basis – the state $|K_{−f_1}⟩$ in the decayed kaon, as a result of the observation of the $f_1$ channel. The “filtering identity” \[20\] defines the precise meaning of the last statement in terms of the decay probability of the decayed state into $f_1$. Here it is worth underlining that physically this corresponds to an irreversible process where the entangled state $|i⟩$ ceases its existence as a two-body state, with an immediate information from the decayed kaon to the survived one, leaving the system in a definite single particle state.

In other words, the $f_1$ decay of a kaon on one side prepares (tags), in the quantum mechanical sense, its partner kaon in the state $|K_{−f_1}⟩$ at a starting time $t = t_1$. Then the $|K_{−f_1}⟩$ state freely evolves in time – and in this sense the information is from past to future – until its decay time at $t_2$, see Eq. (4).

In summary four sequential steps are present in the time history of the entangled state $|i⟩$:

1. the time evolution of the state $|i⟩$ from time $t = 0$ to time $t = t_1$, with definite total width $Γ$;
2. the projection of the state $|i(t = t_1)⟩$ onto the orthogonal pair $|K_{−f_1}⟩/|K_{−f_1}⟩$, filtered by the decay $f_1$, times the decay amplitude of the state $|K_{−f_1}⟩$ into the $f_1$ channel;
3. the time evolution of the surviving (single) kaon state $|K_{−f_1}⟩$ from time $t = t_1$ to time $t = t_2$;
4. the projection at time $t = t_2$ of the evolved state $|K_{−f_1}(Δt)⟩$ onto the state $|K_{−f_2}⟩$ filtered by the decay $f_2$, times the decay amplitude of the state $|K_{−f_2}⟩$ into the $f_2$ channel.

These steps straightforwardly lead to the calculation of the observable double differential decay rate by factorising the amplitudes as follows:

\[ I(f_1, t_1; f_2, t_2)_{TH} = |⟨f_2|T|K_{−f_2}⟩⟨K_{−f_2}⟩|K_{−f_1}(Δt)⟩⟨f_1|T|K_{−f_1}⟩⟨K_{−f_1}⟩|K_{−f_1}|i(t = t_1)⟩|^2 . \tag{9} \]
Expression (10) corresponds to the state of the decayed kaon (particle-1) immediately before its decay at time $t_1$, once $t_2$ and $f_2$ are fixed – the future “fate” of its partner. Keeping $t_2$ and $f_2$ fixed – the observation – and varying the first decay time $t_1$, it corresponds to the single kaon evolved state, before the first decay, from time $t = 0$ to time $t = t_1$ of the state

$$\langle K(1)(t = t_1) = \langle f_2 | T | t_1, t_2 \rangle = \frac{\mathcal{N}}{\sqrt{2}} \langle f_2 | T | K_S \rangle \{ e^{-i\lambda_s t_1} \eta_2 e^{-i\lambda_l t_2} | K_S \rangle - e^{-i\lambda_l t_1} [ e^{-i\lambda_s t_2} | K_L \rangle \} . \tag{10}$$

Expression (10) corresponds to the state of the decayed kaon (particle-1) immediately before its decay at time $t_1$, once $t_2$ and $f_2$ are fixed – the future “fate” of its partner. Keeping $t_2$ and $f_2$ fixed – the observation – and varying the first decay time $t_1$, it corresponds to the single kaon evolved state, before the first decay, from time $t = 0$ to time $t = t_1$ of the state

$$\langle K(1)(t = 0) = \mathcal{N}_1 \{ \eta_2 e^{-i\lambda_l t_2} | K_S \rangle - e^{-i\lambda_s t_2} | K_L \rangle \} . \tag{11}$$

with $\mathcal{N}_1$ a suitable renormalization factor. Contrary to eq. (5) which is independent on the past $t_1$ decay time, eq. (11) shows a dependence not only on the decay channel $f_2$, but also on the future $t_2$ decay time.

This is a striking result which clearly involves a correlation-in-time from the future observation at time $t_2$ to the past, inferring the initial kaon state before its first decay at $t_1$. It becomes well defined during the time evolution of the entangled state $|i\rangle$ described by eq. (1) when the state of particle-1 (and particle-2) should have been undefined in the absence of any observation. We insist that the post-diction implied by Eq. (10) is not an artefact of the formalism but a factual observable accessible to experimental studies and thus it is fully physical. In a time history from future to past, the future observation at time $t_2$ tags particle-1 at the time $t_1 = t_2$ into the state proportional to $\{ \eta_2 | K_S \rangle - | K_L \rangle \}$, the state not decaying to $f_2$. Keeping $t_2$ and $\eta_2$ fixed – the observation –, the backward evolution of this tagged unobserved state to $t_1 < t_2$ leads to Eq. (10).

The interference and decoherence regimes: the $K_S$ tag. As a counterpart of the observability of the predicted Eq. (4) through the $t_2$ time distribution of the second decay, once the first decay to the $f_1$ decay channel at $t_1$ is fixed, the $t_1$ time distribution of the first decay as post-dicted in Eq. (10) is also observable, once the second decay channel $f_2$ and the decay time $t_2$ are fixed. As function of $t_1$, two different regimes can be identified: the generic interference region, in which the $t_2$ dependence of Eq. (10) is apparent, and the decoherence region, in which the relative weight of the $K_L$ component is negligible. Decoherence is reached for large $\Delta t$ satisfying the condition:

$$e^{-\Delta \Gamma \Delta t/2} / |\eta_2| \ll 1 \quad [K_S \text{-tag}] , \tag{12}$$

leading to a pure $K_S$ beam before the first decay. This consequence of the surviving correlation-in-time is most rewarding. Due to $CP$ violation and the non-orthogonality of the stationary states $\langle K_L | K_S \rangle \neq 0$, there is no decay channel able to tag either $K_S$ or $K_L$ on an event-by-event basis. While it is relatively easy to prepare FAPP pure $K_L$ beams, fulfilment of condition (12) constitutes the only known FAPP method to actually post-pare a $K_S$ beam (i.e. the short-lived stationary state) with arbitrary high purity (depending on $\Delta t$ and $\eta_2$), preparation otherwise impossible with other methods. As illustration of the observables in the two different regimes, Figure 1 shows the decay rate distribution into a generic channel $f_1$ of state (10) as a function of $t_1$ in two cases: either observed at $t_2 = 3 \tau_S$ (interference region) or when condition (12) is satisfied (decoherence region), with $f_2 = f_1$ to maximize the interference effects and make visible the difference between the two cases. This choice $f_2 = f_1$ also emphasizes the differing results as due to the dependence on the time of the future observation. Whereas the decoherence case shows a definite width $\Gamma_S$, the future observation in the interference region leads to a $t_1$-distribution with no definite lifetime. In the latter case the $t_1$ distribution does depend on the decay channel. All these results differ from the time distribution, given by the total width $\Gamma$, in the absence of any future observation [37].

In the case of entangled neutral mesons in the $C = (-)$ state, the dynamics before the first decay was considered to be trivial, even with mixing, as corresponding to a definite time evolution with the total width of the system. Hence in the past the experimental studies were concentrated in the observation of the single kaon decay rate distribution between the two decays depending on $\Delta t$. The first decay acts as a filtering preparation of the initial single state of the living partner. Our paper demonstrates the consistency of this description in terms of observables like: What is the state of the living partner at the time $t_2$ of the second decay?, a well defined – speakable – question after the measurement of the first decay channel $f_1$ at $t_1$. Asking What was the state of the first decayed kaon?, however, was considered to be unspeakable. Our study from future to past leads to the conclusion that this last contention is only valid as long as no future observation is made. In our case of a future measurement, the independence on the reference frame is assured when a time-like interval between the two decays is considered.

The present research has gone indeed a step further and seems to recognize that the correlation between the two partners survives their explicit dynamics, with a transition from the quantum correlation of entanglement to a classical correlation of separable $K_S$ $K_L$ states. The information from a measurement on the living partner – $f_2$ decay channel at $t_2$ – to the state of the decayed meson at $t_1$ is most surprising. Entering into the entangled
FIG. 1. The decay rate distribution into a generic channel \( f_1 \) of state (10) as a function of \( t_1 \) for the future observation at \( t_2 = 3 \tau_S \) (solid line), and when condition (12) for decoherence is satisfied (dashed line), with \( f_2 = f_1 \). The last shows a definite lifetime \( \tau_S \) and does not depend on the decay channel \( f_1 \). They differ both from the \( t_1 \)-distribution (dotted line) in the absence of a future measurement (in this case \( \Gamma_L \) has been multiplied by a factor 100 to appreciate graphically the difference between dotted and dashed lines). All distributions are normalised to unity at \( t_1 = 0 \).

region, i.e. just before the first decay, is not an artefact of our formalism, but a precise experimental observable through the \( t_1 \)-distribution of decays in any channel, as shown in Fig. 1. This is so for even situations of decoherence, Eq.(12), a physical situation only reachable for strange entanglement with the two very different lifetimes of \( K_S \) and \( K_L \). The relevance of this result for particle physics is outstanding: the unique way to tag what a \( K_S \) is, i.e. the solution of an open problem since the discovery of CP violation.

Our results seem to confirm the counter-intuitive feature of time in quantum mechanics. The surviving correlation-in-time found here goes beyond other phenomena, like delayed choice experiments, quantum erasers or teleportation, discussed for photons [38–41].

The result is not symmetric when comparing the outcomes in the two senses of “from past to future”, Eq.(4), and “from future to past”, Eq (10). This is characteristic of the neutral K-meson system with flavor mixing, \( \Delta \Gamma > 0 \) and \( \langle K_L|K_S \rangle \neq 0 \), leading at decoherence times to the unique experimental \( K_S \)-tag . These predictions are fully observable through the measurement of \( t_1 \)-distributions as the ones shown in Fig. 1 with no analogue in other physical systems.

The surviving correlation-in-time, by which the future observation influences the past “element of reality”, merits further thinking to unveil what kind of reality is behind the undefined lack of local realism. Our results demonstrate that the nature of the correlation-in-time is definite between the outcome of the observed partner and the state of the unobserved partner, no matter the time ordering of these two facts. This is so for even a non-symmetric time evolution and the time-dependent influence of the future observation on the first decayed state, as actually shown for the neutral K-meson system.

This research has been supported by the FEDER/MClyU-AEI Grant FPA2017-84543-P and Generalitat Valenciana Project GV PROMETEO 2017-033.

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The eigenstates of the effective Hamiltonian describing a single neutral kaon are the short- and long-lived physical states ($|K_S\rangle$ and $|K_L\rangle$). They have definite masses $m_{S,L}$ and lifetimes $\tau_{S,L}$ and evolve as a function of the kaon proper time $t$ as pure exponentials $|K_{S,L}(t)\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}\rangle$ with $\lambda_{S,L} = m_{S,L} - i\Gamma_{S,L}/2$, and $\Gamma_{S,L} = (\tau_{S,L})^{-1}$. They are usually expressed in terms of the flavor eigenstates $|K^0\rangle, |K^0\rangle$ as: $|K_{S,L}\rangle \propto \left[(1 + \epsilon_{S,L})|K^0\rangle + (1 - \epsilon_{S,L})|K^0\rangle\right]$ with $\epsilon_S$ and $\epsilon_L$ two small complex parameters describing the CP impurity in the physical states, making them non-orthogonal, $\langle K_S|K_L\rangle \simeq \epsilon_L + \epsilon_S$. One can equivalently define $\epsilon \equiv (\epsilon_S + \epsilon_L)/2$, and $\delta \equiv (\epsilon_S - \epsilon_L)/2$; adopting a suitable phase convention $\epsilon \neq 0$ implies $T$ violation, $\delta \neq 0$ implies $CPT$ violation, while $\delta = 0$ or $\epsilon = 0$ implies $CP$ violation. It is worth noting that one of the most stringent tests of the fundamental $CPT$ symmetry is performed comparing the measured $CP$ violation in $K_S$ and $K_L$ states [17-28].

This property holds only for the $C = -1$ antisymmetric state, but not for the $C = +1$ symmetric state in which the time evolution would induce $K^0\rightarrow K^0$ and $K^0\rightarrow\bar{K}^0$ terms due to $K^0 - \bar{K}^0$ mixing by weak interactions.

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It is worth mentioning that the concepts of quantum entanglement and decay as a filtering measurement – as used in the TH approach – have been instrumental in recent years [31-33] for observing $T$-Violation for the $B^0$-meson [34], bypassing the well known NO-GO argument for measuring genuine $T$ and $CPT$ asymmetries in transitions for decaying particles. Similar ideas have been formulated for the case of neutral kaons [35, 36].

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The KLOE/KLOE-2 experiment has already used the method based on condition (12) to select a pure $K_S$ beam to study its semileptonic decay [17, 19], to search for its rare $CP$ violating decay [16] into $3\pi^0$, and to study the $K_S$ decay time distribution and $K_{S,L}$ interference with $f_1 = f_2 = \pi^+ \pi^-$. In light of these results, the KLOE-2 experiment is in an unique position to perform a full experimental verification of the surprising phenomenon discussed in this paper about the information from future to past implied by result [10].

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