Goldstino Superfields in Supergravity

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Abstract—We review two off-shell models for spontaneously broken $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity proposed in arXiv:1702.02423 and arXiv:1707.07390. New results on nilpotent $\mathcal{N} = 1$ supergravity are also included.

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1. INTRODUCTION

According to the general relation between linear and nonlinear realisations of $\mathcal{N} = 1$ supersymmetry established by Ivanov and Kapustnikov [1], the Volkov–Akulov Goldstone fermion (Goldstino) [2] may equivalently be described in terms of a constrained superfield. Such a Goldstino superfield is called irreducible [3] since the Goldstino is its only independent component. There also exist reducible Goldstino superfields that contain auxiliary field(s) in addition to the Goldstino. The very first example of an irreducible Goldstino superfield in four dimensions was the nilpotent chiral scalar $\mathcal{X}$ introduced in [1, 4]. Roček [4] defined $\mathcal{X}$ to obey the nilpotency condition $\mathcal{X}^2 = 0$ and nonlinear constraint $f\mathcal{X} = -\frac{1}{4} \mathcal{X} \overline{D}^2 \mathcal{X}$, where $f$ is a real parameter characterising the scale of supersymmetry breaking, and $D_a = (\partial_a, D_a, \overline{D}^a)$ are the covariant derivatives of $\mathcal{N} = 1$ Minkowski superspace. The first reducible Goldstino superfield was proposed by Casalbuoni et al. [5] and rediscovered, in a different framework, by Komargodski and Seiberg [6]. It is a chiral scalar $X$ subject to the only constraint $X^2 = 0$. As argued in [3], every reducible Goldstino superfield may always be represented as an irreducible one plus a ‘matter’ superfield, which contains all the component fields except for the Goldstino. For instance, it was shown in [3] that

\[ X = \mathcal{X} + \mathcal{Y}, \quad f\mathcal{X} := -\frac{1}{4} \mathcal{X} \overline{D}^2 (\Sigma \Sigma), \]

\[ \Sigma := -4f \frac{X}{\overline{D}^2 \mathcal{X}}, \]

(1.1)

where the auxiliary field $F$ of $X$ is the only independent component of the chiral scalar $\mathcal{Y}$. Originally, the irreducible Goldstino superfield $\Sigma$ was introduced in [7] to be a modified complex linear superfield, $-\frac{1}{4} \overline{D}^2 \Sigma = f$, which is nilpotent, $\Sigma^2 = 0$, and obeys the holomorphic nonlinear constraint $fD_a \Sigma = -\frac{1}{4} \Sigma \overline{D}^2 D_a \Sigma$.

These properties follow from (1.1).

Due to the universality of the Volkov–Akulov action [1, 2], all irreducible Goldstino superfield models are equivalent. There exists a computer program created by Tyler [8] to construct the most general (twelve-parameter) field redefinition that relates any Goldstino model to the Volkov–Akulov action. Explicit relations that express every irreducible Goldstino superfield in terms of a given one have been worked out in [3, 7, 9, 10]. All irreducible Goldstino superfields share one remarkable feature discovered in [11]. Each of them may be realised as a composite of $X$ and its conjugate $\overline{X}$ that is invariant under arbitrary local rescalings $X \rightarrow e^{\tau} X$, with the parameter $\tau$ being chiral; an example is provided by $\Sigma$ given by eq. (1.1). This result implies, in fact, that any coupling of $X$ to a supergravity-matter system is dynamically equivalent to the same system coupled to a nilpotent chiral scalar $\mathcal{X}$, $\mathcal{X}^2 = 0$, subject to a suitable deformation of the constraint $f\mathcal{X} = -\frac{1}{4} \mathcal{X} \overline{D}^2 \mathcal{X}$, see also [3].

When a Goldstino superfield is coupled to off-shell supergravity, the local supersymmetry becomes spontaneously broken, in accordance with the super-Higgs effect [12, 13]. This is accompanied by the appearance of a positive contribution to the cosmological constant, which is proportional to $f^2$. The latter phenomenon was first observed in 1977 by Deser and Zumino within on-shell supergravity [13], and a year later by Lindström and Roček [14] who constructed the first off-shell model for spontaneously broken local $\mathcal{N} = 1$ supersymmetry in four dimensions. They coupled the

\[ 1 \] The article is published in the original.
nilpotent chiral scalar $\bar{\chi}$ of [4] to old minimal supergravity, with a supersymmetric cosmological term included. Their work completed the earlier attempt made in [13] to couple the Volkov–Aekulov action [2] to supergravity\(^2\). The coupling of $X$ to old minimal supergravity was worked out in detail by two groups in 2015 [15, 16]. The work by Bergshoeff et al. [15] put forward the concept of de Sitter supergravity, which has renewed interest in spontaneously broken supergravity.

This paper is a review of two models for spontaneously broken $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity proposed in recent publications [17, 18].

2. NILPOTENT $\mathcal{N} = 1$ REAL SCALAR MULTIPLET

The $\mathcal{N} = 1$ Goldstino superfield model proposed in [17] is described in terms of a real scalar superfield $V$ subject to the three nilpotency constraints\(^3\)

\begin{align}
V^2 &= 0, \quad (2.1a) \\
V \bar{\nabla}_a \bar{\nabla}^a V &= 0, \quad (2.1b) \\
V \bar{\nabla}_A \bar{\nabla}^A V &= 0. \quad (2.1c)
\end{align}

It is also necessary to require that the real descendant $\mathcal{D} W := \mathcal{D}^a W_a = \bar{\nabla}_a \bar{\nabla}^a V$ be nowhere vanishing, with

\[ W_a := -\frac{1}{4}(\bar{\chi}^2 - 4R)\bar{\nabla}_a V. \quad (2.2) \]

Here $R$ is one of the torsion tensors $R_a = \bar{\nabla}_a$ and $W_{ab} = W_{(ab)}$ which determine the curved superspace geometry (see [19] for a review), with $R$ and $W_{ab}$ being covariantly chiral. Because of the constraints imposed, $V$ has only two independent component fields, the Goldstino $\psi_a \propto W_a|_{\bar{\nabla} \phi = 0}$ and the auxiliary scalar $D \propto \mathcal{D}^a W_a|_{\bar{\nabla} \phi = 0}$. As shown in [17], the constraints (2) imply the representation

\[ V = -4 W^2 \Pi^2 \quad (\mathcal{D} W), \quad W^2 := W^a W_a. \quad (2.3) \]

which ensures the fact that the constraints (2) hold. The dynamics of this supermultiplet is governed by the super-Weyl invariant action

\[ S[V] = \int d^4 x d^2 \theta d^2 \bar{\theta} E \]

\[ \times \left\{ \frac{1}{16} V \mathcal{D}^a (\bar{\chi}^2 - 4R) \bar{\nabla}^a V - 2 f \Phi \bar{\Phi} \right\}, \quad (2.4) \]

where $\Phi$ is the chiral compensator, $\bar{\nabla}_a \Phi = 0$, for the old minimal formulation for $\mathcal{N} = 1$ supergravity, and $E^{-1} = \text{Ber}(E_A^M)$.

The constraints (2) are invariant under local re-scalings of $V, \Phi \rightarrow e^{\rho} V$, with $\rho$ an arbitrary real scalar superfield. Requiring the action (2.4) to be stationary under such re-scalings leads to the nonlinear constraint

\[ f \Phi \bar{\Phi} V = \frac{1}{16} V \mathcal{D}^a (\bar{\chi}^2 - 4R) \bar{\nabla}^a V, \quad (2.5) \]

which expresses the auxiliary scalar $D$ in terms of the Goldstino. The set of constraints (2.1) and (2.5) defines the irreducible Goldstino superfield $\mathcal{V}$ introduced in [3]. The constraints (2.1a) and (2.5) appeared originally in [14], and later were discussed in [9]. In both papers [9, 14], the Goldstino superfield $\mathcal{V}$ was considered as a composite superfield, $f \mathcal{V} = \bar{\chi} \mathcal{D}$. However, if $\mathcal{V}$ is viewed as a fundamental Goldstino superfield, then the constraints (2.1b) and (2.1c) must be imposed, as was first observed in [3].

We now consider the supergravity-matter action $S = S_{\text{OMSG}} + S[V]$, where $S_{\text{OMSG}}$ denotes the action for the old minimal supergravity with a cosmological term (see [19] for a review),

\[ S_{\text{OMSG}} = -\frac{3}{k^2} \int d^4 x d^2 \theta d^2 \bar{\theta} E \Phi \bar{\Phi} \]

\[ + \left\{ \mu \int d^4 x d^2 \theta d^2 \bar{\theta} \Phi^3 + c.c. \right\}, \quad (2.6) \]

where $\kappa$ is the gravitational coupling constant, and $\mu$ the cosmological parameter. In the second term of (2.6), $\mathcal{C}$ denotes the chiral coupling constant. Varying the action $S$ with respect to the chiral compensator $\Phi$ gives the equation of motion

\[ \mathcal{R} - \mu = \frac{6 \Phi^2 (\bar{\chi}^2 - 4R) \bar{\Phi}}{\Phi}, \quad (2.7) \]

where we have introduced the super-Weyl invariant chiral scalar

\[ \mathcal{R} := -\frac{1}{4} \Phi^2 (\bar{\chi}^2 - 4R) \bar{\Phi}. \quad (2.8) \]

The constraints (2.1) and the equation of motion (2.7) imply the nilpotency condition

\[ (\mathcal{R} - \mu)^2 = 0. \quad (2.9) \]
Making use of (2.7) once more, the functional $S = S_{\text{OMSG}} + S[V]$ can be rewritten as the following higher-derivative supergravity action [20]

$$
S = \left( \frac{3}{2 f \kappa} \right)^2 \int d^4 x d^2 \theta d^2 \bar{\theta} E \bar{\Phi} \Phi \{ R - \mu \}^2 - \left\{ \frac{1}{2 \kappa} \int d^4 x d^2 \theta \bar{\theta} \Phi^3 + c.c. \right\},
$$

(2.10)

where $R$ is subject to the constraint (2.9). This action is formulated purely in geometric terms, for it does not involve the Goldstino superfield explicitly.

The action for nilpotent old minimal supergravity, eq. (2.10), is universal in the sense that it is independent of the Goldstino superfield used to derive it. Indeed, let us consider another Goldstino superfield, the most fashionable one [5, 6]. In curved superspace it is described by a covariantly chiral scalar $\Phi$, $\bar{\Phi} \Phi = 0$, subject to the nilpotency condition

$$
\Phi^2 = 0.
$$

(2.11)

The super-Weyl invariant action for the Goldstino superfield $X$ is

$$
S[X, \bar{X}] = \int d^4 x d^2 \theta d^2 \bar{\theta} E \bar{\Phi} \Phi X,
$$

(2.12)

This action is equivalent to the one used in [15, 16]. Following [20], we vary the supergravity-matter action $S = S_{\text{OMSG}} + S[X, \bar{X}]$ with respect to the chiral compensator $\Phi$, resulting with the equation of motion

$$
R - \mu = -\frac{2}{3} f \kappa^2 X \Phi.
$$

(2.13)

Due to (2.11), the equation of motion tells us that (2.9) holds. Making use of (2.11) once more, the action $S = S_{\text{OMSG}} + S[X, \bar{X}]$ can be recast exactly in the form (2.10).

The constraint (2.9) coincides with the one derived in [21] within the Goldstino brane approach. It is also similar in form to the one postulated in [22]. However, the nilpotent supergravity action (2.10) differs from the one used in [22]. The two actions are actually related, as explained in Appendix D of [17].

The nilpotency condition (2.11) is preserved if $X$ is locally rescaled, $X \rightarrow e^{\tau} X$, where $\tau$ is covariantly chiral, $\bar{\Phi}_{\alpha} \tau = 0$. Requiring the action (2.12) to be stationary under such re-scalings gives the nonlinear equation

$$
\int \Phi^3 X = -\frac{1}{4} X (\bar{\Phi}^2 - 4 R) \bar{X}.
$$

(2.14)

The constraints (2.11) and (2.14) define the chiral Goldstino superfield $\mathcal{X}$ of [14].

If we define $V$ to be a composite superfield,

$$
\int \Phi \Phi V = \mathcal{X},
$$

(2.15)

then the constraints (2.1) are satisfied automatically. Relation $\int \Phi \Phi V = \mathcal{X}$ holds identically for the irreducible Goldstino superfields $V$ and $\mathcal{X}$. Plugging (2.15) into (2.4) gives the higher derivative action

$$
S_{\text{HD}}[X, \mathcal{X}] = \int d^4 x d^2 \theta d^2 \bar{\theta} E \bar{\Phi} \Phi X,
$$

(2.16)

Its important property is that $S_{\text{HD}}[\mathcal{X}, \mathcal{X}] = S[\mathcal{X}, \mathcal{X}] = S[V]$.

Within the new minimal formulation for $N = 1$ supergravity (see, e.g., [19] for a review), the compensator is a real scalar superfield $L = L$ constrained by $(\bar{\Phi}^2 - 4 R) L = 0$. We consider the supergravity-matter action $S = S_{\text{OMSG}} + S[V]$, where $S[V]$ is obtained from (2.4) by replacing $\Phi \Phi \rightarrow L$, and $S_{\text{OMSG}}$ is the action for new minimal supergravity

$$
S_{\text{NMSG}} = \frac{3}{2 \kappa} \int d^4 x d^2 \theta d^2 \bar{\theta} E \bar{\Phi} \Phi L \ln \frac{L}{|\Phi|^2},
$$

(2.17)

in which $\Phi$ is a purely gauge degree of freedom. New minimal supergravity is known to allow no supersymmetric cosmological term. Thus the action $S = S_{\text{NMSG}} + S[V]$ generates a positive cosmological term. Varying $S$ with respect to the compensator $L$ gives the equation

$$
\frac{3}{2 f \kappa} W_{\alpha} = W_{\alpha},
$$

(2.18)

$$
W_{\alpha} = -\frac{1}{4} (\bar{\Phi}^2 - 4 R) \bar{\Phi}_{\alpha} \ln \frac{L}{|\Phi|^2},
$$

where $W_{\alpha}$ is given by (2.2). This equation allows us to eliminate the Goldstino superfield from $S = S_{\text{NMSG}} + S[V]$, and the resulting action takes the following form

$$
S = \left( \frac{3}{4 f \kappa} \right)^2 \int d^4 x d^2 \theta d^2 \bar{\theta} \bar{\Phi} W_{\alpha} W_{\alpha}.
$$

(2.19)

This functional is the action for $R^3$ supergravity within the new minimal formulation [23, 24]. Making use of (2.3) gives

$$
W_{\alpha} = (\bar{\Phi}^2 - 4 R) \bar{\Phi}_{\alpha} \bar{W}_{\alpha} \bar{W}_{\alpha} / (\bar{\Phi} \bar{W}).
$$

(2.20)

The action (2.19) and constraint (2.20) define nilpotent new minimal supergravity.
3. NILPOTENT $\mathcal{N} = 2$ REDUCED CHIRAL MULTIPLET

The zoo of irreducible and reducible $\mathcal{N} = 2$ Goldstino superfields coupled to $\mathcal{N} = 2$ supergravity was described in [17]. A novel feature of $\mathcal{N} = 2 \to \mathcal{N} = 0$ local supersymmetry breaking is that one can consistently define nilpotent Goldstino-matter superfields that contain a physical gauge field (one-form or two-form) in addition to the two Goldstino fields and some auxiliaries [18].

3.1. Reduced Chiral and Linear Multiplets

In $\mathcal{N} = 2$ supersymmetry, the field strength of an Abelian vector multiplet is a reduced chiral superfield [25]. In curved superspace, it is a covariantly chiral superfield $W$, $\tilde{\mathcal{D}}^i \epsilon W = 0$, subject to the Bianchi identity [26]

$$\left( \mathcal{G}^i + 4 \mathcal{S}^i \right) W = (\mathcal{G}^i + 4 \mathcal{S}^i) \Psi, \quad \tilde{\mathcal{D}}^i : = \mathcal{D}^i (\tilde{\mathcal{G}}^i), \quad \tilde{\mathcal{D}}^i : = \mathcal{D}^i (\tilde{\mathcal{G}}^i) \alpha .$$

(3.1)

The superfields $\mathcal{S}^i$ and $\tilde{\mathcal{S}}^i$ in (3.1) are special dimension-1 components of the torsion, see [27] for the technical details of the superspace formulation $\mathcal{N} = 2$ conformal supergravity [26] used. The constraints on $W$ can be solved in terms of Mezincescu’s prepotential, $V_\gamma = V_\gamma$, which is an unconstrained real SU(2) triplet. The curved-superspace solution is [28]

$$W = \frac{1}{4} \tilde{\Delta} (\mathcal{G}^i + 4 \mathcal{S}^i) V_\gamma .$$

(3.2)

Here $\tilde{\Delta}$ denotes the $\mathcal{N} = 2$ chiral projection operator (see, e.g., [28] for the details)

$$\tilde{\Delta} = \frac{1}{96} (\tilde{\mathcal{G}}^i + 16 \tilde{\mathcal{S}}^i) \tilde{\mathcal{D}}_i - (\tilde{\mathcal{G}}^{\alpha} \partial \mathcal{G}_\alpha - 16 \mathcal{S}^{\alpha} \partial \mathcal{G}^\alpha) \tilde{\mathcal{D}}_\alpha .$$

(3.3)

In curved superspace, the $\mathcal{N} = 2$ tensor multiplet is described by its gauge-invariant field strength $G^\gamma$ which is a linear multiplet. The latter is defined to be a real SU(2) triplet (that is, $G^\gamma = \mathcal{G}^\mu$ and $\tilde{G}_\gamma := G^\gamma = G_\gamma$) subject to the covariant constraints

$$\tilde{\mathcal{D}}^i G^{jk} = \tilde{\mathcal{D}}^i G^{jk} = 0,$$

(3.4)

which are solved in terms of a chiral prepotential $\Psi$ (see, e.g., [28] for the details)

$$G^\gamma = \frac{1}{4} (\mathcal{G}^\gamma + 4 \mathcal{S}^\gamma) \Psi + \frac{1}{4} (\tilde{\mathcal{G}}^\gamma + 4 \tilde{\mathcal{S}}^\gamma) \tilde{\Psi}, \quad \tilde{\mathcal{D}}_\alpha \Psi = 0,$$

(3.5)

which is invariant under Abelian gauge transformations

$$\delta_\Lambda \Psi = i \Lambda,$$

(3.6)

with the gauge parameter $\Lambda$ being a reduced chiral superfield.

3.2. Deformed Reduced Chiral Multiplet

As defined in [29, 30], a deformed reduced chiral superfield $\mathcal{F}$ coupled to $\mathcal{N} = 2$ supergravity is described by the constraints

$$\mathcal{D}_\alpha \mathcal{F} = 0, \quad (\mathcal{G}^i + 4 \mathcal{S}^i) \mathcal{F}$$

$$- (\tilde{\mathcal{G}}^i + 4 \tilde{\mathcal{S}}^i) \tilde{\mathcal{D}}_\gamma \mathcal{F} = 4 i G^\gamma .$$

(3.7)

Here $G^\gamma$ is a linear multiplet which obeys the constraints (3.4). In addition, $G^\gamma$ is required to be nowhere vanishing, $G^\gamma G_\gamma \neq 0$. We identify $G^\gamma$ with one of the two conformal compensators of the minimal formulation for $\mathcal{N} = 2$ supergravity proposed in [31].

In the flat limit, a chiral superfield obeying the constraints (3.7) with $G^\gamma = G_\gamma$ appeared in the framework of partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking [32, 33].

3.3. Quadratic Nilpotency Condition

In [30], $\mathcal{F}$ was subject to the quadratic nilpotency condition

$$\mathcal{F}^2 = 0 .$$

(3.8)

The constraints (3.7) and (3.8) imply that, for certain $\mathcal{N} = 2$ supergravity backgrounds, the degrees of freedom described by the $\mathcal{N} = 2$ chiral superfield $\mathcal{F}$ are in one-to-one correspondence with those of an Abelian $\mathcal{N} = 1$ vector multiplet. The specific feature of such $\mathcal{N} = 2$ supergravity backgrounds is that they possess an $\mathcal{N} = 1$ subspace $\mathcal{M}^{\mathcal{N} = 1}$ of the full $\mathcal{N} = 2$ curved superspace $\mathcal{M}^{\mathcal{N} = 2}$. This property is not universal. In particular, there exist maximally $\mathcal{N} = 2$ supersymmetric backgrounds with no admissible truncation to $\mathcal{N} = 1$ [34]. As shown in [30], the superfield constrained by (3.7) and (3.8) is suitable for the description of partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ rigid supersymmetry breaking in every maximally supersymmetric spacetimes $\mathcal{M}^{\mathcal{N} = 1}$ which is the bosonic body of an $\mathcal{N} = 1$
superspace $M^{\dot{4}4}$ described by the following algebra of $\mathcal{N} = 1$ covariant derivatives$^4$

$$
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0, \quad \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0, \\
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -2i\mathcal{D}_{\alpha\beta},
$$

(3.9a)

$$
[\mathcal{D}_\alpha, \mathcal{D}_{\beta\gamma}] = i\epsilon_{\alpha\beta\gamma}, \\
[\mathcal{D}_\alpha, \mathcal{D}_{\beta\gamma}] = -i\epsilon_{\alpha\beta\gamma},
$$

(3.9b)

$$
\{\mathcal{D}_\alpha, \mathcal{D}_{\beta\gamma}\} = i\epsilon_{\alpha\beta\gamma} G^G_{\gamma\delta} \mathcal{D}_\delta, \\
\{\mathcal{D}_\alpha, \mathcal{D}_{\beta\gamma}\} = i\epsilon_{\alpha\beta\gamma} G^G_{\gamma\delta} \mathcal{D}_\delta,
$$

(3.9c)

where the real four-vector $G_a$ is covariantly constant,

$$
\mathcal{D}_a G_b = 0, \quad G_b = \mathcal{G}_b.
$$

(3.9d)

Since $G^2 = G^a G_a$ is constant, the geometry (3.9) describes three different superspaces, for $G_b \neq 0$, which correspond to the choices $G^2 < 0$, $G^2 > 0$ and $G^2 = 0$, respectively. The Lorentzian manifolds $M^4$ supported by these superspaces are $\mathbb{R} \times S^3$, $\text{AdS}_3 \times S^1$ or its covering $\text{AdS}_3 \times \mathbb{R}$, and a pp-wave spacetime isometric to the Nappi–Witten group [36], respectively. For each of the backgrounds (3.9) with $G_a \neq 0$, Ref. [30] constructed the Maxwell–Goldstone multiplet actions for partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking, as a generalisation of the earlier works [37, 38] corresponding to the $G_a = 0$ case.

### 3.4. Cubic Nilpotency Condition

If one is interested in $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ breaking of local supersymmetry, the nilpotency condition (3.8) should be replaced with a weaker constraint

$$
\mathcal{F}^3 = 0.
$$

(3.10)

The action for our supergravity–matter theory involves two contributions

$$
S = S_{SG} + S[\mathcal{F}, \mathcal{F}],
$$

(3.11)

where $S_{SG}$ denotes the pure supergravity action and $S[\mathcal{F}, \mathcal{F}]$ corresponds to the Goldstino superfield. We make use of the minimal formulation for $\mathcal{N} = 2$ supergravity with vector and tensor compensators [31]. In

4 These backgrounds are maximally supersymmetric solutions of pure $R^2$ supergravity [35].

the superspace setting, the supergravity action can be written in the form [28]

$$
S_{SG} = \frac{1}{\kappa^2} \int d^4 x d^4 \theta \bar{\theta} \left\{ \mathcal{W} - \frac{1}{4} W^2 + m \mathcal{W} \right\} + \text{c.c.}
$$

(3.12)

$$
= \frac{1}{\kappa^2} \int d^4 x d^4 \theta \bar{\theta} \left\{ \mathcal{W} - \frac{1}{4} W^2 \right\} + \text{c.c.}
$$

$$
+ \frac{m}{\kappa^2} \int d^4 x d^4 \theta \bar{\theta} \bar{\mathcal{W}} \mathcal{W} + \text{c.c.},
$$

where $m$ is the cosmological parameter. The supergravity action involves the composite

$$
\mathcal{W} := -\mathcal{G}_{\mathcal{F}} (\mathcal{F}_i + 4 \mathcal{G}_i) (\mathcal{F}_i / \mathcal{G}_i),
$$

(3.13)

which proves to be a reduced chiral superfield. Eq. (3.13) is one of the simplest applications of the powerful approach to generate composite reduced chiral multiplets presented in [28].

The action for the goldstino superfield $\mathcal{F}$ in (3.11) is

$$
S[\mathcal{F}, \mathcal{F}] = \int d^4 x d^4 \theta \bar{\theta} \left\{ \frac{1}{4} \mathcal{F}^2 + \frac{1}{2} \mathcal{W} \mathcal{F} + \frac{1}{2} \mathcal{W} \right\} + \text{c.c.},
$$

(3.14)

where $\zeta$ and $\rho$ are complex and real parameters, respectively. The $\rho$-term in (3.14) was introduced in [30], where it was shown to be invariant under gauge transformations (3.6).

In the flat superspace limit, with $G^G = \text{const}$, a chiral superfield $\mathcal{F}$ constrained by (3.7) and (3.10) was considered in [39]. As was demonstrated in [39], for a certain range of parameters $G^G$, $\mathcal{F}$ contains the following independent fields: two Goldstini, a gauge one-form and a real, nowhere vanishing, SU(2) triplet of auxiliary fields.

Ref. [18] also proposed a different $\mathcal{N} = 2$ Goldstino–matter multiplet in order to describe $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ local supersymmetry breaking. It is a linear superfield $\mathcal{H}^i$, $\mathcal{F}^i, \mathcal{H}^i = \mathcal{F}^i, \mathcal{H}^i = 0$, which is subject to the cubic nilpotency condition

$$
\mathcal{H}^{i,j,k} \mathcal{H}^{i,j,k} \mathcal{H}^{i,j,k} = 0,
$$

(3.15)

which proves to express the SU(2) triplet of physical scalars, $\mathcal{H}^i|_{i=0}$, in terms of the other component fields of $\mathcal{H}^i$. Thus the field content of $\mathcal{H}^i$ is as follows: two Goldstini, a gauge two-form, and a complex nowhere vanishing auxiliary scalar. The interested reader is referred to [18] for the complete description of this model.

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