A study of dynamic joint promotion strategies in the distribution channel

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ABSTRACT

This paper is concerned with the optimal decision problems for a class of dynamic joint promotions in the distribution channel. Here, the evolution of brand goodwill is allowed to be influenced by vertical joint promotions (VJP) and horizontal joint promotions (HJP). By applying the Hamilton–Jacobi–Bellman (HJB) equation, the optimal VJP and HJP strategies, the wholesale prices and the VJP participation rates are obtained under the Decentralized-Decentralized (DD) and Decentralized-Centralized (DC) channel structures, respectively. The comparison between the two channel structures shows that channel members are inclined to take the DC channel structure when HJP’s impact factor \( \theta \) is no less than a given constant. Finally, two numerical examples are given to show the changes of manufacturers’ and retailers’ profits with \( \eta \) under the different channel structures.

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1. Introduction

In the past decade, the decision problems of joint promotion programmes have gained considerable research interest due mainly to their widely employed by many famous firms, such as General Electric, Apple and IBM Corporation. To be more specific, the joint promotions in a typical channel are divided into vertical joint promotions (VJP) and horizontal joint promotions (HJP). VJP programmes, also called cooperative advertising, are typically promotional mechanisms between manufacturer and retailer (Javid & Hoseinpour, 2011; Lamberti, 2014; Szmerekovsky & Zhang, 2009). In VJP programmes, a manufacturer supports a fraction of promotional expenditures for his downstream retailer and the fraction shared by the manufacturer is commonly referred to as the participation rate. Additionally, some members also offer a monetary support to their competitors in HJP programmes (Karray, 2015). Through such these programmes, the channel members can induce his cooperators or competitors to increase spending on promotions to boost product demands and strengthen brand goodwill. So far, much work has been shown that the joint promotion programmes can bring more available choices to consumers and improve profitability to joint companies (Pawels, 2007; Yan, 2010; Zhang, Gou, & Liang, 2013).

Existing results on VJP programmes have focused on static and dynamic settings according to whether firms consider the long-term benefits (dynamic) or not (static). The main topics of interest have been the determination of promotion strategies and the design of coordination contracts (see, e.g. Liu, Zhang, & Tang, 2015; Song, Li, Wu, Liang, & Dolgui, 2017; Zaccour, 2008; Zhou & Lin, 2014). However, research on the dynamic framework of HJP programmes is scattered and these related phenomena are frequently occurred in the marketing economy. A case in this point is the horizontal promotional activity held by Thai GAP and 5 retail stores in 2013. The main purpose is to drive the incremental traffic in stores, increase retailers’ visibility and improve the brand goodwill of products. Also, the HJP programmes can improve the brand goodwill by further introducing the specific products and improving after-sales service of products. Considering that there is a consecutive relationship between joint promotion decisions in each period, this paper takes account of the time variation by applying differential game models and addresses the above-mentioned issues by merging three research streams-VJP programmes, HJP programmes and competing supply chains with ‘multiple-manufacturer multiple-retailer’. Although several models have been developed in these disciplines, none explored the comprehensive effect of the whole three factors in one dynamic framework. This study contributes to the literature in two main aspects. (1) It introduces the impact of time into the analysis framework and explores the joint promotion strategies in the competing supply chains with a dynamic way. In our paper, all of the supply

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chain members are far-sighted. When deciding the joint promotion decisions in the current period, they take the further influence into account such that the decision-making progress is iterative. (2) It constructs a comprehensive model including the macroscopic effects of VJP programmes and HJP programmes on the brand goodwill. The macroscopic effects are divided into two parts: the positive effect of own promotion and the negative effect of competitors promotion. It also contributes to practice by advising managers on how to plan efficiently to improve the brand goodwill and drive demand growth.

Motivated by the above discussions, we aim to provide new analytical results how the dynamic joint promotions affect the determination of decision-making and the choice of channel structure. In detail, the optimal VJP and HJP strategies are studied under two channel structures: two competing decentralized systems (DD) and a decentralized system competing with a centralized system (DC). Specifically, the following key questions are addressed in this paper: (1) Under what conditions does the manufacturer pay a fraction of promotional expenditures for his downstream retailer? (2) How do the joint promotion programmes influence members’ strategies? (3) Under what conditions do channel members tend to adopt the DC channel structure or the DD channel structure? To explore these issues, we investigate a dynamic joint promotion in two competing supply chains. Here, the evolution of brand goodwill is allowed to be influenced by VJP and HJP programmes. By applying the Hamilton–Jacobi–Bellman equation, the optimal promotion strategies of each channel member are presented under two different channel structures. The reminder of this paper is organized as follows: In Section II, the related previous literature is briefly introduced. The problem description and the basic model are thoroughly described in Section III. The optimal promotion strategies under two channel structures are presented in Section IV to ensure the profit maximization of each member or supply chain. Some numerical examples are given to show the changes of manufacturers’ and retailers’ profits with $\eta$ and the changes of profit differences with coefficient $\theta$ under the different channel structures in Section V. Finally, the conclusions are summarized.

2. Literature review

The topic of this paper is mainly related to two recent streams in operations management: VJP and HJP programmes. The issue of VJP programmes in the supply chain will be discussed first and HJP programmes second.

Recent studies on VJP (cooperative advertising) programmes are fruitful, and many important results have been reported in the literature. These results make significant contributions for gaining deep insights into both static and dynamic models of cooperative advertising. Earlier research on static cooperative advertising models can be traced back to Berger (Berger, 1972). It is sufficiently found in Berger (1972) that, by using the optimal participation rate, each member could obtain more profits than the simplistic 50-50 cost sharing. Further, most extended static models have been investigated in various business environments, see (Huang & Li, 2001; Roslow, Laskey, & Nicholls, 1993; SeyedEsfahani, Blazaran, & Gharakhani, 2011; Xie & Neyret, 2009; Yue, Austin, & Wang, 2006). To mention just a few, the impact of local advertising on demand has been put particular emphasis in Roslow et al. (1993). By taking retailer’s local advertising and manufacturer’s national advertising into account, the optimal advertising strategy problems have been studied in Huang and Li (2001) under two scenarios: (i) the manufacturer and the retailer are leader and follower, respectively (ii) the members make decisions in a co-op partnership. Based on the work of Huang and Li (2001), the cooperative advertising problem of two-level supply chain has been studied by considering a price discount in demand elasticity market circumstance (Yue et al., 2006). Later, the more general models have been presented in Xie and Neyret (2009), and SeyedEsfahani et al. (2011) including cooperative advertising and pricing decisions.

Unlike static models, dynamic models can portray the various effects of advertising such as long-term effect, short-term effect and forgetting effect. The classical Nerlove-Arrow advertising model has been proposed in Nerlove and Arrow (1962), where the time-dependent demand function is related to some factors such as brand goodwill, price and advertising investment. Later, the Nerlove-Arrow advertising model has been used to the study of dynamic cooperative advertising problems (Jørgensen, Sigue, & Zaccour, 2000; Jørgensen, Taboubi, & Zaccour, 2003; Karray & Zaccour, 2005; Zhang & Zhang, 2006). In Zhang and Zhang (2006), supposed that the local advertising has a positive influence on brand goodwill, the optimal advertising strategies have been obtained under the cooperation and non-cooperation scenarios. An interesting phenomenon has been discussed in Jørgensen et al. (2003), where the retailer’s promotion might damage the brand goodwill and the optimal advertising strategy has been proposed by applying the differential game theory. In addition, the long-term and short-term advertising strategies have been presented in Jørgensen et al. (2000). Extended the work of Jørgensen et al. (2000), optimal advertising strategies have been investigated in Karray and Zaccour (2005), where a retailer simultaneously sells national brand products and their own products with private brand. Very
recently, the dynamic models have been discussed in the ‘multiple-manufacturer single-retailer’ supply chain and ‘single-manufacturer multiple-retailer’ supply chain (Adida & DeMiguel, 2011; Cachon & Kök, 2010; He, Gou, & Wu, 2013; Lu, Tsao, & Chairoensiriwath, 2011). However, few results have been addressed a framework with two competing supply chains.

Moreover, comparing with the literature of VJP programmes, the research of HJP programmes has been explored only in a primary way. To be specific, considering the retailers’ HJP programmes, the promotion strategy problems have been presented and the effect of HJP programmes has been investigated on the optimal promotion strategies and profits (Karray, 2011). Later, the equilibrium strategies have been proposed in Karray (2015) for VJP and HJP programmes in the distribution channel. It is clearly shown that the manufacturers should offer a lower participation rate in VJP programmes when the promotion competitions domain the price competitions. Note that the existing literature on HJP programmes primarily accounts for the static setting. However, to the best of our knowledge, there has been little work undertaken on the dynamic VJP and HJP strategies in the distribution channel, not to mention the case that the HJP programmes would influence the brand goodwill of products. In this paper, we will investigate the optimal VJP and HJP strategies in the distribution channel and study the impact of VJP and HJP programmes on the brand goodwill, profits and channel choice.

3. Notation and problem formulation

This section defines the notations and problem statement used throughout this paper.

3.1. Notations

\[ \Pi_{Mi}, \Pi_{Ri}, \Pi_{ch} \] Profit functions for Mi, Ri and the total channel, respectively
\[ j_{Mi}, j_{Ri} \] Current value of profit functions for Mi and Ri, respectively

\[ G_{i}(t) \] Brand goodwill of the product i at time t, \( i \in \{1, 2\} \)
\[ A_{i}(t) \] Manufacturer i’s national advertising effort at time t
\[ U_{i}(t) \] Retailer i’s local advertising effort at time t
\[ B_{i}(t) \] Retailer i’s HJP effort at time t
\[ S_{i}(t) \] Demand of product i
\[ p_{i}(t) \] Retail price of product i
\[ \omega_{i} \] Wholesale price of product i
\[ \phi_{i} \in [0, 1] \] The participation rate of cooperative advertising
\[ \theta \] The HJP’s impact factor
\[ \lambda \] Discount rate of the manufacturers and the retailers

\[ \frac{\partial G_{i}(t)}{\partial t} = -\delta G_{i}(t) + A_{i}(t) - \alpha A_{3-i}(t) + U_{i}(t) - \beta U_{3-i}(t) + \theta (B_{i}(t) + B_{3-i}(t)), \quad G_{i}(0) = G_{i0} \quad (1) \]

where \( G_{i}(t) \) is the brand goodwill of product i, \( G_{i0} > 0 \) is the initial value of brand goodwill. This specification deserves some comments. First, \( \delta > 0 \) stands for the decay rate or forgetting effect of brand goodwill which captures the idea that consumers may forget the product to some extent. Next, \( 0 < \alpha, \beta < 1 \) are nonnegative constants and reflect that the competitors’ advertising programmes have lower effect on the accumulation of brand goodwill than direct advertising. Finally, the term \( \theta (B_{i}(t) + B_{3-i}(t)) \) indicates that the HJP efforts of retailers also can create additional brand goodwill of each product and a higher \( \theta \) implies that consumers are more sensitive to the HJP programmes.

Generally, the product demand is allowed to depend positively on the brand goodwill and negatively on the...
retail price in a separable multiplicative way. Therefore, the demand function \( S_i(t) \) can be written as follows:

\[
S_i(t) = e_i [1 - b_i p_i(t) + d_i p_{3-j}(t)] G_i(t),
\]

where \( e_i > 0 \) is the contribution of brand goodwill to product demand. \( 1 - b_i p_i(t) + d_i p_{3-j}(t) \) measures the impacts of own and competitor’s retail price on the product demand. The parameters \( b_i \) and \( d_i \) are the sensitivity of product demand to retail price and \( b_i > d_i > 0 \) implies that competitor’s retail price has a lower effect on product demand (Karray & Amin, 2015; Lu et al., 2011). The formulation expands the demand function used in Viscolani and Zaccour (2009), where the demand function is modelled just depending on own brand goodwill without considering the effect of competitor’s retail price.

As discussed in Bass, Krishnamoorthy, and Prasad (2005), the promotional expenditures may be described by the means of convex and increasing functions and given by:

\[
\begin{align*}
C(A_i(t)) &= \frac{1}{2} A_i^2(t), \\
C(U_i(t)) &= \frac{1}{2} U_i^2(t), \\
C(B_i(t)) &= \frac{1}{2} B_i^2(t),
\end{align*}
\]

where the promotional expenditures are taken to be quadratic. We model the distribution channel with DD and DC channel structures. Under the DD channel structure, the channel consists of two competitive supply chains and each member strives to maximize the present value of their own profits. Under the DC channel structure, the channel is formed by a decentralized and a centralized supply chain. The decentralized system consists of manufacturer 1 and retailer 1, which both members control their VJP and HJP efforts independently and manufacturer 1 supports the retailer’s promotion effort. In the centralized system, manufacturer 2 and retailer 2 jointly implement VJP and HJP efforts to maximize the total profit of supply chain. Therefore, at time \( t \), the profit functions of each retailer (\( \Pi_{Rj} \)), manufacturer (\( \Pi_{Mj} \)) and the total channel (\( \Pi_{Ch} \)) are described in Table 1 under the DD and DC channel structures.

In Table 1, \( \phi_j \) is the participation rate of VJP programmes and measures the amount that the manufacturer pays for the VJP expenditures of the retailers. In the following sections, we will obtain the optimal promotion strategies under each channel structure. For ease of illustration, we present this here for production cost \( c_1(t) = c_2(t) = 0 \) in Table 1. Using both wholesale price and retail price as decision variables drastically increase the complexity of game, we then assume that the retail price is a linear and proportional function of the wholesale price.

### Table 1. The profit functions at time \( t \) under the DD and DC channel.

| Chain | Profit | DD channel |
|-------|--------|------------|
| 1     | \( \pi_{R1} \) | \( [p_1(t) - c_1(t)]S_1(t) - \frac{1}{2} (1 - \phi_1) U_1^2(t) - \frac{1}{2} B_1^2(t) \) |
|       | \( \pi_{M1} \) | \( [c_1(t) - c_1(t)]S_1(t) - \frac{1}{2} \phi_1 U_1^2(t) - \frac{1}{2} A_1^2(t) \) |
| 2     | \( \pi_{R2} \) | \( [p_2(t) - c_2(t)]S_2(t) - \frac{1}{2} (1 - \phi_2) U_2^2(t) - \frac{1}{2} B_2^2(t) \) |
|       | \( \pi_{M2} \) | \( [c_2(t) - c_2(t)]S_2(t) - \frac{1}{2} \phi_2 U_2^2(t) - \frac{1}{2} A_2^2(t) \) |
|       | \( \pi_{ch} \) | \( \pi_{R1} + \pi_{M1} + \pi_{R2} + \pi_{M2} \) |

### 4. Main results

In this section, the optimal promotion strategies and optimal pricing decisions are presented for each channel member by applying the two-stage game. In particular, the manufacturer and the retailer are assumed to be leader and follower, respectively. The game sequence is as follows: the manufacturer simultaneously controls the wholesale price decision by the Nash game and separately announces his VJP efforts and the participation rates. This information is considered by retailer who determines her VJP and HJP efforts. Once the information about later decisions is available for the manufacturer to select his optimal promotion strategies.

#### 4.1. The optimal decisions under the DD channel structure

Under the DD channel structure, manufacturer and retailer maximize the present value of their own profits without considering the goal of others. With a common discount rate \( \lambda > 0 \), the objective functions of manufacturer and retailer are expressed:

\[
J_{Mi} = \int_0^\infty e^{-\lambda t} \Pi_{Mi} dt, \quad J_{Ri} = \int_0^\infty e^{-\lambda t} \Pi_{Ri} dt, \quad i = 1, 2.
\]

(4)

To obtain the optimal VJP and HJP efforts, wholesale price and the participation rate under the DD channel structure, we solve the following optimization problems for manufacturer and retailer, respectively.

\[
\begin{align*}
\max_{A_i, \phi_i, c_i} \quad & J_{Mi}, \\
\max_{U_i, B_i} \quad & J_{Ri}, \\
\text{s.t.} (1) \quad & \text{s.t.} (1)
\end{align*}
\]
We get the following proposition to characterize the optimal promotion strategies under the DD channel structure.

**Proposition 4.1:** Under the DD channel structure, the optimal VJP and HJP efforts of channel members are given by:

\[
A_i^{DD} = \frac{\epsilon_i \epsilon_i}{\eta (\lambda + \delta)}, \quad U_i^{DD} = \frac{\eta - 1}{\eta (1 - \phi_i) \lambda + \delta},
\]

\[
B_i^{DD} = \frac{\theta (n - 1) \epsilon_i \epsilon_i}{\eta (\lambda + \delta)}.
\]

The optimal wholesale price and participation rate of manufacturer \(i\) are:

\[
\omega_i^{DO} = \frac{2b_{3-i} + d_i}{\eta (4b_i b_{3-i} - d_i d_{3-i})},
\]

\[
\phi_i^{DO} = \begin{cases} 
\frac{3 - \eta}{1 + \eta}, & 1 < \eta < 3 \\
0, & \eta \geq 3 
\end{cases}.
\]

Further, the optimal profit functions of channel members satisfy:

\[
V_{Mi}^{DO}(G_i, G_{3-i}) = A_i^{DO} G_i + m_{3-i},
\]

\[
V_{Ri}^{DO}(G_i, G_{3-i}) = (1 - \phi_i) U_i^{DO} G_i + r_{3-i},
\]

where \(e_i = (b_i (2b_{3-i} + d_i))^2 / ((4b_i b_{3-i} - d_i d_{3-i})^2)\), \(m_{3-i}\) and \(r_{3-i}\) \((i = 1, 2)\) are obtained from the following equations:

\[
m_{ii} = A_i^{DO}, \quad r_{ii} = (\eta - 1)m_{ii},
\]

\[
m_{3} = \frac{1}{\lambda} \left[ \frac{1}{2} m_{3}^2 - \frac{\phi_i}{2(1 - \phi_i)^2} r_{3-i}^2 - \alpha m_{3-i} m_{3-i} - \frac{1}{2} \theta^2 r_{3-i} \right],
\]

\[
r_{3-i} = \frac{1}{\lambda} \left[ \frac{1}{2} \theta^2 - \frac{1}{2(1 - \phi_i)^2} r_{3-i}^2 + (m_{ii} + \alpha m_{3-i} r_{ii}) \right].
\]

**Proof:** The manufacturer as the leader controls the wholesale price, national advertising and participation rate of VJP programmes, while the retailer determines the local advertising and HJP efforts. We need to establish the existence of bounded and continuously differentiable value functions \(V_{Mi}^{DO}(G_i, G_{3-i})\) and \(V_{Ri}^{DO}(G_i, G_{3-i})\) such that for max \(J_M\) and max \(J_R\) there exists the unique solutions \(G_i^{DD}(t)\) and \(G_{3-i}^{DD}(t)\). Since the problem is played as a two-stage game and the manufacturer is the leader, we first derive the retailer’s decision variables by applying backward induction. The retailer’s HJB equation is:

\[
\lambda V_{Ri}^{DD} = \max_{U_i, B_i} \left\{ \epsilon_i (\eta - 1) \omega_i(t) [1 - \eta b_i \omega_i(t)] + \eta d_{3-i} (t) G_i(t) - \frac{1}{2} B_i^2(t) - \frac{1}{2} (1 - \phi_i) U_i^2(t) + \frac{\partial V_{Ri}^{DD}}{\partial G_i} [- \delta G_i(t) + A_i(t) - \alpha A_{3-i}(t) + U_i(t)] - \beta U_{3-i}(t) + \theta (B_i(t) + B_{3-i}(t)) \right\}.
\]

Performing the maximization of the right-hand side, yields the optimal local advertising and HJP effort:

\[
U_i^{DD}(t) = \frac{1}{1 - \phi_i} \left( \frac{\partial V_{Ri}^{DD}}{\partial G_i} - \beta \frac{\partial V_{Ri}^{DD}}{\partial G_{3-i}} \right),
\]

\[
B_i^{DD}(t) = \beta \frac{\partial V_{Ri}^{DD}}{\partial G_i} + \alpha \frac{\partial V_{Ri}^{DD}}{\partial G_{3-i}}.
\]

Similarly, the manufacturer’s HJB equation is as follows:

\[
\lambda V_{Mi}^{DD} = \max_{A_i, \omega_i, B_i} \left\{ \epsilon_i (\eta - 1) \omega_i(t) [1 - \eta b_i \omega_i(t)] G_i(t) - \frac{1}{2} A_i^2(t) - \frac{1}{2} \phi_i U_i^2(t) + \frac{\partial V_{Mi}^{DD}}{\partial G_i} [- \delta G_i(t) + A_i(t) - \alpha A_{3-i}(t) + U_i(t)] - \beta U_{3-i}(t) + \theta (B_i(t) + B_{3-i}(t)) \right\}.
\]

Substituting (11) into the HJB equation of the manufacturer and performing the maximization of the right-hand side of (12), we obtain:

\[
A_i^{DD}(t) = \frac{\partial V_{Mi}^{DO}}{\partial G_i} - \alpha \frac{\partial V_{Mi}^{DO}}{\partial G_{3-i}},
\]

\[
\phi_i^{DO} = \frac{1}{\lambda} \left[ \frac{1}{2} \theta^2 - \frac{1}{2(1 - \phi_i)^2} r_{3-i}^2 + (m_{ii} + \alpha m_{3-i} r_{ii}) \right].
\]

(13)
while the first-order condition for $\omega_1$ and $\omega_2$ yield
\[ 1 - 2\eta b_1 \omega_1 + \eta d_1 \omega_2 = 0, \quad 1 + \eta d_2 \omega_1 - 2\eta b_2 \omega_2 = 0. \] (14)

Solving the two simultaneous equations in (14), the optimal wholesale price of product $i$ is given
\[ \omega_{iD}^D(t) = \frac{2b_{3-i} + d_i}{(4b_i b_{3-i} - d_i d_{3-i})\eta}, \] (15)
simplifying the price items in (10) and (12), such that
\[ \omega_{iD}^D(1 - \eta b_i \omega_{iD}^D + \eta d_i \omega_{iD}^D) = \frac{1}{\eta} e_i, \] (16)
where $e_i = (b_i(2b_{3-i} + d_i)²) / ((4b_i b_{3-i} - d_i d_{3-i})²).$ □

We may satisfy (10) and (12) by conjecturing linear value functions. Hence, we define $V_{jii}^{DD}(G_{i}, G_{3-j}) = m_i G_i + m_{3-j} G_{3-j} + m_3$ and $V_{jii}^{DD}(G_{i}, G_{3-j}) = r_i G_i + r_{3-j} G_{3-j} + r_3$ where $m_j$ and $r_j$ ($i = 1, 2, j = 1, 2, 3$) are constant parameters. By inserting $V_{jii}^{DD}$, $V_{jii}^{DD}$, their derivatives, (11), (13) and (16) into (10) and (12), we get
\[ m_{i3} = \frac{1}{\lambda} \left[ \frac{1}{2} m_{i3}^2 - \frac{\phi_i}{2(1 - \phi_i)} \right] r_{i3}^2 - \alpha m_{i3} m_{3-i3-i} \]
\[ + \left( \theta^2 + \frac{1}{2(1 - \phi_i)} \right) m_i r_{i3} \]
\[ + \left( \theta^2 - \frac{\beta}{1 - \phi_{3-i}} \right) m_{3-i3-i} r_{i3} - \alpha m_{i3} m_{3-i3-i} \] (17)
\[ r_{i3} = \frac{1}{\lambda} \left[ \frac{1}{2} \theta^2 + \frac{1}{2(1 - \phi_i)} \right] r_{i3}^2 + (m_{i3} - \alpha m_{3-i3-i}) r_{i3} \]
\[ + \left( \theta^2 - \frac{\beta}{1 - \phi_{3-i}} \right) m_{3-i3-i} r_{i3} - \alpha m_{i3} m_{3-i3-i} \] (18)

Finally by combining (11), (13), (15) and (17)–(19), the optimal VJP and HJP efforts of channel members, the optimal participation rate of manufacturer are given by
\[ A_i^{DD} = \frac{\varepsilon_i e_i}{\eta(\lambda + \delta)}, \quad U_i^{DD} = \frac{\eta - 1}{\eta(1 - \phi_i)} \frac{\varepsilon_i e_i}{\lambda + \delta}, \]
\[ B_i^{DD} = \frac{\theta(\eta - 1)}{\eta} \frac{\varepsilon_i e_i}{\lambda + \delta}, \]
\[ \phi_i^{DD} = \begin{cases} \frac{3 - \eta}{1 + \eta}, & 1 < \eta < 3 \\ 0, & \eta \geq 3 \end{cases} \] (20)

From Proposition 4.1, the following corollaries are presented.

**Corollary 4.1:** Only when the condition $1 < \eta < 3$ holds, the manufacturer is willing to pay a part of VJP expenditures for retailer and then the VJP programmes are feasible under the DD channel structure. Moreover, $A_i^{DD}$ and $U_i^{DD}$ are decreasing functions of parameter $\eta$ and $B_i^{DD}$ is an increasing function of parameter $\eta$.

It is interesting to note that the ratio between manufacturer’s marginal profit to the retailer’s is expressed as $1/(\eta - 1)$. When the ratio is larger than 0.5, the manufacturer is willing to undertake a part of the retailer’s promotional expenditures. The same result has been proposed in the previous literature, see (He et al., 2013; Zhang et al., 2013). In addition, the ratio is a decreasing function of parameter $\eta$ and the retailer’s marginal profit is an increasing function of parameter $\eta$. More specifically, a higher parameter $\eta$ indicates that retailer has the power to participate in the HJP programmes with her competitors and manufacturer has no need to pay a higher promotional expenditures for the retailer. Moreover, a higher marginal profit gives the manufacturer a great impetus to participate in the VJP programmes. Hence, the channel members could drive marginal profits through decreasing the production cost or operation cost.

**Corollary 4.2:** $A_i^{DD}$, $U_i^{DD}$ and $B_i^{DD}$ decrease with the sensitivity coefficient $b_i$ and increase with the sensitivity coefficient $d_i$.

It is readily shown that a sharp sensitivity of product demand on retail price ($b_i$) will lead to the less HJP and VJP efforts of channel members. Furthermore, if the market competitiveness of product ($d_i$) is higher, then the channel members will invest more VJP and HJP efforts to strengthen the brand goodwill and promote the product demand.

**Proposition 4.2:** At equilibrium, under the DD channel structure, the optimal brand goodwill of product $i$ is given by:
\[ G_i^{DD}(t) = \frac{\Omega_i^{DD}}{\delta} + \left( G_0 - \frac{\Omega_i^{DD}}{\delta} \right) e^{-\delta t}. \] (21)

The steady state of brand goodwill is:
\[ \lim_{t \to \infty} G_i^{DD}(t) = \frac{\Omega_i^{DD}}{\delta}, \] (22)
where
\[ \Omega_i^{DD} = \frac{1}{\eta} \left[ 1 + \left( \frac{1}{1 - \phi_i} + \theta^2 \right)(\eta - 1) \right] \frac{\varepsilon_i e_i}{\lambda + \delta} \]
\[ - \frac{1}{\eta} \left[ \alpha + \left( \frac{\beta}{1 - \phi_{3-i}} - \theta^2 \right)(\eta - 1) \right] \frac{\varepsilon_{3-i} e_{3-i}}{\lambda + \delta}. \]
It follows from Proposition 4.2, when \( \delta > (\Omega_i^{DD})/\gamma_i \), the brand goodwill of product \( i \) decreases over time, but the value of brand goodwill is always higher than \( (\Omega_i^{DD})/\delta \). However, when \( \delta < (\Omega_i^{DD})/\gamma_i \), the brand goodwill of product \( i \) increases over time, but the value of brand goodwill is always lower than \( (\Omega_i^{DD})/\delta \). Specifically, when the forgotten degree of product is relatively higher (lower), although the firm holds a high (low) initial goodwill, the brand goodwill of product would be decreasing (increasing) without implementing promotional behaviour. Then, when the firm never invests indispensable promotion to enhance the brand goodwill, which would oblige the consumers to convert to the other market or the other brand. Hence, the firm should improve the forms of promotion programmes or further enlarge the promotional investment to maintain the sustainable development of product brand.

### 4.2. The optimal decisions under the DC channel structure

In this case, the decentralized system is formed of manufacturer 1 and retailer 1, in which the channel members strive to maximize the present value of their own profits. However, manufacturer 2 integrates with retailer 2 maximizing the total profit of supply chain 2. The objective functions of channel members are expressed:

\[
J_{M1} = \int_0^\infty e^{-\lambda t} \Pi_{M1} dt, \quad J_{R1} = \int_0^\infty e^{-\lambda t} \Pi_{R1} dt,
\]

and

\[
J_2 = \int_0^\infty e^{-\lambda t} \Pi_{2} dt. \tag{23}
\]

To obtain the optimal VJP and HJP efforts and wholesale prices under the DC channel structure, we solve the following optimization problems of channel members:

\[
\max_{\phi_1, \phi_2, \phi_3} J_{M1}, \quad \max \phi_1, \phi_2, \phi_3, \quad \phi_4, \phi_5, \phi_6, \phi_7.
\]

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Similar to the above part, we get the following proposition to characterize the optimal promotion strategies under the DC channel structure.

**Proposition 4.3:** Under the DC channel structure, the optimal VJP and HJP efforts of channel members are given by:

\[
A_1^{DC} = A_1^{DD}, \quad U_1^{DC} = U_1^{DD}, \quad B_1^{DC} = B_1^{DD}, \tag{24}
\]

\[
A_2^{DC} = \eta A_2^{DD}, \quad U_2^{DC} = \frac{\eta(1 - \phi_2)}{\eta - 1} U_2^{DD}, \quad B_2^{DC} = \frac{\eta}{\eta - 1} B_2^{DD}, \tag{25}
\]

the optimal wholesale price and the participation rate of manufacturer 1 are:

\[
\phi_1^{DC} = \phi_1^{DD}. \tag{26}
\]

Further, the optimal profit functions of channel members satisfy:

\[
V_{M1}^{DC}(G_1, G_2) = m_{11} G_1 + m_{13}, \tag{28}
\]

\[
V_{R1}^{DC}(G_1, G_2) = r_{11} G_1 + r_{13}, \tag{29}
\]

\[
V_2^{DC}(G_1, G_2) = \eta m_{22} G_2 + \nu_{13}, \tag{30}
\]

where \( m_{13} \), \( r_{13} \) and \( \nu_{13} \) are obtained from the following equations:

\[
m_{13} = \frac{1}{\lambda} \left[ \frac{1}{2} m_{11} - \frac{\phi_1}{2(1 - \phi_1)} - \frac{1}{2} \theta^2 \right] + \eta (\theta^2 - \alpha - \beta) m_{11} m_{22}.
\]

\[
r_{13} = \frac{1}{\lambda} \left[ \left( \frac{1}{2} \theta^2 + \frac{1}{2} (1 - \phi_1) \right) + \eta (\theta^2 - \alpha - \beta) m_{22} r_{11} \right].
\]

\[
\nu_{13} = \frac{1}{\lambda} \left[ \frac{\eta^2}{2} \left( 1 + \frac{1}{2} \theta^2 \right) m_{22} - \alpha \eta m_{11} m_{22} \right]
\]

where

\[
\eta = \frac{1}{\lambda} \left[ \frac{1}{2} m_{11} - \frac{\phi_1}{2(1 - \phi_1)} - \frac{1}{2} \theta^2 \right] + \eta (\theta^2 - \alpha - \beta) m_{11} m_{22}.
\]

**Proof:** Under the DC channel structure, supply chains 1 and 2 are assumed to be decentralized and centralized systems, respectively. We need to establish the existence of bounded and continuously differential value functions \( V_{DC}^{M1}(G_1, G_2), V_{DC}^{R1}(G_1, G_2) \) and \( V_{DC}^{2}(G_1, G_2) \), such that there exists unique solutions \( G_{11}^{DC} \) and \( G_{22}^{DC} \). Similar to the proof of Proposition 1, we will obtain the optimal HJP effort and local advertising strategies by applying backward induction. The HJB equations of retailer 1 and supply chain 2 are:

\[
\lambda \frac{\partial V_{R1}^{DC}}{\partial G_1} = \max \left\{ \epsilon_1 (\eta - 1) \omega_1(t) \left[ 1 - \eta \beta_1 \omega_1(t) \right] + \eta d_1 \omega_2(t) \right\} G_1(t) - \frac{1}{2} B_1^2(t) - \frac{1}{2} (1 - \phi_1) U_1^2(t) + \frac{1}{\lambda} \left[ - \delta G_1(t) + A_1(t) - \alpha A_2(t) + U_1(t) - \beta U_2(t) + \theta (B_1(t) + B_2(t)) \right]
\]

\[
+ \frac{1}{\lambda} \left[ - \delta G_2(t) + A_2(t) - \alpha A_1(t) + U_2(t) - \beta U_1(t) + \theta (B_1(t) + B_2(t)) \right], \tag{31}
\]

\[
\lambda \frac{\partial V_{2}^{DC}}{\partial G_2} = \max \left\{ \epsilon_2 (\eta - 1) \omega_2(t) \left[ 1 - \eta \beta_2 \omega_2(t) \right] + \eta d_2 \omega_1(t) \right\} G_2(t) - \frac{1}{2} B_2^2(t) - \frac{1}{2} (1 - \phi_2) U_2^2(t) + \frac{1}{\lambda} \left[ - \delta G_1(t) + A_1(t) - \alpha A_2(t) + U_1(t) - \beta U_2(t) + \theta (B_1(t) + B_2(t)) \right]
\]

\[
+ \frac{1}{\lambda} \left[ - \delta G_2(t) + A_2(t) - \alpha A_1(t) + U_2(t) - \beta U_1(t) + \theta (B_1(t) + B_2(t)) \right]. \tag{31}
\]
\[
\begin{align*}
\lambda V_{2}^{DC} &= \max_{u_2,b_2} \left\{ \varepsilon_2 \eta \omega_2(t) \left[ 1 - \eta b_2 \omega_2(t) + \eta d_2 \omega_1(t) \right] G_2(t) \right. \\
& \quad - \frac{1}{2} b_2^2(t) - \frac{1}{2} A_2^2(t) - \frac{1}{2} U_2^2(t) \\
& \quad + \frac{\partial V_{2}^{DC}}{\partial G_1} \left[ - \delta G_1(t) + A_1(t) - \alpha A_2(t) + U_1(t) \right] \\
& \quad - \beta U_1(t) + \theta (B_1(t) + B_2(t)) \\
& \quad \left. + \frac{\partial V_{2}^{DC}}{\partial G_2} \left[ - \delta G_2(t) + A_2(t) - \alpha A_1(t) + U_2(t) \right] \\
& \quad - \beta U_1(t) + \theta (B_1(t) + B_2(t)) \right\}.
\end{align*}
\]

(32)

Performing the maximization of the right-hand sides of Equations (31) and (32), yields the optimal local advertising and HJP effort:

\[
\begin{align*}
U_1^{DC}(t) &= \frac{1}{1 - \phi_1} \left( \frac{\partial V_{1}^{DC}}{\partial G_1} - \beta \frac{\partial V_{1}^{DC}}{\partial G_2} \right), \\
U_2^{DC}(t) &= \theta \left( \frac{\partial V_{2}^{DC}}{\partial G_1} - \frac{\partial V_{2}^{DC}}{\partial G_2} \right),
\end{align*}
\]

(33)

\[
\begin{align*}
B_1^{DC}(t) &= \theta \left( \frac{\partial V_{1}^{DC}}{\partial G_1} + \frac{\partial V_{1}^{DC}}{\partial G_2} \right), \\
B_2^{DC}(t) &= \theta \left( \frac{\partial V_{2}^{DC}}{\partial G_1} + \frac{\partial V_{2}^{DC}}{\partial G_2} \right).
\end{align*}
\]

(34)

Similarly, the optimal VJP effort and wholesale price of manufacturer satisfy:

\[
\begin{align*}
A_1^{DC}(t) &= \frac{\partial V_{M_1}^{DC}}{\partial G_1} - \alpha \frac{\partial V_{M_1}^{DC}}{\partial G_2}, \\
A_2^{DC}(t) &= \frac{\partial V_{M_2}^{DC}}{\partial G_2} - \alpha \frac{\partial V_{M_2}^{DC}}{\partial G_1},
\end{align*}
\]

(35)

\[
\begin{align*}
\phi_1^{DC} &= \frac{2}{\lambda} \left[ \frac{\partial V_{M_1}^{DC}}{\partial \omega_1} - \beta \frac{\partial V_{M_1}^{DC}}{\partial \omega_2} \right] - \left( \frac{\partial V_{M_1}^{DC}}{\partial \omega_1} - \beta \frac{\partial V_{M_1}^{DC}}{\partial \omega_2} \right), \\
\phi_2^{DC} &= \frac{2}{\lambda} \left[ \frac{\partial V_{M_2}^{DC}}{\partial \omega_1} - \beta \frac{\partial V_{M_2}^{DC}}{\partial \omega_2} \right] + \left( \frac{\partial V_{M_2}^{DC}}{\partial \omega_1} - \beta \frac{\partial V_{M_2}^{DC}}{\partial \omega_2} \right),
\end{align*}
\]

(36)

\[
\omega_i^{DC}(t) = \frac{2b_{3-i} + d_i}{(4b_1 b_{3-i} - d_1 d_{3-i}) \eta}.
\]

(37)

Then, the optimal profit functions of manufacturer 1, retailer 1 and supply chain 2 are given as follows:

\[
\begin{align*}
V_{M_1}^{DC}(G_1, G_2) &= m_{i1} G_1 + m_{13}, \\
V_{M_1}^{DC}(G_1, G_2) &= r_{i1} G_1 + r_{13}, \\
V_{2}^{DC}(G_1, G_2) &= \eta m_{22} G_2 + v_{13}.
\end{align*}
\]

(38) (39) (40)

where

\[
\begin{align*}
m_{13}' &= \frac{1}{\lambda} \left[ \frac{1}{2} m_{i1}^2 - \frac{\phi_1}{2(1 - \phi_1)^2} r_{11} \\
& \quad + (\theta^2 - \alpha - \beta) \eta m_{22} m_{i1} + \left( \theta^2 + \frac{1}{1 - \phi_1} \right) m_{i1} r_{11} \right],
\end{align*}
\]

(41)

\[
\begin{align*}
r_{13}' &= \frac{1}{\lambda} \left[ \left( \frac{1}{2} \theta^2 + \frac{1}{2(1 - \phi_1)^2} \right) r_{11} + m_{i1} r_{11} \\
& \quad + (\theta^2 - \alpha - \beta) \eta m_{22} r_{11},
\end{align*}
\]

(42)

\[
\begin{align*}
v_{i3}' &= \frac{1}{\lambda} \left[ \left( 1 + \frac{\theta^2}{1 - \phi_1} \right) \eta m_{22} - \alpha \eta m_{11} m_{22} \\
& \quad + \left( \theta^2 - \frac{\beta}{1 - \phi_1} \right) \eta m_{22} r_{11} \right].
\end{align*}
\]

(43)

Finally, by combining (33)–(39), the optimal VJP and HJP efforts of channel members, the optimal wholesale price and participation rate of manufacturer 1 are:

\[
\begin{align*}
A_1^{DC}(t) &= \frac{e_1 e_1}{\eta (\lambda + \delta)} = A_1^{DD}(t), \\
A_2^{DC}(t) &= \frac{e_2 e_2}{\lambda + \delta} = \eta A_2^{DD}(t), \\
U_1^{DC}(t) &= \frac{\eta - 1}{\eta (1 - \phi_1)} \frac{\varepsilon_1 e_1}{\lambda + \delta} = U_1^{DD}(t), \\
U_2^{DC}(t) &= \frac{\varepsilon_2 e_2}{\lambda + \delta} = \frac{\eta (1 - \phi_2)}{\eta - 1} U_2^{DD}(t), \\
B_1^{DC}(t) &= \frac{\theta (\eta - 1) e_1 e_1}{\eta (\lambda + \delta)} = B_1^{DD}(t), \\
B_2^{DC}(t) &= \frac{\varepsilon_2 e_2}{\lambda + \delta} = \frac{\eta}{\eta - 1} B_2^{DD}(t), \\
\omega_i^{DC} &= \frac{2b_{3-i} + d_i}{(4b_1 b_{3-i} - d_1 d_{3-i}) \eta} = \omega_i^{DD}, \\
\phi_i^{DC} &= \phi_i^{DD} = \begin{cases} 
3 - \eta & 1 < \eta < 3 \\
0 & \eta \geq 3
\end{cases}.
\end{align*}
\]

Next, we discuss the optimal brand goodwill of product under the DC channel structure in the following proposition.

Proposition 4.4: At equilibrium, under the DC channel structure, the optimal brand goodwill of product 1 is given by:

\[
G_1^{DC}(t) = \frac{\Omega_i^{DC}}{\delta} + \left( \frac{G_0 - \Omega_i^{DC}}{\delta} \right) e^{-\lambda t}.
\]

(44)

The steady state of brand goodwill is:

\[
\lim_{t \to \infty} G_1^{DC}(t) = \frac{\Omega_i^{DC}}{\delta},
\]

(45)

where

\[
\begin{align*}
\Omega_i^{DC} &= \frac{1}{\eta} \left[ 1 + (\eta - 1) \left( \frac{1}{1 - \phi_1} + \theta^2 \right) \right] \frac{e_1 e_1}{\lambda + \delta} \\
& \quad - (\alpha + \beta - \theta^2) \frac{e_2 e_2}{\lambda + \delta}.
\end{align*}
\]
and

\[ \Omega_{2}^{DC} = (2 + \theta^{2}) \frac{\varepsilon_{2} e_{2}}{\lambda + \delta} - \frac{1}{\eta} \left[ \alpha + (\eta - 1) \left( \frac{\beta}{1 - \phi_{1}} - \theta^{2} \right) \right] \frac{\varepsilon_{1} e_{1}}{\lambda + \delta}. \]

4.3. Comparison between DD and DC channel structures

According to the above discussions, the following proposition summarizes the comparison of the optimal promotion strategies and profits under the different channel structures.

**Proposition 4.5**: Comparing the DD and DC channel structures, we obtain: a. under the DC channel structure, the optimal VJP and HJP efforts of channel members are no less than those under the DD channel structure. b. the optimal wholesale prices of product i are the same under both channel structures.

**Proposition 4.6**: When the manufacturers prefer the VJP programmes (1 < \eta < 3), we obtain:

(a) For any \( G_{1}, G_{2} > 0 \), if \( \theta > \sqrt{(\eta - 1)(\alpha + (\beta/2))} \), then \( \Pi_{1}^{DC} > \Pi_{1}^{DD} \). For any \( G_{1}, G_{2} > 0 \), we obtain \( \Pi_{2}^{DC} > \Pi_{2}^{DD} \). Letting \( L_{1} = \eta(\eta - 1)(\alpha + (\beta/2)) - (5\xi/8) \), \( \Pi_{ch}^{DC} > \Pi_{ch}^{DD} \) if \( L_{1} > 0 \). Otherwise, we have \( \Pi_{ch}^{DC} < \Pi_{ch}^{DD} \).

**Proof**: When the manufacturers prefer the VJP programmes, in order to compare the profits of supply chains under the DD and DC channel structures, the profit differences between manufacturer 1 and retailer 1 are given as follows:

\[ \Pi_{\text{M1}}^{DC} - \Pi_{\text{M1}}^{DD} = \frac{\varepsilon_{2} e_{2}}{\lambda(\lambda + \delta)} \theta^{2} - (\eta - 1) \left( \frac{\alpha + (\beta/2)}{2} \right), \]

\[ \Pi_{\text{R1}}^{DC} - \Pi_{\text{R1}}^{DD} = \frac{\varepsilon_{2} e_{2}}{\lambda(\lambda + \delta)} \theta^{2} - (\eta - 1) \left( \frac{\alpha + (\beta/2)}{2} \right). \]

Then, for any \( \theta > \sqrt{(\eta - 1)(\alpha + (\beta/2))} \), we have \( \Pi_{k1}^{DC} - \Pi_{k1}^{DD} > 0 \). Otherwise, for any \( 0 < \theta < \sqrt{(\eta - 1)(\alpha + (\beta/2))} \), we have \( \Pi_{k1}^{DC} - \Pi_{k1}^{DD} < 0 \). Therefore, \( \Pi_{ch}^{DC} - \Pi_{ch}^{DD} = \Pi_{k1}^{DC} - \Pi_{k1}^{DD} \).

Considering the profit of supply chain 2 under the different channel structures, we obtain:

\[ \Pi_{2}^{DC} - (\Pi_{M2}^{DD} + \Pi_{R2}^{DD}) = \frac{\varepsilon_{2} e_{2}}{\lambda(\lambda + \delta)^{2}} \eta^{2} \left[ \frac{5\xi}{8} (\eta^{2} - \eta + 1) + \frac{\xi}{2} \right], \]

since \( \eta > 1 \), it is easy to see that \( \Pi_{2}^{DC} > \Pi_{M2}^{DD} + \Pi_{R2}^{DD} \) for any positive constant \( \theta \).

Similarly, the profit difference of channel is expressed as follows:

\[ \Pi_{ch}^{DC} - \Pi_{ch}^{DD} = \frac{\varepsilon_{2} e_{2}}{\lambda(\lambda + \delta)^{2}} \eta^{2} \left[ \left( \frac{\xi}{2} + \eta \right) \theta^{2} - (\eta - 1) \left( \alpha + \frac{\beta}{2} - \frac{5\xi}{8} \right) \right]. \]

Without loss of generality, let \( L_{1} = \eta(\eta - 1)(\alpha + (\beta/2)) - (5\xi/8) \). If \( L_{1} < 0 \), for any real constant \( \theta > 0 \), we have \( \Pi_{ch}^{DC} - \Pi_{ch}^{DD} > 0 \). Else if \( L_{1} > 0 \), for any real constant \( \theta > \sqrt{2L_{1}/(2\eta + \xi)} \), we have \( \Pi_{ch}^{DC} - \Pi_{ch}^{DD} > 0 \). Otherwise, \( \Pi_{ch}^{DC} - \Pi_{ch}^{DD} < 0 \).

**Remark 4.1**: When \( \theta > \max(\sqrt{(\eta - 1)(\alpha + (\beta/2))}, \sqrt{2L_{1}/(2\eta + \xi)}) \), manufacturer 1, retailer 1 and the total channel are both better off under the DD channel structure. Then, the channel members tend to accept the DD channel structure. Nevertheless, when \( \theta < \min(\sqrt{(\eta - 1)(\alpha + (\beta/2))}, \sqrt{2L_{1}/(2\eta + \xi)}) \), manufacturer 1, retailer 1 and the total channel are both better off under the DD channel structure. Specifically, when consumers are less sensitive to the HJP programmes (a lower \( \theta \)), the members of supply chain 1 prefer the DD channel structure, yet the members of supply chain 2 still prefer the DC channel structure. To ensure the profit maximization of the total channel, the members of supply chain 2 are compelled to accept the DD channel structure. At this moment, if not properly handled, this phenomenon may bring the great threat to the stability and coordination of channel system. Hence, we shall design a feasible contract to coordinate the supply chain in the future.

The following proposition summarizes the results in other scenarios, in which the manufacturers are not willing to pay a part of the promotional expenditures for retailers.
Proposition 4.7: When the manufacturers don’t prefer the VJP programmes \((\eta \geq 3)\), we obtain:

(a) For any \(G_1, G_2 > 0\), if \(\theta > \sqrt{\alpha(\eta - 1) + \beta}\), then \(\Pi^G_{R_1} > \Pi^D_{R_1}\), \(\Pi^G_{M_1} > \Pi^D_{M_1}\).

(b) For any \(G_1, G_2 > 0\), we obtain \(\Pi^G_{2} > \Pi^D_{2} + \Pi^D_{R_2}\).

(c) Letting \(L_2 = (\eta - 1)(\alpha \eta - \beta - \zeta) + \beta - (\zeta/2)\eta^2\), for any \(G_1, G_2 > 0\), if \(L_2 < 0\) then \(\Pi^G_{ch} > \Pi^D_{ch}\), else if \(L_2 > 0\), for \(\theta > \sqrt{2L_2/(2\eta + \zeta)}\), we have \(\Pi^G_{ch} > \Pi^D_{ch}\).

4.4. Numerical analysis

Based on the results proposed in the previous sections of this paper, we use numerical analysis here to illustrate the relationship between the profits of channel members and parameter \(\eta\) first and the effect of coefficient \(\theta\) on the profit difference of channel second.

To explore the relationship between the profits of channel members and parameter \(\eta\), we use these parameters in our analysis: \(\varepsilon_1 = \varepsilon_2 = 1\), \(\lambda = 0.1\), \(\delta = 0.1\), \(b_1 = b_2 = 0.5\), \(d_1 = 0.3\), \(d_2 = 0.2\), \(\theta = 0.83\), \(\alpha = 0.5\), \(\beta = 0.5\) and change parameters \(\eta \in (1, 3)\) and \(\eta \in (3, \infty)\), respectively. Then, the profits of channel members under two channel structures are shown in Figure 1–4.

Figure 1–4 show that whether VJP programmes between manufacturer and retailer exist or not, (i) the profit of manufacturer increases firstly and then decreases under the DD channel structure; (ii) under the DC channel structure, the profit of manufacturer 1 decreases and the profit of supply chain 2 increases with \(\eta\); (iii) no matter which channel structure the members adopt, the profit of each retailer increases with the impact factor of HJP programmes. Similar results are obtained when we illustrate the effect of parameter \(\eta\) on the profits of manufacturer 2 and retailer 2 under the DD channel structure and then we ignore them in Figure 1 and Figure 2.

Next, we illustrate the effect of coefficient \(\theta\) on the profit difference of channel. Based on the above parameters except parameters \(\eta, \alpha, \beta\) and \(\theta\). We use \(\eta = 2.8\) and change parameter \(\alpha \in (0, 1), \beta \in (0, 1)\) and \(\theta \in (0, 2)\). Parametric case calculations show that profit difference of channel have two kinds of variation tendency represented in Figure 5–6. Similar results are obtained with no VJP programmes and then we don’t discuss this case here.

In this numerical analysis, there exists \(\tilde{\alpha} \in (0.5, 0.6)\) such that for any \(\alpha \geq \tilde{\alpha}, \beta \in (0, 1)\) or \(\alpha \leq \tilde{\alpha}, \beta\) is higher, the tendency of channel choice depends on the values \(\theta_1\) and \(\theta_2\). In addition, when \(\alpha \leq \tilde{\alpha}\) and \(\beta\) is less, the tendency of channel choice only depends on the value \(\theta_2\).
In Figure 5, $\Delta \Pi_{M1}$, $\Delta \Pi_{R1}$ and $\Delta \Pi_{ch}$ represent the profit differences of manufacturer 1, retailer 1 and the total channel under two channel structures, respectively. If $\theta < \theta_1$ holds, we obtain $\Delta \Pi_{M1} < 0$, $\Delta \Pi_{R1} < 0$ and $\Delta \Pi_{ch} < 0$. Else if $\theta > \theta_2$ holds, we have $\Delta \Pi_{M1} > 0$, $\Delta \Pi_{R1} > 0$ and $\Delta \Pi_{ch} > 0$. To be specific, when the consumers are less sensitive to the HJP programmes ($\theta < \theta_1$), manufacturer 1, retailer 1 and total channel get less benefits from the DC channel structure. Even though the profit of supply chain 2 is higher under the DC channel structure, the incremental profit of supply chain 2 through selecting the centralized systems can not bring more revenues to the total channel. Therefore, focusing on the total channel profit, the members prefer the DD channel structure in this case. Moreover, when the consumers are more sensitive to the HJP programmes ($\theta > \theta_2$), the profits of manufacturer 1, retailer 1 and total channel are higher under the DC channel structure. Then, all of the channel members are perfectly willing to select the DC channel structure.

When manufacturer’s and retailer’s competition degree are both less intensive ($\alpha \leq \bar{\alpha}$, a less $\beta$), the variation tendency of profit differences are shown in Figure 6. For any $\theta$, we have $\Delta \Pi_{ch} > 0$ and when $\theta < \theta_2$, $\Delta \Pi_{M1} < 0$ and $\Delta \Pi_{R1} < 0$ hold. It means that when $\theta < \theta_2$, the profits of total channel and supply chain 2 are higher and the profits of manufacturer 1 and retailer 1 are less under the DC channel structure. To ensure the profit maximization of channel, the members of supply chain 1 have to select the DC channel structure.

### 4.5. Conclusions

In this paper, the optimal control problem has been investigated for dynamic joint promotions in the distribution channel. Here, the evolution of brand goodwill is allowed to be influenced by VJP and HJP programmes and the distribution channel consists of the DD and DC channel structures. By using the HJB equation, the optimal VJP and HJP efforts of members are no less than those under the DD channel structure. Comparing the two channel structures, the main conclusions have been presented as follows: (1) If the profit rate of retailer $\eta - 1$ is less than 200%, the manufacturer is willing to support the partial VJP expenditures of the retailer. (2) For the DC channel structure, the optimal VJP and HJP efforts of members are no less than those under the DD channel structure. (3) When the sensitivity coefficient $\theta$ satisfies certain conditions, all of the total channel members are better off under the DC channel structure over the DD channel structure.

This research can be extended in several directions. The stochastic noise in the real world is of great importance, which has two forms, namely, internal noise and external noise (Marinelli, 2006; Raman, 2006). In general, the internal random fluctuations in the processing of brand evolution are inevitable as industrial background and human factors. Besides, the external noise originates
in one or more uncontrollable variables of enterprises. Therefore, it is necessary to handle the stochastic noise to improve the channel performance. We are now researching the stochastic optimal control problems of dynamic joint promotion. The corresponding results will appear in the near future.

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