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On 4D, $\mathcal{N} = 1$ Massless Gauge Superfields of Higher Superspin: Integer Case

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ABSTRACT

We present an alternative method of exploring the component structure of an integer super-helicity $Y = s$ (for any integer $s$) irreducible representation of the Super-Poincaré group. We use it to derive the component action and the SUSY transformation laws. The effectiveness of this approach is based on the equations of motion and their properties, like Bianchi identities. These equations are generated by the superspace action when it is expressed in terms of prepotentials. For that reason we reproduce the superspace action for integer superspin, using unconstrained superfields. The appropriate, to use, superfields are dictated by the representation theory of the group and the requirement that there is a smooth limit between the massive and massless case.

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1 Introduction

Higher spin field theory has a very rich history driving the developments of modern theoretical physics and after many decades still remains a very active subject. It started with Dirac [1] trying to generalize his celebrated spin-$\frac{1}{2}$ equation. His comment in that paper “the underlying theory is of considerable interest” still resonates. After the classical work by Fierz and Pauli [2] there was an increasing number of papers formulating the theory of a massive arbitrary spin in four dimensions [3, 4] as well as developments for the massless arbitrary helicities using the ‘principle’ of gauge invariance [5, 6]. Since then there has been tremendous progress with generalizations of these results regarding irreducible representations of the little group in $D$-dimensions [7], derivations of the massive theories by means of dimensional reduction of the massless theories in $D+1$-dimensions [8], Stückelberg formulations [9], BRST [10], quantization and many other things.

The discussion of arbitrary spin gauge fields in the context of simple symmetry in four dimensions parallels this development of the general discussion. At the level of component fields this was initiated by Curtright [11], followed by the superfield discussion at the level of on-shell equations of motion [12], and finally followed by the off-shell discussions in the work of Kuzenko, et. al. [13, 14]. These pioneering works on higher spin 4$D$, $\mathcal{N} = 1$ supermultiplets have also led to the creation of a growing literature [15] on the subject.

A current generator of interest about higher spin theories has been generated by string theory as it low-energy approximation leads to consideration of fields of unbounded spins since the spectrum of string and superstring theory includes an infinite tower of massive spin states. Therefore a limit must exist where (super)string theory is formulated as a field theory of interacting spins. That points to the interesting direction of extending all previous results to include supersymmetry. The tool to build 4$D$, $\mathcal{N} = 1$ manifestly SUSY invariant theories is superspace and the usage of superfields.

For the massless case such a construction exists [13, 14]. The theories presented in these works, were initially described in terms of constrained superfields. The purpose of the differential constraints is to achieve gauge invariance. As they comment in their work these constraints can easily be solved in terms of prepotentials. These prepotentials can play a role in the formulation of massive superspin theories and maybe even spin interaction theories. In a subsequent work [16], these unconstrained prepotentials were introduced and used to show that the works of [17] occur by applying a transformation to the original formulations.

In this current work (and an accompanying one [18]) we would like to show how representation theory of the Super-Poincaré group makes these prepotential variables building blocks for massive and massless theories and then use them to reproduce the realizations of irreducible representations with arbitrary super-helicity.
In the previous works, when discussion about the component field spectrum of the theories was given, it was based on $\theta$-expansion of the superfields in the Wess-Zumino gauge. This implied that by using that ansatz for the components and the usual rules of projection, the component action and the SUSY-transformation laws can be derived.

This process is straightforward but cumbersome. For this reason we exploit an alternative efficient way of defining components, using the superfield equations of motion. The action itself, with the help of the Bianchi identities, will guide us to efficient definitions of the components, derive the component action and the SUSY-transformation laws. This approach builds naturally on [19] for the study of the component structure of super-helicity $Y = 1$ and discussions [20] on old-minimal supergravity.

However there is a key difference with both of these. The first one used the superfield strength as a guide for the definition of the components. This approach can not be generalized for the arbitrary super-helicity because of the mass dimensionality of the superfield strength is proportional to super-helicity. In the second paper components were defined without finding the component action and SUSY-transformation laws. We will do both of these for the arbitrary integer super-helicity case.

In this follow, we focus on arbitrary integer super-helicity irreducible representation of the $4D, \mathcal{N} = 1$ Super-Poincaré group. A discussion for the half-integer super-helicities will presented in a following letter. The presentation is organized as follows: In section 2 we briefly review the representation theory of the little group of the $4D, \mathcal{N} = 1$ Super-Poincaré group, following [10]. This discussion will illuminate the proper superfields one should use in order to construct the desired representations. In section 3 we focus on the massless integer super-helicity case and illustrate how the principle of gauge invariance emerges from the requirement to have a smooth transition between massive and massless theories. In section 4 we find the superspace action of the theory and prove that it describes the desired super-helicity. The last section 5 is a discussion about the off-shell component structure of the theory. We present a self-contained method of defining the components, find the component action and give explicit expressions for the SUSY-transformation laws. The main new results in this (and a companion) work involve the derivation of a complete component-level description that involves no explicit $\theta$-expansion of superfields. The conventions used are the ones of [20].

2 Irreducible Representations

As is well known the Super-Poincaré group has two Casimir operators that label the irreducible representations. The first one is the mass and the other one is a supersymmetric extension of the Poincaré Spin operator.
2.1 Massive Case

For the massive case the second casimir operator takes the form

\[ C_2 = \frac{W^2}{m^2} + \left( \frac{3}{4} + \lambda \right) P_{(o)} \]  

(1)

where \( W^2 \) is the Poincaré Spin operator, \( P_{(o)} \) is a projection operator and the parameter \( \lambda \) satisfies the equation

\[ \lambda^2 + \lambda = \frac{W^2}{m^2} \]  

(2)

In order to diagonalize \( C_2 \) we want to diagonalize both \( W^2, P_{(o)} \). The superfield \( \Phi_{\alpha(n)\dot{\alpha}(m)} \) which does that and describes the highest possible superspin representation

\[ C_2 \Phi_{\alpha(n)\dot{\alpha}(m)} = Y(Y+1)\Phi_{\alpha(n)\dot{\alpha}(m)} \]  

(3)

has to satisfy the following constraints:

1. Symmetrized dotted and undotted indices
   \[ D^2 \Phi_{\alpha(n)\dot{\alpha}(m)} = 0 \]
   \[ \bar{D}^2 \Phi_{\alpha(n)\dot{\alpha}(m)} = 0 \]
   \[ D^\gamma \Phi_{\alpha(n-1)\dot{\alpha}(m)} = 0 \]
   \[ \partial^{\gamma\dot{\gamma}} \Phi_{\alpha(n-1)\dot{\gamma}\dot{\alpha}(m-1)} = 0 \]
   \[ \Box \Phi_{\alpha(n)\dot{\alpha}(m)} = m^2 \Phi_{\alpha(n)\dot{\alpha}(m)} \]  

(4)

All these can be satisfied if

\[ \Phi_{\alpha(n)\dot{\alpha}(m)} \sim D^\gamma W_{\alpha(n)\gamma\dot{\alpha}(m)}, W_{\alpha(n+1)\dot{\alpha}(m)} \sim \bar{D}^2 D_{(\alpha n+1)} \Phi_{\alpha(n)\dot{\alpha}(m)} \]  

(5)

with

\[ \bar{D}_\beta W_{\gamma\alpha(n)\dot{\alpha}(n)} = 0, \text{ chiral} \]
\[ \partial^{\beta\dot{\beta}} W_{\beta\alpha(m)\dot{\beta}\dot{\alpha}(n-1)} = 0 \]
\[ \Box W_{\alpha(m+1)\dot{\alpha}(n)} = m^2 W_{\alpha(m+1)\dot{\alpha}(n)} \]  

(6)

and the spin content of this supermultiplet is \( j = Y + 1/2, Y, Y, Y - 1/2 \).

Therefore the superfield that describes a superspin \( Y \) system, has index structure such that \( n + m = 2Y - 1 \) where \( n, m \) are integers. This Diophantine equation has a finite number of solutions for \( (n, m) \) pairs, but the corresponding superfields are all related because we can use the \( \partial^{\beta\dot{\beta}} \) operator to convert one kind of index to another. So we can pick one of them to represent the entire class.
One last comment has to be made about the reality of the representation. Under a hermitian conjugation a \((n,m)\) representation realized by a superfield like \(\Phi_{\alpha(n)\dot{\alpha}(m)}\) goes to a \((m,n)\) representation, realized by \(\bar{\Phi}_{\alpha(m)\dot{\alpha}(n)}\)

\[
(n,m)^* \rightarrow (m,n)
\]

\[
\begin{cases}
(n,n)^* \rightarrow (n,n) : \text{reality} \\
(n,m)^* \rightarrow (m,n) \neq (n,m)
\end{cases}
\]

to make real representations

we need to consider \((n,m) \oplus (m,n)\)

At the superfield level this mapping can be done by the dimensionless operator \(\Delta_{a\dot{a}} \equiv -i\frac{\partial}{\partial\gamma_1}\) which if used in repetition will convert all the undotted indices to dotted ones and vice versa.

\[
\bar{\Phi}_{\alpha(m)\dot{\alpha}(n)} = \Delta_{a} \gamma_1 \ldots \Delta_{a_m} \gamma_m \Delta_{\dot{a}_1} \ldots \Delta_{\dot{a}_1} \bar{\Phi}_{\dot{\gamma}(m)}(n)
\]

For irreducible representations with \(n = m\) (bosonic superfields) the reality condition becomes \(\Phi_{\alpha(n)\dot{\alpha}(n)} = \bar{\Phi}_{\alpha(n)\dot{\alpha}(n)}\) and for fermionic superfields \(n = m+1\) the reality condition is the Dirac equation \(i\partial_{\alpha} \dot{\alpha} \bar{\Phi}_{\alpha(n)\dot{\alpha}(n)} + m\Phi_{\alpha(n)\dot{\alpha}(n)} = 0\).

### 2.2 Massless Case

For the massless case, the supersymmetric analogue to the Pauli-Lubanski vector takes the form

\[
Z_{\gamma\dot{\gamma}} = W_{\gamma\dot{\gamma}} + \frac{1}{4} [D_{\gamma}, \bar{D}_{\dot{\gamma}}]
\]

and our goal is to make it proportional to momentum. The superfield \(F_{\alpha(n)\dot{\alpha}(m)}\) which does that and describes the highest super-helicity

\[
Z_{\gamma\dot{\gamma}} F_{\alpha(n)\dot{\alpha}(m)} = \left( Y + \frac{1}{4} \right) P_{\gamma\dot{\gamma}} F_{\alpha(n)\dot{\alpha}(m)}, \ Y = \frac{n - m}{2}
\]

must satisfy the following:

symmetrized dotted and undotted indices

\[
\bar{D}_{\dot{\gamma}} F_{\alpha(n)\dot{\alpha}(m)} = 0, \ \text{chiral}
\]

\[
D^\beta F_{\beta\alpha(n-1)\dot{\alpha}(m)} = 0
\]

\[
\partial_{\dot{\gamma}} \dot{\beta} F_{\alpha(n)\dot{\beta}\dot{\alpha}(m-1)} = 0
\]

and the helicity content is \(h = Y + 1/2\), \(Y\)

So the superfield that describes a system with super-helicity \(Y\), must have index structure such that \(n - m = 2Y\). This Diophantine equation has infinite many solutions with an increasing number of indices. Nevertheless all of them can be generated by acting with \(\partial_{\dot{\gamma}} \dot{\beta}\) on the superfield with the fewest indices \(F_{\alpha(2Y)}\).
3 Integer super-helicity, \( Y = s \)

The above discussion suggests that a theory of massive integer superspin \( Y = s \) must be constructed in terms of a fermionic superfield \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) \) and there exists a chiral superfield \( W_{\alpha(s+1)\hat{\alpha}(s-1)} \sim \bar{D}^2D_{\alpha(s+1)}\Psi_{\alpha(s)}\hat{\alpha}(s-1) \).

On the other hand the theory of massless integer super-helicity must be described in terms of a chiral superfield \( F_{\alpha(2s)} \).

Now let us assume we have managed to develop the theory of massive integer superspin. We should be able to take the massless limit of that. It would be nice if such a limit leads to the theory of massless integer super-helicity (plus possibly other sectors that decouple). But we showed that these two theories are described by different objects. How can this be? For something like that to happen we have to able to construct an object like \( F_{\alpha(2s)} \) out of the remaining objects after the limit has been taken. Given the chirality property of \( F \) and \( W \) and their index structure we could guess a mapping that could do the trick.

\[
F_{\alpha(2s)} \sim \partial_{\alpha_2} \hat{\alpha}_s \cdots \partial_{\alpha_{s+1}} \hat{\alpha}_1 \bar{D}^2D_{\alpha(s)}\Psi_{\alpha(s)}\hat{\alpha}(s-1) \tag{10}
\]

But there is a problem with this map. The problem is that \( F_{\alpha(2s)} \) which describes the system and carries the physical degrees of freedom seems to be defined in terms of another object \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) \). Also \( F \) as defined above seems to have the on-shell degrees of freedom of \( \Psi \) which is more than needed. If this is going to work we have to find a way to 1) remove the physical (observable) status of \( \Psi \) and 2) remove its extra degrees of freedom.

There is a mechanism that can do both at the same time. That is to introduce a redundancy. We identify \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) \) with \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) + R_{\alpha(s)}\hat{\alpha}(s-1) \) and instead of talking about \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) \) we talk about equivalence classes. \( \Psi_{\alpha(s)}\hat{\alpha}(s-1) \sim \Psi_{\alpha(s)}\hat{\alpha}(s-1) + R_{\alpha(s)}\hat{\alpha}(s-1) \). This redundancy has to respect the physical - propagating degrees of freedom of \( F \) and leave them unchanged. Hence

\[
\partial_{\alpha_2} \hat{\alpha}_s \cdots \partial_{\alpha_{s+1}} \hat{\alpha}_1 \bar{D}^2D_{\alpha(s)}R_{\alpha(s-1)}\hat{\alpha}(s) = 0 \tag{11}
\]

The most general solution to that is

\[
R_{\alpha(s)}\hat{\alpha}(s-1) = \frac{1}{s!}D_{\alpha(s)}K_{\alpha(s-1)}\hat{\alpha}(s-1) + \frac{1}{(s-1)!}\bar{D}(\hat{\alpha}_{s-1}\Lambda_{\alpha(s)}\hat{\alpha}(s-2)) \tag{12}
\]

where \( K_{\alpha(s-1)}\hat{\alpha}(s-1) \), \( \Lambda_{\alpha(s)}\hat{\alpha}(s-2) \) are completely unconstrained superfields. It is obvious that this redundancy will be the starting point for the gauge invariance story.

4 The Superspace Action

Using the equivalency class characterized by \( \Psi \) and redundancy \( R \) we attempt to construct a superspace action that will describe the irreducible representation of integer super-helicity.
For that Ψ must have mass dimensions \( \frac{1}{2^4} \) and the action must involve two covariant derivatives.\(^5\)

The most general action is

\[
S = \int d^8 z \left[ a_1 \Psi^{\alpha(s)\dot{\alpha}(s-1)} D^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
+ a_2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \\
+ a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{D}^{\dot{\alpha}_s} D_{\alpha_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \\
+ a_4 \Psi^{\alpha(s)\dot{\alpha}(s-1)} D_{\alpha_s} \bar{D}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \right] + c.c.
\]

The goal is to find an action that respects the redundancy. That is the starting point for gauge invariance \( \delta_G S = 0 \). The strategy to obtain this is to pick the free parameters in a special way. If this is not possible then we introduce auxiliary superfields, compensators and/or impose constraints on the parameters of the redundancy (gauge parameters). We also assume it is reasonable to expect any compensators introduced, if necessary, will not introduce degrees of freedom with spin higher or equal than the one we wish to describe. Thus, they must have less indices than Ψ.

For this case we obtain the following expression for the modification of the action due to the redundancy,

\[
\delta_G S = \int d^8 z \left\{ -2a_1 D_{\alpha_s} \Psi^{\alpha(s)\dot{\alpha}(s-1)} \\
+ a_4 \bar{D}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \right\} D^2 \bar{D}_{\dot{\alpha}_{s-1}} A_{\beta\alpha(s-1)\dot{\alpha}(s-2)} \\
+ \left\{ -a_3 \left[ \frac{s-1}{s} \right] \bar{D}^{\dot{\alpha}_s} D_{\alpha_s} \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \\
+ \left[ -a_3 + \frac{s+1}{s} a_4 \right] D_{\alpha_{s-1}} \bar{D}^{\dot{\alpha}_s} \bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)} \right\} D^2 K_{\beta\alpha(s-2)\dot{\alpha}(s-1)} \\
+ \left\{ 2a_2 D_{\alpha_s} \bar{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} - a_3 D_{\alpha_s} D^2 \bar{\Psi}^{\alpha(s-1)\dot{\alpha}(s)} \right\} K_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c.
\]

(13)

Obviously we can not make all this terms vanish just by picking values for the a’s without setting them all to zero and also we can’t introduce compensators with proper mass dimensionality and index structure. The way out is to give some structure to the gauge parameter \( K \). So let us choose

\[
a_1 = a_4 = 0 \\
D^2 K_{\beta\alpha(s-2)\dot{\alpha}(s-1)} = 0 \rightarrow K_{\alpha(s-1)\dot{\alpha}(s-1)} = D^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)}
\]

(14)

\[
2a_2 = -a_3
\]

\(^5\)it’s highest spin component is a propagating fermion.

\(^6\)The action must be quadratic in Ψ and dimensionless.
So we find

\[
\delta_G S = -a_3 \int d^8 z D_{\alpha_1} \bar{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \left( \bar{D}^\dot{\alpha} L_{\dot{\alpha}(s-1)\dot{\alpha}(s-1)} + \bar{D}^\dot{\alpha} \bar{\Lambda}_{\dot{\alpha}(s-1)\dot{\alpha}(s-1)} \right) + c.c.
\]

This suggests we introduce a real bosonic compensator \( V_{\alpha(s-1)\dot{\alpha}(s-1)} \) which transforms like

\[
\delta_G V_{\alpha(s-1)\dot{\alpha}(s-1)} = D^{\alpha s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{D}^{\dot{\alpha} s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}
\]

and couples with the real piece of \( D^{\alpha s} \bar{D}^2 \Psi_{\alpha(s)\dot{\alpha}(s-1)} \).

In order to achieve invariance, we add to the action two new pieces, a coupling term of \( V \) with \( \Psi \) and a kinetic energy term for \( \dot{V} \). The full action takes the form

\[
S = \int d^8 z \left[ \frac{1}{2} a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + c.c. \right] + a_3 \Psi^{\alpha(s)\dot{\alpha}(s-1)} \bar{D}^{\dot{\alpha} s} \bar{D}_{\alpha s} \Psi^{\alpha(s-1)\dot{\alpha}(s)} + c.c. - a_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} D^{\alpha s} \bar{D}^2 \Psi^{\alpha(s)\dot{\alpha}(s-1)} + c.c. + b_1 V^{\alpha(s-1)\dot{\alpha}(s-1)} D^{\gamma} \bar{D}^2 D_{\gamma} V^{\alpha(s-1)\dot{\alpha}(s-1)} + c.c. + b_2 V^{\alpha(s-1)\dot{\alpha}(s-1)} \left\{ D^2, \bar{D}^2 \right\} V^{\alpha(s-1)\dot{\alpha}(s-1)} + c.c. + b_3 V^{\alpha(s-1)\dot{\alpha}(s-1)} D_{\alpha s-1} \bar{D}^2 \bar{D}^{\gamma} V^{\alpha(s-2)\dot{\alpha}(s-1)} + c.c. + b_4 V^{\alpha(s-1)\dot{\alpha}(s-1)} D_{\alpha s-1} \bar{D}^{\dot{\alpha} s-1} \bar{D}^{\gamma} \bar{D}^2 V^{\alpha(s-2)\dot{\alpha}(s-2)} + c.c.
\]

and it has to be invariant under

\[
\delta_G \Psi^{\alpha(s)\dot{\alpha}(s-1)} = -D^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{(s-1)!} \bar{D}_{(\dot{\alpha} s-1)} \Lambda_{\alpha(s)\dot{\alpha}(s-2)}
\]

\[
\delta_G V^{\alpha(s-1)\dot{\alpha}(s-1)} = D^{\alpha s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{D}^{\dot{\alpha} s} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}
\]

The equations of motion of the superfields are the variation of the action with respect to the corresponding superfield

\[
T^{\alpha(s)\dot{\alpha}(s-1)} = \frac{\delta S}{\delta \Psi^{\alpha(s)\dot{\alpha}(s-1)}}
\]

\[
G_{\alpha(s-1)\dot{\alpha}(s-1)} = \frac{\delta S}{\delta V^{\alpha(s-1)\dot{\alpha}(s-1)}}
\]

and the invariance of the action gives the following Bianchi Identities

\[
\bar{D}^2 T^{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{s!} D_{(\alpha s)} G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0
\]

\[
\bar{D}^{\dot{\alpha} s} T_{\alpha(s)\dot{\alpha}(s-1)} = 0
\]

The satisfaction of the Bianchi identities fix all the coefficients

\[
\begin{align*}
b_1 &= \frac{1}{2} a_3 & b_3 &= 0 \\
b_2 &= 0 & b_4 &= 0
\end{align*}
\]
and the action takes the form\(^6\)

\[
S = \int d^8 z \left\{ -\frac{1}{2} c \Psi^{\alpha(s)} \tilde{\partial}^{(s-1)} \tilde{D}^2 \Psi^{\alpha(s)} \tilde{\alpha}(s-1) + \text{c.c.} \\
+ c \Psi^{\alpha(s)} \tilde{\partial}^{(s-1)} \tilde{D} \alpha \tilde{\alpha}(s) \tilde{\Psi}^{\alpha(s-1)} \tilde{\alpha}(s) \\
- c V^{\alpha(s)} \tilde{\partial}^{(s-1)} \tilde{D} \alpha \tilde{\alpha}(s) \tilde{\Psi}^{\alpha(s-1)} \tilde{\alpha}(s-1) + \text{c.c.} \\
+ \frac{1}{2} c V^{\alpha(s)} \tilde{\partial}^{(s-1)} \tilde{D} \gamma \tilde{D}^2 \gamma V^{\alpha(s)} \tilde{\alpha}(s-1) \tilde{\alpha}(s-1) \right\} 
\]

(20)

The equations of motion are

\[
T_{\alpha(s)} \tilde{\alpha}(s-1) = -c \tilde{D}^2 \Psi^{\alpha(s)} \tilde{\alpha}(s-1) + \frac{c}{s!} \tilde{D} \tilde{\alpha}(s) \tilde{\Psi}^{\alpha(s)} \tilde{\alpha}(s) \\
+ \frac{c}{s!} \tilde{D}^2 \alpha \tilde{\alpha}(s-1) \tilde{\Psi}^{\alpha(s)} \tilde{\alpha}(s-1) 
\]

(21a)

\[
G_{\alpha(s-1)} \tilde{\alpha}(s-1) = -c \left( \tilde{D} \tilde{\alpha} \tilde{D}^2 \Psi^{\alpha(s)} \tilde{\alpha}(s-1) + \tilde{D} \tilde{\alpha} \tilde{D} \tilde{\Psi}^{\alpha(s-1)} \tilde{\alpha}(s) \right) \\
+ c \tilde{D} \gamma \tilde{D}^2 \gamma V^{\alpha(s)} \tilde{\alpha}(s-1) \tilde{\alpha}(s-1) 
\]

(21b)

This is exactly the longitudinal-linear theory presented in [14] if we solve the constraint superfield and express their action in terms of the prepotential. Now, however we gain a different understanding of why the action has to be expressed in terms of a superfield like \( \Psi \) and why it has a gauge transformation as it does.

The work in [14] presented a second theory for integer super-helicity, the transverse-linear theory. That theory is most certainly consistent classically, but violates one of our assumptions in that some of its auxiliary fields possess spins greater than that carried by the gauge superfield. To our knowledge, no studies of the quantum behavior of these off-shell supersymmetrical and even free theories has been carried out. If is our suspicion that the presence of auxiliary superfields with a higher superspin than the main gauge superpotential is likely to have a more complicated ghost structure. It would be a very interesting investigation to test this idea.

We have managed to find a superspace action which is gauged invariant but still we haven’t proved that this theory describes an integer super-helicity system. To do so, we must show that there is an object like \( F_{\alpha(2s)} \), it is chiral and on-shell it satisfies the required by representation theory constraints.

Using the equations of motion we can now prove that a chiral superfield \( F_{\alpha(2s)} \) exists

\(^6\text{Here } c \text{ is an overall unconstrained parameter which can be absorbed into the definition of } \Psi. \)

\(^6\text{We leave it as it is for now and fix it later in the component discussion.}\)
and satisfies following Bianchi identity:

\[
\bar{D}\alpha_{2s}\bar{F}_{\dot{\alpha}(2s)} = -\frac{i}{(2s-1)!c}\bar{\partial}^{\alpha_{s}}(\dot{\alpha}_{2s-1} \cdots \partial^{\alpha_{1}}\dot{\alpha}_{s}T_{\alpha(s)\dot{\alpha}(s-1)})
\]

\[
+ \frac{B}{(2s-1)!}\bar{D}^{\alpha_{s-1}}(\dot{\alpha}_{2s-1} \cdots \partial^{\alpha_{1}}\dot{\alpha}_{s+1}\bar{T}_{\alpha(s-1)\dot{\alpha}(s)})
\]

\[
+ \frac{1+2cB}{(2s-1)!2c}\bar{D}(\dot{\alpha}_{2s-1}\partial^{\alpha_{s-1}}\dot{\alpha}_{2s-2} \cdots \partial^{\alpha_{1}}\dot{\alpha}_{s}G_{\alpha(s-1)\dot{\alpha}(s-1)})
\]

\[
+ \frac{1}{(2s-1)!2c}\bar{D}(\dot{\alpha}_{2s-1}\partial^{\alpha_{s}}\partial^{\alpha_{s-1}}\dot{\alpha}_{2s-2} \cdots \partial^{\alpha_{1}}\dot{\alpha}_{s}T_{\alpha(s)\dot{\alpha}(s-1)})
\]

where

\[
\bar{F}_{\dot{\alpha}(2s)} = \frac{1}{(2s)!}\bar{D}^{2}\bar{D}(\dot{\alpha}_{2s}\partial^{\alpha_{s-1}}\dot{\alpha}_{2s-1} \cdots \partial^{\alpha_{1}}\dot{\alpha}_{s+1}\bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)})
\]

and that shows that if \(T_{\alpha(s)\dot{\alpha}(s-1)} = G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0\), we obtain the desired constraints to describe a super-helicity \(Y = s\) system, where \(B\) is a parameter determined by variations and definitions.

Before we start investigating the field spectrum of the above action, one more comment needs to be made. This specific action and superfield configuration is not unique but the simplest representative of a two parameter family of equivalent theories. To see that we can perform redefinitions of the superfields. Dimensionality and index structure allow us to make the following redefinition of \(\Psi\)

\[
\Psi_{\alpha(s)\dot{\alpha}(s-1)} \rightarrow \Psi_{\alpha(s)\dot{\alpha}(s-1)} + \frac{z}{s!}\bar{D}(\alpha_{s}\gamma_{\alpha(s-1)})\dot{\alpha}(s-1)
\]

(23)

where \(z\) is a complex parameter. This operation will generate an entire class of actions and transformation laws which all are related by the above redefinition. The action is

\[
S = \int d^{8}w \left\{-\frac{1}{2}c\Psi^{\alpha(s)\dot{\alpha}(s)}\bar{D}^{2}\Psi_{\alpha(s)\dot{\alpha}(s)} + c.c.
\right.
\]

\[
+ c\Psi^{\alpha(s)\dot{\alpha}(s-1)}\bar{D}\alpha_{s}\bar{\Psi}_{\alpha(s-1)\dot{\alpha}(s)}
\]

\[
+ c(z + \bar{z} - 1)\gamma^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}^{\alpha}\bar{D}^{2}\Psi_{\alpha(s)\dot{\alpha}(s-1)} + c.c.
\]

\[
+ c\bar{z}V^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}^{2}\bar{D}\gamma_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c.
\]

\[
- \left[\frac{s-1}{s}\right]c\bar{z}V^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}_{\alpha(s-1)}\bar{D}^{\alpha}\bar{D}^{\gamma}\bar{\gamma}_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c.
\]

\[
+ \frac{1}{2}c(z + \bar{z} - 1)^{2}V^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}^{2}\bar{D}^{\gamma}\bar{D}_{\gamma}V_{\alpha(s-1)\dot{\alpha}(s-1)}
\]

\[
+ \left[\frac{1}{s}\right]c\bar{z}V^{\alpha(s-1)\dot{\alpha}(s-1)}\left\{\bar{D}^{2},\bar{D}^{2}\right\}V_{\alpha(s-1)\dot{\alpha}(s-1)}
\]

\[
+ \left[\frac{s-1}{2s}\right]c(z + 2\bar{z} - 2)V^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}_{\alpha(s-1)}\bar{D}^{2}\bar{D}^{\gamma}V_{\gamma\alpha(s-2)\dot{\alpha}(s-1)} + c.c.
\]

\[
- \left[\frac{(s-1)^{2}}{2s^{2}}\right]c\bar{z}V^{\alpha(s-1)\dot{\alpha}(s-1)}\bar{D}_{\alpha(s-1)}\bar{D}_{\alpha(s-1)}\bar{D}^{\gamma}\bar{D}^{\gamma}V_{\gamma\alpha(s-2)\dot{\alpha}(s-2)} + c.c.
\]
and the transformation laws are

\[ \begin{align*}
\delta_{\bar{G}} \Psi_{\alpha}(s-1) & = (z - 1) D^2 L_{\alpha(s)\dot{\alpha}(s-1)} - \frac{z}{s!} D(\alpha_{s}) \bar{D} \Lambda_{\alpha(s-2)} \bar{\Lambda}_{\alpha(s-2)} \delta_{\bar{G}} \Psi_{\alpha}(s-1) \\
\delta_{\bar{G}} V_{\alpha(s-1)\dot{\alpha}(s-1)} &= D^{\alpha_{s}} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{D}^{\dot{\alpha}_{s}} \bar{L}_{\alpha(s-1)\dot{\alpha}(s)}
\end{align*} \]

(25a)  

(25b)

5 Projection and Components

Although superspace was developed to describe supersymmetric theories in a more efficient, compact and clear way, there are still some reasons why we would like to study the off-shell component structure of the theory.

1. There are cases where two theories on-shell describe the same physical system. Therefore from the path integral point of view the theories are equivalent. Nevertheless the off-shell structure of the two theories might be completely different. Knowledge of the component formulation of the two theories will help us decide if they are different theories with the same on-shell description or they are the same theory and there is a 1-1 mapping between the two.

2. The off-shell component structure of a supersymmetric theory will give us clues about which theories can be used to realize higher \( N \) and higher \( D \) representations.

For these reasons we would like to extract the component field content of the above superspace action, the number of degrees of freedom involved, their transformation law under supersymmetry and their gauge transformations.

Previous discussion to this use the Wess-Zumino and explicit \( \theta \)-expansions. We propose a different technique that will illuminate a more natural way to define the component structure and make the entire process of finding the component action and SUSY-transformation laws efficiently.

Since we want the auxiliary fields of the final action to be gauge invariant it might be smart to define them using objects that are already gauge invariant. But the superspace action already provides us with two gauge invariant objects, the equations of motion:  

\[ \begin{align*}
T_{\alpha(s)\dot{\alpha}(s-1)} &= \frac{\delta S}{\delta \Psi_{\alpha(s)\dot{\alpha}(s-1)}} , \quad [T_{\alpha(s)\dot{\alpha}(s-1)}] = 3/2 \\
G_{\alpha(s-1)\dot{\alpha}(s-1)} &= \frac{\delta S}{\delta V_{\alpha(s)\dot{\alpha}(s-1)}} , \quad [G_{\alpha(s-1)\dot{\alpha}(s-1)}] = 2 \\
G_{\alpha(s-1)\dot{\alpha}(s-1)} &= G_{\alpha(s-1)\dot{\alpha}(s-1)} 
\end{align*} \]

(26a)  

(26b)

\[ ^7 \text{There is also the superfield strength } F_{\alpha(2s)} \text{ but because of dimensionality reasons we can not write the action in terms of it.} \]
Because they are gauge invariant, if we expand them to components, each one of them will be gauge invariant. Furthermore because they vanish on-shell each one of these components will vanish as well. So it looks like the ideal place to look for the auxiliary component structure.

These superfields satisfy a set of equations that we will discover as we go along, but at the top of the list we have the Bianchi identities\(^8\) and their consequences:

\[
D^2 T_{\alpha(s)\dot{\alpha}(s-1)} + \frac{1}{s!} D_{(\alpha_s G_{\alpha(s-1)})\dot{\alpha}(s-1)} = 0 \leadsto D^2 G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0
\]

\[
\bar{D}^2 G_{\alpha(s-1)\dot{\alpha}(s-1)} = 0 \quad (27a)
\]

\[
\bar{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)} = 0 \leadsto \bar{D}^2 T_{\alpha(s)\dot{\alpha}(s-1)} = 0
\]

The results of these are that most of the components in the expansion of \(T\) and \(G\) vanish and we are left with very few that we can associate with auxiliary fields. For example, the bosonic auxiliary fields (dimensionality 2) have to be related to \(\bar{D}_{(\dot{\alpha}_s T_{\alpha(s-1)})\dot{\alpha}(s-1)}\), \(D^{\alpha_s} T_{\alpha(s-1)}\), \(G_{\alpha(s-1)\dot{\alpha}(s-1)}\) and the fermionic ones (3/2, 5/2) will have to be related to \(T_{\alpha(s)\dot{\alpha}(s-1)}\), \(D^2 T_{\alpha(s)\dot{\alpha}(s-1)}\). So by just looking at the Bianchi identities we find for free the spectrum of the auxiliary fields of the action and because they are gauge invariant we can do a straightforward counting of their degrees of freedom. For the dynamical fields, we can use the superfield strength \(F_{\alpha(2s)}\) to connect them with some components of the superfields. Instead we will let the action, the equations of motion and their properties to guide us to their definition.

But if the equations of motion are the proper objects to define the components and we want to find the component action of the theory we must be able to express the action in terms of the equations of motion. That can be easily done by using the definitions of \(T\) and \(G\) to rewrite the action in the following form

\[
S = \int d^8z \left\{ \frac{1}{2} \Psi^{\alpha(s)\dot{\alpha}(s-1)} T_{\alpha(s)\dot{\alpha}(s-1)} + c.c. \right\}\]

\[
= \int d^4x \frac{1}{2} D^2 \bar{D}^2 \left( \Psi^{\alpha(s)\dot{\alpha}(s-1)} T_{\alpha(s)\dot{\alpha}(s-1)} \right) + c.c.
\]

\[
+ \frac{1}{2} D^2 \bar{D}^2 \left( V^{\alpha(s-1)\dot{\alpha}(s-1)} G_{\alpha(s-1)\dot{\alpha}(s-1)} \right)
\]

and now we distribute the covariant derivatives.

---

\(^8\)The Bianchi identities include the entire information about redundancy and therefore effectively they make everything that could have been gauged away, if we had followed the WZ-gauge path, disappear.
5.1 Fermions

Let us focus on the fermionic action first. After the distribution of D’s and the usage of Bianchi identities we find for the fermionic Lagrangian:

\[
\mathcal{L}_F = \frac{1}{2} \bar{D}^2 D^2 \bar{\Psi}^{\alpha(s)} \bar{\alpha}^{(s-1)} |T_{\alpha(s)}\bar{\alpha}^{(s-1)}|
\]

\[
+ \frac{1}{2} \left( \bar{D}^2 \Psi^{\alpha(s)} \bar{\alpha}^{(s-1)} - \frac{1}{s!} \bar{D}^2 \bar{D}(\alpha, \Psi^{\alpha(s-1)} \bar{\alpha}^{(s-1)}) \right) |D^2 T_{\alpha(s)}\bar{\alpha}^{(s-1)}|
\]

\[
- \frac{1}{2 (s+1)!s!} \bar{D}^{(\alpha+1)} \bar{D}(\alpha, \Psi^{\alpha(s-1)} \bar{\alpha}^{(s-1)}) + \frac{1}{(s+1)!s!} D^{(\alpha+1)} D(\alpha, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})
\]

\[
+ \frac{s}{2 s + 1 s!} \bar{D}^{(\alpha, \bar{\Psi}^{\alpha(s-1)} \bar{\alpha}^{(s-1)})} |D^{\alpha(s-1)} G_{\alpha(s-1)}\bar{\alpha}^{(s-1)}|
\]

At this point we can show that \(T\) and \(G\) satisfy a few more identities:

\[
\frac{1}{(s+1)!s!} D^{(\alpha, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})} = \frac{i c}{(s+1)!} \partial^{(\alpha, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
- \frac{i c}{(s+1)!} \partial^{(\alpha, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
+ \frac{i c}{s! s(s+1)!} \partial^{(\alpha, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
\frac{1}{s!} \bar{D}^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})} = \frac{i s + 1}{s!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
+ \frac{s + 1}{s} \bar{D}^2 T_{\alpha(s-1)}\bar{\alpha}^{(s-1)}
\]

\[
- \frac{i c}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
- \frac{i c}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
\frac{1}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
D^{\alpha(s-1)} G_{\alpha(s-1)}\bar{\alpha}^{(s-1)} = i \partial^{\alpha(s-1)} \bar{\alpha}_{\bar{T}_{\alpha(s-1)}\bar{\alpha}^{(s-1)}}
\]

\[
- \frac{i c}{s! s(s+1)!} \partial^{\alpha(s-1)} \bar{T}_{\alpha(s-1)}\bar{\alpha}^{(s-1)}
\]

\[
+ \frac{i c}{s! s(s+1)!} \partial^{\alpha(s-1)} \bar{T}_{\alpha(s-1)}\bar{\alpha}^{(s-1)}
\]

\[
\bar{D}^2 T_{\alpha(s-1)}\bar{\alpha}^{(s-1)} + \frac{i c}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
= \frac{i c}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
+ \frac{i c}{s! s(s+1)!} \partial^{\alpha(s, \bar{T}_{\alpha(s)}\bar{\alpha}^{(s-1)})}
\]

\[
- c \bar{D}^2 \bar{\Psi}_{\alpha(s-1)}\bar{\alpha}^{(s-1)}
\]

\[
+ \frac{i c (s - 1)}{s! s(s+1)!} \partial^{\alpha(s-1)} \bar{T}_{\alpha(s-1)}\bar{\alpha}^{(s-1)}
\]

\[
13
\]
We notice that in all the above there are some combinations that appear repeatedly. So let us define the following fields:

\[
\frac{1}{s!(s+1)!}D_{(\alpha_{s+1}} \bar{D}_{(\hat{\alpha}_s \Psi_{a(s)})\hat{a}(s-1))} \equiv N_1 \psi_{\alpha(s+1)\hat{a}(s)}
\]

\[
\frac{1}{s!} \bar{D}^\hat{\alpha}_s D_{(\alpha_s \Psi_{a(s-1)})\hat{a}(s)} \equiv N_2 \psi_{\alpha(s)\hat{a}(s-1)}
\]

\[
D^2 \bar{D}^{\hat{\alpha}_s+1} V_{a(s-1)\hat{a}(s-1)} \equiv N_3 \psi_{\alpha(s-1)\hat{a}(s-2)}
\]

where \(N_1, N_2, N_3, N_4\) are some overall normalization, to be fixed later as needed.

Putting everything together we find the fermionic terms of the Lagrangian

\[
L_F = -\frac{1}{2c} T^\alpha( \bar{\phi}(s) \hat{a}(s-1)) \left( 2 \partial^2 T^\alpha( \bar{\phi}(s) \hat{a}(s-1)) + \frac{i}{s!} \partial_{(\alpha_{s+1}} \bar{D}_{\hat{\alpha}_s T^\alpha(\Psi_{a(s)})\hat{a}(s))} \right) + c.c.
\]

The first term in the Lagrangian is the algebraic term of two auxiliary fields and the rest of the terms have exactly the structure of a theory that describes helicity \(h = s + 1/2\)\(^9\). For an exact match we choose coefficients

\[
c = -1 , \quad N_2 = 1
\]

\[
N_1 = 1 , \quad N_3 = -\frac{s}{s - 1}
\]

So the fields that appear in the fermionic action are defined as:

\[
\rho_{\alpha(s)\hat{a}(s-1)} \equiv T_{\alpha(s)\hat{a}(s-1)}
\]

\[
\beta_{\alpha(s)\hat{a}(s-1)} \equiv D^2 T_{\alpha(s)\hat{a}(s-1)} + \frac{i}{2s!} \partial_{(\alpha_{s+1}} \bar{D}_{\hat{\alpha}_s T_{\alpha(s-1))\hat{a}(s)}}
\]

\[
\psi_{\alpha(s+1)\hat{a}(s)} \equiv \frac{1}{s!(s+1)!}D_{(\alpha_{s+1}} \bar{D}_{(\hat{\alpha}_s \Psi_{a(s)})\hat{a}(s-1))}
\]

\[
\psi_{\alpha(s)\hat{a}(s-1)} \equiv \frac{1}{s!} \bar{D}^{\hat{\alpha}_s} D_{(\alpha_s \Psi_{a(s-1)})\hat{a}(s)}
\]

\[
\psi_{\alpha(s-1)\hat{a}(s-2)} \equiv \frac{s - 1}{s} D^2 \bar{D}^{\hat{\alpha}_s+1} V_{a(s-1)\hat{a}(s-1)}
\]

\(^9\)We are following the conventions of [23] which differ from the conventions used in [22].
The Lagrangian is

\[
\mathcal{L}_F = \rho^{(s)} \delta^{(s-1)} \beta_{\alpha(s)} \alpha_{\alpha(s-1)} + c.c.
\]

\[
+ i \bar{\psi}^{(s)} \delta^{(s+1)} \gamma_{\alpha(s-1)} \psi_{\alpha(s-1)} \delta_{\alpha(s-1)} \psi_{\alpha(s-1)} \psi_{\alpha(s-1)} + c.c.
\]

\[
+ i \left[ \frac{s}{s+1} \right] \psi^{(s-1)} \delta^{(s-1)} \partial_{\alpha(s-1)} \psi_{\alpha(s-1)} \psi_{\alpha(s-1)} + c.c.
\]

\[
- i \left[ \frac{s+1}{(s+1)^2} \right] \bar{\psi}^{(s-1)} \delta^{(s-1)} \partial_{\alpha(s-1)} \psi_{\alpha(s-1)} \psi_{\alpha(s-1)} \psi_{\alpha(s-1)} \psi_{\alpha(s-1)} + c.c.
\]

\[
(31)
\]

and the gauge transformations of the fields are

\[
\delta_G \rho_{\alpha(s)} \alpha_{\alpha(s-1)} = 0, \quad \delta_G \psi_{\alpha(s)} \alpha_{\alpha(s-1)} = \frac{1}{s+1} \partial_{\alpha(s)} \xi_{\alpha(s)} \alpha_{\alpha(s-1)}
\]

\[
\delta_G \beta_{\alpha(s)} \alpha_{\alpha(s-1)} = 0, \quad \delta_G \psi_{\alpha(s)} \alpha_{\alpha(s-1)} = \frac{1}{s+1} \partial_{\alpha(s)} \xi_{\alpha(s)} \alpha_{\alpha(s-1)} \psi_{\alpha(s-1)}
\]

\[
\delta_G \psi_{\alpha(s-1)} \alpha_{\alpha(s-1)} = \frac{s+1}{s+1} \partial_{\alpha(s-1)} \xi_{\alpha(s)} \alpha_{\alpha(s-1)} \psi_{\alpha(s-1)}
\]

\[
(32)
\]

with \( \xi_{\alpha(s)} \alpha_{\alpha(s-1)} = -i \mathcal{D}^2 L_{\alpha(s)} \alpha_{\alpha(s-1)} \)

5.2 Bosons

For the bosonic action we follow exactly the same procedure as was presented for the fermionic sector. The fields that appear in the action are defined as:

\[
U_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{1}{s+1} \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)}
\]

\[
u_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{1}{2s+1} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} - \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
v_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{-i}{2s} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} + \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
A_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv G_{\alpha(s)} \alpha_{\alpha(s-1)} \alpha_{\alpha(s-1)} - \frac{s}{2s+1} \left( \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} + \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} \right)
\]

\[
S_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{1}{2} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} + \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
P_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{-i}{2} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} - \mathcal{D}_{\alpha(s)} \mathcal{T}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
h_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{1}{\sqrt{2}} \left\{ \frac{1}{s+1} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} + \frac{1}{s+1} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
- \frac{1}{2s+1} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
(33)
\]

\[
h_{\alpha(s)} \alpha_{\alpha(s-1)} \equiv \frac{1}{\sqrt{2}} \left\{ \frac{1}{s+1} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} + \frac{1}{s+1} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]

\[
- \frac{1}{2s+1} \left\{ \mathcal{D}_{\alpha(s)} \mathcal{D}_{\alpha(s)} \mathcal{V}_{\alpha(s)} \alpha_{\alpha(s-1)} \right\}
\]
the gauge transformations are
\[
\begin{align*}
\delta_G U_{\alpha(s+1)} &= 0, & \delta_G A_{\alpha(s-1)} &= 0 \\
\delta_G u_{\alpha(s)} &= 0, & \delta_G S_{\alpha(s-1)} &= 0 \\
\delta_G v_{\alpha(s)} &= 0, & \delta_G P_{\alpha(s-1)} &= 0 \\
\delta_G h_{\alpha(s)} &= \frac{1}{s!} \partial (\alpha_s \zeta_{(s-1)} \hat{\alpha}(s-1)) \\
\delta_G h_{\alpha(s-2)} &= \frac{s-1}{s^2} \partial^{\alpha(s-1)} S_{\alpha(s-1)} \hat{\alpha}(s-1)
\end{align*}
\]

where
\[
\zeta_{(s-1)} = \frac{i}{2\sqrt{2}} (D^{\alpha_s} L_{\alpha(s)} \hat{\alpha}(s-1) - \bar{D}^{\hat{\alpha}_s} \bar{L}_{\alpha(s-1)} \hat{\alpha}(s))
\]

and the Lagrangian is
\[
L_B = -\frac{1}{2} U^{\alpha(s+1)} U_{\alpha(s+1)} + c.c. + u^{\alpha(s)} u_{\alpha(s)} + v^{\alpha(s)} v_{\alpha(s)}
\]
\[
- \left[ \frac{2s+1}{4s} A^{\alpha(s-1)} A_{\alpha(s-1)} \right] + \left[ \frac{2s+1}{2s+1} S^{\alpha(s-1)} S_{\alpha(s-1)} \right] + \left[ \frac{2s+1}{2s+1} P^{\alpha(s-1)} P_{\alpha(s-1)} \right]
\]
\[
+ h^{\alpha(s)} h(s) \Box h_{\alpha(s)} + \frac{s}{2} h^{\alpha(s)} h(s) \partial \gamma^{\alpha_s} h_{\gamma(s)} h(s) + \frac{s}{2} h^{\alpha(s)} h(s) \partial \gamma^{\alpha_s} h_{\gamma(s)} h(s) + \frac{s}{2} h^{\alpha(s)} h(s) \Box h_{\alpha(s)} + \frac{s}{2} h^{\alpha(s)} h(s) \Box h_{\alpha(s)}
\]

5.3 Off-shell degrees of freedom

Let us count the bosonic degrees of freedom of the theory:


| fields                      | d.o.f       | redundancy | net          |
|-----------------------------|-------------|------------|--------------|
| \( h_{\alpha(s)} \hat{\alpha}(s) \) | \((s + 1)^2\) | \(s^2\)   | \(s^2 + 2\)  |
| \( h_{\alpha(s-2)} \hat{\alpha}(s-2) \) | \((s - 1)^2\) | \(0\)     | \((s + 1)^2\) |
| \( u_{\alpha(s)} \hat{\alpha}(s) \) | \((s + 1)^2\) | \(0\)     | \((s + 1)^2\) |
| \( v_{\alpha(s)} \hat{\alpha}(s) \) | \((s + 1)^2\) | \(0\)     | \((s + 1)^2\) |
| \( A_{\alpha(s-1)} \hat{\alpha}(s-1) \) | \(s^2\)     | \(0\)     | \(s^2\)      |
| \( U_{\alpha(s+1)} \hat{\alpha}(s+1) \) | \(2(s + 2)s\) | \(0\)     | \(2(s + 2)s\) |
| \( S_{\alpha(s-1)} \hat{\alpha}(s-1) \) | \(s^2\)     | \(0\)     | \(s^2\)      |
| \( P_{\alpha(s-1)} \hat{\alpha}(s-1) \) | \(s^2\)     | \(0\)     | \(s^2\)      |
| **Total**                  | **8s^2 + 8s + 4** |           |              |

and the same counting for the Fermionic degrees of freedom:

| fields                      | d.o.f       | redundancy | net          |
|-----------------------------|-------------|------------|--------------|
| \( \psi_{\alpha(s+1)} \hat{\alpha}(s) \) | \(2(s + 2)(s + 1)\) | \(2(s + 1)s\) | \(4s^2 + 4s + 4\) |
| \( \psi_{\alpha(s)} \hat{\alpha}(s-1) \) | \((s + 1)s\)  | \(2(s + 1)s\)  | \(2s + 1s\)  |
| \( \psi_{\alpha(s-1)} \hat{\alpha}(s-2) \) | \(2s(s - 1)\) | \(0\)     | \(2s + 1s\)  |
| \( \rho_{\alpha(s)} \hat{\alpha}(s-1) \) | \((s + 1)s\)  | \(0\)     | \(2s + 1s\)  |
| \( \beta_{\alpha(s)} \hat{\alpha}(s-1) \) | \((s + 1)s\)  | \(0\)     | \(2s + 1s\)  |
| **Total**                  | **8s^2 + 8s + 4** |           |              |

### 5.4 SUSY-transformation laws

The last thing left to do is to find explicit expressions for the SUSY-transformation laws of the fields. The transformation under susy can be easily calculated by the action of the SUSY-generators on the specific component. In terms of the covariant derivatives \( D(\bar{D}) \) we see that

\[
\delta_S \text{Component} = - \left( \epsilon^\beta \bar{D}_\beta + \bar{\epsilon}^{\bar{\beta}} \bar{\bar{D}}_{\bar{\beta}} \right) \text{Component}\]

But not all the fields are on equal footing. The dynamical ones \((\in \mathcal{D})\) are treated as equivalence classes, in other words they have a gauge transformation of the form \(\{\mathcal{D}\} \sim \{\mathcal{D}\} + \partial (\zeta)\). Hence when we apply the susy transformation they will possess an extra term in the gauge parameter space

\[
\delta_S \{\mathcal{D}\} \sim \delta_S \{\mathcal{D}\} + \partial (\delta_S \zeta)
\]

This says that we must identify these two classes as well, therefore we can ignore any terms in the transformation law of the dynamical fields that have the same structure as their gauge transformation.
With all that in mind we find for the transformation of the fermionic fields:

\[ \delta S \rho_{\alpha(s)} \dot{\alpha}(s-1) = -\epsilon^{\alpha+1} U_{\alpha(s+1)} \dot{\alpha}(s-1) \]
\[ + \frac{s}{(s + 1)!} \epsilon^{\alpha} [S_{\alpha(s-1)} \dot{\alpha}(s-1) + i P_{\alpha(s-1)} \dot{\alpha}(s-1)] \]
\[ - \epsilon^{\alpha} \left[ u_{\alpha(s)} \dot{\alpha} + iv_{\alpha(s)} \dot{\alpha}(s) \right] \]

\[ \delta S \beta_{\alpha(s)} \dot{\alpha}(s-1) = -i \epsilon^{\beta} \partial^{\alpha+1} \beta U_{\alpha(s+1)} \dot{\alpha}(s-1) \]
\[ - \frac{i}{2s!} \epsilon^{\alpha+1} \partial_{\dot{\alpha}} \dot{\alpha}(s) \dot{U}_{\alpha(s-1)} \dot{\alpha}(s+1) \]
\[ + \frac{i}{2s!} \epsilon^{\beta} \partial_{\dot{\alpha}} \dot{U}_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ + \frac{i \sqrt{2}}{(s + 1)!} \epsilon^{\alpha} \partial_{\dot{\alpha}} \dot{U}_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ - \frac{i}{2} \frac{(s - 1)!}{s(s + 1)!} \epsilon^{\alpha} \partial_{\dot{\alpha}} \dot{U}_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ + \frac{i}{2} \frac{(s - 1)!}{s(s + 1)!} \epsilon^{\alpha} \partial_{\dot{\alpha}} \dot{U}_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ - \frac{1}{\sqrt{2}} \epsilon^{\alpha} \partial_{\dot{\alpha}} \dot{U}_{\alpha(s-1)} \dot{\alpha}(s-1) \]

\[ \delta S \psi_{\alpha(s+1)} \dot{\alpha}(s) = \frac{1}{s!} \epsilon^{\alpha} U_{\alpha(s+1)} \dot{\alpha}(s-1) \]
\[ - \frac{1}{(s + 1)!} \epsilon^{\alpha} [u_{\alpha(s)} \dot{\alpha}(s) - iv_{\alpha(s)} \dot{\alpha}(s)] \]
\[ + \frac{i \sqrt{2}}{(s + 1)!} \epsilon^{\beta} \partial_{\dot{\alpha}} \dot{h}_{\alpha(s)} \dot{\alpha}(s) \]

\[ \delta S \psi_{\alpha(s)} \dot{\alpha}(s-1) = \epsilon^{\alpha} [u_{\alpha(s)} \dot{\alpha}(s) + iv_{\alpha(s)} \dot{\alpha}(s)] \]
\[ - \frac{1}{s!} \frac{s}{2s + 1} \epsilon^{\alpha} S_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ - \frac{i s}{s!} \epsilon^{\alpha} P_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ - \frac{1}{s!} \frac{s + 1}{2s} \epsilon^{\alpha} A_{\alpha(s-1)} \dot{\alpha}(s-1) \]
\[ + \frac{i s}{\sqrt{2}} \epsilon^{\beta} \partial_{\dot{\alpha}} \dot{h}_{\alpha(s)} \dot{\alpha}(s) \]
\[ + \frac{i(s + 1)s(s - 1)}{\sqrt{2} s!} \epsilon^{\alpha} \partial_{\dot{\alpha}} \dot{h}_{\alpha(s-1)} \dot{\alpha}(s-2) \]

(35) (36) (37) (38)
\[
\delta S \psi_\alpha(s-1) \dot{\bar{\alpha}}(s-2) = \frac{1}{2} \frac{(s-1)(2s+1)}{s^2} e^{\dot{\bar{\alpha}}_s} A_\alpha(s-1) \dot{\bar{\alpha}}(s-1)
\]
\[
+ \frac{i}{\sqrt{2}} \frac{(s-1)^2}{s} \frac{1}{(s-1)!^2} e^{\dot{\bar{\alpha}}_s} \partial_\alpha (\dot{\bar{\alpha}}_s \bar{h}_\alpha(s-2)) \dot{\bar{\alpha}}(s-2)
\]
\[
- i \sqrt{2} \frac{(s-1)^2}{s} \frac{1}{(s-1)!^2} e^{\dot{\bar{\alpha}}_s} \partial_\alpha (\dot{\bar{\alpha}}_s \bar{h}_\alpha(s-2)) \dot{\bar{\alpha}}(s-2)
\]

The SUSY-transformation laws for the bosonic fields are:

\[
\delta S U_\alpha(s+1) \alpha(s-1) = \frac{1}{(s+1)!} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s-1)
\]
\[
- \frac{i}{2 (s+1)!} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s-1)
\]
\[
- \frac{i}{(s+1)!} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s-1)
\]
\[
- \frac{i}{(s+1)!} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s+1)
\]
\[
- \frac{i}{s+1} \frac{1}{(s+1)!^2} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s-1)
\]

\[
\delta S \left( u_\alpha(s) \dot{\alpha}(s) + iv_\alpha(s) \dot{\alpha}(s) \right) = \frac{i}{(s+1)!} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s+1)
\]
\[
- \frac{i}{s+1} \frac{1}{s!^2} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s+1)
\]
\[
+ \frac{i}{s+1} \frac{1}{(s+1)!^2} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s+1)
\]
\[
+ \frac{i}{(s+1)!^2} e^{\dot{\alpha}_s \beta_\alpha(s)} \dot{\alpha}(s-1)
\]
\[
+ \frac{1}{s!} e^{\dot{\alpha}_s \beta_\alpha(s-1)} \dot{\alpha}(s)
\]
\[
+ \frac{i}{2 s!} e^{\dot{\alpha}_s \beta_\alpha(s-1)} \dot{\alpha}(s-1)
\]
\[ \delta_\Sigma A_{\alpha(s-1)\dot{\alpha}(s-1)} = -\frac{i}{2s+1} s! \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} (\dot{\alpha}_s \rho_{\alpha(s)} \dot{\alpha}(s-1)) + c.c. \]
\[ + \frac{i}{s!(s+1)(s+2)} \left( \frac{1}{s!} \epsilon_{(\dot{\alpha}_s+1)\dot{\alpha}} \rho_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \right) + \frac{s^2 + 1}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s+1) \right) + c.c. \]
\[ + \frac{i}{s+1} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s-1) + c.c. \right) + \frac{i}{s} \left( \frac{1}{s!} \epsilon_{(\dot{\alpha}_s+1)\dot{\alpha}} \rho_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \right) \]
\[ \frac{s^2 + 1}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s-1) + c.c. \right) + \frac{i}{s} \left( \frac{1}{s!} \epsilon_{(\dot{\alpha}_s+1)\dot{\alpha}} \rho_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \right) + c.c. \]

\[ \delta_\Sigma \left( S_{\alpha(s-1)\dot{\alpha}(s-1)} + iP_{\alpha(s-1)\dot{\alpha}(s-1)} \right) = \]
\[ = \epsilon^{\dot{\alpha}_s} \beta_{\alpha(s)} \dot{\alpha}(s-1) \]
\[ + \frac{i}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \rho_{\alpha(s-1)\dot{\alpha}(s-1)} \right) + c.c. \]
\[ + \frac{2s+1}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s+1) \right) + c.c. \]
\[ + \frac{i}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s-1) + c.c. \right) + \frac{i}{s} \left( \frac{1}{s!} \epsilon_{(\dot{\alpha}_s+1)\dot{\alpha}} \rho_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \right) \]
\[ + \frac{s^2 + 1}{s} \left( \frac{1}{s!} \epsilon^{\dot{\alpha}_s} \partial^{\alpha_s} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s-1) + c.c. \right) + \frac{i}{s} \left( \frac{1}{s!} \epsilon_{(\dot{\alpha}_s+1)\dot{\alpha}} \rho_{\alpha(s-1)\dot{\alpha}(s-2)} + c.c. \right) \]

\[ \delta_\Sigma h_{\alpha(s)} \dot{\alpha}(s) = -\frac{1}{\sqrt{2s!}} \epsilon(\dot{\alpha}_s \rho_{\alpha(s-1)}) \dot{\alpha}(s) + c.c. \]
\[ + \frac{1}{\sqrt{2}} \epsilon^{\dot{\alpha}_s+1} \dot{\psi}_{\alpha(s)} \dot{\alpha}(s+1) + c.c. \]
\[ - \frac{1}{\sqrt{2(s+1)}} \frac{1}{s!} \epsilon(\dot{\alpha}_s \dot{\psi}_{\alpha(s)} \dot{\alpha}(s-1)) + c.c. \]

\[ \delta_\Sigma h_{\alpha(s-2)} \dot{\alpha}(s-2) = -\frac{1}{\sqrt{2s}} \epsilon^{\dot{\alpha}_s-1} \dot{\psi}_{\alpha(s-1)} \dot{\alpha}(s-2) + c.c. \]

6 Summary

We started with a quick review of the representation theory of the little group of the Super-Poincaré group and then we required the massless limit of an irreducible massive
superspin $Y$ representation give us the massless irreducible representation with the same value of super-helicity. This forced us to promote the fields used to build the theory to equivalence classes and introduce a redundancy. The invariance of the physical degrees of freedom of the theory under this redundancy fully determines the action of the theory. In this way we reproduce the arbitrary integer super-helicity theory but in terms of the prepotentials. We recognized that this action is a member of a larger two parameter family of equivalent actions, all of which are connected through superfield redefinitions.

Then we focussed on the off-shell component structure of this superspace theory. We presented an alternative technique of defining the field content of the theory, using the equations of motion and their Bianchi identities, which encode all the information about invariance. Finally we applied it to the derivation of the component action and the SUSY-transformation laws of the fields involved.

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