SMT Queries Decomposition and Caching in Semi-Symbolic Model Checking

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ABSTRACT
In semi-symbolic (control-explicit data-symbolic) model checking the state-space explosion problem is fought by representing sets of states by first-order formulas over the bit-vector theory. In this model checking approach, most of the verification time is spent in an SMT solver on deciding satisfiability of quantified queries, which represent equality of symbolic states. In this paper, we introduce a new scheme for decomposition of symbolic states, which can be used to significantly improve the performance of any semi-symbolic model checker. Using the decomposition, a model checker can issue much simpler and smaller queries to the solver when compared to the original case. Some SMT calls may be even avoided completely, as the satisfaction of some of the simplified formulas can be decided syntactically. Moreover, the decomposition allows for an efficient caching scheme for quantified formulas. To support our theoretical contribution, we show the performance gain of our model checker SymDIVINE on a set of examples from the Software Verification Competition.

KEYWORDS
Model checking, semi-symbolic model checking, state slicing, caching, formal verification, SMT query decomposition, SymDIVINE.

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1 INTRODUCTION
Automated formal verification of a real-world code is an ultimate goal for both academia and industry. One of the methods that are most suitable for achieving this goal is model checking. Originally, the model checking approach was designed for verification of distributed systems that were modeled in something like a lightweight programming language for the purpose of verification. However, recent achievements made model checking tools more general and applicable directly to source codes of middle-sized software projects. For example, model checker DIVINE [2] allows for a direct model checking of an unmodified C or C++ code. The general applicability and widespread use of model checking is unfortunately limited by the well-known state space explosion problem, i.e. data structures that need to be produced and explored in order to complete the verification process may blow-up exponentially with respect to the size of the model-checked source code.

The exponential growth of data structures comes from two sources – the interleaving of parallel processes in the system being verified (control-flow non-determinism) and from processing of input data (data non-determinism). The control flow non-determinism can be alleviated by state space reduction methods, e.g. τ-reduction [13], or partial order reduction [12]. Data non-determinism, on the other hand, is typically dealt with using abstract interpretation [7], BDD data structures [6], or using formulas in a suitable logic and SMT or SMT solvers [5]. Surprisingly, most model checking tools available to general public focus mainly on a single type of non-determinism. To address this issue, a semi-symbolic approach to model checking has been introduced recently. This approach is also called Control-Explicit Data-Symbolic (CEDS) model checking [1]. CEDS approach basically follows the enumerative model checking scheme with the exception that the data parts of states are represented symbolically, which allows a CEDS model checker to represent multiple states with the same control-flow part as a single compactly represented object – so-called multi-state. This efficiently mitigates data non-determinism blow-up during the state space exploration, but introduces costly operations for working with symbolic parts of states. The CEDS approach is implemented, e.g. within the tool SymDIVINE [11].

SymDIVINE employs first-order formulas and an SMT solver to deal with multiple possible values of symbolic data at one control-flow location. As a result, most of the verification time of SymDIVINE verification process is spent in queries to the SMT solver. In this paper, we introduce a new scheme for slicing of states in CEDS approach, which reduces complexity of issued SMT queries and allows for their caching. Caching of SMT queries has a significant impact on the performance of the whole model checking procedure, and it is not easy to achieve without the state slicing as the queries issued during verification contain universal quantifiers. Furthermore, the state slicing allows for further optimizations in the model checking procedure: in some cases, the equality of symbolic parts of states may be solved purely syntactically without even calling an SMT solver, which brings another performance boost. In the paper, we give the necessary theoretical background for state slicing, as well as a report on our implementation in the tool SymDIVINE and its experimental evaluation on benchmarks from Software Verification Competition (SV-COMP) [4].

The rest of the paper is organized as follows. In Section 2, we give the necessary introduction to the CEDS model checking and context for the state slicing and caching, which are described in Section 3. In Section 4, we describe details of our implementation.
and report on experimental evaluation we performed to measure the benefit of our new approach. Finally, we conclude the paper in Section 5.

2 PRELIMINARIES

The performance of a semi-symbolic model checker relies on a compact representation of the sets of states and on efficient implementations of operations using this representation. Although several representations have been proposed and tested, the first-order formulas over the theory of fixed size bit-vectors have shown to be most efficient in practice. Using this representation, tests for emptiness of a symbolic state and for equivalence of two symbolic states are performed by queries to an smt solver capable of handling quantified bit-vector formulas.

2.1 Theory of Fixed Sized Bit-vectors

In this section, we briefly recall the bit-vector theory, which is used to represent sets of valuations in the latest versions of the tool SymDIVINE.

The theory of fixed sized bit-vectors is a many-sorted first-order theory with infinitely many sorts $\{n\}$ corresponding to bit-vectors of length $n$. Additionally, as in Hadarean [9], we suppose a distinguished sort Boolean and instead of treating formulas and terms differently, we consider formulas as merely the terms of sort Boolean. The only predicate symbols in the BV theory are $=, \leq, \in$, and $\exists_s$, representing equality, unsigned inequality of binary-encoded natural numbers, and signed inequality of integers in $\mathbb{Z}$’s complement representation, respectively. Functions symbols in the theory are $+, \times, \div, |, \&$, $\llbracket, \lllbracket, \rrbracket, \rrrbracket$, representing addition, multiplication, unsigned division, bit-wise and, bit-wise-or, bit-wise exclusive or, left-shift, right-shift, concatenation, and extraction of $n$ bits starting from position $p$, respectively. For the map $\mu$ assigning to each variable a value in a domain of its sort, we denote as $\llbracket \cdot \rrbracket_\mu$ the evaluation function, which to each formula $\phi$ assigns the value $[\phi]_\mu$. This value is obtained by substituting free variables in $\phi$ by values given by $\mu$ and evaluating all functions, predicates, and quantifiers according to their standard interpretation. The formula $\phi$ is satisfiable if $[\phi]_\mu = 1$ for some mapping $\mu$; it is unsatisfiable otherwise. Formulas $\varphi$ and $\psi$ with the same set of free variables are equivalent if $[\varphi]_\mu = [\psi]_\mu$ for all assignments $\mu$. Further, formulas $\varphi$ and $\psi$ are equisatisfiable if both are satisfiable or both are unsatisfiable. If $\Phi$ is a finite set of formulas, we denote as $\wedge \Phi$ the conjunction of all formulas in $\Phi$. A set of free variables of the formula $\phi$ is defined as usual and denoted free($\phi$). Formulas $\varphi$ and $\psi$ are called (syntactically) dependent if they do not share any free variable, i.e., free($\varphi$) $\cap$ free($\psi$) $\neq \emptyset$. The precise description of the many-sorted logic can be found for example in Barrett et al. [3]. For a precise description of the syntax and semantics of the bit-vector theory, we refer the reader to Hadarean [9].

2.2 SymDIVINE

SymDIVINE is a semi-symbolic model checker aiming for verification of real world C and C++ programs featuring parallelism. To achieve precise semantics of the input languages and, at the same time, to ease the parsing, it is built upon the LLVM compiler framework. In order to verify real-world pieces of code, SymDIVINE provides intrinsic implementations of a subset of the pthread library to allow parallelism and also of a subset of SV-COMP interface to allow users to model non-deterministic inputs in their programs. Internally, SymDIVINE relies on the ceds approach, in detail described in [1]. The ceds approach allows SymDIVINE to verify both safety and LTL properties of programs under inspection. In the following subsection, we cover the basics and details relevant to this paper. For further information, we kindly refer the reader to the original paper.

2.3 Control-Explicit Data-Symbolic Model Checking

In the standard explicit state model checking, the state space graph of a program is explored by an exhaustive enumeration of its states, until an error is found or all reachable states have been enumerated. SymDIVINE basically follows the same idea, but instead of enumerating states for all possible input values, it employs a ceds approach in which the inputs of the program are treated in a symbolic manner. While a purely explicit-state model checker has to produce a new state for each and every possible input value, in SymDIVINE a set of states that differ only in data values may be represented with a single data structure, the so-called multi-state.

A multi-state consists of an explicit control location and a set of program’s memory valuations. In practice, a set of memory valuations is usually not listed explicitly, but in a more succinct representation. By providing procedures for deciding whether the set of memory valuations is empty and whether two sets of memory valuations are equal, we can easily mimic most of explicit-state model checking algorithms [1] – from a simple reachability of error states to the full LTL model checking. By operating on multi-states, SymDIVINE can achieve up to exponential time and memory savings, compared to purely explicit approaches.

Although the ceds approach is independent of the multi-state representation, the choice of the representation can have an enormous effect on the verification performance. Most recent versions of SymDIVINE use a first-order formula over the theory of fixed size bit-vectors to represent a multi-state. In this representation, the set of represented memory valuations is precisely the set of satisfying assignments to the given formula.

2.4 Multi-state Representation

Formally, a multi-state in ceds approach is a tuple $(c, m, \phi)$, where $c$ is an explicit control part, $m$ is an explicit memory shape and $\phi$ is a quantifier-free first-order formula over the theory of fixed size bit-vectors.

A control part $c$ is a tuple of call stacks, which contains contents of stack for each active thread of a program under inspection. Note that not all multi-states in a state space have to contain the same number of call stacks, as threads can be spawned and killed during the program execution. For each thread, a call stack is composed of frames corresponding to function calls, which hold a program counter and a reference to the segment in the memory shape $m$ that corresponds to the active function call.
A **memory shape** is a collection of segments, where each segment has a unique identifier and contains a list of corresponding variables. Each variable in a segment has associated several pieces of information:

1. a type determining whether the variable is a pointer or a value, and if it is a value, also its bit-width,
2. explicit/symbolic mark, and
3. a value in case of variables marked as explicit.

Given this setup, each variable can be uniquely identified by a pair \((s, p)\), where \(s\) is a segment identifier and \(p\) is the position of the variable inside that segment.

Note that SymDiViNE distinguishes between the control flow and a memory shape, because LLVM instructions like `alloca` may allocate memory that can escape from the function’s segment. This happens for example when a function obtains an argument that is a value that was obtained by `alloca`. Therefore a single multi-state can contain more segments than frames of call stacks.

Finally, each multi-state contains a quantifier-free formula \(\varphi\) that represents possible values of all variables marked as symbolic in the memory shape. Because a single program variable can be assigned to multiple times, the formula \(\varphi\) may for each program variable \((s, p)\) contain multiple variables of form \((s, p)^{gen}\), where \(gen \in \mathbb{N}\). Variable \((s, p)^{gen}\) represents a value of the program variable \((s, p)\) just after \(gen\)-th assignment to that variable. The number \(gen\) is called the **generation** of the variable \((s, p)\). Let \(prog(s, p)\) denote the variable \((s, p)^{gen}\), where \(gen\) is the greatest generation of all variables \((s, p)^{gen}\) in \(\varphi\). The variable \(prog(s, p)\) intuitively represents a real value of the program variable \((s, p)\) in the multi-state. Therefore, given a model \(\mu\) of a formula \(\varphi\), we can obtain a possible valuation of program variables in a multi-state by restricting \(\mu\) only to variables of form \(prog(s, p)\). Thus a single satisfiability query to an SMT solver is sufficient to determine whether the set of states represented by a multi-state is empty or not. This query is called the **emptiness check** for a multi-state.

In the further text, we refer to the control part \(c\) and the memory shape \(m\) in a multi-state as the **explicit part** of \(s\). Similarly, we refer to \(\varphi\) as the **symbolic part** of \(s\). Furthermore, if the segment identifier, position of the variable in the segment and the generation of the variable are not important, we refer to variables of formula \(\varphi\) only as \(x, y, z, a, b, \ldots\). For the convenience, we suppose that each program variable defined in the program location \(c\) has at least one corresponding variable in the formula \(\varphi\). This assumption is without the loss of generality, as for each program variable \((s, p)\), a vacuous equality \((s, p)^{1} = (s, p)^{i}\) can be conjoined to the formula \(\varphi\).

During the interpretation of the program, the verifier has to be able to compute all successors of a given multi-state in order to construct the complete state-space graph. Successors of a node can arise by two types of operations:

- transformation caused by arithmetic, bitwise and memory instructions and
- pruning caused by control-flow branching and by atomic propositions.

Both of these operations can be modeled by changing the formula \(\varphi\). For a given formula \(\varphi\) and a program variable \((s, p)\), denote as \(gen(s, p)\) the number \(i\) such that \(prog(s, p) = (s, p)^{i}\); this number is called the **last generation of \((s, p)\) in \(\varphi\)**. Suppose we want to compute a successor of a multi-state with a symbolic part \(\varphi\). Then the state resulting from a program instruction \((s, p) = (s_{2}, p_{2})@ (s_{3}, p_{3})\), where @ is a binary arithmetic, bitwise or memory instruction, has a symbolic part 

\[
\varphi' \equiv \varphi \land ((s, p)^{gen(s, p)} + 1 = prog(s_{2}, p_{2}) \oplus prog(s_{3}, p_{3}))
\]

where \(\oplus\) is the corresponding function symbol in the bit-vector theory. Similarly, pruning a multi-state by a binary predicate \((s_{1}, p_{1}) \in (s_{2}, p_{2})\) results in a multi-state with the symbolic part 

\[
\varphi' \equiv \varphi \land (prog(s_{1}, p_{1}) \oplus prog(s_{2}, p_{2}))
\]

where \(\in\) is again the corresponding predicate symbol in the bit-vector theory.

**Example 2.1.** Consider the following single-threaded C program.

```c
int main() {
    int x = nondet();
    int y = x + 5;
    int x = x + 10;
    if (x > y)
        y = y + 1;
}
```

Control parts of all states are straightforward, since they contain only one stack with the associated program counter and a single memory segment. We describe the memory shape and the symbolic part of the multi-state \(s\) that represents the state of the program on the end of the line 6. The single memory segment is labeled by 1 and contains two variables: \(x\) labeled by the index 1 and \(y\) labeled by the index 2. Both these variables are marked as symbolic values. Therefore, the symbolic part contains variables \((1, 1)^{1}\), which represent values of the program variable \(x\), and variables \((1, 2)^{1}\), which represent values of the program variable \(y\). For the sake of readability, we will refer to variables \((1, 1)^{1}\) as \(x^{1}\) and to variables \((1, 2)^{1}\) as \(y^{1}\). In particular, the symbolic part \(\varphi\) of this multi-state is 

\[
(x^{1} = x^{1} + 5) \land (x^{2} = x^{1} + 10) \land (x^{2} \leq y^{1}) \land (y^{2} = y^{1} + 1).
\]

### 2.5 Multi-state Equality Check

We now describe how an SMT solver can be used to decide whether two multi-states represent the same set of concrete states. We further refer to this check as to the **equivalence check**.

Let \(s_{1}\) and \(s_{2}\) be multi-states with the same explicit part. That is, \(s_{1} = (c, m, \varphi)\) and \(s_{2} = (c, m, \psi)\) for a control part \(c\), memory shape \(m\) and formulas \(\varphi\) and \(\psi\). Let \(\text{free}(\varphi) = \{x_{1}, \ldots, x_{n}\}\) and \(\text{free}(\psi) = \{y_{1}, \ldots, y_{m}\}\). Furthermore, let us denote the set of program variables defined at the control location \(c\) as \(\text{vars}(c)\). For each program variable \(p\), there is a variable \(x^{p}\) in \(\varphi\) that represents the last generation of the program variable \(p\) in the multi-state \(s_{1}\). Analogously, the last generation of the program variable \(p\) in \(s_{2}\) is represented by a variable \(y^{p}\) in \(\psi\).

We want to decide whether the sets of states represented by \(s_{1}\) and \(s_{2}\) are equal. To determine this, we define a formula \(\text{notsubseque}(s_{1}, s_{2})\), which is satisfiable precisely if there is a state
represented by $s_1$ that is not represented by $s_2$:
\[
\text{notsubseteq}(s_1, s_2) \overset{\text{df}}{=} \varphi \land \forall y_1 \ldots y_m \left( \psi \Rightarrow \bigvee_{p \in \text{var}(c)} (x^p \neq y^p) \right)
\]

The equality of two multi-states can now be determined by using an smt solver: the states $s_1$ and $s_2$ are equal precisely if both of formulas $\text{notsubseteq}(s_1, s_2)$ and $\text{notsubseteq}(s_2, s_1)$ are unsatisfiable, i.e. there is no memory valuation that is represented only by one of the multi-states. However, the equality check requires a quantified smt query, which is usually more expensive than the quantifier-free one. For example, in the theory of fixed-size bit-vectors, deciding satisfiability of a quantifier-free formula is NP-complete, whereas deciding satisfiability of a formula with quantifiers is PSPACE-complete [10].

3 STATE SLICING AND CACHING

In order to reduce the cost of a quantified smt query, we observe that real-world programs often contain lots of independent variables, i.e. pairs of variables such that a change in any of them does not affect the other. We give two simple examples that illustrate this phenomenon: a sequential program in Figure 1 and a multi-threaded program in Figure 2.

```
int foo( int a, int b ) {
    int result;
    // Store a complicated expression,
    // e.g. modular arithmetics, to result
    return result;
}

int main() {
    int x = foo( nondet(), nondet() );
    int y = 0;
    for ( uint n = nondet(); n % 42 != 0; n++ ) {
        y++;
    }
    return x * y;
}
```

Figure 1: Sequential code demonstrating the motivation for the state slicing.

Let us examine state spaces of these two examples. To keep the explanation simple, we provide a rather high-level description and omit technical details of the real implementation of SymDIVINE that operates on top of the LLVM infrastructure. The example in Figure 1 consists of two functions – main and foo. The function foo represents a function taking two integer arguments and computing an integer result. Effect of this function can be described by a formula $\psi'(a, b, x)$ as a relation between input arguments and the return value.

When this example is examined by SymDIVINE, among others, the following states are produced:

- an initial state (before main starts) $s_{\text{init}}$;
- a state $s_i$ after every cycle iteration;
- a final state $s_{\text{final}}$, corresponding to the location just after line 14 for each number $0 \leq i \leq 42$ of the performed iterations of the for loop.

Note that in this example the cycle has to be unravelled for each iteration. This is due to the presence of a variable $y$, which has different value in every iteration and therefore each iteration can be distinguished from every other.

The symbolic parts of these states are:

$s_{\text{init}} = \top,$

$s_i = \psi(a, b, x^i) \land y^i = 0 \land$

$n^i \mod 42 \neq 0 \land y^2 = y^i + 1 \land n^2 = n^i + 1 \land$

$\cdots$

$n^i \mod 42 \neq 0 \land y^{i+1} = y^i + 1 \land n^{i+1} = n^i + 1$

$s_{\text{final}} = \psi(a, b, x_1) \land y^1 = 0 \land$

$n^1 \mod 42 \neq 0 \land y^2 = y^1 + 1 \land n^2 = n^1 + 1 \land$

$\cdots$

$n^1 \mod 42 \neq 0 \land y^{i+1} = y^i + 1 \land n^{i+1} = n^i + 1 \land$

$n^{i+1} \mod 42 = 0 \land \text{return Value} = x^i \times y^{i+1}$.

During the generation of the state space, SymDIVINE tries to merge newly generated states with already existing states and thus performs the equality check. As merging occurs only on the states with the same control-flow locations, equality checks are issued whenever a state $s_i$ or $s_{\text{final}}$ is produced. If we examine the formulas that are checked for satisfiability during equality checks for $s_i$, we can observe that although the values of variables $x$, $a$, and $b$ do not change during the loop, all queries test their equality. This forces the smt solver to consider the expensive formula $\psi(a, b, x_0)$ for each unrolling of the cycle and thus slows the verification of the program down. This effect is even stronger if there is a large number of states per a control-flow location. The observed pattern of computing multiple independent sub-results in advance and combining them later in the computation of the program is quite common in sequential programs.

Similarly, if we examine the example of a parallel code in Figure 2, we find out that the smt solver again does more work than necessary. If the program under inspection contains threads, they do not necessarily interact in each step of the computation. Some thread interleavings are not interesting from verification point of view and therefore SymDIVINE implements state-space reduction techniques. However, these techniques do not eliminate all states produced by equivalent interleaving of threads. By using a similar approach as in the sequential case, i.e. dividing the formula into unrelated parts, we can decrease the verification time and alleviate r-reduction’s imperfections. In this case, the independent parts of the formula consist of variables local to each of the threads.

These observations motivate the decomposition of multi-states into independent parts, in which each group of independent variables is represented by one first-order formula. We show that such representations can decrease size and in most of the cases also the number of smt queries necessary during the program verification. By decomposing multi-states into multiple independent parts, emptiness and equality checks can be performed independently on
volatile int x;
volatile int y;

void foo( arg ) {
    argy = nondet();
    while( *arg % 5 ) {
        (*arg)++;
    }
}

int main() {
    t1 = new_thread( foo, &x );
    t2 = new_thread( foo, &y );
    join( t1 );
    join( t2 );
    return x + y;
}

Figure 2: Parallel code demonstrating the motivation for the state slicing.

each of these parts. The benefit of thus modified emptiness checks is twofold. First, although the number of performed SMT queries grows, they are much simpler, as the SMT solver does not have to reason about independent variables, which have no effect on the satisfiability. In many cases, such queries can be decided by purely syntactic decision procedures, without even using the SMT solver. Second, when an operation is performed on a subset of program variables, the unrelated part of the multi-state does not change, and the resulting queries can thus be efficiently cached.

3.1 Sliced Multi-states

A sliced multi-state is a tuple \((c, m, \{\varphi_i\}_{1 \leq i \leq k})\), where \(c\) and \(m\) are as before and \(\varphi_i\) are mutually independent formulas. Formulas \(\varphi_i\) are called independent symbolic parts of the state. Intuitively, each \(\varphi_i\) describes possible memory valuations of a set of independent program variables in a given control location. Semantics of sliced multi-states is straightforward - a set of concrete program states represented by a sliced multi-state \((c, m, \{\varphi_i\}_{1 \leq i \leq k})\) is defined as a set of program states represented by the (ordinary) multi-state \((c, m, \bigwedge_{1 \leq i \leq k} \varphi_i)\). We say that an ordinary multi-state \(s_1 = (c_1, m_1, \varphi)\) is syntactically equivalent to the sliced multi-state \(s_2 = (c_2, m_2, \{\varphi_i\}_{1 \leq i \leq k})\) if \(c_1 = c_2, m_1 = m_2\), and \(\varphi\) is equal to \(\bigwedge_{1 \leq i \leq k} \varphi_i\) up to the ordering of the conjuncts.

Although each multi-state \(s\) can be converted to a syntactically equivalent sliced multi-state in many ways, there always exists a syntactically equivalent sliced multi-state with the largest number of independent symbolic parts. Recall that each multi-state created during the interpretation of the program is of form \((c, m, \bigwedge_{1 \leq i \leq k} \varphi_i)\), with possible dependencies among formulas \(\varphi_i\). Let \(\Phi = \{\Psi_1, \ldots, \Psi_l\}\) be the set of equivalence classes of \(\{\varphi_i\}_{1 \leq i \leq k}\) modulo the dependence relation. A sliced multi-state syntactically equivalent to \(s\) with the largest number of independent symbolic parts is then \((c, m, \{\bigwedge \Psi_i | 1 \leq i \leq l\})\).

Example 3.1. Let

\[
(c, m, x = y + z \land b > c \land z = a \land d > 0)
\]

be a multi-state. The syntactically equivalent sliced multi-state with the largest number of independent symbolic parts is

\[
(c, m, \{(x = y + z \land z = a), (b > c), (d > 0)\}).
\]

As before, during the program interpretation a sliced multi-state has to be transformed by conjoining a formula to represent variable assignments or program branching. However, the situation is now more complex, as parts of a sliced multi-state may have to be merged during the computation, because a conjoined formula can introduce new dependencies among the variables and parts of the multi-state may have to be merged together in order for the new sliced multi-state to be correct. In particular, suppose that \(s = (c, m, \{\varphi_i\}_{1 \leq i \leq k})\) is a sliced multi-state and \(\psi\) is the formula to be conjoined to it. We partition the set \(\{\varphi_i | 1 \leq i \leq k\}\) to two sets \(\Phi\) and \(\Psi\) such that all formulas in \(\Phi\) are independent on \(\psi\) and all formulas in \(\Psi\) are dependent on \(\psi\). Then a result of conjoining \(\psi\) to the state \(s\) is the sliced multi-state \((c, m, (\psi \land \Psi) \cup \Phi)\).

Example 3.2. Consider the sliced multi-state from Example 3.1. After pruning caused by interpretation of the branching \(x = b\), the new sliced multi-state is

\[
(c, m, \{(x = b \land x = y + z \land z = a \land b > c), (d > 0)\}),
\]

because symbolic parts \(x = y + z \land z = a\) and \(b > c\) have to be merged.

Moreover, to be able to compare two sliced multi-states, their independent parts have to be in one-to-one correspondence that respects program variables. To define this formally, we introduce a function \(pVars\) that for a given formula \(\varphi\) returns the set of program variables represented by the formula \(\varphi\), i.e. \(pVars(\varphi) = \{(s, p) | (s, p) \in free(\varphi)\) for some \(i \in N\)\}.

Two sliced multi-states \(s_1 = (c_1, m_1, \{\varphi_i\}_{1 \leq i \leq k})\) and \(s_2 = (c_2, m_2, \{\varphi_i\}_{1 \leq i \leq k})\) with the same number of independent symbolic parts are then said to be matching if

- they have the same control part,
- sets \(pVars(\varphi_i)\) and \(pVars(\varphi_j)\) are disjoint for all \(i \neq j\),
- sets \(pVars(\psi_i)\) and \(pVars(\psi_j)\) are disjoint for all \(i \neq j\), and
- there is a bijection \(f\): \(\{\varphi_i | 1 \leq i \leq k\} \rightarrow \{\psi_i | 1 \leq i \leq k\}\) such that \(pVars(\varphi_i) = pVars(f(\varphi_i))\).

It is easy to see that each two sliced multi-states \(s_1 = (c, m, \Phi)\) and \(s_2 = (c, m, \Psi)\) with the same control part can be transformed to equivalent sliced multi-states that are matching: an equivalent pair of matching sliced multi-states can always be obtained by setting \(s'_1 = (c, m, \{\bigwedge \Phi\})\) and \(s'_2 = (c, m, \{\bigwedge \Psi\})\). However, it is often possible to obtain equivalent matching sliced multi-states with more independent groups than one, as the following example shows.

Example 3.3. Let \(s_1 = (c, m, \Phi)\) and \(s_2 = (c, m, \Psi)\), where \(c\) and \(m\) contain only one thread and one segment with variables \(x, y, z, u, v\). Because a segment number is not important for this example, let us denote \(i\)-th generation of the variable \(x\) as \(x^i\). Let

\[
\Phi = \{(x^1 = y^2 \land y^2 = y^1 + 1), (z^2 \geq 0), (u^1 \leq v^1), (u^2 = 5)\}.
\]
We can raise the equality check to sliced multi-states. Let $\phi$ be a satisfiable formula, $\rho$ a formula independent of $\phi$ and $\psi$ an arbitrary formula. Then formulas $\phi \land (\psi \lor \rho)$ and $(\phi \land \psi) \lor \rho$ are equivalently.

Proof. The implication from left to right follows easily from distributivity of conjunction and disjunction. For the converse, suppose $(\phi \land \psi) \lor \rho$ is satisfiable and $\mu$ is its model. If $\mu$ is a model of $(\phi \land \psi)$, it is obviously also a model of $(\phi \land \psi) \lor \rho$. Suppose on the other hand that $\mu$ is a model of $\rho$ and let $\mu'$ be the restriction of $\mu$ to the free variables of $\rho$. As $\phi$ is satisfiable, it has a model $\nu$. Then by the independence of $\phi$ and $\rho$, the map $\mu' \cup \nu$ is well-defined and is a model of both $\phi$ and $\rho$ and therefore also of $(\phi \land \psi) \lor \rho$. \qed

Lemma 3.4. Let $\phi$ be a satisfiable formula, $\rho$ a formula independent of $\phi$ and $\psi$ an arbitrary formula. Then formulas $\phi \land (\psi \lor \rho)$ and $(\phi \land \psi) \lor \rho$ are equivalently.

Proof. First, we show that the formula $\text{nospubseteq}(s_1, s_2)$ is logically equivalent to the formula

$$\bigwedge_{1 \leq i \leq k} \phi_i \land \bigvee_{1 \leq i \leq k} \left( \forall y_1 \ldots y_m (\neg \psi_i \lor \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right)$$

and subsequently we show that this formula is equivalent with $\bigvee_{1 \leq i \leq k} \text{nospubseteq}(s_1, s_2, i)$.

To show the logical equivalence above, we present a sequence of formulas that are logically equivalent due to the definitions used and well-known first-order tautologies.

$$\phi \land \forall y_1 \ldots y_m (\psi \implies \bigvee_{p \in \text{vars}(c)} (x^p \neq y^p)) \quad (1)$$

$$\bigwedge_{1 \leq i \leq k} \phi_i \land \forall y_1 \ldots y_m \left( \forall y_{1 \leq i \leq k} (\psi_i \implies \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right) \quad (2)$$

$$\bigwedge_{1 \leq i \leq k} \phi_i \land \forall y_1 \ldots y_m \left( \forall y_{1 \leq i \leq k} (\neg \psi_i \lor \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right) \quad (3)$$

$$\bigwedge_{1 \leq i \leq k} \phi_i \land \forall y_1 \ldots y_m \left( \forall y_{1 \leq i \leq k} (\neg \psi_i \lor \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right) \quad (4)$$

$$\bigwedge_{1 \leq i \leq k} \phi_i \land \forall y_1 \ldots y_m \left( \forall y_{1 \leq i \leq k} (\neg \psi_i \lor \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right) \quad (5)$$

Some of these steps require explanation. The equivalence of (1) and (2) follows from definitions used and reordering of disjunctions. The equivalence of (3) and (4) follows again by reordering the disjunctions, and the equivalence of (4) and (5) follows from the fact that for independent formulas $\phi$ and $\psi$, the formula $\forall x (\phi \lor \psi)$ is equivalent to $(\forall x \phi) \lor (\forall x \psi)$.

Now, as each $\phi_i$ is satisfiable and independent on $y_1 \ldots y_m (\neg \psi_j \lor \bigvee_{p \in \text{vars}(c,j)} (x^p \neq y^p))$ for all $i \neq j$, a repeated application of Lemma 3.4 shows that the last formula is equivalently

$$\bigwedge_{1 \leq i \leq k} \left( \phi_i \land \forall y_1 \ldots y_m (\neg \psi_i \lor \bigvee_{p \in \text{vars}(c,i)} (x^p \neq y^p)) \right).$$
was implemented and integrated. In the extended version of Sym-
Table 1: Summary results showing effects of PartialStore and caching. *Solved* marks benchmark count, which the configuration solved in the time limit. *Time* is a sum of benchmarks time, which were solved by all the configurations.

| Category      | SMTStore no cache | SMTStore with cache | PartialStore no cache | PartialStore with cache |
|---------------|-------------------|---------------------|-----------------------|-------------------------|
|               | time[s] solved    | time[s] solved      | time[s] solved        | time[s] solved          |
| Concurrency   | 1828              | 40                  | 1344                  | 41                      |
|               |                   |                     | 2144                  | 39                      |
|               |                   |                     |                      | 1506                    |
|               |                   |                     |                      | 42                      |
| DeviceDrivers | 12156             | 241                 | 12154                 | 241                     |
|               |                   |                     | 768                   | 298                     |
|               |                   |                     |                      | 763                     |
|               |                   |                     |                      | 298                     |
| ECA           | 20794             | 230                 | 20388                 | 228                     |
|               |                   |                     | 38907                 | 211                     |
|               |                   |                     |                      | 21606                   |
|               |                   |                     |                      | 211                     |
| ProductLines  | 19571             | 276                 | 19770                 | 275                     |
|               |                   |                     | 25361                 | 272                     |
|               |                   |                     |                      | 11995                   |
|               |                   |                     |                      | 293                     |
| Sequentialized| 3710              | 44                  | 3821                  | 45                      |
|               |                   |                     | 3200                  | 43                      |
|               |                   |                     |                      | 1735                    |
|               |                   |                     |                      | 47                      |
| Summary       | 58061             | 831                 | 57478                 | 830                     |
|               |                   |                     | 70652                 | 863                     |
|               |                   |                     |                      | 37607                   |
|               |                   |                     |                      | 891                     |

Table 2: Summary results showing effects of PartialStore and caching on number of smt solver calls. *Equal checks* is the number of all issued equality checks. *Syntactic equality* is a number of checks in which syntactically equivalent formulas were supplied. *Cached* marks number of cache hits. *Solver calls* is the number of checks, which required a query to an SMT solver.

|           | Equal checks | Syntactic equality | Cached | Solver calls |
|-----------|--------------|--------------------|--------|--------------|
| SMTStore  | 3 110 831    | 2 356 034 (76%)    | 0 (0%) | 754 797 (24%)|
| PartialStore | 81 562 863  | 77 132 869 (95%)  | 0 (0%) | 4 429 994 (5%)|
| SMTStore with cache | 3 620 635 | 2 785 076 (77%) | 136 982 (4%) | 698 577 (19%) |
| PartialStore with cache | 129 798 056 | 122 666 491 (95%) | 6 757 205 (5%) | 374 360 (<1%) |

However, the overhead can be outweighed by:

- simplicity of issued queries, as the solver does not have to deal with irrelevant variables;
- syntactic equality, because the state slicing makes it easier to preserve syntactically equivalent formulas, which helps SymDivine as it can quickly recognize syntactically equal formulas; and
- caching, as an equality query of sliced states is composed of many small, rarely changing queries to an SMT solver.

The reasoning above lines up with results observed: configuration with PartialStore without cache and the configuration with SMTStore with cache rarely bring any speedup. The behavior was different only in categories Concurrency and ECA.

In Concurrency set, the difference is caused by the simplicity of the benchmarks and the presence of diamond-shapes in the state space. Diamonds in the state space of simple benchmarks tend to produce syntactically equivalent formulas and therefore SymDivine can tremendously benefit from their detection. Only less than 2% of the equality queries are handled out to an SMT solver. Therefore, state slicing increases the overhead and does not pay off. Benchmarks in the ECA set are synthetically generated benchmarks with complicated data dependencies and therefore multi-states of these benchmarks are rarely sliceable.

On the other hand, our method significantly improves the performance in the DeviceDriversLinux64 category, where it managed to save 93% of the verification time and managed to verify 57 additional benchmarks compared to the original configuration. The reason for that is as follows: these benchmarks are closest to the real-world code and contain a lot of code irrelevant to potential errors unlike benchmarks in the other categories, which are usually reduced to the bare-bone of the problem. In this environment, where multi-states are large and program interpretation changes them only locally, slicing produces large amount of syntactically equal queries and also allows for a large number of cache hits.

In sum, slicing combined with caching performs best on large benchmarks, as can be seen in Figure 3, and can save roughly 40% of the verification time. If we omit the category ECA, which is in our point of view not-so-relevant for real usage, the time savings go up to 60%. Note that state slicing does not cause any significant memory usage increase as the amount of additional information is rather small compared to the whole multi-state.

5 CONCLUSION

We believe that state slicing combined with caching is a substantial improvement to the Control-Explicit Data-Symbolic approach to automated formal verification. Our experimental measurements confirm a significant performance boost especially if the verified program is similar to the real-world code. We still see some future work that would be of much more technical and implementation nature. For example, we can imagine that an incorporation of other formula simplification methods, which are not present in Z3, could save more verification time. The same goes for storing counter examples for multi-state equality, which could be used to
Figure 3: Comparison of verification time in seconds of basic SMTStore without cache (x axis) and PartialStore with cache (y axis). Diagonal lines denote double, equal and half the verification time.
differentiate between two multi-states without issuing a quantified SMT query.

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