Two-loop renormalization of $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives.

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Abstract

Two-loop $\beta$-function and anomalous dimension are calculated for $N = 1$ supersymmetric quantum electrodynamics, regularized by higher derivatives in the minimal subtraction scheme. The result for two-loop contribution to the $\beta$-function appears to be equal to 0, does not depend on the form of regularizing term and does not lead to anomaly puzzle. Two-loop anomalous dimension can be also made independent on parameters of higher derivative regularization by a special choice of subtraction scheme.

1 Introduction.

Investigation of quantum corrections in supersymmetric theories is a very important and complicated problem. In principle, supersymmetric theories have better ultraviolet behavior, than nonsupersymmetric models. For example, in $N = 2$ Yang-Mills theory perturbative divergences are present only in one-loop diagrams. In principle it follows from the fact, that in supersymmetric theories axial anomaly and anomaly of energy-momentum tensor trace belong to the same supermultiplet [1, 2, 3, 4]. The axial anomaly is known to be completely defined by the one-loop approximation [3, 4], while the trace anomaly is proportional to $\beta$-function [4]. Therefore, due to the supersymmetry the $\beta$-function should be also defined by the one-loop approximation. The same arguments can be applied to $N = 1$ supersymmetric theories. However, explicit calculations of radiative corrections show, that the $\beta$-function in $N = 1$ supersymmetric models has contributions from higher loops [5, 6, 7, 8]. This contradiction is

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usually called "anomaly puzzle" and was investigated in a large number of papers, for example \[12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\].

Usually different proposals to solve anomaly puzzle require to fix the form of the \(\beta\)-function in all orders of perturbation theory. For example, in theories with matter the \(\beta\)-function should be related with the anomalous dimension. For the first time such \(\beta\)-function was obtained by Novikov, Shifman, Vainshtein and Zakharov (NSVZ) from investigation of structure of instanton corrections \[24\]. Later this result was checked by explicit calculations, which were usually made by the dimensional reduction technique \[25\]. Two-loop \(\beta\)-function, obtained in this regularization, is shown to coincide with a prediction, following from NSVZ exact expression. However, three-loop \(\beta\)-function \[26, 27, 28\] does not agree with it. Nevertheless, the deviations can be removed by a redefinition of the coupling constant \[29\], the possibility of such redefinition being highly nontrivial \[30\]. In principle it is possible to relate \(\overline{\text{DR}}\) scheme and NSVZ scheme order by order \[31\] in the perturbation theory.

It is necessary to especially mention paper \[23\], in which \(\beta\)-function is shown to depend on the normalization of matter and gauge superfields. In particular, NSVZ \(\beta\)-function can be obtained after a special rescaling of these superfields, which reduces kinetic terms to the canonical normalization. Otherwise, (without rescalings) \(\beta\)-function is argued to be completely defined by the one-loop approximation. So, it is really quite possible to obtain zero contributions of higher loops. The problem is how to calculate the corrections. In principle, it is possible to look for the regularization or renormalization scheme, in which \(\beta\)-function is equal to the one-loop result or coincides with NSVZ exact \(\beta\)-function. For example, the calculation of super Yang-Mills two-loop \(\beta\)-function in the differential renormalization \[32\] was made in \[33\]. Another interesting possibility is using of higher covariant derivative regularization \[34, 35\]. For the supersymmetric Yang-Mills theory the Lagrangian of the regularized theory was constructed in \[36\]. For electrodynamics construction of the regularized Lagrangian is simpler, because instead of covariant derivatives it is necessary to use usual derivatives. However, calculation of diagrams, regularized by higher covariant derivatives is rather complicated. In particular, explicit calculation of the one-loop quantum correction \[1\] for the (nonsupersymmetric) Yang-Mills theory was made rather recently \[37, 38\] and gives the same result as the dimensional regularization. In principle it is possible to prove, that one-loop calculations using higher covariant derivative regularization (certainly, complemented by the additional regularization for one-loop diagrams) always give the same result as the dimensional regularization \[39\]. Investigation of two-loop corrections in theories, regularized by higher derivatives has not yet been done.

In this paper we try to understand features of higher derivative regularization in supersymmetric theories and calculate two-loop renormgroup functions for massless \(\mathcal{N} = 1\) supersymmetric quantum electrodynamics in this regularization using minimal subtraction scheme.

\[1\]Note, that introducing of a term with higher covariant derivatives does not lead to regularization of one-loop divergences. For the one-loop divergences it is necessary to use one more regularization, for example, introduce Pauli-Villars fields.
The paper is organized as follows:

In Section 2 we introduce notations and remind some information about $N = 1$ supersymmetric electrodynamics. In the next Section 3 the considered model is regularized by adding of higher derivative term. After it we describe the quantization procedure for the constructed theory. Two-loop $\beta$-function and anomalous dimension are calculated in Section 4. Agreement of the results with renormgroup equations is checked in Section 5. A brief summary and discussion are presented in Conclusion. Technical details of calculations, including expressions for all Feynman diagrams, can be found in the Appendix.

2 Supersymmetric quantum electrodynamics.

$N = 1$ supersymmetric massless electrodynamics in the superspace is described by the following action:

$$S_0 = \frac{1}{4e^2} \text{Re} \int d^4x d^2\theta W_a C^{ab} W_b + \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right).$$  \hspace{1cm} (1)

Here $\phi$ and $\tilde{\phi}$ are chiral superfields, which in components can be written as

$$\phi(y, \theta) = \varphi(y) + \bar{\theta}(1 + \gamma_5)\psi(y) + \frac{1}{2} \bar{\theta}(1 + \gamma_5) \theta f(y);$$

$$\tilde{\phi}(y, \theta) = \tilde{\varphi}(y) + \bar{\theta}(1 + \gamma_5)\tilde{\psi}(y) + \frac{1}{2} \bar{\theta}(1 + \gamma_5) \theta \tilde{f}(y),$$  \hspace{1cm} (2)

where $y^\mu = x^\mu + i\bar{\theta}\gamma^\mu\gamma_5\theta/2$ are chiral coordinates, $\varphi$ and $\tilde{\varphi}$ are complex scalar fields, $\psi$ and $\tilde{\psi}$ are Majorana spinors, which can be unified in a Dirac spinor

$$\Psi = \frac{1}{\sqrt{2}} \left( (1 + \gamma_5)\psi + (1 - \gamma_5)\tilde{\psi} \right),$$  \hspace{1cm} (3)

and $f$ and $\tilde{f}$ are auxiliary complex scalar fields.

$V$ is a real abelian superfield, which is a supersymmetric generalization of the gauge field. In components this superfield can be written as

$$V(x, \theta) = C(x) + i\sqrt{2}\theta\gamma_5\xi(x) + \frac{1}{2}(\bar{\theta}\theta)K(x) + \frac{i}{2}(\bar{\theta}\gamma_5\theta)H(x) + \frac{1}{2}(\bar{\theta}\gamma^\mu\gamma_5\theta)A_\mu(x) + \sqrt{2}(\bar{\theta}\theta)\bar{\theta} \left( i\gamma_5\chi(x) + \frac{1}{2}\gamma^\mu\gamma_5\partial_\mu\xi(x) \right) + \frac{1}{4}(\bar{\theta}\theta)^2 \left( D(x) - \frac{1}{2}\partial^2C(x) \right).$$  \hspace{1cm} (4)

The chiral superfield $W_a$ is a supersymmetric generalization of the field strength tensor and in the abelian case is defined as

\[\text{In our notations the metric tensor in the Minkowski space-time has the diagonal elements (1, -1, -1, -1).}\]
\[
W_a = \frac{1}{16} \bar{D}(1 - \gamma_5)D[(1 + \gamma_5)D_aV],
\]  

(5)

where the supersymmetric covariant derivative \(D\) is written as

\[
D = \frac{\partial}{\partial \theta} - i\gamma^\mu \theta \partial_\mu.
\]  

(6)

Model (\[\square\]) is invariant under supersymmetric gauge transformations

\[
V \to V - \frac{1}{2}(A + A^\dagger); \quad \phi \to e^A \phi; \quad \tilde{\phi} \to e^{-A} \tilde{\phi},
\]  

(7)

where \(A\) is an arbitrary chiral superfield. In principle, it is possible to choose Wess-Zumino gauge, in which the superfield \(V\) is written as

\[
V(x, \theta) = \frac{1}{2}(\bar{\theta} \gamma^\mu \gamma_5 \theta)A_\mu(x) + i\sqrt{2}(\bar{\theta} \theta)\bar{\theta} \gamma_5 \chi(x) + \frac{1}{4}(\bar{\theta} \theta)^2 D(x).
\]  

(8)

However, this gauge is not supersymmetric. That is why we do not use it for calculation of quantum corrections.

3 Higher derivative regularization of \(N = 1\) supersymmetric electrodynamics.

To regularize model (\[\square\]) by higher derivatives let us first modify its action by the following way:

\[
S_0 \to S = S_0 + S_\Lambda = 
\]

\[
= \frac{1}{4e^2} \text{Re} \int d^4x \, d^2 \theta \, W_a \bar{C}^{ab}(1 + \frac{\partial_2^a}{A^{2\xi}})W_b + 
\]

\[
+ \frac{1}{4} \int d^4x \, d^4 \theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right).
\]  

(9)

Note, that the considered model is abelian and the superfield \(W^a\) is gauge invariant. Therefore, a regularizing term should contain usual derivatives instead of the covariant ones.

It is convenient to introduce operators

\[
\bar{D}^2 \equiv \frac{1}{2} \bar{D}(1 - \gamma_5)D; \quad D^2 \equiv \frac{1}{2} D(1 + \gamma_5)D; \\
\Pi_{1/2} \equiv -\frac{1}{16\bar{D}^2} D_a \left( C(1 + \gamma_5) \right)^{ab} \bar{D}^2 D_b.
\]  

(10)
which satisfy identities

\[ D^2 \bar{D}^2 + \bar{D}^2 D^2 = -16 \Pi_{1/2} \partial^2 - 16 \partial^2; \]  
\[ \frac{1}{2} \bar{D} \gamma^\mu \gamma_5 D \bar{D} \gamma^\nu \gamma_5 D + \frac{1}{2} \bar{D} \gamma^\nu \gamma_5 D \bar{D} \gamma^\mu \gamma_5 D = -16 \eta^{\mu \nu} \Pi_{1/2} \partial^2 - 16 \partial^\mu \partial^\nu. \]  

(11)  

(12)

Then the first term in action (9) can be presented in the following for \( m \):

\[ S_{\text{gauge}} \equiv \frac{1}{4e^2} \Re \int d^4 x d^2 \theta W_\alpha C^{ab} \left( 1 + \frac{\partial^2}{\Lambda^2} \right) W_\beta = \]
\[ = -\frac{1}{4e^2} \int d^4 x d^4 \theta \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) V. \]  

(13)

The gauge invariance (7) can be fixed by addition of the terms

\[ S_{gf} = -\frac{1}{64e^2} \int d^4 x d^4 \theta \left( V D^2 \bar{D}^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) V + V \bar{D}^2 D^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) V \right). \]  

(14)

which are invariant under supersymmetry transformations. Then due to identity (11) the kinetic term for the gauge field is written in the most simple form:

\[ S_{\text{gauge}} + S_{gf} = \frac{1}{4e^2} \int d^4 x d^4 \theta V \partial^2 \left( 1 + \frac{\partial^2}{\Lambda^2} \right) V. \]  

(15)

Due to the gauge invariance (7) the renormalized action of the considered model (without gauge fixing term) can be presented as

\[ S_{\text{ren}} = \frac{1}{4e^2} Z_3(\Lambda/\mu) \Re \int d^4 x d^2 \theta W_\alpha C^{ab} \left( 1 + \frac{\partial^2}{\Lambda^2} \right) W_\beta + \]
\[ + \frac{1}{4} Z(\Lambda/\mu) \int d^4 x d^4 \theta \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right), \]  

(16)

where \( e = e(\Lambda/\mu) \) is a renormalized coupling constant. A bare coupling constant \( e_0 \) is defined by the equation

\[ \frac{1}{e_0^2} = \frac{1}{e^2} Z_3(\Lambda/\mu) \]  

(17)

and does not depend on \( \Lambda/\mu \). The \( \beta \)-function and anomalous dimension in our notations are defined as

\[ \beta = \frac{d}{d \ln \mu} \left( \frac{e^2}{4\pi} \right); \quad \gamma = \frac{d \ln Z}{d \ln \mu}. \]  

(18)

At the first sight, the generating functional can be written in the following form:
\[ Z = \int DV D\phi D\bar{\phi} \exp \left\{ i \left[ \frac{1}{4e^2} \int d^4x \, d^4\theta \, V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V - \frac{1}{4e^2} (Z_3(\Lambda/\mu) - 1) \int d^4x \, d^4\theta \, V \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \frac{1}{4} Z(\Lambda/\mu) \int d^4x \, d^4\theta \, \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right) + \int d^4x \, d^4\theta \, (j \phi + \bar{j} \bar{\phi}) + \int d^4x \, d^2\theta \, (j^* \phi^* + \bar{j}^* \bar{\phi}^*) \right] \right\} \].

(Below we slightly modify this expression.) Note, that the considered case corresponds to the gauge group \( U(1) \) and, therefore, diagrams with ghost loops are absent.

Taking into account, that
\[ \bar{D}^2 D^2 \phi = -16\partial^2 \phi \]
for any chiral superfield \( \phi \) and that
\[ \int d^4x \, d^2\theta = -\frac{1}{2} \int d^4x \, D^2; \]
generating functional \((19)\) can be presented as
\[ Z = \int DV D\phi D\bar{\phi} \exp \left\{ i \left[ \int d^4x \, d^4\theta \, \left( \frac{1}{4e^2} V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V - \frac{1}{4e^2} (Z_3(\Lambda/\mu) - 1) V \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \frac{1}{4} Z(\Lambda/\mu) \int d^4x \, d^4\theta \, \left( \phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi} \right) + \int d^4x \, d^2\theta \, (j \phi + \bar{j} \bar{\phi}) + \int d^4x \, d^2\theta \, (j^* \phi^* + \bar{j}^* \bar{\phi}^*) \right] \right\} \]
\]

In order to calculate this functional we should present the argument of the exponent as a sum of a part \( S_Q \), quadratic in fields, and interaction \( S_I \), where
\[ S_Q = \frac{1}{4e^2} \int d^4x \, d^4\theta \, \left( V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \frac{1}{4} (\phi^* \phi + \bar{\phi}^* \bar{\phi}) + J V + \phi^* \frac{D^2}{8\partial^2} j + \bar{\phi}^* \frac{D^2}{8\partial^2} \bar{j} + \phi^* \bar{D}^2 \bar{j}^* + \bar{\phi}^* D^2 j^* \right); \]
\[ S_I = \sum_{n=1}^{\infty} \frac{1}{4n!} \int d^4x \, d^4\theta \, (\phi^* (2V)^n \phi + \bar{\phi}^* (-2V)^n \bar{\phi}) - \left( Z_3(\Lambda/\mu) - 1 \right) \frac{1}{4e^2} \int d^4x \, d^4\theta \, V \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \left( Z(\Lambda/\mu) - 1 \right) \frac{1}{4} \int d^4x \, d^4\theta \, (\phi^* e^{2V} \phi + \bar{\phi}^* e^{-2V} \bar{\phi}). \]
(Last two terms of $S_I$ will generate vertexes with insertions of counterterms, obtained in previous orders of perturbation theory.)

Then the generating functional can be written as

$$Z = \exp \left\{ iS_I \left( \frac{1}{i \delta J}, \frac{1}{i \delta j}, \ldots \right) \right\} \intDV D\phi D\tilde{\phi} \exp \left( iS_Q \right). \quad (24)$$

Note, that the differentiation over chiral superfields in our notations is defined as follows:

$$\frac{\delta j(\theta_x, x)}{\delta j(\theta_y, y)} = -\frac{D^2}{2} \delta^4(\theta_x - \theta_y) \delta^4(x - y), \quad (25)$$

so that

$$\frac{\delta}{\delta j(\theta_y, y)} \int d^4 x d^2 \theta_x j(x, \theta_x) \phi(x, \theta_x) = \phi(y, \theta_y). \quad (26)$$

The integral, remaining in equation (24), is Gaussian and can be easily calculated:

$$Z = \exp \left\{ iS_I \left( \frac{1}{i \delta J}, \frac{1}{i \delta j}, \ldots \right) \right\} \times \exp \left\{ i \int d^4 x d^4 \theta \left( \frac{1}{\partial^2} j + \frac{1}{\partial^2} \tilde{j} - J \frac{e^2}{\partial^2 \left( 1 + \partial^{2n}/\Lambda^{2n} \right) J} \right) \right\} \quad (27)$$

Expansion of this expression in powers of $J$, $j$ and $\tilde{j}$ gives a series of perturbation theory.

However, introducing of higher derivative term does not eliminate all divergences. Really, in Appendix A we check, that the superficial degree of divergence for model (9) is equal to

$$\omega_\Lambda = 2 - 2n(L - 1) - E_\phi(n + 1), \quad (28)$$

where $L$ is a number of loops and $E_\phi$ is a number of external $\phi$-lines. Note, that $\omega_\Lambda$ does not depend on a number of external $V$-lines $E_V$. Therefore even after introducing of the higher derivative term with $n \geq 2$, divergences are present in one-loop diagrams. In order to regularize them [6] it is necessary to insert Pauli-Villars determinants in generating functional (19), so that

$$Z = \int DV D\phi D\tilde{\phi} \prod_i \left( \det PV(V, M_i) \right)^{c_i} \exp \left\{ i \left[ \frac{1}{4e^2} \int d^4 x d^4 \theta V \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V - \frac{1}{4e^2} \left( Z_3(\Lambda/\mu) - 1 \right) \int d^4 x d^4 \theta V \Pi_{1/2} \partial^2 \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) V + \ldots \right] \right\}, \quad (29)$$

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\[ + \frac{1}{4} Z(\Lambda/\mu) \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) + \]
\[ \int d^4x d^4\theta \left( j^* \phi + \tilde{j}^* \tilde{\phi} \right) \left( j^* \phi + \tilde{j}^* \tilde{\phi} \right) \right) \right), \quad (29) \]

where

\[ \left( \text{det} PV(V, M) \right)^{-1} = \int D\Phi D\tilde{\Phi} \exp \left\{ i \left[ Z(\Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \Phi^* e^{2V} \Phi + \right. \right. \]
\[ + \tilde{\Phi}^* e^{-2V} \tilde{\Phi} \left. \right) + \frac{1}{2} \int d^4x d^2\theta M \tilde{\Phi} \Phi + \frac{1}{2} \int d^4x d^2\tilde{\theta} M \tilde{\Phi}^* \Phi^* \right\} \right) \right), \quad (30) \]

and the coefficients \( c_i \) satisfy equations

\[ \sum_i c_i = 1; \quad \sum_i c_i M_i^2 = 0. \quad (31) \]

Below we will also assume, that \( M_i = a_i \Lambda, \) where \( a_i \) are constants. Insersion of such Pauli-Villars determinants allows to cancel remaining divergences in all one-loop diagrams, including diagrams with insersions of counterterms.

Repeating the above arguments it is possible to write the Pauli-Villars determinants in the following form

\[ \left( \text{det} PV(V, M) \right)^{-1} = \exp \left\{ i (S_{PV})_I \left( V, \frac{1}{i \delta j} \ldots \right) \right\} \exp \left\{ i \int d^4x d^4\theta \times \right. \]
\[ \times \left( j^* \frac{1}{\partial^2 + M^2 j} + \tilde{j}^* \frac{1}{\partial^2 + M^2 \tilde{j}} + j \left( \frac{M}{\partial^2 + M^2} \frac{D^2}{4\partial^2 j} + j^* \frac{M}{\partial^2 + M^2} \frac{D^2}{4\partial^2 \tilde{j}} \right) \right) \right\} \bigg|_{j=0}, \quad (32) \]

where

\[ (S_{PV})_I = \sum_{n=1}^{\infty} \frac{1}{4n!} \int d^4x d^4\theta \left( \Phi^*(2V)^n \Phi + \tilde{\Phi}^*(-2V)^n \tilde{\Phi} \right) + \]
\[ + \left( Z(\Lambda/\mu) - 1 \right) \frac{1}{4} \int d^4x d^4\theta \left( \Phi^* e^{2V} \Phi + \tilde{\Phi}^* e^{-2V} \tilde{\Phi} \right), \quad (33) \]

that allows to find their perturbative expansions. Then it is possible to calculate the generating functional \( Z \) according to the prescription

\[ Z = \prod_i \left( \text{det} PV \left( \frac{1}{i \delta j}, M_i \right) \right)^{c_i} \exp \left\{ i S_I \left( \frac{1}{i \delta j}, \frac{1}{i \delta \tilde{j}}, \ldots \right) \right\} \times \]
\[ \times \exp \left\{ i \int d^4x d^4\theta \left( j^* \frac{1}{\partial^2} j + \tilde{j}^* \frac{1}{\partial^2} \tilde{j} - J \frac{e^2}{\partial^2 \left( 1 + \partial^2 n/\Lambda^2 n \right)} J \right) \right\}. \quad (34) \]
The generating functional for connected Green functions in our notations is written as

\[ W = -i \ln Z, \tag{35} \]

and the effective action is defined by making a Legendre transformation

\[ \Gamma = W - \int d^4x d^4\theta \, jV - \int d^4x d^2\theta \left(j^* \phi^* + \tilde{j}^* \bar{\phi}^*\right) - \int d^4x d^2\bar{\theta} \left(j \phi + \tilde{j} \bar{\phi}\right), \tag{36} \]

where \( J, j \) and \( \tilde{j} \) should be expressed in terms of \( V, \phi \) and \( \bar{\phi} \) through solving of the equations

\[ V = \frac{\delta W}{\delta J}; \quad \phi = \frac{\delta W}{\delta j}; \quad \bar{\phi} = \frac{\delta W}{\delta \tilde{j}}. \tag{37} \]

Expressions for Feynman diagrams in the coordinate representation can be found expanding generating functional (34) and substituting the result into equations (35) and (36). Certainly, after this procedure \( \Gamma \) will contain only 1PI-diagrams. Expressions for Feynman diagrams in the momentum space can be then obtained by Fourier transformations. Performing the calculations we used this algorithm and tried to avoid direct application of Feynman rules in order to be completely sure in the correctness of numerical factors for all diagrams. However, for the sake of completeness we formulate Feynman rules for the considered theory, which allow to verify structure of expressions for the diagrams.

1. External lines correspond to a factor

\[ \prod_E \int \frac{d^4p_{E_V}}{(2\pi)^4} V(p_{E_V}) \int \frac{d^4p_{E_{\phi}}}{(2\pi)^4} \delta^4(\phi(p_{E_{\phi}})) \cdots (2\pi)^4 \delta^4 \left( \sum_E p_E \right), \tag{38} \]

where the index \( E \) numerates external momentums.

2. Each internal line of \( V \)-superfield corresponds to

\[ \frac{2e^2}{(k^2 + i0)} \left(1 + (-1)^n k^{2n} / \Lambda^{2n}\right) \delta^4(\theta_1 - \theta_2). \tag{39} \]

3. Each internal line \( \phi - \phi^* \) or \( \bar{\phi} - \bar{\phi}^* \) corresponds to

\[ -\frac{1}{4(k^2 + i0)} \bar{D}^2 D^2 \delta^4(\theta_1 - \theta_2). \tag{40} \]

(Note, that in the considered theory the action is quadratic in matter superfields, that allows to formulate Feynman rules in a bit different manner, than for, say, Wess-Zumino model.)

4. Pauli-Villars fields are present only in the closed loops. Each internal line \( \Phi - \Phi^* \) or \( \bar{\Phi} - \bar{\Phi}^* \) corresponds to
− \frac{1}{4(k^2 - M_i^2 + i0)} \bar{D}^2 D^2 \delta^4(\theta_1 - \theta_2). \quad (41)

Internal lines $\Phi - \tilde{\Phi}$ and $\Phi^* - \tilde{\Phi}^*$ corresponds to

\frac{M_i}{k^2 - M_i^2 + i0} \bar{D}^2 \delta^4(\theta_1 - \theta_2) \quad \text{and} \quad \frac{M_i}{k^2 - M_i^2 + i0} D^2 \delta^4(\theta_1 - \theta_2) \quad (42)

respectively. Also it is necessary to add $- \sum c_i$ for each closed loop of Pauli-Villars fields.

5. Each loop gives integration over a loop momentum $\int \frac{d^4 k}{(2\pi)^4}$.

6. Each vertex gives integration over the corresponding $\theta$: $\int d^4 \theta$.

7. There are numerical factors, which can be calculated expanding generating functional (34).

4 Calculation of two-loop renormgroup functions.

Let us calculate two-loop $\beta$-function and anomalous dimension for a model, described by action (9). In the two-loop approximation $\beta$-function can be found after calculation of diagrams with $E_V = 2, E_\phi = 0$, presented at Figure 1. Note, that each graph at this figure corresponds to a diagram with internal $\phi$-line, a diagram with internal $\tilde{\phi}$-line and a set of diagrams with internal lines of Pauli-Villars fields. As an example in Appendix B we present detailed calculation of one-loop diagrams. Expressions obtained for the other diagrams are collected in Appendix C. Each of these diagrams has the following structure:

\int d^4 \theta \frac{d^4 p}{(2\pi)^4} \left( V(-p, \theta) \partial^2 \Pi_{1/2}(p, \theta) f_1(p, \Lambda) + V(-p, \theta)V(p, \theta) f_2(p, \Lambda) \right). \quad (43)

Terms proportional to $\int d^4 \theta V(-p, \theta)V(p, \theta)$ are not gauge invariant and should disappear after summing of all Feynman diagrams, that is very convenient for checking correctness of the calculations. The other terms can be written as

\[- \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_\alpha(-p, \theta) C^{ab} W_b(p, \theta) \sum_{\text{diagrams}} f_1(p, \Lambda). \quad (44)\]

Having performed the calculations, in the Minkowsky space we obtained, that the result for two-loop contribution to the effective action, corresponding to the two-point Green function of the gauge field, can be written as
\[ \Delta \Gamma_V^{(2)} = \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p) \left( f_{1\text{-loop}} + f_{2\text{-loop}} + f_{PV} + f_{\text{Konishi}} \right), \] (45)

where (for simplicity we omit \( +i0 \) in propagators)

\[ f_{1\text{-loop}} = -\frac{i}{2} \left( \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 (k+p)^2} - \sum_i c_i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M_i^2)((k+p)^2 - M_i^2)} \right) \] (46)

is a total one-loop contribution, including contributions of diagrams with internal lines of Pauli-Villars fields;

\[ f_{2\text{-loop}} = -e^2 \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{(k+p+q)^2 + q^2 - k^2 - p^2}{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) (k+q)^2 (k+p+q)^2 q^2 (q+p)^2} \] (47)

is a sum of diagrams (92) – (97) with internal lines of \( \phi \) and \( \tilde{\phi} \) fields;

\[ f_{PV} = e^2 \sum_i c_i \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n})} \times \]
\[ \times \left[ \frac{(k+p+q)^2 + q^2 - k^2 - p^2}{((k+q)^2 - M_i^2)((k+p+q)^2 - M_i^2)(q^2 - M_i^2)((q+p)^2 - M_i^2)} + \frac{4M_i^2}{((k+q)^2 - M_i^2)(q^2 - M_i^2)((q+p)^2 - M_i^2)} \right] \] (48)

is a contribution of diagrams (92) – (97) with internal lines of Pauli-Villars fields;

\[ f_{\text{Konishi}} = -\frac{i e^2}{2\pi^2} \ln \frac{\Lambda}{\mu} \sum_i c_i \int \frac{d^4 k}{(2\pi)^4} \frac{M_i^2}{(k^2 - M_i^2)^2 ((k+p)^2 - M_i^2)} \] (49)

is a total contribution of diagrams (98) – (101) with insertions of one-loop counterterms. (\( M_i = a_i \Lambda \) are masses of Pauli-Villars fields.)

Similarly, anomalous dimension can be found after calculation of diagrams with \( E_V = 0, E_\phi = 2 \), presented at Figure 2. Calculation of the one-loop diagram is described in Appendix E. Results for the other diagrams are presented in Appendix C. The total two-loop contribution in the Minkowsky space can be written in the following form:
\[
\Delta \Gamma^{(2)}_\phi = \int d^4\theta \frac{d^4p}{(2\pi)^4} \left( \phi^*(p, \theta) \phi(-p, \theta) + \tilde{\phi}^*(p, \theta) \tilde{\phi}(-p, \theta) \right) \times \\
\times \left\{ i \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{2k^2(k + p)^2 \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right)} - \\
- \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{e^4}{k^2q^2(k + p)^2(q + p)^2 \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right) \left( 1 + (-1)^n q^{2n}/\Lambda^{2n} \right)} - \\
- \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{e^4}{k^2q^2(q + p)^2(k + q + p)^2 \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right) \left( 1 + (-1)^n q^{2n}/\Lambda^{2n} \right)} + \\
+ \frac{ie^4}{4\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k + p)^2 \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right)} + \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \\
\times \frac{e^4}{k^2(k + p)^2q^2(q + p)^2(k + q + p)^2 \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right) \left( 1 + (-1)^n q^{2n}/\Lambda^{2n} \right)} - \\
- \int \frac{d^4q}{(2\pi)^4} \frac{e^4}{q^2(q + p)^2 \left( 1 + (-1)^n q^{2n}/\Lambda^{2n} \right)^2 \left( \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k + q)^2} \right)} - \\
- \sum_i c_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_i^2)(k^2 - M^2)^2} \right) \right. 
\]

(50)

Results (13) and (15) are evidently invariant under supersymmetry transformations, because they can be written as integrals from products of superfields (2) and (5) over the superspace. The gauge invariance is absent in equation (14) because the calculations were made only for diagrams with \( E_V = 0, E_\phi = 2 \). Adding of terms, corresponding to diagrams with arbitrary \( E_V \) and \( E_\phi = 2 \), will certainly restore the gauge invariance.

Divergent parts of the integrals in equations (16) – (19) are calculated in Appendix D. Using results, obtained there, one can conclude, that the sum of contributions (13) and (17) gives NSVZ result for the \( \beta \)-function. Expression (18) is shown in Appendix D to be a finite constant and does not contribute to the \( \beta \)-function. However, sum of diagrams with insertions of one-loop counterterms (19) is not zero and exactly cancels contribution (17). Actually, equation (19) produces Konishi anomaly (20), calculated by using Pauli-Villars regularization according to a method, described in (21). According to this method, an anomaly is equal to a contribution of diagrams with internal lines of Pauli-Villars fields in the limit \( M_i \to 0 \), while contributions of diagrams with internal lines of usual fields are equal to 0.

Divergent parts of integrals in equation (15) are also calculated in Appendix D.

Using results of Appendix D it is easy to verify, that counterterms, needed to cancel two-loop divergences in the minimal subtraction scheme can be written as

\[
\Delta S = -\frac{1}{16\pi^2} \ln \frac{\Lambda}{\mu} \text{Re} \int d^4x \, d^2\theta \, W_a C^{ab} \left( 1 + \frac{q^{2n}}{\Lambda^{2n}} \right) W_b +
\]
\begin{align*}
+ \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right) \times \\
\times \left\{ \frac{\alpha}{\pi} \ln \frac{\Lambda}{\mu} + \frac{\alpha^2}{\pi^2} \ln^2 \frac{\Lambda}{\mu} - \frac{\alpha^2}{\pi^2} \ln \frac{\Lambda}{\mu} \left( \sum_i c_i \ln \frac{M_i}{\Lambda} + \frac{3}{2} \right) \right\}, \quad (51)
\end{align*}

that corresponds to

\begin{align*}
\frac{4\pi^2}{e_0^2} &= \frac{\pi}{\alpha(\Lambda/\mu)} - \ln \frac{\Lambda}{\mu} + O(\alpha^2); \\
Z(\Lambda/\mu) &= 1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{\mu} + \frac{\alpha^2}{\pi^2} \ln \frac{\Lambda}{\mu} \left( \sum_i c_i \ln \frac{M_i}{\Lambda} + \frac{3}{2} \right) + O(\alpha^3). \quad (53)
\end{align*}

Therefore the two-loop \( \beta \)-function and anomalous dimension of \( N = 1 \) supersymmetric quantum electrodynamics, regularized by higher derivatives, are written as

\begin{align*}
\beta &= \frac{\alpha^2}{\pi} + O(\alpha^4); \\
\gamma(\alpha) &= -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2} \left( \sum_i c_i \ln \frac{M_i}{\Lambda} + \frac{3}{2} \right) + O(\alpha^3). \quad (54)
\end{align*}

In particular, two-loop contribution to the \( \beta \)-function appears to be 0, so that the beta function is completely defined by the one-loop approximation.

The anomalous dimension \( \gamma(\alpha) \) in the two-loop approximation does not depend on \( n \) or, by other words, on a form of regularizing term. However, it depends on the ratios of Pauli-Villars masses to the constant \( \Lambda \). Nevertheless, the dependence on \( M_i/\Lambda \) can be removed by addition of finite counterterms, proportional to \( \ln M_i/\Lambda \):

\begin{align*}
S_{\text{ren}} &= \frac{1}{4e^2} \int d^4x d^2\theta W_a C^{ab} \left( 1 + \frac{\phi^2}{\Lambda^2} \right) W_b - \frac{1}{16\pi^2} \sum_i c_i \ln \frac{M_i}{\Lambda} \times \\
\times \int d^4x d^2\theta W_a C^{ab} W_b + Z(\Lambda/\mu) \frac{1}{4} \int d^4x d^4\theta \left( \phi^* e^{2V} \phi + \tilde{\phi}^* e^{-2V} \tilde{\phi} \right), \quad (55)
\end{align*}

that corresponds to another subtraction scheme, in which anomalous dimension is equal to

\begin{align*}
\gamma(\alpha) &= -\frac{\alpha}{\pi} + \frac{3\alpha^2}{2\pi^2} + O(\alpha^3) \quad (56)
\end{align*}

and does not depend on both \( n \) and \( M_i/\Lambda \). In principle, in this scheme it is possible to consider, that \( \Lambda \to \infty, M_i/\Lambda \to \infty \) instead of \( M_i = a_i \Lambda \).
Comparing of the results with predictions of the renormalization group method.

The obtained results can be checked by the renormalization group method. It is well known [42], that in renormalizable theories terms proportional to $\ln^2 \frac{\mu}{\Lambda}$ are completely defined by one-loop counterterms. Therefore, it is possible to calculate such terms by renormgroup equations and compare them with the result of calculation of Feinman graphs.

Using the notation

$$t = \ln \frac{\mu}{\Lambda}$$

(57)

for the considered model renormgroup equations can be written as

$$Z(t) = \exp \left\{ \int dt \gamma(\alpha(t)) \right\}; \quad t = \int \frac{d\alpha}{\beta(\alpha)}.$$  

(58)

Because in the one-loop approximation the $\beta$-function is equal to

$$\beta(\alpha) = \alpha^2 \beta_1 + O(\alpha^3),$$

(59)

in the lowest order

$$\alpha(t) = \alpha_0 \left( 1 + \beta_1 \alpha_0 t + O(\alpha_0^2) \right),$$

(60)

where $\alpha_0 = \alpha(0)$. Expanding the anomalous dimension in powers of $\alpha$

$$\gamma(\alpha) = \alpha \gamma_1 + \alpha^2 \gamma_2 + O(\alpha^3) = \gamma_1 \left( \alpha_0 + \beta_1 \alpha_0^2 t \right) + \alpha_0^2 \gamma_2 + O(\alpha_0^3)$$

(61)

and substituting it to the first equation of (58), we obtain, that

$$Z(t) = 1 + \gamma_1 \alpha_0 t + \gamma_1 \beta_1 \alpha_0^2 t^2 / 2 + \gamma_2 \alpha_0^2 t + \gamma_1^2 \alpha_0^2 t^2 / 2 + O(\alpha_0^3).$$

(62)

Taking into account, that according to the results of one-loop calculations $\gamma_1 = -1/\pi$ and $\beta_1 = 1/\pi$, the function $Z$ should take the following form:

$$Z(\Lambda/\mu) = 1 + \frac{\alpha_0}{\pi} \ln \frac{\Lambda}{\mu} - \gamma_2 \alpha_0^2 \ln \frac{\Lambda}{\mu} + O(\alpha_0^3) =$$

$$1 + \frac{\alpha}{\pi} \ln \frac{\Lambda}{\mu} + \frac{\alpha^2}{\pi^2} \ln^2 \frac{\Lambda}{\mu} - \gamma_2 \alpha^2 \ln \frac{\Lambda}{\mu} + O(\alpha^3).$$

(63)

Comparing this expression with equation (53) we see, that terms proportional to $\ln^2 \frac{\mu}{\Lambda}$ coincide, that can be considered as a check of performed calculations.
6 Conclusion.

In this paper we calculated two-loop \( \beta \)-function and anomalous dimension for \( N = 1 \) supersymmetric massless quantum electrodynamics, regularized by higher derivatives. In particular, two-loop contribution to the \( \beta \)-function is found to be 0 and not to depend on the form of higher derivative term. As we mentioned above, this result follows from the fact, that the axial anomaly and the anomaly of energy-momentum tensor trace in the considered model belong to one supermultiplet. However, to obtain it we have to perform calculations using higher covariant derivative regularization. In principle, this regularization (complemented by the Pauli-Villars regularization for one-loop diagrams) allows to perform easy calculation of diagrams with insertion of counterterms, which are proportional to Konishi anomaly [40] and have nonzero contribution. Possibly the results of the paper allow to assume, that contributions of all higher loops to the \( \beta \)-function of \( N = 1 \) supersymmetric electrodynamics, regularized by higher derivatives, are also equal to 0. However, to be completely sure in it, it is necessary to calculate scheme dependent three-loop \( \beta \)-function.

Two-loop anomalous dimension is found to be independent on the form of higher derivative term if one renormalizes the coupling constant in this term. However, \( \gamma(\alpha) \) depends on the ratios of Pauli-Villars masses to the constant \( \Lambda \). Nevertheless, it is possible to make anomalous dimension completely independent on parameters of higher derivative regularization (\( n \) and \( M/\Lambda \)) if one introduces some finite counterterms in the renormalized action, that actually corresponds to a different choice of renormalization scheme.

Note, that the result for \( \beta \)-function does not contradict to NSVZ result

\[
\beta(\alpha) = \frac{\alpha^2}{\pi^2} \left( 1 - \gamma(\alpha) \right),
\]

because in considered model \( \beta \)-function depends on the normalization of the matter superfields [23]. In particular, after a scale transformation making matter superfields canonically normalized, it is possible to obtain NVSZ \( \beta \)-function. However, in the present paper this statement was not checked by explicit calculations and we hope to make it later.

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Appendix.
A  Superficial degree of divergence for the supersymmetric quantum electrodynamics, regularized by higher derivatives.

In order to calculate the superficial degree of divergence for an arbitrary diagram in the massless supersymmetric quantum electrodynamics let us introduce the following notations:

- $L$ is a number of loops,
- $I_V$ is a number of internal $V$-lines,
- $I_\phi$ is a number of internal $\phi$-lines,
- $E_V$ is a number of external $V$-lines,
- $E_\phi$ is a number of external $\phi$-lines.

We will start with the result for the superficial degree of divergence of massless supersymmetric electrodynamics without higher derivatives \[43\]

$$\omega = 2 - E_\phi. \quad (65)$$

Adding of the higher derivative term changes only the propagator of $V$-superfield, which will be proportional to $k^{-2-2n}$ instead of $k^{-2}$ in the usual supersymmetric electrodynamics. Therefore, in the regularized theory the superficial degree of divergence is equal to

$$\omega_\Lambda = \omega - 2n I_V. \quad (66)$$

Taking into account, that $\phi$-lines are continuous, it is easy to prove the following identity:

$$L = I_V + 1 - \frac{1}{2} E_\phi. \quad (67)$$

Therefore expression (66) can be finally rewritten as

$$\omega_\Lambda = 2 - 2n(L - 1) - E_\phi(n + 1). \quad (68)$$

In principle this result can be also obtained from the Feinman rules in the superspace, but the derivation is more complicated.

B  Calculation of one-loop $\beta$-function and anomalous dimension.
Expanding generating functional (34) and substituting it to effective action (36) it is possible to find, that a one-loop diagram with $E_V = 0$ and $E_\phi = 2$ in the coordinate representation is written as

\[
= \frac{i}{8} \int d^8x_1 d^8x_2 \phi^*(x_1, \theta_1) \phi(x_2, \theta_2) \frac{e^2}{\partial^2 \left(1 + \partial^2 / \Lambda^2\right)} \delta_{12}^8 D_1^2 \bar{D}^2_1 \delta_{12}^8. \tag{69}
\]

where

\[
\int d^8x \equiv \int d^4x d^4\theta; \quad \delta_{12}^8 \equiv \delta^4(x_1 - x_2) \delta^4(\theta_1 - \theta_2). \tag{70}
\]

After Fourier transformation this expression in the Minkowsky space can be written as

\[
= \frac{i}{2} \int d^4\theta_1 d^4\theta_2 \int \frac{d^4p}{(2\pi)^4} \phi^*(p, \theta_1) \phi(-p, \theta_2) \int \frac{d^4k}{(2\pi)^4} \times \frac{e^2}{k^2 \left(1 + (-1)^n k^2 / \Lambda^2\right)} \delta^4(\theta_1 - \theta_2) \frac{1}{(k + p)^2} D_1^2 \bar{D}^2_1 \delta^4(\theta_1 - \theta_2) = \frac{i}{2} \int d^4\theta \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \phi^*(p, \theta) \phi(-p, \theta) \frac{e^2}{k^2 \left(1 + (-1)^n k^2 / \Lambda^2\right)(k + p)^2}, \tag{71}
\]

where we used the identity

\[
\delta^4(\theta_1 - \theta_2) D_1^2 \bar{D}^2_1 \delta^4(\theta_1 - \theta_2) = 4 \delta^4(\theta_1 - \theta_2). \tag{72}
\]

The integral over $d^4k$ in equation (71) can be calculated after the Wick rotation:

\[
\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{k^2 \left(1 + (-1)^n k^2 / \Lambda^2\right)(k + p)^2} \rightarrow \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{k^2 \left(1 + k^2 / \Lambda^2\right)(k + p)^2}. \tag{73}
\]

Then it is possible to perform calculation in the Eucliedian space and analytically continue the result for imaginary $p^0$.

The diagrams contributing to the one-loop $\beta$-function can be considered similarly.

From equations (34), (35) and (36) it is possible to find, that in the coordinate representation

\[
= -i \int d^8x_1 V(x_1) \frac{D_1^2 \bar{D}_1^2}{4\bar{\partial}^2} \delta_{11}^8 = -i \int d^8x_1 V(x_1) \frac{1}{\bar{\partial}^2} \delta^4(x_1 - x_1) = \]

\[
= i \int d^4\theta \frac{d^4p}{(2\pi)^4} V(p, \theta) V(-p, \theta) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \tag{74}
\]

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where we take into account identity (72). The corresponding diagram with Pauli-Villars fields in the coordinate representation is written as

\[
= i \sum_i c_i \int d^8 x_1 V(x_1)^2 \frac{D_i^2 D_j^2}{4(\partial^2 + M_i^2)} \delta_{ij}^8 =
\]

\[
= -i \sum_i c_i \int d^4 \theta \frac{d^4 p}{(2\pi)^4} V(p, \theta) V(-p, \theta) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - M_i^2}.
\]

(A bar at diagrams with Pauli-Villars fields denotes a part of a vertex corresponding to \(\Phi^*\) or \(\bar{\Phi}^*\).

For the next diagram equations (34), (35) and (36) give the following expression:

\[
= -i \int d^8 x_1 d^8 x_2 V(x_1) V(x_2) \frac{D_i^2 D_j^2}{4\partial^2} \frac{\bar{D}_i^2 D_j^2}{4\partial^2} \delta_{ij}^8 =
\]

\[
= -i \int d^8 x_1 d^8 x_2 V(x_2) \frac{1}{4\partial^2} \delta_{ij}^8 \bar{D}_i^2 D_j^2 (V(x_1) \frac{\bar{D}_i^2 D_j^2}{4\partial^2} \delta_{ij}^8) =
\]

\[
= -i \int d^8 x_1 d^8 x_2 V(x_2) \frac{1}{4\partial^2} \delta_{ij}^8 \left( D_i^2 D_j^2 + [\bar{D}_i^2, D_j^2] \right) (V(x_1) \frac{\bar{D}_i^2 D_j^2}{4\partial^2} \delta_{ij}^8) =
\]

\[
= -i \int d^8 x_1 d^8 x_2 V(x_2) \frac{1}{4\partial^2} \delta_{ij}^8 \left( D_i^2 D_j^2 V(x_1) \frac{\bar{D}_i^2 D_j^2}{4\partial^2} \delta_{ij}^8 + \partial_\mu \left( i \bar{D}_1 \gamma^\mu \gamma_5 D_1 V(x_1) \frac{\bar{D}_1^2 D_2^2}{4\partial^2} \delta_{ij}^8 - 4V(x_1) \partial_\mu \frac{\bar{D}_1^2 D_2^2}{4\partial^2} \delta_{ij}^8 \right) \right).
\]

Here we took into account,

\[
\delta^4(\theta_1 - \theta_2) D_{a_1} \ldots D_{a_k} \delta^4(\theta_1 - \theta_2) = 0
\]

for \(k = 0, 1, 2, 3\) and used identities

\[
[\bar{D}^2, D^2] = 4i \bar{D} \gamma^\mu \gamma_5 D \partial_\mu; \quad \bar{D} \gamma_\mu \gamma_5 D \bar{D}^2 = 4i \partial_\mu \bar{D}^2.
\]

Taking into account equation (72) in the momentum representation expression (76) can be written as

\[
- \frac{i}{16} \int d^4 \theta \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2(k + p)^2} V(p, \theta) \times
\]

\[
\times \left( D^2 \bar{D}^2 + 4D \gamma^\mu \gamma_5 D k_\mu + 16(p + k)^\mu k_\mu \right) V(-p, \theta).
\]

Let us note, that the integral
is proportional to $p_\mu$. Therefore, it can be presented as

$$I_\mu = \frac{p_\mu}{p^2} I_\nu = \frac{p_\mu}{2p^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+p)^2}$$

$$= -\frac{1}{2} p_\mu \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+p)^2} + \frac{p_\mu}{2p^2} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2} - \frac{1}{(k+p)^2} \right).$$  \hspace{1cm} (81)

The last term can be omitted, because with the corresponding contribution of the diagram with Pauli-Villars fields it is proportional to

$$\int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2} - \sum_i \frac{c_i}{k^2 - M_i^2} \right) - \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{(k+p)^2} - \sum_i \frac{c_i}{(k+p)^2 - M_i^2} \right) = 0,$$  \hspace{1cm} (82)

where we take into account, that both integrals are convergent and it is possible to make in the second integral a substitution $k+p \to k$. Therefore, the considered diagram takes the following form:

$$-\frac{i}{16} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+p)^2} V(p, \theta) \left( D_1^2 \bar{D}_1^2 - 2 \bar{D}_1 \gamma^\nu \gamma_5 D p_\mu + 16(k+p)^\mu k_\mu \right) V(-p, \theta) =$$

$$= -\frac{i}{16} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+p)^2} V(p, \theta) \left( D_1^2 \bar{D}_1^2 + \frac{1}{2} [\bar{D}_1^2, D_1^2] + 16(k+p)^\mu k_\mu \right) V(-p, \theta) =$$

$$= \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k+p)^2} V(-p, \theta) \left( \partial^2 \Pi_{1/2} - (k+p)^2 - k^2 \right) V(p, \theta).$$  \hspace{1cm} (83)

This diagram has two corresponding diagrams with the loop of Pauli-Villars fields, presented below. The first diagram is calculated similar to the massless case:

$$= \frac{i}{4} \sum_i c_i \int d^4x_1 d^4x_2 V(x_1) V(x_2) \frac{D_1^2 \bar{D}_1^2}{4(\partial^2 + M_i^2)} \delta_{12} \times$$

$$\times \frac{\bar{D}_1^2 D_1^2}{4(\partial^2 + M_i^2)} \delta_{12} = \frac{i}{2} \sum_i c_i \int d^4\theta \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_i^2)((k+p)^2 - M_i^2)} \times$$

$$\times V(p, \theta) \left( - \partial^2 \Pi_{1/2} + (k+p)^2 + k^2 \right) V(-p, \theta).$$  \hspace{1cm} (84)

Taking into account, that signs in $\phi^* V \phi$ and $\tilde{\phi}^* V \tilde{\phi}$ vertexes are different, the second diagram can be written in the following form:
Collecting the results we see, that the sum of the one-loop diagrams is equal to

$$\frac{i}{4} \sum_i c_i \int d^4 x_1 d^4 x_2 V(x_1) V(x_2) \frac{M_i D_i^2}{\partial^2 + M_i^2} \delta^8_{12} \times$$

$$\times \frac{M_i D_i^2}{\partial^2 + M_i^2} \delta^8_{12} = -\frac{i}{4} \sum_i c_i \int d^4 \theta_1 d^4 \theta_2 \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} V(p, \theta_1) V(-p, \theta_2) \frac{M_i^2}{k^2 - M_i^2} \times$$

$$\times \delta^4(\theta_1 - \theta_2) \frac{1}{(k + p)^2 - M_i^2} \bar{D}_i^2 D_i^2 \delta^4(\theta_1 - \theta_2) = -i \sum_i c_i \int d^4 \theta \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \times$$

$$\times V(p, \theta) V(-p, \theta) \frac{M_i^2}{(k^2 - M_i^2)((k + p)^2 - M_i^2)}. \quad (85)$$

Note, that all noninvariant terms, proportional to \( V^2 \), disappeared, that can be considered as a check of the correctness of the calculations.

After Wich rotation in the Euclidean space the last integral over \( d^4 k \) can be written as

$$\frac{i}{2} \int d^4 \theta \frac{d^4 p}{(2\pi)^4} V(p, \theta) \partial^2 \Pi_{1/2} V(-p, \theta) \times$$

$$\times \int \frac{d^4 k}{(2\pi)^2} \left( \frac{1}{k^2(k + p)^2} - \sum_i c_i \frac{1}{(k^2 - M_i^2)((k + p)^2 - M_i^2)} \right) =$$

$$= -\frac{i}{2} \text{Re} \int d^2 \theta \frac{d^4 p}{(2\pi)^4} W_a(p, \theta) C^{ab} W_b(-p, \theta) \times$$

$$\times \int \frac{d^4 k}{(2\pi)^2} \left( \frac{1}{k^2(k + p)^2} - \sum_i c_i \frac{1}{(k^2 - M_i^2)((k + p)^2 - M_i^2)} \right). \quad (86)$$

Two-loop Feynman diagrams are calculated in the similar way. However, the calculations are much more complicated and we do not describe them in details.
C  Diagrams, giving nontrivial contribution to the two-loop $\beta$-function and anomalous dimension.

Below we present expressions for all Feinman graphs, giving nontrivial contributions to the two-loop $\beta$-function. Each of these graphs corresponds to a set of diagrams, which consists of a diagram with internal $\phi$-line, a diagram with internal $\tilde{\phi}$-line and diagrams with internal lines of Pauli-Villars fields. In order to find contributions to the effective action it is necessary to add the factor

$$\int d^4\theta \frac{d^4 p}{(2\pi)^4}. \quad (89)$$

Note, that for simplicity of notations we also omit $+i0$ in propagators.

\[ \begin{align*}
- V^2 \frac{1}{(k+p)^2} - \sum_i c_i \left( V \partial^2 \Pi_{1/2} V \frac{1}{k^2 - M_i^2} \right) - V^2 \frac{1}{k^2 - M_i^2} - V^2 \frac{1}{(k+p)^2 - M_i^2} \right]; \quad (90)
\end{align*} \]

\[ \begin{align*}
= i \int \frac{d^4 k}{(2\pi)^4} V^2 \left( \sum_i c_i \frac{1}{k^2 - M_i^2} \right); \quad (91)
\end{align*} \]

\[ \begin{align*}
= \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^2} \frac{e^2}{k^2 \left( 1 + (-1)^n k^{2n} / \Lambda^{2n} \right)} \times \left[ V \partial^2 \Pi_{1/2} V \frac{4(k+p+q)^2 - k^2 - p^2}{(k+q)^2(k+p+q)^2q^2(q+p)^2} - V^2 \frac{2}{(k+q)^2q^2} \right] - \sum_i c_i \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^2} \frac{e^2}{k^2 \left( 1 + (-1)^n k^{2n} / \Lambda^{2n} \right)} \left[ V \partial^2 \Pi_{1/2} V \times \frac{4(k+p+q)^2 - k^2 - p^2 - 2M_i^2}{((k+q)^2 - M_i^2)((k+p+q)^2 - M_i^2)(q^2 - M_i^2)((q+p)^2 - M_i^2)} - V^2 \frac{2}{((k+q)^2 - M_i^2)(q^2 - M_i^2)} \right]; \quad (92)
\end{align*} \]
\[
\sum_i \sum \left[ \frac{2}{(k + q)^2(q + p)^2} + \sum \frac{2}{(k + q)^2 - M_i^2} \left( (q + p)^2 - M_i^2 \right) \right];
\]

\[
\sum_i \sum \left[ \frac{2}{(k + q)^2q^2(q + p)^2} - V^2 \left( \frac{2}{q^2(k + q)^2} + \frac{2}{(k + q)^2(q + p)^2} \right) \right] - \sum \left[ \frac{2}{(k + q)^2 - M_i^2} \left( (q + p)^2 - M_i^2 \right) \right];
\]

\[
\sum_i \sum \left[ \frac{4}{(k + q)^2q^2(q + p)^2} + V^2 \left( \frac{4}{q^2(k + q)^2} + \frac{4}{(k + q)^2(q + p)^2} \right) \right] - \sum \left[ \frac{4}{(k + q)^2 - M_i^2} \left( (q + p)^2 - M_i^2 \right) \right] + V^2 \left( \frac{4}{(k + q)^2 - M_i^2} \left( \frac{4}{q^2 - M_i^2} + \frac{4}{(k + q)^2 - M_i^2} \right) \right];
\]

\[
= \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{e^2}{k^2(1 + (-1)^n k^{2n}/\Lambda^{2n})} \times \left[ - \frac{2}{q^2(k + q)^2} + \frac{2}{q^2 - M_i^2} \left( (k + q)^2 - M_i^2 \right) \right];
\]
Expressions for diagrams, giving nontrivial contributions to the two-loop anomalous
dimension are presented below. Again, in order to obtain corresponding contributions to the effective action it is necessary to add the factor

$$
\int d^4\theta \frac{d^4p}{(2\pi)^4} \left( \phi^*(p, \theta) \phi(-p, \theta) + \tilde{\phi}^*(p, \theta) \tilde{\phi}(-p, \theta) \right). \quad (102)
$$

\[ \int d^4k \frac{e^2}{k^2(1 + (-1)^n k^2n/\Lambda^2n)(k + p)^2} \]  \( (103) \)

\[- \int d^4k d^4q \frac{e^4}{(2\pi)^4 (2\pi)^4 q^2(1 + (-1)^n q^2n/\Lambda^2n)^2(q + p)^2} \times \left( \frac{1}{k^2} - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \right) + \]

\[ + 2 \int d^4k \frac{e^4}{(2\pi)^4 (2\pi)^4 q^4(1 + (-1)^n q^2n/\Lambda^2n)^2(q + p)^2} \left( \frac{1}{k^2} - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \right) \]

\[ + \int d^4k d^4q \frac{e^4}{(2\pi)^4 (2\pi)^4 q^4(1 + (-1)^n q^2n/\Lambda^2n)^2(q + p)^2} \left( \frac{1}{k^2} - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \right); \quad (104) \]

\[- \int d^4k \frac{e^4}{(2\pi)^4 (2\pi)^4 q^4(1 + (-1)^n q^2n/\Lambda^2n)^2(q + p)^2} \times \left( \frac{1}{k^2} - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \right); \quad (105) \]

\[- \int d^4k d^4q \frac{e^4}{(2\pi)^4 (2\pi)^4 q^4(1 + (-1)^n q^2n/\Lambda^2n)^2(q + p)^2} \left( \frac{1}{k^2} - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \right) \]

\[ - \sum_i c_i \left( \frac{1}{k^2 - M_i^2} \right) \left( \frac{1}{(k + q)^2 - M_i^2} \right); \quad (106) \]
\[= -2 \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(107)

\[= \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(108)

\[= \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(109)

\[= -\frac{i}{8\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 (q + k + 2p)^2 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (k + q + p)^2 (q + p)^2 (k + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(110)

\[= -\frac{i}{8\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (k + q + p)^2 (q + p)^2 (k + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(111)

\[= -\frac{i}{8\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \]
\[e^4 \]
\[\times \frac{k^2 (1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (k + q + p)^2 (q + p)^2 (k + p)^2}{(1 + (-1)^n k^{2n}/\Lambda^{2n}) q^2 (1 + (-1)^n q^{2n}/\Lambda^{2n}) (q + p)^2 (k + q + p)^2}; \]

(112)
\[
+ \frac{i}{8\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{e^4}{k^2(1 + (-1)^n k^{2n}/\Lambda^{2n})(k + p)^2};
\]

\[
= \frac{i}{4\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{e^4}{k^4(1 + (-1)^n k^{2n}/\Lambda^{2n})};
\]

\[
= \frac{i}{8\pi^2} \ln \frac{\Lambda}{\mu} \int \frac{d^4k}{(2\pi)^4} \frac{e^4}{k^4(1 + (-1)^n k^{2n}/\Lambda^{2n})}.
\]

D Calculation of integrals, regularized by higher derivatives.

Let us calculate divergent parts of integrals, which were encountered in equations (13) and (50). First it is necessary to perform the Wick rotation. After it the integrals, which are present in equations (13) and (50) will be proportional to

\[
I_1 = \int d^4k \frac{1}{k^2(k + p)^2(1 + k^{2n}/\Lambda^{2n})};
\]

\[
I_2 = \int d^4k \frac{1}{k^2(k + p)^2} - \sum_i c_i \int d^4k \frac{1}{(k^2 + M_i^2)((k + p)^2 + M_i^2)};
\]

\[
I_3 = \int d^4k d^4q \frac{(k + p + q)^2 + q^2 - k^2 - p^2}{k^2(1 + k^{2n}/\Lambda^{2n})(k + q)^2(k + p + q)^2q^2(q + p)^2};
\]

\[
I_4 = \int d^4k \frac{M^2}{(k^2 + M^2)^2((k + p)^2 + M^2)};
\]

\[
I_5 = \int d^4k d^4q \frac{1}{k^2(k + p)^2q^2(q + p)^2(1 + k^{2n}/\Lambda^{2n})(1 + q^{2n}/\Lambda^{2n})} = I_1^2;
\]

\[
I_6 = \int d^4k d^4q \frac{(k + q + 2p)^2}{k^2(k + p)^2q^2(q + p)^2(k + q + p)^2(1 + k^{2n}/\Lambda^{2n})(1 + q^{2n}/\Lambda^{2n})};
\]

\[
I_7 = \int d^4k d^4q \frac{1}{k^2q^2(q + p)^2(k + q + p)^2(1 + k^{2n}/\Lambda^{2n})(1 + q^{2n}/\Lambda^{2n})};
\]
\[ I_8 = \int d^4 q \frac{1}{q^2(q+p)^2 \left(1 + q^{2n}/\Lambda^{2n}\right)} I_2(q/M); \]

\[ I_9 = \int \frac{d^4 k \cdot d^4 q}{(2\pi)^4 (2\pi)^4} \frac{1}{k^2 \left(1 + k^{2n}/\Lambda^{2n}\right)} \times \frac{(k+p+q)^2 + q^2 - k^2 - p^2}{((k+q)^2 + M^2)((k+p+q)^2 + M^2)(q^2 + M^2)((q+p)^2 + M^2)}; \]

\[ I_{10} = \int \frac{d^4 k \cdot d^4 q}{(2\pi)^4 (2\pi)^4} \frac{1}{k^2 \left(1 + k^{2n}/\Lambda^{2n}\right)} \times \frac{M^2}{((k+q)^2 + M^2)((q+p)^2 + M^2)(q^2 + M^2)((q+p)^2 + M^2)}. \]

(115)

In order to calculate integral \( I_1 \) it is possible to use four-dimensional spherical coordinates

\[ k_1 = k \sin \theta_3 \sin \theta_2 \sin \theta_1; \]
\[ k_2 = k \sin \theta_3 \sin \theta_2 \cos \theta_1; \]
\[ k_3 = k \sin \theta_3 \cos \theta_2; \]
\[ k_4 = k \cos \theta_3. \]

(116)

and direct fourth axis along \( p^\mu \), so that the integrand will depend only on \( \theta_3 \) and

\[
\int d^4 k = \int_0^\infty k^3 dk \int_0^\pi d\theta_3 \sin^2 \theta_3 \int_0^\pi d\theta_2 \sin \theta_2 \int_0^{2\pi} d\theta_1 = 4\pi \int_0^\infty k^3 dk \int_0^\pi d\theta_3 \sin^2 \theta_3 =
\]

\[
= [x = \cos \theta_3] = 4\pi \int_0^\infty k^3 dk \int_{-1}^1 dx \sqrt{1 - x^2}. \]

(117)

Taking into account, that \( k^\mu p_\mu = kp \cos \theta_3 = kpx \), the integral can be written as

\[
\int d^4 k \frac{1}{k^2(k+p)^2 \left(1 + k^{2n}/\Lambda^{2n}\right)} =
\]

\[
= 4\pi \int_0^\infty k^3dk \int_{-1}^1 dx \frac{\sqrt{1 - x^2}}{(k^2 + 2kpx + p^2)(1 + k^{2n}/\Lambda^{2n})} =
\]

\[
= 2\pi \int_0^\infty k^2 dk \oint_C dx \frac{\sqrt{1 - x^2}}{(k^2 + 2kpx + p^2)(1 + k^{2n}/\Lambda^{2n})}. \]

(118)
where the contour $C$ is presented at Figure [3]. The integrand here has singularities at branch points $x = \pm 1$, a pole $x = \infty$ and a pole

$$x_0 = -\frac{k^2 + p^2}{2kp}.$$  \hspace{1cm} (119)

Then it is easy to see, that

$$\oint_C dx \frac{\sqrt{1-x^2}}{k^2 + 2kpx + p^2} = 2\pi i \text{Res} \left( \frac{\sqrt{1-x^2}}{k^2 + 2kpx + p^2}, x = \infty \right) - 2\pi i \text{Res} \left( \frac{\sqrt{1-x^2}}{k^2 + 2kpx + p^2}, x = x_0 \right) = 2\pi i \left( \frac{k^2 + p^2}{4kp^2} + i \frac{|k^2 - p^2|}{4kp^2} \right).$$  \hspace{1cm} (120)

Therefore, the integral over angles is reduced to

$$\oint dx \frac{\sqrt{1-x^2}}{k^2 + 2kpx + p^2} = \begin{cases} \frac{\pi}{k^2}, & k \geq p; \\ \frac{\pi}{p^2}, & p \geq k \end{cases}$$  \hspace{1cm} (121)

and finally

$$I_1 = 2\pi^2 \int_0^p dk \frac{k}{p^2} \frac{1}{\left(1 + k^2/\Lambda^2n\right)} + 2\pi^2 \int_0^\infty dk \frac{1}{k} \frac{1}{\left(1 + k^2/\Lambda^2n\right)} =$$

$$= \pi^2 + o(1) + \frac{\pi^2}{n} \ln \frac{\Lambda^2n + p^2n}{p^2n} = 2\pi^2 \left( \ln \frac{\Lambda}{p} + \frac{1}{2} \right) + o(1).$$  \hspace{1cm} (122)

Integral $I_2$ can be calculated using standard methods \[14\]. First, using an identity

$$\frac{1}{ab} = \int_0^1 dy \frac{1}{\left(ay + b(1 - y)\right)^2},$$  \hspace{1cm} (123)

it can be written as

$$I_2 = \int_0^1 dy \int d^4k \left( \frac{1}{\left(k^2 + 2kpy + yp^2\right)^2} - \sum_i c_i \frac{1}{\left(k^2 + 2kpy + yp^2 + M_i^2\right)^2} \right).$$  \hspace{1cm} (124)

Each of these integrals diverges, but their difference is finite. Therefore, to simplify calculations it is convenient to use an auxiliary regularization, for example, the dimensional regularization. Then the integrals in equation (124) can be easily taken:
\[
I_2 = \lim_{D \to 4} \pi^2 \int_0^1 dy \frac{\Gamma(2 - D/2)}{\Gamma(2)} \left( (y(1 - y)p^2)^{D/2} - \sum_i c_i \left( y(1 - y)p^2 + M_i^2 \right)^{D/2} \right) = \pi^2 \sum_i c_i \int_0^1 dy \ln \left( 1 + \frac{M_i^2}{y(1 - y)p^2} \right) = \pi^2 \sum_i c_i \left( \ln \frac{M_i}{p} + \sqrt{1 + \frac{4M_i^2}{p^2}} \arctanh \sqrt{\frac{p^2}{4M_i^2 + p^2}} \right).
\]

where we take into account, that \( \sum_i c_i = 1 \).

To calculate divergent part of the integral \( I_3 \) note, that \( I_3 = I_3(p/\Lambda) \) and due to the logarithmical divergence

\[
I_3 = a_1 \ln^2 \frac{\Lambda}{p} + a_2 \ln \frac{\Lambda}{p} + \sum_{i=0}^{\infty} b_i \left( \frac{p^2}{\Lambda^2} \right)^i. \tag{126}
\]

If \( a_1 = 0 \), then it is possible to find

\[
a_2 = \lim_{p \to 0} \frac{dI_3}{d \ln \Lambda} = \int d^4 k \frac{4n k^{2n-2}}{\Lambda^{2n} \left( 1 + k^{2n}/\Lambda^{2n} \right)^2} \frac{q^2 + k\mu q_\mu}{(k^2 + 2 k q x + q^2)^2}. \tag{127}
\]

If this limit does not exist, then \( a_1 \neq 0 \). The integral in the right hand side of equation (127) can be taken, using four-dimensional spherical coordinates:

\[
\int d^4 q \frac{q^2 + k\mu q_\mu}{(k + q)^4 q^4} = 4\pi \int_0^\infty dq \int_{-1}^1 dx \frac{(q + k x)\sqrt{1 - x^2}}{(k^2 + 2 k q x + q^2)^2} = -2\pi \int_{-1}^1 dx \int_0^\infty dq \frac{\sqrt{1 - x^2}}{d q (k^2 + 2 k q x + q^2)} = \frac{2\pi}{k^2} \frac{1}{\sqrt{1 - x^2}} = \frac{\pi^2}{k^2}, \tag{128}
\]

so that

\[
a_2 = 4n \pi^2 \int d^4 k \frac{k^{2n-4}}{\Lambda^{2n} \left( 1 + k^{2n}/\Lambda^{2n} \right)^2} = 4\pi^4, \tag{129}
\]

Therefore, from equation (126) we conclude, that

\[
I_3 = 4\pi^4 \ln \frac{\Lambda}{p} + O(1). \tag{130}
\]
In order to calculate integral $I_4$ let us note, that

$$
\int \frac{d^4k}{(2\pi)^4} \frac{M^2}{(k^2 + M^2)^2((k + p)^2 + M^2)} = f(p/M).
$$

(131)

Therefore, instead of taking the limit $M \to \infty$ it is possible to take the limit $p \to 0$, so that

$$
I_4 = \int d^4k \frac{M^2}{(k^2 + M^2)^3} + o(1) = \frac{\pi^2}{2} + o(1).
$$

(132)

Divergent part of integral $I_5$ can be also easily calculated, because

$$
I_5 = I_1^2 = 4\pi^4 \left( \ln \frac{\Lambda}{p} + \ln \frac{\Lambda}{p} \right) + O(1).
$$

(133)

To find a divergent part of $I_6$ let us consider

$$
\lim_{p \to 0} \Lambda \frac{d}{d\Lambda} (I_5 - I_6) =
= \lim_{p \to 0} \Lambda \frac{d}{d\Lambda} \int d^4k d^4q \left( 1 - \frac{(k + q + 2p)^2}{(k + q + p)^2} \right) \times
$$

$$
\times \frac{1}{k^2(k + p)^2q^2(q + p)^2} \frac{4\pi^2}{(1 + k^2/\Lambda^2n)(1 + q^2/\Lambda^2n)} =
= \lim_{p \to 0} \int d^4k d^4q \frac{-2(k + q)p - 3p^2}{(k + q + p)^2} \times
$$

$$
\times \frac{4\pi^2}{k^2(k + p)^2q^2(q + p)^2} \frac{4\pi^2}{(1 + k^2/\Lambda^2n)(1 + q^2/\Lambda^2n)} = 0.
$$

(134)

(It is important to note, that all integrals here are convergent.) Therefore,

$$
I_6 = I_1^2 + O(1) = 4\pi^4 \left( \ln \frac{\Lambda}{p} + \ln \frac{\Lambda}{p} \right) + O(1).
$$

(135)

A divergent part of $I_7$ can be calculated similarly:

$$
\lim_{p \to 0} \Lambda \frac{d}{d\Lambda} (I_5 - 2I_7) =
= \lim_{p \to 0} \Lambda \frac{d}{d\Lambda} \int d^4k d^4q \left( \frac{1}{(k + p)^2} - \frac{2}{(k + q + p)^2} \right) \times
$$

$$
\times \frac{1}{k^2q^2(q + p)^2} \frac{4\pi^2}{(1 + k^2/\Lambda^2n)(1 + q^2/\Lambda^2n)} =
$$

30
To calculate this integral we again use four-dimensional spherical coordinates and direct fourth axis along \( q^\mu \). Then similar to the case, considered above, the integral over angles is reduced to

\[
\frac{4\pi}{k^4 q^4 (k + q)^2} \Lambda \left[ \frac{1}{(1 + k^2 n/\Lambda^2 n)(1 + q^2 n/\Lambda^2 n)} \right] = 8n \int d^4 q d^4 k \frac{k q}{k^4 q^4 (k + q)^2} \frac{q^{2n}/\Lambda^{2n}}{(1 + k^2 n/\Lambda^2 n)(1 + q^2 n/\Lambda^2 n)^2}. \tag{136}
\]

so that

\[
2\pi \int dx \frac{x \sqrt{1 - x^2}}{k^2 + 2k q x + q^2} = \frac{\pi^2 q}{k^3}, \quad k \geq q;
\]

\[
= \frac{\pi^2 k}{q^3}, \quad q \geq k. \tag{138}
\]

Therefore,
\[
\lim_{p \to 0} \Lambda \frac{d}{d\Lambda} (I_5 - 2I_7) = -16n \pi^4 \int_0^\infty dq \frac{q^{2n}/\Lambda^{2n}}{(1 + q^{2n}/\Lambda^{2n})^2} \times \\
\times \left( \int_0^\infty dk \frac{k^3}{q(1 + k^{2n}/\Lambda^{2n})} + \int_0^\infty dq \frac{k^3}{q^3(1 + k^{2n}/\Lambda^{2n})} \right) = \\
= -4n \pi^4 \int_0^\infty dx \frac{x^n}{(1 + x^n)^2} \int_0^1 \frac{dy}{1 + y^{-n}} - 4n \pi^4 \int_0^\infty dx \frac{x^n}{(1 + x^n)^2} \int_0^\infty \frac{dy}{1 + y^n} = \\
= -4n \pi^4 \int_0^\infty dx \frac{x^{n-1}}{(1 + x^n)^2} = -4\pi^4 
\]

and finally

\[
I_7 = \frac{1}{2} I_1^2 + 2\pi^4 \ln \frac{\Lambda}{p} + O(1) = 2\pi^4 \left( \ln^2 \frac{\Lambda}{p} + 2 \ln \frac{\Lambda}{p} \right) + O(1). 
\]

Using equation (125) integral \( I_8 \) can be written as

\[
I_8 = 2\pi^2 \sum_i c_i \int d^4 q \frac{1}{q^2(q + p)^2(1 + q^{2n}/\Lambda^{2n})^2} \times \\
\times \left( \ln \frac{M_i}{q} + \sqrt{1 + \frac{4M_i^2}{q^2}} \arctanh \sqrt{\frac{q^2}{4M_i^2 + q^2}} \right), 
\]

where \( M_i = a_i\Lambda, \ a_i \) being constants. To calculate the divergent part of this integral let us consider first an integral

\[
I_f \equiv \int d^4 q \frac{1}{q^2(q + p)^2} f(\Lambda/q) = I_f(\Lambda/p), 
\]

where \( f \) is a function. Differentiating \( I_f \) over \( \ln \Lambda \) and setting then \( p = 0 \), we obtain, that

\[
\Lambda \frac{dI_f}{d\Lambda} \bigg|_{p=0} = \int d^4 q \frac{1}{q^4} \Lambda \frac{d}{d\Lambda} f(\Lambda/q) = - \int d^4 q \frac{1}{q^3} \frac{d}{dq} f(\Lambda/q) = \\
= -2\pi^2 \int_0^\infty dq \frac{d}{dq} f(\Lambda/q) = 2\pi^2 (f(\infty) - f(0)). 
\]

So, if the values \( f(\infty) \) and \( f(0) \) are finite, then

\[
I_f = 2\pi^2 (f(\infty) - f(0)) \ln \frac{\Lambda}{p} + O(1). 
\]
If the function \( f \) is taken to be
\[
f \left( \frac{\Lambda}{q} \right) = \sum_i c_i \frac{2\pi^2}{\left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sqrt{1 + \frac{4M_i^2}{q^2}} \arctanh \sqrt{\frac{q^2}{4M_i^2 + q^2}},
\]
then from equation (144) we conclude, that
\[
2\pi^2 \sum_i c_i \int d^4 q \frac{1}{q^2(q + p)^2 \left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sqrt{1 + \frac{4M_i^2}{q^2}} \arctanh \sqrt{\frac{q^2}{4M_i^2 + q^2}} =
\]
\[
= 4\pi^4 \sum_i c_i \ln \frac{\Lambda}{p} + O(1) = 4\pi^4 \ln \frac{\Lambda}{p} + O(1).
\]
However, it is impossible to substitute in equation (144)
\[
f \left( \frac{\Lambda}{q} \right) = \frac{2\pi^2}{\left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sum_i c_i \ln \frac{M_i}{q}
\]
because \( f(\infty) \) does not exist. Nevertheless, the function \( f \) can chosen in following form:
\[
f \left( \frac{\Lambda}{q} \right) = \Lambda \frac{d}{d\Lambda} \left( \frac{2\pi^2}{\left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sum_i c_i \ln \frac{M_i}{q} \right) =
\]
\[
= \frac{8\pi^2 n q^{2n}/\Lambda^{2n}}{\left(1 + q^{2n}/\Lambda^{2n}\right)^3} \sum_i c_i \ln \frac{M_i}{q} + \frac{2\pi^2}{\left(1 + q^{2n}/\Lambda^{2n}\right)^2},
\]
so that \( f(0) = 0 \) and \( f(\infty) = 2\pi^2 \). Then from equation (145) we obtain, that
\[
\Lambda \frac{d}{d\Lambda} 2\pi^2 \int d^4 q \frac{1}{q^2(q + p)^2 \left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sum_i c_i \ln \frac{M_i}{q} = 4\pi^4 \ln \frac{\Lambda}{p} + O(1)
\]
and, therefore,
\[
2\pi^2 \int d^4 q \frac{1}{q^2(q + p)^2 \left(1 + q^{2n}/\Lambda^{2n}\right)^2} \sum_i c_i \ln \frac{M_i}{q} = 2\pi^4 \ln^2 \frac{\Lambda}{p} + O\left( \ln \frac{\Lambda}{p} \right).
\]
Then it is necessary to calculate logarithmical divergences. For this purpose we subtract from integral (150) terms, proportional \( \ln^2 \Lambda/p \) and differentiate the result over \( \ln \Lambda \):
\[
\lim_{p \to 0} \Lambda \frac{d}{d\Lambda} \left[ 2\pi^2 \int d^4q \frac{1}{q^2(q+p)^2(1+q^{2n}/\Lambda^{2n})} \sum_i c_i \ln \frac{M_i}{q} - 2\pi^4 \ln^2 \frac{\Lambda}{p} \right] =
\]

\[
= \lim_{p \to 0} \left\{ -2\pi^2 \int d^4q \frac{1}{q^2(q+p)^2} q \frac{d}{dq} \left( \frac{1}{1+q^{2n}/\Lambda^{2n}} \right)^2 \sum_i c_i \ln \frac{M_i}{q} - 4\pi^4 \ln \frac{\Lambda}{p} \right\} =
\]

\[
= \lim_{p \to 0} \left\{ -2\pi^4 \int_0^\infty dq \frac{q^2}{p^2} \frac{d}{dq} \left( \frac{1}{1+q^{2n}/\Lambda^{2n}} \right)^2 \sum_i c_i \ln \frac{M_i}{q} - 4\pi^4 \ln \frac{\Lambda}{p} \right\} =
\]

\[
= 2\pi^4 + 4\pi^4 \sum_i c_i \ln \frac{M_i}{\Lambda}. \quad (151)
\]

From equations (141), (146), (150) and (151) we see, that the divergent part of \( I_8 \) is equal to

\[
I_8 = 2\pi^4 \left( \ln^2 \frac{\Lambda}{p} + 2 \ln \frac{\Lambda}{p} \left( \sum_i c_i \ln \frac{M_i}{\Lambda} + \frac{3}{2} \right) \right) + O(1). \quad (152)
\]

In order to prove, that integrals \( I_9 \) and \( I_{10} \) are finite at \( \Lambda \to \infty \), first note, that

\[
I_9 = I_9(p/\Lambda); \quad I_{10} = I_{10}(p/\Lambda). \quad (153)
\]

Therefore, it is necessary to prove, that \( I_9(p = 0) \) and \( I_{10}(p = 0) \) are finite constants. Let us set \( p = 0 \) and make a substitution

\[
k^\mu = \Lambda K^\mu; \quad q^\mu = \Lambda Q^\mu. \quad (154)
\]

Taking into account, that \( M = a\Lambda \), where \( a \) is a finite constant, the considered integrals can be written as

\[
I_9 = \int \frac{d^4K}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{2(K+Q)_\mu Q_\mu}{K^2(1+K^{2n})(K+Q)^2(a^2+Q^2)^2}; \quad (155)
\]

\[
I_{10} = \int \frac{d^4K}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{a^2}{K^2(1+K^{2n})(K+Q)^2(a^2+Q^2)^2}. \quad (156)
\]

The integrals over \( Q \) are evidently convergent. However, it is necessary to check, that, after taking these integrals, the remaining integration over \( K \) will be also convergent. Possible divergences can arise at \( K \to 0 \) or at \( K \to \infty \). In the limit \( K \to 0 \)
\[ \int \frac{d^4Q}{(2\pi)^4} \frac{2(K+Q)_\mu Q_\mu}{((K+Q)^2+a^2)^2(Q^2+a^2)^2} \rightarrow \int \frac{d^4Q}{(2\pi)^4} \frac{2Q^2}{(Q^2+a^2)^4} = \frac{1}{24\pi^2a^2}; \quad (157) \]
\[ \int \frac{d^4Q}{(2\pi)^4} \frac{a^2}{((K+Q)^2+a^2)(Q^2+a^2)^3} \rightarrow \int \frac{d^4Q}{(2\pi)^4} \frac{a^2}{(Q^2+a^2)^4} = \frac{1}{96\pi^2a^2}. \quad (158) \]

It means that, in equations (153) and (156) the integration over \( K \) is convergent if \( K \rightarrow 0 \). Similarly, in the limit \( K \rightarrow \infty \)

\[ \int \frac{d^4Q}{(2\pi)^4} \frac{2(K+Q)_\mu Q_\mu}{((K+Q)^2+a^2)^2(Q^2+a^2)^2} \approx \int \frac{d^4Q}{(2\pi)^4} \frac{2(K+Q)_\mu Q_\mu}{(K+Q)^4Q^4} = \frac{1}{8\pi^2K^2}; \quad (159) \]
\[ \int \frac{d^4Q}{(2\pi)^4} \frac{a^2}{((K+Q)^2+a^2)(Q^2+a^2)^3} \approx \int \frac{d^4Q}{(2\pi)^4} \frac{a^2}{(K+Q)^2(Q^2+a^2)^3} = \frac{1}{32\pi^2(K^2+a^2)} \approx \frac{1}{32\pi^2K^2}, \quad (160) \]

where we used equations (128) and (121). Therefore, due to the presence of higher derivative term the integration over \( K \) in equations (155) and (156) is also convergent at \( K \rightarrow \infty \). Thus the integrals \( I_9 \) and \( I_{10} \) are proven to be finite in the limit \( \Lambda \rightarrow \infty \).

Collecting the above results we can finally write integrals (113) in the following form:

\[ I_1 = 2\pi^2 \left( \ln \frac{\Lambda}{p} + \frac{1}{2} \right) + o(1); \]
\[ I_2 = 2\pi^2 \sum_i c_i \left( \ln \frac{M_i}{p} + \sqrt{1 + \frac{4M_i^2}{p^2}} \arctanh \sqrt{\frac{p^2}{4M_i^2 + p^2}} \right); \]
\[ I_3 = 4\pi^4 \ln \frac{\Lambda}{p} + O(1); \]
\[ I_4 = \frac{\pi^2}{2} + o(1); \]
\[ I_5 = 4\pi^4 \left( \ln^2 \frac{\Lambda}{p} + \ln \frac{\Lambda}{p} \right) + O(1); \]
\[ I_6 = 4\pi^4 \left( \ln^2 \frac{\Lambda}{p} + \ln \frac{\Lambda}{p} \right) + O(1); \]
\[ I_7 = 2\pi^4 \left( \ln^2 \frac{\Lambda}{p} + 2 \ln \frac{\Lambda}{p} \right) + O(1); \]
\[ I_8 = 2\pi^4 \left( \frac{\ln^2 \Lambda}{p} + 2 \ln \frac{\Lambda}{p} \left( \sum_i c_i \ln \frac{M_i}{\Lambda} + \frac{3}{2} \right) \right) + O(1); \]

\[ I_9 = O(1); \]

\[ I_{10} = O(1). \]  

(161)

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Figure 1: Feinman graphs, giving nontrivial contributions to the two-loop $\beta$-function of $N = 1$ supersymmetric electrodynamics.
Figure 2: Feynman graphs, giving nontrivial contributions to the two-loop anomalous dimension of $N = 1$ supersymmetric electrodynamics.
Figure 3: Contour $C$ for calculation of integral over $x$. 