Interacting Non-Abelian Anti-Symmetric Tensor Field Theories

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Abstract. Non-Abelian Anti-symmetric Tensor fields interacting with vector fields have a complicated constraint structure. We enlarge the gauge invariance in this system. Relevant gauge invariant quantities including the Hamiltonian are obtained. We also make introductory remarks on a different but more complicated gauge theory.

1. Introduction

The anti-symmetric tensor field \([1, 2]\), in Abelian and non-Abelian versions, free and in interaction with vector fields, presents interesting theories. It has appeared in various scenarios – for instance, in black hole physics, in mass-generating mechanisms other than the Higgs mechanism. In terms of structure, theories involving the tensor field are complicated. In particular, the constraint structure \([4]\), especially of the non-Abelian theories is complicated. Both first class and second class constraints are present, indicating the presence of gauge invariances in the theory.

In this paper the anti-symmetric tensor field interacting with vector fields, both of non-Abelian nature is considered. The system \([3]\) involves invariances under gauge transformations. We however, look for additional hidden gauge invariances, by considering the constraint structure of the system. We present a construction of a particular gauge theory, from the already existing set of second class constraints using a method applicable within the phase space of the original system.

2. Non-Abelian Interacting Theory

The interaction of the non-Abelian anti-symmetric tensor field \((B^a)_{\mu\nu}\) with a vector field \((A^a)^\mu\) is described by

\[
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a \mu\nu} + \frac{1}{12} H^a_{\mu\lambda\nu} H^{a \mu\lambda\nu} + \frac{m}{4} \epsilon_{\mu\nu\lambda\sigma} F^{a \mu\nu} B^a_{\lambda\sigma}
\]
with \((F^a)_{\mu\nu}\) and \((H^a)_{\mu\nu\lambda}\) being the field strengths for the vector field \((A^a)^\mu\) and \((B^a)^{\mu\nu}\) respectively,

\[
(F^a)_{\mu\nu} = \left(\partial^\mu A^\nu\right)^a - \left(\partial^\nu A^\mu\right)^a + g f^{abc} (A^b)^\mu (A^c)^\nu
\]

\[
(H^a)_{\mu\nu\lambda} = \left(\partial^\mu B^\nu\right)^a + \left(\partial^\nu B^\mu\right)^a + \left(\partial^\lambda B^\mu\right)^a.
\]

Here \(a, b, c\) are group indices, and \(\mu, \nu, \lambda = 0, 1, 2, 3\) are Lorentz vector indices. Further \((D^a)_\mu\) is covariant derivative.

The above Lagrangian is invariant under the gauge transformations,

\[
A^a_\mu \rightarrow A^a_\mu + \left(D^a_\mu \omega\right)^a \quad \text{and} \quad B^a_{\mu\nu} \rightarrow B^a_{\mu\nu} + g f^{abc} B^b_{\mu\nu} \omega^c,
\]

where \(\omega^a\) are space-time dependent parameters of the gauge transformations.

However, the Lagrangian is not invariant (unlike the Abelian case) under

\[
B^a_{\mu\nu} \rightarrow B^{a\prime}_{\mu\nu} = B^a_{\mu\nu} + \left(D_\mu \lambda_\nu - D_\nu \lambda_\mu\right)^a
\]

This non-invariance is due to the non-Abelian nature of the antisymmetric tensor fields.

### 3. Phase Space and the Hamiltonian

In the phase space formulation of the theory \[6\], \(\pi^a_\mu\) and \(\pi^{a\prime}_{\mu\nu}\) are the canonical momentum fields conjugate to the \((A^{a\prime})^\mu\) and \((B^{a\prime})^{\mu\nu}\) respectively.

The fundamental Poisson brackets are,

\[
\{A^{a\prime\mu}(\vec{x}), \pi^b_\nu(\vec{y})\} = \delta^{ab} \delta_\mu^\nu \delta(\vec{x} - \vec{y})
\]

\[
\{B^{a\prime\mu\nu}(\vec{x}), \pi^b_\rho_\sigma(\vec{y})\} = \delta^{ab} \left(\delta^\mu_\rho \delta^\nu_\sigma - \delta^\mu_\sigma \delta^\nu_\rho\right) \delta(\vec{x} - \vec{y}),
\]

with all other Poisson brackets being zero. In the second line, the right hand side is due to the antisymmetric nature of the tensor fields \(B^{a\prime\mu\nu}\) and \(\pi^{b\prime\rho_\sigma}\).

#### 3.1. Constraints

The primary field constraints obtained from the Lagrangian are,

\[
\pi^{a\prime}_\mu(\vec{x}) \approx 0, \quad \pi^{a\prime\nu}_\nu(\vec{x}) \approx 0 \quad (i = 1, 2, 3).
\]

The canonical Hamiltonian, from the Lagrangian is,

\[
H_c = \int d^3x H_c = \int d^3x \left(\dot{A}^{a\prime\mu}_\mu \pi^{a\prime}_\mu + \frac{1}{2} B^{a\prime\mu\nu} \pi^{a\prime}_{\mu\nu} - \mathcal{L}\right)
\]

\[
= \int d^3x \left[\frac{1}{4} F^{aij} F^a_{ij} + \pi^{a\prime ij} \pi^{a\prime}_{ij} - \frac{1}{12} H^a ijk H^a ijk - A^a_0 \left(D^i \pi^a_i\right)^a - \frac{g f^{abc}}{2} \pi^{b}_{ij} B^{i j}\right] - \frac{1}{2} \left(\pi^{aij} - \frac{m}{2} \epsilon^{ijk} B^{ajk}\right) (\pi^{aij} - \frac{m}{2} \epsilon^{ijk} B^{ajk})
\]
\[ \Lambda^a(\vec{x}) = (D^a_i \pi_i)^a - g f^{abc} \pi^b_{ij} B^c_{ij} \approx 0 \]

\[ (\Lambda^a)^i(\vec{x}) = (D^a_i \pi^j)^i - \frac{m}{2} \epsilon^{ijk} F^a_{jk} \approx 0 \quad (i, j, k = 1, 2, 3) \quad (5) \]

Further, from the time independence of the \((\Lambda^a)^i(\vec{x})\) we get the tertiary constraint,

\[ \psi^a_i(\vec{x}) = g f^{abc} \left( \frac{(F^b)^{jk}(H^c)_{ijk}}{2} - (\pi^b)_{ij} \left( \pi^c \right)^j - \frac{m}{2} \epsilon^{kl}(B^c)_{kl} \right) \approx 0. \quad (6) \]

There are no more constraints. The full set of field constraints are,

\[ \pi^a_0(\vec{x}) \approx 0 \]
\[ \pi^a_{0i}(\vec{x}) \approx 0 \]
\[ \Lambda^a(\vec{x}) = (D^a_i \pi_i)^a - g f^{abc} \pi^b_{ij} B^c_{ij} \approx 0 \]
\[ \Lambda^a_i(\vec{x}) = (D^a_i \pi^j)^i - \frac{m}{2} \epsilon^{ijk} F^a_{jk} \approx 0 \]
\[ \psi^a_i(\vec{x}) = g f^{abc} \left( \frac{(F^b)^{jk}(H^c)_{ijk}}{2} - (\pi^b)_{ij} \left( \pi^c \right)^j - \frac{m}{2} \epsilon^{kl}(B^c)_{kl} \right) \approx 0. \]

3.2. Classification of Constraints

Following Dirac’s procedure, we classify all these constraints as below:

The Poisson brackets of the primary constraints \( \pi^a_0 \approx 0, \pi^a_{0i} \approx 0 \) with all the constraints vanish. Further, among the remaining constraints we have,

\[ \{ \Lambda^a(\vec{x}), \Lambda^b(\vec{y}) \} = -g f^{abc} \Lambda^c \delta(\vec{x} - \vec{y}) \]
\[ \{ \Lambda^a(\vec{x}), \psi^b_i(\vec{y}) \} = g f^{abc} \psi^c_i \delta(\vec{x} - \vec{y}) \]
\[ \{ \Lambda^a(\vec{x}), \Lambda^b_i(\vec{y}) \} = g f^{abc} \Lambda^c_i \delta(\vec{x} - \vec{y}) \]
\[ \{ \Lambda^a_i(\vec{x}), \Lambda^b_j(\vec{y}) \} = 0, \quad (7) \]
\[ \{ \Lambda^a_i(\vec{x}), \psi^b_j(\vec{y}) \} = (\mathcal{E}^{ab})^i_j(\vec{x}, \vec{y}) \]
\[ = g f^{acd} g f^{bce} \left[ (\pi^d)^{jk}(\pi^e)_{jk} + \frac{\delta^i_j}{2} (F^d)_{kl}(F^e)_{kl} - (F^d)_{jk}(F^e)_{kl} \right] \delta(\vec{x} - \vec{y}) \]

Thus the \( \pi^a_0, \pi^a_{0i} \) and \( \Lambda^a \) are all of the first class and \( \Lambda^a_i \) and \( \psi^a_i \) are of the second class. Further, the constraints \( \psi^a_i \) have non-zero Poisson bracket among themselves,
\{\psi_i^a, \psi_j^b\} = g^2 f^{acd} f^{bed} m \epsilon_{kmn} (\pi^f)_m (\pi^e)_n \left( \delta_i^k \delta_j^l \delta^c^f \delta^g^f - \delta_j^k \delta_i^l \delta^c^g \delta^f^f \right) \delta(\vec{x} - \vec{y})
+ g^2 f^{acd} f^{bef} \left[ - \frac{g f^{dg} g}{2} [F^{ge}_{mn}(\vec{x}) \pi^c_{ji}(\vec{y}) B^g_{mn}(\vec{x}) + F^e_{mn}(\vec{y}) \pi^c_{ij}(\vec{x}) B^g_{mn}(\vec{y})] \right.
+ \left[ (\pi^e)_{im}(\vec{x})(\pi^f)_{jn}(\vec{y})(\pi^g)_{mj}(\vec{y}) \right] (\pi^d)_{in}(\vec{x}) - (\pi^e)_{jm}(\vec{y})(\pi^f)_{in}(\vec{x})(\pi^g)_{mj}(\vec{y}) + g_{ij} [(F^e)^{mn}(\vec{y})(\pi^f)^{d_m}(\vec{x})(\pi^g)^{m_n}(\vec{y})(\pi^d)^{ij}(\vec{y})] \right]
+ \epsilon^{klr} \left[ (F^e)^{nm}(\vec{y})(\pi^f)^{d_m}(\vec{x})(\pi^g)^{m_n}(\vec{x})(\pi^d)^{ij}(\vec{y}) \right] (\pi^i)^{l_k}(\vec{y}) \delta(\vec{x} - \vec{y})

where \((\pi^e)^a_i = \left[ \pi^a_i - \frac{m}{2} \epsilon^{ijk} B^a_{jk} \right] \).

3.3. Total Hamiltonian

In standard Dirac procedure, the total Hamiltonian includes a combination of all constraints,

\[ H_T = H_c + \int d^3x \left( \mu^a_1 \pi^a_0 + \mu^a_2 \pi^a_0 + \mu^a_3 \Lambda^a + \mu^a_4 \Lambda^{ai} + \mu^a_5 \psi^a_i \right), \]

with the coefficients \(\mu^a_1, \mu^a_2, \mu^a_3, \mu^a_4, \mu^a_5\) being Lagrange multipliers.

Now, the entire set of constraints given earlier define the constraint surface \(\Gamma\). The Lagrange multipliers \(\mu^a_4, \mu^a_5\) corresponding to the second class constraints can be determined (on the surface \(\Gamma\)), by demanding time-independence of these constraints. We get

\[ \mu^a_4(\vec{x}) = \int d^3y (\mathcal{E}^{-1})^{ab}_{i j}(\vec{x}, \vec{y}) \{ \psi^b_j(\vec{y}), H_c \} \quad \text{and} \quad \mu^a_5(\vec{x}) \approx 0 \]

The \(\mu^a_1, \mu^a_2, \mu^a_3\) corresponding to the first class constraints, are undetermined; these indicate the presence of gauge invariance in the theory. We thus have,

\[ H_T = H_c + \int d^3x \left( \mu^a_1 \pi^a_0 + \mu^a_2 \pi^a_0 + \mu^a_3 \Lambda^a \right) + \int d^3(x,y) (\mathcal{E}^{-1})^{ab}_{i j}(\vec{x}, \vec{y}) \{ \psi^b_j(\vec{y}), H_c \} \Lambda^{ai}(\vec{x}) \]

The second class constraints can be eliminated by defining and constructing Dirac brackets, and replacing all relevant Poisson brackets by the corresponding Dirac brackets. In canonical quantisation, in place of Poisson brackets, the Dirac brackets are replaced by commutators.

Once the second class constraints are eliminated, only the first class constraints remain, and they generate the gauge transformations and the resulting gauge theory. These are handled using different methods — they can be either eliminated by the gauge fixing method and then defining Dirac brackets, or they can be retained during quantisation.

A more recent approach is to retain the second class constraints also, instead of eliminating them, and converting them into first class constraints. This results in an enlarged gauge theory, with additional gauge invariances brought in by the newly introduced first class constraints.

More than one approach is available for achieving this. We briefly introduce this method here.
4. Gauge Unfixing Method

Consider a system described by a Hamiltonian with two second class constraints $\phi$ and $\psi$ defining a surface $\Sigma_2$ in the phase space.

In the gauge unfixing method [5], either one of the two second class constraints is retained, and the other discarded. The retained constraint is taken to be the first class constraint. Thus we have a system with one first class constraint.

For instance, the constraint $\psi$ may be discarded, and the $\phi$ retained. Next we redefine the $\phi$ as $\xi = C^{-1}\phi$, where $C = \{\phi, \psi\}$. This makes the $\xi$ and the $\psi$ a canonically conjugate pair, $
abla$,$\xi, \psi = 1$, upto terms proportional to the first class $\xi$.

Now the relevant constraint surface is no longer $\Sigma_2$, but a different one, $\Sigma_1$ defined by $\xi \approx 0$. In order to obtain the gauge theory generated by $\xi$, relevant physical quantities must be gauge invariant; their Poisson brackets with $\xi$ must be zero, on the new surface $\Sigma_1$. In particular the Hamiltonian $H_c$ of the original system may have non-zero Poisson brackets with $\xi$ on $\Sigma_1$, $\{\xi, H_c\} \neq 0$.

Now to get gauge-invariant observables, we define the operator,

$$\hat{P} = :e^{-\int d^3x \psi \hat{\xi}} :$$

where for any phase space variable $B$, we have $\hat{\xi}B = \{\xi, B\}$.

In applying the operator (11) we adopt a particular ordering; when $\hat{P}$ acts on any $B$, $\psi$ and its powers should always be outside the Poisson bracket.

We thus have the gauge invariant quantity $\tilde{B}(x)$,

$$\tilde{B}(x) = :e^{-\int d^3x \psi \hat{\xi}} : B(x)$$

$$= B(x) - \int d^3y \psi(y) \{\xi(y), B(x)\}$$

$$+ \frac{1}{2!} \int d^3yd^3z \psi(y)\psi(z) \{\xi(y), \{\xi(z), B(x)\}\} - \ldots + \ldots$$

Hence we find the new gauge-invariant Hamiltonian $\tilde{H}_c$ from the above operator by acting it on $H_c$. It can be verified that $\{\xi(x), \tilde{H}_c\} = 0$.

The $\xi \approx 0$ and $\tilde{H}_c$ describe a consistent new gauge theory, with gauge transformations generated by $\xi$.

It must be noted that instead of choosing and retaining $\xi$, the $\psi$ can instead be retained and the $\phi$ discarded, resulting in a different constraint surface and possible different gauge theory.

For the field system under consideration, we attempt to use a similar method to obtain a theory with enlarged gauge invariance.

5. New Gauge Theories

As mentioned in the previous section, the gauge unfixing method can be applied in different ways, by choosing and retaining constraints to be of first class, and discarding the others. We consider two such possibilities here.
5.1. The First Case

In our tensor field system the second class constraints are $\Lambda^a_i$ and $\psi^a_i$ — $\{\Lambda^a_i, \psi^b_j\} = \langle \xi^{ab}\rangle_i$. Hence one simple choice of first class constraints will be the $\Lambda^a_i$, they have zero Poisson brackets among themselves.

We next rescale the $\Lambda^a_i$ as,

$$\chi^{a\ i}(\vec{x}) = \int d^3y \ (E^{-1})^{ab\ i\ k}(\vec{x}, \vec{y}) \Lambda^b_k(\vec{y})$$  \hspace{1cm} (13)

so that the $\chi^{ai}$ and $\psi^a_i$ are canonically conjugate to each other, up to terms in the $\chi^{ai}$. We take the $\chi^{ai}$ to be the first class constraints, and discard the $\psi^a_i$.

We construct the operator $P$ as,

$$P = \exp \left(-\int d^3x \ \psi^a_i \chi^{ai} \right)$$  \hspace{1cm} (14)

where we take $\chi^{ai}B = \{\chi^{ai}, B\}$. Also the $\psi^a_i$ are taken to be always outside the Poisson bracket. Using the operator $P$, we construct the gauge invariant Hamiltonian,

$$H_{GU} = H_c - \int d^3y \ \psi^a_i(y) \left\{\chi^{ai}(y), H_T\right\} + \frac{1}{2!} \int d^3y d^3z \ \psi^a_i(y)\psi^b_j(z)\left\{\chi^{ai}(y), \chi^{aj}(z), H_T\right\} - \ldots + \ldots$$

$$= H_T + \frac{1}{2} \int d^3y \ (E^{-1})^{ab\ ik} \psi^a_i \psi^b_k + \int d^3y \ \gamma^{cbe} \psi^a_i (E^{-1})^{ab\ i} A^c_i \Lambda^a_k$$ \hspace{1cm} (15)

The new gauge theory is defined by this $H_{GU}$ and the gauge transformation generating constraints $\xi^a_i$. Further the other relevant fields can also be made gauge invariant, by operating $P$ on the quantities $A^{ai}, \pi^a_i, B^{aij}$ and $\pi^a_{ij}$. These quantities are:

$$\tilde{A}^a_i = A^a_i \hspace{1cm} \tilde{\pi}^a_i = \pi^a_i - gf^{ace} \psi^b_j (E^{-1})^{bc\ k} \pi^c_k + m\epsilon_{ikl}(D^j)^{ace} [\psi^b_j (E^{-1})^{bc\ k}]$$

$$\tilde{B}^{aij} = B^{aij} - \epsilon^{ijm} \epsilon_{lmn}(D^b_m)[\psi^c_k (E^{-1})^{ci\ k}] $$

$$\tilde{\pi}^a_{ij} = \pi^a_{ij}$$ \hspace{1cm} (16)

It should be noted that the gauge invariant fields $\tilde{A}^a_i, \tilde{\pi}^a_i$ no longer form a canonically conjugate pair. Similarly with the tensor fields $B^{aij}, \tilde{\pi}^a_{ij}$, in contrast to the corresponding (gauge non-invariant) fields.

We further note that the gauge invariant Hamiltonian $H_{GU}$ obtained in eqn. (15), can also be obtained by substituting the above gauge invariant fields in the original Hamiltonian $H_c$:

$$\tilde{H}_c = \int d^3x \ \left[\frac{1}{4} \Pi^a_{ij}\Pi^{aij} - \frac{1}{2} (\Pi^a_i - \frac{m}{2} \epsilon_{ijk} B^{ajk}) (\Pi^{ai} - \frac{m}{2} \epsilon_{ilm} B^{al}_{im} ) + \frac{1}{4} F^a_{ij} F^{aij} \right.$$

$$\left. - \frac{1}{12} H^a_{ijk} A^{a\ijk} - A^a_i \Lambda^a + B^a_{ik} \Lambda^{ai} + \frac{1}{2} (E^{-1})^{ab\ ij} \psi^a_i \psi^b_j \right] $$ \hspace{1cm} (17)
which is the same as $H_{GU}$ in Eqn. (15), upto terms in the $\xi^i$.

The Lagrangian corresponding to the above gauge invariant Hamiltonian $\tilde{H}_c$ can also be obtained. We write the inverse Legendre transformation and get the Lagrangian as

$$L = \int d^3x (\pi^a_\mu \dot{A}^{a\mu} + \frac{1}{2} \pi^a_\mu B^{a\mu\nu} - \tilde{H}_c). \quad (18)$$

To go further, the equations of motion for the $A^{a\cdot i}$ and the $B^{a\cdot ij}$ fields, with respect to the gauge invariant Hamiltonian given in eqns. (15) and (17) are to be used in the above Lagrangian. The resulting Lagrangian would also be invariant under gauge transformations. However it is to be seen if a manifestly Lorentz invariant Lagrangian can be obtained.

Quantisation can be attempted in many ways. The gauge invariant Hamiltonian $H_{GU}$ can be used as a starting point, and the first class constraints introduced as operators. Another approach is to use gauge fixing conditions appropriately, introduce Dirac brackets and then quantised.

5.2. The Second Case

Another choice of possible first class constraints are the $\psi^a_i$. But the Poisson brackets $\{\psi^a_i, \psi^b_j\}$ among the $\psi^a_i$ themselves are not zero. It may be possible to modify the $\psi^a_i$, to make them first class. For this we use an iteration procedure.

Choosing the pair of constraints $\psi^1_i$ and $\Lambda^{11}$, we have $C_1 = (\mathcal{E})^{111}_1 = \{\psi^1_i, \Lambda^{11}\}$. We then scale $\psi^1_i$ as $\omega^1_i = \frac{1}{C_1} \psi^1_i$. By discarding the $\Lambda^{11} \approx 0$, we set the $\omega^1_i = 0$ as first class. To obtain the gauge theory and gauge invariant fields, we define our operator $\mathbf{P}$ as,

$$\mathbf{P} = \exp(-\int d^3x \Lambda^{11}\omega^1_i) : \quad (19)$$

and operate $\mathbf{P}$ on $A^{ai}$, $\pi^a_i$, $B^{aij}$ and $\pi^a_{ij}$. We get,

$$\tilde{A}^{ai} = A^{ai} - g e^{abc} \epsilon^{1cd} D^{bd}_{a} \left( 1 - \frac{2X}{X^2} \right) \pi^{b_1i}_{\Lambda^{11}} \frac{1}{C_1}$$

$$- g e^{abc} \epsilon^{1cd} D^{bd}_{a} \left( 1 - \frac{2X}{X^2} \right) \frac{1}{X} \frac{F^{cjk} \delta^{1j}_{i} \delta^{1k}_{j} (\Lambda_{11})^2}{1 \ C_1^2}$$

$$+ m g e^{abc} \epsilon^{1cd} \epsilon^{1ef} \epsilon^{1ij} D^{c}_{p} D^{e}_{r} \left( 1 - \frac{2X}{X^2} \right) \frac{1}{X} \pi^c_{ij} (\Lambda_{11})^2 \frac{1}{C_1^2}$$

$$+ m g e^{abc} \epsilon^{1cd} \epsilon^{1ef} \epsilon^{1ij} D^{c}_{p} D^{e}_{r} \frac{(X-2)\sqrt{1-2X} - 3X + 2}{3X^3} \frac{1}{\partial_m \delta^1_{i} \delta^1_{j} \delta^1_{k} \delta^1_{l}}$$

$$- g e^{abc} \epsilon^{1cd} \epsilon^{1ef} \epsilon^{1ij} D^{c}_{p} D^{e}_{r} \frac{(X-2)\sqrt{1-2X} - 3X + 2}{3X^3} \frac{1}{\partial_m \delta^1_{i} \delta^1_{j} \delta^1_{k} \delta^1_{l}}$$

$$+ g e^{abc} \epsilon^{1cd} \epsilon^{1ef} \epsilon^{1ij} D^{c}_{p} D^{e}_{r} \frac{(X-2)\sqrt{1-2X} - 3X + 2}{3X^3} \frac{1}{\partial_m \delta^1_{i} \delta^1_{j} \delta^1_{k} \delta^1_{l}} \frac{(\Lambda_{11})^3}{1 \ C_1^2} \quad (20)$$
\[ +m^2 g^3 \epsilon^{1ab} \epsilon^{1bc} e^{m11} \left( \frac{(X - 2) \sqrt{1 - \frac{2X}{3X^3}}}{3X^3} \right) \pi^e_{1m} (\Lambda^{11})^3 \frac{1}{c^3} \]

\[ + g \epsilon^{1bv} \left[ 2g^4 \epsilon^{1GH} \epsilon^{1de} e^{1fb} \left( 2D^e_{\mu} (D^H)^h b \left[ \frac{1}{3!} + \frac{7}{4!} X + \frac{57}{5!} X^2 + \ldots \right] \right) \pi^G_{1k} \pi^{1m} \right] \]

\[ - D_p ^{\epsilon} D^{\pi H} \left[ \frac{1}{3!} + \frac{7}{4!} X + \frac{57}{5!} X^2 + \ldots \right] \pi^G_{1k} \delta_{[j} \delta^{[1]}_{m]} \delta^{1m}_{[1]} \]

\[ - 8g^4 \epsilon^{1GH} \epsilon^{1be} e^{1jm} D_k ^{\pi H} \left[ \frac{1}{3!} + \frac{7}{4!} X + \frac{57}{5!} X^2 + \ldots \right] \pi^{e1m} F^{Gnq} F^{bjk} \delta^{1m}_{[1]} \delta^{1m}_{[1]} \]

\[ + 32m^2 g^4 (\pi^{b} \pi^{c}) \pi^v_{11} \Lambda^{11} \frac{1}{c^3} + \ldots \ldots \]

The gauge invariant field in this case is a complicated, infinite series, and it is not clear if it can be written down in closed form. This is unlike the first case, where all the relevant gauge invariant fields have finite number of terms.

Other fields, in gauge invariant form, are also seen to be in infinite series form, and it is not clear if they can be written in closed form.

The new gauge invariant Hamiltonian \( \tilde{H}_c \) is also an infinite series,

\[ \tilde{H}_c = \int d^3 x \left[ \frac{1}{2} g \epsilon^{abc} (D^b_i D^c_i - \beta \pi^{a}_{ij}) + \frac{1}{8} g \epsilon^{abc} (D^b_i D^c_i - \beta \pi^{a}_{ij}) \right] \]

\[ - \frac{1}{8} g \epsilon^{abc} (D^b_i D^c_i - \beta \pi^{a}_{ij}) \delta_{[j} \delta^{[1]}_{m]} \delta^{1m}_{[1]} \frac{1}{2} g \epsilon^{abc} (D^b_i D^c_i - \beta \pi^{a}_{ij}) \]

\[ + \frac{1}{6} g^2 e^{ace} \beta H^{a}_{ij} \pi^{c}_{i} B^{b}_{j} \frac{1}{2} e^{alc} \beta H^{c}_{ij} \pi^{a}_{i} B^{b}_{j} \]

\[ - m g \epsilon^{ab}_{i} \beta (\pi^{a} \pi^{b}_{i}) - m \epsilon^{ab}_{i} \beta (\pi^{a} \pi^{b}_{i}) + g \epsilon^{ab}_{i} A_{0} \beta \psi_{i} - B_{0} \beta \psi_{i} \]

\[ + \Lambda^{11} \frac{1}{c^3} + \ldots \ldots \]

where \( \beta = \left( \frac{1 - \sqrt{\alpha - 2X}}{\alpha} \right) \), \( X = g^3 \epsilon^{1JL} \epsilon^{1LM} \epsilon^{1NQ} D^{|e|_{1}^{M}} F^{Quv}_{1m} \delta^{11}_{[1]} \delta^{1w}_{[1]} + 4m g^3 \epsilon^{1JL} \epsilon^{011v} \pi^{11}_{11} L^{1v} \).

The second choice of first class constraints presents a much more complicated gauge theory, even at the first step of iteration. It is to be seen if the various infinite series can be rewritten in closed form.

6. Conclusion

We have considered the constraint structure of the interacting non-Abelian Antisymmetric Tensor field theory in some detail. It is found that both first and second class constraints are present.

By gauge unfixing method we have attempted to convert the second class constraints into first class constraints. This results is an enlarged gauge symmetry in the theory. We have looked at this conversion in two ways.

In the first gauge theory thus obtained, the gauge invariant quantities like the Hamiltonian and the fields have simple form. But the corresponding Lagrangian is not manifestly Lorentz invariant. The resulting new theory has an enhanced gauge invariance.
In the second gauge theory the new gauge invariant Hamiltonian (and other variables) are more complicated in structure, and appear in infinite series form. It is to be seen if these can be rewritten as closed form expressions. Further work in this is in progress.

Acknowledgments
K Ekambaram wishes to thank the Principal of Kanchi Shri Krishna College of Arts and Science, Kanchipuram, India for encouragement. Both the authors thank the Head of the Department of Theoretical Physics, University of Madras, for constant encouragement.

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