**X(1576) and the Final State Interaction Effect**

Xiang Liu, Bo Zhang, Lei-Lei Shen, and Shi-Lin Zhu

*Department of physics, Peking University, Beijing, 100871, China*

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We study whether the broad peak X(1576) observed by BES Collaboration arises from the final state interaction effect of ρ(1540, 1700) decays. The interference effect could produce an enhancement around 1540 MeV in the K⁺K⁻ spectrum with typical interference phases. However, the branching ratio \( B[J/ψ → ρ(1540, 1700)] \cdot B[ρ(1540, 1700) → K⁺K⁻] \) from the final state interaction effect is far less than the experimental data.

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Recently BES Collaboration reported a broad signal X(1576) in the \( K⁺K⁻ \) invariant mass spectrum in the \( J/ψ → π⁰K⁺K⁻ \) channel [1]. Its quantum number and mass are \( J^{PC} = 1^{−} \) and \( m = (1576.49^{+50}_{−45}(\text{stat})^{+56}_{−91}(\text{syst})) \) MeV respectively. The branching ratio is \( B[J/ψ → X(1576)π⁰] \cdot B[X(1576) → K⁺K⁻] = (8.5±0.6^{+5.6}_{−3.6}) \times 10^{-4} \). If one ignores the tiny isospin violation effect and assume both isospin and G-parity are good quantum numbers, \( X(1576) \) is an isovector with even G-parity. However, the most notable character of \( X(1576) \) is its extremely large width around 800 MeV, which motivated theoretical speculations that it could be a \( K^*(892) − \kappa \) molecular state [2], tetraquark [3, 4], diquark-antidiquark bound state [5, 6].

The lowest scalar meson σ is also very broad [7]. After decades of experimental and theoretical investigations, the underlying structure of the σ meson is still elusive now. Although exotic theoretical interpretations such as treating \( X(1576) \) as a tetraquark is quite interesting, one must answer where are those partner states of \( X(1576) \) in the same tetraquark multiplet. On the other hand, it will be worthwhile and equally interesting to explore whether such a broad signal could be produced by more conventional schemes like the final state interaction (FSI) effect.

It's interesting to note that there are two broad overlapping resonances ρ(1450) and ρ(1700) with the same quantum number as \( X(1576) \) around 1600 MeV. Their widths are about 147 MeV and 250 MeV respectively [7]. The FSI effect sometimes plays a very important role in some processes [8]. Hence, we want to take a look at the FSI effect in the \( \rho(1450, 1700) → K⁺K⁻ \) decays and explore whether the FSI effect may produce a similar broad signal in this work.

The intermediate states \( π^+π^−, ωπ^0, ρ⁰η, ρ^+ρ^−, a_1(1260)π^0 \) contribute to \( \rho(1450, 1700) → K⁺K⁻ \) as shown in Fig. 1. The \( K'^+K'^− + c.c. \) intermediate state also contributes to \( ρ(1700) → K⁺K⁻ \). The above intermediate states are of four types: \( P + P, P + V, V + V \) and \( A + P \), where \( P, V \) and \( S \) denote pseudoscalar, vector and axial vector mesons respectively.

![FIG. 1: Some possible intermediate states contributing to the ρ(1450, 1700) → K⁺K⁻ decays.](image)

The effective Lagrangians related to our following calculation read

\[ \mathcal{L}_{V₁→V₂V₃} = ig₁(P₁⁺P₂⁺)V₃⁺, \]
\[ \mathcal{L}_{V₁→V₂P₃} = 2g₂\varepsilon_{µναβ}V¹₂⁺(∂µV₂⁺)(∂νV₃⁻), \]
\[ \mathcal{L}_{V₁→V₂V₃} = ig₃ \left\{ V₁⁺(∂µV₂⁺V₃⁻ + V₂⁻∂µV₃⁺) + (∂µV₁⁺V₂⁺ + V₁⁺V₂⁻ + V₂⁺V₃⁻) \right\}, \]
\[ L_{S→P₁V₁} = g₄P₁V₁⁺S⁺, \]

where \( g_1, g_2, g_3, g_4 \) denote the coupling constants. \( P₁, V₁, V₂, V₃, S \) respectively represent pseudoscalar, vector, axial vector, and pseudoscalar mesons.
vector fields.

Using the Cutkosky rule, one obtains the absorptive contribution to the process of \(\pi^+(p_1)\pi^-(p_2) \rightarrow \rho^+(p_3)\rho^-(p_4)\) in Fig. 4(a)

\[
\text{Abs}^{(a)}[\pi^+,\pi^-,K^0]\n\]
\[
= \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4E_1} \frac{d^4p_2}{(2\pi)^4E_2} (2\pi)^4 \delta^4(M-p_1-p_2) \times \left\{ i\epsilon_{\mu\nu\rho\sigma} (p_1\cdot p_2) \gamma^\nu - \epsilon^\mu (p_1 + p_2) \right\} \times [-i\epsilon_{\nu\rho\sigma} (p_2\mu + p_4\nu) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_{K^*}^2} \right] \times \left[ \frac{i}{q^2 - m_{K^*}^2} \right] F^2(m_{K^*},q^2). \tag{5}
\]

The amplitude corresponding to the process of \(\rho(1450,1700) \rightarrow \omega(p_1)\pi^0(p_2) \rightarrow K^+(p_3)K^-(p_4)\) can be written as

\[
\text{Abs}^{(b)}[\omega\pi^0,K^+] \n\]
\[
= \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4E_1} \frac{d^4p_2}{(2\pi)^4E_2} (2\pi)^4 \delta^4(M-p_1-p_2) \times \left\{ i\epsilon_{\mu\nu\rho\sigma} (p_1\cdot p_2) \gamma^\nu - \epsilon^\mu (p_1 + p_2) \right\} \times \left[ i\epsilon_{\nu\rho\sigma} (p_2\mu + p_4\nu) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_{K^*}^2} \right] \times \left[ \frac{i}{q^2 - m_{K^*}^2} \right] F^2(m_{K^*},q^2). \tag{6}
\]

The amplitudes of \(\rho(1450,1700) \rightarrow \rho^0\eta \rightarrow K^+K^-\) and \(\rho(1700) \rightarrow K^{*+} + c.c. \rightarrow K^+K^-\) can be obtained by replacing the coupling constants and masses in the above formula. \(\rho, \omega, \) and \(\phi\) are the exchanged mesons between \(K\) and \(K^*\).

For the processes of \(\rho(1450,1700) \rightarrow \rho^+(p_1)\rho^-(p_2) \rightarrow K^+(p_3)K^-(p_4),\) the exchanged mesons are \(K^0\) and \(K^{*0}.\) Thus the relevant amplitudes are

\[
\text{Abs}^{(c)}[\rho^+,\rho^-,K^0]\n\]
\[
= \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4E_1} \frac{d^4p_2}{(2\pi)^4E_2} (2\pi)^4 \delta^4(M-p_1-p_2) \times \left\{ i\epsilon_{\mu\nu\rho\sigma} (p_1\cdot p_2) \gamma^\nu - \epsilon^\mu (p_1 + p_2) \right\} \times \left[ i\epsilon_{\nu\rho\sigma} (p_2\mu + p_4\nu) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_{K^*}^2} \right] \times \left[ \frac{i}{q^2 - m_{K^*}^2} \right] F^2(m_{K^*},q^2). \tag{7}
\]

and

\[
\text{Abs}^{(d)}[\rho^+\rho^-,K^{*0}] \n\]
\[
= \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4E_1} \frac{d^4p_2}{(2\pi)^4E_2} (2\pi)^4 \delta^4(M-p_1-p_2) \times \left\{ i\epsilon_{\mu\nu\rho\sigma} (p_1\cdot p_2) \gamma^\nu - \epsilon^\mu (p_1 + p_2) \right\} \times \left[ i\epsilon_{\nu\rho\sigma} (p_2\mu + p_4\nu) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_{K^*}^2} \right] \times \left[ \frac{i}{q^2 - m_{K^*}^2} \right] F^2(m_{K^*},q^2). \tag{8}
\]

The decay amplitude for the process \(\rho(1450,1700) \rightarrow a_1(1260)(p_1)\pi^0(p_2) \rightarrow K^+(p_3)K^-(p_4)\) is

\[
\text{Abs}^{(e)}[a_1\pi^0,K^{*+}] \n\]
\[
= \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4E_1} \frac{d^4p_2}{(2\pi)^4E_2} (2\pi)^4 \delta^4(M-p_1-p_2) \times \left\{ i\epsilon_{\mu\nu\rho\sigma} (p_1\cdot p_2) \gamma^\nu - \epsilon^\mu (p_1 + p_2) \right\} \times \left[ i\epsilon_{\nu\rho\sigma} (p_2\mu + p_4\nu) \left[ -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_{K^*}^2} \right] \times \left[ \frac{i}{q^2 - m_{K^*}^2} \right] F^2(m_{K^*},q^2). \tag{9}
\]

In the above eqs. (5-9), \(F(m_i,q^2)\) etc. denotes the form factors which compensate the off-shell effects of mesons at the vertices and are written as (9-10)

\[
F(m_i,q^2) = \left( \frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2} \right)^n, \tag{10}
\]

where \(\Lambda\) is a phenomenological parameter. As \(q^2 \to 0\) the form factor becomes a number. If \(\Lambda \gg m_i\), it becomes unity. As \(q^2 \to \infty\), the form factor approaches to zero. As the distance becomes very small, the inner structure would manifest itself and the whole picture of hadron interaction is no longer valid. Hence the form factor vanishes and plays a role to cut off the end effect. The expression of \(\Lambda\) is

\[
\Lambda(m_i) = m_i + \alpha \Lambda_{QCD}, \tag{11}
\]

where \(m_i\) denotes the mass of exchanged meson and \(\alpha\) is a phenomenological parameter. Although we use \(\Lambda_{QCD} = 220\) MeV, the range of \(\Lambda_{QCD}\) can be taken into account through the variation of the parameter \(\alpha\). In this work, we adopt the monopole form factor \(F(m_i,q^2) = (\Lambda^2 - m_i^2)/(\Lambda^2 - q^2),\) where \(\alpha\) is of order unity and its range is around \(0.8 < \alpha < 2.2\).

Let’s first focus on the ratio

\[
R^{ABC} = \left| q_{KK} \right| \left| \mathcal{M}[\rho(1450,1700) \rightarrow A + B \rightarrow K^+K^-] \right|^2 / \left| q_{AB} \right| \left| \mathcal{M}[\rho(1450,1700) \rightarrow A + B] \right|^2, \tag{12}
\]
where $A$ and $B$ are possible intermediate states. $C$ denotes the exchanged meson for the scattering process of $A$ and $B$ mesons. $q_{KK}$ and $q_{AB}$ are the decay momenta corresponding to $\rho(1450, 1700) \rightarrow A + B \rightarrow K^+ K^-$ and $\rho(1450, 1700) \rightarrow A + B$ respectively. The uncertainty from the vertex of $\rho(1450, 1700) \rightarrow A + B$ can be eliminated [11]. $R^{AB,C}$ depends on the masses of $\rho(1450, 1700)$. Because $\rho(1450, 1700)$ are broad, this ratio gives us information on the evolution of $R^{AB,C}$ with the masses of $\rho(1450, 1700)$ mesons.

Using $\Gamma(K^*0 \rightarrow K\pi) = 50.3$ MeV and $\Gamma(\phi \rightarrow K^+ K^-) = 2.09$ MeV [7], we obtain $g_{K^*0K^+\pi\pi} = 3.76$ and $g_{\phi K^+ K^-} = 5.55$. In the limit of SU(3) symmetry, we take $g_{K^*0K^+\pi\pi} = \sqrt{2}g_{K^+ K^0\pi^0} = \sqrt{6}g_{K^+ K^0\eta}$ and $\sqrt{2}g_{\omega(\rho^0)K^+\pi^0} = g_{\rho\pi K^+ K^0} = g_{\rho K^+ K^0}$. $g_{\rho K^+ K^0} = \sqrt{2}g_{\omega(\rho^0)K^+\pi^0} = g_{\rho K^+ K^0} = 6.48$ GeV$^{-1}$ [11].

FIG. 2: The dependence of $R^{AB,C}$ corresponding to the different intermediate states on the mass range of $\rho(1450, 1700)$ with monopole form factor.

The dependence of $R^{AB,C}$ on the mass of $\rho(1450, 1700)$ with different intermediate states and $\alpha = 0.8, 1.5, 2.2$ is shown in Fig. 2. For comparison, we illustrate the variation of $R^{AB,C}$ with the mass of $\rho(1450, 1700)$ using the dipole form factor $F(m_i, q^2) = [\Gamma^2 - m_i^2]/(\Gamma^2 - q^2)^2$ in Fig. 3.

In the following we focus on the interference of the dominant amplitudes corresponding to Fig. 1 (a) and (c), which reads

$$\mathcal{M}[\rho(1450, 1700) \rightarrow K^+ K^-] = \mathcal{M}[\rho(1450, 1700) \rightarrow \pi^+ \pi^- \rightarrow K^+ K^-] + e^{i\phi}\mathcal{M}[\rho(1450, 1700) \rightarrow \rho^+ \rho^- \rightarrow K^+ K^-].$$

where $\phi$ denotes the phase between Fig. 1 (a) and (c). The dependence of the decay width from the above interference amplitude on the parent mass and different $\phi$ is presented in Fig. 3 and Fig. 4.

Our numerical results indicate: (1) no enhancement structure exists in the case of no interference as shown in Fig. 2 (2) the interference between Fig. 1 (a) and
(c) leads to an enhancement in a typical range of phase $\phi \sim 120^\circ \sim 180^\circ$. Especially from Fig. 5, we notice that the enhancement with different phases occurs around 15.40 MeV, very close to $X(1576)$. The enhancement depicted in Fig. 5 is similar to the cusp effect discussed in Ref. [12] to some extent. In fact, such an enhancement occurs with the opening of the $\rho \rho$ channel. Although the numerical results depend on the particular parametrization of the form factor, the qualitative features and conclusion remain essentially the same.

The $K^+K^-$ spectrum from the above interference mechanism mimics the observed broad spectrum from BES's measurement. However, basing on the estimate from the calculation with monopole form factor, the decay width of $\rho(1450, 1700) \rightarrow K^+K^-$ from FSI effect is about 0.2 MeV only. Taking the width of $\rho(1450, 1700)$ as 300 MeV, the branching ratio of $\rho(1450, 1700) \rightarrow A + B \rightarrow K^+K^-$ is about $10^{-4}$. If the order of magnitude of $B[J/\psi \rightarrow \pi + \rho(1450, 1700)]$ is roughly $10^{-3}$, the $B[J/\psi \rightarrow \pi^0 + \rho(1450, 1700)] \cdot B[\rho(1450, 1700) \rightarrow AB \rightarrow K^+K^-]$ is about $10^{-7}$, which is far less than experimental value $B[J/\psi \rightarrow \pi + X(1576)] \cdot B[X(1576) \rightarrow K^+K^-] = 8.5 \times 10^{-4}$. The naive interpretation of the extremely broad structure $X(1576)$ arising from the final state interaction effect seems not very favorable. Clearly more experimental information on the exotic structure will be helpful.

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FIG. 3: The dependence of $R^{AB,C}$ corresponding to the different intermediate states on the mass range of $\rho(1450, 1700)$ with the dipole form factor ($\alpha = 1.5$).
FIG. 4: The dependence of the decay width from the interference amplitude [13] on the mass and $\phi$ using monopole form factor with typical value $\alpha = 1.5$.

\(|k| = f(s, m_{a_1}, \Gamma_{a_1}) = \frac{1}{\pi} \frac{m_{a_1} \Gamma_{a_1}}{(s-m_{a_1}^2 + m_{a_1}^2 \Gamma_{a_1})^{1/2}} \times |k| = \frac{\sqrt{(s-(m_{K}+m_{K^*})^2)(s-(m_{K}-m_{K^*})^2)}}{2\sqrt{s}}.\)
FIG. 5: (a) and (b) show the evolution of the decay width with the mass and φ considering the monopole and dipole form factors respectively with α = 1.5.