Nearly Mass-Degenerate Majorana Neutrinos: Double Beta Decay and Neutrino Oscillations

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Abstract

Assuming equal tree-level Majorana masses for the standard-model neutrinos, either from the canonical seesaw mechanism or from a heavy scalar triplet, I discuss how their radiative splitting may be relevant to neutrinoless double beta decay and neutrino oscillations.

• Talk given at the International Conference on Non-Accelerator New Physics, Dubna, Russia (June 28 - July 3, 1999).
1 Introduction

In this talk I will first discuss two equally simple mechanisms for small Majorana neutrino masses, one famous and one not so famous. I will then mention briefly how they are related to neutrinoless double beta decay and neutrino oscillations. My main focus will be on the possibility of nearly mass-degenerate neutrinos and their radiative splitting due to the different charged-lepton masses. In particular, I show how a two-fold neutrino mass degeneracy can be stable against radiative corrections. I finish with three examples: (1) a two-loop explanation of vacuum $(\Delta m^2)_{\text{sol}}$, (2) a one-loop connection between $(\Delta m^2)_{\text{atm}}$ and vacuum $(\Delta m^2)_{\text{sol}}$, and (3) a one-loop explanation of small-angle matter-enhanced $(\Delta m^2)_{\text{sol}}$ with the prediction $0.20 \text{ eV} < m_{\nu} < 0.36 \text{ eV}.$

2 Origin of Neutrino Masses

In the standard model, leptons are left-handed doublets $(\nu_i, l_i)_L \sim (1, 2, -1/2)$ and right-handed singlets $l_i_R \sim (1, 1, -1)$ under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The absence of the gauge singlet $\nu_{iR} \sim (1, 1, 0)$ implies that $m_{\nu_i} = 0$. However, since the Higgs scalar doublet $\Phi = (\phi^+, \phi^0) \sim (1, 2, 1/2)$ exists, there is a unique 5-dimensional operator for nonzero Majorana neutrino masses. The underlying theory which realizes this operator is usually assumed to be that of the seesaw mechanism. In other words, the gauge-invariant operator

$$\Lambda^{-1} \phi^0 \phi^0 \nu_i \nu_j$$ (1)

for nonzero Majorana neutrino masses. The underlying theory which realizes this operator is usually assumed to be that of the seesaw mechanism. In other words, the gauge-invariant operator

$$(\phi^0 \nu_i - \phi^+ l_i)(\phi^0 \nu_j - \phi^+ l_j)$$ (2)

is obtained by inserting a heavy Majorana fermion singlet $N$ as the intermediate state, as illustrated in Fig. 1 below.
The resulting neutrino mass matrix is then given by
\[
(M_\nu)_{ij} = -\frac{f_i f_j v^2}{M},
\]
where \(f_i\) are Yukawa couplings of \(\nu_i\) to \(N\), \(v = \langle \phi^0 \rangle\), and \(M\) is the mass of \(N\). On the other hand, the expression in (2) can be rewritten as
\[
\phi^0 \phi^0 \nu_i \nu_j - \phi^+ \phi^0 (\nu_i l_j + l_i \nu_j) + \phi^+ \phi^+ l_i l_j,
\]
which allows the insertion of a scalar triplet \((\xi^{++}, \xi^+, \xi^0)\) as the intermediate state, as illustrated in Fig. 2 below.

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The neutrino mass matrix is now given by

\[
(M_{\nu})_{ij} = -\frac{2f_{ij}\mu v^2}{M^2},
\]

where \(f_{ij}\) are the Yukawa couplings of \(\nu_i\) to \(\nu_j\), \(\mu\) is the trilinear coupling of \(\xi\) to \(\Phi\Phi\), and \(M\) is the mass of \(\xi\). The alternative way to understand this mass is to note that \(\xi^0\) acquires a nonzero vacuum expectation value in this model given by \(u = -\frac{\mu v^2}{M^2}\). In other words, in the limit where \(M^2\) is positive and large, it is natural for \(u\) to be very small. This method for generating small Majorana neutrino masses is as simple and economical as the canonical seesaw mechanism. To obtain the most general \(3 \times 3\) neutrino mass matrix, we need \(3\) \(N\)'s in the latter, but only one \(\xi\) in the former.

### 3 Neutrinoless Double Beta Decay and Neutrino Oscillations

Let the \((\nu_e, \nu_\mu, \nu_\tau)\) mass matrix \(M\) have eigenvalues \(m_{1,2,3}\) with \(\nu_e = \sum_i U_{ei} \nu_i\), then

\[
M_{ee} = \sum_i U_{ei} m_i U_{ie}^T = \sum_i U_{ei}^2 m_i
\]

is what is being measured in neutrinoless double beta decay. The most recent result from the Heidelberg-Moscow experiment is\([9]\) \(M_{ee} < 0.2\) eV. Note however that since \(U_{ei}^2 m_i\) may be of either sign for each \(i\), \(M_{ee}\) does not constrain \(|m_i|\) without further information. For example, consider

\[
M = \begin{bmatrix}
\cos^2 \theta m_1 + \sin^2 \theta m_2 & \sin \theta \cos \theta (m_2 - m_1) \\
\sin \theta \cos \theta (m_2 - m_1) & \sin^2 \theta m_1 + \cos^2 \theta m_2
\end{bmatrix},
\]

which tells us that \(\nu_e = \cos \theta \nu_1 + \sin \theta \nu_2\). Now if \(m_1 > 0, m_2 > 0\), then \(m_1 < M_{ee}\); but if \(m_1 < 0, m_2 > |m_1|\), then there are no individual upper bounds on \(|m_1|\) or \(m_2\).

In neutrino oscillations, the parameters accessible to experimental determination are \(\Delta m_{ij}^2 = m_i^2 - m_j^2\) and \(U_{ei}\), hence the sign of \(m_i\) is irrelevant there. The sign of \(\Delta m_{ij}^2\) is
important in matter-enhanced oscillations[10] because neutrino and antineutrino forward scattering amplitudes in matter have opposite signs.

4 Nearly Mass-Degenerate Majorana Neutrinos and Their Stability Against Radiative Corrections

Suppose neutrinos are Majorana and are equal in mass:

\[ \nu_i = U_{ie}^T \nu_e + U_{i\mu}^T \nu_\mu + U_{i\tau}^T \nu_\tau, \quad i = 1, 2, 3, \tag{8} \]

and \( m_1 = m_2 = m_3 \). Since \( m_e, m_\mu, \) and \( m_\tau \) are all different, this degeneracy cannot be exact. In other words, splitting must occur, but how? This question has two answers. (1) Depending on the specific mechanism by which the neutrinos become massive, there are finite radiative corrections to the mass matrix itself[4, 5, 6]. (2) There are model-independent wavefunction renormalizations which shift the values of the mass matrix from one mass scale to another[11].

The stability of neutrino mass degeneracy against radiative corrections depends[4, 12] on the symmetry of the mass matrix. Consider

\[ \mathcal{M} = \begin{bmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{bmatrix}, \tag{9} \]

then

\[ \Delta m^2 = (m_{ee} + m_{\mu\mu}) \sqrt{(m_{ee} - m_{\mu\mu})^2 + 4m_{e\mu}^2}. \tag{10} \]

Thus \( \Delta m^2 = 0 \) has two solutions. One is

\[ \mathcal{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}, \tag{11} \]

then the effect of radiative corrections is to shift it by \( 4m^2(\delta_\mu - \delta_e) \). This is inherently unstable. The other is

\[ \mathcal{M} = \begin{bmatrix} m & m' \\ m' & -m \end{bmatrix}, \tag{12} \]
then the shift is $4m\sqrt{m^2 + m'^2} (\delta_\mu - \delta_e)$. This is stable as long as $m << m'$ and is easily understood because the $m = 0$ limit corresponds to the existence of an extra global $L_e - L_\mu$ symmetry for the entire theory.

5 Two-Loop Example

Choose the canonical seesaw mechanism for obtaining neutrino masses. Impose a global $SO(3)$ symmetry so that $(\nu_i, l_i)_L$ and $N_i_R$ with $i = +, 0, -$ are triplets. Invariants are then

$$f[(\bar{\nu}_+ N_+ + \bar{\nu}_0 N_0 + \bar{\nu}_- N_-)\bar{\phi}^0 - (\bar{l}_+ N_+ + \bar{l}_0 N_0 + \bar{l}_- N_-)\phi^-] + h.c. \quad (13)$$

and

$$M(2N_+ N_- - N_0 N_0). \quad (14)$$

Assume $SO(3)$ invariance for $f$ to be valid at the electroweak symmetry breaking scale [i.e. no renormalization correction from different $\nu_i$'s.] Let $m_D = f\langle \bar{\phi}^0 \rangle << M$ and $m_0 = m_D^2/M$, then

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & -m_0 & 0 \\ -m_0 & 0 & 0 \\ 0 & 0 & m_0 \end{bmatrix} \quad (15)$$

in the basis $(\nu_+, \nu_-, \nu_0)$. Now choose $l_+ = e$ so that $\mathcal{M}_{ee} = 0$, and let

$$l_- = c\mu + s\tau, \quad l_0 = c\tau - s\mu, \quad (16)$$

where $c = \cos \theta$, $s = \sin \theta$.

This model\cite{5} differs from the standard model only in the addition of 3 heavy $N$'s. The effective low-energy theory differs at tree level only in the appearance of 3 nonzero, but equal, neutrino masses. This degeneracy is then lifted in two loops\cite{13}, as illustrated in Fig. 3 below.
The leading contribution to the above two-loop diagram is universal, but the effects of the charged-lepton masses show up in the propagators, and since $m_\tau$ is the largest such mass, the radiative splitting is proportional to $m_\tau^2$. The neutrino mass matrix of Eq. (15) is now corrected to read

$$
\mathcal{M}_\mu = \begin{pmatrix}
0 & -m_0 - s^2 I & -scI \\
-m_0 - s^2 I & 0 & scI \\
-scI & scI & m_0 + 2c^2 I
\end{pmatrix},
$$

(17)

where

$$
I = \frac{g^4}{256\pi^4 M_W^2} \left( \frac{\pi^2}{6} - \frac{1}{2} \right) m_0 = 3.6 \times 10^{-9} m_0,
$$

(18)

and the eigenvalues are $-m_0 - s^2 I$, $m_0$, and $m_0 + (1 + c^2)I$. Let $s^2 << 1$, then $\nu_e$ oscillates mostly into $\nu_\mu$ with

$$
P(\nu_e \to \nu_e) = \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\Delta m^2 L}{2E} \right),
$$

(19)

where $\Delta m^2 = 2s^2m_0I = 7.2 \times 10^{-9} s^2m_0^2 \sim 10^{-10}$ eV$^2$, if $s \sim 0.1$ and $m_0 \sim 1$ eV.

This example shows that a minimum splitting of the Majorana neutrino mass degeneracy in the canonical seesaw model is suitable for the vacuum oscillation solution of the solar neutrino deficit[14]. However, other effects may be larger, such as the renormalization of the $\bar{\nu}_L N R \phi^0$ vertex.
Choose the heavy scalar triplet $\xi$ for generating small Majorana neutrino masses. Impose a discrete $S_3$ symmetry, having the irreducible representations $\mathbf{2}$, $\mathbf{1}$, and $\mathbf{1}'$. Let $(\nu_1, \nu_2) \sim \mathbf{2}$, and $\nu_3 \sim \mathbf{1}$, then

\[ \mathcal{L}_{\text{int}} = \xi^0 [f_0 (\nu_1 \nu_2 + \nu_2 \nu_1) + f_3 \nu_3 \nu_3] + \mu \bar{\xi}^0 \phi^0 \phi^0 + \ldots \]  

(20)

Let $\langle \xi^0 \rangle = u = -\mu \langle \phi^0 \rangle^2 / m_\xi^2$, then

\[ \mathcal{M}_\nu = \begin{bmatrix} 0 & m_0 & 0 \\ m_0 & 0 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \]  

(21)

where $m_0 = 2 f_0 u$ and $m_3 = 2 f_3 u$. Now choose $\nu_1 = \nu_e$ so that again $\mathcal{M}_{ee} = 0$, and let $\nu_2 = c \nu_\mu - s \nu_\tau$, $\nu_3 = c \nu_\tau + s \nu_\mu$.

This model\[6\] allows the radiative splitting of the two-fold neutrino mass degeneracy to occur in one loop, as illustrated in Fig. 4 below.

![Fig. 4](image-url)  

**Fig. 4** One-loop radiative breaking of neutrino mass degeneracy.
Hence $\mathcal{M}_\nu$ of Eq. (21) becomes

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & m_0(1 + s^2I) & -scm_0I \\ m_0(1 + s^2I) & 0 & -scm_3I \\ -scm_0I & -scm_3I & m_3(1 + 2c^2I) \end{bmatrix},$$

(22)

whose eigenvalues are $m_3(1 + 2c^2I)$, and

$$\mp m_0(1 + s^2I) \mp \frac{s^2c^2(m_0 \mp m_3)^2I^2}{2(m_0 \pm m_3)},$$

(23)

with

$$I = \left( \frac{1}{4\pi^2} - \frac{1}{16\pi^2} \right) \frac{G_Fm_\tau^2}{\sqrt{2}} \ln \frac{m_\xi^2}{M_W^2},$$

(24)

where the second term inside the parentheses comes from the shift of the neutrino wavefunction renormalization from $m_\xi$ to $M_W$. Numerically, $I^2 < (m_o - m_3)^2/(m_0 + m_3)^2$, hence

$$\Delta m_{12}^2 \simeq \frac{8s^2c^2I^2m_\nu^4}{m_\tau^2 - m_3^2},$$

(25)

where $m_\nu \simeq m_0 \simeq m_3$ has been used. Thus a simple connection between atmospheric and solar neutrino vacuum oscillations is obtained:

$$\frac{(\Delta m^2)_{sol}(\Delta m^2)_{atm}}{m_\nu^4(\sin^2 2\theta)_{atm}} = 2I^2 = 4.9 \times 10^{-13} \left( \ln \frac{m_\xi^2}{M_W^2} \right)^2.$$

(26)

This equality holds for the sample values of $m_\nu = 0.6$ eV, $(\sin^2 2\theta)_{atm} = 1$, $m_\xi = 1$ TeV, $(\Delta m^2)_{sol} = 4 \times 10^{-10}$ eV$^2$, and $(\Delta m^2)_{atm} = 4 \times 10^{-3}$ eV$^2$. [If $m_\xi = 10^{13}$ GeV, then $m_\nu \sim 0.2$ eV.]

This example shows that it is possible to have a one-loop effect, but which appears only in second order because of nondegenerate ($m_0 \neq m_3$) perturbation theory. In the previous example, the effect is two-loop but it occurs in first order because of degenerate perturbation theory.
7 One-Loop Example II

This model\textsuperscript{[7]} is a variation of Example I, with $\nu_1 \nu_1 + \nu_2 \nu_2$ as an invariant, say under $SO(2)$. Hence

$$\mathcal{M}_\nu = \begin{bmatrix}
    m_0 & 0 & 0 \\
    0 & m_0(1 + 2c^2 I) & -sc(m_0 + m_3)I \\
    0 & -sc(m_0 + m_3)I & m_3(1 + 2s^2 I)
\end{bmatrix}, \quad (27)$$

where $\nu_1 = \nu_e$, $\nu_2 = c\nu_\tau - s\nu_\mu$, $\nu_3 = c\nu_\mu + s\nu_\tau$. Now rotate $\nu_1$ and $\nu_2$ slightly by $\theta'$, then the small-angle matter-enhanced solution to the solar neutrino deficit works for $\sin^2 2\theta' \simeq (2 - 10) \times 10^{-3}$ and

$$(\Delta m^2)_{12} = 4c^2 I m_0^2 \simeq (3 - 10) \times 10^{-6} \text{ eV}^2. \quad (28)$$

For $c^2 = 0.7$, i.e. $(\sin^2 2\theta)_{atm} = 0.84$, and $m_\xi = 10^{14}$ GeV, this implies

$$0.20 \text{ eV} < \mathcal{M}_{ee} < 0.36 \text{ eV}. \quad (29)$$

Experimentally, the most recent Heidelberg-Moscow result\textsuperscript{[9]} is $\mathcal{M}_{ee} < 0.2$ eV, but the expected sensitivity is only 0.38 eV, both at 90\% confidence level. More data may see something or rule out the above prediction.

8 Conclusions

- Neutrino mass is equally natural coming from the seesaw mechanism or a heavy scalar triplet.
- If $\nu_{e,\mu,\tau}$ are nearly mass-degenerate, their radiative splitting may be suitable for solar neutrino oscillations. Details depend on the specific model, but the smallness of vacuum $(\Delta m^2)_{sol}$ is only obtained in certain special cases.
Acknowledgements

I thank Sergey Kovalenko and everyone associated with the organization of NANP-99 for their great hospitality. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

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