The calculation of weakly non-spherical cavitation bubble impact on a solid

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Abstract. The effect of small spheroidal non-sphericity of a cavitation bubble touching a solid at the beginning of its collapse on its impact on the solid of a copper-nickel alloy is investigated. The impact on the solid is realized by means of a high-speed liquid jet arising at collapse on the bubble surface. The shape of the jet, its velocity and pressure are calculated by the boundary element method. The spatial and temporal characteristics of the pressure pulses on the solid surface are determined by the CIP-CUP method on dynamically adaptive grids without explicitly separating the gas-liquid interface. The solid surface layer dynamics is evaluated by the Godunov method. The results are analyzed in dimensionless variables obtained with using the water hammer pressure, the time moment and the jet-solid contact area radius at which the jet begins to spread on the solid surface. It is shown that in those dimensionless variables, the dependence of the spatial and temporal characteristics of the solid surface pressure pulses on the initial bubble shape non-sphericity is relatively small. The non-sphericity also slightly influences the main qualitative features of the dynamic processes inside the solid, whereas its effect on their quantitative characteristics can be significant.

1. Introduction

It is widely known that the solid surfaces contacting with a liquid with the variable pressure are often subject to cavitation erosion resulting from the action of cavitation bubbles [1]. One of the main mechanisms of cavitation erosion is considered the impact of the high-speed liquid jet arising on the surface of the bubble at its asymmetrical collapse near a solid [2]. It is shown in [3] that the parameters of such a jet (the velocity and the pressure, the geometry of its end) are strongly dependent on the shape of the bubble at the beginning of its collapse.

The present paper is devoted to investigating the effect of a small non-sphericity of the bubble at the beginning of its collapse on the surface layer of a solid in the case of a cavitation bubble touching the solid surface. The solid material is a copper-nickel alloy (Monel 400), which has high strength, ductility and corrosion resistance in various media, including sea water, hydrofluoric acid, sulfuric acid and alkalis. Such materials are used in ship manufacturing, chemical industry, etc.

2. Problem statement and the main points of the numerical technique

The impact of a cavitation bubble at its collapse in liquid is considered. The liquid is water (the density $\rho_L = 1000$ kg / m$^3$), the solid material is Monel 400 (a copper-nickel alloy with the Young modulus $E = 173$ GPa, the Poisson ratio $\nu = 0.3$, the density $\rho = 7850$ kg / m$^3$, the yield strength...
\( Y_0 = 250 \text{ MPa} \), which is used for corrosion protection. The bubble collapses due to a large difference between the initial pressure in its cavity (0.023 bar, which corresponds to the saturated water vapor at room temperature) and the pressure in the surrounding liquid (4 bar). The bubble touches the solid whose dimensions are much larger than those of the bubble so that the solid surface is considered plane. At the beginning of collapse the bubble and the liquid are at rest, the bubble is spheroidal. The contour of the bubble in its axial cross-section is an ellipse with the semi-axes \( a \) and \( b \), the former being orthogonal to the solid surface. The initial non-sphericity of the bubble is characterized by a parameter \( \varepsilon = 1 - e \) where \( e = a / b \). The influence of the small initial bubble non-sphericity in the range \(-0.02 \leq \varepsilon \leq 0.02\) is studied. At that, \( \varepsilon < 0 \), \( \varepsilon > 0 \), and \( \varepsilon = 0 \) respectively correspond to the slightly elongated, slightly flattened, and purely spherical bubbles.

The study is conducted with the help of a numerical technique, in which the process of computing the impact of a bubble on a solid is divided into three stages, according to the features of the impact process (figure 1).

![Figure 1. Three stages of the impact of a bubble on a solid according to which the numerical technique of the present work is divided into three stages: I – formation of a jet hitting the solid surface (a), II – the impact of the jet on the solid surface resulting in a surface pressure pulse (b), III – the solid surface layer response to the pressure pulse arising on the solid surface (c).](image)

The first stage is devoted to computing the relatively long-lasting bubble collapse until the moment of contact of the end of the jet arising on the bubble surface with the solid surface, i.e. until the beginning of impact on the solid surface. The main results of the first stage are the liquid jet characteristics at the beginning of its impact on the solid, namely, the jet velocity, the jet pressure, the shape of the jet end, which are required for the next stage. At the first stage the effects of the liquid viscosity and compressibility, the inhomogeneity of the vapor in the bubble, the solid surface layer dynamics are not taken into account due to their smallness. The computation is carried out by the numerical technique [4] based on the boundary element method.

The second stage is devoted to computing the spatial and temporal characteristics of the pressure pulse occurring on the solid surface. At this stage, shock waves appear in the jet, the jet surface is strongly deformed. At the same time, in the considered range of the small initial bubble non-sphericity most intensive jet impact on the solid surface is realized in the region of their contact the radius of which is significantly less than the transverse dimension of the bubble. Because of this, the influence of the liquid outside the jet is not taken into account, so that an impact of a cylindrical liquid jet, surrounded by a vapor, with the hemispherical end on a plane rigid solid surface is actually considered at the second stage. The radius of the hemispherical end is taken equal to the radius of the end of the bubble jet. The backward effect of the solid dynamics on the liquid and vapor dynamics is here, as in the previous stage, not taken into account. The computation is carried out by the numerical technique [5] based on the CIP-CUP method on dynamically adaptive grids without explicit separation of the gas-liquid interface [6].

At the third stage, the dynamics of the solid surface layer is determined. To this end, the solid surface load is calculated by approximating the spatial and temporal characteristics of the pressure computed earlier at the second stage. The solid is modeled by an elastic-plastic isotropic semi-space,
in which the deformations and the displacements are assumed small. The Mises yield condition is used in the plastic zones. The solution is found numerically by the numerical technique [7] based on the classic Godunov method [8]. In that technique the non-reflective conditions are posed on the artificial boundaries of the computational domain.

3. Characteristics of the jet at the beginning of its impact
Due to the presence of a solid, the bubble initially close to the spherical collapses strongly non-spherically (figure 1a). In particular, a high-speed liquid jet directed to the solid arises on the bubble surface.

![Figure 2](image-url) Contours of the bubbles in their axial cross-sections at the beginning of impact on the solid surface, the initial bubble non-sphericity $\varepsilon = 0.02$ (a), $\varepsilon = 0$ (b), and $\varepsilon = -0.02$ (c)

Figure 2 illustrates the shape of a bubble at the beginning of its impact on the solid surface in the cases of the bubble initially slightly flattened ($\varepsilon = 0.02$), purely spherical ($\varepsilon = 0$) and slightly elongated ($\varepsilon = -0.02$). One can see that the jet markedly thickens as the non-sphericity of the bubble at the beginning of its collapse varies from 0.02 to -0.02. In particular, the radius of its end curvature $R$ increases from 15 $\mu$m to 116 $\mu$m, the jet end velocity $V$, with which it hits the solid, decreases from 270 m/s to 220 m/s, and the jet end pressure increases from 4.4 bar to 17 bar. The jet end pressure is determined by the vapor pressure in the bubble cavity so that its increase with decreasing $\varepsilon$ is due to the decrease in the volume of the bubble. It should be noted that because of the relatively rapid collapse of the bubble the vapor in its cavity is modeled as a perfect gas.

4. Characteristics of the pressure pulses on the solid surface
As mentioned in the previous section, for all values of the initial bubble non-sphericity $\varepsilon$ the action of the bubble on the solid is simulated in the present technique as impact of a jet with the hemispherical end on a rigid surface. Upon the impact, a shock wave forms in the jet, propagating away from the solid. At the beginning of the impact the pressure behind the front of this wave is approximately equal to the water hammer pressure $p_{wh} = \rho_l V D$ where $D$ is the shock wave velocity (for water $D \approx C_L + 2 V$).

In the initial phase of the impact the shock wave remains attached to the solid surface by its edge on the periphery of the circular rapidly-expanding jet-solid contact area. The jet-solid contact area expansion velocity rapidly decreases so that at $t = \tau_j = RV / (2 D^2)$ (hereinafter, the time $t$ is measured from the beginning of the jet impact) the shock wave edge leaves the solid surface and goes away from it along the jet boundary. At this moment the jet begins to spread radially on the body surface. The radius of the jet-solid contact area at $t = \tau_j$ is $R_j = RV / D$. The mentioned features of the jet impact in the numerical simulations are consistent with those known in the literature [9].

Figure 3 illustrates the spatial and temporal characteristics of the pressure pulses that occur on the solid surface at impact by the jets arising in the bubbles with an initial bubble non-sphericity in the considered range $-0.02 \leq \varepsilon \leq 0.02$. In this figure, the pressure is referred to the water hammer pressure $p_{wh}$, the time $t$ and the radial coordinate $r$ to the time moment $\tau_j$ and the radius $R_j$, respectively, at which the jet begins to spread on the solid surface. One can see that all three solid lines 1 and all three solid lines 2 are in fact graphically coincident. Two solid lines 3 and two solid lines 4 corresponding to the initially non-spherical bubbles are different from those corresponding to the spherical ones by no more than 5% and 10%, respectively. The difference between three solid curves 5
Figure 3. (a) – the time-dependences of the pressure (lines 1-5) at 5 points (shown in (b)) of its radial profiles on the solid surface, corresponding to the 5 values of \(r/R_{\text{max}}(t)\) where \(R_{\text{max}}(t)\) is the radius at which the solid surface pressure at a moment \(t\) is maximum; (b) - the radial solid surface pressure profiles at two moments \(t_1, t_2\) shown in (a) by the vertical lines. In (a): solid lines 1-4 in the interval \(0 \leq t/\tau_j \leq 6\) are the numerical simulation results with \(\varepsilon = -0.02, 0\) and \(0.02\), dashed lines 1-5 are the approximations of the numerical simulation results. In (b): solid lines (their maxima are marked by crosses) are the numerical simulation results with \(\varepsilon = 0.02\), dotted lines are the approximations of the numerical simulation results:

is much larger (to avoid encumbering they are not given in figure 3). But the large difference takes place in only a very small spatial vicinity of the maximum pressure. The smallness of this vicinity allows one to assume that the influence of the large difference on the solid dynamics is small. Taking this into account, we conclude that in terms of dimensionless variables of figure 3 the difference of the solid surface pressure pulse characteristics in the spherical and non-spherical cases is not significant. This allows us to approximate all the numerical simulation results in the range \(-0.02 \leq \varepsilon \leq 0.02\) by the dotted curves of figure 3. Such approximations are utilized in the next stage.

5. Dynamics of the solid surface layer

As mentioned in the previous section, the solid surface layer dynamics is studied with using dimensionless variables obtained with the pressure \(p_{wh}\), the time moment \(\tau_j\), and the radius \(R_j\). In such variables, the spatial and temporal characteristics of the solid surface pressure pulses used in the numerical simulations are the same (in figure 3 they are presented by the dotted lines) for any initial bubble non-sphericity \(\varepsilon\) in the range under consideration. However, the dimensionless plasticity value defined by the ratio \(Y_0/p_{wh}\) becomes variable due to the dependence of \(p_{wh}\) on the velocity \(V\), which, in its turn, depends on \(\varepsilon\). For example, \(p_{wh} \approx 5.6, 4.7,\) and \(4.2\) kbar for \(\varepsilon = 0.02, 0,\) and \(-0.02\), respectively. Note that at one-dimensional impact with these pressures the yield strength of the considered alloy \((Y_0 = 250\) MPa) gets exceeded only when \(p_{wh} = 5.6\) and \(4.7\) kbar.

Numerical experiments show that the dynamic processes realizing in the copper-nickel alloy solid surface layer at impact on the surface by the jets arising in a bubble with an initial non-sphericity in the range \(-0.02 \leq \varepsilon \leq 0.02\) have the same qualitative features. Figure 4 illustrates the change of the stress intensity in the solid surface layer in a small vicinity of the point of impact in the case of the initial bubble non-sphericity \(\varepsilon = 0.02\). With decreasing \(\varepsilon\), the region in which the stress intensity is equal to the yield strength, becomes, in the dimensionless coordinates, smaller, and disappears, in the dimensionless time, faster.
Figure 4. Contours of the stress intensity $\sigma_i$ in the solid in a small vicinity of the jet impact point at the three consecutive time moments $t_1, t_2, t_3$ and the corresponding radial pressure profiles in the case of the initial bubble non-sphericity $\varepsilon = 0.02$. The bold contours and the shaded regions corresponds to $\sigma_i = \sigma_0$. At the moment $t_1 = \tau_j$ the pressure on the periphery of the loaded area is maximum (figure 3).

Quantitative changes in the solid surface layer dynamics with varying the initial bubble non-sphericity in the range $-0.02 \leq \varepsilon \leq 0.02$ is illustrated in figure 5 where the increase in the depth of the micropit on the solid surface during the jet impact is shown for $\varepsilon = -0.02, 0, 0.02$. One can see that for $\varepsilon = 0.02$ ($\varepsilon = -0.02$) the micropit is deeper (shallower) than for $\varepsilon = 0$ by 37.5% (17.5%), though in the former case the solid surface pressure pulse amplitude is larger (smaller) than in the latter case by only 17% (9%). The difference in the pressure pulse amplitude is determined by the difference in the corresponding values of the water hammer pressure $p_{wh}$.

Figure 5. Time-dependences of the axial coordinates of the center of the circular micropit arising on the solid surface at the jet impact, the initial bubble non-sphericity $\varepsilon = 0.02$ (curve 1), 0 (curve 2), and $-0.02$ (curve 3).

6. Conclusion
A numerical study of the influence of small spheroidal non-sphericity of a cavitation bubble at the beginning of its collapse on impact of this bubble on the body of copper-nickel alloy has been performed. The velocity, the pressure and the shape of the jet hitting the solid are determined by the boundary element method. The spatial and temporal characteristics of the solid surface pressure pulses are evaluated by the CIP-CUP method on dynamically adaptive grids without explicitly separating the gas-liquid interface. The dynamics of the solid surface layer in the region of impact is calculated by the Godunov method. The study has been performed in dimensionless variables obtained with using the water hammer pressure, the time moment and the jet-solid contact area radius at which the jet begins to spread on the solid surface. It has been shown that in those dimensionless variables the spatial and temporal characteristics of the solid surface pressure pulses are slightly dependent on the initial bubble non-sphericity. The initial bubble non-sphericity also slightly influence the main features of the dynamic processes inside the solid (for example, the shape of the plastic zone). However, its effect on the quantitative characteristics of those processes can be significant. In particular, the
difference in the depth of the micropits arising on the solid surface due to the jet impact can differ by more than 35%.

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