Tension-controlled switch between collective actuations in active solids

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The recent finding of collective actuation in active solids, namely solids embedded with active units, opens the path towards multifunctional materials with genuine autonomy. In such systems, collective dynamics emerge spontaneously and little is known about the way to control or drive them. Here, we combine the experimental study of centimetric model active solids, the numerical study of an agent based model and theoretical arguments to reveal how mechanical tension can serve as a general mechanism for switching between different collective actuation regimes in active solids. We further show the existence of a hysteresis when varying back and forth mechanical tension, highlighting the non-trivial selectivity of collective actuations.

One important aspect of metamaterial design is multifunctionality — the ability of a system to perform several different tasks. Multifunctional materials are usually actuated from an external source of work, which allows for a good control of the targeted functions. Active matter, the individual components of which perform work, can also be tamed to actuate built-in functions autonomously. It is therefore a promising alternative framework to create multifunctional materials with bona fide autonomy. Active solids hold promise for such multifunctionality, especially when they exhibit emergent collective dynamics which selectively actuate only few deformation modes.

At first, autonomous actuation was demonstrated in the case of active elastic networks comprising zero modes mechanisms. More recently, it was shown both experimentally and numerically that a mechanically stable elastic structure can also exhibit collective actuation (CA), when a nonlinear elasto-active feedback is present. Typical examples of such CA are illustrated on Fig. 1. When a given node of the elastic structure is pinned both in translation and rotation, the structure alternatively rotates clockwise and counterclockwise around this node (Fig 1a and SM Movie 1). When the structure is pinned at its edge, the nodes perform a local but synchronized oscillation that spontaneously breaks chiral symmetry (Fig 1b and SM Movie 2). In the following, we shall call GAR (Global Alternating Rotation), resp. SLO (Synchronized Local Oscillations), these two regimes. Quite remarkably, it was shown that the CA obeys a non trivial selection mechanism in the sense that it does not necessarily correspond to a condensation on the lowest energy modes of the elastic structure. In the above example the two CA regimes are obtained by imposing different pinning conditions. This is not always a convenient way of monitoring the actuation of a structure and, certainly, there are circumstances under which the use of a continuous control parameter is desirable.

In this letter, we demonstrate that mechanical tension is a suitable mechanism for controlling CA in active solids. Tension governs the sound of stringed musical instruments, the growth and response of biological systems, the stability of civil engineering work, and the stability of active solids (Fig. 1e,f and SM Movies 3-4), dissect the underlying mechanism and extend our findings to more...
The dynamics are quantified by computing the mean of the dynamical matrix, \( \mathbf{D} \). At low tension, the dynamics condensate on the translational modes \( \mathbf{u} \), whereas at large tension, they condensate on the rotational modes \( \mathbf{u} \). This nonlinear elasto-active feedback between deformations and polarities is controlled by the ratio \( \pi = l_e/l_a \), with \( l_e = F_0/k \), the typical elastic deformation caused by the active force and \( l_a \), the alignment length over which \( \mathbf{n} \) aligns towards \( \mathbf{u} \).

We start by demonstrating experimentally the possibility of controlling the switch between the two CA regimes described above and so far obtained with different pinning conditions. To do so, we design an active elastic structure, which consists of \( N = 6 \) active particles at the vertices of an inner rigid hexagon, each connected radially to the vertices of an outer pinned hexagon via soft springs of stiffness \( k = 1 \text{ N/m} \) (Fig. 2a), which we call active \( \text{Gerris} \). We control the tension in the springs by elongating homogeneously the radial springs of a factor \( \alpha \). As illustrated in Figs. 1(d-f), the dynamics of the active \( \text{Gerris} \) switches from the GAR regime (Fig. 1g and SI Movie 3) at low tension, to the SLO regime (Fig. 1f and SI Movie 4) at large tension.

These dynamics are best described when decomposed on the elastic modes of the structure, that are the eigen-vectors, \( |\varphi_k\rangle \), associated with the eigenvalues, \( \omega_k^2 \) of the dynamical matrix, \( \mathbf{M} \). More specifically, we shall represent the dynamics in the space spanned by the amplitude of the polarity field projected on the three modes of interest. By convention, the vertical axis represents the normalized projection on the rotation mode \( a_R = (R|\mathbf{n})/\sqrt{N} \), whereas the equatorial plane represents the normalized projections on the two translational like modes \( a_{T_{x,y}} = (T_{x,y}|\mathbf{n})/\sqrt{N} \) (Fig 2c). From the polarity field normalization, the projections are confined inside the 3-sphere of radius \( \sqrt{N} \), normalized to 1. In the GAR regime, obtained from the central pinning condition, the dynamics alternatively condensate on the clockwise and counterclockwise rotation (the poles of the sphere), separated by fast reversal motion (Fig 1a). In the SLO regime, obtained from the edge pinning condition, the dynamics condensate on the translational modes spanning the equator of the sphere (Fig. 1b). Figs. 1e and f convincingly demonstrate that the active \( \text{Gerris} \) explores the same dynamics under the control of tension. The dynamics are quantified by computing the mean square projection of the polarity field on each mode:

\[
\lambda_k = \langle a_k^2 \rangle = \frac{1}{T} \int_0^T \left( \frac{\varphi_k|\mathbf{n}(t)}{\sqrt{N}} \right)^2 \, dt
\]

The active \( \text{Gerris} \) switch is illustrated by the abrupt drop of this condensation fraction on the rotation mode \( \lambda_R \) as tension increases (Fig. 1d).

To investigate the origin of the switch in the active \( \text{Gerris} \), we now proceed to numerical simulations using an agent-based model, which was shown to faithfully describe the dynamics of active elastic structures [20]:

\[
\mathbf{u}_i = \frac{\pi}{\lambda} \mathbf{n}_i + F_{\text{el}}^i,
\]

\[
\mathbf{n}_i = (\mathbf{n}_i \times \mathbf{u}_i) \times \mathbf{n}_i + 2D\xi \mathbf{n}_i^\perp
\]

where \( \mathbf{u}_i \) is the displacement of node \( i \) with respect to its reference position, and \( \xi \) are i.i.d gaussian variables with zero mean and correlations \( \langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t-t') \). The elasto-active feedback, \( \pi \) controls the emergence of CA. We set it to \( \pi = 2.0 \), a value consistent with the experiments, and investigate the effect of tension.

The \( \text{Gerris} \) has six nodes that are connected by a structure, which can safely be considered as rigid (Fig. 2a and 27). It is thus described by three degrees of freedom, the spatial coordinates of its barycenter and its angular orientation, the dynamical equations of which are provided in [27]. The three associated normal modes are two degenerated translation modes \( T_{x,y} \) and one rotation mode \( R \), which are illustrated on Fig. 2b, together with their energies as a function of the imposed tension. Both the rotation and translation energies increase with tension, but the energetic ordering of the modes is preserved, and their geometries are unaffected. The three modes end up degenerated at infinite tension.

We first simulate the noiseless, \( D = 0 \), active \( \text{Gerris} \) equations in the harmonic approximation \( F_{\text{el}}^i = -M_{ij}\mathbf{u}_j \), annealing back and forth between small and large tensions. We find two linearly stable actuation branches, which we respectively denote the \( TT \) and \( RT \) regimes (Fig. 2c, circle markers). The \( TT \) regime is a strict condensation of the polarity field on the equator (Fig. 2d), with \( \lambda_R = 0 \), corresponding to a SLO of the \( \text{Gerris} \). This regime exactly maps to that of a single particle trapped in a parabolic potential [20][27][28]. The \( RT \) regimes consist in a condensation of the polarity field on a plane, defined by the rotation vector \( \mathbf{R} \) and one of the six translational vector \( \mathbf{T} \), pointing toward one of the hexagon’s main axis, in the equatorial plane (Fig. 2d). They correspond to a GAR of the \( \text{Gerris} \). The six possible orientations of this plane, define six equivalent attractors, one of which is selected, spontaneously breaking the 6-fold symmetry of the system [27]. Depending on the tension, we actually report different \( RT \) dynamics, separated by hysteretic transitions, which differ in the precise trajectory of the alternating rotation. These RT regimes and the transitions among them are well captured by the dynamics of a single particle trapped in an
elliptic harmonic potentials [27, 29]. Within the linear level of description, there is however no switch between the coexisting RT and TT regimes.

Including the geometrical non-linearities of the elastic forces, allows for a better description of the experimental observations (Fig. 2c, square markers). Simulating the full expression for central force springs, we find that the TT regime is unaffected, while the stability range of the RT regimes are shifted toward smaller tensions. More significantly, the RT regime destabilizes towards the TT regime for large enough tension. Therefore, geometrical non-linearities allow for a switch between the RT and the TT regimes as tension increases.

The TT regime persists for all values of the tension and coexists with the RT regime. This raises the issue of the relative stability of the two attractors. We address it by adding a small noise, $D = 10^{-2}$, consistent with the experimental values. Starting from the RT regime, the system first remains close to the initial RT attractor, then visits the six equivalent RT attractors, before it eventually undergoes a destabilization event, which drives it into the TT regime at long times (Fig. 3a). The smaller the tension, the longer it takes for this destabilization event to take place. We evaluate the metastability of the RT regime, by performing 80 independent simulations runs with random initial condition, for each value of the tension. At small tension, the probability to end up in a RT regime at $t = 10000$, $P_{RT}$, is close to one and slowly decreases with increasing tension. This is due to both the increasing size of the attraction basin of the TT regime and the decreasing lifetime of the metastable RT regime. For tensions $\alpha \geq 1.2$, $P_{RT}$ vanishes abruptly: all initial conditions end up in the TT regime at long time. Altogether the active Gerris establishes the proof of concept for the experimental control of a switch between two CA regimes using tension.

The Gerris structure, which results from several experimental compromises, is however rather artificial. More specifically the fact that it has only three degrees of freedom raises the question of the possible generalization of our results to larger structures. Furthermore the fact that the two branches of eigenfrequencies, corresponding to the TT and RT modes only meet at infinite tension is likely to be specific.

We now show theoretically that the tension controlled switch is generically expected even in the harmonic approximation. Consider an arbitrary lattice undergoing
homogeneous dilation of factor $\alpha \in [1, +\infty]$, the dynamical matrix of which reads \[ M(\alpha) = \frac{1}{\alpha} M_0 + \left(1 - \frac{1}{\alpha}\right) M_1. \] (3)

$M_0$ is the dynamical matrix of the structure at zero tension, and $M_1$ reads:

\[ M_1 = \begin{pmatrix} M_{1xx} & 0 \\ 0 & M_{1yy} \end{pmatrix}. \] (4)

where $M_{1xx} = M_{1yy}$ is the Laplacian matrix of the structure network $M_{i,j} = Z(i)$, $M_{i,j} = -1$ if $i$ and $j$ are neighbors and zero otherwise. Since $M_1$ decouples the $x$ and $y$ directions, its eigenvectors $\varphi_n$ come in degenerated pairs with identical form, respectively polarized along $x$ and $y$. In particular, as a result of a discrete nodal domain theorem \[27][30][32], the lowest energy modes of $M_1$ have the geometry of translational modes. Increasing the tension, the spectral properties of $M_1$ progressively dictate that of the elastic structure, thereby favoring the emergence of two low energy modes, with geometries akin to translation. These are the perfect conditions for selecting the SLO regime at large tension \[20].

This is why any elastic structure, which, in the absence of tension, exhibits some form of CA, different from the condensation on modes akin to translation, will eventually switch to the SLO regime, when tension is increased. This argument is strictly valid in the case of a homogeneous dilation, but one expect it to persist as a design principle for CA switch in elastic structures which do not dilate homogeneously, as long as tension is evenly distributed. In the case of the Gerris, the specificity of the spectrum prevents the application of the recipe at finite tension; we however saw that the nonlinearities can enforce it at tensions, that can be reached experimentally.

We confirm this design principle by considering a large honeycomb lattice, composed of $N = 180$ nodes, pinned at its hexagonal edges (Fig. 4a). Under small tension this lattice has a rotation mode $|R\rangle$ that lies at the bottom of its vibrational spectrum (Fig. 4b). As tension increases, the energies of both the degenerated translational modes and the rotation mode increase, but at different pace, and eventually cross each other for $\alpha = \alpha^* \simeq 1.1$, as expected from Eq. \[3\]. When simulating the dynamics of the active honeycomb, within the harmonic approximation, with $\pi = 0.055$, we do confirm the presence of a tension controlled switch between two linearly stable actuation regimes, SLO and GAR (Fig. 4c). Note that the condensation of the dynamics taking place on modes that are not fully delocalized, the condensation fraction are here normalized by the participation ratio of the modes: $\tilde{\lambda}_k = \lambda_k/Q_k$, with $Q_k = (\sum |\varphi_k|^2)^2/N$ \[27][33]. The SLO is a TT regime, very similar to the one discussed above (Fig. 4d and SI Movie 5), except for additional fluctuations taking place outside of the equatorial plane. Indeed, the translational modes being not fully delocalized, there is room for a spatial coexistence of a collectively actuated region at the center of the system with a frozen/disordered one close to the boundary. The GAR regimes, with strictly positive $\lambda_R$, exhibit richer dynamics than in the case of the Gerris: for small enough tension, the GAR regimes are aperiodic, because of the many low energy modes, which couple to the rotational and translational modes \[27\]. At large enough tension, one recovers the $RT_1$ regime, condensed on a RT plane in mode space (Fig. 4d and SI Movie 6), modulo some fluctuations of the same origin than in the TT regime. Annealing from small to large tension, the RT regime switches to the TT one for a tension $\alpha > \alpha^*$ (Fig. 4c). Additionally, performing the backward annealing, the TT branch becomes unstable for a tension $\alpha < \alpha^*$. In the absence of geometrical
non-linearities, the observed hysteretic switch must be attributed to the non-trivial selectivity of CA. As shown in [20], the selection of the modes ruling the CA not only depends on the energy level, but also on their geometries. More precisely, CA preferably takes places on a pair of modes that are maximally extended and locally orthogonal. As demonstrated by the open symbols and dashed lines in Fig. 6c, the presence of geometrical non-linearities do not alter the above picture.

Altogether, having unveiled a new CA regime arising in active solids with a low energy rotation mode (GAR), we demonstrate that mechanical tension is a robust control parameter to switch to a regime dominated by a pair of degenerated translational modes (SLO). On the metamaterial science side, our work opens the path towards the study of structures with more low energy modes, and the possible emergence and competition of several actuation branches. In the realm of bio-physics, it suggest the possible emergence and competition of several actuation modes. More precisely, CA preferably takes places on a pair of modes that are maximally extended and locally orthogonal. As shown in [20], the selection of the modes ruling the CA not only depends on the energy level, but also on their geometries.

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