Searching for New Physics using Precision Standard Model Measurements

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ABSTRACT

New physics interactions beyond the Standard Model can make themselves known as small corrections to Standard Model reactions. There is a diverse array of proposals for new physics, and so any parametrization of those effects must be as general and all-inclusive as possible. This can be accomplished by the use of Standard Model Effective Field Theory (SMEFT). In this article, part of the celebration of 50 years of the Standard Model of particle physics, I describe how SMEFT has been applied to search for new physics in fermion-fermion scattering and precision electroweak analysis and how it will be applied in the precision study of the Higgs boson.

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1 Introduction

As has been well documented at this symposium, the Standard Model has been remarkably successful at explaining a wide range of experimental measurements. From low-energy observables in weak interaction decays to multiparticle production at the highest energies of the LHC, the Standard Model seems to give a complete description of the reactions of elementary particles.

Still, there are good reasons to believe that the Standard Model is an incomplete description of nature, and that additional fundamental interactions are waiting to be discovered.

There are many methods to search for these new interactions. One way is to search for new elementary particles that can be produced at high energies. Another way is to search in low-energy processes for specific interactions, for example, flavor- or CP-violating, that are forbidden in the Standard Model. A third way is to use our ability to perform high-precision calculations in the electroweak sectors of the Standard Model to search for small deviations from those predictions. This last method can be sensitive to new interactions well above the accelerator center of mass energy. It can also be remarkably robust, sensitive to a very wide variety of models.

To understand the constraints that come from precision Standard Model tests, it is useful to have a formalism that can describe as large a range of new physics models as possible. This is supplied by Standard Model Effective Field Theory (SMEFT). In this article, I will review some of the applications of SMEFT to the interpretation of precision measurements.

Recently, Brivio and Trott have given a comprehensive review of SMEFT [1]. In this article, I will have relatively little to say about the formalism of SMEFT, its renormalization, and the computation of loop corrections in this framework. Instead, I will emphasize its practical applications to the analysis of electroweak processes.

An outline of this paper is as follows: In Section 2, I will discuss in more detail the need for new interactions beyond the Standard Model. In Section 3, I will review the principles of SMEFT that we will need for our applications. In Section 4, I will describe the use of SMEFT to describe possible quark and lepton compositeness. In Section 5, I will describe the application of SMEFT to the analysis of corrections to precision $Z$ physics measurements. In Section 6, I will pause to briefly review the possible effects of new physics on Higgs boson couplings. In Section 7, I will discuss the measurement of these couplings through the application of SMEFT to the analysis of Higgs boson processes at $e^+e^-$ colliders. In Section 8, I will discuss the prospects for precision Higgs boson measurements at next-generation $e^+e^-$ colliders. Section 8 will give some conclusions.
To begin, I should discuss at greater depth the idea that the Standard Model is incomplete. Though many anomalies are discussed, there is at this time no convincing evidence of a deviation from the predictions of the Standard Model in elementary particle reactions. The deficiencies of the Standard Model are conceptual. Of these, the clearest difficulties are the facts that the Standard Model has no explanation for the dark matter of the universe, or for the observed preponderance of matter over antimatter. However, the Standard Model presents many more challenges to our understanding. For example, precisely because the Higgs-fermion Yukawa couplings are renormalizable couplings, the quark and lepton masses and mixings are inputs to the Standard Model and cannot be explained within that framework.

Most importantly for me, the Standard Model is incapable of explaining the phase transition to an ordered vacuum state that breaks the \( SU(2) \times U(1) \) gauge symmetry. The full explanation for this phase transition within the Standard Model is

1. The most general renormalizable potential for the Higgs field is

\[
V = \mu^2 |\Phi|^2 + \lambda |\Phi|^4. \tag{1}
\]

2. The parameter \( \mu^2 \) satisfies \( \mu^2 < 0 \).

The value of \( \mu^2 \) receives large (divergent) additive radiative corrections with both signs. So it is very difficult to give a coherent explanation for the sign of \( \mu^2 \). Sophisticated theorists call this the “gauge hierarchy problem”. I prefer to state the problem as the fact that we have no idea where the value of \( \mu^2 \) comes from.

It is not like this elsewhere in physics. Condensed matter physics has many examples of order-disorder phase transitions—in magnets, superconductors and superfluids, binary alloys, liquid crystals, and other systems. In all cases, there is a nontrivial and fascinating explanation for the ordering in the ground state. Superconductivity provides an especially interesting example. In 1950, Landau and Ginzburg put forward a phenomenological theory of superconductivity that is the model for the Higgs sector of the Standard Model [2]. This is an extremely powerful and successful theory. It explains the thermodynamics of the phase transition, the presence of a critical magnetic field, the distinction between Type I and Type II superconductors and the existence of the Abrikosov flux state. What is does not do is give a fundamental explanation for why superconductivity occurs. That insight came only 7 years later, with the work of Bardeen, Cooper, and Schrieffer [3].

In the theory of the Higgs field vacuum, we are still at the Landau-Ginzburg stage. To go beyond this stage, we need a theory with new particles and interactions.
beyond those of the Standard Model. There is no rigorous argument that the universe contains this extension, but this logic offers us a remarkable opportunity to discover new, hidden laws of nature. We should not ignore it.

3 Principles of Standard Model Effective Field Theory

If there are new particles and interactions at high energy, how are these reflected in the observables of the electroweak interactions? We would like an answer to this question that is systematic and that uses as few assumptions as possible about the scenario for physics beyond the Standard Model.

One of the unexpected results of the search for physics beyond the Standard Model is that no new particles have yet been discovered in the energy range of the LHC. Although the possibilities for lighter new particles have not yet been exhausted, this suggests that we assume that new particles have masses above a mass scale $M$, where $M \gg m_h$. In this case, we can imagine integrating out the new fields. This will leave behind a local Lagrangian quantum field theory with the gauge symmetries of the Standard Model and built from Standard Model fields only. The integration out of heavy fields may produce terms in this Lagrangian with dimension higher than 4, corresponding to non-renormalizable interactions. Still, it is possible to compute with this Lagrangian in a straightforward way, as long as we treat all of the coefficients of renormalizable and nonrenormalizable operators as free parameters to be determined from experiment [1]. The foundations of this approach were set out in classic papers of Ken Wilson [4] and Steven Weinberg [5]. In the early 1980’s Gasser and Leutwyler demonstrated the power of this approach by working out in detail the application to the low-energy scattering of $\pi$ and $K$ mesons [6].

Without any further assumptions except that $M$ is sufficiently large, we can now construct a general theory of new physics effects on Standard Model precision calculations. Again, we consider the most general Lagrangian with $SU(3) \times SU(2) \times U(1)$ gauge invariance built from Standard Model fields. Consider first the part of the Lagrangian containing operators of dimension 4 and below. In fact, this part is nothing more than the Standard Model itself. The Standard Model is in fact the most general renormalizable Lagrangian consistent with $SU(3) \times SU(2) \times U(1)$ gauge invariance [7,8]. When heavy particles are integrated out, the couplings in the renormalizable part of the Lagrangian are shifted. However, these shifts are unobservable, since in any event the Standard Model couplings are fit to experiment.

Integrating out heavy particles will also generate new terms proportional to higher dimension operators, of dimension 6, 8, ... . Operators of odd dimension lead to baryon- or lepton-number; for example, these give neutrino mass terms. I will ignore these in the rest of this article. Higher-dimension operators come with dimensionful
coefficients, whose size is set by the mass scale \( M \) that is integrated out. Then, the form of the effective Lagrangian is

\[
L = L_{SM} + \sum_i \frac{\tau_i}{M^2} \mathcal{O}_i + \sum_j \frac{\bar{d}_j}{M^4} \mathcal{O}_j + \cdots ,
\]

(2)

where \( \tau_i, \bar{d}_j \), etc., are dimensionless coefficients. In processes at center of mass energy \( \sqrt{s} \), the higher-dimension operators lead to effects of order \( s/M^2 \), \((s/M^2)^2\), and so on. For \( M \gg \sqrt{s} \), this is a systematic approximation scheme. The Lagrangian (2) defines the Standard Model Effective Field Theory (SMEFT).

To guide intuition, consider the case of \( M = 1 \) TeV and \( \tau_i, \bar{d}_j \) of order 1. Under these assumptions, the dimension-6 operators give few-percent corrections to the Standard Model, and the dimension-8 operators give corrections of order \( 10^{-4} \). Then, in practical applications, we can ignore the operators of dimension 8 and higher and concentrate on the effects of operators of dimension 6. The most general models of new physics, subject to the requirement of large \( M \), are described by a finite set of parameters \( \{\tau_i\} \). These assumptions can be excessively strong in some models, but it is difficult for the \( \tau_i \) to take much larger values without violating unitarity [9].

What if \( M \) is not much larger than \( m_h \)? In that case, the approximation that I have described cannot be completely systematic, but there are examples in which it is qualitatively, and even quantitatively, correct. We will see one example in Section 5.

In analyses that involve a large number of higher-dimension operators, especially when the physical significance of the scale \( M \) is not clarified by relation to a model, it is useful to set the scale of the operator coefficients using the Higgs field vacuum expectation value \( v \). Then I will write the SMEFT effective Lagrangian

\[
L = L_{SM} + \sum_i \frac{c_i}{v^2} \mathcal{O}_i + \sum_j \frac{d_j}{v^4} \mathcal{O}_j + \cdots .
\]

(3)

In the intuitive picture suggested above, the \( c_i \) will be small parameters, of order 1\%, the \( d_j \) of order \( 10^{-4} \), and so on. I will use this notation in my discussion beginning with Sec. 5.

4 SMEFT description of lepton and quark compositeness

The approximation scheme suggested by SMEFT is powerful, but it has a difficulty. The number of gauge-invariant dimension-6 operators is large, and this number increases rapidly with the number of generations and with the operator dimension. For 1 generation, the number of independent baryon- and lepton-number conserving operators is 59 [13]; for 3 generations, it is 2499 [14]. The SMEFT approximation
scheme is only useful if there is a subset of operators that can be argued to give a complete description of a particular problem. The earliest examples of the use of SMEFT to parametrize new physics are all of this type.

The first example came as the answer to a question posed at Snowmass 1982: In models in which the quarks and leptons are composite, how should one parametrize the size of composite fermion? Up to that time, compositeness was usually considered as modifying pointlike electroweak couplings by the addition of form factors. In a leading-order scattering process whose Standard Model amplitude would be of order $\alpha$, the compositeness effect would be of order $\alpha \cdot s/M^2$, where $s$ is the CM energy and $M$ is the inverse of the bound state size. However, in addition to weak gauge interactions, composite states bound by a new strong interaction would also have contact interactions involving the exchange of the bound constituents. This strong interaction effect would be of order $s/M^2$, with no weak gauge suppression; see Fig. 1. This observation was described in the Snowmass proceedings [15] and, more formally, in an article by Eichten, Lane, and me [16].

Assuming helicity conservation at short distances, to forbid the generation of large masses for the light fermions, this contact interaction is parametrized by dimension-6 current-current operators. We noted that, for the process of Bhabha scattering, there are exactly 3 such operators, so that the process can be described by the effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{2\pi}{\Lambda^2} \left[ \eta_{LL} \bar{e}_L \gamma^\mu e_L \bar{e}_L \gamma_\mu e_L + \eta_{RR} \bar{e}_R \gamma^\mu e_R \bar{e}_R \gamma_\mu e_R + 2\eta_{LR} \bar{e}_L \gamma^\mu e_L \bar{e}_R \gamma_\mu e_R \right],$$

where $\Lambda$ is interpreted as the scale of compositeness and the operator coefficients $\eta_{IJ}$ are expected to be of order 1. Because the Standard Model amplitude for Bhabha scattering violates parity, the three operator coefficients can be determined independently by fitting to the Bhabha scattering angular distribution. Analysis of the LEP 2 data gives 95% CL limits on the $\Lambda$ parameter ranging from 6 to 16 TeV depending on the choice of nonzero values of the $\eta_{IJ}$ [17]. In the worst case, this corresponds to

Figure 1: Contributions to $e^+e^- \rightarrow \ell \bar{\ell}$ from $s$-channel photon and $Z$ exchange, and from a strong interaction of fermion constituents.
a limit on the size of the electron of

\[ r_e < 3 \times 10^{-18} \text{ cm} \]  \hspace{1cm} (5)

In the case of quark-quark scattering, such a model-independent analysis is not possible. There are 17 possible contact interactions, and these depend on the flavors and chiralities of the interacting species. Typically, limits are quoted under the assumption that one operator, a universal left-handed contact interaction,

\[ \Delta \mathcal{L} = \pm \frac{2\pi}{\Lambda} \sum_{f,f'} \bar{q}_{L,f} \gamma^\mu q_{L,f} \bar{q}_{L,f'} \gamma_\mu q_{L,f'} \]  \hspace{1cm} (6)

is added to the Standard Model. However, these model-dependent limits are very impressive. Using data at 13 TeV, the ATLAS and CMS experiments have set 95\% CL limits on \( \Lambda \) of 13 and 22 TeV for the two possible signs of the contact term \([18,19]\).

5 SMEFT description of corrections to precision electroweak observables

Another situation in which analysis with a reduced set of dimension-6 operators makes sense comes in the study of new physics corrections to precision electroweak interactions. The most general effects of new physics bring in a large number of dimension-6 operators. However, there is a specific interesting circumstance which is described by adding only two new operators to the Standard Model.

In many models of new physics, the new particles couple directly to the Higgs sector but with very small couplings to light quarks and leptons. This applies, for example, to models of an extended or composite Higgs sector, and models with new quarks, leptons, or vectorlike fermions. In the limit in which these light quark couplings can be ignored, the new physics corrections to electroweak reactions at low energies and at the \( Z \) pole come only from vacuum polarization diagrams, as in Fig. 2. Lynn, Stuart, and I described this limit by labelling these diagrams as “oblique corrections” \([20]\). This terminology calls attention to a simple but important class of new physics effects that are amenable to general analysis.
In fact, that analysis turns out to be very straightforward. As Takeuchi and I pointed out [21], it does not require any sophisticated operator counting, but only a glance at the Taylor expansion of the vacuum polarization amplitudes in powers of $q^2$. There are four relevant vacuum polarization amplitudes; define these according to their $SU(2)$ quantum numbers as shown in Fig. 3, in the figure, $s_w = \sin \theta_w$, $c_w = \cos \theta_w$. I have omitted terms proportional to $q^\mu q^\nu$ that, in any event, give zero when contracted with light fermion lines. The subscripts 1 and 3 refer to currents of the gauge $SU(2)$ symmetry and $Q$ refers to the electric charge current.

If we take $M$ to be the scale of new particle masses, the four amplitudes have Taylor expansions in $q^2/M^2$ of the form

\begin{align}
\Pi_{QQ} &= A q^2 + \cdots \\
\Pi_{3Q} &= B q^2 + \cdots \\
\Pi_{33} &= C + D q^2 + \cdots \\
\Pi_{11} &= E + F q^2 + \cdots . \tag{7}
\end{align}

The zeroth-order terms in the first two lines vanish due to electric current conservation. Of the 6 coefficients, 3 linear combinations are fixed by the renormalization of the 3 basic parameters of the Standard Model $g$, $g'$, and $v$. The remaining 3 linear combinations will be finite in a renormalizable extension of the Standard Model. These are canonically defined as

\begin{align}
S &= 16 \pi \frac{m_Z}{m^2_Z} \left[ \Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2) \right] \\
T &= 4 \pi \frac{s^2_w}{m^2_Z} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right] \\
U &= 16 \pi \frac{m_Z}{m^2_Z} \left[ \Pi_{11}(m_Z^2) - \Pi_{11}(0) - \Pi_{33}(m_Z^2) + \Pi_{33}(0) \right] . \tag{8}
\end{align}

Figure 3: The photon, $Z$, and $W$ vacuum polarizations decomposed in terms of their $SU(2) \times U(1)$ components. The notation on the right-hand side is explained further in the text.
The parameters $S$ and $T$ have appealing physical interpretations. $T$ indicates the correction to the Standard Model relation $m_W = m_Z \cos \theta_w$ that reflects its custodial $SU(2)$ symmetry \cite{22}. $S$ indicates in a dimensionless way the size of the (custodial symmetry-invariant) new physics sector.

The leading oblique corrections to electroweak observables can then be expressed as linear combinations of $S$, $T$, and $U$. It is useful to make reference to the value of $\theta_w$ constructed from the best-measured observables $\alpha$, $G_F$, and $m_Z$,

$$\sin^2 2\theta_0 \equiv \frac{\alpha(m_Z^2)}{\sqrt{2}G_F m_Z^2}.$$  \hfill (9)

Then it is possible to represent the effect of general oblique corrections as deviations from the values of observables predicted by the Standard Model with this standardized value of $\sin^2 \theta_w$. For example,

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c_w^2}{c_w^2 - s_w^2} \left(-\frac{1}{2}S + c_w^2 T + \frac{c_w^2 - s_w^2}{4s_w^2} U \right)$$

$$s_w^2 - s_0^2 = \frac{\alpha}{c_w^2 - s_w^2} \left(\frac{1}{4}S - \frac{s_w^2}{s_w^2} T \right), \hfill (10)$$

where $s_w^2$ is the value of $\sin^2 \theta_w$ extracted from the $Z$ resonance polarization asymmetries. By fitting deviations from the Standard Model predictions to these formulae, we can put constraints on the full set of models to which the assumptions of the oblique approximation apply.

In SMEFT, the parameters $S$ and $T$ are represented by adding two dimension-6 operators to the Standard Model Lagrangian,

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_T}{2v^2} (\Phi^\dagger \slashed{D} \mu \Phi)(\Phi^\dagger \slashed{D} \mu \Phi) + \frac{16s_w c_{WB}}{v^2} \Phi^\dagger t^a \Phi W^a_{\mu\nu} B_{\mu\nu}. \hfill (11)$$

Here, the notation is as in (3), $c_T$ and $c_{WB}$ are dimensionless operator coefficients, $\Phi$ is the Standard Model Higgs doublet field, $W^a_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ field strengths, and

$$\Phi^\dagger \slashed{D} \mu \Phi = \Phi^\dagger D_\mu \Phi - D_\mu \Phi^\dagger \Phi. \hfill (12)$$

I will clarify the relation of this truncated Lagrangian to the full SMEFT Lagrangian in Sec. 7. The relation between the SMEFT coefficients and the $S$ and $T$ parameters is

$$\alpha S = 4s_w^2 (8c_{WB}) \quad \alpha T = c_T. \hfill (13)$$

The parameter $U$ is doubly suppressed, requiring both direct effects of heavy new particles and custodial $SU(2)$ violation. In the SMEFT context, $U$ turns out to be
generated by a dimension-8 operator. When $U$ is included in fits to electroweak data in any event, its value is consistent with 0.

Some guidance about the expected sizes of $S$ and $T$ is given by the expressions for these quantities in specific models. For example, for one new heavy electroweak doublet $(N, E)$,

$$S = \frac{1}{6\pi} \quad T = \frac{|m_N^2 - m_E^2|}{m_Z^2}. \quad (14)$$

Since the top quark and the Higgs boson have only tiny direct couplings to the light generations, we can express the contribution to electroweak observables from these Standard Model particles in the $S$, $T$ framework. For the top quark,

$$S = \frac{1}{6\pi} \log \frac{m_t^2}{m_Z^2} \quad T = -\frac{3}{16\pi s_w c_w} \frac{m_t^2}{m_Z^2}; \quad (15)$$

for the Higgs boson

$$S = \frac{1}{12\pi} \log \frac{m_h^2}{m_Z^2} \quad T = -\frac{3}{16\pi c_w^2} \frac{m_h^2}{m_Z^2}. \quad (16)$$

Although the top quark and Higgs boson masses are by no means much larger than $v$, as would be needed to give formal justification to the SMEFT approximation, these formulae turn out to be a very good representation of the effects of the top quark and the Higgs boson on the precision electroweak fit.

The progress of the $S$, $T$ fit to electroweak observables since the days of the earliest LEP and SLC data indicates clearly the impressive growth of our knowledge. Figure 4 shows the 1991 $S$, $T$ fit from [21], a 2008 fit based on the final results of the $Z$ resonance parameters reported in [23], and a 2014 fit by the Gfitter Collaboration [24]. Note the changes in scale in the three plots. The first plot predicts a top quark mass in the range 120–180 GeV. The second plot uses the by-then measured value of the top quark mass and predicts a value of the Higgs boson mass below 140 GeV. The third plot gives the current status of the electroweak fit, in excellent agreement with the Standard Model in accord with the known values of the top quark and Higgs boson masses.

6 The Higgs boson as a probe of physics beyond the Standard Model

The next logical target for electroweak precision measurement is the Higgs boson. It is widely appreciated that the verification of the Standard Model will not be complete without a detailed study of the properties of the Higgs boson. I feel that it is
Figure 4: ST fits to the precision electroweak data from 1991 [21], 2008 (based on the results of [23]) and 2014 [24]. Note the changes of scale and the changing interpretation as new inputs are added.
less well appreciated that precision measurement of the Higgs boson couplings gives a remarkable opportunity for the discovery of new physics, beyond the capabilities of the LHC experiments. In this section, I will explain my viewpoint on this question.

First of all, the discovery of the Higgs boson at the LHC in 2012 [25, 26] puts us in a new situation with respect to the Standard Model. The complete set of particles predicted by the Standard Model have been discovered, and their masses have been measured accurately. In particular, the mass of the Higgs boson has been determined as [27–29]

\[ m_h = 125.10 \pm 0.14 \text{ GeV}. \]  

With this measurement, all of the parameters of the Standard Model are specified to part-per-mil accuracy. From these parameters, we can predict the properties of the Higgs boson in detail without ambiguity. Any deviation from these predictions would be a signal of new interactions beyond the Standard Model.

The fact that the Higgs boson mass is close to 125 GeV has the consequence, according to the Standard Model, that this particle has 10 distinct decay modes with branching ratios greater than \( 10^{-4} \). Five of these modes, the decays to \( ZZ^* \), \( WW^* \), \( b\bar{b} \), \( \tau^+\tau^- \), and \( \gamma\gamma \), and the Higgs boson couplings to \( gg \) and \( t\bar{t} \), have already been observed at the LHC. The couplings are consistent with the Standard Model predictions up to uncertainties of 10-30\% [30,31].

This is a very impressive increase in our knowledge, but it cannot be taken as evidence against physics beyond the Standard Model. In Sec. 3, we saw that observable effects of new physics on the observed Higgs boson are associated with dimension-6 SMEFT operators and are generically of a few percent in size. So, the current level of agreement with the predictions of the Standard Model is just that expected in any extension of the Higgs sector. To use the Higgs boson to probe for new physics, we need to push the measurement uncertainties down below the 1\% level.

However, if we can perform Higgs measurements sufficiently precisely to meet this criterion, considerable insight is available. There are two important points to be made here. First, the study of new physics effects on the Higgs boson couplings gives a window on new physics that is different from the search for new particles. Though it is tempting to compare the “reach” of direct searches and precision measurements, this is too simplistic a view. The point is illustrated in Fig. 5, from [32]. The colored bands show the expected variation of the \( hbb \) coupling from the Standard Model prediction in a class of supersymmetric models with \( b\tau \) Yukawa unification. The region in the upper left-hand corner, bounded by the solid line, is the part of the parameter space excluded by the LHC experiments in Run 2. At the end of the HL-LHC running, the region down to the dotted line is expected to be explored. Below these curves, though, there is a whole space of models in which the \( b \) squarks have multi-TeV masses and cannot be discovered by LHC searches but, at the same time, they produce modifications of the \( hbb \) coupling of 1-3\% that are potentially
Figure 5: Fractional deviations of the $h b \bar{b}$ coupling, in %, from the Standard Model expectation in a class of supersymmetric models studied by Wells and Zhang [32], as a function of model parameters. The models in the upper left, above the solid line, are excluded by LHC searches. The dotted line shows the exclusion contour expected from the HL-LHC. The second point is that different extensions of the Higgs sector have their most important effects on different Higgs boson decay modes. New physics models have parameter freedom, and the effects on the Higgs couplings vary over the parameter space. But, in general,

- Higgs couplings to $b$, $\tau$ are modified by supersymmetric and 2-Higgs-doublet models
- Higgs couplings to $W$, $Z$ are modified by composite Higgs models and mixing with scalar singlets
- Higgs couplings to $g$, $\gamma$, $t$ are modified by top quark partners and symmetry-breaking models based on top condensation

This subject is reviewed in more detail in [33]. The point is illustrated in Fig. 6, from [34], which shows the pattern of deviations of the Higgs couplings from the Standard Model predictions in four specific new physics models. In all four cases, the new particles associated with the model are expected to be out of the reach of the HL-LHC. The error intervals shown are those expected from measurements at the
Figure 6: Fractional deviations of 8 Higgs boson couplings, in %, from the Standard Model expectations, in a variety of new physics models. The four models shown are representative supersymmetric models, two-Higgs doublet models, composite Higgs models, and Little Higgs models. In all cases, the parameters are chosen so that the new particles predicted by these models are not expected to be discovered at the HL-LHC.
International Linear Collider, to be discussed in Sec. 8. With sufficient, achievable, precision, the study of Higgs boson couplings can not only demonstrate that the Standard Model is modified but also can give us guidance on the type of model that solves the conceptual problems of the Higgs theory discussed in Sec. 2.

7 SMEFT analysis of Higgs boson reactions at $e^+e^-$ colliders

I will now discuss how to use SMEFT to extract the values of the Higgs boson couplings from observables measured at colliders. The Higgs couplings cannot be read off directly from measurements because some needed information is missing. In particular, the total width of the Higgs boson is expected in the Standard Model to be about 4.3 MeV, a value too small to be measured directly from the width of the resonance observed in collider detectors. To extract the total width of the Higgs boson, which provides the normalization of all partial widths, we need a framework in which to fit the various measurements. In the best case, this framework would not make strong assumptions about the nature of new physics that generates corrections to the predictions of the Standard Model. I will now present the use of SMEFT to provide that framework.

I will concentrate here on the extraction of Higgs boson couplings from precise measurements of the Higgs boson at next-generation $e^+e^-$ colliders. The extraction of Higgs couplings from LHC data is discussed in [36], together with projections for the results expected from experiments at the HL-LHC. Those experiments will greatly improve our knowledge, but their interpretation will be model-dependent, and they are expected to reach only the few-% level of uncertainty, insufficient to demonstrate the existence of new physics corrections at the size expected from the examples of the previous section. It is the future $e^+e^-$ experiments that will really have the power to challenge the Standard Model.

At this time, there are four proposals for $e^+e^-$ “Higgs factories” under serious consideration at different sites around the world. Two of these are circular $e^+e^-$ colliders of roughly 100 km circumference, CEPC in China [37, 38] and FCC-ee [39] at CERN. The other two are linear $e^+e^-$ colliders, ILC in Japan [40, 41] and CLIC at CERN [42]. The technical implementation differs among the 4 proposals, but all have similar goals, including the high-precision study of the reaction $e^+e^- \rightarrow Zh$. The peak of the cross section for the process $e^+e^- \rightarrow Zh$ is at a center of mass energy 250 GeV. Thus, a 250 GeV $e^+e^-$ collider, well within the capabilities of current technologies, can produce a large sample of events in which the Higgs boson is produced together with a $Z$ boson.

The $e^+e^- \rightarrow Zh$ reaction is an exceptionally clean setting in which to study the Higgs boson. To a first approximation (with a smooth and precisely calculable
background) any Z boson observed at a lab energy of 110 GeV is recoiling against a Higgs boson. One simply needs to remove the Z boson from the event and see what is left to measure the quantities

$$\sigma(e^+e^- \rightarrow Zh)BR(h \rightarrow A\bar{A})$$

for all Higgs boson decay products $A\bar{A}$. The ratios of these quantities give the Higgs boson branching ratios. If we can also determine the Higgs total width, we can find all of the partial widths and use these to extract the Higgs boson couplings.

A simple method to find the Higgs total width $\Gamma(h)$ is to assume that each individual Higgs coupling $g(hA\bar{A})$ is modified from its Standard Model value by a multiplicative constant $\kappa_A$. This parametrization has the appealing property that the quantities

$$\sigma(e^+e^- \rightarrow Zh)$$

and

$$\Gamma(h \rightarrow ZZ^*)$$

are both proportional to $\kappa_Z^2$. The branching ratio of the Higgs boson to $ZZ^*$ is given by

$$BR(h \rightarrow ZZ^*) = \Gamma(h \rightarrow ZZ^*)/\Gamma(h) .$$

We can measure $\sigma(e^+e^- \rightarrow Zh)$ by counting recoil $Z$ bosons, and we can measure $BR(h \rightarrow ZZ^*)$ by identifying $Z$ bosons among the Higgs decay products. Then in the quantity

$$\frac{\Gamma(h \rightarrow ZZ^*)}{BR(h \rightarrow ZZ^*)}$$

our assumption would imply that the factors of $\kappa_Z^2$ cancel out and the result is directly proportional to $\Gamma(h)$.

There is a problem, though, that this strategy is not completely model-independent. In the Standard Model, the $hZZ$ coupling has the structure $hZ_\mu Z^\mu$, but in general two independent Lorentz structures are possible,

$$L_{hZZ} = (1 + \eta_Z) \frac{m_Z^2}{v} hZ_\mu Z^\mu + \frac{1}{2} \zeta_Z \frac{1}{v} hZ_{\mu\nu}Z^{\mu\nu},$$

where $Z_{\mu\nu}$ is the $Z$ field strength tensor. There are two parameters, $\eta_Z$ and $\zeta_Z$, that represent possible new physics corrections. Both can arise from dimension-6 SMEFT operators, so they are arguably on equal footing. The $\zeta_Z$ term leads to a momentum-dependent vertex that gives very different corrections to the two quantities in (19). If $\zeta_Z$ is nonzero, the dependence of the quantities in (19) on new physics is not a simple overall multiplicative factor and the argument in the previous paragraph does not go through. What formalism can replace it?

It would be attractive to use the dimension-6 SMEFT coefficients as the parameters in a framework for fitting the Higgs width and couplings. At first sight, this seems out of reach. I have explained at the beginning of Sec. 4 that the number
of dimension-6 SMEFT coefficients is very large. However, in 2016, Tim Barklow proposed that, since \( e^+e^- \) annihilation processes are sensitive to only a subset of these operators, and since \( e^+e^- \) colliders allow a very large number of independent measurements, a fit to the relevant set of coefficients can be completely constrained.

This approach was worked out in detail in [34, 35]. We are concerned with electroweak processes, so it suffices to consider the new physics corrections at the tree level. We make use of CP-even observables. CP-violating terms contribute to these only in order \( c_2^i \), and it is possible, by measuring CP-odd observables, to check that these coefficients are small enough that their effects can be ignored. The relevant dimension-6 operators will be those that are built from the fields of \( h, W, Z, \gamma, \) and \( e_{L,R} \) and those that contribute to Higgs boson decays at tree level. This set of operators can be reduced using the Standard Model equations of motion. From these considerations, we find a parameter set consisting of the 4 Standard Model parameters \( g, g', v, \) and \( \lambda \), plus 18 dimension-6 operator coefficients.

The dimension-6 terms involving only Higgs fields are

\[
\Delta L_1 = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger D^\mu \Phi) (\Phi^\dagger D_\mu \Phi) - \frac{c_{6} \lambda}{v^2} (\Phi^\dagger \Phi)^3.
\]

The additional terms with gauge fields can be reduced to

\[
\Delta L_2 = \frac{4c_{WW}}{v^2} \Phi^\dagger \Phi W_{\mu \nu} W^{\mu \nu} + 16 \frac{s_w c_{WB}}{v^2} \Phi^\dagger t^a \Phi W_{\mu \nu} B^{\mu \nu} \\
+ \frac{4s_w^2 c_{WW}}{v^2} \Phi^\dagger \Phi B_{\mu \nu} B^{\mu \nu} + \frac{4g c_{3W}}{v^2} W^{\mu \nu} W^{\rho \nu} W^{\epsilon \rho \mu},
\]

The dimension-6 terms with electrons and Higgs fields are

\[
\Delta L_2 = ic_{HL} \frac{1}{v^2} (\Phi^\dagger \gamma_\mu L) (L^\dagger \gamma_\mu L) + ic'_{HL} \frac{1}{v^2} (\Phi^\dagger t^a \gamma_\mu L) (L^\dagger \gamma_\mu t^a L) \\
+ ic_{HE} \frac{1}{v^2} (\Phi^\dagger \gamma_\mu \Phi) (e^\dagger \gamma_\mu e) + \frac{c_{LL}}{v^2} (L^\dagger \gamma_\mu L) (L^\dagger \gamma_\mu t^a L) + \frac{c_{LR}}{v^2} (L^\dagger \gamma_\mu t^a L).
\]

These terms allow explicit violation of the “oblique” assumption mentioned in Sec. 5. The additional terms modifying Higgs decay amplitudes are

\[
\Delta L_4 = -\frac{c_{\tau \Phi}}{v^2} (\Phi^\dagger \Phi) L^\dagger \cdot \Phi^\tau_R + \frac{c_{b \Phi}}{v^2} (\Phi^\dagger \Phi) Q^b_R \cdot \Phi^b_R,
\]

and similar operators for \( c \) and \( \mu \). There are several independent dimension-6 operators that contribute to the amplitude for \( h \to gg \). These are not distinguishable in this analysis, since only the on-shell \( h \to gg \) amplitude contributes to the observables. We can represent this degree of freedom by adding

\[
\delta L_5 = \frac{4c_{gg}}{v^2} G^a_{\mu \nu} G^{a \mu \nu}.
\]
Finally, additional coefficients $c_{Hf}$ similar to those in (25) multiplying dimension-6 operators that couple the Higgs current to other flavors appear in the tree-level expressions for the partial widths $\Gamma(h \rightarrow WW^*)$ and $\Gamma(h \rightarrow ZZ^*)$. Fortunately, only two linear combinations of these coefficients appear, and the same two linear combinations appear in the dimension-6 corrections to the $W$ and $Z$ total widths.

Of the 18 dimension-6 coefficients introduced here, $c_6$ does not appear in single-Higgs boson observables. Its role to shift the Higgs self-coupling. The parameter $c_{LL\mu}$ is related to one of the the $\Lambda$ parameters introduced in Sec. 4. It is already strongly constrained by studies of the reaction $e^+e^- \rightarrow \mu^+\mu^-$, and that constraint is expected to become about 100 times stronger at next-generation $e^+e^-$ colliders [41]. Thus, I will ignore these two parameters here. This leaves 4+16 parameters that need to be determined from $e^+e^-$ collision data.

It is difficult to find 20 independent high-precision measurements of Higgs processes, even at $e^+e^-$ colliders. However, in this formalism, the SMEFT Lagrangian is the Lagrangian for all of electroweak physics, not only for the Higgs sector. In fact, we can use data from all electroweak processes to constrain the 20 parameters. The analysis is worked out in detail in [34]. From precision electroweak measurements of the $Z$ and $W$, we have the 8 well-determined quantities

\[ \alpha, G_F, m_Z, m_W, A_e, \Gamma(Z \rightarrow e^+e^-), \Gamma_W, \Gamma_Z. \]  

Note that, of these quantities, only $G_F$, $\Gamma_W$, and $\Gamma_Z$ make reference to any fermion other than the electron. Thus, this strategy makes no assumption about lepton flavor universality or any other possible regularity of the electroweak couplings. Measurements of the triple gauge vertices in $e^+e^-$ constrain three additional parameters. These are combinations of $c_{WB}$, $c_{WW}$ and the $c_{HL,E}$ distinct from those that contribute to the precision electroweak observables.

From single-Higgs processes, the $e^+e^-$ experiments will separately measure the Higgs mass $m_h$, the total cross section for $e^+e^- \rightarrow Zh$ and the $\sigma \times BR$ for this reaction in the $b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$, $gg$, $WW^*$, $ZZ^*$, $\gamma\gamma$, $Z\gamma$, and $\mu^+\mu^-$ modes. Of these, the last 4 modes have relatively low statistics in planned $e^+e^-$ collider experiments. However, the HL-LHC is expected to make high-precision measurements of the ratios of the $\gamma\gamma$, $Z\gamma$, $\mu^+\mu^-$, and $ZZ^*$ branching ratios. These ratios are especially favorable to measure in the hadronic environment, since the four final states are measured in the dominant central-rapidity production channel $h \rightarrow gg$, and the systematic error from the production cross section can be arranged to cancel to a great extent. Combining this information, we have 6 high-precision measurements from $e^+e^-$ and 3 from HL-LHC. The 20-variable system is now closed. It can be checked that there are no unexpected flat directions in the determination of the SMEFT parameters from these inputs.

Measurements in addition to these overdetermine the fit, and their consistency
provides useful cross-checks. In particular, the measurement of the $\sigma \times BR$ for the final state $ZZ^*$ provides useful additional information. This addition is especially important for the high-luminosity circular $e^+e^-$ colliders. On the other hand, the beam polarization asymmetry of the $e^+e^- \to Zh$ cross section turns out to be exceptionally sensitive to the parameter $c_{WW}$. This gives an important input to the fit at linear colliders with beam polarization [43]. In practice, the two advantages balance to a great extent, predicting similar performance for all four of the currently proposed Higgs factories [44]. Measurements of $\sigma \times BR$ values from the $W$-fusion reaction $e^+e^- \to \nu\bar{\nu}h$ can provide additional independent inputs, especially at energies well above 250 GeV, that further constrain the SMEFT fit.

It is worth saying more about the role of the decays $h \to WW^*$ and $h \to ZZ^*$ in this analysis. Note that, at the level of dimension-6 operators, there are no terms beyond those in (23) and (24) that shift the $hWW$ and $hZZ$ couplings from their Standard Model values. The SMEFT Lagrangian generates both of the tensor structures shown in (22) for the $hZZ$ coupling, and two similar terms for the $hWW$ coupling. Both coefficients are allowed to take different values for $W$ and $Z$, but the differences between these values are constrained by $SU(2) \times U(1)$ symmetry. In particular,

$$\eta_W = -\frac{1}{2}c_H, \quad \eta_Z = -\frac{1}{2}c_H - c_T,$$

so the difference between the coefficients of the first tensor structure is constrained by the precision electroweak constraints on the $T$ parameter. Further,

$$\zeta_W = (8c_{WW}), \quad \zeta_Z = c_w^2(8c_{WW}) + 2s_w^2(8c_{WB}) + \frac{s_w^4}{c_w^2}(8c_{BB}),$$

so these parameters cannot be very different if precision electroweak, $e^+e^- \to WW$, and $h \to \gamma\gamma$ measurements constrain the sizes of $c_{WB}$ and $c_{BB}$. This is the reason that a high-precision measurement of $BR(h \to ZZ^*)$ is not essential for the success of the SMEFT fit.

8 Prospects for precision Higgs boson measurements

Now that I have explained the mechanics of the SMEFT fit to the projected results from $e^+e^-$ Higgs factories, I would like to present the sensitivities predicted for the measurement of Higgs boson couplings. Here I will present the results for ILC presented in [41]. Similar results are expected for any of the four Higgs factory proposals currently under discussion [44].

The proposed ILC program has two stages, the first at 250 GeV, the second at 500 GeV in the center of mass. In principle, a third stage at 1 TeV is also possible with
The analysis put forward by the ILC group includes one more possible type of deviation from the Standard Model. There may exist new particles with masses much lighter than $m_h$, perhaps associated with a dark matter sector. These can lead to invisible or partially invisible decays of the Higgs boson [46]. These decays are actually observable using the reaction $e^+e^- \rightarrow Zh$. For example, an invisible decay of the Higgs boson is indicated by an event with a $Z$ boson at 110 GeV in the lab and nothing else. In the ILC analysis, 2 extra parameters are included, one representing the branching ratio for fully invisible decays and and one representing the branching ratio for other exotic decays that do not fit into any preassigned category. Since the invisible decay rate of the Higgs boson is measured or bounded, these two parameters can be added to the SMEFT fit without affecting its closure. It is assumed that the loop effects of the light particles do not affect precision electroweak observables. This is typically true for models of light dark matter particles.

The results of this 22-parameter fit are shown in Table 1, taken from [41]. Already at the 250 GeV stage of the ILC, the SMEFT fit with the expected experimental precision gives uncertainties less than 1% for the important Higgs boson couplings to $W$, $Z$, and $b$. As data is added at higher energies, using the measurements of the

| coupling | ILC250 full no BSM | ILC500 full no BSM | ILC1000 full no BSM |
|----------|--------------------|--------------------|---------------------|
| $hZZ$    | 0.48 0.38          | 0.35 0.20          | 0.34 0.16           |
| $hWW$    | 0.48 0.38          | 0.35 0.20          | 0.34 0.16           |
| $hbb$    | 0.99 0.80          | 0.58 0.43          | 0.47 0.31           |
| $h\tau\tau$ | 1.1 0.95  | 0.75 0.64  | 0.63 0.52   |
| $hgg$    | 1.6 1.6           | 0.96 0.92          | 0.67 0.59           |
| $hcc$    | 1.8 1.8           | 1.2 1.1            | 0.79 0.72           |
| $h\gamma\gamma$ | 1.1 1.1   | 1.0 0.97  | 0.94 0.89   |
| $h\gamma Z$ | 8.9 8.9  | 6.5 6.5  | 6.4 6.4   |
| $h\mu\mu$ | 4.0 4.0   | 3.8 3.8  | 3.4 3.4   |
| $htt$    | —     | 6.3 6.3  | 1.6 1.6   |
| $hhh$    | —     | 27 27  | 10 10   |
| $\Gamma_{\text{tot}}$ | 2.3 1.3 | 1.6 0.70 | 1.4 0.50 |
| $\Gamma_{\text{inv}}$ | 0.36 — | 0.32 — | 0.32 — |

Table 1: Projected uncertainties in the Higgs boson couplings for the ILC250, ILC500, and ILC1000, with precision LHC input [41]. All values are relative errors, in percent (%). The columns labelled “full” refer to the 22-parameter fit including the possibility of invisible and exotic Higgs boson decays. The columns labelled “no BSM” refer to the 20-parameter fit including only decays modes present in the SM.

The same technology, either with a longer tunnel or with high-gradient accelerating cavities that might be available in the future [40].
Figure 7: Expected precision of the determination of Higgs boson coupling constants at the ILC, after its 250 GeV, 500 GeV, and 1000 GeV stages [41]. Input from specific measurements at the HL-LHC is included, as described in the text. Precisions are given in %, but for the last four couplings, the estimates are rescaled by the factors shown in the figure. The full column heights show the estimates for an analysis that allows exotic Higgs boson decays. When it is assumed that there are no exotic decays, the estimates improve to the heights shown by the light-colored bands.

The independent $e^+e^-\rightarrow \nu\bar{\nu}h$ reaction. the uncertainties on the $c$, $\tau$, $g$, and (with the help of HL-LHC data) $\gamma$ couplings also reach the 1% level of accuracy. Running at 1 TeV would further improve these determinations, and also would bring the uncertainties on the $t$ coupling and the Higgs self-coupling to 1.6% and 10%, respectively. Completing this program would give us experimental determinations of the full suite of Higgs boson couplings, at a level at which the possible effects of new physics would be expected to appear with high significance. The improvement in the Higgs coupling determinations expected from this program is shown graphically in Fig. 7.
9 Conclusions

Though the Standard Model is very successful in explaining current experimental data, we should not claim that it is the final theory of the fundamental interactions. It is easy to call out phenomena in the universe that the Standard Model does not account for. In this article, I have emphasized the mysteries surrounding the most important conceptual feature of the Standard Model physics, the spontaneous breaking of its gauge symmetry, which the model can parametrize but which it is incapable of explaining. To provide that explanation, there must be new interactions lying undiscovered at higher energies. The discovery of these new interactions will be as consequential as the discovery of the Standard Model itself.

Among the methods of searching for new interactions, I have emphasized here the precision study of electroweak and Higgs interactions. A useful tool for understanding the implications of our current level of precision and the significance of future improvements is the Standard Model Effective Field Theory. In this article, I have reviewed the application of SMEFT to a variety of experimental measurements, including the future program of precision measurements of the couplings of the Higgs boson.

I have argued that it is within our current technical capabilities to measure the couplings of the Higgs boson with a precision of 1% or below. This is not only a matter of completing the verification of the Standard Model. If we can reach this high level of precision in the study of the Higgs boson, we will have the possibility of observing with high confidence the characteristic effects of new interactions that could explain the origin of Higgs electroweak symmetry breaking. This study could well be the one that breaks through to the next level of fundamental physics, the level that answers the questions that seem intractable today.

We should not miss this opportunity to move to the next deeper level of our understanding of fundamental physics.

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References

[1] I. Brivio and M. Trott, Phys. Rept. 793, 1 (2019) [arXiv:1706.08945 [hep-ph]].
[2] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Theor. Fiz 20, 1064 (1950).
[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 106, 162, 108, 1175 (1957).
[4] K. G. Wilson, Phys. Rev. 179, 1499 (1969).
[5] S. Weinberg, Physica A 96, 327 (1979).
[6] J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984), Nucl. Phys. B 250, 465 (1985).
[7] S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).
[8] D. V. Nanopoulos, Lett. Nuovo Cim. 8, 873 (1973).
[9] There is an extensive literature on the estimation of SMEFT coefficients. See, for example, [10–12].
[10] G. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP 06, 045 (2007) [arXiv:hep-ph/0703164 [hep-ph]].
[11] I. Low, R. Rattazzi and A. Vichi, JHEP 04, 126 (2010) [arXiv:0907.5413 [hep-ph]].
[12] D. Liu, A. Pomarol, R. Rattazzi and F. Riva, JHEP 11, 141 (2016) [arXiv:1603.03064 [hep-ph]].
[13] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP 1010, 085 (2010) [arXiv:1008.4884 [hep-ph]].
[14] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, JHEP 1404, 159 (2014) [arXiv:1312.2014 [hep-ph]].
[15] M. Abolins, et al., in Proceedings of the 1982 DPF Summer Study on Elementary Particle Physics and Future Facilities (Snowmass 82), R. Donaldson, R. Gustafson, and F. Paige, eds. eConf C8206282, 274 (1982).
[16] E. Eichten, K. D. Lane and M. E. Peskin, Phys. Rev. Lett. 50, 811 (1983).
[17] D. Bourilkov, Phys. Rev. D 62, 076005 (2000) [arXiv:hep-ph/0002172 [hep-ph]].
[18] M. Aaboud et al. [ATLAS Collaboration], Phys. Rev. D 96, 052004 (2017) [arXiv:1703.09127 [hep-ex]].
[19] A. M. Sirunyan et al. [CMS Collaboration], Eur. Phys. J. C 78, 789 (2018) [arXiv:1803.08030 [hep-ex]].

[20] B. W. Lynn, M. E. Peskin and R. G. Stuart, in Physics at LEP, J. Ellis and R. Peccei, eds. CERN Yellow Report 86-02 (1986).

[21] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990), Phys. Rev. D 46, 381 (1992).

[22] P. Sikivie, L. Susskind, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. B 173, 189 (1980).

[23] S. Schael et al. [ALEPH and DELPHI and L3 and OPAL and SLD Collaborations and LEP Electroweak Working Group and SLD Electroweak Group and SLD Heavy Flavour Group], Phys. Rept. 427, 257 (2006) [hep-ex/0509008].

[24] M. Baak et al. [Gfitter Group], Eur. Phys. J. C 74, 3046 (2014) [arXiv:1407.3792 [hep-ph]].

[25] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[26] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[27] M. Aaboud et al. [ATLAS Collaboration], Phys. Lett. B 784, 345 (2018) [arXiv:1806.00242 [hep-ex]].

[28] A. M. Sirunyan et al. [CMS Collaboration], JHEP 1711, 047 (2017) [arXiv:1706.09936 [hep-ex]].

[29] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018), and 2019 update.

[30] A. M. Sirunyan et al. [CMS Collaboration], Eur. Phys. J. C 79, 421 (2019) [arXiv:1809.10733 [hep-ex]].

[31] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 101, 012002 (2020) [arXiv:1909.02845 [hep-ex]].

[32] J. D. Wells and Z. Zhang, JHEP 05, 182 (2018) [arXiv:1711.04774 [hep-ph]].

[33] M. E. Peskin, in Proceedings of the 2016 European School of High-Energy Physics, M. Mulders and G. Zanderighi, eds. CERN Yellow Report CERN-2017-009-SP (2017) [arXiv:1708.09043 [hep-ph]].

[34] T. Barklow, K. Fujii, S. Jung, R. Karl, J. List, T. Ogawa, M. E. Peskin and J. Tian, Phys. Rev. D 97, 053003 (2018) [arXiv:1708.08912 [hep-ph]].
[35] T. Barklow, K. Fujii, S. Jung, M. E. Peskin and J. Tian, Phys. Rev. D 97, 053004 (2018) [arXiv:1708.09079 [hep-ph]].
[36] M. Cepeda, et al., in Physics of the HL-LHC, and perspectives of the HE-LHC, A. Dainese, M. Mangano, A. B. Meyer, A. Nisati, G. Salam and M. Vesterinen, eds. CERN Yellow Report CERN-2019-007 (2019) [arXiv:1902.00134 [hep-ph]].
[37] CEPC Study Group, IHEP Report IHEP-CEPC-DR-2018-01 (2018) [arXiv:1809.00285 [physics.acc-ph]].
[38] J. B. Guimares da Costa et al. [CEPC Study Group], IHEP Report IHEP-CEPC-DR-2018-02 (2018) [arXiv:1811.10545 [hep-ex]].
[39] A. Abada et al. [FCC Collaboration], Eur. Phys. J. ST 228, 261 (2019), CERN-ACC-2018-0057.
[40] P. Bambade, et al., arXiv:1903.01629 [hep-ex].
[41] K. Fujii et al. [LCC Physics Working Group], arXiv:1908.11299 [hep-ex].
[42] P. Burrows et al. [CLICdp and CLIC], CERN Yellow Report CERN-2018-005 (2018) [arXiv:1812.06018 [physics.acc-ph]].
[43] This point, made quite explicitly in [34], seems to have been missed in the comparisons of linear and circular colliders given in [39] and other reports from the FCC-ee group.
[44] These comparisons are shown in detail in the tables of [40], Section 11, and in those from the SMEFT analysis presented in [45].
[45] J. de Blas, et al. [Higgs @ Future Colliders Working Group], arXiv:1905.03764 [hep-ph].
[46] D. Curtin, et al., Phys. Rev. D 90, 075004 (2014) [arXiv:1312.4992 [hep-ph]].