Bounded Area Theorems for Higher Genus Black Holes

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Abstract. By a simple modification of Hawking’s well-known topology theorems for black hole horizons, we find lower bounds for the areas of smooth apparent horizons and smooth cross-sections of stationary black hole event horizons of genus $g > 1$ in four dimensions. For a negatively curved Einstein space, the bound is $\frac{4\pi(g-1)}{-\ell}$ where $\ell$ is the cosmological constant of the spacetime. This is complementary to the known upper bound on the area of $g = 0$ black holes in de Sitter spacetime. It also emerges that $g > 1$ quite generally requires a mean negative energy density on the horizon. The bound is sharp; we show that it is saturated by certain extreme, asymptotically locally anti-de Sitter spacetimes. Our results generalize a recent result of Gibbons.
I. Introduction

Black holes embedded in “locally adS” background spacetimes (backgrounds locally isometric to spacetimes of constant negative curvature; adS = anti-de Sitter) have been seminal to recent developments in black hole physics. Most notably, these black holes have been essential to recent progress in understanding black hole entropy, but they have forced us to revisit other issues as well. For example, in contrast to the situation in asymptotically flat spacetimes, static anti-de Sitter black holes can have horizons of non-zero genus. This observation has led us to improve our understanding of topological censorship [1] and its implications for black hole horizon topology [2].

Related to the locally adS black holes are the “asymptotically locally adS” solutions of Mann [3], and Brill, Louko, and Peldán [4]. These are vacuum, negative scalar curvature spacetimes with topology \( R^2 \times \Sigma_g \), where \( \Sigma_g \) is a Riemann surface of genus \( g \). This integer \( g \) and a real number \( m \) referred to as the mass (or energy) parametrize the family of solutions. The term “mass” reflects the fact that \( m \) is a conserved charge conjugate to a Killing vector field that is timelike at infinity. For \( m \) less than a negative minimum value \( m_0 \), there are no horizons and the solutions are nakedly singular. For \( m > m_0 \), future event horizons exist and for each such value of \( m \) and every \( g \geq 2 \), one can construct a not-nakedly-singular, maximally extended spacetime, even for \( m < 0 \). The zero-mass solution is everywhere locally isometric to adS spacetime, but is also a black hole and has trapped surfaces. The \( m = m_0 \) case has Killing horizons that are not event horizons, so this is a nakedly singular spacetime, not a black hole. We will refer to this case as an “extreme solution” rather than as an extreme black hole.
The identification of \( m \) as mass is contentious. Vanzo [5] gives explicit mass formulae both for the special class of solutions under discussion here and for more general stationary black holes with the same asymptotic behaviour. At issue is the zero of mass-energy, since \( m \) can be negative. Vanzo advocates shifting the zero of energy so that the total energy is \( m - m_0 \), assuming there is suitable shift \( m_0 \) that works not only for the special solutions considered here and by Vanzo, but for all reasonable solutions with the same asymptotic behaviour. Contrary-minded, the parameter \( m \) appears be the gravitational mass; the tidal deformations of rings of particles are those produced by a source whose gravitational mass is \( m \). This is evident from the form of the Weyl spinor, which is type II–II:

\[
\Psi_{ABCD} = kr^{-3}\alpha(A\alpha_B\beta_C\beta_D)
\]  

(1)

for \( r \) an “area radial coordinate” and \( k \) a numerical factor times the mass, whence the geometry is conformally flat iff \( m = 0 \).

Still this interpretation of \( m \) is startling, for assuming the gravitational weak equivalence principle (cf. [6]), then we lose the positive mass theorem. This is not completely unexpected, however. The generator of the time translation symmetry here is not the usual “timelike rotation generator” \((J_{04} \text{ in a common notation})\) of the anti-de Sitter group; it’s a “boost-like generator” \((J_{34})\), and this undermines a key step in the Witten-type derivations such as that of [7].

\[1\]

It is also true that the topological identifications needed to compactify the horizon create obstructions to the global Killing spinors that enter parts of the Witten-type arguments and that generate supersymmetries. Solutions of the Witten equation may equally encounter global obstructions. However, even if this were not the case, the energy defined by \( J_{34} \) would not yield to Witten-type positivity arguments.
Nonetheless, in the class of not-nakedly-singular solutions discussed above, and the slightly generalized class of electrovac solutions discussed below, the mass is bounded below, and very recently a general boundedness argument has been given [8] for solutions that share this asymptotic behaviour. This is reminiscent of a conjecture of Horowitz and Myers [9] made in a different (but perhaps not unrelated) context and suggests that a physical mechanism acts to stabilize the theory and prevent arbitrarily negative energies.

Herein we obtain lower bounds on the area of certain embedded surfaces, including apparent horizons and cross-sections of stationary black hole event horizons, with genus $g > 1$. While these computations are perhaps most relevant to black holes in adS backgrounds, the results apply more generally. The next section contains the derivations. Our main result follows from a variation within a spacelike hypersurface and is applicable to both apparent horizons and stationary event horizons. We also briefly discuss a variation within a null hypersurface of an event horizon cross-section. The discussion section shows that the area bound is saturated by higher genus event horizons of certain extreme solutions. In that section, we also suggest that the area bound plays a role in the stability of solutions with higher genus horizons.

As well as being applicable to both apparent horizons and stationary event horizons, our main result does not require the hypersurface in which the variation takes place to be a moment of time symmetry, a maximal slice, or otherwise special. We note, however, that apparent horizons lying in hypersurfaces that are moments of time symmetry are minimal surfaces. The area bound for them is then a standard result of hyperbolic (Riemannian) geometry [10,11] (but beware [10] erroneously omits a factor of 2 in the final result). Soon after the results of this paper were obtained, the author became aware
of a preprint by Gibbons [8] independently deriving the area bound in this special case. The more general proof given below may well have been known to Gibbons when [8] was written since it is a variation on an argument given in [12], wherein the idea is attributed to Gibbons. Finally, the results herein are closely related to an argument of [13] asserting an upper bound for black hole horizon area in positively curved backgrounds.

II. Bounded Area Theorems

Let Σ denote a compact, orientable, spacelike 2-surface embedded in a spacelike hypersurface \( H \), which itself is embedded in spacetime \( (\mathcal{M}, g_{ab}) \). We require Σ to be smooth, marginally outer trapped (the outgoing null geodesics orthogonal to it must have zero divergence), and without outer trapped surfaces lying outside it in \( H \). Apparent horizons and, under certain global assumptions, cross-sections of stationary event horizons have the last two of these properties, but should be noted that event horizons need not be smooth [14] and that it is not known whether apparent horizons are always smooth. The first fundamental form (or Riemannian metric induced by \( g_{ab} \)) on \( H \) is \( h_{ab} \), while the first fundamental form on \( \Sigma \) is \( \gamma_{ab} \).

The derivation is a modification of an argument given in [12]. As this is not an easily-obtained standard reference, we provide details. We will rely heavily on the text [15], and therefore will use the sign conventions of that text.

We fix a complex null frame at every point of \( \Sigma \) as follows. First, let \( l^a \) and \( n^a \) be real and orthogonal to \( \Sigma \), with \( l^a \) future-outward directed, \( n^a \) future-inward directed, and \( l^a n_a = 1 \). This still leaves the overall freedom \( l^a \mapsto e^w l^a \), \( n^a \mapsto e^{-w} n^a \), with \( w : \Sigma \to \mathbb{R} \). Fix \( w \) by requiring that

\[
\frac{1}{\sqrt{2}} (l^a - n^a) =: r^a \in TH ,
\]

\( \text{(2)} \)
so that $r^a$ is the unit outward-directed normal to $\Sigma$ in $H$. The remaining two elements of the tetrad, $m^a$ and $\overline{m}^a$, are tangent to $\Sigma$ and obey $m_a\overline{m}^a = -1$.

For some *a priori* arbitrary function $y : \Sigma \rightarrow R$, we define

\[ v^a = e^yr^a \in T\Sigma \quad , \tag{3} \]

and use it to construct a variation $\Sigma(r)$, $r \in [0, 1]$, of the surface $\Sigma = \Sigma_0$ by pushing each point $p$ of $\Sigma$ a parameter distance $\nu$ along the geodesic of $h_{ab}$ whose initial point is $p$ and whose initial tangent vector is $v^a|_p$. We will later choose $y$ so that $\Sigma_\nu$ is a particularly useful variation of $\Sigma$, but for now it will remain unspecified. We may extend $m^a$ and $\overline{m}^a$ to complex null fields tangent to $\Sigma_\nu$. To extend $l^a$ and $n^a$ to null fields orthogonal to $\Sigma_\nu$, we need to impose the hypersurface orthogonality conditions

\[
\begin{align*}
  l_a \left( m^b v^a_{;b} - v^b m^a_{;b} \right) &= 0 \quad , \\
  n_a \left( \overline{m}^b v^a_{;b} - v^b \overline{m}^a_{;b} \right) &= 0 \quad . \tag{4a} \\
\end{align*}
\]

Following [12], we will use the Newman-Penrose spin coefficient formalism. The formalism is explained in Chapter 4 of [15]. By giving the quantities appearing in (4) their names in this formalism, the hypersurface orthogonality conditions may be written as

\[
\begin{align*}
  \kappa - \tau - \delta y + \overline{\alpha} + \beta &= 0 \quad , \\
  \nu - \pi + \overline{\delta y} + \alpha + \overline{\beta} &= 0 \quad . \tag{5a} \\
\end{align*}
\]

These are equations (7.1) and (7.2) of [12], respectively.
The Newman-Penrose quantity $\rho := m^a m^b \nabla_b l_a$ is real, in virtue of Prop. (4.14.2) of [15], and represents the convergence of the geodesic congruence tangent at $H$ to the $l^a$ field. We seek to determine the change in $\rho$ along the variation defined above:

$$\frac{d\rho}{dr} := v^a(\rho) = \frac{e^y}{\sqrt{2}} (D\rho - D'\rho) . \quad (6)$$

The derivatives $D\rho := l^a \nabla_a \rho$ and $D'\rho := n^a \nabla_a \rho$ are given by the Newman-Penrose equations (4.11.12a) and (4.11.12f) of [15]:

$$D\rho = \rho (\rho + \epsilon + \tau) + \sigma \sigma + \Phi_{00} - \kappa \tau - \kappa (3\alpha + \beta - \pi) + \delta \kappa \quad , \quad (7a)$$

$$D'\rho = - \rho \mu - \sigma \lambda - \tau \tau + \kappa \nu + \rho (\gamma + \tau) - \tau (\alpha - \beta) - \Psi_2 - 2\Lambda + \delta \tau . \quad (7b)$$

Here we have used the “normalized spin frame conditions” $\beta' = -\alpha$, $\alpha' = -\beta$, and $\Pi = \Lambda$, together with the fact that $\delta' = \delta$. As well, so that our notation follows that of [12], we use the symbols $\pi$, $\lambda$, $\mu$, and $\nu$ for $-\tau'$, $-\sigma'$, $-\rho'$, and $-\kappa'$ respectively.

If we plug expressions (7) into (6) and simplify using (5), we obtain

$$\frac{d\rho}{dr} = \frac{e^y}{\sqrt{2}} \left\{ \rho (\rho + \epsilon + \tau - \gamma - \overline{\tau} + \overline{\mu}) + \sigma \sigma + \lambda \sigma + (\kappa - \tau) (\overline{\kappa} - \overline{\tau}) + \Phi_{00} + \Psi_2 + 2\Lambda + \delta \partial y - \partial (\overline{\alpha} + \overline{\beta}) \right\} , \quad (8)$$

where $\partial := \delta - \overline{\alpha} + \overline{\beta}$. This is equation (7.3) of [12] (except for an inconsequential sign difference in the $\rho \overline{\mu}$ term).

Most terms appearing above have simple interpretations in terms of familiar geometrical quantities. For example, $\rho$, $\sigma$, and $\mu$ are related to the null extrinsic curvatures of $\Sigma$, while $\Lambda$, $\Phi_{00}$, and $\Psi_2$ are built from components of the spacetime Riemann tensor. The Gauss curvature of $\Sigma$ is simply twice the real part (cf. Prop. (4.14.21) of [15]) of the combination

$$K := -\lambda \sigma - \Psi_2 + \rho \mu + \Phi_{11} + \Lambda . \quad (9)$$
where \( K \) is known as the *complex curvature* of \( \Sigma \). Moreover, the term \( \partial \delta y \) is simply the Laplacian \( \Delta y \) in \((\Sigma, \gamma_{ab})\). Hence we can write

\[
\frac{d\rho}{dr} = e^y \sqrt{2} \left\{ \rho (\rho + \epsilon + \tau - \gamma - \mu + \nu) + \sigma \overline{\sigma} + (\kappa - \tau) (\overline{\kappa} - \tau) \right. \\
+ \Delta y + \Phi_{00} + \Phi_{11} + 3\Lambda - K - \overline{\partial} (\overline{\alpha} + \beta) \left. \right\} .
\]

(10)

Every term in this expression is manifestly real except for the last two, whose imaginary parts must therefore cancel (this is a manifestation of Prop. (4.4.43) of [15]).

Finally, we must specify the precise nature of the variation vector field \( v^a \), or in other words we must specify \( y \). We use the trick introduced by Hawking, which was to note that if \( \int_\Sigma f(x) d^2 \Sigma = 0 \) then one can always solve \( \Delta y + f(x) = 0 \) on \( \Sigma \). That is, we choose \( y \) to solve

\[
\Delta y + \rho (\rho + \epsilon + \tau - \gamma - \mu + \nu) + \Phi_{00} + \Phi_{11} + 3\Lambda - K - \overline{\partial} (\overline{\alpha} + \beta) = c ,
\]

(11)

for a constant \( c \) to be determined below. Then (10) becomes simply

\[
\frac{d\rho}{dr} = e^y \sqrt{2} \left[ \sigma \overline{\sigma} + (\kappa - \tau) (\overline{\kappa} - \tau) + c \right] .
\]

(12)

The constant \( c \) is evaluated by integrating (11) over \( \Sigma \). When doing so, note that the \( \partial \)-term appearing there is a divergence, so it does not contribute to the integral. Also, \( \rho \) is zero on \( \Sigma \) by assumption. Moreover, the real part of \( K \) is just one-half the Gauss curvature, so the integral of this term gives \( \pi \) times the Euler characteristic of \( \Sigma \), and for a Riemann surface of genus \( g \) that characteristic is \( 2(1 - g) \). Therefore,

\[
cA(\Sigma) = \int_\Sigma (\Phi_{00} + \Phi_{11} + 3\Lambda) d^2 \Sigma + 2\pi (g - 1) ,
\]

(13)

where \( A(\Sigma) \) is the area of \( \Sigma \).
Hawking proved his topology theorem by next observing that \( c \leq 0 \), for if the first-order change in \( \rho \), as given by (12), were positive everywhere on \( \Sigma \) and since \( \rho|_\Sigma = 0 \), then the variation \( \Sigma(r) \) would be an outer trapped surface just outside \( \Sigma \), contrary to assumption. He then imposed an energy condition so that the integral on the right of (13) was non-negative, whence he concluded that \( g = 0, 1 \). We will reverse this last step and argue instead that the non-positivity of \( c \) forces that integral term to be negative whenever \( g > 1 \). That is, defining

\[
\langle E \rangle = \frac{2 \int_\Sigma (\Phi_{00} + \Phi_{11} + 3\Lambda) \, d^2\Sigma}{A(\Sigma)} ,
\]

then (13), with \( c \leq 0 \), gives

\[
\langle E \rangle A(\Sigma) + 4\pi(g - 1) \leq 0 ,
\]

\[
\Rightarrow \quad \langle E \rangle \leq \frac{4\pi(1 - g)}{A(\Sigma)} < 0 \quad \text{for } g > 1 ,
\]

\[
\Rightarrow \quad A(\Sigma) \geq \frac{4\pi(g - 1)}{-\langle E \rangle} .
\]

The division in (15c) makes sense whenever \( g > 1 \) in virtue of (15b).

From the Einstein equations, which in this signature are

\[
R_{ab} - \frac{1}{2}g_{ab}R + \ell g_{ab} = -8\pi T_{ab} ,
\]

for stress-energy tensor \( T_{ab} \) and cosmological constant \( \ell \), we have (cf. equation (4.6.32) of [15])

\[
\Phi_{ab} = 4\pi \left( T_{ab} - \frac{1}{4}g_{ab}T \right) ,
\]

\[
\Lambda = \frac{\pi}{3} T + \frac{1}{6} \ell ,
\]

\[
\Rightarrow \quad E = 8\pi T_{ab} l^a \left( l^b + n^b \right) + \ell .
\]
For $T_{ab} = 0$, then $\langle E \rangle = \ell$ so (15c) yields the bound quoted in the Abstract.

An area bound can also be obtained by taking the hypersurface $H$ to be null and $\Sigma$ to be its intersection with a future event horizon, by a simple modification of the derivation quoted in [16] of Hawking’s topology theorem for event horizons. Since [16] is a standard reference, we will quote from it, *mutatis mutandis*. We therefore use the sign conventions of [16] from here to the end of this section. These are generally opposite to those of [15] used in the rest of this article.

Using properties of stationary event horizons, it is shown in [16] that the right-hand side of their equation (9.7) cannot be positive. Following their notation, let $\partial B(\tau)$ denote an event horizon cross-section “at time $\tau.$” The Einstein equation appearing below (9.7) of [16] must be corrected by adding the cosmological constant $\ell$ to the right-hand side. Let $\tilde{E} := 8\pi T_{ab}Y_1^aY_2^b + \ell$, where $Y_{1,2}^a$ respectively denote in [16] the fields called $l^a$ and $n^a$ above (so they are null and normalized such that $Y_1^aY_2^a = -1$, using the signature employed in [16]). Let $\langle \tilde{E} \rangle$ be the mean of $\tilde{E}$ over the surface $\partial B(\tau)$:

$$\langle \tilde{E} \rangle = 8\pi \frac{\int_{\partial B(\tau)} T_{ab}Y_1^aY_2^b dS}{A(\partial B(\tau))} + \ell,$$

where $A(\partial B(\tau))$ is the area of the cross-section $\partial B(\tau)$.

By substituting this expression into (9.7) of [16] and using the sign constraint subsequently deduced therein (the argument parallels that used to constrain the sign of $c$ above), we obtain

$$\langle \tilde{E} \rangle A(\partial B(\tau)) \leq 2\pi \chi(\partial B(\tau)) = 4\pi(1 - g),$$

2 This argument is based on the observation that trapped surfaces do not lie outside future event horizons. Fortunately for present purposes, this fact does not depend on asymptotic flatness, as is clear from the phrasing of Prop. (12.2.4) of [17].
for $\chi(\partial B(\tau))$ the Euler characteristic and $g$ the genus of $\partial B(\tau)$.$^3$ From this we recover
the form of (15c), but with $\langle \tilde{E} \rangle$ replacing $\langle E \rangle$. Notice these quantities differ by the
additional $T_{ab}l^al^b$ factor in (17c) that does not appear in $\tilde{E}$.

III. Discussion

We now consider a class of higher genus black holes, essentially those discussed in
[3,4,18]. The metric

$$ds^2 = V(r) dt^2 - \frac{1}{V(r)} dr^2 - r^2 (d\theta^2 + \sinh^2 \theta d\phi^2)$$

(20)

with

$$V(r) = -1 - \frac{\ell}{3} r^2 - \frac{2m}{r} + \frac{Z^2}{r^2}$$

(21)

solves the Einstein-Maxwell system with electromagnetic potential $A_0 = Q/r$, $A_\phi = H \cosh \theta$, $A_r = A_\theta = 0$, where $Q$ is the electric charge, $H$ the magnetic charge, and $Z^2 = Q^2 + H^2$. The non-zero components of the stress-energy tensor for these metrics
can be expressed in the form

$$T_{\mu\nu} = \pm \frac{Z^2}{r^4} g_{\mu\nu}$$

(22)

with the plus sign for $\mu, \nu = 0, 1$ and the minus sign for $\mu, \nu = 2, 3$, using the conventions
of [15].

$^3$ The factor of 2 in the middle term of (19) is missing in [16]. This error originates
in their unnumbered equation preceding (9.7), the Gauss-Bonnet theorem, wherein the
integrand should be the Gauss curvature, not the curvature scalar, the difference being
the factor of 2.
Horizons occur at roots $r = a > 0$ of $V(r)$. Roots are easily located by considering points of intersection of the curves

$$y_1 = -\frac{\ell}{3}r^4 - r^2 \quad \text{(23a)}$$
$$y_2 = 2mr - Z^2 \quad \text{(23b)}$$

One fixes $y_1$ and varies the slope $2m$ and intercept $-Z^2$ of $y_2$ to move about in solution space. For $r > 0$ generically there are either zero or two intercepts, and never three. There is a single intercept if either $Z = 0$ and $m \geq 0$ or, more interestingly, $y_2$ is tangent to $y_1$. The latter case is an extreme solution and the point of intersection is a double root of $V(r)$. Extreme solutions minimize the mass for fixed $Z$ and, as we will see, realize the lower bound on horizon area.

Near a root $r = a$ of $V(r)$, we may expand

$$r = a + \Delta r \quad \text{(24a)}$$
$$V(r) = \Delta r V'(a) + \frac{\Delta r^2}{2} V''(a) + O(\Delta r^3) \quad \text{(24b)}$$

If we replace $\Delta r$ by a new variable $\sigma$ using

$$\sigma := 2\sqrt{\frac{\Delta r}{V'(a)}} \quad \Rightarrow \quad d\sigma = \frac{dr}{\sqrt{V'(a)\Delta r}} \quad , \text{(25)}$$

then the metric near $r = a$ can be written as

$$ds^2 \simeq \left[\frac{1}{2}V'(r)\sigma\right]^2 dt^2 - d\sigma^2 - a^2 (d\theta^2 + \sinh^2 \theta d\phi^2) \quad . \text{(26)}$$

If we treat $t$ as complex, then its imaginary part is a coordinate for a non-singular Euclidean submanifold iff it’s periodic with period $4\pi/V'(a)$. Then continuous Euclidean Green functions must have this period, so by standard arguments the Hawking temperature is

$$T_H = -\frac{\hbar V'(a)}{4\pi k_B} \quad . \text{(27)}$$
As with more familiar black holes, the Hawking temperature vanishes when $V'(a) = 0$; i.e., when $r = a$ is a double root of $V(r)$. Hence the extreme solutions above, and only the extreme solutions, are stable against semi-classical decay by Hawking radiation; in particular, the $m = Z = 0$ black hole is unstable.

For solutions given by (20) and (21), by symmetry the area bound (15c) on the horizon becomes a bound on the horizon “radius” $a$:

$$a^2 \geq \frac{1}{-\langle E \rangle}.$$  \hspace{1cm} (28)

Since $T_{ab}$ is constant over the horizon, $\langle E \rangle$ is easily computed from (22) to give:

$$\langle E \rangle = \ell + \frac{Z^2}{a^4}. \hspace{1cm} (29)$$

Plugging this into (28) and rearranging, we obtain

$$0 \leq -\ell a^4 - a^2 - Z^2,$$

which is saturated iff (recall $\ell < 0$ here)

$$a^2 = \frac{1 + \sqrt{1 - 4\ell Z^2}}{-2\ell}.$$ \hspace{1cm} (31)

But this is the simultaneous root of $V(r) = V'(r) = 0$, so (31) is satisfied iff the metric of form (20, 21) is an extreme solution iff it has zero temperature.

The area bound may point to interesting features of the mechanical laws governing asymptotically locally anti-de Sitter black holes. Consider the semi-classical decay of the horizon by Hawking radiation. If the flux at infinity is positive and if the surface gravity $k$ is positive, then the first law of black hole thermodynamics, $\delta M = k \delta A$, 

13
implies that this process will cause the horizon area to shrink. The horizon area $A(t)$ will be a decreasing function of time, bounded below by a positive number whenever $g > 1$. Thus, $A(t)$ will converge to a positive value, and the spacetime will approach a steady state, just as asymptotically flat charged black hole spacetimes do (but without any restored supersymmetries). This is reminiscent of the conjecture of Horowitz and Myers [9], argued from an entirely different point of view, that the mass-energy of certain 5-dimensional spacetimes with negative scalar curvature is bounded below, even though the positive energy theorem fails for these spacetimes. Gibbons [8] has recently used the mean curvature flow method of Geroch [19] to argue that such a lower bound applies for mass-energy of 4-dimensional asymptotically locally adS black holes on any hypersurface that is a moment of time symmetry.

In certain circumstances, the horizon could still “decay” by first growing until it develops self-intersections. This can result in a decrease in the genus, resulting in turn in a decrease in the area bound (this process can sometimes drive the genus and, hence, the area bound, to zero). So-called temporarily toroidal horizons arising in numerical simulations of asymptotically flat spacetimes change genus in this manner, although the effect depends on a choice of spatial slicing—there are slicings in which these horizons are never toroidal (see [2] for details and further references; also [20]). This process, however, is not available to all non-zero genus horizons. Indeed, for the black holes discussed above, the horizon and the boundary at infinity are linked in such a manner that horizon topology change is precluded.

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References

[1] J.L. Friedman, K. Schleich, and D.M. Witt, Phys. Rev. Lett. 71 (1993), 1486, gr-qc/9305017; Erratum-ibid. 75 (1995), 1872.

[2] G.J. Galloway, K. Schleich, D.M. Witt, and E. Woolgar, preprint (1999), gr-qc/9902061.

[3] R.B. Mann, Class. Quantum Gravit. 14 (1997), 2927, gr-qc/9705007.

[4] D.R. Brill, J. Louko, and P. Peldan, Phys. Rev. D56 (1997), 3600, gr-qc/9705012.

[5] L. Vanzo, Phys. Rev. D56 (1997), 6475.

[6] C.M. Will, Theory and experiment in gravitational physics (Cambridge University Press, Cambridge, 1981), p. 82.

[7] C.M. Hull, Commun. Math. Phys. 90 (1983), 545.

[8] G.W. Gibbons, Class. Quantum Gravit. 16 (1999), 1677.

[9] G.T. Horowitz and R.C. Myers, Phys. Lett. B428 (1998), 297, hep-th/9803066.

[10] Y. Shen and S. Zhu, Math. Ann. 309 (1997), 107

[11] G.J. Galloway, Contemp. Math. 170 (1994), 113.

[12] S.W. Hawking, in Black Holes, eds. C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973), 1.

[13] S.A. Hayward, T. Shiromizu, and K. Nakao, Phys. Rev. D49 (1994), 5080, gr-qc/9309004.

[14] P.T. Chruściel and G.J. Galloway, preprint (1996), gr-qc/9611032.
[15] R. Penrose and W. Rindler, *Spinors and space-time* Vol. I (Cambridge University Press, Cambridge, 1984).

[16] S.W. Hawking and G.F.R. Ellis, *The large scale structure of space-time* (Cambridge University Press, Cambridge, 1973).

[17] R.M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).

[18] R.B. Mann, in *Internal Structure of Black Holes and Spacetime Singularities*, eds. L. Burko and A. Ori, *Ann. Israeli Phys. Soc.* 13 (1998), 311.

[19] R. Geroch, *Ann. N. Y. Acad. Sci.* 224 (1973), 108.

[20] T. Jacobson and S. Venkataramani, *Class. Quantum Gravit.* 12 (1995), 1055.