The optimal design support system for shell components of vehicles using the methods of artificial intelligence

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Abstract. The paper is devoted to an application of the evolutionary methods and the finite element method to the optimization of shell structures. Optimization of thickness of a car wheel (shell) by minimization of stress functional is considered. A car wheel geometry is built from three surfaces of revolution: the central surface with the holes destined for the fastening bolts, the surface of the ring of the wheel and the surface connecting the two mentioned earlier. The last one is subjected to the optimization process. The structures are discretized by triangular finite elements and subjected to the volume constraints. Using proposed method, material properties or thickness of finite elements are changing evolutionally and some of them are eliminated. As a result the optimal shape, topology and material or thickness of the structures are obtained. The numerical examples demonstrate that the method based on evolutionary computation is an effective technique for solving computer aided optimal design.

1. Introduction
Optimal properties of shell components of vehicles can be provided by using the computer aided optimization techniques. For example, an appropriate strength of structures can be established by changing their shape, topology and material properties. The choice of optimal shape and topology are the key elements defining the effectiveness of structures, and thus finding them is the problem of great practical interest. Shape and topology structural optimization is a very active research area. Several competing approaches for topology optimization exist [1][2][3]. More recently, intelligent optimal design techniques based on bio-inspired methods like the evolutionary algorithm (EA), the particle swarm optimizer (PSO) and artificial immune system (AIS) have found applications to structural optimization problems. The main feature of evolutionary method is to simulate of biological processes based on heredity principles (genetics) and the natural selection (the theory of evolution) to creating of optimal individuals (solutions) presented by single chromosomes. Evolutionary algorithms are usually applied in the situations when the optimization problems are too complicated for the traditional gradient optimization methods. The task considered in the present work is related to such a problem. This task consists in creating an effective optimization algorithm for shell-structures in respect of topology [4], shape and material or thickness arrangement. The main advantage of the evolutionary algorithm is the fact that this approach does not need any information about the gradient of the fitness function and gives a strong probability of finding the global optimum. The fitness function is calculated for each chromosome in each generation by solving the boundary-value problem by means of the finite element method (FEM). In order to solve the optimization problem the fitness function,
design variables and constraints should be formulated. Coupling the finite element method and the evolutionary algorithm gives an effective and efficient alternative optimization tool, which enables solving a large class of the optimization problems of mechanical structures for example structure of car wheel. The main feature of the first proposed optimization method is the evolutionary distribution of the material, changing its material properties or by changing its thickness in the construction of car wheel. This process leads to the elimination of the part of material from the wheel construction and as a result the new shape and the topology of the car wheel emerges. Using interpolation surfaces (hyper surface) reduces the number of the design variables and shortens the time of the computation. The application of the professional program of the finite element method MSC NASTRAN in this method enables the optimization of the complex mechanical systems. The numerical examples confirm the efficiency of the proposed optimization method and demonstrate that the method based on evolutionary computation is an effective technique for solving computer aided optimal design problems.

2. Formulation of evolutionary optimization problem

Evolutionary algorithms are algorithms searching the space of solutions and they operate on populations of individuals. Each individual which consists of chromosomes in population represents a single solution. All chromosomes consist of genes which are design variables in optimization problems. Next the fitness function is computed in order to adaptation each individual in population. All genes of an individual decide about the fitness function value. The algorithm of evolutionary optimization has a several stages. First an initial population of individuals is created. The design variables (genes) in randomly way are generated. In the next step, the fitness function value for each chromosome is computed. In the next step genes are changed using evolutionary operators and working on the parent population individuals. The offspring population is created on the bases of modified individuals with previous step. The algorithm is continuing iteratively till the end of the computation. The stop condition of the computation can be formulated as the maximum number of iterations. Two types of operators are used: mutation and crossover. Mutation is represented by uniform mutation, mutation with Gaussian distribution and boundary mutation. The last operator is simple and arithmetical crossover.

Consider a 2D structure which, at the beginning of an optimization process, occupies a domain \( \Omega_0 \), bounded by a boundary \( \Gamma_0 \). The domain of this structure \( \Omega_0 \) is filled by a homogeneous and isotropic material (parameters: Young’s modulus \( E_0 \) and a Poisson ratio \( \nu_0 \)). The thickness of the structure \( g_0 \) is also constant at the beginning of the evolutionary process. The shells are considered in the framework of theory of elasticity. During the optimization process the domain \( \Omega_t \), its boundary \( \Gamma_t \) and the field of Young’s modulus \( E(x) = E_t \), \( x \in \Omega_t \), or the thickness \( g(x) = g_t \), can change for each iteration \( t \) (for \( t=0, E_0=\text{const}, g_0=\text{const} \)). The optimization process proceeds in an environment in which the structure fitness is describing by the objective is to minimize the stress functional

\[
J = \int_{\Omega} \psi(\sigma) d\Omega 
\]

where \( \psi \) is an arbitrary function of stress tensor \( \sigma \), with a constraint imposed on the volume of the structure

\[
V = |\Omega| \leq V_{\text{max}}
\]

In order to solve the formulated problem Finite Element models of the structures are considered [5]. The structure is divided into finite elements \( \Omega_e, e = 1, 2, \ldots, R \), and node displacements are calculated by solving a system of linear algebraic equations
where $U$ is a column matrix of unknown displacements, $F$ is a known column matrix of acting forces and $K$ is a known global stiffness matrix of the structure whose elements are given as follows:

$$K^e = B^TDB^e$$

(4)

where $D$ and $B$ are the known elasticity and geometrical matrices, respectively, $V^e$ represents the volume of the finite element.

3. The idea of evolutionary optimization

The distribution of the Young’s modulus or thickness $E(x), x \in \Omega$, in the shell is described by a hypersurface $W(x), x \in H'$ in the finite element. The hypersurface $W(x)$ is stretched under $H' \subseteq E'$ and the domain $\Omega$ is included in $H'$, i.e. $(\Omega \subseteq H')$ [6].

The shape of the hypersurface $W(x)$ is controlled by genes $h_j, j=1,\ldots,N$, which create a chromosome

$$ch = \{h_1, h_2, \ldots, h_j, \ldots, h_N\}, \quad h'^{\min} \leq h_j \leq h'^{\max}$$

(5)

where $h'^{\min}$ - the minimum value of the gene and $h'^{\max}$ - the maximum value of the gene.

Genes are values of the function $W(x)$ in interpolation nodes $x_j$, i.e. $h_j = W(x_j), j=1,2,\ldots,N$.

The assignment of Young’s moduli or thickness to each finite element $\Omega_e, e=1,2,\ldots,R$ is adequately performed by the mapping:

$$E_e = W(x_e), x_e \in \Omega_e, e=1,2,\ldots,R$$

(6)

$$g_e = W(x_e), x_e \in \Omega_e, e=1,2,\ldots,R$$

(7)

It means that each finite element can have different material. When the value of the Young’s modulus or thickness for the e-th finite element is included in:

- the interval $0 \leq E_e < E_{\min}$ (or $0 \leq g_e < g_{\min}$), the finite element is eliminated and the void is created,
- the interval $E_{\max} \leq E_e < E_{\max}$ (or $g_{\max} \leq g_e < g_{\max}$), the finite element remains having the value of the Young’s modulus from this material (figure 1).

![Figure 1. Requirements for elimination and existence of the finite elements](image-url)

Two types of interpolations which aim at the appropriate selection of mass densities are used. The first of them is the multinomial interpolation, the other the interpolation based on the neighborhood of elements [13]. On the basis of obtained results the interpolation bases on the neighbourhood of
elements is better than multinomial interpolation. In addition to second interpolation bases on the
eighbourhood of elements the optional number of control points can be loaded and all working space
always is used. It is following advantage of this method. In addition to increase number of control
points permit to obtain more accurate distribution of hyper surface but for bigger number of control
points, number of individuals in each generation must be increased.

In order to improve optimization results two different additional have been introduced:
- the additional procedure aiding the topology optimization [9][10],
- the assignation of materials [13].

Using the first method (the additional procedure aiding the topology optimization), one can change
material properties of finite elements during the immune optimization process and some elements are
eliminated. As a result, the optimal shape, topology and material of structure are obtained [9][10].

Using second method the Young’s modulus value should be devided into the enforced number of
subintervals (equal to the number of materials). Each subinterval represents another material whose
the Young’s modulus belongs to this subinterval. The subintervals should have the same length and
their centres correspond with Young’s modulus values of suitable materials [13].

Figure 2. Evolutionary optimization of shells
4. **Evolutionary optimization algorithm of topology, shape and material or thickness**

The minimization of the fitness functions (1) or (2) with respect to the chromosome (5) is performed by means of an evolutionary algorithm [7][8] with the floating point representation (figure 2). After the FEM discretisation the starting population of chromosomes is randomly generated. At the next step the main loop of the optimization algorithm is performed. Operations included in the main loop lead to the calculation of the fitness function. It requires that the boundary-value problem should be solved by the FEM. After the calculation of the fitness function for all of the chromosomes in the population the evolutionary algorithm is applied. The evolutionary algorithm contains the following operators: the ranking selection, the simple and arithmetical crossovers, the uniform and boundary mutations and the cloning. As the result a new offspring population is created. The end of the algorithm’s work, i.e. a break in the main loop activity, occurs after the declared generation number. The algorithm can be also stopped, when after the specified iteration number the change of the fitness function is very small [9].

5. **Numerical example – car wheel optimization**

The problem is to find optimal shape, topology and thickness of a car wheel by the minimization of the stress functional and by the volume constraint. The geometry of a car wheel with characteristic dimensions (Table 1) is presented. Three surfaces of revolution built the structure (figure 3): the central surface with the holes destined for the clamping screws, the surface of the ring of the wheel and the surface connecting the two mentioned earlier. The connecting surface is optimized. The shell-structure is loaded with a pressure $c^0$ and the tangent force $s^0$ (figure 4b). The support is applied on the central surface in the direction of the rotation axis of the wheel. The nodes in the areas near the holes destined for the clamping screws are fixed (figure 4b). The symmetry of the car wheel is imposed (revolution of the 1/5 part of the structure). Figure 4a presents distribution of the control points of the interpolation hypersurface. In this way the number of design variables (genes) are decreased and the symmetrical results are obtained. The input data and the parameters of evolutionary algorithm are presented in the tables 2 and 3, respectively. The maps of thickness presents the results of the optimization after 100th generations (figure 5). All the presented solutions are included in the search space and are optimal – the second optimization criterion is the esthetic aspect dependent on the car user. The results correspond very well with the car wheels produced by different automotive companies (figure 5).

![Figure 3. Geometry and characteristic dimensions of a car wheel](image)

| Table 1. Characteristic dimensions of a car wheel |
|-----------------------------------------------|
| diameter of the wheel LW                     | 355.6 mm |
| width of a tyre LF                           | 175 mm   |
| diameter of the wheels spacing LK            | 110 mm   |
| diameter of a wheel hub LP                   | 60 mm    |
| thickness of the wheel hub                   | 30 mm    |
| thickness of a tyre                          | 8 mm     |
Table 2. Input data to the optimization task

| tangent force \( s' \) [N] | pressure \( p' \) [Mpa] | number of design variables | number of control points | number of chromosomes |
|---------------------------|-------------------------|---------------------------|--------------------------|-----------------------|
| 500                       | 0.22                    | 23                        | 86                       | 100                   |

| material                | range of the change of the genes [mm] | existing of an element | elimination of an element | \( V_{\text{max}} \) [cm\(^3\)] |
|-------------------------|----------------------------------------|------------------------|--------------------------|-------------------------------|
| aluminium               | 4.0\( \leq g_e \leq 20 \)            | \( 4 \leq g_e < 10 \) | \( 10 \leq g_e \leq 20 \) | 5,500                        |

Table 3. Parameters of evolutionary algorithm

| cloning | uniform mutation | boundary mutation | simple crossover | arithmetical crossover |
|---------|------------------|-------------------|------------------|------------------------|
| 2%      | 5%               | 5%                | 10%              | 10%                    |

Figure 4. Car wheel: (a) distributing of the control points of the interpolation hipersurface; (b) boundary conditions
Figure 5. The results of the car wheel optimization: (a), (c), (e), (g), the best obtained solution - maps of thickness, (b), (d), (f), (h) real car wheels which are in good coincidence with obtain solutions

6. Conclusions
An effective tool of evolutionary optimization of shells has been presented. Using this approach shape, topology and material or thickness optimization are performed simultaneously. The important feature of this approach is a strong probability of finding the global optimal solutions received by implementation of the evolutionary algorithm and its universality consisting in application of this approach to the optimization of the 2-D structures in plane stress/strain [10]. Evolutionary method belong to methods based on population of solutions and they have some interesting features which can be considered as alternative to artificial immune system [11][12][13][14] or particle swarm optimizers [10][15][16]. Comparison between AIS, PSO and EA in the paper [6][17][18][19] is presented.

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