Mass Hierarchy Determination for $\theta_{13} = 0$ and Atmospheric Neutrinos

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Abstract

We examine the possibility of determining the neutrino mass hierarchy in the limit $\theta_{13} = 0$ using atmospheric neutrinos as the source. In this limit, in which $\theta_{13}$ driven matter effects are absent, independent measurements of $\Delta_{31}$ and $\Delta_{32}$ can, in principle, lead to hierarchy determination. Since the difference between these two is $\Delta_{21}$, one needs an experimental arrangement where $\Delta_{21} L/E \gtrsim 1$ can be achieved. This condition can be satisfied by atmospheric neutrinos since they have a large range of energies and baselines. In spite of this, we find that hierarchy determination in the $\theta_{13} = 0$ limit with atmospheric neutrinos is not a realistic possibility, even in conjunction with an apparently synergistic beam experiment like T2K or NO$\nu$A. We discuss the reasons for this, and also in the process clarify the conditions that must be satisfied in general for hierarchy determination if $\theta_{13} = 0$.

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1 Introduction

The neutrino mass hierarchy is an important discriminator between large classes of models of unified theories [1]. Its determination remains one of the outstanding problems facing neutrino physics. The term hierarchy refers to the position of the third mass eigenstate with respect to the other two (relatively) degenerate states. When the hierarchy is normal (NH), the third mass eigenvalue ($m^3_{31}$) is the largest and hence $\Delta_{31} = m^2_{3} - m^2_{1}$ and $\Delta_{32} = m^2_{3} - m^2_{2}$ are both positive. On the other hand, when it is inverted (IH), $m^3_{31}$ is the smallest, leading to $\Delta_{31}$ and $\Delta_{32}$ being negative.

The most widely discussed method for hierarchy determination relies on sizable matter effects at long baselines. The baselines of the foreseeable super-beam experiments are 1000 Km or less. Therefore in these experiments appreciable matter effects do not develop, although it may be possible to have some hierarchy sensitivity in the $\nu_\mu \rightarrow \nu_e$ channel [2, 3] provided $\theta_{13}$ is large enough (i.e. $\sin^2 2\theta_{13} \geq 0.04$). However, this channel is seriously compromised by the ($\delta_{CP}$, hierarchy) degeneracy [4, 5]. Thus hierarchy determination via this channel requires data from a number of complementary experiments. The degeneracy problem is reduced in the case of a detector located at the magic baseline of $\sim 7000$ km [5–7]. However, this requires high luminosity sources such as neutrino factories or beta-beams [8], likely to be available only in the far future. Recently synergy in T-conjugated channels were explored to determine hierarchy for shorter baselines with small matter effects for $\sin^2 2\theta_{13} \gtrsim 0.03$ [9].

Independent of the value of $\theta_{13}$, $\Delta_{31}$ is the largest mass squared difference for NH, whereas for IH, $\Delta_{32} = \Delta_{31} - \Delta_{21}$ has the largest magnitude. In principle, one can exploit this difference to determine the mass hierarchy if one can measure $|\Delta_{31}|$ and $|\Delta_{32}|$ individually and precisely [10]. Detection of the interference effect in vacuum neutrino oscillations [11–13] and the measurement of the phase of monoenergetic $\bar{\nu}_e$ [14] have been proposed to determine the hierarchy.

With atmospheric neutrinos as the source, wide ranges in energy ($E$) and baseline ($L$) become available and it is possible to observe large resonant matter effects [15–20]. For values of $\theta_{13}$ much below the CHOOZ upper bound [21], the distance a neutrino needs to travel for these effects to be observable, becomes larger than the diameter of the earth [16]. Therefore, hierarchy determination with atmospheric neutrinos using matter effects also requires a moderately large value of $\theta_{13}$ [18, 19] ($\sin^2 2\theta_{13} \gtrsim 0.05$).

There is a large class of models which predict $\theta_{13}$ to be exactly zero due to some symmetry principle [22]. Recent global analysis of neutrino data indicates a small preference for a non-zero $\theta_{13}$, $\sin^2 2\theta_{13} \simeq 0.05$ [30, 31]. However, at present the statistical significance of this result is only at the level of $\sim 1\sigma$. In addition, the latest results from MINOS show an excess of electron events [32]. This may also be an indication for a non-zero $\theta_{13}$ [33].

In the present work we consider the problem of determination of the sign of atmospheric

\footnote{It is of course likely that such symmetries are broken at low energies and $\theta_{13}$ would acquire a small non-zero value [23–27]. Hierarchy determination using matter effects driven by very small values of $\theta_{13}$ entails measuring the $\nu_e \rightarrow \nu_\mu$ probability at the magic baseline using a beta-beam for $\sin^2 2\theta_{13} \geq 10^{-2}$ [28], or with a neutrino factory beam for $\sin^2 2\theta_{13} \geq 10^{-3}$ [29].}
neutrino mass difference in the limit $\theta_{13} \to 0$. This is a challenging problem requiring high
precision measurements. The general requirements for this were discussed in [34]. It was
argued that one would need measurements of $P_{\mu\mu}$ and $P_{\bar{\mu}\bar{\mu}}$ at different $L/E$ with atleast
one $L/E$ satisfying $\Delta_{21} L/E \geq 1$. In this connection the utility of a broadband accelerator
beam or atmospheric neutrinos capable of providing different values of $L/E$ within the same
experimental set up was pointed out in [34]. Reference [35] made a detailed implementation
of the ideas discussed in [34] in the context of neutrino factories and superbeams using two
narrow band beams at different $L$ or one broad band beam. However, no detailed quantitative
analysis of hierarchy determination for $\theta_{13} = 0$ using atmospheric neutrinos had been done
in the literature so far. In this paper, we undertake this task and perform the analysis
using atmospheric neutrinos in conjunction with long baseline experiments. Our study shows
that despite the fact that atmospheric neutrinos provide a large range of $L$ and $E$, practically
speaking, this is an impossible prospect. We discuss in detail the conceptual and experimental
issues that lead to this conclusion.

2 $P_{\mu\mu}$ in the limit $\theta_{13} = 0$

For $\theta_{13} = 0$, the probabilities involving the electron flavour depend only on $\Delta_{21} = m_2^2 - m_1^2$
because the mass eigenstate $\nu_3$ decouples from $\nu_e$. Hence probabilities involving $\nu_e$ are not
sensitive to the sign of $\Delta_{31}$ and to determine the hierarchy, other oscillation and survival modes
must be explored. In the case of atmospheric neutrinos, the $\nu_{\mu} \to \nu_{\mu}$ survival probability
provides the most effective channel, especially from the detection viewpoint.

As explained above, for different hierarchies the relative magnitudes of $\Delta_{31}$ and $\Delta_{32}$ are
different. The difference in magnitudes of $\Delta_{31}$ and $\Delta_{32}$ for NH and IH is proportional to $\Delta_{21}$.
To exploit the corresponding difference in $P_{\mu\mu}$, one needs values of $L/E$ such that $\Delta_{21} L/E \gtrsim 1$
or $L/E \gtrsim 10^4$. Such values of $L/E$ can be obtained for $E \lesssim 1$ GeV and $L \sim 10^4$ Km. A large
flux of atmospheric neutrinos with these baselines and energies is readily available.

For such long distances, the neutrinos propagating through earth’s matter experience a
potential $V = \sqrt{2} G_F n_e$ ($G_F$ is the Fermi Constant and $n_e$ is the ambient electron number
density). This gives rise to an effective mass squared matter term, $A = 2E V$ which can be
expressed as

$$A = 2 \times 7.6 \times 10^{-5} \rho (\text{g/cm}^3) Y_e E (\text{GeV}) \text{[eV}^2]$$

where $\rho$ is the average density of matter along the neutrino path and $Y_e = 0.5$ is the electron
fraction per nucleon.

Including matter effects, the expression for $P_{\mu\mu}$ for $\theta_{13} = 0$ is

$$P_{\mu\mu}^m = 1 - 4 \cos^4 \theta_{23} \sin^2 \theta_{12} \cos^2 \theta_{12} \sin^2 \left( \frac{\Delta_{21}^m L}{4E} \right)$$

$$- 4 \cos^2 \theta_{23} \sin^2 \theta_{23} \sin^2 \theta_{12} \sin^2 \left( \frac{\Delta_{31}^m L}{4E} \right)$$

$$- 4 \cos^2 \theta_{23} \sin^2 \theta_{23} \cos^2 \theta_{12} \sin^2 \left( \frac{\Delta_{32}^m L}{4E} \right)$$
where, the squared mass-differences in matter are
\[
\Delta_m^{21} = m_{2m}^2 - m_{1m}^2
\]
\[
\Delta_m^{31} = m_{3m}^2 - m_{1m}^2 = \Delta_{31} - \frac{1}{2} \left[ \Delta_{21} + A - \Delta_m^{21} \right]
\]
\[
\Delta_m^{32} = m_{3m}^2 - m_{2m}^2 = \Delta_{31} - \frac{1}{2} \left[ \Delta_{21} + A + \Delta_m^{21} \right]
\]
(3)

and,
\[
\Delta_m^{21} = \sqrt{\left( \Delta_{21} \cos 2\theta_{12} - A \right)^2 + \left( \Delta_{21} \sin 2\theta_{12} \right)^2}
\]
\[
\sin 2\theta_{12} = \sin 2\theta_{12} \frac{\Delta_{21}}{\Delta_m^{21}}
\]
(4)
(5)

From the above equations we see that the condition \( \Delta_{21} \cos 2\theta_{12} = A \) defines a resonance energy related to the solar mass-squared difference \( \Delta_{21} \),
\[
E_s^R(\text{GeV}) = \frac{\Delta_{21} \cos 2\theta_{12}}{0.76 \times 10^{-4} \rho(g/cm^3)}.
\]
(6)

The present best fit value of \( \Delta_{21} \sim 8 \times 10^{-5} \text{eV}^2 \) \([36–38]\) gives \( E_s^R \approx 0.06 - 0.2 \text{ GeV} \).

In the third column of Table 1 we list the values of \( E_s^R \) for the various baselines. Note that for longer baselines, the average density is larger because the neutrinos pass through the inner mantle (5000 \( \leq L \leq 10000 \text{Km} \)) and the core \( (L \geq 10000 \text{Km}) \).

| \( L \text{ (km)} \) | \( \rho_{\text{avg}} \text{ (gm/cm}^3\) | \( E_s^R \text{ (GeV)} \) |
|------------------|-----------------|-----------------|
| 295, 732         | 2.3             | 0.20            |
| 2900             | 3.3             | 0.14            |
| 7330             | 4.2             | 0.11            |
| 12000            | 7.6             | 0.06            |

Table 1: Values of \( E_s^R \) are listed as a function of baseline or the density \( \rho \). See text for details. The value of \( \sin^2 2\theta_{12} = 0.8 \) or \( \cos 2\theta_{12} = 0.43 \) is used for evaluating \( E_s^R \).

For energies \( E << E_s^R \), the matter term is negligible and we obtain the vacuum limit of the survival probability. Its form is similar to eq. (2) with the vacuum angles and mass-square differences taking the place of the corresponding matter dependent quantities. For energies \( E >> E_s^R \) or \( A >> \Delta_{21} \), we get
\[
\Delta_m^{21} \simeq A - \Delta_{21} \cos 2\theta_{12} \text{ and } \theta_m^{12} \simeq \pi/2.
\]
(7)

Substituting these in eq. (2), we get
\[
P_{\mu\mu}^m = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{L}{4E} \right).
\]
(8)

Here we define \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \). We note that the survival probability in matter is independent of the matter term because of cancellations [34]. But it must be stressed that this
probability is in general not the same as the vacuum survival probability. It coincides with vacuum $P_{\mu\mu}$ only in the limit $\Delta_{21} L / 4E \ll 1$ and if only terms linear in the small parameter $\Delta_{21} / \Delta_{31}$ are retained in both expressions. In this limit both $P_{\mu\mu}$ in vacuum and $P_{m}^{\mu\mu}$ from eq. (8) are given by

$$P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \left[ \sin^2 \frac{\Delta_{31} L}{4E} - \frac{c_{12}^2 \Delta_{21} L}{4E} \sin \frac{\Delta_{31} L}{2E} \right]$$

(9)

It is to be noted that the lengths involved in the long baseline experiments T2K [39] ($L = 295$ Km) and MINOS [40, 41] and NO\nu A [42, 43] ($L = 732/810$ Km) are short enough such that the approximation $\Delta_{21} L / 4E \ll 1$ is valid. The energy of T2K ($E \geq 0.5$ GeV) is a few times $E_R$ whereas the energies in MINOS and NO\nu A ($E \geq 2$ GeV) are much larger than $E_R$ as can be seen from the first row of Table 1. Therefore for MINOS and NO\nu A eq. (8) is valid to a very good approximation. It is interesting to consider the precise difference between $P_{m}^{\mu\mu}$ measured by T2K [39] and the corresponding vacuum probability in eq. (9). Expanding terms in eq. (2) to first order in the small parameter $\Delta_{21} L / 4E$, we get

$$P_{m}^{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \left[ \sin^2 \frac{\Delta_{31} L}{4E} - \frac{(c_{12}^m)^2 \Delta_{21} L}{4E} \sin \frac{\Delta_{31} L}{2E} \right]$$

(10)

In this expression, $(c_{12}^m)^2$ is very small ($\sim 0.09$) because $\theta_{12}^m$ is just a little lower than $\pi/2$. This multiplies the small term $\Delta_{31}^{m} L / E$ (which is of the same order as $\Delta_{21} L / E$). Hence the second term in the square bracket can be neglected and we have

$$P_{m}^{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \left[ \sin^2 \frac{\Delta_{31} L}{4E} - \frac{0.55 \Delta_{21} L}{4E} \sin \frac{\Delta_{31} L}{2E} \right]$$

(11)

Comparing this expression with eq. (9) above, we find that the only difference is the replacement of the factor $c_{12}^2 = 0.69$ by 0.55 in the coefficient of the second term. But since this term is suppressed by the small parameter $\Delta_{21} L / 4E = 0.04$, compared to which the contribution of the first term $(\sin^2 \Delta_{31} L / 4E)$ is about 0.9 for T2K, the difference in the coefficient causes a negligible change in the value of $P_{\mu\mu}$. The difference in magnitude between $P_{\mu\mu}$ from eq. (9) and eq. (11) above is less than 0.5%. This makes eq. (11) equivalent (to a very good approximation) to eq. (8) in this limit [34].

3 Hierarchy Sensitivity

It is clear from eq. (8) that if the magnitude of $(\Delta_{31} - \Delta_{21} c_{12}^2)$ is different for NH and IH, $P_{\mu\mu}$ will also be different for the two hierarchies. Before computing the difference in $P_{\mu\mu}$, we need to figure out the difference in the magnitude of the quantity $(\Delta_{31} - \Delta_{21} c_{12}^2)$ with a change in the hierarchy. We investigate this change in the light of various assumptions made in the literature.
First we consider $\Delta_{31}(IH) = -\Delta_{31}(NH)$. This assumption amounts to identifying $\Delta_{31}$ with $\Delta_{m_{21}}^{atm}$ determined by Super-K [44] and is widely used in the literature for situations where the difference between $|\Delta_{31}|$ and $|\Delta_{32}|$ lies well below experimental errors, and is thus immaterial. It leads to

$$P_{\mu\mu}^m(NH) - P_{\mu\mu}^m(IH) =$$

$$\sin^2 2\theta_{23} \left[ \sin^2 \left( (\Delta_{31} + \Delta_{21} c_{12}^2)L/4E \right) \right] - \sin^2 \left( (\Delta_{31} - \Delta_{21} c_{12}^2)L/4E \right). \quad (12)$$

Clearly, this assumption is untenable for the situation under consideration here, where the difference between these two mass differences is crucial.

The other assumption often used is $\Delta_{31}(IH) = -\Delta_{32}(NH) = -\Delta_{31}(NH) + \Delta_{21}$. This is equivalent to the statement that the largest mass squared difference has the same magnitude for both NH and IH. It leads to

$$(\Delta_{31}(IH) - \Delta_{21} c_{12}^2) = (-\Delta_{31}(NH) + \Delta_{21}(1 - c_{12}^2))$$

$$= -(\Delta_{31}(NH) - \Delta_{21}s_{12}^2). \quad (13)$$

Substituting this in eq. (8) gives

$$P_{\mu\mu}^m(NH) - P_{\mu\mu}^m(IH) =$$

$$\sin^2 2\theta_{23} \left[ \sin^2 \left( (\Delta_{31} - \Delta_{21} s_{12}^2)L/4E \right) \right] - \sin^2 \left( (\Delta_{31} - \Delta_{21} c_{12}^2)L/4E \right). \quad (14)$$

This amounts to making an ad hoc assumption which currently is unsupported by experimental evidence.

If we do not make any assumptions but instead consider the question: what information do the experiments give regarding the magnitudes of $\Delta_{31}$ and $\Delta_{32}$ for NH and IH? Ongoing experiments such as MINOS and future experiments such as T2K and NO$\nu$A measure the muon neutrino survival probability. In the two flavour limit, $\theta_{13} = 0$ and $\Delta_{21} = 0$, this is given by

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \left( \frac{\Delta_{31} L}{4E} \right). \quad (15)$$

Analyzing the data of these experiments, in this limit, gives us the magnitude of $\Delta_{31}$. This is the reason behind the widely used assumption $\Delta_{31}(IH) = -\Delta_{31}(NH)$, quoted at the beginning of this section.

The survival probability for T2K and MINOS/NO$\nu$A is given by eq. (8) to a very good approximation as shown in the previous section. The three flavour effects appear in the form of non-zero $\Delta_{21}$. We note that eq. (8) can be obtained from eq. (13) by the replacement $\Delta_{31} \rightarrow (\Delta_{31} - c_{12}^2 \Delta_{21})$. Therefore the "atmospheric" mass-squared difference measured by MINOS or T2K is not $\Delta_{31}$ (or $\Delta_{32}$) but the combination [13, 34],

$$\Delta_{m_{21}}^{atm} = \Delta_{31} - c_{12}^2 \Delta_{21} = s_{12}^2 \Delta_{31} + c_{12}^2 \Delta_{32}. \quad (16)$$
Thus, in the limit $\theta_{13} = 0$, MINOS or T2K measure the magnitude of the mass-squared difference $\Delta m^2_{\text{atm}}$ defined in eq. (16). This quantity is positive for NH and negative for IH. Then from eq. (16) one can easily derive $\Delta_{31}(IH) = -\Delta_{31}(NH) + 2c_{12}^2\Delta_{21}$. With this relation between $\Delta_{31}(NH)$ and $\Delta_{31}(IH)$, we see that $P_{\mu\mu}^m$ in eq. (8) has no hierarchy sensitivity [34]. Atmospheric neutrino data with energies $E >> E^R_s$ measure the above $P_{\mu\mu}^m$, and hence contain no hierarchy sensitivity. This statement is true independent of the precision with which $\Delta m^2_{\text{atm}}$ can be measured. Any deviation from this prediction is likely to come from the regions where the approximations made in obtaining eq. (8) are not exact [34,35]. This is equivalent to saying that a combination of two experiments can give hierarchy sensitivity only if the mass-squared differences (i.e. the frequencies) measured by the two experiments are different.

In Figure 1, the hierarchy difference for $P_{\mu\mu}^m$ with $\theta_{13} = 0$ is plotted as a function of the neutrino energy $E$ for a baseline of 12000 Km using the three definitions of the hierarchy discussed in this section, i.e. (a) $\Delta_{31}(IH) = -\Delta_{31}(NH)$, (b) $\Delta_{31}(IH) = -\Delta_{31}(NH) + \Delta_{21}$ and (c) $\Delta_{31}(IH) = -\Delta_{31}(NH) + 2c_{12}^2\Delta_{21}$. Note that the assumptions a) and b) lead to a large but spurious sensitivity to the hierarchy, as reflected in the large values of the hierarchy difference for $P_{\mu\mu}^m$. This difference is greater for case (a) than case (b). Case (c) gives the hierarchy difference for $P_{\mu\mu}^m$ for the third hierarchy definition, which is based on the experimental measurement of $\Delta m^2_{\text{atm}}$ by accelerator experiments. This implies that we need this measurement from experiments such as T2K or NOνA to explore the hierarchy sensitivity in atmospheric neutrino data. In generating the plots, $P_{\mu\mu}$ is obtained by the numerical integration of the neutrino evolution equations.

We see that the hierarchy sensitivity is quite small for case (c). A difference $\simeq 0.2$ can be observed in the numerical plots at lower energies ($E \simeq 1$ GeV) for this very long baseline of 12000 Km. This is because the expressions given in eq. (2) and eq. (8), derived in the constant density approximation, do not hold good at such values of $E$ and $L$. But this difference is very small and would be washed out even with optimistic values of energy and angular resolution of the detector.

From Table 1, we see that for multi-GeV atmospheric neutrinos the energy $E$ will always be larger than $E^R_s$ and so it will be difficult to find a regime where the probability will be different than that given by eq. (8). Table 1 also shows that $E^R_s$ is larger for shorter baselines. Thus, we expect that a departure from the form given in eq. (8) is most likely for the baselines of $3000 - 6000$ Km and neutrino beam energy of $0.2 - 0.7$ GeV. These scenarios were analysed in [35] for superbeam and neutrino factory experiments.

If we look at the region in atmospheric neutrino experiments where the neutrino energy is moderately larger than $E^R_s$ (in the range 0.2 - 0.6 GeV), the matter modified mixing angle $\theta_{12}^m$ will be less than $\pi/2$. Hence, we can’t use $P_{\mu\mu}$ in eq. (8) but need to go to the full expression given in eq. (2). This equation is in principle sensitive to hierarchy due to the matter term $A$ and the difference between $\Delta_{31}$ and $\Delta_{32}$. However, this difference oscillates rapidly with energy and with baseline. To measure it, one would require extraordinary neutrino energy and angular resolution and very large statistics. Such angular resolution is impossible to achieve in the relevant sub-GeV energy range, because quasi-elastic neutrino scattering, which has a broad distribution in the direction of the final state lepton, dominates in this range. Additionally, most atmospheric detectors with target nuclei have an inherent limit on the energy resolution.
possible, set by Fermi motion of the bound nucleons. Such energies are typically \( \sim 100 \) MeV, and thus are a significant fraction of the energy of the final state lepton in the range of interest here.

In Figure 2, we plot the difference in \( P_{\mu\mu}^{m} \) for NH and IH as a function of baseline for the two energies, 0.5 GeV and 1.0 GeV. These figures show a moderate to substantial difference for the case of no smearing. However, this difference becomes less than 0.02 when we include an angular smearing with \( \sigma_\theta = 5^\circ \). This value of \( \sigma_\theta \) is much smaller than the width of the angular distribution for quasi-elastic events. Note that no energy smearing was included in generating this figure. Thus we see that even with an ideal energy resolution, the difference in \( P_{\mu\mu}^{m} \) for the two hierarchies becomes tiny. If a moderate energy smearing is included the difference is likely to vanish. Hence it is not possible to determine the neutrino mass hierarchy using atmospheric neutrinos as a source, no matter how good the detector is.

To summarize, the general requirements for hierarchy determination for \( \theta_{13} = 0 \) are

1. \( |\Delta m^2_{\text{atm}}| \) should be measured to a precision better than 2%.
2. The neutrino energy \( E \) should be large enough to produce a muon but should not be too large compared to \( E^R_s \).
3. The baseline \( L \) should be such that \( \Delta_{21}L/E \gtrsim 1 \).
4. Excellent energy and angular resolution are necessary, so that rapid oscillations in \( P_{\mu\mu} \) can be resolved.

Atmospheric neutrinos clearly do not satisfy the last criterion. In a beam experiment, however, \( \sigma_\theta \) is zero, so the problem of angular resolution is automatically resolved. However, it must still meet the requirement of superior energy resolution, good enough to resolve the closely spaced peaks shown in case (c) of Figure 1. It is apparent from the size of the peaks that the experiment needs to have the capability to gather high statistics [35].

4 Conclusions

In conclusion, we have studied the feasibility of determining the sign of \( \Delta_{31} \) for \( \theta_{13} = 0 \) using atmospheric neutrinos as the source. When \( \theta_{13} = 0 \), matter effects induced by it are absent. Moreover, \( \nu_e \) is decoupled from \( \nu_3 \), leading to no hierarchy dependence in \( P_{ee} \) or \( P_{\mu e} \). This makes \( P_{\mu\mu} \) the most suitable remaining channel for hierarchy determination. In this limit, \( P_{\mu\mu} \) depends on \( \Delta_{21}, \Delta_{31} \) and \( \Delta_{32} \). For NH (IH), \( \Delta_{31} (\Delta_{32}) \) is the highest frequency. If these two close frequencies can be resolved along with their magnitudes, then the hierarchy can be determined. This requirement is, in practice, difficult to achieve by measurement at a single \( L/E \). Hence it translates to having two experiments, one of which measures a single frequency \((|\Delta m^2_{\text{atm}}|)) \), which is a known combination of the two independent frequencies \((\Delta_{21} \text{ and } \Delta_{31}) \), to high precision. The other experiment then needs to satisfy the requirement \( \Delta_{21}L/E \gtrsim 1 \) and measure a different combination of these frequencies, also with high precision. The precision requirements imply that the energy and angular resolutions have to be exceptionally good.
The $\Delta_{21} L/E \gtrsim 1$ requirement implies long baselines. The condition for measuring a different frequency enforces tapping into $\Delta_{21}$ driven matter effects which in turn needs energies which are not too far removed from the solar resonance energy $E^R_s$.

Atmospheric neutrinos offer a broad range of $L/E$ values including $\Delta_{21} L/E \gtrsim 1$. But the precision requirements preclude any chance of hierarchy determination by a single atmospheric experiment on its own. We have explored if hierarchy determination is possible using atmospheric neutrinos in conjunction with upcoming accelerator experiments such as T2K and NO$\nu$A. For these experiments, $L/E$ values are such that $\Delta_{31} L/E \sim 1$ and hence $\Delta_{21} L/E \ll 1$. In this approximation, $P^\mu_\mu$ depends on the single effective frequency $\Delta_{31} - c_{12}^2 \Delta_{21}$. For atmospheric neutrinos with energies above 1 GeV and satisfying the condition $\Delta_{21} L/E \gtrsim 1$, one will measure this same frequency. Thus, even if one had a high precision atmospheric neutrino experiment detecting multi-GeV neutrinos, this combination would not satisfy the requirement of different frequency measurement discussed above.

If, on the other hand, one had an atmospheric experiment capable of measuring lower energies (which would have the consequence of widening the range of $L$ required to satisfy $\Delta_{21} L/E \gtrsim 1$ and allowing the inclusion of shorter baselines), it is likely that the different frequency condition could be met. However, the precision requirements on energy and angular resolution become very demanding in this case. In particular, the energy resolution has to be better than the typical Fermi energy of a bound nucleon. Also, given that quasi-elastic scattering provides the dominant contribution to the event rates for sub-GeV neutrinos, the precision requirement on angular resolution is impossible to realize. Hence neutrino hierarchy determination using atmospheric neutrinos, even in conjunction with another high precision beam experiment, is not possible.

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Figure 1: Hierarchy difference for $P_{\mu\mu}^m$ (exact numerical muon survival probability in matter) as a function of the neutrino energy $E$ with $L = 12000$ Km and $\theta_{13} = 0$ for the three assumptions relating $\Delta_{31}(NH)$ and $\Delta_{31}(IH)$.

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Figure 2: Hierarchy difference for $P_{\mu\mu}^m$ (exact numerical muon survival probability in matter) as a function of the baseline $L$ for $\theta_{13} = 0$ with the T2K definition of $\Delta_{31}(IH)$, for (a) $E = 0.5$ GeV and (b) $E = 1$ GeV. The difference without angular smearing and with an angular smearing of $5^\circ$ are shown.

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