What do the radiative decays of $X(3872)$ tell us

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Abstract

Since the discovery of $X(3872)$, its structure has been in ceaseless dispute. The data of $X(3872) \rightarrow \pi^{+}\pi^{-}\pi^{0}J/\psi$ suggest that $X(3872)$ may be a high-spin charmonium-like of $2^{-+}$. In terms of the light front quark model (LFQM) we calculate the rates of the radiative decays $X(3872) \rightarrow J/\psi(\psi')\gamma$ supposing $X(3872)$ to be a $2^{-+}$ charmonium. Within this framework, our theoretical prediction on $BR(X(3872) \rightarrow \psi(1S)\gamma)$ is at order of $10^{-3}$ which is slightly lower than the Babar’s data but close to the Belle’s. Our prediction on $BR(X(3872) \rightarrow \psi'\gamma)$ is at order of $10^{-5}$ if $\psi'$ is a pure $2S$ state or $10^{-4}$ if $\psi'$ is a $2S - 1D$ mixture, which does not conflict with the upper bound set by the Belle collaboration, but is much lower than the Babar’s data. Thus if the future measurement decides the branching ratio of $BR(X(3872) \rightarrow \psi'\gamma)$ to be much larger than $10^{-4}$, the $2^{-+}$ assignment for $X(3872)$ should be ruled out.

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I. INTRODUCTION

Very recently, a series of charmonium-like resonances has successively been observed by various experimental collaborations, such as $X(3872)$[1], $X(3940)$[2], $Y(3940)$[3], and several bottomonium-like states were also discovered at the Belle and Babar energy ranges, such as $Z(4430)^[±]$, $Z_b$ and $Z_b^[±]$. It is noted that there is almost no room in the ground state representations of $O(3)\otimes SU_f(3)\otimes SU_s(2)$ to accommodate those newly observed resonances. Not only their mass spectra, but also their behaviors of production and decay may hint if they are exotic states such as hybrids, molecular states, tetraquarks, or just radial and/or orbital excited charmonia.

It would be an interesting task to determine their inner structures from both experimental and theoretical aspects. As a matter of fact, there are different interpretations for these resonances which are of different constituent compositions from the regular mesons, so that for clarifying its structure, all the interpretations should be studied one by one and then we can see if the predictions are in agreement with data. The most reasonable way is to calculate its mass and decay widths by assuming its structure in certain theoretical frameworks, then a comparison of the prediction with data would confirm or negate the assumption about the identity of the resonance.

Among the newly observed resonances, $X(3872)$ which was found a while ago has caused special interests of experimentalists as well as theorists [4-11]. Some authors regard it as a molecular state [12-14], whereas other groups consider it as a tetraquark [15-17]. Instead, the recent data on $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi$ may hint a possible assignment $^1D_2$ (i.e. $2^-$) for $X(3872)$ which has inspired theoretical interests [21-24]. The charmonia mass spectrum has been calculated in the potential model and the theoretical prediction for a $^1D_2$ charmonium is about 3.81GeV [25, 26] or 3.84GeV [27] which is 30-60 MeV lower than the measured value of $X(3872)$. Because of uncertainty existing in the calculation of the binding energies of excited states with the potential model such difference may not be too serious, but the difference might imply that our understanding on its assignment might be not completely correct. To determine its structure one needs more information from other sources, especially its decay modes. The Belle and Babar collaborations also reported their measurements on the radiative decays of $X(3872)$[18, 28]. The data given by the Babar collaboration [29] are $BR(B^+ \rightarrow X(3872)K^+)BR(X(3872) \rightarrow J/\psi\gamma) = (2.8 \pm 0.8(stat) \pm 0.1(syst)) \times 10^{-6}$, $BR(B^0 \rightarrow X(3872)K^+)BR(X(3872) \rightarrow J/\psi\gamma) = (2.3 \pm 0.9(stat) \pm 0.3(syst)) \times 10^{-6}$ and $BR(X(3872) \rightarrow \psi'\gamma) = 3.4 \pm 1.4$. The Belle collaboration also reported their new result as $BR(B^+ \rightarrow X(3872)K^+)BR(X(3872) \rightarrow J/\psi\gamma) = (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6}$ and set an upper bound $BR(B^+ \rightarrow X(3872)K^+)BR(X(3872) \rightarrow J/\psi\gamma) < 3.45 \times 10^{-6}$ and $BR(X(3872) \rightarrow \psi'\gamma) < 2.1$ because they found no evidence for $X(3872) \rightarrow \psi'\gamma$ [30]. The data from the two collaborations are consistent on $X(3872) \rightarrow J/\psi\gamma$ but largely apart on $X(3872) \rightarrow \psi'\gamma$. Obviously the further measurement and theoretical study are badly
needed.

In this work we investigate the radiative decays of \( X(3872) \) which is supposed to be a \( ^1D_2 \) charmonium in the light-front quark model. The results may help us to determine the structure of \( X(3872) \).

The light-front quark model (LFQM) is a relativistic model\[31, 32\] and in this model wave functions are manifestly Lorentz invariant and expressed in terms of the fractions of internal momenta of the constituents which are independent of the total hadron momentum. This approach has been applied to study many processes and thoroughly discussed in literatures \[33–44\]. Generally the results obtained in this framework qualitatively coincide with the experimental observation on the concerned processes and while taking the error ranges into account (both experimental and theoretical), they can be considered to quantitatively agree with the available data.

To evaluate the transition rate in the LFQM one needs to know the wave functions of the parent and daughter hadrons. The wavefunctions for the s-wave and p-wave were given in Ref.\[37\] and we studied the wavefunctions of the d-wave in Ref.\[45\] with which we are able to investigate the transitions involving s-, p- and d-wave mesons. Then for obtaining the transition amplitude, one also needs calculate those form factors in terms of the effective Lagrangian. In this work we will deduce the form factors for the radiative decay of \( 2^{-+} \to 1^{--} \) in the covariant light-front quark model. With those formulas we calculate the rate of the radiative decays of \( X(3872) \) which is supposed to be a pure \( 2^{-+} \) meson. Because of the so-called \( \rho - \pi \) puzzle, namely the branching ratio of \( \psi' \to \rho\pi \) is extremely small while \( J/\psi \to \rho\pi \) is one of the main decay modes of \( J/\psi \), it is suggested \( \psi' \) might not be a pure \( 2S \) state (\( \psi(2S) \)) but a mixture of \( 2S \) and \( 1D \) states (\( \psi(2S - 1D) \))\[46\]. This allegation has not been fully confirmed so far, even though it forms a reasonable interpretation for the \( \rho - \pi \) puzzle and needs more theoretical and experimental tests indeed. In this work, we consider the two possibilities and obtain the branching ratio of the decay \( X(3872) \to \psi'\gamma \) with and without considering the mixing scenario respectively.

In this work after the introduction we derive the form factors for the radiative decay of \( 2^{-+} \to 1^{--}\gamma \) in the covariant light-front approach in section II. Then in section III we present our numerical results about the decay \( X(3872) \to J/\psi\gamma \) and \( X(3872) \to \psi'\gamma \). The section IV is devoted to discussions and our conclusion.

II. THE DECAY OF \( 2^{-+} \to 1^{--}\gamma \) IN THE COVARIANT LIGHT-FRONT APPROACH

In Refs.\[36, 37\] the authors discussed how to calculate the transition matrix in the covariant light-front approach. Following their strategy we formulate the matrix element for \( 2^{-+} \to 1^{--}\gamma \). The relative orbital angular momenta of \( 2^{-+} \) and \( 1^{--} \) are \( L = 2(d \text{ wave}) \) and
Let us list the vertex functions for $2^{-+}$ and $1^{--}$ states as

$$iH(1D_2)\gamma_5K_\mu K_\nu,$$

$$iH_V[\gamma_\mu - \frac{1}{W_V}(p_1 - p_2)_\mu].$$

where $V$ represents the $1^{--}$ state and $K = \frac{p_2 - p_1}{2}$. The amplitude of $1D_2$ state decaying into $1^{--}$ via a photon emission is written as

$$A_\mu = -iee_q\frac{N_c}{16\pi^4} \int d^4p_1 \left\{ \frac{H(1D_2)H_V}{N_1N_2N'_1} s_{a\beta\mu\nu}^a + \frac{H(1D_2)H_V}{N_1N_2N'_2} s_{a\beta\mu\nu}^b \right\} \varepsilon^\nu \varepsilon^{\alpha\beta},$$

where

$$s_{a\beta\mu\nu} = \text{Tr}\{\gamma_5(-\not{p}_2 + m_2)[\not{\gamma}_\nu - \frac{(p_1 - p_2)_\nu}{W_V}](\not{p}'_1 + m_1)\gamma_\mu(\not{p}_1 + m_1)\}K_\alpha K_\beta,$$

$$s_{b\alpha\beta\mu\nu} = \text{Tr}\{\gamma_5(-\not{p}_2 + m_2)\gamma_\mu(-\not{p}'_2 + m_2)[\not{\gamma}_\nu - \frac{(p_1 - p_2)_\nu}{W_V}](\not{p}_1 + m_1)\}K_\alpha K_\beta,$$

$N_1 = p_1^2 - m_1^2 + i\epsilon$, $N'_1 = p'_1^2 - m_1^2 + i\epsilon$, $N_2 = p_2^2 - m_2^2 + i\epsilon$ and $N'_2 = p'_2^2 - m_2^2 + i\epsilon$. The momentum $p_i$ is decomposed as $(p, p_1^\perp)$ in the light-front frame. One needs to integrate over $p_1^\perp$ by a contour integration [36, 37]. Here we have set the relations among all relevant
momenta as $P = P + P'$, $P = p_1 + p_2$, $P' = p'_1 + p'_2 q = P - P'$, $p_2 = p'_2$ for Fig (a); $p_1 = p'_1$ for Fig (b). Then it is easy to find $s^a_{\alpha\beta\mu\nu} = s^b_{\alpha\beta\mu\nu}$. The integration contour is closed in the upper plane of the complex $p'_1$ for the first term in Eq.(2) and in the lower plane for the second term. Then the first term after the integration turns into

$$
\int d^4p_1 \frac{H^{(1)2}H_V}{N_1N_2N_1} s^a_{\alpha\beta\mu\nu} \varepsilon^{\mu} \varepsilon^{\alpha\beta} \to -i\pi \int d^4p_1 \frac{h^{(1)2}h_V}{x_2N_1N_1} s^a_{\alpha\beta\mu\nu} \varepsilon^{\mu} \varepsilon^{\alpha\beta},
$$

where

$$
h^{(1)2} = (M^2 - M_0^2) \sqrt{\frac{x_1x_2}{N_c}} \frac{1}{M_0^2} \phi(nS),
$$

$$
h_V = (M'^2 - M_0'^2) \sqrt{\frac{x_1x_2}{N_c}} \frac{1}{\sqrt{2M_0'}} \phi'(nS) \text{ for } ^3S_1,
$$

$$
(M'^2 - M_0'^2) \sqrt{\frac{x_1x_2}{N_c}} \frac{1}{\sqrt{2M_0'}} \phi'(nD) \text{ for } ^3D_1,
$$

$$
\hat{N}_1^{(i)} = x_1^{(i)} (M'^2 - M_0'^2),
$$

$$
\phi'(nD) = \frac{\sqrt{6[M_0'^2 - (m_1 - m_2)^2][M_0'^2 - (m_1 + m_2)^2]}}{12\sqrt{5}M_0'^2} \phi(nS).
$$

In the formula, $W_V$ and $p_{1\mu}$, $p_{1\mu}p_{1\nu}$, $p_{1\mu}p_{1\nu}p_{1\alpha}$, $p_{1\mu}p_{1\nu}p_{1\alpha}p_{1\beta}$ in $s^a_{\alpha\beta\mu\nu}$ must be replaced by the appropriate quantities as discussed in Ref.[37] to include the contributions of the zero modes, for example

$$
W_V \to w_V = M_0 + m_1 + m_2 \text{ for } ^3S_1,
$$

$$
\frac{M_0^2 - (m_1 + m_2)^2}{2M_0 + m_1 + m_2} \text{ for } ^3D_1,
$$

$$
p_{1\mu} \to \frac{x_1}{2} P_\mu + \left( \frac{x}{2} - \frac{p_1^0 q_1}{q^2} \right) q_\mu,
$$

$$
\text{......}
$$

with $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\mu}$ being identical to $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\mu}$ for the maximally transverse polarized state $(m = \pm J)$ and $s^a_{\alpha\beta\mu\nu}$ changes into $s^a_{\alpha\beta\mu\nu}$. More details about the derivation can be found in Ref.[37]. After the replacement, $s^a_{\alpha\beta\mu\nu}$ is decomposed into

$$
\hat{s}^a_{\alpha\beta\mu\nu} = F_1 (\varepsilon_{\beta\mu\rho\omega} P^\rho q^\omega g_{\alpha\nu} + \varepsilon_{\alpha\mu\rho\omega} P^\rho q^\omega g_{\beta\nu}) + (F_2 + F_3) \varepsilon_{\nu\mu\rho\omega} P^\rho q^\omega q_\alpha q_\beta
$$

$$
+ F_4 (\varepsilon_{\beta\mu\rho\omega} P^\rho q^\omega q_\alpha q_\nu + \varepsilon_{\alpha\mu\rho\omega} P^\rho q^\omega q_\beta q_\nu),
$$

with

$$
F_1 = \frac{-4iA_1^{(4)}}{W_V},
$$

$$
F_2 = 2iA_4^{(2)} + 2iA_2^{(2)} - 4iA_3^{(2)},
$$

5
\[ F_3 = \frac{4iA_4^{(4)} - 4iA_4^{(2)} + 8iA_3^{(4)}}{W_V}, \]
\[ F_4 = \frac{4iA_2^{(3)} - 4iA_3^{(2)} + 4iA_4^{(2)} - 4iA_4^{(4)}}{W_V}, \] (5)

where \( A_i^{(j)}(i = 1 \sim 4, j = 1 \sim 4) \) are defined in the appendix.

We define the form factors as following

\[ f_1 = \frac{eeq}{16\pi^3} \int dx_1 d^2 p_\perp \frac{F_1 \phi'}{\sqrt{2}\beta^2 M_0'M_0'} \frac{(x_1 + x_2)}{x_1 x_2}, \]
\[ f_2 = \frac{eeq}{16\pi^3} \int dx_1 d^2 p_\perp \frac{F_2 \phi'}{\sqrt{2}\beta^2 M_0'M_0'} \frac{(x_1 + x_2)}{x_1 x_2}, \]
\[ f_3 = \frac{eeq}{16\pi^3} \int dx_1 d^2 p_\perp \frac{F_3 \phi'}{\sqrt{2}\beta^2 M_0'M_0'} \frac{(x_1 + x_2)}{x_1 x_2}, \]
\[ f_4 = \frac{eeq}{16\pi^3} \int dx_1 d^2 p_\perp \frac{F_4 \phi'}{\sqrt{2}\beta^2 M_0'M_0'} \frac{(x_1 + x_2)}{x_1 x_2}, \] (6)

which will be numerically evaluated in next section.

With these form factors the amplitude is obtained as

\[ A_\mu = -[f_1(\varepsilon_{\beta\mu\nu}\mathcal{P}^\rho q^\omega g_{\alpha\nu} + \varepsilon_{\alpha\mu\nu}\mathcal{P}^\rho q^\omega g_{\beta\nu}) + (f_2 + f_3)\varepsilon_{\nu\mu\rho\omega}\mathcal{P}^\rho q^\omega q_{\alpha\beta} + f_4(\varepsilon_{\beta\mu\rho\omega}\mathcal{P}^\rho q^\omega q_{\alpha\nu} + \varepsilon_{\alpha\mu\rho\omega}\mathcal{P}^\rho q^\omega q_{\beta\nu})]\varepsilon^\nu \varepsilon^\alpha \varepsilon^\beta. \] (7)

## III. NUMERICAL RESULT FOR \( X(3872) \to J/\psi \gamma \) AND \( \psi(2S)\gamma \)

With the formulas derived in section II we can calculate the decay rates for \( X(3872) \to \psi(1S, 2S)\gamma \) in the light-front quark model. In fact the main task is to evaluate these form factors \( f_1, f_2, f_3 \) and \( f_4 \) at \( q^2 = 0 \).

We set \( m_c = 1.4 \text{ GeV} \) following Ref.\[37] at first and will discuss the dependence of our results on the choice in later part of the paper. Then we need to fix the parameter \( \beta \) in the wavefunction. Generally in LFQM, we can fix the \( \beta \) value by fitting the decay constant of a vector meson, thus in this work, we employ \( J/\psi \) as the meson. The formula for calculating the decay constant of a vector meson was given in Refs.\[36, 37]:

\[ f_V = \frac{\sqrt{N_c}}{4\pi^3 M} \int dx_1 \int d^2 p_\perp \frac{\phi(nS)}{\sqrt{2x(1-x)M_0}} \left[ xM_0^2 - m_1(m_1 - m_2) - p_\perp^2 + \frac{m_1 + m_2}{M_0 + m_1 + m_2}p_\perp^2 \right], \] (8)

where \( m_1 = m_2 = m_c \) and other notations are collected in the appendix.

Using the data of \( J/\psi \to e^+e^- \)\[47] we extract the decay constant of \( J/\psi \): \( f_{J/\psi} = 416 \pm 5 \text{ MeV} \) by which we fix the model parameter \( \beta \) as \( 0.631 \pm 0.005 \text{GeV} \). The masses of \( X(3872) \), \( J/\psi \) and \( \psi(2S) \) are chosen according to Ref.\[47].
TABLE I: the form factors \( f_1, f_2, f_3, \) and \( f_4 \)

| Decay Mode                      | \( f_1 \)         | \( f_2 \)         | \( f_3 \)         | \( f_4 \)         |
|--------------------------------|------------------|------------------|------------------|------------------|
| \( X(3872) \rightarrow J/\psi \gamma \) | -0.0146 ± 0.0002 | 0.146 ± 0.0003  | -0.0092 ± 0.0001 | 0.0180 ± 0.0001  |
| \( X(3872) \rightarrow \psi(2S) \gamma \) | -0.0157 ± 0.0002 | 0.342 ± 0.006   | -0.0144 ± 0.0004 | 0.0352 ± 0.0002  |
| \( X(3872) \rightarrow \psi(2S - 1D) \gamma \) | -0.0594 ± 0.0004 | 0.330 ± 0.005   | -0.0396 ± 0.0007 | 0.0800 ± 0.005   |

The numerical values of the form factors for \( X(3872) \rightarrow J/\psi \gamma \) are listed in Tab. I. Then we obtain \( \Gamma(X(3872) \rightarrow J/\psi \gamma) = (3.54 \pm 0.12) \times 10^{-6} \text{GeV} \). The recent measurement sets only an upper bound on the total width of \( X(3872) \), so that we cannot determine accurate branching ratios for the radiative decays, but only a bound which is listed in Table III.

Theoretically, Jia et.al.\[24\] studied the same transition within the pNRQCD framework where several phenomenological potentials were adopted, in their work the quantum number of \( X(3872) \) was assigned as \( 2^{-+} \). They obtained \( \Gamma(X(3872) \rightarrow J/\psi \gamma) = (3.11 \sim 4.78) \times 10^{-6} \text{GeV} \) corresponding to different potential models which is in accordance with our result. For comparing theoretical results with the data we present the experimental data and theoretical prediction on the branching ratios \( BR(X(3872) \rightarrow J/\psi \gamma) \) in Table III. The theoretical prediction of \( BR(X(3872) \rightarrow J/\psi \gamma) \) is slightly lower than the Babar’s data but quite close to the Belle’s data as long as we suppose \( X(3872) \) as a \( 2^{-+} \) state.

Now we turn to \( X(3872) \rightarrow \psi' \gamma \). We will deal with the cases where \( \psi' \) is regarded as a pure \( 2S \) charmonium \( \psi(2S) \)\[47\] or a mixture of \( 2S \) and \( 1D \) charmonium \( \psi(2S - 1D) \)\[46, 48, 49\] respectively and obtain the corresponding numerical results. First we assume \( \psi' \) to be a pure \( 2S \) state to calculate the branching ratio of the transition \( X(3872) \rightarrow \psi' \gamma \). Here we need to use the wave function for the radial excited state. In Ref.\[41\] we notice that keeping the orthogonality among the \( nS \) states of heavy quarkonia and requiring the theoretical predictions on the decay constants of heavy quarkonia to be in agreement with that obtained from the data of leptonic decays, the wavefunction of the radial excited states \( nS \) \( (n > 1) \) should be modified \[41\]. In Eq.A(2) of the appendix we present the explicit form of the modified \( 2S \) wavefunction. With those wavefunction we obtain the form factors for \( X(3872) \rightarrow \psi(2S) \gamma \) which are listed in Tab. I. Now we proceed to evaluate the decay rate of \( X(3872) \rightarrow \psi(2S) \gamma \). We obtain \( \Gamma(X(3872) \rightarrow \psi(2S) \gamma) = (2.44 \pm 0.10) \times 10^{-8} \text{GeV} \). By the same assumption, the authors of Ref.\[24\] obtained \( \Gamma(X(3872) \rightarrow \psi(2S) \gamma) = 1.7 \sim 2.9 \times 10^{-8} \text{GeV} \).

The predicted branching ratio of \( BR(X(3872) \rightarrow \psi(2S) \gamma) \) presented in Table III is much lower than the Babar’s data, but does not contradict to the upper bound set by the Belle data.

The authors of Ref.\[24\] noticed that the mixing of \( \psi(2S) \) and \( \psi(1D) \) in \( \psi' \) can enhance the rate of \( \Gamma(X(3872) \rightarrow \psi' \gamma) \) remarkably. In this work, we also evaluate the impact of the mixing in \( \psi' \) on \( \Gamma(X(3872) \rightarrow \psi' \gamma) \) with the same mixing scheme and mixing angle \( \theta = 12^\circ \).
TABLE II: Theoretical predictions on the decay widths of the radiative decay of \(X(3872)\), which is regarded as a \(^1D_2\) charmonium

| \(\Gamma(X(3872) \to J/\psi\gamma)\) | \([24]\) | our results |
|----------------------------------|--------|-------------|
| \(1.7 \sim 2.9 \times 10^{-8}\)GeV | \(1.7 \sim 2.9 \times 10^{-8}\)GeV | \(1.7 \sim 2.9 \times 10^{-8}\)GeV |
| \(\Gamma(X(3872) \to \psi(2S)\gamma)\) | \(4.9 \sim 5.6 \times 10^{-7}\)GeV | \(4.9 \sim 5.6 \times 10^{-7}\)GeV |

TABLE III: Experimental data and theoretical predictions on the branching ratios of the radiative decay of \(X(3872)\), which is regarded as a \(^1D_2\) charmonium

| \(BR(X(3872) \to J/\psi\gamma)\) | \([29]\) | \([30]\) | \([24]\) | our results\(^a\) |
|----------------------------------|--------|--------|--------|----------------|
| \(\geq 8.75 \times 10^{-3}\) | \(\geq 5.56 \times 10^{-3}\) | \(\geq 1.35 \sim 2.08 \times 10^{-3}\) | \(\geq 1.59 \times 10^{-3}\) |
| \(\geq 2.97 \times 10^{-2}\) | \(\sim 1.01 \times 10^{-2}\) | \(\geq 7.39 \sim 12.6 \times 10^{-6}\) | \(\geq 10.6 \times 10^{-6}\) |
| \(\geq 2.13 \sim 2.43 \times 10^{-4}\) | \(\geq 2.13 \sim 2.43 \times 10^{-4}\) | \(\geq 1.59 \times 10^{-4}\) |

\(^a\) the total width \(\Gamma(X(3872)) < 2.3 \text{ MeV}\) are used.

as given in \([24]\). We calculate the form factors which are also presented in table I and obtain \(\Gamma(X(3872) \to \psi(2S - 1D)\gamma) = (3.65 \pm 0.11) \times 10^{-7}\)GeV. Our result is in accordance with that of Ref. \([24]\) which is within a range of \((4.9 \sim 5.6) \times 10^{-7}\)GeV. One can notice that even though the mixing of 2S and 1D is taken into account the theoretical prediction on the branching ratio of \(\Gamma(X(3872) \to \psi'\gamma)\) is still lower than the Babar’s data by two orders. Moreover, we obtain the ratios \(\frac{BR(X(3872) \to \psi(2S)\gamma)}{BR(X(3872) \to J/\psi\gamma)} = 0.069\) and \(\frac{BR(X(3872) \to \psi(2S - 1D)\gamma)}{BR(X(3872) \to J/\psi\gamma)} = 0.1\) which obviously contradict to the Babar’s data.

Now we turn to study the dependence of the theoretical predictions on \(m_c\) for the three radiative decay modes. We let \(m_c\) vary within a reasonable range from 1.3GeV to 1.6GeV and see how the widths of \(\Gamma(X(3872) \to J/\psi\gamma)\), \(\Gamma(X(3872) \to \psi(2S)\gamma)\) and \(\Gamma(X(3872) \to \psi(2S - 1D)\gamma)\) change. Indeed, it is noted that when we do so, we need to refit the \(\beta\) parameter for different \(m_c\) values using Eq.\([5]\). The dependence of our results on \(m_c\) is shown in Fig.\([2]\). For example, when \(m_c\) varies from 1.3GeV to 1.6GeV the theoretical prediction on \(\Gamma(X(3872) \to \psi(2S - 1D)\gamma)\) changes from \(4.21 \times 10^{-7}\) to \(2.82 \times 10^{-7}\). It is about 33% change, but the order of magnitude remains unchanged. The main goal of the work is to look for a criterion to determine the spin-orbit-identity of \(X(3872)\). As aforementioned, the Babar datum is several orders larger than the prediction, thus the theoretical uncertainty about \(m_c\) does not influence the qualitative conclusion.
IV. DISCUSSION AND OUR CONCLUSION

Determining the structures of the newly observed resonances is an interesting and difficult job, but the study is highly rewarding, because one can eventually identify if they are exotic states. Retrospecting the history of searching glueballs at the charm energy range, the results are indeed discouraging. The new discovery offers an opportunity to recognize the hadrons which are not existing in the basic representations of $SU(3)$, but as exotic states or even mixtures of glueball and other states. As a matter of fact, the charm energy range may be an abundant mine for such new exotic resonances. However, before one can identify any of them as an exotic state, he has to thoroughly investigate if it is an excited state of regular charmonium no matter it is radially or orbitally excited. $X(3872)$ is such an unknown resonance. Many authors consider $X(3872)$ to be a molecular state or tetraquark, instead, it is also suggested that it may be a $2^{-+}$ charmonium.

In this work we calculate the rates of the radiative decays of $X(3872)$ in the LFQM. We first deduce the amplitude and corresponding form factors in the amplitude of $2^{-+} \rightarrow 1^{--}\gamma$ with the LFQM. Then we suppose the $X(3872)$ is a pure $2^{-+}$ charmonium and calculate the widths of $X(3872) \rightarrow J/\psi\gamma$ and $X(3872) \rightarrow \psi'\gamma$. We obtain $\Gamma(X(3872) \rightarrow J/\psi\gamma) = (3.54 \pm 0.12) \times 10^{-6}\text{GeV}$, $\Gamma(X(3872) \rightarrow \psi(2S)\gamma) = (2.44 \pm 0.10) \times 10^{-8}\text{GeV}$, $\Gamma(X(3872) \rightarrow \psi(2S-1D)\gamma) = (3.65 \pm 0.11) \times 10^{-7}\text{GeV}$.

Our theoretical predictions on $\Gamma(X(3872) \rightarrow \psi'\gamma)$ is obviously inconsistent with the data of the Babar collaboration, but that on $BR(X(3872) \rightarrow J/\psi\gamma)$ is consistent with data within the error tolerance.

As a matter of the fact, the transition matrix element eventually depends on an overlap-
ping integral of the wavefunctions of the initial and final hadrons. Thus the node structure of the wavefunctions is extremely important. Generally, the state with the principal quantum number \( n \) possesses \( n - 1 \) nodes, thus the overlapping integral between \( X(3872) \) which is supposed to be an orbital excited state with \( n = 1 \) does not possess a node, and neither \( J/\psi(1S) \), but \( \psi(2S) \) does have a node. Therefore, the integrand in the overlapping integration of \( X(3872) \) and \( J/\psi \) is always positive, instead, in the overlapping integration between wavefunctions of \( X(3872) \) and \( \psi(2S) \), the integrand is positive on the left side of the node of the 2S state and negative on the right side of the node. Thus a cancellation effect exists. Therefore by a common sense, the transition \( X(3872) \rightarrow \psi(2S) + \gamma \) is suppressed and \( B(X(3872) \rightarrow \psi(2S) + \gamma) \) should be smaller than \( B(X(3872) \rightarrow J/\psi + \gamma) \). When the 2S − 1D mixing for \( \psi' \) is considered, the overlapping integration is decomposed into two integrations as

\[
f_i = \frac{e e_q}{16\pi^3} \int dx_1 d^2 p_\perp F_i \frac{x_1 + x_2}{x_1 x_2} \phi[\cos \theta \phi'(2S) - \sin \theta \phi'(1D)]
\]

where \( \phi'(2S), \phi'(1D) \), \( \phi \) are the radial parts of the wavefunctions for \( \psi(2S), \psi(1D) \) which are the ingredients in \( \psi' \), and \( X(3872), \theta \) is the mixing angle of 2S and 1D states in \( \psi' \). There is a node in \( \phi'(2S) \), but not in \( \phi'(1D) \), so the first overlapping integration is suppressed as discussed before, but not for the second one. Therefore, the predicted branching ratio is enhanced in comparison with the pure 2S state assignment for \( \psi' \).

The data of the Babar and Belle collaborations on \( X(3872) \rightarrow \psi' + \gamma \) are obviously different, so that more accurate measurements are needed.

Our conclusion is that if the branching ratio of \( X(3872) \rightarrow \psi' + \gamma \) is indeed as large as the Babar group measured, the 2−+ charmonium assignment of \( X(3872) \) should be ruled out.

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**Appendix A: Notations**

Here we list some variables appearing in the context. The incoming meson in Fig. 1 has the momentum \( P = p_1 + p_2 \) where \( p_1 \) and \( p_2 \) are the momenta of the off-shell quark and antiquark and

\[
\begin{align*}
p_1^+ &= x_1 P^+, & p_2^+ &= x_2 P^+, \\
p_1^\perp &= x_1 P^\perp + p^\perp, & p_2^\perp &= x_2 P^\perp - p^\perp, & (A1)
\end{align*}
\]
with \(x_i\) and \(p_\perp\) are internal variables and \(x_1 + x_2 = 1\).

The variables \(M_0, \tilde{M}_0\) and \(\hat{N}_1\) are defined as

\[
M_0^2 = \frac{p_\perp^2 + m_1^2}{x_1} + \frac{p_\perp^2 + m_2^2}{x_2},
\]

\[
\tilde{M}_0 = \sqrt{M_0^2 - (m_1 - m_2)^2},
\]

\[
\phi(1S) = 4(\frac{\pi}{\beta^2})^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp(-\frac{p_\perp^2 + p_\perp^2}{2\beta^2}),
\]

\[
\phi(2S) = 4(\frac{\pi}{\beta^2})^{3/4} \sqrt{\frac{\partial p_z}{\partial x_2}} \exp\left(-\frac{2^4 p_\perp^2 + p_\perp^2}{2\beta^2}\right)(a_2 - b_2 \frac{p_\perp^2 + p_\perp^2}{\beta^2}).
\]  
(A2)

with \(p_\perp = \frac{x_2 M_0}{2} - \frac{m_1^2 + m_2^2}{2x_2 M_0} \cdot \delta = 1/1.82\), \(a_2 = 1.88684\) and \(b_2 = 1.54943\).

The \(A_{ij}(i = 1 \sim 4, j = 1 \sim 4)\) are

\[
A_1^{(1)} = \frac{x_1}{2}, \quad A_2^{(1)} = A_1^{(1)} - \frac{p_\perp \cdot q_\perp}{q^2}, \quad A_1^{(2)} = -p_\perp^2 - \frac{(p_\perp \cdot q_\perp)^2}{q^2},
\]

\[
A_2^{(2)} = (A_1^{(1)})^2, \quad A_3^{(2)} = A_1^{(2)} A_2^{(2)}, \quad A_4^{(2)} = (A_2^{(1)})^2 - \frac{A_1^{(2)}}{q^2},
\]

\[
A_1^{(3)} = A_1^{(1)} A_1^{(2)}, \quad A_2^{(3)} = A_2^{(1)} A_2^{(2)}, \quad A_3^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_4^{(3)} = A_2^{(1)} A_2^{(2)},
\]

\[
A_1^{(4)} = \frac{(A_1^{(2)})^2}{3}, \quad A_2^{(4)} = A_1^{(1)} A_1^{(3)}, \quad A_3^{(4)} = A_1^{(1)} A_2^{(3)}, \quad A_4^{(4)} = A_2^{(1)} A_3^{(3)} - \frac{A_1^{(4)}}{q^2}.\]  
(A3)
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