An estimate of a lower bound on the masses of mirror baryons

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Abstract

We consider the most favorable conditions to indirectly observe the mixing of ordinary and mirror hadrons in non-leptonic and weak radiative decays of hyperons. This allows us to set a lower bound on the masses of mirror baryons. This bound turns out to be impressively high, of the order of 10^6 GeV.

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In this paper we shall obtain a lower bound on the masses of mirror hadrons under what are probably the most favorable conditions to indirectly observe such hadrons in low energy physics. Lee and Yang in their pioneering paper [1] about parity violation in nature discussed in detail the existence of mirror matter as a possible framework to help us understand such violation. Ever since, mirror matter has been discussed at different levels, including extensions of the minimal standard model [2]. Not only hadrons and leptons (or quarks and leptons) might have mirror partners but the electroweak gauge bosons too would have mirror partners. The photon might be accompanied by a massless paraphoton, which however could not couple to ordinary fermions [3]. Extensions of the standard model doubling the electroweak gauge group and the fermion content have been discussed in detail in the literature [4]. Such extensions might provide a solution to the problem of strong violation of $\mathcal{C}\mathcal{P}$. Of all the possibilities considered the most attractive one and still in the spirit of Ref. [1], which we shall refer to as manifest mirror symmetry, is the one in which both ordinary matter (hadrons and leptons) and mirror matter (mirror hadrons and leptons) share the same strong and electromagnetic interactions. Other alternatives, even if they keep the same particle content, are intuitively less attractive.

Manifest mirror symmetry opens the possibility that mirror hadrons become observable in low energy physics through their mixing with ordinary hadrons. The reason for this is that, since they share strong and electromagnetic interactions with ordinary hadrons, even if the mixing angles are very small they still could lead to observable effects that might compete with weak interactions process. In previous papers [5,6] we studied how such mixing might mimic non-leptonic and weak radiative decays of hyperons (NLDH and WRDH, respectively). This mechanism, which we referred to as a priori mixing, might explain the $|\Delta I| = 1/2$ rule observed in such decays, in case (for some as yet unknown reason) the enhancement of the $W$-boson contributions to NLDH and WRDH could not be produced within the minimal standard model. We remind the reader that despite the effort invested in this direction no final answer to this problem has been produced so far, although of course the possibility of obtaining it remains quite open. If this last were to be the case then the data on NLDH and WRDH should be saturated by the $W$-boson predictions and then the competing a priori mixing contributions should be suppressed necessarily by reducing the mixing angles. It is this situation that will allow us to set a lower bound to the mass of mirror hadrons in the manifest case.

Unfortunately, the mixing of ordinary and mirror hadrons cannot be derived reliably starting at the quark level with a model, such as the one discussed by Barr, Chang, and Senjanović in Ref. [4], due to our current inability to perform QCD calculations in the non-perturbative regime of low energy physics. One is therefore forced to introduce an ansatz to derive these a priori mixing, as discussed in detail in Ref. [5]. Nevertheless, at least for illustration purposes, mixing at the quark level can be shown to lead to such an ansatz [7].

We shall not repeat the details of the ansatz of Ref. [5]. Instead, we shall discuss in more detail how the mixing angles between ordinary and mirror hadrons appear in the physical hadrons (mass eigenstates) when the mass matrices of hadrons are diagonalized. To be specific and to keep our analysis simple, we shall discuss only the proton and $\Sigma^+$ system. This will be sufficient for our purposes.

In this system the rotation matrix $R$ to get the physical ordinary $p_\text{ph}$ and $\Sigma^+_{\text{ph}}$ and mirror $\hat{p}_\text{ph}$ and $\hat{\Sigma}^{+\text{ph}}$ is a $4 \times 4$ matrix. It can be split into the product of two $4 \times 4$ matrices,
\( R = R^1 R^0. \) \( R^0 \) contains the large Cabibbo-Kobayashi-Maskawa [8,9] angles and \( R^1 \) contains the necessarily small mixing angles that will connect both the ordinary and mirror hadrons. \( R^0 \) is block diagonal and it contains two non-zero \( 2 \times 2 \) submatrices on its diagonal. The upper-left block actually contains only the Cabibbo angle and operates to yield the flavor and parity eigenstates \( p_s \) and \( \Sigma^+_s \). If no mirror hadrons exist at all this would be reduced to two dimensions and the result would be

\[
\begin{pmatrix}
\bar{p}_s \\
\Sigma^+_s
\end{pmatrix}
\begin{pmatrix}
m^0_p & 0 & 0 \\
0 & m^0_{\Sigma^+_s} & 0
\end{pmatrix}
\begin{pmatrix}
p_s \\
\Sigma^+_s
\end{pmatrix}
\]

(1)

The lower-right block operates analogously to yield the mirror flavor and parity eigenstates \( p_p \) and \( \Sigma^+_p \) in a similar \( 2 \times 2 \) matrix. So, in 4 dimensions the result of applying \( R^0 \) is

\[
\begin{pmatrix}
\bar{p}_s \\
\Sigma^+_s \\
\bar{p}_p \\
\Sigma^+_p
\end{pmatrix}
\begin{pmatrix}
m^0_p & 0 & 0 & 0 \\
0 & m^0_{\Sigma^+_s} & 0 & 0 \\
0 & 0 & \tilde{m}^0_p & 0 \\
0 & 0 & 0 & \tilde{m}^0_{\Sigma^+_s}
\end{pmatrix}
\begin{pmatrix}
p_s \\
\Sigma^+_s \\
p_p \\
\Sigma^+_p
\end{pmatrix}
\]

(2)

The indices \( s \) and \( p \) stand for positive and negative parity, respectively, and strong flavor is identified by the particle symbol. The case (2) contemplates ordinary and mirror matter still disconnected. When both worlds are connected then the initial \( 4 \times 4 \) mass matrix contains two \( 2 \times 2 \) off-diagonal submatrices. In this case the action of \( R^0 \) yields

\[
\begin{pmatrix}
\bar{p}_s \\
\Sigma^+_s \\
\bar{p}_p \\
\Sigma^+_p
\end{pmatrix}
\begin{pmatrix}
m^0_p & \Delta_{11} & \Delta_{12} & 0 \\
0 & m^0_{\Sigma^+_s} & \Delta_{21} & \Delta_{22} \\
\Delta_{11} & \Delta_{21} & \bar{m}^0_p & 0 \\
\Delta_{12} & \Delta_{22} & 0 & \tilde{m}^0_{\Sigma^+_s}
\end{pmatrix}
\begin{pmatrix}
p_s \\
\Sigma^+_s \\
p_p \\
\Sigma^+_p
\end{pmatrix}
\]

(3)

instead of (2). We are neglecting CP violation and therefore the \( \Delta_{ij} \) can be taken as real numbers. Notice that the zero entries in the two diagonal submatrices still remain\[.\] However, \( R^0 \) does affect the two off-diagonal \( 2 \times 2 \) submatrices in the initial mass matrix. The effect of \( R^0 \) on them is already incorporated in the \( \Delta_{ij} \). The role of the rotation \( R^1 \) is to finally diagonalize the full \( 4 \times 4 \) mass matrix of Eq. (3) and it is this final step that leads to the physical \( p \) and \( \Sigma^+ \), which contain flavor and parity mixing.

\( R^1 \) is in principle a complicated matrix with many angles. However, one expects that the connection between the ordinary and the mirror worlds be very small, because the mirror

\[1\text{This is a very important point. The CKM rotations cannot lead to flavor and parity violation in strong and electromagnetic interactions. So, for example, when these rotations are performed at the hadron level, as initially proposed by Cabibbo, it is indispensible that the matrix containing magnetic form factors remains diagonal along with the mass matrix. This is possible because CKM rotations connect one kind of matter with itself only and flavor and parity eigenstates can be defined after CKM rotations (\( p_s \), etc.). This is not the case for } R^1 \text{ and the reason is that it only connects worlds of different kind.}\]
world would be far away (that is, with very heavy masses). Therefore, one must necessarily require the inequality

\[ \hat{m}_p, \hat{m}_\Sigma^+ \gg \Delta_{ij}, m_p^0, m_\Sigma^+ \]  

(4)

This allows us then to keep \( R^1 \) to first order in the angles, namely,

\[ (R^1)_{ij} \simeq \delta_{ij} + \epsilon_{ij} \]  

(5)

where \( \epsilon_{ij} = -\epsilon_{ji}, \delta_{ij} \) is the Kronecker delta, and \( i, j = 1, \ldots, 4 \). There are only six relevant angles in \( R^1 \).

The action of \( R^1 \) upon the matrix \( M \) in the sandwich of (3) leads to a diagonal matrix \( M_D \), i.e., \( R^1 M R^1\) = \( M_D \), sandwiched between the physical states and whose eigenvalues are the physical masses, namely,

\[ \begin{pmatrix} \hat{p}_{ph} & \Sigma_{ph}^+ & \hat{p}_{ph} & \Sigma_{ph}^- \end{pmatrix} \begin{pmatrix} m_p & 0 & 0 & 0 \\ 0 & m_\Sigma^+ & 0 & 0 \\ 0 & 0 & \hat{m}_p & 0 \\ 0 & 0 & 0 & \hat{m}_\Sigma^+ \end{pmatrix} \begin{pmatrix} p_{ph} \\ \Sigma_{ph}^+ \\ \hat{p}_{ph} \\ \Sigma_{ph}^- \end{pmatrix} \]  

(6)

where

\[ p_{ph} = p_s + \epsilon_{12} \Sigma_s^+ + \epsilon_{13} p_p + \epsilon_{14} \Sigma_p^+ \]  

(7a)

\[ \Sigma_{ph}^+ = \Sigma_s^+ - \epsilon_{12} p_s + \epsilon_{23} p_p + \epsilon_{24} \Sigma_p^+ \]  

(7b)

and analogous expressions for \( \hat{p}_{ph} \) and \( \Sigma_{ph}^+ \). This diagonalization yields 16 equations. Keeping the lowest relevant order in each equation, remembering that \( \hat{m}_p^0 \) and \( \hat{m}_\Sigma^+ \) are order zero and \( \epsilon_{ij}, \Delta_{ij}, m_p^0, m_\Sigma^+ \) are first order, one obtains \( m_p \simeq m_p^0, m_\Sigma^+ \simeq m_\Sigma^+, \hat{m}_p \simeq \hat{m}_p^0, \hat{m}_\Sigma^+ \simeq \hat{m}_\Sigma^+ \), and

\[- m_p^0 \epsilon_{12} + m_\Sigma^+ \epsilon_{12} + \Delta_{21} \epsilon_{13} + \Delta_{22} \epsilon_{14} + (\Delta_{11} + \hat{m}_p^0 \epsilon_{13}) \epsilon_{23} + (\Delta_{12} + \hat{m}_\Sigma^+ \epsilon_{14}) \epsilon_{24} = 0 \]  

(8a)

\[- m_p^0 \epsilon_{12} + m_\Sigma^+ \epsilon_{12} + \Delta_{11} \epsilon_{23} + \Delta_{12} \epsilon_{24} + (\Delta_{21} + \hat{m}_p^0 \epsilon_{23}) \epsilon_{13} + (\Delta_{22} + \hat{m}_\Sigma^+ \epsilon_{24}) \epsilon_{14} = 0 \]  

(8b)

\[ \Delta_{11} + \hat{m}_p^0 \epsilon_{13} = 0 \]  

(8c)

\[ \Delta_{12} + \hat{m}_\Sigma^+ \epsilon_{14} = 0 \]  

(8d)

\[ \Delta_{21} + \hat{m}_p^0 \epsilon_{23} = 0 \]  

(8e)

\[ \Delta_{22} + \hat{m}_\Sigma^+ \epsilon_{24} = 0 \]  

(8f)
\[- \hat{m}_p^0 \epsilon_{34} + \hat{m}_0^{\Sigma^+} \epsilon_{34} = 0 \quad (8g)\]

Eq. (8g) is just a rearrangement of Eq. (8a), this rearrangement will be useful for a later discussion. The remaining five equations just repeat Eqs. (8c–8g). From all these equations and still to lowest order, one obtains

\[|\epsilon_{12}| = \left| \frac{\Delta_{11} \epsilon_{23} + \Delta_{12} \epsilon_{24}}{m_{\Sigma^+} - m_p} \right| \quad (9a)\]

\[|\epsilon_{12}| = \left| \frac{\Delta_{22} \epsilon_{14} + \Delta_{21} \epsilon_{13}}{m_{\Sigma^+} - m_p} \right| \quad (9b)\]

\[|\epsilon_{13}| = \left| \frac{\Delta_{11}}{\hat{m}_p} \right| \quad (9c)\]

\[|\epsilon_{14}| = \left| \frac{\Delta_{12}}{\hat{m}_p} \right| \quad (9d)\]

\[|\epsilon_{23}| = \left| \frac{\Delta_{21}}{\hat{m}_p} \right| \quad (9e)\]

\[|\epsilon_{24}| = \left| \frac{\Delta_{22}}{m_{\Sigma^+}} \right| \quad (9f)\]

and \(\epsilon_{34} \simeq 0\). The ordering of these equations corresponds to the ordering of Eqs. (8).

We shall need absolute values only. The angles \(\epsilon_{12}, \epsilon_{14},\) and \(\epsilon_{23}\) may give observable effects in NLDH and WRDH. Their values were obtained in Refs. [5] and [6], assuming that the mixings of Eqs. (7) give contributions that saturate the corresponding NLDH and WRDH available data. These are the most favorable conditions to observe the mixing with mirror hadrons. The contributions of the W-boson were assumed not to be enhanced, i.e., its \(\Delta I = 1/2\) contributions were assumed to be at the same level of its \(\Delta I = 3/2\) contributions and, accordingly, were neglected.

The magnitudes of the angles obtained are

\[|\epsilon_{12}| = (4.9 \pm 2.0) \times 10^{-6} \quad (10)\]

\[|\epsilon_{14}| = (0.22 \pm 0.09) \times 10^{-6} \quad (11)\]

\[|\epsilon_{23}| = (0.26 \pm 0.09) \times 10^{-6} \quad (12)\]

In these last two references these angles were identified as \(\sigma, \delta,\) and \(\delta'\), respectively.

Notice that \(|\epsilon_{12}|\) is an order of magnitude larger than \(|\epsilon_{14}|\) and \(|\epsilon_{23}|\). The angles \(|\epsilon_{13}|\) and \(|\epsilon_{24}|\) have not been determined. However, we expect them to be at most of the same
order of magnitude as the first three. To obtain a lower bound on mirror baryon masses we must assume that \(|\epsilon_{13}|\) and/or \(|\epsilon_{24}|\) are of the order of magnitude of \(|\epsilon_{12}|\) namely, \(|\epsilon_{13}| \simeq |\epsilon_{12}|\) and/or \(|\epsilon_{24}| \simeq |\epsilon_{12}|\). Looking back at Eqs. (9a) and (9b) we may conclude then that

\[
\frac{|\Delta_{ab}|}{m_{\Sigma^+} - m_p} \simeq 1
\]  

(13)

where the pair of indices \(ab\), take the values 12 or 21. To get Eq. (13) notice that the first summands in Eqs. (9a) and (9b) involve the smaller angles \(|\epsilon_{14}|\) and \(|\epsilon_{23}|\) of Eqs. (11) and (12). If these summands were to dominate the numerators of Eqs. (9a) and (9b) then the indices \(ab\) would take the values 11 or 22 and the right hand side of (13) would become 10. This option leads to a higher lower bound and we discard it, accordingly.

Using the value of the mass difference \(m_{\Sigma^+} - m_p \simeq 0.25\text{GeV}\), Eq. (13) gives \(|\Delta_{12}| \simeq 0.25\text{GeV}\). Substituting this into Eq. (9d) and using the central value of Eq. (12) we obtain an order of magnitude lower bound for \(m_p\),

\[
m_p \geq \mathcal{O}(10^6\text{GeV}).
\]  

(14)

From Eq. (9e) a similar bound is obtained.

At this point it is important to emphasize in what sense Eq. (14) is to be understood as a lower bound. So far, it is possible to assume that the \(\Delta_{ij}\) of Eq. (13) may become smaller, to the extent \(\Delta_{ij} \to 0\). Then the mirror and the ordinary baryons become decoupled. Of course, once decoupled one cannot set any sort of lower bound on mirror baryon masses using information of the ordinary baryon world. This situation corresponds to requiring that mirror matter does not give observable effects in our ordinary matter world. Eq. (14) is to be understood in the opposite sense. That is, what values of the masses of mirror baryons would lead to observable effects in our world?. This clearly requires \(\Delta_{ij} \neq 0\) as used above and then Eq. (14) tells us that a value of below the bound of Eq. (14) may lead to effects in NLDH and WRDH which exceed the level experimentally observed for these decays, even the equality sign exceeds the level predicted by the \(W\) boson when the observed enhancement of its \(\Delta I = 1/2\) contributions is assumed to arise within the minimal standard model. It is in this sense that Eq. (14) represents a lower bound.

To our knowledge this is the only available lower bound on the mass of mirror hadrons. Other bounds on mirror matter refer to the values of their mixing angle with ordinary matter. Such bounds, from precision tests of the standard model, were thoroughly discussed by Langacker, Luo, and Mann in Ref. [10], but no attempt was made there to get bounds on masses of mirror fermions. The bounds for the mixing angles obtained there are around \(3 \times 10^{-2}\), which are much too high compared with Eqs. (10)–(12).

Besides the (rather trivial) way to avoid the lower bound of Eq. (14) that we just discussed, one other way to avoid it is by relaxing the manifest mirror symmetry assumption that we made. Our bound depends crucially on the assumption that mirror matter shares the same strong and electromagnetic (e.m.) interactions with ordinary matter. The question arises then if it is possible to keep such manifest mirror symmetry while making mirror matter much heavier than ordinary matter. We cannot give a rigorous answer to this question. However, symmetry breaking scenarios are conceivable that allow breaking mirror symmetry without affecting the strong and e.m. effective couplings in the mirror sector. The model of Ref. [4] provides useful guidance in this respect. This model is conceived at the quark
level. But, as we discussed in Ref. [7] our phenomenological group-theoretic ansatz of Ref. [3] can be qualitatively derived from a quark level approach. Using the model of Ref. [4] one can show that after breaking mirror (in this case left-right) symmetry one can reconstruct the e.m. current operator of physical quarks as a proper flavor-conserving four-vector which couples to ordinary and mirror physical matter with common charges. Also, since the QCD interactions are described by the same $SU(3)_C$ in direct product with the electroweak sector, the strong interactions of mirror physical hadrons remain the same ones of ordinary hadrons, after mirror matter was made much heavier. Therefore, the answer to the previous question is qualitatively in the affirmative.

The lower bound of Eq. (14) is impressively high. It means that producing mirror matter on earth is way far into the future. Even if one is willing to abandon the manifest mirror symmetry assumption (and allowing mirror masses to become smaller) would not help, because the coupling through strong and e.m. interactions to ordinary matter would be greatly reduced and, since machines on earth would be made out of ordinary matter, it would still be very difficult to produce mirror matter with them. Also, the bound of Eq. (14) means that mirror matter may not be a very good candidate for dark matter in the universe [11], although there always exists the possibility of detecting it in cosmic rays. Whatever the real situation may be, the possibility exists that mirror matter may give unwanted effects in low energy physics if it is too light, and this is what makes (14) a lower bound.

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