Medium modifications of the bound nucleon generalized parton distributions and the quark contribution to the spin sum rule

V. Guzey,1,* A. W. Thomas,1,2,† and K. Tsushima3,‡

1Theory Center, Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA
2College of William and Mary, Williamsburg, Virginia 23178, USA
3Excited Baryon Analysis Center (EBAC) and Theory Center, Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

(Received 5 February 2009; revised manuscript received 6 March 2009; published 26 May 2009)

We estimate the nuclear medium modifications of the quark contribution to the bound nucleon spin sum rule, \( J^q \), as well as separated helicity, \( \Delta \Sigma^q \), and the angular momentum, \( L^q \), contributions to \( J^q \). For the calculation of the bound nucleon generalized parton distributions (GPDs), we use as input the bound nucleon elastic form factors predicted in the quark-meson coupling model. Our model for the bound nucleon GPDs is relevant for incoherent deeply virtual Compton scattering (DVCS) with nuclear targets. We find that the medium modifications increase \( J^q \) and \( L^q \) and decrease \( \Delta \Sigma^q \) compared to the free nucleon case. The effect is large and increases with increasing nuclear density \( \rho \). For instance, at \( \rho = \rho_0 = 0.15 \text{ fm}^{-3} \), \( J^q \) increases by 7\%, \( L^q \) increases by 20\%, and \( \Delta \Sigma^q \) decreases by 17\%. These in-medium modifications of the bound nucleon spin properties are a general feature of relativistic mean-field quark models and may be understood qualitatively in terms of the enhancement of the lower component of the quark Dirac spinor in the nuclear medium.

DOI: 10.1103/PhysRevC.79.055205

PACS number(s): 12.39.–x, 13.40.Gp, 24.85.+p

Properties of hadrons in a nuclear medium are expected to be modified compared to those in a vacuum. This manifests itself in the modifications of quark and gluon parton distributions of the bound nucleon measured in deep inelastic scattering (DIS) with unpolarized nuclear targets [1–5]. Even stronger medium modifications have been predicted for DIS with polarized nuclear targets [6–8]. Possible medium modifications of the bound-nucleon elastic form factors were probed by the polarization transfer measurement in the \(^4\text{He}(e,e')\overline{p})\text{H}\) reaction at the Hall A Jefferson Lab experiment [9,10]. The results of the experiment have been described by either the modified elastic form factors as predicted by the quark-meson coupling (QMC) model [11] or the strong charge-exchange final-state interaction (FSI) [12]. However, such a strong FSI may not be consistent with the induced polarization data—see Ref. [10] for details. In addition to the modification of structure functions (parton distributions) and elastic form factors of the bound nucleon, various static properties of hadrons (masses, magnetic moments, coupling constants) have been predicted to be modified in a nuclear medium (see, e.g., Ref. [13]).

Generalized parton distributions (GPDs) interpolate between parton distributions and elastic form factors [14–17]. Therefore, it is natural to expect that GPDs of the bound nucleon should also be modified in the nuclear medium. An early investigation [18,19] of such modifications in \(^4\text{He}\) assumed that in-medium nucleon GPDs are modified through the kinematic off-shell effects associated with the modification of the relation between the struck quark’s transverse momentum and its virtuality. Recently, we considered incoherent deeply virtual Compton scattering (DVCS) on \(^4\text{He}\), \( \gamma^*\text{He} \rightarrow \gamma p X \), and suggested a model of the bound nucleon GPDs in \(^4\text{He}\), where the GPDs are modified in proportion to the corresponding bound nucleon elastic form factors [20]. In the present work, we extend our approach to an arbitrary nucleus (any nuclear density) and study the medium modifications of the quark contribution to Ji’s [21] spin sum rule, \( J^q \). As in our recent work [20], the present model of the bound nucleon GPDs is relevant for incoherent DVCS (and other incoherent exclusive processes) with nuclear targets, \( \gamma^*A \rightarrow \gamma p X \), where \( A \) denotes the nucleus, \( N \) is the final detected nucleon, and \( X \) is the undetected product of the nuclear breakup. We find that medium modifications increase \( J^q \) and the effect is quite noticeable. The effect increases with increasing nuclear density \( \rho \). For instance, at \( \rho = \rho_0 = 0.15 \text{ fm}^{-3} \) (\( \rho_0 \) is the density of the nuclear matter or, to a good accuracy, the density in the center of a nucleus), the increase is 7\%. Separating \( J^q \) into the quark helicity contribution, \( \Delta \Sigma^q \), and the quark orbital momentum contribution, \( L^q \), we find that the medium modifications decrease \( \Delta \Sigma^q \) and increase \( L^q \). At \( \rho = \rho_0 = 0.15 \text{ fm}^{-3} \), \( \Delta \Sigma^q \) decreases by 17\% and \( L^q \) increases by 20\%.

Before presenting details of our calculations, we explain that modifications of the bound nucleon spin properties in the nuclear medium may be understood in terms of the enhancement of the lower component of the quark wave function in the nuclear medium, which is a general feature of relativistic mean-field quark models and which has the following consequences.

(i) The axial coupling constant of the nucleon is suppressed in the nuclear medium, \( g_A^* < g_A \), where the quantities with an asterisk refer to the in-medium nucleon and the quantities without one refer to the free nucleon. The suppression of \( g_A \) was deduced from the measurements

*vguzey@jlab.org
†awthomas@jlab.org
‡tsushima@jlab.org
of the nuclear Gamow-Teller beta decay [22–25] and confirmed by theoretical calculations using the Nambu-Jona-Lasinio model [8], the quark-meson coupling model [26,27], and chiral perturbation theory [28]. The suppression of $g_A$ and of the axial vector form factor can be explained by the Lorentz structure of the axial current and by the enhancement of the lower component of the quark spinor in the nuclear medium. In the framework of relativistic mean-field quark models—we use the results of the quark-meson coupling model [26,27]—the mechanism of the suppression is independent of the isospin structure of the corresponding matrix element. Therefore, similarly to the suppression of the isovector axial coupling constant $g_A$ of the Jona-Lasinio model [8], the quark-meson coupling confirmed by theoretical calculations using the Nambu–Jona-Lasinio model [26,27], and chiral perturbation theory [28]. The inverse of the nuclear Gamow-Teller beta decay [22–25] and confirmed by theoretical calculations using the Nambu-Jona-Lasinio model [8], the quark-meson coupling model [26,27], and chiral perturbation theory [28]. The suppression of $g_A$ and of the axial vector form factor can be explained by the Lorentz structure of the axial current and by the enhancement of the lower component of the quark spinor in the nuclear medium. In the framework of relativistic mean-field quark models—we use the results of the quark-meson coupling model [26,27]—the mechanism of the suppression is independent of the isospin structure of the corresponding matrix element. Therefore, similarly to the suppression of the isovector axial coupling constant $g_A$ of the Jona-Lasinio model [8], the quark-meson coupling confirmed by theoretical calculations using the Nambu–Jona-Lasinio model [26,27], and chiral perturbation theory [28].

(ii) The Pauli form factor in medium, $\lim_{t,\xi \to 0} \frac{1}{2} \sum_q \int_{-1}^{1} dx H^{q/N}(x, \xi, t) = F_1^{q/N}(t)$, is enhanced relative to that in the vacuum, $F_2(t)$, while the Dirac form factor remains almost the same ($F_1^{q/N}(t) \simeq F_1(t)$ for $|t| < 2$ GeV$^2$) because of the charge conservation ($F_1^q(0) = F_1(0)$) [27,29]. Recalling the model-independent connection between the elastic form factors and the corresponding generalized parton distributions [15,16],

$$
\int_{-1}^{1} dx H^{q/N}(x, \xi, t) = F_1^{q/N}(t),
$$

$$
\int_{-1}^{1} dx H^{q'/N}(x, \xi, t) = F_1^{q'/N}(t),
$$

$$
\int_{-1}^{1} dx E^{q/N}(x, \xi, t) = F_2^{q/N}(t),
$$

$$
\int_{-1}^{1} dx E^{q'/N}(x, \xi, t) = F_2^{q'/N}(t),
$$

the above observations imply

$$
F_1^q(t) \simeq F_1(t), \quad F_2^q(t) > F_2(t) \rightarrow H^{q/N} \simeq H^{q/N},
$$

$$
E^{q/N}(t) > E^{q/N},
$$

where superscript $q$ denotes the quark flavor, $H^{q/N}$ and $E^{q/N}$ are the quark GPDs of the bound nucleon, and $F_1^{q/N}(t)$ and $F_2^{q/N}(t)$ are the contributions of quark flavor $q$ to the elastic Dirac and Pauli form factors of the bound nucleon, respectively. The corresponding quantities without an asterisk refer to the free nucleon.

Inserting the relations of Eqs. (1) and (3) in the proton spin decomposition relation [14] for the in-medium and vacuum cases and summing over the quark flavors, we obtain

$$
J^{q'} = \frac{1}{2} - J^{q'} = \Delta \Sigma^* + L^{q'}
$$

$$
= \lim_{t,\xi \to 0} \frac{1}{2} \sum_q \int_{-1}^{1} dx x (H^{q/N}(x, \xi, t) + E^{q/N}(x, \xi, t))
$$

$$
> \lim_{t,\xi \to 0} \frac{1}{2} \sum_q \int_{-1}^{1} dx x (H^{q/N}(x, \xi, t) + E^{q/N}(x, \xi, t))
$$

$$
= \Delta \Sigma + L^g = 1/2 - J^g = J^g,
$$

where $(J^q, L^q, J^g)$ [$(J^q, L^q, J^g)$] are the (net quark helicity, net quark orbital angular momentum, gluon total angular momentum) contribution to the proton spin in medium (in vacuum). Equation (4) demonstrates that $J^{q} > J^q$ and $J^{g} < J^g$. In addition, using the fact that $\Delta \Sigma^* < \Delta \Sigma$, Eq. (4) leads to $L^g > L^g$. Below, by an explicit calculation, we demonstrate that these relations are indeed true and quantify the effect of the medium modifications.

We assume that these quark GPDs of the bound nucleon are modified in proportion to the corresponding quark contribution to the bound nucleon elastic form factors,

$$
H^{q/N}(x, \xi, t) = \frac{F_1^{q/N}(t)}{F_1^{N}(t)} H^{q/N}(x, \xi, t),
$$

$$
E^{q/N}(x, \xi, t) = \frac{F_2^{q/N}(t)}{F_2^{N}(t)} E^{q/N}(x, \xi, t).
$$

By construction, the resulting bound nucleon GPDs obey the fundamental property of polynomiality (provided that the free nucleon GPDs obey polynomiality), which is a consequence of Lorentz invariance and which states that the $x$ integrals of $x^n H^{q/N}$ and $x^n E^{q/N}$ are polynomials in $\xi^2$ of order $n$ for even $n$ and of order $n + 1$ for odd $n$. As a particular example of polynomiality, our bound nucleon GPDs are constrained to reproduce the elastic form factors of the bound nucleon [see Eq. (2)]:

$$
\sum_q e_q \int_{-1}^{1} dx H^{q/N}(x, \xi, t)
$$

$$
= \sum_q e_q \frac{F_1^{q/N}(t)}{F_1^{N}(t)} \int_{-1}^{1} dx H^{q/N}(x, \xi, t)
$$

$$
= \sum_q e_q F_1^{q/N} = F_1^{N}(t),
$$

$$
\sum_q e_q \int_{-1}^{1} dx E^{q/N}(x, \xi, t)
$$

$$
= \sum_q e_q \frac{F_2^{q/N}(t)}{F_2^{N}(t)} \int_{-1}^{1} dx E^{q/N}(x, \xi, t)
$$

$$
= \sum_q e_q F_2^{q/N} = F_2^{N}(t),
$$

where $e_q$ is the electric charge of quark flavor $q$. One should emphasize that it is Eqs. (2) and (6) that motivated our model for the bound nucleon GPDs in Eq. (5).

The $t$ dependence of the bound nucleon GPDs comes from the $t$ dependence of the free nucleon GPDs and from the $t$ dependence of the ratio of the quark contribution to the bound and free nucleon form factors. It is important to point out that our assumption for the form of the bound nucleon GPDs neglects the EMC, Fermi motion, nuclear shadowing, and antishadowing effects. We estimated the reliability of this approximation and found that the effect of this approximation on $J^q$ is small.
the EMC and nuclear shadowing effects are counterbalanced by the antishadowing and Fermi motion effects in the integral for $J^P$. For details, see the discussion below.

Provided that the strange quark contribution is small, as shown by recent parity violation experiments [30,31], the $u$ and $d$ quark contributions to the elastic form factors of the proton and neutron, $F_{1,2}^p(t)$ and $F_{1,2}^n(t)$, are

\[
F_{1,2}^p(t) = \frac{2}{3} F_{1,2}^u(t) - \frac{1}{3} F_{1,2}^d(t), \\
F_{1,2}^n(t) = \frac{2}{3} F_{1,2}^d(t) - \frac{1}{3} F_{1,2}^u(t),
\]

(7)

where each flavor is accompanied by its electric charge. In the second line, we used charge symmetry, which relates the quark contributions to the elastic form factors of the neutron to those of the proton, $F_{1,2}^p(t) = F_{1,2}^d(t) \equiv F_{1,2}^u(t)$ and $F_{1,2}^n(t) = F_{1,2}^u(t) \equiv F_{1,2}^d(t)$. Similar relations hold for the bound proton and neutron.

Using Eq. (7) for the bound and free nucleon, our assumption for the form of the quark GPDs of the bound proton reads

\[
H^{u/d,0}(x, \xi, t) = \frac{2 F_{1,2}^u(t) + F_{1,2}^d(t)}{2 F_{1,2}^u(t) + F_{1,2}^d(t)} H^u(x, \xi, t),
\]

\[
H^{d/u,0}(x, \xi, t) = \frac{F_{1,2}^u(t) + 2 F_{1,2}^d(t)}{F_{1,2}^u(t) + 2 F_{1,2}^d(t)} H^u(x, \xi, t),
\]

\[
E^{u/d,0}(x, \xi, t) = \frac{2 F_{1,2}^u(t) + F_{1,2}^d(t)}{2 F_{1,2}^u(t) + F_{1,2}^d(t)} E^u(x, \xi, t),
\]

\[
E^{d/u,0}(x, \xi, t) = \frac{F_{1,2}^u(t) + 2 F_{1,2}^d(t)}{F_{1,2}^u(t) + 2 F_{1,2}^d(t)} E^u(x, \xi, t),
\]

(8)

where we introduced the shorthand notation for the ratio of the bound to free proton and neutron elastic form factors,

\[
r_{1,2}^p \equiv \frac{F_{1,2}^p(t)}{F_{1,1}^p(t)}, \quad r_{1,2}^n \equiv \frac{F_{1,2}^n(t)}{F_{1,1}^n(t)}.
\]

(9)

Note that charge symmetry for the quark contributions to the nucleon elastic form factors and for the free nucleon GPDs leads to charge symmetry for the bound nucleon GPDs [see Eq. (5)]. Therefore,

\[
H^{u/d,0}(x, \xi, t) = H^{d/u,0}(x, \xi, t),
\]

\[
H^{d/u,0}(x, \xi, t) = H^{u/d,0}(x, \xi, t),
\]

\[
E^{u/d,0}(x, \xi, t) = E^{d/u,0}(x, \xi, t),
\]

(10)

where the right-hand side of Eq. (10) is given by Eq. (8).

In addition, we assume that the strange quark GPDs are not modified by the nuclear medium, e.g., $H^{s/d}(x, \xi, t) = H^{s/u}(x, \xi, t) = H^{s/u}(x, \xi, t) = H^{s/u}(x, \xi, t)$. Note also that the model used in our recent analysis of the bound nucleon GPDs in $^4$He [20] is slightly different from our present model given by Eq. (8) and leads to small violations of charge symmetry for the bound nucleon.

In the forward limit, which is relevant for the Ji spin sum rule [21], the bound proton GPDs given by Eq. (8) become

\[
H^{u/d,0}(x, 0, 0) = u(x),
\]

\[
H^{d/u,0}(x, 0, 0) = d(x),
\]

\[
E^{u/d,0}(x, 0, 0) = r_2^u(0) \left( 1 + \frac{1}{2} \frac{k^p}{k^n} \right) \frac{\rho^u}{\rho^n} e^u(x),
\]

\[
E^{d/u,0}(x, 0, 0) = r_2^d(0) \left( 1 + \frac{1}{2} \frac{k^p}{k^n} \right) \frac{\rho^d}{\rho^n} e^d(x),
\]

(11)

which correspond to three quark flavors [32]. For the forward limits of the nucleon GPDs (11),

\[
\rho^u = \frac{2 k^p r_2^u(0) + k^n r_2^u(0)}{k^p + k^n},
\]

\[
\rho^d = \frac{k^p r_2^d(0) + 2 k^n r_2^d(0)}{k^p + k^n}.
\]

(12)

The factors $r^u$ and $r^d$ are linear combinations of the factors $r_2^u(0)$ and $r_2^d(0)$, which characterize the modifications of the Pauli form factor of the nucleon at the zero momentum transfer (the modifications of the nuclear anomalous magnetic moment). For the latter, we used the results of the QMC model [11,26,27]. In the QMC model, medium modifications depend on the nuclear density and the effect increases as the nuclear density is increased.

Figure 1 presents the factors $r^u$ and $r^d$ as a function of $\rho/\rho_0$, where $\rho$ is the nuclear density and $\rho_0 = 0.15$ fm$^{-3}$ is the density of the nuclear matter. Note that the nuclear density at the center of sufficiently heavy nuclei is close to $\rho_0$.

In addition, for the forward limit of the nucleon GPDs (11), we used the following input. The quark parton distributions (PDFs) were taken from the next-to-next-to-leading order (NNLO) parametrization by MRST2002 at $Q^2 = 1$ GeV$^2$, which corresponds to three quark flavors [32]. For the forward limit of the GPDs $E^q$ denoted by $e^u(x)$ and $e^d(x)$, we used the model of Ref. [33], which provides a good description of the
Equation (13) for the quark contributions to the nucleon anomalous magnetic moment and 

d_{u}(x) and 
d_{d}(x) are the valence quark contributions to the net quark and gluon contributions to the bound nucleon spin sum rule, respectively. The free parameters \( \eta_{u} = 1.713 \) and \( \eta_{d} = 0.566 \) are determined from fits to the nucleon elastic form factors. \( N_{u} \) and \( N_{d} \) are the normalization factors, \( N_{u} = \int_{0}^{1} dx (1 - x)^{\eta_{u}} u_{e}(x) \) and \( N_{d} = \int_{0}^{1} dx (1 - x)^{\eta_{d}} d_{e}(x) \). Finally, we assume that the strange quark \( e^{s}(x) = 0 \).

Note that the use of the NNLO MRST2002 parametrization for the quark distributions and the resolution scale \( Q^{2} = 1 \text{ GeV}^2 \) as well as the model for \( e^{u}(x) \) and \( e^{d}(x) \) should be considered as parts of a bigger model [33], whose parameters were adjusted to give the best description of the nucleon (proton and neutron) elastic form factors.

Having fully specified our model for the forward limit of the bound nucleon GPDs, we can examine the influence of the medium modifications on the spin sum rule for the bound nucleon. The quark contribution to the bound proton spin sum rule reads

\[
2 J^{q^{\|}} = \sum_{q = u, d, s} \int_{0}^{1} dx x (H_{q}^{u/p^{\|}}(x, 0, 0) + E_{q}^{u/p^{\|}}(x, 0, 0))
+ \sum_{q = u, d, s} \int_{0}^{1} dx x (q(x) + \bar{q}(x) + r^{u} e^{u}(x) + r^{d} e^{d}(x))
= 0.654 + 0.219 r_{u} - 0.263 r_{d}.
\]

Note that quark contribution to the bound neutron spin sum rule is given by the same expression.

Figure 2 presents the quark contribution to the spin of the bound nucleon, \( 2 J^{q^{\|}} \), as a function of the nuclear density at \( Q^{2} = 1 \text{ GeV}^2 \). The case of the free proton corresponds to \( \rho / \rho_0 = 0 \), for which \( 2 J^{q^{\|}} / 2 J^{p^{\|}} \), is plotted as a function of \( \rho / \rho_0 \). As one can see from Fig. 2 and also from Eq. (14), the medium modifications of the bound nucleon GPDs \( E_{q}/N_{q}^{\|} \) increase the quark contribution to the bound nucleon spin sum rule.

The effect is quite noticeable and increases with increasing nuclear density \( \rho \). This is illustrated in Fig. 3, where the ratio of the quark contribution to the bound nucleon spin sum rule to that of the free nucleon, \( 2 J^{q^{\|}} / 2 J^{p^{\|}} \), is plotted as a function of \( \rho / \rho_0 \). As one can see from the figure, for instance, at \( \rho = \rho_0 = 0.15 \text{ fm}^{-3} \), \( 2 J^{q^{\|}} / 2 J^{p^{\|}} = 1.070 \), i.e., it is a 7% effect. Because the sum of the net quark and gluon contributions to the bound nucleon spin should be one half, the gluon contribution to the bound nucleon spin sum rule, \( J^{g^{\|}} \equiv 1/2 - J^{q^{\|}} \), is decreased in the nuclear medium.

![Figure 1](image1.png)

**FIG. 1.** The factors \( r^{u}\) and \( r^{d}\), which define the medium modification of the forward limit of the GPDs \( E^{q}\) in our model, see Eqs. (11), (12), and (9), as a function of \( \rho / \rho_0 \), where \( \rho \) is the nuclear density and \( \rho_0 = 0.15 \text{ fm}^{-3} \). The medium modifications are calculated using the results of the QMC model.

![Figure 2](image2.png)

**FIG. 2.** The quark contribution to the spin sum rule of the bound nucleon, \( 2 J^{q^{\|}} \), as a function of \( \rho / \rho_0 \) at \( Q^{2} = 1 \text{ GeV}^2 \), where \( \rho \) is the nuclear density and \( \rho_0 = 0.15 \text{ fm}^{-3} \). The medium modifications are calculated using the results of the QMC model.

![Figure 3](image3.png)

**FIG. 3.** The ratio of the quark contribution to the bound nucleon spin sum rule to that of the free nucleon, \( 2 J^{q^{\|}} / 2 J^{p^{\|}} \), as a function of \( \rho / \rho_0 \) at \( Q^{2} = 1 \text{ GeV}^2 \), where \( \rho \) is the nuclear density and \( \rho_0 = 0.15 \text{ fm}^{-3} \). The medium modifications are calculated using the results of the quark-meson coupling (QMC) model.
It is important to point out that in our approach to the bound nucleon GPDs, $H^q$ has the usual unmodified quark distribution $q(x)$ as a forward limit. This is an approximation that neglects the Fermi motion effect and possible medium modifications of the shape of $q(x)$ (see the relevant discussion in Ref. [20]). An estimate of the reliability of this approximation, based on a parametrization of the EMC effect [34], suggests that the effect of this approximation on $2J^q$ is small: even for a nucleus as heavy as $^{208}$Pb, the contributions of $e^d$ and $e^u$ do not change and the contribution of $q(x) + \bar{q}(x)$ changes (decreases) by less than 2% (the EMC and nuclear shadowing effects almost exactly counterbalance the Fermi motion and antishadowing effects).

The quark contribution to the spin sum rule, $J^q$, can be separated in a gauge-invariant way into the contribution of the quark helicity distributions, $\Delta \Sigma$, and the contribution of the quark angular momentum, $L^q$ [21]. Thus, for the bound nucleon,

$$J^q = \Delta \Sigma^0 + L^q,$$

where the quark helicity contribution to the bound nucleon spin is given by the sum of the first moments of the quark helicity distributions in the bound nucleon, $\Delta q^0(x)$,

$$\Delta \Sigma^0 = \frac{1}{2} \sum_{q=u,d,s} \int_0^1 dx (\Delta q^0(x) + \Delta \bar{q}^0(x)).$$

To estimate $\Delta \Sigma^0$, we assume that the contribution of the $u$ and $d$ quarks to $\Delta \Sigma^0$ is modified (suppressed) in proportion to the medium modifications of the axial coupling constant $g_A$ and that the contribution of the strange quark is unmodified,

$$\Delta \Sigma^0 = \frac{g_A^2}{g_A} \sum_{q=u,d} \int_0^1 dx (\Delta q(x) + \Delta \bar{q}(x))
+ \frac{1}{2} \int_0^1 dx (\Delta s(x) + \Delta \bar{s}(x)),$$

where $\Delta q(x)$ are the helicity distributions of the free nucleon. A more detailed treatment of the helicity distributions of the bound nucleon in the framework of the QMC model, which leads to the same result, $\Delta q^0(x) \approx \Delta q(x)$, can be found in Ref. [35]. The assumption of Eq. (17) is consistent with the simultaneous suppression of the axial coupling constant $g_A^2$ and $\Delta \Sigma^0$ because of the enhancement of the lower component of the quark Dirac spinor in the nuclear medium [see Eq. (1) and its qualitative discussion]. Equation (17) is also consistent with the medium modifications of the Bjorken sum rule [36] and, in another language, with the model of the medium modifications of the GPDs $\tilde{H}$ suggested in Ref. [20].

For the medium modifications of the axial coupling constant of the bound nucleon, we use the results of the QMC model [11,26,27]. For the free nucleon helicity distributions $\Delta q(x)$, we used the next-to-leading order (NLO) GRSV2000 parametrization at $Q^2 = 1$ GeV$^2$ [37].

Figure 4 presents the nuclear medium modifications of the quark helicity contribution, $\Delta \Sigma^0$, and the quark angular momentum contribution, $L^q \equiv J^q - \Delta \Sigma^0$, to the bound nucleon spin as a function of $\rho/\rho_0$. The upper panel represents the absolute values; the lower panel gives the ratios with respect to corresponding free nucleon $\Delta \Sigma$ and $L^q$. The medium modifications are calculated using the results of the QMC model.

As one can see from Fig. 4, because of the quenching of the axial coupling constant in the nuclear medium, $\Delta \Sigma^0 < \Delta \Sigma$. As a consequence of the relation $L^q \equiv J^q - \Delta \Sigma^0$ and the fact that $J^q > J^q$, the quark angular momentum contribution to the nucleon spin is larger for the bound nucleon compared to that for the free nucleon, $L^q > L^q$. Both effects are large: at $\rho = \rho_0 = 0.15$ fm$^{-3}$, $\Delta \Sigma^*/\Delta \Sigma = 0.83$ and $L^q/L^q = 1.20$, i.e., these are 17% and 20% effects, respectively.

In summary, assuming that the bound nucleon GPDs are modified in proportion to the corresponding quark contributions to the bound nucleon elastic form factors, we estimated the nuclear medium modifications of the quark contribution to the bound nucleon spin sum rule, $J^q$, as well the separate helicity, $\Delta \Sigma^0$, and the angular momentum, $L^q$, contributions to $J^q$. For the bound nucleon elastic form factors, we used
the results of the quark-meson coupling model. The resulting
density of the bound nucleon GPDs is relevant for incoherent
DDCS (with nuclear breakup) with nuclear targets. We found
that the medium modifications increase $J^q$ and $L^q$ and
decrease $\Delta \Sigma^*$ compared to the free nucleon case. The effect
is large and increases with increasing nuclear density $\rho$. For
instance, at $\rho = \rho_0 = 0.15$ fm$^{-3}$, $J^q$ increases by 7%, $L^q$
increases by 20%, and $\Delta \Sigma^*$ decreases by 17%. These effects
are a general feature of relativistic mean-field quark models
and may be qualitatively explained by the enhancement of the
lower component of the quark wave function of the bound
nucleon.

ACKNOWLEDGMENTS
This work was authored by Jefferson Science Associates,
LLC, under US DOE Contract DE-AC05-06OR23177.