On-line identification of time-varying systems equipped with adaptive control

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Abstract. The present study aims at numerically investigating the feasibility of an adaptive TMD control system applied on lightweight, flexible structures characterized by time-varying inertial properties. The case study will consist of a photovoltaic support structure subject to snow drifting and slippage in windy conditions.

1. Introduction

In its basic arrangement, a passive tuned mass damper (TMD) is a single-degree-of-freedom mechanical appendage of the main structure, schematically consisting in a linear mass/spring/damper system. When optimally tuned to the resonant target mode of vibration of the primary structure, a TMD adds significant damping in the dominant vibration modes, thereby reducing the overall vibration response of the structure. Optimal tuning generally requires that, once a proper mass is chosen for the absorber (usually small if compared to the effective mass in the structural mode), its frequency and damping ratios are set to the respective optimal values such as to minimise a desired norm (typically either the $H_2$ norm or the $H_\infty$ norm) of the input-output transfer functions for the overall dynamical system. De-tuning, including any possible deviation of the TMD parameters from their optimal values, and resulting for example from variations in the structural properties of the primary structure, deterioration of TMD’s mechanical parameters, incorrect design forecasts, etc., may lead to a significant loss of control performance. One possible remedy to de-tuning is represented by the “multiple TMD” [1], constituted by lightly damped oscillators having frequencies scattered over a small range around the target natural frequency. An alternative, more efficient solution will follow in this paper, represented by the so-called “adaptive TMD”, i.e. a TMD where the stiffness and damping parameters can be adjusted, or re-tuned, in real time to their optimal values at a minimum of energy expense [2], depending on some measures of the structural dynamic response.

Although different re-tuning criteria are available in the literature, including data driven, non-model-based algorithms, a model-based method will be presented in the sequel, in which the optimal control parameters are continuously updated based on an on-line tracking of the actual dynamical properties of the overall system, herein performed through the dynamic identification technique known as the Unscented Kalman Filter (UKF) [3], which has been chosen among the others because of its limited computational cost. The UKF has been firstly proposed in 1995 by Julier et al [4] as an improvement of the Extended Kalman Filter (EKF).
2. Model Description
On-line identification for the re-tuning of the TMD, through the recursive algorithm UKF, has been applied to a structure modelled as two degrees of freedom system. This structure is constituted by a steel pillar, clamped at the base, which at the top supports a photovoltaic panel assumed to be infinitely rigid (Figure 1), representing an application of practical relevance that may be subjected to loading by snow in adverse weather conditions.

In practice, the reduced slope of the panel due to efficiency requirements would let snow accumulate on the panel. As the weather conditions change, snow could then slip and detach resulting in a highly time-varying nature of mass loading and hence the inertial properties of the dynamic system.

![Solar panels: example of a typical linear arrangement and model scheme.](image)

Generally, snow tends to accumulate on surfaces and on the ground unevenly depending on the action of the wind $\tau$ and the threshold level $\tau^*$: for $\tau > \tau^*$ there is erosion; instead for $\tau < \tau^*$ there is deposit or transport. In the latter case, in order to establish which of the two phenomena occur, i.e. either transport or deposit, energetic considerations must be taken into account: if the energy of the particle in motion, which corresponds to the sum of the kinetic and wind energies and depends on $\tau$, is greater than the energy corresponding to $\tau^*$, the snow particle will bounce off the surface and continue to move. Conversely, if the overall energy of the particle is insufficient to break the ties among the snow particles at the moment of impact, deposit will occur [5].

The difficulty of determining precisely the areas and the entity of erosion and deposit is correlated with the multiplicity of physical and geometrical parameters influencing the value of $\tau^*$ [6,7,8] which in practice depends on the cohesion, dimension and shape of the snow particles, on the density and evolution of the snowdrift. Initial research on the wind transport of snow dates back to the studies of Owen [9]. Referring to the work conducted by Bagnol (1941) on the wind transport of sand, Owen immediately realised that the analysis of snow was more complex [10] since the particles do not have a nominally spherical shape like sand; the dendritic shape of snowflakes affects the elastic properties and the rebound capabilities of the particles on the surface. Furthermore, due to the subsequent rebounds and variations of temperature, these characteristics tend to modify over time and can do so rapidly. Although several studies have been conducted since 1941 concerning snow drifting, analytical relations available in published literature address the determination of the snowdrift evolution in terms of settling and/or exposure to the wind. On the basis of these considerations, a parabolic distribution of the snow on the panel has been assumed according to Yan et al [11].

Concerning the model of snow fall and detachment, reference has been made to the work of Taylor et al [12]. According to the model proposed by Taylor, the detachment of a block of snow happens when the component tangent to the surface of the force of gravity equals the snow tensile strength and of the attractive and adhesive forces generated at the snow-surface interface. The tensile strength, which depends on the density and on the degree of humidity, has been conformed to values suggested by Zempachi [13].

Considering a temporal interval of approximately 40s, two subsequent detachments are assumed to occur that alter the condition of the panel from the initial condition of complete covering to the
condition of unloaded panel. Figure 2 illustrates the time-variant characteristics of the system, the mass \( m(t) \) and the polar inertia \( I(t) \) calculated with respect to the joint between the panel and the pillar.

The mathematical model, describing the temporal evolution of the displacements of the system under wind action, can be expressed in a matrix form as:

\[
\begin{bmatrix}
  m(t) & 0 \\
  0 & I(t)
\end{bmatrix}
\begin{bmatrix}
  \dot{u} \\
  \dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
  c_{11} & c_{12} \\
  c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
  \dot{u} \\
  \dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
  k_{11} & k_{12} \\
  k_{21} & k_{22}
\end{bmatrix}
\begin{bmatrix}
  u \\
  \theta
\end{bmatrix}
= \begin{bmatrix}
  F \\
  C
\end{bmatrix}
\]  

(1)

where \( u \) and \( \theta \) are the translation in the panel plane and its rotation (Figure 1).

![Figure 2. Time-evolution of mass parameters due to progressive snow drifting and slippage.](image)

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![Figure 3. TMDs configuration, the brown TMD controls the translation, the blue TMDs the rotation.](image)

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![Figure 4. FRFs between force and tangential acceleration.](image)

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![Figure 5. Displacements for the not-controlled structure (grey line) and controlled one (black line).](image)

**Figure 5.** Displacements for the not-controlled structure (grey line) and controlled one (black line).
3. Passive control with TMD and time-variant identification via UKF

Since the system has two degrees of freedom, three TMDs have been used, placed as illustrated in Figure 3 and tuned to the dynamic characteristics of the panel without snow, which is the reference serviceability condition. In order to ensure an appropriate performance of the passive control system even in presence of snow, it was decided to adopt a relatively high mass ratio equal to 1% of the panel mass completely covered in snow (snow level from the ground assumed equal to 0.5 m). In this case the equations of the controlled dynamic system are:

\[
\begin{bmatrix}
    m(t) & 0 & 0 & 0 \\
    0 & I(t) & 0 & 0 \\
    m_1 & 0 & m_2 & 0 \\
    0 & m_2L & 0 & m_2 \\
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_1 \\
    \dot{\theta} \\
    \dot{u}_t \\
    \dot{v}_2 \\
\end{bmatrix}
+ \begin{bmatrix}
    c_{11} & c_{12} & -c_1(t) & 0 \\
    c_{21} & c_{22} & 0 & -c_2(t) \\
    0 & 0 & c_1(t) & 0 \\
    0 & 0 & 0 & 2c_2(t) \\
\end{bmatrix}
\begin{bmatrix}
    u \\
    \dot{\theta} \\
    \dot{u}_t \\
    \dot{v}_2 \\
\end{bmatrix}
= \begin{bmatrix}
    F \\
    C \\
    0 \\
    0 \\
\end{bmatrix}
\]

(2)

where \( u_1 \) and \( v_2 \) are the relative displacements of the TMD with respect to the panel in the tangential and normal directions respectively.

The accumulation of snow on the panel, and the subsequent detachment of blocks, causes sudden changes of the natural frequencies of the structure and the consequent de-tuning and efficiency loss of the passive control system. This is clearly visible from the comparison between the Frequency Response Functions obtained for different temporal intervals (Figure 4). Figure 5 shows that when the panel mass diverges from the value used for the TMDs tuning (without snow), the control efficiency reduces significantly.

Having calculated the output data in terms of accelerations at two opposed corners of the panel, the TMD controlled system has been identified using the UKF, selecting appropriate values of the parameters \( \alpha, \beta \) and \( \kappa \) [14]. The algorithm identifies correctly the time-variant inertial characteristics of the principal structure as highlighted in figure 6.

4. UKF identification and control through the adaptive TMD

In contrast to other identification techniques, the UKF enables the determination of the dynamic characteristics of the principal structure by simplifying significantly the device re-tuning. In fact this can be performed by using the same formulas used for the passive TMDs. After measuring the input and output of the system at regular time intervals and applying the unscented filter, it is possible to determine the time-varying characteristics of the system and consequently the natural frequencies and
the effective masses of the principal system. Simulations have shown that whilst the UKF identifies in an accurate way the characteristics of the principal structure (see figure 7), the quality of the results is directly related to the appropriate selection of the UKF algorithm parameters. The parameters values have been chosen accordingly to previous studies available in the literature [4,15]. Anyway, the topic may represent an interesting future area of research.

Once identification has been performed, the adaptive TMDs with constant mass should be retuned, varying the stiffness and damping. The dynamic behaviour of the panel and of the auxiliary masses continues to be described by a system [2] with time-varying terms \( c_1, c_2, k_1, k_2 \).

In figure 7, the optimal characteristics of TMDs, calculated using Warbuton's formulas [16] as a function of the inertial properties instantaneously identified, are shown together with the respective values identified via the UKF.

![Figure 7. Comparison between the optimal characteristics of translational and rotational TMDs and those obtained with UKF.](image)

Table 1 displays the percentage variation of the inertial characteristics of the system referred to the condition of unloaded panel and the relative variations (\( \Delta M \) mass, \( \Delta I_p \) polar inertia), in terms of variation of TMD stiffness (\( \Delta K_{TMD} \)) and damping (\( \Delta C_{TMD} \)) in the degrees-of-freedom \( u \) and \( \theta \).

| Variation | \( \Delta M \) | \( \Delta K_{TMDx} \) | \( \Delta C_{TMDx} \) | \( \Delta I_p \) | \( \Delta K_{TMD\theta} \) | \( \Delta C_{TMD\theta} \) |
|-----------|----------------|-----------------|-----------------|----------------|----------------|----------------|
| step 1-3  | 173.52%        | -62.00%         | -60.87%         | 128.57%        | -57.31%        | -55.00%        |
| step 1-2  | 48.68%         | -31.49%         | -30.43%         | 24.84%         | -23.02%        | -20.00%        |

The highest effective level of passive damping can be evaluated through Table 2 which reports the Root Mean Square (RMS) values of the displacements \( u \) and \( \theta \) and their relative reductions due to the introduction of the passive (TMD) or adaptive (STMD) devices.

![Table 2. Comparison of results in the cases of structure which is not-controlled, controlled with passive devices, and controlled with adaptive device.](image)
5. Conclusions
This paper proposed an on-line procedure for the identification of a time-variant dynamic system by resorting to the UKF algorithm. The aim of the identification was to determine the time-variant characteristic of the system with accuracy adequate to allow for the re-tuning of adaptive TMDs. Through simulations on a system characterised by sharp variations of dynamic properties, it has been shown that STMDs supported by a UKF identification algorithm can recover their optimal performance with delays fully compatible with practical applications. Future developments may involve the numerical investigation of the performances of the UKF with different excitation sources and the application of the methodology to real-scale structure.

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