On the Origin of Lepton and Quark Masses

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Gauging the flavor (family, generation, horizontal) index of the chiral fermion fields of the Standard model, for anomaly freedom extended by three sterile right-handed neutrino fields, results in asymptotically free, bona fide nonconfining $SU(3)_f$ quantum flavor dynamics. Approximate nonperturbative strong-coupling solutions of the corresponding Schwinger-Dyson (SD) equation for fermion self-energies give rise to the complete flavor symmetry breaking by: (1) Three huge Majorana masses of sterile right-handed neutrinos. (2) Three exponentially light Dirac masses common to all fermion sorts in a family. Masses of charged leptons and quarks are further distinguished from Dirac neutrino masses by the weak hypercharge contributions to the universal $SU(3)_f$ kernel of the SD equation, free of unknown parameters. The $SU(3)_f$ dynamics itself thus gives the neutrino mass spectrum in the seesaw form.

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I. INTRODUCTION

Understanding the wide and wild mass spectrum of quantum fields of neutrinos, charged leptons and quarks is an alluring challenge of theoretical elementary particle physics. The valuable achievement of recent past is its phenomenological, theoretically unobjectionable parametrization: The charged lepton and quark masses are described in the Standard model (SM) by the essentially classical Higgs mechanism [1], and the extremely light neutrino masses are described in its minimal extension by the entirely classical seesaw [2]. With fermion mixing the Lagrangian contains about two dozens of theoretically arbitrary parameters which differ by at least twelve orders of magnitude. Such a state of affairs is unsatisfactory [3]. According to the standard understanding of the energy spectra of the genuine quantum systems like oscillators, hadrons, nuclei, atoms and molecules, the mass spectrum of quantum fields of leptons and quarks should also be calculable.

Calculable mass spectrum of fermion fields viewed as coupled quantum oscillators is conceivable [4] by replacing the Higgs mechanism by the dynamical gauge symmetry breakdown pioneered by Nambu [5]. Necessity of dealing with non-perturbative techniques at strong coupling requires, however, approximations which a priori are not under control [4]. The attempt formulated in [6] and illustrated here, following the suggestion of Yanagida [7], is no exception. We believe that the obtained results might justify the used approximations a posteriori.

The paper is structured as follows. In Sect. II we briefly summarize the main properties of the model described in detail in [6]. The necessary new strong dynamics introduced there is the gauge quantum flavor dynamics of three SM chiral fermion families extended for anomaly freedom by three sterile right-handed neutrinos. Section III demonstrates the universal fermion flavor mass splitting fundamentally different for Majorana and Dirac masses. This analysis fixes the neutrino mass spectrum uniquely in the seesaw form [8]. In Sect. IV we describe how the weak hypercharge contributions to the $SU(3)_f$ kernel of the SD equation can provide large mass splitting observed within each generation between the charged leptons and quarks with different electric charges. In Sect. V we summarize how the strong-coupling quantum flavor dynamics efficiently replaces the weakly coupled Higgs sector of the Standard model, and provide an illustrative fit of the fermion mass spectrum. Sect. VI contains our brief conclusions.

II. QUANTUM FLAVOR DYNAMICS

Gauging the flavor (horizontal, family, generation) symmetry of SM is so natural that it could hardly be new [3]. In the present form the model is defined by gauging the flavor $SU(3)_f$ triplet index of three chiral SM lepton $(l_{fL}, e_{fR})$ and quark $(q_{fL}, u_{fR}, d_{fR})$ families of the $SU(2)_L \times U(1)_Y$ gauge invariant SM. This amounts to introduction of the octet of gauge flavor gluons $C_a^f$, and for anomaly freedom to addition of one triplet of sterile right-handed neutrino fields $\nu_{fR}$. In [7] this $SU(3)_f \times SU(2)_L \times U(1)_Y$ gauge symmetry is spontaneously broken down to $U(1)_{em}$ by an extended sector of elementary scalar Higgs fields. In [6] we argue that no Higgs fields are necessary, i.e., the strong flavor dynamics itself self-consistently completely self-breaks. We believe this is a particular realization of the Nambu’s idea [5] of dynamical gauge symmetry breaking. The resulting anomaly free, asymptotically free gauge $SU(3)_f$ quantum flavor dynamics is characterized by one parameter. It is either the dimensionless gauge coupling constant $h$ or, due to the dimensional transmutation, the theoretically arbitrary scale $\Lambda$. Because the new dynamics results from gauging another fermion index we follow the habit and suggest, as others did already before, to call it the
quantum flavor dynamics’ (QFD).

Both in QCD and in QFD all the left- and the right-handed fermion fields transform as triplets of the gauge group $SU(3)$, i.e., their Lagrangians are formally identical. Consequently, in perturbation theory, i.e., at short distances, these two theories must be identical. In particular, both are asymptotically free. Despite this we believe \[6\] that at the strong coupling they are entirely different. The QCD confines all its colored oscillator-like excitations, whereas the QFD self-consistently generates the masses to all its flavored ones.

The difference is in different electric charges of fermion fields: In QCD, dealing with the electrically charged quark fields, only the Dirac mass terms are possible. The Lagrangian (hard) mass terms are allowed by the color $SU(3)$ symmetry (the product $3 \times 3 = 1 + 8$ does contain unity), and the strong low-momentum QCD corrections to them are harmless for the color confinement (generate the constituent quark masses). In contrast, the QFD deals also with the electrically neutral neutrinos. The $SU(3)_f$ invariant hard Dirac mass term common to all fermion sorts is obviously also allowed, but there is a generically new possibility of the effective Majorana mass of the neutrinos at large distances. For the right-handed ones it has the form

$$L_{\text{Majorana}} = -\frac{1}{2}(\bar{\nu}_R M_R \nu_R)^C + h.c.) .$$

It is, however, strictly prohibited by the flavor $SU(3)$ symmetry: The product $3 \times 3 = 3 + 6$ does not contain unity. As the Dirac mass term also the Majorana mass term connects the right- and the left-handed fermion fields: The charge-conjugate $(\nu_R)^C$ is a left-handed field. Unlike the Dirac mass term the left-handed charge-conjugate right-handed neutrino field transforms as an antitriplet of $SU(3)_f$. Consequently, the QFD is not vector-like as QCD, but at strong coupling it is effectively chiral. The strong QFD quantum corrections, if energetically favorable, generate the Majorana masses dynamically, and this has the far reaching consequence: All eight flavor gluons acquire masses by absorbing eight composite 'would-be' Nambu-Goldstone bosons as their longitudinal polarization states \[10\], and QFD gets completely self-broken.

Ultimately, also the Dirac masses of all fermions have to be generated dynamically. The point is that all hard QCD and QFD Dirac mass terms are strictly prohibited by the chiral gauge electroweak $SU(2)_L \times U(1)_Y$ interactions always present in the game at least as weak external perturbations.

Lack of systematic analytic strong coupling methods both in QCD and QFD implies that for the description of majority of the nonperturbative low-energy phenomena we are sentenced to using models and rough approximations. In QCD the systematic analytic method is the chiral perturbation theory \[11\], and the systematic numerical method is the lattice \[12\]. In the effectively chiral QFD we are not aware of any systematic analytic method, and the lattice methods apparently do not apply \[12\].

### III. NEUTRINO SEESAW MASS MATRIX

Following the suggestion of T. Yanagida we have demonstrated in \[6\] that in the anomaly free gauged three-flavor $SU(3)_f \times SU(2)_L \times U(1)_Y$ model no Higgs fields are needed. Strong flavor gluon interactions themselves, treated in a separable approximation, clearly distinguish between the Majorana and the Dirac masses. They result in the huge Majorana masses of sterile neutrinos, and in naturally light hierarchically split Dirac masses of the electroweakly interacting leptons and quarks.

This can be understood by looking at the Fig.1 of \[8\] as follows:

In flavor space the Majorana mass term transforms in general as

$$3 \times 3 = 3 + 6,$$  \hspace{1cm} (1)

where the subscripts abbreviate the antisymmetric ($a$) and symmetric ($s$) representations. Because the right-handed neutrino fields are sterile, the Pauli principle uniquely selects the symmetric sextet.

In flavor space the Dirac mass term transforms differently:

$$3 \times 3 = 1 + 8.$$  \hspace{1cm} (2)

The difference between $3 \times 3$ and $3 \times 3$ translates into different combinations of the effective low-energy parameters $g_{ab}$ which determine $M_R$ and $m_D$ in the Schwinger-Dyson equation.

Three Majorana masses of the right-handed neutrinos come out huge, of order $\Lambda$

$$M_{fM} \sim \Lambda.$$  \hspace{1cm} (3)

The neutrinos, charged leptons and quarks of three generations acquire the universal Dirac masses

$$m_{fD} = \Lambda \exp (-1/4\alpha_1),$$  \hspace{1cm} (4)

where

$$\alpha_1 = \frac{3}{64\pi} \left( g_{33} + \frac{2}{\sqrt{3}} g_{38} + \frac{1}{3} g_{88} \right),$$
$$\alpha_2 = \frac{3}{64\pi} \left( g_{33} - \frac{2}{\sqrt{3}} g_{38} + \frac{1}{3} g_{88} \right),$$
$$\alpha_3 = \frac{3}{64\pi} \frac{4}{3} g_{88},$$

form the real diagonal matrix $\alpha$ in flavor space. It can be expanded as

$$\alpha = \frac{3}{64\pi} \left[ a_1 + b\lambda_3 + c\lambda_8 \right],$$  \hspace{1cm} (5)
where 
\[
\begin{align*}
a &= \frac{2}{3}(g_{33} + g_{ss}) , \\
b &= \frac{2}{\sqrt{3}}g_{3s} , \\
c &= \frac{1}{\sqrt{3}}(g_{33} - g_{ss}) .
\end{align*}
\]

Because for the neutrinos there are no other contributions to the mass matrices \(M_\nu\) a \(m_D\), they combine into the famous seesaw 6 × 6 symmetric mass matrix 2
\[
\begin{pmatrix}
0 & m_D \\
m_D^T & M_M
\end{pmatrix} .
\tag{6}
\]

After diagonalizing it describes three Majorana neutrinos with huge masses \(M_\nu \sim \Lambda\), and three extremely light Majorana neutrinos with masses \(m_\nu \sim m_D^2/M_M\).

The scale \(\Lambda\) is theoretically arbitrary and must be fixed once for ever from one appropriately chosen experimental datum. In any case it is huge, because the flavor gauge symmetry, if it is real, is badly broken and yet unobserved.

Natural possibility is to relate the new mass scale \(\Lambda\) with the nonzero neutrino masses. The trouble is that their values are not known. From the experimentally available constraints the preferred value is around \(\Lambda \sim 10^{14}\) GeV. In 3 we have argued that \(\Lambda\) can be fixed also from the invisibility of one particular composite pseudo NG boson, the QCD axion, the existence of which the QFD also implies. The resulting numerical value of \(\Lambda\) is similar.

If the hypothesis of the complete dynamical self-breaking of \(SU(3)\) is correct there should be no theoretically arbitrary parameters in \(M_R\) and \(m_R\) except \(\Lambda\). The effective low-energy constants \(g_{ab}\) should be calculable in terms of the pure numbers, like the invariant group characteristics, e.g. the Clebsch-Gordan coefficients or Casimir operators of various \(SU(3)\) representations. Ultimately, the neutrino mass spectrum is completely fixed by the strong \(SU(3)_F\) dynamics.

IV. MASSES OF CHARGED LEPTONS AND QUARKS

Masses of the charged leptons and quarks are not fully determined by the strong flavor gluon exchanges in the SD equation connecting the right-(R) and the left-(L) handed fermion fields.

Both for the charged leptons and for the quarks there are also the Abelian electroweak gauge field \(B^\mu\) exchanges which differ by different numerical values of weak hypercharges (the non-Abelian \(SU(2)\) electroweak gauge fields \(A^\mu_i\) interact merely with the left-handed fermion fields and do not contribute to their Dirac masses):
\[
\begin{align*}
Y(l_L) &= -1, & Y(e_R) &= -2, & Y(\nu_R) &= 0 , \\
Y(q_L) &= \frac{1}{3}, & Y(u_R) &= \frac{4}{3}, & Y(d_R) &= -\frac{2}{3} .
\end{align*}
\]

(1) For charged leptons the \(RLB\) exchange in the SD equation is proportional to \(\bar{g}^2(q)\frac{1}{4}Y(e_R)Y(l_L)\).

(2) For quarks with the charge \(Q = 2/3\) the \(RLB\) exchange in the SD equation is proportional to \(\bar{g}^2(q)\frac{1}{4}Y(u_R)Y(q_L)\).

(3) For quarks with the charge \(Q = -1/3\) the \(RLB\) exchange in the SD equation is proportional to \(\bar{g}^2(q)\frac{1}{4}Y(d_R)Y(q_L)\).

(4) For neutrinos the \(RLB\) exchange in the SD equation is proportional to \(\bar{g}^2(q)\frac{1}{4}Y(\nu_R)Y(l_L)=0\).

The electroweak \(B\) interactions themselves, being weak all the way up to the Planck scale cannot generate the fermion self energies \(\Sigma\) dynamically. But they definitely contribute to the full \(LR\) kernel of the SD equation for \(\Sigma\) as described above. We don’t know at present how to incorporate convincingly these contributions into the separable Ansatz. We expect that in the non-perturbative solution these contributions become amplified by the non-analytic dependence of the result upon the effective couplings.

It is utmost important that these extra contributions are fully determined: The sliding coupling constant \(g'(q)\) is known as are the values of the fermion weak hypercharges.

Another important property of the \(B\) interaction is that it does not feel flavor: It is identical for fermions of the same \(Y\) and \(Q\) charges.

It is then natural to introduce the nine real parameters \(g'_i\), where \(i = e\) abbreviates the charged leptons, and \((f = 1, 2, 3\) or \(c, u, \tau\); \(j = u\) abbreviates the quarks with the charge \(Q = 2/3\), and \((f = 1, 2, 3\) or \(u, c, t\); \(j = d\) abbreviates the quarks with the charge \(Q = -1/3\), and \((f = 1, 2, 3\) or \(d, s, b\). We assume that these parameters modify phenomenologically the universal mass formulas 4 by the electroweak \(B\) contributions as follows:
\[
m_f(i) = \Lambda \exp \left(-1/4\alpha_i(i)\right) ,
\tag{7}
\]
where
\[
\begin{align*}
\alpha_1(i) &= \frac{3}{64\pi^2}\left\{g_{33} + \frac{2}{\sqrt{3}}g_{3s} + \frac{\sqrt{3}}{3}g_{ss}\right\} + g'_1 \}, \\
\alpha_2(i) &= \frac{3}{64\pi^2}\left\{g_{33} - \frac{2}{\sqrt{3}}g_{3s} + \frac{\sqrt{3}}{3}g_{ss}\right\} + g'_2 \}, \\
\alpha_3(i) &= \frac{3}{64\pi^2}\left\{\frac{\sqrt{3}}{3}g_{ss} + g'_3 \right\} .
\end{align*}
\]

This simple parametrization serves merely as a primitive illustration of the more ambitious picture formulated in the Abstract.
V. NUMERICAL ILLUSTRATION

1. General considerations

First we summarize how the QFD at strong coupling serves as a microscopic dynamics underlying the weakly coupled Higgs sector of the Standard model:

I. Primary is the spontaneous, genuinely quantal generation of different huge Majorana masses \( M_{fR} \sim \Lambda \) of three sterile right-handed neutrinos, and of three different exponentially small Dirac masses \( m_{fD} = \Lambda \exp(-1/4 \alpha_{f}) \) common to all fermions \( \nu_f, e_f, u_f, d_f \) in a family. These fermion masses break spontaneously the \( SU(3)_f \times SU(2)_L \times U(1)_Y \) symmetry down to unbroken \( U(1)_{em} \).

II. The underlying Goldstone theorem has two, valuable and firm, gold and stone, consequences:

First, eight 'would-be' NG bosons composed predominantly of sterile neutrinos (six of them have an admixture of the SM fermion composites) give self-consistently rise to different huge calculable masses of all flavor gluons. Here the self-consistency means that for the formation of the longitudinal spin states of flavor gluons their strong dynamics is crucial. The flavor \( SU(3)_f \) gauge symmetry gets dynamically badly completely self-broken, and practically all its consequences are indirect.

Second, three multi-component 'would-be' NG bosons composed by the strong QFD of all SM fermions give rise to the calculable masses of the electroweak gauge bosons \( W \) and \( Z \). Here the electroweak gauge interactions do not play any dynamical role. They are treated merely as weak external perturbations.

III. Masses of the \( W, Z \) bosons \( \xi \) are given in terms of the universal Dirac masses \( m_{fD} \). This implies that the Weinberg relation \( m_W/m_Z = \cos \theta_W \) is exact. Saturation of sum rules for \( m_W, m_Z \) by the mass of the heaviest (third) family implies

\[
m^2_W = \frac{1}{2} g^2 \frac{\Lambda}{\sqrt{\pi}} m^2_{3D},
\]

and enables to fix the mass \( m_{3D} \) as

\[
m_{3D} \approx 390 \text{ GeV} \quad (8)
\]

Obviously, this is the directly unobservable heaviest Dirac neutrino mass entering the famous seesaw mass formula.

IV. Inclusion of quantum effects of the electroweak \( B \) interactions, weakly coupled at \( \Lambda \), results in mass splitting of charged leptons and quarks within a given family. Study of the influence of this weak coupling effect on the masses of the intermediate bosons \( W \) and \( Z \) requires extra work. Our experience with modeling this effect \( 14 \) suggests that it should be small.

V. The model predicts six massive composite \( 0^+ \) particles \( \xi \) as the partners completing the sets of the composite 'would-be' NG bosons into a representation of the corresponding gauge group: (1) There should be one flavorless Higgs-like boson \( h \) completing three multi-component electroweak 'would-be' NG bosons composed of the electromagnetically interacting fermions into a composite (complex) \( SU(2) \) doublet. (2) There should be two flavored, flavor-conserving spinless bosons \( h_3 \) and \( h_8 \) completing six flavored components of the 'would-be' NG bosons composed of the electromagnetically interacting fermions into a composite real flavor octet. (3) There should be three superheavy flavored spinless bosons \( \chi_i \) completing eight components of flavored 'would-be' NG bosons composed of sterile right-handed neutrinos into the composite complex flavor sextet \( (2 \times 6 = 3 + 8 + 1) \) (one pseudo-NG boson remains in the physical spectrum as one of three axions).

It is instructive to compare the steps above with the corresponding steps in the weakly coupled Higgs sector of the Standard model:

ad I. Primary is to arrange the classical Higgs-field potential into the form which allows for the spontaneous breakdown of the gauge \( SU(2)_L \times U(1)_Y \) symmetry down to \( U(1)_{em} \).

ad II. The underlying Goldstone theorem has one valuable consequence: Three elementary 'would-be' NG bosons, pre-prepared in the complex Higgs field doublet become at the tree level the longitudinal spin states of \( W \) and \( Z \) bosons. Their masses are proportional to the Higgs-field condensate.

ad III. Because of the symmetry of the Higgs field kinetic term the Weinberg relation is fulfilled.

ad IV. Inclusion of fermions, i.e. adding their invariant Yukawa couplings with the Higgs field is an independent and fortunate step. These couplings generate at tree level for free the theoretically arbitrary fermion masses. Their quantum effects on the robust tree-level generation of \( m_W, m_Z \) are essentially negligible.

ad V. There is one elementary massive Higgs boson as a remnant of the elementary complex \( SU(2) \) doublet.

2. Fitting the fermion mass spectrum

1. We start by assuming that the heaviest Dirac neutrino mass is \( m_{3D} = 390 \) GeV, fixed from the sum rule for \( W, Z \) masses. For definiteness we set \( \Lambda = 10^{14} \) GeV. From \( 14 \) we easily compute \( g_{88} = 1.502790 \).

2. With the known \( g_{88} \) and with the experimental values of \( m_{\tau}, m_t, m_b \) (TABLE I) we fix \( g_3^3 = -0.341190, g_3^2 = -0.060098, g_3^1 = -0.295027 \).

3. From the experimental values of fermion masses of two lighter families (TABLE I) we fix the right-hand sides
by the seesaw mass formula, the system for the unknown masses are collected in TABLE II. It is gratifying that these are essentially of the same order of magnitude.

These are the six linear inhomogeneous equations for eight unknown parameters $g_f^1, g_f^2, g_f^3, g_{33}, g_{38}, f = 1, 2$.

4. Consequently, there is a two-parameter freedom in fixing $g_{33}$ and $g_{38}$, constrained by the assumption from the point 1: $m_{1D} < m_{3D}, i = 1, 2$. Clearly, would we know $g_{33}$ and $g_{38}$ from the neutrino mass spectrum given by the seesaw mass formula, the system for the unknown $g_f^1, g_f^2, g_f^3$ would be uniquely fixed.

5. For an illustration we fix the remaining universal parameters of QFD as $g_{33} = 1.27262$ and $g_{38} = -0.0594915$, corresponding to $m_{1D} = 3.9$ GeV, $m_{2D} = 39$ GeV and $m_{3D} = 390$ GeV. The resulting values of the weak hypercharge contributions to the charged lepton and quark masses are collected in TABLE II. It is gratifying that they all are essentially of the same order of magnitude.

With these illustrative numbers the masses of three active Majorana neutrinos can be roughly estimated, ignoring the matrix structure of seesaw, as $m_{\nu_e} \sim m_{2D}^2/\Lambda \equiv 1.521$ eV, $m_{\nu_\mu} \sim m_{3D}^2/\Lambda \equiv 1.521 \times 10^{-2}$ eV, $m_{\nu_\tau} \sim m_{2D}^2/\Lambda \equiv 1.521 \times 10^{-4}$ eV.

6. It follows from the explicit illustration presented above that the fermion masses are related with each other in a rather sophisticated way. First, six neutrino masses come from two QFD sources: $M_{FR}$ and $m_{FD}$. Second, masses of the charged leptons and quarks are described in terms of the QFD parameters of $m_{FD}$, and in terms of the parameters associated with the weak hypercharge.

Ultimately, however, the $SU(3)_f \times SU(2)_L \times U(1)_Y$ gauge dynamics of the system of the chiral fermion fields of the Standard model extended by three sterile right-handed neutrino fields gives rise both to the gauge boson and the fermion masses calculable solely in terms of $\Lambda$ and the electroweak couplings.

IV. CONCLUSION

The picture painted in this paper is based on a very strong assumption: The $SU(3)_f$ dynamics apparently identical with the QCD dynamics does not confine at large distances its elementary constituents, but rather self-consistently generates the calculable masses to all of them. Crucial is the possibility of generating the Majorana neutrino masses. The separable Ansatz used for illustrating this, though physically well motivated by the BCS [3, 13], is not under theoretical control. We tend to defend ourselves by F. Wilczek’s [12] Jesuit credo: “It is more blessed to ask forgiveness than permission.” We justify our perseverance by a number of desirable physical phenomena which the present rigid model describes and correlates [6]. Above all, to convert the innocent fermion flavor index into the source of a new force resulting in the universal hierarchical flavor splitting is irresistibly suggestive. Suggestive is also to associate the mass splitting within one family with the known electroweak force [17].

Gauging the family (flavor, horizontal, generation) index promises a hint to a solid theoretical answer to the famous question ‘why three families’ [18]. The LEP experimental proof of the existence of three light neutrinos [10] makes, of course, the question rather academic. Observation of the Higgs-like scalars $h_3$ and $h_8$ with calculable properties [20], which are the clear signature of the dynamical $SU(3)_f$ family picture [6] would be quite intriguing.

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TABLE I: Fermion masses from experiment [21].

| $m_3(e)$ | $m_2(e)$ | $m_1(e)$ |
|---------|---------|---------|
| 1.777 GeV | 105.7 MeV | 0.511 MeV |

| $m_3(u)$ | $m_2(u)$ | $m_1(u)$ |
|---------|---------|---------|
| 173.1 GeV | 1.28 GeV | 2.2 MeV |

| $m_3(d)$ | $m_2(d)$ | $m_1(d)$ |
|---------|---------|---------|
| 4.18 GeV | 96.0 MeV | 4.6 MeV |

TABLE II: The parameters of the theory describing the fermion spectra.

| $g_{33}$ | $g_{38}$ | $g_{88}$ |
|---------|---------|---------|
| 1.27262 | -0.0594915 | 1.502790 |

| $g_1^d$ | $g_2^d$ | $g_3^d$ |
|---------|---------|---------|
| -0.382808 | -0.315776 | -0.341190 |
| -0.332490 | -0.196766 | -0.060098 |

| $g_1^u$ | $g_2^u$ | $g_3^u$ |
|---------|---------|---------|
| -0.305581 | -0.320025 | -0.295027 |

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