The $Z \rightarrow \bar{b}b$ decay asymmetry
and flavor changing neutral currents

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Abstract

The measured value of $A_b$, the $Z\bar{b}b$ asymmetry parameter, disagrees with the Standard Model at 99% confidence level (2.6σ). If genuine the discrepancy could indicate new interactions unique to third generation quarks, implying an enhanced $Z$ penguin amplitude and an enhanced rate for the rare decay $K^+ \rightarrow \pi^+ \nu\bar{\nu}$. Measurements of $\epsilon'/\epsilon$ then suggest there should also be enhanced QCD penguin amplitudes. The Higgs sector of an $SU(2)_L \times SU(2)_R$ gauge theory has some of the features needed to explain these phenomena and would also imply right-handed penguin amplitudes.

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1This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
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**Introduction.** Each successive update of the precision electroweak data tends to reinforce the already spectacular agreement with the Standard Model (SM). An exception emerged in the Summer 1998 update, when new data from SLC on the $b$ quark front-back, left-right polarization asymmetry, $A_{FBLR}^b$, reinforced a possible discrepancy previously implicit in the LEP front-back asymmetry measurement, $A_{FB}^b$. Combined the two measurements implied a value for the $b$ asymmetry parameter $A_b$ three standard deviations ($\sigma$) below the SM value. The discrepancy continues today, though diminished to $2.6\sigma$ in the Spring 1999 data\[^1\], implying a confidence level (CL) for consistency with the SM of 1%.

The convergence of the SLC and LEP determinations of $A_b$ at a value in conflict with the SM could resolve the longstanding disagreement between the SLC and LEP measurements of the effective weak interaction mixing angle, $\sin^2 \theta_W$, a critical parameter that currently provides the most sensitive probe of the SM Higgs boson mass. If $A_b$ is affected by new physics then $A_{FB}^b$ must be removed from the SM fit of $\sin^2 \theta_W$, leaving the remaining measurements in good agreement. This possibility is also consistent with theoretical prejudice that the third generation is a likely venue for the emergence of new physics.

On the other hand the discrepancy could have an experimental origin. The now resolved $R_b$ anomaly illustrates the difficulties, which may be even greater for $A_b$ and $A_{FB}^b$. Or it could be a statistical fluctuation. Unfortunately the study of $Z$ decays is nearing its end. When the dust settles we may still be left wondering about the significance of the discrepancy.

The purpose of this paper is to observe that there is another arena in which the $A_b$ anomaly can be studied. If it is a genuine sign of new physics unique to (or dominant in) the third generation, new phenomena must emerge in flavor-changing neutral current (FCNC) processes. Then if the underlying physics has a mass scale much greater than $m_W$ and $m_t$, $Z$ penguin amplitudes are enhanced by about a factor two.\[^2\] The cleanest tests are in semileptonic decays such as the rare decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$, for which a factor two enhancement over the SM rate is predicted. A single event has been observed\[^3\] with a nominal branching ratio five times the SM prediction.

The $A_b$ anomaly could arise from new physics in the form of radiative corrections or $Z - Z'$ or $b - Q$ mixing. In the first case, but not in the latter two,
there would generically also be enhanced gluon (and photon) penguin amplitudes. This possibility is favored by the recent measurements of $\epsilon'/\epsilon$, since the $Z$ penguin enhancement by itself would exacerbate the existing disagreement with the SM. The gluon penguin enhancement cannot be deduced in a model independent way from the $A_b$ anomaly but can be estimated from $\epsilon'/\epsilon$. The existence of enhanced $Z$ and gluon penguins can be tested in $B$ meson decays and elsewhere. If present they would have a big impact on studies of the CKM matrix and CP violation.

_Fits of the $b$ quark couplings_ In the SM the $b$ quark asymmetry parameter is $A_b = 0.935$ with negligible uncertainty. In terms of the left- and right-handed $Zb\bar{b}$ couplings $g_{bL,R}$ it is

$$A_b = \frac{g_{bL}^2 - g_{bR}^2}{g_{bL}^2 + g_{bR}^2}.$$  

(1)

It is measured directly by the front-back left-right asymmetry, $A_b = A_{FB,LR}^b = 0.898(29)$ [1] and also by the front-back asymmetry using $A_b = 4A_{FB}^b/3A_\ell$ where $A_\ell$ ($\ell = e, \mu, \tau$) is the lepton asymmetry parameter defined as in eq. (1) with $b \to \ell$. Using $A_{FB}^b = 0.0991(20)$ from LEP and $A_\ell = 0.1489(17)$ from the combined leptonic measurements at SLC and LEP, we find $A_b = 0.887(21)$. The two determinations together imply $A_b = 0.891(17)$.

I have performed several fits to the five quantities that most significantly constrain $g_{bL}$ and $g_{bR}$. In addition to $A_b$ and the ratio of partial widths, $R_b = \Gamma_b/\Gamma_h$, they are the total $Z$ width $\Gamma_Z$, the peak hadronic cross section $\sigma_h$, and the hadron-lepton ratio $R_\ell = \Gamma_h/\Gamma_\ell$. The fits assume real shifts in the couplings. [2]

A brief summary is presented here; details will be given elsewhere. [1]

The SM fit assumes $\sin^2 \theta_W = 0.23128(22)$ which follows from $A_\ell$. It has chi-squared per degree of freedom $\chi^2/dof = 10.4/5$ with confidence level $CL = 6.5\%$. In fit 1 $g_{bL}$ and $g_{bR}$ are allowed to vary while all other $Zq\bar{q}$ couplings are held at their SM values, yielding $\chi^2/dof = 3.0/3$ and $CL = 39\%$. In fit 2 only $g_{bR}$ is allowed to vary; the result is $\chi^2/dof = 7.8/4$ with $CL = 10\%$, little better than the SM fit. In fit 3 the couplings of the $b$, $d$ and $s$ quarks are varied equally, $\Delta g_{bL,R} = \Delta g_{dL,R} = \Delta g_{sL,R}$; with a result nearly as good as fit 1. The other fits considered resulted in poorer $CL$’s than the SM fit.

We conclude that positive shifts are preferred for both $g_{bL}$ and $g_{bR}$, either for the $b$ quark alone as in fit 1 or for $b$, $d$ and $s$ equally as in fit 3. The need to
shift both left and right couplings is clear: $\delta A_b \simeq -0.05$ requires positive shifts in $g_{bR}$ and/or $g_{bL}$ (remember that $g_{bL} < 0$) while $g_{bL}^2 + g_{bR}^2$ is tightly constrained by the other measurements, forcing $\delta g_{bR}^2 \simeq -\delta g_{bL}^2$. Fit 3 seems unnatural in that $s, d$ couplings are varied while $u, c$ couplings are not, an issue finessed in fit 1 which presumably reflects physics unique to the third generation quarks, perhaps due to the large value of the top quark mass. The 32% and 5% contours from fit 1 are shown in figure 1, with the SM values, $g_{bL}, g_{bR} = -0.4197, +0.0771$, and the fit central values, $g_{bL}, g_{bR} = -0.4154, +0.0997$.

The Z penguin enhancement We now focus on fit 1 and the FCNC effects it implies. Physics from higher mass scales will couple to the $SU(2)_L$ quark eigenstates, so a nonuniversal $Z\bar{b}_L b_L$ coupling, $\delta g_{bL}$, has its origin in a nonuniversal $Z\bar{b}_L b'_L$ amplitude where $b'_L = V_{tb} b_L + V_{ts} s_L + V_{td} d_L$. As a result $Z\bar{s}s, Z\bar{d}d$, and $Z\bar{s}d$ interactions are induced.

The very same phenomenon occurs in the SM where the leading correction to the $Z\bar{b}b$ vertex arises from $t$ quark loop diagrams. For $m_t \to \infty$ the leading correction is

$$\delta g_{bL}^{SM} = \frac{\alpha_W(m_t)}{16\pi} \frac{m_t^2}{m_W^2}$$

where $\alpha_W = \alpha/\sin^2 \theta_W$. For $m_t = 174.3$ GeV this is $\delta g_{bL}^{SM} \sim 0.0031$. A more careful estimate based on the complete one loop result and with the pole mass $m_t$ replaced by the running $\overline{MS}$ mass, $\overline{m}_t(m_t) \simeq m_t - 8$ GeV, yields a similar result, $\delta g_{bL}^{SM} = 0.0032$, resulting in $g_{bL} = -0.4197$. In fit 1 $g_{bL}$ is shifted by an additional amount, $\delta g_{bL}^{A_b} = 0.0043$. These are large shifts: e.g., $\delta g_{bL}^{SM}$ corresponds to a $3\sigma$ effect in $R_b$.

The same Feynman diagrams responsible for the leading $Z\bar{b}b$ vertex correction also generate the SM Z penguin amplitude and in the limit $m_t \to \infty$ they are identical. Rewriting the one loop Z penguin vertex for $\bar{s}d$ transitions as an effective $\delta g_{\bar{s}dL}^{SM}$ coupling normalized like $g_{bL}$, we have (see eq. (2.18) of [5])

$$\delta g_{\bar{s}dL}^{SM} = \lambda_t \frac{\alpha_W}{2\pi} C_0(x_t)$$

where $\lambda_t = V_{ts}^* V_{td}$, $x_t = m_t^2/m_W^2$, and $C_0$ is

$$C_0(x) = \frac{x}{8} \left( \frac{x - 6}{x - 1} + \frac{3x + 2}{(x - 1)^2} \ln(x) \right)$$
Taking $m_t \gg m_W$ and comparing with eq. (2) we have

$$\delta g_{ZdL}^{SM} = \lambda_t \delta g_{bL}^{SM}. \quad (5)$$

Eq. (5) shows that if $m_t$ were much larger than any other relevant scale we could smoothly extrapolate the on-shell $Z\bar{b}b$ vertex correction to the related, $Zq\bar{q}$ penguin amplitudes. The same would be true of any new physics at a scale $m_X \gg m_W, m_t$, whether it affects the $Z\bar{b}b$ vertex by radiative corrections or by $Z - Z'$ or $b - Q$ mixing. Therefore if the $A_b$ anomaly arises from physics at a very high scale, the additional contribution to the $sd$ $Z$ penguin amplitude would be

$$\delta g_{ZdL}^{A_b} = \lambda_t \delta g_{bL}^{A_b}. \quad (6)$$

$K^+ \to \pi^+\nu\bar{\nu}$. The rare decay $K^+ \to \pi^+\nu\bar{\nu}$ is mediated by $t$ quark induced $Z$ penguin and box amplitudes plus an important contribution from $c$ quark penguins. To compute the enhancement due to eq. (6) it is convenient to rewrite eq. (7.7) from ref. \[6\] as

$$BR(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ [A^4 \left(1 + (\rho^2 + \eta^2) - 2\rho\right) X^2(x_t)$$

$$+ 2A^2(1 - \rho) \left(1 - \frac{\lambda^2}{2}\right) X(x_t) P_0(X) + \left(1 - \frac{\lambda^2}{2}\right)^2 P_0^2(X)] \quad (7)$$

where $A, \lambda, \rho, \eta$ are the Wolfensteinized CKM parameters, $X(x_t)$ contains the $t$ quark penguin and box amplitudes, $P_0$ contains the $c$ quark contribution, and $\kappa_+ = 4.11 \cdot 10^{-11}$ is determined from $K^+ \to \pi^+ e^+ \nu$. The $t$ quark contribution, evaluated at $m_t \to m_{\pi^+}(m_t)$ and with a small additional QCD correction, is $X = 1.49$. For the charm contribution I set $m_c = 1.3$ GeV and choose $\lambda_c^{(4)}$ to agree with $\alpha_S(m_Z) = 0.119$, implying $P_0 = 0.394$. Following ref. \[8\] I use $A = 0.819 \pm 0.035$, $\lambda = 0.2196 \pm 0.0023$ and $\sqrt{\rho^2 + \eta^2} = 0.423 \pm 0.064$. Then for $\rho = 0$ eq. (7) implies the SM value $BR(K^+ \to \pi^+\nu\bar{\nu}) = 0.86 \cdot 10^{-11}$. \[9\]

The SM box amplitude is essential for gauge invariance and it is numerically important in 't Hooft-Feynman gauge in which we work. But in the limit $m_t \gg m_W$ it is suppressed by $m_t^2/m_W^2$ relative to the penguin because of its softer UV behavior. Similarly for new physics at a scale $m_X \gg m_t, m_W$ the penguin amplitude will dominate over the new-physics box by a factor $m_X^2/m_W^2$.\[10\]
The FCNC amplitude $\delta g_{bL}^{Ab}$ then contributes an additional term to the $t$ quark contribution, $X(x_t)$, in eq. (7),

$$\delta X^{Ab} = \frac{2\pi \delta g^{Ab}_{bL}}{\alpha_W}. \quad (8)$$

With $\delta g^{Ab}_{bL} = 0.0043$ from fit 1, we find $\delta X^{Ab} = 0.81$. The enhanced branching ratio follows by replacing $X(x_t)$ in eq. (7) by $X' = X(x_t) + \delta X^{Ab}$. For the parameters given above and $\rho = 0$ the prediction is $BR(K^+ \to \nu \bar{\nu}) = (1.70 \pm 0.32^{+0.33}_{-0.30}) \cdot 10^{-11}$, twice the SM value. The first uncertainty is based on the SM Gaussian fit in ref. [3] and the second reflects the range of $\delta g^{Ab}_{bL} = 0.043 \pm 0.014$ indicated by the $32\% \text{ CL}$ contour in figure 1. Predictions for a range of $\rho$ values are shown in table 1. The SM and fit 1 predictions overlap, so an independent determination of $\rho$ is needed to distinguish them.

The $Z$ penguin enhancement, eq. (6), is by itself consistent with conservative bounds from $K_L \to \mu^+ \mu^-$ and $\epsilon'/\epsilon$ [11] but is disfavored by the most recent measurements of $\epsilon'/\epsilon$ unless QCD penguins are also enhanced. Using the analytical formula and parameters in ref. [11], the SM result is $\epsilon'/\epsilon = 6.9 \cdot 10^{-4}$, in reasonable agreement with the corresponding numerical result [11], $\epsilon'/\epsilon = 7.7_{-3.5}^{+6.0} \cdot 10^{-4}$.

The effect of the $Z$ penguin enhancement follows by replacing the SM $Z$ penguin loop integral $C_0(x_t)$ (which contributes as specified in [11] to the functions $X_0, Y_0, Z_0$ in the analytical formula) by $C_0 + \delta X^{Ab}$ where $\delta X^{Ab}$ is given in eq. (8). The result is $\epsilon'/\epsilon = (-0.4^{+6.0}_{-3.5} \pm 2.4) \cdot 10^{-4}$ where the first error is from the SM fit and the second reflects $\pm 0.0014$ in the fit of $\delta g^{Ab}_{bL}$.

These values are compared with $21.8 \pm 3.0$, the combined value [11] from E731, NA31, and KTeV. Taking the theoretical estimates at face value, consistency requires that gluon penguins are also enhanced. While the effect of gluon penguin enhancements on the determination of the CKM angles should be included, here we consider only rough estimates based on the usual CKM analysis. Enhancing the principal gluon penguin term (proportional to $r_0^{(6)}$ in the analytical formula) by the same factor ($\sim 2$) as the $Z$ penguin, the result is $15 \cdot 10^{-4}$, while a factor 3 enhancement of the $r_0^{(6)}$ term yields $29 \cdot 10^{-4}$.

It is important to keep in mind the theoretical uncertainties that cannot be fully characterized by the error estimates quoted above. The quoted determination of $\epsilon'/\epsilon$ depends sensitively on three theoretical parameters — the
strange quark running mass \( m_s \) and the hadronic matrix elements \( B_6^{1/2} \) and \( B_8^{3/2} \) — estimated by nonperturbative methods not yet under rigorous control. One can imagine plausible values of the parameters that raise the SM or enhanced Z penguin predictions to the preferred range without invoking new physics contributions to the gluon penguin operators. The uncertainties will hopefully be resolved by more powerful lattice simulations.

**Discussion** The \( A_b \) anomaly could be caused by radiative corrections of new bosons and/or quarks, by \( Z - Z' \) mixing, or by \( b - Q \) mixing with heavy quarks \( Q \) in nonstandard \( SU(2)_L \) representations. Generically radiative corrections would also affect gluon and photon penguin amplitudes, by model dependent amounts, while \( Z - Z' \) and \( b - Q \) mixing would only enhance the \( Z \) penguin. With the caveat expressed above, \( \epsilon'/\epsilon \) then favors radiative corrections, since it could be explained if gluon penguins are enhanced by a similar factor to the \( Z \) penguin enhancement deduced from \( A_b \).

The hypothesis that the \( A_b \) anomaly represents the effect of higher energy physics on third generation quarks can be falsified if the predicted \( Z \) penguin enhancements are absent. If they are present the hypothesis remains viable and the \( A_b \) anomaly provides key information beyond the FCNC studies. The large value of \( \delta g_{bR} \) and the condition \( \delta(g_{bR}^2 - g_{bL}^2) \gg |\delta(g_{bL}^2 + g_{bR}^2)| \) would then point to a radical departure from the SM with a sharply defined signature. For instance, the Higgs sector associated with a right-handed extension of the SM gauge sector could shift \( g_{bR} \) and \( g_{bL} \) with little effect on other precision measurements. Depending on the right-handed CKM matrix, there could also be observable right-handed FCNC effects.

The burgeoning program to study CP violation and the CKM matrix must measure \( Z \) and gluon penguin amplitudes in order to fully achieve its goals — an enterprise characterized as controlling “penguin pollution.” In the process we should learn if the FCNC effects implied by fit 1 occur or not. If they do “penguin pollution” would be transformed into a window on an unanticipated domain of new physics, of which the measurement of \( A_b \) would have provided the first glimpse.

*Acknowledgements:* I wish to thank R. Cahn, H. Quinn, P. Rowson, S. Sharpe, and especially Y. Grossman for helpful discussions. This work was supported
by the Director, Office of Science, Office of Basic Energy Services, of the U.S. Department of Energy under contract DE-AC03-76SF00098.

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Figure 1. $\chi^2$ contours for fit 1. The diamond is the SM prediction and the box is the best value from fit 1. The inner contour indicates $\chi^2 = 3.5$ corresponding to CL = 32% for 3 dof. The outer contour indicates $\chi^2 = 7.8$ corresponding to CL = 5% for 3 dof.

Table 1. Predictions for $BR(K^+ \rightarrow \pi^+\nu\bar{\nu})$ in units of $10^{-10}$ as a function of the CKM parameter $\rho$, for the SM and the enhanced $Z$ penguin model.

| $\rho$ | -0.4 | -0.2 | 0.0 | +0.1 | +0.2 | +0.4 |
|--------|------|------|-----|------|------|------|
| SM     | 1.32 | 1.09 | 0.86| 0.75 | 0.64 | 0.41 |
| Enhanced | 2.68 | 2.19 | 1.79| 1.46 | 1.21 | 0.72 |