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Contents
Restoration of QCD classical symmetries in excited hadrons

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Abstract. Restoration of chiral and $U(1)_{A}$ symmetries in excited hadrons is reviewed. A connection of these restorations with the semiclassical regime in highly excited hadron is discussed. A solvable confining field-theoretical toy model that exhibits chiral restoration is presented. Implications of the string description of the highly excited hadrons that suggests an additional dynamical symmetry of the spectra on the top of $U(2)_{L} \times U(2)_{R}$ are presented.

Keywords: Chiral symmetry, Hadrons, QCD, Strings

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PHENOMENOLOGICAL FACTS, CLASSIFICATION AND DEFINITIONS

The experimental spectrum of excited hadrons, both baryons [1, 2, 3] and mesons [4, 5] suggests that the highly excited hadrons in the $u, d$ sector fall into approximate multiplets of $SU(2)_{L} \times SU(2)_{R}$ and $U(1)_{A}$ groups that are compatible with the Poincaré invariance, for a short overview see ref. [6]. If confirmed by discovery of still missing states, this phenomenon is referred to as effective chiral symmetry restoration or chiral symmetry restoration of the second kind.

It is important to precisely characterize what is implied under effective restoration, because sometimes it was (is) erroneously interpreted in the sense that the highly-excited hadrons are in the Wigner-Weyl mode. This confusion was a source of a controversy [7, 8], which has been overcome, however [9]. The mode of symmetry is defined only by the properties of the vacuum. If symmetry is spontaneously broken in the vacuum, then it is the Nambu-Goldstone mode and the whole spectrum of excitations on the top of the vacuum is in the Nambu-Goldstone mode. However, it may happen that the role of the chiral symmetry breaking condensates of the vacuum becomes progressively irrelevant in excited states. This means that the chiral symmetry breaking effects (dynamics) become less and less important in the highly excited hadrons and asymptotically the states approach the regime where their properties are determined by the underlying unbroken chiral symmetry (i.e. by the symmetry in the Wigner-Weyl mode). One of the particular consequences of the chiral symmetry restoration in excited hadrons is that they should gradually decouple from the Goldstone bosons [3, 8, 9, 10, 11, 12]. A hint for such a decoupling is indeed observed phenomenologically since the coupling constant for $h^{*} \rightarrow h + \pi$ decreases very fast higher in the spectrum (because a decay width increases with the mass of the resonance much slower than the phase space factor).

By definition this effective chiral symmetry restoration means the following. All
hadrons that are created by the given interpolator, \(J_\alpha\), appear as intermediate states in the two-point correlator,

\[
\Pi = i \int d^4x e^{iqx}\langle0|T \{J_\alpha(x)J^\dagger_\alpha(0)\}|0\rangle.
\]  

(1)

Consider two interpolating fields \(J_1(x)\) and \(J_2(x)\) which are connected by a chiral transformation (or by a \(U(1)_A\) transformation), \(J_1(x) = UJ_2(x)U^\dagger\). Then, in the Wigner-Weyl mode, \(U|0\rangle = |0\rangle\), it follows from (1) that the spectra created by the operators \(J_1(x)\) and \(J_2(x)\) would be identical. We know that in QCD one finds \(U|0\rangle \neq |0\rangle\). As a consequence the spectra of the two operators must be in general different. However, it happens that the noninvariance of the vacuum becomes unimportant (irrelevant) high in the spectrum. Then the spectra of both operators become close at large masses and asymptotically identical. One could say, that the valence quarks in high-lying hadrons decouple from the quark condensate of the vacuum.

More precisely the effective symmetry restoration is defined to occur if two conditions are satisfied: (i) the states fall into approximate multiplets of \(SU(2)_L \times SU(2)_R\) (and of \(U(1)_A\)) and the splittings within the multiplets (\(\Delta M = M_+ - M_-\)) vanish at \(n \to \infty\) and/or \(J \to \infty\); (ii) the splitting within the multiplet is much smaller than between the two subsequent multiplets \([4, 5, 6]\). This definition is very restrictive, because the structure of the chiral multiplets is nontrivial and is different for mesons with different spins. The latter is a consequence of the requirement that the chiral multiplets must satisfy the Poincaré invariance \([5]\). In particular, given the set of the standard quantum numbers \(I, J^{PC}\) the meson multiplets of \(SU(2)_L \times SU(2)_R\) are

\[
J = 0
\]

\[
\begin{align*}
(1/2, 1/2)_a & : 1,0^{-} \longleftrightarrow 0,0^{++} \\
(1/2, 1/2)_b & : 1,0^{++} \longleftrightarrow 0,0^{-}, \quad (2)
\end{align*}
\]

\[
J = 2k, \; k=1,2,...
\]

\[
\begin{align*}
(0,0) & : 0,J^{--} \longleftrightarrow 0,J^{++} \\
(1/2, 1/2)_a & : 1,J^{-} \longleftrightarrow 0,J^{++} \\
(1/2, 1/2)_b & : 1,J^{++} \longleftrightarrow 0,J^{-} \\
(0,1) \oplus (1,0) & : 1,J^{++} \longleftrightarrow 1,J^{--} \quad (3)
\end{align*}
\]

\[
J = 2k-1, \; k=1,2,...
\]
\[
(0,0) : 0, J^{++} \leftrightarrow 0, J^{--}
\]
\[
(1/2, 1/2)_a : 1, J^{+-} \leftrightarrow 0, J^{--}
\]
\[
(1/2, 1/2)_b : 1, J^{--} \leftrightarrow 0, J^{+-}
\]
\[
(0, 1) \oplus (1, 0) : 1, J^{--} \leftrightarrow 1, J^{++}
\]

The \( U(1)_A \) symmetry connects the opposite parity states with the same isospin from the distinct \((1/2, 1/2)_a \) and \((1/2, 1/2)_b \) multiplets of \( SU(2)_L \times SU(2)_R \).

The recent data on highly excited mesons from the \( \bar{p}p \) annihilation at LEAR \cite{13,14} do support the \( SU(2)_L \times SU(2)_R \) and \( U(1)_A \) restorations as can be seen from the high-lying \( \bar{nn} \) mesons with \( J = 2 \).

Note, that the chiral symmetry requires a doubling of some of the radial and angular Regge trajectories for \( J > 0 \). This is a highly nontrivial prediction of chiral symmetry. For example, asymptotically some of the \( \rho \)-mesons lie on the trajectory that is characterized by the chiral index \((0, 1) \oplus (1, 0)\) and have as their chiral partners the \( a_1 \) mesons, while the other \( \rho \)-mesons have \( h_1 \) mesons as their chiral partners and lie on the other independent trajectory with the chiral index \((1/2, 1/2)_b \).

If we look carefully at the data one notices that all possible different chiral multiplets with the same \( J \) are approximately degenerate \cite{5}. Then it means that all these states fall into a reducible representation

\[
(0, 1/2) \oplus (1/2, 0) \times [(0, 1/2) \oplus (1/2, 0)]
\]
which combines all possible representations for the system of massless quark and anti-quark. Such a degeneracy is consistent with the view of the excited hadron as a string with massless quarks with definite chirality at the end points of the string [3].

There are still some missing states in the multiplets with $J = 0, 1, 3, 4$ [4, 5] and it is a very important experimental task to find them. This can be done in particular with the polarized target formation experiment in $\bar{p}p$ at the NESR low-energy antiproton ring at GSI, which will have similar or better characteristics than LEAR.

**ORIGINS OF CHIRAL AND $U(1)_A$ RESTORATIONS**

An important question is a physical origin of chiral and $U(1)_A$ restorations. If the spectrum is strictly continuous and the function $R$ approaches a constant value at large $s$, then the asymptotic freedom at large space-like momenta together with a dispersion relation do allow us to claim that the chiral symmetry is manifest in such a spectral function, as it is observed e.g. in $e^+e^- \rightarrow \text{jets}$. However, it is a trivial case and not what we actually need. We have to consider a (quasi) discrete spectrum where the given hadron state is isolated. The conjecture of ref. [2] was that may be the chiral restoration is true in the regime where the spectrum is quasidiscrete and saturated mainly by resonances.

One would expect that the Operator Product Expansion (OPE) could help us to find the correct spectrum of the high-lying hadrons. This is not so, however. This is because the OPE is only an asymptotic expansion. While such a kind of expansion is very useful in the space-like region, it does not define any analytical solution which could be continued to the time-like region. This means that while the real (correct) spectrum of QCD must be consistent with the OPE, there is an infinite amount of incorrect spectra that can also be consistent with the OPE. Then, if one wants to get some information about the spectrum, one needs to assume something else on the top of the OPE. Clearly a result then is crucially dependent on these additional assumptions, for the recent activity in this direction see refs. [15, 16, 17]. This implies that in order to really understand chiral symmetry restoration one needs a microscopic insight and theory that would incorporate at the same time chiral symmetry breaking and confinement.

A fundamental insight into phenomenon can be obtained from the semiclassical expansion of the functional integral directly in the time-like region [6]. We know that the axial anomaly as well as the spontaneous breaking of chiral symmetry in QCD is an effect of quantum fluctuations of the quark field. The latter can generally be seen from the definition of the quark condensate, which is a closed quark loop. This closed quark loop explicitly contains a factor $\hbar$. The chiral symmetry breaking, which is necessarily a nonperturbative effect, is actually a (nonlocal) coupling of a quark line with the closed quark loop, which is a graphical representation of the Schwinger-Dyson (gap) equation.

At large $n$ (radial quantum number) or at large angular momentum $J$ we know that in quantum systems the semiclassical approximation must work. Physically this approximation applies in these cases because the de Broglie wavelength of particles in the system is small in comparison with the scale that characterizes the given problem. In such a system as a hadron the scale is given by the hadron size while the wavelength of valence quarks is given by their momenta. Once we go high in the spectrum the size of hadrons increases as well as the typical momentum of valence quarks.
The semiclassical approximation applies when the action in the system $S \gg \hbar$. In this case the whole amplitude (path integral) is dominated by the classical path (stationary point) and those paths that are infinitesimally close to the classical path. In other words, in the semiclassical case the quantum fluctuations effects are strongly suppressed and vanish asymptotically. Then the correlation function can be expanded in powers of $\hbar/S$. The leading contribution is a tree-level contribution to the path integral and keeps chiral symmetries of the classical Lagrangian. It contains no quantum fluctuations of the valence quark lines. Its contribution is of the order $(\hbar/S)^0$. The quantum fluctuations of the quark lines as well as the vacuum fermion loops contribute at the subleading orders in $(\hbar/S)$ and hence are suppressed in hadrons with large intrinsic action $S$. Then it follows that in a hadron with large enough radial quantum number $n$ or $J$, where action is large, the loop contributions must be relatively suppressed and vanish asymptotically. Hence in such systems both the chiral and $U(1)_A$ symmetries should be approximately restored. This is precisely what we see phenomenologically. Note that the semiclassical expansion is not an expansion in the coupling constant, which is large in the nonperturbative regime.

**A SOLVABLE TOY MODEL**

While the argument presented above is general and solid enough, a detailed microscopic picture is missing. Then to see how all this works one needs a solvable field-theoretical model. The model must be chirally symmetric and confining. Such a model is known [22, 23, 24]. This model can be considered as a generalization of the large $N_c$ 't Hooft model (QCD in 1+1 dimensions) [25] to 3+1 dimensions. In both models the only interaction between quarks is the instantaneous infinitely raising Lorentz-vector linear potential. Then chiral symmetry breaking is described by the standard summation of the valence quarks self-interaction loops (the Schwinger-Dyson or gap equations), while mesons are obtained from the Bethe-Salpeter equation for the quark-antiquark bound states. Restoration of chiral symmetry in excited heavy-light mesons has been previously studied with the quadratic confining potential [19].
FIGURE 2. Angular Regge trajectories for isovector mesons with \( M^2 \) in units of \( \sigma \). Mesons of the chiral multiplet \((1/2, 1/2)_a\) are indicated by circles, of \((1/2, 1/2)_b\) by triangles, and of \((0, 1) \oplus (1, 0)\) by squares \((J^{++} \text{ and } J^{--} \text{ for even and odd } J, \text{ respectively})\) and diamonds \((J^{--} \text{ and } J^{++} \text{ for even and odd } J, \text{ respectively})\).

The results for excited light-light mesons with the linear potential are reported in ref. \([26]\) and presented in Fig. 1. The excited states fall into approximate chiral multiplets and a very fast restoration of both \(SU(2)_L \times SU(2)_R\) and \(U(1)_A\) symmetries with increasing of \( J \) and essentially more slow restoration with increasing of \( n \) is seen. In Fig. 2 the angular Regge trajectories are shown. They exhibit deviations from the linear behavior. This fact is obviously related to the chiral symmetry breaking effects for lower mesons. In the limit \( n \to \infty \) and/or \( J \to \infty \) one observes a complete degeneracy of all multiplets, which means that the states fall into representation \([5]\). This means that in this limit the quark loop effects disappear completely \([6, 18]\).

A few comments about physics which is behind these results. The chiral symmetry breaking Lorentz-scalar dynamical mass of quarks arises via loop dressing of quarks and represents effects of quantum fluctuations of the quark field \([18]\). A key feature of this dynamical mass is that it is strongly momentum dependent and vanishes at large quark momenta. When one increases excitation energy of a hadron, one also increases a typical momentum of valence quarks. Consequently, the chiral symmetry violating Lorentz-scalar dynamical mass of quarks becomes small and asymptotically vanishes in highly excited hadrons. Hence, chiral and \(U(1)_A\) symmetries get approximately restored \([1, 18, 10]\).

Exactly the same reason implies a decoupling of these hadrons from the Goldstone bosons \([3, 10]\). The coupling of the Goldstone bosons to the valence quarks is regulated via the axial current conservation by the Goldberger-Treiman relation. Then the coupling constant must be proportional to the Lorentz-scalar dynamical mass of valence quarks and vanishes at larger momenta. This represents a microscopical mechanism of decoupling which is required by the general considerations of chiral symmetry in the
Nambu-Goldstone mode \([8, 11, 12]\).

**CHIRAL MULTIPLETS AND STRING**

There are certain phenomenological evidences that the chiral multiplets of excited baryons and mesons with *different* spins cluster at some energies \([8, 27, 12, 28]\). This implies that one observes higher symmetry that includes chiral \(U(2)_L \times U(2)_R\) as a subgroup. Presumably this additional degeneracy reflects a dynamical symmetry of the string. Indeed, like the Veneziano amplitude, the spectrum of the open bosonic string is degenerate with respect to the orbital momentum of the string \([29]\). A mathematical description of a hadron as a string with quarks at the ends that have definite chirality \([3]\) is an open question.

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