How Thin and Efficient Can a Metasurface Reflector Be? Universal Bounds on Reflection for Any Direction and Polarization

Mohamed Ismail Abdelrahman* and Francesco Monticone*

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Light reflection plays a crucial role in a number of modern technologies. In this paper, analytical expressions for maximal reflected power in any direction and for any polarization are given for generic planar structures made of a single material represented by a complex scalar susceptibility. The problem of optimal light-matter interactions to maximize reflection is formulated as the solution of an optimization problem in terms of the induced currents, subject to energy conservation and passivity, which admits a global upper bound by using Lagrangian duality. The derived upper bounds apply to a broad range of planar structures, including metasurfaces, gratings, homogenized films, photonic crystal slabs, and more generally, any inhomogeneous planar structure irrespective of its geometrical details. These bounds also set the limit on the minimum possible thickness, for a given lossy material, to achieve a desired reflectance. Moreover, the results allow the discovery of parameter regions where large improvements in the efficiency of a reflective structure are possible compared to existing designs. Examples are given of the implications of these findings for the design of superior and compact reflective components made of real, imperfect (i.e., lossy) materials, such as ultra-thin and efficient gratings, polarization converters, and light-weight mirrors for solar/laser sails.

1. Introduction

Advanced nano-fabrication and nano-patterning techniques allow unprecedented control over light-matter interactions, opening vast opportunities for light-based technologies with enhanced performance. Nanophotonic solar cells are a good example of how confining light at subwavelength scales can dramatically increase the absorption enhancement beyond the conventional limit (Yablonovitch limit) for bulky solar cells.[1] Subwavelength patterning allows to realize not only better optical devices, but also novel effects and functions that were not previously thought to be possible with natural materials, such as negative refraction, invisibility, artificial magnetism, and light manipulation over thin surfaces.[2–8]

Because of the capacity to create nano-structures in virtually unlimited forms and with high precision, the design space of all conceivable geometrical configurations for a given volume is vast, possibly spanning thousands to millions of optimization variables. Design methods that involve large-scale simulations, either via brute-force parametric sweeps or using more advanced inverse-design and optimization algorithms, are the typical approach to arrive at components with superior performance. There is no doubt that this approach is successful in achieving efficient designs[9–13]; however, it suffers from a fundamental weakness: no matter how many simulations are performed, there is typically no guarantee that a globally maximal/minimal solution could be identified. If a structure's performance metrics are already close to the optimal solution, significant computing work could be wasted for the sake of finding a better design with no noticeable enhancement.

In this context, the question of how to determine a fundamental bound on a certain physical response in a specific volume (such as absorption, scattering, reflection, etc.), no matter how finely structured the system is, has become critically important both from a scientific and a practical perspective. To obtain universal bounds on optical response, these questions should be approached from a fundamental perspective, by examining basic physics constraints like energy conservation, causality, passivity, and symmetries, which govern the totality of electromagnetic interactions. Several fundamental bounds have already been identified in the literature, such as bounds on the scattering cross section, absorption, near-field radiative heat transfer, antenna performance, the local density of states, the refractive index, and other physical responses.[14–20] In a design approach informed by fundamental bounds, inverse-design methods can then be used to determine an actual feasible design as close as possible to the global optimum.

A particularly significant optical function that has only received marginal attention in this context is the ability to optimally control the reflected field in terms of magnitude, phase, direction, and polarization, which is important for a plethora of
applications, from standard ones related to reflection gratings, polarizers, and mirrors, to more exotic scenarios. For instance, maximizing and controlling reflections with the smallest possible amount of material is of crucial importance for solar/laser sails powered by radiation pressure. A lighter sail can more easily be accelerated to higher speeds, potentially reaching a significant fraction of the speed of light.\[21,22\] Furthermore, channeling the incident light power into the orthogonal polarization state through reflection is critical to create chiral cavity modes (modes with well-defined handedness), which in turn can enhance molecular detection sensitivity.\[23,24\] Many applications, such as holography, light-based radars (lidars), virtual/augmented reality, and solar sail steering, also require the ability to reflect power in a specific direction with very high efficiency using thin platforms (e.g., metasurfaces).\[22,25–31\]

In this paper, we derive an analytical closed-form expression (Equation (10)) for the upper bound on the power reflected, in any direction and polarization, from a generic planar structure of thickness $h$ that is made of a single material characterized by a complex scalar susceptibility $\chi$. No prior assumption about the structural features and details of the optimal design is made, as illustrated in Figure 1. The methodology used to derive the upper bounds is adopted from Ref. [17], where the authors developed a comprehensive framework using convex optimization techniques to determine global bounds on single-frequency light-matter interactions constrained by energy conservation, manifested in the so-called “optical theorem.” Intuitively, this theorem shows how the combined action of both scattering and absorption processes restricts the induced polarization currents $\Phi$, and accordingly, the physical response of the structure. Using this approach, Ref. [17] derived fundamental upper bounds for the total extinction, scattering, and absorption across sections of a material body. When compared to earlier bounds that relied solely on either the scattering or absorption processes to construct energy-conservation constraints, the bounds derived in Ref. [17] have been shown to be tighter and converge at all scales.

![Figure 1](https://www.advancedsciencenews.com)

Figure 1. Our goal is to find an upper bound on the general reflection problem from a generic planar structure of thickness $h$ (incidence and reflection directions and polarizations are arbitrary, and there is no prior assumption on the optimal structure details). The bound indicates the highest possible reflection over all possible structures made of a single material characterized by a complex susceptibility $\chi$. For simplicity, the structure width is assumed to be much larger than the thickness $h$, and the considered material is local, isotropic, and nonmagnetic. (Inset): The optimization problem is formulated using induced polarization currents radiating in vacuum, which results in the same fields originating from the inhomogeneous material distribution. To find an analytical expression for the bound, all the possible induced current distributions are assumed to be bounded by a film of thickness $h$ and centered at $z = 0$.

Based on this approach, the bounds are derived from an optimization problem formulated in terms of the induced currents, with a general form given as follows:

$$\max \Phi^\dagger \bar{A} \Phi \quad \text{given } \Im(\Phi^\dagger \Im(\chi \Phi + \Im Y)) = 0$$

(1)

In this vector notation, the inner product between two vectors indicates the integral over the structure volume: $X : Y = \int_X X(r') \cdot Y(r') \, dV'$, while matrix products act as convolution operators (the integrals and position dependencies may also be entirely removed by assuming any sufficiently good numerical discretization of the problem, in which case $X$ and $Y$ would be vectors of size $pN \times 1$ for $N$ spatial degrees of freedom and $p$ polarization degrees of freedom). The objective function to be maximized represents a general quadratic power-flow response function, defined by the matrix $\bar{A}$ (further details below), while the constraint equation is the volume-integral form of the energy conservation law (optical theorem for real power conservation) that governs the light-matter interaction: the total power extracted from the excitation field $E$ is equal to the sum of total absorbed and scattered power by the material body, proportional respectively to the loss term $\Im(\chi)$ and the electromagnetic free-space Green’s function tensor $\Im \Gamma_0$. Here, and in the rest of the paper, passivity is assumed, i.e., no gain mechanism exists in the medium.

The crucial advantage of formulating the problem as in Equation (1) is that a global maximal solution, i.e., an upper bound, is guaranteed to exist, and it can be readily determined, due to the convexity of the “dual” problem.\[17\] Moreover, the upper bound of the solution in a given domain $V_{\gamma}^r$ is always equal to or smaller than the upper bound in a larger domain $V_{\gamma}^r \supset V_{\gamma}^{r'}$.\[32\]

Thus, the upper reflection bound for a planar optimizable domain of thickness $h$, which can be calculated analytically, is also the upper bound for any other optimizable domain, of any shape, contained in it. Nevertheless, it is important to note that since energy conservation applies to the given structure as a whole, the bound is not necessarily tight, namely, it is not necessarily fulfilled by a particular, physical, excitation field. Adding more constraints, such as local energy conservation or the passive version of the optical theorem, can result in a tighter bound, but at the expense of no available closed-form solution, therefore losing relevant physical insight.\[15,33\]

2. Upper Bounds on the General Reflection Problem: Derivation

The quantity representing reflection must meet two conditions for the application of the optimization approach. Equation (I), discussed in the previous section: it must be expressed in terms of the induced polarization currents and it must be quadratic (or linear) with respect to those currents. An ideal candidate is the far-field directional radiation/scattering intensity, which represents the scattered power in an arbitrarily direction $k^\dagger$ and polarization $\hat{e}^\dagger$ and can be written in terms of the induced polarization currents as\[34,35\]
\[ U(k^*, \hat{e}^*) = \frac{k^4}{32\pi^2} \int \left| \hat{e}^* \cdot J(r) e^{ikr} \right|^2 dV \]  

(2)

where \( k \) is the free-space wavenumber, and the vacuum permeability and permittivity are set to unity. In this paper, we consider nonmagnetic materials, where only electric polarization currents \( \hat{I}(r) = i \hat{e}(r) \) are nonzero. Using the vector notation introduced above, the radiation intensity can then be expressed as

\[ U = \frac{k^4}{32\pi^2} \Phi \hat{F}^* \Phi^\dagger \]  

(3)

where \( \Phi^* = e^{ikr} \cdot \hat{e}^* \) is a vector determined by the direction and polarization of the reflected wave. The quantity in Equation (3) is in the same form as the general objective function in Equation (1), with \( \hat{A} \propto \Phi \hat{F}^* \). Thus, using the convexity of the dual problem, and following the same approach as in Refs. [15, 17], it can be shown that the upper bound on the directional radiation intensity can be determined as

\[ U_{\text{opt}} = \frac{k^4}{128\pi^2} \left( |\alpha| + \sqrt{\beta} \gamma \right)^2 \]  

(4)

where \( \alpha = \Phi^* \cdot \hat{G} \cdot \Phi, \beta = \hat{E}^* \cdot \hat{G} \cdot \hat{E}, \gamma = \Phi^* \cdot \hat{G} \cdot \Phi^\dagger \) while \( \hat{G} \) is a matrix given by \( \left( \text{Im} \zeta + \text{Im} \Gamma_0 \right)^{-1} \). For isotropic materials, \( \text{Im} \zeta \) is a scalar multiplied by the identity matrix, and \( \hat{G} \) can be directly expanded in terms of the eigenvectors of the Green’s function.

As discussed in the previous section, to find analytical expressions for the upper reflection bound for a planar structure of thickness \( h \), instead of evaluating Equation (4) for the specific shape of the given structure, we evaluate it for a high-symmetry geometry enclosing the structure, i.e., a planar bounding volume of the same thickness, for which analytical expressions for the eigenvectors and eigenvalues of the Green’s function operator are available. An important point to mention here is that, while we assume that the structure is laterally infinite, so that we can properly define a scattering intensity in “far-field,” as in (2), we nevertheless expand the Green’s function using the basis functions for an infinitely extended planar geometry, which are simple propagating plane waves. Clearly, this assumption is valid only if the area of the structure \( A \) is much larger than its thickness and the wavelength, which is the case of interest here. Indeed, the validation examples presented in the next section show that this assumption leads to valid results.

The expansion of the imaginary part of the Green’s function into propagating plane wave modes is given by\[5\]

\[ \text{Im} C_\alpha = \sum_{\nu, p, \kappa, \chi} \int \text{v}_{\nu, p}(\kappa, \chi) \text{v}^*_\nu(\kappa, \chi) \frac{dk_\nu}{(2\pi)^2} \]  

(5)

where the index \( s \) represents modes with even(+) or odd(−) parity, the index \( p \) denotes TE (Transverse Electric) or TM (Transverse Magnetic) polarizations, \( \kappa = \kappa_x \hat{e}_x + \kappa_y \hat{e}_y \), and \( \kappa' = \kappa'_x \hat{e}_x + \kappa'_y \hat{e}_y \). The expressions for the modes \( \text{v}_{\nu, p} \) are given in the literature, e.g., in the Appendix of Ref. [17]. These modes form a complete and orthogonal set over the film volume as \( \text{v}^*_\nu(\kappa) \text{v}_{\nu'}(\kappa') = \delta_{\nu, \nu'} \delta_{\kappa, \kappa'} \delta_{\kappa, \kappa'}, \) where \( \delta \) is the Kronecker delta, and the eigenvalues \( \rho_{\nu, \nu'}(\kappa) \) are given by:

\[ \rho_{+\text{TE}}(\kappa) = k^4/\kappa^2 (1 + \sin k_x h)/4, \text{ and } \rho_{+\text{TM}}(\kappa) = \rho_{-\text{TE}}(\kappa) = \rho_{-\text{TM}}(\kappa) = k^4/\kappa^2 \sin k_x h/2. \]  

The same set of propagating plane-wave modes can also be used to expand the incident propagating electric field as

\[ \text{E}^i = \frac{1}{k^{1/2}} \sum_{\nu, p, \kappa, \chi} e^{ikr} \text{v}_{\nu}(\kappa) \frac{dk_\nu}{(2\pi)^2} \]  

(6)

Without loss of generality, however, in the following we consider the incident field to be a single propagating plane wave, \( \text{E}^i = E_0 e^{ikr} e^{i\alpha \hat{e}}, \) corresponding to choosing the expansion coefficients in Equation (6) as \( \text{E}^{\alpha}(\kappa) = C(\kappa, \nu, \mu) = \sqrt{2\pi} k_\nu k \text{E}_0 \text{A} \delta_{\kappa, \kappa'} \delta_{\nu, \nu'} \). The values of \( C \) are as follows: \( C(\kappa, \nu, \mu) = \kappa \mu = i, C(\kappa, \nu, \mu) = \kappa \nu = i, C(\kappa, \nu, \mu) = 1, C(\kappa, \nu, \mu) = -i \). The last two values are multiplied by \( -1 \) if the propagation direction of the incident field is flipped from \( z \) to \( -z \) (backreflection). The reflection vector \( \hat{F} \) can also be expanded similarly.

Using these expansions, the terms in Equation (4) are evaluated as

\[ \beta = 2A \cos(\theta) \frac{E_0^2}{k} \sum_{\nu} \frac{\rho_{\nu, \nu}(\kappa)}{\text{Im} \zeta + \rho_{\nu, \nu}(\kappa)} \]  

(7)

\[ \gamma = 2A \cos(\theta) \frac{1}{k} \sum_{\nu} \frac{\rho_{\nu, \nu}(\kappa)}{\text{Im} \zeta + \rho_{\nu, \nu}(\kappa)} \]  

(8)

\[ \alpha = 2A E_0 \cos(\theta) \frac{\delta_{\kappa, \kappa'} \delta_{\nu, \nu'}}{k} \sum_{\nu, \mu} \frac{\rho_{\nu, \nu}(\kappa)}{\text{Im} \zeta + \rho_{\nu, \nu}(\kappa)} \]  

(9)

where the angle of incidence is \( \cos(\theta) = k_\parallel^2 / k \), and the angle of reflection is \( \cos(\theta') = k_\parallel' / k \). The first two terms characterize the incident and reflected waves independently, while the \( \alpha \)-term depends on the specific polarization and direction of both incident and scattered waves. For a generic incident field, these terms would consist of a summation over the \( k_\parallel \) spectrum of the incident wave.

### 3. General Reflection Bound and Validation

The reliability of Equation (4) as an upper bound on reflection from planar structures, and the validity of our assumptions, can be tested by comparing \( U_{\text{opt}} \) to the standard Fresnel reflectance \( R \) for a homogeneous film for various incidence angles \( \theta = \theta' \), and for both polarizations.\[36\]

For a physically consistent comparison between the far-field scattering intensity and the reflectance, a suitable normalization factor \( U_0 \) is introduced to ensure that \( U_{\text{opt}} = U_{\text{opt}} / U_0 \) does not exceed unity for passive systems. The reference case \( U_0 \) is the far-field scattering intensity, in the desired direction, from a finite perfectly conducting surface of area \( A \), yielding \( U_0 = k^4 A E_0^2 \cos^2 \theta / (8\pi^2) \). Alternatively, the same result can be obtained by setting \( \text{Im} \zeta \rightarrow 0 \) in the bound expression above. For the general case with different incident and reflected
directions, the normalization factor should be slightly modified as $U_0 = k^2 A^2 E_0^2 \cos \theta \cos \theta' / (8 \pi^2)$.

Finally, the normalized upper bound $\tilde{U}_{\text{opt}}$ can be written as

$$\tilde{U}_{\text{opt}} = \frac{1}{4} \left\{ \delta_{k, k'} \delta_{p, p'} \sum_{n \neq 1} \frac{\rho_{n, p}(k)}{\Im \zeta + \rho_{n, p}(k)} + \delta_{k, k'} \delta_{p, p'} \sum_{n \neq 1} \frac{\rho_{n, p}(k)}{\Im \zeta + \rho_{n, p}(k)} \right\}^{\frac{1}{2}} \tag{10}$$

This expression, which represents the main result of the paper, provides a strict upper bound on reflection from a generic planar structure, for any polarization and direction of incidence and reflection. The bound depends directly on the thickness of the structure, which determines the eigenvalues $\rho_{n, p}(k)$, and on the loss factor $\Im \zeta = \Im \chi / |\chi|^2$. The bound converges to unity (perfect reflection) if the thickness diverges, $h \to \infty$, and/or in the lossless limit, $\Im \zeta \to 0$. Indeed, if the material properties were unconstrained, it would be possible to create planar structures with ideal reflectance in any direction/polarization, as demonstrated by recent work on metasurfaces and meta-gratings,[37-40] by relying on perfect conductors or lossless plasmonic materials to create the desired (often resonant) response for any non-zero thickness or to engineer strong nonlocal effects (i.e., spatial dispersion) mediated by guided waves.[37,41]

Instead, since real materials and metamaterials are always imperfect, i.e., they exhibit non-zero dissipation or scattering/disorder losses, it is very relevant to study the fundamental limits to reflection for real lossy materials.

**Figure 2** shows a comparison between the Fresnel reflectance $R$ and the derived bound (10) for a dielectric lossy film illuminated by a TE plane wave, as a function of the film thickness and for different incident angles. The polarization term $\delta_{k, k'}$ equals unity since only reflection with the same polarization as the incident field is considered. For $h \ll \lambda$, $\tilde{U}_{\text{opt}}$ is close to the Fresnel reflectance $R$, which indicates that a homogeneous film is the best design choice for the given lossy material to achieve maximum reflection in the deep-subwavelength region. This result is consistent with the fact that if the structure is thin compared to the wavelength, and the considered material is lossy, then patterning/structuring the thin film would not help, as the structure would not be able to induce any strong resonant response to increase reflection.

The significance of the reflection bound becomes more apparent for larger thicknesses, as the bound becomes distinctly higher than $R$ (Figure 2), and eventually converges to unity when the planar volume is several wavelengths thick. As the designable volume gets larger, it is more likely to find a (possibly resonant) current distribution that produce reflected fields exceeding the homogeneous dielectric film reflection. Similar observations were made in Ref. [17], but for the total scattering/absorption cross sections of thin films (whereas here we are interested in the reflection in a specific direction and polarization), and inverse-design methods were used to find structures that maximize absorption beyond the homogeneous film case. By repeating the comparison for different values of the refractive index and for both polarizations, it is found that, correctly, the bound always exceeds the Fresnel reflectance, i.e., $\tilde{U}_{\text{opt}} \geq R$, for any incidence angle, and the bound always increases with increasing thickness. Moreover, for vanishing thicknesses, $\tilde{U}_{\text{opt}}$ approaches $R$ and goes to zero for any non-zero loss factor $\Im \zeta$.

The case of TM polarization is more subtle and merits further discussion, as the bound significantly deviates from $R$ for large incidence angles, as seen in Figure 3. This is a consequence of the Brewster angle effect, according to which a TM wave is entirely transmitted through a homogeneous interface at the Brewster’s angle $\theta_B = \tan^{-1}(n)$, where $n$ is the refractive index of the material.[34] Intuitively, by disrupting the homogeneity of the slab through patterning/structuring, the Brewster angle condition is violated, and therefore a significantly higher reflection, for the same incident angle, is certainly possible. Consistent with this observation, the bound predicts that it is possible to find a configuration that yields a much higher reflection for the TM case near the Brewster angle, compared to a homogeneous film, even for deeply subwavelength thicknesses. This can be further confirmed by simulating the reflection from a simple configuration different from a homogeneous film, such as a periodic array of disks of the same height and material properties, as shown in Figure 3.

The upper reflection bound can also be validated by comparing it to the results of effective medium theories (EMTs), or homogenization theories, for an inhomogeneous distribution of matter. Homogenization theories show that the collective behaviour of an ensemble of subwavelength, dipolar, polarizable particles, with subwavelength inter-particle distances, can be treated as the response of a homogeneous medium of well-defined effective macroscopic properties. For example, the well-established Maxwell Garnett homogenization theory provides an analytical expression to determine the space of possible values of effective permittivity $\varepsilon_\ell$ that can be obtained from identical spherical inclusions of permittivity $\varepsilon_i$, as a function of the density of polarizable elements or filling factor $\chi$.[43-46] Maximum possible specular reflection from a homogenized film of thickness $h/\lambda$ can then be calculated by considering
all the possible values of \( \varepsilon_r \) predicted by the homogenization formula. As confirmed by Figure 4, the derived upper reflection bound always exceeds the reflectance of a homogenized film. This is consistent with the fact that, while standard homogenization theories assume purely dipolar light-matter interactions, therefore constraining the resulting polarization current, our approach to derive reflection bounds does not make any assumption for the induced current, making it broadly applicable to any structure, not just those that can be homogenized.

![Figure 3](image3.png)

**Figure 3.** Reflectance \( R \) of a homogeneous dielectric film (solid lines) vs. the reflection bound \( U_{\text{opt}} \) (dashed), as a function of incidence angle, for both TE (blue) and TM (red) polarizations. The film thickness is \( h/\lambda = 0.04 \) and the refractive index is \( n = 7 + 2i \). For the TM case, \( R \) is suppressed around the Brewster angle, significantly deviating from the bound. The dots represent the specular reflection from a periodic square array of disks (the periodicity is \( h/\lambda = 0.2 \)) of the same height and material properties, which demonstrates five times the reflectance of the homogenized film at the Brewster angle. COMSOL Multiphysics (v. 5.4) was used for the numerical evaluation of the reflected power for the array.

![Figure 4](image4.png)

**Figure 4.** Comparison between the reflectance of a homogeneous plasmonic film of permittivity \( \varepsilon_r = -3 + 0.1i \) (solid black line), the maximum possible reflectance of a homogenized-material film made of identical spherical inclusions with the same permittivity \( \varepsilon_r \) (dashed red), and the upper reflection bound for a generic planar structure made of the same material (dashed black). All cases are for the same thickness \( h \). The effective permittivity of the homogenized-material film is obtained using the Maxwell Garnett formula while varying the filling factor to determine the maximum possible reflectance for each value of thickness.

**4. Results: Selected Applications**

The fundamental bound on reflection derived in the previous section applies to any planar structure made of lossy materials, including homogeneous and homogenized slabs, multilayer thin films, gratings and meta-gratings, photonic crystal slabs, and metasurfaces. In this section, we discuss the use of these general bounds in three examples of application scenarios where designing efficient reflective structures is a crucial goal.

**4.1. Reflective Mirrors for Solar Sails**

Solar and laser sails have been recently proposed for a variety of novel space science missions ranging from ultra-fast interstellar travel to imaging exoplanets via solar gravity lensing effects.[22] This emerging technology requires minimizing the weight of the sail as a crucial requirement to increase the acceleration produced by the radiation pressure that propel the sail. Moreover, refractory materials are required since they can withstand high temperatures, which however limits the range of available material properties. In this scenario, the design goal is to achieve a certain acceptable reflectance at optical wavelengths using realistic refractory materials and the minimum possible thickness (hence minimum weight). This is exactly the kind of problem that our derived reflection bound, given by Equation (10), can address.

As an example, Figure 5 shows that for a lightweight low-loss refractory material like Aluminium Oxide \( \text{Al}_2\text{O}_3 \), a much higher reflectance may be possible, beyond the homogeneous film case, even in the subwavelength thickness regime. To demonstrate this point, the reflectance of a periodic square array of square prisms has been calculated and optimized using COMSOL 5.4, varying the width \( w \) of the prisms and the lattice periodicity \( a \) to achieve maximum reflection, for a thickness \( h = \lambda/4 \), at the center of the visible spectrum \( \lambda = 500 \) nm. The optimized configuration \( (w/\lambda = 0.575, a/\lambda = 0.95) \) results in 64% reflectance, which is almost three times the maximum possible reflectance from a homogeneous film with the

![Figure 5](image5.png)

**Figure 5.** Normal-incidence reflectance \( R \) of a homogeneous film (solid blue line), upper reflection bound (dashed blue), and reflectance of a periodic array of square prisms, optimized for a quarter-wavelength thickness. The considered material is \( \text{Al}_2\text{O}_3 \) with refractive index \( n = 1.75 + 0.02i \) at 500 nm.[22]
same refractive index, as shown in Figure 5. The bound suggests further enhancement may be possible by considering more complicated structures designed using more advanced optimization techniques.

Moreover, Table 1 summarizes the minimum possible thickness for a reflective film made of selected refractory materials to achieve at least 60% and 90% reflectance at 500 nm. This analysis may be useful as a factor in the selection process of the optimal material for the design of solar and laser sails. We also note that another important question in the study of the fundamental limits of solar sails (powered by a broadband light source) is the issue of the maximum bandwidth over which a high reflectance can be sustained. This will be the subject of future work.

### 4.2. Reflection Gratings

The derived reflection bound given by Equation (10) can also be used to determine the maximum possible efficiency of a reflection grating or meta-grating for a given diffraction order. For instance, the commercially available gold grating in Ref. [47] is optimized to reflect 95% of TM-polarized light at 800 nm, incident at \( \theta_{\text{inc}} = 54^\circ \), to the –1st reflection order at \( \theta_{\text{refl}} = -22^\circ \).

As shown in Figure 6 (blue ‘X’), this optimized grating with thickness \( h = 165 \text{ nm} \) is already close to the maximum possible efficiency for that specific thickness and to the minimum possible thickness for that efficiency level predicted by the bound for the considered material.

Another possible example is a commercially available aluminum blazed grating that is designed to reflect 65% of incident TE-polarized light in Littrow configuration (back-reflection, \( \theta' = -\theta'' = 10.36^\circ \)) for UV wavelengths \( \lambda = 300 \text{ nm} \). The thickness of the aluminum coating is \( 800 \text{ nm} \). While the grating performance may be acceptable for commercial applications, by varying the dimensions of a binary aluminum grating, we arrive at an optimized design (red ‘+’ in Figure 6) that reflects 90% of the incident light with one-fourth the thickness of the original design.

### 4.3. Polarization Converters

Polarization converters are an important class of electromagnetic/optical components that are widely used in several applications ranging from communications and imaging to molecular sensing. Assuming normally incident and reflected light, and specializing the bound for cross-polarized reflection by setting \( \delta_{\text{p}} = 0 \), Equation (10) is reduced to a simplified form that directly bounds the maximum relative amount of energy coupled to the orthogonal polarization in reflection

\[
\hat{U}_{\text{PC}} = \frac{1}{4} \left( \sum_i \frac{\rho_{s,i}\left(k_i\right)}{\text{Im} \rho_{s,i} + \text{Im} \rho_{p,i}} \right)^2
\]

Interestingly, in the small-thickness limit, \( h/\lambda \to 0 \), this expression can be further simplified by using the small-argument approximation of trigonometric functions, yielding

\[
\hat{U}_{\text{PC}} \approx \frac{1}{4} \left( \frac{\pi h}{\lambda} \right)^2
\]

for both TE and TM polarizations. Then, by also letting \( \text{Im} \zeta \to 0 \) (lossless limit), the bound converges to \( \hat{U}_{\text{PC}} \to 1/4 \). This asymptotic result for the small-thickness lossless limit matches the theoretical limit on polarization cross-coupling derived earlier in the literature, from very different considerations, for any infinitesimally thin structure made of a passive lossless material.

Intuitively, this can be explained by the fact that currents induced on a single ultra-thin layer (e.g., a metasurface made of thin, planar, lossless (nano)antennas) will necessarily radiate symmetrically toward both sides, resulting in an unavoidable 50% reduction in reflection efficiency; in addition, the planar polarizable elements on the surface (e.g., obliquely oriented dipoles) can couple, at most, half of the incident field to the orthogonal polarization, resulting in a maximum cross-polarized reflectance of 1/4. This intuitive result is confirmed and generalized by our fundamental bound on reflection.
As an example, Figure 7 shows the cross-polarized reflectance of a reflective microwave polarization converter from the literature\(^{[50]}\) compared against the fundamental bound. Using a low-loss metal like copper, it was possible to create a very thin and efficient polarization-converting metasurface, with \(R_{PC} \approx 0.8\) at 10 GHz, and a thickness of \(h = 1.27\) mm or \(h/\lambda = 0.04\).\(^{[50]}\) Nevertheless, our derived bound in Equation (11) suggests that a further minimization may be achieved. As illustrated in Figure 7, the minimum possible thickness to achieve \(R_{PC} = 0.8\), using the same material, is \(h_{\text{min}} = 0.003\lambda\), which is more than an order of magnitude lower than the proposed design in Ref. [50]. On the other hand, the bound indicates that there is no fundamental constraint that could prevent achieving a perfect polarization conversion since \(U_{PC} \approx 1\) for \(h/\lambda = 0.04\). Rather, the main issue could be the computational resources required to find a better design and the feasibility to fabricate it. Finally, we note that the bound quickly reduces to approximately 1/4 as the thickness is reduced, consistent with the discussion above.

5. Conclusion

In this paper, we have derived analytical upper bounds on the general problem of reflection from complex planar structures, irrespective of their specific design, and for any direction of incidence/reflection and polarization. The only assumptions are that the structure is passive, with a surface area much larger than its thickness and the wavelength, and is made of a single local, isotropic, and nonmagnetic material. The latter two assumptions may be relaxed, but the resulting bounds would no longer be expressible as simple closed-form formulas. We have validated our theoretical results by comparing the derived bounds against the standard Fresnel reflectance \(R\) of homogeneous and homogenized, dielectric and plasmonic films, for both polarizations. For TM polarization, the bound predicts the possibility of achieving significantly higher reflection than \(R\) near the Brewster angle, even for deeply subwavelength films, and we demonstrated this possibility with a numerical example. Interestingly, the derived bound also replicates, confirms, and generalizes the previously derived limit on polarization conversion efficiency for very thin structures (25\%\(^{[50]}\)).

In the second part of the paper, we have applied the derived fundamental bound on reflection to various results from the recent literature and the associated applications, focusing on ultra-thin reflective mirrors, reflection gratings, and polarization converters made of real, imperfect (i.e., lossy) materials. As a relevant example, we have identified, for several promising refractory materials, the minimal possible thickness of highly reflective ultra-light mirrors for solar/laser sails. Furthermore, while we have found that some designs are already optimized and result in performance close to the bound, other designs can be further improved both in terms of compactness and reflectance. Thus, when using optimization techniques and inverse-design methods, the bound can serve as a guideline to minimize computational resources by identifying parameter regions where there is room for further enhancement. For future work, it would be interesting to explore the possibility to derive similar bounds for planar optimizable structures on substrates (e.g., metasurfaces on glass) or layered media, or more broadly in the presence of scatterers outside the designable domain, which could be analyzed by using the Green’s function for the considered environment. Our analysis may also be extended to include more complex material properties, such as magnetic and chiral materials. Moreover, more work is needed to extend these results to the problem of broadband maximization of reflection, establishing fundamental trade-offs between bandwidth, thickness, and reflectance.

To conclude, given the importance of engineering and optimizing light reflection in many scenarios, we believe our results will prove useful to many researchers and engineers working in this area, offering them new analytical tools and relevant insight into the fundamental performance limits of reflective components for various applications.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

fundamental limits, gratings, light sails, metasurfaces, reflection
