GiNaC - Symbolic computation with C++

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We give an introduction to the C++ library GiNaC, which extends the C++ language by new objects and methods for the representation and manipulation of arbitrary symbolic expressions.

1. Introduction

GiNaC is a C++ library that enables the user to perform algebraic operations within the C++ language. It has been created to become the new algebra engine of the xloops project [1] as a replacement for Maple. Version 1.0 released in the year 2001 finally provided all the necessary features for that intended purpose. Since then, GiNaC has been extended and improved continuously and is currently available in version 1.3.2.

A detailed discussion of the reasons why GiNaC was started and how it was designed was given in [2] and doesn’t need to be repeated here. But to point out the role of GiNaC in high energy physics calculations, we will shortly discuss the problem of software complexity.

When doing calculations in the context of perturbation theory, one has to accomplish a number of tasks. The major ones are the following:

- Derivation of Feynman rules
- Creating Feynman diagrams
- Application of Feynman rules
- Dirac algebra and calculating traces
- Loop integration
- Summation of diagrams
- Renormalization
- Phase space integration
- Self-testing and comparisons

Each of these tasks poses certain mathematical or conceptual problems. Most can be addressed with the help of computer programs. In certain cases the use of computers for automated calculation has become essential. The list above can be categorized into a small number of algorithmic tasks:

- Combinatorics
- Algebraic operations
- Data management
- Numerics
- Input / Output

It is common practice to use specialized software tools for each of these tasks, like Maple for algebra and FORTRAN for numerics. The software components then have to be connected to each other by again different software tools to complete a calculation.

Such automated calculations are confronted with a new kind of problem: software complexity. This problem becomes more significant with the increasing scale of the calculations:

- Algorithms themselves are getting more complicated and involved.
- The number of utilized algorithms is growing.
- The number of different software technologies, i.e. programming languages or systems, in use is rising.
• More people are working on the same software project concurrently.

Thus topics like usability, verifiability, maintainability and extensibility become important. Computer science deals with these issues since decades now and various solutions have been put forward. Software engineering principles were invented and C++ has gained widespread acceptance as a means to realize them. The question is, whether C++ can be used for high energy physics calculations in that rôle as well.

C++ excels at combinatorics and input/output operations. It is also very good for doing numerics and data management. Only one of the algorithmic tasks being listed above could not be done in C++ so far and that was algebraic operations. GiNaC filled this gap. With GiNaC it is now possible to write software for physics computations completely in C++ adhering to software engineering principles.

2. Basic concepts

GiNaC is a software library that is used by writing C++ programs. It is not an algebra system with an elaborate user interface. GiNaC defines several new data types for C++ programs. A very important one is the data type for indeterminate mathematical symbols. Corresponding C++ variables can be defined like this

    symbol x("x");

Here a C++ variable x gets defined with the character string "x" as its output name.

A very nice feature of C++ is the ability to re-define the built-in operators for the new data types. For example, the + operator can be used with symbols like

\[ a + b \]

to form a symbolic expression. These expressions can be stored in another data type of GiNaC called ex, which allows for the corresponding variables to contain arbitrary symbolic expressions or numeric values. This can be seen in the following example.

```cpp
#include <iostream>
#include <ginac/ginac.h>
using namespace GiNaC;

int main()
{
    symbol a("a"), b("b");
    ex myterm = sqrt(a*b) + 0.3*b;
    cout << myterm.subs(b==2) << endl;
    return 0;
}
```

After some C++ commands to include the needed libraries, two symbols a and b and a small formula are defined in the main program. The result of the substitution b → 2 is printed on the screen. The output is the expression \( \sqrt{2a} + 0.6 \). Apart from simple operations like multiplication and summation one can see the usage of mathematical functions and the easy mixing of symbolic and numeric terms. The integration into the C++ language is seamless.

3. Feature overview

We already introduced the two most important data types provided by GiNaC. Instead of listing the other data types available, it is more instructive to name the core functionalities of GiNaC.

Expressions in GiNaC may not only consist of symbols but also of numbers. Numbers can be exact including exact rational fractions, as well as floating point numbers with an arbitrary numeric precision. Other objects that can be part of expressions are indexed variables like vectors or matrices. Mathematical functions are another important example for allowed objects.

Expressions can be manipulated in various ways. Mathematical operations like symbolic derivation, series expansion or symmetrization are available. Operations on polynomials like expanding and collecting or square-free decomposition are also present. Pattern matching and algebraic substitutions complete the picture.

Systems of linear equations can be solved. GiNaC also knows about special algebras like the Clifford algebra including the Dirac \( \gamma^\mu \) matrices and the \( SU(3) \) color algebra.
There are a lot of mathematical functions already defined in GiNaC and new ones can easily be added. All functions know their expected algebraic properties and can be evaluated numerically for arbitrary complex arguments. In addition to the standard functions like sin, cos, Euler’s $\Gamma(x)$ function, $\sqrt{x}$, log and many more, GiNaC also provides polylogarithms of all kinds: The classical polylogarithm $\text{Li}_n(x)$, which includes the ubiquitous dilogarithm $\text{Li}_2(x)$, Nielsen’s generalized polylogarithm $S_{n,p}(x)$, the harmonic polylogarithm $H_{m_1,\ldots,m_k}(x)$, the multiple zeta value $\zeta(m_1,\ldots,m_k)$ and the multiple polylogarithm $\text{Li}_{m_1,\ldots,m_k}(x_1,\ldots,x_k)$ are available. Again, all these functions can be evaluated numerically for arbitrary complex arguments.

Finally, another important feature of GiNaC has to be mentioned: GiNaC is open-source software licensed under the GPL. The code is well documented and a comprehensive manual is available.

4. Summary

We have given a short introduction to GiNaC. We highlighted the benefits of using GiNaC in complex software systems written for high energy physics calculations. Features of GiNaC and a programming example have been given. GiNaC can be downloaded from [http://www.ginac.de](http://www.ginac.de).

REFERENCES

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