Thermoeconomic analysis of an irreversible Stirling heat pump cycle

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Abstract

In this paper an analysis of the Stirling cycle in thermoeconomic terms is developed using the entropy generation. In the thermoeconomic optimization of an irreversible Stirling heat pump cycle the F function has been introduced to evaluate the optimum for the higher and lower sources temperature ratio in the cycle: this ratio represents the value which optimizes the cycle itself. The variation of the function F is proportional to the variation of the entropy generation, the maxima and minima of F has been evaluated in a previous paper without giving the physical foundation of the method. We investigate the groundwork of this approach: to study the upper and lower limits of F function allows to determine the cycle stability and the optimization conditions. The optimization consists in the best COP at the least cost. The principle of maximum variation for the entropy generation becomes the analytic foundation of the optimization method in the thermoeconomic analysis for an irreversible Stirling heat pump cycle.

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I. INTRODUCTION

The Stirling cycle is an important model of refrigeration systems and the recent developments in design were proposed after the new concept of finite time thermodynamics came into existence. Blanchard applied the Lagrange multiplier method to find out the $COP$ of an endoreversible Carnot heat pump operating at the minimum power input for a given heating load. Several papers have been devoted to propose mathematical functions to optimize thermodynamics cycles starting from different initial conditions and focusing on the total cost and efficiency. The definition of optimization that we adopt in this paper is the best $COP$ at the least cost. The performance of the different heat engine and refrigeration systems were investigated using the concept of finite time thermodynamics, of the ecological approach and the thermoeconomic analysis. On the other hand, the key-role of entropy generation maximum has been recently demonstrated in thermodynamics analysis of the irreversible processes and it has been shown that it represents a new criterion for determining the conditions for stability. In this paper we show that this criterion represents the thermodynamic foundation for some recent results obtained in thermoeconomic analysis of the Stirling heat pump cycle. We start from studying the time evolution of an open system and we take as working hypothesis that it evolves in the optimization of the irreversibility due to entropy generation. F function is well suited to show the system evolution related to the irreversibility due to entropy generation, our aim is to propose an approach based upon the natural behaviour of the thermodynamics/thermoeconomic system as a groundwork for the optimization analysis. The evolution of an open system is considered natural when it moves in order to get the optimization of the entropy generation.

II. THE THERMODYNAMIC ANALYSIS

The working substance of the Stirling cycle may be a gas, a magnetic material, etc., and for different working fluids the performance of the cycle are quite different. The Stirling cycle with an ideal gas consists of two isothermal and two isochoric processes. It approximates the expansion stroke of the real cycle by an isothermal process to whom heat is added to reach the temperature $T_c$ from a heat source of finite capacity whose temperature varies
from $T_{L1}$ to $T_{L2}$. The heat addition to the working fluid is thought as an isochoric process: heat is going towards the heat sink of finite heat capacity that gets a temperature variation from $T_{H1}$ to $T_{H2}$. The heat rejection from the working fluid to the regenerator is modelled as an isochoric process which completes the cycle itself. Let $Q_c$ and $Q_h$ be the amount of heat absorbed from the sources at the temperature $T_c$ and $T_h$ respectively, during the two isothermal processes \[1\]:

$$Q_h = C_H \epsilon_H (T_h - T_{H1}) t_h$$ \[1\]

$$Q_c = C_L \epsilon_L (T_{L1} - T_c) t_L$$ \[2\]

where $C_H$ is the heat capacitance rate of the sink reservoir, $C_L$ is the heat capacitance rate of the source reservoir, $t_H$ is the heat rejection time, $t_L$ is the heat addition time, $\epsilon_H$ is the effectiveness of the heat exchangers for the hot-side and $\epsilon_L$ is the effectiveness of the heat exchangers for the cold-side. These cycles do not possess the condition of perfect regeneration, hence it is assumed that the loss per cycle, $\Delta Q_R$, is proportional to the temperature difference of the two isothermal processes as follows \[1, 15, 16, 17, 18, 19\]:

$$\Delta Q_R = n c_f (1 - \epsilon_R) (T_h - T_c)$$ \[3\]

where $c_f$ is the molar heat capacity of the working fluid and $n$ is the number of moles. The Gouy-Stodola theorem \[20\] states that the thermodynamic work burnt in the irreversibility due to the entropy generation is equal to the product between the lowest source temperature and the entropy generation, i.e. total entropy is equal to the isolated system entropy plus the irreversibility due to entropy generation. Considering the Gouy-Stodola theorem and the definition of the entropy due to irreversibility $\Delta S_{irr}$ \[13\], the last one can be written as:

$$\Delta S_{irr} = \frac{\Delta Q_R}{T_c} = n c_f (1 - \epsilon_R) \frac{(T_h - T_c)}{T_c} = n c_f (1 - \epsilon_R) (x - 1)$$ \[4\]

with $x = T_h / T_c$. The theorem of maximum entropy generation states that the entropy generation is maximum at stationary state \[9\]. This theorem allows a new approach to irreversible processes as it is proved in a lot of different applications in hydrodynamics \[10\], engineering thermodynamics \[11\], rational thermodynamics \[12\] and biophysics \[13, 14\]. Hence applying it here, we argue that equation \[4\] must be a maximum in the thermodynamics stability: this equation described the natural behavior of the thermodynamics system.
III. THE THERMODYNAMICS FOUNDATION OF THE THERMEOECONOMIC ANALYSIS

The objective function $F$ of the thermoeconomic optimization recently proposed is \[1, 21, 22\]:

$$F = \frac{\dot{Q}_H}{C_i + C_e}$$  \hspace{1cm} (5)

with $\dot{Q}_H$ = heating power, $C_i$ and $C_e$ refer to annual investment and energy consumption costs, and are defined as:

$$C_i = a(A_H + A_L + A_R) + b\frac{Q_h - Q_c}{t_{cycle}}$$  \hspace{1cm} (6)

$$C_e = b\frac{Q_h - Q_c}{t_{cycle}}$$  \hspace{1cm} (7)

where $a$ is a constant directly proportional to the investment cost of the heat exchanger and is equal to the capital recovery factor multiplied by the investment cost per unit heat exchanger area. $A_H + A_L + A_R$ is the heat exchanger total area, with $A_H$ the heating area, $A_L$ the heat source area and $A_R$ the regenerative area. $b$ is the capital recovery factor multiplied by the investment cost per unit power input and $t_{cycle}$ is defined as:

$$t_{cycle} = t_H + t_L + t_R$$  \hspace{1cm} (8)

with

$$t_R = 2\alpha(T_h - T_c) = 2\alpha T_c(x - 1)$$  \hspace{1cm} (9)

where $\alpha$ is a constant that depends upon the kind of working fluid used in the cycle, and shows that the working time of the regenerator (a sort of recovering time towards the initial conditions in thermoeconomics) is proportional to the difference of temperature. In the thermoeconomic analysis of an irreversible Stirling heat pump cycle the function $F$ has been used to evaluate the ratio of the higher and lower source temperature in order to reach the optimization of the cycle itself. The common solution, based upon the application of the variation method, consists in evaluating the maxima of $F$ function, solving the equation $\delta F = 0$ \[1\] applying the variational method.
Now, from 6 and 7 the 5 becomes:

\[ F = \frac{\dot{Q}_H}{a(A_H + A_L + A_R) + (b + b')\frac{Q_h - Q_c}{t_{cycle}}} \]  

(10)

Starting from the relations 4, 5, 8-10 we can argue that the objective function F of the thermoeconomic optimization is related to the entropy generation as follows:

\[ F = \frac{\dot{Q}_H}{a(A_H + A_L + A_R) + (b + b')\frac{Q_h - Q_c}{t_H + t_L + \frac{2\alpha T_c}{n_{cf}(1 - \epsilon_R)}\Delta S_{irr}}} \]  

(11)

which can be easily written after few algebraic operations:

\[ F = \frac{\Gamma_1 + \Gamma_2\Delta S_{irr}}{\Gamma_3 + \Gamma_4\Delta S_{irr}} \]  

(12)

with \[
\begin{align*}
\Gamma_1 &= \dot{Q}_H(t_H + t_L) \\
\Gamma_2 &= \frac{2\alpha T_c\dot{Q}_H}{n_{cf}(1 - \epsilon_R)} \\
\Gamma_3 &= a(A_H + A_L + A_R)(t_H + t_L) + (b - b')(Q_h - Q_c) \\
\Gamma_4 &= \frac{2\alpha a(A_H + A_L + A_R)T_c}{n_{cf}(1 - \epsilon_R)}
\end{align*}
\]

From equation 12 we can argue that the variation of the function F is proportional to the variation of the entropy generation:

\[ \delta F = \frac{\Gamma_2\Gamma_3 - \Gamma_1\Gamma_4}{\Gamma_3 + \Gamma_4\Delta S_{irr}}\delta(\Delta S_{irr}) \]  

(13)

with \[
\Gamma_2\Gamma_3 \neq \Gamma_1\Gamma_4 \]  

(14)

Then it follows that

\[ \delta(\Delta S_{irr}) = 0 \Rightarrow \delta F = 0 \]  

(15)
In this way it has been stressed the relation between the economic analysis and the thermodynamics. In the economical analysis the function $F$ was introduced in several papers: we need to know its upper and lower limits to fulfill the basic conditions of optimization, but no physical explanation has been up to now given about this method. Here we prove that the limits of the $F$ function are directly correlated to the entropy generation in the state of stability and related to the optimization of the cycle. Hence the optimization, which consists in the best COP related to the least cost, can be obtained in the conditions of natural stability for the open systems. The evolution of an open system is defined natural when it moves to get the optimization of entropy. The advantages of this method consist in exploiting the natural dynamics of the system in order to reach, following its natural behaviour, the optimum by the shortest way (i.e. the lower cost).

**IV. CONCLUSIONS**

The thermodynamic and thermoeconomic analysis of the optimization of an irreversible Stirling heat pump cycle is presented in relation with its thermodynamic foundation. We proved that the principle of maximum variation for the irreversible entropy is the analytic foundation for the optimization method recently introduced in the thermoeconomic analysis for an irreversible Stirling heat pump cycle. Of course it represents not only an analytical and mathematical groundwork, but also the physical and thermodynamic foundation for the method itself, as a consequence of the physical meaning of the principle of maximum entropy variation in thermodynamics [9, 10, 11, 12, 13, 14]. The optimization method is a useful tool to design thermodynamics systems characterized by lower working costs. The principle of maximum variation allows a deeper thermoeconomic analysis focused on the stability conditions.
Nomenclature

\( a \) \quad \text{capital recovery factor times cost per unit heat 0 area} \\
\( A \) \quad \text{area} \quad [m^2] \\
\( b \) \quad \text{capital recovery factor times investment cost per unit power input} \\
\( c \) \quad \text{molar heat capacity} \quad [J mole^{-1} k^{-1}] \\
\( C \) \quad \text{heat capacitance rate} \quad [kW K^{-1}] \\
\( COP \) \quad \text{Coefficient of Performance} \\
\( n \) \quad \text{number of moles} \quad [mole] \\
\( Q \) \quad \text{heat} \quad [J] \\
\( S \) \quad \text{entropy} \quad [JK^{-1}] \\
\( t \) \quad \text{time} \quad [s] \\
\( T \) \quad \text{temperature} \quad [T] \\
\( x \) \quad \frac{T_h}{T_c} \\

Greek letters

\( \epsilon \) \quad \text{effectiveness} \\
\( \delta \) \quad \text{differential} \quad 1 \sum \frac{\partial}{\partial z_i} dz_i \\
\( \Delta \) \quad \text{finite variation} \\

Subscripts

\( f \) \quad \text{fluid} \\
\( h \) \quad \text{sink side} \\
\( H \) \quad \text{heating} \\
\( irr \) \quad \text{irreversible which is related to the entropy generation} \\
\( L \) \quad \text{heat source}
[1] S.K. Tyagi, J. Chen, S.C. Kaushik, *Thermoeconomic optimization and parametric study of an irreversible Stirling heat pump cycle*, Int. J. Thermal Sci. 43 (2004) 105-112

[2] F.L. Curzon, B. Ahlborn, *Efficiency of a Carnot engine at maximum power output*, Amer. J. Phys. 43 (1975) 22-24

[3] C. Wu, *Power optimization of a finite time Carnot heat engine*, Energy 13 (1988) 681-687

[4] C.H. Blanchard, *Coefficient of performance for a finite speed heat pump*, J. Appl. Phys. 51 (1980) 2471-2472

[5] J. He, J. Chen, C. Wu, *Ecological optimization of an irreversible Stirling heat engine*, Int. J. Ambient Energy 22 (2001) 211-220

[6] S.K. Tyagi, S.C. Kuashik, R. Salhotra, *Ecological optimization for irreversible Stirling and Ericsson heat engine cycle*, J. Phys. D: Appl. Phys. 35 (2002) 2058-2065

[7] S.K. Tyagi, S.C. Kuashik, R. Salhotra, *Ecological optimization for irreversible Stirling and Ericsson heat engine cycle*, J. Phys. D: Appl. Phys. 35 (2002) 2668-2675

[8] A. Kodal, B. Sahin, I. Ekmekei, T. Yilmaz, *Thermo-economics optimization for irreversible absorption refrigerators and heat pumps*, Energy Conv. Mangt. 44 (2003) 109-123

[9] U. Lucia, *Mathematical consequences and Gyarmati’s principle in Rational Thermodynamics*, Il Nuovo Cimento B110, 10 (1995) 1227-1235

[10] G. Grazzini e U. Lucia, *Global analysis of dissipations due to irreversibility*, Rev. Gén. Thermique 36 (1997) 605-609

[11] U. Lucia, *Maximum principle and open systems including two-phase flows*, , Rev. Gén. Thermique 37 (1998) 813-817

[12] U. Lucia, *Irreversibility and entropy in Rational Thermodynamics*, Ricerche di Matematica, L1 (2001) 77-87

[13] U. Lucia, *Irreversible entropy in biological systems*, EPISTEME 5 (2002) 192-198

[14] U. Lucia e G. Maino, *Thermodynamic analysis of the dynamics of tumor interaction with the host immune system*, Physics A 313, 3-4 (2003) 569-577

[15] D.A. Blank, C. Wu, *Power optimization of an extra-terrestrial solar-radiating Stirling heat engine*, Energy 20 (1995) 523-530

[16] D.A. Blank, C. Wu, *Power limit of an endoreversible Ericsson cycle with regeneration*, Energy
[17] J. Chen, Minimum power input of irreversible Stirling refrigerator for given cooling load, Energy Conv. Mangt. 39 (1998) 1255-1263

[18] S.C. Kaushik, S.K. Tyagi, S.K. Bose, M.K. Singhal, Performance evaluation of irreversible Stirling and Ericsson heat pump cycle, Int. J. Termal Sci. 41 (2001) 193-200

[19] F. Angelo-Brown, An Ecological optimization criterion for finite time heat engine, J. Appl. Phys. 59 (1991) 7465-7469

[20] A. Bejan, Advance Engineering Thermodynamics, John Wiley & Sons, New York, 1988

[21] B. Sahin, A. Kodal, Finite time thermo-economic optimization for endoreversible refrigerators and heat pumps, Energy Conv. Mangt. 40 (1999) 951-960

[22] A. Kodal, B. Sahin, T. Yilmaz, Effects of internal irreversibility and heat leakage on the finite thermo-economic performance of refrigerators and beam pumps, Energy Conv. Mangt. 41 (2000) 607-619