Quasi-linearly polarized hybrid modes in tapered and metal-coated tips with circular apertures: understanding the functionality of aperture tips

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Abstract
In this study, we investigate analytically and experimentally the roles of quasi-linearly polarized (LP), hybrid, plasmonic and photonic modes in optical detection and excitation with aperture tips in scanning near-field optical microscopy. Aperture tips are tapered and metal-coated optical fibers where small circular apertures are made at the apex. In aperture tips, there exist plasmonic modes that are bound at the interface of the metal cladding to the inner dielectric fiber and photonic modes that are guided in the area of the increased index in the dielectric fiber core. The fundamental photonic mode, although excited by the free-space Gaussian beam, experiences cutoff and turns into an evanescent mode. The photonic mode also becomes lossier than the plasmonic mode toward the tip aperture, and its power decay due to absorption and reflection is expected to be at least $10^{-9}$. In contrast, the fundamental plasmonic mode has no cutoff and thus reaches all the way to the tip aperture. Due to the non-adiabaticity of both modes’ propagations through the taper below a core radius of 600 nm, there occurs coupling between the modes. The transmission efficiency of the plasmonic mode, including the coupling efficiency and the propagation loss, is expected to be about $10^{-6}$ that is at least 3 orders of magnitude larger than that of the photonic mode. Toward the tip aperture, the longitudinal field of the photonic mode becomes stronger than the transverse ones while the transverse fields always dominate for the plasmonic mode. Experimentally, we obtain polarization resolved images of the near-field at the tip aperture and compare with the $x$- and $y$-components of the fundamental quasi-LP plasmonic and photonic modes. The results show that not only the pattern but also the intensity ratios of the $x$- and $y$-components of the aperture near-field match with that of the fundamental plasmonic mode. Consequently, we conclude that only the plasmonic mode reaches the tip aperture and thus governs the near-field interaction outside the tip aperture. Our conclusion remains valid for all aperture tips regardless of the cladding metal type that mainly influences the total transmission efficiency of the aperture tip.

1. Introduction

Various types of scanning near-field optical microscopy (SNOM) are used to access evanescent fields of nanoparticles and to obtain both optical and topographic information simultaneously [1–7]. Among many variants of SNOM tips, the aperture tip was the first to be invented and used in near-field optical measurements [1–4]. Figure 1 illustrates the schematics of the aperture tip that is a tapered and metal-coated fiber where a small aperture is made at the end for light transmission or collection. Many studies have been carried out to explain the light transmission or detection processes with aperture SNOM tips [8–26]. For example, dipole and multipole...
models has been widely used for describing the diffracted fields through a sub-wavelength metallic hole as a magnetic dipole oscillating in the plane and compared with Bethe multipoles and including an electric dipole to describe better the diffracted fields in the far-field. These studies showed that the tapered and metal-coated dielectric waveguides support not only photonic modes but also plasmonic modes at the inner and outer interfaces that are dielectric core-metal cladding and plasmonic modes at a core radius of 2 μm, respectively. The blue dashed lines indicate the boundary between the dielectric core and the metal cladding.

Figure 1. Schematics of aperture tips used in scanning near-field optical microscopy (SNOM). Aperture SNOM tips are tapered and metal coated fibers where small apertures are made at the front for in- and out-coupling of light. Due to the metal cladding of the aperture tip, such waveguide can support photonic modes (red color) in the fiber core and plasmonic modes (blue color) at the inner interface (fiber core-metal cladding). Among many different orders of photonic and plasmonic modes, we consider only the fundamental quasi-linearly polarized hybrid modes since they can reach closest to the tip aperture while others experience cutoff and die off before reaching the tip aperture. The red and blue boxed insets show the electric field distributions of the fundamental photonic and fundamental plasmonic modes at a core radius of 2 μm, respectively. The red dashed lines indicate the boundary between the dielectric core and the metal cladding.

1.1. Problem statement
Despite all these studies, there are several open questions yet to be clarified to better understand the aperture tip’s optical characteristics. First, it is often assumed that the evanescent tail after the cutoff of the fundamental photonic mode determines the light throughput of aperture tips. However, this fundamental photonic mode (illustrated with red in figure 1) in the dielectric fiber core becomes highly lossy toward the aperture and experiences cutoff before reaching the tip aperture. The optical transmission efficiency of the photonic mode is extremely weak due to the huge absorption and reflection as we will show later. Thus, it is questionable that the evanescent throughput is the only contribution to the near-field interaction outside the tip aperture. Second, the
existence of the fundamental plasmonic mode (illustrated with blue in figure 1) at the metal cladding and dielectric core interface has been often overlooked and misidentified as photonic modes. The fundamental plasmonic mode is only briefly discussed in [26, 27], yet it was never taken seriously for its role in the near-field interaction of the aperture tip. Such plasmonic mode must exist in aperture tips regardless of the material of the metal coating. To understand the aperture tip’s optical characteristics, we have to study both fundamental plasmonic (illustrated with blue in figure 1) and photonic (illustrated with red in figure 1) modes and their field distributions at the aperture plane. Third, the fundamental HE mode needs to be decoupled into LP degenerate states since the incident beam into the fiber of the aperture tip is often LP. Due to the tapering and the metal coating of the aperture tip, there cannot exist purely LP modes in aperture tips. Thus, we should not use the LP mode approximation that is often used to describe LP modes in optical fibers. To circumvent this issue, it is crucial to adapt the quasi-LP mode formalism that can accurately describe modes in aperture tips and the aperture tip’s near-field free of any approximation. Finally, it is not clearly understood why the circular aperture tips are mostly sensitive to the transverse field components. Many experiments with the circular aperture tips demonstrate that they detect mostly the transverse fields [31, 32], yet on rare occasions, the aperture tips do detect the longitudinal field [10]. To understand this behavior, we need to study the lowest order HE plasmonic and photonic modes and their field components’ strengths at the aperture plane. Such analytical calculation is only possible with the quasi-LP modal analysis. Once the fundamental quasi-LP hybrid plasmonic and photonic modes are calculated at the aperture plane, we can compare them with the experimentally measured aperture tip’s near-field. Finally then, we shall be able to understand the optical excitation and detection characteristic of aperture tips. In this study, we aim to clarify the above mentioned issues.

In the current paper, we study analytically and experimentally the aperture tip and its near-field outside the tip aperture. By using the quasi-LP modal formalism, we explore the behaviors of the quasi-LP, fundamental, hybrid, photonic and plasmonic modes. The quasi-LP hybrid modes are expressed with all 6 vector fields and are especially suited for describing LP modes in tapered and metal coated fibers where the LP mode approximation fails. Based on this analytical model, we discuss their dispersion relation and the evolution of mode fields during the propagation toward the tip aperture. Finally, we experimentally measure the aperture near-field when a LP fiber mode is excited in the aperture tip and obtain polarization resolved images of the aperture near-field. By correlating the polarization resolved near-field images with the different field components of the quasi-LP plasmonic and photonic modes, we intend to determine the roles of these modes in the near-field excitation and detection process outside the tip aperture.

2. Analytical descriptions of quasi-LP, fundamental, hybrid modes

We start our study by establishing an analytical formalism of the quasi-LP guided modes within a cylindrical waveguide with a dielectric core and a metal cladding. The derivation of modes in cylindrical, dielectric waveguides are well described in [33] (see also appendix A), and we adapt this derivation to our study. The adapted formalism, however, allows us to calculate the hybrid (HE or EH) modes that are not LP but are rather a mixture of two orthogonally polarized states [26, 33–35]. As we have mentioned in our problem statement, the incident beam into the fiber of the aperture tips is usually LP; therefore, we have to analyze LP modes in the aperture tips. In purely dielectric fibers with a negligible refractive index difference between the core and the cladding, the LP mode approximation is often used to decouple the hybrid mode and obtain two orthogonally polarized degenerate hybrid modes in a Cartesian coordinate system [35]. Conversely, waveguides with metal claddings and dielectric cores have large difference in the core and cladding refractive indices; thus, the LP mode approximation is not valid for such waveguides. There cannot exist purely LP modes in the aperture tips because of the tapering and the metal cladding, but quasi-LP modes. Hence, we decompose the fundamental hybrid mode into two orthogonally polarized Gaussian-like modes that are degenerate and quasi-LP along the x- and y-axis. The quasi-LP modes have all 6 vector field components. For the quasi-x-polarized mode, the electric field’s x-component is the dominant one, and the y-component is for the quasi-y-polarized mode.

To obtain a formalism on quasi-LP fundamental hybrid modes, we need to make some modifications in the existing formalism (equations (A4) and (A5) in appendix A) to describe the modes in a cylindrical coordinate system (ρ, φ, z). Here, ρ and φ are the radial and the azimuthal coordinates, respectively. First, we set the azimuthal mode number of the HE_{mν} mode to 1 (ν = 1 in equations (A4) and (A5) in appendix A) to restrict the solutions to the lowest order hybrid modes and their radial variations that are HE_{mν} or EH_{mν} with a radial mode number of m. Thus, we are able to obtain HE_{11} mode that is the fundamental photonic Gaussian-like hybrid mode. Second, we replace exp(iνφ + iψ), which describes the azimuthal variation of fields with a constant phase of ψ, with cos(φ + ψ) and sin(φ + ψ) in equations (A4) and (A5) (for details see [36]). Since the exponent function contains both sine and cosine in itself (Euler equation), the hybrid mode cannot be decomposed into two orthogonally polarized degenerate modes. When the constant phase of ψ is 0 or π/2, we
obtain the quasi-\(x\)-polarized HE\(_{im}^a\) mode (\(\psi = 0\)) or the quasi-\(y\)-polarized HE\(_{im}^a\) mode (\(\psi = \frac{\pi}{2}\)). Last, we will assume that the cladding is infinitely thick and consider only the dielectric fiber core and metal cladding in the calculation excluding the surrounding medium. This assumption is valid since the aperture tip has a metal coating thickness of about 150 nm that is an order of magnitude larger than the penetration depth of fields in the metal cladding.

With these simplifications, we find the field components in the core (\(\rho \leq a\)) of a waveguide with a core radius of \(a\) as

\[
E_z = A_1(\zeta \rho) \cos(\varphi + \psi),
\]
\[
H_z = B_1(\zeta \rho) \sin(\varphi + \psi),
\]
\[
E_\rho = -i \frac{\beta}{\zeta^2} \left[ \beta \zeta A_1'(\zeta \rho) + \frac{\omega \mu_0 B_1}{\rho} \right] \cos(\varphi + \psi),
\]
\[
H_\rho = -i \frac{\omega \varepsilon_\text{core}}{\zeta \rho} A_1(\zeta \rho) + \beta \zeta B_1(\zeta \rho) \sin(\varphi + \psi),
\]
\[
E_\varphi = -i \frac{\beta}{\zeta^2} \left[ -\frac{\beta}{\rho} A_1(\zeta \rho) - \omega \mu_0 \zeta B_1(\zeta \rho) \right] \cos(\varphi + \psi),
\]

and

\[
H_\varphi = -i \frac{\omega \varepsilon_\text{core} \zeta A_1'(\zeta \rho) + \beta B_1(\zeta \rho)}{\zeta^2} \cos(\varphi + \psi),
\]

where \(A_1'(\zeta \rho) = \frac{\partial A_1(\zeta \rho)}{\partial \zeta \rho}\) is the derivative of the Bessel function of the 1st kind. Moreover, \(\omega\) is the angular frequency, \(\mu_0\) is the vacuum permeability, \(\varepsilon_0\) is the vacuum permittivity, \(\varepsilon_\text{core} = n_\text{core}^2\) is the dielectric constants of the core, and \(k_0 = 2\pi / \lambda_0\) is the wavenumber with a wavelength of \(\lambda_0\). Additionally, \(\beta\) and \(\zeta = \sqrt{k_0^2 \varepsilon_\text{core} - \beta^2}\) are the propagation constant and the transversal wavevector, respectively, and \(A\) and \(B\) are field amplitudes.

In the cladding (\(\rho > a\)), the field components are given as

\[
E_z = C H_1^{(1)}(i \gamma \rho) \cos(\varphi + \psi),
\]
\[
H_z = D H_1^{(1)}(i \gamma \rho) \sin(\varphi + \psi),
\]
\[
E_\rho = \frac{i}{\gamma^2} \left[ i \beta \gamma C H_1^{(1)}(i \gamma \rho) + \frac{\omega \mu_0 D}{\rho} H_1^{(1)}(i \gamma \rho) \right] \cos(\varphi + \psi),
\]
\[
H_\rho = \frac{i}{\gamma^2} \left[ \omega \gamma \varepsilon_\text{gold} C H_1^{(1)}(i \gamma \rho) + i \beta \gamma D H_1^{(1)}(i \gamma \rho) \right] \sin(\varphi + \psi),
\]
\[
E_\varphi = \frac{i}{\gamma^2} \left[ -\frac{\beta}{\rho} C H_1^{(1)}(i \gamma \rho) - i \omega \mu_0 \gamma D H_1^{(1)}(i \gamma \rho) \right] \sin(\varphi + \psi),
\]
\[
H_\varphi = \frac{i}{\gamma^2} \left[ \omega \gamma \varepsilon_\text{gold} \gamma C H_1^{(1)}(i \gamma \rho) + \frac{\beta}{\rho} D H_1^{(1)}(i \gamma \rho) \right] \cos(\varphi + \psi),
\]

where \(H_1^{(1)}(i \gamma \rho) = \frac{\partial H_1^{(1)}(i \nu \rho)}{\partial \nu \rho}\) is the derivative of the Hankel function of the 1st kind. Furthermore, \(C\) and \(D\) are field amplitudes, and \(\gamma = \sqrt{\beta^2 - k_0^2 \varepsilon_\text{gold}}\) is the transversal wavevector with the metal cladding dielectric constant of \(\varepsilon_\text{gold}\). In calculations, we consider a steady state case and thus drop the function \(\exp(i(\beta z - \omega t))\) for simplicity in equations (1) and (2). To find higher order hybrid modes, one should start with equations (A4) and (A5) in appendix A without setting the azimuthal mode number \(\nu\) to 1.

Expressions (1) and (2) are exact solutions of Maxwell’s equations for a uniform waveguide, and the orthogonality relation is fulfilled between HE\(_{im}^a\) (\(\psi = 0\), quasi-\(x\)-polarized) and HE\(_{im}^a\) (\(\psi = \pi/2\), quasi-\(y\)-polarized) modes. Therefore, with equations (1) and (2), we decompose accurately the hybrid modes into two orthogonally polarized degenerate modes that are likely to be excited by the LP fundamental fiber mode.

Furthermore, any other polarization states (circular polarization or elliptical polarization) of a hybrid mode with an index of \(lm\) can be described by a linear superposition of HE\(_{im}^a\) and HE\(_{im}^a\) modes. Thus, these two quasi-linear modes form the orthonormal basis of HE\(_{im}^a\) mode.
2.1. Dispersion relation

Based on our analytical model, we would like to analyze the modes and their propagation through the adiabatically tapered and metal-coated fiber tip. First, we find the amplitudes \(B, C,\) and \(D\) by using the continuity of the transversal fields \(E_z, H_z, E_m\) and \(H_m\) at the dielectric-metal interface at the core radius of \(a\)

\[
C = A \frac{J_1(\xi a)}{H_1^{(1)}(i\gamma a)},
\]

\[
D = B \frac{J_1(\xi a)}{H_1^{(1)}(i\gamma a)},
\]

and

\[
B = -A \frac{\beta}{\omega \mu a} \left[ \frac{1}{\xi^2} + \frac{1}{\gamma^2} \right] \left[ \frac{J_1'(\xi a)}{\xi J_1(\xi a)} + \frac{1}{\gamma} \frac{H_1^{(1)}(i\gamma a)}{H_1^{(1)}(i\gamma a)} \right].
\]

The constant \(A\) can be determined by making the total time averaged power to unity (see equation (B1)).

From above relations, we find the dispersion relation as

\[
\begin{align*}
\begin{vmatrix}
J_1(\xi a) & \frac{\beta}{\alpha \xi^2} J_1'(\xi a) & \omega \mu a J_1(\xi a) & -H_1^{(1)}(i\gamma a) & 0 \\
\frac{\beta}{\alpha \xi^2} J_1'(\xi a) & 0 & \frac{\beta}{\alpha \gamma^2} H_1^{(1)}(i\gamma a) & -H_1^{(1)}(i\gamma a) & 0 \\
\omega \mu a J_1(\xi a) & 0 & \frac{\beta}{\alpha \gamma^2} H_1^{(1)}(i\gamma a) & 0 & -H_1^{(1)}(i\gamma a) \\
\frac{\omega \mu a \xi e_{core}}{\xi} J_1'(\xi a) & \frac{\beta}{\alpha \gamma^2} J_1(\xi a) & \frac{\omega \mu a \xi e_{gold}}{\gamma} H_1^{(1)}(i\gamma a) & \frac{\beta}{\alpha \gamma^2} H_1^{(1)}(i\gamma a) & 0
\end{vmatrix}
= 0.
\end{align*}
\]

With equation (4), we calculate and plot the dispersion relations of the two hybrid modes of the lowest order in figures 2(a) and (b). The real \((\beta'/k_0)\) and imaginary \((\beta''/k_0)\) parts of the effective indices are in solid and dashed lines, respectively. Inset 1 in figure 2(a) illustrates the geometry of the waveguide structure that is considered in the calculation. The waveguide has a dielectric core with radius of \(a\) and an infinitely large metal cladding. The aperture tip has typically a metal coating with a thickness of about 150 nm or more that is significantly larger than the mode field’s penetration depth in the metal. Thus, the metal cladding can be assumed to be infinitely large. Furthermore, we assume the waveguide is tapered adiabatically, and thus the local modes in the tapered waveguide can be approximated as eigen states that are orthogonal to each other. This means that the modes do not couple to other higher order modes or to the back propagating modes during propagation through the taper. However, this adiabatic condition will no longer hold for a core radius \(a < 600\, \text{nm}\) since the effective indices depending on the core radius \(a\) start to change rapidly despite the small tapering angle. Consequently, for a core radius \(a < 600\, \text{nm}\), the modes are non-orthogonal and can couple to each other. Inset 2 in figure 2(a) zooms into the region where the core radius is \(a < 500\, \text{nm}\). As the inset shows, the slope of the mode’s dispersion curve changes dramatically near the mode’s cutoff. We will discuss the influence of these slope changes on the mode property in sections 2.2 and 2.3. Parameters used in the calculation are a core refractive index of \(n_{core} = 1.4535\), a laser wavelength of \(\lambda = 784\, \text{nm}\), and a gold refractive index of \(n_{gold}^g = -20.95 + 1.68i\) [37].

In a waveguide with a dielectric core and a thick metal cladding, there exist photonic modes that are guided through the dielectric core as well as plasmonic modes that are guided at the dielectric-metal interface. We have mentioned previously that the photonic modes are classified as HE_{\phi m} or EH_{\phi m}, where m and \(\phi\) express the radial and azimuthal variations, respectively. Note that since the plasmonic modes do not have radial variations, we classify them only with their azimuthal variation. Furthermore, surface guided conical plasmonic modes are called conical-surface plasmon (Co-SP) modes and thus denote them as HE_{\phi Co-SP}.

Besides these plasmonic and photonic mode classifications, the effective indices \((\beta'/k_0)\) of these modes change as the core radius \(a\) decreases along the propagation toward the aperture. Thus, depending on the real part of the effective index, a mode transforms from one type to another: from a bound mode into a leaky mode then into an evanescent mode (see also [34]). For photonic modes, when \(0 < \beta'/k_0 < n_{core}\), they are bound modes. A dotted horizontal line in figure 2(a) \((\beta'/k_0 = 1)\) signifies cutoffs of photonic bound modes for normal dielectric waveguides such as optical fibers. However, the photonic modes in aperture tips do not have a leaky mode region since they cannot leak outside of the aperture tip due to the thick metallic cladding. When \(\beta'/k_0 = 0\), the bound photonic modes experience cutoff and become evanescent modes. Note that photonic modes have an effective index that is never larger than the core refractive index.

Meanwhile, for plasmonic modes, they are bound modes when \(n_{core} < \beta'/k_0\) and leaky modes when \(n_{core} > \beta'/k_0\). The boundary between the leaky and bound mode is marked with a dashed horizontal line in figure 2(a). At this core radius \(a = 502.3\, \text{nm}\), there occurs a discontinuity in the blue dashed curve in figure 2(b) that is the imaginary part of the effective index curve of the plasmonic mode. This discontinuity signifies the transition between the bound and leaky mode regions [38]. All modes are lossy due to the metal cladding. As
shown in figure 2(b), the imaginary parts of the effective indices ($\beta''/k_0$) increase several orders of magnitude ($10^{-6}$) toward the small core radius $a$.

Keeping in mind the above mentioned classifications of a mode type, we would like to discuss each mode in detail.

2.2. Fundamental plasmonic mode

We classify the lowest order mode as the fundamental Co-SP mode or HE1Co-SP mode that is shown in blue in figures 2(a) and (b). We calculate absolute of electric field distributions ($|E_x|$, $|E_y|$, and $|E_z|$) of the quasi $x$-polarized HE1Co-SP mode for core radiuses $a = 2 \mu m$, 502.3 nm (bound mode cutoff radius), and 50 nm. The results are presented in figures 2(c)–(e), and each position in the dispersion curve is also noted in figure 2(a). The field distributions in figure 2(c) illustrate the plasmonic nature of the mode. The mode for a core radius $a = 2 \mu m$ is bound at the metal-dielectric interface. Furthermore, the real part of the effective index ($\beta_{\text{HE1Co-SP}}/k_0 = 1.5191 + 0.0063i$) of this mode is larger than the dielectric core’s refractive index ($n_{\text{core}} = 1.4535$). At a sufficiently large radius, the effective index of HE1Co-SP mode will converge to that of planar SPPs that is $\beta_{\text{SPP}}/k_0 = 1.5328 + 0.0069i$ at a planar gold-dielectric interface. Even for a core radius $a = 2 \mu m$, they are quite close. Consequently, it is evident that the lowest order hybrid mode is the fundamental plasmonic mode that has no cutoff in its leaky mode region.

For a core radius $a = 502.3 \text{ nm}$ as shown in figure 2(a), the plasmonic bound mode turns into a leaky mode. Consequently, there occur modifications in the penetration depth in the core and cladding as shown in the electric field distributions for core radiuses $a = 502.3 \text{ nm}$ and $a = 50 \text{ nm}$ in figures 2(d) and (e), respectively. To look at these changes clearly, we calculate the absolute square of electric field components ($|E_x|^2$ and $|E_y|^2$) along $x$-axis at different core radius $a$. The results are presented in figure 3. The penetration depths (1/e of the maximum value) of the normalized $|E_x|^2$ (dashed line) and $|E_y|^2$ (solid line) are noted with blue and red, respectively. As the figure shows, the penetration depth of the $x$-component increases in the dielectric core and decreases in the metal cladding with decreasing the core radius $a$. When the core radius $a$ decreases by 50%
from 1 to 0.5 μm, the penetration depth increases about 90% in the core. Thus, at a small core radius a, the tails of the fields of the x-component start to overlap in the core and create maximum field at the center of the core as shown in figure 2(e). Since the x-component is the dominant one, this field component gives the HE_{1Co-SP} mode an illusion of core guided photonic mode when the core radius a is below ≈400 nm. Due to this phenomenon, the HE_{1Co-SP} mode has been often misidentified as the photonic HE_{11} mode in previous studies [26, 28]. Although the power of the x-component is mostly in the core like a photonic mode, the mode remains plasmonic that can be seen clearly in the y- and z-components that are bound to the metal-dielectric interface (see figure 2(f)). Figure 3 demonstrates it as well with the field profile of |Ex|^2 plotted in a solid line. The penetration depth of the |Ex|^2 decreases about 10% in the cladding when the core radius a decreases by 90% from 1 to 0.1 μm. Meanwhile, in the core, the penetration depth of the z-component slightly increases until a core radius a = 0.5 μm and then decreases. In general, figure 3 shows that the penetration depth of the mode decreases in the metal cladding with the decreasing core radius a. The results demonstrate that the mode size shrinks with decreasing structure dimensions, and this property is unique to plasmonic modes that allows confinement of fields in a subwavelength region.

Finally, the last feature of HE_{1Co-SP} mode that needs to be mentioned is the ratio between |Ex|^2, |Ey|^2 and |Ez|^2 components and their evolution during propagation. Along the dispersion curve of HE_{1Co-SP} mode (blue curve) as shown in inset 2 in figure 2(a), there are two slopes: ‘downward slope’ when the core radius a is larger than ≈120 nm and ‘upward slope’ when the core radius a is below ≈120 nm. The field components ratios, |Ey|^2/|Ex|^2 and |Ez|^2/|Ex|^2, of HE_{1Co-SP} mode change their trend depending on the slopes of the dispersion curve. We calculate |Ey|^2/|Ex|^2 (blue dashed line) and |Ez|^2/|Ex|^2 (blue solid line) ratios depending on the core radius a and present the results in figure 4. Compared with the x-component that is the ever dominant one for the quasi x-polarized fundamental plasmonic mode, the strength of y- and z-components decrease in the downward slope toward the core radius a = 120 nm (see also figures 2(c), (d) and 4). In the upward slope below the core radius a = 120 nm, they start to increase again but never become larger than the x-component. In general, the x-component of the electric field is the dominant one regardless of the core radius a for the HE_{1Co-SP} mode.
The minimum of the $|E_x|^2/|E_y|^2$ curve in figure 4 (blue solid line) can be attributed to the maximum of a mode impedance $\left( |Z|^2 = |E_x|^2/H_x|^2 \right)$ that is the ratio between amplitudes of dominant electromagnetic fields ($E_x$ and $H_x$) for quasi $x$-polarized modes at a given core radius $a$. The mode impedance influences the relative contribution of electromagnetic fields during coupling between HE$_{11}$ mode and the incidence field at the tip aperture [39].

2.3. Fundamental photonic mode

Next, we consider the red curves in figures 2(a) and (b) that are real and imaginary parts of the effective index, respectively. After the fundamental plasmonic mode, this mode is the lowest order photonic mode whose real part of the effective index at a larger core radius $a$ converges to $n_{core}$. Thus, it is the fundamental photonic mode or HE$_{11}$ mode. We calculate the absolute of the electric field distributions ($|E_x|$, $|E_y|$ and $|E_z|$) of the quasi $x$-polarized mode, HE$_{11}$, for core radiiuses $a = 2\ \mu m$, 405 nm, and 200 nm and present the results in figures 2(f)–(h), respectively. Figure 2(f) shows the HE$_{11}$ mode’s field distributions for a core radius $a = 2\ \mu m$ where each component resembles that of the LP (along $x$-axis) Gaussian beam [21, 40]. Thus, the mode can be excited by the Gaussian beam. If figures 2(f)–(h) are compared, the field distributions of the photonic mode do not change much. This trend is drastically different from the plasmonic mode where not only the field distributions but also the penetration depths in the core and the cladding change dramatically toward the aperture.

Now, we look at the ratios between the $|E_x|^2$, $|E_y|^2$ and $|E_z|^2$ components along the propagation direction toward the tip aperture. The field components ratios $|E_x|^2/|E_y|^2$ (dashed red line) and $|E_x|^2/|E_z|^2$ (solid red line), of the HE$_{11}$ mode versus the core radius $a$ are plotted in figure 4. Absolute square of field components, $|E_x|^2$ (dashed lines) and $|E_y|^2$ (solid lines), are calculated at each core radius $a$, and the maxima of them are normalized by that of $|E_y|^2$. As the core radius $a$ decreases, the $y$-component increases in strength compared with the $x$-component but never gets larger than it. Meanwhile, the ratio $|E_y|^2/|E_x|^2$ increases up to about 150 times until core radius $a = 303$ nm where the slope of the effective index curve changes (see the red curve in inset 2 in figure 2(a)). Then, the $z$-component starts to decrease but still stays larger than the $x$-component until core radius $a = 150$ nm where the ratio is 1.6 times. In general, these field components ratios show that the longitudinal field component ($|E_z|^2$) increases in strength compared with the transversal components as the core radius $a$ decreases. This nature is akin to focusing of a Gaussian beam with a lens. As the Gaussian beam is focused with a high numerical aperture lens, the longitudinal component gets stronger at the focal plane [41–43]. However, by guiding a mode through a taper with a metal cladding, we can achieve a ratio ($|E_x|^2/|E_y|^2 > 1$) that is not achievable for a Gaussian beam focused by any physical lens ($|E_x|^2/|E_y|^2 < 1$ always) [44]. Thus, besides the similarity in field profiles between the HE$_{11}$ mode and a free space Gaussian beam, the propagation of the HE$_{11}$ mode through a tapered waveguide with a metal cladding is analogous to focusing of a Gaussian free space beam with a lens.

Finally, we need to discuss about the power decay of the fundamental photonic mode. We calculate the power decay of plasmonic and photonic modes due to the propagation loss in the region where the photonic mode has $\beta_{HE_{11}}/k_0 \ll 1$. The results are shown in figure 5. The power decay $P(a)/P(a_0 = 405 \text{ nm})$ is the ratio between the transversal mode power $P$ at a core radius of $a$ and at $a_0 = 405 \text{ nm}$ (see appendix B). Without including the coupling to other modes that are propagating in forward and in backward directions, the HE$_{11}$ mode suffers from a power loss of about 9 orders of magnitude just in this small region. Thus, even before reaching the evanescent mode region ($a \lesssim 150$), the majority of the mode’s power gets absorbed and reflected.

![Figure 4. Field components ratios versus core radius $a$ for HE$_{11}$-SP (blue) and HE$_{11}$ (red) modes. Absolute square of field components, $|E_x|^2$ (dashed lines) and $|E_y|^2$ (solid lines), are calculated at each core radius $a$, and the maxima of them are normalized by that of $|E_y|^2$. The dominant component of the quasi $x$-polarized modes at large core radius $a$. For HE$_{11}$-SP plasmonic mode, $|E_x|^2$ and $|E_y|^2$ components are always weaker than the $|E_z|^2$ component. For HE$_{11}$ photonic mode, $|E_x|^2$ and $|E_y|^2$ component increases in amplitude compared with the $|E_z|^2$ as the core radius $a$ decreases. The longitudinal component $|E_z|^2$ gets about 150 times greater than the dominant $|E_y|^2$ component for the core radius $a \approx 303$ nm. Meanwhile, $|E_x|^2$ remains smaller than $|E_y|^2$ at any core radius $a$.](image-url)
by the metal layer due to the dramatically increased imaginary part of the effective index and the tapering of the waveguide. Since a typical aperture tip has an aperture size or core diameter of 100–200 nm, we expect that the light transmission efficiency will be even less than this value. Consequently, this mode is highly unlikely to reach the tip’s aperture if the aperture diameter is below ≈300 nm. Unlike the fundamental photonic mode, the fundamental plasmonic mode has no cutoff (for its leaky mode) so it reaches the aperture. Note that the cutoff radius will slightly differ when different metal coating (other than gold) or operation wavelengths (other than 784 nm) are used. The cutoff radius increases with the increasing wavelength while the propagation loss will decrease.

2.4. Coupling between fundamental plasmonic and photonic modes

To calculate the coupling efficiency from the photonic mode (HE11) to the plasmonic mode (HE1Co-SP), we use a coupled mode theory for tapered waveguides [34, 45–48]. The slowly varying amplitudes of the photonic (A_{ph}(z)) and the plasmonic (A_{pl}(z)) modes propagating forward through the taper can be evaluated with the following equations

\[
\frac{dA_{ph}}{dz} = \alpha_{pl-ph}A_{pl} - i\beta_{ph}A_{ph}, \tag{5a}
\]

and

\[
\frac{dA_{pl}}{dz} = \alpha_{ph-pl}A_{ph} - i\beta_{pl}A_{pl}, \tag{5b}
\]

where \( \beta_{ph} \) and \( \beta_{pl} \) are the propagation constants of the photonic and plasmonic modes. Furthermore, the coupling coefficient \( \alpha_{1\rightarrow2} \) describes the coupling strength from one mode to the other mode with the following equation [34, 45]

\[
\alpha_{1\rightarrow2} = \frac{\omega_0 (\varepsilon_{core} - \varepsilon_{\text{gold}}) a \tan \left( \frac{\theta}{2} \right)}{4 \varepsilon_0 \varepsilon_{\text{fract}} (\beta_1 - \beta_2)} \int_{2\pi}^{\phi} \left( \varepsilon_{\text{core}} E_{\rho}^1 E_{\rho}^2 + E_{\varphi}^1 E_{\varphi}^2 + E_{z}^1 E_{z}^2 \right) \left| \varphi = a \right. \, \text{d}\varphi. \tag{6}
\]

Here, \( \theta \) is the tapering angle, and subscripts 1 and 2 can be replaced by either subscripts ph and pl. \( E_{\rho}^1 \) is the electric field that is described by equations (1) and (2) where \( A = 1 \), \( P_{1,2} \) is the mode’s time averaged power at a core radius \( a \) that is given in equation (B1) in appendix B. If the waveguide is uniform where the tapering angle is \( \theta = 0 \), the coupling strength is 0. For non-uniform waveguides with varying cross sections (\( \theta > 0 \)), the continuous taper is described as small discrete steps where perturbations and reflections occur predominantly [34, 45–48]. From equations (5) and (6), one can see that the absolute square of the slowly varying amplitude is essentially the mode power at a particular core radius \( a \). Since the initial photonic mode’s amplitude is 1, this value gives the coupling efficiency of the mode. The mode coupling in tapered waveguides and its derivation are discussed in details in [34, 45].

We calculate the absolute squares of the slowly varying amplitudes of the photonic \( |A_{ph}|^2 \) in red line) and plasmonic \( |A_{pl}|^2 \) in blue line) modes and show them in figure 6. The absolute squares of the slowly varying amplitude of the photonic mode is about \( 10^{-9} \) at a core radius \( a = 150 \) nm and for a tapering angle \( \theta = 20^\circ \).
If we include the coupling of the forward propagating photonic mode to the backward propagating photonic mode, $|A_\text{ph}|^2$ is expected to be much less. Meanwhile, the $|A_\text{pl}|^2$ of plasmonic mode is about $0.9 \times 10^{-6}$ at a core radius $a = 150$ nm so the coupling efficiency is estimated to be about 0.0001%. This coupling efficiency increases if the tapering angle enlarges due to the tangent relation between the tapering angle and the coupling coefficient and also the increased non-adiabaticity. The coupling to the backward propagating mode will increase also in this case.

To understand the increase of the slowly varying amplitude of the plasmonic mode, we need to look at the coupling efficiency. The coupling coefficient $\alpha$ increases mainly due to two reasons that are the increasing non-adiabaticity toward the tip aperture with the decreasing core radius $a$ and the mode’s local resonance at a particular core radius $a$ of the tip. As shown in figures 2(a) and (b), the dispersion curves bend strongly when the core radius $a$ becomes smaller than 600 nm for $\text{HE}_{1\text{Co-SP}}$ and $\text{HE}_{11}$ modes. Due to this steep fall of the effective index’s real part depending on the core radius $a$, the adiabatic condition of a mode propagation through the taper is no longer valid despite the small tapering angle of $\theta$. Hence, we can no longer approximate that the modes are roughly orthogonal to each other. This non-adiabatic propagation of modes creates a favorable condition for the coupling between plasmonic and photonic modes. Furthermore, the waveguide modes, both the photonic and plasmonic, become resonant at a certain core radius $a$ allowing an efficient excitation of modes and coupling between them. For the plasmonic mode, it becomes resonant at a core radius $a$ of about 650 nm; thus, the plasmonic mode’s amplitude increases further at this point as it is shown in figure 6. This phenomenon is much like nanoparticles with different sizes and shapes are resonant at different wavelengths [49, 50]. Only in this case, a particular section of the tapered and metal-coated waveguide acts like a metallic nanohole and interacts resonantly with modes. Cylindrical dielectric or metallic rods are shown to exhibit resonant behaviors [51–53]. In case of tapered waveguides, different parts of the tapered waveguide interact resonantly depending on the incident beam’s wavelength and thus offer a continuum of resonance spectrum [51]. With the increasing wavelength of the incident beam, this resonant core radius of the tapered waveguide moves to a larger core radius $a$ [51]. Moreover, the resonant behavior is more pronounced in a waveguide with a large refractive index difference in the core and the cladding than in a weakly guiding waveguide. Further study is required to understand this effect fully.

3. Which mode participates in the near-field interaction?

So far, we have discussed about fundamental photonic and plasmonic modes labeled as $\text{HE}_{11}$ and $\text{HE}_{1\text{Co-SP}}$, respectively. Now, we would like to understand which mode or modes of an aperture tip participates in detection and illumination processes in SNOM applications. When the tapering of the tip is adiabatic and free of perturbations, the LP fiber mode shall slowly transform into quasi-LP photonic mode in the tapered and metal-coated fiber. Thus, the photonic fundamental mode dominates between the fiber end and a tapered region that is some micrometers away from the aperture. However, this mode cannot and do not take part in near-field detection and excitation processes because of two reasons. First, the $\text{HE}_{11}$ photonic mode suffers from high propagation loss and decays even before reaching the aperture. Just in between core radius $a = 405$ nm and $a = 150$ nm, the power decay is about 9 orders of magnitude for a tapering angle of $20^\circ$ (see figure 5). Thus, for typical aperture tips with diameters of $\approx 100$ nm, the $\text{HE}_{11}$ photonic mode is extremely weak or even absent at the aperture. Second, if the $\text{HE}_{11}$ mode would participate in the near-field interaction outside the tip aperture, circular aperture tips shall be sensitive to the longitudinal z-component (parallel to the fiber axis) since this
component dominates at a small core radius \(a\) (see solid red curve in figure 4). However, many experiments demonstrated that circular aperture tips are sensitive to the transversal field components (parallel to the tip’s aperture plane) unless the sample under investigation supports orders of magnitude larger longitudinal field [10, 31, 32].

Although the HE\(_{11}\) photonic mode decays strongly before reaching the tip aperture, it is not all in vain. The HE\(_{11}\) photonic mode excites the HE\(_{1\text{Co-SP}}\) plasmonic mode before completely vanish. Consequently, at the end of the aperture, The HE\(_{1\text{Co-SP}}\) mode dominates and thus governs in the near-field detection and excitation processes outside the tip’s aperture. Several facts support our hypothesis:

First, the excitation of the plasmonic mode from the photonic mode is most likely to happen due to the vanishing adiabaticity as the core radius \(a\) decreases and the mode’s local resonance at a particular core radius \(a\).

As we have calculated and shown in section 2.4, the total coupling efficiency to the plasmonic mode from the photonic mode is estimated to be about 0.0001%.

Second, the HE\(_{1\text{Co-SP}}\) mode describes well the circular aperture tip’s detection and excitation characteristics. Circular aperture tips have aperture diameters of about 100 nm where the only existing mode is the HE\(_{1\text{Co-SP}}\) mode since it has no cutoff and thus reaches the tip aperture. Furthermore, the transversal field components are always stronger (at least \(10 \times\)) than the longitudinal field component for the HE\(_{1\text{Co-SP}}\) mode that has been shown in figures 2(c)–(e) and figure 4. This behavior of fields gives the circular aperture tip the nature of being sensitive to the transversal field components in the collection SNOM applications [31, 32].

Third, the optical transmission efficiency of circular aperture tips are better explained with the plasmonic mode. As the core radius \(a\) decreases, the imaginary part of the effective index (\(\beta''\)) increases for all modes. Hence, the propagation loss accumulates not only due to the propagation length (distance to aperture) but also due to the increased \(\beta''\). The propagation length can be reduced by increasing the tapering angle, but for a chosen metal and a wavelength, one cannot avoid propagation loss caused by the radius dependent \(\beta''\). Toward the region near the aperture (core radius \(a = 300\) nm to 150 nm), \(\beta''\) is two orders of magnitude smaller for the HE\(_{1\text{Co-SP}}\) mode than the HE\(_{11}\) mode. Therefore, the power decay differs by several orders of magnitude between the HE\(_{1\text{Co-SP}}\) and HE\(_{11}\) modes as shown in figure 5. The HE\(_{11}\) mode’s power reduces at least 9 orders of magnitude as it reaches the core diameter of 300 nm. Since the mode transforms into an evanescent mode after this point, the transmission efficiency through an aperture diameter of 100 nm shall be even much smaller than that. Note that the HE\(_{11}\) mode’s coupling with the back propagating counterpart is not included here. If included, it should reduce the transmission efficiency even further. Nonetheless, the typical measured values of the optical transmission efficiency are about \(10^{-5}\) for tips with an aperture diameter of 100 nm and a tapering angle of about 30° [54]. Meanwhile, the total transmission efficiency of the HE\(_{1\text{Co-SP}}\) mode is estimated to be in the order of 0.9 \(\times\) \(10^{-6}\). This estimation is more realistic and closer to the measured results.

Consequently, based on the above mentioned facts, we believe that both HE\(_{11}\) and HE\(_{1\text{Co-SP}}\) modes participate in different ways during the detection and excitation processes. The HE\(_{11}\) photonic mode mediates between the fiber end and the mid-tapered region that is a couple of micrometers away from the tip aperture. Meanwhile, the HE\(_{1\text{Co-SP}}\) plasmonic mode relays between the mid-tapered region and the aperture of the tip. Hence, the HE\(_{1\text{Co-SP}}\) mode not only participates in the near-field interaction outside the tip aperture but also determines the circular aperture tips behavior. This conclusion once again highlights the importance of plasmonic fields in nano-optics and SNOM where the light needs to be confined below a subwavelength region.

Although we have performed our calculations for aperture tips with gold claddings, we would like to emphasize that our conclusion remains valid for aperture tips with different metal claddings such as aluminum or silver. We calculate the dispersion curves of the fundamental plasmonic (blue lines) and photonic (red lines) modes for silver (solid lines) and aluminum (dashed lines) aperture tips depending on the core radius \(a\) and plot the results in figure C1 in appendix C. Based on these calculations, we can conclude that regardless of the metallic cladding type, there must exist both plasmonic and photonic modes. Furthermore, the plasmonic mode’s propagation loss is always smaller than that of the photonic mode. The material, however, determines the transmission efficiency of the aperture tip.

**4. Experiments**

So far, we have obtained expressions of the quasi-LP modes in aperture tips and shown that it offers thorough understanding on the aperture tips’ behavior. Now, we would like to experimentally measure the near-fields of the aperture tip and compare with the waveguide modal analysis. We explore the near-fields of aperture tips by studying planar SPPs excited by a circular aperture tip located on a gold planar surface. Figure 7(a) illustrates our setup used in this study. We install aperture tips in the SNOM head and use the shear force method to bring the tip in contact with the sample surface containing a Bull’s eye grating structure. After obtaining topographical images of the grating, we place the tip at the center of the grating for further measurements. The Bull’s eye
A grating consists of a circular disc with a diameter of 9 μm and ring gratings with a period of 768 nm. This value equals to the wavelength of the planar SPPs excited by a free space wavelength of 784 nm. The circular disc and the ring gratings are concentric. The grating was prepared by depositing 100 nm of gold on a quartz substrate. The grating consists of a circular disc with a diameter of 9 μm and ring gratings with a period of 768 nm. This value equals to the wavelength of the planar SPPs excited by a free space wavelength of 784 nm. The circular disc and the ring gratings are concentric. The grating was prepared by depositing 100 nm of gold on a quartz substrate and then milling grooves on the gold surface with a focused ion beam (FIB).

In the experiment, a LP Gaussian beam is coupled into the fiber-end of the tip where the continuous-wave laser operates at wavelength of 784 nm. We take particular care for the tapering to be adiabatic and free of perturbations so that the LP fiber mode can transform into the quasi-LP mode and reach the tip aperture. Thus, when the aperture tip is brought in contact with the gold disc’s surface, planar SPPs are excited on the gold surface where the shape of excited SPPs is determined by the waveguide mode in the aperture. Then, the excited SPPs propagate away from the contact point of the tip and sample in an outward direction and get scattered by the circular gratings. Consequently, by observing the positions of the scattered light through the microscope, we can obtain information on the directions to which the planar SPPs are excited by the tip. The scattered light through the grating is collected by a microscope objective and imaged on a charge coupled device where we take a real image of the grating’s back surface. To obtain polarization resolved images, we also place a linear polarization analyzer between the objective and the tube lens. Depending on the input beam’s polarization direction and polarization analyzer’s transmission axis, we shall get different patterns on the gratings back surface.

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We couple an x-polarized free space Gaussian beam into the fiber-end of the fiber. By imaging the tip’s emission with a microscope objective and a polarization analyzer in a separate experiment, we confirmed that the emission of the circular aperture tip has a degree of polarization of ≈0.9. After this, we excite planar SPPs with the aperture tip and observe the scattering image of the excited SPPs through the grating. The experimental results are presented in figures 7(c) and (d). The first and second pictures in figure 7(c) show the total intensity images when the tip is in and out of contact with the grating surface, respectively. The total intensity image shows that when an x-polarized mode is in the aperture tip, the tip excites SPPs along x-axis creating two-lobe pattern as it scatters through the grating. When the tip is retracted (2nd picture in figure 7(c)) and out of contact with the sample surface (at a distance of ≈4 μm), the characteristic two-lobe pattern vanishes, and the total intensity decreases 25 times. These results are consistent with the previous works where they observed similar two-lobe patterns when SPPs were excited by aperture tips [20, 55]. However, the difference of our experiment is that we obtain polarization resolved images of the SPPs excited by the aperture near-field that extend the available information from the measurements. We acquire three sets of data: two images of the aperture near-field that are showing the x- (parallel to the input polarization) and y- (perpendicular to the input polarization) components and the intensity ratio between them. These can be compared and correlated with the calculated x-component and y-component of the quasi-LP hybrid photonic and plasmonic modes. The polarization resolved scattering images are presented in figure 7(d) where the yellow arrows indicate the polarization analyzer’s transmission axis.
axis. The figure shows that along the x-axis (parallel to the input beam polarization direction), the pattern has two lobes, yet along y-axis, it has four lobes.

The results in figures 7(c) and (d) can lead us to several important conclusions. First, we should observe carefully the pattern in figures 7(c) and (d). As shown figure 7(c), the two-lobe pattern is oriented along the input beam polarization direction, and this pattern is identical to the case where the SPPs are excited by an in-plane dipole [19]. Furthermore, the polarization resolved images in figure 7(d) resemble the patterns of the SPPs excited by the transverse field components of the quasi-x-polarized plasmonic and photonic modes (see figure 2). These facts imply that the planar SPPs are excited by fields that are oriented mostly along the x-axis. Second, we can anticipate the pattern of the planar SPPs excited by a longitudinal field to exclude a certain mode. This case would be like exciting planar SPPs with an out-of-plane dipole [19]. These previous studies [19, 56] showed that the longitudinal field should excite SPPs to all radial directions. Since we do not see such a circular pattern in figure 7(d), we can say that there is a negligible longitudinal field contribution from the aperture tip. Based on the First and Second conclusions, we can already exclude the presence of the photonic mode ($HE_{11}^x$) since it possesses a strong longitudinal field at the aperture plane (see figure 4). Third, not only the patterns in figure 7(d) but also the intensity ratios should be compared with that of the fundamental quasi-LP hybrid photonic or plasmonic modes. For the plasmonic mode, the ratios between the x- and y-components in calculation is $|Ey|^2/|Ex|^2 \approx 0.08$ at a core diameter of 150 nm (see figure 4). Meanwhile, in the experiment, the ratios are $Iy/Ix \approx 0.14$ at maximum and $Iy/Ix \approx 0.09$ at minimum as shown in figure 7(c). The intensity ratios between the calculation and the experimental results, thus, are quite close and agree reasonably well. The slight discrepancies can be caused by a tilt of the tip with respect to the grating plane, surface roughness of the grating, or some unwanted dust or particles on the grating surface. On the whole, these results and facts confirm that the near-fields of the aperture tip indeed determined by the quasi x-polarized fundamental plasmonic mode, $HE_{10x}^{\text{SP}}$.

It is important to note, however, that our analysis and conclusion does not contradict with previously reported experimental results [10, 20, 31, 32, 55]. In fact, our result supports and explains why the aperture tip detects mostly transverse fields during measurements. According to our analysis, it is because the fundamental plasmonic mode, that has stronger transverse fields, exists at the tip aperture and participates in the near-field interaction outside the tip aperture.

5. Summary

In this paper, we studied analytically the quasi-LP fundamental modes of aperture tips and experimentally confirmed our findings. First, we obtained a set of analytical equations that describes quasi-LP hybrid modes that are a degenerate, orthonormal basis of the hybrid modes in uniform waveguides. The quasi-LP modes have all 6 vector fields that are directly comparable to the free space LP Gaussian beam. Since the quasi-x- and y-polarized modes form the basis of the original hybrid mode, any other polarization states can be composed with the basis accurately. Next, by studying the dispersion relation, we found that the photonic fundamental mode experiences a cutoff at a core radius $a \approx 150$ nm and turns into an evanescent mode. Meanwhile, the fundamental plasmonic mode has no cutoff. Furthermore, at a core radius $a \leq 500$ nm, the fundamental photonic mode has a larger imaginary part of the effective index than the plasmonic mode. In this region, the photonic mode’s power decay due to the propagation loss alone is about 9 orders of magnitude. We can observe another important feature from the dispersion curves. When the core radius $a$ is smaller than 600 nm, the effective index depending on the core radius $a$ decreases with a fast rate for both photonic and plasmonic modes. This behavior indicates that these modes change no longer propagating in adiabatic manner while propagating along the taper despite having a small tapering angle. Furthermore, a mode can interact resonantly with a particular section of the tapered and metal-coated waveguide as if that part of the waveguide is a locally resonant metallic nanohole. Both the non-adiabaticity and the mode’s local resonance give rise to a coupling between the photonic and plasmonic modes. The coupling efficiency to the plasmonic mode from the photonic mode is estimated to be about 0.0001%. As a consequence, although only the photonic mode gets excited by the free-space Gaussian beam, near the tip aperture, there exist both photonic and plasmonic modes. The plasmonic mode has no cutoff and thus reaches the tip aperture while the photonic mode dies off before reaching the aperture. Therefore, we concluded that the plasmonic mode governs in the near-field excitation and detection processes at the tip aperture.

We also studied the field profiles of the quasi x-polarized plasmonic and photonic modes at different core radius $a$. For the plasmonic mode, we found that the x-component’s penetration depth increases in the core with the decreasing core radius $a$ since the mode enters the leaky mode region (core radius $a < 500$ nm). This results in maximum field amplitude at the center of the dielectric core and makes the mode looking like a photonic one. This effect led many to believe the mode is a photonic one. On the other hand, the penetration depth decreases in the metal for all components; thus, the size of plasmonic mode shrinks with the decreasing dimension of the
waveguide. Meanwhile, the field profiles of the photonic mode do not change as much as the plasmonic mode. Next, we analyzed the ratios between different field components of the quasi-$x$-polarized plasmonic and photonic modes at different core radius $a$. We found that the photonic mode's propagation through the taper resembles focusing of a Gaussian beam in free space with a lens. For the quasi-$x$-polarized photonic mode, the longitudinal $z$-component increases in strength compared with the dominant transversal field component and reaches a ratio of about 150 at core radius $a \approx 300$ nm. Conversely, for the quasi-$x$-polarized plasmonic mode, the $y$- and $z$-components are always weaker than the dominant $x$-component at any core radius $a$. This nature gives the circular aperture tip the characteristics of being mostly sensitive to the transversal fields. Finally, we experimentally measure and obtain polarization resolved images of the aperture near-field. By comparing these images with the field components of the quasi-LP plasmonic and photonic modes, we confirm that indeed, the near-field of the aperture tip is well described by the fundamental plasmonic mode.

In conclusion, based on our analytical and experimental study on aperture tips, we claim that the fundamental plasmonic mode plays a crucial role in the near-field interaction outside the tip aperture and determines optical characteristics of circular aperture tips. We reckon that the plasmonic fundamental mode, though the coupling efficiency may be low, is present in the aperture of tips rather than the evanescent tail of the fundamental photonic mode. Our results emphasize the importance of plasmonic fields in confining light in a subwavelength region. Surface plasmons, being naturally confined in transversal axis, inherently enable the fundamental plasmonic mode to be confined in a space that is not limited by its geometrical dimension. Meanwhile, in aperture tips, the photonic mode, which has natural tendency to expand, experiences strong decay when the structure dimension shrinks and eventually escapes the region either through loss or reflection. Our conclusion remains valid for aperture tips made of different metal cladding such as silver or aluminum. The analytical description of quasi-LP modes in aperture tips can be applied in studying many other cases where it involves complex waveguides or apertures with significant difference in refractive indices of the core and the cladding.

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Appendix A. Derivation of field components

We establish first an analytical description of the guided modes within a cylindrical waveguide with a dielectric core and a metal cladding. Maxwell’s equations in cylindrical coordinate system ($\rho$, $\varphi$, $z$) allow us to express the radial and azimuthal ($\rho$ and $\varphi$, respectively) field components of the electromagnetic field with $z$ components ($E_z$ and $H_z$) as [33]

\begin{align}
E_\rho &= -\frac{i}{\zeta^2} \left[ \frac{\beta}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{\omega \mu_0}{\rho} \frac{\partial H_z}{\partial \varphi} \right], \\
E_\varphi &= -\frac{i}{\zeta^2} \left[ \frac{\beta}{\rho} \frac{\partial E_z}{\partial \varphi} - \frac{\omega \mu_0}{\rho} \frac{\partial H_z}{\partial \rho} \right], \\
H_\rho &= -\frac{i}{\zeta^2} \left[ \frac{\beta}{\rho} \frac{\partial H_z}{\partial \rho} - \frac{\omega \varepsilon_0 \varepsilon_1}{\rho} \frac{\partial E_z}{\partial \varphi} \right],
\end{align}

and

\begin{align}
H_\varphi &= -\frac{i}{\zeta^2} \left[ \frac{\beta}{\rho} \frac{\partial H_z}{\partial \varphi} + \frac{\omega \varepsilon_0 \varepsilon_1}{\rho} \frac{\partial E_z}{\partial \rho} \right].
\end{align}

Here, $\omega$ is the angular frequency, $\mu_0$ is the vacuum permeability, $\varepsilon_0$ is the vacuum permittivity, and $\varepsilon_1 = n_1^2$ is the dielectric constants of a medium $l$ with the refractive index of $n_l$. Moreover, $\beta$ and $\zeta$ are the propagation constant and the transversal wavevector, respectively.

With equation (A1) and Maxwell’s equations, one can find the wave equation as

\begin{align}
\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial F}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 F}{\partial \varphi^2} + \zeta^2 F = 0,
\end{align}

where $F$ is either $E_z$ or $H_z$ fields. With an ansatz $F = A F(r) \exp[i(\nu \varphi + \psi - \omega t + \beta z)]$ where $A$, $\nu$, $\omega$, and $\psi$ are constants, and $t$ is a time variable; we can deduce above equation to
\[
\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial F}{\partial \rho} + \left( \zeta^2 - \frac{\nu^2}{\rho^2} \right) F = 0. \tag{A3}
\]

Physical meanings of the constants are: \(A\) is the field amplitude, \(\nu\) is an azimuthal mode number, \(\omega\) is the angular frequency, and \(\psi\) is a phase constant. In later calculations, we consider a steady state case and thus drop the function \(\exp(i(\beta z - \omega t))\) for simplicity since it can be added any time later. The solutions of equation (A3) are Bessel functions. To have physical solutions in the core, we choose \(I_\nu(r)\) with the azimuthal mode number of \(\nu\) to express core guided photonic modes. When \(r\) is complex valued with a large imaginary part, \(J_\nu(r)\) changes into \(I_\nu(r)\) and thus allows us to calculate also the plasmonic modes at the dielectric-metal inner interface. For the cladding, we choose Hankel function of the 1st kind \((H_\nu^{(1)}(r))\) because it converges to zero when \(r\) goes to infinity. This function is particular suitable for lossy waveguides like a metal cladded dielectric waveguide. Once appropriate functions are chosen, we get a general ansatz in the core as

\[
E_z = A I_\nu(\zeta \rho) \exp(i\nu \varphi + i\psi), \tag{A4a}
\]

and

\[
H_z = B I_\nu(\zeta \rho) \exp(i\nu \varphi + i\psi), \tag{A4b}
\]

where \(A\) and \(B\) are field amplitudes, \(\zeta = \sqrt{k_0^2 \varepsilon_{\text{core}} - \beta^2}\) is the transversal wavevector with the core dielectric constant of \(\varepsilon_{\text{core}}\), and \(k_0 = 2\pi/\lambda_0\) is the wavenumber with the wavelength of \(\lambda_0\). In the cladding, we substitute \(\zeta = i\gamma\) in equation (A3) and obtain the fields as

\[
E_z = C H_\nu^{(1)}(i\gamma \rho) \exp(i\nu \varphi + i\psi), \tag{A5a}
\]

and

\[
H_z = D H_\nu^{(1)}(i\gamma \rho) \exp(i\nu \varphi + i\psi), \tag{A5b}
\]

where \(C\) and \(D\) are field amplitudes, and \(\gamma = \sqrt{\beta^2 - k_0^2 \varepsilon_{\text{gold}}}\) is the transversal wavevector with the cladding dielectric constant of \(\varepsilon_{\text{gold}}\).

We can find \(\rho\) and \(\varphi\) components of the electromagnetic field by using equations (A1), (A4), and (A5). Meanwhile, to find \(x\)- and \(y\)-components of fields, we can use the following relations

\[
F_x = F_\rho \cos(\varphi) - F_\gamma \sin(\varphi), \tag{A6a}
\]

and

\[
F_y = F_\rho \cos(\varphi) + F_\gamma \sin(\varphi), \tag{A6b}
\]

where \(F\) is either electric or magnetic fields.

**Appendix B. Time averaged power of a mode and power decay**

The time averaged power of an eigen mode in a lossy waveguide is given as

\[
P = 0.5 \text{Re} \left[ \iint (E \times H) \cdot z \, dS \right]. \tag{B1}
\]

We calculate the total power decay that occurs during the propagation toward the tip aperture accordingly

\[
\text{Power decay } \approx \frac{P(z)}{P(z_0)} = \exp\left(-2 \int_{z_0}^z \beta''(z) \, dz\right). \tag{B2}
\]

Here, \(P(z)\) is the mode power at an end position of \(z\), \(z_0\) is the starting position, and \(\beta''\) is the imaginary part of the propagation constants. The equation evaluates the mode power at a certain position relative to the starting position.

**Appendix C. Influence of the cladding material**

Besides gold, aluminum or silver can be used as a cladding material. We would like to demonstrate that our conclusion remains valid for aluminum or silver aperture tips. We calculate the dispersion curves of the fundamental plasmonic (blue lines) and photonic (red lines) modes for silver (solid lines) and aluminum (dashed lines) aperture tips depending on the core radius \(a\) and plot the results in figure C1. In the figure, inset 1 illustrates the structure considered in the calculation. Meanwhile, inset 2 zooms into the region where the plasmonic mode changes from a bound to a leaky mode by having an effective index lower than that of the core. Based on the results presented in figure C1, we can conclude three things. First, regardless of the cladding material, the fundamental plasmonic mode does not experience cutoff and reach all the way to the tip aperture.
Second, the fundamental photonic mode always experiences cutoff before reaching the tip aperture. Third, the fundamental photonic mode becomes at several orders of magnitude lossier than the fundamental plasmonic mode toward the aperture.

For aluminum aperture tips the plasmonic mode turns into a leaky mode at a core radius of 1.41 μm that is much sooner than the silver (a = 540 nm) or gold (a = 500 nm) aperture tips. This is the consequence of the poor plasmonic feature of aluminum. Furthermore, aluminum is highly lossy (n_{aluminum} = −63.44 + 43.95i) at our chosen wavelength of 784 nm, and thus we can expect that the transmission efficiency will be smaller than the silver or gold aperture tips. For a full tapering angle of 20°, we can calculate the propagation loss of the fundamental plasmonic and photonic modes just like it was done in figure 5. We estimate that the photonic mode’s propagation loss is about 10⁻¹⁹ while the plasmonic mode has a propagation loss of 10⁻⁶ when the modes propagate from the core radius a of 400 nm to the core radius a of 50 nm.

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