Unitary Higgsless and Higgsful Standard Models from Spontaneous Dimensional Reduction and Weak Boson Scattering at the LHC

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Abstract

Spontaneous dimensional reduction (SDR) is a striking phenomenon predicted by a number of quantum gravity approaches which all indicate that the spacetime dimensions get reduced at high energies. In this work, we construct an effective theory of electroweak interactions based on the minimal Higgsless standard model with nonlinearly realized electroweak gauge symmetry and incorporating the spontaneous reduction of space-dimensions. Such a Higgsless standard model is manifestly renormalizable at high energies. In spite of no Higgs-boson-like excitations in our model, we reveal that the scattering amplitude of longitudinal weak bosons remains unitary at high energies. We further demonstrate that this can be discriminated from the conventional standard model (SM) and its extensions by the $WW$ scattering experiments at the CERN LHC. Finally, we study an extension for the Higgsful SM with TeV-scale SDR, as a natural solution to the hierarchy problem. It contains either a light or heavy Higgs boson with anomalous gauge couplings. We show that the corresponding $WW$ scattering cross sections become unitary at TeV scale under the SDR, but exhibit different behaviors.

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1. Introduction

So far it is well established both theoretically and experimentally that the electroweak interactions fit in the gauge structure of $SU(2)_L \otimes U(1)_Y$, and are mediated by the massless photon plus three massive weak gauge bosons ($W^\pm, Z^0$). In the electroweak standard model (SM) [1], the gauge symmetry $SU(2)_L \otimes U(1)_Y$ is linearly realized and spontaneously broken by the conventional Higgs mechanism [2] in 4-dimensions, which ensures both the renormalizability [3] and the unitarity [4, 5, 6, 7] of the SM. However, in the minimal Higgsless SM with the Higgs boson removed from its particle spectrum, the electroweak gauge symmetry becomes nonlinearly realized and the three Goldstone bosons convert into the longitudinal polarizations of $W^\pm/Z^0$ after spontaneous symmetry breaking. Such a minimal Higgsless SM is necessarily incomplete because of the loss of traditional renormalizability and the violation of unitarity of $WW$ scattering at TeV scale. Any consistent quantum field theory has to maintain the unitarity within the relevant energy ranges of its validity. Different ways of unitarizing the longitudinal $WW$ scattering around the TeV scale reflect the corresponding underlying mechanisms of electroweak symmetry breaking (EWSB), and will be discriminated by the $WW$ scattering experiments as a key task of the CERN LHC [8].

As is known, naively introducing a bare mass-term for the weak gauge bosons or SM fermions into the Lagrangian by hand would ruin both the renormalizability and unitarity of the theory. Such bare mass-terms can be made gauge-invariant via nonlinearly realized electroweak gauge symmetry
in the minimal Higgsless SM, but suffer strong unitarity limits for the \( W/Z \) and the SM fermions, all within the energy range of \( 1.2 - 170 \text{ TeV} \). The SM resolves this problem by invoking a Higgs doublet to linearly realize the electroweak gauge symmetry and its spontaneous breaking via Higgs vacuum expectation value. With this, the weak gauge bosons \( W/Z \) as well as SM fermions acquire the observed masses, and a physical Higgs boson is predicted. Such a theory is indeed renormalizable and unitary as ensured by the presence of the physical Higgs boson, and is usually expected to be ultraviolet (UV) complete up to high energies below the Planck scale \( M_{\text{Pl}} \approx 1.2 \times 10^{19} \text{ GeV} \). However, such a fundamental SM Higgs boson suffers a number of theoretical inconsistencies, including the naturalness and triviality problems. Such a SM Higgs boson has been sought in vain by all known experiments so far, including the current run of the 7 TeV LHC. It is therefore important to keep open mind and seek possible alternatives of the SM Higgs boson, which can be definitively tested at the on-going LHC. Many candidates have been put on market, incorporating new physics such as strong dynamics, supersymmetry, and extra dimensions or deconstructions.

In this work, we present a novel, but conceptually truly simple and attractive approach to the electroweak symmetry breaking via spontaneous dimensional reduction (SDR). In our construction, the electroweak gauge symmetry is nonlinearly realized without any Higgs boson, while the renormalizability and unitarity will be retained, due to spontaneous reduction of space-dimensions in the high energy regime. The concept of dimensional reduction was first conceived by ‘t Hooft in the context of holographic principle, and the phenomenon of SDR has been predicted by a number of quantum gravity approaches. Some well-known and intriguing studies of quantum gravity along this direction include such as the exact renormalization group approach, the causal dynamical triangulation, the loop quantum gravity, the high-temperature string theory, and Hořava-Lifshitz gravity. We note that the possibility of dynamical generation of the space-dimension(s) at low energies has also been stressed and explored via dimensional (de)construction. Despite the differences in their detailed structures, all these studies reveal a common feature, showing that the spacetime dimensions, being \( n = 4 \) in the infrared region, will become reduced and continuously approach \( n = 2 \) in the UV, due to nonperturbative quantum gravity corrections. This essential feature is truly appealing, since in general a field theory exhibits worse UV behavior in higher spacetime dimensions, but displays better UV behavior for lower dimensions. For instance, a gauge theory (including the SM) in \( n \geq 5 \) dimensions is superficially nonrenormalizable. On the contrary, the SDR predicts the spacetime dimension to decrease towards \( n = 2 \) in the UV region, and will thus substantially improve the UV behavior of the theory. At the present, the high energy behavior of the SM is far from being well tested and understood. With the LHC starting its first phase at 7 TeV, we are just entering the TeV scale, and the decisive measurements of the longitudinal weak boson scattering and thus the probe of the UV behavior as well as its underlying EWSB.

\footnote{For convenience, we simply use the symbol \( W \) to denote both the weak bosons \( W^\pm \) and \( Z^0 \) in this paper, unless specified otherwise.}
mechanism have to rely on the second phase of the LHC runs at 13 – 14 TeV. This also means that the energy scale at which the SDR becomes significant can be as low as TeV scale \[26\]. In this work, we conjecture that the TeV Scale SDR can play a key role to make the Higgsless SM unitary as well as renormalizable. This will be definitively tested by the WW scattering experiments at the LHC.

In spite of lacking a complete theory of quantum gravity that could precisely describe the SDR, we will be modest and approach this problem by using the effective theory formulation. So, we will parameterize the spacetime dimension \( n = n(\mu) \) as a smooth function of the energy scale \( \mu \) (which we will call the dimensional flow by following Calcagni \[27\]), such that \( n(\mu) \rightarrow 4 \) under \( \mu \rightarrow 0 \) in the infrared region as confirmed by all low energy experiments, and \( n(\mu) \rightarrow 2 \) at a certain UV scale \( \Lambda_{\text{UV}} \), which will serve as the UV cutoff of our effective theory. For instance, we can make a simple choice for the dimensional flow,

\[
n(\mu) = 4 - 2 \left( \frac{\mu}{\Lambda_{\text{UV}}} \right)^\gamma, \quad (\mu \leq \Lambda_{\text{UV}}), \tag{1.1}
\]

where the index \( \gamma > 1 \) is a model-dependent parameter, arising from the nonperturbative dynamics of quantum gravity. Other variations \[22, 27\] of (1.1) are possible before finding a unique full theory of quantum gravity, but this will not affect the main physics features of our present analysis. In this work, we will show that, if the SDR indeed occurs at the TeV scale, then an effective theory can be consistently constructed for scales below \( \Lambda_{\text{UV}} \), in which the conventional \( SU(2)_L \otimes U(1)_Y \) electroweak gauge symmetry is nonlinearly realized. Here the new physics effect will enter at low energy scales \( \mu < \Lambda_{\text{UV}} \) through the modification of \( n(\mu) \) in (1.1). Practically, we will take \( \Lambda_{\text{UV}} = \mathcal{O}(5 \text{ TeV}) \), as required to ensure the unitarity of WW scattering. Our construction of the minimal Higgsless SM with SDR (HLSM-SDR) contains no SM Higgs boson or any Higgs-like excitation, yet it remains manifestly renormalizable and unitary at scales below \( \Lambda_{\text{UV}} \). Even though our model differs from all other conventional candidates for the EWSB and the new physics at the TeV scale, it does coincide with the intuitions and predictions of many quantum gravity theories \[20, 21, 22, 23, 24, 25\], showing that the UV behavior of a field theory will be improved by lowering the spacetime dimensions.

Finally, we note that the SDR can occur at TeV scales even if there is a light Higgs boson. In fact, such quantum-gravity-induced SDR at TeV scales provides a natural solution to the “hierarchy problem” (naturalness problem) \[11\] that plagues the conventional 4d SM with a Higgs boson. For the TeV-scale SDR, new physics effects arising from quantum gravity are expected in the low energy effective theory. This means that the Higgs boson can be non-SM-like and encode such new physics in its anomalous gauge couplings with \( WW \) and \( ZZ \), as well as in its self-couplings. For the conventional 4d SM with a non-SM Higgs boson \[28\], it was found that the WW scattering has non-canceled \( E^2 \) behavior \[29\] in the TeV range, and can be probed at the LHC \[29, 30\]. But additional new physics is required to unitarize the non-canceled \( E^2 \) contributions to the WW scattering. So, we will study an extension for the Higgsful SM with SDR (HFSM-SDR), which contains either a light or heavy Higgs boson, but with anomalous couplings. We show that under the SDR the corresponding WW
scattering cross sections become unitary at TeV scale, but exhibit different behaviors.

This paper is organized as follows. In Sec. 2, we present the construction of the minimal Higgsless SM incorporating the SDR. Then, in Sec. 3 we apply this model to study the WW scattering, and demonstrate how the unitarity is guaranteed by the SDR even without invoking any Higgs boson. We further show that this can be discriminated from the conventional unitarization schemes via WW scattering at the LHC. For comparison, in Sec. 4 we study the Higgsful SM with SDR containing a non-SM Higgs boson with anomalous gauge couplings, and show that the non-canceled $E^2$ contributions to the WW scattering get unitarized by the SDR at the TeV scale. We finally conclude in Sec. 5. In Appendix-A, we compute the final-state phase space in general $n$-dimensions, and derive the corresponding unitarity bounds for both partial wave amplitudes and cross sections.

2. Higgsless SM with Spontaneous Dimensional Reduction

As an effective theory description of SDR, we choose to encode the information of dimensional flow $n = n(\mu)$ into the measure of the spacetime integral $d\rho$, and replace all integral measure $d^4x$ appearing in the action functional by $d\rho$. A rigorous mathematical construction of $d\rho$ has been proposed in [27], but it is not needed for our present study at tree level. All we need to know is that the mass-dimension of this measure is given by $[d\rho] = -n$, with $n = n(\mu)$ the dimensional flow introduced in Sec. 1. For this purpose, it is enough to define the measure $d\rho$ formally by $d^n x$, with $n$ a scale-dependent quantity. Then the action of the theory can be written as,

$$S = \int d^n x \mathcal{L} = \int d^n x (\mathcal{L}_G + \mathcal{L}_F),$$

(2.1)

where $\mathcal{L}_G$ and $\mathcal{L}_F$ are the gauge and fermion parts of the SM Lagrangian with Higgs boson removed, and are defined under the prescription of conventional dimensional regularization method [31]. We will focus on the gauge sector for the current study,

$$\mathcal{L}_G = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu,$$

(2.2)

with

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g e^{abc} W_{\mu}^b W_{\nu}^c,$$

(2.3a)

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

(2.3b)

$$M_Z = \frac{M_W}{\cos\theta_w},$$

(2.3c)

where $(a, b, c)$ denote the $SU(2)$ gauge-indices, and $\theta_w = \arctan(g'/g)$ is the weak mixing angle connecting gauge-eigenbasis $(W_\mu^3, B_\mu)$ to the mass-eigenbasis $(Z_\mu^0, A_\mu)$. The mass-eigenbasis fields $W_\mu^\pm, Z_\mu^0$ and $A_\mu$ fields are defined as usual,

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2),$$

(2.4a)
\[
\begin{pmatrix}
Z_\mu^0 \\
A_\mu
\end{pmatrix} = \begin{pmatrix}
\cos \theta_w & -\sin \theta_w \\
\sin \theta_w & \cos \theta_w
\end{pmatrix} \begin{pmatrix}
W_\mu^3 \\
B_\mu
\end{pmatrix}.
\]

(2.4b)

For the fermion sector, let us consider a pair of generic SM fermions \((\psi_1, \psi_2)\). They form a left-handed \(SU(2)_L\) doublet \(\Psi_L = (\psi_1, \psi_2)^T\) (with hypercharge \(Y_L\)), and two right-handed weak singlets \(\psi_{1R}\) and \(\psi_{2R}\) (with hypercharges \(Y_{1R}\) and \(Y_{2R}\)). So, \((\psi_1, \psi_2)\) have electric charges \(Q_1 = \frac{1}{2} + Y_L = Y_{1R}\) and \(Q_2 = -\frac{1}{2} + Y_L = Y_{2R}\), respectively. Then we can write down the bare Dirac mass-terms for \((\psi_1, \psi_2)\),

\[
\mathcal{L}_{\text{Fm}} = -m_1 \bar{\psi}_1 \psi_1 - m_2 \bar{\psi}_2 \psi_2.
\]

(2.5)

The Lagrangians (2.2) and (2.5) derive directly from the SM in the unitary gauge, after removing the Higgs boson. We note that (2.2) is just the lowest order of the SM electroweak chiral Lagrangian in the unitary gauge \([33]\). To make this clear, we can rearrange the Higgs doublet \(\phi\), together with its charge conjugation field \(\tilde{\phi}\) into a \(2 \times 2\) matrix \(\Phi = (\tilde{\phi}, \phi)\), which can be parameterized as follows,

\[
\Phi = \frac{1}{2} (v + h) \Sigma, \quad \Sigma = \exp[i \tau^a \pi^a / v],
\]

(2.6)

where \(\tau^a (a = 1, 2, 3)\) are Pauli matrices. Once we simply remove the Higgs boson \(h\), then to the lowest order in derivative expansion \([32]\), we have the following unique kinematic term for \(\Sigma\) field,

\[
\mathcal{L}_\Sigma = \frac{1}{4} v^2 \text{tr} \left[ (D_\mu \Sigma)^\dagger (D_\mu \Sigma) \right],
\]

(2.7)

where the covariant derivative is defined as

\[
D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W^a_\mu \tau^a \Sigma - \frac{i}{2} g' B_\mu \tau^3. \quad \text{(2.8)}
\]

It is clear that for the unitary gauge \(\Sigma = 1\), the above kinematic term directly reduces to the mass terms of \(W^\pm\) and \(Z^0\) bosons with \(M_W = \frac{1}{2} g v\), as shown in (2.2). Under the SM gauge group \(SU_L(2) \otimes U_Y(1)\), the matrix \(\Sigma\) transforms as,

\[
\Sigma \rightarrow \Sigma' = U_L \Sigma U_Y^\dagger, \quad \text{(2.9)}
\]

where \(U_L = \exp[-i \theta_L^a \tau^a / 2] \in SU(2)_L\) and \(U_Y = \exp[-i \theta_Y^a \tau^3 / 2] \in U_Y(1)\). We see that the nonlinear Lagrangian (2.7) is gauge-invariant under the transformation (2.9). Since (2.7) induces the bilinear gauge-Goldstone mixing terms for \(W^\pm - \pi^\pm\) and \(Z^0 - \pi^0\), we can impose the \(R_\xi\) gauge-fixing terms as usual,

\[
\mathcal{L}_{\text{gf}} = - \frac{1}{\xi_W} F_+ F_- - \frac{1}{2 \xi_Z} F_Z^2 - \frac{1}{2 \xi_A} F_A^2 \quad \text{(2.10a)}
\]

\[
F_\pm = \partial^\mu W_\mu^\pm - \xi_W M_W \pi^\pm, \quad F_Z = \partial^\mu Z_\mu - \xi_Z M_Z \pi^0, \quad F_A = \partial^\mu A_\mu,
\]

and the corresponding Faddeev-Popov ghost terms. So, in this gauge all gauges bosons have well-behaved propagators which scale with the momentum as \(1/p^2\) in the high energy limit.
For completeness, we can also use $\Sigma$ field to rewrite the fermion mass-term (2.5) into the gauge-invariant form,

$$L_{F\Sigma} = -\bar{\Psi}_L \Sigma M_f \Psi_R + \text{h.c.},$$ \hspace{1cm} (2.11)

where $M_f = \text{diag}(m_1, m_2)$ and $\Psi = (\psi_1, \psi_2)^T$. Further incorporation of Majorana mass terms for light neutrinos is given in [10].

Such a minimal Higgsless SM is usually viewed as the leading order Lagrangian of a nonlinear effective theory of spontaneous electroweak symmetry breaking with strong dynamics and gauge-invariant higher order nonrenormalizable operators can be systematically formulated \[33\] under derivative expansion \[32\]. These higher order operators will induce unknown coefficients order by order and the unitarity cannot be restored unless a UV completion model is constructed to provide the proper unitarization. This is because the Lagrangians (2.7) and (2.11) in 4-dimensional spacetime will cause bad UV behavior, and thus violate both renormalizability and unitarity. However, as will be shown below, once the SDR is implemented, both troubles disappear. So we turn to the SDR and explain what new features it will bring to our construction.

We first recall that the action functional in $n$-dimensional spacetime with any $n$, should remain dimensionless. This implies that the Lagrangian has the mass-dimension $[\mathcal{L}] = n$. In consequence, the gauge coupling $g$, which has mass dimension $[g] = (4 - n)/2$, becomes dimensionful when SDR takes place. Thus, we can always define a new coupling $\tilde{g}$ which keeps dimensionless at all relevant scales, by transferring the mass-dimension of $g$ to another mass-parameter. Since the only dimensionful parameter appearing in the Lagrangian (2.2) at dimension-4 is the $W$ mass $M_W$, it is reasonable to define the dimensionless $\tilde{g}$ as follows,

$$g = \tilde{g} M_W^{(4-n)/2},$$ \hspace{1cm} (2.12)

and the value of the dimensionless coupling $\tilde{g}$ is given by that of $g$ at dimension $n = 4$. This prescription is a part of the definition of our model, and we will concentrate on the tree-level analysis for the present study.

In fact, the scaling (2.12) is well justified for more reasons. We may easily wonder why we could not use the UV cutoff $\Lambda_{UV}$ in the scaling of $g$ as a replacement of the infrared mass-parameter $M_W$ of the theory. This is because that in spacetime dimension $n < 4$, the gauge coupling $g$ is super-renormalizable with positive mass-dimension $[g] = (4 - n)/2 > 0$. Such a super-renormalizable coupling must be insensitive to the UV cutoff of the theory, contrary to a non-renormalizable coupling with negative mass-dimension (e.g., in $n > 4$ or in association with certain higher-dimensional operators) and naturally suppressed by negative powers of the UV cutoff $\Lambda_{UV}$. It is easy to imagine that for a super-renormalizable theory in dimension $n < 4$, if its coupling $g$ were scaled as $g = \tilde{g} \Lambda_{UV}^{(4-n)/2}$, it would even make tree-level amplitude UV divergent and blow up as $\Lambda_{UV} \to \infty$; this is clearly not true. On the other hand, it is well-known that a non-renormalizable coupling $g$ with
negative mass-dimension \([g] = −p < 0\) should be scaled as \(g = ˜g/Λ_{UV}\), and thus the tree-level amplitude naturally approaches zero when \(Λ_{UV} \to ∞\), as expected.\(^2\)

Now we can easily realize that if spacetime dimension flows to \(n = 2\) in the UV limit, then the theory defined by (2.1) is well-behaved at high energies. This is because all gauge couplings of the Lagrangian (2.1) in \(n < 4\) dimensions becomes super-renormalizable, and the gauge boson propagators scale as \(1/p^2\) in high momentum limit under the \(R_ξ\) gauge-fixing (2.10). The only concern is the nonlinear Lagrangians (2.7) and (2.11) for the gauge boson and fermion mass terms. In 4d this is the origin of nonrenormalizability and unitarity violation, as the expansion of matrix \(Σ\) gives rise to infinite number of higher dimensional operators (involving the Goldstone fields \(π^a\)) whose couplings have negative mass-dimensions. But, in our construction the spacetime dimension flows to \(n = 2\) in high energy limit where the Goldstone fields \(π^a\) have zero mass-dimension and the same is true for the vacuum expectation value (Goldstone decay constant) \(v\). In fact for \(n = 2\) dimensions the Lagrangians (2.7) and (2.11) just describe a 2d gauged nonlinear sigma model including fermions. As is well-known, in 1+1 dimensions, (2.7) is renormalizable and (2.11) is super-renormalizable.

We further note that in 2d, a massless vector particle has no propagating degrees of freedom, as can be easily seen from the Maxwell equation. This means that the propagating degrees of freedom for a vector particle in 2d, if exist, can only be the longitudinal polarization, and the associated vector particle must be massive. We also note that there exist new mechanisms in \(n < 4\) dimensions to generate masses for gauge bosons other than the Higgs mechanism. For instance, consider the 1 + 1 dimensional QED, known as Schwinger model \([35]\). The radiative corrections to the vacuum polarization from a massless-fermion loop can contribute a finite nonzero mass to the gauge boson \([35]\),

\[
m_γ = \frac{e^2}{π},
\]

where \(e\) is the dimension-1 QED gauge coupling. For the 1 + 2 dimensional gauge theories such as the 3d QED, the Chern-Simons term also introduces a topological mass to the corresponding gauge field \([36]\),

\[
m_{cs} = \kappa e^2,
\]

where \(\kappa\) is the dimensionless Chern-Simons coupling and \(e\) is again the QED gauge coupling with dimension \(\frac{1}{2}\). Hence, it is quite natural to have an explicit mass-term for the vector field in a lower dimensional field theory. We also note that in these two examples, the dimensionful gauge couplings are always proportional to the masses of gauge bosons. This further justifies our prescription for

\(^2\)For nonrenormalizable theories such as a massless 5d Yang-Mills theory without compactification, it was explicitly shown \([34]\) that the unitarity violation of the gauge boson scattering occurs at a scale of \(O(1/g_5^2)\) \([cf. Eq. (6) of Ref. \([34]\)]\), where the 5d gauge coupling \(g_5\) has a mass-dimension \(−\frac{1}{2}\), and the UV cutoff scale \(Λ_{UV}\) of this nonrenormalizable theory was naturally identified to be at the order of \(1/g_5^2\), i.e. \(Λ_{UV} \sim 1/g_5^2\) \([34]\), or one can define \(g_5^2 = ˜g^2/Λ_{UV}\) with \(˜g\) being dimensionless. Then, after compactification, it was noted \([34]\) that this theory contains one more scale in the infrared which makes a nontrivial distinction, namely, the compactification scale \(1/R\), which characterizes the mass scale \(M_1\) of the lightest Kaluza-Klein (KK) state. It is thus a well-known fact that the compactified 4d KK theory has a dimensionless gauge coupling \(g\) related to the 5d coupling via, \(g^2 = ˜g^2/(πR) \sim ˜g^2/M_1\), where the lightest KK gauge boson mass \(M_1 \sim 1/R\) is an infrared mass-parameter.
In the next section, we will further demonstrate that such a mass term is indeed harmless for the unitarity of high energy scattering of the longitudinal gauge bosons, due to the realization of the SDR.

Next, we turn to the issue of the dimensional flow \( n = n(\mu) \). As commented in Sec. 1, to derive an explicit form of \( n = n(\mu) \) requires to know a complete theory of quantum gravity which is not yet available so far. However, various constructions of the quantum gravity have already indicated that the dimensional flow \( n = n(\mu) \) approaches \( n = 2 \) at an UV scale \( \mu = \Lambda_{UV} \). An easy choice for \( \Lambda_{UV} \) would be the Planck scale \( M_{Pl} \). But, this is certainly not the only choice, and it is a very interesting and intriguing possibility that the nonperturbative dynamics of the quantum gravity drives \( \Lambda_{UV} \) down to TeV scale \( \Lambda_{UV} \). In the case, a number of difficulties associated with the EWSB and \( W/Z \) mass-generations in the SM can be resolved without introducing additional ad hoc hypothetical dynamics. As we have explained, the EWSB and mass-generations for \( W/Z \) and all SM fermions must be tied to the TeV scale \( \Lambda_{UV} \). So, if the quantum-gravity-induced SDR is going to resolve the EWSB and mass-generations for the SM particles, our construction of the HLSM-SDR thus provides a supporting evidence for the TeV scale SDR.

Therefore, it is reasonable and appealing to design a function \( n = n(\mu) \) such that it mimics the main features of the dimensional flow mentioned above. So we conjecture that \( n = n(\mu) \) is a smooth monotonic function of the relevant energy scale \( \mu \), satisfying

\[
n(0) = 4, \quad \text{and} \quad n(\Lambda_{UV}) = 2,
\]

where \( \Lambda_{UV} \) is the UV cutoff of our effective theory. There certainly exist a lot of functions which fulfill the requirement (2.15) in the effective theory, but they should all share the same qualitative physical implications. A simple choice is given by (1.1). We will adopt this ansatz as an explicit realization of the dimensional flow for the current analysis, together with the prescription that \( n(\mu) = 2 \) for \( \mu \geq \Lambda_{UV} \). This additional prescription is required as an extrapolation of (1.1) beyond the UV scale \( \Lambda_{UV} \). The reason for this is that the SDR itself is an effect induced by quantum gravity, which implies that quantum gravity becomes strong and dominant for \( \mu \geq \Lambda_{UV} \), where the classical concept of spacetime no longer holds and the conventional quantum field theory would break down. Therefore, our choice here is only an effective theory description of the spontaneous reduction of spacetime dimensions as well as the physical degrees of freedom, as a consequence of nonperturbative quantum gravity. We also note that these quantum gravity effects are significant nearby \( \Lambda_{UV} \), where the concept of spacetime (together with its dimension) can be very different from the conventional one. So, we will regard the reduction of spacetime dimensions only as an effective theory description of the low energy quantum gravity effects, since our study does not deal with any full theory of quantum gravity. In summary, we rewrite our ansatz for the dimensional flow as follows,

\[
n(\mu) = \begin{cases} 
4 - 2 \left( \frac{\mu}{\Lambda_{UV}} \right)^\gamma, & (\mu \leq \Lambda_{UV}), \\
2, & (\mu > \Lambda_{UV}),
\end{cases}
\]

(2.16)
where \( \Lambda_{UV} \) is at the TeV scale, and will serve as \textit{the UV cutoff of our effective theory}. The new physics effect will enter at low energy scales \( \mu < \Lambda_{UV} \) by modifying the dimensional flow \( n(\mu) \) as in \( \text{(2.10)} \). For practical applications, we will set \( \Lambda_{UV} = \mathcal{O}(5 \text{ TeV}) \), as required to ensure the unitarity of \( WW \) scattering in Sec. 3. In \( \text{(2.10)} \), the exponential parameter \( \gamma > 1 \) arises from the nonperturbative dynamics of quantum gravity and is thus model-dependent. As the simplest realization we may set, \( \gamma = 2 \) or \( \gamma = 1.5 \), for instance. Possible variations \( \text{[22, 27]} \) of \( \text{(2.10)} \) are all allowed since a unique full theory of quantum gravity is unknown so far, but they will not affect the main physics features of our analysis below. We may note that \( \text{(2.10)} \) is continuous but not differentiable at \( \mu = \Lambda_{UV} \). But this is not essential here since what we are interested in is the feature of dimensional reduction \textit{below} \( \Lambda_{UV} \) in the effective theory formulation, and thus is insensitive to the physics at and above the scale \( \Lambda_{UV} \). As a final remark, one possible concern may be the potential Lorentz violation around the cutoff scale \( \Lambda_{UV} \). But we note that the Lagrangian \( \mathcal{L}_G + \mathcal{L}_F \) above is manifestly Lorentz invariant and thus does not change the particles’ dispersion relations from the conventional 4d forms. Hence, our model is free from the Lorentz-violation constraints in cosmic ray observations, which are derived from modification of photon’s dispersion. We also like to note that as shown by Calcagni \( \text{[27]} \), the construction of \( d^n x \) in \( \text{(2.1)} \) may cause a Lorentz violation due to scale-dependence of \( n \), but this does not affect the particles’ dispersion relations at tree-level and thus is irrelevant to our current unitarity analysis.

3. Unitarity of \( WW \) Scattering in Higgsless SM with SDR

In the previous section, we have set up the formalism of the Higgsless SM with SDR (HLSM-SDR), which is truly simple. In the HLSM-SDR, we have removed the Higgs boson from the conventional 4d SM [cf. \( \text{(2.1)-(2.2)} \)] and nonlinearly realized the \( SU(2)_L \otimes U(1)_Y \) electroweak gauge symmetry [cf. \( \text{(2.7)-(2.8)} \)]. Furthermore, we construct the dimensional flow of spacetime as a proper function of the relevant energy scale to realize the SDR. The physical picture of this construction is simple enough, yet it offers a novel solution to ensure the renormalizability and unitarity without invoking any conventional Higgs boson. In this section, we are going to study the scattering of longitudinally polarized weak gauge bosons. We demonstrate how the \( WW \) scattering amplitudes are unitarized in the HLSM-SDR construction, and how the corresponding cross sections can be discriminated from the conventional 4d SM at the LHC.

3.1. Analysis of \( WW \) Scattering Amplitudes for HLSM-SDR

In conventional 4d quantum field theory, the longitudinal polarization \( \epsilon^\mu_L \) of a massive vector boson increases with its energy,

\[
\epsilon^\mu_L(k) = \frac{1}{M} \left( |\vec{k}|, k^0 \vec{k}/|\vec{k}| \right) = \frac{k^\mu}{M} + \nu^\mu(k),
\]  

(3.1)
where $M$ is the vector boson mass. As a result, the longitudinal scattering amplitude may have dangerous power-law dependence on the scattering energy, and thus may cause the unitarity violation.

The SM Higgs boson plays the key role of “unitarizing” the bad high energy behaviors of longitudinal scattering amplitudes \[4\]. To see how this works, let us take an explicit channel of $W^+_L W^-_L \to Z^0_L Z^0_L$. In the unitary gauge where all would-be Goldstone bosons decouple, there are just four diagrams contributing to the tree-level scattering amplitude, as shown in Fig.1. These diagrams can be readily evaluated in the center-of-mass (c.m.) frame. By counting the powers of c.m. energy $E_{\text{cm}}$, we see that as $E_{\text{cm}}$ becomes much larger than $W$ boson mass $M_W$, the leading energy dependence of the sum of the first three diagrams is of $O(E_{\text{cm}}^4)$. In fact, we can always expand the contribution of each diagram by large $E_{\text{cm}}$ expansion, and deduce their sum in the following form,

\[ T = A \cdot E_{\text{cm}}^4 + B \cdot E_{\text{cm}}^2 + O(E_{\text{cm}}^0). \] (3.2)

Unitarity of the $S$-matrix requires vanishing $E_{\text{cm}}^4$ and $E_{\text{cm}}^2$ terms, i.e., $A = B = 0$. The explicit calculation shows that the $E_{\text{cm}}^4$ terms exactly cancel among the first three diagrams ($A = 0$), while summing up the $E_{\text{cm}}^2$ terms for the first three diagrams yields a nonzero result,

\[ T_2[(a)+(b)+(c)] = \frac{g^2}{4M_W^2} E_{\text{cm}}^2. \] (3.3)

This term is canceled exactly by the $E_{\text{cm}}^2$ term from the last diagram with the SM Higgs boson exchange in the $s$-channel. In the conventional 4d SM, it is this cancelation that plays a key role to ensure the unitarity for the scattering channel $W^+_L W^-_L \to Z^0_L Z^0_L$.

In lower dimensions, the longitudinal amplitude from each diagram remains the same as in the 1+3 dimensions. However, we observe that the condition of the partial wave unitarity changes, or more precisely, the form of the partial wave expansion changes, due to reduction of phase space in the final state. In consequence, the cancellation of $E_{\text{cm}}^2$ terms described above is not required anymore. This is an essential feature of our Higgsless SM with SDR, i.e., the $WW$ scattering amplitudes keep unitary at high energies under SDR, without invoking any conventional Higgs boson.

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Figure 1: Feynman diagrams for longitudinal scattering $W^+_L W^-_L \to Z^0_L Z^0_L$ consist of (a)-(d) in the conventional 4d SM, but include only (a)-(c) in the minimal 4d Higgsless SM and in our HLSM-SDR.
The appearance of the factor $E$ in general.

nevertheless, the expression of particles being nonidentical (identical). For clarity, we present the derivation of (3.7) in Appendix A.

inelastic and elastic scattering processes, respectively, which equal 1! (2!) for the two final state $\rightarrow$

the unitarity condition for partial waves of 2

Hence, the correct partial wave expansion in 3-dimensions reads,

\[
\rho(n > 3) \quad \text{dimensions, we can derive},
\]

\[
T_{el}(E_{cm}, \theta) = \lambda_n E_{cm}^{\delta - \nu} \sum_\ell \frac{1}{N_\ell^\nu} C_\ell^\nu(1) C_\ell^\nu(\cos \theta) a_{\ell}^{\text{el}}(E_{cm}) ,
\]

where

\[
\lambda_n = 2(16\pi)^{n-3} \Gamma(\frac{n}{2} - 1) , \quad \nu = \frac{1}{2}(n - 3) , \quad N_\ell^\nu = \frac{\pi \Gamma(\ell + 2\nu)}{2^{n+1} \ell! (\ell + \nu)! \Gamma^2(\nu)} ,
\]

and $C_\ell^\nu(x)$ is the Gegenbauer polynomial of order $\nu$ and degree $\ell$. Then, from (3.4) one can derive the unitarity condition for partial waves of $2 \rightarrow 2$ elastic and inelastic scattering processes,

\[
|\text{Re} a_{\ell}^{\text{el}}| \leq \frac{\rho_\ell}{2} , \quad |a_{\ell}^{\text{el}}| \leq \rho_\ell , \quad |a_{\ell}^{\text{inel}}| \leq \frac{\sqrt{\rho_\ell \rho_e}}{2} ,
\]

where $\rho_i$ and $\rho_e$ are symmetry factors associated with the final states, for the corresponding $2 \rightarrow 2$ inelastic and elastic scattering processes, respectively, which equal 1! (2!) for the two final state particles being nonidentical (identical). For clarity, we present the derivation of (3.7) in Appendix A.

We see that the unitarity condition (3.7) does not explicitly depend on the spacetime dimension, nevertheless, the expression of $a_{\ell}$ does. In fact, for $n > 3$ dimensions, we can derive,

\[
a_{\ell}^{\text{el}}(E_{cm}) = \frac{E_{cm}^{n-4} \Gamma(n/2) - 1) C_{\ell}^{n/2}(1)}{2(16\pi)^{n/2} \Gamma(n/2 - 1) C_{\ell}^2(1)} \int_0^{\pi} d\theta \sin^{n-3}\theta C_{\ell}^{n/2}(\cos \theta) T_{el}(E_{cm}, \theta) .
\]

The appearance of the factor $E_{cm}^{n-4}$ is expected, since the $2 \rightarrow 2$ $S$-matrix has a mass-dimension $4 - n$ in $n$ spacetime dimensions, and the partial wave amplitude $a_{\ell}$ is dimensionless by definition in general.

The partial wave expansion in $n \leq 3$ dimensions needs a separate treatment, since for $n = 3$ dimensions, the Gegenbauer polynomial $C_{\ell}^{(n-3)/2}(x)$ vanishes identically. In fact, the correct eigenfunctions for partial wave expansion in $n = 3$ spacetime are simply given by a pure phase $e^{i\theta}$. Hence, the correct partial wave expansion in 3-dimensions reads,

\[
\begin{align*}
  T_{el}(E_{cm}, \theta) &= 8E_{cm} \sum_\ell e^{i\theta} a_{\ell}^{\text{el}}(E_{cm}); \\
  a_{\ell}^{\text{el}}(E_{cm}) &= \frac{1}{16\pi E_{cm}} \int_0^{2\pi} d\theta e^{-i\theta} T_{el}(E_{cm}, \theta) , \\
\end{align*}
\]

(n = 3).
For the spacetime dimension $2 \leq n < 3$, it is meaningless to talk about scattering angle and thus no partial wave expansion is needed. Or, we would say that the only nonvanishing partial wave here is the $s$-wave $a_0$, which can be identified as the forward and backward scattering amplitude multiplied by $E_{cm}^{-2}$ to make the partial wave dimensionless,

$$a_0^c(E_{cm}) = \frac{1}{4E_{cm}^2} [T_{el}(E_{cm},0) + T_{el}(E_{cm},\pi)], \quad (2 \leq n < 3). \quad (3.10)$$

The coefficients in expansions (3.9) and (3.10) are chosen such that the $s$-waves obtained from these two equations coincide with that from the analytic continuation of (3.8) as a function of the spacetime dimension $n$.

By naively removing Higgs boson from the SM, we note that the scattering amplitude for $W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ is dominated by (3.3). Substituting this into (3.8) and (3.9) for 4-dimensions and 3-dimensions, respectively, and using the unitarity condition (3.7) for $s$-wave, we infer the upper unitary limits on the scattering energy, $E_{cm} < 1.74 \text{ TeV}$ in 4-dimensions, and $E_{cm} < 6.02 \text{ TeV}$ in 3-dimensions. But for $n = 2$ dimensions, we should use (3.10), and it yields an energy-independent $s$-wave, $a_0 = \frac{1}{8}g^2$, which always satisfies the unitarity condition $|a_0| \leq 1$. So there is simply no unitarity constraint on $E_{cm}$ in 2-dimensions, as we have expected.

This analysis shows that if SDR is indeed responsible for unitarizing the scattering amplitudes of longitudinal weak gauge bosons, then the transition scale $\Lambda_{UV}$ must be on the order of TeV, otherwise, the unitarity would have been violated before the SDR takes place. Actually, we can derive a stronger bound on the transition scale $\Lambda_{UV}$ via coupled channel analysis. Let us consider the Higgsless Lagrangian (2.2)-(2.3), and for current analysis it is enough to set the weak mixing angle $\theta_w = 0$ for simplicity. For the electronically neutral channels, we have two states $|W_L^+ W_L^-\rangle$ and $\frac{1}{\sqrt{2}} |Z_L^0 Z_L^0\rangle$. When the Higgs boson is absent, the tree-level amplitude for the process $Z_0^0 Z_0^0 \rightarrow Z_0 Z_0$ vanishes. So there are only two independent amplitudes for evaluation, corresponding to the scattering channels,

$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-, \quad (3.11a)$$

$$W_L^+ W_L^- \leftrightarrow Z_L^0 Z_L^0. \quad (3.11b)$$

Three diagrams will contribute to the scattering (3.11a), via 4W contact interaction, $s$- and $t$-channel $Z$ exchanges, respectively. We summarize the amplitude for each diagram under the power expansion of the c.m. energy $E_{cm}$,

$$T_{ct} = \frac{g^2}{4} \left[ \frac{1}{4} (-3 + 6 \cos \theta + \cos^2 \theta) \bar{E}_{cm}^4 + (2 - 6 \cos \theta) \bar{E}_{cm}^2 \right] + \mathcal{O}(E_{cm}^0), \quad (3.12a)$$

$$T_{zs} = \frac{g^2}{4} \left[ - \cos \theta \bar{E}_{cm}^4 - \cos \theta \bar{E}_{cm}^2 \right] + \mathcal{O}(E_{cm}^0), \quad (3.12b)$$

$$T_{zt} = \frac{g^2}{4} \left[ \frac{1}{4} (3 - 2 \cos \theta - \cos^2 \theta) \bar{E}_{cm}^4 + \left( \frac{3}{2} + \frac{15}{2} \cos \theta \right) \right] + \mathcal{O}(E_{cm}^0), \quad (3.12c)$$

where $\bar{E}_{cm} \equiv E_{cm}/M_W$. We see that the $\mathcal{O}(E_{cm}^0)$ terms cancel among themselves, as guaranteed by the Yang-Mills gauge symmetry. But the sum of $\mathcal{O}(E_{cm}^2)$ terms gives rise to the following nonzero
result,
\[ \mathcal{T}[W_L^+ W_L^- \rightarrow W_L^+ W_L^-] = \frac{g^2 E_{\text{cm}}^2}{8 M_W^2} (1 + \cos \theta) + \mathcal{O}(E_{\text{cm}}^0). \] (3.13)

Next, we turn to the process (3.11b). The tree-level diagrams for the scattering channel \( W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0 \) are shown in Fig. 1. Since the Higgs boson is absent in our model, only the first three diagrams are relevant, and we compute them as follows,
\[ \mathcal{T}_{ct} = \frac{g^2}{4} \left[ \frac{1}{4} (-6 + 2 \cos^2 \theta) E_{\text{cm}}^4 + 4 E_{\text{cm}}^2 \right] + \mathcal{O}(E_{\text{cm}}^0), \] (3.14a)
\[ \mathcal{T}_{Wt} = \frac{g^2}{4} \left[ \frac{1}{4} (3 - 2 \cos \theta - \cos^2 \theta) E_{\text{cm}}^4 + \left( \frac{3}{2} + \frac{15}{2} \cos \theta \right) E_{\text{cm}}^2 \right] + \mathcal{O}(E_{\text{cm}}^0), \] (3.14b)
\[ \mathcal{T}_{Wu} = \frac{g^2}{4} \left[ \frac{1}{4} (3 + 2 \cos \theta - \cos^2 \theta) E_{\text{cm}}^4 + \left( \frac{3}{2} - \frac{15}{2} \cos \theta \right) E_{\text{cm}}^2 \right] + \mathcal{O}(E_{\text{cm}}^0). \] (3.14c)

Again, all the \( \mathcal{O}(E_{\text{cm}}^4) \) terms sum up to zero, and the nontrivial leading amplitude arises at \( \mathcal{O}(E_{\text{cm}}^2) \),
\[ \mathcal{T}[W_L^+ W_L^- \rightarrow \frac{1}{\sqrt{2}} Z_L^0 Z_L^0] = \frac{g^2 E_{\text{cm}}^2}{4 \sqrt{2} M_W^2} + \mathcal{O}(E_{\text{cm}}^0). \] (3.15)

With (3.13) and (3.15), we can form the \( 2 \times 2 \) matrix amplitude for the initial/final states \( |W_L^+ W_L^-\rangle \) and \( \frac{1}{\sqrt{2}} |Z_L^0 Z_L^0\rangle \),
\[ \mathcal{T}_{\text{coup}} = \frac{g^2 E_{\text{cm}}^2}{8 M_W^2} \begin{pmatrix} 1 + \cos \theta & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}. \] (3.16)

Then, we derive the \( s \)-wave amplitude in \( n \)-dimensions with the aid of (3.8),
\[ a_0^{\text{coup}} = \frac{\tilde{g}^2}{2^{n+2} \pi^{\frac{n-3}{2}} \Gamma(\frac{n-1}{2})} \left( \frac{E_{\text{cm}}}{2 M_W} \right)^{n-2} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}, \] (3.17)
where \( \tilde{g} \) is defined by (2.12). So we can diagonalize this matrix,
\[ a_0^{\text{diag}} = \frac{\tilde{g}^2}{2^{n+2} \pi^{(n-3)/2} \Gamma(\frac{n-1}{2})} \left( \frac{E_{\text{cm}}}{2 M_W} \right)^{n-2} \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \] (3.18)
and extract the maximal eigenvalue,
\[ |a_0^{\text{max}}| = \frac{\tilde{g}^2}{2^{n+1} \pi^{(n-3)/2} \Gamma(\frac{n-1}{2})} \left( \frac{E_{\text{cm}}}{2 M_W} \right)^{n-2}, \] (3.19)
from which we may impose the unitarity condition (3.7), \( |a_0^{\text{max}}| \leq 1 \) (or \( |\text{Re} a_0^{\text{max}}| \leq \frac{1}{2} \)). The same amplitude (3.19) can also be inferred from computing the scattering of spin-0 and gauge-singlet state, \( |0\rangle = \frac{1}{\sqrt{6}} (2 |W_L^+ W_L^-\rangle + |Z_L^0 Z_L^0\rangle) \).

We show the constraint in Fig. 2(a)-(b) for our Higgsless SM with SDR, under the ansatz (2.16) for the dimensional flow with the index \( \gamma = 2 \) [plot-(a)] and \( \gamma = 1.5 \) [plot-(b)]. In each plot of Fig. 2 we have also varied the transition scale \( \Lambda_{\text{UV}} = 4, 5, 6 \) TeV, in red, purple and blue curves

\[ ^{a}\text{We have checked other possible variations of (2.16) and found that our main physics picture remains.} \]
Figure 2: Partial wave amplitude of coupled channel scattering as a function of the c.m. energy $E_{cm}$. The $s$-wave amplitudes for the Higgsless SM with SDR (HLSM-SDR) are shown by red, purple, and blue curves (from bottom to top), corresponding to the transition scale $\Lambda_{UV} = 4, 5, 6 \text{ TeV}$, respectively. Plot-(a) is for the index parameter $\gamma = 2$, and plot-(b) for $\gamma = 1.5$. As comparisons, the $s$-wave amplitudes for the minimal 4d Higgsless SM (4d HLSM) and for the conventional 4d SM (with a 600 GeV Higgs boson) are also shown by the black curves.
Figure 3: Same as Fig. 2, but for the 4d Higgsless SM under the K-matrix and Padé unitarizations in blue and red dashed curves, as comparisons with our HLSM-SDR. The $s$-wave amplitudes for the non-unitarized minimal 4d HLSM and for the conventional 4d SM (with a 600 GeV Higgs boson) are also shown by the black curves as the reference.

(from bottom to top), respectively. The shaded region in yellow is excluded by the unitarity bound $|a_0^{\text{max}}| \leq 1$. From this plot we see that the partial wave always has a rather broad “lump” around $1.5 - 5$ TeV and then falls off very quickly, exhibiting unitary high energy behaviors. For comparisons in the same plot, we also display the results of (i) the 4d SM with a 600 GeV Higgs boson, and (ii) the naive 4d Higgsless SM which breaks the unitarity at $E_{\text{cm}} \simeq 1.74$ TeV.

We stress that our new mechanism of maintaining unitarity is via the phase-space reduction of final states under the SDR. This fully differs from the conventional 4d SM, which ensures the unitarity of $WW$ scattering via exchange of Higgs resonance. In passing, it has also been known before [10] that changing the phase space can strongly affect the unitarity bound. As shown in Ref. [10], enlarged phase space of $2 \rightarrow N$ scattering by proper increase of the number $N$ of final state gauge bosons can enhance the cross section and lead to a new class of unitarity bounds for all light SM fermions (including Majorana neutrinos). Our present study just interestingly shows the other way around: reducing the phase space of final states in $2 \rightarrow 2$ scattering by decreasing the spacetime dimension $n$ can significantly reduce the partial wave amplitudes and cross sections, which leads to the restoration of unitarity.

For comparison with the conventional unitarizations, we also analyze the coupled-channel $s$-wave amplitudes for longitudinal scattering in the 4d Higgsless SM, by using the traditional K-matrix method and Padé method [38], though they are a sort of arbitrary without knowing the actual UV dynamics. This means that we compute the partial waves to the order of $O(E_{\text{cm}}^4)$, and make the
The cross section of $W^+_L W^-_L \rightarrow Z^0_L Z^0_L$ as a function of the c.m. energy $E_{cm}$. The cross sections for the HLSM-SDR are shown by red, purple, and blue curves, corresponding to the transition scale $\Lambda_{UV} = 4, 5, 6$ TeV, respectively. For comparisons, the cross sections for the conventional 4d SM with a light Higgs boson ($M_h = 125$ GeV) and a heavy Higgs boson ($M_h = 600$ GeV) are shown in black dashed-curves; the cross section for the 4d Higgsless SM is depicted by the black solid-curve. The shaded regions with yellow and light-blue colors represent unitarity violation in 4d and in the HLSM-SDR, respectively; and the three blue dashed-lines, from bottom to top, show the unitarity bounds with $\Lambda_{UV} = 4, 5, 6$ TeV, respectively.

Following resummations for the $O(E_{cm}^2)$ and $O(E_{cm}^4)$ partial waves $a^{(2)}$ and $a^{(4)}$,

$$a_K(E_{cm}) = \frac{a^{(2)} + \Re a^{(4)}}{1 - i(a^{(2)} + \Re a^{(4)})}, \quad (K\text{-matrix}), \quad (3.20a)$$

$$a_P(E_{cm}) = \frac{a^{(2)}}{1 - (a^{(4)}/a^{(2)})}, \quad (\text{Padé}). \quad (3.20b)$$

Following the traditional K-matrix and Padé procedures [38], we show the corresponding s-wave amplitudes for the 4d Higgsless SM in Fig. 3. This should be compared with our HLSM-DSR prediction in Fig. 2(a)-(b).

### 3.2. Analysis of $WW$ Scattering Cross Sections for HLSM-SDR

In this subsection, we further analyze the $WW$ scattering sections for the HLSM-SDR. We study the scattering processes: (a) $W^+_L W^-_L \rightarrow Z^0_L Z^0_L$, (b) $W^+_L Z^0_L \rightarrow W^+_L Z^0_L$, (c) $W^+_L W^-_L \rightarrow W^+_L W^-_L$, which will be measured at the LHC. In particular, the scattering (a) contains an s-channel Higgs boson resonance for the conventional SM, while the reactions (b) and (c) do not. However, the new scattering (b) is also expected to be sensitive to s-channel charged new vector bosons such as the $W^\pm_1$ in the extra dimensional Higgsless theories and its deconstruction [39] or techni-rho $\rho_T^\pm$ in technicolor models [15]. The scattering (c) can be used to probe the double-charged Higgs particles in some extensions of the SM [40], or the non-resonance scenario [41], or the anomalous Higgs-gauge couplings of a light Higgs boson [29, 30].

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4The scattering (b) is also expected to be sensitive to s-channel charged new vector bosons such as the $W^\pm_1$ in the extra dimensional Higgsless theories and its deconstruction [39] or techni-rho $\rho_T^\pm$ in technicolor models [15]. The scattering (c) can be used to probe the double-charged Higgs particles in some extensions of the SM [40], or the non-resonance scenario [41], or the anomalous Higgs-gauge couplings of a light Higgs boson [29, 30].
Figure 5: Plot-(a) depicts the cross section of $W_L^\pm Z_0^0 \rightarrow W_L^\pm Z_0^0$ as a function of the c.m. energy $E_{cm}$, and plot-(b) shows the cross section of $W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm$ as a function of $E_{cm}$. In each plot, the cross sections for the HLSM-SDR are shown by red, purple, and blue curves, corresponding to the transition scale $\Lambda_{UV} = 4, 5, 6$ TeV, respectively. For comparisons, the cross sections for the conventional 4d SM with a light Higgs boson ($M_h = 125$ GeV) and a heavy Higgs boson ($M_h = 600$ GeV) are shown in black dashed-curves; the cross section for the 4d Higgsless SM is depicted by the black solid-curve. The shaded regions with yellow and light-blue colors represent unitarity violation in 4d and in the HLSM-SDR, respectively, and the three blue dashed-curves, from bottom to top, show the unitarity bounds with $\Lambda_{UV} = 4, 5, 6$ TeV, respectively.
predictions of our HLSM-SDR arise from the spontaneous reduction of spacetime dimensions at high energies due to the nonperturbative dynamics of quantum gravity, so they are universal and show up in all WW scattering channels. This is an essential feature of our HLSM-SDR and will play an important role in discriminating the HLSM-SDR from all other models of the electroweak symmetry breaking.

To evaluate cross sections in a spacetime with varying dimensions for our HLSM-SDR, we pay special attention to the phase space integral. For clarity, we present the systematical derivations in Appendix-A. In 4-dimensions, the unitarity bounds for cross sections are given by [10, 42],

$$\sigma_{\text{el}} \leq \frac{16 \pi \rho_e}{E_{\text{cm}}^2}, \quad \sigma_{\text{inel}} \leq \frac{4 \pi \rho_e}{E_{\text{cm}}^2}. \quad (3.21)$$

In general n-dimensions, we have derived the formulas in (A.24) of Appendix-A, and find that the above bound should be extended to,

$$\sigma_{\text{el}} \leq \frac{\lambda_n \rho_e}{N_0^n E_{\text{cm}}^{n-2}}, \quad \sigma_{\text{inel}} \leq \frac{\lambda_n \rho_e}{4 N_0^n E_{\text{cm}}^{n-2}}. \quad (3.22)$$

where $\lambda_n$ and $N_0^n$ are defined in (3.6).

With these, we show our predictions for the HLSM-SDR in Fig.4 and Fig.5 for $\gamma = 1.5$. As comparisons, we have computed the cross sections of these processes for the conventional SM with a light Higgs boson ($M_h = 125$ GeV) and a heavy Higgs boson ($M_h = 600$ GeV), depicted by the black dashed-curves. We also evaluate the cross section for the 4d Higgsless SM as shown by the black solid-curve. In each plot, we show our analysis for different transition scales, $\Lambda_{\text{UV}} = 4, 5, 6$ TeV, respectively. The unitarity bound for the cross section is also depicted in Figs.4-5. The regions
shaded by yellow and light-blue colors represent unitarity violation in 4d and in the HLSM-SDR, respectively, and the three blue dashed-curves, from bottom to top, show the unitarity bounds with $\Lambda_{UV} = 4, 5, 6$ TeV, respectively. From these, we see how the SDR works as a mechanism to successfully unitarize the high energy behaviors of cross sections without invoking any hypothesized new particle (such as the SM Higgs boson), as we promised in the previous section. This can be discriminated from other unitarization schemes of $WW$ scattering at the LHC. A systematical collider analysis of the LHC signals and the corresponding SM backgrounds for our HLSM-SDR is fully beyond the current scope and will be presented elsewhere [43].

For comparisons with our HLSM-SDR, we further plot in Fig. 6 the cross section of $W^+_L W^-_L \rightarrow Z^0_L Z^0_L$ in the 4d Higgsless SM under the K-matrix (red-dashed) and Padé (blue-dashed) unitarizations. The shaded area with yellow color denotes the unitarity violation region. The cross section for the conventional 4d SM with a 600 GeV Higgs boson is also shown as a reference. In passing, we would like to note that the unitarity of $WW$ scattering in generic 4d technicolor-type theories was recently studied by Sannino and collaborators [44].

Finally, we clarify how to compare the SDR cross section computed above with what will be measured by the experiments. Note that the cross section $\sigma$ in different spacetime dimensions also has different mass-dimensions, equal to $[\sigma] = 2 - n$, while the experimentally measured cross section $\sigma_{\text{exp}}$ always has mass-dimension $-2$, as the detectors can record events only in 4d. So we need to convert the theory cross section $\sigma$ under the SDR to $\sigma_{\text{exp}}$, where the extra mass-dimensions of $\sigma$ should be scaled by the involved energy scale of the reaction, namely, the c.m. energy $E_{\text{cm}}$,

$$\sigma_{\text{exp}} = \sigma E_{\text{cm}}^{n-4}. \quad (3.23)$$

In Figs. 4-5, the unitarity bound with SDR (the blue region) has also been rescaled by $E_{\text{cm}}^{n-4}$ to keep its mass-dimension in accord with experimental measurements. In passing, we also note that our present unitarity analysis only depends on the total cross sections where the possible azimuthal angles are formally integrated out, as shown in (A.2)-(A.5), by the same way as the conventional dimensional regularization method for doing momentum-space integrals in $n < 4$ dimensions. The dependence of differential cross sections on the azimuthal angle in a theory with SDR relies on the detailed geometrical structure of the model. But this is irrelevant to our current unitarity study, and will be considered in future works.

4. Extension to Higgsful SM with SDR

As demonstrated in the previous section, the SDR can play a unique role to unitarize the $WW$ scattering at TeV scales in the absence of any Higgs boson. We note that the SDR can still occur at TeV scales even if there is a light Higgs boson. Especially, such quantum-gravity-induced SDR at TeV scales provides a natural solution to the “hierarchy problem” (naturalness problem) that plagues the Higgs boson in the conventional 4d SM. For the TeV-scale SDR, new physics effects induced by
the quantum gravity are expected in the low energy effective theory. So the Higgs boson can behave as non-SM-like and encode such new physics in its anomalous couplings with WW and ZZ gauge bosons, as well as in its self-couplings. In the conventional 4d SM with a non-SM Higgs boson [28], it was found that the WW scattering has non-canceled \(E^2\) behavior [29] in the TeV range, and can be probed at the LHC [29, 30]. But additional new physics is required to unitarize the non-canceled \(E^2\) contributions to the WW scattering. So, in this section we study an extension for the Higgsful SM with SDR (HFSM-SDR), which contains either a light or heavy Higgs boson, but with anomalous gauge couplings. We will show that under the SDR, the corresponding WW scattering cross sections become unitary at TeV scale, but exhibit different behaviors.

Let us start from the nonlinear realization of the SM Higgs boson \(h\) in terms of the 2\(\times\)2 matrix field \(\Phi\) as defined in (2.6), where \(h\) is a gauge-singlet. So we can extend (2.7) and write down the general effective Lagrangian for the Higgs sector (up to dimension-4 operators in 4d) [28],

\[
\mathcal{L}_H = \frac{1}{4} \left( v^2 + 2\kappa v h + \kappa' h^2 \right) \mathrm{tr} \left( (D_\mu \Sigma)^\dagger (D_\mu \Sigma) \right) + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} M_h^2 h^2 - \frac{\lambda_3}{3!} v h^3 + \frac{\lambda_4}{4!} h^4 ,
\]

(4.1)

where \(\Delta \kappa \equiv \kappa - 1\) and \(\Delta \kappa' \equiv \kappa' - 1\) represent the anomalous gauge couplings of the Higgs boson \(h\) with WW and ZZ. For cubic and quartic Higgs self-couplings, the conventional 4d SM predicts \(\lambda_3 = \lambda_4 = \lambda_0 = \frac{3M_h^2}{v^2}\), but the general effective Lagrangian (4.1) allows anomalous self-couplings \(\lambda_3, \lambda_4 \neq \lambda_0\). In parallel, we can also extend (2.11) and write down the general effective Lagrangian for the fermion-Higgs sector (up to dimension-4 operators in 4d),

\[
\mathcal{L}_{FH} = -\overline{\Psi}_L \Sigma \left[ M_f + \frac{M_f}{v} (1 + \Delta Y_f) h \right] \Psi_R + \text{h.c.} ,
\]

(4.2)

where \(\Delta Y_f = \text{diag}(\Delta y_1, \Delta y_2)\) denotes the anomalous Yukawa couplings of the Higgs boson. For the current analysis, we will focus on the anomalous gauge coupling \(\Delta \kappa \neq 0\) in (4.1). In the unitary gauge, we can infer from (4.1) the following anomalous cubic and quartic gauge-Higgs interactions,

\[
\Delta \mathcal{L}_H = \left( \Delta \kappa 2vh + \Delta \kappa' h^2 \right) \left[ \frac{2M_W^2}{v^2} W^+_\mu W^-\mu + \frac{M_Z^2}{v^2} Z_\mu Z^\mu \right] .
\]

(4.3)

For our present construction of the Higgsful SM with TeV-scale SDR, the new physics effects will be induced by the non-perturbative dynamics of quantum gravity above the transition scale \(\Lambda_{\text{UV}}\), and can be parameterized via anomalous couplings in the effective Lagrangian, as shown above.

We note that for the conventional 4d SM, besides the hierarchy problem [11], it further suffers constraints from the Higgs vacuum instability [45] and the triviality of Higgs self-interactions [12]. If such a 4d SM would be valid up to Planck scale, then the SM Higgs boson mass is bounded from below [45] and from above [40], 133 GeV \(\lesssim M_h \lesssim 180\) GeV [47]. Hence, any Higgs mass outside this window will indicate a non-standard Higgs boson in association with new physics. The Higgs boson in our present model under the TeV-scale SDR has anomalous couplings induced from quantum gravity and thus should belong to this case. So, for the numerical analysis below, we will set two sample
mass-values of such a non-standard Higgs boson: (i) $M_h = 125$ GeV for a light non-standard Higgs boson; and (ii) $M_h = 600$ GeV for a heavy non-standard Higgs boson. They are both consistent with the current Higgs boson direct searches at the LHC [13,14].

In the high energy region with $E_{\text{cm}} \gg M_W, M_h$, we note that the anomalous cubic interactions in (4.3) induce non-canceled $E_{\text{cm}}^2$ terms in the longitudinal gauge boson scattering amplitudes. For the processes (3.11a)-(3.11b), we find the non-canceled $O(E_{\text{cm}}^2)$ amplitudes,

$$T[W^+_L W^-_L \to W^+_L W^-_L] = -(\kappa^2 - 1) \frac{g^2 E_{\text{cm}}^2}{8 M_W^2} (1 + \cos \theta) + O(E_{\text{cm}}^0), \quad (4.4a)$$

$$T[W^+_L W^-_L \to \frac{1}{\sqrt{2}} Z^0_L Z^0_L] = -(\kappa^2 - 1) \frac{g^2 E_{\text{cm}}^2}{4 \sqrt{2} M_W^2} + O(E_{\text{cm}}^0), \quad (4.4b)$$

They are expected to eventually violate the unitarity as the increase of scattering energy $E_{\text{cm}}$ in the conventional 4d formulation. But, for our present SDR formulation given in Sec. 2-3, we find that such scattering amplitudes and the corresponding cross sections are unitarized at TeV scales in a rather universal way, due to the spontaneous dimensional reduction.

Let us take the process $W^+_L W^-_L \to Z^0_L Z^0_L$ as an example for demonstration. In Fig.7(a)-(b) we show the scattering cross section for this channel, where we consider a light Higgs boson with the sample mass $M_h = 125$ GeV and anomalous couplings $\Delta \kappa = \pm 0.3$. The red, purple and blue curves correspond to the SDR transition scale $\Lambda_{\text{UV}} = 4, 5, 6$ TeV, respectively, where we set the SDR index parameter $\gamma = 1.5$. The shaded area with yellow color represents the unitarity violation region in 4d. The shaded area with light-blue color shows the unitarity violation region for the HFSM-SDR, where the three blue dashed-curves, from bottom to top, display the corresponding (upper) unitarity bounds for $\Lambda_{\text{UV}} = 4, 5, 6$ TeV, respectively. In each plot, the upper black curve (marked by “SM+AC”) denotes the result of conventional 4d SM with the same Higgs mass $M_h = 125$ GeV and the same anomalous coupling $\Delta \kappa = \pm 0.3$. The lower black curve (marked by “SM”) is nearly flat and denotes the usual 4d-SM with the same Higgs mass but zero anomalous coupling $\Delta \kappa = 0$. This process contains an $s$-channel Higgs-exchange as in Fig.1(d), but it does not show up in Fig.7(a)-(b) because the light Higgs boson has its sample mass $(M_h = 125$ GeV) below the $ZZ$ threshold. In contrast, our SDR-unitarized cross sections have sizable excesses above the flat black curve of the 4d-SM with a 125 GeV light Higgs boson and $\Delta \kappa = 0$, but then fall off around the $2-4$ TeV region, consistent with the corresponding unitarity limits. We also note that the usual non-unitarized 4d-SM with nonzero anomalous coupling $\Delta \kappa \neq 0$ has its cross section monotonically increase and eventually violate unitarity around $E_{\text{cm}} = 2$ TeV for this scattering channel.

In Fig.8 we consider the same scattering process $W^+_L W^-_L \to Z^0_L Z^0_L$ as in Fig.7 but with a heavy Higgs boson. We set a sample mass $M_h = 600$ GeV for such a heavy Higgs boson. All the labels have the same meaning as in Fig.7 except the Higgs boson mass. As expected, the $s$-channel peak of this heavy Higgs boson does show up. The plot-(a) has a positive anomalous coupling $\Delta \kappa = 0.3$, and this makes the Higgs peak in our HFSM-SDR become higher and fatter. The predictions of the HFSM-SDR give excesses above the exact SM-Higgs peak for the SDR transition scale $\Lambda_{\text{UV}} = 4, 5, 6$ TeV.
Figure 7: The cross section of $W^+_L W^-_L \rightarrow Z^0 \bar{Z}^0_L$ as a function of the c.m. energy $E_{cm}$. The cross sections for the HFSM-SDR with nonzero anomalous coupling $\Delta \kappa$ are shown by red, purple, and blue curves, corresponding to the transition scale $\Lambda_{UV} = 4, 5, 6$ TeV, respectively. For comparisons, the cross sections for the conventional 4d-SM with $M_h = 125$ GeV (labeled by “SM”) and the 4d-SM with the same anomalous coupling (labeled by “SM+AC”) are shown by the black curves. The shaded area with yellow color denotes the unitarity violation region in 4d, while the shaded area with light-blue color shows the unitarity violation region for the HFSM-SDR, where the blue dashed-curves, from bottom to top, display the corresponding upper unitarity bounds for $\Lambda_{UV} = 4, 5, 6$ TeV, respectively.
Figure 8: Same as Fig. 7 for the cross section of $W_LW_L \rightarrow Z_0Z_0$, except that the Higgs boson mass is set to $M_h = 600$ GeV.
But these curves fall off around $E_{\text{cm}} = 2-4 \text{ TeV}$ region, respecting the corresponding unitarity limits. On the contrary, the naive non-unitarized 4d-SM with nonzero $\Delta \kappa \neq 0$ has monotonically increasing cross section and violates unitarity around 2 TeV. In plot-(b), we have a negative anomalous coupling $\Delta \kappa = -0.3$. As a result, the Higgs peak with anomalous coupling is generally lower and thinner than that of the exact 4d SM with $\Delta \kappa = 0$. Also, for our HFSM-SDR predictions, the tails on the right-hand side of the Higgs peak still show excesses above that of the exact 4d-SM with a 600 GeV heavy Higgs boson, but then fall below that of the 4d-SM around $2-4 \text{ TeV}$ region. This makes it harder to discriminate such a heavy Higgs boson with $\Delta \kappa < 0$ in the HFSM-SDR from that of the exact 4d SM. But, in this case the predictions of the HFSM-SDR still give significant deviations from that of the naive non-unitarized 4d SM with the same $\Delta \kappa < 0$ [marked by “SM+AC” in plot-(b)].

5. Conclusions and Discussions

Spontaneous dimensional reduction (SDR) [20] is a truly simple and attractive concept about the evolution of spacetime in high energies. It is well-motivated from many quantum gravity theories, which support that the dimension of spacetime is scale-dependent, approaching $n = 2$ in the ultra-violet (short distance) and $n = 4$ in the infrared (large distance). The world appears four-dimensional to us so far because we have been dealing with low energy phenomena at relatively large distances, and the nature of spacetime at short distance scales around $\mathcal{O}(\text{TeV}^{-1})$ is still awaiting explorations by the LHC.

In this work, we have studied the exciting possibility that the onset of the SDR happens at the TeV scale, under a proper conjecture of dimensional flow (2.16). We demonstrate that the TeV scale SDR can play a key role to restore the unitarity of $W_L W_L$ scattering as well as ensuring the renormalizability. We have constructed a consistent effective theory of the Higgsless standard model with SDR (HLSM-SDR). It nonlinearly realizes the electroweak gauge symmetry and its spontaneous breaking. The model becomes manifestly renormalizable in high energies by simple power counting. As being Higgsless, this model also simply has no so-called fine-tuning problem associated with the quadratically divergent radiative corrections to the Higgs scalar mass in the usual 4d SM. Then, we analyzed the partial wave unitarity of the $W_L W_L$ scattering (Fig.2), and computed the longitudinal $W_L W_L \to W_L W_L$ scattering cross sections in various channels (Figs.4-5). Our new mechanism of maintaining unitarity is realized by the phase-space reduction of final states under the SDR. These predictions can be tested at the LHC and discriminated from other mechanisms of the electroweak symmetry breaking, such as the conventional 4d SM with/without a Higgs boson and other ways of unitarization. We stress that our new predictions originate from the spontaneous reduction of spacetime dimensions at high energies due to the nonperturbative dynamics of quantum gravity, so they are universal and show up in all $WW$ scattering channels. This is an essential feature of our HLSM-SDR. To definitively test the HLSM-SDR and discriminate it from all other EWSB mechanisms requires full analysis for all possible $WW$ scattering channels at the LHC.
In fact, the HLSM-SDR provides a novel non-resonance unitarization of the WW scattering cross sections via SDR. As shown in Figs. 4-5, our predicted signals display excess above the conventional 4d SM (with a relatively light Higgs boson) as a rather broad “lump” around the $0.5 - 3 \text{ TeV}$ energy range; but as expected, the excess is not as substantial as a conventional sharp “resonance peak” like a 600 GeV Higgs boson of the SM in the same plot. So, according to the experiences with the non-resonance analyses at the LHC [41, 29, 30], it would be quite challenging for the LHC to measure such broad “lumps” in the WW scattering cross sections, and discriminate them from the SM backgrounds. The second phase of the LHC at 13–14 TeV with a higher integrated luminosity around 50 – 100 fb$^{-1}$ or above will be essential for this task. In the past, a non-resonance realization of the WW scattering was thought to be less exciting among all possible new physics scenarios, but our HLSM-DSR provides a truly exciting prospect for the non-resonant WW scattering despite the experimental challenge, because probing such non-resonant WW scattering will open the door to the quantum spacetimes with spontaneous dimensional reduction, as originated from the dynamics of quantum gravity.

We also note that all SM couplings will become super-renormalizable for the spacetime dimension $n < 4$. It is interesting to compare our HLSM-SDR with the existing extra dimensional models under Kaluza-Klein (KK) compactification. In contrast to the KK theories which lead to more space dimensions at higher energies, the SDR suggests that the spacetime dimensions become reduced rather than increased in the ultraviolet. For the KK-type theories, the unitarity of the $S$-matrix for a given longitudinal gauge boson scattering is restored due to a geometric Higgs mechanism under which the extra components of the higher dimensional gauge fields get eaten by the corresponding 4-dimensional KK gauge bosons [48], as characterized by the KK equivalence theorem [48]. In consequence, the $S$-matrix gets unitarized through the exchange of KK states which ensure the exact energy cancellations and thus the good high energy behavior. But including both elastic and inelastic scattering channels in the coupled channel analysis still violates the unitarity at a higher scale which must cut off the intrinsically nonrenormalizable KK theory [48]. Hence a KK theory could only delay the unitarity violation rather than fully removing it [48]. In contrast, for our present Higgsless SM model with SDR, the cross sections get unitarized through the universal suppression in the phase space. In addition, while any KK theory is necessarily nonrenormalizable, our HLSM-SDR naturally becomes renormalizable at high energies as ensured by the spontaneous dimensional reduction. We also note that the constraints on our model from the electroweak precision tests can be checked by calculating the loop corrections, which is expected to exhibit better UV behavior than the conventional 4d SM. For this, a method to quantize field theories with varying spacetime dimension is needed, which is fully beyond the current scope and will be considered elsewhere [43].

Finally, in Sec. 4, we further studied the Higgsful SM with the TeV-scale SDR, which provides a natural solution to the “hierarchy problem” [11] that troubles the conventional 4d SM. For the low

\footnote{For phenomenological studies on TeV-scale quantum gravity in a different context under the asymptotical safety scenario, some interesting papers appeared in Ref. [49].}
energy effective theory with operators up to dimension-4, the quantum-gravity-induced new physics effects are encoded in the anomalous gauge couplings of the non-standard Higgs boson, as well as the anomalous Higgs self-couplings. We found that the non-canceled $E^2$ contributions to the $WW$ scattering can be unitarized by the SDR at the TeV scale, and the scattering cross sections exhibit different behaviors which will be probed at the LHC. In Figs. 7-8, we showed two samples with anomalous gauge coupling $\Delta \kappa \neq 0$, for a light Higgs boson with mass $M_h = 125$ GeV and a heavy Higgs boson with mass $M_h = 600$ GeV, respectively.

A. Unitarity Conditions in $n$-Dimensional Spacetime

In this Appendix we systematically derive the unitarity conditions for the partial waves and cross sections in general $n$-dimensional spacetime. For simplicity and the current analysis, we consider the case in which all initial and final particles have the same spin. Extension to more general case will be given elsewhere.

For the cross section in general $n$ dimensions, we need to evaluate the corresponding phase space integral of the final states. For two-body final states, this integral is given by

$$\int d\Pi^{(n)}_2 |\mathcal{T}(E_{cm}, \theta)|^2 = \frac{1}{\rho_c} \int \frac{d^{n-1}p_1 d^{n-1}p_2}{(2\pi)^{n-2}(2\pi)^{n-1}} \frac{1}{2E_1 E_2} |\mathcal{T}(E_{cm}, \theta)|^2 (2\pi)^n \delta^{(n)}(p - p_1 - p_2)$$

$$= \frac{1}{\rho_c} \int \frac{d^{n-1}p_1}{(2\pi)^{n-1}} \frac{1}{2E_1 E_2} |\mathcal{T}(E_{cm}, \theta)|^2 (2\pi) \delta(E_{cm} - E_1 - E_2), \quad (A.1)$$

where $\rho_c = 1!(2!)$ is a symmetry factor corresponding to the final state particles being nonidentical (identical). To complete the remaining integral, we note that the integral measure with spherical coordinates in $(n-1)$-dimensions can be represented by

$$d^{n-1}p = p^{n-2} \sin^{n-3} \varphi_1 \sin^{n-4} \varphi_2 \cdots \sin \varphi_{n-3} dp d\varphi_1 d\varphi_2 \cdots d\varphi_{n-3} d\varphi_{n-2}. \quad (A.2)$$

The region of integration is given by $0 \leq p < \infty$, $0 \leq \varphi_i \leq \pi$ ($1 \leq i \leq n - 3$) and $0 \leq \varphi_{n-2} \leq 2\pi$.

The $2 \to 2$ scattering amplitude depends on c.m. energy $E_{cm}$ and a single scattering angle $\varphi_1$. Thus, we can always write it in the form $f = f(p, \varphi_1)$, and finish the integration over the rest of angular coordinates $\varphi_i$ ($1 \leq i \leq n - 3$) as follows,

$$\int d^{n-1}p f(p, \varphi_1) = \int_0^\infty dp \int_{\varphi_1}^\infty d\varphi_1 \sin^{n-2} \varphi_1 \int_{\varphi_1}^\pi d\varphi_2 \cdots \int_{\varphi_{n-3}}^\pi d\varphi_{n-2} f(p, \varphi_1)$$

$$= \int_0^\infty dp \int_{\varphi_1}^\infty d\varphi_1 \frac{\sqrt{\pi} \Gamma\left(\frac{n-i-2}{2}\right)}{\Gamma\left(\frac{n-i-1}{2}\right)} \int_0^{\varphi_1} d\varphi_2 \sin^{n-3} \varphi_2 f(p, \varphi_1)$$

$$= \int_0^\infty dp \frac{2\pi^{n/2-1}}{\Gamma\left(\frac{n-i}{2} - 1\right)} \int_0^{\varphi_1} d\varphi_1 \sin^{n-3} \varphi_1 f(p, \varphi_1). \quad (A.3)$$

Substituting this result back to the phase space integral (A.1), we derive,

$$\int d\Pi^{(n)}_2 |\mathcal{T}(E_{cm}, \theta)|^2 = \frac{1}{\rho_c (2\pi)^{n-2}} \frac{p^{n-2}}{2E_1 E_2} \frac{m_1^2}{E_1^2} \frac{m_2^2}{E_2^2} \frac{2\pi^{n/2-1}}{\Gamma\left(\frac{n-i}{2} - 1\right)} \int_0^\pi d\varphi \sin^{n-3} \varphi_1 |\mathcal{T}(E_{cm}, \theta)|^2, \quad (A.4)$$

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where we have replaced the remaining angular variable (scattering angle) $\varphi_1$ by the more familiar notation $\theta$. In the c.m. frame we have, $E_1 = E_2 = E_{cm}/2$ and $p_1 = p_2 = p$, where $p_1 \equiv |\vec{p}_1|$ and $p_2 \equiv |\vec{p}_2|$. So, we have,

$$
\int d\Pi^2_{2(n)} |T(E_{cm}, \theta)|^2 = \frac{1}{\rho_n 2^{n-1} \pi^{n/2-1} \Gamma \left( \frac{n}{2} \right)} \frac{p^{n-3}}{E_{cm}} \int_0^\pi d\theta \sin^{n-3} \theta |T(E_{cm}, \theta)|^2,
$$

(A.5)

which holds for $n \geq 3$. For our purpose, this is not quite enough, since we also need to evaluate the integral over $d^{n-1}p$ with $2 \leq n < 3$. In the case of $2 \leq n < 3$, there is no meaning to talk about the angular dependence of the scattering amplitude, as commented in Sec. 3, since the scattering in spacetime dimensions lower than 3 can be achieved only in the forward or backward direction. So, we can complete the phase space integral,

$$
\int d\Pi^2_{2(n)} \left[ |T(E_{cm}, 0)|^2 + |T(E_{cm}, \pi)|^2 \right] = \frac{C}{\rho_n 2^{n-1} \pi^{n/2-1} \Gamma \left( \frac{n}{2} \right)} \frac{p^{n-3}}{E_{cm}} \left[ |T(E_{cm}, 0)|^2 + |T(E_{cm}, \pi)|^2 \right], \quad (2 \leq n < 3),
$$

(A.6)

where the coefficient $C$ needs an explanation. We note that applying both (A.5) and (A.6) to $n = 3$ would yield different results if simply set $C = 1$, due to the improper normalization of phase space in (A.6) for $2 \leq n < 3$. For consistency, we can normalize the coefficient $C$ by imposing the condition that the cross section is a smooth function of $E_{cm}$. So, we have,

$$
C = \frac{1}{\pi \left[ |T(E_{cm}, 0)|^2 + |T(E_{cm}, \pi)|^2 \right]} \int_0^\pi d\theta |T(E_{cm}, \theta)|^2.
$$

(A.7)

Next, we analyze the unitarity of partial-waves in $n$-dimensions with $n > 3$. From unitary condition $\mathcal{T} \dagger \mathcal{T} = 2 \Im \mathcal{T}$, we take the matrix element on both sides and insert a complete set of intermediate states into its left-hand side. This leads to the condition,

$$
\int d\Pi^2_{2(n)} |T_{el}(2 \rightarrow 2)|^2 + \sum_N \int d\Pi^2_{N(n)} |T_{inel}(2 \rightarrow N)|^2 = 2 \Im \mathcal{T}_{el}(2 \rightarrow 2),
$$

(A.8)

where $T_{el}(2 \rightarrow 2)$ represents the amplitude for $2 \rightarrow 2$ elastic scattering, $T_{inel}(2 \rightarrow N)$ the amplitudes for $2 \rightarrow N$ ($N \geq 2$) inelastic scattering, and the summation on $N$ runs over all possible intermediate states. From (A.8), we have,

$$
- \int d\Pi^2_{2(n)} |T_{el}(2 \rightarrow 2)|^2 + 2 \Im \mathcal{T}_{el}(2 \rightarrow 2) = \sum_N \int d\Pi^2_{N(n)} |T_{inel}(2 \rightarrow N)|^2 \geq 0,
$$

(A.9)

where the right-hand-side (RHS) is nonnegative as it is a sum of squares. Let us expand the elastic amplitude in terms of partial waves for $n > 3$,

$$
T_{el}(E_{cm}, \theta) = \lambda_n E_{cm}^{1-n} \sum_\ell \frac{1}{N_\ell} C_{\ell}^{\nu}(1) C_{\ell}^{\nu}(\cos \theta) a_{\ell}^{el}(E_{cm}),
$$

(A.10a)

$$
a_{\ell}^{el}(E_{cm}) = \frac{E_{cm}^{n-4}}{\lambda_n C_{\ell}^{\nu}(1)} \int_0^\pi d\theta \sin^{n-3} \theta C_{\ell}^{\nu}(\cos \theta) T_{el}(E_{cm}, \theta),
$$

(A.10b)
The function \( C_\nu(x) \) is the Gegenbauer polynomial of order \( \nu \) and degree \( \ell \), and satisfies the following orthogonal condition,

\[
\int_{-1}^{1} dx \ (1 - x^2)^{\nu - \frac{1}{2}} C_\nu(x) C_\nu'(x) = N_\nu^\nu \delta_{\ell\ell'}, \quad (A.11a)
\]

\[
\sum_\ell N_\nu^\nu C_\nu(x) C_\nu(y) = (1 - x^2)^{\nu - \frac{1}{2}} \left( 1 - y^2 \right)^{\nu - \frac{1}{2}} \delta(x - y). \tag{A.11b}
\]

From \((A.10)\), we work out the phase space of the two-body final states,

\[
\int d\Pi_2^{(n)} |T_{el}(2 \rightarrow 2)|^2 = \frac{2 \lambda_n}{E_{cm}} \left( \frac{2 \rho_e}{E_{cm}} \right)^{n-3} E_{cm}^{4-n} \sum_\ell \frac{|C_\nu(1)|^2}{N_\nu^\nu} |a_\ell^{el}|^2. \tag{A.12}
\]

Then we can expand the inequality \((A.9)\) also in terms of partial waves,

\[
0 \leq - \int d\Pi_2^{(n)} |T_{el}(2 \rightarrow 2)|^2 + 2 \Re m T_{el}(2 \rightarrow 2)
= \frac{2 \lambda_n E_{cm}^{4-n}}{\rho_e} \sum_\ell \frac{|C_\nu(1)|^2}{N_\nu^\nu} \left[ \left( \frac{2 \rho_e}{E_{cm}} \right)^{n-3} |a_\ell^{el}|^2 + \rho_e C_\nu^\nu(\cos \theta) |a_\ell^{el}| \right]. \tag{A.13}
\]

The coefficient outside the brackets on the right-hand side is positive definite. So \((A.13)\) results in,

\[
0 \leq - \left( \frac{2 \rho_e}{E_{cm}} \right)^{n-3} |a_\ell^{el}|^2 + \rho_e C_\nu^\nu(\cos \theta) \Im a_\ell^{el}. \tag{A.14}
\]

At high energies where the masses of initial/final state particles are negligible, we have \( E_{cm} \simeq 2 \rho_e \).

Thus, we deduce,

\[
0 \leq - |a_\ell^{el}|^2 + \rho_e \Im a_\ell^{el} = \frac{\rho_e^2}{4} - (\Re a_\ell^{el})^2 - \left( \Im a_\ell^{el} - \frac{\rho_e}{2} \right)^2. \tag{A.15}
\]

where we have made use of the property of Gegenbauer polynomial, \( C_\nu^\nu(1) \gtrsim |C_\nu(x)| \) for \(|x| \lesssim 1\).

From this, we obtain the familiar unitarity condition for each partial wave amplitude,

\[
(\Re a_\ell^{el})^2 + \left( \Im a_\ell^{el} - \frac{\rho_e}{2} \right)^2 \leq \frac{\rho_e^2}{4}. \tag{A.16}
\]

In particular, we have,

\[
|\Re a_\ell^{el}| \leq \frac{\rho_e}{2}, \quad |a_\ell^{el}| \leq \rho_e. \tag{A.17}
\]

Similarly, we can derive a unitarity bound for partial wave amplitudes of \( 2 \rightarrow 2 \) inelastic scattering.

This can be done by expanding the inelastic amplitude \( T_{\text{inel}}(2 \rightarrow 2) \equiv T_{\text{inel}}(E_{cm}, \theta) \) in terms of partial waves,

\[
T_{\text{inel}}(E_{cm}, \theta) = \lambda_n E_{cm}^{4-n} \sum_\ell \frac{1}{N_\nu^\nu} C_\nu^\nu(1) C_\nu^\nu(\cos \theta) a_\ell^{\text{inel}}(E_{cm}). \tag{A.18}
\]

In the high energy region where the masses of initial/final state particles are negligible, we infer,

\[
\int d\Pi_2^{(n)} |T_{\text{inel}}(2 \rightarrow 2)|^2 = \frac{2 \lambda_n E_{cm}^{4-n}}{\rho_i} \sum_\ell \frac{|C_\nu^\nu(1)|^2}{N_\nu^\nu} |a_\ell^{\text{inel}}|^2, \tag{A.19}
\]
where $\rho_i = 1!(2!)$ is a symmetry factor corresponding to the inelastic final state particles being nonidentical (identical). So, from (A.9), (A.13)-(A.15) and (A.19), we have,

$$\frac{2\lambda_n E_{\text{cm}}^{2-n}}{\rho_i} \sum_{\ell} \frac{[C_{\ell}(1)]^2}{N_{\ell}^n} |a_{\ell}^{\text{inel}}|^2 \leq \sum_N \int d\Pi_2^{(n)} |T_{\text{inel}}(2 \to N)|^2$$

$$= -\int d\Pi_2^{(n)} |T_{\text{el}}(2 \to 2)|^2 + 2\Im T_{\text{el}}(2 \to 2) \leq \frac{2\lambda_n E_{\text{cm}}^{2-n}}{\rho_e} \sum_{\ell} \frac{[C_{\ell}(1)]^2}{N_{\ell}^n} \left( \frac{\rho_e^2}{4} \right). \quad (A.20)$$

This leads to the unitarity condition for inelastic partial waves,

$$|a_{\ell}^{\text{inel}}| \leq \frac{\sqrt{\rho_e \rho_e}}{2}. \quad (A.21)$$

On the other hand, if we consider that the amplitude is dominated by $s$-wave, then we can also derive unitarity bounds for both elastic and inelastic scattering cross sections from (A.9), (A.12) and (A.17),

$$\sigma_{\text{el}}(2 \to 2) = \frac{1}{2E_{\text{cm}}^2} \int d\Pi_2^{(n)} |T_{\text{el}}(2 \to 2)|^2 = \frac{\lambda_n |a_0^{\text{el}}|^2}{\rho_e N_0^2} E_{\text{cm}}^{2-n} \leq \frac{\lambda_n \rho_e}{N_0^2} E_{\text{cm}}^{2-n}, \quad (A.22)$$

and

$$\sigma_{\text{inel}}(2 \to N) = \frac{1}{2E_{\text{cm}}^2} \sum_N \int d\Pi_2^{(n)} |T_{\text{inel}}(2 \to N)|^2 = \frac{1}{2E_{\text{cm}}^2} \left[ -\int d\Pi_2^{(n)} |T_{\text{el}}(2 \to 2)|^2 + 2\Im T_{\text{el}}(2 \to 2) \right]$$

$$\leq \frac{\lambda_n}{\rho_e} E_{\text{cm}}^{2-n} \sum_{\ell} \frac{[C_{\ell}(1)]^2}{N_{\ell}^n} \left[ \frac{\rho_e^2}{4} - (\Re a_{\ell}^{\text{el}})^2 - (\Im a_{\ell}^{\text{el}})^2 - \left( \frac{\rho_e}{2} \right)^2 \right]$$

$$\leq \frac{\lambda_n \rho_e}{4N_0^2} E_{\text{cm}}^{2-n}. \quad (A.23)$$

Note that both of the above unitarity bounds are smooth functions of spacetime dimension $n$ when $n \geq 2$ although the original derivation is done with $n > 3$. The validity of this extrapolation is guaranteed by the uniqueness of the analytic continuation, if we view the unitarity bound as an analytic function of the spacetime dimension $n$. So, in summary, we have unitarity conditions for both elastic and inelastic cross sections,

$$\sigma_{\text{el}}(2 \to 2) \leq \frac{\lambda_n \rho_e}{N_0^2} E_{\text{cm}}^{2-n}, \quad \sigma_{\text{inel}}(2 \to N) \leq \frac{\lambda_n \rho_e}{4N_0^2} E_{\text{cm}}^{2-n}. \quad (A.24)$$

In 4-dimensions, we have $n = 4$ and $\nu = \frac{1}{2}$. So, these inequalities reduce to $\sigma_{\text{el}}(2 \to 2) \leq 16\pi \rho_e / E_{\text{cm}}^2$ and $\sigma_{\text{inel}}(2 \to N) \leq 4\pi \rho_e / E_{\text{cm}}^2$, respectively, in accord with the literature \[10, 42\].

**Note Added:**

After the submission of this paper to arXiv:1112.1028 on December 5, 2011, the Atlas and CMS collaborations announced preliminary results of the SM Higgs searches at the LHC (7 TeV) on December 13, 2011 \[40\], which excluded wider mass-ranges of the SM Higgs boson, leaving a small window within $116 - 128$ GeV or above $468 - 525$ GeV. Some excesses of events around the 125 GeV
Higgs mass were found, but it is inconclusive and still consistent with statistical fluctuations \cite{50}. As we commented in Sec. 4, a SM Higgs boson outside the mass-range $133 - 180 \ \text{GeV}$, definitely hints new physics beyond the SM. We note that such a light Higgs boson with mass $M_h = 125 \ \text{GeV}$ was analyzed in our Sec. 4 and Fig.7 in the context of Higgsful SM with SDR (HFSM-SDR), where the new physics effects originate from the non-perturbative quantum gravity above the UV scale $\Lambda_{\text{UV}} = \mathcal{O}(5 \ \text{TeV})$, and generate the anomalous Higgs couplings as in (4.1)-(4.3). Our Fig.7 showed how such a non-standard Higgs boson (125 GeV) can be discriminated from the conventional 4d-SM via high energy $WW$ scattering experiments at the LHC. If the upcoming LHC runs in 2012 do not confirm this preliminary excess of events \cite{50}, then our Higgsless SM with SDR (HLSM-SDR) in Sec. 2-3 provides an attractive construction of new physics beyond the 4d-SM, which can be definitively probed during the next phase of the LHC (14 TeV).

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