Antimagnets: controlling magnetic fields with superconductor–metamaterial hybrids

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Abstract. Magnetism is very important in various areas of science and technology, ranging from magnetic recording through energy generation to trapping cold atoms. Physicists have managed to master magnetism—to create and manipulate magnetic fields—almost at will. Surprisingly, there is at least one property that has been elusive until now: how to ‘switch off’ the magnetic interaction of a magnetic material with existing magnetic fields without modifying them. Here we introduce the antimagnet, a design that conceals the magnetic response of a given volume from its exterior, without altering the external magnetic fields, in some respects analogous to recent theoretical proposals for cloaking electromagnetic waves with metamaterials. However, unlike these devices, which require extreme material properties, our device is feasible and needs only two kinds of available materials: superconductors and isotropic magnetic materials. Antimagnets may have applications in magnetic-based medical techniques such as magnetic resonance imaging or in reducing the magnetic signature of vessels or planes.

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1. Introduction

Cloaking a region in space from electromagnetic waves seemed scientifically impossible until very recently. Then, Pendry et al [1] and Leonhardt [2], using the concepts of metamaterials and transformation optics, theoretically designed an electromagnetic cloak, which would render a given volume ‘invisible’ to electromagnetic waves. Such a device required extreme values of magnetic permeability $\mu$ and electrical permittivity $\epsilon$. Although some experimental results presented partial cloaking in special cases [3–5], no complete broadband cloak has been experimentally achieved until now [6].

In 2007, Wood and Pendry [7] introduced the concept of magnetic cloaking. They showed that in the dc case (electromagnetic waves in the limit of zero frequency), for which the electrical and magnetic effects decouple, a magnetic cloak for concealing static magnetic fields without disturbing the external field needed a material with anisotropic and position-dependent $\mu$ values, smaller than 1 in one direction and larger than 1 in the perpendicular direction (see figure 1(a) and equation (1) for the case of a cylinder). A $\mu < 1$ could be achieved by arrays of superconducting plates [7–9], whereas $\mu > 1$ could be obtained with ferromagnetic materials. However, no method has been presented to achieve the required position-dependent values in perpendicular directions simultaneously in a real case. Because of these difficulties, a magnetic cloak has not yet been designed or fabricated.

This eventual magnetic cloaking would have not only scientific interest but also important technological applications, since magnetic fields are fundamental to many everyday technologies, and in many of them it is necessary to have a precise spatial distribution of the magnetic field, which should not be perturbed by magnetic objects—not only by magnets but also by any material containing iron or steel, for example. Is it therefore possible to build a cloak for static magnetic fields while—very importantly—using only ingredients that are practical and available? In this paper, we demonstrate the affirmative answer to this question, by exploiting the properties of two worlds: metamaterials and superconductors.

2. Definition of the antimagnet

At this point we want to redefine our goal into a more precise and even more ambitious one: instead of a magnetic cloak—null interior field and external field unaffected—we want to design
Figure 1. Magnetic permeabilities and behavior of exact and approximate magnetic cloaks. In panel (a), the radial dependence of radial and angular permeabilities for a cylinder of inner radius $a$ and outer radius $b = 2a$; curves are the values for the exact cloak (equation (1)) and straight lines are the approximate constant values in panel (c). In the rest of the panels, magnetic field lines for (b) the exact cloak, (c) the approximate cloak with constant values $\mu_\theta = 6$ and $\mu_\rho = 1/6 = 0.1667$ and (d) the approximate cloak with constant values $\mu_\theta = 10$ and $\mu_\rho = 1/10$.

an antimagnet, defined as a material forming a shell that encloses a given region in space while fulfilling the following two conditions.

(i) The magnetic field created by any magnetic element inside the inner region—e.g. a permanent magnet—should not leak outside the region enclosed by the shell.

(ii) The system formed by the enclosed region plus the shell should be magnetically undetectable from outside (no interaction—e.g. no magnetic force—with any external magnetic sources).

In this paper, we consider the case of a cylindrical cloak; the results can be extended to other geometries. It was demonstrated in [1, 7, 10] that different sets of radially dependent values of radial and angular permeabilities, $\mu_\rho$ and $\mu_\theta$, yield magnetic cloak behavior. An example is

$$
\mu_\rho = \frac{\rho - a}{\rho}, \quad \mu_\theta = \frac{\rho}{\rho - a},
$$

(1)
represented in figure 1(a), with the resulting field profile shown in figure 1(b). Materials with such fine-tuned values of anisotropic permeabilities do not exist. It is easily seen from equation (1) that $\mu_\theta = \infty$ and at the same time $\mu_\rho = 0$ at the inner surface ($\rho = a$). This makes any practical implementation very difficult [1]. In particular, condition (i) could not be fulfilled because the field from any internal source would leak to the exterior. Therefore, a strong (or an exact) validation of condition (i) for our antimagnet requires a radically new approach. This is what we present in this paper. Two important new ideas would be required in order to achieve the design of a practical antimagnet fulfilling conditions (i) and (ii): firstly, the introduction of a new scheme for cloaking, requiring homogeneous (although anisotropic) parameters, which would involve a new transformation of space, and secondly, the addition of an inner superconducting layer, which in the static case considered here would ensure that $\mu = 0$. In a final step, we present a general means of implementing the antimagnet design in practical cases.

3. A cloaking design with homogeneous parameters

As a first step, we want to study whether the values of $\mu$ derived for the magnetic cloak, which are impractical because they involve a fine-tuned continuous variation of anisotropic permeabilities, can be modified into a simplified scheme. Can a cloaking behaviour (in particular, no field distortion in an exterior region) be produced with simpler permeability arrangements—even with a homogeneous permeability value? In the following, we demonstrate that a whole family of homogeneous magnetic systems exist with the property that they produce a null effect on an externally applied magnetic field. Each of these systems—we call them homogeneous cloaks—is a cylindrical shell composed of a homogeneous (i.e. properties are the same in all the material points) anisotropic material.

Consider one such cylinder, infinitely long along the $z$-axis with radius $b$ and a central coaxial hole of radius $a < b$. This cylinder is made of a magnetic material with homogeneous radial and angular relative permeabilities, $\mu_\rho$ and $\mu_\theta$, respectively. A uniform external field $H_a$ is applied along the $y$-direction. We want to find the analytical expression for the field $H$ in all regions with the condition that the applied field is not modified by the presence of the cylinder, in the region outside it.

Inside the material there are neither free charges nor free currents. Then, $\nabla \cdot B = 0$ and $\nabla \times H = 0$ and given that neither $\mu_\rho$ nor $\mu_\theta$ depend on position, we obtain

$$\mu_\rho \rho \frac{\partial H_\rho}{\partial \rho} + \mu_\rho H_\rho + \mu_\theta \frac{\partial H_\theta}{\partial \theta} = 0,$$

$$\rho \frac{\partial H_\theta}{\partial \rho} + H_\theta - \frac{\partial H_\rho}{\partial \theta} = 0.$$  

The boundary conditions for these equations are set by considering that both $B$ and $H$ are equal to the applied field values at $\rho = b$ and that there is a uniform field inside the hole. Then, by imposing continuity of the normal component of $B$ and the tangential component of $H$, we obtain

$$H_\rho(b) = H_a \frac{1}{\mu_\rho} \sin \theta,$$

$$H_\theta(b) = H_a \cos \theta.$$
The solution for the magnetic field inside the ring ($a \leq \rho \leq b$) is found to be

$$H_\rho(\rho, \theta) = H_a \mu_\theta \left( \frac{\rho}{b} \right)^{1+\mu_\theta} \sin \theta, \quad (6)$$

$$H_\theta(\rho, \theta) = H_a \left( \frac{\rho}{b} \right)^{1+\mu_\theta} \cos \theta. \quad (7)$$

It is important to remark that this solution is valid only when the condition $\mu_\rho \mu_\theta = 1$ is fulfilled; otherwise, the problem has no solution. Therefore, we demonstrate that the condition of null external distortion of magnetic field is directly fulfilled as long as permeabilities $\mu_\rho$ and $\mu_\theta$ are the inverse of each other ($\mu_\rho \mu_\theta = 1$).

It is interesting to note that the presented scheme for a homogeneous cloaking implicitly involves a space transformation different from that of Wood and Pendry [7], which involved expanding a zero-dimensional point into a finite sphere (in our cylindrical case, expanding a central one-dimensional (1D) line into a cylindrical region, that from $\rho' = 0$ to $\rho' = a$). This transformation basically consists of, firstly, compressing the space from $\rho = R_0$ to $\rho = b$ into the space occupied by $\rho' = a$ and $\rho' = b$ using a radially symmetric function, and secondly expanding the space from $\rho = 0$ to $\rho = R_0$ into the space from $\rho' = 0$ to $\rho' = a$, using another radially symmetric function. $R_0$ is a positive constant ($R_0 < a$). A detailed description and discussion of the transformation (which may be used also in the case of electromagnetic waves) will be presented elsewhere.

An example of such a homogeneous cloak is shown in figure 1(c), with the values $\mu_\theta = 6$ and $\mu_\rho = 1/6 = 0.1667$ (straight lines in figure 1(a)), and in figure 1(d), that with $\mu_\theta = 10$ and $\mu_\rho = 1/10$ is shown; in both, the condition $\mu_\rho \mu_\theta = 1$ is enough to ensure that there is no distortion of the magnetic field outside the shell. Interestingly, it can be demonstrated that the null distortion is also obtained for a non-uniformly applied magnetic field. As to the field in their interior, we see that increasing $\mu_\theta$ while fixing the ratio $\mu_\rho \mu_\theta = 1$ causes the magnetic field lines to be concentrated nearer the external surface of the shell. This means that such homogeneous cloaks do not have an exactly zero magnetic field inside, although they do so approximately—the larger the value of $\mu_\theta$, the closer to zero is the internal field.

In spite of achieving magnetic cloaking, these homogeneous cloaks are not antimagnets, because the magnetic field created by a source in its interior will leak to the exterior. To avoid this, we introduce the second key step in our idea: placing a superconducting layer at the inner surface of the cloak. Because $\mu = 0$ for an ideal superconductor, it follows directly from the magnetostatic boundary conditions at the inner boundary of the superconductor that condition (i) is fulfilled. Introducing such a superconducting layer does not substantially modify the property of cloaking, as long as $\mu_\rho$ is sufficiently larger than 1, because in this case, as seen in figures 1(c) and 1(d), magnetic field is excluded from the central part, so a superconducting layer would not practically interact with the field created by external sources. In this way, an antimagnet design is being outlined: an inner superconducting layer and an outer homogeneous shell. However, this scheme alone cannot yet solve our goal of a feasible antimagnet, because the material in the outer shell, even though it would have a constant permeability, would require fine-tuned anisotropic values (with $\mu_\rho = 1/\mu_\theta$); such materials are not available.
4. Antimagnet design and a demonstration of its properties

Therefore, another important step is needed, and again help from metamaterial concepts will be used. Can we transform our homogeneous cloak with uniform and anisotropic parameters into one made with realistic materials? We have attained this by modifying the homogeneous shell with constant anisotropic permeability into a discrete system of alternating layers of two different kinds: one type consisting simply of a uniform and isotropic ferromagnetic material with constant permeability ($\mu_{\rho}^{\text{FM}} = \mu_{\theta}^{\text{FM}} > 1$) and a second type having a constant value of radial permeability $\mu_{\rho}^{\text{SC}}$ ($\mu_{\rho}^{\text{SC}} < 1$) and $\mu_{\theta}^{\text{SC}} = 1$. The first (isotropic) kind of layers could be a superparamagnet (i.e. ferromagnetic nanoparticles embedded in a non-magnetic medium [11]). The second kind could be realized with arrays of superconducting plates (the precise value tunable by changing distances between the plates) [7–9]; one such array has actually been constructed and tested [9]. Now we need to find the values of permeability for the two kinds of layers. To do this, we start with the values for a homogeneous cloak described above (for example, that in figure 1(c)) and then apply the following method: we select the value of the angular permeability ($\mu_{\theta} = 6$) for the isotropic layers ($\mu_{\rho}^{\text{FM}} = \mu_{\theta}^{\text{FM}} = 6$) and then reduce the value of $\mu_{\rho}^{\text{SC}}$ in the superconducting layers to a lower value than $1 / \mu_{\rho}^{\text{FM}}$ (1/6) to compensate for the larger value of $\mu_{\rho}^{\text{FM}}$ in the isotropic layers.

To demonstrate the validity of the method, we show in figure 2(e) the calculated response for a system with an inner superconducting layer surrounded by ten outer alternating layers, half of them with $\mu_{\rho}^{\text{FM}} = \mu_{\theta}^{\text{FM}} = 6$ and the other half with $\mu_{\rho}^{\text{SC}} = 0.104$ (and $\mu_{\theta}^{\text{SC}} = 1$); the scheme cloaks a uniform static magnetic field with impressive quality. This means that our goal of using only realistic material is fulfilled. It is only left to confirm that the resulting scheme indeed acts as an antimagnet and it does so for any applied field. This is demonstrated on the panels of figure 2, where we show the magnetic field of a single small magnet (basically a dipole field), the field of two such magnets (now the field has changed greatly because of the interaction) and, finally, how surrounding one of the magnets with the antimagnet makes the field outside it unaffected—i.e. equal to the field of a single magnet. Besides the constant field and two dipole cases, we also show in figure 2(d) another example corresponding to the field created by a current line. In all cases the antimagnet is the same. We thus confirm that the antimagnet performs as such for any applied field configuration. We also show in figure 2(f) that even when there is a missing portion of the antimagnet, a reasonable shielding inside with a small field modification outside can be obtained.

In order to confirm that the antimagnet design is a good solution—although not an exact one—we can compare the antimagnet performance with that for a single shell of a homogeneous and isotropic material with high permeability that has traditionally been used to achieve magnetic shielding. In figure 3, we show that even for a uniformly applied field the distortion created by the magnetic shell is extremely large compared with that created by the antimagnet, which is practically negligible.

5. Discussion

The presented solution with ten layers and $\mu_{\theta}^{\text{FM}} = 6$ is not unique; starting from a different $\mu_{\theta}^{\text{FM}} = \mu_{\rho}^{\text{FM}}$ value (10, for example, as in figure 1(d)) and even with a different number of layers, one can follow the described method and obtain similarly good antimagnet properties. Actually, although good behavior is obtained for ten layers as shown above, increasing the number of...
Figure 2. Display of the antimagnet behavior. The magnetic properties of an antimagnet are visualized as follows. First, we show in panel (a) the magnetic field lines for a uniformly magnetized cylindrical magnet. When a second magnet is added (b) the magnetic field is distorted owing to the magnetic interaction between the two magnets. When one of the two magnets is covered by the antimagnet (c), the magnetic field outside the region enclosed by the antimagnet is the same as that for a single magnet (as in panel (a)), demonstrating the two antimagnet properties: the field of the inner magnet does not leak outside the antimagnet shell and the field external to the antimagnet remains unperturbed independently of what is contained in its interior. In panels (d) and (e), the same antimagnet is shown to behave as such in the field of a current-carrying wire and a uniformly applied magnetic field, respectively. Panel (f) shows that rather good antimagnet behavior is maintained even when the shell is not closed. The antimagnet is composed of an inner superconducting layer (µ = 0) and ten alternating outer layers of two kinds: one with \( \mu_{\text{FM}} = \mu_{\text{SC}} = 6 \) and the other with \( \mu_{\text{SC}} = 0.104 \).

Layers may be convenient when there is an applied field very spatially inhomogeneous or when we require a certain practical tolerance in the values of the permeabilities (more on this in figure 4).

We would like to make some remarks with practical consequences. First, we have considered that the superconductor is characterized by \( \mu = 0 \), which is a good approximation for superconductors in the Meissner state. This would limit, in principle, the applicability of the antimagnet to applied fields less than the (thermodynamic or lower) critical field of the superconductor. However, a type-II superconductor (like most high-temperature ones) with a high critical-current density can produce a response very similar to a Meissner response (with currents circulating mainly in a thin layer at the surface) up to much larger fields [12].

Finally, constructing the described antimagnet with ten layers may be feasible, but difficult in practice. However, the same strategy can be applied to a much more simple design, by...
Figure 3. Comparison of an antimagnet with a high-permeability isotropic and homogeneous shielding layer. In (a), an antimagnet is simulated when an external uniform field is applied. In (b), an isotropic and homogeneous shell with the same size and high permeability (numerically $\mu = 10^6$) is simulated interacting with a uniform field, showing that it creates a large distortion outside the shell. The region in which the difference between the total field and the applied field exceeds 1% extends until $\sim 10b$, in contrast to the case of the antimagnet, which creates a practically null distortion.

Figure 4. Optimizing the number of layers for the antimagnet. Response of antimagnets with—from left to right—10, 20 and 30 layers to the field created by a near uniformly magnetized cylindrical magnet. The permeabilities of the magnetic layers are in all cases $\mu_0^{FM} = \mu_\rho^{FM} = 6$, whereas those of the superconducting ones are $\mu_\rho^{SC} = 0.104, 0.128$ and 0.136 for the 10-, 20-, and 30-layer cases, respectively. The light pink region indicates the zones for which the difference between the total field and the dipole field exceeds 1% and the darker pink region indicates those for which it exceeds 3%.

substituting all superconducting layers (except the central one) with layers with $\mu = 1$ (the permeability of air or any non-magnetic material such as plastic). This change requires tuning the value of $\mu$ in the magnetic layers to a different value. An example of that is shown
Figure 5. Simplified antimagnet with air ($\mu = 1$) layers. Response of the antimagnet of ten layers (as in figure 1) for a uniform applied field, a dipole-like field and the field of a current-carrying wire (top row, from left to right). For comparison, the same results are plotted in the bottom row for the simplified antimagnet case in which the permeability $\mu_{\rho}^{SC}$ of the superconducting layers—except the central one, which has $\mu = 0$—has been set as 1 (and $\mu_{\rho}^{FM} = \mu_{\rho}^{FM} = 2.405$ in this case). The pink regions indicate the zones for which the difference between the total field and the externally applied field exceeds 1%. Both antimagnet and simplified designs work well for a uniform applied field, but the region of distortion of the external field is large in the latter.

In figure 5. This simplified scheme works as well as the described antimagnet when there is a uniform magnetic field, whereas its response gets worse with increasing applied field inhomogeneity. Moreover, in the latter case, increasing the number of layers does not bring significant improvement of the antimagnet properties. Therefore, this simplified scheme with only a central superconducting layer and layers with homogeneous $\mu$ alternated with air may work well for applications in which the applied field has a small spatial variation.

6. Conclusions

In summary, we have presented a method to design hybrid superconductor–metamaterial devices that prevent any magnetic interaction with its interior while keeping the external magnetic field
unaffected. Two important key ideas have been needed for achieving our goal: the design of a simplified cloak with homogeneous parameters, corresponding to a new space transformation, and the placement of a superconducting layer at the inner surface. Such an antimagnet would be passive and, provided that the superconductors are in the Meissner state [8] and that the isotropic layers have a negligible coercivity (as if using superparamagnetic materials), also lossless. The strategy for antimagnet design presented in this paper can be adapted to other geometries (e.g. spheres) or even to other forms of manipulating magnetic fields, such as magnetic field concentrators [10, 13]. Antimagnet devices may bring important advantages in applications such as reducing the magnetic signature of vessels or allowing patients with pacemakers or cochlear implants to use medical equipment based on magnetic fields, such as magnetic resonance imaging [14] or transcranial magnetic stimulation [15]. Moreover, by tuning one parameter such as the working temperature of the device—below or above the critical temperature of the superconductor, for example—one could ‘switch off and on’ magnetism in a certain region or material at will, opening up room for some novel applications.

7. Methods

The simulations have been performed with Comsol Multiphysics software, using the electromagnetics module (magnetostatics). All the results presented correspond to an infinitely long cylinder (with translational symmetry), with the outer radius equal to twice the inner radius \((b = 2a)\). We have checked that other dimensions yield similar results. Unless otherwise indicated, we have considered ten outer layers plus an inner superconducting layer, all with the same thickness. The outermost layer is of the magnetically isotropic type. The superconductor has been simulated assuming \(\mu = 0\). All permeabilities are understood to be relative permeabilities. The pink region in figures 2–4 denoting the region in which the magnetic induction \(B\) differs from the externally applied magnetic induction \(B_{\text{ext}}\) is calculated as the points in space following the condition

\[
\sqrt{(B_x - B_{x,\text{ext}})^2 + (B_y - B_{y,\text{ext}})^2} > 0.01.
\]

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