Cosmic Solutions in the Einstein-Weinberg-Salam Theory and the Generation of Large Electric and Magnetic Fields

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In the SU(2)\textsubscript{L} \times U(1)\textsubscript{Y} standard electroweak theory coupled with the Einstein gravity, new topological configurations naturally emerge, if the spatial section of the universe is globally a three-sphere (S\textsuperscript{3}) with a small radius. The SU(2)\textsubscript{L} gauge fields wrap the space nontrivially, producing homogeneous but anisotropic space. As the universe expands, large electric and magnetic fields are produced. The electromagnetic field configuration is characterized by the Hopf map.

1. STANDARD MODEL IN GRAVITY

It has been firmly established that the standard model of electroweak interactions based on the SU(2)\textsubscript{L} \times U(1)\textsubscript{Y} gauge symmetry describes the Nature at low energies, and the evolution of the universe is well described by Einstein’s theory of gravity. Given these two facts, it is natural to ask if there was strong interplay between the Einstein gravity and electroweak interactions in the early universe. We report that indeed such strong interplay may have existed, provided the universe is spatially closed.\cite{1}

2. TOPOLOGICAL CONFIGURATIONS

The Lagrangian density is given by
\[ \mathcal{L} = \frac{1}{16\pi G} (R - 2\Lambda) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda (\Phi^\dagger \Phi - \frac{v^2}{2})^2 \]
where \( D_\mu \Phi = (\partial_\mu - i \frac{g}{2} \tau^a A_\mu^a - i \frac{g'}{2} B_\mu) \Phi \). \( \Lambda \) is the effective cosmological constant at the time the universe was wrapped with nontrivial field configurations discussed below.

Recall that the SU(2) Maurer-Cartan 1-forms \( \sigma^j = \sigma^j_\mu dx^\mu \) satisfy \( d\sigma^j = \epsilon^{jkl} \sigma^k \wedge \sigma^l \). The metric of the space-time we consider is
\[ ds^2 = -e^0 \otimes e^0 + \sum_{j=1}^3 e^j \otimes e^j \]
where \( e^0 = dt \), \( e^j = a_j(t) \sigma^j \).

It gives a homogeneous but anisotropic space. The SU(2) and U(1) gauge fields and the Higgs fields take
\[ A = \frac{1}{2g} \sum_{j=1}^3 f_j(t) \sigma^j \tau^j, \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v(t) \end{pmatrix} \]
\[ B = h(t) \sigma^3 \]
where \( \tau^j \)'s are Pauli matrices. The Ricci tensors and the energy-momentum tensors in these tetrads become diagonal and independent of the spatial coordinates. Both the Einstein equations and field equations are consistently solved.

The spatially isotropic universe where all scale factors \( a_j(t) \)'s are equal to each other is not permitted in the presence of the U(1) gauge interactions. It still is legitimate to set \( a_1 = a_2 = a \) and \( f_1 = f_2 \) when the Higgs field and the U(1) gauge field are aligned as in (2). The ansatz of this type has been investigated before in the pure Einstein-Yang-Mills theory in \cite{2}.
3. LOCAL MINIMUM OF THE POTENTIAL

The configuration (2) is topological in the sense that the SU(2) gauge fields wrap the space non-trivially if \( a_j(t) \)'s are sufficiently small. However it is not absolutely stable as it can be continuously deformed to the trivial configuration \( A = B = 0 \). To see how it may become important we insert the ansatz (2) into the potential:

\[
V = \frac{\lambda}{4} (v^2 - v_0^2)^2 + \frac{v^2}{8} \left( \frac{2f_1^2}{a_1^2} + \frac{(f_3 - g'h)^2}{a_3^2} \right) + \frac{(2f_3 - f_3')^2}{2g^2a_1^4} + \frac{2f_3^2(f_3 - 2)^2}{2g^2a_1^2a_3^2} + \frac{2h^2}{a_1^2} . \tag{3}
\]

The global minimum of the potential is always located at \( v = v_0, f_1 = f_3 = h = 0 \). When the scale factors \( a_1 \) and \( a_3 \) are sufficiently small, there appears a new local minimum located at \( 1.5 < f_1, f_3 \leq 2 \) and \( 0 \leq v < v_0 \).

It could well be that the very early universe was settled near the local minimum of the potential. The local minimum has positive energy density so that the Einstein equations drive the universe to expand. As \( a_j \)'s become larger, the terms of \( O(a^{-2}) \) and \( O(a^{-4}) \) in \( V \) become less important. The barrier separating the local minimum from the global minimum quickly disappears.

4. ELECTROMAGNETIC FIELDS

After the barrier disappears, the field configuration starts to roll down the hill of the potential. It is in this period of the evolution of the universe that some of interesting physical consequences emerge. One salient feature of the electroweak theory is that there remains the electromagnetic \( U(1)_{EM} \) gauge invariance after the Higgs field develops a nonvanishing vacuum expectation value, i.e. \( v \neq 0 \). The electromagnetic fields are given by

\[
F_{EM} = \frac{h_{EM}}{a_3} e^0 \wedge e^3 + \frac{2h_{EM}}{a_1^2} e^1 \wedge e^2 ,
\]

\[
h_{EM}(t) = \frac{1}{\sqrt{g''^2 + g'''} \left( \frac{g'h + g'f_3}{g} \right)} . \tag{4}
\]

In the potential (3), the terms of \( O(a^{-4}) \) quickly become irrelevant as \( a_j \)'s increase. The terms of \( O(a^{-2}) \) are independent of \( h_{EM} \), reflecting the \( U(1)_{EM} \) invariance. In other words, the potential has approximately flat direction along \( f_1 = f_3 - g'h = 0 \). During the expansion the field configuration heads for this shallow valley of the potential, but not necessarily for the absolute minimum. \( h_{EM} \) approaches a nonvanishing value, producing large electromagnetic fields. Typical behavior is depicted in fig. 1.

A typical value for the generated magnetic fields is about \( (50 \text{GeV})^2 \). It dies away as \( a(t)^{-2} \) as the universe further expands. The final value of \( h_{EM} \) depends on the initial condition, but the fact \( h_{EM}(t = \infty) \neq 0 \) does not. In fig. 2 the final values of \( h_{EM} \) and \( f_3 \) are plotted as functions of the
initial value of $h_{EM}$. The figure shows how generic it is to generate large electromagnetic fields.

5. FIELDS AS THE HOPF MAP

As is evident in eq. (4), the magnitude of the generated electromagnetic fields is independent of the spatial position. Both the electric and magnetic fields are in the $e^3 = a_3 \sigma^3$ direction where $\sigma^3$ varies as the spatial position. They define smooth, regular vector fields without any singularity.

The space is topologically isomorphic to $S^3$. At each instant $t$, the vector $\vec{E}(x)$ or $\vec{B}(x)$ is on $S^2$ specified in the field space by $|\vec{E}|$ or $|\vec{B}| = \text{constant}$. The vector field defines a map from $S^3$ onto $S^2$. It is nothing but the Hopf map.

This is how it becomes possible to have spatially homogeneous universe with nonvanishing $U(1)$ gauge field strengths. If we were living in two-dimensional space, the gauge field strengths with constant magnitude would necessarily induce singularities, either in the form of sources/sinks or vortices.

6. A COLD ERA OF THE EARLY UNIVERSE

When does the field configuration (2) become important in the history of the universe? In the standard scenario of the early universe, the temperature effect plays an important role. It modifies the effective potential and the time-evolution. What we have in mind here is an era preceding the hot universe which continuously evolves to the current universe. Suppose that at one instant the universe was very small and cold, and the gauge and Higgs fields assumed the nontrivial configuration under discussions. Driven by an effective cosmological constant the universe underwent inflation, and large electromagnetic fields were generated. Eventually the inflation stopped and the universe was reheated to the temperature about $\left(\frac{\Lambda}{8\pi G}\right)^{1/4}$. The universe continued to expand by the radiation dominance since then.

When large electromagnetic fields were generated, quarks and leptons must have been copiously created. As the field configuration was not invariant under $CP$, there may have been asymmetry in the baryon and lepton numbers as well. Such asymmetry could be washed out in the later stage of the evolution of the universe, but a part of it may have survived.

In this report we have assumed the homogeneity of the space. In reality inhomogeneity has to be generated either classically or quantum mechanically. The generated magnetic flux, which is taken to be homogeneous in our ansatz (2), may be squeezed in space. Could such magnetic flux have remnant in the present universe? We do not have an answer at the moment.

REFERENCES

1. H. Emoto, Y. Hosotani and T. Kubota, Prog. Theoret. Phys. 108 (2002) 157.
2. Y. Hosotani, Phys. Lett.B147 (1984) 44; A. Hosoya and W. Ogura, Phys. Lett. B225 (1989) 117; S.J. Rey, Nucl. Phys. B336 (1990) 146; M.S. Volkov and D.V. Gal’tsov, Phys. Rep. 319 (1999) 141.
3. G.W. Gibbons and A.R. Steif, Phys. Lett. B320 (1994) 245.
4. J.D. Barrow, Phys. Rev. D55 (1997) 7451; A.D Dolgov, [hep-ph/010293].