STRANGE MAGNETISM

T. R. HEMMERT\textsuperscript{a}, U.-G. MEI\ss\textsc{ner}, S. STEININGER
FZ Jülich, IKP (Theorie), Jülich, Germany

We present an analytic and parameter-free expression for the momentum dependence of the strange magnetic form factor of the nucleon $G_M^{(s)}(Q^2)$ and its corresponding radius which has been derived in Heavy Baryon Chiral Perturbation Theory. We also discuss a model-independent relation between the isoscalar magnetic and the strange magnetic form factors of the nucleon based on chiral symmetry and SU(3) only. These limits are used to derive bounds on the strange magnetic moment of the proton from the recent measurement of $G_M^{(s)}(Q^2 = 0.1\text{GeV}^2)$ by the SAMPLE collaboration.

1 Introduction

There has been considerable experimental and theoretical interest concerning the question: How strange is the nucleon? Despite tremendous efforts, we have not yet achieved a detailed understanding about the strength of the various strange operators in the proton. A dedicated program at Jefferson Laboratory preceded by experiments at BATES (MIT) and MAMI (Mainz) is aimed at measuring the form factors related to the strange vector current. In fact, the SAMPLE collaboration has recently reported the first measurement of the strange magnetic moment of the proton. To be precise, they give the strange magnetic form factor at a small momentum transfer, $G_M^{(s)}(q^2 = -0.1\text{ GeV}^2) = +0.23 \pm 0.37 \pm 0.15 \pm 0.19$ nuclear magnetons (n.m.). The rather sizeable error bars document the difficulty of such type of experiment. On the theoretical side, there is as much or even more uncertainty. For example, the spread of the theoretical predictions for the strange magnetic moment, $-0.8 \leq \mu_p^{(s)} \leq 0.5$ n.m. underlines clearly the above mentioned statement. In the following we report about a parameter–free prediction for the momentum dependence of the nucleons’ strange magnetic (Sachs) form factor based on the chiral symmetry of QCD solely. In addition, a leading order model–independent relation between the strange and the isoscalar magnetic form factors has been derived, which allows to give an upper bound on the momentum dependence of $G_M^{(s)}(Q^2)$. These two different results can then be combined to extract a range for the strange magnetic moment of the proton from the SAMPLE measurement of the form factor at low momentum transfer.

\textsuperscript{a}Talk given BARYONS 98, Bonn, Sept. 22-26,1998.

email: th.hemmert@fz-juelich.de
2 Strangeness Vector Current

The strangeness vector current of the nucleon is defined as

$$\langle N | \bar{s} \gamma_\mu s | N \rangle = \langle N | \bar{q} \gamma_\mu (\lambda^0/3 - \lambda^8/\sqrt{3}) q | N \rangle = (1/3)J^{0}_\mu - (1/\sqrt{3})J^{8}_\mu , \quad (1)$$

with $q = (u, d, s)$ denoting the triplet of the light quark fields and $\lambda^0 = I(\lambda^a)$ the unit (the $a = 8$ Gell–Mann) SU(3) matrix. Assuming conservation of all vector currents, the corresponding singlet and octet vector current for a spin–1/2 nucleon can then be written as

$$J^{0,8}_\mu = \bar{u}_N(p') \left[ F^{(0,8)}_1(q^2)\gamma_\mu + F^{(0,8)}_2(q^2)\frac{i\sigma_\mu q^\nu}{2m_N} \right] u_N(p) . \quad (2)$$

Here, $q_\mu = p'_\mu - p_\mu$ corresponds to the four–momentum transfer to the nucleon by the external singlet ($v^{(0)}_\mu = v_\mu \lambda^0$) and the octet ($v^{(8)}_\mu = v_\mu \lambda^8$) vector source $v_\mu$, respectively. The strangeness Dirac and Pauli form factors are defined via:

$$F^{(s)}_{1,2}(q^2) = \frac{1}{3} F^{(0)}_{1,2}(q^2) - \frac{1}{\sqrt{3}} F^{(8)}_{1,2}(q^2) , \quad (3)$$

subject to the normalization $F^{(s)}_{1,2}(0) = S_B$, with $S_B$ the strangeness quantum number of the baryon ($S_N = 0$) and $F^{(s)}_{2}(0) = \kappa^{(s)}_B$ with $\kappa^{(s)}_B$ the (anomalous) strangeness moment. In the following we concentrate on the “magnetic” strangeness form factor $G^{(s)}_M(q^2)$, which in analogy to the (electro)magnetic Sachs form factor is defined as

$$G^{(s)}_M(q^2) = F^{(s)}_1(q^2) + F^{(s)}_2(q^2) \quad (4)$$

and for which chiral perturbation theory (CHPT) gives the most interesting predictions.

3 The strange magnetic form factor

To obtain the complete strange magnetic form factor in ChPT one only has to consider the diagrams where the external singlet/octet source couples directly to the nucleon as well as the one where the octet source couples to the intermediate kaon cloud, the pion and the $\eta$ cloud do not contribute to this order. For the proton ($p$) and the neutron ($n$) one finds

$$G^{(s)}_M(Q^2) = G^{(s)}_M(Q^2) = G^{(s)}_M(Q^2) = \mu^{(s)}_N + \frac{\pi m_N M_K}{(4\pi F_\pi)^2} \frac{2}{3} (5D^2 - 6DF + 9F^2) f(Q^2) , \quad (5)$$
with $Q^2 = -q^2$. The strange magnetic moment $\mu_N^{(s)}$ cannot be directly predicted in ChPT due to the influence of poorly known singlet counterterms. However, to $O(p^3)$ in ChPT the momentum dependence is given entirely in terms of well-known parameters and the analytic function

$$f(Q^2) = -\frac{1}{2} + \frac{4 + Q^2/M_K^2}{4\sqrt{Q^2/M_K^2}} \arctan\left(\frac{\sqrt{Q^2}}{2M_K}\right). \quad (6)$$

$f(Q^2)$ is shown in Fig.1. For small and moderate $Q^2$, it rises almost linearly with increasing $Q^2$.

![Figure 1: The function $f(Q^2)$ for small and moderate momentum transfer squared.](image)

4 The isoscalar connection

An SU(3) analysis of the magnetic isoscalar ($I = 0$) form factor of the nucleon $G_{M}^{I=0}(Q^2)$ shows that to $O(p^3)$ it can be expressed via the same function $f(Q^2)$ given in Eq.(6). We can therefore eliminate $f(Q^2)$ from both expressions and derive a model-independent relation between the isoscalar magnetic form factor $G_{M}^{I=0}(q^2)$ of the nucleon and the strange magnetic form factor

$$G_{M}^{(s)}(Q^2) = \mu_N^{(s)} + \mu_s - G_{M}^{I=0}(Q^2) + O(p^4), \quad (7)$$

with $\mu_s = 0.88$ n.m. being the isoscalar nucleon magnetic moment. This relation is exact to $O(p^3)$ in SU(3) heavy baryon CHPT. Possible corrections in higher orders can be calculated systematically. This relation again does not constrain $G_{M}^{(s)}(0) = \mu_N^{(s)}$, but makes new predictions on its $Q^2$-dependence. Utilizing the empirical dipole parameterization for $G_{M}^{I=0}(Q^2)$ instead of the functional form $f(Q^2)$ of Eq.(6) one now obtains the $Q^2$-dependence shown in Fig.2 for vanishing $\mu_N^{(s)}$. Given that there are also non–strange contributions in the physical isoscalar magnetic form factor, which will start to manifest at order $q^4$, we consider Eq.(7) as an upper bound on the strange magnetic form factor.
Figure 2: $G_M^{(s)}(Q^2)$ derived from the isoscalar magnetic form factor with $\mu_N^{(s)} = 0$.

5 Summary

In summary, we have derived two novel relations which constrain the momentum dependence of the strange magnetic form factor in the low energy region. The first one is based on the observation that to one loop order in three flavor ChPT, the strange form factor picks up a momentum dependence which is free of unknown coupling constants. The second one rests upon the observation that the isoscalar magnetic form factor calculated in SU(3) also acquires a momentum dependence which can be related to the one of the strange magnetic form factor. One can now utilize the $Q^2$-dependence from the two bounds, Eqs. (5,7), to extract the strange magnetic moment from the SAMPLE result for the strange magnetic form factor. For $Q^2 = 0.1 \text{GeV}^2$, the correction is -0.06 and -0.20, respectively, i.e. for the mean value of ref. [1] we get

$$\mu_p^{(s)} = 0.03 \ldots 0.18 \text{n.m. ,}$$

which even for the upper value is a sizeable correction. Clearly, these numbers should only be considered indicative since (a) the current experimental errors are bigger than the correction and (b) higher order corrections to the relations derived here should be worked out. Finally, we note that the G0 collaboration at TJNAF will also probe this particular range of momentum transfer.

We would like to thank the organizers of Baryons98 for providing us with the opportunity to present this work to the physics community.

References

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