Molecular description of $X(3872)$ in effective field theory

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Abstract.
The unusual mass of $X(3872)$ strongly suggests that it could be a molecular state of $D^0\bar{D}^{0*}$ mesons. The tiny binding energy of this molecular state introduces an energy scale which is much smaller than the mass of the lightest particle, the pion, whose exchange can provide binding. We have employed effective field theory to describe $X(3872)$ and investigated the existence of bound states in the $D^0\bar{D}^0$ and $D^{*0}\bar{D}^{*0}$ channels.

1. Introduction
The $X(3872)$ was first observed by Belle collaboration through the decay channel $X \rightarrow J/\Psi \pi^+\pi^-$ and subsequently confirmed by CDF, D0, and Babar collaborations. The available experimental information indicates that $J^{PC} = 1^{++}$ for $X(3872)$. Many theoretical investigations have been done trying to interpret $X(3872)$ [1, 2]. The most tempting interpretation is the molecular interpretation due to the fact that the mass of $X$ lies very close to the $D^0\bar{D}^{0*}$ threshold of 3871.2 MeV. The description of $X(3872)$ using the techniques of effective field theory is possible due to the multitude of scales present in QCD. The extreme smallness of the binding energy

$$E_b = (m_{D^0} + m_{D^{0*}}) - M_X = -0.6 \pm 1.1 \text{MeV}$$

suggests that this state can play the role of the deuteron [3] in meson-meson interactions. The nature of this state allows us to use methods similar to those developed for the description of the deuteron [4, 5], with the added benefit of heavy quark symmetry. The tiny binding energy of this state introduces an energy scale which is much smaller than the mass of the lightest particle, the pion, whose exchange can provide binding. Then a suitable effective Lagrangian describing such a system contains only heavy-meson degrees of freedom with interactions approximated by local four-boson terms constrained only by the symmetries of the theory. While the predictive power of this approach is somewhat limited, it is completely model independent. For instance, possible existence of a molecular state in $D^{*0}\bar{D}^0$ channel does not imply a molecular state in the $D^{*0}\bar{D}^0$ channel as we will see.

2. The effective Lagrangian
The effective Lagrangian needed to describe the molecular states of heavy mesons is invariant under both heavy quark and chiral symmetries. Thus this theory is written in terms of superfield
doublet combining the pseudoscalar and vector mesons:
\[ H_a^{(Q)} = \frac{1 + \beta}{2} [P_a^{(Q)} \gamma_\mu - P_a^{(Q)} \gamma_5], \quad \bar{H}^{(Q)\mu} = \gamma^0 H_a^{(Q)\mu} \gamma^0 \] (2)

These fields have the usual transformation properties under both heavy-quark spin symmetry and SU(2)_V flavor symmetry and describe heavy mesons with a definite velocity v. The effective Lagrangian \( \mathcal{L} \) can be written as [6]
\[ \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 \] (3)
where
\[ \mathcal{L}_2 = -i \text{Tr} \left[ \bar{H}^{(Q)} v \cdot D H^{(Q)} - \frac{1}{2m_P} \text{Tr} [\bar{H}^{(Q)} D^2 H^{(Q)}] + \lambda_{2} \frac{\sigma}{m_P} \right] \text{Tr} [\bar{H}^{(Q)} H^{(Q)\mu} H^{(Q)\mu}]
+ \frac{ig}{2} \text{Tr} [\bar{H}^{(Q)} H^{(Q)} \gamma_\mu \gamma_5 [\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger]] \] (4)
describes the strong interactions of the heavy mesons \( P \) and \( P^* \) (\( P = B, D \)) containing one heavy quark \( Q \) [7] where \( g \) is the \( P P^* \pi \) coupling, and \( D^\mu_{ab} = \delta_{ab} \partial^\mu - (1/2) (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) \) is the covariant derivative. The pseudo-Goldstone fields are realized nonlinearly through \( \xi = e^{i M/f}, \) where \( M \) is
\[ \tilde{M} = \left( \begin{array}{cc} \frac{1}{\sqrt{2}} \pi^0 & \pi^+ \\ -\pi^- & -\frac{1}{\sqrt{2}} \pi^0 \end{array} \right), \] (5)
and \( f \simeq 132 MeV \) is the pion decay constant.

The four-meson local interactions piece is
\[ \mathcal{L}_4 = -C_1 \frac{3}{4} \text{Tr} [\bar{H}^{(Q)} H^{(Q)} \gamma_\mu] \text{Tr} [\bar{H}^{(Q)} H^{(Q)} \gamma_\mu] + \frac{C_2}{4} \text{Tr} [\bar{H}^{(Q)} H^{(Q)\mu} \gamma_5] \text{Tr} [\bar{H}^{(Q)} H^{(Q)\mu} \gamma_5] \] (6)

Evaluating the trace yield for the \( P \bar{P}^* \) sector
\[ \mathcal{L}_{4,PP^*} = -C_1 P^{(Q)\dagger} P^{(Q)} P_{\mu}^{(Q)\dagger} P_{\mu}^{(Q)} + C_1 P_{\mu}^{(Q)\dagger} P_{\mu}^{(Q)} P^{(Q)\dagger} P^{(Q)} - C_1 P_{\mu}^{(Q)\dagger} P_{\mu}^{(Q)} P^{(Q)\dagger} P^{(Q)} + C_2 P_{\mu}^{(Q)\dagger} P_{\mu}^{(Q)} P_{\mu}^{(Q)\dagger} P_{\mu}^{(Q)} + \ldots \] (7)

Similarly, one obtains the lagrangian piece governing the interactions of \( P \) and \( \bar{P} \),
\[ \mathcal{L}_{4,PP} = C_1 P^{(Q)\dagger} P^{(Q)} P^{(Q)\dagger} P^{(Q)}, \] (8)

Clearly, one cannot relate the existence of the bound state in the \( P \bar{P}^* \) and \( P \bar{P} \) channels, as the properties of the latter will depend on \( C_1 \) alone, not a linear combination of \( C_1 \) and \( C_2 \).

3. Properties of bound states
The lowest-energy bound state of \( P \) and \( \bar{P}^* \) is an eigenstate of charge conjugation.
\[ |X_\pm\rangle = \frac{1}{\sqrt{2}} [ |D^* \bar{D}^-\rangle \pm |D \bar{D}^+\rangle] \] (9)

To find the bound state energy of X(3872) with \( J^{PC} = 1^{++} \), we shall look for a pole of the transition amplitude \( T_{++} = \langle X_+ | T | X_+ \rangle \).
The transition amplitudes describe $D^0 \bar{D}^0$ scattering are

$$T_{11} = \langle D^* \bar{D} | T | D^* \bar{D} \rangle, \quad T_{12} = \langle D^* \bar{D} | T | D \bar{D}^* \rangle, \quad T_{21} = \langle D \bar{D}^* | T | D^* \bar{D} \rangle, \quad T_{22} = \langle D \bar{D}^* | T | D \bar{D}^* \rangle$$

(10)

Including the resummation of loop contributions \([4]\), these transition amplitudes satisfy a system of Lippman-Schwinger equations \([6]\) depicted in Fig. 1. To diagonalize that system of equations one can put them in an algebraic matrix form

$$\begin{pmatrix}
T_{11} \\
T_{12} \\
T_{21} \\
T_{22}
\end{pmatrix} = \begin{pmatrix}
-C_1 & C_2 & 0 & 0 \\
C_2 & -C_1 & 0 & 0 \\
0 & 0 & C_2 & -C_1 \\
0 & 0 & C_2 & -C_1
\end{pmatrix} \begin{pmatrix}
T_{11} \\
T_{12} \\
T_{21} \\
T_{22}
\end{pmatrix} + i\tilde{A} \begin{pmatrix}
-C_1 & C_2 & 0 & 0 \\
C_2 & -C_1 & 0 & 0 \\
0 & 0 & C_2 & -C_1 \\
0 & 0 & C_2 & -C_1
\end{pmatrix},$$

(11)

and after diagonalization we get

$$T_{++} = \langle X_+ | T | X_+ \rangle = \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) = \frac{\lambda}{1 - i\lambda\tilde{A}}$$

(12)

where $\lambda = -C_1 + C_2$, and $\tilde{A}$ is a divergent integral

$$\tilde{A} = \frac{1}{4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p^2/2m_D^* + q_0 - \Delta - q^2/2m_D^* + i\epsilon)(\bar{p}^2/2m_D - q_0 - \bar{q}^2/2m_D + i\epsilon)}$$

(13)

where $\Delta$ is the mass difference between $D_0^{*0}$ and $D^0$ and $\bar{p}$ is the momentum of one of the mesons in the center of mass system. We chose to define a renormalized $\lambda_R$ within the $MS$ subtraction scheme in dimensional regularization to remove the divergence in the previous integral. Hence the transition amplitude becomes

$$T_{++} = \frac{\lambda_R}{1 + (i/8\pi)\lambda_R\mu_{DD^*} |\bar{p}| \sqrt{1 - 2\mu_{DD^*} \Delta/p^2}}$$

(14)
from which the position of the pole of the molecular state on the energy scale can be read off
\[ E_{Pole} = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3} - \Delta \] (15)
where \( \mu_{DD^*} \) is the reduced mass of the \( D^0 D^{0*} \) system.

This is the amount of energy we must subtract from the constituent mass of the system, in order to calculate the mass
\[ M_X = 2m_D - E_{Pole} \] (16)
Recalling the definition of the binding energy we finally get
\[ E_b = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3} \] (17)
Assuming \( E_b = 0.5MeV \), which is one sigma below the central value \[2\], and the experimental values of the masses \[8\], we obtain
\[ \lambda_R \simeq 8.4 \times 10^{-4}MeV^{-2} \] (18)
The small binding energy of the \( X(3872) \) state implies that the scattering length can be calculated from
\[ a_D = \sqrt{(2\mu_{DD^*}E_b)^{-1}} \] (19)
yielding a numerical value \( a_D = 6.3fm \). Consequently universality implies that the leading order wave function of \( X(3872) \) is known
\[ \psi_{DD^*}(r) = e^{-r/a_D} \sqrt{\frac{2}{\pi a_D r}} \] (20)
which can be used to predict the production and decay properties of \( X(3872) \).

### 4. Some Remarks
One can also study \( D^0 \bar{D}^0 \) and \( B^0 \bar{B}^0 \) states in light of the previous analysis. In this case there is only one coupling constant \( C_1 \) to describe the \( D^0 \bar{D}^0 \) state which leads to solving only one Lippman-Schwinger equation. The resulting binding energy is then
\[ E_b = \frac{256\pi^2}{C_1^2 \lambda_R^4 \mu_{DD^*}^3} \] (21)
Since \( C_1 \) and \( C_2 \) are generally not related to each other, the existence of a bound state in the \( D^{0*} \bar{D}^0 \) channel does not dictate the properties of a possible bound state in the \( D^0 \bar{D}^0 \) or \( B^0 \bar{B}^0 \) channels. Notice also that \( C_1 < 0 \) is not in general excluded, thus allowing the possibility of a bound state in the \( D^0 \bar{D}^0 \) system, but not requiring it.

One can also place some constraints on the renormalized values of \( C_1 \) and \( C_2 \) assuming that the orthogonal state \( J^P \sigma = 1^+- \) is not bound which is consistent with all the experimental observations. The amplitude orthogonal to \( T_{++} \) is
\[ T_{--} = \langle X_-|T|X_- \rangle = \frac{1}{2}(T_{11} - T_{12} + T_{21} - T_{22}) = \frac{\hat{\lambda}_R}{1 - i\lambda_R A_R} \] (22)
where \( \hat{\lambda}_R = -C_1 - C_2 \), \( T_{--} \) does not have a pole corresponds to a bounds state if \( C_1 + C_2 > 0 \). The exclusion of the \( C = -1 \) state together with the assumption of the existence of the \( C = +1 \) state limits the \( (C_1, C_2) \) parameter space as shown in Fig. 2.
Figure 2. The coupling constant $C_2$ is plotted vs. $C_1$. The lightly shaded area shows the region of parameter space allowed by postulating the existence of a $J^{PC} = 1^{++}$ bound state with $E_b = 0.1$ MeV, while excluding the orthogonal bound state with $C = -1$. The darker area becomes allowed in addition if we assume $E_b = 0.5$ MeV.

5. Conclusion
We have used an effective field theory approach in the analysis of the likely molecular state $X(3872)$. Binding interaction was described by contact terms invariant under both chiral and heavy quark symmetries. The flexibility of this description allows us to ignore the details of the interaction and to concentrate on its effects, namely a shallow bound state and a large scattering length. We found that the existence of the bound state in the $D^0 \bar{D}^0$ channel does not in general exclude a possibility of a bound state in the $D_s^0 \bar{D}_s^0$ system, but does not require it. Studies of this state [9] should shed more light on the properties of QCD bound states.

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