Many-body effects in nuclear structure

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We calculate, for the first time, the state-dependent pairing gap of a finite nucleus (120Sn) diagonalizing the bare nucleon-nucleon potential (Argonne v14) in a Hartree-Fock basis (with effective $k-$mass $m_κ ≈ 0.7 m$), within the framework of the BCS approximation including scattering states up to 800 MeV above the Fermi energy to achieve convergence. The resulting gap accounts for about half of the experimental gap. We find that a consistent description of the low-energy nuclear spectrum requires, aside from the bare nucleon-nucleon interaction, not only the dressing of single-particle motion through the coupling to the nuclear surface, to give the right density of levels close to the Fermi energy (and thus an effective mass $m^* ≈ m$), but also the renormalization of collective vibrational modes through vertex and self-energy processes, processes which are also found to play an essential role in the pairing channel, leading to a long range, state dependent component of the pairing interaction. The combined effect of the bare nucleon-nucleon potential and of the induced pairing interaction arising from the exchange of low-lying surface vibrations between nucleons moving in time reversal states close to the Fermi energy accounts for the experimental gap.

In the study of finite many-body systems such as the atomic nucleus with its rich variety of quantal size effects, structural properties, and fluctuations, the central problem has been to identify the appropriate degrees of freedom for describing the phenomena encountered. The complementary concepts referring to the independent motion of the individual nucleons and the collective behaviour of the nucleus as a whole provide the elementary modes of excitation needed to describe the system. The unifying picture emerging from the interweaving of these degrees of freedom and described in terms of nuclear field theory (NFT) [2-6] based on the particle-vibration coupling (for other related approaches, see e.g. [7-10]) and tailored upon QED [11-12], has been applied to a number of schematic models and realistic situations [13-21] and its validity demonstrated. It thus provides a natural framework to assess the role different degrees of freedom play in the nuclear structure.

An important subject presently under intensive study concerns the characterization of an eventual long range component of the pairing interaction in nuclei [22-24]. In what follows we use NFT to assess the importance the exchange of vibrations between pairs of nucleons moving in time reversal states have in building up pairing correlations in nuclei, taking also into account self-energy and vertex corrections (i.e. avoiding using approximations which, in condensed matter literature are connected with the so-called Migdal theorem, cf. ref. [25] and refs. therein).

To this scope, we study the quasiparticle and vibrational spectrum of odd- and even- isotopes of single-closed-shell nuclei, where all the richness of the single-particle and collective degrees of freedom are fully expressed, avoiding the extra complications of static deformations and associated rotations. The spectra of the $^{A}_{3}$Sn isotopes, in particular those with mass number $A=119,120$ and $121$, with their abundance of detailed experimental information, provide an excellent laboratory where to test the importance of the residual pairing interaction and its relation to self-energy processes. To be remembered that BCS theory connects the mass enhancement factor $λ$ associated with the $ω-$mass ($m_ω = m(1 + \lambda)$, cf. e.g. [26]) with the pairing gap $\Delta ≈ 2\omega_c exp(-1/\lambda)$, $\omega_c$ being the energy associated with a typical vibration (boson) of the system [25].

In general one fixes the parameters of the effective interaction of nucleons in the nucleus, by requiring mean field theory, as a rule Hartree-Fock or Hartree-Fock-Bogoliubov theory if the system is superfluid, to reproduce the experimental findings: binding energies, mean square radii, etc. This is equivalent to requiring that the solution of the Schrödinger equation describing the bound states of the electron-proton system, interacting through the Coulomb force, reproduces the energy levels of the hydrogen atom. We know that this is not possible, unless the renormalization effects arising from the electron-photon coupling are properly taken into account as prescribed by QED [11,12]. Similarly, the parameters of the effective nuclear interaction should reproduce the experimental findings only when the particle-vibration coupling is allowed to renormalize, screen and dress the different modes of elementary excitation and the interaction among them in a similar way as, for example, the Lamb shift of the hydrogen atom is accounted for only when the renormalization effects arising from the electron-photon coupling are considered.

The formalism we shall use is based on the Dyson equation [22]. It can describe on equal footing the dressed one-particle state $\tilde{a}$ of an odd nucleon renormalized by the (collective) response of all the other nucleons (Figs.
The renormalization of the energy $\hbar \omega_x$ (Figs. 2(a)-2(b)) and of the transition probability $B(E\lambda)$ (Figs. 2(c)-2(f)) of the collective vibrations of the even system where the number of nucleons remains constant (correlated particle-hole excitations), and the induced interaction due to the exchange of collective vibrations between pairs of nucleons [23], moving in time reversal states close to the Fermi energy (Figs. 1(e)-1(g)). We include both self-energy and vertex correction processes, thus satisfying Ward identities (cf., e.g., [25]). Within this framework, the self-consistency existing between the dynamical deformations of the density and of the potential sustained by "screened" particle-vibrations coupling vertices leads to renormalization effects which make finite (stabilize) the collectivity and the self-interaction of the elementary modes of nuclear excitation, in particular of the low-lying surface vibrational modes, providing an accurate description of many seemingly unrelated experimental findings, in terms of very few (theoretically calculable) parameters, namely: the $k-$mass $m_k$ [26] and the particle vibration coupling vertex $h(ab\nu)$, associated to the process in which a quasiparticle changes its state of motion from the unperturbed quasiparticle state $a$ to $b$, by absorbing or emitting a vibration $\nu$ [1].

The equation describing the renormalization of a quasiparticle $a$, due to this variety of couplings is

$$
\begin{pmatrix}
E_a & 0 \\
0 & -E_a
\end{pmatrix}
\begin{pmatrix}
\Sigma_{11}(E_a) & \Sigma_{12}(E_a) \\
\Sigma_{12}(E_a) & \Sigma_{22}(E_a)
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_a \\
\tilde{y}_a
\end{pmatrix}
=
\begin{pmatrix}
\tilde{E}_a \\
\tilde{E}_a
\end{pmatrix},
$$

where $\Sigma_{ii}$ and $\Sigma_{ij}$, $(i \neq j)$ are the normal and abnormal self-energies.

Eq. (1) is to be solved iteratively, and simultaneously for all the involved quasiparticle states. At each iteration step, the original quasiparticle states $a$ with occupation numbers $u_a$ and $v_a$, become fragmented over the different eigenstates $\tilde{a}$ with probability $\tilde{u}_a^2 + \tilde{v}_a^2$, while the renormalized occupation numbers are obtained from the components of the eigenvectors, $\tilde{x}_a$ and $\tilde{y}_a$, according to the relations $\tilde{u}_a = \tilde{x}_a u_a + \tilde{y}_a v_a$, $\tilde{v}_a = -\tilde{y}_a u_a + \tilde{x}_a v_a$. The quantities $\tilde{u}_a$ and $\tilde{v}_a$ are related to the spectroscopic factors measured in one-nucleon stripping and pick-up reactions, respectively. One can also define [22,25] a renormalized state-dependent pairing gap, through the relation $\Delta_a = 2\tilde{E}_a \tilde{u}_a \tilde{v}_a / (\tilde{u}_a^2 + \tilde{v}_a^2)$, which in the limit of no fragmentation reduces to the usual BCS expression [27].

In the calculations reported below, a Skyrme interaction (Sly4 parametrization, with $m_k \approx 0.7 m[29]$), was solely used to determine the properties of the bare single-particle states and the collective vibrations in the particle-hole channel. Both the bare nucleon-nucleon $v_{14}$ Argonne potential as well as the exchange of collective vibrations were used in the particle-particle (pairing) channel.

As seen from Fig. 3, Hartree-Fock theory is not able to account for the experimental quasiparticle energies of the low-lying states. Diagonalizing the Argonne $v_{14}$ nucleon-nucleon potential in the Hartree-Fock basis, within the framework of the generalized Bogoliubov-Valatin approximation including scattering states up to 800 MeV above the Fermi energy (to achieve convergence) in a spherical box of radius equal to 15 fm, one obtains the state-dependent pairing gap shown in Fig. 4 (labelled $v_{14}$). The resulting pairing gap (average value for levels around the Fermi energy) accounts for about half of the empirical pairing gap value ($\approx 1.4$ MeV) obtained from the odd-even mass difference [30]. In keeping with this result, the quasiparticle spectrum (cf. Fig. 3), although being slightly closer to the experimental findings than that predicted by Hartree-Fock theory, displays large discrepancies with observations. The situation is rather similar concerning the low-lying quadrupole vibration of $^{120}$Sn calculated in the QRPA with standard effective nucleon-nucleon interactions like Gogny or Skyrme forces. While energy is predicted too high, which may not be too important, the $B(E2)$ value is about a factor 3 too small (cf. Table 1), a result which calls for a better theory.

In fact, renormalizing the energy and the transition strength of the $2^+$ phonon, following NFT [2,3], that is, considering the couplings of the type depicted in Fig. 2 (cf. also ref. [32] and refs. therein), one obtains an increase of the $B(E2)$ transition probability which brings theory essentially in agreement with experiment (cf. Table 1) [33]. The most important processes which renormalize the energy of the phonon are shown in Figs. 2(a) and (b). Other graphs which are also of fourth order in the particle-vibration coupling vertex, but contain intermediate states with more than four quasiparticle states, lead to very small contributions. This is because these
FIG. 2: Most relevant processes taken into account in the renormalization of the energy of the phonon (a-b) and of the associated transition strength (c-f).

Because in the above calculations we have included only a partial set (although the most important for the physics under discussion) of the NFT graphs needed to provide a completely consistent description of single-particle and collective vibration renormalizations, the mixing of spurious states with the physical states has to be contemplated. Although it is difficult to give a precise estimate of the error induced by such undesired couplings, 30% effects have been found in the calculation of the energy of the one-phonon state [39].

Making use of phonons which account for the experimental findings, the normal and abnormal self-energies were calculated, and Eq.(1) solved. The average value of the resulting state-dependent pairing gap of $^{120}$Sn is now close to the value $\Delta_{exp} = 1.4$ MeV derived from the odd-even mass difference (cf. Fig. 4) [40]. In Fig. 3 we show the energy of the peaks carrying the largest quasiparticle strength, for the orbitals around the Fermi energy, which provide an overall account of the lowest quasiparticle states measured in the odd systems $^{119}$Sn and $^{121}$Sn.

One can conclude that mean field theory and bare nucleon-nucleon potentials reproduce neither the experimental transition strengths nor the pairing gaps, let alone the density of quasiparticle states close to the ground state. Dressing the single-particle motion, the correlated particle-hole excitations of mean field and the nucleon-nucleon interaction with collective surface vibrations, brings theory in overall agreement with experiment. In particular, about half of the pairing gap arises from the long range component of the pairing interaction associated with the exchange of collective vibrations. To further clarify the interdependence of single-particle and collective degrees of freedom, future studies should, for example, concentrate on the role this interdependence has on the nuclear masses. In particular, whether the
explicit, simplified, inclusion of ground state correlations and of the induced pairing interaction can reduce the present r.m.s. error of 0.674 MeV with which one of the best presently available Hartree-Fock mass formula [24] is able to reproduce the experimental findings.

[1] A. Bohr and B.R. Mottelson, *Nuclear Structure*, Vol. II, Benjamin (1975).
[2] D.R. Bes, R.A. Broglia, G.G. Dussel, R. Liotta, and H.M. Sofia, Nucl. Phys. A260 (1976)1; Nucl. Phys. A260 (1976)27.
[3] D.R. Bes, R.A. Broglia, G.G. Dussel, R. Liotta, and R.J. Perazzo, Nucl. Phys. A260 (1976)77.
[4] R.A. Broglia, B.R. Mottelson, D.R. Bes, R. Liotta and H.M. Sofia, Phys. Lett. 64B (1976)29.
[5] D.R. Bes, G.G. Dussel, R.A. Broglia, R. Liotta and B.R. Mottelson, Phys. Lett. 52B (1974)253.
[6] G.G. Dussel and R.J. Liotta, Phys. Lett. 37B (1971)477.
[7] A.B. Migdal, *Theory of finite Fermi systems and applications to atomic nuclei*, Wiley, New York (1967).
[8] V.G. Soloviev, *Theory of atomic nuclei: quasi-particles and phonons*, IOP, Bristol (1992)
[9] A.V. Avdeenkov and S.P. Kamerdzhiev, Phys. Atom. Nucl. 62 (1999)563
[10] S. Kamerdzhiev, E. Litvinova, and D. Zawitscha, Eur. Phys. J. A12(2001)285.
[11] R.P. Feynman, *The theory of elementary processes*, Benjamin, Readings (1975).
[12] S.S. Schweber, *QED*, Princeton Univ. Press., New Jersey (1994).
[13] P.F. Bortignon, R.A. Broglia, D.R. Bes, R. Liotta, Phys. Rep. 30C (1977)305.
[14] F. Barranco, M. Gallardo, and R.A. Broglia, Phys. Lett. 198B (1987)19.
[15] F. Barranco and R.A. Broglia, Phys. Rev. Lett. 59 (1987)2724.
[16] F.P. Bortignon, R.A. Broglia and D.R. Bes, Phys. Lett. 76B (1978)153.
[17] I. Hamamoto, Nucl. Phys. A126 (1969)545; ibid., A141 (1970)1; ibid., A155 (1970)362; ibid., A196 (1972); ibid., A205 (1973)225.
[18] H. Reinhardt, Nucl. Phys. A251 (1975)317.
[19] I. Hamamoto, Proc. of the International School of Physics “E. Fermi, Elementary modes of excitation in nuclei”, eds. A. Bohr and R.A. Broglia, North Holland (1977)234.
[20] H. Huch et al., Phys. rev. C24 (1981)2227.
[21] I. Dukelsky, G.G. Dussel and H.M. Sofia, J. Phys. G8 (1982)L191
[22] J. Terasaki, F. Barranco, R.A. Broglia, P.F. Bortignon and E. Vigezzi, Nucl. Phys. A697 (2002)126.
[23] F. Barranco et al., Phys. Rev. Lett. 83 (1999)2417.
[24] S. Goriely et al., Phys. Rev. C66 (2002)024326.
[25] J.R. Schrieffer, *Theory of superconductivity*, Addison Wesley, Redwood (1964).
[26] C. Mahaux, P.F. Bortignon, R.A. Broglia and C.H. Dasso, Phys. Rep. 120 (1985)1.
[27] The formalism leading to Eq. (1) is equivalent to that presented in ref. [22], which was based on Green functions, and is closely connected to that of ref. [28], based on the equation of motion method. To solve Eq.(1) we have diagonalized an equivalent energy independent matrix as explained in ref. [28].
[28] V. Van der Sluys et al., Nucl. Phys. A551 (1993)210.
[29] E. Chabanat, P. Bonche, P. Haensel, J. Meyer and R. Schaeffer, Nucl. Phys. A627 (1997)710. We have found it convenient to reduce the strength of the spin-orbit term by 15% in order to obtain a better overall agreement with the experimental data.
[30] This result can be compared to a similar calculation performed with an effective mass equal to one, which gives $\Delta \approx 2.2$ MeV [31], a result which reflects the fact that the gap depends strongly on the density of levels at the Fermi energy.
[31] F. Barranco et al., Phys. Lett. 390B (1997)13.
[32] G.F. Bertsch, P.F. Bortignon and R.A. Broglia, Rev. Mod. Phys. 55 (1983)287.
[33] We have only considered the renormalization of the 2+ low-lying phonon, the properties of the other phonons participating in the renormalization processes being instead taken from experiment. For simplicity, the calculations were carried out making use of a separable multipole-multipole interaction, including the modes with $\lambda^\pi = 2^+, 3^+, 4^+, 5^+$. The coupling constant of the quadrupole-quadrupole component was adjusted so that the bare quadrupole vibration displayed properties similar to those calculated making use of effective interactions (i.e. $\hbar \omega_{2+} = 2$ MeV and $B(E2 \uparrow) = 700 e^2 fm^4$), while the other coupling constants were chosen, so as to give an overall description of the measured energies and transition strengths of the low-lying states [34].
[34] O. Beer et al., Nucl. Phys. A417 (1987)326.
[35] P.F. Bortignon, R.A. Broglia, and C.H. Dasso, Nucl. Phys. A398 (1983)221.
[36] To be noted that the anharmonic effects associated to the finite value of the quadrupole moment produce a shift of the energy of phonon, which is however very small compared to the processes already considered.
[37] P.H. Stelson et al., Phys. Rev. C2 (1970)2015.
[38] R. Graetzter et al., Phys. Rev. C12 (1975)1462.
[39] D.R. Bes and J. Kurchan, *The treatment of collective coordinates in many-body systems. An application of the BRST invariance*, World Scientific, Singapore (1990).
[40] In the case of $d_{5/2}$ orbital, the associated quasiparticle strength is strongly fragmented, and displays three low-energy peaks which collect less than 40% of the single-particle strength. In Fig. 3 and 4 we show respectively the energy and the pairing gap associated to the lowest of these three peaks.
| Method           | $h\omega_{2^+}$ (MeV) | $B(E2 \uparrow)$ ($e^2$ fm$^4$) |
|------------------|------------------------|----------------------------------|
| RPA (Gogny)      | 2.9                    | 660                              |
| RPA (Sly4)       | 1.5                    | 890                              |
| RPA + renorm     | 0.9                    | 2150                             |
| Exp.             | 1.2                    | 2030                             |

TABLE I: The energy and reduced E2 transition strength of the low-lying $2^+$ state, calculated according to different theoretical models, are compared to the experimental values [37].