Calculation method of stiffness of partially fabricated steel reinforced concrete beams

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Abstract: In this paper, two new types of partially precast steel reinforced concrete beams are proposed by combining the advantages of steel reinforced concrete beams, high strength concrete beams and precast assembly structures. Partially fabricated steel reinforced concrete beams are composed of precast high-strength concrete shell and cast-in-place concrete core area. In order to further reduce the dead weight of the members, partially fabricated steel concrete beam sections are hollow or can be filled with thermal insulation materials. In this paper, by using the superposition method and based on the existing test results, two new stiffness calculation methods for partially prefabricated SRC beams are proposed.

1. Introduction
The steel reinforced concrete (SRC) structure has been widely used in high-rise and heavy-load structures due to its higher bearing capacity, better seismic performance and durability. However, the on-site construction of SRC structure involves both the steel structure and the steel concrete structure, so the construction procedure of cast-in-place SRC structure is more complicated. At the same time, precast concrete structures are highly recommended due to their simple on-site construction procedures and good quality control of structural components. By combining the advantages of reinforced concrete and precast concrete, this paper proposes that steel reinforced high-strength concrete composite beam (SRHCC) and hollow steel reinforced high-strength concrete composite beam (HSRHCC) two new types of partially prefabricated steel concrete beams. In order to study the bending stiffness of SRHCC and HSRHCC beams, based on the existing test results, this paper proposed the calculation method of their bending stiffness, so as to provide reference for engineering design.

The section layout is shown in Figure 1. The two kinds of composite beams are composed of precast part of high-strength concrete and cast-in-place part of ordinary concrete, among which precast part is composed of section steel, longitudinal bar, stirrup and high-strength concrete. The cast-in-place section is composed of ordinary concrete, and the HSRHCC beam retains a hollow section to reduce the dead weight of the member. In the construction site only will be assembled, high-strength precast shell and casting internal ordinary concrete can form a complete part of the precast steel reinforced concrete beam. This method not only effectively simplifies the traditional construction technology of reinforced concrete beam, but also simultaneously cast-in-place concrete and adjacent slabs are poured, which has the effect of improving the structural integrity. Thus, the consumption of high strength concrete is reduced and the cost is saved. The basic information of the test specimen is shown in Table 1, and the section of the specimen is shown in Figure 1.
Figure 1 SRHCC specimen (top) and HSRHCC specimen (bottom)
Table 1  Section shape

| Specimen number | Shear span ratio | Specimen length L(mm) | Cross section form | Compressive strength of precast concrete cube f_{cu,out} (MPa) | Compressive strength of cast-in-place concrete cube f_{cu,in} (MPa) |
|-----------------|-----------------|-----------------------|-------------------|-------------------------------------------------|-------------------------------------------------|
| SRHCC-1         | 3.0             | 2600                  | solid             | 54.0                                            | 21.7                                            |
| SRHCC-2         | 3.0             | 2600                  | solid             | 54.0                                            | 38.1                                            |
| SRHCC-3         | 3.0             | 2600                  | solid             | 54.0                                            | 68.0                                            |
| HSRHCC-4        | 3.0             | 2600                  | hollow            | 54.0                                            | 21.7                                            |
| HSRHCC-5        | 3.0             | 2200                  | hollow            | 54.0                                            | 38.1                                            |
| HSRHCC-6        | 3.0             | 2600                  | hollow            | 54.0                                            | 68.0                                            |

2. Materials and methods

The section of the composite steel-concrete beam consists of precast concrete and cast-in-place concrete, so there are two kinds of concrete with different strength grades in the same section. In this paper, the combination of precast high-strength concrete and cast-in-place ordinary concrete was adopted in the tests, and the mechanical properties of hollow beams of bending members were also considered. In order to adapt to the industry standard stiffness calculation method, the three-part superposition calculation method proposed in the design principle of composite structure was adopted for calculation. Equivalent superposition calculation chart shown in figure 2.

As can be seen from Figure 2 left and middle, there are two kinds of concrete combination, so the stiffness B_{rc} of reinforced concrete is calculated again. Due to the difference in elastic modulus between precast and cast-in-place concrete, it is regarded as two kinds of materials, which are converted into the same material by using the converted section method, and the moment of inertia of the converted section is calculated to obtain the stiffness of the beam after the converted section.

3. Results & Discussion

The short-term stiffness of reinforced concrete members under the action of standard combination of loads is calculated according to i-shaped section (figure 3):

$$B_{rc} = \frac{M_e}{\phi_m} = \frac{1}{\eta \frac{1}{\phi_0 e_0} + \frac{1}{\psi \phi_0 e_c}} = \frac{E_s A_s K^2}{\eta \frac{1}{\phi_0 e_0} + \frac{1}{\psi \phi_0 e_c}}$$  \hspace{1cm} (1)

The denominator in the formula (1) the first bits of steel strain in the non-uniform coefficient, reflects the tension of concrete, precast concrete influence on stiffness, paragraph 2 of the denominator reflects
the compressive zone of concrete (cast-in-place concrete) deformation on the influence of the stiffness. \( a_E = E_s/E_c \). Here, \( E_c \) is the modulus of elasticity of the compression zone.

\[
\varepsilon_s = \frac{\varepsilon_c(b-x_c-a_s-a_f^s)}{x_c}
\]

(2)

Here, \( f_{tk} \) is the standard value of concrete (precast) axial tensile strength; \( \sigma_a = E_a\varepsilon_a \) is the ratio of longitudinal tensile reinforcement calculated by the effective tensile concrete section area.

For the calculation of neutralization axis, the calculated section is considered as a rectangular steel concrete beam. The neutralization axis will change with the load under the service load. Therefore, the two cases are considered when calculating the height of the average compression zone.

As shown in figure 3, when the neutralization axis passes through the section steel web, the stress-strain relationship between reinforcement, section steel and concrete is as follows:

\[
\varepsilon_a = \frac{\varepsilon_c(b-x_c-a_s-a_f^s)}{x_c} \quad \varepsilon_a' = \frac{\varepsilon_c(b-x_c-a_s-a_f^s)}{x_c} \quad \varepsilon_s = \frac{\varepsilon_c(x_c-a_s)}{x_c} \quad \varepsilon_s' = \frac{\varepsilon_c(x_c-a_s)}{x_c}
\]

(3)

\[
\sigma_a = E_a\varepsilon_a \quad \sigma_a' = E_a\varepsilon_a' \quad \sigma_s = E_s\varepsilon_s \quad \sigma_s' = E_s\varepsilon_s' \quad \sigma_c = E_c\varepsilon_c
\]

(4)

Here, \( \sigma_s, \varepsilon_s, \sigma_s', \varepsilon_s' \) respectively represent stress and strain of tensile reinforcement and stress and strain of compression reinforcement;

\( \sigma_a, \varepsilon_a, \sigma_a', \varepsilon_a' \) respectively represent stress and strain of tensile flange and stress and strain of compression flange of fractured section steel;

\( x_c \) represents the height of the compression zone of the fracture section at the service stage;

\( t \) denotes the thickness of section steel flange.

According to \( \sum N = 0 \), the force balance formula of fracture section is obtained.

\[
N_c + N_s + N_a - T_s - T_a = 0
\]

(5)

That is to say,

\[
\frac{1}{2}b x_c \sigma_c + \sigma_a' A_s' + \sigma_a' A_{af'} + \frac{1}{2} \sigma_a' t_w (x_c - a_s' - t) - \sigma_a A_{af} = 0
\]

(6)

So, the formula can be derived as,

\[
\alpha_E (A_s + A_s' + A_a)[D - x_c] = \frac{1}{2} b x_c^2
\]

(7)

As shown in figure 4, when the neutralization axis does not pass through the section steel, that is, when the section steel is under full-section tension, the stress-strain relationship between reinforcement, section steel and concrete is as follows:
Figure 4. Stress distribution when neutral axis does not pass steel web

\[
\varepsilon_a = \frac{E_a(a_x - a_x^c)}{x_c} ; \quad \varepsilon'_a = \frac{E_a(a_x^c - a_x)}{x_c} ; \quad \varepsilon_S = \frac{E_S(x_c - a_x^c)}{x_c} \quad \varepsilon'_S = \frac{E_S(x_c - a_x)}{x_c} ; \quad (8)
\]

\[
\sigma_a = E_a \varepsilon_a ; \quad \sigma'_a = E_a \varepsilon'_a ; \quad \sigma_S = E_S \varepsilon_S ; \quad \sigma'_S = E_S \varepsilon'_S ; \quad \sigma_c = E_c \varepsilon_c
\]

Then get the formula:

\[
\alpha_c(A_s + A'_s + A_w)[D - x_c] = \frac{1}{2}b x_c^2
\]

Due to the different bending moments of each section of the beam in the service stage, even if the bending moments of each section are equal within a certain section, the height of the neutralization axis of each section of the beam changes due to the occurrence of cracks, that is, the position of the neutralization axis along the length of the beam is not straight line. According to the existing research literature, the variation of the waveform curve close to the trigonometric function in the middle and axial positions in the crack section of the beam is shown in FIG. 5. The height of the average compression zone of the beam on the concrete section of the shaped steel

\[
\bar{x} = \frac{1}{l_{cr}} \int_0^{l_{cr}} x \, dx = 0.5(x_c + x_{max})
\]

Figure 5. Neutral axis after cracked

In HSRHCC beam, there is no concrete within the "rigid core area" restricted by section steel. Therefore, when calculating the concrete stiffness in the core area of the hollow precast specimen, the influence of that part of concrete stiffness on the whole specimen is approximately ignored. Therefore, the calculation formula of flexural stiffness can be obtained as follows:

Hollow prefabricated specimen: \( B = B_{rc} + B_s = \frac{E_a A_{sh}^2}{1.15y_f + 0.2 + \frac{n_{sh}}{1.5y_f}} + E_c I_{ss} \left[ A_{sso} + A_{ss}(a_s' + \frac{h_s}{2} - x)^2 \right] \)

Solid precast specimen: \( B = B_{rc} + B_c + B_s = \frac{E_a A_{sh}^2}{1.15y_f + 0.2 + \frac{n_{sh}}{1.5y_f}} + E_c I_{ss} \left[ A_{sso} + A_{ss}(a_s' + \frac{h_s}{2} - x)^2 \right] \)
4. Conclusions
In this paper, based on the existing test results, two new calculation methods for the stiffness and deformation of partially prefabricated SRC beams are proposed by using the superposition method. The calculation method considering the superposition of three parts of SRC beams, reinforced concrete and rigid core zone is in good agreement with the test results.

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