Regular Sequential Serializability and Regular Sequential Consistency

Jeffrey Helt  
Princeton University  
United States  
jhelt@cs.princeton.edu

Matthew Burke  
Cornell University  
United States  
matthelb@cs.cornell.edu

Amit Levy  
Princeton University  
United States  
aalevy@cs.princeton.edu

Wyatt Lloyd  
Princeton University  
United States  
wlloyd@princeton.edu

Abstract

Strictly serializable (linearizable) services appear to execute transactions (operations) sequentially, in an order consistent with real time. This restricts a transaction’s (operation’s) possible return values and in turn, simplifies application programming. In exchange, strictly serializable (linearizable) services perform worse than those with weaker consistency. But switching to such services can break applications.

This work introduces two new consistency models to ease this trade-off: regular sequential serializability (RSS) and regular sequential consistency (RSC). They are just as strong for applications: we prove any application invariant that holds when using a strictly serializable (linearizable) service also holds when using an RSS (RSC) service. Yet they relax the constraints on services—they allow new, better-performing designs. To demonstrate this, we design, implement, and evaluate variants of two systems, Spanner and Gryff, relaxing their consistency to RSS and RSC, respectively. The new variants achieve better read-only transaction and read tail latency than their counterparts.

*CCS Concepts: • Information systems → Parallel and distributed DBMSs; Distributed database transactions.

ACM Reference Format:
Jeffrey Helt, Matthew Burke, Amit Levy, and Wyatt Lloyd. 2021. Regular Sequential Serializability and Regular Sequential Consistency. In ACM SIGOPS 28th Symposium on Operating Systems Principles (SOSP ’21), October 26–29, 2021, Virtual Event, Germany. ACM, New York, NY, USA, 35 pages. https://doi.org/10.1145/3477132.3483566

1 Introduction

Strict serializability [75] and linearizability [37] are exemplary consistency models. Strictly serializable (linearizable) services appear to execute transactions (operations) sequentially, in an order consistent with real time. They simplify building correct applications atop them by reducing the number of possible values services may return to application processes. This, in turn, makes it easier for programmers to enforce necessary application invariants.

In exchange for their strong guarantees, strictly serializable and linearizable services incur worse performance than those with weaker consistency [10, 27, 40, 54, 55]. For example, consider a read in a key-value store that returns the value written by a concurrent write. If the key-value store is weakly consistent, the read imposes no constraints on future reads. But if the key-value store is strictly serializable, the read imposes a global constraint on future reads—they all must return the new value, even if the write has not yet finished. Existing strictly serializable services guarantee this by blocking reads [22], incurring multiple round trips between clients and shards [96, 97], or aborting conflicting writes [96, 97]. These harm service performance, either by increasing abort rates or increasing latency.

Services with weaker consistency models [5, 7, 45, 55], however, offer application programmers with a harsh trade-off. In exchange for better performance, they may break the invariants of applications built atop them.

This work introduces two new consistency models to ease this trade-off: regular sequential serializability (RSS) and regular sequential consistency (RSC). They allow services to achieve better performance while being invariant-equivalent to strict serializability and linearizability, respectively. For any application that does not require synchronized clocks, any invariant that holds while interacting with a set of strictly serializable (linearizable) services also holds when executing atop a set of RSS (RSC) services.

To maintain application invariants, a set of RSS (RSC) services must appear to execute transactions (operations) sequentially, in an order that is consistent with a broad set of causal constraints (e.g., through message passing). We prove formally this is sufficient for RSS (RSC) to be invariant-equivalent to strict serializability (linearizability).

To allow for better performance, RSS and RSC relax some of strict serializability and linearizability’s real-time guarantees for causally unrelated transactions or operations, respectively. For example, when a read returns the value written by a concurrent write, instead of a global constraint, RSS
imposes a causal constraint—only reads that causally follow the first must return the new value.

But in addition to helping enforce invariants, strict serializability’s (linearizability’s) real-time guarantees help applications match their users’ expectations. For instance, from interacting with applications on their local machine, users expect writes to be immediately visible to all future reads. Applications built atop weakly consistent services can violate these expectations, exposing anomalies.

Because RSS (RSC) relax some of strict serializability’s (linearizability’s) real-time constraints, applications built atop an RSS (RSC) service may expose more anomalies. But prior work suggests anomalies are rare in practice [58], and further, RSS and RSC include some real-time guarantees to make the chance of observing these new anomalies small. They should only be possible within short time windows (a few seconds). Thus, we expect the difference between RSS (RSC) and strict serializability (linearizability) to go unnoticed in practice.

To compose a set of RSS (RSC) services such that they appear to execute transactions (operations) in some global RSS (RSC) order, each must implement one other mechanism: a real-time fence. We show how the necessary fences can be invoked without changing applications.

Finally, to demonstrate the performance benefits permitted by RSS and RSC, we design, implement, and evaluate variants of two existing services: Spanner [22], Google’s globally distributed database, and Gryff [18], a replicated key-value store. The variants implement RSS and RSC instead of strict serializability and linearizability, respectively.

Spanner-RSS improves read-only transaction latency by reducing the chances they must block for conflicting read-write transactions. Instead, Spanner-RSS allows read-only transactions to immediately return old values in some cases. As a result, in low- and moderate-contention workloads, Spanner-RSS reduces read-only transaction tail latency by up to 49% without affecting read-write transaction latency.

Gryff-RSC improves read latency with a different approach. By removing the write-back phase of reads, Gryff-RSC halves the number of round trips required between application processes and Gryff’s replicas. As a result, for moderate- and high-contention workloads, Gryff-RSC reduces p99 read latency by about 40%. Further, because Gryff-RSC’s reads always finish in one round, it offers larger reductions in latency (up to 50%) farther out on the tail.

In sum, this paper makes the following contributions:

- We define RSS and RSC, the first invariant-equivalent consistency models to strict serializability and linearizability.
- We prove that for any application not requiring synchronized clocks, any invariant that holds with strictly serializable (linearizable) services also holds with RSS (RSC).
- We design, implement, and evaluate Spanner-RSS and Gryff-RSC, which significantly improve read tail latency compared to their counterparts.

![Figure 1. An application deployed in a data center. It comprises the processes running on user devices, Web servers, and asynchronous workers. They are supported by a pair of services. The services’ consistency models significantly impact application correctness and performance.](image)

## 2 Background and Motivation

In this section, we first describe the typical structure of applications and their interaction with supporting services. We then discuss the role consistency models play in these interactions. Finally, we demonstrate how existing consistency models offer difficult trade-offs to application programmers and service designers.

### 2.1 Distributed Applications

Distributed applications can be split into two parts: a set of processes executing application-specific logic and a set of services supporting them. The application-specific processes include those that respond interactively to users, such as those executing on a user’s device and those they cooperate with synchronously or asynchronously, such as Web servers running in a nearby data center. The services provide generic, reusable functionality, such as data storage [16, 22, 54, 55] and messaging [84].

For example, consider a Web application deployed in a data center (Figure 1). A user interacts with a browser on their device. These interactions define the behaviors of the application. Under the hood, the browser sends HTTP requests to a set of Web servers. In processing a request, a server reads and modifies state in a key-value store and renders responses. There are also worker processes that these servers invoke asynchronously to perform longer running tasks [39]. The set of processes running application-specific logic are the clients of the services. The services are responsible for persisting application state, replicating it across data centers, and coordinating between application components.

### 2.2 Motivating Example: Photo-Sharing App

Throughout the paper, we consider a simple but illustrative example: a photo-sharing application. The application allows...
users to backup and share their photos with other users while handling compression and other photo-processing functions.

In our example application, photos are organized into albums. Both photos and albums are stored in a globally distributed, transactional key-value store. Each photo and album has a unique key. A photo’s key maps to a binary blob while an album’s key maps to structured data containing the keys of all photos in the album. If a key is read but is not present, the key-value store simply returns null.

In addition to the transactional key-value store, the application uses a messaging service to enqueue requests for asynchronous processing. For example, when a user adds a high-resolution photo, the application enqueues the photo’s key in the messaging service, requesting some worker process create lower-resolution thumbnails of the image.

When a user adds a new photo to an album, a Web server issues a read-write transaction: it creates a new key-value mapping for the photo, reads the album, and writes back the album after modifying its value to include a reference to the newly added photo. Then it enqueues a request for additional processing.

### 2.3 Consistency Models

The correctness and performance of an application are heavily influenced by the consistency models of its supporting services. A **consistency model** is a contract between a service and its clients regarding the allowable return values for a given set of operations. Services with stronger consistency are generally restricted to fewer possible return values, so it is easier for programmers to build a correct application atop them. Restricting the allowable return values, however, often incurs worse performance in these services and consequently, in the application.

**Invariants and anomalies.** The stronger restrictions of stronger consistency models enable applications to more easily ensure correctness and provide better semantics to users. The correctness of an application is determined by its invariants, which are logical predicates that hold for all states of an application (i.e., the combined states of all application processes). The semantics for users are determined by the rate of anomalies, which are behaviors the user would not observe while accessing a single-threaded, monolithic application running on a local machine with no failures. Table 1 shows some invariants and anomalies for our application.

Application logic relies on invariants to function correctly. For the photo-sharing example, client-side application logic assumes that if an album contains a reference to a photo, the photo exists in the key-value store (\(I_1\)). Similarly, workers that receive a photo’s key through the messaging service assume fetching the key from the key-value store will not return null (\(I_2\)).

Applications also attempt to present reasonable behaviors to users, which is quantified by the rate of anomalies. Unlike an invariant violation, the detection of an anomaly may require information that is beyond the application’s state. For example, once Alice adds a new photo to an album, Bob not seeing it is an anomaly (\(A_3\)). But detecting \(A_3\) requires the application either to have synchronized clocks to record the start and end times of Alice and Bob’s requests or to somehow know that Alice communicated with Bob.

### 2.4 Strict Serializability is Too Strong

Strict serializability [75] is one of the strongest consistency models. A service that guarantees strict serializability appears to execute transactions sequentially, one at a time, in an order consistent with the transactions’ real-time order. As a result, only transactions that are concurrent (i.e., both begin before either ends) may be reordered. Further, strict serializability is composable: clients may use multiple services, and the resulting execution will always be strictly serializable because real-time order is universal to all services [37].

1. This holds only with the reasonable assumption that individual transactions do not span multiple services. This makes each strictly serializable service equivalent to a linearizable “object” [37].
Strict serializability ensures a large set of invariants hold. For example, it ensures both invariants hold for our photo-sharing application. For $I_1$, the application logic writes a photo’s data and adds it to an album in a single transaction $T$. Strict serializability then trivially ensures $I_2$: because transactions appear to execute sequentially, any other transaction will be either before $T$ and not see the photo or after $T$ and see both the photo in the album and its data. For $I_2$, the application logic first executes the add-new-photo transaction and then enqueues a request to process it in the messaging service. The real-time order and composability of strict serializability then ensures $I_3$: the enqueue begins in real time after the add-new-photo transaction ends, and thus, any process that sees the enqueued request must subsequently see the writes of the add-new-photo transaction.

Strict serializability also mitigates anomalies. As shown in Table 1, $A_1$, $A_2$, and $A_3$ never occur with a strictly serializable key-value store. Because strictly serializable transactions appear to execute sequentially, no writes are lost, and its real-time guarantees ensure Bob’s transactions always follow Alice’s after receiving her call.

Yet applications built atop strictly serializability services are not perfect. For instance, asynchronous networks, transient failures, and a lack of fate sharing among components can all cause anomalies that are beyond the scope of a consistency model. $A_4$ in Table 1 shows one example, which could not occur on a local machine with no failures.

**Strict serializability imposes performance costs.** In exchange for ensuring invariants and preventing most anomalies, strict serializability imposes significant performance costs on services. For instance, consider anomaly $A_3$ in Table 1, and assume Charlie is in the middle of adding a photo when Alice sees it. Strict serializability mandates that any subsequent read by any application server includes the photo, even if Bob is on a different continent and Charlie’s transaction has not finished. As a result, the key-value store must ensure Alice’s transaction only includes Charlie’s photo once all subsequent reads will, too.

Existing services provide this guarantee through a variety of mechanisms. Some block read-only transactions during conflicting read-write transactions [22]. Others incur multiple round trips between an application server and a set of replicas [96, 97] or abort concurrent read-write transactions [96, 97]. These mechanisms reduce performance by increasing either read-only transaction latency or abort rates.

### 2.5 Process-Ordered Serializability is Too Weak

Because strict serializability incurs heavy performance overhead, many services provide a weaker consistency model. The next strongest is process-ordered (PO) serializability, which guarantees that services appear to execute transactions sequentially, in an order consistent with each client’s process order [24, 56]. PO serializability is weaker than strict serializability because it does not guarantee that non-concurrent transactions respect their real-time order. Moreover, PO serializability is not composable. Thus, process orders across services can be lost.

Because PO serializability is weaker than strict serializability, it avoids some of its performance costs. For instance, there are read-only transaction protocols that can always complete in one round of non-blocking requests with constant metadata in services with PO serializability, while this is impossible with strict serializability [57].

**PO serializability provides fewer invariants.** In exchange for better performance, weaker consistency models, like PO serializability, present application programmers with a harsh trade-off: fewer invariants will hold with reasonable application logic.² For our photo-sharing example, $I_1$ holds because like with strict serializability, PO-serializable services appear to execute transactions sequentially.

On the other hand, $I_2$ does not hold because PO serializability is not composable. A worker seeing a photo in the message queue does not ensure its subsequent reads to the key-value store will include the writes of a preceding add-new-photo transaction because the message queue and key-value store are distinct services.

### 2.6 Non-Transactional Consistency Models

Our discussion above focuses on transactional consistency models. The same tension between application invariants and service performance exists for the equivalent non-transactional models. Linearizability [37] and sequential consistency [45] are the non-transactional equivalents of strict serializability and process-ordered serializability, respectively. If we temporarily ignore albums and assume application processes issue a single write to add a photo, invariant $I_2$ holds with a linearizable key-value store but not with a sequentially consistent one. But linearizable services must employ mechanisms that hurt their performance to satisfy linearizability’s constraints, for example, by requiring additional rounds of communication for reads (§7).

### 3 Regular Sequential Consistency Models

A consistency model’s guarantees affect both application programmers and users. Stronger models place less burden on programmers (by guaranteeing more invariants) and users (by exposing fewer anomalies) but constrain service performance. In this work, we propose two new consistency models, regular sequential serializability (RSS) and regular sequential consistency (RSC), to diminish this trade-off.

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²We say “reasonable application logic” because one can always write a middleware layer that implements a stronger consistency model, X, atop a weaker one, Y, e.g., by taking the ideas of bolt-on consistency [11] to an extreme. But the resulting X middleware on Y service is simply an inefficient implementation of an X service.
RSS and RSC are invariant-equivalent to strict serializability and linearizability, respectively. Thus, they place no additional burden on application programmers.

While they do allow more anomalies, prior work suggests anomalies with much weaker models (e.g., eventual consistency) are rare in practice (e.g., at most six anomalies per million operations [58]). Thus, we expect the additional burden on users to be negligible.

In this section, we define RSS and RSC and prove their invariant-equivalence to strict serializability and linearizability. We first describe our formal model of distributed applications (§3.1) and the services they use (§3.2). We then define RSS and RSC (§3.3 and §3.4) and finally prove our main result (§3.5). (We demonstrate RSS and RSC allow for services with better performance in later sections.)

3.1 Applications and Executions

We model a distributed application as a collection of $n$ processes. Processes are state machines [60, 61] that implement application logic by performing local computation, exchanging messages, and invoking operations on services.

An application’s processes define a prefix-closed set of executions, which are sequences $s_0, s_1, s_2, \ldots$ of alternating states and actions, starting and ending with a state. An application state contains the state of each process—i.e., it is an $n$-length vector of process states. As part of a process’s state, we assume it has access to a local clock, which it can use to set local timers, but the clock makes no guarantees about its drift or skew relative to those at other processes.

Each action is a step taken by exactly one process and is one of three types: internal, input, or output. Internal actions model local computation. Processes use input and output actions to interact with other processes (e.g., receiving and replying to a remote procedure call) and their environment (e.g., responding to a user gesture). As we will describe in the following section, a subset of the input and output actions are invocations and responses, respectively, which are used to interact with services.

Processes can also exchange messages with one another via unidirectional channels. To send a message to process $P_j$, $P_i$ uses two actions: first, $P_i$ uses an output action $send_{ij}(m)$ and later, an input action $recv_{ij}(m)$ occurs, indicating $m$’s transmission on the network. Similarly, to receive a message from $P_i$, $P_j$ first uses an output action $recv_{ji}(m)$ and later, an input action $recv_{ij}(m)$ occurs, indicating the receipt of $m$.

Given an execution $\alpha$, we will often refer to an individual process’s sub-execution, denoted $\alpha(P_i)$, $\alpha[P_i]$ comprises only $P_i$’s actions and the $i$th component of each state in $\alpha$.

Well-formed. An execution is well-formed if it satisfies the following: (1) Messages are sent before they are received; (2) A process has at most one (total) outstanding invocation (at a service) or $recv_{ij}(m)$ (at a channel); and (3) Processes do not take output steps while waiting for a response from a service. We henceforth only consider well-formed executions.

Equivalence. Two executions $\alpha$ and $\beta$ are equivalent if for all $P_i$, $\alpha[P_i] = \beta[P_i]$. Intuitively, equivalent executions are indistinguishable to the processes.

3.2 Services

Databases, message queues, and other back-end services that application processes interact with are defined by their operations and a specification [37, 60]. An operation comprises a pair of invocations, specifying the operations and their arguments, and matching responses, containing return values. The specification is a prefix-closed set of sequences of invocation-response pairs defining the service’s correct behavior in the absence of concurrency. A sequence $S$ in specification $\mathcal{S}$ defines a total order over its operations, denoted $<_S$.

Several services can be composed into a composite service by combining their specifications as the set of all interleavings of the original services’ specifications. Notably, this means a service composed of constituent services that support transactions include those transactional operations but does not support transactions across its constituent services. In the results below, we assume the processes interact with an arbitrary (possibly composite) service.

3.3 Consistency Models

A consistency model specifies the possible responses a service may return in the face of concurrent operations. Before we define our new consistency models, we must define four preliminaries. For ease of presentation, two of our definitions, conflicts and reads-from, assume a key-value store interface. While these definitions could be made general, we leave precisely defining them for other interfaces to future work.

Complete operations. Given an execution $\alpha$, we say an operation is complete if its invocation has a matching response in $\alpha$. We denote $\text{complete}(\alpha)$ as the maximal subsequence of $\alpha$ comprising only complete operations [37].

Conflicting operations. Given read-write transaction $W$, we say a read-only transaction $R$ conflicts with $W$ if $W$ writes a key that $R$ reads. Given an execution $\alpha$, we denote the set of read-only transactions in $\alpha$ that conflict with $W$ as $C_\alpha(W)$. We define conflicts and $C_\alpha(w)$ analogously for non-transactional reads and writes.

Real-time order. Two actions in an execution $\alpha$ are ordered in real time [37, 75], denoted $\pi_1 \rightarrow_{\alpha} \pi_2$, if and only if $\pi_1$ is a response, $\pi_2$ is an invocation, and $\pi_1$ precedes $\pi_2$ in $\alpha$.

Causal order. Two actions are causally related [5, 7, 44, 45, 54, 55] in an execution $\alpha$, denoted $\pi_1 \rightarrow_{\alpha} \pi_2$, if any of the following hold: (1) Process order: $\pi_1$ precedes $\pi_2$ in a process’s sub-execution; (2) Message passing: $\pi_1$ is a $send_{ij}(m)$ and $\pi_2$ is its corresponding $recv_{ji}(m)$; (3) Reads from: $\pi_1$ is operation $o_1$’s response, $\pi_2$ is $o_2$’s invocation, and $o_2$ reads a value written by $o_1$; or (4) Transitivity: there exists some action $\pi_3$ such that $\pi_1 \rightarrow_{\alpha} \pi_3$ and $\pi_3 \rightarrow_{\alpha} \pi_2$. 
3.4 RSS and RSC

We now define our new consistency models, regular sequential serializability and regular sequential consistency. Their definitions are nearly identical but because supporting transactions has significant practical implications, we distinguish between the transactional and non-transactional versions.

Intuitively, RSS (RSC) guarantees a total order of transactions (operations) such that they respect causality. Further, like prior "regular" models [46, 82, 92], reads must return a value at least as recent as the most recently completed, conflicting write.

**Regular Sequential Serializability.** Let \( \mathcal{T} \) be the set of all transactions and \( \mathcal{W} \subseteq \mathcal{T} \) be the set of read-write transactions. An execution \( \alpha_1 \) satisfies RSS if it can be extended to \( \alpha_2 \) by adding zero or more responses such that there exists a sequence \( S \in \mathcal{S} \) where (1) \( S \) is equivalent to \( \text{complete}(\alpha_2); \) (2) for all pairs of transactions \( T_1, T_2 \in \mathcal{T}, T_1 \sim_{\alpha_1} T_2 \iff T_1 <_S T_2 \); and (3) for all read-write transactions \( W \in \mathcal{W} \) and transactions \( T \in C_{\alpha_1}(W) \cup \mathcal{W}, W \rightarrow_{\alpha_1} T \implies W <_S T \).

**Regular Sequential Consistency.** Let \( \mathcal{O} \) be the set of all operations and \( \mathcal{W} \subseteq \mathcal{O} \) be the set of writes. An execution \( \alpha_1 \) satisfies RSC if it can be extended to \( \alpha_2 \) by adding zero or more responses such that there exists a sequence \( S \in \mathcal{S} \) where (1) \( S \) is equivalent to \( \text{complete}(\alpha_2); \) (2) for all pairs of operations \( o_1, o_2 \in \mathcal{O}, o_1 \sim_{\alpha_1} o_2 \iff o_1 <_S o_2 \); and (3) for all writes \( w \in \mathcal{W} \) and operations \( o \in C_{\alpha_1}(w) \cup \mathcal{W}, w \rightarrow_{\alpha_1} o \implies w <_S o \).

3.5 RSS and RSC Maintain Application Invariants

This section presents a condensed version of the proof. For brevity, we assume here processes do not fail and all operations finish. The full proof is in Appendix C.

**Preliminaries.** The results below reason about an application’s invariants, which are assertions about its states. Formally, we say a state is reachable in application \( A \) if it is the final state of some execution of \( A \). An invariant \( I_\alpha \) is a predicate that is true for all of \( A \)’s reachable states [60].

In the proofs below, it will be convenient to focus on the actions within an execution. Given an execution \( \alpha \), its schedule, \( \text{sched}(\alpha) \), is the subsequence of just its actions.

**Proof intuition.** Our main results follow from two observations. First, Lemma 1 shows we can transform an execution in which the operations respect RSS into an execution in which they respect strict serializability without reordering any actions at any of the processes. Figure 2 shows an example. The key insight is that both RSS and strict serializability guarantee equivalent to a sequence in the service’s specification, which by definition is strictly serializable. Second, Theorem 2 shows that the final states of the two executions related by Lemma 1 are identical. It follows that invariants that hold in the first execution also hold in the second.

![Figure 2. Example transformation from an RSS execution to a strictly serializable one.](image-url)
in $\alpha$ to get $\beta$, $\beta$ is clearly finite, and because $\sim_{\alpha}$ captures the sending and receiving of messages, $\beta$ is well-formed. Finally, since $S$ is a sequence of matching invocation-response pairs, the processes’ interactions with the service in $\beta$ are sequential, not overlapping in real time. Thus, $\beta$ satisfies strict serializability.

**Theorem 2.** Suppose $I_A$ is an invariant that holds for any execution $\beta$ of $A$ that satisfies strict serializability. Then $I_A$ also holds for any execution $\alpha$ of $A$ that satisfies RSS.

*Proof.* Let $\alpha$ be an arbitrary execution of $A$ that satisfies RSS. We must show that $I_A$ is true for the final state $s$ of $\alpha$.

By Lemma 1, there is an equivalent execution $\beta$ that satisfies strict serializability. Let $s'$ be the final state of $\beta$. Because $s'|P_i = \beta|P_i$ for all $P_i$, it is easy to see that $s' = s$. By assumption, $I_A$ is true of $s'$, so $I_A$ is also true of $s$.

We prove similar results for RSC and linearizability in Appendix C.

### 4 Practical Implications

Lemma 1 shows we can transform any RSS execution into an equivalent strictly serializable one. Theorem 2 shows this is sufficient for RSS to maintain application invariants.

While this transformation preserves the order of each process’s actions, however, the order of causally unrelated actions, e.g., the order of Alice and Bob’s Web requests handled by different servers, may not be. In fact, this is why anomalies like $A_2$ and $A_3$ are possible with RSS and RSC.

Further, RSS (RSC) is defined with respect to a potentially composite service. The results above thus assume a set of distinct services can together guarantee RSS (RSC), even if processes interact with multiple services, but they do not specify how this is achieved.

In the remainder of this section, we first describe how multiple services can be composed such that their composition guarantees RSS (§4.1). We then discuss supporting applications whose processes interact via message passing (§4.2). For ease of exposition, the discussion focuses on RSS but applies equally to RSC.

#### 4.1 Composing RSS Services

A set of RSS services must always ensure the values returned by their transactions reflect a global total order spanning all services. This is straightforward with strictly serializable services because real-time order is universal across services.

With RSS, however, some pairs of transactions, such as causally unrelated read-only transactions, may be reordered with respect to real time. As a result, the states observed by processes as they interact with multiple services can form cycles (e.g., $P_1$ reads $x = 1$ then $y = 0$ while $P_2$ reads $y = 1$ then $x = 0$), precluding a total order. Service builders thus must implement one additional mechanism, *real-time fences*, to allow a set of RSS services to globally guarantee RSS.

| Function                              | Description                        |
|---------------------------------------|------------------------------------|
| REGISTERSERVICE(name, fence_f)        | Register new service.              |
| UNREGISTERSERVICE(name)               | Unregister service.                |
| STARTTRANSACTION(name)                | Start txn at service.              |

**Figure 3.** 1ibRSS Interface. 1ibRSS helps RSS service builders implement composition by invoking the necessary real-time fences.

A real-time fence $f_x$ at RSS service $x$ provides the following guarantee: for each pair of transactions $T_1$ and $T_2$ at service $x$, if $T_1 \rightsquigarrow f_x$ and $f_x \rightarrow T_2$, then $T_1 <_{S_x} T_2$, where $<_{S_x}$ is the total order of $x$’s transactions. Every transaction that causally precedes the fence must be serialized before any transaction that follows the fence in real time. Intuitively, a process that issues a real-time fence ensures all other processes observe state that is at least as new as the state it observed. Thus, if each process issues a fence at its previous service before interacting with another, the fences prevent cycles in the states observed by multiple processes as they cross service boundaries. (We discuss the service-specific implementation of real-time fences for Spanner-RSS and Gryff-RSC in Sections 5 and 7.)

Although the need to implement a fence for each RSS service places an additional burden on service builders, using real-time fences to guarantee a global total order across services does not require changes to applications. The client libraries of the RSS services can insert real-time fences as necessary at run time. To this end, we implement a meta-library, 1ibRSS, to aid service builders with composition. Figure 3 shows its interface.

At initialization, an RSS service’s client library registers itself with the 1ibRSS meta-library, passing it a unique name and a callback that implements its fence. The meta-library keeps an in-memory registry of all RSS services. During execution, the client library must simply notify the meta-library before starting a new transaction.

With these calls, the meta-library implements composition without intervention from application programmers. Every time an RSS client starts a transaction, the meta-library checks if the transaction is at the same service as the previous one, if any. If not, 1ibRSS invokes the prior service’s fence. In Appendix C.4, we prove that if each service’s real-time fence provides the guarantee described above and 1ibRSS follows this simple protocol, then the composition of a set of RSS services globally guarantees RSS.

#### 4.2 Capturing Causality

A meta-library that issues real-time fences is sufficient to guarantee RSS for applications whose processes interact solely through a set of RSS services. But for those whose processes also interact through message passing, an RSS service must ensure causality is respected across these interactions.
For instance, recall our photo-sharing application and assume Alice is using her browser, which sends requests to Web servers that interact with an RSS key-value store. If one server reads and transmits a photo to Alice’s browser and the browser subsequently reads the same photo via a second server, the key-value store must ensure causality is respected across the two transactions—the second must not return null. But if the store is unaware of the causal constraint between the two read-only transactions, then this may not be guaranteed.

One approach is to require application processes to issue a fence before such out-of-band interactions. For instance, the Web server must issue one before transmitting the response back to Alice’s browser. Depending on the structure of the application, however, this may be inefficient.

A better approach is to use a context propagation framework [62] to pass metadata between the interacting processes. This would ensure the second Web server has the necessary metadata to convey causality before it interacts with the RSS store. This context must also include the name of the last RSS service the process interacted with, so libRSS can correctly implement composition.

5 Spanner-RSS

Spanner is a globally distributed, transactional database [22]. It uses synchronized clocks to guarantee strict serializability [75]. While Spanner is designed to provide low-latency read-only (RO) transactions most of the time, they may block, increasing tail latency significantly. Such increases in the tail latency of low-level services can translate into increases in common-case, user-visible latency [26].

Our variant of Spanner’s protocol, Spanner-RSS, improves tail latency for RO transactions by relaxing the constraints on read-only transactions in accordance with RSS. (We prove it provides RSS in Appendix D.1.)

**Spanner background.** Spanner is a multi-versioned key-value store. Keys are split across many shards, and shards are replicated using Multi-Paxos [47]. Clients atomically read and write keys at multiple shards using transactions.

Spanner’s read-write (RW) transactions use two-phase locking [15] and a variant of two-phase commit [36]. Each shard’s Paxos leader serves as a participant or coordinator. Further, using its TrueTime API, Spanner gives each transaction a commit timestamp that is guaranteed to be between the transaction’s real start and end times.

During execution, clients acquire read locks and buffer writes. To commit, the client chooses a coordinator and sends its writes to the shards. Each participant then does the following: (1) ensures the transaction still holds its read locks, (2) acquires write locks, (3) chooses a prepare timestamp, (4) replicates the prepare success, and (5) notifies the coordinator. Assuming all participants succeed, the coordinator finishes similarly: It checks read locks, acquires write locks, chooses the transaction’s commit timestamp, and replicates the commit success. Finally, the coordinator releases its locks and sends the outcome to the client and participants.

To guarantee strict serializability, each participant ensures its prepare timestamp is greater than the timestamps of any previously committed or prepared transactions. The coordinator chooses the commit timestamp similarly but also ensures it is greater than the transaction’s start time and greater than or equal to all of the prepare timestamps. Combined with commit wait [22], this ensures the transaction’s commit timestamp is between its start and end times.

**Strict serializability unnecessarily blocks ROs.** Many workloads are dominated by reads [16, 74, 85]. Thus, Spanner also includes an optimized RO transaction protocol to make the majority of transactions as fast as possible. Spanner’s RO transactions are strictly serializable but unlike RW transactions, only require one round trip between a client and the participant shards. As a result, RO transactions have significantly lower latency than RW transactions.

RO transactions, however, must sometimes block to ensure strict serializability. RO transactions in Spanner read at a client assigned timestamp $t_{\text{read}} = TT.now.latest$, which TrueTime guarantees is after $T_{\text{RO}.\text{start}}$. When a read arrives at a shard with $t_{\text{read}}$ greater than the prepare timestamp of some conflicting RW transaction $T_{\text{RW}}$, it must block until the shard learns if $T_{\text{RW}}$ commits at some time $t_c$ or aborts. Otherwise, $T_{\text{RO}}$ risks violating strict serializability: if $t_c < t_{\text{read}}$ because $T_{\text{RW}.\text{end}} < T_{\text{RO}.\text{start}}$ and $T_{\text{RW}.\text{start}} < t_{\text{read}}$, then strict serializability mandates that $T_{\text{RO}}$ includes $T_{\text{RW}}$’s writes. Because they must potentially wait while a RW transaction executes two-phase commit, blocked RO transactions can have significantly higher latency.

One potential optimization to improve RO transaction latency would be to include the earliest client-side end time $t_{\text{ee}}$ for each RW transaction. Then, RO transactions could avoid...
writes from a conflicting RW transaction that could have ended before its read timestamp. Thus, a RO transaction can avoid blocking by observing a write at a timestamp later than the transaction's commit timestamp. Unfortunately, strict serializability disallows this optimization because it requires $T_{RO}$ to observe $T_{RW}$ even before $t_{ee}$ if there is some other RO transaction that finishes before $T_{RO}$ and includes any of $T_{RW}$’s writes.

Figure 4 shows an example. Because $C_{R1}$’s read observes $C_W$’s RW transaction at $S_C$, strict serializability requires all future reads at both shards to include $C_W$’s writes. Thus, $C_{R2}$’s read must block until $C_W$’s RW transaction commits.

In contrast, RSS allows this optimization. $C_{R1}$’s transaction only imposes a constraint on reads that causally follow it, so $C_{R2}$’s read may still return an older value.

**Spanner-RSS.** Spanner-RSS is our variant of Spanner that improves tail RO transaction latency by often avoiding blocking, even when there are conflicting RW transactions. Intuitively, a RO transaction can avoid blocking by observing a state of the database as of some timestamp $t_{snap}$ that is before its read timestamp $t_{read}$ if it can infer the state satisfies regular sequential serializability.

Observing this state from before $t_{read}$ is safe under RSS when three conditions are met: (1) there are no unobserved writes from a conflicting RW transaction that could have ended before $T_{RO}$ started; (2) there are no causal constraints that require $T_{RO}$ to observe a write at a timestamp later than $t_{snap}$; and (3) its results are consistent with a sequential execution of transactions.

Spanner-RSS ensures each of these conditions are met. To ensure (1), RW transactions include a client-side earliest end time $t_{ee}$. To ensure (2), RO transactions include a minimum read time $t_{min}$. Finally, to ensure (3), before completing a RO transaction, clients ensure all returned values reflect precisely the state of the database at $t_{snap}$. Algorithms 1 and 2 show the full protocol.

**Estimating, including, and enforcing $t_{ee}$ for RW transactions.** Each RW transaction includes an earliest client-side end time $t_{ee}$. The client estimates $t_{ee}$ and includes it when it initiates two-phase commit (not shown). The shards then store $t_{ee}$ while the transaction is prepared but not yet committed or aborted (Alg. 2, line 1). The client later ensures $t_{ee}$ is less than the actual client-side end time by waiting until $t_{ee} < TT.now.earliest$.

**Enforcing a minimum timestamp for RO transactions.** Each RO transaction includes a minimum read timestamp $t_{min}$ to ensure it obeys any necessary causal constraints. Each client tracks this minimum timestamp and updates it after every transaction to include new constraints. After a RW transaction, it is set to the transaction’s commit timestamp (not shown). After a RO transaction, it is set to be at least the transaction’s snapshot time (Alg. 1, line 12). $t_{min}$ thus captures the causal constraints on this RO transaction; it must observe a state that is at least as recent as its last write and any writes the client previously observed.

**Avoiding blocking on shards with $t_{ee}$ and $t_{min}$.** Using $t_{ee}$ and $t_{min}$, shards can infer when it is safe for a RO transaction to skip observing a prepared-but-not-committed RW transaction (Alg. 2, line 6). It is safe unless the prepared transaction either must be observed due to a causal constraint.
(t_p ≤ t_min) or could have ended before the RO transaction began (t_re ≤ t_read).

Reading at t_snap. Although each shard can now infer when a RO transaction can safely skip a prepared RW transaction, the values returned by multiple shards may not necessarily reflect a complete, consistent snapshot at t_snap. Thus, clients and shards take four additional steps to ensure a client always returns a consistent snapshot.

First, as in Spanner, a shard waits to process a RO transaction until its Paxos safe time is greater than t_read (Alg. 2, line 4) [22]. As a result, all future Paxos writes, and thus all future RW prepare timestamps, will be larger than t_read. Thus, the shard ensures it is returning information that is valid until at least t_read. (The Paxos safe time at leaders can be advanced immediately if it is within the leader’s lease.) Second, shards include the commit timestamps t_c for the returned values (Alg. 2, lines 2, 8, and 10). Third, they return the prepare timestamp t_p for each skipped RW transaction with t_p ≤ t_read (Alg. 2, lines 9-10). (Because writes use locks, there is at most one per key.) Fourth, when a skipped RW transaction commits, a shard sends the commit timestamp and the written values in a slow path (Alg. 2, lines 13-15).

Before returning, the client examines the commit and prepare timestamps to see if the shards returned values that are all valid at some snapshot time. Specifically, it sets t_snap to the earliest time for which it has a value for all keys (Alg. 1, lines 15-20). Then, it sees if any prepared transactions have timestamps less than t_snap (Alg. 1, lines 22-23). If they all prepared after t_snap, the RO transaction returns immediately (Alg. 1, line 13).

If some transaction prepared before t_snap, however, the client must wait for slow replies from the shards (Alg. 1, lines 9-10). As the client learns of commits and aborts through the slow replies, it moves transactions out of the prepared set (Alg. 1, line 11), updates the values it will return (if t_c ≤ t_snap), and potentially advances the earliest prepared timestamp (Alg. 1, line 22). Note that t_snap remains the same, so the latter continues until t_snap < t_c', which is guaranteed by the time the final slow reply is received.

Performance discussion. Spanner-RSS’s RO transaction latency is never worse and often better than Spanner’s. When there are no conflicting RW transactions, RO transactions in both will return consistent results at t_read. When there are conflicting transactions, however, Spanner-RSS will often send fast replies quickly while Spanner blocks. Further, the fast replies let Spanner-RSS complete the RO transaction right away unless one of the shards returns a value with a commit timestamp that is greater than the prepared timestamp of a skipped RW transaction. Even then, the slow replies from Spanner-RSS’s shards will be sent at the same time Spanner would unblock.

5.1 Real-Time Fences
As described above, to ensure the order of transactions reflects causality, a client tracks and enforces a minimum read timestamp t_min. Using t_min, a client ensures its next transaction will be ordered after any transaction that causally precedes it by ensuring the next transaction reflects a state of the database that is at least as recent as t_min.

A real-time fence must provide a slightly stronger guarantee. It must ensure that all transactions that causally precede it are serialized before any transaction that follows it in real time, regardless of the latter transaction’s originating client. While this is guaranteed for future RW transactions since they already respect their real-time order, the same is not true of future RO transactions. Thus, when executed at a client with a minimum read timestamp t_min, a fence in Spanner-RSS must ensure that all future RO transactions reflect a state that is at least as recent as t_min.

To achieve this, Spanner-RSS’s real-time fences leverage the following observation: If L is the maximum difference between t_c and t_re for any RW transaction, then a RO transaction that starts after t_c + L will reflect all writes with timestamps less than or equal to t_c. After t_c + L, a RO transaction cannot skip reading a RW transaction with commit timestamp t_c (since t_re ≤ t_c + L < t_read).

As a result, fences in Spanner-RSS are simple. To ensure all future RO transactions reflect a state that is at least as recent as t_min, a fence blocks until t_min + L < TT.now.earliest.

6 Spanner-RSS Evaluation
Our evaluation of Spanner-RSS aims to answer two questions: Does Spanner-RSS improve tail latency for read-only transactions (§6.1), and what performance overhead does Spanner-RSS’s read-only transaction protocol impose (§6.2)?

We implement the Spanner and Spanner-RSS protocols in C++ using TAPIR’s experimental framework [96, 97]. Each shard is single-threaded. The implementation reuses TAPIR’s implementation of view-stamped replication [72] instead of Multi-Paxos [47] but is otherwise faithful. Our code and experiment scripts are available online [3].

The implementation includes two optimizations not presented in Section 5: First, a skipped, prepared transaction’s writes are returned in the fast path instead of the slow path, allowing the client to return faster in some cases, e.g., if it learns the transaction already committed at a different shard. Second, when a transaction blocks as part of wound-wait [79], it estimates how long it blocked and advances its local estimate of t_re by that amount. The coordinator then aggregates the shards’ t_re values and returns the maximum to the client, which waits until it has passed. This reduces the chance a RO transaction will be blocked by a RW transaction whose t_re has become inaccurate because of lock contention.

Unless otherwise specified, experiments ran on Amazon’s EC2 platform [1]. Each t2.large instance has 2 vCPUS and EC2 platform [1]. Each t2.large instance has 2 vCPUS and
We first compare the latency distributions for RO and RW workload, we set $p = 0.9$; and if it does, it waits for a think time $H$. The clients use a separate $t_{\text{min}}$ for each session. We set $H = 0$ since this yields the worst performance for Spanner-RSS. Further, we set $p = 0.9$, so the average session length is ten transactions, matching measurements from real deployments [85]. Finally, for each workload, we set $\lambda$ such that the offered load is 70-80% of the maximum throughput. Each data center contributes an equal fraction of the load.

6.1 Spanner-RSS Reduces RO Tail Latency

We first compare the latency distributions for RO and RW transactions with Spanner and Spanner-RSS as skew varies. Spanner-RSS’s RO transactions have lower latency than Spanner’s due to less blocking during conflicting RW transactions. These improvements do not harm RW transaction latency because Spanner-RSS’s protocol simply requires passing around an extra timestamp with RW transactions.

Figure 5 compares the tail latency distributions of RO transactions at three skews. (We omit the distributions for RW transactions after verifying they are identical.) Spanner-RSS reduces RO tail latency in all cases. When contention is low (Figure 5a), Spanner’s RO transactions offer low tail latency; up to $p_{99}$, their latency is bounded only by wide-area latency. Above this, however, it starts increasing. At $p_{99.9}$, Spanner-RSS offers a 14% (38 ms) reduction in tail latency.

Spanner-RSS offers larger improvements at higher skew. In Figure 5b, latency consistently decreases by at least 76 ms above $p_{99.5}$. This is up to a 45% reduction; at $p_{99.9}$, it is a 37% (114 ms) reduction.

With a skew of 0.9, (Figure 5c), Spanner’s RO transaction latency starts increasing at lower percentiles. As a result, Spanner-RSS reduces $p_{99}$ RO latency by 49% (135 ms). With high contention, however, improvements farther out on the tail (e.g., above $p_{99.95}$) are more inconsistent. Increased waiting by RW transactions for wound-wait [79] make the earliest end time estimates less accurate. Further, each session’s $t_{\text{min}}$ advances more rapidly and in turn, increases the chance a RO transaction must block.

6.2 Spanner-RSS Imposes Little Overhead

We now compare the two protocol under heavy load to quantify the overhead Spanner-RSS incurs from its additional protocol complexity. Because the number and size of its additional messages is small, Spanner-RSS’s performance should be comparable to Spanner’s.

To stress the implementations, we use a uniform workload, set the TrueTime error to zero, and place all shards in one data center. Since it does not depend on wide-area latencies, we ran this experiment on CloudLab’s Utah platform [28]. Each m5t10 machines has 8 physical cores, 64 GB RAM, and a 10 Gbit NIC. Inter-data-center latency is less than 200 $\mu$s. We use eight shards, so each leader has a dedicated physical CPU on one server.
Figure 6. Spanner-RSS does not significantly impact Spanner’s performance at high load.

Figure 6 compares the throughput and median latency as we increase the number of closed-loop clients. As shown, Spanner-RSS does not significantly impact the server’s maximum throughput. Spanner-RSS’s is within a few hundreds of transactions per second of Spanner’s, and its latency is within a few milliseconds.

7 Gryff-RSC

Gryff is a geo-replicated key-value store that supports non-transactional reads, writes, and atomic read-modify-writes (rmws) [18]. It provides linearizability using a hybrid shared register and consensus protocol. Reads and writes are executed using a shared register protocol to provide bounded tail latency whereas rmws are executed using a consensus protocol, which is necessary for correctness.

We introduce Gryff-RSC, which provides regular sequential consistency and is able to reduce the bound on tail latency from two round trips to a quorum of replicas to one round trip. This section gives an overview of Gryff-RSC’s design and evaluation. The full design is described in Appendix B, and we prove it guarantees RSC in Appendix D.2.

7.1 Gryff-RSC Design Overview

Read operations in Gryff consist of an initial read phase that contacts a quorum of replicas to learn of the most-recent value they know of for a given key. If the quorum returns the same values, then the read finishes. If the quorum returns different values, however, the read continues to a second, write-back phase that writes the most-recent value to a quorum before the read ends. This second phase is necessitated by linearizability: once this read ends, any future reads must observe this or a newer value.

Regular sequential consistency relaxes this constraint: before the write finishes, only causally later reads must observe this or a newer value. This enables Gryff-RSC’s reads to always complete in one phase. On a read, instead of immediately writing the observed value back to a quorum, a Gryff-RSC client piggybacks it onto the first phase of its next operation. Replicas write the piggybacked value before processing the next operation. Causally later operations by the same client are thus guaranteed to see this or a newer value. By transitivity then, causally later operations at other clients, e.g., by the reads-from relation, will also observe this or a newer value.

Piggybacking a read’s second phase onto the next operation ensures a client’s next operation can be serialized after all operations that causally precede it. Similarly, a real-time fence must ensure all future operations, including those from other clients, are ordered after any operation that causally precedes it. By RSC, future writes and rmws are already required to respect their real-time order, but the same is not true of future reads. Thus, to execute a real-time fence in Gryff-RSC, a client writes back the key-value pair, if any, that would have been piggybacked onto its next operation. This guarantees future reads return values that are at least as recent as any operation that causally precedes the fence.

7.2 Gryff-RSC Evaluation

Our evaluation of Gryff-RSC aims to answer two questions: Does Gryff-RSC offer better tail read latency on important workloads (§7.3), and what are the performance costs of Gryff-RSC’s protocol (§7.4)?

We implement Gryff-RSC in Go using the same framework as Gryff [18], and our code and experiment scripts are available online [2]. We keep all of Gryff’s optimizations enabled. All experiments ran on the CloudLab [28] machines described in Section 6.2, and we emulate a wide-area environment. We use five replicas, one in each emulated geographic region, because with Gryff’s optimizations, reads already always finish in one round trip with three replicas. An equal fraction of the clients are in each region. Table 2 shows the emulated round-trip times.

![Table 2. Emulated round-trip latencies (in ms).](image)

We generate load with 16 closed-loop clients. With this number, servers are moderately loaded. Each client executes the YCSB workload [21], which includes just reads and writes. We vary the rate of conflicts and the read-write ratio.

7.3 Gryff-RSC Reduces Read Tail Latency

Figure 7 compares Gryff and Gryff-RSC’s p99 read latency across a range of conflict percentages and read-write ratios. We omit similar plots for writes because write performance is identical in the two systems.

With few conflicts (Figure 7a), nearly all of Gryff’s reads complete in one round, so Gryff-RSC cannot offer an improvement. p99 latency for both systems is 145 ms.

As Figures 7b and 7c show, however, as the rate of conflicts increases, more of Gryff’s reads must take its slow path, incurring two wide-area round trips. This increases Gryff’s
p99 latency by 61% (from 145 ms to 234 ms). On the other hand, Gryff-RSC’s reads only require one round trip, so p99 latency remains at 145 ms. At lower write ratios, the magnitude of Gryff-RSC’s improvement over Gryff increases with the rate of conflicts.

Further, because reads always finish in one round, Gryff-RSC offers even larger latency improvements farther out on the tail (not shown). For instance, with 10% conflicts and a 0.3 write ratio, Gryff-RSC reduces p99.9 latency by 49% (from 290 ms to 147 ms).

### 7.4 Gryff-RSC Imposes Negligible Overhead

We also quantify the performance overhead of Gryff-RSC’s piggybacking mechanism, but we omit the plots due to space constraints. We compare Gryff and Gryff-RSC’s throughput and median latency as we increase the number of clients. As in Section 6.2, we disable wide-area emulation. With a 10% conflict ratio, we run two workloads: 50% reads-50% writes and 95% reads-5% writes (matching YCSB-A and YCSB-B [21]). In both cases, Gryff-RSC’s throughput and latency are within 1% of Gryff’s, suggesting the overhead from Gryff-RSC’s protocol changes are negligible.

### 8 Related Work

This section discusses related work on consistency models, explicitly reasoning about invariants, equivalence results, and strictly serializable and linearizable services.

**Consistency models.** Due to their implications for applications and services, consistency models have been studied extensively. In general, given an application, more invariants hold and fewer anomalies are possible with stronger models. But weaker models allow for better-performing services.

RSS and RSC are distinct from prior works because they are the first models that are invariant-equivalent to strict serializability and linearizability, respectively. They achieve this by guaranteeing that transactions (operations) appear to execute sequentially, in an order consistent with a set of causal constraints. Prior works are not invariant-equivalent to strict serializability (linearizability) because either they do not guarantee equivalence to a sequential execution or do not capture all of the necessary causal constraints.

The discussion below generally proceeds from the strongest to the weakest consistency models. Since we have already discussed strict serializability [75], process-ordered serializability [24, 56], linearizability [37], and sequential consistency [45] extensively, we focus here on other models. (We also provide a technical comparison between RSS, RSC, and their proximal models in Appendix A.)

Like RSS, CockroachDB’s consistency model (CRDB) [87] lies between strict serializability and PO serializability. CRDB guarantees conflicting transactions respect their real-time order [87], but it gives no such guarantee for non-conflicting transactions, which can lead to invariant violations. For instance, in a slight modification to our photo-sharing application, assume clients issue a single write to add a photo and included in this write is a legal timestamp comprising a user ID and a counter. Further, assume clients can issue a RO transaction for a user’s photos. With CRDB, if Alice adds two photos and those transactions execute at different Web servers, a RO transaction that is concurrent with both may only return the second photo. If the application requires a user’s photos to always appear in timestamp order, then it would be correct with a strictly serializable database but not with CRDB.

Similarly, like RSC, OSC(U) [49] lies between linearizability and sequential consistency. OSC(U) guarantees writes respect their real-time order. Reads, however, may return stale values [49], so some pairs of reads (e.g., those invoked by different processes that also communicate via message passing) may return values inconsistent with their causal order. As a result, the non-transactional version of $I_2$ discussed in Section 2.6 does not hold with OSC(U). On the other hand, OSC(U) allows services to achieve much lower read latency than what is currently achievable with RSC.

PO serializability and sequential consistency impose fundamental performance constraints on services [53], so many weaker models, both transactional [4, 7, 10, 14, 29, 66, 75, 76, 86, 90] and non-transactional [5, 13, 20, 53, 55, 83, 88], have been developed. These weaker models allow for services...
with much better performance than what is currently achievable with RSS or RSC. For example, a causal+ storage system can process all operations without synchronous, cross-data-center communication [55]. But application invariants break with these models because they do not guarantee equivalence to a sequential execution of either transactions or operations. Thus, they present developers with a harsh trade-off between service performance and application correctness.

Based on the observation that some invariants hold with weaker consistency models, other work proposes combinations of weak and strong guarantees for different operations [43, 51, 52, 89]. This allows these services to offer dramatically better performance for a subset of operations. Maintaining application correctness while using these services, however, requires application programmers to choose the correct consistency for each operation.

Finally, three other works use causal or real-time constraints in innovative ways [12, 63, 95]. First, A-causal messaging applies real-time guarantees to a different domain where messages have limited, time-bounded relevance (e.g., video streaming) [12]. Second, real-time causal strengthens causal consistency by ensuring writes respect their real-time order [63]. But because real-time causal does not capture all necessary causal constraints, \( L_2 \) would not hold.

Third, TACT gives application developers fine-grained control over its consistency [95]. For instance, an application can set a different staleness bound for each invocation to control how old (in real time) the values returned by the operation may be. (Setting zero for all operations provides strict serializability.) Compared to RSS, TACT’s fine-grained control allows for services with better performance but requires developers to choose the correct bounds when ensuring their application’s correctness.

**Reasoning about explicit invariants.** Several tools and techniques have been proposed for reasoning about the correctness of applications that run on services with weaker consistency [8, 17, 35, 50, 70, 77]. For example, SIEVE [50] uses static and dynamic analysis of Java application code to determine the necessary consistency level for operations to maintain a set of explicitly written invariants. Brutschy et al. [17] describe a static analysis tool for identifying non-serializable application behaviors that are possible when running on a causally consistent service. Gotsman et al. [35] introduce a proof rule (and accompanying static analysis tool [70]) that enables modular reasoning about the consistency level required to maintain explicit invariants.

These tools and techniques help application programmers ensure explicit invariants hold when using services with weaker consistency. In contrast, RSS (RSC) services ensure the same application invariants as strictly serializable (linearizable) services. This makes it easier to build correct applications because programmers can write code without stating invariants, running static analyses, or writing proofs.

**Equivalence results.** Other works have leveraged the notion of equivalence, or indistinguishability, to prove interesting theoretical results [9, 32, 34, 59]. In fact, our results here are inspired by them. But while we leverage some of their ideas and techniques, these works apply equivalence to different ends, e.g., to prove bounds on clock synchronization [59] or show there are fundamental differences in the performance permitted by different consistency models [9].

**Strictly serializable services.** Spanner is a globally distributed, strictly serializable database [22]. Since its publication, other such services have been developed [42, 64, 68, 69, 78, 87, 91, 93, 96, 97]. These services have largely focused on improving the throughput [78, 91] and latency [42, 64, 78, 93, 96, 97] of read-write transactions, which can incur multiple inter-data-center round trips in Spanner.

Because they only require one round trip between a client and the participant shards, Spanner’s RO transactions continue to perform as well or better than those of other services. These improvements are thus orthogonal to those offered by Spanner-RSS, and in fact, weakening the consistency of these other services to RSS may allow for designs that combine their improved RW transaction performance with RO transactions that are competitive with Spanner-RSS’s.

**Linearizable services.** Gryff is a recent geo-replicated storage system that combines shared registers and consensus [18]. Many other protocols have been developed to provide replicated and linearizable storage [6, 30, 31, 33, 41, 47, 65, 67, 71–73, 98]. Weakening the consistency of these other services to RSC is likely to enable new variants of their designs that improve their performance.

### 9 Conclusion

Existing consistency models offer a harsh trade-off to application programmers; they often must choose between application correctness and performance. This paper presents two new consistency models, regular sequential serializability and regular sequential consistency, to ease this trade-off. RSS and RSC maintain application invariants while permitting new designs that achieve better performance than their strictly serializable or linearizable counterparts. To this end, we design variants of two existing systems, Spanner-RSS and Gryff-RSC, that guarantee RSS and RSC, respectively. Our evaluation demonstrates significant (40% to 50%) reductions in tail latency for read-only transactions and reads.

### Acknowledgments

We thank the anonymous reviewers and our shepherd, Rodrigo Rodrigues, for their helpful comments and feedback. We are also grateful to Khiem Ngo for his comments on an earlier version of this paper. This work was supported by the National Science Foundation under grant CNS-1824130.
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A Comparing RSS and RSC To Their Proximal Consistency Models

As discussed extensively in the main body of the paper, regular sequential serializability lies between strict serializability [75] and process-ordered serializability [24, 56]. While other consistency and isolation definitions lie between or near these points in the consistency spectrum, RSS is the first consistency model that is invariant-equivalent to strict serializability. Existing models either fail to reflect a sequential execution of transactions [25] or respect the broad set of causal constraints necessary to maintain application invariants [87]. As a result, invariants like \( I_1 \) or \( I_2 \), respectively, will not hold.

Similarly, regular sequential consistency lies between linearizability [37] and sequential consistency [45]. While many existing consistency models lie near RSC, RSC is the first to be invariant-equivalent to linearizability. Existing non-transactional models fail to provide one or both of two guarantees: They either do not reflect a sequential execution of operations [63, 81, 82] or do not respect the necessary causal constraints [45, 49, 92].

In the remainder of this section, we compare RSS and RSC to their (respective) proximal consistency models. (For brevity, we focus only on proximal models and refer the reader to other works for more comprehensive surveys of existing transactional [4, 23] and non-transactional [92] consistency definitions.) These comparisons primarily serve to verify that our definitions of RSS and RSC are indeed novel. They also, however, help illuminate how existing models compare to RSS and RSC with regards to application invariants and user-visible anomalies. While we also compare these existing models to each other with informal arguments, we omit formal proofs of these comparisons as they are not the focus of this work.

A.1 Regular Sequential Serializability

Figure 8 compares RSS to its proximal consistency models. **CRDB.** Like RSS, CRDB’s consistency model [87] is stronger than process-ordered serializability; it ensures transactions appear to execute in some total order that is consistent with each client’s process order. Further, it provides some real-time guarantees for writes, which prevent stale reads in many cases.

But CRDB’s consistency model is incomparable to RSS. To show this, we exhibit two schedules: Figure 9 is allowed by CRDB but not by RSS, and the reverse is true of Figure 10.

**Figure 8.** RSS compared to its proximal consistency models: strict serializability [75], CRDB [87], process-ordered serializability [24, 56], and strong snapshot isolation [25].

**Figure 9.** Allowed by CRDB but disallowed by RSS.

**Figure 10.** Allowed by RSS but disallowed by CRDB.

Figure 9 shows a generic version of the example described in Section 8. As described, in a slightly modified version of our photo-sharing example application, the invariant that a user’s photos always appear in order breaks with CRDB. **Strong Snapshot Isolation.** Strong snapshot isolation [25] is weaker than RSS. Briefly, strong snapshot isolation strengthens snapshot isolation [4] by requiring that if a transaction \( T_2 \) follows \( T_1 \) in real time, then \( T_2 \)’s start timestamp must be greater than \( T_1 \)’s commit timestamp. This implies that \( T_2 \)’s reads will observe \( T_1 \)’s writes and \( T_2 \)’s writes will be ordered after \( T_1 \)’s. This guarantee is similar to the real-time guarantee provided by RSS.

Strong snapshot isolation, however, does not guarantee the database’s state reflects a sequential execution of transactions. For instance, like with snapshot isolation, write skew is possible. Figure 11 shows an example. As a result, one can construct an invariant like \( I_1 \) that does not hold with strong snapshot isolation.

**Figure 11.** Allowed by strong snapshot isolation.
Figure 12. RSC compared to its proximal consistency models: linearizability [37], OSC(U) [49], sequential consistency [45], real-time causal consistency [63], VV Regularity [92], and the four regularity definitions proposed by Shao et al. [81, 82].

but violates strong snapshot isolation because \( r_1 \)'s return values imply that \( w_1 \) is serialized after \( w_2 \) even though it precedes \( w_2 \) in real time.

### A.2 Regular Sequential Consistency

Figure 12 compares RSC to its proximal consistency models. OSC(U). Like RSC, ordered sequential consistency [49], denoted OSC(U), is stronger than sequential consistency; it ensures operations appear to execute in some total order that is consistent with each client’s process order. It also imposes some real-time constraints on writes.

But OSC(U) is incomparable to RSC because OSC(U) and RSC’s real-time guarantees differ. Unlike RSC, which requires that all operations following a write are ordered after it, OSC(U) requires that all operations preceding a write are ordered before it.

This difference has practical consequences. As shown in Figure 13, OSC(U) allows stale reads, which are not allowed by RSC. Thus, the non-transactional equivalent of invariant \( I_2 \) does not hold with OSC(U).

**Figure 13.** Allowed by OSC(U) but disallowed by RSC.

On the other hand, Figure 14 shows an execution that RSC allows but OSC(U) does not. OSC(U) forbids it because \( r_1 \) precedes \( w_1 \) in real time, but P4’s reads of \( x = 1 \) and then \( x = 2 \) imply \( w_1 \) precedes \( w_2 \) and thus \( r_1 \) in the total order.

**Real-time Causal.** Real-time causal consistency [63] strengthens causal consistency [5] by also requiring that the order of causally unrelated writes respects their real-time order. Like other causal consistency models, real-time causal does not guarantee that operations appear to execute in some total order. Further, RSC gives the same real-time ordering guarantee for pairs of writes and captures a superset of the causal constraints required by real-time causal. RSC is thus stronger than real-time causal.

By similar reasoning, OSC(U) is stronger than real-time causal. They capture the same causal constraints, and OSC(U) also ensures pairs or writes are serialized in their real-time order. Further, as Viotti and Vukolić show, real-time causal is incomparable to sequential consistency and their definition of regularity (discussed below) [92].

**Viotti-Vukolić Regularity.** Viotti and Vukolić [92] give a definition of regularity that applies for multiple writers. (Lamport’s original definition only applied to a single-writer register [46].) Like RSC, their definition of regularity ensures operations appear to execute in some total order and operations that follow a write in real time must also follow it in the order.

But their consistency is weaker than RSC because the total order is not required to respect the causal constraints imposed by RSC. This can lead to violations of invariants. For instance, if in the execution shown in Figure 10, there was a causal dependency between \( r_1 \) and \( r_2 \) (e.g., through message passing), \( r_2 \) could still return \( x = 0 \) with VV regularity, which could lead to a violation of an invariant. The same is not true of RSC.

As mentioned above, Viotti and Vukolić show their regularity definition is incomparable to real-time causal and sequential consistency [92]. Viotti-Vukolić (VV) regularity is also incomparable to OSC(U): Figure 13 is allowed by OSC(U) but disallowed by VV regularity, and Figure 14 is allowed by VV regularity but disallowed by OSC(U).

**Shao et al. Regularity** Shao et al. [81, 82] propose six new definitions of multi-writer regularity that form a lattice between their weakest consistency definition, MWR-Weak, and linearizability. Above MWR-Weak, MWR-Write-Order (MWR-WO), MWR-Reads-From (MWR-RF), and MWR-No-Inversion (MWR-NI) together form the next level of the lattice. The three intersections of pairs of MWR-WO, MWR-RF, and MWR-NI form the level above that. Finally, linearizability is at the top.

Briefly, MWR-Weak requires that each read returns the value of the most recently completed write or a value of some ongoing, concurrent write. Another way to state this guarantee is that an execution satisfies MWR-Weak if for
each read \( r \), there exists a serialization of \( r \) and all writes in the execution that respects the real-time order of the read and writes. Note, however, that each read has its own serialization, so they may reflect different serializations of concurrent writes. Thus, MWR-Weak does not guarantee that operations appear to execute in some total order.

MWR-WO, MWR-RF, and MWR-NI each strengthen MWR-Weak by imposing additional, incomparable constraints on the serializations for each read [81, 82]. Informally, MWR-WO requires that the serializations for each pair of reads agree on the order of the writes that are relevant to both, where a write is relevant to a read if it is concurrent or precedes the read in real time. MWR-RF requires the serialization of each read to respect a global reads-from relation in addition to the real-time order of read and writes. For instance, MWR-RF forbids the execution in Figure 14 because the facts that \( r_1 \) reads from \( w_2 \) and \( r_1 \) precedes \( w_1 \) in real time imply \( w_1 \) must precede \( w_2 \) in \( r_3 \)’s serialization. Thus, \( r_3 \) must return \( x = 2 \). Finally, MWR-NI strengthens MWR-Weak by requiring that reads executed by the same process agree on the order of writes, although different processes may disagree on the order of concurrent writes.

Both MWR-WO and MWR-NI are weaker than VV regularity and thus RSC. VV regularity gives the same real-time guarantee for operations following writes. Further, the total order guaranteed by VV regularity implies that reads agree on the serialization of their mutually relevant writes (satisfying MWR-WO) and that reads from the same process agree on the order of writes (satisfying MWR-NI).

But neither MWR-WO nor MWR-NI guarantee a total order. Figure 15 shows an example execution allowed by both. MWR-WO allows it because although both \( w_1 \) and \( w_2 \) are relevant to all four reads, MWR-WO does not require that reads respect the process orders. Thus, the serializations for \( r_1 \) and \( r_2 \) can be \( w_1, w_2, r_1 \) and \( r_2, w_1, w_2 \), respectively. MWR-NI allows it by similar reasoning. Thus, MWR-WO and MWR-NI are weaker than VV regularity and RSC.

On the other hand, MWR-RF is incomparable to RSC. As mentioned above, MWR-RF forbids the execution in Figure 14, and Figure 16 shows an execution allowed by MWR-RF but not RSC. In Figure 16, the reads-from relation does not introduce any additional constraints on the values returned by the reads.

![Figure 15](http://example.com/figure15.png)

**Figure 15.** Allowed by MWR-WO and MWR-NI but disallowed by RSC.

![Figure 16](http://example.com/figure16.png)

**Figure 16.** Allowed by MWR-RF and MWR-NI but disallowed by RSC.

Finally, we consider the remaining three consistency models found by intersecting pairs of MWR-WO, MWR-RF, and MWR-NI. Since both MWR-WO and MWR-NI allow Figure 15, their intersection does, too. Further, since VV regularity (and thus RSC) is stronger than both, it is also stronger than their intersection. Similarly, the execution shown in Figure 16 is also allowed by MWR-NI since each process has at most one read. Thus, the intersection of MWR-RF and MWR-NI is also incomparable to RSC. Finally, Shao et al. show that the intersection of MWR-WO and MWR-RF is equivalent to linearizability [82].

Shao et al. show their regularity definitions are incomparable to sequential consistency because they do not guarantee that operations appear to execute in some total order consistent with each client’s process order [82]. By similar reasoning, their definitions are incomparable to OSC(U). Finally, the authors also show their consistency definitions are incomparable to causal consistency. By similar reasoning, they are incomparable to real-time causal.

## B Full Gryff-RSC Design

Gryff is a geo-replicated key-value store that supports non-transactional operations, namely, reads, writes, and atomic read-modify-writes (rmws) of single objects. It provides linearizability using a hybrid shared register and consensus protocol. Reads and writes are executed using a shared register protocol to provide bounded tail latency whereas rmws are executed using a consensus protocol, which is necessary for correctness. We show Gryff can be modified to provide regular sequential consistency to improve tail read latency.

**Gryff background.** Each replica in Gryff maintains a mapping from keys to values and auxiliary state for its underlying consensus protocol. In addition, the value of each key is associated with a consensus-after-register timestamp (carstamp) that denotes the position in the linearizable total order of operations of the last write or rmw to the key.

The read, write, and rmw protocols each follow a common structure that ensures carstamps are observed and updated in an order consistent with the real-time order of operations. For a coordinator executing an operation \( o \) that accesses key
For reads and writes, the client directly performs the Read Phase, so it piggybacks $d$ in the Read Phase of $o$. For reads and writes, the client directly performs the Read Phase. The coordinator gathers the carstamp $c_s$ associated with $k$ from each server $s$ in a quorum $Q$. It then chooses the read phase carstamp $c_s$ to be the maximum over all $s \in Q$ of $c_s$.

2. **Write Phase.** The coordinator determines the operation’s carstamp $c_s$ and write value $v$ based on $o$’s type. For reads, $c_s = c_r$ and $v$ is the value that is associated with $c$. For writes, $c_s$ is chosen to be larger than $c_r$ and $v$ is the new value being written. For rmws, $c_s$ is also chosen to be larger than $c_r$, and $v$ is some user-defined function of the value that is associated with $c$. The coordinator then propagates $c_s$ and $v$ to each server in a quorum $Q'$.

The key to correctness is that quorums are required to have non-empty intersection. This implies that once an operation $o_1$ completes its Write Phase, any subsequent operation $o_2$ will observe the carstamp for $o_1$ on at least one replica during its Read Phase. Further, since the carstamp of $o_2$ is chosen to be at least as large as the largest observed carstamp during the Read Phase, the carstamp for $o_2$ will be at least as large as the carstamp for $o_1$.

This manner of ordering operations enables a performance optimization for reads because they never modify the value of the objects they access. A coordinator of a read can omit the Write Phase entirely while still maintaining linearizability if all of the carstamps it observes in the Read Phase are the same. In this case, the carstamp of the read is already propagated to a quorum, so the Write Phase is not needed to ensure subsequent operations observe the read’s carstamp.

**Gryff-RSC.** Relaxing the consistency model from linearizability to regular sequential consistency allows us to further optimize Gryff’s read protocol. The Write Phase of the read protocol is only necessary to ensure that subsequent reads observe the same or newer values as previously completed reads, which is required for linearizability. Regular sequential consistency, however, only requires this when the reads are causally related.

To take advantage of this weaker requirement, Gryff-RSC always omits the Write Phase for reads and instead tracks a small amount of causal metadata to ensure that causally related reads are ordered properly. Algorithms 3, 4, and 5 show how this metadata is tracked. It is a single tuple $d$ maintained by each client. The tuple comprises the key $k$, carstamp $c_s$, and value $v$ of the most recent read the client completed that has not yet been propagated to a quorum.

The metadata is populated with the carstamp and value of a read when the read completes at the client and it does not have enough information to know that the observed value already exists on a quorum. When the client executes its next operation $o$, it piggybacks $d$ in the Read Phase of $o$. For reads and writes, the client directly performs the Read Phase. For reads, the structure is composed of a Read Phase and a subsequent Write Phase:

1. **Read Phase.** The client executes the Read Phase, so it piggybacks $d$ to the server that coordinates the operation, and the server attaches $d$ to PreAccept messages.

The coordinator then propagates $c_s$ and $v$ to each server in a quorum $Q$.

### Algorithm 3 Gryff-RSC Client

```plaintext
1: state $c$ ← unique client ID
2: state $d$ ← ⊥               // Dependency
3: function Client::Read(k)
4:   send Read(k, d) to all $s \in S$
5:   wait receive ReadReply($v_s$, $c_s$) from all $s \in Q \in Q$
6:   $c_s \leftarrow \max_{s \in Q} c_s$
7:   $v \leftarrow v_s : c_s = c$
8: if $\exists s \in Q : c_s \neq c$ then
9:   $d \leftarrow (k, v, c_s)$
10: return $v$
```

### Algorithm 4 Gryff-RSC Server Read/Write

```plaintext
1: state $V \leftarrow \{⊥, \ldots, ⊥\}$               // Values
2: state $CS \leftarrow \{(0, 0, 0), \ldots, (0, 0, 0)\}$  // Carstamps
3: function Server::ReadRecv(c, k, d)
4:   if $d \neq ⊥$ then
5:     Apply(d, k, d, v, d, cs)
6:   send ReadReply($V[k], CS[k]$) to $c$
7: function Server::Write1Recv(c, k, d)
8:   if $d \neq ⊥$ then
9:     Apply(d, k, d, v, d, cs)
10:    send Write1Reply(CS[k]) to $c$
11: function Server::Write2Recv(c, k, v, cs)
12: Apply(k, v, cs)
13: send Write2Reply to $c$
14: function Server::Apply(k, v, cs)
15: if $cs > CS[k]$ then
16: $V[k] \leftarrow v$
17: $CS[k] \leftarrow cs$
```
Algorithm 5 Gryff-RSC Server RMW

1: state $s \leftarrow$ unique server ID
2: state $\text{prev} \leftarrow ([\bot, (0, 0, 0)], \ldots) \triangleright$ Result of previous rmw for key
3: state $i \leftarrow 0 \triangleright$ Next unused instance number
4: state $\text{cmds} \leftarrow [[\bot, \ldots]], \ldots \triangleright$ Instances:
5: cmd - command to be executed
6: deps - commands that must execute before this one
7: seq - sequence #, breaks cycles in dependency graph
8: base - possible base update for rmw
9: status - status of instance

10: function Server::RMWRecv(c, k, f(\cdot), d)
11: \quad i \leftarrow i + 1 \quad \triangleright$ PreAccept Phase
12: \quad cmd \leftarrow (k, f(\cdot))
13: \quad seq \leftarrow 1 + \max(\{\text{cmds}[j][\ell].\text{seq}(j, \ell) \in I_{\text{cmd}}\} \cup \{0\})
14: \quad deps \leftarrow I_{\text{cmd}}
15: \quad base \leftarrow (V[k], CS[k])
16: \quad \text{cmds}[s][i] \leftarrow (\text{cmd}, \text{seq}, \text{deps}, \text{base}, \text{pre-accepted})
17: \quad \text{send} \text{PreAccept}(\text{cmd}, \text{seq}, \text{deps}, \text{base}, s, i, d) \text{ to all } s' \in F \setminus \{s\} \text{ where } F \in \mathcal{F}
18: \quad \text{wait} \text{ receive PreAcceptOK}(\text{seq}', \text{deps}', \text{base}') \text{ from all } s' \in F \setminus \{s\}
\quad \ldots \quad \triangleright$ Rest of RMW coordinate unchanged

19: function Server::PreAcceptRecv(cmd, seq, deps, base, s', i, d)
20: \quad \text{if } d \neq \bot \text{ then}
21: \quad \quad \text{Apply}(d.k, d.v, d.cs)
22: \quad \quad \text{seq}' \leftarrow \max(\{\text{seq} \cup \{1 + \text{cmds}[j][\ell].\text{seq}(j, \ell) \in I_{\text{cmd}}\})
23: \quad \quad \text{deps}' \leftarrow \text{deps} \cup I_{\text{cmd}}
24: \quad \quad \text{if } cs > \text{base},cs \text{ then}
25: \quad \quad \quad \text{base}' \leftarrow (V[cmd.k], CS[cmd.k])
26: \quad \quad \text{else}
27: \quad \quad \quad \text{base}' \leftarrow \text{base}
28: \quad \quad \text{cmds}[s'][i] \leftarrow (\text{cmd}, \text{seq}', \text{deps}', \text{base}', \text{pre-accepted})
29: \quad \quad \text{send} \text{PreAcceptOK}(\text{seq}', \text{deps}', \text{base}') \text{ to } s'
\quad \ldots \quad \triangleright$ Other message handlers unchanged

Replicas receiving the Read Phase messages first update their key-value stores with the information contained in $d$, overwriting their local carstamp and value for $d.k$ if $d.cs$ is larger than their current carstamp. Then the servers process the Read Phase messages as normal in Gryff. The client clears $d$ as soon as it receives confirmation that it has been propagated to a quorum, either at the end of the Read Phase for reads and writes or when it receives notification that the operation is complete for rmws.

In Algorithm 5, we omit the coordination of a rmw beyond the PreAccept phase because the rest of the processing of PreAccept messages, the recovery procedure, and the execution procedure are unchanged from Gryff. We refer the reader to the complete description of Gryff [18] for more details.

C Full Proof

C.1 Preliminaries

C.1.1 I/O Automata We model each component in our system as an I/O automaton (IOA) [60, 61], a type of state machine. Each transition of an IOA is labeled with an action, which can be an input, output, or internal action. Input and output actions allow the automaton to interact with other IOA and the environment. We assume input actions are not controlled by an IOA—they may arrive at any time. Conversely, output and internal actions are locally controlled—an IOA defines when they can be performed.

To specify an IOA, we must first specify its signature. A signature $S$ is a tuple comprising three disjoint sets of actions: input actions $in(S)$, output actions $out(S)$, and internal actions $int(S)$. We also define $local(S) = out(S) \cup int(S)$ as the set of locally controlled actions, $extacts(S) = in(S) \cup out(S)$ as the set of external actions, and $acts(S) = in(S) \cup out(S) \cup int(S)$ as the set of all actions.

Formally, an I/O automaton $A$ comprises four items:

1. a signature $\text{sig}(A)$,
2. a (possibly infinite) set of states $\text{states}(A)$,
3. a set of start states $\text{start}(A) \subseteq \text{states}(A)$, and
4. a transition relation $\text{trans}(A) \subseteq \text{states}(A) \times \text{acts}(\text{sig}(A)) \times \text{states}(A)$.

Since inputs may arrive at any time, we assume that for every state $s$ and input action $\pi$, there is some $(s, \pi, s') \in \text{trans}(A)$.

An execution of an I/O automaton $A$ is a finite or infinite sequence of alternating states and actions $s_0, \pi_1, s_1, \ldots$ such that for each $i \geq 0$, $(s_i, \pi_{i+1}, s_{i+1}) \in \text{trans}(A)$ and $s_0 \in \text{start}(A)$. Finite executions always end with a state.

Given an execution $\alpha$, we can also define its schedule $\text{sched}(\alpha)$, which is the subsequence of just the actions in $\alpha$. Similarly, an execution’s trace $\text{trace}(\alpha)$ is the subsequence of just the external actions $\pi \in \text{extacts}(A)$.

C.1.2 Composition and Projection To compose two IOA, they must be compatible. Formally, a finite set of signatures $\{S_i\}_{i \in I}$ is compatible if for all $i, j \in I$ such that $i \neq j$:

1. $\text{int}(S_i) \cap \text{acts}(S_j) = \emptyset$, and
2. $\text{out}(S_i) \cap \text{out}(S_j) = \emptyset$.

A finite set of automata are compatible if their signatures are compatible.

Given a set of compatible signatures, we define their composition $S = \prod_{i \in I} S_i$ as the signature with $in(S) = \bigcup_{i \in I} in(S_i) - \bigcup_{i \in I} out(S_i)$, $out(S) = \bigcup_{i \in I} out(S_i)$, and $\text{int}(S) = \bigcup_{i \in I} \text{int}(S_i)$.

The composition of a set of compatible IOA yields the automaton $A = \prod_{i \in I} A_i$ defined as follows:
The states of the composite automaton $A$ are vectors of the states of the composed automata. When an action occurs in $A$, all of the component automata with that action each take a step simultaneously, as defined by their individual transition relations. The resulting state differs in each of the components corresponding to these automata, and the other components are unchanged. We denote the composition of a small number of automata using an infix operator, e.g., $A \times B$.

Given the execution $\alpha$ of a composed automaton $A = \prod_{i \in I} A_i$, we can project the execution onto one of the component automata $A_i$. The execution $\alpha[A_i]$ is found by removing all actions from $\alpha$ that are not actions of $A_i$. The states of $\alpha[A_i]$ are given by the $i$th component of the corresponding state in $\alpha$. The projection of a trace is defined similarly.

Further, we can write the projection of a state $s$ of $A$ on $A_i$ as $s[A_i]$. Finally, we can also project trace($\alpha$) onto a set of actions $\Pi$ where trace($\alpha$)$\Pi$ yields the subsequence of trace($\alpha$) containing only actions in $\Pi$.

### C.1.3 Invariants

Application programmers reason about their applications by reasoning about the invariants that hold during all executions of their application. To formalize this notion in the IOA model, we say a state is reachable in automaton $A$ if it is the final state of some finite execution of $A$. An invariant $I_A$ is a predicate on the states of $A$ that is true for all reachable states of $A$ [60]. Similarly, if $A_i$ is a component of some automaton $A$, then $I_{A_i}$ is an invariant of $A$ if $I_{A_i}$ is true of $s[A_i]$ for all reachable states $s$ of $A$.

### C.1.4 Channels

Each pair of processes in our system communicates via a pair of FIFO channels that are asynchronous, reliable, and buffered. Each channel’s signature, states, and actions are specified in Figure 17.

We denote the channel that process $i$ uses to send messages to process $j$ as $C_{ij}$. $C_{ij}$ has two sets of input actions, sendto$_{ij}(m)$ and recvfrom$_{ij}$, and two sets of output actions, sent$_{ij}$ and received$_{ij}(m)$, for all $m$ in some space of messages $M$. Process $i$ has corresponding output actions, sendto$_{ij}(m)$ and recvfrom$_{ij}$, and input actions, sent$_{ij}$ and received$_{ij}(m)$, for all other processes $j$ and messages $m$. To send a message to process $j$, process $i$ takes a sendto$_{ij}$ step, and $C_{ij}$ subsequently takes a sent$_{ij}$ step. Similarly, to receive a message from $C_{ij}$, process $j$ takes a recvfrom$_{ij}$ step, and $C_{ij}$ subsequently takes a received$_{ij}(m)$ step.

The modeling of buffering in the channels differs from past work [60]. There, receive-from actions are omitted, and received actions are modeled as output actions of channels and corresponding input actions of processes. This implies processes cannot control when they change their state in response to a message.

But in real applications, this is unrealistic. Although the network stack of a machine may accept and process a packet at any time, application code controls when it processes the contained message. For instance, the packet’s contents remain in a kernel buffer until the application performs a read system call on a network socket. This control is essential to our proof as it ensures application processes do not receive messages while waiting for responses from services.

We say an execution $\alpha$ of a channel $C_{ij}$ is well-formed if (1) $\text{trace}(\alpha) \{ \text{sendto}_{ij}(m) \}_{m \in M} \cup \{ \text{sent}_{ij} \}$ is a sequence of alternating send-to and sent actions, starting with a send-to; and similarly (2) $\text{trace}(\alpha) \{ \text{recvfrom}_{ij} \} \cup \{ \text{received}_{ij}(m) \}_{m \in M}$ is a sequence of alternating receive-from and received actions, starting with a receive-from. The following four lemmas show that adjacent pairs of actions are commutative—reordering them in an execution always yields another execution. Lemmas C.1 shows this for adjacent send-to and receive-from actions, Lemma C.2 for adjacent send-to and received actions, Lemma C.3 for adjacent sent and receive-from action, and finally, Lemma C.4 for adjacent sent and received actions.

**Lemma C.1.** Let $\alpha$ be a well-formed, finite execution of $C_{ij}$, and let $\alpha' = \text{trace}(\alpha)$. If there exists some $\pi_s = \text{sendto}_{ij}(m)$ and $\pi_r = \text{recvfrom}_{ij}$ that are adjacent in $\alpha'$, then there exists a well-formed, finite execution $\beta$ of $C_{ij}$ with trace $\beta'$ such that $\beta'$ is identical to $\alpha'$ but with the order of $\pi_s$ and $\pi_r$ reversed.

**Proof.** Let $\alpha$ be a well-formed, finite execution of $C_{ij}$ and let $\alpha' = \text{trace}(\alpha)$. Assume that $\pi_s = \text{sendto}_{ij}(m)$ and $\pi_r = \text{recvfrom}_{ij}$ are adjacent in $\alpha'$. We proceed by cases, so to start, assume $\pi_s$ is before $\pi_r$ in $\alpha'$.
We construct a sequence of alternating states and actions $\beta$ and show that $\beta$ is a well-formed, finite execution of $C_{ij}$.

Define $k \geq 1$ such that $\pi_{k}$ is the $k$th action in $\alpha$. The sequence $\beta$ is identical to $\alpha$ in all states and actions except for the $k$th action, the $k$th state, the $(k+1)$th action, and the $(k+1)$th state. The $k$th action is $\pi_{k}$ and the $(k+1)$th action is $\pi_{k}$. Let $s_{k}$ be the $k$th state in $\beta$. The $k$th state $s_{k}$ is defined such that $(s_{k-1}, \pi_{k}, s_{k}) \in trans(C_{ij})$. Similarly, the $(k+1)$th state $s_{k+1}$ is defined such that $(s_{k}, \pi_{k}, s_{k+1}) \in trans(C_{ij})$. By the definition of the actions of $C_{ij}$ in Figure 17, these transitions must exist because the preconditions of $\pi_{k}$ and $\pi_{k}$ are vacuously true.

We claim that $\beta$ is an execution of $C_{ij}$. Let $s_{1}$ be the $1$th state and $\pi_{1}$ be the $1$th action of $\beta$. Clearly the zeroth state $s_{0}$ in $\beta$ is in $start(C_{ij})$ because $s_{0}$ is identical to the zero state of $\alpha$ and $\alpha$ is an execution of $C_{ij}$. Moreover, for $0 \leq i \leq k$, $(s_{i}, \pi_{i}, s_{i+1}) \in trans(C_{ij})$ because the first $k$ states and $k-1$ actions of $\beta$ are identical to the corresponding states and actions in $\alpha$ and $\alpha$ is an execution of $C_{ij}$. By the definition of $s_{k}, s_{k}, \pi_{k+1}$, and $s_{k+1}$ in $trans(C_{ij})$.

Now consider the state $s_{k+1}$. Let $s_{k+1}'$ be the $i$th state and $\pi_{k+1}'$ be the $i$th action of $\alpha$. Since $\pi_{k}$ only modifies the $Q$ and $e$ variables of $C_{ij}$ and $\pi_{k}$ only modifies the $r$ variable of $C_{ij}$, as defined in Figure 17, the state after executing $\pi_{k}$ then $\pi_{k}$ is the same as the state after executing $\pi_{k}$ then $\pi_{k}$ when starting from the same state. By the previous fact, the fact that $s_{k+1}'$ is the state after executing $\pi_{k}$ then $\pi_{k}$ from $s_{k}'$, the fact that $s_{k}'$ is the state after executing $\pi_{k}$ then $\pi_{k}$ from $s_{k}$, and the definition of $s_{k} = s_{k}', s_{k+1}$ is identical to $s_{k+1}'$.

By the previous fact and the definition of $\beta$, $\beta$ is identical to $\alpha$ for all states and actions after and including $s_{k+1}$. Hence, for all $i \geq 0$, $(s_{i}, \pi_{i}, s_{i+1}) \in trans(C_{ij})$. This implies that $\beta$ is an execution of $C_{ij}$.

Because $\beta$ is identical to $\alpha$ except for the order of $\pi_{k}$ and $\pi_{k}$ and the intervening states and because $\alpha$ is finite, $\beta$ is also finite. Furthermore, $\beta' = trace(\beta)$ is identical to $\alpha'$ but with the order of $\pi_{k}$ and $\pi_{k}$ reversed.

Lastly, $\alpha'[\{send_{i,j}(m)\}_{m \in M} \cup \{send_{i,j}\}]$ is a sequence of alternating send-to- and send actions and similarly $\alpha'[\{recv_{i,j}\} \cup \{recv_{i,j}(m)\}_{m \in M} \cup \{recv_{i,j}\}]$ is a sequence of alternating receive-from and receive actions because $\alpha$ is well-formed. Since $\beta$ is identical to $\alpha$ except for the order of a single pair of receive-from and send actions, $\beta$ is thus also well-formed.

The case where $\pi_{k}$ precedes $\pi_{k}$ in $\alpha'$ can be shown using nearly identical reasoning.

The remaining three proofs employ similar logic as above. For each, we define $\alpha$, $\alpha'$, $\beta$, and $\beta'$ as above and to start, assume $\pi_{k}$ precedes $\pi_{k}$ in $\alpha'$.

Similarly, we define $k$ as above, so $\pi_{k}$ is $\pi_{k}$ in $\alpha$ but $\pi_{k}$ in $\beta$. To conclude each proof, we then simply show that $(s_{k-1}, \pi_{k}, s_{k}) \in trans(C_{ij})$, $(s_{k}, \pi_{k}, s_{k+1}) \in trans(C_{ij})$, and $s_{k+1}$ is identical in $\alpha$ and $\beta$. The remaining reasoning is identical to that above.

**Lemma C.2.** Let $\alpha$ be a well-formed, finite execution of $C_{ij}$, and let $\alpha' = trace(\alpha)$. If there exists some $\pi_{s} = send_{i,j}(m)$ and $\pi_{r} = recv_{i,j}(m')$ that are adjacent in $\alpha'$ with $m \neq m'$, then there exists a well-formed, finite execution of $\beta$ of $C_{ij}$ with trace $\beta'$ such that $\beta'$ is identical to $\alpha'$ but with the order of $\pi_{s}$ and $\pi_{r}$ reversed.

**Proof.** First, by the definitions of $C_{ij}$’s actions shown in Figure 17, $\pi_{s}$ does not modify $r$. Combining this fact with the assumption that $m \neq m'$, $\pi_{r}$’s precondition must hold in $s_{k-1}$. Thus, $(s_{k-1}, \pi_{r}, s_{k}) \in trans(C_{ij})$. Further, $(s_{k}, \pi_{s}, s_{k+1}) \in trans(C_{ij})$ because $\pi_{s}$’s precondition is vacuously true.

Now consider the state $s_{k+1}$. Because $m \neq m'$, $Q$ must not have been empty in $s_{k-1}$. Thus, the value of $Q$ resulting from the enqueue of $m$ by $\pi_{r}$ and the dequeue of $m'$ by $\pi_{r}$ is identical to the value resulting from performing the two operations in the reverse order. Using this fact, since only $\pi_{s}$ sets $e$ and only $\pi_{r}$ sets $r$, $\pi_{k+1}$ must thus be identical in both $\alpha$ and $\beta$.

As above, the case where $\pi_{r}$ precedes $\pi_{s}$ can be shown using nearly identical reasoning.

**Lemma C.3.** Let $\alpha$ be a well-formed, finite execution of $C_{ij}$, and let $\alpha' = trace(\alpha)$. If there exists some $\pi_{s} = send_{i,j}$ and $\pi_{r} = recv_{i,j}$ that are adjacent in $\alpha'$, then there exists a well-formed, finite execution $\beta$ of $C_{ij}$ with trace $\beta'$ such that $\beta'$ is identical to $\alpha'$ but with the order of $\pi_{s}$ and $\pi_{r}$ reversed.

**Proof.** First, by the definitions of $C_{ij}$’s actions shown in Figure 17, $\pi_{s}$’s precondition is vacuously true. Thus, $(s_{k-1}, \pi_{r}, s_{k}) \in trans(C_{ij})$. Further, because $\pi_{s}$ does not modify $e$ and $s_{k-1}$ is identical in $\alpha$ and $\beta$, $\pi_{r}$’s precondition must hold in $s_{k}$. Thus, $(s_{k}, \pi_{s}, s_{k+1}) \in trans(C_{ij})$.

Finally, consider the state $s_{k+1}$. Because $\pi_{s}$ only modifies $e$ and $\pi_{r}$ only modifies $r$, $\pi_{k+1}$ thus must be identical in both $\alpha$ and $\beta$.

As above, the case where $\pi_{s}$ precedes $\pi_{r}$ can be shown using nearly identical reasoning.

**Lemma C.4.** Let $\alpha$ be a well-formed, finite execution of $C_{ij}$, and let $\alpha' = trace(\alpha)$. If there exists some $\pi_{s} = send_{i,j}$ and $\pi_{r} = recv_{i,j}(m)$ that are adjacent in $\alpha'$, then there exists a well-formed, finite execution $\beta$ of $C_{ij}$ with trace $\beta'$ such that $\beta'$ is identical to $\alpha'$ but with the order of $\pi_{s}$ and $\pi_{r}$ reversed.

**Proof.** First, by the definitions of $C_{ij}$’s actions shown in Figure 17, $\pi_{s}$ does not modify $r$ or $Q$. Thus, since $s_{k-1}$ is identical in $\alpha$ and $\beta$, $\pi_{r}$’s precondition must hold in $s_{k-1}$. Thus, $(s_{k-1}, \pi_{r}, s_{k}) \in trans(C_{ij})$. Further, because $\pi_{s}$ does not modify $e$, $\pi_{s}$’s precondition must hold in $s_{k}$, so $(s_{k}, \pi_{s}, s_{k+1}) \in trans(C_{ij})$.

Finally, consider the state $s_{k+1}$. Because $\pi_{s}$ only modifies $e$ and $\pi_{r}$ only modifies $r$ and $Q$, $\pi_{k+1}$ thus must be identical in both $\alpha$ and $\beta$. 
As above, the case where $\pi_i$ precedes $\pi_s$ can be shown using nearly identical reasoning.

C.1.5 Types and Services Processes in our system interact with services, each with a specified type [37, 60]. A service’s type $T$ defines its set of possible values $\text{vals}(T)$, an initial value $v_0 \in \text{vals}(T)$, and the operations $\text{ops}(T)$ that can be invoked on the service. Each operation $o \in \text{ops}(T)$ is defined by a pair of sets of actions: invocations $\text{invs}(o)$ and responses $\text{resps}(o)$. Each contains subscripts denoting a unique service name and a process index. An invocation and response match if their subscripts are equal. Finally, each service has a sequential specification $S$, a prefix-closed set of sequences of matching invocation-response pairs.

For example, consider a read/write register $x$ that supports a set of $n$ processes and whose values is the set of integers. The read operation would then be defined with invocations $\{\text{read}_{lx}(i)\}$ and responses $\{\text{ret}_{lx}(j)\}$ for all $0 \leq i \leq n$ and $j \in N$. Similarly, the write operation would have invocations $\{\text{write}_{lx}(j)\}$ and responses $\{\text{ack}_{lx}\}$. Finally, its sequential specification would be the set of all sequences of reads and writes such that reads return the value written by the most recent write or the initial value if none exists.

We can also compose types $(T_x)_{x \in X}$ and sequential specifications $(S_x)_{x \in X}$. Formally, the values of $T = \prod_{x \in X} T_x$ are vectors of the values of the composed types. $\text{ops}(T)$ is the union of those of the composed types. Finally, the composite sequential specification $S = \prod_{x \in X} S_x$ is the set of all interleavings of the the invocation-response pairs of the component specifications $S_x$.

C.1.6 System Model We model a distributed application as the composition of two finite sets of I/O automata: processes and channels. $n$ denotes the number of processes, so there are $n \times n$ channels. The processes execute the application’s code by performing local computation, exchanging messages via channels, and performing invocations on and receiving responses from services.

In the results below, we are interested in reasoning about which process invariants hold while assuming various correctness conditions of the services they interact with. Thus, we do not model services as IOA. Instead, we assume the processes interact with a (possibly composite) service with an arbitrary type $T$ and $S$ defined for $n$ processes.

For each operation $o \in \text{ops}(T)$, process $P_i$ is then assumed to have an output action for every invocation action in $\text{invs}(o)$ with process index $i$. Similarly, $P_i$ has an input action for every response action in $\text{resps}(o)$ with process index $i$. We refer to these input and output actions as a process’s system-facing actions $\text{sys}(P_i)$.

To model stop failures, we assume each process $P_i$ has an input action $\text{stop}_i$ such that after receiving it, $P_i$ ceases taking steps. If $\text{stop}_i$ occurs while $P_i$ is waiting for a response from a service, then we assume the service does not return a response. But the operation may still cause a service’s state to change, and this change may be visible to operations by other processes.

Finally, to allow the distributed application to receive input from and return values to its environment (e.g., users), we assume each $P_i$ has a set of user-facing actions $\text{user}(P_i)$. Similar to a process’s interactions with services, a user’s interaction with a process is modeled through input-output pairs of user-facing actions.

We make two final assumptions about the processes: First, we assume that while each process has access to a local clock, which is part of its state, and may set local timers, which are internal actions, the process makes no assumptions about the drift or skew of its clock relative to others. Second, processes only invoke an operation on a service when they have no outstanding send-to or receive-from actions at any channel.

Let $P = \prod_{i \in I} P_i$ be the composition of the $n$ processes and $C = \prod_{1 \leq i \leq n} \prod_{1 \leq j \leq n} C_{ij}$ be the composition of $n^2$ channels. Let $a$ be an execution of the distributed application $P \times C$. $a$ is well-formed if it satisfies three criteria: First, for all $P_i$, $\text{trace}(a)|\text{sys}(P_i)$ must be a sequence of alternating invocation and matching response actions, starting with an invocation. Second, for all $C_{ij}$, $a|C_{ij}$ is well-formed. Third, for all $P_i$, $P_i$ does not take an output step while waiting for a response from some service.

C.1.7 Real-Time Precedence Given an execution $a$, let complete($a$) be the maximal subsequence of $a$ comprising only matching system-facing invocations and responses [37]. Further, let $S$ be a sequence of invocation and response actions. We define an irreflexive partial order $\rightarrow_S$ as the real-time order induced by $S$: $\pi_1 \rightarrow_S \pi_2$ if and only if $\pi_1$ is a response action, $\pi_2$ is an invocation action, and $\pi_1 <_S \pi_2$ where $<_S$ is the total order defined by $S$.

Execution $a$ of $P \times C$ thus induces the irreflexive partial order over the invocation and response actions it contains defined by $\rightarrow_{\text{trace}(a)}|\text{sys}(P_i)$. For simplicity, we denote this as $\rightarrow_a$. A well-formed execution $a$ of $P \times C$ satisfies real-time precedence if $a_1$ can be extended to $a_2$ adding zero or more response actions such that there exists a sequence $S \in S$ where (1) for all processes $P_i$, complete($a_2$)|$P_i = S|P_i$, and (2) $a_2 \rightarrow_a \subseteq <_S$.

C.1.8 Potential Causality To define the next notion of precedence, we must first define causality. An execution $a$ induces an irreflexive partial order $\rightarrow_a$ on its actions, reflecting the notion of potential causality [5, 44, 55]. $\pi_1 \rightarrow_a \pi_2$ if one of the following is true:

1. $\pi_1$ precedes $\pi_2$ in some process’s local execution $a|P_i$;
2. $\pi_1$ is a send($o_i$,$j$)($m$) action and $\pi_2$ is its corresponding receive($o_i$,$j$)($m$) action;
3. $\pi_1$ is a response action of operation $o_1$ and $\pi_2$ is an invocation action of operation $o_2$ such that $o_2$’s return value includes the effect of $o_1$; or
4. there exists some action \( \pi_3 \) such that \( \pi_1 \gg \pi_3 \) and \( \pi_2 \gg \pi_3 \).

The meaning of item three depends on the type and specification of the service that the processes interact with through their system-facing actions. For example, for a shared register, \( \pi_1 \gg \pi_2 \) if \( \pi_1 \) is the response of a write and \( \pi_2 \) is the invocation of a read that returns the written value. Similarly, for a FIFO queue, \( \pi_1 \gg \pi_2 \) if \( \pi_1 \) is the response of an enqueue operation and \( \pi_2 \) is the invocation of a dequeue operation that returns the enqueued value.

This definition of potential causality subsumes prior definitions, which either only consider messages passed between processes [44] or only consider causality between operations on a shared data store [5, 55].

C.1.9 Causal Precedence A well-formed execution \( \alpha \) of \( P \times C \) satisfies causal precedence if \( \alpha \) can be extended to \( \alpha' \) adding zero or more response actions such that there exists a sequence \( S \in \mathfrak{S} \) where (1) for all processes \( P_i \), \( \text{complete}(\alpha_i)|P_i| = S|P_i| \), and (2) for all pairs of system-facing actions \( \pi_1 \) and \( \pi_2 \), \( \pi_1 \gg \pi_2 \) \( \iff \) \( \pi_1 \ll \pi_2 \).

C.2 Causal Precedence Maintains Invariants

We now show that for every well-formed, finite execution \( \alpha \) of \( P \times C \) that satisfies causal precedence, there is a corresponding well-formed, finite execution \( \beta \) of \( P \times C \) that satisfies real-time precedence such that each process proceeds through the same sequence of actions and states.

Lemma C.5. Suppose \( \alpha \) is a finite execution of \( P \times C \) that satisfies causal precedence. Then there exists a finite execution \( \beta \) of \( P \times C \) such that \( \beta \) satisfies real-time precedence and for all processes \( P_i \), \( \alpha_i|P_i| = \beta_i|P_i| \).

Proof. We show how to construct \( \beta \) from an arbitrary, finite \( \alpha \). We start by focusing on the actions in \( \alpha \), so let \( \alpha' = \text{sched}(\alpha) \). We first construct a schedule \( \beta' \) from \( \alpha' \) and then replace the states to get \( \beta \).

Since \( \alpha \) satisfies causal precedence, there exists a sequence \( S \in \mathfrak{S} \) such that \( \ll \) respects \( \gg \) and thus \( \ll' \). As defined in Sections C.1.7 and C.1.9, \( \mathfrak{S} \) includes zero or more response actions that are not necessarily in \( \alpha' \). Let \( R \) denote this set of responses. \( \ll \) is thus defined over the set of complete operations in \( \alpha' \) as well as the additional set of invocations completed by responses in \( R \).

As allowed by the definition in Section C.1.9, however, there may be some invocation actions in \( \alpha' \) that do not have matching responses in \( R \), so \( \ll \) does not order them. Thus, we extend \( \ll \) to \( \ll' \) by placing these removed invocation actions at the end of \( \ll \) in an arbitrary order.

Let \( O \) be the totally ordered set defined by the set of all invocation and response actions in \( \alpha' \) and \( \ll' \). To get \( \beta' \), we reorder all of the actions in \( \alpha' \) by ordering each action after the maximal element of \( O \) that causally precedes it.

To do so, we first define two relations: Given two actions \( \pi_1 \) and \( \pi_2 \), let \( \pi_1 \ll \pi_2 \iff \exists \pi_3 \in O, \forall \pi_4 \in O : (\pi_3 \gg \pi_4 \land \pi_4 \gg \pi_3) \implies \pi_4 \ll \pi_3 \). Finally, let \( \pi_1 \equiv \pi_2 \) if \( \pi_1 \neq \pi_2 \) and \( \pi_2 \neq \pi_1 \). It is clear that \( \ll \) is an irreflexive partial order over the actions in \( \alpha' \).

Let \( \ll' \) be the total order of actions given by their order in \( \alpha' \). To define \( \ll \), we use \( \ll' \) to extend \( \ll \) to a total order. In particular, let \( \pi_1 \ll \pi_2 \) if \( \pi_1 \neq \pi_2 \) or \( \pi_1 \equiv \pi_2 \) and \( \pi_1 \ll' \pi_2 \). Thus, \( \ll \) is a total order over the actions in \( \alpha' \). Let \( \beta' \) be the schedule defined by \( \ll \).

We show that this does not reorder any actions at any of the processes, and thus \( \alpha'|P_i = \beta'|P_i \) for all \( P_i \). Assume to the contrary that there exists some pair of actions \( \pi_1, \pi_2 \) from the same \( P_i \) that have been reordered in \( \beta' \). Without loss of generality, assume \( \pi_2 < \pi_1 \) but \( \pi_1 \ll \pi_2 \).

It is clear that \( \pi_1 \neq \pi_2 \) because otherwise by the definition of \( \ll \), \( \pi_1 \) and \( \pi_2 \) would be ordered identically in \( \alpha' \) and \( \beta' \). Thus, by the definition of \( \ll \) and the assumption that \( \pi_1 \neq \pi_2 \), it must be that \( \pi_1 \ll \pi_2 \).

Let \( \pi_i \in O \) be as defined by \( \pi_1 \ll \pi_2 \), so by definition, \( \pi_2 \gg \pi_1 \) and \( \pi_1 \neq \pi_2 \) or \( \pi_2 \neq \pi_1 \) are from the same process, \( \pi_2 \gg \pi_1 \). Thus, \( \pi_1 \gg \pi_2 \) by the transitivity of \( \gg \). Since \( \ll' \) is an irreflexive total order, \( \ll' \) respects \( \ll \), contradicting the definition of \( \ll \). Thus, \( \beta' \) is a schedule such that \( \alpha'|P_i = \beta'|P_i \) for all \( P_i \).

To get \( \beta \) from \( \beta' \), we now just need to replace the states of \( P \times C \). Since we did not reorder any of the actions at any of the processes, we can simply use the same process states from \( \alpha \) for the component processes of the states in \( \beta \).

We just need to fill in the states of each channel \( C_i \). Since \( \alpha'|P_i = \beta'|P_i \) for all \( P_i \) and by the definitions of \( \gg \) and causal precedence, only some pairs of actions may have been reordered when transforming \( \alpha' \) to \( \beta' \). Specifically, the order of a \text{send}_{ij}(m) \) action may be reordered with respect to a \text{recv}_{ij}(m') \) action or a \text{recv}_{ij}(m') \) action for \( m \neq m' \). Similarly, the order of a \text{send}_{ij} \) action may be reordered with a \text{recv}_{ij} \) or a \text{recv}_{ij}(m') \) action.

Let \( k_1 = \alpha|C_{ij} \), \( k_1' = \text{trace}(k_1), \) and \( k_2' = \beta'|C_{ij} \). Since the only possible differences between \( k_1 \) and \( k_2' \) are the reorderings described above, it is possible, using repeated applications of Lemmas C.1 through C.4 if necessary, to find an execution \( k_2 \) from \( k_1 \) such that \( \text{trace}(k_2) = k_2' \). Thus, to fill in the states of each \( C_{ij} \), we first find \( k_2 \) and then use the states from this execution to fill in \( C_{ij} \)'s states in \( \beta \).

Since we previously showed that \( \alpha'|P_i = \beta'|P_i \), it is clear that \( \alpha|P_i = \beta|P_i \) for all \( P_i \) and \( \beta \) is well-formed. Further, because we did not add any states or actions at any of the processes or channels, it is clear that \( \beta \) is finite.

Since \( S \in \mathfrak{S} \) was a sequence of matching invocation-response pairs, by the definition of \( \ll \), \( \text{complete}(\beta) \) is sequential. But \( \text{complete}(\beta)|P_i \) may not equal \( S|P_i \) if \( \text{stop}_i \) occurred while \( P_i \) was waiting for a service response and the operation’s effects affected the responses of operations after other processes. To show that \( \beta \) satisfies real-time precedence, we thus extend \( \beta \) to \( \gamma \) by adding those response actions that are
in $S$ but are not in $β$. These are exactly the response actions in $R$ (defined above) that were originally added to satisfy the definition of causal precedence for $α$. After this addition, it is clear that $γ|P_i = S|P_i$ for all $P_i$, and since $\text{complete}(β)$ is sequential, it respects the real-time precedence of the operations. Thus, $β$ thus satisfies real-time precedence.

**Theorem C.6.** Suppose $I_P$ is an invariant that holds for any execution $β$ of $P \times C$ that satisfies real-time precedence. Then $I_P$ also holds for any execution $α$ of $P \times C$ that satisfies causal precedence.

**Proof.** Let $α$ be an arbitrary, finite execution of $P \times C$ that satisfies causal precedence. We must show that $I_P$ is true for the final state $s$ of $α|P$.

By Lemma C.5, there exists a finite execution $β$ of $P \times C$ that satisfies real-time precedence and for all processes $P_i$, $α|P_i = β|P_i$.

Let $s'$ be the final state of $β|P$. Because $α|P_i = β|P_i$ for all $P_i$, it is easy to see that $s' = s$. By assumption, $I_P$ is true of $s'$, so $I_P$ is also true of $s$.

**C.3 RSC and RSS Maintain Application Invariants**

In this section, we leverage the results above to show that our new consistency models, regular sequential consistency and regular sequential serializability, maintain application invariants that hold with linearizability [37] and strict serializability [75], respectively. We start by defining the two new consistency models and then present the results.

**C.3.1 Regular Sequential Consistency** Regular sequential consistency (RSC) guarantees causal precedence and also requires that writes respect their real-time order. In particular, any conflicting operation that follows a write in real-time must reflect the state change of that write in its result.

Let $W \subseteq O$ be the subset of the (possibly composite) service’s operations that mutate its value. Further, given an execution $α$ and write $w \in W$, define $C_{α}(w)$ as the set of non-mutating operations in $α$ that conflict with $w$. A well-formed execution $α_1$ of $P \times C$ satisfies RSC if $α_1$ can be extended to $α_2$ by adding zero or more response actions such that there exists a sequence $S \in Σ$ where (1) for all processes $P_i$, $\text{complete}(α_2)|P_i = S|P_i$; (2) for all pairs of invocation and response actions $π_1$ and $π_2$, $π_1 \sim_α π_2 \implies π_1 ≤ π_2$; and (3) for all response actions $π_1$ of $w \in W$ and invocation actions $π_2$ of $o \in C_{α_2}(w) \cup W$, $π_1 →_α π_2 \implies π_1 ≤ π_2$.

**C.3.2 Regular Sequential Serializability** Before defining regular sequential serializability, we must first discuss how transactions can be defined within the formal framework presented above. Fortunately, the formalism of types, services, and sequential specifications presented in Section C.1.5 is sufficiently general and can be easily adapted to transactions.

**Transactional services.** We refer to services that support transactions as **transactional services**. For example, consider a transactional key-value store $D$ that stores a mapping from a set of keys $K$ and values $V$ including some initial value $⊥$. The read-only transaction operation is defined with invocations $ρ_{W,D}(K)$ and responses $\text{RET}_{R,D}(V)$ for all $0 \leq i \leq n, K \in 2^K$, and $V \in 2^V$. Let $f : 2^K \times 2^V \times K \to V$ be a function that takes as input a set of keys, their corresponding values, and a single key (that may or may not be in first set) and returns a value.

The transactional key-value store’s sequential specification is the set of all sequences of read-only and read-write transactions satisfying the following: (1) Reads of a key $k$ in both read-only and read-write transactions return the most recently written value for $k$ or $⊥$ if none exists. (2) For each $k \in W$ of a read-write transaction, the transaction writes value $f(R,V,k)$ where $V$ is the set of read values.

**Composition.** Unlike some prior work [37], when composing the types and sequential specifications of transactional services, we do not assume that transactions are extended across multiple services in the composition. For instance, the composition of two transactional key-value stores does not yield a single transactional key-value store whose operations are transactions that possibly span both key sets. As a result, the prior definitions can be used without modification for transactional services.

**Regular Sequential Serializability.** Thanks to the generality of our definition, regular sequential serializability (RSS) is simply regular sequential consistency applied to a transactional service, such as the transactional key-value store described above. The set of writes $W \subseteq O$ is simply the set of read-write transactions, and given a read-write transaction $w \in W$, the set of non-mutating conflicts $C_{α}(w)$ in an execution $α$ is simply the set of read-only transactions that read a key written by $w$.

**C.3.3 Proof Results** Given the definitions above, we are now ready to show that RSC and RSS maintain application invariants. In fact, these results follow as corollaries of Theorem C.6.

**Corollary C.7.** Suppose $I_P$ is an invariant that holds for any execution $β$ of $P \times C$ that satisfies linearizability. Then $I_P$ also holds for any execution $α$ of $P \times C$ that satisfies RSC.

**Proof.** By their definitions, linearizability [37] guarantees real-time precedence and RSC guarantees causal precedence for a set of non-transactional services, respectively. Thus, the corollary follows immediately from Theorem C.6.

**Corollary C.8.** Suppose $I_P$ is an invariant that holds for any execution $β$ of $P \times C$ that satisfies strict serializability. Then $I_P$ also holds for any execution $α$ of $P \times C$ that satisfies RSS.
Proof. By their definitions, strict serializability [75] guarantees real-time precedence and RSC guarantees causal precedence for a set of transactional services, respectively. Thus, the corollary follows immediately from Theorem C.6.  

C.4 RSC Composition Using Real-Time Fences

The definitions and proofs in this section mirror very similar results proved for ordered sequential consistency (OSC) [49]. The differences in the proofs primarily result from differences in the definitions of OSC and RSC and differences in our notation. The techniques and proof steps are nearly identical, but we include them for completeness.

The main result in this section shows that a special mechanism, a real-time fence, can be used to compose a set of RSC services and ensure their composition satisfies RSC. As a result, the results in the previous section regarding application invariants will hold.

C.4.1 Definitions and Assumptions

We focus here on composition, so we need to distinguish between a sequence \( S \in \mathfrak{S} \) of a composite RSC service, as used above, and the corresponding serializations of each service \( x \in X \). We denote such serializations as \( S_x \in \mathfrak{S}_x \).

In some of the definitions and results below, instead of assuming that an execution \( \alpha \) satisfies RSC, we assume each service \( x \in X \) individually satisfies RSC. In other words, we do not assume there is a sequence \( S \in \mathfrak{S} \) of the composite service that satisfies the definition in Section C.3.1, and instead assume each \( S_x \in \mathfrak{S}_x \) satisfies the definition. In the remainder of the section, we make clear which we assume.

C.4.2 Real-Time Fences

Real-time fences are special operations, one per service \( x \in X \), that help compose a set of RSC services. Each fence \( f_x \) has exactly one invocation and response action, which we denote \( i_{f_x} \) and \( r_{f_x} \), and gives the following guarantees: Let \( \alpha \) be a well-formed execution and \( f_x \) be a real-time fence on service \( x \). Then for all system-facing actions \( \pi \in S_x \), (1) if \( \pi \xrightarrow<_{\alpha} i_{f_x} \), then \( \pi \xrightarrow<_{S_x} i_{f_x} \); and (2) if \( r_{f_x} \xrightarrow<_{\alpha} \pi \), then \( r_{f_x} \xrightarrow<_{S_x} \pi \). As a result, any system-facing actions that causally precede the fence are serialized in \( S \) before any that follow it in real time.

Given an execution \( \alpha \) of services \( X \) that individually satisfy RSC, we define a fence \( f_x \)’s past set, denoted \( P(\alpha)(f_x) \), as the set of actions \( \pi \in S_x \) such that \( \pi \xrightarrow<_{S_x} i_{f_x} \) where \( \leq_{S_x} \) extends \( \leq_{S_x} \) in the natural way. Further, define a fence’s last invocation \( L(\alpha)(f_x) \) as the latest invocation in \( P(\alpha)(f_x) \). Note \( L(\alpha)(f_x) \) can be \( i_{f_x} \).

C.4.3 Proof

Before we can prove our main result, we first introduce several lemmas. First, we show that we can define a total order over the set of fences in an execution, even if those fences were issued at different RSC services. Next, we lift this to a total order over all system-facing actions at all services. Finally, we leverage this definitions to order that if processes follow a simple protocol, then the composition of a set of RSC services also satisfies RSC.

Lemma C.9. Given an execution \( \alpha \) of services \( X \) that individually satisfy RSC, for all fences \( f_x \), \( L(\alpha)(f_x) \xrightarrow<_{\alpha} r_{f_x} \), where \( \leq_{\alpha} \) is the strict total order of actions defined by \( \alpha \).

Proof. We prove by contradiction, so suppose \( r_{f_x} \xrightarrow<_{\alpha} L(\alpha)(f_x) \). Then clearly, \( L(\alpha)(f_x) \notin i_{f_x} \). Since \( r_{f_x} \xrightarrow<_{\alpha} L(\alpha)(f_x) \), \( r_{f_x} \xrightarrow<_{\alpha} L(\alpha)(f_x) \) by the definition of \( \leq_{\alpha} \). But then by the definition of \( \leq_{\alpha} \), \( r_{f_x} \xrightarrow<_{S_x} L(\alpha)(f_x) \), contradicting the definition of \( L(\alpha)(f_x) \).

Lemma C.10. Given an execution \( \alpha \) of services \( X \) that individually satisfy RSC, let \( f_x \) and \( f_y \) be two fences in \( S_X \). If \( r_{f_x} \xrightarrow<_{S_x} i_{f_y} \), then \( L(\alpha)(f_x) \leq_{\alpha} L(\alpha)(f_y) \), where \( \leq_{\alpha} \) extends \( \leq_{\alpha} \) in the natural way.

Proof. Since \( r_{f_x} \xrightarrow<_{S_x} i_{f_y} \), \( P(\alpha)(f_x) \subseteq P(\alpha)(f_y) \) by the definition of \( P(\alpha) \). Thus, either \( L(\alpha)(f_x) = L(\alpha)(f_y) \) or there is some later invocation action in \( P(\alpha)(f_y) \) with a later invocation.

As a result, \( L(\alpha)(f_x) \leq_{\alpha} L(\alpha)(f_y) \).

Lemma C.11. \( \leq \) is a strict total order.

Proof. We must prove \( \leq \) is irreflexive, transitive, and total. Irreflexivity and totality follow from the definitions of \( \leq \) and \( \leq_{\alpha} \). We now show \( \leq \) is transitive.

Let \( f_x, f_y, \) and \( f_z \) be fences such that \( f_x \leq f_y \) and \( f_y \leq f_z \). We must show \( f_x \leq f_z \). There are four cases:

If \( x = y = z \), then the transitivity of \( \leq_{S_x} \) implies \( r_{f_x} \leq_{S_x} i_{f_y} \), so \( f_x \leq f_z \). If \( x = y \neq z \), then since \( x = y \) and \( f_x \leq f_y \), \( L(\alpha)(f_x) \leq_{\alpha} L(\alpha)(f_y) \) by the Lemma C.10. By the definition of \( \leq \), since \( y \neq z \), \( L(\alpha)(f_y) \leq_{\alpha} L(\alpha)(f_z) \), so \( L(\alpha)(f_x) \leq_{\alpha} L(\alpha)(f_z) \) and \( f_x \leq f_z \). Similar reasoning applies to the case where \( x \neq y \neq z \). Finally, if \( x \neq y \neq z \), then by the definition of \( \leq \) and the transitivity of \( \leq_{\alpha} \), \( L(\alpha)(f_x) \leq_{\alpha} L(\alpha)(f_y) \). Clearly if \( x \neq z \), then \( f_x \leq f_z \). Further, if \( x = z \), then by the contrapositive of Lemma C.10, \( i_{f_x} \leq_{S_x} r_{f_z} \). Then since \( S_x \) is a sequence of invocation-response pairs, \( r_{f_x} \leq_{S_x} i_{f_z} \), so \( f_x \leq f_z \).

\( \leq \) defines a strict total order over the fences in executions involving multiple RSC services. To extend this to a total order over all system-facing actions, we first define a system-facing action’s next fence, denoted \( n_f(\pi) \). Specifically, given an execution \( \alpha \) of services \( X \) that individually satisfy RSC, a RSC service \( x \in X \), and a system-facing action \( \pi_x \in S_x \), define \( n_f(\pi_x) \) as the earliest fence \( f_x \) such that \( \pi_x \xrightarrow<_{S_x} r_{f_x} \). To ensure \( n_f(\pi) \) is defined for all \( \pi \), we assume \( \alpha \) is augmented with a sequence of fence invocation-response pairs \( \pi \rightarrow r \), one for each \( x \in X \), that are added to the end of \( \alpha \).

We use next fences to lift \( \leq \) to all system-facing actions. Let \( \alpha \) be an execution of services \( X \) that satisfy RSC individually;
We now prove that $S$ respects causality, so let $\pi_1$ and $\pi_2$ be system-facing actions such that $\pi_1 \prec_\alpha \pi_2$. To start, we only consider the case where there is no system-facing action $\pi_3$ such that $\pi_1 \prec_\alpha \pi_3$ and $\pi_3 \prec_\alpha \pi_2$.

If both actions are on the same service $x$, then since $\pi_1 \prec_\alpha \pi_2$ and $S_x$ satisfies RSC, $\pi_1 \leq_{S_x} \pi_2$. By Lemma C.13, $\pi_1 <_{S_x} \pi_2$.

Now suppose $\pi_1$ is on service $x$ and $\pi_2$ is on service $y$ with $x \neq y$. Since processes issue fences between interactions with different services, it must be the case that $\pi_1 = r_{f_x}$ for some fence $f_x$ and $\pi_2$ is an invocation action on $y$. By the definition of $n_{fa}$, $n_{fa}(\pi_1) = \pi_1$. Further, by Lemma C.9, $L_\alpha(f_x) <_a r_{f_x} = \pi_1$.

Since $\pi_1$ and $\pi_2$ are on different services, $\pi_2$ cannot be part of an operation whose return value includes the effects of $\pi_1$’s operation (i.e., $f_x$). Combining this with the assumption that $\pi_1 \prec_\alpha \pi_2$, we get $\pi_1 <_{\alpha} \pi_2$.

Let $f_y = n_{fa}(\pi_2)$. By the definitions of $L_\alpha$ and $n_{fa}$, it must be the case that $\pi_2 <_a L_\alpha(f_y)$; either $\pi_2 = L_\alpha(f_y)$ or there is some later last invocation. Combining this with the facts that $L_\alpha(f_x) <_a \pi_1$ and $\pi_1 <_{\alpha} \pi_2$, we see that $L_\alpha(f_y) <_a L_\alpha(f_y)$. By the definitions of $<$ and $<_\alpha$, $\pi_1 <_\alpha \pi_2$.

We now consider transitivity, so suppose there is some system-facing action $\pi_3$ such that $\pi_1 \prec_\alpha \pi_3$ and $\pi_3 \prec_\alpha \pi_2$. By the reasoning above, $\pi_1 <_\alpha \pi_3$ and $\pi_3 <_\alpha \pi_2$. Then by Lemma C.12, which shows $<$ is transitive, we conclude that $\pi_1 <_\alpha \pi_2$.

Thus, $S$ satisfies the second requirement of RSC.

Finally, since $S$ respects causality and by Lemma C.13, generalizes each $S_x$, it is clear that $S$ respects the order of system-facing invocations and responses at each process. Thus, $S$ satisfies the first requirement of RSC.

### D Proofs of Correctness

In Sections 5 and 7, we presented two new protocols: a variant of Spanner that relaxes its consistency from strict serializability [75] to RSS and a variant of Gryff that relaxes its consistency from linearizability [37] to RSC. The designs are agnostic to the structure of the applications using them, but we make some basic assumptions about the application’s runtime depending on the structure of the application.

If a set of clients (e.g., mobile phones) use the services directly and do not communicate outside of the service via message passing, then it is sufficient for each client to simply use the client libraries to communicate with the services. If clients do communicate via message passing (e.g., a mobile phone proxying its requests through multiple Web servers), then as discussed in Section 4.2, some metadata must be propagated between processes executing on different machines to ensure that the services return values reflecting all causal constraints. Fortunately, existing frameworks, such as Baggage Contexts [62], can automatically propagate this metadata between processes.
In the proofs below, we assume this metadata propagation, if necessary, is implemented correctly within the application’s runtime. For Spanner-RSS, this metadata is the minimum read timestamp \( t_{\text{min}} \), and for Gryff-RSC, it is the dependency tuple \( \text{dep} \) that is piggybacked on the next interaction with Gryff-RSC.

### D.1 Spanner-RSS

We begin with three observations about Spanner’s protocol:

**Observation 1.** If a read-write (RW) transaction has committed to a shard with key \( k \) and timestamp \( t_c \), then there cannot be a current or future prepared transaction that writes \( k \) and has a prepare timestamp less than \( t_c \). This follows from Spanner’s use of strict two-phase locking and the fact that each RW transaction chooses its prepare timestamp to be greater than all previously committed writes at each participant shard [22].

**Observation 2.** If two RW transactions conflict, then their commit timestamps cannot be equal. This follows from Spanner’s use of strict two-phase locking and the fact that each RW transaction chooses its commit timestamp to be greater than the prepare timestamp from each participant shard.

**Observation 3.** The commit timestamp of each RW transaction is guaranteed to be between its real start and end times. This is shown in the original paper [22] and is what makes Spanner strictly serializable.

We now prove several supporting lemmas and then use them to prove the correctness of Spanner-RSS. We define a transaction’s timestamp \( t \) as \( t_c \) if it is a RW transaction and \( t_{\text{map}} \) if it is a read-only (RO) transaction. We denote a transaction \( T \)’s timestamp as \( t_T \). We use the transactions’ timestamps to construct a total order. Lemmas D.1 and D.2 prove properties about the timestamps of transactions related by causality or real time, and we use them to show the constructed total order satisfies RSS. Lemmas D.3 and D.4 are used to show the constructed total order is in Spanner-RSS’s sequential specification (i.e., that the order is consistent with the values returned by each transaction’s reads).

#### Lemma D.1. If \( T_1 \) and \( T_2 \) are transactions such that \( T_1 \rightsquigarrow T_2 \), then \( t_{T_1} \leq t_{T_2} \). Further, if \( T_1 \) and \( T_2 \) are both RW transactions, then \( t_{T_1} \leq t_{T_2} \).

**Proof:** We first consider the four pairs of transactions. For each case, we consider the three direct causal relationships: process order, message passing, and reads-from. We then consider transitivity.

\( \text{RO}_1 \rightsquigarrow \text{RO}_2 \). Observe that because the first transaction is RO, it is not possible for the second to read from the first. If the two RO transactions are causally related by process order or message passing, then the second RO transaction’s \( t_{\text{min}} \) will be greater than or equal to the first’s by the assumption that applications propagate the necessary metadata. As a result, line 6 of Algorithm 2 guarantees the second RO transaction will include any writes with \( t_c \leq t_{\text{min}} \), so \( t_{\text{min}} \leq t_{T_2} \). Thus, \( t_{T_1} \leq t_{T_2} \).

\( \text{RO} \rightsquigarrow \text{RW} \). As above, the RW transaction cannot read from the RO transaction. If the two transactions are causally related by process order or message passing, then the RO transaction must precede the RW transaction in real time. Because RW transactions perform commit wait, if a write is returned in a RO transaction, then that write’s commit timestamp is guaranteed to be in the past before the RO transaction ends. As a result, \( t_{T_1} \) is guaranteed to be less than the RO’s end time. Combined with the fact that a RW transaction’s commit timestamp is guaranteed to be after its start time, this implies \( t_{T_1} \leq t_{T_2} \).

\( \text{RW} \rightsquigarrow \text{RO} \). If the RO transaction reads from the RW transaction, then clearly \( t_{T_1} \leq t_{T_2} \) by the way \( t_{\text{map}} \) is calculated (Alg. 1, lines 15-20). If instead the transactions are causally related by process order or message passing, then because a process sets its \( t_{\text{min}} \) to be at least \( t_c \) after a RW transaction finishes, line 6 of Algorithm 2 guarantees the RO transaction includes any writes with \( t'_c \leq t_{T_1} \), and thus \( t_{T_1} \leq t_{T_2} \).

\( \text{RW}_1 \rightsquigarrow \text{RW}_2 \). If the two RW transactions are causally related by process order or by message passing, then RW1 must precede RW2 in real time. By observation 3 above about Spanner’s RW transactions (and thus Spanner-RSS’s), it must be that \( t_{T_1} < t_{T_2} \). If RW2 reads from RW1, then the two transactions conflict and Spanner-RSS’s use of strict two-phase locking guarantees \( t_{T_1} < t_{T_2} \).

We now consider transitivity. Clearly, if \( t_{T_1} \leq t_{T_2} \) holds for each pair of causally related transactions, then \( t_{T_1} \leq t_{T_2} \) applies for pairs of transactions causally related through transitivity. Further, because in the cases two and four above we have shown \( t_{T_1} < t_{T_2} \), it must be that \( t_{T_1} < t_{T_2} \) for pairs of RW transactions causally related through transitivity.

#### Lemma D.2. If \( T_1 \) is a RW transaction and \( T_2 \) is a conflicting RO transaction such that \( T_1 \rightsquigarrow T_2 \), then \( t_{T_1} \leq t_{T_2} \).

**Proof:** Since \( T_1 \) ends before \( T_2 \) starts, \( t_{T_1} \) must be less than \( T_2 \)’s start time. Further, line 4 of Algorithm 1 guarantees the RO transaction’s \( t_{\text{read}} \) is greater than its start time, so \( t_{T_1} < t_{\text{read}} \).

As a result, since \( T_1 \) has committed and its earliest end time \( t_{ee} \) has passed, when \( T_2 \) executes at any shard with conflicting keys, lines 6-8 of Algorithm 2 ensure it will read \( T_1 \)’s write or one with a greater timestamp. Thus, \( t_{T_1} \leq t_{T_2} \).

#### Lemma D.3. Suppose \( T_2 \) is a RW transaction that commits with timestamp \( t_{T_2} \). Then for each key, \( T_2 \)'s reads return the values written by the RW transaction with the greatest commit timestamp \( t_{T_1} \) such that \( t_{T_1} < t_{T_2} \).

**Proof:** Since Spanner-RSS’s RW transaction protocol is nearly identical to Spanner’s, this follows from the correctness argument for Spanner, which follows from the correctness of strict two-phase locking [15] and Spanner’s timestamp assignment [22].
Lemma D.4. Suppose \( T_2 \) is a RO transaction with a snapshot time of \( t_2 \). Then for each key, \( T_2 \) returns the values written by the RW transaction with the greatest commit timestamp \( t_1 \) such that \( t_1 \leq t_2 \).

Proof. Let \( T_1 \) be an arbitrary RW transaction that conflicts with \( T_2 \) at keys \( K \). Fix a \( k \in K \), and assume \( t_1 \) is the greatest timestamp for a write of \( k \) such that \( t_1 \leq t_2 \). For ease of exposition, assume each key resides on a different shard.

We say a RO transaction begins executing at a shard once it finishes waiting for \( t_{read} \) to be less than the Multi-Paxos maximum write timestamp (i.e., it reaches line 5 of Algorithm 2). There are three cases. In each case, we either derive a contradiction or show that \( T_2 \) returns \( T_1 \)'s write of \( k \).

First, suppose there is at least one \( k' \in K \) such that \( T_1 \) has not prepared at \( k' \)'s shard when \( T_2 \) begins executing there. Then combining the facts that \( \tau_{map} \leq \tau_{read} \), \( T_2 \) waits until \( \tau_{read} \leq \text{MAXWRITE}_T \), and \( T_1 \)'s prepare timestamp at each shard is chosen to be strictly greater than the shard's \( \text{MAXWRITE}_T \), \( T_1 \)'s prepare timestamp \( \tau_p \) at the shard must be strictly greater than \( \tau_{read} \). Since \( \tau_{map} \leq \tau_{read} \) and \( \tau_p \leq \tau_{read} \), this contradicts the assumption that \( t_1 \leq t_2 \).

Next, for each \( k \in K \), suppose \( T_1 \) has prepared but not committed at \( k \)'s shard when \( T_2 \) begins executing there. Because \( \tau_p \leq t_1 \) and \( t_1 \leq t_2 \), \( \tau_p \leq t_2 \). Further since \( t_2 \leq \tau_{read} \), \( \tau_p \leq \tau_{read} \), so lines 5, 9, and 10 of Algorithm 2 ensure \( T_1 \)'s prepare timestamp is returned to \( T_2 \)'s client.

Since \( \tau_p \leq t_1 \leq \tau_{map} = t_2 \) by assumption, lines 22 and 23 of Algorithm 1 ensure the client waits until \( T_1 \) commits. Once \( T_1 \) commits, lines 13-15 of Algorithm 2 and lines 10-11 of Algorithm 1 transmit \( T_1 \)'s values to \( T_2 \)'s client. Since \( t_1 \) was assumed to be the greatest timestamp for key \( k \) such that \( t_1 \leq t_2 \), line 13 of Algorithm 1 returns \( T_1 \)'s write of \( k \).

Finally, suppose \( T_1 \) has prepared at all shards containing keys \( K \) and further, there is at least one \( k' \in K \) such that \( T_1 \) has committed at \( k' \)'s shard when \( T_2 \) begins executing there. There are two sub-cases: If \( T_1 \) has not committed at the shard containing \( k \) when \( T_2 \) begins executing there, then by similar reasoning as in the previous case, \( T_1 \)'s write of \( k \) will ultimately be sent to \( T_2 \)'s client and returned. Now suppose \( T_1 \) has committed at \( k \)'s shard when \( T_2 \) begins executing there. We argue that line 8 of Algorithm 2 must return \( T_1 \)'s write.

Since line 8 returns the latest write with timestamp less than \( \tau_{read} \), the only other possibility is that it returns some write with a timestamp \( t_3 > t_1 \). In this case, since \( T_1 \) already committed, its write would never be returned to \( T_2 \)'s client. As a result, the value of \( \tau_{earliest} \) calculated for key \( k \) (Alg. 1, line 18) will be at least \( t_3 \), so by line 19 of Algorithm 1, \( \tau_{map} \) would ultimately be at least \( t_3 \), contradicting the assumption that \( t_1 \) is the write of \( k \) with the greatest timestamp less than or equal to \( \tau_{map} = t_2 \). Thus, line 8 of Algorithm 2 must return \( T_1 \)'s write of \( k \). By observation 1, there are no prepared transactions that write \( k \) with timestamps less than \( t_1 \), so line 13 of Algorithm 1 ultimately returns \( T_1 \)'s write.

Theorem D.5. Spanner-RSS guarantees RSS.

Proof. Let \( \alpha_1 \) be a well-formed execution of Spanner-RSS. We first construct a sequence \( S \) of transaction invocations and responses from \( \alpha_1 \). We then use \( S \) to extend \( \alpha_1 \) to \( \alpha_2 \) such that \( S \) is equivalent to \( \text{complete}(\alpha_2) \) and finally, show \( S \) satisfies RSS.

To start, define a RW transaction as complete if it has committed at its coordinator and a RO transaction as complete if it has returned to its client. Further, using Observation 2 and the second part of Lemma D.1, observe that the set of transactions with a given timestamp \( t \) comprises a set of non-conflicting, causally unrelated RW transactions and for each RW transaction, a set of causally related RO transactions. Thus, the set of transactions with a given timestamp \( t \) can be arranged into a set of directed acyclic graphs (DAGs). Each DAG's vertices are a RW transaction and its causally related RO transactions, and each DAG's edges are defined by \( \prec \) between the transaction vertices. The directed graphs are acyclic because \( \prec \) is acyclic.

Using this observation, we define a strict total order \( \prec \) over pairs of complete transactions \( T_1 \) and \( T_2 \) in \( \alpha_1 \). Two steps: First, order the sets of transactions according to their timestamps \( t \). Second, for each set of transactions, choose an arbitrary order for its DAGs and then within each DAG, topologically sort the transactions.

To show \( \prec \) is a strict total order, we must show it is irreflexive, total, and transitive. Irreflexivity follows from the irreflexivity of \( \prec \) and the fact that a transaction can only belong to one set (since it only has one timestamp). Totality follows from the fact that timestamps are totally ordered, the fact that the arbitrary order of DAGs is chosen to be total, and the fact that the directed graphs of transactions are acyclic. We now show \( \prec \) is transitive.

Let \( T_1 \), \( T_2 \), and \( T_3 \) be three transactions with timestamps \( t_1 \), \( t_2 \), and \( t_3 \) such that \( T_1 \prec T_2 \) and \( T_2 \prec T_3 \). We show \( T_1 \prec T_3 \).

There are four cases. (1) If \( t_1 < t_2 \) and \( t_2 < t_3 \), then clearly \( t_1 < t_3 \) and \( T_1 \prec T_3 \). Similarly, (2) if \( t_1 < t_2 \) and \( t_2 = t_3 \) or (3) if \( t_1 = t_2 \) and \( t_2 < t_3 \), \( T_1 \prec T_3 \). For the case (4) where \( t_1 = t_2 = t_3 \), there are four sub-cases:

(a) If \( T_1 \prec T_2 \) and \( T_2 \prec T_3 \), then \( T_1 \prec T_3 \), and \( T_1 \) and \( T_3 \) are in the same DAG of causally related transactions with the same timestamp. By the transitivity of \( \prec \), \( T_1 \prec T_3 \), so \( T_1 \) will be topologically sorted before \( T_3 \).

(b) Now suppose \( T_1 \prec T_2 \) and \( T_2 \not\prec T_3 \). \( T_1 \) and \( T_2 \) are in the same DAG, but \( T_3 \) is in a different DAG. Furthermore, because \( T_2 < T_3 \) by assumption, \( T_2 \)'s (and \( T_1 \)'s) DAG is ordered before \( T_3 \)'s. Thus, \( T_1 \prec T_3 \).

(c) Similar reasoning applies to the case where \( T_1 \not\prec T_2 \) and \( T_2 \prec T_3 \).

(d) Finally, if \( T_1 \not\prec T_2 \) and \( T_2 \not\prec T_3 \), all three transactions are in different DAGs. Since \( T_1 < T_2 \) and \( T_2 < T_3 \), \( T_1 \)'s DAG must follow \( T_2 \)'s and thus \( T_1 \)'s, so \( T_1 \prec T_3 \).
Thus, \(<\) is a strict total order.

Let \(S\) be the sequence of transaction invocations and responses defined by \(<\). \(a_1\), however, may not contain some responses that are in \(S\), in particular, those of committed RW transactions whose response did not yet reach the client.

We thus construct \(a_2\) by extending \(a_1\) with responses for these RW transactions. For each key \(k\) read by a RW transaction whose response must be added, let the returned value be that of the most recent write of \(k\) that precedes it in \(S\). Then let \(a_2\) be the extension of \(a_1\) with any necessary response actions with these return values.

To conclude the proof, we first show that \(S\) is in Spanner-RSS’s sequential specification (i.e., the order is consistent with the values returned by each transaction’s reads) and then that it satisfies RSS.

To show \(S\) is in Spanner-RSS’s sequential specification, we use Lemmas D.3 and D.4. First, observe that since a read in Spanner-RSS can only return a transaction’s write after the transaction has committed, any transaction whose writes have been observed will be complete, have a commit timestamp, and thus be ordered by \(<\).

Let \(T_2\) be a RW transaction. Since \(<\) orders the transactions according to their timestamps \(t\), by Lemma D.3, \(T_2\)’s reads include the writes of all transactions \(T_1\) such that \(T_1 < T_2\).

Now let \(T_2\) be a RO transaction. By Lemma D.4, \(T_2\)’s reads reflect all the writes of all RW transactions \(T_1\) such that \(t_1 < t_2\). Further, by Observation 2 about conflicting RW transactions and Lemma D.4, if \(T_1\) is a conflicting RW transaction such that \(t_1 = t_2\), then \(T_2\) reads from \(T_1\), so \(T_1 \rightsquigarrow T_2\).

Then by the definition of \(<\), \(T_1 < T_2\). Thus, the sequence \(S\) is in Spanner-RSS’s sequential specification. We now show it satisfies the three requirements of RSS:

1) By construction, \(S\) contains the same invocations and responses as \(a_2\), which extends \(a_1\) with zero or more response actions. Further, as we show below, \(S\) respects causality, which subsumes the clients’ process orders. Thus, \(S\) is equivalent to complete \((a_2)\).

2) Consider two transactions \(T_1\) and \(T_2\). Assume that \(T_1 \rightsquigarrow T_2\). Lemma D.1 implies that \(t_1 \leq t_2\). If \(t_1 < t_2\), then \(T_1 < T_2\) because \(< \) is first defined on the order of the timestamps of transactions. Otherwise, if \(t_1 = t_2\), then \(T_1\) and \(T_2\) are in the same DAG of causally related transactions. The topological sort of the DAG ensures that \(T_1 < T_2\).

3) By Observation 3, it is clear that if \(T_1\) and \(T_2\) are RW transactions and \(T_1 \rightarrow T_2\), \(t_1 < t_2\), so \(T_1 < T_2\). Further, by Lemmas D.2 and D.4, if \(T_1\) is a RW transaction and \(T_2\) is a conflicting RO transaction, then \(T_2\)’s conflicting reads will return \(T_1\)’s writes or newer versions.

As a result, either \(t_1 = t_2\) and \(T_1 \rightsquigarrow T_2\) or \(t_1 < t_2\). In either case, \(T_1 < T_2\).

D.2 Gryff-RSC

Unless stated otherwise, we consider an arbitrary, well-formed execution \(\sigma\) of a set of application processes interacting with a Gryff-RSC service. In a slight abuse of notation, we define \(o_1 \rightsquigarrow\_\sigma\ o_2\) to mean that \(o_1\)’s response causally precedes \(o_2\)’s invocation, and we define \(o_1 \rightarrow\_\sigma\ o_2\) similarly. For simplicity, we assume real-time values are unique. To reason about the order of operations in Gryff-RSC, we first introduce several definitions:

Given an operation \(o\), we define its decision point, denoted \(dp(o)\), as the time at which the last replica in its first-round quorum chooses a carstamp. If \(o\) is a write or rmw, then \(o\)’s visibility point, denoted \(vp(o)\), is the earliest time at which its write is applied to one replica, and \(o\)’s propagation point, denoted \(pp(o)\), is the earliest time that \(o\)’s write is applied to a quorum of replicas. The latter can occur either as part of \(o\)’s protocol or through dependency propagation.

Given an execution \(\sigma\), we define an operation \(o\) as complete as follows: If \(o\) is a write or rmw, at least one replica has applied its key-value-carstamp tuple (while processing a Write2 message or executing a rmw command, respectively). Note that by definition, all complete operations have a decision point, and all complete writes and rmws have a visibility point. However, not all complete writes and rmws have a propagation point. Unless specified otherwise, we henceforth only consider complete operations.

Recall that each write or read-modify-write (rmw) in Gryff-RSC has a unique carstamp. Further, a read’s carstamp is equal to the carstamp of the write or rmw it reads from. Using these observations, we can define a total order over the operations to a single object \(x \in X\).

Let \(o_1\) and \(o_2\) be complete operations on object \(x \in X\) with carstamps \(cs_1\) and \(cs_2\). We define a strict total order \(<\) as follows: if \(cs_1 \neq cs_2\), then \(o_1 < x o_2\) if and only if \(cs_1 < cs_2\); otherwise, \(cs_1 = cs_2\), and \(o_1 < x o_2\) if and only if \(dp(o_1) < dp(o_2)\).

Lemma D.6. \(<\) is a strict total order.

Proof. We must show \(<\) is irreflexive, total, and transitive. Irreflexivity follows from the fact that each operation has one carstamp and the irreflexivity of \(<\). Totality follows from the fact that carstamps and real-time values are totally ordered. We now show \(<\) is transitive.

Let \(o_1, o_2, o_3\) be three complete operations with carstamps \(cs_1, cs_2, cs_3\) such that \(o_1 < x o_2\) and \(o_2 < x o_3\). We must show \(o_1 < x o_3\).

If \(o_1 < x o_2\) because \(cs_1 < cs_2\) and \(o_2 < x o_3\) because \(cs_2 < cs_3\), then clearly \(o_1 < x o_3\). Similarly, if \(cs_1 < cs_2\) and \(cs_2 = cs_3\) or if \(cs_1 = cs_2\) and \(cs_2 < cs_3\), then \(o_1 < x o_3\). Finally, if \(cs_1 = cs_2 = cs_3\), then \(o_1 < x o_2\) implies \(dp(o_1) < dp(o_2)\) and \(o_2 < x o_3\) implies \(dp(o_2) < dp(o_3)\). Thus, \(dp(o_1) < dp(o_3)\). This implies \(o_1 < x o_3\).
Lemma D.8. The sequence $S_x$ defined by $<_x$ over the invocation-response pairs of complete operations to object $x \in X$ is in the sequential specification $\Sigma_x$.

Proof. Consider a read $r$ and let $w$ be the write or rmw that $r$ reads from. Then by Gryff-RSC’s protocol, $cs_r = cs_w$. Since $r$ reads from $w$, it must the case that $vp(w) < dp(r)$. Otherwise, $w$ would not have been applied at any replica when $r$ read from the replica. By the definition of $dp(w)$, $dp(w) < vp(w)$, so $dp(w) < dp(r)$. Since $cs_r = cs_w$, and $dp(w) < dp(r)$, $w < r$. Further, since writes and rmws have unique carstamps, there does not exist any other write or rmw $w'$ with $cs_w = cs_{w'}$. This implies that for all other writes or rmws $w'$, either $w'< w < r$ or $w < r < w'$. Since $S_x$ is the sequence of invocation-response pairs defined by $<_x$, the same holds for $r$ in $S_x$.

Now consider a rmw $rmw$. Let $w$ be the write or rmw that $rmw$ reads from. We proceed by contradiction.

Assume for a contradiction that there exists a write or rmw $w'$ such that $w <_x w' < rmw$. Since $w <_x w' < rmw$, the definition of $<_x$ implies that $cs_w < cs_{w'} < CS_{rmw}$. Cases 3.2.2, 3.2.3, and 3.2.4 in the proof of Lemma B.10 from the Gryff proof of correctness [18] show that such an ordering of carstamps is impossible when carstamps are assigned to writes and rmws as in Gryff. Since Gryff-RSC uses the same process for carstamp assignment, this impossibility is a contradiction resulting from the earlier assumption. Since $S_x$ is the sequence of invocation-response pairs defined by $<_x$, there is thus also no $w'$ such that $w <_x w' < rmw$. ■

Given an execution $\alpha$, we define $\leadsto^*_{\alpha} \subseteq \leadsto_{\alpha}$ as the relation that omits the reads-from-case. $\leadsto^*_{\alpha}$ thus also omits any pairs derived transitively using one or more reads-from-pairs. We prove three useful lemmas involving $\leadsto^*_{\alpha}$.

Lemma D.9. Given operations $o_1$ and $o_2$ such that $o_1 \leadsto^*_{\alpha} o_2$, $dp(o_1) < dp(o_2)$.

Proof. Since $\leadsto^*_{\alpha}$ omits the reads-from-case, $o_2$ must causally follow $o_1$ through some sequence of one or more actions related by process order or message passing. As a result, for any pair of adjacent operations $o_i, o_{i+1}$ in this sequence, $o_i \rightarrow_{\alpha} o_{i+1}$. The transitivity of $\rightarrow_{\alpha}$ thus implies $o_1 \rightarrow_{\alpha} o_2$.

By the definition of $dp$, for any operation $o$, $dp(o)$ is between $o$’s invocation and response. Since $o_1 \rightarrow_{\alpha} o_2$, $dp(o_1) < resp(o_1) < inv(o_2) < dp(o_2)$. ■

Lemma D.10. Let $o_1$ be a write or rmw, $o_2$ be a read that reads from $o_1$, and $o_3$ be an operation such that $o_2 \leadsto^*_{\alpha} o_3$. Then $pp(o_1) \leq dp(o_3)$.

Proof. There are two cases.

Case 1. Suppose $o_1$’s write is returned by a quorum of replicas in $o_2$. Then clearly $pp(o_1) < dp(o_2)$. By the definition of $\leadsto^*_{\alpha}$, since $o_2 \leadsto^*_{\alpha} o_3$, $o_2 \rightarrow_{\alpha} o_3$, so $resp(o_2) < inv(o_3)$. Further, by the definition of $dp$, $dp(o_2) < resp(o_2)$ and $inv(o_3) < dp(o_3)$. Together, these inequalities imply $pp(o_1) < dp(o_3)$.

Case 2. Now suppose $o_1$’s write is not returned by a quorum of replicas in $o_2$. Then by Gryff-RSC’s read protocol, $o_2$’s client will store $o_1$’s write as its dependency $d$. Since $o_2 \leadsto^*_{\alpha} o_3$, there must exist some sequence of operations that begins with an operation $o$ such that $o_2 \leadsto^*_{\alpha} o \leadsto^*_{\alpha} \ldots \leadsto^*_{\alpha} o_3$. By the definition of $\leadsto^*_{\alpha}$, $o_2 \rightarrow_{\alpha} o$. There are two sub-cases.

Assume $o = o_3$. If $o_1$ finishes applying its write to a quorum before $dp(o_3)$, then we are done, so suppose not. Since $o_2 \leadsto^*_{\alpha} o_3$ and $o_2$ stored $o_1$’s write as a dependency, Gryff-RSC’s dependency propagation ensures that $o_1$’s write is propagated as a dependency to the process invoking $o_3$. And since Gryff-RSC’s dependency propagation includes $o_1$’s write as part of the first round round messages for $o_3$, $pp(o_1) = dp(o_3)$.

Now assume $o \neq o_3$. By similar reasoning about Gryff-RSC’s dependency propagation, $pp(o_1) \leq dp(o)$. By the definition of $\leadsto^*_{\alpha}$ and the transitivity of $\rightarrow_{\alpha}$, $o \rightarrow_{\alpha} o_3$. Thus, $pp(o_1) \leq dp(o) < resp(o) < inv(o_3) < dp(o_3)$. ■

Lemma D.11. Let $o_1, o_2,$ and $o_3$ be operations such that $o_1 <_x o_2$ for some $x \in X$ and $o_2 \leadsto^*_{\alpha} o_3$. Then $dp(o_1) < dp(o_3)$.

Proof. To start, observe that by the definition of $\leadsto^*_{\alpha}$, since $o_2 \leadsto^*_{\alpha} o_3$, the process executing $o_2$ must have either executed another operation or sent a message after $o_2$. This implies $o_2$ executed its entire protocol. Further, by the reasoning in Lemma D.9, $dp(o_2) < resp(o_2) < inv(o_3) < dp(o_3)$. There are two cases.

Case 1. Assume $o_2$ is a write or rmw. Since $o_1 <_x o_2$ and $o_2$ is a write or rmw, $cs_{o_1} < cs_{o_2}$. Further, since $o_2$ executes its entire protocol, it must be the case that $pp(o_2) < resp(o_2)$. Finally, $dp(o_1) < pp(o_2)$ because otherwise $o_1$ would read $o_2$’s carstamp at at least one replica in its first-round quorum. By Gryff-RSC’s protocol, this would force $cs_{o_2} \leq cs_{o_1}$, contradicting the fact that $cs_{o_1} < cs_{o_2}$. Together, these inequalities imply $dp(o_1) < pp(o_2) < resp(o_2) < inv(o_3) < dp(o_3)$.

Case 2. Assume $o_2$ is a read. There are two sub-cases.

(2a) Assume $o_1$ is also a read. Suppose $o_1$ and $o_2$ read from the same write or rmw. Then $cs_{o_1} = cs_{o_2}$, and by the definition of $<_x$, $dp(o_1) < dp(o_2)$. Combined with the inequality above, $dp(o_1) < dp(o_2) < dp(o_3)$. Now suppose $o_1$ reads from $w_1$ and $o_2$ reads from $w_2 \neq w_1$. Since $o_1 <_x o_2$, $cs_{o_1} = cs_{w_1} < cs_{w_2} = cs_{o_2}$. Assume to contradict that $dp(o_1) < dp(o_2)$. By Lemma D.10, because $o_2$ reads from $w_2$ and $o_2 \leadsto^*_{\alpha} o_3$, $pp(w_2) \leq dp(o_2)$, so $pp(w_2) \leq dp(o_1)$. But then $w_2$ would be applied at least one replica before that replica chooses a carstamp for $o_1$, contradicting that $o_1 <_x o_2$. ■
in \( o_1 \)'s first-round quorum. This implies \( cs_{o_2} = cs_{o_1} \leq cs_{o_1}, \) which contradicts the fact that \( cs_{o_1} < cs_{o_2}. \)

(2b) Assume \( o_1 \) is a write or rmw. Suppose \( o_2 \) reads from \( o_1. \) By the definition of visibility point, \( dp(o_1) < \psi(p(o_1)). \) Since \( o_2 \) read from \( o_1, \) clearly \( \psi(p(o_1)) < dp(o_2). \) Combined with the reasoning above, these inequalities imply \( dp(o_1) < \psi(p(o_1)) < dp(o_2) < dp(o_3). \)

Now suppose \( o_2 \) reads from some \( w_2 \neq o_1. \) Since \( o_1 < x_1, o_2, \) \( cs_{o_1} < cs_{w_2} = cs_{o_2}. \) Assume to contradict that \( dp(o_1) < \psi(p(o_1)) \). By Lemma D.10, because \( o_2 \) read from \( w_2 \) and \( o_2 \sim o', \) \( o_3, pp(w_2) \leq dp(o_3), \) so \( pp(w_2) < dp(o_1). \) But then \( w_2 \) would be applied at least one replica before that replica chooses a carstamp for \( o_1 \) in \( o_1 \)'s first-round quorum. As above, this contradicts the fact that \( cs_{o_1} < cs_{o_2}. \)

Theorem 1. There cannot be two consecutive \( \sim o' \) edges in the cycle. Assume to contradict that there are two consecutive \( \sim o' \) edges \( o_j \sim o' \), \( o_{j+1} \sim o' \). By the transitivity of \( \sim o' \), there must exist an edge \( o_j \sim o' \), \( o_{j+2} \), which forms a shorter cycle \( \ldots, o_j, o_{j+2}, \ldots \). This contradicts our choice of a shortest cycle.

Theorem 2. There cannot be two consecutive \( \sim o_\alpha \) edges in the cycle. By similar reasoning as above, \( o_j \sim o_\alpha, o_{j+1} \sim o_\alpha \) implies the existence of a shorter cycle using the edge \( o_j \sim o_{j+2}. \) This contradicts our choice of a shortest cycle.

Theorem 3. There is at most one \( \sim o_\alpha \) edge in the cycle. Assume to contradict that there are two \( \sim o_\alpha \) edges in the cycle. Let them be \( o_j \sim o_\alpha, o_k \sim o_\alpha, o_\alpha \) re-indexing the cycle if necessary such that \( j < k < \ell. \) By the definition of \( \sim o_\alpha, \) \( \psi(p(o_\alpha)) < inv(o_j) \) and \( \psi(p(o_\alpha)) < inv(o_\alpha). \) It must be the case that \( inv(o_j) < inv(o_\alpha) \); otherwise the edge \( o_j \sim o_\alpha \) would exist, which allows for a shorter cycle \( \ldots, o_i, o_\alpha, o_j \). It also must be the case that \( inv(o_j) < inv(o_\alpha) \), since the contrary would similarly imply the existence of a shorter cycle. Together, however, these inequalities imply that \( \psi(p(o_\alpha)) < inv(o_j) < inv(o_\alpha) < \psi(p(o_\alpha)), \) contradicting the irreflexivity of \( \sim. \)

We are now ready to prove \( <_\phi \) is acyclic. Recall that \( <_\phi \) has three types of edges: \( <_x, \sim o_\alpha, \) and \( \sim o_\alpha. \) By Property 3, there are two cases.

Case 1. Assume there are zero \( \sim o_\alpha \) edges in the cycle. Since \( m \geq 2, \) there are at least two edges in the cycle, and by Property 1, at least one is a \( <_x \) edge. Without loss of generality, re-index the cycle \( o_1, o_2, \ldots, o_m, o_1 \) such that \( o_1 < x, o_2. \) By Properties 1 and 2, the sequence \( o_1, \ldots, o_m, o_1 \) must alternate between \( <_x \) and \( \sim o_\alpha \) starting with \( <_x, \) and ending with \( \sim o_\alpha. \) Lemma D.12 thus implies \( dp(o_1) < dp(o_1), \) contradicting the irreflexivity of \( <. \)

Case 2. Assume there is one \( \sim o_\alpha \) edge in the cycle. Without loss of generality, re-index the cycle \( o_1, o_2, \ldots, o_m, o_1 \) such that \( o_1 \sim o_\alpha, o_2. \) Note that since \( o_1 \sim o_\alpha, o_2, \) \( dp(o_1) < dp(o_2). \) We proceed by cases.

(2a) \( o_m \sim o_\alpha o_1. \) From the re-indexing of the cycle and by the assumptions of this case, the sequence of operations from \( o_2, \ldots, o_m, o_1 \) alternates between \( <_x, \) and \( \sim o_\alpha, \) ending with \( \sim o_\alpha. \) Thus, by Lemma D.12, \( dp(o_2) < dp(o_2), \) contradicting the inequality above.

(2b) \( o_m < x, o_1. \) By the assumptions of this case, the sequence of operations from \( o_2, \ldots, o_m, o_1 \) alternates between \( <_x, \) and \( \sim o_\alpha, \) ending with \( \sim o_\alpha. \) Thus by Lemma D.12, \( dp(o_2) < dp(o_2). \) By the definitions of \( <_x, \sim o_\alpha, \) and \( dp(o_2), \) since \( o_1 \sim o_\alpha, o_2, o_1 \in W' \) and \( pp(o_1) < \psi(p(o_1)) < inv(o_2) < dp(o_2). \) This implies \( pp(o_1) < dp(o_m). \)

Since \( pp(o_1) < dp(o_m), \) \( o_1 \)'s write would be applied at at least one replica when that replica replies with a carstamp
in $o_m$’s first-round quorum. This implies $cs_{o_1} < cs_{o_m}$, which contradicts the assumption that $o_m <_{x_m} o_1$. ■

**Lemma D.14.** A topological sort $S$ of $<_{\psi}$ over complete operations to objects $X$ is in the sequential specification $\prod_{x \in X} S_x$.

**Proof.** Since each operation targets a single object $x \in X$, it suffices to only consider the orders of subsets of operations to each $x$. For a fixed $x$, this order is solely dictated by the partial order $<_x$ in the topological sort of $<_{\psi}$. Lemma D.8 shows that the sequence $S_x$ defined by $<_x$ is in $x$’s sequential specification $S_x$. Since $S$ is a topological sort of $<_{\psi}$, which by definition generalizes each $<_x$, it follows that the entire sequence $S$ is in $\prod_{x \in X} S_x$. ■

**Theorem D.15.** Gryff-RSC guarantees RSC.

**Proof.** Let $\alpha_1$ be a well-formed execution of Gryff-RSC. Extend $\alpha_1$ to $\alpha_2$ by adding a response action for any complete operation $o$ that does not have one in $\alpha_1$.

Let $S$ be a topological sort of $<_{\psi}$ on the operations in $\alpha_2$. Lemma D.14 implies that $S \in \prod_{x \in X} S_x$. We now show that $S$ satisfies the three properties of RSC:

1. By construction, $S$ contains the same invocations and responses as $\alpha_2$, which extends $\alpha_1$ with zero or more response actions. Further, as we show below, $S$ respects $\sim_{\alpha_1}$, which subsumes the clients’ process orders. Thus, $S$ is equivalent to $\text{complete}(\alpha_2)$.

2. Consider two operations $o_1$ and $o_2$ such that $o_1 \sim_{\alpha_1} o_2$. By the definition of $\sim_{\alpha_1}, \sim'_{\alpha_1} \subseteq \sim'_{\alpha_2}$. Further, since the definition of $<_{\psi}$ includes $\sim'_{\alpha_2}, <_x$ (including $o_2$ reading from $o_1$), and their transitive closure, $<_{\psi}$ thus also includes $\sim_{\alpha_1}$. Since $S$ is a topological sort of $<_{\psi}$, $o_1 <_S o_2$.

3. Consider two operations $o_1 \in W$ and $o_2 \in C_{\alpha_1}(o_1) \cup W$ such that $o_1 \rightarrow_{\alpha_1} o_2$. By the definition of $<_{\psi}, o_1 <_{S} o_2$, and since $S$ is a topological sort of $<_{\psi}$, $o_1 <_{S} o_2$. ■