Shape effect on information theoretic measures of quantum heterostructures

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Abstract

Theoretic measures of information entropies like Shannon entropy and Fisher information are studied for multiple quantum well systems (MQWS). The effect of shape and number of wells in the MQWS is explored in detail. The shapes taken are: rectangular, parabolic and V-shape. Onicescu energy is an important tool to study the information content stored in the system, which is also found to depend on shape and number of wells of heterostructures. Statistical measure of complexity also shows noticeable dependence on these parameters.

1. Introduction

It is well known that the physical and optical properties [1–6] of quantum heterostructures are generally determined by the spread of position and momentum densities over the space. These properties are studied only through the expectation values of the spatial and momentum coordinates which provides famous Heisenberg uncertainty relation [7]. The study of quantum systems from an information point of view is an important aspect and it has many applications in the field of quantum computation [8] and information technology [9]. Shannon entropy [10–14] and Fisher information [15–17] are the two major information entropies as they are the best estimator for uncertainty measurements. Standard deviation in position and momentum defines spreading of the wave function with respect to a particular point in the domain i.e. the mean value. But Shannon and Fisher entropies do not depend on particular point in the domain. Also, these two entropies show complementary description of spreading of the wave function. Shannon entropy, Fisher information and standard deviation define the spread of probability density. Shannon entropy gives a global measure of concentration of probability density, but Fisher information gives point wise concentration of probability density. It is important to study the information entropies of quantum systems as they are related to other experimentally measurable parameters such as polarizability, susceptibility and kinetic energy [18] etc.

Information measures play an important role in studying quantum systems. These measures have been widely used in explaining many phenomena in atomic physics [19, 20]. Dehesa et al. [21] have calculated Shannon and Fisher information for Hydrogen atom. Romera and Dehesa [22] have introduced a new correlation measure, which is the product of Shannon entropy power and Fisher information of electron density. Nagy [23] has described the relation between Shannon and Fisher information for 2-e entangled artificial atoms. Shannon information is helpful in the study of electronic structure studies in atoms and molecules [24]. Localization of particle in quantum well can be well understood by information theory. For a highly localized probability density Shannon entropy is negative and it can define the stability of the system [25]. Sears et al. [18] have first described the importance of Fisher information by defining the relation between quantum mechanical kinetic energy and Fisher information. Reginatto [26] has used the same to derive equation of non-relativistic quantum mechanics. Fisher information for atoms with \( Z = 1–102 \) has been very well studied by Romera [27] and Sen et al. [28]. Alongwith various information measures, complexity is a very important measure in quantum information [29, 30]. Concept of statistical complexity was initially introduced...
by Lopez-Ruiz and co-workers [31]. It depends on Shannon information and Onicescu energy. It has been shown that for hydrogen atom even the states with the same energy can have different complexities for different orbital angular momentum [32]. From a quantum mechanical point of view, it is important to study complexity for different quantum systems.

In present work, we have explained standard deviation, Shannon entropy, Fisher information and their associated information lengths in position and momentum space for MQWS. Complexity of these heterostructures is also studied. Although the difference in the measure of complexity is not very pronounced, yet it is interesting to note that the shape of the heterostructures and the number of wells in MQWS change the complexity. Another important quantity in information theory is Onicescu energy as it explains the distance from the uniform distribution. It is also important to note that the inverse of Onicescu energy is directly related to information content stored in the system [33, 34]. Quantum heterostructures have been receiving the attention due to their importance in many interdisciplinary fields, such as in the fields of solid state physics, optoelectronics and quantum information. In particular MQWS are quite interesting structures as their energy spectrum and other properties, e.g. dipole matrix element, polarizability and oscillator strength can be controlled by varying the width of the well and barrier width. The momentum entropies for infinite potential well are studied by Majernik et al [35]. Recently Yahya et al [36] have explained the position and momentum entropies for Poschl Teller type potential potential. For particle in a box [37], hyperbolic potential well [38] and squared tangent potential [13] entropic relations have been studied earlier. Recently, entropic measures for asymmetric double quantum well have been examined by Mukherjee and Roy [39]. But to the best of our knowledge, effect of shape and number of quantum well has not been studied earlier. In particular our purpose in this paper is (i) To give an idea about the information theoretic measures for MQWS. (ii) And how these measures get affected by the shape and the number of wells used.

2. Theory

2.1. Information theory

Shannon entropy in position space in defined as

\[ S_x = -\int \rho(x) \ln \rho(x) \, dx. \]  

(1)

And corresponding Shannon entropy in momentum space is defined as

\[ S_p = -\int \rho(p) \ln \rho(p) \, dp, \]  

(2)

where \( \rho(x) = |\psi(x)|^2 \) and \( \rho(p) = |\phi(p)|^2 \) are the probability density of the system in position and momentum space respectively. \( \psi(x) \) and \( \phi(p) \) are the normalized wave functions in position and momentum space. \( \rho_x(x) = \rho(x) \ln \rho(x) \) and \( \rho_p(p) = \rho(p) \ln \rho(p) \) are the Shannon probability density in position and momentum space respectively. \( S_x \) and \( S_p \) define the uncertainty in localization of particle in position and momentum space respectively. They follow the famous BBM inequality given by [40, 41]

\[ S_x + S_p \geq D(1 + \ln \pi), \]  

(3)

where \( D \) is the spatial dimension.

Fisher information in position space is given by

\[ I_x = \int \frac{d\rho_x(x)}{dx} \left( \frac{d\rho_x(x)}{dx} \right)^2 \, dx, \]  

(4)

whereas in momentum space Fisher information is given by

\[ I_p = \int \frac{d\rho_p(p)}{dp} \left( \frac{d\rho_p(p)}{dp} \right)^2 \, dp. \]  

(5)

As Fisher information is derivative of probability density, so it is a property of locality and it measures the sharpness of probability density. \( \rho_x(x) = \frac{1}{\rho(x)} \frac{d\rho(x)}{dx} \) and \( \rho_p(p) = \frac{1}{\rho(p)} \frac{d\rho(p)}{dp} \) are the Fisher probability density in position space and momentum space respectively.
Standard deviation $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ defines the spreading of probability density with respect to mean value. Standard deviation and Shannon entropy are the global measure of spreading, but Fisher information is a local measure. All the three quantities have different units, so it is difficult to compare them, so it is better to use Shannon length $H_x$ and Fisher length $\delta x$ defined as

\[
H_x = \exp(S_x),
\]

\[
\delta x = \frac{1}{\sqrt{I_x}}.
\]

It is well represented that $H_x$, $\delta x$ and $\Delta x$ follow the following inequalities

\[
\delta x < (\Delta x)_n,
\]

\[
H_x < (2\pi \varepsilon)^{1/2}(\Delta x)_n.
\]

Statistical measure of complexity was proposed by Lopez-Ruiz et al [31]. It is defined as

\[
C_x = H_x D_x,
\]

where $H_x$ is the Shannon length, which is a measure of information and Onicescu energy $D_x$ is a measure of the distance of the system from uniform distribution. In position and momentum space, Onicescu energy is defined as

\[
D_x = \int |\rho(x)|^2 dx.
\]

And

\[
D_p = \int |\rho(p)|^2 dp.
\]

Total information energy $D = D_x D_p$ defines the total information in position and momentum space in the quantum system [30]. Lesser the value of $D$ more is the information stored in the system.

2.2. Multiple quantum well system (MQWS)

Time independent Schrodinger equation for MQWS under effective mass approximation is defined as (atomic units are used, otherwise mentioned)

\[
-\frac{1}{2m^*} \frac{d^2}{dx^2} \psi(x) + V(x) \psi(x) = \varepsilon \psi(x),
\]

where $m^*$ is the effective mass of the electron and $\varepsilon$ is the energy of the system. If $ww$ and $bw$ are the well width and barrier width respectively, and $d = ww + bw$, then the potential profile of the MQWS is given by

\[
V(x) = \sum_{k=0}^{N/2} V(x - kd),
\]

where, $N$ is the number of wells and $V(x - kd)$ for different shapes of MQWS is given

(i) Rectangular well

\[
V(x - kd) = \begin{cases} V_0, & |x - kd| > ww/2 \\ 0, & |x - kd| \leq ww/2 \end{cases}
\]

(ii) Symmetric parabolic well

\[
V(x - kd) = \begin{cases} kx^2, & |x - kd| > ww/2 \\ 0, & |x - kd| \leq ww/2 \end{cases}
\]

where $b = 4V_0/ww^2$.

(iii) Symmetric V-shaped well

\[
V(x - kd) = \begin{cases} (2V_0/ww)(x - d/2), & |x - kd| > ww/2 \\ 0, & |x - kd| \leq ww/2 \end{cases}
\]

For different potentials and for different values of $N$ equation (13) is solved numerically over equally spaced fine grid using 8th order finite difference method. The matrix thus obtained is diagonalized using MATLAB.
commands and corresponding eigenvalues and eigenvectors are obtained in position space. $\psi(x)$ is normalized wavefunction in position space and it is momentum space wavefunction is given by it is Fourier transform

$$\phi(p) = \frac{1}{2\pi} \int \exp(-ipx) \psi(x) dx.$$  

(18)

Using $\psi(x)$ and $\phi(p)$, all information theoretic measures and complexities are calculated numerically and their inequality relations are verified in the next section.

3. Results

In this work we have analyzed the information theoretic measures in position and momentum space with respect to the number of wells as well as the shape of the wells. Different quantum well structures considered here are rectangular, parabolic and symmetric V-shaped (defined by equations (15)–(17)). We present numerical results of the study, as it is very difficult to calculate information entropy analytically due to logarithmic term and derivative terms involved in it. The wave function of the system in position space is calculated by diagonalization of the eigenvalue equation resulting from equation (13). So information entropies are obtained by numerically solving Schrodinger equation using 8th order finite difference method over an equally spaced grid. We have used the same method and MATLAB codes previously to find energy levels of confined rotor [42] and also for confined Harmonium [43], which guarantees the accuracy of our results. We have used GaAs/Ga$_{1-d}$Al$_d$As MQWS. Al concentration is kept constant throughout the paper, i.e. $d = 0.25$. The well width $w$ and barrier width $d$ are kept 2.83 $\times$ 35Å and 2.83 $\times$ 22Å respectively, whereas well height $V_0$ is taken to be 300 meV. Shannon entropy, Fisher information and their respective lengths, standard deviation and Onicescu energy are calculated using equations (1)–(12). In order to understand the shape effect, we have shown the energy spectrum of MQWS system for $N = 2, 5$ and $10$ in table 1. As can be seen with the change of shape the energies change quite significantly. The levels are almost degenerate (degeneracy $= N$).

3.1. Shannon entropy

Next, we have presented Shannon entropy in position and momentum space in table 2. The results are presented for low lying five levels. For MQWS with higher number of wells, Shannon entropy in position space gets increased and in momentum space it gets decreased. It can be explained as by increasing the number of wells stability of particle increases and it cannot escape from the well. V-shaped has minimum Shannon entropy in position space and rectangular has maximum for most of the states. It is due to the change in the probability density of the system as the shape of the system changes. Also total Shannon entropy $S_p + S_q$ follows the BBM inequality (defined by equation (3)), which verifies our results. Characteristics feature of Shannon probability density for ground state and 1st excited state in position space and in momentum space are shown in figures 1 and 2 respectively. It has the same importance in information technology as probability density in quantum mechanics. For all the shapes and for any value of $N$, the Shannon probability density is symmetric in position space as well as in momentum space. This is due to the symmetric nature of quantum wells.

| $N$ | $n$ | Rectangular | Parabolic | V-shaped |
|-----|-----|-------------|-----------|----------|
| 2   | 1   | 34.55177    | 82.49923  | 129.49768|
|     | 2   | 34.94869    | 82.70819  | 129.80120|
|     | 3   | 133.22948   | 236.06182 | 280.24807|
|     | 4   | 136.03584   | 241.36659 | 290.89970|
|     | 5   | 270.95087   | 333.42248 | 345.37041|
| 5   | 1   | 34.37494    | 82.42113  | 129.38208|
|     | 2   | 34.51993    | 82.49797  | 129.49449|
|     | 3   | 34.71845    | 82.60263  | 129.64634|
|     | 4   | 34.91746    | 82.70693  | 129.79802|
|     | 5   | 35.06344    | 82.78303  | 129.90771|
| 10  | 1   | 34.32126    | 82.40148  | 129.35395|
|     | 2   | 34.39053    | 82.42615  | 129.38950|
|     | 3   | 34.43992    | 82.46507  | 129.44598|
|     | 4   | 34.55724    | 82.51508  | 129.51847|
|     | 5   | 34.60563    | 82.57211  | 129.60238|
3.2. Fisher information

The information theoretic measures, calculated in this work have been carried out for many time independent potentials. However studies related to Fisher information has been scarce. Romera et al [44] introduced uncertainty relation for quantum systems in central potentials and other quantum mechanical systems such as oscillators. Fisher information, which explains the local spreading of the wave function is presented in table 3.

Fisher information in position and momentum space is presented for $N = 2, 5$ and $10$ number of wells. V-shaped MQWS has maximum while rectangular MQWS has minimum Fisher information in position space, which means V-shaped MQWS has more localized and sharpened probability density in position space. Actually

Table 2. Shannon entropy for rectangular, parabolic and V-shaped MQWS for $N = 2, 5$ and $10$. Results are expressed in atomic unit.

| N  | n  | $S_x$ | $S_p$ | $S_x + S_p$ |
|----|----|-------|-------|-------------|
| 2  | 1  | 5.68277 | -3.13868 | 2.54409 |
| 2  | 2  | 5.66523 | -3.12272 | 2.54251 |
| 3  | 3  | 6.27337 | -3.08749 | 3.18588 |
| 4  | 4  | 6.19106 | -3.09733 | 3.09373 |
| 5  | 5  | 6.20820 | -3.04940 | 3.15880 |
| 5  | 1  | 6.46029 | -3.91604 | 2.54426 |
| 2  | 2  | 6.37613 | -3.40499 | 2.97114 |
| 3  | 3  | 6.08236 | -3.52021 | 2.50035 |
| 4  | 4  | 6.58583 | -3.37981 | 2.97877 |
| 5  | 5  | 6.44787 | -3.88780 | 2.56007 |
| 10 | 1  | 7.08388 | -4.54577 | 2.53811 |
| 2  | 2  | 7.08133 | -4.11932 | 2.96200 |
| 3  | 3  | 7.07900 | -4.05027 | 3.02873 |
| 4  | 4  | 7.07478 | -3.87307 | 3.20172 |
| 5  | 5  | 7.09032 | -3.92484 | 3.16548 |

| N  | n  | $S_x$ | $S_p$ | $S_x + S_p$ |
|----|----|-------|-------|-------------|
| 2  | 1  | 5.69787 | -3.15997 | 2.53791 |
| 2  | 2  | 5.68655 | -3.14901 | 2.53754 |
| 3  | 3  | 6.16147 | -2.86070 | 3.30078 |
| 4  | 4  | 6.06585 | -2.97929 | 3.08656 |
| 5  | 5  | 6.19487 | -3.02348 | 3.17139 |
| 10 | 1  | 6.9740 | -3.92001 | 2.55739 |
| 2  | 2  | 6.9114 | -3.51577 | 2.87537 |
| 3  | 3  | 6.9986 | -3.42274 | 2.67619 |
| 4  | 4  | 6.7381 | -3.49826 | 2.88155 |
| 5  | 5  | 6.4673 | -3.89959 | 2.56776 |
| 10 | 1  | 7.09536 | -4.55920 | 2.53616 |
| 2  | 2  | 7.09548 | -4.14501 | 2.95047 |
| 3  | 3  | 7.09449 | -3.99808 | 3.09641 |
| 4  | 4  | 7.09138 | -3.94059 | 3.15079 |
| 5  | 5  | 7.09934 | -3.88887 | 3.21048 |

Figure 1. Variation of Shannon probability density for $n = 1$ and $n = 2$ in position space with $x$ for rectangular, parabolic and V-shaped MQWS. $N = 2, 5$ and $10$ and all values are in atomic units.

### 3.2. Fisher information

The information theoretic measures, calculated in this work have been carried out for many time independent potentials. However studies related to Fisher information has been scarce. Romera et al [44] introduced uncertainty relation for quantum systems in central potentials and other quantum mechanical systems such as oscillators. Fisher information, which explains the local spreading of the wave function is presented in table 3. Fisher information in position and momentum space is presented for $N = 2, 5$ and $10$ number of wells. V-shaped MQWS has maximum while rectangular MQWS has minimum Fisher information in position space, which means V-shaped MQWS has more localized and sharpened probability density in position space.
Shannon entropy and Fisher information have shown the complementary description of spreading of the wave function, which is clear from tables 2 and 3. Fisher information in momentum space has very high value. To check the correctness of our method used, we have also calculated the Fisher information in position and momentum space for double quantum well defined by Mukherjee et al [45] and our calculation match with their results (not shown in the paper). And recently, Mukherjee et al [46] have presented quantitative results on Fisher information in both the position and momentum space for confined hydrogen like atoms. Their results also show that with confinement Fisher information in one of the space is quite large compared to Fisher information in another.

![Figure 2](image_url). Variation of Shannon probability density for $n = 1$ and $n = 2$ in momentum space with $p$ for rectangular, parabolic and V-shaped MQWS. $N = 2, 5$ and $10$ and all values are in atomic units.

| $N$ | $n$ | $I_x$ | $I_p$ | $I_x$ | $I_p$ | $I_x$ | $I_p$ |
|-----|-----|-------|-------|-------|-------|-------|-------|
| 2   | 1   | 0.000812677 | 57259.71232 | 0.00071584 | 90836.19419 | 0.000514983 | 100534.542 |
| 2   | 2   | 0.000838075 | 75277.62129 | 0.000789289 | 91199.00711 | 0.000541887 | 394616.025 |
| 3   | 1   | 0.00119255  | 91972.44155 | 0.001751071 | 94417.58739 | 0.001842976 | 173876.329 |
| 4   | 1   | 0.001732721 | 133678.3026 | 0.002108372 | 105411.3824 | 0.002031084 | 121715.944 |
| 5   | 1   | 0.001788972 | 323257.0115 | 0.001694862 | 95938.92615 | 0.002918726 | 498818.327 |
| 2   | 1   | 0.000802995 | 385180898  | 0.000764879 | 382293.4119 | 0.000504546 | 740248.116 |
| 2   | 2   | 0.000812335 | 10202071.8  | 0.000771387 | 790307.964  | 0.000514325 | 406110.852 |
| 3   | 1   | 0.000825063 | 4193100199  | 0.000780255 | 825713.2751 | 0.000527754 | 857089.963 |
| 4   | 1   | 0.000837731 | 233152380.3 | 0.000789091 | 801545.8528 | 0.000541258 | 1778967.021 |
| 5   | 1   | 0.000846966 | 122643758.5 | 0.000795336 | 391013.811 | 0.000551189 | 442611.558 |
| 10  | 1   | 0.000800569 | 10407937.72 | 0.000763264 | 2412034.556 | 0.000502509 | 963870.625 |
| 2   | 1   | 0.00083549  | 121458470.1 | 0.000765346 | 569505.672  | 0.000509048 | 1862514.977 |
| 3   | 1   | 0.000808251 | 22747234.05 | 0.000768634 | 4583420.166 | 0.000510385 | 3656795.881 |
| 4   | 1   | 0.000814305 | 7512741.793 | 0.000772862 | 5106844.48  | 0.000517899 | 1089769.28 |
| 5   | 1   | 0.000821216 | 8774479.552 | 0.000777691 | 9676826.077 | 0.000523363 | 5507738.41 |

Table 3. Fisher information for rectangular, parabolic and V-shaped MQWS for $N = 2, 5$ and $10$. Results are expressed in atomic unit.

Shannon entropy and Fisher information have shown the complementary description of spreading of the wave function, which is clear from tables 2 and 3. Fisher information in momentum space has very high value. To check the correctness of our method used, we have also calculated the Fisher information in position and momentum space for double quantum well defined by Mukherjee et al [45] and our calculation match with their results (not shown in the paper). And recently, Mukherjee et al [46] have presented quantitative results on Fisher information in both the position and momentum space for confined hydrogen like atoms. Their results also show that with confinement Fisher information in one of the space is quite large compared to Fisher information...
in other space. Figures 3 and 4 represent Fisher probability density in position space and momentum space for the ground state and for the first excited state.

In table 4 Shannon length $H_x$, Fisher length $x_d$, standard deviation $x_D$, $\Delta x^2$ and $\Delta p^2$ are presented for $N = 2, 5$ and 10. $H_x$, $x_d$ and $x_D$ should follow the inequality represented in equations (8) and (9), which is verified in table 4. Fisher information satisfies the Cramer–Rao inequality [47] and Stem inequality [47] and multiplication of these inequalities [16] produces $\frac{81}{\langle r^2 \rangle \langle p^2 \rangle} \leq I_p I_r \leq 16 \langle r^2 \rangle \langle p^2 \rangle$. In table 4, $\langle r^2 \rangle$ and $\langle p^2 \rangle$ is shown. From tables 3 and 4 above inequality satisfies for Fisher entropy.

3.3. Onicescu energy

Onicescu energy or information energy is another measure of information, which is represented in table 5 for V-shaped, rectangular and parabolic well. As mentioned earlier, Onicescu energy is a measure of the information stored in the system. For constant number of wells in position space rectangular well has minimum information energy which means maximum information content and reverse is the effect in momentum space. V-shaped well has minimum information content of the same value of the number of wells in momentum space. Product $D = D_s D_r$ is also presented and $D$ should be less than $1/2\pi$ [48]. $1/D$ defines the total information in position and momentum space. Total Onicescu energy verifies the inequality. $D$ does not follow any trend w.r.t number of wells and shape of wells.

3.4. Complexity

Statistical complexity is a function of order as well as disequilibrium of the system. Its value is zero at both extreme conditions of probability density, i.e. for a perfectly ordered system as well as for the maximum disordered system. In figure 5 statistical complexities are plotted in position space for $n = 1$ to $n = 9$ states for V-shaped, rectangular and parabolic quantum well. In the graphs * represents the complexity of rectangular quantum well, o for V-shaped and + for parabolic quantum well $C_x$ is plotted as a function of the number of wells of the system $N$. Variation of $C_x$ with $N$ follows oscillatory nature for all the shapes of the system. There is a very interesting behavior followed by a $C_x$ curve that complexity $C_x$ for any state no greater than $N$ is the maximum for V-shaped well and minimum for rectangular except for $N = 10$. Minimum complexity for rectangular well means it is less complex as compared to other two. Complexity depends on both orderness and disorderness. As stated by Rosa et al [49] joint effect of two opposite terms on complexity is dominated by

![Figure 3. Variation of Fisher probability density for $n = 1$ and $n = 2$ in position space with x for rectangular, parabolic and V-shaped MQWS. $N = 2, 5$ and 10 and all values are in atomic unit.](image-url)
disequilibrium \( D \), so minimum complexity implies to uniform distribution. V-shaped is far from uniformity in density. Figure 6 represents the variation of complexity in momentum space as a function of \( N \). Legends used are same as in figure 3. In momentum space also complexity has oscillatory nature, but it does not follow any pattern with respect to the shape of the well. The study of complexity in position as well as momentum space is very helpful to understand structural characteristics of the quantum system.

4. Discussion

Complete study of information theoretic measures and complexity of MQWSs with \( N = 1–10 \) is done. MQWS are important to study as they can be considered one dimensional optical lattice from the semiconductor physics point of view and have many applications in advanced electronic devices, optical components and optoelectronic devices. Results are shown for \( \text{GaAs/\text{Ga}_{1−d}\text{Al}_d}\text{As} \) semiconductor, as it is a most thoroughly studied material and also it is easy to fabricate. It forms a solid solution over the entire \( 0 < d < 1 \) composition range without much affecting the lattice parameters. For Al concentration between 0 and 0.35, direct band gap is obtained, we have done our calculations with \( d = 0.25 \), as for \( d = 0.25 \) well height will remain approximately constant [50]. The Shannon entropy is highlighted in position and momentum space and we have also shown that total Shannon entropy follows BBM inequality. According to this inequality, \( S_p \) and \( S_p \) are not bounded alone, but sum of these two is bounded. And this sum will always be positive. It is impossible to have complete information about position and momentum simultaneously. Difference of Shannon entropy (in same coordinate) will directly give an idea about the absolute change in information, when one is going from one state to another. Shannon entropy also provides information about the extent of the electron delocalization. For a sharp probability distribution, its value is minimum. In position space, V-shaped has minimum value of Shannon entropy for same number of wells. Further, Fisher information is determined in position and momentum space for MQWS. Shape dependence of entropic measures is well observed and it can be explained as follows: as we are changing the shape from rectangular to parabolic to V-shaped, bottom of the well becomes narrow, which affect the localization of the particle. Sen et al [28] have described that a sharp and strongly localized probability density gives rise to larger value of Fisher information in space. For a constant value of number of wells, V-shaped well has maximum Fisher information as compared to other two in position space, which means a more localized
Table 4. Shannon length, Fisher length and standard deviation for rectangular, parabolic and V-shaped MQWS for \(N = 2, 5\) and 10. Results are expressed in atomic unit.

| \(N\) | \(n\) | \(H_x\) | \(\Delta x\) | \(\delta H_x/\Delta x\) | \(H2 = N_e/(2\pi e)^{1/2}\Delta x\) | \(\langle x^2 \rangle\) | \(\langle p^2 \rangle\) |
|-------|-------|---------|-------------|-----------------|-------------------------|------------------|------------------|
|       |       |         |             |                 |                         |                  |                  |
| V shaped |       |         |             |                 |                         |                  |                  |
| 2     | 1     | 293.76262 | 35.07830    | 156.22478      | 0.22454                 | 0.4500           | 24406.18245     | 0.00020         |
| 2     | 2     | 288.65434 | 34.54288    | 156.70834      | 0.22043                 | 0.44571          | 24557.30263     | 0.00021         |
| 3     | 3     | 530.25955 | 28.95754    | 157.32069      | 0.18407                 | 0.81558          | 24749.79993     | 0.00030         |
| 4     | 4     | 488.36192 | 24.02346    | 184.90105      | 0.12993                 | 0.65909          | 34188.39773     | 0.00043         |
| 5     | 5     | 496.80748 | 23.64276    | 188.08392      | 0.12570                 | 0.63914          | 35375.56008     | 0.00039         |
|       |       |         |             |                 |                         |                  |                  |
| Parabolic |       |         |             |                 |                         |                  |                  |
| 1     | 1     | 298.23286 | 36.00030    | 156.42727      | 0.23014                 | 0.46132          | 24469.54375     | 0.00019         |
| 2     | 2     | 294.87415 | 35.39443    | 156.74724      | 0.22708                 | 0.45520          | 24569.82099     | 0.00020         |
| 3     | 3     | 474.12666 | 23.89726    | 162.48340      | 0.14708                 | 0.70607          | 26400.86083     | 0.00044         |
| 4     | 4     | 430.88986 | 21.77842    | 171.92215      | 0.12668                 | 0.60645          | 29557.44687     | 0.00053         |
| 5     | 5     | 490.22770 | 24.29030    | 168.39324      | 0.14408                 | 0.70359          | 28423.90637     | 0.00042         |
|       |       |         |             |                 |                         |                  |                  |
| Rectangular |       |         |             |                 |                         |                  |                  |
| 1     | 1     | 370.48786 | 44.06625    | 158.47115      | 0.27807                 | 0.56570          | 25113.10472     | 0.00013         |
| 2     | 2     | 364.89950 | 42.95852    | 159.25731      | 0.26974                 | 0.55442          | 23562.89176     | 0.00014         |
| 3     | 3     | 422.13392 | 23.29345    | 166.01668      | 0.14031                 | 0.61526          | 27561.53791     | 0.00046         |
| 4     | 4     | 401.35632 | 22.18839    | 168.63730      | 0.13158                 | 0.57589          | 28435.53837     | 0.00051         |
| 5     | 5     | 521.27279 | 18.51384    | 168.62308      | 0.10979                 | 0.74802          | 28433.74321     | 0.00073         |
|       |       |         |             |                 |                         |                  |                  |
| V shaped |       |         |             |                 |                         |                  |                  |
| 5     | 1     | 639.24924 | 35.28934    | 327.41961      | 0.59295                 | 0.47242          | 107203.60348    | 0.00020         |
| 2     | 2     | 587.64940 | 35.08589    | 480.20526      | 0.37207                 | 0.29611          | 230597.08797    | 0.00020         |
| 3     | 3     | 438.14888 | 34.81421    | 498.93429      | 0.26762                 | 0.21249          | 248935.42647    | 0.00021         |
| 4     | 4     | 577.42848 | 34.54997    | 486.31431      | 0.36351                 | 0.28731          | 236501.61223    | 0.00021         |
| 5     | 5     | 631.35754 | 34.36109    | 334.99738      | 0.57536                 | 0.45603          | 112223.24624    | 0.00021         |
|       |       |         |             |                 |                         |                  |                  |
| Parabolic |       |         |             |                 |                         |                  |                  |
| 1     | 1     | 650.27778 | 36.15794    | 329.24240      | 0.10984                 | 0.47797          | 108400.6526     | 0.00019         |
| 2     | 2     | 596.53327 | 36.00508    | 481.46466      | 0.07497                 | 0.29982          | 231808.358      | 0.00019         |
| 3     | 3     | 445.39596 | 35.79988    | 499.04276      | 0.07174                 | 0.21596          | 249034.8914     | 0.00019         |
| 4     | 4     | 589.81455 | 35.39889    | 485.15185      | 0.07337                 | 0.29413          | 235372.6404     | 0.00020         |
| 5     | 5     | 643.77156 | 35.45439    | 333.21281      | 0.10639                 | 0.46743          | 111031.1983     | 0.00020         |
|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   | Hx | d   | 1  | 2  | 3  | 4  | 5  |
| 1 | 1181.55105 | 44.60958 | 486.03010 | 0.09178 | 0.58824 | 236225.26102 | 0.00013 |
| 2 | 1435.41987 | 44.45965 | 902.12999 | 0.04928 | 0.38501 | 813838.51204 | 0.00013 |
| 3 | 1485.50050 | 44.26405 | 948.34594 | 0.04668 | 0.37903 | 899360.03130 | 0.00013 |
| 4 | 1394.91508 | 43.94174 | 1090.71746 | 0.04029 | 0.30946 | 1189664.56899 | 0.00013 |
| 5 | 1184.38154 | 43.71177 | 1095.38800 | 0.03991 | 0.26163 | 1199874.86905 | 0.00013 |
| 6 | 1211.17325 | 24.29030 | 951.44037 | 0.03769 | 0.30803 | 905239.12468 | 0.00019 |

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |

Table 4 (Continued)
probability density. With respect to the number of wells absolute maximum value of Shannon probability density decreases with increasing the number of wells independent of the shape of the well. All the curves are symmetric in position space as well as in momentum space. Also sharpness of the graph is the maximum for V-shaped well. In the case of Fisher information density peaks are more sharp for rectangular quantum well in position space. As shown in Table 3 variation of Shannon entropy with number of wells and with shape of wells in position and momentum space has opposite behavior, same nature is reflected in Shannon probability density graphs in figures 1 and 2. Shannon probability density curves are broad in position space while in momentum space curves are sharp. Fisher probability density also follows the same behavior in position and momentum

| N  | n  | V-shaped       | Parabolic       | Rectangular     |
|----|----|----------------|----------------|----------------|
|    |    | $D_x$, $D_P$, $D$ | $D_x$, $D_P$, $D$ | $D_x$, $D_P$, $D$ |
| 2  | 1  | 0.00411, 0.12713, 0.12713 | 0.00396, 0.12516, 0.12516 | 0.00305, 0.11874, 0.11874 |
| 2  | 2  | 0.00415, 0.11973, 0.11973 | 0.00398, 0.11927, 0.11927 | 0.00308, 0.10777, 0.10777 |
| 3  | 0.00207, 0.06215, 0.06215 | 0.00243, 0.05339, 0.05339 | 0.00275, 0.05836, 0.05836 |
| 4  | 0.00233, 0.07271, 0.07271 | 0.00270, 0.07298, 0.07298 | 0.00287, 0.08272, 0.08272 |
| 5  | 0.00237, 0.06176, 0.06176 | 0.00246, 0.06358, 0.06358 | 0.00212, 0.03956, 0.03956 |
| 5  | 1  | 0.00207, 0.14314, 0.14314 | 0.00199, 0.14198, 0.14198 | 0.00153, 0.13571, 0.13571 |
| 2  | 0.00206, 0.08682, 0.08682 | 0.00198, 0.09284, 0.09284 | 0.00153, 0.08761, 0.08761 |
| 3  | 0.00275, 0.13496, 0.13496 | 0.00265, 0.11306, 0.11306 | 0.00204, 0.10450, 0.10450 |
| 4  | 0.00208, 0.08246, 0.08246 | 0.00199, 0.08918, 0.08918 | 0.00154, 0.08084, 0.08084 |
| 5  | 0.00206, 0.13053, 0.13053 | 0.00198, 0.13181, 0.13181 | 0.00154, 0.11793, 0.11793 |
| 10 | 0.00112, 0.14826, 0.14826 | 0.00108, 0.14647, 0.14647 | 0.00119, 0.16960, 0.16960 |
| 2  | 0.00112, 0.09485, 0.09485 | 0.00108, 0.09749, 0.09749 | 0.00093, 0.10065, 0.10065 |
| 3  | 0.00113, 0.10139, 0.10139 | 0.00108, 0.08731, 0.08731 | 0.00076, 0.06657, 0.06657 |
| 4  | 0.00112, 0.08028, 0.08028 | 0.00108, 0.08424, 0.08424 | 0.00103, 0.08610, 0.08610 |
| 5  | 0.00110, 0.09184, 0.09184 | 0.00107, 0.08348, 0.08348 | 0.00116, 0.09572, 0.09572 |
space in figures 3 and 4. Complexity of the quantum well shows a very unique behavior, it not only defines the orderness of the system but also the disequilibrium of the system. Actually, complexity of a system gives a combine balance of average height of probability density and its total bulk extent. Rectangular MQWS has maximum information content for any number of wells in position space and for all the cases ground state has maximum information. Verification of inequalities related to these three shows the correctness of our method. Although we have not shown here, but all the information measures studied in this paper also depend on the width of the wells. As reducing well width may result in overlapping of the wave functions, which finally affects the locality of given wave function.

5. Conclusion

Shannon entropy, Fisher information and Onicescu energy and statistical measures of complexity are studied for MQWS. Effect of shape and number of wells on these information theoretic measures is very interesting as it enables us to study the localization of particle in the system. Different shapes considered are rectangular, parabolic and V-shaped, which are the basic structures of MQWS. All these information measures are studied in position as well as in momentum space. Localization of particle in quantum well can be well understood by information theory. As we have considered the symmetric quantum well, so Shannon probability density is symmetric in position and in momentum space. We get that Onicescu energy is maximum for the V shape in position space and minimum in momentum space. Variation of complexity with the number of wells for different shapes has very interesting results. The study of information entropies of quantum systems is crucial as it can give an insight to understand the physical aspect of system, which can be helpful for better designing of electronic devices. Also physical and chemical properties of atoms and molecules can be understood in a better way from the information point of view.

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