Switching Energy of Ferromagnetic Logic Bits

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Power dissipation in switching devices is believed to be the single most important roadblock to
the continued downsizing of electronic circuits. There is a lot of experimental effort at this time to
implement switching circuits based on magnets and it is important to establish power requirements
for such circuits and their dependence on various parameters. This paper analyzes switching energy
which is dissipated in the switching process of single domain Ferromagnets used as cascadable logic
bits. We obtain generic results that can be used for comparison with alternative technologies or
guide the design of magnet based switching circuits. Two central results are established. One is that
the switching energy drops significantly if the ramp time of an external pulse exceeds a critical time.
This drop occurs more rapidly than what is normally expected of adiabatic switching for a capacitor.
The other result is that under the switching scheme that allows for logic operations, the switching
energy can be described by a single equation in both fast and slow limits. Furthermore, these
generic results are used to quantitatively examine the possible operation frequencies and integration
densities of these logic bits which show that nanomagnets can have scaling laws similar to CMOS
technology.

I. INTRODUCTION

It has been suggested1 that the use of collective systems like a magnet can reduce the intrinsic switching energy
(that is dissipated throughout switching) significantly compared to that required for individual spins. There is also
a lot of experimental effort2,3,4,5,6,7 at this time to implement switching circuits based on magnets. There has been
some work8 on modeling magnetic circuits like MQCA’s in the atomic scale using quantum density matrix equation
but most of the work9,10,11,12,13 is in the classical regime using the well known micromagnetic simulators (OOMMF)
based on the Landau-Lifshitz-Gilbert (LLG)14,15,16,17 equation. This paper too is based on the LLG equation, but our
focus is not on obtaining the energy requirement of any specific device in a particular simulation. Rather it is to
obtain generic results that can guide the design of magnet based switching circuits as well as providing a basis for
comparison with alternative technologies.

The results we present are obtained by analyzing the cascadable switching scheme illustrated in Fig.1 where the
magnet to be switched (magnet 2) is first placed along its hard axis by a magnetic pulse (see ‘mid state’ in Fig.1).
On removing the pulse, it falls back into one of its low energy states (up or down) determined by the ‘bias’ provided
by magnet 1. What makes this scheme specifically suited for logic operations is that it puts magnet 2 into a state
determined by magnet 1 (thereby transferring information), but the energy needed to switch magnet 2 comes largely
from the external pulse and not from magnet 1. This is similar to conventional electronic circuits where the energy
needed to charge a capacitor comes from the power supply, although the information comes from the previous
capacitors. This feature seems to be an essential ingredient needed to cascade logic units. To our knowledge, the
switching scheme shown in Fig.1 was first discussed by Bennett20 and is very similar to the schemes described in
many recent publications (see e.g Likharev et.al21, Kummamuru et.al22, and Csaba et.al23).

This paper uses the LLG equation to establish two central results. One is that the switching energy drops
significantly as the ramp time \( \tau_r \) of the magnetic pulse exceeds a critical time \( \tau_p \) given by equation (1). This
is similar to the drop in the switching energy of an RC circuit when \( \tau_r \gg RC \). But the analogy is only approximate
since the switching energy for magnets drops far more abruptly with increasing \( \tau_r \). The significance of \( \tau_p \) is that it
tells us how slow a pulse needs to be in order to qualify as “adiabatic” and thereby reduce dissipation significantly.
Considering typical magnets used in the magnetic storage industry, and using ramp times of a few \( \tau_c \), intrinsic
switching frequency of 100 MHz to 1 GHz can easily be in the adiabatic regime of switching where dissipation is very
small.

Interestingly, we find that the switching energy for the trapezoidal pulses investigated in this paper in both
the ‘fast’ and ‘slow’ limits can be described by a single equation \( \text{Eq} [8] \) which is the other central result of this paper. Later in this paper (IV) we will discuss how equations \( \text{Eq} [13] \) and \( \text{Eq} [8] \) can be used to guide scaling and increase switching speeds. Furthermore these equations can be used to compare magnet based switching circuits with
alternative technologies.

It has to be emphasized that dissipation of the external circuitry also has to be evaluated for any new technology.
A careful evaluation would require a consideration of actual circuitry to be used (see e.g.13,14) and is beyond the

scope of this paper. However following Nikonov et.al., if a wire coil is used to produce the pulse, we can estimate the energy dissipated in creating the field $H_{\text{pulse}}$ as $\frac{H_{\text{pulse}}^2}{2} Q V$ in CGS system of units. $Q$ is the quality factor of the circuit and $V$ is the volume over which the field extends. Depending on $Q$, $V$ and $H_{\text{pulse}}$, the dissipated energy can be much larger, comparable to or much smaller than $K u_2 V$ which sets the energy scale for the effects considered here in this paper.

**Overview of the paper**: As mentioned before our results are based on direct numerical simulation of the LLG equation. However we find that in two limiting cases, it is possible to calculate switching energy simply using the energetics of magnetization and these limiting results are described in sections §II (dissipation with fast pulse) and §III (dissipation with adiabatic pulse) which are related to equation (8). In §IV we use the LLG equation to show that the switching energy drops sharply for ramp times larger than the critical time given by equation (14). In section §V using coupled LLG equations we analyze a chain of inverters to show that the total dissipation increases linearly with the number of nanomagnets thus making it reasonable to use the one-magnet results in our paper to evaluate complex circuits, at least approximately. Finally in section §VI practical issues such as dissipation versus speed, increasing the switching speed and scaling are qualitatively discussed in the light of these results.

II. DISSIPATION WITH FAST ($\tau_r << \tau_c$) PULSE

Before we get into the discussion of switching energy, let us briefly review the energetics of a magnet. The energy of a magnet with an effective second order uniaxial anisotropy can be described by $E = -M_s \vec{m} \cdot \vec{H}_{sphere} = K u_2 \sin^2(\theta)$ where $\theta$ measures the deflection from the easy axis which we take as the $z$ axis. All isotropic terms have been omitted because they do not affect dynamics and hence dissipation of the magnet. There are two magnetic fields that control the switching (see Fig.1): The external pulse $H_{\text{pulse}}$ and the bias field $H_{dc}$ due to the neighboring magnet. Including the internal energy and the interaction energy of magnetic moment with external fields, the energy equation reads

$$\frac{E}{V} = -M_s \vec{m} \cdot \vec{H}_{\text{pulse}} + K u_2 \sin^2(\theta) - M_s \vec{m} \cdot \vec{H}_{dc}$$

$M_s$ is the saturation magnetization. If the unit volume is magnetized to saturation, $M_s$ is equivalent to the magnetic moment per unit volume. $\vec{m}$ is a unit vector in the direction of magnetization. $V$ is the volume of the magnet and $K u_2$ is the second order anisotropy constant with dimensions of energy per unit volume. The applied field $H_{\text{pulse}}$ is along the hard axis $\hat{y}$, the bias field $H_{dc}$ is along the easy axis $\hat{z}$ so the energy equation becomes

$$\frac{E}{V} = -M_s H_{\text{pulse}} \sin(\theta) \sin(\phi) + K u_2 \sin^2(\theta) - M_s H_{dc} \cos(\theta)$$

(1)
Where \( \phi \) is defined as in a standard spherical coordinate system. Using equation 1, we will show that dissipation with a fast pulse (small ramp time) can be written as

\[
E_d = \left( \frac{H_{\text{pulse}}}{H_c} \right)^2 (2Ku_2V) \quad \text{for} \quad H_{\text{pulse}} \leq H_c \quad (2a)
\]

\[
E_d = 2Ku_2V \quad \text{for} \quad H_{\text{pulse}} = H_c \quad (2b)
\]

\[
E_d = \left( \frac{H_{\text{pulse}}}{H_c} \right)^2 (2Ku_2V) \quad \text{for} \quad H_{\text{pulse}} \geq H_c \quad (2c)
\]

For reasons to be explained, under the condition of equation 2a, logic device will not work. Nevertheless, it is useful for determining dissipation in the adiabatic limit. In the equations above, \( H_c = \frac{2Ku_2M_s}{3} \) is the minimum field necessary to put the magnet along its hard axis. Notice that the bias field \( H_{\text{dc}} \) is a dc field coming from the neighboring magnet. In practice, whether the bias field is a dc field or not, its magnitude has to be bigger than noise such that when the magnet is put along its hard axis as in Fig. 1, the bias field can deterministically tilt the magnet towards its direction.

We will show in \( \text{II B} \) that for \( H_{\text{dc}} \leq 0.1H_c \), dissipation can still be calculated using equation 2.

To derive equations 2, we find the initial and final state energies under various conditions and evaluate the difference. We have to emphasize that all these states essentially pertain to the energy minima (equilibrium states) i.e. they are either the minimum of energy; or they represent a non-equilibrium state instantaneously after the equilibrium state (minimum of energy) has changed. Since all the fields considered here are in the \( y-z \) plane and no out-of-plane field is considered, the equilibrium states (the energy minima) will always lie in the \( y-z \) plane for which \( \phi = 90^\circ \).

### A. Zero bias field \( (H_{\text{dc}} = 0) \)

Fig. 2 is plotted using equation 1 with \( \phi = 90^\circ \) and \( H_{\text{dc}} = 0 \) which is the first case to be discussed. The different contours correspond to different values of \( H_{\text{pulse}} \).

**Derivation of equation 2b** Let’s start with equation 2b which is the most important and also easiest. Dissipation occurs both during turn-on and turn-off of the pulse and the overall switching energy is sum of the two in general. The dashed contour in Fig. 2 corresponds to \( H_{\text{pulse}} = H_c \) which is the minimum value needed to make \( \theta = 90^\circ \) (point 2) the energy minimum. For a pulse with fast \( (\tau_r << \tau_c) \) turn-on, dissipation can be calculated using equation 1 as the difference between the initial and the final energies which are given by point 1 (or 4) and point 2 on the dashed contour. This value is

\[
E_{1(4)} - E_2 = Ku_2V
\]

For a pulse with fast \( (\tau_r << \tau_c) \) turn-off, the energy contour immediately changes from the dashed one to the uppermost one in Fig. 2. Under any infinitesimal bias, magnetization falls down the barrier to the left (relaxing to

**FIG. 2: Energy landscape of the magnetization under various applied fields. For fast turn-on of the pulse to \( H_c \), dissipation is equal to the barrier height (magnet relaxes from point 1 (or 4) to point 2). When the field is turned off fast, magnet relaxes from point 3 to point 4 or 1 depending on any infinitesimal bias again dissipating an amount equal to the barrier height.**
point 1) or to the right (relaxing to point 4) giving a dissipation of

\[ E_3 - E_{1(4)} = Ku_2V \]
equal to the turn-on dissipation. The switching energy (total dissipation) is sum of the values for turn-on and turn-off which gives us equation 2b.

**Derivation of equation 2b** This is the case with \( H_{\text{pulse}} > H_c \). The bottom most energy contour in Fig 2 shows such a situation as an example. The minimum of energy is still at \( \theta = 90^\circ \) (point 5) however now the energy well is deeper. For a pulse with fast \( (\tau_r \ll \tau_c) \) turn-on, dissipation is the difference between the initial and final state energies

\[ E_{1(4)} - E_5 = (M_s H_{\text{pulse}} - Ku_2)V \]

(Where \( E_5 \) is used as a generic notation for the bottom of any well with \( H_{\text{pulse}} > H_c \)). For a pulse with fast \( (\tau_r \ll \tau_c) \) turn-off, the energy contour immediately changes from the bottom most curve to the uppermost curve in Fig 2. Depending on any infinitesimal bias magnet will relax from point 3 to either point 1 or 4 dissipating the difference

\[ E_3 - E_{1(4)} = Ku_2V \]

The switching energy is sum of the values for turn-on and turn-off which with straightforward algebra gives us equation 2c.

**Derivation of equation 2c** With \( H_{\text{pulse}} < H_c \), magnetization will not align along its hard axis \( (\theta = 90^\circ) \). This can be seen in Fig 2 where for a pulse lower than \( H_c \) there are two minima of energy not located along the hard axis. The logic device will not work in this regime because it needs to be close to its hard axis so that the field of another magnet can tilt it towards one minima deterministically. Nevertheless we derive dissipation for these pulses because we use the results in section III A to show switching energy in the adiabatic limit. For a pulse with fast \( (\tau_r \ll \tau_c) \) turn-on, dissipation is the difference between the initial and final state energies

\[ E_1 - E_6 = \left( \frac{M_s H_{\text{pulse}}}{2Ku_2} \right)^2 (Ku_2V) \]

(3)

For a pulse with fast \( (\tau_r \ll \tau_c) \) turn-off, the energy contour suddenly becomes the uppermost one in Fig 2. At that moment magnetization is still at the same \( \theta \) (point 7). It follows down the barrier with the dissipation given by

\[ E_7 - E_3 = \left( \frac{M_s H_{\text{pulse}}}{2Ku_2} \right)^2 (Ku_2V) \]

(4)

The total dissipation is sum of the values for turn-on and turn-off which gives us equation 2c.

**B. Non-zero bias field \( (H_{dc} \neq 0) \)**

In this section we show that for \( H_{\text{pulse}} = H_c \), so long as \( H_{dc} \leq 0.1H_c \) switching energy can be calculated fairly accurately using equation 2a considering only the effect of \( H_{\text{pulse}} \). For \( H_{\text{pulse}} > H_c \) the effect of \( H_{dc} \) is even less pronounced as compared to \( H_{\text{pulse}} \) and equation 2c can be used to calculate dissipation. Again we are interested in initial and final state energies which can be calculated using equation 1 with \( \phi = 90^\circ \). \( H_{dc} \) can be positive (along \( z \)) or negative (along \( -z \)). Fig 3 shows the energy landscape with an \( H_{dc} \) in the \( -z \) direction. If \( H_{dc} \neq 0 \) then the up and down states (points 1 and 4) of the magnet have different initial energies which result in two different cases to be analyzed. Case 1 designates the situation where initial magnetization (point 1) and \( H_{dc} \) are in the opposite direction. Case 2 designates the situation where initial magnetization (point 4) and \( H_{dc} \) are in the same direction.

For a pulse with fast \( (\tau_r \ll \tau_c) \) turn-on, case 1 dissipates the difference between points 1 and 2 and case 2 dissipates the difference between points 4 and 2. When the pulse is suddenly turned off, in both cases magnetization finds itself at point 3, drops down to point 4 and dissipates the difference. It is not possible to give an exact closed form expression for the value of dissipation with non-zero bias. Instead based on numerical calculations, we show figures that provide useful insight to conclude that for pulses with fast ramp time the effect of bias on switching energy is negligible.

The energy of point 2 (and subsequently point 3) depicted in Fig 3 changes as the relative magnitude of \( H_{dc} \) and \( H_c \) are changed. We like to know how dissipation changes as a function of the ratio \( \frac{H_{dc}}{H_c} \). The numerical results are plotted in Fig 3 using equation 1. Fig 4a shows that for a pulse with fast turn-on and small values of \( \frac{H_{dc}}{H_c} \), both
FIG. 3: Energy landscape of magnetization with bias field $H_{dc}$ in the $-z$ direction for two values of the pulse: 0 and $H_c$. Upon turn-on, if magnetization starts from $\theta = 0^\circ$ (case 1), it drops from point 1 ($E = +M_sVH_{dc}$) to point 2 dissipating the difference. If it starts from $\theta = 180^\circ$, it drops from point 4 ($E = -M_sVH_{dc}$) to point 2 dissipating the difference. Upon turn-off, both cases 1 and 2 drop from point 3 to point 4 dissipating the difference.

FIG. 4: (a) Shows the turn-on dissipation with non-zero bias. Cases 1 and 2 correspond to different initial directions of magnetization (see Fig.3). The dashed line depicts the value of dissipation with zero bias. (b) Shows the turn-off dissipation with non-zero bias. Both cases 1 and 2 dissipate the same amount (see Fig.3). (c) Shows the total dissipation with non-zero bias. Notice that for relevant (small) values of $\frac{H_{dc}}{H_c}$, total dissipation of both cases 1 and 2 is close to the value $2Ku_2V$ which is the same as the case with infinitesimal bias.

III. DISSIPATION WITH ADIABATIC ($\tau_r >> \tau_c$) PULSE

We have seen in section II that for pulses with fast ramp times, the effect of bias ($H_{dc}$) is negligible for $H_{dc} \leq 0.1H_c$ and switching energy is obtained fairly accurately even if we set $H_{dc} = 0$. By contrast for pulses with slow ramp time,
switching energy can be made arbitrarily small for $H_{dc} = 0$ and the actual switching energy is determined entirely by the $H_{dc}$ that is used. In this section we will first show why the switching energy can be arbitrarily small for $H_{dc} = 0$ and then show that for $H_{dc} \neq 0$ it will saturate in case 1 but can be made arbitrarily small in case 2. Two points are in order. First, the analysis presented here is exact in the absence of noise. If thermal noise is present the analysis may not be true in general and needs to be modified accordingly. Second, if in the process of switching, a bit of information is destroyed as in two inputs and one output gates (e.g. AND/OR), then there will be a finite switching energy even for adiabatic switching.

Gradual turn-on of the pulse corresponds to increasing the pulse in many small steps. Fig. 5 shows the energy landscape. As the field is gradually turned-on the energy contours change little by little from top to bottom. The minimum of energy gradually shifts from point 1 (or 4) to point 2. Magnetization hops from one minimum of energy to the other. But why is it that gradual turn-on of the pulse dissipates less than sudden turn-on?

If the external pulse is turned on to $H_c$ in $N$ equal steps, we show that there is equal amount of dissipation at each step. Then total dissipation is $N$ times that of each step. We show that dissipation of each step is proportional to $\frac{1}{N}$; hence as the number of steps increases, dissipation decreases as $\frac{1}{N}$ and in the limit of $N \to \infty$, $E_d \to 0$ (this is not unlike a similar argument that has been given for charging up a capacitor adiabatically). At each step when the pulse is increased by $\Delta H = \frac{H_c}{N}$, the dissipated energy is the difference between initial and final state energies. Such a situation is illustrated in Fig. 5 where $a$ denotes a minimum on an energy contour corresponding to $H_n$ (magnitude of the pulse after $n$ steps). When the pulse is stepped up to $H_{n+1}$, magnetization suddenly finds itself at point $b$ (initial state) and falls down to $c$ (final state). Note that dissipation is $E_b - E_c$ and not $E_a - E_c$. This is because when the field suddenly changes from $H_n$ to $H_{n+1}$, magnet not has had time to relax and dissipate energy. Here we use $E_a$ and $E_c$ as generic notations for initial and final energy of any step. $E_b$ can be found by finding the $\theta$ which corresponds to point $a$ (the minimum of energy with $H_{pulse} = H_n$) and substituting it in equation 1 with $H_{pulse} = H_{n+1}$. With straightforward algebra we get $E_b = -(M_s V)H_{n+1}\left(\frac{n H_{n+1}}{2Ku_2 V}\right) + (Ku_2 V)\left(\frac{\Delta H}{2 Ku_2}\right)^2$. Equation 3 can be used to calculate...
it is evident that there is a discontinuous jump in the \( \theta \) how slow the pulse is turned on and causes the switching energy to saturate so long as point A to B and dissipates the energy difference. This sudden change in the minimum of energy occurs no matter energy barrier which formerly separated the two minima on the two sides disappears. Magnetization falls down from 0 to \( m \) magnetization starts from point 1′ to the next. Increasing the number of steps brings the minima closer to each other so that magnetization stays in its minimum of energy with no discontinuity. Dissipation tends to zero as the pulse is turned off in the limit of \( N \to \infty \), tends to 0 (\( E_d \to 0 \)).

For gradual turn-off consider points c,d and a. When \( H_{pulse} = H_{n+1} \), magnetization is at c and after the pulse is decreased by one step to \( H_n \), it finds itself at d, falls down to a dissipating the difference \( E_d - E_n \). \( E_d \) can be found by finding the \( \theta \) which corresponds to point c (the minimum of energy with \( H_{pulse} = H_{n+1} \)) and substituting it in equation 1 with \( H_{pulse} = H_n \). We get \( E_d = - (M_sV)h_n - (M_sV)(M_sH_{n+1}) \). Again equation 3 can be used to give \( E_d = - (M_sV)h_n - (M_sV)(\frac{M_sH_{n+1}}{Ku_2V}) \). Using the identities \( H_{n+1} = H_n + \Delta H \) and \( \Delta H = \frac{H_d}{N} \), we obtain for the dissipated energy per step

\[
E_d^{step} = E_d - E_a = Ku_2V \left( \frac{1}{N^2} \right)
\]

The switching energy is sum of the dissipation values for turn-on: \( E_d = \frac{Ku_2V}{N} \) and turn-off: \( E_d = \frac{Ku_2V}{N} \) which in the limit of \( N \to \infty \), tends to 0 (\( E_d \to 0 \)).

B. Non-zero bias field (\( H_{dc} \neq 0 \))

For turn-on let’s consider case 1 first where initial magnetization and \( H_{dc} \), are in opposite directions (point 1′ in Fig[5]). As the field is gradually turned-on, magnetization starts from point 1′ and hops from one minimum of energy to the next. Increasing the number of steps brings the minima closer to each other so that magnetization stays in its ground state while being switched. However when magnetization gets to point A, situation changes. At that point the energy barrier which formerly separated the two minima on the two sides disappears. Magnetization falls down from point A to B and dissipates the energy difference. This sudden change in the minimum of energy occurs no matter how slow the pulse is turned on and causes the switching energy to saturate so long as \( H_{dc} \neq 0 \). Quantitatively this can be seen by plotting \( \theta_{min} \) vs. \( H_{pulse} \) (Fig[5]) using equation 1. When the left solid curve is traced from \( \theta_{min} = 0 \), it is evident that there is a discontinuous jump in the \( \theta_{min} \) values which minimize energy when the pulse is increases from 0 to \( H_c \) in infinitesimal steps. This discontinuity goes away only when \( H_{dc} = 0 \) (right solid curve). In case 2, magnetization starts from point 4′, i.e. \( \theta_{min} = 180^\circ \) (see Fig[5]d and c), gets to point B at which there is no sudden change of minimum and as the pulse is increased further to \( H_c \), it gradually moves to point 2′. During turn-off in both cases 1 and 2, magnetization gradually moves from (see Fig[5]) point 2′ to B and then finally to point 4′ all along staying in its minimum of energy with no discontinuity. Dissipation tends to zero as the pulse is turned off in infinitesimal steps.

In the slow limit the entire dissipation is determined by the energy difference between points A and B, \( E_A - E_B \) in Fig[5]. For a given \( H_{dc} \), one has to find that particular value of \( H_{pulse} \) for which the local energy maximum in the middle disappears which means that the second derivative of energy with respect to \( \theta \) must be zero (no curvature). Since magnetization has been in the minimum of energy while getting to point A, first derivative of energy with respect to \( \theta \) must also be equal to zero. Under these conditions, the value of \( \theta \) at A and subsequently \( E_A \) can be found using equation 4. \( E_B \) can be found as the true minimum of energy from equation 5 where the first derivative of energy with respect to \( \theta \) is zero but the second derivative is not. What affects \( E_A - E_B \) is the relative magnitude of \( H_{dc} \) and \( H_c \). It is not possible to give an analytical closed form expression for this saturating value of dissipation. Instead we’ve numerically plotted dissipation versus \( \frac{\Delta H}{H_c} \) (solid curve in Fig[4]). For small values of \( \frac{\Delta H}{H_c} \), dissipation can be written as

\[
E_d = \left( \frac{2H_{dc}}{H_c} \right)^p (2Ku_2V), (p = 1.23)
\]

Where the value of \( p \) is obtained by an almost perfect fit to the solid curve for \( H_{dc} \leq 0.1H_c \). The dashed curve is plotted using equation 7. As is evident from Fig[4] this equation is fairly accurate. There is some digression from the actual value of dissipation for large values of \( \frac{\Delta H}{H_c} \) which are not of practical interest especially \( \frac{\Delta H}{H_c} = 1 \) for which \( H_{dc} \) alone can switch the magnet and is completely an unwanted situation.

It is important to note that the switching energy in the adiabatic limit is case dependent. For case 1, it is given by equation 7 and it is not zero as it might have been expected for dissipation in the adiabatic limit. Interestingly if \( p \)}
FIG. 6: Shows the total dissipation under adiabatic switching with non-zero bias. There is no dissipation associated with case 2 and dissipation of case 1 for small (relevant) values of $H_{dc}$ is less than the barrier height $Ku_2V$. The dashed line is plotted using equation 7.

was equal to 1, the dissipation would be equal to the energy difference between initial and final states (see points 1' and 4' in Fig 5). However the actual value is significantly smaller.

Dissipation in both the fast and slow limits can be casted into a single equation

$$E_d = \left(\frac{\tilde{H}}{H_c}\right)^p (2Ku_2V)$$

(8)

In the fast limit, $\tilde{H}$ is the magnitude of the pulse while in the slow limit, $\tilde{H}$ is related to the magnitude of the small bias field as states above. $Ku_2V$ is the height of the anisotropy energy barrier separating the two stable states of the magnet, and has to be large enough so that the magnet retains its state while computation is performed without thermal fluctuations being able to flip it. The retention time for a given $Ku_2V$ can be calculated using $t_r = t_0e^{-\frac{Ku_2V}{kT}}$ where $t_0^{-1}$ is the attempt frequency with the range $10^9 - 10^{12}s^{-1}$ which depends in a nontrivial fashion on variables like anisotropy, magnetization and damping.

IV. MAGNETIZATION DYNAMICS: SINGLE MAGNET

Thus far we’ve shown switching energy in the two limiting cases of $\tau_r << \tau_c$ and $\tau_r >> \tau_c$. To understand how switching energy changes in between and also how fast it decreases we need to start from the LLG equation which in the Gilbert form reads:

$$\frac{d\vec{M}}{dt} = -|\gamma|\vec{M} \times \vec{H} + \frac{\alpha}{|\vec{M}|}\vec{M} \times \frac{d\vec{M}}{dt}$$

(9)

And in the standard form reads:

$$(1 + \alpha^2)\frac{d\vec{M}}{dt} = -|\gamma||(\vec{M} \times \vec{H}) - \frac{\alpha|\gamma|}{|\vec{M}|}\vec{M} \times (\vec{M} \times \vec{H})$$

(10)

$\gamma$ is the gyromagnetic ratio of electron and its magnitude is equal to $2.21 \times 10^9(rad.m)(A.s)^{-1}$ in SI and $1.76 \times 10^7(rad)(Oe.s)^{-1}$ in CGS system of units. $\alpha$ is the phenomenological dimensionless Gilbert damping constant. $\vec{M}$ is the magnetization. Here $\vec{H} = \vec{H}_{ani} + \vec{H}_{pulse}$ where $\vec{H}_{ani} = \frac{2Ku_2}{M_s}m_z\hat{z}$. In general $\vec{H}$ can be derived as the overall effective field: $\vec{H} = -\frac{1}{\mu_0V}\nabla_m E$.

The following expressions are all equivalent statements of dissipated power:

$$P_d = \vec{H} \cdot \frac{d\vec{M}}{dt} = \frac{\alpha}{|\gamma||\vec{M}|}\left|\frac{d\vec{M}}{dt}\right|^2 = \frac{\alpha|\gamma|}{(1 + \alpha^2)|\vec{M}|}\left|\vec{M} \times \vec{H}\right|^2$$

(11)
FIG. 7: Solid lines show the dissipated power $2\vec{h} \cdot \frac{d\vec{m}}{dt}$ under an instantaneous turn-on of $H_{\text{pulse}}$ to $H_c$ for $\alpha = 0.005$ and $\alpha = 0.5$. Dashed line shows an exponential decay $e^{-t'}$. This figure shows that although the value of $\alpha$ changes the time (with real dimensions) at which the dissipated power decreases to $1/e$ through changing $\tau_c$, it does not affect the functional form of the decay which is more or less an exponential decay even if $\alpha$ changes by 2 orders of magnitude.

The dissipated power has to be integrated over time to give the total dissipation. In general, LLG can be solved numerically using the method. To obtain generic results that are the same for various parameters, we recast LLG and the dissipation rate into a dimensionless form. This will also show the significance of $\tau_c$ and demonstrate why for ramp times exceeding $\tau_c = 1$, there is a significant drop in dissipation.

Using scaled variables $\vec{m} = \frac{\vec{M}}{M_s V}$ and $\vec{h} = \frac{\vec{H}}{H_c}$ equation (10) in dimensionless form can be written as

$$\frac{d\vec{m}}{dt'} = -\frac{1}{2\alpha}(\vec{m} \times \vec{h}) - \frac{1}{2} \vec{m} \times (\vec{m} \times \vec{h})$$

(12)

where $t' = \frac{t}{\tau_c}$ with $\tau_c$ given by equation (14). The energy dissipation normalized to $Ku^2 V$ can be written as

$$\frac{E_d}{Ku^2 V} = \frac{1}{Ku^2 V} \int \vec{H} \cdot \frac{d\vec{M}}{dt} dt = \int 2\vec{h} \cdot \frac{d\vec{m}}{dt'} dt'$$

(13)

To estimate the time constant involved in switching a magnet it is instructive to plot the integrand $2\vec{h} \cdot \frac{d\vec{m}}{dt} = \frac{\tau_c}{Ku^2 V} \left(\vec{H} \cdot \frac{d\vec{M}}{dt}\right)$ appearing above in equation (13) assuming a step function for $H_{\text{pulse}}$ and obtaining the corresponding $\frac{d\vec{m}}{dt}$ from equation (12). Note that the integrands die out exponentially for a wide range of $\alpha$’s from 0.005 to 0.5. In other words, all the curves (ignoring the oscillations) can be approximately described by $e^{-t'} = e^{-t/\tau_c}$ thus suggesting that the approximate time constant is $\tau_c$.

$$\tau_c = \frac{(1 + \alpha^2)}{2\alpha(|\gamma|H_c)}$$

(14)

This is more evident from Fig.8 where we show the energy dissipation for pulses with different ramp times. The dissipated energy drops when $\tau_r$ exceeds $\tau_c$ as we might expect, but the drop is sharper than an RC circuit. Needless to say, the dissipation values calculated from LLG equation for the two limits of fast pulse ($\tau_r << \tau_c$) and adiabatic pulse ($\tau_r >> \tau_c$) are consistent with the values calculated using energetics previously. Fig.9 shows the turn-on dissipation where case 1 has saturated and case 2 goes down as ramp time is increased. The curve in the middle is the case with infinitesimal bias $H_{dc} = 0$ and it is just provided for reference. Fig.9 shows the turn-off dissipation where both cases 1 and 2 dissipate arbitrarily small amounts as the ramp time is increased. With slow pulses, overall switching energy of case 2 is very small and the entire switching energy of case 1 essentially occurs during turn-on which is illustrated in Fig.8. This dissipation was discussed in section III B and it is associated with the sudden fall down from point A to B (see Fig.8, 9, c). It has a saturating nature and will never become zero. As $H_{\text{pulse}}$ is applied more and more gradually, the dissipated power in Fig.8 becomes narrower and taller. In the true adiabatic limit it will become a delta function occurring for one particular value of $H_{\text{pulse}}$. 
FIG. 8: (a) Turn-on dissipation versus ramp time. As ramp time is increased, dissipation in case 2 decreases arbitrary but it saturates in case 1. In both cases there is a significant drop in dissipation once the ramp time exceeds $\tau_c$. (b) Turn-off dissipation versus ramp time. In both cases dissipation can be made arbitrarily small by increasing the ramp time. Again there is a significant drop in dissipation as ramp time exceeds $\tau_c$. (c) Dissipated power vs. ramp time. This figure shows that in the slow limit of switching, for case 1 that has a saturating switching energy, the dissipated power essentially occurs during turn-on. This fact was discussed earlier in Fig.5b,c as the dissipation between points A and B during turn-on. If adiabatic limit of switching is really reached, then the dissipated power in this figure will become a very sharp spike.

V. MAGNETIZATION DYNAMICS: CHAIN OF INVERTERS

Fig.9a shows an array of spherical nanomagnets (MQCA) that interact with each other via dipole-dipole coupling. The objective is to determine the switching energy if we are to switch magnet 2 according to the state of magnet 1. In section V A we will show a clocking scheme under which propagation of information can be achieved and basically shows how magnets can be used as cascadable logic building blocks. In section V B, we briefly go over the method and equations used to simulate the dynamics and dissipation of the coupled magnets. In section V C we analyze the dissipation of the chain of inverters where we show that after cascading the magnetic bits, dissipation changes linearly with the number of magnets that the pulse is exerted on. This shows that the switching energy of larger more complicated circuits can be calculated using the one-magnet results presented in this paper at least approximately.

A. Clocking scheme

In the introduction we mentioned that in the clocking scheme the role of the clock field is to provide energy whereas field of another magnet acts as a guiding input. Using a clock we can operate an array of exactly similar magnets as a chain of inverters. Fig.9a shows a 3 phase inverter chain where the unit cell is composed of 3 magnets. Each magnet has two stable states showed as up and down in the figure. We want to switch magnet 2 according to the state of magnet 1. First consider only magnets 1 and 2. We’ve already explained (see section I) how magnet 1 can determine the final state of magnet 2. But what happens if more magnets are present?

Consider magnets 1, 2 and 3. Just like magnet 1, magnet 3 also exerts a field on magnet 2 and if it is in the opposite direction can cancel out the field of magnet 1. To overcome this, we apply the pulse to magnet 3 as well thereby diminishing the exerted z field of magnet 3 on magnet 2 so that magnet 1 becomes the sole decider of the final state of magnet 2. In the process the data in magnet 3 has been destroyed (it will end up wherever magnet 4 decides). It takes 3 pulses to transfer the bit (in an inverted manner) in magnet 1 to magnet 4. Magnet 4 has been included because it affects the dissipation of magnet 3 through affecting its dynamics. Inclusion of more magnets to the right or left of the array will not change the quantitative or qualitative results of this paper. Next we’ll briefly go over the method used to simulate the chain of inverters.
FIG. 9: (a) An array of identical nanomagnets with uniaxial anisotropy and easy axis along $z$ coupled together via dipolar coupling which can be operated as a 3 phase inverter chain. Initially the 4 magnet array can be randomly in any of the 16 possible states. A unit cell is composed of 3 magnets with the real information stored in magnet 1 in the initial state. A $y$ pulse provides energy and puts magnets 2 and 3 in the mid state thereby shutting off the $z$ field of magnet 3 on 2, so that field of magnet 1 can deterministically tilt magnet 2 downwards. Upon removing the pulse, magnet 2 relaxes down in the final state. (b) LLG simulation of coupled system of Fig.9a. This figure shows the proper operation of the clocking scheme by showing the normalized magnetization of magnet 2 along its easy axis for various initial configurations. (c) Dissipation of the array as a function of ramp time. There are $\binom{4}{2} = 6$ physically distinct configurations out of 16 possible states. The dissipation is lower if the initial configuration minimizes the energy of dipolar interaction. Assigning binary 1 to $\uparrow$ and binary 0 to $\downarrow$ the 6 curves (from highest to lowest) represent these configurations: (1)0,15 (2)1,7,8,14 (3)3,12 (4)2,4,11,13 (5)6,9 (6)5,10

B. Numerical simulation of the chain of inverters

Equations 12 (with $\alpha = 0.1$) and 13 are used to simulate the dynamics and dissipation of each magnet respectively. The overall scaled (divided by $H_c$) magnetic field $\vec{h}$ of equation 12 for each magnet at each instant of time is modified to

$$\vec{h} = \frac{\vec{H}_{\text{pulse}} + \vec{H}_{\text{ani}} + \vec{H}_{\text{dip}}}{H_c}$$

composed of the applied pulse:

$$\vec{H}_{\text{pulse}} = H_{\text{pulse}} \hat{y}$$

the anisotropy (internal) field of each magnet:

$$\vec{H}_{\text{ani}} = \frac{2Ku_2}{M_s} m_z \hat{z}$$

and exerted dipolar fields of other magnets which in general in CGS system of units reads

$$\vec{H}_{\text{dip}}^j = \sum_{n \neq j} 3 (\vec{\mu}_n \cdot \vec{r}_{nj}) \frac{\vec{r}_{nj}}{r_{nj}^5} - \vec{\mu}_n \frac{r_{nj}^2}{r_{nj}^5}$$

All field values are time dependent. Here $j$ denotes any one magnet and $\mu_n$ runs over magnetic moments of the other magnets. Though this equation can be simplified for an array of magnets along the same line, in this form it can be used for more complicated arrangement of magnets. Fig.9 shows the LLG simulations of the chain of inverters where magnet 2 is switched solely according to the state of magnet 1 irrespective of its history or the state of magnets 3 and 4.

C. Dissipation of the chain of inverters with one application of the pulse

Fig.9c shows dissipation of the entire array after one application of the pulse as a function of ramp time. The pulse is exerted on magnets 2 and 3 which accounts for the $4Ku_2V$ value in the fast limit. This essentially points out that
after cascading these logic building blocks, dissipation changes linearly with the number of magnets.

In the slow limit, depending on the initial configuration, dissipation will be affected. The 4 magnet array can initially be in any of its 16 possible states. Some configurations saturate and some don’t. Here the field of magnet 1 plays the role of the bias field $H_{dc}$ for magnet 2 and the field of magnet 4 is like another bias field on magnet 3 which accounts for the 3 groups of curves in Fig.9. The upper curves correspond to the situation where initial magnetization of both magnets 2 and 3 are opposite to the fields exerted from magnets 1 and 4 respectively. The middle curves correspond to only one of magnets 2 or 3 initially being opposite to the exerted fields of magnet 1 or 4 respectively. The lower curves correspond to both magnets 1 and 3 initially being in the same direction as the exerted fields from magnets 2 and 4 respectively.

An added complication is the field of the other neighbor (magnet 3) which is diminished in the $z$ direction but has a non-negligible $y$ component exerted on magnet 2. All this $y$ directed field does is to wash away a tiny bit the effect of the field of magnet 1 which has little bearing on the qualitative or quantitative results as illustrated in Fig.9.

VI. DISCUSSION AND PRACTICAL CONSIDERATIONS

A. Dissipation versus speed

The speed of switching can be increased by increasing the magnitude of the external pulse $H_{pulse}$ above $H_c$. Larger fields will dissipate more energy but have the advantage of aligning the magnet faster during the turn-on segment but are of no use for increasing the speed of the turn-off segment because the magnet relaxes to its stable state under its own internal field. If $H_c = \frac{2Ku_2 V}{M_s}$ can be altered, then it is a better idea to increase $H_c$ and always set $H_{pulse} = H_c$. This way the speed of switching is increased by shortening the time of both turn-on and turn-off segments.

B. Increasing the switching speed

Consider equation (14). Increasing $\alpha$ shortens the switching time constant (note that $\alpha$ is usually less than 1); however this parameter is not very controllable in experiments. $|\gamma|$ is a physical constant and cannot be altered. So to increase the switching speed, one has to increase $H_c = \frac{2Ku_2 V}{M_s}$. Thermal stability of a magnet requires $Ku_2 V$ to be larger than a certain amount for the desired retention time. For instance with an attempt frequency of about 1GHz (see the discussion at the end of III) and $Ku_2 V$ of about 0.5 eV, magnet is stable for about 0.5 seconds which is large enough because switching takes place in the nano-second scale. A higher retention time requires higher $Ku_2 V$.

Once $Ku_2 V$ is set because of stability requirements, the only way to increase $H_c$ is to decrease $M_s V$. Assuming that volume is magnetized to saturation, $M_s V = N_s u_B$ is the magnetic moment of the magnet. $N_s$ is the number of spins giving rise to the magnetization and $\mu_B$ is Bohr magneton. So decreasing $M_s V$ translates to making the magnet smaller or decreasing its saturation magnetization.

The discussion just presented is similar to the theory of scaling in CMOS technology where decreasing the capacitance causes an increase in the switching speed by decreasing the $RC$ time constant. With the same operating voltage, smaller capacitance results in lower number of charges stored on the capacitor. In the case of CMOS, as $C$ decreases, energy dissipated i.e. $0.5CV^2$ also decreases. In the case of magnet however, energy dissipation is fixed around $2Ku_2 V$ so for a lower $M_s V$, dissipation of the Ferro-magnetic logic element (already very small) is not altered; however one might be able to reduce the dissipated energy in the external circuitry since it needs to provide the energy for a shorter period of time. Again we should emphasize that a thorough analysis of external dissipation also has to be done. This has to do with generating the external source of energy for switching. In the case of MQCA circuits this is done by running currents through wires and generating magnetic fields. In principle, spin transfer torque phenomena or electrically controlled multi-Ferrioicty could also be used to provide the source of energy. These methods would also have energy dissipation associated with them.

C. Integration density

A complete lay-out circuit is necessary to properly evaluate the integration density of logic circuits made of magnets. For example fringing fields and unwanted cross talks have to be taken into account. External circuitry will take up space. Efficient methods have to be developed to properly address these issues. One component of the lay-out is the magnetic logic bit itself which we discuss here. The barrier height $E_b = Ku_2 V$ between the stable states of a magnet can be engineered by adjusting $Ku_2$ (anisotropy constant) and $V$ (volume). Increasing the anisotropy constant is of
great interest for the magnetic storage industry because it allows stable magnets of smaller volume that translates to higher densities. Many experiments report $Ku_2$ values on the order of a few $10^7$ erg/cm$^3$. This results in stable magnets with volumes of only 10s of nm$^3$; which means that stable magnets can be made as small as a few nm in each dimension. Even though a complete lay-out is necessary, nevertheless these numbers are very promising and could potentially result in very high integration densities.

VII. CONCLUSION

In this paper we analyzed the switching energy of single domain nanomagnets used as cascadable logic building blocks. A magnetic pulse was used to provide the energy for switching and a bias field was used as an input to guide the switching. The following conclusions can be drawn from this study.

1. Through analyzing the complete dependence of the switching energy on ramp time of the pulse, it was concluded that there is a significant and sharp drop in dissipation for ramp times that exceed a critical time given by equation whose significance is separating the energy dissipation characteristic of a fast pulse (small ramp time) and energy dissipation characteristic of a slow pulse (big ramp time).

2. The switching energy can be described by a single equation (equation 8) in both fast and slow limits for trapezoidal pulses analyzed in this paper. In the fast limit the effect of the bias field or equivalently the field of neighboring magnet in MQCA systems is negligible so long as the bias field is less than 10th of the switching field of the magnet. In the slow limit however, dissipation is largely determined by the value of the bias field.

3. By evaluating switching energy of both one magnet and a chain of inverters for MQCA systems, it was shown that the switching energy increases linearly with the number of magnets so that the one magnet results provided in this paper can be used to calculate the switching energy of larger more complicated circuits, at least approximately.

4. Practical issues such as dissipation versus speed, increasing the switching speed and scaling were discussed qualitatively. It was concluded that by proper designing, Ferromagnetic logic bits can have scaling laws similar to the CMOS technology.

Noise was not directly included in the models; however we took it into account indirectly: thermal noise is the limiting factor on the anisotropy energy $Ku_2V$ (that determines the magnet’s thermal stability) of a magnet which we discussed thoroughly. Thermal noise also limits the lowest possible magnitude of the bias field (or equivalently coupling between magnets in MQCA systems). We’ve provided the results for a wide range of bias values. More thorough discussions of dissipation in the external circuitry can be found in references.

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26 In principle magnet A can switch magnet B unidirectionally with no need for an external pulse given that: $K_{u_A V_A} > (M_{s_A} V_A) (M_{s_B} V_B) > K_{u_A V_B}$ holds. This entails designing circuits with magnets of different parameters (e.g. volume) so no two magnets in the circuit can have the same parameters; not to mention the complexities caused by the fields exerted from other neighbors.

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