Modification of the Turbulent Energy Cascade by Polymer Additives

Nicholas T. Ouellette,1,2† Haitao Xu,1,2 and Eberhard Bodenschatz1,2,3,4,5,6

1International Collaboration for Turbulence Research
2Max Planck Institute for Dynamics and Self-Organization, D-37077 Göttingen, Germany
3Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853, USA
4Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA
5Institute for Nonlinear Dynamics, Universität Göttingen, D-37073 Göttingen, Germany

(Dated: February 1, 2008)

PACS numbers: 47.27.Jv,47.27.Gs,47.57.Ng,47.50.-d

By tracking small particles in the bulk of an intensely turbulent flow, we show that even a very small concentration of long-chain polymers disrupts the usual turbulent energy cascade. The polymers affect scales much larger than their physical size, from the dissipation range to the inertial range. The effect depends strongly on the polymer concentration. While the dissipative-scale statistics change continuously as the polymer concentration is increased, the inertial-range energy transfer rate is only altered by the polymer additives when the concentration is above a threshold (approximately 5 parts per million by weight for the polymer we used).

Minute amounts of long-chain flexible polymers added to a fluid can strongly modify flow properties. In a turbulent wall-bounded flow, for example, they lead to the extraordinary phenomenon of drag reduction [1, 2, 3, 4, 5]. These effects may be qualitatively attributed to the stretching of polymer molecules by the velocity gradients in the flow. Although progress has recently been made in understanding drag reduction at a wall, comparatively little is known about the action of polymers far from the boundaries of a turbulent flow [6, 7, 8, 9].

Fluid turbulence is inherently far from equilibrium: energy continuously passes into and out of the system. In a Newtonian fluid like water, where molecular viscosity provides the sole mechanism for the dissipation of energy, the classical Richardson-Kolmogorov cascade hypothesis [10] states that energy is injected into the flow at large length and time scales, transferred to smaller and smaller scales without loss, and finally dissipated at the smallest scales where viscosity acts. The rates of energy injection $\epsilon_I$, energy transfer $\epsilon_T$, and energy dissipation $\epsilon_D$, are therefore equivalent in Newtonian fluid turbulence. Long-chain polymer molecules, which tend to coil up like balls of thread in their equilibrium state, can be stretched by the straining of the fluid flow to many times their equilibrium length and store elastic energy. In a turbulent flow, the polymer molecules will stretch and recoil in the fluctuating flow field, and this process will dissipate kinetic energy due to interactions between the monomers of one polymer molecule and between the polymers and the fluid. The addition of polymers to a turbulent flow therefore provides a new route by which kinetic energy can flow out of the turbulent phase. The previously unaddressed but fundamental question is then the relationship between $\epsilon_I$, $\epsilon_T$, and $\epsilon_D$ for turbulence in polymer solutions.

In this Letter, we show that a very small concentration of long-chain polymer molecules strongly modifies the turbulent cascade of energy from large to small scales. Our results indicate that the effect of polymers on the energy cascade can be divided into two regimes depending on polymer concentration. For small concentrations, the energy transfer rate $\epsilon_T$ is unchanged and only the viscous dissipation rate $\epsilon_D$ is reduced due to the additional dissipation mechanism provided by the polymers. For large concentrations, however, we find that not only $\epsilon_D$ but also the apparent energy transfer rate $\epsilon_T$ decreases. The observations cannot be explained by current theories.

Newtonian turbulence is described by a single nondimensional parameter, the Reynolds number, which compares the strength of inertial driving forces to viscous damping forces. A large Reynolds number implies that a wide range of length and time scales participate in the energy cascade, where the statistical properties of the flow are expected to be universal and independent of the driving mechanism. This universal regime is known as the inertial range. A polymer solution is additionally characterized by both the polymer concentration and the Weissenberg number $Wi$, which compares $\tau_p$, the relaxation time of a single polymer, to the fastest flow time scale. In turbulent flows, it is defined as $Wi = \tau_p/\tau_\eta$, where $\tau_\eta$ is the Kolmogorov time scale. When the Weissenberg number is less than a critical value (of order unity), the polymer molecules are generally in their coiled state and will be passively advected by the flow. When the Weissenberg number is larger than this value, the polymers will be stretched by the flow and may modify it [3].

Earlier experimental investigations of the interactions of bulk turbulence and polymers have generally either forced the turbulence through a boundary layer [11, 12, 13, 14], or have been performed at relatively low Reynolds number [6, 7], where the turbulence was not fully developed and it was difficult to quantify the effect of polymers on turbulence, particularly for inertial-range quantities. In a water flow between counter-rotating disks with raised vanes, it was observed that the energy injection at the disks remained constant when adding polymers into the flow [15]. A previous experiment with polymers in our apparatus, however, showed that the acceleration statistics of the flow were strongly affected [8, 9]. Numerical simulations of isotropic turbulence...
with polymers have also been performed at small Reynolds number using model equations \cite{16, 17, 18, 19}, but they are very difficult due both to the nature of the equations and to numerical instabilities \cite{20}.

In this Letter, we report results from Lagrangian particle tracking experiments conducted in a water flow between counter-rotating disks with vanes. Flow properties were measured by tracking \cite{21} the simultaneous motion of hundreds of nearly neutrally buoyant 33 μm fluorescent polystyrene tracer particles, excited by a high-power pulsed Nd:YAG laser delivering up to 90 W and recorded with three Phantom v7.1 CMOS cameras from Vision Research, Inc. The polymer used was an 18 × 10^6 a.m.u. molecular weight polyacrylamide (Polysciences 18522) with an equilibrium radius of gyration of 0.5 μm, a fully stretched length of 77 μm, and a relaxation time of τ_p = 43 ms \cite{9}. While our apparatus allows us to reach very high Reynolds numbers \cite{22, 23, 24, 25}, the size and flexibility of the polymer molecules makes them prone to tearing in very intense turbulence; we therefore only consider Reynolds numbers where our results are not affected by polymer degradation \cite{8, 9}.

In our experiments, the Weissenberg and Reynolds numbers are coupled: based on the smallest turbulent time scale, the Weissenberg number ranges from Wi = 1.2 to 6.0. The Taylor-microscale Reynolds number where our results are not affected by polymer degradation \cite{8, 9}. In Fig. 1(a), we plot D_{NN}(r) in pure water before polymer addition, with polymers and the Kolmogorov length scale is \eta_w = 84 μm.

FIG. 1: (color online) The effect of polymer concentration on the Eulerian structure functions. The Reynolds number of the water flow (before adding polymers) is R_\lambda = 350 (corresponding to Wi = 6.0 with polymers) and the Kolmogorov length scale is \eta_w = 84 μm. The structure functions compensated by (a) the inertial-range scaling prediction and (b) the dissipation-range scaling prediction. The dashed line in (b) indicates \epsilon_T(0) as measured in the water flow.

To measure the modification of the turbulent energy cascade by the polymer additives, we use the second-order transverse Eulerian structure function D_{NN}(r) = \langle (\delta_r u)^2 \rangle, which measures the difference in velocity over a separation r, as a probe of the scale-by-scale properties of the cascade. For the transverse structure function, velocities are measured orthogonal to r. The analogous longitudinal structure function D_{LL}(r) is defined with the velocities taken along r. We show here only measurements of D_{NN}(r); our results are equivalent for D_{LL}(r). D_{NN}(r) has three distinct scaling regimes. In the small-scale dissipation range (r \ll \eta), for isotropic turbulence,

\begin{equation}
D_{NN}(r) = \frac{2}{15} \nu r^2, \quad (r \ll \eta).
\end{equation}

At large scales (r \gg L), D_{NN}(r) saturates at twice the velocity variance. At intermediate scales, in the so-called inertial range (\eta \ll r \ll L), the classical Kolmogorov theory \cite{10} predicts that

\begin{equation}
D_{NN}(r) = \frac{4}{3} C_2 (\epsilon_T r)^{2/3}, \quad (\eta \ll r \ll L),
\end{equation}

where C_2 = 2.13 ± 0.22 is a well-known universal constant determined from previous experiments \cite{26}. We note again that in Newtonian turbulence, the energy transfer rate \epsilon_T and the energy dissipation rate \epsilon_D are the same.

We show in Fig. 1(a) the measured D_{NN}(r) for different polymer concentrations at R_\lambda = 350, where R_\lambda was measured without polymers and the corresponding Kolmogorov length scale is \eta_w = 84 μm. In Fig. 1(b), we plot D_{NN}(r) compensated by the Kolmogorov scaling prediction \cite{i.e.}, [(3/4)D_{NN}(r)/C_2]^{5/2}/(\epsilon_T(0)r), where \epsilon_T(0) is the energy transfer rate measured from the water data), so that a plateau at unity indicates inertial-range scaling. We observe that at small concentrations (\phi \leq 5 ppm in our experiments), the shape of D_{NN}(r) remains the same: the curves return to the same plateau value in the inertial range, while the extent of the apparent dissipation range increases with concent-
At higher concentrations ($\phi \geq 7$ ppm), the apparent inertial-range plateau is suppressed, indicating that the energy transfer rate has changed. We also measured the effect on the energy dissipation rate $\epsilon_D$ by plotting in Fig. 1(b) $D_{NN}(r)$ compensated by the dissipation-range scaling prediction (i.e., $[2\nu D_{NN}(r)/15r^2]$). Due to the finite spatial resolution of our measurement system, the very small scales ($r \ll \eta$) are only partially resolved. This effect is more pronounced for pure water case since for polymer solutions the small scales increases. Within experimental uncertainty, for water, $\epsilon_D(0) = \epsilon_F(0)$ as indicated by the dashed line in Fig. 1(b). Nevertheless, we observe that the effect of polymers on dissipation scales is smooth, without any sign of a transition.

To quantify the effect on the energy cascade at each polymer concentration we measured $\epsilon_T$ using Eq. 1 and $\epsilon_D$ using Eq. 2 as shown in Fig. 2. It is clear that $\epsilon_T$ remains approximately unchanged for small concentrations, but drops suddenly when the concentration is above 5 ppm. The energy dissipation rate $\epsilon_D$, on the other hand, decreases smoothly with concentration. We cannot measure the energy injection rate $\epsilon_I$ directly in our current apparatus. Since $\epsilon_I \sim u^3/L$, however, measurements of the root-mean-square turbulent velocity can serve as a surrogate, provided that the integral scale L remains constant. The measured $u'$ decreases slowly with concentration, as shown in Fig. 2. This nearly linear decay cannot account for the change of $\epsilon_T$.

The physical basis for the observed transition at a concentration of roughly 5 ppm remains unclear. It is possible that it may result from polymer-polymer interactions that occur only above the so-called “overlap” concentration. For the polymer used in our experiments, the overlap concentration based on the maximum extension length is $\sim 10^{-4}$ ppm, while it is approximately 200 ppm if based on the radius of gyration. Neither of these estimates coincides with the critical concentration observed in our experiments.

In his pioneering work in the 1970s, Lumley suggested that the length scale of any polymer effect should be determined purely by the time scale on which the polymer recoils [3]. This (inertial-range) scale $r^* = (\epsilon_F^2 \tau_p^2)^{1/2}$ is the scale at which the local Weissenberg number is unity, and at which we expect the polymers to begin to be stretched by the flow. In this theory, there is no concentration effect, and so it cannot fully explain our experimental observations. It may, however, apply below the critical concentration. Since the scale $r^*$ increases with Reynolds number, we tested Lumley’s hypothesis by varying the Reynolds number while keeping the concentration fixed at 5 ppm, just below the concentration where we observe changes in the energy transfer rate. The change of $D_{NN}(r)$ with $R_N$ is shown in Fig. 3(a). As shown in Fig. 3(b), however, scaling by $r^*$ does not collapse our data. We find instead, as illustrated in Fig. 3(c), that the naive normalization by the Kolmogorov length scale $\eta$ (measured from pure water at the same Reynolds number) collapses all the curves. We note that $\eta$ decreases with Reynolds number in our experiments, while the prediction of $r^*$ increases. The behavior we observe is therefore qualitatively different from drag reduction at a boundary, where Lumley’s theory does appear to apply [5].

In contrast to Lumley’s argument, Tabor and de Gennes suggested qualitatively that while the polymers are affected by the flow at $r^*$, it is only at a smaller scale $r^{**}$ that the flow is affected by the polymers [27, 28]. In their framework, $r^{**}$ is determined by balancing the turbulent kinetic energy at a given scale with the elastic energy in the polymer phase. Each polymer molecule can store some elastic energy; increasing the number of polymers therefore increases the energy in the polymer phase. The Tabor-de Gennes picture thus allows for a concentration dependence of the polymer effect. There are, however, many undetermined parameters in their qualitative theory, and further development is needed to make a quantitative experimental test.

In summary, we investigated the effect of very small concentrations of long-chain polymers on the dynamics of turbulence in the bulk of the flow. We observed a strong concentration dependence: the energy dissipation rate appears to change for all polymer concentrations, while the inertial-range energy transfer rate only changes above a critical concentration. At a fixed concentration below the critical concentration, we observed that the scales at which the polymers affects the energy cascade decrease with increasing Reynolds number, in a manner similar to the dependence of the Kolmogorov scale $\eta$ on Reynolds number. Both the concentration effect and the Reynolds number effect can not be explained by the theory (using Lumley’s time criterion) that has been used to explain the drag reduction phenomenon. Our results suggest several
challenges for future research. The qualitative change in the polymer effect above the critical concentration must be explained, and the exact ways in which the polymers change the three energy rates in turbulence must be clarified. And finally, if Lumley’s time criterion does not hold in the bulk, a physical mechanism by which the polymers can affect scales much larger than their size must be identified.

We are grateful to L. Collins and D. Vincenzi for helpful discussions over the course of this work, to M. Gibert, W. Pauls, and M. Torralba for comments and suggestions for the manuscript, and to A. Crawford for developing experimental protocols that give repeatable, robust results. This work was supported by the National Science Foundation under Grants No. PHY-9988755 and No. PHY-0216406 and by the Max Planck Society.

* Present address: Department of Physics, Haverford College, Haverford, PA 19041, USA
† Email: eberhard.bodenschatz@ds.mpg.de

[1] B. A. Toms, Proc. 1st Intern. Rheol. Congr. 2, 135 (1948).
[2] P. S. Virk, E. W. Merill, H. S. Mickley, K. A. Smith, and E. L. Mollo-Christensen, J. Fluid Mech. 30, 305 (1967).
[3] J. L. Lumley, J. Polymer Sci.: Macromolecular Reviews 7, 263 (1973).
[4] K. R. Sreenivasan and C. M. White, J. Fluid Mech. 409, 149 (2000).
[5] V. S. L’vov, A. Pomyalov, I. Procaccia, and V. Tiberkevich, Phys. Rev. Lett. 92, 244503 (2004).
[6] A. Liberzon, M. Guala, B. Lüthi, W. Kinzelbach, and A. Tsinober, Phys. Fluids 17, 031707 (2005).
[7] A. Liberzon, M. Guala, W. Kinzelbach, and A. Tsinober, Phys. Fluids 18, 125101 (2006).
[8] A. M. Crawford, A. La Porta, N. Mordant, and E. Bodenschatz, in Advances in Turbulence IX, edited by I. P. Castro, P. E. Hancock, and T. G. Thomas (2002), p. 306.
[9] A. M. Crawford, Ph.D. thesis, Cornell University (2004).
[10] A. N. Kolmogorov, Dokl. Akad. Nauk SSSR 30, 301 (1941).
[11] W. D. McComb, J. Allan, and C. A. Greated, Phys. Fluids 20, 873 (1977).
[12] P. Tong, W. I. Goldburg, and J. S. Huang, Phys. Rev. A 45, 7231 (1992).
[13] D. Bonn, Y. Couder, P. H. J. van Dam, and S. Douady, Phys. Rev. E 47, R28 (1993).
[14] E. van Doorn, C. M. White, and K. R. Sreenivasan, Phys. Fluids 11, 2387 (1999).
[15] O. Cadot, D. Bonn, and S. Douady, Phys. Fluids 10, 426 (1998).
[16] D. Vincenzi, S. Jin, E. Bodenschatz, and L. R. Collins, Phys. Rev. Lett. 98, 024503 (2007).
[17] E. De Angelis, C. M. Casciola, R. Benzi, and R. Piva, J. Fluid Mech. 531, 1 (2005).
[18] J. Davoudi and J. Schumacher, Phys. Fluids 18, 025103 (2006).
[19] P. Perlekar, D. Mitra, and R. Pandit, Phys. Rev. Lett. 97, 264501 (2006).
[20] T. Vaithianathan, A. Robert, J. G. Brasseur, and L. R. Collins, J. Non-Newtonian Fluid Mech. 140, 3 (2006).
[21] N. T. Ouellette, H. Xu, and E. Bodenschatz, Exp. Fluids 40, 301 (2006).
[22] A. La Porta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz, Nature 409, 1017 (2001).
[23] M. Bourgoin, N. T. Ouellette, H. Xu, J. Berg, and E. Bodenschatz, Science 311, 835 (2006).
[24] G. A. Voth, A. La Porta, A. M. Crawford, J. Alexander, and E. Bodenschatz, J. Fluid Mech. 469, 121 (2002).
[25] N. T. Ouellette, H. Xu, M. Bourgoin, and E. Bodenschatz, New J. Phys. 8, 109 (2006).
[26] K. R. Sreenivasan, Phys. Fluids 7, 2778 (1995).
[27] M. Tabor and P. G. de Gennes, Europhys. Lett. 2, 519 (1986).
[28] P. G. de Gennes, Physica A 140, 9 (1986).

FIG. 3: (color online) Reynolds number effects at fixed concentration. Compensated Eulerian structure functions are shown for four Reynolds numbers in a 5 ppm polymer solution. (a) The length scale of the polymer effects changes with Reynolds number. (b) Scaling by the Lumley scale \( \lambda ^* \) does not collapse the data for the different Reynolds numbers. (c) The small-scale data collapse when \( r / \eta \) is scaled by \( \eta \), the Kolmogorov length scale determined from water data.