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Tse, Siu Keung; Ding, Chang

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Accelerated Life Test Sampling Plans under Progressive Type II
Interval Censoring with Random Removals

Siu Keung Tse¹ & Chang Ding²

¹Department of Management Sciences, City University of Hong Kong, China
²Yunnan University of Finance & Economics, China

Correspondence: Siu Keung Tse, Department of Management Sciences, City University of Hong Kong, China.

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Abstract
This paper investigates the design of accelerated life test (ALT) sampling plans under progressive Type II interval censoring with random removals. For ALT sampling plans with two over-stress levels, the optimal stress levels and the allocation proportions to them are obtained by minimizing the asymptotic generalized variance of the maximum likelihood estimation of model parameters. The required sample size and the acceptability constant which satisfy given levels of producer’s risk and consumer’s risk are found. ALT sampling plans with three over-stress levels are also considered under some specific settings. The properties of the derived ALT sampling plans under different parameter values are investigated by a numerical study. Some interesting patterns, which can provide useful insight to practitioners in related areas, are found. The true acceptance probabilities are computed using a Monte Carlo simulation and the results show that the accuracy of the derived ALT sampling plans is satisfactory. A numerical example is also provided for illustrative purpose.

Keywords: accelerated life test, progressive Type II interval censoring, random removal, sampling plan

1. Introduction
The design of reliability sampling plans under Type II censoring schemes has been studied by many researchers (Fertig & Mann, 1980; Hosono, Ohta, & Kase, 1981; Kocherlakota & Balakrishnan, 1986; Schneider, 1989; Balasooriya, 1995; Wu, Hung, & Tsai, 2003). In practice, it is not uncommon that some units are removed during the test, which leads to progressive censoring schemes. Balasooriya and Saw (1998), Balasooriya and Balakrishnan (2000), and Balasooriya, Saw, and Gadag (2000) discussed reliability sampling plans for the two-parameter exponential, lognormal and Weibull distributions under progressive Type II censoring schemes, respectively.

The number of removals at each failure was assumed to be pre-fixed in those works. However, in practice it might be infeasible to pre-determine the removal pattern and the decision of removing any units is based on the status of the experiment at that specific time, such as excessive heat or pressure, reduction of budget and facility, etc. Therefore, the number of removals should be a random outcome (Yuen & Tse, 1996). Tse and Yang (2003) discussed the design of reliability sampling plans for the Weibull distribution under progressive Type II censoring with random removals, where the number of units removed at each failure was assumed to follow a binomial distribution. In recent years the feature of random removal has been adopted by many researchers in designing various kinds of progressive censoring schemes, such as Ashour and Afify (2007), Wu, Chen, and Chang (2007), and S. Dey and T. Dey (2014).

Units are supposed to run at use condition in traditional reliability sampling plans. When it is desired to test the acceptance of highly reliable products, it is impractical to use such reliability sampling plans due to time constraint. Wallace (1985) stressed the need for introducing ALT into reliability sampling plans. Bai, Kim, and Chun (1993) studied the design of failure censored ALT sampling plans for lognormal and Weibull distributions. Hsieh (1994) investigated reliability sampling plans with ALT under Type II censoring for exponential distribution. The optimal design of ALT sampling plans with a non-constant shape parameter under both Type I and Type II censoring schemes was given by Seo, Jung, and Kim (2009).

Note that continuous inspections were assumed in the above works. Nevertheless, sometimes it is inconvenient to conduct a test with continuous inspections due to the high cost and/or possible danger in monitoring the test continuously. Under these circumstances, the interval inspection schemes, in which only the number of failures between two successive
Consider an ALT with the following settings:

1. A total of \( n \) identical and independent units are available at the beginning of the test.
2. There are \( m \) over-stress levels, i.e., \( s_1, s_2, \ldots, s_m \). Denote \( s_0 \) as the stress level at use condition.
3. Suppose that \( n_i \) units are randomly allocated to the \( i \)-th stress level \((i = 1, 2, \ldots, m)\). Then the allocation proportion to the \( i \)-th stress level is given by \( \alpha_i = n_i/n \).
4. A progressive Type II censoring scheme is employed, and the test on the \( i \)-th stress level will be terminated after \( c_i \) \((i = 1, 2, \ldots, m)\) or more units fail.
5. Interval inspections are conducted at time points \( t_{i1}, t_{i2}, \ldots, t_{ik(i)} \) and the number of failures \( x_{ij} \) between inspection interval \((t_{i,j-1}, t_{ij})\) is recorded. It should be pointed out that both the experiment time \( t_{ik(i)} \) and the number of inspections \( k(i) \) are random variables.
6. Suppose that \( r_{ij} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, k(i) - 1) \) non-failed units are randomly removed at inspection time \( t_{ij} \). To ensure that there are at least \( c_j \) failed units at the end of the test on stress level \( s_j \), \( r_{ij} \) is restricted to be any integer value between 0 and \( n_i - c_i - \sum_{r=1}^{j-1} r_{ir} \). Further assume that \( r_{ij} \) follows a binomial distribution with probability \( p \), then we have \( r_{ij} \sim B(n_i - c_i - \sum_{r=1}^{j-1} r_{ir}, p) \). For notational convenience, denote \( r_{k(i)} = n_i - \sum_{j=1}^{k(i)} x_{ij} - \sum_{j=1}^{k(i) - 1} r_{ij} \) as the number of units left.

The process of this testing scheme is depicted in Figure 1.
Suppose that the lifetime of a unit $T$ follows a Weibull distribution with probability density function (pdf)

$$f(t) = (\delta/\theta)(t/\theta)^{\delta-1} \exp\left[-(t/\theta)^\delta\right], \quad t > 0.$$  

(1)

Further assume that the scale parameter $\theta$ and stress level $s$ are related as

$$\theta = \exp(\beta_0 + \beta_1 s),$$  

(2)

where $\beta_0$ and $\beta_1$ are unknown constants and the shape parameter $\delta$ does not depend on $s$. Define $Y = \ln(T)$, then $Y$ has an extreme value distribution with cumulative distribution function (cdf)

$$G(y) = 1 - \exp\left[-\exp((y - \mu)/\sigma)\right], \quad -\infty < y < +\infty,$$

(3)

where $\mu = \ln\theta = \beta_0 + \beta_1 s$, $\sigma = 1/\delta$.

Given observations $(x_i, r_i, y_i, r_i, k(i))$, the logarithm of the likelihood function can be derived as:

$$\ln L(\beta_0, \beta_1, \sigma, p; x_i, r_i, k(i)) = \sum_{i=1}^{n} \left[ \ln \left( \frac{n_i^{(i)} - x_i - r_i}{n_i^{(i)}} \right) + \ln \left( \frac{n_i^{(i)} x_i}{n_i^{(i)} - x_i - r_i} \right) + \ln \left( \frac{n_i^{(i)} x_i}{n_i^{(i)} - x_i - r_i} \right) \right],$$

(4)

$$= \sum_{i=1}^{n} \left[ \ln \left( \frac{n_i^{(i)} - x_i - r_i}{n_i^{(i)}} \right) + \ln \left( \frac{n_i^{(i)} x_i}{n_i^{(i)} - x_i - r_i} \right) + \ln \left( \frac{n_i^{(i)} x_i}{n_i^{(i)} - x_i - r_i} \right) \right],$$

(5)

where $y_i = \ln(t_i)$.

The maximum likelihood estimates of $\beta_0$, $\beta_1$, $\sigma$ and $P$ (denoted by $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\sigma}$ and $\hat{p}$, respectively) can be solved from equations $\partial \ln L / \partial \beta_0 = \partial \ln L / \partial \beta_1 = \partial \ln L / \partial \sigma = \partial \ln L / \partial p = 0$. Besides, the Fisher information matrix of $(\beta_0, \beta_1, \sigma, p)$ is given by

$$I(\beta_0, \beta_1, \sigma, p) = -E \begin{bmatrix} \frac{\partial^2 \ln L}{\partial \beta_0^2} & \frac{\partial^2 \ln L}{\partial \beta_0 \partial \beta_1} & \frac{\partial^2 \ln L}{\partial \beta_0 \partial \sigma} & \frac{\partial^2 \ln L}{\partial \beta_0 \partial p} \\ \frac{\partial^2 \ln L}{\partial \beta_1 \partial \beta_0} & \frac{\partial^2 \ln L}{\partial \beta_1^2} & \frac{\partial^2 \ln L}{\partial \beta_1 \partial \sigma} & \frac{\partial^2 \ln L}{\partial \beta_1 \partial p} \\ \frac{\partial^2 \ln L}{\partial \beta_0 \partial \sigma} & \frac{\partial^2 \ln L}{\partial \beta_1 \partial \sigma} & \frac{\partial^2 \ln L}{\partial \sigma^2} & \frac{\partial^2 \ln L}{\partial \beta_0 \partial p} \\ \frac{\partial^2 \ln L}{\partial \beta_0 \partial p} & \frac{\partial^2 \ln L}{\partial \beta_1 \partial p} & \frac{\partial^2 \ln L}{\partial \sigma^2} & \frac{\partial^2 \ln L}{\partial p^2} \end{bmatrix} = I_1(\beta_0, \beta_1, \sigma) 0 \quad I_2(p),$$

(5)

where $I_1(\beta_0, \beta_1, \sigma)$ is the upper left $3 \times 3$ sub-matrix of $I(\beta_0, \beta_1, \sigma, p)$ and $I_2(p) = -E(\partial^2 \ln L / \partial p^2)$. The asymptotic covariance matrix of $(\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma})$ is then given by $I^{-1}(\beta_0, \beta_1, \sigma)$. The detailed formulation for the entries of Eq. (5) can be found in the Appendix of Ding and Tse (2013).

Given sample size $n$, use condition $s_0$ and high stress level $s_m$, removal probability $p$, predetermined number of
failures \( c \) and inspections times \( (r_i, i = 1, 2, ..., m, j = 1, 2, ..., k(i)) \) on each stress, the stress levels \( (s_i, i = 1, 2, ..., m-1) \) and the allocation proportions \( (\alpha_i, i = 1, 2, ..., m-1) \) to these stress levels are selected in such a way that the generalized asymptotic variance of \( \hat{\beta}_0, \hat{\beta}_1 \) and \( \hat{\sigma} \), which is given by \[ I_1^{\prime}(\beta_0, \beta_1, \sigma)/I_1(\beta_0, \beta_1, \sigma) \], is minimized.

### 3. Design of ALT Sampling Plans

Suppose that a sample of size \( n \) is randomly drawn from the lot and the test is conducted at the accelerated settings described in Section 2. Assume that the lifetime of a unit \( T \) follows a Weibull distribution \( F(\theta, \delta) \), where the relationship between the scale parameter \( \theta \) and the stress \( s \) is given by Eq. (2) and the shape parameter \( \delta \) does not depend on \( s \). Suppose that a unit with lifetime less than \( \eta \) is considered to be nonconforming. Define \( Y = \ln(T) \), then \( Y \) follows an extreme value distribution \( G(\mu, \sigma) \) and the lower specification limit for the log lifetime is given by \( \zeta = \ln(\eta) \).

Define \( \kappa = \mu_0 - d\sigma \), where \( \mu_0 \) is the location parameter of \( G(.) \) at use condition and \( d \) is the acceptability constant. Since the stresses can be standardized such that \( s_0 = 0, s_m = 1 \) and \( 0 < s_i < 1 \) \( (i = 1, 2, ..., m-1) \), it follows from Eq. (3) that \( \mu_0 = \hat{\beta}_0 + \hat{\beta}_1 s_0 = \hat{\beta}_0 \). By the invariance principle of the maximum likelihood method, the MLE of \( \kappa \) is then given by \( \hat{\kappa} = \hat{\mu}_0 - d\hat{\sigma} = \hat{\beta}_0 - d\hat{\sigma} \). To judge whether a lot should be accepted or not, \( \hat{\kappa} \) is compared with the lower specification limit \( \zeta \). If \( \hat{\kappa} > \zeta \), the lot is accepted; otherwise, it is rejected.

Define the nonconforming fraction of the lot by \( p_f \), which is calculated as
\[
p_f = P(Y < \zeta) = 1 - \exp\left\{ -\exp\left( (\zeta - \mu_0)/\sigma \right) \right\}.
\]

The sample size \( n \) and the acceptability constant \( d \) are determined such that lots with nonconforming fraction \( p_f < p_c \) are accepted with a probability of at least \( 1 - \tilde{\vartheta} \) and lots with \( p_f > p_c \) are rejected with a probability of at least \( 1 - \beta \), where \( \tilde{\vartheta} \) and \( \beta \) are the given levels of producer’s and consumer’s risks, respectively.

It follows from \( \hat{\kappa} = \hat{\beta}_0 - d\hat{\sigma} \) that \( Var(\hat{\kappa}) = Var(\hat{\beta}_0) - 2d \times Cov(\hat{\beta}_0, \hat{\sigma}) + d^2 Var(\hat{\sigma}) \).

Since \( U = [\hat{\kappa} - (\beta_0 - d\sigma)]/[Var(\hat{\kappa})]^{1/2} \) is parameter-free and asymptotically standard normal, the operating characteristic (OC) curve is given by
\[
O(p_f) = P(\hat{\kappa} > \zeta) = 1 - \Phi\left[ \frac{\sigma \ln(-\ln(1 - p_f)) + d\sigma}{\sqrt{Var(\hat{\kappa})}} \right],
\]
where \( \Phi(.) \) is the cdf of the standard normal distribution.

The sample size \( n \) and the acceptability constant \( d \) are determined such that the OC curve goes through two points \( (p_c, 1 - \tilde{\vartheta}) \) and \( (p_c, \beta) \). This implies
\[
1 - \tilde{\vartheta} = 1 - \Phi\left[ \frac{\sigma \ln(-\ln(1 - p_c)) + d\sigma}{\sqrt{Var(\hat{\kappa})}} \right],
\]
\[
\beta = 1 - \Phi\left[ \frac{\sigma \ln(-\ln(1 - p_c)) + d\sigma}{\sqrt{Var(\hat{\kappa})}} \right].
\]

It follows that
\[
d = \left[ u_{\vartheta, p} \left( -\ln(1 - p_c) \right) - u_{\vartheta, p} \left( -\ln(1 - p_c) \right) \right] / \left( u_{\vartheta, p} - u_{\vartheta, p} \right),
\]
\[
Var(\hat{\kappa}) = \sigma^2 \left( -\ln(1 - p_c) + d \right)^2 / u_{\vartheta}^2,
\]
where \( u_{\vartheta} = \Phi^{-1}(\vartheta) \). The acceptability constant \( d \) is calculated directly from the first part of Eq. (9), while the required sample size \( n \) can be obtained by a search method from the second part (the detailed algorithm is provided in Section 4.1).

### 4. Numerical Study

#### 4.1. ALT Sampling Plans with Two Over-stress Levels

The properties of the derived ALT sampling plans under different parameter values are evaluated by a numerical study in this section. The following settings are made:

1. Two over-stress levels \( s_1, s_2 \) are employed, i.e., \( m = 2 \).
2. The inspections on each stress level are equally spaced, i.e., \( t_0 = 0, \ t_j = t_{i,j-1} + l_i \) \((i = 1, 2, ..., m; j = 1, 2, ..., k(i))\), where \( l_i \) is the inspection length on the \( i^{th} \) stress level. Define \( MT_i = \exp(\beta_{0i} + \beta_{1i} \delta) \Gamma(1+1/\delta) \) as the mean of units’ lifetime distribution on the same stress and \( \tau_i = l_i / MT_i \) as the proportion of the inspection length to the corresponding mean. \( \tau_i \), which is proportional to the inspection length \( l_i \), is used in this numerical study since it is more convenient to use a relative value than an absolute one. The case of \( \tau_1 = \tau_2 = \tau \) is considered.

3. Define the censoring fraction on the \( i^{th} \) stress level as \( \hat{f}_i = c_i / n_i \) \((i = 1, 2)\). The cases of both \( \hat{f}_{c1} = f_{c2} \) and \( \hat{f}_{c1} < f_{c2} \) are considered since units are much easier to fail on the high stress level than on the low one.

Without loss of generality, set \( s_1 = 0, \ s_m = 1 \). In practice, it is often difficult for an experimenter to give prior estimates of parameters \( \hat{\beta}_0 \) and \( \hat{\beta}_1 \). On the contrary, based on the experimenters’ experiences and/or the information collected from preliminary or similar studies, the estimation of the probability that a unit falls into a certain interval is much easier. Define \( P_n = P \) (a unit’s lifetime \( T \) falls into \((0, 1)\) at use condition) and \( P_n = P \) (a unit’s lifetime \( T \) falls into \((0, 1)\) on high stress level), then we have

\[
\hat{\beta}_0 = -\sigma \ln \left( -\ln (1 - P_n) \right), \\
\hat{\beta}_1 = \sigma \ln \left( -\ln (1 - P_n) \right) - \ln \left( -\ln (1 - P_n) \right).
\]

In order to obtain an optimal ALT sampling plan under progressive Type II interval censoring with random removals, the values of \( n, d, s_i \) and \( \alpha_i \) have to be determined. The acceptability constant \( d \) depends on \((p_n, 1-\delta)\) and \((p_n, \beta)\) only, and it can be calculated from Eq. (9) directly. The determination of the other three parameters requires the combination of a grid search method and the Monte Carlo simulation. For the sake of simplicity, let \( \Delta \) denote \( \sigma^2 \left( \ln \left( -\ln (1 - P_n) \right) + d \right)^2 / u_\lambda^2 \). Then, \( n, s_i \) and \( \alpha_i \) are calculated using the following algorithm:

1. Set an initial value \( n^{(0)} \) for \( n \). Consider the smallest sample size and set \( n^{(0)} = 2 \). Find the optimal \((s^{i}_1, \alpha^{i}_1)\) which minimizes \( |I_1^{-1}(\beta_0, \beta_1, \sigma)| \) using a grid search method over unit square \((0,1) \times (0,1)\). Calculate the corresponding value of \( \text{Var} \left( \hat{\kappa} \right) \) at \((s^{i}_1, \alpha^{i}_1)\).

2. Set \( n^{(1)} = 2n^{(0)} \), find \((s^{i}_1, \alpha^{i}_1)\) based on sample size \( n^{(1)} \) and compute \( \text{Var} \left( \hat{\kappa} \right) \) accordingly.

3. Repeat step 2 until \( n^{(i)} = \Delta \) and for \( n^{(i+1)} \), \( \text{Var} \left( \hat{\kappa} \right) < \Delta \). Define \( n^{(i)} = n^{(i)} \) and \( n^{(n)} = n^{(n+1)} \).

4. Set \( n^{(i+2)} = (n^{(i)} + n^{(n)}) / 2 \). Find \((s^{i}_1, \alpha^{i}_1)\) and calculate \( \text{Var} \left( \hat{\kappa} \right) \). If \( \text{Var} \left( \hat{\kappa} \right) > \Delta \), set \( n^{(i)} = n^{(i+1)} \); otherwise, set \( n^{(n)} = n^{(i+2)} \).

5. Repeat step 4 until \( \text{Var} \left( \hat{\kappa} \right) = \Delta \) approximately holds or \( n^{(n)} - n^{(i)} \leq 2 \).

For a given parameter values, specifically \( (p_n, 1-\delta) = (0.00041, 0.95) \); \( (p_n, \beta) = (0.01840, 0.10) \); \( P_n = 0.01 \); \( P_n = 0.1 \); \( \delta = 0.5, 1, 2 \); \( (f_{c1}, f_{c2}) = (0.5, 0.5), (0.8, 0.8), (0.5, 0.7), (0.5, 0.9) \); \( p = 0, 0.05, 0.1, 0.3 \) and \( \tau = 0.02, 0.05, 0.1, 0.3 \), the optimal ALT sampling plans \((n, d, s^{*}_1, \alpha^{*}_1)\) are determined using the algorithm described above. A consistent pattern emerged based on the results of these combinations. The required sample size \( n \) decreases as the censoring fractions \( f_{c1}, f_{c2} \) increase. In order to provide a better insight on the effect of \( p \) (the probability of random removal) and \( \tau \) (which is proportional to the inspection length), some cases are selected for illustration and the corresponding results are depicted in Figure 2.
The following patterns are observed:

a. For the cases of $\delta = 0.5$, $n$ increases as $\tau$ increases for all values of $p$. For the cases of $\delta = 1$, when $p=0$, 0.05 and 0.1, $n$ increases as $\tau$ increases; when $p=0.3$, $n$ first decreases and then increases as $\tau$ increases. For the cases of $\delta = 2$, when $p=0$ and 0.05, $n$ increases as $\tau$ increases; when $p=0.1$ and 0.3, $n$ first decreases and then increases as $\tau$ increases. This pattern can be interpreted in this way: Larger $\tau$ means wider inspection intervals, from which the collected information on units’ lifetime is less accurate and thus more units are required to judge whether to accept the lot or not. However, when $p > 0$, a larger $\tau$ also implies that units are less likely to be removed at the early stage of the test. Consequently, more information on the lifetime distribution is collected and the required sample size $n$ is decreased. Taking these two kinds of effect into consideration, shorter inspection intervals doesn’t always yield smaller required sample size for ALT sampling plans under progressive Type II interval censoring with random removals.

b. For the cases of $\delta = 0.5$ and 1, $n$ decreases as $p$ increases for all values of $\tau$. For the cases of $\delta = 2$, when $\tau =0.1$ and 0.3, $n$ decreases as $p$ increases; when $\tau =0.02$ and 0.05, $n$ first decreases and then increases as $p$ increases. This pattern is caused by the two-sided effects of the removal probability $p$. Generally speaking, a test is likely to be prolonged as $p$ increases. Thus more information on the lifetime distribution can be observed and the required sample size $n$ is decreased. Nevertheless, when the inspection intervals are too small, a non-zero removal probability $p$ also causes more units being removed at the early stage of the test. In this case, less data can be collected and thus $n$ is increased. In conclusion, except for several cases ($\delta = 2$ and $\tau =0.02/0.05$), the removal probability $p$ is helpful in reducing the required sample size $n$.

4.2. ALT Sampling Plans with Three Over-stress Levels

ALT plans with three over-stress levels are useful in practice since they can provide a way to check the assumed straight-line relationship between distribution parameter $\mu$ and stress level $s$ by adding a middle stress. The design of three over-stress levels ALT sampling plans under progressive Type II interval censoring with random removals is discussed in this section. They are developed under the following settings:

1. Three over-stress levels, $s_1$, $s_2$ and $s_3$ are employed. In particular, set $s_0 = 0$, $s_3 = 1$ and $s_2 = (s_1 + s_3)/2$.

2. The allocation proportions to three over-stress levels ($\alpha_1$, $\alpha_2$, $\alpha_3$) are set to be $(1/3, 1/3, 1/3)$ and $(0.5, 0.3, 0.2)$.

3. Three settings of censoring fractions are considered, namely, $(f_{i1}, f_{i2}, f_{i3})$ equals $(0.5, 0.5, 0.5)$, $(0.8, 0.8, 0.8)$ and $(0.5, 0.7, 0.9)$.

4. The proportion of the inspection length to the corresponding mean, that is, $\tau_i$ is set to be equal on three over-stress levels, i.e., $\tau_1 = \tau_2 = \tau_3 = \tau$.

A numerical study is conducted to determine ALT sampling plans with three over-stress levels under equally spaced inspection times. For given parameters, the optimal low stress level $s^*_1$ is found by a grid search method over interval $(0,
1). The step size is 0.01. For different parameter values, specifically, \((p, \delta, 1 - \delta) = (0.00041, 0.95)\); \((p, \beta, \beta) = (0.01840, 0.10)\); \(P_u = 0.01\); \(P_n = 0.1\); \(\delta = 0.5, 1, 2\); \(p = 0, 0.05, 0.1, 0.3\) and \(\tau = 0.02, 0.05, 0.1, 0.3\), the optimal low stress level \(s^*\) and the required sample size \(n\) are calculated. The effects of \(p\) and \(\tau\) on the required sample size \(n\) are depicted in Figure 3. We note that:

![Figure 3. Three over-stress levels ALT sampling plans under progressive Type II interval censoring with random removals](image-url)

(a)\(\delta = 0.5\), \(\{q, \alpha, \beta\} = (1, 3, 1/3)\), \(f_3 = 0.5, f_2 = 0.7, f_1 = 0.9\)
(b)\(\delta = 1\), \(\{q, \alpha, \beta\} = (0.5, 0.3, 0.2)\), \(f_3 = f_1 = f_2 = 0.5\)
(c)\(\delta = 2\), \(\{q, \alpha, \beta\} = (1, 0.2, 1/3)\), \(f_3 = f_1 = f_2 = 0.5\)

We note from Table 1 and Table 2 that the simulated acceptance probabilities are close to their nominal values in most cases. This indicates that the optimal ALT sampling plans derived based on asymptotic approximation have satisfactory accuracy.
of the lot. The probabilities for a unit to fail at use condition and high stress level are estimated to be 0.01 and 0.1, respectively. A progressive Type II interval censoring scheme is employed, and the censoring fractions on two stress levels are 0.8. The proportions of the inspection length to the corresponding distribution mean on both stresses are set to be 0.1. Besides, based on prior information, it is assumed that units’ lifetimes are Weibull distributed with shape parameter \( \delta = 1 \) and a unit is likely to be removed at each inspection with probability 0.1. The problem is to determine the number of units used in this ALT sampling plan and to determine the low stress level and the allocation proportions to two stresses so that (1) both the consumer’s risk and the producer’s risk can be satisfied and (2) the maximum amount of information on units’ lifetime distribution can be collected.

The optimal ALT sampling plan is obtained using the proposed method. The required sample size is 17, with 7 and 10 units allocated to the low and high stress levels, respectively. The low stress level should be settled at 0.02 multiplied by the actual high stress. Besides, the acceptability constant which is required to make the decision is 5.6560.

7. Conclusion

The design of ALT sampling plans under progressive Type II interval censoring with random removals was discussed in this paper. For ALT sampling plans with two over-stress levels, the optimal stress levels and the corresponding allocation proportions, which minimize the generalized asymptotic variance of the MLE of model parameters, were found. The sample size and the acceptability constant required to judge the acceptability of the lot were calculated.

The properties of the derived ALT sampling plans were examined by a numerical study. It is shown that generally the removal probability is helpful in reducing the required sample size. More importantly, when there exists random removal, short inspection interval doesn’t always yield small required sample size, which is different from the case of no random removal. These interesting patterns would provide useful insights to experimenter in designing similar ALT sampling plans. The accuracy of the proposed sampling plans was evaluated by a Monte Carlo simulation. The results show the simulated acceptance probabilities are close to their nominal values in most cases, which indicates that the performance of the derived ALT sampling plans is satisfactory.

Table 1. Simulated acceptance probabilities for two over-stress levels ALT sampling plans under progressive Type II interval censoring with random removals (\( m = 2; \delta = 1; p_{\theta} = 0.00041; 1 - \hat{\delta} = 0.95; p_{\theta} = 0.01840; \beta = 0.10 \))

| \( \tau \) | \( n \) | Selected points on OC curve | Simulated probability | \( f_{c1} = f_{c2} = 0.5 \) | 99% | 95% | 99% | 95% | \( f_{c1} = f_{c2} = 0.8 \) | 99% | 95% | 99% | 95% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.02 | 25 | 0.00016 | 0.00039 | 0.987 | 0.973 | 17 | 0.00016 | 0.00037 | 0.986 | 0.967 |
| 0.05 | 29 | 0.00018 | 0.00042 | 0.984 | 0.957 | 19 | 0.00017 | 0.00040 | 0.985 | 0.963 |
| 0.1 | 29 | 0.00016 | 0.00039 | 0.982 | 0.949 | 19 | 0.00015 | 0.00038 | 0.991 | 0.970 |
| 0.3 | 45 | 0.00017 | 0.00042 | 0.966 | 0.923 | 24 | 0.00017 | 0.00040 | 0.989 | 0.953 |
| 0.02 | 21 | 0.00018 | 0.00041 | 0.992 | 0.978 | 17 | 0.00017 | 0.00043 | 0.995 | 0.972 |
| 0.05 | 23 | 0.00016 | 0.00038 | 0.991 | 0.973 | 17 | 0.00015 | 0.00039 | 0.988 | 0.965 |
| 0.1 | 29 | 0.00017 | 0.00038 | 0.989 | 0.966 | 17 | 0.00015 | 0.00040 | 0.982 | 0.972 |
| 0.3 | 44 | 0.00017 | 0.00041 | 0.972 | 0.949 | 24 | 0.00017 | 0.00042 | 0.985 | 0.933 |
| 0.02 | 21 | 0.00019 | 0.00044 | 0.991 | 0.975 | 17 | 0.00018 | 0.00044 | 0.995 | 0.974 |
| 0.05 | 22 | 0.00016 | 0.00039 | 0.994 | 0.973 | 17 | 0.00017 | 0.00041 | 0.985 | 0.971 |
| 0.1 | 25 | 0.00016 | 0.00039 | 0.985 | 0.969 | 17 | 0.00015 | 0.00039 | 0.989 | 0.957 |
| 0.3 | 41 | 0.00016 | 0.00038 | 0.968 | 0.948 | 24 | 0.00018 | 0.00044 | 0.982 | 0.962 |
| 0.02 | 21 | 0.00016 | 0.00042 | 0.992 | 0.979 | 17 | 0.00016 | 0.00040 | 0.992 | 0.970 |
| 0.05 | 20 | 0.00016 | 0.00039 | 0.997 | 0.976 | 15 | 0.00014 | 0.00037 | 0.984 | 0.968 |
| 0.1 | 20 | 0.00014 | 0.00035 | 0.992 | 0.967 | 17 | 0.00017 | 0.00038 | 0.988 | 0.978 |
| 0.3 | 37 | 0.00015 | 0.00039 | 0.973 | 0.945 | 22 | 0.00017 | 0.00042 | 0.982 | 0.955 |
|   | 99%  | 95%  | 99%  | 95%  | 99%  | 95%  | 99%  | 95%  |
|---|------|------|------|------|------|------|------|------|
| $p = 0.0$ |      |      |      |      |      |      |      |      |
| 0.02 | 21   | 0.00015 | 0.00040 | 0.991 | 0.974 | 18   | 0.00017 | 0.00042 | 0.989 | 0.956 |
| 0.05 | 22   | 0.00016 | 0.00037 | 0.989 | 0.969 | 17   | 0.00015 | 0.00036 | 0.992 | 0.967 |
| 0.1  | 24   | 0.00016 | 0.00039 | 0.984 | 0.961 | 19   | 0.00017 | 0.00042 | 0.990 | 0.948 |
| 0.3  | 31   | 0.00016 | 0.00038 | 0.977 | 0.954 | 24   | 0.00017 | 0.00042 | 0.985 | 0.963 |
| $p = 0.05$ |      |      |      |      |      |      |      |      |
| 0.02 | 18   | 0.00017 | 0.00039 | 0.994 | 0.976 | 17   | 0.00019 | 0.00041 | 0.994 | 0.965 |
| 0.05 | 21   | 0.00018 | 0.00043 | 0.984 | 0.967 | 17   | 0.00016 | 0.00041 | 0.992 | 0.962 |
| 0.1  | 21   | 0.00016 | 0.00038 | 0.981 | 0.977 | 19   | 0.00018 | 0.00042 | 0.987 | 0.969 |
| 0.3  | 31   | 0.00016 | 0.00040 | 0.979 | 0.947 | 22   | 0.00016 | 0.00038 | 0.981 | 0.964 |
| $p = 0.1$ |      |      |      |      |      |      |      |      |
| 0.02 | 17   | 0.00015 | 0.00037 | 0.994 | 0.975 | 17   | 0.00015 | 0.00042 | 0.999 | 0.972 |
| 0.05 | 19   | 0.00017 | 0.00041 | 0.997 | 0.973 | 17   | 0.00016 | 0.00042 | 0.994 | 0.968 |
| 0.1  | 19   | 0.00014 | 0.00038 | 0.990 | 0.973 | 17   | 0.00015 | 0.00039 | 0.994 | 0.969 |
| 0.3  | 30   | 0.00017 | 0.00042 | 0.979 | 0.950 | 22   | 0.00017 | 0.00042 | 0.982 | 0.951 |
| $p = 0.3$ |      |      |      |      |      |      |      |      |
| 0.02 | 19   | 0.00017 | 0.00041 | 0.995 | 0.971 | 17   | 0.00017 | 0.00040 | 0.993 | 0.967 |
| 0.05 | 18   | 0.00015 | 0.00038 | 0.995 | 0.978 | 17   | 0.00016 | 0.00038 | 0.993 | 0.975 |
| 0.1  | 19   | 0.00016 | 0.00040 | 0.988 | 0.980 | 17   | 0.00016 | 0.00040 | 0.990 | 0.970 |
| 0.3  | 27   | 0.00016 | 0.00038 | 0.985 | 0.961 | 21   | 0.00017 | 0.00040 | 0.983 | 0.967 |
Table 2. Simulated acceptance probabilities for three over-stress levels ALT sampling plans under progressive Type II interval censoring with random removals ($m = 3; \delta = 1; p_0 = 0.00041; 1 - \delta = 0.95; p_m = 0.01840; \beta = 0.10$)

| $\tau$ | $n$     | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $f_{c_1} = f_{c_2} = f_{c_3} = 0.5$ | $f_{c_1} = f_{c_2} = f_{c_3} = 0.8$ | $f_{c_1} = 0.5, f_{c_2} = 0.7, f_{c_3} = 0.9$ |
|--------|---------|------------|------------|------------|----------------|----------------|----------------|
| $p = 0.0$ |         |            |            |            | Selected simulated probabilities | Selected simulated probabilities | Selected simulated probabilities |
| 0.02   | 2/8     | 0.00016    | 0.00041    | 0.991      | 0.972          | 19              | 0.00019        | 0.00043        | 0.994          | 0.964          | 20              | 0.00017        | 0.00043        | 0.996          | 0.972          |
| 0.05   | 2/9     | 0.00016    | 0.00041    | 0.987      | 0.947          | 19              | 0.00017        | 0.00042        | 0.988          | 0.973          | 21              | 0.00018        | 0.00044        | 0.984          | 0.959          |
| 0.1    | 3/0     | 0.00015    | 0.00040    | 0.991      | 0.947          | 18              | 0.00015        | 0.00038        | 0.988          | 0.952          | 21              | 0.00016        | 0.00042        | 0.987          | 0.963          |
| 0.3    | 4/2     | 0.00016    | 0.00038    | 0.961      | 0.936          | 27              | 0.00018        | 0.00041        | 0.983          | 0.969          | 27              | 0.00017        | 0.00040        | 0.980          | 0.953          |
| $p = 0.05$ |         |            |            |            | Selected simulated probabilities | Selected simulated probabilities | Selected simulated probabilities |
| 0.02   | 2/1     | 0.00019    | 0.00045    | 0.992      | 0.971          | 17              | 0.00017        | 0.00042        | 0.993          | 0.970          | 18              | 0.00017        | 0.00042        | 0.998          | 0.969          |
| 0.05   | 2/5     | 0.00016    | 0.00038    | 0.985      | 0.961          | 17              | 0.00017        | 0.00042        | 0.986          | 0.968          | 19              | 0.00018        | 0.00042        | 0.997          | 0.964          |
| 0.1    | 2/9     | 0.00016    | 0.00039    | 0.989      | 0.960          | 18              | 0.00016        | 0.00039        | 0.988          | 0.973          | 20              | 0.00017        | 0.00042        | 0.989          | 0.978          |
| 0.3    | 4/2     | 0.00016    | 0.00042    | 0.976      | 0.938          | 27              | 0.00019        | 0.00042        | 0.980          | 0.967          | 27              | 0.00017        | 0.00043        | 0.983          | 0.953          |
| $p = 0.1$ |         |            |            |            | Selected simulated probabilities | Selected simulated probabilities | Selected simulated probabilities |
| 0.02   | 1/9     | 0.00016    | 0.00037    | 0.994      | 0.975          | 17              | 0.00017        | 0.00041        | 0.993          | 0.966          | 17              | 0.00015        | 0.00038        | 0.981          | 0.969          |
| 0.05   | 2/2     | 0.00017    | 0.00041    | 0.990      | 0.972          | 17              | 0.00017        | 0.00042        | 0.991          | 0.973          | 18              | 0.00018        | 0.00040        | 0.988          | 0.967          |
| 0.1    | 2/9     | 0.00018    | 0.00043    | 0.990      | 0.963          | 17              | 0.00016        | 0.00039        | 0.990          | 0.964          | 19              | 0.00016        | 0.00039        | 0.996          | 0.951          |
| 0.3    | 4/2     | 0.00016    | 0.00040    | 0.989      | 0.946          | 25              | 0.00016        | 0.00041        | 0.983          | 0.952          | 27              | 0.00018        | 0.00042        | 0.977          | 0.956          |
| $p = 0.3$ |         |            |            |            | Selected simulated probabilities | Selected simulated probabilities | Selected simulated probabilities |
| 0.02   | 2/1     | 0.00016    | 0.00040    | 0.997      | 0.978          | 17              | 0.00017        | 0.00041        | 0.995          | 0.974          | 19              | 0.00017        | 0.00039        | 0.990          | 0.968          |
| 0.05   | 2/1     | 0.00018    | 0.00042    | 0.994      | 0.971          | 17              | 0.00018        | 0.00041        | 0.992          | 0.970          | 18              | 0.00016        | 0.00041        | 0.995          | 0.963          |
| 0.1    | 2/1     | 0.00017    | 0.00038    | 0.985      | 0.967          | 17              | 0.00016        | 0.00040        | 0.993          | 0.954          | 19              | 0.00018        | 0.00042        | 0.991          | 0.977          |
| 0.3    | 3/3     | 0.00016    | 0.00039    | 0.976      | 0.964          | 24              | 0.00016        | 0.00042        | 0.989          | 0.974          | 23              | 0.00016        | 0.00039        | 0.987          | 0.956          |
Table 2. (Cont’d) Simulated acceptance probabilities for three over-stress levels ALT sampling plans under progressive Type II interval censoring with random removals \((m = 3; \delta = 1; p_{10} = 0.00041; 1 - \delta = 0.95; p_\beta = 0.01840; \beta = 0.10)\)

\[ \begin{align*}
\alpha_1 : \alpha_2 : \alpha_3 &= (0.5, 0.3, 0.2) \\

f_{c_1} &= f_{c_2} = f_{c_3} = 0.5 \\
f_{c_1} &= f_{c_2} = f_{c_3} = 0.8 \\
f_{c_1} &= 0.5, f_{c_2} = 0.7, f_{c_3} = 0.9
\end{align*} \]

| \(n\) | Selected points | Simulated probabilities | \(\tau\) | \(p = 0.0\) | \(99\%\) | \(95\%\) |
|---|---|---|---|---|---|---|
| 0.02 | 27 | 0.00017 | 0.00039 | 0.995 | 0.970 | 19 | 0.00018 | 0.00044 | 0.990 | 0.974 | 20 | 0.00018 | 0.00044 | 0.995 | 0.971 |
| 0.05 | 29 | 0.00017 | 0.00043 | 0.984 | 0.962 | 19 | 0.00016 | 0.00042 | 0.991 | 0.961 | 20 | 0.00017 | 0.00042 | 0.993 | 0.966 |
| 0.1 | 31 | 0.00017 | 0.00041 | 0.988 | 0.965 | 20 | 0.00018 | 0.00041 | 0.989 | 0.969 | 22 | 0.00019 | 0.00044 | 0.982 | 0.969 |
| 0.3 | 45 | 0.00016 | 0.00041 | 0.964 | 0.942 | 27 | 0.00018 | 0.00044 | 0.983 | 0.962 | 31 | 0.00017 | 0.00040 | 0.973 | 0.969 |

| \(n\) | Selected points | Simulated probabilities | \(\tau\) | \(p = 0.05\) | \(99\%\) | \(95\%\) |
|---|---|---|---|---|---|---|
| 0.02 | 19 | 0.00016 | 0.00040 | 0.988 | 0.979 | 17 | 0.00018 | 0.00041 | 0.995 | 0.972 | 17 | 0.00018 | 0.00040 | 0.991 | 0.975 |
| 0.05 | 22 | 0.00016 | 0.00041 | 0.989 | 0.966 | 17 | 0.00015 | 0.00039 | 0.989 | 0.974 | 17 | 0.00016 | 0.00040 | 0.990 | 0.965 |
| 0.1 | 28 | 0.00017 | 0.00040 | 0.987 | 0.962 | 19 | 0.00018 | 0.00042 | 0.988 | 0.964 | 20 | 0.00016 | 0.00040 | 0.993 | 0.959 |
| 0.3 | 45 | 0.00017 | 0.00042 | 0.981 | 0.955 | 22 | 0.00015 | 0.00039 | 0.984 | 0.951 | 27 | 0.00015 | 0.00036 | 0.982 | 0.965 |

| \(n\) | Selected points | Simulated probabilities | \(\tau\) | \(p = 0.1\) | \(99\%\) | \(95\%\) |
|---|---|---|---|---|---|---|
| 0.02 | 19 | 0.00017 | 0.00041 | 0.998 | 0.974 | 16 | 0.00017 | 0.00042 | 0.988 | 0.975 | 17 | 0.00018 | 0.00040 | 0.988 | 0.977 |
| 0.05 | 22 | 0.00018 | 0.00044 | 0.991 | 0.971 | 17 | 0.00017 | 0.00041 | 0.987 | 0.974 | 17 | 0.00017 | 0.00039 | 0.991 | 0.966 |
| 0.1 | 27 | 0.00017 | 0.00042 | 0.990 | 0.955 | 18 | 0.00017 | 0.00040 | 0.989 | 0.966 | 20 | 0.00017 | 0.00042 | 0.991 | 0.976 |
| 0.3 | 42 | 0.00015 | 0.00038 | 0.970 | 0.941 | 22 | 0.00016 | 0.00038 | 0.985 | 0.962 | 27 | 0.00015 | 0.00038 | 0.981 | 0.953 |

| \(n\) | Selected points | Simulated probabilities | \(\tau\) | \(p = 0.3\) | \(99\%\) | \(95\%\) |
|---|---|---|---|---|---|---|
| 0.02 | 21 | 0.00016 | 0.00038 | 0.993 | 0.976 | 17 | 0.00016 | 0.00041 | 0.992 | 0.966 | 19 | 0.00017 | 0.00043 | 0.996 | 0.982 |
| 0.05 | 21 | 0.00015 | 0.00041 | 0.994 | 0.974 | 17 | 0.00017 | 0.00041 | 0.989 | 0.969 | 17 | 0.00016 | 0.00039 | 0.991 | 0.972 |
| 0.1 | 21 | 0.00015 | 0.00037 | 0.990 | 0.969 | 17 | 0.00015 | 0.00039 | 0.989 | 0.968 | 19 | 0.00017 | 0.00042 | 0.986 | 0.977 |
| 0.3 | 35 | 0.00016 | 0.00041 | 0.980 | 0.960 | 21 | 0.00016 | 0.00040 | 0.984 | 0.967 | 27 | 0.00017 | 0.00042 | 0.985 | 0.952 |
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