ON DYNAMICS OF RELATIVISTIC SHOCK WAVES WITH LOSSES IN GAMMA-RAY BURST SOURCES

E.V. Derishev, VI.V. Kocharovsky, K.A. Martiyanov
Institute of Applied Physics
46 Ulyanov st., 603950 Nizhny Novgorod, Russia
mca1@appl.sci-nnov.ru

Abstract
Generalization of the self-similar solution for ultrarelativistic shock waves (Blandford & McKee, 1976) is obtained in presence of losses localized on the shock front or distributed in the downstream medium. It is shown that there are two qualitatively different regimes of shock deceleration, corresponding to small and large losses. We present the temperature, pressure and density distributions in the downstream fluid as well as Lorentz factor as a function of distance from the shock front.

Keywords: relativistic shock waves, gamma-ray bursts

Introduction
The progenitors of gamma-ray bursts (GRBs) are believed to produce highly relativistic shocks at the interface between the ejected material and ambient medium (see, e.g., Meszaros, 2002; Piran, 2004 for review). Non-thermal spectra and short duration of GRBs place a firm lower limit to the bulk Lorentz factor of radiating plasma, which must exceed a few hundred to avoid the compactness problem (e.g., Baring & Harding, 1995).

Consider a relativistic spherical blast wave expanding into a uniform ambient medium with the Lorentz factor $\Gamma \sim 300$. The average energy per baryon in the fluid comoving frame behind the shock front is of the order of $\Gamma m_p c^2$ (Taub, 1948), where $m_p$ is proton mass, and the plasma in the downstream presumably forms a non-thermal particle distribution extending up to very high energies. Under these conditions, medium downstream is subject to various loss processes. The non-thermal electrons produce synchrotron radiation, which accounts for GRB afterglow emission, and (at least partially) for the prompt emission. Apart from the synchrotron radiation of charged particles there is another mechanism of energy and momentum losses connected with inelastic interactions of energetic protons with photons. These reactions cause...
proton-neutron conversion as a result of charged pion creation. It should be noticed that for typical interstellar density the Coulomb collisions are inefficient and the charged particles instead interact collectively through the magnetic field. This allows to describe plasma motion using hydrodynamical approach, though it can break for a small fraction of the most energetic particles.

When a proton turns into a neutron or another neutral particle is born, it does not interact with the magnetic field and hence the energy spent for its creation is lost from the hydrodynamical point of view. The synchrotron and inverse Compton emission, as well as energetic photons, neutrinos and neutrons produced via photopionic reactions, escape from downstream giving rise to non-zero divergence of the energy-momentum tensor. The creation of energetic neutrons is also a first step in the production of highest-energy cosmic rays through the converter mechanism (Derishev et al., 2003).

Ejection from the GRB progenitor of a mass $M_0$ with initial Lorentz factor $\Gamma_0$ results in two shocks propagating asunder from the contact discontinuity. The forward shock moves into the external gas and has a much greater compression ratio at its front than the other, reverse shock, which passes through the ejected matter. As the shocked external gas has a temperature much higher than that in the vicinity of the reverse shock, we neglect the losses in the ejecta.

We discuss two models. In the first one we assume the energy losses to be localized close to the shock front, whereas the matter downstream the shock is considered lossless. In another model the shock front is treated as non-dissipative and the losses are distributed all over the shocked gas.

Following the recipe of Blandford and McKee (1976) we generalize their well-known self-similar solutions for relativistic blast waves for the case, where the energy and momentum of the relativistic fluid is carried away by various species of neutral particles.

**Self-similar solutions**

We start from the energy-momentum continuity equations, where in the case of distributed losses a non-zero r.h.s. is included:

\[
\frac{\partial T^{00}}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 T^{0r})}{\partial r} = -\varphi_0 T^{00}, \quad \frac{\partial T^{0r}}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 T^{rr})}{\partial r} - \frac{2p}{r} = -\varphi_1 T^{0r},
\]

\[
T^{00} = w\gamma^2 - p, \quad T^{0r} = w\gamma^2 \beta, \quad T^{rr} = w\gamma^2 \beta^2 + p,
\]

where $T$ is the energy-momentum tensor, $\gamma$ the Lorentz factor, $w = e + p$ the enthalpy density, $p$ the pressure, $e$ the energy density. All quantities are measured in the fluid comoving frame. In the following analysis we use the ultrarelativistic approximation of these equations, obtained by expanding velocity up to the third contributing order in $\gamma^{-2}$ and the equation of state up to the first order. Because of the lack of space, here we consider only equal losses for the energy and momentum ($\varphi_0 = \varphi_1 = \varphi$).
In the case of localized losses we treat them as discontinuities of the energy, momentum and particle number fluxes at the shock front, which are characterized by three parameters $\varepsilon$, $\delta$, $\eta$ equal to the fractions of corresponding fluxes lost at the shock front in the front comoving frame. We obtain the following expressions for the pressure $p_2$, number density $n_2$ and the Lorentz factor $\gamma_2$ immediately behind the shock front:

$$ p_2 = \frac{2}{3} \xi_1 \Gamma^2 w_1, \quad \gamma_2^2 = \frac{1}{2} \xi_2 \Gamma^2, \quad n_2 = 2 \sqrt{2} \xi_3 \Gamma n_1, $$

where $\xi_1 = \frac{3}{2} \frac{(1 - \delta) \xi_2}{\xi_2^2 - \xi_2 + 1}$,

$$ \xi_2 = 1 + \frac{3(1 - (1 - \varepsilon)/(1 - \delta))}{1 + \sqrt{4 - 3(1 - \varepsilon)^2/(1 - \delta)^2}}, \quad \xi_3 = \frac{(1 - \eta)}{\sqrt{2}} \frac{\sqrt{\xi_2}}{1 - \xi_2}. $$

They differ from the Taub adiabat only by numerical factors $\xi_i$, which become unity in the absence of losses. Here $w_1 = n_1 m_0 c^2$ is the enthalpy density and $n_1$ the number density of the external gas.
To find a self-similar solution we assume $\Gamma^2 = t^{-m}$ and introduce the similarity variable $\chi = \sqrt{c_\rho - r}/\sqrt{c_\rho - R}$, where $r$ is distance from the center and $R$ the current radius of the shock front. We find self-similar solutions in the case of localized losses and in the case of $\varphi = \frac{\alpha}{c_\rho}$, $\alpha = \text{const}$, but they also exist for non-uniform distributions of losses if $\alpha = \alpha(\chi)$. The velocity, pressure and particle density are found in terms of variables $\Gamma, \chi$.

From the energy balance equation we find that the power law index $m$ is in the range $3 \leq m \leq 6$. There are two regions in the parameter space where the solutions are qualitatively different. In the case of small losses, $\alpha < 3$ or $\varepsilon + \frac{2 - \sqrt{109}}{14} \delta < 1 + \frac{2 - \sqrt{109}}{14}$, the index $m$ rises from 3 to 6 as the losses increase. The pressure, Lorentz factor and number density of the downstream are proportional to powers of the similarity variable whose indices are different from those in the solution of Blandford and McKee (1976). On the contrary, large losses lead to the universal deceleration law of the shock: $m$ is equal to 6. The problem is fully integrable but solutions can not be written as explicit. In the high-loss solutions there appears an expanding spherical cavity bounded by the contact discontinuity and the temperature at its edge tends to infinity.

The solutions obtained are presented in Fig. 1.

**Conclusion**

We have analyzed the dynamics of relativistic shock wave with losses due to escape of neutral particles from plasma flow. Both for localized and for distributed losses there are self-similar solutions, which are different from those found previously for lossless case. We find that increasing of the losses change the dynamics of the shock deceleration qualitatively. In the case of small losses, the role of ejected material asymptotically vanishes and the Lorentz factor of the shock decreases as $t^{-m}$ with $m$ varying from 1.5 (no losses) to 3. In the opposite case of large losses, the shock decelerates in accordance with universal law $\Gamma \sim t^{-3}$ and the energy content in the ejecta constitutes a significant fraction of the total energy budget. Also, in the presence of large losses, the temperature and the Lorentz factor of the fluid behind the shock can be non-monotonic functions of distance from the shock.

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