Optimization of the shell compliance by the thickness changing using PSO

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Abstract. The paper is devoted to an application of the particle swarm methods and the finite element method to the optimization of 2-D structures (plane stress/strain, bending plates and shells). The shape, topology and material or thickness of the structures are optimised for the stress, strain and volume criteria. The numerical examples demonstrate that the method based on particle swarm computation is an effective technique for solving computer aided optimal design.

1. Introduction

Reinforced structures are often used in practice because they are resistant, stiff and stable. A typical area of application of such structures is an aircraft and automotive industry where light, stiff and highly resistant structures are required. Many car and aircraft elements are made as thin panels reinforced by stiffeners or by changing thickness. Reinforced structures are frequently subjected to static or dynamic loads and it is important to get an information about their response. If the response is not satisfactory, it must be improved in order to satisfy proper requirements. These improvements can be achieved, for instance, by the optimization process [1]. Optimal choice of number of stiffeners and their locations or thickness arrangement in a structure decides about the effectiveness of the reinforcement. In the paper the reinforced structures are subjected to static loads and analyzed by the FEM. The aim of optimization is to find the optimal thickness of the structure by minimize maximal displacements with constrains imposed on volume of the structure.

Strength of structures [2][3] with an arbitrary geometry, material properties and boundary conditions can be obtained by carrying out laboratory tests but they are usually very expensive and time consuming. In order to reduce costs and time, computer simulations are performed instead of experimental investigations. As a result, static or dynamic quantities of interest like displacements, velocities, accelerations, forces, stresses, i.e. can be determined. The most versatile methods of analysis of structures subjected to arbitrary static and time dependent boundary conditions are the finite element method (FEM) or boundary element method (BEM).

In the present paper, FEM with swarm method in optimization of statically loaded reinforced structures is presented. The additional comparisons of the effectiveness of particle swarm optimizer (PSO) with the effectiveness of the evolutionary algorithms (EA) and the artificial immune systems (AIS) are presented in the papers [4][5][6].
2. The particle swarm optimiser

The particle swarm algorithms, similarly to the evolutionary and immune algorithms, are developed on the basis of the mechanisms discovered in the nature. The swarm algorithms are based on the models of the animals social behaviours: moving and living in the groups. The animals relocate in the three-dimensional space in order to change their stay place, the feeding ground, to find the good place for reproduction or to evading predators. We can distinguish many species of the insects living in swarms, fishes swimming in the shoals, birds flying in flocks or animals living in herds.

A simulation of the bird flocking was published by Reynolds [7]. He assumed that this kind of the coordinated motion is possible only when three basic rules are fulfilled: collision avoidance, velocity matching of the neighbours and flock centring. The computer implementation of these three rules showed very realistic flocking behaviour flaying in the three dimensional space, splitting before obstacle and rejoining again after missing it. The similar observations concerned the fish shoals. Further observations and simulations of the birds and fishes behaviour gave in effect more accurate and more precise formulated conclusions [7]. The results of this biological examination where used by Kennedy and Eberhart [8], who proposed Particle Swarm Optimiser – PSO. This algorithm realizes directed motion of the particles in n-dimensional space to search for solution for n-variable optimization problem. PSO works in an iterative way. The location of one individual (particle) is determined on the basis of its earlier experience and experience of whole group (swarm). Moreover, the ability to memorize and, in consequence, returning to the areas with convenient properties, known earlier, enables adaptation of the particles to the life environment. The optimization process using PSO is based on finding the better and better locations in the search-space (in the natural environment that are for example hatching or feeding grounds).

The algorithm with continuous representation of design variables and constant constriction coefficient (constricted continuous PSO) has been used in presented research. In this approach each particle oscillates in the search space between its previous best position and the best position of its neighbours, with expectation to find new best locations on its trajectory. When the swarm is rather small (swarm consists of several or tens particles) it can be assumed that all the particles stay in neighbourhood with currently considered one. In this case we can assume the global neighbourhood version and the best location found by swarm so far is taken into account – current position of the swarm leader (figure 1).

![Figure 1. The idea of the particle swarm](image_url)
The position of the $i$-th particle is changed by stochastic velocity $v_i$, which is dependent on the particle distance from its earlier best position and position of the swarm leader. This approach is given by the following equations:

$$v_i(k+1) = wv_i(k) + \phi_1(k)\left[q_i(k) - d_i(k)\right] + \phi_2(k)\left[\hat{q}_i(k) - d_i(k)\right]$$

$$d_i(k+1) = d_i(k) + v_i(k+1), \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$

where:

$$\phi_1(k) = c_1r_1(k); \quad \phi_2(k) = c_2r_2(k),$$

$m$ – number of the particles,

$n$ – number of design variables (problem dimension),

$w$ – inertia weight,

$c_1, c_2$ – acceleration coefficients,

$r_1, r_2$ – random numbers with uniform distribution $[0,1],$

$d_i(k)$ – position of the $i$-th particle in $k$-th iteration step,

$v_i(k)$ – velocity of the $i$-th particle in $k$-th iteration step,

$q_i(k)$ – the best found position of the $i$-th particle found so far,

$\hat{q}_i(k)$ – the best position found so far by swarm – the position of the swarm leader,

$k$ – iteration step.

The velocity of $i$-th particle is determine by three components of the sum in Eqn 1. The first component $wv_i(k)$ plays the role of the constraint to avoid excessive oscillation in the search space. The inertia weight $w$ controls the influence of particle velocity from the previous step on the current one. In this way this factor controls the exploration and exploitation. Higher value of inertia weight facilitates the global searching, and lower – the local searching. The inertia weight plays the role of the constraint applied for the velocities to avoid particles dispersion and guaranteeing convergence of the optimization process. The second component $\phi_1(k)[q_i(k) - d_i(k)]$ realizes the cognitive aspect. This component represents the particle distance from its best position found earlier. It is related to the natural inclination of the individuals (particles) to the environments where they had the best experiences (the best value of the fitness function). The third component $\phi_2(k)[\hat{q}_i(k) - d_i(k)]$ represents the particle distance from the position of the swarm leader. It refers to the natural inclination of the individuals to follow the other which achieved a success.

The flowchart of the particle swarm optimiser is presented in figure 2. At the beginning of the algorithm the particle swarm of assumed size is created randomly. Starting positions and velocities of the particles are created randomly. In the next step the objective function values are evaluated for each particle. The data necessary for calculation of the objective function are obtained in consequence of solving the boundary value problem by means of the FEM (described in the next section). After obtaining information about fitness of all the particles, the best positions of the particles are updated and the swarm leader is chosen. Then the particles velocities are modified by means of the Eqn 1 and particles positions are modified according to the Eqn 2. The process is iteratively repeated until the stop condition is fulfilled. The stop condition is typically expressed as the maximum number of iterations. The general effect is that each particle oscillates in the search space between its previous best position (position with the best fitness function value) and the best position of its best neighbour (relatively swarm leader), hopefully finding new best positions (solutions) on its trajectory, what in whole swarm sense leads to the optimal solution.

The modified version of PSO algorithm with additional procedure has been used in presented research and was originally proposed by the author. The additional procedure consists of two stages. First the clones of the swarm leader are created. The clones substitute the particles with the worst fitness. Next
the clones parameters are mutated with the declared probability by adding random numbers with uniform distribution from the range of the design parameters variation

$$\hat{q}_j(k+1) = \hat{q}_j(k) + \text{rand}[h_{j\min}, h_{j\max}]$$  \hspace{1cm} (3)

where \(\text{rand}[h_{j\min}, h_{j\max}]\) - function which returns the random numbers with uniform distribution from the range of the design parameters variation \([h_{j\min}, h_{j\max}]\). The prescribed PSO parameters are: number of particles = 30, inertia weight \(w = 0.73\), acceleration coefficients \(c_1, c_2 = 1.47\), number of the clones = 5, probability of mutation = 50%.

**Figure 2.** The flowchart of the swarm algorithm

3. **Formulation of the problem**

Consider a structure (plate in plane stress/strain, bending plate or shell) which, at the beginning of an iterative process, occupies a domain \(\Omega_0, (in E^d, d = 2 or 3)\), bounded by a boundary \(\Gamma_0\). The domain \(\Omega_0\) is filled by a homogeneous and isotropic material of a Young’s modulus \(E_0\) and a Poisson ratio \(\nu\). The thickness of the structure \(g_0\) is also constant at the beginning of the particle swarm process. The 2-D structures are considered in the framework of theory of elasticity. During the particle swarm process the domain \(\Omega_t\), its boundary \(\Gamma_t\), and the field of Young’s modulus \(E(x) = E, x \in \Omega_t\) or the thickness \(g(x) = g_t\), can change for each generation \(t\) (for \(t=0\), \(E_0=\text{const}\), \(g_0=\text{const}\)). The particle swarm process...
proceeds in an environment in which the structure fitness is describing by minimize of the maximal displacements

\[ J = \max |u| \]  \hspace{1cm} (4)

with a constraint imposed on the volume of the structure \( V = |\Omega| \leq V_{\text{max}} \).

In order to solve the formulated problem FE models of the structures are considered [9]. The structure is divided into finite elements \( \Omega, e = 1, 2, \ldots, R \), and node displacements are calculated by solving a system of linear algebraic equation

\[ KU = F \]  \hspace{1cm} (5)

where \( U \) is a column matrix of unknown displacements, \( F \) is a known column matrix of acting forces and \( K \) is a known global stiffness matrix of the structure whose elements are given as follows:

\[ K_e = B^T D B \]  \hspace{1cm} (6)

where \( D \) and \( B \) are the known elasticity and geometrical matrices, respectively, \( V^e \) represents the volume of the finite element.

The distribution of thickness \( E(x), x \in \Omega \) in the structure is describing by a surface \( W(x), x \in H^2 \) (for plate in plane stress/strain, bending plate) or a hipersurface \( W(x), x \in H^3 \) (for shell). The surface (hipersurface) \( W(x) \) is stretched under \( H^d \subset E^d, (d = 2, 3) \) and the domain \( \Omega \) is included in \( H^d \), i.e. \( \Omega \subseteq H^d \).

The shape of the surface (hipersurface) \( W(x) \) is controlled by parameters of particle \( h_j, j = 1, \ldots, N \), which create a particle

\[ \text{par} = \{h_1, h_2, \ldots, h_j, \ldots, h_N\}, \quad h_{\text{min}} \leq h_j \leq h_{\text{max}} \]  \hspace{1cm} (7)

where \( h_{\text{min}} \) - the minimum value of the parameter of particle and \( h_{\text{max}} \) - the maximum value of the parameter of particle.

Parameters of particles are values of the function \( W(x) \) in interpolation nodes \( x_j \), i.e. \( h_j = W(x_j) \), \( j = 1, 2, \ldots, N \).

The assignation of of thickness to each finite element \( \Omega, e = 1, 2, \ldots, R \) is adequately performed by the mapping:

\[ g_e = W(x_e), x_e \in \Omega, e = 1, 2, \ldots, R \]  \hspace{1cm} (8)

It means that each finite element can have different material. When the value of thickness for the e-th finite element is included in:

- the interval \( 0 \leq g_e < g_{\text{min}} \), the finite element is eliminated and the void is created,
- the interval \( g_{\text{min}} \leq g_e < g_{\text{max}} \), the finite element remains (figure 4).

The idea of particle swarm generation for 2-D structure in the figure 3 is presented.
4. Examples of particle swarm optimization of structures
The bracket of the car bumper is considered. The geometry of the structure is shown in the figure 5. The considered construction is stiffly supported as is presented in the figure 6. and is loaded with a pressure 0.02 MPa. It is made from the polypropylene with addition of the glass fibre (E = 6 000 MPa, ν=0.35). The thickness of the shell construction is equal 3 mm and in the region around the hole for the tow hook is equal 6 mm. Calculation parameters of optimization of the bracket of the car bumper in the table 1 are presented. Distribution of control points of interpolation hypersurface for the bracket of the car bumper in the figure 7 is presented. Map of displacements of the bracket of the car bumper in the figure 8 and 9 are presented.
**Figure 5.** Geometry of the bracket of the car bumper

**Table 1.** Calculation parameters of optimization of the bracket of the car bumper

| pressure $c^p$ [MPa] | Number of design variables | Number of particles | $V_{max}$ [cm³] |
|-----------------------|---------------------------|---------------------|-----------------|
| 0.02 MPa              | 31                        | 20                  | 530             |

| material                            | Young module $E$ [MPa] | Poisson ratio | range of parameters of particle [mm] | Thickness of the bracket outside optimization area [mm] |
|-------------------------------------|------------------------|---------------|--------------------------------------|--------------------------------------------------------|
| polypropylene with addition of the glass fibre | 6 000                  | 0.35          | 3.0 ÷ 8.0                            | 3.0                                                    |

**Figure 6.** Boundary condition of the bracket of the car bumper

**Figure 7.** Distribution of control points of interpolation hipersurface for the bracket of the car bumper
Figure 8. Map of displacements of the bracket of the car bumper

(a) 
(b) 
(c) 
(d) 

Figure 9. Results of optimization for the bracket of the car bumper
(a), (b) the best solution in the first iteration; (c), (d) the best solution after optimization process
(a), (c) map of thickness; (b), (d) map of displacements

Figure 10. History of the fitness function for the best particle
5. Conclusions
An effective tool of particle swarm optimization of reinforced structures has been presented. Using this approach shape, topology and thickness optimization are performed simultaneously [10][11]. The important feature of this approach is a strong probability of finding the global optimal solutions received by implementation of the particle swarm algorithm and its universality consisting in application of this approach to the optimization of the 2-D structures [12]. The particle swarm algorithms [13][14], similarly to the evolutionary [15] and immune algorithms [16][17][18], are developed on the basis of the mechanisms discovered in the nature. Comparison between AIS, PSO and EA in the paper [4][5][19] is presented.

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