Dark Matter from freeze-in and its inhomogeneities

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Abstract

We consider generic freeze-in processes for generation of Dark Matter, together with the consequent re-thermalization of the Standard Model fluid. We find that Dark Matter inherits the Standard Model adiabatic inhomogeneities on the cosmological scales probed by current observations, that were super-horizon during freeze-in. Thereby, freeze-in satisfies the bounds on iso-curvature perturbations.

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1 Introduction

Freeze-in is a possible mechanism that could have generated the Dark Matter (DM) cosmological abundance [1]. It assumes that the Standard Model (SM) cosmological thermal plasma was not initially accompanied by any DM abundance. Since all SM components self-interact thermalising to a common temperature, cosmological inhomogeneities were initially adiabatic.

Next, ‘freeze-in’ particle physics processes produce DM particles with mass $M$ out of the SM plasma. For example, one can have decays $\text{SM} \rightarrow \text{DM DM}$ or scatterings $\text{SM SM}$...
DM DM, dominated either at large temperatures $T \gg M$ (‘UV-dominated freeze-in’) or at low temperatures $T \sim M$ (‘IR-dominated freeze-in’). In order to match the observed cosmological DM density [2], the rate of freeze-in processes must be much smaller than the Hubble rate $H$. Freeze-in automatically generates DM inhomogeneities out of SM inhomogeneities.

Observations are consistent with dominant adiabatic inhomogeneities (namely, the SM/DM fluid is the same everywhere), while iso-curvature inhomogeneities (namely, DM inhomogeneities different from SM inhomogeneities) are constrained, on cosmological scales, to be below a few % level [2].

We consider if freeze-in leads to acceptable DM inhomogeneities.

Weinberg answered positively this issue for thermal freeze-out: since freeze-out dominantly happens in the non-relativistic regime, computing inhomogeneities in the DM number density was enough [3]. On the other hand, freeze-in can be relativistic, and the iso-curvature issue started being considered recently: [4] claims that a specific freeze-in model is excluded because it generates too large scale-independent iso-curvature perturbations. The authors of [4] argue that all freeze-in models are similarly problematic. In the model considered in [4] DM has a small electric charge and is thereby produced by IR-dominated scatterings of two SM particles, such as $e^- e^+ \rightarrow$ DM DM. This generates, at any given time, a contribution to the DM density $\rho_{DM}$ proportional to the square of the SM density $\rho_{SM}$, and DM inhomogeneities might be not proportional to SM inhomogeneities. However, one must consider the cumulative cosmological process taking into account that all regions of the Universe undergo a similarly diluting $\rho_{SM}$. As we will see, this leads to negligible iso-curvature effects. A simple argument is presented in section 2, and the general formalism is used in section 3. Section 4 presents our conclusions.

2 Intuitive argument based on ‘separate universes’

We start presenting an intuitive argument. Working in the Newtonian gauge,

$$ds^2 = -[1 + 2\Phi(t, \vec{x})]dt^2 + a^2(t)[1 - 2\Psi(t, \vec{x})]d\vec{x}^2$$

(1)

the primordial adiabatic perturbations $\delta \rho_{\alpha}(t, \vec{x})$ in the density $\rho_{\alpha}(t)$ of a fluid $\alpha$ can be characterised in a simple geometric way as [3, 5–7]

$$\delta \rho_{\alpha} = \frac{d\rho_{\alpha}}{dt} \delta t$$

(2)

working at first-order in the small $\delta \rho_{\alpha} \ll \rho_{\alpha}$. In eq. (2) $\delta t(t, \vec{x})$ is some universal function common to all fluids that can be intuitively thought as a delay in the time evolution of the different regions.

Observation constrain iso-curvature perturbations only on scales comparable to the horizon today, while the freeze-in DM density was generated before matter/radiation
equality (much before in most freeze-in models). This means that we only need to worry
if freeze-in generated iso-curvature perturbations on scales much larger than the small
horizon at freeze-in.

We can thus apply the ‘separate universes’ picture (see e.g. [5]): the very early Universe
at freeze-in can be thought as many homogeneous regions without causal contact, given
that inhomogeneities on different scale evolve independently in first-order approximation.
Freeze-in dynamics produces DM with adiabatic perturbations because all regions undergo
the same dynamics, up to the delay \( \delta t \). So eq. (2) holds for the DM density, no matter
how complicated the freeze-in dynamics is. Explicitly, the Boltzmann equation for the
homogeneous small DM number density \( n_{\text{DM}} \) is \( d(n_{\text{DM}}/s)/d\ln T \approx \gamma/H_s \), where \( s \) is
the entropy density, \( H \) is the Hubble rate, and \( \gamma(T) \) is the space-time density rate of
freeze-in processes that produce one DM particle out of the SM plasma at temperature
\( T \). Integrating this equation leads to

\[
\frac{n_{\text{DM}}}{s} = \int dT \frac{\gamma(T)}{T \cdot H_s}.
\]

(3)

Interpreting eq. (3) in the ‘separate universes’ picture implies that, in regions where
the SM plasma was denser, freeze-in initially produced more DM by some amount that
depends on the freeze-in model, but in these region the DM average density changed
more rapidly leading to adiabatic DM inhomogeneities. The above discussion explicitly
verifies how, in the special freeze-in case, the ‘separate universes’ regions undergo the
same evolution, up to the time delay.

The next section substantiates the above intuitive reasoning by explicit computations.

3 Iso-curvature perturbations during freeze-in?

A general formalism to compute the cosmological evolution of inhomogeneities in inter-
acting fluids was developed in [8,9]. We adopt its presentation as summarized in [10],
that makes more explicit the sources of iso-curvature inhomogeneities.

Simple first-order evolution equations for the various densities are obtained by combin-
ing the Einstein gravity equations into the conservation of the energy-momentum tensor
\( T_{\mu\nu} = \sum_\alpha T_{(\alpha)}^{\mu\nu} \). The energy-momentum tensor \( T_{(\alpha)}^{\mu\nu} \) of fluid \( \alpha \) only is not conserved because
interactions transfer energy-momentum \( Q_{(\alpha)}^\nu \) to other fluids. So one has

\[
\nabla_\mu T_{(\alpha)}^{\mu\nu} = Q_{(\alpha)}^\nu \quad \text{with} \quad \sum_\alpha Q_{(\alpha)}^\nu = 0
\]

(4)

because of total energy conservation. In the homogeneous limit, this implies that the
average densities evolve as \( \dot{\rho}_\alpha + 3H(\rho_\alpha + \varphi_\alpha) = Q^0_{(\alpha)} = Q_{(\alpha)} \), where \( \varphi_\alpha \) is the pressure of
fluid \( \alpha \). The energy component of \( Q_{(\alpha)} \) is expanded in small inhomogeneities as \( Q_{(\alpha)0} =
- Q_{\alpha}(1 + \Phi) - \delta Q_{\alpha} [10] \) so that \( \sum_\alpha \delta Q_{\alpha} = 0 \) by total energy conservation. The total
density is \( \rho = \sum_\alpha \rho_\alpha \).

3
It is useful to write equations in terms of the curvature perturbation \( \zeta = -H[\Psi/H + \delta \rho/\dot{\rho}] \), which is the relative displacement between uniform-density and uniform-curvature surfaces. This curvature perturbation can be defined for each fluid

\[
\zeta_\alpha = -\Psi - H \frac{\delta \rho_\alpha}{\rho_\alpha}
\]  

and it evolves as [10]

\[
\dot{\zeta}_\alpha = -H \frac{\delta \rho_\alpha}{\rho_\alpha} + 3H^2 \frac{\delta \varphi_{\text{intr},\alpha}}{\rho_\alpha} - H \frac{Q_\alpha}{\rho_\alpha} \left( \frac{\delta \rho_\alpha}{\rho_\alpha} - \frac{\delta \rho}{\rho} \right) + O(k^2)
\]  

where \( \delta Q_{\text{intr},\alpha} \) and \( \delta \varphi_{\text{intr},\alpha} \) will be defined later. As usual, small perturbations are conveniently expanded in comoving Fourier modes \( k \), and the ‘separate universe’ argument amounts to consider the limit \( k \to 0 \) of the full equations. We focus on large super-horizon scales, thereby omitting the label \( k \) and neglecting Laplacians and other terms suppressed by \( k^2/a^2H^2 \). Such terms are indeed negligible whenever freeze-in occurs way before matter/radiation equality, for relevant cosmological scales \( k \).

The equations (6) can be written in a slightly more convenient form by avoiding using the total density \( \rho \) and defining instead the iso-curvature relative perturbations \( S_{\alpha\beta} \) between two fluids \( \alpha \) and \( \beta \)

\[
S_{\alpha\beta} \equiv 3(\zeta_\alpha - \zeta_\beta) = -3H \left( \frac{\delta \rho_\alpha}{\rho_\alpha} - \frac{\delta \rho_\beta}{\rho_\beta} \right)
\]  

that evolve as

\[
\dot{S}_{\alpha\beta} = -3H \left( \frac{\delta Q_{\text{intr},\alpha}}{\rho_\alpha} - 3H \frac{\delta \varphi_{\text{intr},\alpha}}{\rho_\alpha} - \frac{\delta Q_{\text{intr},\beta}}{\rho_\beta} - 3H \frac{\delta \varphi_{\text{intr},\beta}}{\rho_\beta} \right) + \dot{\zeta}_{\alpha\beta} + O(k^2).
\]  

We again ignore the terms suppressed by \( k^2 \). We can also ignore the ‘multiplicative’ terms (namely, those proportional to combinations of \( S_{\alpha',\beta'} \) terms) [10]

\[
\dot{S}_{\alpha\beta}^{\text{mul}} = \frac{\dot{H}}{2H} \left[ \left( \frac{Q_\alpha}{\rho_\alpha} + \frac{Q_\beta}{\rho_\beta} \right) S_{\alpha\beta} + \left( \frac{Q_\alpha}{\rho_\alpha} - \frac{Q_\beta}{\rho_\beta} \right) \sum_\gamma \frac{\dot{\rho}_\gamma}{\dot{\rho}} (S_{\alpha\gamma} + S_{\beta\gamma}) \right]
\]  

because we are only concerned in understanding if non-zero iso-curvature perturbations are generated by the ‘source’ terms explicitly shown in eq. (8). The formalism summarized in [10] makes clear that, in the long-wavelength limit \( k \to 0 \), iso-curvature perturbations are only sourced by the non-adiabatic energy transfer \( \delta Q_{\text{intr},\alpha} \) and by the non-adiabatic pressure \( \delta \varphi_{\text{intr},\alpha} \) intrinsic in each fluid \( \alpha \). These terms will be now be defined and evaluated.

### 3.1 Intrinsic non-adiabatic energy transfer

One source of iso-curvature perturbations is the intrinsic non-adiabatic energy transfer, the part of energy transfer \( \delta Q_\alpha \) from fluid \( \alpha \) ‘biased’ with respect to its energy density
\[ \delta Q_{\text{intr},\alpha} \equiv \delta Q_{\alpha} - \frac{\dot{Q}_{\alpha}}{\dot{\rho}_{\alpha}} \delta \rho_{\alpha}. \]  

We next consider its value during freeze-in, where the relevant fluids are \( \alpha = \{\text{SM, DM}\} \).

The rate of freeze-in particle collisions can be computed, in any given particle-physics model, as a function of the local temperature of the SM fluid, that also controls its density. Thereby the energy transfer from the SM fluid only depends on its local density, \( Q_{\text{SM}}(\rho_{\text{SM}}) \). Consequently \( \delta Q_{\text{intr,SM}} = \delta Q_{\text{SM}} - \delta \rho_{\text{SM}} dQ_{\text{SM}}/d\rho_{\text{SM}} = 0 \) vanishes in a generic freeze-in model.

Next, energy conservation demands \( \delta Q_{\text{SM}} + \delta Q_{\text{DM}} = 0 \), so that the intrinsic non-adiabatic energy transfer to the DM fluid can be written as

\[ \delta Q_{\text{intr,DM}} = \delta Q_{\text{DM}} - \frac{\dot{Q}_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \delta \rho_{\text{DM}} = \dot{Q}_{\text{DM}} \left( \frac{\delta \rho_{\text{SM}}}{\dot{\rho}_{\text{SM}}} - \frac{\delta \rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \right). \]  

This potential ‘source’ terms thereby becomes a ‘multiplicative’ term, proportional to the relative entropy \( S_{\text{SM,DM}} \). Since this is assumed to be initially vanishing, \( \delta Q_{\text{intr,DM}} \) generates no isocurvature perturbation.

### 3.2 Intrinsic non-adiabatic pressure

The second kind of source term, the non-adiabatic part of the pressure perturbation intrinsic of each fluid \( \alpha \), is given by [10]

\[ \delta \varphi_{\text{intr,\alpha}} = \delta \varphi_{\alpha} - c_{\alpha}^2 \delta \rho_{\alpha} \text{ where } c_{\alpha}^2 = \frac{\dot{\varphi}_{\alpha}}{\dot{\rho}_{\alpha}} \]  

is its adiabatic speed of sound. This term vanishes when the pressure and energy inhomogeneities respect the equation of state of the fluid, \( \varphi_{\alpha}(\rho_{\alpha}) \).

Freeze-in particle-physics processes contribute as \( \delta \varphi_{\text{intr,SM}} \neq 0 \) because they convert SM particles into DM particles, thereby inducing an energy and momentum loss of the SM fluid, as dictated by the specific freeze-in interaction, that generically does not follow the equation of state of the SM fluid.

As a simple example of this unbalance, freeze-in via the decay into DM particles of some SM particle (or, in SM extensions, of some speculative new-physics particle tightly coupled to the SM) transfers more energy than pressure (\( \dot{\rho}_{\text{SM}}/\dot{\varphi}_{\text{SM}} > \rho_{\text{SM}}/\varphi_{\text{SM}} \)) because the decaying particles must be massive and thereby they decay slower when they have higher relativistic energy. An unbalance also generically occurs in freeze-in scatterings, described by a cross-section \( \sigma(\text{SM SM} \rightarrow \text{DM DM}) \) that only depends on the invariant energy \( \sqrt{s} \) at leading order in the couplings (the motion with respect to the plasma enters at higher orders). The sign of \( \delta \varphi_{\text{intr,SM}} \) is not fixed, as the energy dependence of \( \sigma \) can either result in a larger energy transfer when the colliding SM particles have higher energy \( E \gtrsim T \) (this can happen in UV-dominated freeze-in, via non-renormalizable interactions,
for example gravitational [11]) or when the colliding SM particles have lower energy $E \lesssim T$ (this can happen in IR-dominated freeze-in, via renormalizable interactions). As a possibly relevant special case, $\delta \varphi_{\text{intr,SM}}$ is nearly-vanishing in freeze-in models that only lead to the disappearance of ultra-relativistic SM particles, as they (on angular average) satisfy the same equation of state $\varrho = \rho/3$ as the radiation-dominated SM fluid.

However, the fact that freeze-in processes (decays and scatterings) can contribute as $\delta \varphi_{\text{intr,SM}} \neq 0$ is inconsequential, as we must also take into account the self-interactions of the SM fluid. A multitude of SM particle processes allow the SM fluid to locally re-thermalize to its equation of state with rates $\Gamma$ much faster than the Hubble rate and than the freeze-in rate. Typically $\Gamma \sim g^2 T$ where $g \sim 1$ is a typical SM coupling, such as a gauge coupling. The re-thermalizion processes conserve the SM energy $\rho_{\text{SM}}$: $\delta Q_{\text{SM}}$ remains given by freeze-in processes only, so that $\delta Q_{\text{intr,SM}} = 0$ remains as in section 3.1. On the other hand, the SM pressure $\varrho_{\text{SM}}$ changes such that the combination of the two processes (freeze-in and re-thermalization) leads to $\delta \varphi_{\text{intr,SM}} = 0$.

This leaves $\delta \varphi_{\text{intr,DM}}$ as a possible source of iso-curvatures. A self-thermalization argument parallel to what just discussed for the SM plasma implies $\delta \varphi_{\text{intr,DM}} = 0$ if DM has significant self-interactions just after being produced during freeze-in. This happens, for example, if DM is a multiplet under a dark gauge group [12] that confines at a scale $\Lambda$ and if freeze-in happens at $T \gg \Lambda$. If instead DM self-interactions are negligible, a formalism extended to higher moments may be needed, but the physics is simple: DM particles free stream on sub-horizon scales, but not on large scales $k \to 0$. The non-thermal DM distribution $f(\vec{x}, t, q) = f_0(q) + \delta f(\vec{x}, t, q)$ produced by freeze-in redshifts with scale factor $a$ as [13]

$$\rho_{\text{DM}} = \frac{1}{a^4} \int \frac{d^3 q}{(2\pi)^3} q^2 \sqrt{E f}, \quad \varrho_{\text{DM}} = \frac{1}{a^4} \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{3E} f$$

(13)

where $q$ and $E = \sqrt{q^2 + a^2 M^2}$ are the comoving momentum and energy of the DM particle with mass $M$. Two limits are of special interest. If freeze-in is IR-dominated, DM is only mildly relativistic, so that DM motion is soon red-shifted down to negligible pressure, $\varrho_{\text{DM}} \ll \rho_{\text{DM}}$. UV-dominated freeze-in can produce ultra-relativistic DM with $\varrho_{\text{DM}}/\rho_{\text{DM}} \propto \varrho_{\text{DM}}/\rho_{\text{DM}} \simeq 1/3$, that becomes non-relativistic only later when the SM cools down to temperatures comparable to the DM mass $M$, while the horizon reaches larger scales.

4 Conclusions

We considered generic models of freeze-in (from decays, from scatterings, IR-dominated, UV-dominated...) finding that the generated Dark Matter inherits the Standard Model adiabatic inhomogeneities on the cosmological scales probed by current observations, that were super-horizon during freeze-in. In section 2 we presented an intuitive argument
based on the well-known ‘separate universe’ picture. This was substantiated in section 3 by checking the explicit sources of iso-curvature perturbations on super-horizon scales.

Iso-curvature perturbations can only be generated on small scales that were sub-horizon during freeze-in: this effect can perhaps be relevant in models where freeze-in happens at the lowest possible temperature $T \sim M \sim \text{keV}$, possibly in the presence of dark long-range forces.

In conclusion, freeze-in appears a viable mechanism for generation of the cosmological DM abundance. Similar arguments hold for other particle-physics mechanisms such as ‘cannibalism’ [14] or ‘freeze-out and decay’. Furthermore, baryogenesis mechanisms that involve elements similar to freeze-in (such as leptogenesis from right-handed neutrinos with initially negligible abundance) are similarly compatible with iso-curvature bounds.

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