On the Nature and Location of the Microlenses

Rosanne DiStefano
Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138

ABSTRACT

This paper uses the caustic crossing events in the microlens data sets to explore the nature and location of the lenses. We conclude that the large majority of lenses, whether they are luminous or dark, are likely to be binaries. Further, we demonstrate that blending is an important feature of all the data sets. An additional interpretation suggested by the data, that the caustic crossing events along the directions to the Magellanic Clouds are due to lenses located in the Clouds, implies that most of the LMC/SMC events to date are due to lenses in the Magellanic Clouds. All of these conclusions can be tested. If they are correct, a large fraction of lenses along the direction to the LMC may be ordinary stellar binary systems, just as are the majority of the lenses along the direction to the Bulge. Thus, a better understanding of the larger-than-anticipated value derived for the Bulge optical depth may allow us to better interpret the large value derived for the optical depth to the LMC. Indeed, binarity and blending in the data sets may illuminate connections among several other puzzles: the dearth of binary-source light curves, the dearth of non-caustic-crossing perturbed binary-lens events, and the dearth of obviously blended point-lens events.

Subject headings: dark matter – gravitational lensing – stars: low-mass, brown dwarfs

1. Lens Binarity as a Tool to Study Lens Populations

1.1. Overview

This paper illustrates what the study of an ensemble of binary-lens events can teach us about the total lens population. §1.2 defines useful categories of events, including caustic crossing events. The heart of the paper is §2, where I show that even the relatively small number of caustic crossing events observed to date in Baade’s window and toward the LMC and SMC, indicate that the large majority of lenses are likely to be binaries. The use of caustic crossing events to study blending is also elucidated; the caustic crossing events detected so far indicate that blending is ubiquitous. §3 addresses the likely location of the lenses; in this regard, individual caustic crossing events have already provided valuable information. Indeed, analysis of the caustic crossings indicates that both the LMC and
SMC binary lenses are most likely to be located in the Magellanic Clouds and not in the Galactic Halo (Afonso et al. 1998; Albrow et al. 1998; Alcock et al. 1998; Udalski et al. 1998; Bennett et al. 1996; Alcock et al. 1997a). Coupling these prior analyses with the results of §2 leads to the conclusion that most of the lenses detected to date along directions to the Magellanic Clouds are actually in the Magellanic Clouds. The main results of this paper, that most lenses detected to date are binaries, that a significant portion of the lenses detected toward the Magellanic Cloud are part of the Magellanic Clouds, and that blending is ubiquitous, can be tested. The tests may provide answers to several puzzles presented by the data sets. The puzzles are outlined and discussed in §4. The fourth puzzle we discuss is of central importance to learning about dark matter through the microlensing observations. That is: why are the measured values of the optical depth higher than predicted for both the Bulge and the LMC? If the LMC lenses are ordinary stellar systems, as those along the direction to the Bulge are thought to be, the reasons for the unexpectedly high values may be essentially, or at least partly, the same. In §5 we ask: can blending and binarity themselves lead to overestimates of $\tau$? We sketch the relevant considerations and point out that further work, largely on the part of the observing teams, is necessary to test the hypothesis that the combination of blending and binarity can cause significant mis-estimates of $\tau$ when these effects are unrecognized or only partially taken into account.

1.2. Categories of Binary Lens Light Curves

(1) Point-lens-like light curves: The vast majority of events in which a binary system serves as a lens produce light curves indistinguishable from point-lens light curves. This is because: (a) values of $a$, the orbital separation, are likely to be distributed over a wide range; thus, many binaries are either too close or too wide to yield light curves with measurable evidence of lens binarity, and (b) even in the range in which the influence of lens binarity is most likely to be evident ($0.1 R_E < a < 3 R_E$), a large fraction of light curves can be practically indistinguishable from point-lens light curves (Di Stefano & Perna 1997).

(2) Caustic Crossings: Caustic crossing light curves are distinctive: they exhibit wall-like structures with measured values of the peak magnification $O(10)$. Neither the presence of the perturbation nor, in most cases, its nature as a binary-lens phenomenon are easily obscured by other astronomical effects, such as the blending of light from the lensed star with light from other sources located in the seeing disk.

(3) Other Significantly Perturbed Light Curves: Close approaches to a caustic can lead to striking perturbations; many such events would be included among the “strong” binary events discussed by Mao & Paczyński (1991). Other significant perturbations would be of a more “gentle” nature. Both strong and gentle perturbations can be systematically identified, classified, and studied (Di Stefano & Perna 1997).
On a theoretical level, only members of the second category are well-defined. To distinguish between members of categories (1) and (3), on the other hand, specific criteria (such as the difference between binary-lens and point-lens fits) must be chosen. Observationally, membership among the three categories can be blurred by issues such as the frequency of sampling, photometric uncertainties, the presence of additional light from other (unlensed) stars, and the methods chosen to analyze the data.

Because caustic crossing events are the only types of binary-lens events that have been reliably classified so far, it makes sense to concentrate on what we can learn from them. Fortunately, caustic crossing events provide us with the clearest view of important elements of the data set, including the fraction of all events likely to be due to binary lenses and the role played by blending throughout the data set.

2. What We Learn from Caustic Crossings

We will conclude that it is not presently possible to falsify the hypothesis that all of the lenses observed to date are binaries. Sketched below, the arguments that lead to this conclusion also imply that blending is playing an important role in shaping the data sets.

2.1. The Observed Relative Frequency of Caustic Crossing Events

The published LMC data contain 1 caustic crossing event and 7 point-lens-like events (Alcock et al. 1997a; Bennett et al. 1996). Thus, caustic crossing events constitute slightly more than 10% of the LMC events. This is consistent with the fraction obtained for the Bulge data set. (See, e.g., Udalski et al. 1994; Alard, Mao, & Guibert 1995; Alcock et al. 1997b). For the Bulge, and especially for the LMC, the total number of published caustic crossing events is still small; it is nevertheless worth considering the conclusions we can draw if caustic crossing events continue to constitute at least a few percent of all events.

The SMC has not been observed for as many star-years (the product of the number of stars monitored and the time during which they are monitored) as either the LMC or the Bulge.

\(^1\)At least one perturbation that passes all of the cuts for a point-lens event exhibits some anomalies that have caused it to be excluded from the detailed statistical analysis of the LMC events.

\(^2\)For both the OGLE and DUO teams, caustic crossing events constitute \(\sim 10\%\) of all Bulge events (Udalski et al. 1994; Alard, Mao, & Guibert 1995). The fraction of the MACHO team’s published Bulge events exhibiting caustic crossings is considerably smaller (Alcock et al. 1997b). With regard to the MACHO results we note that (1) the algorithms used to identify the events published so far were designed to exclude perturbations not of the point-lens form, and (2) the present observing strategy of calling an “alert” which institutes frequent high-precision monitoring of many light curves, will increase the rate of binary-lens discoveries. As a matter of fact there are \(\sim 5\) Bulge caustic crossing events listed on the alert web pages covering 1996-1998.
The published SMC data consist of 1 caustic crossing event (Afonso et al. 1998; Albrow et al. 1998; Alcock et al. 1998; Udalski et al. 1998; Rhie et al. 1998) and 1 point-lens-like event (Alcock et al. 1997c; Palanque-Delabrouille et al. 1997; Udalski et al. 1997; Sahu 1998).

2.2. The Predicted Relative Frequency of Caustic Crossing Events

Let \( N_{cc}/N_{tot} \) represent the fraction of all events exhibiting caustic crossings. The larger the predicted value of \( N_{cc}/N_{tot} \), the more likely it is that some of the point-lens-like events already observed are actually due to lensing by a point mass rather than lensing by a binary. To determine a reasonable upper limit for \( N_{cc}/N_{tot} \), I carried out a set of simulations in which all lenses were binaries with orbital separations between 0.1 \( R_E \) and 1.6 \( R_E \). The binaries were also characterized by a value of \( q \), the ratio of the masses of the less massive to more massive component. I considered 20 values of \( q \) (starting with \( q = 0.05 \) and proceeding by intervals of 0.05 to \( q = 1.0 \)), and 5 values of \( a \), the semi-major axis (\( a = [0.1] 2^{i-1} R_E \), with \( i \) ranging from 1 to 5), and computed the rates of events in each of the 3 categories sketched in §1. The bottom-most (thick) curves in Figure 1 show the results. Averaging over the values of \( a \) considered, \( N_{cc}/N_{tot} \) is maximized for values of \( q \) near unity. In fact, \( N_{cc}/N_{tot} \) is approximately equal to 0.15 for \( q > 0.4 \). Thus, even if all lenses are binaries with \( a \) and \( q \) in the ranges that maximize the size of caustic structures, each caustic crossing event should be accompanied by 6 – 7 additional events. 2 – 3 of these additional events are likely to be significantly perturbed from the standard point-lens form; 3 – 5 events will, with the light curve sampling used in the simulations, be indistinguishable from point-lens events.

Of course, even if all lenses are binaries, events in which the orbital separation between the lens components is in the range considered in the simulations will form only a fraction \( \mathcal{F}_{ev} < 1 \) of all events. For populations such as those in our disk, it seems reasonable to estimate \( \mathcal{F}_{ev} \sim 1/3 \), although this number can be smaller or somewhat larger, depending on the distributions of values of the mass ratio and orbital separation. Thus, each caustic crossing event should be accompanied by \( \sim 15 \) or more point-lens-like events. Even more point-lens-like events are expected when the lens population is not entirely composed of binaries.

3It is interesting to note that the lenses used in the simulation are most likely to be representative of a true lens population located in the core of a globular cluster, where (1) the fraction of stars that are in binaries may be large due to the effects of mass segregation and stellar capture processes, and (2) orbital separations are expected to be set by ambient translational velocities on the order of 10 km/s, and to therefore be in the range of several au, fairly close to reasonable values of \( R_E \).
caustic crossing events are overrepresented in the data sets. When, however, we take into account that the fields being monitored are crowded, and that some of the lens systems may themselves be luminous, it is clear that blending of light from different sources along the line of sight to an event may be important. In §2.4 we test the influence of blending on the value of \( N_{cc}/N_{tot} \). In §2.3 we show that caustic crossing events provide a direct measure of the role blending plays in microlensing observations.

2.3. What Caustic Crossing Events Can Teach Us About Blending

Let \( A_{lim} \) be the minimum value of the peak magnification required in order to reliably conclude that an observed light curve perturbation is due to microlensing. If \( f \) is the fraction of the baseline flux provided by a lensed star, then its true magnification must be greater than

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A = 1 + \left[ \frac{A_{lim} - 1}{f} \right],
\]

in order for the event to be correctly identified. Given this, caustic crossing events provide a clear advantage for the detection of heavily blended (i.e., small \( f \)) events. First, the magnification during the caustic crossing can be so large that only the most heavily blended events would not be detected during such a crossing. Second, because there are always 2 crossings (which should generally be resolvable), there are 2 chances to catch even a heavily blended event. Third, the minimum magnification after the first caustic crossing during the interval until the second crossing is 3 (Witt & Mao 1995); this, together with the large caustic crossing magnification, means that, even with \( A_{lim} \) as high as 1.34, caustic crossings should allow for extremely efficient sampling of events with \( f > 0.17 \). In fact, for \( f > 0.17 \), the distribution of \( f \) values in caustic crossing events should mirror the true distribution of values in all lensing events.

The fits to all of the published caustic crossing events require significant blending. With \( f \) defined as the fraction of the baseline flux due to the lensed star: \( f_I = 0.56 \) for the OGLE 7 event (Udalski et al. 1994); \( f_B = 0.73, f_R = 0.70 \) for the DUO team’s caustic crossing event (Alard, Mao, & Guibert 1995); \( f_B = 0.17, f_R = 0.26 \) for the LMC caustic crossing event (Bennett et al. 1996); \( f_V = 0.57, f_R = 0.49 \) for the SMC caustic crossing event (Rhie et al. 1998; but see also the compendium of fits included therein). \(^4\) Thus, although it is too early to extract a distribution of \( f \) values, caustic crossing events give a clear indication that blending is important along each direction presently being monitored. This is consistent with what we have learned about blending in other ways (Alard 1998).

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\(^4\) Note that, among the papers quoted, the subscripts indicating color do no generally refer to the same wavelength ranges.
The apparent importance of blending throughout the data set means that its effects should be included in our estimates of $N_{cc}/N_{tot}$.

### 2.4. Blending and the Value of $N_{cc}/N_{tot}$

The fraction of the baseline flux emanating from the lensed source, $f$, was smaller than unity in 5 simulations: $f = 0.1 + 0.2 (j - 1); j = 1, 5$. The results are summarized in Figure 1. When blending is moderate ($f > 0.6$), the results do not differ significantly from the unblended case. As blending becomes more important, the fraction of events in which there are caustic crossings rises. The results for the real data sets depend on the distribution of values of $f$; there are consistent models with moderate blending in which 1 caustic crossing event is accompanied by $O(1)$ member of category 3, and $\sim 10$ point-lens events.

### 2.5. Conclusions

The work described above indicates that each caustic crossing event should be accompanied by a number of other events, and that the number of other events can be on the order of 10 for moderate blending. Occam’s razor, applied to the observations, thus dictates that these additional events form the major component of the events we have actually already seen. The majority of lenses giving rise to the microlensing events observed so far are almost certainly binaries.

### 3. Where Are the Lenses?

Information gleaned from studying the caustic crossings in both the SMC and LMC data indicate that the lenses in these specific events are most likely located in the Magellanic Clouds themselves (Afonso et al. 1998; Albrow et al. 1998; Alcock et al. 1998; Udalski et al. 1998; Bennett et al. 1996; Alcock et al. 1997a). Should these conclusions be bolstered by future events, and if it is true that the lenses giving rise to point-lens-like events are drawn from the same population as the lenses giving rise to the caustic crossing events, we must conclude that it is likely that all of the events are due to lenses in the Magellanic Clouds rather than in the Halo. Conversely, if the location of the lenses implicated in the caustic crossing events is established to be in the Halo, then we would have discovered that Galactic MACHOs tend to be binary systems.

**Tests:** At present it seems most likely that the lenses are in the Magellanic Clouds. This conclusion carries a number of testable consequences. First, many LMC lenses will be luminous, and the degeneracy inherent in the point-lens light curve is likely to be broken in some cases (Udalski et al. 1994; DiStefano & Esin 1995; Kamionkowski 1995); this can
determine the location of the lenses associated with even some point-like events. Second, if the lenses are in the LMC and are luminous binaries, then there are opportunities for consistency checks between the known binary population (which becomes better known through the frequent observations carried out by the observing teams), the population of detected binary lenses and, eventually, binary sources. We note that there is direct evidence that the single SMC event without the caustic crossing is located in the SMC. Furthermore, there are indirect arguments that the majority of the Bulge lenses are in the Bulge (see, e.g., Kiraga & Paczyński 1994). The open question, and the one with the most important implications vis-a-vis the composition of the Halo, is the location of the LMC lenses. Additional caustic crossing, and/or members of category 3, will therefore play a crucial role in helping us to understand the nature and location of the LMC lenses.

4. Tests and Puzzles

4.1. Tests

The conclusions of the preceding two sections are testable. Some tests were discussed in §3. Here we provide an overview, focusing on the connection between the phenomena of blending and binarity and some puzzles posed by the existing data. First, the overview. If the lenses are indeed binaries, we should continue to discover caustic crossing light curves. Even if all or most lenses are binaries, the average fractional rate could be significantly lower than that observed so far, and will depend on the distribution of values of $q$ and $a$ among the lens population. A second important test is that perturbed events that are not caustic crossing events should be observed. Finally, if the events are primarily due to lenses in a single binary population, several consistency checks are possible. One example: the caustic crossing events can, as a group be used to extract the probability distribution for $q$; this distribution should be consistent with the results of a parallel analysis using only members of category 3. As a larger ensemble of events is collected, the ratio $N_{cc}/N_3$ can be measured as a function of $f$, $q$, and $a$; the results can be compared with theory.

4.2. Puzzles

1. If blending is ubiquitous, why do so few point-lens-like events exhibit evidence of blending? Evidence of blending in the light curve is most noticeably encoded in the discrepancy between the peak and baseline fluxes, relative to a point-lens model (Di Stefano & Esin 1995). It is easier to detect these discrepancies when the peak magnification is large. In fact, depending on the photometry and frequency of sampling, the distance of closest approach to a point lens needs to be $\sim 0.2 - 0.3$ in order for blending to be detected using the light curve alone (Di Stefano & Esin 1995, Wozniak & Paczyński 1997). When the lens is a binary, the enhancement in peak magnification associated with
caustic crossings can provide an ideal opportunity to look for blending. Indeed, during the first few years of observations, caustic crossing events were the only known blended events, and to date, all caustic crossing events are blended.

The importance of the role of blending can be tested in many ways. (See, e.g., Di Stefano & Esin 1995; Kamionkowski 1995; Buchalter, Kamionkowski, & Rich 1996; Wozniak & Paczyński 1997; Goldberg, Wozniak, & Paczyński 1997; Goldberg 1998; Goldberg & Wozniak 1998; Han 1998) Indeed, by using the image subtraction method to re-examine the OGLE team’s Bulge data, Alard finds that the fraction of light curves exhibiting signs of blending is consistent with the fraction predicted by Di Stefano & Esin (1995) and Wozniak & Paczyński (1997). Future spectroscopic and astrometric studies can provide complementary information. Thus, detailed answers to this first puzzle seems well within reach. The next two puzzles may, at this point in time, be compared to Conan Doyle’s dog that didn’t bark in the night.

2. and 3. Where are the binary source events? Where are the members of category 3? In this paper I argue that the presence of caustic crossing events in the data sets implies the presence of members of category 3. Griest & Hu (1992) have argued that 2 – 5% of microlensing light curves should exhibit significant perturbations associated with source binarity. An independent analysis yields comparable results (Han & Jeong 1998), even after taking into account degeneracies inherent in binary-source light curves (Dominik 1998; Han & Jeong 1998). Unfortunately, no perturbed binary source light curves are in the published literature, which is also free of members of category 3. Some of the missing highly perturbed light curves must be in the data set, but are not yet identified as microlens candidates. In fact, when I carried out a set of simulations like those described in §2, but attempting to mimic the MACHO team’s observing pattern (frequency, including gaps, and photometric sensitivity) and algorithmic cuts to eliminate events not deemed to be good candidates for microlensing, the large majority of strongly perturbed binary-lens events did not pass the cuts. It is also important to note that many events with physical parameters normally associated with serious perturbations are likely published among point-lens/point-source events; the light curves show no obvious perturbations because of the blending of light from the source star or stars with light from other stars, possibly including the lens system, along the line of sight. Blending washes out distinctive non-singular features of the perturbed light curves. If, for example,

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5It has been established, however, that in SMC-1 a binary source was magnified; the discovery of the binary nature of the source was based on residuals in the baseline flux.

6It seems to be the case that the Bulge data set contains a number of such light curves (Axelrod 1997).
there is a secondary peak that rises $\Delta A$ above the curve associated with a point-lens fit, blending reduced the difference to $f \Delta A$. Indeed, the second and fourth columns of Figure 2 show that when blending is severe ($f = 0.1$), then the events which are well-fit by point-lens models are primarily drawn from events that were in categories 2 and 3 when there was no blending ($f = 1$). This is because the caustic structures enhance the peak magnification of a significant fraction of the events in categories 2 and 3, so they survive as detectable events, even when there is severe blending, although signatures of their distinctive nature may be washed out. Because binary-source events do not benefit from such local increases in magnification, they are more likely to be classified as Paczyński light curves when blending is moderate, and more likely to be missed altogether when blending is severe. This distinction between binary-lens and binary-source events should have observable consequences. For example, if most source and lens systems are binaries, highly perturbed light curves discovered by additional thorough searches of the data or by the careful observations of the follow-up teams are more likely to be binary-lens events. In addition, perhaps 1% of all light curves should display truly unusual features, due to the lensing of a binary source by a binary.

4. Why Are the Event Rates and the Measured Values of the Optical Depth So High? Along the direction to the LMC the measured value of the optical depth is $2.1^{+1.3}_{-0.8} \times 10^{-7}$, while estimates of the component due to stars is smaller than $0.5 \times 10^{-7}$ (Alcock et al. 1997a, Renault et al. 1997; see also Bennett 1998). It is tempting to ascribe the discrepancy to the presence of dark matter in the Halo (but see Sahu 1994a, 1994b; Wu 1994). Proof that the lenses are in the LMC may be welcome, since Halo models of a MACHO population of masses in the range indicated by the microlensing results have proved difficult to make compatible with other observations that should be have found evidence of this population or its precursor. (See, e.g., Graff & Freese 1996; Adams & Laughlin 1996; Chabrier, Segretain, & Mera 1996; Gibson & Mould 1997; Fields, Mathews, & Schramm 1997.) Problems with the LMC-lens interpretation have, however, also been noted; the most serious of these is that the virial theorem provides an independent measure the optical depth (Gould 1995), and measurements of the speeds of some tracer stars indicate that velocities are too small to be consistent with a large optical depth (Cowley & Hartwick 1991). It is of course possible that additional kinematic studies of LMC populations will indicate that the optical depth of LMC lenses is indeed consistent with the present analysis of the microlensing data. It is also worth considering whether the published microlensing results are too high. A comparison with the Bulge, where the majority of lenses are stellar systems, may be instructive. Indeed, most indications are that the microlensing-measured value of the Bulge optical depth is also higher than predictions based on other types of observations. The OGLE result (based on 9 lenses), and the MACHO result (based on 45
lenses) were $3.3^{+1.2}_{-1.2} \times 10^{-6}$, and $3.9^{+1.8}_{-1.2} \times 10^{-6}$, respectively, while predictions were in the range $0.5 - 1.0 \times 10^{-6}$. While changes in how the Galaxy is modeled can apparently make the predictions come into marginal agreement with the observations (see, e.g., Alcock et al. 1997b), it is far from clear that the problem is solved. In fact, using COBE data, Bissantz et al. 1997 derive values of $0.83 - 0.89 \times 10^{-6}$ for main-sequence stars, and $1.2 - 1.3 \times 10^{-6}$ for red-clump giants. Presumably the discrepancies between predictions and observations will be resolved by an explanation based on features of the stellar population thought to provide most of the lenses. Thus, whatever the solution for the Bulge, it may take us a long way toward the solution for the LMC. Can the combination of binarity and blending provide an important part of this solution?

5. Binarity and the Optical Depth

Blending and binarity both need to be systematically incorporated into the data analysis. Their influence on event detectability can be understood geometrically. Consider events with $A >> 1$. When the lens is a point mass, the high-magnification region is a disk of radius $1/A$ centered on the lens. For blended events, the high-magnification region is a disk of smaller radius, $f/(A - 1) \sim f/A$. When the lens is a binary, the high-magnification regions are closed ribbons which trace the caustic structures; there can be several disconnected, closed high-magnification ribbons. Although the width of each ribbon is smaller for larger values of $A$, the linear size of the region enclosed by the ribbons is not much affected by increasing values of $A$, and is generally significantly larger than $2/A$, even when there is blending. Thus, lens binarity allows us to more often reach deeply into the luminosity function of potential source stars. How we interpret the additional events can influence observation-based estimates of $\tau$.

5.1. Interpretation of the Data

The optical depth has been derived from the data by using the simple relationship

$$\tau = \frac{\pi}{4 N_{\text{obs}} T_{\text{obs}}} \sum_{l=1}^{L} \frac{\hat{t}_{E,l}}{\xi_l}.$$  

$N_{\text{obs}}$ is the number of stars monitored and $T_{\text{obs}}$ is the time during which the stars were monitored; $l$ labels the light curves; $\hat{t}_{E,l}$ is the time it takes for the source star to traverse an Einstein diameter; and $\xi_l$ is the efficiency for detecting a point-lens event with the duration observed for event $l$.

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7This can be derived by considering the appropriate series expansion of the the blended Einstein radius, $R_{E,bl}$, which is smaller than $R_E$ by a factor $\sqrt{\frac{1}{\sqrt{(A_{\text{min}} - 1)^2 + 2f(A_{\text{min}} - 1)}}}$ (Di Stefano & Esin 1995).
Blending and binarity can introduce non-trivial complications into the process of using this equation to compute $\tau$. First, all perturbations from baseline not clearly associated with known types of stellar variability must be considered as microlens candidates. Second, for each candidate light curve, the full complement of acceptable fits which include the effects of binarity and blending must be found; this allows significantly blended point-lens events, and most members of categories 2 and 3 to be included in the sum. Di Stefano & Perna (1997) have shown that (1) although binary-lens light curves are diverse, they are nevertheless distinctive enough that most non-lensing perturbations can be eliminated; and (2) most light curves that can be described by binary-lens models can be well fit by several sets of physical parameters, $\{M_i\}$, and that all acceptable fits can be found in a systematic way. The technique they developed can readily accommodate blending (and other effects, such as source binarity) as well. Each fit provides a value of $\hat{t}_{E,l}$; thus, an important effect of the degeneracies is to increase the uncertainties in estimates of $\tau$. In addition, the value $\xi_l$ may depend on the true nature of the underlying event, e.g., on the size of high-magnification regions, in addition to the size of the Einstein ring. Finally, the number of source stars that can be detectably lensed must be carefully computed. In fact, if blending and binarity play important roles, this single consideration can significantly decrease estimates of $\tau$ based on computations that did not fully consider binarity and blending in concert.

The discussion above makes it clear that a good deal of further work is required to quantify the influence of blending and binarity on estimates of the optical depth. It is, however, useful to note that: (1) Estimates that do not take blending and binarity into account are likely subject to larger uncertainties than those presently quoted. (2) Measurements good to within a few tens of percent will require that blending and binarity be systematically incorporated into the analysis. (3) The total effect may be larger, but its sign and size can only be determined by the observing teams; this is because detailed information about blending in the fields studied, and the details of the analysis of data play such important roles.

In summary, developing an integrated approach to the data analysis that incorporates source binarity, lens binarity, and blending, is necessary if we are to fully understand the

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8An example is provided by blending (even when point sources are lensed by point masses). In this case, the sum can contain several terms per line of sight. Yet, the estimates of the value of individual terms are smaller when there is unrecognized blending (see footnote 7), and in fact are small enough to fairly well balance the influence of the additional terms. The arguments above indicate, however, that this is not necessarily the case when the lenses are binaries. Instead, this question needs to be considered as a function of the true distributions of $f$ values and binary properties to determine if the factor outside the sum (essentially a normalization factor) requires alteration.
results of the microlensing observations. This paper begins to demonstrate, however, that the effort is well worthwhile: apparently anomalous events can, both individually and collectively, encode more information than the standard point-source/point-lens events.

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Fig. 1.— $N_{cc}/N_{tot}$ is the fraction of all detected events in which there was a caustic crossing. The upper left and right panels show $N_{cc}/N_{tot}$ as a function of $q$ and $a$, respectively. (An event was called “detectable” if $A_{peak} > 1.34$, and at least 10 observations were made while the source was within $1.5 R_E$ of the lens) In each panel, the thickest (bottom-most) curve corresponds to the case of no blending; proceeding generally upward to progressively thinner lines, $f = 0.9, 0.7, 0.5, 0.3, 0.1$. Note that the differences between $f = 0.7 - 1$, and even $f = 0.5$, are small. For smaller values of $f$, the role of caustic crossing events becomes relatively more important. For any given value of $f$, caustic crossings are most important for moderate to high values of $q$ and for $a$ near unity. Note that the trend seen here as $a$ increases clearly indicates that caustic crossings continue to play a role, even for larger values of $a$. The lower left and right panels show the ratio, $N_{cc}/N_3$, of caustic crossing to other perturbed events. It is clear that, unless blending is extreme, perturbed events with no caustic crossings dominate.
Fig. 2.— Each point represents a microlensing event; the position of the point is the position of the closest approach to the center of mass of the binary lens. The horizontal and vertical axes represent the $x$– and $y$– axes of the lens plane. **Top row:** All measurable events ($A_{\text{peak}} > 1.34$, and at least 10 observations were made while the source was within $1.5 R_E$ of the lens). **Second row:** All measurable events with acceptable point-lens fits. **Third row:** All measurable events not well-fit by a point-lens model and in which there is no caustic crossing. **Bottom row:** All measurable events not well-fit by a point-lens model and in which there is a caustic crossing. The binary depicted in the two left-most columns has $q = 1.0$, $a = 1.0$. The binary depicted in the two right-most columns has $q = 0.25$, $a = 1.0$. In each set of two columns, there is no blending in the left column, and severe blending ($f = 0.1$) in the right column.