SUB-PARSEC SUPERMASSIVE BINARY QUASARS: EXPECTATIONS AT \( z < 1 \)

M. VOLONTERI, J. M. MILLER, AND M. DOTTI

Department of Astronomy, University of Michigan, 500 Church Street, Ann Arbor, MI 48109 USA

Received 2009 March 19; accepted 2009 August 17; published 2009 September 4

ABSTRACT

We investigate the theoretical expectations for detections of supermassive binary black holes that can be identified as sub-parsec luminous quasars. To date, only two candidates have been selected in a sample comprising 17,500 sources selected from the Sloan Digital Sky Survey (SDSS) Quasar Catalog at \( z < 0.70 \). In this Letter, we use models of assembly and growth of supermassive black holes (SMBHs) in hierarchical cosmologies to study the statistics and observability of binary quasars at sub-parsec separations. Our goal is twofold: (1) to test if such a scarce number of binaries is consistent with theoretical prediction of SMBH merger rates and (2) to provide additional predictions at higher redshifts and at lower flux levels. We determine the cumulative number of expected binaries in a complete volume-limited sample. Motivated by Boroson & Lauer, we apply the SDSS quasar luminosity cut (\( M_r < -22 \)) to our theoretical sample, deriving an upper limit to the observable binary fraction. We find that sub-parsec quasar binaries are intrinsically rare. Our best models predict \( \sim 0.01 \) deg\(^{-2}\) sub-parsec binary quasars with separations below \( \sim 10^4 \) Schwarzschild radii (\( \mu_{\text{orb}} > 2000 \) kms) at \( z < 0.7 \), which represent a fraction \( \sim 6 \times 10^{-4} \) of unabsorbed quasars in our theoretical sample. In a complete sample of \( \sim 10,000 \) sources, we therefore predict an upper limit of \( \sim \) sub-parsec binary quasars. The number of binaries increases rapidly with increasing redshift. The decreasing lifetime with SMBH binary mass suggests that lowering the luminosity threshold does not lead to a significant increase in the number of detectable sub-parsec binary quasars.

Key words: black hole physics – cosmology: theory – galaxies: nuclei – quasars: general

1. INTRODUCTION

Supermassive black holes (SMBHs) appear to inhabit most galaxy centers (e.g., Richstone et al. 1998; Ferrarese & Ford 2005), and in ΛCDM cosmologies galaxies experience multiple mergers during their cosmic assembly. SMBH binaries (SMBHBs) are therefore expected to be recurrent, albeit transient, features of most galactic bulges. Observationally, the paucity of quasar pairs (\( \sim 0.1\% \)) on galactic scales (Foreman et al. 2009) points toward rapid inspiral of the SMBHs down to (sub-)parsec separations. As discussed in Foreman et al. (2009), if we assume that every galaxy hosts an SMBH and that quasar activity is triggered by galaxy mergers, the probability of observing a double quasar scales with the ratio of the quasar lifetime to the total merger timescale as \( \sim (t_{\text{qso}}/t_{\text{merge}})^2 \), if the two quasars light up at different, random times. This is consistent with the fraction of quasar pairs found in the Sloan Digital Sky Survey (SDSS; Hennawi et al. 2006). At a lower level of activity, Comerford et al. (2009) find that about 2% of early-type galaxies host candidate active galactic nucleus (AGN) pairs in the same galaxy. Detecting sub-parsec binaries by imaging techniques is extremely difficult. However, the presence of SMBHBs in AGNs can be discovered spectroscopically, as double broad-line emission systems (see Gaskell 1996 and references therein). The two sets of broad emission lines originate in gas associated with the two SMBHs, and the velocity separation between the two emission line systems traces the projected orbital velocity of the binary. Dotti et al. (2009) and Bogdanovic et al. (2008) extended the pre-existent spectroscopical technique, discovering that, depending on the SMBH mass ratio, the AGN spectrum shows two sets of broad lines (equal mass binaries) or a single set of broad lines and two sets of narrow emission lines at different redshifts (unequal mass binaries). In the latter case, only one of the two SMBHs is active, and the two sets of narrow emission lines correspond to emission from low-density gas in the potential well of the binary and from the “standard” narrow-line region (NLR) of the AGN.

Dotti et al. (2009) and Bogdanovic et al. (2008) apply the binary model to the peculiar quasar SDSS J092712.65+294344.0. This quasar exhibits two distinct sets of lines. The first set of very narrow emission lines is assumed to be emitted in the NLR of the host and traces the redshift of the host galaxy (\( z = 0.713 \)). The second set comprises two blueshifted features featuring different FWHMs: the broad Mg II and Balmer emission lines with FWHM \( \approx 4000 \) km s\(^{-1}\), and narrow lines with FWHM \( \approx 400–2000 \) km s\(^{-1}\), both consistent with a redshift of 0.698. In this model, the source emitting the blueshifted line system is gas inside or near the broad-line region (BLR) of the secondary, comoving with the SMBH with a light-of-sight velocity of 2630 km s\(^{-1}\) relative to the rest frame of the host.

Boroson & Lauer (2009) developed a principal components analysis technique that identifies sources having peculiar spectral characteristics. They applied this procedure to the rest-frame optical spectrum of \( \approx 17,500 \) quasars. Their sample comprises all quasars having \( z < 0.7 \) from the fifth release of the SDSS Quasar Catalog (Schneider et al. 2007) plus all sources classified as quasars in the seventh SDSS data release. Of the 17,500 objects in their entire sample, only two objects have multiple redshift systems consistent with the presence of a candidate SMBHB, SDSS J092712.65+294344.0, and SDSS J153636.22+044127.0 (but see Chornock et al. 2009; Heckman et al. 2009; Shields et al. 2008; Wrobel & Laor 2009; Gaskell 2009).

In this Letter, we assess the expected number of merging SMBHBs based on realistic merger rates of SMBHs in hierarchical cosmologies, and determine an upper limit to sub-parsec binary quasars detectable as double broad-line emission quasars.
2. BLACK HOLE MERGER RATE AND QUASAR ACTIVITY

We trace the evolution of SMBHs within a plausible scenario for the hierarchical assembly, growth, and dynamics of SMBHs in a $\Lambda$CDM cosmology. Our model has been shown to capture many features of the SMBH population (e.g., luminosity function of quasars, X-ray background, SMBH mass density). The main features of the models have been discussed elsewhere (Volonteri et al. 2003, 2005; Volonteri & Rees 2006; Sesana et al. 2007b, and references therein). We summarize in the following the relevant assumptions.

SMBH “seeds” form at high redshift ($z > 15$) in highly biased halos, corresponding to $3.5-4$ the relevant assumptions. We summarize in the following relevant assumptions.

The occupation fraction of SMBHs increases with time, and approaches unity for massive galaxies at low redshift (Marulli et al. 2006; Volonteri et al. 2008). In our scheme, therefore, not all galaxy mergers lead to SMBH mergers, but only those involving two galaxies both hosting SMBHs. We further assume that SMBHs merge within the merger timescale of their hosts, which is a likely assumption for SMBHs formed after gas-rich galaxy mergers (Escala et al. 2004, 2005; Dotti et al. 2006, 2007). This is the most likely scenario in the context of this work, as quasar fueling requires a substantial gas supply. We explored an alternative scenario where at late cosmic times SMBHs shrink via three-body interactions, i.e., by capturing and ejecting at much higher velocities the stars passing by within a distance comparable to the binary separation (Merritt 2006; Volonteri et al. 2003; Sesana et al. 2007a), and we found that the merger rate in the $z < 1$ redshift range is very similar, as already found in previous tests (Sesana et al. 2005). Sesana et al. (2008) compare our theoretical merger rates with the merger rates inferred by observations of the fraction of close galaxy pairs (assuming that SMBH masses scale with bulge masses). Our SMBH merger rate is consistent within a factor of $\lesssim 2$ with the merger rate of massive spheroidal found in Bell et al. (2006), who quote a factor of 2 uncertainty in their rate estimate.

We base our model for SMBH mass growth on a set of simple assumptions, supported by both simulations of AGN triggering and feedback (Springel et al. 2005), and analysis of the relationship between SMBH masses ($M_{\text{BH}}$) and the properties of their hosts (Gebhardt et al. 2000; Ferrarese & Merritt 2000; Ferrarese et al. 2002). SMBHs in galaxies undergoing a major merger (i.e., having a mass ratio $>1:10$) undergo accretion. Each SMBH accretes an amount of mass, $\Delta M = 9 \times 10^7 M_\odot (\sigma/200$ km s$^{-1})^2$, that scales with the $M_{\text{BH}}$–$\sigma$ relation of its hosts (see Volonteri & Natarajan 2009). Accretion starts after a dynamical timescale and lasts until the SMBH has accreted $\Delta M$.

The accretion rate during the active phase is derived from the empirical distribution of Eddington ratios, $\lambda = \log(L_{\text{bol}}/L_{\text{Edd}})$, found in Merloni & Heinz (2008). We adopt a fitting function of the Eddington ratio distribution as a function of SMBH mass and redshift (A. Merloni 2009, in preparation). The distributions of Eddington ratios, $f_\lambda$, are computed in 10 redshift intervals (from $z = 0$ to $z = 5$) for four different mass bins ($6 < \log(M_{\text{BH}}/M_\odot) < 7$, $7 < \log(M_{\text{BH}}/M_\odot) < 8$, $8 < \log(M_{\text{BH}}/M_\odot) < 9$, and $9 < \log(M_{\text{BH}}/M_\odot) < 10$), and then fit with an analytic function which is the sum of a Schechter function and a lognormal. The Eddington ratio distributions are normalized so that at a given mass and redshift, $\int f_\lambda \, \text{d} \log \lambda = 1$. We adopt the bolometric corrections presented in Richards et al. (2006), and we correct for absorbed quasars according to model 4 in La Franca et al. (2005).

Our first check is on the cumulative number counts of quasars, regardless of binarity. We select all accreting SMBHs in our theoretical sample, and apply the same luminosity cut as in the SDSS Quasar Catalog, $M_1 < -22$. This choice is motivated by the SDSS being the largest quasar catalog currently available, however, we stress here that our models provide upper limits to the number of observable binaries, in a complete volume-limited sample. We compare our number counts to the expected number from the integration of the bolometric luminosity function of quasars (Hopkins et al. 2007) at $M_1 < -22$ (dashed curve). In the upper-right panel, we also show the number of unabsorbed quasars when we correct for absorption (top curve with error bars).

3. SUB-PARSEC BINARY QUASARS

To evaluate the expected number of binary quasars, we start from the SMBBH merger rate. Our theoretical models indicate which of these binaries are in an active phase, following a major merger. We initially analyze the complete sample of SMBHBs, that is, we consider all merging SMBHs, regardless of their activity status. This first model (model I) provides a strong upper limit to the number of theoretical sub-parsec binary quasars. To each SMBH in our sample, we randomly assign an Eddington ratio, $\lambda$, from the normalized distribution (A. Merloni 2009, in preparation). Note that, since $-5 \lesssim \lambda_{\text{min}} \lesssim -3$ for most masses/redshift this model does not allow for “quiescent” SMBHs at, e.g., the level of Sgr A* (Quataert et al. 1999).

We further assume that only SMBHBs with a mass ratio, $q = M_{\text{BH,1}}/M_{\text{BH,2}} \lesssim 1$, above a certain threshold ($q > 10^{-1}$ or $q > 10^{-2}$) create distinguishable double broad emission line systems. We regard $q > 10^{-1}$ as our best choice, because of an additional independent motivation, as follows. Callegari...
et al. (2009) study the formation of SMBHBs during galaxy mergers, and how SMBH pairing depends on the interplay between different physical processes (dynamical friction, tidal and ram-pressure stripping). From this analysis, Callegari et al. (2009) find that $q = 10^{-1}$ is the minimum mass ratio between two merging galaxies (and as a consequence between the two SMBHs) to guarantee the formation of a SMBHB. However, tidal and gas-pressure stripping can be reduced for extreme structural properties and orbital parameters of merging galaxies. We consider the very conservative case of SMBHBs with $q > 10^{-2}$ to take into account every possible merger configuration (e.g., plunging radial orbits).

Finally, we assign a lifetime to SMBHBs detectable at sub-parsec separations following the detailed study of SMBHB dynamical evolution in circumbinary disks performed by Haiman et al. (2009). The lifetime corresponds to the time spent by the SMBHB at a separation such that the shift between the BLRs (or the NLR and the BLR) is a few thousand km s$^{-1}$, and the SMBHB at a separation such that the shift between the BLRs is a few thousand km s$^{-1}$, becoming comparable with the widths of typical broad lines (Shen et al. 2008). Blending of profiles with smaller velocity differences would be missed (Gaskell 1996). For $v_{\text{orb}} > 2000$ km s$^{-1}$, the typical separation is $r = 11.25 \times 10^3$ Schwarzschild radii, and for these conditions, Haiman et al. (2009) give a lifetime

$$t_{\text{lfile}} = 6 \text{ Myr} \left(\frac{M_{\text{bin}}}{10^7 M_\odot}\right)^{3/4} \left(\frac{4 q}{1 + q^2}\right)^{3/8} \left(\frac{10^4}{0.1}\right)^{-5/8},$$

where $M_{\text{bin}} = M_{\text{BH1}} + M_{\text{BH2}}$, and we have chosen the longest residence time, thus providing an upper limit to SMBHBs lifetime.

The results of this model (model I) are shown in Figure 1 (bottom panels). We find that at $z < 0.7$ the total expected number of binaries with $M_i < -22$ deg$^{-2}$ is $0.27 \pm 0.27$ for $q > 10^{-2}$ and $0.04^{+0.06}_{-0.04}$ for $q > 10^{-1}$. These detectable binaries represent a fraction $\sim 10^{-2}$ and $\sim 2 \times 10^{-3}$, respectively, of the unabsorbed quasars. Within the sample of 17,500 sources analyzed by Boroson & Lauer (2009), 9985 objects, including the two putative binary SMBHs, belong to the uniformly selected statistical sample of SDSS quasars (T. A. Boroson 2009, private communication). The statistical sample reaches a completeness larger than 90% at $z < 1$ for sources with apparent magnitude $i < 19.1$ (roughly corresponding to $M_i \lesssim 24$ at $z < 0.7$). Given these caveats, our results must be considered upper limits to the number of detectable sub-parsec binary quasars. We therefore find marginal agreement, within the uncertainties, with Boroson & Lauer (2009) findings. If binaries with $q \lesssim 10^{-2}$ do produce distinguishable double broad emission line systems, then the expected SMBHB merger rate must be lower than we predict. The high luminosity selection criterion leads to a sample composed of actively accreting ($\lambda \sim -1$) massive ($\sim 10^8 M_\odot$) SMBHBs.

In a second model (model II), we select only SMBHBs where at least one of the SMBHs is active, according to our merger-driven quasar activity scheme. The results of this model are shown in the top panels of Figure 1. In this case, the expected number of binaries per deg$^2$ is $0.02^{+0.06}_{-0.02}$ for $q > 10^{-2}$ and $0.01_{-0.01}^{+0.05}$ for $q > 10^{-1}$. Detectable binaries represent a fraction $\lesssim 6 \times 10^{-4} - 10^{-3}$ of the unabsorbed quasars, consistent with Boroson & Lauer (2009) findings (a fraction $\lesssim 1 - 2 \times 10^{-4}$).

If we decrease the luminosity threshold, we expect two factors enter into play: on the one hand, the merger rate of SMBHs is expected to increase at lower SMBH masses, where the mass function is less steep (Gültekin et al. 2009). This is indeed what we find in our models (Figure 2; see also Dotti et al. 2009; Sesana et al. 2005). On the other hand, however, the lifetime of detectable binaries decreases with decreasing mass (Haiman et al. 2009), making the detection harder. Using the scaling for lifetimes presented in Haiman et al. (2009), we indeed expect that the number of detectable sub-parsec binary quasars does not increase dramatically with decreasing luminosity, because of the shorter timescale over which they are observable. For instance, if we decrease the flux limit by a factor of 10, we find negligible changes in model II, and a mild increase in the number of binaries by a factor of 3 in model I: $0.17 \pm 0.13$ for $q > 10^{-1}$ and $0.79 \pm 0.64$ for $q > 10^{-2}$.

4. DISCUSSION

Stimulated by the recent putative discovery of two candidate sub-parsec SMBHBs identified as quasars with multiple redshift line systems (BLR and NLR), SDSS J092712.65+294344.0 and SDSS J153636.22+044127.0, we investigate theoretically the occurrence of sub-parsec SMBHBs that can be identified as sub-parsec binary quasars. We study the SMBH cosmic evolution via a Monte Carlo merger tree approach. We trace the growth and dynamical history of SMBHBs from high redshift via physically motivated prescriptions that allow us to reproduce many observational constraints.

Our approach provides us with a catalog of SMBHBs, for which we know the masses and the redshift. We further assume that only SMBHBs with a mass ratio, $q = M_{\text{BH2}}/M_{\text{BH1}} \leq 1$, above a certain threshold ($q > 10^{-1}$ or $q > 10^{-2}$) create distinguishable double broad emission line systems. Motivated by the work by Boroson & Lauer (2009), we apply to our theoretical model the same luminosity cut as in the SDSS Quasar Catalog, $M_i < -22$, thus deriving upper limits to the fraction of detectable binaries. We stress here that the theoretical sample is complete and volume limited. Since the SDSS Quasar Catalog is not a complete sample (Schneider et al. 2007), a direct comparison with Boroson & Lauer (2009) is not appropriate. However, our results are consistent with the binary
fraction derived for the subset of the 17,500 quasars used by Boroson and Lauer that are part of the statistical sample of the SDSS.

Our merger-driven quasar scheme provides us also with an accretion rate, hence a luminosity. We analyze two models that likely bracket the theoretical uncertainties. Model I ignores merger-driven quasar activity and we assume that each SMBH is at some level “active.” Each SMBH in our binary sample is assigned an Eddington ratio, \( \lambda \), from the normalized distribution derived from synthesis model for AGN evolution (Merloni & Heinz 2008). Model I is therefore our strong upper limit to the number of detectable SMBHBs. Model II is more rooted in our quasar activity scheme, as we select SMBHBs where at least one of the SMBHs is active, according to our merger-driven scenario.

Our main findings are as follows.

1. Sub-parsec binary quasars are intrinsically rare due to a combination of strict requirements: the time over which SMBHBs are detectable through line shifts decreases with decreasing binary mass. On the other hand, the merger rate of SMBHBs increases with increasing mass.

2. In a volume-limited, complete sample of \( \sim 10,000 \) sources at \( z < 0.7 \), our best models (II), that relate quasar activity to galaxy mergers, predict an upper limit of \( \sim 5–10 \) sub-parsec binary quasars. Model I, that does not associate quasars to mergers, is only marginally compatible with Boroson & Lauer (2009) who find only two candidate sub-parsec binary quasars in the statistical SDSS quasar sample.

3. Figure 1 extends our predictions out to \( z = 1 \). The number of detectable binaries increases by a factor \( \sim 5–10 \) from \( z = 0.7 \) to \( z = 1 \).

4. The lifetime over which SMBHBs can be detected as sub-parsec quasars decreases with decreasing binary mass (Haiman et al. 2009). This effect is stronger than the increase in the merger rate of SMBHB at lower masses. Lowering the luminosity threshold is unlikely to lead to a large increase in the number of detectable sub-parsec binary quasars.

SDSS-III will increase the spectroscopic quasar sample and will provide a good testing ground for our predictions. The masses of SMBHBs that can be identified as sub-parsec binary quasars are too large for the gravitational waves emitted by these binaries to be detectable at merger by the Laser Interferometer Space Antenna (LISA), which will instead focus on the mass range \( 10^5–10^7 \, M_\odot \). However, such massive binaries (in a later evolutionary stage, when the binary has shrunk by an additional factor of 10 and the dynamical evolution is driven by emission of gravitational radiation) are typical candidates for detection via Pulsar Timing Arrays (PTAs, e.g., the Parkes radio telescope). PTAs rely on the effect of gravitational waves on the propagation of radio signals from a pulsar to the Earth, producing a characteristic signature in the time of arrival of radio pulses. Sesana et al. (2009) find that the mass distribution of the SMBHBs detectable via PTAs peaks at \( \sim 10^6 \, M_\odot \), with most binaries at \( z < 1 \), the same mass range probed by sub-parsec binary quasars identifiable in the SDSS.

We thank Tim McKay and Douglas Tucker for help with the SDSS magnitude system. We also thank Armin Rest for suggestions on the statistical treatment of data. We gratefully acknowledge help from Todd Boroson, Tod Lauer, and Gordon Richards for the comparison between models and data.

REFERENCES

Bell, E. F., Phleps, S., Somerville, R. S., Wolf, C., Borch, A., & Meisenheimer, K. 2006, ApJ, 652, 270
Bogdanovic, T., Eracleous, M., & Sigurdsson, S. 2008, ApJ, 697, 288
Boroson, T. A., & Lauer, T. R. 2009, Nature, 458, 53
Callegari, S., Mayer, L., Kazantzidis, S., Colpi, M., Governato, F., Quinn, T., & Wadsley, J. 2009, ApJ, 696, L89
Chornock, R., et al. 2009, ApJL, submitted (arXiv:0906.0849)
Comerford, J. M., et al. 2009, ApJ, 698, 956
Dotti, M., Colpi, M., & Haardt, F. 2006, MNRAS, 367, 103
Dotti, M., Colpi, M., Haardt, F., & Mayer, L. 2007, MNRAS, 379, 956
Dotti, M., Montuori, C., Decarli, R., Volonteri, M., Colpi, M., & Haardt, F. 2009, MNRAS, 398, L73
Escala, A., Larson, R. B., Coppi, P. S., & Mardones, D. 2004, ApJ, 607, 765
Escala, A., Larson, R. B., Coppi, P. S., & Mardones, D. 2005, ApJ, 630, 152
Ferrarese, L. 2002, ApJ, 578, 90
Ferrarese, L., & Ford, H. 2005, Space Sci. Rev., 116, 523
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Foreman, G., Volonteri, M., & Dotti, M. 2009, ApJ, 693, 1544
Gaskell, C. M. 2009, arXiv:0903.4447
Gaskell, C. M. 1996, in Lecture Notes in Physics, Vol. 471, Jets from Stars and Galactic Nuclei, ed. W. Kundt (Berlin: Springer), 165
Gebehart, K., et al. 2000, ApJ, 539, L13
Gehrels, N. 1986, ApJ, 303, 336
Gültekin, K., et al. 2009, ApJ, 695, 1577
Haiman, Z., Kocsis, B., & Menou, K. 2009, ApJ, 700, 1952
Heckman, T. M., Krollik, J. H., Moran, S. M., Schnittman, J., & Gezari, S. 2009, ApJ, 695, 363
Hennawi, J. F., et al. 2006, AJ, 131, 1
Hopkins, P. F., Richards, G. T., & Hernquist, L. 2007, ApJ, 654, 731
La Franca, F., et al. 2005, ApJ, 635, 864
Marulli, F., Crociani, D., Volonteri, M., Branchini, E., & Moscardini, L. 2006, MNRAS, 368, 1269
Mayer, L., Kazantzidis, S., Madau, P., Colpi, M., Quinn, T., & Wadsley, J. 2007, Science, 316, 1874
Merloni, A., & Heinz, S. 2008, MNRAS, 388, 1011
Merritt, D. 2006, ApJ, 648, 976
Quataert, E., Narayan, R., & Reid, M. J. 1999, ApJ, 517, L101
Richards, G. T., et al. 2006, ApJS, 166, 470
Richstone, D., et al. 1998, Nature, 395, A14
Shen, Y., Greene, J. E., Strauss, M. A., Richards, G. T., & Schneider, D. P. 2008, ApJ, 680, 169
Shields, G. A., Bonning, E. W., & Salvatier, S. 2008, ApJ, 696, 1367
Schneider, D. P., et al. 2007, AJ, 134, 102
Sesana, A., Haardt, F., & Coppi, P. 2007a, ApJ, 660, 546
Sesana, A., Haardt, F., Madau, P., & Volonteri, M. 2005, ApJ, 623, 23
Sesana, A., Vecchio, A., & Colacino, C. N. 2008, MNRAS, 390, 192
Sesana, A., Vecchio, A., & Volonteri, M. 2009, MNRAS, 394, 2255
Sesana, A., Volonteri, M., & Haardt, F. 2007b, MNRAS, 377, 1711
Springel, V., Di Matteo, T., & Hernquist, L. 2005, MNRAS, 361, 776
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Volonteri, M., Lodato, G., & Natarajan, P. 2008, MNRAS, 383, 1079
Volonteri, M., Madau, P., Quataert, E., & Rees, M. J. 2005, ApJ, 620, 69
Volonteri, M., & Natarajan, P. 2009, MNRAS, in press (arXiv:0903.2262)
Volonteri, M., & Rees, M. J. 2006, ApJ, 650, 669
Wrobel, J. M., & Laor, A. 2009, ApJ, 699, L22