A Further Study of Superoutbursts and Superhumps in SU UMa Stars by the Kepler Light Curves of V1504 Cygni and V344 Lyrae

Yoji Osaki
Department of Astronomy, School of Science, University of Tokyo, Hongo, Tokyo 113-0033
osaki@ruby.ocn.ne.jp
and Taichi Kato
Department of Astronomy, Kyoto University, Sakyo-ku, Kyoto 606-8502
tkato@kusastro.kyoto-u.ac.jp

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Abstract

We made a supplemental study of the superoutbursts and superhumps in SU UMa stars by using the recently released Kepler public data of V1504 Cyg and V344 Lyr. One of the superoutbursts in V1504 Cyg was preceded by a precursor normal outburst which was well separated from the main superoutburst. The superhump first appeared during the descending branch of the precursor normal outburst and it continued into quiescence (the deep dip between the precursor and the main superoutburst), and it began to grow in amplitude with the growth of the main superoutburst after quiescence ended. A similar phenomenon was also observed in V344 Lyr. This observation demonstrates very clearly that the superoutburst was triggered by the superhump (i.e., by the tidal instability), supporting the thermal-tidal instability model. Smak (2013, Acta Astron., 63, 109) criticized our previous paper (Osaki and Kato, 2013, PASJ, 65, 50) and challenged our main conclusion that various observational lines of evidence of V1504 Cyg support the thermal-tidal instability model for the superoutbursts of SU UMa stars. We present our detailed accounts to all of his criticisms by offering clear explanations. We conclude that the thermal-tidal instability model is after all only the viable model for the superoutbursts and superhumps in SU UMa stars.

Key words: accretion, accretion disks — stars: dwarf novae — stars: individual (V1504 Cygni) — stars: individual (V344 Lyrae) — stars: novae, cataclysmic variables

1. Introduction

The SU UMa stars are one of the sub-classes of dwarf novae characterized by the so-called “superoutburst” and “superhumps”. It is now well established that the outbursts of dwarf novae are produced by a sudden brightening of the accretion disk around the central white dwarf in the semi-detached close binary systems in which the red-dwarf secondary star supplies mass to the accretion disk. In addition to the ordinary normal outbursts observed in the U Gem type dwarf novae, the SU UMa stars show a longer and brighter outburst called the superoutburst. During a superoutburst, a periodic photometric hump with amplitude 0.2–0.3 mag appears and it is called the superhump whose period is very near to the orbital period of the system but it is longer than that by a few percent. The ordinary superhump mentioned above is also called the “positive” superhump (abbreviated as pSH) because another periodic hump with a period shorter than the orbital period appears in some SU UMa and nova-like variable stars and the latter is called the “negative” superhump (nSH). The origin of the positive superhump is now understood as due to a deformation of accretion disk into eccentric form and a slow apsidal precession of the eccentric disk is responsible for the basic clock of the positive superhump. On the other hand the negative superhump is now thought to be produced by a tilted accretion disk whose nodal line precesses retrograde, giving rise to the underlying clock of the negative superhump. The general reviews on these subjects are found in Warner (1995) and Hellier (2001).

In a series of papers we have studied the superoutbursts and superhumps in SU UMa stars by using the Kepler light curves of SU UMa stars. In the first paper (Osaki, Kato 2013a) (hereafter referred to Paper I) we have studied the Kepler light curve of an SU UMa star, V1504 Cyg, and we have demonstrated that various observational lines of evidence in V1504 Cyg support the thermal-tidal instability (TTI) model for the superoutburst of SU UMa stars proposed by Osaki (1989) (for a review on the TTI model, see, Osaki 1996). In particular, we have shown that the frequency of the negative superhump varies systematically during a supercycle (a cycle from one superoutburst to the next) and by interpreting those variations basically as the result of variations in the disk radius, we have demonstrated that observed disk radius variations fit very well with a prediction of the thermal-tidal instability model. In the second paper (Osaki, Kato 2013b) (Paper II), we have examined the Kepler short-cadence light curves of two SU UMa stars, V344 Lyr and V1504 Cyg. We have analyzed the simultaneous frequency variations of the positive and negative superhumps. We have demonstrated that these
two signals vary in unison, if appropriately converted, indicating the disk radius variation.

The Kepler public data are released every three-months and the new data continue to arrive. As shown below, the newly released data of V1504 Cyg and V344 Lyr have basically supported our main conclusions reached in our Paper I and Paper II. However, the new data also give us some surprise and in section 2 we made a supplemental study to our Paper I and Paper II on the superoutbursts and superhumps of V1504 Cyg and V344 Lyr by using newly released public Kepler data.

Smak (2013) has recently criticized our Paper I by arguing that our interpretation and our main conclusion about the nature of superoutbursts were incorrect. As his preprint already appeared in astro-ph (arXiv:1301.0187) on 2013 January 2, we examined in our Paper II his criticism about our interpretation on variation of the negative superhump periods. Since his paper has finally appeared in a published form in Acta Astronomica, here we present our own accounts once again to almost all his criticisms in section 3.

This paper consists of two parts: the first part concerns a supplemental study of newly released Kepler data of V1504 Cyg and V344 Lyr in section 2, and the second part our reply to Smak’s criticisms in section 3. A summary is given in section 4.

2. A Further Study of the Superoutburst and Superhumps by Using Newly Released Kepler Light Curves of V1504 Cyg and V344 Lyr

When we wrote our Paper II in March 2013, the Kepler data available for the public were those from the quarters 2 to 10 (Q2–10) and the quarter 14 (Q14). Since then, data for two more quarters Q11 and Q15 were released in April 2013 and then further data for two more quarters, Q12 and Q16, released in July 2013. In total, the short-cadence Kepler Data for V1504 Cyg and V344 Lyr available to the public now extend from June 2009 to April 2013, well over three years from Barycentric Julian Date (BJD) 2455002 to 2456390 except for the quarter 13 (Q13), which will be released in October 2013. The photometric observations by the Kepler satellite were unfortunately stopped in May 2013 because of problems in the reaction wheels onboard the satellite. Its recovery is still uncertain. The data from the quarter 16 could be the last.

In this paper, we examine those new data of V1504 Cyg and V344 Lyr for Q10–12 and Q14–16. Analysis of those for Q2–Q10 has been presented in Paper II, which should be consulted whenever it is necessary. We first summarize global light curves and the power spectra for newly released data of V1504 Cyg and V344 Lyr in subsection 2.1. We then discuss three problems in this section. We found that one of superoutbursts in V1504 Cyg was preceded by a precursor normal outburst which was well separated from the main superoutburst. The same type of precursor was also found in V344 Lyr and we study them in subsection 2.2. We then examine the failed superhump which occurred in the descending branch of one of normal outbursts just prior to a superoutburst but failed to trigger a superoutburst in subsection 2.3. In subsection 2.4 we discuss mini-outbursts which occurred exclusively in the later half of the Type S supercycle of V1504 Cyg, a supercycle not accompanied by negative superhumps.

2.1. Global Light Curves and the Power Spectra for the Newly Released Kepler data of V1504 Cyg and V344 Lyr

First we made two-dimensional discrete Fourier transform (FT) power spectra for the newly released data of quarters Q10–12 and Q14–16 of V1504 Cyg and V344 Lyr, and they are shown in figures 1 and 2. The method of analysis was the same as in described in Paper I. The two-dimensional least absolute shrinkage and selection operator (Lasso) power spectra (Kato, Uemura 2012; Kato, Maehara 2013) are also shown in figures 3 and 4 as in paper II.

We summarize the main characteristics of supercycles and superoutbursts in table 1 for V1504 Cyg and table 2 for V344 Lyr as in Paper I and Paper II. The symbols used in these two tables are the same as those given there and no explanations may be needed.

Figures 1 and 3 exhibit light curves and power spectra of V1504 Cyg for supercycles Nos. 8–13 and figures 2 and 4 do those of V344 Lyr for supercycles Nos. 8–13. Most of supercycles in both V1504 Cyg and V344 Lyr shown here are the Type S supercycle (i.e., lacking the negative superhumps) except for the supercycle No. 8 of V344 Lyr in which the ordinary negative superhumps were visible in its early part. The newly released Kepler data of the two stars basically confirm the results of Paper I and Paper II. That is, a good correlation between the appearance of the negative superhumps and the quiescence interval of two consecutive outbursts (and thus the number of normal outbursts in a supercycle), which led us to the classification of supercycles: the Type L supercycle accompanied by the negative superhumps and the Type S supercycle without the negative superhumps.

Let us examine more closely the outburst characteristics of V1504 Cyg which are summarized in table 1. As seen in table 1, the Kepler data used here now cover all 12 superoutbursts of V1504 Cyg and its supercycles 2-12 except for a supercycle 10 which occurred in Q13. As discussed in Paper I, the supercycles No.4 and No. 5 of V1504 Cyg were well defined Type L supercycles in which a strong signal of the negative superhumps appeared. Supercycles No.8-9 and No.11-12 shown in figures 1 and 3 are found to be the Type S supercycles without negative superhumps. The mean supercycle length of V1504 Cyg is found to be 116.3d. As seen in table 1, the supercycle length is well correlated with the supercycle Types: short for the Type L supercycles and long for the Type S. From this correlation, we can guess that the supercycle No. 10 must be the Type S supercycle because of a long supercycle

1 As mentioned in note added in proof, the original figures 1–4 were replaced by new ones, by including data for Q13.
2 As mentioned in note added in proof, the original tables 1-2 were replaced by new ones, by including data for Q13.
Fig. 1. Two-dimensional discrete Fourier power spectrum of the Kepler light curve of V1504 Cyg for supercycles 8–10. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 1. Two-dimensional discrete Fourier power spectrum of the Kepler light curve of V1504 Cyg for supercycles 11–13. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 2. Two-dimensional discrete Fourier power spectrum of the Kepler light curve of V344 Lyr for supercycles 8–10. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 2. Two-dimensional discrete Fourier power spectrum of the Kepler light curve of V344 Lyr for supercycles 11–13. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 3. Two-dimensional lasso power spectrum of the Kepler light curve of V1504 Cyg for supercycles 8–10. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d. Lower: power spectrum the sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 3. Two-dimensional lasso power spectrum of the Kepler light curve of V1504 Cyg for supercycles 11–13. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 4. Two-dimensional Lasso power spectrum of the Kepler light curve of V344 Lyr for supercycles 8-10. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Fig. 4. Two-dimensional Lasso power spectrum of the Kepler light curve of V344 Lyr for supercycles 11–13. From the top to the bottom for each supercycle, Upper: light curve; the Kepler data were binned to 0.005 d, Lower: power spectrum. The sliding window and the time step used are 5 d and 0.5 d, respectively.
Table 1. Superoutbursts and supercycles of V1504 Cyg. *

| SC number | SC start | SC end | SC length | SO start | SO end | SO duration | SC number excluding SO | SH hump |
|-----------|----------|--------|-----------|----------|--------|-------------|------------------------|---------|
| 1         | –        | 74.5   | 88.5      | –        | –      | 14          | >88                    | >8      |
| 2         | 88.5     | 201    | 215       | 112.5    | 14     | 126.5       | 10                     | later half twice |
| 3         | 215      | 312    | 325       | 97       | 13     | 110         | 9                      | partly visible |
| 4         | 325      | 406.5  | 419       | 81.5     | 12.5   | 94          | 6                      | full visible    |
| 5         | 419      | 517    | 530       | 98       | 13     | 111         | 5                      | partly visible  |
| 6         | 530      | 638.5  | 650       | 108      | 12     | 120         | 10                     | early part half |
| 7         | 650      | 750    | 763.5     | 100      | 13.5   | 113.5       | 12                     | yes      |
| 8         | 763.5    | 878    | 890       | 114.5    | 12     | 126.5       | 14                     | yes      |
| 9         | 890      | 994.5  | 1006      | 104.5    | 11.5   | 116         | 14                     | yes      |
| 10        | 1006     | 1115   | 1128      | 109      | 13     | 122         | 12                     | yes      |
| 11        | 1128     | 1232   | 1245      | 104      | 13     | 117         | 11                     | yes      |
| 12        | 1245     | 1355   | 1368      | 110      | 13     | 123         | >10                    | yes      |
| 13        | 1368     | –      | –         | –        | –      | –           | –                      | –       |

*Abbreviations in this table: supercycle (SC), superoutburst (SO), normal outburst (NO), superhump (SH).

†BJD−2455000.

‡Unit: d.

§date guessed because of data gap

∥precursor normal outburst included

Table 2. Superoutbursts and supercycles of V344 Lyr.*

| SC number | SC start | SC end | SC length | SO start | SO end | SO duration | SC number excluding SO | SH hump |
|-----------|----------|--------|-----------|----------|--------|-------------|------------------------|---------|
| 1         | –        | 55.5   | 73        | –        | –      | 17.5        | >70                    | >4      |
| 2         | 73       | 160    | 178       | 87       | 18     | 105         | 7                      | well visible   |
| 3         | 178      | 277    | 294       | 99       | 17     | 116         | 7                      | no         |
| 4         | 294      | 397    | 415       | 103      | 18     | 121         | 10                     | no         |
| 5         | 415      | 527    | 544.5     | 112      | 17.5   | 129.5       | 13                     | no         |
| 6         | 544.5    | 641.5  | 659.5     | 97       | 18     | 115         | ≥10                    | no         |
| 7         | 659.5    | 742    | 760       | 82.5     | 18     | 100.5       | 8                      | well visible   |
| 8         | 760      | 850    | 868       | 90       | 18     | 108         | 10                     | partly visible |
| 9         | 868      | 966    | 985       | 98       | 19     | 117         | 11                     | partly visible |
| 10        | 985      | 1074   | 1092      | 89       | 18     | 107         | 11                     | partly visible |
| 11        | 1092     | 1188   | 1206      | 96       | 18     | 114         | 11                     | partly visible |
| 12        | 1206     | 1301   | 1319.3    | 95.5     | 18     | 113         | 10                     | no         |
| 13        | 1319     | –      | –         | –        | –      | –           | >10                    | no         |

*Abbreviations in this table: supercycle (SC), superoutburst (SO), normal outburst (NO), superhump (SH).

†BJD−2455000.

‡Unit: d.

§date guessed because of data gap
length (~122d), which can be checked in October 2013. Particularly interesting is a supercycle No. 8 of V1504 Cyg where 14 normal outbursts occurred in a supercycle, including two mini-outbursts. This will be discussed in subsection 2.4.

Let us now turn our attention to the case of V344 Lyr. The light curve and power spectra of newly released data of V344 Lyr are shown in figures 2 and 4 and the characteristics of supercycles are summarized in table 2. As seen in these figures, a weak signal of the negative superhump (nSH) is visible in the early part of supercycle No. 8 and also a much weaker signal is barely visible in the early part of supercycle No. 9. We also note from these figures that all superoutbursts of V344 Lyr were preceded by one to three normal outbursts accompanied by the impulsive negative superhump (which was found by Wood et al. (2011) and discussed in Paper II). These observations basically confirm our conclusion reached in Paper II: the occurrence of the negative superhump in V344 Lyr was more patchy than the case of V1504 Lyr, and all superoutbursts of V344 Lyr were preceded by normal outbursts accompanied by the impulsive nSH.

### 2.2. A Precursor Normal Outburst Well Separated from the Main Superoutburst

All superoutbursts in V1504 Cyg and V344 Lyr so far studied in Paper I and Paper II have turned out to be precursor-main types in which the precursor normal outburst merged quite well with the main superoutburst and it formed a part of superoutburst.

However a new type of superoutburst occurred in superoutburst No. 8 at the day 55880 of V1504 Cyg (where the day is counted from BJD 2400000) in which a precursor normal outburst was separated by a deep dip (quiescence) from the main superoutburst so that the precursor normal outburst looked as an isolated normal outburst. Figure 5 (panel b) shows this part of light curve of V1504 Cyg. A normal outburst which occurred at the day 55875 looked like an ordinary normal outburst; there is no clear difference from the preceding two normal outbursts in light curve. The next superoutburst occurred 4 d after the maximum of this normal outburst. At a first look, this superoutburst looked different from those studied before as it looked as if not accompanied by a precursor because the star rose straightly up to a light maximum from quiescence.

In reality, what happened is that the preceding normal outburst at the day 55875 was a precursor of this superoutburst but it was so widely separated from its main part by quiescence that the precursor looked like an isolated normal outburst, as discussed below. Thus this superoutburst has turned out to be an extreme case of precursor-main types in which the dip between the precursor and the main was so deep so as to touch quiescence.

Let us now look at the enlarged light curve in figure 5 (panel a). We can see that a periodic hump most likely of “superhump” origin first appeared on the descending branch of the preceding normal outburst and it continued to quiescence. To confirm that the signal was really of superhump origin, we made the 2D power spectral analysis of the light curve. We note here that frequencies of three periodic signals in V1504 Cyg are \( \nu_{\text{orb}} = 14.38 \text{c/d} \) for the orbital frequency, \( \nu_{\text{nSH}} \sim 13.8 \text{c/d} \), and \( \nu_{\text{SH}} \sim 14.7 \text{c/d} \), respectively. We show the power spectrum in figure 5 (panel d). As seen in the figure, the signal of the positive superhump at a frequency around 13.5 c/d first appeared in the descending branch of the normal outburst at around 55876.5 and it continued till quiescence with a small amplitude. This signal had a lower frequency than the positive superhump during the later course of the superoutburst, and corresponds to stage A (growing stage) superhumps in Kato et al. (2009). A new outburst started at around the day 55879. The existing positive superhump grew in amplitude together with the increase of brightness and both the superhump and the system brightness reached their maxima simultaneously at around the day 55879.5. This demonstrates very clearly that the superoutburst was triggered by the superhump (or by the tidal instability) as the enhanced tidal dissipation in the eccentric (flexing) disk rekindled the thermal instability (from the cold state to hot one) at the outer edge of the disk and a superoutburst was initiated by the tidal instability, giving a strong support to the original thermal-tidal instability model (TTI model) proposed by Osaki (1989).

The same kind of phenomenon occurred in superoutburst No. 9 of V1504 Cyg. A preceding normal outburst occurred at the day 55991 in which the superhump with a frequency \( \nu \sim 13.5 \text{c/d} \) first appeared in its descending branch and it continued to quiescence. The next superoutburst occurred 5 d after this normal outburst. Unfortunately, a strong noise (most likely caused by a solar flare) happened between 55994 and 55995, which deteriorated the Kepler data in the most delicate part of the very dip between the precursor normal outburst and the next superoutburst and it made further analysis impossible.

The same sort of phenomenon was also observed in V344 Lyr for a superoutburst which occurred at around 56193. In figure 6 we show the corresponding one for V344 Lyr. As seen in this figure, the precursor normal outburst, which started around 56187, reached its maximum around 56188.5. The superhump with a frequency around 10.5 c/d appeared in the descending branch of this normal outburst. However, the system brightness did not reach quiescence but it reached a local minimum (the dip) and it began to grow together with the growth of superhump amplitude. The dip between the precursor and the main outburst in this case was slightly shallower than that of V1504 Cyg, not reaching the quiescence.

Let us now discuss a problem why some of precursor normal outbursts are so much separated from the main outburst – by as much as 5 d. This can be understood in the following way. The triggering normal outburst occurs rather randomly with respect to the 3:1 resonance radius (i.e., the tidal instability radius); sometimes the disk ex-
Fig. 5. Kepler light curve of superoutburst No. 8 of V1504 Cyg. (a) Enlargement of the box in panel b. The Kepler data were binned to 0.001 d. Superhumps developed between the short outburst and the superoutburst. (b) Light curve corresponding to the interval of panels c and d; the Kepler data were binned to 0.01 d (c) Two-dimensional power spectrum. The sliding window and the time step used are 5 d and 0.1 d, respectively. (d) Frequency variation of superhumps. The initial low frequency superhumps correspond to the growing stage superhumps (stage A superhumps).
Fig. 6. Kepler light curve of V344 Lyr for superoutburst around BJD 2456190. (a) Enlargement of the box in panel b. The Kepler data were binned to 0.001 d. Superhumps developed between the short outburst and the superoutburst. (b) Light curve corresponding to the interval of panels b and c; the Kepler data were binned to 0.01 d. (c) Two-dimensional power spectrum. The sliding window and the time step used are 5 d and 0.1 d, respectively. (d) Frequency variation of superhumps. The initial low frequency superhumps correspond to the growing stage superhumps (stage A superhumps).
pands well beyond the 3:1 resonance radius \( r_{3:1} \) during the triggering normal outburst so that a large amount of disk mass is pushed into the instability region while sometime the disk expands just beyond the \( r_{3:1} \) so that only a small amount of the disk mass is left within the instability region. In the former case, the tidal eccentric instability is expected to be so strong that the superhump grows rapidly and it triggers the main superoutburst, producing a precursor-main superoutburst with a short interval between the precursor and the main outburst. On the other hand, in the latter case even if the superhump grows in the descending branch of the normal outburst, mass addition with low specific angular momentum from the secondary star pushes the outer disk mass below the \( r_{3:1} \) and it kills the tidal instability (and the superhump), resulting in the failed superhump discussed in the next subsection. These are the two extreme cases and there must be an intermediate case in which the growth of the tidal instability is rather slow but it can survive during quiescence and it eventually triggers the main superoutburst, producing the precursor normal outburst well-separated from the main superoutburst discussed in this subsection. We do not know how much days of separation between the precursor normal outburst and the main superoutburst are possible but observations indicate 4 to 5 d for high-mass-transfer (\( M \)) systems like V1504 Cyg and V344 Lyr. In low-\( M \) systems, the interval between the separated precursor outburst and the main superoutburst can be even longer: 10 d in QZ Vir (1998, Ohshima et al. 2011) and 11 d in V699 Oph (2001, Kato et al. 2009). In all superoutbursts in the Kepler light curves of V344 Lyr studied before in Paper II and by Cannizzo et al. (2010), the dip between the precursor and the main was so shallow that it was called “shoulder” by Cannizzo et al. (2010). The latter authors tried to explain this phenomenon as a “shoulder” based on Cannizzo’s pure thermal disk instability model. It may be clear that the superoutburst shown here with a deep dip can not be explained by Cannizzo et al. (2010) model as already discussed in subsection 3.2 of Paper I.

### 2.3. Failed Superhump in a Normal Outburst Prior to a Superoutburst

Another interesting phenomenon in the light curve of V1504 Cyg was a failed superhump seen in the descending branch of a normal outburst prior to a superoutburst. Figure 7 shows such an example for a normal outburst which occurred around 56221. We can see very clearly that a signal of periodic light hump with a period around 14 c/d first appeared in the descending branch of this outburst but it disappeared before the system brightness reached quiescence. This signal was of the superhump nature because its frequency corresponds to that of the ordinary superhump and we call this the “failed superhump” because it failed to excite a superoutburst. This phenomenon is exactly the same as that of “aborted superhump” discussed by Osaki, Meyer (2003). The next superoutburst occurred at around 56233, 10 d after this normal outburst. It is very interesting to compare two light curves of figure 5 and figure 7 where the former one is for the precursor normal outburst and the latter is for a normal outburst with the failed superhump. The superhump signal appeared in the descending branch of a normal outburst in both figures. It continued until quiescence and it led to the superoutburst in the former case while it died down before quiescence in the latter case.

Figure 8 shows another example of this phenomenon for V344 Lyr in a normal outburst at around 55842, which preceded the next superoutburst No. 8 by 8 d. We also note that the same phenomenon was already noticed by Kato et al. (2012) for a normal outburst at around 50068 just prior to superoutburst No. 1 of V1504 Cyg in the Kepler light curve.

The phenomenon of the failed superhump supports a picture presented by Osaki, Meyer (2003) and thus supports the TTI model.

### 2.4. Mini-outbursts in the Type S Supercycle

One of unsolved problems in the Kepler light curves of V1504 Cyg and V344 Lyr concerns about the outburst intervals during a supercycle in the Type S supercycle. As already pointed out by Cannizzo et al. (2012), the trend
The occurrence of frequent outbursts with low amplitudes in the later half of a supercycle suggests that the disk seems to know somehow its approach to the next superoutburst. A similar tendency is also seen in supercycles No. 9, 11, and 12, and No. 6 in the later half of which the signal of the negative superhump disappeared as shown in figure 7 of Paper II.

By criticizing the TTI model, Cannizzo et al. (2012) have mentioned that the observed trend of quiescence intervals indicates in the TTI model that the triggering radius for normal outburst (NO) does not move uniformly outward with each successive NO, but rather attains a local maximum and then recedes. This was clearly their misinterpretation of the TTI model. The TTI model predicts that the disk radius increases uniformly with each successive NO even in the later half of a supercycle. The origin of decreasing interval between normal outbursts in the later half of the Type S supercycle must be sought in other causes. Observations mentioned above indicate rather that the disk’s outer edge in the later half of a supercycle is approaching either to the 3:1 resonance radius or to the tidal truncation radius. Here we propose a possible scenario for occurrence of frequent normal outbursts with low outburst amplitudes in the later half of the Type S supercycle within the TTI model, although it is still very much speculative.

We seek its cause in the radius dependence of the S-shaped thermal equilibrium curve in the thermal instability model. In the thermal-viscous instability model there are two critical surface densities: \( \Sigma_{\text{max}} \) above which no cold state exists, and \( \Sigma_{\text{min}} \) below which no hot state exists. They are given by Cannizzo et al. (1988) and Warner (1995):

\[
\Sigma_{\text{max}} = 11.4 r_1^{0.5} M_1^{-0.35} \alpha_C^{-0.86} \alpha_H^{-0.86} \text{ g cm}^{-2}, \tag{1}
\]

\[
\Sigma_{\text{min}} = 8.25 r_1^{0.5} M_1^{-0.35} \alpha_H^{-0.8} \text{ g cm}^{-2}, \tag{2}
\]

where \( r_1 \) is the radius in units of 10^{10} cm, \( M_1 \) is the mass of the primary white dwarf in units of the solar mass, and \( \alpha_C \) and \( \alpha_H \) are the viscosity parameters in the cold disk and the hot disk, respectively. It is generally thought that the viscosity in a hot ionized plasma is given by the magneto-rotational instability (MRI, see Balbus, Hawley 1998) and \( \alpha_H \sim 0.1 \). On the other hand, the viscosity in the cold state is still poorly known but in order to reproduce the dwarf nova outburst by the thermal-viscous instability model, we need that \( \alpha_C \) is much smaller than \( \alpha_H \). Although no definite mechanism to produce the viscosity in the cold state is yet known, the only mechanism so far suggested is that of the tidal dissipation (Menou 2000).

If the tidal dissipation is the only source for \( \alpha_C \), we expect that \( \alpha_C \) may be a function of the binary mass ratio, \( q \), and the radial distance, \( r \). A possible mass-ratio dependence of \( \alpha_C \) is used to argue for a low value of \( \alpha_C \) in WZ Sge stars to explain their long recurrence time. Since the tidal torque is strongly increasing with the radial coordinate and it increases rapidly as the disk edge approaches the tidal truncation radius, we expect that...
\( \alpha_C \) is an increasing function toward the tidal truncation radius. The size of the loop of the S-shaped thermal equilibrium curve depends on the ratio of \( \Sigma_{\text{max}} \) to \( \Sigma_{\text{min}} \); the larger this ratio, the larger is the S-shaped loop. If \( \alpha_C \) and \( \alpha_H \) are assumed to be constant and have a definite ratio, the S-shaped curve is self-similar with respect to the radial coordinate, \( r \). This is the standard assumption most researchers have adopted.

On the other hand, if \( \alpha_C \) has a radial dependence while \( \alpha_H \) remains constant with respect to the radius, the S-shaped loop becomes smaller as we approach near to the tidal truncation radius. A similar effect (a reduction in the ratio of \( \Sigma_{\text{max}} \) to \( \Sigma_{\text{min}} \)) may be produced in the vicinity of the mass addition region due to the associated energy input (see, figure 2 of Lin et al. 1985). As a matter of fact, Ichikawa et al. (1993) have made light curve simulations of SU UMa stars by using an \( r \)-dependent viscosity parameter, \( \alpha_C \). The main purpose of their simulations was to reproduce the “outside-in”-type normal outbursts and its \( r \)-dependence is rather mild, i.e., \( \alpha_C \propto r^{0.3} \). In our scenario, we require a steep increase in \( \alpha_C \), when the radius approaches to the tidal truncation radius as compared with that used by Ichikawa et al. (1993).

As discussed in Paper I, the normal outbursts in V1504 Cyg are thought to be of “outside-in” in which the heating transition first starts from the outer edge of the disk, propagating inward. We now come to the main point of our scenario. In the Type S supercycle in V1504 Cyg, as the disk expands outward with a successive normal outburst in a supercycle and approaches to the tidal truncation radius, the loop of S-shaped curve at the disk edge becomes smaller. This may produce much frequent outbursts with a smaller outburst amplitude in the later phase of a supercycle. Thus it is essential that the disk radius increases with advance of supercycle phase in this scenario.

Let us now discuss about mini-outbursts which occurred in the later half of Type S supercycles of V1504 Cyg, that is, two mini-outbursts in supercycle (SC) No. 8, three in SC No. 9, two in SC No. 11, and one in SC No.6 which was not a typical Type S supercycle but in which the negative superhumps disappeared in its later stage. Here we propose a possible model which may explain mini-outbursts along this scenario. In the standard disk-instability model we assume the so-called S-shaped thermal equilibrium curve for the thermal limit cycle instability. However, a much complicated thermal equilibrium curve was obtained in the 1980s by various authors (particularly, Mineshige, Osaki 1985, Meyer-Hofmeister 1987, Cannizzo, Wheeler 1984 among others) and it looked more like the greek letter \( \xi \) rather than the Roman letter S as shown schematically in figure 9 (see figure 4b of Osaki 1996). In this type of thermal equilibrium curve, there are three stable branches; besides the ordinary hot branch and cold branch an intermediate branch which is called the warm branch. The ordinary thermal instability goes around the big loop shown in the figure. Another limit cycle with a smaller loop may occur if the cold branch is somehow elevated by an extra heating possibly by the tidal dissipation.

Let us explain our model for the mini-outburst by using the thermal equilibrium curve shown in figure 9. We understand that the ordinary normal outburst is explained by the thermal limit-cycle instability in which the disk goes around the large loop shown in this figure. However, if the disk’s outer edge approaches to the tidal truncation radius, an increased tidal dissipation may modify the cold branch of the equilibrium curve as shown by the dashed line of figure 9. It will then give rise to a premature ignition of thermal instability. In such a situation the disk may not jump to the hot state but it may be stopped at the intermediate warm branch. The resultant outburst may be of a small scale because the disk goes around the small loop (the small thermal-limit cycle) as shown in figure 9. This is our explanation for mini-outbursts observed in V1504 Cyg. In fact, the Kepler light curve of V1504 Cyg shows that the quiescence level was raised just prior to mini-outbursts, indicating some additional heating in quiescent disk. As discussed above, besides the tidal dissipation, heating by collision of gas stream with the disk edge may also contribute to an additional heating in the type S supercycle.

Observations of V1504 Cyg showed that mini-outbursts occurred rather randomly. This suggests that the ex-
tra heating in quiescent disk discussed above may occur rather randomly. This suggests that the disk sometime goes around the large loop and the other time goes around the small loop. However, we do not know any mechanisms responsible for such random behavior. Furthermore the triggering normal outburst was found to have a rather large amplitude. These two points remain unsolved by this model. We leave them as a future problem to be solved.

3. Reply to Smak’s Criticism to Our Paper

Recently, Smak (2013) published a paper in Acta Astronomica in which he challenged our main conclusions of our Paper I. He concluded in his paper that our main conclusions of Paper I were incorrect. Since his challenge is clearly serious for us, we present our detailed accounts to all of his criticisms in this section, by examining his criticisms one after another.

3.1. Amplitudes of Negative Superhumps

In his subsection 2.2, Smak (2013) has argued that our results on the disk radius variation were inconsistent with observed amplitudes of negative superhumps. We think that amplitudes of negative superhumps have nothing to do with the disk radius variation as discussed below.

Smak (2013) has tried to compare the amplitude of negative superhump during a superoutburst with that of quiescence. To do so, he assumed the light source of the negative superhump was solely due to the gas stream in a tilted disk both in quiescence and in a superoutburst. However we do not agree with him about the origin of light source of negative superhumps. We rather think that the light source of the negative superhump is most likely different between superoutburst and quiescence as evidenced from different wave forms between these two phases as shown in figures 7 and 8 in our Paper I. During superoutbursts the disk component will contribute to the light variation of negative superhump besides the gas stream component and we have discussed a possible origin of the wave form of the negative superhumps during a superoutburst in the Appendix of our Paper I. In fact, we showed in figure 5 of Paper II the variation in amplitude (in flux unit) of negative superhump together with the light curve and its frequency variation in a complete supercycle No. 5 of V1504 Cyg, which we studied in Paper I. As seen in the figure, the amplitude of negative superhump exhibited a characteristic variation. Generally speaking, its amplitude (in flux unit) increases when an outburst occurs, which we interpreted as an evidence of contribution of the disk component to the negative superhump light source.

Furthermore even if we accept Smak’s assumption that the light source of the negative superhump were solely due to the gas stream in a tilted disk both in quiescence and in superoutburst, we reach an opposite conclusion to Smak (2013) about the negative superhump amplitudes as discussed below. Smak (2013) has argued that the negative superhump amplitude must be smaller when the disk is larger and vice versa. However, the nSH amplitude must be determined by difference in the depth of potential well between the deepest arrival point of the gas stream and the shallowest point in a tilted disk. What Smak (2013) referred to was the depth of potential well at the outer disk edge which corresponds to the shallowest point in the above discussion and he did not referred to the deepest arrival point. If the deepest arrival point is assumed to be the same in quiescence and in superoutburst in a tilted disk, the nSH amplitude must be larger when the disk’s outer edge is larger, a conclusion exactly opposite to his.

We think that Smak’s criticism on amplitudes of negative superhump is irrelevant because light source of the negative superhump is most likely different between quiescence and a superoutburst and furthermore because his argument on the depth of potential well was incorrect.

3.2. Frequency Variations of Negative Superhumps during Normal Outburst Cycle

In his subsection 2.3, Smak (2013) criticized our results on frequency variations of negative superhumps during normal outburst cycles shown in our figure 5 in paper I. He stated that in our figure 5 the minima of $\nu_{nSH}$ occurred $\sim 3 \text{ d}$ before the initial rise to outburst and the following increase of $\nu_{nSH}$ till its maximum lasted for $\sim 3 \text{ d}$ while the model calculations show that expansion of the disk occurs nearly simultaneously with the rising light and lasts for only $\sim 0.5 \text{ d}$ (which corresponds to a viscous time scale).

Since this was the most important criticism, we have already presented our detailed account on this criticism in subsection 2.2 of our Paper II. We do not repeat these discussions here but rather we would like to ask the readers to consult on our Paper II. Here we summarize the main points of our discussion of Paper II below.

1. We made a mistake of 2 d in the time axis in figure 5 of the first version of astro-ph, arXiv:1212.1516v1 (a preprint of Paper I) submitted on 2012 December 6, and this error was corrected in the second version submitted on 2013 January 6, and in the published version of PASJ. We realized this error by Smak’s scrutiny of our Paper I in its first version.

2. The reason why the frequency jump in $\nu_{nSH}$, in figure 5 of our Paper I, took 4 d [Smak (2013) said $\sim 3 \text{ d}$] instead of a much shorter time scale of $\sim 0.5 \text{ d}$ is due to a simple artifact in our calculations of local frequency with the window width of 4 d.

3. By taking into account these two points, results shown in figure 5 of our Paper I is consistent with the model calculations quoted by Smak (2013).

3.3. Negative Superhumps during Superoutbursts

In his subsection 2.4, Smak (2013) criticized our results on the decrease in frequency of negative superhumps, $\nu_{nSH}$, during the main part of superoutbursts indicating the decrease in the disk radius during the superoutbursts.
On the other hand, Smak (2013) stated that the disk radius of Z Cha determined by himself from eclipses during superoutbursts remains constant throughout superoutburst.

We believe that this is one of the most crucial issues to be tested in observations. That is, the TTI model predicts the decrease in the disk radius during superoutburst while Smak's enhanced mass-transfer (EMT) model expects a more or less constant disk radius during superoutburst.

We note, however, some shortcomings in Smak (2013). These observations of Z Cha were performed only 1.5–6.5 d after the onset of (different) superoutbursts, and did not cover the full duration (10–12 d) of the plateau phase of superoutbursts in this object. The resultant error in his estimate of the decrease in the disk radius was large [equation (2) in Smak (2013)] due to the short baseline, and the decrease in the disk radius in our Paper I data was only 1.6σ different from his estimate. It means that his data were not statistically significant enough to test our results. Furthermore, Smak (2013) used the times of ingress and egress of the hot spot to estimate the disk radius. Smak himself confessed that ingress and egress times do not always give consistent results, and may not be directly used to estimate the disk radius (Smak 2012).

Eclipsing SU UMa-type dwarf novae, however, are a vital tool in probing the variation of the disk radius in outburst, and we show some clues from our own observations. We used the 2009 observation of the eclipsing SU UMa-type dwarf nova IY UMa (Kato et al. 2010; Kato et al. 2012) and estimated the disk radius by profile fitting. We assumed that the disk is axisymmetric and ignored the thickness. We modeled the surface brightness of the disk as a form of $\propto r^{-n}$, and obtained the model light curve. By using Markov-chain Monte Carlo (MCMC) method, we determined the disk radius and $n$ value. In order to reduce the effect of the strong beat phenomenon between the superhump and orbital periods, we averaged profiles for 4 d, where 2 d is the beat period. We used $q=0.125$ and an inclination of $86^\circ$ (Steeghs et al. 2003). The result is shown in figure 10. Although our treatment is rough and based on an assumption that the mean shape of the disk is axisymmetric, the present result of eclipse analysis supports the decrease in the disk radius during superoutburst. We note that the total duration of the eclipse is basically determined by the radius of (the luminous part of) the disk, and the uncertainty arising from the simplified model is relatively small. This result may suggest that the disk radius did not decrease strongly during the initial stage of the superoutburst, when Smak (2013) observed Z Cha. This issue has to be settled in observations in future.

As stated clearly by Smak (2013), the frequency variation of negative superhumps during the plateau stage of superoutbursts in V1504 Cyg, shown in figure 5 of our Paper I, indicates the decrease in the disk radius. Since the disk radius increases secularly with a succession of normal outbursts from the end of the preceding superoutburst till the start of the next superoutburst as demonstrated in figure 5 of our Paper I, the disk radius must decrease during superoutburst from the standpoint of continuity argument and the observed variations in the nSH frequency mentioned above is consistent with this prediction.

We have two more pieces of evidence for the decrease in disk-radius during superoutburst other than the variation in nSH frequency.

The first evidence concerns the frequency variations of the Positive Superhumps during superoutbursts. In subsection 2.6 of our paper II, we studied the frequency variations of the positive superhumps in V1504 Cyg and V344 Lyr during superoutbursts. As shown in figures 9–12 of our Paper II, the positive superhump periods, $P_{\text{pSH}}$, (or the apsidal precession rates of eccentric disks) in V1504 Cyg and V344 Lyr show a rapid decrease from the highest $P_{\text{pSH}}$ (or the highest apsidal precession rates) at the start of a superoutburst, to a less rapid decrease during the plateau stage of the superoutburst. In Paper II we interpreted the initial rapid decrease is due to propagation of eccentricity wave from the 3:1 resonance region to the inner region of the disk. The most interesting point in these Kepler observations both in V1504 Cyg and V344 Lyr is a slower decrease in the positive superhump period during the plateau stage of the superoutburst. As discussed in subsection 2.6 of Paper II, this is understood to be due to a decrease in the disk radius during the superoutburst.

The second evidence concerns about luminosity level between the precursor maximum and the end of plateau stage of superoutburst when the rapid decline from superoutburst begins. The Kepler light curves of V1504 Cyg and V344 Lyr show that the former (the precursor maximum) is brighter typically by about 0.5 mag than the latter (the end of the plateau stage). According to the
disk instability theory, these two stages are exactly when the cooling transition just starts at the outer edge of the disk. The critical effective temperature of the disk’s outer edge when the cooling transition starts is well specified in the thermal instability theory to be around 7500K. Since the surface brightness in these two stages are more or less the same, difference in brightness by about 0.5 mag between these two stages is explained only by difference in the surface area of the disk. This indicates that the disk radius at the end of superoutburst is about 0.8 of that at the start of superoutburst. We thus reach a conclusion that the disk must contract during superoutburst. This is a very simple argument free from any sophisticated theory and we can easily confirm this just by looking at the Kepler light curves of these stars by eye.

Here we must add the potential caveat of this argument. In the above discussion, we assumed implicitly a steady hot disk both at the precursor maximum and at the end of the plateau stage. The assumption of the steady disk is a good approximation for the end of the plateau stage but it is not for the precursor maximum. At the precursor maximum, the disk is non-steady as the heating front may still be propagating inward in the disk. This affects the above discussion in two different ways; firstly the part of the disk may be still in cold state (i.e., the disk is not fully hot) and the hot part may not be a circular disk but rather a circular ring. The second effect concerns the existence of heating front accompanied with narrow spikes in the surface density and the temperature distribution within the disk may differ from that of the steady disk.

### 3.4. Comparison with Other Systems

In his subsection 2.5, Smak (2013) examined negative superhumps and their variable periods in several other dwarf novae in the literature. He concluded that (1) the decreasing nSH period during supercycles is a common phenomenon among dwarf novae with superoutbursts, while (2) the nSH period variations during their normal outburst cycles occur only in some of them. In particular, nSH period variations observed in V1504 Cyg cannot be considered as representative for all such systems.

Here we study these two points raised by Smak, by examining individual systems in more detail. The systems quoted by him were V503 Cyg, BK Lyn, V344 Lyr, and ER UMa. It has turned out all five systems showing negative superhumps are those SU UMa stars with rather short supercycles. Since the behaviors of the negative-SH period variations in these systems have a strong correlation with the supercycle properties, we first summarize their properties in table 3. The first column (1) of the table is star’s name, (2) $T_S$: the supercycle length, (3) $T_S^\ast$: the supercycle length excluding superoutburst, that is, the interval from the end of a superoutburst to the next superoutburst, (4) $T_N$: the normal-outburst interval or the cycle length of normal outbursts, and (5) $T_Q$: the quiescence interval. All quantities listed in table 3 refer to the Type L supercycle. We make this remark because the normal-outburst intervals are very different between the Type L supercycle and the Type S supercycle as discussed in Paper I.

| name               | $T_S^\ast$ | $T_S^\ast$ | $T_N$ | $T_Q$ |
|--------------------|------------|------------|-------|-------|
| V1504 Cyg          | 110        | 98         | 16    | 12    |
| V344 Lyr           | 102        | 85         | 10    | 6     |
| V503 Cyg           | 89         | 70         | 30    | 25    |
| ER UMa             | 50         | 17         | 7     | 2     |
| BK Lyn             | 45         | 20         | 5     | 0     |

$T_S^\ast$: the supercycle length
$T_S^\ast$: the supercycle length excluding superoutburst
$T_N$: the interval of two consecutive normal outbursts
$T_Q$: the quiescence interval

Let us now examine nSH frequency variations for individual stars by taking into account their supercycle properties listed in table 3. To do so, we use more recent data than those of Smak (2013), when they are available. As discussed by Smak (2013), we discuss two phenomena, separately: (1) secular variation of nSH frequency with supercycle phase, and (2) nSH frequency (or period) variations during normal outburst cycles.

As for the nSH frequency variations during normal outburst cycles, we expect that the frequency difference between an outburst and quiescence, $\delta \nu = \nu_{\text{max}} - \nu_{\text{min}}$, will be larger if the normal outburst interval, $T_N$, (or $T_Q$) is longer because mass added by gas stream having lower specific angular momentum is more accumulated in the disk during quiescence if the quiescence interval is longer. This can be confirmed in figure 5 of Paper I for supercycle No. 5 of V1504 Cyg where five normal outbursts occurred. The longest quiescence interval occurred between the third and the fourth normal outbursts and the jump in the nSH frequency from minimum to maximum was the largest for the fourth normal outburst. If we examine table 3, we find that the cycle length of normal outbursts of ER UMa and BK Lyn are as short as 7 d and 5 d, respectively.

The time variations in nSH frequency (or period) have been studied by two different methods: (1) variations of local frequency of nSH with time, and (2) study of the $O-C$ diagram for times of superhump maxima. The first method is more suitable for frequency variations on a short time-scale while the second method is more suitable for long time-scale frequency variations because the $O-C$ diagram represents an integral form of the frequency variation. We do not expect any large variation in the $O-C$ diagram during an outburst cycle as short as several days seen in ER UMa and BK Lyn.

Let us examine individual stars more closely. We start from V344 Lyr. In our Paper II we have studied frequency variations of negative superhumps by using the Kepler data of V344 Lyr, and we showed our results in Figure 9 of our Paper II for supercycle No. 7 and in Figure 10 for supercycle No. 2 of V344 Lyr. From these two figures we find that V344 Lyr exhibited basically the same pattern of frequency variation in the nSH as that of V1504 Cyg, that is, (1) the nSH frequency increases when an
outburst occurs while it decreases during quiescence, and
(2) the cycle averaged nSH frequency increases secularly
with advance of supercycle phase. However, the ampli-
tude of frequency variation within an outburst cycle in
V344 Lyr is smaller than that of V1504 Cyg. This differ-
tude of frequency variation within an outburst cycle in
with advance of supercycle phase. However, the ampli-

(2) the cycle averaged nSH frequency increases secularly
outburst occurs while it decreases during quiescence, and
these two stars. V1504 Cyg has a longer quiescence interval than V344 Lyr and it exhibited
more cut variation in nSH frequency than V344 Lyr. We also note that our results are consistent with
those of Wood et al. (2011).

As for V503 Cyg, this system has been in the Type S
supercycles in recent years (Kato et al. 2013; Pavlenko
et al. 2012; Kato et al. 2014) and no new data for the neg-
avertheless, it might have been difficult to find its secular
variation in Harvey et al. (1995) because of a shorter su-
percycle length in V503 Cyg. We hope that this point
may be clarified by future observations sometime when
this star enters in the Type L supercycle.

BK Lyn was a novalike variable with a small brightness
variation of about 0.2 mag until early 2000 (see, Skillman,
Patterson 1993; Kato et al. 2013; Patterson et al. 2013).
However, it has begun to show dwarf nova eruptions as
its mass-transfer rate apparently decreased and it is now
classified as an ER UMa star, a subclass of SU UMa
stars with a short supercycle length less than 50 d and ex-
tremely short normal outburst cycle of around 5 d (Kato,
Kunjaya 1995; Robertson et al. 1995). BK Lyn showed
short supercycles of around 45 d. It exhibited both pos-
itive and negative superhumps. The O − C diagram for
negative SH was shown in figure 26 of Kato et al. (2013)
which exhibited a concave upward pattern, indicating that
nSH period decreased (i.e., the frequency increased) be-
tween two successive superoutbursts, as already noted by
Smak (2013). This means that the disk radius increases
secularly with advance of a supercycle phase between two
superoutbursts while it decreases during a superoutburst.
However, no clear evidence is seen in the same diagram
about any decrease in frequency in normal outburst cycles.
This is most likely due to a short interval of two consecu-
tive normal outbursts in BK Lyn as compared with that
of V1504 Cyg.

By using a combined set of the data used in Kato et al.
(2013) and the additional data supplied by E. de Miguel
(private communication 2013), we have examined local
frequency variations of nSH in BK Lyn by using the PDM
method for period determination. Figure 11 illustrates
our results in which a quantity $e^*$ representing the nodal
precession rate over the orbital frequency is shown to-
gether with the light curve. It clearly exhibits that the
nSH frequency increases when an outburst occurs and it
decreases when the star fades in brightness. A Lasso
two-dimensional spectrum of this star will be presented in
Kato et al. (2014).

ER UMa basically exhibits the same pattern as that of
BK Lyn (Ohshima et al. 2012; T. Ohshima et al. in prepa-
ration). Thus we conclude that the disk radii in
BK Lyn and ER UMa secularly increase with advance of
supercycle phase from the end of a superoutburst to the
next while they decrease during superoutbursts. In fact,
de Miguel et al. (2012) demonstrated that the O − C di-
agram for times of maxima of nSH waves of ER UMa
showed a cyclic variation with the same cycle length as
the supercycle itself. If this variation of nSH frequency is
interpreted as the disk radius variation, it indicates that
the disk radius decreases during superoutbursts and it in-
creases secularly during inter-superooutburst, completely
consistent with the prediction of the thermal-tidal insta-
bility model.

For all SU UMa stars, V344 Lyr, BK Lyn, and ER
UMa, observed variations in the nSH frequency exhib-
ted basically the same pattern during supercycles: (1)
the secular increase in nSH frequency between the two
successive superoutbursts. (2) On the other hand, the
decrease in nSH frequency during normal outburst cycles
depends very much on the cycle lengths of normal out-
bursts in a sense that the longer the outburst interval, the
larger the frequency decrease. In particular, two stars, ER
UMa and BK Lyn, did not show any clear variation dur-
ing outburst cycles in the O − C diagrams because these
two stars have extremely short outburst cycles. However
our new analysis for variations of nSH frequency for BK
Lyn by using the PDM method for the period determina-
tion showed clear up and down in nSH frequency during
outburst cycles, which is consistent with that of V1504 Cyg.

We conclude that observed nSH frequency variations
during supercycles in these 5 SU UMa stars are all con-
sistent with the prediction of the thermal-tidal instability
model. Quite recently, yet another example showing the
same pattern (KIC 7524178 = KIS J192254.92+430905.4)
has been identified (Kato, Osaki 2013), strengthening this
conclusion.

3.5. Negative Superhumps and Their Variations

In his subsection 2.6, Smak (2013) discussed the vari-
ation in negative superhump periods and criticized our
Paper I by arguing that many different causes are respon-
sible for the variation in negative superhump periods while
Osaki, Kato (2013a) limited their discussion only to the
disk radius variation. Here we examine his criticism.

Concerning about many different causes, his discussions
consisted of two different points.
1. The first point concerns the nodal precession period, $P_{\text{prec}}$. The retrograde precession period of a tilted disk depends not only on the disk radius but also on the distribution of its surface density $\Sigma(r)$ (cf., Larwood 1998, Montgomery 2009). Thus the variation in the surface density distribution has to be taken into account when variation of the negative superhump period is discussed.

2. The second point concerns the observed nSH period. He argued that the observed nSH period is determined by the interval of time between two successive maxima resulting from the collision of the stream with the surface of the tilted disk and thus it depends also on the flight time of the stream elements from $L_1$ to the effective point of collision. He argued that the effects of the flight time have to be taken into account for the nSH period variation.

We discuss below these two points separately.

### 3.5.1. The effects of mass distribution in the disk on the nodal precession rate in a tilted disk

Concerning about the first point raised by Smak (2013), we completely agree with him and here we discuss it.

We have already discussed the effects of different surface-density [$\Sigma(r)$] distribution on the nodal precession rate of a tilted disk in subsection 2.5 and Appendix of Paper II. There we introduced the precession rate of a tilted disk over the orbital frequency by $\epsilon^* = \nu_{nPR}/\nu_{\text{orb}}$, where $\nu_{nPR}$ and $\nu_{\text{orb}}$ are the nodal precession frequency and the orbital frequency, respectively. This quantity is expressed (see, Larwood 1998) by

$$\epsilon^* = \frac{\nu_{nPR}}{\nu_{\text{orb}}} = 1 - \frac{\nu_{nSH}}{\nu_{\text{orb}}} = -\frac{3}{7} \left(1 + \frac{q}{q + 1}\right)\left(\frac{R_d}{A}\right)^{3/2} \cos\theta. \quad (3)$$

Here the negative sign of $\epsilon^*$ signifies retrograde precession. Since this expression is derived under an assumption of a certain mass distribution in the disk, we introduced in Paper II a correction factor $\eta$ to this expression for allowing a different mass distribution;

$$\frac{\nu_{nPR}}{\nu_{\text{orb}}} = -\frac{3}{7} \left(1 + \frac{q}{q + 1}\right)\left(\frac{R_d}{A}\right)^{3/2} \cos\theta. \quad (4)$$

The correction factor $\eta$ was calculated for several mass distributions in the Appendix of Paper II and it was listed in Table 3 in that paper.

Let us now discuss the variation of nSH frequency in supercycle No. 5 of V1504 Cyg shown in figure 5 of Paper I. Since this figure was shown both in Paper I and Paper II, we do not here reproduce it any more. Instead we summarize its variation in what follows: the nSH frequency jumps from local minimum to local maximum every time when a normal outburst occurs and it decreases monotonically during quiescence between two consecutive normal outbursts. Furthermore the peak frequency just after a normal outburst increases monotonically with a succession of normal outbursts from the end of the previous superoutburst until the next superoutburst.
The most important discovery of Paper I is the secular variation of the peak frequency of nSH during a supercycle of V1504 Cyg. As far as the peak frequency of the nSH is concerned, the correction factor $\eta$ takes a same value and it is given by $\eta \simeq 1.22$ because the surface density distribution just after a normal outburst is self-similar and it is approximately given by $\Sigma \propto r^{-1}$. Thus we can conclude that secular variation of the peak frequency of nSH during a supercycle reflects the disk-radius variation because $\eta$ remains constant and thus the disk radius just after an end of normal outburst secularly increases monotonously with advance of supercycle phase, which is in good agreement with a prediction of the TTI model.

As for the variation in nSH frequency during two consecutive outbursts, we admit some uncertainty in the surface density distribution and so in $\eta$ because we do not know either how mass is supplied to the different part of the disk in a tilted disk nor how the viscous diffusion modifies the surface density distribution in quiescence. Nevertheless we think that the observed variation of nSH frequency may reflect more or less the disk-radius variation even in quiescence. The reason why we think so is two-fold. Firstly, if we interpret the frequency variation of nSH observed in V1504 Cyg solely due to variation in disk radius, it is reminiscent of the disk-radius variation in an outburst cycle of U Gem discovered by Smak (1984), one of the most important discoveries in the history of the disk instability theory. Secondly, the surface density distribution in quiescence has a certain limitation because it can not exceed the local critical surface density $\Sigma_{\text{max}}(r) \propto r^{-1}$ anywhere in a cold disk. Because of this constraint, it is difficult to consider a situation in which the correction factor $\eta$ is very different from that of the start of quiescence. We may therefore conclude that the variation in nSH frequency during a supercycle of V1504 Cyg reflects more or less variation in the disk radius even if the effects of variation in the surface-density distribution are taken into account.

3.5.2. The effect of flight time of gas stream

On the other hand, we believe that the second point raised by Smak (2013) is clearly a problem of different (higher) level of approximations from the first point because the second point goes deep into a particular mechanism for the origin of light source of the negative SH. As discussed in subsection 3.1, we also consider other possibility for light source of the nSH different from the gas stream component. When we study variation in characteristic period (or frequency) of an astrophysical object, the first step will be to examine variation of the underlying clock, in our case the precession rate of the tilted disk. Going deep into the origin of light source will be the next higher step. We believe that our discussion on variation in the nSH frequency is still in the first step.

Furthermore, even if we accept Smak’s view about the origin of negative superhump light source as due to gas stream, we think that the effect of the flight time discussed by Smak (2013) will be negligible in the frequency variation of the negative superhumps as shown below.

If we assume that the light maximum of the negative superhump coincides with the epoch when the gas stream hits the innermost part of a tilted disk. The maximum difference of the flight time of the gas stream then will not exceed the travelling time of the infalling gas across the disk radius. Assuming a maximum disk radius of 0.46 $A$, we can evaluate the time the trajectory falling from the L1 point within this radius to be $0.132/P_{\text{orb}}$, for $q = 0.22$. The time of 0.066 $P_{\text{orb}}$ elapses when the trajectory reaches the periastron since it first enters the radius of 0.46 $A$. We can use the same upper limit of 0.132 $P_{\text{orb}}$ as the variation of the flight time. This value (0.009 d in V1504 Cyg) can be directly compared to the $O-C$ diagram of the negative superhumps (figure 5 in Osaki, Kato 2013b) since this flight time effect corresponds to the global $O-C$ variation. The real $O-C$ variation is much larger, even restricting to the variation in relation to the normal outburst, indicating that Smak’s second effect cannot explain the period variation of negative superhumps.

3.6. Evidence for the Enhanced Mass-transfer Model

In his section 3, Smak (2013) stated that observational evidence accumulated showing prominent hot spots during superoutbursts, in particular in the case of Z Cha (Smak 2008). However, observational evidence for the enhanced mass transfer quoted by Smak (2008) is very much dependent on an assumption used there, that is, an assumption that light source during superoutbursts consisted of the axi-symmetric disk component and of the hot spot. However, quite a different conclusion is obtained by starting from a different assumption.

Osaki, Meyer (2003) questioned the interpretation of evidence for enhanced hot spot and they argued that the so-called orbital hump (or “hot spot”) during superoutbursts simply results from the non-axisymmetric tidal dissipation pattern in eccentric disk and the observed eclipses are not of mass-transfer hot spot but rather of the superhump light source itself. Based on the smoothed-particle hydrodynamics (SPH) simulations of tidally unstable disks, Truss (2005) showed that the brightening can be attributed to tidal heating in eccentric disk, with no need for an increase in mass-transfer rate. We believe that accretion disks during superoutbursts are far from axi-symmetry and the non-axisymmetric tidal dissipation pattern must be taken into account even in superhump phases far away from the superhump light maximum.

This indicates that by starting from a different assumption, one gets a quite different conclusion. In such a situation a real problem will be which assumptions are more appropriate in such a case. We leave the judgment to readers and to future research.

The Kepler light curves of V1504 Cyg and V344 Lyr have demonstrated that the rise to superoutburst maximum and the growth of superhump occurred almost simultaneously and both of them reached their maxima at the same time. The enhanced mass-transfer model seems to contradict with this observation because the enhanced mass transfer causes a contraction of the disk radius away from the 3:1 resonance by adding extra mass with low specific angular momentum, and it kills the tidal instability...
and the superhump as already discussed by Ichikawa et al. (1993), Lubow (1994), and Truss (2005).

4. Summary

(1) We made a supplemental study of the superoutbursts and superhumps of V1504 Cyg and V344 Lyr by using the recently released Kepler data. We have basically confirmed the results given in Paper I and Paper II. In addition to them, we have found that the supercycle lengths of the type L supercycles are shorter than those of the Type S supercycles in V1504 Cyg.

(2) The superoutburst No. 8 of V1504 Cyg was preceded by a precursor normal outburst which was well separated from the main superoutburst. The ordinary superhump first appeared during the descending branch of the normal outburst and it continued to quiescence and it began to grow in amplitude with the growth of the system brightness and these two reached simultaneously their maxima (the main superoutburst maximum). This superoutburst is understood as an extreme case of the precursor-main type in which the precursor and the main were separated by the quiescence. This demonstrates that the superoutburst is triggered by the superhump (i.e., the tidal instability), supporting the thermal-tidal instability model. A similar phenomenon was observed in V344 Lyr.

(3) In one of the normal outbursts just prior to the next superoutburst of V1504 Cyg, a signal of “ordinary superhump” nature appeared in its descending branch but it disappeared before quiescence and thus failed to trigger a superoutburst. This phenomenon called the failed superhump is exactly the same as that of “aborted superhump” discussed by Osaki, Meyer (2003). In V344 Lyr, on the other hand, superoutbursts were found to be preceded always by one to three normal outbursts in which impulsive negative superhumps appeared during the descending branch but they disappeared before quiescence.

(4) We discussed mini-outbursts which occurred exclusively in the later half of the Type S supercycles of V1504 Cyg. We proposed a possible scenario wherein the outburst intervals and the outburst amplitudes decrease in the later half of the Type S supercycles in V1504 Cyg.

(5) Smak (2013) criticized our paper I (Osaki, Kato 2013a) in which he argued that our conclusions of Paper I were incorrect. We presented our replies to almost of all his criticisms by offering clear explanations to his criticisms. In particular, we presented more evidence for the frequency variation in the negative superhump in other SU UMa stars.

(6) The study of Kepler light curves of two SU UMa stars, V1504 Cyg and V344 Lyr presented in this paper and our Papers I & II demonstrates that almost all of the observational evidence supports the TTI model but it seems against the EMT model. We conclude that the TTI model is the only viable model for the superoutbursts and superhumps in SU UMa stars, in particular in V1504 Cyg and V344 Lyr.

Note added in proof (2013 November 9)

The Kepler data for Q13 of V1504 Cyg and V344 Lyr were released to public on October 22, 2013. By taking into account these new Kepler data, we replaced the original figures 1–4 and tables 1–2 to new ones.

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