Coupled Strip Lines with Highly Unbalanced Electromagnetic Coupling and their Use

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Abstract — The coupled lines being considered are with horizontally and vertically positioned strip conductors. Constructions like this help gain vastly different phase velocities of synphase and antiphase wave types. The task of calculating the electric field in the per unit parameters’ cross section was being solved using the grid method. The capacities per unit were being defined as a sum of the partial volumes in the chosen subareas of the strip structure, in which the accumulated energy of the electric field was calculated. The inverse problem of finding the relative permittivity using the set phase velocities relation of synphase and antiphase waves was solved. There are given synthesis examples of the constructions using the set phase velocities relation. Simulation and experimental research of several coupled lines sections with different relation of phase velocities was complete. It was shown that having a substantial difference in the phase velocities, resonant fluctuations appear in the sections with scheme in which one of the unloaded on ports 2 and 4 lines is under the floating induced potential by the second line. That line in such scheme is included between ports 1 and 3 for passage. Herewith depending on the phase velocities relation, pass-bands of differing width and multiplicity are observed.

Index Terms — coupled strip lines, horizontal and vertical position of the strips, phase velocities difference of synphase and antiphase waves, finding the permittivity of the substrates.

I. INTRODUCTION

Questions about the theory of coupled lines (CL), which is based around solving summarized telegraphic equations for the identical lines, were reviewed in article [1]. In this work the introduced coefficients are: coefficient of line coupling by the voltage \( K_U = Y_m / Y \), and the current \( K_I = Z_m / Z \), where \( Z \) and \( Y \) are per unit self-impedance and the conductivity of every line accordingly; \( Z_m, Y_m \) - mutual per unit impedance and per unit conductivity of lines. In general, coefficients \( K_U \) and \( K_I \) may not be equal. In this case the inequality points out the unbalance of electromagnetic coupling between the lines. Using \( K_U \) and \( K_I \) we have four values of the spread coefficients of synphase (index “c”) and antiphase (index “π”) waves:

\[
\gamma_{c,\pi} = \sqrt{Z \cdot Y \cdot (1 \pm K_U) \cdot (1 \pm K_I)}.
\]

The inequality \( \gamma_c \neq \gamma_\pi \) was not used in article [1] for the analysis of the transfer matrix and devices based on coupled lines, because it was relied on that in coupled lines with TEM waves at any frequency the waves move at the same velocity.

In work [2] the matrix ABCD was found, \( Z, Y \) - parameters of the identical coupled lines in a heterogeneous dielectric environment. The heterogeneity of the dielectric environment was being taken into account by introducing an electric length inequality of the coupled lines at their synphase (index “c”) and antiphase (index “π”) excitation. Since in [2] were reviewed coupled lines with the identical parameters, for naming synphase and antiphase types of excitation were used the terms “even” (index “c”) and “odd” (index “π”) wave and electric wave lengths accordingly

\[
\theta_c = \frac{\pi}{2} \cdot \frac{f}{f_{0c}},
\]

\[
\theta_\pi = \frac{\pi}{2} \cdot \frac{f}{f_{0\pi}},
\]

where \( f_{0c} \) and \( f_{0\pi} \) are frequencies, at which the coupled lines have a quarter length of a wave at even and odd excitation.

The found transfer matrices in [2], impedances and conductivities matrices \( Z, Y \) were used for analyzing a number of equivalent schemes reviewed earlier Jones and Boljajla [3]. Zysman and Johnson showed [2] that the frequency characteristics of the schemes change vastly due to the inequality of electric lengths \( \theta_c \neq \theta_\pi \).

Further progress in researching of the coupled lines with unbalanced electromagnetic coupling is associated with articles [4-7]. In those works, using insignificantly different ways the task of calculating matrix parameters of coupled
lines in a heterogeneous dielectric environment and different primary parameters of the lines was solved.

The practical use of the CL with a heterogeneous dielectric filling had stimulated the search and creation of new coupled lines construction variants. The goals of creating those new constructions were sometimes diametrically opposing. Thus, for making directional couplers, publication authors aimed to shorten the gap between phase velocities of synphase and antiphase waves \( v_c = \frac{c}{\sqrt{\varepsilon_{eff}}} \) and \( v_\pi = \frac{c}{\sqrt{\varepsilon_{eff}}} \), where \( c \) is the velocity of light, \( \varepsilon_{eff}, \varepsilon_{eff} \) are effective permittivities of synphase and antiphase modes. This allowed us to avoid the interference of waves in coupled lines, which reduces decoupling and is accompanied by resonant phenomena [8 - 10].

The other direction consists of the search for an optimal ratio of the inequality of \( v_c \) and \( v_\pi \) for solving the problems of improving frequency selective characteristics of devices based on CL [11, 12], and creating short impulse equipment defense devices using modal filters [13, 14], and the designing of trans directional couplers [15, 16].

The purpose of this work consists of researching the influence of the phase velocities inequality \( v_c \) and \( v_\pi \) on the frequency characteristics of sections based on coupled strip lines. The analysis of the unusual interference of normal waves in CL using the variation \( v_c/v_\pi \) is complete, and the additional functional possibilities of the schemes based on coupled lines with \( v_c/v_\pi \neq 1 \) are shown.

II. CONSTRUCTION AND THE MODEL OF COUPLED LINES

For researching the dependence of coupled lines parameters from the ratio \( v_c/v_\pi \), we have taken the construction of coupled strip lines (CSL) with the cross section shown on Fig. 1. This construction represents the modification of coupled lines with a vertically positioned substrate (VIP), which were suggested and researched in articles [17, 18]. Having the gap lets us reduce own capacities of horizontally positioned strips and thus increase the characteristic impedance during synphase excitement of conductors. Besides, there is an emerging possibility of varying electromagnetic coupling’s degree of unbalance.

The modification of VIP with a gap in the ground plane was used for building the C-section with an unbalanced electromagnetic coupling in the correctors of group delay [19, 20].

In articles [17, 18, 21] the calculation of primary and secondary parameters CSL with a vertical substrate using various methods with set geometrical sizes and substrate properties was studied.

But in practice it's often needed to solve an inverse problem, that is to find the geometrical sizes and substrate parameters using the set matrices of the primary parameters in the form of \( C \) capacities’ matrix and \( L \) inductances’ matrix. The initial data may also be secondary parameters in the form of characteristic impedances of synphase \( Z_{0c} \) and antiphase \( Z_{0\pi} \) excitement, and coefficients of the capacitive \( k_C \) and inductive \( k_L \) coupling [7], which define the ratio of synphase and antiphase phase velocities in accordance with (4) [22].

\[
\frac{v_c}{v_\pi} = \sqrt{\frac{(1-k_L)(1+k_C)}{(1+k_L)(1-k_C)}}.
\]

Fig. 1. Cross section of coupled strip lines with a vertically positioned substrate and a gap in the ground plane.

The being examined coupled lines with a cross section according to Fig. 1 own the following distinctive features.

1) The constituents of the strips’ per unit capacities, which are executed on horizontally and vertically positioned substrates are to varying extents dependent from sizes \( w_1 \) and \( w_2 \) having the other conditions equalized. This lets us relatively independently change fractional capacities of the strip structure having variations \( w_1 \) and \( w_2 \).

2) Having the gap in the ground plane and an air gap between it and the screen reduces own fractional capacities of mainly \( w_1 \) sized strips.

3) The orthogonal positioning of the dielectric substrates and accordingly of \( w_1 \) and \( w_2 \) sized strips, while changing \( \varepsilon_2 \) and \( \varepsilon_3 \) lets us to varying extents change \( C_{11} \) and \( C_{12} \) accordingly.

The features listed above were studied by us while changing sizes \( w_1 \) and \( w_2 \) relative permittivities \( \varepsilon_2 \) and \( \varepsilon_3 \). Calculation of the primary parameters was done using the grid method [23]. Herewith the goal was to independently from the chosen method (as a tool) justify the algorithm of the sizes’ synthesis using the set primary parameters, which lets us define the being physically implemented construction of the
CSL’s cross section with unequal phase velocities of normal waves.

The calculation of the electric field during synphase and antiphase excitement of coupled strips (Fig. 1) was being done by transitioning from the Laplace’s differential equation to finite-difference approximation and solving using the iterative procedure on a PC [23] with a deviation of $10^{-6}$ with net size 125×95. The results of the calculation in the form of equipotential lines during synphase and antiphase excitement of coupled lines are shown on Fig. 2 and Fig. 3.

![Fig. 2. Equipotential lines of the electric field at synphase excitement of coupled lines’ horizontal and vertical strips.](image)

![Fig. 3. Equipotential lines of the electric field at antiphase excitement of coupled lines’ horizontal and vertical strips.](image)

Herewith we took the following cross section sizes of strip lines $w_1=1.0$ mm, $w_2=3.8$ mm, $h_1=h_2=h_3=0.8$ mm, $h_4=6$ mm, $a=10$ mm, $d=1.0$ mm and relative permittivity $\varepsilon_1=\varepsilon_4=1.0$, $\varepsilon_2=\varepsilon_3=2.68$.

As a result of solving finite-difference equations for potentials $U_{i,j}$, in the grid’s couplings the electric fields’ projections $E_x$ and $E_y$ on the axes $x$ and $y$ were being calculated. Then, for the synphase and antiphase excitement the overall energy was being found (5), which was stored in the electric field [23]

$$WE = \sum_{i=1}^{\max(i)-1} \sum_{j=1}^{\max(j)-1} \Delta WE_{i,j}, \quad (5)$$

where $\Delta WE_{i,j}$ - stored energy in the element $\Delta x \times \Delta y$.

Calculating $\Delta WE_{i,j}$ is done using the expression (6) [23]

$$\Delta WE_{i,j} = \frac{\varepsilon_0 \varepsilon_r}{4} \left[ (U_{i,j} - U_{i+1,j+1})^2 + (U_{i+1,j} - U_{i,j+1})^2 \right]. \quad (6)$$

where $\varepsilon_0$ – absolute dielectric permeability; $\varepsilon_r$ – relative dielectric permeability of the element $\Delta x \times \Delta y$; $U_{i,j}$ - potential in the node $i, j$. $U_{i+1,j}$, $U_{i,j+1}$, $U_{i+1,j+1}$ are potentials in the nodes of element $\Delta x \times \Delta y$.

If calculating full energy $WE$, the inner area of the strip structure divides into subareas $\Omega_m$ with numbers $m=1,...,6$, in which the relative permittivities $\varepsilon_k$ ($k=1,...,4$) are constant. Subareas $\Omega_m$, in which the constituents of energy $WE_1,...,WE_6$ are calculated, are shown on Fig. 4.

![Fig. 4. Division of CSL cross section into subareas.](image)

$WE_1,...,WE_6$ are determined the following way

$$WE_1 = \frac{\varepsilon_0 \varepsilon_1}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h-1} (E_x^i + E_y^j)^2, \quad (7.1)$$

$$WE_2 = \frac{\varepsilon_0 \varepsilon_2}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h+1} (E_x^i + E_y^j)^2, \quad (7.2)$$

$$WE_3 = \frac{\varepsilon_0 \varepsilon_3}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h+1} (E_x^i + E_y^j)^2, \quad (7.3)$$

$$WE_4 = \frac{\varepsilon_0 \varepsilon_4}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h+1} (E_x^i + E_y^j)^2, \quad (7.4)$$

$$WE_5 = \frac{\varepsilon_0 \varepsilon_5}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h+1} (E_x^i + E_y^j)^2, \quad (7.5)$$

$$WE_6 = \frac{\varepsilon_0 \varepsilon_6}{4} \sum_{i=1}^{l-1} \sum_{j=1}^{h+1} (E_x^i + E_y^j)^2, \quad (7.6)$$

where the electric field’s sum of tension vectors’ projection squares on the axes $x$ and $y$ is calculated as follows
\[ E_x^2 + E_y^2 = (U_{i,j} - U_{i+1,j+1})^2 + (U_{i,j+1} - U_{i+1,j+1})^2, \]  
(8)

indexes \( i, j \) are taken from within their alternation range limits for the corresponding subareas. In expressions (7.1) - (7.6) sum limits contain the \( \Omega_1, \ldots, \Omega_6 \) subareas’ coordinates of limits, which are gained after the strip structure’s cross section sampling (see Fig. 4).

Formulas (7.1) - (7.6) are used while calculating the matrices of per unit capacities and coupled lines’ inductances. For this the current-carrying strips potential is set to +1 at synphase excitement and -1 at antiphase excitement. Then the Laplace’s equation in the finite-difference form is solved and the dependence \( U_{i,j} \) is defined. The distribution of \( U_{i,j}^c \) potentials during filling with \( \varepsilon_1, \ldots, \varepsilon_4 \) relative permittivities subareas and \( U_{i,j}^c(1) \) potentials during air filling is calculated for the synphase excitement. The procedure is then repeated for the antiphase excitement of coupled lines, which results in \( U_{i,j}^\pi \) and \( U_{i,j}^\pi(1) \). For each of the mentioned potentials distribution in the cross section of strip structure \( WE_1, \ldots, WE_6 \) are calculated. Let us supply them with the same indexes of correspondence to synphase and antiphase excitement “c” and “\( \pi \)” and condition of dielectric filling “m”.

\( WE_m^c \) – the accumulated energy at synphase mode while being filled with \( \varepsilon_1, \ldots, \varepsilon_4 \) dielectrics,

\( WE_m^c(1) \) – the accumulated energy at synphase mode while being filled with air,

\( WE_m^\pi \) – the accumulated energy at antiphase mode while being filled with \( \varepsilon_1, \ldots, \varepsilon_4 \) dielectrics,

\( WE_m^\pi(1) \) – the accumulated energy at antiphase mode while being filled with air.

The per unit capacity of a strip for the synphase modes having the identical coupled lines’ sizes filled with \( \varepsilon_1, \ldots, \varepsilon_4 \) dielectrics is defined as follows in (9)-(12)

\[ C^c = \sum_{m=1}^{6} WE_m^c, \]  
(9)

While being filled with air the per unit capacity at synphase excitement is found the following way

\[ C^c(1) = \sum_{m=1}^{6} WE_m^c(1). \]  
(10)

The per unit capacities at antiphase excitement are found similarly:

\[ C^\pi = \sum_{m=1}^{6} WE_m^\pi, \]  
(11)

\[ C^\pi(1) = \sum_{m=1}^{6} WE_m^\pi(1). \]  
(12)

Formulas (9) - (12) let us represent per unit capacities \( C^c \), \( C^c(1) \), \( C^\pi \), \( C^\pi(1) \) as the sums of partial capacities since expressions (7.1) - (7.6) were gotten while summing \( \Delta WE_{i,j} \) in the limits of subareas, every of which represents the cross section of a complex capacitor.

Now let’s write down the matrix coefficients of the considered coupled lines’ capacities and inductances (13) - (17)

\[ C_{11} = C_{22} = 0.5 \left[ \sum_{m=1}^{6} WE_m^c + \sum_{m=1}^{6} WE_m^c(1) \right], \]  
(13)

\[ C_{12} = 0.5 \left[ \sum_{m=1}^{6} WE_m^\pi - \sum_{m=1}^{6} WE_m^\pi(1) \right]. \]  
(14)

The coefficients of capacitances’ matrix while being filled with air are written down based on (10) and (12)

\[ C_{11}(1) = C_{22}(1) = 0.5 \left[ \sum_{m=1}^{6} WE_m^c(1) + \sum_{m=1}^{6} WE_m^c(1) \right], \]  
(15)

\[ C_{12}(1) = 0.5 \left[ \sum_{m=1}^{6} WE_m^\pi(1) - \sum_{m=1}^{6} WE_m^\pi(1) \right]. \]  
(16)

Having the matrix of per unit capacities while being filled with air written down, let’s find the matrix of per unit inductances

\[ L = \frac{1}{c^2} \begin{bmatrix} C_{11}(1) & C_{12}(1) \\ C_{12}(1) & C_{22}(1) \end{bmatrix}^{-1}, \]  
(17)

\( c \) – velocity of light.  

Next let’s define the effective permittivities at synphase excitement

\[ \varepsilon_{eff}^c = \sum_{m=1}^{6} \frac{WE_m^c}{\sum_{m=1}^{6} WE_m^c(1)}, \]  
(18)

and for antiphase excitement

\[ \varepsilon_{eff}^\pi = \sum_{m=1}^{6} \frac{WE_m^\pi}{\sum_{m=1}^{6} WE_m^\pi(1)}. \]  
(19)

Let us define \( \frac{WE_m^c}{\varepsilon_m} = \frac{WE_m^\pi}{\varepsilon_m} \). Then let us assume that \( \varepsilon_{eff}^c \) and \( \varepsilon_{eff}^\pi \) are known. Then (18) and (19) allow us to write down a system of equations, from which the relation between permittivities \( \varepsilon_k \) \( (k = 1, \ldots, 4) \), effective permittivities \( \varepsilon_{effc,\pi} \) and cross section sizes is determined, since \( \frac{WE_m^c}{\varepsilon_m}, \frac{WE_m^\pi}{\varepsilon_m} \) depend on the ratio of strip structure cross sizes. If \( \varepsilon_1 = \varepsilon_4 = 1 \) (filling with air), we get a system of two equations for defining \( \varepsilon_2, \varepsilon_3 \) of the horizontally and vertically positioned substrates (Fig. 2):

\[ \varepsilon_2 \overline{WE}_2 + \varepsilon_3 \overline{WE}_3 = \varepsilon_{eff} \sum_{m=1}^{6} WE_m^c(1) \left[ \overline{WE}_1 + \sum_{m=4}^{6} \overline{WE}_m \right], \]  
(20a)

\[ \varepsilon_2 \overline{WE}_2 + \varepsilon_3 \overline{WE}_3 = \varepsilon_{eff} \sum_{m=1}^{6} WE_m^\pi(1) \left[ \overline{WE}_1 + \sum_{m=4}^{6} \overline{WE}_m \right], \]  
(20b)

from which

\[ \begin{bmatrix} \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} \overline{WE}_2 & \overline{WE}_3 \\ \overline{WE}_2 & \overline{WE}_3 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{eff} \sum_{m=1}^{6} WE_m^c(1) - A \\ \varepsilon_{eff} \sum_{m=1}^{6} WE_m^\pi(1) - B \end{bmatrix}, \]  
(21)

where
Expressions (21) and (23) can give us differing results, if set with arbitrary values of \( \varepsilon_{\text{eff}, \pi} \) and \( Z_{0c, \pi} \). It is because the value \( \varepsilon_{\text{eff}, \pi} \) leads to changing of \( W_{m}^{\pi} \) whilst with unchanged cross section sizes. Therefore, in order to simultaneously get into the permissible neighborhood of the values \( \varepsilon_{\text{eff}, \pi} \) it is advisable to study the dependence \( W_{m}^{\pi} \) on the dimensions of the cross section \( w_{1}, w_{2} \). The following initial parameters of the linked strip lines were taken: \( \varepsilon_{1} = \varepsilon_{2} = \varepsilon_{4} = 1, \quad \varepsilon_{3} = 16, \quad h_{1} = 0.8 \text{ mm}, \quad h_{2} = 0.8 \text{ mm}, \quad h_{3} = 1.0 \text{ mm}, \quad h_{4} = 8.0 \text{ mm}, \quad d = 0, \quad a = 10 \text{ mm} \). Strip sizes varied between \( w_{1} = 0.5...5.0 \) at \( w_{2} = 2.3 \text{ mm} \).

On Fig. 5 and Fig. 6 the dependencies of \( \sum_{m=1}^{6} W_{m}^{\pi}(1) \) and \( W_{m}^{\pi}(1) \) from \( w_{2} \) while having an air filling of the considered coupled line’s cross section and the antiphase excitation of the strips are shown. Figures 7 and 8 illustrate the dependencies of \( \sum_{m=1}^{6} W_{m}^{\pi} \) and \( W_{m}^{\pi} \) from \( w_{2} \) while the value of the permittivity is \( \varepsilon_{1} = 16 \), and with an air filling of the remaining considered coupled lines’ cross section subareas and the antiphase excitation of strips.
The analysis of graphs on Fig. 5-8 shows that the dominant contribution to the total capacity of antiphase fluctuations type is the capacity between vertically positioned strips (coefficient $WE_3^\pi$). Functions which approximate dependencies

$$\sum_{m=1}^{6} WE_m^n (1), \quad WE_3^n (1), \quad \sum_{m=1}^{6} WE_m^n, \quad \text{and} \quad WE_3^n \quad \text{from} \quad w_2 \quad \text{are gotten as} \quad (26)-(30)$$

$$\sum_{m=1}^{6} WE_m^n (1) \approx f_0 \left( w_2 \right),$$

$$f_0 \left( w_2 \right) = \begin{bmatrix} 1.443 & 2.852 & -0.161 & 0.014 \end{bmatrix} \cdot [w_2], \quad (26)$$

$$WE_3^n (1) \approx f_1 \left( w_2 \right),$$

$$f_1 \left( w_2 \right) = \begin{bmatrix} -0.102 & 2.018 & -0.005 & 0.0005 \end{bmatrix} \cdot [w_2], \quad (27)$$

$$\sum_{m=1}^{6} WE_m^n \approx f_2 \left( w_2 \right),$$

$$f_2 \left( w_2 \right) = \begin{bmatrix} -0.548 & 33.197 & -0.266 & 0.025 \end{bmatrix} \cdot [w_2], \quad (28)$$

$$WE_3^n \approx f_3 \left( w_2 \right),$$

$$f_3 \left( w_2 \right) = \begin{bmatrix} -2.231 & 32.386 & -0.118 & 0.012 \end{bmatrix} \cdot [w_2], \quad (29)$$

$$[wr] = \begin{bmatrix} 1 & w_2 & w_2^2 & w_2^3 \end{bmatrix}^T. \quad (30)$$

$WE_m^n (1)$

![Graph](image1)

Fig. 5. Dependency of $\sum_{m=1}^{6} WE_m^n (1)$ and $WE_3^n (1)$ from $w_2$ at antiphase excitation and $\epsilon_3 = 1$.

$WE_m^n (1)$

![Graph](image2)

Fig. 6. Dependence of $WE_m^n (m = 2, 4, 5, 6)$ from $w_2$ at antiphase excitation and $\epsilon_3 = 1$.

Addressed to expression (22), let’s find $WE_3^n$ using the $Z_{\Omega \pi}$ being set

$$WE_3^n = \frac{1}{Z_{\Omega \pi} \epsilon_0 C} \left[ \epsilon_0 Z_{\Omega \pi} \left( \sum_{m=1}^{6} WE_m^n + \sum_{m=3}^{6} \right) \right]. \quad (31)$$

Now we can use (29) and find the width of strips $w_2$, positioned on the vertical substrate by the set $Z_{\Omega \pi}$. This comes down to solving the equation

$$32.386 \cdot w_2 - 0.118 \cdot w_2^2 + 0.012 \cdot w_2^3 = 2.231 = WE_3^n. \quad (32)$$

The approximate value $w_2$ comes out of (32), if we do not take into consideration the coefficients whilst having $w_2^2$ and $w_2^3$

$$w_2 = (WE_3^n + 2.231) / 32.386. \quad (33)$$

Equation (32) or the approximate formula (33) let us determine $w_2$ whilst changing $Z_{\Omega \pi}$.

**Example 1.** The conducted calculation of coupled lines’ primary and secondary parameters while $w_2 = 2$ mm and starting data, mentioned above, gave us the value $Z_{\Omega \pi} = 18.167$ Ohms. Then the correction of $Z_{\Omega \pi} = 20$ Ohms is made, from (31) $WE_3^n = 56.199$ is determined and as a result of solving (32) $w_2 = 1.817$ mm is gained. Approximation by (33) gave us the value $w_2 = 1.804$ mm.
III. SYNTHESIS OF CONSTRUCTIONS WITH A SET RATIO OF SYNPHASE AND ANTIPHASE PHASE VELOCITIES

Constructions with different ratios of synphase and antiphase waves phase velocities are synthesized. As a basis we took a construction with sizes and relative permittivities $w_1 = 0$, $a = 10 \text{ mm}$, $h_4 = 6 \text{ mm}$, $h_l = h_2 = h_3 = 1.0 \text{ mm}$, $d = 0$, $\varepsilon_4 = \varepsilon_4 = 1.0$. During the synthesis, as usual, two iterations were being done. Their purpose came down to on the first step determine by using formulas (21) and (23) the permittivities $\varepsilon_2$ and $\varepsilon_3$, which provide us the set $v_c/v_x$ in the range $0.8...2.5$. After getting the first approximation $\varepsilon_2$ and $\varepsilon_3$ were calculated again using (21) and primary, secondary parameters being determined.

Example 2. $v_c/v_x = 0.8$ is set. During the first iteration using base parameters and having taken $\varepsilon_2 = \varepsilon_3 = 2.68$, we use (21) and come to the conclusion that the relative permittivities should meet the inequality $\varepsilon_2 > \varepsilon_3$. Having taken $\varepsilon_2 = 16$, $\varepsilon_3 = 2.6$ the calculation of primary parameters while $w_2 = 2.5 \text{ mm}$ gave us the result of $v_c/v_x = \sqrt{3.572/5.295} = 0.821$.

The second iteration was conducted by changing $\varepsilon_{eff} = 5.58$. The necessary $\varepsilon_2 = 17.237$, $\varepsilon_3 = 2.414$ were acquired. $\varepsilon_2 = 27.0$, $\varepsilon_3 = 2.42$ were taken for the calculation. The result is:

$v_{eff} = 5.571, v_{eff} = 3.567, v_c/v_x = 0.8$, $C = \begin{bmatrix} 177.5 & -61.47 \\ -61.47 & 177.5 \end{bmatrix}$ pF/m, $L = \begin{bmatrix} 0.3498 & 0.1839 \\ 0.1839 & 0.3498 \end{bmatrix}$ μH/m.

Example 3. $v_c/v_x = 1.0$ is set. During the first iteration using base parameters and having taken $\varepsilon_2 = \varepsilon_3 = 2.68$, $w_2 = 3$, we use (21) and conclude that the relative permittivities should meet the inequality $\varepsilon_2 > \varepsilon_3$, and the condition of equalizing $v_c$ and $v_x$ is satisfied, if $\varepsilon_2/\varepsilon_3 = 4.738/2.289$.

The second iteration was conducted while $\varepsilon_2 = 5.0$, $\varepsilon_3 = 2.4$, but a third iteration was needed after the clarification using (21). Wherein the ratio $v_c/v_x = 1.004$ achieved while $v_{eff} = 2.35$, $v_{eff} = 2.37$, and the primary parameters in the form of capacitances and inductances matrices are as follows

$C = \begin{bmatrix} 117.6 & -65.15 \\ -65.15 & 117.6 \end{bmatrix}$ pF/m, $L = \begin{bmatrix} 0.3212 & 0.1771 \\ 0.1771 & 0.3212 \end{bmatrix}$ μH/m.

The realization of coupled lines constructions with synthesized sizes is inconvenient for the installation of a vertical substrate. To be able to increase the build’s manufacturability, it is desirable for $w_1 > 0$. Then connecting vertically positioned strips with horizontally positioned strips is possible using soldering. But the increase of $w_1$ leads to the rise of self-capacity on the ground plane and the decrease of $Z_{0c}$. To compensate the undesirable changes, we can increase the gap $d$.

We calculated dependencies of $v_c/v_x$ and $\sqrt{Z_{0\pi} Z_{0c}}$ from $d$ whilst having $w_1 = 0.5 \text{ mm}$ and the rest of second iteration parameters. The approximation of functions $v_c/v_x \approx f_4(d)$ and $\sqrt{Z_{0\pi} Z_{0c}} \approx f_5(d)$ was gained in the form of polynomials

$f_4(d) = \begin{bmatrix} 0.944 & 1.49 \cdot 10^{-3} & 0.026 & -3.38 \cdot 10^{-5} \end{bmatrix} \cdot [wd], \quad (34)$

$f_5(d) = \begin{bmatrix} 44.129 & -0.237 & 1.803 & -0.231 \end{bmatrix} \cdot [wd], \quad (35)$

$[wd] = \begin{bmatrix} 1 & d_z & d_z^2 & d_z^3 \end{bmatrix}^T$. \quad (36)$

The joint solving of equations (34) and (35) let us find the gap $d = 1.7 \text{ mm}$ and after the second calculation using software NETEPSILON to get $v_c/v_x = 1.012$ and a satisfactory match while $\sqrt{Z_{0\pi} Z_{0c}} = 48.2 \text{ Ohms}$.

Example 4. $v_c/v_x = 1.7$ is set. The condition of using a dielectric of only one type with $h_3 = 1.5, \varepsilon_3 = 5.0$ was placed. The rest of the space is filled with air. The antiphase wave’s wave impedance should be $Z_{0c} = 25 \text{ Ohm}$, $\sqrt{Z_{0\pi} Z_{0c}} = 50 \text{ Ohms}$. As a basic primary variant, the construction with vertically positioned strips’ size of $w_2 = 3 \text{ mm}$ is chosen. After the first iteration we got $v_c/v_x = 1.642 Z_{0\pi} = 31.24 \text{ Ohm}$, $Z_{0c} = 31.24 \text{ Ohm}$. The second step is done by using formula (31), which let us clarify $w_2 = 4 \text{ mm}$ and as a final result get $v_c/v_x = 1.702, Z_{0\pi} = 24.79 \text{ Ohm}$, $\sqrt{Z_{0\pi} Z_{0c}} = 50.228 \text{ Ohms}$.

Example 5. As a base construction we took the strip structure with $w_1 = 0$, $w_2 = 2 \text{ mm}$, $h_l = 0$, $h_2 = 0.45 \text{ mm}$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_4 = 1$. The objective is to based on this construction, get the ratio $v_c/v_x \geq 2.5$, using the estimations of expressions (21) and (23), and conduct an experimental researches of coupled lines’ sections with synthesized parameters and the strip switch on scheme as on fig. 9. Wherein the condition of $\sqrt{Z_{0\pi} Z_{0c}} \approx 50$ is supposed to be met at the same time. From the conducted analysis (see previous example) it follows that $v_c/v_x > 1$ can be achieved only if $\varepsilon_3 > \varepsilon_2$. As the first iteration $\varepsilon_3 = 20.0$ was taken, which in a marginal case ensures $v_{eff} = 2.4$ and while $v_{eff} = 1.5$ we get $v_c/v_x = 3.65$. 
But the calculation showed that with the chosen strip structure parameters we have \( \nu_c/\nu_\pi = 2.947 \) and \( \sqrt{Z_{\text{on}} \cdot Z_{\text{off}}} = 42.176 \).

The second iteration was conducted with firstly addressing to (23) with \( Z_{\text{on}} = 133.57 \), \( Z_{\text{off}} = 18.73 \) and the calculation \([\varepsilon_2, \varepsilon_3] = [0.703, 16.871] \). This outcome means that with the chosen \( \varepsilon_3 = 20.0 \) and other set parameters the physical realization of \( \sqrt{Z_{\text{on}} \cdot Z_{\text{off}}} \approx 50 \) Ohms is not possible. But at the same time with that it points out the necessity of reducing \( \varepsilon_3 \) to \( \varepsilon_3 < 16.87 \). \( \varepsilon_3 = 16 \) was taken, \( \nu_c/\nu_\pi = 2.704 \).

\( \sqrt{Z_{\text{on}} \cdot Z_{\text{off}}} = 44.959 \) were gotten. A layout with the chosen parameters was made, and its frequency characteristics were experimentally measured using the scheme on Fig. 9. The results of calculating the coefficients of the scattering matrix \( S_{11} \) and \( S_{11} \) the section of the coupled lines showed a significant difference in their frequency dependence from the experimental one. One of the reasons, according to our assumption, was to deviate \( \varepsilon_3 \) the material of the vertical substrate from the specified nominal value. A change of up to 15 resulted in a perfectly satisfactory correspondence between the calculated and experimental dependencies \( S_{11}(f) \), \( S_{11}(f) \) as shown in the following subsection.

IV. FREQUENCY CHARACTERISTICS – DEGRADATION OF ALL-PASS SECTION PROPERTIES

It is known [3], that the coupled lines’ sections with their switch on scheme as on Fig. 9 are all-pass while the electromagnetic couplings are balanced. Next it is shown, that while the synphase and antiphase velocities are unequal and even with the identical lines there are resonant fluctuations in the sections, which lead to losses and reflections in the frequency range with the observed differing periodicity of pass and reflect strips. The calculation of frequency characteristics was done using articles [5-7], which gave identical results.

Fig. 10 shows the frequency dependencies of matrix dispersion coefficients \( S_{31} \) and \( S_{11} \) for coupled lines with lengths \( l = 0.1 \) m, synthesized with the parameter \( \nu_c/\nu_\pi = 0.8 \) (example 2). In frequencies up to 4 GHz we can see the presence of resonates, which repeat from the first one to the second after 764 MHz, but from the third one to the fourth already after 884 MHz. The periodicity’s disturbance is caused by the synphase and antiphase waves interference particularities, which were studied in article [24]. In this case the phase velocities ratio \( \nu_c/\nu_\pi = 0.8 \) conditions the specified resonants’ repetency. Having the multiple ratio of \( \nu_c/\nu_\pi = 0.8 \), not only a “breakdown” of resonants is possible, but also the fusion of neighboring pass bands. This will further be shown using the section example with \( \nu_c/\nu_\pi > 2 \).

The construction’s dispersion matrix coefficients’ frequency dependence, which is characterized by \( \nu_c/\nu_\pi = 1.0 \) (example 3), confirmed the absence of resonants (Fig. 11). From the shown figure, a small development periodicity of the being implemented decay \( S_{11} \) while with fluctuations of the input inductance, which can be observed by increasing the return losses \( S_{11} \). Wherein the phase-frequency characteristic is close to the linear one.

![Fig. 10](image-url)  
Fig. 10. Calculated and experimental frequency dependencies of section scattering matrix coefficients (Fig. 9) at \( \nu_c/\nu_\pi = 0.8 \).

Fig. 11. The simulation and experimental frequency dependencies of the section scattering matrix coefficients (Fig. 9) while \( \nu_c/\nu_\pi = 1.0 \).

The frequency characteristics of coupled lines, which were synthesized and described in example 4 (\( \nu_c/\nu_\pi = 1.7 \)), were calculated and experimentally measured in the frequency range of up to 8 GHz. The lines had a length of \( l = 0.48 \) m, per unit capacities of \( C_{11} = C_{22} = 142.4 \) pF/m, \( C_{12} = C_{21} = 106.7 \) pF/m, per unit inductances of \( L_{11} = L_{22} = 0.2611 \) μH/m. The simulative and experimental dependencies of dispersion matrix coefficients are shown on Fig. 12 – 14. In the listed above dependencies, a decrease with the implemented decay depth frequency growth can be seen due to the bigger synphase and antiphase phase velocities difference comparing to the studied above case of \( \nu_c/\nu_\pi = 0.8 \).

The coupled lines’ section, described in example 5, was theoretically and experimentally researched. The lines had a length of \( l = 0.1 \) m, per unit capacities of \( C_{11} = C_{22} = 142.1 \) pF/m and per unit inductances of \( L_{11} = L_{22} = 0.2611 \) μH/m, simulative ratio of \( \nu_c/\nu_\pi = 2.636 \).
The frequency characteristics are shown on Fig. 15-17. We can observe the alternation of wide and narrow pass bands. Those are formed, as it seems, at the expense of resonant fluctuations' on frequencies 1.5199 GHz and 3.0399 GHz disappearance while putting together constitutive waves with close but being inverted phases with +0 on -0.

On Fig. 15 we can observe an unusual dependency of return losses on the frequency, which is uncharacteristic for coupled lines with not a big phase velocities difference of normal waves.

The primary parameters' analysis, based on the numerical method's usage for the electric's field and accumulated energy in the selected coupled lines' cross section subareas calculation, can be applied to other coupled lines' types. This will allow us go from the heuristic search of optimal solutions while building modal filters [13, 14] to using analytic expressions for finding the relative dielectric permittivities of substrates.
VI. CONCLUSION

The suggested approach, which’s point consists of solving inverse finding dielectric permittivities and strip sizes problems based on the Laplace’s numerical equation and the selected subareas’ electric field’s accumulated energy defining can be used for other coupled lines’ types. The substrates’ values of the relative dielectric permittivities being found cannot correspond to the being manufactured foiled materials’ permittivities. In example 2, the necessity of getting the relative dielectric permittivities \( \varepsilon_g = 17.2 \), \( \varepsilon_e = 2.42 \) is shown this way. Materials with those parameters can be manufactured with the help of additive technologies of multicomponent printing using different dielectrics with specified in advance components’ contents in percent [25]. Another possible way – use multilayered substrates made from different available dielectrics with different thickness and dielectric permittivities, including the ones made with technologies of printing. In this case the presented way of solving the set phase velocities ratio getting problem with a limit for the other parameters is also usable. This way, the conducted in the study CSL synthesis’ possibilities by the criteria of the set ratio of \( V_g/V_e \) make the new problem of dielectric materials with a set dielectric permittivity manufacturing using additive printing methods technological process’s development expedient.

ACKNOWLEDGMENT

The authors are grateful to the Centre of collective usage of «Impulse» science equipment of the Tomsk state university of control systems and radioelectronics staff for the help in conducting the measurements. We also thank Sychev A. N., who provided us one of the coupled lines with a vertically positioned substrate section layouts for conducting measurements and Malyutin Maxim for manuscript preparing.

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