Abstract

An approach for describing the electromagnetic form factors of hadrons, in the space- and time-like regions, within a constituent quark model on the light front is shortly illustrated and calculations for the pion case are reported. Three main ingredients enter our approach: i) the dressed photon vertex where a photon decays in a quark-antiquark pair, ii) the on-shell quark-hadron vertex functions in the valence sector, and iii) the non-valence component of the hadron state.

INTRODUCTION

The experimental study of hadron electromagnetic (em) form factors allows us to gather valuable information on hadron states, and the increasing accuracy of the experiments calls for refined theoretical tools in order to make stringent the interpretation of the data. In particular, the analysis of em form factors both in the space- and time-like regions, within the light-front framework [1], opens a unique possibility to study hadron states in the valence sector and in the non-valence one. As a matter of fact, one can write the states for mesons and baryons, respectively, as follows

\begin{align}
\langle meson \rangle &= |q\bar{q}\rangle + |gq\bar{q}\rangle + |q\bar{q}g\rangle + \ldots \nonumber \\
\langle baryon \rangle &= |qqg\rangle + |qqg\bar{q}\rangle + |qqg\bar{g}\rangle + \ldots \nonumber \\
\end{align}

Indeed, it should be pointed out that only within the light-front approach the Fock expansion becomes meaningful, given the coincidence between the mathematical vacuum and the physical one.

As it is well-known, investigations of the em properties of hadrons in the time-like (TL) region yields the possibility to address a vast phenomenology, and consequently to impose quantitative constraints on dynamical models pointing to a microscopical description of hadrons.

GENERAL FORMALISM

The Mandelstam formula for the matrix elements of the em current [2] is the starting point for the construction of our approach. Following Ref. [2], the matrix elements in the TL region read

\begin{align}
\langle k' | \mathcal{M}_0 | k \rangle = \frac{1}{p - m + i\epsilon} \int \frac{d^4k}{(2\pi)^4} Tr \left[ S_Q(k - P_h) \bar{\lambda}_h(k - P_h) \right] \\
\end{align}

where \( S(p) = \frac{1}{p - m + i\epsilon} \), with \( m \) the mass of the constituent quark struck by the virtual photon, \( S_Q(p) \) is the propagator of the spectator constituent, quark or diquark (in a simple picture of baryons), \( \Gamma^\mu(k, q) \) the quark-photon vertex, \( q^\mu \) the virtual photon momentum, \( \lambda_h(k, P_h) \) the hadron vertex function, \( P^\mu_h \) and \( P^\mu_{h'} \) are the hadron momenta. The "bar" notation on the vertex function means that the associated amplitude is the solution of the Bethe-Salpeter equation, where the irreducible kernel is placed on the right of the amplitude, while in the conventional case it is placed on the left of the Bethe-Salpeter amplitude [3].

For the space-like (SL) region, the crossing symmetry has to be applied, replacing the antihadron vertex with the corresponding hadron one, reverting the sign of the four-momenta and appending the proper number of \( \gamma^5 \).

THE PION EM FORM FACTOR

For the sake of concreteness, let us consider the case of the pion. If both the SL and TL regions have to be investigated, it is necessary to perform the analysis of the em form factor in a frame where the plus component of the four-momentum transfer is nonvanishing, \( q^+ \neq 0 \) [4, 5]. Following [6] one can choose \( q^+ \neq 0 \) and \( q_\perp = 0 \). Figs. 1 and 2 [7] show a diagrammatic analysis of the physical processes pertaining to SL and TL regions, respectively. In particular, Figs. 1(b) and 2(b) illustrate the contribution due to a \( q\bar{q} \) pair, produced by the virtual photon. Such a contribution is dominant for both kinematical regions, if a vanishing pion mass is assumed. In what follows, in order to emphasize the unified description of the em form factor in TL and SL regions, the simplifying assumption of a chiral pion \( (m_\pi = 0) \) will be adopted. From figs. 1 and 2 the following questions immediately arise: i) how to model the quark-photon vertex? ii) how to deal with the amplitude for the emission or absorption of a pion by a quark? iii) how to describe the
$q\bar{q}$-pion vertex? The illustration of our proposal for the construction of a phenomenological model in order to answer such questions is the aim of the present contribution.

$$\psi_n(k^+, k_\perp; P_n^+, P_n^\perp) = \Lambda_n(k, P_n)[|k-\bar{k_n}|] \times$$

$$\frac{P_n^+}{[M_n^2 - M_0^2(k^+, k_\perp; P_n^+, P_n^\perp)]}$$

where $M_0$ is the standard front-form free mass and $\psi_n$ is the eigenfunction, in the $1^-$ channel, of the square mass operator proposed in Refs. [10, 11] within a relativistic constituent quark model. This model takes into account the confinement through a harmonic oscillator potential and the $\pi-\rho$ splitting through a Dirac-delta interaction in the pseudoscalar channel. It achieves a satisfactory description of the experimental masses for both singlet and triplet $S$-wave mesons, with a natural explanation of the "Iachello-Anisovitch law" [12, 13], namely the almost-linear relation between the square mass of the excited states and the radial quantum number $n$. Since the model of Refs. [10, 11] does not include the mixing between isoscalar and isovector mesons, in what follows only contributions from the isovector $p$-like vector mesons are included.

The VM eigenfunction, $\psi_n(k^+, k_\perp; q^+, q_\perp)$, which describes the valence component of the VM state $|n\rangle$, is normalized to the probability of the lowest $(q\bar{q})$ Fock state (i.e. of the valence component). The $q\bar{q}$ probability can be roughly estimated to be $\sim 1/\sqrt{2n + 3/2}$ in a simple model [8] that reproduces the "Iachello-Anisovitch law" [12, 13], and is based on an expansion of the VM state $|n\rangle$, in terms of properly weighted Fock states $|i\rangle_0$, with $i > 0$ quark-antiquark pairs.

If in the calculation of the decay constant, $f_{Vn}$, the following assumptions on the analytic behaviour of the VM vertex function are adopted: i) $\Lambda_n(k, P_n)$ does not diverge in the complex plane $k^- \to \infty$, and ii) the contributions of its singularities in the integration over $k^-$ are negligible, one obtains

$$f_{Vn} = -\frac{N_c}{4(2\pi)^3} \int \frac{dk^+ dk_\perp}{k^+ (P_n^+ - k^+)} \psi_n(k^+, k_\perp, M_n, \bar{q}_\perp)$$

$$Tr \left[ (k + m) \gamma^+(k - P_n + m) \bar{V}_{nc}(k, k - P_n) \right]$$

(6)

The form factor of the pion in the TL and in the SL regions can be obtained from the plus component of the proper current matrix elements: $j_{TL}^+ = (\pi\bar{n}|q\gamma^\mu q|0)$ = $(P_n^+ - P_0^+)$ $F_\pi(q^2)$, and $j_{SL}^+ = (\pi\bar{n}|q\gamma^\mu q|\bar{p}) = (P_n^+ + P_0^+)$ $F_\pi(q^2)$. Since in the limit $m_\pi \to 0$ the form factor receives contributions only from the diagrams of Figs. 1(b) and 2(b), where the photon decays in a $q\bar{q}$ pair,
one can apply our approximation for the plus component of the dressed photon vertex \( \Gamma_n \), both in the SL and in the TL regions. Then the matrix element \( j^+ \) can be written as a sum over the vector mesons and consequently the form factor becomes

\[
F_\pi(q^2) = \sum_n \frac{f_{Vn}}{q^2 - M_n^2 + i M_n \Gamma_n(q^2)} \frac{g_{Vn}^+(q^2)}{g_{Vn}^-(q^2)}
\]

where \( g_{Vn}^+(q^2) \), for \( q^2 > 0 \), is the form factor for the VM decay in a pair of pions.

Each VM contribution to the sum \( \overset{\text{7}}{\text{7}} \) is invariant under kinematical front-form boosts and therefore it can be evaluated in the rest frame of the corresponding resonance. In this frame one has \( q^+ = M_n \) and \( q^- = q^2/M_n \) for the photon and \( P_n^+ = P_n^- = M_n \) for the vector meson. This means that we choose a different frame for each resonance (always with \( q_\perp = 0 \)), but all the frames are related by kinematical front-form boosts along the \( z \) axis to each other, and to the frame where \( q^+ = -q^- = \sqrt{-q^2} \) (\( q_z = \sqrt{-q^2} \)), adopted in previous analyses of the SL region \( \overset{\text{5}}{\text{5}} \). Since in our reference frame one has \( \sum_{n} [\epsilon_n^+(P_n)]^* \epsilon_n^+(P_n) \cdot \tilde{\Gamma}_n = [\epsilon_n^-(P_n)]^* \epsilon_n^-(P_n) \cdot \tilde{\Gamma}_n = -\tilde{\Gamma}_{n,\perp} \), we obtain \( \overset{\text{7}}{\text{7}} \)

\[
\begin{align*}
|g_{Vn}^+(q^2)| &= \frac{N_c}{8\pi^3 P_{\pi\bar{\pi}}} \int_0^{q^2} \frac{dk^+}{(k^+)^2 (q^+ - k^+)} \\
&\times \left( (k^2 - q^2)^{5/2} \right)^{5/2} \\
&\times \psi_n^+(k^+, k_{\perp}^+; \bar{P}_n^+, P_{\pi\perp}) \psi_n(k^-, k_{\perp}^-; q^+, q_{\perp}^-) \\
&\times \frac{[M_n^2 - M_{\pi}^2(k^+, k_{\perp}^+; q^+, q_{\perp}^-)]}{q^2 - M_{\pi}^2(k^+, k_{\perp}^+; q^+, q_{\perp}^-)}
\end{align*}
\]

where \( \Theta^5 = \Gamma_{n\pi^\pm}(k, k - q)^5 \left[ k - P_{\pi\bar{\pi}} + m \right] \gamma^5 \). To obtain Eq. \( \overset{\text{8}}{\text{8}} \) we have i) performed the \( k^- \) integration assuming once more negligible contributions from the singularities in the vertex function, and ii) adopted Eqs. \( \overset{\text{5}}{\text{5}} \) for describing the quark-VM vertex in the valence sector. The same assumption is adopted for the momentum part of the \( \bar{q}q \) pion vertex in the valence sector, namely

\[
\begin{align*}
\psi_n(k^+, k_{\perp}^+; P_{\pi\perp}^+, P_{\pi\perp}) &= \frac{m_{\pi}}{f_{\pi}} \Lambda_\pi(k, P_{\pi}) |_{k^- = k_{\perp}^-} \\
&\times \frac{P_{\pi}^+}{[m_{\pi}^2 - M_{\pi}^2(k^+, k_{\perp}^+; P_{\pi\perp}^+, P_{\pi\perp})]}
\end{align*}
\]

It should be pointed out that we do not distinguish between \( \Lambda_\pi(k, P_{\pi})(n) \) and \( \Lambda_{\pi(n)}(k, P_{\pi}) \) in the range \( 0 < k^+ < P_{\pi\perp}^+ \), given the symmetry between the quark momenta. Finally, we have introduced a third main approximation: following Ref. \( \overset{\text{14}}{\text{14}} \), the momentum part of the quark-pion emission vertex in the non-valence sector, \( \frac{m_{\pi}}{f_{\pi}} \Lambda_\pi(k, P_{\pi}) \), is assumed to be a constant.

![Figure 3. Pion electromagnetic form factor vs the square momentum transfer \( q^2 \). Dashed and solid lines are the results with the asymptotic (see text) and the full pion wave function, respectively. Experimental data are from Ref. \( \overset{\text{16}}{\text{16}} \). (After \( \overset{\text{7}}{\text{7}} \)).](image)

The value of the constant is fixed by the pion charge normalization. The same constant value is assumed for the quark-pion absorption vertex (see the square blob in Fig. 1(b)). It turns out that the same expression for \( g_{Vn}^+(q^2) \) holds both in the TL and in the SL regions \( \overset{\text{7}}{\text{7}} \).

**Results**

Our calculation of the pion form factor contains a very small set of parameters: i) the constituent quark mass, ii) the oscillator strength, \( \omega \), and iii) the VM widths, \( \Gamma_n \), for \( M_n > 2.150 \text{ GeV} \). The up-down quark mass is fixed at 0.265 GeV \( \overset{\text{11}}{\text{11}} \). For the first four vector mesons the known experimental masses and widths are used in the calculations \( \overset{\text{15}}{\text{15}} \). However, the non-trivial \( q^2 \) dependence of \( g_{Vn}^+(q^2) \) in our microscopical model implies a shift of the VM masses, with respect to the values obtained by using Breit-Wigner functions with constant values for \( g_{Vn}^+ \). As a consequence, the value of the \( \rho \) meson mass is moved from the usual one, 775 GeV, to 750 GeV. For the VM, with \( M_n > 2.150 \text{ GeV} \), the mass values corresponding to the model of Ref. \( \overset{\text{11}}{\text{11}} \) are used, while for the unknown widths we use a single value \( \Gamma_n = 0.15 \text{ GeV} \), which presents the best agreement with the compilation of the experimental data of Ref. \( \overset{\text{16}}{\text{16}} \). We consider 20 resonances in our calculations to obtain stability of the results up to \( q^2 = 10 \text{ (GeV/c)}^2 \).

The oscillator strength is fixed at \( \omega = 1.39 \text{ GeV}^2 \). The values of the coupling constants, \( f_{Vn} \), are evaluated from the model VM wave functions through Eq. \( \overset{\text{6}}{\text{6}} \). The corresponding partial decay width, \( \Gamma_{e+e-} = \delta \pi \alpha^2 f_{Vn}^6 / (3M_n^3) \), where \( \delta \alpha \) is the fine structure constant, is compared for the first three VM with the experimental
information in the following table

| VM   | $\Gamma_{e^+e^-}^{\text{th}}$ | $\Gamma_{e^+e^-}^{\exp}$ |
|------|------------------|------------------|
| $\rho(770)$ | 6.37 KeV         | 6.77 $\pm$ 0.32 KeV |
| $\rho(1450)$ | 1.61 KeV         | $> 2.30$ $\pm$ 0.50 KeV |
| $\rho(1770)$ | 1.23 KeV         | $> 0.18$ $\pm$ 0.10 KeV |

We perform two sets of calculations for the pion form factor. In the first one, we use the asymptotic form of the pion valence wave function, obtained with $\Lambda_{\pi}(k, P_\pi) = 1$ in Eq. (9); in the second one, we use the eigenstate of the square mass operator of Refs. [10, 11]. The pion radius for the asymptotic wave function is $r_{\text{asymp}} = 0.65$ fm and for the full model wave function is $r_{\text{model}} = 0.67$ fm, to be compared with the experimental value $r_{\exp} = 0.67$ $\pm$ 0.02 fm [17]. The good agreement with the experimental form factor at low momentum transfers is expected, since we have built-in the generalized $\rho$-meson dominance.

The calculated pion form factor is shown in Fig. 3 in a wide region of square momentum transfers, from $-10$ (GeV/$c$)$^2$ up to $10$ (GeV/$c$)$^2$. A general qualitative agreement with the data is seen, independently of the detailed form of the pion wave function. The results obtained with the asymptotic pion wave function and the full model, present some differences only above $3$ (GeV/$c$)$^2$. The SL form factor is notably well described. It has to be stressed that the heights of the TL bumps directly depend on the calculated values of $f_{Vn}$ and $g_{Vn}^+$. The introduction of $\omega$-like [18] and $\phi$-like mesons could improve the description of the data in the TL region.

THE NUCLEON EM FORM FACTOR IN THE SPACE- AND TIME-LIKE REGIONS: A HEURISTIC INTRODUCTION

A particularly challenging puzzle in the study of hadron em properties is represented by the TL form factors of the nucleon (see e.g. [19] and [20] for theoretical and experimental status reports, respectively).

For $q^2 = 0$ the SU(3) CQ model yields for the ratio $G_p^M(0)/G_n^M(0)$ the well known result, i.e. $G_p^M(0)/G_n^M(0) = -3/2$, that can be easily obtained from the expectation value of a one-body operator, viz.

$$G_N^M(0) \propto \langle N | \sigma_z \left\{ \frac{c_u (1 + \tau_3)}{2} + c_d (1 - \tau_3) \right\} | N \rangle = \langle N | \sigma_z \left\{ \frac{1}{6} + \frac{\tau_3}{2} \right\} | N \rangle$$

where, in the second line, the isoscalar and isovector contributions to the matrix element are put in evidence. In the calculation of the ratio $G_p^M(0)/G_n^M(0)$ the momentum distribution of a quark in the nucleon, i.e. the square modulus of the valence component, is the relevant quantity due to the one-body nature of the current adopted for evaluating the ratio in SU(3) CQ model. By using Eq. (10), for the proton we have the contribution only from a scalar diquark with $I = 0$, while for the neutron we have contributions from both a (0,0)-diquark and a (1,1)-diquark, properly weighted for the spin-isospin part, while the dynamical part turns out to be equal.

In the TL region, (see Fig. 4), a naive perturbative description of the $e^+e^-$ annihilation [19] leads to

$$FENICE \text{ data (} G_n^M(0) \text{)}$$

Figure 4. Diagrammatic representation of the photon decay ($\gamma^* \rightarrow NN$).

Figure 5. Solid line: naive expectation $\equiv c_d/c_u \times \mu_p/\mu_n$ (19). Experimental data from ref. 20.
\(G_n^M(4M_n^2)/G_p^M(4M_n^2)\) \(\equiv |e_d/e_u| = 1/2\), to be compared with an experimental value of the order of 1.4, but with a large error bar. As suggested in [19], a possible solution is given by a strong prevalence of the isovector channel in the \(N\bar{N}\) final state in a two-step production process, where the virtual photon could materialize in a \(\rho\)-like meson.

In our understanding, also a dynamical motivation could play a role in getting close to the experimental result. In the TL region, one has to calculate the expectation value of the decay of the virtual photon in a \(q\bar{q}\) pair dresses the quark-photon vertex. In order to deal with such a process, both valence and nonvalence component of the hadron state (see, e.g., Eq. (3)) are necessary. This implies that the relevant quantity becomes the wave function instead of the momentum distribution. It turns out that a dynamical contribution to the many-body current, like in the case of the pion, where the mechanism of the decay of the virtual photon in a \(q\bar{q}\) pair dresses the quark-photon vertex. It produces an increasing in \(G_n^M(4M_n^2)\) with respect to \(G_p^M(4M_n^2)\).

A very preliminary estimate, without taking into account the possible difference between the isoscalar and isovector contributions of the quark-photon vertex (as suggested in [19]) and any flavour dependence in the diquark-emission vertex, indicates a value of the ratio \(G_n^M(4M_n^2)/G_p^M(4M_n^2)\) \(\equiv e_d/e_u \times \mu_p/\mu_n \sim 0.73\). (while in the naive perturbative description one has \(G_n^M(4M_n^2)/G_p^M(4M_n^2)\) \(\equiv |e_d/e_u| \times \mu_p/\mu_n \sim 0.73\).

**SUMMARY**

The theoretical results presented in this contribution show that a VM dominance ansatz for the dressed photon - (\(q\bar{q}\)) vertex, within a CQ model consistent with the meson spectrum, is able to give a unified description of the pion form factor both in the SL and TL regions. Using the experimental widths for the first four vector mesons and a single free parameter for the unknown widths of the other vector mesons, the model gives a fair agreement with the TL data, while in the SL region it works surprisingly well. These results encourage the investigation of the TL form factors of the nucleon using an approach based on a simple ansatz for the nonvalence component of the nucleon state, following as a guideline the pion case.

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