Thermalization temperature in Pb+Pb collisions at SpS energy from hadron yields and midrapidity $p_t$ distributions of hadrons and direct photons

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In the frame of 2+1 Bjorken hydrodynamics we describe simultaneously hadron yields and midrapidity $p_t$ distributions of direct photons, unflavored, and strange hadrons measured in Pb+Pb collisions at SpS. We find, that a reasonably quantitative description of these data is possible only if we introduce some radial velocity at the beginning of the one-fluid hydrodynamic stage. We fix uniquely all parameters of this model, and estimate an initial temperature of the just thermalized state $T_{in} = 200^{+40}_{-20}$ MeV in the case of EoS with phase transition to Quark-Gluon Plasma, and $T_{in} = 230^{+60}_{-30}$ MeV in the pure hadronic scenario. We conclude that SpS data considered in our study can not distinguish between a production pure hadronic scenario. We conclude that SpS data considered in our study can not distinguish between a production.

A finite volume of rapidly expanding Quark-Gluon Plasma (QGP) can be possibly created for a very short time in ultrarelativistic heavy ion collisions as an almost locally equilibrated thermodynamic state of deconfined quarks and gluons with strong residual interaction. Taking into account a very short time available for QGP formation in this process a necessary condition of its creations is that an initial (thermalization) temperature $T_{in}$ have to be visibly higher than the quark-hadron phase transition temperature, which is estimated from lattice calculations to be $T_c = 150 - 170$ MeV [1]. Therefore an estimation of a temperature at which the local thermal equilibrium is just established – thermalization temperature – is of great importance.

There are a lot of attempts to extract this temperature from direct photon momentum distributions measured in S+Au [2] and Pb+Pb [3] collisions. However, to extract the thermalization temperature from experimental data reasonably well, it is necessary to separate accurately the collective and chaotic motions. To do this one needs to know the evolution of the collision starting from the first interactions till the final particle stage. Unfortunately, the most part of the estimations of the thermalization temperature [4,5] are based on hydrodynamic description with too simplified initial conditions (zero initial radial collective velocity), and other model parameters fixed partially by some experimental data, and partially – ad hoc from more or less reasonable considerations. Only in the recent paper [6] a first attempt to estimate the influence of nonzero initial radial velocity was undertaken.

In contrast to this, in the present letter we use initial conditions for hydrodynamic model in the most general form, describe simultaneously a large amount of available experimental data, measured in Pb+Pb collisions at SpS (hadron yields, midrapidity $p_t$ distributions of direct photons, unflavored and strange hadrons), fix uniquely all model parameters from these data and obtain reliable estimate of the thermalization temperature with its uncertainty concerned with experimental data bars.

In this way we find, that if we use a realistic equation of state (EoS) of hadronic gas incorporating contributions of all known hadrons and resonances, then to describe midrapidity $p_t$ distributions of final hadrons it is necessary to assume an essentially nonzero initial radial collective velocity at the beginning of the one-fluid hydrodynamic stage. This velocity is created during the pre-equilibrium (two-fluid) stage of the collision due to significant radial energy density gradient coming from the shapes of the colliding nuclei. Nevertheless, in all previous estimations of the thermalization temperature this initial radial velocity was absolutely neglected, and only the simplest case (zero initial velocity) was considered.

Evaluating the direct photon yield we assume the following scenario of the collision. Initially, on the pre-equilibrium stage, colliding nuclei penetrate through each other like two fluids with a friction. On this stage the ‘prompt’ photons are emitted and radial collective velocity is created. If the thermalization time is not negligibly small then the both these effects should be taken into account. The next stage is a one-fluid-like expansion of the locally thermalized matter with an emission of the ‘thermal’ photons and its decay onto final hadrons.

To begin with we calculate contribution of the prompt photons. We take into account Compton, annihilation and bremsstrahlung processes. We use GRV-94 structure functions [7], two-loop expression for $\alpha_s$ and K-factor, K=2, accounting higher order corrections. Comparing calculated yield of prompt photons in pp and pA collisions with experimental data at $\sqrt{s} = 19$ GeV – the closest available c.m. energy to Pb+Pb at SpS ($\sqrt{s} = 17.3$ GeV) we find, that both experimental data and theoretical predictions have approximately the same slope, but difference in normalization reaches factor $\approx 7$. Nowadays this difference is attributed to the intrinsic transverse momentum of colliding partons [8]. In this letter we account these corrections by introducing a proper factor, assuming that it is independent on $\sqrt{s}$ while moving from pA to Pb+Pb energy. To go from pA to AA collisions we need the number of nucleon-nucleon collisions occurred during the interpenetration of the colliding nuclei. In our calculations we use Glauber approximation, what gives in the case of Pb+Pb collision $N_{G1} \approx 3 \cdot A$. Comparison with
WA98 data [3] shows, that prompt photons contribute approximately 30% of total direct photon yield.

To evaluate yield of thermal photons we integrate average emission rate of direct photons from unit volume per unit time over a space-time evolution of hot matter calculated in a one-fluid hydrodynamics. In the present analysis we use recently calculated emission rates accounting next to leading effects in QGP [9] and dominating contribution in hadronic gas – reaction $\pi\rho \to a_1 \to \pi\gamma$ [10]. For description of the evolution we use 2+1 Bjorken hydrodynamics, which works sufficiently well at midrapidity. It has several parameters which should be fixed to define the evolution: a) initial temperature $T_{in}$, initial time $\tau_{in}$, initial energy density and velocity distributions; b) EoS of hot matter: transition temperature $T_c$ (if EoS includes phase transition), chemical freeze-out temperature $T_{ch}$ and thermal freeze-out temperature $T_f$.

For the shape of energy density distribution we use Woods-Saxon distribution, integrated along beam axis. As for initial distribution of radial velocity, it deserve separate discussion. As we find, to describe momentum distributions of final hadrons using realistic EoS it is necessary to introduce some radial velocity already at the beginning of the one-fluid hydrodynamics. This velocity is created during interpenetration of colliding nuclei due to significant radial energy density gradient, originated from spherical shapes of colliding nuclei. Two-fluid hydrodynamics or kinetic models might predict value and radial dependence of this collective velocity but, in this letter we use ‘phemenologic’ approach and use a simple linear distribution $v(r) = (r/R) \cdot \Theta(R-r) \cdot V_{\max}$ characterized by one new parameter $V_{\max}$ – the velocity on the surface of the just thermalized system. We find, that our results are not sensitive to reasonable variations of the shape of this distribution.

Concerning equation of state we consider two different cases: EoS with phase transition to QGP, and pure hadronic EoS. In the case of phase transition for QGP phase we use EoS of ideal gas of massless quarks and gluons with degeneracy $g = 41.5$, what corresponds to QGP consisting of 2.5 massless quark flavors and gluons. To calculate EoS of hadronic phase both in the case of phase transition and pure hadronic scenario we take into account contributions of all hadrons listed in the Particle Data Book [11]. Inclusion of heavy hadrons leads to significant reduction of speed of sound in the hadronic gas, the chemical freeze-out takes place before the thermal one. Following paper [13] we use the following values of the temperature of chemical freeze-out, and baryon, strange and $I_3$ chemical potentials – $T_{ch} = 168$ MeV, $\mu_b = 266$ MeV, $\mu_s = 71$ MeV and $\mu_3 = -5$ MeV. As a result we well describe yields of $\pi, K, \eta, \rho, \Lambda, \Xi$ and $\Omega$ hadrons measured by NA44 [14], NA49 [15], WA97 [16] and WA98 [17] collaborations in Pb+Pb collisions at SpS energy.

A description of our results we start from pure hadronic scenario. Despite the belief that hadronic gas can not exist at temperatures higher than $\sim 200$ MeV (because of the essential overlapping of hadrons in it) we adopt its existence and consider the consequences of this assumption from the point of view of description of direct photon production. To begin with, we assume zero initial radial velocity and try to describe momentum distributions of final hadrons. We choose some initial temperature and calculate initial time to describe multiplicity of final $\pi^0$. After that we perform the hydrodynamic calculations with various values of chemical and thermal freeze-out temperatures and obtain $\chi^2$ of the fit of the calculated to the experimental $\pi^0$ distributions as a function of $T_{ch}$ and $T_f$. We find, that it is impossible to describe midrapidity $p_t$ distributions of $\pi^0$ within any reasonable set of model parameters: because of the much steeper slope of the calculated $p_t$ distribution of $\pi^0$ with respect to the experimental one, we obtain $\chi^2 \sim 10^2$/point. We find the same situation for all initial temperatures in the range $160$ MeV $< T_{in} < 300$ MeV. A reasonable way to improve this situation is to introduce some initial radial velocity to the one-fluid hydrodynamic stage.

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On the next step, we repeat calculations with nonzero initial radial velocity. Again we choose initial temperature, fix initial time from the multiplicity considerations, fix temperature of chemical freeze-out $T_{ch} = 168 \text{ MeV}$ to describe hadron yields and plot levels of constant $\chi^2_\pi$ in the $T_f - V_{max}$ coordinates – see fig. 1. The function $\chi^2_\pi(T_f, V_{max})$ has a shape of a valley with steep walls, while any point on its bottom (thick line), corresponds to a good description of midrapidity $p_t$ distributions of $\pi^0$ with $\chi^2 \sim 1/\text{point}$. To fix a point on this line we use spectra of heavier hadrons ($K, p, \Lambda, \Xi$). It was shown [18] that spectra of the heavier hadrons can be described well if we assume freeze-out temperature $T_f = 120 \text{ MeV}$.

The description of the hadron $p_t$ distributions at midrapidity obtained in this way is shown on the fig. 2. One can see, that shapes of all available experimental spectra: $\pi^0$ [17], $K$ and $\Lambda$ [19], protons [20], and $\Xi$ [21] are described very well, although we overestimate total yields of $p$ and $\Lambda$ in this rough model.

![Graph showing hadron $p_t$ distributions](image)

**FIG. 2.** Comparison of experimental $p_t$ distributions of several hadrons with hydrodynamic predictions for pure hadronic gas EoS with $T_{ch} = 230 \text{ MeV}$ and initial radial velocity $V_{max} = 0.26$. To fix finally all model parameters we estimate $T_{in}$ using $p_t$ distribution of direct photons. We depict on the fig. 3 the sum of the $p_t$ distribution of prompt and thermal photons, calculated at three initial temperatures. For each initial temperature we repeat calculations, described above and obtain the following sets of parameters:

| $T_{in}$, MeV | $\tau_{in}$, fm/c | $V_{max}$, c | Curve on fig. 3 |
|---------------|-------------------|--------------|-----------------|
| 200           | 1.4               | 0.36         | solid           |
| 230           | 0.6               | 0.29         | dashed          |
| 300           | 0.12              | 0.23         | dotted          |

We find, an agreement with experimental data at initial temperature $T_{in} = 230^{+60}_{-30} \text{ MeV}$: $T_{in} = 230 \text{ MeV}$ corresponds to $\chi^2_\pi = 3.7/\text{point}$, $\chi^2_\gamma = 0.67/\text{point}$, while curves, calculated for $T_{in} = 200 \text{ MeV}$ ($\chi^2_\pi = 4.9/\text{point}$, $\chi^2_\gamma = 0.67/\text{point}$), and $T_{in} = 300 \text{ MeV}$ ($\chi^2_\pi = 3.5/\text{point}$, $\chi^2_\gamma = 2.3/\text{point}$) are still within experimental errors. The reason of this weak sensitivity to the initial temperature is following. As far as we introduce the initial radial velocity, all time stages contribute with more or less the same effective slopes. But, the difference between initial time for $T_{in} = 300 \text{ MeV} - \tau_{in} = 0.12 \text{ fm/c}$, and for $T_{in} = 230 \text{ MeV} - \tau_{in} = 0.6 \text{ fm/c}$ is not large, and therefore contribution of this stage is not very important with respect to the subsequent evolution.

![Graph showing yield of direct photons in Pb+Pb collisions](image)

**FIG. 3.** Yield of direct photons in Pb+Pb collisions at SpS energy with pure hadronic gas EoS for different $T_{in}$: 200 (solid line), 230 (dashed line) and 300 MeV (dotted line). Dots – WA98 data.

We confirm the result found in [5] – ‘reach’ hadron gas EoS allows to describe the experimental $p_t$ distribution of direct photons [3] within pure hadronic scenario. Our best fit value of $T_{in} = 230^{+60}_{-30} \text{ MeV}$ is smaller, than the initial temperature estimated in [5] $T_{in} \approx 260 \text{ MeV}$. There are two reasons of this discrepancy: first is, the initial radial velocity which we included to describe final hadron spectra. In contrast to our approach, authors of [5] made not attempt to describe hadron spectra, and did not consider a nonzero initial radial velocity. The second reason is different normalization to final hadron multiplicity. The authors of [5] used the well known Bjorken formula, relating initial time and temperature with final multiplicity (formula (5) in [5]). But, this formula is valid only for massless final particles and being applied to real hadronic gas leads to an underestimation of the initial
time and, as a consequence, of the thermal photon yield.

Now let us consider EoS incorporating first order phase transition. In this case there is an additional parameter $T_c$, which value is estimated from lattice QCD simulations to be $150 - 170 \text{ MeV}$ [1]. On the other hand $T_c \geq T_{ch}$. Therefore we use $T_c = T_{ch} = 168 \text{ MeV}$ to consider an extreme case with the most intensive QGP production.

As in the case of pure hadronic gas, first we try to describe experimental $p_t$ distribution of $\pi^0$ with zero initial radial velocity, and find almost the same situation: for any reasonable set of model parameters the calculated $p_t$ distribution of $\pi^0$ goes much steeper, and result in unreasonably large $\chi^2_\pi \sim 10^2$/point. So, we find again, that it is necessary to introduce initial radial velocity to describe momentum distributions of final hadrons. Repeating the same procedure as for pure hadronic gas EoS we obtain the following sets of model parameters:

| $T_{in}$, MeV | $\tau_{in}$, fm/c | $V_{max}$, c | Curve on fig. 4 |
|---------------|------------------|-------------|----------------|
| 180           | 2.0              | 0.40        | solid         |
| 200           | 1.5              | 0.38        | dashed        |
| 230           | 1.0              | 0.29        | dotted        |
| 250           | 0.7              | 0.26        | dash-dotted   |

We find the best agreement with experimental data at $T_{in} = 200^{+40}_{-20}$ MeV ($\chi^2_\pi = 3.3$/point, $\chi^2_\gamma = 0.8$/point), what means very short QGP phase and much more prolonged mixed phase.

To conclude, we use 2+1 Bjorken hydrodynamics to analyze hadron yields, and midrapidity $p_t$ distributions of hadrons and direct photons, measured in Pb+Pb collision at SpS energy. We find, that calculations with zero initial velocity, and EoS of hadronic gas accounting contribution of all known hadrons result in too soft $p_t$ distributions of final hadrons, and a reasonable way to describe experimental slopes is to introduce nonzero radial velocity at the beginning of the one-fluid hydrodynamic stage. This initial radial velocity, which might be produced during the pre-equilibrium stage because of the strong radial energy density gradient originated from spherical shapes of colliding nuclei, is not small ($\approx 0.3 c$ near the surface). Therefore the pre-equilibrium stage is expected to be long enough, and, in particular the ‘prompt photons’ have to be taken into account.

An introduction of the initial radial velocity to this model allows to describe simultaneously direct photon and hadron $p_t$ distributions for the both EoS including phase transition and without it. We estimate an initial temperature of the just thermalized state $T_{in} = 200^{+40}_{-20}$ MeV in the case of EoS with phase transition to QGP, and $T_{in} = 230^{+60}_{-30}$ MeV in the pure hadronic scenario. Because the lower bounds of the estimated thermalization temperature are very close to the expected transition temperature we conclude that the SpS data considered in this letter can not distinguish between a production of QGP, and its absence: The initial temperature in central Pb+Pb collisions at SpS is too small, and experimental errors for the direct photon $p_t$ distributions are too large.

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[1] C. Bernard et al., Phys. Rev. D54 (1996) 4585.
[2] R. Albrecht et al., Phys. Rev. Lett. 76 (1996) 3506.
[3] M.M. Aggarwal et al., e-Print Archive: nucl-ex/0006008.
[4] E.V. Shuryak, L. Xiong, Phys.Lett. B 333 (1994) 316.
[5] D.K. Srivastava, B.C. Sinha, Phys. Rev. Lett. 73 (1994) 2421.
[6] D.K. Srivastava, B.C. Sinha, Eur. Phys. J. C(1999).
[7] D.K. Srivastava, B.C. Sinha, e-Print: nucl-th/0006018.
[8] D.K. Srivastava, B.C. Sinha, e-Print: hep-ph/0008074.
[9] M. Gluck, E. Reya, A. Vogt, Z. Phys. C67 (1995) 433.
[10] Ch.-Y. Wong and H. Wang, Phys. Rev. C 58 (1998) 376.
[11] P. Aurenche et al., Phys.Rev. D58 (1998) 085003.
[12] L. Xiong et al., Phys. Rev. D46 (1992) 3798.
[13] Particle Data Group, Phys. J. C 3, (1998).
[14] D. Cleymans et al., Phys. Rev. C 55 (1997) 1431.
[15] P. Braun-Munzinger et al., nucl-th/9903010.
[16] M. Kaneta, NA44 Coll., Nucl.Phys. A 638 (1998) 419c.
[17] P.G. Jones, NA44 Coll., Nucl.Phys. A 610 (1996) 188c.
[18] G. Roland, NA49 Coll., Nucl.Phys. A 638 (1998) 91c.
Appelshauser, NA49 Coll., Phys. Lett. B444 (1998) 523,
S. Margetis, NA49 Coll., J. Phys. G 25 (1999) 189.
[16] E. Andersen, WA97 Coll., J. Phys. G 25 (1999) 171,
E. Andersen, WA97 Coll., Phys. Lett. B449 (1999) 401.
[17] T. Peitzmann, WA98 Coll., Nucl. Phys. A610 (1996) 200.
[18] B. Kampfer et al., J. Phys. G 23 (1997) 2001.
[19] C. Bormann et al., J. Phys. G23:1817-1825,1997.
[20] I.G. Bearden et al., Phys. Lett. B388, 431(1996).
[21] J. Bachler et al., J. Phys. G25:199-207,1999.