Buckling and Post-buckling Analysis of Flatness Defects in High-strength Automotive Sheet

Dai Jie-tao\textsuperscript{1, a}, Zhou Feng\textsuperscript{2*, b}, LI Huiyuan\textsuperscript{3, c}, Li Liejun\textsuperscript{4, d}

\textsuperscript{1}School of mechanical and electrical engineering, Guangzhou University, Guangzhou, 510006, China
\textsuperscript{2}Foshan Polytechnic, Foshan, Guangdong, 528137, China
\textsuperscript{3}Guangzhou JFE steel plate Co., Ltd., Guangzhou, 511464, China
\textsuperscript{4}South China University of Science and Engineering, Guangzhou, 510641, China
\textsuperscript{a}whzf1234@fspt.edu.cn
\textsuperscript{b}djt66008@163.com, \textsuperscript{b}63419822@qq.com, \textsuperscript{c}gwli98@163.com, \textsuperscript{d}liliejun@scut.edu.cn

Abstract. Shape defect is a common problem in the production of automobile plate. But, the wave type function used by previous scholars cannot reflect the actual situation of the site well. To solve this problem, high-order trigonometric functions are used to describe wave patterns, it expresses the fact that the wave pattern does not cover the whole width. Based on the buckling theory of plates, the shape defect was investigated in this paper. Using S. Timoshenko principle of least work, a new mechanical model of longitudinal buckling was established. And the mathematic mode of critical buckling was achieved by Galerkin’s principle of virtual displacement, the path generation of post-buckling was found with Perturbation-Variational Method, and the mode of post-buckling was achieved. The rolling experiment was carried out in test rolling mill. In this experiment, variety of different shape defect model were obtained by designing different roll-shaped curves for work roll, and the mode was good in according to the analytical and numerical results.

1. Introduction

It is well known in the metal forming community that residual stresses can lead to buckling of the strips during the cold rolling process in high-strength automotive sheet. Fig.1 shows some common flatness defects on site. For this problem, many researchers studied deeply. In Analytical calculations, Wistreich\textsuperscript{[1]} proposed strip flatness defects can be attributed to the stability of elastic plate problems in 60s last century, the critical load is the results of residual stress; Roberts\textsuperscript{[2]} gives the different stress distribution corresponding to buckling mode; A generic deflection function was constructed by using cubic spline function, using large deflection theory and the lowest energy principle studied the vertical buckling in cold-rolled strip in [3]; In\textsuperscript{[4,5]}, the author think that the longitudinal buckling is due to the length direction of elongation uneven distribution along the strip width; In Numerical calculations, The finite strip method was applied to solve the shape discrimination mode in cold rolling strip in [6]; In\textsuperscript{[7,8]}, The whole/local buckling and post-buckling were studied by using finite element method. In those papers, The wave-shaped defect described is full board wide, but the wave shape is not full-width in the actual production process. In this paper, the high-order trigonometric function is used to describe the
flatness defect, deals with the experiment of buckling and the analytical of the post-buckling, and the critical buckling value and post-buckling path of the plate shape are solved.

2. The critical buckling conditions

2.1. Deflection function

According to the field data of the longitudinal buckling, which the deflection of the longitudinal buckling is cyclical in the longitudinal direction, and the deflection is different when the form of buckling model is different. When the buckling region is $\Omega[-b,b,-a,a]$, where $b$ is determined by the width of the longitudinal buckling, $a$ is determined by the half-wavelength of the longitudinal buckling, the deflection function can be expressed as follows:

$$W(x, y) = A(\cos \frac{\pi x}{2b})^n \cos \frac{\pi y}{2a}$$

Where: $A$——Amplitude of the deflection;

$n$——The power index, is determined by the wave form of the field measurement.

For example, when $n=9$, the wave shape deflection function simulation diagram of the middle wave at this time is obtained according to formula (1) as shown in the figure 2.

2.2. Boundary conditions

According to the actual situation on site, the deflection should meet the following conditions:
Both sides of the buckling region should meet:

\[
\begin{align*}
M_x(b, y) &= -D \frac{\partial^2 W}{\partial x^2} = 0 \\
M_x(-b, y) &= -D \frac{\partial^2 W}{\partial x^2} = 0
\end{align*}
\]  

(3)

2.3. Boundary load

In order to take unified load form when the buckling is different, this paper defines the boundary load function of the longitudinal buckling as follows:

\[
\sigma_y = \sigma_0 \cdot f(x) = \sigma_0 \left[ \left( \cos \frac{\pi x}{2b} \right)^n - d_y \right]
\]  

(4)

Where, \( \sigma_0 \) is the peak value of the boundary uneven distribution stress.

To consider the stress should meet the balance condition, so:

\[
\int_{-b}^{b} \left\{ \left[ \frac{1}{2} \left( 1 + \cos \frac{2\pi x}{b} \right) \right]^n - d_y \right\} dx = 0
\]  

(5)

Then the \( d_y \) can be solved. The medium wave plate stress diagram is given in fig3.

Fig3 Plate-shaped stress diagram of integral high-order plate-shaped longitudinal buckling

2.4. Critical stress

According to the small deflection theory and the energy principle, in the slightly curved sheet state, a small curved can not lead the stretch of central layer, only the bending strain energy can be generated. So just consider the energy of bending and the work by caused the force in the central plane. Therefore, the critical buckling stress obtained by the following formula:

\[
T = U
\]  

(6)

On the style, the bending energy can be expressed as:

\[
U = \frac{1}{2} \int_{0}^{b} \int_{a}^{h/2} \left[ (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + r_{xy} \gamma_{xy}) \right] \mathrm{dy} \mathrm{dx}
\]  

\[
= \frac{D}{2} \int_{0}^{b} \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + \left( \frac{\partial^2 W}{\partial y^2} \right)^2 + 2\mu \frac{\partial^2 W}{\partial x \partial y} \frac{\partial^2 W}{\partial y^2} + 2(1-\mu) \left[ \frac{\partial^2 W}{\partial x^2} \right] \mathrm{dy}
\]  

(7)

Where, \( D = \frac{Eh^3}{12(1-\mu^2)} \), stands for the stiffness.

The work by caused the force in the central plane can be expressed as:
Equations (6), (7) and (8) constitute the mathematical formulation for solving the critical stress. So, the $\sigma_c$ can be expressed as:

$$
\sigma_c = \frac{1}{h} \int \sigma y \left( \frac{\partial W}{\partial y} \right)^2 \, dy
$$

Substituting deflection function (1) and plate stress function (4) into the Equations (9), the critical stress load can be obtained.

By seeking the minimum critical stress, that:

$$
\frac{\partial \sigma_c}{\partial \alpha} = 0
$$

Through solving the Equations (10), the critical buckling half-wavelength can be obtained, and then the critical buckling stress can be solved. When the half board width $b = 500\, \text{mm}$, thickness $h = 0.5\, \text{mm}$, Elastic Modulus $E = 210\, \text{Gpa}$, Poisson's ratio $\mu = 0.3$, the table 1 gives the critical stress of typical example of different longitudinal buckling.

| Number | Power exponent $n$ | Width of obvious wave area $b_1$/mm | Wave half wavelength $a_1$/mm | Critical load $\sigma_{cr}$/Mpa |
|--------|-------------------|------------------------------------|-------------------------------|-----------------------------|
| 1      | 1                 | 500                                | 500                           | 0.894412                    |
| 2      | 2                 | 450                                | 329.019                       | 1.03709                     |
| 3      | 3                 | 400                                | 288.675                       | 1.12922                     |
| 4      | 4                 | 350                                | 255.664                       | 1.28863                     |
| 5      | 6                 | 300                                | 212.683                       | 1.63845                     |
| 6      | 9                 | 250                                | 175.587                       | 2.17542                     |
| 7      | 14                | 200                                | 141.825                       | 3.06569                     |

3. Solving post-buckling path

Reference [9] [10], this paper uses perturbation method to solve the post-buckling path. In order to simplify the calculation process, this article does not consider the change of buckling area, assuming that the buckling regional of post-buckling process is the same as the critical buckling.

Using the dimensionless perturbation method, the following type into dimensionless form:

$$
\theta = \frac{a}{b}, \quad \xi = \frac{x}{b}, \quad \eta = \frac{y}{a}, \quad w = \frac{W}{h}
$$

$$
\frac{12(1-\mu^2)F}{Eh^2}, \quad N_0 = \frac{12(1-\mu^2)b^2}{Eh^2}, \quad N_h = \frac{N_h}{h}
$$

Where $N_0 = \sigma_c \cdot h$, unit N/mm.

The Equations (11) substituted into the Karman large deflection equations, then:

$$
\left\{ \begin{align*}
\theta^2 \frac{\partial^4 w}{\partial \xi^4} + 2 \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{1}{\theta^2} \frac{\partial^2 w}{\partial \xi^2 \partial \eta^2} &= \frac{\partial^2 f}{\partial \xi^2 \partial \eta^2} \quad (12)
\end{align*} \right.
$$
Selected the center deflection \( (w_0 = w(0, 0) = \frac{W_0}{h}) \) as perturbation parameters, considered the coordination between the load and the bending displacement, the load and displacement are expanded in power series of the perturbation parameters.

\[
\begin{align*}
N_0 &= r_0 + r_2 w_0^2 + r_4 w_0^4 + r_6 w_0^6 + \cdots \\
w &= w(\xi, \eta)w_0^0 + w(\xi, \eta)w_0^2 + w(\xi, \eta)w_0^4 + \cdots 
\end{align*}
\] (13)

Where, the coefficients of the expansion must satisfy the conditions:

\[
w_0(0, 0) = 1, \quad w_2(0, 0) = w_4(0, 0) = \cdots = 0
\] (14)

Through the process of solving the critical buckling stress obtained \( r_0 = N_{cr} \), therefore, the post-buckling path just need get the relationship between the load factor \( r_2, \ r_4, \ r_6 \cdots \) and the deflection parameters \( w(\xi, \eta), \ w(\xi, \eta), \ w(\xi, \eta) \cdots \). To Equation (13) into Equation (12), Get the coefficients of the same power series in equation's both sides and make them equal, then obtained the perturbed equations, all levels of perturbation equations carry on the Galerkin variational, the Perturbation solution be obtained. Then, according Equation (13), the post-buckling path is obtained.

The fig.4 gives the post-buckling mode and the fig.5 gives post-buckling path of the quarter wave. The mode of post-buckling was in good accordance with the field quarter wave.

![Fig.4 The post-buckling mode of the middle wave](image1)

![Fig.5 The path of load-displacement in post-buckling of the middle wave](image2)

4. Experiments

In order to further verify the analytical results and the numerical results, according to the analytical results, the rolling experiment is designed.

4.1. Rolling experiment program

The rolling strip use aluminum sheet, which width is 270mm, thickness is 0.4mm. In the experiment, the reduction rate is 25%; the thickness of the rolled sheet is 0.3mm. According to the analytical method, the critical stress of the rolled aluminum sheet, on this basis, the profile of work-roll is designed. Last, in the
Four roller mill, various wave-shaped are obtained. The fig.6 gives the program of the rolling experiment.

4.2. Buckling mode of rolling experiment
Through the aluminum rolling, obtained the buckling mode of rolled as fig.7. The comparison in the figure shows the shape defects obtained from the test are in good agreement with the simulation results, it reflects the correctness of the theoretical results.

5. Conclusions
In this paper, based on the buckling theory of plates, the longitudinal buckling was investigated in this paper. The following results are obtained:
(1) High-order trigonometric functions are used to describe wave patterns, it expresses the fact that the wave pattern does not cover the whole width.
(2) Using the stress distribution conditions and S.Timoshenko principle of least work, a new mechanical model of longitudinal buckling was established, and the mathematic model of critical buckling was achieved by Galerkin’s principle of virtual displacement, the path generation of
post-buckling was found with perturbation-variational solution, and the mode of post-buckling was achieved. (3) The rolling experiment was carried out in test rolling mill. In this experiment, variety of different longitudinal buckling model were obtained by designing different roll-shaped curves for work roll, and the mode was good in according to the analytical results.

Acknowledgements
The authors gratefully acknowledge the financial support of Guangzhou Science and Technology Plan Project[grant number: 201804010168]; And Natural Science Foundation of Guangdong Province [grant number: 2018A030313287,2017A030313276].

References
[1] Wistreich J G. Control of strip shape during cold rolling[J]. Journal of the Iron and Steel Institute, 1968, 206(12):1203–1206.
[2] Roberts, W.L., Cold rolling of steel[M]. Dekker, New York, NY, 1978.
[3] Bian Yuhong, Liu Hongmin. Universal method analyzing the large deflection buckling deformation of rolled strip [J]. Chinese Journal of Mechanical Engineering, 1994, 30(Supp): 21–27.
[4] Yang Quan, Study on the cold rolled strip buckling and the target shape in the automatic flatness control[D]. Dissertation: University of Science and Technology Beijing, 1992.5.
[5] Yang Quan, Chen Xianlin. The deforming route of buckled waves of rolled Strip [J]. Journal of University of Science and Technology Beijing [J]. 1994, 16(1): 53-57.
[6] Lin Zhenbo, Finite strip method analysis on shape discrimination model in cold strip rolling mill [D]. Dissertation: Yanshan University, 1993.9.
[7] Qing Weijie, Yang Quan. Study on cold rolled strip global and local buckling, post-buckling using the finite element method[J]. Journal of University of Science and Technology Beijing, 2000, 22(4): 377–380.
[8] Qing Weijie, Study on the target shape in the automatic flatness control of WISGCO1700mm five-stand cold tandem rolling mill [D]. Dissertation:University of Science and Technology Beijing, 2000.3.
[9] Chang Tiezhu, Deformation of Herringbone Buckling and Transverse Buckling for Thin and Wide Strip [D]. Dissertation:University of Science and Technology Beijing, 2009.6.
[10] Chang Tiezhu, Zhang Qingdong, Huang Shiqing. Analysis of transverse buckling for thin strip[J]. China Mechanical Engineering, 2009, 20(18): 2255–2259.