Amplitude scintillation effects on SAR

David P. Belcher, Paul S. Cannon

University of Birmingham, Edgbaston, Birmingham B15 2TT, UK
E-mail: d.belcher@bham.ac.uk

Abstract: Space-based low-frequency synthetic aperture radar (SAR) is affected by the ionosphere, which induces both phase and amplitude fluctuations, known as scintillation, into the radar signal. This paper describes the effect of amplitude scintillation on SAR imagery. The two-way amplitude and intensity probability density functions (pdf) for both monostatic and bistatic SAR are derived from the one-way Nakagami-m distribution. The moments are then used to determine the SAR radiometric calibration error and image contrast from the one-way S4 index. It is also shown that monostatic SAR experiences an S4-dependent radar cross-section (RCS) enhancement that is not experienced by bistatic SAR. The anisotropy of the ionospheric irregularities strongly affects the degree to which amplitude scintillation will be visible in SAR imagery. The description of anisotropic effects is reviewed and extended to cover SAR. The variation over the Earth is illustrated, showing that a sun-synchronous satellite will experience the strongest effect near Brazil. Two PALSAR images of the same area of Brazilian rainforest are compared, one of which shows azimuthal streaking, corresponding to an amplitude modulation of ± 1 dB. The one-way S4 index is determined from this imagery using both the RCS enhancement and image contrast measures of S4, which produce similar results.

1 Introduction

Space-based low-frequency synthetic aperture radar (SAR) is a particularly useful all-weather remote sensing tool. However, it can be affected by the ionosphere, which induces both phase and amplitude fluctuations, known as scintillation, into the radar signal passing through it. The statistics of a signal received on the ground from a space-based transmitter have been studied for many years [1, 2] and can also be applied to the two-way transit through the ionosphere that occurs in radar [3].

Previous work has considered the effect of phase scintillation on the SAR image [3–5] but has not considered the effect of amplitude scintillation, which is one of the most visible effects [6]. Under the weak scattering approximation, phase and amplitude scintillation are separable [7] and can be considered independently. This paper examines the effect of amplitude scintillation on the SAR image, which is usually observed as azimuthal streaking [8].

In Section 2, the traditional Nakagami description of one-way intensity scintillation is reviewed and extended to cover both monostatic and bistatic SAR. It is shown that the ionosphere enhances the apparent radar cross section (RCS) of monostatic backscatter, must attenuate it at small bistatic angles and does not affect the backscatter RCS at large bistatic angles. The intensity modulation that is seen in a SAR image is derived in terms of the one-way S4 index, and shows a strong dependence on the anisotropy of the ionosphere’s irregularities.

In Section 3, the anisotropy of the ionospheric irregularities is reviewed and applied to cover SAR imaging. It is shown that the elongation of ionospheric irregularities along magnetic field lines, and the alignment of the satellite track to the field, strongly determines the effects of the anisotropy. For a sun-synchronous satellite, the effects are most likely to occur near Brazil, close to the geomagnetic South Atlantic anomaly.

In Section 4, two PALSAR images of the same area of Brazilian rainforest are compared, only one of which exhibits azimuthal streaking. It is shown that this streaking must be the effect of ionospheric interference, and that the brightening of the streaking is due to RCS enhancement. The contrast of the smoothed SAR image is measured and the one-way S4 is inferred from that measurement as well as the RCS enhancement.

Finally, it is concluded that although the images were inhomogeneous, the two measures of S4 produced broadly comparable results, the main unknown being the estimates of elongation along field lines. The correlation of the amplitude scintillation over the Fresnel zone should allow correction for amplitude scintillation to be made.

2 Intensity and amplitude scintillation

2.1 One-way scintillation

The statistical properties of the scintillation experienced by a constant signal passing through the ionosphere have been studied for many years. Nakagami [1] showed that, at least approximately, the Nakagami-m distribution arises from the coherent random sum of vectors whose amplitude is Gaussian. It is therefore an excellent fit to the intensity of a trans-ionospheric signal that arises from the addition of
many different signal paths through the ionosphere. This has been confirmed experimentally both by Nakagami and independently by Fremouw [9], who showed that it is the best model of one-way intensity scintillation.

The probability density function (pdf) of one-way signal intensity \( I \) is a special case of the gamma distribution

\[
p_I(I) = \frac{b}{\Gamma(m)} (bI)^{m-1} \exp[-bI]
\]

where \( m \) is the order, shape or fading parameter of the gamma pdf. In the special case of

\[
b = \frac{m}{\mu}
\]

where \( \mu \) is the mean, the pdf is a Nakagami-m distribution, a sub-class of the gamma pdf. This pdf has the convenient property that as \( m \to \infty \), the pdf becomes a delta function centred on the mean intensity \( \mu \) (i.e. no scintillation). At \( m = 1 \), the pdf becomes a negative exponential in intensity, and therefore Rayleigh in amplitude and Gaussian in complex components. This therefore corresponds to saturated scintillation, where the phase is essentially random. It is also worth noting that the underlying RCS of typical radar clutter also has a Nakagami-m distribution.

The pdf can be characterised by its normalised moments, which are given by

\[
\langle I^n \rangle \langle I \rangle^2 = \mu^{-n} \int_0^\infty I^n p_I(I)dI
\]

where \( \langle \rangle \) denotes the ensemble average and \( \mu = \langle I \rangle \). For the gamma pdf, these are

\[
\langle I^n \rangle \langle I \rangle^2 = \frac{\Gamma(m+n)}{m^2 \Gamma(m)}
\]

The moments are independent of both \( b \) and the mean \( \mu \), are a property of any gamma pdf and are not confined to a Nakagami-m distribution.

An important method of characterising the intensity scintillation is by measuring the \( S_4 \) index, which is defined as the standard deviation of intensity divided by the mean intensity [10]

\[
S_4 = \frac{\langle (I - \langle I \rangle)^2 \rangle}{\langle I \rangle^2} = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1
\]

and can therefore be obtained from the normalised second intensity moment. From (4) and (5)

\[
m = \frac{1}{S_4^2}
\]

The value of \( m \) is often estimated from \( S_4 \) using (6), but it is not the only moment that can be used to estimate \( m \) and is not optimum [11]. The one-way amplitude pdf can be derived from the intensity pdf by noting that \( p_A(A) = 2A p_I(A^2) \), where \( p_A(A) \) is the intensity pdf and \( p_A(A) \) is the amplitude pdf, to give a root gamma pdf.

### 2.2 Monostatic radar

In the case of a monostatic radar, where the transmitter and receiver are located on the same spacecraft, the path travelled through the ionosphere down to the ground target is the same as that on the up path back to the radar. Although the spacecraft does move along its orbit during the pulse transit time, for an L-band SAR the difference between the up and down paths is sufficiently small to be ignored. Assuming perfect correlation, the two-way amplitude modulation will therefore be the square of the one-way amplitude. Since the one-way amplitude is root gamma distributed, the two-way amplitude will have a gamma pdf

\[
p_2(A) = \frac{b}{\Gamma(m)} (bA)^{m-1} \exp[-bA]
\]

where \( b = m\mu_A \) and \( \mu_A \) is the mean two-way amplitude. The two-way amplitude distribution is thus a Nakagami-m distribution, and the amplitude moments are as given in (4) for the intensity moments of a gamma pdf.

For a radar performing coherent addition of \( N \) pulses, the addition will take place in amplitude. The resultant pdf will therefore be the \( N \)-fold convolution of the amplitude pdf, or alternatively, the amplitude characteristic function raised to the power \( N \). For the gamma pdf, the addition of \( N \) samples with the same mean results in the same pdf, but with the order parameter \( m \) increased to \( Nm \). However, for an MTI radar that performs integration over short intervals, all of the radar pulses will be very highly correlated in amplitude and thus the effective \( N \approx 1 \). For a SAR though, \( N \) can be large, depending on the synthetic aperture length.

The two-way monostatic intensity pdf can be obtained from the amplitude pdf by noting that \( p_I(I) = p_A(\sqrt{I})/(2\sqrt{I}) \), to give

\[
p_2(I) = \frac{b}{2\sqrt{I} \Gamma(m) } \left( b\sqrt{I} \right)^{m-1} \exp \left[ -b\sqrt{I} \right]
\]

The intensity moments can be obtained from (8) by direct integration, or by noting that

\[
\frac{\langle I^n \rangle}{\langle I \rangle^2} = \frac{\langle A^{2n} \rangle}{\langle A^2 \rangle^2} = \frac{\langle A^2 \rangle^n}{\langle A^2 \rangle^n}
\]

which is a general relationship between amplitude and intensity moments. In the case of a gamma distributed amplitude, the intensity moments are

\[
\frac{\langle I^n \rangle}{\langle I \rangle^2} = \frac{\langle A^{2n} \rangle}{\langle A^2 \rangle^2} = \frac{\Gamma(m+2n) \Gamma(m)}{(\Gamma(m+2))^2}
\]

An important parameter for quantifying the intensity scintillation is the \( S_4 \) index, and using the above, the square of the two-way \( S_4 \) index on a monostatic channel is

\[
S_{4\times2} = \frac{4m+6}{m(m+1)}
\]

which has been previously derived in [12]. Substituting for
the one-way $S_4$

$$S_{4\times 2}^2 = 4S_2^2 + 2\frac{S_4^4}{S_2^4 + 1}$$ (12)

a result previously obtained in [13]. When $S_4$ is small, the two-way $S_4$ is thus twice the one-way $S_4$.

### 2.3 Bistatic radar

Although there are currently no space-based bistatic SARs, airborne imagery has been generated [14] and space-based operation is possible. It is therefore useful to consider the case of a bistatic radar, where the receiver and transmitter are located on different spacecraft. In this geometry, it is highly unlikely that the down and up paths through the ionosphere will be the same, and the amplitude on both paths will be essentially uncorrelated random variables. The two-way intensity will therefore be the product of two independent gamma-distributed random variables, the down path having a mean $\mu_d$ and order $m_d$ and the up path a mean $\mu_u$ and order $m_u$.

The pdf of the product of two gamma distributed random variables is given by [15]

$$p_{2b}(I) = \frac{2(I)^{m_d-1}}{\Gamma(m_d)\Gamma(m_u)} \left(b\sqrt{I}\right)^{m_u+m_d}K_{m_u-m_d}(2b\sqrt{I})$$ (13)

where $K_n(x)$ is the modified Bessel function of order $n$, and in the bistatic case

$$b = \frac{m_um_u}{\mu_d\mu_u}$$ (14)

The characteristic function of this pdf is such that averaging intensity samples does not change the mean or the pdf but just increases $\mu$ and order $m$, respectively [15]. In the special case of $m_d = m_u = 1$, this pdf reduces to that previously derived in [16].

The moments of the bistatic intensity pdf are given by [17]

$$\frac{\langle I^n \rangle}{\langle I \rangle^m} = \frac{\Gamma(m_d+n)}{m_d^n\Gamma(m_d)}\left(\frac{\Gamma(m_u+n)}{m_u^n\Gamma(m_u)}\right)$$ (15)

From this, the two-way bistatic $S_4$ index is

$$S_{4\times 2b}^2 = S_{4u}^2 + S_{4d}^2 + S_{4u}S_{4d}$$ (16)

where $S_{4u} = 1/\sqrt{m_d}$ is the scintillation index on the up path, and $S_{4d} = 1/\sqrt{m_d}$ is the scintillation index on the down path. In the special case of either of the $S_4$ indices on the up or down path equaling zero, the two-way $S_4$ will be equal to the one-way $S_4$. In practice, this means the largest one-way $S_4$ will dominate the two-way $S_4$. When the $S_4$ index is the same on both bistatic paths

$$S_{4\times 2b}^2 = 2S_2^2 + S_4^2$$ (17)

which is the result first derived by [13] for an uncorrelated up and down paths. For small values of $S_4$, the two-way bistatic $S_4$ is thus $\sqrt{2}$ times the one-way $S_4$, and $\sqrt{2}$ less than the equivalent two-way monostatic $S_4$.

From (13), the bistatic amplitude pdf is

$$p_{2b}(A) = \frac{4}{\phi\Gamma(m_d)\Gamma(m_u)} (bA)^{m_{u+d}+m_d} K_{m_{u+d}-m_d}(2bA)$$ (18)

Although similar to the intensity pdf, adding bistatic amplitude samples does not have a simple solution for the pdf. By direct integration, the bistatic amplitude moments are

$$\langle A^n \rangle = \frac{\Gamma(m_d+n/2)}{m_d^{n/2}\Gamma(m_d)}\left(\frac{\Gamma(m_u+n/2)}{m_u^{n/2}\Gamma(m_u)}\right)$$ (19)

The 2$\text{nd}$ amplitude moment is therefore equal to the $n$th moment of intensity.

The general case, when the intensity of the upgoing signal is partly correlated with the downgoing signal, can be accommodated by a linear combination of the two cases. In practice, for a SAR, the intensity is only correlated over small distances (of the order of a Fresnel zone) and so the statistics are usually either fully correlated (monostatic) or fully uncorrelated (bistatic). The case of a ground-based radar imaging a point target (not a diffuse one) under Rayleigh fading conditions ($S_4 = 1$) when partial correlation occurs has been considered by [16].

### 2.4 RCS enhancement

Knapp and Houpt [18] have shown that, for a ground-based radar tracking an orbiting point-like calibration sphere, the mean two-way backscattered intensity is increased under ionospheric scintillation. This is because when scintillation occurs, there are many more ray paths to the same point, as other parts of the radar beam can illuminate the sphere as well as the direct specular ray. The two-way monostatic mean is given by

$$\mu_2 = \int_0^\infty I p_2(I)dI = \mu^2\left(1 + \frac{1}{m}\right)$$ (20)

Since the mean is $\mu^2$ when there is no ionospheric disturbance, the mean RCS is enhanced by a factor of $1 + 1/m = 1 + S_4^2$. This surprising result has been confirmed experimentally [18] and is also observed with optical phase screens [19].

The bistatic mean may be determined in a similar manner

$$\mu_{2b} = \int_0^\infty I p_{2b}(I)dI = \mu_u\mu_d$$ (21)

Thus in the uncorrelated bistatic case, the scintillation does not affect the mean radar signal power.

However, the backscattered power integrated over all angles must be conserved, so therefore anti-correlations must occur between the up and down paths at some bistatic angles. Such anti-correlations cannot occur at large bistatic angles because the radar pulse travels through completely different parts of the ionosphere’s phase screen on the down and up paths, which must therefore be uncorrelated.

Given that the variance of the phase scintillation over the Fresnel zone is twice $1/m$, which is the RCS enhancement, it is likely that the correlated monostatic case applies only to bistatic angles within the cone of the Fresnel zone. Immediately outside the Fresnel zone, the radar’s phase is no longer stationary but adds destructively, and it is likely
that this is the region where anti-correlations occur. A power-conserving RCS reduction will therefore be observed at small bistatic angles. At large bistatic angles, there is no correlation between the up and down paths and therefore no RCS change. It is also worth noting that an X-band SAR (which has a small Fresnel zone) operating at the longest ranges may be in the small bistatic angles regime owing to the movement of the satellite between pulse transmission and reception.

It is not clear whether the RCS enhancement also applies to clutter backscatter as well as point targets, partly because shadowing effects can occur, but monostatic backscatter enhancement seems to be a fundamental property of a phase screen [13, 19]. It should therefore be observable by a monostatic SAR, and therefore provides another method of measuring the one-way $S_4$ from the image.

### 2.5 Application to SAR

For a SAR that integrates many radar pulses to form an image, the effects of amplitude scintillation will be determined by the stochastic sum over many amplitude samples, which are highly correlated over a Fresnel zone. The synthetic aperture will be longer than a Fresnel zone (by definition, because otherwise SAR would merely be Doppler beam sharpening), so the synthetic aperture will effectively integrate $N$ independent Fresnel zones together. For monostatic SAR, whose amplitude is gamma distributed, the new order parameter will therefore be $Nm$, but for bistatic SAR, it is necessary to assume that $N$ is large, so that a Gaussian approximation of adding variances can be used. This approximation is, of course, the same as multi-look averaging of intensity images.

In the monostatic case, from (12), the square of the standard deviation of intensity divided by the mean intensity after integrating $N$ independent samples is

$$c_d^2 = \frac{4S_4^2}{N} + \frac{S_4^4/N}{2S_4^2 + N} \approx \frac{4S_4^2}{N}$$

(22)

The approximation is valid for large $N$ and is less by a factor of 2 for bistatic SAR. If the SAR image has no underlying terrain variation and the speckle has been averaged out, $c_d$ is the same as the image contrast.

A reasonable measure of the effect of amplitude scintillation on the radiometric calibration error is to add two standard deviations to the mean and then normalise by the mean. Under Gaussian statistics, this error value will encompass 95% of values. Thus the radiometric calibration error due to intensity scintillation is, in dB

$$10 \log_{10}(1 + 4S_4/\sqrt{N})$$

(23)

For $N=100$ and a one-way $S_4=0.25$, the radiometric calibration error due to amplitude scintillation effects alone is approximately 0.4 dB. This random error will vary over the image.

The number of independent amplitude scintillation samples is given by

$$N = N_A N_B$$

(24)

where $N_A$ is the number of independent amplitude scintillation samples in the along-track direction, and $N_B$ is the number of samples over the RF bandwidth.

The decorrelation over the signal bandwidth is a function of the scintillation strength, but for weak scattering (not strong [20]) it can be shown that the difference in propagation paths across the bandwidth is negligible and the scintillation can be considered to occur only at the centre frequency [3]. Thus for a SAR, it is reasonable to assume that $N_B = 1$, even for large bandwidths.

The effective number of independent along track amplitude samples is the coherence length in the ionosphere $L_C$ divided by the Fresnel zone diameter

$$N_A = \frac{L_C}{\sqrt{\lambda_0 z_R \sec \theta}} = \frac{L_{SA}}{\gamma \sqrt{\lambda_0 z_R \sec \theta}}$$

(25)

where $z_R$ is the distance to the ionosphere from a ground target, suitably adjusted for wavefront curvature from the satellite, $\lambda_0$ is the centre wavelength and $\theta$ is the incidence angle [2, 3]. For a focused SAR image to be formed, the signal must be coherent over the synthetic aperture length $L_{SA}$, which in turn means that the phase over a shorter distance $L_C$ in the ionosphere must be less than $\pi$, the ratio $L_{SA}/L_C$ being $\gamma$. The value of $\gamma$ is strongly affected by the anisotropy, which is examined in the following section.

### 3 Anisotropy

#### 3.1 Introduction

In an isotropic ionosphere, the variation in the number of free electrons per m$^3$ obeys a three-dimensional power law spectrum. This three-dimensional spectrum can be then integrated in the vertical direction to give a two-dimensional spectrum of total electron content (TEC) per m$^2$, which forms the basis of the phase screen. It can be then integrated again to give the one-dimensional spectrum of in situ electrons per metre, each integration increasing the power law spectral index by one [21]. For this integration to be performed in an anisotropic ionosphere, in which the irregularities are elongated along and across magnetic field lines, a transformation to an isotropic co-ordinate system is performed, using a matrix, $C$ [2, 22, 23]. This allows the projection of a shadow pattern of the three-dimensional electron density irregularity structure onto a two-dimensional horizontal phase screen. The intersection of the line of sight from the radar to a ground target with the phase screen, which occurs at the ionospheric penetration point (IPP), then determines the phase modulation of the radar pulse. As the radar moves along the synthetic aperture at velocity $v_{SAR}$, the IPP moves along the phase screen at velocity $v_{IPP}$ at an angle $\theta_{IPP}$ relative to magnetic North.

The main co-ordinate system used is a Cartesian system centred at the IPP, which moves with the IPP, see Fig. 1. The $z$-axis points vertically downwards, the $x$ and $y$ axes being aligned with the magnetic North and East, respectively, the $xy$ plane being horizontal. The magnetic field lines dip at an angle $\psi$ to the horizontal along the northwards pointing $x$-axis, and at a usually small angle $\delta$ to the $y$-axis. The ionospheric irregularities are elongated along magnetic field lines by a factor of $a$ in the $xz$ plane and a factor of $b$ in the $yz$ plane, both being elongated relative to the vertical direction [22]. Equatorial field aligned rod-like irregularities are therefore represented by large $a$ values, and the sheet-like irregularities that occur...
The radar pulse itself propagates at an incidence angle $\theta$ to the phase screen (located in the $xy$ plane) and at an azimuth angle of $\varphi$ from the $x$-axis, the polar co-ordinates of its propagation vector therefore being $(\theta, \varphi)$. The IPP scans along the phase screen at an angle $\theta_{IPP}$ relative to the $x$-axis, at a velocity of $v_{IPP}$. The anisotropy is accommodated by reducing this velocity to an effective scan velocity $v_{eff}$, thus shifting the phase spectrum of the ionospheric modulation down in frequency. The effective velocity of the scan is given by [23]

$$v_{eff} = \sqrt{\frac{Cv_x^2 - Bv_yv_x + Av_y^2}{AC - B^2/4}}$$

(26)

where $(v_x, v_y)$ are the velocity components of the IPP and the values of $A$, $B$ and $C$ depend only on the elongation and the angles of propagation and magnetic field, and represent the anisotropy. An isotropic ionosphere is represented by $A = C = 1$, $B = 0$ and therefore $v_{eff} = v_{IPP}$. The values of $A$, $B$ and $C$ have a complicated dependence on the angles and are listed correctly in the Appendix of [2].

The velocity vector of a SAR satellite relative to the ground can be calculated from the orbital inclination, latitude and component of Earth rotation [24]. This velocity vector can be then projected into the phase screen at the IPP and the scan angle relative to the local magnetic North, $\theta_{IPP}$, determined. The phase screen velocity components are then

$$v_x = v_{IPP} \cos \theta_{IPP}$$
$$v_y = v_{IPP} \sin \theta_{IPP}$$

(27)

The vertical velocity component of the satellite, $v_z$, which is usually zero for a circularly-orbiting, broadside-looking SAR, can be included by projection into the $(x, y)$ plane [2]. The effective scan velocity can also accommodate the drift velocity of the ionosphere (generally about 10 m/s at mid-latitudes, 100 m/s in the equatorial region and can be higher still at auroral latitudes [25]).

The ionosphere’s effect on a SAR image is usually described not by the frequency spectrum, but by the spatial spectrum [3]. As the SAR moves along the synthetic aperture of length $L_{SA}$, the IPP moves a distance $L_C$ along the phase screen, the ratio between the two being $\gamma = L_{SA}/L_C$. If the time taken by the SAR to travel along the synthetic aperture is $T_{SA}$, then $L_{SA} = v_{SAR} T_{SA}$ and $L_C = v_{eff} T_{SA}$. The ratio between the spatial distance along the aperture and the effective distance along the phase screen is therefore given by

$$\gamma = \frac{v_{SAR}}{v_{eff}}$$

(28)

In an anisotropic ionosphere, $v_{eff}$ is typically smaller than $v_{IPP}$, so $\gamma$ is larger in an ionosphere with irregularities elongated along field lines.

The magnitude of the spectrum is also affected by the anisotropy, which is accommodated by multiplying the spectrum by a geometric factor $G$, as given correctly in the Appendix of [2]

$$G = \frac{ab \sec \theta}{\sqrt{AC - B^2/4}}$$

(29)

As far as the SAR point spread function is concerned, the effect of anisotropy is entirely contained within the values of $G$ and $\gamma$. The anisotropy varies sufficiently slowly that $G$ and $\gamma$ can be considered constant over the synthetic aperture, and indeed over much larger distances as well [26].

### 3.2 Special cases

Although the ionospheric geometry of [2, 22, 23] is completely general, it is instructive to consider certain special cases. In the following, the vertical velocity component of the satellite, the ionospheric drift and Earth rotation will therefore be neglected.

#### 3.2.1 Isotropic irregularities: If the ionospheric irregularities are isotropic ($a = b = 1$), then the geometric factor $G$ is always unity, and the effective velocity $v_{eff}$ reduces to the velocity component perpendicular to the line of sight [23]. The isotropic value of $\gamma$ for a SAR in circular orbit is therefore

$$\gamma_{iso} = \frac{v_{SAR}}{\sqrt{v_x^2 + v_y^2}} = \frac{H_{SAR}}{H_{ion}}$$

(30)

where $H_{SAR}$ is the orbital height of the SAR and $H_{ion}$ is the height of the phase screen.

#### 3.2.2 Equatorial ionosphere: For an ionosphere with a horizontal magnetic field (no dips in either direction), the value of $G$ is given by

$$G = \frac{ab \sec \theta}{\sqrt{a^2 b^2 + (a^2 \sin^2 \varphi + b^2 \cos^2 \varphi) \tan^2 \theta}}$$

(31)

If it is also assumed that the SAR images at broadside, so that
\[ \varphi = \theta_{\text{app}} \pm 90^\circ, \text{ then} \]
\[ \gamma = \gamma_{\text{iso}} \sqrt{\frac{a^2 b^2 + (a^2 \sin^2 \varphi + b^2 \cos^2 \varphi) \tan^2 \theta}{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi + \tan^2 \theta}} \] (32)

In the equatorial ionosphere, the irregularities tend to be rod-like, elongated along the field lines, so \( b \approx 1 \) and typically \( a \approx 30 \). For a SAR flying perpendicular to the magnetic field (i.e. equatorial rather than polar orbit) \( \gamma = \gamma_{\text{iso}} \) regardless of the amount of elongation along field lines, whereas \( G \rightarrow \sec \theta \) as \( a \rightarrow \infty \). In practice, any \( a > 20 \) closely approximates infinite elongation, at which point \( G = \sec \theta \).

For a SAR in polar orbit, flying along the field lines, \( G = 1 \) and \( \gamma = a \gamma_{\text{iso}} \), and the effective velocity is reduced by a factor of \( a \). This results in a considerable reduction in the rate of phase modulation along the synthetic aperture, and a consequent rise in the side lobes closest to the mainlobe peak (located at zero frequency). Since the value of \( \gamma \) will increase by the elongation factor \( a \), the number of independent Fresnel zones decreases by the same amount (causing slow fading conditions). The variance of the amplitude modulation in the image will therefore be at least \( a \) times greater than the anisotropy.

3.2.3 High latitude ionosphere: Nearer the poles, flat sheet-like irregularities tend to form, inclined with the magnetic latitude, so both \( a > 1 \) and \( b > 1 \). This case does not have a simple solution, and the full geometry given in the Appendix of [2] must be used. However, for sheet-like irregularities, the values of \( G \) and \( \gamma \) are generally between their isotropic values and the equatorial values given above with \( a = b > 1 \). As the sheet-like irregularities become square (\( a \rightarrow b \)), the dependence on the propagation angle \( \varphi \) tends to disappear and the effects of anisotropy become independent of the orbital direction of the satellite.

3.2.4 Any ionosphere: The value of \( G \) is not critical since it generally varies between 1 and \( \sec \theta \), whereas \( \gamma \) can be multiplied by the elongation factor \( a \) when flying along field lines.

3.3 Earth’s ionosphere

Although scintillation conditions vary considerably, the anisotropy of the Earth’s ionospheric irregularities is relatively stable, and depends largely on the magnetic field for its shape. Empirical models have been constructed [25] which can be combined with the satellite orbital geometry [24] to determine \( v_{\text{eff}} \) and hence \( \gamma \).

Fig. 2 shows the value of \( \gamma \) as a function of latitude and longitude for PALSAR [27], assuming an ionosphere height \( H_{\text{ion}} \) of 350 km. The magnetic field data was obtained from the International Geomagnetic Reference Field (IGRF) and assumed a magnetic activity index \( K_p = 3.66 \). The anisotropy of the irregularities is only very weakly dependent on \( K_p \) and the local time.

The main features of the variation of \( \gamma \) are essentially the same for all sun-synchronous satellites, since they have similar inclination angles, and the variation with orbital altitude is small. Anisotropic effects have previously been observed over Brazil [6] and this is precisely the area where \( \gamma \) is at its highest over the land. The value of \( \gamma \) is very sensitive to the value of \( a \), and shows different characteristics when imaging is performed on the descending node, the peak \( \gamma \) being over equatorial Asia.

4 Experimental data

4.1 PALSAR radar

PALSAR was an L-band space-based SAR that could produce single-look complex imagery at 5 m resolution. It was in a sun-synchronous orbit with a repeat cycle of 46 days and was normally operated at a fixed incidence angle, the main parameters of which are summarised in Table 1. [27, 28]. During its operational life, it produced many data sets, a tiny minority of which showed ionospheric effects, most noticeably azimuthal streaking [29] producing stripes in the imagery [6], which has also been observed in Radarsat imagery [8].

The repeat ground tracks and fixed incidence angle allow PALSAR data from different passes to be compared. Two
images of the same area of Brazilian rainforest have been identified as suitable for comparison [29]. One was collected on 25 December 2007 and was unaffected by the ionosphere, and the other was collected on 26 March 2008 and shows azimuthal streaking. Part of this full PALSAR scene, 19.2 km by 9.8 km, is shown in Fig. 3.

4.2 Estimating $S_4$

An image region, shown by the box in Fig. 3, was extracted from the PALSAR scene. This region was selected because in the undisturbed image, its cross-section is constant. This region was further subdivided into vertical sub-image strips just 50 pixels (234 m) wide in the range direction and 1024 pixels (3.2 km) long in the along-track (vertical) direction. These sub-images were then averaged to determine their mean backscatter RCS, shown in Fig. 4.

As can be seen, the total backscatter variation of the disturbed image is not large (about ± 1 dB) but clearly shows a streak of decreased RCS as well as streaks of raised RCS relative to the undisturbed scene. This effect can only occur as a result of interference causing amplitude modulation; it cannot be the result of additive noise. Furthermore, the RCS change observed over the streaks in Fig. 3 occur equally across all clutter types, not just forest, but over rivers as well, thus eliminating changes in clutter backscatter as an explanation. It is also clear that the average RCS of the disturbed image is above that of the undisturbed one. This 0.33 dB mean RCS enhancement is to be expected from a monostatic SAR, and from (20) is equivalent to an $S_4$ of 0.28.

The $S_4$ index can also be estimated from the variation in intensity shown in Fig. 4. The contrast (standard deviation of intensity divided by the mean intensity) calculated from the smoothed image intensity data of Fig. 4 is shown in Table 2. As the undisturbed image was of almost constant RCS, no adjustment for variation in the RCS of the underlying terrain was necessary, but calculating the contrast on the undisturbed image nonetheless acts as a control. The estimated values of $N$ and $\gamma$ are also shown in Table 2, and using (22) the value of the one-way $S_4$ can therefore also be estimated. Although the $S_4$ value from the undisturbed image represents the random error, there is considerable uncertainty about the value of $\gamma$ since no actual measurements of the elongation along magnetic field lines were made.

The estimated disturbed one-way $S_4$ from the contrast measure, although statistically accurate to 0.03, depends very heavily on the estimated value of $N$, and also requires the assumption of a statistically invariant ionosphere. From Fig. 3, the ionospheric disturbance does vary over the image, and must therefore vary over the synthetic aperture, thus violating the assumptions. The $S_4$ derived from the backscatter RCS enhancement is also difficult to measure accurately because backscatter changes, for example due to changes in forest moisture, strongly affect the measurement.

---

**Table 1** PALSAR data set parameters

| Operational life          | Jan 2006-Mar 2011 |
|---------------------------|-------------------|
| centre frequency          | 1270 MHz          |
| altitude (nominal)        | 698.546 km        |
| inclination angle         | 98.2°             |
| equator crossing (local time) (ascending node) | 22:25 |
| antenna length            | 8.9 m (azimuth)   |
| look direction            | right             |
| incidence angle           | 34.3°             |
| polarisation              | HH                |
| resolution (rg × az)      | 5.35 × 4.6 m      |
| pixel sampling (rg × az)  | 4.684 × 3.197 m   |
| scene centre (lat ,long)  | 4.052°S, 67.909°W |

**Table 2** $S_4$ derived from the image contrast and related quantities

| Measurement                  | Disturbed | Undisturbed |
|------------------------------|-----------|-------------|
| contrast of smoothed image intensity | 0.114     | 0.018       |
| estimated $\gamma$           | 6.5       | 13.7        |
| estimated $N$                |           |             |
| $S_4$ (one-way)              | 0.21      | 0.03        |

---

Fig. 3  Disturbed PALSAR image of Brazilian rainforest, showing azimuth streaking (slant range horizontal)

Fig. 4  Mean backscatter (dBm²/m²) as a function of range for the undisturbed (+) and disturbed (X) image

---

This is an open access article published by the IET under the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0/).
However, the two measures of $S_3$ are broadly comparable (0.21 and 0.28) and are entirely within the values likely to be encountered during scintillation events. Indeed, it is only the elongation along field lines that makes the effect visible.

5 Conclusions

The one-way intensity scintillation, normally described using Nakagami-m statistics, can be extended to two-way propagation that is experienced by a radar. The amplitude scintillation increases the apparent RCS observed by a monostatic radar, but does not affect the RCS observed by a bistatic radar at large angles. Conservation of energy must therefore occur. The accuracy of radiometric calibration is still affected by the Fresnel zone and therefore the scintillation averages out. The accuracy of other statistical measures, such as the K-distribution order parameter [5]. Correction for amplitude scintillation should be relatively straightforward [30], especially if previous imagery is available of the same place, owing to the high degree of correlation of the amplitude over a Fresnel zone.

The focusing effect of a constant slab of high TEC has not been examined, but this too can be corrected by using image displacement as a measure of TEC. It should therefore be possible to achieve a high quality radiometric calibration by correcting for amplitude scintillation.

6 Acknowledgments

The authors wish to thank JAXA for the PALSAR data used in this paper. This project was funded by EPSRC grant number EP/I013601/1.

7 References

[1] Nakagami, M.: ‘The m-distribution – a general formula of intensity distribution of rapid fading’, in Hoffman, W.C. (ed.): ‘Statistical methods in radio wave propagation’ (Pergamon, 1960)
[2] Rino, C.L.: ‘On the application of phase screen models to the interpretation of ionospheric scintillation data’, Radio Sci., 1982, 17, (4), pp. 855–867
[3] Belcher, D.P.: ‘Theoretical limits on SAR imposed by the ionosphere’, IET Proc. Radar Sonar Navig., 2008, 2, (6), pp. 435–444, doi: 10.1049/iet-rsn:20070188
[4] Belcher, D.P., Rogers, N.C.: ‘Theory and simulation of ionospheric effects on SAR’, IET Proc. Radar Sonar Navig., 2009, 3, (5), pp. 541–551, doi: 10.1049/iet-rsn.2008.0205
[5] Belcher, D.P., Cannon, P.S.: ‘Ionospheric effects on synthetic aperture radar (SAR) clutter statistics’, IET Radar Sonar Navig., 2013, 7, (9), pp. 1004–1011, doi: 10.1049/iet-rsn.2012.0227
[6] Shimada, M., Muraki, Y., Otsuka, Y.: ‘Discovery of anomalous stripes over the Amazon by the PALSAR onboard ALOS satellite’. Proc. IGARSS 2008, Boston, MA, 7–11 July 2008
[7] Blacker, D.P.: ‘Sideband prediction in transionospheric SAR imaging radar from the ionospheric turbulence strength CLI’. Proc. Radar 2008 Conf., Adelaide, Australia, 3–5 September 2008, doi: 10.1109/RADAR.2008.4635891
[8] Gray, A.L., Mattar, K.E., Sohka, G.: ‘Influence of ionospheric electron density fluctuations on satellite radar interferometry’, Geophys. Res. Lett., 2000, 27, (10), pp. 1451–1454
[9] Fremouw, E.J., Livingston, R.C., Miller, D.A.: ‘On the statistics of scintillating signals’, J. Atmos. Solar Terr. Phys., 1980, 42, pp. 717–731
[10] Briggs, B.H., Parkin, I.A.: ‘On the variation of radar star and satellite scintillations with zenith angle’, J. Atmos. Terr. Phys., 1963, 25, pp. 339–366
[11] Zhang, O.T.: ‘A note on the estimation of Nakagami-m fading statistics’, IEEE Commun. Lett., 2002, 6, (6), pp. 237–238
[12] Knepp, D.L., Reinking, J.T.: ‘Ionospheric environment and effects on space-based radar detection’, in Cantafio, L.J. (ed.): ‘Space based radar handbook’ (Artech House Inc., 1989)
[13] Fremouw, E.J., Ishimaru, A.: ‘Intensity scintillation index and mean apparent radar cross section on monostatic and bistatic paths’, Radio Sci., 1992, 27, (4), pp. 539–543
[14] Yates, G., Horne, A.M., Blake, A.P., Middleton, R.: ‘Bistatic SAR image formation’, IEE Proc. Radar Sonar Navig., 2006, 153, (3), pp. 208, doi: 10.1049/ieeprsn:20054091
[15] Blacknell, D.: ‘Conservative prediction in transionospheric SAR imaging’, IEE Proc. Radar Sonar Navig., 1994, 141, (1), pp. 45–52
[16] Dana, R.A., Knepp, D.L.: ‘The impact of strong scintillation on space based radar design II: noncoherent detection’, IEEE Trans. Aerosp. Electron. Syst., 1986, AES-22, (1), pp. 34–46
[17] Oliver, C.J., Quegan, S.: ‘Understanding synthetic aperture radar images’ (SciTech Publishing Inc., 2004), 1-911211-31-6
[18] Knepp, D.L., Houps, H.L.F.: ‘Altair VHF/UHF observations of multipath and backscatter enhancement’, IEEE Trans Antennas Propag., 1991, 39, (4), pp. 528–534
[19] Jakeman, E.: ‘Enhanced backscattering through a deep random phase screen’, J. Opt. Soc. Am. A, 1988, 5, (10), pp. 1638–1648
[20] Cannon, P.S., Groves, K.M., Donnelly, W.J., Perrier, K.,: ‘Signal distortion on V/UHF trans-ionospheric paths: first results from WIDE’, Radio Sci., 2006, 41, doi: RSS540, doi:10.1029/2005RS003369
[21] Yeh, K.C., Liu, C.H.: ‘Radiowave scintillation in the Ionosphere’, Proc. IEEE, 1982, 70, (4), pp. 324–360
[22] Rino, C.L., Fremouw, E.J.: ‘The angle dependence of singly scattered wavefields’, J. Atmos. Terr. Phys., 1977, 39, pp. 859–868
[23] Rino, C.L.: ‘A power law phase function model for ionospheric scintillation. I – Weak scatter’, Radio Sci., 1979, 14, (6), pp. 1135–1145
[24] Wertz, J.R., Larson, W.J.: ‘Space mission analysis and design’ (McGraw-hill & Kluwer Press, 1999, 3rd ed)
[25] Fremouw, E.J., Secan, J.A.: ‘Modelling and scientific application of scintillation results’, Radio Sci., 1984, 19, (3), pp. 687–694
[26] Aarons, J.: ‘Global morphology of ionospheric scintillation’, Proc. IEEE, 1982, 70, (4), pp. 360–377
27 Rosenqvist, A., Shimada, M., Ito, N., Watanabe, M.: ‘ALOS PALSAR: a pathfinder mission for global-scale monitoring of the environment’, *IEEE Transactions on Geoscience and Remote Sensing*, 2007, 45, (11), pp. 3307–3316, doi: 10.1109/TGRS.2007.901027
28 Eriksson, L.E.B., Sandberg, G., Ulander, L.M.H. *et al.*: ‘ALOS PALSAR calibration and validation results from Sweden’. Proc. IEEE Geoscience and Remote Sensing Symposium (IGARSS 2007), Barcelona, Spain, 23–27 July 2007
29 Carrano, C.S., Groves, K.M., Caton, R.G.: ‘Phase screen simulator for predicting the impact of small-scale ionospheric structure on SAR image formation and interferometry’. Proc. IEEE Geoscience and Remote Sensing Symposium (IGARSS), 2010, doi: 10.1109/IGARSS.2010.5651485
30 Roth, A.P., Huxtable, B.D., Chotoo, K., Chotoo, S.D., Caton, R.G.: ‘Detection and mitigation of ionospheric stripes in PALSAR data’, Int. Geoscience and Remote Sensing Symposium (IGARSS), 2012