Aggregation operators for the measurement of systemic risk

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Abstract

The policy objective of safeguarding financial stability has stimulated a wave of research on systemic risk analytics, yet it still faces challenges in measurability. This paper models systemic risk by tapping into expert knowledge of financial supervisors. We decompose systemic risk into a number of interconnected segments, for which the level of vulnerability is measured. The system is modeled in the form of a Fuzzy Cognitive Map (FCM), in which nodes represent vulnerability in segments and links their interconnectedness. A main problem tackled in this paper is the aggregation of values in different interrelated nodes of the network to obtain an estimate systemic risk. To this end, the Choquet integral is employed for aggregating expert evaluations of measures, as it allows for the integration of interrelations among factors in the aggregation process. The approach is illustrated through two applications in a European setting. First, we provide an estimation of systemic risk with a pan-European set-up. Second, we estimate country-level risks, allowing for a more granular decomposition. This sets a starting point for the use of the rich, oftentimes tacit, knowledge in policy organizations.

Keywords: systemic risk, aggregation operators, Fuzzy Cognitive Maps, Choquet integral

JEL codes: E440, F300, G010, G150, C430

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"Clearly, there is widespread awareness that the analytical framework, no matter its level of sophistication, cannot replace the expert knowledge and judgment of the ESRB’s members."
– Vítor Constâncio, Vice-President of the ECB, Frankfurt am Main, 16/11/2010

1. Introduction

Measurement of systemic risk has become a pivotal topic among academics, policymakers, and supervisors. The search for the one unrivaled systemic risk measure has mostly stimulated empirical research for a mechanistic analysis of system-wide risks. Exploiting the fact that macroprudential supervisory authorities possess a variety of specialized domain intelligence and experience, this paper takes a bottom-up approach to address the topic: How do we tap into the expertise of individual supervisors to measure systemic risk?

The current financial crisis has highlighted the importance of a macroprudential approach to ensuring financial stability [9]. In contrast to only being concerned with the stability of individual financial institutions (i.e., microprudential), the shift towards a system-wide perspective has imposed complexity in terms of analysis tasks and the underlying data (see Flood and Mendelowitz [23]). It accentuates the need for an understanding of not only individual financial components, be they economies, markets or institutions, but also interconnectedness among them and their system-wide risk contributions. To this end, analytical tools and models provide ample means for two types of tasks: (i) early identification of vulnerabilities and risks, as well as their triggers, across financial instruments, markets and institutions, and (ii) early assessment of transmission channels of and a system’s resilience to shocks, and potential severity of the risk materialization. Yet, despite the rise of big data and analytics in previous years, macroprudential analysis as a support to policy remains highly dependent upon market intelligence and expert judgment and experience, as is above noted by Vítor Constâncio. An illustrative and intuitive example is the ever increasing shadow banking activities occurring behind the scenes, in which quantitative risk analysis and measurement are challenging tasks. Going beyond lack of data, one could in line with Lucas’ critique and Goodhart’s law also question the use of quantitative models in an ever changing environment, such as the impact of regulation on markets and the endogeneity of risk (e.g., Danielsson and Shin [16]).

Managing knowledge within an organization in an efficient way is an essential capability, not the least for knowledge-producing organizations like macroprudential supervisory bodies. In addition to producing information about systemic risks, a key task is to disseminate it horizontally and vertically within the organization. Yet, expert knowledge is not unproblematic. Beyond common challenges in the incentive structure to share information (e.g., Lin [48]), a large share of challenges relate to the fact that most knowledge possessed by experts is classified as unstructured and tacit (e.g., Haldin-Herrgard [32]). This obviously hinders capturing, representing, and transferring it within the organization. Leveraging on groups of experts knowledgeable in specific topics, a key concern ought to be judging (i) which expert’s knowledge is more relevant or reliable than the others, and (ii) how to combine the knowledge of different experts in a structured way to obtain a unique solution to a problem. One solution to these types of challenges comes from the family of aggregation operators. To this end, we need to answer the remaining question: How do we aggregate expert opinions to measure systemic risk?

The quantification of expert knowledge in risk assessment is not uncommon (e.g., [28]). In this paper, the objective is to provide a framework for measuring systemic risk by aggregating the knowledge of financial supervisors with a set of families of aggregation operators. Since its introduction, fuzzy set theory and fuzzy logic has been applied in numerous contexts and in numerous ways to utilizing expert knowledge. The most traditional way involves creating fuzzy rule-based systems based on the knowledge extracted from the experts. An example of this type of research can be found in León and Machado [46] and León et al. [47], whose focus is closely connected to the focus of this paper. The authors propose to use a fuzzy inference system with linguistic rules to estimate the systemic importance of financial institutions; the rules are extracted from expert’s assessment of the possible combinations of factors describing the system and combined using different approximate reasoning
schemes. A different direction of applications makes use of fuzzy sets and fuzzy measures in representing and aggregating expert knowledge with the main problem being the appropriate choice of aggregation function, as highlighted in Beliakov and Warren [5]. As is pointed out by Moon and Kang [56], the two main issues to consider in these approaches are to decide (i) in what form the information is elicited from the experts, and (ii) how the information is aggregated in the presence of multiple experts.

In this paper, we present an approach that combines Fuzzy Cognitive Maps (FCMs) and aggregation based on Choquet integrals to handle the two above mentioned challenges. The first important issue to tackle as the basis of the aggregation process is to identify a representation of data acquired from experts (i.e., the output of the knowledge elicitation process). The representation of the expert knowledge should not only be such that it provides an appropriate placeholder for the effective use of the aggregation process, but it should also support analyzing the system underlying the problem from different perspectives. The general goal is to estimate the overall level of risk present in a complex system, in our case the vulnerability of the financial system. The system can be described as the hierarchy of components. A network representation of the system, with the components of the system as the nodes and the interrelations between the components as the edges, provides flexibility to analyze different attributes of the system, such as identifying the most critical or central nodes and their interlinkages. In this paper, the FCM [42], as a special type of a weighted graph, is utilized to capture and make use of expert evaluations regarding the interrelation between different sectors of a financial system. As our main goal is to estimate systemic risk and not, as in general applications of FCMs, to identify the “optimal” input vector of the map that results in an equilibrium point, we focus on aggregating the impacts of a set of nodes on a target node of the network. For this purpose, we need to identify an appropriate aggregation operator that can handle the complex interrelations in the underlying system and provide a final value of systemic risk. In this paper, we show how we can model the spread of risk in the system, represented as a FCM, through a fuzzy measure and consequently use the corresponding Choquet integral to aggregate values in the nodes of the map. With our approach, we not only provide a measure of systemic risk but also identify the most central parts of a system, such as the most vulnerable components or countries. We achieve this goal by specifying different fuzzy measures corresponding to different components of the modeled system. The main theoretical contribution of the article lies in combining FCMs and Choquet integrals to represent and analyze complex systems of interrelated objects. Additionally, we propose approaches for assessing the system through quantitative network measures and visual network graphs.

We illustrate the approach for measuring systemic risk from expert opinions through two applications in a European setting, for which we also discuss practical implications and challenges. First, we provide an estimation of systemic risk in a pan-European set-up, where we model systemic risk at the European, country and sectoral level. Second, we also estimate country-level risk, allowing for a more granular decomposition, by modeling risk at the level of the country, its sectors and sub-dimensions of the sectors. For both applications, we also perform quantitative and qualitative analysis of the systemic risk measures as a network of nodes and edges. While the former uses standard measures of network centrality, the latter approach provides visual interactive interfaces for the systemic risk measures. The visualizations are available as a web-based application. The approach overall and these applications in particular set a starting point for the use of the rich, oftentimes tacit, knowledge in a policy setting overall and their organizations in particular.

The rest of the paper is structured as follows. Section 2 links systemic risk to expert knowledge of supervisors and policymakers, as well as introduces aggregation operators. In Section 3, we introduce FCMs and the use of the Choquet integral, particularly for the measurement of systemic risk. Section 4 presents two applications: the case of systemic risk in Europe and in an individual country. Finally, we conclude and discuss future research in Section 5.

1 The complimentary web-based applications are available here: http://vis.risklab.fi/#/fuzzyAgg
2. Systemic risk and aggregation operators

This section discusses the concept of systemic risk, and the use of aggregation operators for the task of its measurement. After providing an overview of systemic risk, we provide a brief overview of aggregation operators, as well as a mapping back into the task of systemic risk measurement.

2.1. Macroprudential supervision and systemic risk

A key task of a macroprudential supervisory body is to internally collect and produce information about systemic risk. The term systemic risk, not to paraphrase Justice Potter Stewart’s definition of explicit content [7], belongs to the group of concepts that are broad and vague, yet implicitly understood. Still, we need a working definition as a basis for measurement and analysis. To start with, financial instability is defined as an event that has adverse effects on a number of important financial institutions or markets (ECB [20]). Systemic risk, as also defined by the ECB, is the risk of widespread financial instability that impairs the functioning of the financial system to the extent that it has severe implications on economic growth and welfare.

The definition used herein is untangled with the help of the systemic risk cube shown in Figure 1. The notion of a risk cube was introduced by the ECB [21], and represents their conceptual framework, but has its origin in a number of works. The three dimensions of the risk cube are the triggers, origins and impacts. The nature of triggers unleashing the crisis could take the form of an exogenous shock, which stems from the outside of the financial system (e.g., macro-economic shocks) or could emerge endogenously from within the financial system (e.g., banks). The origins of the events may be distinguished to limited idiosyncratic shocks and widespread systematic shocks. While the former initially affect only the health of a single financial market, financial intermediary or asset, the latter may in the extreme affect the entire financial system. Further, the impact of the events may cause problems for a range of financial intermediaries and markets in a sequential and simultaneous fashion.

![Systemic risk cube with three forms of risks.](image)

**Notes:** The figure represents the systemic risk cube with three dimensions and systemic risks and is an adapted version of that in ECB [21].

Beyond three dimensions, we herein concretize the notion of systemic risk through the three forms presented by de Bandt et al. [18]. The first form of systemic risk focuses on the unraveling of widespread imbalances in the vein of Kindleberger’s [41] and Minsky’s [54] financial fragility view of a boom-bust credit or asset cycle. Hence, the subsequent abrupt unraveling of the imbalances may be endogenously or exogenously caused by idiosyncratic or systematic shocks, and may have adverse effects on a wide range of financial intermediaries and markets in a simultaneous fashion. Early and later empirical

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2For further information, see de Bandt and Hartmann [17], de Bandt et al. [18], ECB [20], and ECB [21]
literature alike have identified common patterns in underlying vulnerabilities preceding financial crises (e.g., Kaminsky et al. [38] and Reinhart and Rogoff [61]). The second type of systemic risk refers to a widespread *exogenous aggregate shock* with negative systematic effects on one or many financial intermediaries and markets at the same time. These types of aggregate shocks have empirically been shown to co-occur with financial instabilities (e.g., Gorton [29] and Demirgüç-Kunt and Detragiache [19]), and can be exemplified by the collapse of banks during recessions due to the vulnerability to economic downturns. The third form of systemic risk is *contagion and spillover*, which usually refers to an idiosyncratic problem, be it endogenous or exogenous, that spreads in a sequential fashion in the cross section. There is wide evidence of cross-sectional transmission of financial instability (e.g., Upper and Worms [65] and van Lelyveld and Liedorp [67]), such as the failure of one financial intermediary causing the failure of another, which initially seemed solvent, was not vulnerable to the same risks and was not subject to the same original shock as the former. While contagion refers to a situation when the initial failure is entirely responsible for subsequent ones, the term spillover is commonly used when the causal relationship is unknown (e.g., ECB [21]).

Macroeprudential oversight requires a broad toolbox of models for systemic risk measurement. The categorization by ECB [21] elegantly maps the three forms of systemic risk to analytical tools: (i) early-warning models, (ii) macro stress-testing models, and (iii) contagion and spillover models. First, to identify vulnerabilities and imbalances in an economy, early-warning models derive probabilities of the future occurrence of systemic financial crises (e.g., Alessi and Detken [3] and Lo Duca and Peltonen [49]). With a set of vulnerability indicators as an input, the output of such models takes the form of crisis probabilities, which are monitored with respect to critical threshold values. Second, macro stress-testing models provide means to assess the resilience of the financial system to a variety of aggregate shocks (e.g., Castrén et al. [12] and Hirtle et al. [33]). These exercises assess the consequences of assumed extreme, but plausible, shocks for different entities, for which a key question is to find the balance between plausibility and severity of the stress scenarios (e.g., Alfaro and Drehmann [4] and Quagliariello [60]). Third, contagion and spillover models can be employed to assess how resilient the financial system is to cross-sectional transmission of financial instability (e.g., IMF [36]). Hence, they attempt to answer the question: With what likelihood, and to what extent, could the failure of one or multiple financial intermediaries cause the failure of other intermediaries? Beyond measures of systemic risk, coincident indicators measure the contemporaneous level of systemic stress (e.g., Holló et al. [35]), which may be used to identify, signal and report on heightened stress.

The three types of systemic risk, and the corresponding tools for measuring them, provide a starting point for an all-encompassing framework of systemic risk. At a more granular level than the forms of systemic risk, measurement ought to be broken down to specific market segments or economic sectors and countries or other geographical definitions. Hence, for each market segment and economy, and at each point in time, the following characteristics should be measured: (i) specific imbalances building-up in the cross-section and their current state, the likelihood of these imbalances to unravel, and their potential severity; (ii) transmission channels of aggregate shocks, an overview of plausible shocks, impacts on other market segments, and potential severity of and resilience to shocks in case of materialization; and (iii) sources of contagion or spillover at the individual and system-level, as well as potential severity of and resilience to cross-sectional transmission. Mapping these to Borio’s [8] two dimensions of systemic risk, the former relates to time and cyclicality, as risk builds-up in tranquil times and abruptly unravels in times of crisis, whereas the two latter relate to cross-sections and structures, as risk may transmit through various channels in interconnected financial systems. Accordingly, these two dimensions can also be mapped to the two tasks of risk identification and assessment, and are what we ought to be interested in when measuring risk with any approach.

### 2.2. Systemic risk and expert knowledge

Beyond only collecting and producing information about systemic risks, a key internal task of a macroprudential supervisory body is to disseminate it horizontally and vertically within the organization. Yet, to highlight the importance of disseminating information vertically, most if not all of the produced information has an external contact point only through the very top of the organizational
structure. The above discussed analytical approaches, while giving an impression of purely mechanistic analysis, is only a part of the truth. Independent of the rigor of the analytical tools, expert knowledge is always a central part of informed policy decisions. This is obviously due to reasons related to transparency and accountability, as decision clearly are taken by humans rather than models, but relates also to challenges faced by policymakers in the measurement of some, if not all, types of systemic risk.

At an epistemological level, one could even argue that predictions about future human behavior is impossible, where one example is the Lucas critique or Goodhart’s law of regulation impacting behavior and the endogeneity of risk another (e.g., Danielsson and Shin [16]). Further, systemic risk is not restricted to banks [25]. Since McCulley [52] coined the term shadow banking, it has become widely recognized that credit intermediation outside the banking system takes place in an environment where regulation and supervision is applied to a lesser degree [26]. Thus, the lack of reporting standards and obligations for shadow banking activities obviously directly impacts measurability. If data exist, a challenge on its own is its quality, comparability and timeliness. Although the US may have more standardized data, concerns have been arisen about pan-European data comparability [22], such as definitions of non-performing loans and forbearance practices, not to mention transatlantic comparability. Beyond precision, accounting-based data also exhibits long publication lags.

When measurability is questioned, and even though it would not be, an obvious qualitative resource for a policymaker would be individual and organization-wide expert knowledge and judgment. Accordingly, a report by IMF-BIS-FSB [37] stresses that systemic risks cannot be assessed with solely quantitative approaches, and that authorities ought to draw upon intimate knowledge of their own financial system’s functioning. We define the term expert as someone who has long-term experience via research and practice in a specific discipline or task. Given the fact that central banks have a high knowledge concentration due to well-educated and oftentimes narrowly focused work tasks, it can also seen as an organization with a high share of expertise. In addition to well-educated central bank governors, Fox [24] highlights that out of 450 employees in research-oriented divisions about half have a PhD in economics. In addition, central banks have direct and explicit research functions, ideally with the ultimate aim of supporting policy formulation, as discussed in Berc et al. [6]. Except for increases in the number of PhDs, and overall educational standards, Fox [24] also highlights that the share of accountants and lawyers has gradually been taken over by economists in the Federal Open Market Committee. As Goodhart et al. [27] show that central banks employ more economists and fewer lawyers and accountants for their own financial stability and supervisory functions than non-central bank supervisory agencies do, this clearly correlates well with a separation of monetary policy (macro) and bank regulation tasks (micro) at the time. Given current macroprudential initiatives, we are likely to see a shift back through an increase in the share of supervisory tasks within or close to central bank activities. This accentuates the heterogeneity in possessed qualitative knowledge that must be managed at a macroprudential supervisory body.

The wealth of focused expertise, while in isolation only being beneficial in individual in-depth investigations, would ideally be a valuable resource as a support for the entire organization, as many tasks require a multidisciplinary perspective. Within expertise-driven and knowledge-producing organizations like macroprudential supervisory bodies, expertise obviously ought to be disseminated both vertically and horizontally. To this end, this paper taps into expert knowledge of systemic risk through its aggregation, in order to foster and provide means to support knowledge sharing and systemic risk identification and assessment overall.

2.3. Aggregating expert knowledge

Measuring or estimating systemic risk simply means forecasting the likelihood of events that can affect the behavior of the entire system under consideration. In many real life situations, statistical approaches provide an important tool for assessing a system when one possesses sufficient reliable and valuable historical data regarding also future events. Yet, there are several cases when other types of methodologies can be more beneficial. A typical choice of approach discussed extensively for example by Cooke [15] is to rely on expert evaluations, estimations or opinions. Expert opinions can be used most importantly to assess the possibility of events with very low likelihood but high potential
impact. When the process involves multiple experts, rather than opinion elicitation of one expert, the obtained opinions need to be combined into a final assessment. There are several issues to be considered regarding the aggregation procedure of individual expert opinions. On the most general level, Clemen and Winkler [14] classify different approaches to two main groups: mathematical and behavioral. In behavioral approaches, the experts are encouraged to interact among each other with the final goal of obtaining a consensual solution to the given problem (typical in group decision making). By mathematical approaches in the context of risk estimation, the literature mainly refers to the elicitation and aggregation of subjective probabilities. This refers to any formal mathematical procedure for aggregation when the experts’ input is restricted to evaluation without further involvement in the process. In practice, the combination of the two approaches are used most frequently.

From a mathematical perspective, the aforementioned view of considering expert assessments as subjective probabilities already imposes assumptions on the interpretation of an expert opinion as a type of likelihood. Yet, when combined with rigorous mathematical procedures, they can provide essential insights in many applications. Beyond this, there can be different interpretations assigned to the expert opinions regarding even when the nature of uncertainty they try to capture is the focus of the aggregation process, as pointed out in Hoffman and Hammonds [34]. For this reason, the use of traditional statistical methods for handling expert opinions, such as Bayes modeling described by Winkler [68], is not necessarily optimal or even justifiable. Likewise, Mosleh et al. [57] pointed out that in general relying only on the “common sense” of experts in a risk assessment process is not the best choice, but their aggregation can provide essential insights in practice. In many cases, the opinions of decision-makers can be interpreted in the framework of preference and utility theories (Keeney and Raiffa [39]). In these cases, aggregation can be performed using traditional operators used in decision-making problems (see Grabisch et al. [31] for a comprehensive treatise on the topics of aggregation functions). There are several important factors related to the elicitation and aggregation process that can be modeled by choosing an appropriate aggregation operator (e.g., the difference in reliability or importance of experts).

Various domains and their specific problems require performing the task of summarizing a set of numerical values differently. Obtaining a single number that provides a representation of the original set and corresponds to the predefined requirements is not trivial. A large number of aggregation operators exist to summarize numerical and/or non-numerical information into a single value. These operators play a fundamental role in many fields related to different decision-making problems. A typical application is the case when an evaluation has to be performed in the presence of multiple objects to be assessed individually and then the assessment is to be combined into an overall evaluation. In many cases, independence of the object is assumed; the most popular aggregation methods in these situations include a wide range of operators from the simple weighted average to the OWA (ordered weighted average) introduced by Yager [69] and used in a wide range of real life problems. In these operators every evaluation is considered individually in the aggregation process and performed independently from others. Yet, the most general notion of an aggregation operator requires only one property to be satisfied by the operator: monotonicity. It implies that if a set of values to be aggregated dominates a set of values, then the aggregated value for the dominated set cannot be higher. In our context it means that if the risk level of every component in a system increases, then the overall systemic risk cannot decrease. Besides the assumption of monotonicity, every other property can be tailored to the application under consideration. Symmetry is a typically required property of the aggregation process: the final value does not depend on the order in which the input values are aggregated. Many aggregation operators satisfy this property, but for example the weighted average is not a symmetric function. The main application areas of aggregation operators include decision making, pattern recognition, and machine learning. A crucial point in the aggregation process is the type of values to be processed. Depending on the values to be estimated by the experts, we can consider three main types of input data to be aggregated: (i) a value in $[0,1]$, with higher numbers indicating higher potential risk (crisp numbers); (b) an interval in $[0,1]$, in case the expert cannot precisely quantify her opinion (interval). Linguistic expressions modeled as different types of fuzzy sets also work in our platform: for example high risk, low risk, medium risk, very high risk.
3. Measuring systemic risk from expert opinions

This section presents our approach to modeling systemic risk with the use of expert opinions and aggregation operators. As was discussed in the previous sections, the case of assessing systemic risk consists most often of a set of interdependent characteristics. The relations can depend for example on the type of risk or the market segment considered. This ought to also impact how expert evaluations about risk are aggregated. Herein, we present our model for measuring systemic risk in a problem in which the system can be decomposed into a hierarchical form, in which the decomposed characteristics are interdependent. We start by describing the representation of this problem in terms of a Fuzzy Cognitive Map (FCM), which aims at capturing the level of risk and the interdependency among various characteristics. With an FCM as its basis, we design the aggregation procedure to account for levels and interdependencies through the use of the Choquet integral. The outcome of the approach is an aggregation procedure allowing for the measurement of systemic risk at various levels of the system, ranging from continents to economies to market segments.

3.1. Fuzzy Cognitive Maps

As our main purpose is measuring/estimating systemic risk by utilizing expert evaluations and opinions, we need to consider aggregation operators from two perspectives: (i) what are the different means to aggregate data obtained from experts in different formats, and (ii) how can we model the interrelated effects of different factors in a system to obtain an overall risk estimation. To support the above defined tasks in systemic risk measurement, we view the macro-financial environment in terms of multiple market segments, which all exhibit risk levels (i.e., risk identification) and interrelations to other segments (i.e., risk assessment). Accordingly, we aim at describing the system as a network, in which nodes represent risk severity for different segments and edges among nodes represent the level of impact of one component on the other. The values required to construct the map is to be specified by experts by either a numerical value of a linguistic expression (e.g., “strong effect”). The final measure of systemic risk is to be calculated by aggregating the information from the different nodes through the interrelation values. Ideally, we also aim at identifying an approach that not only provides a mathematical model and a numerical risk estimation, but can also be used to offer a visual representation of the different relationships within the considered system. To fulfill these needs, we tap into the concept of Fuzzy Cognitive Maps (FCMs).

In the seminal article, Kosko [42] defined FCMs with the aim of representing complex systems in a structured way, which at the same time allows for assessing the actual state, as well as tracking and estimating possible changes in the system. At the general level, a FCM is a directed graph with nodes as factors related to a general concept and edges defining the relationships among nodes. The main advantages of FCMs over traditional cognitive maps lies in the fact that they can represent different degrees of influence of one factor on another, and can be used to understand the potential consequences of a change in the state of a node of the map (Montibeller and Belton [55]). Consequently, one dominant line of applications of FCMs aims at modeling changes taking place in dynamic and complex systems. A typical example of this is a financial system comprised of a large number of interacting entities. For example, in case of modeling systemic risk in a continent, the system includes several subsystems (i.e., countries), which can be seen as complex and dynamic systems on their own, with interrelated components (i.e., market segments). In case of these dynamically changing systems with highly interrelated components, we can face different limitations by applying traditional modeling and analytical techniques.

In many applications, FCMs are utilized for supervising complex processes taking place in a system. Based on the obtained understanding of the system, the overall goal many times is to translate it into a failure detection or prediction system. Typical cases in the literature mainly concern engineering systems or chemical processes taking place in complex industrial systems (e.g., Mendonça et al. [53]). The common factors among these applications are dynamic, nonlinear relationships between system components. As was already pointed out, the modeling of this type of systems requires methods that can utilize human experience and the knowledge of domain experts (Aguilar [1]). In the above
engineering examples, experts with decades of experience can develop a holistic view of the complex process taking place in the system. The acquired knowledge makes them capable of identifying different relationships between different components of a system that are hardly recognizable with only collected historical data. Additionally, this knowledge is not necessarily expressible in terms of, for example, IF-THEN rules to form a basis of a classical expert-based decision support system. In these cases, FCMs provide a framework to capture experts’ knowledge in a graphical representation that is more intuitive (i) for the experts to communicate their knowledge, and (ii) for the decision maker to understand the behavior of the system as a whole.

The application of FCMs in economics and finance is not widespread yet, although we can identify a handful of contributions. In the context of modeling stock market investments, Lee and Kim [45] developed a new, bi-directional inference system based on FCMs with the main goal of representing highly unstructured decision making problems. Koulouriotis et al. [43] attempt to model the complex system of agents affecting stock-market behavior in national economies to provide organizations. The simulation results show promising forecasting accuracy, although the authors acknowledge a number of shortcomings of their model. Carvalho and Tomé [11] designed a qualitative, rule-based FCM approach to model dynamic economic systems, specifically they use the example of modeling the behavior of the European Central Bank with regard to interest rates.

3.2. Mapping systemic risk from expert opinions

The previous discussion can be formulated in a similar way when considering financial systems with financial supervisors as the experts. The knowledge of experts can be utilized in assessing the relationship between different sectors of a financial system, specifically to what extent a given sector affects another sector. The natural interpretation of a cognitive map corresponds to that of a directed graph: nodes represent concepts (i.e., components of a system) while edges represent causal relationships between nodes. In the FCM approach, weights on the edges are incorporated to represent the strength of the relationship between two components. As our main goal is to estimate the level of systemic risk in a complex system, the FCMs will be of a special type of weighted directed graphs, as they will represent, on the general level, a hierarchical structure of the components of the system. The top level of the hierarchy consists of a single node with the associated value as the level of systemic risk. On the second level of the hierarchy, the main components of the system are included. For instance, in estimating the systemic risk in Europe, this second level would consist of the individual countries. At the next level, the sub-components or sectors of the second-level components are included. In general, the set of components can differ for different higher level nodes, such as including the macroeconomy, banking sector, financial markets, etc. We can define the required number of hierarchy levels in a similar way. In the following formulation and later in the presented examples, we will discuss the case of hierarchies with two levels additionally to the systemic risk parent node for the sake of presentation; the formulation can easily be extended to an arbitrary number of hierarchy levels.

The main component of our hierarchy, and correspondingly the central node of the FCM, is the node $SR$ representing overall systemic risk. The next level consists of the main components of the system, $S_1, \ldots, S_t$. Subsequently, every $S_i$ is associated with a set of sub components $s_{i,1}, \ldots, s_{i,p_i}$. The set of sub components is not necessarily the same for every node on the higher level, so we formulate the problem in a general way. Every node is associated to a corresponding value of the actual level of risk in that specific system-component. Additionally, the connections between the nodes represent the relationship among different sectors of the underlying system. For example, if $S_i$ and $S_j$ are connected with edges with associated weights $w_{i,j}$ and $w_{j,i}$, then the higher the value of the edges, the larger the extent to which risk in the first sector results in increased risk in the second sector. In the basic formulation, the weights of the map are assumed to take a value in the $[0, 1]$ interval with 0 indicating no effect and 1 indicating maximum possible effect (i.e., the two sectors are maximally interconnected). It is important to note that $w_{i,j}$ and $w_{j,i}$ are not equal; for example, if we consider two countries in Europe, the effects on each other are not necessarily symmetrical. According to these observations, the most general representation of a FCM is an adjacency relation matrix.
symmetrical, that defines the weights on the edges with the actual values of estimated risk in different components of the systems in the diagonal \((w_{i,i})\).

\[
S = \begin{pmatrix}
  w(SR,SR) & w(SR,S_1) & \ldots & \ldots & w(SR,s_{t,p_t}) \\
  w(S_1,SR) & w(S_1,S_1) & \ldots & \ldots & \ldots \\
  \ldots & \ldots & \ldots & \ldots & \ldots \\
  \ldots & \ldots & \ldots & \ldots & \ldots \\
  w(s_{t,p_t},SR) & \ldots & \ldots & w(s_{t,p_t},s_{t,p_{t-1}}) & w(s_{t,p_t},s_{t,p_t})
\end{pmatrix}
\]

The process of employing FCMs for assessing systemic risk is performed in the following steps that will be worked out in the subsequent parts of this section:

1. The knowledge captured in the map is provided by experts (i.e., financial supervisors). The experts are asked to fill in the elements of the described adjacency matrix. This step can be performed regularly, with the set of experts not necessarily being the same in every evaluation. Naturally, the experts are not expected to provide estimations about all the values in the matrix, but only of the sectors and dependencies of the system they are confident about. As a result, we obtain a FCM for every expert with associated confidence level for the provided values. Based on these individual FCMs, we need to create an aggregated FCM that incorporates the information by taking into consideration their confidence values and the nature of the underlying hierarchy. At time \(t\), this step will result in the initial FCM, \(S_t^{(0)}\).

2. Based on the FCM \(S_t^{(0)}\) and the FCM \(S_{t-1}\) created in the previous evaluation process, we define \(S_t\) as the actual FCM representing systemic risk at time point \(t\). The values of the nodes will be updated by combining the information from both matrices by using appropriate aggregation functions. In this case, appropriateness refers to the need to aggregate interrelated quantities.

3. Using \(S_t\), we can assess the risk in the system by making use of traditional measures in directed graphs. This includes identifying receiver and transmitter nodes in the map: components of the underlying system that are (i) highly vulnerable to risks taking place in other connected components (receiver), and (ii) components that affect the risk in the system more significantly than others (transmitter). We can also identify central nodes of the map by employing different centrality measures. In our model, we can make use of the special nature of the defined cognitive map, i.e. the underlying hierarchy of the system. Using this, we can define global and local receivers, transmitters and central nodes. To identify locally important nodes, we can restrict the evaluation to a specific level of the hierarchy to identify the component of the system that has the highest impact on the risk on that level. Taking systemic risk in Europe as an example, we can determine countries that contribute most to the risk level of the system, and additionally analyze countries individually to determine which sectors contribute most to the national financial systems.

4. The nature of the FCM offers the possibility to create a graphical representation of the dynamic evolution of the relationships among the system components. We can employ visualization techniques to follow the changes and identify the dependencies in the network that are evolving in a way that points to a potential threat to the system as a whole.

Next, we consider the problem of creating the FCM at a fixed point in time, so we avoid using an additional index for denoting the time parameter. The organizational units are denoted by \(o_1, o_2, \ldots, o_k\). The experts associated to unit \(o_i\) are denoted by \(e_{i1}, \ldots, e_{ik}\). Expert \(e_{ij}\) provides the adjacency matrix \(FS_{ij}\). In the literature, such as Khan and Quaddus [40], it is usually assumed that every expert has a single confidence level, \(r_{ij}\), that can result in either (i) the experts providing information only about a small set of nodes and edges of the FCM about which they are highly confident; or (ii) the experts provide information on a large set of nodes and edges but with low average confidence. While the former situation enables formulating the final results of the systemic risk estimation with fairly high confidence, yet with a small sample, the latter case enables tapping into a larger sample, but includes more uncertain opinions, and might hence hinder extracting useful information. For this reason, a
A fuzzy measure basis for this approach is provided by the construct of a fuzzy measure [58]: an approach is to consider the effect of subsets of information rather than individual elements. The in it. To incorporate interaction of different components in the aggregation process, a key feature of time and different centrality values) to better understand the underlying system and the spread of risk of the structure of a system in analyzing the map but also allows for further analysis (by incorporating of temporal dimension in the map (Zhong et al. [70]). More importantly for our context, in case a node allows for the representation of simple monotonic relationships (Carvalho and Tomé [10]), or the lack of temporal dimension in the map. For this reason, from the broad literature on aggregation operators, we use for systemic risk assessment an approach that enables interdependencies and non-linear behavior to be captured when calculating the initial value of a node in the fuzzy cognitive map. When modeling complex systems, it is not reasonable to assume that the behavior of the components of the system are independent from each other. In our case, we aim at aggregating the components to obtain values for the central nodes, i.e. the overall level of systemic risk. There exists several methods to model the changes taking place in the system through the impact of the nodes on each other on the existing links. The most common method introduced already in the original paper by Kosko [42] is to calculate the weighted average of the values from nodes connected to the target node with a directed edge. This simple aggregation procedure has several drawbacks, for example it only allows for the representation of simple monotonic relationships (Carvalho and Tomé [10]), or the lack of temporal dimension in the map (Zhong et al. [70]). More importantly for our context, in case a node is not assigned an initial value, it is estimated through the weighted average without considering the interrelations between the considered nodes. For this reason, from the broad literature on aggregation operators, we use for systemic risk assessment an approach that enables interdependencies and non-linear behavior to be captured when calculating the initial value of a node in the fuzzy cognitive map. As we will see later, the approach will not only provide a tool to incorporate and model the complexity of the structure of a system in analyzing the map but also allows for further analysis (by incorporating time and different centrality values) to better understand the underlying system and the spread of risk in it. To incorporate interaction of different components in the aggregation process, a key feature of an approach is to consider the effect of subsets of information rather than individual elements. The basis for this approach is provided by the construct of a fuzzy measure [58]:

**Definition 1.** A fuzzy measure \( \mu \) on the finite set \( N = \{1, 2, ..., n\} \) is a set function \( \mu : P(N) \rightarrow [0, 1] \) (where \( P(N) \) is the power set of \( N \)) satisfying the following two conditions:

- \( \mu(\emptyset) = 0, \mu(N) = 1; \)
- \( A \subseteq B \) implies that \( \mu(A) \leq \mu(B) \).

The second condition enables representing in this framework measures that do not satisfy the strong condition of additivity. In our context, this means that we can model situations when the high value of a characteristic of the financial system in itself does not indicate risk unless a set of other characteristics show deviations from their usual values at the same time. One of the most general formulations when using monotone measures as the basis of aggregation can be described by the Choquet integral [13, 51]. To formulate the definition, we suppose that there are \( n \) number of characteristic measures \( (c_1, ..., c_n) \) based on which an evaluation is performed, which results in the corresponding \( (x_1, ..., x_n) \) values.

**Definition 2.** A discrete Choquet integral with respect to a monotone measure \( \mu \) is defined as

\[
C_{\mu}(x_1, ..., x_n) = \sum_{i=1}^{n} (x(i) - x(i-1)) \mu(C(i))
\]

where \( x(i) \) denotes a permutation of the \( x \) values such that \( x(1) \leq x(2) \leq \ldots \leq x(n) \) and \( C(i) = \{c(i), c(i+1), \ldots , c(n)\} \).
Different well-known aggregation operators are special cases of the Choquet integral, but also in its general form it starts to gain popularity in different applications [59]. The basic properties of the operator are determined by the monotone measure, such as symmetry, additivity and linearity. The simplest case is when no interaction is considered among the different evaluation criteria, which is reflected in the monotone measure in a way that it takes only non-zero values on singletons (i.e., individual criteria) and it is zero for all possible combinations. This implies that the criteria are supposed to be independent which is not necessarily a realistic assumption in our problem context, although the aggregations mainly used in practice belong to this group with the simple weighted average being the most prominent one. With simple averages, we would look at the components of our system independently and determine the importance of the components individually with respect to overall systemic risk, whereafter an aggregated value over a threshold would signal the existence of elevated levels of risk. Extreme measures of this type are the maximum and minimum operators. Using the maximum would imply that we consider a situation risky if there is at least one component of the system that is elevated, while minimum is applicable in a situation when we are concerned with the cases where all the components exhibit elevated values. In general, different modifications of the Choquet integral make it appropriate to integrate all the different data types, including crisp numbers, intervals and linguistic expressions. Although difficult to generalize, in some cases it is shown to be the best possible aggregation tool [44]. In the subsequent example cases, we will only use crisp numbers as our main focus is on describing the conceptual framework, but it is important to point out that it is straightforward to incorporate for example fuzzy values in the aggregation as is done in [64].

A more complex special case is the one when the pairwise interaction between components of the model is considered additionally to the individual effects. The Choquet integral in this case is 2-additive and can be formulated as (see Grabisch [30]):

\[
C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} (v(c_i) - \frac{1}{2} \sum I(c_i, c_j))x_i + \sum_{I(c_i, c_j) > 0} I(c_i, c_j) \min(x_i, x_j) + \\
\sum_{I(c_i, c_j) < 0} |I(c_i, c_j)| \max(x_i, x_j) 
\]

(1)

The interaction measure \(I(c_i, c_j)\) can be defined in different ways by transforming the measure for pairs into the \([-1, 1]\) interval. A list of methods can be found for example in Marichal [50]. If the interaction between pairs is zero, then they can be seen as independent from each other, providing us the above described singleton case. If the measure takes the value 1, it indicates that one component strongly affect the other; intermediate values stand for different degrees between no effect and strong effect. In our case, we will restrict the interaction measure to the \([0, 1]\) interval with the maximum operator. In the following, we will describe how Choquet integrals are used to aggregate values in the FCM.

As our main goal is to estimate the level of systemic risk, we will start by describing the procedure of calculating \(SR\) assuming that all the other elements in the adjacency matrix are known. To make use of the Choquet integral, first we need to identify the underlying fuzzy measure. The domain of the defined measure is specified as all the possible combinations of edges in the FCM but with only a small subset of them associated to non-zero values. We will assign positive value of the measure only to the set of paths, \(P\), in the FCM that has length smaller than or equal to 2 and the end-node of the path is \(SR\). This includes three types of paths:

- Edges in the form of \((S_i, SR)\): connections between first-level components and the final node
- Edges in the form of \((S_i, S_j, SR)\): connections between first-level components and the final node mediated by another first-level component
- Edges in the form of \((s_j,p_k, S_j, SR)\): connections between second-level components and the final node mediated by a first-level component
Based on these considerations, we can define a normalized fuzzy measure, i.e., a fuzzy measure when the sum of the measure values is 1. In our case, we will restrict the interaction measure to the $[0, 1]$ interval, but it is important to note that the interpretation of our case corresponds to the negative interactions in eq. 1. If the risk increases on any point in a path leading to the final node, it will result in increased systemic risk. We do not require the risk to be increased in every node on the path to achieve this. Thus, the calculations are performed as follows:

$$C_\mu(x_1, \ldots, x_n) = \sum_{i=1}^{n} (v(c_i) - \frac{1}{2} \sum I(c_i, c_j) x_i + \sum I(c_i, c_j) | \max(x_i, x_j),$$

(2)

with $c_i$ as the edges with the final node as the endpoint and the associated risk level $x_i$ in the starting node. $I(c_i, c_j)$ denotes the overall effect on a path to the final node and is calculated as the function of the values on the two edges forming the path. There are several possibilities for combining the two values reflecting what portion of risk we think is transferred along the path. For example, if we use the product of the two values, it will result in a lower rate of risk spread than using the maximum of the two values. In the following, a numerical example is presented to clarify the use of the Choquet integral in aggregating risk values in the nodes of FCMs.

**Example 3.** To illustrate the use of the Choquet integral in aggregating the values in a FCM, we look at a simple example of a system of two countries. Fig. 2 depicts three different cases based on the assumed relationship of the two countries. In the first map, there is no relationship between the two countries, in this case the aggregation with the Choquet integral is the same as the weighted average. Following the previous discussion, we only have two elements in the set that forms the basis of the fuzzy measure: the two edges of the FCM. In this case, we obtain that

$$SR = \frac{0.3 \times 0.5}{0.3 + 0.8} + \frac{0.8 \times 0.3}{0.3 + 0.8} = 0.35.$$

In the second graph, we assume that there is a connection between the two countries, but only one of the countries affects the other. This model includes three paths in the set $P$: the two edges between the countries ($p_1$ and $p_2$) and the final node and the path from Country 2 to Country 1 to System ($p_3$). Additionally, we have a non-zero measure value for the edge between the two countries but it is not used directly in the systemic risk calculations. The measure of the three paths is as follows:

$$\mu(p_1) = \frac{0.3 + 0.8 + 0.6 = 0.3}{0.3 + 0.8 + 0.6 + 0.3}, \quad \mu(p_2) = \frac{0.3 + 0.8 + 0.6 = 0.3}{0.3 + 0.8 + 0.6 + 0.3}, \quad \mu(p_3) = \frac{0.3 + 0.8 + 0.6 = 0.3}{0.3 + 0.8 + 0.6 + 0.3}.$$

The systemic risk is calculated based on eq. 1 as

$$SR = \left( \frac{0.3 + (0.6 + 0.3)}{0.3 + 0.8 + 0.6 + 0.3} - \frac{0.8}{0.3 + 0.8 + 0.6 + 0.3} \right) \frac{0.5}{0.3 + 0.8 + 0.6 + 0.3} + \frac{0.3 + 0.6}{0.3 + 0.8 + 0.6 + 0.3} \max(0.3, 0.5) = 0.38.$$

We can see that the introduced relationship in the map increases the value of systemic risk. In the third part of Fig. 2, the map is further extended with a new connection between the two countries, but with a lower associated value. An example of this type of a situation is when there are two interconnected economies with different size, such as the case of Germany (Country 2) and Poland (Country 1). A risk in any of them will increase the risk in the other, but as Germany is of greater importance, elevated risks in Germany has more severe effects on Poland than the other way around (i.e., 0.6 compared to 0.2). In this figure, we have four paths to be considered, including the three from the previous example and $p_4$ as the path from Country 1 to the final node through Country 2. They have the following associated measure values:

$$\mu(p_1) = \frac{0.3}{0.3 + 0.8 + 0.6 \times 0.3 + 0.2 \times 0.8}; \quad \mu(p_2) = \frac{0.8}{0.3 + 0.8 + 0.6 \times 0.3 + 0.2 \times 0.8};$$

$$\mu(p_3) = \frac{0.3 \times 0.6}{0.3 + 0.8 + 0.6 \times 0.3 + 0.2 \times 0.8}; \quad \mu(p_4) = \frac{0.2 \times 0.8}{0.3 + 0.8 + 0.6 \times 0.3 + 0.2 \times 0.8}.$$

Hence, systemic risk is calculated as
A more important and novel way of utilizing Choquet integrals in the fuzzy cognitive map is to consider the time dimension in the development of the underlying system. In the above description, the underlying assumption is that the risk present in any segment of the system contributes at the same time to the systemic risk. In reality, we could look at the problem in a way that takes into consideration the length of considered paths in the aggregation. Considering the above example, we could assume that the risk present in Country 2 directly contributes to the present systemic risk, while it takes some time until this risk affects the system as a whole, indirectly through Country 2’s effect on Country 1. In this respect, the aggregated systemic risk obtained in the example evaluates a situation that will take place in a future time-point; if we assume that the delay of spreading risk from one node to a neighboring node is one time unit, then the example estimates the development of the system in two time units from now. With the similar reasoning, we could estimate (or “forecast”) systemic risk at different time points by considering paths with different lengths in the construction of the fuzzy measure. Formally, to estimate the state of the system $k$ time units from now based on the adjacency matrix of the FCM at time point $t$, a sequence of fuzzy measures, $\mu^k_i$ and corresponding Choquet integrals can be defined by considering non-zero values in the measure paths in the FCM that have length not greater than $k$ and has the final node ($SR$) of the map as the endpoint of the path. Based on each measure we can obtain an estimation of the risk values in different nodes of the network at any point between now and $k$ time units later. As in the case of paths of length 2, an even more important problem here is to define the fuzzy measure to combine the value associated to the edges on paths with different lengths. We can perform these operations considering $\mu^k_i$ to be a $k$-additive monotone measures described in [30] and moving beyond the complexity of the 2-additive Choquet integral as discussed above. For instance, considering the product of values as the final path value for large $k$’s in general results in low effect values meaning that after a point we do not gain any new

\[
SR = \left( \frac{0.3 + (0.6 \cdot 0.3)/2}{0.3 + 0.8 + 0.6 \cdot 0.3 + 0.2 \cdot 0.8} - \frac{1}{2 \cdot 0.3 + 0.8 + 0.6 \cdot 0.3 + 0.2 \cdot 0.8} \right) 0.5 + \frac{0.3 \cdot 0.6}{0.3 + 0.8 + 0.6 \cdot 0.3 + 0.2 \cdot 0.8} \max(0.3, 0.5) + \frac{0.2 \cdot 0.8}{0.3 + 0.8 + 0.6 \cdot 0.3 + 0.2 \cdot 0.8} \max(0.3, 0.5) = 0.39
\]
information by increasing the possible length of considered paths. While using the maximum of the 
individual values on the path would increase the systemic risk value significantly after every step. It 
always depends on the understanding of the underlying domain to determine for how many steps it is 
still meaningful to forecast based on the present situation. In highly fluctuating and rapidly changing 
systems the recommended value should be lower than in rather stationary systems.

3.4. Exploiting the mapping and its aggregation

This section discusses practical challenges and considerations when using FCMs and the Choquet 
integral in conjunction with expert evaluations. First, we view requirements on the experts, in order 
to support consistent and meaningful evaluations. Then, we discuss both quantitative and qualitative 
analysis of the expert evaluations and their aggregations.

Requirements on the experts. The approach presented in the previous section offers a relatively straight-
forward procedure to combine several FCMs created by individual experts into an aggregate form to 
represent the overall knowledge of the experts. In a possible setting that can be envisioned as a typical 
application, the approach can be used as the way of facilitating knowledge sharing within organiza-
tions. The experts working in the organizations, are working in different units with each unit focusing 
on a specific topic. These topics (or a subset of them) form a system that is depicted in the form of the 
FCM. Experts are asked to comment on the strength of the relation between some components of the 
system. This information reflects the individual domain knowledge of experts. Consequently, it is not 
rare that the individual FCMs constructed using the matrix provided by an expert cannot be directly 
used to estimate systemic risk. As experts are most often knowledgeable only in a limited number of 
components of a complex system, in order to claim that they are confident enough regarding the esti-
mations, the provided matrices are expected to be sparse (or contain several values with low confidence 
level). For instance, an expert in an organizational unit with a focus on component $S_i$ of the system is 
likely to provide confident estimations regarding the relation between this component and the others, 
but not necessarily between two components different from $S_i$. We can conclude that the involvement 
of multiple experts in a systemic risk evaluation process of a complex system is: (i) sufficient in the 
sense that it can result in higher reliability of the estimations and (ii) necessary in the sense that there 
is no single expert who can provide a credible assessment of the whole system (although the individual 
evaluations can be used independently to estimate the level of risk in components of the system).

The only requirement specified for the experts is to provide the estimations in a way that forms a 
connected network. This is ensured by requiring them to specify a value at a low level of the system 
hierarchy, and that the associated node has to be connected to the main system node through their 
antecedent node in the hierarchy. In practice, this requires experts to connect each node for which 
the risk level is evaluated to the system node highest in the hierarchy. As specified above, experts are 
not required to estimate the risk in sectors that themselves are assumed to be composed of a set of 
segments, although they have to specify the relationship of this node to the other nodes. This in turn 
makes it unnecessary to employ consistency measures for assessing expert evaluations. For instance, 
if the experts are allowed, as in the general description, to specify every element in the adjacency matrix 
of the FCM, they can unintentionally define conflicting information, which would create problems 
in specifying the fuzzy measure in a consistent way. The perspective to expert opinions that we 
take incorporates aspects of a general behavioral approach, but does still not directly incorporate the 
communication and interaction of experts for convincing each other. Yet, we do include feedback, as 
experts may receive the aggregated assessment prior to their judgment, which can make them rethink 
their individual evaluations and indirectly increase the consensus without being of direct focus, as was 
shown in Van de Ven and Delbecq [66]. The feedback can take the form of a simple report containing 
the final FCM or a detailed report containing a comparison of the expert’s evaluation with other 
opinions.

Quantitative and qualitative analysis of the map. The FCM representation allows for various types of 
quantitative analysis of risk present in the underlying system. As the FCM can be seen as a weighted,
directed graph, we can utilize different measures from graph theory. Besides traditional aggregation operators, we can make use of the structure of the aggregation procedure introduced in the FCM to identify the most important or central nodes of the system. The first question to understand about the system may be the overall level of interconnectedness in the FCM. For this purpose, the definition of the density of a graph can be used: in general, if the number of nodes of the map is \( N \) and the number of edges is \( E \), then the density is \( E^2/N(N-1) \). By making use of the special, hierarchical structure of our FCM, we can modify this measure in two ways: (i) decrease the value of the denominator as the hierarchy puts restrictions on the pairs of nodes that can be connected; and (ii) make use of the fact that the FCM is a weighted graph and use the weight of an edge in estimating the overall connectedness.

Two traditional measures for directed graphs have specific meaning in the proposed FCM representation: the out-degree and in-degree of a node. Additionally to the level of systemic risk, two important issues need to be analyzed: (i) which receiver nodes of the FCM (i.e., components of the system) are most vulnerable to risk, and (ii) which transmitter nodes possess the highest threat towards other nodes. The traditional approach to determine these important nodes makes only use of the values of the edges in the map. The nodes can be identified using the in-degree and out-degree of a node, \( s \):

\[
in(s) = \sum_{t \in FCN, t \neq s} w(t, s), \quad out(s) = \sum_{t \in FCN, t \neq s} w(s, t).
\]

The sum of these two quantities specifies the centrality of a node in the map (i.e., degree): \( c(s) = in(s) + out(s) \). There are several ways to improve these measures. First, to assess the level of risk propagation from a node, we can consider not only the direct links from this node to others but also paths with length at least two to estimate the potential effect of the node in its neighborhood. In a similar manner, the evaluation of receiver status can incorporate information on the indirect relationship from nodes that are not directly connected through an edge but through a path with predefined length. This analysis can make use of Choquet integral based aggregation in a similar way as it was described to determine systemic risk. For instance, taking the problem of vulnerability of a node, this concept naturally translates to the final value of systemic risk for the final node, as it is interpreted as how much risk the system receives from its components. For any arbitrary node \( s \), a fuzzy measure and the corresponding Choquet integral, \( C_{s,2} \), can be defined by assigning non-zero weights to paths with length not more than two and having \( s \) as the endpoint. Additionally to the product function that is used in the previous subsections to combine effects on a sequence of edges, we can use for example different \( T \)-norms or triangular norms (for a detailed overview see for example Grabisch et al. [31]) that provide a general framework for the conjunction operation, including the min operator or the product norm:

\[
C^T_{s,2}(x_1, \ldots, x_n) = \sum_{t \in FCN} (v(t) - \frac{1}{2}) \sum_{(q,t) \in FCN} T(t,q)w(t,t) + \sum_{(t,q) \in FCN} \max(w(t,t), w(q,q)).
\]

To incorporate higher levels of indirect effects between nodes, we can use \( C^T_{s,k} \), where paths ending in node \( s \) and with length at most \( k \) are included. Assuming that the risk can be potentially transferred through an edge in one time unit, this definition can provide an estimation to what extent the node can be seen as a \( k \)-receiver, i.e. what is the potential degree of vulnerability after \( k \) time periods.

Beyond quantitative network analysis, we also provide means for visualization. As networks constitute an inherently complex source of information, we ought to support in exploring the data through an overview and and drill-down analysis, as well as overall communication. Drawing upon the visual analytics paradigm, we aim at supporting analytical thinking through interactive visualizations of the map and its information products, where the operative term is interaction. Networks constitute of nodes for entities and relationships among them in matrix form, where an edge could be the link between nodes \( S_1 \) and \( S_2 \), for instance. With matrices of directed and weighted graphs, we have links \( w(S_1, S_2) \) and \( w(S_2, S_1) \) representing the size of the link from \( S_1 \) to \( S_2 \) and from \( S_2 \) to \( S_1 \). The matrix
is of size $n^2$, where $n$ is the number of nodes in the network. Hence, each node is described by its relationship to each other entity, such as $S_1 \in \mathbb{R}^n$. Yet, the complexity of a graph is often decreased by constraining the existence of links. To support qualitative analysis of networks, we follow Sarlin [62] in requiring two features: (i) interactive and (ii) analytical visualizations.

Analytical visualization involves computational approaches for simplifying data in a trustworthy manner. The notion of an analytical technique for visualization differs from data analytics by rather using analytics to reduce the complexity of data, with the ultimate aim of visualizing underlying data structures. A commonly used dimension reduction approach for networks is force-directed layouting, which positions nodes by approximating node distances to their corresponding link strengths. This seeks to uncover the structure of the network in terms of more and less densely connected areas and their relation. Still, it has a number of limitations and remedies, such as hairball visualizations and cases of weighted networks with few strong but many weak connections. The coupling of visual interfaces with interaction techniques goes to the core of information visualization and visual analytics. Rather than a final stop, a visual interface is a mere starting point for data exploration and understanding, and requires hence the support of means for interaction. This becomes evident in the visual information seeking mantra by Shneiderman [63]: "Overview first, zoom and filter, then details-on-demand". The visualization provides merely a high-level overview, and should hence be manipulated through interaction to zoom in on a portion of items, eliminate uninteresting items and obtain further information about requested items. Thus, a large share of the revealed information descends from manipulating the medium, which not only enables better data-driven communication of information, but facilitates also tasks related to data exploration and analysis. Despite the inherent value of visual interfaces to support understanding and communication of data, the use of qualitative representations is by no means a substitute to quantitative approaches to network analysis.

4. Two applications to macroprudential oversight

This section illustrates the application of FCMs and aggregation operators to two cases in macroprudential oversight in Europe. First, we describe a pan-European application, which highlights systemic risk in various countries and market segments. Second, we describe an application within an individual country, which illustrates a more granular measurement of risks.

4.1. Measuring systemic risk in Europe

Measuring systemic risk is a key concern of policymakers and supervisors in Europe. Since the wake of the global financial crisis of 2007–2008, the European institutional model for supervision and regulation has adapted to the need for risk measurement and the implementation of macroprudential policy. As can be exemplified by recently founded supervisory bodies with the mandate of safeguarding financial stability, a system-wide perspective to financial supervision is currently being accepted and implemented as a common objective of governmental authorities and supervisors, such as the European initiatives of the European Systemic Risk Board (ESRB), the Single Supervisory Mechanism and the Financial Stability Committee. Herein, we apply FCMs and aggregation operators with a pan-European perspective to systemic risk. Figure 3 describes the structure of the risk segments that we assess. First, European systemic risk is decomposed to the country-level, with five countries included: $S_1, \ldots, S_5$. Second, for each country-specific segment $S_i$, we measure systemic risk in four different sectors $s_{i1}, s_{i2}, s_{i3}, s_{i4}$. The sectoral risks include the macroeconomy, financial markets, banking sector and other financial institutions.
For a more comprehensible example, we focus on Greece, Italy, Ireland, Portugal and Spain (henceforth the GIIPS). The evaluation process starts with a set of experts expressing their insights on the level of risk in the sectors of each of the five countries and the interdependencies among sectors in individual countries. The experts are also asked to evaluate the interrelations among countries, as well as impacts from sectors to countries and countries to the GIIPS. We define the risk of a sector as its vulnerability, or more precisely probability of distress (i.e., not accounting for the impact of a shock), whereas interrelations are defined as the impact of one segment on another given the occurrence of a shock (i.e., not accounting for the probability of a shock). In general, it would be possible to incorporate more relationships (e.g., among sectors in different countries like the impact of the Spanish banking sector on the Portuguese macroeconomy), but in this example we restrict the input to the specified subset of possible relationships. Moreover, in a more complex hierarchy (e.g., entire Europe), it is not reasonable to expect experts to be knowledgeable or evaluate with a high confidence every country and sector. The example illustrated herein provides thus a case of countries that oftentimes naturally appear together in analysis related to banking and sovereign risk, and consequently a high number of experts can be assumed to have opinions on risks in all of the countries.

The outcome of this initial evaluation is the matrix of the information provided by the experts and the matrix of corresponding confidence values; at this point we obtain a matrix for every expert. The
individual evaluations (i.e., values associated with nodes and edges of the FCM) can be aggregated according to the procedure described in the previous section by taking the confidence weighted average for all the values in the matrix. To obtain this matrix, we first aggregate the evaluation of the experts for every cell using the confidence weighted average of the completed evaluations. These values need to be normalized in order to obtain the final adjacency matrix of the FCM. The following phase is the actual assessment of the systemic risk level. Before employing the Choquet integral, the confidence weighted average of the evaluations is calculated as (for nodes $s$ and $t$):

$$w(s, t) = \frac{1}{\sum_{e \in E} r_{ij}^{e}} \sum_{e \in E} r_{ij}^{e} w^{e}(s, t).$$

Using these values, the final aggregated evaluation can be obtained according to Eq. 3. The data that we utilized to illustrate the use of the approach can be found in Table A.1 in the Appendix. In this table, the diagonal represents the estimated vulnerability of a sector, while the other values reflect the impact of one segment on another. For instance, the value 0.2 in the diagonal implies that this sector is vulnerable only to a low extent, whereas the value 0.8 shows significant vulnerability. The other values indicate the strength of the interconnectedness among two segments.

![Figure 4: A network of systemic risk in Europe.](image)

The results of the hypothetical example can be found in Table 1. As the numbers are not based on real data, although resembling reality, it is worth noticing that the countries with high centrality, e.g. Spain and Greece, have high levels of aggregated vulnerability. In order to represent the results
of expert evaluations and aggregations, we make use of network visualization. Figure 4 illustrates the network of countries and their sectors, as well as the system level (i.e., GIIPS). The sectors, countries and GIIPS are represented by nodes, whose size encodes level of vulnerability, whereas their interrelations are shown through opacity of links. The figure includes a screen capture from the publicly open web-based interactive visualization platform VisRisk, in which this application has been included as an instance.\textsuperscript{3} This type of a visual interface could not only be an end product, but also a means for supporting evaluations by experts and the feedback of previous or aggregated opinions.

![Figure 5: Decomposition of systemic risk in a country.](image)

### 4.2. Measuring systemic risk in a country

The second application follows the previously discussed set-up, but focuses on an individual country. Hence, we further breakdown the risks to the macroeconomy, financial markets, banking sector and other financial institutions into a number of sub-dimensions. In this application, our segments $s_1, \ldots, s_t$ refer to the risk in the sectors. Accordingly, the second level of the hierarchy refers to sub-dimensions pinpointing various types of risks in the sectors in question. Again, to identify interconnectedness among various risks, and individual sectors, experts are asked to represent interdependence on pairs of components of the system. The process of finding the final systemic risk value is the same as was described for the previous application. The assessment of systemic risk in a country can be essential

\textsuperscript{3}The application is available here: http://vis.risklab.fi/#/fuzzyAgg
information on its own, but at the same time it could be used as an input for the first application. From a practical point of view, this would put unreasonable requirements on the experts in terms of evaluations they have to perform which can lead to unreliable results. To avoid this problem, before the evaluation every expert can indicate the sectors and countries she is competent to evaluate. Accordingly, the expert would be presented only with the corresponding questions.

To illustrate the use of aggregation, we again provide a numerical example based on the data provided in Table A.2 in the Appendix. As determining the aggregated importance values requires only the use of a weighted average, we do not include details concerning this step. We suppose that the values are already aggregated from the individual evaluations obtained from experts, including their confidence weights. Again, in Table A.2 in the Appendix, the diagonal represents the estimated vulnerability of a subdimension, while the other values reflect the impact of one segment on another. We consider a differing number of measures for every sector, but the calculations are performed in a similar way according to the Choquet integral formula. The final systemic risk value is 0.49, which indicates a fairly low level of risk in this hypothetical example.

Table 2: Results from the second example

|                      | Macroeconomy | FM | Banking sector | Other | Country |
|----------------------|--------------|----|----------------|-------|---------|
| Level of vulnerability| 0.29         | 0.62| 0.43           | 0.47  | 0.49    |
| Out-degree           | 6.21         | 5.35| 5.17           | 4.94  |         |
| In-degree            | 5.66         | 4.76| 5.69           | 5.50  |         |
| Centrality           | 11.87        | 10.11| 10.86          | 10.44 |         |

5. Conclusion

Multiple recent waves of systemic financial crises have sparked an interest in measuring system-wide risks. The literature has predominantly addressed the task by employing various statistical techniques using market-based or accounting data. Yet, we reason that besides data-driven, numerical approaches to quantifying systemic risk, one should also tap into the knowledge of experts when assessing various dimensions of vulnerability and risk. The experience of experts is a treasure trove for revealing levels and trends of risks that are not necessarily discoverable from available data sources, due to inter alia shadow banking activities or other challenges in measurability of systemic risk.

This paper puts forward a quantitative framework to incorporate the knowledge of experts in the assessment of systemic risk. We propose to model systemic risk as a network of risk segments through the Fuzzy Cognitive Map (FCM), whose risks and interrelations are to be evaluated by experts. The natural interpretation of the FCM corresponds to that of a directed graph: nodes represent concepts (i.e., segments of a system) while edges represent interrelations between nodes. Further, we propose a procedure to aggregating risk to the highest level of the system by incorporating levels of risk and interconnections among the risk segments. The systematic aggregation approach to aggregate expert evaluations of risk is based upon the Choquet integral.

We have exemplified how the approach can aid macroprudential supervisory bodies in making use of the information gathered from experts in their organization, particularly for measuring systemic risk from expert opinions in a European setting. First, we provided an estimation of systemic risk in a pan-European set-up, where we systemic risk is modeled at the European, country and sectoral level. Second, we also illustrated the estimation of country-level risks, allowing for a more granular decomposition. In the latter application, we model risk at the level of one country, including its sectors and sub-dimensions of the sectors. In conjunction with the applications, we also showed both a quantitative and qualitative analysis of the systemic risk measures by treating them as a network of nodes and edges. While the former application uses standard measures of network importance to describe nodes, the latter approach provides a visual interactive interface for the systemic risk measures.
The visualizations are available as a complimentary web-based application.4

The implications of this paper are twofold. First, the general introduction of aggregation operators to systemic risk measurement sets a starting point for the use of the rich, oftentimes tacit, knowledge in a policy setting overall and their organizations in particular. We foresee this to be an area of wide interest given the increased mandates and overall responsibilities of financial supervisors, as well as the discretionary nature of macroprudential policy. For instance, as discretionary systems may be gamed yet rules struggle to properly account for risk, Agur and Sharma [2] advocate a hybrid of discretion and rule-based macro-prudential policy. Second, independent of the domain under analysis, the theoretical contribution of the article provides means for a wide range of applications combining FCMs and Choquet integrals to represent and analyze complex systems of interrelated objects. In an increasingly interlinked world, these types of systems are not a rare occurrence. In light of the present paper, future research ought to tackle two problems in need of further work: i) the use and testing of the proposed framework in practice, including all challenges involved in collecting and using the expert knowledge for macroprudential purposes, and ii) the combination of the collection of expert knowledge with interactive visual interfaces, in order to combine data-driven analysis with expert judgment in a structured and collaborative manner.

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4The complimentary web-based applications are available here: http://vis-risklab.fi/#/fuzzyAgg
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Appendix: Data

Table A.1 and Table A.2 provide the data used in Section 4. In Table A.1, the abbreviations stand for Macroeconomy (M), Financial markets (FM), Banking sector (BS), and Other financial institutions (OFI). In Table A.2, the sectors are as follows:

- Macroeconomy: Employment (E), GDP growth (GDP), Inflation (I), Government budget and debt (GBD), and International trade & finance (ITF).
- Financial markets: Credit imbalances (CI), Asset imbalances (AI), Markets and instruments (MI), and Financial market infrastructures (FMI).
- Banking sector: Capital adequacy (CA), Asset quality (AQ), Management soundness (MS), Earnings and profitability (EP), Liquidity (L), and Sensitivity to market risk (SMR).
- Other financial institutions: Insurance companies (IC), Contractual savings institutions (CSI), Market makers (MM), Specialized sectoral financiers (SSF), and Other financial activities (OFA).

| Sector | M | FM | BS | OFI | Greece |
|--------|---|----|----|-----|--------|
| Greece | M | 0.8| 0.4| 0.9 | 0.7    | 0.8    |
|        | FM| 0.2| 0.4| 0.4 | 0.3    | 0.4    |
|        | BS| 0.7| 0.6| 0.8 | 0.4    | 0.7    |
|        | OFI|0.2 | 0.2| 0.3 | 0.3    | 0.2    |
| Italy  | M | 0.7| 0.3| 0.8 | 0.4    | 0.7    |
|        | FM| 0.3| 0.3| 0.4 | 0.4    | 0.5    |
|        | BS| 0.8| 0.3| 0.6 | 0.4    | 0.6    |
|        | OFI|0.4 | 0.4| 0.4 | 0.4    | 0.6    |
| Ireland| M | 0.7| 0.2| 0.6 | 0.4    | 0.7    |
|        | FM| 0.3| 0.3| 0.3 | 0.5    | 0.6    |
|        | BS| 0.7| 0.3| 0.7 | 0.5    | 0.6    |
|        | OFI|0.5 | 0.5| 0.5 | 0.5    | 0.8    |
| Portugal| M | 0.8| 0.3| 0.7 | 0.3    | 0.7    |
|         | FM| 0.3| 0.3| 0.4 | 0.3    | 0.4    |
|         | BS| 0.8| 0.4| 0.8 | 0.4    | 0.5    |
|         | OFI|0.4 | 0.3| 0.3 | 0.4    | 0.3    |
| Spain  | M | 0.8| 0.2| 0.9 | 0.3    | 0.8    |
|         | FM| 0.3| 0.2| 0.4 | 0.3    | 0.5    |
|         | BS| 0.9| 0.3| 0.9 | 0.3    | 0.8    |
|         | OFI|0.4 | 0.4| 0.5 | 0.6    | 0.7    |

| Country | Greece | Italy | Ireland | Portugal | Spain | Europe |
|---------|--------|-------|---------|----------|-------|--------|
| Greece  | 0.9    | 0.4   | 0.7     | 0.8      | 0.8   |        |
| Italy   | 0.7    | 0.3   | 0.5     | 0.7      | 0.9   |        |
| Ireland | 0.3    | 0.4   | 0.3     | 0.4      | 0.7   |        |
| Portugal| 0.7    | 0.6   | 0.3     | 0.8      | 0.7   |        |
| Spain   | 0.8    | 0.8   | 0.5     | 0.9      | 0.9   |        |
Table A.2: The data used in the second example in Section 4.

| E | GDP | I | GBD | ITF | M |
|---|-----|---|-----|-----|---|
| E | 0.3 | 0.8 | 0.4 | 0.8 | 0.4 |
| GDP | 0.8 | 0.3 | 0.5 | 0.8 | 0.5 |
| I | 0.4 | 0.4 | 0.3 | 0.5 | 0.5 |
| GBD | 0.6 | 0.6 | 0.4 | 0.2 | 0.4 |
| ITF | 0.7 | 0.7 | 0.6 | 0.7 | 0.3 |

**Macroeconomy (M)**

| CI | AI | MI | FMI | FM |
|----|----|----|-----|----|
| CI | 0.3 | 0.8 | 0.5 | 0.3 |
| AI | 0.6 | 0.8 | 0.7 | 0.3 |
| MI | 0.3 | 0.4 | 0.5 | 0.5 |
| FMI | 0.3 | 0.3 | 0.5 | 0.4 |

**Financial markets (FM)**

| CA | AQ | MS | EP | L | SMR | BS |
|----|----|----|----|---|-----|----|
| CA | 0.5 | 0.6 | 0.7 | 0.5 | 0.5 | 0.4 |
| AQ | 0.5 | 0.4 | 0.6 | 0.7 | 0.7 | 0.7 |
| MS | 0.5 | 0.5 | 0.3 | 0.5 | 0.5 | 0.5 |
| EP | 0.7 | 0.7 | 0.5 | 0.4 | 0.6 | 0.5 |
| L | 0.8 | 0.6 | 0.5 | 0.4 | 0.4 | 0.5 |
| SMR | 0.7 | 0.7 | 0.5 | 0.6 | 0.7 | 0.4 |

**Banking sector (BS)**

| IC | CSI | MM | SSF | OFA | OFI |
|----|-----|----|-----|-----|-----|
| IC | 0.6 | 0.4 | 0.4 | 0.4 | 0.3 |
| CSI | 0.3 | 0.3 | 0.2 | 0.2 | 0.2 |
| MM | 0.3 | 0.2 | 0.4 | 0.2 | 0.2 |
| SSF | 0.2 | 0.2 | 0.2 | 0.5 | 0.2 |
| OFA | 0.2 | 0.2 | 0.1 | 0.1 | 0.2 |

**Other financial institutions (OFI)**

| M | FM | BS | OFI | Country |
|---|----|----|-----|---------|
| Macroeconomy (M) | 0.8 | 0.7 | 0.7 | 0.4 |
| Financial markets (FM) | 0.7 | 0.6 | 0.5 | 0.7 |
| Banking sector (BS) | 0.7 | 0.4 | 0.7 | 0.6 |
| Other financial institutions (OFI) | 0.6 | 0.4 | 0.7 | 0.6 |