Lensing of invisible stars by brown dwarfs

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Abstract

To detect brown dwarfs in the dark galactic halo through gravitational lensing, experiments follow the luminosity of millions of stars to observe a few lensing events per year. The luminosity of a star too faint to be continuously followed can be temporarily increased above the detection limit by a lensing. The detection of these invisible stars would increase the number of events by a factor 2 to 7, and moreover their presence would confirm the lensing interpretation of events due to continuously monitored stars.
1 Introduction

Two experiments (Bennett et al. 1990, Vidal-Madjar et al. 1991) try to detect brown dwarfs in the dark halo of our Galaxy (Carr, Bond & Arnett 1984), through gravitational lensing of stars in the Large Magellanic Cloud (LMC). The light of a star is amplified when a brown dwarf gets in the line of sight, and this amplification varies in a characteristic way with time as the brown dwarf moves relative to the line of sight (Paczyński 1986). Due to the low probability of a lensing event, between $10^4$ and $10^6$ stars (depending on the brown dwarf mass) must be monitored for one year to observe one 30% luminosity increase.

To follow several millions objects daily, efficient algorithms are required to process the huge amount of recorded photometric data and to extract the few events looked for. First, a pattern recognition algorithm detects the presence of some stellar object on a picture, and computes its precise position in the sky. Then, its luminosity is extracted from a fit of the light distribution, and compared with previous measurements. To decrease photometric errors and computer time, experiments produce a star catalog from the first pictures, and process the following pictures to extract the luminosity curves of the stars already in the reference catalog.

The drawback of this procedure is that a star absent from this catalog (usually a faint star) will never be “seen” afterwards by the detection algorithm, even if it shows up on a few pictures, when lensed by a brown dwarf for instance. This paper reports the result of a preliminary study, which shows that there should be two to seven times more detected lensings of invisible stars than of monitored stars. The number of detected lensing events is very sensitive to details of the apparatus, to the halo model, to the brown dwarf mass function, etc. However, the relative increase in the number of events due to invisible stars depends only weakly on the brown dwarf mass, and does not depend on halo parameters or detector characteristics. Therefore, it would be a very good test of the lensing interpretation of the luminosity variations of some stars: these additional events must be there!

I therefore suggest to apply the pattern recognition algorithm to all pictures, to detect the apparition of stars. Because the star catalog was used both to improve photometric accuracy and computer time, one could fear that my suggestion will be too costly in computer time, and inefficient because of the loss in photometric accuracy. This does not need to happen, because the only requirement now is that
a new star appears on a succession of pictures, and then disappears. Its precise luminosity does not matter at this stage, and therefore the time consuming reconstruction of the stellar luminosities is not required. It is only needed afterwards, to distinguish a lensed star from a background event such as a variable star, but then the star reconstruction is only needed for the few candidates, and not for the $10^{5\pm 1}$ monitored stars.

2 Basics of lensing

We first recall a few basic notions about micro-lensing (Paczyński 1986). When a brown dwarf of mass $M_{bd}$ comes at a distance $R$ to the line of sight of a star, the star light is amplified by a factor $A$:

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

where

$$u = \frac{R}{R_E} = R \left[ \frac{4GM_{bd}}{c^2} \frac{D_{bd}(D_{\text{star}} - D_{bd})}{D_{\text{star}}} \right]^{-1/2}$$

$R_E$ is the Einstein radius, $D_{bd}$ is the distance between the observer and the brown dwarf, and $D_{\text{star}}$ is the distance between the observer and the star.

The event rate $\Gamma(A)$ is the number of times a given star is amplified by a factor $A$ per unit time. It depends linearly on the dimensionless impact parameter $u(A)$ (Griest 1991):

$$\Gamma(A) = \Gamma_0 u(A) = \Gamma_0 \left[ \frac{2A}{\sqrt{A^2 - 1}} - 2 \right]^{1/2} \simeq \Gamma_0 \frac{1}{A} \text{ for } A \gg 1$$

$\Gamma_0$ depends on the brown dwarf mass, on the halo density and velocity distributions, and on the direction of the star (but does not depend on the nature of the star).

In the procedure followed by present experiments, an “event” is defined by a minimal amplification $A_0 = 1.34$ of the light of a monitored star, the same for all stars. The number $N_{\text{monitored}}$ of detected events from $N_{\text{stars}}$ monitored stars during an observation time $t_{\text{obs}}$ then writes:

$$N_{\text{monitored}} = t_{\text{obs}} \Gamma(A_0) N_{\text{stars}}$$
For light brown dwarfs, lensing events become too short to be detected. The mean duration $t_{\text{event}}$ of a lensing event is:

$$t_{\text{event}} = \frac{\pi R}{2 V_\perp} \approx 95\,000\,\text{s} \times u(A_{\text{min}}) \frac{200\,\text{km/s}}{V_\perp} \left( \frac{M_{\text{bd}}}{10^{-4}\,M_\odot} \frac{D_{\text{bd}}}{10\,\text{kpc}} \right)^{1/2}$$

where $A_{\text{min}}$ is the minimal detectable amplification (for present experiments $A_{\text{min}} = A_0$). We assume for simplicity that all brown dwarfs have the same mass $M_{\text{bd}}$ and the same transverse velocity $V_\perp$. These assumptions can be relaxed, but this would be an unnecessary refinement at this level. Possible brown dwarf masses range from $10^{-1}\,M_\odot$ (hydrogen burning limit) down to $10^{-7}\,M_\odot$ (evaporation limit, De Rújula et al. 1992). The CCD cameras of ongoing brown dwarf searches require exposure times of a few minutes to detect stars up to 19th magnitude, and the lensing event must appear on several consecutive exposures to reconstruct the light curve which is the signature of the lensing event. Therefore the minimal duration $t_{\text{min}}$ of a lensing event to be detected is a few hours. This leads to a steep drop of the detection efficiency for low masses, in which case the only detected lensings are due to brown dwarfs slower than average. This effect is strengthened by the fact that there is a maximal amplification for extended sources, such as red giants, and this finite size effect is stronger for lower brown dwarf masses.

3 Increase in the number of events

We are now interested in invisible stars, too faint to be detected unless they are lensed. The magnitude $m$ of a star amplified by a factor $A$ becomes $m - 2.5 \log A$. If an experiment detects stars up to a limiting apparent magnitude $m_{\text{thresh}}$, a star of magnitude $m$ $m_{\text{thresh}}$ shows up if:

$$A(m) = A_0 \times 10^{0.4(m-m_{\text{thresh}})}$$

(6)

Whereas the minimal amplification is independent of the star for monitored stars, it depends on the stellar magnitude $m$ for invisible stars. The event rate per star $\Gamma(A(m))$ then depends on the magnitude $m$ of the lensed star, through Equations 3 and 6. Large amplifications imply short events, therefore there is an upper bound on the amplification corresponding to the shortest detectable event, and an upper
bound \( m_{\text{max}} \) on the magnitude of a lensed star. From Equations 3, 5 and 6 we get for large amplifications:

\[
m_{\text{max}} \simeq m_{\text{thresh}} + 2.5 \log \left( \frac{95000 \text{s} \ 200 \text{ km/s}}{t_{\text{min}} V_{\perp}} \left[ \frac{M_{\text{bd}}}{10^{-4} M_{\odot}} \frac{D_{\text{bd}}}{10 \text{ kpc}} \right]^{1/2} \right)
\]  

(7)

The number \( N_{\text{invisible}} \) of detected micro-lensings of invisible stars is a sum over all stars of magnitude between \( m_{\text{thresh}} \) and \( m_{\text{max}} \):

\[
N_{\text{invisible}} = t_{\text{obs}} \int_{m_{\text{thresh}}}^{m_{\text{max}}} \Gamma(m) \Phi(m) dm = t_{\text{obs}} \Gamma_0 \int_{m_{\text{thresh}}}^{m_{\text{max}}} u(m) \Phi(m) dm
\]  

(8)

where \( \Phi(m) \) is the luminosity function of the target galaxy (i.e. \( \Phi(m) dm \) is the number of stars of apparent magnitude between \( m \) and \( m + dm \) in the area surveyed).

Let us stress that whereas the number of detected lensing events (for both monitored stars and invisible stars) sensitively depends on detector characteristics, on the normalisation of the luminosity function \( \Phi(m) \), and, through \( \Gamma_0 \), on the halo parameters and on the mass and transverse velocity distributions of the brown dwarfs, all these dependances disappear in the ratio \( N_{\text{invisible}}/N_{\text{monitored}} \): the number of monitored stars is \( N_{\text{stars}} = \int_{-\infty}^{m_{\text{thresh}}} \Phi(m) \ dm \), and we get:

\[
\frac{N_{\text{invisible}}}{N_{\text{monitored}}} = \frac{\int_{m_{\text{thresh}}}^{m_{\text{max}}} u(m) \Phi(m) dm}{u(A_0) \int_{-\infty}^{m_{\text{thresh}}} \Phi(m) dm}
\]  

(9)

If \( \Phi(m) \) increases fast enough with \( m \), there will be more events due to the lensing of faint invisible stars than due to bright monitored stars. Moreover, the interpretation of any variation of the light of monitored stars as lensing events requires the automatic presence of such lensings of invisible stars, which will therefore be a welcome confirmation.

4 Numbers

We take as an example the LMC luminosity function \( \Phi(m) \) given by Ardeberg et al. (Ardeberg et al. 1985) for a small area of the LMC, as representative of the mean LMC luminosity function. It goes approximately as \( \Phi(m) \simeq 10^{0.4(m-15.2)} \)
for $14 < m < 23$, which compensates the $m$ dependence of $u(m)$ since $u(m) \simeq u(A_0)10^{-0.4(m-m_{\text{thresh}})}$ for large amplifications. Then Equation 2 gives:

$$\frac{N_{\text{invisible}}}{N_{\text{monitored}}} \simeq m_{\text{max}} - m_{\text{thresh}}$$

(10)

$$\simeq 2.5 \log \left( \frac{95000 \text{s}}{t_{\text{min}}} \frac{200 \text{ km/s}}{V_{\perp}} \left[ \frac{M_{\text{bd}}}{10^{-4} M_{\odot}} \frac{D_{\text{bd}}}{10 \text{ kpc}} \right]^{1/2} \right)$$

(11)

which depends only logarithmically on the minimal duration $t_{\text{min}}$ or on the brown dwarf parameters, and gives numbers in the range 0-7 for $M_{\text{bd}}$ in the range $10^{-7} - 10^{-1} M_{\odot}$ and $t_{\text{min}} = 1 \text{ h}$.

Equation 11 is derived in the limit $A \gg 1$ and uses an interpolation of the LMC luminosity function which becomes poor for large magnitudes. We can directly compute the ratio $N_{\text{invisible}}/N_{\text{monitored}}$ from Equation 8 using actual data and no approximation. Table 1 shows, as a function of the apparent magnitude $m$, the amplification $A(m)$ corresponding to a detection threshold $m_{\text{thresh}} = 18.9$, the impact parameter $u(A(m))$, the factor $u(m)\Phi(m)$ and the duration $t_{\text{event}}$. For instance, Table 1 shows that lensings of stars of magnitude larger than 21 will not be detected if we require a lensing event to last more than 3 hours. The durations $t_{\text{event}}$ given in Table 1 correspond to a $10^{-4} M_{\odot}$ brown dwarf with a transverse velocity $V_{\perp} = 200 \text{ km/s}$ at a distance $D_{\text{bd}} = 10 \text{ kpc}$, and they scale according to Equation 5.

The number of monitored stars is the sum of the numbers in the second column of Table 1, up to magnitude $m = 18.9$, that is 38 (in actual brown dwarf searches, the surveyed areas are larger and denser than the area surveyed by Ardeberg et al., and the corresponding number of stars is nearly $10^5$). The number of invisible stars which can be detected when lensed is the sum of the fifth column of Table 1 between $m = 18.9$ and $m = 21$ (for $t_{\text{min}} = 3 \text{ hours}$), that is 96. The ratio $N_{\text{invisible}}/N_{\text{monitored}}$ in this case is $96/38 = 2.5$.

This ratio depends on the brown dwarf mass only through the $t_{\text{min}}$ cut-off, and Table 2 shows how this ratio varies as a function of $t_{\text{min}}$ for brown dwarf masses $M_{\text{bd}} = 10^{-1} \text{ to } 10^{-7} M_{\odot}$. We see that this ratio is almost always between 2 and 4, except for very light brown dwarfs. The large magnitude bins and the small number of stars in each bin induce wild fluctuations in the ratio $N_{\text{invisible}}/N_{\text{monitored}}$ when the threshold $m_{\text{thresh}}$ or the minimal duration $t_{\text{min}}$ are changed (this is why we chose a threshold at 18.9 instead of 19). Smoothing and interpolation improve the situation,
but we saw that it does not change the conclusion that invisible stars more than double the expected number of lensing events.

Acknowledgement: While experimentalists involved in ongoing experiments are very busy now, and have no time to spare for funny ideas, I wish to thank brown dwarf teams for their open mind, and in particular Marc Moniez for a first check of the ideas exposed here. So far statistics are too limited to give any answer, positive or negative, but the test revealed no unexpected difficulty.

References

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Table captions

Table 1: The first column shows the apparent magnitude $m$ of a star in the LMC, the second column is the luminosity function given by Ardeberg et al. for a limited area of the LMC, the third column is the minimal amplification $A(m)$ corresponding to a detection threshold $m_{\text{thresh}} = 18.9$, the fourth column is the corresponding impact parameter $u(A(m))$, the fifth column the contribution of stars of magnitude $m$ to lensing events, and the sixth column is the mean duration of the event for a $10^{-4} M_\odot$ brown dwarf at $D_{\text{bd}} = 10$ kpc with transverse velocity $V_\perp = 200$ km/s.

Table 2: Ratio $N_{\text{invisible}}/N_{\text{monitored}}$ as a function of the $t_{\text{min}}$ cut-off and of the brown dwarf mass $M_{\text{bd}}$. Beyond $m = 23$, the LMC luminosity function from Arde-
berg et al. was extrapolated by \( \Phi(m) = 10^{0.36(m-15.2)} \).
## Tables

| Magnitude $m$ | $\Phi(m)dm$ | $A(m)$ | $u(A(m))$ | $u(m)\Phi(m)dm$ | $t_{\text{event}}$ (hours) |
|--------------|-------------|--------|-----------|-----------------|-----------------------------|
| 14.0         | 0           | 1.34   | 1.00      | 0               | 26                          |
| 14.5         | 1           | 1.34   | 1.00      | 1               | 26                          |
| 15.0         | 0           | 1.34   | 1.00      | 0               | 26                          |
| 15.5         | 1           | 1.34   | 1.00      | 1               | 26                          |
| 16.0         | 0           | 1.34   | 1.00      | 0               | 26                          |
| 16.5         | 2           | 1.34   | 1.00      | 2               | 26                          |
| 17.0         | 7           | 1.34   | 1.00      | 7               | 26                          |
| 17.5         | 3           | 1.34   | 1.00      | 3               | 26                          |
| 18.0         | 7           | 1.34   | 1.00      | 7               | 26                          |
| 18.5         | 17          | 1.34   | 1.00      | 17              | 26                          |
| 19.0         | 42          | 1.47   | 0.85      | 36              | 23                          |
| 19.5         | 69          | 2.33   | 0.46      | 32              | 12                          |
| 20.0         | 45          | 3.69   | 0.28      | 13              | 7                           |
| 20.5         | 87          | 5.85   | 0.17      | 15              | 5                           |
| 21.0         | 136         | 9.27   | 0.11      | 15              | 3                           |
| 21.5         | 169         | 14.69  | 0.07      | 12              | 2                           |
| 22.0         | 300         | 23.29  | 0.04      | 13              | 1                           |
| 22.5         | 439         | 36.91  | 0.03      | 12              | 1                           |
| 23.0         | 646         | 58.49  | 0.02      | 11              | 0.5                         |

### Table 1

| $t_{\text{min}}$ | 96 h | 24 h | 12 h | 6 h  | 3 h  | 1 h  | 30 mn |
|------------------|------|------|------|------|------|------|-------|
| $M_{bd} = 10^{-1} M_\odot$ | 2.5  | 3.5  | 4.1  | 4.4  | 5.0  | 5.4  | 5.9   |
| $M_{bd} = 10^{-2} M_\odot$ | 1.8  | 2.9  | 3.2  | 3.9  | 4.1  | 5.0  | 5.2   |
| $M_{bd} = 10^{-3} M_\odot$ | 0    | 1.8  | 2.5  | 2.9  | 3.5  | 4.1  | 4.7   |
| $M_{bd} = 10^{-4} M_\odot$ | 0    | 0    | 1.8  | 2.1  | 2.5  | 3.5  | 3.9   |
| $M_{bd} = 10^{-5} M_\odot$ | 0    | 0    | 0    | 0.9  | 1.8  | 2.5  | 3.2   |
| $M_{bd} = 10^{-6} M_\odot$ | 0    | 0    | 0    | 0    | 0    | 1.8  | 2.1   |
| $M_{bd} = 10^{-7} M_\odot$ | 0    | 0    | 0    | 0    | 0    | 0    | 0.9   |

### Table 2