NEUTRON CHARGE FORM FACTOR AND QUADRUPOLE DEFORMATION OF THE NUCLEON

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A quark model relation between the neutron charge form factor and the $N \rightarrow \Delta$ charge quadrupole form factor is used to predict the $C2/M1$ ratio in the $N \rightarrow \Delta$ transition from the elastic neutron form factor data. Excellent agreement with the electro-pion production data is found, indicating the validity of the suggested relation. The implication of the negative $C2/M1$ ratio for the intrinsic deformation of the nucleon is discussed.

The geometrical shape of the proton can be determined from its intrinsic quadrupole moment

$$Q^p_0 = \int d^3r \rho^p(r) (3z^2 - r^2),$$  \hspace{1cm} (1)

where $\rho^p(r)$ is the not necessarily spherically symmetric charge density of the proton. If $Q^p_0 > 0$, the proton is prolate (cigar-shaped); if $Q^p_0 < 0$, it is oblate (pancake-shaped); and if $Q^p_0 = 0$, the charge density inside the proton is spherically symmetric. The intrinsic quadrupole moment, which is defined with respect to the body-fixed frame, must be distinguished from the spectroscopic quadrupole moment measured in the laboratory. The latter is zero due to angular momentum selection rules. $Q^p_0$ can be inferred by measuring electromagnetic quadrupole transitions between the nucleon ground and its low-lying excited states, or by measuring the quadrupole moment of an excited state, e.g. the $\Delta(1232)$ with $J > 1/2$.

Another possible way to obtain information on the shape of the nucleon has recently been suggested. In the constituent quark model with two-body exchange currents a connection between the neutron charge form factor $G^p_0(q^2)$ and the $N \rightarrow \Delta$ charge quadrupole form factor $G^{p\rightarrow\Delta^+}_{C2}(q^2)$ has been found

$$G^{p\rightarrow\Delta^+}_{C2}(q^2) = -\frac{3\sqrt{2}}{q^2} G^p_0(q^2),$$ \hspace{1cm} (2)

where $q$ is the three-momentum transfer of the virtual photon. Together with the known SU(6) relation $G^{p\rightarrow\Delta}_{M1}(q^2) = -\sqrt{2} G^p_{M1}(q^2)$, which remains valid after adding exchange currents, this provides a determination of the
C2/M1 ratio through the elastic charge and magnetic neutron form factors. The C2/M1 ratio obtained in this way agrees well with C2/M1 data from electro-pion production experiments (see Fig. 1). By comparing the low $q$ expansion of the left and right hand side of Eq. (3), the $N \rightarrow \Delta$ quadrupole moment $Q_{p \rightarrow \Delta}$ can be expressed in terms of the known neutron charge radius $r_n^2$. Likewise, the $N \rightarrow \Delta$ quadrupole transition radius $r_{p \rightarrow \Delta}^2$ is obtained from the fourth moment $r_n^4$ of the neutron’s charge density $\rho_n(r)$

$$Q_{p \rightarrow \Delta} = \frac{1}{\sqrt{2}} r_n^2, \quad r_{p \rightarrow \Delta}^2 = \frac{3}{10} r_n^4. \quad (3)$$

Experimentally one finds $r_n^2 = -0.113(3) \text{ fm}^2$ and $r_n^4 = -0.32(8) \text{ fm}^4$, from which we predict $Q_{p \rightarrow \Delta} = -0.080(2) \text{ fm}^2$ and $r_{p \rightarrow \Delta}^2 = -0.84(21) \text{ fm}^2$.

Quite generally, the baryon’s charge density consists of a sum of one-, two-, and three-quark pieces:

$$\rho(q) = \rho_{[1]}(q) + \rho_{[2]}(q) + \rho_{[3]}(q).$$

The two- and three-quark terms describe the nonvalence quark degrees of freedom (e.g. $q\bar{q}$ pairs) seen by the electromagnetic probe. In deriving Eq. (2) we have made the following assumptions: (i) the baryon wave functions with orbital angular momentum $L = 0$ contain only valence quark degrees of freedom; (ii) for $G_{C0}$ and $G_{C2}$ one- and three-body operators are suppressed in comparison to the two-body term $\rho_{[2]}$. Assumption (i) is not a restriction provided the nonvalence degrees of freedom are included in the form of many-body operators, and assumption (ii) is supported by several investigations. Eq. (2) is then a result of the dominance of $\rho_{[2]}$ for both observables, the spin-flavor structure of $\rho_{[2]}$, and the SU(6) spin-flavor symmetry of the $N$ and $\Delta$ wave functions.

This can be seen as follows. A multipole expansion of, e.g., the gluon exchange charge operator $\rho_{[2]}$ up to quadrupole terms gives with $q = qe_z$ the
Figure 2. Intrinsic quadrupole deformation of the nucleon (left) and ∆ (right) in the pion cloud model. In the N the p-wave pion cloud is concentrated along the polar (symmetry) axis, with maximum probability of finding the pion at the poles. This leads to a prolate deformation. In the ∆, the pion cloud is concentrated in the equatorial plane producing an oblate intrinsic deformation.

following decomposition in spin-isospin space

\[ \rho_{[2]} = -B \sum_{i<j} e_i \left( 2 \sigma_i \cdot \sigma_j - (3 \sigma_{iz} \sigma_{jz} - \sigma_i \cdot \sigma_j) \right) + (i \leftrightarrow j) \]  

where \( \sigma_{iz} \) is the z component of the spin operator of quark i, and \( e_i \) is the quark charge. \( B \) contains the orbital and color part common to the spin-scalar (C0) and spin-tensor (C2) part of the operator. Note that there is a fixed relative strength between the C0 term and the C2 term. Evaluating \( \rho_{[2]} \) between SU(6) spin-flavor N and ∆ wave functions, one obtains \( r_n^2 = 4B \) and \( \sqrt{2}Q_p \rightarrow \Delta^+ = 4B \). This leads to the first equation in Eq.(3). A generalization of this derivation to finite momentum transfers is straightforward and leads to Eq.(4).

In order to calculate \( Q_0^p \) from the observable \( Q_p \rightarrow \Delta^+ \) we need a model. We have calculated \( Q_0^p \) using three different nucleon models. In the quark model we find that Eq.(3) implies

\[ Q_0^p = -Q_0^{\Delta^+} = -r_n^2, \]  
i.e., a prolate intrinsic deformation of the proton and an oblate intrinsic deformation of the \( \Delta^+ \). We also see that the neutron charge radius \( r_n^2 \) and the quadrupole deformation of the nucleon are intimately related phenomena, which reflect the \( q\bar{q} \) degrees of freedom in the nucleon.
Also in the pion cloud model a relation between $r_n^2$ and $Q_{p\rightarrow\Delta^+}$ and between the intrinsic quadrupole moments of the $N$ and $\Delta$ is obtained.

$$Q_{p\rightarrow\Delta^+} = Q_{\Delta^+} = r_n^2, \quad Q_0^0 = -Q_0^{\Delta^+} = -r_n^2.$$  \hspace{1cm} (6)

Even though the valence quark core of the nucleon may have a small oblate deformation (as one would obtain from the small D-state admixture of the valence quarks), the major contribution to the intrinsic quadrupole moment comes from the p-wave coupling of the pion cloud to the valence quark core. From angular momentum coupling the pion cloud in the proton is oriented along the polar axis (see Fig. 2). This leads to an overall prolate deformation of the nucleon. Similarly, for the $\Delta$ with spin $3/2$, angular momentum conservation implies that the pion cloud lies in the equatorial plane characteristic of an oblate intrinsic deformation. In addition, we have calculated $Q_0^0$ and $Q_0^{\Delta^+}$ in the Bohr-Mottelson collective model, with the same qualitative results, namely a prolate shape of the $N$ and an oblate shape of the $\Delta$.

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