Electroweak Baryogenesis in the Adiabatic Limit

MICHAEL DINE AND SCOTT THOMAS
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

Abstract

Electroweak baryogenesis can occur in the “adiabatic limit,” in which expanding bubbles of true vacuum are assumed to have rather thick walls and be slowly moving. Here the problem of calculating the baryon asymmetry in this limit is reconsidered. A simple prescription for obtaining the relevant kinetic equations is given. An additional suppression beyond that usually assumed is found. This arises because the generation of an asymmetry requires violation of approximately conserved currents by Higgs expectation values, which are small near the front of the walls. As applications, the asymmetries in multi-Higgs models and the minimal supersymmetric standard model are estimated to be proportional to $\alpha_8$ and $\alpha_6$ respectively. Also, in this limit the baryon asymmetry in the Minimal Standard Model is extraordinarily small.

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1. Introduction

The origin of the baryon asymmetry of the universe is a long-standing problem.\cite{1} With the recognition that baryon number is badly violated in the standard model at high temperatures,\cite{2} there have been many proposals for how an asymmetry might be generated at the electroweak phase transition.\cite{3} Virtually all of these proposals assume that the electroweak phase transition is first order, providing the required departure from equilibrium. CP violation already exists in the minimal standard model (MSM); extensions of the standard model generically contain further sources of CP violation. Since all three generations of quarks must be involved, any asymmetry which arises in the MSM should be extremely small. Most studies of electroweak baryogenesis have involved extensions of the MSM. However, there have been a number of attempts to explain the asymmetry within the framework of the MSM, the most recent being that of ref. 4.

The electroweak transition, if first order, proceeds by formation of bubbles. The baryon asymmetry is produced in or near the bubble walls, which are the sites of the most significant departures from equilibrium. The various schemes considered to date for producing an asymmetry divide into two classes. The first is referred to as the “adiabatic case.” If a wall is slowly moving and thick, its passage through the plasma is nearly adiabatic, in the sense that all quantities (but the baryon number) should be close to equilibrium. In this case, the time-varying Higgs field can act to bias the baryon-violating processes.\cite{5,6,7} In the second scheme, the wall is assumed to be thin and rapidly moving, so more significant departures from equilibrium can occur. Cohen, Kaplan and Nelson have pointed out that in this limit, scattering of particles from the wall can produce an asymmetry in front of the wall for some (approximately conserved) quantum number. This, in turn, can bias the baryon violating processes.\cite{8} Other non-adiabatic scenarios have been considered in refs. 9 and 10.

In this paper, certain aspects of the adiabatic case will be considered. The adiabatic picture has been most carefully developed in refs. 6 and 7; it is closely
related to the “spontaneous baryogenesis” scheme of Cohen and Kaplan.\footnote{11} In ref. 6, the problem was analyzed by obtaining the leading coupling between the time-varying Higgs field and Chern-Simons number. In order to make the analysis as simple as possible, models were considered in which this coupling arose from loops of heavy fields (massive compared to the temperature of the transition). However, while the analysis is simple in this limit, the resulting asymmetry is quite small, for two reasons. First, there is simply the suppression by the masses of heavy particles. But in addition, there is a suppression by four powers of coupling constant, $g$. This latter suppression results because the operator which gives rise to the coupling is quadratic in $g\phi$ (where $g$ denotes a gauge or Yukawa coupling and $\phi$ denotes the Higgs field). The baryon number violating process cuts off at a value of the scalar field, $\phi_{co}$ (in the notation of ref. 3), for which

$$g\phi_{co} \sim \alpha_w T,$$

i.e. for a rather small value of the Higgs field. There has been some debate in the literature as to how large, numerically, this suppression may be. This question will be taken up later; here the parametric dependence on the couplings, $g$, will be determined.

In many models the coupling of the scalar field to the Chern-Simons number arises due to light fields. While this case is potentially more promising, it is inherently more complicated, involving all the subtleties of real-time, finite temperature field theory. In order to deal with this, an attractive method has been proposed by Cohen, Kaplan and Nelson in ref. 7. The results for the asymmetry are distinctly more promising; in particular, it is not obvious from the analysis of ref. 7 that the suppression by powers of coupling constant mentioned above is obtained.

However, this method has limitations. In the multi-Higgs model treated by many authors, for example, it cannot be valid for very small quark mass, since a non-zero asymmetry results even as the top quark mass tends to zero. This fact has already been remarked by the authors of ref. 7, who argue that for small mass there will be suppression by powers of Yukawa couplings. But a non-zero
asymmetry is also obtained for vanishing Higgs expectation value, implying an additional suppression proportional to powers of $\phi_{co}$ has been neglected. The method also does not lend itself to the treatment of a number of other interesting situations, such as baryogenesis in the MSM.

In this note, baryogenesis in this adiabatic, light field, case will be reexamined. In this discussion, $\phi_{co}$ will be assumed small. While this leads inevitably to rather small asymmetries, it significantly simplifies the analysis. This is because the $B$-violating interactions switch off rapidly, in a time of order

$$t_{co} \sim \frac{\phi_{co}\ell}{v\Delta\phi}$$

where $\ell$ is the wall thickness and $v$ its velocity, and $\Delta\phi$ is the total change in $\phi$ across the wall (typically of order $T$). Even if the wall is rather thick, $t_{co} \sim T^{-1}$.

During this time, the particle number densities can change only by small amounts. The kinetic equations can then be written down and solved almost trivially.

The case where the cutoff is not so small can be analyzed by a more complete set of kinetic equations then will be considered below. This will be postponed for a future publication.$^{[12]}$

Given the assumption of small $\phi_{co}$, we first note that a more direct, naive treatment of the problem leads to the sort of suppression by powers of $\phi_{co}$ observed in the large mass case. This naive treatment involves two steps. The coupling of the time-varying field to the Chern-Simons number is first computed. A rate equation for the baryon number is then developed by determining how this coupling biases the baryon-violating rate. We next consider the approach of ref. 7, in which the theory is rewritten so as to exhibit the coupling of the Higgs field to certain currents. Here the issue will be to derive suitable rate equations. In fact, the results of the naive treatment are recovered.

In the end, then, the asymmetry in the adiabatic limit for the multi-Higgs model is not of order $\alpha_w^4$, but rather $\alpha_w^8$ (times $CP$-violating phases and dynamical
factors involving sphaleron rates, the bubble profile and velocity, etc.\footnote{Here }While this sounds alarmingly small, it may yet correspond to an acceptable asymmetry. For example, general arguments give that the high temperature baryon-violating rate goes as \( \kappa \alpha_w^4 \), but inclusion (or not) of factors of \( 4\pi \) is currently a matter of prejudice. Indeed, the only calculation of this rate is that of ref. 13; the authors of this paper have recently expressed skepticism as to the validity of their result.\footnote{As for the significance of the suppression, some authors have argued, by interpolating a formula valid for large \( \phi \), that \( \phi \sim \alpha_w \sim 0.2T \).} As for the significance of the suppression, some authors have argued, by interpolating a formula valid for large \( \phi \), that \( g\phi \sim 7\alpha_w \sim 0.2T \). \( (1.3) \)

One of us has argued that formulas which interpolate between the small and large \( \phi \) limits are likely to give \( 2 - 3 \) instead of \( 7 \) in eq. \( (1.3) \).\footnote{The same estimate, however, tends to give quite a large value of \( \kappa \). At best, these estimates are just educated guesses. More extensive simulations are essential to give results reliable even as to order of magnitude.} The same estimate, however, tends to give quite a large value of \( \kappa \). At best, these estimates are just educated guesses. More extensive simulations are essential to give results reliable even as to order of magnitude.

Apart from revising earlier estimates in certain models, the methods developed here may be applied to the MSM. Recently it has been suggested that the MSM may lead to a suitable baryon asymmetry.\footnote{Perturbative treatments of the phase transition suggest that the wall is indeed thick and slowly moving, so that the adiabatic treatment should not be unreasonable. In this case, the asymmetry turns out to be extremely small \( (n_B/s \sim 10^{-32} \text{ or so}) \). In ref. 4, however, it was argued that perturbation theory is not a reliable guide to the physics of the phase transition, and that the wall is fast and thin. We will comment on this possibility, but are not in a position to make any definitive statements.} Perturbative treatments of the phase transition suggest that the wall is indeed thick and slowly moving, so that the adiabatic treatment should not be unreasonable. In this case, the asymmetry turns out to be extremely small \( (n_B/s \sim 10^{-32} \text{ or so}) \). In ref. 4, however, it was argued that perturbation theory is not a reliable guide to the physics of the phase transition, and that the wall is fast and thin. We will comment on this possibility, but are not in a position to make any definitive statements.
2. Naive Treatment of the Multi-Higgs Model

For definiteness, following ref. 7, a multi-Higgs model with a CP violating Higgs potential will be considered. Only one Higgs, \( \phi_1 \), couples to quarks (to avoid flavor-changing neutral currents). In the bubble wall, \( \phi_1 = \rho_1(x)e^{i\theta_1(x)} \), where \( \theta_1(x) \) arises from the CP violation. The coupling of \( \theta_1 \) to \( F\tilde{F} \) may be calculated from the triangle diagram of fig. 1. The propagators in the loop are understood as suitable finite-temperature Green’s functions. The real-time expression can be obtained by analytic continuation of the Euclidean time result. In the present case, this is straightforward. The diagram is perfectly finite in the infrared and ultraviolet, and for slowly varying \( \theta_1 \) (compared to \( T^{-1} \)) can be expanded in a power series in the momenta. The result is necessarily quadratic in \( m_t = \lambda_t \rho_1 \) because of the required chirality flip (\( m_t \) is the effective \( t \) quark mass, \( \lambda_t \) is the \( t \) quark Yukawa coupling). The calculation is quite straightforward (related calculations, for example, have been performed in ref. 16) and one obtains

\[
\mathcal{L}_\theta = a\theta_1 m_t^2 T^2 \frac{F\tilde{F}}{32\pi^2} + \mathcal{O}\left(\frac{m_t}{T}\right)^4
\]

where \( a = \frac{14}{3\pi} \zeta(3) \).

How might this coupling bias the sphaleron process? Integrating by parts, \( \mathcal{L}_\theta \) may be written in terms of the Chern-Simons number

\[
\mathcal{L}_\theta = a \left( \frac{m_t^2}{T^2} \right) \partial_\theta \theta_1 n_{cs}.
\]

Suppose that the coefficient of \( n_{cs} \) here is very slowly changing with time. This gives, effectively, a chemical potential for Chern-Simons number,

\[
\mu_{cs} = a \left( \frac{m_t^2}{T^2} \right) \partial_\theta \theta_1.
\]

In an (anti-)sphaleron transition, \( n_{cs} \) changes by \((-1)1\). Suppose \( \Gamma \) is the rate for transitions changing \( n_{cs} \) at equilibrium. Considerations of detailed balance then

* Ref. 16 indeed calculates many of the couplings necessary for the analysis. However, they use the results of ref. 7 without modification to obtain rate equations, and thus their final results differ from ours.
give that, when the particle densities are small compared with the equilibrium values (as is the case for our assumption of short times), the difference in rates for transitions changing $n_{cs}$ by $+1$ and $-1$ is

$$\frac{dn_{cs}}{dt} = \mu_{cs} \beta \Gamma$$

(2.4)

The corresponding change in baryon number is three times larger.

This result will be obvious to many readers, but a brief description of the derivation is perhaps useful. Near equilibrium, for small number densities, states differing in $n_{cs}$ by one unit differ in free energy by an amount $\mu_{cs}$. Yet at equilibrium, the rate of transitions increasing and decreasing the free energy must be equal. Thus the ratio of these rates must equal the ratio of Boltzmann factors for the two states,

$$e^{-\beta \mu}.$$

(2.5)

For small $\mu$, this just gives eq. (2.4).

Since $\mu \propto m_{t}^{2} \propto \rho_{1}^{2}$, this treatment should give a baryon number proportional to $\phi_{co}^{2}$, i.e. the small sort of rate discussed earlier. To make an estimate, we make the simplifying assumption of $\Gamma = \Gamma(\phi)$. We then make the further simplification of taking $\Gamma = \kappa (\alpha_{w} T)^{4}$ for $\phi < \phi_{co}$, and $\Gamma = 0$ for $\phi > \phi_{co}$. This gives

$$n_{B} \sim 3 a \kappa \alpha_{w}^{4} \lambda_{t} \phi_{co}^{2} T \Delta \theta_{1}.$$

(2.6)

Here, $\Delta \theta_{1}$ is the value of the $CP$-violating phase when the baryon-violating process turns off. The baryon to entropy ratio is of order

$$\frac{n_{B}}{s} \approx \kappa \left(\frac{100}{g^{*}}\right) \left(\frac{\Delta \theta_{1}}{\pi}\right) \left(\frac{\lambda_{t} \phi_{co}}{T}\right)^{2} \times (1 \times 10^{-7}).$$

(2.7)

It should be noted that in the multi-Higgs case, $\Delta \theta_{1}$, is itself of order $(\phi_{co}/T)^{2}$ (times coupling constants), since in the absence of quartic couplings the Higgs potential is $CP$-conserving.
The worst-case scenario here, in which $\kappa \sim 1$ and $g\phi_{co} \sim \alpha_w T$, gives an unacceptably small result, of order $10^{-13}$ for $n_B/s$. As remarked above, however, it is conceivable that $\kappa$ is large, and $\phi_{co}$ is 3 to 7 times as large, so the final result could well be large enough, provided CP-violating phases are large.

Finally, we can ask how other processes affect the final asymmetry. For example, processes involving scattering of top quarks and Higgs fields, and QCD sphaleron processes, will change the numbers of left and right-handed fields. Each fermion species obeys a rate equation of the form

$$\frac{dn_i}{dt} \simeq \beta \mu_{cs} d_i \Gamma + \gamma_i$$  \hspace{1cm} (2.8)

Here the first term represents the sphaleron process; $d_i = 0$ for $SU(2)$ singlets, $d_i = 1/2$ for doublets. As before, $\Gamma$ is the sphaleron rate, while $\gamma_i$ denotes other processes which change the number densities. If $\gamma_i$ can be neglected, then summing over the individual rates weighted with the baryon number, eq. (2.8) reproduces eq. (2.4).

The estimate described above will be valid provided that $\gamma_i$ and $n_i$ are not too large. In order to understand this criterion, consider a particular process which has been discussed recently: strong sphalerons in the two Higgs model. In this case, the associated terms in the rate equations are calculated just as for the weak sphalerons. The triangle diagram with the $W$ bosons replaced by gluons gives rise to a chemical potential for “strong Chern-Simons number”, $n_{cs}^{cs}$ precisely as for $n_{cs}$. In this case, one obtains in the effective lagrangian

$$\mathcal{L}_\theta^c = a^c \theta_1 \frac{m_t^2}{T^2} \frac{G \tilde{G}}{32 \pi^2} + \mathcal{O} \left( \frac{m_t}{T} \right)^4$$  \hspace{1cm} (2.9)

where $a^c = -\frac{14}{\pi^2} \zeta(3)$. In other words, a chemical potential for the QCD Chern-Simons number results which is three times as large as that for $SU(2)$. The rate equation for small densities is now

$$\frac{dn_i}{dt} \simeq \mu_{cs} \beta (d_i \Gamma - 3 h_i \gamma_i)$$  \hspace{1cm} (2.10)

where $h_i = 1/3$ for color triplets and anti-triplets, $h_i = 0$ for leptons, and the rate,
\[ \gamma = \kappa c(\alpha sT)^4. \]

In this equation, terms on the right hand side linear in the densities have been neglected; this is correct provided none of the densities are of order \( \mu csT^2 \). This equation can be integrated, as before, up to times for which \( \phi = \phi_{co} \). As argued above this corresponds to a time, \( t_{co} \sim T^{-1} \) (eq. (1.2)). Apparently the strong sphaleron term can be neglected unless \( \kappa c \) is extremely large, of order \( 10^5 \) or so (the precise value depending also on \( \phi_{co} \)). The earlier estimate of the baryon number is then unmodified. Other processes may be treated similarly, such as top quark scattering from Higgs bosons. Again, provided the estimate for \( t_{co} \) is reasonable, there is no effect. The case with more complete rate equations will be discussed in a subsequent publication.\[12\]

3. Spontaneous Baryogenesis

Ref. 7 suggested a different treatment, which avoids considering directly the coupling to Chern-Simons number. These authors perform an anomaly-free re-definition of the fermion fields which eliminates the phase from the fermion mass terms. For example, consider again the multi-Higgs model, in the version where only one Higgs field couples to ordinary quarks and leptons (\( \phi_1 \)). Transforming each fermion by a phase proportional to its hypercharge ("fermionic hypercharge", \( \tilde{Y} \)) eliminates the phase, \( \theta_1 \), from the Yukawa couplings at the price of a coupling

\[ \partial_\mu \theta_1 j^\mu_{\tilde{Y}}. \]  \hspace{1cm} (3.1)

This appears to have induced a chemical potential for fermionic hypercharge, and to bias the sphaleron process. Suppose the fermionic part of the free energy is minimized subject to the constraints of charge conservation and separate \( B - L \) conservation, and including this chemical potential. The minimum lies at a nonzero value of the baryon number. Detailed balance arguments similar to those given above yield an equation for the rate of change of baryon number. Even without writing this equation however, it is clearly not the one above. In particular, it does not involve the modulus of the Higgs field, \( \rho_1 \), at all.
To understand this question better, focus again on short times. In this limit, as before, the non-zero densities of various species may be neglected in writing kinetic equations. But in this limit, it is clear that if the (small) Higgs vev is ignored, a chemical potential for hypercharge cannot bias the sphaleron process, since the sphaleron process does not violate hypercharge. Moreover, scattering of top quarks from the Higgs particles in the plasma (as suggested in refs. 3 and 7) cannot help, even if the scattering is rapid. The problem is that the field redefinition by fermionic hypercharge, while removing phases from the fermion mass terms, induces phases in the Yukawa couplings of the fermions to the fluctuating part of the Higgs field. To avoid these, write the Higgs field as

$$\phi_1 = (\rho_1 + \phi'_1)e^{i\theta_1}$$  \hspace{1cm} (3.2)$$

where $\phi'_1$ represents the (complex) fluctuating field. Neglecting $\rho_1$, the lagrangian in terms of these fields contains the phase, $\theta_1$, only in the coupling

$$\partial_\mu \theta_1 j^\mu_Y$$

where $j^\mu_Y$ represents the full hypercharge current, including the scalar parts. But this current is conserved in any process (in the limit of small $\rho$); the chemical potential has no effect.

In order to obtain any asymmetry at small times terms involving $\rho_1$ must be included. In this case, the results of the naive analysis are recovered. In particular, it is no longer true that $j^\mu_Y$ is conserved; instead, at tree level

$$\partial_\mu j^\mu_Y = m_t \bar{t}i\gamma_5 t + \ldots$$ \hspace{1cm} (3.3)$$

In order to understand how much this is violated in the presence of background gauge fields consider, again, a finite-temperature Feynman diagram. The calculation is identical to that encountered earlier, and gives

$$\partial_\mu j^\mu_Y = \frac{am^2_T}{T^2} \frac{F \tilde{F}}{32\pi^2}$$ \hspace{1cm} (3.4)$$

We interpret this as meaning, on average, the violation of hypercharge in a sphaleron
transition is
\[ \frac{a m_i^2}{T^2} \Delta n_{cs}. \] (3.5)

So the change in free energy, on average, in a sphaleron transition is precisely that encountered in the naive treatment. The rate equation obtained is thus identical. Although the discussion given here is for the particular hypercharge field redefinition of ref. 7, the results are more generally applicable to any model.\textsuperscript{112}

4. Spontaneous Baryogenesis in the MSM and the MSSM

A naive, perturbative treatment of the electroweak phase transition in the MSM, for Higgs masses larger 25 GeV or so (well below the current LEP limits, of course), gives a bubble wall which is thick and rather slowly moving, and therefore in the adiabatic regime.\textsuperscript{18–20} Having acquired some confidence in our understanding of spontaneous electroweak baryogenesis, the case of the minimal standard model may be considered. In the spirit of the naive treatment, the coupling of the Higgs field to Chern-Simons number must be found. Assuming \( g \phi_{co} \sim \alpha T \), the leading operator should be one with a minimal number of external Higgs fields (each additional loop costs roughly a factor of \( \alpha \), whereas a pair of external Higgs fields costs a factor of \( \alpha^2 \)). The simplest such operator is

\[ \mathcal{L}_{\text{MSM}} = \frac{\gamma |\phi|^2}{T^2} F \tilde{F}. \] (4.1)

Such a coupling is, of course, \( CP \)-violating, and so arise only at high orders. Indeed, in a manner similar to the calculation of \( \theta \) renormalization in the standard model,\textsuperscript{21} such a coupling cannot occur before 7-loop order. Six loops are required to obtain the Jarlskog invariant. A seventh loop involving a hypercharge gauge boson is also needed. These diagrams are perfectly finite in the infrared and ultraviolet, and for momenta small compared to \( T \), represent the first term in a Taylor expansion of the amplitude in powers of the momentum. Thus no difficulty with the analytic
continuation of the result is expected, and we estimate

\[ \frac{n_B}{s} \sim \kappa \alpha_w^6 J/g^* \]  \hspace{1cm} (4.2)

where

\[ J = \text{Im} \det \left( \lambda_U \lambda_U^\dagger, \lambda_D \lambda_D^\dagger \right) \sim 10^{-21} \]

and \( \lambda_U \) and \( \lambda_D \) are the up and down type quark Yukawa matrices. We have not attempted to include further suppression factors of \( \pi \), etc., since the answer is already extremely small (one might guess that these will be at least \((2\pi)^{-7} \sim 10^{-5}\), since this is effectively a 21-dimensional Feynman integration). So even if \( \kappa \) is quite large, and the suppression described earlier is small, the asymmetry is extremely tiny; \(10^{-32}\) is probably a quite conservative estimate.

Shaposhnikov and collaborators have recently argued that the phase transition is much more strongly first order than perturbation theory suggests. This is an important ingredient in the analysis of ref. 4, where it is assumed that the appropriate limit is the “thin-wall,” highly non-adiabatic situation, in which scattering of particles from the wall is the most important process. We are rather skeptical of this claim. Moreover, a crude estimate of the mean free path for scattering of top quarks passing through the wall, gives a result of order \( T^{-1} \), so that even for an extremely thin wall, the scattering treatment may not be appropriate. However, it is a crucial assumption in the work of ref. 4 that perturbation theory is an extremely poor guide for all significant questions about the phase transition. This may be the case (though recent studies of two loop thermal effects have yielded only modest corrections\(^{[23]}\)). Still, it would be quite amazing if these non-perturbative effects change the final asymmetry by 22 orders of magnitude! (For a critique of the calculation of ref. 4 see ref. 24.)

We close this section by extending the earlier estimates to the minimal supersymmetric standard model. The scale of superparticle masses is assumed to be of order \( T \). The principle sources of CP violation are assumed to lie in phases of the \( \mu \) term and soft breaking mass terms of squarks and gauginos. The estimate is
very similar to that of ref. 6. Diagrams as in fig. 2 give rise to couplings such as

$$\mathcal{L}_{cs} = a g^2 \sin \delta \frac{H_1 H_2 \bar{F} F}{T^2 32\pi^2}$$

where $\delta$ represents some combination of CP-violating phases. Repeating the earlier estimates, we obtain for the asymmetry

$$\frac{n_B}{s} \sim \kappa \sin \delta \left( \frac{100}{g^*} \right) \left( \frac{g \phi_{\text{co}}}{T} \right)^2 \times (8 \times 10^{-8})$$

Here there is an additional difficulty. From limits on electric dipole moments (edms) of atoms and the neutron it is expected that $\sin \delta < 10^{-2} - 10^{-3}$\cite{25,26}. Again one must be fortunate with $\kappa$ and $\phi_{\text{co}}$ in order to obtain an acceptable asymmetry. Still, without further simulations, it is hard to rule out this possibility. A supersymmetry aficionado might even view this estimate as tantalizingly close, and suggestive that edms should not be far below the current bounds\cite{25,26}.

5. Conclusions

From all this, we conclude that electroweak baryogenesis, in the adiabatic limit, is less efficient than has been widely believed. Due to the existence of approximately conserved quantities, violated by Higgs expectation values, the asymmetries which arise in the adiabatic limit involve additional powers of couplings beyond those usually assumed. On the other hand, while formal arguments can be given to determine the parametric dependence on couplings, there are enormous uncertainties in the final numerical results. Even obtaining order of magnitude estimates requires knowledge of the sphaleron transition rates for small or vanishing Higgs field. These are, by definition, not accessible to semiclassical treatment. Crude estimates give widely varying answers, and existing calculations are, according to their own authors, unreliable even as to order of magnitude. Clearly, improved simulations are necessary if we are to know whether acceptable asymmetries can be obtained in the adiabatic limit.
In the non-adiabatic, thin wall, regime, there is good reason to believe that acceptable asymmetries can be obtained.\cite{8} For these, $\kappa$ does not need to be large, and $\phi_{co}$ can be small, since the asymmetry is produced in front of the wall where the Higgs field essentially vanishes (indeed, this issue was stressed in ref. 8). The MSM and MSSM are probably far from this regime, but multi-Higgs models may well yield sufficiently violent transitions. If the baryon asymmetry were produced at the electroweak phase transition, and if the “worst case” scenario of small $\kappa$ and $\phi_{co}$ is correct, a significant step beyond currently popular theoretical ideas is likely required.

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FIGURE CAPTIONS

1) Leading diagram which couples the phase, $\theta_1$, to the Chern-Simons number.

2) Leading diagram in the MSSM which couples the Higgs fields to the Chern-Simons number.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9401265v1