Matter radii and skins of $^{6,8}$He from reaction cross section of proton+$^{6,8}$He scattering based on the Love-Franey t-matrix model

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Background: For $^{4,6,8}$He, Tanihata et al. determined matter radii $r_m(\sigma)$ = 1.57(4), 2.48(3), 2.52(3) fm from interaction cross sections $\sigma_I$ for $^{4,6,8}$He scattering on Be, C Al targets at 790 MeV/nucleon. Lu et al. measured the atomic isotope shifts (AIS) for $^{4,6,8}$He and determined proton radii $r_p(\mathrm{AIS})$ for $^{4,6,8}$He. As for $^{4}$He, the distance between $^{4}$He and the center of mass of valence four neutrons is 2.367 fm.

Aim: Our aim is to determine matter radius $r_m$ and skins $r_{\mathrm{skin}}$ for $^{6,8}$He from the $\sigma_I(\mathrm{exp})$ for $^{p+}$ $^{6,8}$He scattering at 700 MeV and the $r_p(\mathrm{AIS})$, since the $\sigma_I(\mathrm{exp})$ have small errors of 1.7%.

Method: Our model is the Love-Franey (LF) t-matrix folding model. We have already shown that the folding model based on LF t-matrix [6] is good for $^{4,6,8}$He+$^{12}$C at 790 MeV per nucleon [6] that is to be published in Results in Physics. Results: Our results for $^{6,8}$He are $r_m(\mathrm{exp})$ = 2.48(3), 2.53(2) fm and $r_{\mathrm{skin}}$ = 0.78(3), 0.82(2) fm.

Conclusion: For $^{6,8}$He, our results $r_m(\sigma_I)$ agree with those of Tanihata et al.. For $^{8}$He, the distance between $^{4}$He and the center of mass (cm) of valence four neutrons is 2.367 fm.

1. INTRODUCTION AND CONCLUSION

Background: The matter radius $r_m$, the neutron skin $r_{\mathrm{skin}}$ and halo structure are important properties of nuclei. When a nucleus has one or more loosely-bound nucleons surrounding a tightly bound core, it is considered that the nucleus has a halo structure. Eventually, we may consider that $^{6,8}$He have the halo structure.

Lu et al. measured the atomic isotope shifts (AIS) along $^{4,6,8}$He by performing laser spectroscopy on individual trapped atoms and determined proton radii as $r_p(\mathrm{AIS}) = 1.462(6), 1.934(9), 1.881(17)$ fm for $^{4,6,8}$He [1].

For He isotopes, meanwhile, Tanihata et al. determined $r_m$ from interaction cross sections $\sigma_I$ for $^{4,6,8}$He scattering on Be, C Al targets at 790 MeV/nucleon [2]; their results are $r_m(\sigma) = 1.57(4), 2.48(3), 2.52(3)$ fm for $^{4,6,8}$He in which the the harmonic-oscillator distribution is assumed for the densities for $^{4,6,8}$He. They used the optical limit of Glauber model [3,4]. The folding model is better than the optical limit of the Glauber model, when the incident energy is smaller than nucleon mass.

As for $^{p+}$ $^{4,6,8}$He scattering, the data on reaction cross section $\sigma_R$ are available at 700 MeV [5] with high accuracy of 1.7%. In Ref. [5], absolute differential cross sections for elastic $^{4,6,8}$He small-angle scattering were measured in inverse kinematics.

Aim: Our aim is to determine matter radius $r_m$ and skins $r_{\mathrm{skin}}$ for $^{6,8}$He from the $\sigma_R(\mathrm{exp})$ [5] for $^{p+}$ $^{6,8}$He scattering at 700 MeV and the $r_p(\mathrm{AIS})$, since the $\sigma_R(\mathrm{exp})$ have small errors of 1.7%.

Method: Our model is the Love-Franey (LF) t-matrix folding model. We show the formulation on the LF t-matrix model below. For proton-nucleus scattering, the potential $U(R)$ between a projectile (P) and a target (T) has the direct and exchange parts, $U^{\mathrm{DR}}$ and $U^{\mathrm{EX}}$, as

$$U^{\mathrm{DR}}(R) = \sum_{\mu,\nu} \int \rho_T^\mu(r_T) t^{\mathrm{DR}}_{\mu\nu}(s; \rho_{\mu\nu}) dr_T ,$$

$$U^{\mathrm{EX}}(R) = \sum_{\mu,\nu} \int \rho_T^\mu(r_T) r_T + s \times t^{\mathrm{EX}}_{\mu\nu}(s; \rho_{\mu\nu}) \exp [-iK(R) \cdot s/M] dr_T$$

(1b)

where $R$ is the relative coordinate between P and T, $s = -r_T + \hat{R}$, and $r_T$ is the coordinate of the interacting nucleon.

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from T. Each of \( \mu \) and \( \nu \) denotes the \( z \)-component of isospin. The nonlocal \( U^{EX} \) has been localized in Eq. (15) with the local semi-classical approximation [8] where \( K(R) \) is the local momentum between \( P \) and \( T \), and \( M = A/(1 + A) \) for the target mass number \( A \); see Ref. [9] for the validity of the localization.

The direct and exchange parts, \( t^{DR}_{\mu \nu} \) and \( t^{EX}_{\mu \nu} \), of the \( t \) matrix are described by

\[
\begin{align*}
t^{DR}_{\mu \nu}(s) &= \frac{1}{4} \sum_S \hat{S}^2 t^{S1}_{\mu \nu}(s) \quad \text{for } \mu + \nu = \pm 1, \quad (2) \\
t^{DR}_{\mu \nu}(s) &= \frac{1}{8} \sum_{S,T} \hat{S}^2 t^{ST}_{\mu \nu}(s) \quad \text{for } \mu + \nu = 0, \quad (3) \\
t^{EX}_{\mu \nu}(s) &= \frac{1}{4} \sum_S (-1)^{S+1} \hat{S}^2 t^{S1}_{\mu \nu}(s) \quad \text{for } \mu + \nu = \pm 1, \quad (4) \\
t^{EX}_{\mu \nu}(s) &= \frac{1}{8} \sum_{S,T} (-1)^{S+T} \hat{S}^2 t^{ST}_{\mu \nu}(s) \quad \text{for } \mu + \nu = 0, \quad (5)
\end{align*}
\]

where \( \hat{S} = \sqrt{2S + 1} \) and \( t^{ST}_{\mu \nu} \) are the spin-isospin components of the \( t \) matrix interaction. We apply the LF \( t \)-matrix folding model for \( p+4,6,8 \)He scattering at \( E_{\text{in}} = 700 \) MeV.

As proton and neutron densities, \( \rho_\nu = \rho_{n} \) and \( \rho_\mu = \rho_{p} \), we use the densities calculated with D1S-Gogny HFB (D1S-GHFB) [10]. As a way of taking the center-of-mass correction to the densities, we use the method of Ref. [11]. We scale D1S-GHFB proton and neutron densities, as mentioned below.

We consider proton and neutron densities calculated with D1S-GHFB as the original density \( \rho(r) \). The scaled density \( \rho_{\text{scaling}}(r) \) is determined from the original density \( \rho(r) \) as

\[
\rho_{\text{scaling}}(r) \equiv \frac{1}{\alpha^3} \rho(\frac{r}{\alpha}), \quad r_{\text{scaling}} \equiv \frac{r}{\alpha} \tag{6}
\]

with a scaling factor

\[
\alpha = \sqrt{\frac{\langle r^2 \rangle_{\text{scaling}}}{\langle r^2 \rangle}} \tag{7}
\]

In Eq. (6), we have replaced \( r \) by \( r/\alpha \) in the original density. Eventually, \( r \) dependence of \( \rho_{\text{scaling}}(r) \) is different from that of \( \rho(r) \). We have multiplied the original density by \( \alpha^{-3} \) in order to normalize the scaled density. The symbol means \( \sqrt{\langle r^2 \rangle_{\text{scaling}}} \) is the root-mean-square radius of \( \rho_{\text{scaling}}(r) \).

For later convenience, we refer to the proton (neutron) radius of the scaled proton (neutron) density \( \rho_{\text{scaling}}^p(r) \) (\( \rho_{\text{scaling}}^n(r) \)) as \( r_p(\text{scaling}) \) (\( r_n(\text{scaling}) \)).

**III. RESULTS**

For \( ^{6,8} \)He, we first deduce neutron radius \( r_n(\sigma_1) = 2.71, 2.70 \) fm from the \( r_m(\sigma_1) = 2.48, 2.52 \) fm and the \( r_p(\text{AIS}) = 1.934, 1.881 \) fm. For \( ^4 \)He, we assume \( r_n(\text{AIS}) = r_p(\text{AIS}) \), i.e., \( r_m(\text{AIS}) = r_n(\text{AIS}) = r_p(\text{AIS}) \). For \( ^{6,8} \)He, the \( r_n(\sigma_1) \) and the \( r_p(\text{AIS}) \) yields \( r_m(\text{exp}) = 2.48(3), 2.53(3) \) fm.

For \( ^{4,6,8} \)He, we scale proton and neutron D1S-GHFB densities so as to satisfy \( r_p(\text{scaling}) = r_p(\text{AIS}) \) and \( r_n(\text{scaling}) = r_n(\text{AIS}) \) for \( ^4 \)He and \( r_p(\text{scaling}) = r_p(\text{AIS}) \) and \( r_n(\text{scaling}) = r_n(\sigma_1) \) for \( ^{6,8} \)He. For \( ^{4,6,8} \)He, the reaction cross section \( \sigma_R(\text{scaling}) \) calculated with the scaled densities undershoot the \( \sigma_R(\text{exp}) \) by 12%, as shown in Fig. 1.

For \( ^4 \)He, we introduce the fine-tuning factor \( F \) as \( F = \sigma_R(\text{exp})/\sigma_R(\text{scaling}) = 1.1385 \). This fine-tuning is necessary for light projectiles and targets [7]. The \( F\sigma_R(\text{scaling}) \) reproduce \( \sigma_R(\text{exp}) \) for \( ^{4,6,8} \)He, as shown in Fig. 1 for \( \sigma_R(\text{exp}) \) of \( ^4 \)He+\( ^4 \)He scattering at 700 MeV. For \( ^{6,8} \)He, we scale the proton and neutron D1S-GHFB densities so as to \( F\sigma_R(\text{scaling}) = \sigma_R(\text{exp}) \) and \( r_p(\text{scaling}) = r_p(\text{AIS}) \). Therefore, our results based on the scaling method are \( r_m(\text{exp}) = 2.48(3), 2.53(2) \) fm and \( r_{\text{skin}} = 0.78(3), 0.82(2) \) fm for \( ^{6,8} \)He.

The proton radius of \( ^6 \)He comes from the proton radius of \( ^4 \)He and the distance \( d_{\alpha-2n} \) between \( ^4 \)He and the cm of valence two neutron; namely,

\[
r_p(\text{AIS}, ^6 \text{He})^2 = r_p(\text{AIS}, ^4 \text{He})^2 + \left( \frac{2}{3} \right)^2 r_{\alpha-2n}^2 \tag{8}
\]

The latter term represents the recoil effect of the cm. The resulting \( r_{\alpha-2n} \) is 3.798 fm, while the \( ^4 \text{He}+n+n \) model of Ref. [12] yields 3.79 fm.

For \( ^8 \)He, the relation becomes

\[
r_p(\text{AIS}, ^8 \text{He})^2 = r_p(\text{AIS}, ^4 \text{He})^2 + \left( \frac{4}{8} \right)^2 r_{\alpha-4n}^2 \tag{9}
\]

The resulting \( r_{\alpha-4n} \) is 2.367 fm.

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