Deviations from Tri-bimaximal Neutrino Mixing in Type-II Seesaw and Leptogenesis

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Abstract

Current experimental data allow the zero value for one neutrino mass, either \(m_1 = 0\) or \(m_3 = 0\). This observation implies that a realistic neutrino mass texture can be established by starting from the limit (a) \(m_1 = m_2 = 0\) and \(m_3 \neq 0\) or (b) \(m_1 = m_2 \neq 0\) and \(m_3 = 0\). In both cases, we may introduce a particular perturbation which ensures the resultant neutrino mixing matrix to be the tri-bimaximal mixing pattern or its viable variations with all entries being formed from small integers and their square roots. We find that it is natural to incorporate this kind of neutrino mass matrix in the minimal Type-II seesaw model with only one heavy right-handed Majorana neutrino \(N\) in addition to the \(SU(2)_L\) Higgs triplet \(\Delta_L\). We show that it is possible to account for the cosmological baryon number asymmetry in the \(m_3 = 0\) case via thermal leptogenesis, in which the one-loop vertex correction to \(N\) decays is mediated by \(\Delta_L\) and the CP-violating asymmetry of \(N\) decays is attributed to the electron flavor.

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I. INTRODUCTION

Recent solar [1], atmospheric [2], reactor [3] and accelerator [4] neutrino experiments have convincingly verified the hypothesis of neutrino oscillation, a quantum phenomenon which can naturally happen if neutrinos are slightly massive and lepton flavors are not conserved. The mixing of lepton flavors is described by a $3 \times 3$ unitary matrix $V$, whose nine elements are commonly parameterized in terms of three rotation angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and three CP-violating phases ($\delta, \rho, \sigma$) [5]. The phase parameters $\rho$ and $\sigma$, which have nothing to do with CP violation in neutrino oscillations, are usually referred as to the Majorana phases. A global analysis of current neutrino oscillation data yields $30^\circ < \theta_{12} < 38^\circ$, $36^\circ < \theta_{23} < 54^\circ$ and $\theta_{13} < 10^\circ$ at the 99% confidence level [6], but three phases of $V$ remain entirely unconstrained. While the absolute mass scale of three neutrinos is not yet fixed, their two mass-squared differences have already been determined to a good degree of accuracy [6]:

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = (7.2 \cdots 8.9) \times 10^{-5} \text{eV}^2$$
$$\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \pm (2.1 \cdots 3.1) \times 10^{-3} \text{eV}^2.$$  

The on-going and forthcoming neutrino oscillation experiments will shed light on the sign of $\Delta m_{32}^2$, the magnitude of $\theta_{13}$ and even the CP-violating phase $\delta$.

From a phenomenological point of view, at least two lessons can be learnt from current experimental data:

- The lightest neutrino is allowed to be massless; i.e., either $m_1 = 0$ (normal neutrino mass hierarchy) or $m_3 = 0$ (inverted neutrino mass hierarchy) has no conflict with the present neutrino oscillation measurements. In both cases, the non-vanishing neutrino masses can be determined in terms of $\Delta m_{21}^2$ and $|\Delta m_{32}^2|$: 

$$m_1 = 0 \implies \begin{cases} m_2 = \sqrt{|\Delta m_{32}^2|} \approx 8.94 \times 10^{-3} \text{eV} , \\ m_3 = \sqrt{|\Delta m_{32}^2| + \Delta m_{21}^2} \approx 5.08 \times 10^{-2} \text{eV} ; \end{cases} \tag{1}$$
$$m_3 = 0 \implies \begin{cases} m_1 = \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2} \approx 4.92 \times 10^{-2} \text{eV} , \\ m_2 = \sqrt{|\Delta m_{32}^2|} \approx 5.00 \times 10^{-2} \text{eV} . \end{cases} \tag{2}$$

Whether one of the above two neutrino mass spectra is true or essentially true remains an open question. But we stress that some interesting neutrino models, such as the minimal seesaw model [7], are actually able to predict the neutrino mass spectrum with either $m_1 = 0$ or $m_3 = 0$.

- A special neutrino mixing pattern, the so-called tri-bimaximal mixing [8],

$$V = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \tag{3}$$

is particularly favored. It yields $\tan \theta_{12} = 1/\sqrt{2}$ (or $\theta_{12} \approx 35.3^\circ$) for the large-mixing-angle MSW solution [9] to the solar neutrino problem, $\tan \theta_{23} = 1$ (or $\theta_{23} = 45^\circ$) for the atmospheric neutrino oscillation, and $\theta_{13} = \rho = \sigma = 0^\circ$. As a direct consequence of $\theta_{13} = 0^\circ$, the CP-violating phase $\delta$ is not well defined. This interesting neutrino
mixing pattern is in general expected to result from an underlying flavor symmetry (e.g., the discrete $A_4$ [10], $S_3$ [11] or $\mu$-$\tau$ [12] symmetry) in the lepton sector. Such a symmetry must be broken spontaneously or explicitly, in order to account for both the observed lepton mass spectra and the realistic neutrino mixing pattern.

One purpose of this paper is just to combine both lessons and reconstruct the simplest neutrino mass texture for either $m_1 = 0$ or $m_3 = 0$. Looking back to Eqs. (1) and (2), we find that $m_1 \ll m_2 \ll m_3$ and $m_3 \ll m_1 \approx m_2$ hold in the $m_1 = 0$ and $m_3 = 0$ cases, respectively. This observation implies that a realistic neutrino mass texture can be established by starting from the symmetry limit (a) $m_1 = m_2 = 0$ and $m_3 \neq 0$ or (b) $m_1 = m_2 \neq 0$ and $m_3 = 0$. We shall show that it is possible to introduce a particular perturbation, which ensures the resultant neutrino mass matrix $M_\nu$ to reproduce the tri-bimaximal mixing pattern or its viable variations with all entries being formed from small integers and their square roots.

The second purpose of this paper is to incorporate the texture of $M_\nu$ in the minimal Type-II seesaw model [13], an economical extension of the standard model with only one heavy right-handed Majorana neutrino $N$ in addition to the $SU(2)_L$ Higgs triplet $\Delta_L$. We shall focus our interest on the $m_3 = 0$ case, so as to obtain a non-vanishing CP-violating asymmetry in the lepton-number-violating decays of $N$. Such an asymmetry arises from the interference between the tree-level amplitude of $N$ decays and the one-loop vertex correction mediated by $\Delta_L$. Following the idea of baryogenesis via leptogenesis [14] and taking account of the flavor-dependent effects [15], we shall show that it is possible to interpret the observed baryon number asymmetry of the Universe (i.e., $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$ [16]) via thermal leptogenesis in our model, in which only the electron flavor plays a role in the lepton-to-baryon conversion.

The remaining part of this paper is organized as follows. In section II we describe a purely phenomenological way to get viable variations of the tri-bimaximal neutrino mixing pattern from two simple textures of the neutrino mass matrix $M_\nu$, one with $m_1 = 0$ and the other with $m_3 = 0$. Section III is devoted to incorporating the texture of $M_\nu$ with $m_3 = 0$ and a non-trivial CP-violating phase in the minimal Type-II seesaw model, and to calculating the flavor-dependent leptogenesis in order to account for the cosmological baryon number asymmetry $\eta_B$. A brief summary of our main results is presented in section IV.

II. DEVIATIONS FROM TRI-BIMAXIMAL NEUTRINO MIXING

Let us work in the basis where the flavor eigenstates of three charged leptons are identified with their mass eigenstates (i.e., the charged-lepton mass matrix $M_l$ is diagonal, real and positive). Then the mass eigenstates of three neutrinos ($\nu_1$, $\nu_2$, $\nu_3$) are directly linked to their flavor eigenstates ($\nu_e$, $\nu_\mu$, $\nu_\tau$) through the neutrino mixing matrix $V$. If $V$ is of the tri-bimaximal mixing pattern as given in Eq. (3), it can be decomposed into a product of two Euler rotation matrices: $V = O_{23}O_{12}$, where

$$O_{12} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{2} + x^2} & \frac{x}{\sqrt{2} + x^2} & 0 \\ \frac{-x}{\sqrt{2} + x^2} & \frac{\sqrt{2}}{\sqrt{2} + x^2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
with $x = 1$. Allowing for small deviations of $x$ from unity, we are then left with some variations of the tri-bimaximal neutrino mixing pattern which can fit current or future neutrino oscillation data to a better degree of accuracy. Our strategy of reconstructing the neutrino mass matrix $M_\nu$ is three-fold: (1) we take a proper symmetry limit of $M_\nu$, denoted as $M_\nu^{(0)}$, which can be diagonalized by the orthogonal transformation $O_{23}$; (2) we introduce a particular perturbation to $M_\nu^{(0)}$, denoted as $\Delta M_\nu$, which can be diagonalized by the orthogonal transformation $O_{23}O_{12}$; (3) we require that $M_\nu = M_\nu^{(0)} + \Delta M_\nu$ should also be diagonalized by the transformation $O_{23}O_{12}$. Of course, the texture of $M_\nu$ ought to guarantee either $m_1 = 0$ or $m_3 = 0$.

**A. Texture of $M_\nu$ with $m_1 = 0$**

In the $m_1 = 0$ case, we observe from Eq. (1) that $m_2 \ll m_3$ holds. Hence a reasonable symmetry limit of $M_\nu$ is expected to be

$$M_\nu^{(0)} = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix},$$

where $c$ is assumed to be real and positive. The $S_2$ permutation symmetry in the $(2,3)$ sector of $M_\nu^{(0)}$ assures that this mass matrix can be diagonalized by the $O_{23}$ transformation:

$$O_{23}^T M_\nu^{(0)} O_{23} = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{pmatrix}. \quad (6)$$

In other words, $m_3 = 2c$ and $m_2 = m_1 = 0$ hold in the chosen symmetry limit. A non-vanishing value of $m_2$ and a generalized tri-bimaximal neutrino mixing pattern can result from the perturbation

$$\Delta M_\nu = c \varepsilon \begin{pmatrix} x^2 & x & -x \\ x & 1 & -1 \\ -x & -1 & 1 \end{pmatrix}, \quad (7)$$

where $\varepsilon$ is a small dimensionless quantity, and $x$ is a positive number of $O(1)$. When $x = 1$ holds, $\Delta M_\nu$ has the $S_2$ permutation symmetry in its $(1,2)$ sector. Given the orthogonal transformations in Eq. (4), the diagonalization

$$(O_{23} O_{12})^T \Delta M_\nu (O_{23} O_{12}) = c \varepsilon O_{12}^T \begin{pmatrix} x^2 & \sqrt{2} x & 0 \\ \sqrt{2} x & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} O_{12} = c \varepsilon \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 + x^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

works. The neutrino mass matrix
\[ M_\nu = M_\nu^{(0)} + \Delta M_\nu = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \varepsilon \begin{pmatrix} x^2 & x & -x \\ x & 1 & -1 \\ -x & 1 & 1 \end{pmatrix} \]  

(9)

can then be diagonalized by the unitary matrix \( V = O_{23}O_{12} \):

\[ V^T M_\nu V = O_{23}^T M_\nu^{(0)} O_{23} + (O_{23}O_{12})^T \Delta M_\nu (O_{23}O_{12}) = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & (2 + x^2) \varepsilon & 0 \\ 0 & 0 & 2 \end{pmatrix}. \]  

(10)

Three neutrino mass eigenvalues of \( M_\nu \) turn out to be \( m_1 = 0, m_2 = (2 + x^2) c \varepsilon \) and \( m_3 = 2c \). Taking account of Eq. (1), we immediately obtain the results \( c = m_3/2 \approx 2.54 \times 10^{-2} \text{ eV} \) and \( \varepsilon = 2m_2/ [(2 + x^2) m_3] \approx 0.35/(2 + x^2) \).

**B. Texture of \( M_\nu \) with \( m_3 = 0 \)**

In the \( m_3 = 0 \) case, we observe from Eq. (2) that \( m_1 \approx m_2 \) holds. Thus a reasonable symmetry limit of \( M_\nu \) is expected to be

\[ M_\nu^{(0)} = c \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \]  

(11)

where \( c \) is also assumed to be real and positive. This neutrino mass matrix can similarly be diagonalized by the \( O_{23} \) transformation:

\[ O_{23}^T M_\nu^{(0)} O_{23} = c \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}. \]  

(12)

Namely, \( m_1 = m_2 = 2c \) and \( m_3 = 0 \) hold in the chosen symmetry limit. To break the degeneracy of \( m_1 \) and \( m_2 \), we may introduce the same perturbation to \( M_\nu^{(0)} \) as that given in Eq. (7), which can be diagonalized by the same transformation as that shown in Eq. (8). It is then possible to diagonalize the neutrino mass matrix

\[ M_\nu = M_\nu^{(0)} + \Delta M_\nu = c \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \varepsilon \begin{pmatrix} x^2 & x & -x \\ x & 1 & -1 \\ -x & 1 & 1 \end{pmatrix} \]  

(13)

by using the orthogonal matrix \( V = O_{23}O_{12} \):

\[ V^T M_\nu V = O_{23}^T M_\nu^{(0)} O_{23} + (O_{23}O_{12})^T \Delta M_\nu (O_{23}O_{12}) = c \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 + (2 + x^2) \varepsilon & 0 \\ 0 & 0 & 2 \end{pmatrix}. \]  

(14)

In this case, three neutrino mass eigenvalues of \( M_\nu \) are \( m_1 = 2c, m_2 = [2 + (2 + x^2) \varepsilon] c \) and \( m_3 = 0 \). With the help of Eq. (2), one may easily arrive at \( c = m_1/2 \approx 2.46 \times 10^{-2} \text{ eV} \) and \( \varepsilon = 2 (m_2 - m_1) / [(2 + x^2) m_1] \approx 3.25 \times 10^{-2}/(2 + x^2) \).
C. Neutrino mixing patterns

Although the tri-bimaximal neutrino mixing pattern is of great interest, it is by no means unique in describing current neutrino oscillation data. Hence we have gone beyond this pattern by allowing for \( x \neq 1 \) in the above discussions. In both case (A) and case (B), the neutrino mixing matrix \( V = O_{23}O_{12} \) reads

\[
V' = \begin{pmatrix}
\frac{2}{\sqrt{2}} \frac{1}{\sqrt{2} + x^2} & \frac{x}{\sqrt{2} + x^2} & 0 \\
-x/\sqrt{2} \frac{1}{\sqrt{2} + x^2} & 1/\sqrt{2 + x^2} & 1/\sqrt{2} \\
x/\sqrt{2} \frac{1}{\sqrt{2} + x^2} & -1/\sqrt{2 + x^2} & 1/\sqrt{2}
\end{pmatrix}.
\]  \tag{15}

It is obvious that \( V \) takes the exact tri-bimaximal mixing pattern for \( x = 1 \). The allowed range of \( x \) can be determined from that of \( \theta_{12} \) through the relationship \( x = \sqrt{2} \tan \theta_{12} \). In view of \( 30^\circ < \theta_{12} < 38^\circ \), which is obtained from a global analysis of current neutrino oscillation data [6], we easily arrive at \( 0.82 \lesssim x \lesssim 1.10 \). We see that the possibility of \( x = \sqrt{2} \), which leads \( V \) to the bimaximal neutrino mixing, has clearly been excluded. On the other hand, \( x = 1 \) seems to be the simplest and most favored possibility.

Within the allowed range of \( x \), it is not difficult to find out some viable variations of the tri-bimaximal neutrino mixing pattern. In particular, we pay interest to such a category of neutrino mixing matrices \( V \): the entries of \( V \) are all formed from small integers and their square roots, which are often suggestive of a certain flavor symmetry in the language of group theories. Below are three examples:

- \( x = \sqrt{6}/3 \), corresponding to \( \theta_{12} = 30^\circ \) and

\[
V = \begin{pmatrix}
\sqrt{3}/2 & 1/2 & 0 \\
-\sqrt{2}/4 & \sqrt{6}/4 & 1/\sqrt{2} \\
\sqrt{2}/4 & -\sqrt{6}/4 & 1/\sqrt{2}
\end{pmatrix}; \tag{16}
\]

- \( x = \sqrt{3}/2 \), corresponding to \( \theta_{12} \approx 31.5^\circ \) and

\[
V = \begin{pmatrix}
4/\sqrt{22} & \sqrt{3}/\sqrt{11} & 0 \\
-\sqrt{3}/\sqrt{22} & 2/\sqrt{11} & 1/\sqrt{2} \\
\sqrt{3}/\sqrt{22} & -2/\sqrt{11} & 1/\sqrt{2}
\end{pmatrix}; \tag{17}
\]

- \( x = 2\sqrt{2}/3 \), corresponding to \( \theta_{12} \approx 33.7^\circ \) and

\[
V = \begin{pmatrix}
3/\sqrt{13} & 2/\sqrt{13} & 0 \\
-\sqrt{2}/\sqrt{13} & 3/\sqrt{26} & 1/\sqrt{2} \\
\sqrt{2}/\sqrt{13} & -3/\sqrt{26} & 1/\sqrt{2}
\end{pmatrix}. \tag{18}
\]

The pattern of \( V \) in Eq. (16) is especially interesting, because all of its nine elements are formed from four smallest integers 0, 1, 2, 3 and their square roots. This pattern has actually been conjectured in Ref. [17], but here we illustrate how it can be obtained from \( M_\nu \).
III. MINIMAL TYPE-II SEESAW AND LEPTOGENESIS

Now let us consider how to derive the neutrino mass matrix $M_\nu$ in Eq. (9) or Eq. (13) from a specific seesaw model. One may naively expect that the minimal seesaw model with two heavy right-handed Majorana neutrinos [7] is a good candidate, because it naturally assures that $M_\nu$ is of rank 2 and has a vanishing mass eigenvalue (either $m_1 = 0$ or $m_3 = 0$). Taking account of the fact that $M_\nu$ is composed of two mass matrices $M_\nu^{(0)}$ and $\Delta M_\nu$, however, we find that it is more natural to incorporate $M_\nu$ in the minimal Type-II seesaw model with only one heavy right-handed Majorana neutrino $N$ in addition to the $SU(2)_L$ Higgs triplet [13]. In this case, the neutrino mass term can be written as

$$
-\mathcal{L}_\nu = \frac{1}{2} (\nu_L N_R^c) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.},
$$

(19)

where $\nu_L$ denotes the column vector of $(\nu_e, \nu_\mu, \nu_\tau)_L$ fields, $M_L$ is a $3 \times 3$ matrix arising from the leptonic Yukawa interaction induced by the Higgs triplet $\Delta_L$, $M_D$ is a $3 \times 1$ matrix arising from the leptonic Yukawa interaction induced by the Higgs doublet $H$, and $M_R = M$ is just the mass of the right-handed Majorana neutrino $N$. Provided $M$ is considerably higher than the mass scale of $M_D$, one may obtain the effective (left-handed) Majorana neutrino mass matrix $M_\nu$ from Eq. (19) via the well-known Type-II seesaw mechanism [18]:

$$
M_\nu \approx M_L - M_D M_R^{-1} M_D^T.
$$

Comparing this formula with $M_\nu = M_\nu^{(0)} + \Delta M_\nu$, we arrive at

$$
M_\nu^{(0)} = M_L, \quad \Delta M_\nu = -M_D M_R^{-1} M_D^T.
$$

(20)

The texture of $M_L = M_\nu^{(0)}$ given in Eq. (5) or Eq. (11) may easily be obtained from certain flavor symmetries (such as the discrete $\mu$-$\tau$ [12] or $S_2$ [19] symmetry). On the other hand, the texture of $\Delta M_\nu$ in Eq. (7) can be derived from Eq. (20) with a unique form of $M_D$:

$$
M_D = i \sqrt{c_\varepsilon M} \begin{pmatrix} x \\ 1 \\ -1 \end{pmatrix},
$$

(21)

together with $M_R = M$. We remark that such a seesaw realization of the texture of $M_\nu$ does not involve any parameter fine-tuning or cancellation, and thus it is quite natural.

More interestingly, the minimal Type-II seesaw model under consideration can offer a possibility of understanding the cosmological baryon number asymmetry via thermal leptogenesis [14]. For simplicity, we assume the mass of $\Delta_L$ is much higher than that of $N$ such that the CP-violating asymmetry in the out-of-equilibrium decays of $N$ is in practice the only source of leptogenesis. We allow $x$ to be complex in $M_D$ and its imaginary part is just responsible for CP violation in the model. In the $m_1 = 0$ case, a straightforward analysis shows that $M_D^* M_L M_D = 0$ holds due to the special textures of $M_L$ in Eq. (5) and $M_D$ in Eq. (21), implying the absence of CP violation in the decays of $N$. Hence we shall focus our interest on the $m_3 = 0$ case in the following.

A. Neutrino Mixing

As $x$ is now taken to be a complex parameter, the diagonalization of $M_\nu$ in Eq. (13) turns out to be quite non-trivial. We need a unitary matrix $V$ to make the transformation $V^\dagger M_\nu V^* = \text{Diag}\{m_1, m_2, 0\}$. We obtain two non-vanishing mass eigenvalues as
\[ m_1 = c\sqrt{X - Y} , \quad m_2 = c\sqrt{X + Y} , \]  
where
\[ X = \frac{1}{2} (|x|^2 + 2)^2 \varepsilon^2 + (x^2 + x^*)^2 \varepsilon + 4 (1 + \varepsilon) , \]
\[ Y = \sqrt{X^2 - 4 |2 + (x^2 + x^*) \varepsilon|^2} . \]  

In addition,
\[ V = \begin{pmatrix} \frac{Z_1}{\sqrt{|Z_1|^2 + 2}} & \frac{Z_2}{\sqrt{|Z_2|^2 + 2}} & 0 \\ -1/\sqrt{|Z_1|^2 + 2} & 1/\sqrt{|Z_2|^2 + 2} & 1/\sqrt{2} \\ 1/\sqrt{|Z_1|^2 + 2} & -1/\sqrt{|Z_2|^2 + 2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  
is just the neutrino mixing matrix, where
\[ Z_1 = \frac{2Y - T_1}{T_2} , \quad Z_2 = \frac{2Y + T_1}{T_2} \]  
with
\[ T_1 = (|x|^4 - 4) \varepsilon^2 + 2 (x^2 + x^*)^2 \varepsilon - 8 \varepsilon , \]
\[ T_2 = 2x^* (|x|^2 + 2) \varepsilon^2 + 2 (x + x^*) \varepsilon ; \]  
and
\[ \rho = \frac{1}{2} \arg \left[ (2 + \varepsilon x^2) Z_1^{*2} - 4 \varepsilon x Z_1^* + 4 (1 + \varepsilon) \right] , \]
\[ \sigma = \frac{1}{2} \arg \left[ (2 + \varepsilon x^2) Z_2^{*2} + 4 \varepsilon x Z_2^* + 4 (1 + \varepsilon) \right] . \]  

Three neutrino mixing angles are \( \theta_{12} = \arctan [(|Z_2|/\sqrt{|Z_1|^2 + 2})/(|Z_1|/\sqrt{|Z_2|^2 + 2})], \) \( \theta_{23} = 45^\circ \) and \( \theta_{13} = 0^\circ . \) Because of \( m_3 = 0 , \) only the difference between \( \rho \) and \( \sigma \) is a physical Majorana CP-violating phase. If \( x \) is real, then there will be no CP violation and Eq. (24) will be simplified to Eq. (15).

**B. Leptogenesis**

The lepton-number-violating and CP-violating decay of \( N \) into a lepton \( l_\alpha \) (for \( \alpha = e, \mu, \tau \)) and a Higgs boson \( H^c \) can occur through both tree-level and one-loop Feynman diagrams. The latter is indeed the one-loop vertex correction mediated by the \( SU(2)_L \) triplet \( \Delta_L \) in the minimal Type-II seesaw model [13], because the one-loop vertex correction mediated by \( N \) itself is CP-conserving (so is the self-energy diagram of \( N \) decays). As a result, the CP-violating asymmetry between \( N \to l_\alpha + H^c \) and its CP-conjugate process \( N \to l_\alpha^c + H \) arises from the interference between the tree-level amplitude and the \( \Delta_L \)-induced vertex correction. For each lepton flavor \( \alpha \), the corresponding CP-violating asymmetry is given by [20]
The overall CP-violating asymmetry turns out to be
\[ \epsilon_\alpha \equiv \frac{\Gamma(N \to l_\alpha + H) - \Gamma(N \to l_\alpha^c + H)}{\sum_\alpha [\Gamma(N \to l_\alpha + H^c) + \Gamma(N \to l_\alpha^c + H)]} \]

where \( v \equiv \langle H \rangle \simeq 174 \text{ GeV} \), and \( M \) is the mass of \( N \). Taking account of \( M_L = M_\nu^{(0)} \) given in Eq. (11) and \( M_D \) given in Eq. (21), we explicitly obtain
\[ \epsilon_e = -\frac{3M_c}{8\pi v_2} \frac{\text{Im}[(x^*)^2]}{2 + |x|^2}, \quad \epsilon_\mu = \epsilon_\tau = 0. \] (29)
The overall CP-violating asymmetry \( \epsilon = \epsilon_e + \epsilon_\mu + \epsilon_\tau = \epsilon_e \). This interesting result implies that only the electron flavor contributes to leptogenesis in our model. To be more specific, we assume that \( M \) lies in the region \( 10^9 \text{ GeV} \leq M \leq 10^{12} \text{ GeV} \), in which only the \( \tau \)-lepton Yukawa coupling is in equilibrium [15]. The flavor-dependents effects are therefore relevant to thermal leptogenesis.

The CP-violating asymmetry \( \epsilon = \epsilon_e \) can give rise to a net lepton number asymmetry in the Universe, and this lepton number asymmetry can partially be converted into a net baryon number asymmetry due to non-perturbative sphaleron interactions [15]
\[ \eta_B \simeq -0.96 \times 10^{-2} \sum_\alpha \epsilon_\alpha \kappa_\alpha = -0.96 \times 10^{-2} \epsilon_e \kappa_e, \] (30)
where the efficiency factors \( \kappa_\alpha \) (for \( \alpha = e, \mu, \tau \)) measure the flavor-dependent washout factors associated with the out-of-equilibrium decays of \( N \). To evaluate the size of \( \kappa_e \) in Eq. (30), one may introduce a parameter \( K_e = P_e K \), where \( P_e \equiv |(M_D)_{e1}|^2/(M_D^H M_D)_{11} \) and \( K = \tilde{m}/m_e \) with \( \tilde{m} = (M_D^H M_D)_{11}/M \) being the effective neutrino mass and \( m_e \simeq 1.08 \times 10^{-3} \text{ eV} \) being the equilibrium neutrino mass [21]. Explicitly, \( P_e = |x|^2/(2 + |x|^2) \) and \( \tilde{m} = c \varepsilon (2 + |x|^2) \) hold. Since the relationship between \( \kappa_e \) and \( K_e \) is rather complicated, we do not write it out here but refer the reader to Ref. [15] for details. We just mention that a numerical analysis yields \( K_e \simeq (0.17 \cdots 1.3) \) and \( \kappa_e \simeq (0.467 \cdots 0.64) \) in our model. Hence \( \epsilon_e \) should be of \( \mathcal{O}(10^{-7}) \) and have a minus sign, in order to correctly reproduce \( \eta_B \sim 6 \times 10^{-10} \).

Let us count the number of free parameters relevant to neutrino masses, flavor mixing and leptogenesis. They are \( c, \varepsilon, |x|, \arg(x) \) and \( M \). On the other hand, we have four observable quantities \( m_1, m_2, \theta_{12}, \theta_{13} \) and \( \eta_B \) which depend on the magnitudes of those five parameters. Although five free parameters cannot be fully determined from four measured quantities, it is possible to constrain the former by using the latter. We shall carry out a numerical calculation and illustrate the viable parameter space of this minimal Type-II seesaw model in the next subsection.

C. Numerical results

Given \( m_3 = 0, \theta_{23} = 45^\circ \) and \( \theta_{13} = 0^\circ \) in our model, the inputs of our numerical calculation include \( \Delta m^2_{21} = (7.2 \cdots 8.9) \times 10^{-5} \text{ eV}^2 \), \( \Delta m^2_{32} = -(2.1 \cdots 3.1) \times 10^{-3} \text{ eV}^2 \),
\[ \theta_{12} = (30^\circ \cdots 38^\circ) [6] \text{ and } \eta_B = (5.9 \cdots 6.3) \times 10^{-10} [16]. \] Since \( |x| \) is expected to be of \( \mathcal{O}(1) \), we typically take \( |x| \leq 3 \) as a reasonable upper limit. Then it is straightforward to obtain the allowed ranges of \( c, \varepsilon, |x|, \operatorname{arg}(x) \) and \( M \) with the help of those analytical expressions given in Eqs. (22)–(30). We demonstrate that this minimal Type-II seesaw model can simultaneously account for current neutrino oscillation data and the cosmological baryon number asymmetry. Some results and discussions are in order.

- First of all, we find that \( c \) gradually ranges between \( 2.0 \times 10^{-2} \text{ eV} \) and \( 2.85 \times 10^{-2} \text{ eV} \). There exist a lower bound on \( |x| \) and an upper bound on \( \varepsilon \); namely, \( |x|_{\text{min}} \simeq 0.82 \) and \( \varepsilon_{\text{max}} \simeq 1.9 \times 10^{-2} \). The correlated parameter space of \( |x| \) and \( \varepsilon \) is shown in FIG. 1, from which one can get a number of nearly tri-bimaximal neutrino mixing patterns.

- FIG. 2 illustrates the correlated parameter space of \( M \) and \( \operatorname{arg}(x) \), which can roughly be understood if one takes into account \( \varepsilon_e \propto M \sin[2 \operatorname{arg}(x)] \) as indicated in Eq. (29). Neglecting the influence of \( |x| \) on \( \varepsilon_e \), we find that the lower bound on \( M \) comes out around \( \operatorname{arg}(x) \sim 45^\circ \) (or equivalently, \( \sin[2 \operatorname{arg}(x)] \sim 1 \)): \( M_{\text{min}} \approx 1.3 \times 10^9 \text{ GeV} \). When \( \operatorname{arg}(x) \) approaches zero, \( M \) has to approach infinity in order to assure \( \varepsilon_e \sim \mathcal{O}(10^{-7}) \). In our model with \( |x| \sim \mathcal{O}(1) \), the favored range of \( M \) is actually between \( 10^9 \text{ GeV} \) and \( 10^{11} \text{ GeV} \).

- Note that \( |x| = 1 \) is a particularly interesting possibility, corresponding to \( c \simeq (2.0 \cdots 2.8) \times 10^{-2} \text{ eV}, \varepsilon \simeq (0.76 \cdots 1.75) \times 10^{-2}, \operatorname{arg}(x) \simeq (0.152^\circ \cdots 28.7^\circ) \) and \( M \approx (4.2 \cdots 1000) \times 10^9 \text{ GeV} \). In this special case, the model is simplified to a unique version which only contains four free parameters and they can all be determined from the experimental values of \( m_1, m_2, \theta_{12} \) and \( \eta_B \). More accurate data will impose much narrower constraints on the model parameters \( c, \varepsilon, \operatorname{arg}(x) \) and \( M \).

Finally, it is worthwhile to point out that our neutrino mixing pattern is stable against radiative corrections. Running from the seesaw scale \( \mu = M \) down to the electroweak scale \( \mu = v, m_3 = 0 \) keeps unchanged while other two neutrino masses and three mixing angles can only receive tiny corrections in the standard model [22]. Although the so-called Dirac CP-violating phase \( \delta \) can be generated together with \( \theta_{13} \) from radiative corrections [23], the resultant CP-violating effect in neutrino oscillations (characterized by the Jarlskog invariant of \( \mathcal{O}(10^{-7}) \) or smaller in this model) is too small to be observable.

**IV. SUMMARY**

In summary, we have proposed a new category of neutrino mass ans"atze by starting from a combination of two phenomenological observations: (1) the lightest neutrino mass might be zero or vanishingly small, and (2) the neutrino mixing matrix might be the tri-bimaximal mixing pattern or a pattern close to it. We have shown that a realistic neutrino mass matrix \( M_\nu \) can be established either in the limit of \( m_1 = m_2 = 0 \) and \( m_3 \neq 0 \) or in the limit of \( m_1 = m_2 \neq 0 \) and \( m_3 = 0 \), corresponding to the possibility of \( m_1 = 0 \) or \( m_3 = 0 \). In both cases, it is possible to introduce a particular perturbation which ensures the resultant neutrino mixing matrix to be the tri-bimaximal mixing pattern or its viable variations with all entries being formed from small integers and their square roots. We have incorporated
the texture of $M_{\nu}$ in the minimal Type-II seesaw model with only one heavy right-handed Majorana neutrino $N$ in addition to the $SU(2)_L$ Higgs triplet $\Delta_L$. The $m_3 = 0$ case has been discussed in detail to accommodate CP violation in the lepton-number-violating decays of $N$. We have demonstrated that our model can simultaneously interpret current neutrino oscillation data and the cosmological baryon number asymmetry via thermal leptogenesis, in which only the electron flavor plays a role in the lepton-to-baryon conversion.

Finally let us remark that both the neutrino mass spectrum and the flavor mixing angles are well fixed in the proposed model. It is therefore easy to test them in the near future, when more accurate experimental data are available.

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FIG. 1. Parameter space of $|x|$ and $\varepsilon$ constrained by current experimental data on $m_1$, $m_2$, $\theta_{12}$ and $\eta_B$ in the minimal Type-II seesaw model with $m_3 = 0$.

FIG. 2. Parameter space of $M$ and $\arg(x)$ constrained by current experimental data on $m_1$, $m_2$, $\theta_{12}$ and $\eta_B$ in the minimal Type-II seesaw model with $m_3 = 0$. 