Investigation of the deformation of elastic rods under thermomechanical loading

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Abstract. This article aims to provide information on ongoing research with coverage of preliminary results. Analysis of the state of the rods theory is one of the intensively developing parts of mechanics since the second half of the XIX-th century. Soviet scientists made a significant contribution to the development of rods theory and applied calculation methods. Based on these methods, a number of practically important problems have been solved. The works were mainly related to the theory of elastic rods. From the analysis of the work performed, we proposed a solution to the deformation of elastic rods under thermomechanical loading in a nonlinear formulation from the standpoint of modern systems of numerical analysis. The purpose of this article is to develop the methods for calculating nonlinear problems of the deformation of rods under thermomechanical loading using modern problem-solving systems. The solution of the nonlinear problem in the general setting is extremely hard. Therefore, it is necessary to develop a methodology that takes into account the features of deformation and will be available to specialists when designing the elements of engineering and building of rod structures.

1. Introduction

The theory of rods is one of the rapidly developing parts of mechanics from the second half of the XIX-th century [1-4]. The works were mainly related to the theory of elastic rods, the foundations of which are given in the work of Kirchhoff. Viewing the rod as a one-dimensional continuum, he obtained basic differential equations for the quantities that determine the geometry of the axis of the rod. However, the system was not closed. Kirchhoff recorded the additional ratios without a strict withdrawal. Therefore, in order to clarify Kirchhoff’s theory, most further research is devoted to obtaining the necessary number of equations. Most of these studies have been analyzed in the reviews of S.S. Antman [5], E.I. Grigolyuk and I.T. Seleznev [6].

Since the second half of the 20th century, there has also been an intense development in the direction where the rod appears to be an oriented or directed curve. J. Erickson and C. Truesdel [7] used this idea to derive equations of rods theory. The foundations of this theory are given in work [8] and subsequently developed for problems of statics in works [9-12].

2. Analytical solution of the problem

Power source (drain), i.e. volume density of the heat flow is the amount of heat emitted (absorbed) by the body volume unit per a unit of time. The unit of this value is [J / (m³s)] or [W / m³]. If the power of
the heated source [W] is set, it is necessary to divide the power of the heated source into the volume of the body in order to determine the density of the heat flow (Figure 1)...

![Figure 1. Calculation of the temperature field in the beam.](image)

The heat equation (Fourier equation) in the Cartesian coordinates is as follows:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right) + \frac{1}{c_y} Q$$  \hspace{1cm} (1)

If there are no heat sources in the body $Q = 0$ the equation (1) is simplified.

To integrate the differential equation into the partial derivatives (1) the initial and boundary conditions are needed. Initial condition, for example: when

$$t = 0, T(x, y, z) = T_f = \text{const} \hspace{1cm} (2)$$

For the desired temperature function, the following boundary conditions may be specified [13]:

1. **Type 1 boundary conditions**, when the temperatures on the body-limiting surfaces are set. In general, the temperature at the border may depend on the coordinates of the boundary points and the time.

$$t > 0; T(x, y, z, t)_{z=0,j} = T_1(x, y, z, t)_{z=0,j}, T(x, y, z, t)_{y=0,j} = T_2(x, y, z, t)_{y=0,j}$$
$$T(x, y, z, t)_{y=0,b} = T_3(x, y, z, t)_{y=0,b} \hspace{1cm} (3)$$

2. **Type 2 boundary conditions**, when the heat flux density is set on the surface, i.e. derived from the normal temperature to the surface as a function of time and the coordinates of the surface points.

3. **Type 3 boundary conditions**, in which the heat flux is assumed to be proportional to the difference in temperature of the surface and the environment

$$t > 0; -\lambda \left[\frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=0,b} = \alpha[T(x, y, z, t) - T_f]_{x=0,b}$$
$$t > 0; -\lambda \left[\frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=0,b} = \alpha[T(x, y, z, t) - T_f]_{y=0,b} \hspace{1cm} (4)$$
$$t > 0; -\lambda \left[\frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=0,l} = \alpha[T(x, y, z, t) - T_f]_{z=0,l}$$

4. **Type 4 boundary conditions** (conjugation conditions), which are reduced to the simultaneous determination of the equality of temperatures and heat fluxes at the interface, when the problem of heat exchange of two media is solved (solid-liquid, body-body, liquid-liquid), in each of which heat transfer is described by an equation.
These conditions allow various modifications depending on the physical conditions at the interface. So, for example, if the contact between two solids is not ideal, then the first condition (5) may contain a temperature jump. If there are heat sources at the interface (chemical reaction, phase transition), then the heat flux resulting from the surface source should be included in the second condition (5).

For a one-dimensional problem, the point heat source is recorded using the Dirac function $\delta$ (unit impulse function) as $Q\delta(y-y_0)$, where $y_0$ is the source coordinate. Similarly, heat sources for two-dimensional and three-dimensional problems can be considered using multidimensional functions $\delta$. Multidimensional functions can be represented as the product of one-dimensional functions in an amount equal to the dimension of the space on which the multidimensional function is defined.

For the two-dimensional problem to be taken into account in the thermal conductivity equation of the heat source with coordinates $x_0, y_0$, we have $Q\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$. For a three-dimensional problem with a point heat source with coordinates $x_0, y_0, z_0$, we have: $Q\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$ and the equation (1) will take the form:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right) + \frac{1}{c\gamma} Q\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

The primitive one-dimensional function $\delta$ is the Heaviside function (Heaviside step function) $H(x) = 0, x < 0; H(x) = 1, x \geq 0$. The value of a function in an argument of zero can be explicitly stated in a function entry, for example $H(x) = 1/2, x = 0$. In the Mathcad system, the Heaviside function returns 1 if $x \geq 0$, otherwise 0. The Heaviside step function can be used to create a pulse of width $a$, $H_1(x-a) - H_1(x-a)$ which continues to impact. If the rod is heated on the site $[a,b]$, then the heat flux density in the heat equation can be taken into account using the Heaviside function as follows: $Q \cdot [H_1(x-a) - H_1(x-b)]$ and the thermal conductivity equation will take the form of:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right) + \frac{1}{c\gamma} Q[H_1(x-a) - H_1(x-b)]$$

From the point of view of the automation of engineering calculations, it is currently advisable to use computer technology (personal computers) and modern problem solving systems.

3. Purpose and objectives of research

Rod elements of structures are often subjected to a mixed temperature field and mechanical load during operation. Therefore, stress-deformation and strength assessment should be based on the related heat conductivity and deformable solid mechanics.

In the overall multi-dimensional approach, this is a difficult task. The development of a relatively simple, application model requires the introduction of certain hypotheses. To validate the reliability of the theoretical results obtained, it is necessary to conduct an experimental study, which also makes it possible to verify the validity of the introduced hypotheses. The main results of an experimental study of displacements and the temperature field of the cantilever beam are presented in [14,15].

4. Research methodology. Description of the test stand

In the Materials Resistance Laboratory of the Akaki Tsereteli State University, a continuous rectangular section console beam for thermomechanical loading was tested (Figure 2).
The test stand is shown in Figure 2 Typical geometric dimensions of the stand are: $l_1 = 700\,\text{mm}$, $l_2 = 350\,\text{mm}$, $l_3 = 190\,\text{mm}$, $l_4 = 240\,\text{mm}$. Beam material st. 45 with geometric dimensions 12x24x700 (mm). A 200-watt helix heater was produced to heat a 19-cm long lower surface of the beam. The temperature was measured by a TK - 5.03 digital contact thermometer. Clockwise indicators were used to measure movements. A mechanical load was created using a 1 kg and 2 kg weight (Figure 3).

The experiments were carried out on two cases of loading: the first - when the beam was only heated on a part of the lower surface and the second when thermomechanical loading. During thermomechanical loading, the beam was initially loaded at the free end of the weight, and then heated to parts of the lower surface [16,17]. The beam was heated for 8 minutes. The temperature of the lower heated part and the upper surface at the given point, as well as the vertical and horizontal movement of the free end of the beam, were measured within 40 seconds.

5. The experimental design and measurement results

This is a one-factor experiment. The independent factor is time. Three repeated measurements were performed for each level of the independent factor. Statistical processing of measurements was carried out to graphically present the experimental data. The sharply deviating experimental points were checked by the Smirnov-Grebs criterion:

$$V_{i\text{max}} = \frac{y_{i\text{max}} - \bar{y}_i}{\sigma} \sqrt{\frac{m}{m-1}}, \quad V_{i\text{min}} = \frac{y_{i\text{min}} - \bar{y}_i}{\sigma} \sqrt{\frac{m}{m-1}}$$

where: $y_{i\text{max}}, y_{i\text{min}}$ - is the maximum and minimum values of the measured value at a given level of an independent factor, $\bar{y}_i$ - is an average value of observed values at a given level of an independent factor, $m$ - is a number of repeated measurements, $\sigma$ - is a mean square deviation [18].

If $V_{i\text{max}} > V_i$ or $V_{i\text{min}} > V_i$ then the corresponding experimental points are excluded from further consideration. If $V_{i\text{max}} < V_i$ or $V_{i\text{min}} < V_i$ then the corresponding experimental points remain. $V_i = V_i(p,m)$ - is a theoretical value of the parameter and is selected from the table [2,3] depending on...
the confidence probability (for example = 0.9) and on the number of repeated measurements (m = 3) (Fig. 4, 5).

For each level of independent factor average values of measured values were found. Fig. 4 shows the relationship between the temperature of the lower heated part of the beam and the temperature of the upper surface of the beam at the measured point. Fig. 5 shows the dependencies of vertical and horizontal end-of-pipe movements under thermomechanical loading of the beam [19, 20].

![Graph showing temperature changes over time](image1)

**Figure 4.** Time-dependence temperature of the lower heated surface of the beam ($T_0$) and the upper surface of the measured point ($T_1$).

![Graph showing movement changes over time](image2)

**Figure 5.** The time dependence of horizontal (W) and vertical (V) movement of the free end of the beam under thermomechanical loading.

### 6. Conclusions

1. Figure 4 shows that the temperature at the top surface within the heating device is actually linear in time and at the lower heated surface the temperature is nonlinear and about twice as high.
2. In Figure 5, the vertical displacement from static mechanical load is much larger than horizontal movement. However, due to the effect of the temperature, the picture changes and the vertical movement remains virtually constant, i.e. the movement from the temperature change is substantially less than from the load and the horizontal movement increases substantially at the end of the heating ($t = 440$ sec) becomes commensurate with vertical movement.

### Reference

[1] Svetlitsky V A 1987 *The mechanics of rods*. In 2 h (Moscow: Higher School) pp 1–320
[2] Svetlitsky V A 1982 *The mechanics of flexible rods and threads* (Moscow: Mechanical Engineering) p 279
[3] Svetlitsky V A, Naraikin O S 1989 *The elastic elements of machines* (Moscow: Mechanical Engineering) p 264
[4] Svetlitsky V A 2001 *The mechanics of absolutely flexible rods* (Moscow: Publishing House of MAI (Moscow Aviation Institute)
[5] Antman S S 1972 *The theory of rods. Hand. Phys.* pp 641–703
[6] Grigolyuk E I, Seleznev I T 1973 Non-classical theories of vibrations of rods, plates, and shells (Moscow) The Outcome of Science and Technology. Mechanics of solid deformable bodies. AU ISTI (All-Union Institute of Scientific and Technical Information) 5 p 272

[7] Kikvidze O G, Baisarova G 2014 Non stationary problem of beam’s deformation at thermo mechanical loading Kutaisi. Bulletin of akaki tsereteli state university 2(4) 77–82

[8] Baisarova G, Kikvidze O 2015 Experimental investigation of beam at thermo mechanical loading VI annual meeting of the georgian mechanical union (Tbilisi, Georgia)

[9] Kikvidze O G, Kikvidze L G 2001 Large displacements of thermoelastic rods during plane bending Problems of Applied Mechanics. Publishing House "Committee of IFToMM of Georgia" 4(5) 73–77

[10] Kikvidze O G, Kikvidze L G 2007 The geometrically nonlinear problem of bending thermoelastic rods (Tbilisi: Georgian Technical University) Collection of International Symposium "Problems of Thin-Walled Spatial Structures", July 4-5, pp 28–31

[11] Kikvidze O G 2003 Large displacements of thermoelastic rods during bending Problems of mechanical engineering and machine reliability, RAS (Russian Academy of Sciences) 1 49–53

[12] Bîrsan M, Altenbach H 2011 Theory of thin thermoelastic rods made of porous materials Archive of Applied Mechanics 81(10) 1365–1391

[13] Starovoitov E I, Leonenko D V, Tarlakovskii D V 2016 Thermal-force deformation of a physically nonlinear three-layer stepped rod Journal of Engineering Physics and Thermophysics 89(6) 1582–1590

[14] Baisarova G, Brzhanov R, Kikvidze O G, LahnoV 2019 Computer simulation of large displacements of thermoelastic rods Journal of Theoretical and Applied Information Technology 97(15)

[15] Starovoitov E I, Leonenko D V, Tarlakovskii D V 2016 Thermal-force deformation of a physically nonlinear three-layer stepped rod Journal of Engineering Physics and Thermophysics 89(6) 1582–1590

[16] Greshnov V M 2019 Physico-Mathematical Theory of High Irreversible Strains in Metals (CRC Press)

[17] Bednarcyk B A et al 2019 A multiscale two-way thermomechanically coupled micromechanics analysis of the impact response of thermo-elastic-viscoplastic composites International Journal of Solids and Structures 161 228–242

[18] Qiu B et al 2019 Rate-dependent transformation ratcheting-fatigue interaction of super-elastic NiTi alloy under uniaxial and torsional loadings: Experimental observation International Journal of Fatigue 127 470–478

[19] Rossmann L et al 2020 Method for conducting in situ high-temperature digital image correlation with simultaneous synchrotron measurements under thermomechanical conditions Review of Scientific Instruments 91(3) 33–35

[20] Hu Y J et al 2020 A thermally-coupled elastic large-deformation model of a multilayered functionally graded material curved beam Composite Structures pp 122–141