The Edge Currents and Edge Potentials in IQHE

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Abstract
It is shown that an observed length in the potential drops across IQHE samples
is a universal length for a given value of magnetic field which results from the
quantum mechanical uncertainty relation.
We showed recently that the microscopic theory of IQHE \[1\] can be given by the canonical quantization of a semi-classical theory of the ”classical” Hall-effect CHE \[2\].

The action functional for this is the semi-classical Schroedinger-Chern-Simons action for a 2-D non-interacting carrier system with the usual minimal electromagnetic coupling on a 2+1-dimensional manifold \(M = \Sigma \times \mathbb{R}\) with spatial boundary. We showed also that the constraints of the theory forces the coupled electromagnetic potential to be an almost pure gauge potential, i. e. with an almost vanishing field strength and they forces also the potential to exist only on the edges \[2\]. Thus, according to our model we have to do in IQHE case with an almost pure ”edge” gauge potential \[3\]. Accordingly, in view of the Ohm’s equations the edge currents are the prefered currents under these constraints.

Here we show that the recent results on the potential drops across IQHE samples near the edges \[5\] follow the universal uncertainty relation of quantum mechanics, in view of the universality of the QHE.

To begin, recall that there are two fundamental aspects of potential which has to be considered:

1) that the potential itself is non-observable but some functions of it becomes observable.
2) that a pure gauge potential is according to the quantum mechanics a non-vanishing quantity (see below).

Let us first explain the situation from the more fundamental point of view of quantum mechanics.

For a charged system, e. g. electrons in magnetic fields, the energy uncertainty is given by the minimum amount of energy, i. e. the ground state energy. This amount of energy is proportional to the applied magnetic field strength. On the other hand, an energy uncertainty is correlated with a position uncertainty for electrons. Thus, quantum mechanically there is always an uncertainty of position of the electronic currents on the surface which is related with the width of the electron orbit. Therefore, if we consider the uncertainty of momentum equal to \((2m_e \Delta E)^{\frac{1}{2}}\) with \(\Delta E = E_{n+1} - E_n = \frac{\hbar \omega_c}{2}\) and \(\omega_c = \frac{eB}{m_e}\), then the mentioned position uncertainty in \(2 - D\) is given by \(\Delta Y = \Delta X = \left(\frac{\hbar}{eB}\right)^{\frac{1}{2}}\) which is the magnetic length \(l_B\). Since, the edge current is defined as the current which flows, in the ideal case, close to the edge within the length scale of the magnetic length \[4\]. This means that one should expect that according to Ohm’s equations for QHE, in the ideal case, also the potential distribution on the sample should be close to the boundary of sample within a distance which is proportional to the magnetic length.

Furthermore, one must take into account that despite of classical physics in quantum physics there
are relevant quantities which are prevented to become zero in view of the uncertainty relations. To these relevant quantities in the QHE case it belongs the electromagnetic potential and its field strength. Equivalently, in quantum mechanics only global quantities $\oint A_m dx^m = \int \int B ds$ are relevant but not their local components $A_m$ or $B$.

Thus, if we consider for example $\Delta P_y = \Delta A_y = eA_y$, then there is an uncertainty relation $eA_y \cdot \Delta Y = eA_y \cdot l_B = \hbar$.

Accordingly, a pure gauge potential which should be zero classically for example within an IQHE sample, is however quantum mechanically non-zero and has a value of $\Delta A_y = \frac{\hbar}{e l_B}$.

In view of the relations between the magnetic field strength $B$, magnetic length and the global density of electrons $n$ with the filling factor $\nu$, i.e. $l_B^2 = \frac{\hbar}{eB} = \frac{\nu}{2\pi n}$, it is obvious that a variation of only one of these factors changes the magnetic length and so it changes also the current position and the potential distribution on the sample. However, if $B$ or $\frac{\nu}{n}$ remain the same for various IQHE samples, then the magnetic length should be invariant for all these samples under the IQHE conditions independent of their geometries and other factors.

These are the quantum theoretical basics of what is observed in the mentioned experiments for the potential drops [5], where the authors report that they observed potential drops across the IQHE-samples over a length of $100\,\mu m$ from the edge of samples. We show that this length which has the magnitude of $|l_B^{-1}|$ for the given data in Ref. [5] is indeed a universal quantity for a given $B$ or for a given $\frac{\nu}{n}$ [6].

Furthermore, as we mentioned above the electromagnetic potential is in view of its gauge dependence non-observable. The observables related with the potential or those related with its field strength are phase angles given by the closed path integral of potential or the surface integral of field strength, which are observable by the quantum mechanical interference patterns. Equivalently, a constant potential multiplied by a proper length, e.g. by the circumference of mentioned closed path is also observable.

For example according to the definition of magnetic length $l_B^2 = \frac{\hbar}{eB}$ we have [6] (see also below):

$$l_B^2 B = l_B A = \frac{\hbar}{e}$$  \hspace{1cm} (1)
which is equivalent to the definition of magnetic flux quantum through \( \int \int B ds = \oint A_m dx^m = \frac{h}{e} \), where the potential component \( A \) in (1) is the relevant component of electromagnetic gauge potential according to \( A_m = B\cdot x_n \epsilon_{mn} \) gauge \[7\].

Recall also that, the potential and magnetic field have always dimension \( L^{-1} \) and \( L^{-2} \) respectively. Thus, it is natural that under the IQHE conditions where \( \sigma_H = \frac{ne}{B} \) is quantized according to \( \sigma_H = \nu \frac{e^2}{h} \), one obtains a purely geometrical relation between potential and magnetic length.

Moreover, as a general result let us mention that, if one considers the relation (1) in form \( 2\pi l_B A = 2\pi l_B^2 B = \frac{h}{e} \) as given according to the flux quantization for electrons flow in the IQHE edge current on a ring with radius and width both equal to \( l_B \). Then, one obtains with the given \( l_B \) according to the data in Ref. \[8\] for \( A = \frac{h}{e} l_B^{-1} \) a value about \( 100 \mu m \) for \( A \), which is the mentioned observed length for potential drops \[8\] \[8\].

This result show that in view of the definition of magnetic length the measured value of \( 100 \mu m \) is a fundamental value for IQHE experiments on those samples independent of other sample parameters. It shows also that for a pure "edge potential" \( A \) which should exists classically exactly on the edges of sample and it should be zero in the rest of sample \[8\], we have however \( \Delta A \neq 0 \) in view of \( \Delta A \Delta X = \frac{h}{e} \).

It is also in view of the \( L^{-1} \) dimension of \( A \) that one obtains \( \Delta A \neq 0 \) within a width of \( l_B^{-1} \).

Furthermore, the relation between edge current and the above discussed "edge potential" or the edge potential drops should be understood in the following way with respect to the above considerations:

Obviously, \( B \cdot x^m = \epsilon^{mn} A_n \) is a solution of the Ohm’s equations \( j_m = \epsilon^{mn} \sigma_H E_n \) with \( j^m = ne \frac{dx^m}{dt} \) and \( E_n = \frac{dA_n}{dt} \), if we use as usual \( \sigma_H = \frac{ne}{B} \) in the quantum Hall limit: \( \omega_c \tau \gg 1 \). From this solution it results that the edge current \( j_m \) should flow within a width of \( \Delta X = l_B \) in view of the already mentioned relation \( B \cdot \Delta X = \Delta A \), in agreement with its definition. Moreover, one can prove directly the measured value of potential drops in Ref. \[8\] from the relation \( B \cdot \Delta X = \Delta A \). Thus, for an applied magnetic field \( B \) about \( 1 \) Tesla and for the \( l_B \) value which is known to be \( 10^{-2} \mu m \) from the \( \frac{\nu}{2\pi n} \) value of the given sample, one obtains according to \( B \cdot \Delta X = \Delta A \) for \( \Delta A \) a value of \( 100 \mu m \).

In this way the edge current of charged carriers which flows within a width of \( l_B \) causes a potential drop of the measured width. The same calculation should be done for the experiments with filling factor \( \nu = 4 \).
about which it is reported in Ref. [9]. The theoretical result agrees also in this case with the measured result. Moreover, the same results can be obtained according to the relation \( \frac{\hbar}{e} B = A^2 \) in view of the definition of \( l_B \). For a given value of \( B \) which corresponds with \( \nu \) according to the relation \( \nu = \frac{n\hbar}{eB} \) this is an invariant relation for the potential \( A \) which shows the general validity of the experimental results and also of our theoretical result.

To be precise, let us mention that in other experiments [9], where the electronic concentration is almost the same as in Ref. [5] but the filling factor is \( \nu = 4 \), one observed potential drops of \( \approx 70 \mu m \). This is in good agreement with our theoretical result, since for \( \nu = 4 \) filling factor one obtains according to the data of Ref. [5] a magnetic length \( l_B' \approx 1.4l_B \approx 1.4 \cdot 10^{-2} \mu m \), where \( l_B \approx 10^{-2} \mu m \) is the magnetic length of samples in Ref. [5]. Thus, the theoretical value of \( A = \frac{\hbar}{e}(l_B')^{-1} \) becomes \( \approx 70 \mu m \) which is indeed the measured value according to Ref. [9] (see also [8]).

This circumstance explains why one observes potential drops within such a distances from the edges of the IQHE samples [5] [9].

Therefore, one should claim that the measured penetration length of electromagnetic potential on IQHE samples should depend, according to the theoretical value of \( A = \frac{\hbar}{e}(l_B')^{-1} \), only on the related value of \( l_B^{-1} \) [5].

By theoretical point of view the origin of these empirical results should lie, as it is mentioned already, in the quantum mechanical uncertainty-principle, where a charged particle in presence of magnetic fields acquires a position uncertainty \( \Delta Y = \Delta X = l_B \). Thus, considering \( \Delta P = \Delta A = eA \), we are given under quantum mechanical conditions of QHE, the uncertainty relation \( A \cdot l_B = \frac{\hbar}{e} \). Here \( \frac{\hbar}{e} \) plays the same role in the quantum electrodynamical uncertainty as that played by \( \hbar \) in the quantum mechanical uncertainty.

Therefore, in view of the fact that the value of \( \frac{\hbar}{e} \) is a fixed quantity, the value of potential (drop) under IQHE conditions is always given by \( A = \frac{\hbar}{e l_B} \), as it is confirmed by results in Ref. [5] and [9], no matter what other relevant quantities are.

Thus, in any IQHE sample one should measure for the potential drops on the edges the related value of \( A = \frac{\hbar}{e l_B} \) according to the value of \( l_B \) from the experimental data of sample.

In view of the fact that this is a result from the uncertainty principle and as such it is an invariant result,
it depends only on the basics of "magnetic" quantization, i.e. on the uncertainty principle in quantum electrodynamics.

Furthermore, it is expected that the observed length of the potential drop should be related with parameters of samples. This is indeed true, if one recalls that the concentration of charge carriers is indeed the main parameter of the sample and also the magnetic length depends on it.

In conclusion let us mention that such a penetration length is also comparable with London’s penetration length in superconductivity [1].

Footnotes and references

References

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[3] A pure gauge potential in QHE case is a potential in a multiply connected region, thus it can not be gauged away everywhere in the region. Its field strength is in some parts of the region zero and in some other parts not. Quantum mechanically one can observe the influence of such pure gauge potential also in those regions where the field strength is zero (see Bohm-Aharonov effect).

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[5] W. Dietsche, K. v. Klitzing and K. Ploog, Potential Drops Across Quantum Hall Effect Samples- In the Bulk or Near the Edges? MPI fuer Festkoerperforschung Stuttgart-preprint 1995;

[6] According to the data about the IQHE samples in Ref. [3] the global concentration is

\[ n = 3.7 \cdot 10^{11} \text{cm}^{-2} \text{ and } \nu = 2. \]

Thus, one obtains \( l_B \approx 10^{-2} \mu m \).

The measured penetration length is given to be about 100\( \mu \)m which is almost exactly \(|l_B^{-1}| \mu m\).

[7] In view of the gauge invariance of electrodynamics one is free to choose relevant gauges to retain the true degrees of freedom of the electromagnetic field. We use here for the homogeneous magnetic field \( B \) the so called Landau gauge \( A = A_y = B \cdot x \) for the only relevant component of potential.

Recall further that the uncertainty relation discussed above, i. e. \( e \Delta A_y \Delta Y = \hbar \) together with \( e \Delta A_y = B \cdot \Delta X \) result in the flux quantization relation \( B \cdot \Delta X \Delta Y = \frac{\hbar}{e} \).

[8] Recall that the measured width of the potential drops should be considered theoretically according to the dimensional structure where \( \hbar \) contains \( L^2 \) dimensions according to its definition.

[9] P. F. Fontein, et al., Phys. Rev. B., 43, 12090 (1991). The given data in this report which are relevant for our calculation are \( n = 5.0 \cdot 10^{15} \text{ m}^{-2} \text{ and } \nu = 4. \)

[10] It is well known that superconducting effects can be considered as to be related with the QHE: see Ref. [1f]; R. B. Laughlin: in Ref. [1g]; and A. Karlhede, et al.: in Ref. [1e]; See further for empirical confirmations: D. Jerome, in J. G. Bednorz, K. A. Mueller (Eds), Superconductivity, (Springer-Verlag, Berlin 1990).