SYNCHRONIZATION IN THE IDENTICALLY DRIVEN SYSTEMS

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Abstract: We investigate a transition from chaotic to nonchaotic behavior and synchronization in an ensemble of systems driven by identical random forces. We analyze the synchronization phenomenon in the ensemble of particles moving with friction in the time-dependent potential and driven by the identical noise. We define the threshold values of the parameters for transition from chaotic to nonchaotic behavior and investigate dependencies of the Lyapunov exponents and power spectral density on the nonlinearity of the systems and character of the driven force.

1. Introduction

Trajectories of the nonlinear dynamical systems are very sensitive to initial conditions and unpredictable. The systems exhibit an apparent random behavior. It might be expected that turning on an additional random forces make their behavior even “more random”. However, as it was shown by Fahy & Hamann [1992] and Kaulakys & Vektaris [1995a,b] when an ensemble of bounded in a fixed external potential particles with different initial conditions are driven by an identical sequence of random forces, the ensemble of trajectories may become identical at long times. The system becomes not chaotic: the trajectories are independent on the initial conditions. Here we analyze the similar phenomenon in the ensemble of particles moving with friction in the time-dependent potential and driven by the identical noise. We define the threshold values of the parameters for transition from chaotic to nonchaotic behavior and investigate dependencies of the Lyapunov exponents and power spectral density on the nonlinearity of the systems and character of the driven force.

2. Models and Results

Consider a system of particles of mass $m = 1$ moving according to Newton’s equations in the time dependent potential $V(r, t)$, e.g. in the potential $V(x, t) = x^4 - x^2 - ax \sin \omega t$, and with the friction coefficient $\gamma$. At time intervals $\tau$ the particles are partially stopped and their velocities are reset to the mixture of some part $\alpha$ of the old velocities with
random velocity $\mathbf{v}_{i}^{ran}$: $\mathbf{v}^{\text{new}} = \alpha \mathbf{v}^{\text{old}} + \mathbf{v}_{i}^{ran}$, where $i$ is the stop number. Note that $\mathbf{v}_{i}^{ran}$ depends on the stop number $i$ but not on the particle. The simplest and most natural way is to choose the random values of velocity $\mathbf{v}_{i}^{ran}$ from a Maxwell distribution with $k_{B}T = m = 1$.

A transition from chaotic to nonchaotic behavior in such a system may be detected from analysis of the neighboring trajectories of two particles initially at points $\mathbf{r}_{0}$ and $\mathbf{r}_{0}'$ with starting velocities $\mathbf{v}_{0}$ and $\mathbf{v}_{0}'$. The convergence of the two trajectories to the single final trajectory depends on the evolution with a time of the small variances $\Delta \mathbf{r}_{i} = \mathbf{r}_{i}' - \mathbf{r}_{i}$ and $\Delta \mathbf{v}_{i} = \mathbf{v}_{i}' - \mathbf{v}_{i}$. From formal solutions $\mathbf{r} = \mathbf{r}(\mathbf{r}_{i}, \mathbf{v}_{i}, t)$ and $\mathbf{v} = \mathbf{v}(\mathbf{r}_{i}, \mathbf{v}_{i}, t)$ of the Newton’s equations with initial conditions $\mathbf{r} = \mathbf{r}_{i}$ and $\mathbf{v} = \mathbf{v}_{i}$ at $t = 0$ it follows the mapping form of the equations of motion for $\Delta \mathbf{r}$ and $\Delta \mathbf{v}$. The analysis of dynamics based on these equations has been investigated by Kaulakys & Vektaris [1995a,b].

Here we calculate the Lyapunov exponents directly from the equations of motion and linearized equation for the variances and extend the investigation for the systems with friction in the regular external field and perturbed by the identical for all particles random force. In Fig. 1 we show the dependence on $\tau$ of the Lyapunov exponents for the motion in the nonautonomous Duffing potential with friction described by the equations

$$\begin{align*}
\dot{\mathbf{v}} &= 2x - 4x^3 - \gamma v + a \sin \omega t, \\
\dot{x} &= v.
\end{align*}$$

(1)

For the values of parameters corresponding to the positive Lyapunov exponents, i.e. without the random perturbation ($\tau \to \infty$) the system is chaotic. The negative Lyapunov exponents for small $\tau$ indicate to the nonchaotic Brownian-type motion.

As it was already been observed in [Kaulakys & Vektaris, 1995b] such systems exhibit the intermittency route to chaos which provides sufficiently universal mechanism for $1/f$–type noise in the nonlinear systems. Here we analyze numerically the power spectral density of the current of the ensemble of particles moving in the closed contour and perturbed by the common for all particles noise. The simplest equations of motion for such model are of the form

$$\begin{align*}
\dot{\mathbf{v}} &= F - \gamma v, \\
\dot{x} &= v.
\end{align*}$$

(2)
with the perturbation given by the resets of velocity of all particles after every time interval $\tau$ according to the identical for all particles replacement $\mathbf{v}^{\text{new}} = \alpha \mathbf{v}^{\text{old}} + \mathbf{v}^{\text{ran}}_i$. We observe the current power spectral density $S(f)$ dependence on the frequency $f$ close to the $1/f$–dependence (see Fig. 2).

Our model may be generalized for systems driven by any random forces or fluctuations. On the other hand, the phenomenon when an ensemble of systems is linked with a common external noise or fluctuating external fields is quite usual. Thus, an ensemble of systems in the external random field may provide a sufficiently universal mechanism of $1/f$–noise.

3. Conclusions

From the fulfilled analysis we may conclude that, first, synchronization and transition from chaotic to nonchaotic behavior in ensembles of the identically perturbed by the random force nonlinear systems may be analyzed as from the mapping form of equations of motion for the distance between the particles and the difference of the velocity as well as from the direct calculations of the Lyapunov exponents and, second, an ensemble of systems linked with a common external noise may exhibit the $1/f$–type fluctuations.

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References

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Figure 1: Lyapunov exponents vs the time $\tau$ between resets of the velocity for motion in the Duffing potential according to Eq. (1) with $a = 5$, $\omega = 1$, $\gamma = 0.07$ and different $\alpha$.

Figure 2: The power spectral density of the current of the ensemble of particles moving according to Eq. (2) with $F = 1$, $\gamma = 0.1$ and perturbed by the common for all particles noise $v^{\text{new}} = \alpha v^{\text{old}} + v^{\text{ran}}$ with $\alpha = 1$ and $\tau = 0.1$. The dense line represents the averaged spectrum.