Comment on “Do Earthquakes Exhibit Self-Organized Criticality?”

In a recent Letter, Yang, Du, and Ma [1] study the interesting problem of the temporal structure of seismicity and its relation with self-organized criticality (SOC). Their main finding is that the reshuffling of earthquake magnitudes changes the shape of the earthquake recurrence-time (or first-return-time) distribution when the low-magnitude bound, $M_c$, is raised. Subsequently, they conclude that it is not true that an earthquake cannot “know” how large it will become. First, we show that this important implication is unjustified.

Yang et al. have in mind a fully uncorrelated temporal point process with independent magnitudes as a picture of SOC systems. It is obvious, by construction, that this model is invariant under random rearrangements of the data; as Yang et al. do not find this invariance in Southern California seismicity they claim that “earthquakes do not happen with completely random magnitudes” and therefore they are not a SOC phenomenon. In fact, the only conclusion that can be drawn from this is that the seismicity time series is not uncorrelated, and there exists some dependence between magnitudes and recurrence times. [This conclusion can be obtained directly, from the fact that a scaling law exists for the recurrence-time distributions corresponding to different low-magnitude bounds, with a scaling function that is not a decreasing exponential [2] (characteristic of a Poisson process, the only uncorrelated process which verifies a scaling law).]

The existence of correlations means that, for a given event $i$, its magnitude $M_i$ may depend on the magnitude of the previous event, $M_{i-1}$, as well as on the backwards recurrence time, $T_i = t_i - t_{i-1}$, with $t_i$ and $t_{i-1}$ the time of occurrence of both events. This dependence can be extended to previous magnitudes and recurrence times, $T_{i-1}, M_{i-2}, T_{i-2}$, etc. But further, the recurrence time to the next event, $T_{i+1}$, may depend on the previous magnitudes, $M_j$ and recurrence times $T_j$, $j \leq i$. The reshuffling of magnitudes performed in Ref. [1] breaks (if they exist) the possible correlations of $M_i$ with the previous magnitudes, as well as with the previous recurrence times, and the correlations of $T_{i+1}$ with the previous magnitudes (but not with the previous recurrence times). Therefore, any of the influences $M_{i-1} \rightarrow M_i$, $T_i \rightarrow M_i$, or $M_i \rightarrow T_{i+1}$, may be responsible of Yang et al.’s results.

The most direct way to test the dependence of a given variable, in this case $M_i$, with another variable $X$, is to measure the probability density of $X$ conditioned to different values of $M_i$, $P(X|M_i)$, and compare with the unconditioned probability density of $X$, $P(X)$. This is what Fig. 1(a) displays, using $X = T_i$ and $X = T_{i+1}$ for the same data as Ref. [1], but restricted to pe-
periods of stationary seismicity (otherwise, for strong aftershock sequences the recurrence times are shorter and more sensitive to catalog incompleteness). As $P(T_i|M_i)$ remains practically unchanged for different sets of values of $M_i$, temporal causality leads to the conclusion that $M_i$ is independent on $T_i$. In contrast, $T_{i+1}$ clearly depends on $M_i$, as $P(T_{i+1}|M_i)$ changes for different sets of values of $M_i$. In other words, the larger the magnitude $M_i$, the shorter the time to the next event $T_{i+1}$, but the value of this time has no influence on the magnitude of the event, $M_{i+1}$. On the other hand, Fig. 1(b) shows that $P(M_i|M_{i-1})$ turns out to be indistinguishable from $P(M_i)$, ensuring the independence of $M_i$ and $M_{i-1}$, $\forall i$ if the $T_i$’s are restricted to be larger than 33 min (shorter periods of time are not reliable, due to data incompleteness). So, when an earthquake starts, its magnitude is undetermined (at least from the information available at the catalogs), whereas the time to the next event decreases when that magnitude turns out to be large.

A second, independent point to clarify is the identification of SOC with the total absence of correlations. It is true that the BTW sandpile model displays an exponential distribution of recurrence times, but SOC is much more diverse than the BTW model. For instance, the Bak-Sneppen model or the Oslo-ricepile model are two well recognized examples of SOC with totally different recurrence-time distributions. Finally, it is necessary to stress that the concept of SOC (as it happens with chaos) does not exclude the possibility of prediction, as Ref. [17] of Yang et al. clearly showed. So, nothing in Ref. [1] is against the SOC picture of earthquakes.

Álvaro Corral

Departament de Física, Facultat de Ciències, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain

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[1] X. Yang, S. Du, and J. Ma, Phys. Rev. Lett. 92, 228501 (2004).

[2] A. Corral, Phys. Rev. Lett. 92, 108501 (2004).
FIG. 1: (color online) (a) Probability densities $P(T_i|M_{i-1})$ and $P(T_i|M_i)$ (shifted upwards) compared to $P(T_i)$. (b) Probability density $P(M_i|M_{i-1})$ compared to $P(M_i)$ with $T_i > 2000$ s.