EXIT Analysis of Interleaver Division Multiple Access System with LDPC Code

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Abstract. Extrinsic Information Transfer(EXIT) characteristics are extensively used in iterative detection and decoding of turbo type processing. This paper makes an exploratory study of the issue of EXIT characteristic analysis for Low-Density Parity Check (LDPC) coded Interleaver Division Multiple Access (IDMA) system in Multiple Access Channel (MAC) condition. By using simulation method and extrinsic information calculation, the a priori information of the IDMA detector and the extrinsic information can be gained. In the same way, the a priori information and extrinsic information of the LDPC decoder in the system can be procured. The EXIT characteristic of the detector illustrates the convergence the detection algorithm of IDMA, while the EXIT characteristic of the LDPC decoder illustrates the convergence nature of the optimized degree profile, which shows the reasonability of LDPC coded IDMA system and the reliability of the optimized degree optimized algorithm for multiple access channel.

1. Introduction

Irregular low-density parity-check (LDPC) forward error correcting codes (FEC) with iterative soft decoding are widely used[1-4] due to their capacity approaching performance. Besides, it has been adopted in 5G services, and it is a promising candidate for ultra-reliable and low-latency communications (URLLC)[5].

LDPC code can be applied to Multiple Access Channel (MAC) to enhance the performance. The traditional method of LDPC code design and analysis method including DE[6], GA[7], EXIT[8]. For MAC which is widely used in wireless communications [9], the analysis and design method must be explored. It is hence necessary to enable some mechanism to allow the users to share the MAC while ensuring an reliable transmission of the information. Based on this fact it is necessary to establish a mechanism that allows potential users to share MAC at the same time, while reliable information transmission is enabled.

To counteract and circumvent the influence of multiple access interference (MAI), different approaches have been put forward, including a new multiple access scheme, i.e. IDMA[10], and the extended DE and EXIT chart analysis method[11]. IDMA is one of Non-Orthogonal Multiple Access (NOMA)[12][13] technique that can be used in multi-user communications due to its high spectral efficiency with low complexity, which is prominent at transmitter[10]. IDMA scheme relies on interleaver pattern to differentiate signals of various users [13] and it has the capability of approaching channel capacity by optimizing FEC[14]. The vital point of IDMA principle resides in the fact that it regards the observed interference as useful information, rather than an additive noise. Therefore in the detection process the message for any user is refined for every iteration. To say in other way, the interference is mitigated and the information is prompted and boosted in the detection
and FEC decoding process. Besides, other advantages that IDMA method possesses are the insensitivity to clipping distortion [15], and needless of scheduling with reduced overhead [16].

The analysis of IDMA system can be analyzed by EXIT method. Extrinsic information transfer (EXIT) chart can be used as a visualized tool to analyze the convergence behavior of concatenated turbo components in the iterative-turbo process, which is an evident and convenient analytical technology.

To state even more clearly, EXIT charts are tools that can be applied to evaluate the performance of the concatenated component in the LDPC decoder, i.e. variable nodes decoder (VND) and check nodes decoder (CND). By this EXIT method, the extrinsic mutual information, and the a priori mutual information of VND and CND can be obtained. In the same way, the extrinsic mutual information, and the a priori mutual information of the IDMA detector can be achieved. Besides, the degree profile for LDPC code can be designed by using the method of fitting the EXIT chart of VND and CND.

Inspired by the wide use of EXIT and the ESE detector algorithm, this paper shows the convergence nature of the detector and LDPC decoder by exploring the extrinsic information transfer nature of the LDPC coded IDMA system.

2. System Model

The system consists of transmitter, channel, and receiver.

2.1. Transmitter

![System model of LDPC-Coded IDMA system](image)

The transmitter, which is shown in the upper part of Fig.1, is consisted of LDPC encoder, spreader, scrambler interleaver. For each and every user, the source information is processed, generating the transmitted chip sequence \( x_k = \{ x_k(j), j = 1, 2, ..., J \} \), where \( k \) is the user index [10].

2.2. Iterative Chip-by-Chip Receiver

The lower end of Fig.1 gives the receiver structure. The receiver is composed of turbo iterative detection and FEC decoding. For the detection part, the ESE is used [10], while for the FEC decoding part, the LDPC decoder is applied.

In the turbo iterative process, \( \{ e_{ESE}(x_k(j)) \} \) are extrinsic mutual information with regard to \( \{ x_k(j) \} \) produced by the ESE, and \( \{ e_{DEC}(c_k(n)) \} \) are extrinsic mutual information for \( \{ c_k(n) \} \) produced by
DECs. After the serial operation of deinterleaving, descrambling, despreading, \{e_{ESE}(x_k(j))\} becomes \{L_{DEC}(c_k(n))\}, which is acted as a priori information for DECs. Likewise, after spreading, scrambling and interleaving operation, the \{e_{ESE}(x_k(n))\} changes to \{L_{ESE}(x_k(j))\}, which is acted as a priori information of ESE in the following iteration. Based on the required performance, a preset number of iteration is conducted. In the end, decision is made for every bit of every user’s data. The ESE and DECs function will be discussed next.

3. Detecting and Decoding

3.1. ESE Operation

The ESE gets \{r_j\} from the channel output and \{e_{DEC}(x_k(j))\} from the interleaver \(\pi_k\). By ESE operation the extrinsic information with regard to \(x_k(j)\) is obtained. For the case of synchronous BPSK modulation with AWGN MAC channel the channel output at time instant \(j\) is in the form of

\[ r(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j) \quad j = 1,2,\ldots,J \]  

(1)

where \(x_k(j)\) is the chip sent from user-\(k\) at time \(j\), \(h_k\) means channel coefficient which is assumed to be 1 in this paper, and \(n(j)\) is the AWGN with zero expection and power \(\sigma^2 = N_0/2\). Concentrating on \(x_k(j)\), (1) can be re-written as

\[ r(j) = h_k x_k(j) + \varsigma_k(j) \]  

(2)

where \(\varsigma_k(j) = r(j) - h_k x_k(j) = \sum_{k'=k}^{K} h_{k'} x_{k'}(j) + n(j)\) is the interference term for user-\(k\).

The ESE detection algorithm is given below[10].

\[ e_{ESE}(x_k(j)) = 2h_k \frac{r(j) - \mathbb{E}(\varsigma_k(j))}{\text{Var}(\varsigma_k(j))} \quad \forall k, j \]  

(3)

In this way the output of ESE can be obtained.

3.2. DEC Operation

As shown in the lower part of Fig. 1, the de-interleaved and descrambled version of the output of ESE are de-spread(decoded), generating \{L_{DEC}(c_k(n))\} which is acted as a priori information of LDPC DECs.

Generating the extrinsic information \(\{e_{DEC}(p_k(j))\}\) of \(\{p_k(j)\}\)

\[ e_{DEC}(p_k(j)) = s_k(j) \cdot L_{DEC}(c_k(n)) - e_{ESE}(p_k(j)) \]  

(4)

The extrinsic LLR \(\{e_{DEC}(p_k(j))\}\) generated from DEC and are sent to the ESE, which is the a priori mutual information of ESE. The ESE detection and LDPC decoding are run at a preset number of time, then the DEC will carry out decisions and generate decoded bits.

4. EXIT Analysis

4.1. EXIT Characteristic Analysis of ESE

The concept of EXIT charts is a semi-analytical technique which can be used to evaluate the iterative detection and decoding process. The EXIT chart technique takes advantage of computing the mutual information of the detector, which is ESE in this paper, and of decoder, which is LDPC decoder here. The mutual information transfer function can be expressed as \(I_E = T(I_A, E_b/N_0)\), where \(I_A\) is the a priori mutual information, \(I_E\) is the extrinsic mutual information, and \(T\) is non-linear transfer functions. This
function can measure the refinement of the LLR transformations in the turbo process as the detector and the decoder takes soft LLRs as input, and output soft LLRs.

![Figure 2](image_url)

**Figure 2.** The EXIT curves of ESE over a AWGN 4,16 user MACs with different SNR.

The extrinsic LLRs about $x_k(j)$ is shown in (3), it is actually the extrinsic information, and can be in the form of $L_{E,k}$ by regarding $\sum_{k'\neq k} h_{k'}(j)x_{k'}(j)$ as a complex Gaussian variable. Next we study the EXIT function of the detector ESE. All $h_k$, $k=1, \ldots, K$ are assumed to be 1 in this paper. The a priori mutual LLRs information is expressed as $L_{A,k}$. As in the aforementioned GA method[7], $L_{A,k}$ and $L_{E,k}$ are assumed to be Gaussian distribution with mean $m_A$, $m_E$, and variance $2m_A$, $2m_E$ respectively. The mean of $L_{E,k}$ can be a function of $m_A$. The mentioned expectation can be roughly achieved by Monte Carlo simulation. Using the relation between $I_A$ and $m_A$, $m_A=J^{-1}(I_A)^{2/2}$, the conditional EXIT function can be in the form of $T(I_A, K, N_0 | h_1, h_2, \ldots, h_K) = J\left(\sqrt{2m_E}\right)$, which is a function of $I_A$.

The definition of $J$ can be found in[8]. By using system level Monte Carlo simulation the EXIT function of ESE can be roughly obtained[17]. In fact, this function can be influenced by the number of users $K$ and noise spectrum density $N_0$.

The EXIT curves for the system contains that of ESE and DEC. For the case of 4,16 user AWGN MACs with different $E_b/N_0$, the curve for ESE are illustrated in Fig.2. Given the same user number and $E_b/N_0$ as $I_A$ increases, $I_E$ increases, and the increase rate is larger at higher $I_A$. This indicates that if the input a priori information is sufficient enough, the detector can output sufficient extrinsic information, which can be utilized by DEC in the iterative decoding and detection process.

At the fixed user number and a priori information $I_A$, the extrinsic information $I_E$ of the ESE decreases with the increase of the noise level. When number of the user becomes larger, the impact of $E_b/N_0$ is less obvious, whereas at smaller user number, the impact of $E_b/N_0$ is prominent. The user number can also have an influence on the value of $I_E$, i.e. at lower $I_A$, the impact of user number on $I_E$ is less obvious than that of higher $I_A$, however, as $I_A$ approaches 1, this impact diminishes.

The EXIT functions give us clues for predicting the performance of noisy channel and less noisy channel, and it also tells us that the optimized code should be different for MACchannel and P-2-P channel.

**4.2. EXIT Characteristic Analysis of DEC**
For analysis of the EXIT characteristic of decoder, we use the following degree distribution pair (DDP) obtained by the method used in [18].

\[
\begin{align*}
\lambda(x) &= 0.7279 + 0.0432 x^2 + 0.0884 x^3 + 0.0998 x^4 + 0.0407 x^{10} \\
\rho(x) &= 0.2245 x^3 + 0.7755 x^4
\end{align*}
\]

The EXIT characteristic of the optimized DDP at different $E_b/N_0$ is plotted in Fig.3. We can see that at lower $E_b/N_0$, the EXIT characteristic of VND intersect with that of CND at value less than 1, which means that the iterative decoding process can not function well. In this case, the decoder is unable to quit the channel successfully in the EXIT chart. If the $E_b/N_0$ is larger than a threshold value, there is a channel between VND and CND. After a certain number of iterations, the decoder can correctly decode, and by transferring the decoded information to the detector and performing ESE, further reduction of interference can be achieved. When the received SNR is small enough, there exists no channel channel, namely channel shut down. This means that no matter any number of iteration is performed, the decoder can not arrives at the point to correctly and successfully decode by using the extrinsic information.

![Figure 3. EXIT characteristic of the optimized LDPC code constructed with DDP3 at different SNR](image)

5. Conclusions
Based on the LDPC coded IDMA system, this paper makes an analysis on the EXIT characteristic of the detector and LDPC decoder. EXIT analysis is performance in the iterative process, and extensive simulations are done to get the EXIT characteristic for 4-user and 16-user cases. The convergence of the detector and decoder can be explicitly viewed from the EXIT characteristic.

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