Analysis of queue and performance of automatic toll booths with a normal distribution (case study: automatic booths toll gate muktiharjo)

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Abstract. Queue process is a process related to the arrival of customers in a service facility, waiting in line queue if it cannot be served, get service and finally leaves the facility after being served. Research on the queue process can be seen directly through the queue system. Queue models and their distribution were obtained using the Sigma Magic program. The model of the vehicle queue system at the Muktiharjo Automatic Toll Gate is (NORM/NORM/4):(GD/∞/∞). Based on the values of the queue system performance measures obtained through the MATLAB GUI program as a whole it can be concluded that the queue of vehicles at the Muktiharjo Automatic Toll Gate has a condition where the average number of vehicles estimated in the system every 30 minutes is 99,2564 vehicles. The average number of vehicles in the queue system every 30 minutes is 98,2557 vehicles. The waiting time in the system is estimated to be around 15,51732 seconds. The estimated waiting time in line is around 15,36084 seconds. The queue system has a busy opportunity for 63.2849%. The simulation of the vehicle queue system at the Automatic Toll Gate of Muktiharjo Toll Gate by using ARENA is optimal with 4 automatic toll booths.

1. Introduction
The growth of vehicles number in Indonesia increased every year. This is influenced by human growth today. Based on the projections of the National Development Planning Agency in 2013, the population of Indonesia in 2018 reached 265 million. Increasing human population in Indonesia resulted in increasing transportation needs every year. It was noted that there were 15.493.068 cars, 2.509.258 buses, and 7.523.550 goods vehicles[1]. The imbalance of the human population with technological facilities often lead to a line in the use of common technology facilities. The emergence of the queue theory triggered by the problem of the limited capacity of services to cater to customer demand[2].

Increased usage of roads used for any means of transportation in Indonesia resulted in lots of congestion. To support the motion of economic growth, Indonesia needs a reliable road network. Through Government Regulation Number 4 In 1978, on March 1st, 1978 the Government established PT Jasa Marga (Persero) Tbk. Enforcement cash payments are done by PT Jasa Marga, but now the use of cash transactions began to turn to the use of non-cash transactions. Toll Gate Muktiharjo is a toll gate that provides Automatic Toll substation facilities for vehicles coming from the direction of the Port of Semarang, Demak, and Kudus. The number of vehicles that arrive at a given time and the lack of
information held by the riders on the Automatic Toll substation could cause a prolonged line. How to determine the performance of the automatic toll booth ministry is doing a queue system analysis and system performance on the automatic toll booth and perform system simulation queue. The results of the analysis will be used as a study for the development of decision making toll booth queue system effectively for the future.

2. Literature review

2.1. Public profile jasa marga
Jasa Marga is Indonesian State-Owned Enterprises engaged in the toll road service providers. PT Jasa Marga (Persero) Tbk was established on March 1, 1978, through Law 4 the Year 1978. The establishment of PT Jasa Marga was established with the aim to support the motion of economic growth in Indonesia. Jasa Marga's main task is to plan, build, operate and maintain toll roads and other means of apparatus that can serve as a toll road motorway give higher benefits than the public is not a toll road. Toll Gate Muktiharjo is a toll gate that provides Automatic Toll substation facilities for vehicles coming from the direction of the Port of Semarang, Demak and Kudus[3].

2.2. Description queue
The queue is a waiting line for a number of customers who require the services of one or more service facilities. The queue process is a process associated with the arrival of a customer at a service facility, wait in line if the line can not be serviced, serviced and eventually left the facility after being served[4].

2.3. Factors queue system
There are some important factors that affect the row lines and services with the line that arrival distribution system, distribution of service time, service facilities, disciplined line, inline size and source of the call[4].

2.4. Basic structure system queue
There are four basic models of the structure of a common line in the entire line system[5], among others:
1. One of the Queue Service (Single Channel Single Phase)
   In this system, there is only one entry point service system and there are only one service facility. After receiving customer service will be out of the system. An example of this is the structure of the purchase train tickets that are served by a single counter.
2. One Queue, Multiple Service Series (Single Channel-multiphase)
   This system indicates there is only one entry point service system and there are two or more service facilities in series in the path. An example of this is the structure of the car wash.
3. One Queue Multiple Parallel Services (Multichannel Single Phase)
   This system shows that there are two or more lines enter the service system and there are only one service facility within each track. An example of the structure of this model is that ticket purchases are served by more than one service counter.
4. Multiple Queue Few Parallel Services (multichannel-multiphase)
   In this system, there are two or more lines enter the service system and also there are two or more service facilities in series in each track. For example, the re-registration of students.

2.5. Kendall notation
A combination of the arrival process with the service, in general, is known as the universal standard, namely: (a/b/c): (d/e/f). With the symbols, a, b, c, d, e and f are the basic elements of the model line queue[4]. Explanation of the symbols above are as follows:
  a: Distribution arrival (arrival distribution).
  b: Distribution of service time (service time distribution).
  c: The number of service points (with c = 1, 2, 3, ... ∞).
d: Discipline supposes servicing FIFO, LIFO, SIRO, PS.
e: The maximum number of customers allowed in the system.
f: Source callings.

2.6. The size of the steady-state
Steady-state is a condition when the properties of the system do not change with time. In steady-state conditions, it is expected that the average customer that comes has a value which is comparable to the average customer who has served or it can be said the average customer who comes does not exceed the average customer who has served ($\lambda < c$), so that if $\rho < 1$, it means fulfilling steady-state conditions.

2.7. Poisson and exponential distribution process
Theorem [6]
For a Poisson process, the number of arrivals occurs at time intervals $t$ is a random variable that follows a Poisson distribution with an average $\lambda t$ and probabilities of $n$ arrivals are

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$  \hspace{1cm} (1)

**Theorem 2** [6]
If the number of arrivals follows a Poisson distribution then a random variable inter-arrival time follows an exponential distribution. If the arrivals follow a Poisson process with parameter $\lambda$, Then a random variable in succession will follow the exponential distribution with parameter $\frac{1}{\lambda}$  \hspace{1cm} (2)

2.8. Test match distribution
To know the shapes of the functions of the population under study, the first step is to test whether the population has a tendency to distributed in accordance with the underlying assumptions of the parametric procedure proposed. When the samples were drawn from populations that are not known, it is used alignment methods to determine how far the data samples observed sample matched with models offered [7]. Tests of conformity can be a helpful tool to evaluate the extent to which a model is able to approach the real situation it describes. There are two methods of alignment that are most commonly used, namely the Kolmogorov-Smirnov test alignment and the Kai-Square test. The steps of determining the distribution are as follows:

2.8.1. Kolmogorov-Smirnov test
1. Determining hypothesis
   - $H_0$: Distribution of samples follow the specified distribution
   - $H_1$: Distribution of the sample did not follow the specified distribution
2. Determining the Level of Significance
   - The significance level used was $\alpha = 5\%$
3. Statistical Test
   - $$D = \sup_x |S(x) - F_0(x)|$$  \hspace{1cm} (3)
   - with:
     - $S(x)$: cumulative odds function calculated from the sample data
     - $F_0(x)$: hypothesized distribution function (cumulative odds function)
4. Test criteria
   - Reject $H_0$ if the significance level $\alpha$ if the value of $D \geq$ value $D_{table}(1- \alpha)$ or if sig < value $\alpha$. $D_{table}(\alpha)$ is the critical value obtained from the table Kolmogorov-Smirnov.

2.8.2. Chi-Square test
1. Determining the hypothesis
   - $H_0$: The sample was drawn from a population that follows a set distribution
H1: The sample did not come from a population with a distribution set
2. Determining the level of significance
   The significance level used was $\alpha = 5\%$
3. Statistical Test
   \[ \chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \]  
   With $O_i$ is the observed frequencies, frequencies $E_i$ is hope and $i$ and $k$ is a category
4. Test criteria
   Reject $H_0$ at significance level $\alpha$ if the value of $\chi^2 \geq \chi^2_{\alpha,v}$ with $v = r - 1 - g$ or $p$-value $< \alpha$,
   where $g$ is the number of parameters specified distribution and $r$, is the number of classes

2.9. Normal distribution
In 1733 DeMoivre finds a normal curve mathematical equation on which to base a lot of statistical theory\[^8\]. Normal distribution bell curve shape as shown below

![Normal distribution curve](image)

**Figure 1.** Normal distribution curve

Normal distribution density function with parameters mean and variance are
\[ f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty \]  
\[ E(x) = \mu \]  
\[ E(x - \mu)^2 = \sigma^2 \]  
\[ (-\infty < \mu < \infty) \text{and} (0 < \sigma^2 < \infty) \]

2.10. Model Queue (G/G/c):(GD/∞/∞)
The model line (G/G/c):(GD/∞/∞) is a model line with the general distribution pattern of arrival (general) and the general pattern of distribution services (general) with a number of service facilities as $c$, $c = 1, 2, 3, \ldots$. Queue discipline used in this model are common: FIFO (First In First Out), the maximum allowed capacity in the system is infinite, and have a source of infinite calling\[^4\]. For the calculation of the estimated number of customers in the queue ($L_q$) is based on performance measures on the model (M/M/c):(GD/∞/∞). The formula to figure measures the performance of the models (G/G/c):(GD/∞/∞) is as follows:
\[ L_q = \frac{\rho^{c+1}}{(c-1)!} \frac{P_o \mu^2 v(t) + v(t') \chi^2}{2} \]
with:
$v(t)$ is a variant of the service time
$v(t')$ is a variant of the inter-arrival time
\[ L_s = L_q + \rho \]  \hspace{1cm} (10)

\[ W_q = \frac{L_q}{\lambda} \]  \hspace{1cm} (11)

\[ W_s = W_q + \frac{1}{\mu} \]  \hspace{1cm} (12)

### 2.11. Simulation system

Simulation refers to methods and applications to mimic the behavior of the real system. Simulation has been widely applied in many industrial fields\(^9\). Solving problems with the simulation model is usually done using computers, as much too complicated calculations calculated by hand. Differences in the distribution of the level of service much more easily adopted in the model compared to the rate of arrival. This can be overcome by using simulation\(^{10}\).

### 2.12. Graphic User Interfaces (GUI) MATLAB

MATLAB (Matrix Laboratory) is a program for analysis and numerical computation, an advanced mathematical programming language built on the premise that use of the nature and form of a matrix\(^1\). Matlab is widely used in mathematics and computing, and algorithm development, programming models, simulation, data analysis, exploration, visualization, and application development techniques.

### 3. Research methods

The data used in the writing of this research is the primary data, ie data obtained by direct observation and recording of the object of research. The study was conducted by taking a sample of data for 5 days. Research conducted at Toll Gate Substation Automatic Toll Mukthiharjo the time of execution on December 17, 2018, until December 21, 2018. The study was conducted from 07:00 am until 17:00 pm every day. The tools used to record the time of arrival of vehicles and the length of time the service is XNote Stopwatch software that connects directly to Microsoft Excel 2010. While the resulting output is the result of data processing by using software Statistics, namely IBM SPSS Statistics 22, Sigma Magic, GUI MATLAB, and ARENA.

1. Determining the place of research and research methods. Conducting research directly at Toll Gate Substation Automatic Toll Mukthiharjo to obtain data on the number of vehicles coming and data on the number of vehicles serviced.

2. The data obtained must meet the conditions steady-state (\( \rho = \frac{\lambda}{c \mu} < 1 \)), with \( \lambda \) as average vehicle arrival and \( \mu \) as the average vehicle has been serviced.

3. To test the suitability of the distribution for arrivals and visitors served by using the Kolmogorov-Smirnov test. If the hypothesis for the distribution of the number of arrivals and the number of services received then the distribution follows a Poisson distribution and if the hypothesis is rejected, the arrival distribution General distribution.

4. Determining the appropriate line models. Judging from the distribution of arrival, the distribution of services, the number of services, the queue discipline used is First In First Out (FIFO) capacity in the system and the source of the call.

5. Creating a Queue Simulation.

6. Determine the performance of the system, ie the number of customers expected in the system (\( L_s \)), the estimated number of customers in the queue (\( L_q \)), the waiting time in the system (\( W_s \)), and the time to wait in line (\( W_q \)).

7. Making the results and discussion.
8. Drawing a conclusion about the service at Automatic Booth Toll Gate Muktiharjo overall.

4. Results

4.1. Queue system overview vehicle in automatic booth toll gate muktiharjo

Toll Gate Muktiharjo is a toll gate that provides Automatic Booth Toll facilities for vehicles coming from the direction of the Port of Semarang, Demak and Kudus.

4.2. Data description

The data used is data on the number of vehicles coming in and the number of vehicles serviced on relay services of Automatic Booth Toll Gate Muktiharjo. The amount of data used are as follows:

| Day     | Number of Vehicles are Coming | Number of Vehicles Served |
|---------|-------------------------------|---------------------------|
| Monday  | 6,859                         | 6,851                     |
| Tuesday | 7,099                         | 7,098                     |
| Wednesday | 7,683                        | 7,679                     |
| Thursday | 7,870                         | 7,866                     |
| Friday  | 8,868                         | 8,860                     |
| Total   | 38,379                        | 38,354                    |

4.3. Analysis queue automatic booth toll gate muktiharjo

4.3.1. The size of the steady state. Steady-state condition is met if the value of the degree of usefulness $(\rho) <1$ means the average number of vehicles that come in smaller than the average number of vehicles serviced. To calculate the value of $\rho$ needs to know the value of the average number of vehicles coming and the average number of vehicles serviced. The time interval used was 30 minutes.

1. The average number of vehicles coming in $(\lambda)$ is a vehicle of 383,79 per 30 minutes.
2. The average number of vehicles serviced $(\mu)$ is a vehicle of 383,54 per 30 minutes.
3. The number of the automatic toll booth at Toll Gate Muktiharjo service is 4.

| $c$ | $\lambda$ | $\mu$ | $\rho = \frac{\lambda}{c\mu}$ |
|-----|-----------|------|-----------------------------|
| 4   | 383,79    | 383,54 | 0.250162956                |

From Table 2, it can be seen that the level of utility service facilities Automatic Booth Toll Gate Muktiharjo less than one, then concluded that it is in a steady-state line.

4.3.2. Test distribution. Test matches are used to test the distribution of data on the number of vehicles coming in and the number of service vehicles is Kolmogorov Smirnov. The test will be known whether the data on the number of vehicles coming in and the number of Poisson distributed vehicle serviced.

| Data                            | calculated as | Durable | Decision |
|---------------------------------|---------------|---------|----------|
| Number of Vehicles are Coming   | $\sup_x | S(x) - F_o(x) | = 0.336 | $\frac{1.36}{\sqrt{100}} = 0.136$ | Ho is rejected |
So, at the 5% significance level showed that data on the number of vehicles coming in and the number of vehicles serviced at Automatic Booth Toll Gate Muktiharjo, not Poisson distributed or distributed General.

4.3.3. Queue systems model. Based on Table 4, the model system of line with the distribution of the number of vehicles coming is Normal and the distribution of the number of vehicles serviced is Normal, with service facilities as much as 4, disciplined line used is a first come first serve basis (FIFO), as well as the amount of capacity customers come and a source of unlimited calling.

**Table 4. Determination of distribution model based on sigma magic output**

| Data                          | Statistical Test | Decision         |
|-------------------------------|------------------|------------------|
| Number of Vehicles are Coming | $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ | Ho accepted |
|                               |                  | p-value = 0.877  |
| Number of Vehicles Served     | $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ | Ho accepted |
|                               |                  | p-value = 0.961  |

4.3.4. Queue system simulation. The simulation was performed to determine the image directly on service substation at Toll Gate Automatic Toll Muktiharjo if there is a place of service as much as two substations, substations 3, 4 and 5 substations. Process simulation is done using ARENA software with a given input parameter distribution of Normal in the preparation of the simulation image. The parameters are the number of vehicles coming $\mu = 383.79$ and $\sigma^2 = 82.619$ as well as the parameters of the number of vehicles serviced are $\mu = 383.54$ and $\sigma^2 = 82.945$.

The simulation process is done by replication as much as 10 times the distribution expression is $\text{NORM}(\mu, \sigma^2)$ And obtained the following results:

**Table 5. Results of simulation queue**

| Information        | c = 2 | c = 3 | c = 4 | c = 5 |
|--------------------|-------|-------|-------|-------|
| Figures vehicle entry |       |       |       |       |
| Minimum            | 90.00 | 90.00 | 90.00 | 90.00 |
| Maximum            | 99.00 | 99.00 | 99.00 | 99.00 |
| Average            | 94.50 | 94.60 | 94.60 | 94.60 |
| Figures vehicle out |       |       |       |       |
| Minimum            | 88.00 | 88.00 | 88.00 | 88.00 |
| Maximum            | 99.00 | 98.00 | 98.00 | 98.00 |
| Average            | 93.20 | 93.30 | 93.40 | 93.50 |
| Time vehicle wait  |       |       |       |       |
4.3.5. Queue systems performance measures. Based on the results obtained by using the GUI software MATLAB, then known measures of system performance line Automatic Booth Toll Gate Muktiharjo. The system performance measures are presented in the table and figure below:

Table 6. Performance measures queue system automatic booth toll gate muktiharjo

| c   | λ    | μ   | Ls  | Lq  | Ws  | Wq  | Po   |
|-----|------|-----|-----|-----|-----|-----|------|
| 4   | 383.79 | 383.54 | 99,2564 | 98,2557 | 0,258622 | 0,256014 | 0,367106 |

Based Queue System Performance Measures Table Automatic Booth Toll Gate Substation Muktiharjo can know the following information:
1. The number of services provided to serve substation automatic toll is four.
2. The average number of vehicles that arrive every 30 minutes is \( \lambda = 383.79 \) That is, in intervals of 30 minutes there are on average 383.79 vehicles coming on line Automatic Booth Toll Gate Muktiharjo
3. The average number of vehicles serviced every 30 minutes is \( \mu = 383.54 \). That is, within 15 minutes there is an average of 383.54 vehicles that come to be served on a line Automatic Booth Toll Gate Muktiharjo
4. The number of vehicles is expected in the line system every 30 minutes is \( L_s = 99,2564 \) vehicles. This means that, on average within 30 minutes of total vehicles that are in the queue and the vehicle is being serviced is 99,2564 vehicles.
5. The number of vehicles is expected in line every 30 minutes is \( L_q = 98,2557 \) vehicles. That is, the average vehicle within 30 minutes of vehicles waiting in line to get service is 98,2557 vehicles.
6. The waiting time in the system is estimated at \( W_s = 0.258622 \) in 30 minutes or approximately 15.51732 seconds.
7. The estimated wait time in the queue is \( W_q = 0.256014 \) in 30 minutes or approximately 15.36084 seconds.
8. The probability that the care workers unemployed is \( Po = 0.367106 \), which means the system busy line has a chance of 63.2894% while 36.7106% is the opportunity queue system is not busy.

5. Conclusion
Based on the discussion of the results of research that has been done, it can be concluded that the model system of the queue of vehicles at Automatic Booth Toll Gate Muktiharjo is (NORM/NORM/4):(GD/\infty/\infty). Through the simulation results, when the automatic booth service provided was 4 obtained by the time value of vehicles waiting in line to be the average of 13.39 seconds, the level of activity in the service points are an average of 25.02%. When the substation service provided is 5, the value of time and the level of activity will also automatically go down, but the construction of
the automatic toll booth at the toll booths will also lead to increased maintenance tollbooth itself from PT Jasa Marga. It was concluded that when the substation service provided is 4 is optimal.

References
[1] Arhami M and Desiani A 2005 *Pemrograman MATLAB* Yogyakarta Andi
[2] Siswanto 2006 *Operations Research*, 2nd Jilid Jakarta Erlangga
[3] Jasa Marga Semarang 2018 *Sekilas Jalan Tol Semarang*. http://www. Jasamarga.com/public/id/infolayanan/toll/ruas.aspx?title=Semarang. Date of acces: 15th November 2018
[4] Kakiay T J 2004 *Dasar Teori Antrian Untuk Kehidupan Nyata* Yogyakarta Andi
[5] Subagyo, P Asri M and Handoko T H 1992 *Dasar-dasar Operation Research* Yogyakarta BPFE
[6] Gross D dan Harris C M 1998 *Fundamental of Queueing Theory Third Edition* New York John Willey and Sons Inc
[7] Daniel W W 1989 *Statistik Nonparametrik Terapan (terjemahan)* Jakarta PT. Gramedia
[8] Walpole R E and Myers R H 1995 *Ilmu Peluang dan Statistika untuk Insinyur dan Ilmuwan* Edisi Keempat Translated by RK Sembiring Bandung Penerbit ITB. Translated from Probability and Statistics for Engineers and Scientists
[9] Kelton W D 2004 *Simulation With Arena Third Edition* McGraw-Hill Singapore
[10] Siswanto 2006 *Operations Research*, 2nd Jilid Jakarta Erlangga