A collection of definitions and fundamentals for a design-oriented inductor model

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Abstract—This paper defines and develops useful concepts related to the several kinds of inductances employed in any comprehensive design-oriented ferrite-based inductor model, which is required to properly design and control high-frequency operated electronic power converters. It is also shown how to extract the necessary parameters from a ferrite material datasheet in order to get inductor models useful for a wide range of core temperatures and magnetic induction levels.

Index Terms—magnetic circuit, ferrite core, major magnetic loop, minor magnetic loop, reversible inductance, amplitude inductance

I. INTRODUCTION

Ferrite-core based low-frequency-current biased inductors are commonly found, for example, in the LC output filter of voltage source inverters (VSI) or step-down DC/DC converters. Those inductors have to effectively filter a relatively low-amplitude high-frequency current being superimposed on a relatively large-amplitude low-frequency current. It is of paramount importance to design these inductors in a way that a minimum inductance value is always ensured which allows the accurate control and the safe operation of the electronic power converter. In order to efficiently design that specific type of inductor, a method to find the required minimum number of turns \(N_{\text{min}}\) and the optimum air gap length \(g_{\text{opt}}\) to obtain a specified inductance at a certain current level is needed. This method has to be based upon an accurate inductor model, for which certain inductances and properties need to be defined and explained. Also, those inductance definitions needs to be parametrized, among other things, according to the specific ferrite material employed in the core.

A design-oriented inductor model can be based on the core magnetic model described in this paper which allows to employ the concepts of reversible inductance \(L_{\text{rev}}\), amplitude inductance \(L_{a}\) and initial inductance \(L_{i}\), all described in this paper, to further develop an optimized inductor design method. Those inductances rely on their respective core permeabilities, which are here presented and obtained for two specific ferrite materials: TDK-EPCOS N27 and TDK-EPCOS N87.

![Fig. 1. General magnetic circuit](image)

This paper is organized as follows. In section II a general magnetic core model is described and its associated permeabilities are introduced. In section III definitions of several types of inductances are presented. Section IV shows how to obtain the previously defined permeabilities from the ferrite material datasheet. Section V presents some useful properties of the reversible inductance that could be needed to justify the selection criterion of the inductance value as well as \(N_{\text{min}}\) and \(g_{\text{opt}}\). Finally, conclusions are presented in section VI

II. MAGNETIC CIRCUIT MODEL

In this section, we obtain a model for the general magnetic circuit considered in Figure 1 using an approach depending on integration along the mean magnetic path into the ferrite core, \(L_{c}\) and into the air gap, \(L_{g}\). Suppose that current \(i(t)\) can be easily decomposed into a) a component denoted as \(i_{LF}(t)\) at a relatively low frequency \(f_{LF}\) and b) a component denoted as \(i_{HF}(t)\) at a relatively high frequency \(f_{HF}\), with \(f_{LF} \ll f_{HF}\). At a certain time \(\hat{t}_{LF}\), \(i_{LF}\) reaches its peak value \(\hat{i}_{LF}\) and
then we have
\[ i(t) \approx i_{LF} + i_{HF}(t) \quad t \in \left[ i_{LF} - \frac{1}{f_{HF}}, i_{LF} + \frac{1}{f_{HF}} \right] \]  
(1)

In such a situation, Ampere’s law relates the frequency components of current \( i(t) \) with their corresponding magnetic field strength \( H \) components as follows

\[ \int \vec{H}(t, l) dl = \int_{L_c \cup L_g} \left[ \vec{H}_{LF}(l) + H_{HF}(t, l) \right] dl = N \left[ i_{LF} + i_{HF}(t) \right] \]

where \( \vec{dl} \) is the path vector, parallel to \( \vec{H} \). Separating the frequency components yields

\[ \int_{L_c} \vec{H}_{LF}(l) dl + \int_{L_g} \vec{H}_{LF}(l) dl = Ni_{LF} \quad (2) \]
\[ \int_{L_c} \vec{H}_{HF}(t, l) dl + \int_{L_g} \vec{H}_{HF}(t, l) dl = Ni_{HF}(t) \]
\[ \int_{L_c} \Delta \vec{H}_{HF}(l) dl + \int_{L_g} \Delta \vec{H}_{HF}(l) dl = N \Delta i_{HF} \quad (3) \]

where \( \Delta \vec{H}_{HF} \) is the amplitude of the field strength excursion due to \( \Delta i_{HF} \), the amplitude of the high-frequency current excursion during \( \frac{1}{f_{HF}} \).

The magnetic induction \( B(t, l) = \vec{B}_{LF}(l) + B_{HF}(t, l) \) and its peak-to-peak variation \( \Delta B_{HF} \) determine the peak induction \( B(l) = \vec{B}_{LF}(l) + \Delta B_{HF}(l) \). These are related to their corresponding field strength \( \vec{H}_{LF}(l) \), \( H_{HF}(t, l) \), \( \Delta H_{HF}(l) \) and \( \vec{H}(l) \) according to the medium permeability. Having the air gap paramagnetic properties, along \( L_g \) simply hold

\[ \frac{\vec{B}_{LF}(l)}{\vec{H}_{LF}(l)} = \frac{B_{HF}(t, l)}{H_{HF}(t, l)} = \frac{\Delta B_{HF}(l)}{\Delta H_{HF}(l)} = \frac{\vec{B}(l)}{\vec{H}(l)} = \mu_0 \]

(4)

where \( \mu_0 \) is the vacuum permeability. In the magnetic core path \( L_c \), those relationships depend on the shape of the ferrite magnetization curve which is shown in Figure 2. It is also the specific major loop that characterizes the behaviour of the ferrite when its magnetic induction evolution spans the two extreme points \( \pm B_s \), being \( B_s \) the saturation induction. In any inductor, although this situation can be reached with a current \( i_{LF} \) having a sufficiently high \( i_{LF} \), the actual \( i_{LF} \) has to be set well below that value since beyond that induction level the core ferrimagnetic properties become severely affected. Starting from a demagnetized core, as \( i_{LF} \) is gradually increased from zero towards the maximum value causing saturation, the tipping points \( (\vec{B}_{LF}, \vec{H}_{LF}) \) and \( (-\vec{B}_{LF}, -\vec{H}_{LF}) \) of the ever increasing LF-major loops describe a LF-commutation curve which is also partly shaped by the current magnitude of \( \Delta B_{HF} \), due to the memory properties of the ferrite material.

For any of the points pertaining to that LF-commutation curve, the LF-amplitude permeability \( \mu_{a}^{LF} \) is defined as

\[ \mu_{a}^{LF} \left( \vec{B}_{LF}, \Delta B_{HF} \right) = \frac{1}{\mu_0} \left. \frac{\vec{B}_{LF}}{\vec{H}_{LF}} \right|_{\Delta B_{HF}} \]

(5)

because it relates only the low frequency amplitude of the magnetic induction and field strength in the ferrite material when the low frequency \( i_{LF} \) takes also its amplitude value \( i_{LF} \). The magnetic induction generated by \( i_{LF} \) will not vary much from \( \vec{B}_{LF} \) while \( t \) is into the time span defined in (1).

Hence during \( \frac{1}{f_{HF}} \), \( i_{HF}(t) \) will produce an approximately closed minor magnetic loop of amplitude \( (\Delta B_{HF}, \Delta H_{HF}) \) starting and ending in the neighbourhood of \( \vec{B}_{LF} \), as it is shown in Figure 2. The incremental permeability \( \mu_{\Delta} \) at that quasi-static induction level \( \vec{B}_{LF} \) is then defined as

\[ \mu_{\Delta} \left( \vec{B}_{LF}, \Delta B_{HF} \right) = \frac{1}{\mu_0} \frac{\Delta B_{HF}}{\Delta H_{HF}} \]

(6)

Consequently, on \( L_c \) holds

\[ \vec{H}_{LF}(l) = \frac{\vec{B}_{LF}(l)}{\mu_0 \mu_{a}^{LF} \left( \vec{B}_{LF}, \Delta B_{HF} \right)} \]

(5)
\[ \Delta H_{HF}(l) = \frac{\Delta B_{HF}(l)}{\mu_0 \mu_{\Delta} \left( \vec{B}_{LF}, \Delta B_{HF} \right)} \]

(6)

If \( \Delta B_{HF} \) is made sufficiently small, then \( \mu_{\Delta} \) and the LF-commutation curve start to be practically independent of \( \Delta B_{HF} \). At this point, on the one hand the existing linear relationship between \( B_{HF} \) and \( H_{HF} \) is captured by the so-called reversible permeability at \( \vec{B}_{LF}, \mu_{rev} \)

\[ \mu_{rev} \left( \vec{B}_{LF} \right) = \lim_{\Delta B_{HF} \to 0} \mu_{\Delta} \left( \vec{B}_{LF}, \Delta B_{HF} \right) \]

(5)

On the other hand, the LF-commutation curve tends to the regular commutation curve and their respective amplitude
permeabilities are related as
\[ \mu_a(B) = \frac{1}{\mu_0} \frac{B}{H} = \lim_{\Delta B_{HF} \to 0} \mu_a^{LF}(B_{LF}, \Delta B_{HF}) \]
\[ = \mu_a(\hat{B}_{LF}) \]
Now making \( \hat{B}_{LF} \to 0 \) due to \( \hat{i}_{LF} \to 0 \), the initial permeability \( \mu_i \) is defined as
\[ \mu_i = \lim_{\Delta B_{HF} \to 0} \mu_a(\hat{B}_{LF}) = \mu_{rev}(\hat{B}_{LF} = 0) \]
Note that in the core, the relationship between \( B \) and \( H \) depends not only on the ferrite magnetic characteristics but also on the way in which \( B \) evolves with time.

In the magnetic circuit of Figure 11 a closed surface \( S = S_c \cup S_g \) that intersects both \( L_c \) and \( L_g \) paths will satisfy according to Gauss' law that
\[
\iint_{S_c} \hat{B}(t, l) dS = \iint_{S_g} \hat{B}(t, l) dS
\tag{7}
\]
where \( S_c \) is a core cross-section perpendicular to \( L_c \), \( S_g \) is the remaining surface of \( S \) crossing the air gap and \( dS \) is the area vector of \( S \). The left side of (7) is the core magnetic flux \( \Phi_c \) since the magnetic induction there is mainly concentrated into \( S_c \) because \( \mu_a, \mu \gg 1 \). All the magnetic induction in the air gap will then pass through \( S_g \), thus the right side of (7) is the air gap magnetic flux \( \Phi_g \). Consequently, \( \Phi_c = \Phi_g = \hat{\Phi}_{LF} + \hat{\Phi}_{HF}(t) \). \( L_c \) passes perpendicular through the center of \( S_c \), so the magnetic induction along \( L_c \) will be approximately an average of that existing inside \( S_c \), and equal to
\[
\hat{B}_{LF}(l) + B_{HF}(t, l) = \hat{\Phi}_{LF} + \hat{\Phi}_{HF}(t) \]
\[
A_c(l) = \lim_{\Delta \Phi_{HF} \to 0} \frac{\Delta \Phi_{HF}}{\Delta \Phi_{HF} + \Phi_{HF}(t)} = \frac{\Phi_{LF} + \Phi_{HF}(t)}{A_c(l)}
\tag{8}
\]
where \( A_c(l) \) is the area of \( S_c \) at a certain point \( l \in L_c \). Around the air gap, the magnetic field is far more non-uniform in \( S_g \) than in \( S_c \) due to the fringing flux. Thus, the mean induction on \( L_g \) can be quite different from the actual values at the edges of the gap, but being a paramagnetic region, it suffices to propose an effective gap area \( A_{ge}(l) \) with \( l \in L_g \), as if all the induction were there concentrated.
\[
\hat{B}_{LF}(l) + B_{HF}(t, l) = \hat{\Phi}_{LF} + \hat{\Phi}_{HF}(t)
\tag{9}
\]
Note that \( A_{ge}(l) \) is approximately equal to \( A_{cg} \), the core cross-section in contact with the air gap, if its length is much smaller than the linear dimensions characterizing \( A_{cg} \). Given that the winding turns \( N \) embrace practically all \( \Phi_c \), it follows that the linkage flux \( \Psi \) is
\[
\hat{\Psi}_{LF} + \hat{\Phi}_{HF}(t) = \hat{\Psi}_{LF} + \frac{\Delta \Psi_{HF}}{N}
\tag{10}
\]
The peak linkage flux \( \hat{\Psi} \) and magnetic induction \( \hat{B}(l) \) in the ferrite core are
\[
\hat{\Psi} = \hat{\Psi}_{LF} + \frac{\Delta \Psi_{HF}}{2}
\tag{11}
\]
\[ \hat{B}(l) = \frac{\hat{\Psi}}{N A_c(l)} \]

III. INDUCTANCE DEFINITIONS
Combining (2), (3), (4), (5), (6), (9) and (10) yield the LF-amplitude inductance, \( L_{a LF} \) and the incremental inductance, \( L_{\Delta} \)
\[
L_{a LF}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) = \frac{\hat{\Psi}_{LF}}{\hat{\Psi}_{LF}} \frac{\Delta \Psi_{HF}}{\Delta \Psi_{HF}} \]
\[ = \frac{\hat{\Psi}_{LF}}{\hat{\Psi}_{LF}} \frac{\Delta \Psi_{HF}}{\Delta \Psi_{HF}} \]
\[ = \frac{N^2}{\hat{R}_{c LF}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) + \hat{R}_g} \]
\[ L_{\Delta}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) = \frac{\Delta \Psi_{HF}}{\Delta \Psi_{HF}} \hat{\Psi}_{LF} \]
\[ = \frac{N^2}{\hat{R}_{c LF}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) + \hat{R}_g} \]
with
\[
\hat{R}_{c LF}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) = \int_{L_c} \frac{dl}{\mu_0 \mu_a^{LF}(\frac{\hat{\Psi}_{LF}}{N A_c}, \frac{\Delta \Psi_{HF}}{N A_c}) A_c(l)}
\tag{14}
\]
\[
\hat{R}_{c LF}(\hat{\Psi}_{LF}, \Delta \Psi_{HF}) = \int_{L_c} \frac{dl}{\mu_0 \mu_a^{LF}(\frac{\hat{\Psi}_{LF}}{N A_c}, \frac{\Delta \Psi_{HF}}{N A_c}) A_c(l)}
\tag{15}
\]
\[
\hat{R}_g = \int_{L_g} \frac{dl}{\mu_0 A_{ge}(l)}
\tag{16}
\]
\[
\hat{R}_{c LF}, \hat{R}_{c LF} \text{ and } \hat{R}_g \text{ are the core LF-amplitude reluctance, core incremental reluctance and the air gap reluctance respectively.}
\]

Considering the situation where \( \Delta \Psi_{HF} \to 0 \), \( \mu_a \) and \( \mu_{rev} \) define the amplitude and reversible inductances at \( \hat{\Psi}_{LF}, L_a \) and \( L_{rev} \), as
\[
L_a(\hat{\Psi}_{LF}) = \lim_{\Delta \Phi_{HF} \to 0} \frac{L_{a LF}(\hat{\Psi}_{LF}, \Delta \Phi_{HF})}{\Delta \Phi_{HF} + \Phi_{HF}(t)}
\tag{17}
\]
\[
L_{rev}(\hat{\Psi}_{LF}) = \lim_{\Delta \Phi_{HF} \to 0} \frac{L_{a LF}(\hat{\Psi}_{LF}, \Delta \Phi_{HF})}{\Delta \Phi_{HF} + \Phi_{HF}(t)}
\tag{18}
\]
\[
\hat{R}_{ca} = \int_{L_c} \frac{dl}{\mu_0 \mu_a(\frac{\hat{\Psi}_{LF}}{N A_c}) A_c(l)}
\tag{19}
\]
\[
\hat{R}_{c rev} = \int_{L_c} \frac{dl}{\mu_0 \mu_{rev}(\frac{\hat{\Psi}_{LF}}{N A_c}) A_c(l)}
\tag{19}
\]
\( \hat{R}_{ca} \) and \( \hat{R}_{c rev} \) are the core amplitude reluctance and the core reversible reluctance respectively.

\( L_a \) and \( L_{rev} \) usually have dissimilar values at a same \( \hat{\Psi}_{LF} \) and vary differently as \( \hat{\Psi}_{LF} \) increases from zero to relatively high values. It is then important to find a common situation to relate and relativize their current values with.

In a demagnetized material, \( \mu_a \) and \( \mu_{rev} \) coincide at the
origin which means that \( L_a \) and \( L_{rev} \) converge to the initial inductance \( L_i \)

\[
L_i = \lim_{\Psi_{LF} \to 0} L_{a} \left( \Psi_{LF} \right) = L_{rev} \left( \Psi_{LF} = 0 \right) = \frac{N^2}{R_{co} + R_q} \tag{20}
\]

\[
R_{co} = \int_{L_c} \frac{dl}{\mu_0 \mu_i A_c(l)} \tag{21}
\]

being \( R_{co} \), the core initial reluctance. Note that only \( \mu_i \) does not vary with the core cross-sectional area \( A_c(l) \) along the magnetic path. However, \( \mu_i \) as well as \( \mu_a \) and \( \mu_{rev} \) do depend heavily on the core temperature, as is modeled in the next section.

### IV. PERMEABILITY MODELS

The dependence of \( R_{co} \) and \( R_{rev} \), from core temperature \( T_c \) and magnetic induction \( B_{LF} \), in each particular part of the core, is addressed when the corresponding functions \( \mu_a \left( B_{LF}, T_c \right) \) and \( \mu_i \left( T_c \right) \) are extracted from the ferrite material datasheet \( \square \) \( \square \): \( \mu_a \) as a function of magnetic induction amplitude and core temperature (3-D lookup table); \( \mu_i \) as a function of core temperature (2-D lookup table). To get the best accuracy in the inductor model, the \( \mu_a \) curve given by the ferrite manufacturer should have been obtained at a frequency close to \( f_{LF} \).

The temperature and induction dependence of \( R_{co} \), is subjected to find \( \mu_{rev} \left( B_{LF}, T_c \right) \). In \( \square \) it is concluded that the commutation curve coincides with the so-called initial magnetization curve for soft ferrite materials, that is \( \left( B_{DC}, H_{DC} \right) = \left( B, H \right) \). This means that \( \mu_{rev} \) is equal to DC-biased \( \mu_{rev} \) which can be extracted from a graph or as a function of DC-bias field strength \( H_{DC} \). That curve may not be given in datasheets for a particular ferrite material or for the core temperatures at which \( \mu_{rev} \) has to be obtained, but even if it were available it should be put in terms of \( B_{LF} \) to be employed in \( \square \). To overcome these limitations, we use a permeability model directly relating DC-biased \( \mu_{rev} \) with DC-bias magnetic induction \( B_{DC} \) \( \square \), where all its parameters at the desired core temperature can be entirely obtained from any ferrite datasheet, in the way it is next explained. This approach has been experimentally validated for many ferrite materials operating at different temperatures \( \square \) and it is currently employed by major ferrite manufacturers \( \square \).

Let us first consider the empirical models that curvefit the upper (u) and lower (l) branches of the dynamic magnetization (B-H) curve of Figure \( \square \) \( \square \),

\[
H_u(B) = \frac{B}{\mu_0 \mu_c 1 - \left( \frac{B}{H_c} \right)^{a_u}} - H_c \tag{22}
\]

\[
H_l(B) = \frac{B}{\mu_0 \mu_c 1 - \left( \frac{B}{H_c} \right)^{a_l}} + H_c \tag{23}
\]

with the positive parameters: coercive field strength \( H_c \), coercive permeability \( \mu_c \) and squareness coefficients \( a_u \) and \( a_l \) for each branch. Supposing that \( a_t \approx a_u \) and \( B = \hat{B}_{LF} \), \( \mu_{rev} \left( \hat{B}_{LF}, T_c \right) \) can be expressed as \( \square \)

\[
\mu_{rev} \left( \hat{B}_{LF}, T_c \right) = \left\{ \begin{array}{ll}
1 + (a_t - 1) \left( \frac{\hat{B}_{LF}}{B_s} \right)^{a_t} & \\
\left[ \left[ 1 - \left( \frac{\hat{B}_{LF}}{B_s} \right)^{a_t} \right]^{-2} \right] & \\
\left( 1 - \frac{\hat{B}_{LF}}{B_s} \right) \left( 2 - \left( 1 - \frac{\hat{B}_{LF}}{B_s} \right)^{a_u}B_s \right) \}
\end{array} \right. \tag{24}
\]

Apart from \( \mu_i \), \( \mu_{rev} \) depends on \( T_c \) through \( B_s \), \( H_c \), \( a_t \) and \( \mu_c \) and thus \( \square \) has to be numerically fitted for each particular \( T_c \). The fitting data is obtained from the 3-D lookup table \( H(B, T = T_c) \) based on the corresponding curves from the ferrite material datasheet \( \square \) \( \square \).

The starting guess points for the fitting process are extracted from 2-D lookup tables \( B_s^{*}(T_c), H_s^{*}(T_c), a_t^{*}(T_c) \) and \( \mu_c^{*}(T_c) \). \( B_s^{*}(T_c) \) and \( H_s^{*}(T_c) \) are built to linearly interpolate the two saturation induction and coercive field strength values \( (B_{s1}, B_{s2}, H_{c1} \) and \( H_{c2} \) respectively), that are stated at the two corresponding temperatures \( T_1, T_2 \), in the datasheet of the magnetic material. Lookup tables \( a_t^{*}(T) \) and \( \mu_c^{*}(T) \) are conformed in the following way. Let \( H_{11}(B_{11}, T_1) \) and \( H_{12}(B_{12}, T_1) \) be the field strength at two different induction levels from the lower branch of the B-H curve at temperature \( T_1 \) given by the datasheet. The value \( B_{11} \) could be from the "linear" region of the curve, while \( B_{12} \) could be taken from the "knee" between "linear" and "saturation" regions of the curve at temperature \( T_1 \). The estimations of coefficients \( a_t \) and \( \mu_c \), from \( \square \) at temperature \( T_1 \), \( a_t^{c1} \) and \( \mu_c^{c1} \) respectively, are found numerically solving

\[
\begin{align*}
1 - \left( \frac{B_{11}}{B_{s1}} \right)^{a_t^{c1}} & = \frac{H_{12} - H_{c1}}{H_{11} - H_{c1}} \frac{B_{11}}{B_{12}} \\
1 - \left( \frac{B_{12}}{B_{s1}} \right)^{a_t^{c1}} & = \frac{H_{12} - H_{c1}}{H_{11} - H_{c1}} \\
\mu_{c1} & = \frac{1}{\mu_0} \frac{H_{11} - H_{c1}}{1 - \left( \frac{B_{11}}{B_{s1}} \right)^{a_t^{c1}}} \tag{24}
\end{align*}
\]

| Material | \( T_c \) [°C] | \( a_t \) | \( H_c \) [A/m] | \( \mu_c \) | \( \mu_i \) | \( B_s \) [T] |
|----------|----------------|--------|--------------|--------|--------|--------|
| N27      | 100            | 1.25   | 18.12        | 140/9  | 3231   | 0.4165 |
| N27      | 25             | 2.00   | 24.35        | 11154  | 1700   | 0.4895 |
| N87      | 100            | 8.00   | 10.94        | 4330   | 3976   | 0.3925 |
| N87      | 25             | 3.78   | 21.17        | 6014   | 2210   | 0.4803 |

Using the B-H curve at temperature \( T_2 \), \( a_t^{c2} \) and \( \mu_c^{c2} \) can be also obtained following a similar reasoning. Finally, \( a_t^{c}(T) \) and \( \mu_c^{c}(T) \) are built to linearly interpolate \( a_t^{c1}, a_t^{c2} \) and \( \mu_c^{c1}, \mu_c^{c2} \).
Accordingly,
\[
\frac{d\mu_d}{dB} = -\mu_d^2 \frac{d^2 H_{u}^{LF}}{dB^2}
\]
\[
\frac{d^2 H_{u}^{LF}}{dB^2} = \frac{a_u \left( \frac{B}{B_u} \right)^a u \left( \frac{B}{B_u} \right)^a u + a_u \left( \frac{B}{B_u} \right)^a u }{\mu_0 H_{r} \left[ 1 - \left( \frac{B}{B_u} \right)^a u \right]^{3} B}
\]
\[
+ \left[ \ln \frac{H_{u}(\tilde{B}_{LF}) - H_u(\tilde{B}_{LF})}{H_{u}^{LF} - H_u(\tilde{B}_{LF})} \right] \left( \frac{2 \tilde{B}_{LF}}{H_{u}(\tilde{B}_{LF}) - H_u(\tilde{B}_{LF})} \right) \left( \frac{H_{u}^{LF} - H_u(\tilde{B}_{LF})}{H_{u}(\tilde{B}_{LF}) - H_u(\tilde{B}_{LF})} \right) \alpha
\]
(26)

\( \tilde{H}_{LF} \) is inside the area delimited by the largest major loop, the magnetization curve, that is described by (22)-(23). Consequently, (26) is positive for \( B \in [0, \tilde{B}_{LF}] \) and thus \( \mu_d \) is ever decreasing for increasing values of \( B > 0 \).

If \( \Delta \Psi_{HF} \to 0 \) we can consider

\[
L_{rev}(\tilde{\Psi}) = L_{\Delta} \left( \tilde{\Psi}_{LF}, \Delta \Psi_{HF} \right) = L_{rev}(\tilde{\Psi}_{LF})
\]

Now suppose that gradually \( \Delta \Psi_{HF} \) is increased and \( \tilde{\Psi}_{LF} \) is decreased in such a way that \( \tilde{\Psi} \) remains unchanged, implying that \( \tilde{B} \) and \( \tilde{H} \) are unmodified in all parts of the core. This scenario brings into existence increasingly asymmetric minor loops in the \( \tilde{B} - \tilde{H} \) plane with tipping points

\[
\tilde{B} = \tilde{B}_{LF} + \frac{\Delta B_{HF}}{2}
\]
\[
\tilde{H} = \tilde{H}_{LF} + k_H \Delta H_{HF}
\]
\[
\tilde{B} - \Delta B_{HF} = \tilde{B}_{LF} - \frac{\Delta B_{HF}}{2}
\]
\[
\tilde{H} - \Delta H_{HF} = \tilde{H}_{LF} - (1 - k_H) \Delta H_{HF}
\]

being \( k_H \in [0.5, 1] \) the magnetic field symmetry factor. Considering that along a general magnetic loop \( \mu_d \) increases as \( B \) decreases, it can be stated that

\[
\frac{\tilde{B} - dB}{dH} \bigg|_{\tilde{B}} \Delta H_{HF} \geq \tilde{B} - \Delta B_{HF}
\]

and hence

\[
\mu_{rev} \left( \tilde{B} \right) \leq \mu_d \left( \tilde{B} \right) \leq \mu_{\Delta} \left( \tilde{B}_{LF}, \Delta B_{HF} \right)
\]

Inside the minor loop, we can define the minor-loop amplitude permeability \( \mu_{a}^{MN} \) as

\[
\mu_{a}^{MN} = \frac{1}{\mu_0} \frac{\tilde{B} - \tilde{B}_{LF}}{H - H_{LF}} = \frac{1}{\mu_0} \frac{\Delta B_{HF}}{2 k_H \Delta H_{HF}} = \frac{1}{2 k_H \mu_{\Delta}}
\]

and to note that it holds \( \mu_{rev} \left( \tilde{B}_{LF} \right) < \mu_{rev} \left( \tilde{B}_{LF} \right) \), since when \( B \) is far from the origin, \( \mu_{rev} \) decreases as \( B \) increases. Consequently,

\[
\tilde{B}_{LF} + \mu_0 \mu_{rev} \left( \tilde{B}_{LF} \right) k_H \Delta H_{HF} \geq \tilde{B}_{LF} + \frac{\Delta B_{HF}}{2}
\]

and hence

\[
\mu_{rev} \left( \tilde{B}_{LF} \right) \geq \mu_{a}^{MN} \leq \mu_{\Delta}
\]
If the minor loop keeps some degree of symmetry around \((\hat{B}_{LF}, \hat{H}_{LF})\), then \(\mu_{\alpha}^{\Delta N} \approx \mu_{\Delta}\) and hence

\[
\mu_{\text{rev}} (\hat{B}) \leq \mu_{\Delta} (\hat{B}_{LF}, \Delta B_{HF}) \leq \mu_{\text{rev}} (\hat{B}_{LF})
\]

However, that minor loop with tipping points

\[
(\hat{B}, \hat{H}) : (\hat{B} - \Delta B_{HF}, \hat{H} - \Delta H_{HF})
\]

and an enclosing symmetric major loop with tipping points

\[
(\hat{B}, \hat{H}) : (-\hat{B}, -\hat{H})
\]

coincide in the vicinity of their uppermost tipping point \((\hat{B}, \hat{H})\) \([5][2]\). Hence, \(\mu_{\text{rev}}(\hat{B})\) at the existing minor loop is equal to \(\mu_{\text{rev}}(\hat{B})\) at that hypothetical major loop. In fact, \(\mu_{\text{rev}}(\hat{B}_{DC})\) \([2]\), from which \([24]\) is particularly derived, depends on the current magnetic induction value regardless its previous evolution. Note that from the minor loop standpoint, \(\hat{\psi}\) is given by \([4]\), but it can be also put in terms of the amplitude permeability as

\[
\hat{\psi} = L_{a} \left( \hat{\psi} \right) \hat{i}
\]

\[
\hat{i} = \max_{t} \left( \hat{i}_{LF} + i_{HF}(t) \right)
\]

and thus it is valid

\[
L_{a} \left( \hat{\psi}_{LF} + \frac{\Delta \psi_{HF}}{2} \right) \hat{i} = L_{a}^{LF} \hat{i}_{LF} + \frac{\Delta \psi_{HF}}{2}
\]

\[
L_{a}^{LF} = L_{a}^{LF} \left( \hat{\psi}_{LF}, \Delta \psi_{HF} \right)
\]

On that enclosing symmetric major loop, characterized by parameters \(\hat{\psi}^{MJ}, \hat{i}_{LF}^{M}\) and \(i_{LF}^{M}\), we have that

\[
\hat{\psi}^{MJ} = L_{a} \left( \hat{\psi}^{MJ} \right) \hat{i}^{MJ} = \hat{\psi}
\]

\[
\hat{i}^{MJ} = \hat{i}_{LF}^{M} = \hat{i}
\]

Consequently,

\[
L_{\text{rev}} (\hat{\psi}^{MJ}) = L_{\text{rev}} (\hat{\psi}) < L_{\Delta} < L_{\text{rev}} (\hat{\psi}_{LF}) \quad (27)
\]

\[
L_{\Delta} = L_{\Delta} \left( \hat{\psi}_{LF}, \Delta \psi_{HF} \right)
\]

VI. CONCLUSION

The objective of this paper is to provide a collection of basic definitions and properties to ground a comprehensive ferrite-core based low-frequency-current biased inductor model for an optimized design method, which is a fundamental tool to properly design and control any type of electronic power converter. The same procedures followed to extract the required parameters of N27 and N87 materials can be easily adapted to obtain that data for other ferrite materials.

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