Shower fractal dimension analysis in a highly-granular calorimeter

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Abstract.
We report on an investigation of the self-similar structure of particle showers recorded at a highly-granular calorimeter. On both simulated and experimental data, a strong correlation between the number of hits and the spatial scale of the readout channels is observed, from which we define the shower fractal dimension. The measured fractal dimension turns out to be strongly dependent on particle type, which enables new approaches for particle identification. A logarithmic dependence of the particle energy on the fractal dimension is also observed.

1. Introduction
When an energetic particle impinges on matter, it may interact and produce multiple daughter particles which may themselves interact. This process iterates while daughter particles have sufficient energy. The resulting particle cascade is called a shower, and can be classified into electromagnetic and hadronic types. The development of electromagnetic showers is governed by $e^+e^-$ pair-production and bremsstrahlung interactions. Hadronic showers are composed of long hadron tracks and of localised clusters produced in $\pi^0$ decay ($\pi^0 \rightarrow \gamma\gamma$) or in nuclear breakup. The strong interactions between nuclei and hadrons, particularly those that lead to pion generation, determine the development of hadronic showers. The cascade process gives rise to a self-similar pattern that can be described by shower fractal dimension.

Recent developments of microelectronics have a notable impact on the granularity of future calorimeters. For example, the total number of readout channels reaches $10^8$ in the calorimeters designed for future $e^+e^-$ collider, either the International Linear Collider (ILC) [1] or the Compact Linear Collider (CLIC) [2]. Such high granularity provides an unprecedented level of detail for the reconstruction of calorimeter showers and enables new approaches to shower analysis.

In this paper, we explore the fractal nature of particle showers measured in a calorimeter with ultra-high granularity. Varying the effective granularity of the calorimeter readout and counting the corresponding number of hits, we observe a strong correlation between these two observables, from which we define the shower fractal dimension. We investigate the dependence of the shower fractal dimension on the type and energy of the incident particle, and develop a particle identification algorithm based only on measurements made with the calorimeter. A similar behavior has been observed on both simulated and test beam data.
2. Measurement of shower fractal dimension in a highly-granular calorimeter

Showers originated by different particle types ($\pi^+$, $\mu^+$, $e^+$, $K^0_L$, proton and neutron) with energies between 2 GeV and 40 GeV were simulated with Mokka [3], which is the official full simulation software for ILD based on GEANT4 [4]. The simulated detector was the Digital Hadron Calorimeter (DHCAL). The DHCAL is a sampling Resistive Plate Chamber (RPC)-iron HCAL with 48 longitudinal layers, each 26.5 mm thick with a 20 mm iron absorber and a 6.5 mm RPC sensor layer. In the simulation, each RPC layer is divided into $1 \times 1 \text{mm}^2$ cells (modified from the original design of $10 \times 10 \text{mm}^2$). In the following analysis, only one bit is used for each channel to record whether the channel was hit or not.

The effective cell size can be varied by grouping $\alpha \times \alpha$ nearby cells. The quantity $\alpha$ then defines the scale at which the shower is analyzed. Defining $N_\alpha$ as the number of hits at a scale $\alpha$, the ratio of hit counts at different scales can be written as:

$$R_{\alpha,\beta} = N_\beta / N_\alpha.$$  

In the following discussion $\beta$ will be called the initial scale. Figure 1 shows the correlation between $R_{\alpha,1}$ and the scale $\alpha$ for different samples. An approximate linear correlation with $\alpha$ is observed in a double logarithmic scale (especially for the $e^+$ and $\pi^+$ samples), thus we define the shower fractal dimension as the average slope + 1, where the latter refers to the longitudinal degree of freedom:

$$FD_\beta = \left\langle \log\left(\frac{R_{\alpha,\beta}}{\log(\alpha)}\right) \right\rangle + 1.$$  

The shower fractal dimension also depends on the energy of the primary particle. More energetic showers are typically more compact, and have a larger fractal dimension (except for a minimum ionizing particle). The fractal dimension of electromagnetic and hadronic showers satisfies the following approximate relations (see figure 2):

$$FD_{\text{em},1\text{mm}}(E) = 1.41 + 0.21 \times \log_{10}(E/\text{GeV}),$$  

$$FD_{\text{had},1\text{mm}}(E) = 1.24 + 0.15 \times \log_{10}(E/\text{GeV}),$$  

$$FD_{\text{mip},1\text{mm}}(E) = 1.2.$$
Figure 2. (a) Fractal dimension for $e^+$ samples at different energies. (b) Correlation between fractal dimension and particle energy for $e^+$ samples.

Figure 3. Particle identification with the fractal dimension of shower and the number of hits corresponding to different initial scales: a, 1 mm; b, 10 mm; c, 30 mm

3. Application to particle identification
We will introduce a particle identification algorithm based purely on calorimeter observables, namely the measured fractal dimension of the shower and the number of hits.

This technique has been tested on different simulated data samples at an energy of 40 GeV, see figure 3. To study the dependence of the performance on cell size, the shower fractal dimension and the number of hits were measured at three different initial scales: 1 mm, 10 mm, and 30 mm.

A clear separation between muons, hadrons and positrons is observed at each initial scale. The tables in figure 3 show the output of the particle identification algorithm based on simple cuts, where the rows represent input samples and the columns present the output for 1000 events.

3.1. Performance on real data
Two cubic meter prototypes with Glass Resistive Plate Chamber sensors have been constructed by the CALICE collaboration, namely the Digital HCAL (DHCAL) prototype and the Semi-
Digital HCAL (SDHCAL) prototype [5, 6]. The DHCAL prototype is longitudinally divided into 51 layers (in the complete version). Each layer is composed of one absorber plane (with 16 mm thick steel) and a sensor layer with two covers, one being 2 mm thick copper and the other 2 mm thick steel. A layer consists of two glass plates, held apart using a fishing line with a diameter of 1.15 mm. Each sensor layer is divided into $96 \times 96 = 9216$ readout channels with a size of $1 \times 1 \text{cm}^2$. The SDHCAL prototype has a similar structure with 48 longitudinal layers, and each layer contains 9216 readout pads of $1 \text{cm}^2$. A detailed description can be found in [6].

The shower fractal dimension has been measured on experimental data using the same method. Figure 4 shows shower fractal dimension versus number of hits from the DHCAL test beam data taken at Fermilab. A Scintillator trigger was used to select the events of interest [5]. Figure 5 (a) corresponds to muon samples, while figure 5 (b) corresponds to a mixed sample of 12 GeV muons, electrons, and pions. The beam components are clearly separated, and the results are in agreement with the Monte Carlo prediction.

4. Summary
Cascade interactions between energetic particles and matter lead to the self-similar structure of particle showers. Using a simple box counting method, the shower fractal dimension can be measured at highly-granular calorimeters using only hit-level information. Since the fractal dimension ultimately reflects the underlying physics processes, it is sensitive to the nature of the incident particle. A logarithmic correlation between the number of hits and the shower fractal dimension is also observed.

Reference
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