Markov Regime Switching-Garch Modeling On World Oil Prices

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ABSTRACT. This study aims to explain and test the performance of the best forecasting model for world oil prices. The world oil price is included in a time-series data type that has high volatility and different variants at each point in time. Precise and precise time-series modeling of this type of data is required to properly explain structural changes and explain any shift in volatility. The method that is applied and produces the best model in describing world oil prices is the Markov Regime Switching-GARCH. Modeling results can be used as alternative data for investor’s consideration for determining their investment decisions.

Keywords: Modeling, Oil, Markov Regime-Switching

1. Introduction
The movement of world oil prices is one indicator that can affect the economic conditions of a country. As a country that has a high dependence on exports and consumption of crude oil, the Indonesian nation cannot be separated from this influence. The increase in world oil prices has implications for the price of fuel oil in a society which linearly affects the price of people's needs. One example of this condition was experienced by the Indonesian nation in May 2008 where there was an increase in the price of premium fuel from Rp 4,500 to Rp 6,000 per liter. This increase was triggered by the increase in world oil prices from 65 US $ to above 100 US $ per barrel. The state of the national economy and the rate of economic growth experienced a slowing effect at that time.

The fluctuation of the world oil price deserves the government's attention because the volume of international trade and the growth of the industrial sector is very vulnerable to impact. In 2014 there was a downward trend in world oil prices after previously always perching above 100 US $ per barrel. The sharp decline to the level of 44 US $ in 2015, had continued to touch 26.2 US $ per barrel on 11 February 2016. This point is the lowest value of oil prices in the last 13 years. At the same time, the Central Statistics Agency (BPS) released a report on Indonesia's economic growth reaching 5.02 percent in 2016, an increase from 2015 of 4.88 percent. This situation shows the level of dependence of the national economy on the phenomenon of world oil price movements.

The ability to predict future oil prices is a key requirement. Accuracy and usefulness of a forecasting model are of importance. Two important aspects that are key in a forecast are the validity of the data and the accuracy of the methods used. Time-series data are often not stationary because there are trends and contain periodic seasonal characteristics. This phenomenon is often accompanied by a high level of volatility, especially in time-series data in the business and economic fields. The problem of the investment movement of an asset is an example of a stochastic process that can be modeled mathematically using the Markov chain model. The Markov regime-switching model is an alternative that has the advantage of determining changes in the status of the return level from small
volatility to high volatility or vice versa for certain periods of the investment portfolio. This model is a type of approach that is relatively simple and can capture the nature of the distribution of returns that are not normal, are fat-tails, and a shift in discontinuous patterns.

Engle introduced the autoregressive conditional heteroscedasticity (ARCH) model as a function of heteroscedasticity by considering the conditional variance relationship of the linear-quadratic combination in the past [1]. In the next period, Bollerslev introduced the generalized autoregressive conditional heteroscedasticity (GARCH) model as a development of the ARCH model [2]. This model is simpler with fewer parameters than the high-grade ARCH model. Although ARCH and GARCH are time-series models that can explain heteroscedasticity in data, they are unable to consider the leverage effect. Nelson introduced the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model which can cover the weaknesses of the GARCH model in capturing asymmetries of good news and bad news [3]. The EGARCH model does not account for structural changes in time-series data. This led to the introduction of the GJR-GARCH model independently by Glosten, Jagannathan, and Runkle to account for the leverage effect [4].

Hamilton introduced Markov Switching (MS) as an alternative to modeling time-series data undergoing structural changes [5]. Changes in the structure of the model that occur in Markov switching are not considered the result of deterministic events but as the result of unobserved random variables and in the literature it is often called a state (st) or regime. Hamilton involved Markov switching in an autoregressive model and produced a model that could explain structural changes well, but did not yet explain the shift in volatility [5]. Furthermore, Hamilton and Susmel involved Markov switching in the ARCH model, known as the SWARCH model [6]. Gray introduced the GARCH regime switching model which has characteristics with Markov switching (MS) ARCH but involves simpler parameters [7]. Research on the MRS-GARCH model is widely applied in asset returns on the stock market [8][9]. Marcucci uses the overall data mean as the conditional average of the Markov regime-switching (MRS) –GARCH [8].

2. Stochastics Process
The stochastic process is a collection of several random variables, each of which is equipped with an identity in the form of an index \( \{X_t; t \in T\} \) where \( T \) is the set of indexes in time points. For each \( t \in T \), in this case, \( X(t) \) is a random variable which states the state of a process at time \( t \). A stochastic process \( \{X_t; t \in T\} \) is called a discrete process if \( T \) is the set of indexes over calculated discrete times [10].

3. Markov Processes
A stochastic process with discrete times \( \{X_n; n = 0, 1, 2, \ldots\} \) is said to be a Markov process, if for a given value \( X_t \) then value \( X_m \) with \( m > n \) independent on \( X_k \), \( k < n \). In other words, conditional probability \( X_n \) with condition \( X_1, \ldots, X_{n-1} \) only depends on the value \( X_{n-1} \). For some \( x_1, \ldots, x_{n-1} \) the conditional probability is written in the form

\[
P(X_n = x_n | X_1 = x_1, \ldots, X_{n-1} = x_{n-1}) = P(X_n = x_n | X_{n-1} = x_{n-1})
\]

4. Markov Chain
A Stochastic process \( \{X_n; n \in \mathbb{Z}^+\} \) is a Markov chain that satisfies the following conditions:

i. \( n = 0, 1, 2, 3, \ldots \)

ii. Possible values are finite or countable.

iii. \( P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0) = P(X_{n+1} = j | X_n = i) = P_{ij} \)

Conditional distribution \( X_{n+1} \), given the past state \( X_0, X_1, \ldots, X_{n-1} \) and present state \( X_n \), only depends on the present situation \( X_n \).

iv. Condition (state)\( = i_0, i_1, i_2, \ldots, i_{n-1}, i \)}
5. Transition Probability Matrix

Let \( \{X_n; n = 0, 1, 2, \cdots\} \) be a Markov chain. Matrices

\[
P = \begin{pmatrix}
P_{00} & P_{01} & P_{02} & \cdots \\
P_{10} & P_{11} & P_{12} & \cdots \\
P_{20} & P_{21} & P_{22} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

defined as a probability matrix with \( P_{ij} = P(X_{n+1} = j | X_n = i) \).

\( P_{ij} \) defined the odds that if the process is on state-\( i \) then the next process will move to state-\( j \). Because the non-probability value is negative and must undergo a transition to a state, it applies:

i. \( P_{ij} \geq 0; i, j = 0, 1, 2, \cdots \)

ii. \( \sum_{j=0}^{\infty} P_{ij} = 1, \) for \( i = 0, 1, 2, \cdots \)

6. Markov Regime-Switching (MRS) Model

Suppose \( \{p_t\} \) is the sequence of commodity prices at time \( t \) and \( \{r_t\}_{t=0}^{\infty} \) be a sequence of random variables in the probability space \((\Omega, \mathcal{F}, \mathbb{P})\). For the index \( t \) shows daily observations with \( t = R + 1, \cdots, n \). The sample period consists of estimation and observation period with \( R \) (\( t = -R + 1, \cdots, 0 \)), and samples with \( n \) observations \((t = 1, \cdots, n)\), \( r_t \) to be a log return at time \( t \)

\[
r_t = 100. \ln \left( \frac{p_t}{p_{t-1}} \right) \]

GARCH (1,1) model for return \( r_t \) can be written as

\[
r_t = \delta + \varepsilon_t = \delta + \eta_t \sqrt{h_t}
\]

\[
h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}
\]

with \( \alpha_0 > 0, \alpha_1 \geq 0 \) dan \( \beta_1 \geq 0 \) is a non-negative integer for \( h_t \geq 0 \). By assuming \( \eta_t \) are i.i.d. (independent identically distributed) where the mean and variance are zero. MRS-GARCH model with a two regime on the following form

\[
r_t = \delta + \varepsilon_t = \delta + \eta_t \sqrt{h_t}
\]

and

\[
h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}
\]

in this case \( S_t = 1, 2 \). With \( \delta_t \) are mean \( h_t \), some volatility under regime \( S_t \) on \( F_{t-1} \), both are function \( F_{t-\tau} \) for \( \tau \leq t - 1 \). To ensure that the non-negative conditional variance it is constrained \( \alpha_{0,t} > 0, \alpha_{1,t} \geq 0, \beta_{1,t} \geq 0 \). A result \( \alpha_{1,t} + \beta_{1,t} \) be a measure of the persistence of the shock conditional variance. \( S_t \) governed by the first-order Markov Chain with a constant transition probability given by \( P = (S_t = i | S_{t-1} = j) = P_{ij} \) for \( i, j = 1, 2 \)

In matrix notation \( P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \)

On the MRS-GARCH model with two regimes, is the forecasting of volatility for \( k \) steps ahead using a recursive method as in the standard models GARCH with \( k \) is nonnegative integer (Klassens, 2002). To forecast future values, first, calculate the weighted average several steps ahead with the estimated volatility in each regime where weight is the predicted probability. \( P_t = (S_{T+1} = i | F_{T-1}) \). Because there is no deep serial correlation of return, \( k \) steps on the volatility estimated at time \( T \) depending on the current information \( T - 1 \). Hence \( \hat{h}_{T,T+k} \) shows the estimated time \( T \) of the aggregate volatility for the next step \( k \). This can be calculated as follows:

\[
\hat{h}_{T,T+k} = \sum_{t=1}^{k} \hat{h}_{T,T+t}
\]

\[
= \sum_{t=1}^{k} \left[ \sum_{i=1}^{2} P_t(S_{T+t} = i | F_{T-1}) \hat{h}_{T,T+t|S_{T+t}=i} \right]
\]
with \( \hat{h}_{T>T+\tau} \) showing \( r \) showing \( k \) easy steps forward in estimation regime \( i \) at the time \( T \) it can be recursively calculated as follows:

\[
\hat{h}_{T>T+\tau} = E_{T-1}(\hat{h}_{T+\tau} | S_{T+\tau} = i)
\]

\[
= \alpha_0 S_{T+\tau} + \alpha_1 S_{T+\tau} E_{T-1}(e_{T+\tau-1} | S_{T+\tau} = i) + \beta_1 S_{T+\tau} E_{T-1}(h_{T+\tau-1} | S_{T+\tau} = i)
\]

\[
= \alpha_0 S_{T+\tau} + \alpha_1 S_{T+\tau} E_{T-1}(E_{T-1}[e_{T+\tau-1} | S_{T+\tau} = i]) + \beta_1 S_{T+\tau} E_{T-1}(h_{T+\tau-1} | S_{T+\tau} = i)
\]

\[
= \alpha_0 S_{T+\tau} + \alpha_1 S_{T+\tau} E_{T-1}(h_{T+\tau-1} | S_{T+\tau} = i)
\]

The general probability prediction can be calculated as follows:

\[
\begin{align*}
&\frac{P_i(S_{T+\tau} = 1 | F_{T-1})}{P_i(S_{T+\tau} = 2 | F_{T-1})} = P_{r+1} \cdot \frac{P_i(S_{T+\tau} = 1 | F_{T-1})}{P_i(S_{T+\tau} = 2 | F_{T-1})} \\
&\text{and}
\end{align*}
\]

\[
E_{T-1}(h_{T+\tau-1} | S_{T+\tau} = i) = E[h_{T+\tau-1} | S_{T+\tau} = i, F_{T-1}]
\]

\[
= E[E[r_{T+\tau-1} | S_{T+\tau-1} = j, F_{T-1}] - E[r_{T+\tau-1} | S_{T+\tau-1} = j, F_{T-1}]]^2 | S_{T+\tau} = i, F_{T-1}]
\]

\[
= E[E[r_{T+\tau-1} | S_{T+\tau-1} = j, F_{T-1}] | S_{T+\tau} = i, F_{T-1}]
\]

\[
= \sum_j E[\delta_{T+\tau-1} + 2\delta_{T+\tau-1} e_{T+\tau-1} + e_{T+\tau-1} | S_{T+\tau-1} = i, F_{T-1}]
\]

\[
= \sum_j \bar{p}_{j,T-1}[\delta_{T+\tau-1} + h_{T+\tau-1} S_{T+\tau-1}]
\]

for

\[
\bar{p}_{j,T-1} = P_i(S_{T+\tau} = j | S_{T+\tau} = i, F_{T-1})
\]

\[
= \frac{p_{j,i} P_i(S_{T+\tau} = j | F_{T-1})}{P_i(S_{T+\tau} = i | F_{T-1})}
\]

As well as,

\[
E[|E[r_{T+\tau-1} | S_{T+\tau-1} = j, F_{T-1}]|^2 | S_{T+\tau} = i, F_{T-1}]
\]

\[
= \sum_j \bar{p}_{j,T-1}[\delta_{T+\tau-1} + h_{T+\tau-1} S_{T+\tau-1}]^2
\]

Stands for

\[
E_{T-1}(h_{T,T+\tau} | S_{T+\tau} = i)
\]

\[
= \sum_j \bar{p}_{j,T-1}[\delta_{T+\tau-1} + h_{T+\tau-1} S_{T+\tau-1}]^2
\]
In the next step, probability of regime \( p_{it} = P_t(S_t = i|F_{t-1}) \) for \( i = 1,2 \) with \( f_{xt} := f(r_t|S_t = 1, F_{t-1}), f_{2t} := f(r_t|S_t = 2, F_{t-1}) \). A conditional distribution return \( r_t \) is a common distribution model into the common variable by probability regime \( p_{it} \).

\[
r_t|F_{t-1} \sim \begin{cases} f(r_t|S_t = 1, F_{t-1}) \\ f(r_t|S_t = 1, F_{t-1}) \end{cases}
\]

\( f(r_t|S_t, F_{t-1}) \) shows one of the conditional distributions that he assumes for errors such as the normal distribution, student t distribution, a distribution with only one degree of freedom (t) or two degree of freedom (2t) and Generalized Error Distribution (GED).

7. Modeling Volatility with Markov Regime-Switching

The Probability of a regime can be calculated in the following two steps.

**Step 1**: Supposed \( P_t(S_{t-1} = j|F_{t-1}) \) at the end of time \( t-1 \), then probability of regime \( p_{it} = P_t(S_{t-1} = j|F_{t-1}) \) can be calculated by

\[
P_t(S_{t-1} = j|F_{t-1}) = \sum_{j=1}^{2} P_t(S_t = i, S_{t-1} = j|F_{t-1})
\]

Because the current regime \( S_t \) only depends on the regime one period past \( S_{t-1} \) then

\[
P_t(S_t = i|F_{t-1}) = \sum_{j=1}^{2} P_t(S_t = i, S_{t-1} = j|F_{t-1})
\]

**Step 2**: At the end time-t, when return is observed at time t \( (r_t) \) then the information at the time t is \( F_t = [F_{t-1}, r_t] \).

\[
P_t(S_t = i|F_t) = P_t(S_t = i|F_{t-1}) = \frac{f(r_t, S_t = i|F_{t-1})}{f(r_t|F_{t-1})}
\]

with \( f(r_t, S_t = i|F_{t-1}) \) is the joint density of the returns and regimes observed in state - i for \( i = 1,2 \) it can be written as follows:

\[
f(r_t, S_t = i|F_{t-1}) = f(r_t|S_t = i|F_{t-1})f(S_t = i|F_{t-1})
\]

and \( f(r_t|F_{t-1}) \) is marginal density function of return:

\[
f(r_t|F_{t-1}) = \sum_{i=1}^{2} f(r_t, S_t = i|F_{t-1})
\]

using Bayesian argument

\[
P_t(S_t = i|F_t) = \frac{f(r_t, S_t = i|F_{t-1})}{f(r_t|F_{t-1})} = \frac{f(r_t|S_t = i|F_{t-1})P_t(S_t = i|F_{t-1})}{\sum_{i=1}^{2} f(r_t|S_t = i|F_{t-1})P_t(S_t = i|F_{t-1})} = \frac{\sum_{i=1}^{2} f(r_t, S_t = i|F_{t-1})P_t(S_t = i|F_{t-1})}{\sum_{i=1}^{2} f(r_t|S_t = i|F_{t-1})P_t(S_t = i|F_{t-1})}
\]

For all of regime probability \( p_u \) can be computed by a two-step iteration. However, at the start of the iteration \( P_t(S_0 = 1|F_0) \)

\[
\pi_1 = P_t(S_0 = 1|F_0) = 1 - 1 - \frac{q}{2} = \frac{1}{2} \frac{p}{p - q}
\]

\[
\pi_2 = P_t(S_0 = 2|F_0) = \frac{1 - q}{2 - p - q}
\]
Given the initial value for the possible regime, where the conditional mean and conditional variance in each regime, the MRS-GARCH model parameters can be obtained by maximizing the value of the log-likelihood function. The log-likelihood function is reconstructed recursively similar to the GARCH model.

8. Research Methode
This study aims to determine the performance of the accuracy of the Markov Regime Switching model in forecasting world oil prices. For this purpose, an empirical study is conducted on crude oil price data. The stages carried out in the research can be seen in Figure 1. The performance of the model obtained is then carried out for forecasting world oil prices.

8.1 Data
This study takes time-series data on world oil price movements in the period July 2015 - August 2020 containing 1274 data obtained from the website http://finance.yahoo.com/.

8.2 Normality Test for Time-series Data
Plots of daily and monthly oil price data can be seen in Figure 2.
Visually, it can be seen that the data on the closing value of oil prices is trendy, is not stationary, and has very high volatility. Also, a positive excess kurtosis value indicates abnormal data, it will appear more clearly visually from Figure 3. The statistical value of oil price data is as follows.

| Parameter | Statistics |
|-----------|------------|
| Min.      | 10.01      |
| 1st Qu.   | 44.75      |
| Median    | 51.15      |
| Mean      | 50.91      |
| 3rd Qu.   | 58.32      |
| Max.      | 76.41      |
| Kurtosis  | 0.3740875  |
| Skewness  | -0.4378606 |

Table 1. Statistical value of oil price data

![Histogram of data](image1.png)

![Normal PDF and Histogram](image2.png)
Figure 3 shows that the data is skewed, asymmetrical, and the kurtosis value exceeds 3 so that the data does not describe a normal distribution. Based on the plot, it shows that the normal distribution, in general, cannot be used to describe the data. The condition that the data does not follow a normal distribution is also corrected for the Q-Q plot. After the subjective graphical method, data abnormalities were also corrected from the results of the Goodness of Fit test. To fulfill the data normality assumption, data transformation is carried out with the Box-Cox transformation. This transformation is done by finding the value $\lambda$ which optimizes the correlation between the transformed data and the quantile value of the normal distribution. Obtained the estimated order value optimum $\lambda$ is 1.531795.

### Table 2. Powered Transformation

| Data                  | Metode Uji     | p-value       | Interpretasi     |
|-----------------------|----------------|---------------|------------------|
| Before transformation | Jarque Bera    | 3.021e-11     | Non-normal       |
| After transformation  | Jarque Bera    | 0.4972        | Normal           |

#### 8.3 Stationarity Test

The assumption of stationarity in time-series data analysis is very important. The detection of this is done in two parts, namely visually by looking at the ACF PACF function plot and testing the existence of a unit root with the Augmented Dickey-Fuller test.
Figure 4. Plot ACF and PACF

Table 3. Results of the ACF and PACF plots

| Stationarity detection | Result | Interpretation   |
|------------------------|--------|------------------|
| Plot ACF and PACF      | The data contains a trend component and decays slowly towards zero | Non-stationer |
| ADF Test               | p-value=0.6117, Dickey-Fuller=-1.921 | Non-stationer |

The visual results of the ACF and PACF plots confirmed by the Augmented Dickey-Fuller test show that the data is not stationary because it still contains a linear trend. To fulfill the stationarity assumption, differens and logarithmic transformations are performed on the data. This transformation shows that the analysis carried out is also related to the return value of world oil prices. The data plot after transformation can be seen in Figure 5. It appears that the variance is more stable and the data has been stationary after the transformation is carried out so that the analysis can be continued.
Furthermore, the appropriate Autoregressive Moving Average (ARMA) model is identified to describe the differentiation result data by looking at the previous ACF / PACF plot. Based on the plot, it can be seen that the ACF / PACF function is significant at the 1st lag and decays towards zero for the next lag. Then an estimate is made of the following alternative models. The significance of the estimated values of the coefficients, the standard error of the coefficients, and the statistical values for diagnostic checks for the observed models are presented in the following table 1.

| ARIMA     | ARIMA     | ARIMA     | ARIMA     | ARIMA     | ARIMA     | ARIMA     | ARIMA     | ARIMA     |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| (1,1,0)   | (2,1,0)   | (0,1,1)   | (0,1,2)   | (1,1,1)   | (2,1,1)   | (1,1,2)   | (2,1,2)   | (5,1,0)   |
| a1        | √         |          |          |          |          |          |           |           |
| a2        | √         | √        |          |          |          |          |           |           |
| a3        |           |          |          |          |          |          |           |           |
| a4        |           |          |          |          |          |          |           |           |
| a5        |           |          |          |          |          |          |           |           |
| b1        |           |          |          |          |          |          | √          |          |
| b2        |           |          |          |          |          |          |           |           |
| RMSE      | 0.1068    | 0.0938   | 0.0831   | 0.0758   | 0.0768   | 0.0756   | 0.0757    | 0.0756    |
| AIC       | -2074.08  | -2404.19 | -2704.5  | -2938.6  | -2901.6  | -2941.7  | -2939     | -2941.5   |
| SBC/BIC   | -2063.78  | -2388.74 | -2694.3  | -2923.2  | -2886.7  | -2921.2  | -2918.41  | -2915.7   |

\( √ \): the ARIMA coefficient is significant  
\( X \): the ARIMA coefficient is non-significant

It is found that the ARIMA model (2,1,1) is the best model with the t-test statistical value more than the t-table statistical value for all coefficients and based on the minimum RMSE, AIC, and BIC values supported by the principle of simplicity of modeling.

**Figure 5. Transformation Result**
From the results of diagnostic tests, it shows that the residual is a white noise process. This can be seen in Figure 7 indicated by the ACF plot, there is no lag (≥ 7) that over the interval limit. and the p-value of the Ljung-Box statistic is above the 5% threshold which means there is no serial correlation to the residuals.

8.5 Heteroscedasticity Checking

It is of interest to see the ARCH / GARCH effect in terms of detecting the homoscedasticity of the data. The step taken is by observing the autocorrelation plot and partial autocorrelation of the residual and residual squared of the data as shown in Figure 8.

In the residual plot, it appears in the data that there is no strong indication of serial correlation from the data (except for several large lags above the limit \( \frac{2a}{\sqrt{n-1}} = \frac{1.96}{\sqrt{1220}} = 0.056 \). Based on the QLjung-
Box statistics in Appendix 16, the null hypothesis regarding no correlation is accepted at the 5% test level.

It can be seen that although the residual data are not correlated, the variance of the residuals shows a correlation. The same thing is also shown from the results of the QLjung-Box test at the 5% test level. Based on the PACF plot from the significant squared residual to lag-2, and the ACF plot that decays towards zero, an alternative model that can be chosen to describe the volatility of the data is the ARCH model (2). However, because of the relatively high order of the ARCH model, at a practical level, the GARCH (p, q) model with the small p and q orders will be used (≤2).

![Plot ACF PACF Residual](image1)

![Plot ACF PACF Residual Squares](image2)

**Figure 8. ARCH/GARCH Effect**

### 8.6 Markov Regime Switching GARCH Estimation

Based on the autocorrelation plot of quadratic residuals containing ARCH / GARCH components, a Markov Regime-Switching GARCH model is used for these residuals. The phenomenon of oil price return data shows a significant difference between state 1 for high volatility and state 2 for low volatility. The Markov regime-switching model is a model can detect these volatility conditions. To estimate the MRS-GARCH model (1,1) the MSwM package in R version 4.0.2 was used. The regime model used is the MRS-GARCH (1,1) model. The estimation results using the MRS-GARCH (1,1) are as follows:
Based on the estimation results of the Markov Regime-Switching GARCH (1,1) in Figure 9, a good model is obtained for forecasting world oil prices. From this model, the return value of oil price and the probability transition matrix is obtained as follows.

$$r_t = \begin{cases} -2.4756, & \text{for state 1} \\ -0.1348, & \text{for state 2} \end{cases}$$

$$P = \begin{bmatrix} p_{00} & p_{11} \\ 1 - p_{00} & 1 - p_{11} \end{bmatrix} = \begin{bmatrix} 0.3697067 & 0.002348256 \\ 0.6302933 & 0.997651744 \end{bmatrix}$$

The transition probability matrix $P$ provides an explanation that the probability of low volatility is followed by a period of low volatility is equal to $p_{00} = 0.3697067$ so that the probability of a period of high volatility followed by a period of low volatility is equal to $1 - p_{00} = 0.6302933$. Furthermore, the occurrence of a probability period of the high volatility followed by a period of high volatility of $p_{11} = 0.002348256$ so that the high probability is followed by a period of low volatility of $1 - p_{11} = 0.997651744$. This means that the world oil price will experience high volatility every $\frac{1}{1 - p_{00}} = \frac{1}{0.3697067} \approx 2.61$ days and oil price will experience low volatility every $\frac{1}{1 - p_{11}} = \frac{1}{0.997651744} \approx 1$ day.

8.7 Volatility Modeling

Although the analysis has shown several good models to be used to describe the data, to obtain comprehensive results it is necessary to compare statistical information criteria such as AIC (Akaike) and BIC (Bayes). The model chosen to be the best model in describing the data is the MRS-GARCH model which is shown from the condition of all significant coefficients and the minimum information criterion value.

| Model Estimasi | AIC     | BIC     |
|----------------|---------|---------|
| GARCH(1,1)     | -3.7039 | -3.6715 |
| EGARCH(1,1)    | -3.7039 | -3.6715 |
| MRS-GARCH      | -4048.764 | -3999.577 |

9. Conclusion

Based on the case study, the best model for forecasting world oil price volatility is the MRS-GARCH model. The results of forecasting the volatility of the oil price using the Markov regime-switching is the best model of several models used in this case study. However, in the volatility forecasting that is carried out, there are still obstacles to forecasting the volatility so that other forecasting models are needed to improve the forecast results.
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