Liouville Cosmology

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Abstract. Liouville string theory is a natural framework for discussing the non-equilibrium evolution of the Universe. It enables non-critical strings to be treated in mathematically consistent manner, in which target time is identified with a world-sheet renormalization-group scale parameter, preserving target-space general coordinate invariance and the existence of an S-matrix. We review our proposals for a unified treatment of inflation and the current acceleration of the Universe. We link the current acceleration of the Universe with the value of the string coupling. In such a scenario, the dilaton plays an essential background rôle, driving the acceleration of the Universe during the present era after decoupling as a constant during inflation.

1 Issues in String Cosmology

Formal developments in string theory [1] over the past decade [2], with the discovery of a consistent way of studying quantum domain-wall structures (D-branes), have opened up novel ways of looking at not only the microcosmos, but also the macrocosmos.

In the microcosmos, there are novel ways of compactification, either via the observation [3] that extra dimensions that are large compared to the string scale [3] are consistent with the foundations of string theory, or by viewing our four-dimensional world as a brane embedded in a higher-dimensional bulk spacetime, whose extra dimensions might even be infinite in size [4]. Such ideas are still consistent with the required large hierarchy between the Planck scale and the electroweak- or supersymmetry-breaking scale. In this modern approach, fields in the gravitational (super)multiplet of the (super)string are allowed to propagate in the bulk, but not the gauge fields, which are attached to the brane world. In this way, the weakness of gravity compared to the rest of the interactions is a result of the large compact dimensions, and the compactification is not necessarily achieved through the conventional means of closing the extra dimensions up on spatial compact manifolds, but could also involve shadow brane...
worlds with special reflecting properties, such as orientifolds, which restrict the bulk dimension [5]. In such approaches, the string scale $M_s$ is not necessarily identical to the four-dimensional Planck mass $M_P$. Instead, they are related by

$$M_P^2 = \frac{8M_s^2V_6}{g_s^2}. \quad (1)$$

through the large compactification volume $V_6$.

In the macrocosmos, this modern approach has offered new insights into the evolution of our Universe. Novel ways of discussing cosmology in brane worlds have been discovered over the past five years, which may revolutionise our way of approaching issues such as inflation [6,7]. On the other hand, mounting experimental evidence from diverse astrophysical sources presents some important issues that string theory must address if it is to provide a realistic description of Nature. Observations of distant supernovae [8] and the cosmic microwave background fluctuations, e.g., by the WMAP satellite [9], indicate that the expansion of our Universe is currently accelerating, and that 73% of its energy density consists of some unknown Dark Energy.

This cosmological development may be quite significant for string theory, requiring that we revolutionise the approach usually followed so far. If the dark energy turns out to be an honest-to-God cosmological constant, leading to an asymptotic de Sitter horizon, then the entire concept of the scattering matrix, the basis of perturbative string theory, breaks down. This would cast doubts on the very foundations of string theory, at least in its familiar formulation [10]. Alternatively, one might invoke some quintessential model for the vacuum energy, in which the vacuum energy relaxes to zero at large cosmic time. This may be consistent with the existence of an S-matrix as well as with the astrophysical data, but there is still the open issue of embedding such models in (perturbative) string theory. In particular, one must develop a consistent $\sigma$-model formulation of strings propagating in such relaxing, time-dependent space-time backgrounds.

The world-sheet conformal-invariance conditions of critical string theory, which are equivalent to the target-space equations of motion for the background fields on which the string propagates, are very restrictive, corresponding only to vacuum solutions of critical strings. The main problem may be expressed as follows. Consider the graviton world-sheet $\beta$ function, which is nothing but the Ricci tensor of the target space-time background to lowest order in $\alpha'$:

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu}, \quad (2)$$

in the absence of other fields. Conformal invariance requires the vanishing of $\beta_{\mu\nu}$, which implies that the background must be Ricci-flat, a characteristic of solutions to the vacuum Einstein equations.

The issue then arises how to describe cosmological string backgrounds, which are not vacuum solutions, but require the presence of a matter fluid, yielding a non-flat Ricci tensor. Specifically, a vacuum solution with a cosmological constant is inconsistent with conformal invariance: a de Sitter Universe with a positive cosmological constant $\Lambda > 0$ has a Ricci tensor $R_{\mu\nu} = \Lambda g_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor.
We now discuss how these issues may be addressed in Liouville Cosmology.

2 Universes in Dilaton Backgrounds

A proposal for obtaining a non-zero cosmological constant in string theory was made in [11], according to which dilaton tadpoles in higher-genus world-sheet surfaces produce additional modular infinities. Their regularisation would lead to extra world-sheet structures in the \( \sigma \) model that do not appear at the world-sheet tree level, leading to modifications of the \( \beta \)-function. As a result, the Ricci tensor of the space-time background is now that of a (anti) de Sitter Universe, with a cosmological constant given by the dilaton tadpole graph \( J_D > 0 \) (\( J_D < 0 \)). One problem with this approach is the above-mentioned existence of an asymptotic horizon in the de Sitter case, which prevents the proper definition of asymptotic states, and hence a scattering S-matrix. However, since the perturbative world-sheet formalism is based on such an S-matrix, one may question the fundamental consistency of this approach.

A way out of this dilemma was proposed in [12], namely a dilaton background depending linearly on time in the so-called \( \sigma \)-model frame. Such a background, even when the \( \sigma \)-model metric is flat, leads to solutions of the conformal invariance conditions of the pertinent stringy \( \sigma \)-model that are exact to all orders in \( \alpha' \), thereby constituting acceptable solutions from a perturbative viewpoint. It was argued in [12] that such backgrounds describe linearly-expanding Robertson-Walker (RW) Universes, which were shown to be exact conformal-invariant solutions, corresponding to Wess-Zumino models of appropriate group manifolds. The important novelty in [12] was the identification of target time \( t \) with a specific dilaton background:

\[
\Phi = \text{const} - Q t, \tag{3}
\]

where \( Q \) is a constant.

The square of \( Q \) is the \( \sigma \)-model central charge deficit in a supercritical string theory [12]. Consistency of the underlying world-sheet conformal field theory, as well as modular invariance of the string scattering amplitudes, led to discrete values of \( Q^2 \), when expressed in units of the string length \( M_s \). This was actually the first example of a non-critical string, with the target-space coordinates \( X^i, i = 1, \ldots , D - 1 \) playing the rôles of the pertinent \( \sigma \)-model fields. This non-criticality broke conformal invariance, which was compensated by Liouville dressing [13]. The required Liouville field had time-like signature, since the central-charge deficit was positive: \( Q^2 > 0 \) in the model of [12], and played the rôle of target time.

In the presence of a non-trivial dilaton field, the Einstein term in the effective \( D \)-dimensional low-energy field theory action is conformally rescaled by \( e^{-2\Phi} \). This requires a redefinition of the \( \sigma \)-model-frame space-time metric \( g^{\sigma}_{\mu\nu} \) to become the physical metric in the ‘Einstein frame’, \( g^{E}_{\mu\nu} \):

\[
g^{E}_{\mu\nu} = e^{\frac{2\Phi}{D-2}} g^{\sigma}_{\mu\nu}. \tag{4}
\]
A redefinition of target time is also necessary to obtain the standard RW form of the metric in the Einstein frame:

\[ ds^2_E = -dt_E^2 + a_E^2(t_E) \left( dr^2 + r^2 d\Omega^2 \right), \tag{5} \]

where we assume for definiteness a spatially-flat RW metric. Here \( a_E \) is an appropriate scale factor, which is a function of \( t_E \) alone in the homogeneous cosmological backgrounds we assume throughout. Time in the Einstein-frame is related to time in the \( \sigma \)-model frame \([12]\) by:

\[ dt_E = e^{-\Phi} dt \quad \rightarrow \quad t_E = \int e^{-\Phi(t)} dt. \tag{6} \]

The linear dilaton background (3) therefore yields the following relation between the Einstein and \( \sigma \)-model time variables:

\[ t_E = c_1 + c_0 Q e^{Q t}, \tag{7} \]

where \( c_{1,0} \) are appropriate (positive) constants.

Thus, the dilaton background (3) scales logarithmically with the Einstein-Robertson-Walker cosmic time \( t_E \):

\[ \Phi(t_E) = (\text{const.}') - \ln\left( \frac{Q}{c_0} t_E \right) \tag{8} \]

In this regime, the string coupling, which is defined as [1]:

\[ g_s = \exp(\Phi(t)) \tag{9} \]

varies with the cosmic time \( t_E \) as

\[ g_s^2(t_E) \equiv e^{2\Phi} \propto \frac{1}{t_E^2}. \tag{10} \]

Thus, the effective string coupling vanishes asymptotically in cosmic time.

The effective low-energy action for the gravitational multiplet of the string in the Einstein frame reads \([12]\):

\[ S_{\text{brane}}^{\text{eff}} = \int d^4x \sqrt{-g} \{ R - 2(\partial_\mu \Phi)^2 - \frac{1}{2} e^{4\Phi} (\partial_\mu b)^2 - \frac{1}{3} e^{2\Phi} \delta c \}, \tag{11} \]

where \( b \) is the four-dimensional axion field associated with the antisymmetric tensor and \( \delta c = C_{\text{int}} - c^* \) is the central charge deficit, where \( C_{\text{int}} \) is the central charge of the conformal world-sheet theory corresponding to the transverse (internal) string dimensions, and \( c^* = 22(6) \) is the critical value of this internal central charge of the (super)string theory in a flat four-dimensional space-time. The linear-dilaton configuration (8) corresponds, in this language, to a background charge \( Q \) of the conformal theory, which contributes a term
−3Q^2 (in our normalisation (8)) to the total central charge, which also receives contributions from the four uncompactified dimensions of our world. In the case of a flat four-dimensional Minkowski space-time, one has: \( C_{\text{total}} = 4 - 3Q^2 + C_{\text{int}} = 4 - 3Q^2 + c^* + \delta c \), which should equal 26. This implies that \( C_{\text{int}} = 22 + 3Q^2 (6 + 3Q^2) \) for bosonic (supersymmetric) strings.

An important result in [12] was the discovery of an exact conformal field theory corresponding to the dilaton background (8), i.e., a constant curvature (Milne) (static) metric in the \( \sigma \)-model frame or, equivalently, a linearly-expanding RW Universe in the Einstein frame. This conformal field theory corresponds to a two-dimensional Wess-Zumino-Witten (WZW) model on the world sheet, on a group manifold \( O(3) \) with appropriate constant curvature, whose coordinates correspond to the spatial components of the four-dimensional metric and antisymmetric tensor fields, together with a free world-sheet field corresponding to the target time coordinate. The total central charge in this more general case reads \( C_{\text{total}} = 4 - 3Q^2 - \frac{6k}{k+2} + C_{\text{int}} \) with \( k \) a positive integer, which corresponds to the level of the Kac-Moody algebra associated with the WZW model on the group manifold. The value of \( Q \) is chosen in such a way that the overall central charge \( c = 26 \) and the theory is conformally invariant.

It was observed in [12] that known unitary conformal field theories have discrete central charges, which accumulate to integers or half-integers from below, and hence that the values of the central charge deficit \( \delta c \) are also discrete. From a physical point of view, this implies that the linear-dilaton Universe may either stay at such a state for ever with a fixed \( \delta c \), or else tunnel through different discrete levels before relaxing to a critical \( \delta c = 0 \) theory corresponding to a flat four-dimensional Minkowski space-time.

The analysis in [12] also showed that there were tachyonic mass-squared shifts of order \( -Q^2 \) for the bosonic string excitations, but not for the fermionic ones. This in turn would imply the breaking of target supersymmetry in such backgrounds, as far as the excitation spectrum is concerned, and the appearance of tachyonic instabilities. The latter could trigger these cosmological phase transitions, since they correspond to world-sheet operators that are relevant in the renormalization-group sense. As such, they can trigger the flow of the internal unitary conformal field theory towards minimisation of its central charge, according to the Zamolodchikov \( c \) theorem [14]. As we discuss below, in semi-realistic cosmological models [15], such tachyons decouple from the spectrum relatively quickly. On the other hand, as a result of the form of the dilaton in the Einstein frame (8), we observe that the dark-energy density for this Universe, \( \Lambda \equiv e^{2\Phi} \delta c \), relaxes towards zero as \( 1/t^2 \), for each of the stationary values of \( \delta c \). The breaking of supersymmetry induced by the linear dilaton should therefore be considered an obstruction [16], rather than a spontaneous breaking, in the sense of appearing only in the boson-fermion mass splittings between the excitations, whereas the vacuum energy of the asymptotic equilibrium theory vanishes.

In [17] we went one step further than [12], considering more complicated \( \sigma \)-model metric backgrounds in \( (D + 1) \)-dimensional target space-times, that did not satisfy the \( \sigma \)-model conformal-invariance conditions, and therefore were
in need of Liouville dressing [13]. These backgrounds were even allowed to be
time-dependent. Non-criticality can be introduced in many mathematically con-
sistent ways, for instance via cosmically catastrophic events such as the collision
of brane worlds [18,19], which lead naturally to supercritical $\sigma$ models. The Li-
ouville dressing of such non-critical models results in $D + 2$-dimensional target
spaces with two time directions. The important point of [17] was the identifi-
cation of the world-sheet zero mode of the Liouville field with the target time,
thereby restricting the Liouville-dressed $\sigma$ model to a $(D + 1)$-dimensional hy-
persurface of the $(D + 2)$-dimensional target space-time, maintaining the initial
target space-time dimensionality. We stress once more that this identification
is only possible in cases where the initial $\sigma$ model is supercritical, so that the
Liouville mode has time-like signature [12,13]. Such an identification was shown
in certain models [18,19] to be energetically preferable from a target-space view-
point, since it minimised certain effective potentials in the low-energy field theory
.corresponding to the string theory at hand.

Such non-critical $\sigma$-models relax asymptotically in the cosmic Liouville time
to conformal $\sigma$ models, which may be viewed as equilibrium points in string
theory space. In some interesting cases of relevance to cosmology [15], which
were particularly generic, the asymptotic conformal field theory was that of [12]
with a linear dilaton and a flat Minkowski target-space metric in the $\sigma$-model
frame. In others, the asymptotic theory was characterised by a constant dilaton
and a Minkowski space-time [18]. In what follows, we describe briefly the main
features of such non-critical cosmological string models, and compare them with
recent observations.

3 Non-Critical Liouville String Cosmologies

We now consider in more detail the model of [15]. Although formulated in the
specific framework of ten-dimensional Type-0 [20] string theory. This has a non-
supersymmetric target-space spectrum, thanks to a special projection of the su-
persymmetric partners out of the spectrum. Nevertheless, its basic cosmological
properties are sufficiently generic to be extended to the bosonic sector of any ef-
fective low-energy supersymmetric field theory obtained from a supersymmetric
string model.

The ten-dimensional metric configuration considered in [15] was:

$$G_{MN} = \begin{pmatrix}
g^{(4)}_{\mu \nu} & 0 & 0 \\
0 & e^{2\sigma_1} & 0 \\
0 & 0 & e^{2\sigma_2}I_{5 \times 5}
\end{pmatrix},$$  

(12)

where lower-case Greek indices are four-dimensional space-time indices, and $I_{5 \times 5}$
denotes the $5 \times 5$ unit matrix. We have chosen two different scales for internal
space. The field $\sigma_1$ sets the scale of the fifth dimension, while $\sigma_2$ parametrises
a flat five-dimensional space. In the context of the cosmological models we deal
with here, the fields $g^{(4)}_{\mu \nu}$, $\sigma_i$, $i = 1, 2$ are assumed to depend only on the time $t$. 

Type-0 string theory, as well as its supersymmetric extensions that appear, e.g., in brane models, contains form fields with non-trivial gauge fluxes (flux-form fields), which live in the higher-dimensional bulk space. In the specific model of [20], there is one such field that was assumed to be non-trivial. As was demonstrated in [15], a consistent background choice for the flux-form field has a flux parallel to the fifth dimension $\sigma_2$. This implies that the internal space is crystallised (stabilised) in such a way that this dimension is much larger than the remaining four, demonstrating the physical importance of the flux fields for large radii of compactification.

Considering the fields to be time-dependent only, i.e., considering spherically-symmetric homogeneous backgrounds, restricting attention to the compactification (12), and assuming a RW form of the four-dimensional metric, with scale factor $a(t)$, the generalised conformal invariance conditions and the Curci-Pafutti $\sigma$-model renormalisability constraint [21] imply a set of differential equations which were solved numerically in [15]. The generic form of these equations reads [13,17,15]:

$$\ddot{g}^i + Q(t)\dot{g}^i = -\tilde{\beta}^i,$$

(13)

where the $\tilde{\beta}^i$ are the Weyl anomaly coefficients of the stringy $\sigma$ model on the background $\{g^i\}$. In the model of [15], the set of $\{g^i\}$ includes the graviton, dilaton, tachyon, flux and moduli fields $\sigma_{1,2}$, whose vacuum expectation values control the size of the extra dimensions.

The detailed analysis of [15] indicated that the moduli $\sigma_i$ fields froze quickly to their equilibrium values. Thus, they together with the tachyon field, which also decays rapidly to a constant value, decouple from the four-dimensional fields at very early stages in the evolution of this string Universe 1. There is an inflationary phase in this scenario and a dynamical exit from it. The important point that guarantees the exit is the fact that the central-charge deficit $Q^2$ is a time-dependent entity in this approach, obeying specific relaxation laws determined by the underlying conformal field theory [15,18,19]. The central charge runs with the local world-sheet renormalisation-group scale, namely the zero mode of the Liouville field, which is identified [17] with the target time in the $\sigma$-model frame. The supercriticality [12] $Q^2 > 0$ of the underlying $\sigma$ model is crucial, as already mentioned. Physically, the non-critical string provides a model of non-equilibrium dynamics, which may be the result of some catastrophic cosmic event, such as a collision of two brane worlds [7,18,19], or an initial quantum fluctuation [22,15]. It also provides, as we now discuss briefly, a unified mathematical framework for analysing various phases of string cosmology, including the inflationary phase in the early Universe, graceful exit from it and reheating, as well as the current and future eras of accelerated cosmologies. It is interesting that one can constrain string parameters such as string coupling, the separation

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1 The presence of the tachyonic instability in the spectrum is due to the fact that in Type-0 strings there is no target-space supersymmetry by construction. From a cosmological viewpoint, the tachyon fields are not necessarily bad features, since they may provide the initial instability leading to cosmic expansion [15], as well as a mechanism for step-wise reduction in the central-charge deficit.
of brany worlds at the end of inflation and the recoil velocity of the branes after the collision, by fits to current astrophysical data [19].

4 Liouville Inflation: the Big Picture

As discussed in [22,18,19], a constant central-charge deficit $Q^2$ in a stringy $\sigma$ model may be associated with an initial inflationary phase, with

$$Q^2 = 9H^2 > 0,$$  \hspace{1cm} (14)

where the Hubble parameter $H$ may be fixed in terms of other parameters of the model. One can consider various scenarios for such a departure from criticality. For example, in the specific colliding-brane model of [18,19], $Q$ (and thus $H$) is proportional to the square of the relative velocity of the colliding branes, $Q \propto u^2$ during the inflationary era. As is evident from (14) and discussed in more detail below, in a phase of constant $Q$ one obtains an inflationary de Sitter Universe.

The specific normalization in (14) is imposed by the identification of the time $t$ with (minus) the zero mode of the Liouville field $-\varphi$ of the supercritical $\sigma$ model. The minus sign may be understood both mathematically, as due to properties of the Liouville mode, and physically by the requirement that the deformation of the space-time relaxes following the distortion induced by the recoil. With this identification, the general equation of motion for the couplings $\{g_i\}$ of the $\sigma$-model background modes is [17]:

$$\ddot{g}_i + Q \dot{g}_i = -\beta^i(g) = -G^{ij} \partial C[g]/\partial g^j,$$  \hspace{1cm} (15)

where the dot denotes a derivative with respect to the Liouville world-sheet zero mode $\varphi$, and $G^{ij}$ is an inverse Zamolodchikov metric in the space of string theory couplings $\{g^i\}$ [14]. When applied to scalar inflaton-like string modes, (15) would yield standard field equations for scalar fields in de Sitter (inflationary) space-times, provided the normalization (14) is valid, implying a ‘Hubble’ expansion parameter $H = -Q/3$ \footnote{The gradient-flow property of the $\beta$ functions makes the analogy with the inflationary case even more profound, with the running central charge $C[g]$ [14] playing the rôle of the inflaton potential in conventional inflationary field theory.}. The minus sign in $Q = -3H$ is due to the sign in the identification of the target time $t$ with the world-sheet zero mode of $-\varphi$ [17].

The relations (15) generalize and replace the conformal-invariance conditions $\beta^i = 0$ of the critical string theory, and express the conditions necessary for the restoration of conformal invariance by the Liouville mode [13]. Interpreting the latter as an extra target dimension, the conditions (15) may also be viewed as conformal invariance conditions of a critical $\sigma$ model in $(D+1)$ target space-time dimensions, where $D$ is the target dimension of the non-critical $\sigma$ model before Liouville dressing. In most Liouville approaches, one treats the Liouville mode $\varphi$ and time $t$ as independent coordinates. However, in our approach [17,15,18], as already mentioned, we take the further step of restricting this extended $(D+1)$-dimensional space-time to a hypersurface determined by the identification $\varphi = -\varphi$.}
This means that, as time flows, one is restricted to a D-dimensional subspace of the full (D+1)-dimensional Liouville space-time. This restriction arose in the work of [18,19] because the potential between massive particles in the effective field theory was found to be proportional to \( \cosh(t + \varphi) \), which is minimized when \( \varphi = -t \).

However, the flow of the Liouville mode opposite to that of target time may be given a deeper mathematical interpretation. It may be viewed as a consequence of a specific treatment of the area constraint in non-critical (Liouville) \( \sigma \) models [17], which involves the evaluation of the Liouville-mode path integral via an appropriate steepest-descent contour. In this way, one obtains a ‘breathing’ world-sheet evolution, in which the world-sheet area starts from a very large value (serving as an infrared cutoff), shrinks to a very small one (serving as an ultraviolet cutoff), and then inflates again towards very large values (returning to an infrared cutoff). Such a situation may then be interpreted [17] as a world-sheet ‘bounce’ back to the infrared, implying that the physical flow of target time is opposite to that of the world-sheet scale (Liouville zero mode).

We now become more specific. We consider a non-critical \( \sigma \) model with a background metric \( G_{\mu \nu} \), antisymmetric tensor \( B_{\mu \nu} \), and dilaton \( \Phi \). These have the following \( O(\alpha') \) \( \beta \) functions, where \( \alpha' \) is the Regge slope [1]:

\[
\begin{align*}
\beta^G_{\mu \nu} &= \alpha' \left( R_{\mu \nu} + 2 \nabla_\mu \partial_\nu \Phi - \frac{1}{4} H_{\mu \rho \sigma} H^{\rho \sigma} \right) , \\
\beta^B_{\mu \nu} &= \alpha' \left( -\frac{1}{2} \nabla_\rho H^{\rho \mu \nu} + H^{\rho \mu \nu} \partial_\rho \Phi \right) , \\
\tilde{\beta}^\Phi &= \beta^\Phi - \frac{1}{4} G^{\rho \sigma} \beta^G_{\rho \sigma} = \frac{1}{6} (C - 26) .
\end{align*}
\]

(16)

The Greek indices are four-dimensional, including target-time components \( \mu, \nu, \ldots = 0, 1, 2, 3 \) on the D3-branes of [18], and \( H_{\mu \nu \rho} = \partial_{[\mu} B_{\nu \rho]} \) is the antisymmetric tensor field strength. We consider the following representation of the four-dimensional field strength in terms of a pseudoscalar (axion-like) field \( b \):

\[
H_{\mu \nu \rho} = \epsilon_{\mu \nu \rho \sigma} \partial^\sigma b ,
\]

(17)

where \( \epsilon_{\mu \nu \rho \sigma} \) is the four-dimensional antisymmetric symbol. Next, we choose an axion background that is linear in the time \( t \) [12]:

\[
b(t) = \beta t , \quad \beta = \text{constant},
\]

(18)

which yields a constant field strength with spatial indices only: \( H_{ijk} = \epsilon_{ijk} \beta \), \( H_{0jk} = 0 \). This implies that such a background is a conformal solution of the full \( O(\alpha') \) \( \beta \) function for the four-dimensional antisymmetric tensor. We also consider a dilaton background that is linear in the time \( t \) [12]:

\[
\Phi(t, X) = \text{const} + (\text{const})' t .
\]

(19)

This background does not contribute to the \( \beta \) functions for the antisymmetric and metric tensors.
Suppose now that only the metric is a non-conformal background, due to some initial quantum fluctuation or catastrophic event, such as the collision of two branes discussed above, which results in an initial central charge deficit $Q^2$ (14) that is constant at early stages after the collision. Let

$$G_{ij} = e^{\kappa \varphi + Hct} \eta_{ij}, \quad G_{00} = e^{\kappa' \varphi + Hct} \eta_{00},$$

where $t$ is the target time, $\varphi$ is the Liouville mode, $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric, and $\kappa, \kappa'$ and $c$ are constants to be determined. As already discussed, the standard inflationary scenario in four-dimensional physics requires $Q = -3H$, which stems from the identification of the Liouville mode with time $\varphi = -t$, that is imposed dynamically [18] at the end of our computations. Initially, one should treat $\varphi, t$ as independent target-space components.

The Liouville dressing induces [13] $\sigma$-model terms of the form

$$\int_{\Sigma} R^{(2)} Q \varphi,$$

where $R^{(2)}$ is the world-sheet curvature. Such terms provide non-trivial contributions to the dilaton background in the (D+1)-dimensional space-time $(\varphi, t, X^i)$:

$$\Phi(\varphi, t, X^i) = Q \varphi + (\text{const})' t + \text{const}. \quad (21)$$

If we choose

$$(\text{const})' = Q, \quad (22)$$

we see that (21) implies a constant dilaton background during the inflationary era, in which the central charge deficit $Q$ is constant.

The choices (21) and (22), like the identification $\varphi = -t$, apply to the world-sheet zero modes of the Liouville field and the time coordinate. As such, they imply a constant dilaton at the mean-field (classical) level. World-sheet quantum fluctuations of the dilaton, associated with non-zero modes of these fields, do not cancel, since the identification $\varphi = -t$ is not valid for the fluctuating parts of the respective $\sigma$-model fields. This leads in turn to non-trivial fluctuations of the dilaton field during inflation. The summation over world-sheet genera turns such fluctuations into target-space quantum fluctuations. This allows one [19] to apply the phenomenology of scalar field fluctuations used in conventional inflationary models also in this case, in order to constrain physically important parameters of the non-critical string theory by means of recent cosmological data [9].

We now consider the Liouville-dressing equations [13] (15) for the $\beta$ functions of the metric and antisymmetric tensor fields (16) at the level of world-sheet zero modes of the $\sigma$-model fields. The dilaton equation yields no independent information for a constant mean dilaton field, apart from expressing the dilaton $\beta$ function in terms of the central-charge deficit as usual. For the axion background (18), only the metric yields a non-trivial constraint (we work in units with $\alpha' = 1$ for convenience):

$$\ddot{G}_{ij} + Q \dot{G}_{ij} = -R_{ij} + \frac{1}{2} \beta^2 G_{ij}, \quad (23)$$

where the dot indicates differentiation with respect to the (zero mode of the) world-sheet Liouville mode $\varphi$, and $R_{ij}$ is the (non-vanishing) Ricci tensor of the
(non-critical) $\sigma$ model with coordinates $(t, x)$: $R_{00} = 0$, $R_{ij} = \frac{c^2 H^2}{2} e^{(\kappa - \kappa')}\phi \eta_{ij}$.

One should also take into account the temporal equation for the metric tensor:

$$\ddot{G}_{00} + Q\dot{G}_{00} = -R_{00} = 0,$$

(24)

where the vanishing of the Ricci tensor stems from the specific form of the background (20). The analogue equation is identically zero for the antisymmetric tensor background. We seek metric backgrounds of inflationary (de Sitter) RW form:

$$G_{00} = -1, \quad G_{ij} = e^{2Ht}\eta_{ij},$$

(25)

Then, from (25), (20), (19) and (18), we observe that there indeed is a consistent solution with:

$$Q = -3H = -\kappa', \quad c = 3, \quad \kappa = H, \quad \beta^2 = 5H^2,$$

(26)

corresponding to the conventional form of inflationary equations for scalar fields.

In this talk we do not mention ways of exiting from this inflationary phase and reheating the Universe. These issues may also be approached from a Liouville $\sigma$-model point of view, as we shall report in a forthcoming publication [23].

5 Liouville Dark Energy: the End Game

In the generic class of non-critical string models described in this talk, the $\sigma$ model always asymptotes, for long enough cosmic times, to the linear-dilaton conformal $\sigma$-model field theory of [12]. However, it is important to stress that this is only an asymptotic limit, and the current era of our Universe should be viewed as close to, but still not at the equilibrium relaxation point. Thus the dilaton is almost linear in the $\sigma$-model time, and hence varies almost logarithmically in the Einstein-frame time (8). This slight non-equilibrium would lead to a time dependence of the unified gauge coupling and other constants such as the four-dimensional Planck length (1), mainly through the time-dependence of the string coupling (9) that results from the time dependence of the linear dilaton (3).

The asymptotic-time regime of the Type-0 cosmological string model of [15] has been obtained analytically, by solving the pertinent equations (13) for the various fields. As already mentioned, at later times the theory becomes four-dimensional, and the only non-trivial information is contained in the scale factor and the dilaton, given that the topological flux field remains conformal in this approach, and the moduli and initial tachyon fields decouple very fast at the initial stages after inflation in this model. For times long after the initial fluctuations, such as the present times where the linear approximation is valid, the solution for the dilaton in the $\sigma$-model frame, as follows from the equations (13), takes the form:

$$\Phi(t) = -\ln \left[ \frac{\alpha A}{F_1} \cosh(F_1 t) \right],$$

(27)
where \( F_1 \) is a positive constant, \( \alpha \) is a numerical constant of order one, and
\[
A = \frac{C_5 e^{s_{01}}}{\sqrt{2V_6}}, \tag{28}
\]
with \( s_{01} \) the equilibrium value of the modulus field \( \sigma_1 \) associated with the large bulk dimension, and \( C_5 \) the corresponding flux of the five-form flux field. Notice the independence of \( A \) from this large bulk dimension.

For very large times \( F_1 t \gg 1 \) (in string units), one therefore approaches a linear solution for the dilaton: \( \Phi \sim \text{const} - F_1 t \). From (27), (9) and (1), we thus observe that the asymptotic weakness of gravity in this Universe [15] is due to the smallness of the internal space \( V_6 \) as compared with the flux \( C_5 \) of the five-form field. The constant \( F_1 \) is related to the central-charge deficit of the underlying non-conformal \( \sigma \)-model [15]:
\[
Q = q_0 + \frac{q_0}{F_1} (F_1 + \frac{d\Phi}{dt}), \tag{29}
\]
where \( q_0 \) is a constant, and the numerical solution of (13) studied in [15]) requires that \( q_0/F_1 = (1 + \sqrt{17})/2 \approx 2.56 \). However, we believe that this is only a result of the numerical approximations in the analysis of [15], and for our purposes we consider from now on
\[
F_1 \sim q_0, \tag{30}
\]
in accord with [12], to which the model relaxes for large times. In this spirit, we require that the value of \( q_0 \) to which the central charge deficit (29) asymptotes must be, for the consistency of the underlying string theory, one of the discrete values obtained in [12], for which the string scattering amplitudes factorise. This asymptotic string theory with a time-independent central-charge deficit, \( q_0^2 \propto c^* - 25 \) (or \( c^* - 9 \) for superstring) may therefore be considered as an equilibrium situation, with an S-matrix defined for specific (discrete) values of the central charge \( c^* \), generalizing the standard critical (super)string which corresponds to central charge \( c^* = 25 \) (=9 for superstrings) [13,12].

Defining the Einstein frame time \( t_E \) through (6), we obtain in this case (27)
\[
t_E = \frac{\alpha A}{F_1^2} \sinh(F_1 t). \tag{31}
\]
In terms of the Einstein-frame time one obtains a logarithmic time-dependence [12] for the dilaton
\[
\Phi_E = \text{const} - \ln(\gamma t_E), \tag{32}
\]
where
\[
\gamma \equiv \frac{F_1^2}{\alpha A}. \tag{33}
\]
For large \( t_E \), e.g., now or in the future, one has
\[
a_E(t_E) \simeq \frac{F_1}{\gamma} \sqrt{1 + \gamma^2 t_E^2}. \tag{34}
\]
At very large times \( a(t_E) \) scales linearly with the Einstein-frame cosmological time \( t_E \) [15], and hence there is no cosmic horizon. From a field-theoretical viewpoint, this would allow for a proper definition of asymptotic states and thus a scattering matrix. As we mentioned briefly above, however, from a stringy point of view, there are restrictions in the asymptotic values of the central charge deficit \( q_0 \), and there is only a discrete spectrum of values of \( q_0 \) that allow for a full stringy S-matrix to be defined, respecting modular invariance [12]. Asymptotically, the Universe relaxes to its ground-state equilibrium situation, and the non-criticality of the string caused by the initial fluctuation disappears, giving rise to a critical (equilibrium) string Universe with a Minkowski metric and a linear-dilaton background. This is a generic feature of the models considered here and in [24], allowing the conclusions to be extended beyond Type-0 string theory to incorporate also string/brane models with target-space supersymmetry, such as those in [25,19].

An important comment is in order at this point, regarding the form of the Einstein metric corresponding to (34):

\[
g^E_{00} = -1, \quad g_{ij} = a^2_E(t_E) = \frac{F^2_1}{\gamma^2} + \frac{F^2_1}{\gamma^2} t^2_E. \tag{35}
\]

Although asymptotically for \( t_E \to \infty \) the above metric asymptotes to the linearly-expanding Universe of [12], the presence of a constant \( F^2_1/\gamma^2 \) contribution implies that the solution for large but finite \( t_E \), such as the current era of the Universe, is different from that of [12]. Indeed, the corresponding \( \sigma \)-model metric (4) is not Minkowski-flat, and the pertinent \( \sigma \) model does not correspond to a conformal field theory. This should come as no surprise, because for finite \( t_E \), no matter how large, the \( \sigma \)-model theory requires Liouville dressing. It is only at the end-point of the time-flow: \( t_E \to \infty \) that the underlying string theory becomes conformal, and the system reaches equilibrium.

The Hubble parameter of such a Universe becomes for large \( t_E \)

\[
H(t_E) \simeq \frac{\gamma^2 t_E}{1 + \gamma^2 t^2_E}. \tag{36}
\]

On the other hand, the Einstein-frame effective four-dimensional ‘vacuum energy density’, defined through the running central-charge deficit \( Q^2 \), upon compactification to four dimensions of the ten-dimensional expression \( \int d^{10} x \sqrt{-g} e^{-2\Phi} Q^2(t_E) \), is [15]:

\[
\Lambda_E(t_E) = e^{2\Phi - \sigma_1 - 5\sigma_2} Q^2(t_E) \simeq \frac{q_0^2 \gamma^2}{F^2_1(1 + \gamma^2 t^2_E)} \sim \frac{\gamma^2}{1 + \gamma^2 t^2_E}, \tag{37}
\]

where, for large \( t_E \), \( Q \) is given in (29) and approaches its equilibrium value \( q_0 \), and we took into account (30). Thus, the dark energy density relaxes to zero for \( t_E \to \infty \). Notice an important feature of the form of the relaxation (37), namely that the proportionality constants in front are such that, for asymptotically large \( t_E \to \infty \), \( \Lambda \) becomes independent of the equilibrium conformal field theory central charge \( q_0 \).
Finally, and most importantly for our purposes here, the deceleration parameter in the same regime of $t_E$ becomes:

$$q(t_E) = -\left(\frac{d^2 a_E}{dt_E^2}\right) \frac{a_E}{(da_E/dt_E)^2} \simeq -\frac{1}{\gamma^2 t_E^2}. \tag{38}$$

The important point to make in connection with this expression is that, as is clear from (32) and (9), it can be identified, up to irrelevant constants of proportionality which conventional normalisation sets to one, with the square of the string coupling [24]:

$$|q(t_E)| = g_s^2 \tag{39}$$

To guarantee the consistency of perturbation theory, one must have $g_s < 1$, which can be achieved in our approach if one defines the present era by the time regime

$$\gamma \sim t_E^{-1} \tag{40}$$

in the Einstein frame. This is compatible with large enough times $t_E$ (in string units) for

$$|C_5|e^{-5s_{02}}/F_1^2 \sim |C_5|e^{-5s_{02}}/q_0^2 \gg 1, \tag{41}$$

as becomes clear from (28),(33) and (30). This condition can be guaranteed either by small radii of five of the extra dimensions, or by a large value of the flux $|C_5|$ of the five-form of the Type-0 string, compared with $q_0$. We discuss later concrete examples of non-critical string cosmologies, in which the asymptotic value of the central charge $q_0 \ll 1$ in string units. Recalling that the relatively large extra dimension in the direction of the flux, $s_{01}$, decouples from this condition, we observe that there is the possibility of constructing effective five-dimensional models with a large uncompactified fifth dimension while respecting the condition (40).

The Hubble parameter and the cosmological constant continue to be compatible with the current observations in the regime (40) of Einstein-frame times, while the string coupling (39) is kept finite and of order unity by the conditions (38, 40), as suggested by grand unification phenomenology [1].

6 Dark Energy and the String Coupling

We next turn to the equation of state of our Universe. As discussed in [15], our model resembles quintessence models, with the dilaton playing the role of the quintessence field. Hence the equation of state for our Type-0 string Universe reads [26]:

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{1}{2}(\dot{\Phi})^2 - V(\Phi) \tag{42}$$

where $p_\Phi$ is the pressure and $\rho_\Phi$ is the energy density, and $V(\Phi)$ is the effective potential for the dilaton, which in our case is provided by the central-charge deficit term. Here the dot denotes Einstein-frame differentiation. In the Einstein
frame, the potential $V(\Phi)$ is given by $\Lambda E$ in (50). In the limit $Q \rightarrow q_0$, which we have argued should characterise the present era to a good approximation, the effective potential $V(\Phi)$ is then of order $(q_0^2/2F_1^2)t_E^{-2}$, where we recall (c.f., (30)) that $q_0/F_1$ is of order one. In the Einstein frame the exact normalisation of the dilaton field is $\Phi_E = \text{const} - \ln(\gamma t_E)$. We then obtain for the present era:

$$\frac{1}{2} \dot{\Phi}^2 \sim \frac{1}{2t_E^2}, \quad V(\Phi) \sim \frac{6.56}{2} \frac{1}{t_E^2}. \quad (43)$$

This implies an equation of state (42):

$$w_\Phi(t_E \gg 1) \simeq -0.74 \quad (44)$$

for (large) times $t_E$ in string units corresponding to the present era (40). This number can be pushed lower, towards $w \rightarrow -1$, by a slight adjustment of the various parameters, improving the agreement with current cosmological data [9].

Assuming a conventional effective four-dimensional low-energy fluid Universe, which is a good picture in our situation where the moduli and other fields have decoupled at early stages of the Universe, we have:

$$q = \frac{1}{2} (1 + 3 w_\Phi) \quad (45)$$

from which we obtain

$$q = -0.61. \quad (46)$$

This fixes the string coupling (39) in the perturbative regime consistent with grand unification scenarios extrapolated from low energies.

So far the model does not include ordinary matter: only fields from the string gravitational multiplet have been included. Inclusion of ordinary matter is not expected to change qualitatively the result. We conjecture that the fundamental relation (39) will continue to hold, the only difference being that the inclusion of ordinary matter will tend to reduce the string acceleration:

$$q = \frac{1}{2} \Omega_M - \Omega_A, \quad (47)$$

where $\Omega_M (\Omega_A)$ denotes the total matter (vacuum) energy density, normalised to the critical energy density of a spatially flat Universe.

There is a remarkable coincidence in numbers for this non-supersymmetric Type-0 string Universe with the astrophysical observations, which yield $q$ close to the value (46). The ordinary matter content of the Universe has $\Omega_{\text{ordinary matter}} \simeq 0.04$ and the dark matter content is estimated to have $\Omega_{DM} = 0.23$, while the Dark Energy content is $\Omega_A \simeq 0.73$. This yields $q = -0.595$, which is only a few per cent away from (46). In fact, if one naively used the value (46) for $q$, obtained in our case where ordinary matter was ignored, in the expression (47), one would find $\Omega_A \simeq 0.74$, indicating that the contribution of the dilaton field to the cosmic acceleration is the dominant one.
If the relation (39) holds after the inclusion of matter, even in supersymmetric models, one arrives at an even better value of the string coupling, $g_s^2 \simeq 0.595$, more consistent with the unification prediction of the minimal supersymmetric standard model at a scale $\sim 10^{16}$ GeV. The only requirement for the asymptotic condition (39) to hold is that the underlying stringy $\sigma$ model is non-critical and asymptotes for large times to the linear dilaton conformal field theory of [12]. It should be understood, of course, that the precise relation of the four-dimensional gauge coupling with the ten-dimensional string coupling depends on the details of compactification, which we do not discuss here.

The variation of the dilaton field at late cosmic times implies a slow variation of the string coupling (39), $\dot{g}_s/g_s = 1/t_E \sim 10^{-60}$ in the present era. The corresponding variations of the gauge couplings are too small to affect current phenomenology.

The above considerations are rather generic to models which relax asymptotically to the linear-dilaton conformal field theory solutions of [12], and from this point of view are physically interesting. We did not need to specify above the microscopic theory underlying the deviation from non-criticality. For this one would need some specific example of such a deviation from a conformally-invariant point in string theory space. One such example, with physically interesting consequences, is provided by the colliding brane-world scenario, in which the Liouville string $\sigma$ model describes stringy excitations on the brane worlds, at relatively long times after the collision so that string perturbation theory is valid. We now discuss this briefly.

7 A Concrete Non-Critical String Example: Colliding Branes

We now concentrate on particular examples of the previous general scenario [22], in which the non-criticality is induced by the collision of two branes as seen in Fig. 1. We first discuss the basic features of this scenario. For our purposes below we assume that the string scale is of the same order as the four-dimensional Planck scale, though this is an assumption that may be relaxed, in view of recent developments in strings with large compactification directions, as we mentioned in the Introduction.

Following [18], we consider two 5-branes of Type-II string theory, in which the extra two dimensions have been compactified on tori. On one of the branes (assumed to be the hidden world), the torus is magnetized with a field intensity $\mathcal{H}$. Initially our world is compactified on a normal torus, without a magnetic field, and the two branes are assumed to be on a collision course with a small relative velocity $v \ll 1$ in the bulk, as illustrated in Fig. 1. The collision produces a non-equilibrium situation, which results in electric current transfer from the hidden brane to the visible one. This causes the (adiabatic) emergence of a magnetic field in our world.

The instabilities associated with such magnetized-tori compactifications are not a problem in the context of the cosmological scenario discussed here. As
discussed in [18], the collision may also produce decompactification of the extra toroidal dimensions at a rate much slower than any other rate in the problem. As also discussed in [18], this guarantees asymptotic equilibrium and a proper definition of an $S$-matrix for the stringy excitations on the observable world. We shall come back to this issue at the end of this Section.

The collision of the two branes implies, for a short period afterwards, while the branes are at most a few string scales apart, the exchange of open-string excitations stretching between the branes, whose ends are attached to them. As argued in [18], the exchanges of such pairs of open strings in Type-II string theory result in an excitation energy in the visible world. The latter may be estimated by computing the corresponding scattering amplitude of the two branes, using string-theory world-sheet methods [27]: the time integral for the relevant potential yields the scattering amplitude. Such estimates involve the computation of appropriate world-sheet annulus diagrams, due to the existence of open string pairs in Type-II string theory. This implies the presence of ‘spin factors’ as proportionality constants in the scattering amplitudes, which are expressed in terms of Jacobi $\Theta$ functions. For the small brane velocities $v \ll 1$ we are considering here, the appropriate spin structures start at quartic order in $v$, for the case of identical branes, as a result of the mathematical properties of the Jacobi functions [27]. This in turn implies [18,25] that the resulting excitation energy on the brane world is of order $V = O(v^4)$, which may be thought of as an initial (approximately constant) value of a supercritical central-charge deficit for the non-critical $\sigma$ model that describes stringy excitations in the observable world after the collision:

$$Q^2 = \left( \sqrt{\beta} v^2 + \mathcal{H}^2 \right)^2 > 0. \quad (48)$$
Fig. 2. A model for supersymmetric D-particle foam consisting of two stacks each of sixteen parallel coincident D8-branes, with orientifold planes (thick dashed lines) attached to them [25]. The space does not extend beyond the orientifold planes. The bulk region of ten-dimensional space in which the D8-branes are embedded is punctured by D0-branes (D-particles, dark blobs). The two parallel stacks are sufficiently far from each other that any Casimir contribution to the vacuum energy is negligible. If the branes are stationary, there is zero vacuum energy, and the configuration is a consistent supersymmetric string vacuum. To obtain excitations corresponding to interesting cosmologies, one should move one (or more) of the branes from each stack and then let them collide (Big Bang), bounce back (inflation), and then relax to their original position, where they collide again with the remaining branes in each stack (exit from inflation, reheating).

where, in the model of [25,19], the proportionality factor $\beta$, computed using string amplitude computations, is of order

$$\beta \sim 2\sqrt{3} \cdot 10^{-8} \cdot g_s \ ,$$

with $g_s$ the string coupling $g_s^2 \sim 0.5$ for interesting phenomenological models [1,5], as discussed above. We recall that the supercriticality, i.e., the positive definiteness of the central charge deficit (48), of the model is essential [12] for a time-like signature of the Liouville mode, and hence its interpretation as target time.

At times long after the collision, the branes slow down and the central charge deficit is no longer constant but relaxes with time $t$. In the approach of [18], this relaxation has been computed by using world-sheet logarithmic conformal field theory methods [28], taking into account recoil (in the bulk) of the observable-world brane and the identification of target time with the (zero mode of the) Liouville field. In that work it was assumed that the final equilibrium value of the central charge deficit was zero, i.e., the theory approached a critical string. This late-time varying deficit $Q^2(t)$ scales with the target time (Liouville mode) as (in units of the string scale $M_s$):

$$Q^2(t) \sim \frac{(H^2 + v^2)^2}{t^2} \ .$$

(50)
Some explanations are necessary at this point. In arriving at (50), one identifies the world-sheet renormalisation group scale $T = \ln(L/a)^2$ (where $(L/a)^2$ is the world-sheet area), appearing in the Zamolodchikov $c$-theorem used to determine the rate of change of $Q$, with the zero mode of a normalised Liouville field $\phi_0$, such that $\phi_0 = QT$. This normalisation guarantees a canonical kinetic term for the Liouville field in the world-sheet action [13]. Thus it is $\phi_0$ that is identified with $-t$, with $t$ the target time.

On the other hand, in other models [15] the asymptotic value of the central-charge deficit may not be zero, in the sense that the asymptotic theory is that of a linear dilaton, with a Minkowski metric in the $\sigma$-model frame [12]. This theory is still a conformal model, but the central charge is a constant $q_0$, and in fact the dilaton is of the form $\Phi = q_0 t + \text{const}$, where $t$ is the target time in the $\sigma$-model frame. Conformal invariance, as mentioned previously, requires [12] that $q_0$ takes on one of a discrete set of values, in the way explained in [12]. In such a case, following the same method as in the $q_0 = 0$ case of [18], one arrives at the asymptotic form

$$Q^2(t) \sim q_0^2 + \mathcal{O}\left(\frac{H^2 + v^2}{t}q_0\right)$$ (51)

for large times $t$.

The colliding-brane model of [18] can be extended to incorporate proper supersymmetric vacuum configurations of string theory [25]. As illustrated in Fig. 2, this model consists of two stacks of D8-branes with the same tension, separated by a distance $R$. The transverse bulk space is restricted to lie between two orientifold planes, and is populated by D-particles. It was shown in [25] that, in the limit of static branes and D-particles, this configuration constitutes a zero vacuum-energy supersymmetric ground state of this brane theory. Bulk motion of either the D-branes or the D-particles \(^3\) results in non-zero vacuum energy [25] and hence the breaking of target supersymmetry, proportional to some power of the average (recoil) velocity squared, which depends on the precise string model used to described the (open) stringy matter excitations on the branes.

The colliding-brane scenario can be realized [19] in this framework by allowing (at least one of) the D-branes to move, keeping the orientifold planes static. One may envisage a situation in which the two branes collide at a certain moment in time corresponding to the Big Bang - a catastrophic cosmological event setting the beginning of observable time - and then bounce back. The width of the bulk region is assumed to be long enough that, after a sufficiently long time following the collision, the excitation energy on the observable brane world - which corresponds to the conformal charge deficit in a $\sigma$-model framework [18,25] - relaxes to tiny values. It is expected that a ground-state configuration will be achieved when the branes reach the orientifold planes again (within stringy length uncertainties of order $\ell_s = 1/M_s$, the string scale). In this picture, since observable time starts ticking after the collision, the question how the brane worlds started

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\(^3\) The latter could arise from recoil following scattering with closed-string states propagating in the bulk.
to move is merely philosophical or metaphysical. The collision results in a kind of phase transition, during which the system passes through a non-equilibrium phase, in which one loses the conformal symmetry of the stringy $\sigma$ model that describes perturbatively string excitations on the branes. At long times after the collision, the central charge deficit relaxes to zero [18], indicating that the system approaches equilibrium again. The Dark Energy observed today may be the result of the fact that our world has not yet relaxed to this equilibrium value. Since the asymptotic ground state configuration has static D-branes and D-particles, and hence has zero vacuum energy as guaranteed by the exact conformal field theory construction of [25,19], it avoids the fine tuning problems in the model of [18].

Sub-asymptotically, there are several contributions to the excitation energy of our brane world in this picture. One comes from the interaction of the brane world with nearby D-particles, i.e., those within distances at most of order $O(\ell_s)$, as a result of open strings stretched between them. The other contribution comes from the collision of the identical D-branes. For a sufficiently dilute gas of nearby D-particles we may assume that this latter contribution is the dominant one. In this case, one may ignore the D-particle/D-brane contributions to the vacuum energy, and hence apply the previous considerations on inflation, based on the $O(v^4)$ central charge deficit, with $v$ the velocity of the brane world in the bulk.

The presence of D-particles, which inevitably cross the D-branes in such a picture, even if the D-particle defects are static initially, distorts slightly the inflationary metric on the observable brane world at early times after the collision, during an era of approximately constant central charge deficit, without leading to significant qualitative changes. Moreover, the existence of D-particles on the branes will affect the propagation of string matter on the branes, in the sense of modifying their dispersion relations by inducing local curvature in space-time, as a result of recoil following collisions with string matter. However, it was argued in [29] that only photons are susceptible to such effects in this scenario, due to the specific gauge properties of the membrane theory at hand. The dispersion relations for chiral matter particles, or in general fields on the D-branes that transform non-trivially under the Standard Model gauge group, are protected by special gauge symmetries in string theory, and as such are not modified.

8 Liouville’s Dark Secrets

The use of Liouville strings to describe the evolution of our Universe seems generally appropriate, since non-critical strings are associated with non-equilibrium situations which undoubtedly occurred in the Early Universe, and may still occur today. It is remarkable that the departure from criticality may even enhance the predictability of string theory, although the space of non-critical string theories is much larger that of critical strings, to the point that purely stringy quantities such as the string coupling are accessible to experiment.

We have discussed in this talk Liouville cosmological models based on non-critical strings with various asymptotic configurations of the dilaton, speculating
on the Big Bang itself, on the inflationary phase and the possibility of exit from it, as well the evolution of the Universe at large times, both current and future. A particularly interesting case from the physical point of view is that of a dilaton that is asymptotically linear in cosmic time. This is known to correspond to a proper conformal field theory [12]. We have observed that the string coupling is identified in such a model (up to irrelevant constants of order one) [24] with the deceleration parameter of the Universe through equation (39).

We stress once more the importance of non-criticality in arriving at (39). In critical strings, which usually assume the absence of a four-dimensional dilaton, such a relation is not obtained, and the string coupling is not directly measurable in this way.

The approach of the identification of target time in such a framework with a world-sheet renormalisation group scale, the Liouville mode [17], provides a novel way of selecting the ground state of the string theory, which may not necessarily be associated with minimisation of energy, but could be a matter of cosmic ‘chance’. Indeed, it may be a random event that the initial state of our cosmos corresponds to a certain Gaussian fixed point in the space of string theories, which is then perturbed in the Big Bang by some relevant (in a world-sheet sense) deformation, thereby making the theory non-critical, and hence out of equilibrium from a target space-time viewpoint. Then the theory flows along some renormalisation-group trajectory to some specific ground state, corresponding to the infrared fixed point of this perturbed world sheet $\sigma$-model theory. This approach allows for many parallel universes to be implemented of course, and our world might be just one of these. Each Universe, may flow between different fixed points, perturbed by different operators. Standard world-sheet renormalisation-group arguments imply that the various flow trajectories do not intersect, although this is something that is far from proven in general. It seems to us that this scenario is much more specific than the landscape scenario [30], which has recently been advocated as an attempt to parametrise our ignorance of the true structure of string/M theory.

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