Sound modes at the BCS-BEC crossover

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First and second sound speeds are calculated for a uniform superfluid gas of fermi atoms as a function of temperature, density and interaction strength. The two sound speed is of particular interest as it is a clear signal of a superfluid component and it determines the critical temperature. The sound modes and their dependence on density, scattering length and temperature are calculated in the BCS, molecular BEC and unitarity limits and a smooth crossover is extrapolated. It is found that first and second sound undergo avoided crossing on the BEC side due to mixing. Consequently, they are detectable at crossover both as density and thermal waves in traps.

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I. INTRODUCTION

Recent experiments probe systems of strongly interacting Fermi and Bose atoms and molecular bosons at low temperatures. Evidence for a continuous crossover between BCS to a molecular BEC superfluid is found by measurements of expansion[1, 2, 3, 4, 5], collective modes[6, 7], RF spectroscopy[8], thermal energies and entropies[9], and correlations[10]. The next generation of experiments will measure sound velocities in these systems. Second sound has been studied in He$^3$, He$^4$ and BCS superfluids (see, e.g.,[11, 12]). Second sound is particularly interesting as it is a clear signal of a superfluid component and determines the critical temperature[13]. The dependence of the sound speeds on temperature, density and interaction strength will constrain the equation of state and models.

Eagles[14] and Leggett[15] described the smooth crossover at zero temperature by a mean field gap equation. Extensions to finite temperature especially around the superfluid critical temperature can be found in[16, 17, 18, 19, 20, 21, 22, 23]. The various crossover models differ in the inclusion of selfenergies, whether one or two channels are employed, etc. Recently, the crossover has been calculated in detail by quantum Monte Carlo (QMC) [24, 25] at zero temperature. The experiments are generally compatible with most these calculations and confirm universal behavior[13, 20] in the unitarity limit as well as superfluidity at high temperatures[19, 22, 26, 27, 28].

The purpose of this work is to calculate sound velocities in the superfluid phase and demonstrate that first and second sound undergoes avoided crossing at the BEC-BCS crossover in uniform systems of strongly interacting Fermi atoms. Second sound reveals the presence of a superfluid and its critical temperature. The sound modes provide details about the equation of state at the BEC-BCS crossover and constrain crossover models.

The two-body interaction range is assumed to be short as compared to the s-wave scattering length $a$ and the interparticle spacing or $k_F^{-1}$, where $n = k_F^3/3\pi^2$ is the density of the Fermi atoms with two equally populated spin states. The physics can be expressed in terms the variable $x = 1/(ak_F)$, which varies from $-\infty$ in the dilute BCS limit through the unitarity limit $x = 0$ at the Feshbach resonance to $x \rightarrow +\infty$ in the molecular BEC limit.

The sound modes may be measured in experiments as density waves in the expanding clouds. Density or thermal perturbations in the trapped clouds may be introduced in the center of the cloud, for example, by a pulsed laser before expansion, or by other means[29]. It is important that the two sound modes mix as will shown below because both modes will be excited by either a density or thermal perturbation. As the sound modes travel through the cloud the density and temperature may vary. It may therefore be nontrivial to extract the sound speeds as function of density, temperature and scattering length, and useful to have a model calculation of both sound speeds.

The manuscript is organized such that the sound modes are first described in general, and then calculated in the BEC, BCS and unitarity limits respectively. The three limits are then approximately matched assuming a smooth crossover. Finally, summary and conclusions are given.

II. SOUND MODES

We shall in the following assume that the thermal excitations collide frequently and therefore can be assumed in local thermal equilibrium. This is the collisional limit where hydrodynamics apply. The transition from zero to first sound as systems change from collisionless to collisional is described in, e.g.,[30] and the hydrodynamics of superfluids in[31, 32]. For a Fermi liquid the collision time $\tau \propto T^{-2}$ can become large at low temperatures. The hydrodynamic limit is therefore limited to long wavelength (or equivalently low frequency $\omega$) sound modes such that $\tau \omega \ll 1$. This can always be achieved in a bulk system but not necessarily in a finite system where
collective frequencies $\omega$ are finite. However, it was found in [6, 7, 33] that axial and radial collective modes can be described by hydrodynamics at temperatures well below the superfluid transition. Thus atoms in traps appear collisional in a wide region around the unitarity limit. Yet the collisionless limit is expected in traps at very low temperatures [34].

The presence of a normal and superfluid components leads to two sound modes in the collisional limit referred to as first and second sound. Their velocities $u_1$ and $u_2$ are given by the positive and negative solutions respectively of [32, 35]

$$u^2 = \frac{c_s^2 + c_\gamma^2}{2} \pm \sqrt{\left(\frac{c_s^2 + c_\gamma^2}{2}\right)^2 - c_T^2 c_\gamma^2}.$$  

The thermodynamic quantities entering are the adiabatic $c_s^2 = (\partial P/\partial \rho)_s$, the isothermal $c_T^2 = (\partial P/\partial \rho)_T$, and the “thermal” $c_\gamma^2 = \rho_s s^2 T/\rho_a c_V$ sound speed squared. The latter also acts as a coupling or mixing term. The difference between the adiabatic and isothermal sound speed squared can also be expressed as

$$c_s^2 - c_T^2 = \left(\frac{\partial s}{\partial \rho}\right)_T \frac{T}{\rho c_V}.$$  

Here, $\rho = \rho_n + \rho_s = nn$ are the total, $\rho_n$ the normal, and $\rho_s$ the superfluid mass densities, $s = S/\rho$ the entropy per unit mass and $c_V = T(\partial s/\partial T)_p$ the specific heat per unit mass.

The isothermal sound speed at zero temperature is in both the hydrodynamic and for a superfluid gas given by

$$c_T^{2,0} = \frac{n}{m} \left(\frac{\partial \mu}{\partial n}\right)_T = \frac{1}{3} v_F^2 \left[1 + \beta - \frac{3}{5} \beta' + \frac{1}{10} \beta''\right],$$  

where $v_F = \hbar k_F/m$, $\beta(x) = E_{\text{int}}/E_{\text{kin}}$ is the ratio of interaction to kinetic energy, $\beta' = d\beta/dx$, etc. QMC calculations [24, 27] find $\beta(x = 0) = -0.57$ in the unitarity limit at zero temperature. The continuous crossover found in QMC and crossover models is confirmed experimentally for the pressure, chemical potential, $\beta$, collective modes [1, 2, 5] and recently also for the entropy [3].

In the dilute BCS limit and at low temperature $c_T = v_F \sqrt{1 + (2/\pi)ak_F}/3$. In the unitarity limit $c_T = v_F \sqrt{1 + \beta(0)/3} \approx 0.37 v_F$. In the dilute BEC limit $c_T = \sqrt{\pi/2} \hbar^2 n a_M/m^2$, where $a_M$ is the molecular scattering length. According to QMC [25] and four-body calculations [36] $a_M \approx 0.62 a$, whereas in the Leggett and several other crossover models $a_M = 2 a$.

We are mainly interested in the superfluid state at temperatures below the critical temperature, which generally is less than that in the BEC limit, $T \leq T_c^{BEC} = 0.218 E_F$, the temperatures will be much smaller than the Fermi energy $E_F$ (in units where $k_B = 1$). At such low temperature the isothermal sound speed is given by Eq. 3 in both the BCS and BEC limits. In the crossover model of Leggett the isothermal sound speed is also to a good approximation given by the zero temperature value for $T \lesssim T_c$. We shall therefore use Eq. 3 for $c_T^2$ in the following.

The entropy and mass densities have been calculated in the BEC [32, 35] and BCS [37] limits to leading order in $ak_F$. One should, however, be careful in extrapolating to the unitarity limit. For example a normal BEC is depleted as $a_M \to +\infty$ and eventually quenched [38] whereas in crossover models the condensate remains although it changes character from BEC to BCS superfluid. Likewise, the collective mode frequencies in trapped Fermi atoms increase for a normal BEC as the unitarity limit is approached [39]. Crossover models [33] and experiments [6, 8] however, find the opposite for a molecular BEC. The BEC and BCS limits will now be described followed up with a discussion on the extension and interpolation towards the unitarity limit in various crossover models.

III. BEC LIMIT

Crossover models and calculations find different molecular scattering length in the BEC limit as mentioned above. Also the effective mass $M$ in the dispersion relation $E_q = q^2/2M$ differs between the molecular BEC limit where $M = 2m$ and the unitarity limit, where $M$ becomes very large indicating that the dispersion relation is not quadratic but rather linear as a Bogoliubov phonon. For example, the pair susceptibility $\chi$, which enters selfenergies, is calculated with different combinations of bare ($G_0$) and full ($G$) Green’s functions. With $\chi = GG$ [17, 24] $M$ increases monotonically as the uni-
the first sound speed by the standard result corresponding to \( x = 1/(a_{k_F}) \) be described approximately by a dispersion relation \( E \approx q^{2}/M \) for temperatures below \( T^* = 2\pi a_{M} n/M \). The critical temperature has the opposite term in the superfluid \( \Omega = \frac{1}{g} \sum_{k} \left[ \frac{1}{2E_k} - \frac{m}{\hbar^2 k^2} - \frac{f_k}{E_k} \right] \). As usual \( \varepsilon_k = \hbar^2 k^2/2m - \mu \), \( E_k = \sqrt{\varepsilon_k + \Delta^2} \) and \( \Delta(T) \) the gap. The thermal distribution function is \( f_k = (\exp(E_k/T) + 1)^{-1} \). With the equation for number density conservation

\[
n = \sum_{k} \left[ 1 - \frac{\varepsilon_k}{E_k} + 2 \frac{\varepsilon_k}{E_k} f_k \right],
\]

the gap and chemical potential can be calculated as function of density, temperature and interaction strength. At zero temperature the last terms in Eqs. \( \text{?} \) and \( \text{?} \) vanish and the gap is

\[
\Delta_0 = \frac{8}{(e^2)} E_F \exp(\pi/2a_{k_F}) \).
\]

Gorkov \( \text{?} \) included induced interactions, which reduce the gap to

\[
\Delta_0 = (2/e)^{7/3} E_F \exp(\pi/2a_{k_F}) \).
\]

Induced interaction can be included in the gap equation \( \text{?} \) by replacing \( a^{-1} \rightarrow a^{-1} - 2k_F \ln(4e)/3\pi \) on the left hand side. Hereby, not only the gap is corrected but also the thermodynamic potential \( \text{?} \). In both cases \( T_c = (\gamma/\pi) \Delta_0 = 0.567\Delta_0 \).

The thermodynamic functions can be calculated from the thermodynamic potential per volume \( \Omega = -P \). We make the standard assumption that the Hartree-Fock terms in the superfluid \( \Omega_s \) and normal state \( \Omega_n \) thermodynamic potentials are the same. The difference is then given in terms of the pairing coupling as

\[
\Omega_s = \Omega_n + \int_0^{\Delta} d\Delta' \Delta' d(1/g) d\Delta^2.
\]

FIG. 2: As Fig.1 but with \( c_T \) and \( a_M = 0.62a \) from QMC. Furthermore, induced interactions are included on the BCS side which reduce \( T_c \).

### IV. BCS LIMIT

The thermodynamic quantities and sound speeds in BCS limit have been calculated in \( \text{?} \) from the mean field BCS equations of Leggett \( \text{?} \)

\[
\frac{1}{g} \sum_{k} \left[ \frac{1}{2E_k} - \frac{m}{\hbar^2 k^2} - \frac{f_k}{E_k} \right],
\]

with coupling strength \( g = -4\pi\hbar^2 a/m \). As usual \( \varepsilon_k = \hbar^2 k^2/2m - \mu \), \( E_k = \sqrt{\varepsilon_k + \Delta^2} \) and \( \Delta(T) \) the gap. The thermal distribution function is \( f_k = (\exp(E_k/T) + 1)^{-1} \). With the equation for number density conservation

\[
n = \sum_{k} \left[ 1 - \frac{\varepsilon_k}{E_k} + 2 \frac{\varepsilon_k}{E_k} f_k \right],
\]

the gap and chemical potential can be calculated as function of density, temperature and interaction strength. At zero temperature the last terms in Eqs. \( \text{?} \) and \( \text{?} \) vanish and the gap is

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The thermodynamic functions can be calculated from the thermodynamic potential per volume \( \Omega = -P \). We make the standard assumption that the Hartree-Fock terms in the superfluid \( \Omega_s \) and normal state \( \Omega_n \) thermodynamic potentials are the same. The difference is then given in terms of the pairing coupling as

\[
\Omega_s = \Omega_n + \int_0^{\Delta} d\Delta' \Delta' d(1/g) d\Delta^2.
\]
The first order temperature correction to the thermodynamic potential in the normal phase is \( \Omega_n = -N(0)\pi^2 T^2/3 \), where \( N(0) = m k_F/\hbar^2 \) is the level density.

Inserting the coupling of Eq. (10) into the thermodynamic potential (11) and again exploiting that \( \Delta \ll \mu \), it reduces to

\[
\Omega_s = -N(0) \left[ \frac{\Delta^2}{(1 + \frac{\Delta}{\Delta s})} - 4T \int_0^\infty d\varepsilon_k \ln(1 - f_k) \right]
\]

The crossover model thus arrives at the standard expression for \( \Omega_s \) and therefore also the standard entropy density \( S_s = -(\partial\Omega_s/\partial T)_{\nu, \mu} \), and specific heat \( C_s = T(\partial\Omega_s/\partial T)_{\nu, \mu} \) in superfluid phase. In the normal phase \( S_n = C_n = N(0)2\pi^2 T^3/3 \). Near \( T_c \): \( S_s/S_n(T) = 1 - (1 + \xi/(1 - T/T_c)) \) and \( C_s/C_n(T) = \xi - 3.77(1 - T/T_c) \). Here, \( \xi = 12/\sqrt{\zeta(3)} \approx 2.43 \) is the ratio of superfluid specific heat just below \( T_c \) to the normal one just above \( T_c \). At low temperatures \( S_n = N(0)\sqrt{2\pi\Delta_0^2/\hbar^2} \exp(-\Delta_0/T) \) and \( C_s = N(0)2\pi\Delta_0^2/\hbar^2 T^3 \exp(-\Delta_0/T) \).

Finally, we need the superfluid (London) density

\[
n_s = n \left( 1 + 2 \int_0^\infty d\varepsilon_k \frac{df_k}{d\varepsilon_k} \right),
\]

where \( n = n_s + n_n \) is the total density. At low temperatures \( n_s/n = 1 - \sqrt{2\pi\Delta_0/\hbar^2} \exp(-\Delta_0/T) \) whereas \( n_s/n = 2(1 - T/T_c) \) near \( T_c \).

We now have all the thermodynamic quantities available for calculating the sound speeds in the BCS limit. Since thermal effects are small we find that \( u_1 = cs \simeq ct \). The second sound is a pure thermal wave with velocity

\[
u_s^2 \simeq c_s^2 = \frac{n_s}{n_n} \frac{S_s^2 T}{m n C_s}.
\]

The dependence on temperature and \( x \) is shown in Fig. 3 as well as in Figs. 1 and 2 for three different temperatures.

At low temperatures the second sound is linear in temperature

\[
u_2 \simeq \frac{\sqrt{3}}{2} \frac{T}{E_F} v_F, \quad T \ll T_c.
\]

Around \( T \simeq 0.7T_c \) the second sound speed has a broad maximum of

\[
u_2 \simeq 0.53 \frac{T}{E_F} v_F, \quad T \simeq 0.7T_c.
\]

Near the critical temperature \( T_c - T \ll T_c \)

\[
u_2 = \frac{\pi}{\sqrt{\xi}} \frac{T}{E_F} \sqrt{1 - \frac{T}{T_c}} v_F.
\]

The characteristic \( u_2 \propto \sqrt{1 - T/T_c} \) behavior near \( T_c \) is the same as in the BEC limit, Eq. (4), but with a different prefactor.

![FIG. 3: Thermodynamic quantities for a superfluid as calculated in the BCS limit vs. \( T/T_c \). Shown are the gap \( \Delta(T)/\Delta_0 \), entropy \( S_s(T)/S_n(T) \), specific heat \( C_s(T)/C_n(T) \), and superfluid density \( n_s(T)/n \). The second sound is plotted in units of \( v_F T_c/E_F \).](image)

V. UNITARITY LIMIT AND CROSSOVER

At crossover the entropy receives contributions from thermal Fermi atoms and molecular bosons. The entropies and sound speeds reflect the underlying dispersion relation for low-lying excitations. On the BEC side for \( T \gtrsim T^* \) the quadratic dispersion relation \( E_q = \hbar^2 q^2/2M \) leads to an entropy \( S \propto T^{3/2} \), whereas for \( T \lesssim T^* \) the linear Bogoliubov phonon \( E_q = \hbar q^2 cT \) yields \( S \propto T^3 \). On the BDS side the gap in the quasiparticle energy leads to a more complicated entropy as discussed above.

Generally, for a power law dispersion relation \( E_q = q^\alpha \) the entropy and specific heat scale as \( S \propto \xi^\alpha \propto T^{3/\alpha} \) and the normal mass density \( \rho_n = \rho(T/T_c)^{\xi^\alpha - 1} \). Consequently \( c_s^2 - c_\alpha^2 \propto T^{3/\alpha + 1} \) and \( c_\alpha^2 \propto (\rho_s/\rho)^{T^2/2 - \alpha} \). The resulting temperature dependence of the sound velocities follows from Eq. (14).

Model calculations [14, 15, 16, 17, 18, 19, 20, 21, 22, 23] find a smooth crossover of most thermodynamic quantities as function of density and scattering length at zero and finite temperature except at \( T_c \), where superfluidity vanishes. We will therefore extrapolate the sound modes from the BCS and BEC limits towards crossover such that they are continuous around crossover and meet the sound speeds in the unitarity limit, which now will be calculated independently.

Experimentally, the entropy has recently been measured and found to scale as \( S/N \propto 20(T/T_F)^{1.53} \) in the unitarity limit at temperatures below \( T_c \simeq 0.27E_F \). In the unitarity limit the entropy is independent of scattering length due to universality [13] as other thermodynamic quantities [24]. Consequently, when \( S/N \propto T^{3/2} \), the entropy density is independent of density and is therefore the same in bulk as in a trap at temperatures sufficiently low as compared to the Fermi temperature. Both
the measured $T_c$ and entropy are then slightly larger in the unitarity limit than for the pure BEC of Eq. 4. With these adjustments of the parameters in Eq. 4, the sound velocities are calculated in the unitarity limit as in the BEC limit above as shown in Figs. 1 and 2 for $x = 0$. To guide the eye the unitarity limit point at $x = 0$ is connected smoothly in the crossover regime to the BEC limit, $x \geq 1$, and the BCS limit, $x \leq -1$.

The sound velocities in Figs. 1 and 2 are well behaved in the sense that both the BCS and BCS limits extrapolate continuously towards the unitarity limit. On the BCS side first sound is basically given by the isothermal sound speed, and the second sound increases exponentially with $x$ as $T_c = (\gamma / \pi) \Delta$ as given by Eqs. 9 and 10. Around the unitarity limit their dependence on temperature and $x = 1/k_F a$ is more complicated because the two sound modes mix and the temperature dependence of the entropy is different from the BEC limit. On the BEC side the curves again extrapolate continuously towards the BEC limit, however, with avoided crossing of the two sound modes.

It should be noted that the curves in Figs. 1 and 2 are plotted for constant $T/T_c$. On the BCS side $T_c$ is almost constant from the unitarity to the BCS limit, and the curves are therefore basically for constant temperature. On the BCS side, however, $T_c$ decrease exponentially as $T_c \propto \exp(\pi x / 2)$ (see Eqs. 9 and 10), and therefore the temperatures also decrease. If the temperature were held fixed, superfluidity and the second sound would thus vanish below a certain $x$ where $T \geq T_c$.

The specific heat is a continuous function of temperature around $T_c$ only in the BEC limit. It is discontinuous in the BCS limit by a factor $C_s/C_n = 2.43$. Recent calculations and experimental fits [10] at $x = 0.11$ find $C_s/C_n = 2.51 \pm 0.05$ at $T_c$. These experiments indicate that the discontinuity in the heat capacity has a maximum crossing over from the BCS value towards zero in the BEC limit. $T_c$ also has a maximum at crossover in several crossover models [10, 11, 12].

The second sound speed squared generally vanishes with the superfluid density at the critical temperature, i.e. $c_2 \propto \sqrt{T_c - T}$. This is observed in the BCS limit of Eq. (17), in the BEC limit of Eq. (6) and also in the unitarity limit.

The sound speeds are continuous as function of $x$ at crossover as long as $T/T_c$ is fixed as shown in Figs. 1 and 2. As function of temperature first sound depends only little on temperature whereas second sound vanishes at $T_c$ and the specific heat is discontinuous. Similar temperature dependency were found in Refs. [21, 41] at crossover.

VI. SUMMARY

The first and second sounds have been calculated in the molecular BEC, the BCS and the unitarity limits. It has been argued that the sound modes interpolate continuously between these limits, and that avoided crossing takes place on the BEC side due to mixing.

The mixing has the consequence that both sound modes can be excited and detected both as density and thermal waves in traps. In contrast the second sound is a pure thermal wave in the BCS and BEC Bogoliubov limits. In finite traps the sound waves could be excited in the center and propagate towards the surface so that the local density decreases. Consequently, $|x| = 1/a k_F$ increases and the sound velocities change as shown in Figs. 1 and 2.

The second sound mode reveals superfluidity and the critical temperature. Measurements of both sound modes constrain the equation of state, entropies, specific heat, etc. and thereby crossover models, which predict very different behavior of, e.g., the molecular scattering length in the BEC limit and effective molecular mass, critical temperature, etc. at crossover. The crossover and mixing of the sound modes will also be possible to study in mixtures of bosons and fermions. A number of different supertluids may also be created in optical lattices [42] (with corresponding Mott transitions) and possibly also supersolids.

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