Improved Control of the Web Winding System Based on a Super-Twisting Observer

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Abstract: The web winding system (WWS) presents a difficult problem at the level of control because of the strong nonlinearities of the models and the effects of internal or external disturbances.

To improve the dynamic performance and robustness of the control of WWS, a robust control method based on the first and second order sliding mode algorithm is proposed in this work. The stability of the controllers is proved by the theory of Lyapunov stability using an appropriate switching function. In addition, an observer of super twisting tensions is suggested. The developed method allows a direct estimate "on line" of the tension. Finally, computer simulations are developed to show the performance of the sliding mode control and the proposed nonlinear observer.

Keywords: Web Winding System; Lyapunov; Sliding mode; Super-Twisting

I. INTRODUCTION

Following the continuous development of industry and technology, the precision of the size and shape of the tape are important indicators for measuring the quality of the tape product. In the manufacture and processing of steel webs, it is important to maintain the tension and velocity of the web of the predefined set points to solve both problems. Otherwise, insufficient or excessive tension can cause wrinkling or breaking of the web, deterioration of the quality of the material and deformation of the web which will directly affect the quality and productivity of the final product.

Several control methods to maintain constant tension and velocity are known. The authors in [1] propose an inverse linear quadratic method control (QIL) for the reversible cold-rolling mill velocity and tension system with multivariate uncertainties and strong coupling. Ref. [2], a compound control method based on the principle of invariance has been proposed for the coupling of tensions and velocity of reversible cold rolling mill. The controller design method requires a repeat differential on the virtual control inputs during the derivation process that makes the growing complexity of the controller with the increase of the relative order of the system. That is, a "differential explosion" phenomenon has appeared.

One of the recent advances in nonlinear control theory is the introduction of adaptive backstepping algorithms to control the WWS [3-4] and nonlinear control by backstepping with integral action introduced in [5].

The authors develop in [6] the control without tension meter for a velocity and tension system of a reversing cold rolling mill based on the pre-feedback control strategy and Hamilton's theory. In [7] a high gain nonlinear observer is proposed to estimate the set of state variables of the process of reversible cold rolling mill.

A backstepping control strategy [8] is proposed based on the filter control. The controller of each of the velocity and tensions subsystems is designed on the basis of backstepping, while filters control is adopted to generate the virtual control inputs and their derivatives in the initial time, which avoid the phenomenon of "differential explosion" when using the backstepping controller.

The complexity of control and observation of the WWS requires new design methodologies for controllers and observers such as the sliding mode algorithm. It has grown considerably in recent decades [9-11]. This is mainly due to the simplicity of design and implementation, high precision, fast dynamic response, stability, robustness against disturbances and other invariance of adapted uncertainties [12].

The calculation of the control law of the WWS requires the knowledge of all the states which is interpreted by placing as many sensors on the system. Sensor measurements can be noisy. Observation is used to provide a guaranteed accuracy and an estimation of the current value of the tensions.

In this work, we propose sliding mode control for the WWS and a super twisting observer has been introduced to estimate the web tensions.

A simulation study is conducted on the velocity and tension system of a 1422 mm reversible cold-roll mill by comparing the sliding-mode control strategy with other control techniques.
This document is organized as follows. Section 2 shows the problematic of WWS and the mathematical model of a web winding system. The design of WWS control by sliding mode is developed in section 3. Two observers are designed and the main results are derived from section 4. To illustrate the effectiveness of the proposed observers, some experiments are carried out in section 5. The conclusions are drawn in section 6.

II. WWS MODEL AND PROBLEM FORMULATION

A. Mathematical model of WWS

Because of the complexity of the problem and the coupling of the tensions and velocities of the WWS to study, we use a mathematical model describing the set of physical processes [3]:

\[
\begin{align*}
\text{sys1} & : \quad \frac{dT_1}{dt} = \theta_1 V_2 T_1 + \theta_2 (V_2 - V_1) \\
\text{sys2} & : \quad \frac{dV_2}{dt} = \theta_3 V_1 (t) + \theta_4 T_1 + \theta_5 U_1(t) \\
\text{sys3} & : \quad \frac{dV_3}{dt} = -\theta_6 T_3 - \theta_7 V_3 (t) + \theta_8 U_2 \\
& \quad - \theta_9 V_2 T_3 + \theta_{10} (V_2 - V_3) \\
& \quad - \theta_{11} T_3 - \theta_{12} U_3 (t) + \theta_{13} U_3(t) 
\end{align*}
\]

where:

\[
\begin{align*}
\theta_1 &= -\frac{1}{L} & \theta_2 &= \frac{ES}{L} & \theta_3 &= \frac{-1}{\tau_m 1} & \theta_4 &= \frac{gr^2}{l_1} \\
\theta_5 &= \frac{r_1}{\tau_m 1} & \theta_6 &= \frac{gr^2}{l_2} & \theta_7 &= \frac{1}{\tau_m 2} & \theta_{10} &= \frac{1}{\tau_m 3} & \theta_{11} &= \frac{gr^2}{l_3} & \theta_{12} &= \frac{1}{\tau_m 3} \\
\theta_{13} &= \frac{r_2}{\tau_m 3} & \theta_8 &= \frac{r_2}{\tau_m 2} & \theta_9 &= \frac{1}{\tau_m 1} & & & & \\
\end{align*}
\]

Table I: Model Parameter for the Web Winding System

| Parameter symbol | Parameter name |
|------------------|----------------|
| s                | transverse surface of the web |
| L, R_1, R_2, R_3 | length, radius of the web |
| L                | Length web length |
| E                | Young modulus of the web |
| h_1, h_2         | Armature resistance of DC motor |
| f                | Input and Output web thickness respectively |
| J_1, J_2, J_3    | DC Motor inertia |
| g                | Intensity of gravity |
| k_1, k_2, k_3    | Motor torque coefficient |
| \tau_m 1, \tau_m 2, \tau_m 3 | The electromechanical time constant of the DC motor |
| r_1, r_2         | Unwinding, work rolls and winding radius respectively |
| r_3              | Density of the web |
| P_1, F_2         | Upstream and Downstream Tension |
| V_1, V_2, V_3    | Upstream linear velocity, Work roll velocity and Downstream linear velocity respectively |

The mathematical model is composed of three subsystems: the first (sys1) is composed of the state vector \([T_1, V_1]\), controlled by the voltage \(U_1\), the second (sys2) as a state vector \([V_2]\), is controlled by \(U_2\) and the third subsystem (sys3) as a state vector \([T_3, V_3]\), is controlled by the voltage \(U_3\).

B. Problem formulation and design objectives

The web winding system of a reversible cold rolling mill has several constraints such as the thermal effects caused by the friction and mechanical effects caused by the elongation of the metal, which generates malfunctions due to the influence of process conditions. Thus, the winding system must guarantee the exact values of velocity, tension of traction and pressure force.

The non-linear model of the web winding system is characterized by a relatively strong coupling relationship between the tension of the winder, the velocity of the main mill and the tension of the unwinder, multiple variables, time varying parameters, model uncertainties and external disturbances. This model presents a non-linearity of product type, appeared in the second and the fourth equation of the system. The control of sheet metal drive systems of thin thickness is relatively difficult since the dynamic behavior of the web depends on the one hand on the variations in tension and running velocity, on the other hand on the production conditions (temperature and hygrometry).

The main objectives in the design of the controller by the WWS are to obtain:

- precise thickness, with the best possible regularity in case of sudden stress;
- a stable rolling and ensure the quality of the web;
- therefore, in order to improve the control accuracy of the velocity and tension of cold rolling mill reversible, a new effective decoupling control algorithm is designed for WWS by combining the observer with the control sliding mode to meet the system control requirements. The observer uses the measurements available to estimate the tensions. Then, the estimated values are returned to the sliding mode controller to build a control law.

III. WEB TENSIONS OBSERVER OF WWS

The system of the tensions of the WWS band which we are discussing is a second order system, which has poorly understood nonlinear terms. For this reason, we design a super-twisting sliding mode observer (STSMO) to estimate unmeasured state variables. In general, a nonlinear OMGST is always used to get an accurate estimate of the tensions of the band and to compensate for parametric variations.

A. Generalized Super-Twisting observer

The super-twisting sliding mode observer is developed based on the super-twisting sliding mode control algorithm. The following second order nonlinear system with bounded disturbances is considered for the observer design:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= f(t, x_1, x_2, u) + v(t, x_1, x_2, u) \\
y_{out} &= x_1(t)
\end{align*}
\]

where \(x_1 \in IR\), \(x_2 \in IR\), all disturbances are contained in the perturbation term \(v(t, x_1, x_2, u)\).

We assume that all the system states and the known control input \(u(x_1, x_2)\) are bounded. Only \(x_1\) is measurable and \(x_2\) needs to be observed. The designed super-twisting observer is expressed as [13]:

\[
\begin{align*}
\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + y_1(t) \\
\dot{\hat{x}}_2(t) &= f(t, x_1, \hat{x}_1, u) + y_2(t) \\
\end{align*}
\]

\(\hat{x}_1(t)\) and \(\hat{x}_2(t)\) are the estimated values of states. According to the super-twisting algorithm, the output injection terms \(y_1(t)\) and \(y_2(t)\) are considered as

\[
\begin{align*}
\dot{y}_1(t) &= f(t, x_1, \hat{x}, u) + y_2(t) \\
\dot{y}_2(t) &= g(t, x_1, \hat{x}, u) + y_3(t) \\
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
    y_1(t) = \lambda \dot{x}_1(t) - \dot{x}_2(t) + \frac{1}{\lambda} \text{sgn}(x_1(t) - \dot{x}_1(t)) \\
y_2(t) = \alpha \text{sgn}(x_1(t) - \dot{x}_1(t))
\end{cases}
\end{align*}
\]  

where gain parameters \(\lambda\) and \(\alpha\) are necessary to satisfy a certain condition to guarantee that the system (8) is stable.

The variable \(\tilde{x}\) is defined as the observation error vector, where \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2)^T\). \(\tilde{x}_1 = x_1(t) - \dot{x}_1(t)\) and \(\tilde{x}_2 = x_2(t) - \dot{x}_2(t)\). The differential equations with respect to the observation errors can be obtained as follows:

\[
\begin{align*}
\dot{\tilde{x}}_1(t) &= -\lambda |\tilde{x}_1(t)|^{\frac{1}{2}} \text{sgn}(x_1(t) - \dot{x}_1(t)) + \tilde{x}_2(t) \\
\dot{\tilde{x}}_2(t) &= -\alpha \text{sgn}(\tilde{x}_2(t)) + \Phi(t,x_1,x_2,\tilde{x}_2,u)
\end{align*}
\]

\[\text{(9)}\]

B. Tensions observers

In this section, two observers are introduced to estimate the Tensions by the super-twist algorithm (STA). Assume the velocities are measurable with some encoders.

For the design of a higher order sliding mode observer, consider the system (1) in a canonical form as follows:

\[
\begin{align*}
\begin{cases}
    \dot{x}_1(t) &= x_2(t) \\
    \dot{x}_2(t) &= \theta_1 \dot{V}_2 x_2 + \theta_2 (\dot{V}_2 - \dot{V}_1) + v_1(t,x_1,x_2,u) \\
    y_{out}(t) &= x_1(t)
\end{cases}
\end{align*}
\]

\[\text{(10)}\]

for tension \(T_1\):

\[
\begin{align*}
\begin{cases}
    \dot{x}_3(t) &= x_4(t) \\
    \dot{x}_4(t) &= \theta_3 \dot{V}_3 x_4 + \theta_4 \dot{V}_3 T_3 + \theta_5 (\dot{V}_3 - \dot{V}_2) + v_2(t,x_3,x_4,u) \\
    y_{out}(t) &= x_3(t)
\end{cases}
\end{align*}
\]

\[\text{(11)}\]

Where \(x_i\) are the scalar state variables and \(v_i\) are the perturbation term.

The tensions observer proposed by super-twisting has the following form:

\[
\begin{align*}
\begin{cases}
    \dot{\tilde{x}}_1(t) &= \tilde{x}_2(t) + \lambda_1 \frac{|x_1(t) - \tilde{x}_1(t)|}{\tilde{x}_1(t)} \text{sgn}(x_1(t) - \tilde{x}_1(t)) \\
    \dot{\tilde{x}}_2(t) &= \theta_1 \dot{V}_2 \tilde{x}_2 + \theta_2 (\dot{V}_2 - \dot{V}_1) + v_1(t,x_1,x_2,u) + a_{11} \text{sgn}(x_1(t) - \tilde{x}_1(t)) \\
    \dot{\tilde{x}}_3(t) &= \dot{x}_4(t) + \lambda_3 \frac{|x_3(t) - \tilde{x}_3(t)|}{\tilde{x}_3(t)} \text{sgn}(x_3(t) - \tilde{x}_3(t)) \\
    \dot{\tilde{x}}_4(t) &= \theta_3 \dot{V}_3 \tilde{x}_4 + \theta_4 \dot{V}_3 T_3 + \theta_5 (\dot{V}_3 - \dot{V}_2) + a_{33} \text{sgn}(x_3(t) - \tilde{x}_3(t))
\end{cases}
\end{align*}
\]

\[\text{(12)}\]

\[\text{(13)}\]

where the gain parameters \(\lambda\) and \(\alpha\) are necessary to satisfy a certain condition in order to guarantee the stability of the system (5).

The variable \(\tilde{x}\) is defined by the observation error vector, where \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2)^T\). \(\tilde{x}_1 = x_1(t) - \dot{x}_1(t)\) and \(\tilde{x}_2 = x_2(t) - \dot{x}_2(t)\). Differential equations for observation errors can be obtained as follows:

\[
\begin{align*}
\dot{\tilde{x}}_1(t) &= \tilde{x}_2 - \lambda_1 \frac{|x_1(t) - \tilde{x}_1(t)|}{\tilde{x}_1(t)} \text{sgn}(x_1(t) - \tilde{x}_1(t)) \\
\dot{\tilde{x}}_2(t) &= \theta_1 \dot{V}_2 (x_2 - \tilde{x}_2) - a_1 \text{sgn}(x_1(t) - \tilde{x}_1(t))
\end{align*}
\]

\[\text{(14)}\]

And

\[
\begin{align*}
\dot{\tilde{x}}_3(t) &= \tilde{x}_4(t) - \lambda_3 \frac{|x_3(t) - \tilde{x}_3(t)|}{\tilde{x}_3(t)} \text{sgn}(x_3(t) - \tilde{x}_3(t)) \\
\dot{\tilde{x}}_4(t) &= \theta_3 \dot{V}_3 (x_4 - \tilde{x}_4) - a_3 \text{sgn}(x_3(t) - \tilde{x}_3(t))
\end{align*}
\]

\[\text{(15)}\]

Suppose that the states of the system can be assumed to be bounded, then the existence of a finite constant \(f^+\) is ensured, so that the inequality \(|V_2(x_2 - \dot{x}_2)| < f^+\) with \(f^+ = \max_x|f(x)|\).

It was proven in [13] that, if design parameters \(\alpha\) and \(\lambda\) are selected to satisfy the following condition: \(\alpha > f^+\) and \(\lambda = \frac{2}{|a-f^+|} (1+p)\) such as \(0 < p < 1\) then the observer states \(\tilde{x}_1\) and \(\tilde{x}_2\) will converge \(x_1\) and \(x_2\) in finite time.

IV. DESIGN OF WWS CONTROL BY SLIDING MODE

The theory of sliding mode systems is a nonlinear control technique. It is characterized by a discontinuous feedback control structure that switches as the system crosses certain manifold in the state space to force the system state to reach, and subsequently to remain on a specified surface within the state space called sliding surface. Once the sliding surface is reached, sliding mode control makes the system states reach the equilibrium point in a finite time and weaken the chattering, playing an important role in sliding mode control [14-15].

The design of sliding mode controllers takes into account the problems of stability and good performance in a systematic way in its approach, which is divided into three main stages [16-19]: choice of surfaces; establishing the conditions of existence, convergence and determination of control law.

A. Generalized sliding mode controller

\(\cdot\) Choice of the sliding surface: The general formula of the sliding surface to ensure the convergence of a variable to its desired value is defined according to the order of the system as follows (proposed by J.Slotine) [19-20]:

\[
S(x) = \left(\frac{n}{\lambda} + \lambda\right)^{n-1} e_i(x)
\]

where \(e_i(x) = x_{ref} - x\);

\(\lambda\) is a positive constant which interprets the bandwidth of the error dynamics, and \(n\) is relative degree representing the number of times to derive the output to display the command [15, 16].

\(\cdot\) Convergence conditions: Lyapunov function

The Lyapunov function [22] is used to estimate the performance of the control for the study of the robustness. It guarantees the stability of the nonlinear system and the attraction of the variable to control towards its reference value [17,23-25]. If the surface is attractive, the sliding mode controller must be chosen so that the Lyapunov function satisfies.

The Lyapunov stability criterion given by:

\[
V(S) = \frac{1}{2} S^2
\]

\[\text{(17)}\]

Its derivative is:

\[
\dot{V}(S) = S_i \dot{S}_i
\]

\[\text{(18)}\]

The law of the command must decrease this function \(\dot{V} < 0\); the idea is to choose a scalar function \(S_i(t)\) to guarantee the attraction of the variable to be controlled to its reference value, and to design a command \(U_i\) such that the
square of the surface corresponds to a Lyapunov function. For a convergence in finite time, the condition (18) which only guarantees an asymptotic convergence towards the sliding surface is replaced by a more restrictive condition called η-attractivity (the condition of attractiveness) and given by [07]: 
\[ S \leq -K|s| ; K \geq 0 \]

B. Control strategy of WWS

1) Controller design without WWS

Once the sliding surface has been chosen, as well as the convergence criterion, it remains to be a necessary condition to bring the variable to be controlled to the surface and then to its point of equilibrium by maintaining the existence condition of the sliding modes.

Hence, the control input \( U_1 \) (The control variable) consists of two components, a discontinuous component (the nonlinear component) \( U_{ni} \) to drive the system states on to the sliding surface and a continuous component (the equivalent component) \( U_{eqi} \) which ensures the motion of the system on the sliding surface whenever it is on the surface to force the error variables to the origin. So, the structure of a sliding mode controller is given as a sum of two components \( U_{eqi} \) and \( U_{ni} \) [19]:

\[ U_i(t) = U_{eqi} + U_{ni} \]  

(19)

The discontinuous component \( U_{ni} \) is of the form given by \( U_{ni} = \dot{S}_i - \eta sgn(S_i) \)  

(20)

with \( \eta \) is a positive gain

\[ \eta > 0; S_i(x) = \begin{cases} +1 & \text{if } S > 0 \\ -1 & \text{if } S < 0 \end{cases} \]

and the continuous one \( U_{eqi} \) is of the form which satisfies \( \dot{s}_i = 0 \).

However, in spite of the different advantages of sliding modes control, its use has an important disadvantage which is called chattering. It is carried out using a switching logic \( sgn(S_i) \) which is unfortunately the source of a very high frequency oscillation phenomenon.

These control commutations can degrade the control performance or can initiate high undesirable dynamic frequencies in the system. Different methods to avoid this phenomenon are studied in the literature [17-19]. Chattering can be avoided with this solution but unfortunately sliding mode performances will be compromised. To overcome this problem, the term \( sgn(S_i) \) is replaced by the function of \( \tanh(S_i) \) [16].

a) Sliding-mode velocity controller

Proposition 2.1 [26]: for the velocity of work roll \( V_2 \) defined by (2.8), the principle of the control by sliding modes consists of making the error \( e_2(t) \) asymptotically tends to zero in finite time; the command is in the form

\[ U_2 = U_{eq2} + U_{n2} \]  

(21)

The degree of the sliding surface is equal to one, to control the velocity of work roll with the main motor in web winding system, a first order switching surface \( S_2 \) is chosen as follow

\[ S_2 = e_2 \]  

(22)

with

\[ e_2 = V_2 - V_2^{ref} \]  

(23)

is the tracking error.

Differentiating \( S_2 \) with respect to time and using (3), the following can be obtained

\[ \dot{S}_2 = V_2 - V_2^{ref} \]

\[ \dot{S}_2 = \theta_i(T_3 - T_1) + \theta_2 V_2^\prime + \theta_0 U_2 - V_2^{ref} \]  

(24)

In sliding mode and during steady-state, we have \( S_1 = 0 \), \( S_2 = 0 \)and \( U_n = 0 \). Hence, we deduce the expression of the equivalent control:

\[ U_{eq2} = -\frac{1}{\theta_i}\theta_0 [\theta_i(T_3 - T_1) + \theta_2 V_2 - V_2^{ref}] \]  

(25)

We want to guarantee derivative of the Lyapunov candidate function \( \dot{V}(S_2) \) < 0 for this we define:

\[ \dot{S}_2 = -K \frac{1}{\theta_0 \theta_1} sgn(s) \]  

(26)

So the derivative of Lyapunov is

\[ \dot{V}(S_2) = -K \frac{1}{\theta_0 \theta_1} S_2 sgn(S_2) \]  

(27)

The Lyapunov function is always defined as negative for positive values of \( K \); the Lyapunov stability criterion is therefore satisfied and the form of the controller effort, \( u \) indicated in Eq. (4) is realized.

So the discontinuous part of the command is given by:

\[ U_{n2} = \dot{S}_2 = -K \frac{1}{\theta_0 \theta_1} sgn(S_2) \]  

(28)

The term \( sgn(S_2) \) is replaced by the function of \( \tanh(S_2) \). So the order law is as follows:

\[ U_2 = -\frac{1}{\theta_0} \left[ \theta_0 (T_3 - T_1) + \theta_2 V_2 - V_2^{ref} \right] - K \frac{1}{\theta_0 \theta_1} \tanh(S_2) \]  

(29)

b) Sliding-mode tension controllers

The purpose of the control is to regulate in real time the tension \( T_2 \) and \( T_3 \) using a super-twisting algorithm in second-order sliding mode [21].

Proposition 2.1 [26]: for the system of the tractive force \( T_1 \) and \( T_2 \) defined by (2.8) and (2.9), the principle of the control by sliding modes consists of making the errors \( e_1(t) \) and \( e_3(t) \) tend asymptotically tends to zero in finite time; they have two terms:

\[ U_i = U_{eq1} + U_{n1} \]  

(30)

\[ U_i = U_{eq3} + U_{n3} \]  

(31)

Let’s define the following control errors:

\[ e_1 = T_1 - T_1^{ref} \]  

(21)

with \( e_1 = \theta_1 V_2 T_1 + \theta_0 (V_2 - V_1) - T_1^{ref} \)  

(22)

\[ e_3 = T_3 - T_3^{ref} \]  

(23)

with \( e_3 = \theta_3 V_2 T_3 + \theta_0 (V_2 - V_3) - T_3^{ref} \)  

(24)

For the subsystems sys1 and sys3, the relative degree is \( r = 2 \), for that we choose the sliding surface as follows:

\[ S_1 = e_1 + \alpha_1 e_1 \]  

(34)

and \( S_3 = e_2 + \alpha_3 e_2 \)  

(35)

- Determination of controller \( U_1 \)

We have \( S_1 = e_1 + \alpha_1 e_1 \) with \( \alpha_1 \) is a positive constant, \( \alpha_1 > 0 \).

\[ S_1 = \theta_1 V_2 T_1 + \theta_2 (V_2 - V_1) - T_1^{ref} + \alpha_1 (T_1 - T_1^{ref}) \]  

(36)

The first-time derivative of the sliding surface and using (31), one can obtain

\[ S_1 = \theta_1 V_2 T_1 + \theta_2 (V_2 - V_1) + \theta_3 (V_2 - V_3) - T_1^{ref} - \alpha_1 T_1^{ref} \]  

(37)

\[ - \frac{1}{\theta} \left[ \theta_0 (T_3 - T_1) + \theta_2 V_2 - V_2^{ref} \right] \]  

(38)

\[ - K \frac{1}{\theta_0 \theta_1} \tanh(S_2) \]  

(39)

\[ U_2 = -\frac{1}{\theta_0} \left[ \theta_0 (T_3 - T_1) + \theta_2 V_2 - V_2^{ref} \right] - K \frac{1}{\theta_0 \theta_1} \tanh(S_2) \]  

(40)

\[ U_2 = -\frac{1}{\theta_0} \left[ \theta_0 (T_3 - T_1) + \theta_2 V_2 - V_2^{ref} \right] - K \frac{1}{\theta_0 \theta_1} \tanh(S_2) \]  

(41)

\[ U_2 = -\frac{1}{\theta_0} \left[ \theta_0 (T_3 - T_1) + \theta_2 V_2 - V_2^{ref} \right] - K \frac{1}{\theta_0 \theta_1} \tanh(S_2) \]  

(42)
= (\theta_1 T_1 + \theta_2) \hat{V}_2 + (\theta_1 V_2 + \alpha_1) \hat{T}_1 - \theta_2 \hat{V}_1 - \hat{T}_1^{\text{ref}} - \alpha_1 \hat{T}_1^{\text{ref}}

\dot{S}_1 = (\theta_1 T_1 + \theta_2) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_1 V_2 + \alpha_1) (\theta_1 V_2 T_1 + \theta_2 (V_2 - V_1))
- \theta_2 (\theta_1 V_1 + \theta_1 T_1) - \hat{T}_1^{\text{ref}} - \alpha_1 \hat{T}_1^{\text{ref}}
\quad - \alpha_1 \hat{T}_1^{\text{ref}} (37)

In sliding mode and during steady-state, we have \( S_1 = 0, \) \( \dot{S}_1 = 0 \) with \( U_3 = 0. \) Hence, we deduce the expression of the corresponding command:

\[ U_{eq1} = \frac{1}{\theta_2 \theta_5} \left[ (\theta_1 T_1 \theta_2) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_1 V_2 + \alpha_1) (\theta_1 V_2 T_1 + \theta_2 (V_2 - V_1))
- \theta_2 (\theta_1 V_1 + \theta_1 T_1) - \hat{T}_1^{\text{ref}} - \alpha_1 \hat{T}_1^{\text{ref}} \right] \quad (38)

According to the Lyapunov stability theorem, the inequality must be satisfied for the system converging to the manifold, we want to guarantee \( \dot{V}(S_1) < 0, \) for that we define:

\[ \dot{S}_1 = -K_1 \frac{1}{\theta_2 \theta_5} \text{sign}(S_1) \quad (39) \]

So, the derivative of Lyapunov is

\[ \dot{V}(S_1) = -K_1 \frac{1}{\theta_2 \theta_5} |S_1| < 0 \quad (40) \]

The function of Lyapunov [20]: is always defined as negative for the positive values of \( K_1 \theta_2 \theta_5 \), the Lyapunov stability criterion is therefore satisfied and the form of the controller effort, \( u \) indicated in Eq. (4) is realized.

So, the discontinuous part of the command is given by:

\[ U_{n1} = \dot{S}_1 = -K_1 \frac{1}{\theta_2 \theta_5} \text{sign}(S_1) \quad (42) \]

The term \( \text{sign}(S_1) \) is replaced by the function of \( \tanh(S_1) \). So, the order law is as follows:

\[ U_1 = \frac{1}{\theta_2 \theta_5} \left[ (\theta_1 T_1 + \theta_2) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_1 V_2 + \alpha_1) (\theta_1 V_2 T_1 + \theta_2 (V_2 - V_1))
- \theta_2 (\theta_1 V_1 + \theta_1 T_1) - \hat{T}_1^{\text{ref}} - \alpha_1 \hat{T}_1^{\text{ref}} \right]
- \frac{1}{\theta_2 \theta_5} \tanh(S_1) \quad (43) \]

- Determination of controller \( U_3 \)

We have \( S_3 = e_3 + \alpha_3 e_3 \) where \( \alpha_3 > 0 \) is a positive constant, and \( e_3 > 0 \).

\[ S_3 = \theta_3 V_2, T_3 + \theta_{10} (V_3 - V_2) - \hat{T}_3^{\text{ref}} + \alpha_3 (T_3 - T_3^{\text{ref}}) \]

\[ S_3 = (\theta_3 V_2 + \alpha_3) T_3 + \theta_{10} (V_3 - V_2) - \hat{T}_3^{\text{ref}} - \alpha_3 T_3^{\text{ref}} \quad (44) \]

The first-time derivative of the sliding surface and using (20), one can obtain

\[ \dot{S}_3 = \theta_3 V_2 T_3 + (\theta_3 V_2 + \alpha_3) \hat{T}_3 + \theta_{10} (\hat{V}_3 - \hat{V}_2) - \hat{T}_3^{\text{ref}} - \alpha_3 \hat{T}_3^{\text{ref}} \]

\[ = (\theta_3 T_3 - \theta_{10}) V_2 + (\theta_3 V_2 + \alpha_3) \hat{T}_3 + \theta_{10} \dot{V}_3 - \hat{T}_3^{\text{ref}} - \alpha_3 \hat{T}_3^{\text{ref}} \]

\[ \dot{S}_3 = (\theta_3 T_3 - \theta_{10}) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_3 V_2 + \alpha_3) (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3)
+ \theta_{10} (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3) - \hat{T}_3^{\text{ref}} - \alpha_3 \hat{T}_3^{\text{ref}} \quad (45) \]

In sliding mode and during steady-state, we have \( S_3 = 0, \) \( \dot{S}_3 = 0 \) with \( U_3 = 0. \) Hence, we deduce the expression of the corresponding command:

\[ U_{eq3} = -\frac{1}{\theta_{10} \theta_{13}} \left[ (\theta_3 T_3 - \theta_{10}) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_3 V_2 + \alpha_3) (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3)
+ \theta_{10} (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3) - \hat{T}_3^{\text{ref}} - \alpha_3 \hat{T}_3^{\text{ref}} \right] \quad (46) \]

We want to guarantee derivative of Lyapunov’s candidate function \( \dot{V}(S_3) < 0, \) for that we define:

\[ \dot{S}_3 = -K_3 \frac{1}{\theta_{10} \theta_{13}} \text{sign}(s) \quad (47) \]

So, the derivative of Lyapunov is

\[ \dot{V}(S_3) = -K_3 \frac{1}{\theta_{10} \theta_{13}} S_3 \text{sign}(S_3) \]

\[ \dot{V}(S_3) = -K_3 \frac{1}{\theta_{10} \theta_{13}} |S_3| < 0 \quad (48) \]

The function of Lyapunov [Slotine.1991]: is always defined as negative for the positive values of \( \frac{1}{\theta_{10} \theta_{13}} \) the Lyapunov stability criterion is therefore satisfied and the form of the controller effort, \( u \) indicated in Eq. (4) is realized.

So, the discontinuous part of the command is given by:

\[ U_{n3} = \dot{S}_3 = -K_3 \frac{1}{\theta_{10} \theta_{13}} \text{sign}(S_3) \quad (49) \]

The term \( \text{sign}(S_3) \) is replaced by the function of \( \tanh(S_3) \). So, the order law is as follows:

\[ U_3 = -\frac{1}{\theta_{10} \theta_{13}} \left[ (\theta_3 T_3 - \theta_{10}) (\theta_6 (T_3 - T_1) + \theta_7 V_2 + \theta_8 U_2)
+ (\theta_3 V_2 + \alpha_3) (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3)
+ \theta_{10} (\theta_1 T_3 + \theta_1 V_2 + \theta_1 U_3) - \hat{T}_3^{\text{ref}} - \alpha_3 \hat{T}_3^{\text{ref}} \right]
- K \frac{1}{\theta_{10} \theta_{13}} \tanh(S_3) \quad (50) \]

Although this command has important advantages such as robustness with regard to variable parameters, there are nevertheless disadvantages when it is the need to know precisely a number of derivatives of the organ to be controlled, according to the order of the system. In practice, some tensions derivatives are not easily measurable due to technical and economic constraints when they are not most often contaminated by noise or not at all available. Therefore, it is interesting to consider the regulation of these organs from the observation of non-measurable derivatives. Thus, the use of an observer is indispensable and more particularly the use of a continuous observer in sliding mode.

2) Controller design with observer

The control variable \( U \) should not only guarantee the sufficiently small estimation error of the observer super-twisting, but also make the output \( y \) track the demand input of WWs. In this section, the controller is designed by sliding mode method based on observer super-twisting. The architecture diagram of the control and observer design in this work is illustrated in Fig.1.
Due to unknown variable state $T_1$ and $T_3$, the variable $e_1$ and $e_3$ cannot be obtained and the controller $u$ shown in (43) and (50) cannot be used directly. In this section, the sliding mode controller is further processed by state estimation $\tilde{T}_1$ and $\tilde{T}_3$.

According to the backstepping iteration, the virtual control variables are derivate for the augmented model (sys1) as follows with

- The controller $U_1$

The switching surface $S_1$ is chosen as follow

$$S_1 = (\theta_1 V_2 + \alpha_1)\tilde{T}_1 + \theta_2 (V_2 - V_1) - T_1^{ref} - \alpha_1 T_1^{ref}$$

By using (38), the expression of the equivalent control:

$$U_{eq1} = \frac{1}{\theta_2 \theta_5} \left[ (\theta_1 \tilde{T}_1 + \theta_2) (\theta_6 (\tilde{T}_3 - \tilde{T}_1) + \theta_7 V_2 + \theta_8 \cdot U_2) + (\theta_2 V_2 + \alpha_1) (\theta_1 V_2 \tilde{T}_1 + \theta_5 (V_2 - V_1)) - \tilde{T}_1^{ref} - \alpha_1 \tilde{T}_1^{ref} \right]$$

The discontinuous part of the command is given by:

$$u_{n1} = \dot{S}_1 = -K_1 \frac{1}{\theta_2 \theta_5} \text{sign}(S_1)$$

To obtain the final control $u$, we should compute the derivatives of the equivalent control and discontinuous part of the command as follows

$$U_{eq1} = \frac{1}{\theta_2 \theta_5} \left[ (\theta_1 \tilde{T}_1 + \theta_2) (\theta_6 (\tilde{T}_3 - \tilde{T}_1) + \theta_7 V_2 + \theta_8 \cdot U_2) + (\theta_2 V_2 + \alpha_1) (\theta_1 V_2 \tilde{T}_1 + \theta_5 (V_2 - V_1)) - \tilde{T}_1^{ref} - \alpha_1 \tilde{T}_1^{ref} \right]$$

- The controller $U_3$

The switching surface $S_3$ is chosen as follow

$$S_3 = (\theta_2 V_2 + \alpha_3)\tilde{T}_3 + \theta_10 (V_3 - V_2) - T_3^{ref} - \alpha_3 T_3^{ref}$$

By using (46), the expression of the equivalent control:

$$U_{eq3} = \frac{1}{\theta_10 \theta_{13}} \left[ (\theta_9 \tilde{T}_3 - \theta_{10}) (\theta_8 (\tilde{T}_3 - \tilde{T}_1) + \theta_7 V_2 + \theta_8 \cdot U_2) + (\theta_2 V_2 + \alpha_3) (\theta_12 \tilde{T}_3 + \theta_12 \cdot V_3) + \theta_{13} \cdot U_3 + \theta_{10} (\theta_{11} \tilde{T}_3 + \theta_{12} \cdot V_3) - \tilde{T}_3^{ref} - \alpha_3 \tilde{T}_3^{ref} \right]$$

The discontinuous part of the command is given by:

$$u_{n3} = \dot{S}_3 = -K_3 \frac{1}{\theta_{10} \theta_{13}} \text{sign}(S_3)$$

To obtain the final control $u$, we should compute the derivatives of the equivalent control and discontinuous part of the command as follows

$$U_3 = \frac{1}{\theta_{10} \theta_{13}} \left[ (\theta_9 \tilde{T}_3 - \theta_{10}) (\theta_8 (\tilde{T}_3 - \tilde{T}_1) + \theta_7 V_2 + \theta_8 \cdot U_2) + (\theta_2 V_2 + \alpha_3) (\theta_11 \tilde{T}_3 + \theta_12 \cdot V_3) + \theta_{13} \cdot U_3 + \theta_{10} (\theta_{11} \tilde{T}_3 + \theta_{12} \cdot V_3) - \tilde{T}_3^{ref} - \alpha_3 \tilde{T}_3^{ref} \right]$$

$$- \frac{1}{\theta_{10} \theta_{13}} \text{tanh}(S_3)$$

V. SIMULATION AND DISCUSSION OF RESULTS

In order to evaluate the control law studied in combination with the observer design, numerical simulations using the MATLAB / SIMULINK software was performed on the complete closed-loop system.

The controller and observer gains are selected as follows:

$$\alpha_1 = 0.02, \alpha_3 = 0.02, K_1 = -20, K_2 = 300, K_3 = 1000, \lambda_1 = 0.001, \lambda_3 = 0.01$$

Figure 2 clearly shows that the real web tensions converge towards their estimations and the estimation voltages converge towards their reference trajectory with a response time equal to 3.5 sec.

Figure 3, demonstrated the high performance of the sliding mode control strategy for the velocity $V_2$. Where we clearly see the high tracking performance and the velocity converges to their reference trajectory, with a response time equal to 5 sec without exceeding. So, convergence and sliding mode along the surface is verified.
As seen in Simulation Results, we obtain a good trajectory and a good adaptation of the transient phenomenon despite the strong non-linearity of the WWS and its figures show that the tension and the velocity does not undergo chatter phenomenon.

In order to demonstrate the interest of the proposed strategy, its performances were compared with those of other techniques [12-13] which grouped in the following table:

Table- II: comparison between several control techniques for WWS

| papers            | Control Strategy               | % Peak overshoot | The settling time for 5% |
|-------------------|--------------------------------|-------------------|-------------------------|
| Rabbah et al. (2015) | backstepping $V_2$              | 2%                | 7s                      |
|                   | ($T_1$,$T_3$)                  | 0%                | 15s                     |
| Zheng et al. (2015) | ILQ design method $V_2$    | 2%                | 2s                      |
|                   | ($T_1$,$T_3$)                  | 18%               | 1s                      |
| PID design        | $V_2$                          | 0%                | 2s                      |
|                   | ($T_1$,$T_3$)                  | 10%               | 2.5s                    |
| pole placement    | $V_2$                          | 0%                | 2s                      |
|                   | ($T_1$,$T_3$)                  | 16%               | 1.2s                    |
| akil et al. (2017) | backstepping control with integral action $V_2$ | 13% | 0.2 |
|                   | ($T_1$,$T_3$)                  | 0%                | 0.3s                    |
| In this paper     | Sliding mode $V_2$             | 0.001%            | 0.075                   |
|                   | ($T_1$,$T_3$)                  | 0%                | 0.45                    |

Simulations show that the observer and the controller give satisfactory results and that the behavior of the complete closed-loop system is stable. Finally, it is important to note that the observation process demonstrates good prosecution even under quite severe operating conditions.

The various simulation tests mentioned allowed us to confirm the robustness of the nonlinear control and their controllers. As well as these results obtained by the application of the proposed command gives appreciable performances and a better response time to that of the command ILQ and backstepping.

VI. SIMULATION WITH UNCERTAINTIES

The test of the robustness of the control allows us to verify if the command applied can adapt to the uncertainties of the parameters of system which can be due to the precision of the sensors, the forces of friction, the unforeseeable external factors.

This time we simulate our system, but we add a disturbance to the tensions and the velocity fig.4.

Fig. 4. Disturbance added to velocity and tensions

Fig. 5 shows the simulation results of the sliding mode control in the case of uncertainty.

Fig. 5. Simulation result for the robustness test with respect to uncertainties

Note well from Figure 5 that the system can overcome this uncertainty and cause the system to stabilize more effectively.

The results show that good position tracking performance and robustness can be obtained, the chatter phenomenon can be effectively mitigated. This result allows us to conclude that control and observation by sliding mode presents a very attractive solution for the control of the WWS.

VII. CONCLUSION

The richness of this work is the development of a nonlinear control law based on the theory of sliding mode. The latter gives a better performance for the control of the WWS and it also validated the observer proposed tensions while reducing the phenomenon of chatter. This robust observer shows improved WWS performance.

The simulations have proved that this method is really efficient and very interesting, they show the effectiveness of the proposed approach and it has good performance in terms of robustness and stability of convergence despite the disturbances introduced. And they also show that the proposed observers have a good estimation accuracy.
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