Abstract

We present a relativistic treatment of the problem of soft electromagnetic structure by the modified instant form of relativistic Hamiltonian dynamics. Our approach uses relativistic parametrization and so picks out the relativistic invariant quantities on each stage of the calculation. The electromagnetic current matrix element satisfies the current conservation law automatically. We use relativistic modified impulse approximation. It is constructed in relativistic invariant way. For composite systems (including the spin 1 case) the approach guarantees the uniqueness of the solution and it does not use such concepts as "good" and "bad" current components. The approach describes correctly the spin Wigner rotation and so gives the correct (QCD) asymptotic.

The relativistic description of bound states was always an important problem in nuclear physics and particle physics. This problem became particularly topical in connection with the development of quark physics, in which the relativistic properties of the light quarks play a fundamental role.

In the relativistic theory of the description of composite systems, it is possible to identify two main but very different approaches.

The first is the method of field theory. Based on the principles of quantum field theory – quantum chromodynamics (QCD) – it is rightly regarded as the most consistent approach to the solution of this problem. However, standard perturbative QCD gives sufficiently reliable computational prescriptions only for the description of so-called "hard" processes, which are characterized by large momentum transfers, and it does not permit the calculation of characteristics determined by "soft" processes. Moreover, there are strong indications [1] that perturbative QCD is not valid for the description of the currently existing experimental facts in exclusive processes. This applies, in particular, to the description of the elastic form factors of such well–studied composite systems as the pion, kaon, nucleon and deuteron. Of course, in the framework of field theory itself there exist various approaches to overcoming these difficulties. For example, there are the well–known approaches associated with the use of the Bethe–Salpeter equation (see e.g. [2, 3, 4]), and quasipotential approaches (see e.g. [4, 5, 6, 7, 8]).

The second method in the relativistic theory of composite systems, in the framework of which we shall operate, is based on the direct realization of the algebra of the Poincaré group on the set of dynamical observables on the Hilbert state space of the system. This approach is called the theory of direct interaction, or relativistic
Hamiltonian dynamics (RHD) (for review see \[9\] and references therein). RHD unifies the potential approach to composite systems and the condition of Poincaré–invariance. It should be noted that the establishment of the connection between RHD and field theory is a difficult and as yet unresolved problem. The idea of RHD goes back to a paper of Dirac \[10\], in which he considered the different methods of describing the evolution of classical relativistic systems – differing in the evolution parameter: point form (PF), instant form (IF), and light–front (LF) dynamics.

There now exists a large number of studies of the use of LF dynamics (see, for example, \[11, 12, 13, 14, 15, 16, 17, 18, 19, 20\] and the references given there). Some studies also contain investigations using other form of dynamics. However, most of quantitative investigations of specific systems that have so far been made are associated with LF dynamics. In particular, this is because this form of dynamics has the smallest number (only 3) of generators that contain interaction. There are some other advantages that caused the fact that LF dynamics is widely used. For example it is the possibility of interpreting the results with the help of Feynmann diagrams calculated in the infinite–momentum frame; the antiparticle contributions to Feynman diagrams are suppressed. However, the use of the LF dynamics leads to certain difficulties that are associated with the loss of rotational invariance \[21, 22\], since the generator of the total angular momentum contains the interaction. Moreover, the space reflection and time reversal operators necessarily depend on interactions \[23\].

Some time ago it was proved that S matrices are equivalent in the different dynamics forms \[24\]. This fact is interesting but it does not mean the equivalence of the forms. First, there are problems which can not be reduced to S matrix, e.g. the calculation of form factors. Second, one has to keep in mind that any concrete calculation uses some approximations; the approximations usually used in different forms of dynamics are nonequivalent.

Our point of view is the following. One must not be conservative, one must choose the form adequate to the problem in question and to the approximation to be done. It seems us that this is in the spirit of RHD – the choosing of the adequate degrees of freedom.

Now we present a relativistic treatment of the problem of soft electromagnetic structure in the framework of IF of RHD \[25, 26, 27, 28\]. IF of relativistic dynamics, although not widely used, has some advantages. The calculations can be performed in a natural straightforward way without special coordinates. IF is particularly convenient to discuss the nonrelativistic limit of relativistic results. This approach is obviously rotational invariant, so IF is the most suitable for spin problems.

Our approach to electromagnetic structure of two–particle composite systems has the following advantages.

- The electromagnetic current matrix element satisfies the current conservation law automatically.
- We use relativistic modified impulse approximation (MIA). It is constructed in
relativistic invariant way. This means that our MIA does not depend on the choose of the coordinate frame, and this contrasts principally with the "frame–dependent" impulse approximation usually used in IF dynamics.

- Our approach provides with correct and natural nonrelativistic limit ("the correspondence principle" is fulfilled).

- For composite systems (including the spin 1 case) the approach guarantees the uniqueness of the solution and it does not use such concepts as "good" and "bad" current components.

- The approach describes correctly the spin Wigner rotation and so gives the correct (QCD) asymptotic.

It is also worth to notice that our approach is directly linked with the dispersion approach of quantum field theory \[29, 30, 31\] and that it gives the adequate description of concrete composite systems: \(\pi^-, K^-\) mesons (quarks systems) and deuteron (nucleons system).

Let us describe briefly the main steps of investigation using as an example quark–antiquark system electromagnetic properties.

Let us consider \(\pi\) meson and \(K\) meson as quark \((q)\)– antiquark \((\bar{Q})\) composite system. We shall use different quark masses \(M_q\) and \(M_{\bar{Q}}\) as in \(K\) meson. The results for \(\pi\) meson can be obtained if \(M_q = M_{\bar{Q}}\).

The charge form factor for two-quark system can be obtained from the electromagnetic current matrix element for composite system

\[
< p_c | j_\mu | p'_c > = (p_c + p'_c)_\mu F_c(Q^2),
\]

\(F_c(Q^2)\) – electromagnetic form factor of composite system, \(p – 4\)-momentum of system.

We shall act following the basic assumptions, valid for all forms of dynamics in RHD \[1\]. The RHD is based on the including of the operator, describing \(q\bar{Q}\) interaction in the generators of Poincaré group while the commutation relations of Poincaré algebra are fulfilled. One usually includes \(\hat{U}\) in the mass square operator of the free two particle system in additive way \[3\]: \(P^2 = (p_1 + p_2)^2 \rightarrow \hat{M}^2_I = P^2 + \hat{U}\). In the case of IF dynamics the Poincaré algebra is conserved if \(\hat{U}\) commutes with the total angular momentum operator \(\hat{J} = (\hat{J}_1, \hat{J}_2, \hat{J}_3)\), with the operator of total 3-momentum \(\hat{P}\) and with the operator \(\nabla_P\). The complete set of commuting operators for the two-particle system with interaction contains now: \(\hat{M}^2_I, \hat{J}, \hat{P}\). In the case of IF the operators \(\hat{J}_1, \hat{J}_3, \hat{P}\) coincide with the appropriate operators of the two-particle system without interaction and one can construct the basis in which these three operators are diagonals. While working in this basis to obtain the wave function one needs to diagonalize \(\hat{M}^2_I\).

In RHD the Hilbert space of composite particle states is the tensor product of single particle Hilbert spaces: \(\mathcal{H}_{q\bar{Q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{Q}}\) and the state vector in RHD is a superposition
of two-particle states. As a basis in $\mathcal{H}_{q\bar{q}}$ one can choose the following set of vectors:

$$|\vec{p}_1, m_1; \vec{p}_2, m_2 >= |\vec{p}_1, m_1 > \otimes |\vec{p}_2, m_2 >,$$

$$< \vec{p}, m | \vec{p}', m' >= 2p_0 \delta(\vec{p} - \vec{p}') \delta_{mm'},$$

(2)

Here $\vec{p}_1, \vec{p}_2$ — are particle momenta, $m_1, m_2$ — spin projections.

Since we consider the two-quark system as one composite system, then the natural basis is one with separated center-of-mass motion:

$$|\vec{P}, \sqrt{s}, J, l, S, m_J >,$$

(3)

with $P_\mu = (p_1 + p_2)_\mu, P^2 = s, \sqrt{s}$ — the invariant mass of two-particle system, $l$ — the angular momentum in the center-of-mass frame, $S$ — total spin, $J$ — total angular momentum, $m_J$ — projection of total angular momentum.

The basis (3) is connected with (2) through the Clebsch – Gordan decomposition of the Poincaré group. Now the decomposition of the electromagnetic current matrix element for the composite system (1) in the basis (3) has the form

$$< \vec{P}, \sqrt{s}, J, l, S, m_J | j_\mu | \vec{P}', \sqrt{s}', J', l', S', m_J' >.$$

Here the sum is over the discrete variables of the basis (3).

$$< \vec{P}', \sqrt{s}', J', l', S', m_J' | p_{c'} >.$$

(4)

The vector $A_\mu(s, Q^2, s')$ is defined by the current transformation properties (by the Lorentz–covariance and the current conservation law):
\[ A_\mu = (1/Q^2)[(s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu]. \] (7)

In our parametrization the current is conserved by construction:
\[ A_\mu(s, Q^2, s')Q^\mu = 0. \] (8)

In the frame of basis (2) non-interacting current matrix element has the following form:
\[ <\vec{p}_1, m_1; \vec{p}_2, m_2|j_\mu|\vec{p}'_1, m'_1; \vec{p}'_2, m'_2> =
\[ = <\vec{p}_1, m_1|\vec{p}'_1, m'_1><\vec{p}_2, m_2|\vec{p}'_2, m'_2> + (1 \leftrightarrow 2). \] (9)

This is, as a matter of fact, the relativistic impulse approximation. The one-particle current in (9) is expressed in terms of one-quark form factors. Clebsh-Gordan decomposition of the basis (3) into basis (2) gives the expression of free form factor \( g_0(s, Q^2, s') \) in terms of one-quark form factors:

\[ g_0(s, Q^2, s') = \frac{\sqrt{ss'}}{\sqrt{[s^2 - 2s(M^2_s + M^2_u) + \eta^2][s'^2 - 2s'(M^2_s + M^2_u) + \eta^2]}} 
\cdot \frac{Q^2(s + s' + Q^2)}{2[\lambda(s, -Q^2, s')]} \cdot \left( B^u(s, Q^2, s') + B^s(s, Q^2, s') \right), \] (10)

\[ B^s(s, Q^2, s') = \left[ f_1^{(s)}(s + s' + Q^2 - 2\eta) \cos(\omega_1 + \omega_2) - 
\right. 
\left. - f_2^{(s)} \frac{M_s}{2} \xi(s, Q^2, s') \sin(\omega_1 + \omega_2) \right] \theta(s, Q^2, s') , \]

\[ \xi(s, Q^2, s') = \sqrt{-\lambda(s, -Q^2, s')M^2_s + ss'Q^2 - \eta Q^2(s + s' + Q^2) + Q^2\eta^2} , \]

\[ \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc) , \]

\[ f_1^{(s)} = \frac{2M_s G_E^{(s)}(Q^2)}{\sqrt{4M^2_s + Q^2}} ; \quad f_2^{(s)} = -\frac{4 G_M^{(s)}(Q^2)}{M_s\sqrt{4M^2_s + Q^2}} , \]

\[ \omega_1 = \arctg \left( \frac{\xi(s, Q^2, s')}{M_u[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + (\sqrt{s} + \sqrt{s'})^2 + \eta] \right) , \]

\[ \omega_2 = \arctg[(\sqrt{s} + \sqrt{s'}) + 2M_s] \xi(s, Q^2, s') . \]

\{M_s(s + s' + Q^2)(\sqrt{s} + \sqrt{s'} + 2M_s) + \sqrt{ss'Q^2(4M^2_s + Q^2) - \eta[2M_s(\sqrt{s} + \sqrt{s'}) - Q^2]}\}^{-1} , \]

\[ \theta(s, Q^2, s') = \theta(s' - s_1) - \theta(s' - s_2) , \]

Here \( \theta \) is the standard step function, \( G_E^{(s)}(Q^2) \) and \( G_M^{(s)}(Q^2) - Sachs quark form factors, \( \omega_1 \) and \( \omega_2 \) – are the Wigner rotation parameters.

\[ s_{1,2} = M^2_s + M^2_u + \frac{1}{2M^2_s}(2M^2_s + Q^2)(s - M^2_s - M^2_u) \mp \]
\[
\mp \frac{1}{2m_s^2} \sqrt{Q^2(4m_s^2 + Q^2)[s^2 - 2s(M_s^2 + M_a^2) + \eta^2]}
\]

Function \( B^\nu(s, Q^2, s') \) can be deduced from \( B^s(s, Q^2, s') \) by substitution \( M_s \leftrightarrow M_a \).

Let us return now to the Eq.(4). The current matrix element entering the r.h.s. of Eq.(4) must be interaction dependent. This dependence is known to be a consequence of the current conservation law and of the condition of current relativistic covariance. This means that we can not use in Eq. (4) the parametrization of non-interacting current matrix element (6) directly and need to include the interaction. Let us perform the interaction including in (6) in minimal manner: we shall include the interaction only in the vector function \( A^\mu(s, Q^2, s') \) in Eqs. (6), (7):

\[ A^\mu(s, Q^2, s') \rightarrow \left. \frac{N_C - G}{N_C - G} A^\mu_{\text{int}} \right| \)

\[ A^\mu_{\text{int}} = A^\mu(s, Q^2, s') \bigg|_{p'_\mu \rightarrow p_{\mu}, \ p'_{\mu} \rightarrow p_{\mu}} = (p_c + p'_c)_\mu , \]

\[ g_0(s, Q^2, s') \rightarrow g(s, Q^2, s') = g_0(s, Q^2, s'). \]

The current matrix element in Eq.(4) is a product of a 4-vector and a scalar function (form factor). This form is quite similar to the form of electromagnetic current matrix element for two non-interacting particles (6), or to the pion electromagnetic current matrix element (1) and can differ only by the explicit form of form factors and 4-vectors. The number of form factors is the same, because all these matrix elements are taken between the states with \( J = l = S = m, J = 0 \). Note, that all the normalization constants, which are not invariant, are included in the covariant part.

Let us rewrite the equation (4) using meson wave function (3) and current matrix element explicitly:

\[ (p_c + p'_c)_\mu F_c(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(k) A^\mu_{\text{int}}(s, Q^2, s') g(s, Q^2, s') \varphi(k'). \]

Here we use for simplicity the notation: \( \varphi^I_{ls}(k) \rightarrow \varphi(k) \).

This means that the two 4-vectors are equal and this equality is to be valid for any choice of wave functions \( \varphi(s) \) of the two-particle system internal motion. If the wave function is varied then the scalar part of the l.h.s. (the form factor \( F_c(Q^2) \)) is changed, while the covariant part (the vector \( (p_c + p'_c)_\mu \)) remains unchanged, because the vector \( (p_c + p'_c)_\mu \) describes the system as a whole and does not depend on the interaction inside the system. So, when the wave function is varied the l.h.s. remains to be collinear to the vector \( (p_c + p'_c)_\mu \). In general case the 4-vector in the r.h.s. changes the direction. The equality is valid for an arbitrary choice of wave function only if the vector \( A^\mu_{\text{int}} \) is collinear to the vector \( (p_c + p'_c)_\mu \) in any coordinate system, so that the proportionality factor can be included in the invariant form factor \( g(s, Q^2, s') \). Thus the form (11) for \( A^\mu_{\text{int}} \) is unique and the most general.
The choice (12) for the form factor \( g(s, Q^2, s') \) is not quite general, of course. One can use different physical approximations to evaluate this quantity. The use of \( g_0(s, Q^2, s') \) (10) instead of \( g(s, Q^2, s') \) means relativistic impulse approximation as formulated mathematically in terms of form factors (modified impulse approximation).

The function \( A^\mu_\text{int} \) contains the interaction through the impulses \( p'_\mu \) and \( p_\mu \). Using (4), (6), (11) and (12) we obtain now the following expression for the form factor:

\[
F_c(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(k) g_0(s, Q^2, s') \varphi(k').
\] (14)

For \( \varphi(k) \) one can use any phenomenological wave function, normalized using the relativistic density of states: \( \varphi(k) = \sqrt{s}(1 - \eta^2/s^2) u(k) k \), \( u(k) \) is nonrelativistic phenomenological wave function.

Let us emphasize, that the r.h.s. of Eq.(4) with (11) inserted satisfies the current conservation law: it is orthogonal to the vector \( Q_\mu = (p'_\mu - p_\mu) \). This latter fact is rather noticeable because generally the construction of the conserved current operator for composite systems presents a rather complicated problem which is not solved yet [23]. Thus, the Eq.(14) takes into account the relativistic covariance and the current conservation law. This is right for any choice of the function \( g(s, Q^2, s') \), including the expressions (10), (12) which we use here.

We have obtained the expression (14) for the form factors in the frame of the principally relativistic approach: instant form of RHD. The form factors are expressed in terms of relativistic function \( g(s, Q^2, s') \) and nonrelativistic wave functions \( u(k) \). The behavior of form factor depends essentially on the model type of wave function. The RHD instant form enables one to obtain easily the nonrelativistic limit of Eq.(14). The relativistic effects are important: the difference between relativistic and nonrelativistic form factors for one and the same wave function is very large.

To conclude, we present an approach – the modified IF of RHD – which uses relativistic parametrization and so picks out the relativistic invariant quantities on each stage of the calculation. The approach describes well the data on electromagnetic form factors of \( \pi^- \), \( K^- \) mesons and the deuteron (see the authors’ poster at FBXV).

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