The mass spectra of nucleon resonances with spin 1/2, 3/2, and 5/2 are systematically studied in a constituent quark model with meson-quark coupling, which is inspired by the spontaneous breaking of chiral symmetry of QCD. Meson-quark coupling gives rise not only to the one-meson-exchange potential between quarks but also to the self-energy of baryon resonances, owing to the existence of meson-baryon decay channels. These two contributions are consistently taken into account in the present model. The gross properties of nucleon resonance spectra are reproduced fairly well, although the predicted mass of $N(1440)$ is too large.

§1. Introduction

Although the constituent quark models have been successful in reproducing various static properties of baryons,1,2 there still remain several problems. One of them concerns the mass spectrum of the spin 1/2 nucleon resonances. Conventional models cannot explain the fact that the ground state is remarkably light while the separations among excited states are relatively narrow.3 Furthermore, the predicted mass of $N(1440)$ is too large.

Glozman et al. have recently examined baryon mass spectra by using a constituent quark model with the one-meson-exchange potential (OMEP) as the residual interaction between quarks,4 - 6 while conventional models employ the one-gluon-exchange potential (OGEP) instead.7 The OMEP has been introduced on the basis of spontaneous breaking of chiral symmetry (SBXS) in low-energy QCD. As a direct consequence of SBXS, there appear Nambu-Goldstone bosons (i.e., the flavor-octet pseudoscalar mesons, such as $\pi$, $K$ and $\eta$), and the mass of the light constituent quarks appears.8,9 The flavor-singlet $\eta'$ also has been taken into account in addition to the flavor-octet mesons. Glozman et al. have claimed that their model provides a unified description of the ground states and the excitation spectra of baryon resonances. They have also pointed out that the spin-flavor dependence of the OMEP is important to reproduce the mass spectra.

On the other hand, there exists another type of mesonic effects. The baryon self-energies, which come from the coupling between single baryon states and meson-baryon scattering states, have been studied by one of the authors (M. A.) and his collaborators.10 - 12 They have shown, for example, that the large $\eta N$ decay width of $N(1535)$ as well as the mass splitting between $\Lambda(1405)$ and $\Lambda(1520)$ can be explained by this self-energy effect.
It also has been pointed out by several authors that the static potential models are not able to describe baryon mass spectra satisfactorily. Geiger and Isgur\textsuperscript{13)} have investigated the $s\bar{s}$ contribution generated by hadronic-loops and have found that it plays an important role in the static properties of the proton. Using the unitarized quark model, Zenczykowski\textsuperscript{14)} has also shown that the coupling of baryons to meson-baryon continuum states is crucial in describing baryon spectra.

In this paper, we construct a model consisting of constituent quarks and pseudoscalar mesons with pseudovector meson-quark coupling. The quarks and mesons are treated as the elementary degrees of freedom on the basis of $\text{SBX}\chi\$S. Since the meson-quark coupling gives rise not only to the OMEP but also to the baryon self-energy, these two mesonic effects can be treated on an equal footing. This is quite desirable, since they are both important in describing baryon spectra, as mentioned above.

The OMEP stems mainly from the exchange of an off-energy-shell meson whose energy is nearly equal to zero, while the self-energy results for the most part from an on-energy-shell meson. These two contributions, however, cannot be strictly separated, as is clearly seen in the relativistic formalism, where they correspond to the same diagram.\textsuperscript{15)} Therefore, the problem of double counting should be resolved in order to treat them in a consistent manner.

The purpose of this work is not to obtain a perfect fit to the observed baryon masses but to investigate a consistent model containing the two mesonic effects, i.e., the static one (OMEP) and the dynamic one (self-energy). We do not consider the OGEP, which is the standard interaction between quarks in conventional models, simply because we attempt to construct a consistent quark model with meson-quark coupling and to clarify how the dynamical mesonic correction affects the mass spectra of baryons. We do not, however, dare to say that the OGEP is of no use. It may be the case that the best potential model for constituent quarks includes some contributions from the OGEP.

Specifically, we calculate the energy levels of nucleon resonances $N^*$ in this paper and mainly examine the energy-dependent effect of $N^*$ self-energies, which are absent in conventional calculations. It is also noted that in the present work we explicitly construct the quark wave functions of baryons by utilizing the spatial and spin-flavor symmetries in a systematic way. We therefore can discuss not only the spin-flavor symmetry, which has been emphasized by other studies\textsuperscript{4)},\textsuperscript{14)} but also the spatial distribution of quarks in a baryon.

This paper is organized as follows. In §2, a constituent quark model with meson-quark coupling is presented, and the OMEP and baryon self-energy are derived with an emphasis on the problem of double counting. The method of calculation is also explained. Numerical results for nucleon resonances are given in §3. Section 4 contains a brief summary.

§2. Model

Our model Hamiltonian $H$ consists of the internal Hamiltonian $H_0$ and kinetic energy $T_B$ of the baryon, the total energy of the meson $\omega_M$, and the meson-quark
Nucleon Resonances in a Constituent Quark Model with Chiral Symmetry

Nucleon Resonances in a Constituent Quark Model with Chiral Symmetry

341
coupling $H_I$, as follows:

$$H = H_0 + T_B + \omega_M + H_I ,$$

(2.1)

with

$$H_0 = T_0 + V_{\text{conf}} + M_0 ,$$

(2.2)

where $T_0$ is the kinetic energy of the three quarks in the baryon without their center-of-mass motion (i.e., $T_B$), $V_{\text{conf}}$ is the confinement potential, and $M_0$ is the constant mass parameter. Note that $T_B$ and $\omega_M$ have non-vanishing contributions only for meson-baryon scattering channels.

Non-relativistic kinematics are used for the constituent quarks:

$$T_0 + T_B = \sum_{i=1}^{3} \frac{p_i^2}{2m_i} .$$

Here $m_i$ and $p_i$ are the mass and momentum of the $i$th quark, respectively. The internal and c.m. motion can be easily separated by means of Jacobi coordinates in the non-relativistic kinematics. This property is quite favorable in the investigation of the baryon mass spectra and meson-baryon scattering.

For the confinement potential that ensures three quarks to always be confined in a baryon, the linear form

$$V_{\text{conf}} = c \sum_{i<j}^{3} |r_i - r_j|$$

is employed, where $c$ is the strength parameter and $r_i$ the coordinate of the $i$th quark. This type of confinement is suggested by lattice QCD calculations for heavy-quark systems^16) as well as by the flux tube model.17) The strength parameter $c$, however, is treated as a phenomenological parameter in this work since it has not been calculated yet for light constituent quarks. The constant mass $M_0$ is a free parameter also.

For the meson-quark coupling $H_I$, the non-relativistic form of pseudovector coupling is employed:

$$H_I = \sum_{i=1}^{3} \frac{ig_{Mqq}}{2m_i} \rho(k) \left( \frac{\omega_M}{2m_i} \left( \sigma_i \cdot \hat{p}_i \lambda_i \cdot \phi_i(k) + \lambda_i \cdot \phi_i(k) \sigma_i \cdot \hat{p}_i \right) - \sigma_i \cdot k \lambda_i \cdot \phi_i(k) \right) + \text{h.c.}$$

(2.5)

with

$$\rho(k) = \exp \left( -\frac{k^2}{8a^2} \right) ,$$

(2.6)

$$\phi_i(k) = \frac{1}{\sqrt{2\omega_M}} a_i(k) e^{ik \cdot r_i} ,$$

(2.7)

where $g_{Mqq}$ is the meson-quark coupling constant, and $\sigma_i$, $\lambda_i$, and $\hat{p}_i$ ($\bar{p}_i$) are the spin $SU(2)$ operator, the flavor $SU(3)$ operator, and the initial (final) momentum.
operator of the \( i \) th quark, respectively. The flavor-octet meson field with momentum \( \mathbf{k} = \hat{\mathbf{p}}_i - \hat{\mathbf{p}}_i \) is represented by \( \phi_i(\mathbf{k}) \) with the meson annihilation operator on the \( i \) th quark \( a_i(\mathbf{k}) \). The phenomenological form factor \( \rho(\mathbf{k}) \) is included to cut off high-momentum contributions, which are beyond the scope of our low-energy effective model.

The pseudovector coupling is one of the lowest-order terms with respect to the derivative operator of the meson field in effective theories based on chiral symmetry.\(^{18}\) This type of coupling has favorable properties to reproduce the low-energy \( \pi N \) phase shifts and the \( \eta \) production cross sections around the \( \eta N \) threshold.\(^{10,19}\) Note also that in the static limit the pseudovector coupling and the pseudoscalar coupling bring about the same OMEP between quarks.

In addition to a single baryon described as a three-quark bound state, meson-baryon continuum states must be considered, due to the meson-quark coupling \( H_1 \). Since two-body decay channels, which are open for any nucleon resonances, often play an important role for the dynamical properties of these resonances, multi-meson contributions are simply neglected in this work. It is known, however, that multi-meson contributions have relatively large effects on several partial waves such as \( P_{11} \). A study including many-body corrections is left as a future project.

In this paper we deal with nucleon resonances \( N^* \) only. These consist of \( u \)- and \( d \)-quarks. The mass difference between the \( u \)- and \( d \)-quarks is neglected, and the standard value is used for the mass of these light constituent quarks, \( m = 340 \text{ MeV} \). For the flavor-octet mesons, \( K \) need not be included, but \( \pi \) and \( \eta \) should be taken into consideration; namely, nucleon resonances do not couple to the \( KN^* \) state but to the \( \pi N^* \) and \( \eta N^* \) states. Note also that low-lying resonances \( N^* \) generally have small \( K \)-hyperon decay widths, while the \( \eta N \) decay width of \( N(1535) \) is quite large. Since the flavor \( SU(3) \) symmetry is significantly broken, the \( \eta \)-quark coupling constant \( g_{\pi qq} \) is taken to be different from the \( \pi \)-quark coupling constant \( g_{\pi qq} \). Physical values are used for the meson masses. (It therefore should be understood that the meson energy and meson-quark coupling constant are properly taken for each component in Eq. \( (2.5) \), although it is expressed in a flavor symmetric form.)

Although Glozman et al. have taken account of the flavor-singlet \( \eta' \),\(^5\) we do not include it in the present model. Because of the \( U_A(1) \) anomaly, \( \eta' \) cannot be treated simply as a Nambu-Goldstone boson. Furthermore, we have found through numerical calculations that the inclusion of \( \eta' \) changes the final results very little.

The mass operator for a single baryon can be derived from the model Hamiltonian \( (2.1) \) by using the projection operator method.\(^{10}\) The contribution of meson-baryon continuum states is included in the energy-dependent effective potential \( W(E) \). The mass operator \( H_{\text{eff}}(E) \) is written as

\[
H_{\text{eff}}(E) = T_0 + V_{\text{conf}} + M_0 + W(E) \, , \tag{2.8}
\]

with

\[
W(E) = H_I \frac{1}{E - H_{MB} + i\epsilon} H_I = H_I G(E) H_I \equiv \sum_{i,j} H_I(i) G(E) H_I(j) \, , \tag{2.9}
\]

where \( H_{MB} = H - H_I \) is the total energy of the intermediate meson-baryon states.
Because the energy-dependent effective potential $W(E)$ diverges if an infinite number of intermediate continuum states are rigorously taken into account, a prescription must be introduced in order to perform actual calculations.\textsuperscript{15,20,21)}

First of all we note that the OMEP is expressed as

$$H_{OMEP} = \sum_{i \neq j} \tilde{H}_I(i) \tilde{G} \tilde{H}_I(j) ,$$

with

$$\tilde{G} = -\frac{1}{\omega_M} ,$$

$$\tilde{H}_I = -\sum_{i=1}^{3} \frac{g_{Mqq}}{2m} \rho(k) \sigma_i \cdot k \lambda_i \cdot \phi_i(k) + \text{h.c.} \equiv \sum_{i=1}^{3} \tilde{H}_I(i) .$$

The auxiliary operators $\tilde{G}$ and $\tilde{H}_I$ are obtained by applying the static approximation to $G(E)$ and $H_I$, respectively. In this approximation, baryons have the same energy in the initial, final, and intermediate states.

It can be easily shown that $H_{OMEP}$ has the form

$$H_{OMEP} = \sum_{i<j} \left( \tau_i \cdot \tau_j V_\pi(r_{ij}) + \frac{1}{3} V_\eta(r_{ij}) \right) ,$$

with

$$V_M(r_{ij}) = \frac{g_{Mqq}}{4\pi} \frac{1}{4m^2} \left[ S_M(r_{ij}) \sigma_i \cdot \sigma_j 
+ T_M(r_{ij}) \left( \frac{3(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij})}{r_{ij}^2} - \sigma_i \cdot \sigma_j \right) \right] ,$$

where $r_{ij}$ is the relative separation between the $i$th and $j$th quarks, and $\tau$ is the quark isospin operator. The space part of the spin-spin and tensor interactions of the OMEP are denoted by $S_M$ and $T_M$, respectively. They are explicitly written as follows:

$$S_M(r_{ij}) = \frac{2m^2_M}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + m^2_M} \rho^2(q) j_0(qr_{ij}) - \frac{4}{\sqrt{\pi}} a^3 e^{-a^2 r_{ij}^2} ,$$

$$T_M(r_{ij}) = \frac{2}{\pi} \int_0^\infty dq \frac{q^4}{q^2 + m^2_M} \rho^2(q) j_2(qr_{ij}) .$$

The second term of $S_M$, which stems from the $\delta$-function and is properly normalized, provides short-range interactions (see Eqs. (3) and (4) in Ref. 5)). The OMEP, the static part of the effective potential $W(E)$, is free from divergence, although it includes all intermediate states.

By extracting the OMEP from $W(E)$, it becomes easy to clarify the correspondence between the present model and the static models including the OMEP only.\textsuperscript{5)
The effective potential \( W(E) \) can then be rewritten as

\[
W(E) = \sum_{i,j} H_{1}(i)G(E)H_{1}(j) - \sum_{i,j} \tilde{H}_{1}(i)\tilde{G}\tilde{H}_{1}(j) + \sum_{i,j} \tilde{H}_{1}(i)\tilde{G}\tilde{H}_{1}(j)
\]

\[
= \sum_{i\neq j} \tilde{H}_{1}(i)\tilde{G}\tilde{H}_{1}(j) + \sum_{i=j} \tilde{H}_{1}(i)\tilde{G}\tilde{H}_{1}(j)
\]

\[
+ \left( \sum_{i,j} H_{1}(i)G(E)H_{1}(j) - \sum_{i,j} \tilde{H}_{1}(i)\tilde{G}\tilde{H}_{1}(j) \right)
\]

\[
\equiv H_{\text{OMEP}} + \tilde{M}_{0} + (W(E) - \tilde{W}) . \tag{2.17}
\]

The divergent piece of the state-independent quantity \( \tilde{M}_{0} \) is expected to be canceled by some counter term, so that our model may provide finite masses of baryons. It is therefore considered in this work that the finite contribution of \( \tilde{M}_{0} \) is included in the constant mass parameter \( M_{0} \).

Since the static OMEP is extracted, the last term in Eq. (2.17), \( W(E) - \tilde{W} \), is regarded as the dynamical mesonic effect. The subtraction term \( \tilde{W} \) plays a crucial role in eliminating double counting, which otherwise would become a serious problem. In the rigorous calculation of \( W(E) - \tilde{W} \), we still must consider in principle all intermediate states consisting of a meson and three quarks. In order to avoid divergence, the following truncation scheme of intermediate states is employed. At first the intermediate three-quark states are expanded in terms of the baryon states that correspond to the energy eigenstates of a three-quark system. Then we consider only the meson-baryon continuum states whose energy thresholds are relatively low. In other words, only physical meson-baryon decay channels of nucleon resonances are taken into account. In order to express this truncation clearly, we change the notation: hereafter we use \( \Sigma(E) \) and \( \tilde{\Sigma} \) instead of \( W(E) \) and \( \tilde{W} \), respectively. We then refer to \( \Sigma(E) \) as the self-energy of nucleon resonances, and to \( \tilde{\Sigma} \) as the subtraction term. It is known that the energy-dependent effect of \( \Sigma(E) \) is quite important to reproduce the properties of baryons.\(^{10}\)\textsuperscript{–}\textsuperscript{12}\)

In the present work, we include only \( \Sigma^{\pi N}(E) - \tilde{\Sigma}^{\pi N} \) and \( \Sigma^{\eta N}(E) - \tilde{\Sigma}^{\eta N} \), which come from the \( \pi N \) and \( \eta N \) intermediate states, respectively. Although they are expected to give the largest contributions among the meson-baryon continuum states, there is no a priori reason to exclude other meson-baryon continuum states; for example, the bag model calculation has suggested that the contribution of the \( \pi \Delta \) state is important.\(^{22}\) A more comprehensive study is necessary to understand what kinds of channels and what kinds of symmetries should be taken into account for a good description of baryons. Although this problem is interesting, we consider it to be beyond the scope of the present work and put off its study to the near future.\(^{23}\)

Finally, the mass operator is written as

\[
H_{\text{eff}}(E) = T_{0} + V_{\text{conf}} + M_{0} + H_{\text{OMEP}}
\]

\[
+ \Sigma^{\pi N}(E) - \tilde{\Sigma}^{\pi N} + \Sigma^{\eta N}(E) - \tilde{\Sigma}^{\eta N} . \tag{2.18}
\]

It should be emphasized again that the decomposition of the energy-dependent effective potential \( W(E) \) does not lead to a simple sum of the OMEP \( (H_{\text{OMEP}}) \) and
N* self-energies ($\Sigma^{*N}(E)$ and $\Sigma^{*N}(N)$). In order to deal with meson-quark coupling consistently and to avoid double counting, it is necessary to include the corresponding subtraction terms ($\Sigma^{iN}$ and $\Sigma^{iN}$).

The matrix elements of $\Sigma^{iN}(E)$ and $\Sigma^{iN}$, for example, are explicitly written as

$$\Sigma^{iN}(E) = \int \frac{d^3k}{(2\pi)^3} \frac{\langle \pi^*|H_I|\pi N; k \rangle \langle \pi N; k | H_I | N^* \rangle}{E - \sqrt{m^2_{\pi} + k^2} - \sqrt{m^2_N + k^2 + i\epsilon}},$$  \hspace{1cm} (2.19)

$$\Sigma^{iN} = \int \frac{d^3k}{(2\pi)^3} \frac{\langle N^*|H_I|\pi N; k \rangle \langle \pi N; k | H_I | N^* \rangle}{-\sqrt{m^2_{\pi} + k^2}}.$$  \hspace{1cm} (2.20)

where $|\pi N; k \rangle$ is the $\pi N$ scattering state with relative momentum $k$. In the integrand of Eqs. (2.19) and (2.20), the relativistic form of $H_{MB}$ is used to avoid the unphysical momentum-dependence that the non-relativistic form has in the virtual high-momentum region. For the nucleon mass $m_N$, which should be calculated self-consistently in the present model, the observed value is temporarily used. This is important to obtain the correct energy-dependence of self-energies. The ‘$0\hbar\omega$’ harmonic-oscillator wave function is used for the ground-state nucleon $N$ in the intermediate states. It has been found by numerical calculations that 90% of the three-quark component of $N$ consists of this ‘$0\hbar\omega$’ wave function (see discussion below and in §3).

Before closing this section, we present the method of calculating baryon masses. First, the matrix elements of the mass operator $H_{eff}(E)$ are systematically calculated with basis functions that are the antisymmetrized products of quark wave functions. The three-quark bound-state wave functions consist of the space, spin, flavor and color parts. The spin, flavor and color wave functions are easily constructed, and they have well-defined symmetries. For the space part, the harmonic-oscillator wave functions are used with the range parameter $\beta$. These functions are convenient because they have analytic forms and also because their c.m. motion can be easily removed by using Jacobi coordinates. The antisymmetrization can be carried out by using Talmi-Moshinsky coefficients.\(^{23)-25}\)

The basis functions must be truncated in practical applications. In the calculation with the truncated basis functions, the most appropriate value of $\beta$ is searched for by minimizing the energy eigenvalues of the static part of $H_{eff}(E)$, i.e., $T_0 + V_{conf} + M_0 + H_{OMEP}$. This variational method basically gives a different value of $\beta$ for each baryon state. In the present case, however, a common value of $\beta$ can be used in the following calculations, since the energy eigenvalues of low-lying states change little over a wide range of $\beta$. The value thus obtained depends on the other model parameters, and especially on the strength of the confinement potential. For the parameters given in Table I of §3, the variational method yields $\beta = 3.7$ fm\(^{-2}\). It also has been found that the basis functions up to the $8\hbar\omega$-shell of the harmonic-oscillator wave functions are sufficient to obtain numerical results with good accuracy and stability.

When the dynamical effect of the mass operator $H_{eff}(E)$ is taken into consideration, its energy-dependence and imaginary part prevent us from naively interpreting
its eigenvalues as resonance masses. Resonances correspond to the S-channel poles of the propagator $\hat{G}$ for meson-baryon scattering:

$$\hat{G} \propto \frac{1}{E - H_{\text{eff}}(E)}.$$  \hspace{1cm} (2·21)

Therefore the resonance mass $E_R$ can be approximately determined as a solution of

$$\text{Re}(E_R - H_{\text{eff}}(E_R)) = 0,$$  \hspace{1cm} (2·22)

after the energy-dependent eigenvalues are obtained by diagonalizing the mass operator $H_{\text{eff}}(E)$.

§3. Results and discussion

The present model still has five parameters to be determined: the strength of the confinement potential $c$, the form factor parameter $a$, the constant mass parameter $M_0$, and the $\pi$- and $\eta$-quark coupling constants, $g_{\pi qq}$ and $g_{\eta qq}$. All parameters except $g_{\pi qq}$ are phenomenologically determined, so as to reproduce the prominent feature of the mass spectrum of the spin 1/2 nucleon resonances: the separations among the negative- and positive-parity resonances are relatively small in comparison with the large mass difference between the ground state nucleon and the other resonances. All the parameters thus determined are summarized in Table I.

The $\pi$-quark coupling constant $g_{\pi qq}$ is derived from the $\pi N$ coupling constant $G_{\pi NN}$ by using the spin-isospin symmetry relations of the constituent quark model. The standard value of $G_{\pi NN}$, 14.3, cited in Ref. 26) is used. For the determination of the $\eta$-quark coupling constant $g_{\eta qq}$, the empirical $\eta$-nucleon coupling constant $G_{\eta NN}$ is not used because the flavor $SU(3)$ symmetry is badly broken; in fact, our parameter fitting yields $g_{\eta qq} = 3.52$, which is smaller than the value 4.27 derived from $G_{\eta NN} = 3.68^{26)}$ by using the spin-flavor $SU(6)$ relations.

We here make a few comments on parameter fitting. Excitation energies put a constraint on the confinement strength $c$, which determines the level spacing between the major energy-shells, as is seen in Fig. 1(d). Because Eq. (2·22) is a non-linear equation with respect to energy, the role of $M_0$ is more than just that of an additional constant. Numerical calculations have shown that the phenomenological parameter $a$ has a relatively strong correlation with the meson-quark coupling strengths through the momentum dependence of the form factor.

We now present the mass spectrum of our model. The masses of the spin 1/2 nucleon resonances are shown in Fig. 1. The calculation can reproduce the general features of the observed spectrum fairly well: the excitation energy of the first negative-parity state is calculated to be approximately 500 MeV, and the mass differences among the excited nucleons become relatively small ($\leq 250$ MeV).

Table I. The parameters used in our model. Other parameters are taken as follows: $m = 340$ MeV, $\beta = 3.7$ fm$^{-2}$, and experimentally measured masses are used for $\pi$ and $\eta$.

| Parameter  | Value |
|------------|-------|
| $c$ (fm$^{-2}$) | 1.5 |
| $a$ (fm$^{-1}$) | 2.5 |
| $g_{\pi qq}$ | 2.91 (fixed) |
| $g_{\eta qq}$ | 3.52 |
| $M_0$ (MeV) | 0 |
Nucleon Resonances in a Constituent Quark Model with Chiral Symmetry

Fig. 1. Energy levels of nucleon resonances with spin 1/2. Calculations are compared with experimental data. The corresponding states are connected by dashed lines. The contributions of $H_{\text{OMEP}}, \Sigma(E)$, and $\Sigma$ are also examined. (a) The data are taken from the Particle Data Table. The parities of resonances are shown. (b) The result of the present model. (c) The result without $\Sigma(E)$ and $\Sigma$. (d) The result without $H_{\text{OMEP}}, \Sigma(E)$ and $\Sigma$.

In order to make clear the reason why our model can reproduce these features, each mesonic effect on the mass spectrum is examined. In Fig. 1, we also show the result without $\Sigma(E) - \Sigma$, and the result without $\Sigma(E) - \Sigma$ and $H_{\text{OMEP}}$. The self-energy effect, as well as the OMEP, significantly changes the mass spectrum. In the consistent treatment of meson-quark coupling in this work, the self-energy correction has almost the same magnitude as the OMEP. In the case of the diagonal element for the ground-state nucleon, for example, the OMEP is $-175\,\text{MeV}$, the self-energy $\Sigma(E \approx m_N)$ is approximately $-400\,\text{MeV}$, and the subtraction term $\Sigma$ is $-266\,\text{MeV}$. Note that the subtraction term is comparable with the self-energy and OMEP. Therefore the naive sum of the self-energy and OMEP without this subtraction causes a serious overestimate of mesonic effects. It should be emphasized also that the subtraction term depends on the initial and final baryon states and is not merely a constant parameter.

For the positive-parity states, Fig. 2 shows that the $N^*$ self-energy effect due to the $\eta N$ state is generally smaller than that due to the $\pi N$ state. This is due to the restriction of phase space, since the $\eta N$ threshold is higher than the $\pi N$ threshold. The self-energy effect for the ground-state nucleon $N$ is significantly large because $N$ couples strongly to the intermediate $\pi N$ state without changing the configuration of quarks. Therefore, the mass of the ground-state nucleon is largely pulled down, owing to the inclusion of self-energies. This is understood from the fact that the energy of the ground state is generally lowered when it is coupled to higher energy states. Note, however, that second-order perturbation theory suggests a mass shift of 300 MeV, whereas only two-thirds of this value is obtained in our non-perturbative calculation.

In Fig. 3, a cusp is seen at the $\eta N$ threshold for the negative-parity states due to their $S$-wave coupling to the $\eta N$ state. This is a characteristic feature of self-energies, and this energy dependence is important to the dynamical properties of
resonances. The OMEP does not have such energy dependence.

For the states in the same 'hω'-shell, the non-diagonal elements of self-energies have the same magnitudes as their diagonal elements. On the other hand, the non-diagonal elements of the total mass operator $H_{\text{eff}}(E)$ are in general smaller than the differences among its diagonal elements. Because of this behavior, the state mixing among different shells is relatively small.

In the present calculation, the mass of the ground-state nucleon $N$ is 1007 MeV, which is larger than the observed mass by approximately 10%. Although this disagreement might become smaller with the fine tuning of model parameters, it is thought to be partly due to our approximate treatment of $N$ in the intermediate $\pi N$ and $\eta N$ states. In these decay channels, $N$ is considered to be a three-quark bound state since multi-meson states are neglected in this work. The physical nucleon is, however, the admixture of various states, such as pure three-quark states, states consisting of a meson and three quarks, and so on. The probability of observing three-quark component of the physical nucleon is related to the energy-derivative of $\Sigma(E)$ as

$$Z^{-1} = 1 - \text{Re} \left( \frac{d}{dE} \Sigma^{\pi N}(E) + \frac{d}{dE} \Sigma^{\eta N}(E) \right)_{E=E_R}. \quad (3.1)$$

Due to the strong energy-dependence of $\Sigma(E)$ around $E_R$, the probability $Z$ deviates from unity. We obtain $Z \simeq 0.6$ for $N$. Since a relatively large part of its wave function

Fig. 2. (a) The matrix elements of $\Sigma^{\pi N}(E) - \tilde{\Sigma}^{\pi N}$ for the positive-parity nucleon resonances with spin 1/2. The solid line corresponds to the diagonal element of the 0hω state, and the dashed line to that of the 2hω state. Among the diagonal elements of the four 2hω states, only the largest one is shown here. (b) Same as (a) but for $\Sigma^{\eta N}(E) - \tilde{\Sigma}^{\eta N}$.
Fig. 3. Same as Fig. 2 but for the negative-parity nucleon resonances with spin 1/2. The diagonal and non-diagonal elements for the two 1hω states are shown. The solid line corresponds to the diagonal element of the intrinsic-spin 3/2 state, the dashed line to that of the intrinsic-spin 1/2 state, and the dot-dashed line to the non-diagonal element between these states. Because the ηNN∗ vertices for these negative-parity states are identical, all the matrix elements for the ηN coupling have the same value; that is, the three lines coincide.

is occupied by the meson-nucleon component, further studies of mesonic effects are required in order to describe the properties of N in detail.

We examine the role of the spin-flavor symmetry of the OMEP by comparing it with the color-spin symmetry of the OGEP. To show the difference between these potentials in an explicit way, we consider the 1/2+ and 1/2− excited-states, which are 2hω and 1hω excited-states, respectively. The three-quark bound-state with [3]space ⊗ [21]spin ⊗ [21]flavor ⊗ [111]color symmetry is taken for the 1/2+ state, and that with [21]space ⊗ [21]spin ⊗ [21]flavor ⊗ [111]color symmetry for the 1/2− state. Note that N(1440) is dominated by this 1/2+ state. On the other hand, the symmetry property of the OMEP is characterized by \( f \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j \), where \( f \) stands for the spatial part of the interaction between the \( i \)th and \( j \)th quarks. Similarly, the OGEP has the operator \(-g \sigma_i \cdot \sigma_j\). The diagonal matrix elements of these operators are shown in Table II. Since both the OMEP and OGEP are short-range and attractive, the relations: \( \langle f \rangle_{00} < \langle f \rangle_{10} < \langle f \rangle_{01} < 0 \) and \( \langle g \rangle_{00} < \langle g \rangle_{10} < \langle g \rangle_{01} < 0 \) hold. (Here the diagonal matrix elements of \( f \) and \( g \) for the state with node \( n \) and angular momentum \( l \) are denoted by \( \langle f \rangle_{nl} \) and \( \langle g \rangle_{nl} \), respectively.) In the case of the OMEP, the difference between the 1/2+ and 1/2− states is 5/2\( \langle f \rangle_{10} + 3/5\langle f \rangle_{01} \). Since the two terms are added constructively, the mass of the 1/2+ state is lowered
by a larger amount than that of the 1/2− state due to the OMEP; that is, their energy levels become close. In the case of the OGEP, contrastingly, the difference is 1/2(⟨g⟩10 − ⟨g⟩01), the magnitude of which is small owing to the destructive combination. Therefore the mass difference between the 1/2+ and 1/2− states changes little. A similar argument can also be applied to the other 1/2− state with [21]space ⊗ [3]spin ⊗ [21]flavor ⊗ [111]color symmetry. The property of the spin-flavor symmetry of the OMEP is thus consistent with the fact that the observed mass of \( N(1440) \) is remarkably small.

The spin-flavor symmetry of self-energies also plays an important role in making the 1/2+ resonances come close to the 1/2− resonances, as seen in Fig. 1. The symmetry structure of the meson-quark coupling is characterized by the operator \( \sigma_i \lambda_i \) (see Eq. (2·5)). Taking the same states (used in Table II) as examples we obtain the following relation for the spin-flavor part of the \( \pi NN^* \) vertex:

\[
\frac{5\sqrt{2}}{4} = \frac{1}{2}^+ \rightarrow \pi N \text{ transition} \quad \frac{1}{2}^− \rightarrow \pi N \text{ transition}
\]

Similarly, the ratio is \( 5\sqrt{2}/2 \) for the other 1/2− state with [21]space ⊗ [3]spin ⊗ [21]flavor ⊗ [111]color in the case of spin-nonflip transitions. We can therefore expect that the mass shift of \( N(1440) \) is larger than that of \( N(1535) \) (see Eq. (2·19)). More quantitative discussion requires proper treatment of the space part in each matrix element of self-energies. Numerical calculations show that the mass shifts due to the self-energy effect are attractive, and its spin-flavor symmetry is important to reproduce the energy levels of nucleon resonances.

The present model has succeeded in reproducing various aspects of the mass spectrum in comparison with the simple harmonic-oscillator potential model. This is because our model has consistently included the dynamic effect of self-energies in addition to the static interactions, such as the OMEP and confinement potential. As seen in Fig. 1, however, \( N(1440) \) is still located above the negative-parity resonances, while it is experimentally observed below them. We believe that we first must expand the model space for meson-baryon intermediate states in order to reproduce the correct order of resonances. The dynamic effects due to these states should be treated in a consistent manner, as shown in this paper. On the other hand, Glozman et al. have recently claimed that their OMEP model reproduces the mass of \( N(1440) \) if relativistic kinematics are used for quarks.6) (Although they explain baryon mass spectra with non-relativistic kinematics in their first paper,5) the \( \delta \)-function piece of the OMEP, i.e., the second term of Eq. (2·15), has an artificially large strength.) If the relativistic correction in the kinetic energy operator is so important, the vertex functions should also be modified in a consistent way. We leave such a calculation with these relativistic corrections as a future project.

We show the spectra of the spin 3/2 and 5/2 nucleon resonances in Fig. 4. All the
Fig. 4. Numerical results are compared with observed spectra of nucleon resonances with spin 1/2 (left), 3/2 (center), and 5/2 (right). The dashed lines represent the correspondence between calculations and data. The parities of states are also shown.

parameters for these resonances are the same as those for the spin 1/2 resonances. The calculated masses of these resonances roughly agree with the empirical values, although the mass differences between the positive- and negative-parity states are too large. It should be noted that the common constant mass parameter $M_0$ is used for all the nucleon resonances. The form factor $\rho(k)$ plays an important role in the realization of this situation. For example, the lowest 1/2$^-$ and 3/2$^-$ resonances, i.e., $N(1535)$ and $N(1520)$, are considered. If $\rho(k)$ is removed from the calculations of self-energies, the mass of $N(1520)$ becomes smaller by approximately 60 MeV, while that of $N(1535)$ changes by only a few MeV. This is because the $D$-wave $\pi NN(1520)$ coupling has larger high-momentum components than the $S$-wave $\pi NN(1535)$ coupling. The effect of $\rho(k)$ is favorable, since it reduces the contribution from the virtual high-momentum intermediate states that the present model cannot be applied to. But for $\rho(k)$, the spin-dependence of $M_0$ should be inevitably introduced.

§4. Summary

We have constructed a constituent quark model that contains a meson-quark coupling so as to incorporate spontaneous breaking of chiral symmetry in low-energy QCD, and calculated the masses of nucleon resonances. The pseudovector meson-quark coupling has a spin-flavor dependence that is thought to be important for the low-energy baryon spectra. In the present formalism, our consistent treatment of the meson-quark coupling provides not only the OMEP but also the baryon self-energy with the subtraction term, which is important to avoid the problem of double counting. We should take account of these terms simultaneously in a model of baryons since each of their effects is significantly large. Our model reproduces the gross features of the observed mass spectra of nucleon resonances, whereas the calculated mass of $N(1440)$ is still too large.

In order to refine the calculation, we should include not only the $\pi N$ and $\eta N$ states but also the states that contain other mesons and baryons, such as the $\pi \Delta$
and $\rho N$ states. These states, which are closely related to the $\pi\pi N$ channel, are considered to be important for a detailed study of nucleon resonance spectroscopy. The relativistic corrections in the vertex function as well as in the kinetic energy are to be investigated in the near future also.

Furthermore, we must calculate scattering quantities and compare them directly with experimental data in order to complete the investigation of our model. This type of study is necessary since the masses of nucleon resonances represent only a small part of the information concerning the phase shift analysis of $\pi N$ scattering. Calculations of scattering quantities are now in progress. On the other hand, it is also desirable to study strange baryons in this model, and the results will be given elsewhere.

Acknowledgements

We would like to thank Professor R. Seki for valuable suggestions and discussions. We are much indebted to Professor T. Sato and Dr. T. Ogaito for discussions on the method of calculation. One of the authors (H. K.) would like to thank Professor Y. Kudo and Professor Y. Sakuragi for their continuous encouragement.

References

1) A. J. G. Hey and R. L. Kelly, Phys. Rep. 96 (1983), 71.
2) N. Isgur and G. Karl, Phys. Rev. D18 (1978), 4187; D19 (1979), 2653; D20 (1979), 1191.
3) Particle Data Group, Phys. Rev. D54 (1996), 1.
4) L. Ya Glozman and D. O. Riska, Phys. Rep. 268 (1996), 263.
5) L. Ya. Glozman, Z. Papp and W. Plessas, Phys. Lett. B381 (1996), 311.
6) L. Ya. Glozman, W. Plessas, K. Varga and R. F. Wagenbrunn, hep-ph/9706507.
7) A. De Rújula, H. Georgi and S. L. Glashow, Phys. Rev. D12 (1975), 147.
8) A. Manohar and H. Gerogi, Nucl. Phys. B234 (1984), 189.
9) T. P. Cheng and L. F. Li, Phys. Rev. Lett. 74 (1995), 2872.
10) M. Arima, K. Shimizu and K. Yazaki, Nucl. Phys. A543 (1992), 613.
11) T. Hamaie, M. Arima and K. Masutani, Nucl. Phys. A591 (1995), 675.
12) T. Hamaie, K. Masutani and M. Arima, Nucl. Phys. A607 (1996), 363.
13) P. Geiger and N. Isgur, Phys. Rev. D55 (1997), 299.
14) P. Zenczykowski, Ann. of Phys. 169 (1986), 453.
15) S. A. Chin, Nucl. Phys. A382 (1982), 355.
16) M. Creutz, Quarks, Gluons and Lattices (Cambridge University Press, Cambridge, 1983), p. 54.
17) J. Kogut and L. Susskind, Phys. Rev. D9 (1974), 3501.
18) D. K. Campbell, in Les Houches 1977 Nuclear Physics with Heavy Ions and Mesons, vol. 2 (North-Holland, Amsterdam, 1978), p. 622.
19) H. A. Bethe and F. de Hoffmann, Mesons and Fields (Row, Peterson & Company, Evanston, 1955).
20) E. Oset, Nucl. Phys. A411 (1983), 357.
21) K. G. Horacek, Y. Iwamura and Y. Nogami, Phys. Rev. D32 (1985), 3001.
22) S. Theberge, A. W. Thomas and G. Miller, Phys. Rev. D22 (1980), 2838; D23 (1981), 2106(E).
23) T. Ogaito, T. Sato and M. Arima, in preparation.
24) T. Ogaito and T. Sato, private communication.
25) L. Trlifaj, Phys. Rev. C5 (1972), 1534.
26) G. C. Oades, H. Behrens, J. J. de Swart and P. Kroll, Nucl. Phys. B216 (1983), 277.