Fermionic light in common optical media

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Recent experiments have proven that the response to short laser pulses of common optical media, such as air or Oxygen, can be described by focusing Kerr and higher order nonlinearities of alternating signs. Such media support the propagation of steady solitary waves. We argue by both numerical and analytical computations that the low power fundamental bright solitons satisfy an equation of state which is similar to that of a degenerate gas of fermions at zero temperature. Considering in particular the propagation in both oxygen and air, we also find that the high power solutions behave like droplets of ordinary liquids. We then show how a grid of the fermionic light bubbles can be generated and forced to merge in a liquid droplet. This leads us to propose a set of experiments aimed at the production of both the fermionic and liquid phases of light, and at the demonstration of the transition from the former to the latter.

PACS numbers: 42.65.Tg, 42.65.Jx, 42.62.-b, 03.75.Ss

In suitable optical media, light has been argued to acquire material properties. A long dated example is the equivalence of the paraxial propagation of a laser pulse in a Kerr medium with the time evolution of a superfluid Bose-Einstein condensate, due to the identity of the nonlinear Schrödinger equation with the Gross-Pitaevskii equation[1]. More recently, optical induction has been used to create photonic crystals[2]; a photonic system has been designed that may undergo a Mott insulator to superfluid quantum phase transition[3]; soliton solutions for light propagation in Cubic-Quintic (CQ) nonlinear media have been shown to behave like ordinary liquids[4,5].

On the other hand, recent experimental and theoretical works have proven that the response to ultrashort laser pulses of common optical media, such as air or Oxygen, can be described by focusing Kerr[6] and higher order nonlinearities of alternating signs[7], which have also been argued to provide the main mechanism in filament stabilization, instead of the plasma defocusing[7].

In this letter, we demonstrate by analytical and numerical computations that such media can support the propagation of steady solitary waves that appear in two clearly different phases. The low power solitons are governed by the same equation of state than a degenerate gas of fermions. We will call such a system “Fermionic Light”. On the other hand, the high power localized states satisfy the Young-Laplace equation that governs the formation of droplets in ordinary liquids, similarly to the result that was recently obtained for the CQ model[5]. We also show how to generate a grid of fermionic light bubbles and make it turn into a liquid droplet.

We will consider the (paraxial) propagation along the z-direction of a linearly polarized laser beam, so that the complex electric field component $\Psi(x,y,z)$ satisfies the nonlinear Schrödinger equation

$$i \frac{\partial \Psi}{\partial z} + \frac{1}{2k_0n_0} \nabla_x^2 \Psi + k_0 \Delta n \Psi = 0,$$  

where $n_0$ is the linear refractive index of the medium, $\nabla_x^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplace operator, and $k_0 = 2\pi/\lambda_0$ is the mean wavenumber in vacuum, where $\lambda_0$ is the central wavelength of the laser source which will be fixed to $\lambda_0 = 800 nm$ throughout this paper as in the experiment of Ref.[7]. For the optical media that have been studied in Ref.[7], the nonlinear correction $\Delta n$ to the refractive index can be expanded as

$$\Delta n \simeq n_2|\Psi|^2 + n_4|\Psi|^4 + n_6|\Psi|^6 + n_8|\Psi|^8,$$  

with alternating sign coefficients $n_2, n_6 > 0$ and $n_4, n_8 < 0$, that contribute to focusing and defocusing respectively. Taking into account that the values of the second-order dispersion and multiphoton-absorption coefficients for the air are $k'' = 0.2 fs^2/cm$ and $\beta = 1.27 \times 10^{-126} cm^{17}/W^9$, respectively, we have checked that both effects do not lead to significant corrections in our results. To be concrete, we will perform most of the numerical calculations in the case of Oxygen taking the mean values obtained in the experiment[8], $n_2 = 1.6 \times 10^{-19} cm^2/W$, $n_4 = -5.2 \times 10^{-33} cm^4/W^2$, $n_6 = 4.8 \times 10^{-46} cm^6/W^3$, $n_8 = -2.1 \times 10^{-39} cm^8/W^4$. For comparison, however, we will also mention the results that we obtain for air, using the corresponding values for the coefficients $n_{2q}$ that are also given in Ref.[7].

We will search for finite localized solutions of Eq. (1) of the form $\Psi(x,y,z) = \Phi(r)e^{-inz}$, where $r = \sqrt{x^2+y^2}$ and $\mu$ is the propagation constant. Fig. 1 shows the result of our numerical computation for $\Phi(0)$ in the existence domain of such solitons in Oxygen ($\mu_\infty < \mu < 0$), where $\mu_\infty = -0.096 cm^{-1}$, and for the radial profiles $\Phi(r)$ of three of them (see left inset in Fig.1), corresponding to the low power (black solid line), moderate power (red dashed line) and high power regimes (blue dashed-dotted line) respectively. We express the transverse spatial variables $x,y$ in terms of the adimensional coordinates $\xi,\chi$, measured in units of $(n_4/n_0)^{1/2}(k_0n_2)^{-1}$. The amplitude $|\Phi(0)|$ is measured in units of $(n_2/n_4)^{1/2}$.

Like in the CQ case, $\mu$ can be identified with the chem-
with a or of the central density $\rho$ of the beam intensity),

tions (black solid line), corresponding to small values of
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tional method with the ansatz $\Phi(0)$.

For our optical system, $\mu$, $N$, $\Omega$ and $p$ correspond
to the propagation constant, the power, the Lagrangian leading
to Eq. (4), and the Lagrangian density, respectively.

The right inset of Fig. 1 shows the pressure distributions $p(r)$, measured in units of $k_0n_2^2\hbar^2$, corresponding
to the stationary states discussed above. Notice that all
the radial distributions display both positive and negative
pressure regions, being this a necessary condition for
the existence of solitary waves. For the low power soliton
(black solid line), corresponding to small values of $|\mu|$, we
can obtain an analytical expression by using the varia-
tional method with the ansatz $\Phi(A, r) = A \exp(-r^2/R^2)$.
The values of $A(\mu)$ and $R(\mu)$ that minimize $\Omega$ for a given
value of $\mu$ can then be used to compute the pressure
distribution. Taking the first non-vanishing order in $|\mu|$
and inverting the dependence $R(\mu)$, we find a simple anal-
tycal approximation for the central pressure $p_c$, as a
function either of the radius $R = \sqrt{2/(\mu^2)}$ of the soliton,
or of the central density $p_c = |\Phi(0)|^2$ (corresponding to
the beam intensity).

$$p_c = \frac{a}{k_0n_2^2\hbar^2} \frac{1}{R^2} = bk_0n_2\rho^2,$$  

with $a = 4$ and $b = 1/4$. This relation is similar to the
equation of state of a degenerate gas of fermions of mass
$m$ at zero temperature. In fact, if we apply the general
definition of the Fermi momentum $\mu$ to a two di-
dimensional system, we obtain $\mu = h\sqrt{2\pi\rho}$, with $\rho$ the
density of the Fermi gas. As a consequence, the pres-
sure, defined as the average force on a unit orthogonal
line in the gas, can be obtained from the average kinetic
energy as follows

$$p = \rho\langle E_{\text{kin}} \rangle = \frac{\rho}{2m} \int_0^{\mu} P^2 dP = \frac{\pi \hbar^2}{2m}\rho^2$$  

which shows the same dependence with $\rho^2$ as Eq. (4).

This proves the formal analogy of our low power solitons
with a degenerate Fermi gas in the central region around
$r = 0$, which is arbitrarily large in the limit $\mu \to 0$ (cor-
responding to large radius $R \to \infty$). For these reasons,
we will call ‘Fermionic’ the phase where the pressure is
proportional to $\rho^2$.

FIG. 2: [Color online] Plot of the logarithms of $p_c$ vs. $R$ for
the fundamental solitons (black line). The fermionic behav-
ior of Eq. (4) (red dotted) and the liquid YL equation (green
dashed) are compared with the numerics. The labeled points
are the same eigenstates displayed in Fig. 1. For each value of $R$,
two out of three solutions are possible depending
on the power, corresponding to two different phases of
light. Inset: plot of $\ln\rho$ vs. $\ln\rho$ (black solid) overlapped
with Eq. (4) (red dotted) for comparison.

Note that in the limit $\mu \to 0$ our variational computa-
tion gives a constant $N = \frac{2a}{\sqrt{\pi m n_2^2}}$, which is consistent
with the known result for the power flow leading to the
collapse threshold in a Kerr medium. The magnitude of
this power $N$ lies in the range of few $GW$ in both $O_2$
and air, and can be interpreted as the threshold for the
existence of the Fermionic Light solitons.

Fig. 2 shows the numerical computation of $p_c$ as a function
of either of $R$ (black solid line) or $p_c$ (inset: black solid
line) for all the nodeless solitary states of the model. The
lower branch, corresponding to the low power solitons, is
in excellent agreement with the dependence described by
Eq. (4), as it can be inferred from the fitting (red dotted)
straight line with slope $s_{\text{low}} = -4$. Furthermore, in the

FIG. 1: [Color online] Plot of the central amplitude $\Phi(0)$ vs.
$\mu/\mu_{\infty}$ for the family of fundamental eigenstates of Eq. (4).
Note that $\Phi(0)$ has a maximum for $\mu/\mu_{\infty} \approx 0.7$, due to the
defocusing nonlinearities. Left inset: radial profiles of three
eigenfunctions of Eq. (4) with $\mu/\mu_{\infty} = 0.004$ (black solid),$\mu/\mu_{\infty} = 0.3$ (red dashed) and $\mu/\mu_{\infty} = 0.97$ (blue dashed-
dotted) respectively. Right inset: radial pressure profiles cor-
responding to these solutions. The low-power profile is mag-
nified by a factor $10^3$ for clarity. Labeled points on the main
curve refer to the eigenstates displayed within the insets.
is of usual liquid droplets. The value of the surface tension $\sigma$ is obtained instead of $\sigma/R$ in the $\mathbf{R}^{n}$ equation
\[ \text{[11]} \]
including the higher order nonlinearities, we obtain that these states obey the celebrated Young-Laplace (YL) equation
\[ \text{[11]} \], $p_{c} = 2\sigma/R$, describing the behavior of usual liquid droplets. The value of the surface tension is

\[ \sigma = \frac{n_{2}}{2\sqrt{\rho_{0}\rho_{4}}} \int_{0}^{\infty} \left( -\mu_{\infty}^{2} - k_{0} \sum_{q=1}^{4} \frac{n_{2} \Phi^{2}(q+1)}{q+1} \right)^{1/2} d\Phi, \quad \text{(6)} \]

being $\mu_{\infty}$ and $\Phi_{\infty}$ the asymptotic values corresponding to the $R \to \infty$ droplet, that can be computed by solving the equation $p_{c} = 0$ (neglecting the Laplacian term). For the propagation in Oxygen, we have obtained the following variational estimations: $\mu_{\infty} = -0.247(n_{2} \rho_{4}^{1/2}) = -0.096 \text{ cm}^{-1}$, $\Phi_{\infty}^{2} = 0.712(n_{2} \rho_{4}^{1/2}) = 2.19 \times 10^{13} \text{ W/cm}^{2}$, $\sigma = 0.0715(n_{2} \rho_{4}^{-3/2} \rho_{0}^{1/2}) = 4.88 \times 10^{9} \text{ W/cm}^{2}$. These results, obtained assuming a top-flat function, are in excellent agreement with the computation given in Fig.2.

In Fig.2, we show that our numerical solution in the case of $O_{2}$ satisfy the YL equation (green dashed line) with very good accuracy for a wide range of values of $\mu$. On the other hand, for the propagation in air, the liquid light phase would correspond to a higher intensity, $\Phi_{\infty}^{2}$, with $[\mu_{\infty}] = -0.116 \text{ cm}^{-1}$ and $[\Phi_{\infty}^{2}] = 7.16 \times 10^{9} \text{ W/cm}^{2}$, and the YL equation would still be valid.

As we have seen above, the propagation of self-guided light beams in media like Oxygen can occur in two clearly separated phases, satisfying two different equations of state. In fact, we have checked that qualitatively similar results can also be obtained for the CQ case, and occur whenever the nonlinear refractive index displays a single well-defined maximum as a function of the intensity. It would then be interesting to demonstrate the possibility of a transition between the fermionic bubbles and the liquid droplets of light. A suggestive analogy is that of the collapse of a star, that occurs when the gravitational interaction overcomes the Fermi pressure of the electrons. We can obtain a qualitatively similar result in the case of light propagation in Oxygen, by compressing the fermionic bubbles using a harmonic potential, leading to the generation of a liquid light droplet. Fig. 3 shows the result of our simulation in $O_{2}$. The initial state (see left snapshot in Fig. 3) consists of a regular grid of fermionic light bubbles with $\mu/\mu_{\infty} = 0.3$ (see their radial profile in Fig. 1.), with a separation between nearest neighbors $\Delta_{k} = 40$. We include an external harmonic potential $V(\xi, \chi) = \frac{K}{2} (\xi^{2} + \chi^{2})$ with $K = 5 \times 10^{-5}$, in Eq.1 in order to induce a net force acting on the grid with the aim of making all the fermionic solitons to collide in the center of the computational window $(\xi, \chi) = 0$.

This parabolic potential can be obtained by inducing in the medium a “gas lens” [12], which can be constructed with an electrically heated pipe through which passes a laminar flow of gas. By controlling the differential heating at the boundaries and the velocity of the flow, a parabolic refractive index gradient can be obtained as documented in Ref. [12]. On the other hand, we have also checked numerically that the same qualitative result can be obtained by using a glass lens, provided that its focal...
length $f$ is at least ten times greater than the Rayleigh length of the input beams, which in the case depicted in Fig. 3 corresponds to $f = 5 \text{ m}$.

For simplicity, we have prepared the initial state with exact solutions of Eq. (1) to reduce the excitation of linear radiative modes. We have checked that such an initial guess can be generated experimentally by means of the modulational instability of a low power probe pulse. In fact, Fig. 3 shows the results of a numerical simulation where we have added an initial cosine squared-type phase distribution (see left snapshot in Fig. 4) to a homogeneous plane wave in order to excite a regular grid of solitons, as in Refs. [13]. As shown in snapshots b) to d), this allows us to control the spatial location of the optical filaments generated during the beam breakup due to modulational instability. The outgoing grid of spatial solitons is depicted in snapshot d). Note its similarity with the initial state of Fig. 3.

Let us come back to the results of Fig. 3. The massive coalescence of all the fermionic bubbles occurs after a finite propagation distance $\eta = 200$ (see middle snapshot in Fig. 4), measured in units of $n_2 k_0^{-1} a^2$. As a result, a unique filament structure with large radius arises, as it can be appreciated in Fig. 4. We have checked that this soliton is a flat-top eigenstate with radial perturbations coming from the transition process. In fact, we have estimated its logarithmic radius to be around 2.3 and its peak density $\rho_c \approx 0.76$ (measured in units of $n_2/n_1$), thus corresponding with an intensity $2.34 \times 10^{13} \text{ W/cm}^2$, which is clearly on the liquid light branch displayed in Fig. 2. For Oxygen [4], we have considered a propagation distance of about 13 m. Although such a distance seems to be huge for a real experiment, we have verified that by stretching the external potential this distance can be realistically reduced by almost one order of magnitude. Finally, we note that the use of pressurized $O_2$ or air may also help to enhance the nonlinear optical response. For all these reasons, we conclude that the demonstration of the existence of both Fermionic and Liquid light and the phase transition between them would be an affordable challenge in real experiments.

In conclusion, we have proven that common media (O$_2$, air) can support the propagation of solitary waves that appear in two clearly different phases with unequal physical properties, namely the low power “Fermionic” light, satisfying an equation of state similar to that of a degenerate gas of fermions, and the high power “Liquid” light, obeying the YL equation. We have then shown how a grid of the fermionic light bubbles can be generated and forced to merge in a liquid droplet. We think that the possible experimental validation of our proposal could also provide an independent way to corroborate the deep change in the understanding of the filamentation process in gases that was proposed in Ref. [7]. Furthermore, these results in air pave the way for the improvement of recent experiments on laser-induced water condensation [14], built on top of these new robust light distributions.

This work was supported by MICINN, Spain (project FIS2008-01001). D.N. acknowledges support from Consellería de Economía e Industria-Xunta de Galicia through the “Maria Barbeito” program.

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