TORSIONAL MAGNETIC OSCILLATIONS IN TYPE I X-RAY BURSTS
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ABSTRACT
Thermonuclear burning on the surface of a neutron star causes the expansion of a thin outer layer of the star, \( \Delta R(t) \). The layer rotates slower than the star due to angular momentum conservation. The shear between the star and the layer acts to twist the star’s dipole magnetic field, giving at first a trailing spiral field. The twist of the field acts in turn to “torque up” the layer, increasing its specific angular momentum. As the layer cools and contracts, its excess specific angular momentum causes it to rotate faster than the star, which gives a leading spiral magnetic field. The process repeats, giving rise to torsional oscillations. We derive equations for the angular velocity and magnetic field of the layer, taking into account the diffusivity and viscosity that are probably due to turbulence. The magnetic field causes a nonuniformity of the star’s photosphere (at the top of the heated layer), and this gives rise to the observed X-ray oscillations. The fact that the layer periodically rotates faster than the star means that the X-ray oscillation frequency may “overshoot” the star’s rotation frequency. Comparison of the theory is made with observations of Chakrabarty et al. of an X-ray burst of SAX J1808.4–3658.

Subject headings: stars: magnetic fields — stars: neutron — X-rays: bursts

1 INTRODUCTION
Type I X-ray bursts have been observed in a number of low-mass X-ray binary systems. They are characterized by a rapid \((\sim 1–10 \text{ s})\) rise in the observed flux, followed by a relatively slow \((\sim 10–100 \text{ s})\) decline (see reviews by Lewin et al. 1995; Strohmayer & Bildsten 2003). Some objects show highly coherent oscillations during the burst. The oscillation frequency varies slightly during the burst but asymptotically approaches the neutron star rotation frequency (Strohmayer & Bildsten 2003 and references therein; van der Klis 2000). The oscillations indicate nonuniform emission from the star’s photosphere. The nonuniformity is likely due to the star’s nonaligned dipole magnetic field.

It is generally thought that Type I X-ray bursts are caused by thermonuclear explosions on the surface of a neutron star. If the neutron star’s magnetic field is strong enough to channel accretion onto the star’s surface, then the thermonuclear burning will not be uniform over the surface. It is then likely that the oscillations are produced by spin modulation of the burst flux. The exact mechanism of the oscillations, however, remains unclear in spite of a number of studies (Cumming et al. 2002; Spitkovsky et al. 2002; Bhattacharyya et al. 2005; Piro & Bildsten 2005). A significant difficulty encountered by models is explaining the oscillation frequency at which the actual layer may be an equatorial band (Inogamov & Sunyaev 1999). The heating of the layer occurs rapidly, and it expands rapidly, giving \( \Delta R(t) \). Subsequently, the layer slowly cools, and \( \Delta R(t) \) evolves and then decreases on a timescale of \( 10–100 \text{ s} \). We consider that \( \Delta R(t) \) is a known function of time obtained from the radiative transfer and the radial force balance.

We use a spherical inertial coordinate system \((R, \theta, \phi)\) and a coordinate system \((R, \theta, \phi')\) rotating with the star at the constant angular rate \( \Omega_s \). The flow field in the layer is

\[
v = v_R \hat{R} + v_\phi \hat{\phi},
\]

For the radial velocity we assume

\[
v_R = \frac{R - R_e}{\Delta R(t)} \frac{d\Delta R(t)}{dt}.
\]

The azimuthal velocity can be written as

\[
v_\phi = R_e \Omega_s \sin \theta \hat{\phi} + v'_\phi \hat{\phi},
\]

where \( v'_\phi(R_e, \theta) = 0 \).

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We consider the case of an orthogonal rotator where the magnetic moment $\mu$ is perpendicular to the rotation axis $\Omega$, as sketched in Figure 1. This situation would give X-ray oscillations at about twice the rotation frequency of the star. This case is valuable in that it is amenable to an analytic treatment, and it indicates the behavior for nonorthogonal rotators when the X-ray oscillations are at about the rotation frequency of the star. For a nonorthogonal rotator, we suggest that the value of $\mu$ in the following expressions be replaced by $\mu_\parallel = |\mu \times \Omega|/|\Omega|$. We focus on the equatorial region of the star’s surface, $|\theta - \pi/2| \leq \pi/6$. The evolution of the magnetic field within the layer is described by the induction equation, which follows from Faraday’s law and infinite conductivity,

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B),$$

with $v$ given by equation (1). Below, we include a finite conductivity. Inside the star $R \leq R_*$, the magnetic field is frozen in and rotates with the star. Outside the layer, $R > R_* + \Delta R$, the magnetic field is considered to be a vacuum field that rotates but is otherwise unaffected by processes in the layer. Any field lines linking the star and disk are expected to be opened because of the large difference between the angular velocity of the inner disk and the star (Lovelace et al. 1995).

It is useful to describe the magnetic field in terms of a flux function $\Psi \equiv RA_\theta$, where $A = (0, A_\theta, 0)$ is the vector potential. Therefore, in the equatorial region of the star,

$$B_R = -\frac{1}{R^2} \frac{\partial \Psi}{\partial R}, \quad B_\phi = \frac{1}{R} \frac{\partial \Psi}{\partial \phi},$$

and $B_\theta = 0$. We adopt a reference frame rotating with the star, with $\phi' = \phi - \Omega_\star t$ the azimuth relative to a fixed point on the star. In place of equation (4) we have

$$\frac{\partial \Psi}{\partial t} = -(v' \cdot \nabla)\Psi = -v_\phi' \frac{\partial \Psi}{\partial R} - v_\phi' \frac{\partial \Psi}{\partial \phi'},$$

where $v' = (v_R, 0, v_\phi')$ and $v_\phi' = v_\phi - \Omega_\star R_*$. This equation simply says that the flux function $\Psi$ is advected with the flow. We solve equation (6) by taking

$$\Psi(R, \phi', t) = iB_0 R_*^2 \exp \left[ i \int_{R_*}^R dR' k_R(R', t) + i\phi' \right].$$

The physical solution is the real part of $\Psi$, denoted $\text{Re}(\Psi)$; $B_0$ is a constant field strength at the magnetic poles of the star; and $k_R$ is the radial wavenumber, which remains to be determined. This form of solution was proposed earlier for magnetized disks (Bisnovatyi-Kogan & Lovelace 2000; in eqs. [2] and [3] of this paper there should be only one radial integral of $k_r$). From equation (5) we have

$$B_R = B_0 \frac{R_*^2}{R} \exp(\ldots), \quad B_\phi = -B_0 \frac{k_R R_*^2}{R} \exp(\ldots),$$

where the exponential factors are the same as in equation (7). Initially, there is no toroidal magnetic field so that $k_\theta(t = 0) = 0$. Substitution of equation (7) into (6) gives

$$\int_{R_*}^R dR' \frac{\partial k_R}{\partial t} = -k_R \frac{R - R_*}{\Delta R} \frac{d\Delta R}{dt} - \frac{v_\phi'}{R}.$$

Taking the $R$-derivative of this equation gives

$$\frac{\partial k_R}{\partial t} = -\frac{k_R}{\Delta R} \frac{d\Delta R}{dt} \frac{R - R_*}{\Delta R} \frac{d\Delta R}{dt} \frac{\partial k_R}{\partial R} \frac{\partial (v_\phi')}{\partial R}.$$

In the following we assume that $(|\Delta R/k_R|)(\partial k_R/\partial R) \ll 1$. As a result the middle term of the right-hand side of this equation can be neglected.

Note that $\delta_0(R, t) \equiv \frac{v_\phi'}{R}$ is the difference between the angular velocity of the layer and the star. We let $\ell$ be the specific angular momentum of the layer matter and $\ell_* = R_*^2 \Omega_\star$ the specific angular momentum of matter in the equatorial region of the star. We then have

$$\delta \ell = \ell - \ell_* = 2(R - R_*)R_\Omega + R_*^2 \delta \omega.$$
Therefore, equation (10) becomes
\[ \frac{dk_R}{dt} = -k_R \Delta R \left( \frac{\Omega_s}{R_s} \right) [L(t) - 2]. \] (12)

Introducing
\[ \Delta \omega(t) = \frac{\Delta R(t)}{R_s} [L(t) - 2] \] (14)
allows us to rewrite this equation as
\[ \frac{d(k_R \Delta R)}{dt} = -\Delta \omega. \] (15)

Because \( k_R \propto -B_0(R) \) from equation (8), \( k_R \Delta R \) is proportional to the toroidal flux (in one direction) in the layer. Equation (15) expresses the fact that the rate of change of this flux is proportional to the “rate of twisting,” \( \Delta \omega(t) \).

2.1. Angular Momentum Conservation

The total angular momentum of the heated layer minus \( \Delta M \Omega_s R_s^2 \) is simply
\[ \Delta L = R_s \Omega_s \int_{R_s}^{R_s + \Delta R} dR (R - R_s) L(t) \frac{dM}{dR}, \]
\[ = c_M R_s \Omega_s \Delta M \Delta R(t)L(t). \] (16)

Here, \( c_M \) is a dimensionless constant of order unity, assuming a self-similar mass distribution in the layer, \( dM/dR = f \left( (R - R_s) / \Delta R \right) \).

Changes in \( \Delta L \) are due to twisting of the magnetic field. The vacuum field in the region \( R > R_s + \Delta R \) rotates but is otherwise unaffected by processes within the heated layer. Thus, the change in \( \Delta L \) is due to the outflow of angular momentum from the surface of the star \( R = R_s \). That is,
\[ \frac{d\Delta L}{dt} = -\int dS_R \sin \theta \frac{\text{Re}(B_R) \text{Re}(B_\phi)}{4\pi} \bigg|_{R=R_s}, \]
\[ = \frac{1}{4} k_R R_s^4 B_0^2, \] (17)
where the fields are given by equation (8) and the integration is over the area \( 2\pi R_s^2 \) of the equatorial region of the star. Using equation (16) gives
\[ \frac{d}{dt} (\mathcal{R} \mathcal{L}) = \frac{\omega^2_B}{\Omega_s} \mathcal{K}, \] (18)
where
\[ \mathcal{R} \equiv \frac{\Delta R}{R_s}, \quad \mathcal{K} \equiv k_R \mathcal{R}_s, \quad \omega^2_B \equiv \frac{R_s B_0^2}{4 c_M \Delta M}, \]
where \( \omega_B = \text{const.} \) is an Alfvén frequency of the heated layer. The actual oscillation frequency of a layer of constant thickness \( \Delta R \) is larger than \( \omega_B \) by a factor of \( (R_s / \Delta R)^{1/2} \) (see eq. [21]).

For representative values from Cumming & Bildsten (2000),
\[ \omega_B \approx \frac{3.16 \text{ s}^{-1}}{\sqrt{4 c_M}} \left( \frac{B_0}{10^8 \text{ G}} \right)^{1/2} \left( \frac{10^{21} \text{ g}}{\Delta M} \right)^{1/2} \left( \frac{R_s}{10^6 \text{ cm}} \right)^{1/2}. \] (19)

For a layer of constant thickness, the Alfvén speed in the layer is \( v_A \approx \omega_B / \pi (R_s / \Delta R)^{1/2} \). For the given reference values and \( \Delta R = 10^6 \text{ cm}, \) \( v_A \approx 3.2 \times 10^4 \text{ cm s}^{-1} \).

2.2. Main Equations without Dissipation

Using equation (14), equations (15) and (18) can be written as
\[ \frac{d(R \mathcal{K})}{dt} = -\mathcal{K} \mathcal{W}, \]
\[ \frac{d\mathcal{W}}{dt} = \frac{\omega^2_B}{\Omega_s} \mathcal{K} - 2 \frac{dR}{dt}. \] (20)
where
\[ \mathcal{W} \equiv \frac{\Delta \omega}{\Omega_s}, \]
is the dimensionless rate of twisting. The initial conditions are that \( \mathcal{K}(0) = 0 \), which corresponds to no initial toroidal magnetic field, and \( \mathcal{W}(0) = -2R(0) \), which corresponds to the specific angular momentum of the layer being equal to its equilibrium value [i.e., \( L(0) = 0 \)].

Equation (20) is a linear system for \( (\mathcal{K}, \mathcal{W}) \) in that \( \mathcal{R}(t) \) is a given function. For \( \mathcal{R} = \text{const.} \), the solution of equation (20) is oscillatory with angular frequency
\[ \omega_{\text{osc}} = \frac{\omega_B}{\sqrt{\mathcal{R}}}. \] (21)

Thus, the period of the oscillation is proportional to \( (\Delta R)^{1/2} \).

Equations (20) have the form of Hamilton’s equations, \( dQ/dt = \partial H / \partial P, \) \( dP/dt = -\partial H / \partial Q, \) with \( P = \omega^2_B K \mathcal{R} / \Omega_s \) the canonical momentum, \( Q \equiv \mathcal{W} \) the canonical coordinate, and the Hamiltonian \( H = P^2 / (2 \mathcal{R}) + \omega^2_B Q / 2 - 2 \mathcal{R} dR / dt \). These equations are idealized in the respect that they neglect the magnetic diffusivity and the viscosity of the layer.

2.3. Equations with Magnetic Diffusivity

For finite conductivity \( \sigma \) of the plasma, the term \( \eta_m \nabla^2 \mathbf{B} \) is added to the right-hand side of equation (4), where \( \eta_m = c^2 / (4 \pi \sigma) \) is the magnetic diffusivity. In place of equation (6) we have
\[ \frac{\partial \Psi}{\partial t} = -(\mathbf{a} \cdot \nabla) \Psi + \eta_m \Delta^* \Psi, \] (22)
where \( \Delta^* \equiv \partial^2 / \partial R^2 + (1 / R^2) \partial^2 / \partial \phi^2 + \partial^2 / \partial \phi^2 \) is the adjoint Laplacian operator.

We solve equation (22) with a generalization of equation (7), where the radial wavenumber is complex, \( k_R \rightarrow k_R + i k_{RI} \). We introduce the dimensionless wavenumber \( (K_r, K_i) \equiv (k_R R_s, k_R R_s) \) and \( \tau_m \equiv R_s \eta_m \). Thus,
\[ \frac{d(R K_r)}{dt} = -\mathcal{K} \mathcal{W} - \frac{2}{\tau_m} K_r K_i, \]
\[ \frac{d(R K_i)}{dt} = -\frac{1}{\tau_m} (K^2 + 1 - K^2), \]
\[ \frac{d\mathcal{W}}{dt} = \frac{\omega^2_B}{\Omega_s} K_r - 2 \frac{dR}{dt}. \] (23)
The initial conditions are that $K_e(0) = 0$, $K_i(0) = 0$, and $\mathcal{V}(0) = -2R(0)$. In contrast with equations (20), the evolution equations with dissipation are non-linear.

2.4. Magnetic Diffusivity

Note that $\tau = R^2/\eta_m$ is the ohmic dissipation timescale of the layer multiplied by $(R_i/\Delta R)^2$. It is less than or equal to its value given by the classical Spitzer value of $\eta_m$,

$$ \tau \leq \tau_{Sp} \approx \frac{10^{13}}{Z} \left( \frac{R_s}{10^6 \text{ cm}} \right)^2 \left( \frac{T}{10^9 \text{ K}} \right)^{3/2}, \quad (24) $$

where $T$ is the temperature of the heated layer, $Z$ is an average atomic charge, and the Coulomb logarithm has been taken to be 8 (Cumming & Bildsten 2000).

Instabilities and associated turbulence in the heated layer are expected to give $\tau \ll \tau_{Sp}$. Under different conditions the deep part of the layer may be convective and the top part radiative, as discussed by Cumming & Bildsten (2000). The sound speed in the layer is $c_s \approx 2.9 \times 10^6 \text{ cm s}^{-1} (T/10^9 \text{ K})^{1/2}$ is generally much larger than the Alfvén speed $c_A$, estimated previously as $c_A \approx 3.2 \times 10^6 \text{ cm s}^{-1}$ at the beginning of the burst. Later in the burst the magnetic field is twisted, and the magnitude of the toroidal field increases substantially, mainly due to the field being confined to the thin layer $\Delta R$. Thus, the Alfvén crossing time $t_A = \Delta R/c_A$ decreases from its initial value of $ \sim 0.03 \text{ s}$, assuming $\Delta R = 10^5 \text{ cm}$. The strong toroidal field may lead to a buoyancy instability. Note, however, that the layer has a strong shear with a velocity difference across it of order $u \sim 2\Omega R \sim 5 \times 10^4 \text{ cm s}^{-1}$ so that the shearing time $t_{sh} = \Delta R/u \sim 2 \times 10^4 \text{ s}$ is shorter than $t_A$ in the initial part of a burst. Furthermore, note that the upward buoyant motion of a blob will be strongly influenced by the Coriolis force $2\rho v \times \Omega$, where $v$ is the velocity in the rotating reference frame. This force has a stabilizing effect on the blob motion and tends to give a circular “gyro” motion of radius $r_g = |v|/2\Omega \approx 6 \text{ cm} \left[ |v|/(3 \times 10^6 \text{ cm s}^{-1}) \right]$ about the $z$-direction (Triton 1988) for $\Omega_r/2\Omega = 400 \text{ Hz}$. Thus, $r_g$ for Alfvén speed motions is smaller than the layer thickness $\Delta R$ even for conditions in which the field strongly increased.

Because $v_B \ll c_A$, the layer may also be unstable to the magnetorotational instability (Balbus & Hawley 1998; Velikhov 1959; Chandrasekhar 1960; Moiseenko et al. 2006). The saturation of the instabilities is assumed to give rise to a turbulent diffusivity $\eta_m$ much larger than the Spitzer value and a turbulent viscosity $\eta_l \approx \eta_m$ (Bisnovatyi-Kogan & Ruzmaikin 1974). The viscosity is taken into account in $\mathcal{S}$ 2.5. The turbulent viscosity and diffusivity can be written in a form analogous to the Shakura & Sunyaev (1973) “alpha” prescription for a thin disk, taking into account that the maximum eddy size is $\Delta R$ and the maximum buoyant velocity is $\sim v_B$, with $v_B$ the initial Alfvén speed. Later in a burst, $v_B$ increases substantially, but at the same time the mentioned effects of shear and Coriolis force act to limit the motion of buoyant blobs. Therefore, we estimate $\eta_m = \alpha v_B \Delta R$ with $\alpha \leq 1$, which gives

$$ \eta_m \approx \nu_l \approx 3 \times 10^7 \frac{\text{cm}^2}{\text{s}} \left( \frac{\Delta R}{10^6 \text{ cm}} \right) \left( \frac{\nu_B}{3 \times 10^4 \text{ cm s}^{-1}} \right), \quad (25) $$

Note that for $\eta_m = 3 \times 10^7 \text{ cm}^2\text{s}^{-1}$, the magnetic diffusion time across the layer is $\tau_{diff} = \Delta R^2/\eta_m \approx 0.033 \text{ s}$ and is of order the initial Alfvén crossing time.

We emphasize that the values of $\eta_m$ and $\nu_l$ are highly uncertain in that relevant linear and nonlinear theory and simulations have yet to be done. An extensive literature exists on the theory and simulations of the related problem of the stability and motion of magnetic flux tubes in the convection zone and overshoot region of the Sun (e.g., Schüssler et al. 1994). It appears possible that these methods can be adapted to the magnetic field stability of the X-ray burst sources.

2.5. Equations with Diffusivity and Viscosity

Viscosity of the heated layer is taken into account by including on the right-hand side of equation (17) the viscous angular momentum flux across the surface $R = R_c$. This term is

$$ \int dS_R R \sin \theta T_{\nu} = -2\pi R^4 \rho \frac{d\omega}{dR} \bigg|_{R=R_c}, $$

where $T_{\nu}$ is the viscous contribution to the momentum flux density tensor. Thus, the equations for the layer including ohmic and viscous dissipation are

$$ \frac{d(R\mathcal{K}_c)}{dt} = -\Omega_{\nu} W - \frac{2}{\tau_m} \mathcal{K}_r \mathcal{K}_i, $$

$$ \frac{d(R\mathcal{K}_r)}{dt} = -\frac{1}{\tau_m} \left( \mathcal{K}_r^2 + 1 - \mathcal{K}_i^2 \right), $$

$$ \frac{dW}{dt} = \frac{\omega^2}{\Omega_c} \mathcal{K}_r - \frac{\mathcal{K}_r}{\tau_v} - \frac{2 dR}{dt}, $$

where

$$ \tau_v = c_v \frac{R^2}{\nu} \quad (28) $$

and $c_v = c_M \Delta M/[2\pi R^2 \Delta R \rho(R)]$ is a dimensionless constant of order unity if the mass distribution of the disk is self-similar. Numerical solutions of equation (26) indicate that the influence of viscosity is negligible for $\tau_v \approx \tau_m$.

3. SAMPLE SOLUTIONS

To solve equations (27), we need to know $\mathcal{R}(t) = \Delta R(t)/R_c$, which is not known from observations. A rough estimation of this function can be made assuming $\Delta R \propto t$ and the X-ray intensity $I \propto T^4$. The X-ray intensity of the SAX J1808.4 burst shown in Figure 2 (Chakrabarty et al. 2003) is fitted approximately by $I \propto 1/[1 + (t - 5/6.5)^2]$ for $10 \leq t \leq 40$ s accurately by $I \propto 1/t^2$. The corresponding layer thickness is $R = R(0)\left[1 + (t - 5/6.5)^2\right]^{1/4}$, with $t$ in seconds.

Figure 3 shows a sample case relevant to the observed burst shown in Figure 2. Later, $K_r < 0$, a leading spiral field forms. The ratio $\beta = B_r/B_B$ at $R = R_c$, is $\beta = -K_r$, and this is seen to vary from $-1450$ at $0.42$ s to $420$ at $1.45$ s. The first positive peak of $\Delta \omega/\Omega_c$ occurs at $t_1 \approx 0.25/\omega_B$ for small damping, $\tau_m \geq 10^7 \text{ s}$. The time between the first and second positive peaks of $\Delta \omega/\Omega_c$ is $\Delta t_{12} \approx 0.53/\omega_B$.

Note that $t_m = 10^7 \text{ s}$ corresponds to a diffusivity $\eta_m = R^2/\tau_m = 10^7 \text{ cm}^2\text{s}^{-1}$ for $R_c = 10^6 \text{ cm}$. From Figure 3 it is seen that the twisting amplifies the toroidal magnetic field to a peak value of order $K_r \approx 1400$ larger than the initial poloidal field of
the star. The amplification is due to the field being confined to the thin layer $R$ rather than it being wrapped many times around the star. For the case of Figure 3, the field is wrapped two turns in the clockwise direction during the first $0.7$ s, and then it is subsequently unwrapped, wrapped, etc. The magnetic diffusion time across the layer is $t_{\text{diff}} = \frac{R^2}{\alpha} t = 5.6$ s.

Figure 4 shows the “phase slip,” $\int_0^t \omega(t') dt'$, during a burst for the conditions as Figure 3. Figure 5 shows the nature of the wrapped field at $t = 0.4$ s for the case of Figure 3.

Fig. 4.—Phase slip during a burst for the conditions of Fig. 3, where $\phi(t) = \int_0^t \omega(t') dt'$.

The field line is given by

$$R = K_r(R/R_*)$$

where $K_r = 1447$, $K_i = 597$, and $R/R_* = 0.00678$. The letters $N$ and $S$ indicate the north and south magnetic poles, respectively.

Fig. 5.—Field lines in the heated layer at $t = 0.4$ s for the same case as in Fig. 3. The radial thickness of the layer has been expanded by a factor of 50.
Figure 6 shows $\Delta \omega / \Omega_*$ over a longer time interval for sample cases. The period of the oscillations of $\Delta \omega$ is proportional to $(\Delta R)^{1/2}$, which in this case is proportional to $1/t^{1/4}$.

4. CONCLUSIONS

We have derived a simple model for the influence of a neutron star’s magnetic field on the rotation of its surface layer rapidly heated by thermonuclear burning. The model assumes that the star’s magnetic moment is perpendicular to its rotation axis. The burning causes the expansion of a thin outer layer of the star, $\Delta R(t)$. The layer rotates slower than the star due to angular momentum conservation. The shear between the star and the layer acts to twist the star’s magnetic field giving at first a trailing spiral field. The twist of the field acts in turn to “torque up” the layer, increasing its specific angular momentum. As the layer cools and contracts, its excess specific angular momentum causes it to rotate faster than the star, which gives a leading spiral magnetic field. The oscillation of the angular velocity of the layer is a result of the tension of the twisted magnetic field. Nonuniformity of the star’s photosphere (at the top of the heated layer) is due to the magnetic field, and this gives rise to the observed X-ray oscillations. The fact that the layer periodically rotates faster than the star means that the X-ray oscillation frequency may “overshoot” the star’s rotation frequency. Observations by Chakrabarty et al. (2003) of an X-ray burst of SAX J1808.4−3658 show clear evidence of the overshoot of the frequency of the X-ray oscillations.

The equations of the model are for the difference in angular velocity between the layer and the star and the radial and toroidal components of the magnetic field. The frequency of the oscillations is proportional to the initial poloidal magnetic field of the star, inversely proportional to the square root of the mass of the heated layer, and inversely proportional to the square root of the layer’s thickness. In the absence of magnetic diffusivity and viscosity, the equations are linear and constitute a Hamiltonian system. The equations become nonlinear with the magnetic diffusivity included, but the inclusion of viscosity adds a linear term. The diffusivity and viscosity are probably due to turbulence in the heated layer, but the level of the turbulence is highly uncertain. The model has two important parameters: one is the oscillation frequency proportional to the initial magnetic field, and the other is the damping time due to magnetic diffusivity. The value of the magnetic diffusivity is highly uncertain. We find that the twisting can amplify the toroidal magnetic field to a peak value of order $10^3$ larger than the initial poloidal field of the star. The amplification is due to the field being confined to the thin layer $\Delta R$ rather than it being wrapped many times around the star.

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