Eternal inflation, energy conditions and the role of decoherent histories

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Abstract

Eternal inflation, the idea that there is always a part of the universe that is expanding exponentially, is a frequent feature of inflationary models. It has been argued that eternal inflation requires the violation of energy conditions, creating doubts for the validity of such models. We show that eternal inflation is possible without any energy condition violation, highlighting the important role of decoherence and the selection of states in the inflationary process.

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Classical physics is deterministic. By specifying the initial conditions we can exactly determine the correct solution to the equations of motion. There are quantities such as the total energy that are conserved by the equations of motion and thus are the same at all times in each solution.

In quantum mechanics, the closest we can come to this concept is the set of decoherent histories\(^1\) with corresponding probabilities. Decoherence is usually understood as interaction with the environment but in cosmology we must use a notion of effective decoherence. Here, things are taken to decohere when the information that could restore their coherence is widely scattered through the universe. Starting from conditions at a given time, there are many possible histories that extend these conditions into the future. Conservation laws are more subtle in this scenario. For example, a decoherent history might have different expectation values for energy at different times, even in a model with energy conservation.

As a simple example consider a system with two energy eigenstates \(|E_1\rangle\) and \(|E_2\rangle\) and corresponding eigenvalues \(E_1\) and \(E_2\) that is in state

\[
|\psi\rangle = \epsilon_1 |E_1\rangle + \epsilon_2 |E_2\rangle .
\]

Under unitary evolution, the expectation value \(E = \langle \psi | H | \psi \rangle = |\epsilon_1|^2 E_1 + |\epsilon_2|^2 E_2\) does not change, but this is not the quantity an observer measures. Upon measurement, an observer will find the system in energy eigenstate \(|E_2\rangle\) with probability \(|\epsilon_2|^2\). Now the energy is \(E_2\) instead of \(E\). This change does not need to be small. If \(E_2 \gg E\) the probability to get \(E_2\) is small but not zero. One might ask whether this change in energy is counterbalanced by a change in energy of the measurement apparatus or the environment. Interestingly, this question was recently answered in the negative with the construction of an explicit counterexample [1].

Rather than energy conservation, we can consider the restrictions imposed by energy conditions. For example, the null energy condition (NEC) guarantees that a converging congruence of null geodesics cannot be defocused to become diverging. But even in a theory that obeys NEC, a decoherent history of spacetime may include such a congruence that converges and then diverges due to the selection of a substate later in the decoherent history. This allows eternal inflation to be possible despite seeming to be ruled out by energy conditions, as we now discuss.

Energy conditions were introduced early in the history of general relativity as statements on how “reasonable” matter should behave [2]. Here we are mainly concerned with the NEC which requires the stress-energy tensor \(T_{\mu \nu}\) to obey

\[
T_{\mu \nu} \ell^\mu \ell^\nu \geq 0 ,
\]

for every null vector \(\ell^\mu\). For a perfect fluid stress-energy tensor, Eq. (2) becomes \(\rho + P \geq 0\), where \(\rho\) is the energy density and \(P\) the pressure. The NEC is obeyed by most classical fields, but it can be violated by quantum fields as with all pointwise energy conditions [4].

To examine the application of the NEC in cosmology we briefly introduce the concept of eternal inflation. Inflation is the proposed period of (near) exponential expansion in the early universe [5]. In most inflationary models, this expansion is driven by a scalar field with a potential, the inflaton. Even though classically the inflaton rolls slowly down the

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1 In interpretations without a collapse postulate, the evolution of the wavefunction is deterministic, however this is not very useful as it includes everything that might ever happen.

2 The NEC can be violated in some cases by classical non-minimally coupled scalar fields. For more information see [3] and [2].
potential leading to the end of inflation, its quantum fluctuations can drive the expansion rate up, as shown in Fig. 1. While upwards fluctuations are infrequent, because the inflating volume is exponentially increasing, there are always parts of the universe inflating in many inflationary models [6]. Borde and Vilenkin [7], Vachaspati [8], and Winitzki [9] argued that eternal inflation requires violations of the NEC. These works consider only fluctuations on superhorizon scales, so regions of horizon-size or a few horizon sizes are approximately given by the Friedman-Lemaître-Robertson-Walker (FLRW) metric,

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2,$$

and the equation governing the Hubble expansion, $H = \dot{a}/a$, is

$$\dot{H} = -4\pi G (\rho + P) + \frac{k}{a^2}.$$  

In an inflating spacetime, we can ignore the curvature term, so set $k = 0$. Eternal inflation requires an increase in the expansion rate, meaning $\dot{H} > 0$, so $\rho + P < 0$, and the NEC is violated.

We will show here that the above argument is incorrect. In fact it is possible to have eternal inflation even when the NEC is obeyed. This conclusion is not necessary to show the consistency of eternal inflation with quantum field theory, because quantum fields can easily violate the NEC. However it is obviously sufficient, and once we have made it we no longer need to be concerned with weaker conditions such as the averaged null energy condition. (For details about that condition and how it interacts with eternal inflation, see [10].)

How is it possible to have eternal inflation without NEC violation? The NEC tells us that $\dot{H} \leq 0$, but that is a statement about the evolution of the state of the universe under the equations of motion. Eternal inflation is not driven by such evolution but rather by the selection of substates of the quantum state. Indeed, a state with a given $H$ cannot evolve into one with a larger $H$. Rather, a state with a given $H$ on average can be composed of substates with different expansion rates, some of which are larger than the average. Decoherence will select one of the substates, and if it happens to be one with a high $H$, the resulting decoherent history will have a larger $H$ at later times than it had before.

For inflation to be eternal, such selections must happen repeatedly. This is possible because more and more modes pass outside the horizon as the universe expands. We can
FIG. 2. The wavefunction $\psi_k$ for a single mode of the inflaton field. The broad orange distribution shows the Bunch-Davies wavefunction for a superhorizon mode; the narrow blue distribution shows a possible wavefunction after decoherence (late-time state). The latter is peaked around some particular field value $\phi_k$.

start with the subhorizon modes in the Bunch-Davies vacuum. These modes are stretched by the expansion but while a mode is subhorizon, its frequency changes at a small rate compared to the frequency itself, thus conditions change adiabatically and the mode remains in the Bunch-Davies state. For subhorizon modes, this is close to the Minkowski space ground state and has a well-defined energy density.

However, as the mode crosses the horizon it no longer evolves adiabatically. The field value is frozen while the momentum is damped. The field values are now in a wide Gaussian, with different field values representing different energies (see Fig. 2). Decoherence processes (e.g., see [11]) select a much narrower state peaked around a specific field value and thus a specific energy. While the probability of the state being at a higher energy than the Bunch-Davies vacuum is small, it is non-zero. Such a state has a higher expansion rate.

This process occurs repeatedly as modes cross the horizon and decohere. Roughly each Hubble time there is a new chance for decoherence to increase $H$. These increases are very rare, but they result in faster expansion. In most models the expansion is fast enough that the physical volume of these rare regions is larger than the much more common ones where $H$ decreases as the inflaton rolls down the potential hill. Therefore, inflation continues forever.

Thus the NEC is not required to be violated in eternal inflation. Each state can obey the NEC, while the expansion rate increases through the selection process. Now we can ask whether typical late-time, fast-expanding states obey the NEC. We find [10] that most states leading to eternal inflation do in fact obey NEC. Even though such states expand rapidly at late times, if we follow them back to earlier times we find they were expanding even faster. Furthermore, in the Bunch-Davies vacuum, $T_{\mu\nu} \propto g_{\mu\nu}$, so NEC is obeyed. The typical states and substates involved in eternal inflation all obey NEC, even though the transition by decoherence from one state to another yields an increase in $H$.

In this essay we attempt to clarify how energy conditions should be applied in cosmology. Importantly, they only apply to each quantum state individually and we cannot use them to restrict how a later different state may arise by selection. This observation is similar to the non-conservation of energy during measurement in a quantum mechanical system. In cosmology the measuring device is replaced by decoherence, which can lead to a repeated selection of states. Therefore an eternally inflating spacetime does not require the violation of any energy conditions.
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