Scotogenic dark matter in an orbifold theory of flavor

Francisco J. de Anda,1,* Ignatios Antoniadis,2,3,† José W. F. Valle,4,‡ and Carlos A. Vaquera-Araujo5,6,§

1 Tepatitlán’s Institute for Theoretical Studies, C.P. 47600, Jalisco, México
2 Laboratoire de Physique Théorique et Hautes Energies (LPTHE), UMR 7589, Sorbonne Université et CNRS, 4 place Jussieu, 75252 Paris Cedex 05, France.
3 Albert Einstein Center, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland.
4 AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València, Parc Científic de Paterna. C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia) - SPAIN
5 Consejo Nacional de Ciencia y Tecnología, Avenida Insurgentes Sur 1582. Colonia Crédito Constructor, Alcaldía Benito Juárez, C.P. 03940, Ciudad de México, México
6 Departamento de Física, DCI, Campus León, Universidad de Guanajuato, Loma del Bosque 103, Lomas del Campestre C.P. 37150, León, Guanajuato, México

We propose a flavor theory in which the family symmetry results naturally from a six-dimensional orbifold compactification. “Diracness” of neutrinos is a consequence of the spacetime dimensionality, and the fact that right-handed neutrinos live in the bulk. Dark matter is incorporated in a scotogenic way, as a result of an auxiliary $Z_3$ symmetry, and its stability is associated to the conservation of a “dark parity” symmetry. The model leads naturally to a “golden” quark-lepton mass relation.

I. INTRODUCTION

Two major drawbacks of the standard model is the lack of neutrino mass and dark matter. Indeed, the discovery of neutrino oscillations implies that neutrinos have mass [1–3] and the need to supplement the Standard Model (SM). Likewise, many models adding particle dark matter to the standard model can be envisaged. An interesting idea is that dark matter is the mediator of neutrino mass generation, the corresponding models have been dubbed scotogenic [4–13]. An even tougher challenge in particle physics is understanding flavor, i.e. the pattern of fermion mixings as well as their mass hierarchies. The latter suggests extending the standard model by the imposition of a family symmetry in order to provide them a non-trivial structure. However, there are just too many possibilities to choose from [14, 15].

Rather than imposing a flavor symmetry in an ad hoc fashion, it could arise from extra dimensions [16]. Here we assume the spacetime to be 6-dimensional, where the extra dimensions are orbifolded as $T^2/Z_2$ from which, after compactification, emerges an $A_4$ flavor symmetry [17–19]. We assume a similar setup as in [20, 21]. However, there are crucial differences from the previous work, where the neutrinos were Majorana-type, with masses arising from the type-I seesaw mechanism. The model studied in this paper does not allow Majorana masses for neutrinos, since they are 6-dimensional chiral fermions, therefore they are fixed to be Dirac type when reduced to 4-dimensions. Due to an auxiliary triality or $Z_3$ symmetry, the small neutrino masses are generated only at one-loop level. All the mediators running in the loop are ”dark”, i.e. charged under a $Z_2$ symmetry which makes the lightest of them stable and hence a Dark Matter candidate. Therefore this is a realization of the Dirac scotogenic mass generation
mechanism [5, 8, 9, 12, 13]. The presence of the family symmetry leads naturally to a “golden” quark-lepton mass relation [22–27].

This letter is organized as follows. In Sec. II we briefly describe how we obtain the $A_4$ family symmetry from extra dimensions, in Sec. III we discuss the basic framework and quantum numbers. Fermion masses are discussed in Sec. IV, including both the charged fermions as well as the scotogenic neutrino masses in Sec. IV B.

II. $A_4$ FAMILY SYMMETRY FROM EXTRA DIMENSIONS

We assume the spacetime manifold as $\mathcal{M} = \mathbb{M}^4 \times (\mathbb{T}^2/\mathbb{Z}_2)$, where the torus $\mathbb{T}^2$ is defined by the relations
\begin{align}
z &= z + 1, \\
z &= z + \omega, \\
z &= -z,
\end{align}
where we rescale the original radii of the torus as $2\pi R_1 \Rightarrow 1$ and $2\pi R_2 \Rightarrow 1$ and adopt the complex coordinate notation $z = x_5 + ix_6$. The twist of the torus is the cube root of unity $\omega = e^{i\theta} = e^{2i\pi/3}$. There are four fixed points under these orbifold transformations, that define four invariant 4-dimensional branes
\begin{align}
\bar{z} = \{0, \frac{1}{2}, \frac{\omega}{2}, \frac{1+\omega}{2}\}.
\end{align}

After orbifold compactification, a remnant symmetry of the set of branes is inherited from the Poincaré invariance of the extra dimensional part of the manifold [29–31]. The transformations that permute the four branes leaving the whole brane-set $\bar{z}$ invariant are
\begin{align}
S_1 : z \rightarrow z + 1/2, \quad S_2 : z + \omega/2, \quad R : z \rightarrow \omega^2 z,
\end{align}
which are just translations and rotations of the extra dimensional coordinates. Among this set of transformations there are only two independent ones, since $S_2 = R^2 \cdot S_1 \cdot R$. These symmetry transformations relate to the $A_4$ generators through the identification $S = S_1$, $T = R$, satisfying
\begin{align}
S^2 = T^3 = (ST)^3 = 1,
\end{align}
which is the presentation for the $A_4$ group. Therefore, the brane set is invariant under $A_4$ transformations, which are a subset of the Extra Dimensional part of the Poincaré group.

The fields located on the branes will transform naturally under the remnant $A_4$ group. Following Refs. [20, 21] we identify this remnant symmetry as a family symmetry. The orbifold compactification also fixes the possible representations for the fields localized on the branes. One can write the $S, T$ as matrices acting on $(\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4)^T = (0, 1/2, \omega/2, (1+\omega)/2)^T$ as
\begin{align}
S &= \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}, \quad T = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix},
\end{align}
so that the 4 branes transform as a reducible representation 4 of $A_4$. One can decompose the 4 into irreducible representations $4 \rightarrow 3 + 1$. This can be made explicit with a basis change of the $S, T$ transformations, through a unitary transformation $V$, namely
\begin{align}
S \rightarrow V^\dagger SV = \begin{pmatrix}
S_3 & 0 \\
0 & 1
\end{pmatrix}, \quad T \rightarrow V^\dagger TV = \begin{pmatrix}
T_3 & 0 \\
0 & 1
\end{pmatrix},
\end{align}
where $S_3, T_3$ are the usual $3 \times 3$ matrix representation of $A_4$. In this way the 4 dimensional representation inherited from the branes can be explicitly decomposed as $4 \rightarrow 3 + 1$. Therefore, the fields on the branes must transform under the flavour group $A_4$ as the irreducible representations $3$ or $1$, depending on the location of each component of the field [18, 32]. A brane field $F_i(x, z)$ transforming as a $3$ of the remnant symmetry $A_4$ would be written as a sum of 4-d fields located on different branes

$$F_i(x, z) = \sum_{j=1,2,3} \delta^2(z - \bar{z}_j)V_{ij}^f F_i(x),$$

for $i = 1, 2, 3$ (no sum implied on $i$) which are the components of the triplet. Therefore different components of the triplet are located at different branes, and they transform into each other by the $A_4$ transformations just as the branes do. The singlet $f(x, z)$

$$f(x, z) = \sum_{j=1,2,3,4} \delta^2(z - \bar{z}_j)V_{ij}^s f(x) = \sum_{j=1,2,3,4} \frac{1}{2} \delta^2(z - \bar{z}_j)f(x),$$

is equally located on the four branes. Concerning fields in the bulk, for consistency all 6-dimensional fields should also transform under some irreducible representation of the $A_4$ remnant symmetry, thus, one can also localize fields in the bulk transforming as an arbitrary irreducible representation of $A_4$ [33].

### III. BASIC FRAMEWORK

The field content of our model and its transformation properties is specified in Table I. The model contains the usual standard model fermions $L, d^c, e^c, Q, u^c$, extended by three right handed neutrinos $\nu^c$. All the scalar fields are localized on the branes and consequently transform as flavor triplets. The model includes two electroweak scalar doublets $H_u, H_d$, together with an $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ singlet scalar $\sigma$ that obtains a VEV above the electroweak scale, breaking the $A_4$ flavor symmetry. The scalars are charged under the $Z_3$ symmetry, so that $H_d$ only couples to down-type fermions (charged leptons and down quarks), $H_u$ couples only to up-quarks and $\eta$ only couples to neutrinos, i.e. the $Z_3$ symmetry fixes the the Higgs couplings to be selective, as in type II 2HDM. Notice also that $Z_3$ forbids a renormalizable coupling that would give tree-level Dirac masses to neutrinos. The $Z_2$ is an unbroken symmetry, which stabilizes the lightest “dark” field. In order to generate scotogenic neutrino masses, two sets of inert scalars $\eta$ and $\chi$ are included, together with the electroweak singlet fields $S$.

| Field | $SU(3)$ | $SU(2)$ | $U(1)$ | $A_4$ | $Z_3$ | $Z_2$ | Localization |
|-------|---------|---------|--------|-------|-------|-------|--------------|
| $L$   | 1 2 $-1/2$ | 3 $\omega^2$ | 1 Brane |
| $d^c$ | 3 1 1/3 | 3 $\omega$ | 1 Brane |
| $e^c$ | 1 1 1 | 3 $\omega$ | 1 Brane |
| $Q$   | 3 2 1/6 | 3 $\omega^2$ | 1 Brane |
| $\nu^c_{i,2,3}$ | 3 1 $-2/3$ | 1 $\nu^c$, 1 $\nu'$, 1 $\omega^2$ | 1 Bulk |
| $H_u$ | 1 2 1/2 | 3 $\omega^2$ | 1 Brane |
| $H_d$ | 1 2 $-1/2$ | 3 1 1 | 1 Brane |
| $\sigma$ | 1 1 0 | 3 $\omega$ | 1 Brane |
| $S$   | 1 1 0 | 3 $\omega$ | -1 Brane |
| $\eta$ | 1 2 $-1/2$ | 3 1 | -1 Brane |
| $\chi$ | 1 1 0 | 3 $\omega^2$ | -1 Brane |

**TABLE I.** Field content of our previous model.

All the fields but the $u^c, \nu^c$ are assumed to live in the branes. Therefore, the $L, d^c, e^c, Q, S$ fields are 4-d Weyl left-fermions and $H_u, H_d, \eta, \chi$ are 4-d scalars. These fields on the branes behave as standard 4-d fields, and we restrict them to have only renormalizable couplings in the low energy theory.
In contrast, the $u^c, \nu^c$ are located in the bulk and are assumed to be 6-d Weyl fermions, each of which decomposes into a left-right 4-d Weyl fermion pair. The trivial boundary conditions on the orbifold do not allow a zero mode for the right-handed part. Therefore, after compactification, each of them decomposes into a massless 4-d Weyl left fermion, plus the corresponding Kaluza-Klein (KK) tower.

IV. FERMION MASSES

A. Quarks and charged leptons

The fermions in the bulk are 6-d Weyl fermions whose dimensionality is $[u, \nu] = 5/2$. They can have an explicit mass term which does not affect the zero modes at low energies. Their couplings to the brane fields come from the 6-d lagrangian

$$L_6 = \delta^2 (z - \bar{z}) \left( \frac{\tilde{y}_u^u}{\Lambda} Q H_u u^c_i + \frac{\tilde{y}_\nu^2}{\Lambda} S \chi^c \right),$$

which is suppressed by an effective scale $\Lambda$. The effective 4-d couplings at low energies absorb this scale as $y = \tilde{y}/R\Lambda$, where $R$ denotes the radii of the torus and defines the compactification scale.

The quarks and charged leptons behave exactly as in the previous model described in [20, 21]. After compactification the Yukawa couplings for the up quarks are

$$L_q = y_u^1 (Q H_u)_{1^\prime} u^c_1 + y_u^2 (Q H_u)_{1^\prime} u^c_2 + y_u^3 (Q H_u)_{1^\prime} u^c_3,$$

where the $(1, 2)$ denote the two possible $3 \times 3 \to 3_{1, 2}$ contractions. The Yukawa couplings for the charged leptons and down quarks, after compactification become

$$L_l = y_e^1 (Le^c H_d)_{1^\prime} + y_e^2 (Le^c H_d)_{2^\prime} + y_d^1 (Qd^c H_d)_{1^\prime} + y_d^2 (Qd^c H_d)_{2^\prime}.$$

Since the above interactions are dictated by the same Higgs doublet and being all the fields involved $A_4$ triplets, the golden relation between charged lepton and down quark masses [22–27] emerge as a prediction from the flavor symmetry

$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}},$$

which is consistent with current experimental data, with an excellent fit at the $M_Z$ scale [20]. Furthermore it is stable under RGE running, as can be seen in [28].

B. Scotogenic neutrino masses

Besides the standard model fermions and the enlarged scalar states described above, the particle content is minimally extended by a dark sector consisting of a fermion $S$ and two scalars $\eta, \chi$. In the low energy theory, the Yukawa couplings for neutrinos and fermion electroweak singlets are

$$L_\nu = y_1^1 (L \eta^\dagger S)_{1^\prime} + y_2^2 (S \chi)_{1^\prime} \nu^c_1 + y_2^2 (S \chi)_{1^\prime} \nu^c_2 + y_3^2 (S \chi)_{1^\prime} \nu^c_3 + y^{\nu_3} \sigma S S + h.c.$$

Among the terms in the scalar potential, after compactification we can find the following trilinear couplings:

$$L_{\ell}/\mu = y_1^1 (H_u \eta \chi^\dagger)_{1^\prime} + y_2^2 \sigma^3 + y_3^2 (\sigma H_u H_d)_{1^\prime} + y^{\chi^2} \chi^2 \sigma^1 + h.c.$$

1 In fact the scalar $\eta$ plays the role of the $H_\nu$ scalar present in [20, 21], but now belongs to the “odd” dark matter sector.
These terms are crucial to ensure the mixing of the neutral components of $\eta$ and $\chi$ and to break any possible degeneracy of the scalar mass eigenstates mediating the neutrino mass generation mechanism.

Note that, by choosing the “right-handed” neutrinos to be 6-d fermions, even if they are Majorana, their 4-dimensional zero modes are Dirac-type. Moreover, thanks to the auxiliary symmetries in our model, neutrinos are massless at the tree level. Indeed, the couplings in Eq. 13, together with the first trilinear scalar coupling in Eq. 14, generate Dirac neutrino masses at one loop from the diagram in Fig. 1 in a “scotogenic” fashion [4–13].

After spontaneous symmetry breaking, the electrically neutral components of the $A_4$ triplets $\eta$, denoted $\eta^0$, and the electroweak singlets $\chi$ mix into a total of six complex neutral scalars. Assuming trivial $CP^2$, the mass matrix in the basis $(\chi^\gamma, \eta^0_\delta)$, with $\gamma, \delta = 1, 2, 3$ is real and symmetric and can be diagonalized by an orthogonal transformation that can be written in blocks as

$$
\begin{pmatrix}
\phi_{A\alpha} \\
\phi_{B\alpha}
\end{pmatrix}
= \begin{pmatrix}
U_{A\alpha}^X & U_{A\alpha}^\eta \\
U_{B\alpha}^X & U_{B\alpha}^\eta
end{pmatrix}
\begin{pmatrix}
\chi^\gamma \\
\eta^0_\delta
\end{pmatrix},
$$

with $\alpha = 1, 2, 3$. By orthogonality, the $3 \times 3$ blocks in the above matrix are subject to the relations

\begin{align*}
(U^{A\gamma})^T U^{A\gamma} + (U^{B\eta})^T U^{B\eta} &= 1, \\
(U^{A\gamma})^T U^{A\delta} + (U^{B\eta})^T U^{B\delta} &= 1, \\
(U^{A\gamma})^T U^{A\delta} + (U^{B\eta})^T U^{B\delta} &= 0.
\end{align*}

(16)

Neutrinos acquire a small Dirac mass generated through the scotogenic loop. To first approximation one gets

$$
m_{\nu ij} = \frac{1}{16\pi^2} \sum_{k\alpha\gamma\delta} (y_{1,2}^\nu)^\delta_k M_k (y_{1,2,3}^{\nu^2})_{kj} \left[ (U^{A\gamma})^T U^{A\alpha} m_{A\alpha}^2 - m_{A\alpha}^2 M_k^2 \ln \frac{m_{A\alpha}^2}{M_k^2} + (U^{B\eta})^T U^{B\alpha} m_{B\alpha}^2 - m_{B\alpha}^2 M_k^2 \ln \frac{m_{B\alpha}^2}{M_k^2} \right],
$$

(17)

$^2$ Here we understand trivial $CP$ as the generalized $CP$ transformations of the fields that render all couplings real. See [20, 21] for details.
where $M_k$ with $k = 1, 2, 3$ are the eigenvalues of the mass matrix for the Fermion Singlets $S$, $m_A$ ($m_B$) are the physical masses of the $\phi_A$ ($\phi_B$) complex neutral scalars, and the effective Yukawa couplings $y_{1,2}^{\nu_1}$ and $y_{1,2,3}^{\nu_2}$ are given by

$$
(y_{1,2}^{\nu_1})^1 = \begin{pmatrix} 0 & y_1^{\nu_1} & 0 \\ y_2^{\nu_1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (y_{1,2}^{\nu_2})^2 = \begin{pmatrix} 0 & 0 & y_2^{\nu_2} \\ 0 & 0 & 0 \\ 0 & y_1^{\nu_2} & 0 \end{pmatrix}, \quad (y_{1,2}^{\nu_1})^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_1^{\nu_1} \\ 0 & y_2^{\nu_1} & 0 \end{pmatrix},
$$

(18)

$$
(y_{1,2,3}^{\nu_1})^1 = \begin{pmatrix} y_1^{\nu_1} & y_2^{\nu_1} & y_3^{\nu_1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (y_{1,2,3}^{\nu_2})^2 = \begin{pmatrix} 0 & 0 & 0 \\ y_1^{\nu_2} \omega^2 & y_2^{\nu_2} \omega & y_3^{\nu_2} \\ 0 & 0 & 0 \end{pmatrix}, \quad (y_{1,2,3}^{\nu_2})^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^{\nu_2} \omega & y_2^{\nu_2} \omega^2 & y_3^{\nu_2} \end{pmatrix}.
$$

(19)

Assuming trivial $CP$ symmetry from the start with real dimensionless couplings, and a general complex VEV alignment for $H_u, H_d, \sigma$ which break CP spontaneously together with the flavor symmetry, the model can describe realistic fermion masses. Besides, the model contains dark matter candidates, since the fields running in the loop are charged under the $\mathbb{Z}_2$ symmetry that remains unbroken after electroweak SSB. Our dark matter candidate is the lightest of the complex neutral scalars $\phi_A, \phi_B$ or the majorana electroweak singlet fermions $S$. A discussion on the viability of these kind of DM candidates can be found in [13].

## V. SUMMARY AND OUTLOOK

We have proposed a scotogenic flavor theory in which the $A_4$ family symmetry arises naturally from a six-dimensional spacetime after orbifold compactification. For spacetime dimensionality reasons, neutrinos must be Dirac fermions after compactification, since “right-handed” components live in the bulk. Thanks to the imposition of auxiliary “dark-parity” and triality symmetries the theory incorporates stable dark matter in a scotogenic way. Neutrinos are massless at tree level, with mass calculable from the scotogenic loop in Fig. 1. While the theory is not predictive enough to shed light on the structure of the quark CKM mixing matrix or the lepton mixing matrix, it does predict in a rather natural way the “golden” quark-lepton unification formula, despite the lack of a unification group [22–27]. The present model therefore provides a scotogenic dark matter completion of the orbifold scenario proposed in Ref. [20, 21], retaining its most remarkable prediction, namely the “golden” quark-lepton mass relation.

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## Appendix A: 6-d fermions

The Clifford algebra needs an $8 \times 8$ matrix representation that satisfy

$$\{ \Gamma^M, \Gamma^N \} = 2\eta^{\mu\nu} \Gamma_8, \quad (A1)$$
where one can use the 6-d chiral representation (in terms of the Pauli matrices

\[
\Gamma^0 = \begin{pmatrix}
0 & I_2 & 0 & 0 \\
I_2 & 0 & 0 & 0 \\
0 & 0 & 0 & I_2 \\
0 & 0 & I_2 & 0 
\end{pmatrix}, \quad \Gamma^i = \begin{pmatrix}
0 & \sigma^i & 0 & 0 \\
-\sigma^i & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^i \\
0 & 0 & -\sigma^i & 0 
\end{pmatrix},
\]

(A2)

Furthermore the chiral matrices are useful

\[
\Gamma^4_C = i\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 = \begin{pmatrix}
I_2 & 0 & 0 & 0 \\
0 & -I_2 & 0 & 0 \\
0 & 0 & I_2 & 0 \\
0 & 0 & 0 & -I_2 
\end{pmatrix},
\]

(A3)

\[
\Gamma^6_C = i\Gamma^5 \Gamma^6 = \begin{pmatrix}
I_2 & 0 & 0 & 0 \\
0 & I_2 & 0 & 0 \\
0 & 0 & -I_2 & 0 \\
0 & 0 & 0 & -I_2 
\end{pmatrix},
\]

which define the 6-d and 4-d chirality correspondingly.

In 6-d the Dirac fermion has 8 components. We can write them in terms of 4-d Weyl fermions

\[
\Psi = \begin{pmatrix}
\Psi^+_R \\
\Psi^+_L \\
\Psi^-_R \\
\Psi^-_L 
\end{pmatrix},
\]

(A4)

Where each \(\Psi^\pm\) (eigenstates of \(\Gamma^6_C\)) is composed of a left and right 4-d Weyl fermion (eigenstates of \(\Gamma^4_C\)).

The boundary conditions applied on 6-d fermions are

\[
P \mathcal{P} \Psi(x, z) = P \Gamma^6_C \Psi(x, -z) \rightarrow P \Psi^+_R(x, -z) \\
\rightarrow \Psi^+_L(x, -z) \\
\rightarrow -P \Psi^-_R(x, -z) \\
\rightarrow -P \Psi^-_L(x, -z).
\]

(A5)

The 6-d fermion irreducible representations are 6-d Weyl fermions, eigenstates of

\[
\Gamma^7 = -\Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5 \Gamma^6 = \Gamma^4_C \Gamma^6_C = \begin{pmatrix}
I_2 & 0 & 0 & 0 \\
0 & -I_2 & 0 & 0 \\
0 & 0 & -I_2 & 0 \\
0 & 0 & 0 & I_2 
\end{pmatrix},
\]

(A6)

so that the irreducible representations are

\[
\Psi_{6R} = \begin{pmatrix}
\Psi^+_R \\
0 \\
0 \\
\Psi^-_L 
\end{pmatrix} \quad \text{or} \quad \Psi_{6L} = \begin{pmatrix}
0 \\
\Psi^+_L \\
\Psi^-_R \\
0 
\end{pmatrix}.
\]

(A7)
The 6-d fermions contain both left and right parts. This way, just as in the 5-d case, even if one writes a 6-d Majorana mass term, it decomposes into a 4-d Dirac mass term.

[1] A. B. McDonald, “Nobel Lecture: The Sudbury Neutrino Observatory: Observation of flavor change for solar neutrinos,” Rev. Mod. Phys. 88 no. 3, (2016) 030502.

[2] T. Kajita, “Nobel Lecture: Discovery of atmospheric neutrino oscillations,” Rev. Mod. Phys. 88 no. 3, (2016) 030501.

[3] P. F. de Salas et al., “Status of neutrino oscillations 2018: 3σ hint for normal mass ordering and improved CP sensitivity,” Phys. Lett. B782 (2018) 633–640, arXiv:1708.01186 [hep-ph].

[4] E. Ma, “Verifiable radiative seesaw mechanism of neutrino mass and dark matter,” Phys. Rev. D73 (2006) 077301, arXiv:hep-ph/0601225 [hep-ph].

[5] Y. Farzan and E. Ma, “Dirac neutrino mass generation from dark matter,” Phys. Rev. D86 (2012) 033007, arXiv:1204.4890 [hep-ph].

[6] M. Hirsch, R. A. Lineros, S. Morisi, J. Palacio, N. Rojas, and J. W. F. Valle, “WIMP dark matter as radiative neutrino mass messenger,” JHEP 10 (2013) 149, arXiv:1307.8134 [hep-ph].

[7] A. Merle, M. Platscher, N. Rojas, J. W. F. Valle, and A. Vicente, “Consistency of WIMP Dark Matter as radiative neutrino mass messenger,” JHEP 07 (2016) 013, arXiv:1603.05685 [hep-ph].

[8] C. Bonilla, E. Ma, E. Peinado, and J. W. F. Valle, “Two-loop Dirac neutrino mass and WIMP dark matter,” Phys. Lett. B762 (2016) 214–218, arXiv:1607.03931 [hep-ph].

[9] M. Reig, D. Restrepo, J. Valle, and O. Zapata, “Bound-state dark matter and Dirac neutrino masses,” Phys. Rev. D97 no. 11, (2018) 115032, arXiv:1803.08528 [hep-ph].

[10] I. M. Ávila, V. De Romeri, L. Duarte, and J. W. F. Valle, “Minimalistic scotogenic scalar dark matter,” arXiv:1910.08422 [hep-ph].

[11] S. K. Kang, O. Popov, R. Srivastava, J. W. F. Valle, and C. A. Vaquera-Araujo, “Scotogenic dark matter stability from gauged matter parity,” Phys. Lett. B798 (2019) 135013, arXiv:1902.05966 [hep-ph].

[12] J. Leite, O. Popov, R. Srivastava, and J. W. F. Valle, “A theory for scotogenic dark matter stabilised by residual gauge symmetry,” arXiv:1909.06386 [hep-ph].

[13] J. Leite, A. Morales, J. W. F. Valle, and C. A. Vaquera-Araujo, “Scotogenic dark matter and Dirac neutrinos from unbroken gauged $B – L$ symmetry,” arXiv:2003.02950 [hep-ph].

[14] P. Chen, G.-J. Ding, A. D. Rojas, C. A. Vaquera-Araujo, and J. W. F. Valle, “Warped flavor symmetry predictions for neutrino physics,” JHEP 01 (2016) 007, arXiv:1509.06683 [hep-ph].

[15] H. Ishimori, T. Kobayashi, H. Okada, and M. Tanimoto, “Non-Abelian Discrete Symmetries in Particle Physics,” Prog. Theor. Phys. Suppl. 183 (2010) 1–163, arXiv:1003.3552 [hep-th].

[16] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. B436 (1998) 257–263, arXiv:hep-ph/9804398 [hep-ph].

[17] K. S. Babu, E. Ma, and J. W. F. Valle, “Underlying $A(4)$ symmetry for the neutrino mass matrix and the quark mixing matrix,” Phys. Lett. B552 (2003) 207–213, arXiv:hep-ph/0206292 [hep-ph].

[18] F. J. de Anda and S. F. King, “An $S_4 \times SU(5)$ SUSY GUT of flavour in 6d,” JHEP 07 (2018) 057, arXiv:1803.04978 [hep-ph].

[19] F. J. de Anda and S. F. King, “$SU(3) \times SO(10)$ in 6d,” JHEP 10 (2018) 128, arXiv:1807.07078 [hep-ph].

[20] F. J. de Anda, J. W. F. Valle, and C. A. Vaquera-Araujo, “Flavour and CP predictions from orbifold compactification,” Phys. Lett. B801 (2020) 135195, arXiv:1910.05605 [hep-ph].

[21] F. J. de Anda, N. Nath, J. W. F. Valle, and C. A. Vaquera-Araujo, “Probing the predictions of an orbifold theory of flavor,” arXiv:2004.06735 [hep-ph].

[22] S. Morisi, E. Peinado, Y. Shimizu, and J. W. F. Valle, “Relating quarks and leptons without grand-unification,” Phys. Rev. D84 (2011) 036003, arXiv:1104.1633 [hep-ph].

[23] S. F. King et al., “Quark-Lepton Mass Relation in a Realistic $A_4$ Extension of the Standard Model,” Phys. Lett. B724 (2013) 68–72, arXiv:1301.7065 [hep-ph].

[24] S. Morisi et al., “Quark-Lepton Mass Relation and CKM mixing in an A4 Extension of the Minimal Supersymmetric
Standard Model,” Phys. Rev. D88 (2013) 036001, arXiv:1303.4394 [hep-ph].
[25] C. Bonilla, S. Morisi, E. Peinado, and J. W. F. Valle, “Relating quarks and leptons with the $T_7$ flavour group,” Phys. Lett. B742 (2015) 99–106, arXiv:1411.4883 [hep-ph].
[26] C. Bonilla, J. Lamprea, E. Peinado, and J. W. F. Valle, “Flavour-symmetric type-II Dirac neutrino seesaw mechanism,” Phys.Lett. B779 (2018) 257–261, arXiv:1710.06498 [hep-ph].
[27] M. Reig, J. W. Valle, and F. Wilczek, “SO(3) family symmetry and axions,” Phys.Rev. D98 (2018) 095008, arXiv:1805.08048 [hep-ph].
[28] S. Antusch and V. Maurer, JHEP 11 (2013), 115 doi:10.1007/JHEP11(2013)115 [arXiv:1306.6879 [hep-ph]].
[29] G. Altarelli, F. Feruglio, and Y. Lin, “Tri-bimaximal neutrino mixing from orbifolding,” Nucl. Phys. B775 (2007) 31–44, arXiv:hep-ph/0610165 [hep-ph].
[30] A. Adulpravitchai, A. Blum, and M. Lindner, “Non-Abelian Discrete Flavor Symmetries from $T^\ast Z/N$ Orbifolds,” JHEP 07 (2009) 053, arXiv:0906.0468 [hep-ph].
[31] A. Adulpravitchai and M. A. Schmidt, “Flavored Orbifold GUT - an SO(10) x S4 model,” JHEP 01 (2011) 106, arXiv:1001.3172 [hep-ph].
[32] F. Bazzocchi, L. Merlo, and S. Morisi, “Fermion Masses and Mixings in a S(4)-based Model,” Nucl. Phys. B816 (2009) 204–226, arXiv:0901.2086 [hep-ph].
[33] T. J. Burrows and S. F. King, “A(4) Family Symmetry from SU(5) SUSY GUTs in 6d,” Nucl. Phys. B835 (2010) 174–196, arXiv:0909.1433 [hep-ph].