Colisional Cloud Debris and Propelled Evasive Maneuvers

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Abstract. Space debris clouds exist at various altitudes in the environment outside the Earth. Fragmentation of debris and/or collision between the debris of a cloud increases the amount of debris, producing smaller debris. This event also increases significantly the chances of collision with operational vehicles in orbit. In this work we study clouds of debris that are close to a spacecraft in relation to its distance from the center of the Earth. The results show several layers of colliding debris depending on their size over time of evasive maneuvers of the vehicle. In addition, we have tested such maneuvers for propulsion systems with a linear and exponential mass variation model. The results show that the linear propulsion system is more efficient.

1. Introduction

Space debris are non-operational objects that orbit Earth at various altitudes. They can be captured by the gravitational field, when they come from the interior solar system, or from the outer by slingshot effect of larger bodies (Jupiter, for example) or are by-product of the space missions. According to the European Space Agency [1], since the launch of Sputnik, there have been more than 4,900 launches, which have placed about 6,600 satellites in orbit, including 3,600 remain in space, with about 1,000 operational. Most of this equipment goes into disuse and orbits the Earth. Over time, they suffer from degradations that cause propellant leakage, explosions, fragmentation, and may collide with each other due to mutual collisions and the performance of space-dissipative forces [2]. With shocks, the number of objects increases exponentially in the Earth’s orbit, allowing for more frequent collisions with operational vehicles. Some of the debris is removed by atmospheric drag in LEO, but many of its layers would take decades or centuries to enter and be destroyed by friction [3]. Space cleansing has been discussed through mitigation techniques, but the alternative of reducing spatial activity or reducing the size of operating vehicles prevails. Meanwhile, in LEO, it is estimated that the average risk of catastrophic collisions is around 3% per year [4]. In a collision, the amount of energy produced is very high (~ 104 J), which would cause irreparable damage to an operational vehicle, making a mission impossible and generating more debris [5], [6], [7]. In the computational modeling of debris clouds, the number of individuals, fragmentation and their propagation must be considered to predict collisions with operational objects [2]. In this work, we propose a study of collisions between debris in a cloud that interacts with an orbiting space vehicle, without considering dissipative forces.

2. Mathematical model

The equations of the relative dynamics between a spatial debris and an operational vehicle (space colliding objects), considering them close in relation to the distance of the vehicle to the center of the
Earth can be found in 2012, [8]. The acceleration of the propulsion that will control the evasive maneuver of the object is given by the expression:

\[ \ddot{a}(t) = -v_e \frac{d}{dt} \ln(M(t)) \] (1)

\( M(t) \) is the total mass of the vehicle (mass of the space vehicle body added to the mass of the propellant). The propellant will vary during the operation of the evasive maneuver. In this work, we will use two models for mass variation:

1) Exponential: \( M(t) = m_0 (\chi + e^{-\gamma t}) \) and;
2) Linear: \( M(t) = (M_0 + \dot{m}t) \),

with, exponential technological parameters, (PTE): \( \gamma > 0 \) - motor power factor; \( \chi > 1 \) - mass factor, ie, the ratio between \( M_0 \) (initial vehicle mass), \( m_0 \) (initial mass of the propellant) and \( V_e \) - exhaust gas velocity and, linear technological parameters, (PTL): \( \dot{m} = \frac{dm}{dt} < 0 \) - Rate in time change of mass, \( V_e \) - exhaust gas velocity.

Collision objects are subject to the Earth's gravitational field in LEO and the propulsion force acts only on the vehicle. In addition, we do not consider dissipative forces. The solution of the differential equations of colliding relative dynamics for a propulsion system with exponential mass variation was found in [8].

Figure 1 shows the reference system in Cartesian coordinates centered on the space vehicle. At a distance \( r \) is located a space debris or a cloud of debris. The vehicle orbits around the Earth with circular velocity, \( \vec{\omega} = \vec{\omega} \hat{k} \), and the \( y \)-axis is in the radius \( r \) direction.

The collision between these objects occurs when their respective relative positions are null, for any values of their relative velocities, at a given time, \( t_c \). Thus, for a given pair of angles (\( \theta, \varphi \)) there will be initial relative positions and velocities that are favorable to collision between colliding objects. In general, the operational vehicle would have the time \( t_c \) to escape the collision with a spatial debris, performing an evasive maneuver.
2.1 Collisonal Cloud Debris

The cloud of debris is subject to the terrestrial gravitational field and is spaced by r from the operational vehicle. The relative initial positions between the debris i and j can be written in spherical coordinates centered on the vehicle with radius equal to r, such that:

\[ x_{0(i,j)} = r_0 \cos(\alpha_j) \sin(\beta_j) - \cos(\alpha_i) \sin(\beta_i) \]  
\[ y_{0(i,j)} = r_0 \sin(\alpha_j) \sin(\beta_j) - \sin(\alpha_i) \sin(\beta_i) \]  
\[ z_{0(i,j)} = r_0 \cos(\beta_j) - \cos(\beta_i) \]

The angles \(\alpha_l, \beta_l\) (\(l = i, j\)) are the angular spherical coordinates of each pair \((i, j)\) of cloud debris. The initial velocities between cloud debris are calculated using the same criteria:

\[ \dot{x}_{0(i,j)} = \dot{x}_{0(j)} - \dot{x}_{0(i)} \]  
\[ \dot{y}_{0(i,j)} = \dot{y}_{0(j)} - \dot{y}_{0(i)} \]  
\[ \dot{z}_{0(i,j)} = \dot{z}_{0(j)} - \dot{z}_{0(i)} \]

In this way, the debris may collide with each other during its relative dynamics with the vehicle, provided that these initial conditions are favorable to the collision between them. In this work, we always combine pairs \((i, j)\) of debris, admitting collision between only two debris at a time. The collision possibilities are obtained by combining events with each other, at each instant. If \(p\) is the number of debris that compose the cloud and \(n\) is the number of possible combinations two to two, the number of collision possibilities is:

\[ n_{p,2} = \frac{p!}{2!(p-2)!} \]

With these equations the dynamics of the near colliding objects (debris or cloud debris and operational vehicle) can be analyzed: 1) the distribution of colliding debris in the cloud by size of debris under the action of the Earth's gravitational force; 2) the distribution of colliding debris with respect to operational vehicle, both under the action of the Earth's gravitational force and; 3) the dynamics of the operational vehicle under the action of the forces of gravity and propulsion as a measure to prevent collision with the cloud.

3. Numerical Results

In this section we present the results of the numerical simulations of the dynamics of the cloud of debris under the action of Earth's gravity. In addition we present the relative dynamics between the debris and the vehicle, subject to linear and exponential propulsions.

3.1 Colliding Cloud Debris

We simulated two clouds of 7 and 15 debris and analyzed the distribution of collisions between them subject to field gravitational Earth attraction. The clouds are close to the spacecraft at a distance far below that of the vehicle to the center of the Earth. The graphs of Figures 2 to 4 show the results of the simulations for time intervals of 2,500 s and 3,000 s. This time interval is a safe margin for the time required for the calculations and implementation of the evasive maneuver (minimum of 25 minutes). We observed that debris from a larger cloud is more likely to collide with each other. In both cases, the highest incidence of collision occurs for small and medium size debris.
This result is expected, since the number of individuals is higher in the first cloud (Figure 2). In Figure 3, we observe that the larger debris collide more (11.0 m for a cloud of 15 and 7.5 m in the cloud of 7). The collisions intensify with the increase of the collision time, under the action of the terrestrial gravitational field.

3.2 Dynamics between Debris Cloud and Vehicle
The graph of Figure 4 shows the dynamics of the 15 debris of the cloud relative to the vehicle without propulsion. The initial distance between the cloud debris and the spacecraft is 3.0 km. The $D_i$, $i = 1, 15$ are the cloud debris. Table 1 shows the angular coordinates of the debris in the cloud.
TABLE 1 - Angular Coordinates - Cloud Debris

| Debris | $\theta$   | $\phi$   |
|--------|------------|----------|
| 1      | 165.0°     | 23.0°    |
| 2      | 137.0°     | 38.0°    |
| 3      | 147.0°     | 157.0°   |
| 4      | 120.0°     | 120.0°   |
| 5      | 149.0°     | 140.0°   |
| 6      | 160.0°     | 80°      |
| 7      | 124.0°     | 16.0°    |
| 8      | 135.0°     | 126.0°   |
| 9      | 180.0°     | 75.0°    |
| 10     | 113.0°     | 124.0°   |
| 11     | 48.0°      | 137.0°   |
| 12     | 129.0°     | 136.0°   |
| 13     | 15.0°      | 150.0°   |
| 14     | 135.0°     | 175°     |
| 15     | 153.0°     | 135.0°   |

Figure 4 - Relative Final Position vs. Collision Time

In the course of time, before 500s, we observe that the debris close to the vehicle and does not collide with it. The collision occurs at 3,000s, since there is no propulsion. At this moment, a greater incidence of collision between them may occur, depending on their size. At 1,600 s greater relative distance occurs between the debris and the vehicle. Figure 5 shows this dynamics propelled.
These results show the efficiency of the linear propulsion w.r.t. exponential propulsion system in the implementation of the evasive maneuver in front of the collision with the debris cloud, because it escapes collision with relatively larger debris. We highlight in Figure 6, below, an evasive maneuver of a collision with single cloud debris under the operation of both propulsion systems. (PTL: $\dot{m} = -0.0160 \text{ kg/s, } M_0=49.5 \text{ kg, PTE: } \gamma = 10^{-6} \text{ 1/s, } \chi = 10$)

We observed that the linear propulsion system is more efficient to escape larger debris from the exponential system at any velocity regime. At low velocities (Figure 6b), the exponential system performs collision evasive maneuvers with debris up to 6.57 m, while the linear system with debris up to 18.0 km.

4. Conclusions
We study the dynamics of debris cloud near a operational vehicle in the terrestrial gravitational field. We observed that the larger the debris cloud has the greater chance of collision between those of small and medium sizes, as the time of exposure to the field increases. The evasive maneuvers with linear propulsion are more efficient to escape from growing debris and even in regimes of small velocities, in relation to the exponential propulsion. We do not consider the atmospheric drag in our approach, because it is a basic theoretical investigation to study the effect of earth's gravity on the cloud of
debris, in order to know its scattering. We expect that the action of the atmospheric drag would certainly redistribute the possibilities of collision for relative high speeds.

5. References
[1] ESA Space Debris Working Group. Space Debris, ESA SP-1109, November 1998.
[2] Zhang, Binbin; Wang, Zhaokui; Zhang, Yulin. 2016. An analytic method of space debris cloud evolution and its collision evaluation for constellation satellites. Advances in Space Research, v. 58, n. 6, pp 903-913.
[3] Rossi A 2011 Space debris Scholarpedia, 6(1):10595. Available in: http://www.scholarpedia.org/article/Space_debris. Last access: 27/02/2014.
[4] Lafleur J M 2011. Extension of a simple mathematical model for orbital debris proliferation and mitigation.
[5] Klinkrad H 2006 Space Debris: Models and Risk Analysis. Springer Praxis, Berlin-Heidelberg.
[6] Rossi A Valsecchi G B 2006 Collision risk against space debris in Earth orbits. Cel. Mech. Dyn. Astron. 95, 345–356
[7] Smirnov N N 2002 Space Debris Hazard Evaluation and Mitigation. Taylor & Francis, London-New York.
[8] Jesus A D C et al 2012 Evasive Maneuvers in Space Debris Environment and Technological Parameters. Mathematical Problems in Engineering. Hindawi Publishing Corporation, v2012, pp15