Disentangling positive and negative partisanship in affective polarization using a coevolving latent space network with attractors model

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Abstract

We develop a broadly applicable class of coevolving latent space network with attractors (CLSNA) models, where nodes represent individual social actors assumed to lie in an unknown latent space, edges represent the presence of a specified interaction between actors, and attractors are added in the latent level to capture the notion of attractive and repulsive forces. The models are estimated using Bayesian inference. We apply the CLSNA models to the question of affective polarization, which expects Republicans and Democrats to cohere, favor and interact with their own party and to distance, repel and interact less with the opposing party. Using two different longitudinal social networks from the social media platforms, Twitter and Reddit, we investigate the relative contributions of positive (attractive) and negative (repulsive) affect among political elites and the public, respectively. Our goal is to uncover and quantify polarization—and disentangle the positive and negative forces within and between parties, in particular. Our analysis confirms the existence of affective polarization among both political elites and the public. While positive partisanship remains dominant across the full period of study for both Democratic elites and the public, a notable decrease in Republicans’ strength of in-group affect since 2015 has led to the dominance of negative partisanship in their behavior.

1 Introduction

The increase in partisan polarization over the last five decades is one of the most consequential developments in American politics. Despite continued scholarly debate on the degree of policy polarization, if any, among the public [1, 9], there is little doubt today that the public is polarized in their affections towards the parties [10, 16, 4, 14, 19, 12]. Affective polarization expects Republicans and Democrats to view their own parties favorably and other parties with increasing disdain [15]. Accordingly, it seems, we have witnessed an increase in party loyalty and straight-ticket voting [2], as well as animosity towards candidates [8], not to mention a decrease in ambivalence, indecision and floating in elections [28].

However, a peculiar facet of affective polarization theory has recently emerged. A historic rise in independents identifying as independents and a more recent preference among swaths of the public to avoid politics altogether as well as hold negative evaluations of partisanship in general are not easily explained by group attachment [17, 18, 6]. Instead, the movement in affective polarization has largely been in terms of increasingly negative evaluations of the other party, while the public’s feelings toward their own party has remained fairly stable [3, 8]. Such dynamics suggest an increased role of negative partisanship. Originally noted in studies of multiparty voting [22, 7, 20], the idea is that negative feelings about the other party (i.e., negative

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partisanship)—as opposed to positive feelings about one’s own (i.e., positive partisanship)—are dominant in political behavior and opinions.

While many prior studies investigate the role of positive/negative partisanship in affective polarization through survey-based measures using self-reports on feeling thermometers [13, 2] and multi-item scales [5], our approach provides a new method for measuring polarization with network data. Moreover, our novel model allows us to uncover and quantify affective polarization, as well as disentangle positive and negative partisanship within and between groups in social network interactions. This is an important contribution because the apparent distance between opposing political parties may be driven by different combinations of positive and negative forces within and between them. A hypothesis of negative partisanship, for example, requires an understanding of the strength of identity to the in-group relative to the out-group. Using our approach, we can untangle the direction and context of partisanship to shed light on their relative strengths in social network behavior.

In this article, we use two different longitudinal social networks, where nodes represent individual social actors and edges represent the presence of a specified interaction between actors from the social media platforms Twitter and Reddit, to investigate the relative contributions of positive (attractive) and negative (repulsive) partisanship in the elite and the public, respectively. To do so, we develop a class of coevolving latent space network with attractors (CLSNA) models in which both the links between nodes and certain characteristics (or attributes) of nodes evolve over time, each in a way that impacts the other, for characterizing coevolutionary phenomena in social behaviors such as flocking and polarization. This class of models falls within the subclass of latent space models, of which static versions are now especially well-developed, and progress on dynamic versions has been made in recent years. Existing dynamic latent space network models [23, 24, 25, 26, 27] dictate simply that latent characteristics evolve in time in a Markov fashion and then links between nodes exist with probabilities driven by node distance in the underlying latent space. In contrast, in our framework the temporal evolution of latent characteristics can depend on network connectivity—hence, a coevolving network model. Such a feedback mechanism is embedded into the model via the presence of attractors (a concept fundamental to dynamical systems) at the latent (i.e., unobserved) level. This class of coevolving network models is, to the best of our knowledge, novel.

In order to capture, disentangle, and quantify key components of polarization that may drive social network behavior online in our setting, we define a two-group CLSNA model with two attractors mimicking attractive and repulsive forces, which is a specific version of the aforementioned CLSNA model class. In this version of the model, each node in the network is assumed to fall into one of two groups with known labels, and their movements are influenced by their neighbors through specially defined attractors. We also account for persistence of links in the dynamic evolution of the network, which is a necessary control informed by social network theory.

This article is organized as follows: In the ‘Materials and Methods’ section we provide an overview of the two network data sets used in this study through an exploratory analysis. We also introduce the statistical network model proposed in this work, including a discussion of model behaviors and parameter interpretation. Adopting a Bayesian perspective, we develop a Metropolis-Hastings (MH) within Gibbs MCMC framework for posterior inference. In the ‘Results’ section we present the application of this model to the two data sets and quantitatively analyze the evolution of the key factors in affective polarization. Finally, we conclude the study with a discussion of the results and the model proposed.

2 Materials and Methods

In this section, we introduce data used in this study and outline our specific statistical modeling and inference approach. Data and code are available at https://github.com/KolaczykResearch/CLSNA-2Party-Polarization.

2.1 Data

We construct and explore two different online longitudinal social interaction networks for evidence of polarization, one of the elite, via Twitter, and one of the public, via Reddit.
2.1.1 Twitter congressional hashtag networks

On Twitter, we collected tweets for every US congressperson with a handle from 2010 to 2020. This yielded 796 accounts, 843,907 tweets (after retweets and tweets with no hashtags are removed) and 1,252,455 instances of hashtag sharing. This data was used to construct a binary network for each year, wherein nodes correspond to sitting members of congress and edges between any two nodes indicates that the number of common hashtags tweeted by both members of congress that year was more than the average. The nodes for the resulting networks vary from year to year since some members of Congress were reelected, while others may have joined Twitter at a later stage, left early or both. In our analysis, we focus on 207 members of Congress who served in office and stayed active on Twitter over the entire course of our study from 2010 to 2020, among whom 131 are Democrats and 76 are Republicans.

2.1.2 Reddit comment networks

In the case of Reddit, we collected the full data on submissions and comments since the site’s inception through pushshift.io. We focus on active Reddit users whose ideologies can be identified from their comments or flairs with declarative patterns, e.g., “I am a Republican.”. We thereby selected 102 Republicans and 267 Democrats who made 1) at least one comment in each month during April 2015—March 2020, and 2) more than 50 comments in a year across political subreddits (e.g., ‘politics’, ‘Libertarian’, ‘PoliticalDiscussion’, ‘Conservative’, etc.). We then constructed longitudinal binary networks on these 369 active users for each one-year period based on their interactions in comments, wherein an edge between two active users indicates that they commented on the same submission.

2.2 Exploratory Analysis

Figure 1 and Figure 2 show plots of the density of edges within and between Democrats and Republicans as they evolve over time for the Twitter and Reddit networks, respectively. In Figure 1 for Twitter congressional hashtag networks, while initially growing together in density, over the last four years, we notice divergent trends among the subsets of Democrats and Republicans, with Democrats increasing in their social media connections to each other and Republicans decreasing. While connections have increased overall, the presidential election year of 2016 brought about a drop in inter-party connections. In Figure 2 for Reddit comment networks, while intra-party connections for the two parties share similar trends, with both of them initially increasing then decreasing, they differ in the extent of drop over the last three years, with more interaction ties dissolved among the subset of Republicans than Democrats. The inter-party connections also have decreased following the election year of 2016.

This straightforward analysis is the first hint at a nuanced perspective on the polarization hypotheses from the Twitter congressional network data of the elite, and the Reddit comment network data of the public. Of course, this is only descriptive, and thus we turn to inferential methods with our proposed two-group CLSNA model with attraction and repulsion.

Figure 1: Twitter congressional hashtag networks. Four types of edge densities over years (inter-party, intra-party and overall) for constructed networks on 207 members of Congress who served in office and stayed active in Twitter over the entire course of our study from 2010 to 2020.
2.3 A Two-group Coevolving Latent Space Network with Attractors (CLSNA) Model with Attraction and Repulsion for Affective Polarization

Let $G_t = (V_t, E_t)$ be a network evolving in (discrete) time $t$, with vertex set $V_t$ and edge set $E_t$. For simplicity, assume that $V_t \equiv V$ is fixed over time, of order $N = |V|$. Let $Y_t$ be the (random) adjacency matrix at time $t$ corresponding to $G_t$. Throughout the paper, we use capitals to denote random variables and lower-cases to denote the realizations of them. We assume data come in the form of time series of adjacency matrices \{$y_t : t = 1, \cdots, T$\}, where $y_{t,ij} = 1$ if there is an edge between node $i$ and node $j$ at time $t$ and 0 otherwise.

To model the dynamic evolution of networks in connection with affective polarization, we use the latent space approach and add attractors in the latent level to capture the notion of attractive and repulsive forces specifically for the mechanism of affective polarization. Let $z_{t,i} \in \mathbb{R}^p$ be a time-indexed latent (i.e., unobserved) position for node $i$ in $p$-dimensional Euclidean space, and $z_t = \{z_{t,i}\}$. Assume that each of the $N$ nodes of the network falls into one of two groups, i.e., Democratic and Republican, with known node label $\pi(i) \in C$ for node $i$, where $C = \{1, 2\}$ is the set of group labels. Formally, we define our model as follows:

$$Y_{t,ij} \mid p_{t,ij} \sim \text{Bernoulli}(p_{t,ij}) \quad (1)$$

$$\logit(p_{t,ij}) = \alpha + \delta Y_{t-1,ij} - s(z_{t,i}, z_{t,j}) \quad (2)$$

$$Z_{t,i} \mid Z_{t-1,i} = z_{t-1,i} \sim \text{Normal}(z_{t-1,i} + \gamma_{\pi(i)}^{w} A_w^{w}(z_{t-1}, Y_{t-1}) + \gamma_{\pi(i)}^{b} A_b^{b}(z_{t-1}, Y_{t-1}), \sigma^2 I_p) \quad (3)$$

with initial distribution at time $t = 1$,

$$Z_{1,i} \sim \text{Normal}(0, \tau^2 I_p) \quad (6)$$

$$\logit(p_{1,ij}) = \alpha - s(z_{1,i}, z_{1,j}) \quad (7)$$

Here $s(\cdot, \cdot)$ is a similarity function, and $A_w^{w}$ and $A_b^{b}$ are the two attractor functions for node $i$ in $Y_{t-1}$. Specifically, we set $s(z_{t,i}, z_{t,j}) = ||z_{t,i} - z_{t,j}||_2$, and define the two attractors for node $i$ as follows,

$$A_w^{w}(z_{t-1}, Y_{t-1}) = \tilde{z}_{t-1,i}^{1} - z_{t-1,i}; \tilde{z}_{t-1,i}^{2} = \frac{1}{|S_1^{(i)}|} \sum_{j \in S_1^{(i)}} z_{t-1,j} \quad (8)$$

$$A_b^{b}(z_{t-1}, Y_{t-1}) = \tilde{z}_{t-1,i}^{2} - z_{t-1,i}; \tilde{z}_{t-1,i}^{2} = \frac{1}{|S_2^{(i)}|} \sum_{j \in S_2^{(i)}} z_{t-1,j} \quad (9)$$

which are the discrepancies of $z_{t-1,i}$ from two local averages at time $t - 1$. These latter are the average of latent values of nodes in the following two sets, informed by a combination of group membership, and network connectivity.
1. \( S_1(i) = \{ j \in N \setminus i \mid Y_{ij} = 1, \pi(i) = \pi(j) \} \), neighbors of node \( i \) in the same group

2. \( S_2(i) = \{ j \in N \setminus i \mid Y_{ij} = 1, \pi(i) \neq \pi(j) \} \), neighbors of node \( i \) in a different group.

In this proposed model, we assume that each node lies in a \( p \)-dimensional Euclidean latent space, and the smaller the distance between two nodes in the latent space, the greater their probability of being connected, as in (1), (2). The expressions in Eqs. (8) and (9) capture the discrepancy between the current latent position of node \( i \) and the average of that of its current neighbors in groups 1 and 2, respectively. The corresponding parameters \( \gamma_1^w, \gamma_2^w \), and \( \gamma^b \) represent attractive/repulsive forces, as we discuss below.

In contrast to the existing dynamic latent space network models [23, 24, 25, 26, 27] where the latent process is assumed to evolve over time in a Markov fashion with transition distribution, e.g., \( Z_{t,i}Z_{t-1,i} = z_{t-1,i} \sim \text{Normal}(z_{t-1,i}, \sigma^2 I_p) \), and thus to drive evolution of the networks, as illustrated in Figure 3 (left), our CLSNA model allows the network connectivity to enter the temporal evolution of latent positions in the form of attractors, as illustrated by the blue arrow in Figure 3 (right). Specifically, in our model the evolution of latent positions for each node \( i \) from \( t - 1 \) to \( t \) is modeled by the normal transition distribution in Eq. (3-5), the mean vector of which depends not only on the latent position of itself at time \( t - 1 \), but also on the two local averages, one from its neighbors in the same group, the other from its neighbors in a different group, as captured in (8) and (9). This is an important aspect of our model since it quantifies propensity for attraction/repulsion within/between two groups, and can help us understand how polarization/flocking and interaction co-evolve. Strength of attraction/repulsion toward local averages is therefore summarized by the attractor functions and the associated parameters, the details of which are discussed in the later sections.

We also include an effect for edge persistence, as illustrated by the red arrow in Figure 3 (right), which is a necessary control informed by social network theory. \( \delta \) captures the impact of having an edge at time \( t - 1 \) on whether or not there is an edge at time \( t \). For \( \delta > 0 \), the probability of an edge at time \( t \) will be increased when one exists already at time \( t - 1 \), and hence the model explicitly captures a notion of edge persistence.

\[
Y_{t-1,ij} \quad Y_{t,ij} \quad Y_{t-1,ij} \quad Y_{t,ij}
\]

\[
\{Z_{t-1,i'}\} \rightarrow \{Z_{t,i'}\} \quad \quad Z_{t-1} \rightarrow Z_t
\]

Figure 3: Graphical model representations of existing dynamic latent space network models (left), and proposed CLSNA (right) with both node attraction (blue arrow) and edge persistence (red arrow).

### 2.4 Model Behavior and Parameter Interpretation

Our model incorporates a level of baseline connectivity (\( \alpha \)), edge persistence (\( \delta \)), two separate within-group node attraction for the two groups (\( \gamma_1^w \) and \( \gamma_2^w \), respectively), between-group node attraction (\( \gamma^b \)), and a measure of volatility (\( \tau^2 \), initially, and \( \sigma^2 \) for \( t > 1 \)). A rich set of behaviors can be generated by varying these parameters. The three attraction parameters \( \gamma_1^w, \gamma_2^w \) and \( \gamma^b \) are of particular interest, in that by varying the signs they allow for the possibility of different combinations of attraction and/or repulsion in the evolution of the latent positions. The sign of these parameters encodes the direction of these forces – a positive sign indicates latent positions being pulled toward the direction of local averages, aka attraction, while a negative sign indicates being pushed toward the opposite direction, aka repulsion. For example, when \( \gamma_1^w > 0, \gamma_2^w > 0 \), we can interpret this as two groups flocking together, while for \( \gamma_1^w > 0, \gamma_2^w > 0 \) but \( \gamma^b < 0 \), the two groups are flocking separately—that is, we have a notion of affective polarization.

In Figure 4, we illustrate the behavior of latent positions and network connectivity in simulated models for the two scenarios, one reflecting two group flocking, and the other, polarization among the same two groups. For convenience of visualization, the latent space is taken to be one-dimensional. We can see that initialized with different latent positions, the time courses for positions of the \( N = 10 \) nodes in this network cluster together under flocking. But initialized together, they diverge into two clusters under polarization.
At the same time, while the network becomes ever more densely connected over time under flocking, it evolves towards two fully connected subgraphs under polarization.

Figure 4: Top: Flocking setting \((\gamma^w_1, \gamma^w_2, \gamma^b > 0)\). Bottom: Polarization setting \((\gamma^w_1, \gamma^w_2 > 0, \gamma^b < 0)\). Left: Trajectories of latent positions by time for two groups in 1-dimensional space. Right: Snapshots of the evolution of the probability of connections between different nodes in the graph.

2.5 Quantifying the Extent of Inter-/Intra-party Attraction/Repulsion and Edge Persistence

While the sign of each attraction parameters \(\gamma^w_1, \gamma^w_2, \gamma^b\) encodes attraction (positive sign) or repulsion (negative sign) within group 1, within group 2 and between the two groups, respectively, the absolute value of these parameters can be used to quantify the extent of inter-/intra-party attraction/repulsion. Similarly, the value of \(\delta\) can be used to quantify the relative importance of edge persistence. Accordingly, we take the effect size of inter-party repulsion as a measurement for negative partisanship, and that of intra-party attractions for positive partisanship.

We can answer an array of questions regarding the proposed mechanism for affective polarization by investigating the values of these parameters. For example, does the phenomenon of affective polarization occur in our Twitter and Reddit data? If so, are negative feelings about the other party or positive feelings about one’s own the dominant factor that drives the interaction of affective partisan polarization among elites and the public online? These questions can be answered respectively by assessing whether \(\gamma^b < 0\); and by comparing the values of \(|\gamma^w_1|, |\gamma^w_2|\) with \(|\gamma^b|\). Essentially, positive (negative) \(\gamma^b\) indicates positive (negative) affect towards the out-party, and \(|\gamma^b|\) measures the extent of out-party favorability or disdain. Similarly for \(\gamma^w_1, \gamma^w_2\), the signs encode in-party positive/negative affect, and the magnitude encode the extent of in-party favorability or disdain.

2.6 Bayesian Inference

The parameters in our model are natural and interpretable candidates for statistical inference. Given the hierarchical nature of our model, Bayesian inference based on appropriate interrogation of the posterior distribution makes sense. That is, given an observed network time series \(\{G_t\}_{t=1}^T\) or, more specifically, a time series of the corresponding adjacency matrices \(\{Y_t\}_{t=1}^T\), we can make inference of the latent positions
\( Z_t \) and model parameters \( \theta = (\alpha, \delta, \gamma_w^1, \gamma_w^2, \gamma_b, \tau^2, \sigma^2) \) based on the posteriors \( p(Z_{1:T}, \theta | Y_{1:T}) \). A closed-form expression for this distribution is not available, but we can use Markov chain Monte Carlo. We have implemented an adaptive Metropolis-Hastings (MH) within Gibbs MCMC scheme for posterior sampling. The implementation in non-trivial, as certain issues of scaling (regarding the volatility parameters \( \sigma^2 \) and latent positions) and rotational invariance (in the latent space) must be resolved. Details of the MCMC algorithm are given in the SI Appendix.

3 Results

In this section, we fit our model to both Twitter data and Reddit data, with a latent space of dimension \( p = 2 \), and present the estimates for model parameters and latent positions. The choice of two dimensions is consistent with DW-NOMINATE, one of the most popular established ideal point models of congressional ideology, for which 2 dimensions explain up to 90% of variation in roll call voting [21]. To evaluate how well the model explains the data used to fit the model, we obtain the in-sample edge predictions by plugging the estimates into the linkage probability function and compute the AUC (area under the ROC curve) [11].

3.1 Twitter Data Analysis

We first fit our model with time-invariant parameters to the whole sequence of longitudinal networks in Twitter from year 2010 to 2020. The AUC values computed at each year are all above 0.976, and the overall AUC value computed across all times is 0.988, providing evidence that our model fits the data very well.

The summary statistics for the posterior distribution of model parameters are provided in Table 2. The edge persistence coefficient indicates that the log-odds that an edge appears increase by 1.5 if the same edge appeared in the previous time frame. The between-group coefficient is \(-0.155\), demonstrating polarization across the sets of Republican and Democratic members of Congress. Additionally, the within group coefficient is 0.493 for Democrats, and 0.105 for Republicans, which means that while they have moved away from one another, they generally flocked to their own. Moreover, the Democratic members have a higher extent of intra-party attraction on Twitter, meaning that for Democratic members the positive feelings toward their own party were stronger than that for Republican members. Comparing the magnitude of between-group coefficients to the two within-group coefficients, we can see that for Democrats the positive feelings toward their own party were stronger than the negative feelings toward the other party in the movement of affective polarization, while for Republicans negative partisanship was dominant.

Figure 5 shows the posterior means of latent positions for each member of Congress in the Twitter hashtag networks. The dynamics of the clustering of latent positions exhibits a clear consistency with the evolution of within/between party edge densities seen in Figure 1, with Democratic members of Congress (blue dots) tending to converge over time, while Republican members of Congress (red dots) initially converge then disperse following the presidential election year of 2016. The inflection around 2016 seen in the bottom panel of Figure 5 suggests that dynamics driving partisanship have changed.

Table 1: Summary statistics for the posterior distribution of parameters using the whole sequence of Twitter networks from 2010 to 2020.

|       | \( \hat{\alpha} \) | \( \hat{\delta} \) | \( \hat{\gamma}_w^1 \) | \( \hat{\gamma}_w^2 \) | \( \hat{\gamma}_b \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Mean  | 2.809           | 1.500           | 0.493           | 0.105           | -0.155          |
| SD    | 0.022           | 0.018           | 0.026           | 0.025           | 0.014           |
| 2.5%  | 2.766           | 1.464           | 0.444           | 0.055           | -0.183          |
| 97.5% | 2.850           | 1.537           | 0.543           | 0.153           | -0.127          |

Table 2: 1-Democrats, 2-Republicans. These results indicate presence of edge persistence (\( \hat{\delta} > 0 \)), higher within-group attraction for Democrats than Republicans (\( \hat{\gamma}_w^1 > \hat{\gamma}_w^2 \) (mean difference = .388, SE = .035)), presence of between-group repulsion (\( \hat{\gamma}_b < 0 \)), and greater magnitude of between-group repulsion than within-group attraction for Republicans (\( |\hat{\gamma}_b| > |\hat{\gamma}_w^2| \) (mean difference = .050, SE = .022)), the opposite for Democrats (\( |\hat{\gamma}_b| < |\hat{\gamma}_w^1| \) (mean difference = -.338, SE = .027)).
So far, we have seen that the CLSNA model is quite powerful in terms of revealing polarization in social network interactions, as well as disentangling and quantifying the two sides of polarization: positive and negative partisanship. These results motivate questions about the dynamics of the relationships uncovered above. In particular, given a host of major political, social and economic events over the past decade, can our model help us to pinpoint changes in polarization and edge persistence over this period?

In order to confirm and quantify change in attraction, repulsion and edge persistence, we fit a series of models that allow a single change-point to vary from 2012 to 2019. Specifically, for each choice of change-point, we parameterized our model separately within the two corresponding subperiods of time, thus obtaining a set of parameter values up to the given change-point, and similarly another set of parameter values after the change-point. The resulting eight fitted models with different change-points were compared through deviance information criteria (DIC) [29], and the one with the lowest DIC value was selected (the DIC values for all competing models are provided in SI Appendix). Our modeling identified 2015 as the year in which the network relationships changed the most. The AUC values computed at each year for this model are all above 0.976, and the overall AUC value computed across all times is 0.988. From this model, we obtain the posterior means and 95% credible intervals for the parameters $\delta$, $\gamma^w_1$, $\gamma^w_2$ and $\gamma^b$, for each of the two time periods 2010-2014 and 2015-2020.

Edge persistence appears to be fairly stable in the two time periods (shown in Figure 1 in the SI Appendix). Figure 6 illustrates the evolution of within-group attraction/repulsion $\gamma^w_1$ for Democrats, $\gamma^w_2$ for Republicans, and between-group attraction/repulsion $\gamma^b$. The between-group coefficient $\gamma^b$ (yellow bars) is negative in both time periods, although its magnitude increases a bit (mean increase = .027, SE = .051, $P(\text{increase}>0)=0.701$) in the second time period from 2015 to 2020. This suggests polarization across the sets of Republican and Democratic members of Congress appeared throughout the past decade, with some indication that it started to rise in 2015.

The within-group attraction coefficients for Democrats (blue bars) remain fairly large for the two time periods, albeit with a slight drop (mean decrease = .170, SE = .053) in the second period, while those for Republicans (red bars) exhibit a steeper downward trend falling from positive to negative. That is, for Democratic members of Congress the positive feelings toward their own party have remained fairly strong,
even after 2015 and during unified government under the Trump administration. For Republicans, however, the positive feelings toward their own party are weaker (mean difference = .174, SE = .087) than they are for Democrats prior to 2015. Most intriguing, perhaps, Republican in-party feelings became negative in 2015. That is, during the Trump administration, Republican members of Congress not only remained opposed to Democrats, but also grew in opposition to their own, i.e., a decrease in strength of in-group affect.

By comparing the magnitude of within-group coefficients (blue or red bars) with between-group coefficients (yellow bars), we find that for Democratic members of Congress positive partisanship dominated the entire time period of study from 2010 to 2020. Democrats favorability of their own was a binding feature of their polarization. In contrast, for Republican members of Congress negative feelings about the other party started to dominate positive feelings about their own in 2015 (mean difference = .075, SE = .028). Indeed, Republicans’ feelings towards their own party became negative as well. Although not explicitly tested, the evidence here suggests that Trump’s extensive appearance in social media and candidacy declaration either caused or occurred in conjunction with the take-over of negative partisanship for Republican members of Congress. Whether due to long evolving attitudes within members of Congress or concurrent political trends, or simply reactions to the new presidential candidate, from 2015 to 2020 Republican members of Congress were defining their online partisanship more in terms of their opposition to Democrats than in support of their own.

Figure 6: Evolution of posterior means and 95% CI for within-group attraction/repulsion $\gamma_w^1$ for Democrats, $\gamma_w^2$ for Republicans, and between-group attraction/repulsion $\gamma^b$ in Twitter congressional hashtag networks, the values of which at each time period in the horizontal axis are obtained by fitting the model using separately parameterized networks in the corresponding time periods. Polarization between two parties appeared in both two time periods. The within-group coefficients for both parties decreased. In the second time period, the between-group repulsion is greater than the repulsion within Republicans ($|\hat{\gamma}^b| > |\hat{\gamma}_2^w|$) (mean difference = .075, SE = .028), i.e. for Republican members of Congress, negative feelings about the Democrats started to dominate the feeling about their own in the second time period. For Democratic members of Congress, positive partisanship dominates all the time ($|\hat{\gamma}^w| < |\hat{\gamma}_1^w|$).

We have so far restricted to only one change-point. We could of course continue this analysis with more than one change-point. For example, we have run a series of models with two change-points chosen between 2012 and 2019, and selected that model with the lowest DIC value, which places change-points at year 2014 and 2019. However, the relative improvement of this model over that with a single change-point is quite modest. See SI Appendix for details.

3.2 Reddit Data Analysis

In this section we carry out the same line of analysis on the Reddit comment networks for the public. Recall that the Reddit data collected are from April 2015 to March 2020. Each network constructed represents the interaction during a one-year period from April of a given year to March the year after, and hence there are in total five networks constructed. We fit a model with a single set of parameters for the entire 2015-2020 period. In addition, we fit a series of models with a single change-point and selected the one with lowest
DIC value, which places that change-point at the year 2018, three years after the change-point for elites. Again, the models appear to fit the data quite well, although arguably slightly worse than in the case of the Twitter networks (e.g., with AUC values for the best fitting change-point model computed at each one-year period above 0.840 and overall AUC values above 0.890).

Table 4 and Figure 7 show the results from fitting without a change-point, analogous to Table 2 and Figure 5 for the Twitter data. Figure 8 displays the evolution of the within-group coefficients for the two groups and the between-group coefficient, in analogy to Figure 6 for the Twitter data. (Similarly, Figure 2 in the SI Appendix displays the evolution of edge persistence, in analogy to Figure 1 in the SI Appendix.)

Table 3: Summary statistics for the posterior distribution of parameters using the whole sequence of Reddit networks from 2015 to 2020.

|     | $\hat{\alpha}$ | $\hat{\delta}$ | $\hat{\gamma}_1^w$ | $\hat{\gamma}_2^w$ | $\hat{\gamma}^b$ |
|-----|----------------|----------------|---------------------|---------------------|------------------|
| Mean | 3.079          | 0.937          | 0.748               | 0.401               | -0.128           |
| SD   | 0.011           | 0.010          | 0.032               | 0.041               | 0.027            |
| 2.5% Quantile | 3.056          | 0.917          | 0.686               | 0.321               | -0.182           |
| 97.5% Quantile | 3.101          | 0.957          | 0.811               | 0.482               | -0.075           |

Table 4: 1-Democrats, 2-Republicans. This results indicate presence of edge persistence ($\hat{\delta} > 0$), higher within-group attraction for Democrats than Republicans ($\hat{\gamma}_1^w > \hat{\gamma}_2^w$ (mean difference = .347, SE = .035)), presence of between-group repulsion ($\hat{\gamma}^b < 0$), and less magnitude of between-group repulsion than within-group attraction for both Democratic and Republican Reddit users ($|\hat{\gamma}^b| < |\hat{\gamma}_1^w|$ (mean difference = -.620, SE = .021), $|\hat{\gamma}^b| < |\hat{\gamma}_2^w|$ (mean difference = -.273, SE = .026)).

Figure 7: Top: Posterior means of latent positions for Reddit comment networks. Blue-Democrats, Red-Republicans. The dynamics of the latent positions is consistent with the evolution of within/between party densities seen in Figure 2. Bottom: Mean latent distances within each group and between groups.

Some conclusions regarding evolution across the two time periods: 1) edge persistence increased (mean increase = .165, SE = .020); 2) between-group repulsion was present, demonstrating polarization across the sets of Democratic and Republican users of Reddit, though with some evidence that such polarization was
Figure 8: Evolution of posterior means and 95% CI for within-group attraction $\gamma^w_1$ for Republicans, $\gamma^w_2$ for Democrats, and between-group attraction/repulsion $\gamma^b$ in Reddit comment networks. Polarization across the two sets of Reddit users appeared for both two time periods, and the within-group attraction declined over time for both two groups.

mitigated (mean decrease = .023, SE = .054, P(decrease > 0)=0.668) in the second time period starting in 2018; 3) while the two groups have moved away from one another, both experienced positive partisanship (within-group attraction) and became less concentrated over time, as both groups experienced a decline (Democrats: mean decrease = .291, SE = .066; Republicans: mean decrease = .213, SE = .082) in within-group attraction in the second time period; and 4) positive partisanship dominated the entire time period for both Democrat and Republican users of Reddit. The latter finding is particularly notable, since it suggests different polarization trends among the public than what we found above among members of Congress. Though they are different social media platforms and we have a shorter time-span on Reddit, the consistent dominance of positive partisanship for both Republicans and Democrats among the public and the dominance of negative partisanship among Republican elites over the same period suggests a disconnect between elites and the public, an early focus of debate in the polarization literature [9, 1].

4 Discussion

We develop a two-group coevolving latent space network with attractors (CLSNA) model for characterizing the mechanism of affective polarization using dynamic social networks. This model incorporates the effects of both attraction and repulsion by specifying appropriate attractor functions to explain the factors driving interactions of polarization. This model may be viewed as a type of causal modeling framework, specifically designed to combine dynamical systems from mathematical modeling with principles of hierarchical statistical modeling. The former allows us to incorporate precise notions of attraction/repulsion relevant to polarization, while the latter permits principled and computationally tractable inferences in the form of statistical estimation, testing and prediction.

While we focus on the context of polarization with the two-group version of CLSNA model, our proposed class of CLSNA models is a flexible framework which may incorporate a variety of attractor functions, making it general and quite broadly applicable to other co-evolutionary social dynamics where behaviors and beliefs impact social interactions, and vice versa. One limitation of our model, as implemented here, is that we assume the node set is fixed over time, which restricts our focus on individuals who are active for the entire period of study. Those who come and go and stay active for only a certain period of time, which is common in practice, is not currently accounted for in our model. It is an interesting subject for future research to design dynamic network models allowing for varying set of nodes.

In this article, we focus the application of our model on two online longitudinal social networks, one of the elite via Twitter for Congress, and one of the public, via Reddit. Our model has captured, disentangled and quantified the two key aspects of affective polarization, positive and negative partisanship, as well as a concept in social network theory, edge persistence. Our results show that for members of Congress active on Twitter polarization across the two parties appeared throughout the past decade. For Republican members
of Congress, negative feelings about the other party began to dominate feelings about their own in 2015, while feelings about their own also became more negative at this time. Thus, among Republican members of Congress we find that increasing disdain for the opposing party is not necessarily accompanied by strong in-party attachments. In fact, within-party forces for Republican members of Congress became negative after 2015. In contrast, for Democratic members of Congress positive partisanship was strongest throughout the entire period of study. We also find evidence of affective polarization among the public on Reddit. However, here positive partisanship dominated the full length of study for both Democrats and Republicans. Thus, the results provide only select support for the increasing role of negative partisanship in polarization. In all, through the modeling and analyses of social media data of both the public and the political elite in the US, this work provides new insights into the nature and presence of affective polarization, as well as how positive and negative partisanship play roles and evolve in driving polarization.

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Supplementary Information for ‘Disentangling positive and negative partisanship in affective polarization using a coevolving latent space network with attractors model’

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This supplement contains five sections. In Section 1, we present the details of the Bayesian method and the Markov chain Monte Carlo (MCMC) algorithm used to estimate the parameters. In Section 2, we evaluate the performance of our proposed posterior-based inference approach through simulation and display the results. In Section 3, we evaluate the performance of our proposed parameter estimation procedure under the change-point setting where parameter values are time-varying. In Section 4, we provide two figures mentioned in the main text, for the evolution of edge persistence in Twitter and Reddit data, respectively. In the last section, we provide supplementary results in Twitter/Reddit data analysis, which are DIC values for model selection and a select handful of alternative analysis results for both Twitter and Reddit data with different choices of change-point.

1 Markov chain Monte Carlo Estimation

Given an observed network time series \(\{G_t\}_{t=1}^T\) or, equivalently, a time series of the corresponding adjacency matrices \(\{Y_t\}_{t=1}^T\), we wish to make inference of the latent positions \(Z_t\) and model parameters \(\theta = (\alpha, \delta, \gamma_w, \gamma_w^b, \gamma_b, \tau^2, \sigma^2)\). Given the hierarchical nature of our model, it is natural to adopt a Bayesian approach. In the meanwhile, estimation within Bayesian framework can address the issues of lack of identifiability in the likelihood function of our model. Specifically, in our model, the probability of observing \(Y_{1:T}\) conditional on all unknowns is

\[
P(Y_{1:T} \mid Z_{1:T}, \alpha, \delta) = P(Y_1 \mid Z_1) \prod_{t=2}^T P(Y_t \mid Z_t, Y_{t-1}) = \prod_{t=2}^T P_{\alpha}(Y_{t,ij} \mid Z_{t,i}, Z_{t,j}) \cdot \prod_{t=2}^T \prod_{i \neq j} P(Y_{t,ij} \mid Z_{t,i}, Z_{t,j}, Y_{t-1,ij})
\]

Notice that

\[
P(Y_{t,ij} \mid Z_{t,i} = z_{t,i}, Z_{t,j} = z_{t,j}, Y_{t-1,ij}, \alpha, \delta) = \left(\frac{\exp(\eta_{t,ij})}{1 + \exp(\eta_{t,ij})}\right)^{Y_{t,ij}} \left(\frac{1}{1 + \exp(\eta_{t,ij})}\right)^{1-Y_{t,ij}}
\]

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where
\[\eta_{t,ij} := \logit(p_{t,ij}) = \log\left(\frac{P(Y_{t,ij} = 1|Z_t,Y_{t-1},\theta)}{P(Y_{t,ij} = 0|Z_t,Y_{t-1},\theta)}\right) = \begin{cases} \alpha + \delta Y_{t-1,ij} - s(z_{t,i},z_{t,j}) & \text{if } t > 1 \\ \alpha - s(z_{t,i},z_{t,j}) & \text{if } t = 1. \end{cases} \] (3)

Note that if \(\exists\) a set of distances and \(\alpha, \delta\) s.t. \(\eta_{t,ij} > 0\) when \(Y_{t,ij} = 1\) and \(\eta_{t,ij} < 0\) when \(Y_{t,ij} = 0\), then by rescaling, i.e. let \(\bar{z}_t = \kappa z_t\), and \(\bar{\alpha} = \kappa\alpha, \bar{\delta} = \kappa\delta, \kappa > 0\), the conditional distribution of \(Y_{t,ij}\) becomes
\[P(Y_{t,ij} | Z_{t,i} = \bar{z}_{t,i}, Z_{t,j} = \bar{z}_{t,j}, Y_{t-1,ij}, \bar{\alpha}, \bar{\delta}) = \frac{\exp(Y_{t,ij}\kappa\eta_{t,ij})}{1 + \exp(\kappa\eta_{t,ij})} \rightarrow 1, \kappa \rightarrow \infty. \] (4)

Therefore, the likelihood (1) is not identifiable, as \(z_t, \alpha\) and \(\delta\) can be scaled arbitrarily. However, the posterior distribution \(\pi(Z_{1:T}, \theta | Y_{1:T})\) is identifiable by imposing appropriate prior distributions on the unknown parameters, which can be viewed as a set of constraints to prevent parameters from scaling arbitrarily. To sample from the posterior distribution, we implement an adaptive Metropolis-Hastings (MH) within Gibbs MCMC scheme, where samples are drawn from the full conditional distributions iteratively. These conditional distributions are either known in closed form or up to a normalizing constant. We sample directly if the conditional distribution is known, otherwise we sample via MH using an adaptive normal random walk proposal. We provide details in the following subsections, where two issues are addressed, one regarding the lack of identifiability for the conditional distribution of \(\gamma^w\) and \(\gamma^g\) in connection with the scaling of \(\sigma^2\) and latent positions \(Z_t\), the other one regarding rotational invariance in the latent space.

### 1.1 Posterior and Full Conditional Distributions

The posterior distribution of latent positions and parameters is
\[
\pi(Z_{1:T}, \theta | Y_{1:T}) \propto P(Y_{1:T}, Z_{1:T} | \theta)\pi(\theta)
\]
\[
= \left( \prod_{t=2}^{T} P(Z_t | Z_{t-1}, Y_{t-1})P(Y_t | Z_t, Y_{t-1}) \right) P(Y_1 | Z_1)P(Z_1)\pi(\theta)
\]
\[
= \left( \prod_{t=2}^{T} \prod_{i=1}^{N} P(Z_{t,i} | Z_{t-1,i}, Y_{t-1}) \prod_{j:j \neq i} P(Y_{t,ij} | Z_{t,i}, Z_{t,j}, Y_{t-1,ij}) \right) P(Y_1 | Z_1)P(Z_1)\pi(\theta)
\]
\[
= \prod_{t=2}^{T} \prod_{i=1}^{N} P_{\eta_{t,ij}}(Z_{t,i} | Z_{t-1,i}, Y_{t-1}) \cdot \prod_{t=2}^{T} \prod_{i \neq j} P_{\alpha,\delta}(Y_{t,ij} | Z_{t,i}, Z_{t,j}, Y_{t-1,ij}) \cdot \prod_{i \neq j} P_{\sigma^2}(Z_{1,i}, Z_{1,j}) \cdot \prod_{i=1}^{N} P_{\tau^2}(Z_{1,i}) \cdot \pi(\theta)
\] (5)

We set the priors on the parameters as follows: assume that \(\alpha \sim N(\nu_\alpha, \xi^2_\alpha), \delta \sim N(\nu_\delta, \xi^2_\delta), \gamma^w, \gamma^g \sim N(\nu_{\gamma^w}, \xi^2_{\gamma^w}), \tau^2 \sim IG(\eta_{\tau}, \phi_{\tau}), \) and \(\sigma^2 \sim IG(\eta_\sigma, \phi_\sigma)\). where \(N()\), \(IG()\) are normal and inverse gamma distributions, respectively. The full conditional distributions follow.
1.1.1 Sampling $Z_{t,i}$

The conditional distribution for $Z_{t,i}$ is

$$
\pi(Z_{t,i} | Y_{1:T}, \theta, Z_{\langle t,i \rangle}) \propto \left\{ \begin{array}{ll}
\prod_{j \neq i} \prod_{t,j} s_{t,i,j} s_{t,j,i} & \cdot N(Z_{t+1,i} | Z_{t,i} + \gamma_i^{w}(Z_{t,i}, Y_i) + \gamma^b A^w_i(Z_{t,i}, Y_i), \sigma^2 I_p)
+ \prod_{j \in N_i} \prod_{t,j} N(Z_{t+1,j} | Z_{t,j} + \gamma_{\pi(j)}^{w}(Z_{t,j}, Y_j) + \gamma^b A^w_j(Z_{t,j}, Y_j), \sigma^2 I_p)
\end{array} \right.
$$

if $t = 1$

$$
= \prod_{j \neq i} \prod_{t,j} s_{t,i,j} s_{t,j,i} \cdot N(Z_{t,i} | Z_{t-1,i} + \gamma_{\pi(i)}^{w}(Z_{t-1,i}, Y_{t-1}) + \gamma^b A^w_i(Z_{t-1,i}, Y_{t-1}), \sigma^2 I_p)
\right)
$$

if $1 < t < T$

$$
= \prod_{j \neq i} \prod_{t,j} s_{t,i,j} s_{t,j,i} \cdot N(Z_{t,i} | Z_{t-1,i} + \gamma_{\pi(i)}^{w}(Z_{t-1,i}, Y_{t-1}) + \gamma^b A^w_i(Z_{t-1,i}, Y_{t-1}), \sigma^2 I_p)
\right)
$$

if $t = T$

where $s_{t,i} := P(Y_{t,i} | Z_{t,i}, \mathcal{N}_i)$ is the set of neighbors of node $i$ at time $t$.

1.1.2 Sampling $\alpha$ and $\delta$

The conditional distribution for $\alpha$ is

$$
\pi(\alpha | Y_{1:T}, Z_{1:T}, \theta_{\langle \alpha \rangle}) \propto \left\{ \begin{array}{ll}
\prod_{t=1}^{T} \prod_{i \neq j} s_{t,i,j} & \cdot \pi(\alpha)
\end{array} \right.
$$

(7)

The conditional distribution for $\delta$ is

$$
\pi(\delta | Y_{1:T}, Z_{1:T}, \theta_{\langle \delta \rangle}) \propto \left\{ \begin{array}{ll}
\prod_{t=1}^{T} \prod_{i \neq j} s_{t,i,j} & \cdot \pi(\delta)
\end{array} \right.
$$

(8)

1.1.3 Sampling $\gamma_i^w$, $\gamma_2^w$

Note that for the conditional distribution of $\gamma_i^w$, we have

$$
\pi(\gamma_i^w | Y_{1:T}, Z_{1:T}, \theta_{\langle \gamma \rangle}) \propto \prod_{t=2}^{T} \prod_{i \neq j} P_{\alpha_{\pi(i)}}^{\gamma_i^w}(Z_{t,i} | Z_{t-1,i}, Y_{t-1}) \cdot \pi(\gamma_i^w)
\right.
$$

(9)

and from here we can get a closed form distribution. Similarly for $\gamma_2^w$.

Hence the conditional distributions for $\gamma_1^w$, $\gamma_2^w$ are

$$
\gamma_1^w | Y_{1:T}, Z_{1:T}, \theta_{\langle \gamma \rangle} \sim N\left( \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i \neq j} a^w_{t,i,j} b_{t,i} + \frac{1}{\gamma^{w}} \nu_{\gamma^{w}}, \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i \neq j} b^w_{t,i,j} b_{t,i} + \frac{1}{\gamma_{\gamma^{w}}^2} \right)'
$$

(10)
\[ \gamma_2^w \mid Y_{1:T}, Z_{1:T}, \theta_{\gamma_1^w} \sim N \left( \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i: \pi(i)=2} a_{t,i}^w b_{t,i} + \frac{1}{\xi^w_2} \nu_1^w, \left( \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i: \pi(i)=2} b_{t,i}^w b_{t,i} + \frac{1}{\xi^w_2} \right)^{1/2} \right) \]  

(11)

where \( a_{t,i} = z_{t,i} - z_{t-1,i} - \gamma^b A^b_t(z_{t-1}, Y_{t-1}), b_{t,i} = A^b_t(z_{t-1}, Y_{t-1}) \).

Notice that if we rescale \( \tilde{z}_t = \kappa z_t \), and \( \hat{\sigma}^2 = \kappa^2 \sigma^2 \), we obtain the same conditional distributions of \( \gamma_1^w \) and \( \gamma_2^w \) since their means and variances remain unaltered. Although the joint posterior distribution is identifiable, the lack of identifiability in these two conditional distributions will cause trouble in the convergence of the Gibbs sampler. To address these forms of rescaling, we fix \( \sigma^2 = 1 \).

### 1.1.4 Sampling \( \gamma^b \)

\[
\pi(\gamma^b \mid Y_{1:T}, Z_{1:T}, \theta_{\gamma^b}) \propto \prod_{t=2}^{T} \prod_{i=1}^{N} \gamma^b \sigma^2 \gamma^w(\gamma^w_{\pi(i)} A_{\pi(i)}^w(Z_{t-1}, Y_{t-1}) + \gamma^b A_t^b(Z_{t-1}, Y_{t-1})), \sigma^2 I_p \cdot N(\gamma^b \mid \nu_{\gamma^b}, \xi_{\gamma^b}^2) 
\]

(12)

The conditional distribution for \( \gamma^b \) is

\[
\gamma^b \mid Y_{1:T}, Z_{1:T}, \theta_{\gamma^b} \sim N \left( \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i=1}^{N} \gamma^w_t d_{t,i} + \frac{1}{\xi^b_1} \nu_{\gamma^b}, \left( \frac{1}{\sigma^2} \sum_{t=2}^{T} \sum_{i=1}^{N} d_{t,i}^2 d_{t,i} + \frac{1}{\xi^b_1} \right)^{1/2} \right) \]

(13)

where \( d_{t,i} = z_{t,i} - z_{t-1,i} - \gamma^w_{\pi(i)} A_{\pi(i)}^w(Z_{t-1}, Y_{t-1}), d_{t,i} = A^b_t(z_{t-1}, Y_{t-1}) \).

### 1.1.5 Sampling \( \tau^2 \) and \( \sigma^2 \)

The conditional distributions for \( \tau^2 \) and \( \sigma^2 \) are

\[
\tau^2 \mid Y_{1:T}, Z_{1:T}, \theta_{\tau^2} \sim IG(\eta_\tau + \frac{np}{2}, \phi_\tau + \frac{1}{2} \sum_{i=1}^{N} ||Z_{1,i}||^2) 
\]

(14)

\[
\sigma^2 \mid Y_{1:T}, Z_{1:T}, \theta_{\sigma^2} \sim IG(\eta_\sigma + \frac{np(T-1)}{2}, \phi_\sigma + \frac{1}{2} \sum_{i=1}^{N} ||Z_{t,i} - Z_{t-1,i} - \gamma^w A_t^w(Z_{t-1}, Y_{t-1}) - \gamma^b A_t^b(Z_{t-1}, Y_{t-1})||^2) 
\]

(15)

Note that in our implementation of the posterior sampler, we set \( \sigma^2 = 1 \) to fix the scaling issues in \( \gamma_1^w, \gamma_2^w \) sampling, hence no sample is drawn from the conditional distribution of \( \sigma^2 \).

### 1.2 Rotational Invariance

The last issue to address around posterior sampling is that the posterior distribution is invariant to rotations, reflections and translation of latent positions. Following Sewell et al.\cite{4} and Hoff et al.\cite{1}'s work, we perform a Procrustes transformation to reorient the sampled latent positions.

Recall that \( Z_t \in R^{N \times p} \), where \( N \) is the number of nodes in the network and \( p \) is the dimension of the latent space. Let \( Z = [Z_1^T, Z_2^T, ..., Z_T^T]^T \in R^{(nT) \times p} \), and \( Z^{(k)} \) be the samples at the \( k \)-th iteration. In our posterior sampling, we do the following,
1. Take \( Z^{(0)} \) (centered) as reference positions.

2. For each \( k > 0 \), perform Procrustes transformation on the new draws \( Z^{(k)} \) (centered), i.e.,

\[
Z^{(k)} \leftarrow \arg\min_{Z^*} \text{tr}(Z^{(0)} - Z^*)^T (Z^{(0)} - Z^*),
\]

where \( R \) is a rotation matrix, and \( Z^* \) is some rotation of \( Z^{(k)} \). The transformed latent positions are a reorientation of the previous draws, which preserve the distance between any actors at any time and are the closest to the reference positions compared with other rotations.

1.3 Posterior Sampling via Adaptive Metropolis-Hastings within Gibbs

Combining all the pieces discussed above, our adaptive Metropolis-Hastings within Gibbs sampling algorithm is as follows. Set initial values for \( Z_{1:T}, \alpha, \delta, \gamma_1^w, \gamma_2^w, \tau^2 \), and fix \( \sigma^2 = 1 \). Then for every iteration:

1. For \( t = 1, \cdots, T \) and for \( i = 1, \cdots, N \), draw \( Z_{t,i} \) via MH using an adaptive normal random walk proposal.
2. Draw \( \alpha \) via MH using an adaptive normal random walk proposal.
3. Draw \( \delta \) via MH using an adaptive normal random walk proposal.
4. Draw \( \tau^2 \) directly from its conditional inverse gamma distribution.
5. Draw \( \gamma_1^w \) directly from its conditional normal distribution.
6. Draw \( \gamma_2^w \) directly from its conditional normal distribution.
7. Draw \( \gamma^b \) directly from its conditional normal distribution.

The latent positions as well as \( \alpha, \delta \) are updated using the adaptive normal random walk proposal [2]. As an example, for variable \( \alpha \) being updated at iteration \( k \), we propose a new value \( \alpha' \) drawn from normal distribution centered at \( \alpha^{(k-1)} \), with standard deviation \( \exp(s^{(k)}) \) determined at previous iteration, and calculate the acceptance ratio \( R^{(k)} = \frac{\pi(\alpha')}{\pi(\alpha^{(k-1)})} \), then accept \( \alpha' \) with probability \( \min\{1, R^{(k)}\} \). The notion of ‘adaptive’ comes from the fact that the tuning parameter \( s^{(k)} \) is updated at every iteration with \( s^{(k)} = s^{(k-1)} + \frac{1}{(k-1)^{\alpha}} \cdot \min\{1, R^{(k-1)} - 0.234\} \), which adjusts the scale of the random walk proposal based on the acceptance ratio, and prevents it from being too small causing the chain moving slowly or too large causing very high rejection rate.

The pseudocode for the MCMC algorithm we proposed is provided in Algorithm 1.

1.4 MCMC settings

The priors on \( \alpha \) and \( \delta \) were chosen to be \( N(0,100) \) to keep it flat and uninformative. We chose the priors on \( \gamma_1^w, \gamma_2^w \) to be \( N(0.5,100) \) and \( \gamma^b \) to be \( N(-0.5,100) \), to reflect the prior belief of polarization, however these are also quite uninformative given the large variance. The prior on \( \tau \) was chosen to be \( IG(2.05, 1.05 \sum_{i=1}^{N} ||Z_{1,i}^{(1)}||^2/(Np)) \) to be flat and uninformative following Sewell et al.’s suggestion in [4].

For the initialization, \( \alpha, \delta, \gamma_1^w, \gamma_2^w, \gamma^b \) were initialized to be \( 0, 0, 0.5, 0.5, -0.5 \), respectively. The estimation is quite robust to the initialization of these parameters, in our experience. All latent positions \( Z_{1:T}^{(1)} \) are initialized using the generalized multidimensional scaling (GMDS) method proposed by Sarkar et al.[3], which first initializes \( Z_1^{(1)} \) through classical multidimensional scaling with similarity matrix being the shortest paths in \( Y_1 \), then initializes \( Z_t^{(1)} \), \( t > 1 \) to be consistent with \( Y_t \) and have similar pairwise distances as \( Z_{t-1}^{(1)} \). \( \tau \) was initialized to be \( \frac{1}{Np} \sum_{i=1}^{N} ||Z_{1,i}^{(1)}||^2 \) using the initial latent positions \( Z_1^{(1)} \).

For the results in the article, we set the number of MCMC iterations to be 50,000 and removed a burn-in of 15,000 samples. We set the initial value of the tuning parameter \( s^{(1)} \) for \( \alpha \) and \( \delta \) to be 2, and for \( Z_{1:T}^{(1)} \) to be 4.
Adaptive Metropolis-Hastings within Gibbs Sampler

**Input:** Network time-series $Y_{1:T}$, total number of nodes $N$. Initial values $\theta^{(0)}$, $Z^{(0)}_{1:T}$ for parameters and latent positions.

Number of samples $N_{\text{samples}}$. Tuning parameters $s_1 \in R$, $s_2 \in R$, and $s \in R^{T \times N}$.

**Output:** $\theta^{(1)}, \ldots, \theta^{(N_{\text{samples}})}$

1. Set initial values for $Z_{1:T}$, $\alpha$, $\delta$, $\gamma_1^w$, $\gamma_2^w$, $\tau^2$, and fix $\sigma^2 = 1$.

2. for $k \leftarrow 1$ to $N_{\text{samples}}$ do

3. $\theta^{(k)} \leftarrow \theta^{(k-1)}$, $Z^{(k)}_{1:T} \leftarrow Z^{(k-1)}_{1:T}$

4. for $t \leftarrow 1$ to $T$ do

5. for $i \leftarrow 1$ to $N$ do

6. Generate $Z'_{t,i} = Z_{t,i}^{(k-1)} + N(0, \exp(s_{t,i})), u \sim U(0, 1)$  \textgreater{} Get $k$-th sample for $Z_{t,i}$

7. Calculate acceptance ratio $\log R = \log \pi(Z'_{t,i} | \theta^{(k)}$, $Z^{(k)}_{1:T} \setminus Z_{t,i}^{(k)}) - \log \pi(Z^{(k-1)}_{t,i} | \theta^{(k)}$, $Z^{(k)}_{1:T} \setminus Z_{t,i}^{(k)})$

8. if $\log u < \min(0, \log R)$ then

9. $Z_{t,i}^{(k)} \leftarrow Z'_{t,i}$

10. else

11. $Z_{t,i}^{(k)} \leftarrow Z_{t,i}^{(k-1)}$

12. end if

13. $s_{t,i} \leftarrow s_{t,i} + \frac{1}{\exp(\pi)}, \min[1, \exp(\log R)] - 0.234$ \textgreater{} Update tuning parameter for $Z_{t,i}$

14. end for

15. end for

16. Generate $\alpha' = \alpha^{(k-1)} + N(0, \exp(s_1)), u \sim U(0, 1)$ \textgreater{} Get $k$-th sample for $\alpha$

17. Calculate acceptance ratio $\log R_1 = \log \pi(\alpha' | \theta^{(k)} \setminus \alpha^{(k)}, Z^{(k)}_{1:T}) - \log \pi(\alpha^{(k-1)} | \theta^{(k)} \setminus \alpha^{(k)}, Z^{(k)}_{1:T})$

18. if $\log u < \min(0, \log R_1)$ then

19. $\alpha^{(k)} \leftarrow \alpha'$

20. else

21. $\alpha^{(k)} \leftarrow \alpha^{(k-1)}$

22. end if

23. Generate $\delta' = \delta^{(k-1)} + N(0, \exp(s_2)), u \sim U(0, 1)$ \textgreater{} Get $k$-th sample for $\delta$

24. Calculate acceptance ratio $\log R_2 = \log \pi(\delta' | \theta^{(k)} \setminus \delta^{(k)}, Z^{(k)}_{1:T}) - \log \pi(\delta^{(k-1)} | \theta^{(k)} \setminus \delta^{(k)}, Z^{(k)}_{1:T})$

25. if $\log u < \min(0, \log R_2)$ then

26. $\delta^{(k)} \leftarrow \delta'$

27. else

28. $\delta^{(k)} \leftarrow \delta^{(k-1)}$

29. end if

30. end for

31. $s_1 \leftarrow s_1 + \frac{1}{\exp(\pi)}, \min[1, \exp(\log R_1)] - 0.234$ \textgreater{} Update tuning parameter for $\alpha$

32. Obtain the $k$-th samples for $\tau^2$, $\gamma_1^w$, $\gamma_2^w$ and $\gamma^b$ by sampling directly from its conditional distribution using the most updated parameter values.

33. end for


2 Simulation Results for Time-invariant Parameters

We evaluate the performance of our proposed posterior-based inference approach through simulation. We consider two sets of parameter settings, one for flocking and the other for polarization (similarly to Figure 4 in the main text), and for each setting we simulate 10 data sets with number of actors $N = 100$ and the number of time points $T = 10$. Table 1 displays the posterior-based mean and standard deviation for parameter estimation under the two settings. We can see that the parameter estimates are reasonably accurate compared with the truth.

In our simulation study, we set $\gamma^w_1, \gamma^w_2, \gamma^b \in [-1,1]$, since in our simulation experience we found that setting their magnitudes too large (typically larger than 1) could lead to divergent model behaviors: we lose the attraction effect since nodes are being pulled too far away from the local averages, although the repulsion effect seems to be well-behaved for large magnitude of $\gamma^b$.

Table 1: Posterior-based mean (standard deviation) for parameters in the flocking (top) and polarization (bottom) settings, based on $N = 100$ nodes and $T = 10$ time points. [Based on 20 Monte Carlo trials]

|   | $\hat{\alpha}$ | $\hat{\delta}$ | $\hat{\gamma}^w_1$ | $\hat{\gamma}^w_2$ | $\hat{\gamma}^b$ | $\hat{\tau}$ | AUC |
|---|----------------|----------------|-------------------|-------------------|-----------------|-------------|-----|
|Truth| 1 | 2 | 0.3 | 0.2 | 0.5 | 1 |
|Posterior Mean| 1.122 (0.022) | 2.046 (0.032) | 0.366 (0.085) | 0.273 (0.115) | 0.496 (0.108) | 1.058 (0.035) | 0.830 (0.002) |

|   | $\hat{\alpha}$ | $\hat{\delta}$ | $\hat{\gamma}^w_1$ | $\hat{\gamma}^w_2$ | $\hat{\gamma}^b$ | $\hat{\tau}$ | AUC |
|---|----------------|----------------|-------------------|-------------------|-----------------|-------------|-----|
|Truth| 1 | 3 | 0.7 | 0.2 | -0.5 | 1 |
|Posterior Mean| 1.083 (0.083) | 3.055 (0.047) | 0.777 (0.064) | 0.205 (0.067) | -0.491 (0.035) | 1.039 (0.068) | 0.945 (0.002) |

In the above simulation study, we fix $\sigma^2 = 1$. To determine how sensitive the estimation procedures are to the true values of $\sigma^2$, we simulate network times series with $N = 100$ nodes, $T = 10$ time points, under different values of $\sigma \in \{0.01, 0.05, 0.1, 0.5, 0.8, 1, 1.2, 1.5\}$, and estimate the model parameters with $\sigma^2$ fixed at 1. Such range of $\sigma$ is chosen based on our simulation experience that the nodes would not be able to mimic the desired social dynamics (flocking/polarization) when the noise level is set to be too high, in which case, the nodes movement will be driven predominantly by the noise in the Gaussian autoregressive process in the latent space, not the attractors. This is also consistent with the literature on latent space models of similar type with Gaussian AR process [4, 3], where small values of noise are assumed.

Table 2 shows the results for the polarization setting. We can see that the estimates for $\gamma^b$ are quite accurate and not sensitive at all to the values of $\sigma$, while all the other parameters are, to varying extents, more sensitive to the values of $\sigma$ although the overall accuracy of the model as modeled through the AUC criterion is always quite high. We note, however, that the signs and relative magnitude are correctly estimated in all cases.

Table 2: Sensitivity Study: under an array of values for $\sigma$, posterior-based mean (standard deviation) for parameters in the polarization setting, based on $N = 100$ nodes and $T = 10$ time points. [Based on 20 Monte Carlo trials]

| $\sigma$ | $\hat{\alpha}$ | $\hat{\delta}$ | $\hat{\gamma}^w_1$ | $\hat{\gamma}^w_2$ | $\hat{\gamma}^b$ | $\hat{\tau}$ | AUC |
|---------|----------------|----------------|-------------------|-------------------|-----------------|-------------|-----|
|0.01     | 1.504 (0.038)  | 3.376 (0.034)  | 1.065 (0.055)     | 0.362 (0.036)     | -0.504 (0.016)  | 1.263 (0.054) | 0.960 (0.003) |
|0.05     | 1.494 (0.047)  | 3.365 (0.043)  | 1.073 (0.057)     | 0.370 (0.057)     | -0.505 (0.018)  | 1.258 (0.061) | 0.960 (0.002) |
|0.1      | 1.484 (0.059)  | 3.360 (0.040)  | 1.064 (0.051)     | 0.364 (0.039)     | -0.501 (0.017)  | 1.252 (0.051) | 0.960 (0.002) |
|0.5      | 1.344 (0.061)  | 3.243 (0.043)  | 0.926 (0.048)     | 0.315 (0.059)     | -0.496 (0.014)  | 1.177 (0.054) | 0.955 (0.002) |
|0.8      | 1.195 (0.070)  | 3.127 (0.050)  | 0.814 (0.050)     | 0.264 (0.083)     | -0.497 (0.031)  | 1.099 (0.054) | 0.948 (0.002) |
|1        | 1.083 (0.083)  | 3.055 (0.047)  | 0.777 (0.064)     | 0.205 (0.067)     | -0.491 (0.035)  | 1.039 (0.068) | 0.945 (0.002) |
|1.2      | 0.970 (0.099)  | 2.970 (0.050)  | 0.762 (0.082)     | 0.179 (0.090)     | -0.494 (0.035)  | 1.006 (0.072) | 0.945 (0.002) |
|1.5      | 0.843 (0.094)  | 2.848 (0.064)  | 0.678 (0.099)     | 0.100 (0.115)     | -0.515 (0.045)  | 0.939 (0.051) | 0.947 (0.002) |

|   | 1 | 3 | 0.7 | 0.2 | -0.5 | 1 |
3 Simulation Results for Time-varying Parameters with Change-point

In this section, we conduct simulations to evaluate the empirical performance of the proposed parameter estimation procedure for CLSNA models under the time-varying parameter setting with a change-point. In general, we fit two models with time-invariant parameters to the two subperiods of network time series divided by the change-point, respectively, thus obtaining a set of parameter values up to the given change-point, and another set of parameter values after the change-point. The caveat is that the two models cannot be fitted simultaneously since the fitting for the second period should be conditional on the latent positions of the last network in the first fitted model.

For each trial, we simulate a network time series \( y_{1:T} \) with \( N = 100 \) nodes, and length \( T = 10 \), using the following parameter setting. The parameters are set to change at time 6. In each period: \([1, 5]\), and \([6, 10]\), the parameters are constant respectively. In the first period, \( \gamma^w_1, \gamma^w_2 \) and \( \gamma^b \) are set to be 0, 0.6 and –0.2, respectively. In the second period, \( \gamma^w_1, \gamma^w_2 \) and \( \gamma^b \) are set to be 0.8, 0.2 and –0.5, respectively. \( \alpha, \delta, \tau \) are set to be fixed all the time.

For each simulated network time series, we fit CLSNA models with constant parameters for the two periods, \([1, r - 1]\), and \([r, 10]\), respectively. Since the true change point \( k \) is unknown, we fit models for different values of \( r \) varying from 4 to 9. For each specified change point \( r \), we repeat the trial 20 times and report mean, standard deviation, and DIC based on the 20 replications. The results for parameter estimation are shown in the Table 3. We can see that for model fitting with the change-point specified at the truth (i.e., \( r = 6 \), the bold part), we can get quite good parameter estimates for each period. And such true model has lower DIC value as expected, illustrating that DIC is a useful criteria for model selection when the true change point is unknown.

Table 3: Sub-window analysis with a change-point: posterior-based mean (standard deviation) for parameters estimates, AUC and DIC with specified change-point varying from 4 to 9; the true change-point is set at time 6. The fitting of second subwindow is conditional on the estimated latent positions from the first subwindow. Networks are set to have \( N = 100 \) nodes, and the results are based on 20 Monte Carlo trials.

| Period | Truth | \( \hat{\alpha} \) | \( \hat{\delta} \) | \( \hat{\gamma}^w_1 \) | \( \hat{\gamma}^w_2 \) | \( \hat{\gamma}^b \) | \( \hat{\tau} \) | AUC | DIC |
|--------|-------|----------------|----------------|-----------------|----------------|----------------|----------------|-----|-----|
| 1-3    |       | 1.137 (0.041) | 3.985 (0.046) | 0.571 (0.175)  | 0.668 (0.183)  | -0.146 (0.147) | 1.065 (0.058)  | 0.856 (0.003) | 35115.168 (783.136) |
| 4-10   |       | 1.086 (0.055) | 3.655 (0.045) | 0.832 (0.056)  | 0.422 (0.078)  | -0.461 (0.031) | 0.945 (0.003)  |       |     |
| 1-4    |       | 1.135 (0.019) | 3.947 (0.034) | 0.634 (0.144)  | 0.363 (0.160)  | -0.129 (0.133) | 1.064 (0.060)  | 0.857 (0.003) | 35092.754 (770.129) |
| 5-10   |       | 1.131 (0.056) | 3.076 (0.053) | 0.848 (0.059)  | 0.303 (0.041)  | -0.475 (0.034) | 0.950 (0.003)  |       |     |
| 1-5    |       | 1.128 (0.043) | 3.066 (0.028) | 0.551 (0.089)  | 0.580 (0.117)  | -0.123 (0.086) | 1.061 (0.059)  | 0.878 (0.003) | 35053.386 (766.977) |
| 6-10   |       | 1.113 (0.063) | 3.073 (0.060) | 0.851 (0.056)  | 0.209 (0.046)  | -0.495 (0.029) | 0.956 (0.003)  |       |     |
| 1-6    |       | 1.115 (0.046) | 3.058 (0.031) | 0.666 (0.080)  | 0.545 (0.102)  | -0.253 (0.057) | 1.053 (0.058)  | 0.885 (0.003) | 35101.588 (764.716) |
| 7-10   |       | 1.122 (0.074) | 3.062 (0.073) | 0.842 (0.060)  | 0.195 (0.029)  | -0.502 (0.027) | 0.963 (0.004)  |       |     |
| 1-7    |       | 1.108 (0.039) | 3.054 (0.033) | 0.781 (0.072)  | 0.539 (0.080)  | -0.388 (0.065) | 1.047 (0.058)  | 0.895 (0.002) | 35101.899 (770.538) |
| 8-10   |       | 1.124 (0.077) | 3.078 (0.091) | 0.864 (0.078)  | 0.180 (0.060)  | -0.509 (0.032) | 0.970 (0.004)  |       |     |
| 1-8    |       | 1.105 (0.037) | 3.057 (0.030) | 0.837 (0.047)  | 0.529 (0.061)  | -0.452 (0.046) | 1.044 (0.058)  | 0.905 (0.003) | 35108.659 (768.719) |
| 9-10   |       | 1.122 (0.047) | 3.067 (0.088) | 0.806 (0.096)  | 0.147 (0.101)  | -0.505 (0.037) | 0.974 (0.003)  |       |     |

4 Figures for Edge Persistence

This section provides two figures mentioned in the main text, for the evolution of edge persistence in Twitter and Reddit data, which are shown in Fig.1 and Fig.2.
5 Supplementary Results in Twitter/Reddit Data Analysis

In this section, we provide the DIC values for all of the competing models in the analysis of Twitter and Reddit networks when choosing change-points using DIC criteria. We also present a select handful of alternative analysis results for both networks. Notice that the change-point preferred by DIC for Twitter data is at 2015, which is a bit earlier than the actual election or presidency year of Trump. An alternative hypothesis (a priori) for possible change-point would be 2017, which is the first year of Trump’s presidency. Therefore, we also present the analysis results with change-point set on 2017, and leave the interpretation to readers.

5.1 DIC values

For twitter data, DIC values for a single change-point chosen at 2012, 2013,..., 2019 are shown in Table 4. The lowest DIC value is from model with change-point at 2015.

| Year | DIC   | 2012  | 2013  | 2014  | 2015  | 2016  | 2017  | 2018  | 2019  |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 2012 | 52168.45 | 51186.39 | 50574.23 | 50538.64 | 50671.02 | 50727.15 | 50977.68 | 50836.72 |

For Reddit data, the DIC values for a single change-point chosen at 2017, 2018, 2019 are 254383.18, 253878.32 and 254272.54, respectively. The lowest DIC value is from model with change-points at 2018.
5.2 Alternative results for Twitter data with one change-point at 2017

Although the model with one change-point at 2015 was selected based on DIC values, it is still interesting to see the results with change-point set on 2017 - the first year under Trump’s administration. The results are shown in Table 3, which do not tell a much different story from the model with change-point at 2015: the positive feelings about their own for both parties fell in the second time period, and for Republican members of Congress, negative feelings about the other party started to dominate positive feelings about their own. What’s intriguing is the decreased trend of polarization under the Trump administration years, as opposed to the indication of increased trend from change-point on 2015.

Figure 3: Alternative results for Twitter data with one change-point at 2017: evolution of posterior means and 95% CI for within-group attraction/repulsion $\gamma_w^1$ for Democrats, $\gamma_w^2$ for Republicans, between-group attraction/repulsion $\gamma_b$ and edge persistence $\delta$ in Twitter congressional hashtag networks. In the first period, strong in-party attachment (positive blue, red bar) for both parties, between-group repulsion (yellow bar) appeared but did not dominate. In the second period, within-group attraction decreased for both parties; the out-party negative feelings dominated the in-party positive feelings for Republicans (magnitude of yellow greater than that of red); and the between-group polarization decreased in the second time period.

5.3 Alternative results for Reddit data with one change-points at 2017

The results for Reddit data with one change-point at 2017 are shown in Table 4, which are qualitatively similar to that with change-point in 2015, except the indication of trend for polarization.

Figure 4: Alternative results for Reddit data with one change-points at 2017: evolution of posterior means and 95% CI for within-group attraction $\gamma_w^1$ for Democrats, $\gamma_w^2$ for Republicans, between-group attraction/repulsion $\gamma_b$, and edge persistence $\delta$ in Reddit comment networks. In this first period, strong in-party attachment for both parties, no evidence of between-group polarization. In the second period, within-group attraction decreased for both parties, between-group repulsion (yellow) appeared but did not dominate.

5.4 Alternative results for Twitter data with two change-points at 2014 and 2019

We also present results for Twitter data with two change-points at 2014 and 2019 in Table 5, which is chosen with DIC criteria among $2^8 = 28$ competing models that place two change-points between 2012 and 2019. Notice that the DIC values for the selected models with zero-, one-, and two-change-point are 53221,
50538 and 50249, respectively, which suggests a much smaller improvement from one change-point to two change-points than that from zero change-point to one change-point. Given that the data is only at yearly resolution, it seems more practical to focus on the model with one change-point. However, we present the results here to demonstrate the possibility of a richer story that could be told potentially from models with more change-points, which of course needs deeper analysis with networks at finer resolution.

Figure 5: Alternative results for Twitter data with two change-points at 2014 and 2019: evolution of posterior means and 95% CI for within-group attraction/repulsion $\gamma^w_1$ for Democrats, $\gamma^w_2$ for Republicans, between-group attraction/repulsion $\gamma^b$ and edge persistence $\delta$ in Twitter congressional hashtag networks. In the first period, strong in-party attachment for both parties (positive blue red), less evidence for the existence of polarization. In the second period, both between-group repulsion (negative yellow), and within-group repulsion for Republicans (negative red) appear. In-party negative feelings dominates out-party negative feelings (magnitude of red greater than that of yellow). For the third period, both in-/out-party negative feeling disappear.

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