MAGNON-LIKE DISPERSION RELATION FROM M-THEORY

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Abstract

We investigate classical rotating membranes in two different backgrounds. First, we obtain membrane solution in $\text{AdS}_4 \times S^7$ background, analogous to the solution obtained by Hofman and Maldacena in the case of string theory. We find a magnon type dispersion relation similar to that of Hofman and Maldacena and to the one found by Dorey for the two spin case. In the appendix of the paper, we consider membrane solutions in $\text{AdS}_7 \times S^4$, which give new relations between the conserved charges.

Keywords: M-theory, AdS-CFT correspondence, spin chains.

1 Introduction

The main directions of developments in String/M theory last years are related to their relations to the gauge theories at strong (weak) coupling. A powerful tool in searching for string/M theory description of gauge theories is AdS/CFT correspondence. One of the predictions of the correspondence is the equivalence between the spectrum of free string/M theory on AdS spaces and the spectrum of anomalous dimensions of gauge invariant operators in the planar $\mathcal{N} = 4$ Supersymmetric Yang-Mills (SYM) theory. Since the string/M theory in such spaces is highly non-linear, the check of this conjecture turns out to be very non-trivial. The known tests that confirm the correspondence beyond the supergravity approximation are based on the suggestion by Gubser, Klebanov and Polyakov, that one can look for certain limits where semiclassical approximation takes place and the problem becomes tractable and some comparisons on both sides of the correspondence can be made. From string/M theory point of view this means that one should consider solutions with large quantum numbers, which are related to the anomalous dimensions of gauge theory operators. While to find the spectrum from this side, although complicated, is possible, a reliable method to do it from gauge theory side was needed.
Minahan and Zarembo proposed a remarkable solution to this problem [2] by relating the Hamiltonian of the Heisenberg spin chain with the dilatation operator of \( \mathcal{N} = 4 \) SYM. On other hand, in several papers, the relation between strings and spin chains was established, see for instance [3], [4], [5], [6] and references therein. This idea opened the way for a remarkable interplay between spin chains, gauge theories, string theory, and integrability (the integrability of classical strings on \( \text{AdS}_5 \times S^5 \) was proven in [10]). One of the ways to compare these two sides of the AdS/CFT correspondence is to look for string/M theory solutions corresponding to different corners of the spectrum of the spin chains arising from string and gauge theory sides. Although there is no known direct relation of M-theory in the limit of large quantum numbers to spin chains, one can still directly relate the dispersion relations obtained from M-theory to the spectrum of gauge spin chains.

The most studied cases were spin waves in long-wave approximation, corresponding to rotating and pulsating strings in certain limits, see for instance the reviews [7], [8], [9] and references therein. Another interesting case are the low lying spin chain states corresponding to the magnon excitations. One class of string/membrane solutions already presented in a number of papers is the string/M theory on pp-wave backgrounds. The later, although interesting and important, describe point-like objects which are only part of the whole picture. The question of more general string/membrane solutions corresponding to this part of the spectrum was still unsolved.

Recently Maldacena and Hofman [11] were able to map spin chain "magnon" states to specific rotating semiclassical string states on \( R \times S^2 \). This result was soon generalized to magnon bound states ([12], [13], [14], [15], [16]), dual to strings on \( R \times S^3 \) and \( R \times S^5 \) with two and three non-vanishing angular momenta. The relation between energy and angular momentum for the one spin giant magnon found in [11] is:

\[
E - J = \frac{\sqrt{\lambda}}{\pi} \left| \sin \frac{p}{2} \right|, \tag{1.1}
\]

where \( p \) is the magnon momentum, which on the string side is interpreted as a difference in the angle \( \phi \) (see [11] for details). In the two spin case, the \( E - J \) relation was found both on the string [13], [14], [15] and spin chain [12] sides and looks like:

\[
E - J = \sqrt{J_2^2 + \frac{l}{\pi^2} \sin^2 \frac{p}{2}}, \tag{1.2}
\]

where \( J_2 \) is the second spin of the string.

In this paper, we are looking for membrane solutions analogous to giant magnon strings. Due to the AdS/CFT correspondence, the dispersion relations are expected to give similar result to that in the case of string magnon states. Indeed, for one particular ansatz for the embedding coordinates, we find dispersion relation that is similar to those obtained from strings for the magnon part of the spectrum of the gauge theory spin chain. One may wonder about how general this solution is. Certainly there are more examples of such solutions, which we will report in the near future, rising the conjecture that this kind of dispersion relations captures an essential feature of membrane spectrum.

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[2] For very nice reviews on the subject with a complete list of references see [7], [8], [9].
Our result gives support to the M/gauge theory correspondence previously checked for particular parts of the spectrum [17]-[26]. Apart from this result, we present in the Appendix different solutions for embeddings in $AdS \times S^1$ part of the geometry. The results give different complicated dispersion relations for which, unfortunately, we don’t have conclusive interpretation from gauge theory side. We hope that these solutions might be also useful for establishing M/gauge theory duality in the future.

2 Giant magnons from M-theory

The idea we will follow in this section is inspired from the analogy with the string theory case. In the later case the magnon dispersion relations are found by considering dynamics of an arc-like string with two ends on the equator of the five sphere $S^5$, or spiky strings. To capture the essence of this dynamics it is important to consider the correct embedding of the string configurations. We will look for an embedding which is arc-like but extended on the second spatial coordinate of the membrane. It can be though as a continuous family of arcs parameterized by the spatial coordinate $\xi^2$. This analogy will allow us latter on to make reduction and compare with the string case. We will work with the following gauge fixed membrane action and constraints [26], which coincide with the usually used gauge fixed Polyakov type action and constraints after the identification $2\lambda^0 T_2 = L = \text{const}$:

\begin{align}
S_{gf} &= \int d^3\xi \left\{ \frac{1}{4\lambda^0} \left[ G_{00} - \left(2\lambda^0 T_2\right)^2 \det G_{ij} \right] + T_2 C_{012} \right\}, \quad (2.1) \\
G_{00} + \left(2\lambda^0 T_2\right)^2 \det G_{ij} &= 0, \quad (2.2) \\
G_{0i} &= 0. \quad (2.3)
\end{align}

In (2.1)-(2.3), the fields induced on the membrane worldvolume $G_{mn}$ and $C_{012}$ are given by

\begin{align}
G_{mn} &= g_{MN} \partial_m X^M \partial_n X^N, \quad C_{012} = c_{MNP} \partial_0 X^M \partial_1 X^N \partial_2 X^P, \quad (2.4) \\
\partial_m &= \partial/\partial\xi^m, \quad m = (0, i) = (0, 1, 2), \quad M = (0, 1, \ldots, 10),
\end{align}

where $g_{MN}$ and $c_{MNP}$ are the target space metric and 3-form gauge field respectively. The equations of motion for $X^M$, following from (2.1), are as follows ($G \equiv \det G_{ij}$)

\begin{align}
g_{LN} \left[ \partial_0^2 X^N - \left(2\lambda^0 T_2\right)^2 \partial_i \left( G G^{ij} \partial_j X^N \right) \right] + \Gamma_{L,MN} \left[ \partial_0 X^M \partial_0 X^N - \left(2\lambda^0 T_2\right)^2 G G^{ij} \partial_j X^M \partial_j X^N \right] = 2\lambda^0 T_2 H_{LMNP} \partial_0 X^M \partial_1 X^N \partial_2 X^P, \quad (2.5)
\end{align}

where

\begin{align}
\Gamma_{L,MN} = g_{LK} \Gamma_{MN}^K = \frac{1}{2} \left( \partial_M g_{NL} + \partial_N g_{ML} - \partial_L g_{MN} \right)
\end{align}

See also [27], [28] and [29].
are the components of the symmetric connection corresponding to the metric $g_{MN}$ and $H_{LMN}$ is the field strength of the 3-form field $c_{MNP}$.

Here, we will search for rotating M2-brane configuration, which eventually could reproduce the string theory and spin chain (field theory) results for the two spin giant magnons [12]-[15]. Namely, we are interested in deriving an energy charge relation of the type

$$E - J_2 = \sqrt{J_1^2 + \lambda \pi^2 \frac{p}{2}}$$

for

$$E \to \infty, \quad J_2 \to \infty, \quad E - J_2 - \text{finite}, \quad J_1 - \text{finite}.$$ 

This relation has been established for strings on $R \times S^3$ [13]-[15] and $AdS_3 \times S^1$ [15] subspaces of $AdS_5 \times S^5$. Such subspaces also exist in M-theory backgrounds $AdS_4 \times S^7$ and $AdS_7 \times S^4$. However, we have not been able to find rotating M2-brane configurations with the desired properties on these subspaces of the target spaces. The experience in these computations led us to the conclusion that in order the membrane to have the needed semiclassical behavior, it must be embedded in a space with at least one dimension higher. In the case of $AdS_7 \times S^4$, the task is more difficult to solve, because of the presence of nontrivial 3-form background gauge field for the subspace $R \times S^4$. Hence, to simplify the problem we choose to consider M2-brane moving on the following subspace of $AdS_4 \times S^7$

$$ds^2 = (2l_p R)^2 \left\{ -dt^2 + 4 \left[ d\psi^2 + \cos^2 \psi d\varphi_1^2 + \sin^2 \psi \left( \cos^2 \theta_0 d\varphi_2^2 + \sin^2 \theta_0 d\varphi_3^2 \right) \right] \right\},$$

where the angle $\theta$ is fixed to an arbitrary value $\theta_0$, and for which the background 3-form field on $AdS_4$ vanishes.

We start with the following ansatz for the membrane

$$X^0(\xi^m) \equiv t(\xi^m) = \Lambda_0^0 \xi^0, \quad X^1(\xi^m) = \psi(\xi^2),$$
$$X^2(\xi^m) \equiv \varphi_1(\xi^m) = \Lambda_0^2 \xi^0,$$
$$X^3(\xi^m) \equiv \varphi_2(\xi^m) = \Lambda_0^3 \xi^0,$$
$$X^4(\xi^m) = \varphi_3(\xi^m) = \Lambda_0^4 \xi^1, \quad \Lambda_0^0, \ldots, \Lambda_0^4 = \text{constants}.$$

It corresponds to M2-brane extended in the $\psi$-direction, moving with constant energy $E$ along the $t$-coordinate, rotating in the planes defined by the angles $\varphi_1, \varphi_2$, with constant angular momenta $J_1, J_2$, and wrapped along $\varphi_3$.

It is easy to check that for this choice of the membrane embedding, the constraints (2.3) are identically satisfied. The remaining constraint (2.2), which for the case under consideration is first integral of the equation of motion for $\psi(\xi^2)$ following from (2.5), takes the form

$$K(\psi)\psi^2 + V(\psi) = 0,$$
$$K(\psi) = -2^{10}(l_p R)^4 (\lambda^0 T_2 \Lambda_4 \sin \theta_0)^2 \sin^2 \psi,$$
$$V(\psi) = (2l_p R)^2 \left\{ (\Lambda_0^0)^2 - 4(\Lambda_0^2)^2 - 4 \left[ (\Lambda_0^3)^2 \cos^2 \theta_0 - (\Lambda_0^2)^2 \right] \sin^2 \psi \right\}.$$
From (2.6) one obtains the turning point \((\psi' = 0)\) for the effective one dimensional motion
\[
M^2 = \frac{(\Lambda_0^0)^2 - 4(\Lambda_0^2)^2}{4[(\Lambda_0^3)^2 \cos^2 \theta_0 - (\Lambda_0^2)^2]}.
\] (2.7)

Now, we are interested in obtaining the explicit expressions for the conserved charges, which are given by
\[
P_\mu = \frac{\Lambda_\nu^0}{2\lambda^0} \int \int d\xi^1 d\xi^2 g_{\mu\nu}, \quad \mu, \nu = 0, 2, 3, 4.
\] (2.8)

For the present case \(P_4 = 0\) because \(X^4 = \varphi_3\) does not depend on \(\xi^0\). The computation of the other three conserved quantities leads to the following expressions for them
\[
E = \frac{2^5 \pi T_2 (l_p R)^3 \Lambda_1^4 \sin \theta_0}{[(\Lambda_0^3)^2 \cos^2 \theta_0 - (\Lambda_0^2)^2]^{1/2}} \ln \left(\frac{1 + M}{1 - M}\right),
\] (2.9)
\[
J_1 = \frac{2^7 \pi T_2 (l_p R)^3 \Lambda_1^4 \sin \theta_0}{[(\Lambda_0^3)^2 \cos^2 \theta_0 - (\Lambda_0^2)^2]^{1/2}} \left[\frac{1 - M^2}{2} \ln \left(\frac{1 + M}{1 - M}\right) + M\right],
\] (2.10)
\[
J_2 = \frac{2^7 \pi T_2 (l_p R)^3 \Lambda_1^4 \sin \theta_0}{[(\Lambda_0^3)^2 \cos^2 \theta_0 - (\Lambda_0^2)^2]^{1/2}} \left[\frac{1 + M^2}{2} \ln \left(\frac{1 + M}{1 - M}\right) - M\right].
\] (2.11)

Our next task is to consider the limit, when \(M\) tends to its maximum from below: \(M \to 1_-\). In this case, by using (2.7) and (2.9)-(2.11), one arrives at the energy-charge relation
\[
E - \frac{J_2}{2 \cos \theta_0} = \frac{1}{2} \sqrt{J_2^2 + [2^7 \pi T_2 (l_p R)^3 \Lambda_1^4]^2 \sin^2 \theta_0},
\] (2.12)
which is of the type, expected from gauge theory side.

It is clear that the subtle limit we used to obtain this dispersion relation shares the features of string derivation. Namely, while both, the energy and the momentum \(J_2\) are infinite, their difference is finite giving rise to the magnon-like dispersion relations. Since our formalism is, although equivalent, somehow different in notations and the way of embedding, one may ask whether one can compare this result with the ones obtained in string theory. To facilitate the comparison we will use in the next section the same technique and the same notations to reproduce well know string results and compare them to ours.

Other new solutions for M2-branes living in \(AdS_7 \times S^4\) with different semiclassical behavior are obtained in the Appendix.

## 3 Comparison with strings on \(AdS_5 \times S^5\)

For correspondence with the membrane formulae, we will use the Polyakov action and constraints in diagonal worldsheet gauge
\[
S_{gf} = \int d^2 \xi \frac{1}{4 \lambda^0} \left[ G_{00} - \left(2\lambda^0 T\right)^2 G_{11}\right],
\]
\[
G_{00} + \left(2\lambda^0 T\right)^2 G_{11} = 0,
\]
\[
G_{01} = 0.
\]
where
\[ G_{mn} = g_{MN} \partial_m X^M \partial_n X^N, \quad \partial_m = \partial / \partial \xi^m, \quad m = (0, 1), \quad M = (0, 1, \ldots, 9). \]

The usually used conformal gauge corresponds to \( 2 \lambda^0 T = 1 \).

We parameterize the metric on \( AdS_5 \times S^5 \) as follows
\[
\begin{align*}
    ds^2_{AdS_5} &= R^2 \left[ - \cosh^2 \rho dt^2 + 2 \rho d\rho \Omega_3^2 \right], \\
    ds^2_{S^5} &= R^2 \left[ d\psi^2 + \cos^2 \psi d\phi^2 + \sin^2 \psi \left( d\theta^2 + \cos^2 \theta d\phi^2 + \sin^2 \theta d\varphi_3^2 \right) \right].
\end{align*}
\]

We will consider three examples of rotating strings moving in three different subspaces of the above background.

First, we fix \( \rho = 0, \theta = \theta_0, \varphi_3 = 0 \), and embed the string as follows
\[
\begin{align*}
    X^0(\xi^m) &\equiv t(\xi^m) = \Lambda_0^0 \xi^0, & X^1(\xi^m) &= \psi(\xi^1), \\
    X^2(\xi^m) &\equiv \varphi_1(\xi^m) = \Lambda_0^2 \xi^0, & X^3(\xi^m) &= \varphi_2(\xi^m) = \Lambda_0^3 \xi^0.
\end{align*}
\]

This ansatz corresponds to string extended in the \( \psi \)-direction, moving with constant energy \( E \) along the \( t \)-coordinate, and rotating in the planes defined by the angles \( \varphi_1, \varphi_2 \), with constant angular momenta \( J_1, J_2 \). Calculations show that in the limit
\[
E \to \infty, \quad J_2 \to \infty, \quad E - J_2 - \text{finite}, \quad J_1 - \text{finite},
\]
the above string configuration is characterized by the following energy-charge relation
\[
E - \frac{J_2}{\cos \theta_0} = \sqrt{J_1^2 + (4TR^2)^2}.
\]

This result reproduces the angular dependence on the left hand side of (2.12).

As second example, let us fix \( \rho = 0, \theta = \theta_0, \varphi_2 = 0 \), and use the ansatz
\[
\begin{align*}
    X^0(\xi^m) &\equiv t(\xi^m) = \Lambda_0^0 \xi^0, & X^1(\xi^m) &= \psi(\xi^1), \\
    X^2(\xi^m) &\equiv \varphi_1(\xi^m) = \Lambda_0^2 \xi^0, & X^3(\xi^m) &= \varphi_3(\xi^m) = \Lambda_0^3 \xi^0.
\end{align*}
\]

This string configuration is of the same type as the one just considered. In the above mentioned limit one finds
\[
E - \frac{J_2}{\sin \theta_0} = \sqrt{J_1^2 + (4TR^2)^2}.
\]

For our third example, we fix \( \rho = 0, \psi = \psi_0, \varphi_1 = 0 \), and choose the string embedding
\[
\begin{align*}
    X^0(\xi^m) &\equiv t(\xi^m) = \Lambda_0^0 \xi^0, & X^1(\xi^m) &= \theta(\xi^1), \\
    X^2(\xi^m) &\equiv \varphi_2(\xi^m) = \Lambda_0^2 \xi^0, & X^3(\xi^m) &= \varphi_3(\xi^m) = \Lambda_0^3 \xi^0.
\end{align*}
\]
which is of the same type as the previous ones. Now, in the limit already pointed out, one receives energy-charge relation given by

\[ E - \frac{J_2}{\sin \psi_0} = \sqrt{\left( \frac{J_1}{\sin \psi_0} \right)^2 + (4TR^2)^2 \sin^2 \psi_0}. \]

This reproduces the angular dependence on the right hand side of (2.12).

Let us mention that in the particular cases when \( \cos \theta_0 = 1 \), or \( \sin \theta_0 = 1 \), or \( \sin \psi_0 = 1 \), the obtained dispersion relations reduce to one single formula

\[ E - J_2 = \sqrt{J_1^2 + \frac{4\lambda}{\pi^2}}, \]

where we have taken into account that \( TR^2 = \sqrt{\lambda}/2\pi \). This exactly reproduces the relation (2.33) of [15]. That is why, our three energy-charge relations represent three different generalizations of the result given in (2.33) of [15] for arbitrary values of the angles \( \theta_0 \) and \( \psi_0 \).

4 Concluding remarks

In this paper we studied particular membrane solutions analogous to the giant string magnons. The solution we found is supposed to cover magnon part of the spectrum of gauge spin chain. This interpretation can be think of in the light of M-theory \( \rightarrow \) string theory reduction due to the effective \( R \times S^1 \) topology of our considerations, which make sense in the heavy brane limit\(^4\). The membrane configuration we consider develops spikes on one of the embedding coordinates while on the other depends linearly on membrane worldvolume coordinates. Roughly, this can be though as a continuous family of arcs parameterized by the additional spacial coordinate. Although there is no concrete membrane spin chain our solutions to map to, it was interesting to see whether one can relate the resulting dispersion relations directly to the gauge theory spin chain. Interestingly, we were able to obtain membrane configuration that has analogous to the spin chain dispersion relation. Our result supports the M/gauge theory correspondence. It also rises the question whether there exists a spin chain/ladder to which M-theory can be mapped.

One of interesting directions of development is to look for multi-spin solutions and whether these solutions have the properties analogous to the corresponding string solutions. The example considered in this paper seems to be a particular case of solutions giving rise to such kind of dispersion relations. Other particular or general solutions would shed more light on M/gauge theory duality. It would be interesting also to look for magnon type solutions in the AdS part of geometry. We hope to report on these issues in the near future.

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\(^4\)We thank A.A. Tseytlin for comments on this point.
A Exact membrane solutions on $AdS_7 \times S^4$ and their semiclassical behavior

A.1 First type of membrane embedding

Here, we will use the following coordinates for the background $AdS_7 \times S^4$ metric

$$l_p^{-2} ds^2_{AdS_7 \times S^4} = 4R^2 \left\{ - \cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho \left( d\psi_1^2 + \cos^2 \psi_1 d\psi_2^2 + \sin^2 \psi_1 d\Omega_3^2 \right) \right\} + \frac{1}{4} \left[ d\alpha^2 + \cos^2 \alpha d\theta^2 + \sin^2 \alpha \left( d\beta^2 + \cos^2 \beta d\phi^2 \right) \right],$$  \hspace{1cm} (A.1)

$$d\Omega_3^2 = d\psi_3^2 + \cos^2 \psi_3 d\psi_4^2 + \cos^2 \psi_3 \cos^2 \psi_5 d\psi_5^2, \hspace{1cm} R^3 = \pi N.$$  \hspace{1cm} (A.2)

If we fix

$$\psi_1 = \pi/4, \hspace{0.5cm} \psi_3 = \psi_4 = \beta = \phi = 0,$$

(A.1) reduces to

$$ds^2 = (2l_p R)^2 \left[ - \cosh^2 \rho dt^2 + d\rho^2 + \frac{1}{2} \sinh^2 \rho \left( d\psi_2^2 + d\psi_5^2 \right) + \frac{1}{4} \left( d\alpha^2 + \cos^2 \alpha d\theta^2 \right) \right].$$  \hspace{1cm} (A.2)

Let us consider the M2-brane embedding $x^M = X^M(\xi^m)$ into (A.2) given by\footnote{The background 3-form on $S^4$ is zero for this ansatz.}

$$X^0(\xi^m) \equiv t(\xi^m) = \Lambda_0^0 \xi^0, \hspace{0.5cm} X^1(\xi^m) = \rho(\xi^2),$$

$$X^2(\xi^m) \equiv \psi_2(\xi^m) = \Lambda_0^2 \xi^0 + \Lambda_1^2 \xi^1 + \Lambda_2^2 \xi^2,$$

$$X^3(\xi^m) \equiv \psi_5(\xi^m) = \Lambda_0^3 \xi^0 - \frac{\Lambda_2^3}{\Lambda_0^3} (\Lambda_1^2 \xi^1 + \Lambda_2^2 \xi^2),$$

$$X^4(\xi^m) = \alpha(\xi^2), \hspace{0.5cm} X^5(\xi^m) \equiv \theta(\xi^m) = \Lambda_0^5 \xi^0, \hspace{0.5cm} \Lambda_0^0, \ldots, \Lambda_0^5 = \text{constants}.$$  \hspace{1cm} (A.3)

It corresponds to membrane extended in the $\rho$- and $\alpha$- directions, moving with constant energy $E$ along the $t$-coordinate, rotating in the planes defined by the angles $\psi_2$, $\psi_5$, $\theta$, with constant angular momenta $S_1$, $S_2$, $J$, and also wrapped along $\psi_2$, $\psi_5$. The ansatz (A.3) is a generalization of the M2-brane embedding considered in [18]. For $\alpha = 0$, one obtains the background felt by the membrane configuration considered there.

The relation between the parameters in (A.3) ensures that the constraints (2.3) are identically satisfied. Therefore, we have to solve the equations of motion (2.5) and the remaining constraint (2.2). On the ansatz (A.3), they read (the prime is used for $d/d\xi^2$):

$$4 \left[ K^2(\rho) \right]' - \frac{1}{2} \frac{dK^2(\rho)}{d\rho} \left( 4\rho^2 + \alpha^2 \right) - \frac{1}{2} \frac{\partial V(\rho, \alpha)}{\partial \rho} = 0,$$

$$\left[ K^2(\rho) \alpha \right]' - \frac{1}{2} \frac{\partial V(\rho, \alpha)}{\partial \alpha} = 0,$$

$$K^2(\rho) \left( 4\rho^2 + \alpha^2 \right) - V(\rho, \alpha) = 0,$$  \hspace{1cm} (A.4)
where
\[ K^2(\rho) = 8(\lambda^0 T_2)^2(l_p R)^4 \left[ 1 + \left( \Lambda_0^2 / \Lambda_0^3 \right)^2 \right] (\Lambda_0^2)^2 \sinh^2 \rho, \]
\[ V(\rho, \alpha) = (2l_p R)^2 \left\{ (\Lambda_0^0)^2 \cosh^2 \rho - \frac{1}{2} \left[ 1 + \left( \Lambda_0^2 / \Lambda_0^3 \right)^2 \right] (\Lambda_0^3)^2 \sinh^2 \rho - \frac{1}{4}(\Lambda_0^5)^2 \cos^2 \alpha \right\}. \]

We have not been able to find exact analytical solution of the above system of nonlinear PDEs. That is why, we restrict ourselves to the particular case \( \Lambda_5^0 = 0 \), i.e. \( \theta = 0 \) and \( J = 0 \). Thus, the equations (A.4) reduce to
\[ \rho' = \frac{1}{2K^2(\rho)} \sqrt{K^2(\rho)V(\rho) - A^2}, \quad \alpha' = \frac{A}{K^2(\rho)}, \quad A = \text{const}, \tag{A.5} \]
from where one gets the equation for the membrane trajectory \( \rho(\alpha) \)
\[ \frac{d\rho}{d\alpha} = \frac{1}{2A} \sqrt{K^2(\rho)V(\rho) - A^2}. \tag{A.6} \]

Here, we are interested in those solutions of (A.5) and (A.6), which correspond to closed trajectories (orbiting membrane). For them, \( \rho \in (\rho_{\min}, \rho_{\max}) \), where \( \rho_{\min} \equiv \rho_- \) and \( \rho_{\max} \equiv \rho_+ \) are solutions of the equation \( K^2(\rho)V(\rho) - A^2 = 0 \). In the case under consideration, one obtains
\[ x_\pm \equiv \cosh 2\rho_\pm = 1 + \frac{a^2}{b^2 - a^2} \left[ 1 \pm \sqrt{1 - \left( \frac{2A}{ac} \right)^2 \frac{b^2 - a^2}{a^2}} \right] \geq 1, \]
\[ b^2 - a^2 > 0, \quad A^2 < \left( \frac{ac}{2} \right)^2 \frac{a^2}{b^2 - a^2}, \]
where
\[ a^2 = (2l_p R)^2(\Lambda_0^0)^2, \quad b^2 = \frac{1}{2}(2l_p R)^2 \left[ 1 + \left( \Lambda_0^2 / \Lambda_0^3 \right)^2 \right] (\Lambda_0^3)^2, \tag{A.7} \]
\[ c^2 = 8(\lambda^0 T_2)^2(l_p R)^4 \left[ 1 + \left( \Lambda_0^2 / \Lambda_0^3 \right)^2 \right] (\Lambda_1^2)^2. \]

The solution for the orbit is
\[ x(\alpha) \equiv \cosh 2\rho(\alpha) = 1 + \frac{x_- - 1}{1 - \frac{z_+ - z_-}{x_+ - 1} \, sn^2 \left( \frac{\alpha}{4\pi} \sqrt{(b^2 - a^2)(x_+ - 1)(x_- + 1)} \right)}, \]
where \( sn(u) \) is one of the Jacobian elliptic functions. For the solutions of the equations (A.5), one receives
\[ \xi^2(x) = \frac{2c(x_- - 1)}{\sqrt{(b^2 - a^2)(x_+ - 1)(x_- + 1)}} \Pi (\varphi_1, \nu, k), \]
\[ \xi^2(\alpha) = \frac{2c(x_- - 1)}{\sqrt{(b^2 - a^2)(x_+ - 1)(x_- + 1)}} \Pi (\varphi_2, -\nu, k), \]
where $\Pi(\varphi, \nu, k)$ is one of the elliptic integrals and

\[
\varphi_1 = \arcsin \sqrt{\frac{(x_+ - 1)(x - x_+)}{(x_+ - x_+)(x - x_+)}} , \quad \varphi_2 = \text{am} \left( \frac{c}{4A} \sqrt{(b^2 - a^2)(x_+ - 1)(x_+ + 1)\alpha} \right),
\]

\[
\nu = \frac{x_+ - x_-}{x_+ - 1} , \quad k = \sqrt{\frac{2(x_+ - x_-)}{(x_+ - 1)(x_+ + 1)}}.
\]

The normalization condition

\[
2\pi = \int_0^{2\pi} d\xi^2 = 2 \int_{\rho_{\text{max}}}^{\rho} d\rho = 4 \int_{\rho_{\text{min}}}^{\rho_{\text{max}}} \frac{K^2(\rho)d\rho}{\sqrt{K^2(\rho)V(\rho) - A^2}}
\]

is given by

\[
\frac{1}{\pi} \sqrt{\frac{c^2}{b^2 - a^2}} \int_{x_-}^{x_+} \sqrt{\frac{x - 1}{(x_+ - x)(x - x_+)(x + 1)}} dx = \frac{c^2}{b^2 - a^2} \sqrt{\frac{x_- - 1}{x_- + 1}} \left( \frac{x_+ - x_-}{x_+ - 1} \right)^{1/2} \left( \frac{x_+ - x_-}{x_+ - 1} \right)^{1/2} \times \]

\[
F_1 \left( \frac{1}{1, 2, 1, 2, -1/2, 1; -1/2, 1; -x_+ - x_-}{x_+ + 1}; 0.5, 0.5, 0.5 \right) = 1,
\]

where $F_1(a, b_1, b_2; c; z_1, z_2)$ is one of the hypergeometric functions of two variables [30].

For the conserved quantities $E$, $S_1$ and $S_2$, on the above membrane solution, we obtain the following expressions

\[
\frac{E}{\Lambda_0^2} = \frac{(2l_pR)^2}{2\lambda^0} \int \int d\xi^1 d\xi^2 \cosh^2 \rho(\xi^2) = \frac{(2l_pR)^2\pi^2 c}{2\lambda^0} \sqrt{\frac{(x_+ + 1)(x_+ - 1)}{b_+ - a_2}} F_1 \left( 1/2, -1/2, 1/2, 1; -x_+ - x_- \frac{x_+ - x_-}{x_+ + 1}, -x_+ - x_- \frac{x_+ - x_-}{x_+ - 1} \right) = \]

\[
\frac{(2l_pR)^2\pi^2 c}{2\lambda^0} \sqrt{\frac{(x_+ + 1)(x_+ - 1)}{b_+ - a_2}} \left( \frac{x_+ - x_-}{x_+ + 1} \right)^{1/2} \left( \frac{x_+ - x_-}{x_+ - 1} \right)^{1/2} \times \]

\[
F_1 \left( 1/2, -1/2, 1/2, 1; -x_+ - x_- \frac{x_+ - x_-}{x_+ + 1}, -x_+ - x_- \frac{x_+ - x_-}{x_+ - 1} \right),
\]

\[
\frac{S_1}{\Lambda_0^2} = \frac{S_2}{\Lambda_0^2} = \frac{(2l_pR)^2}{4\lambda^0} \int \int d\xi^1 d\xi^2 \sinh^2 \rho(\xi^2) = \frac{(2l_pR)^2\pi^2 c}{4\lambda^0} \sqrt{\frac{(x_+ + 1)^3}{(b_+ - a_2)^2}} F_1 \left( 1/2, 1/2, 3/2, 1; -x_+ - x_- \frac{x_+ - x_-}{x_+ + 1}, -x_+ - x_- \frac{x_+ - x_-}{x_+ - 1} \right) =
\]

\[
\frac{(2l_pR)^2\pi^2 c}{4\lambda^0} \sqrt{\frac{(x_+ - 1)^3}{(b_+ - a_2)(x_+ + 1)}} F_1 \left( 1/2, 1/2, 3/2, 1; -x_+ - x_- \frac{x_+ - x_-}{x_+ + 1}, -x_+ - x_- \frac{x_+ - x_-}{x_+ - 1} \right) =
\]
\[
\left(\frac{2l_p R}{2\pi c}\right)^2 \frac{\pi^2 c}{4\lambda^0} \sqrt{\frac{(x_+ - 1)^3}{(b^2 - a^2)(x_- + 1)}} \left(1 + \frac{x_+ - x_-}{x_- + 1}\right)^{-1/2} \left(1 + \frac{x_+ - x_-}{x_- - 1}\right)^{3/2} \times 
\]

\[
F_1\left(1/2, 1/2, -3/2, 1; \frac{1}{1 + \frac{x_+ - 1}{x_-}}, \frac{1}{1 + \frac{x_+ - x_-}{x_- + 1}}\right).
\]

In the semiclassical limit (large conserved charges), which in the present case correspond to
\[
x_+ \to \frac{2a^2}{b^2 - a^2} \to \infty, \quad x_- \to 1,
\]
the equalities (A.8), (A.9) and (A.10) simplify to
\[
c^2 = b^2 - a^2, \quad \frac{E}{\Lambda_0^2} = \frac{2\pi l_p R}{2\lambda^0 (b^2 - a^2)}, \quad \frac{S_1}{\Lambda_0^2} = \frac{S_2}{\Lambda_0^2} = \frac{(2\pi l_p R)^2 a^2}{4\lambda^0 (b^2 - a^2)}.
\]

From here, one receives the following energy-charge relation
\[
\left[2(S_1^2 + S_2^2) - E^2\right]^3 - 8(l_p R)^6 (\pi^2 T_2\Lambda_1^2)^2 \left(1 + \frac{S_1}{S_2}\right) E^4 = 0, \quad (A.11)
\]
where \(\Lambda_1^2\) is wrapping parameter with integer values (see (A.3)). (A.11) is third order algebraic equation for \(E^2\), and does not give a 2-spin generalization of the known \(E - S \sim S^{1/3}\) relation for membranes in \(AdS_p \times S^q\) backgrounds. If we set \(\Lambda_1^2 = 0\), this will correspond to string-like ansatz for the membrane, because the \(\xi^1\) worldvolume coordinate drops out from the embedding. Then we get
\[
E = \sqrt{2(S_1^2 + S_2^2)},
\]
which has nothing to do with the 2-spin giant magnon dispersion relations. For say \(S_1 = 0\), it reduces to \(E - \sqrt{2}S_2 = 0\), which should be compared with the spin chain ferromagnetic vacuum characterized by the relation \(E - J = 0\). For \(S_1 \ll S_2\), one can rewrite it as
\[
E - \sqrt{2}S_2 = \frac{S_1^2}{\sqrt{2}S_2} + \ldots.
\]

Obviously, this relation does not correspond to the low energy spin waves states of the spin chain, dual to spinning strings with
\[
E - J \sim \frac{\lambda}{J} + \ldots.
\]

The conclusion is that even this stringy-restricted membrane configuration does not describe any part of the exited spin chain spectrum.

Let us finally recall that the above results are obtained in the framework of the particular case \(\theta = 0\), when the background seen by the membrane is
\[
ds^2 = (2l_p R)^2 \left[-\cosh^2 \rho dt^2 + d\rho^2 + \frac{1}{2} \sinh^2 \rho \left(d\psi_2^2 + d\psi_5^2\right) + \frac{1}{4} d\alpha^2\right], \quad (A.12)
\]
for which the Lagrangian density in the action (2.1), on the ansatz (A.3) (for \(\Lambda_0^5 = 0\), reduces to
\[
\mathcal{L} = -\frac{1}{4\lambda^0} \left[K^2(\rho) \left(4\rho^2 + \alpha^2\right) + V(\rho)\right]. \quad (A.13)
\]
A.2 Second type of membrane embedding

Here, we intend to consider the membrane configuration described in the previous subsection from another viewpoint, by representing it as embedded in flat space-time. To this end, we rewrite the metric (2.4) induced on the membrane worldvolume in the form

\[ G_{mn} = \eta_{MN} \partial_m Z^M \partial_n \bar{Z}^N = \eta_{\alpha\beta} \partial_m Y^\alpha \partial_n \bar{Y}^\beta + \partial_m X \partial_n \bar{X}, \quad \eta_{\alpha\beta} = \text{diag}(-1, 1, 1), \]

where \( Y^\alpha \) and \( X \) are complex coordinates given by

\[ Y_0 = 2l_p \text{Re}^t \cosh \rho, \quad Y^1 = \sqrt{2} l_p \text{Re}^{i\psi_2} \sinh \rho, \quad Y^2 = \sqrt{2} l_p \text{Re}^{i\psi_5} \sinh \rho, \quad X = l_p \text{Re}^{i\alpha}, \]

and satisfying the equalities

\[ \eta_{\alpha\beta} Y^\alpha \bar{Y}^\beta = -(2l_p R)^2, \quad X \bar{X} = (l_p R)^2. \]

In the coordinates (A.14), the flat target space metric

\[ ds_f^2 = \eta_{\alpha\beta} dY^\alpha d\bar{Y}^\beta + dXd\bar{X} \]

coincides with the one in (A.12). In order to reproduce the reduced Lagrangian density (A.13) corresponding to the membrane solution obtained in the previous subsection, one just have to replace \( t, \rho, \psi_2, \psi_5 \) and \( \alpha \) in (A.14) with the expressions for them given in (A.3).

Now, let us consider the membrane configuration given by the embedding

\[ y^0(\xi^0, \xi^2) = 2l_p \text{Re}^{i\Lambda_0^0} r_1(\xi^2), \quad y^1(\xi^m) = \sqrt{2} l_p \text{Re}^{i(\Lambda_2^0 \xi^0 + \Lambda_4^1 \xi^1 + \Lambda_2^3 \xi^2)} r_2(\xi^2), \]
\[ y^2(\xi^m) = \sqrt{2} l_p \text{Re}^{i\left[\Lambda_2^0 \xi^0 - \frac{\Lambda_2^1}{6}(\Lambda_4^1 \xi^1 + \Lambda_2^3 \xi^2)\right]} r_2(\xi^2), \quad y^3(\xi^2) = l_p \text{Re}^{i\alpha(\xi^2)}, \]

where in accordance with (A.15), \( r_1 \) and \( r_2 \) are constrained by

\[ r_1^2 - r_2^2 - 1 = 0. \]

The corresponding reduced Lagrangian density is

\[ \mathcal{L}_f = \frac{1}{4\Lambda_0^0} \left[ c^2 r_2^2 \left[ 4(r_1^2 - r_2^2) - \alpha^2 \right] - a^2 r_1^2 + b^2 r_2^2 \right] - \Lambda(r_1^2 - r_2^2 - 1). \]

Here, the constant coefficients \( a^2, b^2, c^2 \) are introduced in (A.7), and \( \Lambda \) is Lagrange multiplier. For the embedding (A.16), the constraints (2.3) are identically satisfied. The remaining constraint (2.2) takes the form

\[ c^2 r_2^2 \left[ 4(r_1^2 - r_2^2) - \alpha^2 \right] + a^2 r_1^2 - b^2 r_2^2 = 0. \]

\(^6\)The second constraint in (A.15) is identically satisfied for this ansatz.
From (A.18), one obtains the following equations of motion\footnote{At this stage, we fix $\Lambda = \text{constant.}$}

\[
(r_2' r_1')' = -\frac{a^2 + 4\lambda^0 \Lambda}{4c^2} r_1, \quad (A.20)
\]

\[
(r_2' r_2')' = -\left[\frac{b^2 + 4\lambda^0 \Lambda}{4c^2} + r_1'^2 - r_2'^2 - \frac{1}{4} a^2\right] r_2, \quad (A.21)
\]

\[
(r_2' \alpha)' = 0, \quad r_1^2 - r_2^2 - 1 = 0. \quad (A.22)
\]

The solution of the equation for $\alpha'$ is

\[
\alpha' = \frac{2C_\alpha}{r_2^2}, \quad C_\alpha = \text{const.} \quad (A.23)
\]

The replacement of (A.21) into the equation for $r_2'$ and (A.19) gives

\[
(r_2' r_2')' = -\left[\frac{b^2 + 4\lambda^0 \Lambda}{4c^2} + r_1'^2 - r_2'^2 - C_\alpha \frac{r_1^2}{r_2^4}\right] r_2, \quad (A.24)
\]

\[
r_1^2 - r_2^2 = C_\alpha \frac{r_1^2}{r_2^4} - \frac{1}{4c^2} \left(a^2 \frac{r_1^2}{r_2^2} - b^2\right). \quad (A.25)
\]

With the help of (A.23) and $r_1^2 - r_2^2 - 1 = 0$, the equation (A.22) reduces to

\[
r_2' = \frac{1}{2\sqrt{2c} r_2^2} \sqrt{-(2b^2 - a^2 + 4\lambda^0 \Lambda) r_1^4 + 2a^2 r_2^2 + C_{r_2}}, \quad (A.26)
\]

where $C_{r_2}$ is arbitrary integration constant. By using the equality $r_1^2 - r_2^2 = 1$ once again, this time in (A.20), one derives for $r_1'$ the expression

\[
r_1' = \frac{1}{2\sqrt{2c} (r_1^2 - 1)} \sqrt{-(a^2 + 4\lambda^0 \Lambda) [r_1^2(r_1^2 - 2) + C_{r_1}]}, \quad C_{r_1} = \text{const.} \quad (A.27)
\]

In order (A.23) to be identically satisfied on the solutions (A.24), (A.25), the integration constants must be related as follows

\[
8c^2 C_\alpha^2 + C_{r_1} + C_{r_2} = a^2 + 4\lambda^0 \Lambda.
\]

Of course, the condition $r_1^2 - r_2^2 = 1$ will give another relation between $C_\alpha$, $C_{r_1}$ and $C_{r_2}$.

For our further purposes, it is enough to solve one of the differential equations (A.24), (A.25), and we choose the first one. Setting $r_2' = 0$, we observe that there exist two different cases, which correspond to periodic motion, depending on the sign of $C_{r_2}$.

**First case** ($y \equiv r_2'$)

\[
y_1 = -\frac{B^2}{|A|} \left(\sqrt{1 + \frac{2 |A| |C|}{B^4}} - 1\right) < 0,
\]

\[
y_{\max} = y_2 = \frac{B^2}{|A|} \left(1 + \sqrt{1 + \frac{2 |A| |C|}{B^4}}\right), \quad \text{therefore} \quad y_{\min} = 0.
\]
Second case

\[
y_{\text{min}} = y_1 = \frac{B^2}{|A|} \left( 1 - \sqrt{1 - \frac{2 |A||C|}{B^4}} \right) > 0,
\]

\[
y_{\text{max}} = y_2 = \frac{B^2}{|A|} \left( 1 + \sqrt{1 - \frac{2 |A||C|}{B^4}} \right) > y_{\text{min}}.
\]

For convenience, we have introduced here the notations

\[
A = -\frac{2b^2 - a^2 + 4\lambda^0\Lambda}{4c^2} < 0, \quad B^2 = \frac{a^2}{4c^2}, \quad C = \frac{C_{r_2}}{8c^2}.
\]

We begin with considering the first case, for which the solution of (A.24) is

\[
\xi^2(y) = \frac{1}{3} \left( \frac{2y^3}{|A||y_1| y_{\text{max}}} \right)^{1/2} F_1 \left( \frac{3}{2}, \frac{1}{2}, \frac{1}{2}; \frac{5}{2}; \frac{y}{|y_1|}, \frac{y}{y_{\text{max}}} \right).
\]

The normalization condition

\[
2\pi = \int_0^{2\pi} d\xi^2 = 2 \int_{(r_2)_{\text{min}}}^{(r_2)_{\text{max}}} \frac{dr_2}{r_2'},
\]

leads to

\[
\frac{y_{\text{max}}}{(2^3 |A||y_1|)^{1/2} 2 F_1 \left( \frac{3}{2}, \frac{1}{2}, \frac{1}{2}; \frac{5}{2}; \frac{y_{\text{max}}}{|y_1|} \right)} = 1.
\]  \hspace{1cm} (A.26)

For the conserved quantities \(E, S_1, S_2\), canonically conjugated to the coordinates \(t, \psi_2, \psi_5\), we obtain the expressions

\[
E_{\Lambda_0} = \frac{\pi}{2\lambda^0}(2l_pR)^2(I + 2\pi), \quad S_1_{\Lambda_0} = S_2_{\Lambda_0} = \frac{\pi}{4\lambda^0}(2l_pR)^2I,
\]

\[
I = \frac{3\pi y_{\text{max}}^2}{(2^5 |A||y_1|)^{1/2} 2 F_1 \left( \frac{5}{2}, \frac{1}{2}, \frac{3}{2}; \frac{y_{\text{max}}}{|y_1|} \right)}.
\]  \hspace{1cm} (A.27)

In the semiclassical limit, which in the case under consideration correspond to

\[
y_{\text{max}}, |y_1| \to \sqrt{2 \frac{|C|}{|A|}} \to \infty,
\]

the equalities (A.26) and (A.27) simplify to

\[
\frac{2^5 \pi^2 |C|}{\Gamma^8(1/4) |A|^3} = 1, \quad I = \left[ \frac{2^3 \pi^6 |C|^3}{\Gamma^8(3/4) |A|^5} \right]^{1/4},
\]

from where one derives the following dependence of the energy \(E\) on the charges \(S_1, S_2\)

\[
E^2 = 4(S_1^2 + S_2^2) - \frac{\Gamma^8(1/4)(l_pR)^2}{2^4 \lambda^0} |C_{r_2}| + \Lambda \left[ \frac{\Gamma^{32}(1/4) |C_{r_2}|^2}{2^{14} \pi^2 (\lambda^0)^7 (T_2 \Lambda_2^4)^4 (l_pR)^2} \right]^{1/3} \left( 1 + \frac{S_1^2}{S_2^2} \right)^{-2/3}. \]  \hspace{1cm} (A.28)
Here, $\Lambda_1^2$ is a winding number as in the previous solution, while the integration constant $C_{r_2}$ and the Lagrange multipliers $\lambda^0$, $\Lambda$ are free parameters. This is a generalization of the energy-charge relation $E - S \sim S^{-1}$, unknown for M2-branes up to now.

Let us turn to the second case. The solution of (A.24) reads ($\Delta y = y - y_{\min}$, $\Delta y_m = y_{\max} - y_{\min}$)

$$
\xi^2(y) = \left( \frac{2y_{\min}\Delta y}{A \Delta y_m} \right)^{1/2} F_1 \left( 1/2, -1/2, 1/2; 3/2; -\frac{\Delta y}{y_{\min}}, \frac{\Delta y}{\Delta y_m} \right).
$$

The normalization condition gives

$$
\left( \frac{y_{\min}}{2A} \right)^{1/2} \left( 1 + \frac{\Delta y_m}{y_{\min}} \right)^{1/2} 2F_1 \left( 1/2, -1/2; 1; \frac{1}{1 + \frac{y_{\min}}{\Delta y_m}} \right) = 1. \tag{A.29}
$$

The explicit expressions for the conserved quantities can be obtained from (A.27) by the replacement $I \to J$, with $J$ given by

$$
\pi \left( \frac{2y_{\min}^3}{A} \right)^{1/2} \left( 1 + \frac{\Delta y_m}{y_{\min}} \right)^{3/2} 2F_1 \left( 1/2, -3/2; 1; \frac{1}{1 + \frac{y_{\min}}{\Delta y_m}} \right). \tag{A.30}
$$

Taking the semiclassical limit, which now corresponds to

$$
y_{\max} \to \frac{2B^2}{A} \to \infty, \quad y_{\min} \to 0,
$$

one gets the following energy-charge relation

$$
\left[ 1 + \frac{2}{\pi^2} \frac{2l_p R^2 (2\lambda^0 T_2 \Lambda_1^2)^2}{(1 + S_1^2/S_2^2)} \right] E^2 \left( 1 + S_1^2/S_2^2 \right) \Lambda \frac{23/2\pi^3}{3l_p R \lambda^0 T_2 \Lambda_1^2} (1 + S_1^2/S_2^2)^{1/2} \right) - 4(S_1^2 + S_2^2) = 0, \tag{A.31}
$$

where $E(S_1, S_2)$ is given by the positive root of this equation.

An interesting question is whether the membrane configuration just considered can be related to the known integrable systems appearing in the context of rigid string motion on $AdS_5 \times S^5$. To make such comparison, we replace the solution (A.21) for $\alpha'$ into (A.18) and obtain

$$
\mathcal{L} = \frac{1}{4\lambda^0} \left\{ 4c^2 r_2^2 (r_1^2 - r_2^2) - a^2 r_2^4 + b^2 r_2^2 - \frac{4c^2 C_\alpha^2}{r_2^2} \right\} - \Lambda (r_1^2 - r_2^2 - 1).
$$

This Lagrangian is similar to a particular case of the Lagrangian describing the n-dimensional Neumann-Rosochatius-like integrable system with indefinite metric [31]

$$
\mathcal{L}_{NR-l} = \frac{1}{2} \eta^{rs} \left( \eta^{r_r} r_s - \omega^{r_s} r_2 r_s - \frac{u_u u_s}{r_r r_s} \right) - \frac{1}{2} \tilde{\Lambda} (\eta^{r_r} r_r r_s + 1), \quad \eta = diag(-, +, ..., +, -).
$$

The essential difference is that in the membrane case the kinetic term depends on the coordinate $r_2$. However, this does not exclude the existence of M2-brane configurations with Lagrangians of the type $\mathcal{L}_{NR-l}$. 

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