QCD Sum Rule for $S_{11}(1535)$

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Abstract

We propose a new interpolating field for $S_{11}(1535)$ to determine its mass from QCD sum rules. In the nonrelativistic limit, this interpolating field dominantly reduces to two quarks in the s-wave state and one quark in the p-wave state. An optimization procedure, which makes use of a duality relation, yields the interpolating field which overlaps strongly with the negative-parity baryon and at the same time does not couple at all to the low lying positive-parity baryon. Using this interpolating field and applying the conventional QCD sum rule analysis, we find that the mass of $S_{11}$ is reasonably close to the experimentally known value, even though the precise determination depends on the poorly known quark-gluon condensate. Hence our interpolating field can be used to investigate the spectral properties of $S_{11}(1535)$.

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The negative parity nucleon resonance $S_{11}(1535)$ has always been an object of study when constructing models of the baryon. A good fit to its mass and decay widths provides further test of the validity of the model, which in return provides a deeper understanding of the structure of the resonance itself.

Recently, in relation to the present and planned experiments of $\eta$-production on a nucleon from electromagnetic or hadronic probes at MAMI (Mainz) \cite{1}, ELSA (Bonn) \cite{4} and CEBAF, there has been a renewed interest in $S_{11}$. Because the mass of $S_{11}$ is just above the $\eta N$ threshold, $\eta$-productions in these experiments are dominated by $s$-channel $S_{11}$ resonance contribution. As a consequence, a detailed study of its properties, such as the $\eta$-nucleon-$S_{11}$ coupling and transition form factors, can provide important constraints in the analysis of these experimental data. So far, most theoretical works \cite{3} have been focused on reproducing the experimental data by using the empirical parameters related to the $S_{11}$, which have been explicitly introduced in their effective models. To make a connection between these effective models with presumably the fundamental theory of QCD, it is important to understand how the spectral properties of $S_{11}$ are generated from QCD order parameters. In this way, phenomenological aspects of $S_{11}$ can be predicted rather model-independently. In this letter, as a starting point for this kind of analysis, we will introduce a new interpolating field for the $S_{11}$ suitable for conventional QCD sum rule analysis.

The QCD sum rule \cite{4} is a useful tool to study the spectral properties of the hadrons. It has been widely used in a number of occasions; in calculating properties of hadrons, coupling strength between hadrons, form factors \cite{5} and recently medium dependent properties \cite{6,7}. The starting point of the QCD sum rule is to introduce an appropriate interpolating field for the hadron of concern. In general, the interpolating field is constructed from quark and gluon fields, whose specific form is usually determined by considering the hadron’s quantum numbers and its nonrelativistic quark wave functions. Then the two-point correlation function of such interpolating field is introduced and calculated in the operator product expansion. By matching the resulting correlation function with its phenomenological counterpart, one can make prediction on the hadron’s spectral properties in terms of the QCD order parameters. Clearly, the success of the prediction is highly dependent upon the choice of the interpolating field.

Recently, Jido Kodama and Oka \cite{8} have proposed a technique to separate out the contribution of the negative-parity baryon from QCD sum rules using the usual interpolating fields of the nucleon. They succeeded in obtaining a reasonable value for the mass of $S_{11}$ by adjusting the QCD input parameters and at the same time optimizing a linear combination of the two independent nucleon interpolating fields. However, there one has to take the difference between the two independent sum rules of the nucleon each proportional to the Dirac structure of $1$ and $\bar{q}$ so as to subtract out the nucleon contribution, which has a large overlap with the current. This fact makes it difficult to investigate other properties, such as the form factors or the couplings, of the $S_{11}$ resonance within the conventional sum rule approach. The problem is that the two independent nucleon currents used so far either couple strongly or weakly to both the nucleon and the low-lying resonances at the same time, such that it is difficult to select out only the negative parity part of the resonances in either case. To overcome this difficulty, we will construct a current that couples weakly to the nucleon and strongly to the $S_{11}$. It should be noted that the reason why the nucleon couples strongly with the commonly used Ioffe current is that, this current has a strong
overlap with the nonrelativistic quark wave function of the nucleons in which all quarks are in the s-wave state. Therefore, we will construct a current whose nonrelativistic limit has a strong overlap with two quarks in the s-wave state and one quark in the p-wave state. This is the picture for \( S_{11} \) both in the bag model or nonrelativistic quark model. This implies that we have to go beyond the two independent nucleon sum rule and introduce an appropriate covariant derivative in the current, for which there are many choices. We will choose a minimal approach and construct our current so that it does not couple to the low lying positive-parity nucleon but couples strongly to the negative-parity baryon. As will be shown by QCD sum rule analysis, our current gives a reasonably stable plateau for its mass in the Borel curve. Using this current, it is possible to apply conventional sum rule approaches to investigate other properties of the \( S_{11} \) resonance, which will be a subject of future study [9].

First, we start with a short discussion of the proton interpolating field. The interpolating field of the proton is constructed from two u-quarks and one d-quark by assuming that all three quarks are in the s-wave state. Specifically, one up and the down quarks are combined into an isoscalar diquark and the other up quark is attached to the diquark so that the quantum numbers of the proton are carried by the attached up quark. The proton should be a color singlet and its interpolating field should transform as the spinor under the parity transformation. Then, it is found that there are two interpolating fields possible [10–12],

\[
\eta_1 = \epsilon_{abc}(u_a^T C d_b)\gamma_5 u_c , \\
\eta_2 = \epsilon_{abc}(u_a^T C \gamma_5 d_b)u_c ,
\]

where \( \eta_2 \) contains the valence quark wave function in the nonrelativistic limit. In general, the interpolating field for a proton is an arbitrary linear combination of these fields,

\[
\eta(t) = 2\epsilon_{abc}[ t (u_a^T C d_b)\gamma_5 u_c + (u_a^T C \gamma_5 d_b)u_c] .
\]

For \( t \) equal to \(-1\), the resulting interpolating field can be shown to have no direct instanton contribution and, at the same time couple strongly to the chiral symmetry breaking effects [11].

\( S_{11} \) is the lowest resonance of the nucleon with negative parity. As suggested by the bag model [13] or nonrelativistic quark model [14], one quark in \( S_{11} \) is believed to be in the p-wave state relative to the other two quarks. We want to construct a current, which in the nonrelativistic limit overlaps largely with this quark field configuration. One way to accomplish this is to put a covariant derivative \( z \cdot D \) to one of the quarks in Eq. (1). Here, we have introduced a four vector \( z^\mu \) such that \( z \cdot q = 0 \) and \( z^2 = -1 \). The property of \( z^\mu \) is chosen to make the covariant derivative orthogonal to the four vector \( q \) carried by the \( S_{11} \) so that in the rest frame, \( z \cdot D \) reduces to the derivative in the space direction. Of course, there are three possible choices for putting in the derivative inside the nucleon current. In this work, we take the interpolating field with the covariant derivative acting on the d-quark instead of other two u-quarks:

\[
\eta_{N^-}(t) = 2\epsilon_{abc}[ t (u_a^T C(z \cdot D)d_b)\gamma_5 u_c + (u_a^T C\gamma_5(z \cdot D)d_b)u_c] .
\]

This choice is preferred because the resulting correlator gets nonzero contribution from the lowest-order chiral breaking term, \( \langle \bar{q}q \rangle \), which is important in the nucleon sum rule.
With this current, we now move on to calculate the time-ordered correlation function defined as

$$
\Pi(q) = \int dx^4 e^{iq \cdot x} \langle 0 | T[\eta_N^- (x) \bar{\eta}_N^- (0)] | 0 \rangle ,
$$

where $| 0 \rangle$ denotes the QCD vacuum. Implementing the properties of $z_\mu$, it is easy to show that the correlation function should have the following form,

$$
\Pi(q) = \Pi_1(q^2, z^2) + \Pi_q(q^2, z^2) \eta .
$$

Calculation of these two scalar functions is carried out readily in the operator product expansion. After some manipulations, we obtain up to dimension 8 operators,

$$
\Pi_q^{\text{ope}}(q) = - \frac{q^6 \ln(-q^2)}{2^{10} \times 3^2 \times 5 \times \pi^4} (21t^2 + 10t + 21) - \frac{q^2 \ln(-q^2)}{2^{11} \times 9 \times \pi^2} (45t^2 + 10t + 45) \left( \frac{\alpha_s}{\pi} G^2 \right) \\
- \frac{1}{12q^2} \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot G q \rangle (t^2 + t) ,
$$

$$
\Pi_1^{\text{ope}}(q) = \frac{q^4 \ln(-q^2)}{2^5 \times \pi^2} (t^2 - 1) \langle \bar{q}q \rangle - \frac{q^2 \ln(-q^2)}{2^7 \times \pi^2} (3t^2 + 2t - 5) \langle g_s \bar{q} \sigma \cdot G q \rangle \\
+ \frac{\ln(-q^2)}{3 \times 2^6} (t^2 - 1) \left( \frac{\alpha_s}{\pi} G^2 \right) \langle \bar{q}q \rangle .
$$

Note that, in calculating these, we have made use of the properties, $z \cdot q = 0$ and $z^2 = -1$. Few remarks are in order. The Wilson coefficients of the dimension six condensate ($\langle \bar{q}q \rangle^2$) in $\Pi_q^{\text{ope}}(q)$ can be shown to be zero. We added the contribution from the dimension eight operator ($\langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot G q \rangle$), as they contribute in tree graph and hence expected to be the important power correction for $\Pi_q^{\text{ope}}$. For $\Pi_1(q^2, z^2)$, the leading contribution comes from dimension three condensate, $\langle \bar{q}q \rangle$, which constitutes the important chiral symmetry breaking operator in predicting the nucleon mass in the nucleon sum rule. In addition, another chiral breaking operator, $\langle g_s \bar{q} \sigma \cdot G q \rangle$, contributes to the $S_{11}$ sum rule. This operator does not enter to the usual nucleon sum rule where Ioffe current is employed. However, as will be discussed below, our prediction for $S_{11}$ mass is sensitive to the value of this dimension-five condensate. Note also that the factorization hypothesis has been employed in the calculation of higher dimensional operators. We did not include the three-gluon condensate $\langle G^3 \rangle$ which is an order of $\alpha_s^{3/2}$.

After Borel transformation, Eqs. (5) and (6) become,

$$
\hat{\Pi}_q^{\text{ope}}(M) = \frac{M^8}{2^9 \times 3 \times 5 \times \pi^4} (21t^2 + 10t + 21) + \frac{M^4}{2^{11} \times 9 \times \pi^2} (45t^2 + 10t + 45) \left( \frac{\alpha_s}{\pi} G^2 \right) \\
+ \frac{1}{12} (t^2 + t) \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot G q \rangle ,
$$

$$
\hat{\Pi}_1^{\text{ope}}(M) = \frac{M^6}{2^4 \pi^2} \langle \bar{q}q \rangle (t^2 - 1) + \frac{M^4}{2^7 \pi^2} \langle g_s \bar{q} \sigma \cdot G q \rangle (3t^2 + 2t - 5) \\
- \frac{M^2}{192} \left( \frac{\alpha_s}{\pi} G^2 \right) \langle \bar{q}q \rangle (t^2 - 1) ,
$$

where $M$ denotes the Borel mass.
For the phenomenological side of the sum rules, first we consider the matrix element of the interpolating field [Eq. (4)] between the QCD vacuum and a baryon state with momentum \( q \) and spin \( s \). Due to the specific form of the field, we should have

\[
\langle 0|\eta(0)|q, s \rangle = \not\!{q}\lambda_B\ u(q, s),
\]

where \( u(q, s) \) is a Dirac spinor and \( \lambda_B \) is the strength with which the baryon couples to the interpolating field. Normally, the interpolating field couples not only to the positive-parity baryon but also to the negative-parity baryon. This implies that, when combined with Eq. (11), the phenomenological form of the correlator should be

\[
\Pi_{\text{phen}}(q) = -\left[ \lambda_N^2 \frac{q^2 - M_N^2}{q^2 - M_N^2 + i\epsilon} + \lambda_{N^-}^2 \frac{q^2 - M_{N^-}^2}{q^2 - M_{N^-}^2 + i\epsilon} \right] + \text{continuum}.
\]

Here we retain the lowest resonance of positive parity which we identify as the nucleon(\( N \)), and the lowest resonance of negative parity which we identify as \( S_{11} \) (\( N^- \)). We have put all other resonances with higher masses into two separate continua, one for the positive-parity resonances which we denote by \( s_+ \), the other for the negative-parity resonances which we denote by \( s_- \). \( \lambda_N \) and \( \lambda_{N^-} \) represent the strengths with which our interpolating field couples to nucleon and \( S_{11} \), respectively while the corresponding masses are denoted by \( M_N \) and \( M_{N^-} \).

In general, the couplings, \( \lambda_N \) and \( \lambda_{N^-} \) are functions of \( t \). To obtain the sum rule for \( S_{11} \), the parameter \( t \) need to be chosen such a way that the resulting interpolating field does not couple to the positive-parity baryons while strongly couples to \( S_{11} \). This can be achieved by applying the finite energy sum rule as described in Ref. \[8\]. We will come back to this procedure later. However, once \( t \) is chosen this way, there is no contribution from the nucleon, and therefore one can derive the sum rule for \( S_{11} \) in the conventional way. Following the usual steps to include the continuum [see for example Ref. \[10\]] and taking the Borel transformation of the resulting expressions, we get the phenomenological side of the sum rule assuming two different continuum thresholds,

\[
\hat{\Pi}_{\text{phen}}^q(M) = \lambda_N^2 e^{-M_N^2/M^2} + \frac{(21t^2 + 10t + 21)}{2^9 \times 3 \times 5 \times \pi^4} \frac{1}{2} (M_{s_+}^8 + M_{s_+}^6)
\]

\[
+ \frac{\langle \alpha_s G^2 \rangle}{211 \times 9 \times \pi^2} (45t^2 + 10t + 45) \frac{1}{2} (M_{s_-}^6 + M_{s_-}^4),
\]

\[
\hat{\Pi}_{\text{phen}}^1(M) = \lambda_{N^-}^2 e^{-M_{N^-}^2/M^2} - \frac{\langle \bar{q}q \rangle}{2^4 \pi^2} (t^2 - 1) \frac{1}{2} (M_{s_-}^6 + M_{s_-}^6)
\]

\[
- \frac{\langle g_s \bar{q} \sigma \cdot G q \rangle}{2^7 \pi^2} (3t^2 + 2t - 5) \frac{1}{2} (M_{s_+}^4 + M_{s_+}^4)
\]

\[
+ \frac{(t^2 - 1)}{192} \frac{\langle \alpha_s G^2 \rangle}{\pi} \langle \bar{q}q \rangle \frac{1}{2} (M_{s_-}^2 + M_{s_-}^2),
\]

where we have defined,

\[
M_{s_+}^2 = M^2 e^{-s_+/M^2},
\]

\[
M_{s_-}^4 = M^4 (1 + \frac{s_-}{M^2}) e^{-s_-/M^2},
\]
\begin{equation}
M_{s_{\pm}}^6 = M_6^6(1 + \frac{s_{\pm}}{M^2} + \frac{s_{\pm}^2}{2M^4})e^{-s_{\pm}/M^2},
\end{equation}
\begin{equation}
M_{s_{\pm}}^8 = M_8^8(1 + \frac{s_{\pm}}{M^2} + \frac{s_{\pm}^2}{2M^4} + \frac{s_{\pm}^3}{6M^6})e^{-s_{\pm}/M^2}.
\end{equation}

By equating these phenomenological side with the OPE side of the sum rule [Eqs. (9), (10)] and taking the ratio, we obtain the sum rule for \(M_{N^-}\),
\begin{equation}
M_{N^-} = \left[ - \frac{\Delta(M_{s_{\pm}}^6) + \Delta(M_{s_{\pm}}^6)}{2}(t^2 - 1)\langle \bar{q}q \rangle + \frac{\Delta(M_{s_{\pm}}^4) + \Delta(M_{s_{\pm}}^4)}{16}(3t^2 + 2t - 5)\langle g_s \bar{q}\sigma \cdot Gq \rangle \\
- \frac{\Delta(M_{s_{\pm}}^2) + \Delta(M_{s_{\pm}}^2)}{24}\pi^2(t^2 - 1)\frac{\alpha_s}{\pi} G^2 \langle \bar{q}q \rangle \\
\times \left[ \frac{\Delta(M_{s_{\pm}}^8) + \Delta(M_{s_{\pm}}^8)}{960\pi^2}(21t^2 + 10t + 21) \\
+ \frac{\Delta(M_{s_{\pm}}^4) + \Delta(M_{s_{\pm}}^4)}{256 \times 9}\frac{\alpha_s}{\pi} G^2 (45t^2 + 10t + 45) \\
+ \frac{4\pi^2}{3}(t^2 + t)\langle \bar{q}q \rangle \langle g_s \bar{q}\sigma \cdot Gq \rangle \right]^{-1},
\end{equation}

where we have defined
\begin{align}
\Delta(M_{s_{\pm}}^2) &= M^2 - M_{s_{\pm}}^2, \\
\Delta(M_{s_{\pm}}^4) &= M^4 - M_{s_{\pm}}^4, \\
\Delta(M_{s_{\pm}}^6) &= M^6 - M_{s_{\pm}}^6, \\
\Delta(M_{s_{\pm}}^8) &= M^8 - M_{s_{\pm}}^8.
\end{align}

Now, the QCD parameters appearing in Eq. (18) should be properly chosen to predict \(M_{N^-}\). The quark condensate can be deduced from Gell-Mann–Oakes–Renner relation combined with the current quark masses estimated from chiral perturbation theory \(\langle \bar{q}q \rangle = -(230 \pm 20 \text{ MeV})^3\).

The gluon condensate can be estimated either from leptonic decays of vector mesons or from the charmonium spectrum \(\langle \frac{\alpha_s}{\pi} G^2 \rangle = (350 \pm 20 \text{ MeV})^4\).

The most important parameter for our prediction is the quark-gluon condensate which is usually expressed in terms of the quark condensate \(\langle \bar{q}q \rangle\):
\begin{equation}
\langle g_s \bar{q}\sigma \cdot Gq \rangle \equiv 2\lambda_q^2 \langle \bar{q}q \rangle.
\end{equation}

Here, the parameter \(\lambda_q^2\), which represents the average virtual momentum of vacuum quarks, is not precisely known. It has been estimated from a number of studies; \(\lambda_q^2 = 0.4 \pm 0.1 \text{ GeV}^2\) from the standard QCD sum rule estimate \(\lambda_q^2 = 0.7 \text{ GeV}^2\) from QCD sum-rule analysis of the pion form factor \(\lambda_q^2 = 0.55 \text{ GeV}^2\) from lattice calculations, and somewhat
larger value from the instanton liquid model. In our discussion, we will investigate the sensitivity of our prediction by changing $\lambda^2$ between 0.4 – 1 GeV.$^2$

Given the QCD parameters, we need to choose an optimal $t$ such that the interpolating field does not couple to the positive-parity nucleon at all while maximally couples to the negative-parity nucleon. To do this, we use the sum rule derived by JKO in Ref. [8], where they have used the “old-fashioned” correlation function defined by

$$\Pi(q) = i \int d^4x e^{iqx} \theta(x_0) \langle 0|\eta_{N-}(x)\eta_{N-}(0)|0 \rangle .$$

This function is analytic in the upper-half region of the complex $q_0$ plane. Therefore, using the nucleon current and assuming,

$$\text{Im} \Pi(q_0, q = 0) = \gamma_0 A(q_0) + B(q_0),$$

one can derive the following sum rules for a large energy $Q$,

$$\int_0^Q dq_0 [A^{\text{ope}}(q_0) - A^{\text{phen}}(q_0)] W(q_0) = 0$$

$$\int_0^Q dq_0 [B^{\text{ope}}(q_0) - B^{\text{phen}}(q_0)] W(q_0) = 0 ,$$

where $W(q_0)$ is the weighting function. The advantage of using the “old-fashioned” sum rule is that it allows us to determine the optimal $t$ independent of baryon masses which we want to calculate ultimately.

As stated above [Eq.(12)], we assume that our phenomenological side has explicit contributions from the nucleon ($N$) and the $S_{11}(N^-)$. That is, we have

$$\frac{1}{\pi} \text{Im} \left[ \Pi^{\text{phen}}(q_0) \right] = \lambda_N^2 \frac{\gamma_0 - 1}{2} \delta(q_0 - M_N) + \lambda_N^2 \frac{\gamma_0 + 1}{2} \delta(q_0 - M_{N^-}) + \text{continuum} .$$

Then we derive the following sum rule from Eq.(28) with $W(q_0) = 1$

$$A(s_+) - B(s_+) = \lambda_N^2 ,$$

$$A(s_-) + B(s_-) = \lambda_{N^-}^2 ,$$

where in our case,

$$A(s_+) = \int_0^{\sqrt{s_+}} dq_0 \left[ \frac{q_0^7}{210 \times 3^2 \times 5 \times \pi^3} (21t^2 + 10t + 21) + \frac{q_0^3}{211 \times 9 \times \pi} (45t^2 + 10t + 45) \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{1}{12} \pi \delta(q_0) \langle \bar{q}q \rangle \langle g_s \bar{q} \sigma \cdot G q \rangle (t^2 + t) \right] ,$$

$$B(s_+) = \int_0^{\sqrt{s_+}} dq_0 \left[ - \frac{q_0^4}{25 \pi} (t^2 - 1) \langle \bar{q}q \rangle + \frac{q_0^2}{2^2 \pi} (3t^2 + 2t - 5) \langle g_s \bar{q} \sigma \cdot G q \rangle - \frac{\pi(t^2 - 1)}{192} \left( \frac{\alpha_s}{\pi} G^2 \right) \langle \bar{q}q \rangle \right] .$$

Here we fix the positive-parity continuum threshold to the second lowest resonance of its kind, $s_+ = (1.44 \text{ GeV})^2$, and for the negative-parity threshold, we take $s_- = (1.65 \text{ GeV})^2$ as it is the next higher resonance with the negative-parity.
Eq. (30) is a kind of duality relation between the quark and nucleon state. It equates the integrated positive-parity spectral strength within the duality interval \( (0 \sim \sqrt{s_+}) \) of the OPE part to that of the hadronic counterpart, which is the coupling strength of the nucleon. Note that the right hand sides of Eqs. (30), (31) are positive definite. Now, we look for the optimal \( t \) which makes the left hand side of Eq. (30) zero while maximizing \( \lambda^2_{N^-} \) from Eq. (31). Our expectation is that such \( t \) yields the interpolating field which couples strongly to the negative-parity baryon, specifically \( S_{11} \). Since the quark-gluon condensate is not well known, we determine \( t \) for the following quark-gluon parameter, \( \lambda^2_q = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 \text{ GeV}^2 \).

These values are arbitrary chosen from the acceptable range of \( \lambda^2_q \) between 0.4 – 1 GeV². [See Eq. (25) for the definition of \( \lambda^2_q \).] With the following parameter set,

\[
\langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3 \\
\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.35 \text{ GeV})^4 ,
\]

we obtain the optimal \( t \) for each \( \lambda^2_q \) which are listed in Table I. Note that our result is not sensitive to the quark or gluon condensate within the error bars as indicated in Eqs. (23) and (24). In general, there are two values of \( t \) which make the left-hand side of Eq. (30) zero. We choose \( t \) which yields larger value of \( \lambda^2_{N^-} \) in Eq. (31).

With this optimal \( t \) value, we are now in a position to calculate \( M_{N^-} \). Substituting all the parameters into Eq. (18) we obtain the Borel curve for \( M_{N^-} \). Some of them are shown in Fig. 1. The solid line is for \( \lambda^2_q = 0.4 \text{ GeV}^2 \), the dashed line is for \( \lambda^2_q = 0.6 \text{ GeV}^2 \), and the dot-dashed line is for \( \lambda^2_q = 1 \text{ GeV}^2 \). Note that all three curves have the stable region from which the spectral mass can be obtained. At \( \lambda^2_q = 0.6 \text{ GeV}^2 \), our prediction for \( M_{N^-} \) obtained from the stable Borel plateau is 1.59 GeV. Similarly, at \( \lambda^2_q = 1 \text{ GeV}^2 \), our prediction is \( M_{N^-} = 1.42 \text{ GeV} \). All other predictions for \( M_{N^-} \) with different \( \lambda^2_q \) values lie within these two limits as shown in Table I. The best prediction for \( S_{11} \) mass is obtained for \( \lambda^2_q = 0.435 \text{ GeV}^2 \) or \( \lambda^2_q = 0.69 \text{ GeV}^2 \). However, for \( \lambda^2_q = 1 \text{ GeV}^2 \), our prediction is off by 7% from the experimentally known \( S_{11} \) mass, 1.535 GeV. To make more reliable prediction, it is important to narrow down the range of the quark-gluon condensate value from other studies. However, it should be emphasized that our prediction is obscured only by 7% from the barely known quark-gluon condensate. That is, for a given value of \( \lambda^2_q \), the optimal procedure that we have employed chooses the interpolating field which couples very strongly to \( S_{11} \).

As we have succeeded in predicting the \( S_{11} \) at least in qualitative level, it would be interesting to understand the origin of the large mass splitting between \( S_{11} \) and nucleon. It should be noted that the dominant portion of our prediction comes from the dimension-five quark-gluon operator. The contribution from this dimension-five operator is enhanced by the covariant derivative acted on the d-quark of the nucleon interpolating field because the covariant derivative modifies the quark configuration to increase the average virtual momentum of the current quark. However, this is not the case for the nucleon sum rule where the quark configurations are mostly in the s-wave state. Indeed, with Ioffe's choice for the nucleon interpolating field, the quark-gluon condensate does not enter in the prediction of nucleon mass. Therefore, within our picture, the large mass splitting between nucleon
and $S_{11}$ is mainly driven by the enhancement of the quark-gluon condensate which increases the contributions from the chiral-symmetry breaking terms.

Another interesting feature of our result is that, for our current, we can always find suitable linear combination of currents such that it has a strong coupling with the negative-parity baryon and zero coupling to the positive-parity baryon. This justifies the use of Eq.(18) or any other sum rule constructed with this current under the assumption that the lowest mass resonance that couple to this current is the $S_{11}$. This is not always possible for any other interpolating field for the nucleon, especially when the current contains no derivative. For example, using Eq.(3) as an interpolating field, it is not possible to find any value of $t$ such that the current has a strong coupling to the negative-parity baryon and zero coupling to the positive-parity baryon. Therefore, our interpolating field can be used to construct any conventional sum rule to study other properties of the $S_{11}$ relevant in reactions involving electromagnetic or hadronic probes [9].

In summary, we have introduced a new interpolating field for $S_{11}$ motivated by the bag model or nonrelativistic quark model. Using the interpolating field, we have calculated the correlation function in the operator product expansion up to dimension eight. To predict $S_{11}$ mass, we have optimized the linear combination at a given QCD parameter $\lambda^2_q$ while fixing other well known parameters. A good prediction for $S_{11}$ mass is obtained under this formalism and its value contains at most 7% error due to the poorly known QCD parameter $\lambda^2_q$. The best value for $S_{11}$ mass is obtained for $\lambda^2_q = 0.435$ or 0.69 GeV$^2$.

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TABLE I. Our prediction for $S_{11}$ at given $\lambda_q^2$ with optimally chosen $t$. The listed $M_{N^-}$ is obtained from the stable Borel plateau.

| $\lambda_q^2$ (GeV$^2$) | Optimal $t$ | $M_{N^-}$ (GeV) |
|----------------------|-------------|-----------------|
| 0.4                  | 4.97        | 1.49            |
| 0.5                  | -47.52      | 1.59            |
| 0.6                  | -4.59       | 1.59            |
| 0.7                  | -2.91       | 1.53            |
| 0.8                  | -2.40       | 1.48            |
| 0.9                  | -2.21       | 1.44            |
| 1.0                  | -2.04       | 1.42            |
FIGURES

FIG. 1. Our prediction for $S_{11}$ mass versus Borel mass. The solid curve is for $\lambda_q^2 = 0.4 \text{ GeV}^2$, $t = 4.97$ and the dashed curve is for $\lambda_q^2 = 0.6 \text{ GeV}^2, t = -4.59$. The dot-dashed curve is for $\lambda_q^2 = 1 \text{ GeV}^2, t = -2.04$. 