The Effect of Partial Coherence on the Statistics of Single-Photon Pulses Propagating in the Atmosphere

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Abstract

The photon density operator function is used to describe the propagation of single-photon pulses through a turbulent atmosphere. The effects of statistical properties of photon source and the effects of a random phase screen on the variance of photon counting are studied. A procedure for reducing the total noise is discussed. The physical mechanisms responsible for this reduction are explained.

1 Introduction

Fluctuations of the atmospheric refractive index caused by turbulent eddies considerably limit the performance of long-distance optical communication

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systems. An initially coherent laser beam acquires some properties of Gaussian statistics in the case of a long propagation path or strong turbulence. The tendency of the scintillation index (defined as a normalized intensity variance) to approach asymptotically the level of unity in this case is a distinct manifestation of gaussianity of the field statistics. In the literature this effect, known first from experimental studies of Gracheva and Gurvich [1], is referred to as the saturation of fluctuations. (See, for example, [2]-[7].) In that way, the strong influence of the refractive index fluctuations on the radiation field statistics is emphasized.

It has been shown quite generally [8]-[10] that the normalized intensity variance approaches unity for any source distribution as the turbulence strength tends to infinity. This result is valid for any degree of coherence of the source, provided the response time of the recording instrument is short compared with the source coherence time (fast detector). In the opposite case of a slow detector, the scintillation index can decrease for partially coherent beams. The effect is not very strong for temporal partial coherence [1], [11]-[13], but is very pronounced in the case of spatial partial coherence. (See papers [14]-[16], which deal with stationary beams.)

The purpose of the present paper is to study the effects of partial spatial coherence on the statistics of the detected photons when photons are generated as individual pulses of electromagnetic field propagating in the earths atmosphere. Moreover, the effects of the initial statistics of photons on fluctuations of the detector counts will be elucidated.

The case of single-photon pulses is of special interest for quantum cryptography because individual photons are carriers of information bits in several basic strategies for free-space quantum key distribution. (Practical free-space quantum key distribution is described in references [17]-[19].) Fluctuations of the detector counts for the case of single-photon pulses were studied both theoretically and experimentally in [20]. In contrast to [20] where two limiting cases (plane-wave and spherical-wave approximations) were studied, our consideration takes into account the actual beam diameter. Also we will not restrict ourselves with the case of small fluctuations of the radiation field as in [20]. The case of strong fluctuations will be studied as well. The method of photon distribution function in phase space developed in [16] is somewhat generalized here to apply to the case of laser pulses.
2  The photon distribution function, and pulse propagation in the atmosphere

Similar to [16], we proceed from the quantum version of Hamiltonian, $H$, of photons in a medium with a fluctuating refractive index, $n(r)$ \[ n(r) \sim n(r) - 1 \ll 1 \]

\begin{equation}
H = \sum_k \hbar \omega_k b_k^+ b_k - \sum_{k,k'} \hbar \omega_k n_{k'} b_k^+ b_{k+k'}.
\end{equation}

Here the terms describing the zero-point electromagnetic energy are omitted; the two terms on the right-hand side describe photons in a vacuum and the effect of refractive index fluctuations, respectively; $b_k^+$ and $b_k$ are creation and annihilation operators of photons with momentum $k$, $\hbar \omega_k \equiv \hbar ck$ is the photon energy; $c$ is the speed of light in a vacuum, and $n_k$ is the Fourier transform of $n(r)$ defined by

\begin{equation}
n_k = \frac{1}{V} \int dV e^{ikr} n(r),
\end{equation}

where $V \equiv L_x L_y L_z$ is the normalizing volume.

Eq. (1) is valid in the limit of small wave-vectors $k'$ ($k' \ll k$). This means that the scale of spatial inhomogeneity of turbulence is much greater than the wavelength of the radiation. Also we assume here that the initial polarization of light remains unaffected by the turbulence throughout the distance of propagation. The depolarization of light due to atmosphere turbulence is very small. (See, for example, [21] and [22] where this effect was studied.)

The photon distribution function is defined by

\begin{equation}
f(r, q, t) = \frac{1}{V} \sum_k e^{-ikr} b_{q+k/2}^+ b_{q-k/2}.
\end{equation}

The operator function $f(r, q, t)$ is the photon density in six-dimensional $(r, q)$ phase space. We will use it to describe light pulses with characteristic sizes, $l$, much greater than the wavelength of the radiation, $\lambda$. In this case, it is convenient to restrict the sum over $k$ by some $k_0$ ($k < k_0 \ll q_0$, $k_0 > 2\pi/l$, where $q_0$ is the wave vector corresponding to the central frequency $\omega_0$ of radiation, $\omega_0 = cq_0$). Then, the kinetic equation for the distribution function is governed by

\begin{equation}
\{ \partial_t + c_q \partial_r + F(r) \partial_q \} f(r, q, t) = 0,
\end{equation}

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where $F(r) = \omega_0 \partial_r n(r)$ and $c_q = \partial \omega_q / \partial q$. When deriving Eq. (4) we have considered the refractive index to be a slowly varying function of the coordinate, $r$. (See more details in [16].) In this case the effect of the turbulence on $f$ is represented by the random force $F(r)$, that can be seen in Eq. (4).

The general solution of Eq. (4) is

$$f(r, q, t) = \phi \left\{ r - \int_{t_0}^{t} dt' \frac{\partial r(t')}{\partial t'}; q - \int_{t_0}^{t} dt' \frac{\partial q(t')}{\partial t'}; t_0 \right\},$$

(5)

where the functions $r(t')$ and $q(t')$ are photon “trajectories” defined by the equations of motion

$$\frac{\partial r(t)}{\partial t} = c(q(t)), \quad \frac{\partial q(t)}{\partial t} = F[r(t)],$$

(6)

and the corresponding initial conditions. The last equations follow from the requirement that the trajectories in Eq. (5) $r(t')$ and $q(t')$ pass through the point $r, q$ at $t' = t$ [i.e. $r(t') = r, q(t') = q$]. The initial value of $f(r, q, t)$ is $\phi(r, q, t_0)$

$$\phi(r, q, t_0) = f(r, q, t_0) = \frac{1}{V} \sum_k e^{-ikr(b^+_q k/2 b^-_q k/2)} \big|_{t=0} \equiv \sum_k e^{-ikr} \phi(k, q, t_0).$$

(7)

It is convenient to set $t_0$ equal to the instant just after photon exits the source (as shown in Fig. 1). These photons have not been affected by the atmospheric turbulence and their statistics is determined by the source properties only. The operators, $b^+_q$, and $b_q$, describe amplitudes of the field, $E_{\text{atm}}$, outgoing from the source. The correspondence between free-space and generated modes can be established using the following reasoning. The field in the atmosphere is given by

$$E_{\text{atm}}(r) = i \sum_q \left( \frac{2\pi \hbar \omega_q}{V} \right)^{1/2} \left[ e^{iqr} b_q - e^{-iqr} b^+_q \right].$$

(8)

On the other hand, the outgoing field localized in the vicinity of the laser aperture can be expressed in terms of the laser mode creation and annihilation operators, $b^+$ and $b$, and the normalized functions, $\Phi^*(r)$ and $\Phi(r)$, describing the spatial distribution of the field, as in Ref. [23]:

$$E^s(r) = i(2\pi \hbar \omega_0)^{1/2} [\exp(iq_0 z)\Phi(r)b - \exp(-iq_0 z)\Phi^*(r)b^+].$$

(9)
Figure 1: The scheme of the communication channel.

The function, $\Phi(r)$, can be chosen as

$$\Phi(r) = \left(\frac{2}{\pi}\right)^{3/4} (r_0^2 r_z)^{-1/2} \exp \left(-\frac{r_0^2}{r^2} - \frac{(z - z_0)^2}{r_z^2}\right),$$

(10)

where $r_\perp = x^2 + y^2$; $z_0$ is the position of the pulse center at $t = t_0$; and $r_0$ and $r_z$ describe the aperture radius and the length of light pulse, respectively. Then, from the condition $E_{atm}(r, t_0) = E_s(r, t_0)$ we obtain

$$b_q(t_0) = \frac{b(t_0)}{\sqrt{V}} \int dr e^{-i(q - q_0)\cdot r} \Phi(r).$$

(11)

It should be noted that the condition $E_{atm} = E^s$ is not rigorous. We ignore here vacuum fields arising from other modes of the resonator as well as the zero-point free-space waves reflected from the output window. These fields, providing correct commutation relations for the operators $b^+_q$, $b_q$, do not contribute to the detected number of photons. There are no photons in these fields. (Direct detection is assumed.) Thus, the corresponding “vacuum” terms are omitted in Eq. (11) as being irrelevant to our problem.

The effects of the phase screen can be included by introducing the multiplier, $e^{i\varphi(r_\perp)}$, into the integrand of Eq. (11), where $\varphi(r_\perp) = ar_\perp$ and $a$ is a Gaussian random variable with a covariance $\langle (a_{x,y})^2 \rangle = 2\lambda_\perp^{-2}$. This results in the following relation for the average of the exponent $\langle e^{iar_\perp} \rangle = e^{-r_\perp^2\lambda_\perp^2}$. This relation will be useful in our further analysis. Also, the fluctuations of the refractive index are usually considered as Gaussian variables with a spatial Fourier-component of the correlation function $\langle n(r)n(r') \rangle_g \equiv \psi(g)$ given by

$$\psi(g) = 0.033C_n^2 \frac{\exp\left[-(gl_0/2\pi)^2\right]}{[g^2 + l_0^{-2}]^{11/6}},$$

(12)
Eq. (12) is refereed to as the von Karman spectrum. $L_0$ and $l_0$ are the outer and inner scales sizes of the turbulent eddies, respectively. In atmospheric turbulence, $L_0$ can range from 1 to 100 meters, and $l_0$ is of the order of several millimeters. $C_n^2$ is the index-of-refraction structure constant. In most physically important cases the quantity $L_0^{-2}$ in the denominator of Eq. (12) can be omitted. In this case, the von Karman spectrum is reduced to the Tatarskii spectrum [2].

Eqs. (5) and (7, 10-12) are sufficient to determine the beam intensity $\langle I(r, t) \rangle = c \sum_q \bar{\hbar} \omega_q \langle f(r, q, t) \rangle$ (13) at any $r$ and $t$. Represent Eq (13) in the form

$$f(r, q, t) = \sum_k \exp \left\{ -i k \{ r - c \mathbf{q} (t - t_0) + c \int_{t_0}^{t} dt' (t' - t_0) \mathbf{F}_\perp[r(t')] \} \right\} \times \varphi_k \left\{ \mathbf{q} - \int_{t_0}^{t} dt' \mathbf{F}_\perp[r(t')]; t_0 \right\}. \quad (14)$$

This form is more convenient for obtaining the explicit form of Eq. (13). We have neglected the changes of the longitudinal photon momentum $(q_z)$ caused by the turbulence, because of their negligible contribution to the initial momentum $(\approx q_0)$ for almost any reasonable propagation path.

The photon “trajectories” $r(t')$ in the arguments of $\mathbf{F}_\perp$ can be approximated by straight lines, $r(t') = r + c \mathbf{q}(t' - t)$. Thus, we disregard the variation of photon momentum at distances of the order of the turbulence correlation length, $L_0$, which is assumed to be much less than the total propagation path, $c(t - t_0)$. Then, the explicit expression for $\langle I \rangle$ is given by

$$\langle I(r, t) \rangle = \sqrt{\frac{2}{\pi}} \frac{c \hbar \omega_0}{\pi R^2 r_z} \exp \left\{ - \frac{r_1^2}{R^2} - 2 \frac{z_{\text{eff}}}{r_z^2} \right\}, \quad (15)$$

where

$$z_{\text{eff}} = z - z_0 - c(t - t_0), \quad R^2 = \frac{r_0^2}{2} \left\{ 1 + \left[ \frac{2c(t - t_0)}{q_0 r_0 r_1} \right]^2 + \frac{8c^3(t - t_0)^3 T}{r_0^2} \right\},$$

$$r_1^2 = \frac{r_0^2}{1 + 2r_0^2 \lambda c^{-2}}.$$
and
\[ T = 0.558C_n^2 l_0^{-1/3}. \]

\( z_{\text{eff}} \) and \( R \) are the distance to the pulse center at time \( t \) and the beam radius, respectively.

The effects of partial coherence on the beam radius is represented by the quantity, \( r_1 \), which enters the second term in expression for \( R^2 \). This term describes the diffraction broadening of the beam in the course of its propagation. The third term is due to turbulence. It dominates at large distances. It does not depend on the initial partial coherence. Therefore, the beam radius is not sensitive to the presence of phase screen when \( c(t - t_0) \to \infty \). In this connection an important question for our analysis arises: Is there any effect of the partial coherence on the photon-counts statistics at large propagation paths? This issue is considered in the next section.

Eq. (15) is derived for a single-photon pulse, i.e. for \( \langle b^+b \rangle = 1 \). For arbitrary photon number per pulse, \( N_{\text{pulse}} \), the coefficient in front of the exponent in Eq. (15) should be multiplied by \( N_{\text{pulse}} \). The case of a homogeneous beam (see, for example, [16]) can be considered by assuming \( r_z \to \infty \) and by renormalizing the coefficient in front of the exponent. One can easily see that the beam radius is increased with the distance (or propagation time \( t - t_0 \)) similar to the case of the stationary intensity. In contrast, the pulse length remains unaffected by the turbulence, within the formalism described above.

### 3 Fluctuations of photon counting

As before, we assume that the background radiation noise is negligible. Also, the detector area is taken to be small compared with the beam width. This is quite reasonable for systems with long transmission distances. The counting interval, \( T_p \), is much greater than the pulse duration \( \tau_p \) (\( \tau_p \sim r_z/c, T_p \gg \tau_p \)), as shown in Fig. 2. (In experiments [20], the radiation was in the form of 1\,ns laser pulses with a repetition rate 1 MHz and the interval \( T_p = 10\,ms \).)

We start with the definition of the mean-square of the photon counts, \( n \), for the time interval, \( T_p \). Usually, this is represented as a sum of two terms
\[ \langle n^2 \rangle = \langle n \rangle + \langle n(n-1) \rangle, \]
where \( \langle n \rangle = \alpha N \), and \( \alpha \) describes the light collection and the quantum detection efficiency. It is the ratio of the numbers of detected \( \langle n \rangle \) and generated
photon pulses, \( N \). The average number \( \langle n \rangle \) is determined by the integral of light intensity \( I(r, t) \) as

\[
\langle n \rangle = \eta \int_{-T/2}^{T/2} dt' \langle I(r, t') \rangle,
\]

where \( \eta \) describes the detector efficiency. By substituting Eq. (15) into Eq. (17), and assuming the detector is at the beam center, \( r_\perp = 0 \), we have

\[
\langle n \rangle = \eta N \frac{\hbar \omega_0}{\pi R^2}.
\]

Thus, we see that \( \alpha \) and \( \eta \) are related by

\[
\alpha = \eta \frac{\hbar \omega_0}{\pi R^2}.
\]

The second term in Eq. (16) is determined by

\[
\langle n(n - 1) \rangle = \left\langle \left\{ \eta \int_{-T_p/2}^{T_p/2} dt' I(r, t') \right\}^2 \right\rangle,
\]

where the symbol \( \langle : ... : \rangle \) indicates normal ordering of the operators \( b_q^\dagger(t') \) and \( b_q(t'') \) with subsequent averaging. In our case these operators enter the definition of the intensity \( I(r, t') \) and, hence, the right part of Eq. (19).

Each integration over \( t' \) within the total interval \( T_p \) is reduced to a sum of \( N \) independent integrations within much smaller intervals \( \Delta t_i \) \( (r_z/c) \ll \Delta t_i \ll T_p/N, i = 1, 2, \ldots N \), which correspond to instants when the pulses cross the detector plane. Then Eq. (19) is reduced to

\[
\langle n(n - 1) \rangle = \eta^2 N(N - 1) \left( \frac{\hbar \omega_0 c}{V} \right)^2 \sum_{q,k} \sum_{q',k'} \int dt' \int dt'' e^{-i(k+k')r} \times
\]

\[
\sum_{r_z} \frac{1}{c} \left( \int dt \right)
\]

Figure 2: A sequence of single-photon pulses.
where integrations are within any two different intervals $\Delta t_i$. The coefficient $N(N-1)$ indicates the number of such intervals. Coinciding intervals do not contribute to Eq. (19) because of the zero value of the intensity-intensity correlation for a single-photon pulse.

Not all terms in the sum over $q$, $k$, $q'$, $k'$ contribute significantly to $\langle n(n-1) \rangle$. The analysis in [16] shows that at large but finite propagation distances the terms with small values of (i) $k$, $k'$ or (ii) $|q' - q + (k + k')/2|$, $|q - q' + (k + k')/2|$ (almost diagonal terms) are the most important. The evolution of such terms can be described in a manner similar to the case of the evolution of $f$. Then, the explicit form of the expression (20) is given by

$$
\langle n(n-1) \rangle = \eta^2 N(N-1)(2\pi)^5 \left(\frac{\bar{\hbar}\omega_0}{V^2}\right)^2 \sum_{q,k,q',k'} \sum_{q,c,q'c'} e^{-\Delta q^2 + \Delta q'^2 + (k^2 + k'^2)/4} r_z^2/2 \times
\left[\delta(k_z)\delta(k'_z)e^{-(Q^2 + Q'^2)r_z^2/2 - (k^2 + k'^2)r_z^2/8} +
\delta(q_z - q'_z)\delta(k_z + k'_z)e^{-(Q+Q')^2 + (k_z - k'_z)^2/4} r_z^2/2 +
\delta(q_z - q'_z)\delta(k_z - k'_z)e^{-(Q-Q')^2 + (k_z + k'_z)^2/4} r_z^2/2} \right]
\times
\exp\left\{i[k_z[r - c(q)t_z + \frac{\bar{\hbar}}{m} \int_0^{t_z} dt_1 F(r_{\mathbf{q}}(t_1)) + k'_z[r - c(q')t_z + \frac{\bar{\hbar}}{m} \int_0^{t_z} dt_1 F(r_{\mathbf{q'}}(t_1))] + k_z[r - c(q)t_z + \frac{\bar{\hbar}}{m} \int_0^{t_z} dt_1 F(r_{\mathbf{q}}(t_1))] + k'_z[r - c(q')t_z + \frac{\bar{\hbar}}{m} \int_0^{t_z} dt_1 F(r_{\mathbf{q'}}(t_1))]\right\},
$$

where $\Delta q_z = q_z - q_0$; $Q = q_0 - \int_0^{t_z} dt_1 F(r_{\mathbf{q}}(t_1))$; $Q' = q_0 - \int_0^{t_z} dt_1 F(r_{\mathbf{q'}}(t_1))$, and the function $r_{\mathbf{q}}(t_1)$ is the particle trajectory which passes through the point $(r, \mathbf{q})$ at the instant $t_1 = t_z = t - t_0$.

Summation over the variables $q_z$, $q'_z$, $k_z$, $k'_z$ can be easily performed. After that, the remaining sum coincides exactly with the corresponding sum in the right part of Eq. (33) of reference [16]. Hence, we have

$$
\langle n(n-1) \rangle = \alpha^2 N(N-1)(1 + \sigma^2),
$$

where $\sigma^2$ is the scintillation index for the stationary beams calculated in [16] using the formalism of photon distribution function that is similar to the presented here. It follows from Eqs. (20) and (22) that the normalized variance of photon counting can be written in the form

$$
\frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2} = 1 - \alpha \frac{\langle n \rangle}{\langle n \rangle} + \sigma^2 \left(1 - \frac{1}{N}\right).
$$

(23)
Eq. (23) was obtained for the case of long propagation distances. Nevertheless, it has the same form in the opposite limit of short distances (or weak turbulence) when perturbation methods like Rytov’s approach [2], [16] are applicable.

The first term in the right part of Eq. (23) is due to the discrete (quantum) nature of the photon field. When $\alpha \to 0$, it is reduced to $1/\langle n \rangle$ which is known in the literature as the shot-noise limit (or the standard quantum limit). The coefficient $(1 - \alpha)$ in the numerator arises from the $\langle n(n - 1) \rangle$ term in Eq. (16) and is evidence of the nonclassicality of the light. The presence of the factor $(1 - \alpha)$ in Eq. (23) is typical for squeezed photon-number radiation. In our case, the total photon number, $N$, is considered to be a constant. For photocount statistics, this situation is equivalent to the case of photon number state (Fock state) of the light. In the hypothetical case of $\alpha = 1$, the quantum term vanishes. This case corresponds to the physical situation in which all generated photons reach the detector aperture and each photon produces a photocount. Of course, this situation is impossible for long propagation distances because the beam radius becomes much greater than the receiver aperture. In addition, the detection quantum efficiency is always less than 100%.

The second term in Eq. (23) can be interpreted as being caused by atmospheric turbulence. The scintillation index determines its relative contribution to the total noise. In the general case, $\sigma^2$ depends on the propagation distance, the radius of the source aperture, turbulence strength, and the correlation length, $\lambda_c$. Fig. 3 shows that the value of $\sigma^2$ can be suppressed considerably by decreasing the initial coherence length, $\lambda_c$. Significant effect is achieved even for a moderate decrease of the dimensionless parameter $(r_1/r_0)^2$. For example, we see in Fig. 3 an almost 50% decrease in $\sigma^2$ when $(r_1/r_0)^2 = 1/2$.

When deriving Eqs. (22, 23), we have assumed that the total number of photons $N$ in the interval $T_p$ is constant (no fluctuations of $N$). In general, $N$ is a fluctuating quantity. For example, much research uses heavily attenuated laser pulses to approximate a single-photon source [17], [18], [20], [25]. (Usually the photon number per pulse is less than 1.) In this case, the random variable $N$ obeys Poisson statistics resulting in the average value $\langle N(N - 1) \rangle$ equal to $\langle N \rangle^2$. Then, Eq. (22) becomes

$$\langle n(n - 1) \rangle = \alpha^2 \langle N \rangle^2 (1 + \sigma^2).$$  (24)
Instead of Eq. (23) we have
\[
\frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2} = \frac{1}{\langle n \rangle} + \sigma^2.
\] (25)

Comparison of Eqs. (23) and (25) shows the increase of total noise in the latter case that is due to contribution of the generation-rate fluctuations.

4 Conclusion

The central point of the present paper is the possibility of reducing the photon-count fluctuations. It follows from the previous Section that a single-photon-on-demand source or squeezed photon-number light provide lower noise levels than a heavily attenuated laser source. Also, an additional decrease in photocount fluctuations can be achieved by means of a random phase screen.

It can be seen that the phase screen can decrease the count noise for long-distance propagation. In the opposite case a similar effect can be achieved by improving the noise characteristics of the source.
The question arises, “What is the physical nature of the second terms in Eqs. (23) and (25) for the case of single-photon pulses?” These terms describe intensity-intensity correlations. At the same time, one can easily see that this correlation is absent within any given pulse. Also, different pulses are independent events. In principle, they can be generated even by different sources. Different photons do not interact one with another. Nevertheless, $\sigma^2 \neq 0$. This paradox can be explained as follows. Different photons move in a fixed distribution of the refractive index. (It was tacitly assumed that the turbulence does not vary during the integration time $T_p$.) They are affected by the same random force, $F$. Therefore, there is a strong correlation of photon’s trajectories. The probabilities of detection for any photons propagating within small time interval $T_p$ depend on the same refractive-index configuration. Hence, the detection events within this interval $T_p$ are correlated. The main purpose of the phase screen is to destroy this correlation. When the characteristic time of phase variation introduced by the phase screen is of the order $\tau_d \ll T_p$ and, in addition, $\lambda_c \ll r_0$, the scintillation index is decreased considerably for long propagation paths. Thus, a rapid spatio-temporal variation of the phase is an effective method for decreasing the photon-counting noise.

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