Time-Varying $\alpha$ and Particle Physics

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Abstract: We argue that models in which an observable variation of the fine structure constant is explained by motion of a cosmic scalar field, are not stable under renormalization, and require massive fine tuning that cannot be explained by any known mechanism.

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1. Introduction

Recent observations of distant quasars have revived the suggestion that the fine structure constant varies over cosmological time scales. The observations of [1] suggest a variation of \( \frac{\delta \alpha}{\alpha} \sim 10^{-4} \) over the time period since emission of the quasar light.

Within both the contexts of effective field theory and M-theory, it is natural to model the change of the fine structure constant by coupling a dynamical scalar field \( \phi \) to the photon kinetic term in the low energy effective action.

\[
\delta e^{-2} = \epsilon \frac{\phi}{M} F_{\mu\nu}^2
\]  

One can imagine two broad classes of scenarios to explain the putative variation of \( \alpha \). Usually, a slow, secular variation of \( \phi \) is postulated. Another possibility is that there was a first order phase transition between the time at which the quasar light was emitted and the present. In this note we will argue that any such time variation of \( \alpha \) raises difficulties. In particular, such a large variation of \( \alpha \) can only be compatible with basic principles of quantum field theory if there is an extraordinary degree of fine tuning of many parameters of the underlying theory.\(^1\)

Let us first state the basic argument, and later show how this comes out of concrete calculations. It is that the vacuum energy, as computed in the standard model, or in more general low energy effective field theory, must depend on \( \alpha \), even taking into account our ignorance about the resolution of the cosmological constant problem. This dependence can be estimated in various ways, and combining these estimates with the criterion that the new contributions do not dominate the energy densities which appear in standard cosmology, leads to fantastically tight bounds.

\(^1\)This conclusion has also been reached by N. Kaloper and L. Susskind [2].
In generic quantum field theory, varying an arbitrary dimensionless coupling such as $\alpha$ will lead to a variation in the vacuum energy $V$ controlled by the cutoff scale $\Lambda$,

$$\delta V = c \, \delta \alpha \, \Lambda^4$$  \hspace{1cm} (1.2)

with $c$ an $O(1)$ constant. For example, in QED this is $< F_{\mu\nu} F^{\mu\nu} >$ and in perturbation theory $c = 1/\pi + \ldots$. Perturbative corrections and the effects of other matter in the standard model will modify this result, but again lead to $c \sim O(1)$.

Let us be extremely conservative and take $\Lambda$ to be the QCD scale $\sim 100$ MeV. Then a variation of $\alpha$ of order $10^{-4}$ over the course of the matter dominated era of the universe, corresponds to a change in scalar field energy of order

$$\delta V = 10^{28}(\text{eV})^4$$  \hspace{1cm} (1.3)

However, the matter dominated era of conventional cosmology has a maximum energy density of order $10^4\text{eV}^4$. Thus, some of what is supposed to be the matter dominated era, and in particular the period when the quasar light was emitted, was instead dominated by a large scalar field potential energy. This changes classical cosmology completely, and is ruled out by observation. This argument leads to the bound

$$\left| \frac{\delta \alpha}{\alpha} \right| < 10^{-28}. \hspace{1cm} (1.4)$$

As we noted, this followed from an extremely conservative choice of $\Lambda$. Most of us believe that physics is well described by field theory at scales below some higher $\Lambda$, possibly as high as the Planck scale $M_P$ (this would lead to $|\delta \alpha/\alpha| < 10^{-104}$). There is no evidence that this general behavior depends on the form of the theory above the cutoff scale. In particular, corrections of this form invariably arise in string theory, in instances where one can do the calculation.

The situation is not quite so extreme in supersymmetric theories, which do generically cancel the vacuum energy. Of course in the real world supersymmetry is broken, at some scale $M_{\text{SUSY}}$. The generic estimate in this situation is

$$\delta V = c \, \delta \alpha \, \Lambda^2 M_{\text{SUSY}}^2.$$  \hspace{1cm} (1.5)

Supersymmetry generically cancels leading order divergences, but does not do better than that. In the most optimistic scenario, a particular supersymmetric model might indeed cancel all divergences, leading to $\delta V \propto M_{\text{SUSY}}^4$. Since experiment constrains $M_{\text{SUSY}} > 100\text{GeV}$, this does not help.

Note that this argument would apply both to a continuous variation of $\alpha$ (we will study this in more detail below), but also to more drastic modifications of the physics
between early and late times, such as a first order phase transition. Indeed, on general
grounds one expects the vacuum energy to decrease with time; in this situation a phase
transition can only increase the variation of the energy.

Let us now address a possible objection to this argument. It is that normally, we
regard terms such as (1.2) as part of the cosmological constant, and imagine that they
are subtracted off by whatever mysterious agency resolves the cosmological constant
problem.

However, if $\alpha$ varies, general physical principles force us to treat it effectively as a
scalar field. Then, we should not do this subtraction for all values of $\alpha$ but only for a
particular value: its asymptotic value, or perhaps its value at the current era. There
are many reasons to believe this:

- Subtracting the cosmological constant in low energy effective field theory means
  fine tuning the coefficient of a single relevant operator. Subtracting the vacuum
  energy for all values of a scalar field means fine tuning the coefficients of an infinite
  number of relevant and irrelevant operators.

- All theories of inflation and quintessence assume that the values of the potential
  away from the minimum of a scalar field are nonzero. Indeed, if we imagined
  the solution of the cosmological constant problem actually subtracted away the
  potential of all scalar fields, we would not even be able to implement the Higgs
  mechanism.

- Specific proposals e.g. [9] for explaining the mystery of the cosmological constant,
  involve fine tuning only the minimum value of the potential.

Thus, this objection does not invalidate our conclusion, that observable variations
of the fine structure constant are highly unnatural.

To complete the discussion, we now study the cosmology of a field $\phi$ as postulated
in (1.1), to see what type of violations of naturalness are required to evade the bound.
Besides the coupling (1.1), we take $\phi$ to be governed by the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 - V(\phi).$$

(1.6)

The potential energy $V(\phi)$ will be modeled by the generic form $\mu^4 f(\phi/M)$. If we neglect
Hubble friction, the natural time scale for motion of this field is $\frac{M}{\mu^2}$. For plausible values
of the microphysical parameters $\mu$ and $M$, this motion will be too rapid to fit the slow
time variation of $\alpha$. We therefore assume that, until very recently, the motion has
been friction dominated. This requires $MH/\mu^2 >> 1$ and $M >> M_P/48\sqrt{\pi}$. Then
\[ \delta \alpha^{-1} = \frac{4 \pi \epsilon \mu^4}{3 M^2} \int \frac{dt f'(\frac{\phi}{M})}{H} \]  

(1.7)

The integral is over a period in the matter dominated era of the universe. During this period, \( \phi/M \) changes only very little, while \( f \) is a smooth function of order one. Thus

\[ \delta \alpha^{-1} \sim \frac{\epsilon \mu^4}{M^2 \rho_{\text{NOW}}} \]  

(1.8)

The energy density \( \mu^4 \) can at most be of order \( \rho_{\text{NOW}} \), since otherwise we would have seen inflation rather than matter domination and the calculation would not be self consistent. If it is of order \( \rho_{\text{NOW}} \) then \( \phi \) is a form of quintessence [3]. Thus we have:

\[ \delta \alpha^{-1} \leq \frac{\epsilon M_P^2}{M^2} \]  

(1.9)

At this point, one might conclude that \( \mathcal{O}(1) \) values for \( \epsilon \) and \( M/M_P \) could lead to the desired result. However, we still need to consider the effect of \( \epsilon \) on the vacuum energy. This can be estimated along the lines discussed earlier. We assume a supersymmetric theory, to obtain

\[ \delta V = (\Lambda M_{\text{SUSY}})^2 g(\epsilon \phi/M) \]  

(1.10)

in terms of a smooth \( \mathcal{O}(1) \) function \( g \). We furthermore assume \( \Lambda \geq M_{\text{SUSY}} \) and then set \( \Lambda = M_{\text{SUSY}} \) to obtain a conservative bound.

In order that this interaction does not change our estimate of the size of the potential, we must insist that

\[ \epsilon \leq \frac{\sqrt{\rho_{\text{NOW}}}}{M_{\text{SUSY}}^2} \]  

(1.11)

and this gives a stringent bound on the time variation of the fine structure constant:

\[ \frac{\delta \alpha}{\alpha} \leq \alpha \frac{\rho_{\text{NOW}}^{1/2} M_P^2}{M^2 M_{\text{SUSY}}^2} \sim 10^{-28} \left( \frac{M_P}{M} \right)^2. \]  

(1.12)

Thus taking \( \delta \alpha \sim 10^{-4} \alpha \) is inconsistent with our initial assumptions. That is, a field with \( M < 10^7 \) GeV and \( \mu^4 \sim \rho_{\text{NOW}} \) would not undergo friction dominated motion during any part of the matter dominated era of the universe. Restoring consistency by requiring \( M > M_P \) brings us back to the bound (1.4).

The essential point is the same as in the first argument, that for such a variation to be consistent with cosmology requires fine tuning of the potential over a range of
parameters. Indeed, if we want $V(\phi) \sim \rho_{\text{NOW}}$ over the entire range $0 \leq \delta \alpha \leq 10^{-4}\alpha$, we must fine tune away contributions from (roughly) the first ten coefficients of the Taylor expansion of the function $g$ in (1.10).

A potential loophole in the argument as we just formulated it is that we assumed the coupling (1.1), but this precludes the interesting case that $V(\phi)$ vanishes at infinity, and that the scalar is evolving towards arbitrarily large values. Many models of quintessence assume such a potential.

This point can be dealt with by generalizing the Lagrangian to

$$\mathcal{L} = \frac{1}{2} g(\phi)(\partial \phi)^2 - V(\phi).$$  \hspace{1cm} (1.13)

Here $g(\phi)$ is an arbitrary positive function which might be thought of as related to wave function renormalization, or better as a metric on the configuration space. Within this family of Lagrangians, we can perform arbitrary field redefinitions $\phi \rightarrow \phi'(\phi)$.

In fact we can use this freedom to redefine $\phi$ so that the coupling (1.1) is exact; in other words define $\phi$ by the relation $\epsilon \phi/M = \delta \alpha^{-1}$, the variation $\delta \alpha$ around its late time asymptotic value. The possibility we missed of $\phi$ going to infinity becomes, after the field redefinition, the possibility that the integral $\int d\phi g(\phi)^{1/2}$, the invariant measure of distance in field space, could diverge as $\phi \rightarrow 0$. \textsuperscript{2}

Under our previous assumptions, the slow roll analysis goes through in the same way, with the equation of motion

$$\dot{\phi} = -g^{-1}(\phi)V'(\phi)$$

and corresponding modifications to (1.7) and other equations. However, the main point does not require any detailed analysis. It is that the new possibility, in this language that $g(\phi)$ diverges as $\phi \rightarrow 0$, would lead to a slowing down of $\dot{\alpha}$, but does not affect the essential point as seen in the first argument.

In general, the limit $\phi >> \Lambda$, which might be thought to be problematic in effective field theory, in many cases is not. Such field redefinitions can also be used to clarify other limits; for example $M >> M_P$, etc.

All this is not to say that scalars with potentials $V(\phi) \sim \rho_{\text{NOW}}$ are impossible. This still requires an extreme fine tuning, of course, but if we place suitable bounds on $\epsilon$, leading to unobservably small $\delta \alpha/\alpha$, one can argue that one has done only two fine tunings, of $\mu/M_P$ and of $\epsilon$. The most natural way to accomplish the second of these would be to simply postulate a symmetry enforcing $\epsilon = 0$.

\textsuperscript{2}We are not being perfectly general yet as we are assuming that the original relation between $\phi$ and $\delta \alpha$ was one-to-one. One can generalize further, but this does not lead to further illumination.
As far as we know, the only possible explanation of such small numbers would be an axion-like shift symmetry for the scalar field $\phi$. General arguments rule out global continuous symmetries in theories including gravity. However, there are axion like fields that arise from higher dimensional antisymmetric tensor gauge fields. A generic context in which one might expect small axion potentials to be generated, is that of brane world models, like that of Horava and Witten [4]. There, axions arise as would be gauge modes of bulk gauge fields, and potentials are generated by interaction with the boundary. In some Horava-Witten models for the real world, one can obtain axions whose potentials are generated by weak interaction instantons[5]. This can give rise to very small numbers for couplings that would be order one by dimensional analysis.

This reinforces the conclusion of [8] that axions are the only known model of quintessence that might be consistent with the naturalness constraints of quantum field theory. Note however that [8][7] and [6] all point out various problems with the idea of axions as quintessence.

Our overall conclusion is that we do not have any field theoretically natural explanations for a variation of the fine structure constant as large as would be required to explain the observations of [1]. If these observations are confirmed, one will have to invent some very exotic physics to explain them.

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