Mass generation from tachyon condensation for vector fields on D-branes

Washington Taylor

Center for Theoretical Physics
MIT, Bldg. 6-306
Cambridge, MA 02139, U.S.A.
wati@mit.edu

Abstract: The level truncation approach to string field theory is used to study the zero-momentum action for vector excitations on a bosonic D-brane which has been annihilated by tachyon condensation. It is shown that in the true vacuum the translation zero modes associated with transverse scalars on the D-brane are lifted by spontaneous generation of mass terms. Similarly, the U(1) gauge field on the brane develops a nonzero mass term.

Keywords: D-branes, String field theory
1. Introduction

It was recently proposed by Sen [1] that the decay of unstable D-brane configurations in bosonic and type II string theory can be systematically described using string field theory. As a quantitative test of this prediction, Sen suggested that the energy gap between an unstable brane configuration and the vacuum should be precisely computable using tree level string field theory. Subsequent work has verified this conjecture in a variety of contexts. Calculations using level-truncated string field theory to analyze the decay of the bosonic D-brane have given an energy gap which agrees with the brane energy to 99.91% [2, 3, 4]; a suggestion for streamlining these calculations was made in [5]. Similar calculations for unstable D-branes in superstring field theory have shown agreement to a level of approximately 90% [6, 7, 8, 9]. D0/D4 brane configurations in superstring field theory with a background B field were considered in [10]. Lower-dimensional D-branes have been constructed as solitonic tachyon configurations on the bosonic D25-brane [11, 12, 13]. It has been shown that the string field theory analysis simplifies significantly in the limit of a large B field [14, 15, 16, 17] and for p-adic strings [18]. Fundamental string excitations have been constructed in the large B limit [19]. A discussion of background independence in the context of tachyon condensation recently appeared in [20]. It has also been suggested that renormalization group analysis of conformal field theory may provide a simpler approach to some tachyon condensation calculations of this type [21].

This set of results provides some of the first concrete evidence that string field theory can be used as a practical calculational tool to study nonperturbative aspects of string physics, particularly in the case of processes involving creation and annihilation of D-branes, which are essentially nonperturbative phenomena from the string theory point of view. Recent arguments [21, 22, 23, 24] that R-R charge is best
described through K-theory, which arises naturally in the context of the annihilation through tachyon condensation of an infinite number of branes and anti-branes, indicate that further development of the technology of string field theory may provide very useful clues to the fundamental structures underlying string theory and M-theory.

When a single unstable D-brane (or a brane and antibrane of type II string theory) annihilate by tachyon condensation, there is a puzzle involving the fate of the U(1) gauge field living on the brane \[25, 21\]. In particular, from the point of view of the field theory living on the brane there does not seem to be any obvious mechanism for removing this field from the theory after the brane annihilation process has completed. This puzzle has been discussed by numerous authors; one particularly interesting suggestion was made in \[26, 27\], where it was argued that the gauge theory becomes confining in the new vacuum.

In this note we address this puzzle by calculating the term quadratic in the U(1) field \(A_\mu\) on a bosonic D-brane after decay to the stable vacuum using the level truncation approach to string field theory. Surprisingly, we find that this quadratic term has a nonzero coefficient, indicating that stringy effects give a mass to the U(1) vector field in the stable vacuum. The possible appearance of such mass terms was discussed previously in \[28\]. We do not have control at this point over the kinetic term for the vector field, so we phrase our discussion in terms of the lifting of the zero modes on the D-brane associated with translation invariance in a perpendicular direction for a brane of dimension \(p < 25\). These zero modes are T-dual to the zero modes associated with constant gauge fields on the D25-brane. The appearance of a quadratic term for these fields indicates that as the D-brane is annihilated, at some point the translation/gauge zero modes of the configuration are lifted by stringy effects. This gives a natural resolution of the U(1) puzzle, in which stringy effects spontaneously give a mass to the gauge field in the true vacuum.

In Section 2 we review the string field theory analysis of tachyon condensation on a bosonic D-brane and the structure of the effective potential for the tachyon field. In Section 3 we give numerical evidence that the U(1) field on the D-brane has a nonzero mass in the stable vacuum. Section 4 contains a discussion of this result.

2. Tachyon condensation on bosonic D-branes

In this section we briefly review the string field theory description of the annihilation of a bosonic D-brane, following [2, 1, 3, 4]. We follow the notation and conventions of [3, 4].

Witten’s cubic open string field theory action is [29]

\[
S = \frac{1}{2\alpha'} \int \Phi \ast Q \Phi + \frac{g}{3!} \int \Phi \ast \Phi \ast \Phi,
\]  

(2.1)
where \( Q \) is the BRST operator and \( \star \) is the string field theory star product. In Feynman-Siegel gauge, the string field \( \Phi \) has the expansion

\[
\Phi = \left( \phi + A_\mu \alpha^\mu_{-1} + \frac{1}{\sqrt{2}} B_{\mu\nu} \alpha^\mu_{-1} \alpha^\nu_{-1} + \beta b_{-1} c_{-1} + \cdots \right) |0\rangle
\]  

(2.2)

where \( |0\rangle = c_1 |\Omega\rangle \) is the state in the string Hilbert space associated with the tachyon field. Explicit descriptions of the vertex operator for the cubic terms in the action (2.1) were given in the oscillator basis in [30, 31, 32].

For the purposes of determining the Lorentz-invariant stable vacuum, it is sufficient to restrict attention to the scalar fields \( \psi^i \) in the string field, in terms of which we write

\[
\Phi_{\text{scalar}} = \sum_{i=1}^{\infty} \psi^i |s_i\rangle
\]  

(2.3)

where \( |s_i\rangle \) are all the scalar states in \( \mathcal{H} \). (We write the tachyon field as \( \psi^1 = \phi \).) The zero momentum action for the scalar string fields can be written as

\[
V = \sum_{i,j} d_{ij} \psi^i \psi^j + g\kappa \sum_{i,j,k} t_{ijk} \psi^i \psi^j \psi^k
\]  

(2.4)

where \( g \) is the string coupling constant and

\[
\kappa = \frac{3^{7/2}}{2^7}.
\]

The quadratic and cubic coefficients \( d_{ij} \) and \( t_{ijk} \) appearing in this potential can be explicitly computed for any finite number of terms. The formalism needed to perform these calculations is reviewed in [33] using notation and conventions which we follow in this paper.

The level truncation approach to string field theory involves systematically truncating the string field theory action by including fields up to some fixed level \( L \) (where the tachyon is taken to have level 0), and interactions up to a total level \( I \). Level truncation has been shown to give a good systematic approximation to string field theory for particular calculations such as the energy of the stable vacuum. In [4, 5], the terms in (2.4) were calculated up to truncation level (4, 8). The equations of motion for the fields \( \psi^i \) were solved in this approximation, and it was shown that the resulting vacuum energy differs from the perturbative vacuum energy by 98.6% of the 25-brane energy \( (2\pi^2 g^2)^{-1} \) predicted by Sen. All the terms in (2.4) up to truncation level (10, 20) were determined in [4], and it was shown that in this approximation the energy gap between the stable and perturbative vacua agrees with Sen’s prediction to 99.91%. The values \( \langle \psi^i \rangle \) of the scalar fields up to level 10 were determined in truncations up to (10, 20) to 10 decimal places; we will use these values in the calculations of this paper.

One way of thinking about the process of tachyon condensation is in terms of the effective potential \( V(\phi) \) which arises from integrating out all the scalar fields.
ψ^i, i > 1 from (2.4). In terms of this effective potential, the stable vacuum arises at Φ^* where V'(Φ^*) = 0, and Sen's conjecture states that

\[ V(0) - V(\Phi^*) = \frac{1}{2\pi^2 g^2}. \]  

(2.5)

In [4], this effective potential was studied both from the point of view of its perturbative expansion

\[ V(\phi) = \sum_{n=2}^{\infty} c_n (g\kappa)^{n-2} \phi^n = -\frac{1}{2} \phi^2 + g\kappa \phi^3 + \cdots \]  

(2.6)

and nonperturbatively using numerical techniques to solve the equations of motion arising from (2.4) in the level-truncated theory for all fields except ϕ. One particularly interesting feature of this effective potential is that at every level of truncation up to (10, 20) a branch point appears near \( \phi \approx -0.25/g \). The effective potential seems to have a critical point near this value. The physical meaning of this critical point is unclear, although some speculations were made in [4, 34]. The existence of this critical point does, however, indicate a finite radius of convergence for the effective potential \( V(\phi) \). The stable vacuum \( \Phi^* \) lies outside this radius of convergence, although \( \Phi^* \) lies in the opposite direction on the \( \phi \) axis and seems to be within the region where the branch of the effective potential containing \( \phi = 0 \) is well defined.

3. Lifting of zero modes

One of the most important outstanding questions related to D-brane annihilation by tachyon condensation is the structure of the theory in the stable vacuum. Since in this vacuum the D-brane has completely disappeared from the picture, we do not expect to see open string modes in this vacuum. According to Sen’s conjecture the kinetic terms for all the open string modes should vanish in the stable vacuum, essentially decoupling all massive string modes from the theory. How the fate of the massless U(1) field on the brane can be understood is, however, a puzzle which has been discussed from several points of view [25, 21, 26, 27] but not fully resolved.

A full string field theory analysis of the fate of the massless vector field on the brane after tachyon condensation would involve a complicated calculation involving subtle momentum-dependent factors. Even without considering momentum-dependence, however, there is an interesting calculation which seems to shed light on the U(1) puzzle. The significance of this calculation is most clear in a T-dual setting, where we consider a bosonic Dp-brane with \( p < 25 \). Under T-duality in \( k \) compact directions, the components of the \( U(1) \) field in the compact directions become transverse scalar fields \( X^i \). For a Dp-brane in noncompact space, the string field theory action is the same as for a D25-brane, with the differences that the fields all have momentum only in \( p + 1 \) dimensions of space-time and that the transverse
Scalars $X^i$ have the physical interpretation of describing the position of the D-brane in the transverse dimensions.

Shifting the position of a D$p$-brane in one of the transverse directions corresponds, then, to turning on a translation zero mode of the theory associated with a constant field $X^i$. The physical question we will address with the calculation in this section is whether these translation zero modes still exist in the stable vacuum arising from tachyon condensation on a bosonic D$p$-brane with $p < 25$. In order to answer this question, we only need to calculate the coefficient of the quadratic term $X^i X^i$ in the effective action arising from integrating out all massive string fields in the stable vacuum. We find that indeed this term seems to have a nonzero coefficient. This indicates that the translation zero modes are lifted by stringy effects in the stable vacuum. While we perform our calculations in a fixed gauge, this physical result is gauge-independent.

Although the physical interpretation of this mass term is clearest in the T-dual context of a D$p$-brane $p < 25$, we carry out this calculation in the context of the zero-momentum action on a D25-brane. In this picture, the quadratic term we derive represents a spontaneously generated mass for the vector field $A_\mu$ in the stable vacuum. We discuss the physical meaning of this term and the question of gauge invariance in the following section; in this section we simply perform the string field theory calculation of the term in question.

To make the discussion precise, we introduce some new notation. We are interested in the zero-momentum string field theory action for all string fields at odd levels which transform as Lorentz vectors. We write this vector part of the string field as

$$\Phi_{\text{vector}} = \sum_{m=1}^{\infty} \eta^{(m)}_\mu |\nu^{(m)}\rangle$$

$$= A_\mu \alpha^{\mu}_{-1} |0\rangle + \eta^{(2)}_\mu \alpha^{\mu}_{-1} (\alpha_{-1} \cdot \alpha_{-1}) |0\rangle + \eta^{(3)}_\mu \alpha^{\mu}_{-3} |0\rangle + \eta^{(4)}_\mu \alpha^{\mu}_{-1} b_{-1} c_{-1} |0\rangle + \cdots$$

(3.1)

where $\eta^{(m)}_\mu$ are the odd-level vector fields in the theory, with $\eta^{(1)}_\mu = A_\mu$. We are interested in the zero-momentum scalar-vector-vector couplings in the string field theory action

$$\kappa g C_{imn} \psi^i \left( \eta^{(m)} \cdot \eta^{(n)} \right)$$

(3.2)

and the zero-momentum mass terms for the vector fields in the perturbative vacuum

$$D_{mn} \left( \eta^{(m)} \cdot \eta^{(n)} \right)$$

(3.3)

In terms of these couplings, the complete set of zero-momentum quadratic terms for the vector fields in the stable vacuum can be written as

$$S_{v^2} = \sum_{m,n} M_{mn} \left( \eta^{(m)} \cdot \eta^{(n)} \right)$$

(3.4)
where
\[ M_{mn} = D_{mn} + g\kappa \sum_i C_{imn} \langle \psi^i \rangle \] (3.5)
is the mass matrix for the vector fields.

If the zero modes of \( A_\mu \) persist in the stable vacuum, we would expect that in the full theory the mass matrix \( M_{mn} \) would have a vanishing eigenvalue. We can test this using the level truncation method by truncating at a sequence of levels \((L_v, L_s, I)\) where \( L_v \) and \( L_s \) are the maximum levels of vector fields and scalar fields considered, and \( I \) is the maximum total level considered in any interaction. We describe here explicitly the mass matrix in the level truncations \((1, 0, 2)\), \((1, 2, 4)\) and \((3, 4, 8)\).

**Level (1, 0, 2) truncation:**

There is only a single scalar at level zero \((\phi)\) and a single vector at level one \((A_\mu)\). The complete zero-momentum string field theory action for these fields is
\[ S_{1,0,2} = -\frac{1}{2} \phi^2 + \kappa g \phi^3 + \frac{16}{9} \kappa g \phi A_\mu A^\mu. \] (3.6)
At this level the mass matrix is
\[ M = \left( \frac{16}{9} \kappa g \langle \phi \rangle \right) \approx (0.59259) \] (3.7)
when we use the level \((0, 0)\) approximation \( g\kappa \langle \phi \rangle_{(0,0)} = 1/3 \). Thus, in the level \((1, 0, 2)\) truncation the smallest eigenvalue of \( M \) is \( \approx 0.59 \).

**Level (1, 2, 4) truncation:**

There are two additional scalars appearing at level 2. Including these scalars the mass matrix becomes
\[ M = \left( \kappa g \left[ \frac{16}{9} \langle \phi \rangle - \frac{1568}{243} \langle \psi^2 \rangle - \frac{176}{243} \langle \psi^3 \rangle \right] \right) \approx (0.67289) \] (3.8)
using the level \((2, 4)\) approximation to the scalar expectation values. We see that including scalars at higher level has shifted the mass of the vector slightly, but in an upward direction.

**Level (3, 4, 8) truncation:**

Using the level \((4, 8)\) approximations for the 10 scalars at level 4 or less we find a mass matrix for the four vectors appearing in (3.1) given approximately by
\[
M \approx \begin{pmatrix}
0.67910 & -2.82440 & -0.08261 & -0.26102 \\
-2.82440 & 82.15982 & 0.31499 & 1.03418 \\
-0.08261 & 0.31499 & 4.70145 & 0.02849 \\
-0.26102 & 1.03418 & 0.02849 & -0.95150
\end{pmatrix} \] (3.9)
The smallest eigenvalue of this matrix (in absolute value) is \( \approx 0.61206 \), and the associated eigenvector is \( \approx (0.98919, 0.03598, 0.01819, -0.14101) \).
We have continued this analysis of the zero-momentum mass matrix up to level (9, 10, 20). The results for the smallest eigenvalue of $M$ are given in Table 1. As is apparent from the table, there is no sign of a vanishing eigenvalue in the limit of large level. Quite to the contrary, the smallest eigenvalue seems to be converging to a number in the vicinity of $0.59$. For each of the eigenvalues given in Table 1, the associated eigenvector is dominated by the component in the direction of $A_\mu$, just as in the (3, 4, 8) example given explicitly above.

| level   | $N_v$ | $N_s$ | $\mu_{\text{min}}$ | $G(\phi_*)$ |
|---------|-------|-------|---------------------|-------------|
| (1, 2, 4) | 1     | 3     | 0.67289             | 0.67289     |
| (3, 4, 8) | 4     | 10    | 0.61206             | 0.63343     |
| (5, 6, 12)| 17    | 31    | 0.59906             | 0.62738     |
| (7, 8, 16)| 61    | 91    | 0.59333             | 0.62491     |
| (9, 10, 20)| 197   | 252   | 0.59026             | 0.62362     |

Table 1: $N_v, N_s$ are numbers of vector and scalar fields in level truncation ($L_v, L_s, I$). $\mu_{\text{min}}$ is smallest eigenvalue of mass matrix $M_{mn}$. $G(\phi_*)$ is mass of vector field in effective action for $\phi, A_\mu$.

One way of physically interpreting the mass matrix for the vector fields $\eta^{(n)}$ is in terms of the effect of this mass matrix on the effective action for the tachyon field $\phi$ and the massless vector $A_\mu$. Integrating out all fields but $\phi$ and $A_\mu$ gives an effective action of the form

$$S_{\phi,A} = V(\phi) + G(\phi) A_\mu A^\mu + \mathcal{O}(A^4)$$

(3.10)

where $V(\phi)$ is the effective tachyon potential discussed in Section 2 and $G(\phi)$ is another function of $\phi$. The quadratic term for the vector field in the stable vacuum has a coefficient $G(\phi_*)$.

We can determine $G(\phi_*)$ in terms of the matrix $M$ by solving the equations of motion for the vector fields at leading order. These equations of motion state that

$$M_{mn} \eta^{(n)}_\mu = 0, \quad m > 1.$$  

(3.11)

Thus, we have

$$\eta^{(n)}_\mu = (M^{-1})_{n1} \alpha_\mu$$

(3.12)

for some $\alpha_\mu$, so

$$\eta^{(n)}_\mu = \frac{(M^{-1})_{n1}}{(M^{-1})_{11}} A_\mu,$$

(3.13)

so

$$G(\phi_*) = \frac{1}{(M^{-1})_{11}}.$$  

(3.14)

For the examples above with $L_v = 1$, this simply gives $G(\phi_*) = M_{11}$ as expected. For the matrix (3.9) describing level truncation (3, 4, 8), this gives

$$G(\phi_*)_{(3,4,8)} \approx 0.63343.$$  

(3.15)

This quantity was also calculated by Sen and Zwiebach in the course of their analysis of marginal deformations of the bosonic D-brane away from the perturbative vacuum.
Using an independent computational technique they arrived at precisely the same value in this level truncated theory\(^1\). We have extended this calculation of \(G(\phi_s)\) up to the level \((9, 10, 20)\) truncation for which we have determined \(M_{mn}\). The results of this analysis are summarized in Table 1. We see that \(G(\phi_s)\) seems to approach a nonzero number near 0.62 in the limit of large levels, indicating that a mass for the vector field is spontaneously generated in the stable vacuum. The rate at which \(G(\phi_s)\) converges to its limiting value is similar to the rate at which \(V(\phi_s)\) approaches its limiting value as higher levels are incorporated; for example, the difference in \(G(\phi_s)\) between the level \((7, 8, 16)\) and level \((9, 10, 20)\) truncations is less than 0.2\%. This similarity with the rate of convergence of \(V(\phi_s)\) gives us confidence that the level truncation method is giving us a good sequence of approximations to a limiting value for \(G(\phi_s)\) near 0.62.

\section{4. Discussion}

In this note we have described a calculation which shows that the transverse scalars \(X_i\) and \(U(1)\) gauge field \(A_\mu\) get mass terms from stringy effects when a bosonic D-brane annihilates through tachyon condensation. In particular, we have found that in the stable open string field theory vacuum associated with tachyon condensation on a D25-brane, there is a term of the form

\[G(\phi_s)A_\mu A^\mu, \quad G(\phi_s) \approx 0.62\quad (4.1)\]

in the effective potential \(S(\phi, A_\mu)\). While we performed this calculation in a particular gauge, the result that zero modes of transverse scalars and the world-volume gauge field are lifted in the stable vacuum is a physically observable gauge-independent effect.

The appearance of the term (4.1) seems very surprising from a field theory point of view. In the vicinity of the perturbative vacuum, we expect to have a massless \(U(1)\) gauge field on the brane, and gauge invariance under this \(U(1)\) seems to guarantee the absence of such a mass term. How does this field theoretic picture mesh with the structure of the string field theory action? To understand the connection it is helpful to consider the (non gauge-fixed) string field theory action around the perturbative vacuum

\[S = -\frac{1}{2}\phi^2 + \kappa g\phi^3 + \frac{16}{9}\kappa g\phi A_\mu A^\mu + \cdots \quad (4.2)\]

It is interesting to note that the leading terms (4.2) appearing in the string field theory action have also been calculated directly from the point of view of the sigma model using a nonstandard regularization scheme \(^3\). The action (4.2) does not

\(^1\)I would like to thank these authors for comparing the details of this calculation and for extended discussions regarding its significance.
have a gauge invariance under the transformations
\[
\delta \phi = 0 \quad \delta A_\mu = \partial_\mu \Lambda \tag{4.3}
\]
Instead, (4.2) is invariant under the BRST transformation
\[
\delta \phi = \frac{32}{9} \kappa g A^\mu \partial_\mu \Lambda + \cdots \\
\delta A_\mu = \partial_\mu \Lambda + \cdots \tag{4.4}
\]
After integrating out all massive fields in the theory, the effective action \( S(\phi, A_\mu) \) for the tachyon and string vector field continues to have an invariance under a symmetry whose leading terms are given by (4.4). Thus, \( S(\phi, A) \) is not gauge-invariant in the sense of (4.3).

In the neighborhood of the perturbative vacuum, it is possible to find a field redefinition \( \tilde{\phi} = f(\phi, A), \tilde{A} = g(\phi, A) \) to a new set of fields under which the effective action \( S(\tilde{\phi}, \tilde{A}) \) becomes gauge invariant under (4.3). Such a field redefinition was recently discussed explicitly in [36]. Under this field redefinition, the zero-momentum part of \( S(\tilde{\phi}, \tilde{A}) \) must become independent of \( \tilde{A} \) by gauge invariance. From the first few terms in the effective action \( S(\phi, A) \), which coincide with (4.2), and from (4.4), it is clear that the first term in this field redefinition must be
\[
\phi \rightarrow \tilde{\phi} + \frac{16}{9} g \kappa A_\mu A^\mu + \cdots \tag{4.5}
\]
This redefinition cancels the term of the form \( \phi A_\mu A^\mu \) in \( S(\phi, A) \). It was furthermore shown in [33] that \( S(\phi, A) \) contains a term of the form \( A_\mu A^\mu A_\nu A^\nu \) which was determined to within a few percent using level truncation including fields up to level 20; this term is also precisely cancelled by (4.3).

Although the existence of a field redefinition of this type seems to mean that there is a set of variables in terms of which the effective action \( S(\phi, A) \) has the standard gauge invariance (4.3), care must be taken with respect to the range of validity of such a field redefinition, as emphasized in [35]. This field redefinition is only defined in a perturbative fashion, and therefore we expect that it will only be well-defined in a finite sized neighborhood of the perturbative vacuum. The apparent existence of a nonzero mass term (4.1) for \( A_\mu \) in the nonperturbative stable vacuum of the open bosonic string field theory can in fact be interpreted as evidence that the U(1) gauge theory description of the bosonic D-brane theory has a finite range of validity. If this interpretation is correct, it would mean that as the tachyon condenses and the theory moves towards the nonperturbative stable vacuum, at a certain point the theory undergoes a phase transition beyond which an effective gauge field theory description of the system such as suggested in [37, 38, 39] would no longer be applicable. This is clearly an important issue to understand better,
since recent analyses of physics in the nonperturbative vacuum \[15, 40, 41\] rely on extending such a gauge-invariant field theory action all the way to the stable vacuum. Since the results found in these papers are generally physically correct, it may be that even if a mass term such as we have found does exist for the U(1) field in the stable vacuum, some qualitative aspects of the physics in this vacuum are correctly captured by a gauge-invariant field theory.

One possible scenario which seems to be consistent with the picture given here is that the transformation law for $A_\mu$ in the effective theory $S(\phi, A_\mu)$ may be of the form

$$\delta A_\mu = f(\phi)\partial_\mu \Lambda + \cdots$$

where $f(\phi_\ast) = 0$. This seems to be the simplest way to ensure that an action of the form (3.10) retains BRST symmetry at a point where $V'(\phi_\ast) = 0$ and $G(\phi_\ast) \neq 0$. This scenario might also help explain why the locally defined gauge field theory comes close to capturing the physics of the true vacuum correctly, as it is possible that the field theory picture is valid for all $\phi < \phi_\ast$.

A skeptic might argue that the numerical results presented here are rather flimsy evidence on which to base a claim that gauge theory breaks down in the process of tachyon condensation. It is certainly not impossible, for example, that the numbers in Table 1, while appearing to converge to quantities in the neighborhood of 0.6, actually are the beginning of two very long sequences of numbers which converge slowly to 0. This seems unlikely, at least to the present author, since the apparent convergence of the value of $G(\phi_\ast)$, in particular, seems to be proceeding in a closely analogous fashion to the convergence of $V(\phi_\ast)$ as studied in \[4\]. These functions arise in a very parallel fashion in the string field theory picture, so it would seem very surprising if the level truncation approach gives such a good approximation to $V(\phi)$ near the stable vacuum and is nonetheless giving misleading information for $G(\phi_\ast)$. As further evidence that the scenario outlined above is a correct description of the physics, it is also relevant to consider the fact that the perturbative expansion of $V(\phi)$ has a radius of convergence noticeably smaller than $\phi_\ast$. This seems to lend credence to the possibility that the field redefinition needed to give a gauge-invariant formulation of the D-brane theory may have a similar radius of convergence and may not be well-defined in the vicinity of the stable vacuum.

The results presented here seem to give a very simple solution to the puzzle regarding the fate of the massless U(1) field, when viewed from the point of view of string field theory. In the picture we have given, the U(1) field becomes massive due to stringy effects when the tachyon condensation becomes sufficient to bring the theory near the stable vacuum. It is not clear whether there is a way to bring this intrinsically stringy effect back into the domain of field theory, so that this phenomenon might be described either in terms of a Higgs effect or confinement as suggested in \[26, 27\].
In this work we have neglected momentum dependence in the string field theory action. Including all momentum-dependent terms in the action and constructing the full momentum-dependent quadratic string field theory action in the vicinity of the stable vacuum would make possible a detailed study of the pole spectrum of the theory, extending earlier work in [4], which would shed a great deal of light on the fate of the open string excitations in the stable vacuum. In particular, to precisely understand the fate of the U(1) vector field it is clearly desirable to have control over the kinetic terms of the vector fields as well as the mass terms studied here. One likely scenario, which was suggested by Sen [1], is that all the kinetic terms will vanish in the stable vacuum. If this is true and the quadratic matrix $M_{nm}$ for the vector fields indeed has a lower bound on its spectrum as suggested by the results described here, all the vector fields in the theory will become infinitely massive and hence decouple in the stable vacuum. This would fit in very nicely with the expected decoupling of all open string fields in this vacuum. Another, more bizarre, possibility is that the kinetic terms for the gauge field $A_\mu$ become infinite in the effective action $S(\phi, A)$ using the standard string field theory normalization conventions we have used here. This would essentially drive the mass of the gauge field back to zero even in the presence of the mass term we have found. This possibility would seem to be very unlikely as it is difficult to reconcile with the decoupling of the other massive string modes, but it is worth mentioning since it seems to be the only possibility other than an extraordinary conspiracy at high level that would restore the massless U(1) field at the stable vacuum. Work is currently in progress to give a complete analysis of the spectrum of the level-truncated string fields around the stable vacuum, including all momentum-dependent factors. When completed, such an analysis may give new insights as to how the complete set of open string fields including the apparently massive U(1) decouple in the stable vacuum.

Finally, it would clearly be of interest to extend the analysis in this note to the case of the superstring. While there seem to be complications in using Witten’s cubic superstring field theory for this kind of calculation (see [42] and references therein), the approach of Berkovits [43] has led to successful calculations of the energy of unstable D-branes in type II string theory to an accuracy of about 90% [4, 7, 8, 9]. It would be very interesting to see whether the world-volume U(1) field on such a brane has a mass analogous to that computed in this note in the supersymmetric type II vacuum. On the one hand, it is natural to expect that the physics of this U(1) field should not be particularly different in the supersymmetric case from the bosonic case. On the other hand, however, arguments for a simple form of the action describing the tachyon and U(1) field were given in [37, 38, 39]. Some of these arguments relied on supersymmetry. It may be that supersymmetry helps to protect the U(1) gauge field from the spontaneously generated mass we have found here in the bosonic case. More work in this direction is clearly needed to clarify this issue.
Acknowledgements

I would like to particularly thank Ashoke Sen and Barton Zwiebach for extensive discussions on the issues discussed in this paper and for comparing detailed aspects of their own calculations with those I have described here. I would also like to thank Justin David, Kentaro Hori, Albion Lawrence and Leonardo Rastelli for helpful discussions. Thanks also to the Aspen Center for Physics, where this work was completed, and to the participants in the Aspen Summer '00 M-theory and Duality Workshop for many stimulating discussions relevant to the subject matter of this note. This work was supported in part by the A. P. Sloan Foundation and in part by the DOE through contract #DE-FC02-94ER40818.
References

[1] A. Sen, “Universality of the tachyon potential,” *JHEP* **9912** (1999) 027, [hep-th/9911116](https://arxiv.org/abs/hep-th/9911116).

[2] V. A. Kostelecky and S. Samuel, “On a nonperturbative vacuum for the open bosonic string,” *Nucl. Phys.* **B336** (1990) 263-296.

[3] A. Sen, B. Zwiebach “Tachyon condensation in string field theory,” *JHEP* **0003** (2000) 002, [hep-th/9912249](https://arxiv.org/abs/hep-th/9912249).

[4] N. Moeller and W. Taylor, “Level truncation and the tachyon in open bosonic string field theory,” [hep-th/0002237](https://arxiv.org/abs/hep-th/0002237).

[5] L. Rastelli and B. Zwiebach “Tachyon potentials, star products and universality,” [hep-th/0006240](https://arxiv.org/abs/hep-th/0006240).

[6] N. Berkovits, “The tachyon potential in open Neveu-Schwarz string field theory,” *JHEP* **0004** (2000) 022, [hep-th/0001084](https://arxiv.org/abs/hep-th/0001084).

[7] N. Berkovits, A. Sen and B. Zwiebach, “Tachyon condensation in superstring field theory,” [hep-th/0002211](https://arxiv.org/abs/hep-th/0002211).

[8] P.J. De Smet and J. Raeymaekers, “Level four approximation to the tachyon potential in superstring field theory,” *JHEP* **0005** (2000) 051, [hep-th/0003220](https://arxiv.org/abs/hep-th/0003220).

[9] A. Iqbal and A. Naqvi, “Tachyon condensation on a non-BPS D-brane,” [hep-th/0004015](https://arxiv.org/abs/hep-th/0004015).

[10] J. R. David, “Tachyon condensation in the D0/D4 system,” [hep-th/0007235](https://arxiv.org/abs/hep-th/0007235).

[11] J. A. Harvey and P. Kraus “D-branes as unstable lumps in bosonic open string field theory,” *JHEP* **0004** (2000) 012, [hep-th/0002117](https://arxiv.org/abs/hep-th/0002117).

[12] R. de Mello Koch, A. Jevicki, M. Mihaiescu and R. Tatar, “Lumps and p-branes in open string field theory,” *Phys. Lett.* **B482** (2000) 249, [hep-th/0003031](https://arxiv.org/abs/hep-th/0003031).

[13] N. Moeller, A. Sen and B. Zwiebach “D-branes as tachyon lumps in string field theory,” [hep-th/0005036](https://arxiv.org/abs/hep-th/0005036).

[14] K. Dasgupta, S. Mukhi and G. Rajesh, “Noncommutative tachyons,” *JHEP* **0006** (2000) 022, [hep-th/0005006](https://arxiv.org/abs/hep-th/0005006).

[15] J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec “D-branes and strings as noncommutative solitons,” *JHEP* **0007** (2000) 042, [hep-th/0005031](https://arxiv.org/abs/hep-th/0005031).

[16] E. Witten, “Noncommutative tachyons and string field theory,” [hep-th/0006071](https://arxiv.org/abs/hep-th/0006071).

[17] C. Sochichiu, “Noncommutative tachyonic solitons. Interaction with gauge field,” [hep-th/0007217](https://arxiv.org/abs/hep-th/0007217).
[18] D. Ghoshal and A. Sen, “Tachyon condensation and brane descent relations in p-adic string theory,” hep-th/0003278.

[19] N. Seiberg, “A note on background independence in noncommutative gauge theories, matrix model, and tachyon condensation,” hep-th/0008013.

[20] J. A. Harvey, D. Kutasov and E. J. Martinec, “On the relevance of tachyons,” hep-th/0003101.

[21] E. Witten, “D-branes and K-theory,” JHEP 9812:019 (1998), hep-th/9810188.

[22] P. Horava, “Type IIA D-branes, K-theory, and matrix theory,” Adv. Theor. Math. Phys. 2, 1373 (1999), hep-th/9812135.

[23] D.E. Diaconescu, G. Moore and E. Witten, “E (8) gauge theory, and a derivation of K theory from M theory,” hep-th/0005090; “A derivation of K theory from M theory,” hep-th/0005091.

[24] E. Witten, “Overview of K-theory applied to strings,” hep-th/0007175.

[25] M. Srednicki, “IIB or not IIB,” JHEP 08 (1998) 005, hep-th/9807138.

[26] P. Yi, “Membranes from five-branes and fundamental strings from Dp-branes,” Nucl. Phys. B550 (1999) 214, hep-th/9901159.

[27] O. Bergman, K. Hori and P. Yi, “Confinement on the brane,” Nucl. Phys. B580 (2000) 289, hep-th/0002223.

[28] V. A. Kostelecky and S. Samuel, “Photon and graviton masses in string theories,” Phys. Rev. Lett. 66 (1991) 1811.

[29] E. Witten, “Non-commutative geometry and string field theory,” Nucl. Phys. B268 (1986) 253.

[30] D. J. Gross and A. Jevicki, “Operator formulation of interacting string field theory (I), (II),” Nucl. Phys. B283 (1987) 1; Nucl. Phys. B287 (1987) 225.

[31] E. Cremmer, A. Schwimmer and C. Thorn, “The vertex function in Witten’s formulation of string field theory” Phys. Lett. B179 (1986) 57.

[32] S. Samuel, “The physical and ghost vertices in Witten’s string field theory,” Phys. Lett. B181 (1986) 255.

[33] W. Taylor, “D-brane effective field theory from string field theory,” hep-th/0001201.

[34] A. Sen, B. Zwiebach “Large marginal deformations in string field theory,” hep-th/0007153.

[35] V. A. Kostelecky, M. J. Perry and R. Potting, “Off-shell structure of the string sigma model,” Phys. Rev. Lett. 84 (2000) 4541, hep-th/9912243.
[36] J. R. David, “U(1) gauge invariance from open string field theory,” hep-th/0005085.

[37] A. Sen, “Supersymmetric world-volume action for non-BPS D-branes,” JHEP 9910 (1999) 008, hep-th/9909062.

[38] M. R. Garousi, “Tachyon couplings on nonBPSD-branes and Dirac-Born-Infeld action,” hep-th/0003122.

[39] E. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, “T-duality and actions for non-eps D-branes,” hep-th/0003221.

[40] T. Suyama, “Description of intersecting branes via tachyon condensation,” hep-th/0006052.

[41] R. Gopakumar, S. Minwalla and A. Strominger, “Symmetry restoration and tachyon condensation in open string theory,” hep-th/0007226.

[42] P.J. De Smet and J. Raeymaekers, “The tachyon potential in Witten’s superstring field theory,” hep-th/0004112.

[43] N. Berkovits “Super-Poincare invariant superstring field theory,” Nucl. Phys. B450 (1995) 90, hep-th/9503099.