NONLINEAR OPTIMIZATION TO MANAGEMENT PROBLEMS OF END-OF-LIFE VEHICLES WITH ENVIRONMENTAL PROTECTION AWARENESS AND DAMAGED/AGING DEGREES

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ABSTRACT. In the past one decade, an increasing number of motor vehicles necessarily results in huge amounts of end-of-life vehicles (ELVs) in the future. From the viewpoint of environment protection and resource utilization, government subsidy and public awareness of environmental protection play a critical role in promoting the formal recycle enterprises to recycle the ELVs as many as possible. Different from the existing similar models, a mixed integer nonlinear optimization model is established in this paper to formulate the management problems of recycling ELVs as a centralized decision-making system, where damaged and aging degrees, correlation between the recycled quantity and take-back price of ELVs, and the public environmental protection awareness are considered. Unlike the results available in the literature, take-back prices of the ELVs are the endogenous variables of the model (decision variables), which affect the collected quantity of ELVs and the profit of recycling system. Additionally, due to distinct damaged and aging degrees of the ELVs, the refurbished or dismantled amounts of ELVs are also regarded as the decision variables so that the recycle system is more applicable. By case study and sensitivity analysis, validity of the model is verified and impacts of the governmental subsidy and environmental awareness are analyzed. By the proposed model, it is revealed that: (1) Distinct treatment of ELVs with different damaged and aging degrees can increase the profit of recycling ELVs; (2) Compared with the transportation cost, higher processing cost is a main obstacle to the profit growth. Advanced processing technology plays the most important role in improving the ELV recovery efficiency. (3) Both of government subsidy and environmental awareness seriously affect decision-making of recycle enterprises.

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1. Introduction.

1.1. Background. In China, an increasing number of motor vehicles in the past one decade will necessarily result in huge amounts of end-of-life vehicles (ELVs) in the future. From the view point of environment protection and resource utilization, recycle of ELVs is helpful to sustainable circular economic and reduction of carbon emission. Actually, with an explosive growth of China’s automobile industry since 2000, China has become the largest domestic market of vehicles in 2009. The average growth rate of new vehicles in China from 1999 to 2009 was about 22% [13]. The civilian car ownership in China has reached 137 million in 2013, the number of ELVs will exceed 14 million in 2020 [9]. It is not alarmist talk, the increasing ELVs may generate a threat to human life from the view points of environmental protection and resource shortage if no action is taken.

A vehicle, as a hi-tech product, is composed by tens of thousands of components and contains hazardous substances as well as recyclable substances. That is to say, improper treatment of ELVs is not only associated with environment pollution but also resource wasting [6]. Owing to valuable elements in ELVs, the ELVs recycle is often conducted on the black market to reduce recycling cost [23]. Unlike a regular market, the black market business is illegal since it does not pay the transaction tax to governments, and also not pay any fee for waste disposal. In order to break up the black market, it is sure that government intervention, such as government subsidy, may be an effective policy so that recycle of the ELVs is operated by formal recycling enterprises. Actually, since 2009, Chinese government has implemented subsidy measures to encourage vehicle owners to sell their old vehicles to the recycling enterprises. In [21], four subsidy policies were summarized which include initial subsidy, recycling subsidy, research and development subsidy, and production subsidy.

On the other hand, in our previous research [24], it has been shown that raising the take-back prices may significantly increase the recycled amount of solid wastes. Therefore, it is valuable to propose an optimization approach to the management problems of ELVs with government subsidy if the take-back prices are incorporated into the model as endogenous variables, as well as the public awareness of environmental protection.

1.2. Literature review. As a kind of special reverse logistics, recycling ELVs is associated with a management problem of logistics network [19]. There have been an increasing number of research papers focusing on reverse logistics management. For example, in [3], a regional approach of waste electrical and electronic equipments (WEEE) management in France was designed. An internet-based reverse logistics system for the WEEE recycling in China was developed in [18] by introducing digital empowerment. A multi-period reverse logistics network design was presented in [7] for the used refrigerators recovery. A fuzzy model was proposed in [17] to strategic planning problem of the lead/acid battery closed-loop supply chain.

Since vehicles contain thousands of reusable components and materials such as steel, copper, rubber and plastics, etc, reuse of those potential resources could not only save energy but also decrease carbon dioxide emissions during the development of new resources [9]. On March 2, 2008, National Development and Reform Committee approved 14 enterprises as remanufacturers of pilot automotive components and released some relevant administrative policies for remanufacturing automotive components in China. Five assemblies, such as engine, transmission,
steering gear, starter and generator, should be remanufactured, and for the first time, the automotive engine, steering gear and transmission have been permitted to be remanufactured legally [25].

Apart from the above valuable components, the ELVs also consist of some toxic substances such as lubricants, acid solutions, and coolants. Improper and informal ELV treatment in informal recovery enterprises can cause tremendous and long-term damage to the environment [13]. More than 30% of used parts are recycled illegally in China and another 29.4% of vehicles that should be scrapped are still in use [21]. The government subsidy policy could stimulate the ELV take-back and promote more ELVs to be scrapped. Actually, China established the “Waste Electrical and Electronic Products Collection and Use of Management Practices Fund” on July 1, 2012. The government used this fund to give the relevant enterprises a fixed subsidy in accordance with the amount of the dismantled WEEEes [21].

It has been believed that recycling the ELVs depends on establishment of an efficient ELV recycling network, which not only can reduce some negative impacts on the environment generated from the recycling process, but also facilitate the effective reuse of the recycled resources. In particular, construction of optimization models for the management problems of recycling the ELVs is helpful to provide the decision-makers an optimal operation of the recycling system in practice.

Demirel et al. proposed a mixed integer linear programming (MILP) model for the ELV recycling, aiming to reduce the total reverse logistics cost [5]. A fuzzy ELVs recycling model was developed in [14], where a part of ELVs at a high quality level will be sold to the used vehicle market after repairing, while the others be dismantled for recycling components and materials. Vladimir Simic presented an interval-parameter chance-constraint programming model to design the ELV recover network for profit maximization [16]. However, They ignored that buy-back prices should be critical decision variables in the model, which can seriously affect the recycled amount in practice. In other words, as a common phenomena in practice, the recycled amount is only an intermediate variable with respect to the buy-back price, rather than being a given exogenous parameter as done in the existing results [24].

Since government subsidy and buy-back prices are two critical factors to affect the recycled amount, they have been incorporated into construction of model in some existing results. For example, in [12], a dual channel, quality-based price competition model was proposed without constraints for the WEEE recycling market, where the government subsidy was taken into account and the buy-back price was regarded as an endogenous decision variable. A mixed integer nonlinear facility location-inventory-pricing model was presented in [8] to decide on the optimal locations of the facilities, inventory amounts, prices for new products and incentive values for the collection of right amount of used products in order to maximize the total profit of supply chain, and developed a heuristic algorithm to found the solution. However, in the existing ELV models, few constrained nonlinear programming models have been built to describe the ELV management problems.

Very recently, a multi-objective mixed-integer piecewise nonlinear programming model (MOMIPNLP) was built by Wu and Wan for the decentralized supply chain management problem of urban mining system, where the decision variables are associated with the buy-back prices, choices of sites, transportation planning and adjustment of production capacities [24]. For solving this problem, the MOMIPNLP model was first transformed into an ordinary mixed integer nonlinear programming model by variable substitution such that the piecewise feature of the model
is removed. Then, based on technique of orthogonal design, a hybrid heuristic algorithm was developed to find an approximate Pareto optimal solution, where a genetic algorithm was used to optimize the structure of search neighborhood, and both the local branching algorithm and the relaxation induced neighborhood search algorithm were employed to cut the searching branches and reduce the number of variables in each branch.

However, in the existing results for optimizing the ELV recovery system, there are still some deficiencies, which can be briefly summarized as follows.

1. The impact of take-back prices on the recycled amount of ELVs is rarely considered in such a reverse logistics network.
2. It is often ignored what is an optimal government subsidy on the formal ELV collection enterprises. Especially, how much government subsidy is suitable to the recycled amount of ELVs?
3. How to determine an optimal percentage of refurbishment and dismantlement within the total collected ELVs? To our best knowledge, there does not exist any result in the literature.

1.3. Research intention of this paper. Based on the above background and literature review, we intend to build a new mixed integer nonlinear optimization model for a more efficient system of the ELV recovery management. We divide the collected ELVs into three types in accordance with their damaged and aging degrees. The first type is of fine quality situation, which will be directly refurbished, the third type is of poor quality situation, which will be directly dismantled, the others belong to the second type, for which we will answer how many percentages are refurbished and dismantled, respectively.

For simplicity, we suppose that the raising take-back prices could promote the recycled amount of ELVs and the latter linearly depends on the take-back price. The collecting enterprises would get the government subsidy based on the dismantled amount of ELVs. Thus, our investigation proceeds along the following three subsequent steps:

- To reflect the reality of recycling system, we construct a new mixed integer nonlinear optimization model to formulate the management problems of recycling ELVs.
- To verify validity and applicability of the built model, we will reveal its practical managerial insights by case study and sensitivity analysis.
- To more efficiently solve the model, we will compare numerical performance of different solvers to see which one should be used [4,11].

The rest of the paper is organized as follows. Next section is devoted to the description of problem and construction of model. In Section 3, numerical results of case study are reported. In Section 4, sensitivity analysis is conducted, and a number of practical managerial implications are revealed from the constructed model. Some conclusions and suggestions in future research are presented in the last section.

2. Problem description and formulation.

2.1. Problem description. Similar to the setting in [5], the network structure of ELV recovery system to be addressed in this paper is shown in Figure 1.

As shown in Figure 1, the network nodes basically consist of the ELV sources, the collection centers, the repair centers, the dismantlers, the shredders, the recycling
Figure 1. Material flow of the ELV recovery network

facilities, the secondary markets and the landfills. Specifically, the processing flow of the recycling network can be stated as follows. The collection centers first purchase the ELVs from the ELV sources. Then, the ELVs are classify into three types in the collection centers, all the the 1st type of ELVs are transported to the repair centers, all the 3rd type of ELVs are transported to the dismantlers, and the the 2nd type of ELVs are transported to the repair centers or to the dismantlers. The ELVs transported to the repair centers will be refurbished, and then sold to consumers. In the dismantlers, it is first required to remove and store separately the fuel, the motor oil, the oil from transmission system, the hydraulic oil, the cooling liquid, the liquid from the brake system, and the other liquids or hazardous substances if any. Subsequently, the components or materials removed from the scrap car are considered for reuse and recycling. Reusable ferrous and non-ferrous components are sold to the secondary markets, while the recycling materials, such as batteries, tyres, glass, plastics and waste oil are sold to the recycling factories. The remaining hulks are shipped to the shredders for further recycling. In the shredders, some materials can be mechanically recycled by shredder, air suction, magnetic sorters, eddy current sorters. Finally, the hulks are divided into ferrous material, non-ferrous material and auto-shredder residue (ASR). The sorted metals will be allocated to steel mills or non-ferrous smelters for further recycling, while the ASR will be directly shipped to the landfill.

Compared with the proposed ELV recycling network in [5], the network shown in Figure 1 has the following new features: (1) The ELV at a high quality level is considered for refurbishing; (2) There is no transportation between the collection centers and dismantlers; (3) Only ferrous and non-ferrous components for reusing are sold to the secondary markets; (4) It does not need to build the recycling facilities for processing the battery, tyre, glass and plastics. Instead, all of them are separately sold to the existent factories with equipments for recycling. Especially, the above setting more coincides with the practical situations in China.

The goal of this paper is to formulate the above recycling network such that the total system profit is maximized, which is associated with the income from the sale of isolated materials and the subsidy from the government, the costs of repurchasing, transportation and processing. In order to build an optimization model that is more
realistic than those available in the literature, dependence relationship between the collected quantity and repurchasing price must be incorporated into formulation of recycling ELVs. In the existing results, it is inappropriate to assume that the quantity of ELVs to be collected is fixed. In contrast, we assume in this paper that the collected quantity of ELVs is linearly depends on the take-back (repurchase) prices. Additionally, the subsidy from government also depends on the dismantled quantity of ELVs.

2.2. Notations. Before presentation of mathematical model, we introduce the following notations.

Indices

- $i$: label of ELV sources, $i=1,2,\cdots,I$.
- $j$: label of collection centers, $j=1,2,\cdots,J$.
- $k$: label of dismantlers, $k=1,2,\cdots,K$.
- $l$: label of shredders, $l=1,2,\cdots,L$.
- $o$: label of repair centers, $o=1,2,\cdots,O$.
- $s$: label of secondary markets, $s=1,2,\cdots,S$.
- $m$: label of steel mills, $m=1,2,\cdots,M$.
- $n$: label of non-ferrous smelters, $n=1,2,\cdots,N$.
- $p$: label of oil recycling factories, $p=1,2,\cdots,P$.
- $q$: label of battery recycling factories, $q=1,2,\cdots,Q$.
- $r$: label of rubber recycling factories, $r=1,2,\cdots,R$.
- $v$: label of glass recycling factories, $v=1,2,\cdots,V$.
- $w$: label of plastics recycling factories, $w=1,2,\cdots,W$.
- $u$: label of landfills, $u=1,2,\cdots,U$.
- $t$: label of recycled vehicle types, $t=1,2,3$.

Parameters

- $R_{ti}$: stock quantity of the $t$-th type of ELVs in ELV source $i$.
- $a_{tj}$: collected quantity of the $t$-th type of ELVs in collection center $j$ even if the recycling price equals 0.
- $b_{tj}$: price coefficient of the $t$-th type of ELVs in collection center $j$.
- $pc_{1o}$: unit cost of repairing the the 1st type of ELVs at repair center $o$.
- $pc_{2o}$: unit cost of repairing the the 2nd type of ELVs at repair center $o$.
- $pc_{k}$: unit cost of dismantling at dismantler $k$.
- $pc_{l}$: unit cost of shredding at shredder $l$.
- $pc_{u}$: unit cost of disposal at landfill $u$.
- $s$: unit subsidy of dismantler for dismantling vehicles from government.
- $s_{1}/s_{3}$: unit selling price of dismantler for the 2nd/3rd type of ferrous components for reusing.
- $s_{2}/s_{4}$: unit selling price of dismantler for the 2nd/3rd type of non-ferrous components for reusing.
- $s_{3}/s_{5}$: unit selling price of dismantler for the 2nd/3rd type of oil for recycling.
- $s_{4}/s_{6}$: unit selling price of dismantler for the 2nd/3rd type of battery for recycling.
- $s_{5}/s_{7}$: unit selling price of dismantler for the 2nd/3rd type of tyre for recycling.
- $s_{6}/s_{8}$: unit selling price of dismantler for the 2nd/3rd type of glass for recycling.
- $s_{7}/s_{9}$: unit selling price of dismantler for the 2nd/3rd type of plastics for recycling.
\( z'_1/z_1 \): unit selling price of shredder for the 2nd/3rd type of ferrous material for recycling.
\( z'_2/z_2 \): unit selling price of shredder for the 2nd/3rd type of non-ferrous material for recycling.
\( t_{c_{ij}} \): unit transportation cost between ELV source \( i \) and collection center \( j \).
\( t_{c_{jo}} \): unit transportation cost between collection center \( j \) and repair center \( o \).
\( t_{c_{jk}} \): unit transportation cost between collection center \( j \) and dismantler \( k \).
\( t_{c{kl}} \): unit transportation cost between dismantler \( k \) and shredder \( l \) for hulk.
\( t_{c_{lu}} \): unit transportation cost between shredder \( l \) and landfill \( u \) for ASR.
\( d_{ij} \): distance between ELV source \( i \) and collection center \( j \).
\( d_{ik} \): distance between ELV source \( i \) and dismantler \( k \).
\( d_{jk} \): distance between collection center \( j \) and dismantler \( k \).
\( d_{kl} \): distance between dismantler \( k \) and shredder \( l \).
\( d_{lu} \): distance between shredder \( l \) and landfill \( u \).
\( c_{a_{j}} \): capacity of collection center \( j \).
\( c_{a_{o}} \): capacity of repair center \( o \).
\( c_{a_{k}} \): capacity of dismantler \( k \).
\( c_{a_{l}} \): capacity of shredder \( l \).
\( c_{a_{u}} \): capacity of landfill \( u \).
\( \alpha \): weight percentage of hull in ELV.
\( \beta_{1} \): weight percentage of reusable ferrous components in ELV.
\( \beta_{2} \): weight percentage of reusable non-ferrous components in ELV.
\( \beta_{3} \): weight percentage of oil in ELV.
\( \beta_{4} \): weight percentage of batteries in ELV.
\( \beta_{5} \): weight percentage of tyres in ELV.
\( \beta_{6} \): weight percentage of glass in ELV.
\( \beta_{7} \): weight percentage of plastics in ELV.
\( \eta \): weight percentage of ASR in hulk.
\( \eta_{1} \): weight percentage of ferrous material in hulk.
\( \eta_{2} \): weight percentage of non-ferrous material in hulk.

**Decision variables**

\( \rho_{tj} \): unit buy-back price of the \( t \)-th ELV paid by collecting center \( j \).
\( \theta_{tj} \): amount of the \( t \)-th vehicle purchased by collecting center \( j \).
\( A_{1_{i,j}} \): amount of the \( t \)-th type of ELVs transported from ELV source \( i \) to collection center \( j \).
\( B_{1_{j,o}} \): amount of the 1st type of ELVs transported from collection center \( j \) to repair center \( o \).
\( B_{2_{j,o}} \): amount of the 2nd type of ELVs transported from collection center \( j \) to repair center \( o \).
\( C_{2_{j,k}} \): amount of the 2nd type of ELVs transported from collection center \( j \) to dismantler \( k \).
\( C_{3_{j,k}} \): amount of the 3rd type of ELVs transported from collection center \( j \) to dismantler \( k \).
\( E_{2_{kl}} \): transported amount of the second hulk from dismantler \( k \) to shredder \( l \).
\( E_{3_{kl}} \): amount of the third hulk transported from dismantler \( k \) to shredder \( l \).
\( F_{lu} \): amount of ASR transported from shredder \( l \) to landfill \( u \).
\( Q_{1_{k,s}}/Q_{1_{k,s}} \): amount of the 2nd/3rd type of ferrous components transported from dismantler \( k \) to secondary market \( s \).
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subsidy. Mathematically, it is written as:

\[ \text{PUC} = \sum_{ij} (p_{ij} \cdot \theta_{1j} + p_{2j} \cdot \theta_{2j} + p_{3j} \cdot \theta_{3j}), \]

where \( \text{PUC} \) is the following total purchasing cost:

\[ \text{PUC} = \sum_{ij} (p_{ij} \cdot \theta_{1j} + p_{2j} \cdot \theta_{2j} + p_{3j} \cdot \theta_{3j}), \]

\( PC \) is the total processing cost of repair centers, dismantlers, shredders and landfills, which reads

\[ PC = \sum_{ij} (p_{ij} \cdot \theta_{1j} + p_{2j} \cdot \theta_{2j} + p_{3j} \cdot \theta_{3j}), \]

\( TC \) is the total transportation cost on each arc of the network:

\[ TC = \sum_{ij} (A_{ij} + A_{2ij} + A_{3ij} \cdot d_{ij} + \sum_{j} \sum_{ij} (B_{1jo} + B_{2jo}) \cdot d_{ij} + \sum_{k} \sum_{ij} (C_{2jk} + C_{3jk}) \cdot d_{jk} + \sum_{k} \sum_{ij} (E_{2kl} + E_{3kl}) \cdot d_{kl} + \sum_{k} \sum_{ij} (Q_{1ks} + Q_{1'ks}) \cdot d_{ks} + \sum_{k} \sum_{ij} (Q_{2ks} + Q_{2'ks}) \cdot d_{ks} + \sum_{k} \sum_{ij} (Q_{3kp} + Q_{3'kp}) \cdot d_{kp} + \sum_{k} \sum_{ij} (Q_{4kq} + Q_{4'kq}) \cdot d_{kq} + \sum_{k} \sum_{ij} (Q_{5kr} + Q_{5'kr}) \cdot d_{kr} + \sum_{k} \sum_{ij} (Q_{6kv} + Q_{6'kv}) \cdot d_{kv} + \sum_{k} \sum_{ij} (Q_{7kw} + Q_{7'kw}) \cdot d_{kw} + \sum_{i} \sum_{j} \sum_{ij} (Q_{8im} + Q_{8'lm}) \cdot d_{im} + \sum_{i} \sum_{j} \sum_{ij} (Q_{9in} + Q_{9'ln}) \cdot d_{in}, \]

where \( i,j,k,l,m,n,p,q,r,s,t,u,v,w,x,y,z \) are sets representing different types of material or actions in the recycling system.
$RE$ represents the following income from the sale of isolated component, materials and refurbished vehicles:

$$ RE = \sum_{k} \sum_{s} (s_1 Q_{1k} + s_2 Q_{2k} + s_3 Q_{3k}) + \sum_{k} \sum_{p} (s_4 Q_{4k} + s_5 Q_{5k}) + \sum_{k} \sum_{q} (s_6 Q_{6k} + s_7 Q_{7k}) + \sum_{k} \sum_{v} (s_8 Q_{8k} + s_9 Q_{9k}) + s_0 \sum_{o} (B_{1jo} + B_{2jo}), $$

and $TS$ stands for the following total subsidy from the government:

$$ TS = \sum_{j} \sum_{k} s(C_{2jk} + C_{3jk}). $$

2.4. Constraints. Next, we present some practical constraints in minimizing the total recycling expense.

Owing to linear relationship assumption between the take-back price and recycled amount, it can be given by

$$ \theta_{tj} = a_{tj} + b_{tj} \cdot \rho_{tj}, \ \forall j \in J, \ \forall t = 1, 2, 3, $$

where $a_{tj}$ embodies the effects of public awareness of environmental protection on the recycling amount even if no any buy-back price is paid to the ELV owners, and $b_{tj}$ as the price coefficient shows how the buy-back price of the ELVs affects the recycling amount.

Note that the linear model (7) is the simplest regression model to fit the practical data on the relation between the recycled amount and the take-back price. With this simplification, the objective in Model (1) is quadratic function of the decision variables. It is possible that a nonlinear regression model more fits the practical data than (7), but Model (1) will become more complicated for development of efficient algorithms to find its solution.

The second type of constraints is on material flow balance of network. It reads

$$ \sum_{j} A_{1ij} \leq R_{ti}, \ \forall i \in I, \ \forall t = 1, 2, 3, $$

$$ \sum_{i} A_{1ij} = \theta_{tj}, \ \forall j \in J, \forall t = 1, 2, 3, $$

$$ \sum_{i} A_{2ij} = \sum_{o} B_{1jo} + \sum_{k} C_{2jk}, \ \forall j \in J, $$

$$ \sum_{i} A_{3ij} = \sum_{k} C_{3jk}, \ \forall j \in J, $$

$$ \sum_{l} E_{2kl} = \alpha \sum_{j} C_{2jk}, \ \sum_{l} E_{3kl} = \alpha \sum_{j} C_{3jk}, \ \forall k \in K, $$

$$ \sum_{s} Q_{1ks} = \beta_1 \sum_{j} C_{3jk}, \ \sum_{s} Q_{1'ks} = \beta_1 \sum_{j} C_{2jk}, \ \forall k \in K, $$

$$ \sum_{s} Q_{2ks} = \beta_2 \sum_{j} C_{3jk}, \ \sum_{s} Q_{2'ks} = \beta_2 \sum_{j} C_{2jk}, \ \forall k \in K. $$
The third type of constraints is on capacities of collection centers, dismantlers, shredders and landfills. The following inequalities are satisfied:

\[ \sum_j Q_{3k} = \beta_3 \sum_j C_{3jk}, \quad \sum_p Q_{4p} = \beta_3 \sum_j C_{2jk}, \quad \forall k \in K \]  \hspace{1cm} (16)

\[ \sum_q Q_{4q} = \beta_4 \sum_j C_{3jk}, \quad \sum_q Q_{5q} = \beta_4 \sum_j C_{2jk}, \quad \forall k \in K, \]  \hspace{1cm} (17)

\[ \sum_r Q_{5r} = \beta_5 \sum_j C_{3jk}, \quad \sum_r Q_{5r} = \beta_5 \sum_j C_{2jk}, \quad \forall k \in K, \]  \hspace{1cm} (18)

\[ \sum_v Q_{6v} = \beta_6 \sum_j C_{3jk}, \quad \sum_v Q_{6v} = \beta_6 \sum_j C_{2jk}, \quad \forall k \in K, \]  \hspace{1cm} (19)

\[ \sum_w Q_{7w} = \beta_7 \sum_j C_{3jk}, \quad \sum_w Q_{7w} = \beta_7 \sum_j C_{2jk}, \quad \forall k \in K, \]  \hspace{1cm} (20)

\[ \sum_m Q_{8ml} = \eta_1 \sum_k E_{3kl}, \quad \sum_m Q_{8ml} = \eta_1 \sum_k E_{2kl}, \quad \forall l \in L, \]  \hspace{1cm} (21)

\[ \sum_n Q_{9ln} = \eta_2 \sum_k E_{3kl}, \quad \sum_n Q_{9ln} = \eta_2 \sum_k E_{2kl}, \quad \forall l \in L, \]  \hspace{1cm} (22)

\[ \sum_u F_{lu} = \eta \sum_k (E_{2kl} + E_{3kl}), \quad \forall l \in L. \]  \hspace{1cm} (23)

The third type of constraints is on capacities of collection centers, dismantlers, shredders and landfills. The following inequalities are satisfied:

\[ \sum_j (A_{1ij} + A_{2ij} + A_{3ij}) \leq ca_j, \quad \forall j \in J, \]  \hspace{1cm} (24)

\[ \sum_j (B_{1jo} + B_{2jo}) \leq ca_o, \quad \forall o \in O, \]  \hspace{1cm} (25)

\[ \sum_j (C_{2jk} + C_{3jk}) \leq ca_k, \quad \forall k \in K, \]  \hspace{1cm} (26)

\[ \sum_k (E_{2kl} + E_{3kl}) \leq ca_l, \quad \forall l \in L, \]  \hspace{1cm} (27)

\[ \sum_l F_{lu} \leq ca_u, \quad \forall u \in U. \]  \hspace{1cm} (28)

The fourth type of constraints is on non-negativity of decision variables:

\[ A_{1ij}, B_{1jo}, B_{2jo}, C_{2jk}, C_{3jk}, E_{2kl}, E_{3kl}, F_{lu}, Q_{1ks}, Q_{1's}, Q_{2ks}, Q_{2's}, \]  \hspace{1cm} (29)

\[ Q_{3kp}, Q_{3'kp}, Q_{4kp}, Q_{4'kp}, Q_{5kr}, Q_{5'kr}, Q_{6kv}, Q_{6'kv}, \]  \hspace{1cm} (29)

\[ Q_{7kw}, Q_{7'kw}, Q_{8lm}, Q_{9ln} \geq 0, \quad \forall t, i, j, o, k, l, s, m, n, p, q, r, v, w, u. \]  \hspace{1cm} (29)

The last type of constraints is on integer variables:

\[ A_{1ij}, B_{1jo}, B_{2jo}, C_{2jk}, C_{3jk} \] are integer numbers, \( \forall t, i, j, o, k. \)  \hspace{1cm} (30)

Consequently, we obtain a mixed integer nonlinear optimization model for the management problem of the ELV recovery system:

\[ \max \quad \Pi \]  \hspace{1cm} (31)

subject to (7) – (30).

**Remark 1.** Compared with the model built in [5], our model (31) considers the refurbished percentage of ELVs in the repair centers. In other words, Model (31) can provide an optimal decision on the refurbished and the dismantled percentages for the 2nd type of ELVs such that it more conforms to the practical situations. On
the other hand, compared with the model built in [14], our model (31) takes into
account the government subsidy to the formal recycling enterprises. Both of them
are helpful to improve the performance of the ELV recovery system from the view
point of environmental protection and resource utilization.

Remark 2. Different from the linear models available in the literature, Model (31)
is nonlinear because the take-back prices are incorporated into the model as endoge-
nous decision variables. Therefore, it is valuable to develop an efficient algorithm
to solve Model (31).

2.5. Convexity of model and development of algorithm. For development of
an efficient algorithm to solve Model (31) (rather than heuristic algorithms as in [1,
10,24]), we first relax the original problem (31) by removing the integer constraints.
It reads
\[
\begin{align*}
\min & \quad -\Pi = PUC + PC + TC - RE - TS \\
\text{subject to} & \quad (7) - (29).
\end{align*}
\]
(32)

We call (32) the relaxed problem of (31). We first prove that (32) is a convex
quadratic programming problem. Note that the objective function of (32) can be
written as:
\[
\min z = \frac{1}{2}x^T H x + f x,
\]
where \(x\) is the vector of continuous decision variables, \(f\) is a given vector of co-
efficients in the linear terms of the objective function. Denote \(H\) the matrix of
coefficients in the quadratic terms of the objective function in the relaxed problem.
By direct calculation, we have
\[
H = \begin{pmatrix}
H_1 & H_2 \\
H_3 & H_4
\end{pmatrix}, \quad H_1 = \begin{pmatrix}
2b_{11} & \cdots & 2b_{15} \\
\cdots & \ddots & \cdots \\
2b_{15} & \cdots & 2b_{15}
\end{pmatrix}_{5 \times 5},
\]
(33)
\[
H_2 = \begin{pmatrix}
2b_{21} & \cdots & 2b_{25} \\
\cdots & \ddots & \cdots \\
2b_{25} & \cdots & 2b_{25}
\end{pmatrix}_{5 \times 5},
\]
\[
H_3 = \begin{pmatrix}
2b_{31} & \cdots & 2b_{35} \\
\cdots & \ddots & \cdots \\
2b_{35} & \cdots & 2b_{35}
\end{pmatrix}_{5 \times 5},
\]
\[
H_4 = \begin{pmatrix}
0 & \cdots & 0 \\
\cdots & \ddots & \cdots \\
0 & \cdots & 0
\end{pmatrix}_{433 \times 433}.
\]

Since \(b_{ij} > 0\) ( for all \(j \in J\) and \(t = 1, 2, 3\), \(H\) is positive semi-definite. In other
words, the objective function \(z\) is convex. On the other hand, all the constraints in
Model (32) are linear. Therefore, we can draw the following conclusion.

**Theorem 2.1.** Any local optimal solution of Problem (32) is its global one.

**Proof.** The result is directly from the convexity of Problem (32).

By Theorem 2.1, if we find a local optimal solution of (32), then it is the global
one. We now describe a branch and bound algorithm for solving the original problem
(31).

**Algorithm 1.** (An Extended Branch and Bound Algorithm (EBBA))

**Step 1:** Solve the relaxed problem (32). If the obtained optimal solution is
a feasible solution of (31), then the solution is an optimal solution of (31). The
algorithm stops. Otherwise, denote $z_0$ the minimal value of the objective function in (32). Clearly, $z_0$ is a lower bound of the objective function in (31).

**Step 2:** Arbitrarily choose a variable $x_i$ in the optimal solution of (32) that does not satisfy the integer conditions (30). Construct two new constraints $x_i \leq \lfloor b_i \rfloor$ and $x_i \geq \lfloor b_i \rfloor + 1$, where $b_i$ is the value of optimal solution $x_i$ and $\lfloor b_i \rfloor$ is the largest integer less than $b_i$. Add the two constraints to (32), respectively. As a result, we divide the relaxed problem (32) into two subproblems, whose feasible regions are two non-intersected subsets of the original feasible region such that the infeasible point $x_i$ is eliminated.

**Step 3:** Solve the two subproblems, respectively. Replace $z_0$ by the least value of the objective function in all the reserved subproblems to obtain a new lower bound of the objective function. If there exists a feasible solution of Model (31) in the optimal solutions of the two subproblems, denote $z^*$ the least value of the objective function at this feasible solution. Clearly, $z^*$ is an upper bound of the objective function in the original problem (31), which can be used to judge whether the current subproblem does not need to be further branched or not (pruning). When $z^* = z_0$, the algorithm stops; Otherwise, go to Step 4. If there does not exist any feasible solution of Model (31) in the optimal solutions of the two subproblems, go to Step 5.

**Step 4:** For the branch whose value of the objective function is larger than the upper bound $z^*$, cut this branch since its any feasible solution must not be an optimal solution of the original problem (31). For the branch whose value of the objective function is less than the upper bound $z^*$, reserve it. Go to Step 5.

**Step 5:** Choose one of the reserved subproblems, whose value of the objective function is the smallest among all the active subproblems, go to Step 2. If the optimal solution of its subproblems is a feasible solution of Model (31), take the smallest value of the objective function among all the current feasible solutions as a new upper bound $z^*$. When $z^* = z_0$, the algorithm stops; Otherwise, go to Step 4.

**Remark 3.** Algorithm 1 is an extension of the classical branch and bound method (CBB) to solve the mixed nonlinear programming model (31). Different from CBB, the relaxed subproblems in EBBA are nonlinear, rather than a linear programming problem. Fortunately, by Theorem 2.1, all the relaxed subproblems are convex quadratic programming problems. In virtue of convexity, we can always obtain the global optimal solution of subproblems by many powerful solvers for local optimization. On the other hand, compared with a general enumeration method or heuristic algorithms, Algorithm 1 can reduce the number of subproblems and improve search efficiency by pruning strategy.

**3. Case study.** In this section, we will apply the proposed model and algorithm in Section 2 to solve a practical ELV recovery management problem in Hunan, China.

**3.1. Case description.** In China, a number of ELV recovery enterprises has been approved [23]. In this case study, we attempt to deal with the ELV recovery management problem of the five key cities (Changsha, Zhuzhou, Xiangtan, Hengyang, Shaoyang) in Hunan province, considering that the data can be collected completely (see Figure 2).

For convenience, the centers of the five cities are regarded as the ELV sources, and all of the relevant collecting enters, repair centers, dismantlers, shredders, landfills, secondary markets and recycling factories are distributed in these cities with given
locations (see Tables 1–5). It is noted that for some cities, there are more than one dismantler, secondary market, oil or glass factory, or there is no any landfill, secondary market, rubber or plastics factory in practice.

Figure 2. The map and the existent ELV recycling network in Hunan

Table 1. Number of different types of nodes in ELV recovery network

| I | J | O | K | L | U | S | P | Q | R | V | W | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 2 | 6 | 5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

Table 2. Distribution of nodes in ELV recovery network

|                           | Changsha | Zhuzhou | Xiangtan | Hengyang | Shaoyang |
|---------------------------|----------|---------|----------|----------|----------|
| Resource                  | 1        | 2       | 3        | 4        | 5        |
| Collection center         | 1        | 2       | 3        | 4        | 5        |
| Repair center             | 1,2      | -       | -        | -        | -        |
| Dismantler                | 1        | 2,3     | 4        | 6        | 5        |
| Shredder                  | 2        | 1       | 3        | 5        | 4        |
| Landfill                  | 2        | -       | 1        | -        | -        |
| Secondary market          | 1,2      | -       | -        | -        | -        |
| Steel mill                | 1,2      | -       | -        | -        | -        |
| Non-ferrous smeltery      | 1        | -       | -        | 2        | -        |
| Oil factory               | 1,2      | -       | -        | -        | -        |
| Battery factory           | 1        | -       | -        | 2        | -        |
| Rubber factory            | -        | 1       | -        | -        | 2        |
| Glass factory             | 1,2      | -       | -        | -        | -        |
| Plastics factory          | -        | -       | 1        | -        | 2        |
### Table 3. Distance between the nodes of network (km)

| Resources | 1   | 2   | 3   | 4   | 5   |
|-----------|-----|-----|-----|-----|-----|
| 1         | 10  | 67.4| 46.9| 152 | 148.3|
| 2         | 49.8| 20.2| 26.9| 117 | 133.6|
| 3         | 43.7| 27.2| 13.4| 111.4| 122.1|
| 4         | 147.6| 100.7| 104.8| 6.1 | 75.5|
| 5         | 170.2| 170.5| 146.8| 109.9| 43.5|

| Repair center | 1 | 2   | 3   | 4   | 5   |
|---------------|---|-----|-----|-----|-----|
| 1             | 29.4| 40.9| 28  | 131 | 137.6|
| 2             | 155.9| 133.2| 120.6| 51.8 | 26.9|

| Dismantler | 1 | 2   | 3   | 4   | 5   | 1 | 2 |
|-----------|---|-----|-----|-----|-----|---|---|
| 1         | 21.8| 59.8| 46.4| 150.9| 153.1|
| 2         | 60.2| 13.8| 36.3| 116.4| 138.7|
| 3         | 49.8| 22.9| 31.9| 122.2| 139.2|
| 4         | 124.9| 64.8| 81.8| 48.8 | 105.1|
| 5         | 148.8| 143.3| 121.6| 84.3 | 15.5|
| 6         | 144.2| 107.3| 103.6| 16.6 | 53.3|

### Table 4. Distance between the nodes of network (Continued Table 3)

| Dismantler | 1 | 2   | 3   | 4   | 5   | 1 | 2 |
|-----------|---|-----|-----|-----|-----|---|---|
| 1         | 123.2| 25.1| 39.1| 194.6| 42.0| 10.2| 2.3|
| 2         | 75.3| 50.7| 33.4| 182.8| 21.9| 39.5| 46.7|
| 3         | 85.8| 42.6| 27.2| 183.0| 17.8| 28.8| 36.1|
| 4         | 40.4| 129.2| 87.1| 145.2| 80.1| 111.6| 119.8|
| 5         | 160.2| 185.1| 128.4| 31.7| 134.1| 155.2| 160.8|
| 6         | 100.7| 165.6| 110.9| 87.1| 110.3| 140.1| 147.7|

| Landfill | 1 | 2   | 3   | 4   | 5   | 1 | 2 |
|----------|---|-----|-----|-----|-----|---|---|
| 1         | 148.7| 39.4| 59.5| 204.0| 65.3| - | - |
| 2         | 88.1| 50.9| 14.4| 170.2| 4.7 | - | - |

| Steel mill | 1 | 2   | 3   | 4   | 5   | 1 | 2 |
|------------|---|-----|-----|-----|-----|---|---|
| 1         | 117.8| 33.0| 29.7| 185.1| 33.8| - | - |
| 2         | 114.7| 33.3| 27.2| 184.0| 30.8| - | - |

| Non-ferrous smelter | 1 | 2   | 3   | 4   | 5   | 1 | 2 |
|---------------------|---|-----|-----|-----|-----|---|---|
| 1                   | 149.4| 33.0| 65.3| 214.1| 69.3| - | - |
| 2                   | 75.7| 156.5| 106.2| 113.9| 102.7| - | - |

A part of available data on unit transportation and processing costs, selling prices and material weight percentages of ELVs are deregistered from [5] (see Tables 6–9). Referring to the government data on the scrapped vehicles [22], the stock quantities of ELVs from the five sources are estimated by

\[
R_{11} = 643, \quad R_{12} = 352, \quad R_{13} = 250, \quad R_{14} = 652, \quad R_{15} = 646, \\
R_{21} = 858, \quad R_{22} = 469, \quad R_{23} = 334, \quad R_{24} = 869, \quad R_{25} = 861, \\
R_{31} = 1073, \quad R_{32} = 587, \quad R_{33} = 418, \quad R_{34} = 1087, \quad R_{35} = 1077.\]
Table 5. Distance between the nodes of network (Continued Table 4)

|     | Oil   | Battery | Rubber | Glass | Plastics |
|-----|-------|---------|--------|-------|----------|
|     | 1     | 2       | 1      | 2     | 1        |
| ND  | 12.8  | 8.7     | 6.6    | 177.7 | 36.1     |
| 1   | 53.4  | 53.8    | 42.7   | 143.54| 17.3     |
| 2   | 42.8  | 43.6    | 32.1   | 149.3 | 9.8      |
| 3   | 121.9 | 129.2   | 115.2  | 72.9  | 85.9     |
| 4   | 153.6 | 171.4   | 157.5  | 86.9  | 142.9    |
| 5   | 145.1 | 158.6   | 143.4  | 30.7  | 118.5    |
| 6   | ND    | 12.8    | 8.7    | 6.6   | 177.7    |

For the need of scenario analysis, the least collected quantities and the price coefficients are assumed to be

\[ a_{1j} = (0 \ 0 \ 0 \ 0 \ 0), a_{2j} = (0 \ 0 \ 0 \ 0 \ 0), a_{3j} = (321 \ 174 \ 126 \ 324 \ 321), \]

\[ b_{1j} = (0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01), b_{2j} = (0.02 \ 0.02 \ 0.02 \ 0.02 \ 0.02), \]

\[ b_{3j} = (0.7 \ 0.7 \ 0.7). \]

In Table 5, ND stands for the serial number of dismantlers. The aim of this case study is to answer how to determine an optimal transportation plan and optimal take-back prices for the ELV recycling network.

Table 6. Capacity (ton) and unit processing cost (yuan RMB/ton)

| \(c_{a_j}\) | \(c_{a_k}\) | \(c_{a_l}\) | \(c_{a_u}\) | \(p_{c_{1o}}\) | \(p_{c_{2o}}\) | \(p_{c_{k}}\) | \(p_{c_{l}}\) | \(p_{c_{u}}\) |
|------------|------------|------------|------------|-------------|-------------|-------------|-------------|-------------|
| 2000       | 2000       | 1500       | 500        | 2000        | 3000        | 1960        | 270         | 500         |

Table 7. Unit transportation cost (yuan RMB/ton-km)

| \(t_{c_{ij}}\) | \(t_{c_{jo}}\) | \(t_{c_{jk}}\) | \(t_{c_{kl}}\) | \(t_{c_{ku}}\) | \(t_{c_{km}}\) | \(t_{c_{kn}}\) | \(t_{c_{kq}}\) | \(t_{c_{kr}}\) | \(t_{c_{kv}}\) | \(t_{c_{kw}}\) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2              | 1              | 0.8            | 0.4            | 1              | 1.5            | 0.7            | 0.6            | 0.5            | 0.5            | 0.7            |

Table 8. Unit selling prices of recycled components \((\times 10^3\ \text{yuan RMB/ton})\)

| \(s\) | \(s_0\) | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(s_5\) | \(s_6\) | \(s_7\) | \(z_1\) | \(z_2\) |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 3000  | 50000   | 2400    | 12000   | 4000    | 600     | 150     | 450     | 6000    | 500     | 1500    |
| 27360 | 136800  | 45600   | 6840    | 17100   | 5130    | 68400   | 5700    | 17100   |

3.2. Numerical solution of the ELV recycling model. We conduct all the numerical experiments on a private computer with 2.5 GHz Intel Core processor and 6 GB of RAM. The computation time required to solve the optimization model (31) by the LINGO solver or Algorithm 1 is less than 20 CPU seconds, while it is more than 500 CPU seconds by the MATLAB solver. Additionally, the obtained optimal solution of Model (31) by Algorithm 1 or by LINGO solver is better than that by the MATLAB solver. Therefore, we solve Model (31) by Algorithm 1 in
The corresponding total profit is $2.393726 \times 10^7$ Yuan (RMB). The optimal refurbishing rate of the three types of ELVs are 749, 999 and 3078, respectively. The optimal solution in case study is shown in Table 10. The optimal refurbishing rates are 0.81, 0.06, 0.04, 0.017, 0.013, 0.03, 0.015, 0.015, 15/81, 62/81, and 4/81.

Table 9. Weight percentages in the recycled ELVs

| $\alpha$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $\eta_1$ | $\eta_2$ |
|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.81     | 0.06      | 0.04      | 0.017     | 0.013     | 0.03      | 0.015     | 0.015     | 15/81     | 62/81     | 4/81      |

Table 10. Optimal solution in case study

| DV  | OS | DV  | OS | DV  | OS | DV  | OS | DV  | OS |
|-----|----|-----|----|-----|----|-----|----|-----|----|
| $\rho_{1,1}$ | 15000 | $A_{3,1,1}$ | 119 | $E_{3,2,5}$ | 468.2 | $Q_{3,1,1}$ | 92.8 | $Q_{6,2,1}$ | 8.7 |
| $\rho_{1,2}$ | 15000 | $A_{3,2,2}$ | 578 | $E_{3,3,5}$ | 442.3 | $Q_{3,2,1}$ | 106.4 | $Q_{6,3,1}$ | 8.2 |
| $\rho_{1,3}$ | 15000 | $A_{3,2,3}$ | 9 | $E_{3,5,3}$ | 507.1 | $Q_{3,6,1}$ | 111.5 | $Q_{6,3,5}$ | 9.4 |
| $\rho_{1,4}$ | 15000 | $A_{3,3,3}$ | 418 | $E_{3,6,3}$ | 531.4 | $Q_{3,3,2}$ | 34 | $Q_{6,6,1}$ | 9.9 |
| $\rho_{1,5}$ | 14900 | $A_{3,4,4}$ | 656 | $F_{3,2}$ | 352.7 | $Q_{3,2,1}$ | 34 | $Q_{6,1,2}$ | 3 |
| $\rho_{2,1}$ | 10000 | $A_{3,5,5}$ | 626 | $F_{5,2}$ | 228.4 | $Q_{3,3,1}$ | 34 | $Q_{6,2,1}$ | 3 |
| $\rho_{2,2}$ | 10000 | $B_{1,1,1}$ | 150 | $Q_{1,1,2}$ | 40.3 | $Q_{3,6,1}$ | 34 | $Q_{6,3,1}$ | 3 |
| $\rho_{2,3}$ | 10000 | $B_{1,2,1}$ | 150 | $Q_{1,2,1}$ | 34.7 | $Q_{4,1,1}$ | 8.7 | $Q_{6,3,1}$ | 3 |
| $\rho_{2,4}$ | 10000 | $B_{1,3,1}$ | 150 | $Q_{1,3,1}$ | 32.8 | $Q_{4,2,1}$ | 7.5 | $Q_{7,1,1}$ | 10.1 |
| $\rho_{2,5}$ | 9950 | $B_{1,4,2}$ | 150 | $Q_{1,5,1}$ | 37.6 | $Q_{4,3,1}$ | 7.1 | $Q_{7,2,1}$ | 8.7 |
| $\rho_{3,1}$ | 501.4 | $B_{1,5,2}$ | 149 | $Q_{1,6,1}$ | 39.4 | $Q_{4,5,2}$ | 8.1 | $Q_{7,3,1}$ | 8.19 |
| $\rho_{3,2}$ | 577.1 | $B_{2,5,2}$ | 199 | $Q_{1,7,2}$ | 12 | $Q_{4,6,2}$ | 8.5 | $Q_{7,5,2}$ | 9.4 |
| $\rho_{3,3}$ | 600 | $C_{2,1,1}$ | 200 | $Q_{1,8,1}$ | 12 | $Q_{4,1,1}$ | 2.6 | $Q_{7,6,2}$ | 9.84 |
| $\rho_{3,4}$ | 474.3 | $C_{2,2,2}$ | 200 | $Q_{1,9,1}$ | 12 | $Q_{4,2,1}$ | 2.6 | $Q_{7,1,1}$ | 3 |
| $\rho_{3,5}$ | 435.7 | $C_{2,3,3}$ | 200 | $Q_{1,10,1}$ | 12 | $Q_{4,3,1}$ | 2.6 | $Q_{7,2,1}$ | 3 |
| $A_{1,1,1}$ | 150 | $C_{2,4,6}$ | 200 | $Q_{2,1,2}$ | 26.9 | $Q_{4,6,2}$ | 2.6 | $Q_{7,3,1}$ | 3 |
| $A_{1,2,2}$ | 150 | $C_{3,1,1}$ | 672 | $Q_{2,2,1}$ | 23.1 | $Q_{5,1,1}$ | 20.16 | $Q_{7,6,2}$ | 3 |
| $A_{1,3,3}$ | 150 | $C_{3,2,2}$ | 578 | $Q_{2,3,1}$ | 21.8 | $Q_{5,2,1}$ | 17.3 | $Q_{8,3,2}$ | 1210.8 |
| $A_{1,4,4}$ | 150 | $C_{3,3,3}$ | 546 | $Q_{2,5,1}$ | 25.0 | $Q_{5,3,1}$ | 16.4 | $Q_{8,5,2}$ | 696.5 |
| $A_{1,5,5}$ | 149 | $C_{3,4,6}$ | 656 | $Q_{2,6,1}$ | 26.2 | $Q_{5,5,2}$ | 18.8 | $Q_{8,3,2}$ | 247.9 |
| $A_{2,1,1}$ | 200 | $C_{3,5,5}$ | 626 | $Q_{2,7,2}$ | 8 | $Q_{5,6,2}$ | 19.7 | $Q_{8,2,2}$ | 247.9 |
| $A_{2,2,2}$ | 200 | $E_{2,1,3}$ | 162 | $Q_{2,8,1}$ | 8 | $Q_{5,1,1}$ | 6 | $Q_{9,3,1}$ | 79.1 |
| $A_{2,3,3}$ | 200 | $E_{2,2,5}$ | 162 | $Q_{2,9,1}$ | 8 | $Q_{5,2,1}$ | 6 | $Q_{9,3,1}$ | 45.5 |
| $A_{2,4,4}$ | 200 | $E_{2,3,5}$ | 162 | $Q_{2,10,1}$ | 8 | $Q_{5,3,1}$ | 6 | $Q_{9,3,1}$ | 16.2 |
| $A_{2,5,5}$ | 199 | $E_{2,6,3}$ | 162 | $Q_{3,1,2}$ | 114.2 | $Q_{5,6,2}$ | 6 | $Q_{9,5,1}$ | 16.2 |
| $A_{3,1,1}$ | 672 | $E_{3,1,3}$ | 544.3 | $Q_{3,2,1}$ | 98.3 | $Q_{6,1,2}$ | 10.1 |

These numerical experiments, and the optimal solutions are reported in Table 10. The corresponding total profit is $2.393726 \times 10^7$ Yuan (RMB).

In Table 10, DV stands for the decision variables in Model (31), and OS represents the output of the optimal solutions of Model (31). The results in Table 10 indicate that:

(1) All the ELVs from the five sources are transported to Collection centers 1, 2, 3, 4 and 5, where the ELVs from sources 1 needs service of the two collection centers 1 and 3, those from Source 2 needs that of the collection centers 2 and 3 for the 3rd type of ELVs. Clearly, such an optimal decision shown in Table 10 benefits from the proposed model and algorithm in this paper.

(2) The total recycling quantities of the 1st type of , the 2nd type of and the 3rd type of ELVs are 749, 999 and 3078, respectively. The optimal refurbishing rate of
(3) The maximal admissible quantity of ELVs from the ELV sources is 672, which is attained by the first collection center. The minimal quantity of transportation is occurred to the third collection center (9 units of ELVs). It indicates that the proposed model in this paper can identify which one of the collection centers to be more critical in the practical production.

(4) All the ELVs at the collection centers are transported to Repair centers 1 and 2 or Dismantlers 1, 2, 3, 5 and 6, rather than using all of these infrastructures. In other words, in practical operation of the ELV recovery system, our model can answer whether it is necessary to close a part of the repair centers or the dismantlers according to the given collected quantities of ELVs.

(5) All the hulks are transported from the dismantlers to Shredders 3 and 5. Then, from the shredders, the total 2403.0 tons of ferrous material and 157.6 tons of non-ferrous material are sold to the steel mills and the non-ferrous smelters, respectively.

(6) The total disposal quantity is 581.1 tons and all the ASR are disposed in the second landfill.

There is no any doubt that the above optimal management policy can not be obtained if no rational model, just like Model (31) in this paper, had been constructed.

4. Sensitivity analysis. In this section, by sensitivity analysis of model parameters, we attempt to reveal some useful managerial implications from the proposed model and algorithm [19, 20, 26]. Specifically, we want to address the following issues:

(1) How does the public awareness of environmental protection affect the optimal strategy of the ELV recovery management?

(2) What are the impacts of government subsidy on the total profit and on the recycled amount?

(3) For the different cost coefficients, what are the impacts of cost reductions on the profit and on the recycled amount?

4.1. Sensitivity of environmental awareness. Since our model (31) has taken into account the effects of public awareness of environmental protection on the recycling amount, we investigate what is the influence of environmental awareness on the cost structure by numerical methods.

In (7), the public awareness of environmental protection is described by $a_{ij}$. Figure 3 show the changes of different types of the costs if the ELV owners have different environmental awareness. Clearly, a larger the value of $a_{ij}$ means a stronger environmental awareness. Since price sensitivity of the ELV owners from different regions may be not the same, we also take different values of $b_{ij}$ in Model (31). With this consideration, the following three scenarios are designed for the case study:

Scenario 1: $a_3 = (107 \ 58 \ 42 \ 108 \ 107)$, $b_3 = (1.2 \ 1.2 \ 1.2 \ 1.2 \ 1.2)$.

Scenario 2: $a_3 = (321 \ 174 \ 126 \ 324 \ 321)$, $b_3 = (0.7 \ 0.7 \ 0.7 \ 0.7 \ 0.7)$.

Scenario 3: $a_3 = (536 \ 293 \ 209 \ 539 \ 538)$, $b_3 = (0.3 \ 0.3 \ 0.3 \ 0.3 \ 0.3)$.

Clearly, the above three scenarios can stand for the following typical regions:

Scenario 1 (underdeveloped area): lower environmental awareness with higher price sensitivity.
Scenario 2 (developing area): medium environmental awareness with medium price sensitivity.  
Scenario 3 (developed area): higher environmental awareness with lower price sensitivity.

Figure 3. Effect of public environmental protection awareness

It follows from the results in Figure 3 that with improved environmental awareness, just like common belief, all of the total purchasing cost, the processing cost, the transportation cost and the subsidy decrease. Therefore, either for the relevant enterprises or for the government, necessary investments to improve the public’s awareness of environmental protection may play an important role in reducing all kinds of costs for recycling ELVs. In future research, as the proposed model in this paper is applied into the management problem of recycling ELVs for a given region, it is valuable to investigate an appropriate model to estimate those parameters which can describe the public’s awareness of environmental protection in that region.

4.2. Optimal subsidy. In our previous research [24], we have shown that the government subsidy is important to establish an efficient recovery system. Larger subsidy investment will undoubtedly attract more social capital into the recycle of ELVs. However, it brings greater financial burden on the government. Thus, for each specific recovery system, it is an interesting issue to search for an optimal subsidy rate of government.

In virtue of Model (31), we similarly conduct the sensitivity analysis of subsidy so as to reveal what are the impacts on the total profit and recycled amount. Then, we attempt to find out an optimal subsidy rate by observing how the profit and the recycled amount change as we choose different values of subsidies.

By changing the value of subsidy in Model (31) with a step length of 5% increment, we obtain optimal solutions corresponding to these subsidies. Numerical results are presented in Figure 4.

From Figure 4, it is easy to see that:

1. An increment of government subsidies can make the total profit of the system increase. Besides, the growth rate of profit gradually becomes greater (see Figure 4(a)). That is to say, increasing subsidy will bring positive system benefit.
2. With an increasing subsidy, the recycled amount of the 3rd type of ELVs becomes greater gradually. When the value of subsidy reaches about 2800 Yuan
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RMB, all of the the 3rd type of ELVs will be recycled (see Figure 4(b)). From the point of environmental protection, it suggests that the government should raise the subsidy to 2800 Yuan RMB. It implies that there exists an optimal subsidy in the practical management of ELV system, and we can find such a subsidy by construction of more and more realistic models.

(3) A small increment in subsidy causes a rapid decline of the refurbishing ratio of the 2nd type of ELVs. As the value of subsidy exceeds 2100 Yuan RMB, the ratio drops to zero (see Figure 4(b)). That is to say, if the government subsidy is large enough, the recycling company would not like to refurbish the ELVs. The amount of dismantling slowly increases. In this case, the dismantled amount of ELVs slowly increases.

4.3. Sensitivity of cost coefficients. Lower costs often allow the enterprises to get more profits in the ELV recovery system. Whether there exist some differences among different types of cost coefficients?

We change the cost coefficients in Model (31) by step length of 5% decrement. Then, we solve the corresponding models. Numerical results are presented in Figures 5 and 6:

![Figure 5: Impact of different types of costs on profit](image)

From Figures 5 and 6, it is concluded that:
(1) A decline in cost brings growth of the total profit, no matter it is the repair cost, dismantling cost, shredding cost or the transportation cost (see Figure 5). It also coincides with general knowledge. More profound results are also revealed that the contribution of profit growth from repair cost is greater than that from other costs. Therefore, reduction of the repair costs is the priority strategic policy to increase the profit for the recycling system.

(2) The repair cost of the 1st type of ELVs can only affect the recycled amount of this type, and there exists a raising trend with the repair cost reducing. Actually, the recycled amount increases from 774 to 999 if the repair cost cuts by half (see Figure 6(a)).

(3) The repair cost of the 2nd type of ELVs can only affect the repair amount of this type, and it also appears a rising trend with reduction of the repair cost. The repair amount increases from 1071 to 1764 if the repair cost reduces by half (see Figure 6(b)). Comparison between Figures 6(a) and 6(b) reveals that the growth rate of the 2nd type of ELVs is about three times that of the 1st type of ELVs. Thus, the enterprises could focus on reducing the repair cost of the 2nd type of ELVs.

(4) The decline in dismantling cost and shredding cost will not affect the dismantled amount of the 2nd type of ELVs, but increases the recycled amount of the 3rd type of ELVs (see Figures 6(c) and 6(d)). In contrast, the contribution of recycled amount from dismantling cost is greater than that from shredding cost.

5. Conclusions and directions of future research. In this paper, we have built a mixed integer nonlinear optimization model to formulate the management problems of recovering the ELVs, where the take-back prices are endogenous variables of the model.
By case study and sensitivity analysis, a number of practical managerial implications are revealed in virtue of the model. Summarily,

(1) Distinct treatment of ELVs with different damaged and aging degrees can increase the profit of recycling ELVs. The proposed method can provide an optimal percentage of the collected ELVs with medium damaged and aging situation to determine whether these ELVs are refurbished or are dismantled.

(2) Public awareness of environmental protection of citizens affects the total subsidy investment from government. It is suggested that higher environmental awareness of citizens can reduce financial expenditure of governments for recovering the ELVs. Both the environmental protection awareness and the government subsidy seriously affect the optimal decisions of recycle enterprises.

(3) For the entire ELV recovery system, one of the main obstacles to the profit growth is the high processing cost. Compared with the transportation cost, the processing cost plays a greater role in increasing the profit of recycling system. It suggests that adopting advanced processing technology and hi-tech machinery equipments plays the most important role in improving performance of the ELV recovery system.

(4) In our case study, the total quantity of the recycled ELVs greatly raises as the subsidy increases, which suggests that the current government subsidy for recycling the ELVs is insufficient. An optimal governmental subsidy can be obtained by the proposed numerical method in this paper.

For future research, since there exist games between governments and recycling enterprises, or between recycling enterprises and black markets in practice, it is worth further studying new models from the perspective of game theory [15], rather than from the centralized decision-making mode in this paper. In particular, in the case that collection centers and repair are owned by different agents, and all of them seek to maximize their profits, it is necessary to construct a decentralized decision-making model to meet the practical requirements.

Uncertainty of modelling parameters should also be considered in the real-world production [2]. For example, the unit transportation cost and the capacity are often fuzzy or random in practice. In this case, techniques for uncertain optimization are needed to find a robust solution. In addition, as the proposed model in this paper is applied into the management problem of recycling ELVs for a given region, it is valuable to study an appropriate model to estimate the model parameters, such as those which can fit the public’s awareness of environmental protection corresponding to that region.

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