A Simple Empirically Motivated Template for the Unresolved Thermal Sunyaev-Zeldovich Effect

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ABSTRACT

We develop a model for the power spectrum of unresolved clusters of galaxies arising from the thermal Sunyaev-Zeldovich (tSZ) effect. The model is based on a ‘universal’ gas pressure profile constrained by X-ray observations and includes a parameter to describe departures from self-similar evolution. The model is consistent with recent Planck observations of the tSZ effect for X-ray clusters with redshifts \(z \lesssim 1\) and reproduces the low amplitude for the tSZ inferred from recent ground based observations. By adjusting two free parameters, we are able to reproduce the tSZ power spectra from recent numerical simulations to an accuracy that is well within theoretical uncertainties. Our model provides a simple, empirically motivated tSZ template that may be useful for the analysis of new experiments such as Planck.

Key words: cosmology: cosmic microwave background, cosmological parameters.

1 INTRODUCTION

The thermal Sunyaev-Zeldovich signal (Sunyaev and Zeldovich 1972), caused by inverse Compton scattering of cosmic microwave background (CMB) photons by the hot plasma in clusters of galaxies, has been detected convincingly by many experiments (see Carlstrom, Holder and Reese, 2002, for a review). It has long been recognised that the integrated tSZ signal from distant, faint, unresolved clusters of galaxies would make a significant ‘secondary’ frequency-dependent contribution to the CMB temperature power spectrum at high multipoles (Cole and Kaiser 1988). However, there are many other contributors to the anisotropies at multipoles \(\ell \gtrsim 2000\), principally Poisson radio sources at low frequencies \((\nu \lesssim 100\text{GHz})\), clustered and unclustered infra-red galaxies at higher frequencies, together with the frequency-independent secondary anisotropies associated with cluster peculiar motions and inhomogeneous reionization (see e.g. Iliev et al. 2007). Isolating the unresolved tSZ contribution requires disentangling these various contributions.

This has become possible recently using high resolution observations of the CMB by the Atacama Cosmology Telescope (ACT, Dunkley et al. 2010) and by the South Pole Telescope (SPT, Lueker et al. 2010). Both the ACT and SPT teams perform multi-parameter fits to the temperature power spectra using ‘templates’ to model the secondary anisotropies. They find statistically significant evidence for a tSZ contribution, but with an amplitude at a frequency of \(\approx 150\text{ GHz}\) of only a few \((\mu\text{K})^2\) \((4.2 \pm 1.5(\mu\text{K})^2\) at \(\ell = 3000\) for SPT, and \(6.8 \pm 2.9(\mu\text{K})^2\) from ACT for the combined thermal and kinetic SZ effects). These amplitudes are significantly smaller than expected from semi-analytic predictions using the WMAP5 parameters (see e.g. Komatsu and Seljact 2002).

The earliest approaches to computing the unresolved tSZ contribution involved adopting a model for the pressure profiles of clusters combined with a Press-Schechter (Press and Schechter 1974) type theory to compute their spatial abundance as a function of mass and redshift (Cole and Kaiser 1988; Bond and Myers 1996; Cooray 2000, 2001; Komats and Seljact 2002). These calculations established the strong sensitivity of the unresolved tSZ amplitude to the normalization of the spectrum of fluctuations (roughly varying as \(\sigma_8^2\), where \(\sigma_8\) is the rms fluctuation amplitude at the present day in spheres of radius \(8\ h^{-1}\text{Mpc}\)). This led to the hope that observations of the unresolved tSZ effect could be used to constrain the amplitude of scalar fluctuations.

An alternative approach to modeling the tSZ effect is to use numerical hydrodynamic simulations incorporating as much realistic physics as possible (e.g. de Silva et al. 2000; Springel, White and Hernquist 2001; Bond et al. 2005; Lau, Kratsov and Nagai, 2009; Battaglia et al. 2010). Various other approaches have been used, including dark matter simulations (Bode, Ostriker and Vikhlinin 2000; Sehgal et al. 2010; Trac, Bode and Ostriker 2010) or Press-Schechter type calculations (Shaw et al. 2010) combined with semi-analytic prescriptions for assigning pressure profiles to dark matter halos incorporating schematic models for star formation and feedback from supernovae and active galactic nuclei.

The numerical hydrodynamic simulations, in particular, have shown just how sensitive cluster pressure profiles are to complex physics. Within \(\sim 0.2\ \tau_{500}\) \(\tau_{500}\) is the radius

\* \(h\) is the Hubble constant in units of \(100\ \text{km}\text{s}^{-1}\text{Mpc}^{-1}\).
at which the cluster has a mean overdensity of 500 times the critical density at the redshift of the cluster), the pressure profiles are sensitive to the prescriptions for star formation and feedback. At larger radii, the pressure profiles differ from those expected from hydrostatic equilibrium because of the increasing importance of non-thermal motions. Although complex and this is reflected in the relatively large scatter from those expected from hydrostatic equilibrium because of the critical density at the redshift of the cluster), the pressure and feedback. At larger radii, the pressure profiles differ from those expected from hydrostatic equilibrium because of the increasing importance of non-thermal motions. Although there has been remarkable progress in the sophistication of numerical hydrodynamic simulations, the physics involved is complex and this is reflected in the relatively large scatter between predictions of the unresolved tSZ power spectrum (see e.g Fig 3 of Battaglia et al. 2010). Early expectations that measurements of the tSZ effect (in particular, number counts and the power spectrum) could be used for precision cosmology now seem naive. It is more likely that such measurements will provide constraints on the complex physics that structures the intra-cluster medium.

Since the unresolved thermal SZ effect is constrained by fitting a template to the observed power spectra, how should the template be chosen? Should experimentalists adopt one or more highly model specific templates determined from hydrodynamic simulations? Should the uncertainties in the physics be represented by a large number of adjustable parameters? Or should experimentalists adopt a phenomenological model with fewer parameters that may be less closely linked to the physics.

In this short paper, we adopt an empirical approach to computing the tSZ power spectrum. The model is based on the (Komatsu and Seljak 2002) model (with minor modifications) but instead of using the theoretical pressure profiles computed by Komatsu and Seljak (2001) we use the ‘universal’ pressure profiles derived from X-ray observations (Arnaud et al. 2010). Moreover, we introduce an additional parameter, \( \epsilon \), to model deviations from self-similar evolution of the cluster profiles. The resulting model is simple, empirically motivated, and provides a flexible tSZ template that can match the results from recent numerical simulations.

2 THE MODEL

Unless otherwise stated, we adopt the cosmological parameters from the 6 parameter \( \Lambda \) CDM model from Table 3 of (Komatsu et al. 2011), namely \( h = 0.71, \sigma_8 = 0.80, n_s = 0.963, \Omega_L = 0.734, \Omega_b = 0.0448 \). We assume a spatially flat Universe, \( \Omega_k = 0 \), and assume that the dark energy is a cosmological constant with equation of state \( p = -\rho c^2 \).

For a Poisson distribution of clusters of mass \( M \) and comoving number mass-function \( dn/dM \), the power spectrum of the tSZ effect is given by

\[
C_\ell = g(\nu) T_0^2 \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz} \int_{M_{\min}}^{M_{\max}} \frac{dn}{dM}[\tilde{y}_t(M, z)]^2 dM, \tag{1}
\]

(Komatsu and Seljak 2002, hereafter KS02). Here \( g(\nu) \) describes the spectral dependence of the tSZ effect, which in the non-relativistic limit is given by

\[
g(\nu) = \left( x \frac{e^x + 1}{e^x - 1} - 4 \right), \quad x = \frac{h_p \nu}{k T_0}, \tag{2}
\]

where \( T_0 \) is the present temperature of the CMB and \( h_p \) is Planck’s constant. We will show results for a frequency of \( \nu = 143 \) GHz corresponding to Planck’s most sensitive channel for detection of the tSZ effect (Planck Collaboration 2011a, b, c) and close to the frequencies of the tSZ sensitive channels of ACT (148 GHz) and SPT (150 GHz). The remaining terms in (2) are as defined in KS02. In particular, \( \tilde{y}_t \) is the two-dimensional Fourier transform of the Compton y parameter:

\[
\tilde{y}_t = 4 L_{500}^{-1} \int_0^\infty dx x^2 Y_{3D}(x) \frac{\sin((\ell x) T_{500})}{(\ell x) T_{500}}, \tag{3}
\]

† The additional parameter, the optical depth \( \tau \) from late reionization of the inter-galactic medium, is unimportant for this study.

![Figure 1. Gas pressure profiles for clusters with masses \( M_{500} = 3 \times 10^{14} h^{-1} M_\odot \) and \( 3 \times 10^{13} h^{-1} M_\odot \) at \( z = 0 \) (a) and \( z = 2 \) (b). The solid lines show the X-ray ‘universal’ pressure profile of equation (5) and the dashed lines show the analytic profile of Komatsu and Seljak (2002) computed for the cosmological parameters adopted in this paper and the revised halo concentration parameter of equation (8). The X-ray profile in Figure 1(b) is plotted for self-similar evolution, i.e. \( \epsilon = 0 \) in equation (5).](image)
where $Y_{3D}$ is the three-dimensional Compton y-profile,

$$x \equiv \frac{r}{r_{500}}, \quad \ell_{500} \equiv \frac{d_A(z)}{r_{500}},$$

and $d_A(z)$ is the angular diameter distance to a cluster at redshift $z$. Notice that we have used $r_{500}$ as a characteristic radius, rather than the scale radius $r_s$ of the dark matter distribution used by KS02.

The three-dimensional Compton profile is given by

$$Y_{3D}(x) = \frac{\sigma_T}{m_e c^2} P_e(x) = \frac{\sigma_T}{m_e c^2} \left( \frac{X}{\gamma} \right) \rho(x) E(z)^{\frac{3}{2}} h \text{eV cm}^{-3},$$

where $X = 0.76$ is the primordial Hydrogen abundance and $P_e$ and $P_{gas}(x)$ are the electron and gas pressure profiles. X-ray data of the REXCESS cluster sample (Arnaud et al. 2010) suggest that clusters are well described by a ‘universal’ electron pressure profile of the form:

$$P_e(x) = 1.88 \left[ \frac{M_{500}}{10^{14} h^{-1} M_\odot} \right]^{0.787} p(x) E(z)^{\frac{3}{2}} h^2 \text{eV cm}^{-3},$$

where

$$p(x) = \frac{P_{\text{th}} h^{-3/2}}{(c_{500} x)^{\gamma}(1 + [c_{500} x]^{\gamma}/(\gamma - 1)/\sigma)},$$

with the parameters given in the first row of Table 1. The function $E(z)$ in (5) is the ratio of the Hubble parameter at redshift $z$ to its present value,

$$E(z) = \left[ (1 - \Omega_m)(1 + z)^3 + \Omega_m \right]^{1/2},$$

and the scaling $E(z)^{8/3}$ in (5) is appropriate for self-similar evolution. The parameter $\epsilon$ therefore describes departures from self-similar evolution.

The profile (5) is constrained from X-ray observations out to radii $r \sim r_{500}$ but the extrapolation beyond $r_{500}$ was designed to fit results from numerical simulations of relaxed clusters (Nagai, Vikhlinin and Kravtsov 2007). Since a significant fraction of the tSZ signal comes from $r > r_{500}$, it is important to recognise that an unresolved tSZ template based on (5), though empirically motivated, relies on: (a) an extrapolation of the pressure profiles beyond the observed range of radii; (b) a highly uncertain extrapolation of the shapes of the profiles to high redshift; (c) the assumption that well observed X-ray clusters are representative of the cluster population as a whole. These points will be discussed in further detail below.

Reliable measurements of the gas temperature in the faint cluster outskirts have become available only recently, allowing the characterization of the pressure behavior of a few clusters out to the virial radius (approximately $2r_{500}$) or beyond. Examples are the Perseus Cluster (Simionescu et al. 2011), a massive and relaxed cluster observed with the Suzaku satellite, and the dynamically younger and lower mass Virgo Cluster observed with XMM-Newton (Urban et al. 2011). After correcting for clumping of the gas at large radii in Perseus, both clusters show a pressure profile in good agreement with (5) out to $\sim 2r_{500}$. Further evidence in favour of the ‘universal’ pressure profile beyond $r_{500}$ comes from the stacked radial SZ profiles of 15 X-ray selected clusters observed with SPT (Plagge et al. 2010).

A recent analysis by Sun et al. (2011) of nearby ($z < 0.12$) galaxy groups observed with Chandra shows that their median pressure profile is also well described by (5) out to $\sim r_{500}$. However, little is known about the pressure profiles of groups at larger radii, or at higher redshifts.

Figure 1 compares the X-ray inferred gas pressure profile of equation (5) with the analytic pressure profiles used by KS02 for clusters with masses $M_{500} = 3 \times 10^{13} h^{-1} M_\odot$ and $M_{500} = 3 \times 10^{14} h^{-1} M_\odot$ at $z = 0$ and $z = 2$ assuming $\epsilon = 0$. To compute the Komatsu-Seljak profiles we have assumed a Navarro, Frenk and White (1997) dark matter profile,

$$\rho_{\text{H}} \propto \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2},$$

but with the revised concentration parameter relating the scale radius to the virial radius ($r_s = r_V/c$) derived by (Duffy et al. 2008)

$$c(M_V, z) = 5.72 \left( \frac{M_V}{10^{14} h^{-1} M_\odot} \right)^{-0.081} (1 + z)^{-0.71}.$$

The definitions of the virial radius and virial mass used here follow those of KS02. The masses $M_{500}$ inferred from X-ray observations assume hydrostatic equilibrium. Numerical simulations (e.g. Nagai, Vikhlinin and Kravtsov 2007; Lau, Kravtsov and Nagai 2009; Battaglia et al. 2010; Nagai 2011) show that non-thermal pressure becomes significant by $r_{500}$ and that assuming hydrostatic equilibrium underestimates the true mass $M_{500}$ by about 10%. We have therefore assumed a 10% correction factor to relate the X-ray mass to the true mass.

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**Table 1**

| $P_0$ | $c_{500}$ | $\gamma$ | $\alpha$ | $\beta$ |
|-------|-----------|----------|----------|--------|
| All   | 4.921     | 1.177    | 0.3081   | 1.0510 | 5.4905 |
| Cool core | 1.902     | 1.128    | 0.7736   | 1.2223 | 5.4905 |
| Non-cool core | 1.875    | 1.083    | 0.3798   | 1.4063 | 5.4096 |

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**Figure 2.** Comparison of the pressure profiles with parameters listed in Table 1. Solid (blue) lines show the average profile of all REXCESS clusters (first row of Table 1). The dashed and dotted line shows the average profile for cool core and non-cool core clusters respectively (second and third rows of Table 1).
At radii \( r \gtrsim 0.3r_{500} \) (most relevant for the unresolved tSZ effect) the X-ray inferred pressure profiles lie below the KS02 profiles (compare Figure 17 of Komatsu et al. 2011). This is expected because the KS02 profile is derived assuming an equation of state with a constant polytropic index and does not account for the increasing importance of non-thermal pressure at large radii. In fact, the integrated Compton Y-parameter \( \int Y_{\text{pp}} x^2 dx \) does not converge for the KS02 profiles. As in KS02, we arbitrarily truncate the integrals (3) at \( r = 2r_v \) to compute the power spectrum. (In contrast, the Compton Y-parameter for the X-ray pressure profiles converges and we adopt an upper cut-off of 4\( r_v \) for these profiles).

As mentioned above, the profile (6) is adjusted to match the numerical results of Nagai, Vikhlinin and Kravtsov (2007) at radii \( r \sim r_{500} \). The pressure profiles from the AGN feedback simulations of Battaglia et al. (2010) at \( z = 0 \) fall off slightly less rapidly than equation (6) at \( r \gtrsim r_{500} \). However, they find that the outer pressure profiles of clusters at a redshift \( z = 1 \) (which make the dominant contribution to \( C_\ell \) at \( \ell \sim 3000 \)) are steeper and in reasonable agreement with equation (6).

The X-ray inferred pressure profiles for cool core clusters differ systematically from those of non-cool core (often morphologically disturbed) clusters, sometimes differing by more than an order of magnitude at \( r \geq 500 \). However, there is no evidence for systematic differences in the pressure profiles at larger radii. In fact, it is the pressure profiles at \( r \gtrsim 0.2r_{500} \) that dominate the power spectrum. The inner pressure profiles have a relatively small effect on the shape of the power spectrum at high multipoles. To illustrate this, we have computed power spectra using fits of equation (6) to the mean pressure profiles of cool core and non-cool core clusters (Planck Collaboration 2011d). The parameters for these fits are listed in Table 1 and the pressure profiles are plotted in Figure 2.

Computations of the tSZ power spectrum for a frequency of 143 GHz are shown in Figure 3. The dotted (purple) lines in each panel show the KS02 model. As in KS02 we used the Jenkins et al. (2001) mass function in equation (1) and fixed other parameters (e.g. \( M_{\text{min}}, z_{\text{max}} \)) to those used in KS02. The power spectra plotted in Figure 3 therefore differ from those of KS02 only because of our choice of cosmological parameters and concentration relation \( c(M_{\text{vir}}, z) \). The peak amplitude of this model is about 12 \((\mu K)^2\), i.e. about three times higher than the amplitude inferred from ACT and SPT (Dunkley et al. 2010; Lueker et al. 2010).

The predictions of our model are shown by the solid lines in each panel for three values of the evolution parameter \( \epsilon \). The three panels show the sensitivity of the models to the shape of the inner pressure profiles. These are relatively minor, except at multipoles \( \ell \gtrsim 10^4 \), which are extremely difficult to probe experimentally. Significantly, the peak amplitude of these models is in the range \( 3 - 6 \) \((\mu K)^2\), consistent with the constraints from ACT and SPT. The observations are therefore consistent with a model based on the X-ray ‘universal’ pressure profile and nearly self-similar evolution.

All of the models in Figure 3 assume a fluctuation amplitude of \( \sigma_8 = 0.8 \). The power spectra of our models scale with amplitude as \( C_\ell \propto \sigma_8^{5/3} \) and so variations of, say, 10% in \( \sigma_8 \) have a much greater effect on the variations in the shapes of the pressure profiles and \( \epsilon \) explored in Figure 3. Note that if the KS02 model is used to fit the observations, the inferred value of \( \sigma_8 \) would be underestimated by about 14%.

The statistical cross correlation of Planck maps with X-ray detected clusters\(^\dagger\) provides a constraint on the evolution parameter \( \epsilon \) (Planck Collaboration 2011d). These authors applied a multifrequency matched filter algorithm (Melin, Bartlett and Delabrouille 2006) to the Planck data at the positions of the X-ray clusters to determine an integrated Compton parameter \( Y_{500} \). The Planck data follow a relation

\[
\frac{d^2A(Y_{500})}{M_{500}} \propto E(z)^{2/3+\epsilon},
\]

with \( \epsilon = 0.66 \pm 0.52 \) for redshifts \( z < 1 \) (see Figure 6 of Planck Collaboration 2011d). These observations do not extend to the very high redshifts \( z \sim 3 \) that contribute to the tSZ power spectrum at multipoles \( \ell \sim 10^4 \), but they provide partial overlap with the redshift range contributing at lower multipoles. The observations are consistent with self-similar evolution, \( \epsilon = 0 \), with perhaps some indication of weaker evolution. Extending this type of analysis to higher redshifts would clearly help in developing models of the unresolved tSZ effect.

The dashed and dot-dashed lines in Figure 3 show two of the tSZ templates used in the ACT analysis. The (red) dashed line shows the AGN feedback template of Battaglia et al. (2011), while the (green) dot-dashed line shows the ‘nonthermal20’ model of Trac, Bode and Ostriker (2011). Following Dunkley et al. (2010) we will refer to the former as the ‘Battaglia’ template and the latter as the ‘TBO-2’ template. These templates are based on very different approaches. The Battaglia template is derived from hydrodynamic simulations, while the TBO-2 template is based on post-processing dark matter simulations by assigning pressure profiles to dark matter halos. These templates give some indication of the theoretical uncertainties involved in computing the tSZ power spectrum. It is encouraging that both templates have about the same peak amplitude, consistent with the observations, but the slopes at both low and high multipoles differ. (Note that because of the finite computational volumes the variance of these templates at low multipoles is high and not accurately quantified).

The physical processes involved in determining the shape and amplitude of the tSZ power spectrum are complicated, and it will not be easy to improve the accuracy of the simulation templates. As mentioned in the Introduction, the choice faced by experimentalists is either to use a number of templates spanning the range of theoretical uncertainties (with possible uncertain scalings with cosmological parameters) or to adopt a parametric model. In our approach, the parametric model is empirically motivated and has a small number of free parameters (principally the amplitude and the evolutionary parameter \( \delta \)). Nevertheless, to be useful, our model should have sufficient flexibility to match the simulation templates. This is illustrated in Figure 4. For each

\(^\dagger\) The Meta-Catalogue of X-ray detected Clusters (Piffaretti et al. 2011), supplemented with 33 clusters at redshifts \( z > 0.6 \).

\(^\delta\) Other cosmological parameters, such as \( \Omega_\Lambda \) and \( \Omega_\delta \) are now so well constrained that their errors can be ignored.
Figure 3. Predictions for the unresolved tSZ power spectrum. The dotted (purple) lines show the KS02 model computed using the WMAP7 cosmological parameters and the concentration parameter of equation (8). The solid lines (blue) show computations using the X-ray pressure profile of equation (6) for three values of the evolution parameter $\epsilon$: panel (a) shows the mean profile for all REXCESS clusters; panel (b) for cool core clusters; panel (c) for non-cool core clusters. The (green) dot-dashed line and red (long dashed) line in each panel show two templates used by the ACT team (Dunkley et al. 2010): dot-dashed line shows the template from the AGN feedback simulations of Battaglia et al. (2011); long-dashed line shows the TBO-2 template from the numerical simulations of Trac, Bode and Ostriker (2011).

Figure 4. Fits of our template model (blue/solid lines) to the simulation templates (red/dashed lines). Panel (a) shows the Battaglia template and panel (b) shows the TBO-2 template. The best fit parameters $A$ and $\epsilon$ are given in each panel.

Simulation template, $C_{\ell}^{\text{tSZsim}}$, we find the amplitude $A$ and $\epsilon$ parameter that minimises

$$\chi^2 = \sum_{\ell} [C^{\text{tSZsim}}_{\ell} - A C_{\ell}(\epsilon)]^2,$$

where the sum extends over the range $1000 \leq \ell \leq 6000$. As can be seen from Figure 4, our model matches the Battaglia template to an accuracy of better than 10% over the multipole range plotted in the figure and matches the TBO-2 template to even higher accuracy. These errors are considerably smaller than the theoretical uncertainties in the simulation templates. For the Battaglia template, the best fit amplitude is $A = 0.99$, so the use of our model would not bias a measurement of $\sigma_8$. The best fit amplitude for the TBO-2 template is $A = 1.26$. If the TBO-2 template were correct, using our model would lead to a downward bias of 3% in a measurement of $\sigma_8$.

One key point, that is not yet well understood, is whether the pressure profiles of X-ray selected clusters are representative of the cluster population as a whole. As Arnaud et al. (2010) have stressed, although the X-ray luminosities of non-cool core clusters differ systematically from those of cool core clusters of the same mass, their pressure profiles at $r \gtrsim 0.3 R_{500}$ are almost identical (c.f. Figure 2). However, a cross-correlation of Planck maps with rich clusters selected from the Sloan Digital Sky Survey (Planck Col-
laboration (2011e) has revealed a possible discrepancy with X-ray ‘universal’ pressure profile. The observed correlation between \( Y_{500} \) and optical richness, \( N_{200} \), lies below the X-ray model by a factor of \(~ 1.7\) and \(~ 2.2\), depending on which empirical weak-lensing mass calibration is used to convert \( N_{200} \) to mass. It is not yet clear whether this result is indicative of a population of sub-luminous tSZ clusters under-represented in X-ray surveys, whether it is caused by some systematic error in the weak lensing mass estimates or some other systematic error such as optical projection bias. Evidently, this discrepancy needs further investigation both experimentally and via numerical simulations.

3 CONCLUSIONS

The physics required to construct an accurate model of the tSZ power spectrum is complicated. The gas pressure profiles at \( r > r_{500} \) depend on an accurate modeling of non-thermal motions in the intra-cluster medium. The profiles on smaller scales require an accurate model of star formation and various feedback processes. Furthermore, since the amplitude of the tSZ effect is independent of redshift, these processes need to be modeled accurately to high redshift (\( z > 1 \)) to predict the power spectrum at multipoles \( \ell \gtrsim 1000 \).

In this paper, we have presented a simple model for the tSZ power spectrum that is based on the X-ray inferred ‘universal’ gas pressure profile of Arnaud et al. (2010) extrapolated to higher redshift. The model is consistent with the low amplitude of the tSZ power spectrum from recent observations from ACT and SPT and is consistent with recent Planck observations of the tSZ effect for X-ray clusters with redshifts \( z > 0.5 \). Our model suggests that the ‘universal’ pressure profile extrapolated assuming nearly self-similar evolution, provides an acceptable description of the observations.

We have shown that our model provides good fits to the tSZ power spectra from recent numerical simulations, to an accuracy that is well within the theoretical uncertainties involved in such simulations. Our model may therefore be useful as a simple tSZ template, since it has only two key parameters (the overall amplitude and the evolution parameter \( \epsilon \)) and is empirically motivated.

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REFERENCES

Arnaud M., Pratt G.W., Piffaretti R., Böhringer H., Croston J.H., Pointecouteau E., 2010, A&A, 517, 92.
Battaglia N., Bond J.R., Pfrommer C., Sievers J.L., Sijacki D., 2010, ApJ, 725, 91.
Bode P., Ostriker J.P., Vikhlinin A., 2009, ApJ, 700, 989.
Bond J.R., Myers S., 1996, ApJS, 103, 1.
Bond J.R. et al., 2005, ApJ, 626, 12.
Carlstrom J.E., Holder G.P., Reese E.D., 2002, ARAA, 40, 643.
Cole S., Kaiser N., 1988, MNRAS, 233, 637.
Cooray A., 2000, PRD, 62, 3506.
Cooray A., 2001, PRD, 64, 3516.