Distinct signals of the gauge-Higgs unification in $e^+e^-$ collider experiments

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Abstract

Effects of Kaluza-Klein excited neutral vector bosons ($Z'$ bosons) in the gauge-Higgs unification on $e^+e^- \rightarrow \bar{q}q, \ell^+\ell^-$ cross sections are studied, particularly in future $e^+e^-$ collider experiments with polarized beams. Significant deviations in the energy and polarization dependence in $\sigma(\mu^+\mu^-)$, the lepton forward-backward asymmetry, $R_0(\mu) \equiv \sigma(\bar{b}b)/\sigma(\mu^+\mu^-)$ and the left-right asymmetry from the standard model are predicted.
With the establishment of the standard model (SM) by the discovery of the Higgs boson, searching for physics beyond the SM and understanding the electroweak phase transition have become a few of the main topics in particle physics. Not only large hadron colliders, but also $e^+e^-$ colliders play an important role for this purpose. In this letter we study distinct signals of the gauge-Higgs unification (GHU) \cite{1}-\cite{10} in the future $e^+e^-$ collider experiments.

In GHU the Higgs boson is a part of the extra-dimensional component of the gauge potentials, appearing as a fluctuation mode of an Aharonov-Bohm (AB) phase $\theta_H$ in the fifth dimension. As a consequence the Higgs couplings $HWW$, $HZZ$ and Yukawa couplings deviate from those in the SM in a universal manner.\cite{11} They are suppressed by a common factor $\cos \theta_H$:

$$
\frac{g_{HWW}^{GHU}}{g_{HWW}^{SM}}, \frac{g_{HZZ}^{GHU}}{g_{HZZ}^{SM}}, \frac{y_{\bar{f}f}^{GHU}}{y_{\bar{f}f}^{SM}} \simeq \cos \theta_H .
$$

For $\theta_H = \mathcal{O}(0.1)$, probable values in the model, the deviation of the couplings amounts to $1 - \cos \theta_H = \mathcal{O}(0.005)$, and is small. At the ILC at $\sqrt{s} = 250$ GeV, the $ZZH$ coupling can be measured in the 0.6\% accuracy with 2 ab$^{-1}$ data \cite{12}. Another prominent feature of the model is that the first Kaluza-Klein (KK) excited states of the neutral gauge bosons, $Z'$, have large couplings to right-handed components of quarks and leptons, viable signals of which can be seen in hadron collider experiments \cite{8, 10}.

The main purpose of this letter is to check the effect of such $Z'$ bosons using lepton collider experiments in the past and future. We first examine the GHU model with precision measurements in LEP1 experiment at $\sqrt{s} = M_Z$, and LEP2 experiments for $130$ GeV $\leq \sqrt{s} \leq 207$ GeV. Then we predict several signals of $Z'$ bosons in GHU in $e^+e^-$ collider experiments designed for future with collision energy $\sqrt{s} \geq 250$ GeV with polarized electron and positron beams.

The GHU model we consider is the $SO(5) \times U(1)_X$ gauge theory in the Randall-Sundrum warped space with metric $ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$ ($0 \leq |y| \leq +L$) where $k$ is the $AdS_5$ curvature. The warp factor $z_L \equiv e^{kL}$ is large ($\gg 1$). $SO(5)$ symmetry is broken to $SO(4) \simeq SU(2)_L \times SU(2)_R$ by the orbifold boundary conditions at $y = 0$ and $L$. The
SO(5)/SO(4) part of the gauge fields, $A^a_y \ (a = 1 \sim 4)$, plays the role of the Higgs field in the SM. $SU(2)_R \times U(1)_X$ symmetry is spontaneously broken to $U(1)_Y$ by a brane-localized scalar field at $y = 0$. Finally the $SU(2)_L \times U(1)_Y$ symmetry is dynamically broken to $U(1)_{em}$ by the Hosotani mechanism.

5D fields are expanded in KK series. In particular, there are four KK towers of the neutral vector bosons, $\gamma^{(m)}$, $Z^{(m)}$, $Z_R^{(n)}$, and $A^4(n)$ ($m = 0, 1, 2, \cdots$, $n = 1, 2, 3, \cdots$) where $\gamma^{(0)}$ and $Z^{(0)}$ correspond to the photon and $Z$ boson, respectively. These fields except for $A^4$ couple to the SM fields and can be observed as neutral $Z'$ vector bosons.

In addition to the quark-lepton multiplets in the vector representation of $SO(5)$, $N_F$ dark fermions in the spinor representation are introduced. As a consequence the electroweak symmetry breaking is achieved at the one loop level. The Higgs boson, which is massless at the tree level, acquires a finite mass $m_H$, independent of the cutoff scale. The gauge hierarchy problem is thus solved

There remain two free parameters, $N_F$ and $z_L$. Given $N_F$ and $z_L$, the effective potential $V_{\text{eff}}(\theta_H)$ is fixed. From the location of the minimum of $V_{\text{eff}}(\theta_H)$, the value $\theta_H$ is determined. There is the property called the universality such that many of the physical quantities are determined by $\theta_H$, but do not depend on $N_F$ and $z_L$ independently. In the following we take $N_F = 4$ and parameterize the model by $\theta_H$.

We note that some of the composite Higgs models (CHM) have similar features to those in the GHU. In particular, CHM based on $SO(5)$ gauge group has almost the same gauge structure as the $SO(5) \times U(1)_X$ GHU

There are many differences between the two. The 4D Higgs boson in GHU is a fluctuation mode of the AB phase $\theta_H$ in the fifth dimension, but is not a pseudo-Nambu-Goldstone boson supposed in CHM. Secondly, in most of the CHM, $SO(4)$-breaking boundary conditions are imposed on fermion fields by hand to obtain the quark-lepton spectrum. In the GHU theory based on the action principle the $SO(5) \times U(1)_X$ gauge invariance in the bulk and the $SO(4) \times U(1)_X$ gauge invariance on the UV and IR branes are strictly preserved. GHU is more restrictive than CHM, and is powerful to make predictions.
Table 1: Masses and widths of $Z'$ bosons, $Z^{(1)}$, $\gamma^{(1)}$, and $Z_R^{(1)}$ ($N_F = 4$)

| $\theta_H$ [rad.] | $z_L/10^4$ | $m_{KK}$ [TeV] | $m_{Z^{(1)}}$ [TeV] | $\Gamma_{Z^{(1)}}$ [GeV] | $m_{\gamma^{(1)}}$ [GeV] | $\Gamma_{\gamma^{(1)}}$ [TeV] | $m_{Z_R^{(1)}}$ [TeV] | $\Gamma_{Z_R^{(1)}}$ [GeV] |
|-------------------|------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| 0.115             | 10         | 7.41           | 6.00            | 406            | 6.01           | 909            | 5.67           | 729            |
| 0.0917            | 3          | 8.81           | 7.19            | 467            | 7.20           | 992            | 6.74           | 853            |
| 0.0737            | 1          | 10.3           | 8.52            | 564            | 8.52           | 1068           | 7.92           | 1058           |

In GHU the relevant parameter for physics of SM particles is $\theta_H$. With $\theta_H$ given, the KK spectra of various fields, the couplings of quarks and leptons to KK gauge bosons, and the Higgs couplings are all determined. The Higgs boson mass $m_H \sim 125$ GeV and $m_{KK} = 7 \sim 10$ TeV are naturally realized for $\theta_H \sim 0.1$ without fine-tuning of the parameters. It has been shown that corrections to $H \rightarrow \gamma\gamma, Z\gamma$ due to an infinite number of KK states of $W, t$ et al. running in the loop are finite and tiny for $\theta_H \sim 0.1$. It has been recognized that the $SO(5) \times U(1)_X$ GHU in the RS space gives nearly the same phenomenology at low energies as the SM for $\theta_H \lesssim 0.1$.

The phase $\theta_H$ in GHU corresponds to the vacuum misalignment angle in CHM. A bound $\theta_H < 0.3$ has been derived in CHM from the $S$ parameter constraint.[13, 14] In GHU much stronger constraint $\theta_H \lesssim 0.1$ is obtained from the current non-observation of $Z'$ signals at LHC. It should be stressed in this connection that in GHU in the RS space right-handed quarks and leptons and KK gauge bosons are localized near the IR brane whereas left-handed quarks and leptons are localized near the UV brane so that right-handed quarks and leptons have larger couplings to KK gauge bosons than left-handed quarks and leptons.

Masses and widths of $Z'$ bosons are tabulated in Table 1. Fermion couplings to $Z'$ for $\theta_H = 0.115, 0.0917$ and 0.0737 are given in Tables 2, 3 and 4 respectively. In the evaluation $\sin^2 \theta_W = 0.23126$ and $M_Z = 91.1876$ GeV are adopted. The $Z$ couplings of quarks and leptons except for top quark are almost the same as in the SM within the accuracy of one part in $10^4$. The deviation of the $Zt\bar{t}$ couplings are less than 1%, whereas the deviation of the $Zb\bar{b}$ couplings are very tiny in GHU.

We evaluate $e^+e^- \rightarrow f\bar{f}$ cross sections $\sigma(\bar{f}f)$ where $f$ is a lepton or quark. In addition
Table 2: Couplings of neutral vector bosons ($Z'$ bosons) to fermions in unit of $g_w = e / \sin \theta_W$ for $\theta_W = 0.115$. Corresponding $Z$-boson coupling in the SM are $(g^L_{Z\mu}, g^R_{Z\nu}) = (0.57027, 0)$, $(g^L_{Z\tau}, g^R_{Z\nu}) = (-0.30651, 0.26376)$, $(g^L_{Zu}, g^R_{Zu}) = (0.39443, -0.17584)$ and $(g^L_{Zd}, g^R_{Zd}) = (-0.48235, 0.08792)$.

| $f$  | $g^L_{Z\nu}$ | $g^R_{Z\nu}$ | $g^L_{Z(1)\nu}$ | $g^R_{Z(1)\nu}$ | $g^L_{Z\mu}$ | $g^R_{Z\mu}$ | $g^L_{Z(1)\mu}$ | $g^R_{Z(1)\mu}$ |
|------|-------------|--------------|------------------|------------------|-------------|--------------|------------------|------------------|
| $\nu_\tau$ | 0.57041 | 0 | -0.1968 | 0 | 0 | 0 | 0 | 0 |
| $\nu_\tau$ | 0.57041 | 0 | -0.1968 | 0 | 0 | 0 | 0 | 0 |
| $e$ | -0.30659 | 0.26391 | 0.1058 | 1.0924 | 0 | -1.501 | 0.1667 | -1.983 |
| $\mu$ | -0.30659 | 0.26391 | 0.1058 | 1.0261 | 0 | -1.420 | 0.1667 | -1.863 |
| $\tau$ | -0.30658 | 0.26391 | 0.1057 | 0.9732 | 0 | -1.354 | 0.1666 | -1.767 |
| $u$ | 0.39453 | -0.17594 | -0.1361 | -0.7152 | 0 | 0.9846 | -0.1111 | 1.2983 |
| $c$ | 0.39453 | -0.17594 | -0.1361 | -0.6631 | 0 | 0.9205 | -0.1111 | 1.2036 |
| $t$ | 0.39339 | -0.17712 | 0.5068 | -0.4764 | 1.0314 | 0.6899 | 0.4158 | 0.8666 |
| $d$ | -0.48247 | 0.087972 | 0.1665 | 0.3576 | 0 | -0.4923 | 0.05557 | -0.6491 |
| $s$ | -0.48247 | 0.087970 | 0.1664 | 0.3315 | 0 | -0.4602 | 0.05556 | -0.6018 |
| $b$ | -0.48254 | 0.087964 | -0.6303 | 0.2387 | 1.0292 | -0.3446 | -0.2082 | -0.4331 |

to leptonic and hadronic cross sections, forward-backward asymmetry defined by

$$A_{FB} = \frac{\int_{0}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^{0} \frac{d\sigma}{d\cos\theta} d\cos\theta}{\int_{-1}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta},$$

(2)

the ratio of hadronic and leptonic cross sections $R_{\mu} \equiv \sigma(\bar{q}q) / \sigma(\mu^+\mu^-)$, and the asymmetry of $\sigma(\bar{f}f)$ with right- and left-handed polarized electron beams are investigated.\[1\]

Cross sections are evaluated to the leading order, which may receive quantum corrections. Such corrections are parametrised as $\sigma \rightarrow \delta_{QCD} \cdot \delta_{QED} \cdot \sigma + r_{nf}$ where $\delta_{QCD} = 1+O(\alpha_s/\pi)$ and $\delta_{QED} = 1+O(\alpha EM/\pi)$ are factorizable QCD and QED corrections, whereas $r_{nf}$ denotes non-factorizable corrections. In this paper we assume that $\delta_{QCD,QED}^{GHU} \simeq \delta_{QCD,QED}^{SM}$ and $r_{nf}$ for both GHU and SM are small. We have taken only the first KK states into account. The second KK states are approximately twice as heavy as the first KK states. The magnitudes of couplings of the second KK states are at most a half of the couplings of the first KK states. Thus the contributions of the second KK states are expected to be small.

\[1\]In the numerical evaluation in this letter we have used the values of the various couplings obtained for $m_H = 126$GeV. With $m_H = 125$GeV the value of $M_{Z'}$, for instance, decreases by 1.3\%.
Table 3: $Z'$ couplings of fermions for $\theta_H = 0.0917$. Unit is the same as in Table 2

| $f$ | $g^L_{Zf}$ | $g^R_{Zf}$ | $g^{(1)}_{Zf}$ | $g^{R(1)}_{Zf}$ |
|-----|------------|------------|----------------|----------------|
| $\nu_e$ | 0.57037   | 0  | -0.2092   | 0  |
| $\nu_\mu$ | 0.57037   | 0  | -0.2092   | 0  |
| $\nu_\tau$ | 0.57037   | 0  | -0.2092   | 0  |
| $e$ | -0.30656   | 0.26387 | 0.1124   | 1.0443 |
| $\mu$ | -0.30656   | 0.26387 | 0.1124   | 0.9804 |
| $\tau$ | -0.30656   | 0.26387 | 0.1124   | 0.9278 |
| $u$ | 0.39450     | -0.17591 | -0.1447   | -0.6838 |
| $c$ | 0.39450     | -0.17591 | -0.1477   | -0.6328 |
| $t$ | 0.39367     | -0.17678 | 0.5635   | -0.4245 |
| $d$ | -0.48243   | 0.087957 | 0.1769   | 0.3419 |
| $s$ | -0.48243   | 0.087955 | 0.1769   | 0.3164 |
| $b$ | -0.48249   | 0.087951 | -0.6959 | 0.2127 |

In the LEP1 experiment \[15\] at the $Z$-pole ($\sqrt{s} = M_Z$) the measured and fitted values of cross sections, forward-backward asymmetries of charged leptons $A^\ell_{FB}$, $R^0_\ell \equiv \Gamma_{\text{hadrons}}/\Gamma_\ell$ ($\ell = e, \mu$) and $R_b \equiv \Gamma_b/\Gamma_{\text{hadrons}}$ are given by

\[
\sigma^{\text{meas}}(\bar{q}q)/\sigma^{\text{fit}}(\bar{q}q) = 1.00149 \pm 0.00089, \quad (3)
\]

\[
A^\ell_{FB}\text{ meas}/A^\ell_{FB}\text{ fit} = 1.042 \pm 0.058, \quad (4)
\]

\[
R^0_\ell\text{ meas}/R^0_\ell\text{ fit} = 1.0012 \pm 0.0012, \quad (5)
\]

\[
A^b_{FB}\text{ meas}/A^b_{FB}\text{ fit} = 0.956 \pm 0.015, \quad (6)
\]

\[
R_b\text{ meas}/R_b\text{ fit} = 1.002 \pm 0.031, \quad (7)
\]

where $\sigma^{\text{fit}}(\bar{q}q) = 41.478$ nb, $A^\ell_{FB}\text{ fit} = 0.01645$, $R^0_\ell\text{ fit} = 20.742$, $A^b_{FB}\text{ fit} = 0.1038$ and $R_b\text{ fit} = 0.21579$. In GHU, we obtain

\[
\sigma^{\text{GHU}}(\bar{q}q)/\sigma^{\text{SM}}(\bar{q}q) = 1.00143, 1.00098, 1.00073, \quad (8)
\]

\[
A^\ell_{FB}\text{ GHU}/A^\ell_{FB}\text{ SM} = 0.99571, 0.99668, 0.99780, \quad (9)
\]

\[
R^0_\ell\text{ GHU}/R^0_\ell\text{ SM} = 0.99984, 0.99989, 0.99992, \quad (10)
\]

\[
A^b_{FB}\text{ GHU}/A^b_{FB}\text{ SM} = 0.99769, 0.99832, 0.99887, \quad (11)
\]

\[
R_b\text{ GHU}/R_b\text{ SM} = 1.00019, 1.00016, 1.00014, \quad (12)
\]
Table 4: $Z'$ couplings of fermions for $\theta_H = 0.0737$. Unit is the same as in Table 2

| $f$ | $g^L_{Zf}$ | $g^R_{Zf}$ | $g^L_{Z(1)f}$ | $g^R_{Z(1)f}$ | $g^L_{Z(1)f}$ | $g^R_{Z(1)f}$ | $g^L_{f(1)f}$ | $g^R_{f(1)f}$ |
|-----|-----------|-----------|---------------|---------------|---------------|---------------|---------------|---------------|
| $\nu_e$ | 0.57034 | 0 | -0.2225 | 0 | 0 | 0 | 0 | 0 |
| $\nu_\mu$ | 0.57034 | 0 | -0.2225 | 0 | 0 | 0 | 0 | 0 |
| $\nu_\tau$ | 0.57034 | 0 | -0.2225 | 0 | 0 | 0 | 0 | 0 |
| $e$ | -0.30655 | 0.26384 | 0.1196 | 0.9981 | 0 | -1.376 | 0.1880 | -1.817 |
| $\mu$ | -0.30655 | 0.26384 | 0.1196 | 0.9369 | 0 | -1.303 | 0.1880 | -1.705 |
| $\tau$ | -0.30655 | 0.26384 | 0.1195 | 0.8847 | 0 | -1.240 | 0.1879 | -1.610 |
| $u$ | 0.39448 | -0.17589 | -0.1539 | -0.6536 | 0 | 0.9034 | -0.1253 | 1.1896 |
| $c$ | 0.39448 | -0.17589 | -0.1539 | -0.6041 | 0 | 0.8439 | -0.1253 | 1.0994 |
| $t$ | 0.39379 | -0.17661 | 0.6888 | -0.3431 | 1.3208 | 0.5253 | 0.5616 | 0.6258 |
| $d$ | -0.48241 | 0.087947 | 0.1882 | 0.3268 | 0 | -0.4517 | 0.06267 | -0.5948 |
| $s$ | -0.48241 | 0.087946 | 0.1882 | 0.3021 | 0 | -0.4320 | 0.06266 | -0.5497 |
| $b$ | -0.48246 | 0.087941 | -0.8470 | 0.1720 | 1.3189 | -0.2625 | -0.2808 | -0.3129 |

for $\theta_H = 0.115$, 0.0917 and 0.0737, respectively. For $\sqrt{s} = M_Z$, cross section is dominated by the $Z$ boson resonance and effects of $Z'$ are very small. $Z$-boson couplings are very close to the SM value so that the deviation of the cross sections from the SM is very tiny. No significant deviations are seen. In LEP1, the measured $A^h_{FB}$ value deviates from the fit value at nearly $3\sigma$ level. In GHU, $A^h_{FB}$ is close to the SM value.

In the LEP2 experiment, cross sections of $\bar{q}q$, $\mu^+\mu^-$ and $\tau^+\tau^-$ for twelve different collision energies $\sqrt{s}$ ($130 \text{ GeV} \leq \sqrt{s} \leq 207 \text{ GeV}$) were measured. For the energy $120 \text{ GeV} \lesssim \sqrt{s} \lesssim 207 \text{ GeV}$, $\sigma(\bar{q}q)$ is much larger than the cross sections for lepton final states. The average of $\sigma^{\exp}/\sigma^{\SM}(\bar{q}q)$ is $1.0092 \pm 0.0076$ [16]. In GHU, we obtain $\sigma^{\GHU}/\sigma^{\SM}(\bar{q}q) = 0.9975$, 0.9985 and 0.9993 at $\sqrt{s} = 130$ GeV, and 0.9882, 0.9923 and 0.9953 at $\sqrt{s} = 207$ GeV for $\theta_H = 0.115$, 0.0917 and 0.0737, respectively. Using the ratios $\sigma^{\exp}/\sigma^{\SM}$ we perform the $\chi^2$-test for the $\sigma^{\model}/\sigma^{\SM}$. When $\sigma^{\model} = \sigma^{\SM}$, the $\chi^2$-value is $\chi^2/\text{d.o.f.} = 7.3/12$. In GHU ($\sigma^{\model} = \sigma^{\GHU}$), $\chi^2/\text{d.o.f.} = 14.4/12$ ($p$-value is 28%) [11.2/7 ($p$-value is 13%)], 12.0/12 [8.9/7 (26%)] and 10.6/12 [7.6/7 (37%)] for with 12-bin [7-bin] fit for $\theta_H = 0.115$, 0.0917 and 0.0737, respectively. Here in the 7-bin fit we have chosen seven largest energies (189 GeV $\leq \sqrt{s} \leq 207$ GeV). In the fit, correlations among the data are taken into account. GHU is within the allowed range of the experimental uncertainty.
Recently at LHC with 36.4 fb\(^{-1}\) pp-collision data \(pp \rightarrow \mu^+\mu^-\) invariant mass distribution \(d\sigma/dM_{\mu\mu}\) has been obtained\(^1\). The number of observed events and the expected number of the Drell-Yang process in the SM are \(N^{\text{exp}} = 4\) and \(N^{\text{SM}}_{\text{DY}} = 5.4 \pm 0.8\) for 1800 GeV \(\leq M_{\mu\mu} \leq 3000\) GeV, respectively. In GHU with \(\theta_H = 0.115, 0.0917\) and 0.0737 the expected numbers of events are \(N^{\text{GHU}}_{\text{DY}}/N^{\text{SM}}_{\text{DY}} = 1.8, 1.1\) and 0.8 for 1800 GeV \(\leq M_{\mu\mu} \leq 3000\) GeV \(^1\), respectively. For \(\theta_H = 0.115\) the GHU prediction deviates from the SM value (the observed number of events) at 1.8-sigma (2.4-sigma) level. In \[^{10}\], we have evaluated the expected number of events of Drell-Yang process with \(\sqrt{s_{pp}} = 14\) TeV. For 300 fb\(^{-1}\) LHC data, we predict for \(\theta_H = 0.0737\) the excess of Drell-Yang events as \(N^{\text{GHU}}_{\text{DY}}/N^{\text{SM}}_{\text{DY}} = 42/47, 6.9/3.6, 2.6/0.4\) and 1.1/0.04 for bins [2000, 3000], [3000, 4000], [4000,5000] and [5000, 6000], respectively.

At LEP2, experimental values of \(\sigma(\mu^+\mu^-), \sigma(\tau^+\tau^-)\) and \(A^\ell_{\text{FB}}\) have rather large statistical errors and no significant deviations of the ratios \(\sigma^{\text{GHU}}/\sigma^{\text{SM}}\) and \(A^{\text{GHU}}_{\text{FB}}/A^{\text{SM}}_{\text{FB}}\) from the experimental data for these modes are seen.

The LHC results put the limit \(\theta_H \lesssim 0.1\) on GHU. To explore GHU one has to go to future \(e^+e^-\) colliders at higher energies, with 250 GeV \(\leq \sqrt{s} \lesssim \) a few TeV \[^{14, 18, 19, 20, 21}\]. Although with such energy \(Z'\)s cannot be directory produced, the effects of interference among \(\gamma, Z\) and \(Z'\)s can be seen. Furthermore, polarized electron and/or positron beams can be produced at future \(e^+e^-\) colliders. Since right-handed fermions have larger couplings to \(Z'\)s in GHU, right-handed polarized electron beam will be sensitive to the \(Z'\)s effects.

Following Ref. \[^{22}\] we define the longitudinal polarization \(P_{e^\pm} (-1 \leq P_{e^\pm} \leq 1)\) so that the electron [positron] is purely right-handed when \(P_{e^-} = 1 [P_{e^+} = 1]\). When the vector bosons dominate in the mediators, the cross section at the center-of-mass frame is given by

\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{4} \left[ (1 - P_{e^-})(1 + P_{e^+}) \frac{d\sigma_{LR}}{d\cos\theta} + (1 + P_{e^-})(1 - P_{e^+}) \frac{d\sigma_{RL}}{d\cos\theta} \right],
\]

(13)

where \(\sigma_{LR} (\sigma_{RL})\) is \(e^-L\bar{e}_R^+ (e^-R\bar{e}_L^+) \rightarrow f\bar{f}\) scattering cross section. Hereafter we consider \(\sigma(q\bar{q}), A_{\text{FB}}(\mu^+\mu^-)\) and \(R_\mu \equiv \sigma(q\bar{q})/\sigma(\mu^+\mu^-)\). Although these quantities depend on both \(P_{e^-}\) and

\(^2\)In the ratio the K-factors in numerators and denominators are cancelled.
Figure 1: $\sigma(\mu^+\mu^-)$ for the polarized electron and positron beams. Blue and red lines indicates $\theta_H = 0.0917$ and 0.0737, respectively. Solid and dotted lines are for $\sqrt{s} = 250$ GeV and 500 GeV, respectively. The gray band indicates statistical uncertainty at $\sqrt{s} = 250$ GeV with 250 fb$^{-1}$ data set.

$P_{e^+}$, the dependence is parameterized by one effective polarization $P_{\text{eff}} = (P_{e^-} - P_{e^+})/(1 - P_{e^-}P_{e^+})$. When $\sigma$ is given by (13), $\sigma(\mu^+\mu^-)_{\text{GHU}}/\sigma(\mu^+\mu^-)_{\text{SM}}(\mu^+ + \mu^-)$ is satisfied so that one finds that $O(P_{\text{eff}}, 0) = O(P_{e^-}, P_{e^+})$ where $O = \sigma_{\text{GHU}}(\bar{q}q)/\sigma_{\text{SM}}(\bar{q}q)$, $A_{FB}$, $R_\mu$. As typical values one finds $P_{\text{eff}} = \pm 0.887$ for $(P_{e^-}, P_{e^+}) = (\pm 0.8, \mp 0.3)$. In the following study we parameterize the polarization in terms of $P_{\text{eff}}$ instead of $(P_{e^-}, P_{e^+})$.

At $\sqrt{s} = 250$ GeV with unpolarized beam (with polarized beam with $P_{\text{eff}} = 0.877$), $\sigma_{\text{SM}}(\mu^+\mu^-) = 1.87$ pb (2.16 pb). In Figure 1 the relative cross section $\sigma(\mu^+\mu^-)_{\text{GHU}}/\sigma(\mu^+\mu^-)_{\text{SM}}$ is plotted as a function of $P_{\text{eff}}$ at $\sqrt{s} = 250$ GeV and 500 GeV. At $\sqrt{s} = 250$ GeV, $\sigma(\mu^+\mu^-)$ in GHU is smaller than the SM value by 4.0% [2.5%] for $\theta_H = 0.0917$ [0.0737] when $P_{\text{eff}} = 0.877$. At $\sqrt{s} = 500$ GeV with polarization $P_{\text{eff}} = 0.877$, 15% [9%] decrease of $\sigma_{\text{GHU}}(\bar{q}q)/\sigma_{\text{SM}}(\bar{q}q)$ due to the interference will be observed. At $\sqrt{s} = 250$ GeV with 250 fb$^{-1}$ unpolarized $e^+e^-$ beam, we expect $4.66 \times 10^5 \mu^+\mu^-$ events in the SM. In GHU the expected number of events and statistical significance are estimated to be $4.57 \times 10^5$ [4.60 \times 10^5] and 13.3 [8.5] for $\theta_H = 0.0917$ [0.0737].

Systematic errors in the normalization of the cross sections can be reduced by measuring

$$R_{f,RL}(\mathcal{P}) = \frac{\sigma(\bar{f}f; P_{e^-} = +\mathcal{P}, P_{e^+} = 0)}{\sigma(\bar{f}f; P_{e^-} = -\mathcal{P}, P_{e^+} = 0)} \quad (14)$$

where the electron beams are polarized with $P_{e^-} = +\mathcal{P}$ and $-\mathcal{P}$. We note that the left-right asymmetry $A_{LR}^f \equiv [\sigma_{LR} - \sigma_{RL}]/[\sigma_{LR} + \sigma_{RL}]$ is related to $R_{f,RL}$ by $A_{LR}^f = (\bar{\mathcal{P}})^{-1}[1 -$
In Table 5, the effects of GHU on the $R_{f,RL}$ are tabulated. GHU predicts a significant deficit in $R_{f,RL}$ in the early stage of the ILC experiment.

Table 5: $R_{f,RL}$ in the SM, and deviations of $R_{f,RL}(\bar{P})^{GHU}/R_{f,RL}(\bar{P})^{SM}$ from unity are tabulated for $\bar{P} = 0.8$. Statistical uncertainties of $R_{f,RL}^{SM}$ is estimated with $L_{int}$ data for $\sigma(\bar{f}f; P_{e^+} = +\bar{P})$ and $\sigma(\bar{f}f; P_{e^-} = -\bar{P})$, namely with $2L_{int}$ data in all.

| $f$ | $\sqrt{s}$, $L_{int}$ | $R_{f,RL}^{SM}$ (uncertainty) | $R_{f,RL}^{GHU}$ | $\theta_H = 0.0917$ | $\theta_H = 0.0737$ |
|-----|-------------------------|-----------------------------|------------------|------------------|------------------|
| $\mu$ | 250 GeV 250 fb$^{-1}$ | 0.890 (0.3%) | -3.4% | -2.2% |
| | 500 GeV 500 fb$^{-1}$ | 0.900 (0.4%) | -13.2% | -8.6% |
| $b$ | 250 GeV 250 fb$^{-1}$ | 0.349 (0.3%) | -3.1% | -2.1% |
| | 500 GeV 500 fb$^{-1}$ | 0.340 (0.5%) | -12.3% | -8.3% |
| $t$ | 500 GeV 500 fb$^{-1}$ | 0.544 (0.4%) | -13.0% | -8.2% |

In Figure 2, $\sigma(\mu^+\mu^-)^{GHU}/\sigma(\mu^+\mu^-)^{SM}$ up to $\sqrt{s} = 3$ TeV is displayed. For $1$ TeV $\lesssim \sqrt{s} \lesssim 3$ TeV large deficit is expected for right-handed electron (and/or left-handed positron) beams. We have also plotted the case ("e" in the figure) with $P_{eff} = -0.877$, namely the case with left-handed electron (and/or right-handed positron) beams. In this case, the interference effect of $Z$'s is hardly seen for $\sqrt{s} < 2$ TeV. In all cases the ratios grow for $\sqrt{s} \gtrsim 3$ TeV up to the large $Z'$ resonances.

Forward-backward asymmetry (FBA) in the SM at $\sqrt{s} = 250$ GeV is $A_{FB}(\mu^+\mu^-) = 0.522 (0.506)$ for $P_{eff} = 0$ (+0.877). In Figure 3 deviations of $A_{FB}(\mu^+\mu^-)$ from the SM values are plotted as functions of $P_{eff}$. At $\sqrt{s} = 250$ GeV and for $P_{eff} = 0.877$, $A_{FB}$ deviates by $-2.1\% (-1.3\%)$ from the SM for $\theta_H = 0.0917 (0.0737)$. At $\sqrt{s} = 500$ GeV and $P_{eff} = 0.877$, $A_{FB}$ deviates by $-12.0\% (-7.4\%)$ for $\theta_H = 0.0917 (0.0737)$. Signals of GHU will be seen at $2\sigma [4\sigma]$ level at $\sqrt{s} = 250$ GeV with 250 fb$^{-1}$ unpolarized [polarized] beams.

In Figure 4, $A_{FB}(\mu^+\mu^-)$ is displayed up to $\sqrt{s} = 3$ TeV. At $\sqrt{s} = 1 \sim 2$ TeV, the effect of the interference among $\gamma$, $Z$ and $Z'$ becomes maximum. In particular for right-handed polarized electron beams very large deviation from the SM is expected.

One can also measure $A_{FB}(\bar{b}b)$, $A_{FB}(\bar{t}t)$. They are tabulated in Table 6. We note that $A_{FB}(\bar{b}b)$ and $A_{FB}(\bar{t}t)$ become larger than those in the SM, in quite contrast with the $A_{FB}(\mu^+\mu^-)$ case.
Figure 2: $\sigma^{GHU}(\mu^+\mu^-)/\sigma^{SM}(\mu^+\mu^-)$ for the polarized electron and positron beams. “a”, “c” and “e” (“b” and “d”) are for $\theta_H = 0.0917$ (0.0737). “a” and “b” are for unpolarized beams whereas “c” and “d” are for polarized beams with $P_{\text{eff}} = +0.877$. “e” is for $P_{\text{eff}} = -0.877$.

Table 6: $(A_{FB}^{q,GHU} - A_{FB}^{q,SM})/A_{FB}^{q,SM}$ ($q = b, t$)

| $q\bar{q}$ | $\theta_H$ | $\sqrt{s}$ | $\sigma_{FB}^{q,SM}$ | $(P_{e^-}, P_{e^+})$ |
|-------------|-------------|-------------|----------------------|----------------------|
|             |             |             | (0, 0)               | (+0.8, -0.3)         | (-0.8, +0.3)         |
| $bb$        | 0.0917      | 250 GeV     | +0.8%                | +3.3%                | +0.1%                |
|             |             | 500 GeV     | +2.9%                | +12.2%               | +0.2%                |
|             | 0.0737      | 250 GeV     | +0.7%                | +3.2%                | +0.1%                |
|             |             | 500 GeV     | +2.5%                | +11.2%               | +0.2%                |
| $tt$        | 0.0917      | 500 GeV     | +0.9%                | +4.5%                | +0.1%                |
|             | 0.0737      | 500 GeV     | +1.2%                | +4.2%                | +0.2%                |
\frac{(A_{FB}^{GHU} - A_{FB}^{SM})(\mu^+\mu^-)}{A_{FB}^{SM}(\mu^+\mu^-)}(\mu + \mu^-)^{-1}

Figure 3: \(\frac{(A_{FB}^{GHU} - A_{FB}^{SM})(\mu^+\mu^-)}{A_{FB}^{SM}(\mu^+\mu^-)}(\mu + \mu^-)^{-1}\) as functions of the effective polarization \(P_{\text{eff}}\). Solid and dotted lines are for \(\sqrt{s} = 250\text{ GeV}\) and \(500\text{ GeV}\), respectively. Blue-thick and red-thin lines correspond to \(\theta_H = 0.0917\) and \(0.0737\), respectively. The gray band indicates the statistical uncertainty at \(\sqrt{s} = 250\text{ GeV}\) with \(250\text{ fb}^{-1}\) data.

The effect of the differences in the couplings of \(Z'\) to leptons and quarks can be seen in the ratio of the cross sections \(R_b(\mu) \equiv \sigma(\bar{b}b)/\sigma(\mu^+\mu^-)\). In the SM with unpolarized \(e^+e^-\) beams, \(R_b(\mu)^{SM} = 0.95, 0.84\) and \(0.82\) for \(\sqrt{s} = 250\text{ GeV}, 500\text{ GeV}\) and \(\infty\), respectively. In Figure 5, deviations of \(R_b(\mu)\) from the SM value \(R_b(\mu)^{GHU}/R_b(\mu)^{SM}\) are plotted as functions of \(P_{\text{eff}}\). The excess in \(R_b(\mu)\) becomes maximum for \(P_{\text{eff}} \sim 0.3\). At \(\sqrt{s} = 250\text{ GeV}\), 1.1\% [0.8\%] excess for unpolarized beams and 1.3\% [0.9\%] excess for \(P_{\text{eff}} = 0.4\) polarized beams for \(\theta_H = 0.0917\) [0.0737] are expected. In GHU with \(\theta_H \simeq 0.09\), 3\(\sigma\) deviation is expected with \(250\text{ fb}^{-1}\) data. At \(\sqrt{s} = 500\text{ GeV}\), 5.3\% [3.3\%] excess is expected for \(P_{\text{eff}} = 0.4\) polarized beams. In Table 7, deviation of \(R_t(\mu)^{GHU}\) from \(R_t(\mu)^{SM}\) is tabulated. The deviation becomes largest around \(P_{\text{eff}} \simeq +0.3\).

Table 7: Deviations of the ratio \(R_t(\mu)^{GHU}/R_t(\mu)^{SM}\) from the unity.

| \(\theta_H\) | \(\sqrt{s}\) | \((0, 0)\) | \((0, 0)\) | \((-0.3, 0.0)\) |
|---|---|---|---|---|
| 0.0917 | 500 GeV | +2.7\% | +2.7\% | +2.2\% |
| 0.0737 | 500 GeV | +1.8\% | +1.9\% | +1.5\% |

In this letter we have studied the effects of the \(Z'\) bosons in GHU in the \(e^+e^-\) collider experiments. At the \(Z\) pole \((\sqrt{s} = M_Z)\), the effects of the \(Z'\) bosons are small and

\footnote{The \(e^+e^- \rightarrow bb\) scattering process contains not only the process mediated by neutral vector bosons, but also the \(W\)-fusion process \(e^+e^- \rightarrow H\nu\bar{\nu}\) followed by \(H \rightarrow bb\). We have assumed that these processes are efficiently separated and we consider only the vector-boson mediated process.}
Figure 4: $A_{FB}(\mu^+\mu^-)$ for unpolarized and polarized beams. “a”, “b” and “c” (“d” and “e”) are for GHU with $\theta_H = 0.0917$ (0.0737). “a” and “d” are for unpolarized beams, whereas “b” and “e” (“c”) are for polarized beams with $P_{\text{eff}} = +0.877$ (−0.877). “f” [solid-black], “g” [dashed-black] and “h” [dotted-black] correspond to SM with unpolarized, $P_{\text{eff}} = +0.877$ and $P_{\text{eff}} = −0.877$ polarized beams, respectively.

both cross sections and lepton forward-backward asymmetries are consistent with the experiments. At the energies $130 \text{ GeV} \leq \sqrt{s} \leq 207 \text{ GeV}$, $e^+e^- \to \bar{f}f$ cross sections and forward-backward asymmetry in GHU are found to be consistent with the LEP2 results. Recent LHC results put the limit $\theta_H \lesssim 0.1$ in GHU. Large deviations from the SM in $\sigma(\mu^+\mu^-)$, $A_{FB}$, $R_b(\mu)$ and $R_{f,RL}$ are predicted at higher energies. In the future $e^+e^-$ collider experiments, measurements of $\sigma(\mu^+\mu^-)$, $\sigma(\bar{q}q)$, $A_{FB}(\mu^+\mu^-)$ and $R_{f,RL}$ with polarized beams will well discriminate GHU from the SM. In particular, $\sigma(\mu^+\mu^-)$ measurement, even with unpolarized beams, can discriminate the GHU with $\theta_H \simeq 0.09$ (0.07) at 11 (8) times of the statistical uncertainty level at $\sqrt{s} = 250 \text{ GeV}$ with $250 \text{ fb}^{-1}$ data. In the left-right asymmetry $R_{f,RL}$, for which systematic uncertainty is reduced, signals of GHU can be observed at 8 (5) times of the statistical uncertainty level. The characteristic dependence of $A_{FB}^e$ and $R_b(\mu)$ on the electron-positron polarization can also be used to study the couplings of the $Z'$ bosons to quarks and leptons as well.

The gauge-Higgs unification is promising. It predicts many signals in $e^+e^-$ collider experiments. The left-right asymmetry $R_{f,RL} = \sigma(\bar{f}f; P_{e^-} = \overline{P}) / \sigma(\bar{f}f; P_{e^-} = -\overline{P})$ will exhibit a distinct deviation from the SM in the early stage of 250 GeV ILC with polarized
Figure 5: The ratio $R_b(\mu)^{\text{GHU}}/R_b(\mu)^{\text{SM}}$, where $R_b(\mu) \equiv \sigma(\bar{b}b)/\sigma(\mu^+\mu^-)$, is plotted as a function of $P_{\text{eff}}$. Solid and dotted lines are for $\sqrt{s} = 250$ GeV and 500 GeV, respectively. Blue-thick and red-thin lines are for $\theta_H = 0.0917$ and 0.0737, respectively. The gray band indicates the statistical uncertainty at $\sqrt{s} = 250$ GeV with 250 fb$^{-1}$ data.

e^- beams. At 1 TeV ILC or CLIC, clear signals of GHU will be seen in the forward-backward asymmetry $A_{\text{FB}}(\mu^+\mu^-)$.

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