Measurement Error Mitigation for Variational Quantum Algorithms

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(Dated: October 19, 2020)

Variational Quantum Algorithms (VQAs) are a promising application for near-term quantum processors, however the quality of their results is greatly limited by noise. For this reason, various error mitigation techniques have emerged to deal with noise that can be applied to these algorithms. Recent work introduced a technique for mitigating expectation values against correlated measurement errors that can be applied to measurements of 10s of qubits. We apply these techniques to VQAs and demonstrate its effectiveness in improving estimates to the cost function. Moreover, we use the data resulting from this technique to experimentally characterize measurement errors in terms of the device connectivity on devices of up to 20 qubits. These results should be useful for better understanding the near-term potential of VQAs as well as understanding the correlations in measurement errors on large, near-term devices.

I. INTRODUCTION

The development of quantum computers and their applications have rapidly accelerated over the last few years. Several different hardware platforms have been experimentally realized at varying scales [1–4], and there has been an increased focus on studying algorithms and applications that can be run on these noisy near-term devices. Some of the most promising algorithms for near-term devices are Variational Quantum Algorithms (VQAs) [5–12], which use a quantum device to evaluate an objective function that is minimized using a classical optimizer. Instances of this algorithm include the Variational Quantum Eigensolver (VQE) [6–10, 13, 14] and Quantum Approximate Optimization Algorithm (QAOA) [5, 15, 16]. Recent process in this field includes, for example, improvements to the measurement process [17–20], selection of variational ansätze [21–24], and optimization. Moreover, they have been demonstrated experimentally on a variety of physical systems [9, 25–27].

The ability of current experimental implementations of VQAs to produce accurate results is limited by noise on the device, despite these algorithms not explicitly requiring error correction. All proposed platforms for quantum computation experience some combination of different errors including decoherence, calibration errors, leakage, cross-talk, and measurement errors. With superconducting systems, cross-talk and measurement are among the largest sources of errors [28]. Error mitigation techniques have been developed for VQAs to reduce the amount of error on near-term devices in the absence of error correction. For example, *extrapolation to the zero-noise limit* [29, 30] uses pulse-level control to mitigate expectation values against decoherence, requiring only a constant factor of overhead in the number of circuits executed. Techniques have also been developed to characterize and mitigate against measurement errors [31–33]. Moreover, Ref. [34] analyzes the underlying model and provides rigorous improvements to correction techniques. Related techniques have also been used in VQE experiments [9] and are implemented in the IBM Qiskit package [35]. Recently, Ref. [36] has introduced a readout error mitigation technique, which we will call Continuous-Time Markov-Process Error Mitigation (CTMP-EM), that mitigates expectation values against correlated measurement errors. The n-qubit calibration procedure for CTMP-EM requires as few as $n + 2$ circuits to execute and $O(n^2)$ parameters to fit. In this work, they used it to mitigate estimates of the fidelity of graph states using stabilizer measurements, as well as expectation values of stabilizers with respect to Clifford circuits.

In this paper we apply the CTMP-EM technique to experimentally characterize long-range correlations in measurement errors, and to calibrate error mitigated measurements in several quantum computers. In addition to demonstrating the presence of long-range correlations in these devices, we are also able to show that these long-range correlations can be as strong between distant qubits as they are between neighboring qubits. Moreover, rather than only considering the global minimum for the VQE objective function, we consider the objective function holistically. Evaluating the objective function at other parameter values is important for various tasks, for example the application of the ubiquitous parameter shift rule [37] for analytic gradient computation in variational experiments. For the Fermi-Hubbard model, we demonstrate that CTMP-EM can fundamentally improve the shape of the objective function for the VQE not only at its minimum, but globally as well.

This article is structured as follows. In Section II we review VQAs and the CTMP-EM technique. In Section III we demonstrate that applying CTMP-EM to the VQE algorithm changes the shape of the objective function. In Section IV we use the calibration data from CTMP-EM to analyze the long-range correlations in readout er-
rors on devices. We also compare several different IBM Quantum superconducting devices with the CTMP-EM calibration data. In Section V we conclude.

II. BACKGROUND

A. Variational Quantum Algorithms

A VQA is an optimization problem, $\min_{\theta} f(\theta)$ where the objective function $f(\theta)$ is evaluated using a quantum device within the classical optimization loop. Two examples of VQAs are the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA). In both, the objective function is $f(\theta) = \langle \psi(\theta) | H | \psi(\theta) \rangle$ for a Hamiltonian $H$ and parameterized state $\{|\psi(\theta)\rangle\}$. In the case of VQE, $H$ corresponds to the Hamiltonian of some physical system. In the case of QAOA, $H$ corresponds to a cost function with binary variables. In QAOA, the state $|\psi(\theta)\rangle$ is prepared by alternating unitary gates corresponding to evolution of the cost Hamiltonian $H$ and some mixer operator(s). In VQE, the state $|\psi(\theta)\rangle$ takes the form of a specific trial-state ansatz. The objective function $f(\theta)$ is computed by performing measurements to estimate the expectation value $\langle \psi(\theta) | H | \psi(\theta) \rangle$ on a quantum device.

VQAs are appealing for near-term devices as they are agnostic to errors in state preparation so long as the true minimum expectation value can be reached. This has been demonstrated on different hardware platforms, with different instances of VQAs, and varying degrees of accuracy. For the case of VQE it has been shown that, in special cases, the parameters that minimize the objective function are resilient to certain types of noise [38]. In general, however, device noise significantly impacts the value of the objective function at that minimum and other points in parameter space.

B. Continuous Time Markov Process Error Mitigation

Error mitigation techniques generally aim to improve the accuracy of the results obtained from using a noisy quantum device. Typically, each technique mitigates against a certain kind of error. Measurement errors are a large source of error in VQA experiments on near-term devices. Measurement errors are modeled by a stochastic assignment matrix $A$ acting on the state before readout. The elements of the matrix $A_{y,x} = P(y|x)$ are the probability of reading out the basis state $y$ where $x$ was prepared. A general $n$-qubit stochastic matrix has $2^n(2^n - 1)$ independent real parameters and thus can only be completely characterized for a small number of qubits. Once this matrix is determined, applying the inverse matrix to a given probability distribution undoes the effect of readout error, however since the inverse is not in general a stochastic matrix the resulting output is not a valid probability distribution.

The CTMP-EM algorithm of Ref. [36] circumvents dealing with the $A$ matrix directly, and is used to mitigate expectation values computed with a quantum device. This is done by modeling $A = e^G$, where $G = \sum_i r_i G_i$ with rates $0 \leq r_i \in \mathbb{R}$ and operators $G_i$ that generate different readout errors. In particular, for multi-qubit bitstrings $a$, $b$, the readout error $a \rightarrow b$ corresponds to the generator $G_i = |b\rangle\langle a| - |a\rangle\langle b|$.

To investigate the impact of CTMP-EM on variational algorithms, we choose the Fermi-Hubbard model which describes a system of Fermions interacting on a lattice [39]. The Hamiltonian

$$
H = \sum_i (\alpha (\gamma_i + \gamma_i^\dagger) + U n_i n_i) + \sum_{i<j} V (n_i - \frac{1}{2}) (n_j - \frac{1}{2})
$$

where $\alpha$ is the single-particle energy, $U$ is the intra-site interaction, $V$ is the inter-site interaction, and $n_i = |\psi(\theta)\rangle_i$ is the number operator on qubit $i$. The Fermi-Hubbard model is a well-known model for studying strongly correlated electron systems.

The expectation value of the Hamiltonian is computed using a quantum device, and the CTMP-EM algorithm is used to mitigate the measurement error. The resulting expectation value is then used in the classical optimization loop. The CTMP-EM algorithm is designed to be efficient, and has been shown to achieve high accuracy with relatively few parameters.

III. MITIGATING VQA OBJECTIVE FUNCTIONS

A. Ground State Energy Mitigation

One of the main applications of VQE is to estimate the ground state energy of a Hamiltonian by minimizing the measured operator expectation value over the variational parameters. Both the expectation value and gradient estimates are particularly sensitive to measurement errors which can greatly effect the performance of the classical optimizer that depends on these values. This makes measurement error mitigation essential for improving the accuracy of VQE and other VQAs. To investigate the impact of CTMP-EM on variational algorithms, we choose the Fermi-Hubbard model which describes a system of Fermions interacting on a lattice [39]. The Hamiltonian

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H = \sum_i (\alpha (\gamma_i + \gamma_i^\dagger) + U n_i n_i) + \sum_{i<j} V (n_i - \frac{1}{2}) (n_j - \frac{1}{2})
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is
\[
H = -t \sum_{\langle j,k \rangle} \sum_\sigma (a_{j,\sigma}^\dagger a_{k,\sigma} + a_{k,\sigma}^\dagger a_{j,\sigma}) + U \sum_{k} n_{k,\uparrow} n_{k,\downarrow},
\]
where \(t\) is the tunneling parameter, and \(U\) is the interaction parameter between Fermions on the same site, \(a_{k,\sigma}^\dagger\) is the raising operator for site \(k\) with spin \(\sigma \in \{\uparrow, \downarrow\}\), and \(n_{k,\sigma} = a_{k,\sigma}^\dagger a_{k,\sigma}\) is the number operator. The Fermionic operators are mapped to qubit operators using the Bravyi-Kitaev mapping [40]. For all calculations we will assume \(U = 2t\). Energies are expressed in units of \(t\). We assume that the lattice is a 1-D chain coupled by nearest neighbors with periodic boundary conditions.

We chose an \(n\)-qubit variational ansatz \(|\psi(\theta)\rangle\) consisting of an initial state \(|+\rangle^\otimes n\) followed by six repetitions of a layer of parameterized single-qubit \(Y\)-rotations followed by a layer of CZ gates between neighboring qubits.

In Fig. 1 we plot the objective function \(f(\theta_0 + s \phi)\) around the global minimum, where \(s\) is the parameter of the sweep, \(\phi\) is a randomly chosen vector with \(\dim(\theta_0) = \dim(\phi)\), and \(\theta_0\) are the parameters that globally minimize \(f\). Evaluations of \(f\) are repeated three times: in the absence of any noise (“Noiseless”), including measurement error without any mitigation (“Unmitigated”), and including measurement error with CTMP-EM applied (“Mitigated”). We observe that CTMP-EM is able to significantly improve both the estimate of the ground state energy, as well as objective function values around the ground state. Nevertheless, arriving at the correct ground state depends not only on the point containing the ground state itself, but the objective function as a whole.

![FIG. 1. Sweep of the objective function \(f\) through the global minimum \((s = 0)\) computed several ways. Here, \(f\) corresponds to the energy of the Fermi-Hubbard model for the given state. In the “Noiseless” case, there is no error in the simulation and no mitigation. In the “Unmitigated” case, we include readout error, but no mitigation. In the “Mitigated” case, we include readout error and CTMP-EM. Estimates of the ground state energy \((s = 0)\) and surrounding points are significantly improved by applying CTMP-EM.](image)

We find that the standard deviation of the error distribution is significantly reduced when mitigation is applied. Specifically, for 2, 4, 6, and 8 qubits respectively, the remaining source of error in the mitigated case is due to undersampling in the number of shots required to perform CTMP-EM. This emphasizes the importance of the scaling of measurements needed for CTMP-EM with the number of qubits.

![FIG. 2. Comparison of samples of the objective function \(f(\theta)\) for random \(\theta\) and different numbers of qubits. The energy error in panel (a) is the difference between the noisy (mitigated or unmitigated) energy and the exact result. The lefthand distributions are using the unmitigated objective function, and the righthand distributions are using the objective function mitigated with CTMP-EM. The standard deviations in panel (b) are those of the distributions in panel (a). For all system sizes considered, adding CTMP-EM significantly improves the estimate of the objective function. We use 8192 \(n\) shots (for \(n\) qubits) to compensate for the overhead in applying CTMP-EM.](image)

**B. Objective Function Sampling**

In VQE the quantum computer is treated as a black box evaluation of the objective function \(f\), hence it is critical that evaluations of \(f\) are accurate. To investigate the effectiveness of measurement error mitigation for black box evaluations of \(f(\theta)\) we sample points in parameter space \(\theta\) and compare the noiseless, unmitigated, and mitigated cases for Fermi-Hubbard models with 1, 2, 3, and 4 sites (with 2, 4, 6, and 8 qubits respectively). We evaluate the energy with and without CTMP-EM for randomly sampled parameters of the objective function and compare the distribution of values with the noiseless result as shown in Fig. 2.

We find that the standard deviation of the error distribution is significantly reduced when mitigation is applied. Specifically, for 2, 4, 6, and 8 qubits respectively,
the standard deviation is reduced by factors of approximately 7.46, 7.64, 5.18, and 3.40. This demonstrates that in the absence of CTMP-EM, the noisy objective function deviates significantly from its noiseless form which can greatly limit the effectiveness of the VQE algorithm even for a small number of qubits.

IV. CHARACTERIZING READOUT ERRORS

The CTMP-EM method can also be used as a characterization protocol for correlated measurement errors in a quantum device, which we will demonstrate by using it to characterize correlated readout errors in several experimental devices. One method for measurement error characterization involves computing the full $A$ matrix (which has $2^n$ elements) as a form of measurement tomography [41], however this is not possible to do past a small number of qubits. Instead we use the set of rates $\{r_i\}$ of the CTMP-EM generator $G$ to characterize correlated measurement errors. This is scalable in the sense that it requires as few as $n + 2$ circuits, and $G$ is parameterized by the $O(n^2)$ rates of its generator components.

A. Characterizing Correlated Errors

Calibration of $G$ in the CTMP-EM model described in Ref. [36] is done by preparing a input set of computational basis states $\{|a_i\}$, labelled by bitstrings $a_i$, and performing measurements in the computational basis to estimate the assignment probabilities $P(x|a_i)$ for $x = 0, ..., 2^n - 1$. These probabilities are then processed to compute the CTMP-EM generator rates $\{r_i\}$ for each of the 1 and 2-qubit generator terms. The set of input labels $a_i$ is complete if the set of all measurement outcomes contains all 1 and 2-qubit transitions for the CTMP-EM generators, if only 2-qubit correlations are present. This requires at least $n + 2$ generators, though more may be used to provide a more uniform distribution across generator terms. We use the set of all bitstrings that have Hamming weight $\leq 2$, of which there are $(n^2 + n + 2)/2$.

For the purposes of characterizing correlated errors, we focus on the 2 qubit generators, as these generate correlations that cannot be captured in the single-qubit tensor product error model. We compare the distributions of the 2-qubit of rates $r_i$ grouped by the qubit distance, which we define as the shortest path between two qubits in terms of the device connectivity.

We apply this technique to the 20 qubit ibmq_boeblingen device, which has a planar qubit connectivity graph as shown inset in Fig. 3. Here, for example, neighboring qubits have distance 1, and some pairs of qubits in the corners of the layout have distance 7. The histograms of measured 2-qubit generator rates $r_i$ vs qubit distance for the ibmq_boeblingen device are shown inset in Fig. 3. Here we further group the generators into three types: excitation generators ($00 \leftrightarrow 11$), decay generators ($11 \leftrightarrow 00$), and exchange generators ($01 \leftrightarrow 10$).

A natural expectation is that the correlated errors between qubits is dependent on their connectivity, and that correlated errors will be largest on neighboring qubits. However, our results indicate that the correlation in measurement errors between distant qubits is non-trivial, and in some cases comparable to neighboring qubits.

As one may expect, the decay generators have the highest associated generator coefficient. This is to be
expected as thermal relaxation to the ground state is a significant contribution to measurement errors. However, it is surprising that the distribution median does not decay appreciably with qubit distance and is non-negligible even for distantly connected qubits. The excitation and exchange generators on the other hand show reduction with qubit distance on average, however in some cases certain rates can be as large as those between neighboring qubits.

B. Comparing Devices with CTMP-EM Data

To illustrate the usefulness of CTMP-EM for characterization, we use this method to characterize readout errors on several IBM Quantum devices and compare their local 1-qubit and correlated 2-qubit generator coefficients. The distribution in generator values is shown in Fig. 4. For all measured devices the 1-qubit error rates are generally higher than the 2-qubit error rates as expected, and all devices give relatively consistent error rates. The devices shown range from 5-qubit to 20-qubits with the calibrations run using the minimum number of \( n + 2 \) calibration circuits, using the maximum number of shots available for each device. Moreover, CTMP-EM as a characterization technique does not depend on device connectivity, and includes information about long-range correlations. This is advantageous since we saw before that the long-range correlations in readout errors can be non-trivial.

V. CONCLUSION

In this paper, we have shown that CTMP-EM is vital for improving the performance of VQAs on near-term devices, and that it can fundamentally change the shape of the objective function computed by the quantum device. Moreover, we demonstrate that CTMP-EM can efficiently be used to characterize long-range correlations in readout error on near-term devices in terms of the device connectivity, and that these long-range correlations are present on current devices. Nevertheless, an interesting topic for future work would be to expand the CTMP model to generators that act on more than two qubits, or analyze how the calibration parameters drift over time. Additionally, it would be interesting to investigate the impact on performance for QAOA Hamiltonians. For now, our work emphasizes the importance of using CTMP-EM for characterization, because it only requires computing the terms of \( G \), of which there are only \( O(n^2) \) many, instead of \( A = e^\mathcal{H} \), which is dense. Moreover, the calibration technique is efficient in the number of circuits. We believe that our results will be useful to understanding the objective function in VQAs on near-term devices, as well as characterizing near-term devices in terms of readout error.

ACKNOWLEDGEMENTS

The authors thank Sergey Bravyi, Sophia Economou, and Sarah Sheldon for helpful discussions. We also thank the IBM Quantum team for providing access to the devices used in this work. This work was done during G.S.B internship at IBM Quantum during the summer of 2020. G.S.B. thanks the IBM Quantum team for a very enriching internship experience.
[1] Andrew W. Cross, Lev S. Bishop, Sarah Sheldon, Paul D. Nation, and Jay M. Gambetta, “Validating quantum computers using randomized model circuits,” Phys. Rev. A 100, 032328 (2019).

[2] J. M. Pino, J. M. Drelling, C. Figgatt, J. P. Gaebler, S. A. Moses, C. H. Baldwin, M. Foss-Feig, D. Hayes, K. Mayer, C. Ryan-Anderson, and et al., “Demonstration of the qccd trapped-ion quantum computer architecture,” arXiv e-prints, 2003.01293 (2020).

[3] Peter J Karalekas, Nikolas A Tezak, Eric C Peterson, Cilm A Ryan, Marcus P da Silva, and Robert S Smith, “A quantum-classical cloud platform optimized for variational hybrid algorithms,” Quantum Science and Technology 5, 024003 (2020).

[4] Petar Jurcevic, Ali Javadi-Abhari, Lev S. Bishop, Isaac Lauer, Daniela F. Bogorin, Markus Brink, Lauren Capelluto, Oktay G¨ unl¨ uk, Toshinaro Iroko, Naoki Kanazawa, Abhinav Kandala, George A. Keefe, Kevin Kruslich, William Landers, Eric P. Lewandowski, Douglas T. McClure, Giacomo Nannicini, Adinath Narasimhan, Hasan M. Nayfeh, Emily Pritchett, Christopher J. Wood, Jing-Bang Yau, Eric J. Zhang, Oliver E. Dial, Jerry M. Chow, and Jay M. Gambetta, “Demonstration of quantum volume 64 on a superconducting quantum computing system,” (2020), arXiv:2008.08571 [quant-ph].

[5] Edward Farhi, Jeffrey Goldstone, and Sam Gutmann, “A quantum approximate optimization algorithm,” arXiv preprint arXiv:1411.4028 (2014).

[6] A. Aspuru-Guzik, and J. M. Martinis, “Scalable quantum simulation of molecular energies,” Phys. Rev. X 6, 031007 (2016).

[7] Jarrod R McClean, Jonathan Romero, Ryan Babbush, and Al´ an Aspuru-Guzik, “The theory of variational hybrid quantum-classical algorithms,” New J. Phys. 18, 023023 (2016).

[8] Abhinav Kandala, Antonio Mezzacapo, Kristan Temme, Maika Takita, Markus Brink, Jerry M Chow, and Jay M Gambetta, “Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets,” Nature 549, 242–246 (2017).

[9] J. I. Colless, V. V. Ramasesh, D. Dahlen, M. S. Blok, M. E. Kimchi-Schwartz, J. R. McClean, J. Carter, W. A. de Jong, and I. Siddiqi, “Computation of molecular spectra on a quantum processor with an error-resilient algorithm,” Phys. Rev. X 8, 011021 (2018).

[10] G Pagano, A Bapat, P Becker, K S Collins, A De, PW Hess, HB Kaplan, A Kyriienidis, WL Tan, C Baldwin, et al., “Quantum approximate optimization with a trapped-ion quantum simulator,” arXiv preprint arXiv:1906.02700 (2019).

[11] Jonathan Romero and Alan Aspuru-Guzik, “Variational quantum generators: Generative adversarial quantum machine learning for continuous distributions,” arXiv preprint arXiv:1901.00848 (2019).

[12] Pauline J Ollitrault, Abhinav Kandala, Chun-Fu Chen, Panagiotis K. Barkoutsos, Antonio Mezzacapo, Marco Pistoia, Sarah Sheldon, Stefan Woerner, Jay Gambetta, and Ivano Tavernelli, “Quantum equation of motion for computing molecular excitation energies on a noisy quantum processor,” arXiv preprint arXiv:1910.12890 (2019), arXiv:1910.12890 [quant-ph].

[13] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Sergio Boixo, Michael Broughton, Bob B. Buckley, David A. Buell, Brian Burkett, Nicholas Buschman, Yu Chen, Zijun Chen, Benjamin Chiaro, Roberto Collins, William Courtney, Sean Demura, Andrew Dunsworth, Daniel Epps, Edward Farhi, Austin Fowler, Brooks Foxen, Craig Gidney, Marissa Giustina, Rob Graff, Steve Habegger, Matthew P. Harrigan, Alan Ho, Sabrina Hong, Trent Huang, William J. Huggins, Lev Ioffe, Serge V. Isakov, Evan Jeffrey, Zhang Jiang, Cody Jones, Dvir Kafri, Kostyantyn Kechedzhi, Julian Kelly, Seon Kim, Paul V. Klimov, Alexander Korotkov, Fedor Kostritsa, David Landhuis, Pavel Laptev, Mike Lindmark, Erik Lucero, Orion Martin, John M. Martinis, Jarrod R. McClean, Matt McEwen, Anthony Megrant, Xiao Mi, Masoud Mohseni, Wojciech Mruczkiewicz, Josh Mutus, Ofer Naaman, Matthew Neeley, Charles Neill, Hartmut Neven, Murphy Yuezhen Niu, Thomas E. O’Brien, Eric Ostby, Andre Petukhov, Harald Puetterman, Chris Quintana, Pedram Roushan, Nicholas C. Rubin, Daniel Sank, Kevin J. Satzinger, Vadim Smelyanskiy, Doug Strain, Kevin J. Sung, Marco Szalay, Tyler Y. Takeshita, Amit Vainsencher, Theodore White, Nathan Wiebe, Z. Jamie Yao, Ping Yeh, and Adam Zalcman, “Hartree-Fock on a superconducting qubit quantum computer,” arXiv e-prints, arXiv:2004.04174 (2020), arXiv:2004.04174 [quant-ph].

[14] J. S. Otterbach, R. Manenti, N. Alidoust, A. Bestwick, M. Block, B. Bloom, S. Caldwell, N. Didier, E. Schuyler Fried, S. Hong, P. Karalekas, C. B. Osborn, A. Pappgeorge, E. C. Peterson, G. Praweratmodjo, N. Rubin, Colm A Ryan, D. Scarabelli, M. Scheer, A. E. Sete, P. Sivarajah, Robert S. Smith, A. Staley, N. Tezak, W. J. Zeng, A. Hudson, Blake R. Johnson, M. Reagor, M. P. da Silva, and C. Rigetti, “Unsupervised Machine Learning on a Hybrid Quantum Computer,” arXiv e-prints, arXiv:1712.05771 (2017), arXiv:1712.05771 [quant-ph].

[15] Stuart Hadfield, Zhihui Wang, Bryan O’Gorman, Eleanor Rieffel, Davide Venturelli, and Rupak Biswas, “From the quantum approximate optimization algorithm to a quantum alternating operator ansatz,” Algorithms 12, 34 (2019).

[16] Ryan Babbush, Nathan Wiebe, Jarrod McClean, James McClain, Hartmut Neven, and Garnet Kin-Lic Chan, “Low-depth quantum simulation of materials,” Physical Review X 8, 011044 (2018).
[18] Vladyslav Verteletskyi, Tzu-Ching Yen, and Artur F. Izmaylov, “Measurement optimization in the variational quantum eigensolver using a minimum clique cover,” The Journal of Chemical Physics 152, 124114 (2020), https://doi.org/10.1063/1.5141458.

[19] William J. Huggins, Jarrod McClean, Nicholas Rubin, Zhang Jiang, Nathan Wiebe, K. Birgitta Whaley, and Ryan Babbush, “Efficient and Noise Resilient Measurements for Quantum Chemistry on Near-Term Quantum Computers,” arXiv:1907.13117 [physics, physics:quant-ph] (2019), arXiv:1907.13117 [physics, physics:quant-ph].

[20] Andrew Zhao, Andrew Tranter, William M. Kirby, Shu Fay Ung, Akinasa Miyake, and Peter J. Love, “Measurement reduction in variational quantum algorithms,” Phys. Rev. A 101, 062322 (2020).

[21] Harper R. Grimsley, Sophia E Economou, Edwin Barnes, and Nicholas J. Mayhall, “An adaptive variational algorithm for exact molecular simulations on a quantum computer,” Nat. Commun. 10, 3007 (2019).

[22] Panagiotis Kl. Barkoutsos, Jerome F. Gonthier, Igor Sokolov, Nikolaj Moll, Gian Salis, Andreas Fuhrer, Marc Ganzhorn, Daniel J. Egger, Matthias Troyer, Antonio Mezzacapo, Stefan Filipp, and Ivano Tavernelli, “Quantum algorithms for electronic structure calculations: Particle-hole hamiltonian and optimized wavefunction expansions,” Phys. Rev. A 98, 022322 (2018).

[23] Ho Lun Tang, V. O. Shkolnikov, George S. Barron, Harper R. Grimsley, Nicholas J. Mayhall, Edwin Barnes, and Sophia E. Economou, “qubit-adapt-vqe: An adaptive algorithm for constructing hardware-efficient ansatze on a quantum processor,” (2019), arXiv:1911.10205 [quant-ph].

[24] Bryan T Gard, Linghua Zhu, George S Barron, Nicholas J Mayhall, Sophia E Economou, and Edwin Barnes, “Efficient symmetry-preserving state preparation circuits for the variational quantum eigensolver algorithm,” arXiv preprint arXiv:1904.10910 (2019).

[25] D. Zhu, N. M. Linke, M. Benedetti, K. A. Landman, N. H. Nguyen, C. H. Alderete, A. Perdomo-Ortiz, N. Korda, A. Garfoot, C. Brecque, L. Egan, O. Perdomo, and C. Monroe, “Training of quantum circuits on a hybrid quantum computer,” Science Advances 5 (2019), 10.1126/sciadv.aaw9918, https://advances.sciencemag.org/content/5/10/eaaw9918.full.pdf.

[26] E. F. Dumitrescu, A. J. McCaskey, G. Hagen, G. R. Jansen, T. D. Morris, T. Papenbrock, R. C. Pooser, D. J. Dean, and P. Lougovski, “Cloud quantum computing of an atomic nucleus,” Phys. Rev. Lett. 120, 210501 (2018).

[27] N. Kleo, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, “Quantum-classical computation of schwinger model dynamics using quantum computers,” Phys. Rev. A 98, 032331 (2018).

[28] Yanzhu Chen, Maziar Farahzad, Shinjae Yoo, and Tzu-Chieh Wei, “Detector tomography on ibm quantum computers and mitigation of an imperfect measurement,” Phys. Rev. A 100, 052315 (2019).

[29] Kristian Temme, Sergey Bravyi, and Jay M. Gambetta, “Error mitigation for short-depth quantum circuits,” Phys. Rev. Lett. 119, 180509 (2017).

[30] Abhinav Kandala, Kristian Temme, Antonio D Córcoles, Antonio Mezzacapo, Jerry M Chow, and Jay M Gambetta, “Error mitigation extends the computational reach of a noisy quantum processor,” Nature 567, 491–495 (2019).

[31] Kathleen E. Hamilton and Raphael C. Pooser, “Error-mitigated data-driven circuit learning on noisy quantum hardware,” Quantum Machine Intelligence 2, 1–15 (2020).

[32] Michael R. Geller and Mingyu Sun, “Efficient correction of multiqubit measurement errors,” arXiv preprint arXiv:2001.09980 (2020), arXiv:2001.09980 [quant-ph].

[33] Kathleen E. Hamilton, Tyler Kharazi, Titus Morris, Alexander J. McCaskey, Ryan S. Bennink, and Raphael C. Pooser, “Scalable quantum processor noise characterization,” arXiv preprint arXiv:2006.01805 (2020), arXiv:2006.01805 [quant-ph].

[34] Michael R. Geller, “Rigorous measurement error correction,” Quantum Science and Technology 5, 03LT01 (2020).

[35] https://github.com/qiskit/qiskit, “Qiskit: An open-source framework for quantum computing,” (2019).

[36] Sergey Bravyi, Sarah Sheldon, Abhinav Kandala, David C. Mckay, and Jay M. Gambetta, “Mitigating measurement errors in multi-qubit experiments,” arXiv preprint arXiv:2006.14044 (2020), arXiv:2006.14044 [quant-ph].

[37] Maria Schuld, Ville Bergholm, Christian Gogolin, Josh Izaac, and Nathan Killoran, “Evaluating analytic gradients on quantum hardware,” Phys. Rev. A 99, 032331 (2019).

[38] Kunal Sharma, Sumeet Khatri, M Cerezo, and Patrick J Coles, “Noise resilience of variational quantum compiling,” New Journal of Physics 22, 043006 (2020).

[39] J. Hubbard and Brian Hilton Flowers, “Electron correlations in narrow energy bands,” Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences 276, 238–257 (1963), https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1963.0204.

[40] Sergey Bravyi and Alexei Yu. Kitaev, “Fermionic quantum computation,” Annals of Physics 298, 210 – 226 (2002).

[41] R. C. Bialczak, M. Ansmann, M. Hofheinz, E. Lucero, M. Neeley, A. D. O’Connell, D. Sank, H. Wang, J. Wenner, M. Steffen, and et al., “Quantum process tomography of a universal entangling gate implemented with josephson phase qubits,” Nature Physics 6, 409–413 (2010).