Fermi-Bose mixtures and BCS-BEC crossover in high-$T_c$ superconductors

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Abstract: In this review article we consider theoretically and give experimental support to the models of the Fermi-Bose mixtures and the BCS-BEC crossover compared with the strong-coupling approach, which can serve as the cornerstones on the way from high-temperature to room-temperature superconductivity in pressurized metallic hydrides. We discuss some key theoretical ideas and mechanisms proposed for unconventional superconductors (cuprates, pnictides, chalcogenides, bismuthates, diborides, heavy-fermions, organics, bilayer graphene, twisted graphene, oxide hetero-structures), superfluids and balanced or imbalanced ultracold Fermi gases in magnetic traps. We build a bridge between unconventional superconductors and recently discovered pressurized hydrides superconductors $H_3S$ and $LaH_{10}$ with the critical temperature close to room temperature. We discuss systems with line of nodal Dirac points close to the Fermi surface and superconducting shape resonances, and hyperbolic superconducting networks which are very important for the development of novel topological superconductors, for the energetics, for the applications in nano-electronics and quantum computations.

Keywords: high-temperature superconductivity, s-wave and d-wave pairing, Kohn–Luttinger and Migdal–Eliashberg mechanisms, BCS-BEC crossover, Fermi-Bose mixture

1. Introduction

The seminal discovery in 1986 of high-$T_c$ superconductivity at 36 K in a doped cuprate perovskite $La_{2-x}Ba_xCuO_4$ by K. A. Muller and J. G. Bednorz in IBM Zurich [1,2] determined the shift of research for room-temperature superconductors from metallic alloys to complex perovskites. In the first 10 years the critical temperature of gradually increased from 36 K in lanthanum family of superconducting cuprates till 90 K in yttrium family $YBa_{2}Cu_{3}O_{7-d}$ and finally reaching 160 K in $HgBa_{2}Ca_{x}Cu_{4}O_{8}$ under a pressure of $P = 350$ Kbar. This extraordinary progress promised a
technological revolution in electronics, and energetics and provided the driving force for a
tremendous progress in the development of many body theories for condensed matter physics,
inorganic chemistry and material science. Today we see the growing interest on quantum complex
materials. The conventional superconductivity in low-temperature superconductors is described by
the BCS weak coupling theory in a homogeneous metal with pairing on a large Fermi surface with
high Fermi energy. The unconventional high temperature superconductors are characterized by the
breakdown of the standard BCS approximations: 1) a single electronic component, 2) high electron
density and high Fermi energy $E_F$; 3) low values of the ratio $\omega_0/E_F$ between the energy cut off of the
attractive interaction $\omega_0$ and the Fermi energy 4) large Fermi surface and high Fermi momentum $k_F$;
5) superconducting energy gap much smaller than the Fermi energy $\Delta \ll E_F$; 6) large ratio between
the coherence length $\xi$ and the average distance between electrons.

Soon after the discovery of Müller, Anderson and Schrieffer advanced different unconventional
mechanisms for high temperature superconductivity (see e. g. [3, 4, 5]). Many fruitful ideas were
introduced in the first years of HTSC by the leading theorists in USA, Europe, Japan and worldwide
(see e. g. [6,7,8,9,10,11,12,13,14,15,16]). One of the authors of the present review was very active in
unconventional mechanisms focusing on the low density electron systems [17,18,19,20,21,22,23,24]
based on the generalization of Kohn–Luttinger ideas [25,26] in purely repulsive Fermi systems. In
the framework of the Fermi gas model with hard-core repulsion [27], repulsive-U Hubbard model
[28] and the Shubin–Vonsovsky model [29, 30] on different type of 3D and 2D lattices during last 30
years of intensive research we predicted p-, d-, f- and anomalous s-wave pairing in various materials
such as the idealized and bilayer graphene (see Fig. 1), superconducting pnictides and organic
superconductors and superfluid $^3$He.

It has been shown that the critical temperature of the superconducting transition can be
substantially increased at low densities by considering spin-polarization or the two-band
superconductivity scenario [31,32,33]. These results proved to be very important for related systems
as in polarized $^3$He - $^4$He mixtures, imbalanced Fermi-gas in magnetic trap as well as for heavy
cellon superconductors and other mixed valence systems described by the two-band Hubbard
model with one narrow band [34,35,36,37]. Together with T.M. Rice we also considered a
superconductivity scenario in 2D t-J model at low and intermediate electron density [38, 39] with
van der Waals interaction which corresponds to strong onsite Hubbard repulsion and weak intersite

![Figure 1. Phase diagram of superconducting state in idealized bilayer graphene [17,18].](image-url)
antiferromagnetic (AFM) attraction. The phase-diagram of this model shown in Fig. 1 shows regions of superconducting phases with different pairing symmetry: p-wave, s-wave and $d_{x^2-y^2}$-pairing.

For the set electron densities (optimal doping) and the values of $J/t \approx 1/2$ we get the critical temperature of the d-wave pairing typical for high-$T_c$

$$T_C \sim E_F \exp\left(-\frac{\pi t}{2Jn_e}\right) \approx 100K.$$  \hspace{1cm} (Eq. 1).

This result was generalized by N.M. Plakida et al [40,41] on the opposite case of small hole densities $x = (1-n_e)$ using the diagrammatic technique of the Hubbard operators for the pairing of two spin polarons. The same order of magnitude $T_C: 100$ K for the critical temperature can be obtained for the d-wave pairing in the Shubin-Vonsovsky model in the intermediate coupling case for the set of parameters $U \approx 3t, n_e \approx 0.9 [20,21,24]$ on the 2D square lattice.

We would like to summarize here both the early stage [42-53] and recent [54-58] theoretical results of the Russian research school in high-$T_c$ investigations.

The proposed mechanisms of superconductivity in high-$T_c$ cuprates range from BCS scenarios involving electron-phonon or electron-electron interactions, with extended pairs [59] or BEC scenarios with local pairs [60, 61], with condensates of s-wave or d-wave [4,6,12,8] in models of the normal phase described by Landau Fermi liquid [62, 63], Luttinger [64] or marginal [13] Fermi liquid (correspondingly with spinons and holons instead of fermionic quasiparticles), spin-charge separation [3,6,12,64,65] or spin-charge confinement [11,65], with the presence of pseudogap [66] in underdoped state, including interplay between superconductivity and antiferromagnetism [67], formation of stripes [14], the role of the second layer and the c-axis plasmon mode [15] have been subject of an intensive debate.

In 2015 following theoretical predictions [68, 69] for high $T_c$ in the high pressure metallic phase $H_{3}S$ the record [70] for the superconducting temperature of 203 K was reached. The authors of the experimental breakthrough claimed that the discovered superconductivity could be explained by conventional strong-coupling version of the BCS theory [71] constructed by Migdal and Eliashberg [72, 73] used to predict the high $T_c$ phase in the dirty limit [69, 70]. It was rapidly shown that by
changing the pressure to reach the maximum critical temperature the chemical potential in H3S is
driven to an electronic topological transition known as a Lifshitz transition for the appearing of a
new piece of a Fermi surface [74,75]. In this regime the Migdal approximation breakdown [76] and it
is not possible to apply the BCS theory to explain the emergence of high temperature
superconductivity. It has been shown that the emergence of high temperature superconductivity in a
multiband scenario near a Lifshitz transition has been described by numerical solution of
Bogoliubov equations [77] and can be qualitatively predicted in the limit of a steep–flat scenario [79]
where the energy dispersion of the appearing band is pushed down to zero as for the case of an
infinite effective mass for strong correlated localized states [78]. An essential point of this proposed
scenario for the emergence of room temperature superconductivity is the theoretical demonstration
that the Lifshitz transition for the appearing of a strongly correlated band is not of 2,5 order but
becomes of first order with appearing of a frustrated phase separation [79], interface
superconducting [80] and a hyperbolic space of filamentary pathways in two dimensional systems
like it was observed in cuprate superconductors at optimal doping [81].

Recent papers on pressurized hydrides under high pressure support early papers [82] and
support the multigap scenario in the proximity of a Lifshitz transition [83].

Quite recently two experimental groups [84, 85] reported a discovery of even higher critical
temperatures $T_c$: (250–260) K in lanthanum superhydrides $LaH_{10+}$, at high pressures $P$: (170–190)
GPa. In this compound according to the Density Functional Theory (DFT) the host atom of La is at
the center of the cage formed by hydrogen atoms. Moreover the authors of [86] on the basis of
numerical calculations predict the topological Dirac nodal line in $LaH_{10}$ near the Fermi energy $E_F$.
Note that room critical temperatures (RTSC) of the order of 300 K or higher in metallic hydrogen for
very high pressures $P$: (170–190) Mbar and in hydrogen dominant metallic alloys (at smaller
pressures) were predicted by Ashcroft [87, 88]. The large values of $T_c$ in Ashcroft estimates were
facilitated by the high phonon frequencies governed by light mass of hydrogen ions.

Note also that profound numerical calculations of the optimal crystalline structure performed
by Brovman, Yu. Kagan and Kholas [89, 90] show the regions of 3D isotropic phase, quasi-2D planar
phase and quasi-1D filamentary phase on the $P$–$T$ phase-diagram of metallic hydrogen. In particular
they predict at relatively low pressures and $T=0$ a strongly anisotropic phase with the proton
filaments embedded in electron liquid. The filaments can move almost freely relative to each other in
the longitudinal direction similar to vortices in superfluid $^4$He [91] (where we have a vortex crystal
in perpendicular direction to the vortex lines and a free superfluid in parallel direction to the vortex
lines). In the same time the filaments form a rigid triangular lattice in the direction perpendicular to
them which resembles Abrikosov vortex lattice in type-II superconductors [92]. Similarly in the
planar phase the hydrogen layers can move almost freely relative to each other similarly to the
smectic liquid crystals or graphite layers.

2. Superconductivity in the Fermi-Bose mixture model. Theory.

Let us stress that the light mass of hydrogen ions $H^+$ not only guarantees the high values of
Debye frequency but also leads to a large kinetic energy of zero-vibrations and thus to a large value
of DeBoer parameter measuring the ratio of kinetic delocalization energy and potential energy and
responsible for the quantumness of the system [93,39]. This fact brings a highly mobile ionic
subsystem in metallic hydrogen (which resembles superionic crystal but at low temperatures in our
case) close to the limit of the quantum crystal [93,39]. Note that a high mobility of fillaments or layers
can promote also rather large values of Lindemann parameter (Lindemann number [93,39]) . It
measures the ratio of the root mean square of the displacement of ion to the interionic distance in
metallic hydrogen or in hydrogen dominant alloys and shows that our system is close to the
quantum melting limit. In principle, according to the ideas of Andreev, Lifshitz the ionic lattice of
the quantum crystal can become superfluid at low temperatures (due for example to a flow of
vacancies or other defects relative to the lattice or due to delocalization of fillaments themselves
which can form the macroscopic wave function). This phenomenon is nowadays called a supersolidity of the quantum crystal [94]. Experimental indications on available today scenarios for quantum supersolid are discussed by Bianconi group in [81]. The experimental support for filamentary supersolids can be found in [76, 78] having in mind not only "new" hydrogen dominant superconductors but also the "old" cuprate perovskites.

In analogy with neutron stars we can describe the ionic subsystem on the language of biproton superfluidity which coexists with the BCS Eliashberg superconductivity of the electron subsystem (probably modified on frequency dependence of effective mass and nonconstant density of states [95]).

These qualitative considerations press us to consider the superconductivity in metallic hydrogen on the level of Fermi-Fermi mixture [96] of protons and electrons (as in plasma physics or in the two components model with one very narrow band [36, 37]) or on the level of Fermi-Bose mixture of Cooper pairs and protons or even of Bose-Bose mixture [97] of Cooper pairs and biprotons. Note that the ideas of Fermi-Bose mixture were very fruitful in low temperature physics describing the search for fermionic (BCS) superfluidity in dilute 3D and 2D solutions of \(^3\)He in superfluid \(^4\)He [34,98,99,100] as well as \(^6\)Li–\(^7\)Li or \(^40\)K–\(^87\)Rb mixtures of ultracold atoms [101,102,103] in restricted geometry of magnetic and dipole traps or on the optical lattices.

Note that in ultracold Fermi gases there is a very effective way to change the magnitude and the sign of the interaction (more precisely of the s-wave scattering length) between the atoms. This can be done in the resonance magnetic field, in the regime of Feshbach resonance [104] which is important for the physics of BCS-BEC crossover in quantum gases. The Fermi-Bose mixture model corresponds to the two-channel description of the Feshbach resonance [105]. Here we have the attractive Majorana exchange term which transforms the (extended) Cooper pair consisting of two fermions in one channel into a real boson (a local pair or dimer) in the other channel.

In the physics of superconductors the model of Fermi-Bose mixture was firstly proposed by T. D. Lee et al. [106] and Micnas et al. [107] for superconducting cuprates. In this model the Majorana exchange term which transforms the Cooper pair in a boson, is present. Later on Larkin, Geshkenbein and Ioffe [108] phenomenologically advanced this model on the level of Ginzburg–Landau theory [109] for 2D electron system described by Hubbard type models close to van Hove singularities and for two-leg ladder systems [110].
Note that the Fermi-Bose mixture model with an additional requirement of the spatial separation of Fermion and Boson component captures thermodynamic and transport properties as well as a lot of essential features on the phase diagram of superconducting bismuth oxides $\text{Ba}_1x\text{K}_x\text{BiO}_3$ [111,112]. The local pairs are formed in these materials in $\text{BiO}_6$ clusters (see Fig. 1). The critical temperature $T_c$ = 36 K corresponds here to the coherent tunneling of local pairs between neighboring Bose clusters through the effective barriers formed by Fermi clusters.

The Fermi-Bose mixture of spinons and holons interacting via a strong confinement potential [8,65] created by an AFM string [113,114] was considered in [115] for the underdoped high $T_c$ cuprates. The main idea was to describe the superconducting pairing in the strongly underdoped region of the $t$-$J$ model on the language of the BCS-BEC crossover for the pairing in the $d$-wave channel of two spin polarons (two composite holes or two strings), where the critical temperature in the BCS phase is governed by (Eq. 1) and its spin-polaronic generalization [40,41]. The crossover however is probably not smooth, containing quantum critical point (QCP) or even intermediate phases in between BCS and BEC phase [115].

Note that non-phonon mechanisms of superconductivity can easily explain superconductivity with the critical temperatures $T_c$=100 K typical for cuprates. However to get $T_c$ = (250–260) K typical for metallic hydrogen alloys another type of models such as Fermi-Bose mixture model or BCS-BEC crossover within attractive-U Hubbard model is possible.

3. The 3D and 2D models of BCS-BEC crossover.

Note that to some extent the Fermi-Bose mixture model naturally appears in the most difficult intermediate part of the phase diagram for the BCS-BEC crossover [116,21] between extended Cooper pairs (dilute BEC regime) and local pairs (or bipolarons [117,118]) which correspond to the weakly repulsive Bogoliubov gas [119] of composed bosons (see Fig. 4).

Note that the typical phase-diagram of BCS-BEC crossover between extended Cooper pairs and local pairs in the 3D Fermi gas [116, 21].

![Figure 4](image)

**Figure 4.** The typical phase-diagram of BCS-BEC crossover between extended Cooper pairs and local pairs in the 3D Fermi gas [116, 21].

Namely for the dilute Bose gas we have two distinct temperatures $T^*$ which corresponds to the formation of local pairs and $T_c$ of their Bose-condensation. For intermediate temperatures:

$$T_c \leq T \leq T^* \quad \text{(Eq. 2)}$$

where the critical temperature of BEC

$$T_c = \frac{3.31 \left(\frac{n}{2}\right)^{2/3}}{2m} \left[1 + 1.3a_{\text{p}}n^{1/3}\right] \approx 0.2E_F \quad \text{(Eq. 3)}$$
is mostly governed by Einstein formula [120] for the number of pairs density $n_p = n / 2$ and the mass of the pair $m_p = 2m$. However there are non-trivial corrections to Einstein results connected with weakly repulsive interaction between the local pairs (between the dimers) [121]. This interaction is defined by the dimer-dimer scattering length $a_{2-2} = 0.6 |a| > 0$ [122,123], where $a$ is an s-wave scattering length for particle-particle interaction. In the same time the crossover (Saha) temperature [124, 125, 21].

$$T^* = \frac{|E_b|}{3 / 2 \ln\left(\frac{|E_b|}{E_F}\right)}, \quad \text{(Eq. 4)}$$

where $|E_b| = 1 / ma^2$ is an absolute value of the binding energy of a local pair, $E_F = \frac{p_F^2}{2m}$ is the Fermi energy, $p_F$ is Fermi momentum.

Note that in the 2D case we have the similar estimate for the Saha temperature [125,126,127]

$$T^* = \frac{|E_b|}{\ln\left(\frac{|E_b|}{E_F}\right)}, \quad \text{(Eq. 5)}$$

In the same time the BEC critical temperature is given by Fisher-Hohenberg theory [128] for weakly repulsive 2D Bose gas. According to [128]

$$T_C = \frac{E_F}{4 \ln(1 / f_{2-2})}, \quad \text{(Eq.6)}$$

where $f_{2-2} = 1 / \ln(1,6 |E_b| / E_F)$ [129] describes the repulsive interaction between the dimers in 2D.

For the intermediate temperatures in (Eq. 2) we have a new state of matter, namely the normal (non-superconducting) metal. According to [126, 127] it has very peculiar transport and thermodynamic properties with a resistivity behaving in semiconducting fashion $R \propto \sqrt{T}$.

In the same time in the intermediate coupling case (for the values of the gas parameter $1 \leq a p_F \leq 3$ [21]) the binding energy of the pair $|E_b|$ becomes comparable with the Fermi-energy $E_F$. When we increase the density we finally reach the limit $|E_b| \leq 2E_F$, and the local pairs start to touch each other. As a result some of the local pairs are crushed and (in the framework of e.g. 3D Fermi gas with attraction or attractive-U Hubbard model) we have a Fermi-Bose mixture of paired and unpaired electrons already at low temperatures.

4. Fano resonances in multigap BCS-BEC superconductivity at Lifshitz transitions

The attractive Majorana’s exchange interaction between bosonic pairs in Quantum Mechanics was proposed in 1933 [130,131] after Heisenberg proposed the well known repulsive exchange interaction [132-133] between fermionic particles. Majorana demonstrated that symmetry played a vital role in bosonic or fermionic systems. In fact his force and the one of Heisenberg made very different predictions because of their different symmetry properties. The Majorana exchange interaction opened the road map for both the interaction boson model [134] and the interaction
boson fermion model [135], and today it is at the base of the research to extract simplicity in complexity applied to a large spectrum of systems where the focus of the research is to identify the dynamical complex symmetry [136] as shown in Figure 5 for the triangular hyperbolic tiling, and it was extended to the case of high temperature superconductors with multiple gaps [137] and in molecular systems. It is now clear that more complex is the system, the more important it is to search for its symmetry as for example in the vibron model described by the so-called(4)u algebra and in the electron-vibron model described by even more complicated superalgebra (4 / )u1 [136].

Figure 5. Tessellation of the hyperbolic Poincare’ plane: showing the (6,4,2) triangular hyperbolic tiling (left side) that inspired the Escher Circle limit III panting (right side) (From M.C. Escher, Circle Limit III, 1959).

Fano introduced the configuration interaction between a closed channel and an open scattering channel in Quantum Mechanics in his famous paper published by Nuovo Cimento in 1935 [138]. He expanded and improved his theory in the 1961 Physical Review paper [139]. The idea of exchange interaction between pairs formed in the cloud of fermionic particles (which he called “pions”) and a localized boson, an extension of Fano resonance, was developed in the quantum field theory of many body systems by Tomonaga [140]. The Fano resonances have been classified as a “Feshbach resonance” if scattering length of the system in the closed channel is negative and it gives a well localized boson pair at negative energy; or as a “shape resonance” if the scattering length of the system in the closed channel is positive and it is a quasi-stationary state at positive energy degenerate with the continuum [141].

The Fermi-bose model for high temperature superconductors can be described as the case of Feshbach resonance driven by Majorana exchange interaction between cooper pairs and bosons or between a BCS condensate and a BEC condensate [142-144].

In 1994 it was proposed an alternative theoretical scenario for high temperature superconductors driven by a Fano “shape resonance” where the Majorana exchange term is in action between a BCS condensate and the second condensate in the BCS-BEC crossover.

In 1994 the compelling experimental evidence was collected from the study of the structure of the 91K Bi2Sr2CaCuO8+ superconductor that bulk superconductivity emerges in a stack of 2D layers of a strongly correlated electron liquid confined in finite domains of incommensurate modulated aperiodic lattice where two electronic components coexist: a first Fermi liquid and a second generalized Wigner charge density wave [145-147].

The “shape resonance” superconducting phase in proximity of two Lifshitz transitions is supported in cuprates by the fact that the maximum critical temperature occurs at optimum doping d=0.16 where the a Fermi surface is made of disconnected Fermi arcs in the proximity of two
topological electronic transitions. The first one is at $\delta=0.21$ where for larger doping a large circular Fermi surface appears and the second for $\delta=0.125$ where the Fermi surface shrinks to only four very short Fermi arcs in the $(\pi,\pi)$ direction with a maximum isotope coefficient [150]. Complexity emerged in the beginning of the XXI century, as the second key universal feature of cuprate superconducting perovskites [151-153]. In fact three key points were established: i) mesoscopic and nanoscale phase separation [151](see also review article[154]), ii) multiple electronic components in the normal metallic phase [152], iii) the key role of anisotropic lattice strain driving the complexity [153] and coexisting multiple superconducting gaps spatially distributed at nanoscale. The phase separation scenario with spin, charge, lattice nanoscale puddles with the proliferation of interface filamentary space was called a Superstripes scenario i.e., a supersolid with a specific symmetry [155] (see also Fig. 6 for illustration).

After the presentation of these results at the Stripes conference in 2000 in Rome the superconducting properties of MgB$_2$, a binary metallic non-magnetic system known since fifty years, have been measured in Japan by Akimitsu. MgB$_2$ was a layered system not expected to be a BCS superconductor, showing the record of $T_c=40K$ for a binary alloy. It was rapidly claimed that it was a clear case of superconductivity driven by a “Fano resonance” near a Lifshitz transition for the appearing of a new spot of a sigma Fermi surface with the key role of a large amplitude zero point motion at low temperature [156-157]. It was the first verified prediction of the BPV theory in a completely different system [148-150]. The second key feature, nanoscale phase separation, was found by doping MgB$_2$ with aluminum or scandium substitution for Mg [158] which shifts the chemical potential toward the Lifshitz transition.

![Figure 6](image_url) Superstripes landscape made of nanoscale striped puddles with a power law size distribution where the pathways of interface superconductivity connecting two points (where superconducting pairs shown in the figure) can be mapped to a hyperbolic space [81,155,175,176] providing a case of supersolid [39, 94] driven by strain and doping [152,153] near a critical point of a Lifshitz transition with large zero point lattice fluctuations [151,155,157].

The second case of a different practical realization of the “Fano resonance” superconductivity high temperature superconducting which reported in 2008 with the discovery of iron based superconductors [159-162] which are stacks of superconducting iron layers which show a strain doping phase diagram with maximum critical temperature in proximity of a Lifshitz transition like diborides and cuprates [159-162]. After 2008 new microscopes using focused synchrotron radiation have shown an ubiquitous presence of nanoscale phase separation in high temperature superconductors in cuprates [81, 163-165] and in iron based superconductors [166-169] providing the theoretical prediction that nanoscale phase separation is an universal feature of systems showing Fano resonances in proximity of a Lifshitz transition [170-172] in the superstripes scenario [155] which is called by some authors with the generic name of “nematic phase”. The theoretical efforts to understand the nanoscale phase separation in high temperature superconductors has pointed out the complexity of possible tiltings in perovskites [171], the mixing of boson and fermions in the complex symmetry [172,173], the role of anisotropic pseudo-Jahn Teller vibronic interaction in the polaron formation,[174] of these materials with the
most recent results pointing out the role of a non-euclidean hyperbolic space for the filamentary network at the interfaces of striped puddles in high temperature superconductivity [81,175-176].

Conclusions

In conclusion let us stress that recently discovered superconductivity in bilayer twisted graphene possibly also corresponds to the regime of BCS-BEC crossover between local and extended pairs in the d-wave channel (similarly to underdoped cuprates) but with anomalous chiral superconductivity of \( d + id \) type [177] which is in agreement with the phase diagram for idealized bilayer graphene in AB modification at low doping levels [20,22,23]. The discovery of superconductivity in graphene with many properties resembling the cuprates helps us to build the bridge between low temperature topological superconductors based on graphene [177] and bismuth [178-180] and vanadium oxides [181-183] which are very promising for superconducting nanoelectronics and quantum calculations (for creation of topologically protected qubits) and the recently discovered high-temperature superconductor \( \text{LaH}_{10} \) [84, 85, 86] with a nodal line of Dirac points close to the Fermi surface which is very promising for energetics.

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