Data compression on board the PLANCK Satellite Low Frequency Instrument: optimal compression rate

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ABSTRACT Data on board the future PLANCK Low Frequency Instrument (LFI), to measure the Cosmic Microwave Background (CMB) anisotropies, consist of \( N \) differential temperature measurements, expanding a range of values we shall call \( R \). Preliminary studies and telemetry allocation indicate the need of compressing these data by a ratio of \( c_r \gtrsim 10 \). Here we present a study of entropy for (correlated multi-Gaussian discrete) noise, showing how the optimal compression \( c_{r,\text{opt}} \), for a linearly discretized data set with \( N_{\text{bits}} = \log_2 N_{\text{max}} \) bits is given by:

\[
c_r \simeq N_{\text{bits}} / \log_2(\sqrt{2\pi e} \sigma_e / \Delta),
\]

where \( \sigma_e \equiv (\det C)^{1/2N} \) is some effective noise rms given by the covariance matrix \( C \) and \( \Delta \equiv R / N_{\text{max}} \) is the digital resolution. This \( \Delta \) only needs to be as small as the instrumental white noise RMS: \( \Delta \simeq \sigma_T \approx 2mK \) (the nominal \( \mu K \) pixel sensitivity will only be achieved after averaging). Within the currently proposed \( N_{\text{bits}} = 16 \) representation, a linear analogue to digital converter (ADC) will allow the digital storage of a large dynamic range of differential temperature \( R = N_{\text{max}} \Delta \) accounting for possible instrument drifts and instabilities (which could be reduced by proper on-board calibration). A well calibrated signal will be dominated by thermal (white) noise in the instrument: \( \sigma_e \simeq \sigma_T \), which could yield large compression rates \( c_{r,\text{opt}} \simeq 8 \). This is the maximum lossless compression possible. In practice, point sources and \( 1/f \) noise will produce \( \sigma_e > \sigma_T \) and \( c_{r,\text{opt}} < 8 \). This strategy seems safer than non-linear ADC or data reduction schemes (which could also be used at some stage).

KEYWORDS: Data compression; Signal processing; Information Theory.

1. INTRODUCTION

A compression rate of about \( c_r \simeq 10 \) is required on board the PLANCK Satellite LFI (see §2.1 below). The data rate could be reduced by accounting for the relative significance of different bits (large and small temperature differences) in the analogue-to-digital converters ADC (Herreros et al. 1997). A further compression is assumed to be possible with classical lossless data compression techniques.

Typically, standard lossless data compression techniques are applied successfully only to data sets with some redundancy. This redundancy can be formally expressed using the entropy per component (Shannon’s entropy), \( h \). A discretized data set can be represented by \( N_{\text{bits}} \), which for a linear ADC is typically given by the maximum range \( N_{\text{max}}: N_{\text{bits}} = \log_2 N_{\text{max}} \). If we express the joint probability for a set of \( N \) measurements as \( p_{i_1, \ldots, i_N} \), we have that the Shannon entropy per component of the data set is:

\[
h \equiv -\frac{1}{N} \sum_{i_1, \ldots, i_N} 1 \cdot p_{i_1, \ldots, i_N} \log_2(p_{i_1, \ldots, i_N}). \]
Table 1. Parameters for the radiometers: a) central frequency (band width is 20%); b) angular resolution (beam FWHM); c) the RMS thermal noise expected at 6.9 ms (144.9 Hz) sampling; d) range of temperatures expected from the sky (Jupiter, dipole, S-Z); e) number of detectors (2x horns); f) total data rate at 6.9 ms (2.5 arcmin); g) data rate for pixels of length FWHM/2.5 along the scanning circle.

| $\nu$(GHz) | FWHM | $\sigma_T$(ts) | Range $\Delta T$ | Det. | Rate(2.5') | Rate(2.5' FWHM) |
|------------|------|---------------|-----------------|------|-------------|------------------|
| 30         | 33'  | 2.8 mK        | -30-61 mK       | 4    | 9.3 Kb s$^{-1}$ | 1.8 Kb s$^{-1}$ |
| 44         | 23'  | 3.2 mK        | -30-138 mK      | 6    | 13.9 Kb s$^{-1}$ | 3.8 Kb s$^{-1}$ |
| 70         | 14'  | 4.1 mK        | -20-340 mK      | 12   | 27.8 Kb s$^{-1}$ | 12.4 Kb s$^{-1}$ |
| 100        | 10'  | 5.1 mK        | -10-667 mK      | 34   | 78.8 Kb s$^{-1}$ | 49.3 Kb s$^{-1}$ |
| TOTAL      |      |               | -30-667 mK      | 56   | 130 Kb s$^{-1}$  | 67 Kb s$^{-1}$  |
| +LOAD      |      |               |                 | 112  | 260 Kb s$^{-1}$  |                 |

Shannon’s theorem states that $h$ is a lower bound to the average length of the code units. We will define the theoretical (optimal) compression rate as

$$c_{r,\text{opt}} = \frac{N_{\text{bits}}}{h}$$

For a uniform distribution of $N$ measurements we have $p_i = 1/N$ and $h = \log_2 N$, which equals the number of bits per data. Thus: it is not possible to compress a (uniformly) random distribution of measurements. If noise is discretized to a high resolution (as compared to its variance) the resulting distribution of numbers approaches a uniform distribution and it is therefore virtually impossible to compress. This indicates that, to first approximation, it seems difficult to produce a lossless algorithm for compression when the data is dominated by noise, but, as we shall see, the problem depends crucially on the digital resolution and the range of values to be stored.

2. THE COMPRESSION PROBLEM

2.1 Data Rate, Telemetry and compression

Following the PLANCK LFI Scientific and Technical Plan (Part I, §6.3, Mandolesi et al. 1998) the raw data rate of the LFI is $r_d \simeq 260$ Kb s$^{-1}$. This assumes: i) a sample frequency of 6.9 ms or $f_{\text{sampl}} = 144.9$ Hz, which corresponds to 2.5 arcmin in the sky, 1/4 of the FWHM at 100 GHz, ii) $N_{\text{detec}} = 112$ detectors: sky and reference load temperature for 56 radiometers. iii) $N_{\text{bits}} = 16$ bits data representation. Thus that the raw data rate is:

$$r_d = f_{\text{sampl}} \times N_{\text{detec}} \times N_{\text{bits}} \simeq 259.7 \text{ Kb s}^{-1}.$$
The values for each channel are shown in Table 1. A factor of two reduction can be obtained by only transmitting the difference between sky and reference temperature. To allow for the recovery of diagnostic information on the separate stability of the amplifiers and loads, the full sky and reference channels of a single radiometer could be sent at a time (changing the selected radiometer from time to time to cover all channels).

Note that the sampling resolution of 6.9 ms corresponds to 2.5 arcmin in the sky, which is smaller than the nominal FWHM resolution. Adjacent pixels in a circle could be averaged on-board to obtain the nominal resolution (along the circle direction). In this case the pixel size should still be at least \( \approx 2.5 \) smaller that the FWHM to allow for a proper map reconstruction. Note that each circle in the sky will be separated by 2.5' so even after this averaging along the circle scan there is still a lot of redundancy across circles. For pixels of size \( \theta \approx FWHM/2.5 \) along the circle scan the total scientific rate could be reduced to \( r \approx 67 \text{ Kb s}^{-1} \) as shown in Table 1 (or 134 Kb s\(^{-1}\) with some subset information of the ref. load).

The telemetry allocation for the LFI scientific data is expected to be \( r_t = 20 \text{ Kb s}^{-1} \). Thus the target compression rates are about:

\[
c_r = \frac{r_d}{r_t} \approx 3 - 13,
\]

depending on the actual on-board processing and requirements.

### 2.2 Data Structure

Planck’s satellite spins with a frequency \( f_{\text{spin}} = 1 \text{ rpm} \) so that the telescope (pointing at approximately right angle to the spin axis) sweep out nearly great circles in the sky. Each circle is scanned over 1 (or 2) hours, so that there are 60 (or 120) images of the same pixel. Each measurement is mostly dominated by instrumental noise, \( \sigma_T \approx 2mK \) (see Table 1) rather than by the CMB noise (\( \sigma_{\text{CMB}} \approx 10^{-2}mK \)). If this noise (at frequencies smaller than \( f_{\text{spin}} \)) is mostly thermal, there is no redundancy in these images and little hope for compression. But one could then say that in this case there is no need for compression, as we can just average those 60 images of a pixel and only send the mean downwards to Earth. The problem is that one expects \( 1/f \) noise to dominate the instrument noise at frequencies smaller than \( \sim 0.1 \text{ Hz} \). Thus, compression is only required when we want to keep these 60 (120) images in order to correct for the instrument instability in the data reduction process (on Earth). This \( 1/f \) type of noise is more redundant and might be subject to some compression, but even in this case if we keep it to a high resolution (as compared to its \( \text{rms} \)) the resulting probabilities would be close to a uniform distribution and compression would be nearly impossible.

### 2.3 Dynamic range & calibration

The final dynamic range for the measured temperature differences per angular resolution pixel will be \( \Delta T \approx 1\mu K - 1K \). The lowest resolution of \( \approx 1\mu K \) will only be
obtained after averaging all data. The highest $\simeq 1K$ being the hottest source that we want to keep (not saturated) by anyone of the frequencies. Positive signals from Jupiter, which will be used for calibration, can be as large as $\simeq 0.7K$ at 100 Ghz. Other point sources and the Galaxy give intermediate positive values. Negative differences (with respect to the mean CMB $T \simeq 2.7K$), of the order of a few $mK$, can be originated by the dipole, the relative velocity between the satellite velocity and the CMB rest frame. The Sunyaev-Zeldovich effect (towards a total of a few hundreds Clusters of Galaxies) can also give a negative signal of few $10mK$. Thus the overall range of mesurements is $-30mK$ to $1K$.

As pointed out by Herreros et al. (1997) the temperature resolution is given by the receiver noise $\sigma_T$ on the sampling time 6.9 ms (or corresponding value if there is some on-board averaging) and not by the final target sensitivity. At the end of the mission, each FWHM pixel will have been measured $\simeq 10^6$ times. Thus a lower resolution $\Delta T \simeq 1\mu K$ is not necessary on board, given that the raw signal is dominated by the white noise component. This higher resolution will be later obtained by the pixel averaging (data reduction on Earth).

We can distinguish two basic components for the receiver noise: the white or thermal noise, and the instabilities or calibration gains (like the $1/f$ noise). An example is given by the following power spectrum of frequencies $f$:

$$P(f) = A \left(1 + \frac{f_{knee}}{|f|}\right).$$  \hspace{1cm} (5)

The 'knee' frequency, $f_{knee}$, is expected to be $f_{knee} \simeq 0.005$ Hz for a 4K load or $f_{knee} \simeq 0.06$ Hz for a 20K load. The expected RMS thermal noise, $\sigma_T \propto A$ at the sampling frequency (2.5 armin), is listed in Table 1. The lowest value is given by the 30 Ghz channel and could be further reduced to $\simeq 1mK$ if the data is averaged to FWHM/2.5 to obtain the nominal resolution. The larger values in the dynamical range can be affected by the calibration gains. This is important and should be carefully taken into account if a non-linear ADC is used, as gains could then change the relative significance of measurements (eg, less significant bits shifting because of gains). In fact, a $1/f$ power spectrum integrated from the knee-frequency ($f_{knee}$ for a time $T$, gives a diverging rms noise:

$$\sigma^2_{1f} = \frac{\sigma_T^2}{f_{max}} \int_{1/T}^{f_{max}} df \frac{f_{knee}}{f} = \sigma_T^2 \int_{f_{knee}}^{f_{max}} \ln(T f_{max}) \frac{f_{knee}^2}{f_{max}}$$  \hspace{1cm} (6)

For a $T \simeq 1$ year mission the contribution from the $1/f$ noise in pixels averaged after successive pointings $f_{max} \simeq 10^{-4}$ and we have $\sigma^2_{1f} \simeq 10^4 \sigma_T^2$. This illustrates why the calibration problem is so important and makes a large dynamic range desirable. Averaging pixels at the spin rate, $f_{spin} \simeq f_{spin}$, gives $\sigma^2_{1f} \simeq 10\sigma_T^2$. This is not too bad for the dynamic range, but it corresponds to a mean value and there could be more important instantaneous or temporal gains. Drifts with periods longer than the spin period (1 rpm) can be removed by requiring that the average signal over each rotation at the same pointing remains constant. Drifts between
pointings (after 1 or 2 hours) could be reduced by using the overlapping pixels. All this can be easily done on-board, while a more careful matching is still possible (and necessary) on Earth. This allows the on-board gain to be calibrated on timescale larger than 1 min with an accuracy given by $\sigma_T$. Additional and more careful in-flight calibration can also be done using the the signal from external planets and the CMB dipole. In any case we will assume here that instabilities or gains are under control ($\simeq \sigma_T$) for frequencies larger than the spin frequency. For smaller times we will typically use Eq.(5) as a mean value but bearing in mind that larger gains are also possible.

In summary, because of the possible instrument gains, it is important to have a constant resolution of $\simeq \sigma_T \approx 1 mK$ over a large range of values (\(\Delta T \approx 1 K\)) to be able to recover the underlying signal after proper calibration. This could be partially done on board. A constant resolution indicates the need of a linear ADC, which with adequate compression (presented next) will be shown to be a good alternative to non-linear ADC.

3. A SOLUTION TO THE PROBLEM

In a separate paper (Romeo et al. 1998) we have presented a general study of (correlated multi-Gaussian) noise compression by studying Shannon entropies per component $h$, and therefore the optimal compression $c_{r,opt}$ in Eq.(2). For a linearly discretized data with $N_{\text{bits}} = \log_2 N_{\text{max}}$ bits, $h$ in Eq.(1) depends only on the ratio of the digital resolution $\Delta$ to the effective rms noise, $\sigma_e$:

$$h = \log_2 \left( \sqrt{2\pi e} \frac{\sigma_e}{\Delta} \right) \quad (7)$$

with $\sigma_e^2 \equiv (\text{det}C)^{1/N}$, where $C$ is the covariance matrix of the (multi-Gaussian random) field $x_i$: $C_{ij} \equiv \langle x_i x_j \rangle$. As mentioned in the section above, $\Delta$ in Planck only needs to be as small as the instrumental white noise $\sigma_T$. If the data is dominated by thermal instrumental noise we have $\sigma_e \simeq \sigma_T \simeq \Delta$ and the optimal compression is simply:

$$c_{r,max} = N_{\text{bits}} / \log_2 \left( \sqrt{2\pi e} \right) \approx 8. \quad (8)$$

where we have use $N_{\text{bits}} = 16$ as planned for the Planck LFI. This is the maximum lossless compression that can be achieved for a well calibrated signal dominated by instrumental thermal (white) noise. This very large compression rate can be obtained because there is a large range of values $\simeq \Delta 2^{N_{\text{bits}}}$ which has a very small probability, and therefore can be easily compressed (e.g. by Huffman’s or arithmetic coding) but can’t be omitted because they are needed to calibrate the instrument gains (and to measure the point sources and the galaxy). In practice point sources and $1/f$ noise will produce $\sigma_e > \sigma_T$ and $c_{r,opt} < 8$ (note that in general $\sigma_e < \sigma_0$, the one-point RMS fluctuation) and will also need to account for the fact that the galaxy and the point sources can’t be represented by a multi-Gaussian, even if we allow for a different power spectrum. Let’s do a more realistic case with $1/f$ noise, but still without point sources or the galaxy.
### 3.1 Compression with 1/f noise

In the case of the power spectrum in Eq.(5), Romeo et al. (1998) find for the entropy:

\[
h - h_0 \approx \frac{1}{2} \log_2 \left[ 1 + \frac{f_{\text{knee}}}{f_{\text{max}}} \right] + \frac{1}{2} \frac{f_{\text{knee}}}{f_{\text{max}}} \log_2 \left[ \frac{f_{\text{max}} + f_{\text{knee}}}{f_{\text{min}} + f_{\text{knee}}} \right]
\]

(9)

where \( h_0 \) is the white noise (thermal) contribution \( [h_0 = \log_2(\sqrt{2\pi e \sigma_T/\Delta})] \) and \( f_{\text{max}} \) and \( f_{\text{min}} \) are the maximum and minimum frequencies covered with the \( N \) measurements. In our case we are only interested in the contribution within a revolution (as we are assuming a good calibration at smaller frequencies) so that \( f_{\text{max}} = f_{\text{sampl}} \simeq 145 \text{ Hz} \) and \( f_{\text{min}} \simeq f_{\text{spin}} \simeq 1/60 \text{ Hz} \).

Another case which can be of interest is:

\[
P(f) = \begin{cases} 
A', & \text{for } f \leq f_L, \\
A + \frac{f_{\text{knee}}}{|f|}, & \text{for } f_L < f \leq f_{\text{max}}.
\end{cases}
\]

(10)

Taking again as reference \( h_0 \) as the thermal case where \( f_{\text{knee}} = 0 \) and \( A' = A \), we may write

\[
h \simeq h_0 + \frac{1}{2} \log_2 \left[ 1 + \frac{f_{\text{knee}}}{f_{\text{max}}} \right] - \frac{1}{2} \frac{f_L}{f_{\text{max}}} \log_2 \left[ 1 + \frac{f_{\text{knee}}}{f_L} \right]
\]

\[
+ \frac{1}{2} \frac{f_{\text{knee}}}{f_{\text{max}}} \log_2 \left[ \frac{f_{\text{max}} + f_{\text{knee}}}{f_L + f_{\text{knee}}} \right] + \frac{1}{2} \frac{f_L}{f_{\text{max}}} \log_2 \left[ \frac{A'}{A} \right]
\]

(11)

In Table 2 we have presented some compression rate values corresponding to this entropy for different 'knee' frequencies.

In order to quote a more realistic compression rate, it is crucial to have a detailed model of the instrument instabilities (i.e., what is the value of \( f_{\text{knee}} \) frequency? what is the value of the white noise amplitude?), the detailed ADC model, the

| \( f_{\text{knee}} \) | \( f_{\text{min}} \) | \( f_{\text{max}} \) | \( f_L (\text{Hz}) \) | \( A'/A \) | \( C_{\text{opt}} \) |
|---|---|---|---|---|---|
| 0 | 0.017 | 145 | 0 | 1 | 7.816 |
| 0.005 | 0.017 | 145 | 0 | 1 | 7.814 |
| 0.06 | 0.017 | 145 | 0 | 1 | 7.798 |
| 0.06 | 0.017 | 10 | 0 | 1 | 7.754 |
| 1.00 | 0.017 | 145 | 0 | 1 | 7.750 |
| 0.005 | 0 | 145 | 0.017 | 1 | 7.814 |
| 0.06 | 0 | 145 | 0.017 | 1 | 7.799 |
| 0.06 | 0 | 145 | 0.017 | 10 | 7.798 |

Table 2. Compression factors for different instrumental noise.
on-board data and calibration strategy, the pointing and a detailed simulations of
the sky.

5. CONCLUSION

Because of the possible instrument gains, it is important to have a constant resolution
of $\sigma_T \simeq 1 mK$ over a large range of values ($\Delta T \simeq 1 K$). This indicates
the convenience of a linear ADC. Although some compression can be achieved with
non-linear ADC, in this case standard linear lossless data compression techniques
seem safer (because of possible calibration drifts) and more efficient (because of the
larger compression rates).

The maximum lossless compression that can be achieved with a well calibrated
signal is $c_r \simeq 8$ (with data of $N_{\text{bits}} = 16$). Similar values can be obtained ($\S 3.1$)
even for non-thermal instrumental noise. These results assume that the dominant
component of data is multi-Gaussian (correlated) noise. Although this might be true
for the mean instrumental noise and the CMB signal, it is not true for point
sources or the galaxy. Nevertheless, a compression factor of $c_r \simeq 3$ can be easily
obtained if we assume that at least $\simeq 75\%$ of the data is described by well-calibrated
instrumental noise (the CMB is only relevant after averaging over many pixels).

Thus, compression factors in the range $c_r \simeq 3 - 7$ are possible within the approxima-
tion for the data structure we have considered. Compression of the raw
data including the reference load ($r_d \simeq 260 \text{ Kb s}^{-1}$) does not seem possible with
a telemetry rate of $r_t \simeq 20 \text{ Kb s}^{-1}$, but it might be possible for $r_t \simeq 40 \text{ Kb s}^{-1}$.
An alternative is to process on-board some of the current redundancy by sticking
nearby pixels to the level of the nominal resolution (FWHM/2.5), as indicated in
Table 1. The actual compression can be achieved with standard Huffman’s or arith-
metic coding, although other possibilities can also be considered or tailored for this
problem (see Romeo et al. 1998 for more details).

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REFERENCES

Herreros, J.M., Hoyland, R., Rebolo, R., Watson, R.A., 1997 Ref.:LFI-IAC-TNT-001
Mandolesi, N. et al. , 1998, LFI for Planck, a proposal to ESA’s AO.
Romeo, A., Gaztañaga, E., Barriga J., Elizalde, E., 1998, physics-ph/9809004

http://xxx.unizar.es/abs/physics/9809004