The Maximal Wrinkle Angle During the Bubble Collapse and Its Application to the Bubble Electrospinning

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Polymer bubbles are ubiquitously used for the fabrication of nanoﬁbers by the bubble electrospinning. When a bubble is broken, the fragments tend to be wrinkled. The wrinkle angle plays an important in controlling the fiber morphology during the bubble electrospinning. This paper shows the maximal angle is about 49°, which is close to the experimental value of 50°. This maximal angle can be used for the optimal design of the nozzle in the bubble electrospinning for the fabrication of non-smooth nanoﬁbers.

Keywords: bubble, electrospinning, nanoﬁber, wrinkle, instability

INTRODUCTION

Bubble dynamics is an old discipline, and the Young-Laplace equation (Liu and Dou, 2013) is widely used for static analysis. The bubble’s rupture affects many materials’ fabricating processes (Zhang et al., 2021), especially for nanoﬁber fabrication by bubble electrospinning (Kong, 2015; Liu et al., 2020a; Li and He, 2020; Yin et al., 2020). When a bubble is broken, many jets are ejected, or many daughter bubbles are formed (Bird et al., 2010). Oratis et al. (2020) found the wrinkle mechanism of a liquid sheet during a bubble collapse, which has attracted skyrocketing attention from various ﬁelds, including mathematics and material science. Although this mechanism is new and has great promise, a mathematical model lacks for the prediction of the wrinkle angle. Here we establish a simple formula to study the wrinkle angle.

MAXIMAL WRINKLE ANGLE

The bubble collapse and the wrinkle of the liquid sheet (Oratis et al., 2020) play an important role in the bubble electrospinning (Yin et al., 2020), Gratis et al. obtained an unparalleled achievement in the bubble collapse dynamics and found the surface tension drives the collapse and initiates its wrinkle (Oratis et al., 2020).

The surface tension depends upon the liquid sheet’s curvature radius, we assume that the initial surface tension is $F_0$, and the initial angle between the free sheet section and the solid surface is $\alpha_0$, see Figure 1. During the collapse process, the liquid sheet tends to zero when it is completely collapsed, and we assume the surface tension becomes zero at its final collapse. Accordingly, we assume that the surface tension can be expressed as

$$F(\alpha) = \frac{F_0}{\alpha_0} \alpha$$  \hspace{1cm} (1)
From Eq. 1, it is obvious that the surface tension vanishes completely when the bubble is completely collapsed when α = 0°.

The force given in Eq. 1 can be decomposed into the radial and tangential forces, as shown in Figure 1, the former is the main force for collapse, and the latter is perpendicular to the section of the sheet, and it is the main force for liquid sheet’s instability, which leads finally to the wrinkle. According to Figure 1, the tangential force, \( \sigma \), can be expressed as

\[
\sigma = F(\alpha) \cos \alpha = \frac{F_0}{\alpha_0} \alpha \cos \alpha
\]  

(2)

The maximal \( \sigma \) happens when

\[
\frac{d}{d\alpha} \sigma = \frac{F_0}{\alpha_0} (\cos \alpha - \alpha \sin \alpha) = 0
\]  

(3)

That is

\[
\cos \alpha - \alpha \sin \alpha = 0
\]  

(4)

This equation can be solved by the ancient Chinese algorithm (He, 2016). We choose two angles \( \alpha_1 = \pi/3 \) and \( \alpha_2 = \pi/4 \), and the following two residuals are obtained:

\[
R_1 = \cos \alpha_1 - \alpha_1 \sin \alpha_1 = \frac{1}{2} - \frac{\pi}{3} \times \frac{\sqrt{3}}{2} = -0.4069
\]  

(5)

and

\[
R_2 = \cos \alpha_2 - \alpha_2 \sin \alpha_2 = \frac{\sqrt{2}}{2} - \frac{\pi}{4} \times \frac{\sqrt{2}}{2} = 0.1517
\]  

(6)

By the ancient Chinese algorithm (He, 2016), we have

\[
\alpha = \frac{R_1 \alpha_2 - R_2 \alpha_1}{R_1 - R_2} = \frac{-0.4069 \times \frac{\pi}{3} - 0.1517 \times \frac{\pi}{3}}{-0.4069 - 0.1517} = 0.2726\pi
\]  

(7)

The exact root of Eq. 4 is \( \alpha = 0.2737\pi \), the relative error is 0.4%. The ancient Chinese algorithm given in Eq. 7 was further developed into an iteration method, which was called Chun-Hui He iteration method (Khan, 2021).

When \( \alpha = 0.2737\pi \) or 49°, \( \sigma \) reaches its maximum; this maximal surface tension leads to instability of the fragments; as a result, the wrinkle occurs. The instability can be analyzed in a similar way as discussed in Refs. (He et al., 2021; Zuo and Liu, 2021).

**EXPERIMENT**

Bubble electrospinning was originally designed for the fabrication of smooth fibers (Liu et al., 2020; Wan, 2020). In order to verify the theoretical prediction, we carried out an experiment where
the angle between the solution surface and the bubble’s wall is about 49°, see Figure 2.

The spun solution was prepared by adding certain amounts of Lithium chloride (LiCl) dropwise to a 15 wt% Polyacrylonitrile/Polyethersulfone (PAN/PES) solution in N,N-Dimethylacetamide (DMAC) following with ultrasonic agitation. The weight ratio of PAN and PES was controlled at 3/2, and the weight percentage of LiCl in the mixed solution was 1 wt%. Afterward, the PAN/PES/LiCl blend nanofibers were obtained using a high DC voltage power supply at a 20 kV potential and the collector with a distance of 15 cm from the bubble top. The SEM image of PAN/PES/LiCl nanofibers was presented in Figure 3. It is seen that we obtain the non-smooth fibers.

DISCUSSION AND CONCLUSION

This short paper gives a simple mathematical analysis, showing that the maximal wrinkle angle is about 49°, which is much closed to Oratis, et al.’s experimental value, which was about 50°. This angle can be used to design the nozzle angle in the bubble electrospinning; when wrinkled nanofibers are to be fabricated, the nozzle angle should be 49°. If smooth nanofibers are wanted, the nozzle angle should deviate from 49°.

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