Abstract

We consider bosonic atoms with a repulsive contact interaction in a trap potential for a Bose-Einstein condensation (BEC) and additionally include a random potential. The ensemble averages for two models of static (I) and dynamic (II) disorder are performed and investigated in parallel. The bosonic many body systems of the two disorder models are represented by coherent state path integrals \( Z_{I}[\mathcal{J}], Z_{II}[\mathcal{J}] \) on the Keldysh time contour which allow exact ensemble averages for zero and finite temperatures. These ensemble averages of coherent state path integrals therefore present alternatives to replica field theories or super-symmetric averaging techniques. Hubbard-Stratonovich transformations (HST) lead to two corresponding self-energies for the hermitian repulsive interaction and for the non-hermitian disorder-interaction. The self-energy of the repulsive interaction is absorbed by a shift into the disorder-self-energy which comprises as an element of a larger symplectic Lie algebra \( \text{sp}(4M) \) the self-energy of the repulsive interaction as a subalgebra (which is equivalent to the direct product of \( M \) times \( \text{sp}(2) \); \( M \) is the number of discrete time intervals of the disorder-self-energy in the generating function \( Z_{I}[\mathcal{J}], Z_{II}[\mathcal{J}] \)). After removal of the remaining Gaussian integral for the self-energy of the repulsive interaction, the first order variations of the coherent state path integrals \( Z_{I}[\mathcal{J}], Z_{II}[\mathcal{J}] \) result in the exact mean field or saddle point equations, solely depending on the disorder-self-energy matrix. These equations can be solved by continued fractions and are reminiscent to the 'Nambu-Gorkov' Green function formalism in superconductivity because anomalous terms or pair condensates of the bosonic atoms are also included into the selfenergies. The derived mean field equations of the models with static (I) and dynamic (II) disorder are particularly applicable for BEC in \( d = 3 \) spatial dimensions because of the singularity of the density of states at vanishing wavevector. However, one usually starts out from restricted applicability of the mean field approach for \( d = 2 \); therefore, it is also pointed out that one should consider different HST’s in \( d = 2 \) spatial dimensions with the block diagonal densities as 'hinge' functions and that one has to introduce a coset decomposition \( \text{Sp}(4M) \backslash U(2M) \) into densities and anomalous terms of the total disorder-self-energy \( \text{sp}(4M) \) for deriving a nonlinear sigma model.

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1 Introduction

The original Bose-Einstein condensation of atoms in traps has been extended and performed in various manners [1]-[6]. In this article we investigate a BE-system of bosonic atoms with repulsive interactions in a trap potential and include a random potential which represents a model of ensemble averaged disorder. We concentrate on the averaging procedure, derive the exact saddle point equation and describe continued fractions as solutions of the mean field equations which also comprise anomalous terms or pair condensates of the self-energies and the saddle point equations [12, 13]. The self-energy of the pure interaction (as the direct product of $M$ times $sp(2)$ and as subalgebra of the disorder-self-energy $sp(4M)$) can be absorbed by a shift into the 'larger' self-energy $sp(4M)$ of the ensemble averaged random potential ('$M$' denotes the number of discrete time intervals). This disorder-self-energy $sp(4M)$ is represented by a larger group and also follows from a HST, but of a non-hermitian 'interaction' term resulting from the average over the random disorder [11, 32]. After a Gaussian integral of the self-energy of the repulsive interaction, the self-energy of the ensemble averaged disorder only remains as a matrix valued field depending on the two branches of the time contour. The exact saddle point equation, obtained by a first order variation, therefore only consists of the self-energy of the disorder. Apart from the purely bosonic constituents of the disorder models the presented approach is reminiscent of the Bogoliubov-de Gennes equations and the Nambu-Gorkov Green function formalism in the theory of superconductivity [14]-[16]. Solutions of the saddle point equation can be achieved by continued fraction of the disorder-self-energy starting from the free Green function [17, 18]. The iteration process for the continued fractions can be simplified in the presence of spatial symmetries or time independence. It is the imaginary increment $-i \varepsilon_p$ ($\varepsilon_{\pm} = \pm \varepsilon$, $\varepsilon > 0$) in the Green function of the disorder-self-energy which determines the solutions of the self-energy of the saddle point equation. In fact the original coherent state path integral, consisting only of the bosonic fields for the atoms, is only defined by introducing this imaginary increment $-i \varepsilon_p$ on the time contour $t_p$. This imaginary increment can be considered as a kind of regularization and is necessary even without zero eigenvalues in the exponents of the time development operators. This follows because Gaussian like integrations (1.1) as part of a time development operator only lead to the absolute values of eigenvalues with respect to the sign of the considered time intervals. Therefore, the important sign or the information about bounded and unbounded states would be lost under time reversal without the infinitesimal imaginary part $-i \varepsilon_p$

$$
\int d|\psi(x,t)| \exp \left\{ -\frac{i}{\hbar} \int_C dt_p \sum_{\tilde{z}} \bar{\psi}_{\tilde{z}}(t_p) \left(-i \varepsilon_p + h_{\tilde{z}} \right) \psi_{\tilde{z}}(t_p) \right\} \propto
$$

$$
= \left[ -\frac{\varepsilon_p}{\hbar} \frac{\Delta t_p}{\hbar} - \frac{i}{\hbar} \frac{\Delta p_+}{\Delta p_-} h_{\tilde{z}} \right]^{-1} \varepsilon_{p=\pm} = \pm \varepsilon, \ (\varepsilon > 0); \quad \Delta t_{p=\pm} = \pm \Delta t, \ (\Delta t > 0) .
$$

The imaginary increment has therefore also to be taken into account in coherent state path integrals without disorder in order to distinguish between advanced and retarded Green functions which can appear after
transformation and integrations over field variables. One can improve the expansion of the actions in the final path integral to quadratic and higher order in the single self-energy matrix of the disorder for the inclusion of fluctuation properties around the saddle point solution.

In section 2 we describe the average of a coherent state path integral on a time contour and consider the two cases of static and dynamic disorder. In section 3 the Hubbard Stratonovich transformations are given and the exact saddle point equations are derived for static and dynamic disorder. We outline the various steps for solving the saddle point equation via continued fractions and indicate the reminiscence to many-body theory with anomalous terms. Section 4 lists the relevant observables obtained by differentiation of the generating function with respect to a source term. Section 5 contains a summary and points out the extension of the mean field approach in sections 2-4 for many-body theory with anomalous terms. Section 5 contains the disorder-self-energy into densities and anomalous terms with a decomposition into a subgroup $Sp(d) \times U(1)$ and coset part $Sp(d+1)/U(d)$. In the spatial $d = 2$ case different kinds of HST's have to be performed for a corresponding nonlinear sigma model as for the $d = 3$ mean field approach (see section 5). The saddle point equations in $d = 3$ can lead to nonanalytic behaviour because of the singularity of the density of states at vanishing wavevector. In the spatial $d = 2$ case one has to apply the Weyl unitary trick for the parametrization of the disorder-self-energy into densities and anomalous terms with a decomposition into a subgroup $SL(2R)/U(1)$ so that a nonlinear sigma model can be achieved after a suitable HST and gradient expansion. In the strictly one dimensional case $d = 1$ one has to take into account large fluctuations so that the transfer matrix approach of disorder-ensemble-averaged generating functions can be chosen as the prevailing computational method for density and correlation functions.

2 Coherent state path integral

2.1 Averaging methods for zero and finite temperatures

The Hamilton operators for the disordered bosonic systems with the Bose field operators $[\hat{\psi}_x, \hat{\psi}_x^+] = \delta_{x,x'}$ contain the trap potential $u(x)$, the quartic, repulsive contact interaction with parameter $V_0 > 0$ and the kinetic energy term with mass $m$. Two kinds of random potentials $V_I(x)$ (static disorder) and $V_{II}(x,t)$ (dynamic disorder) are introduced separately and result in the two different Hamilton operators $\hat{H}_I(\hat{\psi}_x, \hat{\psi}_x^+, V_I)$ and $\hat{H}_{II}(\hat{\psi}_x, \hat{\psi}_x^+, V_{II})$ so that we consider two models I and II in parallel. We examine these two models of disorder at zero temperature with coherent state path integrals and incorporate a chemical potential or zero energy $\mu_0$. Furthermore, a $U(1)$ symmetry breaking, hermitian source term with $\hat{J}_0^V(x,t)$ is included for the creation of a coherent BE-condensate wavefunction. Since we also consider pair condensates of bosonic atoms on the coset part $Sp(d+1)/U(d)$ of the disorder-self-energy, we have to define a hermitian source term $\hat{J}_{0\psi}(x,t)$ for the creation of anomalous terms as $\langle \hat{\psi}_x(t'_q) \hat{\psi}_x(t_p) \rangle$ and $\langle \hat{\psi}_x(t_p) \hat{\psi}_x^+(t'_q) \rangle$ (for the contour times $t_p, t'_q$ see relations (2.10,2.11))

$$\hat{H}_I(\hat{\psi}_x^+, \hat{\psi}_x, V_I) = \sum_{x} \hat{\psi}_x^+ \left( -\frac{\hbar^2}{2m} \Delta + u(x) - \mu_0 + V_I(x) + V_0 \hat{\psi}_x \right) \hat{\psi}_x + \sum_{x} \left( j^+_{\psi,x}(t) \hat{\psi}_x + \hat{\psi}_x^+ j_{\psi,x}(t) \right)$$

$$\hat{H}_{II}(\hat{\psi}_x^+, \hat{\psi}_x, V_{II}) = \sum_{x} \hat{\psi}_x^+ \left( -\frac{\hbar^2}{2m} \Delta + u(x) - \mu_0 + V_{II}(x,t) + V_0 \hat{\psi}_x \right) \hat{\psi}_x + \sum_{x} \left( j^+_{\psi,x}(t) \hat{\psi}_x + \hat{\psi}_x^+ j_{\psi,x}(t) \right)$$

3
\[
\sum_{\vec{x}} \ldots \sum_{\vec{x}_i} \left( \frac{\Delta x}{L} \right)^d \ldots = \int_{L:d} \frac{d\vec{x}_i}{L^d:} \ldots \quad \Omega = \frac{1}{\Delta t} \quad -\frac{T_0}{2} < t < +\frac{T_0}{2} . \tag{2.3}
\]

Especially, in the case of disordered systems, the second moments of the Gaussian probability distributions of the random potentials \( V_I(\vec{x}) \) \( \text{(2.1)} \) and \( V_{II}(\vec{x}, t) \) \( \text{(2.6)} \) have to be normalized in such a manner that the broadenings of the eigenvalue spectra of the hermitian, ordered parts in \( \hat{V}_{Es} \)\[\text{integrals} \]

\[\sum_{\vec{x}} \text{broadenings of the eigenvalue spectra of the hermitian, ordered parts in } \hat{V}_{Es} \]

\[
\text{Especially, in the case of disordered systems, the second moments of the Gaussian probability distributions}
\]

\[
\text{are represented by coherent state path integrals (2.6,2.7) of the unitary time development operators (2.8,2.9)}
\]

\[
\text{(±disordered systems I, II couple the two 'branches of the contour time in contrast to ordered systems)}
\]

\[
\text{where the self-energy fields depend only on time arguments with a single branch of the contour, respectively}
\]

\[
\text{[37]}
\]

\[
Z[J, V_I] = \langle 0 | \hat{U}_I(-\frac{T_0}{2}, +\frac{T_0}{2}; V_I; J) | \hat{U}_I(+\frac{T_0}{2}, -\frac{T_0}{2}; V_I; J) | 0 \rangle \tag{2.6}
\]

\[
Z[J, V_{II}] = \langle 0 | \hat{U}_{II}(-\frac{T_0}{2}, +\frac{T_0}{2}; V_{II}; J) | \hat{U}_{II}(+\frac{T_0}{2}, -\frac{T_0}{2}; V_{II}; J) | 0 \rangle \tag{2.7}
\]

\[
\hat{U}_I(t, -\frac{T_0}{2}; V_I; J) = \mathcal{T} \exp \left\{ -\frac{i}{\hbar} \int_{-\frac{T_0}{2}}^t d\tau \left[ \hat{H}_I(\hat{\psi}_{\tau}^+, \hat{\psi}_{\tau}; V_I; J) \right] \right\} \tag{2.8}
\]

\[
\hat{U}_{II}(t, -\frac{T_0}{2}; V_{II}; J) = \mathcal{T} \exp \left\{ -\frac{i}{\hbar} \int_{-\frac{T_0}{2}}^t d\tau \left[ \hat{H}_{II}(\hat{\psi}_{\tau}^+, \hat{\psi}_{\tau}; V_{II}; J) \right] \right\} \tag{2.9}
\]

\[
\int_C dt_p \ldots = \int_{-\infty}^{+\infty} dt_p + \ldots + \int_{+\infty}^{-\infty} dt_p + \ldots = \int_{-\infty}^{+\infty} dt_p + \ldots - \int_{-\infty}^{+\infty} dt_p + \ldots \tag{2.10}
\]

\[
\sum_{p=\pm} \int_{-\infty}^{+\infty} dt_p \eta_p \ldots \quad \eta_p = \left\{ \begin{array}{c} \eta_+ = +1 ; \eta_- = -1 \\ p=+ \quad p=- \end{array} \right\} . \tag{2.11}
\]

The coherent state path integrals \( Z[J, V_I], Z[J, V_{II}] \) \( \text{(2.6,2.7)} \) at zero temperature are normalized in the case of vanishing 'exterior' source term \( J \) which allows to obtain observables from differentiating \( Z[J, V_I], Z[J, V_{II}] \) \( \text{(2.6,2.7)} \) by \( J \) (compare section 1). The property of normalization of \( Z[J, V_I], Z[J, V_{II}] \)
is guaranteed by the unitary time development $\hat{U}_I$, $\hat{U}_{II}$ (2.8) in forward 't$_+$' and backward 't$_-$' direction on the time contour (2.10,2.11). The presence of the source fields $j_{\psi;x}(t_p)$ and $j_{\psi;x}(t_p)$ creates Bose particles from the vacuum states $|0\rangle$, $|0\rangle$ with the corresponding coherent state fields $\psi_{\hat{p}}(t_p)$ and $\psi_{\hat{p}}(t_p)$ and nonvanishing anomalous terms $\langle \psi_{\hat{p}}(t_p) \psi_{\hat{p}}(t_p) \rangle$, $\langle \psi_{\hat{p}}(t_p) \psi_{\hat{p}}(t_p) \rangle$. However, we have also to require in final relations for observables that the source terms $j_{\psi;x}(t_p)$, $j_{\psi;x}(t_p)$ have the same values on the two branches of the time contour. This is defined by relation (2.14) with the vertical line and has to be added to the generating functions. Therefore, the required normalization property of ensemble averaged disordered systems is fulfilled and possible problems with limits in replica field theories or super-symmetric extensions are circumvented [33, 59]. According to the property of normalization at zero temperature (2.12,2.13), the Gaussian ensemble averages in model I and II (2.15,2.16) are well defined and can be transferred to other physical problems with disordered parts for generalized coherent states (as e.g. $SU(2)$-coherent states [32]).

\[
Z[J \equiv 0, V_I]_{\{j_\psi,j_{\psi;x}\}} = 1 \quad (2.12)
\]

\[
Z[J \equiv 0, V_{II}]_{\{j_\psi,j_{\psi;x}\}} = 1 \quad (2.13)
\]

\[
\cdots \{j_\psi,j_{\psi;x}\} := \{j_{\psi;x}(t_+) = j_{\psi;x}(t_-) ; j_{\psi;x}(t_+) = j_{\psi;x}(t_-)\} \quad (2.14)
\]

\[
Z_I[J] = \langle 0|\hat{U}_I(-T_0/2,+T_0/2; V_I; J) \hat{U}_I(+T_0/2,-T_0/2; V_I; J)|0\rangle_{\{j_\psi,j_{\psi;x}\}} \quad (2.15)
\]

\[
Z_{II}[J] = \langle 0|\hat{U}_{II}(-T_0/2,+T_0/2; V_{II}; J) \hat{U}_{II}(+T_0/2,-T_0/2; V_{II}; J)|0\rangle_{\{j_\psi,j_{\psi;x}\}} \quad (2.16)
\]

The normalized unitary time development at zero temperature (2.6,2.16) has to be modified in the case of a finite temperature. We briefly describe the suitably normalized generating function for finite temperature in the case of model I (static disorder, compare with the Hamilton and unitary time development operators (2.12,2.8) for model I). The inclusion of the grand canonical statistical operator $exp\{-\beta(\hat{h}_I - \mu \hat{N})\}$ with $\hat{h}_I(\hat{\psi}_x^+, \hat{\psi}_x^-, V_I)$ (2.18) as part of $\hat{H}_I(\hat{\psi}_x^+, \hat{\psi}_x^-, V_I)$ (2.17,2.1) and its appearance in the denominator with the trace $Z_\beta[V_I]$ (2.20) of the total generating function $Z[J, \beta, V_I]$ (2.19) lead to an expansion of a large $(n \rightarrow \infty, n \in N_0 \geq 0)$ limit with $Z_\beta[V_I]$ (2.21,2.23). This follows from the representation of the inverse of $Z_\beta[V_I]$ (2.21) by an exponential integral with auxiliary integration variable $x$ and the Taylor expansion (2.21) of the exponential $\exp\{-x \ Z_\beta[V_I]\}$ in the integrand with $x \in [0, \infty)$.

\[
\hat{H}_I(\hat{\psi}_x^+, \hat{\psi}_x^-, V_I) = \hat{h}_I(\hat{\psi}_x^+, \hat{\psi}_x^-, V_I) + \sum_{x} \left( j_{\psi;x}^+(t) \hat{\psi}_x^+ + j_{\psi;x}^-(t) \hat{\psi}_x^- \right) + \frac{1}{2} \sum_{x} \left( j_{\psi;x}^+(t) \hat{\psi}_x^+ \hat{\psi}_x^+ + j_{\psi;x}^-(t) \hat{\psi}_x^- \hat{\psi}_x^- \right) \quad (2.17)
\]

\[
\hat{h}_I(\hat{\psi}_x^+, \hat{\psi}_x^-, V_I) = \sum_{x} \hat{\psi}_x^+ \left( -\frac{\hbar^2}{2m} \Delta + u(\vec{x}) - \mu_0 + V_I(\vec{x}) + V_0 \hat{\psi}_x^+ \hat{\psi}_x^+ \right) \hat{\psi}_x^- \quad (2.18)
\]

\[
Z[J, \beta, V_I] = \frac{\text{Tr} \left[ \exp\{-\beta(\hat{h}_I - \mu \hat{N})\} \hat{U}_I(-T_0/2,+T_0/2; V_I; J) \hat{U}_I(+T_0/2,-T_0/2; V_I; J) \right]}{Z_\beta[V_I]} \quad (2.19)
\]

\[
Z_\beta[V_I] = \text{Tr} \left[ \exp\{-\beta(\hat{h}_I - \mu \hat{N})\} \right] \quad Z[J \equiv 0, \beta, V_I]_{\{j_\psi,j_{\psi;x}\}} = 1 \quad (2.20)
\]

\[
\frac{1}{Z_\beta[V_I]} = \int_{0}^{\infty} dx \exp\{-x Z_\beta[V_I]\} = \int_{0}^{\infty} dx \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} \left( Z_\beta[V_I]\right)^n. \quad (2.21)
\]
The ensemble averaged generating function $Z_I[J, \beta]$ \eqref{eq:2.22} of $Z[J, \beta, V_I]$ \eqref{eq:2.19} for finite temperatures is given by the sum of averaged generating functions $Z_{I,n}[J, \beta]$ \eqref{eq:2.23} with increasing number of fields in $(Z_{\beta}[V_I])^n$ so that the large ($n \to \infty$, $n \in \mathbb{N}_0 \geq 0$) limit of field theories has to be considered. The unitary time development operator $\dot{U}_I$ is determined by the relation \eqref{eq:2.8} with the operator $\hat{H}_I(\hat{\psi}_t^+, \hat{\psi}_t, V_I)$ \eqref{eq:2.17, 2.21} which includes the symmetry breaking source terms and additionally the 'exterior' source variable $J$ for the observables

$$Z_I[J, \beta] = \int_0^\infty \frac{d\tau}{\tau} \sum_{n=0}^\infty \frac{(-\tau)^n}{n!} Z_{I,n}[J, \beta]$$ \hfill \eqref{eq:2.22}

$$Z_{I,n}[J, \beta] = \text{Tr} \left[ \exp\{-\beta(\hat{h}_I - \mu N)\} \hat{U}_I(-T_0/2, +T_0/2; V_I; J) \times \ldots \right.$$ \hfill \eqref{eq:2.23}

$$\ldots \times \hat{U}_I(+T_0/2, -T_0/2; V_I; J) \left( \text{Tr} \left[ \exp\{-\beta(\hat{h}_I - \mu N)\} \right] \right)^n \bigg|_{(j_0, j_{\psi, \phi})}.$$

However, we restrict in this paper to zero temperature for the models I, II. A coset decomposition into densities and pair condensate terms is preferable in $d=2$ spatial dimensions with a gradient expansion of a determinant leading to the Goldstone modes of a spontaneous symmetry breaking within a nonlinear sigma model \cite{24, 25}.

### 2.2 Ensemble averages in model I and II

We list in relations \eqref{eq:2.24, 2.25} the coherent state path integral representation of the unitary time development operators $\dot{U}_I, \dot{U}_{II}$ \eqref{eq:2.8, 2.21} in $Z[J, V_I], Z[J, V_{II}]$ \eqref{eq:2.16, 2.17} at zero temperature for the disorder models I and II

$$Z[J, V_I] = \int d[\psi_\tau(t_p)] \exp \left\{ -\frac{i}{\hbar} \int_C dt_p \sum_{x, x'} \Psi_{x, x'}^{j_{\psi, \phi}}(t_p) J_{x, x'}^\beta(t_p) \Psi_{x'}^{\psi_\tau}(t_p) \right\}$$ \hfill \eqref{eq:2.24}

$$Z[J, V_{II}] = \int d[\psi_\tau(t_p)] \exp \left\{ -\frac{i}{\hbar} \int_C dt_p \sum_{x, x'} \Psi_{x, x'}^{j_{\psi, \phi}}(t_p) J_{x, x'}^\beta(t_p) \Psi_{x'}^{\psi_\tau}(t_p) \right\}$$ \hfill \eqref{eq:2.25}

The one-particle parts are given by $\hat{h}_p(t_p) + V_I(\vec{x}), \hat{h}_p(t_p) + V_{II}(\vec{x}, t)$ which consist of the common Hamilton operator $\hat{h}_p(t_p)$ \eqref{eq:2.26, 2.27} with the kinetic energy, the trap potential $u(\vec{x})$, the chemical potential $\mu_0$, the energy operator $-i\hbar \partial/\partial t_p$ of the corresponding branch of the contour time and the random potentials.
\( V_f(\vec{x}), V_{ff}(\vec{x}, t) \). Note the inclusion of the small imaginary energy increment \(-i\varepsilon_p\) on both branches of the time contour which allows the selection between advanced and retarded Green functions. This imaginary increment \(-i\varepsilon_p\) determines a direction for the time development so that the coherent state path integrals \((2.24,2.25)\) are well defined for all kinds of effective energies whether vanishing, bounded or unbounded.

\[
\hat{h}_p(t_p) = -i\hbar \frac{\partial}{\partial t_p} - i\varepsilon_p - \frac{\hbar^2}{2m} \Delta + u(\vec{x}) - \mu_0 \tag{2.26}
\]

\[
\hat{h}_{\vec{x},\vec{p}}(t_p, t'_p) = \delta_{p,q} \eta_\varepsilon \delta(t_p - t'_p) \delta_{\vec{x},\vec{p}} \hat{h}_p(t_p) ; \quad \varepsilon_p = \eta_\varepsilon \varepsilon ; (\varepsilon > 0; \quad \eta_\varepsilon = \pm 1) \tag{2.27}
\]

\[
j_{\psi;\vec{x}}(t_+) = j_{\psi;\vec{x}}(t_-) = j_{\psi;\vec{x}}(t_+) = j_{\psi;\vec{x}}(t_-) \tag{2.28}
\]

The symmetry breaking source terms \( j_{\psi;\vec{x}}(t_+), j_{\psi;\vec{x}}(t_-) (2.28) \) for the creation of a coherent BE-condensate wavefunction and pair condensates have to be set to the same values on the two branches of the contour time in the final relations for observables with vanishing 'exterior' source \( J_{\vec{x},\vec{p}}(t'_p, t_p) \). We perform a 'Nambu'-doubling of the coherent state fields \( \psi_{\vec{x}}(t_p) \) on the time contour with its complex conjugated fields \( \psi_{\vec{x}}^*(t_p) \) in order to obtain also anomalous terms as \( \langle \psi_{\vec{x}}(t_p) \psi_{\vec{x}}(t_p) \rangle \) by a single differentiation with \( J_{\vec{x},\vec{p}}(t'_p, t_p) \). We denote this 'Nambu'-doubled field by \( \Psi_{\vec{x}}^{a(1/2)}(t_p(\pm \varepsilon)) \) (2.29) (with capital '\( \Psi \) instead of the lower-case letter '\( \psi \)' and introduce the additional indices \((a, b = 1, 2)\) for referring to \( \psi_{\vec{x}}(t_p) \) \((a = 1)\) or to the complex conjugated part \( \psi_{\vec{x}}^*(t_p) \) \((a = 2)\). There are two possible orders of the four component 'Nambu'-doubled field \( \Psi_{\vec{x}}^{a(1/2)}(t_p(\pm \varepsilon)) \). In the listing \((2.29)\) one gives priority with respect to the two branches of the contour time so that the first two components of \( \Psi_{\vec{x}}^{a(1/2)}(t_p) \) are on the plus branch of \( t_{p=+} \) whereas the listing \((2.29)\) prefers ordering with respect to the anomalous pair condensates

\[
\text{contour time ordering :} \tag{2.29}
\]

\[
\Psi_{\vec{x}}^{a(1/2)}(t_p(\pm \varepsilon)) = \begin{pmatrix}
\psi_{\vec{x}}(t_p) (a = 1) \\
\psi_{\vec{x}}^*(t_p) (a = 2)
\end{pmatrix}
\]

ordering for anomalous terms : \( \Psi_{\vec{x}}^{a(1/2)}(t_p(\pm \varepsilon)) = \begin{pmatrix}
\psi_{\vec{x}}(t_p) (a = 1) \\
\psi_{\vec{x}}^*(t_p) (a = 2)
\end{pmatrix}
\]

In the remainder we partially use a different notation for the coherent state field variables \( \psi_{\vec{x}}(t_p(\pm \varepsilon)), \psi_{\vec{x}}^*(t_p(\pm \varepsilon)) \) in the generating functions of models I, II in order to emphasize the complete independence of the fields \( \psi_{\vec{x}}(t_+) := \psi_{\vec{x}+,\vec{t}}(t), \psi_{\vec{x}}(t-) := \psi_{\vec{x},-,\vec{t}}(t) \) and also \( \psi_{\vec{x}}^*(t_+) := \psi_{\vec{x}+,\vec{t}}^*(t), \psi_{\vec{x}}^*(t-) := \psi_{\vec{x},-,\vec{t}}^*(t) \) concerning the two branches of the time contour \((2.31,2.32)\). However, if classical approximations are implemented in the coherent state path integrals of disorder models I and II, the fields \( \psi_{\vec{x}}(t_p(\pm \varepsilon)), \psi_{\vec{x}}^*(t_p(\pm \varepsilon)) \) may take exactly the same values on both contour time branches \( \psi_{\vec{x}}(t_+), \psi_{\vec{x}}^*(t_-) \) (for classical approximations following from variations with respect to contour fields)

\[
\text{contour time ordering :} \tag{2.31}
\]

\[
\Psi_{\vec{x}}^{a(1/2)}(t_p(\pm \varepsilon)) = \begin{pmatrix}
\psi_{\vec{x}}(t_p) (a = 1) \\
\psi_{\vec{x}}^*(t_p) (a = 2)
\end{pmatrix}
\]
ordering for anomalous terms:

\[
\Psi_{\hat{t}x}^{a(=1/2)}(t_p=\pm) = \Psi_{\hat{t}x}^{a(=1/2)}(t_p=\pm) = \begin{pmatrix}
\psi_{\hat{t}x}(t_p) & (a = 1) \\
\psi_{\hat{t}x}^+(t_p) & (a = 2)
\end{pmatrix} = \begin{pmatrix}
\psi_{\hat{t}x}(t) & (a = 1) \\
\psi_{\hat{t}x}^+(t) & (a = 2)
\end{pmatrix}
\]

Therefore, one can also rewrite the 'Nambu'-doubled bilinear term (2.33) with a matrix \( \tilde{M}_{\hat{t}x,q;\hat{t}x,p}(t', t_p) := \tilde{M}_{\hat{t}x,q;\hat{t}x,p}(t', t) \) and its integrations over contour times \( t'_q, t_p \) as follows (by using the metric \( \eta_p, \eta_q \) (2.10, 2.11))

\[
\int_C dt_p \ dx_p \sum_{a,b=1,2} \Psi_{\hat{t}x}^{a,b}(t'_q) \tilde{M}_{\hat{t}x,q;\hat{t}x,p}(t', t_p) \ \Psi_{\hat{t}x}^{a,b}(t_p) = \int_{-\infty}^{+\infty} dt \ dx_p \sum_{a,b=1,2} \Psi_{\hat{t}x}^{a,b}(t'_q) \eta_q \tilde{M}_{\hat{t}x,q;\hat{t}x,p}(t', t) \ \eta_p \ \Psi_{\hat{t}x}^{a,b}(t_p) .
\]

However, the fields \( \Psi_{\hat{t}x}^{a,b}(t_{-} = +T_0/2) \) and \( \Psi_{\hat{t}x}^{a,b}(t_{+} = +T_0/2) \) have to approach exactly the same values at the time \( +T_0/2 \) for continuity reasons. This must hold in both kinds of expressions with coherent state fields in (2.33) (compare (2.29, 2.31) for contour time ordering and relations (2.30, 2.32) with prevailing order for the anomalous parts).

The Gaussian ensemble averages of \( Z[J_\phi, V_\phi], Z[J, V_{\phi J}] \) (2.24, 2.25) result in relations \( \overline{Z_1[J]} = \int d[\psi_{\hat{t}x}(t_p)] \exp \left\{ -\frac{i}{\hbar} \int_C dt_p \sum_{\hat{t}x} \psi_{\hat{t}x}^+(t_p) \hat{h}_p(t_p) \ \psi_{\hat{t}x}(t_p) \right\} \)

(2.34)

\[
\times \ \exp \left\{ -\frac{i}{2\hbar} \int_C dt_p \sum_{\hat{t}x} \left[ J_{\phi,\hat{t}x}^a(t_p) \ \Psi_{\hat{t}x}^a(t_p) + \Psi_{\hat{t}x}^+(t_p) \ J_{\phi,\hat{t}x}^a(t_p) \right] \right\}
\]

\[
\times \ \exp \left\{ -\frac{i}{2\hbar} \int_C dt_p \sum_{\hat{t}x} \psi_{\hat{t}x}^+(t_p) \ J_{\phi,\hat{t}x}^a(t_p) \ \Psi_{\hat{t}x}^a(t_p) \right\}
\]

\[
\times \ \exp \left\{ -\frac{i}{2\hbar} \int_C dt_p \ dx_p \sum_{a,b=1,2} \Psi_{\hat{t}x}^{a,b}(t'_q) \ J_{\phi,\hat{t}x}^a(t'_q, t_p) \ \Psi_{\hat{t}x}^a(t_p) \right\}
\]

\[
\times \ \exp \left\{ -\frac{i}{\hbar} \int_C dt_p \ \psi_{\hat{t}x}^+(t_p) \ J_{\phi,\hat{t}x}^a(t_p) \ \psi_{\hat{t}x}(t_p) \right\} \left( \int_C dt' \ \psi_{\hat{t}x}^+(t'_q) \ \psi_{\hat{t}x}(t'_q) \right)
\]

(2.35)
The 'Nambu'-doubling of the source term \( j \) in manners of ordering (2.29, 2.30) as the doubled coherent state field \( \Psi \). The source terms of the contour time 'Nambu'-doubling defined by its capital letter \( J \); (2.40-2.43) for as that of the coherent state field \( \psi \) or the field \( \Psi \) (2.36, 2.37) or the field \( \Psi \). The 'Nambu'-doubled source field for \( j \) in the final relations for the observables or the saddle point equation (2.14). In the 'contour time ordering' : (2.36)

\[
J_{j}^{(1/2)}(t_{P(\pm \pm)}) = \begin{pmatrix} j_{j}(t_{P}(a = 1)) & j_{j}^{*}(t_{P}(a = 2)) \\ j_{j}(t_{P}(a = 2)) & j_{j}^{*}(t_{P}(a = 1)) \end{pmatrix}
\]

'ordering for anomalous terms' :

\[
J_{j}^{(1/2)}(t_{P(\pm \pm)}) = \begin{pmatrix} j_{j}(t_{P}(a = 1)) & j_{j}^{*}(t_{P}(a = 2)) \\ j_{j}(t_{P}(a = 2)) & j_{j}^{*}(t_{P}(a = 1)) \end{pmatrix}
\]

The 'Nambu'-doubling of the source term \( j \) for the pair condensate terms yields a matrix \( J_{j}^{ab}(t_{P}) \) with local contour time dependence which can also be ordered in the two analogous kinds as the \( U(1) \) source term \( J_{j}^{ab}(t_{P}) \) or the field \( \Psi \) (2.29, 2.30).

'contour time ordering' :

\[
J_{j}^{ab}(t_{P}) = \begin{pmatrix} 0 & j_{j}(t_{P}(a = 1)) \\ j_{j}^{*}(t_{P}(a = 1)) & 0 \end{pmatrix}
\]

'ordering for anomalous terms' :

\[
J_{j}^{ab}(t_{P}) = \begin{pmatrix} 0 & j_{j}(t_{P}(a = 1)) \\ j_{j}^{*}(t_{P}(a = 1)) & 0 \end{pmatrix}
\]

The source terms \( J_{j}(t_{P}) \) and \( J_{j}^{ab}(t_{P}) \) have to be set to equivalent values concerning the two branches of the contour time \( t_{\pm} \) in the final relations for the observables or the saddle point equation (2.14). In the remainder the equivalent notations (2.40, 2.41) for \( J_{j}^{ab}(t_{P}) \) and \( J_{j}^{ab}(t_{P}) \) will also temporarily occur as the equivalent notations for the coherent state fields and 'Nambu'-doubled matrices 2.33.
'ordering for anomalous terms':

\[ \mathcal{J}_{\psi;\xi}^{a(1/2)}(t_p(=\pm)) = \mathcal{J}_{\psi;\xi}^{a(1/2)}(t_p(=\pm)) = \begin{pmatrix} \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) \\ \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) \\ \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) \end{pmatrix} \]

(2.41)

'contour time ordering':

\[ \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \begin{pmatrix} 0 & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & 0 \\ 0 & 0 & 0 \\ \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & 0 & 0 \end{pmatrix} \]

(2.42)

'ordering for anomalous terms':

\[ \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \begin{pmatrix} 0 & 0 & \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) \\ 0 & 0 & 0 \\ \mathcal{J}_{\psi;\xi}^{\psi;\xi,\xi}(t) & 0 & 0 \end{pmatrix} \]

(2.43)

One has to apply the matrix form \( \mathcal{J}_{\psi;\xi}^{ab}(t_p) \) or its corresponding notation symbol \( \mathcal{J}_{\psi;\xi}^{ab}(t_p) \) \( (2.43) \) in the case of two time contour integrations \( (2.45) \) with the bilinear fields \( \Psi^{ab}(t_p) \) as with the matrix \( \mathcal{M}_{\psi;\xi}^{ab}(t_p) \) in \( (2.43) \)

\[ \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \int_{-\infty}^{\infty} dt_p dt'_q \sum_{x,q} \Psi^{ab}(t'_q, t_p) \mathcal{J}_{\psi;\xi}^{ab}(t_p) = \int_{-\infty}^{\infty} dt_p \sum_{x,q} \Psi^{ab}(t_p) \mathcal{J}_{\psi;\xi}^{ab}(t_p) \Psi^{a}(t_p) = \]

(2.45)

In this section we have achieved the ensemble averages of the disorder models I, II with the coherent state path integrals \( Z_{II}[\mathcal{J}], Z_{II}[\mathcal{J}] \) \( (2.44) \). We have described the various forms and equivalent notations concerning the coherent state fields and matrices \( (2.44) \) on the Keldysh time contour \( t_p = \pm \). The 'Nambu'-doubling of source fields and matrices has also been incorporated for the creation of a coherent BE-wavefunction and pair condensate terms \( (2.44) \).

3 Hubbard-Stratonovich transformations for the repulsive and ensemble-averaged interactions in model I and II

3.1 Hubbard-Stratonovich transformation for repulsive interactions in model I and II

The repulsive interaction with parameter \( V_0 > 0 \) is a common part of the two disorder models I and II. Its Hubbard-Stratonovich transformation (HST) to a density matrix \( \mathcal{M}_{\psi;\xi}^{ab}(t_p) \) \( (5.13.2) \) is accomplished by a
dyadic product of the fields in the repulsive interaction term where we already insert the 'Nambu'-doubled form of the dyadic products with \( \hat{r}^{ab}_{x}(t_p) = \Psi^+_x(t_p) \otimes \Psi^+_x(t_p) \)

\[
\int_C dt_p \sum_x \left( \psi^+_x(t_p) \right)^2 \left( \psi_x(t_p) \right)^2 = \frac{1}{4} \int_C dt_p \sum_x \left( \psi^+_x(t_p) \psi_x(t_p) + \psi_x(t_p) \psi^+_x(t_p) \right)^2
\]

\[
= \frac{1}{4} \int_C dt_p \sum_x \Psi^+_x(t_p) \otimes \Psi^+_x(t_p) \Psi^+_x(t_p) \otimes \Psi^+_x(t_p) = \frac{1}{4} \int_C dt_p \sum_x \sum_{a,b} \left[ \frac{\hat{r}^{ab}_{x}(t_p) \hat{r}^{ba}_{x}(t_p)}{\hat{r}^{ab}_{x}(t_p)} \right].
\]

We introduce the self-energy matrix \( \hat{\sigma}^{ab}_{x}(t_p) \) for the repulsive interaction term with \( V_0 > 0 \) in the models I, II with static and dynamic disorder according to the symmetries of the resulting density matrix \( \hat{r}^{ab}_{x}(t_p) \) in the trace 'tr_{a,b}' over 'Nambu'-indices \( 3.1 \)

\[
\hat{r}^{ab}_{x}(t_p) = \left( \begin{array}{cc} \psi_x(t_p) & \psi^+_x(t_p) \\ \psi^+_x(t_p) & \psi_x(t_p) \end{array} \right),
\]

\[
\hat{\sigma}^{ab}_{x}(t_p) = \left( \begin{array}{cc} b_x(t_p) & c_x(t_p) \\ c_x(t_p) & b_x(t_p) \end{array} \right), b_x(t_p) \in \mathbb{R}, c_x(t_p) \in \mathbb{C}.
\]

This self-energy matrix \( \hat{\sigma}^{ab}_{x}(t_p) \) has a local dependence in the spatial coordinates and also regarding the contour time. Therefore, the HST of the repulsive interaction or of its density matrix form with the trace 'tr_{a,b}' over the 'Nambu'-indices \( a, b = 1, 2 \) yields the relation \( 3.3 \) with hermitian action in a Gaussian factor of the self-energy and the hermitian coupling between density matrix \( \hat{r}^{ab}_{x}(t_p) \) and \( \hat{\sigma}^{ab}_{x}(t_p) \)

\[
\exp \left\{ -\frac{\hbar}{4} \int_C dt_p \sum_x V_0 \left( \psi^+_x(t_p) \right)^2 \left( \psi_x(t_p) \right)^2 \right\} = \exp \left\{ -\frac{i}{4\hbar} V_0 \int_C dt_p \sum_x \sum_{a,b} \left[ \frac{\hat{r}^{ab}_{x}(t_p) \hat{r}^{ba}_{x}(t_p)}{\hat{r}^{ab}_{x}(t_p)} \right] \right\} = \exp \left\{ \frac{i}{2\hbar} \int_C dt_p \sum_x \sum_{a,b} \left[ \frac{\hat{r}^{ab}_{x}(t_p) \hat{r}^{ba}_{x}(t_p)}{\hat{r}^{ab}_{x}(t_p)} \right] \right\}.
\]

The real field \( b_x(t_p) = \sigma^{11}_{x}(t_p) = \sigma^{d2}_{x}(t_p) \) in \( 3.3 \) describes the density term of the self-energy for the repulsive interaction whereas the complex field \( c_x(t_p) = \sigma^{12}_{x}(t_p) \) and its complex conjugate \( c^*_x(t_p) = \sigma^{21}_{x}(t_p) \) in \( 3.3 \) determine the anomalous terms of the interaction with \( V_0 > 0 \).

### 3.2 Hubbard-Stratonovich transformation for the disorder term in model I

and derivation of the mean field equations with the disorder-self-energy

The HST of the quartic, non-hermitian 'interaction term' for the disorder in model I is obtained by the dyadic product of fields in a similar manner as in section \( 3.1 \). The 'Nambu'-doubling with \( \Psi^+_x(t_p) \otimes \Psi^+_x(t_q) \) leads to the disorder-density matrix \( \hat{R}^{ab}_{x}(t_p, t_q) \) with inclusion of the anomalous terms

\[
\int_C dt_p dt_q' \sum_x \left( \Psi^+_x(t_p) \psi_x(t_p) \psi^+_x(t_q') \psi_x(t_q') \right) = \frac{1}{4} \int_C dt_p dt_q' \sum_x \left( \Psi^+_x(t_p) \Psi^+_x(t_q) \right) \left( \Psi^+_x(t_q') \Psi^+_x(t_q') \right)
\]

\[
= \frac{1}{4} \int_C dt_p dt_q' \sum_x \Psi^+_x(t_p) \otimes \Psi^+_x(t_q') \Psi^+_x(t_q) \otimes \Psi^+_x(t_q) = \frac{1}{4} \int_C dt_p dt_q' \sum_{a,b} \left[ \hat{R}^{ab}_{x}(t_p, t_q') \hat{R}^{ba}_{x}(t_q', t_p) \right]
\]

\[
= \frac{1}{4} \int_{-\infty}^{+\infty} dt dt' \sum_{p,q,a,b} \text{Tr} \left[ \hat{R}^{ab}_{x,pq}(t, t') \eta_q \hat{R}^{ba}_{x,qp}(t', t) \eta_p \right].
\]

We explicitly list the spatially local density matrix \( \hat{R}^{ab}_{x}(t_p, t_q') \) in \( 3.5 \) with 'Nambu'-indices \( a, b = 1, 2 \) and nonlocal time contour dependence with \( t_p, t_q' \) (\( p, q = \pm \)) according to the contour ordering of fields.
as for \( \Psi_{\tilde{z}}(t_p) = \Psi_{\tilde{z}}^0(t_p) \). Note that two different notations for the disorder-density matrix can be used as in the cases of \( \Psi_{\tilde{z}}^0(t_p) = \Psi_{\tilde{z}}^0(t_p) \) or \( j^{0a}_{\psi_{\tilde{z}}}(t_p) = j^{0a}_{\psi_{\tilde{z}}}(t_p) \). Therefore, we have also added in relation (5.5) the last line with the trace \( \Tr_{p,a,b} \) and the metric \( \eta_a, \eta_b \) for the disorder-density matrix in notation (3.7). This clarifies the symmetry relations between the matrix elements of \( \hat{R}_{\tilde{z}}^{ab}(t_p, t') = \hat{R}_{\tilde{z}}^{ab}(t_p, t') \) (3.6,3.7). In the listings (3.6,3.7), the contour time ordering prevails for the disorder-density matrices \( \hat{R}_{\tilde{z}}^{ab}(t_p, t') = \hat{R}_{\tilde{z}}^{ab}(t_p, t') \) which follow by the dyadic products \( \Psi_{\tilde{z}}^0(t_p) \otimes \Psi_{\tilde{z}}^0(t') \). \( \Psi_{\tilde{z}}^0(t) \otimes \Psi_{\tilde{z}}^0(t') \) of fields (3.8) also applied with respect to the contour time order.

\[
\hat{R}_{\tilde{z}}^{ab}(t_p, t') = \left( \begin{array}{ccc}
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\end{array} \right) 
\]

(3.6)

\[
\hat{R}_{\tilde{z}}^{ab}(t, t') = \left( \begin{array}{ccc}
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+) & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' & \psi_{\tilde{z}}(t_+), \psi_{\tilde{z}}(t_+)' \\
\end{array} \right) 
\]

(3.7)

\[
\Psi_{\tilde{z}}^{a(=1/2)}(t_{p(=1)}) = \left( \begin{array}{c}
\psi_{\tilde{z}}(t_+), (a = 1) \\
\psi_{\tilde{z}}(t_+), (a = 2) \\
\psi_{\tilde{z}}(t_+), (a = 2) \\
\end{array} \right) = \begin{pmatrix}
\psi_{\tilde{z}}(t_+) \\
\psi_{\tilde{z}}(t_+) \\
\psi_{\tilde{z}}(t_+)
\end{pmatrix}
\]

(3.8)

The corresponding 'Nambu'-doubled disorder-self-energy \( \Sigma_{\tilde{z}}^{ab}(t_p, t') = \Sigma_{\tilde{z}}^{ab}(t_p, t') \) (3.9,3.10) has to fulfill the equivalent symmetry relations between its matrix elements as the disorder-density matrix \( \hat{R}_{\tilde{z}}^{ab}(t_p, t') \), \( \hat{R}_{\tilde{z}}^{ab}(t, t') \) (3.6,3.7). We also consider in (3.9,3.10) the two equivalent notations as for the disorder-density matrix (3.6,3.7). The disorder-self-energy (3.9,3.10) has a nonlocal dependence with respect to the contour times \( t_p, t_q \) and contains density related terms labeled with the capital letter 'B' and pair condensate terms labeled with the capital letter 'C'. The basic matrices of the disorder-self-energy \( \Sigma_{\tilde{z}}^{ab}(t_p, t') \) are \( \hat{B}_{\tilde{z}}(t_+, t_+), \hat{B}_{\tilde{z}}(t_-, t_-) \) and \( \hat{B}_{\tilde{z}}(t, t') \) for density related terms. The basic matrices for the anomalous parts are given by \( \hat{C}_{\tilde{z}}(t_+, t_+'), \hat{C}_{\tilde{z}}(t_-, t_-') \) and \( \hat{C}_{\tilde{z}}(t, t') \). Taking into account the symmetries of \( \hat{R}_{\tilde{z}}^{ab}(t_p, t_q) \), one has to place the basic matrices \( \hat{B}_{\tilde{z}}(t_+, t_+), \hat{B}_{\tilde{z}}(t_- t_-) \) and \( \hat{B}_{\tilde{z}}(t, t') \) for densities and the basic matrices \( \hat{C}_{\tilde{z}}(t_+, t_+'), \hat{C}_{\tilde{z}}(t_-, t_-'), \hat{C}_{\tilde{z}}(t, t') \) for the pair condensates in the disorder-self-energy as in relation (3.9,3.10). Furthermore, one has to require the symmetry restrictions (3.11) for these basic matrices of \( \Sigma_{\tilde{z}}^{ab}(t_p, t_q) \), \( \Sigma_{\tilde{z}}^{ab}(t, t') \). The two equivalent notations concerning the nonlocal contour time dependence are also tabulated in relation (3.11) for the symmetries of the 'B' and 'C' matrices. These two notations clarify the symmetry relations with complex conjugation, transposition and hermitian conjugation (see following examples for complex conjugation \( \hat{B}_{\tilde{z}}(t_p, t_q)^* = \hat{B}_{\tilde{z}}(t_q, t_p) \), transposition \( \hat{B}_{\tilde{z}}(t_p, t_q)^T = \hat{B}_{\tilde{z}}(t_q, t_p) \) and hermitian conjugation \( \hat{B}_{\tilde{z}}(t_p, t_q)^+ = \hat{B}_{\tilde{z}}(t_q, t_p) \) within the two equivalent notations in (3.11).

\[
\Sigma_{\tilde{z}}^{ab}(t_p, t_q) = \left( \begin{array}{ccc}
\hat{B}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+) \\
\hat{C}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+) \\
\hat{C}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+)
\end{array} \right)
\]

(3.9)

\[
\Sigma_{\tilde{z}}^{ab}(t, t') = \left( \begin{array}{ccc}
\hat{B}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+) \\
\hat{C}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+) \\
\hat{C}_{\tilde{z}}(t_+, t_+) & \hat{C}_{\tilde{z}}(t_+, t_+) & \hat{B}_{\tilde{z}}(t_+ t_+)
\end{array} \right)
\]

(3.10)
\[
\psi_\tau(t_+ \tau') \psi_\tau(t_+ \tau') \propto B_\tau(t_+ \tau') = B_\tau(\tau' \tau') = (B_\tau(t_+ \tau'))^+ = (B_\tau(\tau' \tau'))^+ \\
\psi_\tau(t_- \tau') \psi_\tau(t_- \tau') \propto B_\tau(t_- \tau') = B_\tau(\tau' \tau') = (B_\tau(\tau' \tau'))^+\quad (3.11)
\]

We use relations (3.5) to (3.11) for the HST of the ensemble averaged disorder term with nonlocal time dependence and with the symmetries following from the 'Nambu'-doubling. A 'non-hermitian' Gaussian factor of the disorder-self-energy \( \Sigma_{i,\tau}(t, t', q) \) and its coupling to the disorder-density matrix \( \hat{R}_{x}^{ab}(t, t', q) \) result in place of the quartic, non-hermitian interaction of fields derived from the ensemble average with \( V_{\tau}(\vec{x}) \)

\[
\exp \left\{ -\frac{R_{\tau}^{2}\Omega^{2}}{2hN_{x}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \left( \psi_{\tau}^{*}(t_{p}) \psi_{\tau}(t_{p}) \right) \left( \psi_{\tau}^{*}(t'_{q}) \psi_{\tau}(t'_{q}) \right) \right\} = \quad (3.12)
\]

\[
= \exp \left\{ -\frac{R_{\tau}^{2}\Omega^{2}}{2hN_{x}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \text{tr} \left[ \hat{R}_{x}^{ab}(t_{p}, t'_{q}) \hat{R}_{x}^{ba}(t'_{q}, t_{p}) \right] \right\} = \\
= \int d[\Sigma_{i,\tau}(t_{p}, t'_{q})] \exp \left\{ -\frac{1}{8h^{2}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \text{tr} \left[ \Sigma_{i,\tau}(t_{p}, t'_{q}) \Sigma_{i,\tau}(t'_{q}, t_{p}) \right] \right\} \times \exp \left\{ -\frac{1}{4h^{2}\sqrt{\sqrt{N_{x}}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \text{tr} \left[ \Sigma_{i,\tau}(t_{p}, t'_{q}) \Sigma_{i,\tau}(t'_{q}, t_{p}) \right] \right\}.
\]

Substituting the terms of the HST's (3.3-3.12) and the 'Nambu'-doubled symmetry breaking source terms into \( Z_{\tau}(\vec{J}) \) (2.34), we acquire the ensemble averaged coherent state path integral of model I with the self-energy \( \hat{\sigma}_{x}^{ab}(t_{p}) \) of the repulsive interaction and the disorder-self-energy \( \Sigma_{i,\tau}(t_{p}, t'_{q}) \). Moreover, we perform the 'Nambu'-doubling (3.14-3.17) on the one-particle terms in \( Z_{\tau}(\vec{J}) \) so that relation (3.13) for \( Z_{\tau}(\vec{J}) \) only consists of 'Nambu'-doubled parts with \( \Sigma_{i,\tau}(t_{p}, t'_{q}), \hat{\sigma}_{x}^{ab}(t_{p}), \Psi_{x}^{a}(t_{p}) \) and \( J_{\psi,\tau}(t_{p}) \), \( J_{\psi',\tau}(t_{p}) \) as well as \( \hat{H}_{x}^{ba}(t'_{q}, t_{p}) \)

\[
Z_{\tau}(\vec{J}) = \int d[\Sigma_{i,\tau}(t_{p}, t'_{q})] d[\hat{\sigma}_{x}^{ab}(t_{p})] \exp \left\{ -\frac{1}{8h^{2}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \text{tr} \left[ \Sigma_{i,\tau}(t_{p}, t'_{q}) \Sigma_{i,\tau}(t'_{q}, t_{p}) \right] \right\} \\
\times \exp \left\{ \frac{i}{4h} \int_{C} dt_{p} \sum_{\tau} \text{tr} \left[ \hat{\sigma}_{x}^{ab}(t_{p}) \hat{\sigma}_{x}^{ba}(t_{p}) \right] \right\} \quad (3.13)
\]

\[
\times \int d[\psi_{\tau}(t_{p})] \exp \left\{ -\frac{i}{2h} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \Psi_{x}^{+b}(t'_{q}) \Psi_{x}^{a}(t_{p}) \right\} \left[ \hat{H}_{x}^{ba}(t'_{q}, t_{p}) + \frac{J_{\psi,\tau}(t'_{q}, t_{p})}{N_{x}} \right] \\
+ J_{\psi,\tau}(t'_{q}, t_{p}) - \delta(t_{p} - t'_{q}) \delta_{p, q} \Psi_{x}^{a}(t_{p}) + \frac{1}{2h\sqrt{\sqrt{N_{x}}} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \text{tr} \left[ \Sigma_{i,\tau}(t_{p}, t'_{q}) \Sigma_{i,\tau}(t'_{q}, t_{p}) \right] \Psi_{x}^{a}(t_{p})} \Psi_{x}^{a}(t_{p}) \\
\times \exp \left\{ -\frac{i}{2h} \int_{C} dt_{p} dt'_{p} \sum_{\tau} \left[ \hat{J}_{\psi,\tau}(t_{p}) \Psi_{x}^{a}(t_{p}) + \Psi_{x}^{a}(t_{p}) \hat{J}_{\psi,\tau}(t_{p}) \right] \right\}
\]

The 'Nambu'-doubled one-particle Hamiltonian \( \hat{H}_{x}^{ba}(t'_{q}, t_{p}) \) has to take the form as in (3.14) for a chosen contour time ordering with the path \( \hat{p}_{\tau}(t_{p}) \) (3.15) and its transpose \( \hat{p}_{\tau}^{T}(t_{p}) \) (3.16) (compare with the two
notations of contour time ordering in \( \text{I}\) is antisymmetric with respect to transposition whereas the other terms of \( \hat{h}_p(t_p) \) are symmetric under transposition. It has been mentioned in the introduction that the self-energy \( \hat{\sigma}^b(t_p) \) of the repulsive interaction can be considered as a subalgebra (with being the direct product of \( M \) times \( sp(2) \)), concerning the ‘larger’ disorder-self-energy \( \hat{\Sigma}^{ab}(t_p, t'_q) \). This disorder-self-energy can itself be regarded as an element of the symplectic Lie-Algebra \( sp(4M) \), with respect to the number of independent parameters which is given by \( 4M \cdot (4M + 1)/2 \). (The parameter \( M \in \mathbb{N} > 0 \) denotes the number of discrete time intervals or steps during time development between \(-T_{0}/2 < t_{p,j} < +T_{0}/2 \) for times of a single branch of the contour \( p = \text{fixed} \pm \text{value}, 0 < j < M - 1, t_{p,j} = \Delta t \cdot j \).) This important observation allows to shift the disorder-self-energy \( \hat{\Sigma}^{ab}(t_p, t'_q) \) by the self-energy \( \hat{\sigma}^b(t_p) \) of the repulsive interaction \( \text{(3.18)} \). We can also transfer the source term \( \hat{J}^{ab}_{\psi;\xi}(t'_q) \) for the pair condensates as a subset of the coset part \( sp(4M) \backslash u(2M) \) of the symplectic Lie-Algebra \( sp(4M) \) to the disorder-self-energy \( \text{(3.19/3.20)} \). Therefore, \( \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \), coupled to the bilinear fields \( \hat{\Psi}^{+;b}_{\xi}(t'_q) \ldots \hat{\Psi}^{2}_{\xi}(t_p) \) in \( \mathcal{Z}[\mathcal{J}] \) \( \text{(3.15)} \), can absorb the self-energy of the repulsive interaction and the source matrix for the anomalous terms

\[
\begin{align*}
\hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) & \rightarrow \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) + \frac{h}{R_\xi} \frac{\sqrt{N}}{\Omega} \delta_{b,\xi} \delta(t_p - t'_q) \hat{\sigma}^b(t_p) \\
\hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \delta_{\xi,\xi'} & \rightarrow \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \delta_{\xi,\xi'} - \frac{2h}{R_\xi} \frac{\sqrt{N}}{\Omega} \hat{J}^{ab}_{\psi;\xi}(t_p, t'_q) \\
\hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) & \rightarrow \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) + \frac{h}{R_\xi} \frac{\sqrt{N}}{\Omega} \delta_{b,\xi} \delta(t_p - t'_q) \hat{J}^{ab}_{\psi;\xi}(t_p) .
\end{align*}
\]  

After these shifts of \( \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \) in \( \mathcal{Z}[\mathcal{J}] \) \( \text{(3.15)} \), we remove the ‘Nambu’-doubled fields \( \hat{\Psi}^{+;b}_{\xi}(t'_q) \ldots \hat{\Psi}^{2}_{\xi}(t_p) \) by integration. According to the doubling of the fields, we obtain the square root of the determinant with the disorder-self-energy, the one-particle Hamiltonian \( \hat{H}^{bo}_{\xi}(t'_q, t_p) \) and with the ‘exterior’ source term \( \hat{J}^{ab}_{\psi;\xi}(t'_q, t_p) \) for generating observables by differentiation. The shifts of \( \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \) as in \( \text{(3.18/3.20)} \) have eliminated the self-energy \( \hat{\sigma}^b(t_p) \) of the repulsive interaction and the source matrix \( \hat{J}^{ab}_{\psi;\xi}(t_p) \) from the determinant. An additional Gaussian factor of \( \hat{\sigma}^b(t_p) \) and Gaussian coupling terms with \( \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \), \( \hat{J}^{ab}_{\psi;\xi}(t'_q) \) result instead of the appearance in the functional determinant. The Gaussian factors of the self-energy \( \hat{\sigma}^b(t_p) \) disappear completely from the coherent state path integral \( \mathcal{Z}[\mathcal{J}] \) \( \text{(3.21)} \) after integration as in \( \text{(3.22)} \) so that the disorder-self-energy \( \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \) remains as the only integration variable

\[
\mathcal{Z}[\mathcal{J}] = \int d[\hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q)] \exp \left\{ -\frac{1}{8\hbar^2} \int_C dt_p \, dt'_q \sum_{\xi} \text{tr} \left[ \hat{\Sigma}^{ab}_{I;\xi}(t_p, t'_q) \hat{\Sigma}^{ba}_{I;\xi}(t'_q, t_p) \right] \right\}
\]  

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\[
\frac{1}{2 \hbar R_I \Omega} \int dt'_p \sum_{\vec{x}} \text{tr} \left\{ \hat{\Sigma}_{I,\vec{x}}^{ab}(t'_p, t'_p) \hat{j}_{\psi;\vec{x}}^{ba}(t'_p) \right\} \\
- \frac{1}{2} \int \frac{dt_p}{\hbar} \sum_{\vec{x}} \text{tr} \left[ j_{\psi;\vec{x}}^{ab}(t_p) \hat{j}_{\psi;\vec{x}}^{ba}(t_p) \right] \\
- \frac{1}{2} \int \frac{dt_p}{\hbar} \sum_{\vec{x}} \text{tr} \left[ \hat{\Sigma}_{I,\vec{x}}^{ab}(t'_p, t'_p) \right] \\
\times \exp \left\{ \frac{1}{2 \hbar R_I \Omega} \int dt'_p \sum_{\vec{x}} \text{tr} \left[ \hat{\Sigma}_{I,\vec{x}}^{ab}(t'_p, t'_p) \right] \right\} \\
\exp \left\{ - \frac{1}{2} \int \frac{dt_p}{\hbar} \sum_{\vec{x}} \text{tr} \left[ \hat{\Sigma}_{I,\vec{x}}^{ab}(t'_p, t'_p) \right] \right\}
\]

Finally, we obtain the ensemble averaged generating function $Z_{I}[\mathcal{J}]$ (3.22) for the disorder model I at zero temperature. It only contains as single integration variables the disorder-self-energy matrix elements $\hat{\Sigma}_{I,\vec{x}}^{ab}(t_p, t'_p)$. The generating function $Z_{I}[\mathcal{J}]$ (3.22) consists of Gaussian factors with $\hat{\Sigma}_{I,\vec{x}}^{ab}(t_p, t'_p)$, one functional determinant and the bilinear source term with $J_{\psi;\vec{x}}^{ab}(t'_p) \ldots J_{\psi;\vec{x}}^{ba}(t_p)$ for the coherent BE-condensate wavefunction. The final expression for $Z_{I}[\mathcal{J}]$ is listed in Eq. (3.23) with the matrix $M_{I,\vec{x}}^{ab}(t'_p, t_p)$ (3.25) as an important ingredient apart from the Gaussian factors with the complex parameter $\mu_p^{(I)}$ (3.24).
In order to derive the saddle point equation, we have to perform the first order variation of the actions to dimensionless values. This scaling to dimensionless quantities is given in relations (3.26) to (3.37). In principle one can continue the first order variations of the actions in $\tilde{Z}_I$ to second or even higher order variations as a kind of functional Taylor expansion with the disorder-self-energy $\tilde{\Sigma}$. We restrict in model I only to solutions following from the first order variations $\delta \tilde{\Sigma}(t_p, t'_p)$ and have to scale all parameters and self-energy matrix fields to dimensionless values. This scaling to dimensionless quantities is given in relations (3.26) to (3.37)²

\[
\dim[\tilde{H}^{ab}_{t_p, t_q} (t'_p, t_p)] = \frac{[\text{energy}]}{[\text{time}]} 
\]

\[
\tilde{H}^{ab}_{t_p, t_q} (t'_p, t_p) \rightarrow \tilde{H}^{ab}_{t_p, t_q} (t'_q, t'_q, t_p) = \frac{\tilde{H}^{ab}_{t_p, t_q} (t'_q, t'_q, t_p)}{\hbar \Omega} 
\]

\[
\tilde{\Sigma}^{ab}_{t_p, t_q} (t'_p, t_p) \rightarrow \tilde{\Sigma}^{ab}_{t_p, t_q} (t'_q, t'_q, t_p) = \frac{1}{\hbar} \frac{\tilde{\Sigma}^{ab}_{t_p, t_q} (t'_q, t'_q, t_p)}{\Omega} 
\]

\[
R_I \rightarrow \tilde{R}_I = \frac{R_I}{\hbar} 
\]

Moreover, we have to consider a kind of typical ‘level spacing’ ‘e’ which follows from the fundamental discreteness of spatial and time-like variables and fields in $\tilde{Z}_I$. A dimensionless parameter $\tilde{V}_0$ replaces the repulsive interaction strength $V_0$ after scaling with a kind of ‘mean level spacing’ ‘e’

²A tilde ‘’ over the self-energy, the one particle Hamilton operator or other parameters refers to the corresponding dimensionless, scaled quantity.
known from random matrix theories. Additionally, we introduce the dimensionless quantity $\xi_I$ as the quotient of the second moment related disorder parameter $R_I^2$ to the parameter $V_0$ of the repulsive interaction so that the complex parameter $\mu^{(I)}_P$ in $\overline{Z}[J]$ (3.29) is determined by relations (3.32 - 3.33)

$$V_0 \rightarrow \overline{V}_0 = \frac{V_0}{e}$$

$$\epsilon = \frac{\hbar \Omega}{N_x}$$

$$\xi_I = \left( \frac{R_I}{h} \right)^2 \frac{1}{(V_0/e)} = \overline{R}_I^2 / \overline{V}_0$$

(3.32)

$$\mu^{(I)}_P = \frac{1 - \frac{1}{2} \eta_p \xi_I}{1 - \frac{1}{2} \eta_p}$$

(3.33)

$$J^a_{\psi;\overline{z}}(t_p) \rightarrow J^a_{\psi;\overline{z}}(t_{p,k}) = \frac{1}{\sqrt{N_x}} \sum_{q,l} \frac{J^a_{\psi;\overline{z}}(t_{p,k})}{\hbar \Omega}$$

(3.34)

$$J^a_{\psi;\overline{z}}(t_{q,l},t_p) \rightarrow J^a_{\psi;\overline{z}}(t_{q,l},t_{p,q,l,p}) = \frac{1}{\hbar \Omega} \sum_{q,l} \delta(t_{p,q,l,p},t_{q,l}) J^a_{\psi;\overline{z}}(t_{p,k})$$

(3.35)

$$\frac{J^a_{\psi;\overline{z}}(t_{q,l},t_p)}{N_x} \rightarrow \frac{J^a_{\psi;\overline{z}}(t_{q,l},t_{p,q,l,p})}{N_x \hbar \Omega}$$

(3.36)

A scaling of the source terms (3.34 - 3.37) has also been included for the derivation of the saddle equation. We list the ensemble averaged generating function $\overline{Z}[J]$ (3.26 - 3.27) in terms of the scaled disorder-self-energy $\overline{\Sigma}_{I,\overline{x}}(t_{q,l},t_{q,l})$ (3.35) and corresponding scaled operators, fields and parameters (3.26 - 3.37) in discrete space-time coordinates in relation (3.38). The generating function $\overline{Z}[J]$ (3.38) only consists of discrete sums with space-time points $\overline{x}_i$, $\overline{x}_j$ and $t_{p,k}, t_{q,l} (p, q = \pm, t_{p,k} = k \cdot \Delta t_p, t_{q,l} = l \cdot \Delta t_q, k, l = 0, ..., M - 1)$.

$$\overline{Z}[J] = \int d[\tilde{\Sigma}^b_{I,\overline{x}}(t_{p,k},t_{q,l})] \exp \left\{- \frac{1}{2} \sigma \sum_{p=\pm} \sum_{p=\pm} \sum_{t} (1 - \mu^{(I)}_P) \sum_{p=\pm} \sum_{t} \left[ \tilde{J}^a_{\psi;\overline{z}}(t_{p,k}) \tilde{J}^a_{\psi;\overline{z}}(t_{p,k}) \right] \right\}$$

(3.38)

$$\times \exp \left\{- \frac{1}{2} \sum_{p, q = \pm} \sum_{t, t'} \left[ 1 - \delta(t_{p,q,l,p},t_{q,l}) \right] \eta_p \sum_{p, q = \pm} \sum_{t, t'} \left[ \tilde{\Sigma}^b_{I,\overline{x}}(t_{p,k},t_{q,l}) \tilde{\Sigma}^b_{I,\overline{x}}(t_{p,k},t_{q,l}) \right] \right\}$$

(3.39)

3In the remainder the symbol $\delta(t_{p,k},t_{q,l})$ as in $\overline{Z}[J]$ (3.38 - 3.39) denotes the Kronecker-delta for the discrete times $t_{p,k}$ and $t_{q,l}$.

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\[
\int_C dt_p \int dt'_k \frac{d^4x}{L^4} \ldots \text{fields}(\vec{x}, t_p) \ldots \rightarrow \sum_{p=\pm} \sum_{k=p+1}^{M-1} \int_{t_p}^{t_k} \sum_{\vec{x}, p} \eta_p \ldots \text{fields}(\vec{x}_i, t_p, k) \ldots .
\] (3.40)

The sum \(\sum_{p=k} t'_p, t'_q\) over discrete time contour variables in (3.38) is defined without the metric \(\eta_p, \eta_q\) which has therefore to be included separately with the sum \(\sum_{p,q=\pm}\) over the contour time branches. However, the metric \(\eta_p, \eta_q\) has to appear in the appropriate terms of \(Z_l[\mathcal{F}]\) (3.38) where a contour integration is really performed with the negative sign for the propagation in the backward direction of a time development.

The iteration \(m \rightarrow m + 1\) from \(\bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m; t_{p,k}, t'_{q,l})\) to \(\bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m+1; t_{p,k}, t'_{q,l})\) proceeds according to relation (3.43) via continued fraction. One starts from the noninteracting, 'free' Green function with vanishing disorder-self-energy \(\bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m = 0; t_{p,k}, t'_{q,l}) \equiv 0\) and obtains a disorder-self-energy \(\bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m = 1; t_{p,k}, t'_{q,l})\) with non-vanishing non-diagonal `+-' and `-+' parts (see the notations for contour time ordering)

\[
(1 - \delta_{p,q} \delta(t_{p,k}, t'_{q,l}) \mu_p(t)) \bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m + 1; t'_{q,l}, t_{p,k}) =
\]

\[
\text{(3.43)}
\]

\[
= \left(2 \left(1 - \frac{\mu_p(t)}{R_l} \right) \eta_p \delta(t_{p,k}, t'_{q,l}) \bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p,k}) - \left(\tilde{R}_l \bar{M}^{1-ba}_{l,\vec{z},\vec{z}}(m; t_{p,k}, t'_{q,l}) + \right)
\]

\[
= \left(\tilde{R}_l \sum_{p',q'=\pm} \sum_{\gamma',\vec{y}',\gamma',\vec{y}', c,d=1,2} \sum_{\gamma',\vec{y}',\gamma',\vec{y}', c,d=1,2} \right) \bar{J}^{d+}_{\psi_p;\vec{y},\gamma'}(t'_{q,l}, \gamma, \vec{y}) \eta_{p'} \bar{J}^{1-d}_{\psi_p;\vec{y},\gamma'}(t_{q,l}, t''_{p,k}) \times
\]

\[
\bar{M}^{1-bc}_{l,\vec{z},\vec{z}}(m; t_{p,k}, t'_{q,l}) \eta_{p'} \bar{J}^{c+}_{\psi_p;\vec{y},\gamma'}(t_{q,l}, \gamma, \vec{y})
\]

\[
\text{(3.44)}
\]

\[
\bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p=\pm}) = \bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p=\pm}) = \bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p=\pm}) = \bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p=\pm}) .
\]

The nondiagonal parts \(\bar{\Sigma}^{ab}_{l,\vec{z},\vec{z}}(m + 1; t_{p=\pm}, t'_{q=\pm})\) reappear at every iteration step due to the source field \(\bar{J}^{ab}_{\psi_p;\vec{y},\gamma'}(t_{q,l}, \gamma, \vec{y})\) for the creation of a coherent BE-wavefunction. The anomalous parts \(\langle \psi_p(t_p) \psi_p(t_p) \rangle\) follow from the source matrix \(\bar{J}^{ab}_{\psi_p;\vec{z}}(t_{p=\pm})\neq 0\) (for \(a \neq b; a, b = 1, 2\)) which has a diagonal contour time dependence, but nondiagonal terms \((a \neq b)\) for the creation of pair condensates.
In the case of a time independent trap potential, the solution \( \tilde{\Sigma}^{ab}_{t;\xi,p,q} (t_p,k,t'_q) \) simplifies to \( \tilde{\Sigma}^{ab}_{t;\xi,p,q} (\omega) \) whose Fourier transform therefore takes the form \( \tilde{\Sigma}^{ab}_{t;\xi,p,q} (\omega) \) and \( \tilde{H}^{ab}_{t;\xi,p,q} (\omega) \) in the matrix \( M^{ab}_{t;\xi,p,q} (\omega) \) (3.46) and \( H^{ab}_{t;\xi,p,q} (\omega) \) (3.47) in the matrix \( M^{ab}_{t;\xi,p,q} (\omega) \) (3.48).

The Green function \( M^{ab}_{t;\xi,p,q} (\omega) \) is obtained from solving the generalized eigenvalue problem (3.50) with the eigenfunction \( \Psi^{R,a}_{t;\xi,p,q}(\omega_N) \) and eigenvalue \( \omega_N \)

\[
\sum_{q=\pm} \sum_{b=1,2} \begin{pmatrix} \hat{h}(\xi) & 0 & 0 & 0 \\ 0 & h^T(\xi) & 0 & 0 \\ 0 & 0 & -\hat{h}(\xi) & 0 \\ 0 & 0 & 0 & -h^T(\xi) \end{pmatrix} \begin{pmatrix} \Psi^{+}_{\xi,p,q}(\omega_N) \\ \Psi^{+}_{\xi,q,p}(\omega_N) \\ \Psi^{-}_{\xi,p,q}(\omega_N) \\ \Psi^{-}_{\xi,q,p}(\omega_N) \end{pmatrix} = \frac{\tilde{R}_q}{2} \sum_{q=\pm} \sum_{b=1,2} \begin{pmatrix} \psi^{+}_{\xi,p}(\omega_N) \\ \psi^{+}_{\xi,q}(\omega_N) \\ \psi^{-}_{\xi,p}(\omega_N) \\ \psi^{-}_{\xi,q}(\omega_N) \end{pmatrix} \eta_q \left( \begin{pmatrix} \tilde{h}^{+}_{\xi,p}(\omega_N) \\ \tilde{h}^{+}_{\xi,q}(\omega_N) \\ \tilde{h}^{-}_{\xi,p}(\omega_N) \\ \tilde{h}^{-}_{\xi,q}(\omega_N) \end{pmatrix} \right) \]

\[
\eta_p \left( \begin{pmatrix} \psi^{+}_{\xi,p}(\omega_N) \\ \psi^{+}_{\xi,q}(\omega_N) \\ \psi^{-}_{\xi,p}(\omega_N) \\ \psi^{-}_{\xi,q}(\omega_N) \end{pmatrix} \right) = \omega_N \eta_p \left( \begin{pmatrix} \psi^{+}_{\xi,p}(\omega_N) \\ \psi^{+}_{\xi,q}(\omega_N) \\ \psi^{-}_{\xi,p}(\omega_N) \\ \psi^{-}_{\xi,q}(\omega_N) \end{pmatrix} \right) \]

\[
\hat{h}(\xi) = -\partial_{\xi} \cdot \partial_{\xi} + \hat{u}(\xi) - \hat{\mu}_0 ; \quad \hat{h}^T(\xi) = \hat{h}_p(\xi) \]
\[ \delta_{\omega_N, \omega'_{N'}} = \int \mathrm{d}[\xi] \sum_{p=\pm 1,2} \sum_{a=1} \Psi_{L,a}^{(\omega_N)}(\xi_p) \eta_p \Psi_{R,a}^{(\omega'_{N'})}(\xi_p) \]  

(3.52)

Note that the disorder-self-energy \( \tilde{\Sigma}_{ab}^{L,b} \) in (3.50) also depends on the frequency \( \omega \) and therefore has to coincide with the resulting eigenvalue \( \omega_N \) on the right hand side of (3.50). Since the generalized eigenvalue problem (3.50) becomes non-hermitian due to the iterations for the continued fraction, we have to introduce left and right eigenvectors \( \Psi_{L,a}^{(\omega_N)}(\xi_p) \) for the ortho-normalization (3.52).

Assuming completeness of the eigenfunctions \( \Psi_{L,b}^{(R/L),a} \) (3.53), one can construct the non-equilibrium Green function \( \tilde{M}^{-1,ab}_{\xi,\xi';q,q'}(\omega) \) (3.54) from the eigenfunctions \( \Psi_{L,b}^{(R/L),a} \) and eigenvalues \( \omega_N \) of (3.50)

\[ \tilde{M}^{-1,ab}_{\xi,\xi';q,q'}(\omega) = \sum_{(\omega_N)} \Psi_{L,b}^{(R/L),a}(\omega_N) \otimes \Psi_{L,b}^{(R/L),a}(\omega_N) \frac{\delta_{\xi,\xi'} - \delta_{p,q}}{-\omega - i \tilde{\varepsilon} + \omega_N} \quad (\tilde{\varepsilon} > 0) \]  

(3.54)

The generalized eigenvalue problem (3.50) has to be considered at every iteration step \( m \to m + 1 \) of the continued fraction, but reduces to the solution of the radial part of (3.50) for a rotational symmetry. The non-diagonal parts of the disorder-self-energy \( \tilde{\Sigma}_{ab}^{L,b} \) are reminiscent of the Bogoliubov-de Gennes equations for superconductivity where the non-diagonal, anomalous parts are an important ingredient of BCS theory. The nondiagonal 'Nambu' parts \( (a \neq b, a, b = 1, 2) \) in (3.50) correspond to such pair condensates as \( (\psi_{\xi,p}^{+}(\omega) \quad \psi_{\xi,p}^{-}(\omega)) \) (in the bosonic case), and the nondiagonal contour time parts '+−' and '−+' are related to quasiparticles created by defects and disorder (for a classification of various types of disorder see [17]). Since both anomalous parts (pair condensates and anomalous disorder effects) are taken into account in (3.50), we have described the exact mean field theory with the \( 4 \times 4 \) disorder-self-energy \( \tilde{\Sigma}_{ab}^{L,b} \) (\omega).

### 3.3 Hubbard-Stratonovich transformation for the disorder term in model II and derivation of the mean field equations with the disorder-self-energy of a local time dependence

The various steps of the derivation for the saddle point equation (3.31,3.32) in section 3.2 can be conveyed to model II with dynamic disorder. Apart from the common repulsive interaction with strength \( V_0 \) in both models, the 'non-hermitian' ensemble averaged quartic interaction depends only on a single time variable, but includes the averaging effect of \( V_1(\tilde{x}, t) \) in the two time contour indices \( (p, q = \pm) \). The resulting 'Nambu'-doubled density matrix \( \tilde{R}_{\xi,\xi;p,q}(t) \) (3.55,3.56) following from the dyadic products of 'Nambu'-doubled Bose fields, therefore has only a single time variable, but two contour indices \( (p, q = \pm) \) for forward and backward propagation apart from the 'Nambu' indices \( (a, b = 1, 2) \)

\[ \int_{-\infty}^{\infty} \mathrm{d}t \sum_{\tilde{x}} \sum_{p,q=\pm} \left( \psi_{\tilde{x}e}^+(t_p) \eta_p \psi_{\tilde{x}e}^-(t_p) \right) \left( \psi_{\tilde{x}e}^+(t_q) \eta_q \psi_{\tilde{x}e}^-(t_q) \right) = \int_{-\infty}^{\infty} \mathrm{d}t \sum_{\tilde{x}} \sum_{p,q=\pm} \left( \psi_{\tilde{x}e}^+(t_p) \eta_p \psi_{\tilde{x}e}^-(t_p) + \psi_{\tilde{x}e}^-(t_p) \eta_p \psi_{\tilde{x}e}^+(t_p) \right) \times \\
\times \left( \psi_{\tilde{x}e}^+(t_q) \eta_q \psi_{\tilde{x}e}^-(t_q) + \psi_{\tilde{x}e}^-(t_q) \eta_q \psi_{\tilde{x}e}^+(t_q) \right) = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}t \sum_{\tilde{x}} \sum_{p,q=\pm} \left( \psi_{\tilde{x}e}^+(t_p) \eta_p \psi_{\tilde{x}e}^-(t_p) \right) \left( \psi_{\tilde{x}e}^+(t_q) \eta_q \psi_{\tilde{x}e}^-(t_q) \right) = \frac{1}{4} \int_{-\infty}^{\infty} \mathrm{d}t \sum_{\tilde{x}} \sum_{p,q=\pm} \left( \psi_{\tilde{x}e}^+(t_p) \eta_p \psi_{\tilde{x}e}^-(t_p) \right) \left( \psi_{\tilde{x}e}^+(t_q) \eta_q \psi_{\tilde{x}e}^-(t_q) \right) 
\]  

(3.55)

20
\[
\begin{align*}
&= \frac{1}{4} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \sum_{p,q=\pm} \eta_p \frac{\Psi^a_{\vec{x}}(t_p) \otimes \Psi^{\dagger B}_{\vec{x}}(t_q)}{R^{a}_{\vec{x},p}(t)} \frac{\Psi^b_{\vec{x}}(t_q) \otimes \Psi^{\dagger a}_{\vec{x}}(t_p)}{R^{b}_{\vec{x},q}(t)} \\
&= \frac{1}{4} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \sum_{p,q=\pm} \sum_{a,b} \left[ \eta_p \hat{R}^{ab}_{\vec{x},p}(t) \eta_q \hat{R}^{ba}_{\vec{x},q}(t) \right]
\end{align*}
\]

\[\hat{R}^{ab}_{\vec{x},p}(t) = \begin{pmatrix}
\psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) \\
\psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},+}(t) \\
\psi_{\vec{x},-}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},-}(t) & \psi_{\vec{x},-}(t) \\
\psi_{\vec{x},-}(t) & \psi_{\vec{x},+}(t) & \psi_{\vec{x},-}(t) & \psi_{\vec{x},-}(t)
\end{pmatrix}
\]

(3.56)

Consequently, the HST for the disorder term in model II can be performed as in relation (3.57) where the doubled disorder-self-energy \(\Sigma^{ab}_{11,\vec{x},p}(t) \in sp(4)\) has to fulfill the equivalent symmetry relations (3.59, 3.60) as the density matrix \(\hat{R}^{ab}_{\vec{x},p}(t)\) (3.59)

\[
\exp \left\{ - \frac{\hat{R}^{ab}_{\vec{x},p}(t)}{2\hbar} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \sum_{p,q=\pm} \psi^\dagger_{\vec{x}}(t_p) \eta_p \psi_{\vec{x}}(t_p) \psi^\dagger_{\vec{x}}(t_q) \eta_q \psi_{\vec{x}}(t_q) \right\} = \frac{1}{8 \hbar^2} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \sum_{p,q=\pm} \sum_{a,b} \left[ \eta_p \hat{\Sigma}^{ab}_{11,\vec{x},p}(t) \eta_q \hat{\Sigma}^{ba}_{11,\vec{x},q}(t) \right] = \int d[\hat{\Sigma}^{ab}_{11,\vec{x},p}(t)] \exp \left\{ - \frac{1}{8 \hbar^2} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \sum_{p,q=\pm} \sum_{a,b} \left[ \eta_p \hat{\Sigma}^{ab}_{11,\vec{x},p}(t) \eta_q \hat{\Sigma}^{ba}_{11,\vec{x},q}(t) \right] \right\}
\]

(3.57)

(3.58)

(3.59)

Inserting the two HST’s (3.57) and (3.58) into \(Z_{11}[J]\) (2.33), we obtain the ensemble averaged coherent state path integral \(\overline{Z_{11}[J]}\) (3.61) for dynamic disorder and also extend the ’Nambu’-doubling to the source terms and one-particle part \(H^{ga}_{\vec{x},\vec{x}}(t',t)\) (3.34, 3.17)

\[
\overline{Z_{11}[J]} = \int d[\hat{\Sigma}^{ab}_{11,\vec{x},p}(t)] d[\hat{\sigma}^{ab}_{\vec{x}}(t_p)] \exp \left\{ - \frac{i}{4 \hbar V_0} \int dt_p \sum_{a,b} \left[ \hat{\sigma}^{ab}_{\vec{x}}(t_p) \hat{\sigma}^{ba}_{\vec{x}}(t_p) \right] \right\}
\]

(3.61)
\[
\times \exp \left\{ -\frac{i}{2\hbar} \int_C dt_p \sum_\mathcal{\tau} \left[ J_{u,p}^a(t_p) \Psi_{u,p}^a(t_p) + \Psi_{u,p}^b(t_p) J_{u,p}^b(t_p) \right] \right\}.
\]

In analogy to (3.13-3.25) in model I, the disorder-self-energy \( \hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) \) with local time dependence can be shifted by the self-energy \( \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t) \) of the repulsive interaction and by the source term \( J_{u,p}^a(t_p) \) for the pair condensates

\[
\begin{align*}
\hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) &\rightarrow \hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) + 2 \delta_{p,q} \eta_p \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t) \quad (3.62) \\
\hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) &\rightarrow \hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) - 2 \delta_{p,q} \eta_p \hat{J}_{u,p}^a(t_p), \quad (3.63)
\end{align*}
\]

so that the matrix \( \hat{M}_{I_1;\mathcal{\tau}}^{ba}(t') \ (3.65) \) coupled to the bilinear Bose fields \( \Psi_{u,p}^b(t') \ldots \Psi_{u,p}^b(t) \) in \( \overline{Z_{I_1}[J]} \) (3.64) only contains the disorder-self-energy \( \hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) \) (3.58).

The remaining Gaussian factors with the quadratic self-energy \( \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t) \) of the repulsive interaction and its coupling to \( \hat{\Sigma}_{I_1;\mathcal{\tau}}^{ab}(t) \) and \( \hat{J}_{u,p}^a(t_p) \) can be integrated out completely as in section 3.2.

\[
\int d[\hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p)] \exp \left\{ -\frac{1}{2} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_\mathcal{\tau} \left[ \frac{1}{2\hbar} \eta_p \frac{1}{\hbar V_0} \right] \right\} \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p) \}
\]

\[
\times \exp \left\{ -\frac{1}{2} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_\mathcal{\tau} \left[ \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p) \right] \}
\]

\[
\times \exp \left\{ \frac{i}{2} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_\mathcal{\tau} \left[ \frac{1}{2\hbar} \eta_p \frac{1}{\hbar V_0} \right] \right\} \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p) \}
\]

\[
\times \exp \left\{ -\frac{1}{2} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_\mathcal{\tau} \left[ \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p) \right] \}
\]

\[
\times \exp \left\{ \frac{i}{2} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_\mathcal{\tau} \left[ \frac{1}{2\hbar} \eta_p \frac{1}{\hbar V_0} \right] \right\} \hat{\sigma}_{I_1,\mathcal{\tau}}^{ab}(t_p) \}
\]

\[
\]
Substituting (3.66) into (3.64), we finally achieve the ensemble averaged generating function in relations (3.69-3.78) for zero temperature with the disorder-self-energy $\hat{\Sigma}^{ab}_{II,x,pp}(t)$ (3.58) as the only field variable and the matrix $\tilde{M}^{ba}_{II,x,q}(t', t_p)$ (3.65)

$$Z_{II}[\mathcal{F}] = \int d[\Sigma^{ab}_{II,x,pp}(t)] \times$$

$$\times \left\{ -\frac{1}{1 - \eta_p} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_{\vec{x}} \left[ 1 - \delta_{p,q} \mu^{(II)}_p \right] \left[ \eta_p \hat{\Sigma}^{ab}_{II,x,pp}(t) \right] \left[ \eta_q \hat{\Sigma}^{ba}_{II,x,qp}(t) \right] \right\}$$

$$\times \left\{ -\frac{1}{2 R^2_{II}} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_{\vec{x}} \left( 1 - \mu^{(II)}_p \right) \left[ \eta_p \hat{\Sigma}^{ab}_{II,x,pp}(t) \hat{J}^{ba}_{\psi;\bar{x}}(t_p) \right] \right\}$$

$$\times \left\{ -\frac{1}{2 R^2_{II}} \sum_{p=\pm} \int_{-\infty}^{+\infty} dt \sum_{\vec{x}} \left( 1 - \mu^{(II)}_p \right) \left[ \hat{J}^{ab}_{\psi;\bar{x}}(t_p) \hat{J}^{ba}_{\psi;\bar{x}}(t_p) \right] \right\}$$

$$\times \left\{ \frac{\Omega^2}{2 \hbar} \int_C dt_p dt'_q \sum_{\vec{x}, \vec{x}'} N_2 \sum_{a,b=1,2} J^{+b}_{\psi;\bar{x}}(t'_q) \tilde{\mathcal{M}}^{ba}_{II,x,\vec{x}'}(t'_q, t_p) J^{b}_{\psi;\bar{x}}(t_p) \right\}$$

$$\mu^{(II)}_p = \frac{1}{\left( 1 - \frac{1}{2} \eta_p \left( \frac{R^2_{II}}{\hbar^2 N_2} \right) \right)} \left( 0 < \Re(\mu^{(II)}_p) < 1 \right)$$

The scaling of the disorder-self-energy and the other energy parameters to dimensionless quantities is listed in relations (3.69-3.78)

$$\tilde{\Sigma}^{ba}_{II,x,qp}(t) \rightarrow \tilde{\Sigma}^{ba}_{II,x,qp}(t) = \frac{\tilde{\Sigma}^{ba}_{II,x,qp}(t)}{M^2 N_2} \delta(t_k - t'_k) \quad (3.69)$$

$$\delta(t - t') \quad (3.70)$$

$$R^2_{II} \rightarrow R^2_{II} = \frac{R^2_{II}}{\hbar^2 \Omega N_2} \quad (3.71)$$

$$V_0 \rightarrow \tilde{V}_0 = \frac{V_0}{\hbar \Omega N_2} \quad (3.72)$$

$$\xi_{II} = \frac{R^2_{II}}{\hbar V_0} \quad (3.73)$$

$$\mu^{(II)}_p = \frac{1}{1 - \frac{1}{2} \eta_p \xi_{II}} \quad (3.74)$$
The coherent state path integral \( \mathcal{Z}_{II}[\mathcal{F}] \) is transformed to relation (3.79) with discrete spatial and time-like variables of the dimensionless parameters and fields defined in (3.69-3.78).

A functional Taylor expansion can be performed on the actions in (3.79) and (3.80) with respect to \( \delta \tilde{S}_{ab}^{I}_{II,\vec{x},i,j}(t_k) \) as in section 3.2. We restrict to the vanishing of the first order variation in (3.79) and so derive a mean field equation (3.81) for dynamic disorder with a dependence on the disorder-self-energy as the only remaining field variable.

\[
\frac{1}{R_{II}^{1}} \left( 1 - \delta_{p,q} \mu_p^{(II)} \right) \tilde{S}_{ab}^{I}_{II,\vec{x},i,j}(t_k) = 2 \frac{R_{II}^{1}}{R_{II}^{1}} \delta_{p,q} \eta_p \left( 1 - \mu_p^{(II)} \right) \tilde{F}_{ab}^{I}_{\psi,\vec{x},i,j}(t_k) + \frac{1}{2} \sum_{p} \delta(t_k,t'_k) \eta_p \tilde{F}_{ab}^{I}_{\psi,\vec{x},i,j}(t'_k, t_k) + \frac{1}{2} \sum_{p} \delta(t_k,t'_k) \eta_p \tilde{F}_{ab}^{I}_{\psi,\vec{x},i,j}(t'_k, t_k) .
\]

We also have to consider the originally introduced imaginary increment \( -i \varepsilon_p \) (2.25) for time reversal symmetry breaking so that these convergence properties are consequently transferred to the disorder-self-energy in the continued fractions. The iteration \( m \to m+1 \) of \( \tilde{S}_{ab}^{I}_{II,\vec{x},i,j}(m; t_k) \) in the continued fractions follows in analogy to section 3.2 and equations (3.43) to (3.45) starting from the free Green function with \( \tilde{S}_{ab}^{I}_{II,\vec{x},i,j}(m; t_k) \equiv 0 \).

\[
\frac{1}{R_{II}^{1}} \left( 1 - \delta_{p,q} \mu_p^{(II)} \right) \tilde{S}_{ab}^{I}_{II,\vec{x},i,j}(m+1; t_k) = 2 \frac{R_{II}^{1}}{R_{II}^{1}} \delta_{p,q} \eta_p \left( 1 - \mu_p^{(II)} \right) \tilde{F}_{ab}^{I}_{\psi,\vec{x},i,j}(t_k) + \frac{1}{2} \sum_{p} \delta(t_k,t'_k) \eta_p \tilde{F}_{ab}^{I}_{\psi,\vec{x},i,j}(t'_k, t_k) .
\]
\[ - \tilde{M}_{11}^{-1,ab}(m; t_{p,k}, t_{p,k}) = \eta_p \left( \tilde{H}_{\tilde{\xi},\tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \tilde{\mathcal{J}}_{\tilde{\psi} \tilde{\xi}, \tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \frac{1}{2} \delta(t_k, t'_q) \delta_{\tilde{\xi}, \tilde{\eta}} \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; t_k) \right) \eta_q \]

\[ \tilde{M}_{11}^{ab}(m; t_{p,k}, t'_{q,l}) = \frac{\eta_p \left( \tilde{H}_{\tilde{\xi},\tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \tilde{\mathcal{J}}_{\tilde{\psi} \tilde{\xi}, \tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \frac{1}{2} \delta(t_k, t'_q) \delta_{\tilde{\xi}, \tilde{\eta}} \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; t_k) \right) \eta_q}{\eta_p \left( \tilde{H}_{\tilde{\xi},\tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \tilde{\mathcal{J}}_{\tilde{\psi} \tilde{\xi}, \tilde{\eta}}^{ab}(t_{p,k}, t'_{q,l}) + \frac{1}{2} \delta(t_k, t'_q) \delta_{\tilde{\xi}, \tilde{\eta}} \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; t_k) \right) \eta_q} \]  

\[(3.84)\]

The nondiagonal parts \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; t_{p,k}, t'_{q,l}) \) of the contour time also occur at every iteration step due to the source field \( \tilde{J}_{\tilde{\psi} \tilde{\xi}, \tilde{\eta}}^{ab}(\tau_{p', k'}) \) as in the disorder model I of section 3.2, and the source matrix \( \tilde{J}_{\tilde{\psi} \tilde{\xi}, \tilde{\eta}}^{ab}(t_k) \) creates the anomalous terms. The solution \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; t_{p,k}) \) reduces to a time independent function \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \) in the case of a time independent trap potential \( \tilde{u}(\tilde{\xi}) \) where \( \tilde{\xi} \) is the dimensionless spatial vector. In order to obtain the corresponding Green function \( \tilde{M}_{11}^{-1,ab}(m; t_{p,k}, t'_{q,l}) \) \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \), one has to solve the eigenvalue problem \( \tilde{M}_{11}^{ab}(m; t_{p,k}, t'_{q,l}) \) \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \) as in section 3.2, which results from the following relations \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \) for the matrix \( \tilde{M}_{11}^{-1,ab}(m; \omega) \) after Fourier transformation to dimensionless frequency \( \omega \)

\[ \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) = \eta_p \left( \tilde{H}_{\tilde{\xi},\tilde{\eta}}^{ab}(m; \omega) + \frac{1}{2} \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \delta_{\tilde{\xi}, \tilde{\eta}} \right) \eta_q \]

\[ \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) = \begin{pmatrix} B_{\tilde{\xi},++} & C_{\tilde{\xi},++} & B_{\tilde{\xi},+-} & C_{\tilde{\xi},+-} \\ C_{\tilde{\xi},++} & B_{\tilde{\xi},++} & C_{\tilde{\xi},+-} & B_{\tilde{\xi},+-} \\ B_{\tilde{\xi},+-} & C_{\tilde{\xi},+-} & B_{\tilde{\xi},--} & C_{\tilde{\xi},--} \\ C_{\tilde{\xi},+-} & B_{\tilde{\xi},+-} & C_{\tilde{\xi},--} & B_{\tilde{\xi},--} \end{pmatrix} \]

\[ \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) = \eta_p \eta_q \eta_{a,b} \delta_{\tilde{\xi}, \tilde{\eta}} \begin{pmatrix} -\omega \mathbb{I}_{4 \times 4} + \begin{pmatrix} \tilde{h}_+ (\tilde{\xi}) & 0 & 0 & 0 \\ 0 & \tilde{h}_+ (\tilde{\xi}) & 0 & 0 \\ 0 & 0 & \tilde{h}_- (\tilde{\xi}) & 0 \\ 0 & 0 & 0 & \tilde{h}_- (\tilde{\xi}) \end{pmatrix} \end{pmatrix} \]

\[ \tilde{h}_p (\tilde{\xi}) = -i \tilde{\xi}_p - \partial_{\tilde{\xi}_p} \tilde{\xi}_p + \tilde{u}(\tilde{\xi}) - \tilde{\rho}_0 \]

\[ \tilde{h}_p^T (\tilde{\xi}) = \tilde{h}_p (\tilde{\xi}) \]  

\[(3.88)\]

\[(3.89)\]

In comparison to the disorder model I, one has also to compute the right and left eigenfunctions \( \psi_{R/L,a}^{\tilde{\xi},p}(\omega_N) \) and eigenvalues \( \omega_N \), but without a dependence on the eigenvalue of the disorder-self-energy \( \tilde{\Sigma}_{11;\tilde{\xi}, \tilde{\eta}}^{ab}(m; \omega) \) for stationary states

\[ \sum_{q=\pm} \sum_{b=1,2} \begin{pmatrix} \tilde{h}_+ (\tilde{\xi}) & 0 & 0 & 0 \\ 0 & \tilde{h}_+ (\tilde{\xi}) & 0 & 0 \\ 0 & 0 & \tilde{h}_- (\tilde{\xi}) & 0 \\ 0 & 0 & 0 & \tilde{h}_- (\tilde{\xi}) \end{pmatrix} \begin{pmatrix} \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \end{pmatrix} = \begin{pmatrix} \Phi_{a,\tilde{\xi}}^{R/b}(\omega_N) \\ \Phi_{a,\tilde{\xi}}^{L/b}(\omega_N) \end{pmatrix} \]

\[ \begin{pmatrix} \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \end{pmatrix} = \begin{pmatrix} \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{R,a}^{\tilde{\xi},p}(\omega_N) \\ \psi_{L,a}^{\tilde{\xi},p}(\omega_N) \end{pmatrix} \]

\[(3.90)\]
Differentiating the orthonormalized eigenfunctions (3.92) and eigenvalues 1 Determination of the observables from the derivative with the iteration procedure can be further simplified in the case of spatial symmetries. Because the disorder-self-energy is time independent for a trap potential having only a spatial dependence.

At every iteration step of the continued fractions, but is easier to solve as in the case of the static disorder model I. The eigenvalue problem (3.90-3.94) is reminiscent of the Bogoliubov-de Gennes equations and the we also assume the completeness of the orthonormalized eigenfunctions (3.93) and eigenvalues

\[
\sum_{\omega_N} \eta \psi_{\xi(p)}^{R,a}(\omega_N) \psi_{\xi(q)}^{L,b}(\omega_N) = \delta_{\xi(p),\xi(q)} \delta_{p,q} \delta_{a,b} \tag{3.93}
\]

The Green function \( \tilde{M}^{-1}_{II,\xi,\rho}^{ab}(\omega) \) (3.94) is also determined by relations (3.91) to (3.93) in terms of the orthonormalized eigenfunctions (3.92) and eigenvalues

\[
\tilde{h}(\xi) = -\partial_{\xi} \cdot \partial_{\xi} + \bar{u}(\xi) - \bar{\mu}_0 \tilde{h}_{p}(\xi) = \tilde{h}_{p}(\xi) \tag{3.91}
\]

\[
\delta_{\omega_N,\omega_{N'}} = \int d[\xi] \sum_{p=\pm 1,2} \psi_{\xi(p)}^{L,a}(\omega_{N'}) \eta \psi_{\xi(p)}^{R,a}(\omega_{N}) \tag{3.92}
\]

We also assume the completeness of the orthonormalized eigenfunctions (3.93) as in section 3.2 for model I. The eigenvalue problem (3.90-3.94) is reminiscent of the Bogoliubov-de Gennes equations and the Nambu-Gorkov Green functions in the theory for superconductivity. It has to be solved at every iteration step of the continued fractions, but is easier to solve as in the case of the static disorder because the disorder-self-energy is time independent for a trap potential having only a spatial dependence. The iteration procedure can be further simplified in the case of spatial symmetries.

4 Determination of the observables from the derivative with the source term \( \mathcal{J}^{ab}_{\xi,\rho}(t_{p},t'_{q}) \)

The \( U(1) \) invariant density \( \lim_{t_{s} \rightarrow 0_{s}} \langle \Psi_{\xi}^{+b=1}(t_{s}) \quad \Psi_{\xi}^{+b=1}(t_{s} + \delta_{t_{s}}) \rangle \) of non-condensed atoms follows from differentiating \( Z_{\xi}[^{+}] \) \( Z_{\xi}[^{-}] \) with respect to \( \mathcal{J}^{b=1}_{\xi,\rho}(t_{s},t_{s} + \delta_{t_{s}}) \) and the appropriate normalization. Since a field operator \( \hat{\psi}_{\xi}(t_{p}) \) and its hermitian conjugate \( \hat{\psi}_{\xi}^{+}(t_{p}) \) must not act at the same space time point due to the infinite delta function of the corresponding commutator at coincidence of time, a limit process \( \lim_{t_{s} \rightarrow 0_{s}} \ldots \) has to be performed for the density terms at the same branch of the contour time. The corresponding relations for static and dynamic disorder are tabulated in Eqs. (4.1) to (4.3) with the matrices \( \tilde{C}_{\xi,\rho}(t_{p},t'_{q}) = \tilde{M}_{\xi,\rho}^{cd}(t_{p},t'_{q}) \) \( \tilde{C}_{\xi,\rho}(t_{p},t'_{q}) = \tilde{M}_{\xi,\rho}^{cd}(t_{p},t'_{q}) \) and \( \tilde{C}_{\xi,\rho}(t_{p},t'_{q}) = \tilde{M}_{\xi,\rho}^{cd}(t_{p},t'_{q}) \) and the one particle part \( \hat{\mathcal{H}}_{\xi,\rho}(t_{p},t'_{q}) \) is defined in Eqs. (3.11,3.17) (1,2) are 'Nambu'-indices as '1,2')

\[
\text{static disorder : see } Z_{\xi}[\mathcal{J}] \text{ with } \tilde{M}_{\xi,\rho}^{cd}(t_{p},t'_{q}) \tag{4.1}
\]

\[
\text{dynamic disorder : see } Z_{\xi}[\mathcal{J}] \text{ with } \tilde{M}_{\xi,\rho}^{cd}(t_{p},t'_{q}) \tag{8.24}
\]
We can disentangle the general relations (4.1-4.7) by Fourier transformation to energy momentum space
\[
\delta t_+ \rightarrow t_+ \langle M^a_{\vec{k},\vec{p}}(t_+, t'_q) \rangle = 2i\hbar \Omega^2 N_x^2 \left( \frac{\partial Z_{1,1;I}^I}{\partial J^{b=1}_{\vec{x},\vec{p}}} \bigg|_{J^{\sigma} = 0} \right) \]
(4.4)

\[
\delta t_+ \rightarrow t_+ -i\hbar \Omega^2 N_x \hat{\chi}^{-1,\alpha=1,\beta=1}(t_+ + \delta t_+, t_+) + \Omega^4 N_x^2 \int d\tau_p d\tau'_q \sum_{\vec{x}_1, \vec{x}_2, c, d=1,2} \sum_{\vec{k}, \vec{p}} J^{c, d}_{\vec{x}_2, \vec{p}}(\tau''_q) \hat{\chi}^{-1, \alpha=1, c}(t_+ + \delta t_+, t'_p) J^{c, d}_{\vec{x}_1, \vec{p}}(\tau'_p) \]
(4.5)

\[
\hat{M}^{d, c}_{\vec{x}, \vec{p}}(t, t'_q) = \eta_p \left( \hat{H}^{d, c}_{\vec{x}, \vec{p}}(t, t'_q) + \frac{1}{2} \frac{R_l \Omega}{\sqrt{N_x}} \hat{x} \hat{S}^{d, c}_{\vec{x}, \vec{p}}(t, t'_q) \right) \eta_q
\]
(4.6)

\[
\hat{M}^{d, c}_{\vec{x}, \vec{p}}(t, t'_q) = \eta_p \left( \hat{H}^{d, c}_{\vec{x}, \vec{p}}(t, t'_q) + \frac{1}{2} \delta(t - t') \hat{S}^{d, c}_{\vec{x}, \vec{p}}(t) \right) \eta_q
\]
(4.7)

We can disentangle the general relations (4.1-4.7) by Fourier transformation to energy momentum space for a time independent trap potential. The non-equilibrium Green functions $\hat{\chi}^{-1, d}_{\vec{x}, \vec{p}}(t, t'_q)$ become more transparent in energy momentum space $\hat{\chi}^{d}_{\vec{k},\vec{k}'}(\omega)$ (4.8-4.15) and are analogous to the 'Nambu-Gorkov' Green function formalism for superconductivity (14-16, 20-21). In the case of $d = 3$ spatial dimensions, we have to consider the zero momentum state in the summation over wave-vectors $\sum_{\vec{k}} \ldots$ explicitly (18-21). The mean field equations (3.23) (4.11) are mainly applied in three spatial dimensions whereas the two dimensional case should be treated preferably by spontaneous symmetry breaking for the derivation of a nonlinear sigma model and requires different HST transformations than the ones described in this paper (see section 5 and 37-28).
\[\begin{align*}
\hat{M}^{cd}_{\hat{t},\hat{\kappa},\hat{R},\hat{R'}}(\omega) &= \eta_p \left( \hat{H}^{cd}_{\hat{R},\hat{R'}}(\omega) + \frac{1}{2} \hat{\Sigma}^{cd}_{11,\hat{R}-\hat{R'}} \right) \eta_q \\
\hat{H}^{cd}_{\hat{R},\hat{R'}}(\omega) &= -\delta_{p,q} \eta_p \delta_{c,d} \delta_{\hat{R},\hat{R'}} \hbar \omega - 1_{4 \times 4} + \\
&+ \delta_{p,q} \eta_p \delta_{c,d} \begin{pmatrix}
\hat{h}^T_{+}(\hat{R} - \hat{R}') & 0 & 0 & 0 \\
0 & \hat{h}^T_{+}(\hat{R} - \hat{R}') & 0 & 0 \\
0 & 0 & \hat{h}_{-}(\hat{R} - \hat{R}') & 0 \\
0 & 0 & 0 & \hat{h}_{-}(\hat{R} - \hat{R}')
\end{pmatrix}^{cd}_{pq}
\end{align*}\]

If we further assume spatial independence of the source field \(J^c_{\psi;\hat{R}}(\omega_p')\) and of the trap potential \(\pi = u(\hat{R} - \hat{R'} = 0)\) and also a constant creation rate with \(J^c_{\psi;\hat{R}}(\omega_p')\), (4.16), the \(U(1)\) invariant density terms in (4.4.19) reduce to integrals over energy momentum space and a contribution from \(\hat{R} \equiv \hat{0}, \omega \equiv 0\) which is related to the 'negative' density of the coherent BE-wavefunction

\[J^c_{\psi;\hat{R}}(\omega_p') = \frac{2\pi}{\hbar} \int d\omega \delta(\omega) \delta_{\hat{R},\hat{0}} \frac{j}{2}; \quad j \in \mathbb{R}\]

\[\begin{align*}
\lim_{\delta t_+ \to 0^+} \langle \Psi^{+,b=1}_{\hat{R}}(t_+) \mid \Psi^{+,b=1}_{\hat{R}}(t_+ + \delta t_+) \rangle_{\text{static disorder}} &= \int \frac{d\omega}{\hbar \Omega N_x} \sum_{\hat{k}} \frac{d\omega}{\hbar \Omega N_x} \exp\{-i \delta t_+ \omega\} \times \\
&\times \left\{ \delta_{p',q'} \eta_p \delta_{\hat{c}',\hat{d}'} \frac{R_i}{\sqrt{\Omega N_x}} \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right\}^{1-1;\hat{a}=1,b=1}_{++} \left( \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right)_{++}^{1-1;\hat{a}=1,c} \eta_{q'} \left( \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right)_{++}^{1-1;\hat{a}=1,c} \eta_{q'} \frac{R_i}{\sqrt{\Omega N_x}} \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \eta_{q'}
\end{align*}\]

\[\begin{align*}
\lim_{\delta t_+ \to 0^+} \langle \Psi^{+,b=1}_{\hat{R}}(t_+) \mid \Psi^{+,b=1}_{\hat{R}}(t_+ + \delta t_+) \rangle_{\text{dynamic disorder}} &= \int \frac{d\omega}{\hbar \Omega N_x} \sum_{\hat{k}} \frac{d\omega}{\hbar \Omega N_x} \exp\{-i \delta t_+ \omega\} \times \\
&\times \left\{ \delta_{p',q'} \eta_p \delta_{\hat{c}',\hat{d}'} \frac{R_i}{\sqrt{\Omega N_x}} \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right\}^{1-1;\hat{a}=1,b=1}_{++} \left( \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right)_{++}^{1-1;\hat{a}=1,c} \eta_{q'} \left( \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \right)_{++}^{1-1;\hat{a}=1,c} \eta_{q'} \frac{R_i}{\sqrt{\Omega N_x}} \frac{\hat{h}_{\omega} + i \varepsilon_{p'} - \left( \frac{\hbar^2 |\hat{R}|^2}{2m} - \mu_0 + \pi \right)}{\sqrt{\Omega N_x}} \eta_{q'}
\end{align*}\]
The coherent BE-wavefunction $\psi_{\text{BEC}}(\vec{x}, t_+)$ is obtained by differentiating $\frac{Z_{1, II}[J]}{J}$ with respect to the $U(1)$ symmetry breaking source field $J_{\psi, \vec{x}}(t_+)$. The general case for a spatial and time dependent coherency of the BE-wavefunction is listed in the following Eq. (4.19) where the Green function $\hat{O}_{\vec{x}, \vec{x}'}(t_+, t_+')$ refers to $M_{t, k, \vec{p}, \epsilon, \vec{x}, \epsilon}'(\omega)$ (4.10, 4.12) and to $M_{t', k, \vec{p}, \epsilon, \vec{x}, \epsilon}'(\omega)$ (4.11, 4.13).

$$\psi_{\text{BEC}}(\vec{x}, t_+) = \langle \Psi_{\text{BEC}}^{t_+ = 1}(t_+) \rangle = 2\hbar \Omega N_x \left( \frac{\partial Z_{1, II}[J]}{\partial J_{\psi, \vec{x}}(t_+)} \right) \bigg|_{J = 0, \{J_{\psi, \vec{x}, \epsilon}'\}} = N_x \Omega^2 \int_{\mathbb{C}} d\omega' \sum_{\vec{x}, \vec{x}' = c=1,2} \hat{O}_{\vec{x}, \vec{x}'}^{-1,c=1,c}(t_+, t_+') J_{\psi, \vec{x}}(t') .$$

We can transform the above relation (4.19) to energy momentum space which simplifies for a homogenous translation invariant system with $\vec{p} = u(\Delta \vec{k} = 0)$ and $J_{\psi, \vec{x}}(\omega_p) = \frac{2\pi}{\hbar} \delta(\omega) \delta_{\vec{k},0} \frac{1}{2}$, $(j \in \mathbb{R})$ (4.10).

$$\psi_{\text{BEC}}(\vec{x}, t_+) = N_x \Omega T_0 \int_{\mathbb{C}} \frac{d\omega_p}{\Omega} \sum_{\vec{k}, \vec{k}', c=1,2} \exp\{i(\vec{k} \cdot \vec{x} - \omega t_+)\} \hat{O}_{\vec{k}, \vec{k}'}^{-1,c=1,c}(\omega_p) J_{\psi, \vec{k}}(\omega_p).$$

$$\psi_{\text{BEC}}^{\text{static disorder}} = \sum_{p=\pm} \sum_{c=1,2} \left\{ \delta_{p', q} \eta_{p'} - \delta_{c', c} \right\} \frac{\hat{O}_{\vec{k}, \vec{k}'}^{-1,c=1,c}(\omega_p)}{\eta_p} \frac{\exp\{i(\vec{k} \cdot \vec{x} - \omega t_+)\}}{\eta_p} +$$

$$\psi_{\text{BEC}}^{\text{dynamic disorder}} = \sum_{p=\pm} \sum_{c=1,2} \left\{ \delta_{p', q} \eta_{p'} - \delta_{c', c} \right\} \frac{\exp\{i(\vec{k} \cdot \vec{x} - \omega t_+)\}}{\eta_p} +$$

In the thermodynamic limit $j \to 0$, a finite coherent BE-wavefunction $\psi_{\text{BEC}}$ for a homogenous system remains if an 'effective zero eigenvalue' appears in the denominators of (4.21, 4.22). The order of magnitude of the BE-wavefunctions (4.21, 4.22) can then be estimated as follows.

$$\psi_{\text{BEC}} \approx \frac{N_x \Omega T_0 j}{\hbar \omega} \left( \text{('effective zero eigenvalue')} \right) \to \text{finite value} .$$

Using the properties of non-equilibrium Green functions [33], the $U(1)$ invariant density of non-condensed atoms 'lim$_{t \to 0+}$ $\langle \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+) \mid \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+) \rangle$' contains the density $|\psi_{\text{BEC}}|^2$ of the coherent BE-wavefunctions (4.20, 4.22) which is subtracted from the total density given by the first terms in (4.17, 4.18).

$$\lim_{t \to 0+} \langle \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+) \mid \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+ + \delta t_+) \rangle_{\text{static disorder}} = \hbar \Omega N_x \sum_{\vec{k}} \int \frac{d\omega}{\Omega} \exp\{-\imath \delta t_+ \omega\} \times$$

$$\left\{ \delta_{p', q} \eta_{p'} - \delta_{c', c} \right\} \frac{\hbar \omega + c}{4m} \left[ \left[ \frac{\imath \hbar}{2m} \right] \right] - \eta_p \frac{R_I \hat{\mathcal{E}}_{\vec{k}, \vec{k}'}(\omega)}{\Omega \sqrt{N_x}} - |\psi_{\text{BEC}}|^2$$

$$\lim_{t \to 0+} \langle \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+) \mid \Psi_{\vec{x}}^{\epsilon, \delta = 1}(t_+ + \delta t_+) \rangle_{\text{dynamic disorder}} = \hbar \Omega N_x \sum_{\vec{k}} \int \frac{d\omega}{\Omega} \exp\{-\imath \delta t_+ \omega\} \times$$

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The corresponding relations for the bosonic anomalous or pair condensate terms \( \langle \psi_x(t_+) \psi_x(t_+ + \delta t_+) \rangle = \langle \psi_x(t_+) \psi_x(t_+ + \delta t_+) \rangle \) can be taken from Eqs. (4.11) to (4.18) by setting the index 'b = 2' in these equations to the value 'b = 1'. This follows from the 'Nambu' doubling of fields where \( \Psi_x^{\pm} \) is equivalent to \( \psi_x(t_+) \). The limit process \( \lim_{\delta t_+ \to 0} \) need not be considered for the anomalous parts \( \langle \psi_x(t_+) \psi_x(t_+ + \delta t_+) \rangle \) whereas the limit process is of central importance for the density term (4.14) \( \lim_{\delta t_+ \to 0} \langle \psi_x^{a}(t_+) \psi_x(t_+ + \delta t_+) \rangle \) because one must not compute with the operator \( \tilde{\psi}_x(t) \) and its hermitian conjugate \( \tilde{\psi}_x(t) \) at the same space time point in coherent state path integrals of many body theory.

5 Summary and conclusion for \( d = 2 \) spatial dimensions

It has already been mentioned that the derived mean field equations (3.41-3.45), (3.81-3.85) for static and dynamic disorder in sections 3.2, 3.3 are particularly applicable for \( d = 2 \) spatial dimensions because of the singularity of the density of states at \( \tilde{k} \equiv 0 \). Since the mean field approach is less applicable in \( d = 2 \) dimensions, we briefly describe and point out an alternative method [37, 28, 24] which extracts the block diagonal densities as 'hinge'-functions, they can be eventually removed from the determinant and the part for the coherent-BE wavefunction with the bilinear source field \( J_{\psi^a}^x(t) \). They remain in Gaussian integrals and parts of the factorized invariant measure and can be eliminated by integration with a coupling to the source matrix for the anomalous terms. In order to acquire the HST for the disorder-self-energy of model II, we introduce the diagonal self-energy \( \sigma^{(0)}_{R_{11}}(x,t) \) (5.1) and the 'hinge' parts \( \delta \Sigma^{11}_{x,pq}(x,t) \), \( \delta \Sigma^{22}_{x,pq}(x,t) \) \( \delta \Sigma^{12}_{x,pq}(x,t) \) \( \delta \Sigma^{21}_{x,pq}(x,t) \) \( \delta \Sigma^{12}_{x,pq}(x,t) \) for the pair condensates

\[
\begin{align*}
\sigma^{(0)}_{R_{11}}(x,t) &\in \mathbb{R} \quad \text{'hinge' functions} : \delta \Sigma^{11}_{x,pq}(t), \delta \Sigma^{22}_{x,pq}(t) \\
\sigma^{(0)}_{R_{11}}(x,t) &\in \mathbb{C} ; \delta B_{x,+}(t), \delta B_{x,-}(t) \in \mathbb{R} ; \delta B_{x,+}(t) \in \mathbb{C} \\
\delta \Sigma^{11}_{x,pq}(t) &= \begin{pmatrix} \delta B_{x,+}(t) & \delta B_{x,-}(t) & \delta B_{x,-}(t) \\
\delta B_{x,+}(t) & \delta B_{x,+}(t) & \delta B_{x,-}(t) \\
\delta B_{x,-}(t) & \delta B_{x,-}(t) & \delta B_{x,-}(t) \end{pmatrix} \\
\delta \Sigma^{22}_{x,pq}(t) &= \begin{pmatrix} \delta c_{x,+}(t) & \delta c_{x,-}(t) & \delta c_{x,-}(t) \\
\delta c_{x,+}(t) & \delta c_{x,-}(t) & \delta c_{x,-}(t) \\
\delta c_{x,-}(t) & \delta c_{x,-}(t) & \delta c_{x,-}(t) \end{pmatrix} \\
\delta \Sigma^{12}_{x,pq}(t) &= \begin{pmatrix} \delta \Sigma^{11}_{x,pq}(t) \delta \Sigma^{22}_{x,pq}(t) \delta \Sigma^{21}_{x,pq}(t) \delta \Sigma^{12}_{x,pq}(t) \end{pmatrix} \\
\delta \Sigma^{21}_{x,pq}(t) &= \begin{pmatrix} \delta \Sigma^{11}_{x,pq}(t) \delta \Sigma^{22}_{x,pq}(t) \delta \Sigma^{21}_{x,pq}(t) \delta \Sigma^{12}_{x,pq}(t) \end{pmatrix}
\end{align*}
\]

One can generalize from the disorder model II to model I with a double time dependence in the disorder-self-energy. In the case of a stationary state, the self-energy of model I with static disorder obtains a single energy dependence after Fourier transformation because of the reduced dependence to the difference of the two times. Therefore, we can replace the single time dependence of the disorder-self-energy in model II with dynamic disorder with the single frequency dependence of model I with static disorder under restriction to stationary states. The corresponding HST with 'hinge' functions is listed for the ensemble averaged, non-hermitian interaction of dynamic disorder in the following relation

\[
\exp \left\{ -\frac{R^2}{2\hbar} \int_{-\infty}^{\infty} dt \sum_{p,q=\pm \tilde{x}} \left( \psi_x^a(t_p) \eta_p \psi_x(t_p) \right) \left( \psi_x^2(t_q) \eta_q \psi_x(t_q) \right) \right\} = \]
\[
\begin{align*}
&= \int d[\sigma^{(0)}_{R_{11},(\vec{x},t)}] \exp \left\{ -\frac{1}{4R_{11}^2} \int_{-\infty}^{\infty} dt \sum_{x} \sigma^{(0)}_{R_{11},(\vec{x},t)} \sigma^{(0)}_{R_{11},(\vec{x},t)} \right\} \times \\
&\times \int d[\Sigma^{ab}_{p,q}(t) K] \exp \left\{ -\frac{1}{8R_{11}^2} \int_{-\infty}^{\infty} dt \sum_{p,q,a,b} Tr \left[ \delta\Sigma^{ab}_{p,q}(t) K \delta\Sigma^{ab}_{p,q}(t) K \right] \right\} \\
&\times \exp \left\{ -\frac{t}{4h} \int_{-\infty}^{\infty} dt \sum_{p,q,a,b} Tr \left[ \begin{pmatrix} \hat{R}^{11}_{p,q}(t) & \hat{R}^{12}_{p,q}(t) \\ \hat{R}^{21}_{p,q}(t) & \hat{R}^{22}_{p,q}(t) \end{pmatrix} \begin{pmatrix} \hat{\eta} & 0 \\ 0 & \hat{\eta} \end{pmatrix} \right] K \right\}
\end{align*}
\]

Note the minus sign before \( \Sigma^{ab}_{p,q}(t) \) (5.6,5.7,5.12) and the tilde \( \sim \) of \( \delta\Sigma^{ab}_{p,q}(t) \) (5.7) which refers to 'anti-hermitian' anomalous parts \( \delta\Sigma^{ab}_{p,q}(t) = i \delta\Sigma^{ab}_{p,q}(t) \) (a \( \neq \) b) in this section [5]. We have also included a second metric \( \tilde{K} \) (5.6,5.9) apart from the metric \( K_{ab} = \eta_{a,b} \delta_{a,b} \) (5.8) for indefinite orthogonal symmetry concerning the two branches of the contour time. The diagonal matrix \( \tilde{K} \) (5.9) is the appropriate metric for the symplectic Lie algebra which changes the disorder-self-energy \( \Sigma^{ab}_{p,q}(t) \tilde{K} \) to an element of \( sp(4) \), thereby fulfilling the exact commutation relations (with antihermitian coset parts!)

\[
\begin{align*}
\delta\Sigma^{aa}_{p,q}(t) &= \delta\Sigma^{aa}_{p,q}(t) + i \delta\Sigma^{ab}_{p,q}(t) = i \delta\Sigma^{ab}_{p,q}(t) \quad (a \neq b) \quad (5.7) \\
\tilde{K}^{ab}_{p,q} &= \delta_{a,b} \delta_{p,q} \eta_{p} \quad (5.8) \\
\hat{\Sigma}^{11}_{p,q}(t) &= \sigma^{(0)}_{R_{11},(\vec{x},t)} \eta_{p} \delta_{p,q} + i \delta\Sigma^{11}_{p,q}(t) \quad (5.11) \\
\hat{\Sigma}^{22}_{p,q}(t) &= -\sigma^{(0)}_{R_{11},(\vec{x},t)} \eta_{p} \delta_{p,q} + i \delta\Sigma^{22}_{p,q}(t) \quad (5.12)
\end{align*}
\]

In a similar manner the repulsive interaction term can be transformed with a diagonal self-energy \( \sigma^{(0)}_{V_{0},(\vec{x},t_{p})} \), 'hinge' functions \( \sigma^{(1)}_{V_{0},(\vec{x},t_{p})} = \delta\sigma_{V_{0}}^{22}(t_{p}) \) and anti-hermitian anomalous terms \( \delta\sigma^{(a-b)}_{V_{0}}(t_{p}) = i \delta\sigma^{(a-b)}_{V_{0}}(t_{p}) \)

\[
\exp \left\{ -\frac{t}{h} \int_{C} dt_{p} \sum_{\vec{x}} V_{0} \left( \psi_{2}^{2}(t_{p}) \right)^{2} \left( \psi_{2}(t_{p}) \right)^{2} \right\} =
\]

\[
\begin{align*}
&= \int d[\sigma^{(0)}_{V_{0},(\vec{x},t_{p})}] \exp \left\{ \frac{t}{2h} V_{0} \int_{C} dt_{p} \sum_{\vec{x}} \sigma^{(0)}_{V_{0},(\vec{x},t_{p})} \sigma^{(0)}_{V_{0},(\vec{x},t_{p})} \right\} \\
&\times \int d[\delta\sigma^{ab}_{V_{0}}(t_{p}) \tilde{K}] \exp \left\{ \frac{t}{4h} V_{0} \int_{C} dt_{p} \sum_{\vec{x}} \epsilon_{a,b} \left[ \delta\sigma^{ab}_{V_{0}}(t_{p}) \tilde{K} \delta\sigma^{ab}_{V_{0}}(t_{p}) \tilde{K} \right] \right\} \\
&\times \exp \left\{ -\frac{t}{2h} \int_{C} dt_{p} \sum_{a,b} \epsilon_{a,b} \left\{ \begin{pmatrix} \hat{R}^{11}_{V_{0},pp}(t) & \hat{R}^{12}_{V_{0},pp}(t) \\ \hat{R}^{21}_{V_{0},pp}(t) & \hat{R}^{22}_{V_{0},pp}(t) \end{pmatrix} \begin{pmatrix} \sigma^{11}_{V_{0}}(t_{p}) & \delta\sigma^{12}_{V_{0}}(t_{p}) \\ \delta\sigma^{21}_{V_{0}}(t_{p}) & -\delta\sigma^{22}_{V_{0}}(t_{p}) \end{pmatrix} \right\} \right\}
\end{align*}
\]

\[
\begin{align*}
\sigma^{11}_{V_{0}}(t_{p}) &= \sigma^{(0)}_{V_{0},(\vec{x},t_{p})} + \delta\sigma^{11}_{V_{0}}(t_{p}) \quad \delta\sigma^{11}_{V_{0}}(t_{p}), \delta\sigma^{22}_{V_{0}}(t_{p}) \in \mathbb{R} \quad (5.14) \\
\sigma^{22}_{V_{0}}(t_{p}) &= -\sigma^{(0)}_{V_{0},(\vec{x},t_{p})} + \delta\sigma^{22}_{V_{0}}(t_{p}) \quad \sigma^{(0)}_{V_{0},(\vec{x},t_{p})} \in \mathbb{R} \quad (5.15) \\
\delta\sigma^{ab}_{V_{0}}(t_{p}) &= i \delta\sigma^{ab}_{V_{0}}(t_{p}) \quad (a \neq b) \quad (5.16)
\end{align*}
\]
\[
\delta \sigma_{2}^{12}(t_{p}) \in \mathbb{C} ; \quad \left( \delta \sigma_{2}^{3}(t_{p}) \right)^{*} = \delta \sigma_{2}^{12}(t_{p}) ; \quad \delta \sigma_{2}^{11}(t_{p}) = \delta \sigma_{2}^{22}(t_{p}) .
\]

In analogy to relations (3.18), (3.20), (3.62), (3.63), we continue by shifts of the total disorder-self-energy with \( \delta \sigma_{2}^{12}(t_{p}) \) and of the self-energy \( \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \) of the repulsive interaction with \( \sigma_{R_{1}}^{(0)}(\vec{x},t) \) and also include the shift with the source matrix \( J_{\psi;\vec{x}}^{a}(t_{p}) \) for the creation of the bosonic pair condensates

\[
\begin{align*}
\delta \Sigma_{\vec{x},pq}^{ab}(t) & \rightarrow \delta \Sigma_{\vec{x},pq}^{ab}(t) - 2 \delta_{p,q} \eta_{p} \delta \sigma_{2}^{ab}(t_{p}) \quad (5.18) \\
\sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) & \rightarrow \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) - \frac{1}{2} \sigma_{R_{1}}^{(0)}(\vec{x},t) \\
\delta \Sigma_{\vec{x},pq}^{ab}(t) & \rightarrow \delta \Sigma_{\vec{x},pq}^{ab}(t) - 2 \delta_{p,q} \eta_{p} \tilde{J}_{\psi;\vec{x}}^{ab}(t_{p}) \quad (5.20)
\end{align*}
\]

After several transformations we finally achieve a coherent state path integral (5.21) which only depends on the anomalous terms, determined by the matrices \( T_{pq}^{ab}(\vec{x},t) \) of the coset part \( Sp(4) \backslash U(2) \), and on the self-energy \( \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \) of the repulsive interaction. We list as starting point for a gradient expansion the relation (5.21) where the block diagonal ‘hinge’ functions \( \delta \Sigma_{\vec{x},pq}^{ab}(t) \) are still present in Gaussian factors which are to be removed after a change to the corresponding invariant measure for \( Sp(4) \backslash U(2) \). The diagonal self-energy \( \sigma_{R_{1}}^{(0)}(\vec{x},t) \) has already been absorbed by a shift into \( \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \) in the determinant and in the bilinear term with \( J_{\psi;\vec{x}}^{a}(t_{p}) \) so that its remaining in a Gaussian factor has easily been eliminated by integration in \( Z_{II}[\mathcal{J}] \) (5.21)

\[
Z_{II}[\mathcal{J}] = \exp \left\{ - \frac{1}{2 R_{II}} \sum_{p=\pm} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \left( 1 - \mu_{p}^{(II)} \right) \text{tr}_{a,b} \left[ \tilde{J}_{\psi;\vec{x}}^{ab}(t_{p}) \tilde{K} \tilde{J}_{\psi;\vec{x}}^{ba}(t_{p}) \tilde{K} \right] \right\} \\
\times \exp \left\{ - \frac{1}{4 \hbar V_{0}} \int_{C} dp \sum_{\vec{x}} \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \right\} \int d[\delta \Sigma_{\vec{x},pq}^{ab}(t)] \tilde{K} \\
\times \exp \left\{ - \frac{1}{8 R_{II}} \sum_{p,q=\pm} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \eta_{p} \left( 1 - \mu_{p}^{(II)} \right) \text{tr}_{a,b} \left[ \delta \Sigma_{\vec{x},pq}^{ab}(t) \tilde{K} \delta \Sigma_{\vec{x},pq}^{ab}(t) \tilde{K} \right] \right\} \\
\times \exp \left\{ - \frac{1}{2 R_{II}} \sum_{p=\pm} \int_{-\infty}^{\infty} dt \sum_{\vec{x}} \eta_{p} \left( 1 - \mu_{p}^{(II)} \right) \text{tr}_{a,b} \left[ \delta \Sigma_{\vec{x},pq}^{ab}(t) \tilde{K} \delta \Sigma_{\vec{x},pq}^{ab}(t) \tilde{K} \right] \right\} \\
\times \exp \left\{ - \frac{1}{2 \hbar} \sum_{\vec{x},t} \frac{J^{V}}{\Omega^{2}} \int_{-\infty}^{\infty} dt \sum_{\vec{x},t} \sum_{p,q=\pm} \sum_{a',b'=\pm} N_{\vec{x}} \delta_{a,b} \delta_{a',b'} \frac{J_{\psi;\vec{x}}^{a'}(t_{p}) \tilde{K} \tilde{T}_{\psi;\vec{x}}^{a'}(t_{p}) \tilde{K} \delta_{a,b}}{\Delta_{\psi;\vec{x}}^{a'}(t_{p})} \right. \left\} \}
\]

\( \hat{O}_{\vec{x},\vec{z}}^{ba}(t_{p},t_{p}) = \delta_{\vec{x},\vec{z}} \delta(t-t_{p}) \left( \delta(t-t_{p}) \right) \left( \tilde{H}_{p}^{a}(t_{p}) + \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \right) \left( \tilde{T}_{p}^{a}(t_{p}) \tilde{K} \tilde{T}_{p}^{a}(t_{p}) \right) + \delta_{\vec{x},\vec{z}} \delta(t-t_{p}) \left( \tilde{T}_{p}^{a}(t_{p}) \tilde{K} \tilde{T}_{p}^{a}(t_{p}) \right) \delta_{\vec{x},\vec{z}} \delta(t-t_{p}) \left( \tilde{H}_{p}^{a}(t_{p}) + \sigma_{V_{0}}^{(0)}(\vec{x},t_{p}) \right) \left( \tilde{T}_{p}^{a}(t_{p}) \tilde{K} \tilde{T}_{p}^{a}(t_{p}) \right) \right. \left. \right\} \}

\[ (5.22) \]
\[ \hat{H}_{p}^{a=1}(t_p) = \hat{h}_{p}(t_p) = -ih \frac{\partial}{\partial t_p} - \frac{\hbar^2}{2m} \Delta + u(\vec{x}) - \mu_0 \] (5.23)

\[ \hat{H}_{p}^{a=2}(t_p) = \hat{h}^T_{p}(t_p) = +ih \frac{\partial}{\partial t_p} - \frac{\hbar^2}{2m} \Delta + u(\vec{x}) - \mu_0 . \] (5.24)

The matrix \( \hat{T}_{pq}^{ab}(\vec{x}, t) \) (5.25) in \( \hat{O}_{pq}^{ab}(t'_q, t_p) \) (5.22) contains the pair condensates with matrices \( \hat{Y}_{pq}^{ab}(\vec{x}, t) \) (5.25), \( \hat{X}_{pq}(\vec{x}, t) \) (5.27) as the coset part \( Sp(4) \backslash U(2) \) of \( Sp(4) \)

\[ \hat{T}_{pq}^{ab}(\vec{x}, t) = \left( \exp \left\{- \hat{Y}_{pq}^{ab}(\vec{x}, t) \right\} \right)_{pq}^{ab} \] (5.25)

\[ \hat{Y}_{pq}^{ab}(\vec{x}, t) = \left( \begin{pmatrix} 0 \end{pmatrix}_{pq}^{11} \left( \hat{X}_{pq}(\vec{x}, t) \right)^{12} \right)_{pq}^{ab} \] (5.26)

\[ \hat{X}_{pq}(\vec{x}, t) = \left( \begin{pmatrix} -\delta c_{D,++}(\vec{x}, t) & \delta c_{D,+-}(\vec{x}, t) \\
-\delta c_{D,+-}(\vec{x}, t) & \delta c_{D,--}(\vec{x}, t) \end{pmatrix} \right) . \] (5.27)

One can extract the Goldstone modes of a spontaneous symmetry breaking \( Sp(4) \backslash U(2) \) in a gradient expansion of the operator \( \delta \hat{H}(\hat{T}^{-1}, \hat{T}) \) with the matrix \( \hat{T}_{pq}^{ab}(\vec{x}, t) \) (5.25) in \( \hat{O}_{pq}^{ab}(t'_q, t_p) \) (5.22). The relevant parameter for classifying the various terms of the gradient expansion is the number \( N_{\hat{c}} \) of discrete space points. Furthermore, one obtains special properties of the coefficients multiplying the traces of the gradients with the matrix \( \hat{T}_{pq}^{ab}(\vec{x}, t) \) in \( d = 2 \) spatial dimensions. Apart from the conformal invariance of the nonlinear sigma model in \( d = 2 \) [10] [11], the coefficients reduce to one point functions in the spatial isotropic case which allow computations by saddle point approximations for the coefficients containing the self-energy \( \sigma_{V_0}^{(0)}(\vec{x}, t_p) \).

The one dimensional case with white noise disorder can be preferably treated by transfer matrices of ensemble averaged generating functions because large fluctuations about mean field solutions may occur [29-31]. One can also try to extend the transfer matrix approach to \( d = 2 \) spatial dimensions by approximating and restricting to the lowest momentum modes perpendicular to the transfer direction [31].

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