Determining the Muon Mass in an Instructional Laboratory

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I. INTRODUCTION

Earth is continually bombarded by cosmic rays, high-speed subatomic particles produced by astrophysical processes such as supernovae. Many of the first experiments in elementary particle physics used cosmic rays as their particle source, and even today, they are used in experiments that probe the very highest particle energies. While this source of high-speed particles is omnipresent and free of charge, the flux of cosmic rays is relatively low. Thus experiments done with cosmic rays can best probe processes with high intrinsic rates.

For over forty years, the presence of high-speed muons at Earth’s surface has allowed educators to design instructional laboratory experiments that illustrate the concerns and experimental methods of elementary particle physics. These muons are the byproduct of collisions between cosmic rays and gas molecules near the top of Earth’s atmosphere. By far, the most widely adopted such experiment measures the muon’s lifetime [1–6], but others include observation of the time dilation effect [6, 7] and determination of the muon’s magnetic moment. [8] In 1964, researchers at the University of Michigan suggested that the mass of the muon could be measured through an experiment that analyzed tracks produced by muon decay within a spark chamber. [9] These researchers built a prototype system and demonstrated the feasibility of this approach. However, at that time, the necessary setup required too much specialized technical expertise for most educators to copy and the experiment was too cumbersome to perform because track images were captured on photographic film. Thus, this experiment has never been embraced by instructional lab developers. In this paper, we describe the muon mass experiment and how it can be implemented today with relative ease through the use of modern instrumentation, including CCD-based image acquisition. We go on to show how an accurate value for the muon mass can be determined from the data gleaned from this experiment using an easy to replicate software simulation program.

II. THEORY

A muon is a negatively charged elementary particle from the lepton family with a rest mass that is about 200 times that of an electron. With effectively 100% probability, a free muon decays into an electron, an electron antineutrino, and a muon neutrino with a mean lifetime of 2.2 µs. That is, the muon’s principal decay mode is \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \). Similarly, the principal decay mode of the muon’s antiparticle is \( \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \). As a result of the interaction of cosmic rays with air molecules at the top of Earth’s atmosphere, Earth’s surface is bombarded by a stream of high-speed secondary cosmic ray particles, 75% of which are a roughly equal mixture of muons and antimuons. In the vertical direction, the total flux per unit solid angle of these secondary particles is about 0.66/cm² · sr · min. Using this value, along with the empirically determined cosine-squared variation of particle flux with angle from the vertical, it can be shown that the flux of secondary cosmic-ray particles impinging on a horizontally aligned detector is on the order of 1/cm² · min.

The goal of our experiment is to stop a large number of secondary cosmic-ray muons within a spark chamber and observe each decay into an electron (to save words, we will simply describe the decay of a muon into an electron, but our discussion likewise applies to the decay of an antimuon decay into a positron). The spark chamber is a charged-particle detector that makes visible the muon’s path prior to being stopped as well as the path of the product electron after the decay. Two product neutrinos are also produced by the muon’s decay, but because they are neutral their paths are not recorded by the spark chamber. Our chamber consists of a vertical stack of aluminum plates. Most cosmic-ray muons that enter the chamber have energies greater than 1 GeV and so, although slowed by the plates, will pass completely through the chamber. However, a small population of low-energy muons will be effectively “stopped” (i.e., slowed to kinetic energies on the order of the aluminum’s ionization energy) as they pass through the sequence of plates, suppressing the time-dilation effect that allowed these short-lived particles to traverse the height of the atmosphere. Roughly half of these “stopped” muons then decay freely via \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \), while the other half are captured by atoms and undergo the process of inverse beta decay \( \mu^- + \text{Al} \rightarrow e^- + \nu_\mu + \text{Mg}^+ \) (which is undetected by the spark chamber). On the other hand, all of the stopped antimuons decay freely via \( \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \).

Consider the decay of a “stopped” (assumed stationary) muon of rest energy \( m_\mu c^2 \) into an electron, an electron antineutrino, and a muon neutrino. Applying conservation of energy to this process, we find \( m_\mu c^2 = E_e + E_{\bar{\nu}_e} + E_{\nu_\mu} \), where the energies \( E_e, E_{\bar{\nu}_e} \), and \( E_{\nu_\mu} \) of the three product particles can be taken (to a good approximation) to be entirely kinetic energy because each of these particles has a rest energy much less than \( m_\mu c^2 \). Additionally, applying conservation of momentum, we

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Our system consists of four main components: spark chamber, particle-decay detection electronics, high-voltage pulsing electronics, and CCD-based image acquisition (see Fig. 1). A short description of each of these components follows; more detailed descriptions are given in the Appendix.

The spark chamber consists of a stack of 21 disk-shaped aluminum plates. Each plate has thickness $t_p = 0.9525 \text{ cm} (= 3/8 \text{ inch})$ and radius $R = 7.62 \text{ cm} (= 3 \text{ inch})$ and is separated from its neighboring plates by “spark gaps” of thickness $t_g = 0.635 \text{ cm} (= 1/4 \text{ inch})$ filled with a noble gas (in our case, neon). The odd-numbered plates are electrically grounded, while the even-numbered plates are connected to a high-voltage, high-current pulser. When a charged particle moves within the chamber, it ionizes the noble gas along its path, leaving behind a trail of ionized atoms and liberated electrons. If the pulser can be triggered to apply sufficiently high voltage (typically 6000 V) to the even-numbered plates more quickly than the mean ion-electron recombination time (on order of 5 $\mu$s), sparks will form in the gaps between the plates along the low-resistance ionized track, making visible the trajectory of the moving charged particle.

The decay of a cosmic-ray muon within the spark chamber is detected through the use of three scintillation detectors stacked vertically about the chamber. Each detector consists of a horizontally aligned light-tight 20 cm $\times$ 20 cm $\times$ 2 cm plastic scintillator coupled to a blue-light sensitive photomultiplier tube (PMT). When a high-speed charged muon passes through such a detector, it causes the scintillator material to fluoresce and the fluorescent photons are detected and converted to a short ($\approx 5 \text{ ns}$), negative-going electrical signal by the attached PMT. This signal is amplified and then passed through a discriminator which converts the PMT signal into a short digital pulse. Two of these detectors (henceforth called $X$ and $Y$) are placed above the chamber, separated by a vertical distance of approximately 1 meter, while the third detector ($Z$) is placed slightly below the chamber. When a muon with (close to) normal incidence enters the chamber from above, and then subsequently decays within the chamber, digital pulses will be produced by $X$ and $Y$, while a digital pulse will not be produced by $Z$, assuming the decay-product electron is stopped within the chamber. Thus, if the digital outputs of the three discriminators are input to a logic unit that performs the coincidence/anticoincidence operation $X \cdot Y \cdot \overline{Z}$ ($X$ AND $Y$ AND not $Z$), the output of this logic unit signals the muon’s decay within the spark chamber.

The $X \cdot Y \cdot \overline{Z}$ output of the logic unit is connected to the input of a high-speed pulser (“driver”). This driver unit produces the proper voltage pulse to trigger a high-voltage, high-current switch (“thyratron”), which then causes a bank of capacitors to discharge, applying a (negative) pulse of several thousand Volts to the even-numbered plates of the spark chamber.

Finally, a video camera focused on the spark chamber is configured to send its analog video signal to the video input of a frame grabber board within the expansion slot.
of a PC, while the $X \cdot Y \cdot Z$ output of the logic unit is connected to the board’s trigger input. The frame grabber is controlled via a LabVIEW program, which acquires and stores a single image of the chamber whenever the logic unit detects a muon decay event.

IV. EXPERIMENTAL PROCEDURE

With high voltage applied to all three scintillation detector PMTs, each PMT output (after passing through a preamplifier) is input to a separate channel of a quad constant fraction discriminator (CFD). Using an oscilloscope, the width of NIM digital pulses output by each CFD channel (in response to PMT input signal pulses) is adjusted to be 50 ns. In our setup, each detector’s cross-sectional area $A = 20 \text{ cm} \times 20 \text{ cm} = 400 \text{ cm}^2$, so the rate of digital pulses produced by secondary cosmic ray particles on each channel is about $(1/\text{cm}^2 \cdot \text{min})(400 \text{ cm}^2)(1/60 \text{ s}) \approx 7/\text{s}$.

The digital-pulse outputs of the CFD’s three channels $X$, $Y$, and $Z$ are input into the logic unit. The TTL output of this logic unit is connected to the high-speed driver, which triggers the spark chamber operation. Proper functioning of the setup can be verified by programming the logic unit to perform the logic operation $X \cdot Y$, so that the spark chamber is triggered for any charged particle (i.e., not just those that decay) that passes into the chamber through the top two detectors. The expected rate of such $X \cdot Y$ coincidence is found as follows: $X$ and $Y$ each have area $A = 400 \text{ cm}^2$, and are separated by distance $d = 100 \text{ cm}$. Thus, at each point on $Y$, the solid angle subtended by $X$ is $\Delta \Omega \approx \frac{400 \text{ cm}^2}{(100 \text{ cm})^2} = 0.04 \text{ sr}$, and so the expected rate at which vertically downward secondary cosmic ray particles (75% of which are muons and antimuons) will pass through both detectors producing coincidence signals is $(0.66/\text{cm}^2 \cdot \text{sr} \cdot \text{min})(400 \text{ cm}^2)(0.04 \text{ sr}) = 10/\text{min}$.

To execute a data run, the logic unit is configured to perform the logic operation $X \cdot Y \cdot Z$, the video camera is focused on the spark chamber along with its reflection in a nearby mirror angled at $45^\circ$ (to obtain an orthogonal viewpoint of the spark tracks), and a LabVIEW-based image acquisition program is started. Each time the logic unit triggers the spark chamber, it also sends a TTL trigger pulse that triggers the frame grabber board to acquire the video camera’s current frame and save it as a jpeg file on the computer’s hard drive with a unique file name. The experiment is run under computer control in a darkened room overnight, typically resulting in about 100 saved images per hour.

After a data run, all the images are inspected. Fig. 2 shows a sample image, which consists of a direct and a right-angle (produced by the $45^\circ$-angled mirror) view of the chamber. For each decay image, we construct two lines, one delineating the path of the incoming muon, and the other marking the path of the product electron. The intersection of these lines defines the location at which the muon decayed. Once the point of decay is established, one then counts the number of sparks $n_S$ created by the electron as it is brought to rest (or else escapes from the chamber) during its traverse through the aluminum plates. For reasons described in the next section, we analyze only those images for which the muon decay occurs within the chamber’s “fiducial volume,” which we define to be a cylinder in the top half of the chamber with a radius $R_{\text{fid}} = R/2$ measured from the chamber’s central axis. After this selection process, one is left with a total of $N_{\text{exp}}$ images that record muon decays (typically $N_{\text{exp}} \approx 100$). Using these data, we then determine the number of product particles $N(n_S)$ that produce $n_S$ sparks, normalize these values to find the probability $P(n_S) \equiv N(n_S)/N_{\text{exp}}$, and then plot $P(n_S)$ vs. $n_S$ as in Figure 3.

V. ANALYSIS OF DATA

The product electron’s initial kinetic energy $E_e$ (i.e., its energy immediately after the muon decay) can be determined by analyzing its path through the chamber’s sequence of aluminum plates and spark gaps. Assuming a ballpark value of $m_\mu c^2 \approx 100 \text{ MeV}$, Eq. (1) indicates that $E_e$ will be in the range from 0 to 50 MeV. As electrons with initial energies in this range are slowed during their travel through aluminum, the two most significant energy-loss processes are ionization and bremsstrahlung,[11] whose stopping powers $S = -dE/dl$, i.e., energy loss per unit length, are approximately a con-

FIG. 2: This is figure 2.
stated $S_o$ and proportional to instantaneous energy $E$, respectively. In particular, the total stopping power for an electron in aluminum [12, 13] is given by
\[
\frac{-dE}{dr} = S_{\text{ioniz}} + S_{\text{brems}} = S_o + \frac{E}{X_o} \tag{3}
\]
where $S_o = 5.09$ MeV/cm and $X_o = 8.9$ cm is the "radiation length" for aluminum. Note that $S_{\text{ioniz}} = S_{\text{brems}}$ at the "critical energy" $E_{\text{crit}} = S_o X_o = 45$ MeV, and that below (above) this critical energy, ionization (bremsstrahlung) become the larger of the two energy-loss processes. From Eq. (3), it is easy to show that the path length $l$ in aluminum required to bring an electron with initial energy $E_e$ to rest is
\[
l = X_o \ln \left[1 + \frac{E_e}{E_{\text{crit}}}\right] \tag{4}
\]

The ideal experimental procedure would be as follows: For each observed muon decay, measure the length $l$ of the product electron’s path through the chamber, and then use Eq. (4) to calculate the electron’s initial energy $E_e$. Repeating this process for all the observed decays, determine the probability $P(E_e)$ of a product particle being produced with energy $E_e$, then fit the resulting plot of $P(E_e)$ vs. $E_e$ to Eq. (2), and extract a value for the muon mass from the fit. Because we obtain two orthogonal views of the electron’s path, in principle, we could carry out this ideal procedure. However, in practice, it proves very difficult to determine the length of the electron’s path in three dimensions from our data, and so we take another tack.

As mentioned in the previous section, we simply characterize the path of a product electron by counting the number of sparks $n_S$ it creates while moving in the chamber. Clearly, there is not a one-to-one correspondence between $n_S$ and $E_e$, e.g., with the same initial energy $E_e$, an electron passing through the aluminum plates at normal incidence will produce more sparks than one passing through the plates at a large incident angle (because of the increased path length per plate). However, by writing a straightforward Monte Carlo simulation, we can account for all of the possible $n_S$ an electron with $E_e$ can produce and so predict the distribution $P(n_S)$, which can then be compared with the experimentally determined distribution.

One issue that we must accurately address in our simulation is the sub-population of product electrons that escape the chamber before they are completely stopped (e.g., electrons created near the chamber’s edge and moving at large angles relative to the chamber’s axis). Because the number of sparks created by an escaping electron is quite sensitive to its direction of travel, to model this type of event accurately requires our simulation to explore a large parameter space. To minimize the influence of escaping electrons on the outcome of our simulation, we restrict our data set in the following way so that this sub-population of electrons is as small as possible: First, we note from Eq. (4) that the most energetic ($\approx 50$ MeV) product electron will require about 7 cm of aluminum to be stopped. Since each plate is approximately 1 cm thick, about 7 plates will be required to stop such an electron that is directed straight downward. Also, we find empirically that a spark is created only if the electron’s path is inclined up to about $40^\circ$ relative to the chamber’s axis (which is the direction of the applied electric field). Thus, by considering muon decays that occur only with the cylindrical “fiducial volume” defined to be in upper half of chamber with radius $R_{fid}$ equal of half the plate radius $R$, most of the spark-producing electrons will interact with enough aluminum to be stopped before escaping the chamber.

For our simulation, because charged particles lose negligible energy when passing through the chamber’s spark gaps, we assume each muon and resulting product electron are stopped within the interior of a plate. At each muon-decay site, the product electron is assumed to emanate downward, located a radial distance $r$ ($< R_{fid}$) from the chamber’s axis and directed at polar angle $\theta$ relative to the chamber’s (downward) axis and azimuthal angle $\phi$. If the decay occurs a (perpendicular) distance $z_o$ above the plate’ bottom surface and the electron’s path length for being stopped in aluminum is $l$, then the number of sparks produced is given by
\[
n_S = 1 + \text{floor} \left[\frac{l \cos \theta - z_o}{t_p}\right] \tag{5}
\]
where the floor function return the highest integer less than or equal to its argument. Also, if $l > l_{esc}$, the electron will escape the chamber before being stopped, where
\[
l_{esc} \approx \frac{-r \cos \phi + \sqrt{r^2 \cos^2 \phi + (R^2 - r^2)^2}}{\sin \theta} \cdot \frac{t_p}{t_p + t_g} \tag{6}
\]
Hence, the algorithm for our simulation is as follows: Assume a value for $m_\mu c^2$. For the product electron emanating from a muon-decay site, randomly assign $z_0$, $r$, $\theta$ and $\phi$ to be within the range $0—t_p$, $0—R/2$, $0—40^\circ$ and $0—2\pi$, respectively, and also assign the electron to have an initial kinetic energy $E_e$ within the range of $0—m_\mu c^2/2$. Then, using Eq. (4) and (6), calculate $l$ and $l_{esc}$. If $l > l_{esc}$, then the electron’s path length in the chamber is found by setting $l = l_{esc}$. Finally, the number of sparks produced by this electron is found from Eq. (5) and, based on Eq. (2), the number of times this event is expected to occur is assigned the value

$$N(n_S) = r(m_\mu c^2 E_e)^2 (3 - 4E_e/m_\mu c^2)$$

(7)

The factor of $r$ is included because the differential area at radius $r$ is $dA = 2\pi r dr$ and so the number of muon-decays at differing radii is expected to be proportional to a geometric scaling factor of $r$. After iterating this procedure $N_{sim}$ times (typically $N_{sim} = 10000$), the accumulated $N(n_S)$ values are normalized to become the probability $P(n_S)$, and this predicted distribution $P(n_S)$ is compared with that determined experimentally as in Figure 3.

Finally, we run our simulation several times, where we assume a different value for $m_\mu c^2$ each time. For each run, we compare the predicted $P_{sim}(n_S)$ with the experimentally determined $P_{exp}(n_S)$ by calculating a chi-square value

$$\chi^2 = \sum_{n_S} \left[ \frac{P_{sim}(n_S) - P_{exp}(n_S)}{P_{exp}(n_S)} \right]^2$$

(8)

After concluding all of the runs, we plot $\chi^2$ vs. $m_\mu c^2$, and then obtain our “best-fit” value for the muon mass from the minimum of this plot. As shown in Fig. 4, the “best-fit” value derived from our data is $m_\mu c^2 = 95$ MeV.

VI. APPENDIX: CONSTRUCTION DETAILS

Spark Chamber: The design of the spark chamber is an adaptation of a spark chamber constructed by A.M. Sachs in the mid-1960s.[15] The chamber is constructed of 3/8” thick, round aluminum plates. The chamber consists of 20 gaps and 21 plates, and is illustrated in Fig. 4. The 19 inner plates are 6” in diameter; the top and bottom plates are 7” in diameter. The conductive aluminum plates of the chamber are separated by 6” outer diameter, 5 1/4” inner diameter plexiglass spacer rings, 1/4” in thickness. It is essential that the chamber be completely leak tight, and O-rings provide a simple cost effective means of sealing the chamber. Therefore, the gas seals between the aluminum plates and plexiglass spacers are made with O-rings. The plates of the chamber have grooves to accommodate 1/8” thick, 5 1/2” inner diameter O-rings. The 19 inner plates have 1/4” holes to allow gas to flow freely in the chamber. The chamber is held together by four threaded rods; these rods are electrically shielded from the pulsed plates with 1/2” O.D., 1/4” I.D. plexiglass sleeves.

The chamber is filled with pure neon at slightly over atmospheric pressure (approximately 1.1 atm) to ensure that any leakage in the chamber will be outward, not inward. This prevents any atmospheric gas impurities from leaking into the chamber. Electronegative gas impurities such as oxygen have detrimental effects on the chambers operation.

Scintillation Detectors: Each scintillation detector consists of a light-tight 20 cm × 20 cm × 2 cm plastic scintillator (Saint-Gobain BC-408) coupled to a blue-light sensitive photomultiplier tube (Hamamatsu R7400). The detectors were purchased as assembled units (Saint-Gobain 8X8.8BC408/.5L-X). The PMTs of all three detectors are connected in parallel to a high-voltage supply (Stanford Research Systems PS310) and operated at 1000 V. The signal output of each PMT is passed through a fast (1 ns rise time) noninverting preamplifier (Ortec VT120C) with fixed 20× gain.

Coincidence/Anti-Coincidence Electronics: Each of the amplified PMT signals from the X, Y, and Z scintillation detectors is sent to an input of a multi-channel constant fraction discriminator (CFD) with shaping delay set to 4 ns. To avoid incorrect transient logic states, cable delays from detector to CFD are arranged to ensure that $Z$ arrives to the CFD before $X$ and $Y$. We use the quad 200 MHz Ortec 935, a NIM-bin module whose negative-NIM logic output pulse widths are adjustable (typical value in our experiment is 50 ns). The $X$, $Y$, and Z NIM logic output signals from the CFD are input to each of two separate channels of a quad 4-Input Logic Unit (Ortec CO4020), where each channel is configured to perform the logical operation $X \cdot Y \cdot Z$. When this condition is met, each channel outputs a TTL pulse with width of 800 ns, one of which triggers the high-speed driver, while the other triggers the frame grabber board.

High-Voltage Pulsing Electronics: Triggerable high-voltage pulses are applied to the spark chamber plates.
using the circuit shown in Fig. 4. Inside a homemade “capacitor bank,” several thousand (typically 6000) Volts from a 10 kV high-voltage supply (Bertan 230-10R) are applied to the anode of a hydrogen thyratron (Perkin Elmer HY-6) as well as the “high-voltage” side of each $C = 500 \text{ pF}$ capacitor (Sprague 20 DK-T5) in a parallel array of capacitors. The “low-voltage” side of each capacitor is connected to a unique plate of the spark chamber as well as a current limiting $R = 3 \text{ MΩ}$ resistor, whose opposite end is grounded. This circuit is triggered by a commercially available Thyratron Driver (Perkin Elmer TM-27, modified to trigger on 1.5 V).[16] When the Driver receives a TTL pulse from the Logic Unit, it outputs a 2-µs wide pulse of amplitude 800 V with a rise time of less than 150 ns. This trigger pulse is passed into the homemade pulser box, where it is applied to the grid of the hydrogen thyratron, driving the thyratron into conduction for 2 µs and causing the “low-voltage” sides of the capacitors to drop to large negative voltage for a time period of approximately $\tau = RC = (3 \text{ MΩ})(500 \text{ pF}) = 1.5\text{ms}$.

Image Acquisition System: A monochrome progressive scan CCD camera head (Sony XC-55) with zoom lens (Navitar Zoom 7010) is focused on the spark chamber directly as well as its perpendicular reflection in a mirror angled at 45°. With the camera’s shutter speed at 1/30 s, the analog video signal is sent to a frame grabber board (National Instruments PCI-1405) plugged into a PCI expansion slot of a PC. The TTL output from the Logic Unit is connected the frame grabbers TRIGGER input. Under the control of a LabVIEW based program, when each TRIGGER pulse is received, the current frame acquired by the camera is stored under a unique filename as a jpeg file on the PC’s hard drive.

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