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Stability and Hopf bifurcation periodic orbits in delay coupled Lotka-Volterra ring system. (English) Zbl 1426.37042
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Summary: In this paper, we consider a class of delay coupled Lotka-Volterra ring systems. Based on the symmetric bifurcation theory of delay differential equations and representation theory of standard dihedral groups, properties of phase locked periodic solutions are given. Moreover, the direction and the stability of the Hopf bifurcation periodic orbits are obtained by using normal form and center manifold theory. Finally, the research results are verified by numerical simulation.

MSC:
37G15 Bifurcations of limit cycles and periodic orbits in dynamical systems
37C10 Dynamics induced by flows and semiflows
34K20 Stability theory of functional-differential equations
34K13 Periodic solutions to functional-differential equations

Keywords:
Lotka-Volterra system; Hopf bifurcation; periodic solution; normal form

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