The spin-current blockade in Luttinger liquid

Yao Yao and Chang-Qin Wu
Surface Physics Laboratory (National Key Laboratory) and Department of Physics, Fudan University, Shanghai 200433, China
E-mail: cqw@fudan.edu.cn

Abstract. We investigate the spin-charge separation in the Luttinger liquid with a defective bond of controllable spin-dependent hopping by using the adaptive time-dependent density-matrix renormalization group method. The model is the non-half-filled Hubbard chain with this special bond, which is found to block the relevant spin current with little influence on charge current. We have considered the pure spin and charge currents induced by the voltage biases that are applied to the ideal leads attached at the ends of this Hubbard chain. The phenomenon is so robust that one may utilize it to observe the spin-charge separation directly.

It has been definitely established that, in one-dimensional (1d) strongly correlated systems, instead of quasi-particle excitations the collective excitations, such as spinon and holon, play the essential roles and they will separate from each other, [1] which is believed to dominate the 1d transport [2]. However, there are plenty of difficult to study this spin-charge separation (SCS) both in experiment and theory. Although a number of experimental works have sought to observe the phenomenon,[3] they all have their own drawbacks. Rapid progress in ultracold atomic gas experiments[4] makes it possible to see SCS in a new context.[5] Meanwhile, due to the limitations of existing theoretical methods, the discussion in the past has been limited to a few simple cases; an example is the significant work by Kane and Fisher[6] on scaling properties of tunneling through a spin-symmetric point impurity in a fermion system. Hence more powerful methods are needed to compute transport properties beyond scaling and to treat a general interacting Hamiltonian.

Recently, the adaptive time-dependent density-matrix renormalization group (t-DMRG)[7, 8] was developed by combining the DMRG method[9] with quantum information concepts. At the same time, other real-time evolution methods within DMRG[10, 11, 12, 13] were also proposed. By using of these numerical methods, there have been investigations on transport properties in 1d strongly-correlated or impurity systems, including spin-1/2 chains,[14] Bose-Hubbard model,[15] and conductance analysis,[16] An interesting result that partly motivates our study was the study of SCS by Kollath, Schollwoeck, and Zwerger.[17] In conventional treatment with the bosonization method, only low energy excitations were considered. The above study goes beyond the low energy excitation spectrum by considering the evolution of a "big" (multiparticle) wave packet that shows the SCS phenomenon.[17]

In a recent work[18], we have proposed to consider a non-half-filled Hubbard chain in which one special bond has controllable spin-dependent electronic hoppings, motivated by recently fast-development of optical lattices of ultracold atoms, which shows a controllable spin-current blockade phenomenon. In this paper, we show more details on the pure spin and charge currents under a spin-dependent voltage bias applied at the leads attached at the ends of the chain. It’s
The Hamiltonian of the leads, $H_{\text{lead}} \equiv H_L + H_R$ and

$$H_\alpha = -t_0 \sum_{i,\sigma \in \alpha} (c_{i,\sigma}^\dagger c_{j,\sigma} + \text{h.c.}) - \sum_{i,\sigma \in \alpha} V^\alpha (n_{i,\uparrow} \pm n_{i,\downarrow}),$$

with $\alpha = L$ or $R$ and $j$ only taking $i - 1$ or $i + 1$ for $\alpha = L$ or $R$. The sign $\pm$ taking $+(-)$ means the voltage bias inducing a pure charge (spin) current. And the interaction between system and leads $H_{\text{int}} = -t_0 \sum_{i,\sigma} (c_{L,i,\sigma}^\dagger c_{i,\sigma} + c_{L+1,i,\sigma}^\dagger c_{L,i,\sigma} + \text{h.c.})$. The voltage biases $V \equiv V^L - V^R$ applied at the two leads turns on at $t = 0$, so that the relative charge or spin currents appear gradually at the same time and reach constant values finally. In the calculation, we always take $V = 0.1t_0$. 

**Figure 1.** The realization of the special bond of controllable spin-dependent electronic hopping in an optical lattice. The rotation of the left spin-down atoms reduces the spin-down atom hopping at the special bond.
the second non-half-filled Hubbard chain, with a bond of controllable electronic hopping. It’s confirmed only spin-up electrons can hop across the special bond. A virtual state. Then it is clear that spin current is blocked at the special bond when intermediate state, so electronic hoppings for both spins are necessary for the process via a H current. Based on $H$ and that for the usual Hubbard model, $H$ is responsible for the low-energy excitations of the model in Eq. (1). It is well known that the hopping term $t_{ij}$ is responsible for the spin-exchange process that is necessary for an oscillation of incoming currents for a moment, which will finally achieve a steady state. In Fig. 2(a), it shows that, the outgoing charge current is about 70 percent of the incoming current. However, in Fig. 2(b) no remarkable outgoing current is observed. We argue that, we have waited enough time for the outgoing spin current, since the incoming current has been decreasing, which means the reflecting current has transported back to the left end of Hubbard chain.

We argue that, in the following, the spin-current blockade we observe here only happens in a strongly correlated system. It is different from the “spin blockade” effect,[19] which is merely transport back to the left end of Hubbard chain. Meanwhile, the currents undergo the second scattering at the special bond, leading to an oscillation of incoming currents for a moment, which will finally achieve a steady state. In Fig. 2(a), it shows that, the outgoing charge current is about 70 percent of the incoming current. However, in Fig. 2(b) no remarkable outgoing current is observed. We argue that, we have waited enough time for the outgoing spin current, since the incoming current has been decreasing, which means the reflecting current has transported back to the left end of Hubbard chain.

We show in Fig. 2 the pure charge and spin currents at various times for different Hubbard $U$ in the calculations induced by the voltage bias. The incoming and outgoing currents are defined as the corresponding currents at the left and right interfaces according to Eq. (2), respectively. At $U = 0$, i.e., the non-interaction case, it is shown that, both incoming and outgoing currents tend to the same, since half of the total current (spin up) transmit through the special bond, and the other half (spin down) reflect. Finally, steady transport is reached at about $t = 40(\hbar/t_0)$.

For the strong interaction case, i.e., $U = 8$, the incoming current is less than non-interaction case, due to the scattering between lead and Hubbard chain. Meanwhile, the currents undergo the second scattering at the special bond, leading to an oscillation of incoming currents for a moment, which will finally achieve a steady state. In Fig. 2(a), it shows that, the outgoing charge current is about 70 percent of the incoming current. However, in Fig. 2(b) no remarkable outgoing current is observed. We argue that, we have waited enough time for the outgoing spin current, since the incoming current has been decreasing, which means the reflecting current has transported back to the left end of Hubbard chain.

We argue that, in the following, the spin-current blockade we observe here only happens in a strongly correlated system. It is different from the “spin blockade” effect,[19] which is merely spin-related Coulomb blockade. To understand the phenomenon we observed above, we consider the large $U$ limit of the Hubbard-like model in Eq. (1), which leads to the so-called $t - J$ model $H_{t-j} = H_t + H_J$, where $H_t$ is the hopping term in Eq. (1) and

$$H_J = -\frac{1}{U} \sum_{i j k s s'} t_{i j}^s t_{k s'}^s c_{i s}^\dagger c_{j s'}^\dagger n_{j s'} n_{j s} c_{j s'} c_{k s'}$$

with $t_{i j}^s \equiv t_{i,j+1}^s \delta_{i,j-1} + t_{i-1,j}^s \delta_{i,j+1}$.[20] This model works on the space that has projected out all configurations with at least one doubly occupied site for a less-than-half filling system and is responsible for the low-energy excitations of the model in Eq. (1). It is well known that the hopping term $H_t$ is responsible for the charge excitations while $H_J$ controls the spin excitations, and that for the usual Hubbard model, $H_J$ corresponds to a Heisenberg spin chain. Writing

$$\vec{S}_i \cdot \vec{S}_j = S_i^+ S_j^- + S_i^- S_j^+ + \frac{1}{2} (S_i^+ S_j^- + S_i^- S_j^+),$$

the second term is responsible for the spin-exchange process that is necessary for an $S_z$ spin current. Based on $H_J$ in Eq.(4) from the large-$U$ expansion, we argue that there is a virtual intermediate state, so electronic hoppings for both spins are necessary for the process via a virtual state. Then it is clear that spin current is blocked at the special bond when $t_{ii} = 0$ since only spin-up electrons can hop across the special bond.

In summary, we have shown the spin and charge transport in the Luttinger liquid, say, a non-half-filled Hubbard chain, with a bond of controllable electronic hopping. It’s confirmed
that the spin current can be blocked by adjusting the spin-dependent electronic hopping at this special bond while charge current passes through the bond freely. This finding may provide a new possibility to observe the SCS directly in experiments.

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