Water entry behaviours of hemisphere-head projectiles with high velocity

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Abstract. The water-entry behaviours of projectiles with hemisphere-head were studied experimentally and theoretically, focusing on projectile dynamics and drag coefficient. Based on equivalent cone theory proposed by Baldwin, the equation of drag coefficient was developed to describe the resistance of hemispherical head projectiles from the time when the projectile contacts the water to the time the hemispherical head is submerged. A set of experimental apparatus was set up to record the water-entry process by high-speed photography, the variation of resistance during water-entry was obtained experimentally. Compared the acceleration calculated by conventional method, new method and experimental results, the new method is more consistent with the experimental value.

1. Introduction

As a kind of important underwater explosive weapon, air-launched water ammunition has always occupied an important position in the sea war because of its low cost, great power and flexible use. This kind of ammunition has the characteristics of high entry velocity and complex entry trajectory. In order to improve the reliability of the ammunition, the ballistic characteristics of projectile in the process of water-entry should be studied deeply. Water-entry of projectile is a complex physical process involving the coupling of solid, liquid and gas, and the projectile motion features in water are quite different from those in air. Due to the development of the weapons industry and the space industry, the issue of water-entry, especially in high velocity, has received significant attention. Early research dates back to 1929, when Von Karman [1] developed several mathematical methods to describe the impact of water upon rigid bodies. He was the first to use added mass instead of fluid action to analyze the impact of water-entry, and proposed the added mass method to calculate the impact load of water. Using the momentum conservation law to deduce the formula of impact load into water. Over the next hundred years, there was a lot of research undertaken by researchers, such as May, Schnitzer, Abelson, Waugh, and others [2-6]. Because of the complexity of water-entry, finding the motion law of projectiles during water-entry mainly rely on experimental study, and the velocity attenuation law is closely related to the drag coefficient. Therefore, to establish a simple and practical mathematical model to describe the motion characteristics of projectiles during water entry is particularly important. In recent studies, Guo et al [7–9] studied high-speed projectile striking into water horizontally and the effects of projectile nose shapes on velocity attenuations and drag coefficient. They considered the effect of initial velocity and head shape on the drag coefficient and established an empirical formula [9]. The premise of all these researches are assuming drag coefficient is a constant over the water-entry process. Indeed, the drag coefficient of the projectile is approximately a constant in the sailing process after it enters the water.
However, due to the change of the wetting surface during the process from touching the water surface to being submerged, the drag coefficient will change greatly. Actually in the early years, Baldwin [10] tested various nose shapes of projectiles, especially the cone body, and derived a set of equations to calculate the total resistance during the vertical water entry of projectiles with arch nose. In Baldwin’s theory, drag coefficient at the initial stage of water entry is well considered.

In this paper, drag coefficient is analyzed during the water-entry process of hemisphere-head projectile and a new calculation model for drag coefficient was established theoretically. A set of experimental apparatus was set up to record the water-entry process by high-speed photography. The resistance of water during water-entry process was studied experimentally and theoretically.

2. Theoretical analysis
In the following analysis, the water-entry process means the process from projectiles touch the water surface to it is submerged. Previous report [11] showed that at an entry velocity of <8 m/s, the surface forces are important and the behavior of the missile depended greatly on its surface condition. Instead, the projectile velocity considered in this work is much higher (>50 m/s), which make the effect of surface condition negligible in the subsequent analyses. Considering an object with an initial velocity of \( v_0 \) penetrating into water along a straight trajectory in the +x direction (figure 1). The motion equation of the object can be described by Newton’s second law while ignoring gravity and buoyancy:

\[
\sum F_x = \frac{d^2 x}{dt^2} = \frac{dV_b}{dt} = \sum F_x = -\frac{1}{2} \rho \omega A_0 C_d V_b^2
\]

(1)

where \( m_b \) and \( x \) denotes the object mass and penetration axis, respectively. \( V_b \) is the penetration velocity of the body, \( t \) is time, \( F_x \) is the drag force, and \( A_0 \) is defined as the projected frontal area of bodies, \( C_d \) represents the drag coefficient.

In this paper, the resistance to the head can be obtained from Newton’s second law

\[
F = -\frac{1}{2} \rho \omega A_0 C_d V_b^2
\]

(2)

Obviously, the value of \( C_d \) is important to the analysis. In conventional resistance calculation, the drag coefficient is usually regarded as a constant, it is impossible to describe the resistance during water-entry process accurately, especially for the projectiles with arch nose-shape. In order to calculate the total resistance during the vertical water entry of projectiles with arch nose, Baldwin [12] developed the method of equivalent cone. In Baldwin’s theory, the equivalent cone is composed of the cross section of the arch nose and water to a conical surface tangent to the surface of a pointed arch. This paper extends the approach in Baldwin to determine the value of \( C_d \) during the water entry of hemispherical head. The geometrical relationship of the system is showed in figure 2. Similar to arch nose, the hemispherical head will also make surface uplift of water, so there will form an equivalent water surface (ews) higher than the original water surface (ows). The first equivalent cone forms by ews, and the
second equivalent cone forms by ews.

\[ \text{C}_{\text{t}} = \text{C}_{\text{tr}} + \text{C}_{\text{st}} + \text{C}_{\text{sf}} \]  

(3)

Where \( \text{C}_{\text{t}} \) is transient drag coefficient, \( \text{C}_{\text{st}} \) is quasi steady drag coefficient, and \( \text{C}_{\text{sf}} \) is surface friction drag coefficient. \( \text{C}_{\text{t}} \) is calculate by the first equivalent cone, \( \text{C}_{\text{st}} \) and \( \text{C}_{\text{sf}} \) are calculate by the second equivalent cone.

\[ \text{C}_{\text{t}} = \frac{2r}{\rho \times \text{A}_h} \frac{dm}{dx_2} = 2.56(\frac{2r}{d})^2 \tan \left( \frac{\theta_1}{2} \right) \]  

(4)

Where \( m \) is added mass, \( x \) is the penetration distance, \( r_1 \) is radius of the first equivalent cone, and \( \theta_1 \) is the aspect angel of the first equivalent cone.

\[ \text{C}_{\text{st}} = (\frac{2r}{d})^2 \text{C}_{\text{st}}' \]  

(5)

Where \( \text{C}_{\text{st}}' = 0.008088 + 0.122 \theta_2 + 0.4364 \theta_2^2 - 0.2811 \theta_2^3 + 0.0587 \theta_2^4 \), and \( \theta_2 \) is the aspect angel of the second equivalent cone.

\[ \text{C}_{\text{sf}} = \frac{A_w}{A_h} \text{C}_f \]  

(6)

Where unit surface friction coefficient \( \text{C}_f = 0.0031 \), reference area \( A_h = \pi r^2 \), and the wetted cross area \( A_w = \pi r^2 \).

The water wetting factor can be determined as

\[ \omega = \frac{x}{x_e} = 0.9985 - 0.1953 \theta + 0.07047 \theta^2 - 0.01528 \theta^3 \]  

(7)

Where \( \theta \) is the vertex angle of water-entry body, for the hemispherical head \( \theta = \pi/2 \).

According to the geometrical relationship showed in figure 2, \( \theta_1 \), \( \theta_2 \), and \( r_1 \) can be obtained as a function of \( x \)
\[
\theta_i = \pi - 2 \arccos\left(1 - \frac{x}{r}\right) \tag{8}
\]

\[
\theta_i = \pi - 2 \arccos\left(1 - \frac{x}{r \omega}\right) \tag{9}
\]

\[
r_i = (2rx - x^2)^{\frac{1}{2}} \tag{10}
\]

Substituting equations (4)-(10) into equation (3)

\[
C_d = C_{d_{\alpha}}(x) + C_{d_v}(x) + C_{d_t}(x) \quad (x \leq r) \tag{11}
\]

Equation (11) shows that \(C_d(x)\) is displacement-dependent. When \(x>r\), \(C_{d_t}\), \(C_{d_v}\), \(C_{d_t}\) will become constants, so will \(C_d\).

3. Experiments

3.1. Experiment design

The schematic diagram of the test system is shown in figure 3. The experiment involved shooting a hemisphere head projectile into a water tank and recording the whole process with a high-speed camera. The required speed is 50 m/s-100 m/s, the front of the high-speed camera needs to be protected by bullet-proof glass.

![Figure 3. The schematic diagram of the test system.](image)

3.2. Ballistic gun

Dynamic loading device used in this experiment is 40 mm smoothbore gun. Due to the larger specimen caliber and lower required speed compared with cannonball, the specimen is directly put at the muzzle, driven by explosive gas. The acceleration and initial velocity of the projectiles are adjusted by tuning the mass of the propellant in the cartridge.

3.3. High-speed camera and water tank

A Phantom digital high-speed camera was deployed to record the fluid, the dynamic of projectile upon the entry. The sampling rate was adjusted to 10000 FPS, one frame was taken every 100 μs, this sampling rate of exposure and resolution rate can satisfy experiment study. The transparent, impact-resistant water tank was made of polycarbonate composite material, and there was no cover on the tank. There were two holes in the front and the back pieces of the polycarbonate and sealing by two pieces of diaphragms. The projectile was fired into the front hole to penetrate the fluid horizontally and left the tank from the
back hole either by itself or by manual unloading. Before each run, water was replenished and the tank was sealed with fresh diaphragms.

3.4. Projectile
The projectile with hemisphere head was made of 2A12 aluminum (150 mm in diameter, 3 kg mass). To reduce the mass, the hemispherical head is a shell structure. In order to facilitate acceleration, a nylon tail rod is connected to the projectile with a thread. There is a hole in the tail rod, high-density steel balls are stuffed into the holes to ensure the accuracy of the mass. Figure 4 is the sketch of the assembled projectile.

![Figure 4. The sketch of the assembled projectile.](image)

3.5. Water-entry process

![Figure 5. The water-entry process taken by high-speed camera. (a) -0.4 ms, (b) 0 ms, (c) 3.2 ms, (d) 5.5 ms, (e) 9.9 ms and (f) 14.6 ms.](image)

Figure 5 shows the typical pictures taken by the high-speed camera. The projectile hits the water surface at a certain initial speed, and the zero point of time is when the projectile touches the water surface. In figure 5(c), it can be seen that the obvious tail emission and cavitation are formed, and then
the cavitation is further expanded until the projectile penetrates out. Due to the limited volume of the water tank, the closure of the cavitation cannot be observed.

3.6. Experimental data analysis

Figure 6 demonstrates the velocity attenuation over time of the projectile, and figure 7 shows the experimental penetration distance with time. This paper is concerned about the movement at the initial stage of water entry. According to the observation, the projectile in this experiment has submerged the head into water within 1ms, so the collection points in the first 1 ms are denser. Integrating the curve shown in figure 6 to obtain the curve of acceleration over time, and then the resistance of water entry at the corresponding collection point can be calculated. The acceleration variations over time are shown in figure 8. As can be seen from the curve in figure 8, the impact at the initial stage of water entry is quite severe, and then the impact will decrease rapidly.

Figure 6. Velocity attenuation over time of the projectile in experiment.

Figure 7. The experimental penetration distance with time in experiment.

Figure 8. Acceleration variations over time in experiment.

Figure 9. Acceleration variations over penetration distance in experiment.

Figure 9 shows the deceleration variations with penetration distance. The diameter of the hemispherical head used in the experiment was 0.15 m, and when the penetration depth reached 0.075 m, the hemispherical head was completely immersed in water. In figure 9, it can be seen that the curve starts to flatten after \( x = 0.075 \) m. This shows that the water entry of projectiles with arch nose can be divided into two stages. The first stage is from the projectile contact with the water surface to the head...
fully immersed in the water, and the second stage is the projectile sailing in the water.

4. Discussion

4.1. Theoretical calculations

4.1.1. Conventional method. In conventional method, the drag coefficient is regarded as a constant during a water-entry process. This method is usually used in engineering calculation to calculate the resistance of a rigid object when it is running stably in the flow field. The value of the drag coefficient is obtained from a lot of engineering experience and is closely related to wetting surface. For hemisphere-head, the drag coefficient is 0.5 under subsonic velocity.

4.1.2. New method. As a matter of fact, to the hemispherical head projectile, at the initial stage of water entry, wetting surface is changeable. So the drag coefficient can’t be regarded as a constant at this stage. According to the result of section 2, the model factors and calculated values are shown in table 1. The values in the table are based on the parameters of the projectile used in the experiments. The relationship between the drag coefficient and penetration distance is shown in figure 10. The drag coefficient calculated by the new method begins to approximate to a constant after the head is immersed, and is close to the empirical engineering value.

Table 1. Model calculation.

| Factors | Calculated model |
|---------|-----------------|
| $\theta$, rad | $\theta_1 = \pi - 2 \arccos\left(1 - \frac{x}{r}\right)$ $= \pi - 2 \arccos\left(1 - \frac{x}{0.075}\right)$ |
| $r$, m | 0.075 |
| $A_0$, m$^2$ | 0.0177 |
| m, kg | 3 |
| $\theta_1$, rad | $\theta_2 = \pi - 2 \arccos\left(1 - \frac{x}{r\omega}\right)$ $= \pi - 2 \arccos\left(1 - \frac{x}{0.06}\right)$ |
| $\omega$ | $\omega = \frac{x}{x} = 0.9985 - 0.1953\theta + 0.07047\theta^2 - 0.01528\theta^3 = 0.807$ |
| $r_1$, m | $r_1 = \sqrt{2rx - x^2} = \sqrt{0.15x - x^2}$ |
| $C_{dt}$ | $C_{dt} = \frac{2}{\rho \pi d A_0} \int dm = 2.56\left(\frac{2r_1}{d}\right)^2 \tan \frac{\theta_1}{2} = 455.1r_1^2 \tan \frac{\theta_1}{2}$ |
| $C_{dx}$ | $C_{dx} = 0.008088 + 0.122\theta + 0.4364\theta^2 - 0.2811\theta^3 + 0.0587\theta^4$ |
| $C_{dx}'$ | $C_{dx}' = \frac{\partial C_{dx}}{\partial \theta} = 0.0031(\frac{r}{r})^2 = 0.0031(\frac{r_1}{r})^2 = 0.551r_1^2$ |
| $C_{df}$ | $C_{df} = \frac{C_f}{A_0} = \frac{0.0031(\frac{r_1}{r})^2 = 0.551r_1^2}{r_1}$ |
| $C_d$ | $C_d = C_{dr} + C_{ds} + C_{df} = 455.1r_1^2 \tan \frac{\theta_1}{2} + 177.8r_1^2 C_{ds}' + 0.551r_1^2$ |
After immersion
Before immersion
Drag coefficient
Penetration distance (m)
Conventional method
New method

Figure 10. The relationship between the drag coefficient and penetration distance obtained by new and conventional method.

4.2. Compared with the experimental data
According to Newton’s second law,

\[ a = \frac{F}{M} = -\frac{\rho A C D V^2}{2m} \]

(12)

The theoretical acceleration can be calculated by substituting the experimental data into equation (12). The results calculated by traditional methods which regards drag coefficient as a constant are also turned out. The results of experiment and theoretical calculations are shown in figure 11. The new model shows good agreements with experimental result, in contrast, the conventional model is unable to describe the deceleration process of the projectile at the initial stage of water entry. Nevertheless, both the traditional model and the new model can well describe resistance of projectiles after the head immersed in water.

5. Conclusions
In this paper, the water entry process of projectile with hemisphere head has been studied experimentally and theoretically, especially the variation of drag coefficient during entry. The current work, within its scope, allows the following main conclusions to be drawn:
- By extent of the approach in Baldwin (1977), a new model of drag coefficient has been developed.
- It can be seen from the test data that when the projectile head is not fully immersed in water at the initial stage of entry, the drag coefficient will change over penetration depth.
- The comparison between experiments and theoretical analysis proves the deficiency of the conventional calculation method and the validity of the new model.

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