Josephson effect in point contacts between "f-wave" superconductors.

R. Mahmoodi (1), S.N. Shevchenko(2), Yu.A. Kolesnichenko(1,2)

(1) Institute for Advanced Studies in Basic Sciences,
45195-159, Gava Zang, Zanjan, Iran

(2) B.Verkin Institute for Low Temperature Physics and
Engineering,
National Academy of Sciences of Ukraine,
47 Lenin Ave., 61103 Kharkov, Ukraine

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A stationary Josephson effect in point contacts between triplet superconductors is analyzed theoretically for most probable models of the order parameter in $U\text{P}t_3$ and $S\text{r}_2\text{RuO}_4$. The consequence of misorientation of crystals in superconducting banks on this effect is considered. We show that different models for the order parameter lead to quite different current-phase dependences. For certain angles of misorientation a boundary between superconductors can generate the parallel to surface spontaneous current. In a number of cases the state with a zero Josephson current and minimum of the free energy corresponds to a spontaneous phase difference. This phase difference depends on the misorientation angle and may possess any value. We conclude that experimental investigations of the current-phase dependences of small junctions can be used for determination of the order parameter symmetry in the mentioned above superconductors.

I. INTRODUCTION.

Triplet superconductivity, which is an analogue of superfluidity in $^3\text{He}$, was firstly discovered in a heavy-fermion compound $U\text{P}t_3$ more than ten years ago [1,2]. Recently, a novel triplet superconductor $S\text{r}_2\text{RuO}_4$ was found [3,4]. In these compounds, the triplet pairing can be reliably determined, for example, by Knight shift experiments [5,6]; however the identification of a symmetry of the order parameter is much more difficult task. A large number of experimental and theoretical investigations done on $U\text{P}t_3$ and $S\text{r}_2\text{RuO}_4$ are concerned with different thermodynamic and transport properties, but the precise order parameter symmetry remains still to be worked out yet (see, for example, [7,10–12], and original references therein).

Calculations of the order parameter $\hat{\Delta}(\hat{k})$ in $U\text{P}t_3$ and $S\text{r}_2\text{RuO}_4$ as a function of the momentum direction $\hat{k}$ on the Fermi surface is a very complex problem. Some general information about $\hat{\Delta}(\hat{k})$ can be obtained from a symmetry of a normal state: $G_{\text{spin-orbit}} \times \tau \times U(1)$, where $G_{\text{spin-orbit}}$ represents the point group with inver-
sion; $\tau$ is the time-inversion operator, and $U(1)$ is the group of gauge transformation. A superconducting state breaks one or more symmetries. In particular, a transition to the superconducting state implies an appearance of the phase coherence corresponding to breaking of the gauge symmetry. According to Landau theory [13] of second order phase transitions, the order parameter is transformed only on irreducible representations of the symmetry group of the normal state. Conventional superconducting states have a total point symmetry of the crystal and belong to the unitary even representation $A_{1g}$. In unconventional superconductors this symmetry is broken. The parity of a superconductor with inversion symmetry can be specified using the Pauli principle. Because for triplet pairing the spin part of the $\hat{\Delta}$ is a symmetric second rank spinor, the orbital part has to belong to the odd representation. In a general case the triplet paring is described by the order parameter of the form

$$\hat{\Delta}(\hat{k}) = id(\hat{k}) \hat{\sigma}_2,$$

where the vector $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$, and $\hat{\sigma}_i$ are Pauli matrices in the spin space. A vector $d(\hat{k}) = -d(-\hat{k})$ in spin space is frequently referred to as an order parameter or a gap vector of the triplet superconductor. This vector defines the axis along which the Cooper pairs have zero spin projection. If $d$ is complex, the spin components of the order parameter spontaneously break time-reversal symmetry.

Symmetry considerations reserves for the order parameter considerable freedom in the selection of irreducible representation and its basis functions. Therefore in many papers (see, for example: [12, 14, 16]) authors consider different models (so-called scenarios) of superconductivity in $UPt_3$ and $Sr_2RuO_4$, which are based on possible representations of crystallographic point groups. The subsequent comparison of theoretical results with experimental data makes it possible to conclude on the symmetry of the order parameter.

In real crystalline superconductors there is no classification of Cooper pairing by angular momentum ($s$-wave, $p$-wave, $d$-wave, $f$-wave pairing, etc.). However these terms are often used for unconventional superconductors meaning that the point symmetry of the order parameter is the same as one for the corresponding representation of $SO_3$ symmetry group of isotropic conductor. In this terminology conventional superconductors can be referred to as $s$-wave. For example, the "$p$-wave" pairing corresponds to the odd two-dimensional representation $E_{1u}$ of $D_{6h}$ point group or $E_u$ representation of $D_{4h}$ point group. The order parameter for these representations has the same symmetry, as for the superconducting state with angular momentum $l = 1$ of Cooper pairs in an isotropic conductor. If the symmetry of $\hat{\Delta}$ can not be formally related to any irreducible representation of $SO_3$ group, these states are usually referred to as hybrid states.

Apparently, in crystalline triplet superconductors the order parameter has more complex dependence on $\hat{k}$ in comparison with well known $p$-wave order parameter for superfluid phases of $^3He$. The heavy-fermion superconductor $UPt_3$ belongs to the hexagonal crystallographic
point group \( (D_{6h}) \), and it is most likely that the pairing state belongs to \( E_{2u} \) ("f-wave" state) representation. A layered perovskite material \( Sr_2RuO_4 \) belongs to the tetragonal crystallographic point group \( (D_{4h}) \). Initially the simplest "p-wave" model based on \( E_u \) representation was proposed for the superconducting state in this compound \[8,9\]. However this model was inconsistent with available experimental data, and later \[10,11\] other "f-wave" models of pairing state were proposed.

Theoretical studies of specific heat, thermal conductivity, ultrasound absorption for different models of triplet superconductivity show considerable quantitative differences between calculated dependences \[7,10,11,16\]. The Josephson effect is much more sensitive to dependence of \( \Delta \) on the momentum direction on the Fermi surface. One of the possibilities to form a Josephson junction is to create a point contact between two massive superconductors. A microscopic theory of the stationary Josephson effect in ballistic point contacts between conventional superconductors was developed in Ref. \[17\]. Later this theory was generalized for a pinhole model in \(^3\)He \[18,19\] and for point contacts between "d-wave" high-\( T_c \) superconductors \[20,21\]. It was shown that current-phase dependences for the Josephson current in such systems are quite different from those of conventional superconductors, and states with a spontaneous phase difference become possible. Theoretical and experimental investigations of this effect in novel triplet superconductors seem to be interesting and enable one to distinguish among different candidates for the superconducting state.

In Ref. \[22\] the authors study the interface Andreev bound states and their influence on the Josephson current between clean "f-wave" superconductors both self-consistently (numerically) and non-self-consistently (analytically). The temperature dependence of the critical current is presented. However in that paper there is no detailed analysis of the current-phase dependences for different orientations of the crystals in the superconducting banks.

In this paper we theoretically investigate the stationary Josephson effect in a small ballistic junction between two bulk triplet superconductors with different orientations of the crystallographic axes with respect to the junction normal. In Sec.II we describe our model of the junction and present the full set of equations. In Sec.III the current density in the junction plane is analytically calculated for a non-self-consistent model of the order parameter. In Sec.IV the current-phase dependences for most likely models of "f-wave" superconductivity in \( UPt_3 \) and \( Sr_2RuO_4 \) are analyzed for different mutual orientations of the banks. We end in Sec.V with some conclusions.

II. MODEL OF THE CONTACT AND FORMULATION OF THE PROBLEM.

We consider a model of a ballistic point contact as an orifice with a diameter \( d \) in an impenetrable for electrons partition between two superconducting half spaces (Fig.1). We assume that the contact diameter \( d \) is much
FIG. 1. Scheme of a contact in the form of an orifice between two superconducting banks, which are misorientated on an angle $\alpha$.

larger than the Fermi wavelength and use the quasiclassical approach. In order to calculate the stationary Josephson current in point contact we use "transport-like" equations for $\xi$-integrated Green functions

$$\sum_{m} \left( V (\mathbf{k}, \mathbf{k}') g_2 (\mathbf{k}',\mathbf{r},\epsilon_m) \right), \quad (5)$$

where $V (\mathbf{k}, \mathbf{k}')$ is a potential of pairing interaction; $\langle ... \rangle$ stands for averaging over directions of an electron momentum on the Fermi surface; $N (0)$ is the electron density of states.

Solutions of Eqs. (1), (5) must satisfy the conditions for Green functions and vector $\mathbf{d}$ in the banks of superconductors far from the orifice:

$$\langle \hat{\mathbf{g}} (\pm \infty) \rangle = \langle \hat{\mathbf{d}} \rangle, \quad (6)$$

$$\mathbf{d} (\pm \infty) = \mathbf{d}_{1,2} (\mathbf{k}) \exp \left( \mp \frac{i \phi}{2} \right), \quad (7)$$

where $\phi$ is the external phase difference. Eqs. (1) and (5) have to be supplemented by the boundary continuity conditions at the contact plane and conditions of reflection at the interface between superconductors. Below we assume that this interface is smooth and electron scattering is negligible.

III. CALCULATION OF THE CURRENT DENSITY.

The solution of Eqs. (1) and (5) allows us to calculate the current density:
\[ j(\mathbf{r}) = 2\pi e T v_F N(0) \sum_m \langle \hat{k} g_1(\hat{k}, \mathbf{r}, \varepsilon_m) \rangle. \quad (8) \]

We consider a simple model of the constant order parameter up to the surface. The pair breaking and the scattering on the partition and in the junction are ignored. This model can be rigorously found for the calculations of the current density \( (8) \) in ballistic point contacts between conventional superconductors at zero approximation with small parameter \( d/\xi_0 \) (\( \xi_0 \) is the coherence length) \[17\]. In anisotropically paired superconductors the order parameter changes at distances of the order of \( \xi_0 \) even near a specular surface \[25,26\]. Thus for point contacts between "d-wave" superconductors \[20\] and pinholes in \( ^3He \) \[27\]. It was also shown in Ref. \[22\] that for the contact between "f-wave" superconductors and pinholes in \( ^3He \) \[27\]. It was also shown in Ref. \[22\] that for the contact between "f-wave" superconductors there is also good qualitative agreement between self-consistent and non-self-consistent solutions (although, of course, quantitative distinctions, take place).

In a ballistic case the system of 16 equations for functions \( g_i \) and \( g_i \) can be decomposed on independent blocks of equations. The set of equations which enables us to find the Green function \( g_1 \) is

\[ iv_F \hat{k} \nabla g_1 + (g_3 \mathbf{d} - g_2 \mathbf{d}^*) = 0; \quad (9) \]

\[ iv_F \hat{k} \nabla g_- + 2i (\mathbf{d} \times g_3 + \mathbf{d}^* \times g_2) = 0; \quad (10) \]

\[ iv_F \hat{k} \nabla g_3 - 2i \varepsilon_m g_3 - 2g_1 \mathbf{d}^* - i \mathbf{d} \times g_- = 0; \quad (11) \]

\[ iv_F \hat{k} \nabla g_2 + 2i \varepsilon_m g_2 + 2g_1 \mathbf{d} - i \mathbf{d} \times g_- = 0; \quad (12) \]

where \( g_- = g_1 - g_4 \). The Eqs. \[11-12\] can be solved by integrating over ballistic trajectories of electrons in the right and left half-spaces. The general solution satisfying the boundary conditions \[3\] at infinity is

\[ g_1^{(n)} = \frac{i \varepsilon_m}{\Omega_n} + i C_n \exp (-2s \Omega_n t); \quad (13) \]

\[ g_-^{(n)} = C_n \exp (-2s \Omega_n t); \quad (14) \]

\[ g_2^{(n)} = \frac{-2C_n \mathbf{d}_n - \mathbf{d}_n \times C_n \exp (-2s \Omega_n t) - \mathbf{d}_n}{-2s \Omega_n + 2\varepsilon_m}; \quad (15) \]

\[ g_3^{(n)} = \frac{-2C_n \mathbf{d}_n^* + \mathbf{d}_n^* \times C_n \exp (-2s \Omega_n t) - \mathbf{d}_n^*}{-2s \Omega_n - 2\varepsilon_m}; \quad (16) \]

where \( t \) is the time of the flight along the trajectory, \( sgn(t) = sgn(z) = s; \eta = sgn(v_z); \Omega_n = \sqrt{\varepsilon_m^2 + |\mathbf{d}_n|^2} \).

By matching the solutions \[13-16\] at the orifice plane \( t = 0 \), we find constants \( C_n \) and \( C_n \). Index \( n \) numbers left \( (n = 1) \) and right \( (n = 2) \) half-spaces. The function \( g_1(0) = g_1^{(1)}(-0) = g_1^{(2)}(+0) \), which determines the current density in the contact is

\[ g_1(0) = \]
we plot the current-phase dependence for different below mentioned scenarios of "f-wave" superconductivity for two different geometries corresponding to different orientations of the crystals to the right and to the left at the interface (see, Fig.1):

(i) The basal ab-plane to the right is rotated about c-axis by the angle \( \alpha; \hat{c}_1 || \hat{c}_2 \).

(ii) The c-axis to the right is rotated about the contact axis (y-axis in Fig.1) by the angle \( \alpha; \hat{b}_1 || \hat{b}_2 \).

Further calculations require a certain model of the vector order parameter \( d \).

IV. CURRENT-PHASE DEPENDENCE FOR DIFFERENT SCENARIOS OF "F-WAVE" SUPERCONDUCTIVITY.

The model which has been successful to explain properties of the superconducting phases in \( \text{UPt}_3 \) is based on the odd-parity \( E_{2u} \) representation of the hexagonal point group \( D_{6h} \) for the strong spin-orbital coupling with vector \( d \) locked along \( c \) axis of the lattice [10]:

\[ \textbf{d} = \Delta_0 \hat{z} \left[ \eta_1 Y_1 + \eta_2 Y_2 \right], \]

where \( Y_1 = k_z (k_x^2 - k_y^2) \) and \( Y_2 = 2k_x k_y k_z \) are the basis function of the representation [1]. The coordinate axes \( x, y, z \) here and below are chosen along the crystallographic axes \( \hat{a}, \hat{b}, \hat{c} \) as at the

1Strictly speaking, in crystals with a strong spin-orbit coupling the spin is a "bad" quantum number, but electron states are twice degenerated and can be characterized by pseudospins.
left at Fig. 1. This model describes the hexagonal analog of spin-triplet \( f \)-wave pairing. For the high-temperature A-phase \( (\eta_2 = 0) \) the order parameter has an equatorial line node and two longitudinal line nodes. In the low-temperature B-phase \( (\eta_2 = i) \) or the axial state:

\[
d = \Delta_0 2k_z (k_x + ik_y)^2,
\]

the longitudinal line nodes are closed and there is a pair of point nodes. The B-phase \( (22) \) breaks the time-reversal symmetry. The function \( \Delta_0 = \Delta_0 (T) \) in Eq.22 and below describes the dependence of the order parameter \( d \) on temperature \( T \) (carrying out numerical calculations we assume \( T = 0 \)).

Other candidates to describe the orbital states, which imply that the effective spin-orbital coupling in \( U \text{Pt}_3 \) is weak, are unitary planar state

\[
d = \Delta_0 k_z (k_x^2 - k_y^2) + 2k_z k_y,
\]

(or \( d = \Delta_0 (Y_1, Y_2, 0) \)) and non-unitary bipolar state

\[
d = \Delta_0 (Y_1, iY_2, 0) \] [7]. In Fig. 2 we plot the Josephson current-phase dependence \( j_J(\phi) = j_y(\phi = 0) \) calculated from Eq.19 for both the axial (with the order parameter given by Eq.22) and the planar (Eq.23) states for a particular value of \( \alpha \) under the rotation of basal \( ab \)-plane to the right (the geometry (i)). For simplicity we use the spherical model of the Fermi surface. For the axial state the current-phase dependence is just the slanted sinusoid and for the planar state it shows a "\( \pi \)-state". The appearance of \( \pi \)-state at low temperatures is due to the fact that different quasiparticle trajectories contribute to the current with different effective phase differences \( \zeta(\hat{k}) \) (see Eqs.19 and 21) [19]. Such a different behavior can be a criterion to distinguish between the axial and the planar states, taking advantage of the phase-sensitive Josephson effect. Note that for the axial model the Josephson current formally does not equal to zero at \( \phi = 0 \). This state is unstable (does not correspond to a minimum of the Josephson energy), and the state with a spontaneous phase difference (value \( \phi_0 \) in Fig. 2), which depends on the misorientation angle \( \alpha \), is realized.

The remarkable influence of the misorientation angle \( \alpha \) on the current-phase dependence is shown in Fig. 3 for the axial state in the geometry (ii). For some values of \( \alpha \) (in Fig. 3 it is \( \alpha = \pi/3 \) ) there are more than one state, which correspond to minima of the Josephson energy (\( j_J = 0 \) and \( dj_J/d\phi > 0 \)).

Calculated \( x \) and \( z \)-components of the current, which
are parallel to the surface, $J_S(\phi)$ are shown in Fig. 4 and Fig. 5 for the same axial state in the geometry (ii). Note that the tangential to the surface current as a function of $\phi$ is not zero when the Josephson current (Fig. 3) is zero. This spontaneous tangential current (see also in Ref. [22]) is due to the specific "proximity effect" similar to spontaneous current in contacts between "$d$-wave" superconductors [20,28]. The total current is determined by the Green function, which depends on the order parameters in both superconductors. As a result of this, for nonzero misorientation angles the parallel to the surface current can be generated. In the geometry (i) the tangential current for both the axial and planar states at $T = 0$ is absent.

The first candidate for the superconducting state in $Sr_2RuO_4$ was "$p$-wave" model [8]

$$ d = \Delta_0 \hat{z} \left( \hat{k}_x + i\hat{k}_y \right). \quad (24) $$

Recently [11,12] it was shown that the pairing state in $Sr_2RuO_4$, most likely, has lines of nodes. It was suggested that this can occur if the spin-triplet state belongs to a non trivial realization of the $E_u$ representation of $D_{4h}$ group, either $B_{2g} \otimes E_u$ [12] or $B_{1g} \otimes E_u$ [11] symmetry:

$$ d = \Delta_0 \hat{z} \hat{k}_x \hat{k}_y \left( \hat{k}_x + i\hat{k}_y \right), \quad \text{for } B_{2g} \otimes E_u \text{ symmetry}; \quad (25) $$

$$ d = \Delta_0 \hat{z} \left( \hat{k}_x^2 - \hat{k}_y^2 \right) \left( \hat{k}_x + i\hat{k}_y \right), \quad \text{for } B_{1g} \otimes E_u \text{ symmetry}. \quad (26) $$

Note that models (24,26) of the order parameter sponta-
neously break time-reversal symmetry.

Taking into account a quasi-two-dimensional electron energy spectrum in Sr$_2$RuO$_4$, we calculate the current numerically using a model of the cylindrical Fermi surface. The Josephson current for the hybrid "f-wave" model of the order parameter Eq.(26) is compared to the p-wave model (Eq.24) in Fig.6 (for $\alpha = \pi/4$). Note that the critical current for the "f-wave" model is several times smaller (for the same value of $\Delta_0$) than for the "p-wave" model. This different character of the current-phase dependencies enables us to distinguish between the two states.

In Figs. 5 and 6 we present the Josephson current and the tangential current for the hybrid "f-wave" model for different misorientation angles $\alpha$ (for the "p-wave" model it equals to the zero). Just as in Fig.2 for the "f-wave" order parameter (22), in Fig.7 for the hybrid "f-wave" model (25) the steady state of the junction with zero Josephson current corresponds to the nonzero sponta-

FIG. 6. Josephson current versus phase $\phi$ for hybrid "f-wave" and "p-wave" states in the geometry (i); $\alpha = \pi/4$.

FIG. 7. Josephson current versus phase $\phi$ for the hybrid "f-wave" state in the geometry (i) for different $\alpha$.

neous phase difference if misorientation angle $\alpha \neq 0$.

V. CONCLUSION.

We have considered the stationary Josephson effect in point-contacts between triplet superconductors. Our consideration is based on models with "f-wave" symmetry of the order parameter belonging to the two-dimensional representations of the crystallographic symmetry groups. It is shown that the current-phase dependences are quite different for different models of the
order parameter. Because the order parameter phase depends on the momentum direction on the Fermi surface, the misorientation of the superconductors leads to spontaneous phase difference that corresponds to the zero Josephson current and to the minimum of the weak link energy. This phase difference depends on the misorientation angle and can possess any values. We have found that in contrast to "p-wave" model, in the "f-wave" models the spontaneous current may be generated in a direction which is tangential to the orifice plane. Generally speaking this current is not equal to zero in the absence of the Josephson current. We demonstrate that the study of a current-phase dependence of small Josephson junction for different crystallographic orientations of banks enables one to judge on the applicability of different models to the triplet superconductors $UPt_3$ and $Sr_2RuO_4$.

It is clear that such experiments require very clean superconductors and perfect structures of the junction because of pairbreaking effects of non-magnetic impurities and defects in triplet superconductors. The influence of single impurities and roughness of interface in the plane of the contact, which may essentially decrease the critical current of the junction, will be analyzed in our next paper.

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