Magne-to-resistance quantum oscillations in a magnetic two-dimensional electron gas

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Magneto-transport measurements of Shubnikov-de Haas (SdH) oscillations have been performed on two-dimensional electron gases (2DEGs) confined in CdTe and CdMnTe quantum wells. The quantum oscillations in CdMnTe, where the 2DEG interacts with magnetic Mn ions, can be described by incorporating the electron-Mn exchange interaction into the traditional Lifshitz-Kosevich formalism. The modified spin splitting leads to characteristic beating pattern in the SdH oscillations, the study of which indicates the formation of Mn clusters resulting in direct anti-ferromagnetic Mn-Mn interaction. The Landau level broadening in this system shows a peculiar decrease with increasing temperature, which could be related to statistical fluctuations of the Mn concentration.

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I. INTRODUCTION

At low temperatures, the longitudinal resistivity of metallic systems exhibits quantum oscillations when submitted to a sufficiently high magnetic field. These so-called Shubnikov-de Haas (SdH) oscillations are, in particular, characteristic of two-dimensional electron gases (2DEG) confined in semiconducting structures and appear at low magnetic fields prior to the development of the quantum Hall effect in clean systems. The analysis of the magnetic field and temperature dependence of the SdH oscillations\textsuperscript{1,2,3} provides a valuable information on the quantized density of states (e.g. the Landau level shape\textsuperscript{4}), and the associated quantum life times\textsuperscript{5,6,7}. While the most detailed studies were historically undertaken in high mobility GaAs-based 2DEG, other 2D systems of high quality have slowly emerged, enabling us to explore the influence of different parameters such as the valley\textsuperscript{8,9} and spin degrees of freedom\textsuperscript{10,11,12} as well as the effect of magnetism in these systems\textsuperscript{13–16}.

In this work, we present an investigation of the SdH oscillations in a high quality “magnetic 2DEG” in a diluted magnetic semiconductor, CdMnTe\textsuperscript{14,15}. CdMnTe is grown by substituting a small fraction of Cd atoms by Mn in the original (non-magnetic) CdTe II-VI semiconductor. The high quality of these systems was recently demonstrated by the observation of the fractional quantum Hall effect\textsuperscript{12,17,18}. A systematic comparison of the SdH oscillations in both systems is made here to identify the particular effects related to the presence of magnetic Mn ions. The SdH oscillations in CdTe exhibits a behavior similar to the widely studied GaAs-based 2DEG; a field/temperature independent Landau level broadening characteristic of long range scattering mechanism, and an exchange-enhanced spin gap leading to spin-split oscillations (a doubling of the frequency) above a critical magnetic field. In CdMnTe, the SdH oscillation exhibit an additional beating pattern with nodes where the oscillations have a vanishing amplitude. We show that this behavior is a consequence of the giant Giant Zeeman splitting (GZS) resulting from the $s-d$ exchange interaction between electrons and the $S = 5/2$ Mn spins\textsuperscript{19}. The SdH characteristics can be well-described by incorporating the electron-Mn exchange interaction into the traditional Lifshitz-Kosevich formalism\textsuperscript{20}. For a good quantitative description, the formation of Mn pair clusters with direct Mn-Mn antiferromagnetic interactions, which reduce the average Mn spin polarization, has to be considered. Another peculiarity of the magnetic 2DEG is a decrease of the Landau Level broadening with increasing temperature, together with an increase in the broadening with increasing magnetic field. This suggests a connection between the Landau level broadening and the Mn spin polarization, as expected in the presence of local fluctuation in the Mn concentration.

II. SAMPLES

The non-magnetic CdTe sample consist of a 20 nm-wide CdTe quantum well (QW), modulation-doped with iodine on one side, and embedded between Cd$_{0.72}$Mg$_{0.26}$Te barriers. The magnetic sample consist of a 21.1 nm-wide Cd$_{1-x}$Mn$_x$Te QW. The average Mn concentration of $\sim 0.3\%$ is introduced by delta-doping within 7 separate monolayers among the 65 CdTe monolayers composing the QW. The samples, in form of 1.5×6 mm rectangles, were fitted with electrical contacts in a Hall bar-like configuration. Experiments have been carried out in a $^3$He/$^4$He dilution refrigerator inserted into a superconducting magnet. A standard, low frequency ($\approx 10$ Hz) lock-in technique has been applied for the resistance measurements. The samples were illuminated by using the 514 nm-line of a Ar$^+$ laser to increase the
2DEG mobility. The laser illumination was limited to \( \sim 50 \mu W/cm^2 \) but permanently maintained as it was found to assure the most stable conditions over the different experimental runs (the resulting heating effects on the 2DEG were estimated directly from the magneto-resistance). A special attention has been paid to use slow sweeps not to affect the amplitude of fast SdH oscillations. Under our experimental conditions, the CdTe 2DEG density was \( 4.5 \times 10^{11} \text{ cm}^{-2} \) (corresponding to a Fermi energy of \( E_F = 10.8 \text{ meV} \)), with a low temperature mobility of \( \mu = 2.6 \times 10^6 \text{ cm}^2/\text{Vs} \). The CdMnTe 2DEG density was \( 4.0 \times 10^{11} \text{ cm}^{-2} \) (corresponding to a Fermi energy of \( E_F = 9.6 \text{ meV} \)), with a low temperature mobility of \( \mu = 1.2 \times 10^7 \text{ cm}^2/\text{Vs} \). The effective mass and the \( g \)-factor of electrons in CdTe, \( m_e = 0.1 m_0 \) and \( |g_e| = 1.6 \), were determined by far infrared magneto-absorption and Raman scattering spectroscopy.

III. EXPERIMENTAL RESULTS

The magneto-resistance of the 2DEG in CdTe and CdMnTe QWs is shown for four selected temperatures in Fig. 1 (a) and (b), respectively. For the sake of comparison we plot the data as a function of the filling factor \( \nu = B_1/B \), where \( B_1 \) is the magnetic field at filling factor \( \nu = 1 \) (\( B_1 = 18.8 \text{ T} \) in the CdTe and \( B_1 = 16.5 \text{ T} \) in the CdMnTe QWs). The magneto-resistance in the CdTe QW exhibits the well-known SdH oscillations, which amplitude increases (decreases) with magnetic field (temperature). At low magnetic fields, before spin-splitting is observed the minima of the longitudinal resistance \( R_{xx} \) correspond to the situations where the Fermi energy lies between two Landau levels (LL), in a minimum of the total density of states \( G_{tot} \). When the Fermi energy lies in the center of a Landau level, maxima in \( R_{xx} \) are observed. Above a critical magnetic field, electron-electron exchange interactions lift the Landau level spin degeneracy, which leads to alternating odd and even filling factors minima in the SdH oscillations (visible e.g. for \( \nu < 30 \) at \( T = 177 \text{ mK} \) in Fig. 1 (a)).

The magneto-resistance in the CdMnTe QW also exhibits SdH oscillations, as can be seen in Fig. 1 (b). However, an additional beating pattern is observed and “nodes” can be distinguished in the SdH amplitude, as previously observed in Ref. 15. At low magnetic fields, the SdH amplitude tends to zero in the region of the nodes, while at higher fields, they are characterized by a local minimum of the SdH amplitude associated with a doubled SdH oscillation frequency. The presence of a strong electron-manganese exchange interaction gives rise to a Giant Zeeman splitting (GZS) in the 2DEG, which grows quickly as the localized Mn spins are polarized by the applied magnetic field. The GZS saturates when the Mn spin polarization has reached its maximum value, for a magnetic field of typically \( \sim 0.5 \text{ T} \) at low temperatures. This “Brillouin-like” strong field dependence of the GZS, compared to the smaller linear increase of the cyclotron gap (\( \hbar \omega_c = 1.16 \text{ meV/T} \)), leads to the rather unusual situation where the spin gap \( \Delta_s \) can be several times larger than the cyclotron gap. As the magnetic field increases, the conditions \( \Delta_s = n \hbar \omega_c \), where \( n \) is a (decreasing) integer, are successively satisfied. These magnetic field-dependent commensurability of the spin and cyclotron gaps leads to a maximized density of states when the Fermi level lies in the center of coinciding (degenerate) levels. When the spin-resolved Landau levels are all equally-spaced (\( \Delta_s = (n + 1/2) \hbar \omega_c \)), the maximum density of states is a factor of two smaller. The observed SdH beating are therefore a direct manifestation of the GZS in magnetic 2DEGs. We note that the “nodes” conditions (\( \Delta_s = (n + 1/2) \hbar \omega_c \)) should also be accompanied by a doubling of the SdH frequency, sim-

![FIG. 1: Longitudinal magneto-resistance in (a) CdTe and (b) CdMnTe QW for four selected temperatures. The data are shifted along the y-axis for clarity and the units of (a) 100 Ω and (b) 200 Ω are marked along the left side by a double arrow. Minima corresponding to odd, even and alternating odd filling factors are labeled “o”, “e” and “o/e”, respectively. The nodes (condition \( \Delta_s = (n + 1/2) \hbar \omega_c \)) in the beating pattern of magneto-resistance of CdMnTe QW are labeled by integer index \( n \).](image)
ilar to the one observed at high enough magnetic fields in CdTe (Fig. 1 (a)). This is indeed observed for small \( n \) as indicated by the down arrows in Fig. 1 (b). In the lower magnetic field regime, where \( \hbar \omega_c \leq \Gamma \), the density of state modulation is so small that the nodes conditions result in a disappearance of the SdH.

IV. THEORETICAL MODEL

In order to quantitatively describe the data in both CdTe and CdMnTe, we have derived the formula describing the SdH oscillation in the case of an arbitrarily large spin splitting \( \Delta s \). We have used the Kubo-Greenwood expression\(^8,19\)

\[
\sigma(B) = \int_{-\infty}^{\infty} \sigma(E) \left( -\frac{\partial n_{FD}}{\partial E} \right) dE
\]

(1)
to calculate the conductivity \( \sigma(B) \) of electrons, where \( n_{FD} \) is the Fermi-Dirac distribution. The conductivity \( \sigma(E) \) calculated within the Drude model yields \( \sigma(E) = e^2 n_{eff} / m_e \tau_{tr} \omega_c^2 \) in the diffusion limit (\( \omega_c, \tau_{tr} \gg 1 \)), where \( n_{eff} \) is an effective carrier concentration contributing to \( \sigma(E) \), \( m_e \) the effective mass of electrons, \( \tau_{tr} \) the transport life-time and \( \omega_c \) the cyclotron frequency.

The effective carrier concentration \( n_{eff} \) is proportional to the density of states at Fermi level \( G(E_F) \) and can be written as \( n_{eff} \propto G(0) = G_0 = m_e / 2 \pi \ell_0^2 \), where \( G_0 \) is the zero-field density of states and \( \delta G(E) \) is the modulation of \( G(E) \) for \( B > 0 \) \( T \) \( (G(E) = G_0 + \delta G(E)) \) and \( \delta G(E) \ll G_0 \).

The relative change of the conductivity can then be written as \( \left| \frac{\sigma_{xx}(B) - \sigma_0}{\sigma_0} \right| = 2 \pi \left| \frac{G(E)}{G_0} \right| \) where the exponent \( p \) depends on the type of scattering \((p = 1 \text{ for long-range scattering and } p = 2 \text{ for short range scattering where } 1/\tau_{tr} \propto \left| \frac{G(E)}{G_0} \right| \) \) \( G(E) \) has been modeled as a sum of either Lorentzian (Eq 2) or Gaussian (Eq 3) Landau levels:

\[
G(E) = \frac{m_e \hbar \omega_c}{2\pi \hbar^2 \pi \Gamma} \sum_{n=0}^{\infty} \sum_{s=\pm1/2} \frac{1}{(E - E_{n,s})^2 + \Gamma^2} \quad (2)
\]

\[
G(E) = \frac{m_e \hbar \omega_c}{2\pi \hbar^2 \sqrt{2\pi \Gamma}} \sum_{n=0}^{\infty} \sum_{s=\pm1/2} \exp \left[ \frac{-(E - E_{n,s})^2}{2\Gamma^2} \right] \quad (3)
\]

where \( \Gamma \) is the Half-Width at half Maximum (HWHM) of the Landau levels \( (\Gamma = \hbar / 2\tau_q, \text{ where } \tau_q \text{ is the quantum lifetime}) \) and \( E_{n,s} \) is the energy of the LL with orbital (spin) quantum number \( n \) \((s = \pm1/2) \). In order to compare model directly with the resistance data, we have used the relation \( \left| \frac{\sigma_{xx}(B) - \sigma_0}{\sigma_0} \right| = \frac{R_{xx}(B) - R_0}{R_0} \left| \frac{R_{xx}(B) - R_0}{R_0} \right| \) where \( \sigma_0 \) and \( R_0 \) are the zero-field conductivity and the resistance, respectively, valid for a 2DEG in a quantizing magnetic field. Hence, the final expression for the resistance reads in the form of the Fourier series as:

\[
\left| \frac{R_{xx}(B) - R_0}{R_0} \right| = 2\pi \sum_{s=1}^{\infty} (-1)^s \exp \left[ -2 \left( \frac{\pi \Gamma s}{\hbar \omega_c} \right) \right] \frac{s2\pi^2 k_B T_\sigma / \hbar \omega_c \sinh(s2\pi^2 k_B T / \hbar \omega_c)} \right) \left( \frac{2\pi E_F \hbar}{\hbar \omega_c} \right) \left( \frac{\sigma \pi \Delta s}{\hbar \omega_c} \right).
\]

(4)

Eq 4 comprises the case of dominant long-range \((p = 1)\) and short-range \((p = 2)\) scattering mechanism, Lorentzian \((l = 1)\) and Gaussian \((l = 2)\) LL broadening. We note that the LL shape is not unambiguously determined from Eq. 4: a \( B \)-independent Lorenzian broadening \( \Gamma_L \) is equivalent to a \( B \)-dependent \( \left( \Gamma_L \propto \sqrt{B} \right) \) Gaussian broadening \( \Gamma_G \), where \( \Gamma_L = \frac{\Gamma}{\hbar \omega_c} \Gamma_G \). The terms “Lorentzian” and “Gaussian” broadening are used here in the sense of magnetic field independent broadening.

Finally, an arbitrarily large spin splitting \( \Delta s \) is taken into account by the last cosine term\(^9,12\). In CdTe, at low magnetic fields, the spin splitting is much smaller than the cyclotron energy such that \( \cos \left( \frac{2\pi E_F \hbar}{\hbar \omega_c} \right) \approx 1 \), which does not influence much the SdH amplitude. In contrast, in CdMnTe, \( \Delta s \) at low magnetic field can be much larger than \( \hbar \omega_c \), which leads to the beating patterns described in the previous section. In this case, the cosine term in Eq. 4 describes the additional modulation of the envelope of the SdH oscillations.

V. DATA MODELING

A. SdH in CdTe

The amplitude of the SdH oscillations in CdTe is plotted in Fig. 2 for carrier temperatures from 177 mK to 1200 mK. The amplitudes are compared with the model Eq. 4, using a Landau level broadening \( \Gamma = 112 \pm 10 \mu \text{eV}, m_e = 0.1 m_0, R_0 = 65 \Omega, \text{ and } g_e = -1.6 \). The carrier temperature \( T_c \) which differs from the bath temperature in our experimental conditions was determined as a fitting parameter and is used throughout the paper. The results were found to be essentially the same when taking into account one or more terms in the Fourier series of Eq. 4. The data are well-described with a long-range scattering formalism \((p = 1)\) and Lorentzian Landau levels \((l = 1)\). The dominant role of long-range scattering mechanism, as well as the extracted value for the quantum lifetime \( \tau_q = (3 \pm 0.3) \) ps is a fingerprint of a good sample quality, sufficient to observe the integer and fractional quantum Hall effects in this II-VI semiconductor material as reported in our earlier study\(^12\).
section V B 3. The final overall behaviour is summarized in section V B 1), and then discuss the oscillation amplitude (section ∆

plotted versus carrier temperature. Experimental data (points) are compared with the models of GZS including/neglecting Mn pair clusters (black/red curves). The mean AF exchange interaction was included by the parameter $T_0 = (40 \pm 10)$ mK and $T_0 = (90 \pm 10)$ mK in the two models of GZS, respectively.

FIG. 2: Natural logarithm of the amplitude of SdH oscillations in CdTe for several carrier temperatures from 177 to 1200 mK (full circles). Due to the limited range of validity of Eq. (H) and/or the smallness of signal-to-noise ratio, only the black points were used in the fitting procedure. The red curves are the theoretical fit using Eq. (6) with a Landau level broadening $\Gamma = 110 \mu$eV and an effective mass $m_e = 0.1m_0$.

B. SdH in CdMnTe

The SdH oscillations in CdMnTe are analyzed in two steps. We first focus on the position of the nodes (section V B 1), and then discuss the oscillation amplitude (section V B 2). The final overall behaviour is summarized in section V B 3.

1. Characteristics of the SdH “nodes”

The temperature dependence of the magnetic-field position of the SdH nodes is shown in Fig. 3. As explained in section III, the nodes appear when the condition $\Delta_s = (n + \frac{1}{2})\hbar\omega_c$ is fulfilled. The spin splitting $\Delta_s$ in CdMnTe can be modelled in a mean-field approach as:

$$\Delta_s = g_s\mu_B B + \Delta_{\text{exch}}B_{5/2} \left[ \frac{5g_{MN}\mu_B B}{k_B(T_{MN} + T_0)} \right] (1 - P_p) + \frac{\Delta_{\text{exch}}P_p}{2S_0} \sum_{n=1}^{5} \exp \left( \frac{2nJ_{AF} - g_{MN}\mu_B B}{k_BT_{MN}} \right) + 1$$

where the four terms are the bare Zeeman splitting, the giant Zeeman splitting (GZS) due to the $s - d$ exchange interaction between electrons and isolated Mn spins, the contribution of anti-ferromagnetic (AF) interactions within pair clusters of Mn atoms which modifies the average Mn spin polarization, and the contribution of electron-electron interactions respectively. The strength of the $s - d$ electron-manganese interaction $\Delta_{\text{exch}}$ depends in particular on the Mn concentration (nominally, $x_{\text{ave}} = 0.3\%$) and the total Mn spin quantum number $S_0 = 5/2$. $B_{5/2}$ is the Brillouin fonction describing the Mn spin magnetism, where $g_{MN} = 2.0$ is the manganese $g$-factor. $T_0$ is an additional phenomenological temperature which can be introduced to take into account AF Mn-Mn interactions through the (single particle) Brillouin function. $T_{MN}$ is Mn temperature (we note that the best fit to the data were obtained with $T_{MN} = T_x$). The “cluster” term aims at directly including the effect of AF Mn-Mn interactions on the spin polarization of the $S_0 = 5/2$ Mn system. $P_p$ is the probability that Mn is a part of a pair cluster, and $J_{AF}$ is the strength of the direct Mn-Mn AF interaction between two neighbors. $\alpha \equiv \frac{\Delta_{\text{exch}}}{2E_F} = 0.11$ stands for the electron-electron interaction (in our case, the Fermi energy is $E_F = 9.6$ meV and the parameter $\Delta_0 = 2.1$ meV has been determined in previous work).

We have used two different approach to fit the experimental data of Fig. 3. In the first approach, the e-Mn interaction is entirely taken into by the second term of Eq. 6 and the third term is neglected ($P_p = 0$). The best fit to the experimental data are reported as red curves in Fig. 3 and describes the data at the qualitative level. The corresponding parameters are $\Delta_{\text{exch}} = 1.7$ meV and $T_{0-1} = 90$ mK, which phenomenologically takes into account the Mn-Mn interaction by reducing the average Mn spin polarization.
In the second approach, the Mn-Mn AF interaction within pair clusters is directly taken into account by using the third (“cluster”) term of Eq.\textsuperscript{33} In this term, the nearest-neighbor (NN) AF interaction is expected to be rather strong ($J_{NN}/k_B \approx 5K$),\textsuperscript{27-30} such that for magnetic fields lower than 5 T ($=(2J_{NN} - 5k_B T_{c,\text{max}})/(g\mu_B)$), where $T_{c,\text{max}} = 823$ mK, NN are always anti-ferromagnetically coupled and thus do not contribute to the GZS. However, the next nearest neighbors (NNN) of manganese atoms interaction is weaker ($J_{NNN}/k_B = 0.5K$) and can play a role already at $B \sim 0.5$T and $T \sim 200$mK and similar or lower $B/T$ values. We note that the interaction strength between third and higher order NN is generally small and decreases exponentially with distance.\textsuperscript{30,31} Their residual influence is sufficiently well-described by the commonly used $T_0$ phenomenological parameter.\textsuperscript{27} Besides these distant pairs, AF interactions from higher order clusters (triplets, quadruplets, etc.) are also included in $T_0$. The fit to the data obtained in this second approach is plotted as black curves in Fig.\textsuperscript{4} and gives an excellent quantitative description of the data. The fitting parameters are $P_p = 20\%$, $J_{AF}/k_B = 0.5K$, $\Delta_{\text{exch}} = 1.7$ meV and $T_{0-2} = 40 \pm 10$mK. The extracted probability of cluster formation, $P_p = 20\%$, is significantly higher than the one expected from statistical considerations (typically a few percent). This is usually explained in terms of non-homogeneous distribution of Mn.\textsuperscript{27} The value of $J_{AF}/k_B = 0.5K$ suggests that the influence of the Mn pair clusters originates from the next nearest neighbors (NNN) interaction of manganese ions. The small but non-zero value observed for $T_{0-2} = 40 \pm 10$mK can be attributed to higher order AF interactions.

As a conclusion, the beating pattern of the SdH oscillations is profoundly modified by the magnetic sub-system, and therefore constitutes a powerful tool to characterize the e-Mn interaction, the Mn concentration, as well as the Mn-Mn interactions in CdMnTe systems.

2. Oscillation amplitude

In Fig.\textsuperscript{4} we plot the SdH oscillation amplitude in our CdMnTe QW for two representative temperatures, using a reciprocal magnetic field scale. In addition to the previously analyzed beating patterns, which manifest themselves as repeated deviations from the solid black line, we observe that the overall envelope of the amplitude of the oscillations depends only weakly on temperature. This is particularly evident when comparing the almost parallel solid black lines in Fig.\textsuperscript{4} to the case of CdTe (Fig.\textsuperscript{2}) where the $1/B$ slope of the oscillation amplitude is strongly increasing with temperature, a usual consequence of the SdH temperature damping. This suggests that the usual SdH temperature damping, described by the “$x/sin(x)$” function in Eq.\textsuperscript{4}, is compensated by some non-trivial temperature dependence of the disorder damping (exponential term in Eq.\textsuperscript{4}). More precisely, the observed behavior points toward an unusual temperature narrowing of the LL broadening (at thus a reduced “disorder damping” at higher temperature) which cannot be anticipated within the most standard forms of scattering. Another interesting observation is the non-linearity of the overall envelope in the $1/B$ scale, which is indicated at low temperature by the red line in Fig.\textsuperscript{4}. This $1/B$ non-linearity implies a magnetic field dependence of the Landau level broadening $\Gamma$.

We note that the value of quantum lifetimes in CdTe and CdMnTe are here very similar, in agreement with the previous observations.\textsuperscript{12,17} However, the observed field and temperature dependence of this quantity in CdMnTe points out to the occurrence of additional physical effect contributing to the level broadening. Again, the main difference between CdTe and CdMnTe QWs is the presence of manganese spins. As a matter of fact, the observed dependence of the broadening are qualitatively reminiscent of the behavior of the average manganese spin polarization $\langle S_z \rangle$, which increase (decreases) with the magnetic field (temperature), as can be seen by considering the (dominant) Brillouin function in Eq.\textsuperscript{6}. As the spin polarization of the manganese system directly determines the position of the Landau level, a variation of the manganese content $x$ which appears in the prefactor $\Delta_{\text{exch}}$ in Eq.\textsuperscript{4} will shift the energy level position proportionally to $\langle S_z \rangle$. A non-homogenous Mn distri-
bution at the local scale is therefore at the origin of an additional level broadening. The mean energy shift $\Gamma_{Mn}$ can be simply written from Eq.\ref{eq:gamma_mn} as:

$$\Gamma_{Mn} = \left( \frac{\Delta x}{x_{ave}} \right) \left( \frac{1}{2} \frac{\Delta_{\text{exch}}}{\Delta_{\text{add}}} \right) B_{5/2} \left[ \frac{\beta g_{Mn} \mu_B B}{k_B (T_{Mn} + T_B)} \right]. \quad (6)$$

where $\Delta x$ represent the maximum Mn spatial fluctuation around the average value $x_{ave}$ (the extremal values of $x$ are then $x = x_{ave} \pm \Delta x$). The other parameters are the ones defined previously, and the mean energy shift $\Gamma_{Mn}$ can be identified with the Lorentzian HWHM used in our formalism Eqs. \ref{eq:gamma_0} and \ref{eq:gamma_mn}.

We have reproduced our experimental data by taking this effect into account and writing the total LL in CdMnTe broadening as: $\Gamma = \Gamma_0 + \Gamma_{Mn}$, where $\Gamma_0$ is a temperature/field independent broadening, and $\Gamma_{Mn}$ is the “fluctuation-induced” contribution described above. This broadening was directly injected in the previously used SdH formalism (Eq.\ref{eq:gamma_0}). The amplitude of SdH oscillations is shown in Fig.\ref{fig:gamma_mn} together with simulations using a LL broadening $\Gamma = \Gamma_0 + \Gamma_{Mn}$ (red solid curves). As in the case of CdTe, we have used Lorentzian Landau levels in a long-range scattering approximation ($\rho = 1$ in Eq.\ref{eq:gamma_0}). The data were fitted using $\Gamma_0 = 20 \, \mu eV$ and $\frac{\Delta x}{x_{ave}} = (11 \pm 5)\%$ which gives an estimation for the relative mean fluctuation of the Mn concentration. This value is in good agreement with expected statistical fluctuation of number of manganese ions $N_{Mn/e}$ per one electron (or per area defined by de Broglie wavelength). In our sample $N_{Mn/e} \approx 60$, giving fluctuations $\frac{1}{\sqrt{N_{Mn/e}}} = 13\%$.

The resulting total LL broadening $\Gamma$ is plotted as a function of magnetic field and temperature in the inset of figure\ref{fig:gamma_mn}. The extracted “non-magnetic” broadening $\Gamma_0$ is smaller in CdMnTe than in CdTe. This is actually not surprising, because our approach considers an additional source of broadening in CdMnTe while the total broadening are similar in both systems. Physically, a smaller $\Gamma_0$ broadening in CdMnTe could be attributed to a reduction of the intra-Landau level spin-flip scattering in CdMnTe. Indeed, while the opposite spin levels in CdTe always belong to the same Landau level, in CdMnTe the GZS puts into coincidence opposite spins with different orbital quantum number which might affect the spin-flip scattering processes.

We finally note that the value of $\frac{\Delta x}{x_{ave}} = (11 \pm 5)\%$ is obtained by assuming that the temperature/field dependent Landau level broadening originates only from fluctuation in the manganese concentration, and thus constitutes an upper bound for the mean fluctuations. Other mechanisms involving the Manganese spin polarization ($S_z$), such as an anisotropic electron-Mn interaction similar to the Mn-Mn anisotropic Dzyaloshinski-Moryia interaction\cite{moriya1976}, could also contribute to the observed broadening.

![FIG. 5: Natural logarithm of the amplitude of SdH oscillations in CdMnTe QW for electron temperatures from 188 to 823 mK. Experimental data used (unused) in the fitting procedure are plotted by black (green) points and the red curves show the theoretical model including fluctuations of the manganese concentration as a additional source of Landau level broadening. (inset) Resulting total LL broadening $\Gamma$ as a function of the magnetic field $B$ and the carrier temperature $T$.](image)

**3. Overall behaviour**

In Fig.\ref{fig:gamma_mn} we report the experimental data of Fig.1.b, together with the model developed throughout the paper for the temperature and field dependence of the SdH oscillation in CdMnTe.

The oscillations amplitude and the nodes position are well reproduced as shown earlier in the paper. The beating pattern together with the phase shift of the oscillations across each node are correctly described by the last cosine term in Eq. \ref{eq:gamma_0} including the giant Zeeman splitting. We note that in order to reproduce the doubled oscillation frequency at the high-field nodes ($\nu < 50$), the first 2 terms of the Fourier series in Eq. \ref{eq:gamma_0} had to be taken into account. Taking into account higher order terms (up to 100) deepen the splitting of the doubled SdH oscillations, which is nevertheless still weaker than in our experimental observations. This could be related to the
proximity of the Stoner transition where spin splitting develops in a non-linear way.\textsuperscript{6,34}

VI. CONCLUSIONS

Shubnikov De-Haas oscillations have been studied in a high quality magnetic 2DEG formed in a diluted magnetic CdMnTe quantum well. The SdH characteristics can be well-described by incorporating the electron-Mn exchange interaction into the traditional Lifshitz-Kosevich formalism in a “mean field” approach. A more detailed analysis reveals the role of antiferromagnetic Mn-Mn interactions in this system, as well as a non trivial reduction of the Landau level broadening with increasing temperature which could be accounted for by fluctuations in the manganese concentration.

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FIG. 6: Longitudinal resistance $R_{xx}$ in the CdMnTe QW (black curves) compared with our model (red curves) assuming manganese pair clusters and fluctuations of the manganese concentration.
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