Effects of color superconductivity on the structure and formation of compact stars

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We show that if color superconducting quark matter forms in hybrid or quark stars it is possible to satisfy most of recent observational boundaries on masses and radii of compact stellar objects. An energy of the order of \(10^{53}\) erg is released in the conversion from a (metastable) hadronic star into a (stable) hybrid or quark star in presence of a color superconducting phase. If the conversion occurs immediately after the deleptonization of the proto-neutron star, the released energy can help Supernovae to explode. If the conversion is delayed the energy released can power a Gamma Ray Burst. A delay between the Supernova and the subsequent Gamma Ray Burst is possible, in agreement with the delay proposed in recent analysis of astrophysical data.

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The new accumulating data from X-ray satellites provide important information on the structure and formation of compact stellar objects. Concerning the structure, these data, are at first sight difficult to interpret in a unique and self-consistent theoretical scenario, since some of the observations are indicating rather small radii and other observations are indicating large values for the mass of the star. Concerning the formation scenario, crucial information are provided by the very recent observations of Gamma-Ray Bursts (GRB), indicating the possibility that some of the GRBs are associated with a Supernova (SN) explosion. It has not yet been clarified if the two explosions are always simultaneous or if, at least in a few cases, a time delay can exist, with the SN preceding the GRB\(^{1,2,3,4,5}\).

The effect of the transition to deconfined Quark Matter (QM) on explosive processes like SNs and GRBs has been discussed by many authors. In particular, the possibility that deconfinement takes place during the core-collapse of massive stars at the moment of the bounce, has been discussed e.g. in Refs.\(^6,7\) and this mechanism could help the SN to explode by increasing the mechanical energy associated with the bounce. However, it seems more plausible that deconfinement takes place only when the proto-neutron star (PNS) has deleptonized and cooled down to a temperature of a few MeV\(^8,9\). The energy released in the conversion to QM produces a refreshed neutrino flux which can help the supernova to explode in a neutrino-driven scheme. Finally, another scenario is possible in which neutron stars having a small enough mass can exist as metastable Hadronic Star (HS) if a non-vanishing surface tension is present at the interface between Hadronic Matter (HM) and QM. The process of quark deconfinement can then be a powerful source for GRBs and it can also explain the possible delay between a SN explosion and the subsequent GRB\(^10\).

In recent years, many theoretical works have investigated the possible formation of a diquark condensate in QM, at densities reachable in the core of a compact star\(^11,12,13\). The formation of this condensate can deeply modify the structure of the star\(^14,15,16,17\). We present here an extension of the previous works, showing that it is possible to satisfy the existing boundaries on mass and radius of a compact object if a diquark condensate forms in a Hybrid Star (HyS) or a Quark Star (QS). Moreover, we show that the formation of diquark condensate can significantly increase the energy released in the conversion from a purely HS into a more stable star containing deconfined QM.

To describe the high density Equation of State (EOS) of matter we adopt standard models in the various density ranges. Concerning the hadronic phase we use relativistic non-linear models\(^18,19\). At very low density we use the standard EOSs of Refs.\(^20,21\). For the QM phase we adopt a MIT-bag like model in which the formation of a diquark condensate is taken into account. To connect the two phases of our EOS, we impose Gibbs equilibrium conditions.

It is widely accepted that the Color-Flavor Locking phase (CFL) is the real ground state of QCD at asymptotically large densities. We are interested in the bulk properties of a compact star and we adopt the simple scheme proposed in Refs.\(^14,17\) where the thermodynamic potential is given by the sum of two contributions. The first term corresponds to a “fictional” state of unpaired QM in which all quarks have a common Fermi momentum chosen to minimize the thermodynamic potential. The other term is the binding energy \(\Delta\) of the diquark condensate expanded up to order \((\Delta/\mu)^2\). In Ref.\(^14\) the gap is assumed to be independent on the chemical potential \(\mu\). In the present calculation we consider a \(\mu\) dependent gap resulting from the solution of the gap equation\(^11\). The resulting QM EOS reads:

\[
\Omega_{\text{CFL}} = \frac{6}{\pi^2} \int_0^\nu k^2(k - \mu) \, dk + \frac{3}{\pi^2} \int_0^\nu k^2(\sqrt{k^2 + m_2^2} - \mu) \, dk - \frac{3\Delta^2 \mu^2}{\pi^2} \tag{1}
\]

with \(\nu = 2\mu - \sqrt{\mu^2 + m_2^2}\), and the quark density \(\rho\) is calculated numerically by deriving the thermodynamic potential respect to \(\mu\). Pressure and energy density read:

\[
P = -\Omega_{\text{CFL}}(\mu) - B - \Omega^c(\mu_c) \tag{2}
\]
correspond to the scenario we are proposing. More
since the effect of the gap is to increase the maximum
quark condensate helps in circumventing this difficulty,
see, the existence of an energy gap associated with the di-
ving both large masses and very small radii. As we will
is in general not easy to obtain stellar configurations hav-
configuration when deconfined QM forms inside the star.
Finally, constraints ("c"[41] and "d"[42]) do not pro-
three representative theoretical curves: thick solid line indicates CFL quark
stars, thick dot-dashed line CFL hybrid stars, thick-dashed line
hadronic stars (see text). Observational limits from: (a) Sanwal et al. 2002 22, (b) Cottam et al. 2002 23, (c) Quaintrell et al. 2003 24, (d) Heinke et al. 2003 22, (e),(g) Dey et al. 1998 26, (f) Li et al. 1999 27, (h) Burwitz et al. 2002 28.

\[
E/V = \Omega_{CFL}(\mu) + \mu \rho + B + \Omega^2(\mu_e) + \mu_e \rho_e. \tag{3}
\]

In Fig.1 we have collected most of the analysis of data from X-ray satellites, concerning masses and radii of com-
 pact stellar objects 22, 23, 24, 25, 26, 27, 28. Observing
Fig.1, we notice that the constraints coming from a few data sets (labeled “e”, “f”, “g” and maybe also con-
straint “h”[10]) indicate rather unambiguously the existence of very compact stellar objects, having a radius
smaller than \(\sim 10\) km. At the contrary, at least in one case ("a" in the figure), the analysis of the data suggests
the existence of stellar objects having radii of the order of 12 km or larger, if their mass is of the order of \(1.4\ M_\odot\).

We recall that it is difficult from an astrophysical view-
point to generate compact stellar objects having a mass
smaller than \(1\ M_\odot\). Therefore the most likely interpre-
tation of constraint “a” is that the corresponding stellar
object does not belong to the same class of objects which
have a radius smaller than \(\sim 10\) km. Concerning con-
straint “b”, it can be satisfied both with a very compact
star or with a star having a larger radius. The appar-
ent contradiction between the constraints “e”, “f”, “g”
and the constraint “a” can be easily accommodated in
our scheme, since it can be the signal of the existence
of metastable purely HS which can collapse into a stable
configuration when deconfined QM forms inside the star.

Finally, constraints ("c"[41] and "d"[42]) do not pro-
vide stringent limits on the radius of the star, but they put strong constraints on the lower value of its mass.

In general not easy to obtain stellar configurations hav-
ing both large masses and very small radii. As we will
see, the existence of an energy gap associated with the di-
quark condensate helps in circumventing this difficulty,
since the effect of the gap is to increase the maximum
mass of stars having a huge content of pure QM.

In Fig.1 we show a few theoretical M-R relations which
 correspond to the scenario we are proposing. More pre-
cisely, we show a thick-dashed line corresponding to HSs
(GM1), a thick dot-dashed line corresponding to HySs
(GM1, \(B^{1/4} = 170\) MeV, \(\Delta_2\)) and a thick solid line corre-
sponding to QSs (\(B^{1/4} = 170\) MeV, \(\Delta_1\)). Similar shapes
can be obtained using the EOS of Ref.19. Both the HyS
and the QS lines can satisfy essentially all the constraints
derived from observations. The shapes of the gaps \(\Delta_i\)
are shown in Fig. 2. In conclusion, in our scheme most
of the compact stars are either HySs or QSs having a
mass in the range \(1.2 \sim 1.8 M_\odot\) and a radius \(\sim 8.5 \sim 10\)
kmc. Stars having a significantly larger radius (as the one
suggested by constraint “a”) correspond in our scheme
to metastable HSs which can exist if their mass is not too
large, as we show in the following.

Let us now discuss \(\Delta E\), the energy released in the con-
version from HS to HyS or QS. \(\Delta E\) is the difference be-
tween the gravitational mass of the HS and that of the
final HyS or QS having the same baryonic mass. As
mentioned in the introduction, a possibility is that de-
confinement takes place a few seconds after the bounce,
when the PNS has deleptonized and its temperature has
dropped down \(\lesssim 10^8\). In particular, for stars having a
small mass the formation of QM takes place only at \(T \lesssim
\) few MeV. Notice that for a star having a mass of order
\(1.4 M_\odot\) and using the relativistic EOSs discussed in this

\[
\begin{array}{cccccccc}
\text{Hadronic} & [MeV] & \Delta E & \Delta E & \Delta E & \Delta E & \Delta E & \Delta E \\
\text{Model} & B^{1/4} & \Delta_0 & \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 & \Delta_5 \\
\hline
\text{GM1} & 160 & 95 & 172 & 178 & 204 & 327 & \\
\text{GM3} & 170 & 40 & 83 & 89 & 133 & 236 & \\
\text{GM3} & 180 & 10 & 29 & 31 & 79 & - & \\
\text{GM1} & 160 & 101 & 178 & 184 & 210 & 333 & \\
\text{GM1} & 170 & 42 & 89 & 95 & 138 & 242 & \\
\text{GM1} & 180 & 6 & 28 & 31 & BH & - & \\
\end{array}
\]

TABLE I: Energy released \(\Delta E\) (measured in foe=10^{51} \ erg) in
the conversion from a \(1.4 M_\odot\) hadronic star into the hybrid or
quark star having the same baryonic mass (labeled with a \(\bullet\),
for various sets of model parameters. BH indicates that the
hadronic star collapses to a Black Hole. A dash (\(\sim\)) indicates
situations in which the Gibbs construction does not provide
a mechanically stable EOS.
paper, hyperons are present in the initial configuration, since the typical mass at which hyperons start forming is $\sim 1M_\odot$. The energy released during quark deconfinement powers a new neutrino flux which can be useful to make the supernova explode. $\Delta E$ is shown in Table I, for a PNS having a mass of $1.4M_\odot$. As it can be seen, $\Delta E$ can be as large as $10^{53}$ erg, if the final configuration corresponds to an HS and three times as large if a QS is obtained. The effect of the gap is to increase the energy released and to allow QS configurations in cases where an HS would be obtained in the absence of quark pairing. Let us now remark that the deconfinement transition can be delayed if a non-vanishing surface tension at the interface between HM and QM exists and if the mass of the HS is not too large. This possibility was not discussed in Ref. [10] and it is the main ingredient of our model. To compute the time needed to form QM we use the technique of quantum tunneling nucleation. We can assume that the temperature has no effect in our scheme because, as discussed above, when QM forms the temperature is so low that only quantum tunneling is a practicable mechanism.

In Ref. [10] it was proposed that the central density of a pure HS (containing hyperons) can increase, due to spin pairing. Let us now remark that the deconfinement transition can be delayed if a non-vanishing surface tension at the interface between HM and QM exists and if the mass of the HS is not too large. This possibility was not discussed in Ref. [10] and it is the main ingredient of our model. To compute the time needed to form QM we use the technique of quantum tunneling nucleation. We can assume that the temperature has no effect in our scheme because, as discussed above, when QM forms the temperature is so low that only quantum tunneling is a practicable mechanism.

The calculation proceeds in the usual way: after the computation (in WKB approximation) of the ground state energy $E_0$ and of the oscillation frequency $\nu_0$ of the virtual QM drop in the potential well $U(R)$, it is possible to calculate in a relativistic frame the probability of tunneling as

$$p_0 = \exp\left[-2\frac{A(E_0)}{\hbar}\right]$$

where

$$A(E) = \int_{R_1}^{R_2} dR \sqrt{2M(R) + E - U(R)} [U(R) - E]$$

Here $M(R) = 4\pi \rho_h \left(1 - \frac{n_\rho}{n_b}\right)^2 R^3$, $\rho_h$ is the hadronic energy density and $n_b$, $n_q$ are the baryonic densities at a same and given pressure in the hadronic and quark phase, respectively. Finally, $R_\pm$ are the classical turning points. The nucleation time is then equal to $\tau = (\nu_0 \rho_0 N_c)^{-1}$, where $N_c$ is the number of centers of droplet formation in the star, and it is of the order of $10^{48}$ [33]. $\tau$ can be extremely long if the mass of the metastable star is small enough but, via mass accretion, it can be reduced from values of the order of the age of the universe down to a value of the order of days or years. We can therefore determine the critical mass $M_{cr}$ of the metastable HS for which the nucleation time corresponds to a fixed small value (1 year in Tab. 1).

In Table II we show the value of $M_{cr}$ for various sets of model parameters. In the conversion process from a metastable HS into an HyS or a QS a huge amount of energy $\Delta E$ is released. We see in Table II that the formation of a CFL phase allows to obtain values for $\Delta E$ which are one order of magnitude larger than the corresponding $\Delta E$ of the impaired QM case ($\Delta = 0$). Moreover, we can observe that $\Delta E$ depends both on magnitude and position of the gap.

In the model we are presenting, the GRB is due to the cooling of the justly formed HyS or QS via neutrino - anti-neutrino emission. The subsequent neutrino-antineutrino annihilation generates the GRB. In our scenario the duration of the prompt emission of the GRB is therefore regulated by two mechanisms: 1) the time needed for the conversion of the HS into a HyS or QS, once a critical-size droplet is formed and 2) the cooling time of the justly formed HyS or QS. Concerning the time needed for the conversion into QM of at least a fraction of the star, the seminal work by [36] has been reconsidered by [37], where it has been shown that the stellar conversion is a very fast process, having a duration much shorter than 1s. On the

| Hadronic $B^{174}$ | $\sigma$ | $M_{cr}/M_\odot$ | $\Delta E [\text{MeV}]$ | $\Delta E [\text{MeV/fm}^2]$ | $\Delta_1$ | $\Delta_2$ | $\Delta_3$ | $\Delta_4$ |
|-------------------|---------|-----------------|-----------------|-----------------|----------|----------|----------|----------|
| GM3 170 10 1.12 18 52 57 86 178 | 1 | | |
| GM3 170 20 1.25 30 66 72 106 206 | 2 | | |
| GM3 170 30 1.33 34 75 81 120 221 | 3 | | |
| GM3 170 40 1.39 38 82 88 131 234 | 4 | | |
| GM3 180 10 1.47 35 38 BH BH BH | 5 | | |
| GM3 180 20 1.50 38 40 BH BH | 6 | | |
| GM3 180 30 1.52 40 42 BH BH | 7 | | |
| GM1 170 10 1.16 18 58 64 94 180 | 8 | | |
| GM1 170 20 1.30 30 75 81 119 219 | 9 | | |
| GM1 170 30 1.41 43 90 96 141 244 | 10 | | |
| GM1 170 40 1.51 105 111 163 267 | 11 | | |
| GM1 180 10 1.56 52 54 BH BH | 12 | | |
| GM1 180 20 1.61 65 65 BH BH | 13 | | |
| GM1 180 30 1.65 BH BH BH BH | 14 | | |

TABLE II: Energy released $\Delta E$ in the conversion to hybrid or quark star, for various sets of model parameters, assuming the hadronic star mean life-time $\tau = 1 \text{yr}$ (see text). $M_{cr}$ is the gravitational mass of the hadronic star at which the transition takes place, for fixed values of the surface tension $\sigma$ and of the mean life-time $\tau$. Notations as in Tab. 1.
other hand, the neutrino trapping time, which provides the cooling time of a compact object, is of the order of a few ten seconds. If deconfinement occurs immediately after deleptonization, the energy released can help the SN to explode. If, at variance, the transition is delayed, a metastable hadronic star can form. Its subsequent transition to a stable configuration, containing deconfined quark matter, can power a GRB via the annihilation of neutrinos and antineutrinos emitted during the cooling of the newly formed compact star. The energy released is significantly increased by the effect of the chemical-potential dependent superconducting gap and it can reach a value of the order of $10^{53}$ erg. The proposed mechanism could explain recent observations indicating a possible delay between a Supernova and the subsequent Gamma Ray Burst.

In conclusion, comparing the theoretical mass-radius curves with recent observational data, we find that color superconductivity is a crucial ingredient in order to satisfy all the constraints coming from observations. The difficult problem posed by astrophysical data indicating the existence of stars which are both very compact and rather massive can be solved either with hybrid or quark stars. Concerning hybrid stars, the gap increases the maximum mass of the stable configuration, while keeping the corresponding radius $\lesssim 10$ km.

The superconducting gap affects also deeply the energy released in the conversion from hadronic star into hybrid or quark star. We assume that the deconfinement transition only takes place when the star has deleptonized and cooled down, in agreement with the results of Ref.\[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42\].

If deconfinement occurs immediately after deleptonization, the energy released can help the SN to explode. If, at variance, the transition is delayed, a metastable hadronic star can form. Its subsequent transition to a stable configuration, containing deconfined quark matter, can power a GRB via the annihilation of neutrinos and antineutrinos emitted during the cooling of the newly formed compact star. The energy released is significantly increased by the effect of the chemical-potential dependent superconducting gap and it can reach a value of the order of $10^{53}$ erg. The proposed mechanism could explain recent observations indicating a possible delay between a Supernova and the subsequent Gamma Ray Burst.

**References**

[1] L. Amati et al., Science 290 (2000) 953.
[2] J.N. Reeves et al., Nature 416 (2002) 512.
[3] J. Hjorth et al., Nature 423 (2003) 847.
[4] R.E. Rutledge, M. Sako, Mon. Not. R. Astron. Soc. 339 (2003) 600.
[5] J.N. Reeves et al., Astron.Astrophys. 401 (2003) 313.
[6] A. Drago, U. Tambini, J. Phys. G. 25 (1999) 971.
[7] D.K. Hong, S.D.H. Hsu, F. Sannino, Phys. Lett. B 516 (2001) 362.
[8] O.G. Benvenuto, G. Lugones, Mon. Not. R. Astron. Soc. 304 (1999) L25.
[9] J.A. Pons et al., Phys. Rev. Lett. 86 (2001) 5223.
[10] Z. Berezhiani et al., Astrophys. J. 586 (2003) 1250.
[11] M.G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537 (1999) 443.
[12] M.G. Alford, J. Berges, K. Rajagopal, Nucl. Phys. B 558 (1999) 219.
[13] T. Schafer, F. Wilczek, Phys. Rev. D 60 (1999) 074014.
[14] M.G. Alford, S. Reddy, Phys. Rev. D 67 (2003) 074024.
[15] M. Baldo et al., Phys. Lett. B 562 (2003) 153.
[16] D. Blaschke et al., AIP Conf. Proc. 660 (2003) 209.
[17] G. Lugones, J. E. Horvath, Phys. Rev. D 66 (2002) 074017; Astron.Astrophys. 403 (2003) 173.
[18] N.K. Glendenning, S.A. Moszkowski, Phys. Rev. Lett. 67 (1991) 2414.
[19] H. Mülle, B.D. Serot, Nucl. Phys. A 606 (1996) 508.
[20] J. Negele, D. Vautherin, Nucl. Phys. A 207 (1973) 298.
[21] G. Baym, C. Pethick, D. Sutherland, Astrophys. J. 170 (1971) 299.
[22] D. Sanwal et al., Astrophys. J. 574 (2002) L61.
[23] J. Cottam, F. Paerels, M. Mendez, Nature 420 (2002) 51.
[24] H. Quaintrell et al., Astron.Astrophys. 401 (2003) 313.
[25] C.O. Heinke et al., Astrophys. J. 588 (2003) 452.
[26] M. Dey et al., Phys. Lett. B 438 (1998) 123.
[27] X.D. Li et al., Phys. Rev. Lett. 83 (1999) 3776.
[28] V. Burwitz et al., Astron. Astrophys. 399 (2003) 1109.
[29] J. Poutanen, M. Gierliński, Mon. Not. R. Astron. Soc. 343 (2003) 1301.
[30] R. Turolla, S. Zane, J.J. Drake, astro-ph/0308326 in print on Astrophys.J.
[31] I.M. Lifshitz, Yu. Kagan, Zh. Eksp. Teor. Fiz. 62 (1972) 385 [Sov. Phys. JETP, 35, 206].
[32] M.S. Berger and R.L. Jaffe, Phys. Rev. C 35 (1987) 213.
[33] D.N. Voskresensky, M. Yasuhira, T. Tatsumi, Nucl. Phys. A 723 (2003) 291.
[34] M.G. Alford et al., Phys. Rev. D 64 (2001) 074017.
[35] K. Iida and K. Sato, Phys. Rev. C 58 (1998) 525.
[36] A. Olinto, Phys. Lett. B 192 (1987) 71.
[37] J.E. Horvath, O.G. Benvenuto, Phys. Lett. B 213 (1988) 516.
[38] M. Prakash et al., Phys. Rept. 280 (1997) 1.
[39] A very recent reanalysis of the data of the pulsar SAX J1808.4-3658, discussed in Ref.\[27, 28\], seems to indicate slightly larger radii, of the order of 9-10 km for a star having a mass of 1.4-1.5 $M_\odot$\[29\].
[40] In Ref.\[30\] an indication for an even more compact stellar object can be found. Anyway, the so-called thermal radius obtained in these analysis could be significantly smaller than the total radius of the star.
[41] The result of Ref.\[30\] is $M/M_\odot = 1.88 \pm 0.13$. In Fig.1 only the lower limit is displayed.
[42] If the observed X-ray emission is due to continuing accretion, a smaller mass is allowed, $M/M_\odot = 1.4$. 
