Microscopic theory for radiation-induced Zero-Resistance States in 2D electron systems: Franck-Condon blockade

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We present a microscopic model on radiation-induced zero resistance states according to a novel approach: Franck-Condon physics and blockade. Zero resistance states rise up from radiation-induced magnetoresistance oscillations when the light intensity is strong enough. The theory starts off with the radiation-driven electron orbit model that proposes an interplay of the swinging nature of the radiation-driven Landau states and the presence of charged impurity scattering. When the intensity of radiation is high enough it turns out that the driven-Landau states (vibrational states) involved in the scattering process are spatially far from each other and the corresponding electron wave functions do not longer overlap. As a result, it takes place a drastic suppression of the scattering probability and then current and magnetoresistance exponentially drop. Finally zero resistance states rise up. This is an application to magnetotransport in two dimensional electron systems of the Franck-Condon blockade, based on the Franck-Condon physics which in turn stems from molecular vibrational spectroscopy.

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Radiation-induced magnetoresistance ($R_{xx}$) oscillations (RIRO) turn up in high mobility two-dimensional electron systems (2DES) under illumination at low temperature ($T \sim 1K$) and low magnetic fields ($B$) perpendicular to the 2DES. When increasing radiation power ($P$), maxima and minima oscillations increase but the latter evolve into zero resistance states (ZRS). Many experiments have been proposed to understand these effects but no consensus among the people devoted to this field has been reached yet. As an example of this lack of consensus, the two, in principle accepted theories on RIRO are no longer so relevant because they cannot explain basic features such as the 1/4-cycle phase shift in the oscillation minima, or the sublinear power law dependence of RIRO. And they can not explain either recent experimental evidence, for instance, about polarization. Therefore, we have to admit that to date, RIRO and ZRS are still open issues that remain in the cutting edge of condensed matter physics regarding radiation-mater interaction. And this is especially true in the case of ZRS, maybe the most intriguing and challenging effect that shows up in this field. Despite the fact that plenty of theories have been developed for RIRO, when it comes to ZRS only a few theoretical models have been put forward. In general they predict negative $R_{xx}$, while it was not experimentally confirmed. On the other hand, the most accepted theory on ZRS is based on the formation of current and electrical field domains; the key is the existence of an inhomogeneous current flowing through the sample due to the presence of a domain structure. Yet, this is a macroscopical model that overlooks any microscopic approach on ZRS.

In this letter we develop a microscopic theory for ZRS that is based on the radiation-driven electron orbit model. The model, in turn, is based on the exact solution of the electronic wave function in the presence of a static magnetic field interacting with radiation and a perturbation treatment for elastic scattering due to randomly distributed charged impurities. This scattering between Landau states, LS, (vibrational states) is successfully completed when there is a net overlap between the initial and final wave functions (see Fig. 1). In this model the LS semiclassically describe orbits driven by radiation, driven LS, whose center positions oscillate according to the radiation frequency. This radiation-driven oscillations alter dramatically the scattering conditions. In some cases the LS advance during the scattering jump and on average the advanced distance by electrons is going to be bigger than in the dark giving rise to peaks in RIRO (see Fig. 2a). In others the LS go backward during the jump and the net distance is smaller obtaining valleys (see Fig. 2b). But in all of them there must be a net overlap of wave functions in order to have important and valuable contributions to $R_{xx}$.

This idea is similar to the one in Franck-Condon physics, and extensively used in vibrational spectroscopy and molecular quantum mechanics. ZRS turn up when the radiation intensity is high enough. Then, it can happen that the final LS ends up behind the initial position of the scattering jump. Although this process corresponds to a good overlap between LS, the average advanced distance is equal to zero and does not contribute to $R_{xx}$. Then, we can consider other final LS much further with respect to the scattering initial position that could end up ahead of it, even at very high light intensities. Nevertheless, these LS do not significantly overlap and the corresponding contribution to $R_{xx}$ exponentially drops (see Fig. 3). As a result, scattering rate, current and $R_{xx}$ are dramatically suppressed, electrons remain in their initial LS and ZRS rise up. This effect is known as Franck-Condon blockade and it is at the heart of the
physical origin of ZRS

In the radiation-driven electron orbits model, the electron time-dependent Schrödinger equation with a time-dependent force and magnetic field is exactly solved to study the magnetoresistance of a 2DES subjected to radiation at low \( B \) and temperature, \( T \). Accordingly, the exact expression of the obtained electronic wave function reads \( \Psi_n(x, t) \propto \phi_n(x - X_0 - x_{cl}(t), t) \), where \( \phi_n \) is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function (Landau state or Landau orbit) is the same as the one of the standard quantum harmonic oscillator where the guiding center, \( X_0 \) without radiation, is displaced by \( x_{cl}(t) \). \( x_{cl}(t) \) is the classical solution of a negatively charged, forced and damped, harmonic oscillator:

\[
x_{cl}(t) = \frac{-eE_0}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^2}} \cos(wt - \beta) - A \cos(wt - \beta)
\]

where \( E_0 \) the intensity of radiation, \( w \) the radiation frequency and \( w_c \) the cyclotron frequency. \( \gamma \) is a phenomenologically introduced damping factor for the electronic interaction with acoustic phonons. \( \beta \) is the phase difference between the radiation-driven guiding center and the driving radiation itself and it is given by \( \tan \beta = \frac{\omega_c^2 - w^2}{\omega} \). Thus, the guiding center lags behind radiation a phase constant of \( \beta \). When the damping parameter \( \gamma \) is important, \( (\gamma > w \Rightarrow \gamma^2 >> w^2) \), then \( \tan \beta \to \infty \) and \( \beta \to \frac{\pi}{2} \).

Now, the time-dependent guiding center is, \( X = X_0 + x_{cl} = X_0 - A \sin wt \). This physically implies that the orbit guiding centers oscillate harmonically at the radiation frequency \( w \), but radiation leads the guiding center displacement in \( \frac{\pi}{2} \).

The longitudinal conductivity \( \sigma_{xx} \) in the 2DES is obtained applying the Boltzmann transport theory. With this theory and within the relaxation time approximation, \( \sigma_{xx} \) is given by the following equation:

\[
\sigma_{xx} = e^2 \int_0^\infty dE \rho_i(E)(\Delta X)^2 W_f \left( -\frac{df(E)}{dE} \right) \tag{2}
\]

being \( E \) the energy and \( \rho_i(E) \) the density of initial Landau states. \( W_f \) is the remote charged impurity scattering rate, given, according to the Fermi’s Golden Rule, by

\[
\rho_i(E) = \sum_n \left( \frac{2m^*}{\pi\hbar^2} \right)^{1/2} \delta(E - E_n)
\]

and

\[
W_f = \frac{1}{\hbar} \sum_{n} \left| \langle \phi_n | V_{imp} | \phi_n \rangle \right|^2
\]

FIG. 1: Schematic diagrams of electron scattering between Landau states. In Fig.(1.a) there is an important overlap between Landau states. The case of \( \Phi_{13} \) and \( \Phi_{14} \) is shown as an example. Then the charged impurity scattering is very likely to occur. For this to happen it is essential that the distance between the guiding center of the Landau states is around twice the cyclotron radius or less. In the Fig. (1.b) we observe the opposite situation. Now the distance is bigger than twice the cyclotron radius and the overlap between the Landau states does not exist. Then, the scattering process is extremely unlikely to happen. The circles represent the guiding center of the Landau states.

FIG. 2: Schematic diagram for scattering between Landau states in the presence of radiation. Under radiation the Landau states are harmonically driven in a swinging motion with the radiation frequency. In Fig. 2.a the Landau states move forward and on average the electrons advance further than the dark case (peaks). In Fig. 2.b the Landau states move backward and on average the electrons advance less than in the dark case (valleys). For both panels dotted parabolas represent the initial driven Landau states and the solid ones the final states after the scattering event. The circles represent the corresponding guiding center positions of the Landau states before (dotted) and after (solid) scattering.
\[ W_I = \frac{2\pi}{\sigma} |< \phi_m | V_a | \phi_n >|^2 = \beta (E_m - E_n), \]

where \( E_n \) and \( E_m \) are the energies of the initial and final LS respectively. \( V_a \) is the scattering potential for charged impurities, \( \Delta X \) is the average distance advanced by the electron between orbits in every scattering jump in the \( x \) direction and is given by \( \Delta X = \Delta X^0 - A \sin(2\pi \frac{w}{\hbar v_c}) \), where \( \Delta X^0 \) is the distance between the guiding centers of the LS involved in the scattering event. Since all LS oscillate in phase this distance remains constant during the driving motion and is the same with or without radiation.

After some algebra we get to an expression for \( \sigma_{xx} \)\(^{58-60} \):

\[
\sigma_{xx} = e^2 m^* \frac{4\pi h^2}{\Delta X^0 - A \sin(2\pi \frac{w}{\hbar v_c})} \left[ 1 + \frac{2X_s \sinh(X_s)}{\sinh(X_s) e^{2\pi E_F / \hbar v_c}} \right] \]

where \( X_s = \frac{2e^2 \mu F}{\hbar v_c} \), \( \Gamma \) is the Landau level width and \( E_F \) the Fermi energy. To find the expression of \( R_{xx} \) we use the well-known tensorial relation \( R_{xx} = \frac{\sigma_{xx}}{\sigma_{xy}} \approx \frac{\sigma_{xx}}{\sigma_{xy}} \), where \( \sigma_{xy} \simeq \frac{n_e e}{B} \), \( n_e \) being the electron density, and \( \sigma_{xx} \ll \sigma_{xy} \).

To apply the Franck-Condon physics to the problem of ZRS we need to properly develop the matrix element inside the scattering rate \( W_I \). This matrix element can be expressed as \(^{54-58} \):

\[
|< \phi_m | V_a | \phi_n >|^2 = \sum_q |V_q|^2 |I_{nm}|^2 \delta_{k_p k_q + q} \]

where \( V_q = \frac{e^2}{\epsilon q^2} \), \( \epsilon \) the dielectric constant and \( q \) is the Thomas-Fermi screening constant. And the integral \( I_{nm} \) is given by:

\[
I_{nm} = \int_{-\infty}^{\infty} e^{iqx} \phi_n(x - X') \phi_n(x - X) dx
\]

where \( X = X_0 - A \sin wt \) and \( X' = X'_0 - A \sin wt \) are the guiding centers of \( \phi_n \) and \( \phi_m \) respectively. Expanding the exponential in the integral in powers of \( q x \):

\[
e^{iqx} = 1 + iqx - \frac{1}{2} q^2 x^2 - ...\]

On the other hand, \( q_x \sim q \sim 2k_F \sin \frac{\theta}{2} \)

where \( \theta \) is the scattering angle and \( k_F \) is the Fermi wave vector. For high mobility samples the scattering is mainly described by long range, small angle (charged impurity) scattering. Then, we assume that for the samples used in experiments this angle is small or very small\(^{55} \). We have taken an average scattering angle of \( \theta \leq 10^\circ \) and for the Fermi wave vector \( 2k_F \sim (3 - 1) \times 10^8 m^{-1} \) for a 2DES with the experimental electron density. This gives for \( q_x \sim 10^6 - 10^7 m^{-1} \) and then \( q_x x \sim 10^{-3} - 10^{-2} << 1 \).

We therefore make a good approximation retaining only the first term in the above expansion: \( e^{iqx} \rightarrow 1 \). The final outcome is that the integral \( I_{nm} \) becomes an overlap integral of the LS involved in the scattering process:

\[
I_{nm} = \int_{-\infty}^{\infty} \phi_m(x - X') \phi_n(x - X) dx
\]

This result implies that an important overlap between the initial and final LS will give, through the term \( |I_{nm}|^2 \), an intense scattering and in turn an intense \( R_{xx} \). This principle is known in Franck-Condon physics and extensively used in molecular vibrational spectroscopy\(^{27,48} \). We translate it now into magnetotransport in 2DES, and calculate the square of the vibrational overlap integral, \( |I_{nm}|^2 \), the Franck-Condon factor. The expression for the Franck-Condon factor (FC) reads\(^{51,52} \):

\[
|I_{nm}|^2 = \frac{n!}{m!} \left[ \frac{\Delta X^0}{2R^2} \right]^{m-n} e^{-\frac{\Delta X^0}{2R^2}} L_n^{m-n} \left( \frac{\Delta X^0}{2R^2} \right)^2
\]

where \( m \geq n, \quad R^2 = \frac{\hbar^2}{\mu e^2} \) is the square of the magnetic length and \( L_n^{m-n} \) the associate Laguerre polynomials.

In Fig. 4 we exhibit the calculated FC factor versus \( \Delta X_0 \) in units of cyclotron radius \( (R_c) \) for three different \( B \), : \( B = 0.1, 0.2 \) and 0.3T. For each case we also present the Landau level index for the Fermi energy and the scattering process considered in the simulation. We observe, as expected, that the FC factor presents important values.

FIG. 3: Schematic diagram explaining the physical origin of ZRS. In Fig. 3a, when the intensity of radiation is high enough and the Landau states move backwards it may happen that the final Landau state, initially at a distance of twice the cyclotron radius \( (2R_c) \), ends up behind the scattering initial position. Now and although the overlap is important, the cyclotron radius \((2R_c)\) remains constant during the driving motion and there is a positive advanced distance. However the overlap between these involved Landau states is negligible and the final magnetoresistance exponentially drops and ZRS show up.
only when $\Delta X_0 \leq 2R_c$ (important overlap between LS) and exponentially drops when $\Delta X_0 > 2R_c$ (negligible overlap). As in vibrational transitions in infrared molecular spectroscopy with the spectroscopic lines, here the FC factor defines the intensity of the scattering. Thus, when the LS involved in the scattering event are at a distance of $2R_c$ or less the FC factors (see Fig. 4) and in turn $W_I$ give important and non-negligible values. Now, with a not very intense radiation, the final driven LS always ends up ahead of the LS initial position of scattering giving rise to bigger or smaller $\Delta X$: peaks or valleys respectively in $R_{xx}$. This is described in Fig. 2. We can get to a totally different scenario if we further increase $P$ reaching a situation where the final LS ends up behind the initial scattering jump position and then although with an important value for the FC factor, the average advance distance is zero. Nevertheless, we can consider further away LS at more distance than $2R_c$ so that they end up, even at high $P$, ahead of the initial scattering position giving a net advanced distance. Yet, there is no overlap now and the FC factor turns out to be negligible. This physical scenario corresponds to the rise up of ZRS. This situation is described in Fig. 3.

Figure 5 shows calculated $R_{xx}$ vs $B$ for different radiation frequencies. ZRS positions move according to the changing of radiation frequency $w$, keeping the ratio, $\frac{w}{w_c} = j + \frac{1}{4}$. Simulated ZRS are very clearly obtained for $j = 1$. In Fig. 6, we exhibit the dependence of $R_{xx}$ on $P$ (Fig. 6a) and on $T$ (Fig. 6b) vs $B$ for a frequency.
of 103.5 GHz. The rise of ZRS for both can be understood in terms of amplitude of RIRO, $A$. In the first case, $P \propto \sqrt{E_0}$ and then, an increasing $P$ makes a bigger $A$ through the radiation electric field $E_0$. For a certain high $E_0$ we will get the condition $\Delta x^0 = A \sin(2\pi \frac{w}{w_0})$ and ZRS will begin to show up. On the other hand as $T$ increases from the lowest $T = 1 K$, $R_{xx}$ is softened and eventually almost disappears. The explanation can be readily obtained through the damping parameter $\gamma$ and its influence on $A$. The damping parameter $\gamma$ is linear with $T^2$, then when increasing $T$ the amplitude $A$ gets smaller, wiping out first ZRS and later RIRO.

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