The asymptotic solution of model equations for heat capacities of fluidized bed phases

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Abstract. This paper is aimed at solving a singularly perturbed system of equations, describing the balance of mass and heat in a two-phase fluidized bed, using asymptotic methods. The time dependencies of heat capacities and temperatures of solid and gaseous phases have been obtained. The dependencies of relaxation times of these characteristics upon kinetic and thermophysical parameters of bed phases have also been found.

1. Introduction. The creation of model equations
It is known, that obtaining exact solutions of equations in a rather general theoretical model of heat- and mass transfer processes when drying particulate materials is quite problematic. In some cases, this is due to non-linearity of equations, caused by the dependence of coefficients upon the sought values. In other cases, this is due to impossibility to record the boundary conditions in the analytical form. In view of these circumstances, it is advisable to resort either to numerical solutions of model equations, or to creating simplified models, adapted to specific conditions of process flow or to design features of devices [1], [2].

At the same time, a number of models, describing the transfer processes during drying materials in the dynamic bed, besides numerical solutions, permit the use of approximate analytical methods [3], including asymptotic ones [4].

The present paper is devoted to constructing the asymptotic solution of the system of differential equations for one model of heat- and mass transfer processes in the dynamic bed with directed movement of moisture-containing particulate material. The model, adopted in the work, belongs to the semi-empirical class [5], [6]. The basic model equations, describing the dynamics of temperatures and heat capacities of the bed subsystems are derived from the phenomenological relations of heat-moisture balance for particulate and gaseous bed phases [1], [7], [8], as well as from the semi-empirical correlation by A.V. Lykov, simulating the change in moisture content of the material during the period of declining speed of mass transfer [9]. The assumptions, simplifying the model and given in the work [7], are also accepted.

As a rule, during drying the particulate material in conditions of fluidization, the ratio between the masses of material and gas, contained in the bed, is low. The indicated mass ratio (the gas content of a bed) is contained in equations for temperature and moisture content of the particulate component as a factor for derivatives from the corresponding variables. This circumstance permits to use the method of boundary functions, developed in the paper [10], for constructing the approximate solution of model equations. Having reformulated the ratios of mass balance, based on moisture content [7], as equations
for heat capacities of particulate and continuous bed subsystems, we will obtain the following initial task for a vector-matrix equation of the type:

$$\hat{E}_y \oplus \hat{E}_y \hat{W}(\tau) = \hat{B} \oplus \hat{A}\hat{W}(\tau) + \hat{q}(\tau),$$

$$\hat{W}(0) = \text{colon} \{1,1,1,1\}.$$  

Here, the dot above the letter denotes the derivative from the dimensionless time variable

$$\tau = j(t - t_0).$$

where $t$ is current time, sec.; $t_0$ is the starting time of the 2nd drying period, sec.; the sign $\oplus$ denotes the direct sum of matrices;

$$\hat{E}_y = \text{diag}(1, \gamma); \quad \gamma = M_2/M_1; \quad \hat{W}(\tau) = \text{colon} \{C^0_y(\tau), C^0_e(\tau), \Theta_i(\tau), \Theta_e(\tau)\};$$

$$C^0_i(\tau) = \frac{C_i(\tau)}{c_{i0}}; \quad C^0_e(\tau) = \frac{C_e(\tau)}{c_{e0}}; \quad C^0_w(\tau) = \frac{C_w(\tau)}{c_{w0}}; \quad \Theta_i(\tau) = \frac{T_i(\tau)}{T_i(0)}; \quad \Theta_e(\tau) = \frac{T_e(\tau)}{T_e(0)}; \quad \Theta_s(\tau) = \frac{T_s(\tau)}{T_s(0)};$$

$$\hat{q}(\tau) = \text{colon} \{q_1(\tau), q_2(\tau), q_3(\tau), q_4(\tau)\}; \quad q_1 = -K_0 \{1 - C^0_n(\tau)\}; \quad q_2 = 1 + \beta_0; \quad K_0 = \frac{kS}{G_2};$$

$$\beta_i = \beta_0 \frac{c^0_i}{c_{i0}}; \quad \beta_e = \frac{G_1}{G_2}; \quad \beta_w = \frac{c_{w0}}{c_{w0}}; \quad \beta_s = \frac{c_{w0}}{c_{w0}}; \quad \beta_t = \frac{c_{w0}}{c_{w0}};$$

$$q_4(\tau) = 1 + r_2 \{1 - C^0_e(\tau)\} - \gamma C^0_e(\tau) + \beta_0 \{1 - C^0_i(\tau)\}; \quad r_1 = \frac{r}{c_{e0}T_e(0)}; \quad r_2 = \frac{r}{c_{i0}T_i(0)};$$

$$\hat{A} = \|A\|; \quad a_{11}(\tau) = \frac{-\beta_0 + f_0 + C^0_i(\tau)}{C^0_i(\tau)}; \quad a_{12} = \frac{\delta_{11} f_0}{C^0_i(\tau)}; \quad a_{21} = \frac{f_0 + \beta_0 \{1 - C^0_i(\tau)\}}{\delta_{21} C^0_i(\tau)};$$

$$a_{22} = \frac{f_0}{\delta_\gamma} + C^0_i(\tau) + \gamma C^0_e(\tau) \frac{C^0_i(\tau)}{C^0_i(\tau)}; \quad f_0 = \beta_0 \frac{S}{S_0}; \quad S_0 = G_1 \frac{c_{w0}}{\alpha_{21}}; \quad \delta_{21} = \frac{T_2(0)}{T_1(0)}; \quad \delta_\gamma = \frac{c_{w0}}{c_{w0}};$$

$$\hat{B} = \|B\|; \quad b_{11} \equiv -\beta; \quad b_{21} = K_0; \quad b_{22} = -\beta_0; \quad b_{22} = -1; \quad \beta = \beta_0 + K_0.$$

The values, introduced in formulas (3), have the following meaning: $M_1$, $M_2$ are masses of the material and gas in a bed, kg; $c_1$, $c_2$, $c_v$, $c_i$ are specific heat capacities of the dry material, dry gas, liquid and vapour, J/(kg K); $r$ is the specific heat of evaporation, J/kg; $w_1(\tau)$, $w_2(\tau)$ are current values of moisture content in material and gas, kg/kg; $w_{eq}$ is the equilibrium moisture content in the material, kg/kg; $G_i$; $G_2$ are mass consumptions of the material and gas, kg/sec; $S$ is the total area of the surface of material particles in the bed, m²; $T_i(\tau)$, $T_e(\tau)$ are current temperatures of the material and gas in the bed, K; $\alpha_{21}$ is the coefficient of heat exchange between the material and gas, W/(m² K); $k$ is drying intensity, kg/(m² sec); $\gamma$ is gas content in the bed.

The parameter $K_0$ represents the dimensionless drying coefficient, and $S_0$ is the thermodynamic contact surface of the material.
The condition of low gas content in the bed $\gamma \ll 1$ is subsequently assumed to be satisfied. In this case, the equation (1) represents the singularly perturbed system of four differential equations for given heat capacities $C_i^0(\tau)$ and dimensionless temperatures $\Theta_k(\tau)$ ($k = 1, 2$) of the bed subsystems.

The solution of this system will be obtained in the first approximation of the perturbation parameter $\gamma$.

2. The asymptotic solution of model equations

Using the method of boundary functions, we will get the principal terms of the asymptotic expansion of system solution (1).

The subsystem of equations in (1) for given heat capacities of the material and gas does not include the temperature terms

$$
C_i^0(\tau) = \beta \left[ 1 - C_i^0(\tau) \right] - K_i \left[ 1 - C_p^0 \right];
$$

$$
\gamma \dot{C}_i^0(\tau) = \beta \dot{C}_i^0(\tau) + \left[ 1 - C_i^0(\tau) \right];
$$

$$
C_i^0(0) = 1, C_i^0(0) = 1.
$$

(4)

Herewith, the first equation of the subsystem is autonomous and its solution is as follows

$$
C_1^0(\tau) = 1 - K_1 \left[ 1 - e^{-\beta \tau} \right], \quad K_1 = \frac{K_p}{\beta} \left[ 1 - C_p^0 \right].
$$

(5)

The approximate solution of the second equation of the system (4) will be obtained in the form of expansion:

$$
C_2^0(\tau, \gamma) = \sum_{n=0}^{N} \gamma^n \left( C_2^0(\tau) + \pi_{2n}(s) \right),
$$

(6)

where $N$ is the specified level of approximation; $\pi_{2n}(s)$ are boundary functions; $s = \tau/\gamma$.

Successively solving the differential equations for functions $C_2^0$ and $\pi_{2n}$ on the basis of methodology, described in the monograph [10], we will get

$$
C_2^0(\tau, \gamma) = 1 + \beta \dot{C}_i^0(\tau) + \frac{\gamma \beta}{\beta} \left[ C_i^0 \left( \frac{s}{\beta} \right) - C_i^0(\tau) \right] + ... .
$$

(7)

It is easy to show that the series in the expression (7) converges under condition of $\gamma \beta < 1$. Therefore, the given heat capacity of the gaseous bed phase is expressed through heat capacity of the particulate phase.

The approximate solution of the subsystem of temperature equations

$$
\dot{\Theta}_1(\tau) = a_{11}(\tau)\Theta_1(\tau) + a_{12}(\tau)\Theta_2(\tau) + q_1(\tau);
$$

$$
\gamma \dot{\Theta}_2(\tau) = a_{21}(\tau)\Theta_1(\tau) + a_{22}(\tau)\Theta_2(\tau) + q_2(\tau);
$$

$$
\Theta_1(0) = 1, \Theta_2(0) = 1
$$

(8)

is carried out in a similar way. We seek the solution of the system (8) in the form of expansion with respect of the perturbation parameter

$$
\Theta_k(\tau, \gamma) = \sum_{n=0}^{N} \gamma^n \left( \Theta_{kn}(\tau) + \Pi_{kn}(s) \right), \quad (k = 1, 2).
$$

(9)
After substituting (9) into the equations of the system (8), and subsequent equating the coefficients for similar degrees of the $\gamma$ parameter (separately for functions depending on «slow» ($\tau$) and «fast» ($s$) arguments), we will obtain the differential equations for principal and boundary functions. The equations for the first terms of expansion (8) have the form

$$
\begin{align*}
\dot{\Theta}_{10}(\tau) &= a_{11}(\tau)\Theta_{10}(\tau) + a_{12}(\tau)\Theta_{20}(\tau) + q_{1}(\tau); \\
0 &= a_{21}(\tau)\Theta_{10}(\tau) + a_{22}(\tau)\Theta_{20}(\tau) + q_{2}(\tau); \\
\Theta_{10}(0) &= 1,
\end{align*}
$$

where $a_{ij}(s)$ are the first terms of expansions of the respective functions in series with reference to degrees $\gamma$.

The value of the function $\Theta_{20}(\tau)$ for $\tau = 0$ is obtained from the second equation of the system (10). On the basis of equations (10), taking expressions (3) into account, we will get:

$$
\begin{align*}
\Theta_{20}(\tau) &= \frac{f_{0} + \beta_{0} \left(1 - C_{1}^{0}(\tau)\right)}{1 + f_{0}/\delta' + \beta_{0}' \left(1 - C_{1}^{0}(\tau)\right)}, \\
\dot{\Theta}_{10}(\tau) + P(\tau)\Theta_{10}(\tau) &= Q(\tau),
\end{align*}
$$

where $Q(\tau) = -\frac{a_{12}(\tau)q_{2}(\tau) + a_{22}(\tau)q_{1}(\tau)}{a_{22}(\tau)}$; $P(\tau) = -\frac{\Delta(\tau)}{a_{22}(\tau)}$; $\Delta(\tau)$ is the coefficient matrix determinant in the system (10).

Let us present the solution of the differential equation (12) in the Cauchy form.

$$
\Theta_{10}(\tau) = \exp \left[-\int_{0}^{\tau} P(x)dx \right] \exp \left[\int_{0}^{\tau} Q(\xi)exp \left[-\int_{\xi}^{\tau} P(x)dx \right]d\xi \right].
$$

The solution of the homogeneous equation, corresponding to the non-homogeneous equation in (12), has the form

$$
\exp \left[-\int_{0}^{\tau} P(x)dx \right] = e^{-\mu_{0}x} \left[ C_{1}^{0}(\tau) \right]^{1+\mu_{0}} \left( \frac{1 + f_{0}/\delta'}{1 + f_{0}/\delta' + \beta_{0}' \left(1 - C_{1}^{0}(\tau)\right)} \right)^{\mu_{0}},
$$

where the following variables are entered:

$$
\mu_{0} = \frac{\beta_{0}}{1 - K_{0}^{*}} \left(1 - K_{0}^{*} \right) + \left(1 + f \right) \left(1 + \frac{\beta_{0}}{\delta'} \delta'_{\mu} K_{0}^{*} \right) \left(1 + \frac{\beta_{0}}{\delta'} \delta'_{\mu} K_{0}^{*} \right),
$$
\[ \mu_1 = \frac{\beta_0}{1 - K_0^*} \left( 1 + f \right) \left( 1 + \frac{\beta_\mu}{\delta} \delta_w \right), \]  
\[ \mu_2 = \frac{\beta_0^2 f}{\delta_c} \left[ 1 + \frac{\beta_\mu}{\delta} \left( f + \delta_w \right) \right] \left[ 1 + \frac{\beta_\mu}{\delta} \left( f + \delta_w K_0^* \right) \right]. \]

Here \( f = S/S_0 \), \( \delta_w = c_v/c_w \).

From the form \( \mu_1 \) in formulas (15) it follows that the parameter \( 1 - \mu_1 \) in expressions (13), (14) can take either positive or negative values depending on correlations between regulative parameters \( \beta_0 \) and \( f \). In particular, if inequalities
\[
\begin{cases}
\beta_0 \geq \frac{\delta_c - K_0 C_0^0}{1 - \delta_w} \\
f \geq 0
\end{cases}
\]

or
\[
\begin{cases}
0 < \beta_0 < \frac{\delta_c - K_0 C_0^0}{1 - \delta_w} \\
0 < f < \frac{K_0 C_p^0 (\delta_c + \beta_0 \delta_w)}{\beta_0 [\delta_c - K_0 C_0^0 - \beta_0 (1 - \delta_w)]}
\end{cases}
\]

are satisfied, then the parameter \( 1 - \mu_1 > 1 \). It should be noted that \( \mu_1 < 1 \) for all physically acceptable values in expressions (15). Later on, we will consider the case of \( \mu_1 < 1 \) and certain additional conditions.

The integral in the right part of the solution (13) is not given in the closed form for arbitrary parameter values, but is expressed as series in special functions with arguments \( K_0^* \left( 1 - e^{-\beta_\tau} \right) \). However, in some particular cases, the integral (13) allows for simple evaluation, with certain correlations between the parameters of flow \( \beta_0 \) and the surface factor \( f \).

Let us present the integrands (13) in the form
\[
\exp \left( \int_0^\tau P(\lambda) d\lambda \right) Q(\tau) = e^{u_0} \sum_{n=1}^3 A_n Z_n(y(\tau)),
\]
where
\[
a = 1 + \frac{f_0}{\delta_c} ;
\]
\[
Z_1(y) = (a + \beta_0 y)^{\mu_0} \left( 1 - y \right)^{\alpha_{10}} ;
\]
\[
Z_2(y) = Z_1(y) \left( 1 - y \right)^{\alpha_{11}} ;
\]
\[
Z_3(y) = (1 - y)^{\alpha_{12}} (a + \beta_0 y)^{\mu_{10}} ;
\]
\[
A_1 = \beta_\tau \delta_c ;
\]
\[
A_2 = \beta_0 \left[ 1 - \eta + \delta_{21} f \right] \left( 1 + \frac{(1 - r_1) \beta_0}{a} \right) ;
\]
\[
A_3 = \beta_0 \delta_{31} f \beta_0 \left[ 1 - (1 - r_1) a \right].
\]
Under conditions, additional to (16)

$$\mu_0 < \beta, \quad f > \frac{1 - \mu_\beta}{1 - \mu_\delta_{\infty}}$$

(19)

all functions $Z_n(y(\tau))$ increase monotonically. This implies the following above evaluation of the solution (14):

$$\Theta_{10}(\tau) \approx a^{\mu_0} \frac{e^{-\gamma_0}}{a + \beta_0(1 - C_1(\tau))} + \frac{1 - e^{-\gamma_0}}{\beta(\mu_1 + \mu_2)} \left[ A_1 + \frac{A_2}{C_1(\tau)} + \frac{A_3}{a + \beta_0(1 - C_1(\tau))} \right].$$

(20)

On completing the construction of the approximate solution of temperature equations in the form (9), derived from equations (11), the boundary functions will be obtained. We have

$$\Pi_{10}(s) = 0,$$

$$\Pi_{20}(s) = \frac{f_0(\delta_{21} - 1)}{\delta_{21} (f_0 + \delta_1)} \exp\left( - \frac{f_0 + \delta_1}{\delta_1} \cdot \frac{\tau}{\gamma} \right).$$

(21)

Therefore, under conditions (16) and (19), the correlations (5), (7), (9), (12), (20) и (21) determine the main approximation to solving the problem of the evolution of heat capacities and temperatures of materials and transfer media in the dynamic bed with directed movement of the particulate phase and low gas content in the bed.

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