Genuine twist 3 in exclusive electroproduction of transversely polarized vector mesons

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We present the detailed analysis of genuine twist-3 contributions to the exclusive electroproduction amplitude of transversely polarized vector mesons. Using the formalism based on the QCD factorization in the momentum representation we calculated all the genuine twist-3 terms and found the total expression of this amplitude at $1/Q$ level. Generally speaking, these terms violate standard factorization owing to the existence of the infrared divergencies in the amplitude of hard sub-processes, although the strongest divergencies cancel due to the QCD equations of motion. We discuss the possible treatment of surviving divergencies.

I. INTRODUCTION

Hard exclusive reactions provide important information for unveiling the composite structure of hadrons. Moreover, a self-consistent description of the hard exclusive reactions is one of main goals of the QCD application. The perturbative theory allows to implement the systematical calculations providing that, first, it is possible to single out the amplitude of the short-distance sub-process and, second, to prove the infrared finiteness of this amplitude. Technically it corresponds to the factorization of large (hard) momenta from small (soft) momenta domains. Besides, the factorized pieces of amplitude depend on the dynamics that is typical at a given scale, i.e. the amplitude becomes the convolution of hard and soft parts. However, the electroproduction of transversely polarized vector mesons

$$\text{hadron}(p_1) + \gamma^*(q) \to \rho(p) + \text{hadron}(p_2) ,$$

provides the well-known example of QCD factorization breaking. Indeed, the factorization is valid only in the case of longitudinally polarized $\rho$-meson production [1,2], when the factorization assert that the photon-parton hard, perturbatively calculable, sub-processes are parted from the nonperturbative matrix elements.

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Unfortunately, it is not so encouragingly simple for the case of transversely polarized meson production. Its description is strongly complicated due to the existence of infrared divergencies in amplitudes, breaking down the factorization (see e.g. [3], [4] and references therein).

The amplitude of transverse vector meson production corresponds to the contributions suppressed as $1/Q$ in comparison with the longitudinal vector meson case [1]. At the same time, the recent experiments show that the transverse mesons production amplitudes provide a sensible contributions even at moderate virtualities $Q^2$ [5]. So, to describe these processes one should take into account the $1/Q$ terms.

In Ref. [2], the analysis of twist-2 amplitudes for hard exclusive electroproduction of mesons in terms of generalized parton distributions (GPD) was presented. Later, the authors of [3] have discussed the factorization problems in the electroproduction of light vector mesons from transversely polarized photons. They have taken into account the kinematical twist-3 terms within the Wandzura–Wilczek (WW) approximation. Also, the helicity flip amplitude of transversely polarized vector mesons production was considered within the same approximation [6].

Although the kinematical and dynamical (genuine) higher twists contributions are, generally speaking, independent, there are notable exceptions in some kinematical regions. In deep inelastic scattering at $x_B \to 1$ the kinematical higher twist terms, described by Nachtmann variable, lead to inconsistencies unless the genuine higher twists are taken into account [7]. One cannot exclude, that the genuine higher twists may cure, at least partially, the problem arising from the treatment of the end-point regions in hard electroproduction.

Thus, in this paper our goal is to study the role of genuine twist-3 contributions in the factorization theorem breaking. We adhere the approach based on the momentum representation, the basic stages of which are expounded in previous papers [8]-[11]. The essence of this approach constitutes of the generalization of Ellis–Furmanski–Petronzio factorization scheme [13]. These authors have considered the twist-4 effects in the processes of unpolarized deep inelastic scattering. At the same time, our formalism is more closely following the approach [14] developed by A.V. Efremov and one of the authors, where the role of twist-3 terms in polarized deep inelastic scattering was studied in detail.

In this paper we have computed the total expression for transverse meson production amplitude comprising the quark and gluon leading twist GPDs in nucleons and the genuine twist-3 contributions to the $\rho$-meson wave function. As a cross-check, we have reproduced the gluon contributions to the transverse meson production amplitude in WW approximation obtained in [3]. We also have discussed the possible ways to treat the infrared singularities and their partial cancellation.

The structure of paper is as follows: in Sections II and III we will introduce the kinematics and parameterization of matrix elements. Also we will present the total amplitude of quark and gluon distribution diagrams in Sections IV and V. We make the conclusion in the last section.

II. KINEMATICS

Let us start with the description of our kinematics: the $p$ is momentum of transversely polarized $\rho$-meson and its polarization vector is $e^T$; the momentum of virtual photon is denoted by $q$ ($Q^2 = -q^2$). As the hard exclusive production kinematics is similar to the DVCS kinematics, it is natural to use the analogous notations (cf. [8]). By making use of initial ($p_1$) and final ($p_2$) nucleon momenta we construct the average momentum $\overline{P}$ and transferred momentum $\Delta$: 
\[ \mathcal{T} = \frac{P_2 + P_1}{2}, \quad \Delta = p_2 - p_1, \quad \Delta^2 = t. \]  

It is convenient to write up Sudakov decompositions for all the relevant particles. We choose the light-cone basis composed by the physical vectors: \( \mathcal{T}, \ p \). Such a choice is possible because in this paper we assume that the initial hadron momentum \( p_1 \) and final hadron momentum \( p_2 \) are collinear, \( i.e. \Delta^T \to 0, \) and \( p_1^2 = p_2^2 = t = 0, \) neglecting all the relevant higher twists contributions arising from the nucleon matrix elements. In addition, we neglect squares of meson masses. Therefore the Sudakov decomposition up to kinematical twist-3 terms takes the form:

\[
\Delta = -2\xi \mathcal{T}, \quad e = e \cdot np + e^T, \quad n = \frac{\mathcal{T}}{p \cdot \mathcal{T}}; \\
p = q - \Delta = p \cdot \mathcal{T} \bar{n}; \quad n \cdot \bar{n} = \frac{4\xi}{Q^2},
\]

where we introduced the normalized vectors \( n \) and \( \bar{n} \).

### III. \( \rho \)-MESON MATRIX ELEMENTS AND THEIR PROPERTIES

We introduce the parameterizations of the \( \rho \)-meson–to–vacuum matrix elements needed for calculation of amplitudes (cf. \([16]\)). Keeping all the terms up to the twist-3 order and using the axial (light-like) gauge

\[ n \cdot A = 0, \]

these matrix elements can be written in terms of the light-cone basis vectors (3):

\[
\langle p \rangle \tilde{\psi}(y) \gamma_\mu \psi(z) | 0 \rangle \equiv \tilde{F}_\mu \phi(y) \langle e \cdot n \rangle p_\mu + \phi_3(y) e_\mu^T, \\
\langle p \rangle \tilde{\psi}(y) \gamma_5 \gamma_\mu \psi(z) | 0 \rangle \equiv \tilde{F}_5 \phi(y) \langle e \cdot n \rangle p_\mu e_\mu^T,
\]

\[
\langle p \rangle \tilde{\psi}(y) \gamma_\mu g A_\rho^T (z_2) \psi(z_1) | 0 \rangle \equiv \tilde{F}_\rho \phi(y_1, y_2) p_\mu \varepsilon_\rho \varepsilon_\alpha \varepsilon_\beta \varepsilon_\delta p_\alpha n_\beta n_\delta,
\]

\[
\tilde{\Phi}(y_1, y_2) = \frac{1}{2} \left( \phi_1^T(y_1) + \phi_1^T(y_2) \right) \delta(y_1 - y_2) + \Phi(y_1, y_2), \\
\tilde{J}(y_1, y_2) = \frac{1}{2} \left( \phi_A^T(y_1) + \phi_A^T(y_2) \right) \delta(y_1 - y_2) + J(y_1, y_2)
\]

Here \( \frac{1}{2}(\partial_\rho - \partial_\rho) \) is the standard antisymmetric derivative and \( \tilde{F} \) denotes the Fourier transformation with measure \( (z_1 = \lambda_i n) \):

\[
\frac{dy}{2} e^{-i y \cdot p z} \quad \text{for quark correlators}, \quad \\
\frac{dy_1 dy_2}{2} e^{-i (y_1 \cdot p z_1 - i(y_2 - y_1) \cdot p z_2)} \quad \text{for quark–gluon correlators}.
\]

Note that the function \( \phi_1 \) corresponds to the twist-2; the functions \( \phi_1^T, \phi_A^T \) correspond to the WW twist-3, functions \( \Phi \) and \( J \) – to the genuine (dynamical) twist-3, while functions \( \phi_3, \phi_A, \tilde{\Phi}, \tilde{J} \) contain both.
In (5)–(7) the functions \( \varphi_1, \varphi_3 \) and \( \varphi_A \) parameterizing the two-particle correlators obey the following symmetry properties:

\[
\begin{align*}
\varphi_1(y) &= \varphi_1(1-y), & \varphi_3(y) &= \varphi_3(1-y), & \varphi_A(y) &= -\varphi_A(1-y). 
\end{align*}
\] (9)

At the same time, the symmetry properties of the functions \( \Phi \) and \( J \) parameterizing the three-particle correlators are:

\[
\Phi(y_1, y_2) = \Phi(1-y_2, 1-y_1), \quad J(y_1, y_2) = -J(1-y_2, 1-y_1). \] (10)

The relations (10) represent the particular case of the relations for the functions parameterizing the analogous matrix elements in the production of pion pair with the arbitrary angular momentum \( j \) [10], while the case under consideration corresponds, obviously, to \( j = 1 \). In the case of arbitrary \( j \) the two-body functions depend on the extra parameter \( \xi = (p_\pi - p'_\pi) \cdot n \) characterizing the skewedness and they have the following properties \(^2\): \( \Phi^{\pi\pi}(y_1, y_2; \xi) = \Phi^{\pi\pi}(1 - y_2, 1 - y_1; -\xi), \quad J^{\pi\pi}(y_1, y_2; \xi) = -J^{\pi\pi}(1 - y_2, 1 - y_1; -\xi). \) (11)

For the sake of comparison, we show the relations between our parameterizing functions introduced in (5) and functions used in [16]:

\[
\varphi_1(y) \leftrightarrow f_\rho m_\rho \phi_\parallel(y), \quad \varphi_3(y) \leftrightarrow f_\rho m_\rho g^{(0)}(y), \quad \varphi_A(y) \leftrightarrow \frac{1}{4} f_\rho m_\rho \frac{\partial g^{(0)}(y)}{\partial y}. \] (12)

Let us remind that the functions \( \varphi_3(y) \) (\( g^{(0)}(y) \)) and \( \varphi_A(y) \) (\( g^{(0)}(y) \)) can be expressed through the \( \varphi_1(y) \) (\( \phi_\parallel(y) \)) owing to the WW-type relations [10] ( [16]). The WW approximations take the especially simple form in terms of functions [10]

\[
\varphi_{\pm}(y) = \varphi_3(y) \pm \varphi_A(y). \] (13)

WW relations for \( \rho \) meson represent the particular case of these relations for pion pair with arbitrary angular momentum [10] and take the form:

\[
\varphi_{+}^{WW}(x) = -\int_{y}^{x} \frac{dy}{y} \varphi_1(y), \quad \varphi_{-}^{WW}(x) = \int_{x}^{0} \frac{dy}{y-1} \varphi_1(y). \] (14)

Let us turn to the integral relations arising from the QCD equations of motion, which may be derived closely following [8] and [10]:

\[
\int_{0}^{1} dy \left( \tilde{\Phi}^{(S)}(x, y) - \tilde{J}^{(A)}(x, y) \right) = \left( x - \frac{1}{2} \right) \varphi_3(x) + \frac{1}{2} \varphi_A(x),
\]

\[
\int_{0}^{1} dy \left( \tilde{\Phi}^{(A)}(x, y) - \tilde{J}^{(S)}(x, y) \right) = - \left( x - \frac{1}{2} \right) \varphi_A(x) - \frac{1}{2} \varphi_3(x), \] (15)

where symmetric and anti-symmetric functions, whose appearance is crucial for the compatibility with the symmetry properties (9, 10), are defined as \( (f = \tilde{\Phi}, \tilde{J}) \):

\(^2\)Here we correct the misprints in [10], formula (48). Notations: \( \Phi^{\pi\pi} \) and \( J^{\pi\pi} \) correspond to \( \tilde{B} \) and \( \tilde{D} \), respectively.
\[ f^{(S,A)}(x, y) = \frac{1}{2} (f(x, y) \pm f(y, x)). \]  

(16)

Moreover, the symmetry properties make two equations equivalent to the single one, which may be rewritten in the simple form (c.f. [14]):

\[ \int_{0}^{1} dy_2 \tilde{F}_-(y_1, y_2) = (1 - y_1) \varphi_-(y_1). \]  

(17)

Here and below we use the obvious similar notations for quark-gluon correlators:

\[ F_{\pm}(y_1, y_2) = J(y_1, y_2) \pm \Phi(y_1, y_2), \]
\[ \tilde{F}_{\pm}(y_1, y_2) = \tilde{J}(y_1, y_2) \pm \tilde{\Phi}(y_1, y_2). \]  

(18)

As we will see in the next section, the QCD equations of motion play the important role in the cancellation of some leading infrared divergencies of the transverse \( \rho \)-meson production amplitudes.

**IV. AMPLITUDE OF QUARK-PHOTON SCATTERING**

At first we dwell on the computation of the quark contributions to the production amplitude. As we would like to concentrate on the twist-3 effects in \( \rho \)-meson blob in Fig.1, it is sufficient to keep the twist-2 GPD in the parameterization of quark matrix elements over nucleon states. Hence, taking into account (2) and (3) the parameterization acquires the following form

\[ \langle N(p_2) | \bar{\psi}(0) \gamma_\mu \psi(\tilde{z}) | N(p_1) \rangle F^{(x)} = H(x) U(p_2) \gamma_\mu U(p_1) = \sqrt{1 - \xi^2} H(x) \tilde{\mu} \]  

(19)

where, as it was above, \( \tilde{F} \) denotes the Fourier transformation, except that the measure now is \( (\tilde{z} = \lambda \tilde{n}) \)

\[ dx e^{-i(x \tilde{\mu} + \Lambda/2) \tilde{z}}. \]  

(20)

Further, using the parameterization (5)–(7), (19) and calculating the traces, the amplitude given by the simplest Feynman diagrams, Figs. 2a and 2b, reads (cf. [2]):

\[ \mathcal{A}_{1, \mu}^{(q), \gamma_5 \rightarrow \rho T} = 8 \sqrt{1 - \xi^2} C_F \frac{\epsilon_\mu^T}{N_c Q^2} \int_{-1}^{1} dx H(x) \left[ \frac{1}{x - \xi + i \epsilon} - \frac{1}{x + \xi - i \epsilon} \right] S_2^{(q)}(y), \]  

(21)

where

\[ S_2^{(q)} = \int_{0}^{1} dy \frac{y}{y} \varphi_+(y). \]  

(22)

Note that the double poles in \( x \) cancel due to the use of the gluon propagator in the axial gauge.

It is convenient to organize genuine twist 3 diagrams according to the insertions of extra gluon to the diagrams of Fig.2. Summing the diagrams of Figs. 3 and 4, we finally obtain the expression for the quark distribution amplitude that includes all the WW and genuine twist-3 contributions:

\[ \mathcal{A}_{2, \mu}^{(q), \gamma_5 \rightarrow \rho T} = 8 \sqrt{1 - \xi^2} C_F \frac{\epsilon_\mu^T}{N_c^2 - 1} \frac{1}{Q^2} \left\{ \mathcal{H}_1 \times T_1^{(q)} + \mathcal{H}_2 \times T_2^{(q)} \right\}, \]  

(23)

3Recently we computed this amplitude using another parameterization [12]
where
\[
\mathcal{H}_1 = \xi \int_{-1}^{1} dx H(x) \left[ \frac{1}{(x + \xi - i\epsilon)^2} + \frac{1}{(x - \xi + i\epsilon)^2} \right], \quad \mathcal{H}_2 = \int_{-1}^{1} dx H(x) \left[ \frac{1}{x - \xi + i\epsilon} - \frac{1}{x + \xi - i\epsilon} \right]
\]  
and
\[
\mathcal{I}^{(q)}_1 = \int_{0}^{1} dy_1 dy_2 \left\{ \frac{4C_F \tilde{\Phi}^-(y_1, y_2)}{(1 - y_2)(1 - y_1)} + \frac{C_A \tilde{\Phi}^-(y_1, y_2)}{(1 - y_2)(1 - y_1)} + \frac{2C_F \tilde{\Phi}^-(y_1, y_2) - C_A \tilde{\Phi}^-(y_1, y_2)}{y_1(1 + y_1 - y_2)} \right\};
\]
\[
\mathcal{I}^{(q)}_2 = \int_{0}^{1} dy_1 dy_2 \left\{ \frac{C_F \tilde{\Phi}^-(y_1, y_2)}{(1 - y_2)(1 - y_1)} + \frac{2C_F \tilde{\Phi}^-(y_1, y_2) - C_A \tilde{\Phi}^-(y_1, y_2)}{(1 + y_1 - y_2)(1 - y_2)} \right\}.
\]
Note that the entire result is expressed in terms of "-" combinations of twist-3 functions (which may be transformed to "+") ones by making use of symmetry properties (10)). Such combinations of axial and vector twist-3 matrix elements are known to appear also in inclusive case [14,17,18].

The $\mathcal{H}_1$- and $\mathcal{I}^{(q)}_1$-structure integrals possess the poles of second order. This fact leads to the vulnerability of factorization theorem. However, one can see that the first term in (25) may be reduced to the two-particle function $\varphi^-(y_1)$ by making use of the QCD equations of motion (17). As a result, the power of infrared divergence is reduced due to the factor $1 - y_1$ in its r.h.s. Moreover, in the case of the gauge different from the axial one (Feynman gauge, in particular) the emerging double pole in $x$ in (21) is cancelled by the corresponding additional contribution to (23) (together with the dependence on the gauge fixing parameter) provided the equation of motion are taken into account. This is quite natural: while the longitudinal amplitude is gauge invariant by itself, the gauge invariant set of diagrams for the transverse amplitude contains also quark gluon diagrams, related to the quark ones by the equations of motion.

Still, the remaining single poles would lead to infrared divergencies unless functions $\Phi, \tilde{\Phi}, J, \tilde{J}$ vanish at $y_i \to 1$ or $y_i \to 0$ at least as the first power of $(1 - y_i)$ or $y_i$. This behaviour is quite reasonable for genuine twist term. However, as the corresponding integral (with a colour factor $C_A$) becomes finite providing they have the corresponding behaviour at the edge points, it cannot be used to cancel the infrared divergence of WW contribution in the term with the factor $C_F$.

To be more specific, let us calculate this WW contribution explicitly. Within the WW approximation where all the quark-gluon parameterizing functions $J(y_1, y_2)$ and $\Phi(y_1, y_2)$ (see, (7)) are equal to zero, the integrals at the structures $\mathcal{H}_1$ and $\mathcal{H}_2$ in the sum of (21) and (23) take the following forms:
\[
\mathcal{K}^{(q)}_1 = \mathcal{I}^{(q), WW}_1 = -\frac{1}{2} \int_{0}^{1} dy \varphi^-(y) \left( 1 + \frac{2}{y} \right);
\]
\[
\mathcal{K}^{(q)}_2 = \mathcal{S}^{(q)}_2 + \mathcal{I}^{(q), WW}_2 = \int_{0}^{1} dy \left( \varphi^+(y) + \frac{\varphi^+(y) + \varphi^-(y)}{2y} \right).
\]

One can see from (14) that the simple pole in $\mathcal{K}^{(q)}_1$ of (27) is cancelled by the zero boundary condition for the function $\varphi^-(y) : \varphi^-(0) = 0$. This is quite similar to the cancellation providing the finiteness of the two pions production amplitude in the collision of real and virtual photons [10]. This makes finite the coefficient of the potentially dangerous (due to the double pole in $x$) integral $\mathcal{H}_1$. At the same time, there is no such effect for $\mathcal{K}^{(q)}_2$. 
V. AMPLITUDE OF GLUON-PHOTON SCATTERING

In this section we turn to the calculation of gluon distribution amplitude. Since we will deal with the two-gluon GPD, it is natural to choose the gauge-fixing condition for the gluons in the form:

\[ \tilde{n} \cdot A = 0, \]  

(28)

where \( \tilde{n} \)-vector (3) corresponds to the "minus" component of the light-cone basis where "plus" component is provided by nucleon (average) momentum. In this case, the gluon fields \( A_\mu \) can be expressed through the gauge-invariant field-strength tensor \( G_{\mu\nu} \). Consequently, one can parameterize the nucleon matrix elements of two gluon fields in terms of the gauge-invariant gluon distribution:

\[
\langle N(p_2)|A^a_\alpha(0)A^b_\beta(\tilde{z})|N(p_1)\rangle_F = \delta^{ab}N^2c^{-1}\Bigg( g_{\alpha\beta} - \slashed{p}_\alpha \tilde{n}_\beta - \slashed{p}_\beta \tilde{n}_\alpha \Bigg) \left( \frac{G(x)}{(x+\xi-i\epsilon)(x-\xi+i\epsilon)} \right). \tag{29}
\]

Note that the gauge condition we use here is different from the one used for parameterization of vector meson matrix elements (see, (5)-(7)); so the question arises, is it possible to use these expressions here. Therefore, let us discuss this problem in more detail.

In principle, owing to the Lorentz and gauge invariance, our physical amplitudes should be independent on the explicit choice form of \( n \) and \( \tilde{n} \). In the general case, the vectors \( n \) and \( \tilde{n} \) can be chosen in an arbitrary way [10]. Now we would like to make the comparative analysis of two gauges: (4) and (28), used for \( \rho \)-meson matrix elements. In the case of gauge (4) that was also used for the quark-gluon scattering calculation, the vector \( n \) in (5)-(7) play the double role. Namely, it fixes the gauge and defines the longitudinal momentum fraction \( x = kn \) carried by the active quark with the momentum \( k \) in \( \rho \)-meson. At the same time when the gauge (28) is adopted, these two roles are distributed between the vectors \( n \) and \( \tilde{n} \). Namely, \( n \) defines the longitudinal momentum fraction whereas \( \tilde{n} \) defines the gauge-fixing condition. Of course, in this case the parameterization of relevant \( \rho \)-meson matrix elements should, generally speaking, look more complicated due to the added \( \tilde{n} \)-terms. However, for "physical" \( n, \tilde{n} \) (3), one may explicitly check that the parameterization (6)-(7) is still self-consistent. This is actually due to the twist-3 approximation, while there is no hope to describe twist-4 terms in such a simple way.

In the gauge (4) only the transverse (physical) gluons exist. In contrast, in the gauge (28) the transverse gluon fields are constructed as

\[
A^T_\rho = A_\rho - p_\rho n \cdot A, \tag{30}
\]

where the second term in (30) corresponds to the twist-2 contribution. As a result, the use of gauge (28) leads to the appearance of twist-2 functions in the parameterization of three-particles \( \rho \)-meson matrix elements, for instance

\[
\langle \rho(p)|\bar{\psi}(0)\gamma_\mu gA_\rho(z_2)\psi(z_1)|0\rangle_F = \Phi_0(y_1, y_2)(e \cdot n)p_\mu p_\rho + \Phi(y_1, y_2)p_\mu e^T_\rho. \tag{31}
\]

The new twist-2 terms, proportional to \( n \cdot A \), should be absorbed to the standard P-ordered exponent (the gauge link) of the matrix elements of leading twist-2 quark operators [9].

Thus, with the help of parameterization (5)-(7) and (29)), we sum the amplitudes given by simplest diagrams of Figs. 2c and 2d, with the amplitudes corresponding to diagrams of Figs. 5 and 6. The last sort of amplitudes
includes both the kinematical and dynamical (genuine) twist-3 in \( \rho \)-meson-to-vacuum matrix elements. We derive the following expressions: the simplest diagrams contributions reads

\[
A_{1,\mu}^{(g), \gamma^* \rightarrow \rho_T} = \frac{C_F}{N_c - 1} \frac{e_\mu^T}{Q^2} \int_{-1}^{1} dx G(x) \left[ \frac{1}{(x + \xi - i\epsilon)^2} + \frac{1}{(x - \xi + i\epsilon)^2} \right] S_1^{(g)} + \frac{1}{\xi} \int_{-1}^{1} dx G(x) \left[ \frac{1}{x - \xi + i\epsilon} - \frac{1}{x + \xi - i\epsilon} \right] S_2^{(g)} \]

(32)

where

\[
S_1^{(g)} = \int_{0}^{1} \frac{dy}{y} \left( \frac{1}{2} \varphi^+(y) - \frac{3}{2} \varphi^-(y) \right), \quad S_2^{(g)} = \int_{0}^{1} dy \frac{\varphi^+(y) + \varphi^-(y)}{y}
\]

(33)

and the genuine twist-3 contributions take the form

\[
A_{2,\mu}^{(g), \gamma^* \rightarrow \rho_T} = \frac{C_F N_c}{(N_c - 1)^2} \frac{e_\mu^T}{Q^2} \left\{ G_1 \times T_1^{(g)} + G_2 \times T_2^{(g)} \right\},
\]

(34)

where

\[
G_1 = \int_{-1}^{1} dx G(x) \left[ \frac{1}{(x + \xi - i\epsilon)^2} + \frac{1}{(x - \xi + i\epsilon)^2} \right], \quad G_2 = \frac{1}{\xi} \int_{-1}^{1} dx G(x) \left[ \frac{1}{x - \xi + i\epsilon} - \frac{1}{x + \xi - i\epsilon} \right]
\]

(35)

and

\[
T_1^{(g)} = \int_{0}^{1} dy_1 dy_2 \frac{C_F F_{-}(y_1, y_2)}{(1 - y_2)(1 - y_1)}, \quad T_2^{(g)} = \int_{0}^{1} dy_1 dy_2 \frac{C_F F_{-}(y_1, y_2)}{(1 - y_2)(y_1 - 1)}.
\]

(36)

Note that the integral \( T_1^{(g)} \) is essentially non-Abelian and contains only genuine twist-3 contribution. Consequently, the assumption on the linear decrease of \( F_{-}(y_1, y_2) \) leads to its finiteness. At the same time, the integral \( T_2^{(g)} \) becomes divergent due to WW term with the coefficient \( C_F \).

To study these effects and to reproduce the results obtained in [3] we address WW approximation. As in the quark case, this approximation implies that all the quark-gluon parameterizing functions \( J(y_1, y_2) \) and \( \Phi(y_1, y_2) \) (see, (7)) are equal to zero. Using the results of [10] after some computations we derive the following expression:

\[
A_{WW,\mu}^{(g), \gamma^* \rightarrow \rho} = \frac{4}{N_c} \frac{e_\mu^T}{Q^2} \left\{ G_1 \times N_1^{(g)} + G_2 \times N_2^{(g)} \right\},
\]

(37)

where

\[
N_1^{(g)} = S_1^{(g)} = \int_{0}^{1} dy y \left( \frac{1}{2} \varphi^+(y) - \frac{3}{2} \varphi^-(y) \right),
\]

\[
N_2^{(g)} = S_2^{(g)} + T_2^{(g), WW} = \int_{0}^{1} dy y \left( \frac{3}{2} \varphi^+(y) - \frac{1}{2} \varphi^-(y) \right).
\]

(38)

If we take into account (12) then the integrals (38) will be rewritten in the forms which completely coincide with [3] 4:

\[
N_1^{(g)} = \int_{0}^{1} dy y \left( 2g^{(v)}(y) + \frac{g^{(a)}(y)}{2y(1 - y)} \right),
\]

\[
N_2^{(g)} = \int_{0}^{1} dy y \left( 4g^{(v)}(y) - 2 \frac{\Phi(y)}{y} + \frac{g^{(a)}(y)}{2y(1 - y)} \right).
\]

(39)

4 The definition of function \( \Phi \) can be found in [16].
Similar to the quark distribution case, the integrals (38) (and, consequently, (39)) are not free from the infrared divergencies owing to non-zero boundary values [3,16].

VI. DISCUSSION AND CONCLUSIONS

Let us stress that we calculated the full set of genuine twist 3 diagrams of transverse vector meson electroproduction. We also present, for the first time, the WW contribution proportional to quark GPD and reproduce the WW contribution due to the gluon GPD calculated earlier by the different method. As a result, we observed the cancellation of the few types of the leading infrared divergencies:

i) The double poles in $x$ of the quark contribution (Fig 2a,b) in the non-axial gauge cancel with the contributions of quark-gluon diagrams (Fig 3,4), provided the QCD equations of motion are taken into account.

ii) Making use of the equation of motion allows also to eliminate the double poles in $y$ of the same quark-gluon diagrams, surviving in the axial gauge, which are reduced to the single poles.

iii) The single pole in $y$ of the WW contribution proportional to the integral $H_1$ of quark GPD cancels between the vector ($\phi_3$) and axial ($\phi_A$) distributions.

Let us start the analysis of potential surviving divergencies from the double poles in $x$. Consider first the imaginary parts of $H_1, G_1$ (providing in the gluon case the dominant contribution at large energies), which are easily calculated and is represented by the derivatives $\frac{\partial}{\partial x} H(x, \xi)|_{x=\xi}$, $\frac{\partial}{\partial x} G(x, \xi)|_{x=\xi}$ in the transition point $x = \xi$ from DGLAP to ERBL region. One may worry, that for the quark case this derivative is not continuous just at that transition point due to the existence of so-called Polyakov-Weiss term in the two-component model of GPD [21], which is non-zero only for $|x| < \xi$. However, this term does not give any contribution to the imaginary part of amplitude (like any meson exchange term), and should not therefore be taken into account when derivative is calculated, so that it should be understood as $\frac{\partial}{\partial x} H(x, \xi)|_{x=\xi+\epsilon}$. Another way of PW term treatment [22] is the consideration of its most general form, occupying the whole region $|x| < 1$, and being therefore smooth at $x = \pm \xi$. It may be reduced to the original PW form by the sort of "gauge transformation" [22], generating the irregularities both in PW term and another component of GPD, so that they are cancelled in their sum. The possible discontinuity of derivative in the gluon case may be treated by adding the small mass to the quark propagators [17], resulting in the similar expression $\frac{\partial}{\partial x} H(x, \xi)|_{x=\xi+\epsilon}$. Anyway, the imaginary parts of $H_1, G_1$ are well defined, while the real parts may be restored by making use of the dispersion relations [23].

Let us now discuss the surviving divergencies arising from the integration over $y$. The truly non-Abelian higher twist contributions ($C_A$ terms) may be assumed finite, as it is sufficient to have the functions $\Phi(y_1, y_2), J(y_1, y_2)$ which are going to zero when $y_{1,2}, \bar{y}_{1,2} \to 0$. The counterpart of this assumption is, however, the impossibility to cancel the divergencies of WW terms, which contain another colour factor $C_F$.

The only real danger is therefore coming from the WW terms, which have the non-zero values at of $\phi_3, \phi_A$ at $y = 0, 1$. Even if they are assumed to be zero at some reference point $Q^2_0$, this property would be (rather slowly) broken by the QCD evolution.

A visible way to regularize these singularities is to keep the transverse momentum $k^T_i$ in loop integrations, which is in fact required for the quantitative description of the longitudinal amplitude as well [20]. At large $Q^2$ the behavior of structure integral is determined by the region where, for instance, $1 - y$ is of the same order as $\langle k^2_1 \rangle/Q^2$. In order to estimate the relevant contribution, staying in the collinear approximation, one may implement the corresponding infrared cutoff in the integration over $y$. As a result, the transverse amplitude
is logarithmically enhanced, so that this non-factorizable contribution defines the asymptotic behaviour of the ratio of transverse and longitudinal amplitudes:

\[
\frac{|A_{\mu}^{(q), \gamma^\rho_T \rightarrow \rho_T}|}{|A_{\mu}^{(q), \gamma^\rho_L \rightarrow \rho_L}|} \sim \frac{m_\rho \ln Q}{Q},
\]

(40)

where power suppression comes from the standard kinematical enhancement of longitudinal polarization.

To make the more quantitative estimates one should take into account the factorizable contribution (involving the parameterization of genuine twist matrix elements) and include the \(k_T\)-dependent distributions to non-factorizable one. The consistent account for all powers of \(k_T\) would be equivalent to the summation of all kinematical high twist terms.

To summarize, in this paper we have computed both gluon and quark contributions to the transversely polarized \(\rho\)-meson electroproduction up to genuine twist-3 accuracy. We observed a number of interesting cancellations of leading infrared divergencies due to the gauge invariance manifested through QCD equations of motions and suggested the possible ways of treatment of the surviving infrared contributions.

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FIG. 1. General structure of factorized electroproduction amplitude

FIG. 2. Simplest diagrams with quark and gluon GPD
FIG. 3. Genuine twist-3 diagrams with quark GPD: insertions to the diagrams of Fig. 2a

FIG. 4. Genuine twist-3 diagrams with quark GPD: insertions to the diagrams of Fig. 2b
FIG. 5. Genuine twist-3 diagrams with gluon GPD: insertions to the diagrams of Fig 2c

FIG. 6. Genuine twist-3 diagrams with gluon GPD: insertions to the diagrams of Fig 2d