The Quark-Gluon-Plasma Liquid

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Abstract

The quark-gluon plasma close to the critical temperature is a strongly interacting system. Using strongly coupled, classical, non-relativistic plasmas as an analogy, we argue that the quark-gluon plasma is in the liquid phase. This allows to understand experimental observations in ultrarelativistic heavy-ion collisions and to interpret lattice QCD results. It also supports the indications of the presence of a strongly coupled QGP in ultrarelativistic heavy-ion collisions.

Ultrarelativistic heavy-ion collision experiments at the accelerators SPS (CERN) and RHIC (Brookhaven) are performed to search for a new state of matter, the so-called quark-gluon plasma (QGP), in which the quarks and gluons are deconfined [1]. The early Universe should have been in this phase for the first few microseconds after the Big Bang. In high-energy nucleus-nucleus collisions a hot and dense fireball is created which is able to reach temperatures above the critical one for the transition from the hadronic to the QGP phase of the order of $T_c = 170$ MeV [2]. First indications for the discovery of the QGP in these experiments have been reported [3–5]. However, the QGP cannot be observed directly, as the tiny fireball (diameter about 10 fm) expands and cools down rapidly, allowing the QGP phase to exist only for less than about 10 fm/c. Only hadrons, leptons, and photons emitted from this little bang are registered in the detectors. Hence, only by comparing experimental data with theoretical predictions for signatures of the QGP formation the QGP can be detected by circumstantial evidence. Unfortunately, the theoretical description of the QGP from first principles is extremely difficult as the interaction between quarks and gluons in the QGP, described by QCD, is strong. Therefore, perturbative QCD is not applicable. Only at extremely high temperatures the interaction becomes weak due to asymptotic freedom, allowing a perturbative treatment of the QGP [6]. The only so far well-established non-perturbative method, lattice QCD, is not capable so far to compute dynamical quantities, such as the most QGP signatures\(^1\).

Consequently, one has to adopt appropriate models or analogies from other fields in physics [4] to describe the QGP. Here we propose to consider strongly coupled, classical,

\(^1\)An exception is the dilepton production rate using the maximum entropy method [7].
non-relativistic plasmas as model systems to learn about qualitative features of the strongly coupled QGP\(^2\). Such plasmas are well studied experimentally as well as theoretically [8]. A non-relativistic plasma is called strongly coupled if the interaction energy (Coulomb energy) between the particles is larger than the thermal energy of the plasma particles, i.e., if the Coulomb coupling parameter \(\Gamma = q^2/(dT) > 1\), where \(q\) is the charge of the particles, \(d\) the interparticle distance, and \(T\) the plasma temperature (\(\hbar = c = k_B = 1\)). A strongly coupled plasma exhibits peculiar properties, in particular liquid and even solid or crystal phases [8]. For example, in the case of a one-component plasma with unscreened Coulomb interaction, Monte Carlo simulations of the free energy revealed a phase transition from the liquid to the solid phase (Coulomb crystal) if the critical value \(\Gamma_c = 172\) is exceeded [9].

We define the Coulomb coupling parameter of the QGP in analogy as \(\Gamma = C g^2/(dT)\) [10], where \(C = 4/3\) or 3 is the Casimir invariant for the quarks or gluons, respectively, and \(g\) the strong coupling constant, related to the strong fine structure constant by \(\alpha_s = g^2/(4\pi)\). For an estimate of the parton-parton interaction strength we have replaced here the square of the electric charge \(e^2\) in the Coulomb potential by the square of the strong charge \(C g^2\). Note, however, that the heavy quark potential, for example in the case of a charmonium, is given by \(V(r) = C\alpha_s/r +\) confining potential, i.e. the Coulomb part differs by a factor of \(4\pi\) from our parton interaction energy entering \(\Gamma\). For light quarks and gluons, however, we consider the scattering amplitude for estimating the interaction strength, where the electric charge \(e\) in QED is replaced by \(g\) times a color factor in QCD [11].

For typical temperatures\(^3\) that can be reached in heavy-ion collisions, \(\Gamma\) is larger than 1. For example, for \(T = 200\) MeV the temperature dependent strong fine structure constant \(\alpha_s\) is between 0.2 and 0.5, corresponding to \(g = 1.5 - 2.5\). For simplicity, we assume for the particle density the ideal gas result (for 2 active quark flavors in the QGP) \(n = (34/\pi^2) \zeta(3) T^3\), leading to \(d \simeq n^{-1/3} \simeq 0.5\) fm. This gives \(\Gamma = 10 - 30\). Note that we have neglected the chromomagnetic interaction here, which is only possible in the non-relativistic case. Hence, the parton interaction is larger and the QGP even stronger coupled. Plasmas with such a value of \(\Gamma\) are known to be in the liquid phase, i.e., the plasma behaves rather like a liquid than a gas due to the strong interaction between the charged particles. Therefore we suggest that the QGP, at least close to the critical temperature, is also in the liquid phase. This was also suggested by Gyulassy and Heinz [3,5] considering the success of hydrodynamical models for describing RHIC data.

Relativistic effects do not alter this picture as they even increase the interaction energy.

\(^2\)It is quite natural to compare the QGP with usual plasmas as it shows many features known from classical electrodynamic plasmas such as plasmons or Debye screening.

\(^3\)Here we consider only an equilibrated QGP. However, in ultrarelativistic heavy-ion collisions the fireball is not in equilibrium at the beginning. Starting from a color glass condensate in the high energy nucleons, the fireball is expected to end up in an equilibrated QGP phase via a pre-equilibrium stage. How this transformation from the weakly interacting color glass condensate to a strongly coupled QGP proceeds by initial hard and secondary parton collisions, including parton production by bremsstrahlung and pair creation, is the subject of intense and on-going investigations [12].
What about quantum effects? In a plasma quantum effects become important if the thermal de Broglie wave length \( \lambda_{th} \) exceeds the interparticle distance. In the QGP we estimate this wave length by \( \lambda_{th} = 1/m^* \), where \( m^* \) is the effective parton mass in the QGP. For gluons we take the effective mass from perturbative QCD, \( m^* = gT/\sqrt{3} \) (for quarks, \( m^* = gT/\sqrt{6} \)), which is a good estimate even at \( T \) not too far from \( T_c \) \[13\]. Then we find \( \lambda_{th} \approx 1 \text{ fm} \), which is of the same order as \( d = 0.5 \text{ fm} \), showing that quantum effects are important. However, they do not destroy the assumption of a QGP liquid because of the following argument: quantum effects are dominant in degenerate plasmas, such as quark matter. In this case the thermal energy in the denominator of the Coulomb coupling parameter \( \Gamma \) has to be replaced by the Fermi energy (\( E_F \approx 350 \text{ MeV} \)), reducing \( \Gamma \) by less than a factor 1/2 compared to a QGP at \( T = 200 \text{ MeV} \). In a degenerate electron plasma Wigner crystallization sets in if \( \Gamma \approx 15 \) \[8\], indicating that also quark matter is in the liquid or even solid phase. Of course, non-abelian contributions to the parton interaction will be important for quantitative considerations. Hence we conclude that for a qualitative understanding of the QGP at temperatures, reached in heavy-ion collisions, the comparison with strongly coupled, classical, non-relativistic plasmas can be useful.

However, strongly coupled plasmas are difficult to produce in the laboratory and rarely exist in nature. The reason is simple: a strongly coupled plasma requires a high density corresponding to a short interparticle distance and/or a low temperature. Under these conditions the ions and electrons recombine quickly and the plasma vanishes\[4\]. Exceptions are, for example, the ion component in a white dwarf (\( \Gamma = 10 - 200 \)) \[8\] or dense, short-living plasmas created by heavy-ion beams on solid state targets \[14\]. Another possibility are so-called complex or dusty plasmas \[15,16\]. These are multi-component low-temperature plasmas, created by external electromagnetic fields (rf or dc fields), in which, in addition to ions, electrons, and neutral atoms, microparticles, i.e., micron size particles like dust, are present. These microparticles get charged by collecting electrons and ions on their surface. Due to the high mobility of the electrons compared to the ions the net charge of the particles can be of the order of \( 10^3 \) to \( 10^5 \) electron charges, depending on the size of the particles and the electron temperature. Owing to this high charge, the microparticles interact strongly by the Yukawa force with each other, corresponding to a large \( \Gamma \gg 1 \). As a matter of fact, the formation of a plasma crystal in complex plasmas has been predicted \[17\] and experimentally verified \[18,19\] as shown in Fig.1. Another advantage of complex plasmas is their easy observability. Using an illumination laser and a CCD camera the microparticles can be observed directly, providing informations of the system on the microscopic and kinetic level.

Now let us discuss two possible consequences of our assumption that the QGP is in the liquid phase.

1. A liquid shows a pronounced peak in the pair correlation function\[5\], defined in coor-
g(r) = \left\langle \frac{1}{N} \sum_{i \neq j}^{N} \delta(r - r_i - r_j) \right\rangle, \quad (1)

where r_i and r_j are the position vectors of 2 particles and N the total particle number. An example for the pair correlation function of the liquid phase of a complex plasma is shown in fig. 2.

In QCD lattice simulations of the temporal meson correlators a strong correlation of the propagating q\bar{q}-pair above T_c was found [21]. These features of the meson correlators cannot be explained in a weakly coupled QGP [22] but have been interpreted instead as the existence of bound states in the QGP [4]. Note, however, that there is a qualitative change if T_c is exceeded: the sharp \delta-like peak below T_c in the spectral function of the correlators, corresponding to mesonic bound states, turns into a broad “resonance” above T_c [21]. Therefore, we suggest that the meson correlators above T_c rather indicate a liquid QGP than a bound state in the QGP. To prove this assumption QCD lattice calculations of static parton pair correlators would be helpful.

2. Strong interactions in a plasma imply that the standard Coulomb scattering theory is not valid because standard Coulomb scattering theory works only if the Coulomb radius, \rho = q^2/E, of a particle with energy E is much smaller than the Debye length \lambda_D = 1/\mu. In the QGP close to T_c we use the non-perturbative estimate \mu = 6T [23] for the Debye screening mass corresponding to a Debye screening length of less than 0.2 fm at T = 200 MeV. The Coulomb radius for a thermal parton with energy 3T is given by \rho = C g^2/3T = 1 - 6 fm. Therefore we get \beta = \rho/\lambda_D = 5 - 30. Consequently the Debye screening length cannot be used as cutoff for calculating the transport cross section [24] from the differential t-channel cross section. This means that in a strongly coupled plasma the interaction range is much larger than the Debye screening length. This has been considered in the case of the interaction of the ions with the microparticles in a complex plasma [25], where ions with an impact parameter larger than the Debye screening length have been shown to be important. As a result a modified Coulomb logarithm \Lambda^* was derived by considering all ions that approach the microparticle closer than \lambda_D, yielding a cutoff for the impact parameter which can be several times the Debye screening length. This leads to a transport cross section (\sigma_t \sim \Lambda^*), which can be more than an order of magnitude larger than predicted by the standard Coulomb scattering theory. Whereas the standard Coulomb logarithm in the transport cross section is given by \Lambda = (1/2) \ln(1+1/\beta^2), the modified reads \Lambda^* = \ln(1+1/\beta), which is valid for about \beta < 10 [25]. For instance, for \beta = 10 \Lambda = 0.005 and \Lambda^* = 0.1 follows, leading to an enhancement of the transport cross section by a factor of 20.

This could explain the enhancement of the parton transport cross sections, which was concluded from the elliptic flow data [26], and the fast thermalization of the fireball in relativistic heavy-ion collisions which is required to explain the success of the hydrodynamical

neighbor, and so on. Also in a liquid 2 or 3 peaks can be observed in the pair correlation function or the structure factor, related by Fourier transformation. However, only the first one corresponding to the nearest neighbors is significant, whereas the following ones are small and broad [8,20].
description of hadron spectra [5], even if perturbative results are used. In order to estimate this effect in the QGP, we consider the transport cross section for elastic gluon scattering given in Eq. (3) of Ref. [26]:

$$\sigma_t(s) = 4 \sigma_0 z (1 + z) [(2z + 1) \ln(1 + 1/z) - 2],$$

(2)

where \(\sigma_0 = \frac{9\pi \alpha_s^2}{2(2\mu^2)}\) and \(z = \mu^2/s\) with the Debye mass \(\mu\) and the square of the center of mass scattering energy \(s\). The Debye screening mass \(\mu\) was used as an IR cutoff to obtain finite results. In a strongly coupled plasma with \(\beta = 10\), the maximum impact parameter is given by \(4.6\lambda_D\) (see Eq. (9) in Ref. [25]). Assuming now this value also for the QGP, we replace \(\mu\) in (2) by \(\mu/4.6 \simeq 1.3T\). Assuming, in addition, thermal energies for the gluons, leading to \(s \simeq 15T^2\) [27], we find a cross section enhancement by a factor of 13.

For higher parton energies \(E\) the Coulomb radius \(\rho = C g^2 / E\) becomes smaller and the transport cross section reduces to the perturbative results [27]. Of course, their might be additional sources for a cross section enhancement, such as higher order and non-perturbative contributions to the scattering amplitudes. Furthermore, non-linear screening effects in strongly coupled plasmas can lead to an interparticle potential different from a Yukawa potential, which results in a reduced screening and therefore enhanced scattering cross sections. Non-linear short range modifications of the screening potential are known, for instance, to play an important role in the enhancement of thermonuclear reaction rates in the late stages of stellar evolution [8].

Summarizing, strongly coupled plasmas have peculiar properties, in particular liquid or solid phases. Comparing with strongly-coupled, classical non-relativistic plasmas, such as complex plasmas, we conclude that the QGP is in the liquid phase. This may help to discover the QGP rather than looking for high-temperature limit (ideal QGP) properties from perturbative QCD or static properties from lattice QCD. Indeed, the QGP liquid explains qualitatively observed features of relativistic heavy-ion collisions, such as flow, cross section enhancement, and fast thermalization, and can be used to interpret lattice QCD results, e.g., meson correlators. Presumably, the examples discussed above are only a small selection for comparing properties of the QGP with liquid systems. We expect that there will be a number of interesting features of the data observed in ultrarelativistic heavy-ion collisions which can be understood by the analogy to liquid plasmas. For example, the equation of state and the quark number susceptibilities should be analyzed in the liquid phase, although the deviation from the weak coupling regime is not so pronounced in this case [28,29].

For a more quantitative description, we propose to use molecular dynamics simulations, similar as in the case of strongly coupled plasmas, but now for a classical, relativistic Yukawa system consisting of partons with an effective, temperature dependent mass. In this way, quantitative results for the equation of state, correlation functions, and other interesting quantities may be derived. Thus lessons and methods from the physics of strongly coupled plasmas may open up a new field of “high-energy plasma physics” or “plasma-particle physics” besides the transport description (Vlasov equation) of weakly coupled plasmas used for relativistic plasmas (electron-positron plasma, high-temperature QGP) already since the sixties [30,31].

The analogy to complex plasmas supports the evidences found so far in ultrarelativistic heavy-ion collisions for the discovery of a strongly coupled QGP. Such a system is even more
interesting than an ideal or perturbative QGP, testing a new phase of non-perturbative QCD besides the confinement phase.

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FIGURES

Fig.1: Hexagonal structures in a complex plasma (plasma crystal). The microparticles of radius 3.4 microns are illuminated by a laser and recorded by a CCD camera. The interparticle distance is about 200 microns.

Fig.2: Pair correlation function in the liquid phase of a complex plasma. The pronounced peak at $r = 0.2$ mm indicates the interparticle spacing. The other structures are statistical fluctuations.
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