Magnetization screening from gluonic currents and scaling law violation in the ratio of magnetic form factors for neutron and proton

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Abstract

The ratio $\mu_p G_E^p / G_M^p$ exhibits a decrease for four-momentum transfer $Q^2$ increasing beyond 1 GeV$^2$ indicating different spatial distributions for charge and for magnetization inside the proton. One-gluon exchange currents can explain this behaviour. The $SU(6)$ breaking induced by gluonic currents predicts furthermore that the ratio of neutron to proton magnetic form factors $\mu_p G_M^p / \mu_n G_M^n$ falls with increasing $Q^2$. We find that the experimental data are consistent with our expectations of an almost linear decrease of the ratio $\mu_p G_M^p / \mu_n G_M^n$ with increasing $Q^2$, supporting the statement that the spatial distributions of magnetization are different for protons and for neutrons.
The electromagnetic (e.m.) structure of the nucleon is currently subject to a renewed theoretical research. At four-momentum transfer $Q^2 > 1$ GeV$^2$, this interest is motivated by the recent measurements at JLab of the ratio $R_P = \mu_P G^p_E/Q^2 / G^p_M(Q^2)$ between the electric $G^p_E(Q^2)$ and magnetic $G^p_M(Q^2)$ form factors of the proton. While the previous extractions of the two form factors relied on the Rosenbluth separation, and thus on the scaling laws:

$$G^p_E(Q^2) = G^p_M(Q^2)/\mu_P = G^n_M(Q^2)/\mu_n = G_D(Q^2)$$  \hspace{1cm} (1)

using the dipole form factor

$$G_D(Q^2) = [1 + Q^2/0.71\text{GeV}^2]^{-2}$$  \hspace{1cm} (2)

the recent analysis is based on the measured recoil proton polarization in elastic scattering of polarized electrons up to $Q^2 \sim 5.5$ GeV$^2$. Methodologically the discrepancy between the results from Rosenbluth separation and polarization measurements is a quite interesting problem. A global re-analysis of the elastic electron-proton scattering data by Ref. \cite{2} confirms a self-consistent interpretation of the individual cross section data when using the Rosenbluth technique, but the inconsistency with polarization data is not resolved. An attractive idea which can potentially solve this problem was recently proposed \cite{3}. The remarkable decrease of the ratio $R_P$ from unity indicates not only a significant deviation from the simple scaling law but also from the non-relativistic constituent quark model (NRCQM). The new data show that the charge and magnetization inside the proton is distributed differently, which could arise \cite{4} through one-gluon exchange (OGE) currents dominant at high $Q^2$.

By now several calculations for the proton ratio $R_P$ within different hadronic models are available \cite{5, 6, 7, 8, 9, 10}. We refer to the recent review \cite{11}, where one can find a discussion of the most recent calculations which agree reasonably well with the trend of the experimental data and which will allow for predictions at higher $Q^2$ than presently accessible. We only note, that the implementation of relativity is an common feature of all these models and all emphasize the necessity of relativistic effects generated by both kinematical and dynamical $SU(6)$ breaking for the interpretation of the decrease of the ratio $R_P$.

In this paper we wish to discuss other ratios of form factors in order to find further support for the scaling law violation. In particular, we will investigate the ratio $R_M = \mu_P G^p_M/Q^2 / \mu_n G^p_M$. To this purpose we have plotted in Fig. 1, left, the reduced nucleon magnetic form factors $G^N_M = G^N_M/Q^2 / \mu G_D$, both normalized by their magnetic moment $\mu$ and $G_D$ given by

$$G_D(Q^2) = [1 + Q^2/0.71\text{GeV}^2]^{-2}$$  \hspace{1cm} (2)
Eq. (2). Exactly the same data sets as select in Ref. [12] are being used here (see Table 1 of [12]). Within errors, the neutron magnetic form factor $G_M^{n}$ follows the proton one up to $\sim 1 \text{ GeV}^2$, thereafter $G_M^{n}$ decreases faster than $G_M^{p}$. The left panel of Fig. 1 clearly shows a different shape for $G_M^{p}$ and $G_M^{n}$ in the $Q^2$ range above $1 \text{ GeV}^2$. The different $Q^2$-dependences result in a deviation from unity for the ratio $R_M$ as seen in Fig. 1 right. This ratio $R_M$ was obtained by interpolating the more numerous $G_M^{p}$ data to the obtain the divisor needed at the proper $Q^2$ of the $G_M^{n}$ values.

In this paper we will not follow the discussion of Ref. [12] concerning features below $Q^2 \sim 1 \text{ GeV}^2$ where pionic degrees of freedom contribute. Note, that the dip in both form factors at $Q^2 \sim 0.2 \text{ GeV}^2$ is purely a pion-cloud phenomena. A dip at low $Q^2$ was noted in the calculations of Ref. [6].

As well known, the nucleon e.m. form factors are functions of the square of the four-momentum transfer in the scattering process: $Q^2 = -q^\mu q_\mu$. The Sachs form factors, $G_{E(M)}$, can be obtained from the Dirac and Pauli form factors $F_1$ and $F_2$, respectively, which in turn are defined through the nucleon e.m. operator $J_{em}^\mu(x)$ satisfying the requirements of

![Figure 1: The reduced magnetic form factors $G_M^{N} = G_M^{n}/\mu G_D$ for proton and neutron (left). For the ratio $R_M = \mu_p G_M^{n}/\mu_n G_M^{p}$ the experimental results are compared to the present model calculation (right).](image-url)
relativistic covariance and the condition of gauge invariance. The Sachs form factors fully characterize the charge and current distributions inside the nucleon [13] and within the Breit frame the nucleon electric $G_E$ and magnetic $G_M$ form factors can be interpreted as Fourier transforms of the distributions of charge and magnetization, respectively,

$$\langle N_s'\left(\frac{q}{2}\right)|J_{\text{em}}(0)|N_s\left(-\frac{q}{2}\right)\rangle = \chi_{s'}^\dagger \frac{i\sigma \times q}{2M_N} \chi_s G_M(q^2)$$

$$\langle N_s'\left(\frac{q}{2}\right)|J^0_{\text{em}}(0)|N_s\left(-\frac{q}{2}\right)\rangle = \chi_{s'}^\dagger \chi_s G_E(q^2)$$

(3)

where $\chi_{s'}$ and $\chi_s$ are Pauli spinors for the initial and final nucleons. Note, that in the Breit frame the energy transfer vanishes and the incoming and outgoing momenta are $p = -q/2$ and $p' = q/2$; and thus $Q^2 = q^2$ follows.

We start our consideration of nucleon e.m. form factors from the non-relativistic constituent quark model [14], where the effective degrees of freedom are the massive quarks moving in a self-consistent potential whose specific form is dictated by QCD. Other degrees of freedom like Goldstone bosons or gluons are not considered in its original version and effectively absorbed into the constituent quarks. In its simplest version with the harmonic oscillator confining potential, the nucleon e.m. form factors $G_N^E$ and $G_N^M$ are

$$G_N^E(q^2) = e_N \cdot \exp\left(-\frac{q^2 b^2}{6}\right),$$

(4)

$$G_N^M(q^2) = \mu_N \cdot \exp\left(-\frac{q^2 b^2}{6}\right)$$

(5)

where $e_N$ and $\mu_N$ are the charge and magnetic moment of the nucleon

$$e_N = \frac{1}{2} \langle N|(1 + \tau_3)|N\rangle,$$

(6)

$$\mu_N = \frac{M_N}{m_q} \frac{1}{6} \langle N|(1 + 5\tau_3)|N\rangle$$

(7)

here $M_N$ and $m_q$ are the nucleon and quark mass, respectively. The constant $b$ in Eqs. (4) and (5) determines the average hadronic size of the baryon and usually is called the quark core radius. Due to the same momentum dependence, Eqs. (4) and (5) lead to the scaling law noted in Eq. (1). A ratio of unity is predicted by this ansatz for $R_p$ as well as for $R_M$. Clearly, the scaling law is in contradiction for the ratio $R_p$, considering the recent experiments [1], and to $R_M$ of Fig. 1 right.

The failure of the simplest non-relativistic concept, which otherwise is quite successful in spectroscopy, calls for other mechanisms or other degrees of freedom which can generate
the necessary effect. As emphasized in our recent study \[8\], the standard Isgur-Karl phenomenology provides such a possibility. The starting point is the short-ranged residual $qq$ interaction.

The phenomenological residual interaction $V^{res}$ can be based on various $qq$ potentials \[7, 14\], constrained by symmetries and the properties of QCD; still, its dynamical origin is rather uncertain. But since the effect of the residual $qq$ interaction is clearly seen in the excitation spectra of hadrons, one expects the corresponding interaction currents to play an important role for various e.m. properties of hadrons. In the presence of residual interactions between the quarks, current conservation requires that the total e.m. current operator of the hadron cannot simply be a sum of free quark currents, but must be supplemented by corresponding interaction currents. These currents are closely related to the $qq$ potential from which they can be derived by minimal substitution.

The OGE short-range potential between constituent quarks, $V^{res} = V^{OGE}$, can be derived from the QCD interaction Lagrangian:

$$L^{QCD}(x) = -\frac{\alpha_s}{2} \bar{\psi}(x) \lambda_\alpha \gamma^\mu G_\mu^\alpha(x) \psi(x)$$

(8)

where $G_\mu^\alpha$ and $\psi$ are the gluon the quark fields, respectively, and $\alpha_s$ is the strong coupling constant. The explicit non-relativistic form of $V^{OGE}$ can be found in Ref \[15\]. In the presence of the OGE force both, the photon and the gluons interacting with the quarks, can produce $q\bar{q}$ pairs leading to a pair-current contribution to the e.m. quark current operator. The e.m. currents we consider are depicted in Fig. 2. The non-relativistic reduction of these diagrams leads to the following OGE-induced configuration-space current operators \[16\] for charge $p_{3q}^{OGE}$ and magnetization $j_{3q}^{OGE}$

$$p_{3q}^{OGE} = -i \frac{\alpha_s}{16\pi^2 q} \sum_{i<j} \lambda_i \cdot \lambda_j \frac{Q_i}{r_{ij}^3} e^{iq \cdot (r_i - r_j)}$$
\[ j_{3q}^{OGE} = -\frac{\alpha_s}{8m_q^2} \sum_{i<j} \lambda_i \cdot \lambda_j \frac{Q_i}{r_{ij}^3} \times e^{i\mathbf{q} \cdot \mathbf{r}_i} \left[ (\sigma_i + \sigma_j) \times (\mathbf{r}_i - \mathbf{r}_j) \right] + (i \leftrightarrow j) \]

(10)

where \( \lambda_i \) are the Gell-Mann SU(3) colour matrices of the \( i \)-th quark normalized to \( \langle \lambda_i \cdot \lambda_j \rangle = -8/3 \) for a \( qq \) pair in a baryon, \( \sigma_i \) are Pauli matrices, \( \mathbf{r}_i \) is the coordinate of the \( i \)-th quark and \( Q_i \) is its charge in units of \( e \): \( Q_i = 1/2 \left[ 1/3 + \tau_i^3 \right] \). These OGE currents describe a \( q\bar{q} \) pair creation process induced by the external photon with subsequent annihilation of the \( q\bar{q} \) pair into a gluon, which is then absorbed by an another quark. It is evident, that production of additional \( q\bar{q} \) pairs will screen (distort) the primary distribution of the charge and magnetization originating due to the constituents.

As shown in Ref. [4], Eqs. (9) and (10) lead to the following electric

\[
G^{OGE}_{E_p}(q^2) = -\frac{\alpha_s}{m_q^3} q e^{-q^2b^2/24} \left\{ \begin{array}{c} \frac{1}{3} \\ -\frac{2}{9} \end{array} \right\} \mathcal{K}(q)
\]

(11)

and magnetic contributions to the form factors

\[
G^{OGE}_{M_p}(q^2) = \frac{\alpha_s}{m_q^2} \frac{M_N}{q} e^{-q^2b^2/24} \left\{ \begin{array}{c} \frac{2}{3} \\ -\frac{2}{9} \end{array} \right\} \mathcal{K}(q)
\]

(12)

where the function \( \mathcal{K}(q) \) can be expressed analytically in terms of generalized Hyper-Geometrical functions \( _sF_t(\alpha_1, \ldots, \alpha_s; \beta_1, \ldots, \beta_t; z) \):

\[
\mathcal{K}(q) = \frac{q}{12b} \sqrt{\frac{2}{\pi}} \left[ 3_2F_2(1, 1; \frac{3}{2}, 2; z) - 2F_2(1, 1; 2, \frac{5}{2}; z) \right]
\]

with \( z = -b^2q^2/8 \) and \( q = |\mathbf{q}| \).

Recently [4] we have suggested, that the dynamical SU(6) breaking induced by gluonic currents and the resulting screening corrections can be considered as a possible mechanism of the scaling law violation in the proton ratio \( R_p \) at momentum transfers \( Q^2 \) beyond 1 GeV\(^2\), where the soft pion cloud is assumed to be of lesser importance. Using the prescriptions of Ref. [17] for the Lorentz boost of the nucleon wave function, Eqs. (4), (5), (11) and (12) were taken to show [4], that the effect of gluonic currents to the CQM is important, and that the ratio \( R_p \) can be well reproduced by the \( SU(6) \times O(3) \) wave function from the NRCQM. For the neutron ratio \( R_n = \mu_n G_E^n/G_M^n \), this mechanism leads to the prediction, that \( R_n \) rises
when the proton ratio \( R_p \) falls with increasing momentum transfer \( Q^2 \) obeying our scaling relation \( \mu_n G_E^n / G_M^n \simeq 2/3 \) \( 1 - \mu_p G_E^p / G_M^p \) \[4\]. Obviously, due to the recent progress in using polarized nuclear targets, this statement can be verified experimentally.

Now we come to the another aspect of gluonic currents, namely to the scaling law violation in the ratio \( R_M = \mu_p G_M^n / \mu_n G_M^p \). As one can see from Eqs. (11) and (12), the key for the decrease of the proton ratio \( R_p \) relies on the opposite signs of the screening corrections to the electric (lead to reduction, i.e. charge screening) and magnetic (increase, i.e. magnetization anti-screening) form factors. Considering now \( R_M \), the gluonic currents contribute again with opposite signs: they are positive for \( G_M^p \) and negative for the \( G_M^n \). Because the bare nucleon magnetic momenta, \( \mu_p \) and \( \mu_n \) in Eq. (7), have also opposite signs (positive for the proton), the resulting effect is “anti-screening” in both cases. At the same time the magnitude of the gluonic corrections are larger by a factor 3 for the proton than for the neutron Eq. (12), which implies that

\[
\mu_p G_M^n / \mu_n G_M^p < 1 \tag{13}
\]

for \( Q^2 > 0 \). Clearly, the scaling law \( G_M^n / \mu_n = G_M^p / \mu_p \) is violated. We use Eqs. (11), (12), (11) and (12) to examine the resulting effect and particularly its \( Q^2 \) dependence. Our results for the ratio \( R_M \) are shown by the full line in Fig. 1 (right), where we have used the model parameters obtained from the best fit to the proton ratio \( R_p \) with \( \alpha_s = 0.4 \), \( b = 0.5 \) and \( m_q = 400 \text{ MeV} \). The trend of the data is reproduced.

One can argue that the dynamical \( SU(6) \) breaking induced by gluonic currents and the resulting e.m. screening are idealized pictures which can be justified only in the momentum transfer region where soft pionic effects are small \[6\]. We agree and our approach clearly indicates the presence of virtual \( |qqq + q\bar{q} \rangle \) (soft) Fock components in the nucleon wave function due to the small value of \( \alpha_s \). The fitted value is in the range of \( 0.4 < \alpha_s < 0.6 \), which however is not able to reproduce the \( N - \Delta \) mass splitting in the case of pure OGE exchange \[4\]. Note, that \( \alpha_s \) in perturbative QCD (pQCD) should go to zero for large inter-quark momenta. Here and in Ref. \[4\], we use \( \alpha_s \) as an effective and momentum independent constant. Following Ref. \[15\], we have proposed that the observed mass splitting can be the result of a linear combination of the pion-loop and OGE contributions. Consequently, the pionic contributions can produce the desirable effect of reducing the size of the strong coupling constant \( \alpha_s \).
Another important statement which can be verified, if the ratio $R_M$ will be measured with much higher precision, is the role of quark configurations with non-zero orbital angular momentum or equivalently the possible role of the nucleon deformation \[18\]. Note, that the dynamical $SU(6)$ breaking induced by OGE do not contradict hadron helicity conservation – this holds at least for our framework, because of the non-relativistic structure of nucleon wave function. At the same time, when imposing Poincaré invariance in a relativistic CQM causes substantial violation of the helicity conservation rule \[10\], and the resulting asymptotic behaviour of form factors differs from that expected in pQCD \[19\]. By now it is well established that the behavior of the ratio of Pauli to Dirac form factors, $Q F_2(Q^2)/F_1(Q^2)$, which is approximately constant in this $Q^2$ range, indicates the presence of quarks in the proton with non-zero orbital angular momentum. We propose to consider the ratio $R_M$ and its decrease as an additional test for the models where relativistic effects are generated by kinematical $SU(6)$ breaking due to the Melosh rotation of the constituent spins \[8, 10\].

![Diagram](image)

**FIG. 3:** The data and theoretical curves show a nearly linear decrease of the ratio $R_M = \mu_p G_M^n / \mu_n G_M^n$ with $Q^2$. The grey area between the two solid curves indicates the range of $R_M$ due to the variation of parameters obtained from Ref. \[4\] by fitting the ratio $R_p = \mu_p G_E^n / G_M^n$. Furthermore, the nearly linear decrease of $R_M$ with increasing $Q^2$ is demonstrated by the dash-triple-dotted line obtained from a fit to the data in Fig. [3]. This decrease is slower than that for $R_p$ (dot-dashed line). Finally to give a measure of sensitivity, the shaded area indicates the variation of the calculated ratio for values of $\alpha_s$ within the range of 0.2 to 0.6, which are the same around the optimum value of 0.4 as used in the analysis of $R_p$ \[1\].
Future studies of contributions due to the pion cloud are expected to reduce the spread of parameters.

In summary, we have noted a scaling law violation in the ratio of the neutron to proton magnetic form factors $R_M$, deviating from unity for $Q^2 > 1 \text{ GeV}^2$. We have proposed that $SU(6)$ breaking induced by OGE and the resulting e.m. screening produces this decrease, which is less pronounced but of the same origin as that for the proton ratio $R_p$ seen in the recent polarization transfer measurements from JLab. The good description by the model supports the idea of a different spatial distribution of magnetization inside neutron and proton. We also mention, that our results are instructive, both theoretically and experimentally, suggesting further checks of our statements against other hadronic models. Certainly, to confirm our results, new data on neutron and proton magnetic form factors with much higher precision in the momentum transfer region $Q^2 > 1 \text{ GeV}^2$ are needed. This will allow to employ the ratio $R_M$ in combination with $R_p$ as a powerful test for any theoretical models of nucleon structure.

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