ADAFs – Models, Observations and Problems

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Abstract

We review some of the properties of Advection-Dominated Accretion Flow (ADAF) models and show that they successfully describe many astrophysical systems. Despite these successful applications some fundamental problems still remain to be solved, the most important one being the physics of the transition between an ADAF and a geometrically thin Keplerian disc.

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1 Introduction

‘Advection Dominated Accretion Flows’ (ADAFs) is a term describing infall of matter with angular momentum, in which radiation efficiency is very low. In their applications, ADAFs are supposed to describe inflows onto compact bodies, such as black holes or neutron stars; but very hot, optically thin flows are bad radiators in general so that, in principle, ADAFs are possible in other contexts. Of course in the vicinity of black holes or neutron stars, the virial (gravitational) temperature is $T_{\text{vir}} \approx 5 \times 10^{12} r^{-1}$ K, where $r$ is the radial distance measured in the units of the Schwarzschild radius $R_S = 2GM/c^2$, so that in optically thin plasmas, at such temperatures, both the coupling between ions and electrons and the efficiency of radiation processes are rather feeble. In such a situation, the thermal energy released in the flow by the viscosity, which drives accretion by removing angular momentum, is not going to be radiated away, but will be advected towards the compact body. If this compact body is a black hole, the heat will be lost forever, so that advection, in this case, acts like a ‘global’ cooling mechanism. In the case of infall onto a neutron star, the accreting matter lands on the star’s surface and the (reprocessed) advected energy will be radiated away. There, advection may act only as a ‘local’ cooling mechanism. (One should keep in mind that, in general, advection may also be responsible for heating, depending on the sign of the temperature gradient - for example,
according to Nakamura et al. (1997), in some conditions, near the black hole, advection heats up electrons in a two-temperature ADAF).

The role of advection in an accretion flow depends on the radiation efficiency, so that it depends on the microscopic state of matter and on the absence or presence of a magnetic field. If, for a given accretion rate, radiative cooling is not efficient, advection is necessarily dominant, assuming that a stationary solution is possible (see below).

1.1 Simple ADAF solutions

One can illustrate fundamental properties of ADAFs by a simple example. The advection ‘cooling’ (per unit surface) term in the energy equation can be written as

\[ Q_{\text{adv}} = \frac{\dot{M}}{2\pi R^2} c_s^2 \xi \]  

(1)

where \( \dot{M} \) is the accretion rate, \( c_s \) the (adiabatic) speed of sound and \( \xi = - \left[ (\gamma - 1)^{-1} (d \ln T / d \ln R) - (d \ln \Sigma / d \ln R) \right] \) is the dimensionless advection factor characterizing the entropy gradient (Abramowicz et al. 1995). \( \gamma \) is the ratio of specific heats and we assume that the pressure is provided only by gas.

Using the hydrostatic equilibrium equation

\[ \frac{H}{R} \approx \frac{c_s}{v_K} \]  

(2)

where \( H \) is the disc semi-thickness and \( v_K = \sqrt{GM/R} \equiv c/\sqrt{2r} \) the Keplerian velocity, one can write the advection term as

\[ Q_{\text{adv}} = \frac{\dot{m} \xi}{2\kappa_{\text{es}} R S} \left( \frac{c}{r} \right)^3 \left( \frac{H}{R} \right)^2 \]  

(3)

whereas the viscous heating term can be written as

\[ Q_{\text{vis}} = \frac{3\dot{m}}{8\kappa_{\text{es}} R S} \left( \frac{c}{r} \right)^3 \]  

(4)

where \( \dot{m} = (\dot{M} \kappa_{\text{es}} / 4\pi GM) \) and \( \kappa_{\text{es}} \) is the electron-scattering opacity coefficient. Therefore, for very high temperatures, when \( (H/R) \sim 1 \) the advection term is comparable to the viscous term and cannot be neglected.

From Eqs. (3) and (4) one can easily obtain an advection dominated solution by writing

\[ Q_{\text{vis}} = Q_{\text{adv}} \]  

(5)

and using

\[ \left( \frac{H}{R} \right)^2 = \frac{\sqrt{2}}{\kappa_{\text{es}}} \dot{m} (\alpha \Sigma)^{-1} r^{-1/2}. \]  

(6)
An ADAF is then described by the relation
\[ \dot{m} = 0.53 \kappa_c \alpha r^{1/2} \Sigma. \] (7)

In Eq. (6) we use the mass and angular momentum conservation equations and the viscosity prescription \( \nu = (2/3)c_s^2/\Omega_K \) \( (\Omega_K = v_K/R) \).

1.2 Maximum accretion rate

Of course, the energy equation is
\[ Q_{\text{vis}} = Q_{\text{adv}} + Q_{\text{rad}} \] (8)
where \( Q_{\text{rad}} \) is the radiative cooling (per unit surface), so that the existence of ADAF solutions depends on \( Q_{\text{rad}} \). There is no universal form for this term, which depends on the microscopic state of the matter, its content and the optical thickness. In the simplest case of an optically thin, one-temperature flow, cooled by non-relativistic free-free processes,
\[ Q_{\text{rad}} \sim \rho^2 T^{3/2} \sim \alpha^{-2} \dot{m}^{-2} r^{-2} \] (9)
so that the cooling term has a steeper dependence on \( \dot{m} \) than both \( Q_{\text{vis}} \) and \( Q_{\text{adv}} \). One can see therefore that there exists a maximum value of \( \dot{m} \) (Abramowicz et al. 1995) for which an ADAF solution is possible: \( \dot{m} \sim \alpha^2 r^{-1/2} \). We have assumed here that Eq. (9) applies for this value of \( \dot{m} \) (see below).

Abramowicz et al. (1995) found, in the non-relativistic free-free cooling case, that the maximum accretion rate for an ADAF solution is
\[ \dot{m}_{\text{max}} = 1.7 \times 10^3 \alpha^2 r^{-1/2}. \] (10)
The value of \( \dot{m}_{\text{max}} \) depends on the cooling in the flow and non-relativistic free-free cooling is not a realistic description of the emission in the vicinity \( r \approx 10^3 \) of a black hole. The flow there will most probably form a two-temperature plasma (Narayan & Yi 1995). More realistic calculations in a 2T flow by Esin et al. (1997) give \( \dot{m}_{\text{max}} \approx 10 \alpha^2 \) with almost no dependence on radius. For larger radii \( \dot{m}_{\text{max}} \) decreases with radius.

1.3 Maximum \( \alpha \)

The existence of a maximum accretion rate for an ADAF is, as explained above, due to the form of the cooling law. For radii \( r \approx 10^3 \) optically thin cooling of gas pressure dominated plasma implies the existence of such a critical accretion rate. If, however, effects of optical thickness and of radiative pressure become important, the slope of \( Q_{\text{rad}}(\dot{m}) \) will change and, as shown by Chen et al. (1995), one may get families of solutions in which an ADAF is possible for all values of \( \dot{m} \) because \( Q_{\text{rad}}(\dot{m}) \) has a maximum for \( Q_{\text{rad}} < Q_{\text{adv}} \). The existence
of such solutions depends on the value of $\alpha$: $\dot{m}_{\text{max}}$ exists only for $\alpha < \alpha_{\text{max}}$, where the value of $\alpha_{\text{max}}$ depends on the cooling mechanism and radius. As a simple example one can use a 1T disc and consider that the simple free-free cooling formula is not valid for $\tau_{\text{eff}} \gtrsim 1$, where the effective optical depth $\tau_{\text{eff}} = \sqrt{\kappa_{\text{ff}}(\kappa_{\text{ff}} + \kappa_{\text{es}})} \Sigma / 2$ and $\kappa_{\text{ff}}$ is the free-free opacity. Using a Planck mean value for this optical depth, one obtains from $\dot{m}_{\text{max}} < \dot{m}(\tau_{\text{ff}} = 1)$ a condition for the viscosity parameter:

$$\alpha < \alpha_{\text{crit}} \approx r$$  \hspace{1cm} (11)

Chen et al. (1995) obtained values of $\alpha_{\text{crit}}$ between 0.2 and 0.4 for $r = 5$, depending on the model, and Björnsson et al. (1996) get $\alpha_{\text{crit}} > 1$. If the viscous stress is assumed to be proportional to the gas, and not to the total pressure, the value of $\alpha_{\text{crit}} > 1$ (Lovás 1998) and is of no physical interest. It seems that for physically interesting configurations ($\alpha \lesssim 1$) there is always a $\dot{m}_{\text{max}}$ for ADAFs but one should not forget that the existence of such a maximum accretion rate is not a generic property of advective flows.

### 1.4 Slim disc solutions

For $\alpha < \alpha_{\text{crit}}$ there exists a separate branch of solutions which represents for $\dot{m} \lesssim 1$ the standard Shakura-Sunyaev discs. Higher accretion rates are represented by the so-called ‘slim discs’ (Abramowicz et al. 1988). It is sometimes said that slim discs represent solutions in which advection dominates because the optical depths are so high that photons are trapped in the flow. It is easy to see, however, that the optical depth of slim discs, for a rather large range of parameters, is rather low. In fact, it is the decrease of the optical depth with increasing accretion rate that is at the origin of the slim disc branch of solutions. Slim discs are only asymptotically advection dominated (see Fig 1. in Abramowicz et al. 1988).

### 2 Global solutions

The so called ‘self-similar’ solutions found by Narayan & Yi (1994) played a very important role in the development of ADAF astrophysics because they allowed Narayan and collaborators (see Narayan, Mahadevan and Quataert 1998, for a recent review) to construct models which could be quickly compared with observations. These comparisons showed that ADAFs provide an excellent description of such systems as the Galactic Center source Sgr A* (Narayan, Yi & Mahadevan 1995) and quiescent soft X-ray transients (SXTs) (Narayan, McClintock & Yi 1995). Of course, these models were rather crude and not self-consistent (for example, the ‘advection parameter’ $f$, which must be constant for a ‘self-similar’ solution varied with radius) but after more refined and consistent models had been calculated, they confirmed most of the prediction of the ‘self-similar’ ones. In fact, major revisions of the early models concerned
only the value of the transition radius between the ADAF and the outer geometrically thin disc, whose presence is required by observations of SXTs and the AGN NGC 4258 (see next section).

The first 2T, global, optically thin ADAF solutions were obtained by Matsumoto, Kato & Fukue (1985). In Chen, Abramowicz & Lasota (1997) and Narayan, Kato & Fukue (1997) one finds the first detailed study of the properties of global ADAF solutions. The Narayan et al. (1997) work deals with pure advection dominated solutions whereas Chen et al. (1997) consider more general solutions which allow for bremsstrahlung cooling.

Figs. (1) and (2) show the characteristic properties of global ADAF solutions. They share some properties with slim disc solutions, such as the sub-Keplerian character of the flow for ‘high’ values of $\alpha$ and a super-Keplerian part of the flow for low values of this parameter. These features are related to the existence of a maximum pressure in the flow. Narayan et al. (1997a) argue that a pressure maximum is necessary for the existence of ‘funnels’ that appear in ‘iron tori’ (Rees et al. 1982). Since no flow models for iron tori exist it is difficult to tell the difference between these structures and ADAFs.

Figs. (1) and (2) show also the influence of the maximum accretion rate for ADAFs: for low $\alpha$’s and/or high $\dot{m}$’s a flow which is advection dominated at small radii ceases to be an ADAF at larger radii.

First studies of global ADAF solutions used the Paczyński-Wiita (1980) approximation of the black effective potential, but such an approach is not satisfactory in the case of a rotating black hole. After the first ADAF solutions in a Kerr spacetime were found by Abramowicz et al. (1996), several other groups produced such ADAF models (Peitz & Appl 1997; Gammie & Popham 1998; Popham & Gammie 1998). Jaroszyński & Kurpiewski (1997) are the only authors who also calculated ADAF spectra in the framework of General Relativity. Fig (3) show Abramowicz et al. (1996) solutions for three values of the black hole angular momentum. One can see that these solutions are very similar to the “pseudo-relativistic” ones. In fact for a non-rotating black hole the Paczyński-Wiita (1980) ansatz is an excellent approximation.

3 Applications

Despite several rather serious uncertainties about the physics of ADAFs, their models are enjoying a growing field of application. The reason is simple. In many astrophysical systems powered by accretion (see below), the X-ray luminosity, which is supposed to probe accretion onto the central compact object, is very weak compared with the expectations based on independent estimates of the accretion rate and the assumption of high ($\sim 0.1$) radiative efficiency. The only model which predicts such properties of accreting systems is the ADAF model, of which a low radiative efficiency is a fundamental feature.

Most models which reproduce observed spectra by ADAFs use the approach
pioneered by Ramesh Narayan and collaborators. It consists in fixing ‘microscopic’ parameters of the flow, such as the ratio of the magnetic to gas pressure, the fraction of viscous dissipation that goes directly to electrons, the $\alpha$-parameter, the thermodynamical parameters, etc. Then the accretion rate is determined by adjusting to the observed X-ray flux. In a pure ADAF solution (no external disc) this determines the whole spectrum so that no additional freedom is allowed. The ‘microscopic parameters’ are considered to be universal so this procedure is, in fact, an one-parameter ‘fit’ (assuming of course that the black hole mass is given by independent observations). In the case of a two-component flow (ADAF + accretion disc) a second ‘parameter’ has to be fixed: the transition radius between the two flows. In principle this should not be a free parameter, but should be given by the physics of this transition. This is however a weak (the weakest according to the present author) point of the ADAF ‘paradigm’: the mechanism of transition between the two flows is unknown. There exist the ‘usual suspects’, such as evaporation, but a consistent physical model is yet to be found. A principle according to which the flow will be an ADAF when, at a given radius, an ADAF and an accretion disc solutions are possible has been used (see e.g. Menou, Narayan & Lasota 1998) but the case of NGC 4258 seems to contradict this principle (see Sect. 3.3).

3.1 Sgr A*  
Rees (1982) was the first to suggest that accretion onto the Galactic Center black hole might be advection-dominated. Detailed models by Narayan et al. (1995,1998) and Manmoto, Mineshige & Kusunose (1997) provide rather impressive fits to observations from radio to X-ray frequencies. In this case the model is a pure ADAF; no outer accretion disc is present.

3.2 Soft X-ray Transients and ‘Low state X-ray binaries’  
Soft X-ray Transients (SXTs) are a natural field for application of ADAF models since an accretion disc model is unable to fit the observations of these systems in quiescence (Lasota 1996). In the first ADAF model of quiescent SXTs proposed by Narayan, McClintock & Yi (1996) an outer stationary accretion disc was responsible for the observed emission in optical and UV frequencies. It was shown however, by Lasota, Narayan & Yi (1996), that it is impossible to find an ADAF + accretion disc configuration which would be consistent both with observations and with the requirements of the disc instability model, which is supposed to describe SXT outbursts. A new version of the model was proposed by Narayan, Barret & Yi (1997) in which the contribution of the outer disc was negligible. This was achieved by increasing the transition radius and increasing the magnetic to gas pressure ratio so that the optical/UV emission is due to synchrotron radiation from the ADAF.
The validity of this model has been independently confirmed by Hameury et al. (1997). They reproduced the multi-wavelength properties of the rise to outburst of GRO J1655-40, including the observed 6 day delay between the rise in optical and X-rays, by using the disc instability code of Hameury et al. (1998). As initial conditions they used parameters given by an ADAF fit to the quiescent spectrum. They also showed that it is very difficult (if not impossible) to reproduce these observations with different initial conditions.

Narayan (1996) proposed to interpret various spectral and luminosity states observed in X-ray binaries using ADAF + accretion disc models with varying accretion rate and transition radius. This idea has been applied to the black hole XRT X-ray Nova Muscae 1991 (Esin et al. 1997) and to Cyg X-1 (Esin et al. 1998; the first to apply an ADAF model to this system was Ichimaru 1984). The existence of a two-component flow with varying accretion rate and transition radius in the XNova Muscae 1991 was confirmed by observations of the X-ray reflected component in the Ginga spectra of this object, but these observations suggest (Zycki, Done & Smith 1998) that the Esin et al. (1997) model requires some modifications.

### 3.3 NGC 4258

This LINER is a very important testing ground for the ADAF models. The black hole mass in this system ($3.6 \times 10^7 M_\odot$) is very well determined due to the presence of narrow, water maser lines (Miyoshi et al. 1997). The observed X-ray luminosity is $\sim 10^{-5} L_{\text{Eddington}}$ and the bolometric luminosity is no more than an order of magnitude larger. Lasota et al. (1996b) proposed, therefore, that the inner accretion in this system proceeds through an ADAF. New observations in infrared (Chary & Becklin 1997) and a new upper limit on the radio flux (Herrnstein et al. 1998) constrain the transition radius to be $r_\text{tr} \sim 30$ (Gammie, Blanford & Narayan 1998). In this model the accretion rate would be $\dot{m} = 9 \times 10^{-3}$, much higher than the value of $\dot{m} \sim 10^{-5}$ proposed by Neufeld & Maloney (1995) in their model of the masing disc, but in agreement with values obtained by Maoz & McKee (1997) and Kumar (1997) for the same disc. It seems that the value proposed by Neufeld & Maloney (1995) is excluded by the IR and X-ray data. In NGC 4258, an ADAF solution is possible for $r > r_\text{tr}$, so that the validity of the principle according to which an ADAF is preferred over an accretion disc whenever the two solutions are possible seems questionable.

### 3.4 LINERs and weak AGNs

Lasota et al. (1996b) suggested that also other LINERS and ‘weak’ AGNs could contain ADAFs in their inner accretion regions. A recent analysis of the variability of such systems (Ptak et al. 1998) brings new arguments in favour of this hypothesis.
3.5 Nuclei of giant elliptical galaxies

Fabian & Rees (1995) proposed that the well known problems with understanding the properties of nuclei of giant elliptical galaxies, which emit much fewer X-rays than one would expect from independent estimates of the accretion rate, could be solved if the accretion flow formed an ADAF. Reynolds et al. (1996) and di Matteo & Fabian (1997) applied ADAF models to the nuclei of M87 and M60 respectively. The latter authors point out that observations of the predicted spectral turnover at $\sim 3 \times 10^{11}$ Hz puts constraints on the values of the ‘microscopic’ parameter of the ADAF.

3.6 BL Lac’s

In BL Lac’s the emission is dominated by the jet but several properties of these systems points out to the possible presence of an ADAF (Madejski 1996; Celotti, Fabian & Rees 1997).

3.7 Remnant and MACHO black holes in the ISM

Fujita et al. (1998) noticed that if black holes accreting from the interstellar medium formed an ADAF, the characteristic ADAF spectral form would help in their detection.

4 Conclusion

ADAF models find successful applications in many domains of accretion astrophysics. They are the only models which describe both the dynamics of the accretion flow and its emission properties. In any case, all models which require the presence of optically thin, very hot plasmas (such as ‘coronal’ models) must take into account advective heat transport in order to be self-consistent (see Section 1.2). ADAFs are the only solutions that satisfy this requirement.

Of course, as mentioned above, there are still serious problems to be solved before one can conclude that ADAFs, in their present form, are the models of accretion flows around black holes at low accretion rates. The main problem to be solved is that of the transition between an accretion disc and an ADAF. Furthermore, in order to reproduce data, ADAF should form two-temperature plasmas with ions at virial temperature and much cooler electrons. This requires weak coupling between ions and electrons and implies that viscosity heats mainly the ions. Some recent studies suggest that it is difficult to achieve this together with equipartition between magnetic and gas pressures (Quataert & Gruzinov 1998). Such calculations however, concern extremely complicated physical processes so that it is not clear that results of numerical simulations apply to real systems. One should therefore apply a more pragmatic attitude, that the apparently successful applications of ADAFs to many astrophysical systems suggests that no
mechanisms coupling ion and electron exist in real astrophysical configurations (Fabian & Rees 1995).

The validity of ADAF models will be decided by observations.

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Figure 1: Disk solutions for $\alpha = 0.1$ and $\dot{m} = 10^{-5}$ - solid lines and $\alpha = 0.1$ and $\dot{m} = 10^{-2}$ - dashed lines. The mass of the black hole is $10M_\odot$. $\mathcal{M}$ is the Mach number, $\Omega$ the angular frequency of the flow and $\xi$ the advection parameter. The heavy lines are the global solution and the thin lines are the corresponding self-similar solution. Note that $\mathcal{M}$, $\Omega$, and $\xi$ depend on $\dot{m}$ very weakly.
Figure 2: Solutions for $\alpha = 0.01$ and $\dot{m} = 10^{-5}$ - solid lines and $\alpha = 0.001$ and $\dot{m} = 10^{-5}$ - dashed lines. The black hole mass is the same as in Fig. (1). Heavy lines correspond to global solutions and thin lines correspond to self-similar solutions. Note the super-Keplerian angular momentum and the maximum pressure near the transonic region. Note also that for $\alpha = 0.001$ and $\dot{m} = 10^{-5}$, the local cooling becomes important for large radii.
Figure 3: The radial structure of the Mach number ($\mathcal{M}$), the pressure ($p$ in cgs unit), the angular momentum ($\mathcal{L}$ in unit of $M$), and the sound speed ($c_s$ in unit of the speed of light). The mass of the black hole is $10 M_\odot$, $\alpha = 0.1$ and $\dot{M}/M_E = 10^{-5}$. The solid, dotted, and dashed lines represent the cases of $a/M = 0, 0.5, $ and $0.99$ respectively. The heavy dots represent solutions obtained with the pseudo-Newtonian potential. These solutions are excellent approximation to the solutions representing Schwarzschild black hole flows (in the case $a = 0$). The corresponding Keplerian angular momenta of test particles around Kerr black holes are also shown for comparison (the thin lines).