Lorentz-violating contributions of the Carroll-Field-Jackiw model to the CMB anisotropy

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We study the finite temperature properties of the Maxwell-Carroll-Field-Jackiw (MCFJ) electrodynamics for a purely spacelike background. Starting from the associated finite temperature partition function, a modified black body spectral distribution is obtained. We thus show that, if the CMB radiation is described by this model, the spectrum presents an anisotropic angular energy density distribution. We show, at leading order, that the Lorentz-breaking contributions for the Planck’s radiation law and for the Stefan-Boltzmann’s law are nonlinear in frequency and quadratic in temperature, respectively. Using our results, we set up bounds for the Lorentz-breaking parameter, and show that Lorentz violation in the context of the MCFJ model is unable to yield the known CMB anisotropy (of 1 part in 10^5).

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I. INTRODUCTION

In his seminal work [1], Maxwell proposed that light requires a medium to travel, in a full analogy to the experiences involving waves propagation in fluids. Such medium was named as ether. In view of the already observed light properties, it was assumed that the ether permeated the whole space, was of a negligible density, and had imperceptible interaction with matter. However, the ether was abandoned with the advent of the special theory of relativity [2] that established Lorentz covariance as one of the fundamental symmetries of nature. Nowadays, the Lorentz covariance pervades all the field theories describing fundamental interactions and has the status of a cornerstone in the construction of all modern physical theories. The present experiments confirm Lorentz invariance to a very high precision at currently accessible energy scales that goes up to 2 TeV. The new experiments to be performed in the Large Hadron Collider (LHC) at CERN, that will extend the energy scale to approximately 14 TeV, should test the Lorentz symmetry to confirm that it still remains unspoiled or to reveal some indications about its violation. At the moment, the possibility is discussed of Lorentz and CPT symmetry breaking at Planck scale (or in the very early Universe when energies are close to the Planck scale). One such scenario is suggested by string theory [3] and it is a key feature of noncommutative field theories [4].

The researches about Lorentz and CPT violation are commonly performed under the framework of the standard model extension (SME) developed by Colladay and Kostelecky [5]. The SME is an enlarged version of the usual standard model that embraces all Lorentz-violating coefficients (generated as vacuum expectation values of tensor quantities belonging to a fundamental theory defined at Planck scale) that yield Lorentz scalars (as tensor contractions) in the observer frame. Such coefficients rule Lorentz violation in the particle frame, where they are seen as sets of independent numbers, whereas they work out as genuine tensor in the observer frame. A strong motivation to study the SME is the necessity to get some information about underlying physics to the Planck scale where the Lorentz symmetry may be broken due to quantum gravity effects. The photon sector of the SME has been extensively studied with a double purpose: the determination of new electromagnetic effects induced by the Lorentz-violating (LV) interactions and the imposition of stringent upper bounds for the magnitudes of the LV coefficients. Lorentz violation has been investigated in a broad perspective in the latest years [6, 7].

The research about LV effects on classical electromagnetism was started by Carroll-Field-Jackiw [8], who studied the Maxwell electrodynamics in the presence of the assigned Carroll-Field-Jackiw (CFJ) term \( \epsilon^{\mu
u\kappa\lambda} (k_{AF})_{\mu} A_{\nu} F_{\kappa\lambda} \), with \((k_{AF})_{\mu}\) standing for the LV fixed background. The gauge sector of the SME embodies a CPT-odd CFJ term and the CPT-even one, \( W^{\mu\nu\kappa\lambda} F_{\mu\nu} F_{\kappa\lambda} \), both of which imply vacuum birefringence [8, 9, 10], which takes place whenever the light velocity depends on the polarization mode, amounting to a rotation in the polarization plane. While the CFJ...
term yields a causal, stable, and unitary electrodynamics only for a purely spacelike background \cite{11}, the CPT-even term provides an electrodynamics not plagued with stability illness. Regarding that birefringence increases linearly with the distance traveled, the analysis of this effect over cosmological scales offers an exceptionally sensitive signal for Lorentz violations. Within this context, as the cosmic microwave background (CMB) is partially polarized, it can be considered an experimental optical probe able to catch minuscule Lorentz violations \cite{12,13}. In \cite{13}, the spacelike sector of the CFJ background, $k_{AF}$, has been analyzed and has been shown that experimental data from the Boomerang experiment and the 5-year Wilkinson Microwave Anisotropy Probe (WMAP) survey are consistent with weak Lorentz violation at the 1-sigma level. Other issues concerning LV corrections to the Planckian spectrum were addressed in Ref. \cite{14}, where the emission and absorption of radiation by nonrelativistic electrons in the SME framework was properly regarded to derive such effects.

The CMB, which is the oldest thermal radiation available to observation, constitutes a good scenario to be described by the Lorentz-violating photon sector of the SME at finite temperature. The detected CMB is interpreted as compelling evidence for the big bang, since theoretical nucleosynthesis calculations foresees the existence of a cosmic background radiation at a temperature of some kelvins \cite{12}. The data coming from the Cosmic Background Explorer (COBE) and WMAP revealed that the CMB is a perfect Planckian black body distribution at 2.73 K with high precision of one part in $10^5$, which bounds the anisotropies to this small extent \cite{16,17}.

Considering that the light propagation is affected by the Lorentz violation, it is probable that its thermodynamical properties and spectral distribution are also altered. Therefore the black body pattern of the CMB is an interesting phenomenon where Lorentz-violating effects may play a relevant role, mainly when concerned with the anisotropies of the spectrum, an issue that captures broad attention nowadays \cite{16,17,18,19}. A natural framework to deal with black body radiation and Lorentz violation is the finite temperature field theory \cite{20}. The aim of the present work is to study the finite temperature properties of the MCFJ electrodynamics for the case of a purely spacelike background (for which the model provides a positive-definite Hamiltonian). Indeed, taking as starting point the MCFJ Lagrangian, we can construct the partition function for this gauge model (after the constraints structure is well determined). Such partition function provides all thermodynamical information required, including the energy density distribution. We thus show that if the CMB radiation is described by the MCFJ model, the radiation presents an anisotropic angular energy density distribution. Consequently, it is possible to obtain the Lorentz-breaking contributions to the Planck’s radiation law and to the Stefan-Boltzmann’s law.

This paper is outlined as follows. In Sec. II we develop the Hamiltonian analysis of the constraints structure of the MCFJ model by following the Dirac formalism for constrained systems. In Sec. III we construct the partition function (into the functional formalism) and study the thermodynamical properties of the model, including the modified energy density distribution and the modified Stefan-Boltzmann’s law. In the last section, we present our conclusions and final remarks.

II. THE MCFJ ELECTRODYNAMICS: HAMILTONIAN STRUCTURE

The Maxwell-Carroll-Field-Jackiw model is defined by the following Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \epsilon^{\mu\nu\rho\lambda} (k_{AF})_{\rho\lambda} A_\mu F_{\nu\lambda},$$

\hspace{1cm} (1)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic stress tensor, $(k_{AF})_{\rho\lambda}$ is the Lorentz-breaking vector background, and $\epsilon^{\mu\nu\rho\lambda}$ is the totally antisymmetric Levi-Civita tensor with $\epsilon^{0123} = 1$. The corresponding Euler-Lagrange equation for the vector field is

$$\partial_\nu F^{\nu\mu} + (k_{AF})_{\mu} \tilde{F}^{\mu\nu} = 0,$$

\hspace{1cm} (2)

where $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\nu\mu\rho\beta} F_{\rho\beta}$ is the dual tensor. Since the pioneering work of Carroll-Field-Jackiw \cite{3}, the properties of the MCFJ electrodynamics were extensively investigated in several distinct respects \cite{11,21,22}. In the present work, the goal is to evaluate the LV corrections to the Planck black body distribution, which will be done by means of the imaginary-time formalism for finite temperature field theory. Once the partition function is carried out, the entire thermodynamics of the model becomes available. For it, we should first try to understand the constraint structure of the model, unveiled by a careful Hamiltonian analysis.

In order to accomplish the Hamiltonian analysis of this model, we begin defining the canonical conjugate momentum

$$\pi^\mu = -F^{\rho\mu} - \frac{1}{2} \epsilon^{0\mu\alpha\beta} (k_{AF})_{\alpha\beta} A_\beta,$$

\hspace{1cm} (3)

with which we can write the fundamental Poisson brackets (PB): \(\{A_\mu (x), \pi^\nu (y)\} = \delta^\nu_\mu (x - y)\).
From Eq. (3), it is easy to note that \( \pi^0 = 0 \); such a null momentum yields a primary constraint \( \dot{\phi}_1 = \pi^0 \approx 0 \) (into the Dirac formalism, the symbol \( \approx \) denotes a weak equality). Also, the momenta \( \pi^k \) are defined via the following dynamic relation:

\[
\pi^k = \dot{A}_k - \partial_k A_0 - \frac{1}{2} \epsilon^{0kij} (k_{AF})_i A_j,
\]

while the canonical Hamiltonian density is explicitly written as

\[
\mathcal{H}_C = \frac{1}{2} \left( \pi^k \right)^2 + \pi^k \partial_k A_0 + \frac{1}{4} (F_{jk})^2 + \frac{1}{2} \pi^k \epsilon^{0kij} (k_{AF})_i A_j + \frac{1}{8} \left[ \epsilon^{0kij} (k_{AF})_i A_j \right]^2 + \frac{1}{4} \epsilon^{0kij} (k_{AF})_0 A_k F_{ij} - \frac{1}{4} \epsilon^{0kij} (k_{AF})_k A_0 F_{ij}.
\]

Following the usual Dirac procedure, we introduce the primary Hamiltonian (\( H_P \)) by adding to the canonical Hamiltonian all the primary constraints, \( H_P = H_C + \int d^3 y \ C \pi^0 \), where \( C \) is a bosonic Lagrange multiplier. The consistency condition of the primary constraint, \( \pi^0 = \{ \pi^0, H_P \} \approx 0 \), gives a secondary constraint

\[
\phi_2 = \partial_k \pi^k + \frac{1}{4} \epsilon^{0kij} (k_{AF})_k F_{ij} \approx 0,
\]

which reveals that the usual Gauss’s law is modified by an arbitrary Lorentz-breaking background. It can be written in terms of the electric and magnetic fields: \( \nabla \cdot \mathbf{E} + (k_{AF}) \cdot \mathbf{B} = 0 \). Therefore, for the case of a pure timelike background, no modification is implied on the usual Gauss law. The situation changes for a pure spacelike case, for which the Gauss’s law is modified by the presence of the background. Such modification reflects the coupling between the electric and magnetic sectors in the MCFJ electrodynamics [22].

The consistency condition of the modified Gauss’s law gives \( \phi_2 = \{ \phi_2, H_P \} = 0 \). Thus, the secondary constraint is automatically conserved and there are no more constraints in this model. The bosonic multiplier of the primary constraint remains undetermined. It is an evidence of the existence of first-class constraints, such as it can be verified by computing the PB between the constraints \( \{ \phi_1, \phi_2 \} = 0 \). Therefore, the set of constraints

\[
\phi_1 = \pi^0 \approx 0, \quad \phi_2 = \partial_k \pi^k + \frac{1}{4} \epsilon^{0kij} (k_{AF})_k F_{ij} \approx 0,
\]

is a first-class one.

### A. Equations of motion and gauge fixing conditions

Following the Dirac conjecture, we define the extended Hamiltonian (\( H_E \)) by adding all the first-class constraint to the primary Hamiltonian,

\[
H_E = H_C + \int d^3 y \ [C \phi_1 + D \phi_2].
\]

Under this Hamiltonian, we compute the time evolution of the canonical variables of the system:

\[
\dot{A}_0 = \{ A_0, H_E \} = C,
\]

\[
\dot{A}_k = \pi^k + \partial_k A_0 + \frac{1}{2} \epsilon^{0kij} (k_{AF})_i A_j - \partial_k D,
\]

showing that the dynamics of \( A_0 \) and \( A_k \) remain arbitrary. For \( \pi^0 \) and \( \pi^k \), it is attained

\[
\pi^0 = \partial_k \pi^k + \frac{1}{4} \epsilon^{0kij} (k_{AF})_k F_{ij} = \phi_2 \approx 0,
\]

\[
\pi^k = -\frac{1}{2} \epsilon^{0ijk} (k_{AF})_j \partial_i F_{kj} - \frac{1}{4} \epsilon^{0ijl} \epsilon^{0klm} (k_{AF})_l (k_{AF})_m A_j - \frac{1}{2} (k_{AF})_0 \epsilon^{0kij} F_{ij} - \frac{1}{2} \epsilon^{0kij} (k_{AF})_i \partial_l A_0 + \frac{1}{2} \epsilon^{0kij} (k_{AF})_j \partial_l D.
\]
Making use of Eq. (10), these expressions can be rewritten as
\[
\partial_k F^{0k} + \frac{1}{4} \epsilon^{0ijk} (k_{AF})_i F_{kj} + \partial_k \partial_k D = 0,
\]
\[
\partial_\alpha F^{\alpha k} + \frac{1}{2} \epsilon^{\alpha k\mu\nu} (k_{AF})_\alpha F_{\mu\nu} + \epsilon^{0kl} (k_{AF})_l \partial_i D - \partial_0 \partial_k D = 0. \tag{13}
\]

We can see that Eqs. (10) and (13) are similar to the Lagrangian equations (4) and (2), respectively, if and only if \(D = 0\). Thus, we should impose a gauge condition in such a way to fix \(D = 0\). It is known that the Dirac algorithm requires a number of gauge conditions equal to the number of first-class constraints in the theory. However, those gauge conditions must be compatible with the Euler-Lagrange equations, such that they should fix \(D = 0\) and determine the Lagrangian multiplier \(C\). Such that the gauge conditions together with the first-class constraints should form a second-class set.

1. Radiation gauge

Equation (10) suggests the following gauge fixing condition \(\psi_1 = \partial_k A_k \approx 0\), whose consistency relation, \(\dot{\psi}_1 = \{\psi_1,H_E\} \approx 0\), gives
\[
\nabla^2 A_0 - \frac{1}{2} \epsilon^{0kij} (k_{AF})_k F_{ij} - \nabla^2 D \approx 0. \tag{14}
\]

In order to get an equation only for \(D\), we impose
\[
\psi_2 = \nabla^2 A_0 - \frac{1}{2} \epsilon^{0kij} (k_{AF})_k F_{ij} \approx 0, \tag{15}
\]
as a second gauge condition, such that \(\nabla^2 D = 0\) is used to fix \(D = 0\). The consistency condition of the second gauge condition, \(\dot{\psi}_2 = \{\psi_2,H_E\} \approx 0\), implies
\[
\nabla^2 C - \frac{1}{2} \epsilon^{0kij} (k_{AF})_k \dot{F}_{ij} = 0. \tag{16}
\]

Thus, we have determined all the Lagrange multipliers. Therefore, the set \(\Sigma_a = \{\phi_1, \phi_2, \psi_1, \psi_2\}\) is a second-class one while the corresponding PB matrix, defined as \(M_{ab}(x,y) = \{\Sigma_a(x), \Sigma_b(y)\}\), explicitly reads as a nonsingular matrix,

\[
M(x,y) = \begin{pmatrix}
0 & 0 & 0 & -\nabla^2 \\
0 & 0 & \nabla^2 & 0 \\
0 & -\nabla^2 & 0 & 0 \\
\nabla^2 & 0 & 0 & 0
\end{pmatrix} \delta(x-y),
\]

whose inverse
\[
M^{-1}(x,y) = \begin{pmatrix}
0 & 0 & 0 & -G(x-y) \\
0 & 0 & G(x-y) & 0 \\
0 & -G(x-y) & 0 & 0 \\
G(x-y) & 0 & 0 & 0
\end{pmatrix},
\]

is written in terms of the Green function for the Poisson equation, \(G(x-y)\), given by
\[
\nabla^2 G(x-y) = -\delta(x-y), \quad \nabla^2 G(x-y) = -\delta(x-y), \tag{19}
\]
\[
G(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{x}}}{\mathbf{p}^2} = \frac{1}{4\pi \|\mathbf{x}\|}. 
\]
At this level, it is necessary to assure that the set of first-class constraints and gauge fixing conditions become strong equalities. Such requirement is fulfilled by defining a new bracket operation, the Dirac brackets, $\{\cdot , \cdot \}_D$, as

$$\{A(x), B(y)\}_D = \{A(x), B(y)\} - \int du \int dv \{A(x), \Sigma_c(u)\} \times [M^{-1}(u,v)]_{cd} \{\Sigma_d(v), B(y)\},$$

where $\Sigma_a = \{\phi_1, \phi_2, \psi_1, \psi_2\}$ and $M^{-1}(x,y)$ is the inverse matrix defined in Eq. (18).

Thus, the non-null Dirac brackets for the physical variables

$$\{A_k(x), \pi_j(y)\}_D = -\left[\delta_{kj} - \frac{\partial_k \partial_j}{\nabla^2} \right] \delta(x-y),$$

$$\{\pi^k(x), \pi^j(y)\}_D = \frac{1}{2} \epsilon^{0 k l i} (k_{AF})_i \partial^x_k \partial^y_l \Gamma(x-y),$$

$$\{A_0(x), \pi^k(y)\}_D = -\epsilon^{0 k l i} (k_{AF})_i \partial^x_k \Gamma(x-y),$$

should be compared with the algebra of the pure Maxwell electrodynamics (in the radiation gauge), for which the only non-null Dirac bracket is given by Eq. (21). Also, the MCFJ algebra establishes a noncommutative relation for the transverse momenta which can be contrasted with the noncommutative gauge field algebra $[A_k(x), A_j(y)] = i\ell_{jk} \delta(x-y) \cdot [A_k(x), \pi_j(y)] = i\delta_{jk} \delta(x-y) \cdot [\pi_k(x), \pi_j(y)] = 0$, proposed in Ref. [23] for studying black body radiation in a Lorentz-breaking context. In a definitive way, we can infer that the physical properties of the MCFJ model are very different from the Maxwell theory and from the noncommutative gauge field approach. Consequently, these models should have different thermodynamical properties such as will be clearly shown in the last section.

Under the Dirac brackets, the canonical Hamiltonian [14] reads as

$$H = \int d^3y \left[ \frac{1}{2} \mathbf{E}^2 + \frac{1}{2} \mathbf{B}^2 + \frac{1}{2} (k_{AF})_0 \mathbf{A} \cdot \mathbf{B} \right].$$

While a pure timelike background does not guarantee a positive-definite Hamiltonian, it can be obtained for the case of a pure spacelike background, for which a well-defined quantum theory may be constructed. Indeed, in this case the model can be quantized in the canonical formalism, once the canonical commutation relations for the quantum fields are obtained from the Dirac brackets (by means of the correspondence principle). It may be also quantized via the functional integral formalism. We follow the last quantization procedure to compute the partition function and to analyze the thermodynamical properties of the MCFJ model.

### III. THE PARTITION FUNCTION

In this section, we study the thermodynamical properties of the MCFJ model. The fundamental object for this analysis is the partition function. The Hamiltonian analysis performed in the previous section allows one to define in a correct way the functional integral representation of the partition function, which is given by

$$Z(\beta) = \int DA_\mu D\pi^\mu \delta(\phi_1) \delta(\phi_2) \delta(\psi_1) \times \delta(\psi_2) \left|\det \{\Sigma_a(x), \Sigma_b(y)\}\right|^{1/2} \times \exp \left\{ \int_\beta dx \left( i \pi^\mu \partial_x A_\mu - H_C \right) \right\},$$

where $\Sigma_a = \{\phi_1, \phi_2, \psi_1, \psi_2\}$ is a second-class set formed by the first-class constraints and the gauge fixing conditions, $M_{ab}(x,y) = \{\Sigma_a(x), \Sigma_b(y)\}$ is the constraint matrix given in Eq. (17), whose determinant is $\det M(x,y) = \det (-\nabla^2)^4$. Given the bosonic character of the gauge field, its functional integration can be performed over all the fields satisfying periodic boundary conditions in the $\tau$ variable: $A(\tau, x) = A(\tau + \beta, x)$. The short notation $\int_\beta dx$ denotes $\int_0^\beta d\tau \int d^3x$, and $H_C$ is the canonical Hamiltonian given by Eq. (5).
We first compute the integration on the field $\pi^0$. Using a Fourier representation for $\delta (\phi_2)$,

$$\delta (\phi_2) = \int D\Lambda \exp \left\{ i \int dx \Lambda \left[ \partial_k \pi^k + \frac{1}{4} \epsilon^{0kij} (k_{AF})_k F_{ij} \right] \right\}, \tag{26}$$

doing the change $\Lambda \to \Lambda + i A_0$, and performing the integration over the $\pi^k$ field, the partition function reads as

$$Z (\beta) = \det (-\nabla^2)^2 \int D\pi D\Lambda \delta (\psi_1) \delta (\psi_2) \times \exp \left\{ \int dx - \frac{1}{2} (\partial_r A_k - \partial_k \Lambda)^2 - \frac{i}{2} (\partial_r A_k - \partial_k \Lambda) \epsilon^{0kij} (k_{AF})_i A_j \right\} \times \exp \left\{ \int dx \frac{i}{4} \nabla A_{ij} \epsilon^{0kij} (k_{AF})_k F_{ij} - \frac{1}{4} (F_{jk})^2 - \frac{1}{4} \epsilon^{0kij} (k_{AF})_0 A_k F_{ij} \right\}. \tag{27}$$

The integration of the $A_0$ field gives the contribution $[\det (-\nabla^2)]^{-1}$. At once, we rename $\Lambda = A_r$ and $(k_{AF})_0 = i (k_{AF})_r$, and setting $\epsilon^{0kij} = -i \epsilon_{r k ij}$, $\epsilon_{r 123} = 1$, we get the partition function for the MCFJ model in the Coulomb gauge:

$$Z (\beta) = N \det (-\nabla^2) \int D\pi A \delta (\partial_k A_k) \times \exp \left\{ \int dx - \frac{1}{4} F_{ab} F_{ab} - \frac{1}{4} \epsilon_{abcd} (k_{AF})_a A_b F_{cd} \right\}, \tag{28}$$

where $a, b, c, d = r, 1, 2, 3$.

The partition function in the Coulomb gauge is not explicitly covariant. It is well known that if the covariance is explicit, the calculation process becomes more manageable. The procedure to pass from a noncovariant gauge to a covariant one, like the Lorentz gauge $\partial_A A_a = 0$, can be implemented using the Faddeev-Popov ansatz, which is defined by

$$\int D\omega (x) \delta (G [A_\omega]) \det \left| \frac{\delta G [A_\omega]}{\delta \omega} \right|_{\omega = 0} = 1, \tag{29}$$

where $\omega (x)$ is the gauge parameter, $D\omega$ is a gauge group measure, $G [A_\omega]$ is a covariant gauge fixing condition, $A_\omega$ is the gauge-transformed field ($A_\omega = A_a + \partial_a \omega$), and det $\left| \frac{\delta G [A_\omega]}{\delta \omega} \right|_{\omega = 0}$ is the so-called Faddeev-Popov determinant, which is gauge invariant. We thus choose the Lorentz gauge

$$G [A_\omega] = -\frac{1}{\sqrt{\xi}} \partial_\omega A_a + f, \tag{30}$$

$f$ being an arbitrary scalar function and $\xi$ a gauge parameter. In this way, we have $G [A_\omega] = G [A_a] - \Box \omega / \sqrt{\xi}$, which implies

$$\det \left| \frac{\delta G [A_\omega]}{\delta \omega} \right|_{\omega = 0} = \det \left| \frac{-\Box}{\sqrt{\xi}} \right|, \tag{31}$$

where $\Box = \partial_a \partial_a = (\partial_r)^2 + \nabla^2$. As the partition function is independent of $f$, such a factor can be eliminated by integrating it with the weight $\exp \left\{ \frac{-1}{2} \int dx f^2 \right\}$. Thus, after an integration by parts, the partition function takes
\[
Z(\beta) = \int DA_a \det\left|\frac{-\Box}{\sqrt{\beta}}\right| \exp\left\{ \int dx - \frac{1}{2} A_a \left[ -\Box \delta_{ab} - \left( \frac{1}{\xi} - 1 \right) \partial_a \partial_b - S_{ab} \right] A_b \right\}.
\]

(32)

where have defined the operator \(S_{ab} = \epsilon_{acdb} (k_{AF}) c \partial_d\). For convenience, we choose the Feynman gauge \(\xi = 1\), and the integration over the gauge field gives

\[
Z(\beta) = \det (-\Box) \left[ \det (-\Box \delta_{ab} - S_{ab}) \right]^{-1/2}.
\]

(33)

After some algebra, we obtain

\[
\det (-\Box \delta_{ab} - S_{ab}) = \left[ \det (-\Box) \right]^2 \det (-\Box^2 + (k_{AF})^2 \Box - ((k_{AF}) \cdot \partial)^2).
\]

Replacing it in the partition function (33), we obtain

\[
Z(\beta) = Z_A(\beta) Z_{LV}(\beta).
\]

(34)

Here, the quantity \(Z_A(\beta) = \exp \{ -\text{tr} \ln (-\Box) \}\) is the partition function of the usual electromagnetic field (without Lorentz violation), while

\[
Z_{LV}(\beta) = \exp \left\{ -\frac{1}{2} \text{Tr} \ln \left[ 1 + \frac{(k_{AF})^2 \Box}{\Box^2 - ((k_{AF}) \cdot \partial)^2} \right] \right\},
\]

(35)

is the contribution stemming from the Chern-Simon-like LV term. We can compute the involved trace writing the gauge field in terms of a Fourier expansion,

\[
A_a(\tau, x) = \left( \frac{\beta}{V} \right)^{1/2} \sum_{n, p} e^{i(\omega_n \tau + x \cdot p)} \tilde{A}_a(n, p),
\]

(36)

where \(V\) represents the system volume and \(\omega_n\) are the bosonic Matsubara’s frequencies, \(\omega_n = \frac{2n\pi}{\beta}\), for \(n = 0, 1, 2, \ldots\).

**A. The pure electromagnetic contribution**

We start computing the pure electromagnetic contribution,

\[
\ln Z_A(\beta) = -\text{tr} \ln (-\Box) = -\sum_{n, p} \ln \left( \beta^2 \left[ p^2 + (\omega_n)^2 \right] \right).
\]

(37)

The sum in \(n\) is evaluated as

\[
\sum_{n= -\infty}^{+\infty} \ln \left[ (2\pi n)^2 + \beta^2 \omega_p^2 \right] = \beta \omega_p + 2 \ln \left[ 1 - e^{-\beta \omega_p} \right],
\]

(38)

where \(\omega_p = \|p\|\). Therefore, the contribution of the pure electromagnetic field is

\[
\ln Z_A(\beta) = -2V \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\beta \omega_p}{2} + \ln (1 - e^{-\beta \omega_p}) \right],
\]

(39)

where the sum in \(\|p\|\) was replaced by an integral. The latter integral can be explicitly evaluated in spherical coordinates, \(p = (\omega, \theta, \phi)\), where \(\omega = \omega_p = \|p\|\), \(\theta\) is the angle between the background \(k_{AF}\) and the photonic momentum \(p\), while \(\phi\) is the azimuthal angle. In this way, the partition function reads as

\[
\ln Z_A(\beta) = -\frac{2V}{(2\pi)} \int d\Omega \int_0 ^\infty d\omega \left[ \frac{\beta \omega^3}{2} + \omega^2 \ln (1 - e^{-\beta \omega}) \right],
\]

(40)
with \( d\Omega = \sin \theta d\theta d\phi \) being the differential solid-angle element. Neglecting the vacuum contributions, the partition function is exactly carried out

\[
\ln Z_A(\beta) = V \frac{\pi^2}{45\beta^4}.
\]

(41)

The energy density \( (u_A = U_A/V) \) for the pure electromagnetic field which represents the expectation value of the energy per unit volume (over the thermodynamical ensemble) can be easily obtained \( (u_A = -V^{-1} \partial \ln Z_A/\partial \beta) \), yielding:

\[
u_A = \int_{0}^{\infty} d\omega \frac{1}{\pi^2} \frac{\omega^3}{e^{\beta \omega} - 1}.
\]

(42)

Without integrating in frequency, we obtain the energy density of radiation per frequency unity, that is, the well-known Planck distribution for the black body spectrum:

\[
u_A(\omega) = \frac{1}{\pi^2} \frac{\omega^3}{e^{\beta \omega} - 1}.
\]

(43)

Now, performing the integral in frequency in Eq. (42), we get the total energy density in the cavity

\[
u_A = \frac{\pi^2}{15\beta^4} = aT^4,
\]

(44)

which corresponds to the Stefan-Boltzmann law, while the constant \( a^1 \) is related to the Stefan-Boltzmann’s constant \( (\sigma) \) by \( a = 4\sigma \). The energy density per solid-angle element is

\[
u_A(\beta, \Omega) d\Omega = \frac{\pi}{60\beta^4} d\Omega,
\]

(45)

which stands for a perfect isotropic distribution.

B. The CPT-odd and Lorentz-violating contribution

The Lorentz-breaking contribution, \( Z_{LV}(\beta) \), to the partition function is computed for the case of a pure spacelike background \( k_{AF} = (0, k_{AF}) \), once it is known that the Hamiltonian is positive-definite (stable) only for this background configuration. Such a feature guarantees the existence of the functional integral from which is attained the partition function, thus, the partition function reads

\[
\ln Z_{LV} = -\frac{1}{2} \sum_{n,p} \ln \left( 1 - \frac{k_{AF}^2}{(\omega_n)^2 + p^2} + \frac{(k_{AF} \cdot p)^2}{[(\omega_n)^2 + p^2]^2} \right). \]

(46)

Now, we consider the spacelike background as a weak coupling, \( \|k_{AF}\| \ll 1 \), then we get at order \( k_{AF}^2 \)

\[
\ln Z_{LV} = -\frac{1}{2} \sum_{n,p} \left( \frac{k_{AF}^2}{(\omega_n)^2 + p^2} - \frac{(k_{AF} \cdot p)^2}{[(\omega_n)^2 + p^2]^2} \right).
\]

(47)

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1 In SI units the relation is \( \sigma = \frac{ac}{4} \) with \( c \) the vacuum light velocity and \( a = \frac{\pi^2 k_B}{15(\hbar c)^3} = 7.565604554 \times 10^{-16} \text{ Jm}^{-3}\text{K}^{-4} \), \( \sigma = 5.670277968 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \), where \( k_B \) is the Boltzmann’s constant and \( \hbar \) the Planck’s constant.
The series above can be computed easily by using expression (38). Performing the sum and expressing the resultant integrals in spherical coordinates, the partition function takes the form

\[ \ln Z_{LV} = \frac{1}{2} \frac{k_{AF}^2}{(2\pi)^3} V \int d\Omega \int_0^\infty d\omega \left\{ \frac{\beta \omega}{2} + \frac{\beta \omega}{e^{\beta \omega} - 1} \right\} \]

\[ \times \frac{1}{2} \frac{k_{AF}^2}{(2\pi)^3} V \int d\Omega \cos^2 \theta \int_0^\infty d\omega \left\{ \frac{\beta \omega}{2} + \frac{\beta \omega}{e^{\beta \omega} - 1} \right\} \].

(48)

Here, the integrals in the frequency (\(\omega\)) can be performed exactly, implying

\[ \ln Z_{LV} = V \frac{k_{AF}^2}{96\pi^2} \int d\Omega - V \frac{k_{AF}^2}{64\pi^2} \int d\Omega \cos^2 \theta \].

(49)

Performing now the angular integrations, we obtain the Lorentz-violating contribution to the partition function,

\[ \ln Z_{LV} = V \frac{k_{AF}^2}{48\beta} \] (50)

where we have neglected vacuum contributions.

From Eq. (48), the expectation value of the energy is achieved by unit volume (over the thermodynamical ensemble) for the Lorentz-breaking contribution:

\[ u_{LV} = -\frac{k_{AF}^2}{4} \int_0^\infty d\omega \frac{1}{\pi^2} \frac{\omega^3}{e^{\beta \omega} - 1} \left\{ \frac{5}{6} \frac{1}{\omega^2} - \frac{7}{6} \right\} \]

\[ \times \frac{e^{\beta \omega}}{(e^{\beta \omega} - 1)} + \frac{1}{6} \beta^2 e^{\beta \omega} \left( \frac{e^{\beta \omega} + 1}{(e^{\beta \omega} - 1)^2} \right) \].

(51)

The integrand gives the LV corrections to the Planckian energy density distribution to be

\[ u_{LV}(\omega) = -\frac{k_{AF}^2}{4} \frac{1}{\pi^2} \frac{\omega^3}{e^{\beta \omega} - 1} \left\{ \frac{5}{6} \frac{1}{\omega^2} - \frac{7}{6} \beta \right\} \]

\[ + \frac{1}{6} \beta^2 e^{\beta \omega} \left( \frac{e^{\beta \omega} + 1}{(e^{\beta \omega} - 1)^2} \right) \],

(52)

where nonlinear contributions in the frequency \(\omega\) appear.

The Lorentz-breaking contribution to the Stefan-Boltzmann law can be achieved by integrating Eq. (51), which yields

\[ u_{LV} = \frac{k_{AF}^2}{48\beta^2} = \frac{k_{AF}^2}{48} T^2 \].

(53)

From Eq. (49), we can also determine the LV contribution to the energy density in each solid angle (\(d\Omega = \sin \theta d\theta d\phi\)):

\[ u_{LV}(\beta, \Omega) d\Omega = \frac{k_{AF}^2}{96\pi^2} \left[ 1 - \frac{3}{2} \cos^2 \theta \right] d\Omega \],

(54)

in which it is manifest the presence of the anisotropy factor \((\cos^2 \theta)\). This reveals that LV is a mechanism that can play an important role in explaining CMB anisotropies.

C. The MCFJ thermodynamics

The energy density of the MCFJ model is the one associated with the full partition function (33), given by the sum of the contributions (44) and (53), namely,

\[ u_{MCFJ} = aT^4 + \frac{k_{AF}^2}{48} T^2 \].

(55)
The expression above shows a LV correction to the Stefan-Boltzmann law at $k_{AF}^2$ order with a dependence in the temperature as $T^2$. Such a correction is potentially more significant at low temperatures. The result (55) can be rewritten in two ways. The first one is

$$u_{MCFJ} = \bar{a}(T) T^4,$$

with $\bar{a}(T)$ being an effective coefficient that retains the temperature and LV modifications:

$$\bar{a}(T) = a + \frac{k_{AF}^2}{48 T^2}.$$

It affords an opportunity to establish a first bound for the spacelike LV background by using the experimental data for the Stefan-Boltzmann constant $\sigma = (5.67040 \pm 0.00004) \times 10^{-8}$ W m$^2$K$^{-4}$. Thus, considering that the corrections to the Stefan-Boltzmann law are of the order of the experimental error, we get $|k_{AF}| \lesssim 3.6 \times 10^{-15}$ GeV for $T = 2.73$ K (the black body temperature of the CMB radiation).

The second form to express Eq. (55) is by considering the $a$ constant as fixed and attributing the small variations on the energy density to temperature fluctuations ($\delta T$). The expression (55) can be then written as

$$u_{MCFJ} \approx a (T^4 + 4 T^3 \delta T),$$

which gives the temperature fluctuation $\delta T$ with respect to the black body temperature $T$ without LV interactions. In others words, we assume that the Stefan-Boltzmann law $u \propto T^4$ remains valid for both models [19]. Considering Eq. (55), we write the first order temperature corrections ($\delta T$) as

$$\frac{\delta T}{T} = \frac{u_{MCFJ} - u_M}{4u_M}.$$ 

Such expression allows one to extract the temperature offsets from the MCFJ (model following the method developed in Ref. [19]), which can be compared with the experimental data coming from the Far-InfraRed Absolute Spectrophotometer (FIRAS) and the WMAP. Thus, we see that Lorentz violation is a mechanism that can play an important role in explaining CMB anisotropies. Physically, the term $\delta T$ in Eq. (55) stands for the temperature offsets of the MCFJ integrated spectra (integration over all frequencies) compared to the conventional Maxwell black body integrated spectra ($u_M$).

The expression for $\delta T$ leads to a second (but similar) bound for the LV parameter if we compare the quadrupole fluctuation implied by Lorentz violation [see Eq. (55)] with the quadrupole temperature fluctuation of the CMB $\delta T / T \sim 10^{-6}$, with $T = 2.73$ K. In this case, we obtain $|k_{AF}| \lesssim 2.75 \times 10^{-15}$ GeV.

Also, from Eqs. (43) and (52) we derive the energy density of the radiation per frequency for the MCFJ electrodynamics

$$u_{MCFJ}(\omega) = \frac{1}{\pi^2} \frac{\omega^4}{e^{\beta \omega} - 1} \left\{ 1 - \frac{k_{AF}^2}{4} \left[ \frac{5}{6} \frac{1}{\omega^2} - \frac{7}{6} \beta \omega e^{\beta \omega} \left( e^{\beta \omega} - 1 \right) \right] \right\}.$$ 

(60)

From the angular energy distribution expressions (55) and (54), we write the MCFJ energy density per solid-angle element:

$$u_{MCFJ}(\beta, \Omega) d\Omega = \left[ \frac{\pi}{60} \frac{1}{\beta^4} + \frac{k_{AF}^2}{96 \pi} \left( 1 - \frac{3}{2} \cos^2 \theta \right) \right] d\Omega.$$ 

(61)

Thus, the angular energy distribution at $k_{AF}^2$ order provides a quadrupole ($l = 2$) contribution to the power angular spectrum, revealing an interesting feature: the LV contribution to the spectrum is anisotropic and gives a maximal contribution in the plane perpendicular to background direction. From Eq. (54), it is easy to show that a contribution at $(k_{AF}^2)^n$ order to the power angular spectrum of the black body radiation may be considered. It gives contributions until the order $l = 2n$ at the same time it is associated with a $T^{4-2n}$ temperature dependence. This guarantees that, for high temperatures, the relevant contributions stem only from the first terms of the expansion.
IV. CONCLUSIONS AND REMARKS

In this work, we have initially established the constraint structure of the MCFJ electrodynamics having as the main goal the correct evaluation of the partition function of this model (at the finite temperature regime). With the partition function, the thermodynamics properties of the model were determined, revealing the LV corrections to the black body spectral distribution. As the CMB map is in fact nothing more than a black body radiation pattern only slightly perturbed by fluctuations, our purpose involves an attempt of relating the CMB anisotropies with the LV corrections here evaluated. Indeed, we have addressed what is expected to appear as anisotropies in the CMB map if the photonic sector in the lately universe is described by the finite temperature CFJ electrodynamics. Such calculation shows that the LV CFJ term modifies (in leading order) the monopole and quadrupole moments of angular power spectrum in a proper way. This is ascribed to the form of the MC FJ field algebra, that is different from the Maxwell and from the noncommutative gauge field (and space-time) approaches proposed in Refs. [23, 25]. Such difference leads to very distinct black body spectra. In fact, at order \( n \geq 1 \) the temperature corrections to the integrated spectra are proportional to \( T^{4+4n} \), for the noncommutative space-time approach, \( T^{4+2n} \), for the noncommutative gauge field model approach, and \( T^{4−2n} \), for the finite temperature MCFJ model.

Finally, we have seen that the CFJ term is able to induce anisotropic contributions to the CMB. Although, we should mention that the background magnitude for yielding a CMB anisotropy of 1 part in \( 10^{15} \) is approximately 10\(^{-15}\) GeV. Considering that birefringence data constrain such background as tightly as \( 10^{-33} \) eV, we conclude that Lorentz violation, as set up in the MCFJ model, cannot be used to explain such anisotropies.

In a forthcoming work, we intend to present the thermodynamical contributions for the black body radiation provided by the less constrained coefficients of the CPT-even term of the gauge sector of the SME. Such work is in progress.

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