Interaction potential between heavy $Q\bar{Q}$ in color octet configuration in QGP

Dibyendu Bala and Saumen Datta

We investigate the interaction between a heavy quark-antiquark pair in color octet configuration in gluon plasma. We calculate nonperturbatively an effective thermal potential for such a pair through the study of the correlation function of a hybrid state with $Q\bar{Q}$ octet and an adjoint gluon source in the static limit. We discuss the extraction of an octet potential, and present results for the effective thermal potential between octet $Q\bar{Q}$ pair in gluon plasma for moderately high temperatures $\lesssim 2T_c$. The implications of our result are discussed.

PACS numbers: 11.15.Ha, 12.38.Gc, 12.38.mh, 25.75.Nq

I. INTRODUCTION

Quarkonia, mesonic bound states of heavy quark and antiquark, provide one of the most important signatures of the formation of quark-gluon plasma (QGP) in relativistic heavy ion collision experiments. It was suggested in the early days of collider studies of QGP [1] that the screening of the color charge inside QGP will lead to dissolution of $J/\psi$ states, which can be observed by modification of the dilepton peak. Various theoretical approaches have been formulated to study the behavior of $Q\bar{Q}$ bound states in the plasma. See for a recent comprehensive review [2]. In particular, an effective thermal potential approach to the problem has been formulated in [4] in perturbation theory, in the hard thermal loop (HTL) approximation. The behavior of such a potential has been studied in [5], in the open quantum system language [6]. It is possible to evaluate this potential nonperturbatively, from numerical calculations of thermal Wilson loop data [8]. A new method, based on splitting of the thermal Wilson loop, has led to a well-controlled extraction of the potential from Wilson loop data [11].

For the phenomenology of quarkonia in plasma, one needs to know the interaction between the $Q\bar{Q}$ not only in the singlet channel but also in the octet configuration. The interaction of the $Q\bar{Q}$ with the medium will change its color configuration from singlet to octet and vice versa, and therefore study of the $Q\bar{Q}$ pair in plasma involves both color configurations. The interaction of the $Q\bar{Q}$ with the medium is described by the effective thermal potential [4]. The behavior of such a potential has been studied in [5].

For the phenomenology of quarkonia in plasma, one needs to know the interaction between the $Q\bar{Q}$ pair. The interaction of the $Q\bar{Q}$ with the medium is described by the effective thermal potential [4]. The behavior of such a potential has been studied in [5].

Here $U$ is a suitable gauge connection such that the current is gauge invariant. $\chi, \psi$ are nonrelativistic fields that annihilate a quark and create an antiquark, respectively, and the angular brackets denote thermal average. The quantum state of the quark is $|\Omega\rangle$, and the potential describes the time evolution of the thermal correlator of the nonrelativistic vector current.

\begin{equation}
C_s(t, r) = \int d^3x \langle \chi(t, x) \psi(t, x + r) \rangle.
\end{equation}

\begin{equation}
\left( \frac{1}{2} \nabla^2 + \frac{M_Q^2}{M_Q^2} \right) C_s(t, r) = V(r) C_s(t, r).
\end{equation}

\textbf{1. INTRODUCTION}

Quarkonia, mesonic bound states of heavy quark and antiquark, provide one of the most important signatures of the formation of quark-gluon plasma (QGP) in relativistic heavy ion collision experiments. It was suggested in the early days of collider studies of QGP [1] that the screening of the color charge inside QGP will lead to dissolution of $J/\psi$ states, which can be observed by modification of the dilepton peak. Various theoretical approaches have been formulated to study the behavior of $Q\bar{Q}$ bound states in the plasma. See for a recent comprehensive review [2]. In particular, an effective thermal potential approach to the problem has been formulated in [4] in perturbation theory, in the hard thermal loop (HTL) approximation. The behavior of such a potential has been studied in [5], in the open quantum system language [6]. It is possible to evaluate this potential nonperturbatively, from numerical calculations of thermal Wilson loop data [8]. A new method, based on splitting of the thermal Wilson loop, has led to a well-controlled extraction of the potential from Wilson loop data [11].

For the phenomenology of quarkonia in plasma, one needs to know the interaction between the $Q\bar{Q}$ not only in the singlet channel but also in the octet configuration. The interaction of the $Q\bar{Q}$ with the medium will change its color configuration from singlet to octet and vice versa, and therefore study of the $Q\bar{Q}$ pair in plasma involves both color configurations. The interaction of the $Q\bar{Q}$ with the medium is described by the effective thermal potential [4]. The behavior of such a potential has been studied in [5].

For the phenomenology of quarkonia in plasma, one needs to know the interaction between the $Q\bar{Q}$ pair. The interaction of the $Q\bar{Q}$ with the medium is described by the effective thermal potential [4]. The behavior of such a potential has been studied in [5].

Here $U$ is a suitable gauge connection such that the current is gauge invariant. $\chi, \psi$ are nonrelativistic fields that annihilate a quark and create an antiquark, respectively, and the angular brackets denote thermal average. The quantum state of the quark is $|\Omega\rangle$, and the potential describes the time evolution of the thermal correlator of the nonrelativistic vector current.

\begin{equation}
C_s(t, r) = \int d^3x \langle \chi(t, x) \psi(t, x + r) \rangle.
\end{equation}

\begin{equation}
\left( \frac{1}{2} \nabla^2 + \frac{M_Q^2}{M_Q^2} \right) C_s(t, r) = V(r) C_s(t, r).
\end{equation}
We then define a potential \( V \) by equating the left hand side of Eq. (2) to \( V(t, \vec{r}) C_{\sigma}(t, \vec{r}) \) (staying within leading order of \( 1/M_Q \)), where the interaction effects are summarized in a time-dependent \( V(t, \vec{r}) \). An effective thermal potential, \( V_{\sigma}(\vec{r}) \), can then be defined in the large \( t \) limit, if the limit exists: \( V_{\sigma}(\vec{r}) = \lim_{t \to \infty} V(t, \vec{r}) \). In the static limit, modulo renormalization factor, \( C_{\sigma}(t, \vec{r}) \) reduces to a Minkowski-time Wilson loop:

\[
W_M(t, \vec{r}) = \frac{1}{3} \text{Tr} \, \mathcal{P} e^{i \int_0^t dt_1 A_0(t_1, \vec{r})} \mathcal{U}(t, \vec{r}, \vec{0}) \, \mathcal{P} e^{i \int_0^t dt_2 A_0(t_2, \vec{0})} \mathcal{U}(0, \vec{0}, \vec{r})
\]

and Eq. (2) reduces to

\[
i \partial_t \log W_M(t, \vec{r}) \xrightarrow{t \to \infty} V_{\sigma}(\vec{r}),
\]

which defines the effective thermal potential for the singlet channel \( C \).

Eq. (2) describes the time evolution of a thermal correlation function, and not of a \( QQ \) wave function. It reduces naturally to Eq. (4) in the static limit, which can lead to a nonperturbative calculation of the potential \( \hat{\mathcal{V}} \). The effective thermal potential defined by Eq. (2) is complex in general \( [4] \). It has been argued that the potential Eq. (2) describes the time evolution of a thermal correlation function, and not of a \( QQ \) wave function. It reduces naturally to Eq. (4) in the static limit, which can lead to a nonperturbative calculation of the potential \( \hat{\mathcal{V}} \). The effective thermal potential remains an essential ingredient in such open quantum system studies of quarkonia in plasma.

For the octet potential, one can proceed in a similar way, starting with a point-split nonrelativistic current

\[
J^a(\vec{r}, \vec{r}, \vec{x}_0, t) = \psi^\dagger(\vec{x} + \vec{r}, t) \sigma_a \psi(\vec{x} + \vec{r}, \vec{x}_0; t) T^a \mathcal{U}(\vec{x}_0, \vec{x}; t) \chi(\vec{x}, \overline{\vec{x}}, t).
\]

and looking at the time derivative of the correlator \( C_{\sigma}^{a} \sigma^a(t, \vec{r}) \) la Eq. (4). The current \( J^a \) is gauge dependent and the correlator \( C_{\sigma}^{a} \sigma^a(t, \vec{r}) \) needs to be defined in a fixed gauge. Unfortunately, standard gauge fixed definitions of \( C_{\sigma}^{a} \sigma^a(t, \vec{r}) \) lead to a system which may be very different from what was intended: e.g., in the temporal gauge \( C_{\sigma}^{a} \sigma^a(t, \vec{r}) \) actually describes, in the static limit, a \( Q\bar{Q}_{adj} \) system \( [15] \). In the literature one usually employs the Coulomb gauge; nonperturbatively, \( C_{\sigma}^{a} \sigma^a(t, \vec{r}) \) is not defined in the Coulomb gauge, and a further fixing of temporal gauge along \( \vec{x}_0 \) gets us back to a \( Q\bar{Q}_{adj} \) system \( [15] \).

When we talk about \( QQ \) in color octet combination in the context of quarkonia, we have in mind a system where \( QQ \) is interacting with an adjoint gluonic source. To mimic this system, we could start with a trial current like

\[
J_G(\vec{r}, \vec{r}, \vec{x}_0, t) = \psi^\dagger(\vec{x} + \vec{r}, t) \mathcal{U}(\vec{x} + \vec{r}, \vec{x}_0; t) C^a(\vec{x}_0, \vec{x}; t) T^a \mathcal{U}(\vec{x}_0, \vec{x}; t) \chi(\vec{x}, \overline{\vec{x}}, t),
\]

which is a color singlet combination of the color-octet quark-antiquark system and an adjoint gluonic source at a time slice \( t \), and then look at the correlator \( C_G(t, \vec{r}) = \langle J_G(t) J_G^G(0) \rangle \). With a judicious choice of \( G \), it is possible to ensure that \( J_G \) does have overlap with a configuration where the quark-antiquark system is in color-singlet state. A color singlet state consisting of \( Q\bar{Q}_{adj} \) and adjoint gluon source is called a hybrid state. At zero temperature, hybrid potentials have been studied in detail in the literature \( [18] \). For us, the important information is that in certain regimes, the hybrid current can be used to define an octet potential \( [17, 18] \).

In this work, we study the thermal effect on the hybrid Wilson loop and extract information about the thermal modification of the interaction potential between static \( QQ \) in color octet configuration in gluon plasma. To our knowledge, this is the first study of the effective thermal color octet potential, though a related quantity, the color octet free energy, has been studied before \( [19, 20] \). Preliminary results of this study were presented in \( [21] \). The crux of the problem is to extract \( V_{\sigma}(\vec{r}) \); this is discussed in Sec. II. We discuss our method in detail in Sec. III. Details of the numerical calculation are given in Sec. IIII. Our results for the potential are given in Sec. IV and in Sec. IV we summarize and discuss the results. Some technical details are relegated to the appendix: Appendix A discusses role of smearing in our study, some details relevant for Sec. IV can be found in Appendix B and various systematics of our extraction of the potential can be found in Appendix C.

II. OBSERVABLES AND ANALYSIS

For our study, we use the hybrid current operator Eq. (6) with the chromomagnetic field operator for \( G = G^a T^a \): we use the two choices \( B_z \) and \( B_x = B_z + i B_y \). Here \( z \) is taken to be the separation between the quark and the antiquark. In the static limit, one gets Wilson loop with insertion of the \( G \) field:

\[
W^G_M(\tau, \vec{r}, \vec{x}) = \frac{1}{3} \text{Tr} \, \mathcal{P} e^{i \int_0^\tau d\tau_1 A_0(\tau_1, \vec{r})} \mathcal{U}(\tau, \vec{r}, \vec{0}) \, \mathcal{P} e^{i \int_0^\tau d\tau_2 A_0(\tau_2, \vec{0})} \mathcal{U}(0, \vec{0}, \vec{x}) G(\tau, \vec{r}) \mathcal{U}(0, \vec{x}, \vec{0}) \mathcal{U}(0, \vec{0}, \vec{r}) \cdot
\]
In order to define a potential, we will need to go to Minkowski time and take long time derivative, similar to Eq. (4). While the aim of the paper here is to calculate the effective thermal potential nonperturbatively, in order to understand the method, it helps to think in terms of perturbation theory. In leading order (LO) of perturbation theory, the effect of the insertion $G$ isolates and one gets the octet potential from the long time behavior of Wilson loop:

$$
\begin{pmatrix}
\phantom{G} \\
\phantom{G}
\end{pmatrix} \\
= \\
\begin{pmatrix}
\phantom{G} \\
\phantom{G}
\end{pmatrix} + \\
\begin{pmatrix}
\phantom{G} \\
\phantom{G}
\end{pmatrix} + \\
\begin{pmatrix}
\phantom{G} \\
\phantom{G}
\end{pmatrix} + \cdots
\tag{8}
\end{equation}

Here time direction is shown vertically, the grey circles indicate the magnetic fields, and the empty dots and dashed lines on the Wilson loop indicate color matrix insertion $T^a$ and 00 component of the gluon propagator, respectively. The leading order potential comes from a ladder sum of diagrams like those explicitly shown inside the parentheses in Eq. (8), where the effect of the $G$ insertion is merely a change in color factor due to the color matrix insertions in the Wilson loops. Such factorization will not hold nonperturbatively, where $J^G$ for the $B_+$ operator will give rise to potential for the $L=0$ state $\Sigma^-$ and the $B_-$ operator, that for the $L=1$ state $\Pi$. Here $L$ refers to angular momentum around the axis joining the quark and antiquark, $u$ denotes CP odd, and $-$ refers to parity for reflection about a plane passing through this axis. The potential for these operators have been studied [14], and its connection to the octet potential has been explored in detail [18]. In the deconfined phase, such hybrid states are not expected to survive. However, we will sometimes refer to the potentials obtained with the two operator insertions as $L=0$ and $L=1$ potentials, respectively.

At short distances, we expect the potential for a state like Eq. (6) to give the potential for the color octet $Q\bar{Q}$ state, modulo a constant term capturing the effect of the $G$ insertion [17, 18]:

$$
V_G(r) \sim V_O(r) + \Lambda_G + \mathcal{O}(r^2)
\tag{9}
$$

Clearly, for the two operator choices here, $\Lambda_G$ is identical. So we expect $V_G$ to be same for the two operators in the short distance regime, modulo $\mathcal{O}(r^2)$ effects. This behavior was tested in detail in [18]: while the convergence of $V_G$ extracted from the two components of the magnetic field was verified, it becomes difficult to isolate the color octet potential due to the quick onset of the nonperturbative effects.

At finite temperatures, we can define a thermal Wilson loop similar to Eq. (4), except the $\tau$ extent is now finite: $0 < \tau < \beta = 1/T$. Just as in the case of the singlet [4], one can define in HTL perturbation theory an effective thermal potential by continuing to Minkowski time and taking a long time derivative a la Eq. (4). For completeness, we outline the steps in Appendix [3] In this order, the factorization of Eq. (8) holds and we get, in the Debye screening regime, a thermal potential

$$
V^a(r; T) = V^a_{\text{im}}(r; T) - i V^a_{\text{re}}(r; T) \tag{10}
$$

$$
V^a_{\text{im}}(r; T) = \frac{g^2}{2N_c} \frac{e^{-m_D r}}{4\pi} - \frac{g^2 C_F}{4\pi} m_D
$$

$$
V^a_{\text{re}}(r; T) = \frac{g^2 T}{2\pi} \left[ \frac{N_c}{2} \int_0^\infty \frac{dz}{(z^2 + 1)^2} - \frac{1}{2N_c} \int_0^\infty \frac{dz}{(z^2 + 1)^2} \left( 1 - \frac{\sin zx}{zx} \right) \right]
$$

where $m_D$, the Debye mass, $= gT$ in this order of perturbation theory (for gluon plasma), and $x = m_D r$. It is interesting to compare it to the thermal singlet potential [3]:

$$
V^T_{\text{im}}(r) = \frac{g^2 C_F}{4\pi} e^{-m_D r} - \frac{g^2 C_F}{4\pi} m_D
$$

$$
V^T_{\text{re}}(r) = \frac{g^2 C_F T}{2\pi} \int_0^\infty \frac{dz}{(z^2 + 1)^2} \left( 1 - \frac{\sin zx}{zx} \right)
\tag{11}
$$

Both $V^a_{\text{im}}(r; T)$ and $V^T_{\text{im}}(r) \to T C_F g^2/4\pi$ as $r \to \infty$. $V^a_{\text{im}}(r)$ corresponds to the usual physics of Debye screening in medium, such that for sufficiently large screening, the bound states will not form. On the other hand, $V^T_{\text{im}}(r)$ clearly leads to a broadening of the spectral function peak. It captures the physics of collision with the thermal particles leading to a decoherence of the $Q\bar{Q}$ wave function [6, 14].

The expressions Eq. (10) and Eq. (11) are valid only in the HTL limit $T \gg 1/r$. For the distance regime $rT \ll 1$, the thermal correction to the singlet potential has been calculated using effective field theory techniques [3]. At short
At large distances, $V^{im}_r(\vec{r}) \sim \vec{r}^2$ [5]. This is indeed the parametric behavior seen in the short distance regime nonperturbatively [11]. Eq. (10) is not expected to give us the correct potential, even qualitatively, at all distance scales; they, however, are compact and are useful in understanding certain features of the thermal potential.

Our aim here is to extract the potential nonperturbatively from the Euclidean Wilson loop Eq. (7). For the singlet, nonperturbative assessment of $V_r(\vec{r})$ has been done, e.g., in [9, 11, 22], and recently in [11]. The potential, in particular $V^{im}_r(\vec{r})$, is very different from Eq. (11) in the temperature range $\sim \ll 2T_c$. Here for the analysis of $W_G$ we will follow the strategy of [11], which we outline below. See [11] for a more detailed discussion.

At zero temperature, modulo renormalization factors, the Minkowski space Wilson loop has the asymptotic time behavior $W_M(\vec{r}) \sim e^{-iV(\vec{r})t}$ (Eq. (4)), leading to the Euclidean time behavior $W_E(\vec{r}) \sim e^{-V(\vec{r})r}$. Going to sufficiently long $\tau$, this behavior is indicated by a plateau in $\log W_{\tau,\vec{r}}(\vec{r})$, which we extract below. See [11] for a more detailed discussion. The plateau from $W^{\pi}_\tau(\tau, \vec{r})$ gives the real part of the potential, while $W^{\pi}_T(\tau, \vec{r})$ contributes to the imaginary part. This can be understood by writing a spectral decomposition for $P(\tau) = \log W^{\pi}_\tau(\tau, \vec{r})$:

$$P(\tau) = \int_{-\infty}^{\infty} d\omega \sigma(\omega; T) \frac{1}{2} \left( e^{-\omega \tau} + e^{-\omega(\beta - \tau)} \right) + \tau - \text{independent terms}$$

$$\tau \to it \Rightarrow \partial_t P(it) = \int_{-\infty}^{\infty} d\omega \sigma(\omega; T) \frac{\omega}{2} \left( e^{-i\omega t} - e^{-\omega T} e^{i\omega t} \right).$$

At large $t$, the oscillating factors $\exp(\pm i\omega t)$ ensure that only the $\omega \to 0$ contribution to the integral in Eq. (10) survives. Then $\exp(\beta \omega) \to 1$ and Eq. (10) shows that $P(it)$ leads to an imaginary potential.

To proceed further, we note that $P(-i\omega t) - \exp(i\omega t - \omega \beta) \stackrel{t \to \infty}{\sim} -2\pi i \omega \delta(\omega)$. Then in order to get a finite potential $-i V^{im}_r(\vec{r}) = \lim_{t \to \infty} i\partial_t P(it)$ we need $\sigma(\omega; T) \sim (1 + n_\omega(\omega))$. Since we expect thermal physics to introduce a distribution function $(1 + n_\omega(\omega)) \omega \to 0 \frac{T}{\omega} \text{in} P(\tau)$ (see Appendix B), the existence of a potential requires a low $\omega$ structure of $\sigma(\omega; T) \sim (1 + n_\omega(\omega))$. This leads to the following behavior of the Wilson loop near $\beta/2$:

$$W_T(\tau, \vec{r}) = e^{-iV^{\pi}(\vec{r})/2} - \frac{\beta}{\pi} V^{im}_r(\vec{r}) \log \sin\left( \frac{\pi \tau}{a} \right) - \ldots W_T(\beta/2, \vec{r})$$

where the higher order terms ... are non-potential terms. For the periodic part, expanding $\sigma(\omega; T) ((1 - \exp(-\beta \omega))$
In the right panel we show the
sampling, so that only the potential part Eq. (14), contributes, as is supported by the data. In the left plot we show the fitted value for $V_{\text{im}}(\tau, \vec{r})$ on top of the 'local values' obtained from $W_{\text{im}}^\mu(\tau, \vec{r})$. In the right panel we show $P(\tau)$, defined above Eq. (13), along with the contribution of the $V_{\text{im}}^\mu(\tau, \vec{r})$ term in Eq. (14) and that of the $c_1$ term. The $V_{\text{im}}^\mu(\tau, \vec{r})$ term captures the behavior of the data near $\beta/2$ very well. The first term of Eq. (15) has a very different behavior near the center. Its addition does not significantly improve the fit quality ($\chi^2$); however, it can sometimes destabilize the plot if the interval is small, or if the data is not very accurate, as is often the case for $W_G$. Therefore for the octet, we stick to just the form in Eq. (14), and choose a suitable interval so that the fit quality is good. In Sec. IV we will show how well Eq. (14) explains the data, by examining plateaus for the local values of the potential.

The discussion of the potential requires a low-$\omega$ peak. In order to successfully determine the potential, it is necessary to get a region in $\tau$ where this peak dominates the contribution to the potential. In earlier literature, this peak has been hypothesized to have a Lorentzian or Gaussian structure. However, the form in Eq. (14) leads to an asymmetric peak: while for $\omega \approx V_r$ it gives a Breit-Wigner structure, the fall-off from the peak is very different in the large-$\omega$ and small-$\omega$ side. The limiting behaviors are

\[ \rho_{\text{low}}(r; \omega) \approx \sqrt{\frac{\beta}{\pi}} \frac{V_{\text{im}}}{(V_{\text{re}} - \omega)^2 + V_{\text{im}}^2}, \quad |V_{\text{re}} - \omega|, \quad V_{\text{im}} \ll T \]

\[ \approx (\omega - V_{\text{re}})^{-1} \left(1 - \frac{V_{\text{im}}}{V_{\text{re}}} \right), \quad \omega - V_{\text{re}} \gg T \]

\[ \approx e^{-\beta(V_{\text{re}} - \omega)} (\omega - V_{\text{re}})^{-1} \left(1 - \frac{V_{\text{im}}}{V_{\text{re}}} \right), \quad \omega - V_{\text{re}} \ll -T \]

An asymmetric peak structure has also been suggested in Ref. [23], where the spectral function for thin Wilson loops was discussed from HTL perturbation theory. It agrees with Eq. (16) near the peak, but starts disagreeing away from it. The peak in Eq. (16) takes into account that the non-potential modes have been sufficiently suppressed by smearing, so that only the potential part Eq. (14) contributes, as is supported by the data.

### III. DETAILS OF THE CALCULATION

As mentioned in the previous section, our primary quantity is the nonperturbatively estimated value of the Wilson loop $W_G$, Eq. (7), in a gluonic plasma, at moderately high temperatures $\lesssim 2T_c$. Since we need a very fine grid of points for the extraction of the potential from the Wilson loop, we have used a space-time anisotropic discretization, with $\xi = a_s/a_\tau = 3$. We have generated lattices with the anisotropic Wilson action

\[ S_W = -3V(\beta_s P_s + \beta_t P_t), \quad \text{where} \quad P_s = \frac{1}{3VN_c} \sum_x \sum_{i<j} \text{Re Tr } U_{ij}(x), \quad P_t = \frac{1}{3VN_c} \sum_x \sum_i \text{Re Tr } U_{ii}(x) \]

are the spatial and temporal plaquette variables, $V = N_s^3 \times N_\tau$, $x$ runs over all space-time points, $i$ runs over all spatial indices, and $U_{mm}(x)$ is the gauge connection around the $mn$ direction plaquette at $x$. We follow the method of Ref. [24] to get $\beta_s, \beta_t$; $\xi$ is estimated from comparison of spatial and temporal Wilson loops. The lattice parameters we use are given in Table I for each set, short Monte Carlo runs are made at closely spaced $N_\tau$ to find the $N_\tau$ for deconfinement transition, and then other temperatures are obtained by varying $N_\tau$. The value of $N_\tau$ at $T_c$, determined by the peak of the Polyakov loop susceptibility, is shown in Table I. For the measurements we ran multiple Monte Carlo chains: first #init number of well-decorrelated, thermalized configurations were generated. From each of these starting configurations long Monte Carlo chains were constructed: each chain was further thermalized, and then #meas measurements made, two measurements being separated by (100 timelink multilevel [25] hits + 100 update sweeps). Each update step consisted of 1 heatbath + 3 overrelaxation steps, whereas for the multilevel only heatbath steps were used.

We therefore have #init $\times$ #meas measurements of timelike Wilson loops, i.e., the Euclidean time version of $W_M$, with magnetic field insertions (Eq. (4)) at the middle of the spatial connection. All averages and errors are done via a bootstrap analysis; for the bootstrap, we have further averaged the $O(10^3)$ measurements into $O(10^2)$ blocks ($O(10)$
for set 1). The magnetic field operators have been implemented using the clover construction. As in the singlet case \cite{11}, we do APE smearing \cite{26} of the spatial links to reduce the non-potential effects. At each APE step, a spatial gauge link is replaced by \text{Proj}_{SU(3)} (α × link + \sum \text{ spatial staples}), where we kept α = 2.5. We have looked at data from a number of APE steps (shown in Table I). With higher number of APE steps, the data quality decreases, but the effect of the non-potential terms also decreases, making it easier to reach a plateau and extract a potential.

The use of spatial gauge link smearing is well-known in potential studies, and detailed nonperturbative studies exist. In our context, it is instructive, however, to understand its effect in the leading order for the Wilson loop. this is discussed in Appendix A. Some illustrations of its effect on the extracted potential can be found in Appendix C.

In this work, we have given all physical quantities in temperature units. Conversion to physical units, if needed, can be done by setting \( T_\text{c} \) to 280 MeV, which is the value obtained by fixing the string tension: \( \sqrt{\sigma} = 0.44 \) GeV. The spatial size of the lattices are > 1.8 fm, with the lattice of set 1 being 3.4 fm.

IV. \( W_G \) AND THE OCTET QQ INTERACTION POTENTIAL

Before presenting the results for the effective potential for \( C_G(t, \vec{r}) \), Eq. (14), we illustrate how well the form Eq. (14) explains the data, by doing the equivalent of a local mass plot: we extract the “local potential” from a subset of data points. We find \( V(r; \tau) \) from the data \( W(r; t) \) with \( t = \tau, \tau + 1, N_\tau - \tau, N_\tau - \tau - 1 \). If the data is dominated by the potential term near \( \beta/2 \), we will expect a plateau near \( \beta/2 \). We give two examples of such an effective potential plateau in Figure 2. To get the results, we have done a bootstrap analysis, where the parameter values within each bootstrap sample are obtained by a \( \chi^2 \) fit with the full covariance matrix. The data is seen to show a plateau behavior in a region around \( \beta/2 \). The final value of the potential (within each bootstrap sample) is then obtained by doing a fit to Eq. (14) over this plateau range. The statistical error for a given fit range is the (16,84) percentile band of the bootstrap distribution. The quoted errors in Sec. IV.A and Sec. IV.B also include effect of varying the fit range within the plateau region, and spread over smearing levels (see Sec. C).

We present results for the real part of the potential in Sec. IV.A and in Sec. IV.B the results for the imaginary part are shown. Discussion of various systematics related to the results presented in this section have been put in Appendix C.

A. \( V^o_{\text{re}}(\vec{r}; T) \)

As discussed in Sec. I1 the results for \( V^o_{\text{re}}(\vec{r}; T) \) are obtained from \( W^o_\tau(\tau, \vec{r}) \) with the hybrid current operators. At \( T = 0 \) the hybrid potential has been studied in detail in the literature. In Figure 3 we show the potential obtained by us below \( T_c \) for the two operator insertions \( B_z \) and \( B_\lambda \). Strictly speaking the lattices here are at 0.75 \( T_c \) (see Table I), but in gluon plasma one expects very little temperature effect at this temperature, and we indeed checked that our results are in very good agreement with a recent analysis of T=0 hybrid potential, Ref. 16. Since this is, in effect, a zero temperature potential, we obtained the potential from a conventional exponential fit.

From Eq. (9) we would expect that, at short distance, the potentials extracted for the two channels would agree and give the octet potential, modulo an additive constant. As the figure shows, the potentials do seem to agree at very short distances \( \lesssim 0.1 \) fm, showing the repulsive behavior expected of the octet channel. A detailed analysis of the short distance part has been made in 18. At larger distances, nonperturbative effects start dominating, and the potentials for the two channels have a different nature, giving the hybrid potentials. While both channels are

| set | \( \beta_1 \) | \( \beta_2 \) | \( N_\tau(T_c) \) | size | \#init | \#meas | Smearing |
|-----|-------------|-------------|----------------|------|--------|--------|----------|
| 1   | 2.53        | 15.95       | 30             | 48^3 \times 20 | 47     | 25     | 200      |
| 2   | 2.57        | 16.53       | 38             | 32^3 \times 25 | 91     | 100    | 150, 200, 250, 300 |
|     |             |             |                | 32^3 \times 32 | 91     | 100    | 150, 200, 250, 300 |
| 3   | 2.60        | 16.98       | 45             | 40^3 \times 23 | 91     | 90     | 150, 200, 250, 300, 400 |
|     |             |             |                | 40^3 \times 30 | 91     | 90     | 150, 200, 250, 300, 400 |
|     |             |             |                | 40^3 \times 38 | 90     | 90     | 150, 200, 250, 300, 400 |
|     |             |             |                | 30^3 \times 60 | 91     | 140    | 200      |
FIG. 2: “Local values” of the potential, $V_{re}^{\alpha}(\vec{r}; T)$ and $V_{im}^{\alpha}(\vec{r})$, for Set 3, at $1.2 T_c$. Results for smearing level 300 are shown. The results quoted in Sec. IV A and Sec. IV B obtained from fits over the plateau range, are shown with horizontal bands. The filled diamonds and dotted lines show results for $V_{im}^{\alpha}(\vec{r})$ while the empty circles and dashed lines show results for $V_{re}^{\alpha}(\vec{r})$. (Left panel) results for $L=0$, at $r = 10 a_s$. (Right) Those for $L=1$, at $r = 12 a_s$.

FIG. 3: Hybrid potentials below $T_c$ for the $L=0$ ($B_z$) and $L=1$ ($B_+^*$) channels. The results are obtained from the $30^3 \times 60$ lattices in Table I. The results for $L=1$ channel have been shifted slightly along x-axis in the plot, for ease of viewing.

Attractive at long distances, supporting bound states for $\Sigma^{-}$ and $\Pi_u$ respectively, quantitatively the long distance attractive part is very different for the two channels, and also from the long distance part of the singlet.

At finite temperatures, the long-distance nonperturbative behavior is suppressed, and one may expect to be able to extract information about the octet potential over longer distances. This is exactly what we found for $V_{re}^{\alpha}(\vec{r}; T)$. We extract the potential from $W_a^{\alpha}(\tau, \vec{r})$, as explained in Sec. II.

In Figure 4 we compare the results of $V_{re}^{\alpha}(\vec{r}; T)$ extracted from the Wilson loop with $B_z$ and $B_+^*$ insertions. Note that the potential can be extracted from the Wilson loop modulo an additive renormalization constant (see Appendix B). For the results in this section, we have fixed the additive renormalization constant by demanding that the $T=0$ singlet potential at the shortest distance $r = a_s$ matches the lattice discretized Coulomb potential:

$$V_s(r = a_s, T = 0) = -g^2 C_F \int \frac{d^3 k}{8 \pi^3} \frac{\cos k_3 a_s}{4 \sum_i \sin^2 (k_i a_s/2)}$$

where for the coupling $g^2$ we have used the “boosted coupling” $g^2(r \sim a) = \frac{6}{\sqrt{\beta_s \beta_T}} \sqrt{P_s P_t}$; $P_s, P_t$ are the plaquette variables defined in Eq. (17). The choice of the coupling and the matching point is somewhat arbitrary, and no detailed study of systematics of the subtraction was done; so the potentials should be taken to be defined modulo a small, temperature-independent additive constant. We stress that the additive normalization of the singlet at $T = 0$ fixes the renormalization both $V_{re}^{\alpha}(\vec{r}; T)$ and $V_{im}^{\alpha}(\vec{r}; T)$ at all temperatures.

The difference between Figure 4 and Figure 3 is stark: there is no nonperturbative rising part of the potential above $T_c$, and the potentials for $L=0$ and $L=1$ agree very well (within errors) to the distance studied. Here, therefore, free from any dominant effects of the gluon string, one can talk about a “octet” potential, which is related to the interaction between the heavy quark and antiquark in the color octet configuration.
In the asymptotic values at two temperatures is larger than what Eq. (10) predicts. However, there are some quantitative differences from perturbation theory: in particular, the differences due to nonperturbative screening, and is qualitatively similar to the behavior of the potential; see [11] for a comparison of the two for the singlet channel. The color octet free energy defined in Coulomb gauge shows screening, and is qualitatively similar to the behavior of $V^{re}_{Q}(\vec{r}; T)$ shown in Figure 5. A detailed analysis of the short distance behavior of the Coulomb gauge fixed color octet free energy has been done in Ref. [20], and has been found to be in excellent agreement with perturbation theory.

In Figure 4 we display together the octet and singlet potentials above $T_c$. The octet potential is much flatter than the singlet potential. Also at each temperature, the two potentials approach the same temperature-dependent constant. Physically one expects this; at sufficiently long distance the interaction between the $Q$ and the $\bar{Q}$ is expected to vanish; the remnant constant then may be interpreted as a thermal correction to mass of the quark. In the right panel of Figure 4 we check this behavior down to longer distances using the larger, but coarser, set 1 data.

To further check the conformity with the perturbative behavior, we look at $\delta V^{re}_{T} = V^{re}_{T}(r+1) - V^{re}_{T}(r)$. In leading order perturbation theory, the ratio of this quantity in singlet and octet channels is $-\left(N_c^2-1\right)$. As Figure
The effective thermal potential is in general complex [4], with the imaginary part of the potential related to damping and decoherence mechanisms [6]. In the case of singlet channel, where the potential is attractive and leads to spectral function peaks for sufficiently massive quarks, the imaginary part controls the width of the spectral function peak. Such an interpretation is not available here, and the imaginary part is to be understood as introducing decoherence.

Figure 6 shows, our data is consistent with this.

**B. \(V^o_{\text{im}}(\vec{r}; T)\)**

The extraction of the imaginary part from the hybrid operators of the sort used here turns out to be more problematic, and we can only get partial information about them. In particular, we were not able to get reliable results for \(V^o_{\text{im}}(\vec{r}; T)\) at 2 \(T_c\), and show results only for 1.2 \(T_c\) and 1.5 \(T_c\).

In Figure 7 we compare the results for \(V^o_{\text{im}}(\vec{r}; T)\) with insertion of the \(B_z\) and \(B_+\) operators. Here we see a different behavior from that seen in Figure 6 for \(V^o_{\text{re}}(\vec{r}; T)\): while at short distances, the results for the two insertions agree within statistical error, at longer distances \(rT \gtrsim 1\) they have statistically significant differences. At such distances, it is clearly not viable to talk of the extracted \(V^o_{\text{im}}(\vec{r}; T)\) as color octet potential, as it has contributions from the gluonic operator insertion.

In Figure 8 we show the temperature dependence of \(V^o_{\text{im}}(\vec{r}; T)\). Below \(T_c\) \(V^o_{\text{im}}(\vec{r}; T)\) is consistent with zero. Above \(T_c\) it is very different from zero. In particular, the most striking behavior of \(V^o_{\text{im}}(\vec{r}; T)\) is that at \(r \to 0\) it approaches a finite value. It is interesting to see that this behavior is qualitatively consistent with the behavior suggested in Eq. (10), though of course, Eq. (10) is based on HTL perturbation theory and is not valid in short distances.

This behavior is very different from that of the singlet channel, where \(V^s_{\text{im}}(\vec{r}; T)(r \to 0) \to 0\) [11]. In Figure 8 we have also shown the imaginary part of the singlet potential. We find that \(V^s_{\text{im}}(\vec{r}; T)\) vanishes at \(r \to 0\), but rises \(\sim r^2\).
at small $r \ [11]$. The slope of $V_{\text{in}}^o(\vec{r}; T)$ at small $r$ is smaller than that of $V_{\text{im}}^o(\vec{r}; T)$. Within the accuracy of our results, $V_{\text{in}}^o(\vec{r}; T)$ for $rT < 1$ is consistent with a constant function. We do not find evidence of $V_{\text{im}}^o(\vec{r}; T)$ decreasing with $r$, as suggested by Eq. (10).

V. DISCUSSION

One way to understand the behavior of a heavy $Q\bar{Q}$ pair in quark-gluon plasma is through the introduction of an effective thermal potential [4, 5, 14]. In order to understand the evolution of quarkonia in plasma, we need to know the effective potential of $Q\bar{Q}$ pair in both singlet and octet color configurations.

Nonperturbative information about the singlet potential is available in the literature [11]. The real part of the potential shows the expected medium screening, but at a quantitative level, differs from the leading order perturbative potential even at $2 T_c$. The deviation from perturbation theory is even stronger in the imaginary part of the potential.

In contrast to the singlet potential, very little is known nonperturbatively about the in-medium interaction of $Q\bar{Q}$ in a color octet configuration. One reason for this is the difficulty in nonperturbatively defining a color octet potential. In this paper, we have made the first nonperturbative study of the effective interaction potential for $Q\bar{Q}$ in color octet configuration in the plasma. The color-octet state is studied by looking at gauge invariant states formed by combining gluonic operators with color octet static $Q\bar{Q}$ source. At $T = 0$ potentials for such states, called hybrid states, have been studied in detail: in the perturbative regime at small $r$, they are expected to give information about the octet potential, while at longer distances, where nonperturbative effects dominate, the potential becomes dependent on details of the gluonic operator.

In contrast, we find that the in-medium potential $V_{\text{in}}^o(\vec{r}; T)$ above $T_c$ remains independent of the specific hybrid channel, and gives information about the interaction between the octet $Q\bar{Q}$ pair. This is illustrated in Figure 4. Our
results for the color octet potential is summarized in Figure 5. The color octet potential is found to be screened above $T_c$, and is repulsive at all distances. This indicates that there will not be any bound states of the heavy $QQ$ in the plasma in the color octet configuration. At long distances, the potential agrees with the color singlet potential (Figure 4). Within the accuracy of our data, the data is also consistent with the leading order scaling behavior $\delta V_\text{im}^o(\vec{r};T) = -\frac{C_A/2 - C_F}{C_F}$; this is demonstrated in Figure 7.

The thermal effective potential is known to have an imaginary part \cite{4}. The imaginary part is related to the physics of Landau damping and decoherence of the wave function of the $QQ$ state. The effective potential obtained from the hybrid state also has an imaginary component. The extraction of this part is more difficult, and our results for $V_\text{im}^o(\vec{r})$ above have large errors. The imaginary part of the extracted potential is more difficult to interpret in terms of a color octet potential. For the two hybrid operators we looked at, we found agreement in

$$V_\text{im}^c(\vec{r};T) \sim \frac{1}{T^2} \sum_{i,j} q_i q_j \delta_{ij} \left\langle \hat{A}^i(\vec{r},\tau) \hat{A}^j(\vec{r},\tau) \right\rangle$$

...the (limited) accuracy of our calculation, $V_\text{im}^o(\vec{r};T)$ is consistent with a constant in this region.

**Acknowledgements:** We would like to thank Gunnar Bali, Nora Brambilla, Peter Petreczky, Anurag Tiwari and Antonio Vaio for discussions. This work was carried out under the umbrella of ILGTI. The computations reported here were performed on the clusters of the Department of Theoretical Physics, TIFR. We would like to thank Ajay Salve and Kapil Ghadiali for technical support.

**Appendix A: APE smearing**

As discussed in Sec. \ref{sec:smear} the extraction of the potential from thin link Wilson loops is difficult, and we do APE smearing \cite{26} of the spatial gauge links. This consists of replacing the spatial gauge links

$$U_i(x,\tau) \rightarrow \text{Proj}_{SU(3)} \left( \alpha U_i(x,\tau) + \sum_{1 \leq j \leq 3 \atop j \neq i} \left\{ U_j(x,\tau) U_i(x + a_s \hat{a}_j,\tau) U_j^\dagger(x + a_s \hat{a}_j,\tau) + U_i^\dagger(x - a_s \hat{a}_j,\tau) U_i(x - a_s \hat{a}_j + a_s \hat{a}_i,\tau) \right\} \right)$$

iteratively. While the quality of the signal for the Wilson loop deteriorates with the number of smearing steps, the effect of non-potential terms also decrease, making extraction of potential easier. For this work, we have taken $\alpha = 2.5$, and have done up to 400 steps of APE smearing.

It is instructive to see the effect of APE smearing on the leading order expressions for the Wilson loops. Following \cite{27} we write the effect of smearing on the gauge fields, $A_\mu$, where $V(x, x + a\mu) = e^{i a A_\mu(x)}$. To linear order,

$$A_\mu^N(Q) = \left\{ f^N(\hat{q}) P_{ij}^T(\hat{q}) + P_{ij}^L(\hat{q}) \right\} A_j(Q)$$

where

$$f(\hat{q}) = (1 - \frac{c}{4} \hat{q}^2) \sim e^{-\frac{\alpha}{4} \hat{q}^2}, \quad \hat{q}_i = \sum_{i=1}^3 q^2_i, \quad \hat{q}_i = 2 \sin q_i a_s/2,$$

and the projection operators are defined above. For small $q_i a_s$, $\hat{q}_i \rightarrow q_i a_s$ and the projection operators

$$P_{ij}^{T,L}(q)$$

Then the propagator of the smeared fields,

$$G_{ij}^{\text{sm}}(Q) = \langle A_i^N(Q) A_j^N(-Q) \rangle \sim f^{2N}(q) \frac{P_{ij}^T(q)}{Q^2 + \Pi_T(\omega_q, q)}$$

...the medium particles, which will not be able to resolve its structure. On the other hand, the medium particles will interact strongly with the color octet $QQ$, leading to damping. The $r$ dependence of the imaginary part is much milder than that of the singlet in the region $rT \lesssim 1$; within the (limited) accuracy of our calculation, $V_\text{im}^o(\vec{r};T)$ is consistent with a constant in this region.

...the medium particles, which will not be able to resolve its structure. On the other hand, the medium particles will interact strongly with the color octet $QQ$, leading to damping. The $r$ dependence of the imaginary part is much milder than that of the singlet in the region $rT \lesssim 1$; within the (limited) accuracy of our calculation, $V_\text{im}^o(\vec{r};T)$ is consistent with a constant in this region.
leads to the spectral function representation

\[ G_{ij}^{N}(q) = \int_{-\infty}^{\infty} \frac{d\rho}{\pi} \frac{\tilde{\rho}_{T}(\rho_{0}, \tilde{\rho}_{0})}{\rho_{0} - i\omega_{k}}, \quad \tilde{\rho}_{T}(\rho_{0}, \tilde{\rho}_{0}) \sim f^{2N}(q) \rho_{T}(\rho_{0}, \tilde{\rho}_{0}). \]  

(A4)

This results in a suppression of the nonpotential contribution to Wilson loop, as we discuss in the next section.

Appendix B: LO calculation of potential in HTL

Various strategies in our nonperturbative calculation of the potential has been motivated by insights from perturbation theory and in particular, the expression for the Wilson loop in LO HTL approximation. Here we put together the leading order results for the thin Wilson loop, \[3\] and \[7\] in this approximation. This section follows Ref. \[4\].

We use the Coulomb gauge. Then the gluon propagators are:

\[
D_{00}(\omega_{n}, \vec{k}) = \frac{1}{K^{2} + \Pi_{F}(K)}, \quad D_{ij}(\omega_{n}, \vec{k}) = \frac{1}{K^{2} + \Pi_{T}(K)} \left( \delta_{ij} - \frac{k_{i}k_{j}}{K^{2}} \right).
\]

(B1)

Here \(K\) refers to the Euclidean four-momenta \((\omega_{n}, \vec{k})\). The spectral functions \(\rho_{E}(k_{0}, \vec{k}), \rho_{T}(k_{0}, \vec{k})\), introduced through the integral relations

\[
\frac{1}{K^{2} + \Pi_{T,E}(K)} = \int_{-\infty}^{\infty} \frac{dk_{0}}{\pi} \frac{\rho_{T,E}(k_{0}, \vec{k})}{k_{0} - i\omega_{k}},
\]

(B2)

provide the connection to Minkowski momenta.

For the singlet channel, the potential in LO will come from diagrams for ordinary Wilson loop similar to the ones shown within parentheses of Eq. \[8\]. They add up to

\[
g^{2}C_{F} \int \frac{d^{3}k}{8\pi^{3}} (\cos k_{3}r - 1) \left\{ \frac{\tau}{k^{2} + \Pi_{F}(0, \vec{k})} + \int_{-\infty}^{\infty} \frac{dk_{0}}{\pi} \rho_{E}(k_{0}, \vec{k}) \left( 1 + n_{a}(k) \right) \left( \frac{1}{k^{2}} - \frac{1}{k_{0}^{2}} \right) (1 + e^{-\beta k_{0}} - \mathcal{F}(k_{0}, \tau)) \right\}
\]

(B3)

where we define the symmetric and antisymmetric functions

\[
\mathcal{F}(k_{0}, \tau) = e^{-k_{0}\tau} + e^{-(\beta - \tau)k_{0}}, \quad \mathcal{G}(k_{0}, \tau) = e^{-k_{0}\tau} - e^{-(\beta - \tau)k_{0}}.
\]

(B4)

The term linear in \(\tau\) in Eq. \[B3\] survives in \(W_{\tau}^{a}(\tau, \vec{r})\), leading to the potential

\[
V_{\tau} \equiv g^{2}C_{F} \int \frac{d^{3}k}{8\pi^{3}} \cos k_{3}r - 1 \frac{\tau}{k^{2} + \Pi_{F}(0, \vec{k})} = -\frac{g^{2}C_{F}}{4\pi r} e^{-m_{D}r} + g^{2}C_{F} \left( -\frac{m_{D}}{4\pi} + I_{a} \right)
\]

(B5)

where the additive divergent term \(I_{a} = \int \frac{dk}{8\pi^{3}} \frac{1}{k^{2}} \sim \frac{1}{a} \) in Eq. \[15\] results from defining the potential through Wilson loop, forcing \(V_{\tau}(r \to 0) \to 0\). The standard convention of defining potential, used in Eq. \[14\], sets \(I_{a} \to 0\), so that we get the familiar Coulomb potential at short distances. This has been done in Sec. \[14\] by fixing the T=0 singlet potential at \(r = a_{s}\) through Eq. \[18\].

Going to Minkowski time and taking large \(t\), using the relation

\[
\lim_{t \to -\infty} i\partial_{t} \mathcal{F}(k_{0}, \tau)|_{\tau \to it} = \lim_{t \to \infty} k_{0} \left( e^{-i\omega_{0}t} - e^{\beta k_{0}e^{i\omega_{0}t}} \right) \to -k_{0}^{2} 2\pi\delta(k_{0}).
\]

(B6)

we see that the potential picks up contribution from \(\rho_{E}(k_{0} \to 0, k_{0} \to 0)\). In leading order of HTL perturbation theory, for \(k_{0} \ll |\vec{k}|\), \(\rho_{T}(k_{0}, \vec{k}), \rho_{E}(k_{0}, \vec{k})\) in Eq. \[12\] behave like \[2\]

\[
\rho_{E}(k_{0}, \vec{k}) \approx -\frac{\pi m_{D}^{2}}{2|\vec{k}|} \frac{k_{0}}{(k_{0}^{2} + m_{D}^{2})^{2}}, \quad \rho_{T}(k_{0}, \vec{k}) \approx \frac{\pi m_{D}^{2}}{4|\vec{q}|^{3}} \omega.
\]

(B7)

The term with \(1/k_{0}^{2}\) in the second term of Eq. \[13\] then leads to \(V_{\tau}^{im}(\vec{r})\) in Eq. \[11\]. The \(1/k_{0}^{2}\) term does not lead to a potential; Figure \[1\] indicates that the contribution of this term is small near \(\tau = \beta/2\). As discussed in Sec. \[3\] smearing will suppress \(\rho_{T}(k_{0}, \vec{k})\).
The diagrams

(B8)

add up to

\[
g^2 C_F \int \frac{d^3 k}{8 \pi^3} (\cos k_3 r - 1) \left( \frac{1}{k^2_3} - \frac{1}{k^2} \right) \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_T(k_0, \tilde{k}) \left( 1 + n_B(k_0) \right) \left( 1 + e^{-\beta k_0} - F(k_0, \tau) \right). \tag{B9}
\]

Here the gluon lines correspond to transverse gluon propagators. Eq. (B9) does not contribute to the potential, as can be seen using Eq. (B6). However, they will contribute to the fit near \( \tau \sim \beta/2 \). These terms, however, have \( \rho_T(k_0, \tilde{k}) \); as explained in Sec. A, smearing leads to a strong suppression of these terms. When the results for potential stabilize with number of smearing steps, it indicates that the contribution of these terms have become negligible and we are getting contribution from the potential terms only.

The discussion for the hybrid Wilson loop is similar. The potential contributions in LO come from the diagrams explicitly shown in the rhs of Eq. (8), summing up to

\[
\langle BB \rangle \times \left\{ 1 + \int \frac{d^3 k}{8 \pi^3} \left( g^2 C_F + \frac{g^2}{2 N_c} e^{ik_3 r} \right) \left[ -\frac{\tau}{k^2 + \Pi_E(0, \tilde{k})} \right] + \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_E(k_0, \tilde{k}) \left( \frac{1}{k^2_0} - \frac{1}{k^2} \right) \left( 1 + n_B(k_0) \right) \left( 1 + e^{-\beta k_0} - F(k_0, \tau) \right) \right\} \tag{B10}
\]

Renormalizing in the same way as the singlet leads to the potentials Eq. (10). We reiterate that the additive renormalization we have used is fixed by matching of the T=0 singlet potential at \( r/a_s = 1 \): no separate additive renormalization is used for the octet.

Other diagrams included in \( \cdots \) in Eq. (8) are

(B11)

and variations: where the gluon lines are at \( \tau = 0 \) or to the right of \( B \), etc. The sum of their contributions is

\[
\langle BB \rangle \times \left\{ \frac{g^2}{2 N_c} \int \frac{d^3 k}{8 \pi^3} \left( 1 - \cos k_3 r \right) - \frac{g^2 N_c}{2} \int \frac{d^3 k}{8 \pi^3} \frac{2 - \cos k_3 x - \cos k_3 (r - x)}{k^2_3} \right\} \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_T(k_0, \tilde{k}) \left( 1 + n_B(k_0) \right) \left( 1 + e^{-\beta k_0} - F(k, \tau) \right). \tag{B12}
\]

Using Eq. (B6) we see that they do not contribute to the potential. In the Euclidean time data, smearing suppresses their contribution, due to the \( \rho_T(k_0, \tilde{k}) \) terms.

Diagrams that do not satisfy the factorization behavior of Eq. (B4) are

(B13)

The left set involves only the symmetric function \( F(k, \tau) \) whereas the right set involves both \( F(k, \tau) \) and \( G(k, \tau) \) of Eq. (B4). The expressions are straightforward, if unilluminating; it is easy to check that they do not contribute to the potential. Also they both involve two or more factors of \( \rho_T(k_0, \tilde{k}) \) and can be suppressed by smearing.
FIG. 10: Smearing dependence of our $V_{re}(\vec{r}; T)$ extracted from the smeared Wilson loops $W_G$. The top row shows results at 1.2 $T_c$ while the bottom row shows results at 1.5 $T_c$. The panels to the left are for $B_z$ and those to the right are for $B_\perp$ insertions. For ease of viewing, some sets have been slightly shifted along x axis in the plot.

The diagrams Eq. (B12) and Eq. (B13) do not contribute to the potential; however, they contribute in the finite $\tau$ Euclidean Wilson loops. For a successful extraction of the potential, we need to identify a plateau where their contributions are negligible. As the structures of these terms demonstrate, smearing lead to their suppression; that is why we get the plateaus demonstrated in Figure 2, from where we can extract the potential.

Appendix C: Systematics in potential estimation

In this section we discuss the effect of smearing on our extracted potential, and the size of the discretization error in our results.

1. $V_{re}(\vec{r}; T)$

As discussed in the text, for the spatial gauge connections $U$ in Eq. (7) we have used APE smeared links. Smearing also affects the $G$ field. For the singlet potential, it was noticed that the extraction of the potential depends on the smearing level to some extent [11]: one gets a better identified plateau, and the extracted potential seems to change with the smearing level at small levels of smearing, before stabilising at some level. A similar trend is seen in the octet case, except the effect is somewhat enhanced, and one needs to go to higher levels of smearing before the result becomes insensitive to the smearing level (within our errors). At higher levels of smearing, a better plateau is obtained; at the same time the statistical noise increases. In Figure 10 we show the potential extracted from Wilson loops with different levels of APE smearing, for set 3. We find that the potential saturates only at 300 smearing steps at this cutoff. For the results quoted, we have included a systematic error covering the spread between results from 300 and 400 levels of smearing. For comparison, for the same set, the singlet potential stabilised by 200 smearing steps. For set 2, we find that 200 smearing steps is enough to stabilise the potential, and the systematic error covers data with 200 and 250 smearing steps. For Figure 10 as well as for other figures shown in cutoff units, we show the unrenormalized data (i.e., the matching to Eq. (18) is not done).

The lattice-discretized results will have discretization errors, which go to 0 as one takes the continuum limit. Our lattices are quite fine-grained, so discretization effects are expected to be small. In order to estimate the size of the discretization error, in Figure 11 we show the results for $V_{re}(\vec{r}; T)$ extracted from two different lattice spacings. Here
FIG. 11: The results for $V^\alpha_{re}(\vec{r}; T)$ extracted from lattices with different discretization levels. Shown are the results at 1.2 $T_c$ (top panels) and 1.5 $T_c$ (bottom panels). The left panels correspond to the L=0 channel while the right panels show the L=1 channel results.

we added a small overall additive constant to the results at the coarser lattice. We see that the discretization error is much smaller compared to the other uncertainties in our calculation. We therefore take the results from our finest set, set 3, as indicative of the continuum results. These are the results shown in Sec. IV A.

2. Systematics for $V^\alpha_{im}(\vec{r}; T)$

For $V^\alpha_{im}(\vec{r}; T)$ the smearing dependence of the extracted results is shown in Figure 12. For the $B_\pm$ operator, the results stabilize quickly: for set 3, already by 200 smearing steps the results seem to have stabilized. For $B_\pm$ it is similar, except at 1.2 $T_c$ at longer distances. Like for $V^\alpha_{re}(\vec{r}; T)$, the error bands quoted in Sec. IV B include the spread between smearing levels 300 and 400 for set 3, and that between smearing levels 200 and 250 for set 2.

In comparison to the large errors associated with the extraction of $V^\alpha_{im}(\vec{r}; T)$, the cutoff effects do not seem to be significant. In Figure 13 we compare the results for $V^\alpha_{im}(\vec{r}; T)$ from Set 1 and set 2. Given the small cutoff dependence, we treat the results from our finest lattices as indicative of continuum results. These are the results shown in Sec. IV B.

[1] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
[2] A. Rothkopf, Phys. Rept. 858 (2020) 1.
[3] For compact reviews, with also discussion of phenomenology, see
A. Mocsy, P. Petreczky & M. Strickland, Int. J. Mod. Phys. A 28 (2013) 1340012.
S. Datta, Pramana 84 (2015) 881.
[4] M. Laine, O. Philipsen, P. Romatschke & M. Tassler, J. H. E. P. 0703 (2007) 054.
[5] N. Brambilla, J. Ghiglieri, A. Vairo & P. Petreczky, Phys. Rev. D 77 (2008) 014017.
[6] Y. Akamatsu, Phys. Rev. D 87 (2013) 045016.
[7] Y. Akamatsu, Phys. Rev. D 91 (2015) 056002.
J.P. Blaizot, D. De Boni, P. Faccioli & G. Garberoglio, Nucl. Phys. A 946 (2016) 49.
N. Brambilla, M. A. Escobedo, J. soto & A. Vairo, Phys. Rev. D 97 (2018) 074009.
R. Sharma & A. Tiwari, Phys. Rev. D 101 (2020) 074004.
FIG. 12: Smearing dependence of the imaginary part of the potential extracted from the hybrid operator inserted Wilson loops. The top row shows results at 1.2 $T_c$ while the bottom row shows results at 1.5 $T_c$. The panels to the left are for $B_z$ and those to the right are for $B_+$ insertions. For viewing purpose some sets have been slightly shifted along x axis in the plot.

FIG. 13: Comparison of results for $V_{\alpha\mu}(\vec{r}, T)$ at two different lattice spacings. The top row shows results at 1.2 $T_c$ while the bottom row shows results at 1.5 $T_c$. The panels to the left are for $B_z$ and those to the right are for $B_+$ insertions.
[8] A. Rothkopf, T. Hatsuda & S. Sasaki, Phys. Rev. Lett. 108 (2012) 162001.
[9] Y. Burnier, O. Kaczmarek & A. Rothkopf, Phys. Rev. Lett. 114 (2015) 082001.
    Y. Burnier & A. Rothkopf, Phys. Rev. D 95 (2017) 054511.
[10] P. Petreczky & J. Weber, Nucl. Phys. A 967 (2017) 592.
[11] D. Bala & S. Datta, Phys. Rev. D 101 (2020) 034507.
[12] R. Sharma & I. Vitev, Phys. Rev. C 87 (2013) 044905.
[13] Y. Akamatsu & A. Rothkopf, Phys. Rev. D 85 (2012) 105011.
[14] S. Kajimoto, Y. Akamatsu, M. Asakawa & A. Rothkopf, Phys. Rev. D 97 (2018) 014003.
[15] O. Philipsen & M. Wagner, Phys. Rev. D 89 (2014) 014509.
[16] K. J. Juge, J. Kuti & C. Morningstar, Phys. Rev. Lett. 90, 161601 (2003).
    S. Capitani, O. Philipsen, C. Reisinger, C. Riehl & M. Wagner, Phys. Rev. D 99 (2019) 034502.
[17] N. Brambilla, A. Pineda, J. Soto & A. Vairo, Nucl. Phys. B 566 (2000) 275.
[18] G. Bali & A. Pineda, Phys. Rev. D 69 (2004) 094001.
[19] F. Zantow, O. Kaczmarek, F. Karsch & P. Petreczky, Proceedings, SEWM 2002 (hep-lat/0301015).
[20] A. Bazavov, N. Brambilla, P. Petreczky, A. Vairo and J.H. Weber, Phys. Rev. D 98 (2018) 054511.
[21] D. Bala & S. Datta, Proceedings, Lattice 2019 (arxiv:1912.04826).
[22] P. Petreczky, A. Rothkopf & J. Weber, Nucl. Phys. A 982 (2019) 735.
[23] Y. Burnier & A. Rothkopf, Phys. Rev. D 87 (2013) 114019.
[24] T. R. Klassen, Nucl. Phys. B 533 (1998) 557.
[25] M. Lüscher & P. Weisz, J. H. E. P. 0109 (2001) 010.
[26] M. Albanese, et al. (APE Collaboration), Phys. Lett. B192 (1987) 163.
[27] C. Bernard and T. DeGrand, Nucl. Phys. B (proc. suppl.) 83 (2000) 845.