Application of the elastoplastic model of fatigue cracks growth for calculation of the residual resource of steps shafts

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Abstract. By the example of a shaft loaded with bending and torsional moments, a method for estimating the residual life in the presence of a fatigue crack is presented. It is shown that a crack under conditions of complex loading in the process of growth is oriented normally to the greatest tensile stresses. The crack originates in the place of greatest stress concentration. Surface crack was modeled by the finite element method in the ANSYS Workbench program. It has been determined that after the initiation of a crack, a detail has a durability resource, which can be estimated by applying an elastoplastic model of fatigue crack growth.

1. Introduction
In the practice of working machines, mechanisms, and other engineering structures, the most of emergency destruction occurs due to the presence of cracks in the details. Under the impact of loads, cracks develop, grow and, when they reach critical sizes, lead to sudden breakage. As a rule, a crack originates on the surface of a detail, and its development plane is oriented perpendicular to the largest tensile normal stresses, i.e. main stress.

Physical phenomena that occur at the tip of a crack and cause an increase in its size are determined by the magnitude of the stresses directly in the area of the crack. For cracks of normal breaking, the stress state at the tip is described using the stress intensity factor $K_I$ [1]. In practice, it is necessary to solve two major problems related to the problem of crack resistance: determining the size of the crack, its shape and location using non-destructive testing methods; study of the stress state in a detail with a crack in order to determine the residual resource [2].

Fatigue fracture occurs due to the gradual accumulation of damage in the material under the action of alternating stresses. The danger lies in the fact that the magnitude of exploitative stresses in the case of fatigue fracture is much less than the strength characteristics of the material, that determined by standard mechanical tests.

There are two main approaches to assess the durability of details under cyclic loads:
- determination of the safety factor of fatigue strength for a detail without a crack;
- assessment of the bearing capacity of the structure in the presence of a crack with a known shape and size.

The first approach is based on the experimental determination of the durability limit of materials on standard smooth samples without cracks and the calculation of the fatigue strength factor for a given cycle.

The second approach is based on advances in the area of fracture mechanics and is widely used today. In [3], the stress intensity factor was determined by numerical methods and the growth of the existing cracks in the disk mounted on the shaft was predicted. An acceptable defective level of details was determined; the maximum values of workloads were calculated to ensure an adequate safety margin.

The article [4] analyzed the torsion shafts of the tracked chassis of transport machines. The study was performed for a crack located on the cylindrical part of the torsion shaft, the plane of which is oriented at an angle to the axis of the torsion and coincides with the position of the principal stress planes of the stress state.
The article [5] describes the process of fatigue fracture and patterns of the fatigue cracks development based on the life test of samples cut from the blades of a helicopter. It is noted that the resistance to fatigue fracture is characterized by endurance limit $\sigma_{\text{f}}$, but during exploitation it is not always possible to control the stresses level arising in the structure. Therefore, the question arises: if a crack in the structure though arose, for how long does its growth rate remain stable and relatively low, which makes it possible to safely exploit the detail with an already existing crack?

In [6], the fact is noted that to expand the scope of applicability of classical fracture mechanics, and to define fracture criteria, it is necessary to introduce additional parameters that more fully characterize the stress-strain state and reflect the local constraint of deformations (or triaxial stress state) in the vicinity of the crack tip.

In article [7], it was determine that for the case of stress studies in the elastic formulation of the problem, the effect of biaxial loading on the fatigue cracks growth rate is associated with a change in the triaxial stress coefficient $\sigma_{01} = \sigma_{01}/[\sigma_{l}]$ at the crack tip.

2. The problem formulation

A brief overview of the research subject, made in the introduction, allows us to conclude that the problem of fatigue fracture in mechanical engineering is currently relevant. One of the important problems is to assess the residual life of the loaded details in the presence of material defects. It is proposed to solve this problem from the position of the recent research results in the area of fracture mechanics.

A stepped shaft from 40X steel, widely used in mechanical engineering for the transmission of rotational motion, was chosen as the subject of study. As a rule, shafts have a complex form of loading and transmit considerable forces. In this case, the shaft was loaded with torsional and bending moments. The torsional moment created a constant stress field, and the bending moment created a stress field that cyclically varied sinusoidally. The design of the stepped shaft assumes the presence of stress concentrators, which create prerequisites for the initiation of fatigue cracks. To study the stress-strain state, the finite element method is used. Geometry modeling, static structural model creation and solutions are performed in the ANSYS Workbench program.

A distinctive feature of this work is that to describe the growth of fatigue cracks, an elastoplastic model is used, which allows to consider the complex type of loading, and, in particular, to consider the compressive part of the shaft loading cycle in the process of crack growth.

To estimate the residual life of a shaft with a crack, it is necessary to determine the size of the crack by non-destructive testing, calculate the stress field at the crack tip for the loading cycle and use the crack growth rate equation to calculate the number of loading cycles to the critical crack size [8, 9].

3. Theory

The authors of this article previously proposed a method for determining the growth rate of surface cracks in thick-walled structures under the action of variable biaxial cyclic loading [10]. The analysis of the three-dimensional elastoplastic stress state at the top of surface cracks under various types of biaxial loading is performed. It was determined that the crack growth rate is influenced by the degree of metal embrittlement before the crack front. In articles [11-13], the change in normal stresses ahead of the crack front during loading of samples to the maximum load and their following unload to zero was investigated. It was determined that when unloading samples with a crack at the crack tip residual stresses arise due to plastic deformations that occurred at the loading stage. Thus, the fracture at the crack tip depends both on the stresses arising under the load of a detail with a crack, and on stresses with opposite sign, arising from the effect of residual deformations. The characteristic of the stress state at the crack tip is proposed, which correlates with the crack growth rate under various types of loading: the difference between the maximum tensile and compressive average stresses per loading cycle. A formula is derived that describes the fatigue cracks growth rate as a function of the change in average stresses ahead of the crack front.

The described theoretical premise is proposed to apply to determine the fatigue cracks growth rate in the case of symmetric loading. A shaft that transmitting torsional moment, as a rule, is also
subjected to bending loads, i.e. in sections of the shaft is realized complex stress state. Consequently, the fatigue crack will arise in the place of the greatest stresses, and in the process of subsequent growth its plane will be oriented normally to the maximum principal stresses.

The proposed elastoplastic model for the fatigue cracks growth on the example of the study of a stepped shaft allows us to explain the reasons why the symmetric loading cycle is the most dangerous and is used to determine the endurance limit.

4. Calculation results

In this article, using a stepped shaft as an example, we consider a method for estimating its efficiency when a fatigue crack is detected. Modeling of a shaft with a defect and calculations of the stress-strain state are performed by the finite element method in the ANSYS Workbench program.

Figure 1 shows the design scheme of the shaft. Shaft material - steel 40X: yield strength $\sigma_y = 400$ MPa; tensile strength $\sigma_s = 650$ MPa; relative residual elongation at break $\delta = 13\%$. When modeling the finite element mesh (Figure 1b), in the places of stress concentration, the sizes of the finite elements were reduced so as to reflect the high gradient of the calculated values.

![Figure 1.](image)

The shaft is loaded simultaneously a bending moment $M_u = 500$ N \cdot m and torsional moment $M_s = 500$ N \cdot m (Figure 1c). Thus, in a rotating shaft, constant stresses from torsional moment and cyclically varying stresses from bending moment arise (Figure 2).

![Figure 2.](image)
strength factor for a given loading cycle. However, this technique does not allow estimating the residual life of the detail in the presence of some kind of metal defect: technological, exploitative, corrosion, etc. In the case of cyclic loading of the detail, the presence of a defect leads to stress concentration and the origin of a fatigue crack.

In the considered example, the shaft defect was modeled as a spherical pore that appeared at the site of the highest stress concentration at the junction of two sections of the stepped shaft (Figure 3).

![Figure 3. Maximum principal stresses in the area of the shaft defect](image)

We will assume that the greatest probability of the initiation of a fatigue crack exists at the point with the maximum principal stresses $\sigma_1$ (Figure 3). In this case, the crack propagation plane during growth will be oriented normally to these stresses. To determine the position of the principal planes, it is necessary to determine the direction of the vector $\vec{b}(l,m,n)$, where $l,m,n$ - the direction cosines relative to the initial coordinate axes. To calculate the direction cosines, we can use known dependencies, which include the elements of the stress tensor relative to the initial coordinate axes and the principal stresses:

$$
\begin{align*}
    l^2 &= \left(\frac{\sigma_1 - \sigma_2 + \sigma_3}{2}\right)^2 + \tau^2 - \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2, \\
    n^2 &= 1 - l^2 - m^2, \\
    m^2 &= \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2 + \tau^2 - \left(\frac{\sigma_3 - \sigma_1}{2}\right)^2.
\end{align*}
$$

The stress state components are defined for the shaft model using the ANSYS Workbench software.

Figure 4 shows the position of the principal plane $\sigma_1$, passing through the point with the highest stresses, and the model of a circular crack originated in this plane. For local modeling of the shaft area near the crack, the Submodeling procedure was used, which makes it possible to obtain a more accurate solution at the next step of studying the stress state by cutting out a small element with known solutions at the boundaries of this element (Figure 5a) and the possibility of significantly reducing the size of the finite element mesh (Figure 5b).
Figure 4. Modeling of cracks in the plane of greatest stresses

Figure 5. Application of the computational procedure Submodeling for the study of local stresses
a) the values of displacements on the edges of the cut out element; b) finite element mesh

In [13-15], an analysis of the three-dimensional elastoplastic stress state at the top of surface cracks was performed for various types of biaxial loading. The change in the field of normal stresses in front of the crack front under loading of samples up to the maximum load and their subsequent unloading to zero is investigated. In [13], a characteristic of the stress state at the crack tip was proposed, which correlates with the crack growth rate under various types of loading. The formula describing the growth of fatigue cracks is obtained.

The stress state at the crack tip has a complex shape and changes in one shaft rotation from the maximum tensile stresses to the maximum compressive stresses. For such a case of shaft loading, it was proposed to determine the fatigue crack growth rate using the formula obtained for steel 40X [15]

$$\frac{da}{dN} = 8.12 \cdot 10^{-12} (\Delta K_{\sigma 0})^{3.01},$$

where $\Delta K_{\sigma 0}$ - the change average stresses coefficient of per loading cycle.

This formula differs in that it takes into account elastic-plastic deformations and residual stresses developing at the crack tip. The fatigue cracks growth rate depends on the difference between the maximum and minimum average stresses $\Delta \sigma_0$ calculated during one loading cycle.
Figure 6 shows how the zone of plastic deformations changes at the tip of a crack in one cycle of shaft rotation. At the compression stage (Figure 6 b), the shape of the plastic deformation zone is determined by the residual deformations obtained at the tension stage (Figure 6 a) during crack opening.

\[ \sigma_0 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \]  

(3)

The average stress is considered to be the measure of brittle fracture at the crack tip [13]

Figure 7 shows the procedure for determining stresses at a point located at a distance of 0.1 mm in front of the crack front, for the case of maximum tensile and compressive stresses.

\[ \Delta \sigma_0 = \sigma_0^{(t)} - \sigma_0^{(c)} \]  

(4)

The change average stress coefficient per loading cycle is calculated by the following formula [15]

\[ \Delta K_{\sigma 0} = \Delta \sigma_0 \sqrt{2\pi r} \]  

(5)

where \( r \) - the distance from the crack tip to the point at which the value \( \Delta \sigma_0 \) is calculated.

Table 1 shows the results of calculating the growth rate of fatigue cracks for different crack sizes.
### Table 1. Determination of the cracks growth rate

| Crack size, mm | Principal stresses, MPa | Δσ₀, MPa | ΔKᵥ₀, MPa·m⁻⁰·⁵ | da/dN, mm/cycle |
|----------------|-------------------------|----------|-----------------|----------------|
| 3              | 558 363 190 -38 -124 -311 | 527      | 13.2            | 1.8·10⁻⁵       |
| 5              | 661 431 293 -72 -241 -441 | 712      | 17.8            | 4.6·10⁻⁵       |

5. Discussion results

To estimate the residual life of machines and structures, in details and elements of which there is a crack, the formula for crack growth rate (2) is used, which describes the growth of the crack front in one loading cycle. If in the equation (2) to separate the variables, then we can form an integral expression

$$
\int_0^{N_c} dN_k = \int \left( \frac{0.12 \cdot 10^{-12} (\Delta K_{\sigma 0})^{0.01}}{a_0 - a_c} \right) da,
$$

where $a_0$ - the crack size registered at the moment of its detection; $a_c$ - critical crack size; $N_c$ - the number of loading cycles from the moment of crack detection to its critical size.

Integration of equation (6) can be performed numerically. The initial size and shape of the crack should be detected with the required accuracy by physical methods of non-destructive testing.

The critical size of a crack can correspond to the increasing of a crack to a certain depth established by the regulatory documents for the industries in which this structure is used. From the point of view of the structures calculation on the limiting state, it is necessary that to perform the condition

$$
K_i \leq K_{IC},
$$

were $K_i$ - stress intensity factor, defined for a body with a crack without plastic deformations; $K_{IC}$ - metal fracture toughness - mechanical characteristic determined by the standard method.

For the numerical determination of the stress intensity factor, a diagram of the distribution of normal stresses at the tip of a crack 5 mm in size was constructed in the elastic formulation of the problem (Figure 8).

![Figure 8. Distribution of elastic stresses at the crack tip](image-url)
For the case under consideration, at $r^* = 0.1$ mm, the value $\sigma^* = 780$ MPa and therefore, according to the formula (8) $K_I = 19.5$ MPa m$^{0.5}$. If for steel 40X a fracture toughness value $K_{IC} = 50$ MPa m$^{0.5}$ is taken with a good reserve of safety, then it can be concluded that for a crack of 5 mm in size there is still a reserve of safe work of detail in accordance with condition (7).

6. Conclusion

Using the study of cyclic loading of a stepped shaft as an example, it is shown that the elastoplastic model of the fatigue cracks growth makes it possible to determine the crack growth rate under various types of loading. Such a model makes it possible to take into account fracture processes at the crack tip for a symmetric cycle of bending stresses, both in the tensile part of the cycle and in the compressive part of the cycle.

When modeling a crack using the finite element method, the most dangerous shaft point was determined in the ANSYS program, and the crack plane was chosen perpendicular to the largest principal stresses. In order to reflect the high stress gradients at the crack tip, the local part of the shaft with a crack was considered. For this, the Submodeling procedure is used.

The elastoplastic model of the fatigue cracks growth in determining the crack rate for a particular shaft loading made it possible to take into account the real stress state ahead of the crack front, i.e. in the zone of the most brittle state of the metal. In this case, residual deformations were taken into account, which in the period of the compressive part of the cycle created high compressive stresses at the crack tip.

The calculation of the residual life of the shaft when a crack is detected in it should be carried out in order to assess the risk of fracture during its further working or to estimate the reduction in the loading capacity of the detail from the very fact that a fatigue crack appears.

7. References

[1] Mikheevskiy S, Glinka G, Lee E 2013 Fatigue Crack Growth Analysis Under Spectrum Loading in Various Environmental Conditions Metalurical and Materials Transactions a Volume vol 44A pp 1301-1310
[2] Ragupathy K and Ramesh K 2014 Analytical Prediction of Fatigue Crack Growth Behavior Under Biaxial Loadings Pressure Vessel Technol. № 136(2) p 11
[3] Mirsalimov V M 2009 The inverse problem of fracture mechanics for a disk mounted on a rotating shaft Applied Mechanics and Theoretical Physics vol 50 No 4 pp 201-209
[4] Pokrovsky A M and Dubin D A 2018 Analysis of crack resistance of torsion shafts of tracked vehicles under exploitative loads News of higher educational institutions. Engineering № 1 (694) pp 37-44
[5] Bokhoeva L A, Kurokhtin V Yu, Rogov V E 2016 The study of the growth of cracks in aircraft products based on natural tests Bulletin of Buryat State University. Chemistry. Physics №2-3 pp 63-68
[6] Matvienko Yu G 2015 Trends in nonlinear fracture mechanics in mechanical engineering problems Izhevs: Institute of Computer Science 56 p
[7] Vansovich K A and Aistov I P 2011 Criterion for estimating the cracks growth rate under biaxial loading Modern technologies. System analysis. Modeling № 3 (31) pp 57-61
[8] Ling Y, Mahadevan S 2012 Integration of structural health monitoring and fatigue damage prognosis Mechanical Systems and Signal Processing vol 28 pp 89–104
[9] Giancane S, Nobile R, Panella F W, Dattoma V 2010 Fatigue Life Prediction of Notched Components Based on a New Nonlinear Continuum Damage Mechanics Model Procedia Engineering vol 2 № 1 pp 1317–1325
[10] Vansovich K A, Aistov I P, Beselia D S 2018 The method of estimating the residual resource of the main pipeline in the presence of surface cracks under exploitative loads Engineering magazine: science and innovation No 5 (77) pp 1-17
[11] Zhang J, He X D, Suo B, Du S Y 2008 Elastic–plastic finite element analysis of the effect of compressive loading on crack tip parameters and its impact on fatigue crack propagation rate Eng. Fract. Mech. vol 75 pp. 5217–5228
[12] Sun Y, Choo H, Liaw P K, et al 2005 Changes in elastic-strain profiles around a crack tip during tensile loading and unloading cycles Int. Centre for Diffraction Data 2005 vol 48 pp 117–122
[13] Vansovich K A 2017 The growth model of fatigue surface cracks during the loading-unloading cycle The Journal Omsk Scientific Bulletin №3 pp 49-53
[14] Sander M, Richard H A 2006 Fatigue crack growth under variable amplitude loading Part I: experimental investigations Fatigue Fract. Engng. Mater. Struct. vol 29 pp 291–301
[15] Vansovich K A, Aistov I P 2017 Analysis of the three-dimensional stress state at the top of surface fatigue cracks Modern technologies. System analysis. Modeling. Science Magazine No 4 (56) pp 27-33
[16] Rajaram H, Socrate S, Parks D M 2000 Application of domain integral methods using tetrahedral elements to the determination of stress intensity factor Engineering Fracture Mechanics vol 66 pp 455–482
[17] Sahu Y, Moulick S 2015 Analysis of Semi-elliptical Crack in a Thick Walled Cylinder Using FEM International Journal of Advanced Engineering Research and Studies vol IV pp 231–235