Optomechanical Transduction and Characterization of a Silica Microsphere Pendulum via Evanescent Light

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Transduction of the motion of a micron- or nano-sized object to an optical signal is essential for optomechanical systems. Here, we study the optical response of a cantilever-like, silica, microsphere pendulum, evanescently coupled to a fiber taper. In this system, the optical coupling element also acts as the mechanical motion transducer and the pendulum’s oscillations modulate the optical whispering gallery modes (WGMs) both dispersively and dissipatively. This unique mechanism leads to an experimentally-observable, asymmetric response function of the transduction spectrum. This phenomenon is explained by using coupled mode theory with Fourier transforms. The optomechanical transduction and its relation to the external coupling gap is experimentally investigated in depth and shows good agreement with the theory. A deep understanding of this mechanism is necessary in order to explore cooling and trapping of a micropendulum system.

Keywords: whispering gallery mode; optomechanics; micropendulum; transduction; microsphere

An object in a steady state, quadratic potential field, e.g. an electric field or gravity, will describe simple harmonic motion if acted upon by small perturbations from equilibrium. A well-known example of such behavior is that of the simple pendulum, which consists of a weight suspended on a pivot. When subjected to a perturbation, the pendulum oscillates at a certain frequency called the fundamental frequency or mode. Recently, simple pendulum-like motion at either the micro- or nanoscopic scale has generated interest in both fundamental and applied physics. Though challenging to measure, the motion can be transduced by means of light. Objects on these scales may be affected by light fields through radiation pressure and optical gradient forces. These can also be exploited in order to achieve optical trapping. These progresses rely mainly on light-enhanced optomechanical interactions and may find practical applications, such as the actuation of optomechanical systems through attractive or repulsive optical forces. Cooling this motion is intriguing and several proposed mechanisms may be utilized. Mesoscopic object cooling to the vacuum ground state can have significant impact on realizing cavity quantum optomechanics, coherent manipulation, state preparation, and even on a reappraisal of quantum mechanics. As for recent progress in low frequency devices, the mechanical motional state of a micromechanical cantilever has been cooled using a cooling-by-measurement technique via pulsed optomechanics.

Among the proposed and/or implemented systems that can be used for optomechanics, we have previously focused on a micropendulum system. This is a very simple configuration, with quite low fundamental frequencies (in the Hz-kHz range) and relatively large amplitude (typically nm) oscillations. The micropendulum under consideration here consisted of a microsphere, that acted as a bob, attached to a thin stem so that it could swing freely. The microsphere supported high quality (Q) factor whispering gallery modes (WGM). A tapered optical fiber was used for evanescent coupling of light into the WGMs, as shown in Fig. 1(a). When probe light in the coupling fiber was tuned near a WGM resonance (see Fig. 1(b)) the large amplitudes of the mechanical oscillations of the micropendulum produced a variation in the fiber transmission signal which was high above the noise level. The signal strength was related to the detuning of the laser relative to the WGM resonances. Previously, it was shown that the signal should be stronger if the laser were red-detuned from resonance. In this paper, we will explain this effect in more detail and discuss the mechanisms involved from both a theoretical and experimental viewpoint. A coupled mode theoretical framework can be used to describe the system depicted in Fig. 1(a). The slowly varying term of the normalized intracavity electromagnetic (EM) field, \(a\), is given by:

\[
\frac{da}{dt} = -(\kappa_0 + \kappa_s + \kappa_e + i\Delta)a + \sqrt{2\kappa_e}a_{in},
\]

where \(\Delta = (\omega_l - \omega_0)\) is the optical detuning, \(\omega_l\) and \(\omega_0\) representing the laser and cavity WGM resonant frequencies, respectively. \(\kappa_0\), \(\kappa_s\), and \(\kappa_e\) are decay rates representing the intrinsic, scattering, and external quality factors, respectively. \(\kappa_e\) and \(\kappa_s\) are related to the coupling gap between the micropendulum and the tapered fiber. \(\kappa_s\) represents the light scattered at the taper coupling region and limits the ideality of the taper-coupled system. The input laser source with power, \(P_{in}\), determines the input normalized amplitude, \(a_{in}\), such that \(a_{in} = \sqrt{P_{in}/\hbar\omega_l}\). The detected signal at the output of the fiber satisfies the input-output relationship:

\[
\begin{align*}
S(t) &= \sqrt{2\kappa_e}a_{in}e^{i\omega_l(t - t_0)} + \text{noise}, \\
\text{where } t_0 &= \text{the delay time.}
\end{align*}
\]
and all the following we apprised is a numerical simulation of the trans-
1chanical oscillation; hence, µ
m
lum with 88 μm sphere diameter, 130 μm long and 2 μm in diameter stem, and is at 1.05 kHz. Therefore, the steady state condition is always satisfied during the mechanical oscillation; hence, da/dt = 0 in Eq. 1. Now the steady state intracavity field can be expressed as 
\[ a(t) = e^{i(\beta t + \delta t)} \] . The final transmission through the fiber, 
\[ T = 1 - \frac{2\kappa_e}{(1 + \kappa_s + \kappa_e + i\Delta)^2} \] .

Here, in Eq 2 and all the following we apprised \( \kappa_e, \kappa_s \) and \( \Delta \) as normalized entities with \( \kappa_0 \). Essentially, \( \kappa_s, \kappa_e \) and \( \Delta \) are all coupling gap \( d \) dependent, as the pendulum moves, hence, their values are also related to time. The contribution from scattering is small compared to that from the other. Therefore, for the following simplified theoretical discussion we disregard \( \kappa_s \).

The relative motion of the fiber and the microsphere causes a frequency shift of the WGMs and this contributes a dispersive perturbation to the detuning term as \( \Delta(t) = \Delta_0 - g \cos(\Omega_m t/\kappa_0) \). Here \( \Delta_0 = \Delta/\kappa_0 \) is the unperturbed detuning term. Supposing that the micropendulum’s displacement and frequency are \( \delta d \) and \( \Omega_m \), the dispersion is defined as 
\[ g = \frac{\partial \Omega_m}{\partial d} \times \delta d \].

The external coupling, \( \kappa_e(t) \), is defined by 
\[ \kappa_e(t) = \kappa_e^0 \exp(\gamma \delta d \cos(\Omega_m t)) \], where \( \gamma \) is a coefficient related to the taper properties and \( \kappa_e \) is the external coupling rate for the pendulum’s equilibrium position, \( d_0 \). This can be expanded as a series of harmonics, \( \cos(n \Omega_m t) \), where \( n = 1, 2, 3 \ldots \). Since \( \delta d \) generally small, \( \kappa_e(t) \) can be linearized to its first order term, corresponding to \( \Omega_m \), i.e. \( \kappa_e(t) \approx \kappa_e^0 (1 + 2\beta \cos(\Omega_m t)) \), where \( \beta = I_1^0 (\gamma \delta d) \). \( I_1^0 \) represents the modified Bessel function of the first kind. Substituting \( \Delta(t) \) and \( \kappa_e(t) \) into Eq. 2 the Fourier transformed (FFT) transmission spectrum, \( \tilde{T} \), at \( \Omega_m \) is:

\[
\tilde{T} \approx \frac{4\kappa_e^0}{(\kappa_e^0 + 1)\Delta_0^2 + \Delta_0^2 + F(\kappa_e^0) \times 2\Delta_0 g - 2\beta(\kappa_e^0 \Omega_m^2 - \Delta_0^2 - 1)}.
\]

Here, \( F(x) = x^4 + 4x^3 + 6x^2 + 4x + 1 \) is a polynomial expression. For conventional optomechanical systems the major contributor to the optomechanical transduction is dispersion and the effect of \( \beta \) is normally omitted. Hence, the resulting spectrum \( \tilde{T} \) is symmetric around \( \Delta_0 = 0 \). For the micropendulum system, the motion induces a dissipative oscillation on the optical coupling. One can see from Eq. 3 that the symmetry is broken if \( \beta \) is non-negligible (typically like that shown in Fig 1(e)). In contrast, when \( g = 0 \), i.e. when the contribution from dispersion is negligible compared to that from dissipation, \( \tilde{T} \) produces a symmetric response around zero detuning. In optomechanical systems such as ours, where the mechanical oscillator is also the optical coupler, the motion induced dissipation and dispersion of the WGMs distorts the optomechanical frequency response spectrum (as shown in Fig. 1(c)-(e)). This is due to that the mechanical oscillator is also the optical coupler and hence both effects are considered as shown in Fig 1(c)-(e)). Recently, a similar mechanism was discussed for a photonic crystal nanocavity. Furthermore, since \( \kappa_e^0 \) and \( g \) vary with \( d_0 \), the transduction spectrum changes with the coupling gap. Fig. 2 is a numerical simulation of the trans-
duction at different coupling gaps and detunings. This was obtained by directly solving Eq. 2 using the same parameters as in our experiments, which will be described later. It can be seen that, due to the two different mechanisms, the transduction signal should be higher when the laser is red-detuned relative to the WGM resonance. Also, if the micropendulum is closer to the fiber, the transduced signal becomes stronger, with the strongest signal being achieved at critical coupling.

Previously, in a similar experimental system, we observed that the signal peak was higher when the laser was red-detuned relative to the WGM resonance arising from thermal effects. For a high input power, a silica sphere can absorb light and generate heat, subsequently shifting the WGMs. This is similar to the effect of dispersion on the system, so the frequency response should be asymmetric around zero detuning, as discussed above. In order to uncover the fundamental dissipative and dispersive mechanisms in the micropendulum system, we redid the earlier experiments using an input power of about 100 nW. The laser detuning was scanned in steps of 0.6 MHz spanning more than one full WGM line width and FFT peaks were recorded for each value of $\Delta_0$. The measurement was repeated for different coupling gaps. The frequency response was then reconstructed from the data. To isolate the system from environmental disturbances, the experiment was conducted in vacuum at 40 Pa. Typical response spectra are shown in Fig. 3. Because of the influence of $\kappa_\text{c}(t)$, red detuning yielded a higher optomechanical transduction signal. It is worth noting that, for other systems, a similar spectrum only occurs when back action is present. For those systems, the signal on the red side is lower due to the cooling effect. However, in our experiment, the input power was not enough to generate neither thermal effect nor back action. The obtained results are evidence that the micropendulum is a unique system where both dissipative and dispersive modulations caused by the pendulum motion play dominant roles.

To further confirm the observed phenomena, the experimental results were fitted with the discussed theory. All the parameters for fitting were extracted from the experimental data. The mechanical mode frequency was observed to be $\Omega_m/2\pi = 1.05$ kHz and the oscillator displacement was estimated as $\approx 1$ nm. By fitting the experimentally measured $T$, at $\Delta_0 = 0$, for different coupling gaps, the general relation for $\kappa_\text{c}$, was reconstructed as an exponentially decaying function of $d$, such that $\kappa_\text{c} = A \exp(-\gamma(d - C))$ with fitting parameters, $A = 4.1504$, $\gamma = 14.12892 \, \mu\text{m}^{-1}$ and $C = 0.12553\mu\text{m}$. Similarly, an independent experimental measurement was taken for the optical mode spectral position for different gaps. The derivative of the above yields a reconstructed dispersion relation, which is an exponentially rising function of $d$, as $g = g'\exp(-\zeta d)$, where $g'/2\pi = 58.515$ MHz and $\zeta = 12 \, \mu\text{m}^{-1}$ are the fitting parameters. The intrinsic loss, $\kappa_0$, was estimated from the optical line width of the WGM when it was far undercoupled, so that $\kappa_0/2\pi \approx 13$ MHz. We found that, by considering the coupling ideality, a better fit to the curves could be obtained, especially for the critically coupled and overcoupled regimes, when the fiber taper scattered significant amounts of light. In order to determine the profile of

![Fig. 3. Transduction response profiles. From top to bottom: (a) undercoupled ($d_0 = 0.384 \, \mu\text{m}$), (b) critically coupled ($d_0 = 0.210 \, \mu\text{m}$) and (c) overcoupled ($d_0 = 0.119 \, \mu\text{m}$). The solid red lines are theoretical fits. Each data point for the critical (overcoupled) coupled plot is an average of 6 (20) data points.](image)
the normalized total decay rate, $\kappa_\epsilon$, the loaded $Q$-factor (given by $1/Q = 1/Q_0 + 1/Q_\epsilon + 1/Q_s$) was measured at different coupling gaps. With $\kappa_\epsilon$ as mentioned above, the scattering rate can be calculated from $\kappa_s = \kappa_\ell - \kappa_\epsilon - 1$. The calculated $\kappa_s$ was fitted with an exponentially decaying profile as a function of $d$, similar to $\kappa_\epsilon$. To introduce the pendulum dynamics, $d = d_0 + \delta d \cos (\Omega_m t)$, was substituted in to the general functions $\kappa_\epsilon$, $g$ and $\kappa_s$ and were then substituted into Eq. 2 which was also used to plot Fig. 2. For Fig. 3 the coupling conditions varied from undercoupled (Fig. 3a) to overcoupled (Fig. 3c)). To fit them, approximated coupling gaps to theoretical equilibrium ones were chosen. We then plotted all detuned maxima of the response curves for coupling gaps ranging from 0.1 to 0.45 $\mu$m with the respective theoretical prediction curves presented Fig. 4. Good agreement with experiments is evident. The discrepancies in the fit may originate from (i) the difficulty in controlling the coupling gap, leading to errors on this value, (ii) an excessive noise level in the low frequency FFT spectrum, (iii) thermal fluctuations, and (iv) laser noise. Regardless of these sources of error, both theory and experiments show a maximum transduction for red detuning at the critical coupling gap, which is depicted as the dashed line in Fig. 4 and a clear contrast between the detuned transductions.

In conclusion, the optomechanical transduction of a micropendulum taper system was measured using weak probe light. Large displacements of the mechanical oscillations modulated the WGMs both dissipatively and dispersively. This, in turn, affected the transmission through the fiber. A model based on coupled mode theory and FFT explained the experimentally-observed, WGM detuned asymmetric response quite well. Maximum sensitivity for probing the system requires that the system operate under the critically coupled condition with the laser properly red-detuned from the WGM resonances. This transduction mechanism, which is by now well understood, can provide a foundation for future work on cooling and trapping of a micropendulum and optomechanical sensing applications.

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