QUANTUM TRANSPORT THEORY
Beyond Molecular Dynamics

Peter A. Henning
Theoretical Physics, Gesellschaft für Schwerionenforschung GSI, P.O.Box 110552
D-64220 Darmstadt, Germany
E-mail: P.Henning@gsi.de

ABSTRACT
The problem of thermalization of a quark–gluon plasma is addressed in the framework of thermal field theory. Within a simple approximation, the full quantum relaxation problem is solved and compared to the Boltzmann solution. Memory effects and a slowdown of the relaxation process are the results, they can be partially described by using a generalized kinetic equation without quasi-particle approximation.

1. Introduction
In the past decade the effort invested in ultrarelativistic heavy ion collisions (URHIC) has grown considerably. The general hope is, that at some time in the near future one may be able to observe an excursion of strongly interacting matter from the state of hadrons before the collision into the phase of a quark–gluon plasma (QGP). Consequently, the discussion of possible signals from such a shortlived state is quite vivid: Weakly interacting probes like photons or lepton pairs, as well as strongly interacting signals like those presented by quark flavors of higher mass have been proposed. Similar to most of these investigations is the assumption of a thermalized plasma phase, followed by the calculation of the time evolution along one or the other line of physical reasoning.

The present paper is a study of the time scales necessary for such a thermalization. Ultimately, it is the goal to investigate a physical scenario that one may reach in future URHIC: A sea of gluons, initially at low temperature, is heated to a very high temperature over a short time. In this hot glue, quark-antiquark pairs are popping up – until at the very end a thermal equilibrium in the sense of a degenerate plasma is reached. For the purpose of this conference contribution however, the calculations will be presented on a more abstract level. The full consideration is the subject of an extended paper.

The primary motivation for this study are serious doubts that the requirements for the applicability of standard transport theory (= kinetic gas theory) are fulfilled in a QGP: The thermal scattering of constituents occurs so frequently, that subsequent collisions overlap quantum mechanically. This implies, that a treatment...
in terms of quasi-particles is inadequate, one has to account for a nontrivial spectral
function of the system components.\[4\]

The use of a finite temperature field theoretical formulation with continuous
spectral function is also suggested by the Narnhofer-Thirring theorem\[5\], which states,
that interacting systems at finite temperature cannot be described by particles with
a sharp dispersion law. As an additional benefit, this approach is free of unphysical
infrared singularities occurring in standard perturbation theory.

The paper is organized as follows: In the next section a brief introduction to the
formalism necessary for non-equilibrium quantum fields is given. In section 3 the
approximate spectral function is discussed, followed by a solution of the quantum
transport equation in section 4. In section 5 a generalized kinetic equation is solved
which stands between the usual Boltzmann equation and the quantum transport
equation of section 4. Conclusions are drawn in the final section of the present
work.

2. Matrix-valued Schwinger-Dyson equation

As has been pointed out by various authors, the description of dynamical (time
dependent) quantum phenomena in a statistical ensemble necessitates a formalism
with a doubled Hilbert space.\[6\],\[7\],\[8\]. For the present purpose the relevant content of
this formalism is, that its two-point Green functions are $2 \times 2$ matrix-valued. It is
left to the reader to choose either the conventional Schwinger-Keldysh, or Closed-
Time Path (CTP) Green function formalism\[9\], or the technically simpler method of
Thermo Field Dynamics (TFD)\[10\].

Within this matrix formulation, consider the Schwinger-Dyson equation for the
full quark propagator $S = S_0 + S_0 \otimes \Sigma \otimes S$. Here $S_0$ is the free and $S$ the full two-point
Green function of the quark field, $\Sigma$ is the full self energy and the generalized product
of these is to be understood as a matrix product (thermal and spinor indices) and an
integration (each of the matrices is a function of two space coordinates). Throughout
this paper the convention is used to write space-time and momentum variables also
as lower indices, e.g. $\Sigma_{xy} \equiv \Sigma(x, y)$.

In the CTP formulation as well as in the $\alpha = 1$ parameterization of TFD\[11\], the
matrix elements of $S$, $S_0$ and $\Sigma$ obey

$$S_{11}^{11} + S_{22}^{22} = S_{11}^{12} + S_{22}^{21} \quad \Sigma_{11}^{11} + \Sigma_{22}^{22} = -\Sigma_{12}^{12} - \Sigma_{21}^{21}. \quad (1)$$

Therefore the four components of the Schwinger-Dyson equation are not indepen-
dent, the matrix equation can be simplified by a linear transformation which one
may conveniently express as a matrix product.\[12\]. It achieves a physical inter-
pretation only in the TFD formalism, see ref. \[4\]. The transformation matrices $B$
are

$$B(n) = \begin{pmatrix} (1-n) & -n \\ 1 & 1 \end{pmatrix}, \quad (2)$$
depending on one parameter only. For example, the third term in the Schwinger
Dyson equation becomes
\[ B(n) \tau_3 S_0 \circ \Sigma \circ S (B(n))^{-1} = \left( \begin{array}{c}
S^R_0 \circ \Sigma^R \circ S^R \\
S^A_0 \circ \Sigma^A \circ S^A
\end{array} \right). \] (3)

Here, \( \tau_3 = \text{diag}(1, -1) \), \( \Sigma^{R,A} \) are the retarded and advanced full self energy function, and \( S^{R,A} \) are the retarded and advanced full propagator (similarly for \( S_0 \))
\[
\Sigma^R = \Sigma^{11} + \Sigma^{12}, \quad \Sigma^A = \Sigma^{11} + \Sigma^{21}
S^R = S^{11} - S^{12}, \quad S^A = S^{11} - S^{21}.
\] (4)

The diagonal elements of the transformed equation therefore are \textit{retarded} and \textit{advanced} Schwinger-Dyson equation. The off-diagonal element is a \textit{transport equation}.

Now one switches to the mixed (or Wigner) representation of functions depending on two space-time coordinates:
\[
\hat{\Sigma}_{XP} = \int \! d^4(x - y) \exp \left(iP_\mu (x - y)^\mu\right) \Sigma_{xy} \quad \text{with} \quad X = (x + y)/2,
\]
the \(-\) sign will be dropped henceforth. The Wigner transform of the convolution \( \Sigma \circ G \) is a nontrivial step. Formally it may be expressed as a gradient expansion
\[
\int \! d^4(x - y) \exp \left(iP_\mu (x - y)^\mu\right) \Sigma_{xz} \circ G_{zy} = \exp \left(-i\hat{\Theta}\right) \hat{\Sigma}_{XP} \hat{G}_{XP}.
\] (5)

\( \hat{\Theta} \) is a 2nd order differential operator acting on both functions appearing behind it, \( \hat{\Theta} A_{XP} B_{XP} = \frac{1}{2} (\partial_X A_{XP} \partial_Y B_{XP} - \partial_Y A_{XP} \partial_X B_{XP}) \). Obviously, this first-order term in the application of the infinite-order differential operator \( \exp(-i\hat{\Theta}) \) is the Poisson bracket. Henceforth this operator is formally split into \( \cos \hat{\Theta} - i \sin \hat{\Theta} \). Similarly, one defines real Dirac matrix-valued functions as real and imaginary part of propagator and self energy:
\[
S^{R,A}_{XP} = G_{XP} \mp i\pi A_{XP} \quad \Sigma^{R,A}_{XP} = \text{Re} \Sigma_{XP} \mp i\pi \Gamma_{XP}.
\] (6)

\( A_{XP} \) is the generalized spectral function of the quantum field.

Now consider the equations obtained by action of Dirac differential operators (= \textit{inverse free propagators}) on the matrix-transformed Schwinger-Dyson equation. The diagonal components are
\[
\begin{align*}
\text{Tr} \left[ (P^\mu \gamma_\mu - m) A_{XP} \right] &= \cos \hat{\Theta} \text{Tr} \left[ \text{Re} \Sigma_{XP} A_{XP} + \Gamma_{XP} G_{XP} \right] \\
\text{Tr} \left[ (P^\mu \gamma_\mu - m) G_{XP} \right] &= \text{Tr} \left[ 1 \right] + \cos \hat{\Theta} \text{Tr} \left[ \text{Re} \Sigma_{XP} G_{XP} - \pi^2 \Gamma_{XP} A_{XP} \right].
\end{align*}
\] (7)

Two important facts about these equations have to be emphasized. First notice that these equations do not in general admit a \( \delta \)-function solution for the spectral function \( A_{XP} \) even in zero order of the gradient expansion. This has led to erroneous statements in papers deriving transport equations from the Schwinger-Dyson equation, because the right side of (7) may not be disregarded. In short terms, there is not such thing as a mass shell constraint in \textit{quantum} transport theory!
Secondly, the equations do not contain odd powers of the differential operator $\diamondsuit$. This implies, that when truncating the Schwinger-Dyson equation to first order in this differential operators (the usual order for the approximations leading to kinetic equations), the spectral function $A_{XP}$ may still be obtained as the solution of an algebraic equation.

The off-diagonal component of the transformed Schwinger-Dyson equation reads, after acting on it with the inverse free propagator $\hat{S}_0^{-1}$,

$$\hat{S}_0^{-1} S^K_{zy} = \Sigma^R_{xz} \otimes S^K_{zy} - \Sigma^K_{xz} \otimes S^A_{zy},$$

with kinetic components $S^K = (1 - n) S^{12} + n S^{21}$ and $\Sigma^K = (1 - n) \Sigma^{12} + n \Sigma^{21}$. Inserting the real functions defined before, this leads to a differential equation, which henceforth is labeled quantum transport equation $\hat{S}_0^{-1}$:

$$\text{Tr} \left[ (\partial^\mu \gamma_\mu + 2 \sin \diamondsuit \Re \Sigma_{XP} + \cos \diamondsuit 2\pi \Gamma_{XP}) S^K_{XP} \right] = 2i \text{Tr} \left[ i \sin \diamondsuit \Sigma^K_{XP} G_{XP} - \cos \diamondsuit \Sigma^K_{XP} i \pi A_{XP} \right].$$

Note, that here even as well as odd powers of the operator $\diamondsuit$ occur. The solution in zero order $\diamondsuit$ is not trivial, since it leads to the diagonalization of the propagator in equilibrium states $\hat{S}_0^{-1}$.

3. Effective fermion propagator and spectral functions

In a thermal equilibrium state at temperature $T$, the full propagator of a fermionic quantum field has to obey the Kubo-Martin-Schwinger boundary condition $\hat{S}_0^{-1}$:

$$(1 - n_F(E)) S_{eq}^{12}(E, \vec{p}) + n_F(E) S_{eq}^{21}(E, \vec{p}) = 0 .$$

$n_F(E)$ is the Fermi-Dirac equilibrium distribution function at temperature $T$, $n_F(E) = (e^{\beta(E-\mu)} + 1)^{-1}$. As seen above, the matrix valued propagator has only three independent components, two of which are furthermore complex conjugate. One may now use the KMS condition to eliminate the off-diagonal component of the equilibrium propagator in favor of $n(E)$ $\hat{S}_0^{-1}$:

$$S_{eq}^{(ab)}(p_0, \vec{p}) = \int dE A(E, \vec{p}) \times$$

$$\tau_3 (B(n(E)))^{-1} \begin{pmatrix} \frac{1}{p_0 - E + i\epsilon} & 0 \\ 0 & \frac{1}{p_0 - E - i\epsilon} \end{pmatrix} B(n(E)).$$

Here $A(E, \vec{p})$ is the spectral function of the quark field, properly normalized and approaching a $\delta$-function for vanishing interaction.
With the present paper one is addressing non-equilibrium states. For such states one may not derive a spectral representation of the propagator in general, but one may still exploit the fact that retarded and advanced propagator are by definition analytical functions of the energy parameter in the upper or lower complex energy half plane.

Hence, even for non-equilibrium states one may write in the mixed (or Wigner) representation

\[ S^{R,A}(E, \vec{p}, X) = \text{Re} G_{XP} \mp \pi i \mathcal{A}_{XP} = \int_{-\infty}^{\infty} dE' \mathcal{A}(E', \vec{p}, X) \frac{1}{E - E' \pm i \epsilon}, \]  

since this is nothing but the Wigner transform of \( S^{R,A}_{xy} = \mp 2\pi i \Theta (\pm (x_0 - y_0)) \mathcal{A}_{xy}. \)

By inspection of eq. (7) one finds, that only a self energy function is needed for a full determination of the function \( \mathcal{A}_{XP} \). This self energy function is in general a functional of \( \mathcal{A}_{XP} \) again – which then leads to a complicated set of integro-differential equations for the self consistent determination of the retarded and advanced propagator.

For the limited purpose of the present paper however, one makes some physically motivated assumptions:

1. The self energy function for the quarks is dominated by gluonic contributions. This is justified because the quark-quark scattering cross section is much smaller than the quark-gluon cross section.

2. The gluon background is dominated by external conditions, i.e., we neglect the back-reaction of quarks on the gluon distribution.

3. The external conditions determining the gluon field are changing in a short time interval, and the system is translationally invariant in 3-dimensional coordinate space.

4. One neglects the influence of anti-quarks in the spectral function. This restriction is removed in the extended version of this paper, ref. 2.

For these assumptions also exists a practical reason: They allow a clean separation of various aspects of the quantum transport problem, whereas this separation is difficult (if not impossible) when considering more realistic systems.

These assumptions lead to the following ansatz for the imaginary part of the self energy function:

\[ \pi \Gamma_{XP} \equiv \gamma^0 \Gamma_t = \gamma^0 g T(t) = \gamma^0 g \left\{ \begin{array}{ll}
T_i & \text{if } -\Lambda > t \\
\frac{(t + \Lambda) T_f - t T_i}{\Lambda} & \text{if } 0 > t > -\Lambda \\
T_f & \text{if } t > 0
\end{array} \right. \]

Within this ansatz the limit of \( \Lambda \to 0 \) is discussed separately, it corresponds to instantaneous heating of the gluon background. Furthermore one ensures causality.
by calculating the real part of the self energy through a dispersion integral. This integral is divergent, hence in principle one also needs a regularization procedure – but the effects of this divergence cancel in the equations.

For the quark spectral function, one uses the simple form

\[ \mathcal{A}(E, \vec{p}, t) = \frac{\gamma^0}{\pi} \frac{\gamma_t}{(E - \omega_t)^2 + \gamma_t^2} \quad \text{(14)} \]

Hence, one approximates the quark spectral function by two time-dependent parameters \( \omega_t \) and \( \gamma_t \), which may be interpreted as effective mass and effective spectral width. One may argue about the validity of this approach, in particular whether not a momentum dependent spectral width is an absolute necessity for a realistic calculation.

However, first of all one may safely assume that the quarks appearing in the hot medium are slow – hence the properties of the quark distribution may be approximated by those of quarks at rest. A second argument in favor of this ansatz is the question of causality: The expectation value of the anti-commutator of two quark fields is nothing but the Fourier transform of the spectral function. Hence, while for some more general spectral function causality may be violated, the above ansatz guarantees it when supplemented with a corresponding antiparticle piece.

With the above spectral function the coupled system reduces to a single nonlinear equation for \( \gamma_t \), plus the condition \( \omega_t^2 = \omega_0^2 = \vec{p}^2 + m^2 \). This latter condition is more complicated, when the anti-particle piece of the spectral function

Figure 1: Time dependent spectral width parameter \( \gamma_t \).
Parameters are \( g=0.12, T_i=1 \text{ MeV}, T_f=200 \text{ MeV}, m=10 \text{ MeV} \).
Thin lines: \( \Gamma_t \) from eq. (13), thick lines: \( \gamma_t \) from eqs. (15), (16);
continuous lines: \( \Lambda = 0 \), dashed lines: \( \Lambda = 4 \text{ fm/c} \).
is taken into account. The energy parameter is chosen as \( E = \omega_0 \), which yields instead of eq. (7) as the Schwinger-Dyson equation for the retarded (or advanced) two-point function of the quarks:

\[
\gamma_t = g T_i + g (T_f - T_i) \Theta(t) + g (T_f - T_i) \left( \frac{t + \Lambda}{\Lambda} - \frac{1}{2 \gamma_\Lambda t} \right) \Theta(-t) \Theta(t + \Lambda) - \frac{g (T_f - T_i)}{2 \gamma_\Lambda t} \left( \Theta(t) e^{-2 \gamma_\Lambda t} - \Theta(t + \Lambda) e^{-2 \gamma_\Lambda (t + \Lambda)} \right) \tag{15}
\]

In the limit \( \Lambda \to 0 \), this becomes even simpler:

\[
\gamma_t = g T_i + g (T_f - T_i) \Theta(t) (1 - e^{-2 \gamma_\Lambda t}) \tag{16}
\]

In Fig. 1, the solution of these equations is plotted in comparison to the time dependent imaginary part of the self energy function from eq. (13). It is obvious, that the solution of the nonlinear equations (15) resp. (16) approaches the imaginary part of the self energy function with a characteristic delay time. Simply using \( \Gamma_t \) from eq. (13) instead of \( \gamma_t \) – which would correspond to an adiabatic approximation – therefore ignores this delay time. In ref. it is discussed how this delay time is calculated from the system parameters.

4. Transport equation

As was stated above, the off-diagonal component of the transformed Schwinger-Dyson equation is a transport equation. To see this more clearly, define the generalized covariant distribution function \( N_{XP} \) through the equation

\[
(1 - N_{XP}) S_{12}^{XP} + N_{XP} S_{21}^{XP} = 0. \tag{17}
\]

Note the similarity with eq. (10): The above equation indeed ensures, that in the limit of thermal equilibrium one achieves \( \lim_{\text{eqil}} N_{XP} = n_F(E) \). For the purpose of the present paper \( N_{XP} \) is taken as a scalar function. The description of phenomena like spin diffusion requires to use a Dirac matrix valued \( N_{XP} \). It follows that \( S_{XP}^K = 2 \pi i (N_{XP} - n) A_{XP} \). From this step the mathematical interpretation of the generalized distribution function \( N_{XP} \) is obvious: It is the parameter which diagonalizes the the full non-equilibrium matrix-valued propagator through the Bogoliubov matrix \( B \) from (2):

\[
B(N_{XP}) \tau_3 S_{XP} (B(N_{XP}))^{-1} = \begin{pmatrix} G_{XP} - i \pi A_{XP} \\ G_{XP} + i \pi A_{XP} \end{pmatrix}. \tag{18}
\]

For the following, one furthermore defines a “pseudo-equilibrium” distribution function: The 2×2 matrix structure of self energy function allows to diagonalize it also by a Bogoliubov transformation with a parameter \( N_{XP}^0 \) such that

\[
\Sigma_{XP}^{12} = 2 \pi i N_{XP}^0 \Gamma_{XP} \quad \Sigma_{XP}^{21} = 2 \pi i \left( N_{XP}^0 - 1 \right) \Gamma_{XP}. \tag{19}
\]
In the present approach $N_{XP}^0$ is determined by the hot gluon gas acting as background, hence without the back-reaction it is equal to the equilibrium function, $N_{XP}^0 ≡ n_F(E, T(t))$ with a time dependence due to the time dependence of the temperature. Looking at slow quarks with $E = m$, one furthermore replaces $N(X; m, \vec{p})$ by $N_t$ and neglects the energy derivative of $n_F(E, T(t))$. The resulting quantum transport equation according to (9) then is:

$$\frac{d}{dt}N_t = -2 \gamma_t (N_t - n_F(m, T(t)))$$

with $T(t)$ as defined in eq. (13). This equation looks surprisingly similar to a kinetic equation in relaxation time approach. However, this similarity is superficial: The kinetic equation, or Boltzmann equation, derived for this simple model system reads

$$\frac{d}{dt}N_t^B = -2 \Gamma_t \left( N_t^B - n_F(m, T(t)) \right) ,$$

with the imaginary part of the self energy $\Gamma_t$ from eq. (13) instead of the spectral width parameter $\gamma_t$. That these differ substantially in the beginning of the relaxation process has been shown in the previous section.

Fig. 2 depicts the influence of this difference on the solution of the transport equation. The result is that the relaxation process is slowed by the inclusion of the spectral function of the system components. Please observe, that the curves of Fig. 2 employ the same behavior as seen in Fig.1: The relaxation rate is similar
in the quantum transport and the Boltzmann equation, but the former experiences a characteristic delay time with respect to the latter. This delay time is almost doubled with respect to the delay time occurring in the spectral width parameter $\gamma_t$, an asymptotic calculation is carried out in ref. 2.

5. Gradient expansion

One may now raise the question, whether one can produce an equation which at least takes some of the quantum features of particles into account in an otherwise kinetic picture. The reason for this is, that in a general non-equilibrium system one cannot hope to reduce the equations (7) and (9) to such simple forms as obtained above. Even a purely numerical solution of these equations seems to be impractical if not impossible.

Therefore, to answer the question, consider the two steps which are between the equations (21) and (24): First of all a quasi-particle approximation, secondly an expansion of the operator $\exp(-i\Box)$ to first order, i.e., replacing it by $1 - i\Box$. The first of these steps would be in contradiction to the philosophy outlined in the introduction to this work. The second step however may be kept: To expand the diagonal as well as the off-diagonal pieces of the original matrix-valued Schwinger-Dyson equation to first order in the operator $\Box^4$.

The necessary differential equation for $N_{XP}$ has been derived in ref. 12, correct to first order in the gradient expansion it reads

$$\begin{align*}
\text{Tr} \left[ A_{XP} \left\{ \left( P_\mu \gamma^\mu - m - \text{Re}\Sigma_{XP} \right) ; N_{XP} \right\} \right] \\
=i\text{Tr} \left[ A_{XP} \left( N_{XP}\Sigma_{XP}^{21} - (N_{XP} - 1) \Sigma_{XP}^{12} \right) \right] \\
-\int_{-\infty}^{0} d\tau \int \frac{dE}{2\pi} \sin(\tau E) \text{Tr} \left\{ A(X; P_0 + E, \vec{P}), \right. \\
\left. \left( N_{XP}\Sigma_{XP}^{21}(t + \tau/2, \vec{X}; P) - (N_{XP} - 1) \Sigma_{XP}^{12}(t + \tau/2, \vec{X}; P) \right) \right\}. \tag{22}
\end{align*}$$

In this equation, $\{\cdot, \cdot\}$ denotes the Poisson bracket, the index $N$ means that the derivatives are not acting on $N_{XP}$. Here, as outlined before, one may use a spectral function which is the solution of an algebraic equation. For the present simple model this means to replace $\gamma_t$ by $\Gamma_t$ in the function $A$. Note, that the above equation is strictly causal: It involves a time integral only over the past history of the system, and its derivation is based on the dispersion integral (12). The problem of unphysical singularities in the propagator therefore does not occur.

Furthermore, replacing $N_{XP}$ by the unknown function $N_t^G$ and inserting all the
Figure 3: Normalized time dependent fermionic distribution function for slow quarks. Parameters as in Fig. 1; thin lines: left $N_t^B/n_F(m, T_f)$ from the Boltzmann equation (21), right $N_t/n_F(m, T_f)$ from the quantum transport equation (20); thick lines: $N_t^G/n_F(m, T_f)$ from the generalized kinetic equation (23); continuous lines: $\Lambda = 0$, dashed lines: $\Lambda = 4 \text{ fm/c}$.

Previous definitions, one obtains the nonlinear equation

$$\frac{d}{dt} N_t^G = -2 \Gamma t \left( N_t^G - n_F(m, T(t)) \right)$$

$$+ 2 g (T_f - T_i) \left[ \Theta(t) \left( \frac{t}{\Lambda} + \frac{1}{2 \Gamma t \Lambda} \right) \exp(-2 \Gamma t t) \right]$$

$$- \Theta(t + \Lambda) \left( \frac{t + \Lambda}{\Lambda} + \frac{1}{2 \Gamma t \Lambda} \right) \exp(-2 \Gamma (t + \Lambda))$$

$$+ \Theta(-t) \Theta(t + \Lambda) \frac{1}{2 \Gamma t \Lambda} \left[ N_t^G - \frac{n_F(m, T_f) T_f - n_F(m, T_i) T_i}{T_f - T_i} \right]. \quad (23)$$

In the limit $\Lambda \to 0$ this may be simplified to

$$\frac{d}{dt} N_t^G = -2 \Gamma t \left( N_t^G - n_F(m, T(t)) \right)$$

$$+ 4 t \Theta(t) (g (T_f - T_i))^2 \exp(-2 \Gamma t t) \left[ N_t^G - \frac{n_F(m, T_f) T_f - n_F(m, T_i) T_i}{T_f - T_i} \right]. \quad (24)$$

Shown in Fig.3 is the numerical solution for $N_t^G$ in comparison to the Boltzmann solution $N_t^B$ as well as the full quantum transport solution $N_t$. 
6. Discussion and Conclusion

The comparison of the three methods to describe the relaxation problem of a quark–gluon plasma (QGP) shows, that the full quantum transport equation results in a much slower equilibration process than the Boltzmann equation. This result is in agreement with other attempts to solve the quantum relaxation problem\cite{6,16}. The quantum system exhibits a memory, it behaves in an essentially non-Markovian way.

In particular, for the physical scenario studied here, the system “remembers” that it has been equilibrated some time ago. The relaxation rate then is very similar to the Boltzmann rate, but the system follows with a characteristic delay time. This delay time depends on the system parameters in a non-algebraic way, hence one may be subject to surprises for physical examples.

In the present quantum transport example for the QGP, the time to reach 1-1/e^2 ≈ 86 % of the equilibrium quark occupation number is almost doubled (14.7 fm/c as compared to 8.2 fm/c in the Boltzmann case). Thus, it may be carefully stated, that the question of the applicability of standard transport theory with quasi-particles needs further investigation: It might turn out, that quantum effects (= memory as described in this contribution) substantially hinder the thermalization of a QGP over long time scales.

One also finds, that this result holds for instantaneous as well as fast (Λ = 4 fm/c) heating of the bosonic background. Without elaboration at this point it may be stated that the inclusion of antiquarks into the spectral function does not change these figures substantially; it only leads to small oscillations of the relaxation rate around the value given in Fig. 1.

The calculated numerical value of 14.7 fm/c for the thermalization time of slow quarks is certainly so large, that the cooling of the bosonic background has to be taken into account for realistic estimates. Thus however one runs into the principal problem of non-equilibrium quantum field theory: The solution of time-dependent coupled equations for the Green’s functions, hardly possible in any concrete case. A way out of this dilemma might be offered by the generalized kinetic equation (22), which is related to the quantum transport equation as well as to the Boltzmann equation: It does not contain the convolutions over coordinate space that are hidden in the Schwinger-Dyson equation.

However, it does contain the gradient approximation of standard transport theory - and thus its applicability to the system studied here is questionable, since a step function in time certainly involves large gradients. The present comparison is therefore justified only through its results: The fact, that with the generalized transport equation one does at least partially describe the memory effects in a quantum system (the characteristic time now is 11.4 fm/c) is encouraging. Applications of this transport equation to more complicated systems seem to be possible, at least in cases where one previously has used Boltzmann-like or Vlasov-like equations which also contain this gradient expansion to first order.

As a more general remark at the end of this paper it might be added, that the
present results certainly demonstrate the importance of solving all three components
of the matrix-valued Schwinger-Dyson equation on the same level of approximation.
Using only a trivial approximation to the diagonal equations, i.e., replacing the
spectral functions of the model by some “mass-shell constraint”, is not justified for
strongly interacting hot systems.

1. Proc. of Quark Matter ’95, Monterey, 1995 (to be published);
Quark Matter 93 Proceedings, Nucl. Phys. A 566, 1c (1994)
2. P.A.Henning, E.Quack and P.Zhuang,
   Thermalization of a Quark-Gluon Plasma
   GSI-Preprint 1995 in preparation
3. N.P.Landsman, Ann.Phys. 186 (1988) 141
4. P.A.Henning, Phys.Rep. 253 (1995) 235 – 380
5. H.Narnhofer, M.Requardt and W.Thirring,
   Commun.Math.Phys. 92 (1983) 247
6. P.Danielewicz, Ann.Phys. 152 (1984) 239 and 305
7. J.Rammer and H.Smith, Rev.Mod.Phys. 58 (1986) 323
8. N.P.Landsman and Ch.G.van Weert, Phys.Rep. 145 (1987) 141
9. J.Schwinger, J.Math.Phys. 2 (1961) 407;
   L.V.Keldysh, Zh.Exsp.Teor.Fiz. 47 (1964) 1515 and JETP 20 (1965) 1018
10. H.Umezawa,
    Advanced Field Theory: Micro, Macro and Thermal Physics
    (American Institute of Physics, 1993)
11. P.A.Henning and H.Umezawa,
    Phys.Lett. B303 (1993) 209 and Nucl.Phys. B417 (1994) 463
12. P.A.Henning, Nucl.Phys. A582 (1995) 633
    (Erratum: Nucl.Phys. A586 (1995) 777)
13. S.Mrowczynski and U.Heinz, Ann.Phys. 229 (1994) 1
14. R. Kubo, J.Phys.Soc. Japan 12 (1957) 570;
   C.Martin and J.Schwinger, Phys.Rev. 115 (1959) 1342
15. P.A.Henning, E.Poliatchenko and T.Schilling,
    Effective spectral functions in thermal field theory
    GSI-Preprint 1995 in preparation
16. P.A.Henning, Nucl.Phys. A567 (1994) 844