Higher twist jet broadening and classical propagation

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The transverse broadening of jets produced in deep-inelastic scattering (DIS) off a large nucleus is studied in the collinear limit. A class of medium enhanced higher twist corrections are re-summed to calculate the transverse momentum distribution of the produced collinear jet. In contrast to previous approaches, re-summation of the leading length enhanced higher twist corrections is shown to lead to a two dimensional diffusion equation for the transverse momentum of the propagating jet. Results for the average transverse momentum obtained from this approach are then compared to the broadening expected from a classical Langevin analysis for the propagation of the jet under the action of the fluctuating color Lorentz force inside the nucleons. The set of approximations that lead to identical results from the two approaches are outlined. The relationship between the momentum diffusion constant $D$ and the transport coefficient $\hat{q}$ is explicitly derived.

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I. INTRODUCTION

The study of dense matter through its effect on hard partonic jets is now an established science. Experiments at the Relativistic Heavy-Ion collider (RHIC) and the upcoming Large Hadron Collider (LHC) have considerable resources dedicated to the study of the modification of high transverse momentum particles produced in the fragmentation of partonic jets which originated within and propagated through the produced dense matter [1]. There currently exists a large number of theoretical models which provide quantitative estimates of the observed data [2, 3, 4, 5, 6].

While the different models have rather diverse origins, within the approximations made, similarities are pervasive. A well known example is the quantitative similarity between the schemes of Refs. [2] and [3] as demonstrated in Ref. [7]. The object of this article is two fold: primarily, the aim is a re-derivation of the transverse broadening of a hard jet brought about by its propagation in dense matter with fluctuating color fields. Secondly, this represents the first effort to find a physical resemblance between the Higher-Twist and Finite temperature field theory jet-quenching schemes Refs. [3, 4]. The focus will lie predominantly on the Higher-Twist approach of Ref. [4]. In this scheme of energy loss, one tends to re-sum a class of diagrams which encapsulate the leading medium length enhanced power corrections to the basic process of a hard parton radiating a soft collinear gluon. Such gluon showers, emanating from hard partons in vacuum, often referred to as vacuum energy loss [5] (or zeroth-order energy loss [5]), lead to the DGLAP evolution [5] of parton-hadron fragmentation and structure functions. Such showers, in a medium, are influenced by the time (or length) dependent transverse momentum fluctuations experienced by the hard propagating jet and its radiated gluons and lead to a loss of the energy of the propagating parton. The effect of the transverse “kicks” from the medium on the systematics of energy loss may be divided into two parts. The effect on the propagation of the jet as well as its radiated gluons and the effect on the radiation vertex. The current work, intends to focus exclusively on the former i.e., the effect of the color fields of the medium on the transverse momentum fluctuations or broadening experienced by a single hard parton as it traverses the medium. Radiation and energy loss will be dealt with in a future effort.

It is well known that hard partons, traversing dense matter, tend to pick up a transverse momentum which depends on the length of the medium [10]. The calculation of this length dependent broadening is achieved via the re-summation of a class of higher twist diagrams [11] which are enhanced by the length traversed by the jet in the medium. Within the assumptions made in such a calculation, the broadening may also be obtained from a purely classical analysis of a hard charged particle moving under the influence of a color Lorentz force. In this sense, there exists a physical similarity with the basic picture underlying the jet quenching scheme of Ref. [2] which also admits a kinetic theory description of hard partons moving under the influence of soft fields.

However, the existing literature on the inclusion of power corrections to energy loss or jet broadening processes has yet to yield such a simple physical picture. This will be the object of the current article. This is hardly the first attempt to identify and re-sum the class of higher twist corrections required for the computation of jet broadening in extended media. A thorough derivation is provided to elucidate the basic assumptions and approximations made. Similar to previous derivations, the original parton emanating from the hard scattering is assumed to possess a vanishing transverse momentum distribution ($\delta^2(\vec{p}_T^0)$). The derivation is carried out in the high energy limit; as a result, the coupling of the hard parton with the medium is assumed to be weak. A derivation of the transverse broadening, when the parton couples strongly with the medium (albeit for a heavy quark) has been presented in Ref. [12]. In contrast to previous derivations, re-summation of the leading higher twist corrections, leads to a diffusion equation for the transverse...
momentum distribution of the final parton on exit from the dense medium. The diffusion equation is solved and the relationship between the diffusion tensor and the energy loss transport coefficient \( \hat{\gamma}_L \) is then re-derived within a classical approximation where a colored charge moves under the influence of the fluctuating Lorentz force within nucleons. This calculation is carried out using a simple Langevin analysis. The similarities between the two results and its implications are discussed.

The paper is organized as follows: in the next section, the leading twist parton distributions will be factorized at leading order. In Sect. III, the class of higher twist diagrams which are length enhanced at a given order \( m \) will be identified and the leading contributions to the hadronic tensor will be calculated. In Sec. IV, the hadronic tensor will be factorized into hard and soft piece and the \( m^{th} \) transverse momentum derivative of the hard part computed. In Sec. V, an all order resummation is carried out and the transverse momentum diffusion equation is derived. The diffusion equation will be solved and the moments of the transverse momentum distribution computed and related with the energy loss parameter \( \hat{\gamma}_L \). In Sect. VI, the Langevin analysis is carried out. Concluding discussions and future directions will be presented in Sect. VII.

II. LEADING TWIST AND PARTON DISTRIBUTION FUNCTIONS

The focus of this article is restricted to the semi-inclusive process of DIS off a nucleus in the Breit frame where one jet with a transverse momentum \( l_\perp \) is produced,

\[
e(L_1) + A(p) \rightarrow e(L_2) + J(l_\perp) + X. \tag{1}
\]

In the above equation, \( L_1 \) and \( L_2 \) represent the momentum of the incoming and outgoing leptons. The incoming nucleus of atomic mass \( A \) is endowed with a momentum \( A p \). In the final state, all hadrons \( (h_1, h_2, \ldots) \) with momenta \( p_1, p_2, \ldots \) are detected and their momentum summed to obtain the jet momentum and \( X \) denotes that the process is semi-inclusive.

The kinematics is defined in the Breit frame as sketched in Fig. 1. In such a frame, the virtual photon \( \gamma^* \) and the nucleus have momentum four vectors \( q, P_A \) given as,

\[
q = L_2 - L_1 \equiv \left[ -\frac{Q^2}{2q^-}, q^- , 0, 0 \right] , \quad P_A \equiv A [p^+, 0, 0, 0].
\]

In this frame, the Bjorken variable is obtained as \( x_B = Q^2/2p^+ q^- \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The Lorentz frame of the process where a nucleon in a large nucleus is struck by a hard space-like photon.}
\end{figure}

The differential cross section of the semi inclusive process with a jet with transverse momentum \( l_\perp \) and four-momentum \( l \) may be expressed as

\[
\frac{E_{L_2} d\sigma}{d^3L_2 d^2l_\perp} = \frac{\alpha_s^2}{2\pi Q^4 L_{\mu\nu}} \frac{dW^{\mu\nu}}{d^2l_\perp}, \tag{2}
\]

where \( s = (p + L_1)^2 \) is the total invariant mass of the lepton nucleon system. The reader may have already surmised the form of the lepton tensor as,

\[
L_{\mu\nu} = \frac{1}{2} \text{Tr} [L_1 \gamma_{\mu} L_2 \gamma_{\nu}]. \tag{3}
\]

In the notation used in this paper, \( |A; p \rangle \) represents the initial state of an incoming nucleus with \( A \) nucleons with a momentum \( p \) per nucleon. The general final hadronic or partonic state is defined as \( |X \rangle \). As a result, the semi-inclusive hadronic tensor may be defined as

\[
W^{\mu\nu} = \sum_X (2\pi^\delta)\delta^4(q + P_A - p_X) \\
\times \langle A; p | J^{\mu}(0) | X \rangle \langle X | J^{\nu}(0) | A; p \rangle \\
= 2\text{Im} \left[ \int d^4 y e^{-iqy} \langle A; p | J^{\mu}(y) J^{\nu}(0) | A; p \rangle \right] \tag{4}
\]

where the sum \( \sum_X \) runs over all possible hadronic states and \( J^{\mu} \) is the hadronic electromagnetic current \( (J^{\mu} = Q_q \bar{\psi}_q \gamma^{\mu} \psi_q) \), where \( Q_q \) is the charge of a quark of flavor \( q \) in units of the positron charge \( e \). It is understood that the quark operators are written in the interaction picture, and factors of the electromagnetic coupling constant have already been extracted and included in Eq. (2). The leptonic tensor will not be discussed further. The focus in the remaining shall lie exclusively on the hadronic tensor. This tensor will be expanded order by order in a partonic basis and leading twist and maximally length enhanced higher twist contributions will be isolated.

The leading twist contribution is obtained by expanding the products of currents at leading order. This contribution may be expressed diagrammatically as Fig. 2.
In the above equation, $W_0^{A\mu\nu}$ quantitatively expressed as (we also take the average over initial states and sum over final states to obtain)

$$W_0^{A\mu\nu} = C_p^A W_0^{\mu\nu}$$

$$= C_p^A \frac{2\pi x_B}{2Q^2} \text{Tr} \left[ p\gamma^\mu (q + x_B \not{p}) \gamma^\nu \right] \sum_q Q^2 f_q(x_B)$$

$$= C_p^A 2\pi [g^{\mu\nu} - g^{\mu\nu} + g^{\mu\nu} - g^{\mu\nu}] \sum_q Q^2 f_q(x_B)$$

$$\times \int \frac{dy^-}{2\pi} e^{-ix_B p^+ y^-} \frac{1}{2} \langle p|\bar{\psi}(y^-)\gamma^+\psi(0)|p\rangle$$

In the above equation, $C_p^A$ expresses the probability to find a nucleon state inside a nucleus with $A$ nucleons and $f_q(x_B)$ is the quark structure function within a single nucleon state, this represents the expectation value of a twist 2 operator: the quark number operator. Throughout this paper, the light-cone component notation for four vectors ($p \equiv (p^+, p^-)$) will be used, where,

$$p^+ = \frac{p^0 + p_z}{2}; \quad p^- = p^0 - p_z. \quad (6)$$

In the collinear limit, the incoming parton is assumed to be endowed with very high forward momentum ($p_0^+ = x_0 p^+ \to 0$) with negligible transverse momentum $p_0 \perp \to p_0 \perp$. Within the kinematics chosen, the photon also has no transverse momentum. As a result, the produced final state parton also has a vanishingly small transverse momentum (i.e., with a distribution $\delta^2(p_\perp^2)$). As a result, the transverse momentum distribution of the produced parton is obtained as the differential hadronic tensor,

$$\frac{d^2 W_0^{\mu\nu}}{d^2 l_\perp} = C_p^A W_0^{\mu\nu} \delta^2 (l_\perp) \quad (7)$$

![FIG. 2: The Lowest order and leading twist contribution to $W^{\mu\nu}$.](image)

It should be pointed out that in DIS on a nucleon, the final transverse momentum distribution of the produced jet is never a strict $\delta$-function but rather a Gaussian with a width that depends on the scale and $x_B$ of the scattering. Neither is the final outgoing quark strictly on shell as the cut line in Fig. 2 would indicate. The outgoing quark will eventually fragment into a jet of hadrons and thus has a positive virtuality $m^2$ which is dependent on the scale $Q^2$ of the hard scattering. In a high energy scattering event $\Lambda_{QCD} < m < Q$, which allows further re-scatterings of the quark to be treated perturbatively, yet $m << q^-$ which leads to its identification as a single jet. The scale $m$ may also be used to define the scale of the final fragmentation function. In the remaining sections, both the virtuality of the produced quark and its initial transverse momentum distribution will be taken to be vanishingly small. These approximations are carried out in the interest of simplicity and both quantities should be understood to be present.

### III. HIGHER TWIST AND TRANSVERSE BROADENING

Higher twist contributions are obtained from diagrams which include expectation values of more partonic operators in the medium e.g., the gluon field strength operator product $F^{+\nu}(y) F^{\nu}_{+}(0)$. To obtain higher twist contributions, higher orders need to be included. A diagram with 2n gluon insertions may contribute to twist $m \leq 2n$. Issues relating to the generalized factorization of such contributions will not be dealt with in this effort; the focus will be to obtain an effective description of the propagation of a hard parton in transverse color fields. In all calculations, the high energy and hence small $q$ limit will be assumed, as a result all diagrams containing the four-gluon vertex as in the left panel of Fig. 3 are suppressed compared to diagrams with the same number of gluon insertions on the hard line which are directly connected to the soft matrix elements. As a result, all such diagrams will be ignored.

![FIG. 3: A higher order contribution suppressed by $g^2$ due to four gluon coupling.](image)

The aim is to isolate the higher twist contributions at a given order and twist that carry the largest multiple of length $L \sim A^{1/3}$. In the language of power counting, one looks at the combination $\alpha^m L^n$ (usually $n \leq m$)
and focuses on diagrams with the maximum \( n \). The multiples of \( L \) are obtained by insisting on conditions that lead to the largest number of propagators going close to their on-shell conditions. This is easily achieved by the hard parton having the maximum number of space-like exchanges with the medium. Such diagrams may or may not include radiated gluons. The introduction of radiated gluons leads to the calculation of energy loss of the parent parton. Computation of diagrams without radiated gluons deal with the propagation of the parent parton without radiative energy loss. Diagrams where a single gluon line originates in the medium and then splits into two prior to attaching with the hard parton (left panel of Fig. 4) represent virtual corrections to radiative diagrams and fulfill unitarity conservation at higher order. While radiative contributions usually require at least one propagator to be off-shell, they are enhanced by large logarithms and thus represent a somewhat different power counting. Such diagrams along with their real counterparts will be dealt with in a future publication.

Diagrams where multiple gluon lines fuse into a single gluon prior to attaching with the hard parton (right panel of Fig. 4) do not produce the same length enhancement as that of the individual gluons attaching directly to the hard parton. The reason behind this is that if the intermediate gluon line is on-shell then it forces the next quark propagator to go off-shell or vice-versa. In either case, it is suppressed by one factor of \( L \) (this is treated in more detail in Appendix A). There is an exception to this condition that occurs in the region where the forward momentum fraction of the gluon is very small. In such cases, due to the saturation mechanism, soft gluon populations may be enhanced and such diagrams may indeed produce considerable contributions. The current analysis will be assumed to be carried out outside the saturation regime. As a result, such gluon fusion contributions will be ignored.

Note, there is no particular ordering of the gluon lines. Also, the gluon lines are not meant as propagators but rather as field insertions at a point \( y_i \); hence there is no meaning associated with crossed gluon lines. The entire set of \( n + n' \) vertex insertions (with the gluon vector potentials contracted with the nucleus) may then be connected by quark propagators in \((n+n')!\) ways (this overall combinatoric factor is removed by the \((n+n')!\) which appears in the denominator from the perturbation expansion). The reader may question the focus on this sub-class of possible diagrams at order \( n+n' \), where all gluon field operators are contracted with the nuclear state. Diagrams of the same order, where the gluon field operators are contracted with each other will contain cut (or uncut) gluon lines, and these will represent real (or virtual) contributions to gluon radiation diagrams. A sub-class of contributions such as those in Fig. 4 may indeed be included by a redefinition of the effective gluon vector potential. As such, they would represent small additive contributions which do not influence the leading length enhanced behavior of the diagrams of Fig. 5 and will not be discussed further.

![Diagram](image-url)

**FIG. 4:** Left panel shows a virtual contribution to a radiative correction. Right panel is a gluon fusion contribution, which is suppressed outside the saturation region.

The diagrams under discussion have the general form of Fig. 5. The ellipses in between gluon lines in Fig. 5 are meant to indicate an arbitrary number of insertions.

![Diagram](image-url)

**FIG. 5:** An order \( 2n \) contribution to \( W^{\mu\nu} \). This contributes to twist \( m \leq 2n \).

At the \((n+n')\) order, we envisage a general contribution where there are \( n \) incoming gluon lines originating in the locations \( y_1, \ldots, y_n \) in the amplitude, whereas there are \( n' \) gluon lines at locations \( y'_1, \ldots, y'_{n'} \) in the complex conjugate. The cut line is generically labelled as \( l \). The Feynman rule for such a contribution to the hadronic tensor is given as,
\[
W^{\mu \nu} = \int d^4y_1 \frac{d^4l}{(2\pi)^4} \prod_{i=1}^{n} \prod_{j=1}^{n'} \left\{ d^4y_i d^4y_j \frac{d^4q_i d^4q_j}{(2\pi)^8} \right\} \\
\times g^{n+n'}(A_0; p_0)\psi(0)\bar{\psi}(y)\gamma^\mu \Pi_{i=1}^{n} \left[ \frac{q_i \gamma^\alpha}{q_i^2 - i\varepsilon} \right] \\
\times \int 2\pi \delta(2l^2 - l_0^2) \Pi_{j=n'}^{1} \left[ \gamma_{j'} q_{j'}^\alpha \right] (q_{j'}^2 + i\varepsilon)^{\alpha} A_{\alpha}^{\mu}(y_i) \\
\times t^\alpha A_{\alpha}^{\mu}(y_j) e^{iq_{j'}y_0} e^{-i\sum_{i=1}^{n} q_i \cdot (y_i - y_{i+1})} \\
\times e^{-i\sum_{j=n'}^{1} q_{j'} \cdot (y_j - y_{j+1})} e^{-il_0 \cdot (y_n - y_{n+1})}. \tag{8}
\]

The momenta, \(q_i, q_j\), label the momenta of the various fermion lines. Note, there is overall momentum conservation within the diagram. As a result, a shift in all momenta may be executed as a consequence of momentum conservation at each vertex: \(q_i = \sum_{m=0}^{i-1} p_m + q\) and \(q_j = \sum_{k=0}^{j-1} p_k + q\), where \(p_1, p_j\) are the momenta brought in by the \(i^{th}\) line in the amplitude and the \(j^{th}\) line in the complex conjugate. It should be pointed out that all such \(p_1, p_j\) integrals run from \(-\infty\) to \(\infty\) and thus may indicate both incoming and outgoing momenta. Using these substitutions, the phase factors in Eq. 8 may be written as

\[
\Gamma = \prod_{i=0}^{n-1} \exp[-ip_i \cdot y_i] \prod_{j=0}^{n'-1} \exp[ip_j' \cdot y_j'] \\
\times \exp \left[ -i y_n \cdot \left( l - \left( q + \sum_{i=0}^{n-1} p_i \right) \right) \right] \\
\times \exp \left[ iy_n' \cdot \left( l - \left( q + \sum_{j=0}^{n'-1} p_j' \right) \right) \right], \tag{9}
\]

where, it is understood that \(y_0'\) is the origin and \(y_0 \equiv y\). At this point an \(n^{th}\) momentum may be introduced, via

\[
1 = \int d^4p \delta^4 \left( l - \sum_{k=0}^{n} p_k - q \right). \tag{10}
\]

This leads to a considerable simplification of the phase factor as

\[
\Gamma = \exp \left[ -\sum_{i=0}^{n} ip_i \cdot y_i + \sum_{j=0}^{n'-1} ip_j' \cdot y_j' \right] \\
+ iy_n' \cdot \left( \sum_{i=0}^{n} p_i - \sum_{j=0}^{n'-1} p_j' \right). \tag{11}
\]

Note the complete absence of the hard photon momentum \(q\) from the phase factor.

The approximations stemming from collinear dynamics may now be instituted. The calculation is carried out in the Breit frame at very high energy. As a result, all momentum lines that originate in the target are dominated by the large + components of their momentum, followed by their transverse coordinates, i.e.,

\[
p_i^+ >> p_i \perp >> p_i. \tag{12}
\]

In most cases the above condition will allow us to practically drop all – components of momentum from the expression for the hadronic tensor and focus solely on the + and \(\perp\) components. This procedure will be carried out not only for the momenta but also for the field operators. In this effort, calculations will be carried out in the covariant gauge. In covariant gauge calculations in the Breit frame at very high energy, the dominant components of the vector potential are the forward or (+) components \[10\], i.e., \(A^\tau \sim g^{\tau\alpha}A^\alpha\). As there are no cancellations involving the vector fields, there will arise no further need to consider the \(\perp\) components. In the corresponding calculation in light-cone gauge \(A^+ = 0\) and the largest components are the \(A^\perp\) components. It should be pointed out that the – components are only being dropped from locations where they appear in addition to the larger +, \(\perp\) components, i.e., they are not dropped from the phase factors. The collinear approximation may also be used to simplify the quark operators, as

\[
\langle \psi(0)\bar{\psi}(y) \rangle \approx \frac{1}{4} \left[ \gamma^- \langle \bar{\psi}(y)\gamma^+ \psi(0) \rangle \right] + \gamma^+ \langle \bar{\psi}(y)\gamma_\perp \psi(0) \rangle \tag{13}
\]

where, once again due the high energy limit, all terms collinear with the + direction are enhanced by boost compared to the \(\perp\) direction and thus the second term above maybe dropped.

The hadronic tensor of Eq. 8 may be expressed as the formal convolution of four terms,

\[
W^{\mu \nu} = \int d^4l_\perp Dg Dp T(p) \Gamma(p, y) M(y) \\
\times \delta^2 \left( l_\perp - \sum_{i=0}^{n} \vec{p}_i \right), \tag{14}
\]

where, \(T(p)\) denotes the pure momentum component of the integrand, \(M(y)\) is the pure position dependent multi-operator matrix element and \(\Gamma(p, y)\) is the phase factor of Eq. 11 that convolutes positions and momenta. The bold face quantities \(p, y\) represent an array of momenta and positions,

\[
p \equiv [p_0, \ldots, p_n; p'_0, \ldots, p'_{n-1}; l],
\]
\[ Y \equiv [y_0, \ldots, y_n; y'_1, \ldots, y'_{n'}]. \]

The integration measures \( \mathcal{D}p \) and \( \mathcal{D}y \) denote a product of integrals over the different four vectors contained in the arrays above. In Eq. (14), the integrations over the lightcone components of the cut line \( l \) have been performed using two of the four delta functions introduced in Eq. (11). The remaining two components of the transverse momentum integration are

\[
\int d^2 l_\perp \delta^2 (l_\perp - \vec{K}_\perp) = \int d^2 l_\perp \delta^2 \left( l_\perp - \sum_{i=0}^{n} p_{i\perp}^0 \right) \tag{15}
\]

where, \( \vec{K}_\perp \) is a representative of the sum of the transverse momenta brought in by the \( n \) gluon insertions. The solely momentum dependent terms in the integrand, post collinear approximation, may be expressed as,

\[
T(p) = \text{Tr} \left[ \gamma^- \gamma^-(p_i^+ + q^+) + \gamma^+ q^- - \gamma^+ \cdot p_i^0 \right] \ldots \times \ \gamma^- \left( \sum_{i=0}^{n-1} p_i^+ + q^+ \right) + \gamma^+ q^- - \gamma^+ \cdot \left( \sum_{i=0}^{n-1} p_i^+ \right) \right]
\times \gamma^- \left( \sum_{i=0}^{n} p_i^+ + q^+ \right) + \gamma^+ q^- - \gamma^+ \cdot \left( \sum_{i=0}^{n} p_i^+ \right) \right] \times 2\pi \delta \left\{ 2q^- \left( \sum_{i=0}^{n} p_i^+ + q^+ \right) - \left( \sum_{i=0}^{n} p_i^+ \right)^2 \right\} \times \gamma^- \left( \sum_{j=0}^{n'} p_j^+ + q^+ \right) + \gamma^+ q^- - \gamma^+ \cdot \left( \sum_{j=0}^{n'} p_j^+ \right) \right] \times \gamma^- \left( \sum_{j=0}^{n'-1} p_j^+ + q^+ \right) + \gamma^+ q^- - \gamma^+ \cdot \left( \sum_{j=0}^{n'-1} p_j^+ \right) \right) \times 2\pi \delta \left\{ 2q^- \left( \sum_{j=0}^{n'} p_j^+ + q^+ \right) - \left( \sum_{j=0}^{n'} p_j^+ \right)^2 \right\} + i\epsilon
\times \ldots \frac{\gamma^- \left( p_i^0 + q^- \right) + \gamma^+ q^- - \gamma^+_0 \cdot \left( p_i^0 \right)}{2p_i^0 q^- - \left( p_i^0 \right)^2 + i\epsilon}, \tag{16}
\]

which is the short distance momentum dependent part of the integrand. The ellipses in the above equation are meant to indicate the presence of propagators with a growing number of additive momentum factors brought in by the gluon insertions. Note that all appearances of the (--) components of the momenta \( p_i^+ \) have been ignored in the above equation. The long distance non-perturbative position dependent part of the integrand is given as

\[
M(y) = \text{Tr} \left[ A; p \right| \bar{\psi}(y)\gamma^+ \psi(0) \prod_{i=1}^{n} t^{a_i} A^+_i(y_i) \right] \times \prod_{j=n'}^{n} t^{a_j} A^+_j(y_j) \right| A; p \right]. \tag{17}
\]

While, the phase factor which convolutes both these terms is essentially contained in Eq. (11).

The complete absence of the (--) components of the momentum, from all expressions except for the phase factors allows for the \( p^- \) and \( p'^- \) integrations to be done, resulting in the localization of the process on the negative light-cone, i.e.,

\[
\Gamma^- = \prod_{i=0}^{n} \prod_{j=0}^{n'-1} \int dp_i^- dp_j^- e^{-\frac{\xi}{2} (y_i^+ - y_j^+)} \times e^{-\sum_{i=0}^{n} ip_i^- (y_j^+ - y_j^+)} \prod_{j=0}^{n'-1} \delta(y_j^+ - y_j^+) \tag{18}
\]

In the above equation, \( y_0^+ = 0 \) and as may be noted from the definition of the location arrays is not being integrated over. As a result, this constrains all the negative light cone locations in the above equation to the origin.

The terms in the numerators of Eq. (16) may be simplified with observation that \( \gamma^- \gamma^- = \frac{1}{2} g^- g^- = 0 \), as a result only the \( \gamma^+ \) and \( \gamma^+ \) terms survive the spin sum. In the extremely high energy limit where \( q^- \gg p_i^+ p_j^+ \), all terms proportional to the transverse components of the \( \Gamma \) matrices may also be ignored. The terms in the denominators may be simplified further with the replacements

\[
Q^2 = 2x_B p^+ q^-; p_i^+ = x_i p^+; p_j^+ = x_j p^+ \tag{19}
\]

\[
\sum_{k=0}^{i} 2p_k^+ \cdot p_k^+ \left| p_k^+ \right|^2 = 2x_B p^+ q^-; \tag{20}
\]

\[
\sum_{l=0}^{j} 2p_l^+ \cdot p_l^+ \left| p_l^+ \right|^2 = 2x^j_B p^+ q^- \tag{21}
\]

In the above equation, \( i \), the index of the unprimed momenta (both for the longitudinal and transverse components) runs from \( 0 \) to \( n \), whereas, \( j \), the index of the primed momenta runs from \( 0 \) to \( n' - 1 \), i.e., one less than maximum.

With the above simplifications, the momentum dependent part of the hadronic tensor \( W^{\mu \nu} \), as sketched in Eq. (15) assumes the form,
\[
T(p) = 4g^{\mu\nu} - g^{\mu\nu} + g^{i\mu}g^j_\nu - g^{i\nu} \frac{(2p^\mu)n+n^\prime + 1}{n} 2^{n+n^\prime} \\
\times \left[ x_0 - x_B - x_0^D - i\epsilon \right]^{-1} \cdots \\
\times \left[ \sum_{i=0}^{n-1} (x_i - x_D^i) - x_B - i\epsilon \right]^{-1} \\
\times 2\pi\delta \left\{ \sum_{i=0}^{n} (x_i - x_D^i) - x_B \right\} \\
\times \left[ \sum_{j=0}^{n^\prime - 1} (x_j' - x_D^j) - x_B + i\epsilon \right]^{-1} \cdots \\
\times \left[ x_0^* - x_B - x_0^D + i\epsilon \right]^{-1} .
\]

(22)

The integrals over the momenta \( p_i^+, p_j^\perp \) may be re-expressed in terms of momentum fractions, \( n = \frac{p_i^+}{p^+} dx_i \). The \( n+n^\prime + 1 \) integrals over \( (+) \) momenta in \( \mathcal{D}p \) lead to an overall factor of \( (p^+)^{n+n^\prime+1} \), which is cancelled by the similar factor appearing in the denominator of Eq. (22). As a result, the integral measure now has the appearance,

\[
\mathcal{D}y \mathcal{D}p = \prod_{i=0}^{n} dy_i^\perp d^2 y_{i\perp} \prod_{j=1}^{n^\prime} dy_j^\perp d^2 y_{j \perp} \\
\times \prod_{i=0}^{n} dx_i d^2 p_{i\perp} \prod_{j=0}^{n^\prime - 2} dx_j' d^2 p_{j \perp}' \frac{2\pi}{(2\pi)^2} .
\]

(23)

The remnant phase factor may be decomposed into a longitudinal piece and a transverse piece, which depends on the corresponding components of the momentum appearing as arguments, \( \epsilon \),

\[
\Gamma = \Gamma^+ \Gamma^\perp = \prod_{i=0}^{n} e^{-ix_i p^+(y_i^- - y_i^+)} \prod_{j=0}^{n^\prime - 1} e^{ix_j p^+(y_j^+ - y_j^-)} \\
\times \prod_{i=0}^{n} e^{i p_i^+ (y_i^- - y_i^+)} \prod_{j=0}^{n^\prime - 1} e^{-i p_j^\perp (y_j^+ - y_j^-)}
\]

(24)

The cut line, in the diagram of Fig. 5 is indicated by the delta function appearing in Eq. (22). Integrating over the last momentum fraction \( x_n \), with the aid of the delta function leads to the condition that

\[
x_n = x_B + \sum_{i=0}^{n} x_D^i - \sum_{i=0}^{n^\prime - 1} x_i.
\]

(25)

Instituting this condition leads to the separation of the longitudinal phase factor \( \Gamma^+ \) into a left and right piece, \( \epsilon \),

\[
\Gamma^+ = \exp \left[ -i \left( x_B + \sum_{i=0}^{n} x_D^i \right) p^+(y_n^- - y_n^+) \right] \\
\times \prod_{i=0}^{n^\prime - 1} \exp [ -i x_i p^+(y_i^- - y_i^+) ] \\
\times \prod_{j=0}^{n-1} \exp [ i x_j p^+(y_j^- - y_j^+) ]
\]

(26)

where, the second line in the equation above involves only momentum fractions and locations from the left-hand side of the cut, where as the last line involves momentum fractions and locations from the right hand side of the cut line. The integrations over the remaining longitudinal momentum fractions, may now be performed starting from the propagators adjacent to the cut and proceeding to the propagators adjacent to the photon vertices.

The first such integration, involves the propagator from the 2nd line of Eq. (22). Isolating the piece that depends on the fraction \( x_{n-1} \) yields the integral, which may be performed by closing the contour of \( x_{n-1} \) with a counterclockwise semi-circle in the positive imaginary direction,

\[
\int \frac{dx_{n-1}}{2\pi} e^{-ix_{n-1} p^+(y_{n-1}^- - y_{n}^+)} \\
= i\theta(y_{n}^- - y_{n-1}^-) \\
\times \int \left[ x_D^1 + x_B - \sum_{i=0}^{n-1} (x_i - x_D^i) \right] p^+(y_{n-1}^- - y_n^+) .
\]

(27)

The effect of performing the above integration is the incorporation of both the real and imaginary parts of the above propagator into the overall expression of the hadronic tensor. It has the physical effect of propagating the quark from \( y_{n-1}^- \) to \( y_n^- \). Similarly, the integration over the propagator to the immediate right of the cut line may be carried out by closing the contour of \( x_{n-1}^\prime \) with a clockwise semi-circle in the negative imaginary direction:

\[
\int \frac{dx_{n-1}^\prime}{2\pi} e^{ix_{n-1}^\prime p^+(y_{n-1}^- - y_{n}^+)} \\
= -i\theta(y_{n-1}^- - y_{n}^-) \\
\times i \left[ x_D^1 + x_B - \sum_{i=0}^{n^\prime-1} (x_i' - x_D^i) \right] p^+(y_{n}^- - y_{n-1}^-) .
\]

(28)

Incorporation of the results of the above two integrals into the longitudinal phase factors leads to the expression,
The expression derived above is completely general, in the sense that no assumption regarding the nature of the nuclear state has been made. In the next section, a factorization of the above hadronic tensor into a part that is solely dependent on hard momenta and a part dependent on soft momenta will be carried out and simplifying assumptions regarding the nuclear state made.

IV. FACTORIZATION AND GRADIENT EXPANSION

Up to this point, no approximation regarding the nature of the state \(|A; p\rangle\) or the action of the quark and gluon operators on this state has been made. We now approximate the nucleus as a weakly interacting homogeneous gas of nucleons. Such an approximation is only sensible at very high energy, where, due to time dilation, the nucleons appear to travel in straight lines almost independent of each other over the interval of the interaction of the hard probe. In a sense, all forms of correlators between nucleons (spin, momentum, etc.) are assumed to be rather suppressed. As a result, the expectation of the \(n + n' + 2\) operators in the nuclear state may be decomposed as

\[
\langle A; p|\bar{\psi}(y^-, y_\perp)\gamma^+\psi(0) \prod_{i=1}^{n+n'} A_i^+(y_i)|\alpha; p\rangle
\]

\[
= AC_{p_1}^A \langle p_1|\bar{\psi}(y^-, y_\perp)\gamma^+\psi(0) \prod_{i=1}^{n+n'} A_i^+(y_i)|p_1\rangle
\]

\[
+ C_{p_1,p_2}^A \langle p_1|\bar{\psi}(y^-, y_\perp)\gamma^+\psi(0)|p_1\rangle \times \langle p_2| \prod_{i=1}^{n+n'} A_i^+(y_i)|p_2\rangle + \ldots ,
\]

(32)

where, the factor \(C_{p_1}^A\) represents the probability to find a nucleon in the vicinity of the location \(\vec{y}_i\), which is a number of order unity (it is the probability to find one of \(A\) nucleons distributed in a volume of size \(cA\) within a nucleon size sphere centered at \(\vec{y}_i\)). The remaining coefficients \(C_{p_1,p_2}^A\) represent the weak position correlations between different nucleons. The overall factor of \(A\) arises from the determination of the origin (the location 0 in the equation above) in the nucleus, which may be situated on any of the \(A\) nucleons.

It is clear from the above decomposition that the largest contribution arises from the term where the expectation of each partonic operator is evaluated in separate nucleon states as the \(\vec{y}_i\) integrations may be carried out over the nuclear volume. As a nucleon is a color singlet, any combination of quark or gluon field strength insertions in a nucleon state must itself be restricted to a color singlet combination. As a result, the expectation of single partonic operators in nucleon states is vanishing. The first (and hence largest) non-zero contribution.

\[
\Gamma^+ = \exp \left[ -ix_D n^p y_i^n + ix_D n^p y_i^n \right] - i x_B p^+ y_{n-1} + i x_B p^+ y_{n-1}'
\]

\[
+ \sum_{i=0}^{n-1} x_D p^+ y_{n-1} + i \sum_{j=0}^{n-1} x_D p^+ y_{n-1}'
\]

\[
\times \prod_{i=0}^{n-2} \exp \left[ -ix_D p^+(y_i^n - y_{n-1}^-) \right] \prod_{j=0}^{n-2} \exp \left[ ix_D p^+(y_j^n - y_{n-1}^-) \right]
\]

(30)
emanates from the terms where the quark operators in the singlet color combination are evaluated in a nucleon state and the \( n + n' \) gluons are divided into pairs of singlet combinations, with each singlet pair evaluated in a separate nucleon state. This requires that \( n + n' \) is even and may lead to a maximum overall factor of

\[
C_{p_1, p_2, \ldots}^A \sim A^{[(n+n'+2)/2]}, \tag{33}
\]

in the large \( A \) limit. It should be pointed out that large contributions may also arise, in principle, when \( n + n' \) is odd. In this case, the two quarks and a gluon are considered in the singlet combination with the remaining gluons evaluated in singlet pairs in the remaining nucleons. Here we institute the experimental observation that \( (n) \)-parton observables are much smaller than \( (n-1) \)-parton observables. This is only true, once again, outside the saturation regime, as discussed at the beginning of this section. Such odd parton expectations also become important in cases where nucleon polarization effects are being studied where the spins of the nucleons are not averaged over \( W_{\perp} \). In this effort, the focus remains exclusively outside such regions, as a result we ignore all terms with more than two quarks or two gluons per nucleon.

Further simplifications arise in the evaluation of gluon pairs in a singlet combination in the nucleon states by carrying out the \( y_{\perp} \) integrations. The basic object under consideration is (ignoring the longitudinal positions and color indices on the vector potentials)

\[
\int d^2 y_{\perp}^i d^2 y_{\perp}^j \langle p| \mathcal{A}_+^i(y_{\perp}^i) \mathcal{A}_+^j(y_{\perp}^j)|p\rangle \times e^{-ix_{\perp}^i p_{\perp}^i y_{\perp}^i} e^{ix_{\perp}^j p_{\perp}^j y_{\perp}^j} e^{-ip_{\perp}^i y_{\perp}^i} = (2\pi)^2 \delta^2(p_{\perp}^i - p_{\perp}^j) \int d^2 y_{\perp} \ e^{-ix_{\perp}^i p_{\perp}^i (y_{\perp}^i - y_{\perp}^j)} \times e^{ip_{\perp}^i y_{\perp}^i} \langle p| \mathcal{A}_+^i(y_{\perp}^i/2) \mathcal{A}_+^j(-y_{\perp}^j/2)|p\rangle \tag{34}
\]

where, \( y_{\perp} \) is the transverse gap between the two gluon insertions and \( p_{\perp} = (p_{\perp}^1 + p_{\perp}^2)/2 \). The physics of the above equation is essentially the transverse translation symmetry of the two gluon correlator in a very large nucleus. One will note that the two dimensional delta function over the transverse momenta has removed an integration over the transverse area of the nucleus thus reducing the overall \( A \) enhancement that may be obtained. This is then used to equate the transverse momenta emanating from the two gluon insertions in the amplitude and complex conjugate amplitude. This also simplifies the longitudinal phase factors which now depends solely on the difference of the longitudinal positions of the two gluon insertions.

Up to this point, the collinear approximation has been used to simplify the expressions for the hadronic tensor, without the introduction of factorization. The separation of the hadronic tensor into a hard short distance piece and a soft long distance contribution may now be accomplished. All factors in Eq. (31) which contain the hard scales \( p^+, q^- \) constitute the hard part. All factors that depend solely on the soft \( \perp \) momenta and distances along with the matrix element constitute the long distance element, \( \ldots \) the first part of the integrand in the 3rd and 4th lines in Eq. (31) belongs in the hard part along with all phase factors which contain a factor \( x'^2 p^+ \) or \( x'^2 p^+ \) as part of their arguments. The purely transverse phase factors such as \( \exp(i\vec{y}_{\perp} \cdot \vec{y}_{\perp}) \) belong in the soft part along with the matrix elements. Thus, we may decompose the hadronic tensor as,

\[
W^{\mu\nu} = \int \mathcal{D}y_\perp \mathcal{D}p_\perp H(p^+, q^-, p_\perp, y) S(p_\perp, y). \tag{35}
\]

In the above equation, \( y \) and \( p_\perp \) are representative of the entire set of distances and transverse momentum that appear in Eq. (31). One now invokes the collinear approximation in expanding the hard part as a Taylor expansion in transverse momenta around the origin \( p_{\perp}^0 \rightarrow 0 \). In the case of a symmetric cut with \( n = n' \) there are \( 2n \) gluon insertions and as a result, as many derivatives, which involve \( n \) different transverse momenta. All terms of the form

\[
\prod_{i=1}^{m} \frac{1}{2} \partial_{p_{\perp}^i} \partial_{p_{\perp}^j} H(p^+, q^-, p_{\perp}^0) \Big|_{p_{\perp}^0=0}^{p_{\perp}^0} p_{\perp}^0 \alpha p_{\perp}^0 \beta,
\]

where, \( m < n \), yield gauge corrections for the contributions with \( 2m \) gluon insertions and as many derivatives \( H \). The genuine \( 2n \) twist correction at this order has \( m = n \). Expanding to this order, one obtains the generic term,

\[
\prod_{i=1}^{n} \frac{1}{2} \partial_{p_{\perp}^i} \partial_{p_{\perp}^j} H(p^+, q^-, p_{\perp}^0) \big|_{p_{\perp}^0=0} \times p_{\perp}^0 \alpha p_{\perp}^0 \beta A_{\alpha}^i(y_{\perp}^i, y_{\perp}^j/2) A_{\beta}^j(y_{\perp}^j, -y_{\perp}^j/2). \tag{36}
\]

Where, we have assumed the result of Eq. (34) and reintroduced the color indices and longitudinal locations.

Using integration by parts over the transverse distance \( y_{\perp}^i \) one may convert the product \( p_{\perp}^0 \alpha A_{\alpha}^i(y_{\perp}^i, y_{\perp}^j/2) \rightarrow \frac{1}{2} \partial_{y_{\perp}^i} A_{\alpha}^i(y_{\perp}^i, y_{\perp}^j/2) \). In the extreme collinear limit, in the presence of a hard scale such that \( g \) is small, one may make the approximation,

\[
\partial_{y_{\perp}^i} A_{\alpha}^i \simeq F_{\alpha}^+, \tag{37}
\]

where, \( F_{\alpha}^+ \) represents the gluon field strength. Carrying this out consistently on the two gluon operator in the nucleon state of Eq. (31) and ignoring derivatives of the
field strength \([\partial^\alpha F^{\beta} + g(m^2)\gamma^{\alpha\beta}J^+ \to 0]\) we obtain,
\[
\int d^2y_1 p_1^{\alpha} \partial^\beta F_{\perp \perp} e^{ip_1 \cdot y_1} (p | A^+ (\bar{y}_1 / 2) A^+ (-\bar{y}_1 / 2) | p) = \int d^2y_1 e^{ip_1 \cdot y_1} \frac{1}{2} (p | F^{+\alpha} (\bar{y}_1 / 2) F^{+\beta} (-\bar{y}_1 / 2) | p) = \int d^2y_1 e^{ip_1 \cdot y_1} \frac{-g^{\alpha\beta}}{4} (p | F^{+\alpha} (\bar{y}_1 / 2) F^{+\beta} (-\bar{y}_1 / 2) | p). (38)
\]

In the last line of the above equation we have averaged over the spins in the two field strength expectation in the nucleon state with the constraint that the operator being evaluated in the nucleon be a spin singlet. The nucleon states are always assumed to be spin singlets or in spin averaged states.

The hard part which consists of 2n transverse momentum derivatives may now be simplified further
\[
\prod_i \frac{\partial^2}{\partial p_i^2} \delta^2 (\bar{I}_i - \bar{K}_i) = (\nabla_{\perp i}^2)^n \delta^2 (\bar{I}_i - \bar{K}_i). \tag{39}
\]
With the derivative expansion (at vanishing transverse momenta \(p_i^\perp \to 0\)) imposed on the hard part \(H\), it no longer has any functional dependence on the transverse momenta. The integrations over the transverse momenta may now be included completely into the soft part. The action of the derivatives remains on the transverse \(\delta\)-function as the only non-vanishing contribution from the collinear expansion on Eq. (61).

In the interest of simplicity, we have considered the symmetric case of gluon insertions above, where \(n = n'\). The other cases may be easily computed in similar fashion (the case where \(n = n' - 2\) is considered in Appendix B). The longitudinal integrals, due to color confinement yield the requirement that the longitudinal locations of the two gluons which act on the same nucleon state be in close proximity. One now tries to identify the most length enhanced term by isolating the maximum number of unconstrained \(dy^-\) integrals. Note that, due to color confinement (ignoring color and spin indices),
\[
\int dy^- dy^- dy^- d^2p_\perp (2\pi)^2 e^{-ix_D p^+ (y^- - y^-')} \times e^{ip_\perp \cdot y_\perp} (p | F(y^-_1, y^-_2 / 2) F(y^-_2, -y^-_2 / 2) | p) = \int dy^- dy^- dy^- d^2p_\perp (2\pi)^2 e^{-ix_D p^+ \delta y^-} \times e^{ip_\perp \cdot y_\perp} (p | F(-\delta y^-_1 / 2, y^-_2 / 2) F(-\delta y^-_2 / 2, -y^-_2 / 2) | p)
\]
where, \(x_D = \frac{p_\perp^2}{2p_\perp^2}\) is a function of the \(p_\perp\) itself.

In the equation above, all three integrals over \(\delta y^-\), \(\bar{y}_\perp\) are limited by the nucleon size. However, the integral over \(Y^-\), under the assumption of longitudinal translation invariance of the two point correlator, may span the entire length of the nucleus. Each such integral yields a factor of \(L^- \sim A^{1/3}\) from the unconstrained \(Y^-\) integration. The equating of the pairs of transverse momenta that appear in each two-gluon correlation, as well as the relation between the longitudinal momenta from the \(\theta\)-functions in Eq. (61), require that the largest transverse momentum broadening and largest length enhancement arises from the terms where the gluon correlations are built up in a mirror symmetric fashion, \(i.e.,\) where the gluon insertion at \(y'\) is contracted with that at \(y''\). This reduces the focus on terms where \(n = n'\) which produce a transverse momentum broadening of order \(A^{n/3}\) and provides an a-posteriori justification for our discussion of symmetric terms.

One may average colors of the quark and gluon field operators,
\[
\langle p | F^a F^b | p \rangle = \delta^{ab} \langle p | F^a F^a | p \rangle \tag{41}
\]
\[
\langle p | \bar{\psi}_i \gamma^\alpha \psi_j | p \rangle = \frac{\delta_{ij}}{N_c} \langle p | \bar{\psi}_i \gamma^\alpha \psi_j | p \rangle. \tag{42}
\]
This reduces the overall trace over color factors to
\[
\frac{1}{N_c (N_c^2 - 1)^n} \text{Tr} \left[ \prod_{i=1}^n \frac{1}{t^{a_i}} \prod_{j=n}^{1} t^{a_j} \right] = \frac{C_F^n}{(N_c^2 - 1)^n} = \frac{1}{(2N_c)^n}. \tag{43}
\]
If the original parton was a gluon, one would simply replace \(C_F\) with \(C_A\) in the above equation.

The remaining \(n\) longitudinal position integrals for the gluon insertions may be simplified as
\[
\int \prod_{i=1}^n dy^- \theta (y^-_i - y^-_{i-1}) = \frac{1}{n!} \int \prod_{i=1}^n dy_i^- \tag{44}
\]

Invoking the above simplifications, the leading length enhanced contribution at order \(2n\) to the differential hadronic tensor is obtained as (in the following we have replaced \(\delta y^-\) with \(y^-\) and translations made to simplify the resulting expression),
\[
\frac{d^2 W^{\mu \nu}}{d^2 I_\perp} = C^{A}_{p_0, \ldots, p_n} W^{\mu \nu}_{\perp / 2} \frac{1}{n!} \left( \nabla_{\perp i}^2 \right)^n \delta^{2} (\bar{I}_i) \times \left[ \frac{\pi^2 \alpha_s}{2N_c} \right] L^- \int d\hat{y}^- d^2y_\perp \frac{\bar{p}_\perp}{(2\pi)^3} e^{-ix_D p^+ \hat{y}^-} \times e^{ip_\perp \cdot y_\perp} (p | F^{\alpha + \alpha} (y^-, -y^-) F^{\alpha + \alpha} (0) | p). \tag{45}
\]
In the next section, a re-summation of all such contributions of arbitrary order \(n\) will be carried out to calculate the leading differential distribution of a hard jet in transverse momentum as a function of the length traversed in the medium.
V. RE-SUMMATION AND TRANSVERSE MOMENTUM DIFFUSION

In the preceding section, the leading length enhanced twist-2 n contribution was evaluated. As is obvious from Eq. (45), the essential combination of factors is that included in the square brackets. When this factor is not very small, all such terms (for all n) need to be re-summed into the generalized differential hadronic tensor, which will include contributions at all-twist.

To simplify the re-summation, one further approximation is required. In the preceding section, a model of the nucleus as a weakly interacting homogeneous gas of nucleons was used. The formal expression of this assumption is hidden within the dimensionful parameter $C_{p_0 \ldots p_n}^A$. The precise evaluation of such combinatoric coefficients is rather complicated, even for the case of next-to-leading twist \[20, 21\]. From general dimensional arguments, in the case of non-interacting nucleons, this factor may be approximated as,

$$C_{p_0 \ldots p_n}^A \simeq C_p^A \left( \frac{\rho}{2p^+} \right)^n,$$

(46)

where, $\rho$ is the nucleon density inside the nucleus and $1/2p^+$ originates in the normalization of the nucleon state. The remaining unknown coefficient $C_p^A$ is now considered to be independent of the order $n$. Straightforward substitution the above approximation into Eq. (45), leads to a simplified form for the 2 n th order contribution to the differential hadronic tensor. The full hadronic tensor, which includes contributions from all orders, may be expressed as,

$$\frac{d^2 W_{\mu \nu}}{d^2 l_\perp} = \sum_{i=0}^{\infty} \frac{d^2 W_{\mu \nu}^{(i)}}{d^2 l_\perp}.$$  

(47)

Note that while $i$ runs over all integers, each term refers to an even number ($2i$) of gluon insertions. Using the simplifications mentioned above and Eq. (45) for the 2 n th order term, we obtain the generalized differential hadronic tensor,

$$\frac{d^2 W_{\mu \nu}}{d^2 l_\perp} = e^{(DL^-)\nabla_l^2} \frac{d^2 W_{\mu \nu}^{(0)}}{d^2 l_\perp},$$

(48)

where, $d^2 W_{\mu \nu}^{(0)}/d^2 l_\perp$ is given by Eq. (4), and the constant $D$ is given as,

$$D = \frac{\pi^2 \alpha_s}{2N_c} \rho \int \frac{d^3 y d^2 p_\perp}{(2\pi)^3 2p^+} (p | F_{\alpha \alpha^c}^a (y) F_{\alpha \alpha^c}^a (0) | p) \exp \left\{ -\frac{\alpha_s}{\pi} \int d^2 y \frac{p_\perp \cdot y_\perp}{2y^+} \right\} \simeq \frac{\pi^2 \alpha_s}{4N_c} \rho x_T G(x_T).$$

(49)

In the above equation, $x_T = (p_\perp^2)/(2p^+ q^-)$, where $(p_\perp^2)$ is the average transverse momentum that a nucleon may impart to a hard parton passing through it and $G(x_T)$ is the longitudinal gluon distribution function of the nucleon at $x_T$, where

$$G(x) = \int \frac{dy^- e^{-ixp^+y^-}}{2\pi x^+} \langle |F_{\alpha \alpha^c}^a (y) F_{\alpha \alpha^c}^a (0)| p \rangle.$$  

(50)

The scale of the gluon distribution is essentially the same as that used for the scatterings as well as for the final fragmentation i.e., $m^2$.

Following the form of Eq. (45), the re-summed differential hadronic tensor may be expressed in terms of a product of the leading order, leading twist, integrated hadronic tensor and a length dependent transverse momentum distribution function $\phi(L^- , l_\perp)$,

$$\frac{d^2 W_{\mu \nu}}{d^2 l_\perp} = C_p^A W_{\mu \nu}^{(0)} \phi(L^- , l_\perp).$$

(51)

While our analysis has focused on the hadronic tensor, it should be pointed out that $W_{\mu \nu}$ may be immediately supplanted with the differential cross section to produce a jet with a transverse momentum $l_\perp$ using Eq. (2). In a sense, the entire process of the production and subsequent propagation of the hard quark through the nucleus has been factorized into two parts: $C_p^A W_{\mu \nu}^{(0)}$ represents the initial production in a hard scattering with a virtual photon and $\phi(L^- , l_\perp)$ represents its propagation and transverse momentum broadening. The entire length ($L^-$) and transverse momentum ($l_\perp$) dependence is contained entirely within the factor $[\phi(L^- , l_\perp)]$. Differentiating Eq. (45), with respect to $L^-$ suggests the obvious relation for the differential transverse momentum distribution,

$$\frac{\partial \phi(L^- , l_\perp)}{\partial L^-} = D \nabla_{l_\perp}^2 \phi(L^- , l_\perp),$$

(52)

which is a two dimensional diffusion equation in the vector $l_\perp$ with the trace of the diffusion tensor given by Eq. (49). The initial condition is discerned from Eq. (4) as,

$$\phi(L^- = 0, l_\perp) = \delta^2(l_\perp).$$

(53)

The two dimensional diffusion equation, with the above mentioned initial condition, has the well known solution \[22\],

$$\phi(L^- , l_\perp) = \frac{1}{4\pi DL^-} \exp \left\{ -\frac{l_\perp^2}{4DL^-} \right\}.$$  

(54)

Using the above diffusion equation, all moments of the transverse momentum distribution may be calculated.
The first non-zero moment is the average squared transverse momentum distribution at a length $L$,

$$\langle l^2 \rangle_L = \int d^2l_\perp l^2_\perp \phi(L^-, \vec{l}_\perp) = 4DL^-.$$  \hfill (55)

As a result, the transport coefficient of energy loss defined as the average transverse momentum squared gained by the jet per unit length traversed in the medium is given as (note: $L = 2L$ the actual length traversed),

$$\hat{q} = \frac{2\langle l^2 \rangle_{L^-}}{L^-} = 8D = \frac{2\pi^2\alpha_s}{N_c} \rho xT G(xT, m^2) |_{x=0},$$  \hfill (56)

It should be pointed out that in an earlier work where a diffusion equation was used to understand the systematics of transverse momentum broadening of jets \cite{22}, the diffusion tensor $\hat{D}$ was defined to be four times that in Eq. (49), such that one obtains, the mean square transverse momentum broadening as $\langle l^2 \rangle = DL^- \text{ and } \hat{q} = 2D$.

The final numerical value of the scale of the gluon distribution function deserves some discussion. We have computed a class of all twist corrections to what amounts to a leading order (LO) hard scattering process. In the numerical implementation of such processes, there exists only one large scale i.e., $Q^2$. Thus, while in principle the multiple scattering and final fragmentation occur at a softer scale, in LO calculations one may set $m^2 = Q^2$ for both the gluon distribution as well as the final fragmentation. If the hard scattering were computed at NLO this may no longer be the case.

VI. LORENTZ-LANGEVIN ANALYSIS AND CLASSICAL PROPAGATION

The field theory calculation presented in the preceding sections demonstrates that, in the limit of no radiation, the transverse dynamics of hard collinear jets passing through dense matter may be understood in terms of a transverse momentum diffusion equation. As such, no clear physical picture has emerged from all the approximations carried out. The situation at this point is not dissimilar to that encountered in the hard thermal loop (HTL) effective theory of QCD \cite{21, 23} where hard momenta ($p \sim T$) were integrated out to find effective propagators for soft modes ($p \sim gT$). Identical results were also derived if one assumed the more physical picture of the hard modes to be classical particles moving under the influence of a soft color field which represented the soft modes \cite{23}. In this section, we demonstrate that an almost identical situation is true for the case of higher twist jet broadening.

In the following, an alternative calculation of the transverse broadening of an energetic jet as it passes through a dense colored medium will be carried out classically. It is not a complete surprise that such a derivation yields a similar result as the rigorous quantum mechanical analysis performed above. In this case, one computes the propagation of a colored particle as it propagates under the action of fluctuating color fields which have a short color correlation length. This last condition is meant to mimic the effect of the color confinement at nucleon like distances as used in the preceding derivation.

Imagine, as in the preceding section, that a colored particle moves under the influence of a color Lorentz force in the $(-)$ direction. The equation of motion for a classical particle is,

$$\frac{dp_\mu}{dt} = gQ^a F_{\mu\nu}^a (y) v^\nu,$$  \hfill (57)

where, $t$ is the time in the observers rest frame, $Q^a(y)$ is the color charge expectation of the particle at location $y$, $F_{\mu\nu}^a(y)$ is the color field strength at that location and $v$ is the three velocity written in four component form i.e., $v \equiv [1, \vec{v}]$ in time-space components. In light cone components the vector $v$ is given as, $v \equiv [(1 + v_z)/2, 1 - v_z, \vec{v}_\perp]$. Note that for a light like particle $v_z^2 + \vec{v}_\perp^2 = 1$, thus $v$ is akin to a unit vector. For a particle moving predominantly in the negative $z$ direction as in the case of the preceding sections, $v_z < 0$ and $|v_z| >> |v_\perp|$. Hence, one may make the approximation,

$$F_{\mu\nu} v^\nu \simeq 2F_\mu^+.$$  \hfill (58)

As a result, the squared transverse broadening in momentum experienced by the light-like particle is given as

$$|\Delta p_\perp(y)|^2 = \left| -g_\perp^{\alpha\mu} \Delta p_\mu(y) \Delta p_\nu(y) \right|^2 \hfill \text{(59)}$$

$$\approx g_\perp^2 \int_{y_0}^y dy^-_1 dy^-_2 Q^a F_\alpha^a (y_1^-) F^{\perp\alpha}_\perp(y_2^-),$$

where, the substitution $y^- = 2t$ has been made for a light like particle traveling in the negative $z$ direction. Taking the expectation of the product of gluon field strength operators, one may now institute the condition that the medium has a short color correlation length, i.e.,

$$\int dy^-_1 dy^-_2 \langle F_\alpha^a (y_1^-) F^{\perp\alpha}_\perp(y_2^-) \rangle \sim \int dy^-_1 \langle F_\alpha^a + F^{\perp\alpha}_\perp \rangle y^-_{\text{conf}}.$$  \hfill (60)

The average over the color of the gluon field correlators may now be carried out (ignoring position coordinates),

$$\langle F^a F^b \rangle = \frac{\delta^{ab}}{(N_c^2 - 1)} \langle F^c F^c \rangle.$$  \hfill (61)

The color charges, $Q^a$ represent the expectation of the color matrices in the state of the colored parton. As a
result $\delta^{ab}Q^aQ^b = C_F$ for a quark parton. Hence, one obtains the mean square transverse momentum per unit length as obtained from the higher twist analysis. This leads to the contention that the re-summing of length enhanced twist corrections on to an outgoing parton line is effectively described as the classical propagation of hard colored partons in soft fields, as long as the correlation lengths of the soft fields are short.

\( |\Delta p_{\perp}(y)|^2 = \frac{4\pi^2\alpha_s t}{N_c} \times \int \frac{dy}{2\pi} \left\langle F_{\alpha^+}^a + \left( y - \frac{1}{2} \right) F_{\alpha^+}^a \left( -y - \frac{1}{2} \right) \right\rangle. \)  

Using the same normalization as in the preceding section for turning a nuclear state into a gas of nucleon states \( i.e., \) one obtains identical expressions for the mean square transverse momentum per unit length as obtained from the higher twist analysis. This places the effective picture of higher twist re-summation on the same footing as that of the HTL effective theory which also admits an almost similar picture. This suggests the possibility of constructing an effective field theory applicable to the propagation of hard jets in soft dense matter.

The derivations in this effort have been restricted to parton propagation. The construction of an effective theory will require also the use of re-summed vertices. These contributions will allow for an all-twist calculation of radiative energy loss and comparisons with the energy loss formalism of Ref. 3. The inclusion of radiative energy loss in an all twist formalism will allow, for the first time, a means to study the evolution of virtuality of the hard partons as they originate in hard collisions, traverse dense matter, escape and fragment into hadrons.

Confrontation with experiment 22 will require a convolution of the above formalism with both single 4 and multi-hadron 28, 29 fragmentation functions. Even without the inclusion of energy loss, the above formalism may be directly applied to the measured transverse broadening of hard jets in cold nuclear matter as measured by the HERMES experiment at DESY 30. Such comparisons will be carried out in an upcoming publication.

\section{VII. DISCUSSIONS AND CONCLUSIONS}

The propagation and energy loss of hard partons in dense matter is a topic of much current interest. In the current effort, a detailed analysis of the transverse momentum diffusion of jets produced in DIS off large nuclei, as they pass through cold nuclear matter was carried out. Radiation and radiative energy loss were ignored, though, in reality, such partons will tend to radiate and lose a substantial fraction of their energy as they traverse the dense matter. Complications arising from energy loss were ignored in an effort to isolate the systematics of the propagation of a single parton in a fluctuating color field without radiation. Radiation will be included in a subsequent effort to complete the formulation of the propagation of hard partons through extended dense matter.

The leading length enhanced higher twist corrections which produce maximal broadening were identified and re-summed onto the outgoing parton line. The final transverse momentum distribution at a given length traversed was shown to obey a two-dimensional diffusion equation. The diffusion equation was solved and the mean square transverse momentum per unit length \( \langle \hat{q} \rangle \) calculated. This is the first effort, to the knowledge of the authors, which presents a first principles derivation of the transverse momentum distribution function (within the higher twist formalism) and a relation between the diffusion tensor and the transport coefficient \( \hat{q} \). Our results differ from those of an earlier attempt 11 to resum length enhanced multiple scattering diagrams, which found a shift, rather than a broadening, of the transverse momentum distribution. We believe that our result is more broadly applicable.

The results for \( \hat{q} \) obtained from the higher twist analysis were shown to be identical to that obtained for a point like colored particle moving in a fluctuating color field if the color correlation length of the fields is assumed to be short. This places the effective picture of higher twist resummation on the same footing as that of the HTL effective theory which also admits an almost similar picture. This suggests the possibility of constructing an effective field theory applicable to the propagation of hard jets in soft dense matter.

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\section{APPENDIX A: GLUON FUSION DIAGRAMS}

In this appendix, we briefly sketch the calculation of diagrams that include gluon fusion contributions such as in the right panel of Fig. 4. Such diagrams originate from the inclusion of interaction terms from the QCD Lagrangian density, of the form,

\[ -\frac{g}{2} \left( \partial_\alpha A_\beta^a(y_0) - \partial_\alpha A_\beta^a(y_0) \right) f^{abc} A^{a\alpha}(y_g) A^{b\beta}(y_g). \]  

A term in the expansion of the hadronic tensor that would contain a fusion contribution will have the general form \( \langle i | p | O | p \rangle \),


\[ W_{3g}^{\mu\nu} = \int dy_1 dy_2 dy_3 \left( J_\mu(y) \bar{\psi}(y_p) \gamma^\mu A_\mu^a(y_p) \psi(y_p) \right) \]

\[ \times \left( \partial_\alpha A_\alpha^a(y_g) - \partial_\alpha A_\alpha^a(y_g) \right) f^{abc} A_\alpha^b(y_g) A_\beta^c(y_g) J_\nu(y_p) \]  

(A2)

In the above equation, \( J_\mu \) and \( J_\nu \) are the currents at the locations \( y \) and 0 which couple to the photon and there is a contraction between the gluon vector potentials appearing in the first line with any of the three gluon vector potentials in the second line. This contraction represents the one gluon propagator in the left panel of Fig. 4. There are multiple reasons why such contributions are small. One reason is that unlike the ladder diagrams of Fig. 5, the introduction of an extra coupling constant is not accompanied with the introduction of an extra length integral. As the momentum in the one gluon propagator is set by requiring that the two quark lines it connects go nearly on shell, the gluon propagator cannot itself go on shell and as a result there is no length enhancement from the gluon propagator.

This contribution is also small due to color confinement and the condition that the forward momenta of all partons originating in the nucleus are not small enough to be in the saturation regime. The contraction of one of the vector potentials located at \( y_g \) [in the second line of Eq. (22)] with that at \( y_p \), requires that the remaining two vector potentials be contracted into the nuclear state. As both vector potentials are at the same location, they must be contracted into the same nucleon. However, due to Eq. (11), this requires both gluons to be in a color singlet state. The presence of the antisymmetric factor \( f^{abc} \) renders such contributions vanishing. Diagrams with three gluon vertices produce non-vanishing contributions if another gluon (at location say \( y' \)) were to be contracted with the two gluons (which fuse into one gluon), into the same nucleon state. Such contributions automatically require that another space integral over \( y' \) be restricted to be in close proximity to \( y_p \). Also, such contributions necessarily require the introduction of the expectation of 3-parton operators in a nucleon state, which, as we have argued previously, are small compared to the expectation of 2-parton operators outside the saturation regime.

**APPENDIX B: NON-CENTRAL CUT DIAGRAMS**

In Sect. IV, the derivation of the transverse momentum diffusion of a hard part was isolated to the case of a central cut. In Eq. (38) and the ensuing discussion, this was identified as the dominant contribution to the differential hadronic tensor at a given order. In this appendix, we focus on diagrams at order \( n + n' = 2m \) in terms of the strong coupling constant \( g \), where \( n \neq n' \). In particular, we focus on the case where \( n = m - 1 \) and \( n' = m + 1 \). The starting point of this analysis is Eq. (31). As in Sect. IV, the hard part is expanded in a Taylor series in transverse momentum as in Eq. (38), resulting in the integrand (written without phase factors and light-cone time ordering \( \theta \)-functions as),

\[ H(p^+, q^-, p_\perp, y) S(p_\perp, y) \]

\[ \rightarrow \prod_{i=1}^{m-1} \prod_{j=1}^{m+1} \frac{\partial}{\partial p_{i,\perp}} \frac{\partial}{\partial p_{j,\perp}} H(p^+, q^-, p_{i,\perp}^{i+}, p_{j,\perp}^{j+}) \bigg|_{p_{\perp} = 0} \]

\[ \times p_i^i (-p_{i,\perp}^{i+}) A_+^a(y_i^-) A_+^b(y_j^+) \bar{y}_{j,\perp}^{j+}. \]  

(B1)

As in the case of the central cut, the \( \theta \)-functions time order the gluon insertions in incremental order from the photon vertex to the cut line. Each occurrence of the factor \( e^{ip_{\perp} \cdot q_\perp} p_{i,\perp}^{i+} A_+^a(y_i) \) may be converted, using integration by parts, to \( -ie^{ip_{\perp} \cdot q_\perp} \partial_{i,\perp}^{i+} A_+^a(y_i) \). As a result we obtain an overall sign of \((-i)^{(m-1)(m+1)} = -1\) compared to the central cut.

**FIG. 6:** An order 2m, contribution to \( W_{3g}^{\mu\nu} \) from a non-central cut

The nuclear state is decomposed as before into a pseudo-free gas of nucleons and pairs of gluon insertions in singlet combinations is contracted into the \( m \) chosen nucleons. Unlike the central cut, there is an excess of two gluon insertions on the left hand side of the cut as shown in Fig. 4, where the rectangular blobs represent single nucleon states. This extra pair is contracted into the same nucleon. For this specific example, the \( m \)th and \((m+1)\)th gluon on the left hand side will be contracted into the same nucleon, as required for maximum length enhancement. Contractions within the same nucleon, due to color confinement, tend to constrain the locations of the field operators \( F_{\mu\nu}^{-}\)(\( y_m \)) and \( F_{\mu\nu}^{-}\)(\( y_{m+1} \)) to a region \( y_{m+1} - y_m \leq y_{conf} \), the confining distance. Thus contractions between non-adjacent gluons, due to light-cone ordering \( \theta \)-functions will constrain the length integrations of all the intermediate gluons to be within the size of a nucleon.
The transverse momentum integrations may now be carried out with the observation that in a very large nucleus, two point correlators only depend on the relative transverse distance between the two insertions. The final result is similar to that in Eq. (33) for $i \leq m - 1$ [see Eq. (31)]. The resulting $\delta$-functions for the case of the $m$th and $(m + 1)$th gluon which are both on the left of the cut, are slightly different, i.e.,

$$\int dy_{m}^{\perp} dy_{(m+1)}^{\perp} e^{-i(p_{m}^{\perp} + p_{(m+1)}^{\perp}) \cdot y_{m}^{\perp} + e^{-i(p_{m}^{\perp} - p_{(m+1)}^{\perp}) \cdot y_{(m+1)}^{\perp}}/2$$

$$\times \left\langle t^{m} F_{\perp}^{\alpha_{m} +} (y_{m}^{\perp} + \delta y_{m}^{\perp}/2) t^{b_{m}} F_{\perp}^{\beta_{m} +} (y_{(m+1)}^{\perp} - \delta y_{(m+1)}^{\perp}/2) \right\rangle$$

$$= (2\pi)^{2} \delta^{2} \left( p_{m}^{\perp} + p_{(m+1)}^{\perp} \right) \int dy_{m}^{\perp} e^{i(p^{\perp} - \delta y_{m}^{\perp} - \delta y_{(m+1)}^{\perp})/2}$$

$$\times \left\langle t^{m} F_{\perp}^{\alpha_{m} +} (\delta y_{m}^{\perp}/2) t^{b_{m}} F_{\perp}^{\beta_{m} +} (-\delta y_{(m+1)}^{\perp}/2) \right\rangle.$$  

(B2)

The transverse momentum $\delta$-function in the above equation imposes the condition that the transverse momentum brought in by one gluon is taken out by the other. Thus a pair of adjacent gluon insertions, in the limit of transverse position invariance, infuses no net transverse momentum, into the final cut line.

Incorporating the $m$ transverse momentum delta functions, leads to the following simplified expression for the hard part of the integrand,

$$H = \left\{ \nabla_{L_{\perp}}^{2} \right\}^{(m-1)} \left[ \delta^{2} \left( \bar{l} - \sum_{i=0}^{m-1} \bar{p}_{i}^{\perp} \right) \right]$$

$$\times \nabla_{F_{\perp}^{\alpha_{m}} e^{ix_{\perp}^{\alpha_{m}}} p_{\perp}^{\alpha_{m}} \delta y_{m}^{\perp}}$$

(B3)

where, $\delta y_{m}^{\perp} = y_{m}^{\perp} - y_{(m+1)}^{\perp}$.

One notes the absence of the two transverse momenta $p_{m}^{\perp}, p_{(m+1)}^{\perp}$ from the argument of the $\delta$-function which determines the momenta of the cut-line ($l_{\perp}$). This will also be the case if the extra gluons were instead considered on the right hand side of the cut line. The essential point is that in the case of translational invariance in a large nucleus, the leading length enhanced contribution from double scattering off a single nucleon produces no net transverse momentum broadening. As a result, such diagrams contribute to terms with a lower number of transverse momentum derivatives. The transverse momentum derivatives that act on the phase factors lead to suppressed contributions. An evaluation of the double derivatives on the phase factors, in the limit of vanishing transverse momenta, will leave an overall factor of $i\delta y_{m}^{\perp} / (q^-)$. As $\delta y_{m}^{\perp}$ is restricted to the size of the nucleon and the entire contribution is suppressed by a factor of $q^-$, such terms are parametrically suppressed in the evaluation of the transverse momentum distribution. Such terms are however important in terms of unitary corrections to the total integrated cross section as will be demonstrated in the next appendix.

APPENDIX C: UNITARITY CORRECTIONS

In the evaluation of the transverse momentum distribution $|\phi(L_{-}, t_{\perp})|$ in Eq. (54), as given by the solution of the diffusion equation [Eq. (52)], the focus was limited to the central cut diagrams as in Fig. 5. It was argued that diagrams with non-central cuts do not contribute to the leading behavior of the transverse momentum distribution. These diagrams, however, play an important role in the unitarization of the integrated cross-section. The unitarity requirement states that

$$\partial \int d^{2}l_{\perp} \phi(L_{-}, t_{\perp}) = 0.$$  

(C1)

One notes that the above requirement is satisfied by Eq. (54). The unitarity requirement was tacitly assumed in formulating the overall normalization factor of the Gaussian distribution in Eq. (53). In what follows, we discuss the origin of such a constraint.

The starting point is that of Eq. (51). In the interest of simplicity we focus on the case of $n + n' = 2$. Cases with a higher number of gluon insertions follow a similar, albeit a more complicated pattern. For the case of two gluon insertion there are three cuts: the central cut with $n = n' = 1$, the left cut with $n = 0, n' = 2$ and the right cut with $n = 2, n' = 0$. The contribution to the central cut may be immediately evaluated from Eq. (51), by setting $n = n' = 1$. Integrating out the transverse momentum of the cut-line $l_{\perp}$ we obtain the hadronic tensor as

$$W^{\mu \nu} = \int \prod_{i=0}^{1} dy_{i} \int d^{2}p_{i} \int d^{2}q_{\perp}$$

$$\times \{ \frac{1}{2} (g_{\mu \nu} - g^{\mu \nu}) + g^{\mu \nu} g_{\rho \sigma} - g^{\mu \rho} g^{\nu \sigma} \}

$$\times e^{-ix_{\perp}^{\mu} p_{\perp}^{\nu} y_{\perp}^{\mu}} \prod_{i=0}^{1} e^{-ix_{\perp}^{\mu} y_{\perp}^{\nu}} \theta(y_{1}^{\perp} - y_{-1}^{\perp}) \theta(y_{-1}^{\perp} - y_{1}^{\perp})$$

$$\times \langle A; p | \psi(y_{1}^{\perp}, y_{\perp}) \gamma^{\mu} \psi(0) | A; p \rangle + \frac{i}{4} A_{s}^{\perp} (y_{1}^{\perp}, y_{\perp})$$

$$\times \frac{i}{4} A_{b}^{\perp} (y_{-1}^{\perp}, y_{\perp}) | A; p \rangle.$$  

(C2)

The nuclear state is now decomposed into a gas of nucleon states with the quark wave function insertions restricted to one nucleon and the gluon vector potential insertions restricted to another nucleon.

In what follows we focus solely of the matrix element of the two gluon insertions in a nucleon, which is referred to as the soft part $S$. 

\[ S = \langle p|A_+^+(y_1', y_1)A_-^-(y_1, y_1')|p\rangle e^{ip_1\cdot y_1' - ip_1\cdot y_1} \\
= e^{i(p_1'\cdot \delta y)} e^{ip_1\cdot y_1'} \langle p|A_+^+(\delta y/2)A_-^-(\delta y/2)|p\rangle, \quad (C3) \]

where, the transverse positions have been recast as \( y + \delta y/2 \) and \( y - \delta y/2 \). Transverse position invariance in a large nucleus is invoked in the last line of the above equation where the two point correlator is set to be independent of the mean transverse location \( \bar{y}_\perp \). The integration over the mean transverse location can be carried out to obtain the two dimensional delta function \( \delta \)-function \( \delta^2(p_1^2 - p_1'^2) \). The hard part, which essentially consists of phase factors may now be expanded in a Taylor series in the transverse momenta \( p_1, p_1' \) as in Eq. \((C3)\).

Carrying out the derivatives and setting the remaining transverse momenta to zero, gives the part that depends on the gluon insertions as

\[ G = \int dy_1 dy_1' \theta(y_1' - y_1)\theta(y_1 - y) \]
\[ \times \frac{1}{q^-} \langle p|F_\alpha^+(y_1') F^{+\alpha}(y_1)|p\rangle. \quad (C4) \]

In the above equation, due to color confinement, \( y_1' - y_1 \) is always restricted to be smaller than the size of a nucleon. However, the mean location \( (y_1' + y_1')/2 \) is unrestricted and as a result this length integration provides an overall factor of \( L^- \) and is the origin of the length enhancement of the part of the cross section which corresponds to the central cut.

Setting \( n = 0, n' = 2 \) and \( n = 2, n' = 0 \), and following an almost similar method as above for the central cut yields the contributions from the left and right cuts. As shown in the preceding appendix, the terms originating from the left and right cuts provide almost identical contributions, except for an over all sign and the argument of the \( \theta \)-functions. Relabeling the locations: e.g., \( y_1 = y_1' \) in the case of the left cut (and similarly for the right cut to have the same set of position labels for all three cuts), yields the combination of \( \theta \)-functions as

\[ \Theta = \theta(y_1' - y)\theta(y_1 - y_1')\theta(y_1 - y) - \theta(y_1' - y_1')\theta(y_1 - y). \quad (C5) \]

As has been demonstrated earlier \([4, 10, 11, 13]\), the above combination, along with the constraint from confinement destroys the length enhancement by restricting the average longitudinal location \( (y_1' + y_1')/2 \) to also lie within the size of a nucleon. The combination of all three cuts, in the evaluation of the integrated distribution, thus removes the overall factor of \( L^- \) and as a result the integrated distribution obeys the condition of Eq. \((C1)\). The general case with \( n \)-gluon insertions may also be demonstrated to contain such unitarity corrections by a similar method of combining contributions from all cuts.

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