The mechanism of domain walls and strings formation in the early Universe

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Abstract. A classical dynamics of two real scalar fields within a model with a saddle point potential in (2+1)-space-time is discussed. We show that in this model, solitons may be formed both as domain walls and strings in three dimensional physical space. The formation and evolution of these field configurations are considered.

1. Introduction
There are a lot of inflation models possessing potentials with complicated forms containing one or even set of minima and saddle points (see, for example, [1–4]). Meanwhile, if potential of scalar fields contains even one saddle point and a minimum, solitons may be formed in such a theory [5,6]. Under certain conditions, these topological nontrivial structures might lead to primordial black holes production in the early Universe due to collapse of domain walls [7,8] or loops of cosmic strings [9]. Moreover, the solitons may dramatically affect on the early Universe [10,11].

In our work, we study the solitons production within models with potentials possessing at least one saddle point and one minimum in (2+1) space-time. We take the potential in a general form to approximate some inflation models. The (1+1) case describing the domain walls formation in the early Universe was previously discussed in [12,13].

2. Numerical simulation
We consider the model of two real scalar fields $\varphi$, $\chi$ where the potential $V$ is chosen in the form

$$V = d(\varphi^2 + \chi^2) + a \exp \left[-b(\varphi - \varphi_0)^2 - c(\chi - \chi_0)^2\right], \quad a > 0, \quad b > 0, \quad c > 0, \quad d > 0,$$

The potential has the minimum in point $(\varphi_0, \chi_0) = (0,0)$ with exponentially small errors. Parameters $b$ and $c$ set the shape of the local maximum, and $a$ describes its height. Parameter $d$ sets a common slope of the potential.

The Lagrangian of two real scalar fields is given by

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} (\partial^\mu \varphi \partial^\nu \varphi + \partial^\mu \chi \partial^\nu \chi) - V(\varphi, \chi),$$

$$\text{(2)}$$
where $g_{\mu\nu}$ is the Friedmann-Robertson-Walker metric tensor. After the inflation ends, the fields begin to obey the classical motion equations which for (2+1) space-time are as follows

$$
\varphi_{tt} - 3H \varphi_t - \varphi_{xx} - \varphi_{yy} = -\frac{\partial V}{\partial \varphi},
$$

$$
\chi_{tt} - 3H \chi_t - \chi_{xx} - \chi_{yy} = -\frac{\partial V}{\partial \chi}.
$$

(3)

Here $H = \dot{a}/a$ is the Hubble parameter, $H_I \sim 10^{13}$ GeV for the inflation stage and becomes smaller after the inflation ends. In our estimations, we neglect the time dependence of the Hubble parameter for the sake of simplicity due to $-3H \varphi_t$ and $-3H \chi_t$ are friction terms and the values define the oscillations rate, but do not influence the final fields distribution. Note, the value of $H_I$ specifies the natural energy scale for the system. Hereinafter, all values are in $H_I$ units.

Energy density $\rho$ of the fields configuration is determined by the energy-momentum tensor $\rho = T^{00}$, where $T^{\mu\nu}$ is given by

$$
T^{\mu\nu} = \frac{\partial L}{\partial \left( \partial_\mu \varphi_a \right)} \partial_\nu \varphi_a - g^{\mu\nu} L.
$$

(4)

Thus, for (2), the energy density $\rho$ is as follows

$$
\rho = \frac{1}{2} \sum_{i=t,x,y} (\varphi_i^2 + \chi_i^2) + V(\varphi, \chi).
$$

(5)

To find the solution of (3), we take the following initial conditions:

$$
\varphi(x, y, 0) = R \cos \Theta + \varphi_1, \quad \varphi_t(x, y, 0) = 0;
$$

$$
\chi(x, y, 0) = R \sin \Theta + \chi_1, \quad \chi_t(x, y, 0) = 0,
$$

(6)

where

$$
R(r) = R_0 \cosh^{-1} \frac{r_0}{r}, \quad \Theta = \theta.
$$

(7)

$R_0 > 0, r_0 > 0, \varphi_1, \chi_1$ are parameters, $r = \sqrt{x^2 + y^2}$ and $\theta$ are the distance from the coordinate origin and the polar angle, respectively.

The natural boundary conditions are free:

$$
\varphi_x(\pm \infty, y, t) = 0, \quad \varphi_y(x, \pm \infty, t) = 0;
$$

$$
\chi_x(\pm \infty, y, t) = 0, \quad \chi_y(x, \pm \infty, t) = 0.
$$

(8)

After the initial and the boundary conditions are defined in (6)–(8), the equations (3) may be numerically solved.

In our work, we present the results for two cases to demonstrate two principal possibilities. In both ones, the potential parameters defined in (1) were chosen as follows: $d = 0.005, a = 2, b = 1, c = 1, \varphi_0 = -5, \chi_0 = 0$, and the parameters of the initial conditions (6) are $R_0 = 1, r_0 = 1, \chi_1 = 0$. In the first case, the location of the initial conditions defined by the parameter $\varphi_1 = -8$. It is depicted in figure 2. The initial location of fields $\varphi$ and $\chi$ is near the saddle point. The final stable distribution establishing during the classical evolution is shown in figure 2. The fields distribution tends to the potential minimum, but the local maximum prevents it and splits the solution into two trajectories. The energy density may be calculated using (5). It is depicted
Figure 1. The first case: potential (1) with the parameters values \( d = 0.005, a = 2, b = 1, c = 1, \varphi_0 = -5, \chi_0 = 0 \) and the initial conditions (6) with \( R_0 = 1, r_0 = 1, \varphi_1 = -8, \chi_1 = 0 \) are shown.

Figure 2. The final energy density distributions for discussing cases are shown.

Figure 3. The second case: potential (1) with the parameters values \( d = 0.005, a = 2, b = 1, c = 1, \varphi_0 = -5, \chi_0 = 0 \) and the initial conditions (6) with \( R_0 = 1, r_0 = 1, \varphi_1 = -5, \chi_1 = 0 \) are shown.

for the final state in figure 2. It corresponds the domain wall in the 3-dimensional space and qualitatively coincides with the results of [12,13] for the winding number \( N = 1 \).

The second case differs from the first one only by the value of the parameter defining the initial conditions location \( \varphi_1 = -5 \). It is shown in figure 2. The parameter are chosen in such a way that the initial fields distribution includes the peak of the potential. Figure 2 illustrates the final state of the fields obtained by solving the motion equations. The fields distribution tends to minimize its energy which causes the fields to roll off the potential peak. The final energy density distribution is shown in figure 2. It corresponds the string with the ridge formation in 3-dimensional space. The ridge appears due to the slope of the potential.
3. Conclusion
The classical dynamics of the inflation-like scalar fields with the potential having at least one minimum and a saddle point after the inflation ends is considered. Our numerical analysis shows the different types of solitons (domain walls and strings) may be formed within the model with the same potential and various initial conditions. As mentioned in the introduction, both types of solitons can produce primordial black holes and seriously affect the evolution of the early Universe. Thus, the possibility of solitons (and consequently PBHs) production in specific inflation theories having the similar potential to that considered in this work should be checked.

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