Phase-selective reversible quantum decoherence in cavity QED experiment

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I. INTRODUCTION

Quantum interference is essential phenomenon to understand the quantum-mechanical effects. The reason why the quantum interference is hard to observe experimentally, is decoherence arising from an interaction of the system with environment. In the last decade, the QED cavity experiments [2] or experiments with trapped ions [3] were performed to investigate the decoherence phenomenon from point of view reversible and irreversible nature. The essential principle of decoherence can be generalized in the following way. Due to system-environmental interaction, mutual quantum entanglement generates a stronger correlations between system and environment than it can be achieved in classical domain. Then, if one treats the system separately to their environment, these strong correlations are reduced and quantum interference vanishes in the system.

Exhibitions of quantum decoherence are often experimentally observed in a decreasing of the visibility of interference fringes in particular measurement. Thus, the vanishing of quantum interference is treated as basis-dependent problem. On the other hand, due to progressive system-environmental entanglement, the particular entanglement (purity) of the system substantially increases and fidelity between states before and after decoherence process decreases. Contrary, this point of view is independent of a choice of basis and is not related to any particular measurement.

In this Paper, based on experimental arrangement with reversible decoherence [2], we propose new feasible cavity QED experiment to analyse the decoherence from both points of view: visibility and entanglement, using the directly measurable quantities. We can conclude that whereas the visibility in particular measurement can be preserved by environmental state squeezing, the state-independent characteristics of decoherence (entropy, purity, fidelity) show that the decoherence is more progressive as the visibility is pronouncedly preserved. The preserving of visibility is closely related to the complementarity between visibility and distinguishability. On the other hand, the pronounced decoherence observed in the entropy and purity is connected with a large entanglement between system and environment. Due to unitary evolution of total system, the decoherence process can be considered ideally reversible. However, it is hard to obtain this case in realistic experiment and therefore the cavity losses and coupling to other modes must be considered. To observe both the effects experimentally, direct measurements of the visibility, negativity of Wigner function, purity and fidelity are proposed for cavity QED experiments.

II. STATE PREPARATION AND MEASUREMENT

The basic experimental scheme is depicted on Fig. 1. It modifies a previously used experimental arrangement [2] to test the reversibility of quantum decoherence. The arrangement utilizes the Ramsey atomic interferometer where a two-level atom cross the high-Q cavity \( C_0 \). This single circular Rydberg atom is treated to prepare the coherent state superposition in cavity \( C_0 \). Then the cavity \( C_0 \) is coupled during an appropriate time to auxiliary resonant cavity \( C_1 \) in the quadrature squeezed state and quantities of interest are measured using atomic probe.

The principle of the coherent state preparation is discussed in detail in [4]. It is generated by the dispersive, nonresonant coupling of a single circular Rydberg atom to the cavity mode, in which a coherent field \( |\xi(0)\rangle \) is initially prepared. Before the atoms enter the cavity, they are excited by \( \pi/2 \)-pulse in first Ramsey zone to balanced superposition of levels \( |b\rangle \) and \( |c\rangle \). An atom crossing the cavity in the state \( |b\rangle \) induces appreciable phase shift on the field in the cavity. Let us assume that this shift can be adjusted to a value exactly equal to \( \pi \) by proper selection of the atomic velocity (about 100m/s). Then a coherent field \( |\xi(0)\rangle \) in cavity is transformed into \( |\xi(0)\rangle \). The phase shift is negligible, if the atom crosses the cavity in state \( |c\rangle \). The atom crosses centimeter-sized cavity in a time of the order of \( 10^{-4} \)s, much shorter than the field relaxation time (typically \( 10^{-3} \) – \( 10^{-2} \)s), and to
the atomic radiative damping time ($3 \times 10^{-7}$ s). After the atom has crossed the cavity, it enters second Ramsey microwave $\pi/2$ pulse and after detection of the atom in the state $|c\rangle$ (or $|b\rangle$), the following Schrödinger-cat states are conditionally prepared

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{N_\pm}}(|\xi(0)\rangle \pm -|\xi(0)\rangle),$$

$$N_\pm = 2\left(1 \pm \exp(-2\xi^2(0))\right).$$

A sign $\pm$ in the superposition can be obtained by post-selection in dependence on output atomic states $|c\rangle$ and $|b\rangle$.

Analogously to experiment [3], we consider a controlled resonant coupling preformed by superconducting waveguide between the cavities $C_0$ and $C_1$. In addition, we simplify the situation by the assuming that the coupling interaction can be effectively reduced to two modes $S$ and $E$, representing the system and environment particularly. Then, the coupling can be simply described by the following interaction Hamiltonian $H = \hbar\kappa(\hat{a}_S^\dagger \hat{a}_E + \hat{a}_E^\dagger \hat{a}_S)$, where $\kappa$ is effective coupling constant and $\hat{a}_S$ and $\hat{a}_E$ are annihilation operators of particular modes. Similarly to experimental arrangement [3], this interaction could be tuned by adjusting the cavities coupling through a superconducting waveguide. We will consider a strongly slowed down the relaxation processes for the both cavities (as well as for the atoms during the state preparation). This assumption is realistic for the quality factor of cavities at least $Q \approx 10^9$ [3] (corresponding photon lifetimes are order of a few ms). Let us assume that the coupling between the cavities plays no role during the cat generation in cavity $C_0$. In addition, the state preparation in cavity $C_1$, provided the preparation times are much shorter than $1/\kappa$. In summary, the coupling constant $\kappa$ must chosen such that the inequalities $t_{\text{int}} \ll 1/\kappa \ll T$, where $t_{\text{int}}$ is atomic transit time through the cavity ($\approx 20\mu$s) and $T$ is photon lifetime ($\approx 1\mu$s) [3].

Whereas in the experiment [3] the cavity $C_1$ was considered in the vacuum state, we will assume that the mode $E$ in cavity $C_1$ is prepared before coupling in the quadrature squeezing vacuum state $|0, r\rangle$. This squeezed state can be prepared by a nonlinear medium coherently performing a reduction of fluctuations in certain quadrature of field. Nonlinear degenerate parametric process described by the effective Hamiltonian $H = 2\hbar\Gamma(\hat{a}_1^2 - \hat{a}_1^\dagger^2)$, where $\Gamma$ is nonlinear coupling constant and starting from vacuum state can be appropriated. The squeezed state is obtained after effective interaction time $\tau = \frac{1}{\Gamma}$. For $r > 0$ the fluctuations are reduced in the $Q$ quadrature, as the Schrödinger-cat state is orientated, whereas for $r < 0$ in the complementary quadrature $\langle (\Delta Q_E(0))^2 \rangle = 1/4 \times \exp(-2r)$, $\langle (\Delta P_E(0))^2 \rangle = 1/4 \times \exp(2r)$. Squeezing in degenerate parametric amplifier was demonstrated [3] using MgO:LiNbO$_3$ crystal putting inside a high-Q cavity and pumped by the second harmonic generation of a 1.06µm Nd:YAG laser. The noise level measured by balanced homodyne detector was larger than 50 percent below the vacuum limit, which is corresponding to the squeezing parameter $r = 2\Gamma t \approx 1$. To obtain the considered interaction, we need to adjust the frequency of light in cavity $C_0$, effectively interacting with atomic probe, with a mean frequency of maximal squeezing in parametric process.

In QED cavity experiments we are not able to directly measure the fields in high-Q cavities. It is only possible to study statistical properties of the cavity field from the statistical properties of the auxiliary atoms that are crossing the cavity. To observe the values of the Wigner function in phase-space points, the direct measurement of the Wigner function in the optical cavity was proposed [3]. We will use a simplified version of the measurement to utilize the identical experimental setup as for state preparation. A probe atom in the excited state $|c\rangle$ is sent through the system. After the atom crosses the apparatus, the internal state of the atom is detected by two field ionization detectors. This experiment is repeated many times, starting each run with the same field, and probabilities $p_b$ and $p_c$ of detecting the probe atom in the state $|b\rangle$ and $|c\rangle$ are determined. From the difference of probabilities, a value of Wigner function in origin of the phase space can be determined from this simple relation $W(0) = 2(p_c - p_b)$. An important feature is insensitivity to the detection efficiency of the atomic counters. However the measurement accuracy depends on the detector’s selectivity, i.e. the ability to distinguish between two atomic states and the velocity spread of atomic beam. It will be proposed in next section, how the visibility of interference, distinguishability and negativity of Wigner function parameters can be directly measured using this setup.

To observe the purity of state and fidelity between initial prepared coherent state superposition and state during evolution the scheme proposed in [3] can be used. Using it we are able to measure the overlap $\text{Tr}[\rho_1 \rho_2]$ between separable states in two cavities by means of their mutual coupling and atomic probe measurement performed by previously discussed Ramsey interferometer. The overlap is given by the visibility of interference of repeatedly performed interferometric experiments. To measure the purity and fidelity, we need to prepare two cavities $C_0$ and $C'_0$ in the same initial coherent state superposition. The purity can be measured, if we perform the proposed experiment immediately in two identical setup and measure the overlap between cavities $C_0$ and $C'_0$ evolving in the same state. On the other hand, to measure the fidelity one cavity is affected by coupling with cavity $C_1$ in proposed experimental setup, whereas the initial state in second cavity is prevented from any changes. The suggested measurements offer a possibility to experimentally observe the decoherence from the both different points of view, as will be discussed in the following sections.
III. EVOLUTION OF QUANTUM INTERFERENCE

Quantum interference effects can be well illustrated in behavior of Wigner function and density matrix in coordinate representation. After coupling time $t$, Wigner function corresponding to the particular states of signal mode $S$ in cavity $C_0$ and environmental mode $E$ in cavity $C_1$ can be found in the following form

$$W_{\pm i}(Q, P, t) = 2N_\pm^{-1} \Gamma_i(P) \Gamma_i(Q) \times$$
$$\times \left[ O_i \cos \left( \frac{\xi_i(t)P}{2((\Delta \hat{P}_i(t))^2)} \right) \pm$$
$$\pm R_i \cos \left( \frac{\mu_i(t)P}{2((\Delta \hat{P}_i(t))^2)} \right) \right],$$

(2)

where $i = S, E$ and $\pm$ corresponds to particular states [3] and

$$\Gamma_i(P) = \frac{1}{\sqrt{2\pi((\Delta \hat{P}_i(t))^2)}} \exp \left( -\frac{P^2}{2((\Delta \hat{P}_i(t))^2)} \right),$$

$$R_i = \exp(-2\xi^2) \exp \left( \frac{\mu_i^2(t)}{2((\Delta \hat{P}_i(t))^2)} \right),$$

$$\Gamma_i(Q) = \frac{1}{\sqrt{2\pi((\Delta \hat{Q}_i(t))^2)}} \exp \left( -\frac{Q^2}{2((\Delta \hat{Q}_i(t))^2)} \right),$$

$$O_i = \frac{\exp(-2\xi^2(0))}{D_i},$$

$$D_i = \exp(-2\xi^2(0)) \exp \left( -\frac{\xi_i^2(t)}{2((\Delta \hat{Q}_i(t))^2)} \right).$$

(4)

Density operator of mode in cavity in the coordinate representation may be calculated as the Fourier transform of the Wigner function $W(Q, P, t)$ and expressed in the following form

$$p_{\pm i}(Q, Q') = \frac{1}{N_\pm \sqrt{2\pi((\Delta \hat{Q}_i(t))^2)}} \times$$
$$\times \left[ \exp \left( -2\left( \frac{Q - Q'}{2} \right)^2((\Delta \hat{P}_i(t))^2) \right) \times$$
$$\times \left( \exp \left( -\frac{(Q + Q' - \xi_i(t))^2}{2((\Delta \hat{Q}_i(t))^2)} \right) +$$
$$+ \exp \left( -\frac{(Q + Q' + \xi_i(t))^2}{2((\Delta \hat{Q}_i(t))^2)} \right) \right) \times$$
$$\pm R_i \exp \left( -\frac{(Q - Q')^2}{2((\Delta \hat{Q}_i(t))^2)} \right) \times$$
$$\times \left( \exp \left( -2\left( \frac{\mu_i(t)}{2((\Delta \hat{P}_i(t))^2)} \right) +$$
$$+ \frac{Q - Q'}{2})^2((\Delta \hat{P}_i(t))^2) \right) +$$
$$+ \exp \left( -2\left( \frac{\mu_i(t)}{2((\Delta \hat{P}_i(t))^2)} \right) -$$
$$\frac{Q - Q'}{2})^2((\Delta \hat{P}_i(t))^2) \right) \right].$$

(5)

For the proposed experiment operating with initial squeezed state in cavity $C_1$, after some calculations incorporating a damping in both the cavities $C_0$ and $C_1$, the evolution of particular time-dependent parameters for both the modes $S, E$ can be expressed in following form

$$\xi_S(t) = \mu_S(t) = \frac{\exp(-\gamma_+t/2)}{\lambda} f(t) \xi(0),$$

$$\xi_E(t) = -\mu_E(t) = \frac{\exp(-\gamma_+t/2)}{\lambda} g(t) \xi(0),$$

$$\langle(\Delta \hat{Q}_S(t))^2 \rangle = \frac{\exp(-\gamma_+t)}{\lambda^2} \left( f^2(t) \langle(\Delta \hat{Q}_S(0))^2 \rangle +$$
$$+ g^2(t) \langle(\Delta \hat{P}_E(0))^2 \rangle \right) + F_S(t),$$

$$\langle(\Delta \hat{P}_S(t))^2 \rangle = \frac{\exp(-\gamma_+t)}{\lambda^2} \left( f^2(t) \langle(\Delta \hat{P}_S(0))^2 \rangle +$$
$$+ g^2(t) \langle(\Delta \hat{Q}_E(0))^2 \rangle \right) + F_S(t),$$

$$\langle(\Delta \hat{Q}_E(t))^2 \rangle = \frac{\exp(-\gamma_+t)}{\lambda^2} \left( f^2(t) \langle(\Delta \hat{Q}_E(0))^2 \rangle +$$
$$+ g^2(t) \langle(\Delta \hat{P}_S(0))^2 \rangle \right) + F_E(t),$$

$$\langle(\Delta \hat{P}_E(t))^2 \rangle = \frac{\exp(-\gamma_+t)}{\lambda^2} \left( f^2(t) \langle(\Delta \hat{P}_E(0))^2 \rangle +$$
$$+ g^2(t) \langle(\Delta \hat{Q}_S(0))^2 \rangle \right) + F_E(t),$$

$$f(t) = \gamma_- \sin \lambda t/2 + \lambda \cos \lambda t/2,$$

$$g(t) = 2\kappa \sin \lambda t/2,$$

$$F_{S,E}(t) = \frac{1}{2} \left[ (\eta_{S,E}^2)(\lambda^2 + \lambda_+^2 + \gamma_+^2) +$$
$$+ 4\eta_{S,E} \lambda^2 \kappa^2 (1 - \exp(-\gamma_+t)) +$$
$$+ \eta_{S,E}^2 \lambda^2 \gamma_+(\gamma_+ + 2\gamma_-) \times$$
$$\times (1 - \exp(-\gamma_+t) \cos \lambda t) -$$
$$- \gamma_+^2 (\eta_{S,E}^2 \gamma_2 + 4\eta_{S,E} \kappa^2) \times$$
$$\times (1 - \cos \lambda t) \exp(-\gamma_+t) +$$
$$+ (\eta_{S,E} \lambda \gamma_-(\lambda^2 - \gamma_2^2 - 2\gamma_+ \gamma_- -$$
$$- 4\eta_{S,E} \kappa^2 \lambda_+) \exp(-\gamma_+t) \sin \lambda t],$$

$$\lambda = \sqrt{4\kappa^2 - \gamma_2^2},$$

$$\gamma_- = \gamma_{E,S} - \gamma_{S,E},$$

$$\eta_{S,E} = \gamma_{S,E}(\eta_{S,E} + \frac{1}{2}),$$

$$\langle(\Delta \hat{Q}_S(0))^2 \rangle = \langle(\Delta \hat{P}_S(0))^2 \rangle = 1/4,$$

$$\langle(\Delta \hat{Q}_E(0))^2 \rangle = 1/4 \times \exp(-2\kappa),$$

$$\langle(\Delta \hat{P}_E(0))^2 \rangle = 1/4 \times \exp(-2\kappa),$$

$$\langle(\Delta \hat{P}_S(0))^2 \rangle = \langle(\Delta \hat{Q}_E(0))^2 \rangle = 1/4.$$
\[ \langle (\Delta \hat{P}_E(0) )^2 \rangle = 1/4 \times \exp(2r), \]  

where \( \gamma_S \) and \( \gamma_E \) are damping constant of particular modes and \( \langle n_S \rangle \) and \( \langle n_E \rangle \) are corresponding mean numbers of thermal reservoir fluctuations.

The decoherence effect exhibits in the decreasing of visibility in particular field quadrature measurements. For given coherent state superposition, particular interference effect can be observed in marginal distribution of quadrature \( \hat{P} \)

\[ p_i(P) = \frac{2}{N_\pm} \Gamma_i(P) \left[ 1 \pm R_i \cos \left( \frac{\mu_i P}{\langle (\Delta P_i)^2 \rangle} \right) \right], \]

where \( i = S, E \). This equation represents an analogue of interference rule in any point of phase space. To measure the visibility \( R_i \), we can consider probability \( p_i(0) \) in origin of distribution for both the states \( |\Psi_\pm \rangle \). Then interference formula (9) can be simply expressed in following form \( p_\pm(0) = 2/N_\pm (1 \pm R_i) \), where \( R_i \) is visibility defined in the following way

\[ R = \frac{N_+ p_+(0) - N_- p_-(0)}{N_+ p_+(0) + N_- p_-(0)}. \]

The normalization factors \( N_\pm \) must be introduced in definition of visibility due to a non-orthogonality of coherent states. They can be approximately considered as unity if amplitude \( \xi(0) \) is sufficiently large \( (\xi(0) > 2) \). Then Eq. (10) approaches commonly used definition of visibility.

Immediately, the marginal distribution for \( \hat{Q} \) quadrature

\[ p_i(Q) = \frac{1}{N} \exp(-2\xi^2(0)) \Gamma_i(Q) \times \]

\[ \left[ D^{-1}_i \cosh \left( \frac{\xi_i Q}{\langle (\Delta Q_i)^2 \rangle} \right) \pm 1 \right] \]

consists (for sufficiently large \( \xi(0) \)) essentially of two Gaussian peaks symmetrically distant from the origin. It can be evident from another form of Eq. (11)

\[ p_i(Q) = \frac{1}{N} (p_L(Q) + p_R(Q) \pm 2 \exp(-2\xi^2(0)) \Gamma_i(Q)), \]

where

\[ p_{L,R}(Q) = \frac{1}{\sqrt{2\pi \langle (\Delta Q_i)^2 \rangle}} \exp \left( -\frac{(Q \pm \xi_i)^2}{2\langle (\Delta Q_i)^2 \rangle} \right) \]

is probability distribution of the marginal peaks and \( \Gamma_i(Q) \) represents the vacuum noise influence. Analogically to interference parameter \( R_i \), the overlap parameter \( O_i \) can be proposed. It can be defined as square root of normalized overlap between separated marginal probability distributions corresponding to particular initial coherent states \( |\alpha \rangle \) and \( | - \alpha \rangle \)

\[ O^2 = \frac{\int_{-\infty}^{\infty} p_L(Q)p_R(Q) dQ}{\int_{-\infty}^{\infty} p_L(0)p_R(0) dQ}, \]

where \( \exp(-2\xi^2(0)) \leq O \leq 1 \) and \( p_{L,R}(Q) = p_{L,R}(Q) \equiv \Gamma(0) \) are the considered Gaussian distributions \( p_L(Q), p_R(Q) \) shifted to origin and having the same variance. The visibility \( R_i \) and overlaps \( O_i \) are connected by a relation \( R_i R_{\xi} \leq O_i O_{\xi} \), where equality occurs for unitary evolution of total system. In cavity QED experiments, the visibility \( R_i \) and overlap \( O_i \) parameters can be straightforwardly calculated from the measurement of Wigner function \( W_i(0,0) \) in origin of phase space

\[ R = \frac{N_+ W_+(0,0) - N_- W_-(0,0)}{4W_0(0,0)}, \]

\[ O = \frac{N_+ W_+(0,0) + N_- W_-(0,0)}{4W_0(0,0)}, \]

where \( W_0 \) is Wigner function of the particular mode for initial vacuum state. A value of Wigner function can be directly determined by atomic-probe measurement as was referred in the previous section. If \( \xi(0) \) is sufficiently large \( (\xi(0) > 2) \), then the relations (15) can be simplified to \( R \approx (W_+(0,0) - W_-(0,0))/2W_0(0,0) \) and \( O \approx (W_+(0,0) + W_-(0,0))/2W_0(0,0) \).

To simply define a distinguishability between marginal peaks, the parameters \( D_i \) inversely related to overlap \( O_i \) can be used. This choice of distinguishability parameter is appropriate for homodyne measurement of particular quadrature operators, where both the parameters \( D_i \) and \( R_i \) can be determined from values of marginal probability distribution of \( \hat{Q} \) and \( \hat{P} \) operators in the origin, using (11) in both the particular cases. In cavity QED experiments, the distinguishability parameter \( D_i \) can be determined from overlap \( O_i \). The distinguishability and interference parameters in the particular modes are connected by the relation

\[ R_S D_S R_E D_E \leq \exp(-4\xi^2(0)), \]

where equality is achieved for unitary evolution of system and environmental modes. Particularly, the products \( R_i D_i \) satisfy the following inequalities

\[ \exp(-4\xi^2) \leq R_E D_E \leq \exp(-2\xi^2) \leq R_S D_S \leq 1. \]

Another definition of distinguishability parameter leading to different form of relation between visibility and distinguishability is proposed in Appendix.

It can be simply found that the first part of the right side of Wigner function in Eq. (13) is connected with distinguishable peaks, and is always positive, whereas the second part is connected to interference oscillations in phase space, and can be negative. In a particular point of
phase space, the negative part can be larger and Wigner function becomes negative. An occurring of these negative values depends only on sign of expression in square brackets in Eq. (3). For initial state $|\Psi_+\rangle$, the expression in square brackets has the maximal negative value in the points $(0, \frac{\pi(\Delta P_0)^2}{\mu_0})$ of the phase space. After some calculations, the necessary and sufficient condition for the occurring of a negative value of the Wigner function can be found in the following form

$$D_i R_i > \exp(-2\xi^2(0))$$

(18)

for the particular mode $i = S, E$. A positive Wigner function can be treated semiclassically, whereas for Wigner function with the negative values no semiclassical theory can be constructed. Irrespective to, the Glauber-Sudarshan quasidistribution cannot be positive or exist everyone. To measure product $D_i R_i$, the atomic-probe measurement of Wigner function can be used. We define a new parameter $N_i = D_i R_i \exp(2\xi^2(0))$ and it can be determined from the values of Wigner functions in origin of phase space by the following relation

$$N = \frac{N_+ W_+(0,0) - N_- W_-(0,0)}{N_+ W_+(0,0) + N_- W_-(0,0)}.$$  

(19)

As negative values of Wigner function are suppressed, the parameter $N_i$ decreases to unity. For sufficiently large $\xi(0)$ ($\xi(0) > 2$), the normalization factors can be approximately considered as unity and thus the relation can be simplified. The negativity parameters $N_i$ satisfy simple relation $N_S N_E \leq 1$, which describe exchange of nonclassicality between the system and environment mode.

Assuming unitary evolution of the total system, the relation (13) can be rewritten for system mode to the following form

$$R_S D_S = \exp(-2\xi^2(0)) N_S.$$  

(20)

Thus, a complementarity between visibility and distinguishability is sharper as the nonclassical character of the state vanishes.

Quantum decoherence process is assisted by an entanglement between system and environment. As the entanglement increases, the state of the system becomes more mixed. Therefore as an appropriate measurable degree of quantum decoherence, the purity of particular system state can be used. In addition, if the total system is in a pure state, then the purity can be immediately used as the measure of entanglement. The purity parameter $P = \text{Tr}\rho^2(t)$ of the state can be expressed by means of the Wigner function $W(Q, P)$ and for initial state $|\Psi_+\rangle$ is given by

$$P_i = \pi \int_{-\infty}^{\infty} W_{S}^2(Q, P, t) dQdP =$$

$$= \frac{1}{N^2 \sqrt{((\Delta Q_i(t))^2)((\Delta P_i(t))^2)}} \left[1 + R_i^2(t) + \exp(-4\xi^2(0)) \left(1 + \frac{1}{D_i^2(t)}\right) + 2\exp(-2\xi^2(0)) \sqrt{\frac{R_i(t)}{D_i(t)}} \right].$$  

(21)

From a value of purity, the Renyi entropy $S_i = -\ln P_i$ can be simply calculated. If state of total system is pure in the initial time and mutual interaction is unitary, the decreasing of purity in particular mode is exhibition of pronounced quantum entanglement and immediately leads to intensive decoherence. Note, for the total pure state case, the particular purity parameters are identical for both the signal and idler modes. To compare the initial state superposition $|\Psi_+\rangle$ with decohered state $\rho$, the fidelity $F_i = \langle \Psi_+ | \hat{\rho}(t) | \Psi_+ \rangle$ can be used. It can be calculated form the Wigner function in the following form

$$F_i = \pi \int_{-\infty}^{\infty} W_i(Q, P, 0) W_i(Q, P, t) dQdP =$$

$$= \frac{2N^{-2}}{\sqrt{((\Delta Q_i(t))^2) + \frac{1}{4}}} \times$$

$$\left(\frac{\exp(-4\xi^2(0))}{2D_i(t)}\right) \times$$

$$\left(\exp \left(2 \left((\Delta Q_i(t))^2\right) + \frac{\xi(t)}{4}\right)\right)$$

$$+ \exp \left(2 \left((\Delta Q_i(t))^2\right) - \frac{\xi(t)}{4}\right)$$

$$\times \frac{R}{2} \left(\exp \left(-2 \left((\Delta P_i(t))^2\right) + \frac{\mu(t)}{4}\right)\right)$$

$$+ \exp \left(-2 \left((\Delta P_i(t))^2\right) + \frac{\mu(t)}{4}\right)$$

$$+ R_i(t) \exp(-2\xi^2(0)) \exp \left(2\xi^2(0)((\Delta Q_i(t))^2)\right) \times$$

$$\exp \left(-\frac{\mu(t)}{8((\Delta P_i(t))^2) + \frac{1}{4}}\right)$$

$$+ \exp \left(-\frac{\mu(t)}{8((\Delta P_i(t))^2) + \frac{1}{4}}\right) \times$$

$$\exp \left(\frac{\xi^2(t)}{8((\Delta Q_i(t))^2) + \frac{1}{4}}\right).$$  

(22)

As fidelity decreases, the state in cavity mode is far from initial coherent state superposition. Note that fi-
duality between initial coherent state superposition and corresponding mix of coherent states is \( F = 1/2(1 + \exp(-2\xi^2(0))) \). The purity and fidelity parameters can be directly measured, as has been pointed out in the previous section.

IV. COMPLEMENTARITY AND ENTANGLEMENT IN DECOHERENCE PROCESS

In this section, we will analyse the quantum decoherence from point of view the complementarity and quantum entanglement in dependence on squeezing of environmental mode. In order to keep the discussion as simple as possible, we assume now the coupling between cavities without an influence of damping (\( \gamma_S = \gamma_E = 0 \)). System mode \( S \) is prepared in coherent state superposition \( |\Psi_S\rangle \) and squeezed state \((0, r)\) is generated in environmental mode \( E \) before coupling with the system. After coupling time rescaled to \( G = \kappa t \), decay of the visibility \( R_S \) in the system mode can be slowed down, if the squeezing \( r \) is sufficiently positive. The decay exhibits a non-exponential character with pronounced offset, as can be seen in Fig. 4(a). Thus appropriate squeezing vacuum state prepared in environment can lead to qualitative change of the exponential to nonexponential vanishing of visibility. This effect was previously mentioned \[\] in efficient homodyne detection and in works \[\] related to phase-sensitive damping and amplification. As amplitude of coherent state \( \xi(0) \) increases, still large squeezing of vacuum fluctuations is needed for a visibility preserving.

Let us now more precisely analyse this slowing down of decoherence using the measurable quantities in proposed experiment. First, it can be found that distinguishability \( D_S \) in complementary quadrature is immediately quickly vanished, as can be observed in Fig. 2. This effect looks particularly simple when the notion of quantum complementarity is considered: due to enhanced noise in \( Q_E \) quadrature (\( r \) is sufficiently positive), the coherent peaks in system mode become more indistinguishable and immediately, due to relation between distinguishability and interference \[\], the visibility is preserved in the measured quadrature \( P_S \). Thus an indistinguishability of superposed coherent states, introduced by noise enhancement, can lead to preservation of visibility in particular measurement. The complementary situation occurs for \( r \) negative, only with a mutually exchange \( R_S \leftrightarrow D_S \). For a region near to \( r = 0 \), both the distinguishability and interference parameters are vanishing, as was previously experimentally analysed \[\].

Irrespective to preserving of the visibility, the state is strongly different from the initial coherent state superposition due to loss of distinguishable peaks and the non-classical negative nature of the Wigner function, as it can be seen in Fig. 3(a,b). From Fig. 4(b), one can find that the suppression of negative values is independent of the sign of squeezing parameter \( r \). However, it can be simply found that Glauber-Sudarshan distribution is still not positive and thus this interference cannot be treated classically. In the corresponding density matrix \( p(Q, Q') \) in coordinate representation the off-diagonal peaks are reduced only to half of initial height and is not completely vanished, as can be seen in Fig.3(d). This is residuum of quantum interference which is sufficient to generate the same visibility as for coherent state superposition. The diagonal peaks become indistinguishable due to appropriate enhanced noise in environmental mode. Thus the interference in quadrature \( P \) is preserved for sufficiently negative \( r \), but some signatures of quantum interference are vanished or degraded. It can lead to two different levels of interference effect: first is connected with pronounced negative character of Wigner function and distinguishable peaks and second with vanishing negative character and undistinguishable peaks corresponding to particular coherent states.

As can be seen in Fig. 4(c), an increase of squeezing in the environmental mode leads to purity vanishing in the system independently of the directions of squeezing. Whereas the visibility is preserved for a long time, if \( r \) is more negative, the state in cavity \( C_0 \) is more mixed. Immediately, it is a signature of pronounced entanglement between cavities, thank to the total system is in pure state. Due to pronounced decreasing for large positive \( r \), strong arising entanglement is signature of quantum decoherence, in contrary to the conjecture obtained from the visibility preserving. However, the slowest decreasing of purity is not found for \( r = 0 \), but it is slightly shifted in positive \( r \) region. Thus for considered superposition with \( \xi(0) = 2 \), the squeezing parameter \( r \approx 1 \) in environmental mode can lead to partial slowing down (not pronounced) of entanglement between system and environmental modes. The effect of decoherence can be described by time evolution of fidelity between initial coherent state superposition and the actual state, which exhibits similar nonexponential behavior as for the purity (Fig. 4(d)). Similarly to evolution of purity, the fidelity decreasing can be slightly slowed down if the squeezing parameter \( r \approx 1 \), however, the fidelity is vanished for sufficiently enhanced positive \( r \).

In ideal case, due to unitary evolution of total system and environment, the decoherence effects are reversible with period \( G = \pi \), as was shown in \[\]. In realistic experiment, we must consider exponential damping in cavity \( C_0 \) which leads to degradation of effect of preserving visibility, as can be seen in Fig. 5(a) for \( \gamma_S = 0.05 \) and \( \langle n_S \rangle = 0 \). Naturally, the damping can be induced by the continuum of modes in cavity \( C_1 \) which are weakly coupled to system mode \( S \) in cavity \( C_0 \). Whereas for visibility \( R \) the degradation is strong for \( G < \pi/4 \), for the other parameters \( R_D, P \) and \( F \) of decoherence, the difference to unitary case is only small. In course of time, all the parameters are vanished, how can be expected. However, the analysis of damping case shows that the proposed ex-
experiment is feasible due to contemporary high-Q cavity arrangements. Note, that for same damping in cavity $C_1$, the exhibitions in particular parameters are very similar, except for slowly decreasing of the visibility $R$.

V. CONCLUSION

In this paper, a feasible cavity QED experiment was proposed to analyse quantum decoherence nature using squeezing of fluctuations in the environment. The results of experiments can be formulated in terms directly measurable quantities and were analysed from a different view of quantum decoherence. Irrespective to different nature of decoherence process, distinction between particular preserving of visibility and pronounced quantum decoherence can be observed. The reason is that the quantum decoherence is rather connected with degree of entanglement than only with vanishing of visibility in particular measurement. Therefore the quantum coherence is vanishing irrespective to behavior in particular quadrature. It is connected with vanishing of nonclassical negative character of Wigner function in the system as nonclassicality of the state is transformed to entanglement between system and environment. On the other hand, the preserving of visibility is immediately connected with pronounced indistinguishability in complementary variable, which can be analysed using the complementarity principle in the experiments. Thus, the quantum mechanical complementarity and entanglement immediately exhibit in phase-sensitive decoherence process causing the different observable effects.

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VI. APPENDIX

The distinguishability parameter can be also defined in a different way as $D = \sqrt{1 - O^2}$ from measurable overlap $O$. Using this definition, it is fixed that $D^2 + O^2 = 1$ and relations between visibility $R_i$ and distinguishability $D_i$ can be found, if the total system evolves unitarily, in the following form similar to commonly used visibility-distinguishability relations

$$\frac{R_i^2}{N_i^2} + D_i^2 = 1, \quad N_S^2 N_E^2 = 1,$$

where $i = S, E$ and $N_i$ is a parameter of negativity of Wigner function. In the analysed cases, it is only matter of convenience which kind of distinguishability definition is used for the description.

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FIG. 1. The proposed experimental setup: O – oven, V – velocity selection, B – excitation box, R₁, R₂ – Ramsey zones, C₀ and C₁ – high-Q cavities, Dₑ, Dᵦ ionization detectors. The atoms are initially excited into upper Rydberg state by a laser. They cross the cavity between two Ramsey microwave fields R₁ and R₂. After that, an ionization detector determines the state of the atom. The cavity C₀ is coupled by the superconducting waveguide to cavity C₁, where the field quadrature squeezing is performed.

FIG. 2. Initial squeezed state - complementarity between visibility and distinguishability parameters $R_S$, $D_S$, $R_E$ and $D_E$ in course of rescaled time $G = κt$: (A) $γ_0 = 0$, (B) $γ_0 = 0.05$; $ξ(0) = 2$, $r = 2$, $γ₁ = 0$, $n₁ = n₀ = 0$.

FIG. 3. Initial squeezed state - decoherence-free case: Wigner function $W_{±S}(Q, P)$ and density matrix elements $p_{±S}(Q, Q')$ in coordinate representation for (a,c) quantum interference ($G = 0$) and (b,d) semiclassical interference exhibitions ($G = \frac{π}{4}$); $r = 2$, $ξ(0) = 2$, $γ₀ = γ₁ = 0$, $n₁ = n₀ = 0$.

FIG. 4. Initial squeezed state - decoherence-free case: (a) visibility $R$, (b) nonclassicality parameter $R_D$, (c) purity $P$ and (d) fidelity $F$ of internal cavity $C₀$ state in dependence on squeezing parameter $r$ in course of rescaled time $G = κt$; $ξ(0) = 2$, $γ₀ = γ₁ = 0$, $n₁ = n₀ = 0$.

FIG. 5. Initial squeezed state - decoherence in cavity $C₀$: (a) visibility $R$, (b) nonclassicality parameter $R_D$, (c) purity $P$ and (d) fidelity $F$ of internal cavity $C₀$ state in dependence on squeezing parameter $r$ in course of rescaled time $G = κt$; $ξ(0) = 2$, $γ₀ = 0.05 γ₁ = 0$, $n₁ = n₀ = 0$. 
