Quantum Max-min Ant Colony Algorithm For QoS Routing Problem

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Abstract. Aiming at the ant colony algorithm is weak to solve the problem of QoS routing, this paper proposes a routing algorithm that combines quantum computing with the max-min ant colony algorithm. The algorithm introduces a quantum selection strategy, uses qubit probability amplitude to encode the path, and dynamically adjusts the quantum rotation gate to update the path quantum coding on the basis of the minimum cost to meet the constraints and other optimal conditions, combined with the maximum and minimum pheromone determination mechanism to find the optimal path, effectively control the evolution speed and direction, and accelerate the algorithm convergence. Experimental results show that the algorithm is effective in controlling the speed and direction of evolution, and has better optimization ability and convergence performance than ant colony algorithm.

1. Introduction
With the rise of big data and cloud platforms and the continuous expansion of Internet network applications and scale, advanced multimedia services such as images and videos have been widely used in the earthquake, meteorology, transportation, and other departments. The big data of real-time streaming services have higher requirements for service quality (QoS) [1] of network parameters. How to choose the route with the best-cost under meeting the parameter constraints is the research content of the QoS routing problem.

In the QoS routing problem, constraint parameters [2] determine the solution's quality and accuracy. For routing NP problems with multiple index constraints, an intelligent bionic algorithm can be used to solve it. Many scholars have applied various intelligent bionic algorithms to the QoS routing problem's solution and achieved certain results [3-5]. As a commonly used bionic algorithm, the ant colony algorithm has many advantages. However, in solving the NP problem, there are many iterations, and the solution obtained is a local solution with high probability, so the convergence performance of the algorithm is low. Given these shortcomings, many scholars have proposed different optimization algorithms. For example, Document [6] adds genetic operators to the ant colony algorithm to solve the QoS anycast routing problem. Document [7] improves ants through pheromone optimization mechanism to make it suitable for solving the QoS unicast routing problem with bandwidth and delay constraints. For solving the large-scale QoS routing problem, the algorithm has more iterations and is easy to converge prematurely.

Quantum computing [8] arose in the 1980s, using the superposition, entanglement, interference, and other quantum states' characteristics to solve classical computing problems. Shor proposed a large number factorization algorithm based on quantum computing in 1994, which was implemented in polynomial time on quantum computers[9], making the NP problem evolve into a P problem. In 2002, Kuk-Hyun Han et al. proposed quantum evolutionary algorithms, which use quantum revolving doors
to complete evolutionary search. It has the characteristics of a small population size and does not affect the performance of the algorithm[10]. In recent years, quantum computing has attracted wide attention in the scientific and technological circles because of its unique computing performance.

This paper proposes a quantum max-min ant colony algorithm aiming at the shortcomings of the existing ant colony algorithm. It uses the state vector in quantum computing to encode the links in the network topology, updates the pheromone through the quantum rotation entanglement feature combined with the max-min test mechanism, and adds a quantum selection strategy to the roulette link selection, which can effectively control the evolution speed and direction and accelerate the convergence of the algorithm under the condition of little difference in fitness. According to the requirements of multi-constrained QoS, the network model is established and compared with the traditional ant colony algorithm from three aspects: global search ability, convergence performance, and the influence of network scale. The results show that the algorithm is superior to the traditional ant colony algorithm in three aspects, and it is feasible and effective for solving the NP problem of large-scale QoS routing.

2. QoS Routing Problem
The network diagram of the QoS routing problem is represented by $G(V, E)$, all network nodes in $G$ are represented by set $V$, the link set is represented by $E$, and there is at most one direct link between two nodes. The number of nodes in $G$ is $n$, $n = |V|$.

It is assumed that there is a routing task in the network $G$, with a starting node of $s \in V$, and $t \in V - \{S\}$ is the ending nodes to be reached. The key to solving the routing problem is to find a path $p(s, t)$ that satisfies the constraint conditions and optimize the cost of this path.

1. In this routing request, the delay from the starting node to the ending nodes must meet:

$$\sum_{e \in p(s,t)} \text{Delay}(e) + \sum_{v \in p(s,t)} \text{Delay}(v) \leq D$$

The time delay caused by transmitting information of link $e$ is represented by $\text{Delay}(e)$, and that by node $v$ is represented by $\text{Delay}(v)$, the link set in $P(s,t)$ is represented by $E_p$, $e \in E_p$, the node sets in $P(s,t)$ is represented by $V_p$, $v \in V_p$, and the current routing requires that the total time delay is not greater than $D$.

2. In order to ensure the smooth transmission of data, the bandwidth of the link $e$ passed through must meet the following requirements in the routing:

$$\min_{e \in p(s,t)} \left\{ \text{Bandwidth}(e) \right\} \geq B$$

In the formula, $\text{Bandwidth}(e)$ represents the bandwidth of link $e$ in link set $E_p$, and $B$ represents the lower limit of bandwidth in the routing.

3. In this routing request, the delay jitter from the starting node to the ending nodes must meet:

$$\sum_{v \in p(s,t)} \text{Delay}_\text{jitter}(v) \leq D_j$$

$\text{Delay}_\text{jitter}(v)$ represents the delay jitter when the data information passes through the node, and $D_j$ represents the delay jitter limit of the current routing request.

4. In this routing request, the packet loss rate from the starting node to the ending nodes must meet:
\[ 1 - \prod_{v \in P(s,r)} (1 - \text{Packet Loss}(v)) \leq P_l \]  

(4)

(5) The total cost calculation is equal to the sum of the costs from the starting node to all the ending nodes. Among the paths satisfying the above conditions, the total cost of Cost is the minimum, and the function of the cost of \( P(s,t) \) is defined as:

\[
\text{Cost}(P(s,t)) = \sum_{v \in P(s,t)} \text{Cost}(e) + \sum_{v \in P(s,t)} \text{Cost}(v)
\]

(5)

3. Quantum Max-Min Ant Colony Algorithm (QMMAS)

Multi-constrained QoS routing problem is to find the best path in a given network topology. In QMMAS, the preprocessing mechanism is adopted, and according to the constraint conditions, the network topology paths are screened to remove invalid paths. Establish the routing tables of the starting node and the ending node, which are used to temporarily store the solutions satisfying the constraint conditions in the operation process. The route between nodes is represented by \( \text{Rou}_{i,j} (i, j \in V) \), which requires each ant to start from the starting node and search for the next node according to the probability transfer formula defined in the QMMAS algorithm until it reaches the ending node.

3.1. Quantum Coding

A qubit can be represented by probability amplitude \( \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \), then the individual probability amplitude with \( n \) qubits can be represented as:

\[
\begin{bmatrix}
\alpha_1 \\
\beta_1 \\
\vdots \\
\alpha_n \\
\beta_n
\end{bmatrix}
\]

Where, \( \alpha_i \) and \( \beta_i \) satisfy \( |\alpha_i|^2 + |\beta_i|^2 = 1 \), \( i = 1, 2, \ldots, n \), and the quantum individual can represent any quantum superposition state.

In QMMAS, quantum bits are used to represent the information coding on each link, and the quantum information coding on the path can be represented as:

\[
\text{Qbit} = \begin{bmatrix}
\alpha_{i,j} \\
\beta_{i,j} \\
\vdots \\
\alpha_{n,n} \\
\beta_{n,n}
\end{bmatrix}
\]

(6)

The number of nodes in the network topology is \( n \), the quantum information on the link between node \( i \) and node \( j \) is represented as \( \begin{bmatrix} \alpha_{i,j} \\ \beta_{i,j} \end{bmatrix} \), and when \( i \neq j \), \( |\alpha_{i,j}|^2 + |\beta_{i,j}|^2 = 1 \); When \( i = j \), \( |\alpha_{i,j}|^2 = |\beta_{i,j}|^2 = 0 \) (1 \( \leq i, j \leq n \)). When the optimal ant passes through node \( i \) to node \( j \) directly, the value of quantum information \( \beta_{i,j} \) on the link between node \( i \) and node \( j \) increases, and the link's tendency to be selected increases. On the contrary, it decreases, and the value of quantum information \( \beta_{i,j} \) on the link decreases. See Section 3.4 for quantum information update rules.

The binary code of each link is obtained by measuring the population, which can be represented as:
\[
\begin{bmatrix}
  b_{11} & b_{12} & \cdots & b_{1n} \\
  b_{21} & b_{22} & \cdots & b_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  b_{m1} & b_{m2} & \cdots & b_{mn}
\end{bmatrix}
\]  

(7)

The measurement criteria are represented by the following pseudocodes:

Begin (start of measurement)

Enter the random number \( rd \) between \( \alpha_j \) and [0, 1]

IF \( rd > \alpha_j^2 \), then \( b_j = 1 \)

Otherwise \( b_j = 0 \)

Print \( b_j \)

End (end of measurement)

3.2. Probabilistic Decision Rule

It is assumed that \( m \) is the number of ants, \( n \) is the number of network nodes, the cost between node \( i \) and node \( j \) is \( C_{ij} \), the pheromone on link \((i, j)\) is \( \tau_{ij} \), and the probability of ants reaching node \( j \) through node \( i \) is \( P_{ij}^k \).

\[
P_{ij}^k = \begin{cases} 
\frac{\tau_{ij}^\alpha \eta_{ij}^\beta}{\sum_{u \in N_i^k} \tau_{iu}^\alpha \eta_{iu}^\beta} & j \in N_i^k \\
0 & j \notin N_i^k 
\end{cases}
\]

(8)

The set of nodes that ant \( k \) can choose when it is at node \( i \) is represented by \( N_i^k \). \( \alpha' \) represents the weight of pheromone accumulation degree, heuristic factor is represented by \( \eta_{ij} \), and \( \beta' \) represents the weight of heuristic factor.

When \( \alpha' = 0 \), the probability transfer formula degenerates into a greedy search. When \( \beta' = 0 \), the next network node's attraction is ignored, and the efficiency of exploration will decrease. The heuristic information \( \eta_{ij} \) enhances the tendency to attractive solutions, which is represented by formula (9).

\[
\eta_{ij} = 1/C_{ij}
\]

(9)

When ants go directly to node \( j \) through node \( i \), pheromones on the link will accumulate, and quantum information \( \beta_{ij} \) will increase. On the contrary, it will decrease. See section 3.3 for the update method of pheromones.

3.3. Pheromone Update Rule

After all the ants reach the set endpoint, the pheromones on each link are needed to update. The pheromones on the link are reduced by volatilization. Then, the corresponding pheromones are increased according to the link through which the optimal ants pass. The volatilization amount of pheromone is executed according to the following formula:

\[
\tau_{ij} = (1 - \rho) \tau_{ij}, \forall (i, j) \in E
\]

(10)

The volatilization coefficient of pheromones is represented by \( \rho \), \( 0 < \rho \leq 1 \), which can effectively control the accumulation of pheromones on the link. The ant with the least routing cost is selected to update the global pheromone.
\[ \tau_y = \tau_y + \Delta \tau_y, \forall (i, j) \in E \tag{11} \]

Where, \( \Delta \tau_y \) is the number of pheromones released by the ant looking for the optimal path to the link \((i, j)\) through that it passes. \( \Delta \tau_y \) is defined as:

\[
\Delta \tau_y = \left\{ \begin{array}{ll}
\left( |\beta_y^i| \right)^{1/C_y}, & \text{if } (i, j) \in P_{best} (s,t) \ Down\\
0 & \text{else}
\end{array} \right.
\tag{12}
\]

Where, \( \beta_y^i \) represents the quantum pheromone strength on the link from \( i \) to \( j \), \( |\alpha_y^i|^2 + |\beta_y^i|^2 = 1 \); \( \gamma \) is the weight of quantum information \( \beta_y^i \) on the link between node \( i \) and node \( j \). After the pheromone update is completed, the pheromone test is carried out. If it is greater than the maximum value \( \tau_{max} \), make it equal to \( \tau_{max} \); if it is less than the minimum value \( \tau_{min} \), make it equal to \( \tau_{min} \). \( \tau_{max} \) and \( \tau_{min} \) are calculated by formulas (13) and (14).

\[
\tau_{max} = \frac{1}{p \cdot L_{best}} \tag{13}
\]

\[
\tau_{min} = \frac{Q \cdot \tau_{max} (1 - P_{dec})}{(\text{avg} - 1) P_{dec}} = \frac{Q \cdot \tau_{max} (1 - \sqrt[4]{P_{best}})}{(n-2) \sqrt[4]{P_{best}}} \tag{14}
\]

Where, the cost of the path \( P(s,t) \) constructed by the optimal ant is \( L_{best} \), the relative coefficient of pheromone increase is \( Q \), and the optimal probability of the path found by the ant in a single search is \( P_{best} \); \( P_{dec} \) is the probability of selecting the optimal solution; \( \text{avg} \) is the number of optional paths. When the algorithm converges, the probability \( P_{best} \) of the optimal solution is a value greater than 0.

### 3.4. Quantum Update Rule

It is assumed that there are \( m \) ants, the matrix \( R \) of \( n \times n \) is a solution path from the starting node to the ending node of the solution in a network of \( n \) nodes. \( R[i, j] = 1 \) means an edge from node \( i \) to node \( j \) in the path \( R \), and when \( i = j \), there must be \( R[i, j] = 0 \). The optimal solution obtained by the previous generation of ants is expressed by \( R \), \( R_{best} \) represents the optimal solution obtained by the current generation of ants. The quantum revolving door updates the quantum information in the network link through Equation (15):

\[
\begin{pmatrix}
\alpha_y^{i+1} \\
\beta_y^{i+1}
\end{pmatrix} =
\begin{pmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{pmatrix}
\begin{pmatrix}
\alpha_y^i \\
\beta_y^i
\end{pmatrix}
\tag{15}
\]

In the formula, \( i, j = 1, 2, 3 \cdots n \), \( (\alpha_y^i, \beta_y^i)^T \) is the quantum coding information on the link between node \( i \) and node \( j \) in the \( t \) iteration, and the quantum rotation angle of the link \( i \) to \( j \) is represented by \( \theta \), and its value is obtained by looking up Table.

| \( \theta[i,j] \) | \( \alpha_{y[i,j]} \) | \( \beta_{y[i,j]} \) | \( \Delta \theta_g \) | \( s(\alpha_g, \beta_g) \) |
|------------------|------------------|------------------|------------------|------------------|
|                  |                  |                  |                  |                  |
| 0                | 0                | false            | 0                | 0                |
|                  | \( \alpha_g > 0 \)                          | \( \alpha_g < 0 \)                          | \( \alpha_g = 0 \)                          | \( \beta_g = 0 \)                          |

Table 1 Rotation Angle Strategy
Where,  \( f(x) \) is an objective function, which is the cost to complete the routing solution for ants in this paper;  \( s(\alpha_y, \beta_y) \) represents the direction of rotation angle offset, which is used to control the progress and direction of convergence of the algorithm.

### 3.5. QMMAS Algorithm Steps

**Step 1 Initialization:** Randomly generate a network topology with parameters, give constraints of various parameters in QoS routing, preprocess the randomly generated network topology, and set link connection nodes that do not meet bandwidth to be unreachable to each other.

**Step 2** Set the value of each parameter \( \alpha, \beta, \rho, P_{best}, Q, \gamma \), with the number of ants of \( m \), the upper limit of iterations of NMAX, the starting node of \( s \), the ending point of \( t \). All \( \alpha_y, \beta_y \) values in the ant quantum pheromone coding are \( \frac{1}{\sqrt{2}} \).

**Step 3** Place an ant at the starting point, initialize the taboo table, total cost, total delay, total delay jitter, and packet loss rate. Add the starting node to the taboo table, construct the optional node set according to the taboo table and node and link parameter information. Use a greedy algorithm to construct the solution \( P(s,t) \) independently according to the ending node \( t \) and QoS constraint conditions. Use formula (13) to obtain the \( \tau_{\text{max}} \) and initialize the pheromone \( \tau = \tau_{\text{max}} \) in the link. The current iteration number is \( C \).

**Step 4** Place \( m \) ants at the starting point, initialize the taboo table, total cost, total delay, total delay jitter, and packet loss rate. Add the starting node to the taboo table, construct the optional node set according to the taboo table and node and link parameter information.

**Step 5** According to the optional node set, establish the probability distribution according to formula (8), and uses the "roulette selection method" to determine the next node \( nc \). If the binary coding of the link between the current node and the \( nc \) is 1, remove the \( nc \) from the optional node set. Otherwise, determine the next node \( nc \) by the "roulette selection method" and remove \( nc \) from the optional node set. Update the optional node set, path cost accumulation, delay accumulation, delay jitter accumulation, packet loss rate accumulation, if the condition is met, modify the current node to \( nc \). If the ant reaches the ending node \( t \) or the search is in a dormant state, then jump out of search, otherwise repeat the above steps.

**Step 6** If all \( m \) ants complete the solution, update the quantum coding information on the network link with Equation (15), and turn to Step 7; otherwise, turn to Step 5.

**Step 7** Update the pheromones on the link with Equations (10) to (12). Check and update the pheromones on the link with Equations (13) and (14).

**Step 8** If \( \text{count} \geq \text{NMAX} \), turn to step 9, otherwise \( \text{count} = \text{count} + 1 \), turn to step 4.

**Step 9** According to the ant's path from the starting node to the ending nodes, construct the optimal solution, and output it.

### 4. Simulation Experiment and Analysis

The network topology of this paper is randomly generated according to the Salama model. In order to make the topology diagram data clear, only some parameters of the node are displayed. Set the QoS
delay constraint of $D = 95$, the bandwidth in the link of $B = 70$, the delay jitter of $D_j = 1000$, the packet loss rate of $P_l = 1000(10e^{-5})$, and the other parameters of $\alpha' = 1$, $\beta' = 2$, $\rho = 0.02$, $\gamma = 2$, $P_{best} = 0.05$, $Q = 5$, $m = 50$, and the Maximum Generation of $N_{MAX} = 100$. The experiments are carried out from three aspects: the global searchability, the convergence performance, and the network size's influence on the algorithm.

4.1. Global Searchability

The randomly generated network topology by selecting 50 network nodes is shown in Figure 1. QMMAS algorithm was used to carry out 50 separate experiments on each routing request, and the optimal solution found in the 50 experiments, the required cost, and the corresponding delay and bandwidth were recorded.

![Figure 1: Randomly Generated Network Topology Diagram](image1)

Figure 1: Randomly Generated Network Topology Diagram

![Figure 2 (1, 50) Best Path](image2)

Figure 2 (1, 50) Best Path

Figure 2 is the best path generated by solving the routing request (1,50) in the network topology diagram of Figure 1 after executing the algorithm in this paper. The conclusions obtained by several independent experiments were consistent with the result in Figure 2.

| Routing request (Starting node, ending node) | Path Selection | Cost | Delay | Minimum Bandwidth |
|---------------------------------------------|----------------|------|-------|-------------------|
| (1,50)                                      | 1-18-31-42-41-50 | 32   | 92    | 64                |
| (2,46)                                      | 2-12-20-31-42-46 | 31   | 76    | 61                |
| (3,48)                                      | 3-16-28-38-49-48 | 22   | 76    | 66                |
| (4,39)                                      | 4-19-20-7-22-30-39 | 35   | 95    | 65                |
| (5,43)                                      | 5-16-28-38-49-48-43 | 29   | 93    | 66                |
Taking different starting nodes and ending nodes to calculate the algorithm proposed in this paper, the simulation results are shown in Table 2. It can be seen from the above that the algorithm proposed in this paper can effectively solve the QoS routing problem under constraints in the global scope.

4.2. Convergence Performance
The random topology of 100 network nodes is selected to verify the convergence performance of the algorithm proposed in this paper. The constraint conditions and algorithm parameters are initially set, and the obtained results are compared with the traditional ant colony algorithm. Experiments are carried out for routing requests (3,99). The iterative solution of the two algorithms is shown in Figure 3.

![Figure 3 Comparison of Convergence Performance](image)

The above experimental results show that the optimal path cost and convergence time obtained by QMMAS are better than those of the traditional ant colony algorithm. The ant colony algorithm converges faster in the early stage, slows down in the later stage, does not converge after 72 generations, and the obtained solution is an effective solution in the local range. The algorithm proposed in this paper effectively controls the direction of optimization through quantum rotation has a fast early convergence speed, and has a good effect on solving multi-constraint QoS routing problems globally.

4.3. Influence of Network Size on Algorithm
To study the influence of network scale on the algorithm, in the experiment, the constraints and algorithm parameters were initially set. The initial number of network nodes was 15, which increased by 10 in turn, and the upper limit of the number of nodes was set to 100. Simultaneously, QMMAS and traditional ant colony algorithm were used to solve and compare, and the results are shown in Figure 4.
As can be seen from the figure, when the network node size is small (the number of nodes is below 25), the traditional ant colony algorithm and QMMAS algorithm's solution effect is flush. With the increase of network nodes, the cost of the solution obtained by the QMMAS algorithm is always better than that of the traditional ant colony algorithm.

5. Conclusions
In this paper, a quantum max-min ant colony algorithm is proposed to solve the QoS routing problem by combining quantum computation with a max-min ant colony algorithm. The state vector in quantum computation is used to encode the links in the network topology, and the pheromone is updated through the entanglement of quantum rotation combined with the max-min test mechanism, which effectively controls the evolution speed and direction. Simulation results show that the algorithm can effectively control the evolution speed and direction, and is superior to the traditional ant colony algorithm in terms of optimization ability and convergence performance, and can effectively and accurately solve the QoS routing problem under the constraint conditions.

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