V,W AND X IN TECHNICOLOUR MODELS

by

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Abstract

Light techni-fermions and pseudo Goldstone bosons that contribute to the electroweak radiative correction parameters V,W and X may relax the constraints on technicolour models from the experimental values of the parameters S and T. Order of magnitude estimates of the contributions to V,W and X from light techni-leptons are made when the the techni-neutrino has a small Dirac mass or a large Majorana mass. The contributions to V,W and X from pseudo Goldstone bosons are calculated in a gauged chiral Lagrangian. Estimates of V,W and X in one family technicolour models suggest that the upper bounds on S and T should be relaxed by between 0.1 and 1 depending upon the precise particle spectrum.
1 Introduction

Precision measurements of electroweak parameters are now sufficiently accurate to test the
effects of radiative corrections. Heavy fermions do not decouple from these radiative
effects in broken gauge theories and hence these measurements provide a probe of new
physics, such as technicolour, at mass scales greater than $M_Z/2$. Peskin et al. have
developed an elegant description of the effects of such new physics when the new particles
have large masses of order $1 TeV$ in terms of the three parameters $S$, $T$ and $U$

$$\alpha S = 4e^2 \frac{d}{dq^2} [\delta \Pi_{33} - \delta \Pi_{3Q}]|_0$$

$$\alpha T = \frac{e^2}{s_W c_W M_Z^2} [\delta \Pi_{11}(0) - \delta \Pi_{33}(0)]$$

$$\alpha U = 4e^2 \frac{d}{dq^2} [\delta \Pi_{11} - \delta \Pi_{33}]|_0$$

where: $\delta \Pi_{ab}$ are the contributions to the standard electroweak theory gauge bosons’ vacuum
polarizations, given by $\Pi_{ab}^{\mu\nu}(q^2) = \Pi_{ab}(q^2) g^{\mu\nu} + (q^\mu q^\nu$ terms), from the new heavy particles;
$s_W$ and $c_W$ are the sine and cosine of the weak mixing angle; $q$ is the gauge boson momenta.
A global fit to current experimental data provides the upper bounds $S \leq 0.5$ and $T \leq 0.6$
at the 95% confidence limit. A naive estimate of the contributions to $S$ and $T$ from a
technicolour model of electroweak symmetry breaking can be made by scaling up QCD or in chiral models of the strong interactions. These custodial SU(2) preserving estimates give $S \sim 0.1 N_{TC}/doublet$ and $T \sim 0$ where $N_{TC}$ is the number of technicolours.
All but the most minimal technicolour models are ruled out by this estimate of $S$. However,
estimates of $S$ and $T$ in one family technicolour models with custodial isospin breaking in
the technilepton sector, which manifests as either Dirac mass splittings between the techni-electron and techni-neutrino or as a Majorana mass for the right handed techni-neutrino, suggest that models can be built with both $S$ and $T \leq 1$.

Recently the $S$, $T$ and $U$ formalism has been extended to incorporate the effects of new
particles with masses $O(M_Z)$. In the orginal formalism the new fermions were assumed
to have large masses so that their self energies could be described well by a Taylor expansion

to linear order:
\[ \delta \Pi_{ab} \approx A_{ab} + B_{ab} q^2. \]
Errors of order \((M_Z/M_{new})^2\) were neglected. Burgess et al. [7] have shown that this is equivalent to assuming that three parameters vanish, V, W and X:

\[
\begin{align*}
\alpha V &= \left. \frac{\partial^2 \Pi_{ZZ}(M_Z^2)}{\partial q^2} \right|_{q^2 = 0} \delta \Pi'_{ZZ}(M_Z^2) - \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \\
\alpha W &= \left. \frac{\partial^2 \Pi_{WW}(M_W^2)}{\partial q^2} \right|_{q^2 = 0} \delta \Pi'_{WW}(M_W^2) - \frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2} \\
\alpha X &= -s_W c_W \left[ \frac{\delta \Pi_{ZA}(M_Z^2)}{M_Z^2} - \Pi'_{ZA}(0) \right]
\end{align*}
\]

where the prime indicates differentiation with respect to \(q^2\). If these parameters are non-zero then the S, T and U parameters are given by the more general forms:

\[
\begin{align*}
\alpha S &= 4s_W^2 c_W^2 \left[ \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \right] - 4s_W c_W (c_W^2 - s_W^2) \delta \Pi'_{ZA}(0) - 4s_W^2 c_W^2 \delta \Pi_{ZA}(0) \\
\alpha T &= \frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2} \\
\alpha U &= 4s_W^2 \left[ \frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2} \right] - 4s_W^2 c_W \left[ \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \right] \\
&\quad - 4s_W^4 \delta \Pi'_{AA}(0) - 8s_W^3 c_W \delta \Pi'_{ZA}(0)
\end{align*}
\]

A global fit to the experimental data in which all six parameters, S, T, U, V, W and X, are allowed to vary simultaneously gives the one standard deviation bounds [8]

\[
\begin{align*}
S &\sim -0.93 \pm 1.7 & V &\sim 0.47 \pm 1.0 \\
T &\sim -0.67 \pm 0.92 & W &\sim 1.2 \pm 7.0 \\
U &\sim -0.60 \pm 1.1 & X &\sim 0.1 \pm 0.58
\end{align*}
\]

The inclusion of V, W and X weakens the bounds on S, T and U considerably. This analysis raises the possibility that a technicolour model with new light particles with masses \(\mathcal{O}(M_Z)\) may be experimentally viable. There are two possible sources of such light particles, light
techni-fermions and the pseudo-Goldstone bosons (PGB) that occur in many technicolour models with large global symmetries. An order of magnitude estimate of the contributions to $V, W,$ and $X$ from light techni-fermions can be obtained by calculating the one loop contribution from a weakly interacting doublet \cite{7}. This naive estimate will be sufficient to show whether $V, W$ and $X$ are large enough relative to $S$ and $T$ to violate the assumption of the original global fit \cite{3} that $V, W$ and $X$ are all $\sim 0$. In section 2 of this paper we review this calculation and extend it to find the contributions to $V, W,$ and $X$ from a techni-lepton doublet with a right handed Majorana mass. The PGB contributions to $S$ and $T$ have been calculated before \cite{9, 5}. In section 3 we review the construction of a Chiral Lagrangian of the interactions of the PGBs below the technicolour confinement scale and present expressions for their contributions to $S, T, U, V, W,$ and $X$. Finally we calculate $V, W$ and $X$ in one family technicolour models with realistic mass spectra (section 4).

2 Light Techni-Fermions

We consider a technicolour model with $N$ left handed Weyl technifermions $\psi^i_L$ and $N$ right handed Weyl technifermions $\psi^i_R$ that transform either under a real or complex representation of a technicolour group, $G$, such as those in ref \cite{10}. In addition to their interactions under the technicolour group and the usual electroweak interactions the techni-fermions may have extended technicolour interactions from physics above the scale $\Lambda_{TC}$ which can be represented by 4-Fermi operators. These extended technicolour interactions may be strong at the technicolour confinement scale \cite{11}. When the technicolour group becomes strongly interacting at the scale $\Lambda_{TC}$ the technifermions’ approximate chiral symmetry is dynamically broken. If the technifermions transform under a complex representation of $G$ then the maximal global symmetry breaking pattern is given by

\[
SU(N)_L \otimes SU(N)_R \rightarrow SU(N)_V
\]  

(2.1)
and if they transform under a real representation of $G$ by

$$SU(2N) \rightarrow O(2N)$$ (2.2)

If the extended technicolour interactions are sufficiently strong to distinguish different techni-fermions at the scale $\Lambda_{TC}$ then the approximate global symmetries will be some sub-group of these groups.

The techni-fermions acquire dynamical masses of order $\Lambda_{TC}$; in practice the precise values of the techni-fermion masses will be dependent on the strengths of the extended technicolour interactions. These techni-fermions can potentially be light (relative to $M_Z$) if the scale $\Lambda_{TC}$ is low ($\sim M_Z$) as in, for example, one family technicolour models with separate technicolour interactions on the techni-quarks and techni-leptons where one or other sector dominates electroweak symmetry breaking, or models with other low scale, strongly interacting fermions in addition to the electroweak symmetry breaking interactions [18]. The techni-fermions must be integrated from the effective theory of the technicolour interactions at current collider energies. We are interested in the contribution to gauge boson self energies (and hence $V,W,$ and $X$) from strongly interacting loops of these techni-fermions. Such a non-perturbative calculation is beyond current technology especially when the techni-fermions are light and custodial isospin is broken by the extended technicolour interactions. We shall naively estimate the order of magnitude of these contributions by calculating the one-loop perturbative contributions to $V,W$ and $X$ of a weakly interacting fermion doublet with momentum independent mass.

We shall express our results in terms of the well known one-loop results for an incoming gauge boson coupling to left handed fermions and an outgoing gauge boson coupling to left and right handed fermions.
\[
\Pi_{LL}(m_1, m_2, q^2) = -\frac{1}{4\pi^2} \int_0^1 \! dx \left( x(1 - x)q^2 - \frac{m_1^2}{2} \right) \ln \left( \frac{\Lambda^2}{m^2-x(1-x)q^2} \right)
\]
\[
\Pi_{LR}(m_1, m_2, q^2) = -\frac{m_1 m_2}{8\pi^2} \int_0^1 \! dx \ln \left( \frac{\Lambda^2}{m^2-x(1-x)q^2} \right)
\]

where \(m^2 = xm_1^2 + (1 - x)m_2^2\).

The contributions to the gauge boson self energies from a fermion doublet, \(\psi = (\psi_+, \psi_-)\), are thus given by the standard electroweak couplings and we find

\[
\delta \Pi_{AA} = \sum_{a=\pm} 2N_{TC} e^2 q_a^2 \left[ \Pi_{LL}(m_a, m_a, q^2) + \Pi_{LR}(m_a, m_a, q^2) \right]
\]

\[
\delta \Pi_{ZA} = \sum_{a=\pm} \frac{e^2 N_{TC}}{s_W c_W} q_a (T_{3a} - 2 s_W^2 Q_a) \left[ \Pi_{LL}(m_a, m_a, q^2) + \Pi_{LR}(m_a, m_a, q^2) \right]
\]

\[
\delta \Pi_{WW} = \frac{e^2 N_{TC}}{2 s_W^2} \Pi_{LL}(m_+, m_-, q^2)
\]

\[
\delta \Pi_{ZZ} = \sum_{a=\pm} \frac{e^2 N_{TC}}{s_W^2 c_W^2} \left[ (T_{3a}^2 - 2 s_W^2 T_{3a} Q_a + 2 s_W^4 Q_a^2) \Pi_{LL}(m_a, m_a, q^2) \right.
\]
\[
-2 s_W^2 Q_a (T_{3a} - s_W^2 Q_a) \Pi_{LR}(m_a, m_a, q^2) \left. \right]
\]

\(V, W\) and \(X\) are given in terms of these contributions to the gauge boson self energies by Eqn(1.2). As an example of the contributions to \(V,W\) and \(X\) from light techni-fermions \((m_\psi \sim M_Z/2)\) we plot the contribution of a techni-lepton doublet (we take \(N_{TC} = 1\)) with \(m_\nu = 50 GeV\) in Fig 1. The maximal values of \(V,W\) and \(X\) result when both the techni-neutrino and the techni-electron are light and are given by \(V \sim -0.15 N_{TC}, W \sim -0.1 N_{TC}\) and \(X \sim 0\).

We may similarly estimate the contributions to \(V,W\) and \(X\) from a techni-lepton doublet, \(\Psi_L = (N, E)_L, E_R, N_R\) with a right handed Majorana neutrino mass. Writing the left and right handed degrees of freedom of \(N\) as two Majorana (self conjugate, \(\psi^M = C(\bar{\psi}^M)^T\)) fields \(N_1^0\) and \(N_2^0\) all possible gauge invariant mass terms are then given by \([12]\)
\[
\mathcal{L}_{\text{mass}} = -\frac{1}{2} (\bar{N}_0^0 \bar{N}_0^{0C}) \left( \begin{array}{cc} 0 & m_D \\ m_D & -M \end{array} \right) \left( \begin{array}{c} N_1^{0C} \\ N_0^0 \end{array} \right) \tag{2.9}
\]

where \( m_D \) is the Dirac mass and \( M \) is the Majorana mass. The Majorana mass eigenstate fields, \( N_1 \) and \( N_2 \) with masses \( M_1 \) and \( M_2 \) are given by

\[
\left( \begin{array}{c} N_1^0 \\ N_2^{0C} \end{array} \right) = \left( \begin{array}{cc} ic_\theta \gamma_5 & s_\theta \\ -is_\theta \gamma_5 & c_\theta \end{array} \right) \left( \begin{array}{c} N_1 \\ N_2 \end{array} \right) \tag{2.10}
\]

where

\[
c_\theta^2 = \frac{M_2}{M_1 + M_2}, \quad s_\theta^2 = \frac{M_1}{M_1 + M_2} \tag{2.11}
\]

and

\[
M = M_2 - M_1, \quad m_D = \sqrt{M_1 M_2} \tag{2.12}
\]

and where we have made an axial \( U(1) \) transformation on the \( N_1 \) field. Using the self conjugacy properties of the Majorana fields we may rewrite the charged and neutral weak currents in terms of the mass eigenstate fields \([13]\) and obtain

\[
\bar{N}_L \gamma^\mu E_L = -ic_\theta \bar{N}_{1L} \gamma^\mu E_L + s_\theta \bar{N}_{2L} \gamma^\mu E_L \tag{2.13}
\]

\[
\bar{N}_L \gamma^\mu N_L = -c_\theta^2 \bar{N}_{1L} \gamma^\mu \gamma_5 N_L - s_\theta^2 \bar{N}_{2L} \gamma^\mu \gamma_5 N_2 + is_\theta c_\theta \bar{N}_{2L} \gamma^\mu N_1 \tag{2.14}
\]

We thus obtain expressions for the gauge boson self energies (remembering to include the additional non-zero Wick contraction \( < \psi^M \psi^M > \) \([13]\)). The neutrino only contributes to \( \delta \Pi_{ZZ} \) and \( \delta \Pi_{WW} \) and we find
\[ \delta \Pi_{WW} = \frac{e^2 N_{TC}}{2 s_W^2} \left[ c_\theta^2 \Pi_{LL}(M_1, m_e, q^2) + s_\theta^2 \Pi_{LR}(M_2, m_e, q^2) \right] \] (2.15)

\[ \delta \Pi_{ZZ} = \frac{e^2 N_{TC}}{4 s_W^2 c_W^2} \left[ c_\theta^4 \Pi_{LL}(M_1, M_1, q^2) + \Pi_{LR}(M_1, M_1, q^2) \right.
\quad + s_\theta^4 \left( \Pi_{LL}(M_2, M_2, q^2) - \Pi_{LR}(M_2, M_2, q^2) \right)
\quad + 2 s_\theta^2 c_\theta^2 \Pi_{LL}(M_1, M_2, q^2) + \Pi_{LR}(M_1, M_2, q^2)
\quad + (1 - 4 s_W^2 + 8 s_W^4) \Pi_{LL}(m_E, m_E, q^2)
\quad + (2 s_W^2 + 4 s_W^4) \Pi_{LR}(m_E, m_E, q^2) \right] \] (2.16)

Only V and W, which are given in terms of Eqn(2.15) and Eqn(2.16) by Eqn(1.2), are changed by the inclusion of the Majorana mass. As an example of the effects of including the Majorana mass on S,T,V and W these quantities are plotted in Fig 2 for a Dirac mass degenerate techni-lepton doublet \((m_D = 200 GeV, N_{TC} = 1)\) for varying Majorana mass. The regions of negative S and T have been reported before \([14]\). Both V and W are slightly enhanced by the inclusion of a Majorana mass. The maximal realistic values of V and W (corresponding to a lightest mass eigenstate \(> M_Z/2\)) are of order \(-0.04 N_{TC}\) and \(-0.01 N_{TC}\) respectively.

3 Pseudo Goldstone Bosons

When the technicolour group becomes strongly interacting at the scale \(\Lambda_{TC}\) it breaks the approximate global symmetry of the techni-fermions according to the symmetry breaking pattern in Eqn(2.1), Eqn(2.2) or some appropriate sub-group of these patterns. There will be a Goldstone boson (GB) associated with each of the broken generators. These bound states remain in the effective theory below the scale at which the techni-fermions have been integrated out. The low energy interactions of these GBs may be described by a gauged Chiral Lagrangian \([15]\). In the notation of Peskin \([13, 16]\) we write the techni-fermion fields
as

\[ \Psi_L = \begin{pmatrix} \psi^i_L \\ \psi^i_C \end{pmatrix}, \quad \Psi_R = \Psi_L^C \] (3.1)

where \( \psi^C = -i\sigma_2\psi^* \). The global symmetry generators can then be written as \( 2N \times 2N \) matrices \( L \) acting on \( \Psi_L \) and \( R \) acting on \( \Psi_R \). The construction

\[ U = \exp \left( \frac{2i\pi^a X^a}{f_\pi} \right) \] (3.2)

where \( \pi^a \) are the GBs, \( X^a \) the \( 2N \times 2N \) broken generators of the global symmetry group of \( \Psi_L \) and \( \Psi_R \), and \( f_\pi \) the pion decay constant, transforms under the global symmetry transformations as

\[ U \longrightarrow LUR^\dagger \] (3.3)

The effective theory of the GBs is then constructed from all chirally invariant terms. At low energies we may perform an expansion in \( q/f_\pi \) and keep only terms of lowest order in the covariant derivative. The first non trivial term in the expansion is given by

\[ \mathcal{L}_\chi = \frac{f_\pi}{4} \text{tr} \left[ (D_\mu U)^\dagger D^\mu U \right] \] (3.4)

with

\[ D^\mu U = \partial^\mu U + ig_L A^\mu_a G^a_L U - ig_R A^\mu_a U G^a_R \] (3.5)

where we have written the gauge generators in terms of components that act on \( \Psi_L, G^a_L \), and those that act on \( \Psi_R, G^a_R \). We define \( G^a_L \) and \( G^a_R \) to contain the gauge couplings.

Footnote 1: We note that an \( SU(N)_L \) global transformation on the techni-fermion fields will have both components L and R since \( \Psi_L \) and \( \Psi_R \) both contain the left handed degrees of freedom.
Gauge boson-PGB vertices may be obtained by explicitly expanding the exponential; for example we obtain

\[
\Gamma_{\pi^a \pi^b A^\mu c} = i(k + q)^\mu tr \left\{ X^a [G^c_L, X^b] + X^a [G^c_R, X^b] \right\}
\]

\[
\Gamma_{\pi^a \pi^b A^\mu c A^\nu d} = tr \left\{ X^a [G^c_L, [G^d_R, X^b]] + X^a [G^d_R, [G^c_L, X^b]] \right\} + c \leftrightarrow d
\]

In a realistic technicolour model the techni-fermions will decompose into \(n SU(2)_L\) doublets and \(2n\) right handed singlets. The standard model electroweak gauge generators may then be split into components L and R

\[
\begin{array}{ccc}
\text{GaugeBoson} & \text{L} & \text{R} \\
A^\mu & \left( \begin{array}{cc} eQ & 0 \\ 0 & -eQ \end{array} \right) & \left( \begin{array}{cc} -eQ & 0 \\ 0 & eQ \end{array} \right) \\
Z^\mu & \frac{e}{s_W c_W} \left( \begin{array}{cc} \tau_3 \otimes \mathbb{I}_n - s_W^2 Q \\ 0 \end{array} \right) & \frac{e}{s_W c_W} \left( \begin{array}{cc} -\tau_3 \otimes \mathbb{I}_n + s_W^2 Q \\ 0 \end{array} \right) \\
W^\pm_\mu & g \left( \begin{array}{cc} \tau_\pm \otimes \mathbb{I}_n \\ 0 \end{array} \right) & g \left( \begin{array}{cc} -\tau_\pm \otimes \mathbb{I}_n \\ 0 \end{array} \right)
\end{array}
\]

(3.7)

where \(Q\) is the \(2n \times 2n\) charge matrix of the Dirac fermions \(\psi\) and \(\mathbb{I}_n\) is the \(n \times n\) unit matrix. Inserting these matrices into Eqn(3.6) we obtain the vertices

\[
\Gamma_{\pi^a \pi^b W^{\pm}_\mu W^{\pm}_\nu} = 0 \quad \Gamma_{\pi^a \pi^a Z^{\pm}_\mu} = \frac{ie}{2s_W} I_{3a} (p + p')^\mu
\]

\[
\Gamma_{\pi^a \pi^a Z^{\mu}_\nu} = \frac{2s_W^2}{c_W} (Q^a_s s_W^2 - Q_a I_{3a}) \quad \Gamma_{\pi^a \pi^a Z^a} = \frac{ie}{2s_W c_W} (I_{3a} - 2s_W^2 Q_a) (p + p')^\mu
\]

\[
\Gamma_{\pi^a \pi^a A^\mu A^\nu} = 2Q^2_a c^2 \quad \Gamma_{\pi^a \pi^a A^\mu} = i e Q_a (p + p')^\mu
\]

\[
\Gamma_{\pi^a \pi^a A^\mu Z^\nu} = \frac{e^2}{s_W c_W} (I_{3a} Q_a - 2s_W Q^2_a)
\]

(3.8)

where \(I_3\) is the custodial \(SU(2)\) isospin quantum number of the GB.
Three of the GBs associated with the breaking of the gauged electroweak symmetry groups will be strictly massless and are “eaten” by the electroweak gauge bosons. These unphysical states may be removed by working in Unitary gauge. The remaining Goldstone modes only correspond to approximate global symmetries and hence will acquire small masses ($\ll \Lambda_{TC}$) through their weak gauge and extended technicolour interactions. Estimates of the masses of these Pseudo Goldstone Bosons (PGB) have been made in one family technicolour models (see for example Ref [16]). We shall simply introduce explicit mass terms into the Lagrangian for these fields.

The PGBs contribute to gauge boson self energies through the loops [9, 17]

\[
\text{Fig 4a} \sim \frac{g_{\alpha\beta}}{16\pi^2} \left[ (C_{UV} + 1)(m_a^2 + m_b^2 - \frac{q^2}{3}) - 2F(m_a, m_b, q^2) \right] + q_\alpha q_\beta \text{ terms}
\]

\[
\text{Fig 4b} \sim \frac{g_{\alpha\beta}}{16\pi^2} \left[ F(m_a, m_a, 0) - (C_{UV} + 1)m_a^2 \right] + q_\alpha q_\beta \text{ terms}
\]

where

\[
F(m_a, m_b, q^2) = \int_0^1 dx \left[ m_a^2(1-x) + m_b^2x - q^2x(1-x) \right] \ln \left( \frac{m_a^2(1-x) + m_b^2x - q^2x(1-x)}{\Lambda^2} \right)
\]

\[
C_{UV} = \frac{1}{\epsilon} - \gamma + 4\pi
\]

Loop diagrams involving higher numbers of GB fields are suppressed at low energies by powers of $q/f_\pi$.

Using the vertices in Eqn(3.8) and the loop results in Eqn(3.9) and Eqn(3.10) we obtain
expressions for the gauge boson vacuum polarizations and hence V, W and X. For completeness we also give expressions for S, T and U.

\[ S = \frac{1}{4\pi} \sum_a \left\{ \frac{I^2_{3a} - 4s^2_W I_{3a} Q_a}{2} \left[ \frac{F(m_a, m_a, 0) - F(m_a, m_a, M^2_Z)}{M^2_Z} \right] + (2s^4_W Q^2_a + (c^2_W - s^2_W) Q_a I_{3a}) F'(m_a, m_a, 0) \right\} \]  (3.13)

\[ T = \frac{1}{8\pi s^2_W c^2_W M^2_Z} \sum_{\text{mult}} \left( F(m_+, m_-, 0) - \frac{1}{2} F(m_+, m_3, 0) - \frac{1}{2} F(m_-, m_3, 0) \right) \]  (3.14)

\[ U = \frac{1}{2\pi} \left\{ \sum_{\text{mult}} \left[ \frac{F(m_+, m_3, 0) + F(m_-, m_3, 0) - F(m_+, m_3, M^2_W) - F(m_-, m_3, M^2_W)}{M^2_W} \right] \right. \\
\left. + \sum_a (I^2_{3a} - 4s^2_W I_{3a} Q_a + 4s^4_W Q^2_a) \left[ \frac{F(m_a, m_a, M^2_Z) - F(m_a, m_a, 0)}{M^2_Z} \right] \right. \\
\left. + \sum_a 4s^2_W Q_a (I_{3a} - s^2 Q_{3a}) F'(m_a, m_a, 0) \right\} \]  (3.15)

\[ V = \frac{1}{8\pi s^2_W c^2_W} \sum_a (I^2_{3a} - 4s^2_W I_{3a} Q_a + 4s^4_W Q^2_a) \left[ \frac{F(m_a, m_a, M^2_Z) - F(m_a, m_a, 0)}{M^2_Z} \right. \\
\left. - F'(m_a, m_a, M^2_Z) \right] \]  (3.16)

\[ W = \frac{1}{8\pi s^2} \sum_{\text{mult}} \left[ \frac{F(m_+, m_3, M^2_W) - F(m_+, m_3, 0)}{M^2_W} - F'(m_+, m_3, M^2_W) \right. \\
\left. + \frac{F(m_-, m_3, M^2_W) - F(m_-, m_3, 0)}{M^2_W} - F'(m_-, m_3, M^2_W) \right] \]  (3.17)

\[ X = \frac{1}{4\pi} \sum_a Q_a (I_{3a} - 2s^2_W Q_a) \left[ \frac{F(m_a, m_a, M^2_Z) - F(m_a, m_a, 0)}{M^2_Z} \right. \\
\left. - F'(m_a, m_a, M^2_Z) \right] \]  (3.18)

where the sum over “a” and “mult” are sums over the full PGB spectrum and custodial SU(2) I = 1 multiplets respectively.

In Fig 3 the contributions to V, W, and X are plotted for a mass degenerate I = 1 PGB
multiplet \( (\pi^+, \pi^0, \pi^-) \). The largest contributions to \( V,W \) and \( X \) are obtained when the PGB masses are of order \( M_Z/2 \) and are given by \( V \sim -0.02, W \sim -0.05 \) and \( X \sim -0.02 \).

4 One Family Technicolour

One family technicolour models \cite{10} are particularly appealing since they naturally give rise to extended technicolour interactions that provide masses for the light fermions. These models with a full techni-family have 16 Weyl fermionic degrees of freedom, \( \Psi_L = (U^\alpha_L, D^\alpha_L, E_L, N_L, U^\alpha_R, D^\alpha_R, E_C, N_C) \), and are hence very tightly constrained by the original S and T analysis.

We shall consider the contributions to \( V,W \) and \( X \) in two one family technicolour scenarios, when the techni-fermions transform under a complex and under a real representation of the technicolour group.

4.1 Complex Representation

If the techni-fermions transform under a complex representation of the technicolour group (eg an \( SU(N)_{TC} \) group) the approximate global symmetry breaking pattern is \( SU(8)_L \otimes SU(8)_R \rightarrow SU(8)_V \). The precise techni-fermion mass spectra will depend upon perturbations to this symmetry breaking pattern from extended technicolour interactions. We might expect the techni-fermion masses to have the same characteristics as the light fermions intra-family spectra. Thus we expect the techni-quarks to be substantially heavier than the techni-leptons and the techni-neutrino to be very light. Mass splitting between the techni-quarks is severely constrained by the experimental value of the T parameter hence we shall assume that they are degenerate with \( m_{t-q} > 300 \text{GeV} \). The techni-quarks contribution to \( V,W \) and \( X \) will therefore be effectively zero (the linear Taylor expansion of the contributions to the \( \Pi_{ab} \)s is a good approximation). Mass splitting between the techni-leptons is much less well constrained if both are light \cite{9}. As an example we take \( m_N = 50 \text{GeV} \) and \( m_E = 150 \text{GeV} \) and find
\( V_{t-l} \sim -0.13N_{TC} \quad W_{t-l} \sim -0.01N_{TC} \quad X_{t-l} \sim 0 \quad (4.1) \)

We note that the value of \( V \) drops by a factor of ten as we increase the techni-neutrino mass to 100\,\text{GeV}.

In addition to the techni-fermions there are 63 (P)GBs associated with the broken generators of \( SU(8)_{\text{axial}} \)

\[
X^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \Lambda^a & 0 \\ 0 & -\Lambda^a \end{pmatrix}
\]

(4.2)

where \( \Lambda^a \) are the generators of \( SU(8) \). The GBs can be classified by the generators \( X^a \) and according to their electroweak and colour quantum numbers

\[ \Theta^\alpha \] colour octets \[ \left\{ \begin{pmatrix} 1 & \lambda^\alpha \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} \tau^a & \lambda^\alpha \\ 0 & 0 \end{pmatrix} \right\} \quad m_{\Theta} \sim 250\text{GeV} \]

\[ T \] colour triplet \[ \left\{ \begin{pmatrix} 0 & \Sigma \otimes \zeta \\ \Sigma \otimes \zeta & 0 \end{pmatrix} \right\} \quad m_T \sim 150\text{GeV} \]

\[ P \] colour singlets \[ \left\{ \begin{pmatrix} \tau^a \otimes \mathbb{1}_3 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & \tau^a \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \mathbb{1}_6 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix} \right\} \quad m_P \sim 0 - 100\text{GeV} \quad (4.3) \]

where \( \tau^a \) are the generators of \( SU(2) \), \( \lambda^\alpha \) the generators of \( SU(3) \), \( \mathbb{1}_n \) the \( n \times n \) unit matrix, \( \zeta \) is a 3-vector of colour, and \( \Sigma \) are the \( 2 \times 2 \) matrices

\[
\Sigma = \{ \mathbb{1}_2, \tau^a, i \mathbb{1}_2, i\tau^a \} \quad (4.4)
\]

The mass estimates are taken from reference \[ 16 \]. Amongst the \( I = 1 \) Ps the PGBs with techni-quark constituents will have the largest couplings to the electroweak gauge bosons \( (F_\mu) \) and hence will constitute the majority of the admixture “eaten” by the \( W^\mu \) and \( Z^\mu \)
gauge bosons. The remaining physical PGBs will be largely techni-lepton bound states with light masses. To maximize the possible contributions to V, W and X we take $m_P = 50GeV$

$$V_{PGB_c} \sim -0.05 \quad W_{PGB_c} \sim -0.08 \quad X_{PGB_c} \sim -0.01 \quad (4.5)$$

The contributions to V, W and X in one family technicolour models with techni-fermions transforming under a complex representation of the technicolour group are typically of order a few hundredths. If one or more techni-fermion is very light (< 100GeV) then the value of V can rise to the order of a few tenths.

### 4.2 Real Representation

If the techni-fermions transform under a real representation of the technicolour group (e.g. an $SO(N)_{TC}$ group) then the global symmetry breaking pattern is given by $SU(16) \rightarrow O(16)$. The gauged symmetries of such a one family model do not forbid a Majorana mass for the right handed techni-neutrino and hence this is an alternative explanation for the proposed isospin splitting in the techni-lepton sector. The techni-lepton’s maximum contributions to V, W and X are then typically (for $m_D \sim 150 - 200GeV$ and $M \sim 400 - 700GeV$)

$$V_{Maj} \sim -0.04N_{TC} \quad W_{Maj} \sim -0.01N_{TC} \quad X_{Maj} \sim 0 \quad (4.6)$$

There are also 36 PGB anti-PGB pairs in addition to those discussed for the complex representation corresponding to the additional broken generators

$$X^a = \begin{pmatrix} 0 & D \\ D^\dagger & 0 \end{pmatrix} \quad (4.7)$$

Classifying these additional GBs by their electroweak and colour quantum numbers we find
where $s$ are the 6 components of the $3 \times 3$ symmetric tensor, $a$ the 3 components of the $3 \times 3$ antisymmetric tensor, $\sigma$ the 3 components of the $2 \times 2$ symmetric tensor and $\epsilon$ the $2 \times 2$ antisymmetric tensor.

The total contributions to $V, W$ and $X$ from PGBs in real representation models is thus

$$V_{PGB_r} \sim -0.14 \quad W_{PGB_r} \sim -0.15 \quad X_{PGB_r} \sim -0.05$$

(4.9)

When the techni-fermions transform under a real representation of the technicolour group one family technicolour models predict that $V, W$ and $X$ are typically of order $-0.1$. Since the minimal one family $SO(N)_{TC}$ technicolour model that is asymptotically free is $SO(7)$ these values can be greatly enhanced by a light techni-fermion. For example $V$ can receive a contribution as large as -0.3 if the techni-neutrino’s light mass is a result of a right handed Majorana mass or -0.8 if the techni-neutrino has a small Dirac mass.

## 5 Conclusions

The non-decoupling of heavy physics ($m_{\text{new}} \sim 1 \text{TeV}$) beyond the Standard Model of electroweak interactions was orginally parameterized in terms of three variables $S, T$ and $U$ \[3\]. A global fit to experimental data tightly constrained technicolour models with large numbers of new strongly interacting doublets such as one family technicolour models. Recently it has
been pointed out that the effects of new physics at mass scales close to the Z mass could be included into the analysis through three additional variables V, W and X. A global fit including these three new parameters places much weaker bounds on the values of S, T and U. The parameters U and W only enter into measurements made at the mass of the W boson and are hence less well bounded than S, T, V and X.

In this paper we have naively estimated the contributions to V, W and X in technicolour models with light techni-fermions by calculating the contributions from a weakly interacting heavy fermion doublet. Whilst not a quantitatively accurate calculation of V, W and X these results provide an order of magnitude estimate of the contributions. These parameters typically are of order a few hundredths unless the techni-fermions' masses fall below 100GeV.

In a one family technicolour model in which the techni-neutrino's Dirac mass falls to 50GeV we find $V \sim -0.15 N_{TC}$ which is comparable to S and T in models with realistic techni-fermion mass spectra (typically S and T $< 1$). Estimates have also been made for the contributions from a techni-lepton doublet with a light component resulting from a see-saw mechanism in the techni-neutrino’s mass matrix due to a right handed Majorana mass. V and W are both enhanced in such a scenario and for example we find $0 > V \geq -0.04 N_{TC}$.

Many technicolour models have large approximate chiral symmetries which are broken by the strong technicolour interactions resulting in a plethora of light pseudo Goldstone bosons ($m_{PGB} \ll \Lambda_{TC}$). The low energy interactions of these PGBs have been described by a gauged chiral Lagrangian and estimates made of their contributions to V, W and X. We have estimated the contributions from PGB spectra in one family technicolour models in which the techni-fermions transform under both real and complex representations of the technicolour group. We conclude that both $V_{PGB}$ and $W_{PGB}$ lie between 0 and -0.15.

The variables S, T, U, V, W and X are normalized so that physical observables are linear functions of the variables with coefficients of the same order of magnitude [7]. Therefore, in the absence of a global fit to the data in which V, W and X are allowed to vary over their
typical values in technicolour models, we can treat $V,W$ and $X$ as indicative of the errors in the estimated values of $S,T$ and $U$ when compared with the original global fit for these three variables alone. In the absence of light techni-fermions the major contributions to $V,W$ and $X$ are from PGBs and we find that the errors in the estimates of $S$ and $T$ are $\sim |V|$ (since $X < V$ and $W$ does not contribute to measurements on the Z mass) which we estimate to be at most a few tenths. However, in one family technicolour models or low scale technicolour models \cite{18} with light techni-fermions ($m_{t,f} < 100\text{GeV}$) the contribution to $V$ can rise to a few tenths per technicolour. Thus, for example, in an $SO(7)_{TC}$ technicolour model the upper bounds on $S$ and $T$ must be relaxed by $\sim 1$. We conclude that it is premature to rule technicolour models out based solely on the global fit to $S,T$ and $U$ until up coming accelerators have performed a search for new light techni-fermions and scalars.

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