Quantum Gravity Effect on the Tunneling Particles from 2+1 dimensional New-type Black Hole

Ganim Gecim and Yusuf Sucu
Department of Physics, Faculty of Science, Akdeniz University, 07058 Antalya, Turkey
E-mail: gecimganim@gmail.com and ysucu@akdeniz.edu.tr

Abstract. We investigate the Generalized Uncertainty Principle (GUP) effect on the Hawking temperature for the 2+1 dimensional New-type black hole by using the quantum tunneling method for both the spin-1/2 Dirac and the spin-0 scalar particles. In computation of the GUP correction for the Hawking temperature of the black hole, we modified Dirac and Klein-Gordon equations. We observed that the modified Hawking temperature of the black hole depends not only on the black hole properties, but also on the graviton mass and the intrinsic properties of the tunneling particle, such as total angular momentum, energy and mass. Also, we see that the Hawking temperature was found to be probed by these particles in different manners. The modified Hawking temperature for the scalar particle seems to be lower compared to its standard Hawking temperature. Also, we find that the modified Hawking temperature of the black hole caused by Dirac particle’s tunnelling rises by the total angular momentum of the particle. It is diminishable by the energy and mass of the particle and graviton mass as well. These intrinsic properties of the particle, except total angular momentum for the Dirac particle, and graviton mass may cause screening for the black hole radiation.

1. Introduction

Black hole radiation is theoretically very important phenomenon for researchers who attempts to merge the gravitation with the thermodynamics and the quantum mechanics [1, 2, 3, 4, 5, 6, 7, 8]. With the formulation of the quantum field theory in curved spacetime based on the framework of the standard Heisenberg uncertainty principle, it was proved that a black hole can emit particles that are created by the quantum vacuum fluctuation near its outer horizon [6, 7, 8]. Since then, many alternative methods have been proposed to derive the black hole radiation known as Hawking radiation in the literature. For instance, the semi-classical method, based on quantum tunneling process of a particle across the outer horizon of a black hole from inside to outside, can be used to derive the Hawking radiation. The method implies two different approaches to compute the imaginary part of the action ($S$), which is the classically forbidden trajectory of a particle across the outer horizon: the null
geodesic \[9, 10, 11, 12\] and the Hamilton-Jacobi \[13, 14, 15\]. In both approaches, the tunneling probability of a particle from a black hole, \(\Gamma\), is defined in terms of the classical action, as \(\Gamma = e^{-\frac{\text{Im}S}{\hbar}}\) \[9, 10, 11, 12, 13, 14, 15\]. By using the semi-classical method, a lot of studies about the Hawking radiation of a black hole as quantum tunneling process of a point-like particle have been carried out in the literature \[18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36\]. On the other hand, above mentioned studies on this method provide no specific information on type of the particle that is tunnelled from a black hole. That is because the Hawking radiation does not dependent on the intrinsic properties, such as mass, total (orbital+spin) angular momentum, energy and charge, of the tunneling point-like particle.

The existence of a minimal observable length which can be identified by the order of the Planck scale is a characteristic of the candidate theories of quantum gravity, such as string theory, loop quantum gravity and noncommutative geometry \[37, 38, 39, 40, 41\]. This length lead us to a generalized uncertainty principle (GUP) instead of the standard Heisenberg uncertainty principle. Because a particle is not a point-like particle in the context of these candidate theories anymore. Therefore, the uncertainty on the momentum of a particle increases and thus the standard Heisenberg uncertainty principle can be generalized as follows;

\[
\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 + \beta (\Delta p)^2\right], \tag{1}
\]

where \(\beta = \beta_0/M_p^2\), the \(M_p^2\) is the Planck mass, \(\beta_0\) is the dimensionless parameter \[42, 43, 44, 45\]. The commutation relations between a particle position, \(x\), and momentum, \(p\), are modified in the following way;

\[
[x_i, p_j] = i\hbar \delta_{ij} \left[1 + \beta p^2\right], \tag{2}
\]

where \(x_i\) and \(p_j\) represent the modified position and momentum operators, respectively, and their definitions are as follows;

\[
x_i = x_{0i}, \tag{3}
\]

\[
p_j = p_{0j}(1 + \beta p_0^2). \tag{3}
\]

The \(x_{0i}\) and \(p_{0j} = -i\hbar \partial_j\) in Eq.(3) are the standard position and momentum operators, respectively, and \(p_0^2 = p_{0j}p^{0j}\) \[53\]. Then, the modified energy expression becomes

\[
\tilde{E} = E \left(1 - \beta E^2\right) = E \left[1 - \beta \left(p^2 + m_0^2\right)\right], \tag{4}
\]

for which the energy mass shell condition, \(E^2 = p^2 + m_0^2\), is used. From these relations, the square of the momentum operator can be derived by the following way,

\[
p^2 = p_ip^i \simeq -\hbar^2 \left[\partial_i \partial^i - 2\beta \partial_j \partial^j \left(\partial_i \partial^i\right)\right], \tag{5}
\]

where the higher order terms of the \(\beta\) parameter are neglected.

The GUP relations are of great help to understand the nature of a black hole since quantum effects are the essential effects near the event horizon of a black hole. Recently, to investigate the quantum effects under the GUP relations, the thermodynamics properties of various black holes have been studied by using the quantum tunneling
process of particles with various spins \[46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64\]. These studies indicate that the modify Hawking radiation depends not only on the black hole’s properties but also on the intrinsic properties of the tunneling particle.

The New-type black hole is one of the important results of the New Massive Gravity, which is 2 + 1 dimensional gravity and graviton in this theory has a mass \[65\]. In the framework of the standard Heisenberg uncertainty principle, the Hawking radiation of the New-type black holes had been studied by using the quantum tunneling process of the scalar, Dirac and vector boson particles \[24, 36\]. In these studies, it was shown that the Hawking radiation only depends on the properties of the black hole and is independent from the properties of the tunneling point-particles, that is, all these particles tunnel from the black holes in the same way. This indicate that, even if an observer in enough or safe (i.e infinite) distant from a black hole may detect Hawking radiation of the black hole, the observer can not determine what kind of particles compose of the radiation. Therefore, in this study, we will investigate whether the properties of the tunneling particles will affect the Hawking radiation of the black hole by using the quantum tunneling process of both the scalar and Dirac particles in the framework of the GUP.

The organization of this work are as follows: In the Section 2, we modify the Dirac equation with respect to the GUP relations. Subsequently, using the modified Dirac equation, we calculate the tunneling possibility of the Dirac particle by using the Hamilton-Jacobi method, and, then, we find the modify Hawking temperature of the black hole. In the Section 3, the standard Klein-Gordon equation is rewritten under the GUP for the 2 + 1 dimensional New-type black hole. Subsequently, the tunneling probability of the scalar particle from the black hole and the modify Hawking temperature of the black hole are calculated, respectively. In conclusion, we evaluate and summarize the results.

2. Dirac particle’s tunneling in the New-type Black Hole

Using the GUP relations, the standard Dirac equation given in Ref. \[66\] can be modified as follows;

\[-i \sigma^0(x) \partial_0 \bar{\Psi} = \left( i \sigma^i(x) \partial_i - i \sigma^\mu(x) \Gamma_\mu - \frac{m_0}{\hbar} \right) \left( 1 + \beta \hbar^2 \partial_j \partial^j \right) \bar{\Psi}, \quad (6)\]

and its explicit form is

\[\sigma^0(x) \partial_0 \bar{\Psi} + i \sigma^i(x) \left( 1 - \beta m_0^2 \right) \partial_i \bar{\Psi} + i \beta \hbar^2 \sigma^i(x) \partial_i \left( \partial_j \partial^j \bar{\Psi} \right) - \frac{m_0}{\hbar} \left( 1 - \beta m_0^2 \right) \bar{\Psi} = 0, \quad (7)\]

where the \(\bar{\Psi}\) is the modify Dirac spinor, \(m_0\) is mass of the Dirac particle, \(\sigma^\mu(x)\) are the spacetime dependent Dirac matrices, the \(\Gamma_\mu(x)\) are the spin affine connection for spin-1/2 particle \[66\]. The spacetime background of the New-type black hole is given
by

\[ ds^2 = L^2 \left[ f(r) dt^2 - \frac{1}{f(r)} dr^2 - r^2 d\phi^2 \right], \]

where \( L \) is the AdS3 radius defined as \( L^2 = \frac{1}{2m^2} = \frac{1}{2\Lambda} \) where \( \Lambda \) is cosmological constant and \( m \) is graviton mass, and \( f(r) = (r-r_+)(r-r_-) \) is defined in terms of the outer, \( r_+ \), and inner, \( r_- \), horizons radius, respectively. The black hole’s horizons are located at \( r_\pm = \frac{1}{2}(-b \pm \sqrt{b^2 - 4c}) \), where \( b \) and \( c \) are two constant parameters \([24, 65]\). Using the Eq. (8), the spinorial affine connections are derived as follows \([24]\):

\[
\Gamma_0 = -\frac{i}{4} f'(r)\sigma^3\sigma^1, \quad \Gamma_1 = 0, \quad \Gamma_2 = \frac{1}{2} \sqrt{f(r)\sigma^1\sigma^2}.
\] (9)

To calculate the tunneling probability of a Dirac particle from the black hole, we use the following ansatz for the modified wave function;

\[
\tilde{\Psi}(x) = \exp \left( \frac{i}{\hbar} S(t, r, \phi) \right) \begin{pmatrix} A(t, r, \phi) \\ B(t, r, \phi) \end{pmatrix}
\] (10)

where the \( A(t, r, \phi) \) and \( B(t, r, \phi) \) are the functions of space-time. The \( S(t, r, \phi) \) is the classical action term for particle trajectory. Inserting the Eqs. (9) and (10) in Eq. (7), we obtain the following equations for the leading order in \( \hbar \) and \( \beta \) as

\[
A \left[ \frac{1}{L\sqrt{f}} \frac{\partial S}{\partial t} + m_0 \left( 1 - \beta m_0^2 \right) + \frac{\beta m_0}{L^2 r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 + \frac{\beta m_0 f}{L^2} \left( \frac{\partial S}{\partial r} \right)^2 \right] + B \left[ i\beta \sqrt{f} \left( 1 - \beta m_0^2 \right) \frac{\partial S}{\partial r} + \frac{1 - \beta m_0^2}{L r} \frac{\partial S}{\partial \phi} + i\beta f^{3/2} \frac{L^3}{L^3} \left( \frac{\partial S}{\partial r} \right)^3 \right] + B \left[ i\beta \sqrt{f} \frac{\partial S}{L^3 r^2} \frac{\partial S}{\partial \phi} + \beta f \frac{\partial S}{L^3 r} \frac{\partial S}{\partial r} + \frac{\beta}{L^3 r^3} \left( \frac{\partial S}{\partial \phi} \right)^3 \right] = 0,
\]

\[
A \left[ -i\beta \sqrt{f} \left( 1 - \beta m_0^2 \right) \frac{\partial S}{L} + \frac{1 - \beta m_0^2}{L r} \frac{\partial S}{\partial \phi} - i\beta f^{3/2} \frac{L^3}{L^3} \left( \frac{\partial S}{\partial r} \right)^3 \right] + A \left[ -i\beta \sqrt{f} \frac{\partial S}{L^3 r^2} \frac{\partial S}{\partial \phi} + \frac{\beta}{L^3 r^3} \left( \frac{\partial S}{\partial \phi} \right)^3 \right] + B \left[ \frac{1}{L\sqrt{f}} \frac{\partial S}{\partial t} - m_0 \left( 1 - \beta m_0^2 \right) - \frac{\beta m_0}{L^2 r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 - \frac{\beta m_0 f}{L^2} \left( \frac{\partial S}{\partial r} \right)^2 \right] = 0.
\] (11)

These two equations have nontrivial solutions for the \( A(t, r, \phi) \) and \( B(t, r, \phi) \) when the determinant of the coefficient matrix is vanished. Accordingly, neglecting of the terms containing higher order of the \( \beta \) parameter, provides

\[
\beta \left[ 2m_0^4 - \frac{4f}{L^4 r^2} \left( \frac{\partial S}{\partial r} \right)^2 \left( \frac{\partial S}{\partial \phi} \right)^2 - \frac{2}{L^4 r^4} \left( \frac{\partial S}{\partial \phi} \right)^4 - \frac{2f^2}{L^4} \left( \frac{\partial S}{\partial r} \right)^4 \right] + \frac{1}{L^2 f} \left( \frac{\partial S}{\partial t} \right)^2 - f \frac{L^2}{L^2} \left( \frac{\partial S}{\partial r} \right)^2 - \frac{1}{L^2 r} \left( \frac{\partial S}{\partial \phi} \right)^2 - m_0^2 = 0.
\] (12)
Due to the commuting Killing vectors $(\partial_t)$ and $(\partial_\phi)$, we can separate the $S(t,r,\phi)$, in terms of the variables $t$, $r$ and $\phi$, as $S(t,r,\phi) = -Et + j\phi + K(r)$, where, $E$ and $j$ are the energy and angular momentum of the particle, respectively, and $K(r) = K_0(r) + \beta K_1(r)$ [57]. Using these definitions in Eq.(12), the integral of the radial equation, $K(r)$, becomes

$$K_\pm(r) = \pm \int \frac{\sqrt{E^2 - f(r) (m_0^2 L^2 + j^2 / r^2)}}{f(r)} [1 + \beta \chi] \, dr,$$

where $\chi$ is an abbreviation and it is

$$\chi = \frac{E^2 [E^2 - 2m_0^2 L^2 f(r)]}{L^2 f(r) [E^2 - f(r) (m_0^2 L^2 + j^2 / r^2)]}.$$

Then, by integrating the radial equation, $K_\pm(r)$ are obtained as

$$K_\pm(r) = \pm i\pi \frac{E}{r_+ - r_-} [1 + \beta \Pi] ,$$

where the abbreviation $\Pi$ is

$$\Pi = \frac{(r_+ - r_-)^2 [3 L^2 m_0^2 r^2 - j^2] + 4 E^2 r^2}{2 L^2 r^2 (r_+ - r_-)^4}.$$

On the other hand, the tunneling probabilities of particles crossing the outer horizon are given by

$$P_{\text{out}} = \exp \left[ -\frac{2}{\hbar} Im K_+ (r) \right],$$

$$P_{\text{in}} = \exp \left[ -\frac{2}{\hbar} Im K_- (r) \right].$$

Hence, the tunneling probability of the Dirac particle is given by

$$\Gamma = e^{-\frac{2}{\hbar} Im S} = \frac{P_{\text{out}}}{P_{\text{in}}} = \exp \left( -\frac{4\pi E}{\hbar (r_+ - r_-)} [1 + \beta \Pi] \right),$$

where $Im S(t,r,\phi)=Im K_+ (r) - Im K_-(r)$ [67] [68] and $Im K_+ (r)=-Im K_-(r)$. Then, the modify Hawking temperature of the Dirac particle, $T_H^D$, is obtained as

$$T_H^D = \frac{\hbar (r_+ - r_-)}{4\pi} \frac{1}{[1 + \beta \Pi D]},$$

where to find the temperature it is used the following relation [69] [70] [71] [72]:

$$\Gamma = e^{-\frac{2}{\hbar} Im S} = e^{-\frac{T_H}{\hbar}}.$$  (18)

If the $T_H^D$ are at first expanded in terms of the $\beta$ powers and second neglected the higher order of the $\beta$ terms, then the modify Hawking temperature of the black hole is obtained as

$$T_H \approx \frac{\hbar (r_+ - r_-)}{4\pi} [1 - \beta \Pi]$$

From these results, we see that the modified Hawking temperature includes not only the mass parameter of the black hole, but also the AdS$_3$ radius, $L$, (and, hence, the graviton mass) and the angular momentum, energy and mass of the tunnelled Dirac particle. On the other hand, in the case of $\beta = 0$, the modified Hawking temperature is reduced to the standard temperature obtained by quantum tunneling process of the point particles with spin-0, spin-1/2 and spin-1, respectively [24] [36].
3. Scalar particle’s tunneling in the New-type Black Hole

To investigate the quantum gravity effects on the tunneling process of the scalar particles from the black hole, by using the GUP relations, the standard Klein-Gordon equation are modified as

\[-(i\hbar)^2 \partial_t \partial_t \Phi = \left[(-i\hbar)^2 \partial_t \partial_t - M_0^2 \right] \left[1 - 2\beta \left(-\hbar^2 \partial_r \partial_r + M_0^2 \right) \right] \Phi, \]  

and its explicit form of the modified Klein-Gordon equation is written as follows;

\[\hbar^2 \partial_t \partial_t \bar{\Phi} + \hbar^2 \partial_r \partial_r \bar{\Phi} + 2\beta \hbar^4 \partial_r \partial_r (\partial_t \partial_t \bar{\Phi}) + M_0^2 (1 - 2\beta M_0^2) \bar{\Phi} = 0, \]  

where \(\bar{\Phi}\) and \(M_0\) are the modify wave function and mass of the scalar particle, respectively. Then, the modified Klein-Gordon equation in the New-type black hole background becomes as follows:

\[\bar{\Phi} (t, r, \phi) = A \exp \left(\frac{i}{\hbar} S (t, r, \phi) \right), \]  

where \(A\) is a constant. Substituting the Eq.(23) into the Eq.(22) and neglecting the higher order terms of \( \hbar \), we get the equation of motion of the scalar particle as

\[\left( \frac{\partial S}{\partial t} \right)^2 - f(r)^2 \left( \frac{\partial S}{\partial r} \right)^2 - \frac{f(r)}{r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 - M_0^2 L^2 f(r) - \beta \frac{2f(r)}{r^4 L^2} \left( \frac{\partial S}{\partial \phi} \right)^4 + \beta \left[ M_0^2 L^2 f(r) - \frac{2f(r)^3}{L^2} \left( \frac{\partial S}{\partial r} \right)^4 \right] = 0 \]  

Using \( S (t, r, \phi) = -Et + j\phi + W(r) \), where \(E\) and \(j\) are the energy and angular momentum of the particle, respectively, and \(W(r) = W_0(r) + \beta W_1(r)\), then, the radial integral, \(W(r)\), becomes as follows;

\[W_{\pm}(r) = \pm \int \sqrt{E^2 - f(r) (M_0^2 L^2 + j^2/r^2)} \frac{1 + \beta \Omega}{f(r)} dr, \]  

where the abbreviation \(\Omega\) is

\[\Omega = \frac{f(r)^2 (M_0^2 L^2 - j^4/r^4) - \left[ E^2 - f(r) (M_0^2 L^2 + j^2/r^2) \right]^2}{L^2 f(r) \left[ E^2 - f(r) (M_0^2 L^2 + j^2/r^2) \right]} . \]

And, \(W_{\pm}(r)\) are computed as

\[W_{\pm}(r) = \pm i\pi \frac{E}{r_+ - r_-} [1 + \beta \Sigma], \]  

where the abbreviation \(\Sigma\) is

\[\Sigma = \frac{(r_+ - r_-)^2 \left[ 3L^2 M_0^2 r_+ + 3j^2 \right] + 4E^2 r_+^2}{2L^2 r_+^2 (r_+ - r_-)^4} , \]
where $W_+(r_h)$ is outgoing and $W_-(r_h)$ is incoming solutions of the radial part. Then, using the Eq.(15), the tunneling probability of the scalar particle is calculated as

$$
\Gamma = \exp \left(-\frac{4\pi E}{\hbar (r_+ - r_-)} [1 + \beta \Sigma]\right)
$$

and, subsequently, using the Eq.(18), the modify Hawking temperature of the scalar particle, $T_{HG}^{KG}$, becomes as

$$
T_{HG}^{KG} = \frac{\hbar (r_+ - r_-)}{4\pi} \frac{1}{[1 + \beta \Sigma]}
$$

Furthermore, as neglecting the higher order of the $\beta$ terms in the expanding form of $T_{HG}^{KG}$ in terms of the $\beta$, we find the modify Hawking temperature of the black hole as follows;

$$
T_{HG}^{KG} \simeq \frac{\hbar (r_+ - r_-)}{4\pi} [1 - \beta \Sigma]
$$

From these results, it can seen that the modified Hawking temperature is related not only to the mass parameter of the black hole, but also to the AdS$_3$ radius, $L$, (and, hence, to the graviton mass) and angular momentum, energy and mass of the tunneling scalar particle. Furthermore, as can be seen from Eq.(19) and Eq.(29), the Hawking temperature probed by a Dirac particle is higher than that of a scalar particle: $T_{HG}^D = T_{HG}^{KG} + \frac{\hbar^2 m^2}{4\pi r_+(r_+ - r_-)}$ for $M_0 = m_0$ and $L^2 = \frac{1}{2m^2}$. On the other hand, in the case of $\beta = 0$, the modify Hawking temperature reduced to the standard temperature obtained by quantum tunneling process of the point particles with spin-0, spin-1/2 and spin-1, respectively [24, 36].

4. Concluding remarks

In this study, we investigated the quantum gravity effect on the tunnelled both spin-0 scalar and spin-1/2 Dirac particles from New-type black hole in the context of 2+1 dimensional New Massive Gravity. For this, at first, using the GUP relations, we modified the Klein-Gordon and Dirac equations that describe the spin-0 scalar and spin-1/2 Dirac particles, respectively. Then, using the Hamilton-Jacobi method, the tunneling probabilities of the these particles are derived, and subsequently, the corrected Hawking temperature of the black hole is calculated. We find that the modified Hawking temperature not only depends on the black hole’s properties, but also depends on the emitted particle’s mass, energy and total angular momentum. Also, it is worth to mention that, the modified Hawking temperature depends on mass of the graviton, i.e. quantum particle which mediates gravitational radiation in the context of New Massive Gravity. As can be seen from Eq.(19), the Hawking temperature of the Dirac particle increase by the total angular momentum of the particle while it decreases by the energy and mass of the particle and the graviton mass.

In addition, we can summarize some important results as follows:
• In Eq. (29), the modify Hawking temperature of the scalar particle is lower than the standard Hawking temperature.

• However, in Eq. (19), as $4E^2r_+^2 + \frac{3m_0^2}{2m^2}r_+^2(r_+ - r_-)^2 < j^2(r_+ - r_-)^2$, the modify Hawking temperature of the Dirac particle is higher than the standard Hawking temperature. Furthermore, when $4E^2r_+^2 + \frac{3m_0^2}{2m^2}r_+^2(r_+ - r_-)^2 > j^2(r_+ - r_-)^2$, the modify Hawking temperature is lower than the standard Hawking temperature. If $4E^2r_+^2 + \frac{3m_0^2}{2m^2}r_+^2(r_+ - r_-)^2 = j^2(r_+ - r_-)^2$, then the GUP effect is canceled, and the Hawking temperature of the Dirac particle reduces to the standard Hawking temperature.

• According to Eq. (19) and Eq. (29), the modify Hawking temperature of the New-type black hole probed by tunneling Dirac particle is higher than that of scalar particle:

$$T_D^H = T^KG_H + \beta \frac{\hbar j^2m^2}{\pi r_+^2(r_+ - r_-)},$$

where we adopt that the mass of the Dirac particle is equivalent to the mass of the scalar particle, i.e. $m_0 = M_0$.

• The New-type black hole is classified as six classes according to the signatures of the parameters $b$ and $c$, and, hence, it exhibits different physical and mathematical properties. For example, it reduced to the static BTZ black hole in the case of $b = 0$ and $c < 0$. In this context, according to tunneling of the scalar and Dirac particles, the modify Hawking temperature of the static BTZ black hole is

$$T^KG_H = T_H \left[ 1 - \alpha m^2 \frac{4 \left( \frac{3m_0^2}{2m^2} \right) |c| + 3j^2}{c^2} \right] + 4E^2$$

and

$$T_D^H = T_H \left[ 1 - \alpha m^2 \frac{4 \left( \frac{3m_0^2}{2m^2} \right) |c| - j^2}{c^2} \right] + 4E^2,$$

respectively. Here, $r_+ = -r_- = \sqrt{|c|}$ is used and the $T_H = \frac{\hbar \sqrt{|c|}}{2\pi}$ is the standard Hawking temperature of the static BTZ black hole in the context of the 2+1 dimensional New Massive Gravity theory [24].

• In the absence of the quantum gravity effect, i.e. $\beta = 0$, the modify Hawking temperature is reduced to the standard temperature obtained by quantum tunneling of the massive spin-0, spin-1/2 and spin-1 point particles [24, 36].

Finally, in the context of GUP, we have seen that the graviton and the tunneling particle masses have an affect decreasing the Hawking temperature in both scalar and Dirac particle tunneling processes. On the other hand, the total angular momentum has different effect on the Hawking temperature for a type of tunneling particle. For a scalar particle, it results in decrease in the temperature whereas it provides an increase in the temperature for a Dirac particle. These results show that the intrinsic properties of the
particle, except total angular momentum for the Dirac particle, and graviton mass may cause screening for the black hole radiation.

Acknowledgements

This work was supported by Akdeniz University, Scientific Research Projects Unit.

References

[1] J. M. Greif, Junior thesis, Princeton University, (unpublished) (1969).
[2] B. Carter, "Rigidity of a black hole", *Nature Physical Science*, vol. 238, pp. 71-72, 1972.
[3] J.D. Bekenstein, "Black Holes and Entropy", *Physical Review D*, vol. 7, no. 8, pp. 2333-2346, 1973.
[4] J.D. Bekenstein, "Generalized second law of thermodynamics in black-hole physics", *Physical Review D*, vol. 9, no. 12, pp. 3292-3300, 1974.
[5] J.M. Bardeen, B. Carter and S.W. Hawking, "The Four Laws of Black Hole Mechanics", *Communications in Mathematical Physics*, vol. 31, pp. 161-170, 1973.
[6] S.W. Hawking, "Black hole explosions", *Nature*, vol. 248, pp. 30-31, 1974.
[7] S.W. Hawking, "Particle creation by black holes", *Communications in Mathematical Physics*, vol. 43, pp. 199-220, 1975.
[8] S.W. Hawking, "Black holes and thermodynamics", *Physical Review D*, vol. 13, no. 2, pp. 191-197, 1976.
[9] P. Kraus and F. Wilczek, "Self-interaction correction to black hole radiance", *Nuclear Physics B*, vol. 433, pp. 403-420, 1995.
[10] P. Kraus and F. Wilczek, "Effect of self-interaction on charged black hole radiance", *Nuclear Physics B*, vol. 437, pp. 231-242, 1995.
[11] M.K. Parikh and F. Wilczek, "Hawking Radiation As Tunneling", *Physical Review Letters*, vol. 85, no. 24, pp. 5042-5045, 2000.
[12] M.K. Parikh, "New coordinates for de Sitter space and de Sitter radiation", *Physics Letters B*, vol. 546, pp. 189-195, 2002.
[13] M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, "Hawking radiation as tunneling for extremal and rotating black holes", *Journal of High Energy Physics*, vol. 2005, no. 5, article 014, 2005.
[14] K. Srivivasan and T. Padmanabhan, "Particle production and complex path analysis", *Physical Review D*, vol. 60, Article no. 024007, pp. 1-20, 1999.
[15] S. Shankaranarayanan, K. Srivivasan and T. Padmanabhan, "Method of complex paths and general covariance of Hawking radiation", *Modern Physics Letters A*, vol. 16, no. 9, pp. 571-578, 2001.
[16] E.C. Vagenas, "Generalization of the KKW analysis for black hole radiation", *Physics Letters B*, vol. 559, pp. 65-73, 2003.
[17] Y.P. Hu, J.Y. Zhang and Z. Zhao, "A note on the Hawking radiation calculated by quasi-classical tunneling method", *Modern Physics Letters A*, vol. 25, no. 4, pp. 295-308, 2010.
[18] I. Sakalli and A. Ovgun, "Black hole radiation of massive spin-2 particles in (3+1) dimensions", *The European Physical Journal Plus*, vol. 131, Article no. 84, pp. 1-13, 2016.
[19] A. Yale and R.B. Mann, "Gravitinos tunneling from black holes", *Physics Letters B*, vol. 673, pp. 168-172, 2009.
[20] R. Kerner and R.B. Mann, "Tunnelling, temperature, and Taub-NUT black holes", *Physical Review D*, vol. 73, Article no. 104010, pp. 1-11, 2006.
[21] R. Kerner and R.B. Mann, "Fermions tunnelling from black holes", *Classical and Quantum Gravity*, vol. 25, no. 9, pp. 1-17, 2008.
[22] R. Kerner and R.B. Mann, "Tunnelling from Godel black holes", *Physical Review D*, vol. 75, Article no. 084022, pp. 1-9, 2007.
G. Gecim and Y. Sucu, "Hawking Radiation of Topological Massive Warped-AdS3 Black Hole Families", arXiv:1406.0290v2, 2014.

G. Gecim and Y. Sucu, "Tunnelling of relativistic particles from new type black hole in new massive gravity", Journal of Cosmology and Astroparticle Physics, vol. 2013, no. 02, Article no. 023, 2013.

G. Gecim and Y. Sucu, "Dirac and scalar particles tunnelling from topological massive warped-AdS3 black hole", Astrophysics and Space Science, vol. 357, article 105, 2015.

S.I. Kruglov, "Black hole radiation of spin-1 particles in (1 + 2) dimensions", Modern Physics Letters A, vol. 29, no. 39, Article no. 1450203, pp. 1-5, 2014.

S.I. Kruglov, "Black hole emission of vector particles in (1+1) dimensions", International Journal of Modern Physics A, vol. 29, no. 22, Article no.1450118, pp. 1-10, 2014.

H. Gursel and I. Sakalli, "Hawking Radiation of Massive Vector Particles From Warped AdS3 Black Hole", Canadian Journal of Physics, vol. 94, no. 2, pp. 147-149, 2016.

I. Sakalli and A. Ovgun, "Hawking radiation of spin-1 particles from three dimensional rotating hairy black hole", Journal of Experimental and Theoretical Physics, vol. 121, no. 3, pp. 404-407, 2015.

G.R. Chen, S. Zhou and Y.C. Huang, "Vector particles tunneling from BTZ black holes", International Journal of Modern Physics D, vol. 24, Article no. 1550005, pp. 1-7, 2015.

G.R. Chen, Y.C. Huang, "Hawking radiation of vector particles as tunneling from the apparent horizon of Vaidya black holes", International Journal of Modern Physics A, vol. 30, no. 15, Article no. 1550083, pp. 1-9, 2015.

X.Q. Li and G.R. Chen, " Massive vector particles tunneling from Kerr and Kerr-Newman black holes", Physics Letters B, vol. 751, pp. 34-38, 2015.

I. Sakalli and A. Ovgun, "Quantum tunneling of massive spin-1 particles from non-stationary metrics", General Relativity and Gravitation, vol. 48, no. 1, Article no. 1, 2016.

G.R. Chen, S. Zhou and Y.C. Huang, "Vector particles tunneling from four-dimensional Schwarzschild black holes", Astrophysics and Space Science, vol. 357, article 51, 2015.

R. Li and J. Zhao, " Hawking radiation of massive vector particles from the linear dilaton black holes", The European Physical Journal Plus, vol. 131, no. 7, Article no. 249, 2016.

G. Gecim and Y. Sucu, "Massive vector bosons tunnelled from the (2+1)-dimensional black holes", The European Physical Journal Plus, vol. 132, no. 3, Article no. 105, 2017.

H. Hinrichsen and A. Kempf, "Maximal localization in the presence of minimal uncertainties in positions and in momenta", Journal of Mathematical Physics, vol. 37, no. 5, pp. 2121-2137, 1996.

A. Kempf, "On quantum field theory with nonzero minimal uncertainties in positions and momenta", Journal of Mathematical Physics, vol. 38, no. 3, pp. 1347-1372, 1997.

A.F. Ali, S. Das and E.C. Vagenas, "Discreteness of space from the generalized uncertainty principle", Physics Letters B, vol. 678, pp. 497-499, 2009.

S. Das, E.C. Vagenas and A.F. Ali, "Discreteness of space from GUP II: Relativistic wave equations", Physics Letters B, vol. 690, pp. 407-412, 2010. [Erratum ibid. 692 (2010) 342].

B.J. Carr, J. Mureika and P. Nicolini, "Sub-Planckian black holes and the Generalized Uncertainty Principle", Journal of High Energy Physics, vol. 2015, no. 07, Article no. 052, 2015.

A. Kempf, G. Mangano and R.B. Mann, "Hilbert space representation of the minimal length uncertainty relation", Physical Review D, vol. 52, no. 2, pp. 1108-1118, 1995.

S. Hossenfelder et al., "Signatures in the Planck Regime", Physics Letters B, vol. 575, pp. 85-99, 2003.
K. Nozari and M. Karami, "Minimal Length and Generalized Dirac Equation", *Modern Physics Letters A*, vol. 20, no. 40, pp. 3095-3103, 2005.

X.Q Li, "Massive vector particles tunneling from black holes influenced by the generalized uncertainty principle", *Physics Letters B*, vol. 763, pp. 80-86, 2016.

D. Chen, H. Wu and H. Yang, "Fermions Tunnelling with Effects of Quantum Gravity", *Advances in High Energy Physics*, vol. 2013, Article no. 432412, 2013.

D. Chen, Q.Q. Jiang, P. Wang and H. Yang, "Remnants, fermions' tunneling and effects of quantum gravity", *Journal of High Energy Physics*, vol. 2013, no. 11, Article no. 176, 2013.

D.Y. Chen, H.W. Wu and H. Yang, "Observing remnants by fermions' tunneling", *Journal of Cosmology and Astroparticle Physics*, vol. 2014, no. 03, Article no. 036, 2014.

D. Chen, H. Wu, H. Yang and S. Yang, "Effects of quantum gravity on black holes", *International Journal of Modern Physics D*, vol. 29, no. 26, Article no. 1430054, 2014.

X.X. Zeng and Y. Chen, "Quantum gravity corrections to fermions tunnelling radiation in the Taub-NUT spacetime", *General Relativity and Gravitation*, vol. 47, no. 4, Article no. 47, 2015.

H.L. Li, Z.W. Feng and X.T. Zu, "Quantum tunneling from a high dimensional Gudel black hole", *General Relativity and Gravitation*, vol. 48, no. 2, Article no. 18, 2016.

P. Wang, H. Yang and S. Ying, "Quantum gravity corrections to the tunneling radiation of scalar particles", *International Journal of Theoretical Physics*, vol. 55, no. 5, pp. 2632642, 2016.

Z.Y. Liu and J.R. Ren, "Fermions Tunnelling with Quantum Gravity Correction", *Communications in Theoretical Physics*, vol. 62, no. 6, pp. 819-823, 2014.

B. Mu, P. Wang and H. Yang, "Minimal Length Effects on Tunnelling from Spherically Symmetric Black Holes", *Advances in High Energy Physics*, vol. 2015, Article ID 898916, 2015.

G. Li and X. Zu, "Scalar Particles Tunneling and Effect of Quantum Gravity", *Journal of Applied Mathematics and Physics*, vol. 03, pp. 134-139, 2015.

M. A. Anacleto, F.A. Brito and E. Passos, "Quantum-corrected self-dual black hole entropy in tunneling formalism with GUP", *Physics Letters B*, vol. 749, pp. 181-186, 2015.

H. Li and X. Zu, "Black hole remnant and quantum tunnelling in three-dimensional Gudel spacetime", *Astrophysics and Space Science*, vol. 357, Article no. 6, 2015.

Z. Feng, L. Zhang and X. Zu, "The remnants in Reissner-Nordstrom-de Sitter quintessence black hole", *Modern Physics Letters A*, vol. 29, no. 26, Article no. 1450123, 2014.

G. Gecim and Y. Sucu, "The GUP effect on tunnelling of massive vector bosons from the 2+1 dimensional black hole" [arXiv:1704.03537v2], (2017).

G. Gecim and Y. Sucu, "The GUP effect on Hawking Radiation of the 2+1 dimensional Black Hole", *Physics Letters B*, vol. 773, pp. 391-394. [arXiv:1704.03530v1], 2017.

Y. Kwon, S. Nam, J. Park and S.H. Yi, "Quasinormal modes for new type black holes in new massive gravity", *Classical and Quantum Gravity*, vol. 28, no. 14, Article no. 145006, 2011.

Y. Sucu and N. Unal, "Exact solution of Dirac equation in 2+1 dimensional gravity", *Journal of Mathematical Physics*, vol. 48, no. 5, Article no. 052503, 2007.

K. Lin and S.Z. Yang, "A simpler method for researching fermions tunneling from black holes", *Chinese Physics B*, vol. 20, no. 11, Article no. 110403, 2011.

Y.P. Hu, J.Y. Zhang and Z. Zhao, "A note on the Hawking radiation calculated by the quasiclassical tunneling method", *Modern Physics Letters A*, vol. 25, pp. 295-308, 2010.

R. Di Crescienzo and L. Vanzo, "Fermion tunneling from dynamical horizons", *Europhysics Letters*, vol. 82, no. 6, Article no. 60001, 2008.

G.E. Volovik, *Exotic properties of superfluid 3He*, World Scientific, Singapore, 1992.

G.E. Volovik, "Simulation of Panleve-Gullstrand black hole in thin 3He-A film", *Journal of Experimental and Theoretical Physics Letters*, vol. 69, pp. 705-713, 1999.

G.E. Volovik, *The Universe in a Helium Droplet*, Clarendon Press, Oxford, 2003.