The effect of bias and redshift distortions on a geometric
test for the cosmological constant

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ABSTRACT

We revisit the feasibility of a cosmological test with the geometric distortion focusing on an ambiguous factor of the evolution of bias. Starting from defining estimators for the spatial two-point correlation function and the power spectrum, in a rigorous manner, we derive useful formulas for the two-point clustering statistics which take the light-cone effect and the redshift-space distortions into account. Then we investigate how the predicted correlation functions are affected by the redshift-space distortions and the bias assuming quasar samples which roughly match the 2dF survey. Effect of the bias is crucial, in particular, on anisotropic component of the clustering statistics in redshift-space. By surveying behavior of the predicted correlation functions to parameters of a phenomenological model for the evolution of bias, it is shown that the correlation functions on the light-cone is sensitive to a mean amplitude of the bias and rather insensitive to the speed of its redshift-evolution. This feature is useful for an analysis of the geometric test.

Key words: cosmology: theory — large scale structure — bias — quasars
1 INTRODUCTION

The wide-field redshift survey projects, Two-degree Field (2dF) and Sloan Digital Sky Survey (SDSS), are upcoming, and large numbers of quasars and galaxies are expected to be detected. In particular, the 2dF group has recently reported their preliminary results about the evolution of the quasar luminosity function and the spatial correlation function based on the several thousands quasars detected from the survey so far (Shanks et al. 2000; Boyle et al. 2000). The SDSS survey project will also provide a set of high precision data for the distribution of quasars in near future. These surveys will provide very important clues on the origin and the evolution of the quasars, and will put constraints on theoretical models of the structure formation of the universe. However, there seems to remain room for discussion as to how we compare the observational data with theoretical models and what information is drawn out from them. Concerning the latter subject, various cosmological tests with the future data have been proposed. Among such cosmological tests, the geometric test using the cosmological redshift-space (geometric) distortion is quite unique.

Alcock and Paczynski first pointed out the possibility of the geometric test (1979). They pointed out that if spherically distributed structures of cosmological objects were distributed in the universe, the observation of such structures at various redshift in redshift space offers a plausible test for the cosmological model of the universe, which is sensitive to a cosmological constant. This test is based on the fact that the shape of the spherically distributed structure is observed distorted in the cosmological redshift-space, where an incorrect distance-redshift relation is assumed. We refer to the distortion as the cosmological redshift-space (geometric) distortion. The geometric distortion is traced back to a coordinate transformation of the distance between real space and redshift space. Several authors have extensively discussed about possible tests for cosmological models with the geometric distortion with the clustering statistics of high-redshift objects (Ryden 1995; Matsubara & Suto 1996; Ballinger et al. 1996; Popowski et al. 1998; Nair 1999; Hui et al. 1999).

In principle, the geometric test is a unique test for the cosmological model, however, a measurement of the geometric distortion suffers from various other observational effects. In particular the peculiar motion of the cosmological objects causes additional distortion in the

* After we have completed the present manuscript, the most recent result on the 2dF quasar survey has been reported (Croom et al. 2000), in which a clustering analysis of the quasars is presented in detail.
distribution in redshift space. Especially the linear distortion, which is due to the bulk motion of the cosmological objects, can be influential even at high-redshift on large scales. The net of the effect depends on the amplitude of the bias. Furthermore, the light-cone effect would become an important effect when the statistical quantities like the correlation function and the power spectrum are computed using the data in a wide range of the redshift. In this case, the evolution of the bias is an important but ambiguous factor in predicting the correlation function and the power spectrum (Suto, Magira, Yamamoto 2000; see also references therein). The theoretical studies based on the Press-Schechter theory and the extensive numerical works recently show that the time evolution of clustering bias for the galaxies and the clusters of galaxies strongly depends on the cosmological models (e.g, Mo & White 1996; Blanton et al. 1999; Somerville et al. 2000; Taruya & Suto 2000), however, the quantitative prediction for the evolution of galaxy clustering is still under consideration. As for the quasars, the uncertainty of the formation mechanism makes for further difficulty in predicting the evolution of bias.

In the present paper we consider the details of the theoretical predictions for quasar two-point statistics focusing on the evolution of the bias factor. We discuss how we can draw out information about the evolution of the bias from the two-point statistics of cosmological objects and how the evolution of bias alters the prediction for the two-point statistics, which determines feasibility of the cosmological test with the geometric distortion. This paper is organized as follows: In section 2, we summarize theoretical formulas for the two-point correlation function and the power spectrum which incorporate the light-cone effect and the redshift-space distortion effects. The final expression of the formula has been presented in reference (Suto, Magira, Yamamoto 2000). On the contrary to this previous paper, we here derive the theoretical formulas in a rigorous manner by defining estimators for the two-point correlation function and the power spectrum on a light cone. This makes clear how our theoretical two-point correlation function and power spectrum are related to the estimators in data processing. Then we investigate the details of the correlation functions assuming quasar samples which roughly match the 2dF survey in section 3. Section 4 is devoted to summary and discussions. Throughout this paper we use the unit in which the light velocity $c$ equals 1.
2 FORMALISM

In this section we summarize formulas for the two-point statistics of the distribution of cosmological objects in redshift space incorporating the light-cone effect. On the contrary to the previous paper by Suto, Magira, & Yamamoto (2000), in which the final expression for the two-point correlation function is presented, we here present a review to derive the formula starting from estimators for the anisotropic correlation function and the power spectrum in cosmological redshift-space.

We start from reviewing an idealized data processing to compute the two-point correlation function and the power spectrum from a map of cosmological objects. Assuming a redshift survey, data is given in terms of a set of pairs of the direction vector $\gamma$ and the redshift $z$ of the cosmological objects. When constructing the three dimensional map of the cosmological objects, we must introduce a radial coordinate $s$ by assuming some distance-redshift relation $s = s(z)$. Because the cosmological model of our universe is not definitely determined, we must assume the distance-redshift relation $s = s(z)$. Frequently the distance extrapolating the Hubble law, $s = s_{\text{Hb}}(z)$, or the comoving distance in the Einstein de Sitter universe, $s = s_{\text{Ed}}(z)$, have been used for the relation, where we defined

$$s_{\text{Hb}}(z) = \frac{z}{H_0},$$

$$s_{\text{Ed}}(z) = \frac{2}{H_0}(1 - 1/\sqrt{1 + z}),$$

with the Hubble constant $H_0 = 100h\text{km/s/Mpc}$. We refer to the three dimensional space $(s, \gamma)$ as the cosmological redshift-space. For a sample at high-redshift $z \gtrsim 1$, extrapolation of the Hubble law, $s = s_{\text{Hb}}(z)$, is unphysical. However, for comparison, we consider the both $s = s_{\text{Hb}}(z)$ and $s = s_{\text{Ed}}(z)$ in sections 3.1 and 3.2, according to the previous investigations (Matsubara & Suto 1996; Ballinger et al. 1996). After section 3.3 we adopt $s = s_{\text{Ed}}(z)$.

Next we assume that the number density field $n(s)$ with $s = s\gamma$ can be constructed from the map. By introducing a synthetic catalogue $n_{\text{syn}}(s)$, we construct the density field:

$$F(s) = \frac{n(s) - n_{\text{syn}}(s)}{\left[ \int ds \bar{n}(s)^2 \right]^{1/2}},$$

where $\bar{n}(s)$ denotes the mean number density at the distance $s$. With the use of $\bar{n}(s)$, we write

$$n(s) = \bar{n}(s)(1 + \Delta(s)),$$

$$n_{\text{syn}}(s) = \bar{n}(s)(1 + \Delta_{\text{syn}}(s)).$$

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The synthetic field \(n_{\text{syn}}(s)\) satisfies
\[
\langle n_{\text{syn}}(s_1)n_{\text{syn}}(s_2) \rangle = \langle n(s_1)n_{\text{syn}}(s_2) \rangle = \tilde{n}(s_1)\tilde{n}(s_2),
\]
which lead
\[
\langle \Delta_{\text{syn}}(s_1)\Delta_{\text{syn}}(s_2) \rangle = \langle \Delta_{\text{syn}}(s_1)\Delta(s_2) \rangle = 0.
\] (6)

Then the two-point correlation function can be computed by averaging the products of \(F(s)\) over all the possible pairs of two-points separated by the distance \(R\). A mathematical expression for this process can be explicitly written
\[
\xi_l(R) = \int \frac{d\Omega}{4\pi} \int ds_1 \int ds_2 F(s_1)F(s_2)\delta^{(3)}(s_1 - s_2 - R)L_l(\hat{s}_h \cdot \hat{R})(2l + 1),
\] (7)

where \(R = |R|, \hat{R} = R/R, s_h = (s_1 + s_2)/2, s_h = |s_h|, \tilde{s}_h = s_h/s_h,\) and \(L_l(\mu)\) is the Legendre polynomial of the \(l\)-th order. (see figure 1.) In the case \(l = 0\), where \(L_0(\mu) = 1\), \(\xi_0(R)\) represents the isotropic (angular averaged) correlation function. On the other hand, other cases measure the anisotropies of the correlation function owing to the factor \(L_l(\hat{s}_h \cdot \hat{R})\). In the particular case \(l = 2\), \(\xi_2(R)\) expresses the quadru-pole anisotropy of the correlation function. We adopt this estimator for the isotropic and the anisotropic correlation functions.

Similar to the case of the correlation function we define the following estimator for the power spectrum (cf., Yamamoto, Nishioka, & Suto 1999)
\[
P_l(k) = \int \frac{d\Omega_k}{4\pi} \int ds_1 \int ds_2 F(s_1)F(s_2)e^{i\mathbf{k} \cdot (s_1 - s_2)}L_l(\hat{s}_h \cdot \hat{k})(2l + 1),
\] (8)

where \(k = |k|, \hat{k} = k/k,\) and \(k\) is the wave number vector. When adopting the above estimators for the correlation function and the power spectrum, which we have defined independently, we can show that a familiar relation holds between them. Namely, by using the mathematical formulas,
\[
\delta^{(3)}(s_1 - s_2 - R) = \frac{1}{(2\pi)^3} \int dk e^{i\mathbf{k} \cdot (s_1 - s_2 - R)},
\] (9)

and
\[
e^{i\mathbf{k} \cdot R} = 4\pi \sum_{l,m} i^l j_l(kR)Y_l^m(\Omega_k)Y_l^m(\Omega_R),
\] (10)

where \(Y_l^m(\Omega)\) and \(j_l(x)\) denote the spherical harmonics on an unit sphere and the spherical Bessel function, respectively, we can verify that the following relation holds
\[
\xi_l(R) = \frac{1}{2\pi^2 l^2} \int dk k^2 j_l(kR)P_l(k).
\] (11)

The inverse transformation leads
\[
P_l(k) = 4\pi i^l \int dRR^2 j_l(kR)\xi_l(R).
\] (12)
As is well recognized as the problem of the cosmic variance, expectation values of the estimators are only predictable theoretically. Then we consider the ensemble average of the estimators, i.e.,

\[ \xi_{LC}^{i}(R) = \langle \xi_{i}(R) \rangle, \]  
\[ P_{LC}^{i}(k) = \langle P_{i}(k) \rangle, \]  

which define the correlation function and the power spectrum on a light cone.

In Appendix we show that, applying the small angle approximation, \( P_{LC}^{i}(k) \) is reduced to

\[ P_{LC}^{i}(k) = \frac{\int dz \left( \frac{dN}{dz} \right)^{2} \left( \frac{s^{2}ds}{dz} \right)^{-1} P_{\text{crd}}^{i}(k, z)}{\int dz \left( \frac{dN}{dz} \right)^{2} \left( \frac{s^{2}ds}{dz} \right)^{-1}}, \]  

where \( dN/dz \) denotes the number count of the objects per unit redshift and per unit solid angle, \( P_{\text{crd}}^{i}(k, z) \) is defined:

\[ P_{\text{crd}}^{i}(k, z) = \frac{2l + 1}{c_{\perp}^{2}c_{\parallel}} \int_{0}^{1} d\mu L_{i}(\mu)b(q[k], z)P_{\text{mass}}(q[k], z) \]
\[ \times \left\{ \frac{1 + \mu^{2}((1 + \beta(z))\omega^{2} - 1)}{1 + \mu^{2}(\omega^{2} - 1)} \right\}^{2} D\left[ \frac{\mu k \sigma_{p}(z)}{c_{\parallel}} \right], \]  

where \( P_{\text{mass}}(q, z) \) denotes the power spectrum of the total mass distribution in real space with the wave number \( q \), \( D[\mu k \sigma_{p}(z)/c_{\parallel}] \) describes the damping function due to the Finger of God effect, and with denoting the comoving radial distance in real space \( r(z) \) and the distance in cosmological redshift-space \( s(z) \) (see equations (1), (2) and (A27) for the definitions), we defined

\[ c_{\perp} = \frac{r(z)}{s(z)}, \]  
\[ c_{\parallel} = \frac{dr(z)}{ds(z)}, \]  
\[ \omega = \frac{c_{\perp}}{c_{\parallel}}, \]  
\[ q[k] = k \sqrt{\frac{1 - \mu^{2}}{c_{\perp}^{2}} + \frac{\mu^{2}}{c_{\parallel}^{2}}}, \]  

and

\[ \beta(z) = \frac{1}{b(q[k], z)} \frac{d\ln D_{1}(z)}{d\ln a(z)}, \]  

with the linear growth rate \( D_{1}(z) \) and the scale factor \( a(z) \). The function \( b(q, z) \) denotes the linear bias factor defined in real space (see Appendix). Expression (17) is correct only for the case.
when the real space is spatially flat. (see equation (A28) for generalization to an open universe)

The correlation function $\xi^{LC}_l(R)$ can be computed from $P^{LC}(k)$ with the use of equation (11).

The expression $P^{crd}_l(k, z)$ is well-known as a power spectrum on a constant time hypersurface in the cosmological redshift-space (e.g., Ballinger et al. 1996; Suto et al. 1999; Magira, Jing, & Suto 2000). The term including the $\beta(z)$-factor in the expression $P^{crd}_l(k, z)$ is originated from the Kaiser factor which represents the effect of the linear distortion (Kaiser 1987). In the present paper we adopt the exponential model for the distribution of the pairwise peculiar velocity, then the damping factor is

$$D[\mu k \sigma_p(z)/c_\parallel] = \frac{1}{1 + \mu^2 k^2 \sigma_p(z)^2/2c^2},$$

where $\sigma_p(z)$ is the pairwise velocity dispersion. For $P_{mass}(q, z)$, the power spectrum in real space at the redshift $z$, we adopt the linear power spectrum with the CDM transfer function (Bardeen et al. 1986) and the Peacock & Dodds formula for the nonlinear power spectrum (Peacock & Dodds 1994; 1996). The essence of the geometric distortion is only the coordinate transformation between $(s(z), \gamma)$ and $(r(z), \gamma)$, which is described by the coefficients $c_\perp$ and $c_\parallel$.

Equation (15) indicates that the light-cone effect is incorporated by averaging the local power spectrum $P^{crd}_l(k, z)$ by weighting the factor in the formula:

$$W(z) = \left(\frac{dN}{dz}\right)^2 \left(\frac{s^2 ds}{dz}\right)^{-1}.$$  

The formula (13) is obtained under the small angle approximation. Hence the validity of the use is limited to small scales. For large scales, $R \gtrsim s_{\max}$, where $s_{\max}$ is the size of a survey area, the finite size effect of the survey area affects the proper estimation of the correlation function and the power spectrum. (e.g., Yamamoto & Suto 1999; Nishioka & Yamamoto 1999; 2000).

## 3 QSO TWO-POINT STATISTICS AND THE EVOLUTION OF BIAS

In this section we discuss the detail of the two-point correlation function focusing on how the quasar two-point statistics depend on the evolution of bias. Motivated from the recent report by Shanks et al. (2000), we assume a sample which roughly corresponds with the 2dF quasar survey.
3.1 The Quasar Luminosity Function

For simplicity we here adopt the B-band quasar luminosity function according to Wallington & Narayan (1993). We set the limiting magnitude $B_{\text{lim}} = 20.85$, which is useful to obtain a sample which roughly matches the 2dF quasar survey. According to Wallington & Narayan (1993; see also Nakamura & Suto 1997), we adopt

$$\Phi(M_B, z) = \Phi^* \times [10^{0.4(M_B - M_B(z))/(\kappa_1 + 1)} + 10^{0.4(M_B - M_B(z))/(\kappa_2 + 1)}]^{-1},$$

where

$$\Phi^* = 6.4 \times 10^{-6} h^3 \text{Mpc}^{-3} \text{mag}^{-1},$$

$$M_B(z) = M_B^* - 2.5k_L \log(1 + z),$$

$$M_B^* = -20.91 + 5 \log h,$$

with $k_L = 3.15$, $\kappa_1 = -3.79$, $\kappa_2 = -1.44$, which is applicable for $0.3 < z < 2.2$. The B-band apparent magnitude is computed from a quasar of absolute magnitude $M_B$ at $z$,

$$B = M_B + 5 \log \left[ \frac{d_L(z)}{10 \text{pc}} \right] - 2.5(1 - p) \log(1 + z),$$

where $d_L(z)$ is the luminosity distance and we adopt the quasar energy spectrum $L(\nu) \propto \nu^{-p}$ ($p=0.5$) for the K-correction. Because the luminosity function is constructed under the assumption of the Einstein de Sitter universe, we here use the luminosity distance

$$d_L(z) = \frac{2}{H_0} (1 + z - \sqrt{1 + z}),$$

which holds in the Einstein de Sitter universe.

The number count of the sample with the B-band limiting magnitude $B_{\text{lim}}$ per unit redshift and unit solid angle is expressed

$$\frac{dN}{dz} = \frac{r^2 dr}{dz} \int_{M_B_{\text{lim}}}^{\infty} dM_B \Phi(M_B, z),$$

with $r$ being the comoving distance. Because of the same reason for the luminosity distance $d_L$, we adopt the comoving distance in the Einstein de Sitter universe being $r = d_L(z)/(1 + z)$.

Figure 2 plots the weight factor $W(z)$ as the function of redshift with adopting cosmological redshift-space extrapolating the Hubble law $s = s_{Hb}(z)$ (dotted line) and Einstein de Sitter universe $s = s_{Ed}(z)$ (solid line). From this figure we see that the choice of the cosmological redshift-space significantly alters the weight factor. After completing almost of the present paper, the preliminary report on the luminosity function from the 2dF quasar survey (Boyle
et al. 2000) has been announced. They have found the consistency of the luminosity function with the previous results. The use of the new luminosity function does not alter our conclusions described below.

### 3.2 QSO Spatial Clustering and Evolution of Bias

Recently bias models for the quasar distribution have been proposed (e.g., Martini & Weinberg 2000; Haiman & Hui 2000; Fang & Jing 1998). Construction of theoretical models for the quasar bias is a challenging problem, and we need more investigations. However, the bias mechanism of the quasar distribution contains many uncertain factors at present. In the present paper we consider one of the phenomenologically simplest models of the bias with the form,

$$b(z) = \alpha + (b(z_*) - \alpha) \left( \frac{1 + z}{1 + z_*} \right)^\beta,$$

where $\alpha$, $\beta$, and $b(z_*)$ are the constant free-parameters (cf., Matarrese et al. 1997). Throughout the present paper we use $\beta$ to denote the free parameter of the bias evolution, which should be distinguished from $\beta(z)$ which we use to denote $\beta(z)$-factor for the linear distortion. In equation (31), $\beta$ specifies the rate of evolution of bias, $b(z_*)$ does the amplitude of bias at some redshift $z_*$, and $\alpha$ corresponds to the amplitude of bias at $z = 0$ in the limit $z_* \gg 1$. We have assumed the bias model is independent of scales $k$. This assumption relies on the recent numerical simulations that the scale-independent bias can be valid on the linear and quasi-linear scales as long as the mechanism of bias is dominantly affected by the gravity (e.g., Mann, Peacock & Heavens 1998). Although the realistic bias model taking account of the nonlinearity and stochasticity should be considered correctly (Dekel & Lahav 1999; Fry 1996; Tegmark & Peebles 1998; Taruya, Koyama & Soda 1999; Taruya & Suto 2000), the deterministic linear bias parameterized by the one parameter $b$ can become a good approximation even in the quasi-linear regime for galaxies and clusters (Taruya et al. 2001; see also Scherrer & Weinberg 1998).

As noted above, both of the power spectrum $P_{iLC}(k)$ and the correlation function $\xi^{LC}_{i}(R)$, in principle, carry the same information because they are connected by the Fourier transformation. As the light-cone effect can be regarded as an averaging process, therefore, it is important to understand the evolution of $P_{i}^{\text{crd}}(k, z)$ (or $\xi^{\text{crd}}_{i}(R, z)$) at different redshift.

Figure 3 shows typical behaviors of the power spectrum $P_{0}^{\text{crd}}(k, z)$ and $P_{2}^{\text{crd}}(k, z)$ at different redshifts adopting the cosmological redshift-space with $s = s_{Hb}(z)$. The upper pan-
els show $P_{0}^{\text{crd}}(k, z)$, the middle panels show $P_{2}^{\text{crd}}(k, z)$, and the lower panels show the ratio $P_{2}^{\text{crd}}(k, z)/P_{0}^{\text{crd}}(k, z)$. The left panels show the case for $z = 0.5$, the center for $z = 2.0$, and the right for $z = 4$. Here we adopted the bias model given by equation (31) with $\alpha = 0.5$, $\beta = 1$, $b(z_*=2) = 2$, and the real space of the CDM model with a cosmological constant, to be specific, $\Omega_0 = 0.3$, $\Omega_{\Lambda} = 0.7$, $h = 0.7$, $\sigma_8 = 1.0$. The $\Lambda$CDM model reproduces the observed cluster abundance (Kitayama & Suto 1997). Figure 4 is same as figure 3, but is the case adopting the cosmological redshift-space with $s = s_{Ed}(z)$.

In each panel, the dashed line plots the case when the distortion due to the peculiar motion of the sources are neglected, to be specific, the Kaiser factor, the nonlinear and the finger of god effect are neglected in the formula. Hence the dashed line shows the case when only the geometric distortion is taken into account. The dotted line shows the case when the nonlinear and the finger of God effects are neglected, and the solid line plots the case obtained from the formula (15), in which all the effects are incorporated. These two figures 3 and 4 indicate how the three kinds of the distortion effects, i.e., the geometric distortion, the linear distortion, the finger of god effect, are effective at different redshifts and on different scales. The three lines, the dashed line, the dotted line, and the solid line, in upper panels behave in a similar way. Therefore, the effect from the peculiar motion is small for the amplitude $P_{0}^{\text{crd}}(k, z)$. The geometric distortion significantly affects the amplitude of the power spectrum $P_{0}^{\text{crd}}(k, z)$. Note also that the effect acts in different ways for the cases $s = s_{Hb}(z)$ and $s = s_{Ed}(z)$. As the redshift becomes higher, the amplitude of $P_{0}^{\text{crd}}(k, z)$ becomes larger (smaller) in the case with $s = s_{Hb}$ ($s = s_{Ed}$). This difference is caused by the scaling effect through the coefficients $c_\perp$ and $c_\parallel$ (See also Taruya & Yamamoto 2000). Thus, for the power spectrum, $P_{0}^{\text{crd}}(k, z)$, the geometric distortion and the bias are the most important effects for the amplitude.

We read the following behaviors from the middle and lower panels of figures 3 and 4. For the anisotropic power spectrum, the redshift-dependence is remarkable when adopting $s = s_{Hb}(z)$. In this case, the geometric distortion is a minor effect, however, the linear distortion and the finger of god effect are effective on the large scales and on the small scales, respectively, at low redshifts. At the higher redshifts, the geometric distortion also becomes influential. On the contrary to this, in the case adopting $s = s_{Ed}(z)$, the geometric distortion effect is a minor effect compared with the linear distortion for $P_{2}^{\text{crd}}(k, z)$ even at high-redshift $z = 4$. Since we here fixed the bias parameter, this feature may be strongly dependent on the behavior of the
bias parameters. Therefore we investigate the bias-dependence of the correlation function in detail in the next subsections.

From figures 3 and 4 it is apparent that the choice of the cosmological redshift-space affects the behavior of the power spectrum (the correlation functions). To avoid the unphysical increase of the clustering amplitude at high-redshift in the case $s = s_{Hb}(z)$ and to compare with the observational result (Shanks et al. 2000), in which QSO correlation function is reported by assuming the Einstein de Sitter universe, hereafter we adopt $s = s_{Ed}(z)$. Though the behavior of the correlation functions depends on the choice of the cosmological redshift-space, sensitivity of the ratio of the correlation functions on a light-cone to the cosmological constant will not be significantly altered (see subsections 3.3 and 3.4), neither will be the feasibility of the geometric test.

3.3 Characteristic Correlation Length Determines QSO Bias

In this subsection we consider how the amplitude of the correlation functions which incorporate the light-cone effect depends on the evolution of bias. Here we determined $z_\ast$ of the bias model by the formula

$$z_\ast = \frac{\int dz z W(z)}{\int dz W(z)},$$

(32)

which describes a mean redshift. Assuming the quasar sample with $B_{\text{lim}} = 20.85$ in the range $0.3 < z < 2.3$, we have $z_\ast = 1.3$ adopting $s = s_{Ed}(z)$. We adopt this set of the values below.

Now we define the characteristic correlation length $R_c$ by

$$\xi_0^{LC}(R_c) = 1.$$  

(33)

Figure 5 shows contour of $R_c$ on the plane of the bias parameters $b(z_\ast)$ and $\beta$, which is computed from the definition (33) with the theoretical correlation function $\xi_0^{LC}$. Hereafter, unless otherwise stated, the correlation functions are computed by taking into account all the effects, i.e., the linear distortion, the geometric distortion, the nonlinear and the light-cone effects described in section 2. The left panels show the case $\alpha = 0$, the center $\alpha = 0.5$, and the right $\alpha = 1.0$, respectively. In the upper panels, the cosmological model with a cosmological constant, $\Omega_0 = 0.3, \Omega_\Lambda = 0.7, h = 0.7, \sigma_8 = 1$ is assumed as the model for real space, and the Einstein de Sitter universe is adopted as the cosmological redshift-space. The lower panels show the case assuming the real space of an open universe $\Omega_0 = 0.3, \Omega_K = 0.7, h = 0.7, \sigma_8 = 1$. It is notable
that $R_c$ does not significantly depend on the parameters $\alpha$ and $\beta$, but is sensitive to $b(z_*)$, the amplitude of the bias at $z_*$. This feature is caused because $z_*$ is roughly the mean value of the redshift for averaging the correlation function.

Shanks et al. (2000) have reported that the QSO correlation function is consistent with being $(r/r_0)^{-1.8}$ with $r_0 = 4h^{-1}\text{Mpc}$. This result is consistent with the previous paper (Croom & Shanks 1996), in which the correlation function was fitted in the similar form with the same power index with $r_0 = 6h^{-1}\text{Mpc}$. At present it seems that there exist not small statistical errors in the observed correlation function of the quasar distribution. If the value of $R_c$ was fixed, it will put a significant constraint on the evolution of bias. Though we need to assume the value of $\sigma_8$, the characteristic correlation length determines the QSO bias. For example, if we seriously take the result $r_0 = 4 \sim 6h^{-1}\text{Mpc}$ by Shanks et al. (2000) and Croom & Shanks (1996), figure 5 suggests that the bias of the quasar sample has the amplitude of factor $1 \sim 2$ at $z_* = 1.3$. However, this statement might not be taken so seriously, because the result is preliminary and the sample is not homogeneous at present. And also note that the estimators to evaluate the correlation function should be taken in a consistent way in data processing according to the theoretical computations.

### 3.4 Feasibility of the Geometric Test

The geometric distortion will be detected by measuring $\xi_2(R)$, so we next consider the anisotropic part of the correlation function, $\xi^{LC}_2(R)$. The anisotropic power spectrum $P^{LC}_2(k)$ can also measure the geometric distortion (Ballinger et al. 1996). In the present paper, however, we consider the ratio $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ (cf., Matsubara & Suto 1996). Figure 6 shows the contour of the ratio $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ on the plane of the bias parameters $b(z_*)$ and $\beta$, with fixed $R = 5h^{-1}\text{Mpc}$ (upper panels), $R = 10h^{-1}\text{Mpc}$ (middle panels), and $R = 20h^{-1}\text{Mpc}$ (lower panels). The left panels show the case $\alpha = 0$, the center $\alpha = 0.5$, and the right $\alpha = 1.0$. In this figure the $\Lambda$CDM model is assumed as the model of real space, where the same cosmological parameters are taken as those in figure 5, and the Einstein de Sitter universe is chosen for the cosmological redshift-space. It can be read from this figure that the dependence on $\alpha$ is weak, and that the dependence on $\beta$ becomes rather weak for $R \gtrsim 10h^{-1}\text{Mpc}$. The amplitude of $\xi^{LC}_2/\xi^{LC}_0$ is most sensitive to the parameter $b(z_*)$. The reason would be same as that described in subsection 3.3 for contour of $R_c$. 
To show the importance of the linear distortion, we show the contour of the ratio $\xi'^{LC}/\xi'^{LC}$ with fixing $R = 10h^{-1}\text{Mpc}$, in figure 7. In this figure the same model parameters as those in figure 6 are adopted, however, the distortion effects due to the peculiar motions are omitted on purpose in the upper and the middle panels. The upper panels show the case only the geometric distortion is incorporated but the distortion effect due to the peculiar motion of sources, i.e., the linear distortion, nonlinear and the finger of god effects, are neglected. The middle panels show the prediction within linear theory, that is, the case the linear distortion effect is incorporated but the nonlinear and the finger of God effects are omitted in the computation. The lower panels show the case all the effects are incorporated (Same as the middle panels in figure 6). From figure 7 it is apparent that the middle and the lower panels are significantly different from the upper panels. This means that the linear distortion, the nonlinear and the finger of god effects are the dominant effects for $\xi'^{LC}$ at this scale. In a redshift survey, the peculiar motion of sources inevitably contaminate the map. These facts limit the ability of the cosmological test with the geometric distortion because the linear distortion effect is sensitive to the amplitude of bias.

Nevertheless we would like to discuss that the cosmological test with the geometric distortion might be a useful tool for testing the cosmological model of our universe with the precise measurement of the correlation functions. In order to discuss the feasibility of the geometric test, we also consider the case that the real space is an open hyperbolic universe. Prescription to compute the correlation functions for the open universe is described in Appendix. Figure 8 shows the contour of $\xi'^{(LC)}(R)/\xi'^{(LC)}(R)$ to demonstrate the feasibility of the geometric test. The solid lines show contour of $\xi'^{(LC)}(R)/\xi'^{(LC)}(R)$ with fixed $R = 10h^{-1}\text{Mpc}$ for various cosmological models (a) $\Lambda$CDM model with $\Omega_0 = 0.3$; (b) $\Lambda$CDM model with $\Omega_0 = 0.4$; (c) open CDM model with $\Omega_0 = 0.3$; (d) open CDM model with $\Omega_0 = 0.4$. In each model we adopted $h = 0.7$, $\alpha = 0.5$, and $\sigma_8$ normalized by cluster abundance (Kitayama & Suto 1997). The region between the dashed lines in each panel satisfies $4 \leq R_c \leq 6$ for each model. Therefore $\xi'^{(LC)}/\xi'^{(LC)}$ at $R = 10h^{-1}\text{Mpc}$ is in the range $-0.8 \sim -0.5$ for the $\Lambda$CDM model taking the range of the bias parameter consistent with $R_c = 4 \sim 6h^{-1}\text{Mpc}$ into account. While $\xi'^{(LC)}/\xi'^{(LC)} \sim -0.5 \sim -0.3$ is predicted in the open CDM models. In this figure we fixed $\alpha = 0.5$, however, the dependence on $\alpha$ is weak for $0 \lesssim \alpha \lesssim 1$.

Figure 9 shows the same figure 8 but with fixed $R = 20h^{-1}\text{Mpc}$. In this case we find that
$\xi_2^{LC}/\xi_0^{LC}$ is in the range $-2.0 \sim -1.2$ ($-2.0 \sim -1.4$) for the ΛCDM with $\Omega_0 = 0.3$ ($\Omega_0 = 0.4$) taking the range of the bias parameter consistent with $R_c = 4 \sim 6h^{-1}\text{Mpc}$ into account. While $\xi_2^{LC}/\xi_0^{LC} \sim -1.2 \sim -0.8$ ($-1.4 \sim -1.0$) is predicted in the open CDM model with $\Omega_0 = 0.3$ ($\Omega_0 = 0.4$). This figure demonstrates that the difference in $\xi_2^{LC}/\xi_0^{LC}$ between the ΛCDM model and the open model appears irrespective of the scale $R$. At the same time this figure shows that the predictions slightly depend on the value of $\Omega_0$. If the constraint on $R_c$ becomes tight, the difference in $\xi_2^{LC}/\xi_0^{LC}$ becomes remarkable. The ratio $\xi_2^{LC}/\xi_0^{LC}$ does not depend on $\sigma_8$ at large length scales unless the nonlinear effect becomes influential. However, we should remind that the assumption of the value of $\sigma_8$ is needed to constrain the bias parameters by the characteristic correlation length.

This difference in $\xi_2^{LC}/\xi_0^{LC}$ between the ΛCDM model and the open model occurs due to the difference of the linear growth rate and the geometric distortion effect. The scaling effect due to the geometric distortion alters the amplitude of $\xi_0^{LC}(R)$ through the coefficient $c_\perp$ and $c_\parallel$. The evolution of $c_\perp$ and $c_\parallel$ in the ΛCDM model is quite different from those in the open model. The main reason for the difference in $\xi_2^{LC}/\xi_0^{LC}$ is from the difference in $\xi_0^{LC}$ due to the scaling effect of the geometric distortion. The difference appears for wide range of the parameters of the bias evolution. In order to check the model dependence of the bias, we have computed the ratio $\xi_2^{LC}/\xi_0^{LC}$ with the following parameterization for the bias instead of (31),

$$b(z) = \alpha + (b(z_*) - \alpha)\left(\frac{D_1(z_*)}{D_1(z)}\right)^\beta.$$ (34)

This alternation does not change the result significantly. These situation suggest that precise measurements of correlation functions might offer a unique and useful test of the cosmological model of our universe, however, further investigation should be needed for the definite conclusion.

4 SUMMARY AND CONCLUSIONS

In the present paper we have revisited the feasibility of the geometric distortion focusing on the ambiguous factor of the evolution of bias. We have derived the useful formulas for the correlation function and the power spectrum which incorporate the light-cone effect and the redshift-space distortions in the rigorous manner by defining the estimators for the anisotropic correlation function and the power spectrum. The final expressions are consistent with those
presented in the previous paper by Suto, Magira, and Yamamoto (2000). Our investigation makes clear how the theoretical two-point correlation function and the power spectrum in our paper correspond to estimators in data processing.

For a quasar sample like the 2dF survey, whose distribution is extended to high-redshift, the light-cone effect becomes very important. In the correlation functions, the light-cone effect is incorporated by averaging a local correlation function over the redshift with the weight factor $W(z)$. With this formula we have investigated the theoretical predictions assuming a simple model of quasar sample which roughly match with the 2dF survey. Adopting a model for the evolution of bias $b(z_*)$, which is phenomenologically parameterized in the simple form, we have examined the theoretical predictions for the two-point correlation functions. As pointed out by Ballinger et al. (1996), the linear distortion is the dominant effect even at the high-redshift on the anisotropic correlation function $\xi^{LC}_2$. The linear distortion is sensitive to the amplitude of bias, which limits the feasibility of the cosmological test with the geometric distortion.

We have shown that the predicted correlation functions on the light-cone are sensitive to the amplitude of bias $b(z_*)$ at the mean redshift $z_*$, and rather insensitive to the other parameters which specify the speed of its redshift-evolution due to the averaging process from the light-cone effect. The characteristic correlation length of the isotropic correlation function can determine the mean amplitude of the bias, though we need to assume the value of $\sigma_8$. If we take the discussions in subsection 3.3 seriously, the quasar correlation function is consistent with the ΛCDM model with the amplitude of the bias being $1.0 \sim 2.0$ at $z_* = 1.3$. We have also shown that $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ is only sensitive to the mean amplitude of bias, not to the speed of its redshift-evolution. Taking the constraint on the mean amplitude of the bias from the characteristic correlation length into account, the measurement of $\xi_2(R)/\xi_0(R)$ can be a useful tool for testing the cosmological model of the universe. Conversely if the cosmological model is fixed, the precise measurement of $\xi_2(R)$ and $\xi_0(R)$ will provide a clue for the bias model.

Finally we mention about other possible effects which affect measurements of statistical quantities. In general, the sampling noise and the shot noise limit the precise measurements of the statistical quantities. The shape of a survey area might affect a proper estimation of the correlation functions too. Those effects depend on observational strategies. We cannot conclude whether the 2dF quasar sample will detect the cosmological constant at present stage and a numerical approach is needed to examine those observational effects to draw definite
conclusions. Furthermore, though we have assumed the time-independent cosmological constant and the deterministic bias, the possibilities of the time-dependent cosmological constant like the quintessential CDM model and the stochasticity of the QSO clustering bias would make the feasibility of the geometric test more complex (Yamamoto & Nishioka 2000). N-body numerical simulations of a cosmological horizon size, which is the frontier of numerical cosmology (e.g., Pearce et al. 2000; Hamana et al. 2000), is another useful approach to investigate the geometric test.

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APPENDIX A:

In this Appendix we give an explicit derivation of the formula (13). Though the formula is presented in the paper (Suto et al. 2000), the explicit derivation is not presented there. We believe that it is useful to present the way to derive the formula from the estimators $P_l(k)$ and $\xi_l(R)$ defined explicitly. From the definition of the estimators, we write

$$\xi_{LC}^l(R) = \int \frac{d\Omega}{4\pi} \int ds_1 \int ds_2 \langle F(s_1) F(s_2) \rangle \delta^{(3)}(s_1 - s_2 - R) L_l(\hat{s}_h \cdot \hat{R})(2l + 1). \quad (A1)$$

With the aid of equation (6), $\langle F(s_1) F(s_2) \rangle$ in (A1) reduces to

$$\langle F(s_1) F(s_2) \rangle = \tilde{n}(s_1) \tilde{n}(s_2) \langle \Delta(s_1) \Delta(s_2) \rangle \int ds\tilde{n}(s)^2. \quad (A2)$$

Instead of the variables $s_1$ and $s_2$, we introduce

$$x = s_1 - s_2, \quad (A3)$$

$$s_h = \frac{1}{2}(s_1 + s_2), \quad (A4)$$

then we have

$$\xi_{LC}^l(R) = [4\pi \int ds^2 \tilde{n}(s)^2]^{-1} \int d\Omega \int dx \int ds_h \tilde{n}(s_1) \tilde{n}(s_2) \langle \Delta(s_1) \Delta(s_2) \rangle$$

$$\times \delta^{(3)}(x - R) L_l(\hat{s}_h \cdot \hat{R})(2l + 1), \quad (A5)$$

where $s_h = |s_h|$ and $\hat{s}_h = s_h/|s_h|$. Exact calculations for $\xi_{0C}^l(R)$ and $P_{0C}^l(k)$ are performed within linear theory of density perturbations in references (Suto & Yamamoto 1999; Nishioka & Yamamoto 1999; 2000). Though all the redshift distortions are not incorporated simultaneously in those works, however, those investigations have shown the validity for using the distant observer approximation. Then we here use the distant-observer approximation:

$$\tilde{n}(s_1) \tilde{n}(s_2) \simeq \tilde{n}(s_h)^2, \quad (A6)$$

$$\langle \Delta(s_1) \Delta(s_2) \rangle \simeq \xi(s_h, x, \hat{s}_h \cdot \hat{x}), \quad (A7)$$

where $\hat{x} = x/|x|$ and $x = |x|$. Here we have approximated that $\langle \Delta(s_1) \Delta(s_2) \rangle$ is a function of $s_h$, $x$, and $\hat{s}_h \cdot \hat{x}$. © 2000 RAS, MNRAS 000, 1–21.
Now we consider the expression for $\xi(s_h, x, \hat{s}_h \cdot \hat{x})$. We introduce the following coordinates to describe real universe,

$$
ds^2 = a(\eta)^2 \left(-d\eta^2 + dr^2 + f_K(r)^2 d\Omega^2(2)\right),
$$

(A8)

where

$$
f_K(r) = \begin{cases} 
\sin (\sqrt{K} r) / \sqrt{K} & (K > 0) \\
r & (K = 0) \\
\sinh (\sqrt{-K} r) / \sqrt{-K} & (K < 0)
\end{cases},
$$

(A9)

with the spatial curvature $K = H_0^2(\Omega_0 + \Omega_\Lambda - 1) = -H_0^2\Omega_K$. Then the number conservation means

$$
n(s, \gamma)s^2 ds = n^R(r, \gamma)f_K(r)^2 dr,
$$

(A10)

where $n^R(r, \gamma)$ denotes the number density field in real space. Introducing the mean number density $\tilde{n}(r)^R$ at the distance $r$ in real space, we write

$$
n^R(r, \gamma) = \tilde{n}^R(r)(1 + \Delta^R(r, \gamma)),
$$

(A11)

where $\Delta^R(r, \gamma)$ denotes the density contrast in real space. Combining (A10) and (A11), we have

$$
\tilde{n}(s) s^2 ds = \tilde{n}^R(r)f_K(r)^2 dr,
$$

(A12)

$$
\tilde{n}(s) \Delta(s, \gamma)s^2 ds = \tilde{n}^R(r)\Delta^R(r, \gamma)f_K(r)^2 dr,
$$

(A13)

and

$$
\langle \Delta(s_1)\Delta(s_2) \rangle = \langle \Delta^R(r_1)\Delta^R(r_2) \rangle,
$$

(A14)

where $r$ denotes the point specified by $r$ and $\gamma$. When the distance $|r_1 - r_2|$ is small compared with the size of the survey volume, we may write

$$
\langle \Delta^R(r_1)\Delta^R(r_2) \rangle
= \int \frac{dq}{(2\pi)^3} e^{iq(q_1 - q_2)} P(q_{||}, q_{\perp}, z)
$$

$$
= \int \frac{dq_{||} dq_{\perp}}{(2\pi)^3} \exp[iq_{||}(r_1 - r_2) + i\hat{q}_{\perp} \cdot (r_{\perp,1} - r_{\perp,2})] P(q_{||}, q_{\perp}, z),
$$

(A15)

where $P(q_{||}, q_{\perp}, z)$ denotes the power spectrum of cosmological objects in real space (including the effect of the peculiar motion) on a constant time hypersurface at redshift $z$, and we introduced $\mathbf{r} = (r, \mathbf{r}_{\perp})$, $\mathbf{q} = (q_{||}, \mathbf{q}_{\perp})$, where the subscript $||$ denotes the component of the line of sight and $\perp$ denotes the components perpendicular to the vector of the line of sight. Here the redshift
z in $P(q_1, q_\perp, z)$ should be understood as a function of the comoving distance $r_h = (r_1 + r_2)/2$. Even in the case of the open universe, we can expand the correlation function in terms of the usual Fourier modes, as in (A15), because we are considering the case the distance $|\mathbf{r}_1 - \mathbf{r}_2|$ is sufficiently small compared with the horizon (curvature) scale. Under the distant observer approximation we have

$$r_{1,1} - r_{1,2} = \frac{f_K(r_h)}{s_h}(s_{1,1} - s_{1,2}),$$

(A16)

$$r_1 - r_2 = \frac{dr_h}{ds_h}(s_1 - s_2),$$

(A17)

where $s_h = \frac{|s_1 + s_2|}{2}$. Introducing the new variables,

$$k_\perp = \frac{f_K(r_h)}{s_h}q_\perp = c_\perp q_\perp,$$

(A18)

$$k_\parallel = \frac{dr_h}{ds_h}q_\parallel = c_\parallel q_\parallel,$$

(A19)

we have

$$\langle \Delta(s_1)\Delta(s_2) \rangle = \frac{1}{c_\perp^2 c_\parallel} \int \frac{dk}{(2\pi)^3} e^{ik\cdot(s_1 - s_2)} P(q_\parallel \to k_\parallel, q_\perp \to k_\perp, z),$$

(A20)

where we used (A15) and (A14). Here the redshift $z$ should be understood as a function of $s_h$ instead of $r_h$. Inserting this equation into (A3), we have

$$\xi_{LC}^1(R) = \frac{4\pi}{4^2} \sum_{l} \int d\Omega_{s_h} \int ds_h \frac{2}{c_\perp^2 c_\parallel} \int \frac{dk}{(2\pi)^3} j_l(kR) \times P(q_\parallel \to k_\parallel, q_\perp \to k_\perp, z) L_l(\hat{\mathbf{s}}_h \cdot \hat{\mathbf{k}})(2l + 1),$$

(A21)

where we used the mathematical formulas,

$$e^{ik\cdot x} = 4\pi \sum_{l} \sum_{m=-l} d^l j_l(kx) Y_l^m(\Omega_\mathbf{k}) Y_l^m(\Omega_\mathbf{x}),$$

(A22)

$$L_l(\hat{\mathbf{s}}_h \cdot \hat{\mathbf{k}}) = \frac{4\pi}{2l + 1} \sum_{m=-l} Y_l^m(\Omega_\mathbf{s}_h) Y_l^m(\Omega_\mathbf{k}).$$

(A23)

By introducing $\mu = k_\parallel/k$, we have

$$\xi_{LC}^1(R) = \int d\Omega_{s_h} \frac{2}{c_\perp^2 c_\parallel} \frac{2l + 1}{2\pi^2} j_l(kR) \int_0^1 d\mu \ni P(q_\parallel \to k_\parallel, |q_\perp| \to k\sqrt{1 - \mu^2}/c_\perp, z) L_l(\mu).$$

(A24)

The transformation (12) leads

$$P_{1,LC}^1(k) = \int d\Omega_{s_h} \frac{2}{c_\perp^2 c_\parallel} \frac{2l + 1}{2\pi^2} j_l(kR) \int_0^1 d\mu L_l(\mu).$$
\[ \times P(q_i \to \frac{k\mu}{c_i}, |q_\perp| \to \frac{k\sqrt{1-\mu^2}}{c_\perp}, z). \]  

(A25)

The effects of the linear distortion, the nonlinear effect, and the finger of god effect have been investigated for the two-point statistics on a constant time hypersurface (Peacock & Dodds 1994; 1996, Ballinger et al. 1996, Magira, Jing, & Suto 2000). We adopt the following formula,

\[ P(q_i, |q_\perp|, z) = \left\{ 1 + \beta(z) \left( \frac{q_i}{q} \right)^2 \right\}^2 b(q, z)^2 P_{\text{mass}}(q, z) D[|q_i\sigma P(z)|], \]  

(A26)

with \( q = \sqrt{q_i^2 + |q_\perp|^2} \), mass power spectrum \( P_{\text{mass}}(q, z) \), the bias factor \( b(q, z) \), the damping factor \( D[|q_i\sigma P(z)|] \), and the \( \beta(z) \) factor defined by equation (21). Here we have assumed that the number density fluctuation of cosmological objects is simply proportional to the mass density fluctuation in real space by introducing the linear bias factor \( b(q, z) \). Then we obtain the expression (15), using the relation \( dN = dss^2\tilde{n}(s) \).

For convenience we summarize the explicit formulas for the factors \( c_i \) and \( c_\perp \). Throughout this paper we assume that the correct model of our universe is the CDM model. Then the comoving distance in the real space is expressed as

\[ r(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_0(1+z')^3 + \Omega_K(1+z')^2 + \Omega_{\Lambda}}} \]  

(A27)

With this formula, \( c_\perp \) is given by

\[ c_\perp = \frac{f_K(r(z))}{s(z)}, \]  

(A28)

and

\[ c_i = \frac{1}{\sqrt{\Omega_0 + \Omega_K(1+z)^{-1} + \Omega_{\Lambda}(1+z)^{-3}}}, \]  

(A29)

for the case \( s = s_{Ed}(z) \), and

\[ c_i = \frac{1}{\sqrt{\Omega_0(1+z)^3 + \Omega_K(1+z)^2 + \Omega_{\Lambda}}}, \]  

(A30)

for the case \( s = s_{Hb}(z) \), respectively.

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FIGURE CAPTIONS

Fig. 1— A sketch to illustrate variables defined in this paper.

Fig. 2— The weight function $W(z)$ for the quasar sample in section 3.1. The dashed line shows the case adopting $s = s_{Hb}(z)$, and the solid line does the case $s = s_{Ed}(z)$.

Fig. 3— Power spectra on a constant-time hypersurface. The upper panels show $P^0_{crd}(k, z)$ in unit of $h^{-3}\text{Mpc}^3$, the middle panels show $P^{2}_{crd}(k, z)$, and the lower panels show the ratio $P^{2}_{crd}(k, z)/P^{0}_{crd}(k, z)$. The left panels show the case $z = 0.5$, the center $z = 2$, and the right $z = 4$. We assumed the ΛCDM model with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\sigma_8 = 1.0$. In this figure the bias parameter is fixed as $\alpha = 0.5$, $\beta = 1$, and $b(z_*) = 2$. Here $s = s_{Hb}(z)$ is adopted. In each panel, the solid line plots the case obtained from the formula (15), while the dotted line plots the case when the nonlinear and the finger of God effects are neglected, and the dashed line plots the case when the distortion due to the peculiar motion of the sources and the nonlinear effect are neglected. The dashed line shows the case that only the geometric distortion is taken into account.

Fig. 4— Same as Fig.3 but with adopting the cosmological redshift-space $s = s_{Ed}(z)$.

Fig. 5— Contour of the characteristic correlation length $R_c$ on the bias-parameter plane $\beta$ and $b(z_*)$ with $z_* = 1.3$. We fixed $\alpha = 0$ in the left panels, $\alpha = 0.5$ in the center panels, and $\alpha = 1.0$ in the right panels. The upper panels show the case when the ΛCDM model, $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $\sigma_8 = 1.0$, is assumed as the cosmological model of the real space. While the lower panels show the case that the open CDM model with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.0$, $h = 0.7$, $\sigma_8 = 1.0$, is adopted as the real space. We adopted the cosmological redshift space $s = s_{Ed}(z)$. In each panel, the lines show the contour of the levels, $R_c = 10, 8, 6, 4, 3 \, h^{-1}\text{Mpc}$, from right to left. The solid lines are the contours $R_c = 6$ and $4 \, h^{-1}\text{Mpc}$.

Fig. 6— Contour of $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ in the ΛCDM model with fixed $R = 5h^{-1}\text{Mpc}$ (upper panels), $R = 10h^{-1}\text{Mpc}$ (middle panels), and $R = 20h^{-1}\text{Mpc}$ (lower panels). Similar to figure 5, we chose $\alpha = 0$ (left panels), 0.5 (center panels), and 1.0 (right panels). In this figure all the distortion effects are taken into account. The cosmological parameters, $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $\sigma_8 = 1.0$, are taken, and we adopted the cosmological redshift space $s = s_{Ed}(z)$.
Fig. 7— Contour of $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ with fixed $R = 10 h^{-1}\text{Mpc}$ on the bias-parameter plane $\beta$ and $b(z_\star)$. We fixed $\alpha = 0$ (left panels), 0.5 (center panels), and 1.0 (right panels). The upper panels show the case that the peculiar motion of sources is neglected, that is, the linear distortion and the nonlinear and the finger of god effects are omitted. The upper panels show the effect only from the geometric distortion. The middle panels show the case the geometric distortion and the linear distortion are incorporated but the nonlinear and the finger of god effects are omitted. The lower panels show the case all the distortion effects are incorporated. In this figure the $\Lambda$CDM model with the same cosmological parameters as those in figure 6 is assumed.

Fig. 8— Contour of $\xi^{LC}_2(R)/\xi^{LC}_0(R)$ with fixed $R = 10 h^{-1}\text{Mpc}$ for various cosmological models. (a) $\Lambda$CDM model with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$; (b) $\Lambda$CDM model with $\Omega_0 = 0.4$, $\Omega_\Lambda = 0.6$; (c) open CDM model with $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.0$; (d) open CDM model with $\Omega_0 = 0.4$, $\Omega_\Lambda = 0.0$. In each model we adopted $h = 0.7$ and $\alpha = 0.5$. The contour lines are $-0.8$, $-0.5$, $-0.3$ from left to right for the $\Lambda$CDM models. While the contour levels are $-0.5$, $-0.3$, $-0.1$ for the open CDM models from left to right. The region between the dashed lines in each panel satisfies $4 \leq R_c \leq 6$. Here $\sigma_8$ is determined from the cluster abundance (Kitayama & Suto 1997). We assumed the cosmological redshift space with $s = s_{Ed}(z)$.

Fig. 9— Same figure as figure 8 but with fixed $R = 20 h^{-1}\text{Mpc}$. The contour-levels drawn by solid lines are shown in each panel.
