I. INTRODUCTION

Production of six pions in $e^+e^-$ annihilation, studied at DM2 [1], showed a “dip” in the cross section at about 1.9 GeV, confirmed later by the FOCUS Collaboration in photoproduction [2,3], and with a much larger effective integrated luminosity at BaBar [4] using initial-state radiation (ISR). Even earlier, a narrow structure near the proton-antiproton threshold has been also observed in the total cross section of $e^+e^-$ annihilation into hadrons in the FENICE experiment [5]. The measurement of the CMD-3 Collaboration [6] confirmed these observations and demonstrated that the drop in the $e^+e^- \to 3(\pi^+\pi^-)$ cross section occurred in the narrow energy range of less than 10 MeV width. The origin of the “dip” remains unclear, but one of the explanations suggests a presence of the below-threshold proton-antiproton ($p\bar{p}$) resonance [7]. Alternatively, in Ref. [8,9] the “dip” is due to the strong interaction in virtual nucleon-antinucleon ($N\bar{N}$) production, and is related to fast rise of the $e^+e^- \to N\bar{N}$ cross section and $N\bar{N}$ annihilation to hadrons. This hypothesis is supported by the fast increase of the $p\bar{p}$ [10,11] and $n\bar{n}$ [12] form factors near threshold, and explains a similar drop in the $\eta'(958)\pi^+\pi^-$ spectrum, observed by the BES-III Collaboration [13]. The authors of Ref. [8] consider the two-step process $e^+e^- \to N\bar{N} \to$ multipoions and evaluate the total reaction amplitude for various intermediate mechanisms of the $e^+e^- \to 5\pi, 6\pi$ reactions. In Ref. [9] the authors go even further taking into account the proton-neutron mass difference and $p\bar{p}$ Coulomb interaction.

However, mass-energy resolution of the previous experiments does not allow to study a fine structure of the “dip” or the rise of the $e^+e^- \to N\bar{N}$ cross section. In this paper we present the analysis of the data sample based on 50 pb$^{-1}$ of the integrated luminosity collected with the CMD-3 detector [14] in the 1.5–2.0 GeV center-of-mass energy ($E_{c.m.}$) range. These data were collected in the energy scan at 29 c.m. energy points, performed at...
the VEPP-2000 collider with the upgraded injection com-
plex [15]. The scan of the $N\bar{N}$-threshold energy range
was performed with a fine step, corresponding to the
C.M. energy spread. The beam energy and energy spread
have been monitored by the back-scattering-laser-light
system [16]. During data taking the $E_{\text{c.m.}}$ stability was
at the level of 0.1 MeV at each energy point, while the
energy spread, $\sigma_{E_{\text{c.m.}}}$, was 1.2 MeV at the
$N\bar{N}$ threshold.

The luminosity was measured using events of Bhabha
scattering at large angles [17].

II. THE $e^+e^- \rightarrow 3(\pi^+\pi^-)$ CROSS SECTION

![Graph of the cross section for $e^+e^- \rightarrow 3(\pi^+\pi^-)$]

FIG. 1: The $e^+e^- \rightarrow 3(\pi^+\pi^-)$ Born cross section measured
with the CMD-3 detector in the 2017 run (squares). The
results of the previous CMD-3 measurement [6] are shown by
dots and those of BaBar [4] by open circles. The inset shows
the visible cross section with the fit described in the text. The
lines show the $N\bar{N}$ thresholds.

The analysis of the $e^+e^- \rightarrow 3(\pi^+\pi^-)$ process was
-described in detail in Ref. [6]. For the new data we have
reproduced all steps for selection of five and six charged
tracks, and the calculation of the efficiency and radiative
corrections. With the new data sample the number of signal
events with six charged tracks increased to 10155 (compared to
2887 events in the previous analysis) and that with one missing
track to 17822 (5069) events. The cross section obtained from
the new data is shown in Fig. 1 by squares, while the BaBar [4]
and previous CMD-3 [6] data are shown by open and closed circles,
respectively. Our previous result is confirmed with better
statistical accuracy, while a systematic uncertainty is estimated at
the same 6% level. The “dip” at the $N\bar{N}$ threshold is also confirmed
and is studied in more detail (see below).

III. THE $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ CROSS SECTION

![Graph of the cross section for $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$]

FIG. 2: The $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ Born cross section mea-
sured with the CMD-3 detector in the 2017 run (squares). The
results of the BaBar [19] measurements are shown by
open circles. The inset shows the visible cross section with
the fit described in the text. The lines show the $N\bar{N}$ thresholds.

The analysis of the $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ process was
described in detail in Ref. [18]. For the new data we have
reproduced all steps for selection of four charged tracks,
pion-kaon separation procedure, and the calculation of
the efficiency and radiative corrections. Only events
with exactly four charged tracks which have practically
no background are used. Nevertheless, the same overall
statistical accuracy is achieved since the scan around
the $N\bar{N}$ threshold allows us to select about 1500 signal
events per energy point. The cross section obtained from
the new data is shown in Fig. 2 by squares, while the BaBar [19]
data are shown by open circles. Our previous
result is confirmed with better statistical accuracy, while
IV. THE $e^+e^- \rightarrow p\bar{p}$ CROSS SECTION AT THE $N\bar{N}$ THRESHOLD

The analysis procedure is described in our previous publication [11]. At the energies near threshold, $E_{\text{c.m.}} < 1900$ MeV, protons and antiprotons from the reaction $e^+e^- \rightarrow p\bar{p}$ stop in the material of the beam pipe because of very low momentum. To select these events, we look for the products of antiproton annihilation with more than two charged tracks coming from the aluminum beam pipe. Comparison of the calorimeter response for such events below and above the $N\bar{N}$ threshold yields the number of $p\bar{p}$ events. Points below the production thresholds, where we assume no signal from the $e^+e^- \rightarrow p\bar{p}$ reaction, are used for background normalization and we find $490 \pm 30$ signal events in the energy range from the production threshold to 1900 MeV. Starting from $E_{\text{c.m.}}=1900$ MeV, protons have enough energy to penetrate the beam pipe, and above this energy no annihilation of antiprotons at the beam pipe is observed. Protons and antiprotons are detected as collinear tracks with large specific energy losses, $dE/dx$, in the drift chamber (DC) of the CMD-3: we detect 4770 signal events.

V. THE $N\bar{N}$ THRESHOLD REGION

At each energy a visible cross section is calculated as the number of selected events divided by the detection efficiency and integrated luminosity. The obtained $e^+e^- \rightarrow p\bar{p}$ visible cross section is shown in Fig. 3. The cross section exhibits very sharp steplike behavior close to the $N\bar{N}$ threshold. The Born cross section cannot be obtained without taking into account its smearing due to radiation of real photons by initial electrons and positrons, and the energy spread of the collision energy with $\sigma_{\text{thr}} = 1.2 \pm 0.2$ MeV. The visible cross section is described by a convolution of the radiative cross section, $\sigma_{\gamma}(E_{\text{c.m.}})$, with the c.m. energy spread function:

$$\sigma_{\text{vis}}(E_{\text{c.m.}}) = \frac{1}{\sqrt{2\pi} \sigma_{\text{thr}}} \int dE'_{\text{c.m.}} \sigma_{\gamma}(E'_{\text{c.m.}}) \cdot \exp \left[ -\frac{(E_{\text{c.m.}} - E'_{\text{c.m.}})^2}{2\sigma_{\text{thr}}^2} \right],$$

where $\sigma_{\gamma}(E_{\text{c.m.}})$ is a convolution of the Born cross section with the radiator function $F(E_{\text{c.m.}}, E_{\gamma})$ [20]:

$$\sigma_{\gamma}(E_{\text{c.m.}}) = \int_0^{E_{\gamma,\text{max}}} dE_{\gamma} \cdot \sigma_{\text{Born}} \left( E_{\text{c.m.}} \sqrt{1 - E_{\gamma}/E_{\text{c.m.}}} \right) \cdot F(E_{\text{c.m.}}, E_{\gamma}),$$

where $E_{\gamma}$ is the radiative photon energy, and $E_{\gamma,\text{max}}$ is a maximum allowed photon energy for the reaction.

For a demonstration of very fast change of the cross section, $\sigma_{\text{Born}}(E_{\text{c.m.}})$ is described with an exponentially saturated function,

$$\sigma_{\text{Born}}(E_{\text{c.m.}}) = A + B \left[ 1 - \exp \left( -\frac{(E_{\text{c.m.}} - E_{\text{thr}})}{\sigma_{\text{thr}}} \right) \right],$$

where $E_{\text{thr}}$ ($E_{\text{c.m.}} > E_{\text{thr}}$) and $\sigma_{\text{thr}}$ are the energy threshold and a variation scale of the Born cross section change, respectively. Parameters $A, B$ are the cross section values below and above the $p\bar{p}$ threshold.
First, the $e^+e^- \rightarrow p\bar{p}$ visible cross section of Fig. [3] is fit to Eq. [3] with all parameters floating except the $A$ value fixed at zero, assuming no signal below the threshold. The fit yields $E_{\text{thr}} = 1875.8 \pm 0.5$ MeV, consistent with the $p\bar{p}$ production threshold within uncertainties in the energy measurement. The obtained value $\sigma_{\text{thr}} = 1.76 \pm 0.58$ MeV is about half of the difference between the neutron and proton production thresholds. Since no $p\bar{p}$ events are expected below the threshold, $E_{\text{thr}}$ is fixed at 1876.54 MeV (the doubled proton mass), and the fit yields $\sigma_{\text{thr}} = 0.95 \pm 0.25$ MeV. Figure [3] shows the visible $e^+e^- \rightarrow p\bar{p}$ cross section with the fit result. Lines show the $p\bar{p}$ and $n\bar{n}$ threshold positions. An expanded view of the visible $e^+e^- \rightarrow p\bar{p}$ cross section around the $N\bar{N}$ threshold is shown in the inset in Fig. [3].

Similarly, the $e^+e^- \rightarrow 3(\pi^+\pi^-)$ visible cross section is fit to the above functions with all parameters floating in the energy range $E_{\text{c.m.}} = 1835–1945$ MeV, where the cross section can be considered flat. The fit yields $E_{\text{thr}} = 1873.7 \pm 0.6$ MeV, and $\sigma_{\text{thr}} = 3.1 \pm 0.9$ MeV. The fit with fixed $E_{\text{thr}} = 1876.54$ MeV yields $\sigma_{\text{thr}} = 0.29 \pm 0.73$ MeV, with a good $\chi^2/ndf = 22/21$ value: our statistical accuracy allows a drop with a zero width. The result of the latter fit is shown as an inset in Fig. [4] by a solid line, while a dashed line shows the expected Born cross section. The obtained $\sigma_{\text{thr}}$ value is consistent with that obtained for the $e^+e^- \rightarrow p\bar{p}$ and $e^+e^- \rightarrow 3(\pi^+\pi^-)$ reactions.

The results of the fit are summarized in Table I, and demonstrate that the observed behavior of the cross sections has similar origin, and the “dip” in the hadron cross section can be interpreted as due to opening of the direct production of the $N\bar{N}$ channel. Unfortunately, the accelerator-induced energy spread and relatively low statistical accuracy do not allow us to directly observe a possible structure of this rise (drop) due to the proton-neutron interaction, which could be expected in the studied reactions.

In a recently published paper [9], the authors use the optical potential to make a prediction of the $p\bar{p}$ and $n\bar{n}$ cross section behavior at very small energies above the production thresholds. Figure [4] for the $e^+e^- \rightarrow p\bar{p}$ Born cross section shows good agreement of available data with the theoretical prediction. But for very small deviations from the threshold, energy spread and radiative effects must be taken into account: the result of this convolution for the theoretical function is shown in the inset in comparison with our visible cross section. Note, the suggested model of the final-state interaction of a very slow $N\bar{N}$ pair predicts a nonzero cross section at the $p\bar{p}$ threshold, but experimental effects and limited accuracy do not allow us to prove that.

| Reaction | A, nb | B, nb | $E_{\text{thr}}, \text{MeV}$ | $\sigma_{\text{thr}}, \text{MeV}$ | $\chi^2/ndf$ |
|----------|-------|-------|--------------------------|-------------------------|--------------|
| $p\bar{p}$ | 0 ± fxd | 0.91±0.01 | 1875.8±0.5 | 1.76±0.58 | 34/25 |
| $p\bar{p}$ | 0 ± fxd | 0.91±0.01 | 1876.54±fxd | 0.95±0.25 | 35/26 |
| $6\pi$ | 1.49±0.02 | 0.40±0.03 | 1873.7±0.6 | 3.1±0.9 | 17/20 |
| $6\pi$ | 1.47±0.03 | 0.36±0.03 | 1876.54±fxd | 0.29±0.73 | 22/21 |
| $2K2\pi$ | 4.68±0.07 | 0.41±0.14 | 1878.7±0.5 | 0.11±2.22 | 7/9 |
| $2K2\pi$ | 4.69±0.07 | 0.43±0.11 | 1876.54±fxd | 2.82±2.18 | 7/10 |

$E_{\text{thr}} = 1878.7±0.5$ MeV, the value close to the $n\bar{n}$ threshold. The value $\sigma_{\text{thr}} = 0.11±2.22$ MeV indicates that the observed effect is dominated by an energy spread and radiative effects, consistent with a sharp drop in the Born cross section. The fit with fixed $E_{\text{thr}} = 1876.54$ MeV yields $\sigma_{\text{thr}} = 2.82±2.18$ MeV, with a good $\chi^2/ndf = 7/10$ value. The result of the latter fit is shown as an inset in Fig. [2] by a solid line, while a dashed line shows the expected Born cross section. The obtained $\sigma_{\text{thr}}$ value is consistent with that obtained for the $e^+e^- \rightarrow p\bar{p}$ and $e^+e^- \rightarrow 3(\pi^+\pi^-)$ reactions.
VI. THE $e^+e^- \rightarrow 2(\pi^+\pi^-)$ CROSS SECTION AT THE $NN$ THRESHOLD

As suggested in Ref. [9], the total hadronic cross section is strongly affected by virtual production and annihilation of the $NN$ pairs. The calculation predicts an about 7 nb “bump” in the total cross section (of about 40 nb at this energy), and should be seen in every specific $e^+e^- \rightarrow$ hadrons final state. A naive expectation suggests that the effect could be proportional to the probability of $p\bar{p}$ annihilation into the studied final state.

To test that, we analyze data at the $NN$ threshold selecting events for the reaction $e^+e^- \rightarrow 2(\pi^+\pi^-)$ according to the procedure described in Ref. [21], and show the obtained cross section in Fig. 5 together with the most precise measurement by BaBar [22]. No structure exceeding the level of 0.1 nb is observed at the $NN$ threshold in both measurements. According to Ref. [23], the $p\bar{p}$ annihilation probability (with isospin one) to four charged pions is 14.6%, while for six charged pions it is about 6%.

If a cross section drop in the hadronic channel is related to virtual $NN$ annihilation [9], for four-pion production one could expect an about 0.5–0.8 nb drop in the cross section, which is not supported by our data. Note that according to Ref. [23] the probability of $N\bar{N}$ annihilation to the $K^+K^-\pi^+\pi^-$ final state is very low.

Conclusion

Using the improved performance of VEPP-2000 the scan of the $e^+e^-$ c.m. energy in the 1680 – 2007 MeV range has been performed. A detailed study of the $NN$ threshold region confirms a fast drop (rise) in the $e^+e^- \rightarrow 3(\pi^+\pi^-)$ $(e^+e^- \rightarrow p\bar{p})$ cross section observed previously. For the first time a width of this structure is measured: the $\sigma_{\text{thr}} = 0.95 \pm 0.25$ MeV value is smaller than the difference between the $p\bar{p}$ and $n\bar{n}$ production thresholds. The “dip” in the $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ cross section is observed for the first time with similar parameters. No structures in the $e^+e^- \rightarrow 2(\pi^+\pi^-)$ cross section have been found at the $NN$ threshold. A search for a structure at the $NN$ threshold in other multihadronic final states is continued.

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