Reducing pure dephasing of quantum bits by collective encoding in quantum dot arrays

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Abstract. We show that phonon-induced pure dephasing of an excitonic (charge) quantum bit in a quantum dot (QD) may be reduced by collective encoding of logical qubits in QD arrays. We define the logical qubit on an array of 2, 4 and 8 QDs, connecting the logical $|\text{0}\rangle$ state with the presence of excitons in the appropriately chosen half of dots and the logical $|\text{1}\rangle$ state with the other half of the dots occupied. We give quantitative estimates of the resulting total error of a single qubit operation for an InAs/GaAs system.

The recent experimental demonstration of an optically controlled two-qubit conditional quantum gate implemented on a bi-exciton system confined in a quantum dot (QD) [1] shows that these semiconductor systems are promising candidates for implementation of quantum information processing schemes. The idea of encoding quantum logical values in charge states of confined carriers [2, 3] is motivated by the discrete, atomic-like spectrum of confined carrier states in QDs that allows one to single out a pair of optically active levels that may be assigned the logical values of 0 and 1. The most natural choice is the ground crystal state and a single confined exciton. The relatively large separation of levels in a self-assembled structure (at least several meV) seems to allow for ultrafast control of such a qubit with sub-picosecond optical pulses. Apparently, this should make it possible to perform of order of $10^4$ operations within the lifetime of a confined exciton which is usually around 1 ns.

However, optical experiments [4] have demonstrated that the coherence of an exciton state is destroyed to a large extent within a few picoseconds after optically creating the state. This results from a pure dephasing process due to carrier-phonon interaction [5, 6]: spontaneous relaxation of the lattice after an optically induced change of a confined charge distribution is accompanied by emission of phonon wave packets [7, 8], which leaves a kind of which way trace in the crystal, distinguishing between the qubit states and thus correlating the subsystems. This short-time dephasing is a purely non-Markovian process resulting from the fact that an abrupt change of a charge distribution cannot be followed adiabatically by the lattice ion positions (deformation). If the operation is performed extremely slowly a local polaron-like deformation forms around the confined charge distribution in a reversible, adiabatic manner, without perturbing the remote parts of the system, thus without dephasing. To be more specific, for a given rate of switching the carrier state, the faster modes follow adiabatically while the slower ones stay behind and then relax spontaneously, contributing to dephasing. From this discussion it is clear that the degree of this dynamically-induced dephasing is reduced for slower driving (longer control pulses), as was indeed confirmed by a formal analysis [9]. Obviously, in a real system the duration of a control operation cannot be extended endlessly since for longer times the exponentially accumulating
Figure 1. (a) The collective encoding of logical qubit in an array of 2, 4 and 8 QDs. Dark dots are occupied in the \( |0 \rangle \) logical state and the white ones in the \( |1 \rangle \) state. In the calculations, the dot sizes were 1 nm along \( z \) and 5 nm in the \( xy \) plane, while the separation between the dots was, respectively, 5 nm and 20 nm. (b) The spectral function corresponding to performing a \( \pi/2 \) qubit rotation by a Gaussian pulse (dotted) and the spectral densities for the original physical qubit (1 dot, solid line) and for the logical qubit encoded in 2, 4 and 8 dots (lines as shown). (c,d) The total (dynamical and Markovian) error \( \delta = \delta_d + \tau/\tau_X \), with the exciton lifetime \( \tau_X = 2 \) ns and 0.5 ns for \( T = 0 \) K and \( T = 10 \) K, respectively [13].

Markovian dephasing processes (e.g., related to a finite exciton lifetime) become important. This leads to a tradeoff situation where the optimum pulse duration must be sought against the two opposite dephasing mechanisms [9] (some further reduction of dephasing may also be achieved by pulse shaping [10]). As a result, maximum fidelity of a control operation on a given system is limited.

In this contribution we use a perturbative theory describing the open system evolution under arbitrary driving [9, 11, 12] to show that pure dephasing may be reduced by collective encoding of logical qubits in an array of QDs. The idea relies on the fact that, as discussed above, for sufficiently slow rotation in the qubit space, only a certain long wavelength (low frequency, i.e. slow modes) sector of the phonon reservoir (selected dynamically, independently of the system geometry) contributes to dephasing. The quantum logical \( |0 \rangle \) state is defined as a state of an array of \( 2N \) QDs in which appropriately selected \( N \) dots are occupied by excitons, while in the \( |1 \rangle \) state the other half of dots is occupied (see Fig. 1a). As we will show, the proper definition of logical qubits leads to destructive interference of low-frequency phonon excitations induced by qubit switching and, in consequence, to a reduction of the low-frequency part of the phonon spectral density, which leads to reduced dephasing for sufficiently slow operations. Although this scheme works only for relatively slow evolution and the resulting fidelity gain is restricted by finite lifetimes, experimentally measured exciton lifetimes allow one to reduce the overall error.

The QDs are approximated by two-level systems corresponding to the absence or presence of an exciton with a fixed polarization, with transition energies \( \epsilon_n, n = 1, \ldots, 2N \). The wave functions of carriers confined in different dots do not overlap so that no phonon-assisted transfer is possible. The Hamiltonian of this system is

\[
H_0 = \sum_n \epsilon_n a_n^\dagger a_n + \sum_k \hbar \omega_k b_k^\dagger b_k + \sum_{n,k} a_n^\dagger a_n f_n (k) \left( b_k^\dagger + b_{-k} \right),
\]

where \( a_n^\dagger, a_n \) are fermionic creation and annihilation operators for the exciton in the \( n \)th dot, \( b_k^\dagger, b_k \) are bosonic operators for phonon modes, \( \omega_k \) are the corresponding frequencies and \( f_n (k) = f_n (-k) \) are the carrier-phonon coupling constants for an exciton in the \( n \)th dot. We restrict our discussion to the deformation potential coupling. Assuming, for simplicity, that the exciton state in the \( n \)th dot is described by a product of identical electron and hole wave functions \( \psi_n (r - r_n) \), where \( r_n \) is the position of the dot, the coupling constants may be written
where $\rho = 5360 \text{ kg/m}^3$ denotes the crystal density, $V$ is the system volume, $c = 5150 \text{ m/s}$ is the sound speed and $\sigma_{e/h}(\sigma_e - \sigma_h = 8 \text{ eV})$ are the deformation potential constants for electrons and holes (the material parameters correspond to GaAs). Note that $F(k) = 1 + O(k^2)$, so that for wavelengths much larger than the dot size the coupling is independent of size and shape and our simplifying assumptions do not play any role.

In our discussion we will assume that a control Hamiltonian of the form

$$H_C = \frac{1}{2}f(t)e^{-i\Delta E t/\hbar}a_{n_1}\cdots a_{n_N}^\dagger a_{n_1'}\cdots a_{n_N'} + \text{H.c.}$$

is available, that couples the relevant pair of states of the multi-QD register by transferring $N$ excitons from one subset of dots to another without leaving the subspace spanned by the logical qubit states. For 2 QDs this can be achieved using the Förster coupling and the optical Stark effect [14, 15].

The Hamiltonian $H_0$ [Eq. (1)] has the structure of the independent boson model and may be diagonalized exactly [16]. To this end, one defines the operator

$$\mathcal{W} = \exp \left[ \sum_{n,k} a_n^\dagger a_n g_n^*(k) b_k - \text{H.c.} \right],$$

where $g_n(k) = f_n(k)/(\hbar \omega_k)$ and writes the Hamiltonian $H_0$ in terms of the operators $\alpha_n = \mathcal{W}^\dagger a_n \mathcal{W} = a_n W_n$ and $\beta_k = \mathcal{W}^\dagger b_k \mathcal{W} = b_k + \sum_n a_n^\dagger \alpha_n g_n(k)$, where $W_n = \exp(\sum_k g_n^*(k)b_k - \text{H.c.})$ (note that $[W_n, W_m] = 0$ for non-overlapping exciton states). The result is

$$H_0 = \sum_n E_n \alpha_n^\dagger \alpha_n + \sum_k \hbar \omega_k \beta_k^\dagger \beta_k, \quad (4)$$

where $E_n = \epsilon_n - \sum_k |f_n(k)|^2/(\hbar \omega_k)$.

In terms of the redefined degrees of freedom, the control Hamiltonian (3) to the leading order in phonon coupling reads

$$H_C = \frac{1}{2}f(t)e^{-i\Delta E t/\hbar}\alpha_{n_1}^\dagger \cdots \alpha_{n_N}^\dagger \alpha_{n_1'} \cdots \alpha_{n_N'} \left[ 1 + \sum_k G(k)(\beta_{-k}^\dagger - \beta_k) \right] + \text{H.c.}, \quad (5)$$

where $G(k) = [g_{n_1'}(k) + \cdots + g_{n_N'}(k)] - [g_{n_1}(k) + \cdots + g_{n_N}(k)]$. The phonon perturbation in Eq. (5) does not drive the system out of the two-level subspace but leads to reservoir excitations accompanying any operation on the carrier subsystem. The effect of the phonon coupling on the driven dynamics is calculated using the second-order (Born) expansion of the evolution equation for the density matrix [17, 12], including the first term in Eq. (5) exactly and the second one as a perturbation. After tracing out the reservoir degrees of freedom one finds the phonon-induced correction $\Delta \tilde{\rho}$ to the reduced density matrix of the carrier subsystem in the interaction picture. For a pure initial state $|\psi_0\rangle$, the fidelity of an operation on the logical qubit is then $F = \sqrt{1-\delta}$, with the error $\delta = \langle \psi_0 | \Delta \tilde{\rho} | \psi_0 \rangle$. The dynamical error, averaged over initial states of the qubit, may be represented as [9]

$$\delta_d = \int_0^\infty d\omega J(\omega) \coth \left( \frac{\hbar \omega}{2k_B T} \right) S(\omega), \quad S(\omega) = \frac{|F(\omega)|^2 + |F(-\omega)|^2}{12}, \quad (6)$$
where \( J(\omega) = \sum_k |G(k)|^2 \omega_k^2 \delta(\omega - \omega_k) \) is the spectral density of the phonon reservoir and the function \( F(\omega) \) is a nonlinear spectral characteristics of the driving,

\[
F(\omega) = \frac{1}{\hbar} \int_{-\infty}^{\infty} dt e^{i\omega t} f(t) e^{i\Phi(t)}, \quad \Phi(t) = \frac{1}{\hbar} \int_{-\infty}^{t} f(\tau)d\tau
\]

(see Fig. 1b). For a family of control pulses \( f(t) = (\hbar/\tau) \tilde{f}(t/\tau) \) (\( \tilde{f} \) defines the pulse shape and \( \tau \) is the pulse duration), the spectral function scales as \( F(\omega) = F(\tau \omega) \), where \( F(x) \) is fixed for a given pulse shape. Thus, for large \( \tau \), \( S(\omega) \) is concentrated in the low frequency sector, in accordance with the qualitative discussion presented above.

For the coupling constants of Eq. (2) one finds the low-frequency behavior of the spectral densities \( J(\omega) \sim \omega^5, \omega^7, \omega^9 \) for \( 2N = 2, 4, 8 \), respectively (the full dependence is shown in Fig. 1b). Thus, collective encoding indeed reduces the spectral function at low frequencies. However, it is clear from Fig. 1b that this reduction appears at lower and lower frequencies as the size of the logical qubit is increased. Hence, taking advantage of the collective encoding requires using longer and longer pulses which leads to growing contribution of the Markovian component. The resulting total error is plotted as a function of the pulse duration \( \tau \) in Fig. 1c,d, using exciton lifetimes from the experimental data [13]. It is clear that for the system geometry assumed here considerable reduction of the error is possible, although not much is gained by extending the collective encoding beyond two QDs.

The proposed collective encoding is similar to the earlier idea of implementing logical qubits robust against real transitions on QD arrays [2] (noiseless encoding, closely related to the effect of sub-radiance). However, our scheme does not assume that the individual dots in the register are identical, which is essential for its possible implementation in artificial semiconductor structures.

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