Thermodynamics of pairing transition in hot nuclei

Lang Liu (刘朗)
School of Science, Jiangnan University, Wuxi 214122, China.
State Key Laboratory of Nuclear Physics and Technology,
School of Physics, Peking University, Beijing 100871, China.

Zhen-Hua Zhang (张振华)
State Key Laboratory of Nuclear Physics and Technology,
School of Physics, Peking University, Beijing 100871, China.
Mathematics and Physics Department,
North China Electric Power University, Beijing 102206, China.

Peng-Wei Zhao (赵鹏巍)
Yukawa Institute for Theoretical Physics,
Kyoto University, Kyoto 606-8502, Japan.
State Key Laboratory of Nuclear Physics and Technology,
School of Physics, Peking University, Beijing 100871, China.

Abstract

The pairing correlations in hot nuclei $^{162}$Dy are investigated in terms of the thermodynamical properties by covariant density functional theory. The heat capacities $C_V$ are evaluated in the canonical ensemble theory and the pairing correlations are treated by a shell-model-like approach, in which the particle number is conserved exactly. A S-shaped heat capacity curve, which agrees qualitatively with the experimental data, has been obtained and analyzed in details. It is found that the one-pair-broken states play crucial roles in the appearance of the S shape of the heat capacity curve. Moreover, due to the effect of the particle-number conservation, the pairing gap varies smoothly with the temperature, which indicates a gradual transition from the superfluid to the normal state.

*pwzhao@pku.edu.cn
I. INTRODUCTION

A phase transition is well defined for infinite systems, while for finite many-body systems, its realization is often obscured due to the surface effects and statistical fluctuations. The ground states of most nuclei, i.e., at zero temperature, are superfluid states, but in warm nuclei, the superfluidity tends to be vanishing when the temperature increases. Such superfluid-to-normal transition has attracted wide attentions and, in the past decades, great progress has been achieved in the experimental aspects, thanks to the accurate measurements of the level density. From these investigations, the so-called S-shaped curve of heat capacity has been found as a function of temperature. This was regarded as a fingerprint of the superfluid-to-normal (pairing) phase transition. Based on this picture, the critical temperature has been estimated from the experimental data as $T_c \simeq 0.5$ MeV for $^{161,162}$Dy, $^{171,172}$Yb, and $^{166,167}$Er.

Similar S shapes come out in many theoretical calculations as well and the nature of the S-shaped heat capacity has also been discussed for many years in the framework of shell model, mean field models and other models see, e.g., Refs. 13, 14. In particular, within the mean-field picture, one could define the superfluid and the normal-fluid phases in nuclei with Bardeen-Cooper-Schrieffer (BCS) theory or the Hartree-Fock-Bogoliubov theory. Clear signatures of pairing phase transition have been provided by the fact that the S shapes of heat capacity could be reproduced by most mean-field calculations including finite-temperature BCS, finite-temperature HFB with a pairing-plus-quadrupole Hamiltonian, as well as the self-consistent mean-field models in both non-relativistic and relativistic form. With a variety of quantum fluctuations, it has been found that the critical temperatures for the pairing phase transition given by most mean-field models locate in the interval of $0.5-0.6\Delta(0)$, where $\Delta(0)$ is the pairing energy gap at zero temperature.

Breakdown of a certain symmetry is often associated with phase transitions. For the pairing phase transition, within the mean-field theories, the particle number conservation is violated in the superfluid phase while preserved in the normal-fluid phase. The number conservation effects on the nuclear heat capacity has been investigated through the particle-number projection methods based on the finite-temperature BCS or HFB approaches. Due to the restoration of particle number conservation, the calculated heat capacity varies smoothly with the temperature, indicating a gradual transition from the superfluid
to the normal phase.

However, none of the above models has treated the pairing correlations in hot nuclei exactly. Therefore, it is imperative to investigate the pairing transition in hot nuclei and the nature of the S-shaped heat capacity curve by the shell-model-like approach (SLAP) [22–24]. In this approach, the particle number is exactly conserved and the blocking effects are also treated exactly. It has been employed successfully for describing odd-even differences in moments of inertia (MOI’s) [25], the nonadditivity in MOI’s [26], the identical bands [27, 28], the nuclear pairing reduction under rotation [29], the high-spin states and high-$K$ isomers in the rare-earth, the actinide region and superheavy nuclei [30–34], the $\alpha$-cluster structures of light nuclei [35, 36] and the nuclear antimagnetic rotation [37].

In this work, the shell-model-like approach will be applied to the investigation of the S-shape of the heat capacity in hot nuclei $^{162}$Dy in the framework of covariant density functional theory (CDFT), which includes the complicated interplay between the large Lorentz scalar and vector self-energies induced on the QCD level, and naturally treats the spin degrees of freedom. Due to the successful description of many nuclear phenomena, CDFT has been one of the most important microscopic models for nuclear structure [38–40].

II. THEORETICAL FRAMEWORK

The CDFT starts from a Lagrangian and the corresponding Kohn-Sham equations have the form of a Dirac equation for nucleons with effective fields $S(r)$ and $V(r)$ derived from this Lagrangian

$$\{\boldsymbol{\alpha} \cdot \boldsymbol{p} + V(r) + \beta [M + S(r)]\} \psi_i = \varepsilon_i \psi_i. \tag{1}$$

Here, the scalar $S(r)$ and vector $V(r)$ potentials are connected in a self-consistent way to various densities through the Klein-Gordon equations for the meson fields $\sigma(r)$, $\omega(r)$, and $\rho(r)$ and the photon fields $A(r)$,

$$\begin{cases}
-\Delta + m_{\sigma}^2 \sigma(r) = -g_{\sigma} \rho_s(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), \\
-\Delta + m_{\omega}^2 \omega(r) = g_{\omega} \rho_\omega(r) - c_3 \omega^3(r), \\
-\Delta + m_{\rho}^2 \rho(r) = g_{\rho} \rho_3(r), \\
-\Delta A(r) = e \rho_p(r).
\end{cases} \tag{2}$$
The iterative solution of these equations yields the total energy, quadrupole moments, single-particle energies, etc. For details see Refs. [38, 39, 41].

For open shell nuclei, one needs to take into account the pairing correlations. In the present work, the SLAP is implemented in the framework of CDFT to treat the pairing correlations. The total Hamiltonian reads

$$H = H_{s.p.} + H_p$$

$$= \sum_i \epsilon_i a_i^\dagger a_i - G \sum_{i,j>0} a_i^\dagger a_j^\dagger a_j a_i,$$  \hspace{1cm} (3)

where $\epsilon_i$ is the single-particle energy obtained from the Dirac equation (1), $\bar{i}$ is the time-reversal state of $i$, and $G$ represents a constant pairing strength. This Hamiltonian is diagonalized in a space constructed with a set of multi-particle configurations (MPCs). For system with an even particle number $N = 2n$, the MPCs could be constructed as follows:

1. fully paired configurations (seniority $s = 0$):

$$|c_1 \bar{c}_1 \cdots c_n \bar{c}_n\rangle = a_{c_1}^\dagger a_{\bar{c}_1}^\dagger \cdots a_{c_n}^\dagger a_{\bar{c}_n}^\dagger |0\rangle; \hspace{1cm} (4)$$

2. configurations with two unpaired particles (seniority $s = 2$)

$$|ij \bar{c}_1 \cdots c_{n-1} \bar{c}_{n-1}\rangle = a_i^\dagger a_{\bar{j}}^\dagger a_{c_1}^\dagger a_{\bar{c}_1}^\dagger \cdots a_{c_{n-1}}^\dagger a_{\bar{c}_{n-1}}^\dagger |0\rangle \hspace{1cm} (i \neq j); \hspace{1cm} (5)$$

3. configurations with more unpaired particles (seniority $s = 4, 6, \ldots$), see e.g., Ref. [22].

The Hamiltonian (3) have the good quantum numbers of the parity $\pi$ and the seniority $s$. As a result, the MPC space could be written as:

$$\text{MPC space} = (s = 0, \pi = +) \oplus (s = 0, \pi = -) \oplus$$

$$\hspace{1cm} (s = 2, \pi = +) \oplus (s = 2, \pi = -) \oplus$$

$$\hspace{1cm} \cdots$$  \hspace{1cm} (6)

In the practical calculations, the MPC space has to be truncated with an energy cutoff $E_c$, i.e., the configurations with energies $E_i - E_0 \leq E_c$ are used to diagonalize the Hamiltonian (3), where $E_i$ and $E_0$ are the energies of the $i$th configuration and the ground-state configuration, respectively.
After the diagonalization of the Hamiltonian (3), one could obtain the nuclear many-body wave function $|\Psi_\beta\rangle$, and thus the pairing gap energy could be evaluated [42–44]

$$\Delta_\beta = G \left[ -\frac{1}{G} \langle \Psi_\beta | H_p | \Psi_\beta \rangle \right]^{1/2},$$

(7)

where $\beta = 0$ for the ground state, and $\beta = 1, 2, 3, \ldots$ for the excited states.

The thermodynamic properties of the pairing interaction are calculated here in the canonical ensemble [45], whose canonical partition function $Z$, average energy $\langle E \rangle$ and heat capacity $C_V$ are defined with the following equations,

$$Z = \sum_{\beta=0}^{\infty} \lambda(E_\beta) e^{-E_\beta/T},$$

(8)

$$\langle E \rangle = Z^{-1} \sum_{\beta=0}^{\infty} E_\beta \lambda(E_\beta) e^{-E_\beta/T},$$

(9)

$$C_V = \frac{\partial \langle E \rangle}{\partial T},$$

(10)

where $E_\beta$ is the excitation energy which could be obtained from the SLAP method with CDFT, and the corresponding level density $\lambda(E_\beta)$ is taken as $2^\alpha$, i.e., the degeneracy of each state. By means of the partition function, one can also evaluate the ensemble average pairing gap energy as

$$\tilde{\Delta} = Z^{-1} \sum_{\beta=0}^{\infty} \Delta_\beta \lambda(E_\beta) e^{-E_\beta/T}.$$  

(11)

III. NUMERICAL DETAILS

In this work, the axial symmetry is imposed, and the Dirac equation (1) is solved in a space of axially deformed harmonic oscillator basis with 14 major shells [46]. The effective interaction PK1 is adopted [47]. In the construction of the multi-particle configurations, twenty single particle levels around Fermi surfaces and six pairs of valence particles are included for both neutrons and protons. The effective pairing strengths $G_p = 0.32$ MeV for protons and $G_n = 0.29$ MeV for neutrons are determined by reproducing the odd-even differences of the experimental binding energies. These two strengths are connected with the dimension of the truncated MPC space. The MPC space is truncated with $E_c = 30$ MeV, and the corresponding dimension of the proton and neutron MPC spaces is $3 \times 10^5$ and $5 \times 10^5$, respectively.
IV. RESULTS AND DISCUSSION

FIG. 1. (Color online). Neutron (a), proton (b), and the total (c) heat capacities for $^{162}\text{Dy}$ as functions of the temperature with (solid lines) and without pairing (dashed lines).

The heat capacity can be evaluated from the partial derivative of the average energy with respect to the temperature as expressed in Eq. (10). Because the proton and neutron degrees of freedom are treated separately in the Hamiltonian of Eq. (3), the heat capacity could be straightforwardly divided into two parts which correspond to the proton and neutron excitations, respectively. In Fig. 1, the neutron, proton as well as the total heat capacities for $^{162}\text{Dy}$ are shown as functions of temperature. For comparison, the results calculated without pairing correlations ($G = 0$ MeV) are shown as dashed lines. It can be seen that the heat capacities without pairing increase almost linearly with the temperature. This linear tendency is very analogous to the results of a pure Fermi gas model \cite{2}, while the gentle fluctuations shown in the proton heat capacity result from the shell structures in the single proton levels of CDFT. The observed S shape in the experimental heat capacity is reproduced from our calculations when the pairing correlations are taken into account. The calculated heat capacities are nearly zero at low temperature ($T \leq 0.35$ MeV) due to the
large energy gap between the ground state and two-quasi-particle excited states induced by the inclusion of the pairing correlations. When the temperature grows up, many pair-broken excited states with seniority $s = 2, 4, \cdots$ appear, and thus the heat capacity increases rapidly till the inflection point at $T \approx 0.75$ MeV. Above this point, the heat capacity increases much slower with the temperature.

![Graph showing heat capacity vs. Temperature for Neutron, Proton, and Total for $^{162}$Dy](image)

**FIG. 2.** (Color online). Neutron (a), proton (b), and the total (c) heat capacities for $^{162}$Dy calculated with different number of valence particle pairs $N=2$ (dotted lines), 4 (dashed lines) and 6 (solid lines) in the model space as functions of the temperature.

It should be mentioned that the truncation of the model space may influence the behavior of the heat capacity at very high temperature. This could be clearly seen in Fig. 2 where the heat capacities calculated with different number of valence particle pairs ($N = 2, 4, 6$) in the model space are shown as functions of the temperature. The heat capacity curves are almost identical for $N = 2, 4, 6$ below $T \sim 0.2$ MeV since almost no pair broken happens at such low temperature. From there on, these heat capacity curves start to deviate from each other. In particular, the heat capacity for $N = 2$ drops at $T \sim 0.6$ MeV. This is due to the fact that the two valence proton and neutron pairs in the model space are exhausted and not able to continue to absorb energy with the same rate. Moreover, it is found that the
values of the heat capacity obtained in this case are roughly 50% lower than the observed ones. It has been known that if too few pairs are contained, one may easily misinterpret the S shape of the heat capacity curve and underestimate the value of the heat capacity [13]. In order to avoid such a misinterpretation of the S shape here, more Cooper pairs are included in the model space. It turns out here that for $^{162}$Dy one could get reasonable values of the heat capacity after six valence proton and neutron pairs are taken into account. This is also consistent with the previous work as in Ref. [13].

![Graph showing neutron and proton pairing gap energies](image)

**FIG. 3.** (Color online). Neutron (a) and proton (b) pairing gap energies for $^{162}$Dy as functions of the temperature calculated by SLAP (solid lines). For comparison, the neutron pairing gap energies obtained from finite temperature BCS (dotted line) and finite temperature variation after projection (VAP) BCS (dashed line) approaches [17] are also shown.

The transition of the pairing correlations with the temperature is characterized by the pairing gap energies. Figure 3 shows how the neutron and proton pairing gaps vary with the temperature. The fact that the pairing gap is almost constant below $T \sim 0.35$ MeV is connected with the nearly zero heat capacity as shown in Fig. 1. Above $T \sim 0.35$ MeV, the neutron pairing gap calculated by SLAP decreases smoothly with the temperature, while it does not vanish at high temperature up to 1 MeV. This indicates a gradual pairing transition from the superfluid state to the normal state in the hot nucleus $^{162}$Dy. The drop of the pairing gap results in an increasing number of the Cooper-pair-broken excited states, and thus a rapid increase of the heat capacity. In this way, the S shape of heat capacity curves shown in Fig. 1 is provided by the competition between the effects from temperature...
and pairing correlations.

For comparison, the neutron pairing gap energies calculated in the finite-temperature BCS and variation after projection BCS approaches \cite{17}, are also shown in Fig. 3. These results are systematically smaller than those obtained from SLAP, since as in Ref. \cite{17}, the pairing strength $G$ there is fixed to have a pairing BCS gap of 0.8 MeV. Nevertheless, it is not the magnitude but its variation tendency with the temperature is the main focus here. One could see that the finite-temperature BCS predicts a sharp transition from the superfluid to the normal phase. As it is well known, this sharp transition is connected to the particle number violation. Due to the restoration of particle number conservation, the pairing gap calculated in the VAP approach varies smoothly with the temperature, which is very similar to the present SLAP results. It should be noted that the present SLAP calculation has provided an exact treatment of pairing correlations without any particle number fluctuations, and it could be implemented easily and effectively to the self-consistent framework of the density functional theory \cite{24}. The extension of such model and its application in the investigation of the nuclear pairing in hot nuclei are in progress.

![Graph](image-url)

**FIG. 4.** (Color online). Neutron (a) and proton (b) seniority components $S_s$ for $^{162}$Dy with different seniority numbers $s = 0, 2, 4, 6, 8, 10, 12$ as functions of the temperature.

In order to provide a microscopic picture of the nuclear pairing transition, it is interesting to explore how many Cooper pairs would be broken with the increasing temperature in the
Here, we define the so-called \textit{seniority component}

\[ S_s = Z^{-1} \sum_{\beta \in \{s\}} \lambda(E_\beta) e^{-E_\beta/T}, \]  

where the summation runs over only the excited states with their seniority number \( s = 0, 2, 4, \cdots \). With such definition, the average energy could be rewritten as \( \langle E \rangle = \sum_{s, \beta} S_s E_\beta \in s \).

Here, it is very clear that the seniority component just represents the contribution of the excited states with each seniority number to the total average energy. In Fig. 4, the neutron and proton seniority components \( S_s \) with different seniority numbers \( s = 0, 2, 4, 6, 8, 10, 12 \) are shown as functions of the temperature, respectively. One could see that the \( s = 0 \) states contribute almost 100\% below \( T \sim 0.35 \) MeV, and this is again consistent with the vanishing heat capacity as shown in Fig. 1. With the temperature \( T \geq 0.35 \) MeV, the contribution of the \( s = 0 \) states fall down, while the contribution of the \( s = 2 \) states go up. This corresponds to the first inflexion point in the heat capacity curve. Above \( T \sim 0.6 \) MeV, the \( s = 4 \) states start to contribute and its contribution keeps increasing. Note that at \( T \sim 0.8 \) MeV, the contribution from the high energy \( s = 2 \) (one pair broken) states start to be extremely suppressed, and thus the increase of corresponding seniority component become slower. This is just the reason for the \( s = 2 \) states play crucial roles in the appearance of the S shape of the heat capacity curve. It might be possible that at higher temperature, where the high energy \( s = 2 \) (one pair broken) states are suppressed, a second S shape would be shown. Apart from these states, the contribution of all the other states with \( s \geq 6 \) is negligible. Furthermore, it is worthwhile to mention that only part of the Cooper pairs are broken at the high temperature \( T = 1 \) MeV, and this is just the reason for the nonvanishing pairing gap energies at high temperature as shown in Fig. 3.

V. SUMMARY

In summary, the pairing correlations in hot nuclei \(^{162}\text{Dy}\) have been investigated by covariant density functional theory, and the paring correlations have been treated by a shell-model-like approach, in which the particle number is conserved exactly. The heat capacities have been evaluated in the canonical ensemble theory, and a clear S shape of the heat capacity curve with respect to the temperature has been presented. It is found that the \( s = 2 \) (one pair broken) states play crucial roles in the appearance of the S shape of the heat
capacity curve. Due to the effect of the particle-number conservation, the pairing gap vary smoothly with the temperature, indicating a gradual transition from the superfluid to the normal state.

It should be noted that the present SLAP calculation has provided an exact treatment of pairing correlations without any particle number fluctuations. In the future, it will be also very interesting to investigate the odd-A hot nuclei with the present framework since the blocking effects here can be treated exactly. Moreover, the SLAP could be implemented easily and effectively to the self-consistent framework of the density functional theory, to develop a self-consistent finite temperature CDFT+SLAP model would also provide significant insights in the investigation of hot nuclei. The related work are still in progress.

ACKNOWLEDGMENTS

The authors are grateful to Shuangquan Zhang, Neculai Sandulescu, and Jie Meng for helpful discussions. This work was partly supported by “the Fundamental Research Funds for the Central Universities” (JUSRP1035), National Natural Science Foundation of China under Grant No. 11305077.

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. I (World Scientific, 1998).
[2] A. Schiller, A. Bjerve, and M. Guttormsen, Phys. Rev. C 63, 1 (2001).
[3] E. Melby, M. Guttormsen, J. Rekstad, A. Schiller, and S. Siem, Phys. Rev. C 63, 044309 (2001).
[4] E. Melby, L. Bergholt, M. Guttormsen, M. Hjorth-Jensen, F. Ingebrøtson, S. Messelt, J. Rekstad, A. Schiller, S. Siem, and S. W. Ødegård, Phys. Rev. Lett. 83, 3150 (1999).
[5] M. Guttormsen, R. Chankova, M. Hjorth-Jensen, J. Rekstad, S. Siem, A. Schiller, and D. J. Dean, Phys. Rev. C 68, 034311 (2003).
[6] S. Rombouts, K. Heyde, and N. Jachowicz, Phys. Rev. C 58, 3295 (1998).
[7] S. Liu and Y. Alhassid, Phys. Rev. Lett. 87, 1 (2001).
[8] K. Langanke, D. Dean, and W. Nazarewicz, Nuclear Physics A 757, 360 (2005).
[9] J. L. Egido, L. M. RoRobledo, and V. Martin, Phys. Rev. Lett. 85, 26 (2000).
[10] B. K. Agrawal, Sil Tapas, J. N. De, and S. K. Samaddar, Phys. Rev. C 62, 044307 (2000).
[11] N. Sandulescu, O. Civitarese, and R. Liotta, Phys. Rev. C 61, 044317 (2000).
[12] Y. F. Niu, Z. M. Niu, N. Paar, D. Vretenar, G. H. Wang, J. S. Bai, and J. Meng, Phys. Rev. C 88, 034308 (2013).
[13] M. Guttormsen, M. Hjorth-Jensen, and E. Melby, Phys. Rev. C 64, 034319 (2001).
[14] M. Guttormsen, M. Hjorth-Jensen, E. Melby, J. Rekstad, A. Schiller, and S. Siem, Phys. Rev. C 63, 044301 (2001).
[15] M. Sano and S. Yamasaki, Theor. Phys. 29, 397 (1963).
[16] A. L. Goodman, Nucl. Phys. A 352, 30 (1981).
[17] D. Gambacurta, D. Lacroix, and N. Sandulescu, Phys. Rev. C 88, 034324 (2013).
[18] A. L. Goodman, Phys. Rev. C 34, 1942 (1986).
[19] C. Reiss, M. Bender, and P.-G. Reinhard, Eur. Phys. J. A 6, 157 (1999).
[20] C. Esebbag and J. L. Egido, Nucl. Phys. A 552, 205 (1993).
[21] K. Esashika, H. Nakada, and K. Tanabe, Phys. Rev. C 72, 044303 (2005).
[22] J. Y. Zeng and T. S. Cheng, Nucl. Phys. A 405, 1 (1983).
[23] J. Y. Zeng, T. H. Jin, and Z. J. Zhao, Phys. Rev. C 50, 1388 (1994).
[24] J. Meng, J. Guo, L. Liu, and S. Zhang, Front. Phys. China 1, 38 (2006).
[25] J. Y. Zeng, Y. A. Lei, and T. H. Jin, Phys. Rev. C 50, 746 (1994).
[26] S. X. Liu and J. Y. Zeng, Phys. Rev. C 66, 067301 (2002).
[27] J. Y. Zeng, S. X. Liu, and Y. A. Lei, Phys. Rev. C 63, 024305 (2001).
[28] S. X. Liu, J. Y. Zeng, and E. G. Zhao, Phys. Rev. C 66, 024320 (2002).
[29] X. Wu, Z. H. Zhang, J. Y. Zeng, et al., Phys. Rev. C 83, 034323 (2011).
[30] S. X. Liu, J. Y. Zeng, and L. Yu, Nucl. Phys. A 735, 77 (2004).
[31] Z. H. Zhang, Y. A. Lei, J. Y. Zeng, et al., Phys. Rev. C 80, 034313 (2009).
[32] X. T. He, S. Y. Yu, J. Y. Zeng, et al., Nucl. Phys. A 760, 263 (2005).
[33] Z. H. Zhang, X. T. He, J. Y. Zeng, et al., Phys. Rev. C 85, 014324 (2012).
[34] Z. H. Zhang, J. Meng, E. G. Zhao, et al., Phys. Rev. C 87, 054308 (2013).
[35] L. Liu, J. Meng, and S. Q. Zhang, Chinese Phys. C 30, 299 (2006).
[36] L. Liu and P. W. Zhao, Chinese Phys. C 36, 818 (2012).
[37] Z. H. Zhang, P. W. Zhao, J. Meng, et al., Phys. Rev. C 87, 054314 (2013).
[38] P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996).
[39] J. Meng, H. Toki, S. G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).

[40] D. Vretenar, A. V. Afanasiev, G. A. Lalazissis, and P. Ring, Phys. Rep. 409, 101 (2005).

[41] Y. K. Gambhir, P. Ring, and A. Thimet, Ann. Phys. (N. Y.) 194, 132 (1990).

[42] L. F. Canto, P. Ring, and J. O. Rasmussen, Phys. Lett. B 161, 21 (1985).

[43] J. L. Egido, P. Ring, J. Iwasaki, and H. J. Mang, Phys. Lett. B 154, 1 (1985).

[44] Y. R. Shimizu, J. D. Garrett, R. A. Broglia, M. Gallardo, and E. Vigezzi, Rev. Mod. Phys. 61, 131 (1989).

[45] T. Sumaryada and A. Volya, Phys. Rev. C 76, 024319 (2007).

[46] P. Ring, Y. Gambhir, and G. Lalazissis, Comput. Phys. Commun. 105, 77 (1997).

[47] W. H. Long, J. Meng, N. Giai, and S. G. Zhou, Phys. Rev. C 69, 034319 (2004).