Mathematic modelling of filtering data of an oil reservoir based on successive random additions

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Abstract: The article presents a method of successive random additions for modelling intermediate values between wells, the data of which are obtained from deposits by special geographic probes. The main idea is to identify the relationship between the data obtained and further calculations in the form of pseudo-values that can be used for further research. Fractal data filtering allows modelling pseudo-random values based on correlated values of known data, simulating random motion. This approach will allow a more detailed study of the petrophysical properties of individual sections of the formation.

1. Introduction
The period of recovering "light" oil is almost over. Extraction is hampered by various complications [1]. Specialists in the oil industry are making great efforts to design new methods of developing hard-to-reach oil deposits [2]. But the extraction of hard-to-reach reserves does not always bring profit. One such example is oil shale deposits which are difficult to reach. Therefore, reservoir development technologies with shale reservoirs can be effectively implemented only after a comprehensive study of the filtration mechanisms. The relevance of this work is the possibility of applying the method for the study of shale deposits, the method of successive random additions makes it possible to compensate for the missing data and conduct a detailed study of the formation.

The purpose of this study is to apply mathematical modelling using computer technology and develop a general method and description of the formation that can be used for further research in the oil and gas industry. The development of the reservoir description technique was achieved due to the study of geostatistics, which allows modelling the phenomena of processes in a geographic space. The study of fractal geometry made it possible to apply the method of successive random additions to model intermediate values between wells, the data of which were obtained from deposits by special geographic probes. The main idea of this method is to identify the relationship between the data obtained and further calculations in the form of pseudo-values that can be used for further research. Fractal data filtering allows modelling pseudo-random values based on correlated values of known data, simulating random motion.

2. Research
For the purpose of further research, the application to the method is the use of such known methods as the Archie, Waxman-Smits model, which allow an evaluation of the petrophysical properties of the obtained pseudo-values between the wells [3], [4]. This solution allows building gamma-ray logs, investigating permeability, building a lateral section of layers of the formation at a given depth. Application of these methods will allow presenting the obtained data in the form of blocks into which
the investigated oil-bearing layer is divided. This approach will allow a more detailed study of the petrophysical properties of individual sections of the formation.

A comprehensive application of numerous research methods allows for a detailed study of the reservoir and identification of potential porous rocks.

In recent years, geostatistical and fractal methods of geometry have made a significant contribution, which allows performing a qualitative description of oil reservoirs.

In this study, methods of geostatistics and fractal geometry were used, which are the main research methods. Application of these methods allowed developing a quantitative and computerized method of reservoir description. In this method, basic data are used to determine the lithology of the reservoir and the petrophysical properties. Fractal geometry concepts are used to estimate the fractal character (transmission coefficient, H) of logs (gamma logarithm, porosity of logarithm and resistance). The estimation of coefficients H allows generating pseudo (or fractal) distributions of these logs throughout the reservoir.

3. Results and discussion

The reliability of the developed algorithms was verified on the basis of data obtained from wells, the construction of fractal interpolation at a given depth between two wells.

Figure 1 shows fractal well interpolation, where Z1 and Z2 are the sources of data obtained from real wells, Z3 is a pseudo-well, the data were obtained using the method. Possible pseudo-wells are at points Z4, Z5.

**Figure 1.** Fractal interpolation of wells (depth 2879.1-2979.2 m).

An additional implementation of the method is the construction of an image without loss of quality, the missing intermediate data is calculated, and a new image is formed from the obtained data (Figure 2).
Figure 2. Fractal interpolation of wells (depth 2919-2929 m).

The method of interpolating successive random additions is based on the fact that the variance of increments (variograms) of random fractals is given as:

$$2\gamma(l) = \sum \{|Z(x+l) - Z(x)|^2\} = V_H l^{2H}.$$  

where $V_H$ is equivalent to $\sigma$.

Below are the steps used in this method of stochastic interpolation, with the correlation structure of fractal Brownian motion:

- a variance is calculated that shows the changes in the data from the received wells;
- values are calculated in the middle of the interval between wells using linear interpolation or kriging (linear regression);
- a random number is added to the approximate interpolated values taken from a normal (Gaussian) distribution with a mean value and variance:

$$\sigma_1 = \frac{\sigma_0}{2^H}, \quad \sigma_1^2 = \frac{\sigma_0^2}{2^{2H}}.$$  

This equation follows the power law:

$$\gamma(rl) = r^{2H} \gamma(l),$$  

where $r = \frac{1}{2}$.

Then:

$$\gamma(\frac{1}{2}l) = \frac{\gamma(l)}{2^{2H}}.$$  

Iterations must be repeated with all interpolated values until the optimal value is obtained. For each $n$-th iteration, a random value is added to each interpolated value with variance $\sigma_n^2$.
\[
\sigma_n^2 = \frac{\sigma_{n-1}^2}{2^{2nH}} = \frac{\sigma_0^2}{2^n2^{2H}}. 
\] (5)

Next, iterations of equations with a value \( R_{ij} \) are given that denote a random number taken from a normal distribution with a mean value and variance.

**Iteration 1:**

\[
\begin{align*}
\sigma_1 &= \frac{\sigma_0}{2^H}; \\
Z_{1,1} &= Z_{1,0} + \sigma_1 R_{1,1}; \\
Z_{2,1} &= Z_{2,0} + \sigma_1 R_{2,1}; \\
Z_{3,1} &= \left[ \frac{Z_{1,0} + Z_{2,0}}{2} \right] + \sigma_1 R_{3,1}.
\end{align*}
\] (6)

**Iteration 2:**

\[
\begin{align*}
\sigma_2 &= \frac{\sigma_1}{2^H}; \\
Z_{1,2} &= Z_{1,1} + \sigma_2 R_{1,2}; \\
Z_{2,2} &= Z_{2,1} + \sigma_2 R_{2,2}; \\
Z_{3,2} &= Z_{3,1} + \sigma_2 R_{3,2}; \\
Z_{4,2} &= \left[ \frac{Z_{1,1} + Z_{3,1}}{2} \right] + \sigma_2 R_{4,2}; \\
Z_{5,2} &= \left[ \frac{Z_{2,1} + Z_{3,1}}{2} \right] + \sigma_2 R_{5,2}.
\end{align*}
\] (7)

**Iteration 3:**

\[
\begin{align*}
\sigma_3 &= \frac{\sigma_2}{2^H}; \\
Z_{1,3} &= Z_{1,2} + \sigma_3 R_{1,3}; \\
Z_{2,3} &= Z_{2,2} + \sigma_3 R_{2,3}; \\
Z_{3,2} &= Z_{3,1} + \sigma_3 R_{3,3}; \\
Z_{4,3} &= Z_{4,2} + \sigma_3 R_{4,3}; \\
Z_{5,3} &= Z_{5,2} + \sigma_3 R_{5,3}; \\
Z_{6,3} &= \left[ \frac{Z_{1,2} + Z_{4,2}}{2} \right] + \sigma_3 R_{6,3}.
\end{align*}
\] (8)
\[ Z_{7,3} = \left[ \frac{Z_{4,2} + Z_{3,2}}{2} \right] + \sigma_{3} R_{7,3}; \]
\[ Z_{8,3} = \left[ \frac{Z_{3,2} + Z_{5,2}}{2} \right] + \sigma_{3} R_{8,3}; \]
\[ Z_{9,3} = \left[ \frac{Z_{5,2} + Z_{2,2}}{2} \right] + \sigma_{3} R_{9,3}. \]

Iteration 4:

\[ \sigma_{4} = \frac{\sigma_{3}}{2^H}; \]
\[ Z_{1,4} = Z_{1,3} + \sigma_{4} R_{1,4}; \]
\[ Z_{2,4} = Z_{2,3} + \sigma_{4} R_{2,4}; \]
\[ Z_{3,4} = Z_{3,4} + \sigma_{4} R_{4,4}; \]
\[ Z_{4,4} = Z_{4,4} + \sigma_{4} R_{4,4}; \]
\[ Z_{5,4} = Z_{5,4} + \sigma_{4} R_{5,4}; \]
\[ Z_{6,4} = Z_{6,4} + \sigma_{4} R_{5,4}; \]
\[ Z_{7,4} = Z_{7,4} + \sigma_{4} R_{7,4}; \]
\[ Z_{8,4} = Z_{8,3} + \sigma_{4} R_{8,4}; \]
\[ Z_{9,4} = Z_{9,3} + \sigma_{4} R_{9,4}; \]
\[ Z_{10,4} = \left[ \frac{Z_{1,3} + Z_{6,3}}{2} \right] + \sigma_{4} R_{10,4}; \]
\[ Z_{12,4} = \left[ \frac{Z_{4,3} + Z_{7,3}}{2} \right] + \sigma_{4} R_{12,4}; \]
\[ Z_{13,4} = \left[ \frac{Z_{7,3} + Z_{3,3}}{2} \right] + \sigma_{4} R_{13,4}; \]
\[ Z_{14,4} = \left[ \frac{Z_{5,3} + Z_{9,3}}{2} \right] + \sigma_{4} R_{14,4}. \]

Where: \( \sigma_{i} \) - root-mean-square deviation for the given iteration, \( R_{j,i} \) - a random number taken from the normal distribution for the j-well, i - iteration number, \( Z_{j,i} \) - data sources of the j-well, i iteration.

This method introduces small changes to the initial data (input values), the final result of distribution corresponds to the input values and during the construction the correlation structure is preserved.
In the study, the Archie [3], Simandoux [4], Waxman [9] models were used to evaluate reservoir petrophysical parameters from logs, both at input wells and in generated pseudo-wells. The application of these models depends on the type of rock lithology. The Archie model is used mainly for pure reservoir rocks with an effective shale content of less than ten percent. The Waxman-Smith and Simandoux models are applied to rocky rocks (sandy sands) with an effective shale content of more than ten percent.

Four modules of programs were developed using geostatistical, fractal geometry and methods of interpreting wells. In these programs, geostatistical methods for constructing variograms, kriging, and conditional simulation were implemented, following the recommendations given in the references.

The method of R/S-analysis of fractal geometry and the method of successive random additions was implemented in these programs, following the recommendations in [5-8].

The Archie, Simandoux and Waxman-Smith models designed to study wells have been implemented. Program modules were written in the MATLAB language and are named VARGRAM, GEOFRACT, GEOSTAT1 and GEOSTAT2, respectively. The developed programs in the MATLAB language can be transferred to the source code C++ using the built-in generator.

4. Conclusion
The formation description technique developed in this study offers many applications for solving problems that arise in the study of hydrocarbon reservoirs. Graphs constructed at a given depth are important for well experts because their study can give a broad idea of the distribution of gamma radiation, resistivity and porosity in the area of missing wells, which allows a petrophysical study of hydrocarbon reservoirs.

Application of mathematical modelling with the use of computer technologies allowed developing a computer program for the analysis of well data and subsequent interpretation in the form of graphs, sections of the reservoir, areal maps. The study revealed an increase in the accuracy of generated data with an increase in the wells in question.

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