HZ Recoil Mass and Cross Section Analysis in ILD

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Abstract

This note describes the details of a simulation study of the Higgs boson production for processes in which the Higgs is produced together with a well measurable di-lepton system using the proposal of the ILD detector for its Letter of Intent \cite{ILD-LI}. The analysis is optimised for the measurement of the Higgs-strahlung process, i.e. $e^+e^- \rightarrow HZ$. The cross section can be determined with a precision of 2-3\% and by combining the decay channels a precision of $\sim 30$ MeV is obtained for the mass of the Higgs boson. The background can be largely reduced and the analysis exhibits a sensitivity to the configuration of the accelerator.

1 Introduction

The understanding of electro-weak symmetry breaking is intimately coupled to the study of the Higgs boson. It arises as a consequence of the observation of massive gauge bosons which can be generated by the spontaneous breaking of the $SU(2) \times U(1)$ symmetry of the electroweak Lagrangian.

If existing, a Higgs boson with a mass $M_H$ of 120 GeV as favoured by recent analyses of electro-weak data \cite{electroweak-data} will be discovered at the LHC or even at the TEVATRON. The ILC will allow for the detailed investigation of the nature of the Higgs boson as has been demonstrated in \cite{ILC-Higgs} and references therein. The relevant processes for the present study are the recoil reaction $e^+e^- \rightarrow HZ \rightarrow Hf\bar{f}$ (where $f=\text{leptons and quarks}$), also called Higgs-strahlung, or $e^+e^- \rightarrow He^+e^-$, also called ZZ fusion. The Feynman diagrams are shown in Figure \ref{fig:1}. Please note that the cross section of the Higgs-strahlung dominates largely over that of the ZZ fusion. Hence, the analysis will be optimised for the measurement of the Higgs-strahlung process.

By detecting the decay products of the Z boson, the introduced processes and in particular the Higgs-strahlung process allow for the search of Higgs signals without any further assumption on its decay modes. In contrast to the LHC, the initial state is very well known at the ILC. These two items together allow for an unbiased search for the Higgs boson also called Model Independent Analysis which is only possible at a Lepton Collider such as the

\cite{ILC-LI}.
ILC. The presumably cleanest way to study the Higgs is given by the process \( e^+e^- \rightarrow HZ \) and the subsequent decay \( Z \rightarrow \mu^+\mu^- \) or \( Z \rightarrow e^+e^- \), i.e. searching for the decay leptons of the well known \( Z \) boson. These channels, also named \( \mu\mu X \)-channel and \( eeX \)-channel hereafter, will be examined in detail in this note for a centre-of-mass energy of \( \sqrt{s} = 250 \text{ GeV} \) as proposed in the definition of the benchmark scenarios for the Letter of Intent Studies for ILC detectors [6].

2 ILD Detector

A detailed description of the current model of the ILD detector can be found elsewhere [1]. The \( z \)-axis of the right handed co-ordinate system is given by the direction of the incoming electron beam. Polar angles given in this note are defined with respect to this axis. The most important sub-detectors for this study are described in the following.

- The vertex detector consists of three double layers of silicon extending between 16 mm and 60 mm in radius and between 62.5 mm and 125 mm in \( z \) direction. It is designed for an impact parameter resolution of \( \sigma_{r\phi} = \sigma_{rz} = 5 \pm 10/(p\sin^2\theta) \mu\text{m} \).

- The measurement of charged tracks is supported by an inner Silicon Tracker (SIT) in the central region and by a set of silicon disks in forward direction, i.e. towards large absolute values of \( \cos\theta \).

- The ILD detector contains a large Time Projection Chamber (TPC) with an inner sensitive radius of 395 mm and an outer sensitive radius of 1743 mm. The half length in \( z \) is 2250 mm. Recent simulation studies confirm that the momentum of charged particle tracks can be measured to a precision of \( \delta(1/P_T) \sim 2 \times 10^{-5} \text{ GeV}^{-1} \). Here \( P_T \) denotes the transverse component of the three momentum \( P \) of the particles.

- The electromagnetic calorimeter is a SiW sampling calorimeter. Its longitudinal depths of 24 \( X_0 \) allows for the complete absorption of photons with energies of up to 50 GeV.
as relevant for the studies here. The simulated energy resolution of the electromagnetic calorimeter is $\Delta E = 15\%/\sqrt{E[\text{GeV}]}$

- The hadronic calorimeter surrounds the electromagnetic calorimeter and comprises 4.5 interaction length $\lambda_I$. Two proposals exist for the hadronic calorimeter. A digital variant consisting of steel absorbers and gas RPC chambers with a pixel size of $1 \times 1 \text{cm}^2$ as active material. The second one features scintillating tiles with size of $3 \times 3 \text{cm}^2$ as active material. The latter option is employed in the present work.

In the current design of the ILC the initial beams enter with a crossing angle of 14 mrad. This crossing angle is not taken into account in the present study.

3 Event Generation, Detector Simulation and Event Reconstruction

All data analysed for this note have been centrally produced by the ILD Group in autumn/winter 2008/09 based on generator files known as SLAC samples. For the event generation the version 1.40 of the event generator WHIZARD [7] has been used. The incoming beams have been simulated with the GUINEA-PIG package [8]. The energy of the incoming beams is smeared with an energy spread of 0.28% for the electron beam and with 0.18% for the positron beam. In addition the energy is modulated by beamstrahlung. The impact on the precision of the physics result of this uncertainty will be discussed below. The generated signal and background samples are given in the Table 1 for the beam polarisation mode $e^-e^L e^+e^R$:

- $P_e^- = -80\%$ and $P_e^+ = +30\%$

and in Table 2 for the beam polarisation mode $e^-e^R e^+e^L$:

- $P_e^- = +80\%$ and $P_e^+ = -30\%$.

The initially generated samples of the signal are combined such that they yield $L = 10 \text{ab}^{-1}$ in each of the polarisation modes. For background samples the integrated luminosity is mostly larger than 250 $\text{fb}^{-1}$. Where it is smaller, it is still provided that the samples contain considerable statistics. Note, that in Tables 1 and 2 the background samples have been grouped by the resulting final state.

Due to the large cross section of the Bhabha Scattering, i.e. $e^+e^- \rightarrow e^+e^-$ and muon pair production, i.e. $e^+e^- \rightarrow \mu^+\mu^-$, pre-cuts have been applied in order to reduce the simulation time. These cuts are given in Table 3 and will be later on referred to as Pre-cuts.

Here, $M_{e^+e^-}$ and $M_{\mu^+\mu^-}$, respectively, are the invariant mass of the di-lepton system for signal events, while $P_{T_{e^+e^-}}$ and $P_{T_{\mu^+\mu^-}}$ denote the transverse momentum calculated from the vectorial sum of the two leptons.

The generated events are subject to a detailed detector simulation. The simulation is performed with the MOKKA [10] software package which provides the geometry interface to the GEANT4 [11] simulation toolkit. The event reconstruction is performed using the MarlinReco framework. For this study the versions as contained in the Software Package

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4 Please note, that due to a bug in the luminosity spectrum in the initial generation, the background has been re-weighted to the correct spectrum according to [9].
Table 1: Processes and cross sections for polarisation mode $e^{-}e^{+}$. The signal is indicated by bold face letters; the cross-section in the parentheses of $e^{+}e^{-}$ and $\mu^{+}\mu^{-}$ are that after Pre-Cuts, see Table 3 for the Pre-Cuts definition.

| Process | Cross-Section |
|---------|---------------|
| $\mu\mu X$ | 11.67 fb |
| $\mu\mu$ | 10.44 pb (88.46 fb) |
| $\tau\tau$ | 213.22 fb |
| $\mu\mu\nu\nu$ | 481.68 fb |
| $\mu\mu ff$ | 1196.79 fb |
| $ee X$ | 12.55 fb |
| $ee$ | 17.30 nb (357.14 fb) |
| $\tau\tau$ | 6213.22 fb |
| $ee\nu\nu$ | 648.51 fb |
| $ee ff$ | 4250.58 fb |

Table 2: Processes and cross sections for polarisation mode $e^{-}e^{+}$. The signal is indicated by bold face letters; the cross-section in the parentheses of $e^{+}e^{-}$ and $\mu^{+}\mu^{-}$ are that after Pre-Cuts, see Table 3 for the Pre-Cuts definition.

| Process | Cross-Section |
|---------|---------------|
| $\mu\mu X$ | 7.87 fb |
| $\mu\mu$ | 8.12 pb (58.26 fb) |
| $\tau\tau$ | 4850.05 fb |
| $\mu\mu\nu\nu$ | 52.37 fb |
| $\mu\mu ff$ | 1130.01 fb |
| $ee X$ | 12.55 fb |
| $ee$ | 17.30 nb (357.14 fb) |
| $\tau\tau$ | 6213.22 fb |
| $ee\nu\nu$ | 648.51 fb |
| $ee ff$ | 4250.58 fb |

Table 3: List of Pre-cuts applied to Bhabha scattering and muon pair production in order to reduce the simulation time.

| Pre-Cuts | Condition |
|----------|-----------|
| $e^{+}e^{-} \rightarrow e^{+}e^{-}$ | $|\cos\theta_{e^{+}e^{-}}| < 0.95$ |
| $Me^{+}e^{-} \in (71.18, 111.18)$ GeV | $P_{Te^{+}e^{-}} > 10$ GeV |
| $M_{recoil} \in (105, 165)$ GeV | $M_{\mu^{+}\mu^{-}} \in (71.18, 111.18)$ GeV |
| $P_{T\mu^{+}\mu^{-}} > 10$ GeV | $M_{recoil} \in (105, 165)$ GeV |

$ILCsoft$ $v01-06$ [10] are employed. The main output of this framework for the present study are the so-called $LDC$ $Tracks$ which is a combination of track segments measured in the vertex detector, the Silicon Inner Tracker and the TPC or Forward Tracking Disks. Their momenta are compared with the energy of calorimeter clusters composed from signals in the electromagnetic and hadronic calorimeter for the particle identification.

4 Signal Selection and Background Rejection

The signal is selected by identifying two well measured leptons in the final state which yield the mass of the Z boson. The mass $M_{recoil}$ of the system recoiling to the di-lepton system is computed using the expression:

$$M_{recoil}^2 = s + M_{Z}^2 - 2E_{Z}\sqrt{s}$$

Here $M_{Z}$ denote the mass of the Z boson as reconstructed from the di-lepton system and $E_{Z}$ its corresponding energy. A number of background processes have to be suppressed. Techniques of background suppression similar to those presented in this note were already introduced in [12]. This section firstly defines the criteria of lepton identification and then
addresses the means to suppress the background. This will be done under two assumptions: 1) model independent 2) model dependent, i.e. assuming a Higgs boson as predicted by the Standard Model. The latter excludes decay modes in which the Higgs boson decays invisibly.

4.1 Lepton Identification

The task is to identify the muons and electrons produced in the decay of the Z boson. In a first step, the energy deposition in the electromagnetic calorimeter ($E_{\text{ECAL}}$), the total calorimetric energy $E_{\text{total}}$ and the measured track momentum $P_{\text{track}}$ are compared accordingly for each final state particle. The lepton identification is mainly based on the assumption that an electron deposits all its energy in the electromagnetic calorimeter while a muon in the considered energy range, see Figure 2, passes both the electromagnetic and the hadronic calorimeter as a minimal ionising particle. The observables and cut values are summarised in Table 4.1. The motivation of the cut values can be inferred from Figure 3 where the spectra for the corresponding lepton type in the relevant momentum range $P > 15$ GeV are compared with those from other particles are displayed.

![P of lepton candidates](image)

Figure 2: Momentum range of the final state leptons as produced in $Z \rightarrow \mu^+ \mu^-$ decays from $e^+e^- \rightarrow HZ$ events at $\sqrt{s} = 250$ GeV.

| $E_{\text{ECAL}}/E_{\text{total}}$ | $\mu$-Identification | $e$-Identification |
|----------------------------------|----------------------|-------------------|
| $E_{\text{total}}/P_{\text{track}}$ | $< 0.5$               | $> 0.6$            |
|                                  | $< 0.3$               | $> 0.9$            |

The criteria to estimate the quality of the lepton identification and hence the signal selection are the Efficiency and Purity. These are defined as follows:

$$\text{Efficiency} = \frac{N_{\text{true}} \cap \text{iden}}{N_{\text{true}}}$$
Figure 3: Distributions of the variables for lepton identification of lepton candidates and other particles with $P > 15$ GeV.

$$\text{Purity} = \frac{N_{\text{true} \cap \text{idem}}}{N_{\text{idem}}}$$

Here $N_{\text{true}}$ defines the generated number of the corresponding lepton type and $N_{\text{idem}}$ defines the reconstructed number of the corresponding lepton type according to the selection criteria. For electrons and muons with $P > 15$ GeV in the signal samples the obtained values are listed in Table 4.

|       | $\mu$ ID in $\mu\mu X$ | e ID in eeX |
|-------|-------------------------|-------------|
| $N_{\text{true}}$ | 31833                   | 34301       |
| $N_{\text{true} \cap \text{idem}}$ | 31063                   | 33017       |
| $N_{\text{idem}}$ | 33986                   | 34346       |
| Efficiency | $97.6\%$                | $96.3\%$    |
| Purity    | $91.4\%$                | $96.1\%$    |

Table 4: Lepton ID Efficiency and Purity for reconstructed particles with $P > 15$ GeV.

The efficiencies and purities are well above 95% except for the purity of the muon identification. This is caused by final state charged pions which pass the detector as minimal ionising...
particles and which are indistinguishable from muons with the applied selection criteria. This deficiency is partially balanced by the fact that two leptons of the same type with opposite charge are required for the reconstruction of the Z boson and that they should yield the mass of the Z boson. Indeed, using the above selection cuts, the efficiency to identify both leptons from the Z boson decay is 95.4% for the case $Z \rightarrow \mu\mu$ and 98.8% for the case $Z \rightarrow ee$. Note, that the cut on $P > 15$ GeV has been omitted in this case.

### 4.2 Track Selection

As the invariant mass of the Z boson and thus the recoil mass will be calculated from the four momenta of the LDC Tracks, badly measured LDC Tracks need to be discarded from the analysis. The track quality can be estimated by the ratio $\Delta P/P$ where the uncertainty $\Delta P$ is derived from the error matrix of the given track by error propagation.

Figure 4: 2D $\Delta P/P^2$ distribution vs. $\cos \theta$ (left) and $\Delta P/P^2$ distribution vs. track momentum (right) of muon candidates

Figure 5: 2D $\Delta P/P^2$ distribution vs. $\cos \theta$ (left) and $\Delta P/P^2$ distribution vs. track momentum (right) of electron candidates

The Figures 4 and 5 show, for muons and electrons separately, the dependency of $\Delta P/P^2$ on the polar angle $\cos \theta$ and on the track momentum $P$. For reasons discussed in the following the latter has been restricted to $|\cos \theta| < 0.78$, i.e. the central region. For both variables the distributions exhibit for muon tracks a narrow band with well measured momenta equivalent to small $\Delta P/P^2$. The track quality decreases as expected towards large $|\cos \theta|$, i.e. towards the acceptance limits of the TPC which motivates the restriction to the central region when displaying $\Delta P/P^2$ versus $P$. These distributions show a decrease in track quality towards small particle momenta as expected from multiple scattering effects. Beyond that, it is clearly visible that for electrons the situation is much more diluted and the number of badly
measured tracks is significantly higher than that for muons. This can be explained by the 
Bremsstrahlung of the electrons in the detector material.

The procedure for track rejection is developed for the better measured muon induced 
tracks:

- For $|\cos \theta| < 0.78$ the shape of $\Delta P/P^2$ versus $P$ is approximated by:

$$\delta(1/P) = \Delta P/P^2 = a \oplus b/P = c(P);$$

with $a = 2.5 \times 10^{-5} \text{GeV}^{-1}$ and $b = 8 \times 10^{-4}$. (1)

Tracks are rejected if $\delta(1/P) > 2c(P)$.

- For $|\cos \theta| > 0.78$ tracks are rejected if $\Delta P/P^2 > 5 \times 10^{-4} \text{GeV}^{-1}$.

The cuts are indicated in Figure 4 and 5 and underline that tracks created by electrons 
are rejected considerably more often which will reduce the number of reconstructed Z bosons 
in the corresponding channel.

4.3 Background Rejection

The recoil analysis is based on the identification of the di-lepton system as produced by the 
decay of the Z boson. It is thus necessary to distinguish the processes which lead to two 
leptons in the final state as given in Table 1 and Table 2 from the ones produced in the 
Higgs-strahlung process.

For the Higgs-strahlung process the invariant mass of the di-lepton system $M_{dl}$ should be 
equal to the Z boson mass while the invariant mass of the recoiling system, $M_{recoil}$ is expected 
to yield the introduced mass of the Higgs boson of 120 GeV. It is unlikely that combinations 
of background processes fulfil both conditions at once. This argumentation is supported by 
Figures 6 and 7 which show the invariant mass distributions for the di-lepton system and 
the recoil mass for both, the di-lepton system consisting of muons and the di-lepton system 
consisting of electrons. These distributions suggest to restrict the analysis to the following 
mass ranges:

- $80 < M_{dl} < 100 \text{GeV}$
- $115 < M_{recoil} < 150 \text{GeV}$

In a next step the selection is to be made by means of the different kinematic properties. 
In the following the variables used to distinguish signal events from background events will 
be introduced.

- Acoplanarity $acop$, see Figure 8: As for $e^+e^-$ collisions with beams of equal energy the 
centre-of-mass system is at rest, it is expected that in processes in which the leptons 
are produced at the $Z^*$ vertex these two leptons are back-to-back in azimuth angle. 
The distance in azimuth angle is expressed by the acoplanarity $acop$, defined as 
$acop = |\phi_{e^+} - \phi_{e^-}|$, where $\phi_{e^\pm}$ is the azimuth angle of the an individual lepton of the di-
lepton system. If the particles are produced from a intermediate particle with a given 
transverse momentum, the exact back-to-back configuration is modulated. Therefore a 
cut on $0.2 < acop < 3$ is applied.
Figure 6: Normalised signal and background distributions of the invariant mass of the di-lepton system $M_{dl}$ for the $\mu^+\mu^-X$ (top) and $e^+e^-X$ Channel (bottom). Here, $\tau\tau$ refers to the $\mu\mu$ or $ee$ created in the decay of $\tau\tau$. Note that the Pre-cuts defined in Section 3 have been applied to the $\mu\mu$ background sample.
Figure 7: Normalised signal and background distributions of the recoil mass $M_{\text{recoil}}$ distributions for the $\mu^+\mu^- X$ (top) and the $e^+e^- X$ Channel (bottom). Here, $\tau\tau$ refers to the $\mu\mu$ or $ee$ created in the decay of $\tau\tau$. Note that the Pre-cuts defined in Section 3 have been applied to the $\mu\mu$ background sample.
Figure 8: Normalised signal and background distributions of the acoplanarity \( \text{acop} \) for the \( \mu^+\mu^- X \)-Channel (top) and \( e^+e^- X \)-Channel (bottom). Here, \( \tau\tau \) refers to the \( \mu\mu \) or \( ee \) created in the decay of \( \tau\tau \). Note that the Pre-cuts defined in Section 3 have been applied to the \( \mu\mu \) background sample.
• Transverse Momentum $P_{Tdl}$ of the di-lepton system, see Figure 9. As the Higgsstrahlung process can be interpreted as a two body decay, both bosons gain equal transverse momentum which is conserved by their decay products. The total final state for muon pair production or Bhabha Scattering has in first approximation no transverse momentum. In order to suppress this background, a cut $P_{Tdl} > 20$ GeV of the di-lepton system is applied. This cut cannot suppress events in which initial state radiation of the incoming beams leads to a transverse momentum of the colliding system. This case will be discussed separately.

• $\cos\theta_{missing}$: this cut discriminates events which are unbalanced in longitudinal momentum, essentially, those of the type $e^+e^- \rightarrow l^+l^-\gamma$. The distributions in Figure 10 motivate a cut on $|\cos\theta_{missing}| < 0.99$.

The last introduced cut also suppresses events with initial state radiation happening approximately collinear with the incoming beams. The final state in $e^+e^- \rightarrow \mu^+\mu^- (e^+e^-)$ can, however, gain sizeable transverse momentum by initial state radiation of a high energetic photon. Figure 11 shows the correlation between the transverse momentum $P_{T\gamma}$ of a detected high energetic photon, assumed to be created by initial state radiation, and the transverse momentum $P_{Tdl}$ of the di-lepton system for both, events in which only a muon pair is created at the $Z^*$- Boson vertex and signal events. The first type shows a clear correlation in transverse momentum. In order to suppress this background the variable $\Delta P_{Tbal.} = P_{Tdl} - P_{T\gamma}$ is introduced which is shown in Figure 12 for signal events and background events superimposed with each other. By selecting events with $\Delta P_{Tbal.} > 10$ GeV, a considerable fraction of background can be suppressed. It should finally be noted that background events of type $e^+e^- \rightarrow \mu^+\mu^- (e^+e^-)$ which are undergoing final state radiation are suppressed by the requirement that the lepton should yield the $Z$ boson mass.

The number of events remaining after each of these cuts for signal and backgrounds are given in Tabs. 5 through 8 for two beam polarisations and the different compositions of the di-lepton system. The combination of cuts will be later referred as MI Cuts. Please note that the cut variables $f_L$ and $|\Delta\theta_{2\ell k}|$ will be introduced later.

From the tables the following conclusions can be drawn

• The requirement to have two well measured leptons retains always more than 95% of the signal while it suppresses in most of the cases the largest part of background events.

• The requirement of a minimum $P_{Tdl}$ of the di-lepton system is very efficient for events in which the di-lepton system is produced directly at the $Z^*$ vertex, see Figure 1. This type of background is further reduced by comparing the transverse momentum of the di-lepton system with the transverse momentum of a radiative photon. The cut is particularly efficient to suppress background events generated by Bhabha Scattering.

• Although largely suppressed, the number of events generated by Bhabha background still exceeds the number of signal events. This remains an irreducible background.

• The acoplanarity $acop$ is particularly efficient against background in which the di-lepton system is composed by $\tau$- Leptons. The larger mass of this particle reduces the phase space for radiative processes. Hence this lepton type is more often produced in a back-to-back configuration than the lighter lepton types.

The tables demonstrate that mostly events in which the di-lepton system is produced at the $Z^*$ vertex can be efficiently rejected by the defined cuts. The background by events
Figure 9: Normalised signal and background distribution of the transverse momentum $P_{Tdl}$ of the dilepton system for the $\mu^+\mu^-X$ Channel (top) and the $e^+e^-X$-Channel (bottom). Here $\tau\tau$ refers to the $\mu\mu$ or $ee$ created in the decay of $\tau\tau$. Note that the Pre-cuts defined in Section 3 have been applied to the $\mu\mu$ background sample.
Figure 10: Normalised signal and background distributions of the $|\cos \theta_{\text{missing}}|$ of the system of undetected particles for the $\mu^+ \mu^- X$-Channel (top) and $e^+ e^- X$-Channel (bottom).
The \( \gamma \)-pair production leads to a flat distribution in the di-lepton mass spectrum in the Z-mass region. The shape of the invariant mass \( M_{dl} \) of the di-lepton system and hence also that of the transverse momentum \( P_{Tdl} \) of the di-lepton system can be employed to suppress background from \( \gamma \)-pair production.

- The production of Z boson and W boson pairs happens predominantly via exchange reactions which lead to a strong increase of the differential cross section towards large absolute values of the cosine of the polar angle. On the contrary, the Higgs-strahlung process is expected to decrease towards the forward and backward direction. Therefore the polar angle spectrum as shown in Figure 13 of the di-lepton system is expected to discriminate between signal events and background from Z and W pair production.

- The acollinearity, defined as \( acol = acos(\mathbf{P}_{\ell^+} \cdot \mathbf{P}_{\ell^-}/|\mathbf{P}_{\ell^+}||\mathbf{P}_{\ell^-}|) \), is sensitive to the boost of the di-lepton system. In case of Z pair production the decay products are expected to be boosted more strongly than in the case of Higgs-strahlung. This results in a
Figure 13: Normalised signal and background distribution of the cosine of the polar angle \( \cos \theta_{dl} \) of the di-lepton system for the \( \mu^+\mu^- X \) Channel (top) and the \( e^+e^- X \)-Channel (bottom). Here \( \tau\tau \) refers to the \( \mu\mu \) or \( ee \) created in the decay of \( \tau\tau \). Note that the Pre-cuts defined in Section 3 have been applied to the \( \mu\mu \) background sample.
different position of the Jacobian Peak in the $d\sigma/d(acol)$ differential cross section as demonstrated in Figure [14].

The likelihood of an event to be the signal is defined as $L_S = \prod P_{S_i}$, where the $P_{S_i}$ is the probability of the event to be the signal according to the PDF of the signal of the $i$th selection variable. Similarly, the likelihood of an event to be the background is defined as $L_B = \prod P_{B_i}$. Hereafter, the Likelihood Fraction is defined as $f_L = L_S/(L_S + L_B)$, which is within $(0, 1)$. The Figures [15] through [18] outline the optimisation procedure in the likelihood analysis separately for the two analysis channels and polarisation modes using the four variables introduced above, for details see [13]. It is clearly visible that the separation between signal and background improves towards small values of $f_L$. The cut on $f_L$ is optimised according to the maximum in the significance $S/\sqrt{S+B}$ where $S$ and $B$ are the number of remaining signal and background events, respectively. The cut on $f_L$ is adjusted for each polarisation mode of the incoming beams and the type of the di-lepton system under study.

The final number of events also included in Tables [5] through [8] shows that with the multivariate analysis the number of background events are further reduced by roughly 50% while the major part of the signal events is kept.

5 Model Dependent Analysis

If the analysis of the Higgs-strahlung process is restricted to modes in which the Higgs can solely decay into charged particles as e.g. suggested by the Standard Model, hence introducing a Model Dependency, the different track multiplicities can be used for the separation of signal and background events. The Higgs boson decays into oppositely charged particles such that events with less than four tracks can be considered as background. Figure [19] shows the number of reconstructed tracks beside the ones from the di-lepton system for final states of the types $\mu\mu X$, $\mu\mu$, $\tau\tau$ and $\mu\mu\nu\nu$.

As expected, the Higgs-strahlung process leads to a considerable amount of charged particles while processes with a low multiplicity of charged particles also create only a small number of tracks. The distributions tell that a large fraction of events have exactly two additional tracks beside those of the di-lepton system. The two additional tracks originate from two sources.

- Tracks created by charged particles by $H \rightarrow \tau^+\tau^-$ and the subsequent decays of the $\tau$-Leptons into charged particles.
- Tracks created by photon conversion. This photon may be created by initial state radiation.

The first type of events need to be kept in the signal as the $\tau$-Leptons constitute an important analyser to determine e.g. quantum numbers like $CP$ of the Higgs boson [3]. The second type of events can be rejected by taking into account that the opening angle of $e^+e^-$ pair created by photon conversion is expected to be very small. This is underlined by Figure [20] which shows the angular difference $\Delta\theta_{2tk}$ between the two additional tracks. While the signal events result in a flat distribution, the background events exhibit a strong maximum around $\Delta\theta_{2tk} = 0$. This observation motivates the a cut $|\Delta\theta_{2tk}| > 0.01$. The di-lepton system of a given type might be contaminated from particles of the other type. Therefore, the polar angle of each of the two particles of the di-lepton system is also compared with the polar
Figure 14: Normalised signal and background distribution of angle $\text{acol}$ between the partners of the di-lepton system for the $\mu^+\mu^- X$ Channel (top) and the $e^+e^- X$-Channel (bottom). Here $\tau\tau$ refers to the $\mu\mu$ or $ee$ created in the decay of $\tau\tau$. Note that the Pre-cuts defined in Section 3 have been applied to the $\mu\mu$ background sample.
angle of the additional tracks, defining the observable $\Delta \theta_{\text{min}}$ as shown in Figure 21. Again, a strong maximum around $\Delta \theta_{\text{min}} = 0$ can be observed which suggests the cut $\Delta \theta_{\text{min}} > 0.01$.

Tables 9 through 12 give the resulting number of events after each cut applied under the assumption that the Higgs boson decays into Standard Model particles. Note, that the cut on the transverse momentum of the di-lepton system has been omitted in order to maximise the number of signal events. The combination of cuts will be later referred as $\text{MD Cuts}$. The numbers in the tables show that the cuts introduced for the additional tracks allow for an entire suppression of backgrounds with a small multiplicity of charged particles in the final
Figure 18: The distributions of the Likelihood Fraction $f_L$ (left), the number of remaining events versus the cut on $f_L$ (middle), and the significance versus $f_L$ cuts (right). The distributions are shown for the $eeX$-channel in the Model Independent Analysis and for the polarisation mode $e_R^e e_L^+$. 

Figure 19: Number of additional tracks ($N_{\text{add.TK}}$) for $\mu\mu X$, $\mu\mu$, $\tau\tau$ and $\mu\mu\nu\nu$ final states.

state. It has to be pointed out that in particular the background from Bhabha events can be removed almost completely. On the other hand at least 50% of the signal is retained by the cuts.

Again the remaining major background is given by events in which vector bosons pairs are produced. This background is further reduced by a likelihood analysis as described above. The results of this likelihood analysis is also given in Tables 9 through 12. From these tables, it can be deferred that the $f_L$ cuts reject the background from $Z$ pair production by a factor of two, and reduce the signal by only 10%. At the same time, the background $\mu\mu$, $ee$, $\tau\tau$, $\mu\mu\nu\nu$ and $ee\nu\nu$ is entirely suppressed.
Figure 20: Distribution of $\Delta \theta_{2tk}$, which is the $\Delta \theta$ between two additional tracks for $N_{add.TK} = 2$, for signal events ($\mu\mu X$) and background by muon pair production ($\mu\mu$).

5.1 Tables of Background Rejection

| $N_{evt,R}$ Remained | $\mu^+\mu^-X$ | $\mu^+\mu^-$ | $\tau^+\tau^-$ | $\mu^+\mu^-\nu\nu$ | $\mu^+\mu^-f\bar{f}$ |
|----------------------|----------------|----------------|----------------|---------------------|----------------------|
| Before any restriction | 2918 (100.0%) | 2.6M | 1.6M | 111k | 317k |
| + Lepton ID          | 2472 (84.72%) | 9742 | 4582 | 9268 | 8175 |
| + Tightened Pre-Cuts | 2408 (82.50%) | 7862 | 3986 | 8462 | 7222 |
| + $P_{T,dl} > 20$GeV | 2292 (78.54%) | 6299 | 2679 | 5493 | 5658 |
| + $M_{dl} \in (80,100)$GeV | 2148 (73.61%) | 5182 | 112 | 5179 | 5083 |
| + $acop \in (0.2,3.0)$ | 2107 (72.20%) | 335 | 80 | 4705 | 4706 |
| + $|\Delta P_{T,bal}| > 10$GeV | 2104 (72.11%) | 149 | 80 | 4647 | 4676 |
| + $|\cos \theta_{missing}| < 0.99$ | 2046 (70.09%) | 82 | 80 | 4647 | 3614 |
| + $M_{recoil} \in (115,150)$GeV | 2028 (69.48%) | 75 | 80 | 3642 | 2640 |
| + $f_L > 0.26$ | 1596 (54.68%) | 41 | 0 | 1397 | 1125 |

Table 5: Number of events left after each cut for the $\mu^+\mu^-X$ channel in the MI Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_L^\pm e_R^\mp$. 
| $N_{\text{evts}}$ Remained | $e^+e^-X$ | $e^+e^-$ | $\tau^+\tau^-$ | $e^+e^-\nu\nu$ | $e^+e^-\bar{f}f$ |
|-----------------------------|-------------|-----------|-----------------|-----------------|-----------------|
| Before any restriction      | 3138 (100.0%) | 4.3G      | 1.6M            | 147k            | 110k            |
| + Lepton ID                 |             |           |                 |                 |                 |
| + Tightened Pre-Cuts        | 2019 (64.33%) | 43607     | 6422            | 13196           | 12548           |
| + $P_{T\text{dil}} > 20$GeV | 1962 (62.50%) | 39152     | 5551            | 12054           | 10583           |
| + $M_{d1} \in (80,100)$GeV  | 1755 (55.93%) | 25501     | 3806            | 7786            | 7509            |
| + $acop \in (0.2,3.0)$      | 1645 (52.41%) | 23228     | 245             | 7239            | 6739            |
| + $\Delta P_{T\text{bal.}} > 10$GeV | 1606 (51.16%) | 1725      | 157             | 6286            | 5904            |
| + $|\cos\theta_{\text{missing}}| < 0.99$ | 1564 (49.83%) | 679       | 157             | 6149            | 4643            |
| + $M_{\text{recoil}} \in (115,150)$GeV | 1539 (49.04%) | 576      | 41             | 4824            | 3335            |
| + $f_L > 0.28$              | 1153 (36.74%) | 243       | 29             | 2019            | 1217            |

Table 6: Number of events left after each cut for the $e^+e^-X$ channel in the MI Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_L^-e_R^+$. 

| $N_{\text{evts}}$ Remained | $\mu^+\mu^-X$ | $\mu^+\mu^-$ | $\tau^+\tau^-$ | $\mu^+\mu^-\nu\nu$ | $\mu^+\mu^-\bar{f}f$ |
|-----------------------------|-------------|-----------|-----------------|-----------------|-----------------|
| Before any restriction      | 1967 (100.0%) | 2.0M      | 1.2M            | 9k              | 291k            |
| + Lepton ID                 |             |           |                 |                 |                 |
| + Tightened Pre-Cuts        | 1667 (84.73%) | 6696     | 3471            | 1048            | 5324            |
| + $P_{T\text{dil}} > 20$GeV | 1623 (82.48%) | 5419     | 3037            | 957             | 4600            |
| + $M_{d1} \in (80,100)$GeV  | 1544 (78.47%) | 4347     | 2092            | 702             | 3530            |
| + $acop \in (0.2,3.0)$      | 1448 (73.60%) | 3592     | 113             | 656             | 3169            |
| + $\Delta P_{T\text{bal.}} > 10$GeV | 1421 (72.21%) | 229      | 81             | 632             | 2873            |
| + $|\Delta\theta_{2\text{tik}}| > 0.01$ | 1419 (72.10%) | 101     | 81             | 625             | 2851            |
| + $|\cos\theta_{\text{missing}}| < 0.99$ | 1379 (70.10%) | 54       | 81             | 625             | 2065            |
| + $M_{\text{recoil}} \in (115,150)$GeV | 1367 (69.49%) | 50      | 81             | 487             | 1506            |
| + $f_L > 0.19$              | 1165 (59.20%) | 28       | 0              | 243             | 752             |

Table 7: Number of events left after each cut for the $\mu^+\mu^-X$ channel in the MI Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_R^-e_L^+$. 

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| $N_{\text{evts}}$ Remained | $e^+e^-X$ | $e^+e^-$ | $\tau^+\tau^-$ | $e^+\nu\nu$ | $e^+e^-f\bar{f}$ |
|--------------------------|-----------|-----------|----------------|-------------|----------------|
| Before any restriction   | 2107 (100.0%) | 4.3G | 1.2M | 17k | 1.1M |
| + Lepton ID              | 1352 (64.16%) | 40896 | 5257 | 1469 | 10198 |
| + Tightened Pre-Cuts     | 1313 (62.33%) | 36742 | 4546 | 1351 | 8430 |
| + $P_{T\text{dil}} > 20$ GeV | 1177 (55.88%) | 23993 | 3051 | 943 | 5909 |
| + $M_{dl} \in (80,100)$ GeV | 1103 (52.36%) | 21846 | 107 | 881 | 5266 |
| + $\Delta P_{\text{Tbal}} > 10$ GeV | 1077 (51.11%) | 1612 | 92 | 805 | 4517 |
| + $|\Delta \theta_{2\text{tk}}| > 0.01$ | 1076 (51.05%) | 927 | 92 | 799 | 4465 |
| + $|\cos \theta_{\text{missing}}| < 0.99$ | 1050 (49.82%) | 638 | 92 | 799 | 3484 |
| + $M_{\text{recoil}} \in (115,150)$ GeV | 1033 (49.04%) | 539 | 12 | 586 | 2521 |
| + $f_L > 0.16$            | 909 (43.14%) | 326 | 4 | 368 | 1294 |

Table 8: Number of events left after each cut for the $e^+e^-X$ channel in the MI Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_R^-e_L^+$. 

| $N_{\text{evts}}$ Remained | $\mu^+\mu^-X$ | $\mu^+\mu^-$ | $\tau^+\tau^-$ | $\mu^+\nu\nu$ | $\mu^+\mu^-f\bar{f}$ |
|--------------------------|-----------|-----------|----------------|-------------|----------------|
| Before any restriction   | 2918 (100.0%) | 2.6M | 1.6M | 111k | 317k |
| + Lepton ID              | 2472 (84.72%) | 9742 | 4582 | 9268 | 8175 |
| + Tightened Pre-Cuts     | 2453 (84.05%) | 604 | 842 | 145 | 6321 |
| + $N_{\text{add.TK}} > 1$ | 2449 (83.91%) | 63 | 816 | 14 | 6254 |
| + $|\Delta \theta_{2\text{tk}}| > 0.01$ | 2417 (82.81%) | 38 | 261 | 1 | 5711 |
| + $\Delta \theta_{\text{min}} > 0.01$ | 2256 (77.29%) | 32 | 0 | 1 | 5051 |
| + $|\cos \theta_{\text{missing}}| < 0.99$ | 2189 (75.00%) | 16 | 0 | 1 | 3843 |
| + $M_{\text{recoil}} \in (115,150)$ GeV | 2154 (73.81%) | 15 | 0 | 1 | 2830 |
| + $f_L > 0.17$            | 1911 (65.49%) | 11 | 0 | 0 | 1387 |

Table 9: Number of events left after each cut for the $\mu^+\mu^-X$ channel in the MD Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_L^-e_R^+$. 

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Table 10: Number of events left after each cut for the $e^+e^-X$ channel in the MD Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_L^+e_R^-R$.

| $N_{\text{evts}}$ Remained | $e^+e^-X$ | $e^+e^-$ | $\tau^+\tau^-$ | $e^+e^-\nu\nu$ | $e^+e^-ff$ |
|---------------------------|-----------|---------|-------------|----------------|---------|
| Before any restriction    | 3138 (100.0%) | 4.3G    | 1.6M        | 147k           | 110k    |
| + Lepton ID               |           |         |             |                |         |
| + Tightened Pre-Cuts      | 2019 (64.33%) | 43607   | 6422        | 13196          | 12548   |
| + $N_{\text{add.TK}} > 1$| 2004 (63.87%) | 3136    | 1740        | 374            | 10202   |
| + $|\Delta\theta_{2tk}| > 0.01$| 2001 (63.77%) | 655     | 1073        | 79            | 10095   |
| + $|\Delta\theta_{\text{min}}| > 0.01$| 1969 (62.75%) | 155     | 128         | 6             | 9271    |
| + $acop \in (0.2, 3.0)$   | 1840 (58.62%) | 134     | 0           | 6             | 8366    |
| + $|\cos\theta_{\text{missing}}| < 0.99$| 1792 (57.11%) | 91      | 0           | 6             | 6696    |
| + $M_{\text{recoil}} \in (115, 150)\text{GeV}$ | 1731 (55.16%) | 73      | 0           | 1             | 4950    |
| + $f_L > 0.27$            | 1378 (43.90%) | 27      | 0           | 0             | 1652    |

Table 11: Number of events left after each cut for the $\mu^+\mu^-X$ channel in the MD Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_R^-e_L^+L$.

| $N_{\text{evts}}$ Remained | $\mu^+\mu^-X$ | $\mu^+\mu^-$ | $\tau^+\tau^-$ | $\mu^+\mu^-\nu\nu$ | $\mu^+\mu^-ff$ |
|---------------------------|----------------|-------------|-------------|----------------|---------|
| Before any restriction    | 1967 (100.0%) | 2.0M        | 1.2M        | 9k            | 291k    |
| + Lepton ID               |               |             |             |                |         |
| + Tightened Pre-Cuts      | 1667 (84.73%) | 6696        | 3471        | 1048          | 5324    |
| + $N_{\text{add.TK}} > 1$| 1654 (84.07%) | 415         | 391         | 9             | 4160    |
| + $|\Delta\theta_{2tk}| > 0.01$| 1651 (83.93%) | 41         | 379         | 0             | 4108    |
| + $|\Delta\theta_{\text{min}}| > 0.01$| 1629 (82.81%) | 22         | 105         | 0             | 3739    |
| + $acop \in (0.2, 3.0)$   | 1522 (77.34%) | 20         | 0           | 0             | 3312    |
| + $|\cos\theta_{\text{missing}}| < 0.99$| 1476 (75.03%) | 11         | 0           | 0             | 2438    |
| + $M_{\text{recoil}} \in (115, 150)\text{GeV}$ | 1453 (73.85%) | 10         | 0           | 0             | 1803    |
| + $f_L > 0.17$            | 1289 (65.53%) | 8           | 0           | 0             | 875     |
Figure 21: Distribution of $\Delta \theta$ between muon candidates and additional tracks, for the signal events ($\mu\mu X$) and background by muon pair creation ($\mu\mu$).

| $N_{\text{evts Remained}}$ & $e^+e^-X$ & $e^+e^-$ & $\tau^+\tau^-$ & $e^+e^-\nu\nu$ & $e^+e^-ff$ |
|-----------------------------|-------------|-------------|-------------|-------------|-------------|
| Before any restriction | 2107 (100.0%) | 4.3G | 1.2M | 17k | 1.1M |
| + Lepton ID | | | | | |
| + Tightened Pre-Cuts | 1352 (64.16%) | 40896 | 5257 | 1469 | 10198 |
| + $N_{\text{add.TK }}> 1$ | 1342 (63.69%) | 2935 | 1500 | 22 | 8227 |
| + $|\Delta \theta_{2tk}| > 0.01$ | 1340 (63.60%) | 617 | 859 | 4 | 8133 |
| + $|\Delta \theta_{\text{min}}| > 0.01$ | 1319 (62.59%) | 146 | 57 | 0 | 7388 |
| + $\cos \theta_{\text{missing}} < 0.99$ | 1232 (58.47%) | 125 | 0 | 0 | 6651 |
| + $M_{\text{recoil}} \in (115,150)\text{GeV}$ | 1201 (57.00%) | 84 | 0 | 0 | 5265 |
| + $f_L > 0.32$ | 1161 (55.10%) | 67 | 0 | 0 | 3886 |

Table 12: Number of events left after each cut for the $e^+e^-X$ channel in the MI Analysis. Fractions of number of events remained of the Higgs-Strahlung process are given inside parentheses, the last one gives the efficiency of signal selection. The polarisation mode is $e_R^+e_L^-$. 
6 Extraction of Higgs Mass and the Higgs production Cross Section

In the previous section the criteria to select the signal events and to suppress the background from various sources have been introduced and applied. The remaining spectra are a superposition of signal and background events convoluted with beam effects. In the following, the relevant observables as the Higgs boson mass \( M_H \) and the total Higgs-strahlung cross section \( \sigma \) are extracted. Note in passing, that the results for the \( eeX \)-channel will always contain a small admixture of the ZZ fusion process.

As indicated above, the resulting spectrum is composed by several components. This motivates to approximate this spectrum in a non-parametric way using a Kernel Estimation as introduced in [14] and applied in [15, 16]. In order to reduce the effort of finding a parent function using either the already simulated data set or by simulating another independent set of data a so-called Simplified Kernel Estimation is proposed.

The signal spectrum is approximated by the following function:

\[
F_S(x) = \frac{1}{N} \sum_{j=1}^{m} n_j G(x; t_j; h_j)
\]

with

\[
h_j = \left(\frac{4}{3}\right)^{1/5} N^{-1/5} \Delta x \sqrt{\frac{N}{n_j}},
\]

Here \( G \) is a Gaussian with the parameters \( \mu = t_j \) where \( t_j \) is the center of the \( j \)th bin of a histogram with \( m \) bins and \( \sigma = h_j \) where \( h_j \) is the smoothing parameter of bandwidth of the individual Gaussians placed around the bin centers. The parameter \( \Delta x \) is assumed to the the standard deviation in each bin and \( \frac{n_j}{\Delta x N} \) is an estimate for the parent distribution. By the transformation \( x \to x' = x - M_H \), the approximated function becomes sensitive to the value of the Higgs-Mass.

The background is approximated by a second order Chebyshev polynomial. By this statistical fluctuations in the remaining background events are smoothened. Using this polynomial as input the background is generated again with 40 times higher statistics. Therefore, statistical uncertainties are nearly excluded. The combination of signal and background is finally fitted by the sum of the signal and the background functions.

The simulated signal sample is separated into two sets of data. One of them, the Reference Sample, is employed to determine all fit parameters except the normalisation \( N \) of the signal signal and the actual Higgs mass, \( M_H \). The normalisation \( N \) and the Higgs mass \( M_H \) enter as free parameters of the fit to the second sample, the Result Sample. The spectra of the Result Sample, scaled to a luminosity of \( L=250 \text{fb}^{-1} \), including the defined fitting function are displayed in Figures 24 and 25 for the Model Independent Analysis and in Figures 26 to 27 for the Model Dependent Analysis. The fit based on the Kernel estimation for the signal part describes the shape of the mass spectra very well and are therefore suited for the extraction of the relevant parameters which are listed in Table 13 for the Model Independent Analysis and in Table 14 for the Model Dependent Analysis. In [13] alternative fit methods are discussed which lead to nearly identical results.
Table 13: Resulting Higgs mass $M_H$ and cross section $\sigma$ of the MI Analysis using Kernel Estimation.

| Pol. $e_R e_L^+$ | Ch. $\ell^+ \ell^-$ | $M_H$ (GeV) | $\sigma$ (fb)  |
|------------------|---------------------|-------------|----------------|
| $\mathcal{L} = 250 \text{ fb}^{-1}$ | $\mu^+ \mu^- X$ | 120.006 $\pm$ 0.039 | 7.89 $\pm$ 0.28 (3.55\%) |
|                  | $e^+ e^- X$  | 120.005 $\pm$ 0.092 | 8.46 $\pm$ 0.43 (5.08\%) |
|                  | merged       | 120.006 $\pm$ 0.036 | 8.06 $\pm$ 0.23 (2.91\%) |

Table 14: Resulting Higgs mass $M_H$ and cross section $\sigma$ of the MD Analysis using Kernel Estimation.

| Pol. $e_R e_L^+$ | Ch. $\ell^+ \ell^-$ | $M_H$ (GeV) | $\sigma$ (fb)  |
|------------------|---------------------|-------------|----------------|
| $\mathcal{L} = 250 \text{ fb}^{-1}$ | $\mu^+ \mu^- X$ | 120.008 $\pm$ 0.037 | 11.70 $\pm$ 0.39 (3.33\%) |
|                  | $e^+ e^- X$  | 119.998 $\pm$ 0.085 | 12.61 $\pm$ 0.62 (4.92\%) |
|                  | merged       | 120.006 $\pm$ 0.034 | 11.96 $\pm$ 0.33 (2.76\%) |

6.1 Discussion of the Results

The Higgs mass can be determined to a precision of the order of 0.03% when the $eeX$ channel and the $\mu\mu X$ are combined. Regarding the individual results, it can be deferred that the precision in the $\mu\mu X$ channel is more than two times smaller than that of the $eeX$ Channel. This increase of the error is mainly induced by bremsstrahlung of the electrons in the detector material. This can be concluded by comparing the results between Tables 13 and 14 as for the latter the background of Bhabha scattering is suppressed while the difference in the precision between the two decay modes of the $Z$ bosons remains the same. The precision on the cross section is less sensitive to this experimental drawback as it is derived within a basically arbitrary mass window. The derived values for the Model Dependent Analysis are consistently slightly more precise. The relatively small difference of the results confirms that the methods employed for background suppression in the Model Independant Analysis are already very efficient.

| $\Delta M_{\text{tot.}}$ (MeV) | $\Delta M_{\text{mac.}}$ (MeV) | $\Delta M_{\text{dec.}}$ (MeV) |
|-------------------------------|-------------------------------|-------------------------------|
| $\mu\mu X$                   | 650                           | 560                           | 330                           |
| $eeX$                         | 750                           | 560                           | 500                           |

Table 15: Mass Resolution with contributions by machine ($\Delta M_{\text{mac.}}$) and detector ($\Delta M_{\text{det.}}$) separated.

The width of the Higgs boson mass as shown before is mainly given by a convolution of detector uncertainties and uncertainties on the energy of the incoming beams. Uncertainties on the energy of the incoming beams are imposed by accelerator components such as the initial linac, the damping rings or, in the case of electrons, by a tentative undulator in the electron beam line. Another source of uncertainty is the beamstrahlung when particles of a
Figure 22: Comparisons of recoil mass spectra in generator level and after full simulation, for the $\mu\mu X$-channel (left) and the $ee X$-channel (right).

beam bunch interact in the electromagnetic field of the opposite one. The Figure 22 shows the Higgs mass spectrum before and after full detector simulation. The detector response leads only to small additional widening of the maximum of the recoil mass distribution. Using a Gaussian fit to the left side of the recoil mass distribution, the width before detector simulation can be quantified to be 560 MeV while it increases to 650 MeV for the $\mu\mu X$ channel after detector simulation, see Table 15. For the given configuration, the uncertainty on the incoming beams remains the dominant contribution to the observed width even for the $ee X$ channel.

**On the Control of Systematic Errors**

Naturally, the measurement of the Higgs mass is sensitive to the calibration of the detector and the beam energies as the Higgs mass is directly computed from the four momenta of the particles composing the di-lepton system and the centre-of-mass energy. Both uncertainties can be controlled by the measurement of the $e^+e^- \rightarrow ZZ$ process as the Z mass is known to a few MeV and the cross section for Z pair production is approximately 40 times higher than that of the Higgs-strahlungs process. Once the detector is calibrated the Higgs-strahlung process can be used to determine, within reasonable limits, arbitrary Higgs masses. The algorithms presented in this note are also suited for the quantification of the systematic error. Note, that the systematic error of the cross section determination might be easier to control by using a smaller set of cut variables than those presented above. Such a set could comprise only the invariant mass and the transverse momentum of the di-lepton system, $M_{dl}$, $P_T^{dl}$ or, in case of the model dependant analysis, the number of additional tracks, $N_{add,TK}$. The expected increase of the statistical error is only about 10%.

**6.1.1 Recovery of Bremsstrahlungs Photons**

The lower precision obtained in the $ee X$-Channel is due to the Bremsstrahlung of the final state electrons in the passive material of the detector. In the following an attempt is made to improve the precision in that channel by recovering the bremsstrahlungs photons [17]. The four momenta of the selected electrons are combined with those of photons which have a small angular distance to the electrons. If these combined objects together with the corresponding other electron candidate form the Z mass, they are included in the Z reconstruction. The inclusion of low energetic photons leads to a penalty in the momentum reconstruction due to
the poor energy resolution of the electromagnetic calorimeter for low energetic particles. This drawback might get counterbalanced by the gain in statistics due to the described recovery of the energy loss.

Figure 23 shows the recoil mass spectrum after the recovery of the Bremsstrahlung photons. The worse resolution around the mass maximum is clearly visible. The corresponding results are given Tables 16 through 17 and the fitted spectra in Figures 28 through 29. The numbers show that the mass resolution is improved by 10% while the precision in the cross section is improved by 20%. The cross section benefits directly from the gain in statistics while the determination of the recoil mass suffers from the reduced momentum resolution.

![Figure 23: Comparison of the Higgs recoil mass distributions of e⁺e⁻X channel with and without the bremsstrahlung photons recovery.](image)

| Ana. | Pol. | Ch.          | S (%)  | B   |
|------|------|--------------|--------|-----|
| MI   |      | e⁺e⁻ X      | 1029 (48.84%) | 1408 |
|      |      | e⁺e⁻ X      | 1491 (41.51%) | 3394 |
| MD   |      | e⁺e⁻ X      | 1152 (54.66%) | 1114 |
|      |      | e⁺e⁻ X      | 1724 (54.94%) | 1513 |

Table 16: Resulting Number of Signal (S) and Number of Background (B), and the efficiencies of signal selection (in the parentheses) after background rejection, for eeX channel with Bremsstrahlung Photons Recovery

7 Conclusion and Outlook

Using mainly the Higgs-strahlung process with the Z boson decaying leptonically and a Higgs boson mass of 120 GeV as input, the current design of the ILD detector promises to determine the mass of the Higgs boson to a precision of the order of 30 MeV. According to [18]...
and references therein, such a precision renders sensitivity to effects from super-symmetric extensions to the Standard Model. Assuming a heavier Higgs, the precision might allow also for the determination of the Higgs boson mass width at centre-of-mass energies higher than 250 GeV. Staying with small Higgs masses, it has been demonstrated semi-analytically [18] and confirmed with full simulation studies [19] that the precision can be further increased by working at a centre-of-mass energy close to the HZ production threshold, i.e. at $\sqrt{s}=230$ GeV.

In the present study, the cross section and therefore the coupling strength at the HZZ vertex is determined to a precision of the order of 2-3% which might already be sufficient to get sensitive to contributions to this coupling from physics beyond the Standard Model.

The signal to background ratio can be enhanced to a value of at least 30% even if the cross sections of the background processes are several orders of magnitudes higher. Note, that this ratio is way higher in the region around the signal maximum. The background suppression exploits the considerable capabilities of track recognition as allowed by the current design of the ILD detector. The precision of the measurement can be improved by a better muon recognition by e.g. including a muon system in the analysis which has not been done so far. The precision obtained in the branch in which the Z boson decays into electrons might gain considerably from a revision of the amount of passive material in the detector. Both decay modes may gain also from an exploitation of the particle identification of the ILD detector by means of a $dE/dx$ measurement in the TPC. For this, further studies are needed. A future study clearly will have to quantify the systematic uncertainties and to identify those which have the largest influence on the systematic errors. This would give important directions on the final detector layout and the precision needed for e.g. alignment systems. For such a study realistic inputs on e.g. the uncertainty of drift times in the TPC or residual misalignments after detector movements are needed.

The analysis has proven that the results are sensitive to details of the accelerator configuration. Using the set of parameters as has been chosen for the SLAC samples which in turn correspond to the current best knowledge of the beam parameters, approximately half of the statistical error is generated by uncertainties caused by beamstrahlung and the energy spread of the incoming beams. The Higgs-strahlung process constitutes an important benchmark for the optimisation of the accelerator performance.

| Ana. | Pol. | $M_H$ (GeV) | $\sigma$ (fb) |
|------|------|-------------|---------------|
| MI   | $e_R e_L$ | 120.003 ± 0.081 | 8.41 ± 0.36 (4.28 %) |
|      | $e_L e_R$ | 119.997 ± 0.073 | 12.52 ± 0.49 (3.91 %) |
| MD   | $e_R e_L$ | 119.999 ± 0.074 | 8.41 ± 0.31 (3.69 %) |
|      | $e_L e_R$ | 120.001 ± 0.060 | 12.51 ± 0.38 (3.04 %) |

Table 17: Resulting Higgs mass $M_H$ and cross section $\sigma$ for the Model Independent Analysis and Model Dependent Analysis of the Higgs-Strahlung process of $e^+ e^- X$ channel with Bremsstrahlung photons Recovery.
Figure 24: Reconstructed Higgs mass spectrum together with the sum of underlying background for the Model Independent Analysis for the $\mu\mu X$-channel (top) and $eeX$-channel (bottom). The polarisation mode is $e_L^-e_R^+$. The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
Figure 25: Reconstructed Higgs mass spectrum together with the sum of underlying background for the Model Independent Analysis for the $\mu\mu X$-channel (top) and $eeX$-channel (bottom). The polarisation mode is $e^-_Re^+_L$. The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
Figure 26: Reconstructed Higgs mass spectrum together with the sum of underlying background for the Model Dependent Analysis for the $\mu\mu X$-channel (top) and $ee X$-channel (bottom). The polarisation mode is $e^+_L e^-_R$. The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
Figure 27: Reconstructed Higgs mass spectrum together with the sum of underlying background for the Model Dependent Analysis for the $\mu\mu X$-channel (top) and $ee X$-channel (bottom). The polarisation mode is $e^{-}e^{+}L$. The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
Figure 28: Reconstructed Higgs mass spectrum after the recovery of Bremsstrahlungs photons together with the sum of underlying background for the Model Independent Analysis for eeX-channel. The polarisation mode is $e^-_Le^+_R$ (top) and $e^-_Re^+_L$ (bottom). The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
Figure 29: Reconstructed Higgs mass spectrum after the recovery of Bremsstrahlung photons together with the sum of underlying background for the Model Dependent Analysis for $eeX$-channel. The polarisation mode is $e_L^+e_R^-$ (top) and $e_R^+e_L^-$ (bottom). The lines show the fits using the Simplified Kernel Estimation fitting formula to the signal and a polynomial of second order to the background as explained in the text.
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