Gauge invariance of the resummation approach to evolution equations

Hsiang-nan Li

Department of Physics, National Cheng-Kung University,
Tainan, Taiwan, Republic of China

Abstract

We show that the Collins-Soper-Sterman resummation approach to the derivation of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation is gauge invariant. The special gauge-dependent parton distribution function employed in the resummation technique is expressed as the convolution of an infrared finite function with the standard distribution function. By means of this convolution relation, we explain how the technique works in summing large logarithmic corrections, and how the gauge invariance of the special distribution function is restored.
1. Introduction

It is known that a quark distribution function for a hadron in the minimal subtraction scheme is defined by

\[
\phi(\xi, \mu/m) = \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{q}(y^-)\gamma^+ P e^{i\int_0^{y^-} dz' v\cdot A(z')} q(0)|p\rangle ,
\]

(1)

where \(\gamma^+\) is a Dirac matrix, and \(|p\rangle\) denotes the incoming hadron with the momentum \(p^\mu = p^+ \delta^\mu_+\). Averages over spins and colors are understood. The above expression, with the presence of the path-ordered exponential \(P e^{i\int dz' v\cdot A(z')}\), \(v^\mu = \delta^\mu_-\) being a light-like vector, is gauge invariant. Through this exponential, \(\phi\) collects the collinear divergences of radiative corrections to a QCD process. While the infrared finite piece of radiative corrections is absorbed into a hard scattering amplitude. The factorization formula for a cross section (or a structure function) is then expressed as the convolution of these two factors. \(\phi(\xi, \mu/m)\) describes the probability that a parton carries the fractional momentum \(\xi p\) at the factorization (or renormalization) scale \(\mu\), with \(m\) an infrared cutoff for the collinear divergences, such as quark mass. The argument \(\mu/m\) denotes the large logarithms \(\ln(\mu/m)\) from the collinear divergences, which will be summed by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [1].

To organize the large logarithms contained in the distribution function using the Collins-Soper-Sterman resummation technique [2], the vector \(v'\) is replaced by an arbitrary vector \(n\), \(n^2 \neq 0\), leading to a \(n\)-dependent distribution function,

\[
\phi^{(n)}(\xi, \nu/\mu, \mu/m) = \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{q}(y^-)\gamma^+ P e^{i\int_0^{y^-} dz n\cdot A(zn)} q(0)|p\rangle .
\]

(2)

We have made explicit the additional argument \(\nu = \sqrt{(p \cdot n)^2/|n^2|}\) of \(\phi^{(n)}\), which appears as a ratio because of the scale invariance of Eq. (3) in \(n\) [3]. Unfortunately, this replacement spoils the gauge invariance of Eq. (1), since, with \(n\) being arbitrary, the end point of the path, on which the gauge field \(A\) is evaluated, does not coincide with the coordinate \(y^-\) of the quark field \(\bar{q}\). It will be shown that the scale \(\nu\) serves as an ultraviolet cutoff for the loop integrals associated with the collinear gluons, so that \(\phi\) and \(\phi^{(n)}\) possess the same infrared structures but different ultraviolet structures.
We shall demonstrate that the evolution kernel of the DGLAP equation, obtained from the resummation technique, turns out to be $n$-independent, i.e., gauge invariant. After deriving the equation, $n$ is brought back to the light cone. In the $n \to v'$ limit $\nu$ diverges, and $\ln(\nu/\mu)$ corresponds to an ultraviolet pole in dimensional regularization. This pole is removed in a renormalization scheme, and $\phi^{(n)}$ reduces to the gauge invariant distribution function $\phi$. We also show that $\phi^{(n)}$ approaches $\phi$ as $\nu = \mu$, for which the extra logarithms $\ln(\nu/\mu)$ vanish. That is, the arbitrary vector $n$ appears only at the intermediate stage of the formalism, and as an auxiliary tool. The gauge invariance of the resummation approach to the evolution equation is thus guaranteed. To make our reasoning concrete, we derive the DGLAP equation in both covariant and axial gauges. The above discussion can be extended to the gluon distribution function directly, whose evolution in the small momentum fraction is governed by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [4].

We explore the relation between the working definition $\phi^{(n)}$ and the standard definition $\phi$: the former is expressed as the convolution of an infrared finite $\nu$-dependent function with the latter, because they contain the same nonperturbative information. Hence, varying $n$ can be understood as different ways to partition the infrared finite contribution to a cross section between the parton distribution function and the hard scattering amplitude. The $n$ dependence is then in fact a factorization scheme dependence. By means of this convolution relation, we analyze in details how the resummation for $\phi^{(n)}$ is accomplished, and how $\phi^{(n)}$ becomes $\phi$ in the limit $n \to v'$.

2. The DGLAP Equation

Take deep inelastic scattering of a hadron as an example. We first explain how to factorize collinear gluons into the quark distribution function, and justify the replacement of $\nu'$ by $n$ in the collinear kinematic region [5]. It is known that the collinear region with loop momenta of radiative gluons parallel to $\nu$ is important, from which large logarithms arise [5]. In this region the partial integrand for a loop diagram, with the radiative gluon
attaching the scattered quark line, is approximated by

\[
\gamma^\mu \frac{p' + l}{(p' + l)^2} \approx \gamma^\mu \frac{p'_\mu}{p' \cdot l} = \gamma^\mu \frac{v'_\mu}{v' \cdot l},
\]

where the scattered quark momentum \(p'\) possesses a large minus component \(p'_-\) in the large \(x\) limit. The preceding factor \(\gamma^\mu\) comes from the final-state cut. Equation (3) indicates that the scattered quark line, the collinear gluons attach, can be replaced by an eikonal line in the direction \(v'\): \(1/(v' \cdot l)\) is the Feynman rule for an eikonal propagator, and \(v'_\mu\) for a vertex on the eikonal line, \(l\) being the momentum flowing through it. It is straightforward to show that these rules are exactly produced by the path-ordered exponential in Eq. (1). With the eikonalization, the collinear gluons are factorized into the quark distribution function \(\phi\).

We may employ the further approximation,

\[
\frac{v'_\mu}{v' \cdot l} = \frac{\delta_{-\mu}}{l^+} = \frac{n^- \delta_{-\mu}}{n^- l^+} \approx \frac{n^- \delta_{-\mu} + n^+ \delta_{+\mu} + n_T \delta_{T\mu}}{n^- l^+ + n^+ l^- - n_T \cdot l_T} = \frac{n_\mu}{n \cdot l}.
\]

The smaller components \(l^-\) and \(l_T\) have been added to the denominator, and the terms proportional to the components \(n^+\) and \(n_T\) of an arbitrary vector \(n\), which give vanishing contributions when contracted with a vertex in the distribution function, have been included in the numerator. Similarly, the path-ordered exponential in \(\phi^{(n)}\) generates the Feynman rules \(n_\mu/(n \cdot l)\) in Eq. (4). Because \(\phi^{(n)}\) and \(\phi\) coincide with each other in the collinear region, they contain the same nonperturbative information, and their difference is infrared finite as stated in the Introduction. A general diagram for \(\phi^{(n)}\) is exhibited in Fig. 1(a), where the exponential is represented by an eikonal line along \(n\) with collinear gluons attaching it.

In the resummation framework the DGLAP equation can be obtained by studying the derivative \(p^+ d\phi^{(n)}/dp^+\) in the covariant gauge \(\partial \cdot A = 0\). Due to the argument \(\nu\), we have the relation

\[
p^+ \frac{d}{dp^+} \phi^{(n)} = -\frac{n^2}{v \cdot n} \frac{d}{dn_\alpha} \phi^{(n)}.
\]

with \(v^\mu = \delta^\mu\) a vector along \(p\). Since \(p\) flows through all the quark and gluon lines in the distribution function, while \(n\) appears only in the exponential, Eq. (5) simplifies the analysis of the \(p^+\) dependence of \(\phi^{(n)}\). This is the
motivation to introduce the arbitrary vector \( n \). The differentiation of Eq. (4) with respect to \( n_\alpha \),

\[
- \frac{n^2}{v \cdot n} \frac{d}{dn_\alpha} \frac{n_\mu}{n \cdot l} = \frac{n^2}{v \cdot n} \left( \frac{v \cdot l}{n \cdot l} n_\mu - v_\mu \right) \frac{1}{n \cdot l}
\]

\[
\equiv \frac{\hat{n}_\mu}{n \cdot l},
\]

gives

\[
p^+ \frac{d}{dp^+} \phi^{(n)} = 2 \tilde{\phi}^{(n)},
\]

which is described by Fig. 2(a). A summation over different attachments of the symbol \( \times \), which represents the special vertex \( \hat{n}_\mu \) defined by the second expression in Eq. (6), is understood. The coefficient 2 comes from the equality of the new functions \( \tilde{\phi}^{(n)} \) with the special vertex on either side of the final-state cut.

If the loop momentum \( l \) is parallel to \( p \), the factor \( v \cdot l \) vanishes, and \( \hat{n}_\mu \) is proportional to \( v_\mu \). When this \( v_\mu \) is contracted with a vertex in \( \tilde{\phi}^{(n)} \), where all momenta are mainly parallel to \( p \), the resultant contribution diminishes. Therefore, the leading regions of \( l \) are soft and hard, in which the subdiagram containing the special vertex can be factorized, leading to

\[
\tilde{\phi}^{(n)}(x, \nu/\mu, \mu/m) = \int_x^1 d\xi [K(x/\xi, \nu/\mu) + G(x/\xi, \nu/\mu)] \phi^{(n)}(\xi, \nu/\mu, \mu/m),
\]

with the functions \( K \) and \( G \) absorbing the soft and ultraviolet divergences, respectively. The \( O(\alpha_s) \) contributions to \( K \) from Fig. 2(b), where the eikonal approximation for the valence quark propagator has been made, and to \( G \) from Fig. 2(c), where the soft subtraction ensures a hard loop momentum flow, are written as

\[
K = -ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon} l}{(2\pi)^{4-\epsilon}} n \cdot l v_\mu \left[ \frac{\delta(\xi - x)}{l^2} + 2\pi i \delta(l^2) \delta \left( \xi - x - \frac{l^+}{p^+} \right) \right],
\]

\[
G = -ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon} l}{(2\pi)^{4-\epsilon}} n \cdot l v_\mu \left[ \frac{\mu + l}{(p + l)^2} \gamma^\mu - \frac{v_\mu}{v \cdot l} \right] \delta(\xi - x),
\]

\( C_F = 4/3 \) being a color factor.
A straightforward calculation gives

\[ K = \frac{\alpha_s(\nu)}{\pi \xi} C_F \left[ \frac{1}{(1 - x/\xi)_+} + \ln \frac{\nu}{\mu} \delta(1 - x/\xi) \right], \]  

(11)

\[ G = -\frac{\alpha_s(\nu)}{\pi \xi} C_F \ln \frac{\xi \nu}{\mu} \delta(1 - x/\xi), \]  

(12)

where constants of order unity have been dropped. In the considered region with \( x \to 1 \) the logarithm \( \ln(\xi \nu/\mu) \) in \( G \) can be replaced by \( \ln(\nu/\mu) \). Adding the above expressions, we have

\[ K(x/\xi, \nu/\mu) + G(x/\xi, \nu/\mu) = \frac{\alpha_s(\nu)}{\pi \xi} C_F \frac{1}{(1 - x/\xi)_+}, \]  

(13)

where the argument of \( \alpha_s \) has been set to the characteristic scale \( \nu \) of the evolution kernel \( K + G \). It is observed that the gauge factors \( \nu \) have cancelled between \( K \) and \( G \).

Inserting Eq. (13) into (8), Eq. (7) becomes

\[ p^+ \frac{d}{dp^+} \phi^{(n)}(x, \nu/\mu, \mu/m) = \frac{\alpha_s(\nu)}{\pi} C_F \int_x^1 \frac{d\xi}{\xi} \frac{2}{(1 - x/\xi)_+} \phi^{(n)}(\xi, \nu/\mu, \mu/m). \]  

(14)

The solution to the above differential equation can be written as

\[ \phi^{(n)}(x, 1, \mu/m) = \phi^{(n)}(x, \Lambda/\mu, \mu/m) \]

\[ + \int_{\Lambda}^\mu \frac{d\bar{\mu}}{\bar{\mu}} \alpha_s(\bar{\mu}) C_F \int_x^1 \frac{d\xi}{\xi} \frac{2}{(1 - x/\xi)_+} \phi^{(n)}(\xi, \bar{\mu}/\mu, \mu/m), \]  

(15)

with \( \Lambda \) an arbitrary scale. We shall explain in Sect. 3. that \( \phi^{(n)} \) coincides with \( \phi \) as \( \nu = \mu \), i.e., as the logarithms \( \ln(\nu/\mu) \) vanish:

\[ \phi^{(n)}(x, 1, \mu/m) = \phi(x, \mu/m). \]  

(16)

Differentiating Eq. (13) with respect to \( \mu \), and employing the RG equation shown in Sect. 3,

\[ \mu \frac{d}{d\mu} \phi^{(n)}(x(\xi), \Lambda(\bar{\mu})/\mu, \mu/m) = -2 \lambda_q \phi^{(n)}(x(\xi), \Lambda(\bar{\mu})/\mu, \mu/m), \]  

(17)
with \( \lambda_q = -\alpha_s/\pi \) the quark anomalous dimension, we obtain

\[
\mu \frac{d}{d\mu} \phi(x, \mu/m) = \frac{\alpha_s(\mu)}{\pi} C_F \int_x^1 \frac{d\xi}{\xi} \frac{2}{(1 - x/\xi)_+} \phi(\xi, \mu/m) - 2\lambda_q(\mu) \phi(x, \mu/m) .
\]

(18)

At this step, the \( n \) dependence disappears completely. The above formula is then identified as the DGLAP equation,

\[
\mu \frac{d}{d\mu} \phi(x, \mu/m) = \frac{\alpha_s(\mu)}{\pi} \int_x^1 \frac{d\xi}{\xi} P(x/\xi) \phi(\xi, \mu/m) ,
\]

(19)

where the evolution kernel \( P \) is given by

\[
P(x) = C_F \left[ \frac{2}{(1 - x)_+} + \frac{3}{2} \delta(1 - x) \right] .
\]

(20)

Next we derive the DGLAP equation in the axial gauge \([5, 6]\), in which the relevant definitions for the quark distribution function are

\[
\phi^{(n)}(\xi, \mu/m) = \int \frac{dy}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{q}(y^-)\gamma^+ q(0)|p\rangle|_{\nu' A=0} ,
\]

(21)

\[
\phi^{(n)}(\xi, \nu/\mu, \mu/m) = \int \frac{dy}{2\pi} e^{-i\xi p^+ y^-} \langle p|\bar{q}(y^-)\gamma^+ q(0)|p\rangle|_{\nu A=0} ,
\]

(22)

as described by Fig. 1(b). The path-ordered exponential is equal to the identity in this gauge, and eikonal lines collecting the collinear gluons are absent. Similarly, we work on \( \phi^{(n)} \) when performing the resummation. The \( n \) dependence goes into the gluon propagator, \((-i/\ell^2)N^{\mu\nu}(l)\), with

\[
N^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l} + n^2 \frac{l^\mu l^\nu}{(n \cdot l)^2} .
\]

(23)

Because of the scale invariance of \( \phi^{(n)} \) in \( n \) as indicated by Eq. (23), \( \phi^{(n)} \) depends on the scale \( \nu \), and thus Eq. (3) holds. The operator \( d/dn_\alpha \), now applying to the gluon propagator, gives

\[
- \frac{n^2}{v \cdot n v_\alpha} \frac{d}{dn_\alpha} N^{\mu\nu} = \frac{n^2 v_\alpha}{v \cdot n n \cdot l} (l^{\mu} N^{\alpha\nu} + l^{\nu} N^{\mu\alpha}) ,
\]

\[
\equiv \hat{v}_\alpha (l^{\mu} N^{\alpha\nu} + l^{\nu} N^{\mu\alpha}) .
\]

(24)
The loop momentum \( l^\mu (l^\nu) \) carried by the differentiated gluon contracts with the vertex the gluon attaches, which is then replaced by the special vertex \( \hat{v}_\alpha \) defined by the second line of Eq. (24).

The contraction of \( l^\nu \) hints the application of the Ward identities [5, 6], such as

\[
\frac{i(k^\mu + l^\mu)}{(k + l)^2} \frac{i(k^\nu)}{k^2} \frac{i(k^\nu)}{(k + l)^2} = \frac{i(k^\nu)}{k^2} - \frac{i(k^\nu)}{(k + l)^2},
\]

for the quark-gluon vertex, and

\[
\frac{iN^{\alpha\mu}(k + l)}{(k + l)^2} \Gamma_{\mu\nu\lambda}^\alpha - \frac{iN^{\lambda\gamma}(k)}{k^2} = -i \left[ \frac{-iN^{\alpha\gamma}(k)}{k^2} - \frac{-iN^{\alpha\gamma}(k + l)}{(k + l)^2} \right]
\]

for the triple-gluon vertex \( \Gamma_{\mu\nu\lambda} \). Similar identities for other types of vertices can be derived easily. Summing all the diagrams with different differentiated gluons, those embedding the special vertices cancel by pairs, leaving the one with the special vertex moving to the outer end of the quark line [4]. We obtain the same formula in Eq. (7), described by Fig. 3(a), where the new function \( \tilde{\phi}^{(n)} \) contains one special vertex represented by a square. The coefficient 2 comes from the equality of the new functions with the special vertex on either of the two quark lines.

The important regions of the loop momentum \( l \) flowing through the special vertex are also soft and hard, since the vector \( n \) does not lie on the light cone, and the collinear enhancements are suppressed. Similarly, in the above leading regions \( \tilde{\phi}^{(n)} \) can be factorized into the convolution of the sub-diagram containing the special vertex with the original distribution function \( \phi^{(n)} \), leading to Eq. (8). In this case the functions \( K \) and \( G \), corresponding to Figs. 3(b) and 3(c), are written as

\[
K = i g^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^{4-\epsilon}} v \cdot l \left[ \frac{\delta(\xi - x)}{l^2} + 2\pi i \delta(l^2) \delta \left( \xi - x - \frac{l^+}{p^+} \right) \right] \frac{N^{\mu\nu}}{l^2},
\]

\[
G = -i g^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^{4-\epsilon}} v \cdot l \left[ \frac{\xi \cdot \slashed{p} - l^\nu}{(\xi p - l)^2} + \frac{v_\nu}{v \cdot l} \right] \frac{N^{\mu\nu}}{(\xi \cdot x)}.
\]

It is easy to show that Eqs. (27) and (28) reduce to Eqs. (11) and (12), respectively. Then following the steps from Eq. (13) to Eq. (19), we derive the same DGLAP equation in the axial gauge. This is natural, because
the evolution of a parton distribution function is measurable, and should be independent of the gauge we adopt.

3. Relation of $\phi^{(n)}$ and $\phi$

In this section we explore the relation between the working definition $\phi^{(n)}$ of the quark distribution function employed in the resummation technique and the standard definition $\phi$. As explained in Sect. 2, $\phi^{(n)}$ and $\phi$ contain the same collinear divergences from gluon momenta parallel to the hadron momentum, and thus the same nonperturbative information. Take the $O(\alpha_s)$ correction with one end of a real gluon attaching the quark line and the other end attaching the eikonal line as an example. The loop integral associated with $\phi$ is written as

$$I = g^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^4} \frac{1}{2} (x \cdot b + f) f' l' 2\pi \delta(l^2) \delta \left(1 - x - \frac{l^+}{p^+}\right),$$

$$= \frac{\alpha_s}{\pi} (2\pi)^{3-\epsilon} \frac{1}{2(1-x)} \int d^2-l_T \frac{1}{l_T^2 + m^2}.$$  \hspace{1cm} (29)

where $m$ is the infrared cutoff mentioned before, and $\mu$ is the renormalization scale. The corresponding integral $I^{(n)}$ associated with $\phi^{(n)}$ is obtained by replacing $v'$ in Eq. (29) by $n$, which is given by,

$$I^{(n)} = \frac{\alpha_s}{2\pi} \int d^2-l_T \frac{2x(1-x)\nu^2}{l_T^2 + m^2(1-x)^2\nu^2}.$$  \hspace{1cm} (30)

The collinear divergences come from the region with small $l_T$, in which $I^{(n)}$ reduces to $I$ as indicated by Eqs. (29) and (30). However, $I$ is ultraviolet divergent, while $I^{(n)}$ is not, because of the extra denominator $l_T^2 + 4(1-x)^2\nu^2$. Therefore, $\phi^{(n)}$ and $\phi$ possess the same collinear structure but different ultraviolet structure.

Performing the integrations over $l_T$, we obtain

$$I = \frac{\alpha_s}{2\pi} \frac{1}{1-x} \left(1 + \ln \frac{\mu}{m}\right),$$

$$I^{(n)} = \frac{\alpha_s}{2\pi} \frac{1}{1-x} \frac{\nu^2}{m}.$$  \hspace{1cm} (31)

\hspace{1cm} (32)
where constants of order unity have been dropped. It is found that \( I \) contains a logarithm \( \ln(\mu/m) \), and \( I^{(n)} \) contains \( \ln(\nu/m) \) with the same coefficient. Corrections in \( \phi^{(n)} \) and \( \phi \) without gluons attaching the eikonal line, such as the self-energy corrections, give the same logarithms \( \ln(\mu/m) \). By splitting the logarithm in \( I^{(n)} \) into

\[
\ln(\nu/m) = \ln(\nu/\mu) + \ln(\mu/m) ,
\]

it is easy to understand why \( \phi^{(n)} \) depends on the argument \( \nu/\mu \) due to the replacement of \( \nu' \) by \( n \). The resummation technique, summing the first logarithm in the above expression to all orders, leads to Eq. (14) in the same form as the (partial) DGLAP equation, that sums the second logarithm, since the two terms possess the same coefficients. The choice \( \nu = \mu \), diminishing the first term, renders \( \phi^{(n)} \) coincide with \( \phi \) as in Eq. (13), because the second term, combined with those from other corrections without gluons attaching the eikonal line, gives the complete logarithms \( \ln(\mu/m) \) in \( \phi \). It is also easy to realize why the RG method, applying to \( \phi^{(n)}(x(\xi), \Lambda(\bar{\mu})/\mu, \mu/m) \), sums only \( \ln(\mu/m) \) from other corrections as in Eq. (17), since the corrections with gluons attaching the eikonal line, proportional to \( \ln(\nu/m) \), are \( \mu \)-independent. At last, in the limit \( n \to \nu' \) the cutoff \( \nu \) diverges, and \( \ln(\nu/\mu) \) corresponds to the ultraviolet pole in \( I \). This pole is removed by renormalization, and \( \phi^{(n)} \) approaches \( \phi \):

\[
\lim_{n \to \nu'} \phi^{(n)}(x, \nu/\mu, \mu/m) = \phi(x, \mu/m) .
\]

The infrared finite difference between \( \phi^{(n)} \) and \( \phi \), indicated by \( I^{(n)} - I \propto \ln(\nu/\mu) \) from Eqs. (31) and (32), can be computed perturbatively. In terms of Feynman diagrams in the covariant gauge, it is the difference between the diagrams with radiative gluons attaching the eikonal line along \( n \) and the diagrams with gluons attaching the eikonal line along \( \nu' \). The above argument leads to the convolution formula,

\[
\phi^{(n)}(x, \nu/\mu, \mu/m) = \int_x^1 \frac{d\xi}{\xi} D(x/\xi, \nu/\mu) \phi(\xi, \mu/m) ,
\]

where the infrared finite piece \( D \), independent of the infrared regulator \( m \), denotes the difference mentioned above. The graphic definition of \( D \) is given in Fig. 4, where the denominator, with the eikonal line along \( \nu' \), subtracts the collinear divergences involved in the numerator with the eikonal line.
along \( n \). All the diagrams without gluons attaching the eikonal lines cancel between the numerator and the denominator. The bubbles contain infinite many gluon exchanges, and the external lines represent quarks. Because of \( D \), \( \phi^{(n)} \) absorbs additional finite contributions compared to \( \phi \). Hence, they can be regarded as the distribution functions defined in different factorization schemes. Varying the vector \( n \) then means varying the factorization scheme.

Based on Eq. (35), the differentiation \( d/dp^+ \) or \( d/dn \) in fact applies to \( D \):

\[
p^+ \frac{d}{dp^+} \phi^{(n)}(x, \nu/\mu, \mu/m) = \int_x^1 \frac{d\xi}{\xi} p^+ \frac{d}{dp^+} D\left(x/\xi, \nu/\mu\right) \phi(\xi, \mu/m) .
\] (36)

The similar resummation procedures give

\[
p^+ \frac{d}{dp^+} \phi^{(n)}(x, \nu/\mu, \mu/m) = \int_x^1 \frac{d\xi}{\xi} \int_{x/\xi}^1 \frac{dz}{z} \frac{2}{[1 - x/(\xi z)]_+} D(z, \nu/\mu) \phi(\xi, \mu/m) .
\] (37)

Using the variable change \( z = y/\xi \), and interchanging the integrations over \( \xi \) and over \( y \), the above equation is reexpressed as

\[
p^+ \frac{d}{dp^+} \phi^{(n)}(x, \nu/\mu, \mu/m) = \int_x^1 \frac{dy}{y} \int_y^1 \frac{d\xi}{\xi} \frac{2}{(1 - x/y)_+} D(y/\xi, \nu/\mu) \phi(\xi, \mu/m) .
\] (38)

Employing the definition in Eq. (36), we arrive at Eq. (14). The remaining steps of the resummation and Eq. (34) in the \( n \to n' \) limit then lead to the DGLAP equation. The above formulas elucidate the statement that the vector \( n \) is an auxiliary tool, and appears only at the intermediate stage of the derivation. Hence, the gauge invariance of the resummation technique is guaranteed.

The discussion presented here can be generalized to the resummation-based derivation of the BFKL equation for the gluon distribution function \( F(x, k_T) \) easily, which describes the probability that a gluon from the hadron carries the longitudinal momentum \( xp^+ \) and the transverse momentum \( k_T \). However, the relation between the working definition \( F^{(n)} \) and the standard definition \( F \) is a \( k_T \)-factorization formula \[7\], instead of a collinear factorization formula in Eq. (35). The details will be published elsewhere.

4. Conclusion
In this paper we have demonstrated how to derive the DGLAP equation using the Collins-Soper-Sterman resummation technique, and that the results are independent of the gauge we employed. Though we started with the $n$-dependent parton distribution function, the evolution kernel turns out to be gauge invariant. We have also explored the relation between the $n$-dependent and standard distribution functions. From this relation, we explained why the resummation technique is a successful method to the summation of large logarithms, and showed that the $n$-dependent definition reduces to the standard one by either taking the $n \rightarrow v'$ limit or setting the gauge factor $\nu$ to the renormalization scale $\mu$. This work provides a solid ground for our previous studies on the resummation approach to evolution equations [3, 4].

This work was supported by the National Science Council of R.O.C. under the Grant No. NSC-87-2112-M-006-018.
References

[1] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 428; G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298; Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.

[2] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381.

[3] H-n. Li, Phys. Lett. B 369 (1996) 137; Phys. Rev. D 55 (1997) 105.

[4] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199; Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822; L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904.

[5] H-n. Li, Phys. Lett. B 405 (1997) 347.

[6] H-n. Li, Report No. hep-ph/9703328 (unpublished); Phys. Lett B 416 (1998) 192.

[7] T. Jaroszewicz, Acta. Phys. Pol. B 11 (1980) 965; S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B 242 (1990) 97; Nucl. Phys. B366 (1991) 657; S. Catani and F. Hautmann, Nucl. Phys. B427 (1994) 475.
Figure Captions

FIG. 1. Definition of the quark distribution function in (a) the covariant gauge and in (b) the axial gauge.

FIG. 2. (a) The derivative $p^+ d\phi^{(n)}/dp^+$ in the covariant gauge. (b) The $O(\alpha_s)$ function $K$. (c) The $O(\alpha_s)$ function $G$.

FIG. 3. (a) The derivative $p^+ d\phi^{(n)}/dp^+$ in the axial gauge. (b) The $O(\alpha_s)$ function $K$. (c) The $O(\alpha_s)$ function $G$.

FIG. 4. The graphic definition of the function $D$. 
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9803202v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9803202v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9803202v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9803202v1