Digital Identification and Control of Multivariable Plants Using Markov Parameters

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Abstract: The algorithm of the digital adaptive controller is offered in this article for control systems with identification of uncertain multivariable continuous-time objects. The adaptive controller includes unit of digital identification and unit of digital control. The algorithm of the identification unit allows to define the transfer matrix of the uncertain continuous-time plant. Unlike traditional methods of identification, the offered analytical algorithm uses Markov parameters not of the uncertain continuous-time plant, but virtual discrete-like plant. Markov parameters are determined by data of I/O of the uncertain continuous-time plant. The algorithm of the control unit is developed with use of control of two types: decomposition control and digital control on the output and impacts. The order and parameters of the control unit are determined by the method of the analytical design of digital control systems. Normalized standard transfer functions are used to create the transfer functions of the adaptive system according to required values of astatism order, settling time and overshoot. The effectiveness of the proposed algorithm of adaptive controller is illustrated by a numerical example. The proposed approach can be applied to create control systems for agricultural, food, mining, and other industries.

Keywords: uncertain multivariable continuous-time plant, identification, Markov parameters, adaptive control, design, digital adaptive controller, standard transfer function, astatism, settling time, overshoot.

1. INTRODUCTION

Requirements to the quality of control systems increase and their area of use expands continuously. Moreover, plant models are often absent. Therefore, the problem of control of uncertain, continuous-time multivariable (MIMO) plants is considered in many papers. To solve this problem, adaptive systems are usually used. Adaptive control systems are designed without using a model of an uncertain plant often. Adaptive MIMO systems are designed based on learning control in Radac et al. (2015), Xu et al. (2014). An adaptive controller is designed according to minimum of a certain functional by Zhu and Hou (2015), and Zhu et al. (2016) use the backstepping method to design adaptive feedbacks of an output. Practically, adaptive design methods assume that the order of an uncertain plant is known and constant.

Adaptive systems with identification of models are more efficient. They can be used when the order and parameters of a plant change in steps at some points in time. Constancy intervals of the order and the parameters can be short Ning et al. (2019) or rather long Xu et al. (2014). To create adaptive systems of this type, digital algorithms for identification of the plant model and for the control design are needed.

The frequency identification method based on the Wiener-Hopf equation is one of the earliest. This method is statistical; therefore, the identification process has long duration. Currently, for identification, harmonious signals are used more often. A model of an object is created on the basis of sinusoidal test signals Gerasimov et al. (2018), Ahmed et al. (2009). In Voevoda et al. (2019) this problem is solved on the basis of the amplitude-frequency characteristic of a closed system. However, this approach can be implemented if changing parameters and ranges of their changes are known a priori. That is a significant drawback of this approach.

One of the earliest non-frequency identification methods of B. Ho was represented in Kalman et al. (1971). This method was developed based on the Markov parameters of dynamical systems Chen (1999) for the identification of discrete plants with the use of pulse transient functions. But these functions are almost impossible to obtain; therefore, the B. Ho’s method is not applied practically.

Identification is often carried out based on the optimal filtering theory. Model coefficients and boundaries of their constancy are determined by solving an optimization problem with constraints in Aida-zade and Ragimov (2017). Parameters of an uncertain plant model are determined by integrating the highest derivative variables, which are measured with some noise, in Shao et al. (2017). In Genov et al. (2017), the state of a complex system is identified under ambiguity measurement conditions.
At present, the LQG method is the most common method for the analytical control systems design (Chen, 1999; Ghaffar and Richardson, 2015). However, this method is difficult to apply when the plant order is uncertain, therefore, adaptive control systems for uncertain plants are often designed using PI or PID controllers, see He and Wang (2006).

The paper is devoted to the development of an algorithm of a digital adaptive control with identification of uncertain multivariable continuous-time (UC MIMO) plants. The algorithm of the identification has been developed using the Markov parameters of a virtual discrete-like plant (DLP). The Markov parameters are calculated based on continuous-time plant reaction to a step test action. The algorithm of the adaptive control is designed using decomposition, digital control on an output and impacts, and standard normalized transfer functions. The offered controller can be implemented using parallel digital processing of experimental data by multiprocessor controllers, see Levin (2015), Minaev (2016).

This paper is structured into seven sections. In the sections 2 and 3 the problem statement and the virtual discrete-like plant are introduced. In the section 4 the connection of the Markov parameters of any dynamic plant with its transfer function and the algorithm of the identification unit following from them are considered. The design problem of adaptive control is formulated in the section 5. The algorithm as its solution is offered in the section 6. Finally, in the section 7 a numerical example shows the performance of the proposed algorithms.

2. PROBLEM STATEMENT

We assume that an uncertain continuous-time plant is described by the equations

\[ w = P w + Q u, \quad y = K w + D u. \]

where \( w \in \mathbb{R}^n \) is the state vector; \( u \in \mathbb{R}^n \) is the vector of controls; \( y \in \mathbb{R}^m \) is the vector of output variables of the plant; \( P, Q, K, D \) are numeric matrices of the corresponding dimensions. The order \( n \) of the plant and the parameters of its model (1) are unknown a priori. They can suddenly change and then remain unchanged for quite a long time. At any time UC MIMO plant (1) is full, i.e. it has only a controllable and observable path between the input and the output of the plant Stengel (1994), Gaiduk (1998); but its order is \( n < n_{\text{max}} \), where \( n_{\text{max}} \) is a priory known value. All the output variables \( y_i = y_i(t), i = 1, 2, \ldots, q \) are measured.

The task consists in the development of the digital identification algorithm to define the current model (1) and the control algorithm of the adaptive digital controller (ADC) based on the model (1). This controller must be realizable and to provide a required astatism order to reference and external disturbances; the settling time and overshoot should have values less than required, Raven (1983), Gaiduk (2012).

3. VIRTUAL DISCRETE-LIKE PLANT

Let \( T_{\text{sp}} \) be the sampling period of the variables \( y = y(t) \) at the identification, then \[ y_i^j = y_i^j(T_{\text{sp}}) = [y_{i1}^j \ y_{i2}^j \ \ldots \ y_{iq}^j]^T \] is the vector of discrete values of the deviations of the output variables caused by a test action \( u_j(t) = \{0 \ldots 0 \ u_{j0} \ 0 \ldots 0\} \) at \( w(0) = 0; \ u_{j0} = \text{const}, j = 1, m \). The identification algorithm is based on virtual discrete plants which have the sampling period \( T_{\text{sp}} \). These plants are described by the equations:

\[ \hat{\bar{w}}_{i,k+1} = \hat{\bar{P}}_{i,k} \hat{\bar{v}}_{i,k} + \hat{\bar{Q}}_{i,k} \hat{\bar{y}}_{i,k} \ \ \ \hat{\bar{y}}_{i,k} = \hat{\bar{K}}_{i,k} \hat{\bar{v}}_{i,k} + \hat{\bar{D}}_{i,k}. \]

Here \( \bar{v}_{i,k} = [\bar{v}_{i1} \ \bar{v}_{i2} \ \ldots \ \bar{v}_{iq}]^T \) and \( \hat{\bar{y}}_{i,k} = [\hat{\bar{y}}_{i1} \ \hat{\bar{y}}_{i2} \ \ldots \ \hat{\bar{y}}_{iq}]^T \) are the deviations vectors of the state and output variables; \( \bar{u}_{ik} = [\bar{u}_{i1} \ \bar{u}_{i2} \ \ldots \ \bar{u}_{iq}]^T \) are the discrete controls, \( k = 0, 1, 2, \ldots; \) \( \hat{\bar{P}}, \hat{\bar{Q}}, \hat{\bar{K}}, \hat{\bar{D}} \) are the numerical matrices of the plants (2), \( v = \{1 \ldots n_{\text{max}}\} \), see: Gaiduk and Plaksienko (2016).

Let \[ \hat{\bar{y}}_{i}^j = [\hat{\bar{y}}_{i1}^j \ \hat{\bar{y}}_{i2}^j \ \ldots \ \hat{\bar{y}}_{iq}^j]^T \] be the deviations of the output variables of the plant (2) caused by the control \( \bar{u}_{i} = [0 \ldots 0 \ \bar{u}_{i0} \ 0 \ldots 0]^T \) at \( \bar{v}_{i0} = 0 \) and

\[ \hat{\bar{u}}_{i0} = \bar{u}_{i0}, \quad \hat{\bar{y}}_{i}^j = y_i^j, \quad j = \{1 \ldots m\}, \quad k = 0, 1, 2, \ldots. \]

Definition 1. The virtual plants (2) are called discrete-like plants (DLP) to the continuous plant (1) if the conditions (3) are fulfilled.

The test action \( u(t) = u_{j0} \) can be represented as a sequence of rectangular pulses having the duration \( T_{\text{sp}} \) and amplitude \( u_{j0} \). Therefore, the transfer functions (TF) \( \Theta_{j,n}(s) \) of the channels \( u_i \rightarrow y_i \) of the plant (1) are connected with the TF \( \hat{\Theta}_{j,n}(z,T_{\text{sp}},\nu) \) of the channels \( \bar{u}_i \rightarrow \bar{y}_i \) of the DLP (2) with \( v = n \) by the special \( Z \)-transform:

\[ \Theta_{j,n}(s) = Z\{\Theta_{j,n}(z,T_{sp},\nu)\}. \]

\[ \Theta_{j,n}(z,T_{sp},\nu) = Z^{-1}\{\Theta_{j,n}(s)\}, \quad n_y = \bar{n}_y, \]

where \( \bar{n}_y, n_y \) are the degrees of the denominators of the TF \( \hat{\Theta}_{j,n}(z,T_{sp},\nu) \) and \( \Theta_{j,n}(s) \) respectively, \( j = 1, 2, \ldots, m; \ i = 1, 2, \ldots, q \). Evidently, the matrices \( \hat{\bar{P}}, \hat{\bar{Q}}, \hat{\bar{K}}, \hat{\bar{D}} \) depend on \( T_{\text{sp}} \).

The transforms (4) are carried simply, see Moore (2013). However, if the continuous-time plant (1) has “left” poles, then the possibility of its identification based on the DLP (2) is lost if \( T_{\text{sp}} \) is too large. In this regard, the following definition is introduced. Let \( \bar{n}_y^0 \) be the free coefficient of the denominator of the TF \( \hat{\Theta}_{j,n}(z,T_{sp},\nu) \) of the system (2) and \( \bar{n}_y^0 \) be the computational error of the controller used.

Definition 2. If the sampling period \( T_{\text{sp}} = T'_{\text{sp}} \), and the TF \( \Theta_{j,n}(z,T_{sp},\nu) \) is such that

\[ \bar{n}_y^0 \gg \bar{n}_y, \quad v = \{1 \ldots n_{\text{max}}\}, \]

then the DLP (2) is correct. If \( T_{\text{sp}} > T'_{\text{sp}} \) and \( \bar{n}_y^0 \ll \bar{n}_y \), then the DLP (2) is incorrect.
4. MARKOV PARAMETERS AND TRANSFER FUNCTIONS

The Markov parameters $\hat{m}_{ij}$ of each channel $\hat{u}_j \rightarrow \hat{y}_i$ of the DLP (2) with the order $v$ are defined by the expressions:

$$\hat{m}_{ij}^v = \tilde{u}_{ij} \cdot \hat{m}_{ij}^0 = \tilde{K}_v \tilde{P}_v^{-1} \tilde{Q}_j, \quad \zeta = 1, 2, 3, \ldots, (6)$$

where $\tilde{Q}_j$ is the element of the matrix $\tilde{D}$; $\tilde{K}_v$ is the $i$-row of the matrix $\tilde{K}$, $i = \text{I}_v \tilde{Q}_j$, $\tilde{P}_v$ is the $j$-column of the matrix $\tilde{Q}_v$, $j = \text{I}_v$, see: Chen (1999), Gaiduk and Plaksienko (2016). The values $\hat{y}_{ij}$ of the DLP (2) are associated with the values $\tilde{u}_{ij}$ by the expressions:

$$\hat{y}_{ij} = \hat{K}_{ij} \hat{w}_{ij} + d_{ij} \tilde{u}_{ij},$$

$$\hat{y}_{ij} = \hat{K}_{ij} \hat{w}_{ij} + \sum_{\zeta=0}^{\infty} \hat{K}_{ij} \hat{P}_v^{-1} \hat{Q}_j \hat{u}_{ij} + d_{ij} \hat{u}_{ij}, \quad k = 1, 2, 3, \ldots, (7)$$

From the expressions (6) and (7) it follows that if $\hat{y}_0 = 0$, and the condition (3) is satisfied, then the Markov parameters can be calculated by the formulas:

$$\hat{m}_{ij}^v = y_{ij} / u_{ij}, \quad \hat{m}_{ij}^0 = y_{ij}^0 / u_{ij} + \sum_{\zeta=1}^{\infty} \hat{m}_{ij}^\zeta, \quad \zeta = 1, 2, 3, \ldots, (8)$$

As shown above, the virtual DLP (2) includes the systems of the order $v=1,2,\ldots,n_{\text{max}}$. Evidently, all these systems have the same Markov parameters $\hat{m}_{ij}$. (8). The transfer function of the channel $\hat{u}_j \rightarrow \hat{y}_i$ of the system (2) with the order $v$ is

$$\hat{\Theta}_{yj}(z,v) = \tilde{P}^{-1}(z,v) \{ \hat{K}_v \cdot \text{Ad}(zI - \hat{P}_v) \hat{Q}_j + \hat{d}_{ij} \hat{P}(z,v) \}, (9)$$

$$\hat{P}(z,v) = \text{det}(zI - \hat{P}_v) = \hat{n}_{yv} z^v + \hat{n}_{yv-1} z^{v-1} + \cdots + \hat{n}_{y0} = \hat{n}_y v, (10)$$

where $\hat{n}_y = 1$; $\text{Ad}(zI - \hat{P}_v)$ is the adjoint $v \times v$-matrix, which can be represented as follows, Gantmakher (1988, p. 88):

$$\text{Ad}(zI - \hat{P}_v) = \hat{n}_{yv} z^{v-1} + (\hat{n}_{yv-1} \hat{P}_v^2 + \hat{n}_{yv-2} z^{v-2}) + \cdots + (\hat{n}_{y0} \hat{P}_v^{-1} + \hat{n}_{y1} z + \hat{n}_{y2} \hat{P}_v^{-2} + \cdots + \hat{n}_{y0} z^{v-1}).$$

(11)

According to (10), (11) the coefficients of the TF $\hat{\Theta}_{yj}(z,T_{\text{sys}},v)$ (9) depend on $\hat{m}_{ij}^0$ (8) and $T_{\text{sys}}$. This TF can be found from the input–output equations of the DLP (2) in the form:

$$\hat{\Theta}_{yj}(z,T_{\text{sys}},v) = \frac{\sum_{\varphi=0}^{v} \hat{n}_{y\varphi} z^{v-\varphi}}{\hat{n}_{y0} z^{v} + \hat{n}_{y1} z^{v-1} + \cdots + \hat{n}_{yv} z^{v-1} + \hat{n}_{y0} z^{v-2}}.$$ (12)

Two systems of algebraic equations relate the coefficients of the transfer functions (9) – (11), (12) and the parameters $\hat{m}_{ij}^0$, Gaiduk and Plaksienko (2016). The first system allows us to estimate the degree $\hat{n}_y$ of the denominator of the TF (12) with the corresponding TF $\hat{\Theta}_{yj}(s)$ according to (4):

$$\hat{n}_y = \max \{ v | \hat{m}_{ij}^v \neq 0 \}, \quad \hat{s}_y^v = \text{det} \hat{M}_y^v.$$ (13)

Here $\beta$ is an arbitrary integer from the interval $[0, \beta_{\text{max}}]$. Then we can calculate the coefficients $\hat{\eta}_y^\varphi$, $\rho = [0,1,\ldots,\hat{n}_y-1]$ of the TF (12) with $v = \hat{n}_y$ using the formulas:

$$\hat{\eta}_y^0 = -\left( \hat{M}_y^0 \right)^{-1} \hat{M}_y^1, (15)$$

$$\hat{\eta}_y^\rho = \left[ \hat{\eta}_y^0, \hat{\eta}_y^1, \ldots, \hat{\eta}_y^{\hat{n}_y-1} \right]^T.$$ (16)

If the condition (5) is satisfied, the coefficients $\hat{\eta}_y^\rho$, $\rho = [0,1,\ldots,\hat{n}_y-1]$ of the TF (12) with $v = \hat{n}_y$ can be calculated as

$$\hat{\eta}_y^\rho = \hat{m}_{ij}^\rho - \sum_{\mu=0}^{\hat{n}_y-1} \hat{m}_{ij}^\mu \hat{m}_{ij}^{\rho-\mu},$$

$$\rho = [0,1,\ldots,\hat{n}_y-1], \quad \hat{\eta}_y^\rho = \hat{m}_{ij}^0.$$ (17)

Application of the $Z$-transform (4) to (12) with the extension $\text{zoh}$ yields the TF $\hat{\Theta}_{yj}(s) = H_y(s) / P_y(s)$. The estimate $\hat{n}$ of the plant (1) order is determined by the degree of the estimate of its characteristic polynomial:

$$\hat{P}(s) = \text{c.m.} [ P_y(s) ], \quad \hat{n} = \text{deg} \hat{P}(s).$$ (18)

The transfer matrix estimate of the plant (1) is determined using the expression

$$\hat{\Theta}_{yj}(s) = \tilde{P}^{-1}(s) \left[ H_y(s) \hat{Q}_{1j}(s) \ldots H_m(s) \hat{Q}_{mj}(s) \right],$$

$$H_y(s) \hat{Q}_{1j}(s) \ldots H_m(s) \hat{Q}_{mj}(s) \hat{Q}_{mj}(s),$$ (19)

where $\hat{Q}_{ij}(s) = \hat{P}(s) / P_y(s)$. The identification algorithm using the expressions above allows you to identify UC MIMO continuous plants. Information processing, when separate channels are identified, can be performed in parallel. It is advisable to implement the identification algorithm based on the expressions (3) - (19) using computing tools oriented on parallel information processing, see: Levin (2015), Minaev (2016). The proposed DLP and expressions (4), (5) ensure the high efficiency of this identification method using the Markov parameters.

5. ADAPTIVE CONTROL DESIGN

The adaptive control system with identification of the UC MIMO plant (1) with $m = q$ has been designed using the decomposition control, which forms $q$ independent channels (plants) channels with single output, proposed in Gaiduk (2012). In this case, the control units of ADC of the system for each this plant are created considering the influence of other controls $u_j$ as disturbances. We consider analytical
Let \( T_{eq} \) be the sampling period of the digital control unit and \( T_{eq} \gg T_{np} \). The matrix \( \bar{\Sigma}_{m}(z) \) is the result of the direct \( z \)-transform (4) with \( T = T_{np} \) of the \( q \times q \)-transfer matrix \( \bar{\Theta}_{m}(s) \) (19). Application of the transformation, which corresponds to the decomposition control, to the transfer matrix \( \bar{\Sigma}_{m}(z) \) gives \( q \) independent plants. Suppose each such plant is described by the equation:

\[
P(z) y(z) = H(z)u(z) + F(z)f(z)
\]

(20)

where \( y(z), u(z) \) and \( f(z) \) are \( z \)-images of the controlled variable, the control and the external non-measurable disturbance; \( P(z) \), \( H(z) \), \( F(z) \) are some polynomials. The coefficients of the polynomials in (20) have known values; and \( P(z) \) and \( H(z) \) do not have common factor. The digital control unit of ADC is described by the equation:

\[
\Delta(z)u(z) = \Gamma(z)r(z) - \Lambda(z)y(z)
\]

(21)

where \( r(z) \) is the \( z \)-image of the reference signal \( r = r(t) \) of control system of plant (20), see Gaiduk and Plaksienko (2018); \( \Delta(z), \Gamma(z), \Lambda(z) \) are required polynomials whose degrees are such that

\[
\mu_{dc} \geq \mu_{sc}
\]

(22)

\[
r_{dc} = \deg \Delta(z), \gamma = \deg \Gamma(z), l = \deg \Lambda(z).
\]

Usually \( \mu_{dc}=1 \) or \( \mu_{dc} = r_{dc}/T_{np} \) where \( r_{dc} \) is a time delay. The system (20), (21) must have the order of atastism \( \zeta'_{dc} = 1 \) to the reference \( r = r(t) \) and \( \zeta'_{sc} = 1 \) to the disturbance \( f = f(t) \); settling time less than \( T_{np} \) s; overshoot less than \( \sigma' \% \) and the roots stability margin \( \beta' \leq 1 \).

6. DESIGN PROBLEM SOLUTION

Let the condition on stability of the designed system be

\[
\beta_{m} = \left(1 - \frac{1}{p} \right) \bar{k}_{c} \geq \beta',
\]

(23)

where \( \bar{k}_{c} \) are the roots of the characteristic polynomial \( D(z) \) of the system (20), (21). This condition defines a region \( \Omega \) on the complex \( z \)-plane, i.e. an area of permissible location of the roots of the polynomial \( D(z) \). The polynomial \( P(z) \) from (20) is factorized as \( P(z) = P_{1}(z)P_{2}(z) \), where the polynomial \( P_{1}(z) \) includes all the roots of the polynomial \( P(z) \) which satisfy condition (23), and \( P_{2}(z) = P(z)/P_{1}(z) \). Suppose, in (20) \( H(z) = h_{0} \) or if \( H(z) = h_{0} \prod_{i=1}^{l} (z - \zeta'_{i}) \) then the roots \( \zeta'_{i} \) satisfy to the condition (23), i.e. \( H(z) = h_{0}(z) \). In this case, the polynomial \( D(z) = P_{2}(z)H_{0}(z)\bar{D}(z) \), where \( H_{0}(z) = h_{0}^{l}H(z) \), and \( \bar{D}(z) = \bar{\Theta}_{0} + \bar{\Theta}_{1}z + \ldots + \bar{\Theta}_{n}z^{n} \) is some polynomial.

In (20), let \( P(z) = (z^{-1})^{n} P_{1}(z) \) and \( F(z) = (z^{-1})^{n} F_{1}(z) \), where \( P_{1}(1) \neq 0, F_{1}(1) \neq 0 \). In this case, the system (20), (21) will have a required order of atastism, if

\[
\nu = \max \{ \zeta'_{dc} - n_{r}; \zeta'_{sc} - n_{f}; 0 \},
\]

(24)

\[
\Delta(z) = H_{0}(z)(z^{-1})^{\nu} \hat{\Lambda}(z) + \Lambda(z) = P_{2}(z)\hat{\Lambda}(z),
\]

(25)

where \( \hat{\Lambda}(z) \) and \( \hat{\Lambda}(z) \) are some polynomials with degrees:

\[
\tilde{r} = r_{dc} - \nu - \nu, \quad \tilde{t} = r_{dc} - n_{r} - n_{f}; \quad n_{d} = \deg P_{2}(z).
\]

The coefficients of the polynomials

\[
\hat{\Lambda}(z) = \sum_{p=0}^{\nu} \theta_{p}z^{p}, \quad \hat{\Lambda}(z) = \sum_{p=0}^{\nu} \theta_{p}z^{p}
\]

(26)

are determined by the solution of the next algebraic system:

\[
\begin{align*}
[ h_{0} & 0 & 0 & \cdots & \lambda_{0} & \bar{\theta}_{0} & \bar{\Sigma}_{0}^{+} & \bar{\Sigma}_{0}^{-} & \bar{\Sigma}_{0}^{0} & \bar{\Sigma}_{0}^{-} & \bar{\Sigma}_{0}^{0} & \bar{\Sigma}_{0}^{-} & \bar{\Sigma}_{0}^{0} & \bar{\Sigma}_{0}^{-} \bigg| \bigg. \\
0 & h_{0} & 0 & \cdots & \lambda_{1} & \theta_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & \cdots & \lambda_{l} & \theta_{l} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \bigg. \\
0 & \cdots & 0 & \cdots & \theta_{l} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \bigg. \\
0 & \cdots & 0 & \cdots & 0 & \theta_{l} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{align*}
\]

(27)

The matrix of system (27) has \( l+1 \) columns composed by the coefficient \( h_{0} \) and \( r+1 \) columns composed by the coefficients \( \eta_{p} \) of the \( P(z) = (z^{-1})^{l} P_{2}(z) = \eta_{0}z^{l} + \ldots + \eta_{l} \). The degrees of the polynomials \( \hat{\Lambda}(z) \) and (25), (26) are calculated as follows

\[
r_{dc} = n_{p} + \nu + \mu_{dc} - 1, \quad \tilde{r} = n_{p} + \mu_{dc} - \nu - 1, \quad \tilde{n} = n_{d} + \nu,
\]

(28)

where \( n_{p} = \deg P(z) \). The formulas (28) follow from solvability conditions of the system (27), the conditions (23), (24), and the equality \( D(z) = P_{2}(z)H_{0}(z)\hat{D}(z) \).

The coefficients \( \bar{\Theta}_{0} \) of the polynomial \( \hat{D}(z) \) are assigned using the standard normalized transfer functions (SNTF) \( \Theta_{m}(s) \) of continuous systems, see Gaiduk (2012, p. 344-346). For this purpose, the coefficients of the required SNTF are stored in the ADC microprocessor memory. As you know, the functions \( \hat{\Theta}(z) \), which are the result of the conversion \( \hat{\Theta}(z) = Z_{T} \{ \Theta(s) \} \), usually have a relative degree \( \mu_{sc} \) equal to zero or one. Therefore, the denominator degree of the TF \( \Theta_{m}(s) \) is taken equal to \( n_{d} = n_{d} = \mu_{sc} \), where

\[
\mu_{dc} = \mu_{dc} + \mu_{dc} - \mu_{dc}, \quad \mu_{dc} = \deg P(z) - \deg H(z).
\]

The coefficients \( \Delta_{0} \) and the settling time \( t_{s} \) of the suitable SNTF are selected from the microprocessor memory by using the values \( \zeta'_{dc} = 1, n_{d} = \mu_{dc} \), and \( \sigma' \% \). Then the coefficients of the denominator of the sought TF \( \Theta_{m}(s) \) are calculated using the formula:

\[
\delta_{p} = \Delta_{p} \omega_{p}^{\nu}, \quad \rho = [n_{d}, 0],
\]

(29)

where \( \omega_{s} = \frac{t_{s}}{T_{np}} \).
If $z^* = 1$, then $\Theta(s) = \delta_0 / \left( s^n + \delta_{n-1}s^{n-1} + \ldots + \delta_0 s^0 \right)$. Its Z-transform with $T=\psi_p$ yields $\Theta_p(s) = z^* / (\zeta / z^*)$. The condition $\zeta^* = \zeta$ must be met. The coefficients of the polynomials $D(z) = z^n + \delta_{n-1}z^{n-1} + \ldots + \delta_0$ and $P(z) = (z - \mathbf{1})P_B(z) = \eta_0z^{\mathbf{n}} + \ldots + \eta_1z + \eta_0$ are calculated. The coefficients $\delta_0$, $\eta_0$ and also $\delta_0$ are substituted into the system (27); its solution yields to the polynomials: $\Delta_1(z)$, $\Delta_2(z)$ (26).

The polynomials $\Delta_1(z)$ and $\Delta_2(z)$ are calculated using (25); the polynomial $\Gamma(z) = h_0^i P_B(z) \zeta_2(z)$ is calculated. Substituting these polynomials in (21), we find the equation of the control unit of the desired ADC. The calculation results are correct, if the polynomials $D(z) = H_{21}(z)P_0(z)\hat{D}(z)$ and $P(z)\Delta_1(z) + H(z)\Delta_2(z)$ have similar coefficients Gaiduk, and Plaksienko (2018).

If the polynomial $\Delta_1(z) = (z - \mathbf{1})^\mathbf{n} \Delta_0(z)$, then we take the equations of the control unit (21) as

$$
\hat{\Delta}_1(z) \hat{\xi}(z) = \Gamma(z) r(z) - \Delta_1(z) y(z) - (z - \mathbf{1})^\mathbf{n} u(z) = \hat{\xi}(z),
$$

where $\hat{\xi}(z)$ is a z-image of an additional variable $\hat{\xi}_z$. We have formed the algorithm for calculating the values $u_i$, $k = 0, 1, 2, ...$ by converting the equations (30) to the original variables $u_k$, $\xi_{z-k}$. Expressions (20) – (30) allow to calculate parameters of the digital control unit (21) for each channel of the UC MIMO (1) automatically. The desired connections between the channels are provided by the method presented in Gaiduk (1998).

7. EXAMPLE

Assume, that the order of an uncertain plant is not more than $n_{\max} = 4$; the plant has one output and two inputs: control $u$ and disturbance $f=\mathfrak{f}(t)$. If $\gamma_{\beta} = \infty$, the duration of each test $t_{\psi} = N_{\psi}T_{\psi} = 10$ s; test action $u_{\beta} = 3-1(\gamma); \hat{\Delta}_0 = 0.01$; number $\hat{\xi}_z = 10$. The identification algorithm of ADC includes the following steps:

**Step 1.** First $u_1(t)$, and then $u_2(t)$ operates on the plant in steady state 3-4 times. The output variable is measured over $t_{test}$ with the sampling period $T_{\psi} = 1$ s. The averaged values $\hat{y}_1(kT_{\psi}) = \hat{y}_{11}$, $\hat{y}_1(kT_{\psi}) = \hat{y}_{12}$ and the Markov parameters, calculated with using (8), are listed in Table 1.

**Step 2.** Let $\hat{m}_{i1} = \hat{m}_{i1}$, then using (14) with $\beta = 0, \nu = 1.2, 3, 4$ we find:

| $k$ | $\hat{y}_{11}$ | $100\hat{m}_{i1}^1$ | $\hat{y}_{12}$ | $\hat{m}_{i1}^2$ |
|-----|---------------|-----------------|--------------|---------------|
| 0   | 0.0000        | 0.7000          | 1.4          |                |
| 1   | 0.5045        | 3.3632          | 0.7706       | 0.1413         |
| 2   | 0.5918        | 0.5821          | 0.7873       | 0.0334         |
| 3   | 0.6360        | 0.2946          | 0.7981       | 0.0215         |
| 4   | 0.6785        | 0.2837          | 0.8082       | 0.0202         |
| 5   | 0.7241        | 0.1821          | 0.8182       | 0.0200         |
| 6   | 0.7733        | 0.2425          | 0.8282       | 0.0200         |
| 7   | 0.8262        | 0.3164          | 0.8382       | 0.0200         |
| 8   | 0.8830        | 0.4065          | 0.8482       | 0.0200         |
| 9   | 0.9436        | 0.5167          | 0.8582       | 0.0200         |
| 10  | 1.0083        | 0.6512          | 0.8682       | 0.0200         |

**Step 2.** The determinants $\det \tilde{M}_{01}^0 = 6.518 \cdot 10^{-5}$, $\det \tilde{M}_{11}^0 = 0.034$, $\det \tilde{M}_{21}^0 = -8.651 \cdot 10^{-10}$, and $\det \tilde{M}_{31}^0 = -2.767 \cdot 10^{-26}$ are calculated using (14) with $\hat{m}_{i1} = \hat{m}_{i1} = 0$, $\nu = 1, 2, 3, 4$; the order $\hat{m}_{i1} = 3$ is determined using (13). Then we go to step 3. **Step 3.** The vector $\tilde{y}_{11}^0 = [-0.113 1.244 -2.131]$ is calculated using (15), (16) with $i=j=1, \hat{m}_{i1} = 3, \beta = 0$. The value $\tilde{y}_{11}^1 = 0.113 > \hat{\Delta}_0 = 0.01$; i.e. condition (5) is satisfied, and DLP (2) is correct with $T_{\psi} = 0.1$ s; therefore, we can go to next step. **Step 4.** The factors $\tilde{y}_{11}^3 = 0.0324, \tilde{y}_{11}^4 = -0.0658, \tilde{y}_{11}^5 = 0.0336, \tilde{y}_{11}^6 = 0$ are calculated using (17). Then the transfer function $\tilde{\Theta}_1(z, 0.1, 3) = 0.0336z^2 - 0.0658z + 0.0324 / (z^2 - 2.131z + 1.244z - 0.113)$ can be written using (12) and we go to step 5. **Step 5.** The unity-transform (4) of the TF $\tilde{\Theta}_1(z, 0.1, 3)$ with $T_{\psi} = 0.1$ yields to

$$
\hat{\Theta}_1(s) = \frac{0.85z + 0.3s + 0.4}{s^3 + 21.8s + 4.8s + 3.54 \cdot 10^{-4}}.
$$

and we go to step 6. **Step 6.** Performing step 2 – step 5 with $i=1, j=2, \nu = 1, 2, 3, 4$ gives...
Then we go to Step 9. Let the control unit (21) should have sampling period \( T_{spc} = 0.6 \) s and \( \mu_{\sigma_{z}} = 1 \). The control system should have \( \zeta' = \zeta = 1 \), \( \beta' \geq 0.11 \); \( \zeta' \leq 5.6 \); \( \sigma' \leq 5 \% \); the reference \( r(t) \) and output \( y(t) \) are measured. The algorithm of the control unit of the ADC includes the next steps:

**Step 9.** The \( Z_{r} \)-transform (4) of the matrix \( \Theta_{sp}(z) \) (33) with \( T_{spc} = 0.6 \) s, see Moore, (2013), gives:

\[
\Theta_{sp}(z) = \left[ \begin{array}{c}
0.05155 z^{-2} - 0.08458 z^{-0.03998} \\
-2.128 z^{-1} + 1.128 z^{-2} - 0.8087 \times 10^{-6}
\end{array} \right]^{T}.
\]

In (34) the factor 2.087\times 10^{-6} is replaced by zero, and the polynomials of the equation (20) are written:

\[
P(z) = z^{3} - 2.128 z^{2} + 1.128 z,
\]

\[
H(z) = 0.05155 (z^{2} - 1.6407 z + 0.7756),
\]

\[
F(z) = 1.44 z^{-3} - 5.804 z^{-2} + 7.376 z^{-1} - 2.946.
\]

**Step 10.** In this case, \( \mu_{\sigma_{z}} = 0.0516 ; \) \( \zeta' = 0 \), \( \zeta' = 1.128 ; \) \( \zeta' = 1.0 ; \) \( n_{p} = 3, n_{q} = 1, F(1) = 0, n_{q} = 0 ; \) \( \mu_{\sigma_{p}} = 3, 2 = 1 ; \) condition (23) is satisfied, since 1 - 0.8807\times 0.11; factorization of the polynomial \( P(z) \) gives:

\[
P_{Q}(z) = z^{3} - 2.128 z^{2} + 1.128 z + n_{q} = 1, n_{p} = 2, \text{ number } \nu = \max\{1; 1 - 0; 0\} = 1. \]

Calculations by formulas (28) give:

\[
r_{\Delta} = 4, r = 1, \hat{h} = 3, \hat{r} = 2 \text{ and } \mu_{\sigma} = 4. \]

Let \( \mu_{n} = 1, \text{ then } \mu_{a_{n}} = 1 + 1 = 1, n_{a_{n}} = 4 + 1 = 3. \)

**Step 11.** The coefficients \( \Delta_{p} \) and \( \Delta_{q} \) are selected from the memory of the controller using values:

\[
\left[ \begin{array}{c}
\zeta' = 1, n_{a_{n}} = 3, \sigma = 0.55, \Delta_{p} = 1, \Delta_{q} = 3, \Delta_{n} = 1, r_{\Delta} = 6.31 s.
\end{array} \right.
\]

\( n_{a_{n}} = 3, \Delta_{p} = 1, \Delta_{q} = 3, \Delta_{n} = 1, t_{a} = 6.31 s. \) The coefficients \( \omega_{a} = 6.31(5.6 - 0.6) = 1.262 \) and \( \delta_{a} \) are calculated using (29): \( \delta_{1} = 1, \delta_{2} = 3.786, \delta_{3} = 4.7779, \delta_{4} = 0.0099. \) These coefficients lead to transfer function: \( \Theta_{s}(z) = 2.01 z^{2} + 3.786 z + 4.778 z^{2} + 2.01. \) \( Z_{r} \)-transform of this \( \Theta_{s}(s) \) with \( T_{spc} = 0.6 \) s gives the polynomials:

\[
\gamma_{sp}(z) = 0.04155 z^{2} + 0.095 z^{2} + 0.0133 \text{ and } \zeta(z) = z^{3} - 1.407 z^{2} + 0.66 z^{2} - 0.103 z.
\]

The roots of the polynomial \( \zeta(z) \) satisfy to the condition (23). Therefore, in this case

\[
\hat{D}(z) = z^{3} - 1.407 z^{2} + 0.66 z^{2} - 0.103 z.
\]

**Step 12.** System (27) is composed using the coefficients of the polynomial \( \hat{D}(z) = z^{3} - 3.128 z^{2} + 3.256 z - 1.128 \) and the numbers \( h_{0} = 0.0516, i = 2 \) and \( r = 1 \). Solution of this system and expressions (25), (26) yield to the polynomials:

\[
\Lambda(z) = 54.0688 z^{-3} - 88.8233 z^{-2} + 37.6594 z^{-1}.
\]

\[
\Gamma(z) = 0.8045 z^{-3} + 1.8423 z^{-2} + 0.2582 z^{-1}.
\]

**Step 13.** Polynomials:

\[
D_{1}(z) = z^{3} - 0.681 z^{-0.08 z^{2}} + 3.744 z^{-2.777 z^{-4}} + 0.681 z^{-0.08 z^{2}} - 0.001 z .
\]

The parameters of the control unit are calculated correctly, since the coefficients of the polynomials \( D_{1}(z) \) and \( D_{2}(z) \) are close to each other. **Step 14.** The equations of this control unit are written according to (30) considering \( \nu = 1 \) and the polynomials (36):

\[
\hat{D}(z) = z^{3} - 0.0804 z^{-2} - 2.0482 z + 1.3349 \hat{z}(z) =
\]

\[
= (0.8045 z^{-3} + 1.8423 z^{-2} + 0.2582 z^{-1}) (z - 1) u(z) + \hat{z}(z).
\]

Step 15. The digital controller algorithm of the digital controller is formed by converting the equations (37) to the origin

\[
\hat{\xi}(k) = 0.804 z^{-3} + 2.048 z^{-4} - 1.335 z^{-4} + 0.805 z^{-3} + 1.842 z^{-4} + 0.258 z^{-5} - 54.069 z^{-4} + 0.888 z^{-3} - 37.659 z^{-4} \cdot u_{n} = u_{k} + \hat{\xi}(k).
\]

The rounded values of the coefficients are given above to be short. Let's note that the adaptive digital controller will correctly work if not less than 13-15 digits are considered in the given above expressions of its algorithm. In Fig. 1 shows the response to a step input \( r(t) = 1(t) \) of the designed adaptive control system (33), (38).

![Step Response](image-url)

Fig. 1. System response to the input \( r(t) = 1(t) \). Obviously, the astigmatism orders, the overshoot and the settling time of the system are not worse than the required values. The expressions (6) – (30) allow to analytically find the ADC
algorithm for each channel of the UC MIMO plant (1) if a part of the SNTF is stored in the memory of the controller.

8. CONCLUSIONS

The offered algorithm of digital controllers for systems of adaptive control with the identification of uncertain plants can be implemented with minimal prior information. This algorithm can be applied only to fully plants, the order and parameters of which jump quite rarely. The digital controllers provide required values of the quality criteria of a closed adaptive control system. The problem of design of adaptive control systems for uncertain plants is currently relevant, since it is increasingly necessary to control complex plants whose mathematical models are a priori complex or impossible to find.

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REFERENCES

Aida-zade, K.R., and Ragimov, A.B. (2017). Identification of piecewise constant filtration parameters and boundaries of their constancy domains, Automation and Remote Control, vol. 78, no. 8, pp. 1404-1416.

Ahmed, S., Huang, B., and Shah, S. (2009). Process identification from sinusoidal test data by estimating step response, Preprints IFAC Symposium System Identification, Saint-Malo, France, pp. 396-401.

Chen, C.T. (1999). Linear systems theory and design, Oxford University Press, New York.

Gaiduk, A.R. (1998). Synthesis of control systems of multivariable objects, Automation and Remote Control, vol. 59(12), pp. 5-13.

Gaiduk, A.R. (2012). Theory and methods of analytical design of automatic control systems (polynomial approach). Phizmatlit Publisher, Moscow.

Gaiduk, A.R., and Plaksienko E.A. (2016). Analysis and synthesis of control systems using system invariants in MATLAB, Palmarium Academic Publishing, Deutschland.

Gaiduk, A.R., and Plaksienko, E.A. (2018). Analyze and analytical design of digital control systems, chap. 6, Lan Publisher, Sankt Pietersburg.

Gantmakher, F.R. (1988). Matrix theory, p. 88, Nauka Publisher, Moscow.

Genov, A.A., Rusakov, K.D., and Hill, S.S. (2017). Identification of a Complex Technical System Functional State under Conditions of Measurement Data Ambiguity, Software & Systems, no. 3, pp. 373-377.

Gerasimov, D., Ortega, R., and Nikiforov, V. (2018). Relaxing the High frequency Gain Sign Assumption in Direct Model Reference Adaptive control, European Journal of Control, vol.43, pp.12-19.

Ghaffar, A.A., and Richardson, T. (2015). Model Reference Adaptive Control and LQR Control for Quadrotor with Parametric Uncertainties. International Journal of Mechanical and Mechatronics Engineering, vol. 9, no. 2, pp. 244-250.

He, Y., and Wang, Q.-G. (2006). An improved ILMI method for static output feedback control with application to multivariable PID control, IEEE Trans. Automatic Control, vol. 51, no. 10, pp. 1678-1683.

Kalman, R., Falb, P., and Arbib, M. (1971). Essays on the mathematical theory of systems, World, Moscow.

Levin, I.I., Dordopulo, A.I., Doronchenko, Yu.I., Kaliaev, I.A., and Raskladkin, M.K. (2015). Modern and advanced high-performance computing systems with reconﬁgurable architecture, SUSU Bulletin. Series Computational Mathematics, Computer Science, Volume 4, No. 3, p. 24–39.

Minaev, I.G., Samoylenko, V.V., Ushkor, D.G., and Fedorenko, I.V. (2016). Freely programmable devices in automated control systems, AGRUS, Stavropol.

Moore, H. (2013) MATLAB for Engineers, 3rd ed., Pearson, New York.

Ning, Z., Zhang, L., and Zheng, W.X. (2019). Observer-Based Stabilization of Nonhomogeneous Semi-Markov Jump Linear Systems With Mode-Switching Delays, IEEE Transactions on Automatic Control, vol. 64, no. 5, pp. 2029-2036.

Radac, M., Precup, R., and Petriu, E.M. (2015). Model-free primitive-based iterative learning control approach to trajectory tracking of MIMO systems with experimental validation. IEEE Trans Trans Neural Network Learning System, no. 26(11), pp. 2925-2938.

Raven, F. H. (1983). Automatic control engineering, third edition, McGraw-Hill IBC, London.

Shao, Z., Zheng, G., Efimov, D., and Perruquetti, W. (2017). Identification, estimation and control for linear systems using measurements of higher order derivatives. Journal of Dynamic Systems, Measurement, and Control, American Society of Mechanical Engineers, vol. 139(12), pp.1-6.

Stengel, R.F. (1994). Optimal control and estimation, Dover Publication, Inc., New York.

Voevoda, A.A., Bobobekov, K.M., and Troshina, G.V. (2019). The parameters identification of the automatic control system with the controller, Journal of Physics: Conference series, vol. 1210, Art. 012021.

Xu, D., Jiang, B, and Shi, P. (2014). A novel model-free adaptive control design for multivariable industrial processes. IEEE Transaction Industrial Electronics, no. 61(11), pp. 6391-6398.

Zhu, Y., and Hou, Z. (2015). Controller dynamic linearisation-based model-free adaptive control framework for a class of non-linear system. IET Control Theory & Applications, vol. 9, no. 7, pp. 1162-1172.

Zhu, Y., Krstic, M., Su, H., and Xu, C. (2016). Linear backstepping output feedback control for uncertain linear systems, International Journal of Adaptive Control and Signal Processing, vol. 30, pp. 1080-1098.