Numerical simulation of hydraulic bulging using uniaxial and biaxial flow curves and different yield criteria

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Abstract. In sheet metal forming industry, finite element analysis has become a necessary step before actual production of the parts. It helps in the improvement of part quality as well reduces the effort applied in experimental trials. However, the success of finite element simulation mainly depends on the constitutive model used for defining the plastic deformation behaviour of the sheet material. Material characterization in biaxial stress state helps in selecting advanced constitutive models in numerical simulations. In this work, aluminium alloy AA5083 sheet material is characterized in uniaxial and biaxial tension conditions. Hydraulic bulge tests are performed to obtain the flow curves and material properties which are then used to find the coefficients in different yield criteria. These coefficients and the flow curves, fitted using different work hardening laws, are used in finite element simulation of the hydraulic bulge test and the results are validated by experiments.

1. Introduction
Design and analysis of sheet metal components through finite element analysis (FEA) is frequently done prior to real forming to reduce the time and cost involved in experimental trials. Even though the multiaxial state of stress is involved in forming most of the sheet metal parts, uniaxial tensile properties are commonly used for defining the material behaviour in FEA. Hence an approximation has to be made while modelling the material behaviour to higher plastic strain and the results predicted are not accurate. An improvement in predicted results can be achieved if material properties obtained from standard tests performed under a similar state of stress are used [1]. In FE simulation, the laws used for defining yielding followed by hardening of material have a significant effect on the predicted results. It is suggested to use flow curve up to larger plastic strain obtained experimentally along with advanced yield criterion [2]. Hydraulic bulge test (HBT) is one of the test methods of characterization of sheet material in biaxial stress state condition. It is a biaxial stretch forming process, carried out under the pressure applied by a fluid medium. Apart from biaxial yield strength and biaxial anisotropy coefficient, flow curve up to a much higher degree of deformation is obtained and thus may eliminate the need for any approximation in defining hardening behaviour. This much amount of deformation achieved in HBT is due to biaxial stress states, which increases the possibility of uniform deformation
leading to delay in neck formation and fracture [3]. The membrane theory is commonly used to determine the flow stress and is applicable to only thin sheets as it neglects bending stresses [4]. In this work, AA5083-O aluminium alloy sheets of 1.24 mm thickness are characterized in uniaxial and biaxial stress states. Hydraulic bulge tests at constant strain rate are performed for characterizing the material in biaxial stress state. The flow curves and the obtained material properties are used in FEA of hydraulic bulging using Hill90 [5] and Yld2000-2d [6] yield criteria. Predicted dome height vs time curves from the numerical simulations are compared with experimental results.

2. Material characterization

2.1. Uniaxial tensile test

Uniaxial tensile tests (UAT) are conducted as per ISO-6892 standard on a universal testing machine (Z100, Zwick Roell AG) of capacity 100kN. In order to identify the anisotropic coefficients (r-values), the specimens are laser cut in direction 0°, 45° and 90° to the rolling. The local strain is measured by digital image correlation using ARAMIS system. The tests are carried out at a constant strain rate of 0.00667 s⁻¹.

2.2. Hydraulic bulge test

The bulge tests are performed on a hydraulic press at LFT Erlangen, Germany. The diameter of the blanks used is 395 mm and the inner diameter of the die is 200 mm. The flow rate of the hydraulic fluid is regulated with respect to signal obtained from an online 3-D strain measurement system (ARAMIS) to achieve a constant strain rate (0.00667 s⁻¹). Hydraulic fluid pressure during the test is continuously measured using a pressure sensor. Pressure vs test time and dome height vs test time plots of the hydraulic bulge experiments for three repetitions are shown in Fig. 1(a) and 1(b), respectively.

![Figure 1](image_url)

Figure 1. (a) Pressure vs test time and (b) dome height vs test time curves obtained from the hydraulic bulge experiments for three repetitions.

The von Mises equivalent stress in equibiaxial stress state is evaluated using the Eq. (1) [4]:

\[ \sigma_b = \sigma_1 = \sigma_2 = \frac{p \times \rho}{2 \times t} \]  

(1)
where \( \rho \), \( p \) and \( t \) represents the radius of curvature of bulge dome, fluid pressure acting and actual sheet thickness respectively. \( \sigma_1 \) and \( \sigma_2 \) are the two equal principal stresses. The von Mises equivalent strain which is equal to true plastic thickness strain is given by Eq. (2):

\[
\varepsilon_{3,pt} = -\varepsilon_1 - \varepsilon_2 - \frac{2(1+\nu)}{E} \sigma_b
\]  

(2)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the principal strains measured by digital image correlation. \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio of sheet material. However, the flow curve obtained using Eq. (1) and Eq. (2) are not suggested to be used directly as they don’t represent correct hardening nature of anisotropic materials \([7][8]\) and hence the obtained curve was scaled as per the methodology suggested in DIN EN ISO 16808-2014. The flow curves obtained from UAT in the rolling direction (UAT-RD), HBT using Eq. (1) and Eq. (2) (HBT-Mises) and scaled using ISO standard (HBT-ISO) are shown in Fig. 2. However, the difference between the directly obtained and the scaled flow curves is not significant for the sheet material investigated in this work.

2.3. Modelling of flow curves

The flow curves obtained from uniaxial tensile tests represents the true hardening nature of the material, and hence this is commonly used in the numerical simulations of all the forming processes. As the total plastic strain achieved in a uniaxial tensile test is much less than what is imparted in the actual forming process, an approximation has been done by using Hockett-Sherby (HS) and Swift hardening laws extrapolating the curve to higher plastic strain before using it in numerical simulations. The flow curves obtained from HBT are also modelled using these hardening laws, however, HS hardening law is found to be resulting into a better fit than Swift law and therefore only HS hardening law is used in this case. The Hockett-Sherby and the Swift hardening laws are given by Eq. (3) and Eq. (4) respectively \([9]\).

\[
\sigma_f = A - B \left( e^{-Ce_p^q} \right) 
\]  

(3)

\[
\sigma_f = K(\varepsilon_0 + \varepsilon_p)^n
\]  

(4)

where \( \sigma_f \) is the flow stress corresponding true plastic strain (\( \varepsilon_p \)). All other parameters are material constants and identified using a curve fitting technique (listed in Table 1). The flow curves extrapolated up to a plastic strain of 1.0 is shown in Fig. 3.

| Parameters | UAT | HBT-ISO | Parameters | UAT | HBT-ISO |
|------------|-----|---------|------------|-----|---------|
| \( K \) (MPa) | 627.21 | 512.05 | \( A \) (MPa) | 378.22 | 417.01 |
| \( \varepsilon_0 \) | 0.0123 | 0.0031 | \( B \) (MPa) | 234.38 | 279.24 |
| \( n \) | 0.3387 | 0.2382 | \( C \) | 10.275 | 6.008 |
| \( q \) | | | \( q \) | 0.9628 | 0.8376 |

Table 1. Coefficients of Hockett-Sherby and Swift hardening laws.
2.4. Determination of coefficients of Yld2000-2d and Hill90 yield criteria

Yield stress in $0^\circ$, $45^\circ$ and $90^\circ$ with respect to the rolling ($\sigma_0$, $\sigma_{45}$ and $\sigma_{90}$), anisotropy coefficients in $0^\circ$, $45^\circ$ and $90^\circ$ with respect to rolling ($r_0$, $r_{45}$ and $r_{90}$), biaxial yield stress ($\sigma_b$) and biaxial anisotropy coefficient ($r_b$) are used to determine the eight anisotropic coefficients of Yld2000-2d yield criterion which in turn used in numerical simulations. The biaxial anisotropy coefficient ($r_b$), is analytically evaluated using the Barlat’s Yld96 yield criterion [10]. The biaxial yield stress ($\sigma_b$) for the case of HBT is determined in accordance with DIN EN ISO 16808-2014. The determined anisotropic coefficients of Yld2000-2d are listed in Table 2.

The coefficients of Hill90 yield criterion can be determined in two ways. One is by using the material properties $\sigma_0$, $\sigma_{45}$, $r_0$, $r_{45}$ and $r_{90}$ (here called R-based) and other is by using the material properties $\sigma_0$, $\sigma_{45}$, $\sigma_{90}$, $c$ and $r_{45}$ (here called S-based). The determined coefficients of both the form Hill90 yield criterion (Hill90-R and Hill90-S) are listed in Table 2. These coefficients are later used in the numerical simulations of hydraulic bulging.

Table 2. Anisotropy coefficients of Yld2000-2d and Hill90 yield criteria.

| Yld2000-2d (m=8) | Hill 90 |
|------------------|---------|
| $\alpha_1$ | 0.9165 | $\alpha_5$ | 1.022 |
| $\alpha_2$ | 1.044 | $\alpha_6$ | 0.9935 |
| $\alpha_3$ | 0.958 | $\alpha_7$ | 1.024 |
| $\alpha_4$ | 1.030 | $\alpha_8$ | 1.093 |
| R-based | m | 1.8409 | m | 1.8409 |
| S-based | a | -0.0522 | a | 0.0491 |
| | b | -0.3966 | b | -0.1525 |
| | c | 1.7406 | c | 1.7406 |

3. Numerical analysis of the hydraulic bulge tests of AA5083-O

A quarter symmetry numerical model (Fig. 4) is used in the preprocessor of Dynaform and the solver is LS-Dyna. The geometrical features of the tools modelled in numerical simulation is same as used in the hydraulic bulge experiments. The tools are modelled as rigid bodies with shell elements of element size 3 mm. The Belytschko-Tsay shell element of size 1.24 mm with 5 integration point is considered for deformable blank. The contact between the tools and blank is modelled with one way surface to surface contacts. A constant static friction coefficient of 0.15 is used between the blank and the dies.
Hill90 and Yld2000-2d yield criteria are used to define yielding whereas hardening is defined by using different hardening laws. The Young’s modulus of the sheet material is kept constant to 70 GPa and the Poisson’s ratio is considered as 0.33. The average of the three pressure vs time curves obtained from the experiments is used as input in the simulations.

4. Results and discussion

4.1. Comparison of yield loci

Experimentally measured values in uniaxial and biaxial stress states and the yield surfaces predicted by Hill90 and Yld2000-2d yield criteria are shown in Fig. 5. The yield surface predicted by von Mises yield criterion with the corresponding plane strain (Mises-PS) points is also shown. It is evident from the figure that both Hill90 and Yld2000-2d have reproduced the equibiaxial stress state accurately. However, Hill90-R is found to be less accurate in predicting uniaxial stress values when compared to Hill90-S and Yld2000-2d. Even though the experimentally obtained uniaxial and biaxial stress values have been used in the formulation of both Hill90-S and Yld2000-2d, a significant variation between them is observed in plane-strain region.

Figure 4. A quarter symmetry numerical model of hydraulic bulge test

Figure 5. Comparison of the initial yield surfaces of AA5083-O predicted by different yield criteria.

Figure 6. Comparison of dome height vs time curves obtained using uniaxial flow curves and different yield criteria with the experimental curve.
4.2. Numerical simulations using uniaxial flow curves

The influence of hardening behaviour on the predicted dome height evolution has been studied by modelling the uniaxial flow curve with Hockett-Sherby and Swift hardening laws. Dome height vs standardized test time predicted from numerical simulations is compared with experimental results and shown in Fig.6. The two cases of Hill90 (Hill90-R & Hill90-S) and Yld2000-2d yield criteria have not matched with the experimental results when used with uniaxial flow curves modelled using HS (UAT-HS) and swift (UAT-Swift) hardening laws. Uniaxial flow curve modelled with UAT-HS combined with these yield criteria highly overestimated the dome height and the simulations aborted due to excessive thinning at a much lower pressure than the experiment. This indicates that HS hardening law underestimated the strength of the material at higher plastic strains. The opposite is the case with the Swift hardening law. Moreover, when UAT-Swift is used in the numerical simulations, the predicted curves for all the yield criteria are found to be almost the same. This could be due to overestimation of the strength of the material at higher plastic strain to a very high extent by Swift hardening law, which diminishes the effect of yield criterion on dome height evolution.

4.3. Numerical simulations using hydraulic bulge flow curve

The evolution of dome height vs time predicted from numerical simulations using HBT flow curve combined with Hill90-(R&S) and Yld2000-2d yield criteria are shown in Fig 7. Hill90-R is found to be least accurate among the three and predicted larger dome height. This was quite expected as the Hill90-R yield criterion was incompetent in predicting uniaxial stress values accurately. A small deviation was observed with Yld2000-2d yield criterion which can be attributed to the assumption of the value of exponent \( m=8 \) as suggested in literature [6] for FCC materials. Hill90-S yield criterion is found to be more accurate but took almost double the computation time than with Yld2000-2d because of the complex formulation. A comparison of Von Mises equivalent strain (of the element lying at the centre of the bulge dome) vs time curve predicted from numerical simulations with that obtained from the experiment is shown in Fig 8. It is evident that the Yld2000-2d and Hill90-R estimated an early and late beginning of plastic deformation respectively.

![Figure 7](image7.png)  ![Figure 8](image8.png)

**Figure 7.** Comparison of dome height vs time curves obtained using biaxial flow curves and different yield criterion with the experimentally obtained curve.

**Figure 8.** Comparison of Von Mises equivalent strain vs time curves obtained using biaxial flow curves and different yield criterion with the experimentally obtained curve.

4.4. Improvement in material modeling with Yld2000-2d yield criterion

It is observed in the previous section that Yld2000-2d, formulated with \( m=8 \) underestimated the thinning and dome height evolution. Even though this value is suggested in general for all the FCC
materials, it should be calibrated using the experimentally obtained stress values in plane strain condition. As the experimental yield stress for the plane strain condition is not available, an attempt has been made to recalculate ‘m’ by reducing it in a step size of 0.25 until the dome height evolution matches with the experiments. From a number of trials, it is observed that at m=5.25, the dome height evolution is in very good agreement with the experiment (Fig. 9). Anisotropic coefficients of Yld2000-2d yield criterion for m=5.25 are listed in Table 3. For comparison, the yield loci of Yld2000-2d at m=8 and m=5.25 are also plotted (Fig. 10). It can be clearly seen that with a decrease in ‘m’ value, yield surface has expanded and at m=5.25, it moved closer to Hill90-S yield surface.

However, the methodology used to find the new value of exponent of Yld2000-2d yield criterion is based on iterative procedure with no consideration of the lattice structure of the metal, but it suggests the use of experimentally determined stress values in plane strain state in the numerical simulations using any suitable yield criterion.

| m    | α1  | α2  | α3  | α4  | α5  | α6  | α7  | α8  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|
| 5.25 | 0.8670 | 1.048 | 0.9471 | 1.036 | 1.034 | 0.9689 | 1.015 | 1.109 |

5. Conclusion
AA5083-O aluminium alloy sheets were characterized in uniaxial and biaxial stress state (hydraulic bulging). The material properties and flow curves obtained were used in numerical simulations of hydraulic bulging using Hill90-(R&S) and Yld2000-2d yield criteria combined with two different hardening laws. Predicted dome height vs standardized test time from the numerical simulations was compared with the experimental results. The following conclusions have been made from this work:

- Uniaxial flow curves modelled using both Hoekett-Sherby and Swift hardening law combined with all the investigated yield criteria showed a significant deviation from the experiments.
- Flow curve obtained from hydraulic bulge tests modified as per the ISO standard and fitted using Hockett-Sherby hardening law, when used in numerical simulations is found to have predicted results closer to the experimental observation than the uniaxial flow curve.
- Dome height vs time curve predicted using Hill90-S yield criterion is found to be in better agreement with experiments than those predicted using Hill90-R.
- Yld2000-2d, when formulated using the suggested value of exponent (m=8) underestimated the dome height and the equivalent plastic strain. A significant improvement in the predicted accuracy of dome height evolution and maximum thinning has been observed when m=5.25 is used.

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