Scaling of gamma-spectra registered by semiconductor detectors

E. G. Obrazovskii*
Novosibirsk State University, 630090, Novosibirsk, Russia
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Abstract
The scaling properties of gamma-spectra recorded by semiconductor detectors are investigated. For practical purposes the method of simulation multicomponent spectra using single experimental spectrum are suggested.

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High-resolution gamma-spectrometry is a powerful tool in different fields of science and technology: from astrophysics [1] and nuclear spectrometry [2] to gamma-ray imaging spectrometers [3], environmental science [4] and neutron activation analysis [5]. The Ge-detectors are most common used in gamma-spectrometry due its high energy resolution and sufficiently high efficiency registration [6]. In recent years significant progress has been made in the technology of semiconductor detectors made of materials with high atomic numbers (CdTe, HgI2) [7].

The spectrum registered by semiconductor detectors (the detector response to a gamma-ray) consist of not only full-energy peak, but also continuum distribution resulting from Compton scattering of primary gammas and subsequent escaping of the scattered gammas from detector [6]. This continuum severely limits the accuracy of the determination of peak areas in low-energy part in multicomponent gamma spectra. The main physical processes of energy dissipation in the semiconductor detectors are well-known [6] and are the basis for numerous methods for Monte Carlo simulation gamma-spectra [8]. To apply these methods require detailed information

*E-mail: e.obrazovskii@ngs.ru
about the exact geometrical parameters of both the detector crystal and surrounding material [9].

In this paper we study the scaling properties of gamma-spectra registered by semiconductor detectors. For practical purposes the method of simulation multi-component spectra with using single experimental spectrum is suggested.

The most simplest scaling properties have so-called planar detectors with sensitive volume $\sim 1 \text{cm}^3$, since the main contribution to the formation of a continuous distribution gives a single Compton scattering, while the multiple scattering contributes only a small correction. The single Compton scattering dominates in region from 0 to $E_g^{(1)} = E_0/(1 + mc^2/2E_0)$, where $E_0$ is the energy of primary gammas. The energy of scattered gamma ray is determined by the well-known equation [10]

$$\frac{1}{\omega'_1} - \frac{1}{\omega_{01}} = 1 - \cos \theta,$$

where for simplicity $\omega_{01} \equiv E_0/mc^2$ and $mc^2 = 511.0 \text{keV}$ is the energy rest of the electron, $\theta$ is the angle of scattering. We can connect the probability of Compton scattering and total attenuation of gammas for two different energies of primary gammas, $E_{01}$ and $E_{02}$, for the same scattering angles, which corresponds to the condition

$$\frac{1}{\omega'_1} - \frac{1}{\omega_{01}} = \frac{1}{\omega'_2} - \frac{1}{\omega_{02}}.$$

Then the count in gamma spectra with energy $E_1 = \omega_{01} - \omega'_1$, deposited in detector for primary gammas with energy $E_{01}$, corresponds to the count with energy $E_2 = \omega_{02} - \omega'_2$, deposited in detector for primary gammas with energy $E_{02}$, where

$$E_2 = \omega_{02} - \omega'_2 = \frac{E_1 \omega_{02}^2}{\omega_{01} \omega_{02} + (\omega_{01} - E_1)(\omega_{01} - \omega_{02})}. \quad (3)$$

The ratio of the number of counts per unit energy interval of the two spectra (for the same activity of primary gammas) is determined as the ratio of differential cross-section for Compton scattering [10]

$$\frac{d\sigma}{d\omega'} = \frac{\pi r_e^2}{\omega_0} \left[ \frac{\omega_0}{\omega'} + \frac{\omega'}{\omega_0} + \left( \frac{1}{\omega'} - \frac{1}{\omega_0} \right)^2 - 2 \left( \frac{1}{\omega'} - \frac{1}{\omega_0} \right) \right], \quad (4)$$

where $r_e = e^2/mc^2$, and the ratio of the total attenuation coefficients of the incident and scattering gammas by detector material. For angles of scattering $\theta = 0$ and $\theta = \pi$ these coefficients are determined only by the height of the detector $h$

$$K(\theta = 0) = e^{-\mu_{\text{tot}}(\omega_0)h},$$

$$K(\theta = \pi) = \frac{1 - e^{-(\mu_{\text{tot}}(\omega_0) + \mu_{\text{tot}}(\omega'))h}}{h(\mu_{\text{tot}}(\omega_0) + \mu_{\text{tot}}(\omega'))}, \quad (6)$$

where $\mu_{\text{tot}}(\omega)$ is the linear coefficient attenuation for energy $\omega$, and $\omega_0$ is the energy of the incident gammas, $\omega' = \omega_0/(1 + 2\omega_0)$. 

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Figure 1: Data numerical simulation of a single Compton scattering spectra for different energies of incident gammas (top) and its scaling transformation (bottom)

Fig. 1 shows the data of numerical simulation of the spectra of single Compton scattering of the planar Ge-detector ($\varnothing = 1.0 \text{ cm}$, height $h = 0.7 \text{ cm}$), and the scaling transformation of these spectra according to the

\begin{align}
E_1 \rightarrow E_2 &= \frac{E_1 \omega_2^2}{\omega_1 \omega_2 + (\omega_1 - E_1)(\omega_1 - \omega_2)}, \\
N_2(E_2) &= N_1(E_1) \frac{d\sigma/d\omega_2}{d\sigma/d\omega_1} \frac{K(E_2)}{K(E_1)},
\end{align}

where the values of attenuation coefficients $K(E)$ for intermediate energy values $E$ obtained by linear interpolation of the coefficients (5) and (6).

In the region gamma-spectra from $E^{(1)}_g = \omega_0/(1 + 1/(2\omega_0))$ to $E^{(2)}_g = \omega_0/(1 + 1/(4\omega_0))$ the double Compton scattering is dominated. Continuum of double scattering has a maximum at $E^{(1)}_g$ and approximately scales with
Figure 2: Data numerical simulation of a double Compton scattering spectra for different energies of incident gammas (top) and its scaling transformation (bottom).

The energy of the incident gammas as $1/\omega_0$ that can be seen in fig. 2, which shows the simulation data (top) and scaling transformation (bottom).

The continuum from $E^{(2)}_0$ to $E_0$ associated with the scattering of the incident gamma radiation on the material of the enclosure and the detector entrance window at a small angles with complete absorption of the scattered gamma-ray photon by the detector.

Compton-scattered gammas in the material surrounding detector at angles $\theta \rightarrow \pi$, then completely absorbed by the detector, lead to backscatter peak. This peak is scaled approximately as

$$E_{1b} \rightarrow E_{2b} = \frac{E_{1b} \omega_{01} \omega_{02}}{\omega_{01} \omega_{02} + E_{1b} (\omega_{01} - \omega_{02})},$$  \hspace{1cm} (9)

$$N_2(E_{2b}) \rightarrow N_1(E_{1b}) \frac{d\sigma/d\omega_2}{d\sigma/d\omega_1}.$$  \hspace{1cm} (10)
For many applications, the electron-positron pair production gives a negligible contribution to the Compton background Ge-detector for energies of gammas less than $3 - 5$ MeV [6].

For practical application of established scaling laws the method for simulation multicomponent spectra, using the experimental spectrum of a monoenergetic source and the detection efficiency of total absorption peaks, are suggested.

As an example, the fig. 3 shows a comparison of the experimental spectrum of the radionuclide $^{187}$W, measured using Ge-detector (ORTEC, 1013-10190, $\varnothing = 1.0 \text{ cm}$, height $h = 0.7 \text{ cm}$) and its simulation with spectrum of the radionuclide $^{198}$Au. For this purpose, from the spectrum of the radionuclide $^{198}$Au was removed as the backscattered peak (by subtracting Compton background), as part of double Compton scattering, which has energy higher than $E_g^{(1)}$. These components of the spectrum were transformed using established for each of them above scaling laws. The peak of total absorption was converted in accordance with the detection efficiency, as measured with the use of the radionuclide $^{182}$Ta. Then all the contributions were summed for each gamma-line of the simulated spectrum.

In conclusion, the scaling properties of gamma-spectra registered by semiconductor detectors are investigated. A method for simulation multicomponent spectra using scaling transformation of the experimental spectrum of a monoenergetic radiation is suggested. Technical details of modeling the spectra and discussion of the approximations made will be the subject of a separate publication.

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