Nonabelian topological mass mechanism for a three-dimensional 2-form field

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Abstract

Starting from a recently proposed abelian topological model in (2+1) dimensions, we use the method of the consistent deformations to prove that a topologically massive model involving the Kalb-Ramond two form field does not admit a nonabelian generalization. The introduction of a connection-type one form field keeps the previous result. However we show that the goal is achieved if we introduce a vectorial auxiliary field, exhibiting a nonabelian topological mass generation mechanism in \( D = 3 \), that provides mass for the Kalb-Ramond field. Further, we find the complete set of BRST and anti-BRST equations using the horizontality condition, suggesting a connection between this formalism and the method of the consistent deformations.

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1 Introduction

Antisymmetric tensor gauge fields provide a natural extension of the usual vector gauge fields, appearing as mediator of string interaction and having an important key role in supergravity. Also, they are fundamental to the well known topological mass generation mechanism \[1\] for abelian vector boson in four dimensions, through a BF term \[4\]. This term is characterized by the presence of an antisymmetric gauge field \( B_{\mu\nu} \) (Kalb-Ramond field) and the field strength \( F_{\mu\nu} \). Nonabelian extensions of models involving antisymmetric gauge fields in four dimensional space-time were introduced by Hwang and Lee \[3\] and Lahiri \[4\], in the context of topologically mass generation models. Both procedures requires the introduction of an auxiliary vector field, justified by the need to untie the constraint between two and three form curvatures \( F \) and \( H \). A nonabelian theory involving an antisymmetric tensor field coupled to a gauge field appears as an alternative mechanism for generating vector bosons masses, similar to the theory of a heavy Higgs particle \[1\]. It is worth to mention a generalization to a compact nonabelian gauge group of an abelian mechanism in the context of nonabelian quantum hair on black holes \[1\].

Kalb-Ramond fields arise naturally in string coupled to the area element of the two-dimensional worldsheet \[7\] and a string Higgs mechanism was introduced by Rey in ref. \[8\].

On the other hand, using the technique of consistent deformation, Henneaux et al. \[9\], have proved that is not possible to generalize the topological mass mechanism pointed out above to its nonabelian counterpart with the same field contents and fulfilling the power-counting renormalization requirements. In this way, they put in more rigorous grounds the need to add an auxiliary field.

Recently, we have shown a topological mass generation in an abelian three-dimensional model involving a two-form gauge field \( B_{\mu\nu} \) and a scalar field \( \varphi \), rather than the usual Maxwell-Chern-Simons model \[10\]. Also we have proved the classical duality between a massless scalar field and a vector gauge field. The action for the model just mentioned reads as

\[
S_{inv}^{A} = \int d^3 x \left( \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} + \frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi + \frac{m}{2} \epsilon^{\mu\nu\alpha} B_{\mu\nu} \partial_{\alpha} \varphi \right),
\]

where \( H_{\mu\nu\alpha} \) is the totally antisymmetric tensor

\[
H_{\mu\nu\alpha} = \partial_{\mu} B_{\nu\alpha} + \partial_{\nu} B_{\mu\alpha} + \partial_{\alpha} B_{\mu\nu}.
\]
The action (1), is invariant under the transformation
\[ \delta \varphi = 0, \quad \delta B_{\mu \nu} = \partial_{[\mu} \omega_{\nu]} , \]
and its equations of motion give the massive equations
\[ (\Box + m^2) \partial_{\mu} \varphi = 0 \] (4)
and
\[ (\Box + m^2) H_{\mu \nu \alpha} = 0 \] (5)

The model described by action (1) can be consistently obtained by dimensional reduction of a four-dimensional \( B \wedge F \) model if we discard the Chern-Simons-like terms [10].

The purpose of the present work is to construct a nonabelian version of the action (1). We begin making an analysis of the possibility to construct the nonabelian action with \( \varphi \to \varphi^a \) and \( B_{\mu \nu} \to B_{\mu \nu}^a \), i.e, with the same field content, and the same number of local symmetries, by making use of the method of consistent deformation [11]. As will be proved, there is a no-go theorem for this construction. The same occur with an introduction of a connection-type one-form gauge field. The only possibility is via an introduction of an auxiliary vector field. This introduction is made and we show a nonabelian topological mass generation mechanism for the Kalb-Ramond field in three dimensions.

This paper is organized as follows. In section 2 we apply the method of consistent deformations to an abelian topological three-dimensional model involving a two-form gauge field \( B_{\mu \nu} \) and a scalar field \( \varphi \) in order to study possible nonabelian generalizations. Then, a no-go theorem is established. In section 3 we obtain the BRST and anti-BRST equations by applying the horizontality condition, including an auxiliary vectorial field, which allows the sought nonabelian generalization. Section 4 presents a nonabelian topological mass generation mechanism and finally we draw our conclusions in section 5.

2 Deforming consistently the abelian model

Let us now apply the consistent deformation method described in [11]. We shall start therefore with the following invariant action
\[ S'_0 = \int d^3x \left( \frac{1}{12} H_{\mu \nu \alpha}^a H_{\mu \nu \alpha}^a + \frac{1}{2} \partial_{\mu} \varphi^a \partial^\mu \varphi^a \right) , \] (6)
where now $\phi^a$ and $H^a_{\mu\nu}$ are scalar fields and the abelian curvature tensor \( (2) \) for a set of \( N \) fields. All fields are valued in the Lie algebra \( \mathcal{G} \) of some Lie group \( G \). Since we are interested if the mass term can exist in abelian extension of \( (1) \), the mass parameter will be considered as a deformation parameter. The action \( (3) \) is invariant under the transformations

\[
\delta \phi^a = 0 , \quad \delta B^a_{\mu\nu} = \partial_\mu \omega^a_\nu - \partial_\nu \omega^a_\mu .
\]  

(7)

Since the transformation of \( B^a_{\mu\nu} \) is reducible, we introduce a set of ghosts \( (\eta^a_\mu, \rho^a) \), where \( \eta^a_\mu \) is a ghost for the gauge transformation of \( B^a_{\mu\nu} \), and \( \rho^a \) the ghost for ghost for taking into account this reducibility. For all fields of the model we introduce the corresponding antifields \( (B^*_a, \phi^{*a}, \eta^{*a}, \rho^{*a}) \). The antifields action reads

\[
S'_{\text{ant}} = \int d^3 x \left( \frac{1}{2} B_{\mu\nu}^a \partial_\mu \eta^a_\nu + \eta^{*a}_\mu \partial_\mu \rho^a \right) .
\]  

(8)

The free action

\[
S_0 = S'_0 + S'_{\text{ant}},
\]  

(9)

is solution of the master equation

\[
(S_0, S_0) = 0 ,
\]  

(10)

with

\[
(S_0, S_0) = \int d^3 x \left( \frac{\delta S_0}{\delta \phi^a} \frac{\delta S_0}{\delta \phi^{*a}} + \frac{1}{2} \frac{\delta S_0}{\delta B^a_{\mu\nu}} \frac{\delta S_0}{\delta B^{*a}_{\mu\nu}} + \frac{\delta S_0}{\delta \eta^a_\mu} \frac{\delta S_0}{\delta \eta^{*a}_\mu} + \frac{\delta S_0}{\delta \rho^a} \frac{\delta S_0}{\delta \rho^{*a}} \right) .
\]  

(11)

The nilpotent BRST transformation \( s \) on all fields and antifields is

\[
s \phi^a = 0 , \quad s \phi^{*a} = - \partial^2 \phi ,
\]  

\[
s B^a_{\mu\nu} = \partial_\mu \eta^a_\nu - \partial_\nu \eta^a_\mu ; \quad s B^{*a}_{\mu\nu} = - \partial_\rho H^a_{\rho\mu\nu} ,
\]  

\[
s \eta^a_\mu = \partial_\mu \rho^a ; \quad s \eta^{*a}_\mu = \partial_\rho B^{*a}_{\rho\mu} ,
\]  

\[
s \rho^a = 0 , \quad s \rho^{*a} = - \partial_\mu \eta^{*a}_\mu .
\]  

(12)
We shown in the table below, the canonical dimension and the ghost number for all fields and antifields of the model

|   | $\Phi^a$ | $B^a_{\mu\nu}$ | $\eta^a_{\mu}$ | $\rho^a$ | $\Phi^{*a}$ | $B^{*a}_{\mu\nu}$ | $\eta^{*a}_{\mu}$ | $\rho^{*a}$ |
|---|---------|-----------------|-----------------|---------|------------|-----------------|-----------------|---------|
| $N_g$ | 0 | 0 | 1 | 2 | -1 | -1 | -2 | -3 |
| $dim$ | 1/2 | 1/2 | -1/2 | -3/2 | 5/2 | 5/2 | 7/2 | 9/2 |

Table 1: Ghost numbers and dimensions.

Having the ghost number and dimension of all fields and antifields at hand, we are now able to solve our problem using the consistent deformation method. The action (9) will be deformed to a new action $S$ in powers of the deformation parameters:

$$S = S_0 + \sum_i g_i S_i + \sum_{i,j} g_i g_j S_{ij} + \cdots,$$

where $S_i, S_{ij}..$ are local integrated polynomials with ghost number zero and dimension bounded by three, and $g_i$ are the deformed parameters with nonnegative mass dimension. The action (13) must satisfy the master equation

$$(S, S) = 0.$$ (14)

Expanding the master equation (14) in powers of the deformation parameters, we have

$$(S_0, S_0) = 0,$$ (15)

$$(S_0, S_i) = 0,$$ (16)

$$2(S_0, S_{ij}) + (S_i, S_j) = 0.$$ (17)

The equation (15) is the the master equation for the $S_0$, and not gives any additional information. The equation (16) tell us that $S_i$ has to be a BRST invariant under (12). We must neglect BRST exacts, since this correspond to fields redefinitions. The last equation (17) is satisfied only if the antibracket $(S_i, S_j)$ is a trivial cocycle.

Let us now construct all $S_i$ solution of equation (16). First we focus our attention to terms that do not deform the gauge symmetry, i.e, terms constructed with the fields only. Due to trivial BRST transformation of $\varphi^a$, the all possible terms with this field are

$$S_1 = \int d^3x \ (\alpha_a \varphi_a), \quad S_2 = \int d^3x \ (\alpha_{ab} \varphi_a \varphi_b),$$ (18)
\[ S_3 = \int d^3x \ (\alpha_{abc} \varphi_a \varphi_b \varphi_c), \quad S_4 = \left( \int d^3x \ \alpha_{abcd} \varphi_a \varphi_b \varphi_c \varphi_d \right), \] (19)

\[ S_5 = \int d^3x \ (\alpha_{abcde} \varphi_a \varphi_b \varphi_c \varphi_d \varphi_e), \quad S_6 = \int d^3x \ (\alpha_{abcde} \varphi_a \varphi_b \varphi_c \varphi_d \varphi_e \varphi_f), \] (20)

where \( \alpha \)'s are parameters. The most general invariant local integrable terms that can be constructed with \( B_{\mu\nu}^a \) and \( \varphi^a \) mixed are

\[ S_7 = \int d^3x \ (m_{ab} \epsilon^{\mu\alpha} H_{a\mu
u\alpha} \varphi_b), \quad S_8 = \int d^3x \ (m_{abc} \epsilon^{\mu\alpha} H_{a\mu
u\alpha} \varphi_b \varphi_c) \] (21)

\[ S_9 = \int d^3x \ (m_{abcd} \epsilon^{\mu\alpha} H_{a\mu
u\alpha} \varphi_b \varphi_c \varphi_d), \] (22)

with \( m_{ab} \) having dimension of mass, \( m_{abc} \) of dimension \( 1/2 \) and \( m_{abcd} \) a dimensionless parameter.

Observing the table (1), it is easy to see that it is impossible to construct invariant local integrated polynomials with dimension bounded by three with the antifields. This means that the algebra of the gauge symmetry is undeformed, i.e., we do not have a non-abelian generalization of the action (1), the only possibility being with an introduction of extra fields or non-renormalizable couplings.

Let us now introduce a set of abelian vectorial gauge field in order to implement the possible nonabelian generalization of (1). We take the mass dimension of all vector equal to one, to make an auxiliary character of those fields. The BRST transformation are

\[ sA^a_\mu = \partial_\mu c^a, \quad sc^a = 0, \] (23)

where \( c^a \) are the ghost for the abelian transformation of \( A^a_\mu \). We must add to the action (9) the corresponding antifield action

\[ S_{\text{ant}}^a = \int d^3x \ A^*_a \partial^\mu c_a. \] (24)

The new antifields have the following BRST transformations

\[ sA^*_\mu = 0, \quad sc^*_a = \partial_\mu A^*_a \mu \] (25)

We show in the table below the ghost number and dimension for new fields (antifields)
The all possible invariant integrated local polynomials that can be constructed with all fields and antifields are

\[ S_{10} = g \int d^3 x \, f_{abc} (\varphi_a^{*} \varphi_b c_c - \partial^\mu \varphi_a \varphi_b A_{c\mu}) , \quad S_{11} = \mu_{ab} \int d^3 x \, (A_{a\mu} A_{c\mu} - c_{a}^{*} A_{b}) , \quad (26) \]

\[ S_{12} = h \int d^3 x \, k_{abc} \left( A_{a}^{*} A_{b}^{\mu} c_{c} - \frac{1}{2} c_{a}^{*} c_{b} c_{c} \right) , \quad (27) \]

where \( g, h \) are dimensionless parameters, \( \mu \) is a matrix with dimension 3/2, and \( f_{abc}(k_{abc}) \) are dimensionless parameters antisymmetric in its first(last) two indices. Now we perform the calculation of the antibrackets \((S_i, S_j)\), with \( i, j = 1, 2, \ldots, 12 \), in order to fit the second order consistency condition. As we have already seen above, this antibrackets must be a BRST exact. The antibrackets \((S_m, S_n)\), for \( n, m = 1, 2, \ldots, 9 \) is identically zero, due to absence of antifields in \( S_n, n = 1, 2, \ldots, 9 \). The antibracket \((S_{10}, S_{10})\) is

\[
(S_{10}, S_{10}) = g^2 \int d^3 x \, f_{abc} f_{ab'c'} (\varphi_a^{*} \varphi_b c_c c_{c'}/c_{c'} + \varphi_{[b} \partial_{\mu} \varphi_{b']} A_{\mu}^{c} c_{c'}) - \frac{g^2}{2} s \left( \int d^3 x \, f_{abc} f_{ab'c'} \varphi_{b} \varphi_{b'} A_{\mu} A_{c'}^{\mu} \right),
\]

(28)

where, \( \varphi_{[b} \partial_{\mu} \varphi_{b']} = \varphi_{b} \partial_{\mu} \varphi_{b'} - \varphi_{b'} \partial_{\mu} \varphi_{b} \). The first term in (28), is not a BRST trivial and it could jeopardize the nonabelian implementation. In order to circumvent this, we must have the identification \( h k_{abc} = g f_{abc} \), and \( f_{abc} \) being the structure constant of a Lie group. Therefore the \( S_{10} \) and \( S_{12} \) are replaced by the sum

\[ S'_{10} = g \int d^3 x \, f_{abc} \left( \varphi^{*a} \varphi^{b} c^{c} - \partial^\mu \varphi^{a} A_{\mu}^{b} c^{c} + A^{*a} A^{b} c^{c} - \frac{1}{2} e^{*a} e^{b} c^{c} \right). \]

(29)

It is easy to see that now \((S'_{10}, S'_{10})\) is BRST trivial

\[
(S'_{10}, S'_{10}) = -\frac{g^2}{2} s \left( \int d^3 x \, f_{abc} f_{ab'c'} \varphi_{b} \varphi_{b'} A_{\mu} A_{c'}^{\mu} \right).
\]

(30)
The antibrackets \((S'_{10}, S_n)\), with \(n = 1, 2, \ldots, 6\), gives us constraints for the parameters 
\(\alpha: \alpha_a = \alpha_{abc} = \alpha_{abcde} = 0, \alpha_{ab} = a_1\delta_{ab}, \alpha_{abcd} = a_2\delta_{ab}\delta_{cd}, \alpha_{abcde} = a_3\delta_{ab}\delta_{cd}\delta_{ef}\), i.e., only 
the terms \(\varphi^2 = \varphi_a\varphi_a\), \((\varphi^2)^2\) and \((\varphi^2)^3\) are permitted. The last antibrackets reads 
\((S_{11}, S_{11}) = 0,\) 
\((S'_{10}, S_{11}) = g \int d^3x \ f_{abc} \mu_{ab'} \left( \rho_{b'\varphi_b^a\varphi_c} + \rho_{b' A_{b\mu}^{\ast} A_{c}^{\mu}} - \rho_{b' c_{b}^{c} c_{c}} \right. \) 
\left. - \eta_{b'}^{b} \partial_{\mu} \varphi_{b} \varphi_{c} - \eta_{b'}^{b} A_{b \mu}^{\ast} c_{c} \right),\) (31) 
\((S'_{10}, S_{7}) = g \int d^3x \ f_{abc} m_{b' a} \varepsilon_{\mu \nu \alpha} H_{b \nu}^{\mu \alpha} \varphi_{b} c_{c},\) 
\((S'_{10}, S_{8}) = g \int d^3x \ f_{abc} (m_{b' c'd'} a + m_{b' ac'd'}) \varepsilon_{\mu \nu \alpha} H_{b \nu}^{\mu \alpha} \varphi_{b} \varphi_{c} \varphi_{c} c_{c},\) 
\((S'_{10}, S_{9}) = g \int d^3x \ f_{abc} (m_{b' c'd' a} + m_{b' c'd' a} + m_{b' ac'd'}) \varepsilon_{\mu \nu \alpha} H_{b \nu}^{\mu \alpha} \varphi_{b} \varphi_{c} \varphi_{c} c_{c}.\) 

The last four antibrackets are not BRST trivial, representing thus an obstruction to the deformation of the master equation. The only way to remedy this is setting \(g = 0\), or 
setting \(S_7 = S_8 = S_9 = S_{11} = 0\). In the case \(g = 0\) we have lost the deformation of the abelian algebra, i.e., we have a set of abelian fields not representing a nonabelian generalization of (1). In the case in which \(S_7 = 0\), we have lost the mass generation of the model. We have thus proved that there are no nonabelian generalization of the action (1), even with an addition of an auxiliary vector gauge field.

3 BRST and anti-BRST symmetry

It is interesting to remark that the introduction of an one form gauge connection \(A\) is required to go further in the nonabelian generalization of our model (1), although our original abelian action (1) does not contain this field. Note that, as pointed out by Thierry-Mieg and Ne’eman [12] for the nonabelian case, the field strength for \(B\) is 1

\(^1\)Here and in the rest of the paper, in order to handle BRST transformations, we use differential forms formalism for convenience.
where $d = dx^\mu (\partial / \partial x^\mu)$ is the exterior derivative.

Taking into account the no-go theorem shown in the previous section, we must add an auxiliary field. Resorting to ref. [12], we can define a new $\mathcal{H}$ given by

$$\mathcal{H} = dB + [A, B] + [F, C] ,$$

where $C$ is the one form auxiliary field required and $F = dA + A \wedge A$.

The obstruction to the nonabelian generalization lies only on the kinetic term for the antisymmetric field, but the topological term must be conveniently redefined. So the nonabelian version of (1) can be written as

$$\int_{M^3} Tr \left\{ 1/2 \mathcal{H} \wedge \ast \mathcal{H} + m \mathcal{H} \wedge \varphi + 1/2 D\varphi \wedge \ast D\varphi \right\} ,$$

where $\ast$ is the Hodge star operator.

The action above is invariant under the following transformations:

$$\delta A = -D\theta ,$$

$$\delta \varphi = [\theta, \varphi] ,$$

$$\delta B = D\Lambda + [\theta, B] ,$$

and

$$\delta C = \Lambda + [\theta, C]$$

where $\theta$ and $\Lambda$ are zero and one-form transformation parameters respectively.
Here we shall use a formalism developed by Thierry-Mieg et al. \cite{12, 13} in order to obtain the BRST and anti-BRST transformation rules. In general lines, we follow closely the treatment of refs. \cite{12} or \cite{3}, since the new object introduced here, namely the scalar field, does not modify the approach.

The presence of a scalar field in topological invariants is not so uncommon. A three-dimensional Yang-Mills topological action was proposed by Baulieu and Grossman \cite{14} for magnetic monopoles by gauge fixing the following topological invariant:

\[ S_{\text{top}} = \int_{M_3} \text{Tr} \{ F \wedge D\phi \} . \] (39)

In the work of Thierry-Mieg and Ne’eman \cite{12}, a geometrical BRST quantization scheme was developed where the base space is extended to a fiber bundle space so that it contains unphysical (fiber-gauge orbit) directions and physical (space-time) directions. Using a double fiber bundle structure Quiros et al. \cite{15} extended the principal fiber bundle formalism in order to include anti-BRST symmetry. Basically the procedure consists in extending the space-time to take into account a pair of scalar anticommuting coordinates denoted by \( y \) and \( \bar{y} \) which correspond to coordinates in the directions of the gauge group of the principal fiber bundle. Then the so-called ”horizontality condition” is imposed. This condition enforces the curvature components containing vertical (fiber) directions to vanish. So only the horizontal components of physical curvature in the extended space survive.

Let us define the following form fields in the extended space and valued in the Lie algebra \( \mathcal{G} \) of the gauge group:

\[ \tilde{\phi} = \varphi , \] (40)

\[ \tilde{A} \equiv A_\mu dx^\mu + A_N dy^N + A_{\bar{N}} d\bar{y}^{\bar{N}} \equiv A + \alpha + \bar{\alpha}, \] (41)

\[ \tilde{B} \equiv \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu + B_{\mu N} dx^\mu \wedge dy^N + B_{\bar{N}\mu} dx^\mu \wedge d\bar{y}^{\bar{N}} + \frac{1}{2} B_{MN} dy^M \wedge dy^N \]
\[ + B_{M\bar{N}} dy^M \wedge d\bar{y}^{\bar{N}} + \frac{1}{2} B_{MN\bar{N}} dy^M \wedge d\bar{y}^{\bar{N}} \]
\[ = B - \beta - \bar{\beta} + \gamma + h + \bar{\gamma}, \] (42)
and

\[ \tilde{C} \equiv C_\mu dx^\mu + C_N dy^N + C_N \tilde{d} \tilde{N} \equiv C + c + \tilde{c}. \]  \hfill (43)

Note that we identify the components in unphysical directions with new fields, namely, \( \alpha, \beta \) and \( c (\tilde{\alpha}, \tilde{\beta} \) and \( \tilde{c} \)) as anticommuting ghosts (antighosts) and the commuting ghosts (antighost) \( \gamma \) and \( h (\tilde{\gamma}) \).

The curvatures 2-form \( \tilde{F} \) and 3-form \( \tilde{H} \) in the fiber-bundle space are

\[ \tilde{F} \equiv \tilde{d} \tilde{A} + \tilde{A} \wedge \tilde{A} \] \hfill (44)

and

\[ \tilde{H} \equiv \tilde{d} \tilde{B} + [\tilde{A}, \tilde{B}] + [\tilde{F}, \tilde{C}], \] \hfill (45)

where \( \tilde{d} = d + s + \tilde{s} \). The exterior derivatives in the gauge group directions are denoted by \( s = dy^N (\partial/\partial y^N) \) and \( \tilde{s} = d\tilde{y}^N (\partial/\partial \tilde{y}^N) \).

It is important to remark here that since we are focusing a mass generation mechanism or, in other words, the action (34), the extra symmetries which appear in the pure topological model have no room in the present discussion.

The horizontality condition, or equivalently, the Maurer-Cartan equation for the field strength \( F \) can be written as

\[ \tilde{F} \equiv \tilde{d} \tilde{A} + \tilde{A} \wedge \tilde{A} = F, \] \hfill (46)

and for the 3-form \( H \) is

\[ \tilde{H} \equiv \tilde{d} \tilde{B} + [\tilde{A}, \tilde{B}] + [\tilde{F}, \tilde{C}] = H. \] \hfill (47)

Also we can impose the horizontality condition for the one form \( D\varphi \), which may be written as

\[ \tilde{D}\tilde{\varphi} = \tilde{d}\varphi + [\tilde{A}, \varphi] = D\varphi. \] \hfill (48)
By expanding both sides of (46) over the pairs of two forms, one can obtain the following transformation rules:

\[ sA_\mu = D_\mu \alpha , \quad \overline{s}A_\mu = D_\mu \overline{\alpha} , \]

\[ s\alpha = -\alpha \wedge \alpha , \quad \overline{s}\alpha = -\overline{\alpha} \wedge \overline{\alpha} , \quad (49) \]

\[ \overline{s}\alpha + \overline{s}\alpha = -\alpha \wedge \alpha \]

In order to close the algebra, we introduce an auxiliary scalar commuting field \( b \) valued in the Lie algebra \( G \) such that

\[ \overline{s}\alpha = b , \quad (50) \]

and consequently

\[ \overline{s}b = -\overline{\alpha} \wedge b , \quad sb = 0 . \quad (51) \]

On the other hand, expanding (47) over the basis of 3-forms yields

\[ sB_{\mu \nu} = -[\alpha, B_{\mu \nu}] - D_{[\mu} \beta_{\nu]} + [F_{\mu \nu}, c] , \quad \overline{s}B_{\mu \nu} = -[\overline{\alpha}, B_{\mu \nu}] - D_{[\mu} \overline{\beta}_{\nu]} - [F_{\mu \nu}, \overline{c}] , \]

\[ s\beta_\mu = -[\alpha, \beta_\mu] + D_\mu \gamma , \quad \overline{s}\beta_\mu = -[\overline{\alpha}, \overline{\beta}_\mu] + D_\mu \overline{\gamma} \]

\[ \overline{s}\beta_\mu + \overline{s}\beta_\mu = -[\alpha, \overline{\beta}_\mu] - [\overline{\alpha}, \beta_\mu] + D_\mu h \]

\[ s\gamma = -[\alpha, \gamma] , \quad \overline{s}\gamma = -[\overline{\alpha}, \overline{\gamma}] \]

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\[
\bar{s}\gamma + sh = -[\alpha, h] - [\bar{\alpha}, \gamma] , \quad s\bar{\gamma} + \bar{s}h = -[\bar{\alpha}, h] - [\alpha, \bar{\gamma}]
\]

Note that when we treat two odd forms, the [ , ] must be reading as an anticommutator.

The action of \( s \) and \( \bar{s} \) upon \( c \), \( \bar{c} \) and \( C \) is not defined in eq.(52). However, the condition (17) leads us to

\[
\tilde{B} + \tilde{D}\tilde{C} = B + DC .
\]  (53)

The condition (53) yields the BRST and anti-BRST transformations for the auxiliary field \( C \) and its ghosts \( c \) and \( \bar{c} \):

\[
sC_{\mu} = -[\alpha, C_{\mu}] + D_{\mu}c + \beta_{\mu} , \quad \bar{s}C_{\mu} = -[\bar{\alpha}, C_{\mu}] + D_{\mu}\bar{c} + \bar{\beta}_{\mu} ,
\]

\[
sc = -[\alpha, c] - \gamma , \quad \bar{s}c = -[\bar{\alpha}, \bar{c}] - \bar{\gamma} ,
\]  (54)

\[
\bar{s}\bar{c} + \bar{s}c = -[\bar{\alpha}, c] - [\alpha, \bar{c}] - h .
\]

However, as usual, the action of \( s \) and \( \bar{s} \) on the ghosts and antighosts is not completely specified by eqs. (52) and (54). Therefore, a set of auxiliary fields is required, namely, a commuting vector field \( t_{\mu} \), two anticommuting scalar fields \( \omega \) and \( \bar{\omega} \) and a commuting scalar field \( n \). These fields are used to solve eqs. (52). Then, we get

\[
s\bar{\beta}_{\mu} = t_{\mu} , \quad \bar{s}\beta_{\mu} = -t_{\mu} - [\alpha, \bar{\beta}_{\mu}] - [\bar{\alpha}, \beta_{\mu}] + D_{\mu}h ,
\]

\[
sh = \omega , \quad \bar{s}\gamma = -\omega - [\alpha, h] - [\bar{\alpha}, \gamma] ,
\]

\[
\bar{s}\bar{\gamma} = \bar{\omega} , \quad \bar{s}h = -\bar{\omega} - [\alpha, \bar{\gamma}] - [\bar{\alpha}, h] ,
\]

13
\[ s\bar{c} = n, \quad \bar{s}c = -n - [\alpha, \bar{c}] - [\bar{\alpha}, c] - h, \]  
(55)

\[ st_\mu = s\omega = s\bar{\omega} = sn = 0, \]

\[ \bar{s}t_\mu = -[\bar{\alpha}, t_\mu] - [D_\mu \alpha, \bar{\gamma}] - D_\mu \bar{\omega} - [\bar{\beta}_\mu, t], \quad \bar{s}n = -[\bar{\alpha}, n] - [\bar{\alpha}, b] + \bar{\omega}, \]

\[ \bar{s}\omega = -[\bar{\alpha}, \omega] - [\alpha, \bar{\omega}] - [\alpha, \omega] - [h, b], \quad \bar{s}\bar{\omega} = -[\bar{\alpha}, \bar{\omega}] - [\bar{\omega}, b]. \]

The nilpotency of the \( s \) and \( \bar{s} \) operators was used to obtain the last eight relations.

Finally, by expanding (48), we obtain

\[ s\varphi = [\alpha, \varphi], \quad \bar{s}\varphi = [\bar{\alpha}, \varphi]. \]  
(56)

Therefore, a complete set of BRST and anti-BRST equations, namely, eqs. (49-51), (54-56), and (52), associated with the classical symmetry (35-37) was obtained.

In this confusion of auxiliary fields it is important to point out the difference between the fields which do not belong to the principal fiber bundle expansion of the ”physical” fields (\( b, t_\mu, n, \omega \) and \( \bar{\omega} \)) (introduced in order to complete the BRST/anti-BRST algebra) and the auxiliary one form field \( C \) introduced in order to overcome the obstruction to the nonabelian generalization. Note that here the \( a \) priori introduction of the auxiliary field \( C \), was necessary in order to fix the BRST and anti-BRST transformation rules. This suggests an interesting and remarkable connection between the technique of consistent deformation and the horizontality condition.

It is worth to mention that in \( D = 3 \) a purely topological model involving a mixed Chern-Simons term (two different one form fields) and the term \( B \wedge D\varphi \) was discussed in ref. [14], and its finiteness was proved in the framework of algebraic renormalization.

We end up this section by observing that the obstruction to nonabelian generalization of the four-dimensional BF model, namely, the existence of the constraint \([F, H] = 0\), appears in the context of our model as \([F, H - m\varphi] = 0\), as can be seen from the equations of motion of the action (34), considered in the absence of the auxiliary field.
4 Nonabelian Topological Mass Generation

The simplest scenario to study mass generation is to consider the equations of motion of the action (34). For convenience, we define a new one form field as

\[ K \equiv D\varphi \]  

(57)

Therefore, the equations of motion can be written as

\[ D^* \mathcal{H} = mK \]  

(58)

and

\[ D^* K = -m\mathcal{H}. \]  

(59)

Equations (58) and (59) can be combined into the following second order equations:

\[ (D^* D^* + m^2) \mathcal{H} = 0 \]  

(60)

\[ (D^* D^* + m^2) K = 0, \]  

(61)

Considering only linear terms for the fields, we get

\[ (d^* d^* + m^2) H = 0, \]  

(62)

\[ (d^* d^* + m^2) d\varphi = 0. \]  

(63)

which are similar to the eqs. (4) and (5), and exhibit mass generation for \( H \) and \( \varphi \).

On the other hand, by looking to the pole structures of the propagators of the model, mass generation can also be established. In order to obtain them, we use the action (34) added with convenient gauge fixing terms, namely
\[ S_T = \int_{M_3} \text{Tr} \left\{ \frac{1}{2} \mathcal{H} \wedge^* \mathcal{H} + m \mathcal{H} \wedge \varphi + \frac{1}{2} D \varphi \wedge^* D \varphi + \mathcal{J} \wedge^* B + j \wedge^* \varphi + J \wedge^* M + J_p \wedge^* p + p^* dM + M \wedge^* dB \right\}, \tag{64} \]

where \( \mathcal{J}, J, J_p \) and \( j \) are currents related to the fields \( B, M, p \) and \( \varphi \) respectively, which generate propagators in the path integral formulation. The auxiliary fields \( M \) and \( p \) are introduced in order to implement the Landau gauge fixing.

Therefore, the tree-level effective propagators for the Kalb-Ramond and scalar fields are

\[ \langle \varphi \varphi \rangle_{a,b} = -\frac{\delta_{ab}}{p^2 - m^2} \tag{65} \]

and

\[ \langle BB \rangle_{a\mu\nu, b\rho\sigma} = \frac{\delta_{ab}}{p^2 - m^2} \left[ g_{\mu\rho} g_{\sigma\nu} - \frac{g_{\mu\rho} P_{\sigma\nu}}{p^2} + \frac{g_{\nu\rho} P_{\sigma\mu}}{p^2} \right], \tag{66} \]

where \( a \) and \( b \) are group indices, and \( \mu, \nu, \rho \) and \( \sigma \) are space-time indices.

It is interesting to note that, here, the gauge field \( B \) "eats" the scalar field (not a Higgs field, however) and acquires a longitudinal degree of freedom and a mass. The inverse process is possible too.

## 5 Conclusions

In this work we have succeeded in extending a tridimensional abelian topological model to the nonabelian case. The model considered here couples a second rank antisymmetric tensor field and a scalar field in a topological way. Initially we use the method of consistent deformations to analyze upon what conditions this generalization can be implemented. Then we have shown that if we require power-counting renormalizable couplings and the same field content, the nonabelian extension is forbidden.

We overcome this obstruction by introduction of two new fields in the model in order to obtain the pursued nonabelian version. One field is a one form gauge connection \( A \) which allows us to define a Yang-Mills covariant derivative. The other auxiliary field
(C') is a vectorial one, which is required in order to resolve the constraint that prevents the correct nonabelianization.

A formal framework to consider the introduction of these fields and the consequent new symmetries, is furnished by BRST and anti-BRST transformation rules, which are obtained using the horizontality condition. Although quite similar to other topological models, it is worth to mention that, in this case, we have constructed transformation rules for the Kalb-Ramond field, for two one form fields and for a scalar field.

Finally, the topological mass generation mechanism for an abelian model found out in a previous paper was extended for the nonabelian case, and we end up with an effective theory describing massive Kalb-Ramond gauge fields in $D = 3$ space-time.

We conclude mentioning the possible relevance of the present discussion to string theory. Indeed, the Kalb-Ramond field couples directly to the worldsheet of strings, and bosonic string condensation into the vacuum realize the Higgs mechanism to the Kalb-Ramond gauge field [8]. Therefore an alternative scenario to give mass to the Kalb-Ramond field in the context of strings may be an interesting continuation of our present results.

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