CP Violation – An Essential Mystery in Nature’s Grand Design

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Abstract

CP violation has so far been observed in one system only, namely in the decays of neutral kaons, and it can still be described by a single real quantity corresponding to a superweak scenario. In these lectures I describe why limitations on CP invariance are a particularly fundamental phenomenon and what experimental information is available. The KM ansatz constitutes the minimal implementation of CP violation: without requiring unknown degrees of freedom it can reproduce the known CP phenomenology. It unequivocally predicts large or even huge CP asymmetries of various kinds in the decays of beauty hadrons. New theoretical technologies will enable us in the foreseeable future to express at least some of these predictions in a quantitatively reliable fashion. There is tremendous potential for discovering New Physics in beauty transitions. Continuing efforts in strange decays and further dedicated searches for electric dipole moments and for CP asymmetries in charm decays are likewise essential for discovering crucial elements that still are missing in the puzzle that is Nature’s Grand Design.

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Schläft ein Lied in allen Dingen,
die da träumen fort und fort,
und die Welt hebt an zu singen,
findst Du nur das Zauberwort.

Sleeps a song in all things
that dream on and on
and the world will start to sing
if you find the magic word.

J. v. Eichendorff

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1 Introduction

Very few symmetries in nature are manifestly realized. Why do I think then that the breaking of CP invariance is very special – more subtle, more fundamental and more profound than parity violation?

- Parity violation tells us that nature makes a difference between "left" and "right" – but not which is which! For the statement that neutrinos emerging from pion decays are left- rather than right-handed implies the use of positive instead of negative pions. "Left" and "right" is thus defined in terms of "positive" and "negative", respectively. This is like saying that your left thumb is on your right hand – certainly correct, yet circular and thus not overly useful.

On the other hand CP violation manifesting itself through

\[
\frac{\text{BR}(K_L \rightarrow l^+\nu\pi^-)}{\text{BR}(K_L \rightarrow l^-\bar{\nu}\pi^+)} \simeq 1.006 J \neq 1
\]

(1)

allows us to define "positive" and "negative" in terms of observation rather than convention, and subsequently likewise for "left" and "right".

- The limitation on CP invariance in the $K^0 - \bar{K}^0$ mass matrix

\[
\text{Im}M_{12} \simeq 1.1 \cdot 10^{-8} \text{ eV} \simeq \frac{\text{Im}M_{12}}{m_K} \simeq 2.2 \cdot 10^{-17}
\]

(2)

represents the most subtle symmetry violation actually observed to date.

- CP violation constitutes one of the three essential ingredients in any attempt to understand the observed baryon number of the universe as a dynamically generated quantity rather than an initial condition [1].

- Due to CPT invariance – on which I will not cast any doubt during these lectures – CP breaking implies a violation of time reversal invariance [2]. That nature makes an intrinsic distinction between past and future on the microscopic level that cannot be explained by statistical considerations is an utterly amazing observation.

\[\text{Operationally one defines time reversal as the reversal of motion: } \vec{p} \rightarrow -\vec{p}, \vec{j} \rightarrow -\vec{j} \text{ for momenta } \vec{p} \text{ and angular momenta } \vec{j}.\]
• The fact that time reversal represents a very peculiar operation can be expressed also in a less emotional way, namely through Kramers’ Degeneracy [2]. The time reversal operator $T$ has to be anti-unitary; $T^2$ then has eigenvalues $\pm 1$. Consider the sector of the Hilbert space with $T^2 = -1$ and assume the dynamics to conserve $T$; i.e., the Hamilton operator $H$ and $T$ commute. It is easily shown that if $|E\rangle$ is an eigenvector of $H$, so is $T|E\rangle$ – with the same eigenvalue. Yet $|E\rangle$ and $T|E\rangle$ are – that is the main substance of this theorem – orthogonal to each other! Each energy eigenstate in the Hilbert sector with $T^2 = -1$ is therefore at least doubly degenerate. This degeneracy is realized in nature through fermionic spin degrees. Yet it is quite remarkable that the time reversal operator $T$ already anticipates this option – and the qualitative difference between fermions and bosons – through $T^2 = \pm 1$ – without any explicit reference to spin!

1.1 General Description of Particle-Antiparticle Oscillations

A symmetry $S$ can be manifestly realized in two different ways:

• There exists a pair of degenerate states that transform into each other under $S$.

• When there is an unpaired state it has to be an eigenstate of $S$.

The observation of $K_L$ decaying into a $2\pi$ state – which is CP even – and a CP odd $3\pi$ combination therefore establishes CP violation only because $K_L$ and $K_S$ are not mass degenerate.

In general, decay rates can exhibit CP violation in three different manners, namely through

• the existence of a reaction, like $K_L \rightarrow \pi\pi$,

• a difference in CP conjugate rates, like $K_L \rightarrow l^-\bar{\nu}\pi^+$ vs. $K_L \rightarrow l^+\nu\pi^-$,

• a decay rate evolution that is not a purely exponential function of the proper time of decay; i.e., if one finds for a CP eigenstate $f$

$$\frac{d}{dt} e^{\Gamma t} \text{rate} (K_{\text{neutral}}(t) \rightarrow f) \neq 0$$

for all (real) values of $\Gamma$, then CP symmetry must be broken. This is easily proven: if CP invariance holds, the decaying state must be a CP eigenstate like the final state $f$; yet in that case the decay rate evolution must be purely exponential – unless CP is violated. Q.E.D.

The whole formalism of particle-antiparticle oscillations is actually a straightforward application of basic quantum mechanics. I will describe it in terms of strange mesons;
the generalization to any other flavour or quantum number (like beauty or charm) is obvious. In the absence of weak forces one has two mass degenerate and stable mesons \( K^0 \) and \( \bar{K}^0 \) carrying definite strangeness +1 and −1, respectively, since the strong and electromagnetic forces conserve this quantum number. The addition of the weak forces changes the picture qualitatively: strangeness is no longer conserved, kaons become unstable and the new mass eigenstates – being linear superpositions of \( K^0 \) and \( \bar{K}^0 \) – no longer carry definite strangeness. The violation of the quantum number strangeness has lifted the degeneracy: we have two physical states \( K_L \) and \( K_S \) with different masses and lifetimes: \( \Delta m_K = m_L - m_K \neq 0 \neq \Delta \tau = \tau_L - \tau_S \).

If CP is conserved in the \( \Delta S = 2 \) transitions the mass eigenstates \( K_1 \) and \( K_2 \) have to be CP eigenstates as pointed out above: \( |K_1\rangle = |K_+\rangle, |K_2\rangle = |K_-\rangle \), where \( \text{CP}|K_\pm\rangle \equiv \pm |K_\pm\rangle \).

Using the phase convention

\[
|\bar{K}^0\rangle \equiv -\text{C}|K^0\rangle
\]

the time evolution of a state that starts out as a \( K^0 \) is given by

\[
|K^0(t)\rangle = \frac{1}{\sqrt{2}} e^{im_1 t} e^{-\frac{\Gamma_1}{2} t} \left( |K_+\rangle + e^{i \Delta m_K t} e^{-\frac{\Delta \Gamma}{2} t} |K_-\rangle \right)
\]

The intensity of an initially pure \( K^0 \) beam traveling in vacuum will then exhibit the following time profile:

\[
I_{K^0}(t) = |\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4} e^{-\Gamma_1 t} \left( 1 + e^{\Delta \Gamma_K t} + 2 e^{\Delta \Gamma_K t} \cos \Delta m_K t \right)
\]

The orthogonal state \( |\bar{K}^0(t)\rangle \) that was absent initially in this beam gets regenerated spontaneously:

\[
I_{\bar{K}^0}(t) = |\langle \bar{K}^0|K^0(t)\rangle|^2 = \frac{1}{4} e^{-\Gamma_1 t} \left( 1 + e^{\Delta \Gamma_K t} - 2 e^{\Delta \Gamma_K t} \cos \Delta m_K t \right)
\]

The oscillation rate expressed through \( \Delta m_K \) and \( \Delta \Gamma_K \) is naturally calibrated by the average decay rate \( \bar{\Gamma}_K \equiv \frac{1}{2} (\Gamma_1 + \Gamma_2) \):

\[
x_K \equiv \frac{\Delta m_K}{\bar{\Gamma}_K} \simeq 0.95 , \quad y_K \equiv \frac{\Delta \Gamma_K}{2 \bar{\Gamma}_K} \simeq 1
\]

Two comments are in order at this point:

- In any such binary quantum system there will be two lifetimes. The fact that they differ so spectacularly for neutral kaons – \( \tau(K_L) \sim 600 \cdot \tau(K_S) \) – is due to a kinematical accident: the only available nonleptonic channel for the CP odd kaon is the 3 pion channel, for which it has barely enough mass.

- \( \Delta m_K \simeq 3.7 \cdot 10^{-6} \text{ eV} \) is often related to the kaon mass:

\[
\frac{\Delta m_K}{m_K} \simeq 7 \cdot 10^{-15}
\]
which is obviously a very striking number. Yet Eq.(9) somewhat overstates the point. The kaon mass has nothing really to do with the $K_L - K_S$ mass difference \(^2\) and actually is measured relative to $\Gamma_K$. There is however one exotic application where it makes sense to state the ratio $\Delta m_K/m_K$, and that is in the context of antigravity where one assumes matter and antimatter to couple to gravity with the opposite sign. The gravitational potential $\Phi$ would then produce a relative phase between $K^0$ and $\bar{K}^0$ of $2m_K\Phi t$. In the earth’s potential this would lead to a gravitational oscillation time of $10^{-15}$ sec, which is much shorter than the lifetimes or the weak oscillation time; $K^0 - \bar{K}^0$ oscillations could then not be observed \cite{3}. There are some loopholes in this argument – yet I consider it still intriguing or at least entertaining.

2 CP Phenomenology in $K_L$ Decays

2.1 General Formalism

Oscillations become more complex once CP symmetry is broken in $\Delta S = 2$ transitions, as seen from solving the (free) Schrödinger equation

$$\frac{i}{\hbar} \frac{d}{dt} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{*12} - \frac{i}{2} \Gamma_{12}^* & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

(10)

CPT invariance imposes

$$M_{11} = M_{22}, \ \Gamma_{11} = \Gamma_{22}.$$  

(11)

\[ \text{Homework Problem } \#1: \]

Which physical situation is described by an equation analogous to Eq.(10) where however the two diagonal matrix elements differ without violating CPT?

\[ \text{Homework Problem } \#2: \]

The mass eigenstates obtained through diagonalising this matrix are given by (for details see \cite{4, 5})

$$|K_S\rangle \equiv |K_1\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left( p|K^0\rangle + q|\bar{K}^0\rangle \right)$$

(12)

$$|K_L\rangle \equiv |K_2\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left( p|K^0\rangle - q|\bar{K}^0\rangle \right)$$

(13)

with

$$q \over p = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}}$$

(14)

\[ ^2\text{It would not be much more absurd to relate } \Delta m_K \text{ to the mass of an elephant!} \]
and eigenvalues

\[ M_S - \frac{i}{2} \Gamma_S = M_{11} - \frac{i}{2} \Gamma_{11} - \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \]  

(15)

\[ M_L - \frac{i}{2} \Gamma_L = M_{11} - \frac{i}{2} \Gamma_{11} + \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \]  

(16)

These states are conveniently expressed in terms of the CP eigenstates \(|K_\pm\rangle\):

\[ |K_S\rangle = \frac{1 + q/p}{\sqrt{2(1 + |q/p|^2)}} (|K_+\rangle + \bar{\epsilon}|K_-\rangle) \]  

(17)

\[ |K_L\rangle = \frac{1 + q/p}{\sqrt{2(1 + |q/p|^2)}} (|K_-\rangle + \bar{\epsilon}|K_+\rangle) \]  

(18)

where the parameter

\[ \bar{\epsilon} \equiv \frac{1 - q/p}{1 + q/p} \]  

(19)

reflects the CP impurity in the state vector.

A few comments – some technical, some not – might elucidate the situation:

- If there is no relative phase between \(M_{12}\) and \(\Gamma_{12}\)

\[ \arg \frac{M_{12}}{\Gamma_{12}} = 0 \]  

(20)

then \(q/p = 1\) and the state vectors conserve CP: \(\bar{\epsilon} = 0\).

- Yet for our later discussion one should take note that \(q/p\) – and therefore also \(\bar{\epsilon}\) – by itself cannot be an observable. For a change in the phase convention adopted for defining \(\bar{K}^0\) does not leave it invariant:

\[ |\bar{K}^0\rangle \rightarrow e^{i\xi}|\bar{K}^0\rangle \implies (M_{12, \Gamma_{12}}) \rightarrow e^{i\xi}(M_{12, \Gamma_{12}}) \implies \frac{q}{p} \rightarrow e^{-i\xi} \frac{q}{p} ! \]  

(21)

On the other hand \(|q/p|\) is independent of the phase convention and its deviation from unity is one measure of CP violation.

- On very general grounds – without recourse to any model – one can infer that CP violation in the neutral kaon system has to be small. CP invariance implies the two mass eigenstates \(K_L\) and \(K_S\) to be orthogonal – as can be read off explicitly from the general expression

\[ \langle K_L|K_S\rangle = \frac{1 - |q/p|^2}{1 + |q/p|^2} \]  

(22)
The Bell-Steinberger relation allows to place a bound on this scalar product from inclusive decay rates [1,2]:
\[
\langle K_L|K_S \rangle \leq \sqrt{2} \sum_f \sqrt{\frac{\Gamma_f^{K_L}}{\Gamma_f^{K_S}}} \leq \sqrt{2} \sqrt{\frac{\Gamma_L}{\Gamma_S}} \simeq 0.06
\] (23)

There is no input from any CP measurement. What is essential, though, is the huge lifetime ratio.

• There are actually two processes underlying the transition $K_L \to 2\pi$: $\Delta S = 2$ forces generate the mass eigenstates $K_L$ and $K_S$ whereas $\Delta S = 1$ dynamics drive the decays $K \to 2\pi$. Thus CP violation can enter in two a priori independent ways, namely through the $\Delta S = 2$ and the $\Delta S = 1$ sector. This distinction can be made explicit in terms of the transition amplitudes:

\[
\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \epsilon_K + \epsilon', \quad \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \epsilon_K - 2\epsilon'
\] (24)

The quantity $\epsilon_K$ describes the CP violation common to the $K_L$ decays; it thus characterizes the decaying state and is referred to as CP violation in the mass matrix or superweak CP violation; $\epsilon'$ on the other hand differentiates between different channels and thus characterizes decay dynamics; it is called direct CP violation.

• **Maximal** parity and/or charge conjugation violation can be defined by saying there is no right-handed neutrino and/or left-handed antineutrino, respectively. Yet maximal CP violation cannot be defined in an analogous way: for the existence of the right-handed antineutrino which is the CP conjugate to the left-handed neutrino is already required by CPT invariance.

### 2.2 Data

The data on CP violation in neutral kaon decays are as follows:

1. **Existence of $K_L \to \pi\pi$:**

\[
\begin{align*}
\text{BR}(K_L \to \pi^+\pi^-) &= (2.067 \pm 0.035) \cdot 10^{-3} \\
\text{BR}(K_L \to \pi^0\pi^0) &= (0.936 \pm 0.020) \cdot 10^{-3}
\end{align*}
\] (25)

2. **Search for direct CP violation:**

\[
\frac{\epsilon'}{\epsilon_K} \simeq \text{Re}\frac{\epsilon'}{\epsilon_K} = \begin{cases} 
(2.3 \pm 0.65) \cdot 10^{-3} & NA 31 \\
(1.5 \pm 0.8) \cdot 10^{-3} & PDG '96 average \\
(0.74 \pm 0.52 \pm 0.29) \cdot 10^{-3} & E 731
\end{cases}
\] (26)
3. Rate difference in semileptonic decays:

\[ \delta_l \equiv \frac{\Gamma(K_L \rightarrow l^+\nu\pi^-) - \Gamma(K_L \rightarrow l^-\bar{\nu}\pi^+)}{\Gamma(K_L \rightarrow l^+\nu\pi^-) + \Gamma(K_L \rightarrow l^-\bar{\nu}\pi^+)} = (3.27 \pm 0.12) \cdot 10^{-3}, \tag{27} \]

where an average over electrons and muons has been taken.

4. \( T \) violation:

\[ \frac{\Gamma(K^0 \Rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \Rightarrow K^0)}{\Gamma(K^0 \Rightarrow K^0) + \Gamma(\bar{K}^0 \Rightarrow K^0)} = (6.3 \pm 2.1 \pm 1.8) \cdot 10^{-3} \quad CPLEAR \tag{28} \]

from a third of their data set. It would be premature to claim this asymmetry has been established; yet it represents an intriguingly direct test of time reversal violation and is sometimes referred to as the Kabir test. It requires tracking the flavour identity of the decaying meson as a \( K^0 \) or \( \bar{K}^0 \) through its semileptonic decays \( \bar{K}^0 \rightarrow l^-\bar{\nu}\pi^+ \) vs. \( K^0 \rightarrow l^+\nu\pi^- \) and also of the \textit{initially produced} kaon. The latter is achieved through correlations imposed by associated production. The CPLEAR collaboration studied low energy proton-antiproton annihilation

\[ p\bar{p} \rightarrow K^+K^0\pi^- \quad \text{vs.} \quad p\bar{p} \rightarrow K^-\bar{K}^0\pi^+; \tag{29} \]

the charged kaon reveals whether a \( K^0 \) or a \( \bar{K}^0 \) was produced in association with it. In the future the CLOE collaboration will study \( T \) violation in \( K^0\bar{K}^0 \) production at DAΦNE:

\[ e^+e^- \rightarrow \phi(1020) \rightarrow K^0\bar{K}^0 \tag{30} \]

2.3 Phenomenological Interpretation

2.3.1 Semileptonic Transitions

CPT symmetry imposes constraints well beyond the equality of lifetimes for particles and antiparticles: certain subclasses of decay rates have to be equal as well. For example one finds

\[ \Gamma(\bar{K}^0 \rightarrow l^-\bar{\nu}\pi^+) = \Gamma(K^0 \rightarrow l^+\nu\pi^-) \tag{31} \]

The rate asymmetry in semileptonic decays listed in Eq.(27) thus reflects pure superweak CP violation:

\[ \delta_l = \frac{1 - |q/p|^2}{1 + |q/p|^2} \tag{32} \]

From the measured value of \( \delta_l \) one then obtains

\[ \left| \frac{q}{p} \right| = 1 + (3.27 \pm 0.12) \cdot 10^{-3} \tag{33} \]
Since one has for the $K^0 - \bar{K}^0$ system specifically
\[
\left| \frac{q}{p} \right| \simeq 1 + \frac{1}{2} \arg \frac{M_{12}}{\Gamma_{12}} \tag{34}
\]
one can express this kind of CP violation through a phase:
\[
\Phi(\Delta S = 2) \equiv \arg \frac{M_{12}}{\Gamma_{12}} = (6.54 \pm 0.24) \cdot 10^{-3} \tag{35}
\]
The result of the Kabir test, Eq.(28), yields:
\[
\Phi(\Delta S = 2) = (6.3 \pm 2.1 \pm 1.8) \cdot 10^{-3}, \tag{36}
\]
which is of course consistent with Eq.(35).

Using the measured value of $\Delta m_K/\Delta \Gamma_K$ one infers
\[
\frac{M_{12}}{\Gamma_{12}} = -(0.4773 \pm 0.0023) \left[ 1 - i(6.54 \pm 0.24) \cdot 10^{-3} \right] \tag{37}
\]

### 2.3.2 Nonleptonic Transitions

From Eq.(25) one deduces
\[
|\eta_{+-}| = (2.275 \pm 0.019) \cdot 10^{-3} \\
|\eta_{00}| = (2.285 \pm 0.019) \cdot 10^{-3} \tag{38}
\]
As mentioned before the ratios $\eta_{+-,00}$ are sensitive also to direct CP violation generated by a phase between the decay amplitudes $A_{0,2}$ for $K_L \to (\pi\pi)_I$, where the subscript $I$ denotes the isospin of the $2\pi$ system:
\[
\Phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0} \tag{39}
\]
One finds
\[
\eta_{+-} \simeq \frac{i\tilde{x}}{2\tilde{x} + i} \left[ \Phi(\Delta S = 2) + 2\omega \Phi(\Delta S = 1) \right], \tag{40}
\]
with
\[
\tilde{x} \equiv \frac{\Delta m_K}{\Delta \Gamma_K} = \frac{\Delta m_K}{\Gamma(K_S)} = \frac{1}{2} x_K \simeq 0.477, \quad \omega \equiv \left| \frac{A_2}{A_0} \right| \simeq 0.05 \tag{41}
\]
where the second quantity represents the observed enhancement of $A_0$ for which a name – ”$\Delta I = 1/2$ rule” – yet no quantitative dynamical explanation has been found. Equivalently one can write
\[
\frac{\epsilon'}{\epsilon_K} \simeq 2\omega \frac{\Phi(\Delta S = 1)}{\Phi(\Delta S = 2)} \tag{42}
\]
The data on $K_L \to \pi\pi$ can thus be expressed as follows \[7\]
\[
\Phi(\Delta S = 2) = (6.58 \pm 0.26) \cdot 10^{-3} \\
\Phi(\Delta S = 1) = (0.99 \pm 0.53) \cdot 10^{-3} \tag{43}
\]
2.3.3 Resume

The experimental results can be summarized as follows:

- The decays of neutral kaons exhibit unequivocally CP violation of the super-weak variety, which is expressed through the angle $\Phi(\Delta S = 2)$. The findings from semileptonic and nonleptonic transitions concur to an impressive degree.

- Direct CP violation still has not been established.

- A theorist might be forgiven for mentioning that the evolution of the measurements over the last twenty odd years has not followed the straight line this brief summary might suggest to the uninitiated reader.

3 Theoretical Implementation of CP Violation

3.1 Some Historical Remarks

Theorists can be forgiven if they felt quite pleased with the state of their craft in 1964:

- The concept of (quark) families had emerged, at least in a rudimentary form.

- Maximal parity and charge conjugation violations had been found in weak charged current interactions, yet CP invariance apparently held. Theoretical pronouncements were made ex cathedra why this had to be so!

- Predictions of the existence of two kinds of neutral kaons with different lifetimes and masses had been confirmed by experiment [8].

That same year the reaction $K_L \to \pi^+\pi^-$ was discovered [3]! Two things should be noted here. The Fitch-Cronin experiment had predecessors: rather than being an isolated effort it was the culmination of a whole research program. Secondly there was at least one theoretical voice, namely that of Okun [10], who in 1962/63 had listed a dedicated search for $K_L \to \pi\pi$ as one of the most important unfinished tasks. Nevertheless for the vast majority of the community the Fitch-Cronin observation came as a shock and caused considerable consternation among theorists. Yet – to their credit – these data and their consequence, namely that CP invariance was broken, were soon accepted as facts. This was phrased – though not explained – in terms of the Superweak Model [11] later that same year.

In 1970 the renormalizability of the $SU(2)_L \times U(1)$ electroweak gauge theory was proven. I find it quite amazing that it was still not realized that the physics known at that time could not produce CP violation. As long as one had to struggle with infinities in the theoretical description one could be forgiven for not worrying unduly about a tiny quantity like $\text{BR}(K_L \to \pi^+\pi^-) \simeq 2.3 \cdot 10^{-3}$. Yet no such excuse existed any longer once a renormalizable theory had been developed! The
existence of the Superweak Model somewhat muddled the situation in this respect: for it provides merely a classification of the dynamics underlying CP violation rather than a dynamical description itself.

The paper by Kobayashi and Maskawa \[12\], written in 1972 and published in 1973, was the first

- to state clearly that the $SU(2)_L \times U(1)$ gauge theory even with two complete families \[3\] is necessarily CP-invariant and
- to list the possible extensions that could generate CP violation; among them – as one option – was the three (or more) family scenario now commonly referred to as the KM ansatz. They also discussed the impact of right-handed currents and of a non-minimal Higgs sector.

3.2 The Minimal Model: The KM Ansatz

Once a theory reaches a certain degree of complexity, many potential sources of CP violation emerge. Popular examples of such a scenario are provided by models implementing supersymmetry or its local version, supergravity; hereafter both are referred to as SUSY. In my lectures I will however focus on the minimal theory that can support CP violation, namely the Standard Model with three families. All of its dynamical elements have been observed – except for the Higgs boson, of course.

3.2.1 Weak Phases like the Scarlet Pimpernel

Weak interactions at low energies are described by four-fermion interactions. The most general expression for spin-one couplings are

$$\mathcal{L}_{V/A} = (\bar{\psi}_1 \gamma_\mu (a + b\gamma_5) \psi_2) (\bar{\psi}_3 \gamma_\mu (c + d\gamma_5) \psi_4) +$$

$$+ (\bar{\psi}_2 \gamma_\mu (a^* + b^*\gamma_5) \psi_1) (\bar{\psi}_4 \gamma_\mu (c^* + d^*\gamma_5) \psi_3)$$

(44)

Under CP these terms transform as follows:

$$\mathcal{L}_{V/A} \xrightarrow{CP} CP \mathcal{L}_{V/A}(CP)^\dagger = (\bar{\psi}_2 \gamma_\mu (a + b\gamma_5) \psi_1) (\bar{\psi}_4 \gamma_\mu (c + d\gamma_5) \psi_3) +$$

$$+ (\bar{\psi}_1 \gamma_\mu (a^* + b^*\gamma_5) \psi_2) (\bar{\psi}_3 \gamma_\mu (c^* + d^*\gamma_5) \psi_4)$$

(45)

If $a, b, c, d$ are real numbers, one obviously has $\mathcal{L}_{V/A} = CP \mathcal{L}_{V/A}(CP)^\dagger$ and CP is conserved. Yet CP is not necessarily broken if these parameters are complex, as we will explain specifically for the Standard Model.

Weak Universality arises naturally whenever the weak charged current interactions are described through a single non-abelian gauge group – $SU(2)_L$ in the case

\[3\] Remember this was still before the $J/\psi$ discovery!
under study. For the single self-coupling of the gauge bosons determines also their couplings to the fermions; one finds for the quark couplings to the charged $W$ bosons:

$$\mathcal{L}_{CC} = g\bar{U}_L^{(0)}\gamma_\mu D_L^{(0)}W^\mu + \bar{U}_R^{(0)}M_UU_L^{(0)} + \bar{D}_R^{(0)}M_DD_L^{(0)} + h.c.$$  \hspace{1cm} (46)

where $U$ and $D$ denote the up- and down-type quarks, respectively:

$$U = (u, c, t), \quad D = (d, s, b)$$  \hspace{1cm} (47)

and $M_U$ and $M_D$ their 3×3 mass matrices. In general those will not be diagonal; to find the physical states, one has to diagonalize these matrices:

$$M_U^{\text{diag}} = K^U_R M_U(K^U_L)\dagger, \quad M_D^{\text{diag}} = K^D_R M_D(K^D_L)\dagger$$  \hspace{1cm} (48)

$$U_{L,R} = K^U_{L,R}U^{(0)}_{L,R}, \quad D_{L,R} = K^D_{L,R}D^{(0)}_{L,R}$$  \hspace{1cm} (49)

with $K^U_{L,R}$ representing four unitary 3×3 matrices. The coupling of these physical fermions to $W$ bosons is then given by

$$\mathcal{L}_{CC} = g\bar{U}_L(K^U_L)\dagger K^D_L\gamma_\mu DW^\mu + \bar{U}_R M_U^{\text{diag}} U_L + \bar{D}_R M_D^{\text{diag}} D_L + h.c.$$  \hspace{1cm} (50)

and the combination $(K^U_L)\dagger K^D_L \equiv V_{CKM}$ represents the KM matrix, which obviously has to be unitary like $K^U$ and $K^D$. Unless the up- and down-type mass matrices are aligned in flavour space (in which case they would be diagonalized by the same operators $K_{L,R}$) one has $V_{CKM} \neq 1$.

In the neutral current sector one has

$$\mathcal{L}_{NC} = g'\bar{U}_L^{(0)}\gamma_\mu U_L^{(0)}Z_\mu = g'\bar{U}_L\gamma_\mu U_L Z_\mu$$  \hspace{1cm} (51)

and likewise for $\bar{U}_R$ and $D_{L,R}$; i.e. no flavour changing neutral currents are generated, let alone new phases. CP violation thus has to be embedded into the charged current sector.

If $V_{CKM}$ is real (and thus orthogonal), CP symmetry is conserved in the weak interactions. Yet the occurrence of complex matrix elements does not automatically signal CP violation. This can be seen through a straightforward (in hindsight at least) algebraic argument. A unitary $N \times N$ matrix contains $N^2$ independant real parameters; $2N - 1$ of those can be eliminated through re-phasing of the $N$ up-type and $N$ down-type fermion fields (changing all fermions by the same phase obviously does not affect $V_{CKM}$). Hence there are $(N - 1)^2$ real physical parameters in such an $N \times N$ matrix. For $N = 2$, i.e. two families, one recovers a familiar result, namely there is just one mixing angle, the Cabibbo angle. For $N = 3$ there are four real physical parameters, namely three (Euler) angles – and one phase. It is the latter that provides a gateway for CP violation. For $N = 4$ Pandora’s box opens up: there would be 6 angles and 3 phases.
PDG suggests a "canonical" parametrization for the $3 \times 3$ CKM matrix:

\[
V_{CKM} = \begin{pmatrix}
V(ud) & V(us) & V(ub) \\
V(cd) & V(cs) & V(cb) \\
V(td) & V(ts) & V(tb)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{13}c_{23}
\end{pmatrix}
\]

where

\[c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}\]

with $i, j = 1, 2, 3$ being generation labels.

This is a completely general, yet not unique parametrisation: a different set of Euler angles could be chosen; the phases can be shifted around among the matrix elements by using a different phase convention. In that sense one can refer to the KM phase as the Scarlet Pimpernel: "Sometimes here, sometimes there, sometimes everywhere!"

Using just the observed hierarchy

\[|V(ub)| \ll |V(cb)| \ll |V(us)|, |V(cd)| \ll 1\]

one can, as first realized by Wolfenstein, expand $V_{CKM}$ in powers of the Cabibbo angle $\theta_C$:

\[
V_{CKM} = \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \frac{\lambda}{1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4} & A\lambda^3(\rho - i\eta + \frac{i}{2}\eta\lambda^2) \\
\frac{\lambda}{A\lambda^3(1 - \rho - i\eta)} & 1 - \frac{1}{2}\lambda^2 & A\lambda^2(1 + i\eta\lambda^2) \\
-A\lambda^2 & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^6)
\]

where

\[\lambda \equiv \sin \theta_C\]

For such an expansion in powers of $\lambda$ to be self-consistent, one has to require that $|A|, |\rho|$ and $|\eta|$ are of order unity. Numerically we obtain

\[\lambda = 0.221 \pm 0.002\]

from $|V(us)|$,

\[A = 0.81 \pm 0.06\]

from $|V(cb)| \simeq 0.040 \pm 0.002|_{\text{exp}} \pm 0.002|_{\text{theor}}$ and

\[\sqrt{\rho^2 + \eta^2} \sim 0.38 \pm 0.11\]

from $|V(ub)| \sim (3.2 \pm 0.8) \cdot 10^{-3}$.

We see that the CKM matrix is a very special unitary matrix: it is almost diagonal, it is almost symmetric and the matrix elements get smaller the more one moves away from the diagonal. Nature most certainly has encoded a profound message in this peculiar pattern. Alas – we have not succeeded yet in deciphering it! I will return to this point at the end of my lectures.
3.2.2 Unitarity Triangles

The qualitative difference between a two and a three family scenario can be seen also in a less abstract way. Consider $\bar{K}_0 \rightarrow \pi^+\pi^-$; it can proceed through a tree-level process $[s\bar{d}] \rightarrow [d\bar{u}] [u\bar{d}]$, in which case its weak couplings are given by $V(us)V^*(ud)$. Or it can oscillate first to $K_0$ before decaying; i.e., on the quark level it is the transition $[s\bar{d}] \rightarrow [d\bar{s}] [s\bar{u}] \rightarrow [d\bar{u}] [u\bar{d}]$ controlled by $(V(cs))^2 (V^*(cd))^2 V^*(us)V(u)$. At first sight it would seem that those two combinations of weak parameters are not only different, but should also exhibit a relative phase. Yet the latter is not so – if there are two families only! In that case the four quantities $V(u)\bar{u}, V(u)\bar{s}, V(c)\bar{d}$ and $V(n)\bar{q}$ have to form a unitary $2 \times 2$ which leads to the constraint

$$V(u)\bar{u}V^*(u) + V(c)\bar{c}V^*(c) = 0 \quad \text{(60)}$$

Using Eq.(60) twice one gets

$$(V(cs)V^*(cd))^2 V^*(us)V(u) = -|V(cd)V(cs)|^2 V^*(us)V*(cd) =$$

$$= |V(cd)V(cs)|^2 V^*(ud)V(u); \quad \text{(61)}$$

i.e., the two combinations $V^*(ud)V(u)$ and $(V(cs))^2 (V^*(cd))^2 V^*(us)V(u)$ are actually parallel to each other with no relative phase. A penguin operator with a charm quark as the internal fermion line generates another contribution to $K_L \rightarrow \pi^-\pi^+$, this one controlled by $V(cs)V^*(cd)$. Yet the unitarity condition Eq.(60) forces this contribution to be antiparallel to $V^*(ud)V(u)$; i.e., again no relative phase.

The situation changes fundamentally for three families: the weak parameters $V(ij)$ now form a $3 \times 3$ matrix and the condition stated in Eq.(60) gets extended:

$$V(u)\bar{u}V^*(u) + V(c)\bar{c}V^*(c) + V(t)\bar{t}V^*(t) = 0 \quad \text{(62)}$$

This is a triangle relation in the complex plane. There emerge now relative phases between the weak parameters and the loop diagrams with internal charm and top quarks can generate CP asymmetries.

Unitarity imposes altogether nine algebraic conditions on the matrix elements of $V_{CKM}$, of which six are triangle relations analogous to Eq.(62). There are several nice features about this representation in terms of triangles; I list four now and others later:

1. The shape of each triangle is independent of the phase convention adopted for the quark fields. Consider for example Eq.(62); changing the phase of any of the up-type quarks will not affect the triangle at all. Under $|s\rangle \rightarrow |s\rangle e^{i\phi_s}$, the whole triangle will rotate around the left end of its base line by an angle $\phi_s$ – yet the shape of the triangle – in contrast to its orientation in the complex plane – remains the same! The angles inside the triangles are thus observables; choosing an orientation for the triangles is then a matter of convenience.
2. It is easily shown that all six KM triangles possess the same area. Multiplying Eq.(62) by the phase factor $V^*(ud)V(us)/|V(ud)V(us)|$, which does not change the area, yields

$$|V(ud)V(us)| + \frac{V^*(ud)V(us)V(cd)V^*(cs)}{|V(ud)V(us)|} + \frac{V^*(ud)V(us)V(td)V^*(ts)}{|V(ud)V(us)|} = 0$$

(area of triangle of Eq.(62)) = $\frac{1}{2}|\text{Im}V(ud)V(cs)V^*(us)V^*(cd)| = \frac{1}{2}|\text{Im}V(ud)V(ts)V^*(us)V^*(td)|$ (63)

Multiplying Eq.(62) instead by the phase factors $V^*(cd)V(ts)/|V(cd)V(ts)|$ or $V^*(td)V(ts)/|V(td)V(ts)|$ one sees that the area of this triangle can be expressed in other ways still. Among them is

(area of triangle of Eq.(62)) = $\frac{1}{2}|\text{Im}V(cd)V(ts)V^*(cs)V^*(td)|$ (64)

Due to the unitarity relation

$$V^*(cd)V(td) + V^*(cb)V(tb) = -V^*(cs)V(ts)$$

one has

(area of triangle of Eq.(62)) = $\frac{1}{2}|\text{Im}V(cd)V(tb)V^*(cb)V^*(td)|$ (65)

– yet this is exactly the area of the triangle defined by Eq.(66)! This is the re-incarnation of the original observation that there is a single irreducible weak phase for three families.

3. In general one has for the area of these triangles

$$A_{\text{CPV}}(\text{every triangle}) = \frac{1}{2}J$$

$$J = \text{Im}V^*(km)V(lm)V(kn)V^*(ln) = \text{Im}V^*(mk)V(ml)V(nk)V^*(nl)$$

irrespective of the indices $k,l,m,n$; $J$ is obviously re-phasing invariant.

4. If there is a representation of $V_{\text{CKM}}$ where all phases were confined to a $2 \times 2$ sub-matrix exactly rather than approximately, then one can rotate all these phases away; i.e., CP is conserved in such a scenario! Consider again the triangle described by Eq.(62): it can always be rotated such that its baseline $V(ud)V^*(us)$ is real. Then $\text{Im}V(td)V^*(ts) = -\text{Im}V(cd)V^*(cs)$ holds. If, for example, there were no phases in the third row and column, one would have $\text{Im}(V(td)V^*(ts)) = 0$ and therefore $\text{Im}V(cd)V^*(cs) = 0$ as well; i.e., $V(ud)V^*(us)$ and $V(cd)V^*(cs)$ were real relative to each other; therefore $J = 0$, i.e. all six triangles had zero area meaning there are no relative weak phases!
3.3 Evaluating $\epsilon_K$ and $\epsilon'$

In calculating observables in a given theory – in the case under study $\epsilon_K$ and $\epsilon'$ within the KM Ansatz – one is faced with the ‘Dichotomy of the Two Worlds’, namely

- one world of short-distance physics where even the strong interactions can be treated perturbatively in terms of quarks and gluons and in which theorists like to work, and
- the other world of long-distance physics where one has to deal with hadrons the behaviour of which is controlled by non-perturbative dynamics and where, by the way, everyone, including theorists, lives.

Accordingly the calculational task is divided into two parts, namely first determining the relevant transition operators in the short-distance world and then evaluating their matrix elements in the hadronic world.

3.3.1 $\Delta S = 2$ Transitions

Since the elementary interactions in the Standard Model can change strangeness at most by one unit, the $\Delta S = 2$ amplitude driving $K^0 - \bar{K}^0$ oscillations is obtained by iterating the basic $\Delta S = 1$ coupling:

$$\mathcal{L}_{\text{eff}}(\Delta S = 2) = \mathcal{L}(\Delta S = 1) \otimes \mathcal{L}(\Delta S = 1)$$  \hspace{1cm} (69)

There are actually two ways in which the $\Delta S = 1$ transition can be iterated:

(A) The resulting $\Delta S = 2$ transition is described by a local operator. The celebrated box diagram makes this connection quite transparent. The contributions that do not depend on the mass of the internal quarks cancel against each other due to the GIM mechanism. Integrating over the internal fields, namely the $W$ bosons and the top and charm quarks \footnote{The up quarks act merely as a subtraction term here.}, then yields a convergent result:

$$\mathcal{L}_{\text{eff}}^{\text{box}}(\Delta S = 2, \mu) = \left(\frac{G_F}{4\pi}\right)^2 \cdot \left[\xi_c^2 E(x_c) \eta_{cc} + \xi_t^2 E(x_t) \eta_{ct} + 2\xi_c \xi_t E(x_c, x_t) \eta_{ct}\right] \cdot [\alpha_S(\mu^2)]^{-6/27} (\bar{s}\gamma_\mu(1 - \gamma_5)d)^2$$  \hspace{1cm} (70)

with $\xi_i$ denoting combinations of KM parameters

$$\xi_i = V(is)V^*(id), \ i = c, t;$$  \hspace{1cm} (71)

$E(x_i)$ and $E(x_c, x_t)$ reflect the box loops with equal and different internal quarks, respectively \footnote{\cite{13}}:

$$E(x_i) = x_i \left(\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2}\right) - \frac{3}{2} \left(\frac{x_i}{1-x_i}\right)^3 \log x_i$$  \hspace{1cm} (72)
Table 1: Values of $B_K$ from various theoretical techniques

| Method                                      | $B_K$       |
|--------------------------------------------|-------------|
| Large $N_C$ Expansion                      | $\frac{2}{3}$ |
| Large $N_C$ Chiral Pert. with loop correction | $0.66 \pm 0.1$ |
| Lattice QCD                                | $0.84 \pm 0.2$ |

$E(x_c, x_t) = x_c x_t \left[ \left( \frac{1}{4} + \frac{3}{2(1-x_t)} - \frac{3}{4(1-x_t)^2} \right) \frac{\log x_t}{x_t-x_c} + (x_c \leftrightarrow x_t) - \frac{3}{4} \frac{1}{(1-x_c)(1-x_t)} \right]$ \hspace{1cm} (73)

$x_i = \frac{m_i^2}{M_W^2}$ \hspace{1cm} (74)

and $\eta_{ij}$ containing the QCD radiative corrections from evolving the effective Lagrangian from $M_W$ down to the internal quark mass. The factor $[\alpha_S(\mu^2)]^{-6/27}$ reflects the fact that a scale $\mu$ must be introduced at which the four-quark operator $(\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma_\mu(1-\gamma_5)d)$ is defined. This dependence on the auxiliary variable $\mu$ drops out when one takes the matrix element of this operator (at least when one does it correctly). Including next-to-leading log corrections one finds (for $m_t \simeq 180$ GeV) \hspace{1cm} (B)

$$\eta_{cc} \simeq 1.38 \pm 0.2, \quad \eta_{tt} \simeq 0.57 \pm 0.01, \quad \eta_{cc} \simeq 0.47 \pm 0.04$$ \hspace{1cm} (75)

Even for a local four-fermion operator it is non-trivial to evaluate an on-shell matrix element between hadron states since that is clearly controlled by non-perturbative dynamics. Usually one parametrizes this matrix element as follows:

$$\langle \bar{K}^0|(\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma_\mu(1-\gamma_5)d)|K^0 \rangle =$$

$$= \frac{4}{3} B_K \langle \bar{K}^0|(\bar{s}\gamma_\mu(1-\gamma_5)d)|0\rangle \langle 0|(\bar{s}\gamma_\mu(1-\gamma_5)d)|K^0 \rangle = \frac{4}{3} B_K f_K^2 m_K$$ \hspace{1cm} (76)

The factor $B_K$ is - for historical reasons of no consequence now - often called the bag factor; $B_K = 1$ is referred to as vacuum saturation or factorization ansatz since it corresponds to a situation where inserting the vacuum intermediate state into Eq.(76) reproduces the full result after all colour contractions of the quark lines have been included. Several theoretical techniques have been employed to estimate the size of $B_K$; their findings are listed in Table 1. These results, which are all consistent with each other and with several phenomenological studies as well, can be summarized as follows:

$$B_K \simeq 0.8 \pm 0.2$$ \hspace{1cm} (77)

\hspace{1cm} 5This can be inferred from the observation that $|e'/\epsilon_K| \ll 0.05$
Since the size of this matrix element is determined by the strong interactions, one indeed expects $B_K \sim 1$.

We have assembled all the ingredients now for calculating $\epsilon_K$. The starting point is given by [6]:

$$|\epsilon_K| \simeq \frac{1}{\sqrt{2}} \left| \frac{\text{Im} M_{12}}{\Delta m_K} \right|$$

(78)

The CP-odd part $\text{Im} M_{12}$ is obtained from

$$\text{Im} M_{12} = \text{Im} \langle K^0| \mathcal{L}_{eff} (\Delta S = 2)|\bar{K}^0 \rangle$$

(79)

whereas for $\Delta m_K$ one inserts the experimental value, since the long-distance contributions to $\Delta m_K$ are not under theoretical control. One then finds

$$|\epsilon_K|_{KM} \simeq |\epsilon_K|_{\text{box}} \simeq$$

$$\simeq \frac{G_F^2 M_W^2 m_K f_K^2 B_K}{6\sqrt{2}\pi^2 \Delta m_K} \left[ \text{Im} \xi_c^2 E(x_c) \eta_{cc} + \text{Im} \xi_t^2 E(x_t) \eta_{tt} + 2\text{Im}(\xi_c \xi_t) E(x_c, x_t) \eta_{ct} \right]$$

$$\simeq 1.9 \cdot 10^4 B_K \left[ \text{Im} \xi_c^2 E(x_c) \eta_{cc} + \text{Im} \xi_t^2 E(x_t) \eta_{tt} + 2\text{Im}(\xi_c \xi_t) E(x_c, x_t) \eta_{ct} \right] \simeq$$

$$\simeq 7.8 \cdot 10^{-3} \eta B_K (1.3 - \rho)$$

(80)

where I have used the numerical values for the KM parameters listed above and $x_t \simeq 5$ corresponding to $m_t = 180$ GeV.

To reproduce the observed value of $|\epsilon_K|$ one needs

$$\eta \simeq \frac{0.3}{B_K} \frac{1}{1.3 - \rho}$$

(81)

For a given $B_K$ one thus obtains another $\rho - \eta$ constraint. Since $B_K$ is not precisely known [7] one has a fairly broad band in the $\rho - \eta$ plane rather than a line. Yet I find it quite remarkable and very non-trivial that Eq.(81) can be satisfied since

$$\frac{0.3}{B_K} \sim 0.3 \div 0.5$$

(82)

without stretching any of the parameters or bounds, in particular

$$\sqrt{\rho^2 + \eta^2} \sim 0.38 \pm 0.11$$

(83)

While this does of course not amount to a prediction, one should keep in mind for proper perspective that in the 1970’s and early 1980’s values like $|V(cb)| \sim 0.04$ and $|V(ub)| \sim 0.004$ would have seemed quite unnatural; claiming that the top quark

6The exact expression is $|\epsilon_K| = \frac{1}{\sqrt{2}} \left| \frac{\text{Im} M_{12}}{\Delta m_K} - \xi_0 \right|$ where $\xi_0$ denotes the phase of the $K^0 \rightarrow (\pi\pi)_{I=0}$ isospin zero amplitude; its contribution is numerically irrelevant.

7Some might argue that this is an understatement.
mass had to be 180 GeV would have been outright preposterous even in the 1980’s! Consider a scenario with $|V(cb)| \simeq 0.04$ and $|V(ub)| \simeq 0.003$, yet $m_t \simeq 40$ GeV; in the mid 80’s this would have appeared to be quite natural (and there had even been claims that top quarks with a mass of $40 \pm 10$ GeV had been discovered). In that case one would need

$$\eta \sim \frac{0.75}{B_K}$$

(84)
to reproduce $|\epsilon_K|$. Such a large value for $\eta$ would hardly be compatible with what we know about $|V(ub)|$.

Homework Problem # 2:

Eq.(80) suggests that a non-vanishing value for $\epsilon_K$ is generated from the box diagram with internal charm quarks only — $\text{Im}\xi^2 \frac{E(x_c)}{x_c} = -\eta A^2 \lambda^6 E(x_c) \neq 0$ — without top quarks. How does this match up with the statement that the intervention of three families is needed for a CP asymmetry to arise?

3.3.2 $\Delta S = 1$ Decays

At first one might think that no direct CP asymmetry can arise in $K \rightarrow \pi\pi$ decays since it requires the interplay of three quark families. Yet upon further reflection one realizes that a one-loop diagram produces the so-called Penguin operator which changes isospin by half a unit only, is local and contains a CP odd component since it involves virtual charm and top quarks. With direct CP violation thus being of order $\bar{h}$, i.e. a pure quantum effect, one suspects already at this point that it will be reduced in strength.

The quantity $\epsilon'$ is suppressed relative to $\epsilon_K$ due to two other reasons:

- The GIM factors are actually quite different for $\epsilon_K$ and $\epsilon'$; in the former case they are of the type $(m_t^2 - m_c^2)/M_W^2$, in the latter log$(m_t^2/m_c^2)$. Both of these expressions vanish for $m_t = m_c$, yet for the realistic case $m_t \gg m_c$ they behave very differently: $\epsilon_K$ is much more enhanced by the large top mass than $\epsilon'$. This means of course that $|\epsilon'/\epsilon_K|$ is a rather steeply decreasing function of $m_t$.

- There are actually two classes of Penguin operators contributing to $\epsilon'$, namely strong as well as electroweak Penguins. The latter become relevant since they are more enhanced than the former for very heavy top masses due to the coupling of the longitudinal virtual $Z$ boson (the re-incarnation of one of the original Higgs fields) to the internal top line. Yet electroweak and strong Penguins contribute with the opposite sign!

For some time it was thought that $B_K \simeq 0.3 \div 0.5$ was the best estimate. This would make satisfying Eq.(84) completely out of the question!
CPT invariance together with the measured $\pi\pi$ phase shifts tells us that the two complex quantities $\epsilon'$ and $\epsilon_K$ are almost completely real to each other; i.e., their ratio is practically real:

$$\frac{\epsilon'}{\epsilon_K} \simeq 2\omega \frac{\Phi(\Delta S = 1)}{\Phi(\Delta S = 2)}$$

where, as defined before,

$$\omega \equiv \frac{|A_2|}{|A_0|} \simeq 0.05, \; \Phi(\Delta S = 2) \equiv \arg \frac{M_{12}}{\Gamma_{12}}, \; \Phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0}$$

Eq. (85) makes two points obvious:

- Direct CP violation – $\epsilon' \neq 0$ – requires a relative phase between the isospin 0 and 2 amplitudes; i.e., $K \rightarrow (\pi\pi)_0$ and $K \rightarrow (\pi\pi)_2$ have to exhibit different CP properties.

- The observable ratio $\epsilon'/\epsilon_K$ is artifically reduced by the enhancement of the $\Delta I = 1/2$ amplitude, as expressed through $\omega$.

Several $\Delta S = 1$ transition operators contribute to $\epsilon'$ and their renormalization has to be treated quite carefully. Two recent detailed analyses yield [15, 16]

$$-2.1 \cdot 10^{-4} \leq \frac{\epsilon'}{\epsilon_K} \leq 13.3 \cdot 10^{-4}$$

$$\frac{\epsilon'}{\epsilon_K} = (4.6 \pm 3.0 \pm 0.4) \cdot 10^{-4}$$

These results are quite consistent with each other and show

- that the KM ansatz leads to a prediction typically in the range below $10^{-3}$,

- that the value could happen to be zero or even slightly negative and

- that large theoretical uncertainties persist due to cancellations among various contributions.

This last (unfortunate) point can be illustrated also by comparing these predictions with older ones made before top quarks were discovered and their mass measured; those old predictions [17] are very similar to Eqs. (87, 88), once the now known value of $m_t$ has been inserted.

Two new experiments running now – NA 48 at CERN and KTEV at FNAL – and one expected to start up soon – CLOE at DAΦNE – expect to measure $\epsilon'/\epsilon_K$ with a sensitivity of $\simeq \pm 2 \cdot 10^{-4}$. Concerning their future results one can distinguish four scenarios:

1. The ‘best’ scenario: $\epsilon'/\epsilon_K \geq 2 \cdot 10^{-3}$. One would then have established unequivocally direct CP violation of a strength that very probably reflects the intervention of new physics beyond the KM ansatz.
2. The ‘tantalizing’ scenario: $1 \cdot 10^{-3} \leq \epsilon'/\epsilon_K \leq 2 \cdot 10^{-3}$. It would be tempting to interpret this discovery of direct CP violation as a sign for new physics – yet one could not be sure!

3. The ‘conservative’ scenario: $\epsilon'/\epsilon_K \simeq \text{few} \cdot 10^{-4} > 0$. This strength of direct CP violation could easily be accommodated within the KM ansatz – yet no further constraint would materialize.

4. The ‘frustrating’ scenario: $\epsilon'/\epsilon_K \simeq 0$ within errors! No substantial conclusion could be drawn then concerning the presence or absence of direct CP violation, and the allowed KM parameter space would hardly shrink.

4 ‘Exotica’

In this section I will discuss important possible manifestations of CP and/or T violation that are exotic only in the sense that they are unobservably small with the KM ansatz.

4.1 $K_{3\mu}$ Decays

In the reaction

$$K^+ \rightarrow \mu^+ \nu \pi^0$$

one can search for a transverse polarisation of the emerging muons:

$$P^K_{\perp} (\mu) \equiv \langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi^0)) \rangle$$

where $\vec{s}$ and $\vec{p}$ denote spin and momentum, respectively. The quantity $P_{\perp}(\mu)$ constitutes a T-odd correlation:

$$\begin{cases} \vec{p} \Rightarrow -\vec{p} \\ \vec{s} \Rightarrow -\vec{s} \end{cases} \rightarrow P_{\perp}(\mu) \xrightarrow{T} -P_{\perp}(\mu)$$

Once a non-vanishing value has been observed for a parity-odd correlation one has unequivocally found a manifestation of parity violation. From $P_{\perp}^{K+}(\mu) \neq 0$ one can deduce that T is violated – yet the argument is more subtle as can be learnt from the following homework problem.

Homework Problem #3:

Consider

$$K_L \rightarrow \mu^+ \nu \pi^-$$

Does $P_{\perp}^{K}\,(\mu) \equiv \langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi^-)) \rangle \neq 0$ necessarily imply that T invariance does not hold in this reaction?
Data on $P_{\perp}^{K^+}(\mu)$ are still consistent with zero [18]:

$$P_{\perp}^{K^+}(\mu) = (-1.85 \pm 3.60) \cdot 10^{-3};$$

(93)

yet being published in 1981 they are ancient by the standards of our disciplin.

On general grounds one infers that

$$P_{\perp}^{K^+}(\mu) \propto \text{Im} \frac{f^*}{f^+}$$

(94)

holds where $f_- [f_+]$ denotes the chirality changing [conserving] decay amplitude. Since $f_-$ practically vanishes within the Standard Model, one obtains a fortiori $P_{\perp}^{K^+}(\mu)|_{KM} \simeq 0$.

Yet in the presence of charged Higgs fields one has $f_- \neq 0$. CPT implies that $P_{\perp}^{K^+}(\mu) \neq 0$ represents CP violation as well, and actually one of the direct variety. A rather model independant guestimate on how large such an effect could be is obtained from the present bound on $\epsilon'/\epsilon_K$:

$$P_{\perp}^{K^+}(\mu) \leq 20 \cdot (\epsilon'/\epsilon_K) \cdot \epsilon_K \leq 10^{-4}$$

(95)

where the factor 20 allows for the ‘accidental’ reduction of $\epsilon'/\epsilon_K$ by the $\Delta I = 1/2$ rule: $\omega \simeq 1/20$. This bound is a factor of 100 larger than what one could obtain within KM. It could actually be bigger still since there is a loophole in this generic argument: Higgs couplings to leptons could be strongly enhanced through a large ratio of vacuum expectation values $v_1$ relative to $v_3$, where $v_1$ controls the couplings to up-type quarks and $v_3$ to leptons. Then

$$P_{\perp}^{K^+}(\mu) |_{Higgs} \leq O(10^{-3})$$

(96)

becomes conceivable with the Higgs fields as heavy as 80 - 200 GeV [19]. Such Higgs exchanges would be quite insignificant for $K_L \rightarrow \pi\pi$!

Since $K_{\mu3}$ studies provide such a unique window onto Higgs dynamics, I find it mandatory to probe for $P_{\perp}(\mu) \neq 0$ in a most determined way. It is gratifying to note that an on-going KEK experiment will be sensitive to $P_{\perp}(\mu)$ down to the $10^{-3}$ level – yet I strongly feel one should not stop there, but push further down to the $10^{-4}$ level.

### 4.2 Electric Dipole Moments

Consider a system – such as an elementary particle or an atom – in a weak external electric field $\vec{E}$. The energy shift of this system due to the electric field can then be expressed through an expansion in powers of $\vec{E}$ [20]:

$$\Delta E = \vec{d} \cdot \vec{E} + d_{ij} E_i E_j + O(|\vec{E}|^3)$$

(97)
where summation over the indices $i, j$ is understood. The coefficient $\vec{d}$ of the term linear in $\vec{E}$ is called electric dipole moment or sometimes permanent electric dipole moment (hereafter referred to as EDM) whereas that of the quadratic term is often named an induced dipole moment.

For an elementary object one has

$$\vec{d} = d\vec{j}$$

(98)

where $\vec{j}$ denotes its total angular momentum since that is the only available vector. Under time reversal one finds

$$\vec{j} \xrightarrow{T} -\vec{j}$$

$$\vec{E} \xrightarrow{T} \vec{E}.$$ (99)

Therefore

$$T \text{ invariance } \rightsquigarrow d = 0;$$

(100)
i.e., such an electric dipole moment has to vanish, unless T is violated (and likewise for parity).

The EDM is at times confused with an induced electric dipole moment objects can possess due to their internal structure. To illustrate that consider an atom with two nearly degenerate states of opposite parity:

$$|\pm\rangle = \pm|\pm\rangle, \quad H|\pm\rangle = E_{\pm}|\pm\rangle, \quad E_+ < E_-, \quad \frac{E_- - E_+}{E_+} \ll 1$$

(101)

Placed in a constant external electric field $\vec{E}$ the states $|\pm\rangle$ will mix to produce new energy eigenstates; those can be found by diagonalising the matrix of the Hamilton operator:

$$H = \begin{pmatrix} E_+ & \Delta \\ \Delta & E_- \end{pmatrix}$$

(102)

where $\Delta = \vec{d}_{\text{ind}} \cdot \vec{E}$ with $\vec{d}_{\text{ind}}$ being the transition matrix element between the $|+\rangle$ and $|−\rangle$ states induced by the electric field. The two new energy eigenvalues are

$$E_{1,2} = \frac{1}{2}(E_+ + E_-) \pm \sqrt{\frac{1}{4}(E_+ - E_-)^2 + \Delta^2}$$

(103)

For $E_+ \simeq E_-$ one has

$$E_{1,2} \simeq \frac{1}{2}(E_+ + E_-) \pm |\Delta|;$$

(104)
i.e., the energy shift appears to be linear in $\vec{E}$:

$$\Delta E = E_2 - E_1 = 2|\vec{d}_{\text{ind}} \cdot \vec{E}|$$

(105)

Yet with $\vec{E}$ being sufficiently small one arrives at $4(\vec{d}_{\text{ind}} \cdot \vec{E})^2 \ll (E_+ - E_-)^2$ and therefore

$$E_1 \approx E_- + \frac{(\vec{d}_{\text{ind}} \cdot \vec{E})^2}{E_- - E_+}, \quad E_2 \approx E_+ - \frac{(\vec{d}_{\text{ind}} \cdot \vec{E})^2}{E_- - E_+};$$

(106)
i.e., the induced energy shift is *quadratic* in $\vec{E}$ rather than linear and therefore does *not* imply T violation! The distinction between an EDM and an induced electric dipole moment is somewhat subtle – yet it can be established in an unequivocal way by probing for a linear Stark effect with weak electric fields. A more careful look at Eq.(105) already indicates that. For the energy shift stated there does not change under $\vec{E} \Rightarrow -\vec{E}$ as it should for an EDM which also violates parity!

The data for neutrons read:

$$d_n = \begin{cases} 
(-3 \pm 5) \cdot 10^{-26} \text{ ecm} & \text{ILL} \\
(2.6 \pm 4 \pm 1.6) \cdot 10^{-26} \text{ ecm} & \text{LNPI}
\end{cases}$$

(107)

These numbers and the experiments leading to them are very impressive:

- One uses neutrons emanating from a reactor and subsequently cooled down to a temperature of order $10^{-7}$ eV. This is comparable to the kinetic energy a neutron gains when dropping 1 m in the earth’s gravitational field.

- Extrapolating the ratio between the neutron’s radius – $r_N \sim 10^{-13}$ cm – with its EDM of no more than $10^{-25}$ ecm to the earth’s case, one would say that it corresponds to a situation where one has searched for a displacement in the earth’s mass distribution of order $10^{-12} \cdot r_{\text{earth}} \sim 10^{-3}$ cm = 10 microns!

A truly dramatic increase in sensitivity for the *electron’s* EDM has been achieved over the last few years:

$$d_e = (-0.3 \pm 0.8) \cdot 10^{-26} \text{ e cm}$$

(108)

This quantity is searched for through measuring electric dipole moments of *atoms*. At first this would seem to be a losing proposition theoretically: for according to Schiff’s theorem an atom when placed inside an external electric field gets deformed in such a way that the electron’s EDM is completely shielded; i.e., $d_{\text{atom}} = 0$. This theorem holds true in the nonrelativistic limit, yet is vitiated by relativistic effects. Not surprisingly the latter are particularly large for heavy atoms; one would then expect the electron’s EDM to be only partially shielded: $d_{\text{atom}} = S \cdot d_e$ with $S < 1$. Yet amazingly – and highly welcome of course – the electron’s EDM can actually get magnified by two to three orders of magnitude in the atom’s electric dipole moment; for Caesium one has $^{[20]}$

$$d_{\text{Cs}} \simeq 100 \cdot d_e$$

(109)

This enhancement factor is the theoretical reason behind the greatly improved sensitivity for $d_e$ as expressed through Eq.(108); the other one is experimental, namely the great strides made by laser technology applied to atomic physics.

The quality of the number in Eq.(108) can be illustrated through a comparison with the electron’s magnetic moment. The electromagnetic form factor $\Gamma_\mu(q)$ of a particle like the electron evaluated at momentum transfer $q$ contains two tensor terms:

$$d_{\text{atom}} = \frac{1}{2m_e} \sigma_{\mu\nu} q^\nu \left[ i F_2(q^2) + F_3(q^2) \gamma_5 \right] + \ldots$$

(110)
In the nonrelativistic limit one finds for the EDM:

\[ d_e = -\frac{1}{2m_e} F_3(0) \]  

(111)

On the other hand one has

\[ \frac{1}{2} (g - 2) = \frac{1}{e} F_2(0) \]  

(112)

The precision with which \( g - 2 \) is known for the electron – \( \delta [(g - 2)/2] \simeq 10^{-11} \) – (and which represents one of the great success stories of field theory) corresponds to an uncertainty in the electron’s magnetic moment

\[ \delta \left[ \frac{1}{2m_e} F_2(0) \right] \simeq 2 \cdot 10^{-22} e cm \]  

(113)

that is several orders of magnitude larger than the bound on its EDM!

Since the EDM is, as already indicated above, described by a dimension-five operator in the Lagrangian

\[ \mathcal{L}_{EDM} = -\frac{i}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu} \]  

(114)

with \( F^{\mu\nu} \) denoting the electromagnetic field strength tensor, one can calculate \( d \) within a given theory of CP violation as a finite quantity. Within the KM ansatz one finds that the neutron’s EDM is zero for all practical purposes [9]

\[ d_N|_{KM} < 10^{-30} e cm \]  

(115)

and likewise for \( d_e \). Yet again that is due to very specific features of the KM mechanism and the chirality structure of the Standard Model. In alternative models – where CP violation enters through right-handed currents or a non-minimal Higgs sector (with or without involving SUSY) – one finds

\[ d_N|_{New\ Physics} \sim 10^{-27} - 10^{-28} e cm \]  

(116)

as reasonable benchmark figures.

5 The Strong CP Problem

5.1 The Problem

It is often listed among the attractive features of QCD that it ‘naturally’ conserves baryon number, flavour, parity and CP. Actually the last two points are not quite

\footnote{I ignore here the Strong CP Problem, which is discussed in the next section.}
true, which had been overlooked for some time although it can be seen in different ways [21]. Consider

$$\mathcal{L}_{\text{eff}} = \sum_q \bar{q} (i \not{D} - m_q) q - \frac{1}{4} G \cdot G + \frac{\theta g_s^2}{32\pi^2} G \cdot \tilde{G}$$  \quad (117)$$

where $D_\mu$, $G$ and $\tilde{G}$ denote the covariant derivative, the gluon field strength tensor and its dual, respectively:

$$D_\mu = \partial_\mu + ig S A^i_\mu t^i$$ \quad (118)

$$G_{\mu\nu} \equiv G^a_{\mu\nu} t^a, \quad G^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g S t_{ijk} A^j_\mu A^k_\nu, \quad \tilde{G}_{\mu\nu} \equiv \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}$$ \quad (119)

$$G \cdot G \equiv G_{\mu\nu} G^{\mu\nu}, \quad G \cdot \tilde{G} \equiv G_{\mu\nu} \tilde{G}^{\mu\nu}$$ \quad (120)

In adding the operator $G \cdot \tilde{G}$ to the usual QCD Lagrangian we have followed a general tenet of quantum field theory: any Lorentz scalar gauge invariant operator of dimension four has to be included in the Lagrangian unless there is a specific reason – in particular a symmetry requirement – that enforces its absence. For otherwise radiative corrections will resurrect such an operator with a (logarithmically) divergent coefficient!

Such an operator exists also in an abelian gauge theory like QED where the field strength tensor takes on a simpler form: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. One then finds

$$F_{\mu\nu} \tilde{F}_{\mu\nu} = \partial_\mu K^{\text{QED}}_\mu, \quad K^{\text{QED}}_\mu = 2 \epsilon_{\mu\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma;$$ \quad (121)

i.e., this extra term can be reduced to a total derivative which is usually dropped without further ado as physically irrelevant.

For nonabelian gauge theories one obtains

$$G_{\mu\nu} \tilde{G}_{\mu\nu} = \partial_\mu K_\mu, \quad K_\mu = 2 \epsilon_{\mu\alpha\beta\gamma} \left( A_\alpha \partial_\beta A_\gamma + \frac{2}{3} i g S A_\alpha A_\beta A_\gamma \right).$$ \quad (122)

The extra term is still a total derivative and our first reaction would be to just drop it for that very reason. Alas this time we would be wrong in doing so! Let us recapitulate the usual argument. If a term in the Lagrangian can be expressed as a total divergence like $\partial_\mu K_\mu$ than its contribution to the action which determines the dynamics can be expressed as the integral of the current $K$ over a surface at infinity. Yet with physical observables having to vanish rapidly at infinity to yield finite values for energy etc., such integrals are expected to yield zero. The field strength indeed goes to zero at infinity – but not necessarily the gauge potentials $A_\mu$! The field configuration at large space-time distances has to approach that of a ground state for which $G_{\mu\nu} = 0$ holds. Yet the latter property does not suffice to define the ground state uniquely: it still allows ground states to differ by pure gauge configurations which obviously satisfy $G_{\mu\nu} = 0$. This is also true for abelian gauge theories, yet remains without dynamical significance. The structure of nonabelian gauge theories on the other hand is much more complex and they possess
an infinity of *inequivalent* states defined by $G_{\mu\nu} = 0$. Their differences can be expressed through topological characteristics of their gauge field configurations. To be more precise: these states can be characterised by an integer, the so-called *winding number*; accordingly they are denoted by $|n\rangle$. They are *not* gauge invariant. Not surprisingly then transitions between states $|n_1\rangle$ and $|n_2\rangle$ with $n_1 \neq n_2$ can take place. The net change $\Delta n$ in winding number between $t = -\infty$ and $t = \infty$ is described by their $K$ charge, the space integral of the zeroth component of the current $K_{\mu}$ defined in Eq. (122). A gauge invariant state is constructed as a linear superposition of the states $|n\rangle$ labeled by a real parameter $\theta$

$$|\theta\rangle = \sum_n e^{-i\theta n}|n\rangle.$$ (123)

One easily shows that for a gauge invariant operator $O_{\text{g.inv.}}$, $\langle \theta | O_{\text{g.inv.}} | \theta' \rangle = 0$ holds if $\theta \neq \theta'$. We thus see that the state space of QCD consists of *disjoint* sectors built up from ground states $|\theta\rangle$.

For vacuum-to-vacuum transitions one then finds

$$\langle \theta_+ | \theta_- \rangle = \sum_{n,m} e^{i\theta (m-n)} \langle m+|n_- \rangle = \sum_{\Delta n} e^{i\Delta n \theta} \sum_n \langle (n+\Delta n)+|n_- \rangle , \quad \Delta n = m-n ,$$ (124)

which can be reformulated in the path integral formalism

$$\langle \theta_+ | \theta_- \rangle = \sum_{\Delta n} \sum_{\text{fields}} e^{i\int d^4x L_{\text{eff}} \delta(\Delta n - \frac{g_5^2}{16\pi^2} \int d^4x G \cdot \tilde{G})};$$ (125)

i.e., the $G \cdot \tilde{G}$ term in Eq. (117) acts as a Lagrangian multiplier implementing the change in winding number $\Delta n$.

This is easily generalized to any transition amplitude, and the situation can be summarized as follows:

- There is an infinity set of *inequivalent* groundstates in QCD labeled by a real parameter $\theta$.
- The dependance of observables on $\theta$ can be determined by employing the *effective* Lagrangian of Eq. (117).

The problem with this additional term in the Lagrangian is that $G \cdot \tilde{G}$ – in contrast to $G \cdot G$ – violates both parity and time reversal invariance! This is best seen by expressing $G_{\mu\nu}$ and its dual through the colour electric and colour magnetic fields $\vec{E}$ and $\vec{B}$, respectively:

$$G \cdot G \propto |\vec{E}|^2 + |\vec{B}|^2 \quad \xrightarrow{\text{PT}} \quad |\vec{E}|^2 + |\vec{B}|^2$$ (126)

$$G \cdot \tilde{G} \propto 2\vec{E} \cdot \vec{B} \quad \xrightarrow{\text{PT}} \quad -2\vec{E} \cdot \vec{B}$$ (127)
since
\[ \vec{E} \xrightarrow{P} -\vec{E} \quad , \quad \vec{B} \xrightarrow{P} \vec{B} \] \quad (128)
\[ \vec{E} \xrightarrow{T} \vec{E} \quad , \quad \vec{B} \xrightarrow{T} -\vec{B} \] \quad (129)
i.e., for \( \theta \neq 0 \) neither parity nor time reversal invariance are fully conserved by QCD. This is the Strong CP Problem.

The problem which resides in gluodynamics spreads into the quark sector through the ‘chiral’ anomaly [23]:
\[ \partial_{\mu} J_{\mu}^5 = \partial_{\mu} \sum_{q} \bar{q} L \gamma_{\mu} q L = \frac{g_s^2}{32\pi^2} \vec{G} \cdot \vec{G} \neq 0 \] \quad (130)
i.e., the axial current of massless quarks, which is conserved \textit{classically}, ceases to be so on the quantum level [25]. This chiral anomaly is also called the ‘triangle’ anomaly because it is produced by a diagram with a triangular fermion loop.

There are two further aspects to the anomaly expressed in Eq. (130):

- The anomaly actually solves one long standing puzzle of \textit{strong} dynamics, the ‘U(1) Problem’: In the limit of massless u and d quarks QCD would appear to have a \textit{global} \( U(2)_L \times U(2)_R \) invariance. While the vectorial part \( U(2)_{L+R} \) is a manifest symmetry, the axial part \( U(2)_{L-R} \) is spontaneously realized leading to the emergence of four Goldstone bosons. In the presence of quark masses those bosons acquire a mass as well. The pions readily play the part, but the \( \eta \) meson does not [27]. Yet from the anomaly one infers that due to quantum corrections the axial \( U(1)_{L-R} \) was never there in the first place even for massless quarks: therefore only three Goldstone bosons are predicted, the pions!

- On the other hand the anomaly aggravates the Strong CP Problem when electroweak dynamics are included. For the quarks acquire their masses from the Higgs mechanism driving the phase transition
\[ SU(2)_L \times U(1) \sim U(1)_{QED} \] \quad (131)
The resulting quark mass \( \mathcal{M}_{\text{quark}} \) matrix cannot be expected to be diagonal and Hermitian ab initio; it will have to be diagonalized through chiral rotations of the quark fields:
\[ \mathcal{M}_{\text{quark}}^{\text{diag}} = U_R^\dagger \mathcal{M}_{\text{quark}} U_L \] \quad (132)
Exactly because of the axial anomaly this induces an additional term in the Lagrangian of the Standard Model:
\[ \mathcal{L}_{\text{SM,eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{SU(2)_L \times U(1)} + \frac{\bar{\theta} g_s^2}{32\pi^2} \vec{G} \cdot \vec{G} \] \quad (133)
\[^{10}\text{This is why it is called an anomaly.}\]
\[^{11}\text{An analogous discussion can be given with } s \text{ quarks included. The spontaneous breaking of the global } U(3)_{L-R} \text{ symmetry leads to the existence of nine Goldstone bosons, yet the } \eta' \text{ meson is far too heavy for this role.}\]
where
\[ \bar{\theta} = \theta_{QCD} + \Delta\theta_{EW}, \quad \Delta\theta_{EW} = \arg \det U_R^t U_L \]  
(134)

Since the electroweak sector has to contain sources of CP violation other than \( G \cdot \tilde{G} \), the second term in \( \bar{\theta} \) has no a priori reason to vanish.

### 5.2 The Neutron Electric Dipole Moment

Since the gluonic operator \( G \cdot \tilde{G} \) does not change flavour one suspects right away that its most noticeable impact would be to generate an electric dipole moment (EDM) for neutrons. This is indeed the case, yet making this connection more concrete requires a more sophisticated argument. In the context of the Strong CP Problem one views the neutron EDM \( d_N \) – as due to the photon coupling to a virtual proton or pion in a fluctuation of the neutron: \( n \rightarrow p^* \pi^* \rightarrow n \). Of the two effective pion nucleon couplings in this one-loop process one is produced by ordinary strong forces and conserves \( P, T \) and CP; the other one is induced by \( G \cdot \tilde{G} \). The first estimate was obtained by Baluni \[24\] in a nice paper using bag model computations of the transition amplitudes between the neutron and its excitations: \( d_N \simeq 2.7 \cdot 10^{-16} \bar{\theta} \, e\text{cm} \). In \[23\] chiral perturbation theory instead was employed: \( d_N \simeq 5.2 \cdot 10^{-16} \bar{\theta} \, e\text{cm} \). More recent estimates yield values in roughly the same range: \( d_N \simeq (4 \cdot 10^{-17} \div 2 \cdot 10^{-15})\bar{\theta} \, e\text{cm} \) [21]. Hence
\[ d_N \sim \mathcal{O}(10^{-16}\bar{\theta}) \, e\text{cm} \]  
(135)

and one infers
\[ d_N \leq 1.1 \cdot 10^{-25} \, e\text{cm} \quad 95\% \, C.L. \quad \sim \quad \bar{\theta} < 10^{-9\pm1} \]  
(136)

Although \( \theta_{QCD} \) is a QCD parameter it might not necessarily be of order unity; nevertheless its truly tiny size begs for an explanation. The only kind of explanation that is usually accepted as ‘natural’ in our community is one based on symmetry. Yet before we start speculating too wildly, we want to see whether there are no more mundane explanations.

### 5.3 Are There Escape Hatches?

One could argue that the Strong CP Problem is fictitious using one of two lines of reasoning:

- Being the coefficient of a dimension-four operator \( \bar{\theta} \) can in general \[3\] be renormalized to any value, including zero. This is technically correct; however \( \theta \leq \mathcal{O}(10^{-9}) \) is viewed as highly ‘unnatural’:

\[12\] Exceptions will be mentioned below.
– A priori there is no reason why $\theta_{QCD}$ and $\Delta \theta_{EW}$ should practically vanish.

– Even if $\theta_{QCD} = 0 = \Delta \theta_{EW}$ were set by fiat quantum corrections to $\Delta \theta_{EW}$ are typically much larger than $10^{-9}$ and ultimately actually infinite.

– To expect that $\theta_{QCD}$ and $\Delta \theta_{EW}$ cancel as to render $\bar{\theta}$ sufficiently tiny would require fine tuning of a kind which would have to strike even a skeptic as unnatural. For $\theta_{QCD}$ reflects dynamics of the strong sector and $\Delta \theta_{EW}$ that of the electroweak sector.

– In models where CP symmetry is realized in a spontaneous fashion one has $\theta_{tree} = 0$ and

$$\bar{\theta} = \delta \theta_{ren}$$

(137)

turns out to be a finite and calculable quantity that has no apparent reason to be smaller than, say, $10^{-4}$.

– A more respectable way out is provided by the following observation: if one of the quark masses vanishes the resulting chiral invariance would remove any $\bar{\theta}$ dependance of observables by rotating it – through the anomaly – into the quark mass matrix with its zero eigenvalue. However most authors argue quite forcefully that neither the up quark nor a fortiori the down quark mass can vanish [21]:

$$m_d(1 \text{ GeV}) > m_u(1 \text{ GeV}) \simeq 5 \text{ MeV}$$

(138)

where the notation shows that one has to use the running mass evaluated at a scale of 1 GeV.

5.4 Peccei-Quinn Symmetry

As just argued $\bar{\theta} \leq \mathcal{O}(10^{-9})$ could hardly come about accidentally; an organizing principle had to arrange various contributions and corrections in such a way as to render the required cancellations. There is the general philosophy that such a principle has to come from an underlying symmetry. We have already sketched such an approach: a global chiral invariance allows to rotate the dependance on $\bar{\theta}$ away; we failed however in our endeavour because this symmetry is broken by $m_q \neq 0$. Is it possible to invoke some other variant of chiral symmetry for this purpose even if it is spontaneously broken? One particularly intriguing ansatz is to re-interpret a physical quantity that is conventionally taken to be a constant as a dynamical degree of freedom that relaxes itself to a certain (desired) value in response to forces acting upon it. One early example is provided by the original Kaluza-Klein theory [26] invoking a six-dimensional ‘space’-time manifold: two compactify dynamically and thus lead to the quantization of electric and magnetic charge.

Something similar has been suggested by Peccei and Quinn [27]. They augmented the Standard Model by a global $U(1)_{PQ}$ symmetry – now referred to as the Peccei-Quinn symmetry – that is axial. The spontaneous breaking of this symmetry gives rise to a Goldstone boson – named the axion – with zero mass on the Lagrangian
level. Goldstone couplings to other fields usually have to be derivative, i.e. involve $\partial_{\mu}a(x)$, but not $a(x)$ directly. Since $U(1)_{PQ}$ is axial it exhibits a triangle anomaly again; this is implemented in the effective Lagrangian by having a term linear in the axion field coupled to $G \cdot \tilde{G}$:

$$L_{\text{eff}} = L_{\text{SM}} + \frac{g_S^2}{32\pi^2}G \cdot \tilde{G} + \frac{g_S^2}{32\pi^2} \frac{\xi}{\Lambda_{PQ}} aG \cdot \tilde{G} - \frac{1}{2} \partial_{\mu}a\partial_{\mu}a + L_{\text{int}}(\partial_{\mu}a, \psi)$$ (139)

The size of the parameters $\Lambda_{PQ}$ and $\xi$ and the form of $L_{\text{int}}(\partial_{\mu}a, \psi)$ describing the (purely derivative) coupling of the axion field to other fields $\psi$ depend on how the Peccei-Quinn symmetry is specifically realized.

The term $aG \cdot \tilde{G}$ represents an explicit breaking of the $U(1)_{PQ}$ symmetry. This gives rise to an axion mass. Yet the primary role of $aG \cdot \tilde{G}$ is to make the $G \cdot \tilde{G}$ term disappear from the effective Lagrangian. $U(1)_{PQ}$ invariance being realized spontaneously means that $a(x)$ acquires a vacuum expectation value ($=\text{VEV}$) $\langle a \rangle$; the physical axion excitations are then described by the shifted field $a_{\text{phys}}(x) = a(x) - \langle a \rangle$ and one rewrites Eq.(139) as follows:

$$L_{\text{eff}} = L_{\text{SM}} + \frac{g_S^2}{32\pi^2} \theta G \cdot \tilde{G} - \frac{1}{2} \partial_{\mu}a_{\text{phys}}\partial_{\mu}a_{\text{phys}} + \frac{g_S^2}{32\pi^2} \frac{\xi}{\Lambda_{PQ}} a_{\text{phys}}G \cdot \tilde{G} + L_{\text{int}}(\partial_{\mu}a_{\text{phys}}, \psi)$$ (140)

where now

$$\theta = \theta_{\text{QCD}} + \text{arg det}U_R^1U_L - \langle a \rangle / f_a, \quad f_a = \frac{\Lambda_{PQ}}{\xi};$$ (141)

i.e., the size of the coefficient of the $G \cdot \tilde{G}$ operator is determined by the VEV of the axion field.

The term $aG \cdot \tilde{G}$ generates an effective potential for the axion field; its minimum defines the ground state:

$$\langle \frac{\partial V_{\text{eff}}}{\partial a} \rangle \equiv - \frac{\xi}{\Lambda_{PQ}^2} \frac{g_S^2}{32\pi^2} \langle G \cdot \tilde{G} \rangle = 0$$ (142)

That means that the term $aG \cdot \tilde{G}$ – the reincarnation of the anomaly – singles out one of the previously degenerate $\theta$ states as the true ground state. This is not surprising since $aG \cdot \tilde{G}$ is not invariant under $U(1)_{PQ}$. It is a pleasant surprise, though, that for this lowest energy state $G \cdot \tilde{G}$ settles into a vanishing expectation value thus banning the Strong CP Problem dynamically.

### 5.5 The Dawn of Axions – and Their Dusk?

Rather than ending here the story contains another twist or two. The breaking of $U(1)_{PQ}$ gives rise to a Nambu-Goldstone boson. Actually the axion is, as already mentioned, a pseudo-Nambu-Goldstone boson; for it acquires a mass due to the anomaly:

$$m_a^2 \sim \frac{G \cdot \tilde{G}}{\Lambda_{PQ}^2} \sim \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^4}{\Lambda_{PQ}^4}\right)$$ (143)
Since one expects on general grounds $\Lambda_{PQ} \gg \Lambda_{QCD}$ one is dealing with a very light boson. The question is how light would the axion be.

The electroweak scale $v_{EW} = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \simeq 250$ GeV provides the discriminator for two scenarios:

- $\Lambda_{PQ} \sim v_{EW}$ 
  
  In that case axions can or even should be seen in accelerator based experiments. Such scenarios are referred to as visible axions.

- $\Lambda_{PQ} \gg v_{EW}$ 
  
  Such axions could not be found in accelerator based experiments; therefore they are called invisible scenarios. Yet that does not mean that they necessarily escape detection! For they could be of great significance for the formation of stars, whole galaxies and even the universe.

### 5.5.1 Visible Axions

The simplest scenario involves two $SU(2)_L$ doublet Higgs fields that possess opposite hypercharge $1^3$. They also carry a $U(1)$ charge in addition to the hypercharge; this second (and global) $U(1)$ is identified with the PQ symmetry, and the axion is its pseudo-Nambu-Goldstone boson. The anomaly induces a mass for the axion:

$$m_a \simeq \frac{m_\pi F_\pi}{v} N_{fam} \left( x + \frac{1}{x} \right) \frac{\sqrt{m_u m_d}}{(m_u + m_d)} \simeq 25 N_{fam} \left( x + \frac{1}{x} \right) \text{KeV}$$

where $N_{fam}$ denotes the number of families. Such an axion is almost certainly much lighter than the pion. Depending on the axion’s mass two cases have to be decided:

- If $m_a > 2m_e$, the axion decays very rapidly into electrons and positrons:
  $$\tau(a \rightarrow e^+e^-) \simeq 4 \cdot 10^{-9} \left( \frac{1 \text{ MeV}}{m_a} \right) \frac{x^2 \text{ or } 1/x^2}{\sqrt{1 - \frac{4m_e^2}{m_a^2}}} \text{ sec}$$

- If on the other hand $m_a < 2m_e$ then the axion decays fairly slowly into two photons:
  $$\tau(a \rightarrow \gamma\gamma) \sim \mathcal{O} \left( \frac{100 \text{ KeV}}{m_a} \right) \text{ sec}$$

I have presented here a very rough sketch of scenarios with visible axions since we can confidently declare that they have been ruled out experimentally. They have been looked for in beam dump experiments – without success. Yet the more telling blows have come from searches in rare decays:

---

13 In the Standard Model the Higgs doublet and its charge conjugate fill this role.
• For long-lived axions \( m_a < 2m_e \) one expects a dominating contribution to \( K^+ \to \pi^+ + \text{nothing} \) from

\[
K^+ \to \pi^+ + a
\]  

(149)

with the axion decaying well outside the detector. For the two-body kinematics of Eq.(149) one has very tight bounds from published data [28]:

\[
BR(K^+ \to \pi^+X^0) < 5.2 \cdot 10^{-10} \quad 90\% \text{ C.L.}
\]  

(150)

for \( X^0 \) being a practically massless and noninteracting particle. Theoretically one would expect:

\[
BR(K^+ \to \pi^+a) \bigg|_{\text{theor}} \sim 3 \cdot 10^{-5} \cdot (x + 1/x)^{-2}
\]  

(151)

Although Eq.(151) does not represent a precise prediction the discrepancy between expectation and observation is conclusive.

• One arrives at the same conclusion that long-lived visible axions do not exist from the absence of quarkonia decay into them: neither \( J/\psi \to a\gamma \) nor \( \Upsilon \to a\gamma \) has been seen.

• The analysis is a bit more involved for short-lived axions \( m_a > 2m_e \). Yet again their absence has been established through a combination of experiments. Unsuccessful searches for

\[
\pi^+ \to ae^+\nu
\]  

(152)

figure prominently in this endeavour. Likewise the absence of axion driven nuclear de-excitation has been established on a level that appears to be conclusive [29].

5.5.2 Invisible Axions

Invisible axion scenarios involve a complex scalar field \( \sigma \) that (i) is an \( SU(2)_L \) singlet, (ii) carries a PQ charge and (iii) possesses a huge VEV \( \sim \Lambda_{PQ} \gg v_{\text{ew}} \).

The reasons underlying those requirements should be obvious; they can be realized in two distinct (sub-)scenarios:

1. Only presumably very heavy new quarks carry a PQ charge. This situation is referred to as KSVZ axion [30]. The minimal version can do with a single \( SU(2)_L \) Higgs doublet.

2. Also the known quarks and leptons carry a PQ charge. Two \( SU(2)_L \) Higgs doublets are then required in addition to \( \sigma \). The fermions do not couple directly to \( \sigma \), yet become sensitive to PQ breaking through the Higgs potential. This is referred to as DFSZ axion [31].
From current algebra one infers for the axion mass in either case

$$m_a \simeq 0.6 \text{ eV} \cdot \frac{10^7 \text{ GeV}}{f_a}$$

(153)

The most relevant coupling of such axions is to two photons

$$\mathcal{L}(a \rightarrow \gamma\gamma) = -\tilde{g}_{a\gamma\gamma} \frac{\alpha a(x)}{\pi f_a} \vec{E} \cdot \vec{B},$$

(154)

where $\tilde{g}_{a\gamma\gamma}$ is a model dependant coefficient of order unity.

Axions with such tiny masses have lifetimes easily in excess of the age of the universe. Also their couplings to other fields are so minute that they would not betray their presence – hence their name invisible axions – under ordinary circumstances! Yet in astrophysics and cosmology more favourable extra-ordinary conditions can arise.

Through their couplings to electrons axions would provide a cooling mechanism to stellar evolution. Not surprisingly their greatest impact occurs for the lifetimes of red giants and the supernovae like SN 1987a. The actual bounds depend on the model – whether it is a KSVZ or DFSZ axion – but relatively mildly only. Altogether astrophysics tells us that if axions exist one has

$$m_a < 3 \cdot 10^{-3} \text{ eV}$$

(155)

Cosmology on the other hand provides us with a lower bound through a very intriguing line of reasoning. At temperatures $T$ above $\Lambda_{QCD}$ the axion is massless and all values of $\langle a(x) \rangle$ are equally likely. For $T \sim 1 \text{ GeV}$ the anomaly induced potential turns on driving $\langle a(x) \rangle$ to a value as to yield $\bar{\theta} = 0$ at the new potential minimum. The energy stored previously as latent heat is then released into axions oscillating around its new VEV. Precisely because of the invisible axion’s couplings are so immensely suppressed the energy cannot be dissipated into other degrees of freedom. We are then dealing with a fluid of axions. Their typical momenta is the inverse of their correlation length which in turn cannot exceed their horizon; one finds

$$p_a \sim \left( 10^{-6} \text{ sec} \right)^{-1} \sim 10^{-9} \text{ eV}$$

(156)

at $T \simeq 1 \text{ GeV}$; i.e., the axions despite their minute mass form a very cold fluid and actually represent a candidate for cold dark matter. Their contribution to the density of the universe relative to its critical value is

$$\Omega_a = \left( \frac{0.6 \cdot 10^{-5} \text{ eV}}{m_a} \right)^2 \cdot \left( \frac{200 \text{ MeV}}{\Lambda_{QCD}} \right)^2 \cdot \left( \frac{75 \text{ km/sec} \cdot \text{Mpc}}{H_0} \right)^2;$$

(157)

$H_0$ is the present Hubble expansion rate. For axions not to overclose the universe one thus has to require:

$$m_a \geq 10^{-6} \text{ eV}$$

(158)
or

\[ \Lambda_{PQ} \leq 10^{12} \text{ GeV} \quad (159) \]

This means also that we might be existing in a bath of cold axions still making up a significant fraction of the matter of the universe.

Ingenious suggestions have been made to search for such cosmic background axions. The main handle one has on them is their coupling to two photons. They can be detected by stimulating the conversion

\[ \text{axion} \rightarrow \vec{B} \rightarrow \text{photon} \quad (160) \]

in a strong magnetic field \( \vec{B} \): the second photon which is virtual in this process effects the interaction with the inhomogeneous magnetic field in the cavity. The available microwave technology allows an impressive experimental sensitivity. No signal has been found yet, but the search continues and soon should reach a level where one has a good chance to see a signal \[33\].

5.6 The Pundits’ Judgement

The story of the Strong CP Problem is a particularly intriguing one. We – like most though not all of our community – find the theoretical arguments persuasive that there is a problem that has to be resolved. The inquiry has been based on an impressive arsenal of theoretical reasoning and has inspired fascinating experimental undertakings.

Like many modern novels the problem – if its is indeed one – has not found any resolution. On the other hand it has the potential to lead the charge towards a new paradigm in high energy physics.

6 Summary on the CP Phenomenology with Light Degrees of Freedom

To summarize our discussion up to this point:

- The following data represent the most sensitive probes:

\[ \text{BR}(K_L \rightarrow \pi^+\pi^-) = 2.3 \cdot 10^{-3} \neq 0 \quad (161) \]

\[ \frac{\text{BR}(K_L \rightarrow l^+\nu\pi^-)}{\text{BR}(K_L \rightarrow l^-\nu\pi^+)} \approx 1.006 \neq 1 \quad (162) \]

\[ \text{Re} \left( \frac{\epsilon'}{\epsilon_K} \right) = \left\{ \begin{array}{l} (2.3 \pm 0.7) \cdot 10^{-3} \quad N.A.31 \\ (0.6 \pm 0.58 \pm 0.32 \pm 0.18) \cdot 10^{-3} \quad E.731 \end{array} \right. \quad (163) \]

\[ \text{Pol}^{K+}_{\perp}(\mu) = (-1.85 \pm 3.60) \cdot 10^{-3} \quad (164) \]
\[ d_N < 12 \cdot 10^{-26} \text{ e cm} \]  
\[ d_{Tl} = (1.6 \pm 5.0) \cdot 10^{-24} \text{ e cm} \quad (165) \]
\[ d_e = (-2.7 \pm 8.3) \cdot 10^{-27} \text{ e cm} \]  
\[ (166) \]

- An impressive amount of experimental ingenuity, acumen and commitment went into producing this list. We know that CP violation unequivocally exists in nature; it can be characterized by a single non-vanishing quantity:

\[ \text{Im} M_{12} \simeq 1.1 \cdot 10^{-8} \text{ eV} \]  
\[ \text{Im} M_{12}^\text{exp} \neq 0 \]  
\[ (167) \]

- The ‘Superweak Model’ states that there just happens to exist a \( \Delta S = 2 \) interaction that is fundamental or effective – whatever the case may be – generating \( \text{Im} M_{12} = \text{Im} M_{12}^\text{exp} \) while \( \epsilon' = 0 \). It provides merely a classification for possible dynamical implementations rather than such a dynamical implementation itself.

- The KM ansatz allows us to incorporate CP violation into the Standard Model. Yet it does not regale us with an understanding. Instead it relates the origins of CP violation to central mysteries of the Standard Model: Why are there families? Why are there three of those? What is underlying the observed pattern in the fermion masses?

- Still the KM ansatz succeeds in accommodating the data in an unforced way: \( \epsilon_K \) emerges to be naturally small, \( \epsilon' \) naturally tiny (once the huge top mass is built in), the EDM’s for neutrons [electrons] naturally (tiny) \(^2 \) [(tiny)\(^3 \)] etc.

7 CP Violation in Beauty Decays – The KM Perspective

The KM predictions for strange decays and electric dipole moments given above will be subjected to sensitive tests in the foreseeable future. Yet there is one question that most naturally will come up in this context: ”Where else to look?” I will show below that on very general grounds one has to conclude that the decays of beauty hadrons provide by far the optimal lab. Yet first I want to make some historical remarks.

7.1 The Emerging Beauty of B Hadrons

7.1.1 Lederman’s Paradise Lost – and Regained!

In 1970 Lederman’s group studying the Drell-Yan process

\[ pp \to \mu^+ \mu^- X \]  
\[ (168) \]
at Brookhaven observed a shoulder in the di-muon mass distribution around 3 GeV. 1974 saw the ‘Octobre Revolution’ when Ting et al. and Richter et al. found a narrow resonance – the $J/\psi$ – with a mass of 3.1 GeV at Brookhaven and SLAC, respectively, and announced it. In 1976/77 Lederman’s group working at Fermilab saw a structure – later referred to as the Oops-Leon – around 6 GeV, which then disappeared. In 1977 Lederman et al. discovered three resonances in the mass range of 9.5 - 10.3 GeV, the $\Upsilon$, $\Upsilon'$ and $\Upsilon''$! That shows that persistence can pay off – at least sometimes and for some people.

7.1.2 Longevity of Beauty

The lifetime of weakly decaying beauty quarks can be related to the muon lifetime

$$\tau(b) \sim \tau(\mu) \left( \frac{m_\mu}{m_b} \right)^5 \frac{1}{9} \frac{1}{|V(cb)|^2} \sim 3 \cdot 10^{-14} \left| \frac{\sin \theta_C}{V(cb)} \right|^2 \text{ sec} \quad (169)$$

for a $b$ quark mass of around 5 GeV; the factor 1/9 reflects the fact that the virtual $W^-$ boson in $b$ quark decays can materialize as a $d\bar{u}$ or $s\bar{c}$ in three colours each and as three lepton pairs. I have ignored phase space corrections here. Since the $b$ quark has to decay outside its own family one would expect $|V(cb)| \sim \mathcal{O}(\sin \theta_C) = |V(us)|$. Yet starting in 1982 data showed a considerably longer lifetime

$$\tau(\text{beauty}) \sim 10^{-12} \text{ sec} \quad (170)$$

implying

$$|V(cb)| \sim \mathcal{O}(\sin^2 \theta_C) \sim 0.05 \quad (171)$$

The technology to resolve decay vertices for objects of such lifetimes happened to have just been developed – for charm studies!

7.1.3 The Changing Identity of Neutral $B$ Mesons

Speedy $B_d - B_d$ oscillations were discovered by ARGUS in 1986:

$$x_d \equiv \frac{\Delta m(B_d)}{\Gamma(B_d)} \sim \mathcal{O}(1) \quad (172)$$

These oscillations can then be tracked like the decays. This observation was also the first evidence that top quarks had to be heavier than originally thought, namely $m_t \geq M_W$.

7.1.4 Beauty Goes to Charm (almost always)

It was soon found that $b$ quarks exhibit a strong preference to decay into charm rather than up quarks

$$\left| \frac{V(ub)}{V(cb)} \right|^2 \ll 1 \quad (173)$$
establishing thus the hierarchy
\[ |V(ub)|^2 \ll |V(cb)|^2 \ll |V(us)|^2 \ll 1 \quad (174) \]

### 7.1.5 Resume

We will soon see how all these observations form crucial inputs to the general message that big CP asymmetries should emerge in $B$ decays and that they (together with interesting rare decays) are within reach of experiments. It is for this reason that I strongly feel that the only appropriate name for this quantum number is beauty! A name like bottom would not do it justice.

### 7.2 The KM Paradigm of Huge CP Asymmetries

#### 7.2.1 Large Weak Phases!

The Wolfenstein representation expresses the CKM matrix as an expansion:

\[
V_{CKM} = \begin{pmatrix}
1 & O(\lambda) & O(\lambda^3) \\
O(\lambda) & 1 & O(\lambda^2) \\
O(\lambda^3) & O(\lambda^2) & 1
\end{pmatrix}, \quad \lambda = \sin \theta_C \quad (175)
\]

The crucial element in making this expansion meaningful is the 'long' lifetime of beauty hadrons of around 1 psec. That number had to change by an order of magnitude – which is out of the question – to invalidate the conclusions given below for the size of the weak phases.

The unitarity condition yields 6 triangle relations:

\[
V^*(ud)V(us) + V^*(cd)V(cs) + V^*(td)V(ts) = \delta_{ds} = 0 \\
O(\lambda) + O(\lambda) + O(\lambda^5)
\]

\[
V^*(ud)V(cd) + V^*(us)V(cs) + V^*(ub)V(cb) = \delta_{uc} = 0 \\
O(\lambda) + O(\lambda) + O(\lambda^5)
\]

\[
V^*(us)V(ub) + V^*(cs)V(cb) + V^*(ts)V(tb) = \delta_{sb} = 0 \\
O(\lambda^4) + O(\lambda^2) + O(\lambda^3)
\]

\[
V^*(td)V(cd) + V^*(ts)V(cs) + V^*(tb)V(cb) = \delta_{ct} = 0 \\
O(\lambda^4) + O(\lambda^2) + O(\lambda^3)
\]

\[
V^*(ud)V(ub) + V^*(cd)V(cb) + V^*(td)V(tb) = \delta_{db} = 0 \\
O(\lambda^3) + O(\lambda^3) + O(\lambda^3)
\]

\[
V^*(td)V(ud) + V^*(ts)V(us) + V^*(tb)V(ub) = \delta_{ut} = 0 \\
O(\lambda^3) + O(\lambda^3) + O(\lambda^3)
\]

where below each product of matrix elements I have noted their size in powers of $\lambda$.

We see that the six triangles fall into three categories:
1. The first two triangles are extremely ‘squashed’: two sides are of order \( \lambda \), the third one of order \( \lambda^5 \) and their ratio of order \( \lambda^4 \simeq 2.3 \cdot 10^{-3} \); Eq.(176) and Eq.(177) control the situation in strange and charm decays; the relevant weak phases there are obviously tiny.

2. The third and fourth triangles are still rather squashed, yet less so: two sides are of order \( \lambda^2 \) and the third one of order \( \lambda^4 \).

3. The last two triangles have sides that are all of the same order, namely \( \lambda^3 \). All their angles are therefore naturally large, i.e. \( \sim \) several \( \times 10 \) degrees! Since to leading order in \( \lambda \) one has

\[
V(ud) \simeq V(tb) \, , \, V(cd) \simeq -V(us) \, , \, V(ts) \simeq -V(cb)
\]

we see that the triangles of Eqs.(180, 181) actually coincide to that order.

The sides of this triangle having naturally large angles are given by \( \lambda \cdot V(cb), V(ub) \) and \( V^*(td) \); these are all quantities that control important aspects of \( B \) decays, namely CKM favoured and disfavoured \( B \) decays and \( B_d - \bar{B}_d \) oscillations!

### 7.2.2 Different, Yet Coherent Amplitudes!

\( B^0 - \bar{B}^0 \) oscillations provide us with two different amplitudes that by their very nature have to be coherent:

\[
B^0 \Rightarrow \bar{B}^0 \rightarrow f \leftarrow B^0
\]

On general grounds one expects oscillations to be speedy for \( B^0 - \bar{B}^0 \) (like for \( K^0 - \bar{K}^0 \)), yet slow for \( D^0 - \bar{D}^0 \). Experimentally one indeed finds

\[
\frac{\Delta m(B_d)}{\Gamma(B_d)} = 0.71 \pm 0.06
\]

(184)

\[
\frac{\Delta m(B_s)}{\Gamma(B_s)} \geq 10
\]

(185)

While Eq.(184) describes an almost optimal situation the overly rapid pace of \( B_s - \bar{B}_s \) oscillations will presumably cause experimental problems.

The conditions are quite favourable also for direct CP violation to surface. Consider a transition amplitude

\[
T(B \rightarrow f) = \mathcal{M}_1 + \mathcal{M}_2 = e^{i\phi_1}e^{i\alpha_1}|\mathcal{M}_1| + e^{i\phi_2}e^{i\alpha_2}|\mathcal{M}_2|
\]

(186)

The two partial amplitudes \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \) are distinguished by, say, their isospin – as it was the case for \( K \rightarrow (\pi\pi)_{I=0,2} \) discussed before; \( \phi_1, \phi_2 \) denote the phases in

\( ^{14}T^0 - \bar{T}^0 \) oscillations cannot occur since top quarks decay before they hadronize [34].
the weak couplings and \( \alpha_1, \alpha_2 \) the phase shifts due to strong final state interactions. For the CP conjugate reaction one obtains
\[
T(\bar{B} \to \bar{f}) = e^{-i\phi_1}e^{i\alpha_1}\vert M_1 \vert + e^{-i\phi_2}e^{i\alpha_2}\vert M_2 \vert .
\]
(187)
since under CP the weak parameters change into their complex conjugate values whereas the phase shifts remain the same; for the strong forces driving final state interactions conserve CP. The rate difference is then given by
\[
\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) \propto |T(B \to f)|^2 - |T(\bar{B} \to \bar{f})|^2 = \\
= -4 \sin(\phi_1 - \phi_2) \cdot \sin(\alpha_1 - \alpha_2) \cdot M_1 \otimes M_2
\]
(188)
For an asymmetry to arise in this way two conditions need to be satisfied simultaneously, namely
\[
\phi_1 \neq \phi_2 \quad \alpha_1 \neq \alpha_2
\]
(189)
I.e., the two amplitudes \( M_1 \) and \( M_2 \) have to differ both in their weak and strong forces! The first condition implies (within the Standard Model) that the reaction has to be KM suppressed, whereas the second one require the intervention of nontrivial final state interactions.

There is a large number of KM suppressed channels in \( B \) decays that are suitable in this context: they receive significant contributions from weak couplings with large phases – like \( V_{ub} \) in the Wolfensteinn representation – and there is no reason why the phase shifts should be small in general (although that could happen in some cases).

### 7.2.3 Resume

Let me summarize the discussion just given and anticipate the results to be presented below.

- Large CP asymmetries are predicted with confidence to occur in \( B \) decays. If they are not found, there is no plausible denialability for the KM ansatz.
- Some of these predictions can be made with high parametric reliability.
- New theoretical technologies have emerged that will allow us to translate this parametric reliability into numerical precision.
- Some of the observables exhibit a high and unambiguous sensitivity to the presence of New Physics since we are dealing with coherent processes with observables depending linearly on New Physics amplitudes and where the CKM ‘background’ is (or can be brought) under theoretical control.
7.3 General Phenomenology

Decay rates for CP conjugate channels can be expressed as follows:

\[
\begin{align*}
\text{rate}(B(t) \to f) &= e^{-\Gamma_B t} G_f(t) \\
\text{rate}(\bar{B}(t) \to \bar{f}) &= e^{-\Gamma_B t} \bar{G}_{\bar{f}}(t)
\end{align*}
\]  (190)

where CPT invariance has been invoked to assign the same lifetime $\Gamma_B^{-1}$ to $B$ and $\bar{B}$ hadrons. Obviously if

\[
\frac{G_f(t)}{G_f(t)} \neq 1
\]  (191)

is observed, CP violation has been found. Yet one should keep in mind that this can manifest itself in two (or three) qualitatively different ways:

1. 

\[
\frac{G_f(t)}{G_f(t)} \neq 1 \quad \text{with} \quad \frac{d}{dt} \frac{G_f(t)}{G_f(t)} = 0 ;
\]  (192)

i.e., the asymmetry is the same for all times of decay. This is true for direct CP violation; yet, as explained later, it also holds for CP violation in the oscillations.

2. 

\[
\frac{G_f(t)}{G_f(t)} \neq 1 \quad \text{with} \quad \frac{d}{dt} \frac{G_f(t)}{G_f(t)} \neq 0 ;
\]  (193)

here the asymmetry varies as a function of the time of decay. This can be referred to as CP violation involving oscillations.

Quantum mechanics with its linear superposition principle makes very specific statements about the possible time dependence of $G_f(t)$ and $\bar{G}_{\bar{f}}(t)$; yet before going into that I want to pose another homework problem:

♠ ♠ ♠

Homework Problem # 4:

Consider the reaction

\[
e^+e^- \to \phi \to (\pi^+\pi^-)_K(\pi^+\pi^-)_K
\]  (194)

Its occurrence requires CP violation. For the initial state $-\phi$ - carries even CP parity whereas the final state with the two $(\pi^+\pi^-)$ combinations forming a P wave must be CP odd: $(+1)^2(-1)^l = -1$! Yet Bose statistics requiring identical states to be in a symmetric configuration would appear to veto this reaction; for it places the two $(\pi^+\pi^-)$ states into a P wave which is antisymmetric. What is the flaw in this reasoning? The same puzzle can be formulated in terms of

\[
e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to (\psi K_S)_B(\psi K_S)_B.
\]  (195)

\[\text{This nomenclature falls well short of Shakespearean standards.}\]
A straightforward application of quantum mechanics yields the general expressions:

\[ G_f(t) = |T_f|^2 \left[ \left( 1 + \frac{q}{p} |\bar{\rho}_f|^2 \right) + \left( 1 - \frac{q}{p} |\rho_f|^2 \right) \cos \Delta m_B t + 2(\sin \Delta m_B t)\text{Im}\frac{q}{p} \bar{\rho}_f \right] \]
\[ \bar{G}_f(t) = |\bar{T}_f|^2 \left[ \left( 1 + \frac{q}{p} |\rho_f|^2 \right) + \left( 1 - \frac{q}{p} |\bar{\rho}_f|^2 \right) \cos \Delta m_B t + 2(\sin \Delta m_B t)\text{Im}\frac{q}{p} \rho_f \right] \]

The amplitudes for the instantaneous \( \Delta B = 1 \) transition into a final state \( f \) are denoted by \( T_f = T(B \rightarrow f) \) and \( \bar{T}_f = T(\bar{B} \rightarrow f) \) and

\[ \bar{\rho}_f = \frac{\bar{T}_f}{T_f}, \rho_f = \frac{T_f}{\bar{T}_f}, \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \]

(197)

Staring at the general expression is not always very illuminating; let us therefore consider three very simplified limiting cases:

- \( \Delta m_B = 0 \), i.e. no \( B^0 - \bar{B}^0 \) oscillations:

\[ G_f(t) = 2|T_f|^2, \bar{G}_f(t) = 2|\bar{T}_f|^2 \sim \frac{\bar{G}_f(t)}{G_f(t)} = \left| \frac{T_f}{\bar{T}_f} \right|^2, \frac{d}{dt} G_f(t) \equiv 0 \equiv \frac{d}{dt} \bar{G}_f(t) \]

(198)

This is explicitly what was referred to above as direct CP violation.

- \( \Delta m_B \neq 0 \) and \( f \) a flavour-specific final state with no direct CP violation; i.e., \( T_f = 0 = \bar{T}_f \) and \( \bar{T}_f = T_f \) \( \dagger \dagger \):

\[ G_f(t) = \left| \frac{q}{p} \right| |\bar{T}_f|^2(1 - \cos \Delta m_B t), \bar{G}_f(t) = \left| \frac{q}{p} \right| |T_f|^2(1 - \cos \Delta m_B t) \sim \frac{\bar{G}_f(t)}{G_f(t)} = \left| \frac{q}{p} \right|^4, \frac{d}{dt} \frac{\bar{G}_f(t)}{G_f(t)} \equiv 0, \frac{d}{dt} \bar{G}_f(t) \neq 0 \neq \frac{d}{dt} G_f(t) \]

(199)

This constitutes CP violation in the oscillations. For the CP conserving decay into the flavour-specific final state is used merely to track the flavour identity of the decaying meson. This situation can therefore be denoted also in the following way:

\[ \frac{\text{Prob}(B^0 \Rightarrow \bar{B}^0; t) - \text{Prob}(\bar{B}^0 \Rightarrow B^0; t)}{\text{Prob}(B^0 \Rightarrow B^0; t) + \text{Prob}(\bar{B}^0 \Rightarrow \bar{B}^0; t)} = \left| \frac{q}{p} \right|^2 - \left| \frac{p}{q} \right|^2 = 1 - \left| \frac{q}{p} \right|^4 \]

(200)

- \( \Delta m_B \neq 0 \) with \( f \) now being a flavour-nonspecific final state – a final state common to \( B^0 \) and \( \bar{B}^0 \) decays – of a special nature, namely a CP eigenstate –

---

\( \dagger \dagger \) For a flavour-specific mode one has in general \( T_f \cdot \bar{T}_f = 0 \); the more intriguing case arises when one considers a transition that requires oscillations to take place.
\[ |\bar{f}\rangle = CP |f\rangle = \pm |f\rangle - \text{without direct CP violation - } |\bar{\rho}_f| = 1 = |\rho_f|: \]

\[
G_f(t) = 2|T_f|^2 \left[ 1 + (\sin \Delta m_B t) \cdot \text{Im} \frac{q}{p} \bar{\rho}_f \right] \\
G_f(t) = 2|T_f|^2 \left[ 1 - (\sin \Delta m_B t) \cdot \text{Im} \frac{q}{p} \bar{\rho}_f \right] \\
\sim \frac{d G_f(t)}{dt} \neq 0
\]

is the concrete realization of what was called CP violation involving oscillations.

### 7.3.1 CP Violation in Oscillations

Using the convention blessed by the PDG

\[ B = [\bar{b}q], \quad \bar{B} = [\bar{q}b] \]

we have

\[
T(B \to l^- X) = 0 = T(\bar{B} \to l^+ X) \\
T_{SL} \equiv T(B \to l^+ X) = T(\bar{B} \to l^- X)
\]

with the last equality enforced by CPT invariance. The so-called Kabir test can then be realized as follows:

\[
\frac{\text{Prob}(B^0 \to \bar{B}^0; t) - \text{Prob}(\bar{B}^0 \to B^0; t)}{\text{Prob}(B^0 \to B^0; t) + \text{Prob}(\bar{B}^0 \to B^0; t)} = \\
= \frac{\text{Prob}(B^0 \to \bar{B}^0 \to l^- X; t) - \text{Prob}(\bar{B}^0 \to B^0 \to l^+ X; t)}{\text{Prob}(B^0 \to B^0 \to l^- X; t) + \text{Prob}(\bar{B}^0 \to B^0 \to l^+ X; t)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}
\]

Without going into details I merely state the results here [38]:

\[
1 - \left| \frac{q}{p} \right| \sim \frac{1}{2} \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right) \sim \begin{cases} 
10^{-3} & \text{for } B_d = (bd) \\
10^{-4} & \text{for } B_s = (bs)
\end{cases}
\]

i.e.,

\[
a_{SL}(B^0) \equiv \frac{\Gamma(\bar{B}^0(t) \to l^+ \nu X) - \Gamma(B^0(t) \to l^- \bar{\nu} X)}{\Gamma(\bar{B}^0(t) \to l^+ \nu X) + \Gamma(B^0(t) \to l^- \bar{\nu} X)} \sim \begin{cases} 
\mathcal{O}(10^{-3}) & \text{for } B_d \\
\mathcal{O}(10^{-4}) & \text{for } B_s
\end{cases}
\]

The smallness of the quantity \(1 - |q/p|\) is primarily due to \(|\Gamma_{12}| \ll |M_{12}|\) or \(\Delta \Gamma_B \ll \Delta m_B\). Within the Standard Model this hierarchy is understood (semi-quantitatively at least) as due to the hierarchy in the GIM factors of the box diagram expressions for \(\Gamma_{12}\) and \(M_{12}\), namely \(m_e^2/M_W^2 \ll m_t^2/M_W^2\).

For \(B_s\) mesons the phase between \(\Gamma_{12}\) and \(M_{12}\) is further (Cabibbo) suppressed for reasons that are peculiar to the KM ansatz: for to leading order in the KM parameters quarks of the second and third family only contribute and therefore \(\text{arg}(\Gamma_{12}/M_{12}) = 0\) to that order. If New Physics intervenes in \(B^0 - \bar{B}^0\) oscillations,
it would quite naturally generate a new phase between $\Gamma_{12}$ and $M_{12}$; it could also reduce $M_{12}$. Altogether this CP asymmetry could get enhanced very considerably:

$$a_{SL}^{New\ Physics}(B^0) \sim 1\%$$

Therefore one would be ill-advised to accept the somewhat pessimistic KM predictions as gospel.

Since this CP asymmetry does not vary with the time of decay, a signal is not diluted by integrating over all times. It is, however, essential to ‘flavour tag’ the decaying meson; i.e., determine whether it was produced as a $B^0$ or $\bar{B}^0$. This can be achieved in several ways as discussed later.

### 7.3.2 Direct CP Violation

Sizeable direct CP asymmetries arise rather naturally in $B$ decays. Consider

$$b \to s\bar{u}u$$

(208)

Three different processes contribute to it, namely

- the tree process
  $$b \to uW^* \to u(\bar{u}s)_W,$$
  (209)

- the penguin process with an internal top quark which is purely local (since $m_t > m_b$)
  $$b \to s\bar{g}^* \to s\bar{u}u,$$
  (210)

- the penguin reaction with an internal charm quark. Since $m_b > 2m_c + m_s$, this last operator is not local: it contains an absorptive part that amounts to a final state interaction including a phase shift.

One then arrives at a guestimate [37, 38]

$$\frac{\Gamma(b \to s\bar{u}u) - \Gamma(\bar{b} \to \bar{s}\bar{u}u)}{\Gamma(b \to s\bar{u}u) + \Gamma(\bar{b} \to \bar{s}\bar{u}u)} \sim O(\%)$$

(211)

Invoking quark-hadron duality one can expect (or at least hope) that this quark level analysis – rather than being washed out by hadronisation – yields some average asymmetry or describes the asymmetry for some inclusive subclass of nonleptonic channels. I would like to draw the following lessons from these considerations:

- According to the KM ansatz the natural scale for direct CP asymmetries in the decays of beauty hadrons (neutral or charged mesons or baryons) is the $10^{-2}$ level – not $10^{-6} \div 10^{-5}$ as in strange decays!
• The size of the asymmetry in individual channels – like \( B \to K\pi \) – is shaped by the strong final state interactions operating there. Those are likely to differ considerably from channel to channel, and at present we are unable to predict them since they reflect long-distance dynamics.

• Observation of such an asymmetry (or lack thereof) will not provide us with reliable numerical information on the parameters of the microscopic theory, like the KM ansatz.

• Nevertheless comprehensive and detailed studies are an absolute must!

Later I will describe examples where the relevant long-distance parameters – phase shifts etc. – can be measured independently.

### 7.3.3 CP Violation Involving Oscillations

The essential feature that a final state in this category has to satisfy is that it can be fed both by \( B^0 \) and \( \bar{B}^0 \) decays. However for convenience reasons I will concentrate on a special subclass of such modes, namely when the final state is a CP eigenstate. A more comprehensive discussion can be found in [38, 5].

Three qualitative observations have to be made here:

• Since the final state is shared by \( B^0 \) and \( \bar{B}^0 \) decays one cannot even define a CP asymmetry unless one acquires independent information on the decaying meson: was it a \( B^0 \) or \( \bar{B}^0 \) or – more to the point – was it originally produced as a \( B^0 \) or \( \bar{B}^0 \)? There are several scenarios for achieving such flavour tagging:

  – Nature could do the trick for us by providing us with \( B^0 - \bar{B}^0 \) production asymmetries through, say, associated production in hadronic collisions or the use of polarized beams in \( e^+e^- \) annihilation. Those production asymmetries could be tracked through decays that are necessarily CP conserving – like \( \bar{B}_d \to \psi K^-\pi^+ \) vs. \( B_d \to \psi K^+\pi^- \). It seems unlikely, though, that such a scenario could ever be realized with sufficient statistics.

  – Same Side Tagging: One undertakes to repeat the success of the \( D^* \) tag for charm mesons – \( D^{**} \to D^0\pi^+ \) vs. \( D^{*-} \to \bar{D}^0\pi^- \) – through finding a conveniently placed nearby resonance – \( B^{-*} \to \bar{B}_d\pi^- \) vs. \( B^{**} \to B_d\pi^+ \) – or through employing correlations between the beauty mesons and a ‘nearby’ pion (or kaon for \( B_s \)) as pioneered by the CDF collaboration. This method can be calibrated by analysing how well \( B^0 - \bar{B}^0 \) oscillations are reproduced.

\footnote{17 Obviously no such common channels can exist for charged mesons or for baryons.}
- **Opposite Side Tagging:** With electromagnetic and strong forces conserving the beauty quantum number, one can employ charge correlations between the decay products (leptons and kaons) of the two beauty hadrons originally produced together.

- If the lifetimes of the two mass eigenstates of the neutral \( B \) meson differ sufficiently from each other, then one can wait for the short-lived component to fade away relative to the long-lived one and proceed in qualitative analogy to the \( K_L \) case. Conceivably this could become feasible – or even essential – for overly fast oscillating \( B_s \) mesons \[39\].

The degree to which this flavour tagging can be achieved is a crucial challenge each experiment has to face.

- The CP asymmetry is largest when the two interfering amplitudes are comparable in magnitude. With oscillations having to provide the second amplitude that is absent initially at time of production, the CP asymmetry starts out at zero for decays that occur right after production and builds up for later decays. The (first) maximum of the asymmetry

\[
\left| 1 - \frac{1 - \text{Im} q \bar{p}_j \sin \Delta m_{BT}}{1 + \text{Im} q \bar{p}_j \sin \Delta m_{BT}} \right| = 1
\]

is reached for

\[
\frac{t}{\tau_B} = \frac{\pi \Gamma_B}{2 \Delta m_B} \simeq 2
\]

in the case of \( B_d \) mesons.

- The other side of the coin is that very rapid oscillations – \( \Delta m_B \gg \Gamma_B \) as is the case for \( B_s \) mesons – will tend to wash out the asymmetry or at least will severely tax the experimental resolution.

### 7.3.4 Resume

Three classes of quantities each describe the three types of CP violation:

1. \[
\left| \frac{q}{p} \right| \neq 1
\]

2. \[
\left| \frac{T(\bar{B} \to \bar{f})}{T(B \to f)} \right| \neq 1
\]

3. \[
\text{Im} \frac{q T(\bar{B} \to \bar{f})}{p T(B \to f)} \neq 0
\]
These quantities obviously satisfy one necessary condition for being observables: they are insensitive to the phase convention adopted for the anti-state.

### 7.4 Parametric KM Predictions

The triangle defined by

$$\lambda V(cb) - V(ub) + V^*(td) = 0$$  \hspace{1cm} (217)

to leading order controls basic features of $B$ transitions. As discussed before, it has naturally large angles; it usually is called the KM triangle. Its angles are given by KM matrix elements which are most concisely expressed in the Wolfenstein representation:

$$e^{i\phi_1} = -\frac{V(td)}{|V(td)|}, \quad e^{i\phi_2} = \frac{V^*(td) V(ub)}{|V(td)| V(ub)}, \quad e^{i\phi_3} = \frac{V(ub)}{|V(ub)|}$$  \hspace{1cm} (218)

The various CP asymmetries in beauty decays are expressed in terms of these three angles. I will describe ‘typical’ examples now.

#### 7.4.1 Angle $\phi_1$

Consider

$$\bar{B}_d \rightarrow \psi K_S \leftarrow B_d$$  \hspace{1cm} (219)

where the final state is an almost pure odd CP eigenstate. On the quark level one has two different reactions, namely one describing the direct decay process

$$\bar{B}_d = [bd] \rightarrow [c\bar{c}][s\bar{d}]$$  \hspace{1cm} (220)

and the other one involving a $B_d - \bar{B}_d$ oscillation:

$$\bar{B}_d = [bd] \Rightarrow B_d = [\bar{b}\bar{d}] \rightarrow [c\bar{c}][\bar{s}\bar{d}]$$  \hspace{1cm} (221)

Homework Problem # 5:

How can the $[s\bar{d}]$ combination in Eq.(220) interfere with $[\bar{s}\bar{d}]$ in Eq.(221)?

Since the final state in $B/\bar{B} \rightarrow \psi K_S$ can carry isospin 1/2 only, we have for the direct transition amplitudes:

$$T(\bar{B}_d \rightarrow \psi K_S) = V(cb) V^*(cs) e^{i\phi_1/2} |M_{1/2}|$$  \hspace{1cm} (222)

$$T(B_d \rightarrow \psi K_S) = V^*(cb) V(cs) e^{i\phi_1/2} |M_{1/2}|$$
and thus
\[ \bar{\rho}_{\psi K_S} = \frac{V(cb)V^*(cs)}{V^*(cb)V(cs)} \] (223)
from which the hadronic quantities, namely the phase shift \( \alpha_{1/2} \) and the hadronic matrix element \( |\mathcal{M}_{1/2}| \) – both of which \emph{cannot} be calculated in a reliable manner – have dropped out. Therefore
\[ |\bar{\rho}_{\psi K_S}| = \left| \frac{T(\bar{B}_d \to \psi K_S)}{T(B_d \to \psi K^*_S)} \right| = 1 ; \] (224)
i.e., there can be \emph{no direct} CP violation in this channel.

Since \( |\Gamma_{12}| \ll |M_{12}| \) one has
\[ \frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \frac{V^*(tb)V(td)}{V(tb)V^*(td)} \] (225)
which is a pure phase. Altogether one obtains
\[ \text{Im} \frac{q}{p}\bar{\rho}_{\psi K_S} = \text{Im} \left( \frac{V^*(tb)V(td)}{V(tb)V^*(td)} \right) \approx \text{Im} \left| \frac{V^2(td)}{|V(td)|^2} \right| = -\sin^2\phi_1 \] (226)
That means that to a very good approximation the observable \( \text{Im} \frac{q}{p}\bar{\rho}_{\psi K_S} \), which is the amplitude of the oscillating CP asymmetry, is in general given by \emph{microscopic} parameters of the theory; \emph{within} the KM ansatz they combine to yield the angle \( \phi_1 \).

Several other channels are predicted to exhibit a CP asymmetry expressed by \( \sin^2\phi_1 \), like \( B_d \to \psi K_L \), \( B_d \to D\bar{D} \) etc.

### 7.4.2 Angle \( \phi_2 \)

The situation is not quite as clean for the angle \( \phi_2 \). The asymmetry in \( \bar{B}_d \to \pi^+\pi^- \) vs. \( B_d \to \pi^+\pi^- \) is certainly sensitive to \( \phi_2 \), yet there are two complications:

- The final state is described by a superposition of \emph{two} different isospin states, namely \( I = 0 \) and 2. The spectator process contributes to both of them.
- The Cabibbo suppressed Penguin operator
  
  \[ b \to d g^* \to du\bar{u} \] (227)

will also contribute, albeit only to the \( I = 0 \) amplitude.

\(^{18} \)The next-to-last (approximate) equality in Eq. (226) holds in the Wolfenstein representation, although the overall result is general.

\(^{19} \)Keep in mind that \( \text{Im} \frac{q}{p}\bar{\rho}_{\psi K_L} = -\text{Im} \frac{q}{p}\bar{\rho}_{\psi K_S} \) holds because \( K_L \) is mainly CP odd and \( K_S \) mainly CP even.
The direct transition amplitudes are then expressed as follows:

\[ T(\bar{B}_d \to \pi^+\pi^-) = V(ub)V^*(ud)e^{i\alpha_2}|M^\text{spect}_2| + \\
+ e^{i\alpha_0} \left( V(ub)V^*(ud)|M^\text{spect}_0| + V(tb)V^*(td)|M^\text{Peng}_0| \right) \] (228)

\[ T(B_d \to \pi^+\pi^-) = V^*(ub)V(ud)e^{i\alpha_2}|M^\text{spect}_2| + \\
+ e^{i\alpha_0} \left( V^*(ub)V(ud)|M^\text{spect}_0| + V^*(tb)V(td)|M^\text{Peng}_0| \right) \] (229)

where the phase shifts for the \( I = 0, 2 \) states have been factored off.

If there were no Penguin contributions, we would have

\[ \text{Im} \frac{q_{\bar{B}_d\pi\pi}}{p_{\bar{B}_d\pi\pi}} = \text{Im} \frac{V(td)V^*(tb)V(ub)V^*(ud)}{V^*(td)V(tb)V^*(ub)V(ud)} = -\sin^2 \phi_2 \] (230)

without direct CP violation – \( |\bar{\rho}_{\pi\pi}| = 1 \) – since the two isospin amplitudes still contain the same weak parameters. The Penguin contribution changes the picture in two basic ways:

1. The CP asymmetry no longer depends on \( \phi_2 \) alone:

\[ \text{Im} \frac{q_{\bar{B}_d\pi\pi}}{p_{\bar{B}_d\pi\pi}} \approx -\sin 2\phi_2 + \left| \frac{V(td)}{V(ub)} \right| \left[ \text{Im} \left( e^{-i\phi_2} \frac{M^\text{Peng}}{M^\text{spect}} \right) - \text{Im} \left( e^{-3i\phi_2} \frac{M^\text{Peng}}{M^\text{spect}} \right) \right] + \mathcal{O}(\frac{|M^\text{Peng}|^2}{|M^\text{spect}|^2}) \] (231)

where

\[ M^\text{spect} = e^{i\alpha_0}|M^\text{spect}_0| + e^{i\alpha_2}|M^\text{spect}_2| \ , \ M^\text{Peng} = e^{i\alpha_0}|M^\text{Peng}_0| \] (232)

2. A direct CP asymmetry emerges:

\[ |\bar{\rho}_{\pi\pi}| \neq 1 \] (233)

Since we are dealing with a Cabibbo suppressed Penguin operator, we expect that its contribution is reduced relative to the spectator term:

\[ \left| \frac{M^\text{Peng}}{M^\text{spect}} \right| < 1 \] (234)

which was already used in Eq.(231). Unfortunately this reduction might not be very large. This concern is based on the observation that the branching ratio for \( \bar{B}_d \to K^-\pi^+ \) appears to be somewhat larger than for \( \bar{B}_d \to \pi^+\pi^- \) implying that the Cabibbo favoured Penguin amplitude is at least not smaller than the spectator amplitude.

Various strategies have been suggested to unfold the Penguin contribution through a combination of additional or other measurements (of other \( B \to \pi\pi \) channels or of \( B \to \pi\rho, B \to K\pi \) etc.) and supplemented by theoretical considerations like \( SU(3)_F \) symmetry \[40\]. I am actually hopeful that the multitude of exclusive non-leptonic decays (which is the other side of the coin of small branching ratios!) can be harnessed to extract a wealth of information on the strong dynamics that in turn will enable us to extract \( \sin^2 \phi_2 \) with decent accuracy.
7.4.3 The $\phi_3$ Saga

Of course it is important to determine $\phi_3$ as accurately as possible. This will not be easy, and one better keep a proper perspective. I am going to tell this saga now in two installments.

(I) CP asymmetries involving $B_s - \bar{B}_s$ Oscillations: In principle one can extract $\phi_3$ from KM suppressed $B_s$ decays like one does $\phi_2$ from $B_d$ decays, namely by measuring and analyzing the difference between the rates for, say, $\bar{B}_s(t) \to K_S\rho^0$ and $B_s(t) \to K_S\rho^0$: $\text{Im}^2 \rho_{K_S\rho} \sim \sin 2\phi_3$. One has to face the same complication, namely that in addition to the spectator term a (Cabibbo suppressed) Penguin amplitude contributes to $\bar{\rho}_{K_S\rho}$ with different weak parameters. Yet the situation is much more challenging due to the rapid pace of the $B_s - \bar{B}_s$ oscillations. A more promising way might be to compare the rates for $\bar{B}_s(t) \to D_s^+ K^- + S$ with $B_s(t) \to D_s^- K^+$ as a function of the time of decay $t$ since there is no Penguin contribution. The asymmetry depends on $\sin \phi_3$ rather than $\sin 2\phi_3$.

(II) Direct CP Asymmetries: The largish direct CP asymmetries sketched above for $B \to K\pi$ depend on $\sin \phi_3$ – and on the phase shifts which in general are neither known nor calculable. Yet in some cases they can be determined experimentally – as first described for $B_{\pm} \to D_{\text{neutral}} K^\pm$. There are four independent rates that can be measured, namely

$$
\Gamma(B^- \to D^0 K^-), \Gamma(B^- \to \bar{D}^0 K^-), \Gamma(B^- \to D_+ K^-), \Gamma(B^+ \to D_+ K^+) \quad (235)
$$

The flavour eigenstates $D^0$ and $\bar{D}^0$ are defined through flavour specific modes, namely $D^0 \to l^+ X$ and $\bar{D}^0 \to l^- X$, respectively; $D_{\pm}$ denote the even/odd CP eigenstates $D_{\pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$ defined by $D_+ \to K^+ K^-, \pi^+ \pi^-, \text{etc.}$, $D_- \to K_S \pi^0, K_S \eta$, etc. [1].

From these four observables one can (up to a binary ambiguity) extract the four basic quantities, namely the moduli of the two independent amplitudes ($|T(B^- \to D^0 K^-)|, |T(B^- \to \bar{D}^0 K^-)|$), their strong phasishift – and $\sin \phi_3$, the goal of the enterprise!

7.4.4 A Zero-Background Search for New Physics: $B_s \to \psi\phi, D_s^+ D_s^-$

The two angles $\phi_1$ and $\phi_2$ will be measured in the next several years with decent or even good accuracy. I find it unlikely that any of the direct measurements of $\phi_3$ sketched above will yield a more precise value than inferred from simple trigonometry:

$$
\phi_3 = 180^\circ - \phi_1 - \phi_2 \quad (236)
$$

Eq.$(236)$ holds within the KM ansatz; of course the real goal is to uncover the intervention of New Physics in $B_s$ transitions. It then makes eminent sense to

---

20 Both $D_s^+ K^-$ and $D_s^- K^+$ are final states common to $B_s$ and $\bar{B}_s$ decays although they are not CP eigenstates.
search for it in a reaction where Known Physics predicts a practically zero result. $B_s \to \psi \phi, \psi \eta, D_s \bar{D}_s$ fit this bill \cite{36}: to leading order in the KM parameters the CP asymmetry has to vanish since on that level quarks of the second and third family only participate in $B_s - \bar{B}_s$ oscillations – $[s \bar{b}] \Rightarrow t^* \bar{t} \Rightarrow [b \bar{s}]$ – and in these direct decays – $[b \bar{s}] \to c \bar{c} s \bar{s}$. Any CP asymmetry is therefore Cabibbo suppressed, i.e. $\leq 4\%$. More specifically

$$\text{Im} \frac{q}{p} \rho_{B_s \to \psi \phi, D_s \bar{D}_s} \bigg|_{KM} \sim 2\% \quad (237)$$

Yet New Physics has a good chance to contribute to $B_s - \bar{B}_s$ oscillations; if so, there is no reason for it to conserve CP and asymmetries can emerge that are easily well in excess of 2\%. New Physics scenarios with non-minimal SUSY or flavour-changing neutral currents could actually yield asymmetries of $\sim 10 \div 30\%$ \cite{43} – completely beyond the KM reach!

### 7.4.5 The HERA-B Menu

Quite often people in the US tend to believe that a restaurant that presents them with a long menu must be a very good one. The real experts – like the French and Italians – of course know better: it is the hallmark of a top cuisine to concentrate on a few very special dishes and prepare them in a spectacular fashion rather than spread one’s capabilities too thinly. That is exactly the advice I would like to give the HERA-B collaboration, namely to focus on a first class menu consisting of three main dishes and one side dish, namely

1. measure $\Delta m(B_s)$ which within the Standard Model allows to extract $|V(td)|$ through

$$\frac{\Delta m(B_d)}{\Delta m(B_s)} \simeq \frac{Bf_B^2}{Bf_{B_s}^2} \left| \frac{V(td)}{V(ts)} \right|^2 ; \quad (238)$$

2. determine the rates for $\bar{B}_d \to \psi K_S$ and $B_d \to \psi K_S$ to obtain the value of $\sin^2 \phi_1$;

3. compare $\bar{B}_s \to \psi \phi, D_s \bar{D}_s$ with $B_s \to \psi \phi, D_s \bar{D}_s$ as a clean search for New Physics and

4. as a side dish: measure the $B_s$ lifetime separately in $B_s \to l \nu D_s^{(*)}$ and $B_s \to \psi \phi, D_s \bar{D}_s$ where the former yields the algebraic average of the $B_{s,\text{short}}$ and $B_{s,\text{long}}$ lifetimes and the latter the $B_{s,\text{short}}$ lifetime. One predicts for them \cite{44}:

$$\frac{\tau(B_s \to l \nu D_s^{(*)}) - \tau(B_s \to \psi \phi, D_s \bar{D}_s)}{\tau(B_s \to l \nu D_s^{(*)})} \simeq 0.1 \cdot \left( \frac{f_{B_s}}{200 \text{ MeV}} \right)^2 \quad (239)$$

If the HERA-B chefs succeed in preparing one of these main dishes, then they have achieved three star status!
### 7.5 Theoretical Technologies in Heavy Flavour Decays

One other intriguing and gratifying aspect of heavy flavour decays has become understood just over the last several years, namely that the decays in particular of beauty hadrons can be treated with a reliability and accuracy that before would have seemed to be unattainable. These new theoretical technologies can be referred to as *Heavy Quark Theory* which combines two basic elements, namely an asymptotic symmetry principle on one hand and a dynamical treatment on the other, which tells us how the asymptotic limit is approached. The symmetry principle is Heavy Quark Symmetry stating that all sufficiently heavy quarks behave identically under the strong interactions. The dynamical treatment is provided by $1/m_Q$ expansions allowing us to express observable transition rates through a series in inverse powers of the heavy quark mass. This situation is qualitatively similar to chiral considerations which start from the limit of chiral invariance and describe the deviations from it through chiral perturbation theory. In both cases one has succeeded in describing nonperturbative dynamics in special cases.

The lessons we have learnt can be summarized as follows [45]: we have

- identified the sources of the non-perturbative corrections;
- found them to be smaller than they could have been;
- succeeded in relating the basic quantities of the Heavy Quark Theory – KM parameters, masses and kinetic energy of heavy quarks, etc. – to various a priori independent observables with a fair amount of redundancy;
- developed a better understanding of incorporating perturbative and nonperturbative corrections without double-counting.

It has been shown that the heavy quark expansion has to be formulated in terms of short distance masses rather than pole masses. One finds

$$m_b - m_c = 3.50 \pm 0.04 \text{ GeV}$$
$$m_b(1 \text{ GeV}) = 4.64 \pm 0.05 \text{ GeV}$$

This information is then used to extract $|V(cb)|$ from the observed semileptonic $B$ width with the result

$$|V(cb)|_{incl} = 0.0412 \cdot \sqrt{\frac{\text{BR}(B \rightarrow lX)}{0.105}} \cdot \sqrt{\frac{1.6 \text{ psec}}{\tau_B}} \cdot (1 \pm 0.05|_{\text{theor}})$$

Alternatively one can analyze the exclusive mode $B \rightarrow l\nu D^*$ and extrapolate to the kinematical point of zero recoil to obtain

$$|F_{D^*}(0)V(cb)| = 0.0339 \pm 0.0014$$
From Heavy Quark Theory one infers [13]

\[ F_{D^+}(0) = 0.91 \pm 0.06 \]  

(243)

to arrive at

\[ |V(cb)|_{\text{excl}} = 0.0377 \pm 0.0016|_{\text{exp}} \pm 0.002|_{\text{theor}} \]  

(244)

The two determinations in Eqs.(241) and (244) are systematically very different both in their experimental and theoretical aspects. Nevertheless they are quite consistent with each other with the experimental and theoretical uncertainties being very similar. A few years ago it would have seemed quite preposterous to claim such small theoretical uncertainties! I am actually confident that those can be reduced from the present 5% level down to the 2% level in the foreseeable future.

\[ |V(ub)| \text{ (or } |V(ub)/V(cb)|) \] is not known with an even remotely similar accuracy, and so far one has relied on models rather than QCD proper to extract it from data. Yet we can be confident that over the next ten years \[ |V(ub)| \] will be determined with a theoretical uncertainty below 10%. It will be important to obtain it from systematically different semileptonic distributions and processes; Heavy Quark Theory provides us with the indispensable tools for combining the various analyses in a coherent fashion.

This theoretical progress can embolden us to hope that in the end even \[ |V(td)| \] can be determined with good accuracy – say \( \sim 10 \div 15\% \) – from \( \Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \), \( \Delta m(B_s) \) vs. \( \Delta m(B_d) \) or \( \Gamma(B \rightarrow \gamma \rho/\omega) \) vs. \( \Gamma(B \rightarrow \gamma K^*) \) etc.

### 7.6 KM Trigonometry

One side of the triangle is exactly known since the base line can be normalized to unity without affecting the angles:

\[ 1 - \frac{V(ub)}{\lambda V(cb)} + \frac{V^*(td)}{\lambda V(cb)} = 0 \]  

(245)

The second side is known to some degree from semileptonic \( B \) decays:

\[ \frac{|V(ub)|}{V(cb)} \simeq 0.08 \pm 0.03 \]  

(246)

where the quoted uncertainty is mainly theoretical and amounts to little more than a guestimate. In the Wolfenstein representation this reads as

\[ \sqrt{\rho^2 + \eta^2} \simeq 0.38 \pm 0.11 \]  

(247)

The area cannot vanish since \( \epsilon_K \neq 0 \). Yet at present not much more can be said for certain.

In principle one would have enough observables – namely \( \epsilon_K \) and \( \Delta m(B_d) \) in addition to \( |V(ub)/V(cb)| \) – to determine the two KM parameters \( \rho \) and \( \eta \) in a
redundant way. In practise, though, there are two further unknowns, namely the size of the $\Delta S = 2$ and $\Delta B = 2$ matrix elements, as expressed through $B_K$ and $B_B f_B^2$. For $m_t$ sufficiently large $\epsilon_K$ is dominated by the top contribution: $d\bar{s} \rightarrow t^*\bar{t}^* \rightarrow s\bar{d}$. The same holds always for $\Delta m(B_d)$. In that case things are simpler:

$$\frac{|\epsilon_K|}{\Delta m(B_d)} \propto \sin^2 \phi_1 \simeq 0.42 \cdot \text{UNC}$$

with the factor $\text{UNC}$ parametrising the uncertainties

$$\text{UNC} \simeq \left( \frac{0.04}{|V_{cb}|} \right) \left( \frac{0.72}{x_d} \right) \left( \frac{\eta_{QCD}^{(B)}}{0.55} \right) \left( \frac{0.62}{\eta_{QCD}^{(K)}} \right) \left( \frac{2B_B}{3B_K} \right) \left( \frac{f_B}{160 \text{ MeV}} \right)^2$$

where $x_d \equiv \Delta m(B_d)/\Gamma(B_d)$; $\eta_{QCD}^{(B)}$ and $\eta_{QCD}^{(K)}$ denote the QCD radiative corrections for $H(\Delta B = 2)$ and $H(\Delta S = 2)$, respectively; $B_B$ and $B_K$ express the expectation value of $H(\Delta B = 2)$ or $H(\Delta S = 2)$ in units of the ‘vacuum saturation’ result which is given in terms of the decay constants $f_B$ and $f_K$ (where the latter is known). The main uncertainty is obviously of a theoretical nature related to the hadronic parameters $B_B$, $B_K$ and $f_B$: as discussed before, state-of-the-art theoretical technologies yield $B_B \simeq 1$, $B_K \simeq 0.8 \pm 0.2$ and $f_B \simeq 180 \pm 30 \text{ MeV}$ where the latter range might turn out to be anything but conservative! Eq.(248) represents an explicit illustration that some CP asymmetries in $B^0$ decays are huge.

For $m_t \simeq 180$ GeV the $c\bar{c}$ and $c\bar{t} + t\bar{c}$ contributions to $\epsilon_K$ are still sizeable; nevertheless Eq.(248) provides a good approximation. Furthermore $\sin^2 \phi_1$ can still be expressed reliably as a function of the hadronic matrix elements:

$$\sin^2 \phi_1 = f(B_B f_B^2 / B_K)$$

It will become obvious why this is relevant.

The general idea is, of course, to construct the triangle as accurately as possible and then probe it; i.e. search for inconsistencies that would signal the intervention of New Physics. A few remarks on that will have to suffice here.

As indicated before we can expect the value of $|V_{ub}/V_{cb}|$ to be known to better than 10% and hope for $|V_{td}|$ to be determined with decent accuracy as well. The triangle will then be well determined or even overdetermined. Once the first asymmetry in $B$ decays that can be interpreted reliably – say in $B_d \rightarrow \psi K_S$ – has been measured and $\phi_1$ been determined, the triangle is fully constructed from $B$ decays alone. Furthermore one has arrived at the first sensitive consistency check of the triangle: one compares the measured value of $\sin^2 \phi_1$ with Eq.(248) to infer which value of $B_B f_B^2$ is thus required; this value is inserted into the Standard Model expression for $\Delta m(B_d)$ together with $m_t$ to see whether the experimental result is reproduced.

A host of other tests can be performed that are highly sensitive to

- the presence of New Physics and
• to some of their salient dynamical features.

Details can be found in the ample literature on that subject.

8 Oscillations and CP Violation in Charm Decays – The Underdog’s Chance for Fame

It is certainly true that

• $D^0 - \bar{D}^0$ oscillations proceed very slowly in the Standard Model and

• CP asymmetries in $D$ decays are small or even tiny within the KM ansatz.

Yet the relevant question quantitatively is: how slow and how small?

8.1 $D^0 - \bar{D}^0$ Oscillations

Bounds on $D^0 - \bar{D}^0$ oscillations are most cleanly expressed through ‘wrong-sign’ semileptonic decays:

$$r_D = \frac{\Gamma(D^0 \rightarrow l^- X)}{\Gamma(D^0 \rightarrow l^+ X)} \approx \frac{1}{2} \left( x_D^2 + y_D^2 \right)$$  \hspace{1cm} (251)

with $x_D = \Delta m_D/\Gamma_D$, $y_D = \Delta \Gamma_D/2 \Gamma_D$. It is often stated that the Standard Model predicts

$$r_D \leq 10^{-7} \overset{\approx}{=} x_D, \; y_D \leq 3 \times 10^{-4}$$  \hspace{1cm} (252)

I myself am somewhat flabbergasted by the boldness of such predictions. For one should keep the following in mind for proper perspective: there are quite a few channels that can drive $D^0 - \bar{D}^0$ oscillations – like $D^0 \rightarrow K \bar{K}$, $\pi \pi \rightarrow \bar{D}^0$ or $D^0 \rightarrow K^- \pi^+ \rightarrow \bar{D}^0$ – and they branching ratios on the (few) $\times 10^{-3}$ level. In the limit of $SU(3)_{FI}$ symmetry all these contributions have to cancel of course. Yet there are sizeable violations of $SU(3)_{FI}$ invariance in $D$ decays, and one should have little confidence in an imperfect symmetry to ensure that a host of channels with branching ratios of order few $\times 10^{-3}$ will cancel as to render $x_D, y_D \leq 3 \times 10^{-4}$. To say it differently: The relevant question in this context is not whether $r_D \sim 10^{-7} \div 10^{-6}$ is a possible or even reasonable Standard Model estimate, but whether $10^{-6} \leq r_D \leq 10^{-4}$ can reliably be ruled out! I cannot see how anyone could make such a claim with the required confidence.

The present experimental bound is

$$r_D|_{\text{exp}} \leq 3.4 \times 10^{-3} \overset{\approx}{=} x_D, \; y_D \leq 0.1$$  \hspace{1cm} (253)

\footnote{For the $K^- \pi^+$ mode this represents the average of its Cabibbo allowed and doubly Cabibbo suppressed incarnations.}
to be compared with a conservative Standard Model bound
\[ r_D|_{SM} < 10^{-4} \cong y_D, \quad x_D|_{SM} \leq 10^{-2} \]
(254)
New Physics on the other hand can enhance \( \Delta m_D \) (though not \( \Delta \Gamma_D \)) very considerably up to
\[ x_D|_{NP} \sim 0.1, \]
(255)
i.e. the present experimental bound.

### 8.2 CP Violation involving \( D^0 - \bar{D}^0 \) Oscillations

One can discuss this topic in close qualitative analogy to \( B \) decays. First one considers final states that are CP eigenstates like \( K^+K^- \) or \( \pi^+\pi^- \) \[16\]:
\[
\text{rate}(D^0(t) \to K^+K^-) \propto e^{-\Gamma_D t} \left( 1 + (\sin \Delta m_D t) \cdot \text{Im} \frac{q}{p} \hat{\rho}_{K^+K^-} \right) \approx \\
\approx e^{-\Gamma_D t} \left( 1 + \frac{\Delta m_D t}{\Gamma_D} \cdot \frac{t}{\tau_D} \cdot \text{Im} \frac{q}{p} \hat{\rho}_{K^+K^-} \right) 
\]
(256)
With \( x_D|_{SM} \leq 10^{-2} \) and \( \text{Im} \frac{q}{p} \hat{\rho}_{K^+K^-}|_{KM} \sim \mathcal{O}(10^{-3}) \) one arrives at an asymmetry of around \( 10^{-5} \), i.e. for all practical purposes zero, since it presents the product of two very small numbers. Yet with New Physics one conceivably has \( x_D|_{NP} \leq 0.1 \), \( \text{Im} \frac{q}{p} \hat{\rho}_{K^+K^-}|_{NP} \sim \mathcal{O}(10^{-1}) \) leading to an asymmetry that could be as large as of order 1%. Likewise one should compare the doubly Cabibbo suppressed transitions \[17, 18\]:
\[
\text{rate}(D^0(t) \to K^+\pi^-) \propto e^{-\Gamma_D t \tau_D} \tan^4 \theta_C |\hat{\rho}_{K\pi}|^2 .
\]
\[
\cdot \left[ 1 - \frac{1}{2} \Delta \Gamma_D t + \frac{(\Delta m_D t)^2}{4 \tan^4 \theta_C |\hat{\rho}_{K\pi}|^2} + \frac{\Delta \Gamma_D t}{2 \tan^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Re} \left( \frac{p}{q} \hat{\rho}_{K\pi} \right) - \\
\cdot \frac{\Delta m_D t}{\tan^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Im} \left( \frac{p}{q} \hat{\rho}_{K\pi} \right) \right] 
\]
(257)
\[
\text{rate}(\bar{D}^0(t) \to K^-\pi^+) \propto e^{-\Gamma_D t \tau_D} \tan^4 \theta_C |\hat{\rho}_{K\pi}|^2 .
\]
\[
\cdot \left[ 1 - \frac{1}{2} \Delta \Gamma_D t + \frac{(\Delta m_D t)^2}{4 \tan^4 \theta_C |\hat{\rho}_{K\pi}|^2} + \frac{\Delta \Gamma_D t}{2 \tan^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Re} \left( \frac{p}{q} \hat{\rho}_{K\pi} \right) + \\
\cdot \frac{\Delta m_D t}{\tan^2 \theta_C |\hat{\rho}_{K\pi}|} \text{Im} \left( \frac{p}{q} \hat{\rho}_{K\pi} \right) \right] 
\]
(258)
where
\[
\tan^2 \theta_C \cdot \hat{\rho}_{K\pi} = \frac{T(D^0 \to K^+\pi^-)}{T(D^0 \to K^-\pi^+)} \quad \tan^2 \theta_C \cdot \hat{\rho}_{K\pi} = \frac{T(\bar{D}^0 \to K^-\pi^+)}{T(\bar{D}^0 \to K^+\pi^-)} ;
\]
(259)
in such New Physics scenarios one would expect a considerably enhanced asymmetry of order 1%/\( \tan^2 \theta_C \sim 20\% \) at the cost of smaller statistics.

Effects of that size would unequivocally signal the intervention of New Physics!
8.3 Direct CP Violation

As explained before a direct CP asymmetry requires the presence of two coherent amplitudes with different weak and different strong phases. Within the Standard Model (and the KM ansatz) such effects can occur in Cabibbo suppressed modes, yet not in Cabibbo allowed or doubly Cabibbo suppressed modes. There is a subtlety involved in this statement. Consider for example $D^+ \to K^0\pi^+$. At first sight it appears to be a Cabibbo allowed mode described by a single amplitude without the possibility of an asymmetry. However

- due to $K^0 - \bar{K}^0$ mixing the final state can be reached also through a doubly Cabibbo suppressed reaction, and the two amplitudes necessarily interfere;
- because of the CP violation in the $K^0 - \bar{K}^0$ complex there is an asymmetry that can be predicted on general grounds

$$\frac{\Gamma(D^+ \to K^0\pi^+) - \Gamma(D^- \to K^0\pi^+)}{\Gamma(D^+ \to K^0\pi^+) + \Gamma(D^- \to K^0\pi^+)} \approx -2\text{Re}\epsilon_K \approx -3.3 \cdot 10^{-3} \approx \frac{\Gamma(D^+ \to K_L\pi^+) - \Gamma(D^- \to K_L\pi^+)}{\Gamma(D^+ \to K_L\pi^+) + \Gamma(D^- \to K_L\pi^+)};$$

(260)

- If New Physics contributes to the doubly Cabibbo suppressed amplitude $D^+ \to K^0\pi^+$ (or $D^- \to \bar{K}^0\pi^-$) then an asymmetry could occur quite conceivably on the few percent scale;
- such a manifestation of New Physics would be unequivocal; against the impact of $\epsilon_K$, Eq. (260) it could be distinguished not only through the size of the asymmetry, but also how it surfaces in $D^+ \to K_L\pi^+$ vs. $D^- \to K_L\pi^-$: if it is New Physics one has

$$\frac{\Gamma(D^+ \to K^0\pi^+) - \Gamma(D^- \to K^0\pi^+)}{\Gamma(D^+ \to K^0\pi^+) + \Gamma(D^- \to K^0\pi^+)} = -\frac{\Gamma(D^+ \to K_L\pi^+) - \Gamma(D^- \to K_L\pi^+)}{\Gamma(D^+ \to K_L\pi^+) + \Gamma(D^- \to K_L\pi^+)}$$

(261)

i.e., the CP asymmetries in $D \to K^0\pi$ and $D \to K_L\pi$ differ in sign – in contrast to Eq. (260).

9 Baryogenesis in the Universe

9.1 The Challenge

One of the most intriguing aspects of big bang cosmology is to ‘understand’ nucleosynthesis, i.e. to reproduce the abundances observed for the nuclei in the universe as dynamically generated rather than merely dialed as input values. This challenge

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22 The effect could well reach the $10^{-3}$ and exceptionally the $10^{-2}$ level.
has been met successfully, in particular for the light nuclei, and actually so much so that it is used to obtain information on dark matter in the universe, the number of neutrinos etc. It is natural to ask whether such a success could be repeated for an even more basic quantity, namely the baryon number density of the universe which is defined as the difference in the abundances of baryons and antibaryons:

\[
\Delta n_{\text{Bar}} \equiv n_{\text{Bar}} - n_{\text{Bar}'}
\]  

(262)

Qualitatively one can summarize the observations through two statements:

- The universe is not empty.
- The universe is almost empty.

More quantitatively one finds

\[
r_{\text{Bar}} \equiv \frac{\Delta n_{\text{Bar}}}{n_\gamma} \sim \text{few} \times 10^{-10}
\]  

(263)

where \(n_\gamma\) denotes the number density of photons in the cosmic background radiation. Actually we know more, namely that at least in our corner of the universe there are practically no primary antibaryons:

\[
n_{\text{Bar}'} \ll n_{\text{Bar}} \ll n_\gamma
\]  

(264)

It is conceivable that in other neighbourhoods antimatter dominates and that the universe is formed by a patchwork quilt of matter and antimatter dominated regions with the whole being matter-antimatter symmetric. Yet it is widely held to be quite unlikely – primarily because no mechanism has been found by which a matter-antimatter symmetric universe following a big bang evolution can develop sufficiently large regions with non-vanishing baryon number. While there will be statistical fluctuations, they can be nowhere near large enough. Likewise for dynamical effects: baryon-antibaryon annihilation is by far not sufficiently effective to create pockets with the observed baryon number, Eq.(263). For the number density of surviving baryons can be estimated as \(50\)

\[
n_{\text{Bar}} \sim \frac{n_\gamma}{\sigma_{\text{annih}} m_N M_{\text{PL}}} \simeq 10^{-19} n_\gamma
\]  

(265)

where \(\sigma_{\text{annih}}\) denotes the cross section of nucleon annihilation, \(m_N\) and \(M_{\text{Pl}}\) the nucleon and Planck mass, respectively. Hence we conclude for the universe as a whole

\[
0 \neq \frac{n_{\text{Bar}}}{n_\gamma} \simeq \frac{\Delta n_{\text{Bar}}}{n_\gamma} \sim \mathcal{O}(10^{-10})
\]  

(266)

which makes more explicit the meaning of the statement quoted above that the universe has been observed to be almost empty, but not quite. Understanding this double observation is the challenge we are going to address now.
9.2 The Ingredients

The question is: under which condition can one have a situation where the baryon number of the universe that vanishes at the initial time – which for all practical purposes is the Planck time – develops a non-zero value later on:

\[ \Delta n_{\text{Bar}}(t = t_{\text{Pl}} \simeq 0) = 0 \implies \Delta n_{\text{Bar}}(t = 'today') \neq 0 \] (267)

One can and should actually go one step further in the task one is setting for oneself: explaining the observed baryon number as dynamically generated no matter what its initial value was!

In a seminal paper that appeared in 1967 Sakharov listed the three ingredients that are essential for the feasibility of such a program [51, 1]:

1. Since the final and initial baryon number differ, there have to be baryon number violating transitions:

\[ \mathcal{L}(\Delta n_{\text{Bar}} \neq 0) \neq 0 \] (268)

2. CP invariance has to be broken. Otherwise for every baryon number changing transition \( N \rightarrow f \) there is its CP conjugate one \( \bar{N} \rightarrow \bar{f} \) and no net baryon number can be generated. I.e.,

\[ \Gamma(N \xrightarrow{\mathcal{L}(\Delta n_{\text{Bar}} \neq 0)} f) \neq \Gamma(\bar{N} \xrightarrow{\mathcal{L}(\Delta n_{\text{Bar}} \neq 0)} \bar{f}) \] (269)

is needed.

3. Unless one is willing to entertain thoughts of CPT violations, the baryon number and CP violating transitions have to proceed out of thermal equilibrium. For in thermal equilibrium time becomes irrelevant globally and CPT invariance reduces to CP symmetry which has to be avoided, see above:

\[ \text{CPT invariance} \xrightarrow{\text{thermal equilibrium}} \text{CP invariance} \] (270)

It is important to keep in mind that these three conditions have to be satisfied simultaneously. The other side of the coin is, however, the following: once a baryon number has been generated through the concurrence of these three effects, it can be washed out again by these same effects.

9.3 GUT Baryogenesis

Sakharov’s paper was not noticed (except for [52]) for several years until the concept of Grand Unified Theories (=GUTs) emerged starting in 1974 [53]; for those naturally provide all three necessary ingredients:
1. Baryon number changing reactions have to exist in GUTs. For placing quarks and leptons into common representations of the underlying gauge groups – the hallmark of GUTs – means that gauge interactions exist changing baryon and lepton numbers. Those gauge bosons are generically referred to as $X$ bosons and have two couplings to fermions that violate baryon and/or lepton number:

$$X \leftrightarrow qq, \ q\bar{l}$$  \hspace{1cm} (271)

2. Those models are sufficiently complex to allow for several potential sources of CP violation. Since $X$ bosons have (at least) two decay channels open CP asymmetries can arise

$$\Gamma(X \to qq) = (1 + \Delta q)\Gamma_q, \quad \Gamma(X \to q\bar{l}) = (1 - \Delta l)\Gamma_l$$  \hspace{1cm} (272)

$$\Gamma(\bar{X} \to \bar{q}\bar{q}) = (1 - \Delta q)\Gamma_q, \quad \Gamma(\bar{X} \to \bar{q}l) = (1 + \Delta l)\Gamma_l$$  \hspace{1cm} (273)

where

$$\begin{align*}
\text{CPT} & \implies \Delta_q\Gamma_q = \Delta_l\Gamma_l \\
\text{CP} & \implies \Delta_q = 0 = \Delta_l \\
\text{C} & \implies \Delta_q = 0 = \Delta_l
\end{align*}$$  \hspace{1cm} (274, 275, 276)

3. Grand Unification means that a phase transition takes place around an energy scale $M_{GUT}$. For temperatures $T$ well above the transition point – $T \gg M_{GUT}$ – all quanta are relativistic with a number density

$$n(T) \propto T^3$$  \hspace{1cm} (277)

For temperatures around the phase transition – $T \sim M_{GUT}$ – some of the quanta, in particular those gauge bosons generically referred to as $X$ bosons acquire a mass $M_X \sim O(M_{GUT})$ and their equilibrium number density becomes Boltzmann suppressed:

$$n_X(T) \propto (M_XT)^{3/2}\exp\left(-\frac{M_X}{T}\right)$$  \hspace{1cm} (278)

More $X$ bosons will decay according to Eq.(271) than be regenerated from $qq$ and $q\bar{l}$ collisions ultimately bringing the number of $X$ bosons down to the level described by Eq.(278). Yet that will take some time; the expansion in the big bang cosmology leads to a cooling rate that is so rapid that thermal equilibrium cannot be maintained through the phase transition. That means that $X$ bosons decay – and in general interact – out of thermal equilibrium [1].

61
To the degree that the back production of $X$ bosons in $qq$ and $q\bar{q}$ collisions can be ignored one finds as an order-of-magnitude estimate

$$r_{\text{Bar}} \sim \frac{\frac{4}{3} \Delta_q \Gamma_q - \frac{2}{3} \Delta_l \Gamma_l}{\Gamma_{\text{tot}}} \frac{n_X}{n_0} = \frac{\frac{2}{3} \Delta_q \Gamma_q n_X}{\Gamma_{\text{tot}} n_0}$$

(279)

with $n_X$ denoting the initial number density of $X$ bosons and $n_0$ the number density of the light decay products. The three essential conditions for baryogenesis are thus naturally realized around the GUT scale in big bang cosmologies, as can be read off from Eq. (279):

- $\Gamma_q \neq 0$ representing baryon number violation;
- $\Delta_q \neq 0$ reflecting CP violation and
- the absence of the back reaction due to an absence of thermal equilibrium.

The fact that this problem can be formulated in GUT models and answers obtained that are very roughly in the right ballpark is a highly attractive feature of GUTs, in particular since this was not among the original motivations for constructing such theories.

On the other hand it would be highly misleading to claim that baryogenesis has been understood. There are serious problems in any attempt to have baryogenesis occur at a GUT scale:

- A baryon number generated at such high temperatures is in grave danger to be washed out or diluted in the subsequent evolution of the universe.
- Very little is known about the dynamical actors operating at GUT scales and their characteristics – and that is putting it mildly. Actually even in the future we can only hope to obtain some slices of indirect information on them.

Of course it would be premature to write-off baryogenesis at GUT scales, yet it might turn out that it is best characterised as a proof of principle – namely that the baryon number of the universe can be understood as dynamically generated – rather than as a semi-quantitative realization.

### 9.4 Electroweak Baryogenesis

Baryogenesis at the electroweak scale is the most actively analyzed scenario at present. For it possesses several highly attractive features:

- We know that dynamical landscape fairly well.
  - In particular CP violation has been found to exist there.

\[23\text{Due to thermalization effects one can have } n_0 \gg 2n_X.\]
- A well-studied phase transition, namely the spontaneous breaking
\[
SU(2)_L \times U(1) \longrightarrow U(1)_{QED}
\] (280)
takes place.

- Future experiments will certainly probe that dynamical regime with ever increasing sensitivity, both by searching for the on-shell production of new quanta – like SUSY and/or Higgs states – and the indirect impact through quantum corrections on rare decays and CP violation.

However at this point the reader might wonder: "What about the third required ingredient, baryon number violation? At the electroweak scale?" It is often not appreciated that the electroweak forces of the Standard Model by themselves violate baryon number, though in a very subtle way. We find here what is called an anomaly: the baryon number current is conserved on the classical, yet not on the quantum level:
\[
\partial_\mu J^\text{Bar}_\mu = \partial_\mu \sum_q (\bar{q}_L \gamma_\mu q_L) = \frac{g^2}{16\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu} \neq 0
\] (281)

where \( G_{\mu\nu} \) denotes the electroweak field strength tensor
\[
G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]
\] (282)
and \( \tilde{G}_{\mu\nu} \) its dual:
\[
\tilde{G}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}
\] (283)
The right hand side of Eq.(281) can be written as the divergence of a current
\[
G_{\mu\nu} \tilde{G}_{\mu\nu} = \partial_\mu K_\mu, \quad K_\mu = 2\epsilon_{\mu\nu\alpha\beta} \left( A_\nu \partial_\alpha A_\beta + \frac{2}{3} i g A_\nu A_\alpha A_\beta \right)
\] (284)

A total derivative is usually unobservable since partial integration allows to express its contribution through a surface integral at infinity. The field strength tensor \( G_{\mu\nu} \) indeed vanishes at infinity – but not necessarily the gauge potential \( A_\mu \). To be more specific: The field configuration at infinity is that of a ground state for which \( G_{\mu\nu} = 0 \) holds. Yet that property does not define it uniquely: ground states get differentiated by the value of their \( K \) charge, i.e. the space integral of \( K_0 \), the zeroth component of the current \( K_\mu \) constructed from their gauge field configuration. This integral reflects differences in the gauge topology of the ground states and therefore is called the topological charge. While this charge is irrelevant for abelian gauge theories where the last term in Eq.(284) necessarily vanishes, it becomes relevant for non-abelian theories. We have encountered this phenomenon already in our discussion of the Strong CP Problem that is driven by the axial quark current not being conserved in the strong interactions of QCD. It is often referred to as ‘Chiral’ Anomaly since it breaks chiral invariance, or ‘Triangle’ Anomaly since it is produced
by a triangular fermion loop diagram or ‘Adler-Bell-Jackiw’ Anomaly named after
the authors who discovered it.

The concrete impact of the triangle anomaly on the physics depends on the
specifics of the theory: here because of the chiral nature of the weak interactions
it induces baryon number violation. Eq.(281) and Eq.(284) show that the difference $J_{\mu}^{\text{Bar}} - K_{\mu}$ is conserved. The transition from one ground state to another which
represents a tunneling phenomenon is thus accompanied by a change in baryon num-
ber. Elementary quantum mechanics tells us that this baryon number violation is
described as a barrier penetration and exponentially suppressed at low temperatures
or energies \[55\]:

$$\text{Prob}(\Delta n_{\text{Bar}} \neq 0) \propto \exp\left(-\frac{16\pi^2}{g^2}\right) \sim \mathcal{O}(10^{-160})$$

– a suppression that reflects the tiny size of the weak coupling.

There is a corresponding anomaly for the lepton number current implying that
lepton number is violated as well with the selection rule

$$\Delta n_{\text{Bar}} - \Delta n_{\text{lept}} = 0 . \quad (285)$$

This is usually referred to by saying that $B - L$, the difference between baryon and
lepton number, is still conserved.

At sufficiently high energies this huge suppression of baryon number changing
transition rates will evaporate since the transition between different ground states
can be achieved classically through a motion over the barrier. The question then is
at which energy scale this will happen and how quickly baryon number violation will
become operative. Some semi-quantitative observations can be offered and answers
given \[56, 1\].

There are special field configurations – called sphalerons – that carry the topolog-
ical $K$ charge. In the Standard Model they induce effective multistate interactions
among left-handed fermions that change baryon and lepton number by three units
each:

$$\Delta n_{\text{Bar}} = \Delta n_{\text{lept}} = 3 \quad (286)$$

At high energies where the weak bosons $W$ and $Z$ are massless, the height of the transition barrier between different ground states vanishes likewise and the change of baryon number can proceed in an unimpeded way and presumably faster than
the universe expands. Thermal equilibrium is then maintained and any baryon
asymmetry existing before this era is actually washed out \[24\]. Rather than generate
a baryon number sphalerons act to drive the universe back to matter-antimatter
symmetry at this point in its evolution.

At energies below the phase transition, i.e. in the broken phase of $SU(2)_L \times U(1)$
baryon number is conserved for all practical purposes as pointed out above.

The value of $\Delta n_{\text{Bar}}$ as observed today can thus be generated only in the transition
from the unbroken high energy to the broken low energy phase. With $\Delta n_{\text{Bar}} \neq 0$
processes operating there the issue now turns to the strength of the phase transition:

\[24\] To be more precise, only $B + L$ is erased within the Standard Model whereas $B - L$ remains unchanged.
is it relatively smooth like a second order phase transition or violent like a first order one? Only the latter scenario can support baryogenesis.

A large amount of interesting theoretical work has been on the thermodynamics of the Standard Model in an expanding universe. Employing perturbation theory and lattice studies one has arrived at the following result: for light Higgs masses up to around 70 GeV, the phase transition is first order, for larger masses it is second order [57]. Since no such light Higgs states have been observed at LEP, one infers that the phase transition is second order thus apparently foreclosing baryogenesis occurring at the electroweak scale.

We have concentrated here on the questions of thermal equilibrium and baryon number while taking CP violation for granted since it is known to operate at the electroweak scale. Yet most authors – with the exception of some notable heretics – agree that the KM ansatz is not at all up to this task: it fails by several orders of magnitude. On the other hand New Physics scenarios of CP violation – in particular of the Higgs variety – can reasonably be called upon to perform the task.

9.5 Leptogenesis Driving Baryogenesis

If the electroweak phase transition is indeed a second order one, sphaleron mediated reactions cannot drive baryogenesis as just discussed and they will wipe out any pre-existing $B + L$ number. Yet if at some high energy scales a lepton number is generated the very efficiency of these sphaleron processes can communicate this asymmetry to the baryon sector through them maintaining conservation of $B - L$.

There are various ways in which such scenarios can be realized. The simplest one is to just add heavy right-handed Majorana neutrinos to the Standard Model. This is highly attractive in any case since it enables us to implement the see-saw mechanism for explaining why the observed neutrinos are (practically) massless; it is also easily embedded into $SO(10)$ GUTs.

The basic idea is the following [58]:

- A primordial lepton asymmetry is generated at high energies well above the electroweak phase transition:
  - Since a Majorana neutrino $N$ is its own CPT mirror image, its dynamics necessarily violate lepton number. It will possess at least the following classes of decay channels:
    \[
    N \rightarrow lH , \bar{l}H
    \]  

    with $l$ and $\bar{l}$ denoting a light charged or neutral lepton or anti-lepton and $H$ and $\bar{H}$ a Higgs or anti-Higgs field, respectively.

- A CP asymmetry will in general arise
  \[
  \Gamma(N \rightarrow lH) \neq \Gamma(\bar{N} \rightarrow \bar{l}H)
  \]  

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through a KM analogue in the neutrino mass matrix (which can be quite different from the mass matrix for charged leptons).

– These neutrino decays are sufficiently slow as to occur out of thermal equilibrium around the energy scale where the Majorana masses emerge.

• The resulting lepton asymmetry is transferred into a baryon number through sphaleron mediated processes in the unbroken high energy phase of $SU(2)_L \times U(1)$:

$$\langle \Delta n_{lep} \rangle = \frac{1}{2} \langle \Delta n_{lep} + \Delta n_{Bar} \rangle + \frac{1}{2} \langle \Delta n_{lep} - \Delta n_{Bar} \rangle \implies \frac{1}{2} \langle \Delta n_{lep} - \Delta n_{Bar} \rangle$$

(289)

• The baryon number thus generated survives through the subsequent evolution of the universe.

9.6 Wisdom – Conventional and Otherwise

We understand how nuclei were formed in the universe given protons and neutrons. Obviously it would be even more fascinating if we could understand how these baryons were generated in the first place. We do not possess a specific and quantitative theory successfully describing baryogenesis. However leaving it at that statement would – we believe – miss the main point. We have learnt which kinds of dynamical ingredients are necessary for baryogenesis to occur in the universe. We have seen that these ingredients can be realized naturally:

• GUT scenarios for baryogenesis provide us with a proof of principle that such a program can be realized. In practical terms however they suffer from various shortcomings:

  – Since the baryon number is generated at the GUT scales, very little is and not much more might ever be known about that dynamics.

  – It appears quite likely that a baryon number produced at such high scales is subsequently washed out.

• The highly fascinating proposal of baryogenesis at the electroweak phase transition has attracted a large degree of attention – and deservedly so:

  – A baryon number emerging from this phase transition would be in no danger of being diluted substantially.

  – The dynamics involved here is known to a considerable degree and will be probed even more with ever increasing sensitivity over the coming years.

However it seems that the electroweak phase transition is of second order and thus not sufficiently violent.
A very intriguing variant turns some of the vices of sphaleron dynamics into virtues by attempting to understand the baryon number of the universe as a reflection of a primary lepton asymmetry. The required new dynamical entities – Majorana neutrinos and their decays – obviously would impact on the universe in other ways as well.

The challenge to understand baryogenesis has already inspired our imagination, prompted the development of some very intriguing scenarios and thus has initiated many fruitful studies – and in the end we might even be successful in meeting it!

10 The Cathedral Builders’ Paradigm

10.1 The Paradigm

The dynamical ingredients for numerous and multi-layered manifestations of CP and T violations do exist or are likely to exist. Accordingly one searches for them in many phenomena, namely in

- the neutron electric dipole moment probed with ultracold neutrons at ILL in Grenoble, France;
- the electric dipole moment of electrons studied through the dipole moment of atoms at Seattle, Berkeley and Amherst in the US;
- the transverse polarization of muons in $K^- \rightarrow \mu^- \bar{\nu}\pi^0$ at KEK in Japan;
- $\epsilon'/\epsilon_K$ as obtained from $K_L$ decays at FNAL and CERN and soon at DAΦNE in Italy;
- in decay distributions of hyperons at FNAL;
- likewise for $\tau$ leptons at CERN, the beauty factories and BES in Beijing;
- CP violation in the decays of charm hadrons produced at FNAL and the beauty factories;
- CP asymmetries in beauty decays at DESY, at the beauty factories at Cornell, SLAC and KEK, at the FNAL collider and ultimately at the LHC.

A quick glance at this list already makes it clear that frontline research on this topic is pursued at all high energy labs in the world – and then some; techniques from several different branches of physics – atomic, nuclear and high energy physics – are harnessed in this endeavour together with a wide range of set-ups. Lastly, experiments are performed at the lowest temperatures that can be realized on earth – ultracold neutrons – and at the highest – in collisions produced at the LHC. And all of that dedicated to one profound goal. At this point I can explain what
I mean by the term "Cathedral Builders’ Paradigm". The building of cathedrals required interregional collaborations, front line technology (for the period) from many different fields and commitment; it had to be based on solid foundations – and it took time. The analogy to the ways and needs of high energy physics are obvious – but it goes deeper than that. At first sight a cathedral looks like a very complicated and confusing structure with something here and something there. Yet further scrutiny reveals that a cathedral is more appropriately characterized as a complex rather than a complicated structure, one that is multi-faceted and multi-layered – with a coherent theme! One cannot (at least for first rate cathedrals) remove any of its elements without diluting (or even destroying) its technical soundness and intellectual message. Neither can one in our efforts to come to grips with CP violation!

10.2 Summary

- We know that CP symmetry is not exact in nature since $K_L \rightarrow \pi\pi$ proceeds and presumably because we exist, i.e. because the baryon number of the universe does not vanish.

- If the KM mechanism is a significant actor in $K_L \rightarrow \pi\pi$ transitions then there must be large CP asymmetries in the decays of beauty hadrons. In $B^0$ decays they are naturally measured in units of 10%!

- Some of these asymmetries are predicted with high parametric reliability.

- New theoretical technologies will allow us to translate such parametric reliability into quantitative accuracy.

- Any significant difference between certain KM predictions for the asymmetries and the data reveals the intervention of New Physics. There will be no ‘plausible deniability’.

- We can expect 10 years hence the theoretical uncertainties in some of the predictions to be reduced below 10%.

- I find it likely that deviations from the KM predictions will show up on that level.

- Yet to exploit this discovery potential to the fullest one will have to harness the statistical muscle provided by beauty production at hadronic colliders.

10.3 Outlook

I want to start with a statement about the past: The comprehensive study of kaon and hyperon physics has been instrumental in guiding us to the Standard Model.
• The $\tau - \theta$ puzzle led to the realization that parity is not conserved in nature.

• The observation that the production rate exceeded the decay rate by many orders of magnitude – this was the origin of the name ‘strange particles’ – was explained through postulating a new quantum number – ‘strangeness’ – conserved by the strong, though not the weak forces. This was the beginning of the second quark family.

• The absence of flavour-changing neutral currents was incorporated through the introduction of the quantum number ‘charm’, which completed the second quark family.

• CP violation finally led to postulating yet another, the third family.

All of these elements which are now essential pillars of the Standard Model were New Physics at that time!

I take this historical precedent as clue that a detailed, comprehensive and thus necessarily long-term program on beauty physics will lead to a new paradigm, a new Standard Model!

CP violation is a fundamental as well as mysterious phenomenon that we have not understood yet. This is not surprising: after all according to the KM mechanism CP violation enters through the quark mass matrices; it thus relates it to three central mysteries of the Standard Model:

• How are fermion masses generated? 

• Why is there a family structure?

• Why are there three families rather than one?

In my judgement it would be unrealistic to expect that these questions can be answered through pure thinking. I strongly believe we have to appeal to nature through experimental efforts to provide us with more pieces that are surely missing in the puzzle. CP studies are essential in obtaining the full dynamical information contained in the mass matrices or – in the language of v. Eichendorff’s poem quoted in the beginning, ”to find the magic word” that will decode nature’s message for us.

Considerable progress has been made in theoretical engineering and developing a comprehensive CP phenomenology from which I conclude:

• $B$ decays constitute an almost ideal, certainly optimal and unique lab. Personally I believe that even if no deviation from the KM predictions were uncovered, we would find that the KM parameters, in particular the angles of the KM triangle, carry special values that would give us clues about New Physics. Some very interesting theoretical work is being done about how GUT dynamics in

\[25\] Or more generally: how are masses produced in general? For in alternative models CP violation enters through the mass matrices for gauge bosons and/or Higgs bosons.
particular of the SUSY (or Supergravity) variety operating at very high scales would shape the observable KM parameters.

• A comprehensive analysis of charm decays with special emphasis on $D^0 - \bar{D}^0$ oscillations and CP violation is a moral imperative! Likewise for $\tau$ leptons.

• A vigorous research program must be pursued for light fermion systems, namely in the decays of kaons and hyperons and in electric dipole moments. After all it is conceivable of course that no CP asymmetries are found in $B$ decays on a measurable level. Then we would know that the KM ansatz is not a significant actor in $K_L \rightarrow \pi\pi$, that New Physics drives it – but what kind of New Physics would it be? Furthermore even if large CP asymmetries were found in $B$ decays, it could happen that the signals of New Physics are obscured by the large ‘KM background’. This would not be the case if electric dipole moments were found or a transverse polarization of muons in $K_{\mu3}$ decays.

• Close feedback between experiment and theory will be essential.

As the final summary: insights about Nature’s Grand Design that can be obtained from a comprehensive and detailed program of CP studies

• are of fundamental importance,

• cannot be obtained any other way and

• cannot become obsolete!

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