Improved understanding of the peaking phenomenon existing in the di-$J/\psi$ invariant mass spectrum newly from the CMS Collaboration

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Very recently, the CMS Collaboration reported the peaking phenomenon existing in the di-$J/\psi$ invariant mass spectrum from $pp$ collision, by which the $X(6900)$ structure announced by the LHCb Collaboration was confirmed, but also more enhancement structures were discovered. Facing such novel phenomenon, in this work we indicate that these new features reflected from the CMS measurement provide a good implication for a dynamical mechanism which reproduces the novel peaking phenomenon in the reported $J/\psi$-pair mass spectrum well. This mechanism is due to the special reactions, where different charmonium pairs directly produced by $pp$ collision may transit into final state of $J/\psi J/\psi$. The present work provides a special viewpoint to decode these observed fully-charm structures in the $J/\psi$-pair invariant mass spectrum.

I. INTRODUCTION

The discovery of a series of new hadron states has stimulated theorists’ extensive interest in the study of exotic hadronic configuration with more constituent quarks and gluons since 2003. With the continuous efforts from experimentalists and theorists, more and more manifestly exotic structures have been identified, which include hidden-charm pentaquark $P_c(4312)^+, P_c(4440)^+, P_c(4457)^+$ [1], a series of charmoniumlike $XYZ$ states as hidden-charm tetraquark candidates [2–5], and doubly charmed tetraquark $T_{cc}^{++}$ and so on (see review articles for more progresses [7–12]). Therefore, how to decode the nature of these exotic hadronic matters has become an extremely interesting research topic in hadron physics.

Different from other new hadronic states, the $X(6900)$ was firstly reported in the di-$J/\psi$ invariant mass spectrum from LHCb [13], where its minimal quark constituents is fully-charm $cc\bar{c}\bar{c}$. At the ICHEP 2022 conference, the ATLAS Collaboration reported the evidence of a four-charm-quark excess [14]. Due to the absence of light flavor degrees of freedom, the $X(6900)$ structure can provide a unique platform to study the dynamics inside multi-body heavy flavor system. In addition to the discovery of the di-$J/\psi$ structure around 6.9 GeV, the LHCb’s data also implies the existence of a broad enhancement structure near the production threshold of the $J/\psi$ pair [13]. Thus, these novel fully-heavy enhancement phenomena have inspired numerous theoretical explanations, which include the mainstreaming fully-charm tetraquark states (compact or di-quark type) [15–44], the dynamical mechanism from the scattering of double charmonia [45–50], the gluonic tetracharm hybrid [51] and a Higgs-like boson [52], etc. Additionally, the production property and decay behavior of fully-charm tetraquark states were also discussed by several research groups [53–59]. We should pay more experimental and theoretical efforts to clarify the nature of peaking phenomenon in the di-$J/\psi$ mass distribution.

Very recently, the CMS Collaboration released their measurements on the $J/\psi$-pair mass spectrum by using 135 fb$^{-1}$ proton-proton data at center of mass energies of 13 TeV [60], which not only confirmed the existence of the $X(6900)$ reported by LHCb with significance 9.4σ, but also found the signals of some new peaking structures. By using a relativistic S-wave Breit-Wigner function, the resonance parameters of two new structures, the $X(6600)$ and $X(7300)$, were obtained [60]:

\[ m_{X(6600)} = 6552 \pm 10 \pm 12 \text{ MeV}, \]
\[ \Gamma_{X(6600)} = 124 \pm 19 \pm 34 \text{ MeV}, \]
\[ m_{X(7300)} = 7287 \pm 19 \pm 5 \text{ MeV}, \]
\[ \Gamma_{X(7300)} = 95 \pm 46 \pm 20 \text{ MeV}. \]

Undoubtedly, the new CMS measurement can provide more refine hints to decode the novel peaking phenomena appeared in the double $J/\psi$ mass spectrum.

Inspired by the experimental results newly from CMS, in this work, we further apply a dynamical mechanism to understand these observed fully-charm enhancement structures. Here, the adopted dynamical mechanism is based on a special reaction that different charmonium pairs from direct hadroproduction may transit into final states of di-$J/\psi$ [15]. This dynamical mechanism has succeed in explaining the $X(6900)$ as a threshold cusp structure resulted from the intermediate $\chi_{c0}\chi_{c1}$ scattering [45]. As suggested in Ref. [61], we firstly extend this
dynamical mechanism by considering higher order multiloop contributions. By applying the extended dynamical model to fit the line shape of the di-$J/\psi$ mass distribution measured by both CMS and LHCb, we demonstrate that in addition to the intermediate $J/\psi J/\psi$ (3686) and $\chi_{c0} \chi_{c1}$ scattering which are enough to explain the LHCb data, the contributions of the remaining two allowed characteristic intermediate channels $\chi_{c2} \chi_{c1}$ and $\chi_{c2} \chi_{c2}$ in the measured energy region can be explicitly revealed by the features appeared in the CMS measurement, which can correspond to the newly observed fully-charm enhancement structure around 6.6 GeV and 7.3 GeV, respectively. Thus, this is a direct evidence to support this dynamical production mechanism based on a double charmonia scattering. Furthermore, we discuss the origins of these di-$J/\psi$ peaking structures in the dynamical mechanism under two fitting schemes. The present study is helpful to improve the understanding of the peaking phenomena in the di-$J/\psi$ invariant mass spectrum.

This paper is organized as follows. After Introduction, in Sec. II we present the theoretical framework of the extended dynamical mechanism of producing the fully-charm enhancement structures in di-$J/\psi$ mass spectrum. In Sec. III we discuss the improved understanding for the peaking phenomena in double $J/\psi$ mass spectrum based on the CMS measurement. Finally, this paper ends with the discussion and conclusion in Sec. IV.

II. THE EXTENDED DYNAMICAL MECHANISM

The hadroproduction of a double charmonium in high energy proton-proton collider is usually achieved by the $gg \rightarrow (cc)(cc) + X$ process in the single parton scattering (SPS) and the $gggg \rightarrow (gg \rightarrow cc)(gg \rightarrow cc) + X$ in the double parton scattering (DPS). This production mechanism usually plays a main role for producing the continuum distribution in the invariant mass spectrum of a double charmonium. Whereas, the observed novel enhancement phenomenon in di-$J/\psi$ mass spectrum by recent LHCb and CMS measurement tell us that a new origin of a double $J/\psi$ hadroproduction should exist, which has stimulated some theoretical discussions on a dynamical production mechanism of $J/\psi$ pair.

This new dynamical mechanism is based on a special reaction, where various combinations of double charmonia that are directly produced via both the SPS and DPS processes are transferred into final states of $J/\psi J/\psi$. The combination selection of intermediate charmonium pair depends on the quantum number conservation of reaction system. This dynamical mechanism was proposed in our previous work for the first time and have been found to produce the $X(6000)$ structure well. In this work, we further extend this reaction picture to the multiloop case, where the higher order coupling contribution of an intermediate double charmonia scattering to the same intermediate double charmonia will be taken into account. These contributions have been demonstrated to provide more dynamical information for the intermediate scattering process. The corresponding schematic diagrams are presented in Fig. 1 where the interaction between an intermediate charmonium pair and a transferred double charmonia is absorbed into a vertex. Here, it is worth mentioning that we have to ignore the coupled channel effects in the subsequent analysis because of the absence of the relevant experimental data.

The concrete theoretical calculations on the direct production of a double charmonium in high energy proton-proton collisions by the SPS and DPS mechanisms are quite challenged. Fortunately, here we only focus on the line shape of their contribution in the di-$J/\psi$ invariant mass spectrum. So the S-wave direct production amplitude of a double charmonium $H_{cc}^i H_{cc}^j$ marked in Fig. 1 can be parameterized as

$$A_{direct}^{ij} = \left( g_{direct}^{ij} \right)^2 e^{i \alpha_{m_{ij}}} \frac{1}{8\pi} \sqrt{\lambda(m_{ij}^2, m_i^2, m_j^2)} \left( \frac{m_{ij}^2}{m_i^2} \right)^{1/2}$$

(1)

where $m_{ij}$ is the invariant mass of double charmonia $H_{cc}^i H_{cc}^j$. The Kållen function is defined to be $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. For the one-loop process with an $S$-wave interaction between intermediate charmonium pairs shown in Fig. 1 the scattering amplitude of producing a $J/\psi J/\psi$ becomes the one proportional to the scalar two-point loop integral, whose analytical form within a nonrelativistic form can be given by, in the rest frame of di-$J/\psi$,

$$L_{ij}(m_{J/\psi J/\psi}) = \int \frac{dq^4}{(2\pi)^4} \frac{i2\sqrt{2e^{-2q^2/\alpha^2}}}{(q^2 - m_i^2 + i\epsilon)(q^2 - m_j^2 + i\epsilon)}$$

$$\cong \frac{-1}{4m_i m_j} \left[ -2\mu \alpha + \frac{2\mu \sqrt{2\alpha m_0} \left( \text{erf} \left( \frac{x_{p} m_0}{\alpha} \right) - i \right)}{2\pi e^{-\frac{x_{p} m_0}{\alpha}}} \right]$$

(2)

where $\mu = (m_i m_j)/(m_i + m_j)$, $m_0 = m_{J/\psi J/\psi} - m_i - m_j$, and $m_i (m_j)$ is the hadron mass of an intermediate charmonium $H_{cc}^i (H_{cc}^j)$. Here, an exponential form factor $e^{-2q^2/\alpha^2}$ with a cutoff parameter $\alpha$ is introduced to avoid the ultraviolet divergence of scalar two-point loop

FIG. 1. The schematic diagrams for the dynamical production mechanism of a double $J/\psi$, where $H_{cc}^i$ stands for allowed intermediate charmonium states. Here, the gray rectangle represents the direct production of a double charmonium in hadron colliders.
integral. In our previous work [45], the intermediate charmonium pairs \( H_{cc} J_{cc} \), \( J/\psi J/\psi \), \( \eta_c \chi_{c1} \), \( J/\psi \phi_c \), and \( \chi_{c2} \chi_{c1} \) with \( J = 0, 1, 2 \) were considered, which are the charmonium combination composed of radially ground states and completely cover the concerned energy region from 6.194 GeV to 7.300 GeV. Here, an important evidence supporting these combinations is that the direct hadroproduction rates of these radially ground charmonia have been proved to be comparable with that of \( J/\psi \) by both experiments [80][81] and theoretical estimations from nonrelativistic QCD (NRQCD) [82][90]. Of course, if one further extends the selection criterion to radially exciting charmonium states, there should be two more channels \( J/\psi J/\psi(3686) \), \( J/\psi J/\psi(3770) \) in the same energy region, whose possible contributions will be also investigated in this work. It is worth noticing that we include the width effects of intermediate charmonium states by replacing \( m_i \) in Eq. (2) with \( (m_i - i\Gamma_i / 2) \).

When considering all of combinations of charmonium pair, one can find that there exist thirteen intermediate channels in the di-\( J/\psi \) energy region from 6.194 GeV to 7.300 GeV. Obviously, it is impossible to include so many dynamical reactions in a practical theoretical analysis. Fortunately, we found that their threshold positions are mainly concentrated in five local energy regions, which provides us convenience to deal with this problem. Considering the fact that the contributions of intermediate channels in the same local energy region may overlap and then behave like one peak structure, we can choose a representative channel to absorb the contributions from other nearby scattering channels. Thus, all of possible dynamical scattering channels for double \( J/\psi \) hadroproduction in the energy range below 7.3 GeV include \( J/\psi J/\psi \), \( \eta_c \chi_{c1} \), \( J/\psi J/\psi(3686) \), \( \chi_{c2} \chi_{c1} \), and \( \chi_{c2} \chi_{c2} \).

The two-loop and three-loop production amplitudes in Fig. 2 are proportional to \( C_{ij} L_{ij}(m_J J/\psi) \) and \( -C_{ij}^3 L_{ij}(m_J J/\psi) \), respectively, and higher order loop contributions can be written accordingly, where \( C_{ij} \) represents the coupling strength of intermediate charmonium pairs scattering to the double charmonia. If we sum up all loop diagram contributions, the line shape of the invariant mass distribution of di-\( J/\psi \) caused by the extended dynamical mechanism are given by

\[
\mathcal{A}_{ij}^2(m_{J/\psi J/\psi}) = \frac{g_{ij}^2 L_{ij}(m_{J/\psi J/\psi})^2}{(1+C_{ij} L_{ij}(m_{J/\psi J/\psi}))^2} e^{i\phi_{mn} m_{J/\psi J/\psi} p_{J/\psi J/\psi}} \tag{3}
\]

and

\[
\mathcal{A}_{ij}^3(m_{J/\psi J/\psi}) = \frac{g_{ij}^2 L_{ij}(m_{J/\psi J/\psi})^2}{(1+C_{ij} L_{ij}(m_{J/\psi J/\psi}))^2} e^{i\phi_{mn} m_{J/\psi J/\psi} p_{J/\psi J/\psi}} \tag{4}
\]

for two types of system parity \( P = + \) and \( P = - \), respectively, in which \( p_{J/\psi} \) is the momentum of a final state \( J/\psi \), and the factor \( e^{i\phi_{mn} m_{J/\psi J/\psi}} = e^{i\phi_{mn} m_{ij}} \) is introduced to describe the energy dependence of the direct hadroproduction amplitude of intermediate charmonium pairs \( H_{cc} J_{cc} \), as shown in Eq. (1). Here, it is worth noting that the coupling constant \( g_{ij} \) involves two contributions, the production ratio and the transition from \( H_{cc} J_{cc} \) to the di-\( J/\psi \) channel. The \( c_0 = -1.5 \) and \( c_0' = -1.0 \) are extracted by fitting the LHCB data [43][45]. Here, the involved intermediate channels \( H_{cc} H_{cc}^J = J/\psi J/\psi, \chi_{c2} \chi_{c1}, J/\psi J/\psi(3686), J/\psi J/\psi(3770) \) and \( H_{cc}^J H_{cc}^J = \eta_c \chi_{c1}, J/\psi \phi_c, \chi_{c2} \chi_{c1} \) are related to the parity-even and parity-odd, respectively, and the parity-odd amplitude corresponds to the hadroproduction of a \( P \)-wave double \( J/\psi \).

The line shape for the total invariant mass spectrum of producing a double charmonium \( J/\psi J/\psi \) in high energy proton-proton colliders can be written as

\[
\mathcal{A}^2 = |\mathcal{A}^{J/\psi J/\psi}_{\text{direct}}(m_{J/\psi J/\psi}) + \sum_{mn} e^{i\phi_{mn}} \mathcal{A}_{mn}(m_{J/\psi J/\psi})|^2 \\
+ |\mathcal{A}^{J/\psi J/\psi'}_{\text{direct}}(m_{J/\psi J/\psi}) + \sum_{mn} e^{i\phi_{mn}} \mathcal{A}'_{mn}(m_{J/\psi J/\psi})|^2, \tag{5}
\]

where \( \phi_{mn} \) is the phase between direct contribution and the corresponding rescattering dynamical process, and a background term \( \mathcal{A}^{J/\psi J/\psi'}_{\text{direct}} = \frac{g_{ij}^2 \lambda_{ij}(m_{J/\psi J/\psi})^2}{\sum m_{J/\psi J/\psi}^2} e^{i\phi_{mn} m_{J/\psi J/\psi} p_{J/\psi J/\psi}} \) describes the direct production of double \( J/\psi \) with \( P = -1 \). Generally, the coupled channel effect in \( T \) matrix may bring some imaginary part term for amplitude, which can be partially absorbed in phase angle factor \( e^{i\phi_{mn}} \).
In addition, there may exist an inherent phase difference between the production vertex of different intermediate charmonium pairs. Here, it is worth mentioning that the $J/\psi J/\psi$ spectrum production in high energy $pp$ collision is from a complex inclusive reaction $pp \to J/\psi J/\psi X$, where the unitarity may be violated. Here, we only choose the phase angle $\phi^{mn} = 0$ and $\pi$ for the consideration of reducing the fitting parameters, which can relate to the constructive and destructive situation, respectively.

In the extended dynamical mechanism, there exist an integral singularity at the threshold of $(m_i + m_j)$ appearing at the on-shell of two intermediate charmonia because of a square root branch point $\sqrt{m_{J/\psi J/\psi} - m_i - m_j}$ from the scattering amplitude. From Eqs. (3)-(4), it can be seen that the extended dynamical amplitude returns to the single loop situation when $C_{ij} L_{ij} << 1$ as shown in Ref. [45], in which this integral singularity of amplitude will cause a non-resonant cusp line shape on the distribution of the $m_{J/\psi J/\psi}$ and its peak position is almost exactly at the threshold of inducing the on-shell intermediate charmonium pairs. Interestingly, if the coupling constant $C_{ij}$ is enough strong, the threshold cusp may transform into a pole structure, whose configuration should be quite novel and is totally different from the compact type of $ccc \bar{c}$.

**III. WHAT CAN WE LEARN FROM THE DI-$J/\psi$ SPECTRUM REPORTED BY CMS?**

With the above preparations, we can study the di-$J/\psi$ mass spectrum based on the extended dynamical model. In our previous research work, we have indicated that an underlying broad enhancement near threshold and the reported $X(6900)$ structure in LHCb data can be reproduced well by the parity-odd channel $\chi c_1 \eta_c$ and parity-even channel $\chi c_0 \chi c_1$, respectively [15]. However, the authors in Ref. [14] found that the broad enhancement near threshold can also be explained by the destructive contribution from the parity-even channel $J/\psi \psi(3686)$, whose threshold just exactly locates at an obvious dip position between two peaking structures. This fact means that the LHCb data cannot distinguish these two channel contributions. Fortunately, the new CMS measurement can reveal more critical information to clarify this problem, where more details on the line shape of novel fully-charm enhancement structures were presented compared with previous LHCb data.

By checking the relevant experimental data of CMS, we found a new data accumulation in the vicinity of 6.6 GeV, which is not far from the threshold of the $\chi c_1 \eta_c$ channel. In order to demonstrate that this accumulation may be caused by the parity-odd contribution of $\chi c_1 \eta_c$ instead of $J/\psi \psi(3686)$, we performed an independent fit to the CMS and LHCb data by considering two parity-even intermediate channels $J/\psi \psi(3686)$ and $\chi c_0 \chi c_1$, whose fitting results are shown in Fig. 2. It can be seen that the line shape of di-$J/\psi$ invariant mass spectrum measured by LHCb can be reproduced well in the present scenario, which is consistent with conclusion of Ref. [16]. However, it is apparent that the CMS data cannot be described well in the same scheme, especially for two energy regions with a large divergence. The first one is the energy range between 6.6 GeV and 6.7 GeV, which just corresponds to the threshold of the rescattering channel $\chi c_1 \eta_c$. The second region is around 7.1 GeV. As a matter of fact, our former work has given the predictions for the existence of possible fully-charm structures in this energy region although the LHCb experiment does not show any obvious hints [15, 14]. It seems that $\chi c_1 \chi c_1$, $\chi c_1 \chi c_2$ and $\chi c_2 \chi c_2$ contribute to the energy position of $(7.03 \sim 7.13)$ GeV. Hence, in the following, we will explore whether the CMS data on these critical energy region of the double $J/\psi$ mass spectrum can be reproduced by the inclusion of two new intermediate double charmonium channels $\chi c_1 \eta_c$ and $\chi c_2 \chi c_2$.

Based on the extended dynamical production mechanism, the complete theoretical analysis to the CMS experimental data of invariant mass spectrum of $J/\psi J/\psi$ is presented in Fig. 3. And the corresponding fitted parameters are summarized in Table I. One can see that the novel peaking structures shown in the CMS data can be reproduced well by the red solid line in the fit-I scheme, where the fitting $\chi^2 / d.o.f. = 0.657$ is obtained. Specifically, the experimental data around 6.6 and 7.1 GeV can indeed be described by the contribution from the $\chi c_1 \eta_c$ and $\chi c_2 \chi c_2$ channel, respectively. This means that the CMS measurement provides a definite evidence for confirming the contribution of parity-odd channel $\chi c_1 \eta_c$ in
the $J/\psi$-pair mass spectrum for the first time. From Table I, one can find that the central values of the fitted coupling constants $C_{\chi_{c1}\eta_c} = 342$, $C_{\chi_{c0}\chi_{c1}} = 380$ and $C_{\chi_{c2}\chi_{c2}} = 145$ with a cutoff $\alpha = 0.871$ are relatively larger than $C_{J/\psi\psi}(3686) = -20$ in the fit-II scheme. A such strong interaction may convert the threshold singularity to a dynamically generated pole structure. By performing a corresponding pole analysis, we found that the $J/\psi\psi(3686)$ channel produces a threshold cusp structure and three resonance poles from the rescattering channels $\chi_{c1}\eta_c$, $\chi_{c0}\chi_{c1}$ and $\chi_{c2}\chi_{c2}$ really appear in the second Riemann sheet, whose pole positions are determined to be

$$E_{\chi_{c1}\eta_c} = (6.625 - 0.107i) \text{ GeV},$$

$$E_{\chi_{c0}\chi_{c1}} = (7.050 - 0.089i) \text{ GeV},$$

$$E_{\chi_{c2}\chi_{c2}} = (7.170 - 0.108i) \text{ GeV},$$

respectively. Their individual line shape on the invariant mass spectrum of double $J/\psi$ can be found in Fig. 3 (b), in which there is an obvious line shape difference between cusp and resonance solution. Although the resonance solution is from the best $\chi^2$ fitting for the experimental data, we must point out that it is difficult to understand a such strong coupling constant $C$ for a double charmonium scattering to a charmonium pairs, where the long-distance interaction from the direct exchange of light medium mesons should be relatively suppressed due to the absence of light quark freedom in the fully-heavy reaction system. Here, some interesting unknown non-perturbative dynamics should play important role, which may be reliably revealed by the lattice QCD in the future.

Beforehand, we further perform an analysis of the fit-II scheme to the CMS’s experimental data by assuming the coupling strengths $C$ for three channels of $\chi_{c1}\eta_c$, $\chi_{c0}\chi_{c1}$ and $\chi_{c2}\chi_{c2}$ are not powerful enough to generate resonance poles in the dynamical mechanism. The corresponding best fitting results are presented in Fig. 3 (a) with a dashed green line, where a $\chi^2/d.o.f.$ value is 0.699. It can be seen that the fitted $\chi^2$ value in fit-II scheme is slightly larger than that in fit-I scheme but there is no essential difference between two fitting line shapes of di-$J/\psi$ mass spectrum, where several enhancement or dip structures in di-$J/\psi$ spectrum can be reproduced in both of the fit-I and fit-II scenarios. By a pole analysis, we found that there are no any pole structures in the fit-II scheme and the corresponding central values of the coupling constants $C_{\chi_{c1}\chi_{c1}} = -21.3$, $C_{J/\psi\psi(3686)} = -32.0$, $C_{\chi_{c0}\chi_{c1}} = 20.0$ and $C_{\chi_{c2}\chi_{c2}} = -41.4$ with a cutoff $\alpha = 1.813$ shown in Table II will induce a threshold cusp line shape on the double $J/\psi$ spectrum. This finding actually means that the threshold cusp or resonance pole solution cannot be definitely distinguished from the present experimental precision, which should depend on the concrete coupling strength of double charmonia scattering.

In addition to the resonance poles, ignoring small width effect from intermediate charmonium states, we found several virtual poles below the respective threshold in two fit schemes, which are

$$E_{J/\psi\psi(/2S)}^I = 6.191 \text{ GeV}, \quad E_{J/\psi\psi(/2S)}^I = 6.718 \text{ GeV},$$

$$E_{J/\psi\psi(/2S)}^{II} = 6.188 \text{ GeV}, \quad E_{J/\psi\psi(/2S)}^{II} = 6.687 \text{ GeV},$$

where superscript I and II represent the fit-I and fit-II scheme, respectively. It can be seen that these virtual pole effects produce the obvious threshold cusp structures.

We also studied the scattering lengths of each intermediate charmonium channels in two fit schemes. The interaction property of double charmonia scattering can be reflected by scattering length $a_0$, i.e., positive $a_0$ and negative $a_0$ correspond to attractive and repulsive interaction in the absence of a bound state pole, respectively [9]. By the effective range expansion, the scattering amplitude in the immediate vicinity of the threshold can be written as

$$A_0^{-1} = \frac{1}{a_0} - i\sqrt{2\mu E},$$

where $\mu = m_i m_j / (m_i + m_j)$ and $E$ is the energy relative to the two-body threshold. Obviously, the scattering length can be obtained by $a_0 = A_0(E)|_{E \to 0}$. Without considering the width effect from intermediate charmonium states, we calculated the scattering length $a_0(ij)$ for each scattering channels and summarized them in Table II. It can be seen that their scattering lengths imply an attractive interaction, which are not contradictory with the produced threshold cusps or pole structures.

Anyway, it can be found that the threshold positions of all of allowed combinations of intermediate charmonium pairs in the di-$J/\psi$ energy region from 6.20 GeV to 7.30 GeV can be assigned to four main regions, which are (6.45 $\sim$ 6.64) GeV, (6.783 GeV, (6.87 $\sim$ 7.00) GeV and (7.03 $\sim$ 7.13) GeV. Very interestingly, we can notice that two observed enhancements and two dips in the CMS’s measurement data can exactly correspond to the above four characteristic energy regions in sequence as shown in Fig. 4. This perfect agreement should provide a very strong hint to support our proposed dynamical

| Parameters | Fit I | Fit II |
|------------|------|-------|
| $g_{\chi_{c1}\chi_{c1}}/g_{\text{direct}}$ | 0.0575 $\pm$ 0.0009 | 0.0699 $\pm$ 0.0010 |
| $g_{\chi_{c0}\chi_{c1}}/g_{\text{direct}}$ | 125 $\pm$ 26 | 28.2 $\pm$ 2.7 |
| $g_{\chi_{c2}\chi_{c2}}/g_{\text{direct}}$ | -26.1 $\pm$ 4.7 | -16.4 $\pm$ 1.7 |
| $g_{\chi_{c0}\chi_{c1}}/g_{\text{direct}}$ | 32.9 $\pm$ 5.8 | 16.2 $\pm$ 1.4 |
| $g_{\chi_{c2}\chi_{c2}}/g_{\text{direct}}$ | -15.3 $\pm$ 6.1 | -12.3 $\pm$ 1.4 |
| $C_{J/\psi\eta_c}$ | -144 $\pm$ 14 | -82.3 $\pm$ 1.8 |
| $C_{\chi_{c1}\chi_{c1}}$ | 342 $\pm$ 107 | 213 $\pm$ 18.7 |
| $C_{J/\psi\psi(3686)}$ | -20 $\pm$ 40 | -32.0 $\pm$ 5.5 |
| $C_{\chi_{c0}\chi_{c1}}$ | 380 $\pm$ 54 | 20.0 $\pm$ 50.6 |
| $C_{\chi_{c2}\chi_{c2}}$ | 145 $\pm$ 175 | -41.4 $\pm$ 13.3 |
| $\alpha$ (GeV) | 0.871 $\pm$ 0.046 | 1.813 $\pm$ 0.030 |
| $\chi^2/d.o.f.$ | 0.657 | 0.699 |
mechanism for explaining these novel fully-charm peaking structures in the di-$J/\psi$ mass spectrum. We expect that this mechanism can be confirmed in future precise experimental measurements, especially at the Run III of LHC.

TABLE II. The calculated center value of scattering length for each intermediate channels by using parameters in the fit-I and fit-II scheme.

| Intermediate Channel | Fit I | Fit II |
|----------------------|-------|--------|
| $a_0(J/\psi J/\psi)$ (fm) | 2.25 | 1.64 |
| $a_0(\eta_c\chi_{c1})$ (fm) | 0.33 | 0.48 |
| $a_0(J/\psi\psi(3686))$ (fm) | 0.87 | 0.53 |
| $a_0(\chi_{c0}\chi_{c1})$ (fm) | 0.32 | 0.33 |
| $a_0(\chi_{c2}\chi_{c2})$ (fm) | 0.51 | 0.58 |

IV. DISCUSSION AND CONCLUSION

Very recently, the CMS Collaboration reported the measurement results of the invariant mass spectrum of $J/\psi$ pair from $pp$ collisions, where several peaking phenomena were observed [60]). Compared with previously reported $X(6900)$ structure by LHCb [13], the new CMS measurement bring us more important information. In this work, we have studied these newly observed fully-charm enhancement structures in an extended dynamical mechanism. Our basic idea is based on a reaction that different combinations of the intermediate double charmonia directly produced in high energy proton-proton collisions are transferred into final states of $J/\psi J/\psi$. This reaction picture can be further extended to the contribution involving higher order loops.

By employing the extended dynamical model to describe the line shape of the invariant mass spectrum of double $J/\psi$ newly measured by CMS, we have demonstrated that the contributions of all four characteristic intermediate channels $\eta_c\chi_{c1}$, $J/\psi\psi(3686)$, $\chi_{c0}\chi_{c1}$, and $\chi_{c2}\chi_{c2}$ are required in order to reproduce the CMS distribution. This fact means that these new features from the CMS measurement provide a strong evidence to support the dynamical interpretation for the observed fully-charm enhancement structures. Furthermore, we adopted two fitting schemes to explore the origin of the fully-charm peaking phenomena in the dynamical mechanism, in which we concluded that the threshold cusp and resonance pole solution cannot be distinguished from the present experimental precision.

In order to better solve this problem, we suggest two accessible ways here. One can find there exist many combinations of on-shell charmonium pairs in the di-$J/\psi$ mass spectrum because of an approximate heavy quark symmetry in charm sector, whose interference effect usually causes the difficulty of identifying the origin of these novel fully-charm enhancements. Thus, an available method is to measure the invariant mass spectrum of the intermediate channel itself in hadron colliders, such as $\eta_c\chi_{c1}$, $J/\psi\psi(3686)$, $J/\psi\psi(3770)$, $\chi_{c0}\chi_{c1}$, and so on, where the line shape measurement should be helpful to identify the threshold cusp or resonance solution because the contributions of some off-shell channels should be suppressed by the phase space. We noticed that the ATLAS Collaboration recently released the preliminary result of $J/\psi\psi(3686)$ mass spectrum, which just can test our dynamical production mechanism of double charmonia. In Fig. 5 we presented the comparison between the ATLAS data and our predictions for the line shape of the invariant mass spectrum of $J/\psi\psi(3686)$ from the $J/\psi\psi(3686)$ channel in two parameter schemes.

**FIG. 5.** The comparison between the ATLAS data and our predictions for the line shape of the invariant mass spectrum of $J/\psi\psi(3686)$ from the $J/\psi\psi(3686)$ channel in two parameter schemes.
nity determining the coupling behavior of $J/\psi\psi(3686)$ channel.

The second approach is to quantitatively estimate the magnitude of coupling strength of double charmonia scattering by the first principle Lattice QCD theory. Coincidentally, we noticed a recent research work to discuss the property of a dibaryon scattering of $\Omega_{cc}^{++}\Omega_{cc}^{++}$ near unitarity region from lattice QCD [92], which is another novel fully-charm system. We hope that our analysis in this work can stimulate activities from lattice group to study double charmonia scattering, which should be worth expecting in the future.

Finally, we strongly call for more theoretical and experimental studies to concentrate on this kind of novel fully-heavy system, which should be a new important frontier to investigate non-perturbative behavior of strong interaction in the future.

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