The statistics of wide-separation lensed quasars

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ABSTRACT
The absence of any wide-separation gravitational lenses in the Large Bright Quasar Survey is used to place limits on the population of cluster-sized haloes in the universe, and hence constrain a number of cosmological parameters. The results agree with previous investigations in strongly ruling out the standard cold dark matter model but they are consistent with low-density universes in which the primordial fluctuation spectrum matches both cluster abundances and cosmic microwave background measurements. These conclusions are essentially independent of the cosmological constant, which is in stark contrast to the statistics of galaxy lenses. The constraints presented here are nullified if clusters have core radii of \(<10\) kpc, but are free of a number of potential systematic errors, owing to the homogeneity of the data.

Key words: galaxies: clusters: general – cosmology: theory – gravitational lensing.

1 INTRODUCTION
The fraction of quasars which are gravitationally lensed by intervening mass concentrations is a function of the deflector population and the underlying cosmological model, which can then be constrained from the observed lensing frequency (e.g. Kochanek 1993, 1996). In most cases of quasar lensing known to date the principal deflectors are elliptical galaxies (e.g. Keeton, Kochanek & Seljak 1997), and the number of such lenses, together with the distribution of image separations and deflector redshifts, has been used to place quite stringent limits on the galaxy population (e.g. Kochanek 1993) and the cosmological model (e.g. Kochanek 1996; Falco, Kochanek & Muñoz 1998). Unfortunately, the dependencies of the expected lensing probability on the galaxy population and the cosmology cannot be easily separated. For instance, cosmological constant-dominated models overpredict the number of lenses if the local and cosmological galaxy populations are similar, but can be made consistent with observations if high-redshift galaxies are optically thick due to dust (e.g. Kochanek 1996; Malhotra, Rhoads & Turner 1997).

One way to bypass the lack of knowledge about the high-redshift galaxy population is to concentrate on lenses with larger image separations (i.e. \(\approx3\) arcsec). These can only be produced by groups and clusters of galaxies, a population which is more closely linked to the underlying cosmological model. Just four such wide-separation lenses have been confirmed: Q 0957+561 (Walsh, Carswell & Weymann 1979); MG 2016+112 (Lawrence et al. 1984); HE 1104–1805 (Wisotzki et al. 1993); and RXJ 0911+0551 (Bade et al. 1997). There are more than 10 other candidates, but statistical arguments suggest that almost all of them are physically distinct binary quasars (Kochanek, Falco & Muñoz 1999; Mortlock, Webster & Francis 1999). If this interpretation is correct, less than 0.1 per cent of all quasars are lensed with such large image separations. Conversely, theoretical calculations based on N-body simulations (e.g. Wambsganss et al. 1995) and analytical models (e.g. Narayan & White 1988; Kochanek 1995; Mortlock, Hewett & Webster 1996) predict more wide-separation lenses than are observed, if a standard cold dark matter (CDM) cosmology is assumed. However models with either a non-zero cosmological constant or strong biasing (allowing the underlying dark matter distribution to be much smoother than the galaxy distribution) are consistent with both lensing results and cosmic microwave background (CMB) anisotropies (Kochanek 1995).

The main limitations on these results are the lack of knowledge of cluster core radii (If they are greater than \(\approx10\) kpc, the standard models can be reconciled with the data.) and the heterogeneous nature of quasar catalogues used in previous analyses. Both these points are addressed here, with the use of a homogeneous lens survey (Section 2), and the possibility of a finite core included explicitly in the lens model (Section 3). The absence of any lenses in the data can then be used to directly constrain a number of cosmological parameters (Section 4). The accuracy of these inferences is limited by both the simple model used for the deflector population and the size of the quasar sample, as discussed in Section 5.

2 THE LARGE BRIGHT QUasar SURVEY
The Large Bright Quasar Survey (LBQS; Hewett, Foltz & Chaffee
such, the quasar catalogue was generated without any explicit objective-prism plates to generate a list of quasar candidates. As limited object catalogues were then used in combination with Schmidt Telescope (UKST) fields. The resultant magnitude-to-plate, ranging from $m_{\text{lim}} = 18.41$ to 18.85, and there is also a low redshift cut-off at $z = 0.2$.

The specific form of the differential number counts used in the calculation of the lensing magnification bias (see Section 3.3) is

$$\frac{d^2N_q}{dm dz} \propto \frac{1}{10^{\alpha_q (m-m_0)} - 10^{\beta_q (m-m_0)}}.$$

where $m_0 = 19.0 \pm 0.2$ is the quasar break magnitude, $\alpha_q = 0.9 \pm 0.1$ the bright-end slope and $\beta_q = 0.3 \pm 0.1$ the faint-end slope. This functional form is taken from Boyle, Shanks & Peterson (1988) and Kochanek (1996), but the parameter values are chosen to match the LBQS (for $m \leq 19$) and the faint-end slope of the compilation of data presented by Hartwick & Schade (1990). Whilst Hewett, Foltz & Chaffee (1993) found some discrepancies between the LBQS counts and the Boyle et al. (1988) sample, the simple parameterization of equation (1) is sufficiently accurate in the context of this calculation.

As part of the LBQS there was a systematic search for companions within $\sim 10$ arcsec of each quasar. The quality of the data used – the same UKST plates – is such that most companions with $m \leq 21.5$ are found (Hewett et al. 1998), but the search was incomplete for image separations $\Delta \theta \leq 3$ arcsec due to the point spread function of the plates. Given the LBQS magnitude limit of $\sim 18.5$, the companion search has sufficient dynamic range to easily pick out most secondary lensed images, and so can be considered complete in the annulus between 3 and 10 arcsec. It is possible for galactic lenses to produce image separations in this range, but only if they are both nearby and very massive, and thus easily detectable. Hence the LBQS can be used to unambiguously constrain the population of galaxy group and cluster lenses.

The search for neighbouring images has yielded five quasar pairs so far, and it is ‘unlikely that further pairs will be identified’ (Hewett et al. 1998). Two of these pairs have vastly different redshifts, and are not the result of lensing (although their existence is evidence of the effectiveness of the companion search); the other three are either lenses or physical binary quasars. Q 1009–0252 (Hewett et al. 1994) is a gravitational lens, but, with $\Delta \theta = 1.5$ arcsec, it cannot be included in the lens calculation as the small-separation lens search is not well characterized. The second pair, Q 1429–0053 (Hewett et al. 1989) is a potential wide-separation lens, with an image separation of 5.1 arcsec and apparently similar spectra. However principal components analysis (e.g. Murtagh & Hecht 1987) shows that the two spectra are no more alike than a pair of spectra chosen at random from the survey (Mortlock et al. 1999). Combined with the absence of any potential deflector and the statistical arguments of Kochanek et al. (1999), it is highly unlikely that Q 1429–0053 is a lens. The third pair, Q 2153–2056 (Hewett et al. 1998), has an image separation of 7.8 arcsec, but very different spectra, and is even less likely to be a lens. Thus the LBQS almost certainly represents a sample of over 1000 quasars that is devoid of wide-separation lenses.

### 3 THE CALCULATION

A given world model can be characterized by one number: $p_0$, the probability that no wide-separation lenses are observed in the LBQS. This, in turn, is simply the product of the probabilities that each individual quasar is un lensed, so that

$$P_0 = \prod_{q=1}^{N_q} (1 + p_q) = (1 + \langle p_q \rangle)^{N_q},$$

where $N_q$ is the number of quasars in the survey, $p_q$ the probability that the $q$th quasar is lensed, and $\langle p_q \rangle$ the lensing probability averaged over the survey. Note that this is a cumulative probability, and so the normalization is unambiguous.

The likelihood of a given model is identified with $P_0$, thus ignoring any prior information. Hence, from equation (2), the only models that can be rejected at the 99 per cent level by the absence of lenses in the LBQS are those for which $\langle p_q \rangle \approx 0.004$. This is true of a number of popular cosmological scenarios, and so the data at hand are far from redundant in this context.

Having set out the statistical framework for the calculation, the next step is to define the deflector populations and mass distributions (Sections 3.1 and 3.2, respectively), from which the lensing probability can be calculated (Section 3.3).

#### 3.1 Deflector population

The population of collapsed haloes can be estimated most accurately from $N$-body simulations (e.g. White et al. 1987; Efstathiou et al. 1988, but this approach is too computationally expensive to explore a wide range of cosmological models. The obvious alternative is the analytical Press & Schechter (1974) formalism (e.g. Peacock 1999). The halo population is normally given as a co-moving mass function, but the isothermal sphere lens model used in Section 3.2 is parameterized by its line-of-sight velocity dispersion, $\sigma$, rather than mass. Equating the total mass to that inside the virial radius of the isothermal sphere (e.g. Narayan & White 1988; Kochanek 1995) results in the conversion

$$M(\sigma, z) = \frac{(2/1)^3}{10GH(z)} ,$$

where $G$ is Newton’s constant and $H(z) = H_0[\Omega_{\Lambda} (1 + z)^3 + \Omega_m -(\Omega_{\Lambda} + \Omega_m - 1)(1 + z)^2]^{1/2}$. The final expression for the co-moving halo population is then (Mortlock 1999)

$$\frac{d\sigma_q}{d\sigma} = \frac{15\Omega_{m0} H_0^2 H(z)/H_0 \delta_c(z)}{8\pi^3 / 2 \sigma^2 \Delta[M(\sigma, z)]} \times \frac{d \ln \Delta[M(\sigma, z)]}{d \ln \sigma} \exp \left( \frac{\delta_c(z)}{2\Delta[M(\sigma, z)]} \right),$$

where $M(\sigma, z)$ is given in equation (3). The cosmological model enters equation (4) only through $\delta_c(z)$, the extrapolated linear overdensity that would have collapsed at redshift $z$ (e.g. Peebles 1980). The present-day variance on mass scale $M$, $\Delta(M)$, is determined by the power spectrum of density fluctuations. The approximate CDM power spectrum of Efstathiou, Bond & White (1992) is adopted here. Most of the power spectrum parameters have little influence on the lensing likelihood (Kochanek 1995).

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1 All magnitudes are in the $B_1$ system, but this subscript is omitted for brevity.

2 The cosmological model is defined by $\Omega_m$, the present day normalized density of the universe, $\Omega_{\Lambda}$, the current value of the similarly normalized cosmological constant, and Hubble’s constant, $H_0 \approx 70$ km s$^{-1}$ Mpc$^{-1}$. 

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Mortlock 1999), and so only the primordial power-law slope, \( n \), and the normalization, \( \Delta_0 \) (the present-day variance in spheres of radius 8 Mpc), are allowed to vary.

### 3.2 Lens model

The mass distributions of galaxy clusters – the inner regions in particular – are not well constrained by observations or theory: CDM-based N-body simulations suggest that they are singular (e.g. Navarro, Frenk & White 1997); the properties of giant arcs imply they have small core radii (e.g. Fort et al. 1992; Smail et al. 1995, but see also Bartelmann 1996) or very massive central galaxies (Wilkins, Navarro & Bartelmann 1999); and other observational data span the possibilities (e.g. Narayan, Blandford & Nityananda 1984; Mohr et al. 1996; Pointecouteau et al. 1999).

A simple isothermal sphere (e.g. Turner, Ostriker & Gott 1984; Binney & Tremaine 1987) is adopted here because the principal analytic alternative – the Navarro et al. (1997) profile – is inconsistent with the strong lensing properties of clusters (Williams et al. 1999). However the main shortcoming of both these models is the absence of any substructure, the inclusion of which increases image separations by \( \sim 50 \) per cent on average (Bartelmann, Steinmetz & Weiss 1995).

The isothermal profile, parameterized primarily by its velocity dispersion, is assumed to have a core radius, \( r_c \), which scales as \( r_c = r_c(\sigma/\sigma_{c})^{k} \), where \( \sigma_{c} = 1000 \text{ km s}^{-1} \) is chosen arbitrarily. Its lensing properties are usually cast in the language of the singular model’s Einstein radius, \( \theta_{E} \) (which is dependent on the cosmological model; Schneider, Ehlers & Falco 1992), and the critical radius in the lens plane, \( \beta_{\text{crit}} \) (which decreases as the core radius increases). For the definitions of these terms and a further discussion of this lens model, see Hinshaw & Krauss (1987), Kochanek (1996) or Mortlock & Webster (2000).

### 3.3 Lensing probability

From the properties of the lens model it is possible to calculate the probability, \( p_{\text{Q}} \), that a quasar of magnitude \( m_{\text{Q}} \) (in the parent survey) and redshift \( z_{\text{Q}} \) is found to be lensed. This is derived rigorously for a typical lens survey in Kochanek (1996) and Mortlock & Webster (2000), but there are some important differences in the calculation of the magnification bias here. In most lens surveys, the angular separation of the images is less than the angular resolution of the parent survey, and so, for a lensed quasar, \( m_{\text{Q}} \) is given by summing the fluxes of all the images. However, in the case of the UKST data from which the LBQS was selected, only images separated by \( \lesssim 5 \) arcsec appear merged (Webster, Hewett & Irwin 1988, although Hewett et al. 1995 give \( \sim 6 \) arcsec; this discrepancy leads to a \( \sim 10 \) per cent uncertainty in the calculated lensing probability), whereas the search annulus of the lens survey extends from \( \Delta \theta_{\text{min}} = 3 \) arcsec to \( \Delta \theta_{\text{max}} = 10 \) arcsec. This is an unusual situation – the lens survey improves on the parent survey in depth, not resolution. For any lenses with \( \Delta \theta \leq 5 \) arcsec the standard magnification bias is correct, but for wider-separation multiples \( m_{\text{Q}} \) is determined only by the magnification of the brightest individual image.\(^3\) This is often considerably less than the total magnification, and the resultant reduction in the lensing probability can be quite marked, as shown below.

The first step in evaluating \( p_{\text{Q}} \) is to calculate \( p_{\text{Q,d}} \), the probability that a given quasar is lensed by a particular halo, defined by its redshift, \( z_{\text{d}} \), and its velocity dispersion, \( \sigma \). With the normal total magnification bias, this is given by

\[
\rho_{\text{Q,d}} = \frac{\int_{0}^{\frac{\Delta m_{\text{min}}}{2}} 2\pi \beta S(\beta) d^{2}N_{\text{Q}}}{\int_{0}^{\frac{\Delta m_{\text{max}}}{2}} 4\pi d^{2}N_{\text{Q}}},
\]

where the quasar luminosity function, \( d^{2}N_{\text{Q}}/dz dm \), is given in equation (1) and \( S(\beta) \) is the selection function (cf. Kochanek 1996). The latter is approximated by

\[
S(\beta) = H[\Delta \theta_{\text{max}} - \Delta \theta_{\text{min}}] \times H[\Delta \theta(\beta) - \Delta \theta_{\text{min}}] H[\Delta \theta_{\text{max}} - \Delta \theta(\beta)].
\]

where \( H(x) \) is the Heavyside step function.

Equation (5) can be converted to the single image magnification bias simply by replacing \( \mu_{\text{tot}}(\beta) \) with the magnification of the brightest image, \( \mu_{\text{max}}(\beta) \). As \( \mu_{\text{max}} < \mu_{\text{tot}} \) in all cases of multiple imaging, the single image magnification bias always results in a lower value of \( p_{\text{Q,d}} \). The ratio of the two probabilities is particularly simple to calculate if the deflectors are assumed to be singular: the integration variable in equation (5) can be changed to \( \mu_{\text{max}} \) and

\[
\frac{p_{\text{Q,d single}}}{p_{\text{Q,d total}}} = \frac{\int_{2}^{\infty} \frac{2}{(\mu_{\text{max}} - 1)^{2}} d^{2}N_{\text{Q}}}{\int_{2}^{\infty} \frac{8}{\mu_{\text{tot}}^{2}} d^{2}N_{\text{Q}}}.
\]

where \( \Delta m_{\text{min}} \to \infty \) for simplicity. Fig. 1 shows how this ratio varies with \( \Delta m_{\text{min}} \). The main difference between the two situations is the reduction in the amplitude of the high-magnification tail in the case of the single image magnification bias, which in turn has a greater effect on \( p_{\text{Q,d}} \) if the luminosity function is steeper. Thus the wide-separation

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\(^3\)Merging images are not explicitly treated here.
lensing probability for bright quasars (including the LBQS) is up to a factor of three lower than previous calculations would suggest.\footnote{Most relevantly, Kochanek (1995) used the normal magnification bias, but due to the image separations under consideration (up to 1 arcmin), the weaker, single image bias would have given a more realistic value of the lensing probability. For example, the two standard deviation result that weakening the normal bias would give a factor of three lower than previous calculations would suggest.\footnote{Kochanek (1995). For example, the two standard deviation result that}\

The probability that a quasar is lensed by any halo is obtained by integrating $p_{q,d}$ over the deflector population, to give

$$p_q = \int_0^\infty \int_0^\infty \frac{dV}{dz} \frac{dn_d}{d\sigma(z)} p_{q,d} \, d\sigma \, dz,$$

where $dn_d/d\sigma(z)$, the co-moving density of deflectors, is given in Section 3.1.

This must be calculated separately for each quasar in the LBQS, mainly due to their large redshift range, and the resultant likelihoods can be averaged to give $\langle p_q \rangle$. This is shown in Figs 2 and 3 as a function of several model parameters. In Fig. 2(a), the lensing probability initially increases very rapidly with $\Delta_8$, as the velocity dispersion of the largest haloes depends exponentially on $\Delta_8$ (Kochanek 1995). For $\Delta_8 \geq \delta_{\text{crit}}$, however, the lensing likelihood flattens off as the largest haloes produce image separations of $\simeq 10\text{arcsec}$, and so cannot contribute to $\langle p_q \rangle$. As shown in Fig. 2(b), the number of lenses decreases with cluster core radii, as expected (e.g. Kochanek 1995). However the dependence is very weak if $\delta_c = 4$. For a given $\delta_c$, the smaller deflectors (which are capable of producing lenses with $\Delta\theta = 3\text{arcsec}$) are more effective, being nearly singular. Unfortunately this degeneracy means that any limits placed on the other model parameters are weakened by the fact that $\delta_c$ is essentially unconstrained. The situation with respect to $n$ is more promising – as can be seen from Fig. 2(c) the slope of the power spectrum has only a small effect on the lens statistics.

The dependence of $\langle p_q \rangle$ on the cosmological model is illustrated in both Figs 2 and 3. The most striking aspect of these plots is that $\Omega_{m_0}$ is considerably more important than $\Omega_{\Lambda_0}$, whereas it is the cosmological constant that dominates the statistics of lensing by galaxy-scale objects. For a given $\Omega_{m_0}$ the volume element is greater in high-$\Omega_{m_0}$ universes, but clusters form at later times, so the increase of $\langle p_q \rangle$ with $\Omega_{\Lambda_0}$ is only mild. The strong dependence on $\Omega_{m_0}$ comes about primarily as haloes are heavier for a fixed $\Delta_8$. A cluster that has collapsed from an initial perturbation of a given co-moving scale has mass $M \propto \Omega_{m_0}$, velocity dispersion $\sigma \propto \Omega_{m_0}^{-1/3}$ (from equation 3), and hence a lensing cross-section that increases as $\Omega_{m_0}^{-1}$ (e.g. Turner et al. 1984; Kochanek 1995). This dependence is apparent for low-density models (see Fig. 3), but as $\Omega_{m_0} \rightarrow 1$ the predominant haloes have typical image separations that are greater than $\Delta\theta_{\text{max}}$. A lens survey with a broader search annulus than that of the LBQS would be required to probe models with either $\Delta_8$ or $\Omega_{m_0}$ of order unity.

4 RESULTS

The selection criteria for a lens to appear in the LBQS (Section 2) and the lensing calculation described in Section 3 can now be combined to give $P_0$, the probability that the LBQS contains no wide-separation lenses. This is shown for several combinations of model parameters in Fig. 4. The contours shown are for $P_0 = 0.01$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{The average probability that any quasar in the LBQS is observed to be lensed, $\langle p_q \rangle$. The dependence on the normalization, $\Delta_8$, is shown in (a); the variation with the canonical core radius, $r_\ast$, is shown in (b); and the dependence on the slope of the power spectrum, $n$, is shown in (c). The ‘default’ values are $\Delta_8 = 1$, $r_\ast = 0$ and $n = 1$ and the three lines represent different cosmological models: $\Omega_{m_0} = 1.0$ and $\Omega_{\Lambda_0} = 0.0$ (solid lines); $\Omega_{m_0} = 0.3$ and $\Omega_{\Lambda_0} = 0.7$ (dashed lines); and $\Omega_{m_0} = 0.3$ and $\Omega_{\Lambda_0} = 0.7$ (dot-dashed lines). Models with $\langle p_q \rangle > 0.004$ (the horizontal dashed line) can be rejected at the 99 per cent level.}
\end{figure}
(i.e. 99 per cent limits), 0.05 and 0.5, but are only one-sided as models which predict an arbitrarily low number of lenses are perfectly consistent with the data.

Fig. 4(a) shows that $D_8 \approx 0.4$ (with 99 per cent confidence) in an Einstein–de Sitter (EdS) cosmology with a standard CDM spectrum. This is considerably lower than the value of $1.4^{+0.1}_{-0.1}$ inferred from the Cosmic Background Explorer (COBE) data (Smoot et al. 1992). Standard COBE-normalized CDM models can be made consistent with the data if $r_{c*} \geq 10$ kpc and $u_c = 2$, as shown in Fig. 4(b). Although such models are somewhat contrived, the scaling of cluster core radii is so poorly constrained that they cannot be completely ruled out. However it is the same low-density models implied by observations of high-redshift supernovae (e.g. Schmidt et al. 1998; Perlmutter et al. 1999) and the position of the first CMB Doppler peak (e.g. Efstathiou et al. 1999; de Bernardis et al. 2000) that are favoured by the data at hand. There may be a slight discrepancy if measurements of $\Delta_b$ are also considered, as the values inferred from the local cluster population (e.g. Peebles 1989; Frenk et al. 1990; Bahcall, Fan & Cen 1997) and the galaxy–galaxy correlation function (e.g. Maddox et al. 1990; Efstathiou et al. 1992) are slightly higher than allowed in the $\Omega_m = 0.3$ model here. However, if $\Omega_m = 0.2$ and the universe is flat then the greatest source of disagreement with the absence of wide-separation lenses in the LBQS is, somewhat ironically, the low number of observed small-separation lenses that precludes $\Omega_m = 0.8$ (e.g. Kochanek 1996).

It is possible, although very unlikely, that there is a wide-separation lens in the LBQS, as Q 1429–0053 has not been conclusively proved to be a binary quasar and there is still a small amount of the companion search to be completed (Hewett et al. 1998). If such a lens were to be discovered then the limits on the model parameters would be shifted somewhat. Performing a calculation similar to that described in Section 3 gives the limits on the model parameters from the absence of lenses in the LBQS. The solid contours are at $P_0 = 0.01$; the dashed contours are at 0.05 and 0.5. In all panels the vertical axis gives the normalization of the power spectrum, $D_8$. The horizontal axes are: $\Omega_m$ (for both $\Omega_m = 0$ and flat models) in (a); the canonical core radius, $r_{c*}$ (for $u_c = 2$ and $u_c = 4$) in (b); and the slope of the power spectrum, $n$, in (c). In panels (b) and (c) an EdS cosmology is assumed, and $r_{c*} = 0$ and $n = 1$ are assumed except where these parameters are explicitly varied.

There must also be some nearly-singular clusters to account for the morphology of the confirmed wide-separation lenses listed in Section 1. Observations of giant arcs (e.g. Smail et al. 1995; Bartelmann 1996) also argue against such high values of $r_{c*}$.
probability of finding one lens in the LBQS as
\[ P_1 = N_q p_q (1 - \langle p_q \rangle)^{N_q - 1} = N_q \frac{p_q}{N_q} (1 - p_q)^{1/N_q}, \]
where \( N_q = 1055 \) is the number of quasars in the survey. From this conversion, the 1, 5 and 50 per cent contours for \( P_0 \) shown in Fig. 4 become 5, 15 and 35 per cent contours for \( P_1 \), respectively. In other words, the 99 per cent limits would be weakened to 95 per cent limits. The constraints would also become two-sided, but the models that would be ruled out by the presence of a lens have \( \langle p_q \rangle = 10^{-5} \), and would not be considered plausible a priori. Thus the presence of a wide-separation lens in the LBQS would broaden the constraints on the model parameters, but would not qualitatively change the conclusions.

5 CONCLUSIONS
The fact that no wide-separation (\( \Delta \theta \gtrsim 3 \) arcsec) lenses have been discovered in the LBQS places tight constraints on the population of cluster-mass objects in the universe. Assuming a hierarchical theory of halo formation, one-sided limits can then be placed on the number of cluster-mass objects in the universe. Assuming a hierarchical theory of halo formation, one-sided limits can then be placed on the number of cluster-mass objects in the universe. Assuming a hierarchical theory of halo formation, one-sided limits can then be placed on the number of cluster-mass objects in the universe. Assuming a hierarchical theory of halo formation, one-sided limits can then be placed on the number of cluster-mass objects in the universe.

The most straightforward result is that the standard CDM model (i.e. an EdS cosmology with a power spectrum index of \( n = 1 \)) is completely ruled out unless the normalization is very low (\( \Omega_m \Delta \approx 0.4 \) at the 99 per cent level). Only if the core radii of \( \sigma = 1000 \text{km s}^{-1} \) clusters are \( \gtrsim 10 \text{kpc} \), and \( r_c \propto \sigma^2 \) can these limits be avoided. For non-EdS cosmologies, the lensing results are in agreement with most a priori reasonable models (as discussed in Section 4), although \( \Omega_m \lesssim 0.2 \) is implied unless \( \Delta_k \) is low or, again, clusters have large core radii.

Whilst the conclusions are in agreement with most expectations, it is important to consider the various approximations in this calculation, and their effects on the parameter constraints. Fortunately, the lensing probability varies as \( \Omega_m^{4/3} \) and increases exponentially with \( \Delta_k \) (provided \( \Delta_k \lesssim 1 \)), whereas the use of the Press & Schechter (1974) formalism, the calculation of \( \delta_{\text{crit}} \), the choice of a smooth, spherical lens model, and the treatment of the magnification bias all lead to uncertainties of tens of per cent at most. Hence the limits on \( \Omega_m \) and \( \Delta_k \) could be changed by similar amounts. The choice of the mass profile used for the lenses is more important (as illustrated by the effect of \( r_c \)) and it would be especially interesting to perform the above calculation using the Navarro et al. (1997) halo model, as the mass distribution of the haloes in their prescription is assumed to be directly related to the underlying cosmological model.

The LBQS represents an ideal type of data-set for this kind of analysis, as it has well-characterized selection effects and the companion search was both deep and systematic; the only real shortcoming is the size of the survey. Both the Jodrell Bank–Very Large Array Astrometric Survey (JVAS; Patnaik et al. 1992) of \( \sim 2500 \) sources and the smaller Hubble Space Telescope Snapshot Survey (Maoz et al. 1997) of \( \sim 500 \) quasars also contain no confirmed lenses with \( \Delta \theta \gtrsim 5 \) arcsec (Marlow et al. 1998), and so would imply similar one-sided limits to those presented here. The situation should be improved with the advent of much larger surveys, such as the 2-degree Field (2dF) quasar survey (e.g. Boyle et al. 1999a,b), with \( \sim 3 \times 10^5 \) quasars, and the Sloan Digital Sky Survey (SDSS; e.g. Lovey & Pier 1998; Szalay 1998), with \( \sim 10^5 \) quasars. The 2dF survey’s companion search is not particularly deep, and will only include lenses with image separations greater than 8 arcsec. Nonetheless, the sheer size of the data set will allow a lensing determination of \( \Omega_m \) and \( \Delta_k \) to within \( \sim 10 \) per cent. The SDSS quasar survey includes high-resolution imaging, and should thus provide a large sample of both galactic and wide-separation lenses. This will allow \( \Omega_m \), \( \Omega_k \) and \( \Delta_k \) to be constrained simultaneously to within several per cent from lensing statistics alone.

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