Phenomenological Consequences of Non-commutative QED

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Abstract
In the context of the noncommutative QED we consider few phenomena which reflect the noncommutativity. In all of them the new interactions in the Feynmann diagrams that are responsible for the deviation from the standard QED results. These deviations appear as the violations of Lorentz symmetry. We suggest experimental situations where these effects may be observed. The extra phases have far reaching consequences including violation of crossing symmetry. Considering the $e p$ scattering and Compton scattering the electric dipole moments of the electron and the photon is calculated.

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1 Introduction

Noncommutative gauge theories has been the center of extensive attention[1-11]. This interest was generated after the work of CDS [1] and acquired boost after its derivation from string theory [2]. The main property of the noncommutative gauge theory is the existence of a non-zero constant background gauge field induced from the nonzero antisymmetric form in the ten dimensional space where the closed strings live. In the decoupling limit where the open strings propagate only on the branes the effective theory is a noncommutative gauge theory or noncommutative open string theory where the parameters characterizing noncommutativity is derived from the background field [2] [4] [5]. There are two essential points that such theories deviate from the standard gauge theories. One is the breakdown of Lorentz invariance, since obviously a non-zero gauge field strength shows preferred directions, and the other is the introduction of new interaction (three photon vertex ) and modification of the standard ones. This two properties have common origin and will obviously appear in a number of phenomena. In this article we shall consider certain scattering processes where the Lorentz symmetry breaking will manifestly appear . We will choose the simplest set up, the U(1) theory and fermions coupled to it, i.e. noncommutative QED [5] [8]. We follow the notation of [8] where the Feynmann rules are also derived. We consider four different phenomena; $ee, e^+e^-$ scattering, correction to the electrons magnetic moment, and the Compton scattering . An interesting result is that the extended nature of the strings is revealed by observing the dipole dipole interaction of electrons. Scattering of photon on electron shows that they also carry electric dipole proportional to its momentum and square of the noncommutativity parameter. In the following we shall briefly review the Feynman rules and establish our notation. Section 2 presents the results of the application of noncommutative QED to $ee$ and $e^+e^-$ scattering . In section 3, we study electron proton interaction and derive the low energy amplitude from which the effective potential is derived. It shows a correction to the Coulomb potential which is interpreted as a velocity dependent dipole moment of the electron. This velocity dependent dipole can be understood in the frame work of string theory where the
non-trivial background stretches the moving string. Section four is devoted to the Compton scattering. In this case the exchange of photon results in a t-channel contribution to the amplitude which is a characteristic of noncommutative QED. In this case we observe a momentum dependent dipole moment for the photon which is twice as large as that of the electron.

As an example of radiative correction we consider the anomalous magnetic moment of electron. We see that $\mu^a$ acquires a spin independent term proportional to noncommutative parameter. The conclusion is devoted to discussion on possible experiments that may test the Lorentz symmetry breaking and the new interactions.

We start with the noncommutative action for electromagnetic field,

$$S_G = -\frac{1}{4} \int d^Dx \ F_{\mu\nu} \ (x) \ast F^{\mu\nu} \ (x) . \quad (1.1)$$

where $F_{\mu\nu}$ define by

$$F_{\mu\nu} \ (x) \equiv \partial_{\mu} A_{\nu} \ (x) - \partial_{\nu} A_{\mu} \ (x) + i g [A_{\mu} \ (x), A_{\nu} \ (x)]_{\ast} . \quad (1.2)$$

The noncommutativity is coded in the star product given by

$$f \ (x) \ast g \ (x) \equiv e^{\frac{i\theta_{\mu\nu}}{2}} \left. \frac{\partial}{\partial \xi^\mu} \frac{\partial}{\partial \zeta^\nu} \ f \ (x + \xi) \ g \ (x + \zeta) \right|_{\xi = \zeta = 0} , \quad (1.3)$$

$$[x^\mu, x^\nu] = i\theta_{\mu\nu} , \quad (1.4)$$

The fermion fields are introduced by the action [3]:

$$S_F[\bar{\psi}, \psi] = \int d^Dx \left[ i \bar{\psi} \ (x) \ \gamma^\mu \ast D_\mu \psi \ (x) - m \bar{\psi} \ (x) \ast \psi \ (x) \right] , \quad (1.5)$$

where the covariant derivative is defined by:

$$D_\mu \psi \ (x) \equiv \partial_\mu \psi \ (x) + ig A_\mu \ (x) \ast \psi \ (x) . \quad (1.6)$$
For completeness we quote the Feynman rules.

Fermion propagator:

\[
\frac{i}{p^2 - m^2}.
\]

Photon propagator:

\[
-\frac{ig_{\mu\nu}}{k^2}.
\]

Fermion photon vertex:

\[
ie_G^\mu \exp \left( -\frac{i}{2} p_1 \times p_2 \right) \tag{1.7a}
\]

Three photon vertex:

\[-2e \sin \left( \frac{i}{2} k_1 \times k_2 \right) \]
\[
\times \left[ g_{\mu_1\mu_2} (k_1 - k_2)_{\mu_3} + g_{\mu_1\mu_3} (k_3 - k_1)_{\mu_2} + g_{\mu_2\mu_3} (k_2 - k_3)_{\mu_1} \right]
\]

Four photon vertex:

\[
4ie^2 \left[ g_{\mu_1\mu_3} g_{\mu_2\mu_4} - g_{\mu_1\mu_4} g_{\mu_2\mu_3} \right] \sin \left( \frac{1}{2} k_1 \times k_2 \right) \sin \left( \frac{1}{2} k_3 \times k_4 \right)
\]
\[
+ (g_{\mu_1\mu_4} g_{\mu_2\mu_3} - g_{\mu_1\mu_3} g_{\mu_2\mu_4}) \sin \left( \frac{1}{2} k_3 \times k_1 \right) \sin \left( \frac{1}{2} k_2 \times k_4 \right)
\]
\[
+ (g_{\mu_1\mu_2} g_{\mu_3\mu_4} - g_{\mu_1\mu_4} g_{\mu_3\mu_2}) \sin \left( \frac{1}{2} k_1 \times k_4 \right) \sin \left( \frac{1}{2} k_2 \times k_3 \right)
\]

In above expression we have , \( p \times q := p^\mu \theta_{\mu\nu} q^\nu \). Now we consider standard processes but in the new noncommutative context.
\section{\textit{e} \textit{e} and \textit{e}^{+}\textit{e}^{-} scattering}

As our first example consider $ee \rightarrow ee$. The following diagrams take part in the $ee$ scattering

\begin{equation}
M_t = -\frac{e^2}{t} \left( \overline{U}(p_4)\gamma^\mu U(p_1) \right) \left( \overline{U}(p_3)\gamma_\mu U(p_2) \right) \exp i \frac{1}{2}(p_4 \times p_1 + p_2 \times p_3)
\end{equation}

\begin{equation}
M_u = -\frac{e^2}{u} \left( \overline{U}(p_3)\gamma^\mu U(p_1) \right) \left( \overline{U}(p_4)\gamma_\mu U(p_2) \right) \exp i \frac{1}{2}(p_3 \times p_1 + p_2 \times p_4)
\end{equation}

$p_1, p_2, p_3$ and $p_4$ are the momenta of the electorns as show in the above diagrams.

The only new factors are the phases which changes the interference terms in the cross section. Then in high energy region partial cross sections is proportional to:

\begin{equation}
|M|^2 = 2e^2 \left( \frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} + \frac{2s^2}{ut} \cos(p_3 \times p_4) \right)
\end{equation}

As one can see the interference term makes a difference when the outgoing momenta are not along the total incoming momentum $P = p_1 + p_2$. The origin of this interference is that in the two Feynman diagrams the outgoing electrons are relatively crossed and their phases are different. Unfortunately this difference vanishes in the center of mass frame. Therefore to set up an experiment to observe the interference one can devise a situation where the two beams are of different energy or are not colinear.

If $\Theta_i := \epsilon_{ijk}\theta_{jk}$ and $\overrightarrow{p}_{r\perp}$ is the projection of momentum $\overrightarrow{p}_r$ on the plane perpendicular
to $\Theta$, 

$$\cos(p_3 \times p_4) = \cos(p_3 \times p) = \cos(\theta p_3 p_\perp \sin \alpha) \quad (2.12)$$

where $\vec{p} = p_1 + p_2$ and $\sin \alpha$ is angle between $\vec{p}_\perp$ and $\vec{p}_3^\perp$. Hence the cross section goes under periodic change when the angle of the perpendicular component of the outgoing electron changes. If $\mathcal{M}_0$ is the amplitude in the standard QED then

$$\Delta |\mathcal{M}|^2 = |\mathcal{M}_0|^2 - |\mathcal{M}|^2 = \frac{2s^2}{ut} \left( \cos \theta \ p_3^\perp p_\perp \sin \alpha \right). \quad (2.13)$$

One can see that the partial cross section decrease in the noncommutative case. In the forward direction the cross section is unchanged. An interesting and important point is that the change in noncommutative cross section depends on the frame of reference. In the center of mass frame $\vec{p} = \vec{p}_1 + \vec{p}_2 = 0$ and hence $\Delta |\mathcal{M}|^2 = 0$ so we do not observe any effect. In the laboratory frame when the target is stationary it is essentially observable.

The dependence on the reference frame is a reflection on breaking of Lorentz invariance group $SO(3,1)$ to $SO(2) \times SO(1,1)$. The $e^+ e^-$ scattering shows no such interference term as can be seen in the following expressions for the amplitude.

$$|\mathcal{M}|^2 = 2e^2 \left( \frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} + \frac{2s^2}{ut} \right) \quad (2.14)$$

The difference appears because in this case we have only planner diagrams where the phases are the same. This difference is an example of the violation of simple crossing symmetry! Crossing symmetry break down can be understood since $s$, $t$, and $u$ are no longer the invariant parameters, the Lorentz symmetry is broken.

### 3 electron proton scattering

In this part we consider the scattering of two fermions, one heavy and one light which we refer to as proton and electron. The main object from which we may draw how energy observable is the scattering amplitude. To see the effect of noncommutativity the tree diagram is sufficient in contrast to the problem of anomalous magnetic moment where
we have to consider the one loop diagrams. Using the Feynman rules given in previous section, the tree amplitude picks up a phase with respect to the commutative case giving.

\[ \mathcal{M}_{NC} = e^{i \frac{\mu}{2} p_1 \times p_2} \mathcal{M}_C, \]

where

\[ \mathcal{M}_C = i e \gamma^\mu \frac{q_{\mu\nu}}{q^2} j^\nu \]

where the kinematical parameters are shown on the Feynman diagram. \( j^\nu \) is the external current. In the limit where \( q \rightarrow 0 \) this amplitude gives the potential of an stationary charged particle which deviates from the Coulomb potential, this deviation is a function of the momentum of the lighter particle.

\[ V = e^2 \int \frac{d^3 q}{(2\pi)^3} e^{i q \cdot (r + \tilde{p})} = \frac{e^2}{4\pi} \frac{1}{|r + \frac{\tilde{p}}{2}|} \]  

(3.16)

This potential is very different from the usual concept. It is hamiltonian between electron and proton in the sense that the amplitude calculated in Born approximation yields the tree amplitude. The shift of the singularity from \( r = 0 \) to \( r = -\frac{\tilde{p}}{2} \) can be explained by taking the string theory point of view. Open string moving with the ends on the brane are stretched, the amount of which is proportional to its momentum \[ \Theta \]. The direction of the stretch is perpendicular to \( \Theta \) and \( \tilde{p} \). So effectively the electric charge is moved \( \frac{1}{2} \tilde{p} \times \Theta \) with respect to the center of mass of the string. Obviously such potential violates rotational invariance. The background \( \theta^{ij} \) specifies a particular direction in space given by vector \( \Theta_i = \epsilon_{ijk} \theta^{jk} \). We still have rotational invariance around \( \Theta \). For a given value of momentum the interaction strongly depends on the direction of the momentum of the projectile. If it moves parallel to the vector \( \Theta = p_i \theta^{ij} \) it sees only the standard Coulomb
force because $\vec{p}$ which sets the strength isof the dipole is zero. The extra term is maximum when the momentum is in the plane perpendicular to the $\vec{\Theta}$. If we expand this potential we have:

$$V = \frac{e^2}{4\pi} \left[ \frac{1}{|r|} - \frac{\vec{r} \cdot \vec{p}}{2r^3} \right]$$

(3.17)

which can be interpreted to the projectile having dipole moment proportional to its momentum. Let us naively calculate the force due to this term, i.e dipole force

$$F = \frac{e^2}{8\pi r^5} \left[ r^2 \theta_{ij} p_j - 3r_k \theta_{kj} p_j r_i \right]$$

(3.18)

The first term looks like a magnetic field acting on the electron. But the second term gives a central force proportional to the momentum perpendicular to the background field $\vec{\Theta}$. Therefore the signature of noncommutativity in $e\,p$ scattering is velocity dependent radial force! in which varies as the inverse third power of the distance between the two particles. Of course the potential which is useful and reliable for long separation shows contribution from higher modes. Certainly short distance behavior of the potential will be affected by correction to the propagator from photon and electron loops which we defer to further investigation. Such contributions are of orders $\theta^2$ which must be very small.

4 Compton Scattering

Another phenomenon that may reveal the non commutativity of space is the electron photon (Compton) scattering.

We assume that the charged particle e.g electron is massive. The noncommutative theories predict direct interaction of three abelian gauge particle which give rises to a t-channel amplitude to the Compton scattering via the following Feynman diagram:
\[ M_t = -2i e^2 \epsilon^a(k_1)\epsilon^b(k_2) \sin \left( \frac{k_1 \cdot k_2}{2} \right) \exp \frac{i}{2}(p_1 \times p_2) \gamma_\mu U(p_2) \gamma_\nu U(p_1) C^{\mu\nu}, \]

where

\[ C^{\mu\nu} = (k_2 - 2k_1)\nu g^{\rho\mu} + (k_1 - 2k_2)\mu g^{\rho\nu} + (k_1 + k_2)^{\rho\omega} g^{\mu\nu}. \]

This diagram has contribution to the amplitude where its leading share to the cross section come from an interference with the standard term which are represented in the following diagram:

\[ M_s = \frac{e^2}{s - m^2} \epsilon^a(k_1)\epsilon^\nu(k_2) U(p_2) \gamma^\beta (\not{p}_1 + \not{k}_1 + m) \gamma^\alpha U(p_1) \exp \frac{i}{2}(p_1 \times k_1 + p_1 \times p_2 + k_1 \times p_2) \]

\[ M_u = \frac{e^2}{u - m^2} \epsilon^\beta(k_1)\epsilon^a(k_2) U(p_2) \gamma^\gamma (\not{p}_1 - \not{k}_2 + m) \gamma^\alpha U(p_1) \exp \frac{i}{2}(p_1 \times k_1 + p_1 \times p_2 + k_1 \times p_2) \]

the amplitude cross section is proportional to

\[ |M_{NC}|^2 = |M_C|^2 + \delta |M|^2, \tag{4.22} \]

which \(|M_C|^2\) term is same as commutative case, and \(\delta |M|^2\) is additional contribution in NCQED.

\[ \delta |M|^2 = 4e^4 \sin^2 \left( \frac{k_1 \times k_2}{2} \right) \left( \frac{s^2 + 2st - 3m^2s - 2m^2t + 4m^4}{st} - \frac{u^2 + 2ut - 3m^2u - 2m^2t + 4m^4}{ut} \right) \]
\[
+ 16e^4 \sin^2 \left( \frac{k_1 \times k_2}{2} \right) \left( 1 - \frac{5us}{4} - \frac{5m^2}{4t} + \frac{5m^4}{4t^2} \right) \\
+ \frac{4e^4}{us} (1 - \cos(k_1 \times k_2)) \left( m^2(s + u) + 2m^4 \right)
\]

(4.23)

In high energy limit, the leading term is
\[
e^4(k_1 \times k_2)^2 \left( \frac{s}{t} + \frac{u}{t} - \frac{4}{t^2} - \frac{5us}{t^2} \right)
\]

(4.24)

although classically it is not appropriate to use the concept of potential between proton and electron, we consider the Fourier transformation of the partial amplitude as an effective potential that may shed light on the electromagnetic interaction of photon and electron. As in the case of fermions we take the situation where the polarization is unchanged. It turns out that such effective potential is
\[
V = \frac{2e^2k_0 \cos(\alpha)}{4\pi} \left( \frac{1}{|r + \frac{P}{2}|} - \frac{1}{|r - \frac{P}{2}|} \right)
\]

(4.25)

this clearly shows that the photon is effectively seen by the electron as two separated charges with values \( \pm e \) and separation \( \tilde{P} \). In the large \( r \) approximation that we are considering it is natural to expand the potential in powers of \( \frac{1}{r} \) to final the dipole moment of the photon to be:
\[
e\tilde{P} = e\varepsilon_{ij}\tilde{p}_j = e\varepsilon_{ijk}\tilde{\Theta}_{k\tilde{p}_j} = e\vec{P} \times \vec{\Theta}.
\]

(4.26)

Note that \( P \) in the photon momentum. It is interesting that the photons electric dipole moment is twice as big as that of an electron for the same momentum \( P \). Again we may use the string theory point of view to understand the two terms in above formula. The two ends of the photon are opposite charges separated from each other by \( \tilde{p} \). This separation gives a dipole moment \( \vec{p} \times \vec{\Theta} \). To identify a signature of noncommutativity in Compton scattering we look at the cross section. Breakdown of rotational invariance is obvious from the contribution of \( k \) to the amplitude (or equivalently the effective potential). If the photon beam is rotated we see a change in the Compton cross section. The interference is maximum when \( p \) is perpendicular to \( \vec{\Theta} \) and goes to zero when \( p \) is parallel to it. Therefore a change in the Compton cross section with rotation of the photon beam is a possible signature of the space-space noncommutativity. the variation with orientation of the photon beam is independent of polarization when the \( \cos(\alpha) \) in (4.25) is averaged out.
5 Anomalous magnetic momentum of electron

The effects we have discussed in previous sections are all from leading tree diagrams. In this section we consider a one loop effect, i.e. electron’s anomalous magnetic moment. We will find that electron’s magnetic moment does not receive any correction from non-commutativity of space. The correction to the vertex is relativistic effect. To the lowest order the contribution come from

\[ e \vec{p}' \times \vec{U}(p) \gamma_\mu U(p') \]

whose value is proportional to

\[ q \]

the correction to the vertex comes from the diagrams,

\[ (b) \quad (a) \]

where the corresponding contributions to the vertex function are:

\[ \text{when our work on this part was completed the paper [0009037] appear with the same ? section IV of our work} \]
\[ i e \Gamma^\mu_{a} e^{\left(\frac{i}{2p' \times p}\right)} = e^{\left(\frac{i}{2p' \times p}\right)} \int \frac{d^4k}{(2\pi)^4} \mathcal{U}(p') \left[ i e \gamma^\alpha \frac{i(p' - k' + m)}{(p' - k)^2 - m^2} i e \gamma^\mu \frac{i(p - k' + m)}{(p - k)^2 - m^2} i e \gamma^\beta \frac{-i g_{\alpha \beta}}{k^2} \right] \mathcal{U}(p) \]

\[ i e \Gamma^\mu_{b} e^{\left(\frac{i}{2p' \times p}\right)} = e^{\left(\frac{i}{2p' \times p}\right)} \int \frac{d^4k}{(2\pi)^4} (1 - e^{ik \times q}) \mathcal{U}(p') \left[ i e \gamma^\alpha \frac{i(p' - k' + m)}{(p' - k)^2 - m^2} i e \gamma^\beta \frac{-i g_{\beta \nu}}{k^2} \frac{-i g_{\alpha \beta}}{(k + q)^2} \right] \]

\[ i e C^{\nu \mu \rho} \mathcal{U}(p) \] (5.29)

which \( C^{\nu \mu \rho} = (k - q)^\rho g^{\mu \nu} (2q + k)^\nu g^{\mu \rho} - (2k + q)^\mu g^{\rho \nu}. \)

The diagram 'a' except for an overall phase is the same as in the commutative case. Its phase vanishes when \( q \to 0. \) Diagram 'b' dose not appear in commutative QED and add a new contribution to electron magnetic dipole.

\[ < \vec{\mu} > = \frac{e}{2m} \left(2 + \frac{\alpha}{2\pi}\right) \vec{S} + \frac{e\alpha E m}{6\pi} \vec{\theta} \] (5.30)

This extra term is a constant independent of electron’s kinematical state, spin or momentum. To see such independent magnetic moment we may use a Stern-Gerlach apparatus. The two electron beams of two different spin respond differently to the external non-uniform magnetic field. The two eigen state of spin have the magnetic moments \( \mu_\pm = \pm \frac{e}{4m} (2 + \frac{\alpha}{2\pi}) \hbar + \frac{e\alpha E m}{6\pi} \theta. \) It is worth noting that spin resonance experiments can not reveal the noncommutativity because they are sensitive to the energy difference of the two states. This difference is independent of \( \theta. \)

### 6 Conclusion

We have considered a number of processes in which the noncommutativity manifests itself. This manifestation is through breaking of Lorentz invariance (both rotation and boost). This symmetry breaking cannot be observed in the scattering processes unless we are in a frame different from the center of mass frame and the momenta are not colinear. As we discussed in the scattering of \((ee, e^+ e^-, \gamma e, e p, \) ) the signature for noncommutativity is the change of the partial and total cross section with rotation of the incident beams or out
going detected particle. This change occurs mainly when the scattered electron makes a large angle, close to $\pi/2$ with the direction of the incident beam. The characteristic of this change is decrease in the cross section which may distinguish it from other sources of anisotropy such as dipole effect due to motion with respect to the microwave background radiation. The other difference with such anisotropy is that the noncommutativity may specify a direction which is different from the detected motion of earth in the cosmic thermal background. The change in the cross section is very small, of the order of $\Theta^2$ which is not easy to measure. One may set a bound for it by relating it to other physical quantities such as axion expectation value. In all the scattering cases change in the direction of the beam may not be possible to perform in the laboratory. An obvious suggestion is the use of earth’s rotation. Hence a comparison of the measurements of $ee$ or $e^+e^-$ and also Compton scattering partial cross sections in different times of the day and year may indicate anisotropy due to noncommutativity.

In the case of electron anomalous magnetic moment, although noncommutativity predicts a constant magnetic moment independent of the spin, shift in the frequency of spin resonance remains unchanged. This happens because the spin dependent part of the anomalous magnetic moment turns out to be the same as in the as the commutative case. This forces the spin flip energy to remain unchanged. In this case an experiment like that of Stern-Gerlach may be useful. The magnetic force on an electron is proportional to its magnetic moment which are different in magnitude for the two spin orientations. Therefore asymmetry in the deviation of the up and down beams, after correction for other effects is sensitive to noncommutativity.

Of course to higher order of perturbation more intricate dependence on $\theta$ may be discovered which is of higher order of $\alpha$ and $\theta$. Such effects are theoretically interesting and are under investigation by the authors but certainly are far too small to be detected or sought for before observation of leading effects.
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