0^{++}-Glueball/\bar{q}q-State Mixing in the Mass Region near 1500 MeV

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Abstract

Basing on the results of the K-matrix fit of \(IJ^{PC} = 00^{++}\) wave \([1]\), we analyze analytic structure of the amplitude and \(q\bar{q}/\text{glueball}\) content of resonances in the mass region 1200-1900 MeV, where an extra state for \(q\bar{q}\)-systematics exists being a good candidate for the lightest scalar glueball. Our analysis shows that the pure glueball state dispersed over three resonances: \(f_0(1300), f_0(1500)\) and \(f_0(1530^{+90}_{-250})\), while the glueball admixture in \(f_0(1750)\) is small. The broad resonance \(f_0(1530^{+90}_{-250})\) is the descendant of the lightest pure glueball. The mass of pure glueball is \(1630 \pm 70\) MeV, in agreement with Lattice calculation results \([2, 3]\).
In ref. [1] the K-matrix analysis of $0^0^{++}$-wave has been performed in the mass region 500-1900 MeV for the channels $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and $\pi\pi\pi\pi$. Simultaneous fit of the data of refs. [4-8] fixes five K-matrix poles (or five bare states, in the terminology of ref. [1]). Only two of them are definitely $s\bar{s}$-rich states: $f_0^{bare}(720 \pm 100)$ and $f_0^{bare}(1830 \pm 30)$. For other three states, $f_0^{bare}(1230 \pm 50)$, $f_0^{bare}(1260 \pm 30)$ and $f_0^{bare}(1600 \pm 50)$: two of them are natural $q\bar{q}$-nonet partners for $f_0^{bare}(720)$ and $f_0^{bare}(1830)$, while one state is extra for $q\bar{q}$-systematics. The $q\bar{q}$/glueball content of meson states reveals itself in coupling ratios for decays into channels $\pi\pi$, $K\bar{K}$, $\eta\eta$ and $\eta\eta'$ [9, 10, 11]. By use of these coupling ratios, the analysis of ref. [1] gives two solutions which describe the data set well:

**Solution I:**

- $f_0^{bare}(720)$ and $f_0^{bare}(1260)$ are $1^3P_0$ nonet partners,
- $f_0^{bare}(1600)$ and $f_0^{bare}(1810)$ are $2^3P_0$ nonet partners,
- $f_0^{bare}(1230)$ is a glueball;

**Solution II:**

- $f_0^{bare}(720)$ and $f_0^{bare}(1260)$ are $1^3P_0$ nonet partners,
- $f_0^{bare}(1230)$ and $f_0^{bare}(1810)$ are $2^3P_0$ nonet partners,
- $f_0^{bare}(1600)$ is a glueball.

Physical states are mixtures of bare states which occur in the K-matrix formalism via transitions of bare states into meson channels (in the analysis of ref. [1]: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$ and $4\pi$). In the mass region 1200-1650 MeV which is key region for determination of the lightest glueball, the $00^{++}$-amplitude has four poles at the following complex masses (in MeV) [1]:

$$
\begin{align*}
(1300 \pm 20) & \quad \rightarrow \quad f_0(1300) \\
(1499 \pm 8) & \quad \rightarrow \quad f_0(1500) \\
(1530^{+90}_{-250}) & \quad \rightarrow \quad f_0(1530^{+90}_{-250}) \\
(1780 \pm 30) & \quad \rightarrow \quad f_0(1780)
\end{align*}
$$

Each of these states is a mixture of $q\bar{q}$ and glueball components. In order to reconstruct $q\bar{q}$/glueball content of the resonances, we have performed here a re-analysis of the $00^{++}$-amplitude in the region 1200-1650 MeV using the language of $q\bar{q}$ and glueball states.
1 Glueball propagator

Mixing of $q\bar{q}$-states with glueball is due to the processes shown in fig. 1a,b: gluons of the glueball produce a $q\bar{q}$-pair, fig. 1a; the produced quarks interact by gluon exchanges, fig. 1b. According to the rules of the $1/N$-expansion [12], the main contribution into the interaction block is given by planar diagrams. Saturating the $q\bar{q}$-scattering block of fig. 1b by $q\bar{q}$ states, we represent the diagrams of fig. 1b as a set of diagrams of the type shown in figs. 1d and 1e. The sum of diagrams of fig. 1c, 1d, 1e, and so on gives the glueball propagator with $q\bar{q}$-state mixing taken into account. The mixtures of the pure glueball state and input $q\bar{q}$-states are determined by the quark loop transition diagrams, $B_{ab}(s)$, ($s = p^2$ is glueball four-momentum squared) which have the following form in the light cone variables:

$$B_{ab}(s) = \frac{1}{(2\pi)^3} \int_0^1 \frac{dx}{x} \int d^2k_\perp \frac{g_a(s')g_b(s')}{s' - s - i0} \frac{2(s' - 4m^2)}{2}.$$  \hspace{1cm} (2)

Here $s' = \frac{m^2 + k^2}{x(1-x)}$, $g_a$ and $g_b$ are the vertices of the transitions state $a \rightarrow q\bar{q}$ and state $b \rightarrow q\bar{q}$, and $m$ is quark mass. Factor $2(s' - 4m^2)$ is determined by the spin structure of the quark loop diagram: $Tr[(\hat{k} + m)(-\hat{p} + \hat{k} + m)] = 2(s' - 4m^2)$.

To analyse analytic structure of the $00^{++}$-amplitude in the mass region of the resonances of eq. (1), let us introduce a $4 \times 4$ propagator matrix, $D_{ab}(s)$, which describes the transition state $a \rightarrow state$ $b$ with $a, b = 1, 2, 3, 4$, in accordance with the number of investigated states. The diagrams of fig. 1 type give:

$$D_{ab}^{-1}(s) = (m_a^2 - s)\delta_{ab} - B_{ab}(s)$$ \hspace{1cm} (3)

Here $m_a$ is an input mass for the state $a$; in the case of the glueball it is the mass of pure gluonic glueball, $\delta_{ab}$ is unit matrix, and $B_{ab}(s)$ is given by eq. (2). The zeros of the determinant

$$\Pi(s) = \text{det}|(m_a^2 - s)\delta_{ab} - B_{ab}(s)|$$ \hspace{1cm} (4)

determine the complex masses of physical states: in the case under investigation, they are given by eq. (1). Let us denote them as $M_A$, $M_B$ $M_C$ and $M_D$. Then, in the vicinity of $s = M_A^2$, $D_{ab}(s)$ is described by the pole term only:

$$D_{ab}(s \sim M_A^2) \simeq N_A \frac{\alpha_a \alpha_b}{M_A^2 - s},$$ \hspace{1cm} (5)

where four coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ satisfy the constraint

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = 1$$ \hspace{1cm} (6)
and determine the probabilities of the input states (1,2,3,4) in the physical state $A$; $N_A$ is a normalization factor common for all $D_{ab}$.

In order to take into account the flavor content in the quark loop diagrams which is omitted in eq. (2), the following replacement should be done:

$$B_{ab}(s) \rightarrow \cos \phi_a \cos \phi_b B_{ab}^{(nn)}(s) + \sin \phi_a \sin \phi_b B_{ab}^{(ss)}(s),$$

(7)

where $|a\rangle = \cos \phi_a \, n\bar{n} + \sin \phi_a \, s\bar{s}$ and $|b\rangle = \cos \phi_b \, n\bar{n} + \sin \phi_b \, s\bar{s}$, while $B_{ab}^{(nn)}$ and $B_{ab}^{(ss)}$ refer to loop diagrams with non-strange and strange quarks, $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$. For pure glueball state, $a = glueball$, the effective mixing angles are determined by relative probabilities of the production of non-strange and strange quarks by gluons, $u\bar{u} : d\bar{d} : s\bar{s} = 1 : 1 : \lambda$, so that $\tan \phi_{\text{glueball}} = \sqrt{\lambda}/2$. Experimental data give $\lambda \simeq 0.5$ [11, 13]: this value corresponds to $\phi_{\text{glueball}} \simeq 25^\circ$. Mixing angles for $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ states were found in ref. [1].

2. **Fit of the 00++ amplitude**

For the calculation of loop diagrams, eq. (2), we should fix the vertices $g_a(s)$. We parametrize the vertices for the transition state $a \rightarrow n\bar{n}$ in a simple form:

$$1^3P_0 \, q\bar{q} \rightarrow \text{state} : \quad g_1(s) = \gamma_1 \sqrt{s} \frac{k^2 + \sigma_1}{k^2 + \sigma_1};$$

$$2^3P_0 \, q\bar{q} \rightarrow \text{first state} : \quad g_2(s) = \gamma_2 \sqrt{s} \left[\frac{k^2 + \sigma_2}{k^2 + \sigma_2} - d_{ab} \frac{k^2 + \sigma_2}{k^2 + \sigma_2 + h}\right];$$

$$\text{Glueball} : \quad g_3(s) = \gamma_3 \sqrt{s} \frac{k^2 + \sigma_3}{k^2 + \sigma_3};$$

$$2^3P_0 \, q\bar{q} \rightarrow \text{second state} : \quad g_4(s) = g_2(s).$$

Here $k^2 = \frac{s}{4} - m^2$ and $k^2_0 = m_b^2 - m^2_a; \, m_a, \gamma_a$ and $\sigma_a$ are parameters. Factor $d$ is due to orthogonality of the $1^3P_0q\bar{q}$ and $2^3P_0q\bar{q}$ states: we put $Re \, B_{12}(s_0) = 0$ at $\sqrt{s_0} = 1.5$ GeV. (In the case of $s$-dependent $B$-functions the orthogonality requirement for loop transition diagrams cannot be fixed at all values of $s$).

The parameters $m_a, \sigma_a, h$ and $\gamma_a \,(a = 1,2,3)$ are to be determined by masses and widths of the physical resonances of eq. (6). However, the masses $m_a$ are approximately fixed by the K-matrix fit of ref. [3], where masses of the K-matrix poles, $M_a^{\text{bare}}$, are determined: $(M_a^{\text{bare}})^2 \simeq m_a^2 - B_{aa}^2 ((M_a^{\text{bare}})^2)$. Let us stress that $m_3$ is the mass of pure gluonic glueball which is a subject of Lattice QCD calculation.

Parameters which are found in our fit of the 00++ amplitude in the mass region 1200-1900 MeV are given in Table 1. Using these parameters, we calculate the
couplings $\alpha_a$ which are introduced by eqs. (5) and (6): these couplings determine relative weight of the initial state $a$ in the physical resonance $A$:

$$W_a(A) = |\alpha_a|^2$$  \(9\)

The probabilities $W_a$ are given in Table 2 together with masses of physical resonances, $M_A$, and masses of input states, $m_a$.

3 Glueball/$q\bar{q}$-state mixing

In order to analyze the dynamics of the glueball/$q\bar{q}$ mixing, we use the following method: in the final formulae the vertices are replaced in a way:

$$g_a(s) \rightarrow \xi g_a(s),$$  \(10\)

with a factor $\xi$ running in the interval $0 \leq \xi \leq 1$. The case $\xi = 0$ corresponds to switching off mixing of the input states. The input states are stable in this case, and corresponding poles of the amplitude are at $s_a = m^2_a$. Fig. 2 shows the pole position at $\xi = 0$ for solution I (fig. 2a) and solution II (fig. 2b). For glueball state $m_3$ is the mass of a pure glueball, without $q\bar{q}$ degrees of freedom. In solution I the pure-glueball mass is equal to

$$m_{\text{pure glueball}}(\text{Solution I}) = 1225 \text{ MeV},$$  \(11\)

that definitely disagrees with the Lattice-Gluodynamics calculations for the lightest glueball. In solution II

$$m_{\text{pure glueball}}(\text{Solution II}) = 1633 \text{ MeV}.$$  \(12\)

This value is in a good agreement with recent Lattice-Gluodynamics results: $1570 \pm 85(\text{stat}) \pm 100(\text{syst}) \text{ MeV}$ \cite{2} and $1707 \pm 64 \text{ MeV}$ \cite{3}. With increasing $\xi$ the poles are shifted into lower part of the complex mass plane. Let us discuss in detail the solution II which is consistent with Lattice result.

At $\xi \simeq 0.1 - 0.5$ the glueball state of solution II is mixing mainly with $2^3P_0$ $q\bar{q}$-state, at $\xi \simeq 0.8 - 1.0$ the mixture with $1^3P_0$ $q\bar{q}$-state becomes significant. As a result, the descendant of the pure glueball state has the mass $M = 1450 - i450$ MeV. Its gluonic content is 47\% (see Table 1). We should emphasize: the definition of $W_a$ suggests that $\sum_{A=1,2,3,4} W_{\text{glueball}}(A) \neq 1$ because of the $s$-dependent $B_{ab}$ in the propagator matrix.

Hypothesis that the lightest scalar glueball is strongly mixed with neighbouring $q\bar{q}$ states was discussed previously (see refs. \cite{14}, \cite{15}, and references therein).
However, the attempts to reproduce a quantitative picture of the glueball/\(q\bar{q}\)-state mixing and the mass shifts caused by this mixing could not be successful within standard quantum mechanics approach that misses two phenomena:

(i) Glueball/\(q\bar{q}\)-state mixing described by propagator matrix can give both a repulsion of the mixed levels, as in the standard quantum mechanics, and an attraction of them. The latter effect may happen because the loop diagrams \(B_{ab}\) are complex magnitudes, and the imaginary parts \(Im B_{ab}\) are rather large in the region 1500 MeV.

(ii) Overlapping resonances yield a repulsion of the amplitude pole positions along imaginary-s axis. In the case of full overlap of two resonances the width of one state tends to zero, while the width of the second state tends to the sum of the widths of initial states, \(\Gamma_{first} \simeq 0\) and \(\Gamma_{second} \simeq \Gamma_1 + \Gamma_2\). For three overlapping resonances the widths of two states tend to zero, while the width of the third state accumulates the widths of all initial resonances, \(\Gamma_{third} \simeq \Gamma_1 + \Gamma_2 + \Gamma_3\).

Therefore, in the case of nearly overlapping resonances, what occurs in the region near 1500 MeV, it is inevitable to have one resonance with a large width. It is also natural that it is the glueball descendant with large width: the glueball mixes with the neighbouring \(1^3P_0 q\bar{q}\) and \(2^3P_0 q\bar{q}\) states, which are both \(n\bar{n}\) rich, without suppression.

4 Problems

Despite the fact that the lightest scalar glueball is now understandable in its principle points, there are problems which need to be clarified. First, it is necessary to estimate the influence of the more distant resonance, \(f_0(980)\). Second, the analysis should be repeated in terms of hadron states, without using the language of \(q\bar{q}\)-states. Such an analysis would give the possibility to check the idea of quark—hadron duality in the mass region 1000-2000 MeV which is used here.

There is one more problem: K-matrix analysis of \(00^{++}\) wave \([1]\) provides two solutions. Correspondingly, analyzing here two variants we rejected one solution basing on the results of Lattice-Gluodynamics calculations. The problem is if it is possible to discriminate between solution I and solution II and what type of experimental data are needed for that.

5 Conclusion

The lightest ghudynamic glueball is dispersed over neighbouring resonances mixing mainly to \(1^3P_0 q\bar{q}\) and \(2^3P_0 q\bar{q}\) states. With this mixing the glueball descendant
transforms into broad resonance, $f_0(1530^{\pm 90}_{-250})$. This resonance contains (40-50)% of the glueball component. Another part of the glueball component is shared between comparatively narrow resonances, $f_0(1300)$ and $f_0(1500)$ which are descendants of $1^3 P_0 \ q\bar{q}$ and $2^3 P_0 \ q\bar{q}$ states.

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Table 1
Masses of the initial states, coupling constants and $q\bar{q}$/glueball content of physical states.

|               | $1^4P_0$          | $2^4P_0$          | Glueball | $2^4P_0$          |
|---------------|-------------------|-------------------|----------|-------------------|
| Initial state | $n\bar{n}$-rich   | $n\bar{n}$-rich   | Glueball | $s\bar{s}$-rich   |
|               | $\phi_1 = 18^\circ$ | $\phi_2 = -6^\circ$ | $\phi_3 = 25^\circ$ | $\phi_4 = 84^\circ$ |
| $m_i$ (GeV)   | 1.457             | 1.536             | 1.230    | 1.750             |
| $\gamma_i$ (GeV$^{3/4}$) | 0.708           | 1.471             | 0.453    | 1.471             |
| $\sigma_i$ (GeV$^2$) | 0.075           | 0.225             | 0.375    | 0.225             |
| $W[f_0(1300)]$ | 32%               | 12%               | 55%      | 1%                |
| 1.300 $- i0.115$ (GeV) | 25%               | 70%               | 3%       | 2%                |
| $W[f_0(1500)]$ | 44%               | 24%               | 27%      | 4%                |
| 1.500 $- i0.065$ (GeV) | 1%               | 1%               | –        | 98%               |
| $h = 0.25$ GeV$^2$, $d = 1.01$ |                     |                    |          |                    |

|               | $1^4P_0$          | $2^4P_0$          | Glueball | $2^4P_0$          |
|---------------|-------------------|-------------------|----------|-------------------|
| Initial state | $n\bar{n}$-rich   | $n\bar{n}$-rich   | Glueball | $s\bar{s}$-rich   |
|               | $\phi_1 = 18^\circ$ | $\phi_2 = 35^\circ$ | $\phi_3 = 25^\circ$ | $\phi_4 = -55^\circ$ |
| $m_i$ (GeV)   | 1.107             | 1.566             | 1.633    | 1.702             |
| $\gamma_i$ (GeV$^{3/4}$) | 0.512           | 0.994             | 0.446    | 0.994             |
| $\sigma_i$ (GeV$^2$) | 0.175           | 0.275             | 0.375    | 0.275             |
| $W[f_0(1300)]$ | 35%               | 26%               | 38%      | 0.4%              |
| 1.300 $- i0.115$ (GeV) | 1%               | 64%               | 35%      | 0.4%              |
| $W[f_0(1500)]$ | 12%               | 41%               | 47%      | 0.3%              |
| 1.500 $- i0.065$ (GeV) | 0.1%             | 0.2%              | 0.2%     | 99.5%             |
| $h = 0.625$ GeV$^2$, $d = 1.16$ |                     |                    |          |                    |
Fig. 1. Diagrams which provide the glueball/$q\bar{q}$ mixing.
Fig. 2. Complex-$\sqrt{s}$ plane ($M = \Re \sqrt{s}$, $-\Gamma / 2 = \Im \sqrt{s}$): location of 00++ amplitude poles after replacing $g_a \rightarrow \xi g_a$. The case $\xi = 0$ gives the positions of masses of input $q\bar{q}$ states and gluodynamic glueball; $\xi = 1$ corresponds to the real case.