The Gamow-Teller States in Relativistic Nuclear Models

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The Gamow-Teller (GT) states are investigated in relativistic models. The Landau-Migdal (LM) parameter is introduced in the Lagrangian as a contact term with the pseudo-vector coupling. In the relativistic model the total GT strength in the nucleon space is quenched by about 12% in nuclear matter and by about 6% in finite nuclei, compared with the one of the Ikeda-Fujii-Fujita sum rule. The quenched amount is taken by nucleon-antinucleon excitations in the time-like region. Because of the quenching, the relativistic model requires a larger value of the LM parameter than non-relativistic models in describing the excitation energy of the GT state. The Pauli blocking terms are not important for the description of the GT states.

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I. INTRODUCTION

For the last 30 years it has been shown that relativistic models work very well phenomenologically to explain various nuclear phenomena[1]. Most of them assume that the nucleus is a relativistic system composed of the Dirac particles in the Lorentz scalar and vector potentials.

In the present paper, we study the excitation energy and strength of the Gamow-Teller (GT) states in the relativistic models. As far as the authors know, the GT states have not been studied in detail so far[2]. We will discuss those mainly in nuclear matter, since we can obtain analytic expressions of the excitation energy and strength which make clear the structure of the relativistic model and the difference between the relativistic and non-relativistic models.

In the next section we will present our relativistic framework to discuss the GT states. The Landau-Migdal (LM) parameter will be introduced in the Lagrangian as a contact term to take into account particle-hole correlations. In the section III the transverse correlation function will be calculated explicitly, from which an analytic expression of the excitation energy will be obtained in section IV. In section V the GT strength will be calculated. We will show that the total GT strength is quenched by about 12% in nuclear matter and by about 6% in finite nuclei, compared with the non-relativistic sum rule value. The quenched strength is taken by the nucleon-antinucleon excitations in the time-like region, which can not be excited with charge-exchange reactions. Effects of the Pauli blocking terms on the excitation energy and strength will be shown to be negligible in section IV and V. In section VI we will show that the way to add the LM parameter to the relativistic meson propagator, which was frequently used in a description of high-momentum transfer reactions[3], can not describe the GT states. The last section will be devoted to a brief summary of the present work.

II. RELATIVISTIC MODEL

We assume that the mean field is provided by the Lorentz scalar and vector potential. The RPA correlations are described using the basis given in this mean field, and are assumed to be induced through the Lagrangian:

\[ L = -g_\pi \bar{\psi} \Gamma_i^{\mu} \psi \partial_\mu \pi_i + \frac{g_5}{2} \bar{\psi} \Gamma_i^{\mu} \psi \Gamma_\mu \psi \]

(1)

with

\[ \Gamma_i^{\mu} = \gamma_5 \gamma^\mu \tau_i, \quad g_\pi = \frac{f_\pi}{m_\pi}, \quad g_5 = \left( \frac{f_\pi}{m_\pi} \right)^2 g', \]

The first term stands for the usual PV coupling between the pion and nucleon, and the second term is to take into account the LM parameter \(g'\). Although it is model-dependent how to introduce \(g'\) in the relativistic model, we will show that the above Lagrangian yields the known expression for the excitation energy of the GT state in the non-relativistic limit. The first term, in fact, is not relevant for the GT states in nuclear matter. It is, however, kept in order to show later that if the LM parameter is put into the meson propagator, we can not describe the GT state.
For the Lagrangian Eq. (1), the RPA correlation function $\Pi_{\text{RPA}}$ is written in terms of the mean field one $\Pi_{\text{MF}}$,

$$\Pi_{\text{RPA}}(\Gamma_A, \Gamma_B) = \Pi(\Gamma_A, \Gamma_B) + \chi_\pi(q) \Pi(\Gamma_A, \Gamma_i^\prime; q) \Pi_{\text{RPA}}(\Gamma_i^\prime; q, \Gamma_B) + \chi_5 \Pi(\Gamma_A, \Gamma_i^{\mu};) \Pi_{\text{RPA}}(\Gamma_{\mu i}, \Gamma_B),$$

where the following notations are employed,

$$\chi_\pi(q) = \frac{g^2}{(2\pi)^3} \frac{1}{m^2 - q^2 - i\varepsilon}, \quad \chi_5 = \frac{g_5}{(2\pi)^3}.$$

For isospin-dependent excitations, the mean field correlation function is given by

$$\Pi(\Gamma_\alpha, \Gamma_\beta) = \frac{1}{2\pi i} \int d^4p \ Tr_\alpha Tr_\tau \left( \Gamma_\alpha G_F(p + q) \Gamma_\beta G_D(p) + \Gamma_\alpha G_D(p + q) \Gamma_\beta G_F(p) \right),$$

with

$$G_D(q) = G_F(q) + G_D(q), \quad G_D(q) = G(k_p; q) \frac{1 - \tau_x}{2} + G(k_n; q) \frac{1 + \tau_z}{2},$$

where we have defined the isospin operator $\tau_a$: $\tau_x = \frac{\tau_x \pm i\tau_y}{\sqrt{2}}$, $\tau_0 = \tau_z$, and the propagator:

$$G_F(q) = \frac{q + M^*}{q^2 - M^{*2} + i\varepsilon}, \quad G(k_i; q) = \frac{i\pi}{E_q} (q + M^*) \delta(q_0 - E_q) \theta^{(i)}(\tau_z), \quad (i = p, n).$$

Here, we have also used the abbreviation for the step function: $\theta^{(i)} = \theta(k_i - |q|)$, $k_p$ and $k_n$ being the Fermi momentums of the protons and neutrons, respectively. Moreover, $E_q$ is equal to $\sqrt{M^{*2} + q^2}$, where the Lorentz scalar potential is included in the nucleon effective mass $M^*$. The Lorentz vector potential does not show up explicitly in the present discussion of the nuclear matter. In Eq. (3), the first three terms are density-dependent, including the Pauli blocking terms, while the last one is density-independent and divergent. The last term is usually neglected, but we keep it for later discussions. Effects of the Pauli blocking terms on the excitation energy and strength of the GT state will be also discussed later.

For the $\tau_\pm$ excitations, the RPA correlation function in Eq. (2) is described as

$$\Pi_{\text{RPA}}(\Gamma^a_+, \Gamma^b_\mp) = |U^{-1}|^{ab} \Pi(\Gamma^{a}_{\nu_+}, \Gamma^{b}_{\mp}),$$

where $U$ denotes the dimesic function of the $5 \times 5$ matrix:

$$U^{ab} = g^{ab} - \chi_b \Pi(\Gamma^a_+, \Gamma^b_-)$$

with the notations for $a = -1, 0, \cdots, 3$,

$$\Gamma^a_\pm = \gamma^a \tau_\pm, \quad \gamma^a = \left\{ \begin{array}{ll} \gamma \cdot q & , \chi_\pi \quad , \quad a = -1 \\ \gamma^\mu & , \chi_5 \quad , \quad a = \mu. \end{array} \right.$$
where $t^{ab}(p, q)$ is given by

$$t^{\mu\nu}(p, q) = 4\left(g^{\mu\nu} (M^*^2 + p^2 + p \cdot q) - 2p^\mu p^\nu - p^\mu q^\nu - p^\nu q^\mu\right),$$  \hspace{1cm} (8)

$$t^{-1\mu}(p, q) = g_{\mu\nu} t^{\mu\nu}(p, q) = 4\left(q^\nu (M^*^2 + p^2) - p^\nu (q^2 + 2p \cdot q)\right),$$  \hspace{1cm} (9)

$$t^{-1\nu}(p, q) = 4\left(q^2 (M^*^2 + p^2) - p \cdot q (q^2 + 2p \cdot q)\right).$$  \hspace{1cm} (10)

When we define the three dimensional axes as $q^\mu = (q_0, q_x, 0, 0)$, we can show that $\Pi(\Gamma^a_+, \Gamma^b_-)$ has a structure as shown in Table II (a), where open boxes mean $\Pi(\Gamma^a_+, \Gamma^b_+)$ to be non-zero. Thus, the transverse parts of $\Pi(\Gamma^a_+, \Gamma^b_+)$ are decoupled from the pion($a = -1$)-, time($a = 0$)- and longitudinal($a = 1$) ones. In the present paper, we are interested in the GT states excited at $q = 0$. In this case, the longitudinal part is also decoupled from the pion- and time-component(PT), as in Table II (b). Consequently, the determinant of the dimesic function is factorized into four parts,

$$\det U = -(D_T)^2 D_T D_{L},$$  \hspace{1cm} (11)

where the transverse, longitudinal and PT dimesic functions are written as, respectively,

$$D_T = 1 + \chi_5 \Pi(\Gamma^2_+, \Gamma^2_+), \quad D_L = D_T,$$  \hspace{1cm} (12)

$$D_{PT} = \left(1 - \chi_5 \Pi(\Gamma^{-1}_+, \Gamma^{-1}_-)\right)\left(1 - \chi_5 \Pi(\Gamma^0_+, \Gamma^0_-)\right) - \chi_5 \chi_5 \Pi(\Gamma^{-1}_+, \Gamma^0_-)\Pi(\Gamma^0_+, \Gamma^{-1}_-).$$  \hspace{1cm} (13)

### III. THE TRANSVERSE CORRELATION FUNCTION

In this section we will derive more explicit form of the transverse correlation function at $q = 0$ which is used for description of the GT states. First we will calculate the real and imaginary part of the density-dependent transverse correlation function separately, and next the density-independent part.

According to Eq. (13), the real part of the density-dependent transverse correlation function is written as

$$\text{Re } \Pi_D(\Gamma^2_+, \Gamma^2_+) = J^{22}(k_n, q_0) + J^{22}(k_p, -q_0),$$  \hspace{1cm} (14)

where $J^{22}(k, q)$ represents

$$J^{22}(k, q) = \int d^4 p \frac{\delta(p_0 - E_p)}{E_p} \frac{t^{22}(p, q)}{(p + q)^2 - M^*^2} \varphi^{(i)}.$$  \hspace{1cm} (15)

Since $t^{22}$ at $q = 0$ is given by

$$t^{22}(p, q) = -4\left(2M^*^2 + E_p q_0 + 2p^2\right),$$  \hspace{1cm} (16)

the real part Eq. (14) is described as

$$\text{Re } \Pi_D(\Gamma^2_+, \Gamma^2_+) = \frac{-4}{q_0} \int d^3 p \frac{M^*^2 + p^2}{E_p^2} \left(\varphi^{(n)}_p - \varphi^{(p)}_p\right)$$

$$-4 \int d^3 p \frac{p^2 - p^2}{E_p^2} \left(\varphi^{(n)}_p \frac{2E_p - q_0}{2E_p + q_0} + \varphi^{(p)}_p \frac{2E_p - q_0}{2E_p - q_0}\right).$$  \hspace{1cm} (17)

The imaginary part of the density-dependent correlation function at $q = 0$ is given by Eq. (10) as

$$\text{Im } \Pi_D(\Gamma^2_+, \Gamma^2_-) = -\pi \int d^4 p \frac{\delta(p_0 - E_p)}{E_p} \left[t^{ab}(p, q) \delta(q^2_0 + 2p_0 q_0) \varphi^{(n)}_p\right.$$  

$$+ t^{ab}(p, -q) \delta(q^2_0 - 2p_0 q_0) \varphi^{(p)}_p - \frac{\delta(q_0)}{E_p} t^{ab}(p, q) \varphi^{(n)}_p \varphi^{(p)}_p\right].$$  \hspace{1cm} (17)
Using
\[
\delta(q_0^2 + 2p_0q_0) = \frac{1}{2p_0} \left( \delta(q_0) + \delta(q_0 + 2p_0) \right),
\]
the above equation is rewritten as
\[
\text{Im} \, \Pi_D(G^+, G^-) = -\pi \delta(q_0) \int d^4p \, \frac{\delta(p_0 - E_p)}{2E_p^2} \left( t^{ab}(p, q) \theta_p^{(n)} + t^{ab}(p, -q) \theta_p^{(p)} - 2t^{ab}(p, q) \theta_p^{(n)}\theta_p^{(p)} \right) + R_{N\bar{N}}.
\]

The last term \( R_{N\bar{N}} \) comes from the \( N\bar{N} \) excitations,
\[
R_{N\bar{N}}(q_0) = -\pi \int d^4p \, \frac{\delta(p_0 - E_p)}{2E_p^2} \left( t^{ab}(p, q) \delta(q_0 + 2E_p) \theta_p^{(n)} + t^{ab}(p, -q) \delta(q_0 - 2E_p) \theta_p^{(p)} \right).
\]

Inserting Eq. (16) into the above equations, the imaginary part of the density-dependent transverse correlation function is obtained as
\[
\text{Im} \, \Pi_D(G^2, G^2) = 4\pi \delta(q_0) \int d^4p \, \frac{M^{*2} + p_y^2}{E_p^2} \left( \theta_p^{(n)} + \delta(q_0 + 2E_p) \theta_p^{(n)} + \delta(q_0 - 2E_p) \theta_p^{(n)} \right) + R_{N\bar{N}},
\]
where \( R_{N\bar{N}} \) is given by
\[
R_{N\bar{N}} = -4\pi \int d^4p \, \frac{p^2 - p_y^2}{E_p^2} \left( \delta(q_0 + 2E_p) \theta_p^{(n)} + \delta(q_0 - 2E_p) \theta_p^{(n)} \right).
\]

From Eqs. (17), (18) and (19), the density-dependent part of the transverse correlation function is described as
\[
\Pi_D(G^2, G^2) = -4 \int d^4p \, \frac{M^{*2} + p_y^2}{E_p^2} \left( \frac{\theta_p^{(n)}(1 - \theta_p^{(p)})}{q_0 + i\epsilon} - \frac{\theta_p^{(p)}(1 - \theta_p^{(n)})}{q_0 - i\epsilon} \right)
- 4 \int d^4p \, \frac{p^2 - p_y^2}{E_p^2} \left( \frac{\theta_p^{(p)}}{2E_p - q_0 - i\epsilon} + \frac{\theta_p^{(n)}}{2E_p + q_0 - i\epsilon} \right).
\]

The density-independent part of the transverse correlation function is calculated in the same way. From Eqs. (17) and (18), we obtain
\[
\Pi_F(G^2, G^2) = 4 \int d^4p \, \frac{p^2 - p_y^2}{E_p^2} \left( \frac{1}{2E_p - q_0 - i\epsilon} + \frac{1}{2E_p + q_0 - i\epsilon} \right).
\]

The sum of Eqs. (20) and (21) provides us with the full transverse correlation function \( \Pi(G^2, G^2) \). It is also expressed as a sum of contributions from particle–hole and \( N\bar{N} \) excitations:
\[
\Pi(G^2, G^2) = \Pi_{ph}(G^2, G^2) + \Pi_{N\bar{N}}(G^2, G^2),
\]
where each term is described as
\[
\Pi_{ph}(G^2, G^2) = -4 \int d^4p \, \frac{M^{*2} + p_y^2}{E_p^2} \left( \frac{\theta_p^{(n)}(1 - \theta_p^{(p)})}{q_0 + i\epsilon} - \frac{\theta_p^{(p)}(1 - \theta_p^{(n)})}{q_0 - i\epsilon} \right),
\]
\[
\Pi_{N\bar{N}}(G^2, G^2) = 4 \int d^4p \, \frac{p^2 - p_y^2}{E_p^2} \left( \frac{1 - \theta_p^{(p)}}{2E_p - q_0 - i\epsilon} + \frac{1 - \theta_p^{(n)}}{2E_p + q_0 - i\epsilon} \right).
\]

**IV. THE EXCITATION ENERGY OF THE GT STATE**

The eigenvalues of the excitation energies are given by the real part of the dimesic function,
\[
\det \text{Re} \, U = 0.
\]

The excitation energy of the GT state is estimated with use of the transverse part of the dimesic function in Eq. (12). The real part of the transverse correlation function, which we need in the dimesic function, is obtained from Eqs. (22) to (24).
A. The Excitation Energy in the Nucleon Space

In this subsection, we will calculate the excitation energy of the GT state, neglecting perfectly the antinucleon degrees of freedom. In this case, according to Eqs. (12) and (29), the real part of the transverse dimesic function is described as

\[
\text{Re } D_T = 1 + \chi_5 \text{Re } \Pi_{ph}(\Gamma_+^2, \Gamma_-^2),
\]

\[
\text{Re } \Pi_{ph}(\Gamma_+^2, \Gamma_-^2) = -\frac{16\pi Q(k_n) - Q(k_p)}{3q_0},
\]

where \(Q(k_i)\) is given by,

\[
Q(k_i) = \frac{3}{4\pi} \int_0^{k_i} d^3 p \frac{M^2 + p_i^2}{E_p^2} = \frac{k_i^3}{3} + 2k_i M^* - 2M^* \tan^{-1} \frac{k_i}{M^*}.
\]

From \(\text{Re } D_T = 0\), finally we obtain the relativistic expression of the excitation energy in nuclei with \(k_n > k_p\),

\[
\omega_0 = \frac{2g_5}{3\pi^2} (Q(k_n) - Q(k_p)).
\]

Relativistic effects on Eq. (29) can be seen more transparently, by defining Fermi momentum \(k_F\) as usual

\[
k_n^3 = \frac{2N}{A} k_F^3, \quad k_p^3 = \frac{2Z}{A} k_F^3.
\]

These yield a relationship for \((N - Z)/A \ll 1\),

\[
k_n - k_p \approx \frac{2}{3} \frac{k_F N - Z}{A}.
\]

By using the equation:

\[
Q'(k_F) = \frac{dQ(k_F)}{dk_F} = \frac{k_F^2(3M^* + k_F^2)}{M^* + k_F^2} = 3k_F^2 \left(1 - \frac{2}{3} \frac{v_F^2}{M^* + k_F^2}\right), \quad v_F = \frac{k_F}{\sqrt{M^* + k_F^2}},
\]

we expand \((Q(k_n) - Q(k_p))\) in Eq. (29) up to first order of \((k_n - k_p)\). Then replacing \((k_n - k_p)\) by Eq. (31), we obtain the relativistic expression as

\[
\omega_0 \approx \left(1 - \frac{2}{3} \frac{v_F^2}{v_F^2}\right) \frac{g_5}{3\pi^2} \frac{8k_F^3 N - Z}{2A}.
\]

The first factor of the r.h.s., depending on the Fermi velocity \(v_F\), shows relativistic effects on the excitation energy. In the non-relativistic limit \(v_F^2 \ll 1\), Eq. (33) becomes to be

\[
\omega_0 = \frac{g_5}{3\pi^2} \frac{8k_F^3 N - Z}{2A}.
\]

This result can be also obtained without using the approximation Eq. (31). In the non-relativistic limit \(p^2 \ll M^*^2\), Eq. (29) becomes

\[
Q(k_i) \approx k_i^3.
\]

This, together with Eqs. (24) and (30), yields the same result as Eq. (34).

Eq. (34) is just the one obtained previously in non-relativistic models with \(g_5 = g'(f_\pi/m_\pi)^2\) [7]. In relativistic models, the excitation energy of the GT state in nuclear matter is thus given by the transverse part of the dimesic function, and is independent of the pion exchange, even when its energy-dependence is taken into account.

We will show later that the relativistic factor \((1 - 2v_F^2)/3\) in Eq. (33) stems from the quenching of the GT strength in the nucleon space. In most of the relativistic models, the nucleon effective mass is about \(0.6M\) [8], which yields \(v_F = 0.43\) for \(k_F = 1.36 \text{fm}^{-1}\). This value implies that we must use a larger value of \(g'\) by 14% in the relativistic model than that in non-relativistic models. Non-relativistic models require the value of \(g'\) to be about 0.6 in order to reproduce experimental data [9]. In this case the relativistic model needs to use \(g' = 0.68\).
B. Effects of the Pauli Blocking Term

It is known in the relativistic model that the antinucleon degrees of freedom play an important role for some physical quantities\cite{10,11}. Even in the RPA based on the mean field approximation, a part of the antinucleon excitations should be taken into account in order to keep the continuity equation\cite{5}. It is the density-dependent part in the antinucleon excitations, which is usually called the Pauli blocking term. Without the Pauli blocking term, for example, the orbital part of the magnetic moment and giant multiple resonance states are not described correctly\cite{10,11}. In the case of the GT state at $q = 0$, it is not clear whether or not the Pauli blocking term should be taken into account. Let us study, however, their effects on the excitation energy of the GT state.

The Pauli blocking term in the present case is given by the density-dependent parts of Eq.(24)

$$\Pi_{\text{Pauli}} (\Gamma^2_{\frac{1}{2}}, \Gamma^2_{\frac{3}{2}}) = -4 \int d^3p \frac{p^2 - \rho^2_n}{E_p^2} \left( \frac{\theta^{(p)}_p}{2E_p - q_0 - i\varepsilon} + \frac{\theta^{(n)}_p}{2E_p + q_0 - i\varepsilon} \right).$$

Its real part is written as

$$\text{Re} \Pi_{\text{Pauli}} (\Gamma^2_{\frac{1}{2}}, \Gamma^2_{\frac{3}{2}}) = \frac{16\pi}{3} \kappa, \quad \kappa = - P_N(k_n, q_0) - P_N(k_p, -q_0), \quad (36)$$

where we have defined

$$P_N(k_F, q_0) = \frac{3}{4\pi} \int_0^{k_F} d^3p \frac{p^2 - \rho^2_n}{E_p^2} \frac{1}{2E_p + q_0}. \quad (37)$$

For $q_0 \ll M^*$, as in the GT state, $P_N$ is approximately given by

$$P_N(k_F, q_0) \approx P(k_F) = \frac{3}{4\pi} \int_0^{k_F} d^3p \frac{p^2 - \rho^2_n}{E_p^2} \frac{1}{2E_p + q_0} \approx E_F^2 \left( 3v_F^3 - \frac{3}{4} \log \frac{1 + v_F}{1 - v_F} \right) = k_F^3 \frac{v_F^3}{5} \left( 1 + \frac{3}{7} v_F^2 + \cdots \right). \quad (38)$$

where $E_F$ denotes $\sqrt{M^* + k_F^2}$. In taking the above contribution to Eq.(24), the excitation energy of the GT state is obtained as

$$\omega_0 \approx 1 - \frac{2}{3} v_F^2 \approx g_s \frac{8k_F^3 N - Z}{1 + \frac{2g_s}{3\pi^2} \frac{3k_F^2}{2A}}. \quad (39)$$

The result shows that when we use the values of parameters as mentioned at the end of the previous subsection, the effect of the Pauli blocking terms is negligible. Even if we should take into account the Pauli blocking terms, their effects are less than 0.5% on the excitation energy.

V. THE GT STRENGTH

In this section, first we will discuss the total GT strength in nuclear matter where we can obtain its analytic expression and understand the structure of the relativistic model. Next we will investigate effects of finiteness in nuclei.

A. The GT Strength in Nuclear Matter

The total GT strength is calculated by integrating the response function $R$ over the excitation energy. The relationship of the response function to the correlation function $\Pi$ is given by\cite{22}

$$R = \frac{3}{16\pi^2} A \frac{N}{k_F^3} \text{Im} \Pi.$$

\[\text{Im} \Pi = \sum_{\Gamma} \text{Im} \Pi_{\Gamma} = \frac{3}{16\pi^2} A \frac{N}{k_F^3} \text{Im} \Pi_{\Gamma} \quad (27)\]
First we investigate the total GT strength in the mean field approximation. For this purpose we can employ the imaginary parts of Eqs. (22) to (24). The total strength for the $\beta^-\tau\beta$ transitions in the nucleon space is given by the first term in the parentheses of Eq. (23),

$$S_{\text{ph}}^- = \frac{3}{4\pi} \frac{A}{k_F^3} \int d^3p \frac{M^* + p_y^2}{E_p^2} \left( \phi_p^{(n)} - \phi_p^{(p)} \right) = \frac{A}{k_F^3} \left( Q(k_n) - Q(k_p) \right).$$

(40)

When we expand $Q$ in terms of $(k_n - k_p)$ as before, we obtain the value of the total strength in the nucleon space,

$$S_{\text{ph}}^- \approx \left( 1 - \frac{2}{3} v_F^2 \right) 2 (N - Z).$$

(41)

In the present definition, Ikeda-Fujii-Fujita sum rule in non-relativistic models[12] is written as

$$\langle |Q^+Q^-| \rangle - \langle |Q^-Q^+| \rangle = 2(N - Z),$$

(42)

for

$$Q^\pm = \sum_i (\tau_i \pm \sigma_y) \cdot$$

This is nothing but the result of the commutation relation:

$$[\tau_+ \sigma_y, \tau_- \sigma_y] = 2\tau_z.$$ 

If we assume that there is no ground-state correlation,

$$Q_+ \rangle = 0,$$

we have simply from Eq.(43)

$$\langle |Q^+Q^-| \rangle = 2(N - Z)$$

in non-relativistic models. Comparing Eq. (43) with the above equation, it is seen that the relativistic sum value is quenched by the factor $(1 - 2v_F^2/3)$, which is about 0.88 for the previous value $v_F = 0.43$.

The strength of the $\beta^+$ transition in the nucleon space is given by the second term in the parentheses of Eq.(23) with replacing $q_0$ by $-q_0$, but its value is zero for $k_n > k_p$ as in Eq.(43). The quenched strength in the nucleon space in Eq.(41) is not taken by the $\beta^+$ transition, but by the antinucleon degrees of freedom. This fact is shown as follows. According to the first term of Eq.(23), the strength of the $\beta^-$ transition in the nucleon-antinucleon excitations is given by

$$S_{\text{NN}}^- = \frac{3}{4\pi} \frac{A}{k_F^3} \int d^3p \frac{P^2 - P_y^2}{E_p^2} \left( 1 - \phi_p^{(n)} \right),$$

while the one of the $\beta^+$ transitions is provided by the second term with replacing $q_0$ by $-q_0$.

$$S_{\text{NN}}^+ = \frac{3}{4\pi} \frac{A}{k_F^3} \int d^3p \frac{P^2 - P_y^2}{E_p^2} \left( 1 - \phi_p^{(p)} \right).$$

(45)

(46)

The above two equations are both divergent, but their difference is finite as

$$S_{\text{NN}}^- - S_{\text{NN}}^+ = \frac{3}{4\pi} \frac{A}{k_F^3} \int d^3p \frac{P^2 - P_y^2}{E_p^2} \left( \phi_p^{(n)} - \phi_p^{(p)} \right).$$

(47)

The sum of the above equation and Eq.(41) provides us with sum rule corresponding to Eq. (42),

$$S_{\text{ph}}^- + S_{\text{NN}}^- - S_{\text{NN}}^+ = 2(N - Z).$$

(48)

In order to obtain the sum rule value $2(N - Z)$, we need a complete set of the nuclear wave functions. This fact requires both the nucleon and the antinucleon space in relativistic models. Since the nucleon-antinucleon states are in the time-like region, the GT strength for charge-exchange reactions which excite nuclear states in the space-like
region is quenched by the amount of Eq. (47). This quenching can be also discussed by calculating GT matrix elements directly, as we have done in ref. [13].

Next we calculate the strength of the GT state in RPA, using the RPA correlation function \( \Pi_{\text{RPA}}(\Gamma^2_+, \Gamma^2_-) \). When using the abbreviations \( \Pi_{\text{RPA}}(\Gamma^2_+, \Gamma^2_-) = \Pi_{\text{RPA}}(q_0) \) and \( \Pi(\Gamma^2_+, \Gamma^2_-) = \Pi(q_0) \), \( \Pi_{\text{RPA}}(q_0) \) is written as

\[
\Pi_{\text{RPA}}(q_0) = \frac{\Pi(q_0)}{D_T(q_0)}, \quad D_T(q_0) = 1 + \chi_5 \Pi(q_0).
\]

Expanding \( D_T(q_0) \) at \( q_0 = \omega_0 \), we have

\[
\Pi_{\text{RPA}}(q_0) = \left( \frac{dD_T}{d\omega_0} \right)^{-1} \frac{\Pi(q_0)}{\omega_0 - \omega_0 + i\varepsilon}.
\]

When keeping only the density-dependent part of the correlation function, the imaginary part of the above equation gives the strength of the GT state,

\[
S_{\text{GT}} = \frac{A}{k_F} \frac{Q(k_n) - Q(k_p)}{1 + 2g_5\kappa/(3\pi^2)} \approx \frac{1 - 2v_F^2/3}{1 + 2g_5\kappa/(3\pi^2)} 2(N - Z).
\]

The \( \kappa \)-dependent term stems from the Pauli blocking effects, and is negligible, as mentioned before. Thus in the present model, the GT state exhausts the total strength in the nucleon space. Comparing Eq. (49) with the above equation, we can see that the factor \( (1 - 2v_F^2/3) \) in the expression of the excitation energy Eq. (33) is due to the quenching of the GT strength in the nucleon space, but not from the relativistic kinematics.

### B. The GT Strength in Finite Nuclei

We have shown analytically that the GT strength, which is responsible for the giant GT resonance, is quenched by about 12% in nuclear matter. The quenched amount, however, depends on the momentum distribution and the value of the nucleon effective mass near the nuclear surface, as seen in Eq. (49). Therefore let us estimate numerically the GT strength for finite nuclei in the mean field approximation.

We write the four-component nucleon spinor as

\[
\psi_{am} = \begin{pmatrix}
\frac{iG_s(r)}{r} |\ell jm\rangle \\
- \frac{G_a(r)}{r} |\ell jm\rangle
\end{pmatrix},
\]

where \( a \) stands for the quantum numbers \( \{n\ell j\} \), and \( \ell \) is given by \( \ell = j \pm 1/2 = \ell \pm 1 \) for \( j = \ell \pm 1/2 \). We define the GT strength as follows,

\[
T_{aa'}(\sigma \mu) = 2 \sum_{mm'} |\langle a' m' | \sigma \mu | a m \rangle|^2 = \frac{2}{3} |\langle \ell' j' || \sigma || \ell j \rangle_{\text{rel}}|^2,
\]

using the notations:

\[
\langle \ell' j' || \sigma || \ell j \rangle_{\text{rel}} = \delta_{\ell \ell'} \langle \ell j' || \sigma || \ell j \rangle g(a, a') + \delta_{\ell \ell'} \langle \ell' j' || \sigma || \ell j \rangle f(a, a'),
\]

\[
g(a : a') = \int_0^\infty dr G_s(r)G_{a'}(r), \quad f(a : a') = \int_0^\infty dr F_a(r)F_{a'}(r).
\]

If we calculate the strengths for the transition from \( j = \ell + 1/2 \) to \( j' = \ell \pm 1/2 \) (\( n'=n \)) only, as in non-relativistic models for subshell closed shell nuclei, the sum of the GT strengths is given by

\[
\sum_{a'} T_{aa'}(\sigma \mu) = \frac{4(\ell + 1)(2\ell + 3)}{3(2\ell + 1)} \left( g_+ - \frac{2\ell + 1}{2\ell + 3} f_+ \right)^2 + \frac{16\ell(\ell + 1)}{3(2\ell + 1)} g^2,
\]
with
\[ g_\pm = g(n, \ell, \ell \pm 1/2 : n, \ell, \ell \pm 1/2), \quad f_\pm = f(n, \ell, \ell + 1/2 : n, \ell, \ell \pm 1/2). \]

In assuming the proton wave function is the same as the neutron wave function, we have \( g_+ + f_+ = 1 \) from the normalisation of the wave functions. Moreover, it may be reasonable to assume that \( g_- \approx 1 - f_- \). Then, the sum of the GT strengths is approximately given by
\[ \sum_{a'} T_{aa'}(\sigma_\mu) \approx 2(2j + 1) \left( 1 - \frac{8}{3} f_+ \right). \]

Since most of the relativistic models provides us with \( f_+ \approx 0.02 \), the above equation shows that the GT strength is quenched by about 5%, compared with the non-relativistic sum value \( 2(2j + 1) \).

In relativistic models, there are other transitions even in sub-shell closed nuclei, like \(^{48}\text{Ca}\). Table \( \text{II} \) shows that their contributions to the total GT strength. In order to calculate the GT strengths, we have employed the relativistic model which is named NL-SH[14]. We calculate the only strength between the bound states. Contributions from the continuum states are expected to be small. In the Table, the top one shows the results in using the neutron wave functions for the initial and final state, and the bottom one those obtained using the proton wave functions for the final state. These calculations are performed to see effects of the Coulomb force. The non-relativistic sum value for \(^{48}\text{Ca}\) is 16 in the present definition. The Table shows that the relativistic sum value is quenched by about 6%, compared with the non-relativistic one. This reduction of the quenched amount, compared with the one in nuclear matter, was expected from the value of the nucleon effective mass near the nuclear surface, as mentioned before. Since the total GT strength in the nucleon space is quenched in the mean field approximation, we expect that the sum of the RPA strengths in finite nuclei is also quenched, as in the case of nuclear matter.

\[ \text{VI. THE PION- AND TIME-PART OF THE CORRELATION FUNCTION} \]

From section \( \text{II} \) to the last section we have discussed the problems related to the transverse part of the correlation function. In this section let us briefly discuss the structure of the pion- and time-component, mainly in order to study the way to use \( g' \) in relativistic models.

Since \( \mathbf{q} = 0 \), the correlation functions satisfy
\[ \Pi(\Gamma^{-1}_+, \Gamma^{-1}_-) = \frac{q_0^2}{\Gamma^0_0} \Pi(\Gamma^0_+, \Gamma^0_+) \text{,} \quad \Pi(\Gamma^{-1}_-, \Gamma^0_+) = \frac{q_0}{\Gamma^0_0} \Pi(\Gamma^0_+, \Gamma^0_+) \text{,} \]

the pion-and time-component of the dimesic function Eq.(13) is rewritten in terms of \( \Pi(\Gamma^0_+, \Gamma^0_+) \),
\[ D_{\text{PT}} = 1 - \frac{1}{(2\pi)^3} \left( g_5 + g_2 \frac{q_0^2}{m_e^2 - q_0^2} \right) \Pi(\Gamma^0_+, \Gamma^0_+) \text{.} \]

The function \( t^{(0)}(p, q) \) at \( \mathbf{q} = 0 \) in \( \Pi(\Gamma^0_+, \Gamma^0_+) \) is calculated according to Eq.(53) as,
\[ t^{(0)}(p, q) = 4 \left( 2M^* - E_p(2E_p + q_0) \right) \text{.} \]

In taking into account the density–dependent parts only, we have the real part of the time–component as
\[ \text{Re}\Pi(\Gamma^0_+, \Gamma^0_+) = J^{(0)}(k_n, q_0) + J^{(0)}(k_p, -q_0), \]

where \( J^{(0)} \) is given by
\[ J^{(0)}(k_i, q_0) = -\frac{4}{q_0^2} \int d^3p \frac{P_p^2}{\mathbf{E_p}^2} \Phi^{(i)} - 4M^* \int d^3p \frac{\Phi^{(i)}}{\mathbf{E_p}(2\mathbf{E_p} + q_0)} \text{.} \]

For \( q_0 \ll M^* \), neglecting the \( q_0 \)-dependence of the second term, we obtain
\[ J^{(0)}(k_i, q_0) \approx 8\pi M^* \left( \frac{Q_0(k_i)}{q_0} + P_0(k_i) \right), \]
with
\[
P_0(k_F) = -\frac{1}{2\pi} \int_0^{k_F} d^3p \frac{1}{2E_p^3} = \frac{k_F}{E_F} - \log \frac{E_F + k_F}{M^*} = -\frac{v_F^3}{3} \left(1 + \frac{3}{5}v_F^2 + \cdots\right),
\]
\[
Q_0(k_F) = -\frac{1}{2\pi M^*} \int_0^{k_F} d^3p \frac{p^2}{E_F^2} = 2k_F - 2M^* \tan^{-1} \frac{k_F}{M^*} - \frac{2k_F^3}{3M^*}. \tag{54}
\]

From the above equations, the real part of \( \Pi(I^0_+, I^0_0) \) is described as
\[
\text{Re} \, \Pi(I^0_+, I^0_0) \approx 16\pi M^* \left(P_0(k_F) + \frac{k_n - k_p}{2q_0} Q_0'(k_F)\right), \tag{55}
\]
where \( Q_0'(k_F) \) denotes the derivative of \( Q_0(k_F) \) with respect to \( k_F \),
\[
Q_0'(k_F) = -\frac{2k_F^4}{M^* E_F^2} = -\frac{2v_F^4}{1 - v_F^2}. \tag{56}
\]

Finally the real part of the PT dimers function is given by
\[
\text{Re} \, D_{PT} \approx 1 - \frac{2M^*}{\pi^2} \left(g_0 + g_2 \frac{q_0}{m^2 - q_0^2}\right) \left(P_0(k_F) + \frac{k_n - k_p}{2q_0} Q_0'(k_F)\right). \tag{57}
\]

The structure of \( \text{Re} \, D_{PT} \) is similar to \( \text{Re} \, D_T \) in Eq. (27) to which Eq. (60) is added. Eqs. (57) and (58), however, show that the quantity in the second parenthesis in the above equation is negative. Therefore, the excitation energy given by \( \text{Re} \, D_{PT} = 0 \) should be higher than the pion mass \( q_0 > m_\pi \).

The first parenthesis of Eq. (57) may be obtained by the insertion of \( g' \) into the pion propagator as
\[
\frac{1}{m^2 - q^2} \to \frac{1}{m^2 - q^2} + \frac{g'}{q^2}, \tag{58}
\]
which was frequently used in relativistic description of high-momentum transfer reactions. Eq. (57), however, shows that the way to put \( g' \) in the meson propagator cannot describe the GT states. In order to show this fact, we have used the Lagrangian form in Eq. (1), although the GT state can be described only by the contact term in nuclear matter. The above modification of the meson propagator in Eq. (58) was introduced from non-relativistic models. Those models use a static potential and modify the meson propagator so as to cancel the short range part of the interaction as
\[
\frac{q^2}{m^2 + q^2} \to \frac{q^2}{m^2 + q^2} - g'. \tag{59}
\]

Eq. (58), however, is not a reasonable extension of Eq. (50) for description of the GT state.

The last statement, of course, does not mean that the Lagrangian form in Eq. (1) provides us with a correct four-momentum dependence of \( g' \). In non-relativistic models also we do not know the dependence so well. The Lagrangian form in Eq. (1) can describe the GT state at \( q = 0 \), and cancel the short range part of the interaction, but yields an additional four-momentum transfer dependence of the dimers function. In fact, the dimers function except for the transverse part is written at the static limit \( q_0 = 0 \) as,
\[
-D_{PTL} = (1 - \chi_5 \Pi^{00}) (1 + (\chi_5 - \chi_5 q_0^2) \Pi^{11}) + \chi_5 (\chi_5 - \chi_5 q_0^2) (\Pi^{10})^2,
\]
where we have used the abbreviation: \( \Pi^{ab} = \Pi(I^+_a, I^-_b) \). More detailed investigation on \( g' \) is necessary for discussions of high momentum transfer phenomena.

Finally we note effects of the Pauli blocking terms. When we take into account the only particle-hole excitations, we have \( \text{Re} \, \Pi(I^0_+, I^0_0) \) as
\[
\text{Re} \, \Pi(I^0_+, I^0_0) = \frac{1}{q_0 + \imath\varepsilon} \int d^3p \frac{\theta^{100}_p - \theta^{01}_p}{2E_p^2} = \frac{8\pi M^*}{q_0 + \imath\varepsilon} \left(Q_0(k_n) - Q_0(k_p)\right).
\]

This shows that the term \( P_0(k_F) \) in Eq. (55) comes from the Pauli blocking terms. The effects of the Pauli blocking terms are not small in the present case, compared with those in the transverse mode. In fact, the relationship between contributions from the particle-hole term to the Pauli blocking one is given by
\[
Q_0'(k_F) \approx -2v_F^4, \quad P_0(k_F) \approx -\frac{v_F^3}{3} \approx \frac{1}{6v_F} Q_0'(k_F),
\]
in the present case, while in the transverse mode, we have from Eqs. (22) and (38)
\[ Q'(k_F) \approx 3k_F^2, \quad P(k_F) \approx k_F^2 \frac{v_F^3}{3} \approx \frac{v_F^3}{15} Q'(k_F). \]

VII. SUMMARY

In 1980’s, analytic expressions of the excitation energies for the giant monopole and quadrupole resonance states were derived in the relativistic model [17]. When they are expressed in terms of the Landau-Migdal (LM) parameters, they are formally equal to the non-relativistic expressions, in spite of the fact that the LM parameters are strongly dominated by relativistic effects. In this paper, we have obtained the relativistic expression of the excitation energy for the Gamow-Teller (GT) state in nuclear matter. It is described in terms of the LM parameter \( g' \) which is introduced in the Lagrangian as a contact term. Compared with the corresponding non-relativistic one, the relativistic expression has an additional factor of \( (1 - 2v_F^2/3) \), \( v_F \) being the Fermi velocity. This means that in order to reproduce the same excitation energy as in non-relativistic models, the present relativistic model requires a larger value of \( g' \) by this factor.

The above relativistic factor comes from the quenching of the GT strength in the nucleon space. A part of the GT strength is taken by the nucleon-antinucleon states in the time-like region which are not excited in usual charge-exchange reactions. This quenching is thus peculiar to the relativistic models. The quenched amount is estimated to be 12% of the classical Ikeda-Fujii-Fujita sum rule value in nuclear matter, and 6% in finite nuclei.

Recently experiment has observed 90% of the classical Ikeda-Fujii-Fujita sum rule value in \(^{90}\text{Zr}\) [17], although the data were analysed in non-relativistic models. So far the quenching of 10% has been considered to be due to the coupling of the particle-hole states with \( \Delta \)-hole states. Under this assumption, the LM parameter \( g'_{\Delta N} \) for the coupling is estimated to be about 0.2 to 0.3, depending on the model [8]. The determination of the value of \( g'_{\Delta N} \) is very important for studies of nuclear magnetic moments and pion condensation. In particular, the critical density of the pion condensation is dominated by the value of \( g'_{\Delta N} \). It has been shown that if its value is about 0.2, a rough calculation yields the critical density to be about 2 times of the normal density [13]. In the present relativistic model the nucleon-antinucleon excitations are also responsible for the quenching. If a half of the quenching is owing to the nucleon-antinucleon excitations, the value of \( g'_{\Delta N} \) becomes to be about a half of the above value, and consequently the critical density becomes lower. More detailed investigation on the observed quenching is required in the relativistic model.

In this paper we have also discussed whether or not it is appropriate for the relativistic model to insert the LM parameter \( g' \) into the meson propagator. This method was frequently employed for the study of high-momentum reactions, but we have shown that this method can not describe the GT states.

Furthermore, it has been shown that the Pauli blocking terms are not important for discussions of the GT states.

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TABLE I: The structure of the correlation function $I(I_a^+, I_{b+}^-)$. The first column and row indicate the values of $a$ and $b$ of $I(I_a^+, I_{b+}^-)$. The open boxes mean that $I(I_a^+, I_{b+}^-)$ has non-zero value. Table (a) is for $\mathbf{q} \neq 0$, while (b) for $\mathbf{q} = 0$.

| a | 1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| −1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| b | 1 | 0 | 2 | 3 |
|−1| 0 0 0 0 |
| 0 | 0 0 0 0 |
| 1 | 0 0 0 0 |
| 2 | 0 0 0 0 |
| 3 | 0 0 0 0 |

TABLE II: The GT strength of the single-particle transition in $^{48}$Ca. The top table shows the results obtained using neutron wave functions for the initial and final states, while the bottom one those using the proton wave functions for final states. The value in the parentheses following the single-particle quantum number shows the binding energy in MeV. $T_{aa'}$ is the value of the GT strength, and $g(a : a')$ and $f(a : a')$ show the overlap of the radial wave functions, as defined in the text. The values indicated by the underline do not contributed to the GT strength.

\[
\begin{array}{ccccccc}
\text{n} \rightarrow \text{n} & n\ell j & n'\ell' j' & T_{aa'} & g(a : a') & f(a : a') \\
1p_{3/2}(-39.41) & 2p_{3/2}(-3.97) & 0.001 & 0.0110 & -0.0110 & \\
1p_{1/2}(-39.41) & 1f_{5/2}(-2.09) & 0.002 & 0.0765 & -0.0175 & \\
1p_{3/2}(-39.41) & 2p_{1/2}(-2.74) & 0.004 & 0.0323 & -0.0091 & \\
1p_{1/2}(-36.23) & 2p_{3/2}(-3.97) & 0.001 & -0.0128 & 0.0116 & \\
1p_{1/2}(-36.23) & 2p_{1/2}(-2.74) & 0.001 & 0.0102 & -0.0102 & \\
1f_{7/2}(-10.00) & 1f_{5/2}(-2.09) & 8.411 & 0.9592 & -0.0150 & \\
1f_{7/2}(-10.00) & 1f_{7/2}(-10.00) & 6.390 & 0.9805 & 0.0195 & \\
\text{Total} & & & 14.810 & & \\
\end{array}
\]

\[
\begin{array}{ccccccc}
\text{n} \rightarrow \text{p} & n\ell j & n'\ell' j' & T_{aa'} & g(a : a') & f(a : a') \\
1p_{3/2}(-39.41) & 2p_{3/2}(-1.09) & 0.000 & -0.0113 & -0.0116 & \\
1p_{3/2}(-39.41) & 1f_{5/2}(-1.16) & 0.002 & 0.8084 & -0.0180 & \\
1p_{1/2}(-36.23) & 2p_{3/2}(-1.09) & 0.005 & -0.0366 & 0.0117 & \\
1f_{7/2}(-10.00) & 1f_{5/2}(-1.16) & 8.629 & 0.9715 & -0.0148 & \\
1f_{7/2}(-10.00) & 1f_{7/2}(-9.59) & 6.361 & 0.9787 & 0.0200 & \\
\text{Total} & & & 14.997 & & \\
\end{array}
\]