Structure of virtual photon polarization in ultrarelativistic heavy-ion collisions

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Abstract

Anisotropy in heavy-ion collisions leads to polarization of both direct and virtual photons, the latter detected via internal conversion to dileptons pairs. Thus measurement of photon polarization probes the anisotropy at all stages of collisions. In order to characterize the polarization of virtual photons, we derive here the general structure of the photon polarization operator, $\rho_{\mu\nu}$, in an anisotropic medium, in terms of the four spectral functions, two transverse and one longitudinal, as usual, plus a new spectral function that reflects anisotropy in asymmetric collisions; and derive the local production rate of dilepton pairs in terms of these four functions.

Keywords: Ultrarelativistic heavy-ion collisions, anisotropy, photon polarization, dilepton pairs

1. Introduction

Direct photons, both real and virtual, are valuable probes of the dynamics of ultrarelativistic heavy-ion collisions, comparable to the way the cosmic microwave background (CMB) probes the early universe. In the early universe, momentum anisotropy of photons at their last scattering with charged particles – at the time of decoupling of radiation and matter – leads to polarization of the CMB, as a consequence of transverse photon polarization being preserved in Thomson scattering [1, 2]. Similarly, direct and virtual photons in heavy-ion collisions provide information on initial as well as later stages of the collisions, e.g., deviations from thermal equilibrium in the hot plasma as reflected in the unpolarized photon spectrum [3]. Measurement of photon polarization in collisions, most promisingly that of virtual photons, measured in the angular distribution of the dileptons they produce [4, 5, 6], provides information on the anisotropy of the quark-gluon plasma. In this note we summarize the general framework, based on the photon spectral functions in the plasma, for analyzing the angular distribution and thus the polarization of dileptons in terms of the plasma anisotropies; a more detailed analysis is given in Ref. [7].

Recently two of us [8] pointed out how anisotropy in heavy-ion collisions leads to polarization of the direct photons produced in collisions; the principle photon sources are Compton scattering of gluons on quarks and quark-anti quark annihilation into a gluon-photon pair, Fig. 1. In general, the photon is polarized along the component of the charge current induced in the collision that is perpendicular to the photon...
momentum. In Compton scattering of a gluon against a quark at rest, the photon is preferentially polarized in the direction perpendicular to the scattering plane, since the charge current is along the direction of the gluon polarization. Similarly, in pair annihilation of an antiquark with a quark at rest the photon is basically polarized in the direction parallel to the scattering plane, since the charged current is along the incident antiquark momentum. The two processes tend to produce orthogonal photon polarizations; to see which is dominant requires detailed calculations. Reference [8] – in a schematic model in which the quarks were replaced by heavy scattering centers, with the gluon anisotropy described by an anisotropic temperature and the collision dynamics described by Bjorken expansion – estimated a net photon polarization of order 10% or larger. However, more realistic calculations [9] showed the final polarization to be only on the order of 3.5%.

Furthermore, detection of polarization of direct photons produced in heavy-ion collisions is very difficult. Essentially one must convert the direct photons in a foil to produce dilepton pairs. The small opening angle of the pairs plus their subsequent rescattering in the foil makes it not possible practically to identify pairs and reconstruct the polarization. Much more promising is to take advantage of internal conversion of photons into dileptons, whose angular distribution is more readily measured. The question is how is the anisotropy reflected in the angular distribution of dileptons from internal conversion. The answer is given in the structure of the photon polarization or self-energy operator to which we now turn.

2. Dilepton production rate and the photon polarization operator

The production rate \( R \) of dilepton pairs of 4-momentum \( p \) and \( p' \) by a virtual photon of 4-momentum \( q \), at a given point in space-time in the collision volume, is \( dR_{\Pi \gamma} / d^3p' dp = (\alpha^2/4\pi^2 Q^2) \rho_{\mu\nu}(q) L_{\mu\nu}(p, p') \), where \( \rho_{\mu\nu} \) is the spectral function of the in-medium photon self-energy, illustrated in Fig. 2.

\[
\Pi_{\mu\nu}(q, z) = \varepsilon^2 \int_{-\infty}^{\infty} \frac{dq^0}{2\pi} \frac{\rho_{\mu\nu}(q^0, q)}{z - q^0},
\]

and \( L_{\mu\nu}(q, s) = 2(q^\mu q^\nu - g^\mu\nu Q^2 - s^\mu s^\nu) \) is the squared matrix element for a virtual photon of 4-momentum \( q \) to produce the lepton pair. Here \( d^3p \equiv d^3p/2E_p \), \( d^3p' \equiv d^3p'/2E_{p'} \), \( q = p + p' \), \( s = p - p' \), \( O^2 \equiv q^2 a_s > 0 \), \( s^{00} = 1 \), \( Q^2 + s^2 = 4m^2 \), and \( m \) is the lepton mass.

To determine the structure of \( \rho_{\mu\nu}(q) \) in an anisotropic medium, we first construct a basis of polarization vectors, working in the local rest frame of the matter, and assuming that the anisotropy is along one axis \( \hat{n} \), generally the beam axis. We define the two polarizations transverse to the photon momentum \( q \): \( e_1^\mu = (0, \hat{\epsilon}_1, 0) \), where \( \hat{\epsilon}_1 \equiv (\hat{n} \times \hat{\mu})/|\hat{n} \times \hat{\mu}| \) and \( \hat{\epsilon}_2 \equiv \hat{n} \times \hat{\mu}/|\hat{n} \times \hat{\mu}| \). In addition, we define the longitudinal polarization vector \( e_0^\mu \equiv (q^\mu, 0, 0) \), where \( q^\mu \equiv q^0 / \sqrt{Q^2} \). These three polarization vectors have norm \( e_0^\mu e_0_\mu = -1 \), as does the 4-vector axis \( n^\mu \equiv (0, \hat{n}) \). The geometry of the polarization vectors and dilepton momenta are shown in Fig. 3. Together with \( q^\mu \) the polarization vectors form an orthonormal basis obeying \( g_{\mu\nu} = \delta_{\mu\nu} q^\mu q^\nu - e_1^\mu e_1^\nu - e_2^\mu e_2^\nu - e_0^\mu e_0^\nu \).
Thus $\rho^{\mu\nu}$ is composed of terms $\sim \delta^{\mu}_1 \delta^{\nu}_1$, $\delta^{\mu}_1 \delta^{\nu}_2$, $\delta^{\mu}_2 \delta^{\nu}_1$, as well as $\delta^{\mu}_1 \delta^{\nu}_2$, but not $\delta^{\mu}_2 \delta^{\nu}_1$ by symmetry, or $\bar{q}^T \bar{q}^T$ since $q_\mu$ is orthogonal to $\rho^{\mu\nu}$. With a little algebra we find the general structure,

$$\rho^{\mu\nu} = \delta^{\mu}_1 \delta^{\nu}_1 + \delta^{\mu}_1 \delta^{\nu}_2 + \delta^{\mu}_2 \delta^{\nu}_1 + N^{\mu} N^{\nu} \rho_n = -(\bar{q}^T \bar{q}^T) \rho^{\mu\nu} + \delta^{\mu}_1 \delta^{\nu}_2 (\rho^1_2 - \rho^2_1) + \delta^{\mu}_2 \delta^{\nu}_1 (\rho^1_2 - \rho^2_1) + N^{\mu} N^{\nu} \rho_n,$$

(2)

where $N^{\mu} \equiv n^{\mu} - (\bar{q} n) \bar{q}^T$, with $(ab) \equiv a^T b_n$. Compared to the structure of the photon polarization operator in an isotropic medium, the momentum-space anisotropy of the system leads to an extra structure function, $\rho_n$ term, and in addition a difference of $\rho^1_1$ and $\rho^2_2$ in general. The new spectral function, $\rho_n$, vanishes when the quark and gluon distribution functions are even under parity, so that $\bar{n}$ enters only as a special axis, and not as a special direction. Under the successive transformations of parity followed by letting $\bar{n} \rightarrow -\bar{n}$, the polarization vector $\hat{e}^n_1$ transforms as a vector, while $\hat{e}^n_1$ transforms as a pseudovector; thus the cross term $\sim \delta^{\mu}_1 \delta^{\nu}_2$ does not occur in $\rho^{\mu\nu}$, and hence neither does $\rho_n$. In symmetric collisions of two identical nuclei, there should not be a special direction in the local rest frame of the matter; however, for asymmetric collisions, e.g., $A$ on $A'$, one expects a non-zero $\rho_n$ term in the photon spectral function.

The various scalar spectral functions in Eq. (2) depend separately on the local $q_1$, $q_2$, and $\bar{n} \cdot \bar{q}$, where $q_\perp$ is the magnitude of the component of $\bar{q}$ orthogonal to $\bar{n}$; or covariantly, they depend on $Q^2$, $(qn)$, as well as on $(qn)$, where $u_n$ is the 4-velocity of the local rest frame.

The production rate of dilepton pairs is then proportional to

$$\frac{1}{2} \rho^{\mu\nu} L_{q\bar{q}} = Q^2 (\rho^1_2 + \rho^2_1 + \rho_n) + 4m^2 \bar{q}^T (\rho^1_2 - \rho^2_1) - \bar{s}^2 (\rho^1_2 - \rho^2_1) + ((qn)^2 - (sn)^2) \rho_n,$$

(3)

where we use $Q^2 + s^2 = 4m^2$ and $(sq) = 0$, and we define $s_j \equiv (s_{2j})$ ($i = 1, 2$), the components of $s$ transverse to $\bar{q}$ in the local rest frame: $s_{1j} = s_1 \delta_{1j} + s_2 \delta_{2j}$. This equation gives the dilepton production rate in terms of the projections of $s$ along $\hat{e}^n_1$ and $\hat{e}^n_2$, and $n$. The $s^2_j$ terms contain the anisotropy produced by transverse virtual photons; the $s^2_j$ term arises from the mixing of longitudinal and transverse ($\hat{e}^n_1$) virtual photons. Again, in symmetric collisions with parity invariance in the local matter rest frame, the final $\rho_n$ term is not present.

Writing $s_1 = |\vec{s}_1| \cos \phi_s$ and $s_2 = |\vec{s}_1| \sin \phi_s$ to bring out the anisotropy, we see that the production rate becomes

$$\frac{dR_{\ell^+\ell^-}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{2\pi^3 Q^2} \left[ 2Q^2 \bar{\rho}^T + (s^2_1 + 4m^2) \rho^1_1 + (Q^2 + (qn)^2 - (sn)^2) \rho_n - |\vec{s}_1|^2 (\bar{\rho}^T + \bar{\delta}^T \cos 2\phi_s) \right],$$

(4)

where $\bar{\delta}^T \equiv (\rho^1_1 + \rho^2_2)/2$ and $\bar{\delta}^T \equiv (\rho^1_2 - \rho^2_1)/2$. The $\cos 2\phi_s$ and $\rho_n$ terms are the effects of anisotropy. Equation (4) is the principal result of our analysis of the structure of the dilepton rate in the presence of anisotropy.1 We emphasize that this structure is valid for virtual photon production processes in all types of collisions, not simply production via interacting quarks and gluons.

For $m = 0$ with $\rho_n$ absent, we find the dilepton production rate,

$$\frac{dR_{\ell^+\ell^-}}{d^3 p_1 d^3 p_2} = \frac{\alpha^2}{2\pi^3 Q^2} \xi \bar{\zeta} \left( 1 + \lambda_s \cos^2 \theta_s + \mu_s \sin 2\theta_s \cos \phi_s + (n_s/2) \sin^2 \theta_s \cos 2\phi_s \right),$$

(5)

with $\lambda_s = (\bar{\rho}^T + \bar{\delta}^T)/\xi$, $n_s = -2\delta T/\xi$, and $\mu_s = 0$, where $\theta_s$ is the angle between $\bar{q}$ and $\vec{s}$, and $s = \bar{\rho}^T (1 - 2s^2_1/s^2) + \rho^T$.

By comparison the production rate of real photons ($Q^2 = 0$) of polarization $\epsilon^\mu$ is [3, 8, 12] is

$$\frac{dR_\gamma}{d^3 q} = \frac{\alpha}{2\pi^2} \epsilon^\mu (\bar{\rho}^T + \bar{\delta}^T \cos 2\phi_s),$$

(6)

1Reference [10] arrives at a similar result, but does not include the anisotropic terms $\bar{\delta}^T$ and $\rho_n$.

2This distribution is similar to that fitted in the NA60 analysis of dimuon pairs produced in In-In collisions at 158 GeV-A at the CERN SPS [11], where there the angles are defined in the Collins-Soper frame; for NA60 data averaged over all lab directions of the virtual photons, the corresponding $\lambda, \mu$, and $n$ are consistent with zero.
where \((\varepsilon_1) \equiv -\cos \phi, (\varepsilon_2) \equiv -\sin \phi,\) and 
\[
d^3 \bar{q} = d^3 q / |\vec{q}|.\]
The anisotropy for real photons arises entirely from the difference, \(\delta \rho^T\), of \(\rho^T_1\) and \(\rho^T_2\): the spectral function \(\rho_n\) does not enter.

### 3. Total dilepton rates

Equation (4) is not the end of the story. To derive the total dilepton production rate in an anisotropic ultrarelativistic heavy-ion collision, it is necessary first to calculate the individual spectral functions, \(\rho^T_1, \rho^T_2, \rho^T_n\), and then integrate in space and time over the full collision. Such a calculation, a generalization of prior calculations for real photon production in an anisotropic quark-gluon plasma [3, 12], is currently in progress [13]. The principle ingredients being included, in addition to simple Drell-Yan processes in the medium [14, 7], are the nominally second order processes in the strong interaction coupling \(\alpha_s^2\), as illustrated in Fig. 4, corresponding to Compton and pair annihilation production of virtual photons (cf. Fig. 1), with full space and time dependent anisotropic quark, antiquark, and gluon distributions, hard thermal loops, and soft scale processes, together with a description of the space-time evolution using full three dimensional anisotropic hydrodynamics [15].

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