Distributed Differentially Private Ranking Aggregation

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Abstract. Ranking aggregation is commonly adopted in cooperative decision-making to assist in combining multiple rankings into a single representative. To protect the actual ranking of each individual, some privacy-preserving strategies, such as differential privacy, are often used. This, however, does not consider the scenario where the curator, who collects all rankings from individuals, is untrustworthy. This paper proposed a mechanism to solve the above situation using the distribute differential privacy framework. The proposed mechanism collects locally differential private rankings from individuals, then randomly permutes pairwise rankings using a shuffle model to further amplify the privacy protection. The final representative is produced by hierarchical rank aggregation. The mechanism was theoretically analysed and experimentally compared against existing methods, and demonstrated competitive results in both the output accuracy and privacy protection.

Keywords: Ranking Aggregation, Distributed Differential Privacy, HRA Algorithm, Shuffle Model

1 Introduction

Cooperative decision-making \cite{14} is pervasive in business management, because of its superiority in providing information from different aspects for better decision-making. As an essential step in cooperative decision-making, aggregation combines all individual preferences into a representative output. In daily life, individuals often rank all available alternatives to reveal the preference relation of multiple alternatives, hence ranking aggregation has become essential for society, and many researchers focus on its two requirements, which are hard to satisfy simultaneously: privacy and utility. Preference data in ranking has sensitive information, and the leaking of it may make individuals susceptible to coercion. Utility represents whether the aggregation result stands for the majority preference. Consequently, the ability to effectively aggregate private ranking into
a representative result is important in ranking aggregation. In the past few years, substantial research efforts have been devoted to ranking aggregation.

Traditional anonymizing methods such as anonymization hardly solves the problem. For example, Hugo Awards 2015 incident [8] shows that the anonymized preferences could result in the re-identification of individuals because the adversary who has background knowledge are able to launch a linkage attack. According to the weakness of the traditional anonymizing method, many researchers draw attention to differential privacy (DP) [5].

DP is an effective method to provide a rigorous privacy guarantee, and it can defend against various attacks, no matter how much background knowledge the adversary has. As a lightweight methodology to protect privacy, many current works address the ranking aggregation problems with DP. Shang et al. [16] designed a privacy-preserving rank aggregation algorithm, and whatever the ranking rules, the algorithm adds noise on votes and returns the histogram of rankings. Based on Quicksort [10], Hay et al. [9] proposed three differentially private rank aggregation algorithms includes P-SORT, a pairwise comparison method about private ranking aggregation. The benefit of using DP to protect individual sensitive information is that the adversary is unlikely to obtain sensitive information by observing the releasing results. In the meantime, the results have high availability. Nevertheless, DP is not without limitations: in real-world applications, the curator may collude with the adversary to leak some information before perturbing.

Local differential privacy (LDP) [7] alleviates this issue through adding noise locally and uploading noisy data to the untrusted curator. Yan et al. [18] proposed LDP-KwikSort algorithm, and they use the number of queries K to trade-off between utility and privacy. Besides privacy and utility, Wang et al. [17] studied another property, soundness, and then proposed the weighted sampling mechanism and the additive mechanism to improve the ranking utility. Unfortunately, LDP needs a large amount of data to achieve an acceptable utility. Moreover, existing approaches that use pairwise comparison information to rank [9, 18] share a common limitation: they introduce additional errors by pivot random selection. In conclusion, two obstacles need to be overcome simultaneously. Firstly, the ranking algorithm needs to output an aggregation result with utility as high as possible. Secondly, in order to protect individuals’ sensitive data, the untrusted curator should not receive the original raw preferences.

With the increased awareness of privacy protection, many researchers are interested in distributed differential privacy (DDP) [15] to amplify the privacy. DDP builds on LDP but further protects privacy using an intermediate node. This may mitigate the problem of poor utility in LDP. Besides, recently advances in ranking aggregation such as the algorithm HRA [4], which takes advantage of pairwise comparisons to aggregate ranking, and provides one way to eliminate the errors of random selecting pivot. However, as this algorithm applies Borda count [2] and pairwise comparison method, it costs too much privacy budget if it is directly combined with DP, and results in very poor performance.
In this paper, we propose a novel algorithm DDP-Helnaksort to meet the requirements of privacy and utility in ranking aggregation. The contributions of this algorithm are two-fold:

- **DDP-Helnaksort** employs a new ranking aggregation that avoids random pivot selection as in quicksort-based LDP methods. Moreover, Borda count in HRA was replaced by a new method that scores the alternatives to reduce the noise effect caused by small privacy budget. Experiments show that it performs better than some pairwise comparison-based DP rank aggregations.

- We firstly adopt DDP to deal with the ranking aggregation problem. This was achieved by combining LDP with a shuffle model \[1\] that randomly permutes the preferences in order to amplify privacy before submitting rankings to the untrusted curator. This provides a stronger DP guarantee, which can be measured by calculating the amplification bound.

The rest of this paper is organized as follows. Section 2 provides the preliminary of ranking aggregation, differential privacy and shuffle model. Section 3 presents the DDP-Helnaksort algorithm and gives the privacy guarantee. ?? reports the comparison results with baseline algorithms and analyzes the effect of adjusting parameters. Final conclusions and future directions are shown in section 5.

## 2 Preliminaries

### 2.1 Ranking aggregation

**Conception and Measurement** In a ranking scenario, an agent \( u \) is asked to rank a set \( A = \{a_1, a_2, ..., a_m\} \) of alternatives and to provide the order of preferences, denoted by \( p_u = [x_1, x_2, ..., x_m] \), where \( x_i \) is the ranking index of \( a_i \), and \( x_i = 1 \) means that agent \( u \)'s favourite alternative is \( a_i \). A curator then collects the order from each agent and uses a ranking aggregation algorithm to output a representative ranking \( R \) based on \( \{p_u\} \). In this paper, \( a_i \succ a_j \) means that \( a_i \) is preferred than \( a_j \).

Ranking aggregation aims to find the most representative ranking \( R^* \). *Kemeny optimal aggregation* (KOA) \[6\] is used to find \( R^* \) by minimising the average Kendall Tau distance \( \bar{K} \). Kendall tau distance \[11\] measures the distance between two rankings by counting the number of inconsistent pairs among all pairs of alternatives: \( K(R, p_u) = \frac{1}{(\frac{m}{2})} \sum_{i \neq j, i, j \in [m]} \kappa_{ij}(R, p_u) \), \( \kappa_{ij}(R, p_u) \) is 1 when the pair \( a_i \) and \( a_j \) is ordered differently in rankings \( R \) and \( p_u \), otherwise it is 0. The average Kendall tau distance is then computed over the rankings \( \{p_u\} \) from all agents: \( \bar{K}(R, p_u) = \frac{1}{n} \sum_{u \in [n]} K(R, p_u) \).

**Hierarchical Ranking Aggregation** Hierarchical ranking aggregation (HRA) \[4\] algorithm can consolidate all agents’ rankings into a total order. It is a recursive process like Quicksort, but does not rank alternatives based on the random pivot selections as in Quicksort, which is likely to reduce the utility of a private ranking besides the impact of additive noise such as in DP.
Given \( m \) ranking alternatives, the HRA algorithm first computes an \( m \) by \( m \) pairwise comparison matrix (PCM) \( M \), where each entry \( M(i,j) = \frac{1}{n} \sum_{u=1}^{n} t_{ij}^u \) is a comparison score for \((a_i, a_j)\) over all \( n \) rankings.

\[ t_{ij}^u = \begin{cases} 1 & \text{if } a_i \succ a_j \\ 0 & \text{if } a_j \succ a_i \\ 0.5 & \text{otherwise} \end{cases} \]

Then, the algorithm computes an \( m \) by \( m \) pairwise preference relation (PPR) matrix \( D \). Each entry \( D(i,j) = 1 \) or \( 0 \) when \( M(i,j) \) is greater or smaller than \( M(i,j) \) respectively, and \( 0.5 \) for equality. Third, every alternative is allocated to a different level according to its score \( L(i) \) that is the row sum of \( D(i,j) \). As multiple alternatives can be allocated at the same level, they are further compared and allocated into sublevels using a sub-PCM that only includes the rankings of the corresponding alternatives. If alternatives have the same score in sublevel, Borda count is used to select a winner. Finally, the algorithm finishes when each level contains only one alternative.

### 2.2 Differential privacy

Differential privacy (DP) \([5]\) is a privacy protection model that adds calibrated noise to query outputs to ensure an adversary having negligible chance of guessing the sensitive information in a database. Formally, DP can be defined as:

**Definition 1.** \((\epsilon, \delta)\)-Differential Privacy. A random algorithm \( \mathcal{M} \) provides \((\epsilon, \delta)\)-differential privacy if for any two datasets \( D \) and \( D' \) that differ in at most a single record, and for all outputs \( A \in \text{Range}(O) \):

\[
Pr[\mathcal{M}(D) \in O] \leq e^{\epsilon} Pr[\mathcal{M}(D') \in O] + \delta.
\]

The parameter \( \epsilon \) is defined as the privacy budget, which controls the privacy guarantee level of the mechanism. Another parameter \( \delta \) is responsible for the probability that \( \epsilon \) does not hold. DP assumes that there is a trusted curator, but in reality, the adversary has possibility to collect information from the curator. Hence, the local differential privacy (LDP) \([7]\) has been utilized. In LDP model, each agent uses an algorithm \( \mathcal{M} \) to perturb data locally and then upload the noisy one to the untrusted curator. The definition of LDP is as follows:

**Definition 2.** \((\epsilon, \delta)\)-Local Differential Privacy. A local algorithm \( \mathcal{M} \) provides \((\epsilon, \delta)\)-local differential privacy if for any two value \( x \) and \( x' \), and for every output \( y \):

\[
Pr[\mathcal{M}(x = y)] \leq e^{\epsilon} Pr[\mathcal{M}(x' = y)] + \delta.
\]

Although LDP solves the problem that the curator may disclose information, it requires a huge amount of data to achieve a satisfactory utility \([1]\). Based on LDP, distributed differential privacy (DDP) \([15]\) can improve the data utility. In DDP, every agent adds noise locally, and uploads the data to a trusted intermediate node to protect privacy further, finally sends the results to the curator. On the one hand, we do not need to worry about the privacy leakage from the curator in DDP. On the other hand, it has a higher utility than LDP. In this paper, we apply DDP model to aggregate ranking. And we use Gaussian mechanism \([7]\) to perturb preferences. An application of Gaussian mechanism satisfies \((\epsilon, \delta)\)-DP, if variables drawn from the Gaussian distribution with \( \mu = 0 \) and

\[
\sigma = \Delta g f \sqrt{\frac{2 \log(\frac{1}{\delta})}{\epsilon}}.
\]

where \( \Delta g f \) is the global sensitivity of the function \( f \).
2.3 Shuffle model

Shuffle model can be used in intermediate nodes to realise DDP, and the protocol $P$ was proposed in [1]. The protocol has three components: a randomizer $R$, a shuffler $S$ and an analyzer $A$. First, $R$ applies LDP to perturb data to get $(\epsilon,\delta)$ protection. Then, $S$ chooses a random permutation $\pi$ to shuffle the data, and cut the connection between the outputs and their sources. Finally, $A$ analyses the data and gets the query result. The shuffle step can amplify the privacy, and the following theorem [3] quantifies the amplification bound of shuffling:

Theorem 1. If every agent sends a message to the shuffle model, and the randomizer $R$ satisfies $(\epsilon + \ln n,\delta)$-local differential privacy, then the protocol $P = (R,S,A)$ satisfies both $(\epsilon,\delta)$-differential privacy and $(\epsilon + \ln n,\delta)$-local differential privacy, where $\epsilon'$ is smaller than $\epsilon$, and $n$ is the number of agents.

3 Ranking Aggregation Algorithm Under DDP

In this section, we propose an algorithm DDP-Helnaksort to solve the private ranking aggregation problem. It can be formalised as follows: given $m$ alternatives to be ranked by $n$ agents, a curator need to present a final ranking that represents most agents’ preferences. In addition, each agent $u$’s ranking $p_u$ must not reveal to the curator his true preferences over the alternatives.

The DDP-Helnaksort algorithm consists of three steps, as shown in fig. 1. These steps are discussed respectively in section 3.1, section 3.2 and section 3.3. The first step (3-4) is ranking preference collection, in which each agent, before submitting the answers to the curator, adds the Gaussian noise to the rank of each pair $(a_i,a_j)$ that being queried. The second step (5-6) is a shuffling process, which collects the ranking of $(a_i,a_j)$ from the corresponding agents that answered the query, in order to further reduce the risk of privacy breach. The third step (7) aggregates to generate a final ranking of all $m$ alternatives.

3.1 Ranking Preference Collection

The first step collects $K$ private pairwise rankings from each agent, where $K$ is an input parameter. A larger $K$ leads to a more accurate aggregated ranking, because each pair $(a_i,a_j)$ will be answered by more agents. The drawback is the partition of the privacy budget into a tiny piece for each query, which results in adding large noise that diminishes the utility. A smaller $K$ can guarantee the utility, but the curator may end up with a less representative final ranking. We explore the optimal $K$ in section 4.2. The ranking preference collection step is shown in Algorithm algorithm 1. $l_{ij} \leftarrow p_u$ represents the preference in agent $u$’s ranking of a randomly selected pair $(a_i,a_j)$. This algorithm uses the Gaussian mechanism for noise addition (other mechanisms can be used too).
Algorithm 1 Ranking Preference Collection

Input: Agent $u$’s ranking $p_u$, $K$ queries, privacy parameter $\epsilon$ and $\delta$

Output: Private pairwise preferences $Q$

1: $Q_u = \emptyset$
2: for $k \in [K]$ do
3: \quad $l_{ij} \leftarrow p_u$
4: \quad $\tilde{l}_{ij} = l_{ij} + \text{Gau}(\frac{K\Delta f \sqrt{2 \ln \frac{2e}{\delta}}}{\epsilon})$
5: \quad if $\tilde{l}_{ij} > 0.5$ then
6: \quad \quad $\tilde{l}_{ij} = 1$
7: \quad else
8: \quad \quad $\tilde{l}_{ij} = 0$
9: \quad end if
10: \quad $Q_u = Q_u \cup \{\tilde{l}_{ij}\}$
11: end for

Fig. 1: Overview of DDP–Helensort
3.2 Shuffling

Shuffling before aggregation can amplify privacy without affecting the output utility. In DDP-Helnaksort, each pair \((a_i, a_j)\)'s answers from the corresponding agents are collected and randomly permuted at an intermediate node, so that when the private rankings are submitted, the curator is unable to guess the source of an answer with a non-negligible probability. The shuffle model finally provides a protection of DP with a smaller \(\epsilon\), which is further discussed in section 3.4.

3.3 Ranking Aggregation

Once all the private rankings are submitted, the DDP-Helnaksort algorithm goes into the final stage, ranking aggregation. This step is based on the HRA algorithm, but with a different fallback to sort equal alternatives in sublevels in order to reduce the noise effect. The algorithm is shown in algorithm 2. \(M\) is the number of alternatives in the unsorted sublevel, and \(C_{a_i, a_j}\) is the number of agents who voted \(a_i \succ a_j\). The method uses pairwise preference to calculate a score for \(a_j\)

\[
C_{a_j} = \sum_{j \in [M]} (C_{a_j, a_i} - C_{a_i, a_j}),
\]

hence avoids splitting some privacy budgets as in Borda count.

This RA(ranking aggregation) algorithm mainly adopts a separate-layer ranking thought to generate the aggregation ranking, which uses the information about \(C_{a_i, a_j}\) and \(C_{a_j, a_i}\). The calculations of PCM and PPR matrix happen at Line 6-8 and Line 9, respectively. After that, we can count the scores of every alternative in \(M\) (Line 10-12). And if the scores are same in two rounds, we calculate the \(C_{a_j}\), and then put the highest one in a high level and others at a low level to do the next round (Line 13-16). The algorithm iterates until \(M = 1\) in each level, and finally the aggregated ranking \(\tilde{R}\) is generated (Line 17-20).

3.4 Privacy Guarantee

**Theorem 2.** DDP-Helnaksort satisfies \((\epsilon, \delta)\)-local differential privacy and \((\epsilon - \ln \frac{n}{(2)^2}, \delta)\)-differential privacy when \(K = 1\).

**Proof.** In the ranking preference collection phase, Gaussian mechanism is used to add noise into every agent’s answers. Because are \(K\) rounds, \(\epsilon_k = \frac{\epsilon}{K}\) in each round. In Gaussian mechanism, we set

\[
\delta = \Delta g f \sqrt{2 \ln 1.25} = \frac{\epsilon}{\epsilon_k} = \frac{\epsilon}{K} \Delta g f \sqrt{2 \ln 1.25}
\]

And DDP-Helnaksort executes the post-processing procedure after applying Gaussian mechanism, hence it satisfies \((\epsilon, \delta) - LDP\). Besides, \(K = 1\) means that every agent answers once and uploads a single message (latter experiments confirm the algorithm utility is the highest when \(K = 1\)). In the shuffling phase,
Algorithm 2 RA

**Input:** Agents pairwise aggregation $C_{a_i,a_j}$ and $C_{a_j,a_i}$

**Output:** Aggregate ranking $\tilde{R}$

1. $M =$ number of alternatives needed to rank
2. $L = \{0\} * M$
3. if $M = 1$ then
4. return
5. end if
6. for each $i, j \in [m]$ do
7. Calculate $PCM(i,j) = \frac{C_{a_i,a_j}}{C_{a_i,a_j} + C_{a_j,a_i}}$
8. end for
9. Calculate $PPR$ according to $PCM$
10. for $j, i \in [M]$ do
11. Calculate alternatives’ level score $L(j)+ = PPR(i,j)$
12. end for
13. if $L(1) = L(2) = \ldots = L(M)$ then
14. put the $C_{a_j}$ winner into a high-ranking level and others into a low level
15. end if
16. for $l = 1$ to the number of different levels do
17. ranking of $l$-th level = RA (input ranking about the alternatives in $l$-th level)
18. end for
19. Rank the alternatives according to their levels to get aggregate ranking $\tilde{R}$

there are $\binom{m}{2}$ pairs of alternatives, so the number of same pair and the size of set $S$ in shuffle model is

$$n' = \frac{n}{\binom{m}{2}}$$  \hspace{1cm} (3)

Therefore, by using theorem 1, the algorithm DDP-Helnaksort satisfies $(\epsilon - \ln \frac{n}{\binom{m}{2}}, \delta)$-DP when $K = 1$.

4 Experiments

In this section, we evaluate the performance of DDP-Helnaksort, and compare it with benchmark methods on both real and synthetic datasets. All algorithms were implemented in Python and executed 300 times to get the result.

4.1 Experiment Settings

Datasets The experiments were conducted on synthetic datasets and a real-world dataset TurkDots [13]. By using R package PerMallows 1.13, we obtained four synthetic datasets with $n \in \{100, 1000, 2500, 5000\}$, $\theta = 0.25$, and $m = 15$ from Mallows model [12]. The dispersion parameter $\theta$ represents the distance between the generated ranking and ground truth ranking. The generated ranking is closer to the ground truth ranking when $\theta$ is larger. TurkDots is from Amazon Mechanical Turk, and it contains $m = 4$ alternatives rankings.
Baseline Algorithms

- **LDP-Kwiksort [18]**. It has $K$ rounds’ interactions between every agent and the untrusted curator. In each round, the curator random selects paired alternatives to ask agents preference and receives noisy answers from agents (queries to an agent are not the same), then uses the *Kwiksort* algorithm to get the aggregate ranking. Its utility is the highest when $K = 1$.

- **LDP-Quicksort**. Compared with LDP-Kwiksort, it only differs in when a new pivot random chosen in *Quicksort*, the curator will query the preference between the pivot and other alternatives. This setting is only to collect preference used in *Quicksort*, and avoid the waste of privacy budget for other pairs. $K$ in this algorithm represents the times of the agent’s answers. Finally, when the *Quicksort* algorithm is finished, the curator gets an aggregated ranking.

Utility Metric - **Average Kendall tau distance** We use the *average Kendall tau distance* to measure the accuracy of the aggregated ranking. The larger the average Kendall tau distance, the worse the algorithm performance. We normalise this distance by $\frac{2}{nm(m−1)}\sum_{u \in N} K(R, Ru)$. Hence, the average Kendall tau distance can be calculated as $K(R, Ru) = \frac{2}{nm(m−1)}\sum_{u \in N} K(R, Ru)$.

4.2 **Performance of DDP-Helnaksort**

Comparison between DDP-Helnaksort and Baseline algorithms We ran three algorithms LDP-Quicksort, LDP-Kwiksort, DDP-Helnaksort with Gaussian noise. Here $\epsilon$ is the parameter in LDP. We set $K \in \{1, m, max\}$ to observe the performance of different algorithms in different $K$, and $m$ is the number of alternatives. When $K = max$, the maximum value of $K$ in LDP-Kwiksort and DDP-Helnaksort is $\binom{n}{2}$, but in LDP-Quicksort, the value is according to the chosen pivot, and it is $(m−1)\log m$ in general. We did the experiment on TurkDots with $n = 100$. With $\epsilon = 1$, $\delta = 10^{-4}$ in local differential privacy, the average Kendall tau distance of LDP-Quicksort, LDP-Kwiksort and DDP-Helnaksort are shown in fig. 2.

The results in fig. 2 proves our algorithm outperforms others across different $K$. When we add the same scale of noise to these algorithms, the average Kendall tau distance of DDP-Helnaksort is the shortest. The cause is when adding same scale of noise, DDP-Helnaksort uses more pairwise alternatives’ information (the comparison information provided by pairwise comparisons) to rank, which leads to a more accuracy result. Besides, it keeps away from the error of pivot random selection, which can not be avoided by the other algorithms.

**Impact of Query Amount to every agent** Different number of the queries has different ranking aggregation results. More information can be obtained when increasing the number of queries, but at the same time, the privacy budget of
each round becomes smaller, and the larger noise is added to every answer. In order to get the best performance with the best $K$, we ran DDP-Heinaksort on dataset TurkDots and the synthetic dataset with 100 agents. We set $\delta = 10^{-4}$, $\epsilon \in \{0.5, 1\}$ (this $\epsilon$ is the parameter of DP, also means that it is the amplification result of local randomizer, and $\epsilon$ in following experiment is the same) as well as varying the number of queries $K$ to observe the performance of DDP-Heinaksort. The results are shown in fig. 3.

![Fig. 2: Comparison of algorithms according to average Kendall tau distance on TurkDots across different K](image)

It is apparent that as the decreasing of $K$, the performance of DDP-Heinaksort is better. The average Kendall tau distance reaches the minimum when $K = 1$. This experiment result is the same as [18], which reveals the best performance is achieved when $K = \frac{\epsilon}{2}$. The reason of this phenomenon is large $K$ leads to a small $\epsilon$ in each round, and large scale of noise has a great impact on results. Although some information about agents’ preferences is lost when $K$ is small, a small noise is added to each answer, and the impact is smaller than large noise with more information. The result also implies if we want to further improve performance of the algorithm, we can do some works about handling $\epsilon$ such as implementation of personalised differential privacy which can release some needless privacy.
Ablation Study: Impact of shuffle model and privacy budget As seen in section 3.4, shuffle model turns LDP to DDP and amplifies the privacy. When every agent gives his noisy answers, a shuffling mechanism used before aggregation can offer another protection. After using the shuffle model, the algorithm satisfies DP with a smaller $\epsilon$ than before. In order to demonstrate the privacy amplification of shuffling, we compared the algorithm with and without shuffle model in a same $\epsilon$. Besides, $\epsilon$ reflects the level of privacy protection of every agent. We varied $\epsilon$ to observe the changes in average Kendall tau distance. We set $k = 1$, and other experimental setup is unchanged.

![Fig. 4: Comparison of DDP-Helnaksort with and without shuffle model according to average Kendall tau distance on TurkDots (a) and a synthetic dataset (b) across different $\epsilon$ when $K = 1$ and $n = 100$](image)

We can conclude from fig. 4 that adding the shuffle step results in a better utility. The reason is that shuffling is equivalent to adding another noise on data. Consequently, when we compared the algorithm with and without shuffling at a certain $\epsilon$, the second one has a large $\epsilon$ locally, so it perturbs less on data and performs better. In fig. 4, the distance average increases more in TurkDots than the synthetic dataset from with shuffling to without shuffling, and this mainly relates to different number of alternatives $m$. The synthetic dataset has more alternatives than TurkDots, thus the synthetic dataset has more alternative pairs and it has fewer collected preferences about a certain pair. Therefore, the shuffle model offers a smaller amplification on the synthetic dataset. This phenomenon is consistent with the theorem 2 that the amplification bound is proportional to the amount of data about a certain pair. Moreover, when decreasing the privacy budget, the average Kendall tau distance increases due to large scale of noise, which make the final aggregation ranking further to the representative ranking. Furthermore, in DDP, we can choose alternative methods, such as some cryptography tools, to amplify the privacy.
5 Conclusions

In order to improve the utility of private ranking aggregation, we proposed a new algorithm DDP-Helnaksort, which avoids the issue of random pivot selection which appears in other private ranking algorithm using the pairwise method. We designed a new method to give alternatives’ score according to preference in pairs, which can save some privacy budget and lead to a higher utility. Experimental results indicate that our algorithm achieves a better performance. Besides, we’re first applying the DDP mechanism shuffle model to amplify the privacy. Theoretical analysis of amplification bound of shuffle model and experimental results all confirm that the shuffle model is valid.

In the future, we will further improve the ranking utility, such as using some cryptography tools. Besides, this algorithm can be further optimised if it could apply personalised DP, which can release some redundant privacy budget to achieve a higher utility.

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