Finite Element Model Updating and Verification of Satellite-borne Slot Waveguide Based on Random Vibration Test

Sun Weimin¹, Cao Weizhong¹, Tong Mingbo²
¹ Nanjing Research Institute of Electronics and Technology, Nanjing 210039, China
² Nanjing University of Aeronautics & Astronautics, Nanjing 210016, China

Abstract. For an axial free-floating slot waveguide, finite element model updating and verification based on the vibration data are studied. The slot waveguide and the free-floating side are modeled by shell element and spring element respectively. The two slot waveguide structures are applied to modal and random vibration test getting the modal frequencies, root mean square of acceleration. The finite element are first verified by tested modal data, and then the connecting stiffness will be updated by random vibration test data. These results show that the analytical dynamic responses of the updated finite element are well compared with the test responses, and the maximum error is 3.8%. The spring element could also model the free-floating side well and verify the accuracy for dynamic analysis.

1. Introduction
The slot waveguide is widely used in Satellite-borne SAR antennas[1]. As an important part of SAR, the slot waveguide is complex[2], and its machining accuracy and welding process requirements are strict. The satellite is subjected to complex alternating thermal loads in the orbital operation[3, 4] so that the floating property of slot waveguide must be taken into consideration during installation to avoid thermal stress caused by thermal deformation. At the same time, the antenna bears a large vibration load when the satellite is launching [5], and the antenna deformation caused by the vibration will affect the operation of the satellite to some extent. High rigidity and strength are needed to ensured that the waveguide does not deform when the satellite is working, which contradicts the floating requirement of the waveguide.

Han Jun et al[6] constructed a semi-rigid structural joint unit, which was applied to the model of structural connection parameter identification, and attributed the parameter identification to the nonlinear least squares problem solving. The trust region method was used to solve the problem of too small selection range for initial value of the least squares solution.; an improved Iwan phenomenological model proposed by Wang Dong et al. [7] can better describe the nonlinear characteristics of the joint interface; Ritto et al. [8] applied the Bayesian method to update the probability model with joint stiffness; Lyu et al. [9] identified the bilinear joint stiffness of a semi-rigid joint model from a finite test strain response by using a temperature-based response sensitivity analysis method. Based on the dynamic test model updating method, the joint stiffness of the structure can be effectively identified. Due to the immature of the method to simplify finite element modeling of the floating end of the slot waveguide, the matching between the slot waveguide floating and the stiffness requirement can be determined by the model updating method.

Based on modal parameters, the model updating technique constructs a target residual is using test data and analysis data to update the model parameters, aiming at obtaining an accurate finite element
analysis model[10]. The conventional model updating method focuses on the modification of the structural modes (frequencies and shapes) [11-16], ensuring that the results of structural dynamics of simulation are consistent with the test. On this basis, relevant scholars have investigated on model updating methods for structural response. Canbaloglu et al. [17] used the pseudo-admittance difference method to solve the structural frequency response function, and regarded it as a modified feature quantity to update the structure using model updating method. Wang et al. [18] taken the local connection relationship of structures into consideration and proposed a four-step strategy for updating the connection structure by using the measured frequency response function: localization, characterization, quantization and verification, which can update some simple local-nonlinear structures. Wang et al. [19] used analytical modal decomposition and Hilbert transform to extract transient features in dynamic response data, and update the finite element model with hinged connection structure.

In this paper, two finite element models of a satellite-borne SAR lightweight slot waveguide structure were established under compression and floating case. The simplified analysis model was updated by vibration modes and random vibration test. A finite element modeling method for floating slot waveguide structure was proposed.

2. Satellite-Borne Slot Waveguide Structure

![Figure 1. Slot waveguide structure](image)

The slot waveguide is assembled by a plurality of high-precision thin-wall aluminum alloy cavity waveguide components with complex shapes as shown in Figure 1. The thickness of the main body is 1.0mm, and the material is made of aluminum alloy 3003(H12). The material parameters are shown in Table 1. Its geometric size is 296x37x19 mm.

| Material properties | Type (3003 H12) |
|---------------------|-----------------|
| Elastic Modulus     | 70000 (MPa)    |
| Poisson's ratio     | 0.33            |
| σb                  | 130 (MPa)       |
| σ0.2                | 125 (MPa)       |
| σ-1                 | 55 (MPa)        |

The slot waveguide is connected to the antenna frame through mounting screws at both ends. In order to reduce the influence of the thermal deformation of the slot waveguide on the flatness of the antenna array in the space environment, the slot waveguide is fixed on one side and free on another side. To achieve zero-friction floating, the floating end screws are installed in a state between compression and relaxation. In order to analyze the dynamic characteristics of this floating state more effectively, the simplified finite element analysis model of the floating slot waveguide is established in two steps and then updated:

1. The finite element model of the slot waveguide with the floating end compressed was established, and the modal test was performed to verify the accuracy of the corresponding finite element model.

2. A finite element model of the slot waveguide under floating state was built, and random vibration test of the real structure under floating state was carried out. The model was updated and verified according to this test data.

3. Dynamics Modeling and Testing
3.1 Dynamics model

The slot waveguide finite element model was built by MSC.PATRAN, in which the waveguide cavity take the mid-surface position of each part and is simulated by shell element; the mounting boss and cable mounting flange are simulated by solid elements, the total number of units is 10580, and the finite element model of slot waveguide is shown in Figure 2.

The coordinate system is defined as: x-axis is the slot waveguide length direction (axial direction), y-axis is the slot waveguide width direction, and the z-axis is the slot waveguide vertical direction.

When the floating end is pressed, the movement of the floating end of the slot waveguide in the yoz plane is constrained, so the same constraint boundary condition as the fixed end is adopted.

Figure 2. Finite element model of slot waveguide structure

When the floating end is under the free state, the slot waveguide will have a translational motion in the z direction, as shown by the arrow near the floating end in Figure 2. In the finite element model, two spring unit are used at the floating end of the slot waveguide to simulate the connection stiffness of the floating end in y-direction and z-direction. The stiffness in the y direction is infinite, simulating its displacement constraint. The z-direction stiffness is used to simulate the actual floating of the slot waveguide, which needs to be updated according to the vibration test data, and the root of the spring unit is fixed.

4. Finite Element Model Updating Based on Random Vibration

Figure 3 is a schematic diagram showing the process of updating and verification of the finite element model of the slot waveguide based on dynamic characteristics and random vibration. Firstly, the modal analysis of the finite element model under compression is carried out, and the experimental modal frequencies and mode shapes are used to update and verify it. Secondly, on the basis of the verified finite element model, considering the influence of the floating end, a finite element model under floating state is established. The random vibration test data is used to update the spring stiffness at the floating end. Finally, the updated finite element model under floating state is verified by using the random response data.

For systems with N degrees of freedom, the equation of motion can be expressed as:

$$M \ddot{x} + C \dot{x} + Kx = F$$  \hspace{1cm} (1)
M, C, and K are collectively referred to as structural physical matrices, which are structural mass matrix, damping matrix, and stiffness matrix, respectively. In the modal analysis, the slot waveguide is regarded as an undamped free vibration system, and its characteristic equation is:

\[(K - \omega^2M)x = 0\]  \hspace{1cm} (2)

In order to solve the eigenvalues and eigenvectors, the Block Lanczos method\cite{20} with high precision and calculation speed is adopted. This method with high precision uses recursive algorithm.

Before the random vibration analysis, frequency response analysis of the slot waveguide is firstly carried out to calculate the vibration response of the structure under steady state excitation. The modal method is used for frequency response analysis. Decomposing the dynamic equations using the regular modes of the structure can reduce the scale of the problem and make the solution more efficient. In the frequency response analysis, according to engineering experience, the structural damping coefficient is generally taken as 3%.

When the structure is subjected to a single random excitation, the response power spectral density \(S_x(\omega)\) has the following relationship with the excitation power spectral density \(S_F(\omega)\):

\[S_x(\omega) = H^*(\omega)SH^T(\omega)\]  \hspace{1cm} (3)

\(H(\omega)\) is the frequency response function of the response relative to the excitation and can be obtained from the previous frequency response analysis. \(H^*(\omega)\) is a conjugate matrix of \(H(\omega)\), and \((\cdot)^T\) represents the transpose of the matrix. On this basis, the finite element model of the slot waveguide is updated.

It is known that there are a total of \(m\) parameters to be updated, and the parameters to be updated change the structural response results by affecting the structural physical matrix. The parameter to be updated can be expressed as:

\[p = \begin{bmatrix} \ p_1 & \cdots & \ p_m \end{bmatrix}^T\]  \hspace{1cm} (4)

Then the physical matrix of the structure can be expressed as a function of the variable \(p\) to be updated:

\[
M = g_M(p) \\
C = g_C(p) \\
K = g_K(p)
\]  \hspace{1cm} (5)

The corresponding feature quantity can also be expressed as a function of the variable to be updated:

\[f = F(M, C, K) = F(g_M(p), g_C(p), g_K(p)) = f(p)\]  \hspace{1cm} (6)

Where \(f\) can be any feature quantity, such as modal frequency, mode shape or random response. The finite element model updating can be transformed into a parameter optimization problem:

\[
\begin{align*}
\text{Min} & \left\| W_f (f - f_p) \right\|_2 \\
\text{s.t.} & \ V_{lb} \leq \ p \leq \ V_{ub}
\end{align*}
\]  \hspace{1cm} (7)

Where \(f_e\) and \(f_a\) represent the experimental value and the analytical value of the structural feature quantity, respectively, \(J(p)\) is called residual term, \(V_{lb}\) and \(V_{ub}\) represent the upper and lower limits of the change of the structure to be updated respectively , and \(W_f\) is the weight matrix among the various feature quantities of the structure.

In general, the feature quantity function \(f(p)\) is a nonlinear function of the parameter to be updated. To linearize the nonlinear optimization problem, at the initial updating point, \(f(p)\) is expanded to the first-order Taylor expression of the parameter to be updated:

\[f(p) = f(p_0) + G\Delta p\]  \hspace{1cm} (8)

Where \(p_0\) is the initial value of the parameter to be updated, and \(G\) is the sensitivity matrix, indicating the sensitivity of the feature quantity to the parameter to be updated, which is expressed as:
Using the Lagrangian multiplier method, the extreme value problem of the equation is transformed into the following linear problem:

$$ W_i G \Delta p = W_j (f_j - f_j(p)) $$  \hspace{1cm} (10)

The above equation is a common model updating problem.

### 4.1 Test content and method

The slot waveguide test pieces are numbered 03# and 05#. Two sets of test pieces were placed on the test bench at the same time. In order to study the vibration characteristics under the compression state and the floating state, the vibration modal test and the random vibration test were performed on the two sets of test pieces respectively.

1. **Vibration modal test under compression state:** Identify the modal parameters of the two sets of slot waveguide specimens, and obtain an accurate dynamic analysis model; use PCB-086B80 micro-hammer to excite the test product; Response is measured by the 352C22 micro-acceleration sensor; the VXI dynamic signal analysis system collects and analyzes the excitation signal and the acceleration response signal to obtain the frequency response function of the test piece; Curve fitting and data synthesis of the frequency response function is performed using the I-deas modal parameter identification software to obtain the modal parameters (natural frequencies and mode shapes) of the test piece. Due to the uncertainty of the floating end of the slot waveguide, the floating end of the slot waveguide is pressed by bolts during modal test, eliminating the uncertainty of the model caused by the influence of the floating end.

2. **Random vibration test in floating state:** According to the requirements of random vibration test conditions, random vibration tests of three parts in x, y and z directions were carried out, and the acceleration response of the test piece was measured. The floating ends of the two sets of slot waveguide test pieces are crimped by the studs to ensure that their zero-frictional-force floats. Finally, the random vibration response data is used to update the stiffness of the floating end and verify the response.

**Figure 4.** Arrangement of the accelerometers

The slot waveguide was mounted on a test fixture, the test fixture is mounted on a UD-H560B-16 electric vibration table with a 3-point average control. The controller performs random vibration in three directions of x, y, and z according to the test conditions of Table 2.

Acceleration monitoring points are arranged on the slot waveguide. The arrangement are shown in Figure 4. PCB-352C22 miniature acceleration sensor and LDS dynamic signal analyzer are used for acceleration response measurement.
Table 2. Random vibration test condition

| Frequency range (Hz) | Acceleration PSD | Total root mean square of acceleration | Time |
|----------------------|-------------------|----------------------------------------|------|
| 20~100               | +3dB/oct          |                                        |      |
| 100~600              | 0.25g^2/Hz        |                                        |      |
| 600~2000             | -9dB/oct          | 14.33g_{rms}                           | 2min |

5. Dynamic Model Correction and Verification

5.1 Modal parameter verification

According to the dynamic signal analyzer, the response signal of the acceleration sensor is collected and analyzed, and the frequency response function of the slot waveguide is obtained, as shown in Figure 5.

![Figure 5. Frequency response function of the slot waveguide](image)

The measured values of the slot waveguide and the calculated values of the finite element model under the compressed state are shown in Table 3. The relative error of the frequency test values between two sets of slot waveguides is small, indicating that the test pieces are consistent.

Table 3. Comparison between tested and numerical modal frequencies

| Mode | Test (Hz) | Simulation (Hz) | Error(%) | Shape |
|------|-----------|-----------------|----------|-------|
| 03#  | 389.3     | 370.9           | 4.73     | 1st Bending |
| 05#  | 394.4     |                 | 5.96     |       |
| 1st  |           |                 |          |       |
| 2nd  | 790.0     | 779.98          | 1.27     | 1st Torsion |
|      | 813.7     |                 | 4.14     |       |
| 3rd  | 1149.0    | 1141.5          | 0.65     | 2nd Bending |
|      | 1197.7    |                 | 4.69     |       |
| 4th  | 1643.6    | 1694.8          | 3.12     | 2nd Bending |
|      | 1689.2    |                 | 0.33     |       |

It can be seen from the comparison of the above data that the calculation error of the slot waveguide is about 5%, which has a high degree of coincidence. It is indicated that the simplified finite element model of the slot waveguide under compressed state meets the engineering accuracy requirements, which could be employed in the following studies.

5.2 Acceleration response correlation and verification
The acceleration response signals of the stationary segments of each monitoring point are divided into five samples, and the root mean square value of each sample is calculated. The average of the root mean square values of the five samples is shown in Table 4. According to the finite element model under the compressed state, the root mean square value of the acceleration response under the corresponding state is also calculated.

**Table 4. Acceleration response of compaction state**

| No. | Test 1σ(g) | Simulation 1σ(g) | Error(%) |
|-----|------------|------------------|----------|
| 1#  | 41.8       | 55.3             | 32.3     |
| 3#  | 49.3       | 48.98            | 0.65     |
| 4#  | 44.5       | 14.68            | 67.01    |
| 5#  | 16.6       | 16.31            | 1.7      |
| 6#  | 33.6       | 38.08            | 13.3     |

It can be seen from Table 4 that there are three deviations between the calculated values and the measured values of the acceleration monitoring points (1#, 4#, 6#). The 4# monitoring point is close to the floating end of the slot waveguide, and its root mean square value of measured acceleration response in z-direction reaches 44.5g. Since the slot waveguide is designed to ensure frictionless floating, the state at the floating end in z direction is in a non-compessed state. In the finite element model under compressed state, the z-direction freedom of the floating end is constrained. Therefore, the finite element model under compressed state is directly used for calculation, which results in a large deviation between the calculated result and the measured value.

The finite element model under free case is adopted, then the root mean square value of the acceleration response under three different z-direction spring stiffnesses is calculated. The results are shown in Table 5. It can be seen from Table 5 that when the spring stiffness in z direction is 130 N/mm, the deviation between the calculated values and the test results of the acceleration monitoring points is the smallest, and both are less than 4%, which meets the engineering precision requirements. From the above comparison, the spring element could model the free-floating side well and improve the accuracy of dynamic analysis.

**Table 5. Acceleration response of float state**

| Point | Test 1σ(g) | UZ=180 N/mm | Error (%) | UZ=140 N/mm | Error (%) | UZ=130 N/mm | Error (%) |
|-------|------------|-------------|-----------|-------------|-----------|-------------|-----------|
| 1#    | 41.8       | 50.12       | 19.9      | 44.79       | 7.15      | 42.9        | 2.8       |
| 3#    | 49.3       | 55.2        | 11.97     | 52.35       | 6.19      | 51.1        | 3.8       |
| 4#    | 44.5       | 36.8        | -17.3     | 43.29       | -2.72     | 45.9        | 3.3       |
| 5#    | 16.6       | /           | /         | /           | /         | 16.2        | 2.3       |
| 6#    | 33.6       | /           | /         | /           | /         | 33.4        | 0.32      |

6. Conclusion

In this paper, the simplified structural dynamics finite element model of the slot waveguide under compression and floating state is established respectively, and the modal test and random vibration test are carried out on the two sets of test pieces. Based on the vibration modal test, the dynamics model of the waveguide under compressed state is verified. The deviation between the calculated values and the experimental values of the first four-order frequencies is about 5%, which is in good agreement with each other. On this basis, to gain the root mean square value of the acceleration response measured in random vibration, the support stiffness of the slot waveguide under floating state in the z direction is determined. The result shows that the spring element with 130N/mm stiffness in the z direction can...
simulate the slot waveguide floating effect well. The deviation between the measured value and the calculated value of the acceleration response at each monitoring point is about 3%, which meets the engineering analysis accuracy requirements. This simplified modeling method can be applied to finite element analysis of floating waveguide in the future, a plenty of test costs can be saved.

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