Evaporating bubble in a heated capillary: effect of passage on the temperature field of the external wall

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Abstract. The nonisothermal Taylor liquid-slug-vapor-bubble problem, occurring inside a capillary of circular cross-section, is investigated numerically. The underlying hydrodynamic and mass transfer phenomena are considered the major heat transfer means in pulsating heat pipes. The temperature signature at the outer side of the capillary, inside which the bubble travels, is particularly examined. It is shown that for typical flow conditions, i.e. for liquid flow velocity and applied heat flux about 0.1 m s$^{-1}$ and $10^5$ W m$^{-2}$, respectively, wall thickness effects on capillary wall temperature are negligible in terms of diffusion and lag. In addition, the larger the liquid flow velocity, the more likely the bubble grows (due to evaporation) axially. This investigation opens new avenue to inverse methods where the bubble position is identified only through the temperature profile at the outer side of PHPs channels wall.

1. Introduction

Pulsating heat pipes (PHP) are very effective two-phase heat transfer devices, widely studied in the last twenty years [1, 2]. A PHP consists of a meandering capillary partially filled with a working fluid. It differs from traditional heat pipes by the fact that there is no wick structure at the inner wall of the capillary. In addition, heat transfer occurs not only by latent heat exchange but also, and mainly as suggested by some studies, by sensible heat. The spontaneous formation of vapor bubbles in the evaporator (hot section) and their condensation at the condenser (cold section) creates the oscillating motion of the bubble-liquid slug structure, which characterizes the PHP operation.

Both PHP’s simplicity and high efficiency, especially compared with other kinds of heat pipes, make it very promising. Nonetheless, its industrial application is still limited because its operation is both partly unpredictable and not totally understood. Modeling of heat and mass transfers remains complex, due to the chaotic behavior and high number of parameters involved in such an oscillating fluid flow. Many – rather phenomenological – attempts have been performed in order to establish global models of PHP [1], but none of them gave absolute satisfaction. In order to better fit reality, experts agree that an accurate understanding of the local physics is essential. Thus, the current work focuses on few physical phenomena on which the information is not yet available in the literature: vapor hydro- and thermodynamics, viscous dissipation during the bubble motion, and in some extent the behavior of the contact line.

It is well known from general considerations that, extending from the contact line, the evaporation of wetting films into elongated bubbles takes an important part in the heat and
mass transfer mechanism in these flows. So no realistic model can be achieved without taking this parameter into account. One can notice that some attempts, integrating the wetting film, have already been performed in order to model the oscillation of a bubble in a PHP [3, 4]. Nevertheless, as mentioned above, the results are limited because of lack of information on the local behavior of the thin wetting film during its evaporation. Particularly, Dobson [3] and Das et al. [4] studies do consider the film thickness constant.

Since the latter parameter is one of the important factors for the prediction of heat transfers in capillaries, the estimation of initial film thickness is essential, before considering bubble evaporation. To enlighten this last point, the study can be based – in first approach – on the original Bretherton problem, which involves the traveling of a long-semi-infinite vapor bubble inside a capillary [5]. While moving inside the liquid flow, the bubble deposits a liquid film on the wall of the capillary, whose thickness depends on the capillary number. When the bubble is finite, the so-called Taylor flow is recovered [6]. It has been shown experimentally that, in some cases at low Reynolds number, the inertia effects can be neglected, so the liquid film thickness is determined only by capillary number [7]. Nevertheless, the correlations used in this case are no more reliable at high flow speeds, when inertia effects can no longer be neglected. Liquids presenting lower viscosity, which some of them are used in PHPs, can present this characteristics. In that case, other parameters have to be integrated with the initial Bretherton problem in order to predict liquid film thickness, bubbles formation and flow behavior [8, 9]. Among these are the dynamic contact angle and the bubble velocity and shape [2, 10]. It has been proved and numerically quantified that the latter, in combination with the hydrodynamics of slug flow inside capillaries, do also influence the heat and mass transfers. Taha and Cui [11] especially succeed in predicting velocity and bubble shape, as functions of the capillary number at isothermal conditions.

The considerations mentioned above are mandatory to elaborate accurate liquid film evaporation models. Thereby, and assuming that the dominant transfer mechanism in this device is the liquid film evaporation, some models describing the dynamics of the film during those heat and mass transfers have already been proposed. Jacobi and Thome [12] have suggested an analytical model to describe the mechanism of cross evaporation of the liquid film. Considering a two-zone heat transfer model, it is based on a simplified physical description: a liquid slug and a vapor bubble. In order to describe even more accurately the physics of this flow, Thome et al. [13] have considered a three-zone flow boiling model in order to mimic closer the evaporation of elongated bubbles in capillaries.

It is worth noticing that most studies mentioned here are based on capillaries of circular cross-section. Indeed, this represents the simplest variant of heat transfer devices, such as PHPs. Therefore, the present study focuses on a similar configuration. The details of the configuration investigated are presented in section 2. In order to simulate the hydrodynamic and heat transfer problem combined with moving boundaries, a phase-field tracking approach, based on the Cahn-Hilliard equation, is adopted. The coupled problem, defined in section 2, is solved using a finite element solver. Then, results are presented and discussed in section 3. In this work the effect of the thickness of the capillary wall on its temperature profile disturbance, when bubble travels, is examined. In fact, a mid-term objective of this investigation is to help identifying bubble displacement through infrared thermography. Both wall diffusion and lag effects are especially addressed. The influence of the inflow velocity and the applied heat flux density on the capillary wall temperature profile is also investigated.

2. Theoretical

2.1. The configuration

A bubble traveling inside a capillary is depicted in Figure 1. The copper capillary inner diameter is 2.5 mm, it is 15 mm long and its wall is $e = 0.25$ mm thick. A constant heat source term is
applied over the thickness of the capillary wall. At one side of the capillary, a constant liquid temperature and a velocity profile are imposed. At the other side, an ambient pressure and no conductive heat flux conditions are applied. The distance \( \delta_0 \) at the thin film region (see Figure 1) is about 25 \( \mu \)m.

![Figure 1](image_url) A representation of the bubble-liquid-slug system considered. Because of the (axi-)symmetry of the problem, only one half-plane is simulated. The sketch is not to scale.

2.2. The model

2.2.1. The phase-field problem In order to perform the mathematical model of the matter at hand, a parameter \( \phi \) is defined such that \( \phi = -1 \) is associated with a phase completely made of particles of fluid \( v \) (for vapor), and \( \phi = 1 \) is associated with a phase completely made of particles of fluid \( l \) (for liquid), with a monotonic interpolation between those phases. Fluids \( v \) and \( l \) are assumed immiscible. This means that the interface, which separates both phases, is penalized by an unfavorable energy of magnitude \( \gamma \). This energy must be associated with the gradient in the \( \phi \) field, therefore. In the frame of Landau theory, this interaction can be approximated by the following expression of the free energy \( F \)

\[
F[\phi(x, t)] = \frac{1}{4} \left( a - (b \phi)^2 \right)^2 + \frac{\gamma}{2} \| \nabla \phi \|^2, \tag{1}
\]

where the parameters \( a \equiv a(T) \) and \( b \equiv b(T) \) are connected with the Hamiltonian of the system. The “phobic” effect produces the well-known sharp-segregation picture [14]. As \( \phi \) is related to the number of particles, it follows that

\[
\frac{\partial F}{\partial \phi} = \mu^{ch}, \tag{2}
\]

where \( \mu^{ch} \) is the chemical potential of the system.

If Fick’s law, relating the flux \( J \) of particles to the gradient of the chemical potential, is recalled

\[
J = -D \nabla \mu^{ch}, \tag{3}
\]

then the conservation of particles leads to the following continuity equation

\[
\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = D \nabla^2 \mu^{ch}, \tag{4}
\]

where \( \mathbf{u} \) is the velocity field and \( D \) is the diffusivity (or the mobility) of the particles. The combination of equations (1), (2) and (4) leads to the so-called Cahn-Hilliard equation [15],

\[
\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = D \nabla^2 \left[(b^4 \phi^3 - a b^2 \phi) - \gamma \nabla^2 \phi \right]. \tag{5}
\]

This equation tracks a diffuse interface separating the immiscible phases \( v \) and \( l \).

The interface surface tension \( \sigma \) is recovered by identifying the total free energy and the surface energy [14], i.e.

\[
\sigma = \int \gamma F \, dv. \tag{6}
\]
2.2.2. The conservation equations

With equation (5) are associated the classical mass, momentum and energy equations, which read for the incompressible flow of Newtonian fluids

\[ \nabla \cdot \mathbf{u} = \left( \frac{1}{\rho_v} - \frac{1}{\rho_l} \right) \dot{m}_{\text{vol}} \delta \]
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = - \nabla p + \nabla \cdot \Sigma + \rho g + \mathbf{F}_\sigma, \]
\[ \rho c_p \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (\lambda \nabla T) + \Sigma : \nabla \mathbf{u} + \dot{q}_{\text{vol}} \delta, \]

where \( p \) is the pressure, \( T \) the temperature and \( g \) the acceleration of gravity. \( \Sigma \) is the deviatoric part of the stress tensor \( \Sigma = \mu \left( \nabla \mathbf{u} + \left( \nabla \mathbf{u} \right)^T \right) \).

(7)

(8)

\( F_\sigma \) is the capillary force distribution and \( (\Sigma : \mathbf{u}) \) is the viscous dissipation term. Because of possible evaporation/condensation effect at the liquid-vapor interface, mass and energy source (or sink) terms, \( \dot{m}_{\text{vol}} \) and \( \dot{q}_{\text{vol}} \), have also been considered in the mass and energy equations, in consequence of which a source term, of the form \( \dot{m}_{\text{vol}} \left[ (1 - \phi)/\rho_v + (1 + \phi)/\rho_l \right] \delta \), must be considered in the phase transport equation (5) as well. By means of a function \( \delta \equiv \delta(x,t) \), which definition is postponed, these source terms are applied only at the diffuse interface.

At a given spot of the flow field, the density \( \rho \), the dynamic viscosity \( \mu \), the heat capacity \( c_p \) and the thermal conductivity \( \lambda \), used in (7), are function of the phase-field parameter through the relation

\[ \alpha = \alpha_v + \frac{1}{2}(1 + \phi)(\alpha_l - \alpha_v), \]

where \( \alpha = \rho, \mu, c_p \) or \( \lambda \).

(9)

As suggested by Yue et al. [14], the capillary force distribution \( \mathbf{F}_\sigma \) can be computed readily from the chemical potential \( \mu^{\text{ch}} \) and the gradient of the phase parameter \( \phi \)

\[ \mathbf{F}_\sigma = \mu^{\text{ch}} \cdot \nabla \phi. \]

(10)

This approach avoids using the interface curvature, which is both difficult to compute with good accuracy and highly sensitive to flow disturbances, and thus enhances the stability of the computational scheme.

2.2.3. Initial, boundary and interfacial conditions

Initially, a spherical-static inclusion (vapor bubble) is inserted inside the capillary filled with liquid. The flow field is at rest and no heating is applied, that is

\[ \phi(x, t = 0) = \phi_0(x) \]
\[ \mathbf{u}(x, t = 0) = \mathbf{0} \]
\[ p(x, t = 0) = \begin{cases} p_0 & \text{if } \phi_0(x) > 0, \\ p_0 + \frac{2}{r_0^{\text{bub}}} \sigma & \text{if } \phi_0(x) \leq 0, \end{cases} \]
\[ T(x, t = 0) = T_0 \]

where \( r_0^{\text{bub}} \) is the initial bubble radius, \( p_0 \) is a reference pressure, \( T_0 \) is the initial temperature and \( \phi_0 \) is phase function corresponding to the initial liquid-vapor bubble configuration.

For \( t > 0 \), and considering the capillary wall infinitely thin, constant heat flux density \( q_0 \) and no-slip conditions are applied at the capillary wall,

\[ \mathbf{u} = \mathbf{0} \quad \text{and} \quad - \lambda \nabla T \cdot \mathbf{n} = q_0, \]

(11)

(12)

where \( \mathbf{n} \) is the inward unit vector normal to the considered boundary. A constant temperature \( T_{\text{in}} \) and a parabolic velocity profile are imposed at one side of the capillary,

\[ T = T_{\text{in}} \quad \text{and} \quad \mathbf{u} \cdot \mathbf{n} = 2 v_{\text{in}} \left( 1 - \left[ \frac{r}{r_{\text{in}}} \right]^2 \right), \]

(13)
where \( v_{in} \) is the liquid superficial velocity and \( r_{in} \) is the capillary radius, while at the other side, flow is open to atmosphere and pure convective heat transfer occurs there, that is

\[
p = p_0 \quad \text{and} \quad -\lambda \nabla T \cdot \mathbf{n} = 0.
\]

(14)

The mass source term \( \dot{m}_{vol} \), given by

\[
\left( \frac{1}{\rho_v} - \frac{1}{\rho_l} \right) \dot{m}_{vol} = (\mathbf{u}_v - \mathbf{u}_l) \cdot \mathbf{n},
\]

(15)

is linked to the heat sink term \( \dot{q}_{vol} \), given by

\[
\dot{q}_{vol} = (-\lambda_v \nabla T_v - (-\lambda_v \nabla T_v)) \cdot \mathbf{n},
\]

(16)

through the expression

\[
\dot{q}_{vol} = L_v \dot{m}_{vol},
\]

(17)

where \( L_v \) is the enthalpy of vaporization.

The unit vector normal to the interface used in equations (15) and (16) (directed towards the vapor-phase) reads

\[
\mathbf{n} = -\frac{1}{\|\nabla \phi\|} \nabla \phi.
\]

(18)

The \( \delta \) function that defines the extent of applicability of the source terms \( \dot{m}_{vol} \) and \( \dot{q}_{vol} \) in equations (7), namely at the diffuse interface, reads [16]

\[
\delta(x, t) = \frac{1}{4}(1 + \phi)(1 - \phi)\|\nabla \phi\|.
\]

(19)

If a contact line forms, due to excessive vapor-bubble growth, then a temperature-independent contact angle is prescribed, \( \theta = \theta_0 \), which is quite a rough approximation [17]. In fact in a fully developed model, not only the contact angle \( \theta_0 \) does depend on the local temperature conditions but also additional heat source terms and Marangoni effects have to be taken into account due to the so-called micro-region [18]. These considerations are out of the scope of the present article.

Because of the axisymmetry of the configuration, only one-half two-dimensional domain is considered. At the symmetry axis, slip-tangential and no-heat-flux conditions are applied

\[
\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{n} \cdot \Sigma \cdot \mathbf{t} = 0 \quad \text{and} \quad -\lambda \nabla T \cdot \mathbf{n} = 0,
\]

(20)

where \( \mathbf{t} \) is the unit vector tangential to the symmetry axis.

3. Results and discussion

In this section, the effects of the capillary wall thickness, the inflow liquid velocity and the applied heat flux density on the heat transfer and wall temperature profile – as the bubble travels inside this capillary – are analyzed. Otherwise specified, the initial temperature is set to 100\( ^\circ \)C, the inflow velocity to 0.1 m s\(^{-1}\), the applied heat flux density to \( 4 \times 10^4 \) W m\(^{-2}\), the interface surface tension to 72 mN m\(^{-1}\) and the reference pressure to 1 atm.

In order to examine the filtering effect of the finite capillary wall thickness on the temperature, a simulation was performed where the solid wall was considered. The heat flux density was then introduced as a volume heat source term, \( q_{vol} = q_0/e \), where \( e \) is the wall thickness. For this configuration, a constant external heat transfer coefficient \( h_0 = 10 \) W m\(^{-2}\) K\(^{-1}\) was applied at the outer side of the capillary.

Figure 2 shows the temperature profile at the inner and outer sides of the capillary wall (of thickness 0.2 mm). Clearly, very small discrepancy occurs in both magnitude and phase as long as the temperature signature is concerned.
For this configuration the filtering behavior of the (metallic) wall is therefore negligible. In fact a glance over the energy equation applied for each phase (i.e. vapor bubble, liquid and solid wall) allows writing the following time scales

\[ \tau_s = \frac{e^2 \rho_s c_p_s}{\lambda_s}, \quad \tau_l = \frac{\delta_0^2 \rho_l c_p_l}{\lambda_l}, \quad \tau_v = \frac{r_{\text{bub}}}{v_{\text{in}}}, \]

(21)

where the index \( s \) refers to the capillary wall material. Then the filtering ineffectiveness of the capillary wall is reflected by the following criterion involving only these time scales

\[ \tau_s + \tau_l \lesssim \tau_v. \]

(22)

Applied to the problem considered here, this yields \( e \lesssim 1 \text{ mm} \) and \( \delta_0 \lesssim 100 \mu \text{m} \). Since \( e = 0.2 \text{ mm} \) and \( \delta_0 \approx 25 \mu \text{m} \), both of these criteria are satisfied and hence no noticeable heat diffusion nor lag wall effects are expected to occur as corroborated by Figure 2. In what follows, simulations are performed setting the capillary wall thickness to zero, therefore.

Figure 3 shows the growing and displacement of the bubble for different wall heat fluxes \( q_0 \) and inlet velocity \( v_{\text{in}} \). For sufficiently large wall heat fluxes (\( q_0 \gtrsim 4 \times 10^4 \text{ W m}^{-2} \)), heating leads quickly to drying out the capillary wall and contact lines develop. At these contact lines, a contact angle was set to 60°, behind which no adsorbed film physics is introduced at this stage of modeling developments as previously mentioned. In addition, the larger the inlet velocity, the more the bubble extends axially. Actually, assuming the bubble pressure \( p_{\text{bub}} \) varying slowly, writing the global mass and momentum conservation laws between the thin film (or liquid neck) region and the capillary exit and setting the pressure for the former to \( \approx p_{\text{bub}} - 2 \sigma/r_{\text{in}} \) yields

\[ \frac{1}{2} \rho_l v_{\text{in}}^2 \left( \frac{S_{\text{film}}}{S_{\text{out}}} \right)^2 - 1 \right) \approx A, \]

(23)

where \( S_{\text{film}} \) is the cross-sectional area at the thin film region and \( A \) a constant. It follows that the larger \( v_{\text{in}} \), the larger \( S_{\text{film}} \) must be. If the bubble has to grow by liquid evaporation at the interface, then it will more likely elongate as the liquid velocity gets larger and vice-versa.

Figures 4 and 5 show the wall temperature profiles and domain temperature snapshots, respectively, for different inlet velocities and wall heat fluxes. As the bubble moves along the capillary, a significant disturbance occurs in the vicinity of the thin film region with a particular pattern: two local minima separated by a local maximum. The local maximum may be explained
as follows. Is it well-known [5] that the bubble velocity is larger than the liquid flow mean velocity (here by about 10%). Because of viscos effects, liquid at the thin film region, close to the interface, is accelerated. In order to satisfy mass balance, fluid at the same region, but in the vicinity of the wall this time, must slow down, which decreases the local convective effects and hence increases the local temperature. In addition, the wall heat flux seems to play a pure amplifying role as long as the magnitude of the wall temperature disturbance is concerned while the inlet velocity both reduces this magnitude and spreads even larger the disturbance region.

\[ q_0 = 2 \times 10^4 \text{ W m}^{-2} \]
\[ q_0 = 8 \times 10^4 \text{ W m}^{-2} \]
\[ q_0 = 16 \times 10^4 \text{ W m}^{-2} \]

Figure 3. Bubble deformation and displacement (from left to right for each snapshot). Between each successive shapes, \( \Delta t = 10 \text{ ms} \). Left column: \( v_{in} = 0.1 \text{ m s}^{-1} \), and right column: \( v_{in} = 0.2 \text{ m s}^{-1} \).

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\[ q_0 = 16 \times 10^4 \text{ W m}^{-2} \]

Figure 4. Wall temperature profile. (a) and (b): \( q_0 = 2 \times 10^4 \text{ W m}^{-2} \) and (c) and (d): \( q_0 = 8 \times 10^4 \text{ W m}^{-2} \).
Figure 5. Temperature snapshots. $q_0 = 8 \times 10^4 \text{W m}^{-2}$ and $v_{\text{in}} = 0.1 \text{m s}^{-1}$. The right inset is a close up – with streamlines – of snapshot (c).

4. Conclusions
The deformation and displacement of a vapor bubble inside a nonisothermal capillary partially filled with liquid was investigated numerically. The first objective of this analysis was to estimate the temperature signature disturbance at the outer side of the capillary wall when the bubble travels inside this capillary. It was found that such a disturbance is strongly correlated to the position of the bubble inside the wall. Moreover for practical values of flow velocity, applied heat flux and wall thickness, no diffusion nor lag effects between the wall inner and outer temperature profiles were noticed.

Albeit only hydrostatic contact line physics was considered, this investigation strongly suggests the possibility to develop effective inverse methods for which the bubble position inside pulsating heat pipes channels is identified reliably solely through the instantaneous temperature profile of the outer side of their wall.

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