Introducing holographic flavor in an intensely magnetized quark-gluon plasma

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We present a construction that makes possible the application of gauge/gravity methods to study fundamental degrees of freedom in a quark-gluon plasma subject to a magnetic field as intense as that expected in high energy collisions. This is achieved by the confection of a ten-dimensional background that is dual to the magnetized plasma and nonetheless permits the embedding of D7-branes in it. For such a background to exist, a scalar field has to be present and hence a scalar operator of dimension 2 appears in the gauge theory. In this letter we present the details of the background and of the embedding of flavor D7-branes in it. We use this system to study the impact of such an intense magnetic field on the mesons melting transition and over a sector of the mesons spectrum. We find that the magnetic field lowers the temperature at which the mesons melt and decreases the mass gap of the spectrum along with their masses. Since our results are obtained from the gravity dual of the gauge theory, the analysis is also interesting from the gravitational perspective.

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INTRODUCTION AND MAIN RESULTS

It has become increasingly accepted that an intense magnetic field is produced in high energy collisions and that understanding its effects is relevant to properly analyze experimental observations [1–3]. A tool that has proven to be very useful to study the quark-gluon plasma produced in these collisions is the gauge/gravity correspondence [4], so the introduction of a magnetic field in this context is necessary. A way to incorporate this field using holographic methods was implemented in [5], where the plasma contained massless matter in the adjoint representation. Another approach was followed in [6], where the magnetic field was introduced as an excitation of a D7-brane considered to be a probe in a fixed ten-dimensional background. For the probe approximation to hold, it is necessary that the ratio $b/T^2$, of the intensity of the magnetic field on the brane and the temperature of the black hole, remains much smaller than the unit, so that the back reaction to the brane can be ignored. Given that the phenomena we are interested in take place when $b$ is of the same order of magnitude as $T^2$, we can not use this setting.

It then becomes necessary to find a manner in the gauge/gravity correspondence to study the effects of an intense magnetic field over a quark-gluon plasma with matter in the fundamental representation and nonvanishing mass. In this context, adding $N_f$ massive flavor degrees of freedom to a gauge theory corresponds to embedding $N_f$ D7-branes in its gravitational dual [3]. Trying to implement this embedding in the ten-dimensional uplift of the five-dimensional geometry employed in [6] turns out to be highly complicated because the compact part of the resulting geometry warps in a way that prevents an easy identification of the right 3-cycle that the D7-brane must wrap [8]. Given this complication, our approach is to find a family of solutions to ten-dimensional type IIB supergravity that accommodates a magnetic field and has a compact five-dimensional space that factors as a warped 3-sphere, a 1-cycle, and an angular coordinate $\theta$ that determines the volume of these two spaces. By keeping the 1-cycle and the $\theta$ direction perpendicular to the rest of the spacetime, we can successfully proceed as in previous approaches [6], and, as we will see below, have the D7-brane naturally wrapping the 3-cycle. We found that the construction of a ten dimensional background with the desired properties requires the excitation of a degree of freedom that is dual to a scalar operator of dimension 2 in the gauge theory, and hence saturates the BF bound [10].

In [11] we adopted a five-dimensional effective perspective to report the rich physics that this family of backgrounds possesses, including the existence of a critical intensity $b_c$ for the magnetic field, above which the solutions become unstable, while below it, two branches of solutions appear, one of them being thermodynamically favored.

The principal purpose of this letter is to present the details of the ten-dimensional family of backgrounds and of the embedding of D7-branes into it, both of which we have constructed numerically, since analytic solutions have eluded our treatment. To show the physical relevance of this construction we use such system to study, in the dual gauge theory, how an intense magnetic field affects the meson melting transition and the spectrum of a sector of the mesons themselves. The computational details of what we present here, along with further in-
formation about physical quantities affected by the magnetic field in this context, will be available in [12].

Aside from the implementation of the embedding itself, one of our main results is that a magnetic field lowers the temperature at which the mesons melt, and seems to smooth this transition, bring it closer to be of second order. We then focus on the mesons that are dual to perturbations on the embedding of the brane and find that their masses decrease with the intensity of the magnetic field, as does the mass gap of the spectrum.

**THE GRAVITATIONAL BACKGROUND**

What we are targeting is a solution to ten-dimensional type IIB supergravity with a metric that asymptotically approaches AdS$_5 \times S^5$, and accommodates a deformation that encodes the dual of a magnetic field in the gauge theory, while still permits to factor out the compact part of the metric in the way described in the introduction.

As it turns out, a general line element that allows such a solution is given by

\[
\begin{align*}
\text{ds}^2 &= \frac{\text{dr}^2}{U(r)} - U(r) \text{d}t^2 + V(r)(\text{dx}^2 + \text{dy}^2) + W(r) \text{dz}^2, \\
\end{align*}
\]

while the one of the 3-cycle is given by

\[
\begin{align*}
\text{d}\Sigma_3^2 &= \text{d}\vartheta_1^2 + \sin^2 \vartheta_1 (\text{d}\vartheta_2 + \sqrt{2A})^2 + \cos^2 \vartheta_1 (\text{d}\vartheta_3 + \sqrt{2A})^2.
\end{align*}
\]

The coordinate $r$ measures a radial distance, and we expect (1) to approach AdS$_5 \times S^5$ as $r \to \infty$, making the directions $t, x, y,$ and $z$, dual to those in which the gauge theory lives.

We see that the 1-form $A$ parametrizes an infinitesimal rotation involving a periodic direction of the compactifying manifold that, in turn, codifies the internal degrees of freedom of the dual gauge theory. If we keep $A$ and its exterior derivative in the cotangent space to the directions dual to those of the gauge theory, it will represent a U(1) vector potential that allows the introduction of the desired magnetic field in the latter. In the family of solutions that will provide the background for our current calculation we set $A = bx \, dy$, automatically satisfying Maxwell equations [11] and introducing a constant magnetic field $F = b \text{dx} \wedge \text{dy}$ in the gauge theory.

The degrees of freedom carried by $\varphi$ are dual to the aforementioned scalar operator of dimension 2 in the gauge theory that saturates the BF bound.

All that is left to do to find a solution is to obtain the metric potentials $U, V,$ and $W,$ from the equations of motion and to provide an expression for the 5-form field strength of type IIB supergravity. Apart from the AdS$_5$ asymptotic behavior of the metric potentials, we only need to mention for our current purposes that they are such that there is a regular horizon at some finite $r_h,$ providing the gauge theory with a finite temperature $T = \frac{3r_h}{2\pi}$. The 5-form can be consulted in [11], but given that it does not couple to the D7-brane, its explicit form is not relevant to our current discussion, so we will omit it in this letter.

In part, the embedding is simple because (1) allows for the D7-brane to be placed at fixed $\phi$. This is consistent since the metric does not depend on the coordinate $\phi$, and because the direction that this coordinate represents remains orthogonal to the rest of the spacetime [13].

Concerning the 3-cycle in (1), we notice that its volume depends on the position $\theta$ and, regardless of the value of $b$, this cycle becomes maximal at $\theta = 0$, while for $b = 0$ it reduces to $S^3$. For non-vanishing $b$, the 3-cycle gets tilted towards the five dimensional non-compact part of the spacetime in a manner that is volume preserving within the eight dimensions of this two spaces together.

**EMBEDDING OF THE D7-BRANE**

To find how a D7-brane is embedded in our background we must extremize the Dirac-Born-Infeld action given by

\[
S_{DBI} = -T_{D7} N_f \int d^8 x \sqrt{-\det(g_T)},
\]

where $T_{D7}$ is the tension of the D7-brane, $g_T$ the metric induced over it, and the integration is to be performed over its world volume.

Given the final observations of the previous section we see that a D7-brane can consistently extend along the directions of $\text{ds}_5^2$ and $\text{d}\Sigma_3^2$, and since the volume of this subspace depends solely on $\theta$ and $r$, an embedding that extremizes (4) can be found by setting $\phi$ to a constant and determining the right profile for $\theta(r)$. This embedding becomes supersymmetric at zero temperature when we turn off $\varphi$ and $A$.

The expressions to follow simplify significantly if they are written in terms of $\chi(r) \equiv \sin \theta(r)$, so that the line element induced on the D7-brane is

\[
\begin{align*}
\text{ds}_{D7}^2 &= \Delta^\frac{1}{2} \left[ -U \text{d}t^2 + V (\text{dx}^2 + \text{dy}^2) + W \text{dz}^2 + \frac{1 - \chi^2}{U(1 - \chi^2)} \text{d}r^2 \right] + \frac{1 - \chi^2}{\Delta^\frac{1}{2}} \text{d}\Sigma_3^2(A),
\end{align*}
\]

where $\Delta = \cos^2 \vartheta_1 V + \sin^2 \vartheta_1 W$.
where the wrapping factor simplifies to $\Delta = X + \chi^2(X^{-2} - X)$.

The embedding can be found by varying (4) with respect to $\chi$ after substitution of (5) and solving the resulting equation.

The behavior of the embedding close to the boundary is given by $\chi = M \pi / \sqrt{2r} + ...$ where $M$ is related to the quark mass by $M_q = M \sqrt{X}/2$, with $\lambda$ the 't Hooft constant. For quark masses that are not too large in comparison to the temperature, the embedding at any $r$ remains close enough to the equator, $\chi(r) = 0$, and the brane falls through the horizon as $r \rightarrow r_h$, receiving the name of black hole embedding. On the contrary, for large enough quark masses the embedding stays distant from the equator and the brane does not touch the horizon, so it is referred to as a Minkowski embedding. There is an intermediate range of masses for which there are both types of embeddings, and a thermodynamic analysis is necessary to determine which one is favored.

Once a D7-brane has been introduced in this background, open strings with both ends on it can exist. The low energy states of these strings are dual to mesons, i.e. quark-antiquark bound states, in the gauge theory. These states are codified as excitation of the D7-brane governed by the DBI action, and their spectrum can be determined by finding the stable vibrational and U(1) perturbations of the brane. It turns out that for Minkowski embeddings the spectrum is discrete and has a non-vanishing mass-gap, while for black hole embedding it is continuous and gapless. From this it is concluded that Minkowski embeddings are dual to a phase of the gauge theory where stable mesons exist, while black hole embeddings correspond to a different phase in which the mesons have dissociated or melted.

If the transition between embeddings is thought of at fixed quark mass but changing temperature, we see that at low temperatures, represented by small $r_h$, the D7 does not fall through the horizon, and stable mesons exist. As the temperature increases the system gets to a point in which the brane does fall through the horizon and the mesons melt.

**NUMERICAL RESULTS FOR THE EMBEDDINGS**

To provide a first visualization of the effect of the magnetic field over the embedding, in Fig. 1 we display several profiles at the same temperature $T = \frac{b}{4\pi}$, grouped by their value for $M$. Our first finding is that for stronger magnetic fields the brane bends closer to the horizon, hence lowering the temperature at which the meson melting occurs. A secondary observation is that there are values for $M$, 0.38 serving as an example, for which the intensity of the magnetic field can change the embedding from Minkowski type to black hole. This means that working at certain fixed temperatures, we can use the magnetic field to parametrize the melting and find a critical intensity at which the transition happens. It is important to notice that, since the intensity is bounded by $b_c$, this will only happen for a limited range in $M$.

To study this transition more carefully we compute the free energy associated to a number of embeddings of both types covering those close to the transition. The free energy is given by the product $F = TS_{D7}$ of the temperature $T$ characterizing the background and the renormalized Euclidean continuation of the DBI action $S_{D7}$ evaluated on the corresponding solution $\chi(r)$.

The details of the renormalization for $S_{D7}$, indicate the existence of some freedom in choosing the renormalization scheme. Since all the qualitative results are scheme independent, in this letter we work in a fixed scheme of which the particulars are not relevant, and leave other instances for [12].

In Fig. 2 we show the behavior of the free energy as a function of $T/M$ for four intensities of $b/T^2$, starting with $b/T^2 = 0$. The insets show the detail for some of these cases close to the melting point of the mesons.

Plots at a fixed value of $T/M$ for which the transition can be undergone by changing $b/T^2$ will be presented in [12] along with other thermodynamic quantities, such as the entropy and energy densities.

**MESON SPECTRUM**

The spectrum of the mesons in the gauge theory can be determined by finding the stable normal modes of the Minkowski embeddings [14] [18] for excitations of either vibrational modes or those of the world volume U(1) field.
To exemplify the effect that the magnetic field has over the meson spectrum, in this letter we will only study a fraction of it that corresponds to perturbations of the embedding in directions perpendicular to it and leave other sectors to be presented in [12].

A general excitation of this kind can be implemented by writing \( \chi(X) = \chi_0(r) + \delta \chi_1(X) \) and \( \phi(X) = \phi_0 + \delta \phi_1(X) \), where the naughted functions are the solutions discussed earlier and the perturbations, denoted by a subindex 1, are free to depend on any coordinate of the ten-dimensional space. The equations of motion for these perturbations show that \( \chi_1(X) \) and \( \phi_1(X) \) decouple from each other, and furthermore, their dependence on the 3-cycle coordinates, on \( r \), and on the gauge theory directions, can be factored. Once reduced over the 3-cycle, the dependence of the perturbations on its coordinates are related to the states with different spins in a Kaluza-Klein tower. For concreteness we will focus on perturbations \( \chi_1(X) \) that do not depend on the coordinates of the 3-cycle and leave other examples for [12].

Even if the magnetic field makes our gauge theory not isotropic, it remains invariant under translations, so we can write \( \chi_1(X) = e^{i\omega t-k_\mu x^\mu} \chi_1(r) \) in the bulk. Lorentz invariance is broken by the non-vanishing temperature, so what is understood as the mass of a meson is frame specific. Our choice follows [9], and it is to consider the mass to be defined in the rest frame of the mesons, which is then given by \( \omega \) with vanishing three-momentum \( k_\mu = 0 \). The allowed values for \( \omega \) are then determined by the requirement for \( \chi_1 \) to remain normalizable near the boundary. The final result [12] is a discrete spectrum of frequencies from which we can compute the corresponding masses \( m_n \), that have a non vanishing value \( m_0 \) for the ground state, establishing the existence of a mass gap.

To graphically display the impact of the magnetic field on the spectrum, in Fig. 3 we present a plot of its first three masses as a function of \( T/\bar{M} \) for three different values of \( b/T^2 \).

![Figure 2](image_url)

**FIG. 2.** \( S_{D^7}/N \) as function of \( T/\bar{M} \). Red, blue and green (bottom to top) correspond to \( b/T^2 = \{0.5, 4.5, 11.04\} \) respectively. Dashed segments correspond to Minkowski embeddings, while continuous segments correspond to black hole embeddings.

![Figure 3](image_url)

**FIG. 3.** First three meson masses \( \omega^2/\bar{M}^2 \) as functions of \( T/\bar{M} \). Red, blue and green curves (top to bottom) correspond to \( b/T^2 = \{0.5, 4.5, 11.04\} \) respectively.

**DISCUSSION**

The construction of a solution to type IIB supergravity that on the one hand possesses a regular horizon, asymptotically approaches AdS, and accommodates a constant magnetic field, but on the other is ideally suited for the embedding of flavor D7-branes made it possible to use the gauge/gravity correspondence to study some relevant physical properties of the fundamental degrees of freedom.

The first conclusion we can draw from our calculations is that the presence of an intense magnetic field lowers the temperature at which the mesons melt. This simple observation implies that the magnetic field modifies the phase diagram of the strongly coupled theory over which it is applied.

From our results for the free energy we notice that, as it should be, the plots for \( b/T^2 = 0 \), for which \( \varphi = 0 \), are identical to those presented in [9], where we identify the typical pattern of a first order phase transition, with a range of values for \( T/\bar{M} \) for which there are solutions in both phases. We see that as \( b/T^2 \) increases, the transition appears to become smoother, perhaps indicating a change to second order. A similar behavior was observed in [19], where an anisotropic theory was considered and the phase transition seemed to change from first to second order as the anisotropy was increased. The task to determine whether or not the transition is so drastically changing is lengthily, and we will present some progress in this direction in [12].

The results in Fig. 3 for \( b/T^2 = 0 \) coincide with those in [9]. We also notice that regardless of the value of \( b/T^2 \), all the plots reach the same point at \( T/\bar{M} = 0 \).
for each state of the spectrum. This is to be expected, as the graphs are done at constant $b/T^2$, so in this limit $b$ has to be very small in comparison to $M$, making its effect negligible, and returning the spectrum to that of a vanishing magnetic field. Still from Fig. 3 we see that the magnetic field lowers the temperature at which the mass gap vanishes, which is another indicator of the melting of the mesons.

Finally, the most relevant physical result obtained from our novel construction, is the shift in the mass spectrum of the mesons, including its mass gap, that can be appreciated in Fig. 3 where plots with higher $b/T^2$ accommodate in a lower position in the graph. If this is also true in QCD, it implies that the masses at which some resonances are detected experimentally in non-central collisions, are shifted by the effect of the magnetic field produced in them.

In [12] we will include further evidence to support the observations already indicated in this letter, but we will also include further results. One of them will be the presence of a conformal anomaly in the gauge theory with fundamental degrees of freedom, that is somewhat a consequence of that found in the background itself. This anomaly is not only responsible for the renormalization scheme dependence exhibit by some physical quantities, but it also opens up the possibility to study the enhancement of direct photon production that a magnetic field in its presence has been speculated to cause [2].

The lines of research that can be followed using our results are numerous. As we just mentioned, one example is the impact of a magnetic field on the luminous spectrum of a QGP of fundamental matter with non-vanishing mass, which we are currently investigating as an extension to the results in [20, 21]. Another example of the topics that can be studied using our gravitational dual is the effect of this magnetic field on jet quenching or the drag force over quarks on the plasma. It is not our intention to present a comprehensive list of the projects that we are pursuing following the construction that we presented, but we would like to finish by mentioning that we expect many results to derive from our current study.

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