Event Rates for Binary Inspiral

Vassiliki Kalogera* and Krzysztof Belczynski*,†

*Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138
†Nicolaus Copernicus Astronomical Center, 00-716 Warszawa, Poland

Abstract.
Double compact objects (neutron stars and black holes) found in binaries with small orbital separations are known to spiral in and are expected to coalesce eventually because of the emission of gravitational waves. Such inspiral and merger events are thought to be primary sources for ground based gravitational–wave interferometric detectors (such as LIGO). Here, we present a brief review of estimates of coalescence rates and we examine the origin and relative importance of uncertainties associated with the rate estimates. For the case of double neutron star systems, we compare the most recent rate estimates to upper limits derived in a number of different ways. We also discuss the implications of the formation of close binaries with two non–recycled pulsars.

INTRODUCTION

Compact objects, neutron stars (NS) or black holes (BH), formed from relatively massive stars can spiral in and coalesce when found in tight binaries, the orbital evolution of which is driven by gravitational radiation. As angular momentum losses dominate, the orbit shrinks and the two compact objects can eventually merge as they revolve in orbit around each other. The prototype progenitor system of such inspiral events is the binary pulsar PSR B1913+16 (the “Hulse–Taylor” pulsar [1]). Sensitive pulsar timing measurements have revealed that the orbital period decreases at a rate comparable (to better than 1%) to that predicted by general relativity for the emission of gravitational waves [2], [3]. The ultimate coalescence of the two neutron stars seems inevitable.

Although PSR B1913+16 will not reach coalescence for another 300 Myr, similar inspiraling systems in the Milky Way and nearby galaxies are thought to be primary sources of gravitational radiation for ground–based interferometric gravitational–wave detectors, currently under construction or commissioning (e.g., LIGO, VIRGO, GEO600). In addition to NS–NS close binaries, NS–BH and BH–BH binaries are also expected to form through the evolution of massive binaries and to contribute to the detection of inspiral events.
The expected detection rate of inspiral events depends on (i) the strength of the expected gravitational-wave signal, (ii) the gravitational-wave detector sensitivity, and (iii) the coalescence rate of each binary population. The first two considerations define a maximum distance $D_{\text{max}}$, out to which different types of inspiral events and mergers could be detected. The coalescence rate for each population is estimated in two steps: first, the Galactic rate, and then its extrapolation out to the maximum distance of interest. Based on the current understanding of the LIGO sensitivities, the maximum distances out to which inspiral events could be detected by LIGO II (and LIGO I) are (approximately), 350 Mpc (20 Mpc) for NS–NS binaries, 700 Mpc (40 Mpc) for NS–BH binaries, and 1500 Mpc (100 Mpc) for BH–BH binaries (assuming 1.4 M$_{\odot}$ NS and 10 M$_{\odot}$ BH; Sam Finn, private communication). Given our current best knowledge (based on recent redshift surveys) of galaxy distributions out to those distances [4], it can be estimated that, for a LIGO II detection rate of 1 event per year, the following Galactic coalescence rates are required: $\sim 5 \times 10^{-7}\text{ yr}^{-1}$ for NS–NS binaries, $\sim 5 \times 10^{-8}\text{ yr}^{-1}$ for NS–BH binaries, and $\sim 5 \times 10^{-9}\text{ yr}^{-1}$ for BH–BH binaries.

Formation rates of coalescing compact binaries (systems with tight enough orbits that merge within a Hubble time $\sim 10^{10}\text{ yr}$) have been calculated so far using two very different methods: either entirely theoretically, based on binary evolution models, or, for NS–NS binaries, empirically, based on the observed NS–NS sample. In what follows we present an up-to-date review of current rate estimates, addressing in detail the most important uncertainties associated with them. We also discuss independent ways of obtaining upper limits to the coalescence rate of NS–NS binaries and possible implications of the formation of systems without recycled pulsars.

THEORETICAL RATE ESTIMATES

The formation rate of coalescing binary compact objects can be calculated, given a sequence of evolutionary stages leading to binary compact object formation. Over the years, a relatively standard picture has been formed describing the birth of such systems based on considerations of NS–NS binaries [5]. More recently, variations of the standard evolutionary channel have also been discussed and suggested [6], mainly based on worries about the fate of neutron stars in situations of hypercritical accretion (not limited to the photon Eddington rate), and their possible collapse into black holes. In all versions, however, the main picture remains the same: the initial binary progenitor consists of two binary members massive enough to eventually collapse into a NS or a BH. The evolutionary path involves multiple phases of stable or unstable mass transfer, common-envelope phases (where one or possibly two stellar cores spiral in the envelopes of evolved stars and eventually lead to the ejection of these envelopes), and accretion onto compact objects, as well as two core collapse events. The final outcome of interest is the formation of binary compact objects in close binary orbits.
Such theoretical modeling has been undertaken by a number of different groups by means of population syntheses. This provides us with \textit{ab initio} predictions of coalescence rates. Monte Carlo numerical techniques are employed in following the evolution of a large ensemble of primordial binaries with certain assumed initial properties through a multitude of channels until compact object binaries are formed. The changes in the properties of the binaries at the end of each stage are calculated based on our current understanding of the various evolutionary processes involved: wind mass loss from massive hydrogen– and helium–rich stars, mass and angular–momentum losses during mass transfer phases, dynamically unstable mass transfer and common–envelope evolution, effects of highly super–Eddington accretion onto NS, and supernova explosions with kicks imparted to newborn NS or even BH. Given our limited understanding of some of these phases, the results of population synthesis are expected to depend on the assumptions made in the treatment of the various processes. Therefore, exhaustive parameter studies are required by the nature of the problem.

Recent studies of the formation of compact objects and calculations of their Galactic coalescence rates ([7], [8], [9], [10], [11], [12]) have explored the input parameter space and the robustness of the results at different levels of (in)completeness. Almost all of these groups have studied the sensitivity of the predicted coalescence rates to the average magnitude of the kicks imparted to compact objects at birth. The range of predicted NS–NS Galactic rates obtained by varying the kick magnitude alone is found in the range $< 10^{-7} - 5 \times 10^{-4}$ yr$^{-1}$. This large range indicates the importance of supernovae (two in this case) in the evolution of massive binaries. Variations in the assumed mass–ratio distribution for the primordial binaries can further change the predicted rate by about a factor of 10, while assumptions of the common–envelope phase add another factor of about $10 - 100$. Variation in other parameters typically affects the results by factors of two or less. Predicted rates for BH–NS and BH–BH binaries lie in the ranges $< 10^{-7} - 10^{-4}$ yr$^{-1}$ and $< 10^{-7} - 10^{-5}$ yr$^{-1}$, respectively when the kick magnitude to both NS and BH is varied. Other uncertain factors such as the critical progenitor mass for NS and BH formation lead to variations of the rates by factors of $10 - 50$.

It is evident that recent theoretical predictions for coalescence rates cover a wide range of values (typically 3–4 orders of magnitude), because the various input parameters and assumptions affect strongly the absolute normalization (birth rate) of the modeled populations. Given these results, it seems fair to say that, at least at present, population synthesis calculations have a rather limited predictive power and provide fairly loose constraints on coalescence rates. One way to improve the reliability of such predictions is to study a number of different binary populations (with or without compact objects) and incorporate a number of independent observational constraints, such as star formation rate, supernova rates of different types, binarity of Wolf–Rayet stars, and others. A number of constraints on the population synthesis models could help restricting the predicted coalescence rates in narrower ranges [13].
The large range of theoretically predicted Galactic coalescence rates of double compact objects motivates us to examine other ways of obtaining rate estimates. The observed sample of coalescing NS–NS binaries found in the Galactic field (PSR B1913+16 and PSR B1534+12) provides us with alternative estimates of their coalescence rate. “Empirical” estimates can be obtained using the observed pulsar and binary properties along with models of selection effects in radio pulsar surveys [14], [15]. For each observed object, a scale factor can be calculated based on the fraction of the Galactic volume within which pulsars with properties identical to those of the observed pulsar could be detected by any of the radio pulsar surveys, given their detection thresholds. This scale factor is a measure of how many more pulsars like the ones detected in the coalescing NS–NS systems exist in our galaxy. The coalescence rate can then be calculated based on the scale factors and estimates of detection lifetimes summed up for all the observed systems. Based on this method the first two studies concluded that the NS–NS Galactic coalescence rate is \( \approx 10^{-6} \text{ yr}^{-1} \).

Since then, estimates of the NS–NS coalescence rate have known a significant downward revision primarily because of (i) the increase of the Galactic volume covered by radio pulsar surveys with no additional coalescing NS–NS being discovered [16], (ii) the increase of the distance estimate for PSR B1534+12 based on measurements of post-Newtonian parameters [17] (iii) revisions of the lifetime estimates [18], [19]. Recent estimates place the NS–NS rate for our Galaxy in the range \( \approx 1 - 3 \times 10^{-7} \text{ yr}^{-1} \). Further, it has been realized that a number of upward correction factors must be included, most importantly to account (i) for the beamed nature of pulsar emission and correct for all the binary pulsars with beams that our line of sight does not intersect, and (ii) for the faint end of the pulsar luminosity function and correct for those systems that are too faint to be detected. These two correction (multiplication) factors have so far typically been assumed to be \( \approx 3 \) and \( \approx 10 \), respectively.

In a study just recently completed [4], we especially focused on all the uncertainties associated with these empirical estimates. We found that the upward correction factor for the faint end of the pulsar luminosity is the most important source of uncertainty. However, it is highly sensitive to the number of observed objects and its distribution function widens dramatically for small–number samples. For a sample of two objects (as the observed one) the faint–pulsar correction factor can vary from very small (close to unity) to as high as \( \approx 200 \) (see following subsection). Beyond the issue of faint pulsars, we considered a number of uncertainties and correction factors. Based on recent observational data for both PSR B1913+16 and PSR B1534+12, we found that the beaming correction factor is higher than previously thought (\( \approx 6 \)) but with a rather small uncertainty (\( \approx 10\% \)). Other factors, such as pulsar ages and lifetimes, and spatial distribution, lead to an uncertainty factor of about 2. We estimate the Galactic NS–NS coalescence rate in the range \( \approx 10^{-6} - 5 \times 10^{-4} \text{ yr}^{-1} \), which is still narrow compared to the range covered by the
Small Number Sample and Pulsar Luminosity Function

One important limitation of empirical estimates of the coalescence rates is that they are derived based on only two observed NS–NS systems, under the assumption that the observed sample is representative of the true population, particularly in terms of their radio luminosity. Assuming that the recycled pulsars in NS–NS binaries follow the radio luminosity function of young pulsars and that therefore their true Galactic population is dominated in number by low–luminosity pulsars, it can be shown that the current empirical estimates most probably underestimate the true coalescence rate. If a small–number sample is drawn from a parent population dominated by low–luminosity (hence hard to detect) objects, it is statistically more probable that the sample will actually be dominated by objects from the high–luminosity end of the population. The result is that the population overall is thought to be brighter than it really is, and therefore, detectable over a larger Galactic volume. Consequently, the empirical estimates based on such a sample will tend to overestimate the detection volume for each observed system, and therefore underestimate the scale factors and the resulting coalescence rate.

This effect can be clearly demonstrated with a Monte Carlo experiment [4] using simple models for the pulsar luminosity function and the survey selection effects. As a first step, the average observed number of pulsars is calculated given a known “true” total number of pulsars in the Galaxy (thick-solid line in Figure 1). As a second step, a large number of sets consisting of “observed” (simulated) pulsars are realized using Monte Carlo methods. These pulsars are drawn from a Poisson distribution of a given mean number ($< N_{\text{obs}} >$) and have luminosities assigned

![Figure 1](image-url)
according to the assumed luminosity function. Based on each of these sets, one can estimate the total number of pulsars in the Galaxy using empirical scale factors, as is done for the real observed sample. The many (simulated) “observed” samples can then be used to obtain the distribution of the estimated total Galactic numbers ($N_{\text{est}}$) of pulsars. We find that these $N_{\text{est}}$ distributions are very strongly skewed and lead to possible correction factors for the faint pulsars in a wide range of values (covering typically a couple of orders of magnitude). The median and 25% and 75% percentiles of this distribution are plotted as a function of the assumed number of systems in the (fake) “observed” samples in Figure 1 (thin–solid and dashed lines, respectively).

It is evident that, in the case of small–number observed samples (less than $\sim 10$ objects), the estimated total number, and hence the estimated coalescence rate, can be underestimated by a significant factor. For observed samples with an expected number of objects equal to two, for example, the true rate may be much higher by more than a factor as high as $\approx 200$. This underestimation factor represents an upward correction factor that must be applied to the rate estimated using the observed sample of coalescing NS–NS binaries. However, we note that distribution of this correction factor covers a wide range and becomes highly skewed for small number samples (less than about 10 objects), and therefore it is currently quite uncertain. We conclude that correcting for the undetected, faint pulsars in the population cannot be decoupled from the problems of a small–number sample because of the assumption of the observed sample being representative of the population, implicit in the method.

**UPPER LIMITS ON THE NS–NS COALESCENCE RATE**

Observations of NS–NS systems and isolated pulsars related to NS–NS formation allow us to obtain upper limits on their Galactic coalescence rate in a number of different ways. Depending on how their value compares to the Galactic rate required for a LIGO II detection rate of 1 event per year, such limits can in principle provide us with valuable information about the prospects of gravitational–wave detection.

The absence of any young pulsars detected in NS–NS systems was used to obtain a rough upper limit to the rate of $\sim 10^{-5} \text{yr}^{-1}$ [20]. Recently the same basic argument was reexamined in more detail and a more robust upper limit of $\sim 10^{-4} \text{yr}^{-1}$ was derived [19].

An upper bound to the NS–NS coalescence rate can also be obtained by combining our theoretical understanding of orbital dynamics (for supernovae with NS kicks in binaries) with empirical estimates of the birth rates of other types of pulsars related to NS–NS formation [21]. Progenitors of NS–NS systems experience two supernova explosions. The second supernova explosion (forming the NS that is not observed as a pulsar) provides a unique tool for the study of NS–NS formation, since the post–supernova evolution of the system is simple, driven only by gravitational–wave radiation. There are three possible outcomes after the second
FIGURE 2. Maximum probability ratio for the formation of coalescing NS–NS systems and the disruption of binaries as a function of the kick magnitude at the second supernova.

supernova: (i) a coalescing NS–NS is formed (CB), (ii) a wide NS–NS (with a coalescence time longer than the Hubble time) is formed (WB), or (iii) the binary is disrupted (D) and a single pulsar similar to the ones seen in NS–NS systems is ejected. Based on supernova orbital dynamics we can accurately calculate the probability branching ratios for these three outcomes, $P_{\text{CB}}$, $P_{\text{WB}}$, and $P_{\text{D}}$. For a given kick magnitude, we can calculate the maximum ratio $(P_{\text{CB}}/P_{\text{D}})^{\text{max}}$ for the complete range of pre-supernova parameters defined by the necessary constraint $P_{\text{CB}} \neq 0$ (Figure 2). Given that the two types of systems have a common parent progenitor population, the ratio of probabilities is equal to the ratio of the birth rates $(BR_{\text{CB}}/BR_{\text{D}})$.

We can then use (i) the absolute maximum of the probability ratio ($\simeq 0.27$ from Figure 2) and (ii) an empirical estimate of the birth rate of single pulsars similar to those in NS–NS based on the current observed sample to obtain an upper limit to the coalescence rate. The selection of this sample involves some subtleties [21], and the analysis results in $BR_{\text{CB}} < 1.5 \times 10^{-5} \text{yr}^{-1}$. Note that this number could be increased because of the small-number sample and luminosity bias, which this time affects the empirical estimate of $BR_{\text{D}}$ by a factor of $\simeq 2 - 6$. Such an upward correction can bring the upper limit in the range $3 - 9 \times 10^{-5} \text{yr}^{-1}$.

This is an example of how we can use observed systems other than NS–NS to improve our understanding of their coalescence rate. A similar calculation can be done using the wide NS–NS systems instead of the single pulsars [21].

**NON–RECYCLED DOUBLE NEUTRON STARS**

We have already pointed out that the empirical methods employed to obtain rate estimates for NS–NS coalescence include the implicit assumption that the proper-
ties of the observed sample are representative of the Galactic NS–NS population. This assumption extends to the pulsar properties and their evolutionary history of recycling (spin–up by accretion). Consistent with the pulsars observed in the detected NS–NS systems, it turns out that so far theoretical studies of NS–NS formation have considered systems where one of the neutron stars had the opportunity to be recycled, at least in principle (through stellar winds, Roche-lobe overflow accretion, or even possibly in a common-envelope phase).

Here, we report on a new evolutionary path leading to the formation of close NS–NS binaries, with the unique characteristic that none of the two NS ever had the chance to be recycled by accretion. As we will discuss in more detail, such NS–NS systems have a negligible probability of being detected as binary pulsars, and could represent a “dormant” NS–NS population in galaxies with important implications for gravitational-wave detection of NS–NS inspiral events. The existence of this recently identified [22] evolutionary channel stems from the evolution of helium-rich stars (cores of massive NS progenitors), which has been neglected in most previous studies of double compact object formation. We find that these non-recycled NS–NS binaries are formed from bare carbon–oxygen cores in tight orbits, with formation rates comparable to or maybe even higher than those of recycled NS–NS binaries.

The Method

We study NS–NS binaries formed through a multitude of evolutionary sequences that are not predefined, but instead are realized in Monte Carlo population synthesis calculations.

To describe the evolution of single stars (hydrogen– and helium–rich) from the zero age main sequence (ZAMS) to carbon–oxygen (CO) core formation, we employ analytical formulae from stellar evolution fits [23] However, we have adopted a prescription for the masses of compact objects formed at core-collapse events, based on the relation between CO core masses and final FeNi core masses [24].

Concerning the evolution of interacting binaries, we model the changes of mass and orbital parameters taking into account mass and angular momentum transfer between the stars or loss from the system during Roche-lobe overflow, tidal circularization, rejuvenation of stars due to mass accretion, wind mass loss from massive and/or evolved stars, dynamically unstable mass transfer episodes leading to common-envelope (CE) evolution and spiral-in of the stars. We also account for the possibility of hyper-critical accretion onto compact objects during CE phases [6] and effects of asymmetric supernovae (SN) on a binary orbit (mass loss and a kick velocity a newly born compact object receives in SN). More details about the treatment of various evolutionary processes are presented elsewhere [13].

In the synthesis calculations, we typically evolve a few million of primordial binaries to satisfy the requirement that the statistical (Poisson) fractional errors \( \propto 1/\sqrt{N} \) of the final NS–NS population are lower than 10%. The formation rates
are calibrated using the latest Type II SN empirical rates and normalized to our Galaxy [25].

In our standard model, primordial binaries follow given distributions: for primary masses \((5 - 100 \, M_{\odot})\), \(\propto M_1^{-2.7} dM_1\); for mass ratios \((0 < q < 1)\), \(\propto dq\); for orbital separations (from a minimum, so both ZAMS stars fit within their Roche lobes, up to \(10^5 \, R_{\odot}\)), \(\propto dA/A\); for eccentricities, \(\propto 2e\). Each of the models is also characterized by a set of assumptions, which, for our standard model, are: (1) Kick velocities. We use a weighted sum of two Maxwellian distributions with \(\sigma = 175 \, \text{km s}^{-1} \,(80\%)\) and \(\sigma = 700 \, \text{km s}^{-1} \,(20\%)\) [26]; (2) Maximum NS mass. We adopt a conservative value of \(M_{\text{max}} = 3 \, M_{\odot}\) [27]; (3) Common envelope efficiency. We assume \(\alpha_{\text{CE}} \times \lambda = 1.0\), where \(\alpha\) is the efficiency with which orbital energy is used to unbind the stellar envelope, and \(\lambda\) is a measure of the central concentration of the giant; (4) Non–conservative mass transfer. In cases of dynamically stable mass transfer between non–degenerate stars, we allow for mass and angular momentum loss from the binary [28], assuming that half of the mass lost from the donor is also lost from the system \((1 - f_a = 0.5)\) with specific angular momentum equal to \(\beta 2\pi A^2/\mathcal{P} \,(\beta = 1)\); (5) Star formation history. We assume that star formation has been continuous in the disk of our Galaxy for the last 10 Gyr [29].

**Results**

We use our population synthesis models to investigate all possible formation channels of NS–NS binaries realized in the simulations. We find that a significant fraction of *coalescing* NS–NS systems are formed through a new, previously not identified evolutionary path. The evolution along this new channel begins with two phases of Roche–lobe overflow. The first, from the primary to the secondary, involves non–conservative but dynamically stable mass transfer and ends when the hydrogen envelope is consumed. The second, from the initial secondary to the helium core of the initial primary, involves dynamically unstable mass transfer, i.e., CE evolution. The post–CE binary consists of two bare helium stars of relatively low masses. As they evolve through core and shell helium burning, the two stars develop ‘giant–like” structures, with clear CO cores and convective envelopes. Their radial expansion eventually brings them into contact and the system evolves through a double CE phase (similar to Brown [1995], for hydrogen–rich stars). During this double CE phase, the combined helium envelopes are ejected at the expense of orbital energy. The tight, post–CE system consists of two CO cores, which eventually end their lives as Type Ic supernovae leaving double neutron star system.

The unique qualitative characteristic of this NS–NS formation path is that both NS have avoided recycling. Based on comparison of non–recycled NS–NS relative to that of recycled pulsars, for each of our models, we derive a correction factor for empirical estimates of the Galactic NS–NS coalescence rate. Since these estimates account only for NS–NS systems with recycled pulsars, they must be increased
to include any non–recycled systems formed. We have performed an extensive parameter study to assure robustness of our results. In Table 1 we present the formation rates of non–recycled NS–NS binaries and the total NS–NS population with merger times shorter than 10 Gyr, along with the upwards correction factor for the Galactic empirical rate estimates. Results are shown only for models where the derived factor differs from our standard model by more than 25%. We find that these factors are typically $\approx 1.5$ but can be higher for some models.

We note that the identification of the formation path for non–recycled NS–NS binaries stems entirely from accounting for the evolution of helium stars and for the possibility of double CE phases, both of which have typically been ignored in previous calculations.

**CONCLUSIONS**

The current theoretical estimates of NS–NS coalescence rates appear to have a rather limited predictive power. They cover a range of values in excess of 3 orders of magnitude. Most importantly, this range includes the value of $\approx 5 \times 10^{-7}$ yr$^{-1}$ required for a LIGO II detection rate of 1 event per year. This means that at the two edges of the range the conclusion swings from no detection to many per month, and therefore the detection prospects of NS–NS coalescence cannot be assessed firmly. On the other hand, empirical estimates based on the observed sample of coalescing NS–NS systems appear to be more robust. Taking into account recent empirical estimates and the associated uncertainties [4], we find the Galactic NS–NS inspiral rate in the range $10^{-6} - 5 \times 10^{-4}$ yr$^{-1}$. If we also include the independently derived upper limit of $10^{-4}$ yr$^{-1}$, we expect a detection rate of 2–300 events per year for LIGO II.

It is important to note here that another implicit assumption in derivation of the empirical estimates is that all NS–NS binaries have at some point in their lifetime contained a recycled pulsar with rather long lifetimes ($\approx 10^9$ yr). However,
recent models of NS–NS formation [22] show that there may exist a significant NS–NS population with neutron that never had the chance to be recycled and therefore have very short lifetimes (by 2-3 orders of magnitude, thus preventing their detection). For a variety of population synthesis models, the birth rate of this separate population of coalescing NS–NS binaries is typically comparable or higher than that of the systems with one recycled NS. The total number of coalescing NS–NS systems could be higher by factors of at least 50%, and up to 10 or even higher. Such an increase has important implications for prospects of gravitational wave detection by ground–based interferometers. Using the recent results on the empirical NS–NS coalescence rate [4], we find that the most optimistic prediction for the LIGO I detection rate could be raised to at least 1 event per 2–3 years, and the most pessimistic LIGO II detection rate could be raised to 3–6 events per year or even higher.

Estimates of the coalescence rate of BH–NS and BH–BH systems rely solely on our theoretical understanding of their formation. As in the case of NS–NS binaries, the model uncertainties are significant and the ranges extend to more than 2 orders of magnitude. However, the requirement on the Galactic rate is less stringent for 10 M⊙ BH–BH binaries, only ≃ 5 × 10⁻⁹ yr⁻¹. Therefore, even with the pessimistic estimates for BH–BH coalescence rates (∼ 10⁻⁷ yr⁻¹), we would expect at least a few to several detections per year (with LIGO II), which is quite encouraging. We also point out that that a recent examination of formation of close BH-BH through dynamical processes (stellar interactions) in globular clusters leads to detection rates as high as a few per day for LIGO II and 1 event per 2 years for LIGO I [30].

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