Hydrostatic buoyancy correction and calibration points interpolation using tare weights

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Abstract. Some weighing results requires high accuracy. Users frequently perform manual procedures to make these corrections. Two of these corrections are the hydrostatic buoyancy force and the balance calibration certificate corrections, the last one can be influenced by the tare weights use, and the tare function, in the weighing procedure. To provide an easy way to consider both simultaneously a spreadsheet was developed to identify and interpolate the right values obtained in the calibration certificate data and to use these data to correct the buoyancy force. In a first step, it will be distributed to the Inmetro’s laboratories.

Keywords: hydrostatic buoyancy, mass corrections, weighing modeling.

1. Introduction

The weighing procedures on high-resolution balances (accuracy class I and II [1]) often use solids and liquids as weighing objects (weighing samples). These materials generally have a very low density when compared to the standards weights densities, typically used in these equipment calibrations. When the weighing process occurs in presence of some kind of atmosphere and where the measurement results have a high accuracy requirement, the hydrostatic buoyancy correction is necessary to mitigate the effects of density differences between the measurand and the used calibration mass standards. Variants of these corrections are found in the academic literature but commonly are manually applied or even neglected sometimes. Another weighing important aspect, on high-resolution equipment, is that the corrections and uncertainties concerning to the balance calibration points, must be applied to the measurement results. This is easily applicable when the weighing support is not used as it is sufficient to directly relate the weighing indication (reading) to the respective balance calibration points, thus obtaining the correction values relative to the weighing indicated value as well as its expanded measurement uncertainty. But in the case of liquids, and many solids, a variety of glassware or containers are used, their use is intended for the simple balance protection, to facilitate the products handling or even just to contain these materials, especially for the liquids.

When using this kind of weighing procedure, the empty container is usually weighed and the tare function, available on these balances, is used to return to the null indication value, thus disregarding the mass value of the container. After this, the same container is filled with the material or materials to...
be weighed. This apparently harmless procedure requires a special care when considering balance’s
calibration points corrections as well as uncertainties related to each respective calibration point. For
simple consideration of these two aspects, and with the minimization of errors by manual calculations
and manual records, a spreadsheet was developed to correct the effects of hydrostatic buoyancy by two
distinct adapted corrections [2] [3], and also a linear interpolation of the calibrations points, and their
respective expanded measurement uncertainties, so that each measurement result have the right
correction provided by the balance calibration certificate data. Just these two corrections were chosen
because are commonly applied in analytical chemistry and other knowledge areas and we choose to
keep its mathematical formalism but both are compatible with the metrological convectional mass [1]
[4] [5], indeed one of them is exactly the metrological correction of conventional mass.

For while was adopted the linear interpolation approximation, after the spreadsheet beta test, it
could be changed or include another available way to consider the balance’s measurements
uncertainties and its respective corrections. The spreadsheet was validated by direct results
comparisons and initially will be distributed for use in Inmetro’s laboratories. After this step will be
distributed to metrological and research laboratories wishing to improve the quality of their own
results. The objective of this work was to develop a simple and automatic procedure to simultaneously
perform hydrostatic buoyancy corrections and calibrations point interpolation to improve the quality of
weighing results on high-resolution balances in analysis and research laboratories. The whole budget
of the measurement uncertainties are not shown here because it is specific of each weighing procedure
and laboratory quality system.

2. Buoyancy and tare use corrections

As one application example and one way to see the correction effects, we will consider a weighing
procedure of a 220 g sample solution mass with a known density of 1200 kg/m$^3$ on a calibrated
balance with 8000 kg/m$^3$ density mass standards. Thus, we have a difference about 6.6 times between
the both density magnitudes. If a 150 g mass Becker vessel (tare) will be used as the liquid container,
the respective balance calibration point to be consider is no longer 220 g but 370 g or its nearest
nominal value.

Table 1 considers the simulated balance calibration data using standard weights with density 8000
kg/m$^3$ to the balance calibration, where $NV$ is the nominal value of the calibration point, $E$ is the error
of the balance indication and $U$ is the expanded uncertainty for each respective calibration point.

It is important to emphasize that the uncertainties values were written with all indication digits for
simple convenience, usually we just use two significant algorisms, but to identify the interpolation
effect in these values we will keep all numerical digits of the results.

| NV(g) | E(g)   | U(g)   |
|-------|--------|--------|
| 100   | 0.094324 | 0.000300 |
| 200   | 0.253892 | 0.000500 |
| 300   | 0.123456 | 0.000900 |
| 400   | 1.003445 | 0.001200 |
| 500   | -1.993454 | 0.001500 |

To correct the balance indications we must use the equation (1).

$$M = I - E$$

Where $M$ is the corrected mass and $I$ is the balance indication (reading). With this data, to the 220 g
mass sample, we have the correction close to $-0.25389$ g in the balance indication value and an
expanded uncertainty corresponding to $\pm 0.0005$ g. These values are related to the 200 g calibration
point, but when using a tare support, for instance 150 g mass, the actual point to be considered to the correction is the value of 370 g, which has an approximate correction of – 1.0 g and uncertainty corresponding to a value of ± 0.0012 g, related to its nearest nominal value, 400 g. Comparing both situations we can see that its values are different, emphasizing the importance of this care when correcting the weighing indication and to consider the correct measurement uncertainties involved in the measurement result.

For a better calibration data use, averages, weighted averages, quadratic sum, interpolations or even the balance uncertainty correction obtained by minimum squares linearization can be used. As a first approach, we will use just a linear interpolation to the weighing correction and uncertainties values to the desired measuring point.

2.1. Typical linear interpolation effect
Before show the equations applied to these corrections, we will use some results to exemplify its effects. With a linear interpolation the calibration point and uncertainty correction for the 220 g point, which actually uses the 370 g weighing data we can compare both situations (see table 1 data),

With linear interpolation  \( CI_{370} = -0.6579 \) g  
Without linear interpolation  \( C_{220} = -0.2538 \) g

With linear interpolation  \( UI_{370} = \pm 0.0011 \) g  
Without linear interpolation  \( U_{220} = \pm 0.0005 \) g

In this simple example it can be noted that using the balance calibration certificate data but not identifying the correct values to be used will lead to a procedure error that will spread across all quantities that use these data. Actually applying a wrong correction may be worse than not applying any correction.

2.2. Typical buoyancy corrections effect
To analyses the hydrostatic buoyancy effects we can use the air density based on the available equations in OIML D28 [1] [4] [5] or the dry air correction as a function of atmospheric pressure and temperature. To simplify, and without substantial correction accuracy loss, the standard air density (conventional air density, 1.2 kg/m\(^3\)) can be used. Applying the hydrostatic buoyancy correction, using the conventional mass equation [1], considering, at this moment, the value not corrected by the interpolation, thus we have,

With the hydrostatic buoyancy correction  \( CI_{220} = 220.18718 \) g  
Without the hydrostatic buoyancy correction  \( I_{220} = 220.00000 \) g

To the hydrostatic buoyancy correction case, the balance indication value for the material, i.e. 220 g of the liquid solution was corrected. Actually, in the next sections we will perform the hydrostatic buoyancy correction over the already interpolated data, which improves the result accuracy of the buoyancy corrections.

3. Corrections modeling
To perform the previously mentioned corrections in mass value we have two basic procedures, the first one is interpolate the calibration data to consider the tare mass use. The second is to correct the buoyancy effect using these previous corrected data. The buoyance correction will be performed in two ways, using the equation adapted of Skoog [2] and Harris [3], both commonly used in academic texts books, indeed the last one is the metrological conventional mass equation [1]. As previously mentioned we choose to keep the mathematical formalism adopted by the users intended for this work.
3.1. Linear interpolation

For the linear interpolation was used the adapted interpolation equation (2).

\[ M = I + (I - NV_1) \frac{(C_2 - C_1)}{(NV_2 - NV_1)} \]  

(2)

Where,

- \( M \) is the corrected mass.
- \( I \) is the balance indication (reading).
- \( C_2 \) is the correction of the first greater value than \( I \) in the calibration certificate.
- \( C_1 \) is the correction of the first lower value than \( I \) in the calibration certificate.
- \( NV_2 \) is the first nominal value greater than \( I \) in the calibration certificate.
- \( NV_1 \) is the first nominal value lower than \( I \) in the calibration certificate.

This relation describes the balance indication interpolated correction value. To the measurement uncertainties interpolated correction, simply we need to change \( C_1 \) and \( C_2 \) for the respective calibration uncertainties values for each respective point, \( U_1 \) and \( U_2 \).

Thus we can estimate the indications corrections and uncertainties for any weighing point due to the balance’s calibration certificate data. This estimative can be modified according to the user’s needs, in general the hypothesis of the interpolated correction for the indication error is quite adequate while the interpolation of uncertainty sometimes underestimate the weighing point uncertainty. However there are alternatives based on the balance’s linearization uncertainty (for all calibrations points) or the addition of an uncertainty contribution term associated with the interpolated indication correction and the interpolated calibration uncertainty.

3.2. Conventional mass correction

The conventional mass [1] is a buoyancy correction commonly used in metrological texts and procedures. The conventional mass is given by equation (3).

\[ m_c = m \left(1 - \frac{\rho_0}{\rho}\right) \left(1 - \frac{\rho}{\rho_c}\right) \]  

(3)

Where,

- \( m_c \) is the conventional mass.
- \( m \) is the true mass (mass).
- \( \rho_0 \) is the dry air density (1.2 kg/m\(^3\)).
- \( \rho \) is the sample density.
- \( \rho_c \) is the reference mass standard density (8000 kg/m\(^3\)).

The conventional mass, in other words, is different of true mass, if the weighing process occurs in the air, the true mass is going to be different from the conventional mass for all the samples with density different of 8000 kg/m\(^3\) or in different environmental conditions. The true mass \( m \) is the value of mass after the buoyancy correction or in the vacuum weighing case.

3.3. Skoog adapted equation

Buoyancy corrections was performed by two distinct equations commonly used in analytical chemistry and that are based on air density, the calibration mass standards and samples densities. The first one, adapted of Skoog [2], is shown in equation (4).

\[ M = I \left[1 + \frac{\rho_{air}}{\rho_{obj}} - \left(\rho_{air}/\rho_{std}\right)\right] \]  

(4)

Where,

- \( M \) is the corrected sample (or object) mass.
- \( I \) is the balance indication (reading).
- \( \rho_{air} \) is the air density.
- \( \rho_{obj} \) is the object (sample) density.
- \( \rho_{std} \) is the mass standard density.
3.4. Harris adapted equation
The correction adapted of Harris [3] is the most common in academic texts books and is given by,

\[ M = I \left( 1 - \left( \frac{\rho_{\text{air}}}{\rho_{\text{std}}} \right) \left( 1 - \frac{\rho_{\text{air}}}{\rho_{\text{obj}}} \right) \right) \]  

(5)

where,

- \( M \) is the corrected sample mass.
- \( I \) is the balance indication (reading).
- \( \rho_{\text{air}} \) is the air density.
- \( \rho_{\text{obj}} \) is the object density.
- \( \rho_{\text{std}} \) is the mass standard density.

Equation (3) is commonly used in metrological knowledge, indeed the equations (3) and (4) are the same just write in a different way. However both, equations (4) and (5), are widely used in analytical chemistry and other knowledge areas. Its advantages are the application simplicity and the accessible information use. There are more accurate buoyancy corrections, but moist air density [6] measurement is required for these, which requires measuring equipment generally not available in laboratories. In other wise, the most precisely corrections need the standards and samples volume information, that generally it is not available information. If there is equipment to measure the laboratory environmental conditions, temperature and atmospheric pressure, the dry air density can be obtained improving the correction result. Another important question is the measurement uncertainties available informations, for instance, to the buoyancy correction generally only the sample and air densities uncertainties and the balance calibration data are known, and its respective estimated values have an enough magnitude to make these corrections, equation (3) (4) and (5), be totally equivalent in mass measurements results in laboratory with common accuracy requirements.

4. Results
To be possible to use the spreadsheet some simple informations are needed:

i) Sample(s) density(ies).
ii) Mass standard(s) density(ies).
iii) Air density.
iv) Used tare mass (zero if not used).
v) Calibration certificate data (Including the zero mass indication).
vi) Sample mass measurement results, \( I \).

It is important to pay attention that the interpolation correction do not be applicable in uses out of the balance calibration data range, thus is necessary to keep in mind that this values range including the tare support mass. In the same way, is important to emphasize that the zero calibration point is not shown usually in calibration certificate data, but if will need to interpolated results in this result range we must to attribute values to the balance error and its respective uncertainty. In this case, we use the null value for the balance error and the uncertainty term must be carefully analysed, we chose use the same value of the nearest calibration point.

To exemplify in a complete measurement corrections set, will be use the same case early described, a set of sample mass measurement results, samples densities 1200 kg/m\(^3\), support tare mass 150 g and the balance calibration mass standard with density 8000 kg/m\(^3\). Table 2 shows the simulated calibrations data. We do not use real data because want to show the enlarged corrections to better realise its effects.
Using the table 2 data and the conditions previously described, we obtained a complete corrected set data to the $I$ measurements results (see table 3), where $IC$ is the interpolated correction, $IU$ is the interpolated expanded uncertainty, $M$ is the interpolated corrected mass, $MH, MS$ and $m_c$ are the hydrostatic buoyancy corrections due to Harris, Skoog and conventional mass equations respectively.

The simulated measurements (balance readings) corresponds to $I$ column, the others columns are the calculated data. The uncertainties (in table 2) were overestimate to better exemplify the interpolation.

Table 3 summarises all the final corrected measurement results obtained by the spreadsheet users.

| $N^I$ (g) | $U$ (g) | $-E$ (g) | $M$ (g) | $MH$ (g) | $MS$ (g) | $m_c$ (g) | $IU$ (g) |
|-----------|---------|----------|---------|---------|---------|---------|---------|
| 0         | 0.1     | 0.00000  | 5.155155| 5.159541| 5.159537| 5.159541| ±1.275775|
| 1         | 0.1     | -0.00030 | 11.961961| 11.972139| 11.972129| 11.972139| ±1.309809|
| 2         | 0.2     | -0.00340 | 42.792792| 42.829203| 42.829166| 42.829203| ±1.463963|
| 5         | 0.2     | -0.05000 | 70.613742| 70.673824| 70.673764| 70.673824| ±1.568712|
| 10        | 0.4     | -0.10000 | 85.223482| 85.295994| 85.295922| 85.295994| ±1.617411|
| 20        | 0.5     | 0.10000  | 120.148839| 120.148737| 120.148839| 120.148839| ±1.733488|
| 50        | 0.9     | 0.20000  | 219.913275| 220.100021| 220.100021| 220.100021| ±2.066377|
| 100       | 1.0     | -0.30000 | 320.340289| 320.340562| 320.340562| 320.340562| ±2.520068|
| 200       | 1.5     | -0.20000 | 454.854235| 454.854235| 454.854235| 454.854235| ±2.654425|
| 500       | 2.5     | 0.00000  | 747.283891| 747.283891| 747.283891| 747.283891| ±2.896648|
| 1000      | 3.0     | 1.70000  | 1000.00000| 1000.00000| 1000.00000| 1000.00000| ±3.000000|

Based on the calculated corrections we can observe that each initial balance indication value was modified twice times, in the first one by the interpolation of the balance calibration certificate correction data, and at the second time this value was corrected by the hydrostatic buoyancy effect. Notwithstanding the both shown corrections, $MH$ and $m_c$, are easy to realize that are exactly the same results.

5. Conclusion
The spreadsheet developed to high-resolution balances users with high accuracy results requirement is simple to use and to understand. The informations needed to fulfil the required data is easily to find in the research laboratories. The most important advantages of this development are the optimization of the correction calculation time and the calculation errors elimination. The interaction with the users will provide a possibility for future improvements based on the received information such as the automatic data collection and wireless communication in the Metrology 4.0 scope. Finally, this initiative intends to disseminate metrological knowledge to laboratories where this knowledge can be used to improve their measurement processes and results.
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