KINEMATICS OF THE STELLAR HALO AND THE MASS DISTRIBUTION OF THE MILKY WAY USING BLUE HORIZONTAL BRANCH STARS

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ABSTRACT

Here, we present a kinematic study of the Galactic halo out to a radius of \( \sim 60 \) kpc, using 4664 blue horizontal branch stars selected from the SDSS/SEGUE survey to determine key dynamical properties. Using a maximum likelihood analysis, we determine the velocity dispersion profiles in spherical coordinates \((\sigma_r, \sigma_\theta, \sigma_\phi)\) and the anisotropy profile \(\beta\). The radial velocity dispersion profile \((\sigma_r)\) is measured out to a galactocentric radius of \(r \sim 60\) kpc, but due to the lack of proper-motion information, \(\sigma_\theta\), \(\sigma_\phi\), and \(\beta\) could only be derived directly out to \(r \sim 25\) kpc. From a starting value of \(\beta \approx 0.5\) in the inner parts \((9 < r/\text{kpc} < 12)\), the profile falls sharply in the range \(r \approx 13\)–\(18\) kpc, with a minimum value of \(\beta = -1.2\) at \(r = 17\) kpc, rising sharply at larger radius. In the outer parts, in the range \(25 < r/\text{kpc} < 56\), we predict the profile to be roughly constant with a value of \(\beta \approx 0.5\). The newly discovered kinematic anomalies are shown not to arise from halo substructures. We also studied the anisotropy profile of simulated stellar halos formed purely by accretion and found that they cannot reproduce the sharp dip seen in the data. From the Jeans equation, we compute the stellar rotation curve \((v_{\text{circ}})\) of the Galaxy out to \(r \sim 25\) kpc. The mass of the Galaxy within \(r \lesssim 25\) kpc is determined to be \(2.1 \times 10^{11} M_\odot\), and with a three-component fit to \(v_{\text{circ}}(r)\), we determine the virial mass of the Milky Way dark matter halo to be \(M_{\text{vir}} = 0.9^{+0.4}_{-0.3} \times 10^{12} M_\odot\) \((R_{\text{vir}} = 249^{+34}_{-21} \text{kpc})\).

Key words: galaxies: individual (Milky Way) -- Galaxy: halo -- stars: horizontal-branch -- stars: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION

Understanding the formation of stellar halos provides vital insight into the formation history and the evolution of galaxies (Majewski 1993; Freeman & Bland-Hawthorn 2002; Helmi 2008). Under the currently favored ΛCDM model of galaxy formation, the stellar halos are thought to have been built up, at least in part, by accretion of satellite galaxies (White & Rees 1978; Searle & Zinn 1978; Helmi & White 1999). Discoveries of structures like the Sagittarius dwarf galaxy (Ibata et al. 1994, 1995; Majewski et al. 2003; Belokurov et al. 2006), the Virgo overdensity (Jurić et al. 2008), the Triangulum–Andromeda structure (Rocha-Pinto et al. 2004; Majewski et al. 2004; Martin et al. 2007), and the low-latitude Monoceros ring (Newberg et al. 2002) lend further support to the idea of the stellar halo being formed by accretion. Other than accretion, a part of the stellar halo could also be formed by in situ stars. Recent hydrodynamical simulations (Abadi et al. 2006; Zolotov et al. 2009; Font et al. 2011; McCarthy et al. 2012) of galaxy formation suggest that in the inner regions, the stellar halo might be dominated by in situ stars, whose kinematic properties are distinct from the accreted stars.

Observational evidence of multi-component halos has been found in dynamical studies of Sloan Digital Sky Survey (SDSS) calibration stars (Carollo et al. 2007, 2010; Beers et al. 2012), in rotational behavior in metallicity bins (Deason et al. 2011a), in kinematics of the Galactic anti-center and North Galactic Pole population (Kinman et al. 2007), in chemical properties (de Jong et al. 2010), and also in the age difference between in situ and accreted halos (Kalirai 2012). There also exists a counterclaim by Schönrich et al. (2011), who demonstrate that the evidence of a retrograde signal in the outer halo in the Carollo et al. (2010) sample is weak and is due to a manifestation of incorrect distance estimates. Investigating whether any signal of a multi-component halo could also be seen in the dispersions of the velocity of the halo population is important. Ultimately, studying the velocity dispersion profiles of the halos and comparing them with simulations might help to constrain the model of galaxy formation.

The lack of proper motions and the slightly off-centered position of the Sun with respect to the galactic center poses a unique challenge in studying the kinematics of the stellar halo. At distances much larger than R⊙, line-of-sight velocity is the same as radial velocity with respect to the galactic center. Hence, the σr profile is easy to compute at large distances, a fact that has been well studied; Battaglia et al. (2005) studied the line-of-sight velocity dispersion \(\sigma_{\text{los}}\) of a mixture of 240 halo objects and found that \(\sigma_{\text{los}}\) decreases monotonically beyond \(r \sim 30\) kpc. In the outermost halo at \(r \sim 100\) kpc, Battaglia et al. (2005) and recently Deason et al. (2012b) both see a significant drop in the \(\sigma_{\text{los}}\) value to \(\sim 50\) km s\(^{-1}\).

Conversely, De Propris et al. (2010) studied 666 blue horizontal branch (BHB) stars from the 2QZ Redshift Survey and found that the velocity dispersion profile increases at large distances. However, using 910 distant halo A-type stars, Brown et al. (2010) found that there is a mean decline of \(-0.38 \pm 0.12\) km s\(^{-1}\) kpc\(^{-1}\) in σr over \(15 < r/\text{kpc} < 75\). More recently, Xue et al. (2008) used 2401 BHB halo stars within 60 kpc and measured a slower decline in \(\sigma_{\text{los}}\) compared to earlier studies. At small \(r\), it has been difficult to derive the σr profile, and the only attempt to measure σr in the inner halo was undertaken by Sommer-Larsen et al. (1997). They found a sharp fall in σr, from \(\sim 140\) to \(\sim 100\) km s\(^{-1}\) at \(r \approx 12\) kpc, although they assumed the circular velocity to be constant.

In a solar neighborhood, one can get useful proper motions, with which all three velocity dispersions \((\sigma_r, \sigma_\theta, \sigma_\phi)\) can be measured. Smith et al. (2009b) used the full phase space information of \(\sim 1700\) halo subdwarfs from the solar neighborhood...
(<5 kpc) and determined the velocity dispersions to be $(\sigma_r, \sigma_\theta, \sigma_\phi) = (143 \pm 2, 82 \pm 2, 77 \pm 2)$ km s$^{-1}$. Also, Bond et al. (2010) analyzed the proper motions of a large sample of main-sequence stars within the solar neighborhood (<10 kpc) and found $\sigma_r = 141$ km s$^{-1}$, $\sigma_\theta = 75$ km s$^{-1}$, and $\sigma_\phi = 85$ km s$^{-1}$.

A summary of the estimated values of the velocity dispersions found in the literature is given in Table 1.

Due to the lack of reliable proper motions of the halo field stars, even fundamental quantities like the tangential components of the velocity dispersions as well as the anisotropy ($\beta$) are still poorly understood beyond the solar neighborhood. The situation is not, however, as hopeless as it might seem. At small $r$, using only the line-of-sight velocity it is possible to put some constraints on these quantities using maximum likelihood techniques, where a model or a distribution function (DF) needs to be specified a priori. In an analysis of 1170 BHB stars ranging from $3 \lesssim d/kpc \lesssim 96$, where $d$ is now the stellar distance rather than a radius, Sirko et al. (2004b) fit an ellipsoidal distribution of velocities and find that the halo is isotropic. Similarly, Deason et al. (2011a, hereafter D11) fit a constant anisotropy model (power-law distribution function) to 3549 BHB stars constructed from the SDSS Data Release 7 (DR7) and find that the halo between $r = 10$ and 25 kpc is tangential, whereas the distant halo within 25 < $r/kpc$ < 50 is radial. D11’s claims for the tangential inner halo contrast with the result by Bond et al. (2010), who found the inner halo to be radial in the same region ($d \approx 10$ kpc). D11 assume the DF to be such that the tracer density and the potential are both power laws. Without a priori knowledge of the density slope, their estimates will have some systematics. Additionally, the potential was also kept constant in their analysis and thus can bias the results due to the degeneracy between the potential and the anisotropy.

In their most recent work, Deason et al. (2012a, hereafter D12) allow both the potential and $\beta$ to vary, and thereby break the degeneracy, and find that the outer halo within 16 < $r/kpc$ < 48 is radial with $\beta = 0.5^{+0.4}_{-0.2}$. Previous estimates of the velocity anisotropy ($\beta$) in the solar neighborhood and the nearby halo are summarized in Table 1. All the above estimates of an anisotropy of the distant halo are done in a large radial bin. Their results might be accurate for the given radial bins and could also be the actual anisotropy of the halo, provided the anisotropy remains nearly constant or monotonic throughout. On the other hand, if the actual $\beta$ of the system is not monotonic but has a more complex shape, then estimating it in the larger bin will just capture an overall property of that bin.

Theoretical, there are families of the DF of the Henon (1973) type that result in a constant anisotropic system, as well as families of models that have their own anisotropy profiles, which include the Osipkov–Merritt model (Osipkov 1979; Merritt 1985a, 1985b), Gerhard (1991), Cuddeford (1991), Baes & van Hese (2007), and few more with the Hernquist potential-density model in Baes & Dejonghe (2002). The question we ask is how well do these anisotropy profiles match the anisotropy of the Galaxy? More fundamentally, how well do we know the anisotropy of the halo? To this end, we compute the beta profile with much finer spatial resolution and without any prior assumptions about the density or the potential.

Another use of studying the kinematics of the stellar halo is to constrain the mass and the potential of the Milky Way. Most of the methods to estimate the mass require knowledge of the anisotropy parameter $\beta$. Without the unbiased estimate of the velocity anisotropy, constraining the mass of the system via the Jeans equation could be uncertain by 73% (for $-4.5 < \beta < 0.44$), as found by Watkins et al. (2010) in their estimates of the mass of the Galaxy. Several other authors have also used this assumption to estimate the mass of the galaxy (Dehnen et al. 2006; Gnedin et al. 2010; Deason et al. 2012b). Precautions need to be taken when making an arbitrary assumption about the anisotropy. It has been found that halo stars, satellites, and the dark matter halo have different orbital properties (Abadi et al. 2006; Sales et al. 2007). Hence, assuming a constant anisotropy for both field stars and satellites (Battaglia et al. 2005; Dehnen et al. 2006) could introduce systematic uncertainties in the mass estimate. Ideally, in order to break the degeneracy, we must have a separate estimation of the radial velocity dispersion, the velocity anisotropy, and the underlying density of the population, as pointed out by Dehnen et al. (2006).

The orbital evolution of the Magellanic clouds (Lin & Lynden-Bell 1982;Besla et al. 2007), the local escape speed (Smith et al. 2007), the timing argument (Li & White 2008), and the study of the kinematics of the tracers population (Kochanek 1996; Xue et al. 2008; Gnedin et al. 2010; Watkins et al. 2010) are the methods undertaken in order to constrain the mass of the Galaxy. Summarizing all these attempts to constrain the mass of the Galaxy, the consensus can be found between $0.5 - 3.5 \times 10^{12} M_\odot$. Recently, using BHB stars, Xue et al. (2008) estimate the mass of the Milky Way to be $0.91^{+0.25}_{-0.18} \times 10^{12} M_\odot$. However, they make an assumption that the variation of $(v_{circ}/v_{max})$, the ratio of the circular to the line-of-sight velocity, with radius is the same as in simulations. In this paper, we estimate $v_{circ}$ as far out as possible without any assumption and then use it to estimate the dark matter halo mass.

This work focuses mainly on the study of the kinematics of the stellar halo in order to present an unbiased estimation of the velocity dispersions, anisotropy parameter, and circular velocity as a function of radius to the extent the data will support. We use a DF which does not require any assumptions to be made a priori about the density profile or the potential. We then use our measurements of velocity dispersions to estimate the rotation curve of the Galaxy. With the disk and bulge mass already constrained from Sofue et al. (2009), we focus on constraining the dark matter halo mass. Using the circular velocity curve

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**Table 1**

| Sample (Number) | Distance (kpc) | $\sigma_r, \sigma_\theta, \sigma_\phi$ (km s$^{-1}$) | Anisotropy ($\beta$) | Reference |
|-----------------|----------------|---------------------------------|---------------------|-----------|
| BHB (~700)      | $(r \lesssim 20, r \gtrsim 45)$ | $\sigma_r = (140, 90-110)$ (Radial (0.5), tangential (~1.3)) | Sommer-Larsen et al. 1997 |
| BHB (1170)      | $5 \lesssim d \lesssim 96$ | 101.4 ± 2.8, 97.7 ± 16.4, 107.4 ± 16.6 (Nearly isotropic) | Sirko et al. 2004b |
| BHB (1933)      | $16 < r < 48$ | Radial (0.9$^{+0.08}_{-0.28}$) | Deason et al. 2012a |
| Subdwarfs (~1700) | $d < 5$ | 143 ± 2, 82 ± 2, 77 ± 2 (Radial (~0.69)) | Smith et al. 2009a |
| MS (10$^5$)     | $d \leq 10$ | 141,758,5 (Radial (~0.67)) | Bond et al. 2010 |
| BHB (3549)      | $(10 < r < 25, 25 < r < 50)$ | ⋯ | Deason et al. 2011a |
Using halo stars. A GVE DF with rotation about the z-axis is given by
\[ f(r, v) = \frac{\rho(r)}{(2\pi)^{3/2}\sigma_r\sigma_\theta \sigma_\phi} \exp\left[-\frac{1}{2} \left(\frac{v_r^2}{\sigma_r^2} + \frac{v_\theta^2}{\sigma_\theta^2} + \frac{(v_\phi - v_{rot})^2}{\sigma_\phi^2}\right)\right]. \]  

The DF as given by Equation (4) assumes that the velocity ellipsoid is perfectly aligned with the spherical coordinates, but in general the velocity ellipsoid can have a tilt. Using halo subdwarf stars, Smith et al. (2009b) and Bond et al. (2010) have found that the tilt is small and consistent with zero. Hence, it is safe to ignore the tilt while computing velocity dispersions.

2.2. Parameter Estimation

The information on the proper-motion of the stars in the stellar halo beyond the solar neighborhood \( r \gtrsim 10 \) kpc is not accurate enough to properly constrain the tangential motions. Nevertheless, our position in the Galaxy still makes it possible to constrain these quantities by utilizing the tangential information carried by the line-of-sight velocities of the stars. For that, we need to marginalize the DF over the unknown quantities, which in this case are the tangential components \((v_t, v_\theta)\). The expression for the marginalized DF is given by

\[ F(l, b, d, v_{los}|\sigma_r, \sigma_\theta, \sigma_\phi, v_{rot}) = \int \int f dv_t dv_\theta. \]  

We use the maximum likelihood method to estimate the model parameters. The log-likelihood function which we maximize is given by

\[ \mathcal{L}(l, b, d, v_{los}|\sigma_r, \sigma_\theta, \sigma_\phi, v_{rot}) = \sum_i^n \log F(l_i, b_i, d_i, v_{los}), \]  

where \( n \) is the number of stars in the system under study. We use Markov Chain Monte Carlo (MCMC) with the Metropolis–Hasting algorithm to obtain the posterior distribution. We quote the central values of the velocity dispersions \((\sigma_r, \sigma_\theta, \sigma_\phi)\) as our initial estimates with 16 and 84% error. Note that for the GVE DF, the density term \( \rho(r) \) in Equation (4), is not a function of model parameters and hence does not have any effect on the likelihood analysis.

Once the radial velocity dispersion \( \sigma_r \) and the anisotropy parameter \( \beta \) are evaluated, the Jeans equation (Jeans 1915) can be used to estimate the circular velocity \( v_{circ} \) of the spherical system in equilibrium using the relation

\[ v_{circ}^2(r) = -\sigma_r^2 \left[ \frac{d \ln \rho}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2\beta \right]. \]  

where \( \rho \propto r^{-\alpha} \) is the density of the tracer population, which implies \( d \ln \rho/d \ln r = -\alpha \). Throughout the analysis, we assume the density to be a double power law with \( \alpha = 2.4 \) \((r \lesssim 27 \) kpc) and \( \alpha = 4.5 \) \((r > 27 \) kpc), in agreement with the recent works by Deason et al. (2011b) and Watkins et al. (2009).

For systems with a constant anisotropy and a given \( v_{circ} \), the solution to the differential equation (7) subject to the boundary condition \( \lim_{r \to \infty} \sigma_r^2 = 0 \) reads

\[ \sigma_r^2(r) = \frac{1}{r^{2\beta} \rho(r)} \int_r^\infty dr' r'^{2\beta} \rho(r')(d\Phi/dr'). \]  

Assuming that density and anisotropy are known, we can use this solution to estimate \( \sigma_r \) as a function of \( r \).
Figure 1. Radial distribution of BHB stars in galactocentric coordinates. The distribution has a peak at around 16 kpc. Most of the stars are found to lie in the range 10 < r/kpc < 25.

2.3. DATA: BHB Stars

Since they are luminous and have nearly constant magnitude, BHB stars are ideal for studying the stellar halo, and this is what we use in our study. We use the BHB catalog published by Xue et al. (2011) for our analysis. The catalog is composed of 4985 BHB stars obtained from SDSS DR 8 (Aihara et al. 2011). The stars were selected by imposing limits on color and Balmer line profile measurements. Imposing limits on Balmer line profile measurements allows one to remove the main-sequence stars and Blue Stragglers. Further details on BHB candidate selection can be found in Xue et al. (2008) and references therein. To avoid contamination from the disk stars, we restrict our analysis to stars with a distance of |z| > 4 kpc from the galactic midplane. As mentioned earlier, due to the way our likelihood function (Equation (5)) is laid out, this cut in distance above the plane will not introduce any bias. No velocity limits have been imposed to obtain the sample and, thus, for the purpose of kinematic studies, the population of BHB stars we select can be considered to be kinematically unbiased.

For the stars that we study, the angular position is known very accurately, but the distance and radial velocity have some uncertainty associated with them. To get more accurate distances, we recalibrate the X11 distances using a color–magnitude relation derived for the same population from Deason et al. (2011b). The estimated dispersion in g-band magnitudes is 0.13, equivalent to a distance uncertainty of 6%. For the SEGUE (Yanny et al. 2009) radial velocity measurements, 94% of our sample have an uncertainty of less than 8 km s\(^{-1}\).

The galactocentric radial distribution of the final BHB samples is shown in Figure 1. One can see that the distribution peaks at around 16 kpc. Most of the stars are found to lie in the range 10 < r/kpc < 25.

3. VELOCITY DISPERSION PROFILE OF THE HALO

We study the kinematics of the halo in radial bins to obtain the radial profile of the model parameters (σ\(_r\), σ\(_θ\), σ\(_φ\)) and also β. Using only the line-of-sight velocity information, the tangential components, σ\(_θ\) and σ\(_φ\), are difficult to constrain except in the very inner regions of the halo. However, σ\(_r\) can be well constrained in both the inner and outer halos. This means that a relatively larger number of stars (>1000) per bin are required to estimate σ\(_θ\) and σ\(_φ\) as compared to σ\(_r\). Given that we only have about 4000 stars, this means that we cannot measure the σ\(_θ\) and σ\(_φ\) profiles with sufficient spatial resolution. Hence, we employ two different binning schemes or estimators, one for radial velocity and the other for tangential velocity. The estimators are the equi-populated estimator (EPE) and the central moving estimator (CME). In EPE, the data are binned radially with each bin containing an equal number of particles, and this is used for computing σ\(_r\)(r). In CME, a set of equi-spaced positions in r are chosen and then at each position an equal number of points on either side of the chosen central value are used to estimate the desired quantity. We use the CME for computing σ\(_θ\)(r) and σ\(_φ\)(r). The crucial difference between the two schemes is that while the bins are non-overlapping in the former, they can be overlapping in the latter. In EPE, the spacing between the bins is directly proportional to the number of particles in each bin n\(_{\text{bin}}\). Hence, if the desired quantity can be estimated with sufficient accuracy by employing small n\(_{\text{bin}}\), then EPE is the desired method. However, if this is not the case, then it is better to use the CME method as the spatial resolution is not directly dependent on n\(_{\text{bin}}\).

Finally, for our data, the number density of points in r is highly non-uniform, and hence it is not accurate to assume that the desired quantity has been estimated at the center of the bin. To alleviate this number density bias, for both schemes we compute the final position of the bin as the mean r of the points in the bin.

3.1. Radial Velocity Dispersion Profile (σ\(_r\)(r))

Here, we focus on the nature of the σ\(_r\)(r) profile of the halo for which we adopt the EPE method with n\(_{\text{bin}}\) = 400. As explained previously, σ\(_r\) can be measured out to the extent of the data (r ∼ 60 kpc). The values of σ\(_r\) obtained from the likelihood analysis are given in Figure 2 and the error bars represent the 1σ confidence interval and are determined from the likelihood fitting.

We find that the radial velocity dispersion, σ\(_r\), at the Sun’s position (R\(_\odot\) = 8.5) is 145.6 km s\(^{-1}\). However, beyond the
solar neighborhood, \( \sigma_r \) sharply decreases until \( r \sim 15 \) kpc, after which it decreases much slowly and approaches a value of \( \sim 100 \) km s\(^{-1} \) at 56 kpc. The error bar in \( \sigma_r \) for the inner halo population is large and is mainly due to the fact that \( v_{los} \) contains less radial velocity information compared to outer parts. The overlaid black dashed line is the linear approximation \((\approx 111-0.31r)\) for the \( \sigma_{los} \) profile from Xue et al. (2008). Additionally, other previous attempts to fit the profile for \( \sigma_r \) in the outer parts of the halo (Battaglia et al. 2005; Gnedin et al. 2010; Brown et al. 2010) also found profiles similar to Xue et al. (2008), with a slightly varying slope and normalization. All of these profiles are reasonable estimates of \( \sigma_r \) for the outer halo \((d \gg R_\odot)\), where the assumption \( \sigma_r \approx \sigma_{los} \) holds. In the inner halo \((r \lesssim 15 \) kpc\), however, the approximation breaks down and \( \sigma_r \) strongly deviates from \( \sigma_{los} \). One can see that the deviation of \( \sigma_r \) from \( \sigma_{los} \) increases as one approaches the center, and at \( R_\odot \) the deviation is as high as \( \sim 40 \) km s\(^{-1} \).

Sommer-Larsen et al. (1997) provide a functional form for fitting the \( \sigma_r \) profiles, which is given by

\[
\sigma_r^2 = \sigma_0^2 + \frac{\sigma_\pi^2}{\pi} \left[ \frac{\pi}{2} - \tan^{-1}\left( \frac{r-r_0}{l} \right) \right].
\] (9)

This has a shape that is similar to our \( \sigma_r(r) \) profile and we fit it to find \( \sigma_0 = 94.5 \) km s\(^{-1} \), \( \sigma_\pi = 122.3 \) km s\(^{-1} \), \( r_0 = 13.2 \) kpc, and \( l = 2.6 \) kpc. In this function, the fit parameter \( \sigma_0 \) gives the asymptotic value that \( \sigma_r \) achieves in the outer halo, whereas \( (\sigma_0^2 + \sigma_\pi^2)^{1/2} \) gives the approximate value for \( \sigma_r \) in the inner halo. The fit parameters \( r_0 \) and \( l \) determine the turnoff point and the steepness of the transition of the profile, respectively. A lower value of \( l \) gives a steeper transition; this can be seen from the comparison between the Sommer-Larsen fit \((l = 7.5 \) kpc\) and the red line in Figure 2, which is our fit with a smaller \( l \).

### 3.2. Tangential Velocity Dispersion and Anisotropy Profiles \((\sigma_\theta (r), \sigma_\phi (r), \text{and } \beta (r))\)

There have been few attempts to constrain the tangential velocity dispersions and most of the studies are either restricted to the solar neighborhood (Smith et al. 2009a; Bond et al. 2010) or for the overall system (Sirko et al. 2004a). The tangential velocity dispersions not only provide information about the anisotropy of the stellar population (through Equation (1)), but together with \( \sigma_r \) also help to measure the mass distribution of the Galaxy.

We estimate the tangential velocity dispersions \((\sigma_\theta \text{ and } \sigma_\phi)\) using CME with \( n_{bin} = 1200 \) stars. Given the quality of the data, we are only able to measure \( \sigma_\theta \) and \( \sigma_\phi \) out to \( \sim 25 \) kpc. Our estimates of \( \sigma_\theta \) and \( \sigma_\phi \) are shown in Figures 3(b) and (c) by the black dots with error bars. For uniformity, we also estimate \( \sigma_r \) with this binning scheme, which is shown (only out to \( \sim 25 \) kpc) in Figure 3(a) by the black dots with error bars.

In general, the tangential components \( \sigma_\theta \text{ and } \sigma_\phi \) near the solar neighborhood are found to be comparatively lower \((\sigma_\theta = 85^{+8}_{-9} \) km s\(^{-1} \), \( \sigma_\phi = 95^{+8}_{-9} \) km s\(^{-1} \)) than the radial dispersion \( \sigma_r \). Figures 3(b) and (c) show that there is a sharp rise in the values of \( \sigma_\theta \) and \( \sigma_\phi \) at \( r = 17 \) kpc. Beyond this, \( \sigma_\phi \) falls, whereas \( \sigma_\theta \) rises; given the large uncertainties and low number of independent bins, it is unclear if this is a real or a spurious trend.

By substituting the estimates of the tangential and the radial velocity dispersions obtained using CME \((n_{bin} = 1200)\) from the above analysis into Equation (1), we compute the corresponding values of the anisotropy constant in the respective bins. As shown in Figure 3(d), the halo within 12 kpc has \( \beta \sim 0.5 \),
whereas the outer halo beyond the turnoff point is nearly isotropic. We discover a significant drop in the $\beta$ profile at $r = 17$ kpc. Here, the halo is strongly tangential with $\beta = -1.2$. We later confirm that the trend observed in anisotropy is neither due to the manifestation of the systematics introduced by the chosen model (Section 3.3) nor due to the presence of the halo substructures (Section 3.4). It is also shown in Appendix B that the assumed $\Omega_{\rm LS}$ and $R_{\odot}$ have negligible effects upon these estimates. The probable reasons for this sudden turnover in the properties of the halo are discussed in the conclusion.

We know that the consecutive CME bins overlap in radius, and thus the dispersion profiles demonstrated in Figure 3 are a smoothed version of the actual dispersion profiles of the halo. However, to check for any systematic associated with the choice of the binning scheme, we also estimate $\sigma_r$, $\sigma_\theta$, $\sigma_\phi$, and $\beta$ in traditional equi-populated (EPE) bins ($n_{\text{bin}} = 700$). The measured values in these bins are shown by the diamond points in Figure 3. If the number of stars per bin is fewer than 700, then it is difficult to constrain $\sigma_\theta$ and $\sigma_\phi$. Even with $n_{\text{bin}} = 700$, we were only able to constrain the tangential motion until 16 kpc (first three diamond points). Hence, we construct the last bin by grouping all the stars beyond 16 kpc into one bin (rightmost diamond point). More importantly, with this binning scheme, we are only interested to see whether we obtain the corresponding dip or rise (depending on the parameter of interest) seen in Figure 3 or not. We find that except for the rightmost diamond points, all of the other diamond points in the figure are in agreement with our previous estimates of the dispersions (given by black dots). The rightmost diamond points are calculated in a huge bin with more than 50% of the total sample. Particularly for $\sigma_\theta$, $\sigma_\phi$, and $\beta$, given their non-monotonic trend, we do not claim that the last diamond point is the correct estimate of anisotropy at $r \sim 35$ kpc.

None of the uncertainties quoted in the above estimates of $\sigma_r$, $\sigma_\theta$, and $\sigma_\phi$ include the uncertainties in distances and radial velocities. As mentioned earlier, the errors in distance and radial velocity are quite small, and convolving the model (Equation (4)) with the error functions should not change the results.

3.3. $\beta(r)$ from Fitting Distribution Function ($f(E,L)$)

It would be interesting to see whether the $\beta$ profile presented above, in particular the dip seen at $r = 17$ kpc, is an artifact of our chosen model (GVE) or a real inherent feature of the Galactic halo. In order to pursue it, here we explore the effect of the chosen model on the determination of the velocity anisotropy ($\beta$). For the comparative study, the alternative model we choose is the D11 DF (Equation (3)). We consider anisotropy ($\beta$) and potential ($=\Phi_0 r^{-\gamma}$) as free model parameters and constrain them using the maximum likelihood method.

First, we compare the theoretical properties of the DFs at our disposal, namely GVE and D11 DF. In Figure 4, we plot the theoretical line-of-sight velocity distributions (LOSVDs) of models along two separate lines of sight. We also show the LOSVDs in Figure 4 for two different distances representing the inner halo ($d = 15$ kpc, given by red lines) and the outer halo ($d = 50$ kpc, given by blue lines). For the inner halo, we assume $\alpha = 2.4$ and assign radially biased anisotropy ($\beta = 0.4$), whereas for the outer halo we assume $\alpha = 4.5$ and assign tangentially biased anisotropy ($\beta = -2.5$), in accordance with the Watkins et al. (2009) and Deason et al. (2011b) estimates for $\alpha$. Note that the density normalization at the break radius ($r = 27$ kpc) is assumed to be equal. For the assumed constant potential, the solid lines in Figure 4 (both left and right panels) are the LOSVDs obtained by adopting the D11 model and the dashed lines are the LOSVDs of our GVE model. Recall that our model does not take $\beta$ directly but demands the information of the velocity dispersion components ($\sigma_r$, $\sigma_\theta$, $\sigma_\phi$) individually. To make the LOSVDs obtained from both the models comparable, we estimate $\sigma_r$, $\sigma_\theta$, and $\sigma_\phi$ from the set of values of $\beta$, $\alpha$, $\Phi_0$, and $\gamma$ chosen to obtain LOSVDs of D11 DF. For an assumed potential power law, we use Equation (8) to first calculate $\sigma_r$, to put in our model. For an assumed $\beta$, substituting this $\sigma_r$ in Equation (1) gives the corresponding value for $\sigma_\theta (=\sqrt{\sigma_r^2+\sigma_\phi^2})$. The figure shows that for all four cases, LOSVDs obtained from both models match well. Naively, from these perfect matches of the LOSVDs at different lines of sight and distances, it can be anticipated that the estimation of $\beta(r)$ with both models should also match.

Now, we estimate the $\beta(r)$ profile using exactly the same sample of BHB stars in the same radial bins as in Section 3.2 (CME, $n_{\text{bin}} = 1200$) but with D11 DF. Blue points in Figure 5 demonstrate the $\beta$ profile obtained by fitting D11 DF. Here, we use brute-force grid-based analysis to constrain the model parameters $\beta$, $\Phi_0$, and $\gamma$. To facilitate the comparison, our estimates from Figure 3(d) are overplotted in Figure 5 and are shown by the black points. Figure 5 shows that within the range of uncertainties, the measured values of $\beta$ agree with both models (D11 DF and GVE models). However, a slight bias
Anisotropy ($\beta$) estimates in the CME using the D11 model (blue points) and the GVE model (black points). Each bin consists of 1200 stars. Anisotropy estimates with the GVE distribution are done with the MCMC technique, whereas estimates with the D11 model are done with the brute-force grid-based analysis. Assigned asymmetric uncertainties are 1$\sigma$ confidence intervals obtained from likelihood fitting.

There exists, in the sense that $\beta$ obtained from GVE DF is higher generally. The reason for this discrepancy lies in the fact that $\beta$ obtained from D11 DF has a dependence on $\alpha$. Hence, unless the underlying $\alpha$ value of the sample is known exactly, a mismatch is expected. The estimated value of $\beta$ increases with the adopted value of $\alpha$ (see Figure 3 of D11). This suggests that the actual value of $\alpha$ is even higher than the one adopted here (2.4).

In order to estimate the quality of the constraints obtained from the likelihood analysis, we display the likelihood contours in the parameter space in the top left and bottom two plots of Figure 6 for a bin centered at $r = 16.93$ kpc, where the maximum dip in the $\beta$ profile was seen in Figure 5. Additionally, in the corresponding bin, the top right plot in the figure demonstrates the posterior distributions of the model parameters $\sigma_r$, $\sigma_\theta$, and $\sigma_\phi$ of the GVE model obtained from $5 \times 10^5$ MCMC random walks. One can see that even at a distance of just twice that of $R_\odot$, the $\sigma_\theta$ and $\sigma_\phi$ distributions are quite broad when compared to the $\sigma_r$ distribution; this is the reason for the large uncertainty in the value of $\beta$ as we move outward in $r$.

### 3.4. Effect of the Halo Substructures

There is now enough observational evidence to suggest that the stellar halo is highly structured, particularly as one moves outward into the halo (Bell et al. 2008). Using clustering algorithms on simulated $N$-body stellar halos, Sharma et al. (2011b) find that the fraction of material in substructures increases monotonically as a function of distance from the center and at around 65 kpc can be as high as 50%. Hence, should we include the substructures or exclude them while studying the kinematic properties of the halo? If the kinematic properties of...
a sample are dominated by a few massive accretion events, then one should exclude them. However, in spite of being highly structured, if the sample is a superposition of a large number of events with none of them being individually too dominant, then it is best to include them. Results of Sharma et al. (2011b) on simulated halos show that for the range of radii that spans our BHB stars \((r < 40 \, \text{kpc})\), the amount of material in the substructure should be less than 20\%. So we expect the substructures to have a minor effect on the kinematic properties that we have derived. However, it is still important to check if this is true.

To study the effect of substructures on our estimation of the dispersions, we mask two prominent features of the halo that contaminate our sample, namely, the Sagittarius stellar stream and the Virgo overdensity. The cuts we impose include the Sagittarius dwarf galaxy, which reduces the original sample size by 40\% to \(\sim 2500\) stars. A proper phase-space masking of these structures will be revised in future work.

In Figure 7, we present our results obtained after masking the substructures. As masking reduces the sample size almost by half, we employ CME with \(n_{\text{bin}} = 500\) stars only, instead of \(n_{\text{bin}} = 1200\) as was done earlier with the unmasked data, to avoid excessive smoothing of the estimated profiles. Figure 7 shows that the masking of substructures has little effect on the estimation of velocity dispersion profiles, and the \(\beta\) profile is almost unchanged. This alleviates the concern that perhaps the turnover points in the velocity dispersion profiles and the dip in the \(\beta\) profile, seen in Figure 3, in the region \(r \approx 13-18 \, \text{kpc}\) could be due to the dominance of halo substructures.

**4. COMPARISON OF ANISOTROPY ESTIMATES IN D11 AND D12 RADIAL BINS**

In their recent works, D11 and D12 fitted a DF of the form given by Equation (3) to the BHB samples obtained from the SEGUE survey in order to estimate the model parameters. The models adopted are constant anisotropy models given by Equation (2). In D11, the potential is assumed to be a power law \((\sigma_r \propto r^{-\gamma})\) with a constant index \((\gamma = 0.35)\). Later in D12, they break the degeneracy in their model and consider the potential normalization (\(\Phi_0\)), potential slope (\(\gamma\)), and anisotropy (\(\beta\)) as free parameters. Note that in D11, there is an additional parameter specifying rotation (odd part of the DF), but this was dropped in D12 as they were not focusing on rotation. The methodology applied to measure the model parameters is similar to ours, which involves marginalizing the DF over the tangential velocities to derive the LOSVD: fitting the LOSVD to the data using the maximum likelihood method and in return obtaining the best estimates of the model parameters.

**4.1. Anisotropies of Inner and Outer Halos by D11**

D11 study the rotation and anisotropy of the BHB samples taken from SDSS DR7 (Abazajian et al. 2009). First, in order to construct their sample, we query the SDSS DR7 database to select the candidate BHB stars using the color and stellar parameter ranges given in D11. Like them, we also mask the Sagittarius dwarf galaxy, which reduces the original sample size by 40\% to \(\sim 3500\).

D11 measure the anisotropy of the halo in radial and metallicity bins. In the inner halo \((16 < r/\text{kpc} < 25)\), both metal-rich ([Fe/H] > −2) and metal-poor ([Fe/H] < −2) stars are found to be tangential with \(\beta \sim −1.2\) and \(\sim −0.2\), respectively. Since they do not give the estimates of \(\beta\) in combined metallicity bins for the inner and outer halos, we estimate them here using the same methodology they adopted. For the inner halo we find \(\beta = −0.62\) (tangential), and for the outer halo we find \(\beta = 0.41\) (radial). These estimates are consistent with D11 results if we combine their low and high metallicity \(\beta\) values in each radial bin.

The inner halo within the solar vicinity has been found to be radial in studies of halo subdwarfs by Smith et al. (2009a) and in studies of \(10^5\) main-sequence stars by Bond et al. (2010; see also Figure 3(d)). Hence, a tangential inner halo as predicted by D11 is surprising. The first thing to check is if the D11 result...
is due to some of their assumptions. For example, in D11, the logarithmic density slope was assumed to be constant and equal to $-3.5$. Later on, Deason et al. (2011b) conducted a detailed analysis of the BHB stars to estimate their density profile and found that the profile is of the form of a broken power law, the inner halo ($\langle 27$ kpc) having a profile index of $-2.3$ and the outer halo of $-4.6$. This is in good agreement with the findings of Watkins et al. (2009) that the halo within 25 kpc has a profile index of $-2.4$, beyond which it is steeper with an index of $-4.5$.

If we update the density profile index in the D11 case with the above values, then we expect the inner halo, which is already tangential, to become even more tangential, and the outer halo, which is already radial, to become even more radial. This is because $\beta$ has a dependence on the adopted value of $\alpha$ as shown by D11 (Figure 5). In general, $\beta$ increases with an increase in the adopted value of $\alpha$. Another effect that can potentially bias the results is the fact that the potential parameters ($\Phi_0$, $\gamma$) have been kept fixed in the D11 analysis, i.e., the degeneracy between $\beta$ and the potential has not been broken. After analyzing data in finer bins and breaking the degeneracy among the model parameters, we see that in Figure 5 the inner halo is radial (from Section 3.3), as was also found with the GVE model. Hence, the assumption of a fixed potential can bias the estimation of $\beta$. However, we present much more apparent reason for the discrepancy. Clearly, from Figure 3, the radial bin from 10 to 25 kpc will encompass the stars within 13–17 kpc which are predominantly tangential. Since the probability density of stars in radius also peaks at around 16 kpc (see Figure 1), we anticipate the overall $\beta$ to be tangential. To conclude, the tangential behavior of the inner halo seen by D11 is most likely due to the large radial bin size they adopted.

4.2. Anisotropy at $16 < r / {\text{kpc}} < 48$ Seen by D12

D12 re-calibrate the distances of BHB stars using the color–magnitude relation given by Deason et al. (2011b) and then select stars within $16 \lesssim r / {\text{kpc}} \lesssim 48$ (1933 stars) from the X11 BHB samples. They fit the D11 model to study the nature of $\beta$ in the outer halo. Unlike D11, as mentioned previously, here they break the degeneracy found in their model and consider $\beta$, $\Phi_0$, and $\gamma$ as free parameters. Thus, while they fit the model, they simultaneously estimate all three parameters. They find $\beta = 0.4 \pm 0.2$ for $\alpha = 4.6$. Using a model allowing for oblateness ($q = 0.59$), they find $\beta = 0.5 \pm 0.2$. If we apply the GVE DF to stars in the range $16 < r / {\text{kpc}} < 48$, we find $\beta = -0.14^{+0.52}_{-0.66}$ ($\sigma_r = 97.3^{+22.9}_{-3.0}$ km s$^{-1}$, $\sigma_\theta = 122.7^{+26.4}_{-33.0}$ km s$^{-1}$, and $\sigma_\phi = 78.5^{+34.3}_{-43.0}$ km s$^{-1}$, posterior distributions are shown in Figure 8). This more or less looks like the mean value of $\beta$ in this range (see Figure 5), provided we take into account the fact that the number density of stars peaks around $r = 16$ kpc. Although the D12 value is still within our $1\sigma$ region, our predicted value is lower than that in D12. Figure 5 shows that $\beta$ is not constant in the range $16 < r / {\text{kpc}} < 23$. It increases from being tangential to isotropic. Using the D11 DF also gives similar results (Section 3.3). Beyond this range, we cannot directly measure $\beta$, but by deriving a best-fit circular velocity profile and making use of the $\sigma_r$ profile which is available until $r = 56$ kpc, we can predict $\beta$, and we return to this in Section 5. However, an assumption about $\alpha$ also has to be made. Beyond $r > 27$ kpc, the density slope has been shown to change from $-2.4$ to $-4.5$. Adopting a steeper density slope increases the value of $\beta$. For $\alpha = 4.5$, we find $\beta \sim 0.5$ for $r > 27$ kpc; this is more in agreement with D12. To conclude, the D12 value of $\beta = 0.4$ for $16 < r / {\text{kpc}} < 48$, although derived for a sample which is dominated by stars within $r < 27$ kpc, is not appropriate for the range $18 < r / {\text{kpc}} < 23$, instead it is correct for the range $23 < r / {\text{kpc}} < 48$.

Finally, we check how we can best measure $\beta$ in the outermost parts, $35 < r / {\text{kpc}} < 84$, using the D11 DF and the data in hand. This region consists of 762 stars, and we assume $\alpha = 4.5$ and $\gamma = 0.35$ and repeat the analysis as in D12. The likelihood distributions of model parameters are shown in Figure 9. The mass–anisotropy degeneracy is clearly visible here, suggesting that it is very difficult to directly measure $\beta$ unless an explicit or implicit assumption about the potential is made.

5. CIRCULAR VELOCITY CURVE OF THE GALAXY

Here, we use the measured values of $\sigma_r(r)$ and $\beta(r)$ from our analysis given in Figure 3 to determine the circular velocity curve of the Galaxy ($v_{\text{circ}}$) through the Jeans equation (Equation (7)). Besides anisotropy and radial velocity dispersion information, we also need to adopt some density profile for the tracer population but not of the spectroscopic sample. We adopt a value of $\alpha = 2.4$, as suggested by recent works of Watkins et al. (2009) and Deason et al. (2011b), for the range of distance explored here ($r < 25$ kpc). The blue dots with error bars in Figure 10 are our estimates of $v_{\text{circ}}$ using CME with $n_{\text{bin}} = 1200$. The uncertainties on $v_{\text{circ}}$ were computed using a Monte Carlo-based scheme from uncertainties in $\sigma_r$ and $\beta$. The leftmost and rightmost points have comparatively larger error bars than intermediate points. For the leftmost point, the large error bar is mainly due to the large error in the value of $\sigma_r$. On the other hand, for the rightmost point, the large error bar is mainly due to large error in the values of $\sigma_\theta$ and $\sigma_\phi$.

Figure 10 shows that the circular velocity profile derived from our analysis (blue points) displays prominent features. We now check if such features are also observed in other studies using tracers other than BHB stars. For this, we overplot the $v_{\text{circ}}$ compiled by Sofue et al. (2009) as black dots, obtained from
Figure 10. Circular velocity curve of the Galaxy and their individual components along a galactocentric distance ($r$). The blue marker represents the value of $v_{\text{circ}}$ obtained in the CME bins in $r$. The red solid line is our fit of the total potential. Black dotted and dot–dashed lines are the fixed disk and the bulge circular velocity profile for a set of adopted values of masses and scale radii. The dashed line is the fitted NFW profile. Black dots with error bars are the collated $v_{\text{circ}}$ values given by Sofue et al. (2009) whereas the yellow solid line is the average of the given observed values.

(A color version of this figure is available in the online journal.)

several references (Burton & Gordon 1978; Blitz et al. 1982; Clemens 1985; Fich et al. 1989; Honma & Sofue 1997a, 1997b; Honma et al. 2007; Demers & Battinelli 2007). Further details about the source of each individual point can be found in Sofue et al. (2009) and references therein. Note that the $v_{\text{circ}}$ values of Sofue et al. (2009) are computed for ($R_\odot$, $v_{\text{LSR}}$) = (8.0 kpc, 200.0 km s$^{-1}$). Correcting $v_{\text{circ}}$ for our adopted values of ($R_\odot$, $v_{\text{LSR}}$) = (8.5 kpc, 220.0 km s$^{-1}$) is beyond the scope of this work and, hence, we simply overplot these published values in Figure 10. One can see that there is a prominent dip at 9 kpc in the Sofue et al. (2009) compiled $v_{\text{circ}}$ profile. They explain this dip by introducing massless rings on top of a disk with exponential surface density. We also find a similar dip in our $v_{\text{circ}}$ profile, but at around 10–12 kpc. The slight shift in the position of the dip could be due to the large width of our bins and also the fact that unlike Sofue et al. (2009), who measure $v_{\text{circ}}$ in the midplane of the Galaxy, we measure $v_{\text{circ}}$ over a spherical shell that intersects with the SDSS footprint.

We now estimate the mass of the dark matter halo of the Milky Way, assuming a three-component model of the Galaxy consisting of the bulge, the disk, and the halo. The bulge is modeled as a Hernquist sphere and the disk is assumed to follow an exponential profile (Xue et al. 2008). The parameters for the bulge and disk are taken from Sofue et al. (2009) and are kept fixed. Although Sofue et al. (2009) use massless rings, here we have ignored them since our main aim is to fit the dark matter halo. We model the dark matter halo using the Navarro–Frenk–White (NFW) density profile (Navarro et al. 1996). Here, we consider both the halo and the bulge to be spherically symmetric. The non-axisymmetric effect due to a bar-shaped bulge is neglected here. Potentials for the bulge, the halo, and the exponential disk can be expressed, respectively, as

$$\Phi_{\text{bulge}}(r) = -\frac{G M_{\text{bulge}}}{r + a},$$  \hspace{1cm} (11)

$$\Phi_{\text{NFW}}(r) = -\frac{G M_{\text{vir}} \ln(1 + r c/R_{\text{vir}})}{g(c) r} ,$$  \hspace{1cm} (12)

where $M_{\text{disk}} = 6.5 \times 10^{10} M_\odot$, $b = 3.5$ kpc, $M_{\text{bulge}} = 1.80 \times 10^{10} M_\odot$, and $a = 0.5$ kpc (Sofue et al. 2009).

Note that the disk potential as given by Equation (10) is spherically symmetric. It means that the disk is considered to be a spherical body with an exponential surface density falloff. To get an idea of the error that is incurred due to the assumption of the disk as a spherical body with the same mass as the flattened disk, we refer the reader to Binney & Tremaine (2008, Figure 2.17). The maximum error in $v_{\text{circ}}$ is roughly 13%, which is at a distance of about twice that of the disk scale length. However, at the larger distances along the midplane, the discrepancy is smaller. In the general case of triplanar symmetry (elliptic disk), in reality, the disk potential has to be the function of both polar and azimuthal coordinates, and in the special case of axial symmetry (circular disk) it has to be the function of sole polar coordinates, in addition to the radial coordinates ($r$). We use the spherically symmetric form here for two reasons. First, we make use of the spherical form of the Jeans equation given by Equation (7), which demands a spherical potential. Second, it is to ease the comparison with earlier studies, e.g., Xue et al. (2008), that adopt a similar definition. However, later on, we consider a three-dimensional (3D) disk potential and discuss its consequences for the estimation of mass. The function $g$ in the NFW potential is given by

$$g(c) = \ln(1 + c) - \frac{c}{1 + c}$$
and
\[ R_{\text{vir}} = \left( \frac{2 M_{\text{vir}} G}{H_0^2 \Omega_m \Delta_{\text{th}}} \right)^{1/3}. \]

The total potential \( \Phi(r) \) of the Galaxy is then simply
\[ \Phi(r) = \Phi_{\text{bulge}}(r) + \Phi_{\text{disk}}(r) + \Phi_{\text{NFW}}(r). \] (13)

We adopt the value of the Hubble constant, \( H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_m = 0.3 \) (Komatsu et al. 2011), and \( \Delta_{\text{th}} = 340 \) (Bryan & Norman 1998).

A NFW halo has two free parameters, the mass \( M_{\text{vir}} \) and the concentration \( c \). Since we do not have enough data points spanning a wide range in radius, we avoid fitting both the parameters simultaneously. Instead, we use the concentration–mass relation
\[ c = 327.3 M_{\text{vir}}^{-0.12}, \quad 10^{11} \leq M_{\text{vir}}/M_\odot \leq 10^{13}, \] (14)
as has been reported in \( N \)-body simulations of dark matter halos (Macciò et al. 2007).

Finally, we can derive the resultant circular velocity (\( v_{\text{circ}} \)) from the total potential (Equation (13)) by computing \( (d\Phi/dr)^{1/2} \). We fit the obtained theoretical rotation velocity curve to our observed values of \( v_{\text{circ}} \) shown by the blue dots in Figure 10, and the red line is our best-fit circular velocity curve. The \( v_{\text{circ}} \) profiles for the different components are also shown separately. The dashed black line is the corresponding best-fit NFW halo profile. The best-fit value for the fit parameter, \( M_{\text{vir}} \), for our three-component baryon and dark matter mass distribution is \( 0.9^{+0.4}_{-0.3} \times 10^{12} M_\odot \). The corresponding values of \( R_{\text{vir}} \) and \( c \) derived from the best-fit value of \( M_{\text{vir}} \) are \( 249^{+31}_{-33} \) kpc and \( 12.0^{+0.6}_{-0.5} \), respectively. We estimate the mass of the Galaxy within \( r \leq 25 \) kpc to be \( 2.1 \times 10^{11} M_\odot \). Assuming a functional form for \( P(v_{\text{los}}/v_{\text{circ}}) \) obtained from simulations, Xue et al. (2008) derive \( v_{\text{circ}} \) from the \( v_{\text{los}} \) of BHB stars. The derived \( v_{\text{circ}} \) is then used to estimate the virial mass of the dark matter halo. They find \( M_{\text{vir}} = 0.91^{+0.27}_{-0.18} \times 10^{12} M_\odot \), which is in good agreement with our result; note that uncertainties are, however, slightly larger in our estimates. Unlike them, we do not make any assumption about the functional form of \( P(v_{\text{los}}/v_{\text{circ}}) \).

Here, we study the effect of chosen disk models for which we consider a more realistic 3D potential for the disk by Miyamoto & Nagai (1975), which holds for the special case of a circular disk. The expression for this potential is given by
\[ \Phi_{\text{disk}}(R, z) = -\frac{G M_{\text{disk}}}{\sqrt{R^2 + (\alpha + \sqrt{z^2 + \beta^2})^2}}. \] (15)

Here, again, the disk parameters are obtained from the best-fit values along the galactic midplane (\( z = 0 \)), which reproduces the \( v_{\text{circ}}(R) \) profile for the Sofue et al. (2009) razor-thin exponential disk (Freeman 1970), i.e., \( b = 0.0 \text{ kpc} \).\(^2\) Hence, the best-fit value for the disk parameter is \( a = 2.5 \text{ kpc} \), whereas the mass \( M_{\text{disk}} = 6.5 \times 10^{10} M_\odot \) is the same as in Sofue et al. (2009). Since we assume the disk potential to be three-dimensional here, for the purpose of computing the total \( v_{\text{circ}} \), we consider the component of the force along the radial direction (\( r \)) only. The bulge and the halo models are the same as earlier. The best-fit values for the NFW halo parameters with the updated disk model are found to be \( M_{\text{vir}} = 1.2^{+0.5}_{-0.4} \times 10^{12} M_\odot \), with \( R_{\text{vir}} = 274^{+35}_{-30} \) kpc. Instead, if we consider the total magnitude of the force in order to compute \( v_{\text{circ}} \), then we estimate \( M_{\text{vir}}(R_{\text{vir}} = 269^{+34}_{-33} \text{ kpc}) = 1.1^{+0.5}_{-0.4} \times 10^{12} M_\odot \). Historically, large values of \( a \) have been assumed while modeling the disk (Wolfire et al. 1995; Bland-Hawthorn 2009). We find that this leads to a much more massive dark matter halo—for \( a = 4.5 \text{ kpc} \) we find \( M_{\text{vir}}(R_{\text{vir}} = 299^{+36}_{-33} \text{ kpc}) = 1.6^{+0.6}_{-0.5} \times 10^{12} M_\odot \) and for \( a = 6.5 \text{ kpc} \) we find \( M_{\text{vir}}(R_{\text{vir}} = 321^{+35}_{-34} \text{ kpc}) = 1.9^{+0.7}_{-0.6} \times 10^{12} M_\odot \).

There are few things that have insignificant or unexplored effects on our mass estimation, e.g., we do not take into account the mass of the supermassive black hole, which is \( \sim 4 \times 10^8 M_\odot \) (Schödel et al. 2002; Eisenhauer et al. 2005; Gillessen et al. 2009), and is approximately \( 1/1000 \) of the mass of the bulge. Moreover, its effect is like that of a point mass and can be easily absorbed into the bulge mass. Another effect that is not considered is the tidal effect on the primary object that has been qualitatively studied to find the mass ratio between the Galaxy and M31 (Baiessi Pillastrini 2009) and depends strongly on different impact parameters (Eneev et al. 1973). We find that the \( R_{\text{vir}} \) of the Galaxy is \( \sim 250 \) kpc and for M31 it is \( \sim 260 \) kpc (Seigar et al. 2008; Majewski et al. 2007). Given that the distance between these two galaxies is \( \sim 780 \) kpc (Ribas et al. 2005; McConnachie et al. 2005; Karachentsev & Kashibadze 2006), which is more than double the sum of their virial radii, we believe that the tidal effect of M31 on the overall mass estimate of the Galaxy, if any, should be negligible. The tidal effects of the LMC and the SMC on the Galaxy have not been explored in this paper.

Note that our mass modeling of the Milky Way does not make any assumption about the value of \( \beta \), instead we use the value of \( \beta \) directly computed from the data. The only assumption that we make is that the density of the dark matter halo follows an NFW profile. As long as that assumption holds, our estimates for \( v_{\text{circ}} \) and the mass of the Milky Way should also be valid in the outer parts \( r > 25 \) kpc, where we cannot directly measure \( \beta \). If one wants to directly measure \( v_{\text{circ}} \) in the outer parts using only line-of-sight velocities, then one has to make an assumption about the underlying \( \beta \). Several attempts have been made in this regard, with each of them making different assumptions about \( \beta \), and hence introducing a bias into the estimated mass. Below, we compare these with our prediction for the mass of the Galaxy.

The dashed line in Figure 11 is our best-fit model of \( v_{\text{circ}} \). Since literature sources mostly report mass within a certain radius, to facilitate comparison we convert it to \( v_{\text{circ}} \) using the relation
\[ M(<r) = v_{\text{circ}}^2 r/G. \]

In Figure 11, the plotted \( v_{\text{circ}} \) from different sources span a wide range in radii and were computed using different types of tracer populations. By fitting a model to the kinematics of the satellite galaxies and globular clusters, Wilkinson & Evans (1999) measure the mass to be \( M(50 \text{ kpc}) \sim 5.4^{+0.2}_{-0.3} \times 10^{11} M_\odot \). This agrees with estimates by Kochanek (1996) (\( M(50 \text{ kpc}) = (4.9 \pm 1.1) \times 10^{11} M_\odot \) and Sakamoto et al. (2003) (\( M(50 \text{ kpc}) = 1.8\sim 2.5 \times 10^{11} M_\odot \)). Watkins et al. (2010) apply the tracer mass estimator formalism to 26 satellite galaxies and find that \( M(300 \text{ kpc}) = (0.9 \pm 0.3) \times 10^{12} M_\odot \). Their mass estimate is, however, prone to the systematics introduced by using an assumed \( \beta \), as duly mentioned by them. One can see in the figure that at \( r = 100 \text{ kpc} \), depending upon the chosen anisotropy, their mass could vary anywhere between \( 0.3 \times 10^{12} \) and \( 1.4 \times 10^{12} M_\odot \). Studying the hypervelocity

\(^2\) The disk model with \( b = 0 \) is also otherwise known as Toomre’s model or the Kuzmin disk (Toomre 1963; Binney & Tremaine 2008).
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Figure 11. Dashed black line is our best-fit model of \( v_{\text{circ}} \) given by the red line in Figure 10. Black dots are the values from the literature (Wilkinson & Evans 1999; Xue et al. 2008; Sofue et al. 2010; Watkins et al. 2010; Gnedin et al. 2010; Samurović & Lalović 2011; Deason et al. 2012b) labeled as W99, X08, S09, W10, G10, S11, and D12b, respectively. To make the plot less obscure, we do not include similar findings from the literature. For details about similar results, refer to the text.

stars within 80 kpc, and assuming \( \beta = 0.4 \), Gnedin et al. (2010) estimate \( M = 6.9^{+1.7}_{-1.0} \times 10^{11} M_\odot \), which is slightly higher than our estimate. Using BHB stars and the Watkins et al. (2010) tracer formalism, Samurović & Lalović (2011) estimate the mass at \( r = 85 \text{kpc} \) to be \( 8.83 \pm 0.73 \times 10^{11} M_\odot \). This is slightly higher than our estimate, probably because they assume \( \beta = 0 \). With the mixed sample of tracers (BHB and CN stars) populating the outermost halo (\( r \sim 50–150 \text{kpc} \)), Deason et al. (2012b) estimated the mass of the Galaxy to be \( M(150 \text{kpc}) = (5–10) \times 10^{11} M_\odot \). The variation is mainly due to the uncertainty related to the adopted potential and density slopes and anisotropy. Their range of mass at the outermost halo falls within our estimation.

6. KINEMATICS OF THE SIMULATED STELLAR HALO

We now study the kinematic properties of simulated stellar halos in which halos are formed purely by accretion of satellite galaxies. For this, we use the simulations of Bullock & Johnston (2005). In order to construct a synthetic sample of BHB stars from these simulations, we use the code GALAXIA (Sharma et al. 2011a). Figure 12 shows the velocity dispersion and anisotropy profiles of 11 different \( \Lambda \)CDM halos as a function of galactocentric radius \( r \). The mean of all the halos is also shown alongside the thick red line. In general, the velocity dispersions fall off with radius. At small \( r \), the fall is rapid, but at large \( r \) it is much slower. Asymptotically, the ratio \( \sigma_r / V_{\text{vir}} \) approaches a value of around 0.8. Note that \( \beta \) rises rapidly from a value of zero in the center to about 0.8 at \( r \sim 10 \text{kpc} \) and, thereafter, shows very little change. These results are in good agreement with results of Abadi et al. (2006) and Sales et al. (2007), who study stellar halos formed in cosmological hydrodynamical simulations including star formation and feedback.

First, note that \( \Lambda \)CDM halos are rarely tangential for any given radius. For most of the range of \( r \), \( \beta \) is in general greater than 0.5. We fit an analytic function of the Cuddeford (1991) form given by

\[
\beta(r) = \beta_0 \frac{r^2}{r^2 + r_0^2}
\]

Figure 13. Mean \( \beta \) profile of BHB stars in 11 simulated \( \Lambda \)CDM stellar halos of Bullock & Johnston (2005). Shown alongside as the dashed line is the best-fit analytic function of the form given by Cuddeford (1991).

to the mean \( \beta \) profile of the 11 \( \Lambda \)CDM halos. The best-fit values of the free parameters were found to be \( \beta_0 = 0.765 \) and \( r_0 = 0.00843 R_{\text{vir}} \). Figure 13 shows that the fit is quite good for a wide range of \( r \). The slight mismatch at \( r < 1 \text{kpc} \) could be due to issues related to force resolution. In the outer parts, most
of the mass is in bound structures and hence is not smoothly distributed. This is probably responsible for the non-smooth behavior in $\beta$ in the outer parts.

Figure 14 presents the beta profiles for halos with a non-$\Lambda$CDM accretion history, i.e., halos having accretion history significantly different from that predicted by the $\Lambda$CDM model of galaxy formation. Six halos that we consider have accretion events that are dominated by (1) radial orbits, (2) circular orbits, (3) old events, (4) recent events, (5) higher luminosity, and (6) low luminosity, and these are from simulations of Johnston et al. (2008). Signatures of different accretion events can be seen in the $\beta$ profiles. The most significant difference is between the radial and the circular halo, which is expected since the orbital properties of the satellites were different to begin with. Note that the circular halo is the only one among all the simulated halos that can have $\beta < 0$. The old halo almost perfectly follows the mean profile that we had derived for the 11 $\Lambda$CDM halos and has the smoothest profile. This is due to the fact that the stars in this halo are completely phase mixed and have no structures of any kind. The young halo has very few stars in the inner regions and shows non-smooth behavior due to the presence of a significant number of structures. The high-luminosity halo is also very similar to the old halo. However, the low-luminosity halo has higher $\beta$ for $r < 0.01 R_{\text{vir}}$. This is most likely due to circularization of orbits when acted upon by dynamical friction. Orbit circularization has also been reported by Sales et al. (2007) in their simulations. Note that the effect of dynamical friction is strongest for high-luminosity events and weakest for low-luminosity events. Moreover, when acted upon by dynamical friction, satellites lose energy and move toward the inner regions of the halo. This partly explains why the high-luminosity halo has low $\beta$ in the inner regions as compared to the low-luminosity halo.

In Section 3.2, we measured $\beta$ until $r = 23$ kpc. Beyond this, we can measure $\sigma_r$ out to $r = 56$ kpc, but not $\sigma_\phi$ and $\sigma_\theta$. By using the circular velocity curve that we derived in Section 5, we can predict $\beta$ beyond $r = 56$ kpc by making use of the Jeans equation (Equation (7)). To proceed, we need to make an assumption about the density slope ($\alpha$); beyond $r > 27$ kpc, it has been shown that $\alpha$ is around 4.5. Assuming this value, the predicted $\beta$ is plotted in Figure 15. One can see that there is a slight jump in the value of $\beta$ passing from $r = 23$ to 27 kpc, and beyond this the value of $\beta$ is around 0.4. The sudden jump in $\beta$ occurs via the Jeans equation (Equation (7)), due to the discontinuity in $\alpha (= -d \ln \rho / d \ln r)$. Note that an assumption of the steeper density slope would increase $\beta$ and vice versa. The red line in the figure shows the anisotropy profile fitted to the simulated $\Lambda$CDM halos given by Equation (16) from Section 6. It can be seen that accretion-based models cannot explain the dip in the observations, especially the profile in the region $12 < r / \text{kpc} < 23$. However, outside this region, the simulations are roughly in agreement with observations. For $r < 12$ kpc, the observations match the value of around $\beta = 0.5$ seen in simulations. For $r > 23$ kpc, the overall value of $\beta$ in observations is slightly low but the profile is flat as in the simulations. The outer halo at $r = 56$ kpc is radial with $\beta = 0.55$. 

Figure 15. Black solid line is the observed $\beta$ profile in the outer halo (assumed $\alpha = 4.5$) whereas the red solid line is the $\beta$ profile of the simulated halo. The blue dots with error bars are the observed values of $\beta$ from Section 3. The sudden jump in the $\beta$ profile passing from $r = 23$ to 27 kpc could be a spurious effect due to our assumption of the broken power law with a break at $r = 27$ kpc. (A color version of this figure is available in the online journal.)
7. CONCLUSION AND DISCUSSION

We study the kinematics of \(~4500\) BHB stars to obtain the velocity dispersion profiles along three orthogonal axes in spherical polar coordinates using the GVE DF. GVE as an estimator of the velocity dispersion has the advantage that no assumptions about potential or density are needed a priori. From the estimated velocity dispersion profiles using maximum likelihood analysis, we also derive the anisotropy profile of the Galactic halo and compare it to the simulated ΛCDM halos. Finally, using the radial velocity dispersion profile, anisotropy profile, and density power law, we constrain the mass of the Milky Way using the Jeans equation.

We measure the \(\sigma_r\) profile of the halo out to \(r \approx 60\) kpc. At large distance (\(d \gg R_0\)), \(\sigma_r\) can be approximated by \(\sigma_{los}\). Thus, in the outskirts, the \(\sigma_{los}\) profile given by Xue et al. (2008) converges to our \(\sigma_r\) profile. At \(r \approx 60\) kpc, \(\sigma_r\) attains \(\lesssim 100\) km s\(^{-1}\). However, in the inner halo, the approximation (\(\sigma_r \approx \sigma_{los}\)) is invalid and we find that the deviation of \(\sigma_{los}\) from \(\sigma_r\) is as high as \(\approx 40\) km s\(^{-1}\). We obtain a \(\sigma_r(r)\) profile with a plateau in the inner halo, a sudden fall at \(r \approx 15\) kpc, and a gradual decline outward. Qualitatively, a similar profile is also found by Sommer-Larsen et al. (1997). However, our \(\sigma_r\) profile sharply falls at \(r \approx 15\) kpc, whereas they find a gentle transition. This is probably due to the fact that they make an assumption that \(v_{circ}(r)\) is constant, which we find is not completely true.

Next, we estimate the tangential velocity dispersions, \(\sigma_\theta\) and \(\sigma_\phi\). Using these estimates, we are able to measure \(\beta(r)\) until \(r = 25\) kpc. Astonishingly, we discover a dip in the \(\beta\) profile at \(r = 17\) kpc, where \(\beta \approx -1.2\). We find that the inner halo (\(r < 12\) kpc) is radial with \(\beta \approx 0.5\). This result of the radially biased inner halo concurs with the recent results by Smith et al. (2009a) and Bond et al. (2010) using the proper motions. Beyond the switchover point in the range \(18 < r < 25\) kpc, the anisotropy rises slightly and becomes isotropic to mild radial.

We also verify the result using an alternative DF, namely, the D11 DF. A small systematic in the \(\beta(r)\) profiles is seen from these two models, which is mainly due to the assumption about the density slope (\(\alpha\)) that needs to be made a priori for D11 DF. We check for the contribution of the halo substructures, namely, the Virgo overdensity and Sagittarius stellar stream, and find that they have little effect on the anisotropy profile. The effects of \(v_{LSR}\) and \(R_0\) upon our velocity dispersions and anisotropy estimates are also found to be negligible.

D11 study the BHB stars in the radial bin (\(10 < r/kpc < 25\)) and find the halo to be tangential. After reanalyzing the D11 sample in this bin, we find that this is mainly because of the choice of their bin size that encompasses the transition region (\(13 < r/kpc < 17\)) where we detected a dip in the \(\beta(r)\) profile. Possibly, it could also be because of the degeneracy between the potential and anisotropy in their model. However, in their recent work, D12 break the degeneracy among their model parameters and measure \(\beta \approx 0.4^{+0.2}_{-0.1}\) with \(\alpha = 4.6\) in the region \(16 < r/kpc \approx 48\). Within the range of uncertainty, our value for \(\beta\) is\(\approx 0.4\) using the GVE model agrees with them. We find that the D12 value of \(\beta \approx 0.4\) in this bin, although derived from a sample which is dominated by stars within \(r < 27\) kpc, is not appropriate for the range \(18 < r/kpc < 23\), instead it is appropriate for the range \(23 < r/kpc < 48\). Finally, we check how well we can estimate the mass and anisotropy together using D11 DF in the outermost region (\(35 < r/kpc < 84\)). We find that due to the lack of tangential information, a degeneracy between mass and anisotropy cannot be broken.

Substituting the estimates of \(\sigma_r(r)\) and \(\beta(r)\) in the Jeans equation, we then calculate the circular velocity profile of the Galaxy (\(v_{circ}(r)\)). We detect the dip in the \(v_{circ}\) profile at 10–12 kpc, also seen by Sofue et al. (2009) at 9 kpc, which is attributed to the massless ring as a perturbation to the disk. Finally, we fit the three-component (exponential disk, Hernquist bulge, and NFW halo) galaxy model to the observed \(v_{circ}\) profile in order to obtain the mass distribution of the Galaxy. From our best-fit model, we calculate \(M_{\text{tot}}\) of the halo to be \(0.9^{+0.4}_{-0.3} \times 10^{12} M_\odot\) with \(R_{\text{fl}} = 249^{+34}_{-31} \text{kpc}\) and the concentration parameter, \(c = 12.0^{+0.6}_{-0.4}\). The mass of the Galaxy, within the extent that we are able to constrain all the three components of velocity dispersions, is estimated to be \(M(r \lesssim 25\text{kpc}) = 2.1 \times 10^{11} M_\odot\). Our estimate for \(M_{\text{vir}}\) is in good agreement with most of the recent estimates, namely by Watkins et al. (2010), Kochanek (1996), Wilkinson & Evans (1999), Sakamoto et al. (2003), Sofue et al. (2009), and Deason et al. (2012b), as demonstrated in Figure 11. In their studies of the same population of stars (BHB), Xue et al. (2008) also fit a three-component galaxy model and calculate \(M_{\text{vir}}\) to be \(0.91^{+0.27}_{-0.18} \times 10^{12} M_\odot\). Our result for \(M_{\text{vir}}\) is in very good agreement with their estimate but our uncertainty is slightly larger. Note that they make an assumption about \(v_{circ}/v_{los}\) with radius from the simulations, whereas we do not make any such assumption. Additionally, we also consider a more realistic 3D disk model that is found to predict a slightly higher Galactic mass \(M_{\text{vir}} = 1.2^{+0.5}_{-0.4} \times 10^{12} M_\odot\) with \(R_{\text{fl}} = 274^{+35}_{-30}\) kpc for flattening constant \(a = 2.5\).

In the end, we used the measured quantities \(\sigma_r\) and \(v_{circ}\) to extend the \(\beta\) profile beyond \(r \approx 25\) kpc up to the distance where \(\sigma_r(r)\) can be confidently measured (\(r \approx 60\) kpc). The only assumption that we make here is about the density profile which we set at \(\alpha = 4.5\) in concurrence with the recent results by Watkins et al. (2009) and Deason et al. (2011b). We find that the outer halo is radial and attains \(\beta = 0.55\) at \(r \approx 60\) kpc.

We also compare our result with the simulated stellar halo, which is formed purely by accretion (Bullock & Johnston 2005). This simulated halo is found to be in rough agreement with the observed halo in the inner region \(r < 12\text{kpc}\). One can see that none of the simulations the \(\beta\) profiles obtained could predict a tangential halo at any distance, and thus fails to explain a dip seen at \(r = 17\text{kpc}\) in the observed \(\beta\) profile. In contrast, in the outer region (\(r > 25\text{kpc}\)), simulations and observations both agree in an overall sense with the anisotropy and predict a flat anisotropy profile.

In all of the observed quantities \(\sigma_r, \sigma_\theta, \sigma_\phi,\) and \(\beta\), we see a dramatic shift in properties at \(r \approx 17\) kpc. We noted that these undulations in the profiles are translated into our \(v_{circ}\) estimation, resulting in a varying \(v_{circ}\) profile. It could be true the other way around, in the sense that the non-monotonic trends seen in all of our kinematic profiles could be due to the presence of so far unaccounted features in the Milky Way potential.

Alternatively, the shift in the properties seen in the observed profiles could possibly be an indication of a complex multi-component halo. The studies of the calibration stars by Carollo et al. (2007, 2010) from the SDSS survey and the follow-up studies by Beers et al. (2012) have shown that the halo has at least two distinct components. They determined the inner halos to be formed in situ, whereas the outer halos are considered to be formed by accretion. Carollo et al. (2010) and de Jong et al. (2010) have found that the population fraction inversion point between the inner and outer halos lies between...
~15 and 20 kpc. Kinman et al. (2012) studied the BHB and RR Lyrae stars toward the Galactic anti-center and North Galactic Pole and found that the retrograde component of the halo dominates for $r > 12.5$ kpc. It seems that this transition between the inner to the outer halos is recorded in the $\beta$ of the halo as well. Additionally, duality in the formation history of the halo has also been seen in the recent smooth particle hydrodynamics and $N$-body simulations by Zolotov et al. (2009), McCarthy et al. (2012), and Font et al. (2011). Kalirai (2012) recently attributed the age difference of two billion years in the halo components to in situ accretion. On the contrary, Schönrich et al. (2011) reanalyzed the calibration stars from Carollo et al. (2010) and found no reliable evidence for the existence of an outer retrograde halo.

In a nutshell, the stellar halo is a test bed to understand the formation history of the galaxy. Even in this era, where we have access to a huge volume of spectroscopic and photometric data, the crucial physical quantities like velocity dispersions and anisotropy are not completely understood due to the lack of proper motions. With the advent of data in the coming decades from the magnificent next generation of spectroscopic surveys like LEGUE (Deng et al. 2012) and, especially, unprecedented proper motions from an astrometric mission like Gaia (Perryman 2002), we will be able to put strong constrains on these fundamental quantities. Additionally, in order to see the bigger picture, we will need to confirm the results with different stellar types or an alternative tracer. Moreover, exploring the southern sky is equally important in order to complete the picture, for which upcoming spectroscopic surveys like GALAH$^3$ will also play an important role.

$^3$ http://www.aao.gov.au/HERMES/GALAH/Home.html
APPENDIX A

BINNING AND EFFECT OF THE BIN SIZE

Here, we investigate an effect of $n_{\text{bin}}$ on our analysis. Figure 16 shows the velocity dispersion profiles and the anisotropy profile for the same sample of stars but with different particles in each bin. We see that with the decrease in the number of stars in each bin, the uncertainties in the result increase. However, the overall trend remains unaltered.

APPENDIX B

EFFECT OF $v_{\text{LSR}}$ AND $R_\odot$

In the literature, there are varied claims about the value of $v_{\text{LSR}}$ ranging from 184 to 270 km s$^{-1}$ (Olling & Merrifield 1998; Méndez et al. 1999; Reid et al. 2009; Bovy et al. 2009; Koposov et al. 2010). Similarly, the value of $R_\odot$ is also debatable within 8–8.5 kpc (Reid 1993; Ghez et al. 2008; Gillessen et al. 2009). McMillan & Binney (2010) found that the ratio of $v_{\text{LSR}}/R_\odot$ can be better constrained than each of them alone and should range between 29.9 and 31.6 km s$^{-1}$ kpc$^{-1}$. Distressingly, there is still no consensus upon the values of $(v_{\text{LSR}}, R_\odot)$. To study the effect of chosen values of $(v_{\text{LSR}}, R_\odot)$ upon our estimates of dispersion profiles, we repeat the same analysis performed to obtain the black diamond points in Section 3 (Figure 3). In Figure 17, we show the results for different values of $(v_{\text{LSR}}, R_\odot)$. Here, again, the black diamond markers are obtained for a case $(v_{\text{LSR}}, R_\odot) = (220.0 \text{ km s}^{-1}, 8.5 \text{ kpc})$, and the figure is thus a replica of the diamond points from Figure 3, repeated for ease of comparison. The red and black markers in the figure show the effect of chosen $R_\odot$ values upon our estimates, whereas the red, blue, and cyan markers demonstrate the effect of chosen $v_{\text{LSR}}$. Within the range of $(v_{\text{LSR}}, R_\odot)$ investigated, the figure clearly demonstrates a negligible effect upon our estimates, given the range of uncertainties.

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