Propagation of optical waves in planar periodically inhomogeneous chiral structures

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Abstract. The analytic solution of the problem of propagation of eigen modes in planar periodically inhomogeneous optical waveguides based on alternating dielectric and chiral (or with optical activity) layers is examined. The method is based on the solution of the Hill equations for the electric field strengths of modes with circular polarizations. As a result, a dispersion equation for the eigen modes of a planar periodically inhomogeneous chiral-dielectric waveguide was obtained and its numerical solution was performed. As a result of the analysis of numerical results, it is shown that the structure under study can operate in different modes: propagation of modes with right and left circular polarizations, propagation of a mode with only one type of circular polarization and the optical key mode.

1. Introduction
At present, artificial composite structures (metamaterials) are actively studied, exhibiting properties in different frequency ranges that are not typical for natural media. The properties of metamaterials are determined primarily not by the properties of elements, but by an artificially created periodic structure. Metamaterials with unique properties for natural media appeared as a result of significant development of such fields of science as physics, materials science, engineering and chemistry. There is a sufficient number of scientific publications and articles [1, 2, 3, 4], in which the unique electromagnetic properties of metamaterials are discussed in detail.

It should be noted that at the beginning of this century the theory of metamaterials of the microwave range was developed. At present the technology of creating metamaterials has stepped to terahertz and optical frequencies.

An interesting electromagnetic property is possessed by a chiral metamaterial, which is a collection of conducting microelements of mirror-like asymmetric shape, uniformly placed and randomly oriented in a homogeneous magneto-dielectric medium [5, 6, 7, 8, 9, 10, 11]. The main properties of the chiral media are the propagation of waves right- (RCP) and left-circular (LCP) polarizations and cross polarization of incident wave.

Another direction of the electrodynamics of chiral media is the investigation of the eigen waves (modes) of waveguides with chirality ("chirowaveguides"). The first work on chiral waveguides was published in 1988 [12]. In this paper the modes of a plane chiral waveguide bounded by ideally conducting planes were investigated. The propagation of eigen waves in open and closed circular uniformly filled chiral waveguides has been studied in detail in scientific literature [13, 14, 15]. The electromagnetic theory of propagation of eigen waves in optical fibers with a chiral core is developed in [16]. Analysis of mode propagation in planar homogeneous chiral
waveguides is performed in [17]. A model of a planar periodically inhomogeneous chiral structure (planar optical waveguide) is proposed that allows obtaining the properties of frequency and polarization selectivity for modes with right- and left-circular (or elliptical) polarizations.

Thus, the study of optical structures based on mirror-asymmetric compositions possessing chirality (optical activity) [5, 6, 7] is of considerable interest. The main properties of such media are frequency and polarization selectivity. Most metamaterials are macroscopically homogeneous, which imposes certain limitations on the expansion of their use. The electromagnetic properties of reflection of a plane electromagnetic wave from an inhomogeneous chiral layer are considered in [18].

In this paper, we investigate periodically inhomogeneous chiral metamaterials in order to obtain a wide variety of frequency and polarization selective electromagnetic properties.

Chiral (or optically active) media are generally described assuming $e^{i\omega t}$ as time dependence by the following material equations [5, 6]:

$$D = \varepsilon E - i\chi H; \quad B = \mu H + i\chi E,$$

where $\varepsilon, \mu$ are the relative permittivity and permeability of the chiral layer; $\chi$ is the relative parameter of chirality (optical activity) of the medium; $E, H$ are vectors of electric and magnetic fields; $B, D$ are the vectors of induction of electric and magnetic fields; $i = \sqrt{-1}$ is the imaginary unit. The material equations (1) are written in the Gaussian system of units. Chirality parameter $\chi < 0$ for the left forms of chiral composites and $\chi > 0$ for the right forms.

2. Statement of the problem and method of solution

The problem of determining the dispersion equation of the structure under study is solved in three stages. At the first stage, the electromagnetic fields in regions I and II are determined from the solution of the system of Maxwell’s equations (figure 1). In the second stage, using the Floquet theorem [19], a field is found in region III, taking into account the periodic heterogeneity of the structure. In the third stage, the solutions obtained in regions I, II and III crosslink at the boundaries between these layers and from the obtained system of linear algebraic equations a dispersion equation for the eigenwaves of the inhomogeneous structure with a period of stepwise variation of the chirality parameter $d = l_1 + l_2$.

It is known that in the presence of chiral layers, the modes with right- and left-circular polarizations will be eigen modes. Thus, it is necessary to determine the dispersion characteristics of waves with RCP and LCP in a periodically inhomogeneous structure with a period of stepwise variation of the chirality parameter $d = l_1 + l_2$.

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The Maxwell equations for region I, taking into account the material relations (1), are written as follows:

$$\text{rot} E^{(1)} = k_0 \left( -i\mu_1 H^{(1)} + \chi_1 E^{(1)} \right); \quad \text{rot} H^{(1)} = k_0 \left( i\varepsilon_1 E^{(1)} + \chi_1 H^{(1)} \right),$$

where $k_0$ is the wave number for a plane homogeneous electromagnetic wave in a vacuum.

Applying operation rot to both sides of both equations (2), we obtain vector differential equations of the second order with respect to the vectors $E^{(1)}$ and $H^{(1)}$:

$$\nabla^2 E^{(1)} + k_0^2 (\varepsilon_1 \mu_1 + \chi_1^2) E^{(1)} - 2ik_0^2 \mu_1 \chi_1 H^{(1)} = 0;$$

$$\nabla^2 H^{(1)} + k_0^2 (\varepsilon_1 \mu_1 + \chi_1^2) H^{(1)} + 2ik_0^2 \varepsilon_1 \chi_1 E^{(1)} = 0,$$

(3)
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Figure 1. Dependence of the chirality parameter on the longitudinal coordinate.

where $\nabla^2$ is the Laplace operator in a Cartesian coordinate system.

We use the well-known representation of the electromagnetic field in a chiral medium in the form of a superposition of RCP and LCP waves (Beltrami fields) [5, 6]:

$$E^{(1)} = E_R + E_L; \quad H^{(1)} = i(\varepsilon_1/\mu_1)^{1/2} (E_R - E_L),$$

where $E_R$ is the electric field intensity of the RCP wave; $E_L$ is the electric field intensity of the LCP wave.

When substituting relations (4) in bounded differential equations (3) on the newly introduced functions $E_R$ and $E_L$ we obtain homogeneous unbounded differential equations of the second order:

$$\nabla^2 E_R + k_R^2 E_R = 0; \quad \nabla^2 E_L + k_L^2 E_L = 0,$$

where $k_R = k_0 (n_1 + \chi_1)$ is the wave number for a RCP wave in an unbounded chiral medium; $k_L = k_0 (n_1 - \chi_1)$ is the wave number for the LCP wave in an unbounded chiral medium; $n_1 = (\varepsilon_1 \mu_1)^{1/2}$ is the refractive index of the chiral layer.

By virtue of condition $\chi_1 \ll n_1$ and taking into account $\nabla^2 = d^2/dz^2$, equations (5) can be represented in the following form:

$$\frac{d^2E_{R,L}}{dz^2} + k_1^2 (1 \pm \alpha \chi_1) E_{R,L} = 0,$$

where $\alpha = 2/n_1 \ll 1$ is a small parameter; $k_1 = k_0 n_1$ is the wave number of the region I.

Equations (6) are known in the theory of ordinary differential equations and in the case of a periodic dependence $\chi_1$ from coordinate $z$, are called the Hill equations.

The general solutions of the Hill equations (6) in regions I and II have the following form:

$$E_{R,L}^{(1)}(z) = C_{R,L}^{(1)} e^{ik_{R,L}^{(1)} z} + D_{R,L}^{(1)} e^{-ik_{R,L}^{(1)} z} (-l_1 < z < 0);$$

$$E_{R,L}^{(2)}(z) = C_{R,L}^{(2)} e^{ik_{R,L}^{(2)} z} + D_{R,L}^{(2)} e^{-ik_{R,L}^{(2)} z} (0 < z < l_2),$$

where $C_{R,L}^{(1,2)}$ and $D_{R,L}^{(1,2)}$ are the unknown constants, determined hereafter from the boundary conditions; $k_{1}^{(R,L)} = k_0(\varepsilon_1 \mu_1)^{1/2} (1 \pm \alpha \chi_1); \quad k_{2}^{(R,L)} = k_0(\varepsilon_2 \mu_2)^{1/2}.$
To determine the electromagnetic field on a segment \([l_2 < z < l_1 + l_2]\) we use the Floquet theorem \([19]\), according to which for one of the two waves the solution of the equation \((\text{ref} \ 6)\) can be written as following:

\[
E_{R,L}^{(3)}(z) = F_{R,L}(z) E_{R,L}^{(1)}(z - d),
\]

where \(F_{R,L}(z)\) are the functions that are periodic with respect to the coordinate \(z\) with period \(d = l_1 + l_2\).

For functions \(F_{R,L}(z)\) it is convenient to choose harmonic functions of the following:

\[
F_{R,L}(z) = e^{j\gamma_{R,L}d},
\]

where \(\gamma_{R,L}\) are the propagation constants of the RCP and LCP modes in a periodically inhomogeneous structure, which are subsequently found as functions of wave numbers \(\tilde{k}_{1,2}^{(R,L)}\) and layer thicknesses \(l_{1,2}\).

Using expressions (8) and (9), determine the intensity of the electric field of the RCP and LCP waves in region III:

\[
E_{R,L}^{(3)}(z) = C_{R,L}^{(1)} e^{j\gamma_{R,L}d} e^{i\tilde{k}_{1}^{(R,L)}(z-d)} + D_{R,L}^{(1)} e^{j\gamma_{R,L}d} e^{-i\tilde{k}_{1}^{(R,L)}(z-d)}.
\]

To determine the unknown constants \(C_{R,L}^{(1,2)}\) and \(D_{R,L}^{(1,2)}\) we use the boundary conditions for \(z = 0\) and \(z = l_2\):

\[
E_{R,L}^{(1)}(z = 0) = E_{R,L}^{(2)}(z = 0); \quad E_{R,L}^{(2)}(z = l_2) = E_{R,L}^{(3)}(z = l_2);
\]

\[
\left. \frac{dE_{R,L}^{(1)}}{dz} \right|_{z=0} = \left. \frac{dE_{R,L}^{(2)}}{dz} \right|_{z=0}, \quad \left. \frac{dE_{R,L}^{(2)}}{dz} \right|_{z=l_2} = \left. \frac{dE_{R,L}^{(3)}}{dz} \right|_{z=l_2}.
\]

When substituting expressions (7) and (10) in the boundary conditions (11)-(12) we obtain a system of linear algebraic equations with respect to unknown coefficients \(C_{R,L}^{(1,2)}\) and \(D_{R,L}^{(1,2)}\):

\[
\cos(\gamma_{R,L}d) = \cos(\tilde{k}_1^{(R,L)} l_1) \cos(\tilde{k}_2^{(R,L)} l_2) - \sigma_{12} \sin(\tilde{k}_1^{(R,L)} l_1),
\]

where

\[
\sigma_{12} = \frac{(\tilde{k}_1^{(R,L)})^2 + (\tilde{k}_2^{(R,L)})^2}{2\tilde{k}_1^{(R,L)} \tilde{k}_2^{(R,L)}} \sin(\tilde{k}_1^{(R,L)} l_1).
\]

The dispersion equations (13)-(14) describe eigen modes propagating in a planar periodically heterogeneous structure of alternating chiral and dielectric layers.

3. Analysis of numerical results
Figure 2 shows the dependencies of the normalized propagation constants \(\gamma_{R}d\) of RCP modes in a periodically inhomogeneous chiral-dielectric metastructure on the normalized frequency \(k_0 l_1\).

Figure 3 shows the dependencies of the normalized propagation constants \(\gamma_{L}d\) of LCP modes in a periodically inhomogeneous chiral-dielectric metastructure on the normalized frequency \(k_0 l_1\).

The following values of the structure parameters were chosen for calculation: \(\varepsilon_{1,2} = 3.5; \mu_{1,2} = 1; \chi_1 = 0.3; k_0 l_2 = 2\).
As can be seen from figure 2, the spectrum of eigen modes with right-circular polarization in a periodically inhomogeneous chiral-dielectric metastructure consists of a set of Hartree spatial harmonics (solid lines). Harmonics, in which the propagation constant increases with increasing frequency, are direct. Harmonics, in which the propagation constants decrease with increasing frequency, are called inverse. Zero harmonic $p = 0$ is a direct. Direct harmonics have positive indices $p > 0$, the inverse ones are negative $c p < 0$. Any harmonic can not propagate throughout the entire range of frequencies studied and the distribution area is divided into transparency and opacity windows. As one would expect, since all harmonics in the superposition form a single RCP wave, then all the frequency intervals of propagation and non-propagation coincide. As can be seen from figure 3, in the areas of change in the normalized frequency from 0.8 to 1.3 and from 2.4 to 2.9, the RCP wave in the metamaterial is not propagated. At other values of the normalized frequency, its propagation takes place.

![Figure 2. Dispersion characteristics of RCP eigen modes.](image)

It can be seen from figure 3 that the spectrum of eigenwaves with left-handed polarization in a periodically inhomogeneous chiral-dielectric metastructure also consists of a set of Hartree spatial harmonics (dashed lines). Similarly, as for the case of RCP waves, each harmonic can not propagate in the entire range of frequencies studied and the region of propagation is divided into transparency and opacity windows. The intervals of propagation and non-propagation of all Hartree harmonics coincide. As can be seen from figure 4, in the regions of variation in the normalized frequency from 0.8 to 1.5 and from 2.4 to 3.1, the LCP mode in the metamaterial under study does not propagate. At other values of the normalized frequency, propagation takes place. Note that for the LCP modes, the opacity windows are wider than for the RCP modes.

As can be seen from the comparison of the characteristics in the figures 2 and 3, the RCP and LCP eigen waves have various ”opacity bands”. The frequency bands in which the RCP and LCP modes do not propagate also exist. It can be seen from a comparison of the dispersion characteristics in the figures 2 and 3 that ”widths” of ”opacity bands” of RCP and LCP modes differ and depend on the types and forms of chiral elements.
4. Conclusion
In conclusion, let us dwell on the main conclusions of the work done:
- A one-dimensional chiral structure based on periodically alternating chiral and dielectric layers can serve as a frequency selective filter for modes with right- and left-circular polarizations. It was found that the RCP and LCP modes in this structure have non-coinciding frequency intervals of opacity.
- The appearance of the mirror type of the chiral composite and their concentration affect the width of the opacity band: an increase in the concentration of chiral elements of the left form leads to a sharp narrowing of the opacity band of the LCP mode and a sharp increase in the opacity band of the RCP mode. Also, an increase in the concentration of chiral elements of the right form leads to a sharp narrowing of the opacity band of the RCP mode and a sharp increase in the opacity band of the LCP mode.
- In a periodically heterogeneous structure of alternating chiral layers on the basis of the right and left forms of chiral composites, frequency range intervals were found in which none of the two modes with circular polarizations can be propagated. In this mode, the structure can be used as an optical key.

Thus, the paper shows that the structure under study can operate in different modes – propagation of eigen modes with right and left circular polarizations, propagation of a wave with only one type of circular polarization and the optical key mode.

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