RELATIVISTIC ELECTRONS AND MAGNETIC FIELDS OF THE M87 JET ON THE ~10 SCHWARZSCHILD RADIi SCALE

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ABSTRACT

We explore energy densities of the magnetic fields and relativistic electrons in the M87 jet. Since the radio core at the jet base is identical to the optically thick surface against synchrotron self-absorption (SSA), the observing frequency is identical to the SSA turnover frequency. As a first step, we assume the radio core has a simple uniform sphere geometry. Using the observed angular size of the radio core measured by the Very Long Baseline Array at 43 GHz, we estimate the energy densities of magnetic fields \(U_B\) and relativistic electrons \(U_e\) on the basis of the standard SSA formula. Imposing the condition that the Poynting power and kinetic power of relativistic electrons should be smaller than the total power of the jet, we find that (1) the allowed range of the magnetic field strength \(10^6 \leq B_{\text{tot}} \leq 15 \times 10^6\) G and that (2) \(1 \times 10^{-5} \leq U_e/U_B \leq 6 \times 10^2\) holds. The uncertainty of \(U_e/U_B\) comes from the strong dependence on the angular size of the radio core and the minimum Lorentz factor of non-thermal electrons \(\gamma_{e,\text{min}}\) in the core. It is still unsettled whether resultant energetics are consistent with either the magnetohydrodynamic jet or the kinetic power dominated jet even on the ~10 Schwarzschild radii scale.

Key words: galaxies: active – galaxies: jets – radiation mechanisms: non-thermal – radio continuum: galaxies

Online-only material: color figures

1. INTRODUCTION

The formation mechanism of relativistic jets in active galactic nuclei (AGNs) remains a longstanding, unresolved problem in astrophysics. Although the importance of energy densities of magnetic fields \(U_B\) and relativistic electrons \(U_e\) for resolving the formation mechanism has been emphasized (e.g., Blandford & Rees 1978), it is not observationally clear whether either \(U_B\) or \(U_e\) is dominant at the jet base. Relativistic magnetohydrodynamic models for relativistic jets generally assume highly magnetized plasma at the jet base (e.g., Koido et al. 2002; Vlahakis & Königl 2003; McKinney & Gammie 2004; Kroll et al. 2005; McKinney 2006; Komissarov et al. 2007, 2009; Tchechlovskoy et al. 2011; Toma & Takahara 2013; Nakamura & Asada 2013), while an alternative model assumes a pair plasma dominated “fireball”-like state at the jet base (e.g., Iwamoto & Takahara 2002; Asano & Takahara 2009 and reference therein). Although a deviation from equi-partition (i.e., \(U_e/U_B \approx 1\)) is essential for the investigation of relativistic jet formation, no one has succeeded in obtaining a robust estimation of \(U_e/U_B\) at the jet base.

M87, a nearby giant radio galaxy located at a distance of \(D_L = 16.7\) Mpc (Jordán et al. 2005), hosts one of the most massive supermassive black holes, \(M_\bullet = (3–6) \times 10^9 M_\odot\) (e.g., Macchetto et al. 1997; Gebhardt & Thomas 2009; Walsh et al. 2013). Because of the largeness of the angular size of its central black hole, M87 is well known for being the best source for imaging the deepest part of the jet base (e.g., Junor et al. 1999). Furthermore, M87 has been well studied at wavelengths from radio to very high energy (VHE) \(\gamma\)-ray (Abramowski et al. 2012; Hada et al. 2012 and references therein), and causality arguments based on the VHE \(\gamma\)-ray outburst in 2008 February indicate that the VHE emission region is less than \(\sim 5\sigma R_s\), where \(\delta\) is the relativistic Doppler factor (Acciari et al. 2009). The Very Long Baseline Array (VLBA) beam resolution at 43 GHz typically attains about 0.21 × 0.43 mas, which is equivalent to 5.3 × 10^{-16} × 1.1 × 10^{-17} cm. When \(M_\bullet = 6 \times 10^9 M_\odot\) holds (Gebhardt & Thomas 2009), then VLBA beam resolution approximately corresponds to 30 × 60 \(R_s\). Recent progresses of very long baseline interferometry (VLBI) observations have revealed the inner jet structure, i.e., frequency and core size relation, and distance and core size relation down to the ~10 Schwarzschild radii (\(R_s\)) scale (Hada et al. 2011, hereafter H11). Thus, the jet base of M87 is the best laboratory for investigating \(U_e/U_B\) in the real vicinity of the central engine.

Recently, two significant steps forward have been made regarding M87 observations, which motivated the present work. First, Hada et al. (2011) succeeded in directly measuring core shift phenomena at the jet base of M87 at 2, 5, 8, 15, 24, and 43 GHz. The radio core position at each frequency has been obtained by the astrometric observation (H11). Since the radio core surface corresponds to the optically thick surface at each frequency, the synchrotron self-absorption (SSA) turnover frequency \(v_{\text{ssa}}\) is identical to the observing frequency itself.6 Second, we recently measured core sizes in Hada et al. (2013a, hereafter H13). Hereafter we focus on the radio core at 43 GHz. In H13, we select VLBA data observed after 2009 with sufficiently good qualities (all 10 stations participated and good uv coverages). To measure the width of the core, a single FWHM Gaussian is fitted for the observed radio core at 43 GHz in the perpendicular direction to the jet axis, and we derive the width of the core \(\delta\) (FWHM). We stress that the core width is free from

6 Difficulties in applying the basic SSA model to real sources have already been recognized by several authors (Kellermann & Pauliny-Toth 1969; Burbidge et al. 1974; Jones et al. 1974a, 1974b; Blandford & Rees 1978; Marscher 1987) because of inaccurate determination of \(v_{\text{ssa}}\) and \(\delta\).
uncertainty of the viewing angle. Therefore, using $\theta_{\text{FWHM}}$ at 43 GHz, we can estimate values of $U_e/U_B$ in the 43 GHz core of M87 for the first time.

In Section 2, we derive an explicit form of $U_e/U_B$ by using the standard formulae of synchrotron absorption processes. As a first step, we simplify the geometry of the radio core by assuming a single uniform sphere although the real geometry is probably more complicated. In Section 3, we estimate $U_e/U_B$ in the M87 jet base by using the VLBA data at 43 GHz obtained in H13. In Section 4, we summarize the result and discuss relevant implications. In this work, we define the radio spectral index $\alpha$ as $S_\nu \propto \nu^{-\alpha}$ and assume $M_\star = 6 \times 10^9 M_\odot$.

2. MODEL

Here we derive explicit expressions of the strength of the total magnetic field $B_{\text{tot}}$ and $U_e/U_B$. Several papers have extensively discussed the determination of magnetic field strength $B_{\text{tot}}$. We follow the fundamental formulae of SSA processes from Ginzburg & Syrovatskii 1965, hereafter GS65; Blumenthal & Gould 1970, hereafter BG70; Pacholczyk 1970; and Rybicki & Lightman 1979, hereafter RL79. Here we will show a simple derivation of the explicit expressions of $B_{\text{tot}}$ and $U_e/U_B$ with sufficient accuracy.

2.1. Method

For clarity, we briefly summarize the method for determining $B_{\text{tot}}$ and $U_e/U_B$ in advance. The four theoretical unknowns related to the magnetic fields and relativistic electrons in the observed radio core with its angular diameter $\theta_{\text{obs}}$ are as follows: $B_{\text{tot}}$, $K_e$ (the normalization factor of non-thermal electron number densities), $\gamma_{e,\text{min}}$ (the minimum Lorentz factor of non-thermal electrons)$^7$, and $p$ (the spectral index of non-thermal electrons). Among them, $\gamma_{e,\text{min}}$ and $p$ are directly constrained by radio observations at millimeter and submillimeter wavebands. The remaining $B_{\text{tot}}$ and $K_e$ can be solved by using the two general relations that hold at $\nu = \nu_{\text{ssa}}$, shown in Equations (3) and (4). The solved $B_{\text{tot}}$ and $K_e$ are written as functions of $\theta_{\text{obs}}$, $\gamma_{e,\text{min}}$, $\nu_{\text{ssa}}$, and the observed flux at $\nu = \nu_{\text{ssa}}$.

Lastly, we further impose that the total jet power constraint not overproduce the Poynting power or kinetic power shown in Equation (21). This constraint can partially exclude larger values of $\theta_{\text{obs}}$. Then, we can determine $B_{\text{tot}}$ and $U_e/U_B$ of M87 consistently.

2.2. Assumptions

The following assumptions are adopted in this work.

1. We assume a uniform and isotropic distribution of relativistic electrons and magnetic fields in the emission region. For M87, the polarized flux does not seem very large. Therefore, we assume isotropic tangled magnetic fields in this work. Hereafter, we denote $B$ as the magnetic field strength perpendicular to the direction of electron motion. Then, the total field strength is

$$B_{\text{tot}} = \sqrt{3}B. \quad (1)$$

Hereafter, we define $U_B \equiv B_{\text{tot}}^2/8\pi$.

2. We assume the emission region is spherical with its radius $R$ measured in the comoving frame. The radius is defined as

$$2R \equiv \theta_{\text{obs}} D_A. \quad (2)$$

where $D_A = D_L/(1+z)^2$ is the angular diameter distance to a source (e.g., Weinberg 1972). Because M87 is the very low redshift source, we only use $D_A$ throughout this paper. There might be a slight difference between $\theta_{\text{FWHM}}$ and $\theta_{\text{obs}}$. VLBI measured $\theta_{\text{FWHM}}$ is conventionally treated as $\theta_{\text{obs}} = \theta_{\text{FWHM}}$, while Marscher (1983) pointed out a deviation expressed as $\theta_{\text{obs}} \approx 1.8\theta_{\text{FWHM}}$, which is caused by a forceable fitting of Gaussians to a non-Gaussian component. In this work, we introduce a factor $A$ defined as $\theta_{\text{obs}} = A\theta_{\text{FWHM}}$, and $1 \leq A \leq 1.8$ is assumed.

We stress that the uniform and isotropic sphere model is a first step simplification and the realistic jet base probably contains more complicated geometry and nonuniform distributions in magnetic fields and electron densities. We will investigate these complicated cases in the future.

2.3. Synchrotron Emissions and Absorptions

In order to obtain an explicit expression of $B$ and $K_e$ in terms of $\theta_{\text{obs}}$, $\nu_{\text{ssa}},\text{obs}$, and $S_{\nu_{\text{ssa}},\text{obs}}$, here we briefly review synchrotron emissions and absorptions. At the radio core, $\tau_{\text{ssa}}$ becomes an order of unity at $\nu = \nu_{\text{ssa}}$,

$$\tau_{\text{ssa}} = 2\alpha_{\nu_{\text{ssa}}} R, \quad (3)$$

where $\tau_{\text{ssa}}$ and $\alpha_{\nu_{\text{ssa}}}$ are the optical depth for SSA and the spectral index of SSA, respectively. We impose that the optically thin emission formula is still applicable at $\nu_{\text{ssa}}$, therefore:

$$\frac{4\pi}{3} R^3 \epsilon_{\nu_{\text{ssa}}} = 4\pi R^2 S_{\nu_{\text{ssa}}}, \quad (4)$$

where $\epsilon_{\nu_{\text{ssa}}}$ and $S_{\nu_{\text{ssa}}}$ are the emissivity and flux per unit frequency, respectively. Combining Equation (4) and the approximation of $\tau_{\text{ssa}} = 1$, we can solve $B$ and $K_e$. This derivation is much simpler than previous studies of Marscher (1983) and Hirotoni (2005, hereafter H05). We will compare the derived $B_{\text{tot}}$ in this work, Marscher (1983), and H05, and they will coincide with each other with a small difference in the range of $2.5 \leq p \leq 3.5$.

Next, let us break down relevant physical quantities. The term $K_e$, the normalization factor of the electron number density distribution $n_e(\gamma)$, is defined as (e.g., Equation (3.26) in GS65)

$$n_e(E_e)dE_e = K_e E_e^{-p} dE_e \quad (E_{e,\text{min}} \leq E_e \leq E_{e,\text{max}}),$$

$$\frac{K_e}{(m_e c^2)^{p-1}} \gamma_e^{-p} d\gamma_e \quad (\gamma_{e,\text{min}} \leq \gamma_e \leq \gamma_{e,\text{max}}), \quad (5)$$

where $E_e = \gamma_e m_e c^2 p = 2a + 1$, $E_{e,\text{min}} = \gamma_{e,\text{min}} m_e c^2$, and $E_{e,\text{max}} = \gamma_{e,\text{max}} m_e c^2$ are the electron energy, the spectral index, minimum energy, and maximum energy of relativistic (non-thermal) electrons, respectively. Let us further review optically thin synchrotron emissions. The maximum in the spectrum of synchrotron radiation from an electron occurs at the frequency (Equation (2.23) in GS65)

$$\nu_{\text{syn}} = 1.2 \times 10^6 B \gamma_e^2. \quad (6)$$
The SSA coefficient measured in the comoving frame is given by (Equations (4.18) and (4.19) in GS65; Equation (6.53) in RL79)

\[ \alpha_\nu = \frac{\sqrt{3} e^3}{8 \pi m_e c} \left( \frac{3 e}{2 \pi m_e c^3} \right)^{p/2} c_1(p) \times K_e B^{(p+2)/2} \nu^{-(p+4)/2}, \]

where the numerical coefficient \( c_1(p) \) is expressed by using the gamma functions as follows: \( c_1(p) = \Gamma(3p + 2)/12 \Gamma(3p + 2 + 1)/12 \). For convenience, we define \( \alpha_\nu = X_1 c_1(p) B^{(p+2)/2} K_e \nu^{-(p+4)/2} \).

The optically thin synchrotron emissivity per unit frequency \( \epsilon_\nu \) from the uniform emitting region is given by (Equations (4.59) and (4.60) in BG70; see also Equations (3.28), (3.31), and (3.32) in GS65)

\[ \epsilon_\nu = 4 \pi \frac{\sqrt{3} e^3}{8 \pi m_e c^2} \left( \frac{3 e}{2 \pi m_e c^3} \right)^{(p-1)/2} c_2(p) \times K_e B^{(p+1)/2} \nu^{-(p-1)/2}, \]

where the numerical coefficient is \( c_2(p) = \Gamma(3p + 19)/12 \Gamma(3p - 1)/12 \Gamma(p + 5)/4 \)/\( \Gamma(p + 7)/4 \)/\( (p + 1) \). For convenience, we define \( \epsilon_\nu = 4 \pi X_2 c_2(p) B^{(p+1)/2} K_e \nu^{-(p-1)/2} \).

### 2.4. Relations between Quantities Measured in Source and Observer Frames

Let us summarize the Lorentz transformations and cosmological effect using the Doppler factor \( \delta \equiv 1/(\Gamma(1 - \cos \theta_{\text{LOS}})) \), where \( \theta_{\text{LOS}} \) is the angle between the jet and our line of sight) and the redshift (\( z \)). Hereafter, we use the subscript (obs) for quantities measured at the observer frame

\[ \nu_{\text{obs}} = \nu \frac{\delta}{1 + \nu} \tag{9} \]

The observed flux from an optically thin source at a large distance is given by (Equation (1.13) in RL79; Equation (7) in H05; see also Equation (C4) in Begelman et al. 1984)

\[ S_{\text{obs,obs}} = \left( \frac{\delta}{1 + \nu} \right)^3 S_b \left( \frac{\theta_{\text{obs}}}{2} \right)^2 \tag{10} \]

### 2.5. Obtained \( B \) and \( K_e \)

Combining the above shown relations, we finally obtain

\[ B = b(p) \nu_{\text{ssa,obs}}^{5} \left( \frac{\theta_{\text{obs}}}{1 \text{ mas}} \right)^4 \times \left( \frac{S_{\text{ssa,obs}}}{1 \text{ Jy}} \right)^{-2} \left( \frac{\delta}{1 + \nu} \right), \tag{11} \]

where the numerical values of \( b(p) = [(2 \times 3)/(4 \pi)]^2 (c_2(p) X_2/c_1(p) X_1)^2 \times 2^{-4} \) are shown in Table 1. In the table, we also note \( b(p) \) obtained in Marscher (1983) and H05. From this, we see that the derived \( B_{\text{tot}} \) values in this work coincide with each other with a small difference.

Inserting Equation (11) into Equation (3) or Equation (4), we then obtain \( K_e \) as

\[ K_e = k(p) \left( \frac{D_A}{1 \text{ Gpc}} \right)^{-1} \left( \frac{\nu_{\text{ssa,obs}}}{1 \text{ GHz}} \right)^{-2p-3} \left( \frac{\theta_{\text{obs}}}{1 \text{ mas}} \right)^{-2p-5} \times \left( \frac{S_{\text{ssa,obs}}}{1 \text{ Jy}} \right)^{p+2} \left( \frac{\delta}{1 + \nu} \right)^{-p-3}, \tag{12} \]

where \( k(p) = b(p)^{(p-2)/2} X_1 c_1(p)^{-1} D_A/(1 \text{ Gpc})^{-1} S_{\text{ssa,obs}}^{-1} \nu_{\text{ssa,obs}}^{-1} \).

The cgs units of \( K_e \) and \( k(p) \) depend on \( p \): erg\(^{-1}\) cm\(^{-3}\). The numerical values of \( k(p) \) are summarized in Table 1 and are similar to the ones in Marscher (1983), which are \( k(2.5) = 1.2 \times 10^{-5} \) and \( k(3.0) = 0.59 \times 10^{-3} \). Using the obtained \( K_e \), we can evaluate \( U_e \) as

\[ U_e = \int_{E_{\text{min}}}^{E_{\text{max}}} E \nu E(e_\nu) d(e_\nu) = K_e E_{e, \text{min}}^{p+2}/p - 2 \] (for \( p > 2 \)). \tag{13} \]

Then, we can obtain the ratio \( U_e/U_B \) explicitly as

\[ \frac{U_e}{U_B} = \frac{8 \pi}{3 b^2(p) (p - 2)} \left( \frac{D_A}{1 \text{ Gpc}} \right)^{-1} \left( \frac{\nu_{\text{ssa,obs}}}{1 \text{ GHz}} \right)^{-2p-13} \times \left( \frac{\theta_{\text{obs}}}{1 \text{ mas}} \right)^{-2p-13} \left( \frac{S_{\text{ssa,obs}}}{1 \text{ Jy}} \right)^{p+6} \left( \frac{\delta}{1 + \nu} \right)^{-p-5} \] (for \( p > 2 \)). \tag{14} \]

From this, we find that \( S_{\text{ssa,obs}} \) and \( \theta_{\text{obs}} \) have the same dependence on \( p \). Using this relation, we can estimate \( U_e/U_B \) without a minimum energy (equipartition \( B \) field) assumption. It is clear that the measurement of \( \theta_{\text{obs}} \) is crucial for determining \( U_e/U_B \). In the next subsection, we discuss more details pertaining to this issue. It is also evident that a careful treatment of \( \nu_{e, \text{min}} \) is crucial for determining \( U_e/U_B \) (Kino et al. 2002; Kino & Takahara 2004).

### 3. APPLICATION TO M87

On the basis of recent VLBA observations of M87 at 43 GHz (see Figure 1), we derive \( U_e/U_B \) at the base of the M87 jet. Here we set the typical index of electrons as \( p = 3.0 \) (Doi et al. 2013).

#### 3.1. Electrons Emitting 43 GHz Radio Waves

First, let us constrain \( \nu_{e, \text{min}} \). At the least, \( \nu_{e, \text{min}} \) should be smaller than the Lorentz factor of electrons radiating synchrotron emission at 43 GHz. Therefore, the minimum Lorentz factor of electrons is constrained as

| \( p \) | \( c_1(p) \) | \( c_2(p) \) | \( b(p) \) | \( b(p) \) by Hiotani (2005) | \( b(p) \) by Marscher (1983) | \( k(p) \) |
|---|---|---|---|---|---|---|
| 2.5 | 1.516 | 0.405 | 3.3 \times 10^{-5} | 2.36 \times 10^{-5} | 3.6 \times 10^{-5} | 1.4 \times 10^{-2} |
| 3.0 | 1.490 | 0.303 | 1.9 \times 10^{-5} | 2.08 \times 10^{-5} | 3.8 \times 10^{-5} | 2.3 \times 10^{-3} |
| 3.5 | 1.520 | 0.245 | 1.2 \times 10^{-5} | 1.78 \times 10^{-5} | \ldots | 3.6 \times 10^{-4} |
This typical $U_e/U_B$ apparently shows the order of unity but has strong dependencies on $\theta_{\text{obs}}$ and $\nu_{\text{ssa}}$. Regarding $\nu_{\text{ssa,obs}}$, an uncertainty only comes from the bandwidth. In our VLBA observation, the bandwidth is 128 MHz with a central frequency of 43.212 GHz. Therefore, it causes only a very small uncertainty $\sim(43.276/43.148)^{19} = 1.06$. The accuracy of the flux calibration of the VLBA can be conservatively estimated as 10%. An intrinsic flux of the radio core at 43 GHz also fluctuates with an order of 10% during a quiescent phase (e.g., Acciari et al. 2009; Hada et al. 2012). Therefore, the flux term also causes an uncertainty of $\sim 1.21^9 = 5.6$. The angular size $\theta_{\text{obs}}$ and $\gamma_{e,\text{min}}$ have much larger ambiguities than those evaluated above, and we derive $U_e/U_B$ by taking these ambiguities into account in the next subsection. At the same time, we again emphasize that $\theta_{\text{obs}}$ obtained in H13 can be sufficient for estimating $U_e/U_B$ in spite of such a strong dependence on $\theta_{\text{obs}}$.

### 3.3. On $\theta_{\text{obs}}$, $p$, and $L_{\text{jet}}$

The most important quantity for the estimate of $U_e/U_B$ is $\theta_{\text{obs}}$. On the basis of VLBA observation data with sufficiently good qualities, here we set

$$0.11 \text{ mas} \leq \theta_{\text{obs}} \leq 0.20 \text{ mas},$$

where we use the average value $\theta_{\text{FWHM}} = 0.11$ mas from H13, and the maximum of $\theta_{\text{obs}}$ is 0.11 mas $\times 1.8 = 0.198$ mas. We note that the measured core’s FWHM overlaps with the measured width of the jet (length between the jets limb structure) in H13. Therefore, we consider $\Lambda \approx 1$ more likely for the M87 jet base. From Equation (15), the maximal value of $\gamma_{e,\text{min}}$ is given by $\sim 2 \times 10^2$ when $\theta_{\text{obs}} = 0.11$ mas.

Regarding the value of $p = 2\alpha+1$, a simultaneous observation of the spectrum measurement at a submillimeter wavelength range is crucial, since most of the observed fluxes at the submillimeter range come from the innermost part of the jet. It has been indeed measured by Doi et al. (2013) by conducting a quasi-simultaneous multifrequency observation with the Atacama Large Millimeter/submillimeter Array (ALMA) observation (in cycle 0 slot), and the observational result shows that $\alpha > 0.5$ at $> 200$ GHz, where synchrotron emission becomes optically thin against SSA. By maximally taking uncertainties into account, we set the allowed range of $p$ as

$$2.5 \leq p \leq 3.5,$$

in this work.

We further impose the condition that the time-averaged total jet power ($L_{\text{jet}}$) inferred from its large-scale jet properties should not be exceeded by the kinetic power of relativistic electrons ($L_e$) and Poynting power ($L_{\text{poy}}$) at the 43 GHz core

$$L_{\text{jet}} \geq \max[L_{\text{poy}}, L_e],$$

$$L_e = \frac{4\pi}{3} \Gamma^2 \beta R^2 c U_e,$$

$$L_{\text{poy}} = \frac{4\pi}{3} \Gamma^2 \beta R^2 c U_B,$$

where $L_{\text{jet}}$ at a large scale is estimated maximally a few $\times 10^{44}$ erg s$^{-1}$ (e.g., Reynolds et al. 1996; Bicknell & Begelman 1996; Owen et al. 2000; Stawarz et al. 2006; Rieger & Aharonian 2012). Hereafter, we conservatively assume $\Gamma \beta = 1$, and a slight deviation from this does not influence the main results in this work. Regarding $L_{\text{jet}}$ in the M87 jet, we set

$$1 \times 10^{44} \text{ erg s}^{-1} \leq L_{\text{jet}} \leq 5 \times 10^{44} \text{ erg s}^{-1}.$$
Here we include an uncertainty due to the deviation from time-averaged $L_{\text{jet}}$ at a large scale, which may attribute to flaring phenomena at the jet base. The X-ray light curve at the M87 core over 10 years showed a flux variation by a factor of several except for exceptionally high X-ray flux during giant VHE flares that occurred in 2008 and 2010 (Figure 1 in Abramowski et al. 2012). On the basis of this information, we set the largest jet that occurred in 2008 and 2010 (Figure 1 in Abramowski et al. 2012). On the basis of this information, we set the largest jet kinetic power case as $L_{\text{jet}} = 5 \times 10^{44}$ erg s$^{-1}$.

### 3.3.1. On Jet Speed

Jet speed in the vicinity of the M87’s central black hole is quite an issue. Ly et al. (2007) and Kovalev et al. (2007) show subluminal speed proper motions of the M87 jet base, which the recent study by Asada et al. (2014) also support. Hada (2013b) also explores the proper motion near the jet base with the VERA (VLBI Exploration of Radio Astrometry) array. The VERA observation has been partly performed in the GENJI Programme (Gamma-ray Emitting Notable AGN Monitoring with Japanese VLBI), aiming for densely sampled monitoring of bright AGN jets (see Nagai et al. 2013 for details), and the observational data obtained by VERA also shows a subluminal motion at the jet base. Furthermore, Acciari et al. (2009) report that the 43 GHz core is stationary within $\sim 6R_s$ on the basis of their phase reference observation at 43 GHz. Therefore, currently there is no clear observational support of superluminal motion within the 43 GHz radio core. The brightness temperature $T_b = (1 + z/\delta)S_{\text{obs}}/2\pi k v^2_\text{obs}(\theta_{\text{obs}}/2)^2$ (e.g., Lähdeennäki et al. 1999; Doi et al. 2006) of the 43 GHz radio core is evaluated as

$$T_b \sim 6 \times 10^{10} \text{K} \left(\frac{S_{\text{obs}}}{0.7 \text{Jy}}\right) \left(\frac{\theta_{\text{obs}}}{0.11 \text{mas}}\right)^{-2},$$

which is below the critical temperature $\sim 10^{11}$ K limited by the inverse Compton catastrophe process (Kellermann & Pauliny-Toth 1969). Because of these two reasons, we assume $\delta \approx 1$ throughout this paper.

## 4. RESULTS

Here we examine the three cases of electron indices as $p = 2.5$, 3.0, and 3.5 against the two cases of the jet power as $L_{\text{jet}} = 1 \times 10^{44}$ erg s$^{-1}$ and $L_{\text{jet}} = 5 \times 10^{44}$ erg s$^{-1}$.

### 4.1. Allowed $B$ Strength

First, it should be noted that $B$ is primarily determined by a value of $\theta_{\text{obs}}$ since $\nu_{\text{obs}}$ is exactly identical to the observing frequency. By combining Equations (16), (21), and (22), we obtain the allowed range of the magnetic field strength in the 43 GHz core. We summarize the obtained maximum and minimum values of $B_{\text{tot}}$ in Table 2. An upper limit of $B$ is governed by the constraint of $L_{\text{jet}} \geq L_{\text{poy}}$. From Equations (11) and (21), it is clear that $L_{\text{poy}}$ behaves as

$$L_{\text{poy}} \propto \theta_{\text{obs}}^{10} \propto B_{\text{tot}}^{5/2}$$

apart from a weak dependence on $p$ originating in $b(p)$. We thus obtain the allowed range $1 \text{G} \leq B_{\text{tot}} \leq 15 \text{G}$ in the 43 GHz core. This is a robust constraint on the M87 core’s B strength.

### 4.2. Allowed $U_e/U_B$ with $p = 3.0$

In Figure 2, we show the allowed region in the $\gamma_{e,\text{min}}$ and $B_{\text{tot}}$ plane (the red boxed region) and the corresponding log($U_e/U_B$) values with $L_{\text{jet}} = 5 \times 10^{44}$ erg s$^{-1}$ and $p = 3.0$. The log($U_e/U_B$) value is obtained from Equation (14). The boundary of the allowed region is determined by Equations (15), (21), and (27). (Short stray lines from the box should be ignored.)

(A color version of this figure is available in the online journal.)

Here we include an uncertainty due to the deviation from time-averaged $L_{\text{jet}}$ at a large scale, which may attribute to flaring phenomena at the jet base. The X-ray light curve at the M87 core over 10 years showed a flux variation by a factor of several except for exceptionally high X-ray flux during giant VHE flares that occurred in 2008 and 2010 (Figure 1 in Abramowski et al. 2012). On the basis of this information, we set the largest jet kinetic power case as $L_{\text{jet}} = 5 \times 10^{44}$ erg s$^{-1}$.

| $p$ | $L_{\text{jet}}$ (erg s$^{-1}$) | Minimum $B_{\text{tot}}$ (G) | Maximum $B_{\text{tot}}$ (G) | Minimum $\theta_{\text{obs}}$ (mas) | Maximum $\theta_{\text{obs}}$ (mas) |
|-----|-------------------------------|-------------------------------|-------------------------------|-----------------------------------|-----------------------------------|
| 2.5 | $1 \times 10^{44}$           | 2.5                           | 7.7                           | 0.11                              | 0.15                              |
| 2.5 | $5 \times 10^{44}$           | 2.5                           | 14.7                          | 0.11                              | 0.17                              |
| 3.0 | $1 \times 10^{44}$           | 1.5                           | 6.9                           | 0.11                              | 0.16                              |
| 3.0 | $5 \times 10^{44}$           | 1.5                           | 13.3                          | 0.11                              | 0.19                              |
| 3.5 | $1 \times 10^{44}$           | 0.93                          | 6.3                           | 0.11                              | 0.18                              |
| 3.5 | $5 \times 10^{44}$           | 0.93                          | 9.6                           | 0.11                              | 0.20                              |

\[ 0.11 \text{mas} \leq \theta_{\text{obs}} \leq 0.19 \text{mas}. \]
The maximum $\theta_{\text{obs}}$ is derived from $L_{\text{jet}} = L_{\text{poy}} = 5 \times 10^{44} \text{ erg s}^{-1}$. Then, the factor $(0.19/0.11)^{18} \sim 2 \times 10^6$ makes the allowed $U_e/U_B$ range broaden. Independent of this factor, $\gamma_{e,\text{min}}$ has an uncertainty factor of $2 \times 10^5$. These factors govern the overall allowed $U_e/U_B$ range of the order of a few times $10^6$, which is presented in Table 3. Additionally, we note that the top right part is dropped out according to Equation (15). This changes the minimum values of $U_e/U_B$ by a factor of a few.

Additionally, we show the allowed $\log(U_e/U_B)$ and $B$ with $L_{\text{jet}} = 1 \times 10^{44} \text{ erg s}^{-1}$ and $p = 3.0$ in Tables 2 and 3. Compared with the case in Figure 2, the upper limit of $L_{\text{poy}}$ becomes smaller. Then the allowed $\theta_{\text{obs}}$ becomes

$$0.11 \text{ mas} \lesssim \theta_{\text{obs}} \lesssim 0.16 \text{ mas}. \quad (26)$$

The maximum $\theta_{\text{obs}}$ is also derived from $L_{\text{jet}} = L_{\text{poy}} = 1 \times 10^{44} \text{ erg s}^{-1}$. The decrease of the maximum $\theta_{\text{obs}}$ value leads to the increase of $U_e/U_B$ correspondingly.

4.3. Allowed $U_e/U_B$ with $p = 3.5$

In Figure 3, we show the allowed region in the $\gamma_{e,\text{min}}$ and $B_{\text{tot}}$ plane (the red boxed region) and the corresponding $\log(U_e/U_B)$ values with $L_{\text{jet}} = 5 \times 10^{44} \text{ erg s}^{-1}$ and $p = 3.5$. Compared with the case with $p = 3.0$, the $U_e/U_B > 1$ region increases in the allowed parameter range according to the relation of $U_e/U_B \propto \theta_{\text{obs}}^{-2} p^{-3} \gamma_{e,\text{min}}^{-p+2}$. The allowed $\theta_{\text{obs}}$ in this case is

$$0.11 \text{ mas} \lesssim \theta_{\text{obs}} \lesssim 0.20 \text{ mas}, \quad (27)$$

which remains the same as Equation (19) because both $L_{\text{poy}}$ and $L_{\text{jet}} = 5 \times 10^{44} \text{ erg s}^{-1}$ in this case. The $\theta_{\text{obs}}$ factor leads to $(0.196/0.11)^{20} \approx 1 \times 10^5$ uncertainty, while the $\gamma_{e,\text{min}}$ factor has $\sim 2 \times 10^5$ uncertainty. Therefore, the allowed $U_e/U_B$ in this case has a few times $10^5$ of uncertainty, which is shown in Table 3. The bottom left part is slightly deficient because of $L_{\text{poy}} > L_{\text{jet}}$.

In Tables 2 and 3, we show the allowed $\log(U_e/U_B)$ and $B$ with $L_{\text{jet}} = 1 \times 10^{44} \text{ erg s}^{-1}$ and $p = 3.5$. In this case, the allowed $\theta_{\text{obs}}$ is

$$0.11 \text{ mas} \lesssim \theta_{\text{obs}} \lesssim 0.18 \text{ mas}. \quad (28)$$

The relation of $L_{\text{jet}} = L_{\text{poy}} = 1 \times 10^{44} \text{ erg s}^{-1}$ leads to the value of maximum $\theta_{\text{obs}}$. This upper and lower $U_e/U_B$ are governed in the same way as in Figure 3.

4.3.1. Allowed $U_e/U_B$ with $p = 2.5$

In Figure 4, we show the allowed region in the $\gamma_{e,\text{min}}$ and $B_{\text{tot}}$ plane (the red boxed region) and the corresponding $\log(U_e/U_B)$ values with $L_{\text{jet}} = 5 \times 10^{44} \text{ erg s}^{-1}$ and $p = 2.5$. In this case, the allowed $\theta_{\text{obs}}$ is

$$0.11 \text{ mas} \lesssim \theta_{\text{obs}} \lesssim 0.17 \text{ mas}. \quad (29)$$

The relation of $L_{\text{jet}} = L_{\text{poy}} = 5 \times 10^{44} \text{ erg s}^{-1}$ determines the maximum $\theta_{\text{obs}}$. The allowed $B_{\text{tot}}$ is in the narrow range of $2.5 \text{ G} \lesssim B_{\text{tot}} \lesssim 14.7 \text{ G}$. It should be stressed that this case shows the magnetic field energy dominance in all of the allowed $B_{\text{tot}} \gamma_{e,\text{min}}$ ranges.

In Tables 2 and 3, we show the allowed $\log(U_e/U_B)$ and $B$ with $L_{\text{jet}} = 1 \times 10^{44} \text{ erg s}^{-1}$ and $p = 2.5$. The basic behavior is similar to the case shown in Figure 4. In this case, the allowed $\theta_{\text{obs}}$ is

$$0.11 \text{ mas} \lesssim \theta_{\text{obs}} \lesssim 0.15 \text{ mas}. \quad (30)$$

The relation of $L_{\text{jet}} = L_{\text{poy}} = 1 \times 10^{44} \text{ erg s}^{-1}$ determines the maximum $\theta_{\text{obs}}$. Corresponding to the narrow allowed range of $\theta_{\text{obs}}$, the allowed field strength resides in the narrow range of $2.5 \text{ G} \lesssim B_{\text{tot}} \lesssim 7.7 \text{ G}$.  

5. SUMMARY AND DISCUSSIONS

On the basis of VLBA observation data at 43 GHz, we explore $U_e/U_B$ at the base of the M87 jet. We apply the standard theory...
of synchrotron radiation to the 43 GHz radio core together with the assumption of a simple uniform sphere geometry. We impose the condition that the Poynting power and relativistic electron kinetic power should be smaller than the total power of the jet. Obtained values of $B_{\text{tot}}$ and $U_e/U_B$ are summarized in Tables 2 and 3, and we find the following.

1. We obtain the allowed range of magnetic field strength in the 43 GHz core as $1 \text{ G} \leq B_{\text{tot}} \leq 15 \text{ G}$ in the observed radio core at 43 GHz with its diameter $0.11-0.20 \text{ mas}$ (15.5–28.2 $R_\bullet$). Our estimate of $B$ is basically close to the previous estimate in the literature (e.g., Neronov & Aharonian 2007), although fewer assumptions have been made in this work. We note that even if $\delta$ of the 43 GHz core becomes larger than unity, the field strength only changes according to $B_{\text{tot}} \propto \delta$.

It is worth comparing these values with independently estimated $B_{\text{tot}}$ in previous works more carefully. Abdo et al. (2009) has estimated the Poynting power and kinetic power of the jet by the model fitting of the observed broad band spectrum and derive $B_{\text{tot}} = 0.055 \text{ G}$ with $R = 1.4 \times 10^{16} \text{ cm} = 0.058 \text{ mas}$, although they do not properly include the SSA effect. Acciari et al. (2009) a predict field strength of $B_{\text{tot}} \sim 0.5 \text{ G}$ on the basis of the synchrotron cooling argument. Since smaller values of $B_{\text{tot}}$ lead to smaller $\theta_{\text{obs}}$, if we assume $\theta_{\text{obs, min}}$ by a factor of $\sim 3$ than the true $\theta_{\text{obs}} = 0.11 \text{ mas}$, the predicted $B_{\text{tot}}$ lies between 0.05 and 0.5 G, which seems to be in a good agreement with previous work. However, for such a small core, the electron kinetic power far exceeds the observed jet power.

Our result excludes a strong magnetic field such as $B_{\text{tot}} \sim 10^{3-4} \text{ G}$, which is frequently assumed in previous works in order to activate the Blandford–Znajek process (Blandford & Znajek 1977; Thorne et al. 1986; Boldt & Loewenstein 2000). Although M87 has been a prime target for testing relativistic MHD jet simulation studies powered by black hole spin energy, our results throw out the caveat that the maximum $B_{\text{tot}}$, one of the critical parameters in the relativistic MHD jet model, should be smaller than $\sim 15 \text{ G}$ for M87.

2. We obtain the allowed region of $U_e/U_B$ in the allowed $\theta_{\text{obs}}$ and $\gamma_{e, \text{min}}$ plane. The resultant $U_e/U_B$ contains both the region of $U_e/U_B > 1$ and the region of $U_e/U_B < 1$. We find that the allowed range is $1 \times 10^{-5} \leq U_e/U_B \leq 6 \times 10^2$. The uncertainty of $U_e/U_B$ is caused by the strong dependence on $\theta_{\text{obs}}$ and $\gamma_{e, \text{min}}$. Our result gives an important constraint against relativistic MHD models in which they postulate very large $U_B/U_e$ at a jet base (e.g., Vlahakis & Königl 2003; Komissarov et al. 2007, Komissarov et al. 2009; Tchekhovskoy et al. 2011). To realize a sufficiently magnetic dominated jet such as $U_B/U_e \sim 10^{5-4}$, relatively large $\gamma_{e, \text{min}}$ of the order of $\sim 10^2$ and a relatively large $\theta_{\text{obs}}$ are required. Thus, the obtained $U_e/U_B$ in this work gives a new constraint on the initial conditions in relativistic MHD models.

Last, we note key future works.

1. Observationally, it is crucial to obtain resolved images of the radio cores at 43 GHz with space/submillimeter VLBI, which would clarify whether there is a substructure or not inside the $\sim 16 \text{ R}_\bullet$ scale at the M87 jet base. Toward this final observational goal, as a first step, it is important to explore the physical relations between the results of the present work and observational data at higher frequencies such as 86 GHz and 230 GHz (e.g., Krichbaum et al. 2005; Krichbaum et al. 2006; Doelman et al. 2012). Indeed, we conduct a new observation of M87 with the VLBA and the Green Bank Telescope at 86 GHz, and we will explore this issue by using the new data. The Space VLBI program also could play a key role since a lower frequency observation can attain higher dynamic range images with a high resolution (e.g., Dodson et al. 2006, 2013; Asada et al. 2009; Takahashi & Mineshige 2011). If more compact regions inside the 0.11 mas region are found by Space VLBI in the future, then $U_e/U_B$ in the compact regions are larger than the ones shown in the present work.

2. Theoretically, we leave the following issues for future work.

(1) Constraining plasma composition (i.e., electron/proton ratio) is one of the most important issues in AGN jet physics (Reynolds et al. 1996; Kino et al. 2012), and we will study it in the future. Generally, the inclusion of proton powers ($L_p$) will simply reduce the upper limit of $B_{\text{tot}}$ because $L_p \approx L_e + L_p \approx L_{\text{poy}}$ would hold. (2) On a $\sim 10 \text{ R}_\bullet$ scale, general relativistic (GR) effects can be important and will induce non-spherical geometry. For example, if there is a Kerr black hole at its jet base, the following GR-related phenomena may happen: (1) a magneto-spin effect that aligns a jet base along the black hole spin, which leads to asymmetric geometry (McKinney et al. 2013), and (2) possible warping of the accretion disk by the Bardeen & Petterson effect that is caused by the frame dragging effect (Bardeen & Petterson 1975; Hatchett et al. 1981). Although recent research by Dexter et al. (2012) suggests that the core emission is not dominated by the disk but instead by the jet component, the disk emission should be taken into account if accretion flow emission is largely blended in the core emission in reality (see also Broderick & Loeb 2009). We should take these GR effects into account when they are indeed effective. (3) Apart from the GR effect, the pure geometrical effect between the jet opening angle and the viewing angle may cause a partial blending of the SSA thin part of the jet. It might also cause non-spherical geometry, and their inclusion is also important.

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