\( \mathcal{N} \) D0-branes and \( \mathcal{\overline{D}}0 \)-branes in the background independent string field theory.

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Abstract: In this paper we extend our previous work hep-th/0102063 to the case of \( \mathcal{D}\mathcal{D}0 \)-brane system.

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1. Introduction

Around the year '92-'93 the formulation of the string field theory known as a background independent string field theory (BSFT) \(1, 2, 3, 4, 5\) was proposed. In the recent papers \(6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\) (For the world-sheet approach to this problem, see \(21, 22, 23\)) the BSFT was used in the analysis of the tachyon condensation on the unstable D-brane systems in string theories (For review and extensive list of references, see \(24\)). It was shown that BSFT is very effective tool for the study of this problem since in the process of the tachyon condensation only tachyon field acquires nonzero vacuum expectation value. In particular, it was shown that:

- The condensation to the closed string vacuum and to lower dimensional branes involves excitations of only one mode of string field-tachyon.

- The exact tachyon potential can be computed in BSFT and its qualitative features agree with the Sen conjecture \(25\).

- The exact tachyon profiles corresponding to the tachyon condensation to the lower dimensional D-brane give rise to descent relations between the tensions of various branes \(14\) which again agree with those expected \(25\).

The problem of tachyon condensation was also studied from the point of view of Witten’s open string field theory \(27\) (For recent discussion see \(28, 29\) and for the review and extensive list of references, see \(30\)). In contrast with BSFT, the tachyon condensation in general involves giving the expectation values of infinite number of component fields. As a consequence, only approximate results are available.
In [41] the problem of the tachyon condensation was studied from different point of view. Namely, we started with the system of $N$ D-instantons in bosonic theory or with the system of $N$ non-BPS D-instantons in Type IIA theory and studied the emergence of higher dimensional D-branes in this system. We have shown that all even dimensional D-brane arise naturally through the standard matrix theory construction [36, 37, 38, 39]. However to describe odd dimensional D-branes the tachyon field should be turned on and then in the process of the tachyon condensation [7, 11] all odd dimensional D-branes emerged.

In this paper we extend this approach to the case of the system of $N$ D0+D-system.) The general $D\bar{D}$ system has been recently studied in [18, 19] and we use results obtained there in our calculations. In fact, $D\bar{D}$ system seems to contain much more information about the tachyon condensation than non-BPS D-branes since all non-BPS D-branes can arise as a particular projection from $D\bar{D}$ system [24]. For that reason it seems to be natural to ask the question whether it is possible to describe all D-branes from $N$ D0-branes in type IIA theory. We will show in this paper that this is really possible.

The organisation of the paper is as follows. In the next section we present the partition function on the disk for $N$ D9-branes which, thanks to the recent conjecture [11], should be identified with the space-time action in BSFT. Our form of the boundary interaction can be considered as a slight modification of the action given in [18]. In order to show that this boundary term gives correct results we will argue that the action for $D\bar{D}$ system reduces to the action for D9-branes without the tachyon field and to the action for non-BPS D9-brane after orbifolding projection [24].

In (3) section we will review the approach presented in [41] on the example of $N$ non-BPS D0-branes.

In (4) section we will turn to the main theme of this paper that is the emergence of all D-branes from the system of $N$ D0-branes in type IIA theory. We will show that all Dp-branes naturally emerge either directly in the standard matrix theory construction or through the tachyon condensation worked out in [18, 19]. We will also discuss other possibilities of the emergence of D-branes that are based on nontrivial gauge fields on the world-volume of $D\bar{D}$ system. Unfortunately, this possibility does not seem to lead to the free boundary theory so that it is difficult to study it in the BSFT theory.

In conclusion (5) we will discuss some open problems and possible extension of this work.

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1The same problem has been already studied in terms of the low energy effective action in [32] and in the framework of the effective action obtained from BSFT theory in [34].
2. $\mathcal{D}\mathcal{D}$-system

In this section we will consider $\mathcal{D}\mathcal{D}$ system from the point of view of BSFT following recent papers [18, 19]. The general construction of the BSFT for $N$ D-branes and antibranes was given in [18] which we will closely follow. The starting point of the BSFT for $\mathcal{D}\mathcal{D}$ system is the partition function on the disk

$$S(A, A', T) = Z(A, A', T) = \langle \text{Tr} \hat{P} \exp \left( \int_0^{2\pi} d\hat{\tau} M(A, A', T) \right) \rangle ,$$

(2.1)

where the symbol $\hat{P}$ is defined as

$$\hat{P} \exp \left( \int d\hat{\tau} \Theta(\hat{\tau}) \Theta(\hat{\tau}_2) \Theta(\hat{\tau}_{N-1,N}) M(\hat{\tau}_1) M(\hat{\tau}_2) \ldots M(\hat{\tau}_N) \right),$$

(2.2)

and where $d\hat{\tau} = d\tau d\theta$, $\hat{\tau}_{12} = \tau_1 - \tau_2 - \theta_1 \theta_2$ and $\Theta$ is a step function whose expansion is equal to $\Theta(\hat{\tau}_1 - \hat{\tau}_2) = \theta(\tau_1 - \tau_2) - \delta(\tau_1 - \tau_2) \theta_1 \theta_2$. The matrix $M$ describes general boundary interaction. In situation, when we confine to the gauge fields and tachyon, it has a form [18]

$$M(A, A', T) = \left( -iA_\mu(X) DX^\mu \quad \frac{T(X)}{T(X)} \quad \right), \quad X^\mu = X^\mu + \sqrt{\alpha'} \theta \psi^\mu,$$

(2.3)

where $A_\mu$ is a gauge field in the adjoint representation of the gauge group $U(N)$ living on the world-volume of $N$ D9-branes, $A'_\mu$ is a gauge field in the adjoint representation of the gauge group $U(N)$ living on the world-volume of $N$ D9-branes and $T$ is a tachyon field in $(N, \overline{N})$ representation of the gauge group $U(N) \times U(N)$. The previous form of the boundary interaction corresponds to the system of $N$ D9-branes and antibranes, generalisation to the lower dimensional case is straightforward. In (2.3) we presume that off-diagonal terms have odd grading. In other words, the form of (2.3) has a property similar to the superconnection [43] (For detailed discussion of the relation between superconnection and BSFT, see [18, 19, 44]). In order to show that (2.3) defines correctly BSFT action for $\mathcal{D}\mathcal{D}$-system we express the partition function in the more familiar form using elegant formalism reviewed in [21] which allows us to rewrite the path-ordered trace in terms of the integral over fermionic fields

$$S = \langle \exp \left( \int d\hat{\tau} \left[ \overline{\eta}_a D\eta^a + \overline{\eta}_a M_0^a \eta^b \right] \right) \rangle = \langle e^{-I} \rangle ,$$

(2.4)

where we have introduced fermionic superfields $\overline{\eta}_a = \eta_a + \theta \chi_a$, $\hat{\eta}^a = \eta^a + \theta \chi^a$, $a = 1, \ldots, 2N$ living on the boundary of the disk. Firstly we perform the integration over $\theta$. In order to do that we expand $M$ as $M = M_0 + \theta M_1$ with

$$M_0 = \left( \begin{array}{cc} -i\sqrt{\alpha'} A_\mu \psi^\mu & \frac{T(X)}{T(X)} \\ T(X) & -i \sqrt{\alpha'} A'_\mu \psi^\mu \end{array} \right),$$

(2.5)

For simplicity we will consider the configurations with the equal number branes and antibranes.

The generalisation to the more general case is trivial.
and

\[
M_1 = \begin{pmatrix}
-i(A_\mu(X)\partial_\tau X^\mu + \frac{\alpha'}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\psi^\mu\psi^\nu) & -\partial_\mu \overleftrightarrow{T}(X)\sqrt{\alpha'}\psi^\mu \\
-\partial_\mu T(X)\sqrt{\alpha'}\psi^\mu & -i(A'_\mu(X)\partial_\tau X^\mu + \frac{\alpha'}{2}(\partial_\mu A'_\nu - \partial_\nu A'_\mu)\psi^\mu\psi^\nu)
\end{pmatrix} .
\]  

(2.6)

Then

\[
-\int d\tau d\theta \eta_A M_\alpha^a \hat{\eta}^b = \int d\tau (-\eta_A M_{0b}^a \chi^b - \chi A_{M_0} M_0 \eta^b + \eta_A M_1 \eta^b) .
\]

(2.7)

In the previous expression we have used the crucial fact that \( M_0 \) anticommutes with \( \theta \) which is a consequence of the presence of the fermionic field \( \psi \) on the main diagonal of \( M_0 \) and the presumption that off-diagonal elements have odd grading so that anticommute with fermionic fields. We must also stress that in \( M_1 \) the off-diagonal terms have a even grading thanks to odd grading of \( T \) and odd grading of \( \psi \). We then obtain

\[
I = \int d\tau \left( \eta_a \partial_\tau \eta^a - \chi A_{M_0} M_0 \eta^b - \chi A_{M_0} M_0 \eta^b + \eta_A M_1 \eta^b \right) .
\]

(2.8)

We can integrate out the auxiliary fields \( \chi, \chi^a \) with the result

\[
\chi_b = -\eta_A M_0^a , \chi^a = -M_0^a \eta^b ,
\]

(2.9)

and consequently (2.8) is equal to

\[
I = \int d\tau \left( \eta_A \partial_\tau \eta^a + \eta_A (M_{0c} M_0^c + M_{1b}^a) \eta^b \right) ,
\]

(2.10)

with

\[
M_{0c} M_0^c = (M_0^b)^a = \begin{pmatrix}
-\frac{-\alpha'}{2}[A_\mu, A_\nu]\psi^\mu\psi^\nu + \overleftrightarrow{T} T & -i\sqrt{\alpha'}(TA_\mu - A_\mu T)\psi^\mu \\
-i\sqrt{\alpha'}(TA_\mu - A_\mu T)\psi^\mu & -\frac{-\alpha'}{2}[A'_\mu, A'_\nu]\psi^\mu\psi^\nu + \overleftrightarrow{T} T
\end{pmatrix} .
\]

(2.11)

We must stress that odd grading of off-diagonal elements \( T \) is crucial for the emergence of the correct form of the covariant derivative. Then (2.10) is equal to

\[
I = \int d\tau \left( \eta_A \partial_\tau \eta^a + \eta_A M_0^a \eta^b \right) , M_0^a = M_{1b}^a + (M_0^2)^a_b ,
\]

\[
\mathcal{M} = \begin{pmatrix}
-i(A_\mu \partial_\tau X^\mu + \frac{\alpha'}{2} F_{\mu\nu} \psi^\mu \psi^\nu) + \overleftrightarrow{T} T & -\sqrt{\alpha'} D_\mu T \psi^\mu \\
-\sqrt{\alpha'} D_\mu T \psi^\mu & -i(A'_\mu \partial_\tau X^\mu + \frac{\alpha'}{2} F'_{\mu\nu} \psi^\mu \psi^\nu) + \overleftrightarrow{T} T
\end{pmatrix} ,
\]

(2.12)

with

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] , F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu - i[A'_\mu, A'_\nu] ,
\]

\[
D_\mu T = \partial_\mu T + i(TA_\mu - A'_\mu T) , \overleftrightarrow{D_\mu T} = \partial_\mu T + i(TA'_\mu - A_\mu T) .
\]

(2.13)
As in \cite{19} we perform some simple tests that will show that \eqref{2.12} could really be the correct form of the action for $D\overline{D}$ system. Firstly, for $T = 0$ we expect that the partition function will be equal to the sum of two independent partition functions for $N$ $D$-9-branes and $N$ $\overline{D}$-9-branes. In this case $\mathcal{M}$ is block diagonal and when we express the integral over $\eta, \overline{\eta}$ in terms of the path-ordered trace then the partition function is a sum of two independent terms corresponding to $N$ $D$-9-branes and $N$ $\overline{D}$-9-branes respectively

$$S_{D+\overline{D}, T=0} = \sum_{D, \overline{D}} <\text{Tr} P \exp \left( i \int d\tau \left[ A_\mu(X) \partial_\tau X^\mu + \frac{\alpha'}{2} F_{\mu\nu} \psi^\mu \psi^\nu \right] \right) > . \quad (2.14)$$

The more nontrivial task is to show that \eqref{2.12} reduces to the partition function for a non-BPS D9-brane through the orbifolding projection $(-1)^F_L$ \cite{24} that acts on the tachyon and gauge fields as

$$T = \overline{T}, A_\mu = A'_\mu \Rightarrow D_\mu T = \partial_\mu T - i[A_\mu, T], \overline{D}_\mu T = \partial_\mu T - i[A_\mu, T], \quad \mathcal{M} = \begin{pmatrix} -i(A_\mu \partial_\tau X^\mu + \frac{\alpha'}{2} F_{\mu\nu} \psi^\mu \psi^\nu) + TT & -\sqrt{\alpha'} D_\mu T \psi^\mu \\ -\sqrt{\alpha'} D_\mu T \psi^\mu & -(A_\mu \partial_\tau X^\mu + \frac{\alpha'}{2} F_{\mu\nu} \psi^\mu \psi^\nu) + TT \end{pmatrix} . \quad (2.15)$$

We again rewrite the path integral over $\eta, \overline{\eta}$ as a path-ordered trace and then express $\mathcal{M}$ as

$$\mathcal{M} = 1_{2\times 2} \otimes \mathcal{M}_1 + \sigma_1 \otimes \mathcal{M}_2, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \quad (2.16)$$

It is clear that the trace over $2N \times 2N$ matrices is the same as the trace over $N \times N$ matrices $\mathcal{M}_{0,1}$ and the trace over $2 \times 2$ matrices 1, $\sigma_1$

$$\text{Tr} \rightarrow \text{Tr}_N \text{Tr}_{2\times 2} . \quad (2.17)$$

When we convert the trace over $N \times N$ matrices into the path integral over fermions it is clear that now we only need $N$ fermionic fields $\eta^a, \overline{\eta}_a$. To proceed further we must express the $2 \times 2$ matrices as fermionic fields. We will mainly follow \cite{12}. Consider the basic fermionic quantum system, two states $|\rightarrow\rangle, |\rightarrow\rangle$ with raising operator $\xi$ and lowering operator $\overline{\xi}$

$$\xi |\rightarrow\rangle = 0, \overline{\xi} |\rightarrow\rangle = |\rightarrow\rangle, \xi |\rightarrow\rangle = |\rightarrow\rangle, \overline{\xi} |\rightarrow\rangle = 0, \{\xi, \overline{\xi}\} = 1, \xi^2 = \overline{\xi}^2 = 0 . \quad (2.18)$$

Then immediately follows that $\xi, \overline{\xi}$ have a matrix representation in the basis $|\rightarrow\rangle, |\rightarrow\rangle$

$$\xi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_1 + i\sigma_2) , \overline{\xi} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_1 - i\sigma_2) . \quad (2.19)$$

$^3$\(\sigma_1, \sigma_2\) are ordinary Pauli matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$
We see that we can replace $\sigma_1$ in the partition sum with $\xi + \bar{\xi}$ and the trace with the path-integral over fermionic fields $\xi, \bar{\xi}$. More precisely, when we insert $\int d\tau \bar{\xi} \partial_\tau \xi$ into the action $I$ we obtain

$$I = \int d\tau \left( \Pi_a \partial_\tau \eta^a + \bar{\eta} \partial_\tau \xi - i \Pi_a \left( A^a_{\mu b} \partial_\tau X^\mu + \frac{\alpha'}{2} F^a_{\mu b} \psi^\mu \psi^b \right) \eta^b + \Pi_a T^a_c \eta^c b - (\xi + \bar{\xi}) \Pi_a \sqrt{\alpha'} D_\mu T^a_b \psi^\mu \eta^b \right).$$

It is important to stress that thanks to the replacement $\sigma_1$ with the fermionic operator $\xi + \bar{\xi}$ the tachyon field $T(X)$ should be regarded as a field with even grading. The integration over $\xi, \bar{\xi}$ gives

$$\xi = \sqrt{\alpha'} \frac{1}{\partial_\tau} \left( \Pi_a D_\mu T^a_b \psi^\mu \eta^b \right), \quad \bar{\xi} = \sqrt{\alpha'} \frac{1}{\partial_\tau} \left( \Pi_a D_\mu T^a_b \psi^\mu \eta^b \right),$$

so that

$$\bar{\xi} \partial_\tau - (\xi + \bar{\xi}) \Pi_\mu T^\mu \eta = -\alpha' \frac{1}{\partial_\tau} (\Pi \mu T^\mu \eta) (\Pi \nu T^\nu \eta) = \alpha' (\Pi \mu T^\mu \eta) \frac{1}{\partial_\tau} (\Pi \nu T^\nu \eta),$$

where in the last step we have used the anticommuting property of fermionic fields. Finally, $S$ is equal to

$$S = < e^{-I} > = \exp \left( - \int d\tau \left\{ \Pi_a \partial_\tau \eta^a - i \Pi_a \left( A^a_{\mu b} \partial_\tau X^\mu + \frac{\alpha'}{2} F^a_{\mu b} \psi^\mu \psi^b \right) \eta^b + \Pi_a T^a_c \eta^c b + \alpha' \left( \Pi_a D_\mu T^a_b \psi^\mu \eta^b \right) \frac{1}{\partial_\tau} \left( \Pi_a D_\nu T^a_b \psi^\nu \eta^b \right) \right\} \right),$$

which is in agreement with the partition function for $N$ non-BPS D9-branes given in [11, 21]. In the next section we use this partition function for $N$ non-BPS D0-branes to review the results presented in [11].

### 3. D-branes from $N$ non-BPS D0-branes

In this section we will construct all D-branes from $N$ non-BPS D0-branes in type IIB theory in the same way as in [11]. We start with the action (2.23). Using transformation properties of the gauge fields and covariant derivatives under T-duality

$$A_I(X^\mu) \to \Phi_I(X^0), \quad I, J = 1, \ldots, 9, \quad \partial_\tau X^I \to \partial_\eta X^I, \quad T(X^\mu) \to T(X^0), \quad F_{IJ} \to -i[\Phi_I, \Phi_J], \quad F_{0I} \to D_0 \Phi_I, \quad D_I T = -i[\Phi_I, T],$$

(3.1)
we obtain the partition function for \( N \) non-BPS D0-branes. It is important to stress that all background fields \( A_0, \Phi_I, T \) are functions of \( X^0(\tau) \) only since this is the string coordinate with the Neumann boundary condition. In the following we will work with the time independent background field so that they are not functions of \( X^0 \) and we also use gauge invariance to eliminate the gauge field \( A_0 \). Then the BSFT action for \( N \) non-BPS D0-branes is

\[
S(\Phi, T) = \langle e^{-I} \rangle = \langle \exp \left( -\int d\tau \left( \Pi_0 \partial_\tau \Pi^0 - i \Pi_a \left( \Phi^a_{\mu} \partial_n X^I - i \frac{\alpha'}{2} [\Phi_I, \Phi_J]_b^a \psi^I \psi^J \right) \eta^b + \Pi_a T^c_\mu \eta^b \right) \left( \Pi_a [\Phi_I, T]^c_\mu \psi^I \psi^J \right) \frac{1}{\partial_\tau} \left( \Pi_a [\Phi_J, T]^c_\mu \psi^I \psi^J \right) \right) \rangle .
\]  

(3.2)

We will show that the tachyon condensation in the action given (3.2) can describe all D-branes in type IIB theory, following [41]. Let us consider the background configuration in the form

\[
T = 0, \ [\Phi_a, \Phi_b] = i \theta_{ab}, \ a = 1, \ldots, 2p, \ \Phi_\alpha = 0, \ \alpha = 2p + 1, \ldots, 9 .
\]  

(3.3)

Then the fermionic term in (3.2) is equal to

\[
- i \frac{\alpha'}{2} [\Phi_a, \Phi_b] \psi^a(\tau) \psi^b(\tau) = \frac{\alpha'}{2} \theta_{ab} \psi^a(\tau) \psi^b(\tau) 1_{N \times N} .
\]  

(3.4)

We see that this term is proportional to the unit matrix so that the following expression can be taken out the trace

\[
\exp \left( i \int d\tau \frac{\alpha'}{2} \theta_{ab} \psi^a(\tau) \psi^b(\tau) \right) .
\]  

(3.5)

The bosonic part can be worked out exactly as in [41] and we briefly review this calculation. From (3.3) we see that \( \Phi_a \) are equivalent to the quantum mechanics operators with nontrivial commuting relations. Then we can use the well known relation between the trace over Hilbert space and the path integral

\[
\text{Tr} P \exp \left( -i \int d\tau H(\tau) \right) = \exp \left( i \int d\tau [p(\tau) \dot{q}(\tau) - H(p(\tau), q(\tau))] \right) ,
\]  

(3.6)

with \( \dot{q} = \partial_\tau q \) and where \( p \) is a momentum conjugate to \( q \). In our case, the Hamiltonian is

\[
H(\tau) = -\Phi_a \partial_n X^a(\tau) .
\]  

(3.7)

with the operators \( \Phi_a \). Then we can rewrite the trace in (3.2) as a path integral (For the time being we omit the fermionic term which is proportional to the unit matrix)

\[
\int \prod_{a=1}^{2p} [\phi_a] \exp \left( i \int d\tau \left( \frac{1}{2} \phi_a(\tau) \theta_{ab} \phi_b(\tau) + \phi_a(\tau) \partial_n X^a(\tau) \right) \right) ,
\]  

(3.8)
with \( \theta_{ac} \theta^{cb} = \delta^b_a \). We can easily perform the integration over \( \phi \)
\[
\int [d\phi_a] \exp \left( - \int d\tau d\tau' \left[ \frac{1}{2} \phi_a(\tau') \Delta(\tau', \tau)^{ab} \phi_b(\tau) - \phi_a(\tau) i \partial_n X^a(\tau) \right] \right) = \exp \left( - \frac{1}{2} \int d\tau d\tau' \partial_n X^a(\tau) \Delta(\tau', \tau)^{ab} \partial_n X^b(\tau') \right), \Delta(\tau', \tau)^{ab} = i \delta(\tau' - \tau) \theta^{ab}
\]
with
\[
\Delta(\tau, \tau')^{-1} = \theta^{ab} \frac{1}{2} \sum_n \frac{1}{n} e^{in(\tau - \tau')}.
\]

We expand the string field as
\[
X^a(\tau, \rho) = \sqrt{\frac{\alpha'}{2}} \sum_{n=-\infty}^{\infty} \rho^m X^a_n e^{in\tau}, \quad \partial_n X^a(\tau, \rho = 1) = \sqrt{\frac{\alpha'}{2}} \sum_{n=-\infty}^{\infty} n X^a_n e^{in\tau}.
\]
Note that there is no zero mode thanks to the Dirichlet boundary conditions. Then (3.9) is equal to
\[
\frac{1}{2} \int d\tau d\tau' \partial_n X^a(\tau) \Delta(\tau, \tau')^{-1} \partial_n X^b(\tau') = \alpha' \pi \sum_{m=1}^{\infty} m \theta^{ab} X^a_n X^b_n,
\]
that agrees precisely with the expression [18]
\[
S = \frac{i}{2} \int_0^{2\pi} d\tau F_{ab} X^a(\tau) \dot{X}^b(\tau) = \pi \alpha' \sum_{n=1}^{\infty} n F_{ab} X^a_n X^b_n, \quad F_{ab} = \theta_{ab},
\]
arising from the term
\[
S_A = -i \int_0^{2\pi} d\tau A_a(\tau) \partial_\tau X^a(\tau).
\]
As a result, we have obtained the partition sum on the disk for a D2p-brane in the presence of the gauge field \( F_{ab} \)
\[
e^{-I} = \exp \left( i \int d\tau \left[ A_a(\tau) \partial_\tau X^a(\tau) + \frac{\alpha'}{2} F_{ab} \psi^a(\tau) \psi^b(\tau) \right] \right), \quad F_{ab} = \partial_a A_b - \partial_b A_a = \theta_{ab}.
\]
Consequently the action is equal to [21] [10] \( ^4 \)
\[
S(F) = Z(F) = < e^{-I} > = - \sqrt{2T_2} \int dtd^{2p}x \sqrt{\det(\delta_{ab} + 2\pi \alpha' F_{ab})}.
\]
In order to describe odd dimensional D-branes we should include the tachyon in the boundary action and calculate its condensation. Let us consider the ansatz
\[
T = u \Phi_2, \quad [\Phi_x, \Phi_y] = i \epsilon_{xy} \theta, \quad x, y = 1, 2, \quad [\Phi_i, \Phi_j] = i \theta_{ij}, \quad i, j = 3, \ldots, 2p,
\]
\( ^4 \)We consider Minkowski space-time with the metric \( \eta_{\mu\nu} = \text{diag}(-1, 1, \ldots, 1) \).
so that $\theta$ in $[\Phi_a, \Phi_b] = i\theta_{ab}$ has a form

$$ \theta = \begin{pmatrix} 0 & \theta & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & \theta_{ij} \end{pmatrix}, i, j = 3, \ldots, 2p . $$ (3.18)

Then (3.2) is equal to

$$ S = < e^{-I} > = \int [d\phi_a] \exp \left( \int d\tau \left[ i\frac{1}{2} \phi_a \theta^{ab} \dot{\phi}_b + -\alpha' u^2 \theta^2 \psi^1 \frac{1}{\partial_r} \psi^1 - u^2 (\phi_2)^2 + i\phi_a \partial_n X^a + \frac{i\alpha'}{2} \theta_{ab} \psi^a \psi^b \right] \right) > . $$ (3.19)

We can take out the following expression from the path integral over $\phi$

$$ \exp \left( \int d\tau \left[ \alpha' u^2 \theta^2 \psi^1 \frac{1}{\partial_r} \psi^1 + \frac{i\alpha'}{2} \theta_{ab} \psi^a \psi^b \right] \right) . $$ (3.20)

Next calculation is the same as the case with vanishing tachyon field. The $i, j = 3, \ldots, 2p$ components give the same result as in the case of even dimensional D-brane without exciting tachyon and the integration over $x, y = 1, 2$ gives

$$ \exp \left( -\frac{1}{2} \int d\tau d\tau' \partial_n X^x(\tau) \Delta(\tau - \tau')^{-1}_{xy} \partial_n X^y(\tau') \right) , $$

$$ \Delta(\tau - \tau')^{-1}_{xy} = \sum_n (E_n)^{-1}_{xy} e^{i(n(\tau - \tau'))}, (E_n^{-1})_{xy} = \frac{\theta^2}{n^2} \left( \frac{n}{\theta} - 1 \right) , $$ (3.21)

which together with the $F_{ij}$ and $F_{xy}$ terms gives the result \[11, 12\]

$$ S = < e^{-I} > = \exp \left( \int d\tau \left[ -(u^2 (X^1)^2 + \alpha' u^2 \psi^1 \frac{1}{\partial_r} \psi^1 + +iA_a(X^a) \partial_r X^a + \frac{i\alpha'}{2} F_{ab} \psi^a \psi^b \right] \right) > , F_{ab} = \theta_{ab} , $$ (3.22)

where we have made replacement $u \theta \rightarrow u$. Then it is easy to see that the tachyon condensation really leads to the emergence of odd dimensional D-brane exactly in the same way as in \[11\]. The partition function is equal to \[11, 12, 16\]

$$ Z = K \int dt d^{2p-1}x Z(a, v)_{\text{fermi}} \sqrt{\text{det}(\delta_{ij} + 2\pi \alpha' F_{ij})} , v = \frac{4\pi \alpha' u^2}{1 + (2\pi \alpha')^2} . $$ (3.23)

with \[11\]

$$ Z(a, v)_{\text{fermi}} = 4^v \frac{Z_1(v)^2}{Z_1(2v)} , Z_1(v) = \sqrt{v} e^\gamma v \Gamma(v) , $$ (3.24)

9
and where \( K \) is a numerical factor that will be determined as in [11]. When we calculate the partition sum for constant tachyon \( T = a_{1N \times N} \) we obtain the exact tachyon potential \( e^{-2\pi a^2} \) multiplied with the DBI term \( \sqrt{\det(\delta_{ab} + 2\pi \alpha' F_{ab})} \) arising from the partition sum calculated in the pure gauge field background. Then we can expect that for a slowly varying tachyon field the action corresponds to the non-BPS D2p-brane action

\[
S = -\sqrt{2} T_{2p} \int dt d^{2p} x e^{-2\pi T^2} \sqrt{(1 + (2\pi \alpha' \theta)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})} + O(\partial T),
\]

This action evaluated on the tachyon profile \( T = u x^1 \) should be equal to the partition sum in the limit \( u \to 0 \). From this requirement we can determine the overall normalisation constant \( K \). Then the action (3.25) evaluated on the tachyon profile \( T(x) = u x^1 \) is equal to

\[
-\sqrt{2} T_{2p} \int dt d^{2p-1} x \sqrt{(1 + (2\pi \alpha' \theta)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})} \int d x^1 e^{-2\pi u^2(x^1)^2} =
\]

\[
= -\frac{1}{u} T_{2p} \int d^{2p-1} x \sqrt{(1 + (2\pi \alpha' \theta)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})}.
\]

(3.26)

On the other hand, in the limit \( u \to 0 \) we have

\[
Z(v)_{\text{fermi}} = \sqrt{\frac{2}{v} + O(v)}, \quad v \sim 0,
\]

(3.27)

so that the partition function is equal to

\[
S = K \frac{1}{\sqrt{2\pi \alpha' u^2}} \int dt d^{2p-1} x \sqrt{(1 + (2\pi \alpha' \theta)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})}
\]

(3.28)

and consequently

\[
K = -T_{2p} \sqrt{2\pi \alpha'}.
\]

(3.29)

Using this result it is easy to see that the action arising from the tachyon condensation is equal to (In this case the tachyon condensation corresponds to \( u \to \infty \) [11])

\[
S = Z(\infty) = -\sqrt{2\pi \alpha'} T_{2p} \sqrt{2\pi} \int dt d^{2p-1} x \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})} =
\]

\[
= -T_{2p-1} \int dt d^{2p-1} x \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})},
\]

(3.30)

where we have used

\[
Z(v) \sim \sqrt{2\pi} + O(v^{-1}), \quad u \to \infty.
\]

(3.31)

The result (3.30) is a correct value of the action for D(2p-1)-brane with the background gauge field strength \( F_{ij} \). However, we must stress one important thing. It
seems that in this approach we cannot construct space-time filling D9-brane as can be seen from (3.17). On the other hand we can construct the space-time filling unstable $D\overline{D}9$ system with equal gauge field background. Let us consider formally the same ansatz as (3.17)

$$T = u\Phi_{10}, \quad [\Phi_x, \Phi_y] = i\epsilon_{xy}\theta, \quad x, y = 9, 10, \quad [\Phi_i, \Phi_j] = i\theta_{ij}, \quad i, j = 1, \ldots, 8,$$

so that $\theta$ in $[\Phi_a, \Phi_b] = i\theta_{ab}$ has a form

$$\theta = \begin{pmatrix} \theta_{ij} & 0 & 0 \\ 0 & 0 & \theta \\ 0 & -\theta & 0 \end{pmatrix}, \quad i, j = 1, \ldots, 8. \quad (3.32)$$

Now $\Phi_{10}$ is quantum mechanical operator without any space-time meaning. We see that we can perform the same calculation as above with the result

$$S = \langle e^{-T} \rangle = \langle \exp \left( \int d\tau \left[ -(u^2(X^9)^2 + \alpha' u^2 \psi^9 \frac{1}{\partial_c} \psi^9) + \right. \\
+ iA_i(X^i) \partial_c X^i + \frac{i\alpha'}{2} F_{ij} \psi^i \psi^j \right] \right) >, \quad F_{ij} = \theta_{ij}, \quad (3.34)$$

where we have used the fact that there is no term proportional to $\partial_n X^{10}$. Naively we could say that this action corresponds to space-time filling D9-brane. However we know that D9-brane is stable so that it does not contain tachyon in its spectrum. Moreover, we know that D8-brane that arises from the tachyon condensation in the previous expression in the limit $u \to \infty$ is unstable so that its tension is $\sqrt{2}T_8$. We also know that in the limit $u \to \infty$ we get the numerical value (3.31). From these facts we conclude that the normalisation constant in (3.34) is equal to $2T_9$. In other words, the action (3.34) corresponds to the partition sum for $D\overline{D}9$ system with equal gauge field background on their world-volumes as it will be clear from the analysis in the next section.

In order to describe single D9-brane in this approach we propose other process which unfortunately does not seem to have simple description in BSFT. We construct an infinite number of D7-branes through the tachyon condensation and then single D9-brane arises by ordinary matrix theory construction reviewed above. Unfortunately, the tachyon condensation in BSFT describing general configuration of D-branes seems to be difficult problem since the boundary theory is not free [7]. For that reason we leave this issue for the future research.

4. $D\overline{D}$-system

The action proposed in (2) section corresponds to the space-time filling $D\overline{D}9$ system, for lower dimensional ones with $x^\mu, \mu = 0, \ldots, p$ Neumann boundary conditions and
then as a field with even grading. Then we obtain [18] Note that now the tachyon field is on the main diagonal so that it should be regarded where now $\Phi_i$ describe transverse fluctuations of $N$ Dp-branes and $\overline{D}p$-branes respectively and where $\tilde{D} = \partial_b + \theta \partial_n$.

In the following we will consider the configuration with the tachyon fields equal to the constant $M$ gauge invariance. Then $M$ has a form

$$M(\Phi, \Phi', T) = \begin{pmatrix} -i \Phi_I(\Phi^0)D X^I & \overline{T}(X^0) \\ T(X^0) & -i \Phi'_I(\Phi^0)\tilde{D} X^I \end{pmatrix}, \quad I = 1, \ldots, 9. \quad (4.1)$$

Performing the same calculation as in (2) section we obtain T-dual version of the action (2.12) (We consider time-independent background fields)

$$S(\Phi, \Phi', T) = < \exp (-I(\Phi, \Phi', T)) >, \quad I = \int d\tau \left( \eta_a \partial_\tau \eta^a + \pi_a M_a^b \eta_i \right),$$

$$M = \begin{pmatrix} -i(\Phi_I \partial_a X^I - i \frac{\alpha}{2} [\Phi_I, \Phi_J] \psi^I \psi^J) + \overline{T} T & i \sqrt{\alpha^I} (\Phi_I T - T \Phi'_I) \psi^I \\ -i \sqrt{\alpha^I} (T \Phi_I - \Phi'_I T) \psi^I & -i(\Phi_I \partial_a X^I - i \frac{\alpha}{2} [\Phi_I, \Phi_J] \psi^I \psi^J) + \overline{T} T \end{pmatrix}. \quad (4.3)$$

To begin with, consider the configuration with the tachyon fields equal to the constant value $T$ and with all D0-branes sitting in the origin which corresponds to $\Phi^I, \Phi'^I = 0$. Then

$$\mathcal{M} = \begin{pmatrix} \overline{T} T & 0 \\ 0 & \overline{T} T \end{pmatrix}. \quad (4.4)$$

Note that now the tachyon field is on the main diagonal so that it should be regarded as a field with even grading. Then we obtain [18]

$$S = 2 T_0 \text{Tr}_{N \times N} e^{-2\pi i \overline{T} T}. \quad (4.5)$$

We see that the unstable $D\overline{D}0$-system corresponds to the tachyon value $T = 0$ and the closed string vacuum to the value $T = \infty$. It is also clear that we can construct many configurations around the perturbative unstable vacuum $T = 0$, for example

$$[\Phi^1_1, \Phi^1_j] = i \theta^1_{ij}, \quad i, j = 1, \ldots, 2p_1, \quad [\Phi^2_i, \Phi^2_j] = i \theta^2_{ij}, \quad i, j = 1, \ldots, 2p_2 \quad (4.6)$$

with the first commutator corresponding to D0-branes and the second one to $\overline{D}0$-branes. When we insert this ansatz into (2.13) we see that $\mathcal{M}$ is again block diagonal and consequently the partition function reduces to two independent parts corresponding to the partition sum for $D2p_1$-brane and $D2p_2$-brane respectively

$$S_{D2p_1 + D2p_2} = \sum_{k=1}^{2} < \text{Tr} P \exp \left( i \int d\tau \left[ \Phi^k_i \partial_n X^i + \frac{\alpha'}{2} \theta^k_{ij} \psi^i \psi^j \right] \right) >. \quad (4.7)$$
Each partition function is equivalent to the partition function for $D2p_{1,2}$-brane with the background gauge field $\theta_{ij}^{1,2}$ as we have shown in [41]. As we can expect from the general theory [26] this configuration either annihilates to the closed string vacuum or to the state with the lower dimensional D-brane charge. The final state of the configuration depends on the form of the background field strengths $F_{ij}^{1,2} = \theta_{ij}^{1,2}$.

It would be nice to study this process in the BSFT theory however it seems to be difficult task since the world-sheet theory is not free even in the case of the constant tachyon field. On the other hand, in [18, 19] the lower dimensional D-branes emerge naturally from $D0\overline{D}$ system even in the case of the vanishing gauge field. We can then expect that higher dimensional D-branes arise from $D0\overline{D}$ system in the similar manner.

To see directly the emergence of D-branes we should turn on the tachyon field. We restrict ourselves on the linear profile of the tachyon as in [18]. Let us consider the ansatz

$$[\Phi_a, \Phi_b] = i\theta_{ab}, \ a, b = 1, \ldots, 2p, \ [\Phi'_i, \Phi'_j] = i\theta_{ij}, \ a, b = 1, \ldots, 2p, \ T = u^a\Phi_a, \ (4.8)$$

where $u^a$ are complex numbers. In other words, we presume that $\Phi, \Phi'$ are the same quantum mechanical operators. Then $\mathcal{M}$ is equal to

$$\mathcal{M} = \left( -i(\Phi_a \partial_n X^a + \frac{\alpha'}{2} \theta_{ab} \psi^a \psi^b) + u^a u^b \Phi_a \Phi_b \frac{\sqrt{\alpha'} \theta_{ab} u^a \psi^b}{\sqrt{\alpha'} \theta_{ab} u^a \psi^b} \right) \left( -i(\Phi_a \partial_n X^a + \frac{\alpha'}{2} \theta_{ab} \psi^a \psi^b) + u^a u^b \Phi_a \Phi_b \right) \ .$$

(4.9)

We rewrite the path integral over $\eta$ as the path-ordered trace. Then as in (2) section we can write $\mathcal{M}$ as

$$\mathcal{M} = 1_{2\times 2} \otimes \mathcal{M}_1(\Phi)_{N_x N} + \mathcal{M}_2,2 \otimes 1_{N_x N} , \ (4.10)$$

Now we can easily rewrite the first term in the form of the path integral over $\phi_a$ so we obtain

$$\langle e^{-I} \rangle = \text{Tr}_{2 \times 2} P \int [d\phi_a] \exp \left( \int d\tau \left[ 1_{2 \times 2} \otimes \left( \frac{1}{2} \phi_a \theta_{ab} \phi_b \right) - u^a u^b \phi_a \phi_b + i\phi_a \partial_n X^a \right) - \frac{\alpha'}{2} \theta_{ab} u^a \psi^b \right) \right) > . \ (4.11)$$

We propose the ansatz

$$T = \sum_{x=1}^{4} u^x X_x, \ u^1 = u_1, \ u^2 = u_4 = 0, \ u^3 = i u_3, \ [\Phi_a, \Phi_b] = i\theta_{ab}, \ a, b = 1, \ldots, 2p, \ (4.12)$$

(4.11)
with
\[
\theta_{ab} = \begin{pmatrix}
0 & \theta_1 & 0 & 0 \\
-\theta_1 & 0 & 0 & 0 \\
0 & 0 & \theta_2 & 0 \\
0 & -\theta_2 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad i, j = 5, \ldots, 2p .
\] (4.13)

We must stress that this is not the most general ansatz, rather this is the background for which we can explicitly calculate the BSFT action. Then
\[
u_i \bar{u}_j^* \phi \bar{\phi} \to u_i^2 \phi_1^2 + u_2^2 \phi_3^2
\] (4.14)

and also
\[
(E_n)_{xy} = \begin{pmatrix}
-u_1^2 & \frac{n}{\theta_1} & 0 & 0 \\
-\frac{n}{\theta_1} & 0 & 0 & 0 \\
0 & 0 & -u_2^2 & \frac{n}{\theta_2} \\
0 & 0 & -\frac{n}{\theta_2} & 0
\end{pmatrix}, \quad \det (E_{xy}) = \frac{n^2}{\theta_1^2 \theta_2^2}, \quad (E_n)^{-1} = \begin{pmatrix}
0 & -\frac{\theta_2}{n} & 0 & 0 \\
\frac{\theta_2}{n} & -\frac{n^2 u_2^2}{\theta_2^2} & 0 & 0 \\
0 & 0 & 0 & -\frac{\theta_2}{n} \\
0 & 0 & -\frac{n^2 u_2^2}{\theta_2^2}
\end{pmatrix}.
\] (4.15)

Terms proportional to \( \theta_{ij} \) give the standard result reviewed in previous parts. On the other hand, for \( x, y = 1, \ldots, 4 \) we obtain
\[
-\frac{1}{2} \int d\tau d\tau' \partial_{\tau} X^x \Delta (\tau - \tau')_{xy}^{-1} \partial_\tau X^y (\tau') = \frac{\pi \alpha'}{2} \sum_n \theta_{xy} n X_n^x X_n^y - \\
-\frac{\pi \alpha'}{2} \sum_{n, n \neq 0} (\bar{u}_2^2 X_n^2 X_n^2 + \bar{u}_4^4 X_n^2 X_n^4) = \frac{i}{2} \int_0^{2\pi} d\tau \theta_{xy} X^x (\tau) X^y (\tau) - \int_0^{2\pi} d\tau T (X)^2 ,
\]
\[
T (X) = \bar{u}_2^2 (X^2 (\tau))^2 + \bar{u}_4^4 (X^4 (\tau))^2, \quad \bar{u}_2 = \theta_1 u_1, \quad \bar{u}_4 = \theta_2 u_3 ,
\] (4.16)

so we have
\[
< e^{-F} > = < \text{TR}_{2 \times 2} P \exp \left( \int d\tau \left[ 1_{2 \times 2} \otimes (i A_a (X) \partial_\tau X^a + \frac{i \alpha' F_{ab}}{2} \phi^a \phi^b - T (X)^2) - \\
-\sqrt{\alpha'} \left( \begin{pmatrix}
0 & \theta_{12} u_1 \psi^2 + i \theta_{34} u_3 \psi^4 & 0 \\
\theta_{12} u_1 \psi^2 + i \theta_{34} u_3 \psi^4 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix} \right) \right] > \exp \left( \int d\tau [i A_a (X) \partial_\tau X^a + \\
+ \frac{i \alpha' F_{ab}}{2} \psi^a \psi^b - T (X)^2 - \bar{\xi} \xi - (\xi + \bar{\xi}) \sqrt{\alpha'} \bar{\psi}_2 \psi^2 - i \sqrt{\alpha'} (\xi - \xi) \bar{\psi}_4 \psi^4] \right) > ,
\] (4.17)

where we have used the correspondence between fermionic fields \( \xi, \bar{\xi} \) and Pauli matrices which was explicitly demonstrated in (3) section. It is now easy to study the tachyon condensation, following [18, 13]. Firstly we integrate out the fermionic field \( \xi, \bar{\xi} \) which gives
\[
\xi = -\sqrt{\alpha'} \frac{1}{\partial_\tau} (u_2 \psi^2 + i u_4 \psi^4), \quad \bar{\xi} = -\sqrt{\alpha'} \frac{1}{\partial_\tau} (u_2 \psi^2 - i u_4 \psi^4),
\] (4.18)
where we have renamed $\tilde{u}_x \to u_x$. Consequently we have
\[
< e^{-T} >= \exp \left( \int dt \left[ iA_a \partial_x X^a + \frac{i\alpha'}{2} F_{ab} \psi^a \psi^b - T(X)^2 - \alpha' \left( u_2^2 \psi^2 \frac{1}{\partial_x} \psi^2 + u_4^2 \psi^4 \frac{1}{\partial_x} \psi^4 \right) \right] \right) > .
\]

(4.19)

Now the calculation is the same as in the original papers [11, 18, 19]. Thanks to the special form of the ansatz (4.8, 4.12) the partition sum factorises into the subspaces labelled with the coordinates $x_{1,2}, x_{3,4}$ and $x_{5,\ldots,2p}$. Then we can use the approach outlined in the previous section with the result
\[
S = K \int dt d^{2p-2} x Z(v_1, v_2) e^{-2\pi T} \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})},
\]
\[
Z(v_1, v_2) = 2 \prod_{i=1}^2 4^{v_{2i}} \frac{Z_1(v_{2i})^2}{Z_1(2v_{2i})}, Z_1(x) = \sqrt{x} e^{\gamma T} \Gamma(x), \quad v_{2i} = \frac{4\pi \alpha' u_{2i}}{1 + (2\pi \alpha' \theta_i)^2}.
\]

(4.20)

As usual we can expect that for a slowly varying tachyonic field this action reduces to the sum of two DBI actions for a D-brane and an anti-D-brane respectively. However thanks to the particular choose of the background fields these two actions are equal so that we expect that the action will have a form
\[
S = -2T_{2p} \int dt d^{2p-2} x \sqrt{\det(\delta_{ab} + 2\pi \alpha' F_{ab}) + O(\partial T)} = -2T_{2p} \int dt d^{2p-2} x \sqrt{(1 + (2\pi \alpha' \theta_1)^2)(1 + (2\pi \alpha' \theta_2)^2)} \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij}) + O(\partial T)}.
\]

(4.21)

As in the previous section we should work out this action on the tachyon profile $T \mathcal{T} = u_2^2(x^2)^2 + u_4^2(x^4)^2$ which gives
\[
S = -2T_{2p} \int dt d^{2p-1} x \sqrt{(1 + (2\pi \alpha' \theta_1)^2)(1 + (2\pi \alpha' \theta_2)^2)} \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})} \times \int dx^2 e^{-2\pi u_2^2(x^2)^2} \int dx^4 e^{-2\pi u_4^2(x^4)^2} = -\frac{T_{2p}}{u_2 u_4} \int dt d^{2p-2} x \sqrt{\det(\delta_{ab} + 2\pi \alpha' F_{ab})}.
\]

(4.22)

On the other hand, for $u_i \sim 0$ we have $Z(v_1, v_2) \sim \frac{2}{\sqrt{u_2 u_4}}$ and consequently (4.20) is equal to
\[
S = K \int dt d^{2p-2} x \sqrt{(1 + (2\pi \alpha' \theta_1)^2)(1 + (2\pi \alpha' \theta_2)^2)} \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})}.
\]

(4.23)

Comparing these two expressions we obtain the value of the normalisation constant
\[
K = -2\pi \alpha' T_{2p}.
\]

(4.24)
Then it is easy to study the tachyon condensation. For \( u_2 \to \infty \) and \( u_4 \to \infty \) we have

\[
4 v Z_1(v)^2 \sim \sqrt{2\pi} v \to \infty
\]

and consequently the action is equal to

\[
-T_{2p-2} \int dt d^{2p-2}x \sqrt{\det(\delta_{ij} + 2\pi \alpha' F_{ij})}
\]

which is the correct value of the action for \( D_{2p-2} \)-brane with the background gauge field \( F_{ij} \). On the other hand, for \( u_2 \to 0, u_4 \to \infty \) we have

\[
Z(v_1, v_2) \to \frac{\sqrt{4\pi}}{\sqrt{v_2}} = \frac{\sqrt{1 + (2\pi \theta_1)^2}}{\sqrt{\alpha' u_2}}
\]

so that

\[
S = -\sqrt{2} T_{2p-1} \int dt d^{2p-2}x \sqrt{(1 + (2\pi \alpha' \theta_1)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})} = -\sqrt{2} T_{2p-1} \int dt d^{2p-2}x \sqrt{(1 + (2\pi \alpha' \theta_1)^2) \det(\delta_{ij} + 2\pi \alpha' F_{ij})}
\]

which is a correct action for non-BPS \( D(2p-1) \)-brane with the background gauge field \( F_{12} = \theta_1, F_{ij} \). To conclude, we have shown in this section that \( D_p \)-branes, \( p \leq 6 \) naturally arise from \( N \) \( \overline{D0} \) and \( \overline{D0} \)-branes in the limit \( N \to \infty \) through the particular ansatz (4.12). We also see that (4.12) cannot lead to the emergence of \( D7, D8 \) and \( D9 \)-brane since the maximal number of \( 2p = 8 \). In order to describe these \( D \)-branes we should proceed in the same way as was outlined in the end of the previous section. In order to describe non-BPS \( D9 \)-brane we take the ansatz

\[
[\Phi_a, \Phi_b] = i\theta_{ab}, a, b = 1, \ldots, 8, \quad [\Phi'_a, \Phi'_{-a}] = i\theta_{ab}, a, b = 1, \ldots, 8,
\]

\[
T = u\Phi_{10}, \quad u = u^*, \quad [\Phi_9, \Phi_{10}] = i\theta,
\]

with \( \theta \) the same as in (3.33). When we insert this configuration into (4.3) \( \mathcal{M} \) is equal to

\[
\mathcal{M} = \begin{pmatrix}
-i(\Phi_a \partial_n X^a + \frac{\alpha'}{2} \theta_{ab} \psi^a \psi^b) + u^2 \Phi_{10}^2 & \sqrt{\alpha' \theta} u \psi^9 \\
\sqrt{\alpha' \theta} u \psi^9 & -i(\Phi_a \partial_n X^a + \frac{\alpha'}{2} \theta_{ab} \psi^a \psi^b) + u^2 \Phi_{10}^2
\end{pmatrix}
\]

We see that the previous expression is a special form of \( \mathcal{M} \) which we have discussed in (3) section and which leads to the partition sum for non-BPS \( D \)-brane through
the $(−1)^{F_L}$ orbifold projection. Then the previous expression naturally leads to the partition function for D9-brane whose boundary theory has two fix points: one corresponding to non-BPS D9-brane and the second one to BPS D8-brane. We see that D8 and D9-branes naturally arise from the $D\overline{D}0$ system. And finally, to describe D7-brane we can use a modification of (4.3) where we replace one part of the tachyon field (say $T = u_9\Phi^3$) with $T = u_9\Phi^{10}$ with the commutation relations between various $\Phi$' the same as in (4.29). Then it is easy to see that the limit $u_1, u_9 \to \infty$ leads to D7-brane exactly in the same way as we have shown in previous parts. We can then claim that all D-branes naturally emerge from $D\overline{D}0$ system.

5. Conclusion

In this paper we have studied the emergence of D-branes from $N$ of $D\overline{D}0$ system in type IIA theory, following our recent paper [41]. We have seen that the $D\overline{D}$-system is very interesting and seems to be the most general unstable configuration in type IIA, IIB theories. Many problems and issues with these configurations have been considered recently in the papers [45, 46, 47]. In this paper we have tried to show that the BSFT theory is also very useful tool for study the emergence of higher dimensional D-branes from lower dimensional ones.

However, many open question remains. We have seen that there is a problem with the tachyon condensation on the $D\overline{D}$-system with different gauge fields background. From the general arguments regarding tachyon condensation [24, 26] we can expect that after the tachyon condensation either lower dimensional D-brane emerges or the system ends in the closed string vacuum according to the value of background gauge field. It would be nice to study this process directly in BSFT theory. It would be also nice to study the fluctuations around various nontrivial configurations which arise from the tachyon condensation in the BSFT approach. And finally, it should be addressed the question of the relation BSFT to the Matrix theory.

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