Holographic Abelian Higgs model and the Linear confinement

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We consider the holographic abelian Higgs model and show that, in the absence of the scale symmetry breaking effect, chiral symmetry breaking gives linear confinement where the slope is given by the value of the chiral condensation. The model can be considered either as the theory of superconductivity or as the axial sector of QCD depending on the interpretation of the charge. We also provided a few models with linear confinement.

I. INTRODUCTION

One of most spectacular phenomena provided by the strong interaction is the stringy structure out of systems of particles, and the leading guidance in strong interaction has been the chiral symmetry. The string theory was born by the observation \([1, 2]\) that a linear relation between the mass squared and spin in the data of hadrons, the Regge trajectory, can be realized by the spectrum of a vibrating string. Naturally, in the era of the AdS/CFT \([3, 4]\), two of the leading questions in its application were how to utilize the chiral symmetry \([5]\) in the new context and whether holography can produce the linear trajectory for quantum chromodynamics (QCD).

When the vacuum expectation value of \(\bar{q}q\) is non-zero, chiral symmetry is broken and QCD is in the confining phase. Therefore these two are likely related and therefore linear Regge trajectory and the chiral symmetry breaking should be so also. However this has not been so clear even in the AdS/QCD era not alone in the perturbative field theory period, although the linear spectrum for the vector meson sector was given in \([6]\) using a dilaton configuration.

In this paper, we will point out that the holographic abelian Higgs model has linear confining spectrum. That is, the chiral symmetry breaking is enough to establish the Regge trajectory in the axial sector of the QCD. We consider one flavor case for simplicity so that the theory becomes an abelian gauge theory. Notice that even for the multi-flavor case, the non-linearity due to the non-abelian structure is irrelevant in leading order discussion on the spectrum and transports, which ensures that the linearity of the spectrum remains for multi-flavor case.

The left and right handed quarks, \(q_L, q_R\), have axial charge \(-1\) and \(+1\) respectively under the axial \(U(1)\) global symmetry. Invoking the holographic principle we have axial gauge field \(A_\mu\) and a complex scalar field \(\Phi\) which is the dual to the \(q_L q_R\) of charge 2. Because the presence of pion indicates that the chiral symmetry is broken spontaneously, so is the promoted chiral \(U(1)\) gauge symmetry. Thus we are naturally lead to the abelian Higgs model where \(\Phi\) plays the role of Higgs field. We will show below that this model has the linearly confining spectrum.

On the other hand, we will see that when there is a dilatonic effect like in the soft-wall model \([7]\) spontaneous chiral symmetry breaking (ChSB) will not give any significant contribution to the Regge slope, instead it can contribute to the Regge intercept. Namely the Regge slope is predominantly determined by the gluon condensation only and this explains why all the Regge slopes are the same. This can be traced back to the fact that the complex scalar field is dominated by the dilaton configuration in the infrared regime. Therefore in this paper we do not use overall dilaton factor \(e^{-\phi}\), and will give a phenomenological QCD model where spontaneous chiral symmetry is not disturbed by dilaton and linear confinement is respected in all sectors.

We will also give a various models with linear confinement properties which may and may not be related to the real QCD, because our observation on the stringy spectrum is very universal not necessarily attached to the phenomena of QCD.

II. STRINGS IN HOLOGRAPHIC ABELIAN HIGGS MODEL

We start from the canonical action of the gauge field \(A_\mu\) and the complex boson \(\Phi\) in a fixed metric background.

\[
S = \int d^{d+1}x \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu}^2 - |D_\mu \Phi|^2 - m_\Phi^2 |\Phi|^2 \right),
\]

where \(D_\mu = \nabla_\mu - i q A_\mu\) is the covariant derivative in the \(AdS_{d+1}\) of radius \(L = 1\) whose metric is

\[
d s^2 = (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)/z^2, \quad \text{with} \quad \eta^{00} = -1.
\]

Bulk mass \(m_\Phi^2\) is given in terms of the conformal dimension of the dual operator: \(m_\Phi^2 = \Delta(\Delta - d)\). We will fix it such that \(\Delta = 2\), which is natural in \(d = 2+1\) dimension. For \(d = 3+1\), we need to choose \(m_\Phi^2 = -4\) for \(\Delta_{qq} = 2\), which is realized at the left boundary value of conformal window of \(N_f/N_c\) \([8]\). Since we are applying the AdS/CFT in the confined phase at the low energy where the conformality is lost, the boundary value 2 is better than the free fermion value 3. The field equation

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\end{itemize}
then gives
\[ \Phi = M_0 z + M z^2, \quad \text{in AdS}_4, \]
\[ \Phi = M_0 z^2 \ln z^{-1} + M z^2, \quad \text{in AdS}_5. \]
which are exact solutions of the scalar field equations in
the probe limit. Since we look for dynamically generated
gap, we set the source \( M_0 = 0 \) so that \( \Phi = M z^2 \).

Now the Maxwell equation is given by
\[ \nabla^\mu F_{\mu\nu} = J_\nu \]  
and for the real solution of \( \Phi \), the current is simplified to
the London equation similarly to the superconductivity,
\[ J_\mu = q^2 \Phi^2 A_\mu. \]  

For the transverse components with \( \vec{k} \cdot \vec{A} = 0 \), it can be
rewritten as Schrödinger equation [2] via \( \Psi = \frac{A_{n \pi}}{z(\pi - 3/2)} \):
\[ -\Psi''_n + V \Psi_n = E_n \Psi_n, \] 
\[ V = \frac{p^2 - 4}{2} + q^2 M^2 z^2 \]
\[ E_n = qM(4n + 2p + 2), \]
with \( p = (d - 2)/2, \) and \( E_n = \omega^2 - k^2 \equiv m_n^2. \) The

For tensor with rank \( s \), there are a few possible models
according to the permutation symmetry of the index and
gauge symmetry of the theory. As we have shown in the
appendix, some of them has spectrum
\[ m_{n,s}^2 = 4qM(n + d/4). \]  

For \( d = 4 \), these results coincide with those of [2], where
vector meson spectrum was discussed using the dilaton.
Notice that here we did not use the dilaton. The
reason for such coincidence is just because the equations of
motion of two models turns out to be the same when
they are expressed in Schrödinger form in spite of the
difference in the degree of freedom. However, this is
because we use the scalar solution in the probe limit at zero
temperature. When we consider the effect of the finite
temperature or back reaction or chemical potential, the
difference will be manifest. For \( d = 3 \), the 1/\( z^2 \)-potential
is accidentally cancelled, but the spectrum is still given
by above formula because we need to impose the boundary
condition \( A_\mu = 0 \) at the boundary of AdS.

For general spin \( s \), we need to choose the mass term of
the higher spin fields properly to get eq. (12). That spin
dependent mass is necessary for the spectral formula has
been known from the original paper [2] but has not been
so clear. Notice that in string theory the action encodes
all the spin simultaneously while in field theory the action
for each spin should be considered one by one. Now how
to add up such spin dependent field theories to describe
the holographic image of the bulk fundamental string? While
the kinetic terms are canonical, the mass term and
interaction term of spin \( s \) excitation are ambiguous. We
suggest that reproducing the linear spectrum can be used
as a guiding principle to determine them especially if our
purpose is to describe a theory whose spectrum follows
Regge trajectory. Then statement is that, for any spin of
given symmetry, there exist a choice of mass term such
that the resulting spectrum is given by eq.(12).

Notice that the spectrum is linear in both spin \( s \) and
vibrational quantum number \( n \) and therefore the model
has a spectrum of open string whose string tension is
\[ T = 1/(2\pi \alpha'), \quad \text{with } \alpha' = 1/(4qM). \] 

In \( M \to 0 \) limit, that is, in the tensionless limit, the
whole tower of string spectrum is reduced to that of a
massless particle.

Although we considered the only abelian theory, the
same spectrum will be obtained for nonabelian theories.
This is because holographic spectrum analysis depends
only on the quadratic part of the field variation’s action.
Therefore, when we consider SU(\( N \)) and we perturb
around 0 background gauge field, the non-linear terms
induced by the non-abelian-ness does not affect the
spectrum. Therefore Non-linear chiral dynamics in hologra-
phy will also give linear spectrum. That is, our spectrum
is the same as that of the 2+1 dimensional version of
EKSS model [6] if boundary condition is the same. In
fact, the important difference is soft wall boundary condi-
tion (BC) installed by scalar condensation. The authors
of [6] did not get linear spectrum because they assumed
the hard wall BC. In [2], the authors introduced softwall
by hand using the dilaton dressing which is not supported
by an equation of motion. Then in terms of the QCD,
what we did is to use the action of the hardwall model
but install the soft wall BC by scalar condensation.

III. PHYSICAL MEANING OF STRINGY
EXCITATIONS

So far, we have seen that the abelian Higgs model con-
sidered as the axial part of the QCD has a linear spec-
trum. In our field theory description, we have seen that
each spin \( s \) excitation of the string in AdS, which we
called ’spin \( s \) particle in AdS’, creates a tower of linear
spectrum in the boundary and summing all of them with
different \( s \) should be interpreted as the string spectrum.

What is the origin of the string? AdS/CFT is the

The Mandelstam-‘t Hooft (MT) duality [2, 6, 10] which
conjectured that QCD vacuum realizes a dual superconductivity: just as magnetic flux is confined in superconducting vacuum electric flux is confined in the vacuum. It would be good if we can understand this more explicitly.

Notice that the abelian Higgs model has long been identified as the holographic theory of superconductivity \[11, 12\] in 2+1 dimensional system, although it also has an interpretation as a theory of superfluidity \[12\].

Notice that both the color flux tube and the stringy spectrum we obtained are in the boundary theory. Therefore it is natural to identify our spectrum obtained from the abelian Higgs model with the string of confined color flux. As we described earlier, the reason why U(1) theory captured the confinement dynamics is because only quadratic fluctuation is relevant in the spectral analysis: the non-abelian nature of SU(N) theory is washed out together with the non-abelian-ness created by the former. Notice also that the abelian Higgs model in AdS was obtained by promoting the global chiral symmetry in the boundary. Therefore, the chiral symmetry describes the confinement dynamics through holography when chiral condensation \( M \) is assumed.

The fact that both the theory of confining dynamics and theory of superconductivity are described by the same theory, the holographic abelian higgs model, can be considered as an explicit demonstration of the Mandelstam- ‘t Hooft duality, which is nothing more than the statement that the way QCD vacuum works is mimicking the superconductivity. Here we are saying that they are not just mimicking each other: they are described by the same theory.

\section{IV. TWO COMPETING ORDERS IN QCD: SCALE AND CHIRAL SYMMETRY BREAKINGS}

So far we have not considered scale symmetry breaking. We now consider how including it can change the behavior of the theory. In fact, one of the important mechanism of mass generation in the QCD is the scale symmetry breaking. The dilaton field has been usually considered as the dual of the gluon operator. In ref. 2 of the softwall model, the dilaton factor \( e^{-\varphi} \). In this paper, we identify \( \varphi \) as the square root of the gluon operator: \( \varphi^2 \sim \text{Tr} F_{\mu \nu} F^{\mu \nu} \). Since we do not want to modify the AdS metric in Einstein frame, we determine the dilaton in the AdS background. Then \( \varphi \) is of dimension 2 and we can write down the bulk action of it using the bulk mass \( m_\varphi^2 = -4 \). Setting the source part of \( \varphi \) to be zero as before, we get

\[ \varphi = G z^2, \quad \text{with } G^2 = \langle \text{Tr} F_{\mu \nu} F^{\mu \nu} \rangle \]  

(14)

Now the action of the softwall model is

\[ S = -\frac{1}{4} \int \sqrt{g} e^{-\varphi} \left( F_A^2 + F_V^2 + 4 |D_A \Phi|^2 \right). \]  

(15)

The Schrödinger form of the equation of transverse component of \( V_i, A_i \) is \( -\psi'' + V \psi = m_\psi^2 \psi \) with

\[ V = G^2 z^2 + \frac{3/4}{z^2} \quad \text{for vector } V_i, \]  

\[ = G^2 z^2 + \frac{1 + |\Phi|^2 - 1/4}{z^2} \quad \text{for axiaval } A_i \]  

(16)

where \( \psi = A_\perp / (\sqrt{z} e^{G z^2/2}) \) with \( A = V_i, A_i \). If the behavior of \( \Phi = M z^2 \) were maintained, the Regge slope would be \( 4 \sqrt{G^2 + q^2 M^2} \). However, it can not be so: the equation for the chiral scalar \( \Phi \) in the presence of \( \varphi \) is

\[ -\Phi''(z) + \left( \varphi'(z) + \frac{3}{z} \right) \Phi' - \frac{4 \Phi}{z^2} = 0. \]  

(18)

Notice that the behavior of \( \Phi \) in large \( z \) is dominated by \( \varphi = G z^2 \), because \( \varphi' >> 3/z \) there. Therefore asymptotic behavior of \( \Phi \) is either \( \Phi \approx \exp(G z^2) \) or \( \Phi \approx M_1 \exp(-1/G z^2) \) with dimensionless parameter \( M_1 \).

Since we should take the finite solution, we have

\[ \Phi \approx M_1 \quad \text{for } z \rightarrow \infty. \]  

(19)

For large quantum number \( n \),

\[ m_\psi^2 = G(4n + 2 \sqrt{1 + M_1^2}). \]  

(20)

For small \( n \), configuration of \( \Phi \) can make a small non-linear component to the trajectory.

Notice that the chiral symmetry breaking, although its breaking is spontaneous, does not contribute to the Regge slope, so that the Regge slope is determined only by the scale symmetry breaking scale. Indeed, the Regge slopes of all the meson family are the same and the we point out that this makes the softwall model explain why this is so. The chiral symmetry breaking contributes to Regge intercept by the parameter \( M_1 \), which is expected to be zero in the limit of chiral symmetry restoration. This explains why the vector and axiaval mesons will be the same, which is another phenomenological fact.

In summary, both chiral symmetry breaking and non-trivial dilaton configuration discussed in this section are natural ways to introduce a physical scale. The issue here was whether two mechanism can co-exist or compete. For the former case, we would have two independent scales in QCD. What we found here is that interestingly they compete and only one mechanism survives and as a consequence we have only one scale.

\section{V. OTHER MODELS WITH LINEAR CONFINEMENT}

In the rest of this paper, we provide other models without the over all dilaton factor \( e^{-\varphi} \) yet having linear Regge trajectories for the future model building for QCD and condensed matter. In the confined phase, we should treat particle of each spin individually.

\[ S = \sum_{s \geq 2} S_{V, s} + S_{A, s}, \]  

(21)
where index $s$ is for spin $s$.

The vector meson can not couple to $\Phi$ because $\Phi$ does not have the vector charge. Usually the dilaton, the Goldstone boson of the scale symmetry, is introduced as a real massless scalar which is dual to the gluon operator $\text{Tr} \, G_{\mu \nu}^2$ whose non-zero vacuum expectation value breaks the scale symmetry. For our purpose we identify it as a square root of the gluon operator. It should couple to the vector meson otherwise the latter will be massless. The action of the vector meson is $S_V = \int d^{d+1}x \sqrt{-g} \mathcal{L}_V$ with

$$\mathcal{L}_V = -\frac{1}{4} F_{\mu \nu}^2 - \frac{1}{2} \nabla \varphi \nabla \varphi - g^2 \varphi^2 V_\mu V^\mu$$

(22)

The (square root of) dilaton has following solution $\varphi = G z^2$ as before. The equation for the transverse vector meson is still given by the Schrödinger Eq. (7) with $qM$ replaced by $gG$. Therefore the spectrum of the vector meson is again a linear tower given by

$$m_{n, \text{vector}}^2 = 4gG(n + d/4).$$

(23)

If we have added $-m_A^2 V_{\mu_1 \ldots \mu_s}$ term to the Lagrangian of spin $s$ vector meson, the spectrum would change to

$$m_{s,n}^2 = 4gG(n + p_\nu + 1/2).$$

(24)

with $p_\nu = 2 \sqrt{(s - 1 + d/4)^2 + m_A^2}$. To fit the data for $\rho$ meson with $d = 4$, $s = 1$, we can take $p \simeq -1$ which can be done most naturally by setting $m_V^2 = 0$. That is, for the phenomenology, it is better not to introduce the bulk mass of the vector meson.

Gluon condensation and axial mesons: The anomaly of the $U(1)_A$ can be considered as a part of the spontaneous breaking of the axial symmetry and we should open the possibility that the promoted bulk gauge invariance can be broken explicitly at the bulk level, because the bulk theory should include the quantum dynamics of the boundary theory at the classical level. This implies that the axial symmetry could have been further broken by adding the bulk mass term $-m_A^2 A_{\mu_1 \ldots \mu_s}$ to the Lagrangian of spin $s$ axial vector meson.

Then the spectrum would change to

$$m_{s,n}^2 = 4qM(n + p_A + 1/2).$$

(25)

with $p_A = 2 \sqrt{(s - 1 + d/4)^2 + m_A^2}$. Again when one consider the equation of motion in the Schrodinger form, they become similar to the model having the dilaton softwall model, and according to refs. [14, 15] $4qM = 1.25 (GeV)^2$ and $m_A = 0.5$ can fit the data well.

It could have been broken even further by dilaton coupling $-\varphi A_\mu A^\mu$. However, then the spectrum is changed by $qM \rightarrow \sqrt{(qM)^2 + (gG)^2}$ so that the slope of Regge trajectory of the axial vector is bigger than that of vector meson, which is not consistent with the data in [14].

Therefore we do not add dilaton coupling of the axial meson. To make two slope equal, we need

$$gG = qM.$$  

(26)

However, this is not a good consequence for the QCD, because it means a fine tuning is necessary for the universality of the Regge slope.

Gluon condensation: To understand the color confinement, it is good idea to look at the behavior of a gauge invariant version of color fields, say $\mathcal{O} = \text{Tr} (G_{\mu \nu})^n$, under the gluon condensation. Let $\phi$ be the scalar field in the bulk which is dual to the scalar field $\mathcal{O}$ of dimension $\Delta$. Then the dynamics of $\phi$ ‘inside’ the bag can be studied by

$$S_\phi = \frac{1}{2} \int d^4x \sqrt{-g} (-\nabla \phi \nabla \phi - m^2 \phi^2 - gS_{\phi} \phi^2)$$

(27)

Using the solution $\varphi = G z^2$ as before, the Schrödinger form of the scalar equation is given by Eq. (4) with $p_S^2 = m^2 + 4$, and the scalar meson spectrum is given by

$$m_{n, \text{scalar}}^2 = 4gsG(n + 1/2 (p_S + 1)).$$

(28)

The linear spectrum of the glueball is interesting but what is more important for us here is the behavior of the wave function Eq. (8) which says that the color flux outside the bag, $z > z_m$ is exponentially suppressed, proving the color confinement within the bag under the presence of the gluon condensation.

Notice that in many of our models, we need to choose the bulk mass of the theory properly to get the promised combination $n + s$. That spin dependent mass is necessary for the spectral formula has been known from the original paper [7] but has not been clear so far. Notice that string theory encode all the spin simultaneously while in field theory the action for each spin should be considered one by one. Now how we add up such spin dependence field theories to describe the holographic image of the bulk fundamental string? While the kinetic terms are canonical, it is not surprising to have ambiguities in the mass term of spin $s$ excitation. We suggest that reproducing the linear spectrum can be used as a guiding principle to determine them.

VI. DISCUSSION

We finish the paper by summary and a few remarks. First one may ask the problem of blowing up of the scalar solution in IR region ($z \rightarrow \infty$). This is precisely the problem of probe approach where we assume that the gravity background is fixed as AdS. There is a known resolution to this: in reality, the back-reaction of AdS will either create horizon, a natural IR cut off, or smooth out the solutions. Whether the probe solution is useful or not depends on what we do with it. If we evaluate the thermodynamic quantity, we would fail. But for the
spectrum, the background will be useful because true solution will be similar to the probe solution away from the singular region \((z \to \infty)\), which is forbidden for the wave function of excitations anyway: we are looking for equation of motion of the vector field’s perturbation, which was shown to be written (after a change of variable) as a Schrödinger equation whose potential contains \(z^2\) term with \(z = 1/r\). Such configuration provide a softwall providing the barrier so that the wave function will not penetrate to the IR region \(z \to \infty\). This is an effective way to cut out the IR region and justify the use of the probe solution for the problem of spectrum. One can show that when we consider the back reaction, a horizon is developed and as the horizon grows the the potential’s \(z^2\) regime will retract. The potential will not grow like \(z^2\) indefinitely but collapse to to \(-\infty\) at the horizon, so that the higher quantum number of the linear spectrum will be deformed and disappear. Details of such effect is a complicated correction to the simple phenomena described here.

Next, in a theory where symmetry breaking does not enter, chiral symmetry breaking can contribute to the slope of axial vector meson. But when scale symmetry breaking comes with coupling of overall \(e^{-\phi}\) coupling, like soft-wall model, the theory changes its face: the chiral symmetry breaking effect is eaten by that of the scale symmetry breaking scales. The linearity of the Regge trajectory is explicitly so that the universlaity of the Regge slope could be but treat the vector and axial vector sector symmetrically so that the universality of the Regge slope could be shown.

Two models with and without the overall dilaton \(e^{-\phi}\) coupling, have pros and cons: The model without such coupling makes the Mandelstam–’t Hooft duality is manifest but we should break vector gauge symmetry explicitly to give vector meson mass and universlaity of the Regge slope need fine-tuning. On the other hand, the original soft-wall model does not have a manifest duality but treat the vector and axial vector sector symmetrically so that the universality of the Regge slope could be shown.

We identified the origin of the Regge slopes as the condensation of order parameter that controls the symmetry breaking scales. The linearity of the Regge trajectory is generated because the potential is the same form as that of 3-dimensional isotropic harmonic oscillator where \(z\) play the role of the radial coordinate. The ‘centrifugal’ term is due to the confining gravity of the AdS space, while quadratic potential is by gluon and chiral condensations. The latter provides infinite “soft wall” and it can be attributed as a nature of the vacuum of such condensation. Our results suggest that the color confinement and the Regge slope is consequence of gluon condensation. Therefore by measuring the Regge slopes, we can determine it, but we can not determine the chiral condensation so easily.

### Appendix A: Models with higher rank tensors

In the first two subsections of this appendix, we study models with diffeomorphism invariance but without gauge invariance. In the final subsection we study the theory with gauge invariance as well as diffeomorphism invariance.

#### 1. Rank-s totally Antisymmetric Tensor without gauge symmetry

We may start with field equation

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} g^{\alpha_1 \beta_1} \ldots g^{\alpha_s \beta_s} \partial_{\nu} A_{\beta_1 \ldots \beta_s} \right) = (\Phi^2 + m_A^2) g^{\alpha_1 \beta_1} \ldots g^{\alpha_s \beta_s} A_{\beta_1 \ldots \beta_s} \tag{A1}
\]

With axial gauge choice \(A_{x_1 \ldots x_s} = 0\), the equation of motion of \(A_{x_1 \ldots x_s} := B e^{i(k \cdot x - \omega t)}\) takes the form,

\[
-z^{-\alpha} \partial_z (z^\alpha \partial_z B) + (\Phi^2 + m_A^2) z^{-2} B = m_n^2 B, \tag{A2}
\]

where \(\alpha = -d + 2s + 1\) and \(m_n^2 = \omega^2 - k^2\). Using the identity

\[
\partial_z (z^\alpha \partial_z B) = z^{\alpha/2} \left( \phi'' - \frac{(\omega^2 - \frac{1}{z^2})^2 - \frac{1}{4}}{z^2} \phi \right) \tag{A3}
\]

with \(B = z^{-\alpha/2} \phi\), we get

\[
-\phi'' + \left( \frac{p^2 + m_A^2 - \frac{1}{z^2}}{z} + M^2 z^2 \right) \phi = m_n^2 \phi \tag{A4}
\]

\[
E_{n,s} = M (4n + 2 \sqrt{p^2 + m_A^2 + 2}), \tag{A5}
\]

with \(p^2 = (s - d)^2\). Then the desired spectrum

\[
E_{n,s} = M (4n + 4s - 4 + d), \tag{A6}
\]

can be obtained for \(m_A^2 = 3(s - 1)(s + d - 3)\).

#### 2. Rank-s totally Symmetric Tensor without gauge symmetry

For the same gauge choice and the variable, the equation of Motion is

\[
V(z) = \left( \frac{\omega}{z^2} \right)^2 + s - \frac{1}{4} + m_A^2 + M^2 z^2 \tag{A7}
\]

The spectrum

\[
E_{n,s} = M (4n + 4s + d - 4), \tag{A8}
\]

can be obtained if \(m_A^2 = (2s - 3)(2s - 3 + d) - s\).

If there were overall dilaton factor \(e^\varphi\) with \(\varphi = M z^2\) in the action,

\[
V(z) = \left( \frac{\omega}{z^2} \right)^2 + s - \frac{1}{4} + m_A^2 + M^2 z^2 + d - 2 \tag{A9}
\]

then we can have

\[
E_{n,s} = M (4n + 4s - 4 + d), \tag{A10}
\]

by choosing \(m_A^2 = 4s^2 - 9s + 4 - d^2/4\).
3. Higher spin Theory

In \([2]\), the rank-s totally symmetric Tensor with gauge symmetry \(A_{\mu_1...\mu_s} \rightarrow A_{\mu_1...\mu_s} + \nabla_{(\mu_1} \xi_{\mu_2...\mu_s)}\) was identified as the spin-s theory. The residual gauge transformation which leaves \(A_{\mu_1...\mu_s}\) invariant \([2]\) is determined by

\[
\nabla_{(\mu_1} \xi_{\mu_2...\mu_s)} = 0. \tag{A11}
\]

Using \(\Gamma^\mu_{\mu_1} = -1/z, \Gamma^z_{\mu_1} = 1/z, \Gamma^z_{\mu_2} = -1/z\), we get

\[
\partial_\xi \xi + 2s - 2 \frac{z}{\xi} \xi = 0, \tag{A12}
\]

namely \(z^{2s-2} \xi(z, x) := \tilde{\xi}_{x_1,...,x_s}(x)\) is \(z\) independent. Introducing the scaled variable \(A_{x_1,...,x_s} := z^{2s-2} A_{x_1,...,x_s}\), the residual gauge transformation in terms of the tilde variable is nothing but the shifting: \(\hat{A}_{x_1,...,x_s} \rightarrow \hat{A}_{x_1,...,x_s} + \hat{\xi}_{x_1,...,x_s}\). The action can be written as

\[
S = \int z^s e^\xi [\langle \partial_\mu \hat{A}_{x_1,...,x_s} \rangle^2, \tag{A13}
\]

where with \(\varphi = M z^2\) and \(\alpha = -(1+d) + 2(s+1) - 2(2s - 2) = 4 - d + 1 - 2s\). In other words, the action should be designed such that Eq. (A13) is hold using covariant derivatives. Now using the methods which is by now familiar, we have

\[
E_{n,s} = M(4n + 4s - 4 + d). \tag{A14}
\]

as it was described in \([2]\) for \(d = 4\).

One should notice that the mass term is not invariant and therefore the invariance under the residual gauge transformation should determine the mass \([2]\) to be \(m^2 = s^2 - 4\). For the same reason, the naive scalar coupling term such as \(\Phi^2(A_{x_1,...,x_s})^2\) is not allowed.

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