Research on Evaluation Methods of Firing Precision of Trajectory Correction Projectile

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Abstract. As a new type of ammunition, the trajectory correction projectile has much different ballistic characteristics than traditional uncontrolled ammunition and missile. The firing precision of trajectory correction projectile is significantly higher than that of uncontrolled projectile and lower than missile. Also the evaluation method for firing precision is different from that of uncontrolled projectile and missile. In this paper, by analyzing the calculation methods and characteristics of different firing precision evaluation indicators, the method suitable for evaluating the precision of trajectory correction projectile is determined, which provides theoretical and technical support for acceptance of trajectory correction projectile firing precision.

1. Introduction

The weapon system firing precision, also known as the firing error, is the statistic of the distance between the projectile's impact point and the target point, which is used to characterize the weapon system's ability to hit the target. During the flight, the projectile will be affected by various disturbances such as initial disturbance and wind disturbance, so that the actual trajectory of the projectile deviates from the ideal trajectory, and finally the impact point is deviated from the target point [1]. For a single shot, the firing error is the distance vector between the actual impact point and the target point. For a batch of projectiles or a type of projectile, the firing error can only be obtained by statistics [2].

For traditional uncontrolled projectiles, bullets and missiles, there are different methods for assessing the precision of firing [3-8]. For the new ammunition, trajectory correction projectile, there is no mature systematic firing precision evaluation index and evaluation method. However, the development and acceptance of the ballistic correction project also put forward the demand for its firing precision evaluation.

2. Basic concepts in firing precision evaluation

2.1. Element error and dispersion error

The accuracy and density of the impact point are two aspects of firing precision. Accuracy, also known as the element error, systematic error, refers to the distance between the center of the impact point and the target point. It is generated when the shooting elements are determined. When shooting with the
same element, it is repeated regardless of how many projectiles are fired. Accuracy, the element error, is the systematic error of the artillery weapon system. Density, also known as dispersion error and random error, refers to the distribution of the impact point around the distribution center point. It is a random error caused by various random factors. It is the independent error of each projectile, and is expressed by the deviation of the impact point relative to the average impact point [9]. The accuracy and density of firing can be expressed by the distance vector. It can also be divided into longitudinal and lateral axes according to the shooting distance and direction for statistical analysis [10]. In general, the accuracy and density of the impact point obey some certain probability distribution [11].

Take the plane rectangular coordinate system $Oxz$ for the target, and take the target point $O$ as the coordinate origin, $OX$ as the shooting direction, called the longitudinal direction, $OZ$ is perpendicular to $OX$ and points to $OX$ right, called the lateral direction. The deviation of the landing point $S$ of any projectile from the target point is called the firing error of it. The deviation of dispersion center $C$ from the target point is the element error. The deviation of impact point $S$ from the dispersion center $C$ is the dispersion deviation, as shown in Figure 1.

![Figure 1. Schematic diagram of firing error](image)

The following formula can be seen from Figure 1.

$$X = X_c + X_s, \quad Z = Z_c + Z_s, \quad \mathbf{R} = \mathbf{R}_c + \mathbf{R}_s$$ (1)

Firing multiple shells with the same firing elements, each shell has a different firing error ($X, Z$) and a dispersion error ($X_s, Z_s$), but the same element error ($X_c, Z_c$). With different firing elements, the firing error, element error and dispersion error are all different from each other.

### 2.2. Root mean square error

The formula for the root mean square error is as follows.

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$ (2)
The root mean square error is also called standard error, standard deviation, and RMS error for short. It reflects the dispersion degree of a data set. In theories of firing, it specifically indicates the distribution of impact point.

2.3. Mean error
The mean error is also called calculate probable error, probable error and probable deviation. It is mathematically interpreted as: In a set of data \( A = (a_1, a_2, ..., a_n) \), centered on the mean \( \bar{a} \), the interval \((\bar{a} - E_a, \bar{a} + E_a)\) contains 50% of the data, then \( E_a \) is the mean error of the data, and its mathematical expression is \( P(\bar{a} - E_a < a < \bar{a} + E_a) = 0.5 \). On the scattered surface of the falling point, two parallel boundaries are equally spaced on both sides of the center axis of the dispersion. 50% of the impact point is located between the two boundaries, and half of the width between the two lines is the mean error, as shown in Figure 2. The mean error is a representation of the degree of dispersion from the center axis for impact point. It is a characterization of the intensity of the firing. The larger the mean error (smaller), the smaller the density (larger), the more dispersed (concentrated) the drop point.

![Figure 2. Schematic diagram of mean error](image)

For a normal distribution, there is a formula as following.

\[
E = \sqrt{2} \rho \sigma = 0.6745 \sigma
\]

(3)

In the formula, the normal constant \( \rho = 0.476936 \), which is also called the artillery constant in shooting.

2.4. Density
The shooting precision of the uncontrolled bomb is usually given as the ratio of the distance mean error to the range. For example, the shooting precision index of a certain type of uncontrolled rocket is: \( E_x = 1/200 \), \( E_z = 1/150 \), which is to say the mean error of the longitudinal deviation is 1/200 of the range, and the mean error of the lateral deviation is 1/150 of the range. This indicates that the uncontrolled bomb has a larger spread as the shooting distance increases.
2.5. Circle error probability

CEP (circle error probability), also known as circular probable deviation, radius of half dispersion, impact radius of half number of shots, 50% hit zone, is mainly used for the accuracy evaluation of the weapon system, which is defined as: the radius of the circle which center on the impact point center and contain half of the impact point. The formula for it is as follows [12, 13].

$$P\left\{ (X)^2 + (Z)^2 \leq CEP^2 \right\} = 0.5$$

(4)

3. Evaluation methods of firing precision

According to different analytical purposes and statistical methods, there are several different representations of firing precision [14].

Method 1. Expressed by the mean error of distance and direction deviation from the target, \( E_{X\Sigma} \) , \( E_{Z\Sigma} \).

Method 2. Expressed by the limit error of distance and direction deviation from the target, \( |\Delta X_{\text{max}}| \), \( |\Delta Z_{\text{max}}| \).

Method 3. Expressed by the distance and direction deviation of impact point center from the target \( X, Z \), and the distance and direction mean error of impact point from the impact point center \( E_x, E_z \).

Method 4. Expressed by the distance and direction mean error of impact point center from the target \( E_x, E_z \), and the distance and direction mean error of impact point from impact point center \( E_x, E_z \).

Method 5. Expressed by CEP.

The following is a detailed analysis of the different evaluation indicators of shooting precision. The range and direction deviation data of each group of \( n \) projectiles in \( m \) groups have been obtained.

1st group \( X_{11}, \ldots, X_{1j}, \ldots, X_{1n} \) \( \vdots \) \( \vdots \) \( Z_{11}, \ldots, Z_{1j}, \ldots, Z_{1n} \)

i-th group \( X_{ij}, \ldots, X_{ij}, \ldots, X_{in} \) \( \vdots \) \( \vdots \) \( Z_{ij}, \ldots, Z_{ij}, \ldots, Z_{in} \)

m-th group \( X_{mj}, \ldots, X_{mj}, \ldots, X_{mn} \) \( \vdots \) \( \vdots \) \( Z_{mj}, \ldots, Z_{mj}, \ldots, Z_{mn} \)

The distance and direction density mean error of impact point from itself impact point center of each group \( E_{Xj}, E_{Zj} \)

\[
\begin{align*}
\bar{X}_i &= \frac{\sum_{j=1}^{n} X_{ij}}{n}, \quad \bar{Z}_j = \frac{\sum_{j=1}^{n} Z_{ij}}{n} \\
E_{Xj} &= 0.6745 \sqrt{\frac{\sum_{j=1}^{n} (X_{ij} - \bar{X}_j)^2}{n-1}} \\
E_{Zj} &= 0.6745 \sqrt{\frac{\sum_{j=1}^{n} (Z_{ij} - \bar{Z}_j)^2}{n-1}}
\end{align*}
\]
The distance and direction mean error of impact point center from the target $E_x, E_z$

$$
\begin{align*}
\bar{X} &= \frac{\sum_{i=1}^{m} X_i}{m}, \quad \bar{Z} = \frac{\sum_{i=1}^{n} Z_i}{n} \\
E_x &= 0.6745 \sqrt{\frac{\sum_{i=1}^{m} (X_i - \bar{X})^2}{m-1}} \\
E_z &= 0.6745 \sqrt{\frac{\sum_{i=1}^{n} (Z_i - \bar{Z})^2}{m-1}}
\end{align*}
$$

The distance and direction mean error of impact point from the impact point center $E_x, E_z$ and the mean error of distance and direction deviation from the target, $E_{x\Sigma}, E_{z\Sigma}$.

$$
\begin{align*}
E_x &= \sqrt{\frac{\sum_{i=1}^{m} E_{x_i}^2}{m}} \\
E_z &= \sqrt{\frac{\sum_{i=1}^{n} E_{z_i}^2}{m}} \\
E_{x\Sigma} &= \sqrt{E_x^2 + E_{x_i}^2} \\
E_{z\Sigma} &= \sqrt{E_z^2 + E_{z_i}^2}
\end{align*}
$$

In the above calculation, we can get three representation methods: Method 1 $E_{x\Sigma}, E_{z\Sigma}$, Method 3 $\bar{X}, \bar{Z}, E_x, E_z$, Method 4 $E_x, E_z, E_{x\Sigma}, E_{z\Sigma}$.

Method 2 is calculated by

$$
\begin{align*}
\Delta X_{\text{max}} &= |X_{\text{max}} - X_{\text{min}}| \\
\Delta Z_{\text{max}} &= |Z_{\text{max}} - Z_{\text{min}}|
\end{align*}
$$

Where, $X_{\text{max}}$ and $X_{\text{min}}$ are the maximum range and minimum range with the same shooting element; $Z_{\text{max}}$ and $Z_{\text{min}}$ are the maximum and minimum lateral deviations with the same shooting element.

$CEP$ in Method 5 is calculated as follows.

Taking the target point as the origin of the coordinate, the probability density function of element error $(\bar{X}, \bar{Z})$ is as follows.
Taking the impact point center as the origin of the coordinate, the probability density function of dispersion error is as follows.

\[
f(\bar{X}_i, \bar{Z}_i) = \frac{\rho^2}{\pi E_X E_Z} \exp \left[ -\rho^2 \left( \frac{\bar{X}_i^2}{E_X^2} + \frac{\bar{Z}_i^2}{E_Z^2} \right) \right]
\]  

(9)

Taking the target point as the origin of the coordinate, the probability density function of firing error is as follows.

\[
f(X_i, Z_i) = \frac{\rho^2}{\pi E_{xi} E_{zi}} \exp \left[ -\rho^2 \left( \frac{X_i^2}{E_{xi}^2} + \frac{Z_i^2}{E_{zi}^2} \right) \right]
\]  

(10)

Where, \((X_i, Z_i)\) is the dispersion error of \(i\)-th group when taking itself impact point center as the origin of the coordinate.

Taking the target point as the origin of the coordinate, the probability density function of firing error is as follows.

\[
f(X, Z) = \frac{\rho^2}{\pi E_{x} E_{z}} \exp \left[ -\rho^2 \left( \frac{X^2}{E_{x}^2} + \frac{Z^2}{E_{z}^2} \right) \right]
\]  

(11)

Equation (4) is converted to

\[
\int \int \exp \left[ -\rho^2 \left( \frac{X^2}{E_{x}^2} + \frac{Z^2}{E_{z}^2} \right) \right] d(X) d(Z) = 0.5
\]  

(12)

From equation (12), CEP has an approximate relationship with \(E_{x}, E_{z}\) as follows [15].

\[
CEP = \begin{cases} 
0.912E_{x} + 0.833E_{z} & E_{x} < E_{z} \\
1.746E_{x} & E_{x} = E_{z} \\
0.833E_{z} + 0.912E_{x} & E_{x} > E_{z}
\end{cases}
\]  

(13)

For the case where the element error \(E_{x}, E_{z}\) is small, the dispersion error \(E_{x}, E_{z}\) can be directly used instead of the integrated error \(E_{x}, E_{z}\). So we can get the following formula.

\[
CEP = \begin{cases} 
0.912E_{x} + 0.833E_{z} & E_{x} < E_{z} \\
1.746E_{x} & E_{x} = E_{z} \\
0.833E_{z} + 0.912E_{x} & E_{x} > E_{z}
\end{cases}
\]  

(14)

From \(E = 0.6745\sigma\) and equation (15), the CEP has an approximate relationship with the longitudinal and lateral RMS error \(\sigma_{x}, \sigma_{z}\) as follows [16].
The methods 1 and 2 are essentially the same, all focusing on the shooting results. These two representation methods are intuitive and operability, and are often used for military training assessment. The artillery equipment used is stable in state and there is no large fixed system deviation. Its shortcoming is that it cannot distinguish the influencing factors of firing precision, and it is not easy to find out the cause after the problem occurs.

Method 3 is widely used in the development of artillery weapons. The object of this representation method is the unarmed artillery weapon system. Many factors affecting the precision of shooting are not determined. In order to ensure that the artillery weapon system can meet the requirements of use after the stereotype, reasonable limits \( X, Z \) and \( E_1, E_2 \) must be specified. According to the test results, it is easier to find out whether the system has large fixed deviations, and it is convenient to analyse the reasons from the overall scheme and the distribution factors.

Method 4 is mainly used in shooting methods and shooting theory research. The core problem is to study the element error, in order to make the impact point center as close as possible to the target [17]. This kind of representation method cannot find fixed large deviation, and the amount of ammunition used in the test is also large. It is not suitable for precision evaluation.

Method 5 is similar to methods 1, 2, and generally does not distinguish between element error and dispersion error, and is mainly used for missile shooting precision calculation.

4. Conclusion

For different types of ammunition, different precision assessment methods are used for different purposes. Generally speaking, there are two ways. One is to separate the element error and the dispersion error. It is mainly used for the uncontrolled projectile shooting method study and the precision evolution. The second is to combine the errors to measure the firing precision with high precision such as missiles [18]. From the view of hit effect, it is really not necessary to distinguish between systematic errors and random errors. The distinction between these two types of errors is to provide feedback information for ammunition system design and system improvement. Therefore, from the perspective of precision assessment, no matter what kind of error, it is always measured with the target point as the center. For the trajectory correction projectile, its element error is small, and the firing precision is high. So it can be concluded that CEP is more suitable as the firing precision index.

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