Mechanism for the Difference in Lifetimes of Charged and Neutral $D$ Mesons

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The reaction $D^\pm \to s + \bar{d} + \text{gluon}$ is proposed as a source for the difference in the lifetimes of the charged and neutral $D$ mesons. In a nonrelativistic bound-state model the rate for the reaction is found to depend on the ratio $f_\rho/m_\rho$. For reasonable values of this ratio the observed difference in the lifetimes may be accounted for.

A number of experiments\textsuperscript{1} have recently reported a significant difference in the lifetimes of the charged and neutral $D$ mesons, with $\tau_{D^+}$ perhaps as much as six times as large as $\tau_{D^0}$. It has been argued that mesons containing a heavy quark $c$, $b$, or $t$ will decay through a mechanism where the light quark acts as a spectator\textsuperscript{2} [Fig. 1(a)]. The process depicted in Fig. 1(b) can contribute only to the decay of the $D^0$.\textsuperscript{3} However, by the usual helicity arguments the contribution of Fig. 1(b) is suppressed by the square of the ratio of light-to-heavy-quark masses and by $f_D/m_\tau^2$, $f_D$ being the pure leptonic decay constant of the $D$ defined by

$$\langle D(\rho) | J_\mu A^\mu | 0 \rangle = \frac{-i}{(2\pi)^{3/2}} \frac{f_D f_D}{(2\omega_D)^{1/2}},$$

(1)

where $J_\mu$ is the weak hadronic axial-vector current. The spectator graph leads to equal charged and neutral decay rates given by\textsuperscript{4}

$$\Gamma_\mu = \Gamma_\mu (m_c/m_\rho)^2 [2 + 3a_\rho],$$

(2)

where $\Gamma_\mu = G_F^2 m_\mu^2/192\pi^2$ is the rate for muon decay $\mu^- \to e^- \nu_\mu \bar{\nu}_\mu$. The factor of 2 is for leptons, and 3 for colors, and $a_\rho = (2f_c^2 + f_\rho^2)/3$. The coefficients $f_c$ and $f_\rho$ incorporate renormalization effects due to gluon exchange on the terms in the weak Lagrangian transforming as the 20 and 84 of SU(4), respectively.\textsuperscript{5} Using $\alpha_s(m_Z^2) = 0.6$, we obtain $f_c \sim 2$ and $f_\rho \sim 0.7$, leading to $a_\rho = 1.7$.

In this note, we propose a mechanism that may account for the observed difference in lifetimes. It is the one depicted in Fig. 2, namely

$$D^0 \to s + \bar{d} + \gamma_\rho (\text{gluon}).$$

(3)

We have calculated the contribution of this process by considering the $D^0$ meson (mass $= 1.86$ GeV) as a nonrelativistic bound state of $c$ and $u$ quarks with "constituent" quark masses of $m_c \sim 1.55$ GeV and $m_u \sim 0.3$ GeV. The momentum variation of the bound-state wave function is faster than that

\begin{align*}
\text{FIG. 1. Graphs contributing to $D$-meson decays.} & \\
& \text{(a) The "spectator" graph that contributes to the non-leptonic and semileptonic decays of both the charged and neutral $D$ mesons.} \quad \text{(b) This contributes to the decays of the $D^0 (\bar{D}^0)$ only. See Ref. 3.}
\end{align*}
of the amplitude multiplying it and thus the total amplitude is proportional to the wave function at the origin and, in turn, to $f_D^6$.

The gauge-invariant amplitude for the contribution of Figs. 2(a) and 2(b) can be written as (color indices are suppressed)

$$A = \frac{G_F^2}{q^2} \left[ F_A(q^2) \delta_{\mu\nu} - q^2 \cdot q \delta_{\mu\nu} \right] + i F_Y \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta,$$

$$\times \frac{\epsilon^\nu(q)}{[2\omega_f(2\pi^3)]^{1/2}},$$

(4)

where $\epsilon^\nu$ is the polarization of the gluon and $l^\mu$ is the weak current of the light quarks:

$$l^\mu = \bar{u}\gamma^\mu(1 - \gamma_5)v_d(q).$$

(5)

Since we are dealing with gluon emission from a color-neutral state the gauge-invariant amplitude (4) is infrared finite. Note that the contribution for gluon emission from final-state light-quark lines will be suppressed by powers of $m_s^2$ and/or $m_u^2$ and is therefore neglected.

In the nonrelativistic model that we have adopted we find

$$F_Y = \frac{\psi(0)}{m_s m_c} (2m_D/m_{D_s})^{1/2} \frac{f_D}{6} \frac{m_{D_s}}{m_u m_c},$$

(6)

$$F_A = \frac{\psi(0)}{m_s m_c} \frac{m_s - m_c}{m_u m_c} (2m_D/m_{D_s})^{1/2}$$

$$= \frac{m_s - m_c}{6} \frac{f_D}{m_u m_c},$$

(7)

The decay rate, $\Gamma_s$, for the process (3) is then found to be

$$\Gamma_s = G_F^2 a_0^+ a_s \left[ |F_Y|^2 + |F_A|^2 \right] m_{D_s}^3/108\pi^2$$

$$= G_F^2 a_0^+ a_s f_D^2 m_{D_s}^5/324\pi^2 m_u^2,$$

(8)

(9)

where $a_0^+ = (f_s + f_c)/4$. This leads to a ratio of lifetimes:

$$R = \frac{\tau_{D_s}^u}{\tau_{D_s}^s} = 1 + \left( \frac{m_{D_s}}{m_c} \right)^2 \frac{16\alpha_s^g}{27} \frac{f_D^2}{m_u^2} \frac{a_0^+}{2 + 3a_0^+}.$$

(10)

With $\alpha_s = 4\pi/[9 (m_{D_s}^2/\Lambda^2)]$, and $\Lambda = 0.5$ GeV, we obtain

$$R = 1 + 0.7(f_D^2/m_u^2).$$

(11)

Both $f_D$ and $m_u$ are not accurately known. In the literature estimates for $f_D$ range from about 150 to 800 MeV. In fact, a nonrelativistic-potential model calculation based on the potential $V(r) = -4\alpha_s/3r + r/a^2$, with $a = 1.95$ GeV$^{-1}$, yields

$$1 \leq f_D/m_u^2 \leq 2.$$

(12)

Using $m_u = 300$ MeV and the values of $f_D$ from the literature quoted above, we find that $R$ varies from 1.2 to 7. The larger values of $f_D/m_u$ could therefore account for a significant difference in the lifetimes of neutral and charged $D$ mesons.

Our method of calculating $F_Y$ and $F_A$ of Eqs. (6) and (7) based on a nonrelativistic bound-state model are expected to work, at best, for heavy-quark systems. They are totally unreliable for $\pi$ or $K$ mesons. Analogous form factors exist for $\pi (K) \to \nu\bar{\nu}$ (also $\pi \to \gamma\gamma$), but they are smaller by a factor of 10 for the $\pi$ case and a factor of 10 for the case of $K$ mesons than a model as ours would suggest. For light mesons these form factors can be understood on the basis of partial conservation of axial-vector current (PCAC) arguments. We do not expect soft-$D$-meson limits to work. On the other hand, nonrelativistic bound-state models have had considerable success in the heavier systems.\cite{10}

We expect an analogous mechanism to be important in other heavy-meson decays. Some consequences are as follows:

(1) The contribution of the gluon mechanism of Fig. 2 to the width of the charmed $F$ meson can be obtained by replacing $a_0^+$ with $a_0^+ = (f_s - f_c)/4$, $m_D$ with $m_F$, $m_u$ with $m_s$, and $f_D$ with $f_F$ in Eq. (9). Note that since the $W$ carries no color, the renormalization of the weak four-fermion vertex via gluon exchange is crucial to this contribution and it vanishes in the limit of $f_s = f_c = 1$. We thus obtain

$$\frac{\tau_{F_s}^u}{\tau_{F_s}^s} \approx 1 + \left[ \frac{m_s}{m_c} \right]^2 \frac{16\alpha_s^g}{27} \frac{f_F^2}{m_s^2} \frac{a_0^+}{2 + 3a_0^+}$$

$$+ \left[ \frac{24\pi^2}{2 + 3a_0^+} \frac{m_s m_c^2}{m_F^2} \left( 1 - \frac{m_s^2}{m_F^2} \right) \frac{f_F^2}{m_c^2} \right]$$

$$\approx 1 + 0.2(f_F^2/m_c^2) + 2.4(f_F^2/m_c^2).$$

(13)

(14)
In Eqs. (13) and (14) the last factors are for the pure lepton mode $F - \tau + \nu_\tau$. Thus the lifetime of the $F$ meson and its semileptonic branching ratio would be somewhat smaller than that of the charged $D$ meson.\(^{11}\)

(2) The lifetimes of the neutral mesons containing $b$ and $s$ quarks and their semileptonic branching ratios will also be smaller than those of their charged isospin counterparts.

(3) Of course, the strongest prediction of our model is the existence of a gluon jet in the decays of heavy mesons. Anticipating an ability to distinguish gluon from quark jets (for instance by $\langle p_T \rangle$ or multiplicities), we give the energy ($\omega$) distribution of the gluon as

$$\Gamma_\varepsilon^{-1} \frac{d\Gamma_\varepsilon}{dr} = 6r(1-r),$$

where $r = \omega/\omega_{\text{max}}$.\(^{25}\)

(4) Similar considerations should apply to radiative leptonic decays of $D$ (Cabibbo suppressed) and $F$ (Cabibbo allowed) decays. The rate for $D^+ (F^+) - e^+ \nu_e$ should be $10^4$ times that for $D^+ (F^+) - e^+ \nu_e$.

(5) As the gluon carries no isospin our mechanism indicates that isospin-$\frac{1}{2}$ final states may dominate Cabibbo-allowed $D^0$ decays. It is not clear whether this dominance would extend to the exclusive two-body channels. If it does, then it is worth pointing out that the mechanism of Fig. 2 yields

$$\Gamma(D^0 \rightarrow K^0 \pi^0)/\Gamma(D^0 \rightarrow K^- \pi^+) = \frac{1}{2}.\quad(16)$$

Recall that the contribution to this ratio from the spectator graph [Fig. 1(a)] is highly suppressed and amounts to $\frac{1}{30^5}$. Experimentally this ratio is $0.7 \pm 0.35$.\(^{12}\) If our mechanism is important for the above two-body modes, then it will also be important to Cabibbo-suppressed decays such as $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^- \pi^-$.

In short, even a large difference in the lifetimes of charged and neutral $D$ mesons can be explained without requiring a revision of the underlying gauge model and/or invoking exotic new interactions, provided $f_D/m_D \approx 2$. The critical point in our calculation is the observation that in the rate for the reaction $D^0 \rightarrow s + \bar{d} + \text{gluon}$, the dependence on $f_D^2$ is compensated for by the appearance of $m_D^2$ in the denominator.

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\(^{2}\) See, for example, M. K. Gaillard, in \textit{Proceedings of the Summer Institute on Particle Physics, Stanford Linear Accelerator Center, July, 1978}, edited by M. C. Zipf (Stanford Univ. Press, Stanford, Cal., 1978); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B190, 313 (1979); N. Cabibbo and L. Maiani, Phys. Lett. 72B, 418 (1979).

\(^{3}\) A crossed-channel version of Fig. 1b exists for charged-$D$-meson decays but it is Cabibbo suppressed. Throughout this paper we will assume the “standard” Weinberg-Salam and Glashow-Iliopoulos-Maiani model for weak interactions and set the Cabibbo angle in charm decays to zero.

\(^{4}\) See, for example, Cabibbo and Maiani, Ref. 2.

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\(^{7}\) These parameters are taken from the charmonium model of E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. - M. Yan, Phys. Rev. D 17, 3090 (1978).

\(^{8}\) See, for example, R. Montemayor and M. Moreno, Phys. Rev. D 20, 250 (1979); T. Goldman and W. J. Wilson, Phys. Rev. D 15, 709 (1977). References to earlier literature may be found in these two works.

\(^{9}\) See, for example, Eichten et al., Ref. 8.

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