Relations between the Local Weight Distributions of a Linear Block Code, Its Extended Code, and Its Even Weight Subcode

Kenji Yasunaga* and Toru Fujiwara†
Graduate School of Information Science and Technology
Osaka University
1-5 Yamadaoka, Suita, Osaka 565-0871, Japan
E-mail: {k-yasunaga*, fujiwara†}@ist.osaka-u.ac.jp

Abstract—Relations between the local weight distributions of a binary linear code, its extended code, and its even weight subcode are presented. In particular, for a code of which the extended code is transitive invariant and contains only codewords with weight multiples of four, the local weight distribution can be obtained from that of the extended code. Using the relations, the local weight distributions of the (127, k) primitive BCH codes for \( k \leq 50 \), the (127, 64) punctured third-order Reed-Muller code, and their even weight subcodes are obtained from the local weight distribution of the (128, k) extended primitive BCH codes for \( k \leq 50 \) and the (128, 64) third-order Reed-Muller code. We also show an approach to improve an algorithm for computing the local weight distribution proposed before.

I. INTRODUCTION

In a binary linear code, a zero neighbor is a codeword whose Voronoi region shares a facet with that of the all-zero codeword [1]. The local weight distribution [2], [3] (or local distance profile [1], [4], [5], [6], [7]) of a binary linear code is defined as the weight distribution of zero neighbors in the code. Knowledge of the local weight distribution of a code is valuable for the error performance analysis of the code. For example, the local weight distribution could give a tighter upper bound on error probability for soft decision decoding over AWGN channel than the usual union bound [7].

Formulas for local weight distribution are only known for certain classes of codes, Hamming codes and second-order Reed-Muller codes. Although an efficient method to examine zero neighborhood of codeword is presented in [1], the computation for obtaining the local weight distribution is a very time-consuming task. As Agrell noted in [1], the automorphism group of the code can help reduce the complexity. Using the automorphism group of cyclic codes, i.e., cyclic permutations, Mohri et al. devised a computation algorithm using a method for obtaining the representative codewords with respect to the cyclic permutations. The algorithm examines the zero neighborhood only for the representative codewords. By using the algorithm, they obtained the local weight distributions of the binary primitive BCH codes of length 63 [5], [6].

When the automorphism group properly contains cyclic permutations, for example the affine group, an efficient way of finding the representative codewords is unknown. We proposed an algorithm for computing the local weight distribution of a code using the automorphism group of the set of all cosets of a subcode in the code [4]. Using the algorithm, we obtained the local weight distributions of the (128, k) extended primitive BCH codes for \( k \leq 50 \) and the (128, 64) third-order Reed-Muller code [3], [4]. For extended primitive BCH codes, which is closed under the affine group of permutations, our proposed algorithm has considerably smaller complexity than the algorithm in [5] and [6].

However, for cyclic codes, the complexity is not reduced. Then the local weight distributions of the (127, k) primitive BCH codes for \( k \geq 36 \) were not obtained although those of the corresponding (128, k) extended primitive BCH codes are known. A method for obtaining the local weight distribution of a code from that of its extended code should be considered.

In this paper, a relation between local weight distributions of a binary linear code and its extended code is given. A more concrete relation is presented for the case that the extended code is transitive invariant and contains only codewords with weight multiples of four. Extended binary primitive BCH codes and Reed-Muller codes are transitive invariant codes. A relation between local weight distributions of a binary linear code and its even weight subcode is also given. By using the relations, the local weight distributions of the (127, k) binary primitive BCH codes for \( 36 \leq k \leq 50 \), the (127, 64) punctured third-order Reed-Muller code, and their even weight subcodes are obtained from the local weight distributions of the (128, k) primitive BCH codes for \( 36 \leq k \leq 50 \) and the (128, 64) third-order Reed-Muller code. Finally, we give an approach to improve the algorithm proposed in [4].

II. LOCAL WEIGHT DISTRIBUTION

Let \( C \) be a binary \((n, k)\) linear code. Define a mapping \( s \) from \( \{0, 1\} \) to \( \mathbb{R} \) as \( s(0) = 1 \) and \( s(1) = -1 \). The mapping \( s \) is naturally extended to one from \( \{0, 1\}^n \) to \( \mathbb{R}^n \). A zero neighbor of \( C \) is defined [1] as follows:

**Definition 1 (Zero neighbor):** For \( v \in C \), define \( m_0 \in \mathbb{R}^n \) as \( m_0 = \frac{1}{2}(s(0) + s(v)) \) where \( 0 = (0, 0, \ldots, 0) \). The
A codeword \( v \) is a zero neighbor if and only if
\[
d_E(m_0, s(v)) = d_E(m_0, s(0)) < d_E(m_0, s(v')),
\]
for any \( v' \in C \setminus \{0, v\} \),
(1)
where \( d_E(x, y) \) is the Euclidean distance between \( x \) and \( y \) in \( \mathbb{R}^n \).

A zero neighbor is also called a minimal codeword in \([8]\).

The following lemma is useful to check whether a given codeword is a zero neighbor or not \([1]\).

**Lemma 1:** \( v \in C \) is a zero neighbor if and only if there is not \( v' \in C \setminus \{0\} \) such that \( \text{Supp}(v') \subseteq \text{Supp}(v) \). Note that \( \text{Supp}(v) \) is the set of support of \( v \), which is the set of positions of nonzero elements in \( v = (v_1, v_2, \ldots, v_n) \).

The local weight distribution is defined as follows:

**Definition 2 (Local weight distribution):** Let \( L_w(C) \) be the number of zero neighbors with weight \( w \) in \( C \). The local weight distribution of \( C \) is defined as the \((n+1)\)-tuple \((L_0(C), L_1(C), \ldots, L_n(C))\).

On the local weight distribution, we have the following lemma \([2], [8]\).

**Lemma 2:** Let \( A_w(C) \) be the number of codewords with weight \( w \) in \( C \) and \( d \) be the minimum distance of \( C \).
\[
L_w(C) = \begin{cases} 
A_w(C), & w < 2d, \\
0, & w > n - k + 1.
\end{cases}
\]
(2)

When all the weights \( w \) in a code are confined in \( w < 2d \) and \( w > n - k + 1 \), the local weight distribution can be obtained from the weight distribution straightforwardly. For example, the local weight distribution of the \((n, k)\) primitive BCH code of length 63 for \( k \leq 18 \), of length 127 for \( k \leq 29 \), and of length 255 for \( k \leq 45 \) can be obtained from their weight distributions.

III. RELATIONS OF LOCAL WEIGHT DISTRIBUTIONS

A. General relation

Consider a binary linear code \( C \) of length \( n \), its extended code \( C_{ex} \), and its even weight subcode \( C_{even} \). For a codeword \( v \in C \), let \( \text{wt}(v) \) be the Hamming weight of \( v \) and \( v^{(ex)} \) be the corresponding codeword in \( C_{ex} \), that is, \( v^{(ex)} \) is obtained from \( v \) by adding the over-all parity bit. We define a decomposable codeword.

**Definition 3 (Decomposable codeword):** \( v \in C \) is called decomposable if \( v \) can be represented as \( v = v_1 + v_2 \) where \( v_1, v_2 \in C \) and \( \text{Supp}(v_1) \cap \text{Supp}(v_2) = \emptyset \) (see Figure 1).

From Lemma 1, \( v \) is not a zero neighbor if and only if \( v \) is decomposable. For even weight codewords, we introduce an only-odd-decomposable codeword and an even-decomposable codeword.

**Definition 4:** Let \( v \in C \) be a decomposable codeword with even \( \text{wt}(v) \). That is, \( v \) is not a zero neighbor in \( C \). \( v \) is said to be only-odd-decomposable, if all the decomposition of \( v \) is of the form \( v_1 + v_2 \) with the odd weight codewords \( v_1, v_2 \in C \). Otherwise, \( v \) is said to be even-decomposable.

When \( v \) is even-decomposable, there is a decomposition of \( v \), \( v_1 + v_2 \) such that both \( \text{wt}(v_1) \) and \( \text{wt}(v_2) \) are even.

The relation between \( C \) and \( C_{ex} \) with respect to zero neighbor is given in the following theorem, which is also summarized in Table I.

**Theorem 1:**
1) For a zero neighbor \( v \) in \( C \), \( v^{(ex)} \) is a zero neighbor in \( C_{ex} \).

2) For a codeword \( v \) which is not a zero neighbor in \( C \), the following a) and b) hold.

a) When \( \text{wt}(v) \) is odd, \( v^{(ex)} \) is not a zero neighbor in \( C_{ex} \).

b) When \( \text{wt}(v) \) is even, \( v^{(ex)} \) is a zero neighbor in \( C_{ex} \) if and only if \( v \) is only-odd-decomposable in \( C \).

**Proof:**
1) Suppose that \( v^{(ex)} \) is not a zero neighbor in \( C_{ex} \). Then \( v^{(ex)} \) is decomposable into \( v_1^{(ex)} + v_2^{(ex)} \), and hence \( v \) is decomposable into \( v_1 + v_2 \). This contradicts the indecomposability of \( v \).

2) Suppose that \( v \) is decomposed into \( v = v_1 + v_2 \). a) Since \( \text{wt}(v) \) is odd, the sum of the parity bits in \( v_1^{(ex)} \) and \( v_2^{(ex)} \) is one. Also, the parity bit in \( v_1^{(ex)} \) is one. Then \( v^{(ex)} \) is decomposable into \( v_1^{(ex)} + v_2^{(ex)} \), and \( v^{(ex)} \) is not a zero neighbor in \( C_{ex} \). b) Since \( \text{wt}(v) \) is even, the parity bit in \( v_1^{(ex)} \) is zero. (If part) Suppose that \( v^{(ex)} \) is not a zero neighbor in \( C_{ex} \). Then there exists a decomposition \( v^{(ex)} = v_1^{(ex)} + v_2^{(ex)} \). Because the parity bit in \( v_1^{(ex)} \) is zero, the parity bits in \( v_1^{(ex)} \) and \( v_2^{(ex)} \) must be zero. This implies that \( v \) is even-decomposable into \( v_1 + v_2 \), and contradicts the assumption that \( v \) is only-odd-decomposable. (Only if part) Suppose that \( v \) is even-decomposable. Then there is a decomposition such that the parity bits in both \( v_1^{(ex)} \) and \( v_2^{(ex)} \) are zero. For such the decomposition, \( v^{(ex)} \) is decomposable into \( v_1^{(ex)} + v_2^{(ex)} \), and \( v^{(ex)} \) is not a zero neighbor in \( C_{ex} \) (see Figure 2). \( \square \)

From 2)-b) of Theorem 1 there may be codewords that are not zero neighbors in \( C \) although their extended codewords are zero neighbors in \( C_{ex} \). Such codewords are the only-odd decomposable codewords. For investigating relations of local weight distributions between a code and its extended code, only-odd decomposable codewords are important.

The following theorem is a direct consequence of Theorem 1.

---

**Fig. 1.** Examples of a decomposable codeword and an indecomposable codeword.
Theorem 2: For a code $C$ of length $n$,
\[ L_{2i}(C_{\text{ex}}) = L_{2i-1}(C) + L_{2i}(C) + N_{2i}(C), \quad 0 \leq i \leq n/2, \] (3)
where $N_j(C)$ is the number of only-odd-decomposable codewords with weight $j$ in $C$.

From Theorem 2 if there is no only-odd decomposable codeword in $C$, the local weight distributions of $C_{\text{ex}}$ are obtained from that of $C$. Next, we give a useful sufficient condition under which no only-odd-decomposable codeword exists.

Theorem 3: If all the weights of codewords in $C_{\text{ex}}$ are multiples of four, no only-odd-decomposable codeword exists in $C$.

Proof: If $v \in C$ is an only-odd-decomposable codeword and decomposed into $v_1 + v_2$, the weights of $v_1$ and $v_2$ can be represented as $\text{wt}(v_1) = 4i - 1$ and $\text{wt}(v_2) = 4j - 1$ where $i$ and $j$ are integers. Then $\text{wt}(v) = \text{wt}(v_1 + v_2) = \text{wt}(v_1) + \text{wt}(v_2) = (4i - 1) + (4j - 1) = 4i + 4j - 2$. This contradicts the fact that $\text{wt}(v)$ is a multiple of four. \qed

For example, all the weights of codewords in the $(128, k)$ extended primitive BCH code with $k \leq 57$ are multiples of four. The parameters of Reed-Muller codes with which all the weights of codewords are multiples of four are given by Corollary 13 of Chapter 15 in [9]. From the corollary, the third-order Reed-Muller codes of length 128, 256, and 512 have only codewords whose weights are multiples of four.

Although the local weight distribution of $C_{\text{ex}}$ for these codes can be obtained from that of $C$ by using Theorem 2 what we need is a method for obtaining the local weight distribution of $C$ from that of $C_{\text{ex}}$. We need to know the number of zero neighbors with parity bit one. In Section III-B we will show a method to obtain the number of zero neighbors with parity bit one for a class of transitive invariant codes.

A similar relation to that between $C$ and $C_{\text{ex}}$ holds between $C$ and $C_{\text{even}}$. This relation is given in Theorem 4 without proof (see Table I).

Theorem 4: 1) For an even weight zero neighbor $v$ in $C$, $v$ is a zero neighbor in $C_{\text{even}}$.
2) For an even weight codeword $v$ which is not a zero neighbor in $C$, $v$ is a zero neighbor in $C_{\text{even}}$ if and only if $v$ is only-odd-decomposable in $C$.

From Theorem 4 we have Theorem 5.

Theorem 5: For a code $C$ of length $n$,
\[ L_{2i}(C_{\text{even}}) = L_{2i}(C) + N_{2i}(C), \quad 0 \leq i \leq n/2. \] (4)

B. Relation for transitive invariant extended codes

A transitive invariant code is a code which is invariant under a transitive group of permutations. A group of permutations is said to be transitive if for any two symbols in a codeword there exists a permutation that interchanges them [10]. The extended primitive BCH codes and Reed-Muller codes are transitive invariant codes. For a transitive invariant $C_{\text{ex}}$, a relation between the (global) weight distributions of $C$ and $C_{\text{ex}}$ is presented in Theorem 8.15 in [10]. A similar relation holds for local weight distribution.

Lemma 3: If $C_{\text{ex}}$ is a transitive invariant code of length $n + 1$, the number of zero neighbors with parity bit one is
\[ \frac{n}{n+1}L_w(C_{\text{ex}}). \]

Proof: This lemma can be proved in a similar way as the proof of Theorem 8.15. Arrange all zero neighbors with weight $w$ in a column. Next, interchange the $j$-th column and the last column, which is the parity bit column, for all these codewords with the permutation. All the resulting codewords have weight $w$ and must be the same as the original set of codewords. Thus, the number of ones in the $j$-th column and that in the last column are the same. Denote this number $l_w$, which is the same as the number of zero neighbors of weight $w$ with parity bit one. Then the total ones in the original set of codewords is $(n + 1)l_w$, or $L_w(C_{\text{ex}})$ times the weight $w$. Thus, $(n + 1)l_w = wL_w(C_{\text{ex}})$, and $l_w = \frac{n}{n+1}L_w(C_{\text{ex}})$. \qed

It is clear that there are $\frac{n+1-w}{n+1}L_w(C_{\text{ex}})$ zero neighbors with weight $w$ whose parity bit is zero from this lemma. The following theorem is obtained from Theorem 1 and Lemma 3.

Theorem 6: If $C_{\text{ex}}$ is a transitive invariant code of length $n + 1$,
\[ L_w(C) = \frac{n+1-w}{n+1}L_{w+1}(C_{\text{ex}}), \quad \text{for odd } w, \] (5)
\[ L_w(C) = \frac{n+1-w}{n}L_w(C_{\text{ex}}) - N_w(C), \] (6)
\[ \leq \frac{n+1-w}{n+1}L_w(C_{\text{ex}}), \quad \text{for even } w. \] (7)

If there is no only-odd-decomposable codeword in a transitive invariant code $C$, the equality of (7) holds. That is, in this case, we have that
\[ L_w(C) = \frac{n+1-w}{n}L_w(C_{\text{ex}}), \quad \text{for even } w. \] (8)

Therefore, for a transitive invariant code $C_{\text{ex}}$ having no only-odd-decomposable codeword in $C$, the local weight distributions of $C$ can be obtained from that of $C_{\text{ex}}$ by using (5) and (8) in Theorem 6. After computing the local weight distribution of $C_{\text{even}}$, that of $C_{\text{even}}$ can be obtained by using Theorem 5.
IV. Obtained Local Weight Distributions
As discussed in the previous section, the local weight distributions of the \((127, k)\) primitive BCH codes for \(k \leq 57\), the punctured third-order Reed-Muller codes of length 127, 255, and 511, and their even weight subcodes are obtained from those of the corresponding extended codes by using Theorems 5 and 6. Since the local weight distribution for the \((128, 57)\) extended primitive BCH code and the third-order Reed-Muller codes of length 256 and 512 are unknown, only the local weight distributions of the \((127, k)\) primitive BCH codes for \(k = 36, 43, 50\), the \((127, 64)\) punctured third-order Reed-Muller code, and their even weight subcodes are obtained. These local weight distributions are presented in Table I. The local weight distributions of the corresponding even weight subcodes are obtained straightforwardly from the distributions given in the table by using Theorem 5.

V. Computation Algorithm for Longer Codes
As mentioned in Section II, we have computed the local weight distributions of several extended BCH codes and the third-order Reed-Muller code of length 128 by using our computation algorithm in [4]. In the algorithm, the time complexity is reduced by reducing the number of codewords whose zero neighborship should be checked. However, it is still time consuming to compute the local weight distribution of a \((n, k)\) code with larger \(k\) and/or larger \(n\). In improving the computation algorithm, the next target codes include the \((256, 93)\) Reed-Muller code and the \((128, 57)\) extended BCH code.

In this section, we give an approach for improving the algorithm proposed in [4]. First, we briefly review this algorithm. For a binary \((n, k)\) linear code \(C\) and its linear subcode with dimension \(k'\), let \(C/C'\) denote the set of cosets of \(C'\) in \(C\), that is, \(C/C' = \{v + C' : v \in C \setminus C'\}\). Then,
\[
|C/C'| = 2^{k-k'}, \quad \text{and} \quad C = \bigcup_{D \in C/C'} D. \quad (9)
\]

**Definition 5 (Local weight subdistribution for cosets):**
Let \(D\) be a coset in \(C/C'\) and \(LS_w(D)\) be the number of zero neighbors of \(C\) in \(D\) with weight \(w\). The local weight subdistribution for a coset \(D \in C/C'\) is the \((n + 1)\)-tuple \((LS_0(D), LS_1(D), \ldots, LS_n(D))\).

Then
\[
L_w(C) = \sum_{D \in C/C'} LS_w(D). \quad (10)
\]

The following theorem gives an invariance property under permutations for cosets.

**Theorem 7 (Invariance property for cosets):** For \(D_1, D_2 \in C/C'\), the local weight subdistribution for \(D_1\) and for \(D_2\) are the same if there exists \(\pi \in \text{Aut}(C)\) such that \(\pi[D_1] = D_2\), where \(\text{Aut}(C)\) is the automorphism group of \(C\) and \(\pi[D_1] = \{\pi v : v \in D_1\}\).

In the algorithm in [4], given a subgroup \(P\) of \(\text{Aut}(C)\), cosets in \(C/C'\) are partitioned into equivalence classes such that two cosets \(D_1\) and \(D_2\) are in the same class if and only if there is \(\pi \in P\) with \(\pi[D_1] = D_2\). Then if the local weight subdistributions only for the representative cosets of the classes and the sizes of the classes are obtained, we could obtain the local weight distribution of \(C\).

Next, we show the approach. To reduce the complexity more, we consider using the invariance property for cosets in computing the local weight subdistributions for the representative cosets. This means that we consider a coset \(v + C' \in C/C'\) as the set of cosets of \(C''\), where \(C''\) is a linear subcode of \(C'\).

For a coset \(v + C' \in C/C'\), let \((v + C')/C''\) denote the set of all cosets of \(C''\) in \(v + C'\), that is, \((v + C')/C'' = \{v + u + C' : u \in C' \setminus C''\}\). We also call the weight distribution of zero neighbors in \(E \in (v + C')/C''\) the local weight subdistribution for \(E\). The following theorem gives an invariance property for cosets in \((v + C')/C''\).

**Theorem 8:** For \(E_1, E_2 \in (v + C')/C''\), the local weight subdistribution for \(E_1\) and for \(E_2\) are the same if there exists \(\pi \in \{\rho : \rho v \in v + C', \rho \in \text{Aut}(C) \cap \text{Aut}(C')\}\) such that \(\pi[E_1] = E_2\), where \(\pi[E] = \{\pi v : v \in E\}\).

We consider partitioning \((v + C')/C''\) into the equivalence classes. Permutations which are used to partition cosets into equivalence classes are presented in the following lemma.

**Lemma 4:** For a coset \(v + C' \in (v + C')/C''\),
\[
\{\pi : \pi[E] \in (v + C')/C'' \text{ for any } E \in (v + C')/C''\} = \{\rho : \rho v \in v + C', \rho \in \text{Aut}(C) \cap \text{Aut}(C') \cap \text{Aut}(C'')\}.
\]
In order to partition cosets into equivalence classes, we should use permutations presented in Lemma 4. Even if \(\text{Aut}(C), \text{Aut}(C'),\) and \(\text{Aut}(C'')\) are known, we should obtain permutations \(\pi\) that satisfy \(\pi v \in v + C'\).
**TABLE II**

The local weight distributions of the (127, k) primitive BCH codes for k = 36, 43, and 50 and the punctured third-order Reed-Muller code of length 127.

| (127, 36) BCH code | (127, 43) BCH code | (127, 50) BCH code | (127, 64) punct. RM code |
|--------------------|--------------------|--------------------|--------------------------|
| \( w \) | \( L_w \) | \( w \) | \( L_w \) | \( w \) | \( L_w \) | \( w \) | \( L_w \) |
| 31 | 2,667 | 31 | 31,115 |
| 32 | 8,001 | 32 | 93,345 |
| 35 | 4,572 | 36 | 2,418,024 |
| 36 | 11,088 | 36 | 6,332,728 |
| 39 | 641,080 | 39 | 82,556,360 |
| 40 | 1,408,176 | 40 | 181,185,312 |
| 43 | 12,220,936 | 43 | 1,554,145,736 |
| 44 | 23,330,916 | 44 | 2,967,005,496 |
| 47 | 132,360,260 | 47 | 393,384,394 |
| 48 | 220,934,280 | 48 | 40,559,071,892 |
| 51 | 823,921,644 | 51 | 10,538,703,840 |
| 52 | 1,204,193,172 | 52 | 80,188,856 |
| 55 | 3,157,055,916 | 55 | 1,306,771,964,441,395,200 |
| 56 | 3,059,076,464 | 56 | 515,097,101,376 |
| 59 | 7,022,797,740 | 59 | 519,285,716,792 |
| 60 | 7,759,170,772 | 60 | 16,837,453,752 |
| 63 | 9,742,180,368 | 63 | 132,560,568 |
| 64 | 9,742,066,368 | 64 | 27,185,148,448 |
| 67 | 7,959,170,772 | 67 | 380,144,258,760 |
| 68 | 7,022,797,740 | 68 | 905,990,552,400 |
| 71 | 4,059,071,892 | 71 | 515,097,101,376 |
| 72 | 3,157,055,916 | 72 | 400,051,016,884 |
| 75 | 1,204,193,172 | 75 | 155,191,535,184 |
| 76 | 823,921,644 | 76 | 191,183,681,968 |
| 79 | 217,627,200 | 79 | 26,980,367,680 |
| 80 | 130,576,320 | 80 | 16,188,220,608 |
| 83 | 23,530,976 | 83 | 1,617,588,890 |
| 84 | 12,220,936 | 84 | 847,308,440 |
| 87 | 1,408,176 | 87 | 31,115 |
| 88 | 640,080 | 88 | 31,115 |

Let \( \text{RM}(r, m) \) denote the \( r \)-th order Reed-Muller code of length \( 2^m \). We consider the case of the \( (256, 93) \) third-order Reed-Muller code, denoted by \( \text{RM}(3, 8) \). The equivalent cosets in \( \text{RM}(3, 8)/\text{RM}(2, 8) \) are presented in [11], and there are 32 equivalence classes. We choose \( \text{RM}(1, 8) \) as a subcode of \( \text{RM}(2, 8) \). Then the general affine group [9] is a subgroup of \( \text{Aut}(\text{RM}(3, 8)) \cap \text{Aut}(\text{RM}(2, 8)) \cap \text{Aut}(\text{RM}(1, 8)) \). For each cosets in \( \text{RM}(3, 8)/\text{RM}(2, 8) \), the estimated time for computing the local weight subdistribution is about 54 days with the algorithm in [4]. The total estimated time is about 1700 days. To compute the local weight distribution of \( \text{RM}(3, 8) \) in practical time, we should find the permutations \( \pi \) that satisfy \( \pi v \in v + \text{RM}(2, 8) \) for each coset \( v + \text{RM}(2, 8) \) in \( \text{RM}(3, 8)/\text{RM}(2, 8) \). If we could find more than 50 such permutations for each cosets, the local weight distribution of \( \text{RM}(3, 8) \) may be computable.

**VI. Conclusion**

In this paper, some relations between local weight distributions of a binary linear code, its extended code, and its even weight subcode are presented. The local weight distributions of the \( (127, k) \) primitive BCH codes for \( k = 36, 43, 50 \), the \( (127, 64) \) punctured third-order Reed-Muller code, and their even weight subcodes are obtained. If the local weight distribution of the \( (128, 57) \) extended primitive BCH code and the \( (256, 93) \) third-order Reed-Muller code are obtained, we can obtain the local weight distributions of the \( (127, 57) \) primitive BCH code, the \( (255, 93) \) punctured third-order Reed-Muller code, and their even weight subcodes.

**References**

[1] E. Agrell, “Voronoi regions for binary linear block codes,” *IEEE Trans. Inform. Theory*, vol.42, no.1, pp.310–316, Jan. 1996.

[2] E. Agrell, “On the Voronoi Neighbor Ratio for Binary Linear Block Codes,” *IEEE Trans. Inform. Theory*, vol.44, no.7, pp.3064–3072, Nov. 1998.

[3] K. Yasunaga and T. Fujiwara, “The local weight distributions of the \( (128, 50) \) extended binary primitive BCH code and \( (128, 64) \) Reed-Muller code,” IEICE Technical Report, IT2004-19, Jul. 2004.

[4] K. Yasunaga and T. Fujiwara, “An algorithm for computing the local weight distribution of binary linear codes closed under a group of permutations,” *Proc. of ISITA2004*, pp.846–851, Oct. 2004.

[5] M. Mohri, and M. Morii, “On computing the local distance profile of binary cyclic codes,” *IEICE Trans.Fundamentals(Japanese Edition)*, vol.J86-A, no.1, pp.60–74, Jan. 2003.

[6] G.D. Forney, Jr., “Geometrically uniform codes,” *IEEE Trans. Inform. Theory*, vol.37, no.5, pp.1241–1260, Sept. 1991.

[7] A. Ashikhmin and A. Barg, “Minimal vectors in linear codes,” *IEEE Trans. Inform. Theory*, vol.44, no.5, pp.2010–2017, Sept. 1998.

[8] F.J. MacWilliams and N.J.A. Sloane, *The theory of error-correcting codes*, North-Holland, 1977.

[9] W.W. Peterson and E. J. Weldon, Jr., *Error-correcting codes*, 2nd Edition, MIT Press, 1972.

[10] X. Hou, “GL(m,2) acting on \( R(r,m)/R(r-1,m) \),” *Discrete Mathematics*, 149, pp.99–122, 1996.