Joint Subcarrier and Power Allocation Methods in Wireless Powered Communication Network for OFDM systems

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Abstract

In this paper, we investigate wireless powered communication network for OFDM systems, where a hybrid access point (H-AP) broadcasts energy signals to users in the downlink, and the users transmit information signals to the H-AP in the uplink based on an orthogonal frequency division multiple access scheme. We consider a full-duplex H-AP which simultaneously transmits energy signals and receives information signals. In this scenario, we address a joint subcarrier scheduling and power allocation problem to maximize the sum-rate under two cases: perfect self-interference cancelation (SIC) where the H-AP fully eliminates its self interference (SI) and imperfect SIC where the residual SI exist. In general, the problems for both cases are non-convex due to the subcarrier scheduling, and thus it requires an exhaustive search method, which is prohibitively complicated to obtain the globally optimal solution. In order to reduce the complexity, for the perfect SIC scenario, we jointly optimize subcarrier scheduling and power allocation by applying the Lagrange duality method. Next, for the imperfect SIC case, the problem is more complicated due to the SI at the H-AP. To solve the problem, we propose an iterative algorithm based on the projected gradient method. Simulation results show that the proposed algorithm for the case of perfect SIC exhibits only negligible sum-rate performance loss compared to the optimal algorithm, and...
the proposed iterative algorithm for the imperfect SIC case offers a significant performance gain over conventional schemes.

**Index Terms**

Wireless powered communication network (WPCN), wireless energy transfer (WET), wireless information transmission (WIT), orthogonal frequency division multiple access (OFDMA), full-duplex (FD).

**I. INTRODUCTION**

Recently, energy harvesting (EH) has been regarded as a promising technique which can replace traditional energy sources (e.g. batteries), since it offers more cost-effective energy supplies to wireless networks [1]. Especially, harvesting the radio frequency (RF) signal has drawn an enormous amount of attention due to its dual usage, called wireless information transmission (WIT) and wireless energy transfer (WET) [2]. Among various energy transfer systems, simultaneous wireless information and power transfer (SWIPT) systems and wireless powered communication network (WPCN) have been widely investigated. In the SWIPT system, where the WIT and the WET signals are simultaneously transmitted in the downlink, many works identified a trade-off between the achievable sum-rate performance and the harvested energy for various situations [3]–[8].

Unlike the SWIPT system which is confined to the downlink network, in the WPCN, an energy access point radiates RF signals intended for the downlink WET, while the users harvest those energy to transmit the WIT signals to a data access point in the uplink. In [9], a single user WPCN was considered, and the optimal power allocation policy to maximize sum-rate was proposed. The work in [9] was extended to orthogonal frequency division multiplexing (OFDM)-systems in [10] by jointly optimizing subcarrier scheduling over time and power allocation in a single user scenario.

For the multi-user case, the authors in [2] introduced a time division multiple access harvest-then-transmit protocol in the WPCN where the downlink WET and the uplink WIT are implemented sequentially, and proposed the optimal time allocation solution for the sum-rate maximization. Also, in order to maximize
the minimum sum-rate of a space division multiple access WPCN, the optimal energy beamforming, time allocation, and power allocation method has been developed in [11]. However, an orthogonal frequency division multiple access (OFDMA) based WPCN has not yet been studied in the literature to our best knowledge.

Meanwhile, full-duplex (FD) based wireless systems, where a transceiver can transmit and receive signals on the same frequency at the same time, have attracted growing interest due to its potential for increasing spectral efficiency [12]. Although the FD system is capable of doubling the spectral efficiency in the ideal case, strong self-interference (SI) generated by simultaneous transmission and reception at the same node degrades the spectral efficiency in the practical system. Recently, this FD protocol was applied to the WPCN [13], [14] where the downlink WET and the uplink WIT are concurrently carried out and thus the system performance can be significantly improved. The authors in [13] provided the optimal time allocation algorithms to maximize the sum-rate and minimize the total transmission time under a perfect self-interference cancelation (SIC) assumption. In addition, joint power and time allocation for the sum-rate maximization problem for the perfect SIC and the imperfect SIC cases have been investigated in [14], and it was verified that FD-WPCN outperforms the half-duplex WPCN.

In this paper, we study WPCN for OFDM systems where a hybrid-access point (H-AP) operates in a FD mode and all users transmit their information signals to the H-AP based on the OFDMA scheme. For this configuration, we propose joint subcarrier scheduling and power allocation algorithms to maximize the sum-rate under two scenarios: the ideal case where perfect SIC is performed at the H-AP, and the practical case where the residual SI exists.

First, for the perfect SIC case, the sum-rate maximization problem becomes non-convex owing to a subcarrier scheduling function, and thus an exhaustive search method is required to obtain the optimal solution. Since this incurs high computational complexity for comparing the subcarrier candidates, we propose a joint subcarrier scheduling and power allocation algorithm based on the Lagrange duality method. The simulation results confirm that the proposed algorithm shows a negligible performance loss
compared to the optimal exhaustive search method with much reduced complexity.

Then we examine the practical imperfect SIC case where the residual SI degrades the sum-rate performance. In this case, due to the SI, the sum-rate maximization problem becomes more complicated. To solve the problem, we provide an algorithm which first optimizes subcarrier scheduling and the uplink power allocation with given downlink power. Then, we compute a downlink power allocation solution based on the projected gradient method with the given subcarrier scheduling and uplink power. The simulation results show that the proposed algorithm outperforms conventional schemes.

The remainder of this paper is organized as follows: Section II introduces the multiuser WPCN for OFDM systems and formulate the sum-rate maximization problem. In Sections III and IV, the joint subcarrier scheduling and power allocation algorithms for the perfect and the imperfect SIC cases are proposed, respectively. Then, we evaluate the average sum-rate performance of the proposed algorithms in Section V. Finally, the paper is ended with conclusions in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Figure 1, we consider a WPCN for OFDM systems which employs WET in the downlink and WIT in the uplink. The FD H-AP, equipped with single dedicated transmit and receive antennas, broadcasts energy signals to \( K \) users denoted by \( k, (k \in K \triangleq \{1, 2, \cdots, K\}) \), and at the same time, receives information signals transmitted by users. In contrast, each user has a single antenna and operates in half-duplex (HD) mode where SCs for energy harvesting and information transmission are separated. We define the subcarrier set by \( \mathcal{N} = \{1, \ldots, N\} \) and assume that the total bandwidth is equally allocated to \( N \) SCs. The frequency selective channel at subcarrier \( n \) of user \( k \) for downlink and uplink are denoted as \( h_{D,k}[n] \) and \( h_{U,k}[n], (k \in \mathcal{K}, n \in \mathcal{N}) \), respectively, and it is assumed that all the channels are known at the H-AP.

In the OFDMA based uplink WIT, each subcarrier is scheduled to at most one user during the same transmission period. Let us denote \( \Pi(n) \) as the subcarrier scheduling function which represents the user to which subcarrier \( n \) is assigned, and \( S(k) \) as the set of SCs assigned to user \( k \). For example, when the
subcarrier scheduling function is given by $\Pi(1) = 1$, $\Pi(2) = 2$, $\Pi(3) = 2$, and $\Pi(4) = 2$, the subcarrier set can be defined as $S(1) = \{1\}$ and $S(2) = \{2, 3, 4\}$, as shown in Figure 1.

In the downlink of the WPCN for OFDM systems, the H-AP broadcasts wireless energy to all users with transmit power $P_D[n]$ at subcarrier $n$. Let us define the total transmit power and the peak power at the H-AP as $P_T$ and $P_{peak}$, respectively. Then, the power constraint can be expressed as $\sum_{n=1}^{N} P_D[n] \leq P_T$ and $P_D[n] \leq P_{peak}$ for $n \in \mathcal{N}$. The received signal of user $k$ at subcarrier $n$ can be written as

$$y_k[n] = \sqrt{P_D[n]} h_{D,k}[n] x_A[n] + z_k[n], \text{ for } k \in \mathcal{K} \text{ and } n \in \mathcal{N},$$

where $x_A[n]$ indicates the transmitted energy signal of the H-AP at subcarrier $n$ and $z_k[n]$ represents the circularly symmetric complex Gaussian (CSCG) noise at subcarrier $n$ of user $k$ with zero mean and variance $\sigma_k^2$. We assume that $x_A[n]$ and $z_k[n]$ are independent over SCs and $\mathbb{E}[|x_A[n]|^2] = 1$. Then, the

1Throughout this paper, we normalize the time duration to unity, so that the terms power and energy are used interchangeably.
TABLE I
EXAMPLE OF SUBCARRIER SCHEDULING FOR WPCN SYSTEMS BASED ON OFDMA UPLINK TRANSMISSION

|     | 1   | 2    | 3    | 4    |
|-----|-----|------|------|------|
| User 1 | uplink | downlink | downlink | downlink |
| User 2 | downlink | uplink | uplink | uplink |

amount of energy harvested by user $k$ is given by

$$E_k = \zeta \sum_{n \not\in S(k)} P_D[n]|h_{D,k}[n]|^2,$$

(2)

where $0 < \zeta < 1$ is a conversion efficiency of the energy harvesting process. In (2), the downlink power is sufficiently larger than the noise power so that we can ignore it as in [4]. Note that since users operate in the HD mode, user $k$ cannot receive the downlink energy signal through the SCs included in $S(k)$.

Then, in the OFDMA-based uplink WIT, each user transmits information signals to the H-AP through the assigned SCs \{S(k)\} by using the harvested energy $E_k$ in (2). The received signal of the H-AP at subcarrier $n$ can be expressed as

$$\bar{y}_A[n] = \sqrt{P_{U,n}[n]h_{U,n}[n]}x_{U,n}[n] + \beta \sqrt{P_D[n]}\hat{h}_A[n]x_A[n] + z_A[n], \text{ for } n \in \mathcal{N},$$

(3)

where $P_{U,n}[n]$ stands for the uplink transmit power of user $k$ at subcarrier $n$, $x_k[n]$ denotes the information signal which user $k$ transmits through subcarrier $n$ with $\mathbb{E}[|x_k[n]|^2] = 1$, $\hat{h}_A[n]$ accounts for the complex coefficient of the SI channel at subcarrier $n$, and $z_A[n]$ represents the CSCG noise of the H-AP at subcarrier $n$ with zero mean and variance $\sigma_A^2$. We model the residual SI by multiplying the attenuation factor $\beta$ on the downlink signal and set $\mathbb{E}[|h_A[n]|^2] = 1$ as in [14] and [15]. Subsequently, the achievable rate of user
\( R_k = \sum_{n \in S(k)} \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma(\sigma^2 + \beta P_D[n])} \right), \)  

where we specify \( \Gamma \) as the gap between the achievable rate and the channel capacity due to a practical modulation and coding scheme (MCS).

In this paper, we investigate a joint subcarrier scheduling and power allocation problem to maximize the sum-rate, which is given as follows:

\[
\max_{\{S(k)\}, \{P_D\}, \{P_{U,k}\}} \sum_{k=1}^{K} R_k \\
\text{s.t.} \sum_{n=1}^{N} P_D[n] \leq P_T, \tag{6} \\
0 \leq P_D[n] \leq P_{\text{peak}}, \quad n \in N, \tag{7} \\
\sum_{n \in S(k)} P_{U,k}[n] \leq E_k, \quad k \in K, \tag{8}
\]

where (6) and (7) represent the total and peak power constraint at the H-AP, respectively, and (8) means that each user cannot use the power more than the harvested energy \( E_k \) for the uplink transmission. In the following sections, we solve problem (5) for two different cases. First, the perfect SIC is assumed at the H-AP. Then, the case where the residual SI exists is considered.

### III. Perfect SIC Case

In this section, we solve the sum-rate maximization problem (5) in the perfect SIC case, i.e., \( \beta = 0 \). Owing to the subcarrier scheduling variable \( \{S(k)\} \), problem (5) is non-convex. Therefore, to identify the globally optimal solution, exhaustive search over \( K^N \) possible subcarrier candidates is required, and thus the computational complexity burden becomes prohibitively high with large \( N \) and user \( k \). To reduce complexity, we provide an efficient algorithm which jointly optimizes the subcarrier scheduling and the power allocation.

Given that the SI is perfectly canceled at the H-AP, the achievable rate of user \( k \) in (4) can be rewritten
as

\[ R_k^{P-SIC} = \sum_{n \in S(k)} \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma \sigma^2} \right). \]  

(9)

By plugging (9) into problem (5), we can reformulate following optimization problem.

\[
\max_{\{S(k)\}, \{P_D[n]\}, \{P_{U,k}[n]\}} \sum_{k=1}^{K} R_k^{P-SIC}
\]

s.t. (6), (7), and (8)  

(10)

Although the above problem (10) is generally non-convex due to \(\{S(k)\}\), it has been shown that the duality gap of this kind of problem converges to zero as \(N\) increases to infinity \[16\]. Thus, we solve problem (10) using the Lagrange duality method with the zero duality gap.  

The Lagrangian of problem (10) is given by

\[
\mathcal{L}(\{S(k)\}, \{P_D[n]\}, \{P_{U,k}[n]\}, \{\lambda_k\}, \mu) = \sum_{k=1}^{K} R_k - \mu \left( \sum_{n=1}^{N} P_D[n] - P_T \right) - \sum_{k=1}^{K} \lambda_k \left( \sum_{n \in S(k)} P_{U,k}[n] - \zeta \sum_{n \notin S(k)} P_D[n]|h_{D,k}[n]|^2 \right),
\]

where \(\mu\) and \(\{\lambda_k\}\) are the non-negative dual variables related to the constraint (6) and (7), respectively. To obtain the dual function \(g(\{\lambda_k\}, \mu)\), we need to solve the following problem

\[
\max_{\{S(k)\}, \{P_D[n]\}, \{P_{U,k}[n]\}} \mathcal{L}(\{S(k)\}, \{P_D\}, \{P_{U}\}, \{\lambda_k\}, \mu)
\]

s.t. \(0 \leq P_D[n] \leq P_{peak}, \ n \in \mathcal{N},\)

\(P_{U,k}[n] \geq 0, \ n \in \mathcal{N}, k \in \mathcal{K}.\)  

(12)

For fixed \(\{S(k)\}\) (or equivalently \(\{\Pi(n)\}\)), problem (12) is jointly convex with respect to \(\{P_D[n]\}\) and \(\{P_{U,k}[n]\}\). Based on this fact, we first compute \(\{S(k)\}\) and, thereafter, optimize the uplink and the downlink power allocation with the given \(\{S(k)\}\). In the following Lemma 1, we derive the optimal subcarrier scheduling solution of problem (12).

\(^2\)In our simulations in Section V, we have verified that the duality gap of problem (10) is negligible at \(N = 8\) and \(N = 16\). Thus, in this paper, we assume that the duality gap of problem (10) can be ignored for practical size of \(N\).
Lemma 1: With a given set of \(\{\lambda_k\}\) and \(\mu\), the optimal subcarrier scheduling function \(\Pi(n)\) which maximizes the Lagrangian (11) should be chosen as

\[
\Pi(n) = \arg \max_k \log \left(1 + \frac{|h_{U,k}[n]|^2 \hat{P}_{U,k}[n]}{\Gamma \sigma^2}\right) - \lambda_k \hat{P}_{U,k}[n] + \hat{P}_D[n] \left(-\mu + \zeta \sum_{s \neq k} \lambda_s |h_{D,s}[n]|^2\right),
\]

(13)

where \(\hat{P}_{U,k}[n]\) and \(\hat{P}_D[n]\) are determined by

\[
\hat{P}_{U,k}[n] = \left(\frac{1}{\lambda_k} - \frac{\Gamma \sigma^2}{|h_{U,k}[n]|^2}\right)^+, \quad n \in \mathcal{N} \text{ and } k \in \mathcal{K},
\]

(14)

\[
\hat{P}_D[n] = \begin{cases} 
  P_{\text{peak}}, & \zeta \sum_{s \neq k} \lambda_s |h_{D,s}[n]|^2 - \mu > 0, \\
  0, & \text{else}
\end{cases} \quad n \in \mathcal{N}.
\]

Here \((x)^+ \triangleq \max(0, x)\).

Proof: See Appendix A.

Now, with the subcarrier scheduling function \(\{\Pi(n)\}\) (or equivalently \(\{S(k)\}\)) computed in Lemma 1, \(\{P_{U,k}[n]\}\) and \(\{P_D[n]\}\) which maximize the Lagrangian (11) can be obtained by setting \(\frac{\partial}{\partial P_{U,n(n)[n]}} \mathcal{L} = 0\) and \(\frac{\partial}{\partial P_D[n]} \mathcal{L} = 0\), respectively. Then, the optimal uplink and downlink power allocations for problem (12) become

\[
P_{U,k}[n] = \begin{cases} 
  \hat{P}_{U,k}[n], & k = \Pi(n) \\
  0, & \text{else}
\end{cases} \quad n \in \mathcal{N} \text{ and } k \in \mathcal{K},
\]

(15)

\[
P_D[n] = \begin{cases} 
  P_{\text{peak}}, & \zeta \sum_{k \neq \Pi(n)}^K \lambda_k |h_{D,k}[n]|^2 - \mu > 0 \\
  0, & \text{else}
\end{cases} \quad n \in \mathcal{N}.
\]

(16)

Notice that if \(\zeta \sum_{k \neq \Pi(n)}^K \lambda_k |h_{D,k}[n]|^2 - \mu = 0\), the optimal \(P_D[n]\) of problem (12) is not unique and can take any non-negative value. Thus, we take \(P_D[n] = 0\) for such \(n\) which satisfy \(\zeta \sum_{k \neq \Pi(n)}^K \lambda_k |h_{D,k}[n]|^2 - \mu = 0\), only for solving problem (12).

From Lemma 1, (15), and (16), we can obtain the dual function \(g(\{\lambda_k\}, \mu)\) with given dual variables \(\{\lambda_k\}\) and \(\mu\). Then, the dual problem is defined as \(\min_{\{\lambda_k\}, \mu} g(\{\lambda_k\}, \mu)\) and this can be solved by the
ellipsoid method [17]. The sub-gradient of the dual function are expressed by 
\[ \nu = [\nu_{\lambda_1}, \ldots, \nu_{\lambda_K}, \nu_\mu]^T, \]
where
\[
\nu_{\lambda_k} = \zeta \sum_{n \notin S(k)}^N P_D[n] |h_{D,k}[n]|^2 - \sum_{n \in S(k)} P_{U,k}[n], \quad \text{for } k \in K,
\]
\[
\nu_\mu = P_T - \sum_{n=1}^N P_D[n].
\]

Then, the optimal solution \{S(k)\}, \{P_{U,k}[n]\}, and \{P_D[n]\} for problem (10) are calculated with the optimal \{{\lambda_k}^*\} and \{\mu^*\}. It is worth noting that the objective function of problem (10) is an increasing function of each individual \(P_{U,k}[n]\). Therefore, the inequality constraints in (6) and (8) hold with equality at the optimal \{P_{U,k}[n]\} and \{P_D[n]\}. However, a solution from (16) may not achieve the equality in (6), since it is determined as either \(P_{peak}\) or 0. Thus, defining the set \(D_1 = \{n_1 \in \mathcal{N} | \zeta \sum_{k \notin \Pi^* (n_1)}^K \lambda_k^* |h_{D,k}[n_1]|^2 - \mu^* = 0\}, D_2 = \{n_2 \in \mathcal{N} | \zeta \sum_{k \notin \Pi^* (n_2)}^K \lambda_k^* |h_{D,k}[n_2]|^2 - \mu^* > 0\}, and \(D_3 = \{n_3 \in \mathcal{N} | \zeta \sum_{k \notin \Pi^* (n_3)}^K \lambda_k^* |h_{D,k}[n_3]|^2 - \mu^* < 0\}\) where \({\lambda_k}^*\) and \{\mu^*\} are the optimal solutions of the dual problem and \{\Pi^* (n)\} is the corresponding optimal solution obtained from the ellipsoid method, \(P_{D}[n]\) for \(n \in D_1 = P_{peak}\) and \(P_{D}[n]\) for \(n \in D_3 = 0\) can be set as the optimal downlink power allocation.

To satisfy the constraint (6) with equality, \(\{P_D[n]\}_{n \in D_1}\), on the other hand, can be determined by solving the following problem:
\[
\text{max}_{\{P_D[n]\}_{n \in D_1}, \{P_{U,k}[n]\}} \sum_{k=1}^K \sum_{n \in S(k)} \log(1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma \sigma^2}) \quad (17)
\]
\[\text{s.t.} \quad P_D[n] = P_T - P_{peak}[D_2], \]
\[\quad P_{U,k}[n] = \zeta \sum_{n \in S(k)} P_D[n] |h_{D,k}[n]|^2, \quad k \in K, \]
\[\quad 0 \leq P_D[n] \leq P_{peak}, \quad n \in \mathcal{N}. \]

With fixed \{S(k)\}, problem (17) is jointly convex over \{P_{U,k}[n]\} and \{P_D[n]\}_{n \in D_1}. This convex problem (17), which contains the reduced number of the optimization variables\(^3\) can now be efficiently solved.

\(^3\)Considering the property of the OFDMA user scheduling function \(\Pi[n]\) and the fact that the total amount of the harvested energy in (2) is a linear function of \(P_D[n]\), we can readily verify that \(|D_1|\) is less than user \(K\). For the sake of brevity, we omit the proof.
by the existing software, e.g., CVX [17]. To summarize, our algorithm to solve problem (10) is given in Algorithm 1.

Algorithm 1: Joint subcarrier scheduling and power allocation algorithm of WPCN for OFDM systems with perfect SIC

Initialize \( \{ \lambda_k > 0 \} \) and \( \mu > 0 \).

Repeat

- Compute \( \{ \hat{P}_{U,k}[n] \} \) and \( \{ \hat{P}_D[n] \} \) by (14).
- Obtain the scheduling function \( \Pi(n) \) for \( n \in \mathcal{N} \) by (13).
- Calculate \( \{ P_{U,\Pi(n)}[n] \} \) and \( \{ P_D[n] \} \) for \( n \in \mathcal{N} \) by (15).
- Update \( \{ \lambda_k \} \) and \( \mu \) by using the ellipsoid method.

Until convergence.

Set \( \Pi^*(n) = \Pi(n) \) for \( n \in \mathcal{N} \).

Compute the set \( \mathcal{D}_1, \mathcal{D}_2, \) and \( \mathcal{D}_3 \).

Set \( P^*_D[n] = \begin{cases} P_{\text{peak}}, & n \in \mathcal{D}_2 \\ 0, & n \in \mathcal{D}_3 \end{cases} \)

Obtain \( \{ P^*_U,k[n] \} \) and \( \{ P^*_D[n] \}_{n \in \mathcal{D}_1} \) by solving (17).

IV. IMPERFECK SIC CASE

Next, we consider a more practical case where the SI is not totally canceled at the H-AP, i.e., \( \beta > 0 \) in problem (5). Unlike the previous perfect SIC case, problem (5) is generally non-convex even when the subcarrier scheduling function \( \{ S(k) \} \) is fixed, since the objective function is non-convex with respect to \( \{ P_D[n] \} \). Thus, it is more difficult to find the globally optimal solution for problem (5) in an efficient manner. However, a locally optimal solution can be found by updating \( \{ P_{U,k}[n] \}, \{ S(k) \}, \) and \( \{ P_D[n] \} \) iteratively. To be specific, we first simultaneously update \( \{ P_{U,k}[n] \} \) and \( \{ S(k) \} \) with fixed \( \{ P_D[n] \} \), similar to the perfect SIC case, using the Lagrangian duality method. Then, \( \{ P_D[n] \} \) are updated with
fixed \( \{ P_{U,k}[n] \} \) and \( \{ S(k) \} \) based on the projected gradient method. The above procedure is alternated until the sum-rate converges.

Let us denote \( P_{U,k}[n] \) and \( P_{D}[n] \) as the uplink and downlink power allocation obtained at the \( i \)-th iteration, respectively. Given \( \{ P_{D}^{(i-1)}[n] \} \), we can simplify problem (5) as

\[
\begin{align*}
\max_{\{ P_{U,k}[n] \}} & \quad \sum_{k=1}^{K} R_k \\
\text{s.t.} & \quad \sum_{n \in S(k)} P_{U,k}[n] \leq E_k, \quad k \in \mathcal{K}, \\
& \quad P_{U,k}[n] \geq 0, \quad n \in \mathcal{N}, k \in \mathcal{K}.
\end{align*}
\]

With given \( \{ P_{D}^{(i-1)}[n] \} \), the above problem (18) is similar to that of the perfect SIC case. Although problem (18) is non-convex due to the subcarrier scheduling function \( \{ \Pi(n) \} \), we can solve it by exploiting the Lagrange duality method with the zero duality gap. The Lagrangian of problem (18) is given by

\[
\mathcal{L}_SI(\{ S(k) \}, \{ P_{U,k}[n] \}, \{ v_k \}) = \sum_{k=1}^{K} R_k - \sum_{k=1}^{K} v_k \left( \sum_{n \in S(k)} P_{U,k}[n] - \zeta \sum_{n \notin S(k)} P_{D}^{(i-1)}[n] | h_{D,k}[n] |^2 \right),
\]

where \( v_k, \forall k \in \mathcal{K} \), denote the non-negative dual variables associated with the constraint (19).

The dual function \( g(\{ v_k \}) \) can be obtained by solving the following problem

\[
\max_{\{ S(k) \}, \{ P_{u} \}} \mathcal{L}_SI(\{ S(k) \}, \{ P_{u,k}[n] \}, \{ v_k \}) \tag{21}
\]

\[
\text{s.t.} \quad P_{U,k}[n] \geq 0, \quad \forall k, n.
\]

We will sequentially compute the optimal \( \{ S(k) \} \) and \( \{ P_{U,k}[n] \} \) of problem (21). First, we provide the optimal subcarrier scheduling function of problem (21) with given \( \{ P_{D}^{(i-1)}[n] \} \) in the following lemma 2.

**Lemma 2**: With a given set of \( \{ P_{D}^{(i-1)}[n] \} \) and \( \{ v_k \} \), the optimal subcarrier scheduling function \( \Pi(n) \) that maximizes the Lagrangian (20) can be obtained as

\[
\Pi^{(i)}(n) = \arg \max_{k} \log \left( 1 + \frac{|h_{U,k}[n]|^2 \hat{P}_{U,k}^{(i)}[n]}{\Gamma(\sigma^2 + \beta P_{D}^{(i-1)}[n])} \right) - v_k \hat{P}_{U,k}^{(i)}[n] + P_{D}^{(i-1)}[n] \left( \zeta \sum_{s \neq k} v_s | h_{D,s}[n] |^2 \right), 
\]

\[
(22)
\]
where $\hat{P}^{(i)}_{U,k}[n]$ is

$$\hat{P}^{(i)}_{U,k}[n] = \left( \frac{1}{v_k} - \frac{\Gamma(\sigma^2 + \beta P_D^{(i-1)}[n])}{|h_{U,k}[n]|^2} \right)^+, \text{ for } n \in \mathcal{N} \text{ and } k \in \mathcal{K}. \quad (23)$$

Proof: The proof is similar as that of Lemma 1, and thus is omitted. ■

With the subcarrier scheduling function $\Pi^{(i)}(n)$ determined in (22), $\{P_{U,k}[n]\}$ maximizing the Lagrangian (20) can be found by setting $\frac{\partial}{\partial P_{U,k}^{(i)}[n]} \mathcal{L} = 0$. Then, the optimal uplink power allocation is given by

$$P_{U,k}^{(i)}[n] = \begin{cases} \hat{P}^{(i)}_{U,k}[n], & \text{for } k = \Pi^{(i)}(n) \\ 0, & \text{else} \end{cases} \quad (24)$$

Subsequent to identifying the dual function $g(\{v_k\})$ by using Lemma 2 and (24), the dual problem is then defined as $\min_{\{v_k\}} g(\{v_k\})$ and this can be solved by the ellipsoid method where the sub-gradient of the dual function can be computed as $\nu = [\nu_1, \cdots, \nu_K]^T$, where

$$\nu_v = \zeta \sum_{n \notin S^{(i)}(k)} P_D^{(i-1)}[n]|h_{D,k}[n]|^2 - \sum_{n \in S^{(i)}(k)} P_{U,k}^{(i)}[n], \text{ for } k \in \mathcal{K}.\)$$

After the ellipsoid method converges, the optimal $\{S^{(i)}(k)\}$ and $\{P_{U,k}^{(i)}[n]\}$ of problem (21) is obtained corresponding to the optimal dual variable $\{v_k^*\}$.

Once $\{S^{(i)}(k)\}$ and $\{P_{U,k}^{(i)}[n]\}$ are found, $\{P_D^{(i)}[n]\}$ can be identified by exploiting the projected gradient method [17]. The gradient of the objective function $R(\{P_D^{(i-1)}[n]\}) = \sum_{k=1}^K \sum_{n \in S^{(i)}(k)} \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}^{(i)}[n]}{\Gamma(\sigma^2 + \beta P_D^{(i-1)}[n])} \right)$ is expressed as $\nabla r = [r_1^{(i)}, \cdots, r_N^{(i)}]^T$, where

$$r_n^{(i)} = \frac{-P_{U,k}^{(i)}[n]|h_{U,k}^{(i)}[n]|^2 \beta}{\Gamma(\sigma^2 + \beta P_D^{(i-1)}[n])^2}, \text{ for } n \in \mathcal{N}. \quad (25)$$
For simplicity, we denote \( \mathbf{P}_D^{(i)} = [P_D^{(i)}[1], \ldots, P_D^{(i)}[N]]^T \). Then, by applying the above gradient in (25) as a descent direction, \( \mathbf{P}_D^{(i)} \) can be updated as

\[
\mathbf{P}_D^{(i)} = \mathcal{P}_E(\mathbf{P}_D^{(i-1)} + t^{(i)} \nabla \mathbf{r})
\]

(26)

where \( t^{(i)} \) is a small step size and \( \mathcal{P}_E(x) \) represents the projection operation of \( x \) onto a feasible set

\[
\mathcal{E} = \{ \mathbf{P}_D | \sum_{n=1}^{N} P_D[n] = P_T, 0 \leq P_D[n] \leq P_{peak} \}
\]

For the imperfect SIC case, the above procedure is repeated until the sum-rate

\[
R(\{P_{U,k}[n]\}, \{P_D[n]\}, \{S(k)\})
\]

converges. Note that this method yields a locally optimal solution, depending on the choice of initial values of \( \{P_D^{(0)}[n]\} \). Thus, we randomly generate \( M \) feasible \( \{P_D^{(0)}[n]\} \) as the initialization points. For each initialization, a locally optimal solution is obtained and the final solution can be chosen as the one that achieves the best sum-rate. Finally, we summarize the proposed algorithm which solves problem (5) for the imperfect SIC case in Algorithm 2.

\[\text{In general, the total power constraint (6) may not hold with equality at the optimal \{P_D[n]\}, but this is usually desirable since there is no energy waste at the H-AP.}\]
Algorithm 2: Subcarrier scheduling and power allocation algorithm of WPCN for OFDM systems with the residual SI

Set $i = 0$ and $\{P^{i(0)}_{U,k}\} = 0$, for $n \in \mathcal{N}$ and $k \in \mathcal{K}$.

Repeat

Set $i \leftarrow i + 1$ and initialize the dual variables $\{\upsilon_k > 0\}$.

Repeat

Compute $\{\hat{P}_{U,k}[n]\}$ by (23).

Obtain scheduling function $\Pi^{(i)}(n)$ for $n = 1, \ldots, N$ by (22).

Compute $\{P^{(i)}_{U,\Pi(n)}\}$ for $n = 1, \ldots, N$ by (24).

Update $\{\upsilon_k\}$ by using the ellipsoid method.

Until $\{\upsilon_k\}$ converges.

Update $P^{(i)}_{D}[n]$ for $n = 1, \ldots, N$ by applying the projected gradient method (26).

Until $R(\{P^{(i)}_{U,k}[n]\}, \{S^{(i)}(k)\}, \{P^{(i)}_{D}[n]\})$ converges.

V. SIMULATION RESULTS

In this section, we evaluate the average sum-rate performance of the WPCN for OFDM systems in the perfect and the imperfect SIC cases. Throughout simulations, the total bandwidth is set to be 10 MHz, which is equally divided by $N$ SCs. The frequency selective uplink and downlink channels for different users are independently generated by the 6 tap exponentially distributed power profile. Also, the distance from the H-AP to all users is 1 meter which results in $-30$ dB path-loss for all subcarrier channels and the noise power spectral density equals -112 dBm/Hz as in [6]. In addition, the energy harvesting efficiency, the MCS gap, and the number of initializations $M$ are set to be $\zeta = 0.5$, $\Gamma = 9$ dB, and $M = 20$ respectively.

In this Section, we compare the average sum-rate performance with following two conventional schemes.

*We confirm through our simulation setup that $M = 20$ is sufficient for increasing overall performance.*
Fig. 2. Average sum-rate of the proposed algorithm and the exhaustive search for the perfect SIC with $K = 2$ and $P_{\text{peak}} = \frac{2P_T}{N}$

- **Equal downlink power allocation**: evenly assigns the downlink power, i.e., $P_D[n] = \frac{P_T}{N}$, $n \in \mathcal{N}$.

Then, the subcarrier scheduling function $\{\Pi(n)\}$ is obtained by using our solution with given $\{P_D[n]\}$, and the uplink power allocation is computed by the water-filling algorithm.

- **Channel-based subcarrier scheduling**: the subcarrier scheduling function $\{\Pi(n)\}$ is chosen by selecting the user with the largest uplink channel at subcarrier $n$, i.e., $\Pi(n) = \arg \max_k h_{U_k}[n]$, $n \in \mathcal{N}$. Then, the downlink and the uplink power allocations are implemented by employing the proposed algorithms.

Figure 2 illustrates the average sum-rate of the WPCN for OFDM systems in the perfect SIC case with $K = 2$ and $P_{\text{peak}} = \frac{2P_T}{N}$. Here, we also plot the performance of the optimal scheme which finds the optimal subcarrier scheduling function $\{\Pi^*(n)\}$ by the exhaustive search. Then, the downlink and the uplink power allocation can be computed from the proposed algorithm with given $\{\Pi^*(n)\}$. As shown in Figure 2, no performance difference is observed between the exhaustive search and the proposed algorithm, which verifies the zero duality gap. Note that the exhaustive search method requires $K^N$ comparison for
the subcarrier scheduling, while the proposed algorithm only compares $KN$ candidates. Therefore, when $N = 16$, the number of candidates for the subcarrier scheduling for the proposed algorithm and the exhaustive search become $2^{16}$ and $2 \times 16$, respectively, where the latter amounts to just 0.0488% of the former. This infers that our proposed algorithm exhibits a near-optimal performance with dramatically reduced complexity.

In Figure 3, the average sum-rate of WPCN for OFDM systems with $N = 64$ is demonstrated for the perfect SIC case. We can observe that the proposed algorithm outperforms the conventional equal downlink power allocation schemes, and the performance gap increases as the available total power at the H-AP grows. With $P_T = 35$ dBm, the proposed algorithm provides 64% and 50% gains over the downlink equal power allocation scheme at $K = 2$ and $K = 4$, respectively.

In Figure 4, we illustrate the convergence behavior of the proposed algorithm for the imperfect SIC case with different system parameters. We observe that for all cases, the average sum-rate converges within...
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iteration stage
0 10 20 30
Avg. Sum-rate (bps/Hz)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
K=4, P_T=40dBm, N=64
K=4, P_T=40dBm, N=32
K=4, P_T=25dBm, N=64
K=2, P_T=40dBm, N=64

Fig. 4. Average sum-rate with respect to the iteration number of the proposed algorithm for the imperfect SIC case

15 iterations. In addition, it is shown that the average sum-rate converges faster for small user $k$ and $P_T$, while $N$ does not affect the number of iterations for the convergence.

Next, by fixing $K = 2$, $P_{Peak} = \frac{2P_T}{N}$, and $N = 64$, Figure 5 depicts the average sum-rate performance for the imperfect SIC case with different $\beta$, which indicates the level of the SIC. Since $\beta = 0$ indicates no self interference at the H-AP which is equivalent to the perfect SIC case, we plot the performance of it with our proposed algorithm for perfect SIC. It is observed that the average sum-rate increases as $\beta$ decreases and the performance gap between the perfect and imperfect SIC cases increases as $P_T$ grows, since the SI significantly degrades the achievable sum-rate performance at high $P_T$ regime.

Figure 6 compares the average sum-rate of the proposed algorithm with conventional schemes in the imperfect SIC case. First, we can see that the proposed algorithm exhibits a significant performance enhancement compared to the channel-based subcarrier scheduling scheme, since SI is not considered when obtaining the subcarrier scheduling. In addition, it is seen that the performance of the equal downlink power allocation scheme is saturated as $P_T$ increases. This is due to the fact that the SI have more influence
Fig. 5. Average sum-rate of the proposed algorithm for different values of $\beta$ with $K = 2$, $N = 64$, and $P_{\text{peak}} = \frac{2P_T N}{N}$.

Fig. 6. Average sum-rate of WPCN for OFDM systems with $\beta = -60$ dBm, $N = 64$, and $P_{\text{peak}} = \frac{2P_T N}{N}$.
on the sum-rate than the uplink power allocation. This result shows contrast to the perfect SIC case where the SI is not reflected in the sum-rate which increases as the $P_T$ grows.

VI. CONCLUSION

In this paper, we have investigated joint subcarrier scheduling and power allocation algorithms of WPCN for OFDM systems where the FD H-AP is employed. We have considered two different scenarios according to the level of the SIC. First, for the perfect SIC case, a joint subcarrier scheduling, downlink and uplink power allocation algorithm has been proposed based on the Lagrange duality method, and we have proven that it achieves the near-optimal performance with much reduced complexity. Next, this work has been extended to the practical imperfect SIC case, and an iterative algorithm has been introduced by using the projected gradient method. Simulation results have confirmed that the proposed algorithm outperforms the conventional schemes, and shown that the downlink power allocation plays a key role when maximizing the sum-rate of WPCN for OFDM systems with the FD H-AP.
Appendix A

Proof of Lemma 1

To obtain the dual function of problem (12), we consider the following Lagrangian maximization problem.

\[
\max_{\{S(k)\}, \{P_D[n]\}, \{P_{U,k}[n]\}} \mathcal{L}(\{S(k)\}, \{P_D[n]\}, \{P_{U,k}[n]\}, \{\lambda\}, \mu)
\]

\[
= \max_{\{\Pi(n)\}, \{P_D[n]\}, \{P_{U,k}[n]\}} \sum_{n=1}^{N} \left( \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma \sigma^2} \right) - \lambda_{\Pi(n)} P_{U,\Pi(n)} \right.
\]

\[
+ P_D[n] \left( -\mu + \zeta \sum_{k \neq \Pi(n)}^K \lambda_k |h_{D,k}[n]|^2 \right) + \mu P_T
\]

\[
= \sum_{n=1}^{N} \max_k \left\{ \max_{\{P_D[n]\}, \{P_{U,k}[n]\}} \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma \sigma^2} \right) + P_D[n] \left( -\mu + \zeta \sum_{s \neq k}^K \lambda_s |h_{D,s}[n]|^2 \right) - \lambda_k P_{U,k} \right\}
\]

\[
+ \mu P_T
\]

\[
= \sum_{n=1}^{N} \max_k \left\{ \max_{\{P_D[n]\}, \{P_{U,k}[n]\}} \hat{\mathcal{L}}_{n,k} \right\} + \mu P_T = \sum_{n=1}^{N} \arg \max_k \mathcal{L}_{n,k} + \mu P_T
\]

\[
= \sum_{n=1}^{N} \mathcal{L}_n + \mu P_T,
\]

where \( \hat{\mathcal{L}}_{n,k} \), \( \mathcal{L}_{n,k} \), and \( \mathcal{L}_n \) are defined as

\[
\hat{\mathcal{L}}_{n,k} \triangleq \log \left( 1 + \frac{|h_{U,k}[n]|^2 P_{U,k}[n]}{\Gamma \sigma^2} \right) + P_D[n] \left( -\mu + \zeta \sum_{s \neq k}^K \lambda_s |h_{D,s}[n]|^2 \right) - \lambda_k P_{U,k},
\]

\[
\mathcal{L}_{n,k} \triangleq \max_{\{P_D[n]\}, \{P_{U,k}[n]\}} \hat{\mathcal{L}}_{n,k}
\]

\[
\mathcal{L}_n \triangleq \max_k \mathcal{L}_{n,k}.
\]

As we can see, Lagrangian (11) can be expressed as sum of individual \( \mathcal{L}_n \) which is obtained by finding the maximum value of \( \mathcal{L}_{n,k} \) over \( k \in \mathcal{K} \). To calculate \( \mathcal{L}_{n,k} \), we first set \( \Pi(n) = k \), and then use the zero gradient condition \( \frac{\partial}{\partial P_{U,k}[n]} \hat{\mathcal{L}}_{n,k} = 0 \), and \( \frac{\partial}{\partial P_D[n]} \hat{\mathcal{L}}_{n,k} = 0 \). Finally, we can respectively obtain \( \hat{P}_{U,k}[n] \) and \( \hat{P}_D[n] \) that maximize \( \hat{\mathcal{L}}_{n,k} \) as shown in (14).

References

[1] C. K. Ho and R. Zhang, “Optimal Energy Allocation for Wireless Communications With Energy Harvesting Constraints,” *IEEE Trans. Signal Process.*, vol. 60, pp. 4808 – 4818, Sep. 2012.
[2] H. Ju and R. Zhang, “Throughput maximization in wireless powered communication networks,” *IEEE Trans. Wireless Commun.*, vol. 13, pp. 418–428, Jan. 2014.

[3] X. Zhou, R. Zhang, and C. K. Ho, “Wireless Information and Power Transfer: Architecture Design and Rate-Energy Tradeoff,” *IEEE Trans. Wireless Commun.*, vol. 61, pp. 4754–4767, Oct. 2013.

[4] R. Zhang and C. K. Ho, “MIMO broadcasting for simultaneous wireless information and power transfer,” *IEEE Transac. Signal Process.*, vol. 12, pp. 1989–2001, May 2013.

[5] J. Xu, L. Liu, and R. Zhang, “Multiuser MISO Beamforming for Simultaneous Wireless Information and Power Transfer,” *IEEE Trans. Wireless Commun.*, vol. 62, pp. 4798–4810, Sep. 2014.

[6] X. Zhou, R. Zhang, and C. K. Ho, “Wireless Information and Power Transfer in Multiuser OFDM Systems,” *IEEE Trans. Wireless Commun.*, vol. 13, pp. 2282–2294, Apr. 2014.

[7] K. Huang and E. Larsson, “Simultaneous Information and Power Transfer for Broadband Wireless Systems,” *IEEE Trans. Signal Process.*, vol. 61, pp. 5972–5986, Dec. 2013.

[8] H. Lee, S.-R. Lee, H.-B. Kong, and I. Lee, “Optimal Beamforming Designs for Wireless Information and Power Transfer in MISO Interference Channels,” *submitted to IEEE Trans. Wireless Commun.*, Aug. 2014.

[9] X. Zhou, C. K. Ho, and R. Zhang, “Wireless power meets energy harvesting: A joint energy allocation approach,” in *Proc. IEEE Global Conf. on Signal and Information Processing*, pp. 198–202, Dec. 2014.

[10] X. Zhou, C. K. Ho, and R. Zhang, “Wireless Power Meets Energy Harvesting: A Joint Energy Allocation Approach in OFDM-based System,” *submitted to IEEE Transactions Information Theory*.

[11] L. Liu, R. Zhang, and K.-C. Chua, “Multi-Antenna Wireless Powered Communication With Energy Beamforming,” *IEEE Trans. Commun.*, vol. 62, pp. 4649 – 4361, Dec. 2014.

[12] M. Ahn, H.-B. Kong, H. M. Shin, and I. Lee, “A Low Complexity User Selection Algorithm for Full-Duplex MU-MISO Systems,” *submitted to IEEE Trans. Wireless Commun.*, Mar. 2015.

[13] X. Kang, C. K. Ho, and S. Sun, “Full-Duplex Wireless-Powered Communication Network with Energy Causality,” *submitted to IEEE Trans. Wireless Commun.*, Apr. 2014.

[14] H. Ju and R. Zhang, “Optimal Resource Allocation in Full-Duplex Wireless-Powered Communication Network,” *IEEE Trans. Commun.*, vol. 62, pp. 3528–3540, Oct. 2014.

[15] W. Li, J. Lilleberg, and K. Rillinen, “On Rate Region Analysis Of Half-and Full-Duplex OFDM Communication Links,” *IEEE J. Sel. Areas Commun.*, vol. 32, pp. 1688–1698, Jun. 2014.

[16] W. Yu and R. Lui, “Dual Methods for Nonconvex Spectrum Optimization of Multicarrier Systems,” *IEEE Trans. Commun.*, vol. 54, pp. 1310–1322, Jul. 2006.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.