Cepheids as Distance Indicators

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We review the use of Cepheids as distance indicators with particular emphasis on the methods which have been applied to HST observations of Cepheids. The calibration of the period-luminosity relations is examined in detail and we identify possible problems with the existing calibrations. New V- and I-band period luminosity relations are presented based on a sample of 53 Cepheids in the LMC with photometry drawn from the literature. These revised PL relations result in a systematic decrease of $\approx 0.1$ magnitudes in distance moduli derived using the standard method of extinction correction. Hence estimates of $H_0$ based on such distances should be increased by $\sim 5\%$. Other aspects of Cepheid distance determination, specifically incompleteness bias, metallicity dependence, the effects of crowding and contamination of samples by non-Cepheids are also discussed. We conclude that current HST distance estimates to individual galaxies are probably good to about 10%, but that much of this error is systematic. Efforts to reduce the systematics, therefore, for example by improving the photometric calibration, refining the distance to the LMC, and reobserving the Cepheid galaxies in the infrared with NICMOS, will give large returns.

1. Introduction

Cepheid variables are the most important primary distance indicators and form the foundation of the extragalactic distance scale. It was Henrietta Leavitt who in 1912 first recognised their potential as standard candles from her observations of variables in the Small Magellanic Cloud (SMC). Subsequently Hubble (1924) used Cepheids to find distances to M31 and M33, based on Shapley’s (1918) period–luminosity (PL) relation, thus proving the “island–universe” hypothesis of the nature of the spiral nebulae. In 1952 Baade fundamentally revised the Cepheid distance scale, differentiating for the first time between Cepheids of populations I and II, with the result that distance estimates to external galaxies were increased by a factor $\sim 2$ bringing them close to the modern values. Recently the large investment of time in Cepheid observations by the Hubble Space Telescope (HST) has led to something of a renaissance of interest in the field. By the end of cycle 6 more than 20 new galaxies will have been surveyed for Cepheids, which it is hoped will lead to the Hubble constant being established to better than 10%. It is now more important than ever to examine the basis of Cepheid distance determination and to rigorously evaluate the reliability of Cepheid distances.

Classical (population I) Cepheids are relatively massive stars, and hence are short lived. Consequently they are only found in late-type, spiral and irregular, galaxies in regions of moderately recent star formation. As distance indicators they have many desirable properties: they are bright compared to most other stellar distance indicators, reaching absolute visual magnitudes of $M_V \sim -6.5$; they are easy to identify from their regular variability and characteristic light-curves; and the main distance independent parameter, period of oscillation, can be obtained with high precision.

Importantly Cepheids are also well understood theoretically. Briefly: having exhausted its hydrogen core, a star will evolve off the main-sequence and move rapidly across the Hertzsprung-Russell diagram to the red-giant branch. A sufficiently massive star will subsequently begin to burn helium in its core and move back again to higher temperature executing a “blue loop” (see e.g. Chiosi 1990). This second crossing of the HR diagram...
proceeds at a slower pace and the star may spend a significant time in the so-called “instability strip”. Possible causes of instability are many and complex (e.g. Cox 1985), but the major contributor in the case of Cepheids is thought to be the He$^+$ ionization zone which, if it occurs at the appropriate level in the star, will drive oscillations at the star’s natural frequency (or, in some circumstances, higher harmonics - see section 1.2). In fact, most of the variation in luminosity is due to changes in temperature with only comparatively small changes in radius. The natural frequency itself depends on the mean density or, via the Stefan-Boltzmann law, on the mass, luminosity and effective temperature of the star (Sandage 1958). An implication of this is that the Cepheid PL relation is actually a projection of a more general period–luminosity–colour (PLC) relation, and the spread of the PL is determined by the width of the instability strip.

From the ground, reasonable samples of well-observed Cepheids have only been obtained for galaxies at distances $\lesssim 4$Mpc, which encompasses just the local group and its few nearest neighbours. With HST this distance range has been increased to $> 20$Mpc, representing an expansion of more than 2 orders of magnitude in the available volume, and bringing into play a number of rich groups and clusters of galaxies. Thus it is now becoming possible to calibrate a host of secondary indicators using samples of many galaxies which have direct Cepheid distances and a much larger number which have distances by association (as witnessed by several other contributions to these proceedings).

In this review we concentrate mainly on the procedures which have been used to obtain Cepheid distances with the HST, and try to assess the remaining uncertainties particularly the systematics. These procedures are outlined in section 2 where we describe the means of correcting for reddening and their consequences. In section 3 we investigate the reliability of the period-luminosity relations in the $V$- and $I$-bands, and present new calibrations based on a large sample of 53 Cepheids. In section 4 we explore in some detail other potential problems in using Cepheids, such as the effect of metallicity differences between the target galaxy and the calibrators; the influence of contamination of Cepheid samples by overtone pulsators and non-Cepheid variables; and statistical biases. Section 5 examines some new developments which may lead to improvements in Cepheid distance determination. Finally, in section 6 we attempt to draw together these threads to give a realistic estimate of the full uncertainty on HST Cepheid distances. There are a number of excellent reviews which also examine other issues: Feast & Walker (1987), Madore & Freedman (1991), Caldwell & Laney (1991) and Welch (in Jacoby et al. 1992).

2. Cepheid studies with HST

All Cepheid studies with HST to date have adopted essentially the same observing strategy. $V$-band (WFPC2 F555W filter) images are obtained at 12 or more suitably spaced epochs spanning at least 8 weeks. The spacing of the epochs can be optimised to search for variables in the range of interest, typically $1 < \log(P) < 1.8$ (see Freedman et al. 1994). Photometry is then found for all stars on each frame down to some magnitude limit. Details of the standard photometric calibration are given by Holtzman et al. (1995) and discussed further by Hill et al. (1996), who also investigate the so-called “long–vs–short exposure” correction, which is due to a small but poorly understood non-linearity type problem with the WFPC2 chips. These data are used to identify variables and measure their periods. Intensity mean $V$-band magnitudes, denoted $\langle V \rangle$ are calculated, usually with phase-weighting as recommended by Saha & Hoessel (1990).
\[ \langle m \rangle = -2.5 \log_{10} \sum_{i=1}^{n} 0.5(\phi_{i+1} - \phi_{i-1})10^{-0.4m_i} \]  

(2.1)

where \( \phi_i \) and \( m_i \) are the phase and magnitude of the \( i^{th} \) epoch after folding on the best period.

Near infrared, \( I \)-band (WFPC2 F814W or WF/PC1 F785LP filters, which are close to Cousins \( I \)), observations are also obtained, although at fewer, typically 4, epochs. These data are combined with the knowledge of the light curve shape and amplitude from the \( V \)-band to determine intensity mean \( I \)-band magnitudes, \( \langle I \rangle \). The transformation between \( V \) and \( I \) light curves is discussed further in appendix A.

2.1. Obtaining true distance moduli

Usually the \( \langle V \rangle \) and \( \langle I \rangle \) magnitudes are combined to estimate the reddening of the Cepheids themselves, and hence the correction required to account for dust extinction. This is done by fitting the \( V \)- and \( I \)- PL relations separately to find apparent moduli, \( \mu_{AV} \) and \( \mu_{AI} \), and then calculating the true distance modulus for the sample (e.g. Freedman et al. 1994), i.e.

\[ \mu_0 = \mu_{AV} - R(\mu_{AV} - \mu_{AI}) + (\text{correction terms}) \]  

(2.2)

Alternatively, we can estimate true distance moduli for each Cepheid individually by equation (2.2) and average the results (e.g. Tanvir et al. 1995a; Saha et al. 1996). We shall call this method 2 for future reference. Although these two methods are mathematically identical, method 2 can more easily incorporate a weighting scheme since it naturally handles the fact that the residuals from the PL relations in each band are correlated. Note that here we define \( R = A_v/(A_v - A_i) \), where \( A \) is the extinction in the given band. From the extinction curve of Cardelli et al. (1989), we find \( R \approx 2.45 \), is appropriate for Cepheids. Other “correction terms”, e.g. for metallicity differences, are discussed below.

This general approach to the extinction problem has the benefit that it corrects explicitly for the reddening to each Cepheid based on the colour of the star itself. Importantly, the correction for extinction also takes out much of the intrinsic PLC correlation between colour and residual-magnitude of Cepheids alluded to above. Put simply, at a given period a Cepheid which appears redder than the average may be so because it suffers from high extinction or because it is towards the red edge of the instability strip. In both cases it will also appear fainter than an average Cepheid would at that period and hence its magnitude should be corrected brighter. Similarly if a Cepheid is blue for whichever reason, it is likely to be brighter than average. The net result is that after applying the reddening correction we are effectively dealing with an intrinsically tighter relation, similar to the PLC, and hence obtain greater accuracy than would naively be expected. Rather ironically this means that the value of the extinction, \( A_v \), is actually less well determined than the value of the true distance modulus, \( \mu_0 \).

It is interesting to note that this prescription is also essentially the same as fitting an appropriate relation to the Wesenheit “reddening-free magnitudes” of the Cepheids, the \( BV \) version of which has been investigated in detail by Madore (1982) and Freedman (1988), and which are discussed further here in section 3. We should emphasize that there is no need for us to actually know the coefficients of the PLC relation with any precision in order to apply this method. The fact that a PLC relation exists and roughly produces
N. R. Tanvir: Cepheids as Distance Indicators

the same correlation between colour residual and magnitude residual from the two PL relations as reddening, means that the reddening correction is doubly beneficial.

2.2. The error budget

Method 2 provides an elegant way of handling the errors since the spread in the sample of $\mu$ estimates, which we shall call $\sigma_{\text{internal}}$, should be a fair estimate of all the internal uncertainties due to random noise from photon statistics, sampling of the light curves and the intrinsic dispersion (expected to be small as noted above). This automatically accounts for the correlation between the residuals from the $V$- and $I$-band PL relations which arise from the intrinsic PLC and also the coupling of errors due to a common estimate of $\log(P)$. So a fairly complete error estimate for a sample of $n$ Cepheids can be written down simply by adding the various sources of systematic error to this:

$$
\sigma_{\text{total}}^2 = \frac{\sigma_{\text{internal}}^2}{n} + [R\sigma_j]^2 + [(R-1)\sigma_V]^2 + [\delta(\mu_{AV} - \mu_{AI})\sigma_R]^2 + \sigma_{PL}^2 + \sigma_{Z}^2 + \sigma_{\text{systematic}}^2 \quad (2.3)
$$

The two terms, $\sigma_{PL}$, the uncertainty associated with the zero-point of the PL relations themselves, and $\sigma_{Z}$, the uncertainty associated with any metallicity correction, are discussed in more detail in sections 3 and 4 respectively. The term $\sigma_{\text{systematic}}$ is a catch-all and is intended to account for any remaining systematic uncertainties in the procedure for obtaining photometry such as acquiring aperture corrections (e.g. Tanvir et al. 1995b), application of the “long–vs–short exposure” correction (Hill et al. 1996) etc. Although at the moment these uncertainties are not well defined, those which have been considered are expected to be small and correlated between the bands and so should not be a serious problem. The observed interstellar extinction curve is fairly constant for low density ISM and, in any case, the uncertainty in the value of $R$ for the target galaxy won’t be important providing the estimate of colour excess relative to the LMC $\delta(\mu_{AV} - \mu_{AI})$ is not very high. This is an advantage of observing in fields of reasonably low extinction.

The main drawback of this reddening correction procedure, as compared to having an independent estimate of the reddening, say, is that it is more sensitive to uncorrelated photometric zero-point uncertainties, $\sigma_{V}$ and particularly $\sigma_{I}$. This is illustrated schematically in figure [ ] which shows that the lever-arm for $VI$ observations is large, although note that it is much smaller for infrared $H$-band observations (see section 5).

Now the WFPC2 calibration uncertainty for $V$ and $I$ is usually estimated to be 0.02–0.04 mag. It is probable that there is also some zero-point uncertainty on the ground-based measurements of the LMC Cepheids in the region of 0.01–0.02 mag, so we allow $\sigma_{V} \approx \sigma_{I} \approx 0.04$ mag for the combined photometric calibration uncertainty. Thus these sources alone contribute a total error on the reddening corrected distance modulus of $\sim 0.11$ mag. While this represents an uncertainty of only $\sim 6\%$ in distance, it is important because it is a systematic error affecting all HST Cepheid distance determinations which use this technique.

We shall return to equation [2.3] in section [ ] to bring together our estimates for the other terms.

3. Calibration of the PL Relations

The standard PL relations which have hitherto been applied to HST Cepheids are those of Madore & Freedman (1991, hereafter MF91), based on a set of 32 Cepheids in
Figure 1. Schematic figure illustrating how the true distance modulus may be obtained by combining the apparent distance moduli in one or more bands. The lever arm operates such that for $VI$ observations photometric uncertainties, which propagate directly to uncertainties in the apparent moduli $\mu_{AV}$ and $\mu_{AI}$, are amplified in the error on the true distance modulus. This produces both increased random scatter for individual Cepheids, and importantly, a large systematic uncertainty due to the photometric zero-point errors. By contrast, the addition of infrared $H$-band magnitudes would result in a very well constrained value for the true modulus.

| Reference                  | $\mu_0$ | Method                                      |
|----------------------------|---------|---------------------------------------------|
| Schommer et al. (1984)     | 18.2    | Main-sequence fitting                       |
| Walker (1992)              | 18.22   | RR Lyraes calibrated by Milky Way RR Lyraes  |
| Fernley (1994)             | 18.43   | RR Lyraes using Baade-Wesselink method       |
| Eastman & Krishner (1989)  | 18.45   | Expanding photosphere of supernova 1987A     |
| Stier et al. (1994)        | 18.47   | Cepheids using Baade-Wesselink (VSB method)  |
| Laney & Stobie (1994)      | 18.53   | Infrared Cepheid observations calibrated in Milky Way |
| Panagia et al. (1996)      | 18.54   | Ring around supernova 1987A                  |
| Feast (1995)               | 18.57   | Cepheids calibrated in Milky Way             |
| Hughes & Wood (1990)       | 18.66   | Mira variables calibrated against Miras in 47Tuc |

Table 1. Compilation of distance estimates to the Large Magellanic Cloud, intended to be representative rather than exhaustive. Although there is a fairly large range, the established techniques mostly give distance moduli around 18.50 which corresponds to 50kpc.

the Large Magellanic Cloud. The advantages of calibrating in the LMC are that the distance to the LMC can be obtained in different ways, including by Cepheids which are calibrated in the galaxy or using Baade-Wesselink type methods. The SMC is less useful in this regard because of its much greater depth along our line of sight. A representative (but by no means exhaustive) selection of recent LMC distance determinations is given in table 1. The question of whether there may be a small systematic discrepancy between the RR Lyrae and Cepheid distance scales has been addressed most recently by van den Bergh (1995), Feast (1995) and Catalan (1996).

MF91 adopted a true distance modulus to the LMC of 18.5 ± 0.1 and a reddening of $E_{B-V} = 0.1$. This distance modulus still seems like a reasonable compromise between the different methods. In fact, the adopted reddening is irrelevant to the derivation of true distance moduli, if they are calculated as described in section 2 (e.g. Freedman et al. 1994). The reddening to the LMC is, of course, still important for several of the distance estimates in table 1, but we assume that the uncertainty on the reddening is included in the ±0.1 error on the true modulus.
In addition to the uncertainty in the distance of the LMC, this calibration will also propagate some uncertainty from the fit to the given sample of Cepheids. In section 3.2 we present new PL calibrations for which the uncertainty on the fit is very small. First we examine a potentially serious problem, identified by Simon & Young (1996; hereafter SY96), which leads us to ask the question whether we have confidence in the LMC Cepheids as calibrators at all?

3.1. Are the LMC Cepheids normal?

Clearly the assumption underpinning the use of Cepheids as distance indicators is that the Cepheids in a target galaxy are similar to those in the calibrating galaxy, in this case the LMC. SY96 have pointed out possible differences from galaxy to galaxy in the distributions of long-period Cepheids in the colour-magnitude diagram. In particular they find the LMC Cepheids to have a steep blue edge to the instability strip when compared to the, apparently more normal, SMC Cepheids. This, they argue, can be explained if the LMC stars have a different mass-luminosity relation from the SMC. Such variations in the M-L relation could produce significant errors in distance modulus of up to 0.25 mag when the LMC calibration is applied to another galaxy with more “SMC-like” Cepheids.

But, the LMC Cepheid sample analyzed by SY96 consists of just the 22 LMC Cepheids from the Madore (1985) compilation which have both $V$ and $I$ photometry and are in the period range 10 to 50 days. The properties of this sample are investigated in figure 2 which shows the $\langle V \rangle$ vs $\langle B \rangle - \langle V \rangle$ colour-magnitude diagram for all the LMC Cepheids from Madore (1985) in the period range 5 to 50 days, highlighting the Cepheids used by SY96. We see immediately that these Cepheids are not a fair sample of the full population. In particular, bluer Cepheids are under-represented in the magnitude range $\langle V \rangle > 13$ mag. Presumably this is just a consequence of having such a small numbers of variables. A simple-minded analysis suggests that this offset of the 22 Cepheid sample from the mean of the larger set is only a 1σ event, although this increases to more than 2σ if, a posteriori, we consider only the 11 Cepheids with $\log(P) < 1.45$ for which the discrepancy is most pronounced. In any case we conclude that the effect noted by SY96 is due to an unfortunate anomaly of their sample.

This is reassuring for the use of the Cepheid PL relations in general since the evidence for a varying M-L goes away. However, these same 22 Cepheids form a large proportion of the sample used by MF91 in calibrating their PL relations, and are also important, but to a lesser extent, in the new calibrations presented in this paper in section 3.2. It is therefore relevant to ask how bad a calibration would have been constructed using these 22 Cepheids alone?

In this context the sampling problem manifests itself as an deficiency of Cepheids in the range $1 < \log(P) < 1.45$ which lie above the mean PL relation. The result is that both the estimated slopes and zero-points of the $P_{V}$ and $P_{I}$ relations will be significantly in error. This unusual slope can be seen, incidentally, in most direct comparisons of LMC Cepheids with those in other galaxies (e.g. see figure 10 of Freedman et al. 1994, or figure 7 of Kelson et al. 1996). For distance determination, the consequences of this depend in general on the period distribution of the Cepheids in the particular target galaxy. However, even for a “bad” case, where the target Cepheids are concentrated around $\log(P) = 1.2$, we find that the estimate of true distance modulus by equation 2.2 is only out by 0.01–0.03 mag for either forward or reverse regression fits to the data. This is a remarkably small error and again comes down to the small intrinsic dispersion of the reddening corrected magnitudes. Inevitably, the estimate of reddening itself is worse affected and can be out by as much as 0.1 mag, but this just illustrates the point made...
earlier, that the estimate of true distance modulus should be taken more seriously than the estimate of the reddening. In fact for this reason, and also because (a) the nominal $E_{B-V}$ reddening to the LMC Cepheids is itself uncertain, and (b) the reddening is also affected by photometric errors, we should not be afraid to apply a “bluing” correction if that is what is found. Indeed if we forbid negative reddening, at least if we are to work consistently within the framework described in section 2, then we run the risk of creating a bias on the resultant distance estimates.

The result of calibrating with the 22 Cepheid sample would have been considerably worse had we departed from the “standard” method. If, for example, we have an external estimate of the reddening not derived from the $VI$ photometry, then the error in distance modulus would have been $\sim 0.14$ mag. If we had adopted a $PL_V$ relation based on the larger sample of Cepheids with $V$-band photometry, and used this in conjunction with the 22 Cepheid $PL_I$ relation to correct for reddening, then an error in distance modulus of 0.3 mags would have resulted.

3.2. New calibrations of the PL relations
Several hundred Cepheids have already been observed in distant galaxies with HST and it is clearly desirable to increase the size of the LMC $VI$ sample to minimize any systematic

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**Figure 2.** Colour–magnitude diagram for LMC Cepheids with the sample used by Simon & Young (1996), namely those with $I$-band photometry and periods in the range 10–50 days, displayed as solid squares, and Cepheids not in their sample displayed as crosses. We see that the unexpected vertical slope of the blue-edge noted by Simon and Young is only an artifact of the unrepresentative distribution of their sample compared to the whole population. Presumably this is just an unfortunate consequence of small number statistics, but in any event it is reassuring that the larger sample shows no cause for concern with the LMC Cepheids. These data were taken from the compilation of Madore (1985).
error on the calibration. With this in mind we have re-examined the published V and I photoelectric photometry for LMC Cepheids. By combining all the available data sets (post–1975) we obtain a sample of 53 Cepheids. The period-luminosity relations for these Cepheids are shown in figure 3. We also plot the Wesenheit magnitude (see Madore 1982) which is constructed from the V I photometry, $W_{V I} = \langle V \rangle - R(\langle V \rangle - \langle I \rangle)$, and is explicitly reddening independent. Notice that the Wesenheit magnitudes produce a very tight PL relation with a dispersion of $< 0.12$ mag. As noted above, this tightening is expected as a result of the fact that the intrinsic PLC relation works in the same sense as reddening. Nonetheless, such a low dispersion seems remarkable given the inhomogeneous collection of data sets used in creating the sample, and recalling also that the dispersion must include contributions from random photometric errors and the depth of the LMC in addition to the intrinsic scatter around the relation. On this point, we might expect the scatter to improve still further if we remove the effect of the tilt of the LMC in the plane of the sky, as determined by Caldwell & Laney (1991). In fact, doing this we find a small reduction in the dispersion around the $V$- and $I$- PL relations, to 0.226 and 0.159 mag respectively, but essentially no change in the dispersion for the $W_{V I}$ relation. This probably indicates that photometric errors are becoming dominant.

If we follow MF91 and adopt a distance modulus to the LMC of $\mu_0 = 18.5$ and reddening $E_{B-V} = 0.1$ (and therefore $A_V = 0.33$ and $A_I = 0.195$), we obtain the following calibrations:

\[
\begin{align*}
M_V &= -2.774(\pm 0.083) [\log(P) - 1.4] - 5.262(\pm 0.040) ; \quad \sigma_{rms} = 0.233 \\
M_I &= -3.039(\pm 0.059) [\log(P) - 1.4] - 6.054(\pm 0.028) ; \quad \sigma_{rms} = 0.164 \\
M_{W} &= -3.423(\pm 0.042) [\log(P) - 1.4] - 7.202(\pm 0.020) ; \quad \sigma_{rms} = 0.117
\end{align*}
\]

which are fits to all the variables with $\log(P) < 1.8$, avoiding potential problems with the longest period Cepheids (MF91), and are referenced to a pivot $\log(P)$ of 1.4, typical of the HST Cepheid samples. Note that the uncertainty on the zero-point of the fitted PL relation is only 1% in distance.

We may ask whether even these 53 Cepheids form a fair sample. To test this we plot in figure an additional sample of 58 Cepheids with V-band but no I-band photometry, also drawn from the literature. From the figure we see that while there is still some deficiency in the number of brighter Cepheids with I-band photometry in the range $1 < \log(P) < 1.4$, nonetheless, the fit to the combined sample of 111 Cepheids is extremely close to the fit for the 53 Cepheid subset:

\[
M_V = -2.756(\pm 0.054) [\log(P) - 1.4] - 5.269(\pm 0.031) ; \quad \sigma_{rms} = 0.219
\]  

Restricting to a subset of 74 longer period variables ($1.0 < \log(P) < 1.8$) leads to an almost identical relation:

\[
M_V = -2.780(\pm 0.084) [\log(P) - 1.4] - 5.263(\pm 0.035) ; \quad \sigma_{rms} = 0.245
\]

which shows that the LMC PL$_V$ relation is linear, to this level of precision, and that the version given in equation 3.4 is representative. Thus there is no compelling reason to
Figure 3. Period-luminosity relations for a sample of 53 LMC Cepheids (solid squares) and 5 long period Cepheids (open squares) with $V$- and $I$-band photometry from the literature. Data were taken from Madore (1975), Eggen (1977), Martin & Warren (1979), Martin (1980), Dean (1981), van Genderen (1983), Harris (1983), Freedman et al. (1985), Walker (1987), Welch et al. (1991), Caldwell et al. (1986), Gieren (1993) and Sebo & Wood (1995). The magnitudes for the 7 Cepheids from Sebo & Wood (1995) were used as given. Many other Cepheids turned out to have been studied by more than one group, so those data were combined and obviously discrepant data sets and data points removed by hand. The three panels are for (a) intensity mean $\langle V \rangle$ magnitudes which were calculated according to equation 2.1, (b) intensity mean $\langle I \rangle$ magnitudes which were calculated using $\langle I \rangle = \langle I \rangle' + 0.6(\langle V \rangle - \langle V \rangle')$, where the primes indicate that the intensity means (not phase weighted) were taken using just those epochs which had both $V$- and $I$-band photometry (see appendix A for justification of the coefficient 0.6), and (c) $W_{VI}$, the reddening free Wesenheit function (e.g. Madore 1982) which is defined here as $W_{VI} = (\langle V \rangle - 2.45(\langle V \rangle - \langle I \rangle))$. The solid lines are least-squares fits to the Cepheids with $0.4 < \log(P) < 1.8$. Note the small dispersion around the mean relation, particularly for $W_{VI}$, and the absence of deviation from linearity. The apparent tendency for points to scatter preferentially below the mean line in the period range $1 < \log(P) < 1.4$ is explained in section 3.1 and has no significant influence on the calibrations.
Figure 4. The period-luminosity plot for the sample of 53 LMC Cepheids (plus 5 with 
$log(P) > 1.8$) which have both $V$- and $I$-band photometry are again shown as solid squares 
and solid line, as in figure 3a. This is compared to a sample of 58 Cepheids (plus 1 
with $log(P) > 1.8$) with $V$ but no $I$ photometry (crosses) which have been compiled 
from: Martin (1981), van Genderen & Nitihardjo (1989), Mateo, Olszewski & Madore (1990), 
Bertelli et al. (1993), and Welch et al. (1993), in addition to the sources referenced in the cap-
tion to figure 3. Variables identified as overtones and the highly reddened Cepheid HV2549 
have not been included. The least-squares fit for the entire sample of 111 variables (dashed line) 
is barely distinguishable, particularly in the period range, $log(P) > 1$, of interest for distant 
studies.

restrict the period range of the calibrating sample to be the same as the period range in 
any particular target sample.

For comparison, recent studies of galactic Cepheids using both Baade-Wesselink meth-
ods and in clusters with main-sequence distances have tended to give somewhat steeper 
$V$-band PL slopes, usually in the range 2.9–3.0 (see Gieren et al. 1993). The theoreti-
cal metallicity dependence found by Chiosi, Wood & Capitanio (1993) (and summarized 
here in equation 4.7) predicts that there should be an increase in slope, but by only 
$\sim 0.06$, however this is quite consistent within the errors.

3.3. Implications of the new calibrations

Our PL$_v$ relation agrees well with MF91, who found $M_v = -2.76[log(P) - 1.4] - 5.27$, 
indicating that the lack of brighter Cepheids in the range $1 < log(P) < 1.45$ does not 
have much influence on their fit either. However there is disagreement in the I-
band, where MF91 gave $M_i = -3.06[log(P) - 1.4] - 6.09$. The chief reason for the 
discrepancy can be traced back to the $V - I$ colours listed by Madore (1985) for the 
Martin, Warren & Feast (1979) Cepheids, which are actually straight magnitude means, 
$i.e. V - I$. The $\langle I \rangle$ magnitudes going into the MF91 calibration appear to have been 
estimated by subtracting these colours from the $\langle V \rangle$ magnitudes, and this produces 
small but systematic errors of 0.02–0.09 mag for Cepheids of normal amplitudes. Thus 
the difference between the two calibrations stems largely from the re-analysis of the 
photometry for the variables we have in common, rather than the increased sample used 
here.

The net result is that distance moduli calculated with MF91 relations, using the $V$ and 
$I$ results to give the reddening via equation 2.2, should be reduced by $\approx 0.1$ mag. Note 
how the small revision in the $I$-band zero-point is magnified in the reddening correction 
procedure. This reduction in distance translates to a $\sim 5\%$ increase in inferred values of 
the Hubble constant. Although small, this revision is significant given the hoped for level 
of precision in determining $H_0$. We should point out that for some of the Sandage et al.
SNIIa host galaxies, for which it was assumed that the reddening to the Cepheids is the same as that to the supernova (e.g. Saha et al. 1994), the effect on the $H_0$ determinations will be less.

In passing we also note that the ground-based Cepheid distances to more nearby galaxies will be less seriously affected since none rely on only $VI$ photometry to give reddening.

4. Some other concerns in applying Cepheids

Apart from the calibration questions dealt with in section 3, there are a number of other issues we should address regarding the application of Cepheids in other galaxies. To preempt the gory details, a common conclusion of the first three topics is that imposing a reasonably conservative lower $\log(P)$ limit on a Cepheid sample will help remove potential systematic problems which can appear at faint magnitudes, but will not usually limit the accuracy of the distance estimate.

4.1. Statistical biases

Much has been written about the effects of statistical biases on Tully-Fisher and $D_n - \sigma$ distance estimates (e.g. Hendry & Simmons 1994; Triay et al. 1994) but less attention has been paid to the equivalent problems for Cepheids. Sandage (1988) demonstrated that Cepheid samples which are truncated by a detection limit do indeed show flatter PL slopes than complete samples and hence produce biased distances. The affect of this, and other sample selection effects, has been analyzed by Hendry & Kanbur (1996) for Sandage et al.’s HST $V$-band Cepheid observations of NGC5253 and IC4182. They identify bias at a low level, but conclude that it is not a significant source of error in determining $H_0$. Feast (1995) pointed out that applying reverse rather than direct regression in some circumstances helps avoid problems of this kind. This argument was also made by Kelson et al. (1996) who adopted reverse fitting in determining the HST distance to M101.

There are some issues of principle here, though, which should give us pause for thought. The LMC Cepheid calibrating sample as it stands is limited in period, not magnitude, and indeed several of the studies of LMC Cepheids from which the photometry is derived have selected samples in particular period ranges. Furthermore, the period distribution of Cepheids in a given galaxy will depend at some level on its star formation history, and the efficiency with which variables will be found in the target galaxy will be a function of period due to the particular spacing of the observations. Additional sources of scatter, such as differential reddening and photometric errors, will work to broaden the distribution in magnitude and will therefore also affect the reverse fit given the various forms of period selection we have identified. If we assume that the only important period selection effect is the upper cut off, which of course applies to both the LMC and the target galaxy, then it may be possible to proceed by imposing a magnitude selection function on the LMC sample which is at least close to that for the target galaxy. The alternative of imposing a bright magnitude limit on both samples seems rather wasteful. In light of these concerns, we feel a better policy, if one is needed, would be to continue with forward regression but to impose a short period cut-off on the target galaxy sample to remove any range in $\log(P)$ which appears to be badly incomplete in magnitude.

But does any of this matter in practice? Distance indicators with large dispersion are more prone to selection biases, thus PL$_V$ used alone, for example, will produce distance estimates which are systematically too short in the presence of a detection limit. However, we have seen that the reddening corrected $W_{VI}$ magnitudes form a linear PL$_W$ relation with very small intrinsic scatter, so as an indicator PL$_W$ should be much less biased
than PL$_V$. Random photometric noise will of course broaden the relation, but because sample selection is done basically on the $\langle V \rangle$ mags, rather than on $\langle I \rangle$ mags, it turns out that net bias for $\mu_0$ estimates actually starts to go in the opposite sense, i.e. upward. On the other hand random but correlated source of error for each Cepheid, which will be present since the $I$ light curve uses information from the $V$ light curve and crowding errors affect both bands, can move the bias down again as with $\mu_{AV}$. We demonstrate these points in figure 5, where we assume the relations given in equation 3.4, and apply them to the original LMC sample degraded by noise and a sharp artificial magnitude limit. Although not entirely realistic, these experiments serve to illustrate that selection effects can produce complicated biases, but overall, for typical levels of noise, residual bias should be small. This explains why Kelson et al. (1996) did not find the difference between forward and reverse fitting in M101 that they were expecting.

Figure 5. This figure illustrates the effects of incompleteness bias in the presence of noise. The PL relations given in equation 3.4 are applied to artificially degraded LMC samples and the results for $\mu_{AV}$ (boxes), $\mu_{AI}$ (circles) and $\mu_0$ (triangles) can be compared to the nominal values (dashed lines). In panel (a) the photometry for the entire sample, $V$ and $I$, has been degraded by the addition of 0.1 mag of random noise, to simulate the higher measurement errors for the distant Cepheid samples, and the bias is created by introduction of a sharp artificial $V$-band magnitude limit. The error bars show the distribution over different realizations. This demonstrates that while PL$_V$ and PL$_I$ alone can give biased results from severely incomplete samples, the reddening corrected modulus, although noisy, is not biased. In panel (b) we instead fix the magnitude limit at $\langle V \rangle = 14.5$, which corresponds to log($P$) $\approx 1$ typical of the distant Cepheid surveys, and plot the recovered distance moduli as a function of the added noise. The purpose of this is to show that random, uncorrelated (between $V$ and $I$) photometric noise increases the bias in determining $\mu_{AV}$, but actually produces a bias on the reddening corrected modulus which is in the opposite sense! The effect of correlated noise depends on the nature of the coupling, but will usually bring the bias on $\mu_0$ down again. The conclusion is that statistical biases can produce complex effects, but are likely to be small for realistic levels of photometric noise.

In conclusion, then, normal slide fitting of the forward PL relations should not produce any strong bias due to incompleteness, although if the random (uncorrelated) photometric
noise near the magnitude limit is high then distances can be biased too far. More thorough appraisal of selection biases would require realistic simulations of artificial Cepheids being added to the images and put through the same reduction procedure as the real data. On a different but related issue, Malmquist bias, used here in the sense of the bias resulting from the non-constant radial distribution of the target galaxies, will depend in general on the scatter, or internal error if you like, of the Cepheid distances. Since these errors are usually small, compared to indicators like the Tully-Fisher and $D_n - \sigma$ relations, Malmquist bias should also be minimal.

4.2. Contamination by overtone pulsators and non-Cepheids

Could there be contamination of the distant samples by Cepheids pulsating in higher harmonic modes or by variables which are not classical Cepheids at all? Such contamination could produce systematic errors in distance determination. In fact, the only plausible non-Cepheid interlopers which might appear in the same colour, magnitude and period range with anything like Cepheid light curves, are the W Virginis stars. These variables, also known as population II Cepheids, follow a PL relation about 1.5 mags fainter than the classical Cepheid relation. However, the statistics of variables discovered in ground-based surveys of the Magellanic Clouds, M31 and NGC300, compiled by Madore & Freedman (1985), show that whilst more than 70% were classical Cepheids, only $\sim$2% were W Virginis stars, making serious contamination unlikely. A further test was performed by Simon & Clement (1993) who compared the light curve shapes of 5 Cepheids in IC4182 (from Saha et al. 1994) with galactic Cepheids using Fourier decomposition. They concluded that the samples are the same within the errors and, in particular, there is no indication that any of their IC4182 Cepheids are in fact W Virginis stars.

Cepheids pulsating in first harmonic mode on the other hand should follow a PL relation which is displaced to lower period and hence apparently to brighter magnitude than the PL relation for fundamental mode Cepheids. Böhm-Vitense (1994) has suggested that a majority of classical Cepheids with periods less than $\log(P) < 0.9$ are 1H pulsators. Overtone pulsators may be distinguished on the basis of their light curves, although this would be difficult in any quantitative sense for the relatively poorly sampled, noisy light curves of Cepheids in distant galaxies. Nonetheless, strongly saw-tooth light curves provide some reassurance that the variables are classical Cepheids pulsating in fundamental mode.

In fact, the results from the microlensing surveys of the LMC appear to show that while overtone pulsators do exist with periods as long as $\log(P) = 0.8$ they form a relatively small proportion of all Cepheids even at $\log(P) = 0.5$ (see figure 3 of Cook et al. (1995) for results from the first year of MACHO data). Moreover, given that most HST samples are restricted to $\log(P) > 0.8$ anyway, simply because of the magnitude limit of the observations, contamination by overtone pulsators is unlikely to be a problem.

Finally we mention a category of non-Cepheid interloper which is frequently overlooked, namely the non-variable stars! At sufficiently faint magnitudes, given the relatively small number of epochs used, random photometric errors will cause some stars to appear as “variable” as true variables. These usually will not fold to produce Cepheid light curves and hence will be rejected, but it is possible that some may slip through if the search for variables is pushed to faint enough magnitudes and low enough amplitudes. The scale of this effect for any particular field can be best judged by simulations. Here we simply make the point again that a conservative cut in $\log(P)$ will exclude the fainter “variables” for which there is less assurance that they are genuine Cepheids.
4.3. The effects of crowding

The photometry for many HST target galaxies is made difficult by stellar crowding and rapid variations in background intensity. Profile fitting photometry software has been shown to work quite well despite the poor sampling of the WFPC2 CCDs. Such algorithms attempt to use knowledge of the point-spread-function to fit for the magnitudes of many stars simultaneously, thus effectively correcting for the influence of light from the wings of neighboring stars.

However, in severe cases problems are bound to arise. The key-project group have found some discrepancies at a fairly low level between the results obtained with the DAOPHOT/ALLFRAME software and those obtained with the DoPHOT software in the rather crowded M100 fields (Hill et al. 1996). Saha et al. (1996) have investigated this problem in NGC4536 by assigning a “quality index” to each variable based on an inspection of the light curves. They find that variables with a low quality index are more likely to be recovered with brighter magnitudes at a given period, which may well indicate the effects of crowding. Indeed crowding errors clearly become dominant for \( \log(P) < 1.2 \) in their data, especially in the \( I \)-band.

Interestingly, if the photometry for a Cepheid is contaminated by the light of an overlapping blue main-sequence star then the reddening correction (equation 2.2) again works in the sense to minimize its influence. Contamination by young blue stars is not improbable given that Cepheids tend to be found in regions of moderately recent star formation. Conversely, of course, contamination by red stars can produce larger errors in the reddening corrected distance estimates. The tip of the red giant branch for an old population has \( M_I \approx -4 \) so Cepheids with \( \log(P) \leq 0.8 \) will be more subject to contamination by such stars.

Experiments with adding simulated stars to the data frames should give a good idea of the scale of the problem in a particular galaxy, and may allow some form of empirical correction to the magnitudes to be estimated. A signature that crowding errors are becoming dominant would be if the distance moduli of the individual Cepheids (using method 2) show a trend with period in the sense of short period Cepheids appearing systematically closer. Possible ways to minimize the effects of crowding are to (a) reject any stars which appear at all broader than the psf, (b) cut the sample in period so as to only include the longer period Cepheids, or (c) cut according to amplitude since contamination by a non-variable star will always result in a reduction in the amplitude. Given the small intrinsic dispersion of the Wesenheit magnitudes, cutting the sample size will not be detrimental for the distance estimate, but including faint Cepheids with biased photometry, due to crowding could be.

4.4. Differences due to metallicity

In principle metallicity may affect the Cepheid PL relations due to changes in stellar evolution, pulsation or atmospheres. Theoretical studies (e.g. Stothers 1988; Chiosi, Wood & Capitanio 1993, hereafter CWC93) suggest that metallicity effects are very important in the \( B \)-band, but less so in \( V \) and \( I \). For example, using the results given by CWC93 we compute, for a change in metallicity \( \delta Z \), the measured apparent \( V \)-band distance modulus will change by:

\[
\delta \mu_{AV} = \left[ 7.73 \log(P) - 0.49 \right] \delta Z \tag{4.7}
\]

where we are additionally assuming that (a) the helium abundance goes as \( \delta Y = 3.58 \delta Z \) (Peimbert 1986), (b) there are no changes in convective overshoot properties with metallicity (the CWC93 models fit the available Milky-Way, LMC and SMC data best if
overshoot is zero or small), and (c) the Cepheids more or less uniformly fill the instability strip, which may be the weakest assumption particularly given the rather uncertain prescription for defining the red-edge of the strip. Anyway, this amounts to 0.08 mag for a Cepheid sample centred around \( \log(P) = 1.4 \) and \( \delta Z = 0.008 \) which is appropriate for the difference between LMC and galactic metallicity for example. In other words, if we measure the distance to a target galaxy of galactic metallicity using the LMC Cepheids as calibrators, then we should apply a correction to the distance modulus of \(-0.08\) mag. However the method of extinction correction outlined in section 2 actually produces a cancelation of most of this dependency:

\[
\delta \mu_0(W_{V_I}) = [3.27\log(P) - 1.79]\delta Z
\]

which is only a 0.02 mag effect for the same assumptions as above.

By contrast the equivalent formula for \( BV \) Wesenheit magnitude implies a metallicity induced error of \( \delta \mu_0 \sim -0.21 \) under the above assumptions, similar to the \( \text{Stothers (1988)} \) prediction, and a much more serious effect. In drawing conclusions about the effect on HST distance determination based on \( V I \) photometry we should be aware that these predictions are made for the standard Johnson \( V \)- and Cousins \( I \)-bands. In fact the central wavelengths of the WFC2 F555W and F814W filters are close to these standard passbands, but the F555W filter, in particular, is considerably wider than Johnson \( V \). This is likely to result in a greater influence of line blanketing in that filter, and perhaps therefore, a better prediction for the metallicity dependence would be intermediate between that indicated by equation (4.8) and the equivalent formula for \( BI \) data. Without going into details, we find the mean of these two to be an even more negligible \( \delta \mu_0 \sim -0.01 \) mag for the \( \delta Z = 0.008 \) we have been using for comparison.

How do these predictions compare to empirical attempts to quantify the metallicity dependency of the PL relations? To date, such investigations have proved controversial. Freedman and Madore (1990, hereafter FM90) found no evidence for a metallicity effect in their multicolour data for Cepheids in three fields in M31. However, Feast (1991) and Stift (1995) have argued that this test is not sufficiently sensitive to rule out an effect of the size predicted by Stothers (1988, hereafter S88). Tanvir (1992), from an independent analysis of M31 Cepheids, in which the reddening was estimated from the locus of the main sequence stars, found that the S88 metallicity correction did give a consistent distance modulus when applied separately to \( B \) and \( V \) data. Gould (1994) reanalyzed the FM90 data set, taking account of the correlations among the \( BVI \) magnitudes, and claimed to find, contrary to the original conclusion, a significant effect of \( \delta \mu_0 = -(0.56 \pm 0.20)\delta[\text{Fe/H}] \), which amounts to \( \delta \mu_0 = 0.17 \) for the difference between the galaxy and the LMC. Most recently, Sasselov et al. (1996) have analyzed a large sample of \( \sim 500 \) LMC and SMC Cepheids from the EROS microlensing experiment. The large number of variables and high quality of the data allows them to simultaneously determine the differential distance, reddening and metallicity effects. They conclude that a fairly large change in distance modulus of \( \delta \mu_0 = -25.4^{+6.2}_{-12}\delta Z \) would occur for the HST \( V I \) data, i.e. a correction of 0.11–0.25 mag is required for \( \delta Z = 0.008 \). They point out that this correction would go some way to bringing the individual \( H_0 \) estimates based on the distances to IC4182, M96 and M100 into agreement. This is much larger than the theoretical prediction given above by equation (4.8) and, if correct, implies a problem with one of the assumptions made above or with the CWC93 models themselves, possibly with the M–L relation at low metallicity (D. Sasselov priv. comm.) or the use by CWC93 of LAOL rather than more modern opacities.

Clearly this is an area where further work is required to refine both the observational
and theoretical results. The key-project group are attempting an empirical study of the metallicity dependence in M101 (Kennicutt et al. 1995) which, since they are using HST, at least removes the uncertainties associated with photometric transforms. For the present, then, the best empirical metallicity calibration comes from the EROS data, but this seems to disagree with the existing theory (at least under the assumptions made here), and itself has a high internal uncertainty. The most secure distances therefore will be for Cepheid fields with metallicities similar to the LMC, and we may take $\sigma_z = 0.06$ mag as a typical uncertainty with the proviso that high metallicity galaxies in particular, will give more uncertain distances.

5. Future directions

From the above deliberations it should be clear that there are several important sources of systematic uncertainty which need to be addressed. Not least of these are the various WFPC2 calibration issues, indeed there would be merit in obtaining a WFPC2 photometric calibration specifically for Cepheids if we want to achieve the highest accuracy. In addition, if we are to continue to use the LMC Cepheids as the primary calibration, then its distance needs to be tied down more precisely. In this regard, results from the microlensing surveys and further ground based observations of long-period LMC Cepheids will be valuable; these are already under way at SAAO (D. Laney and J. Caldwell, priv. comms.) and by the key-project team (Kennicutt et al. 1995). The more local scale within the Milky-Way is of relevance for some of the LMC distance determinations and observations of galactic Cepheids, the results from the Hipparcos mission and improvements to the Baade-Wesselink method (see Krockenberger et al. 1996) will be important here.

In the remainder of this section we highlight two new developments which will hopefully lead to better Cepheid distance estimates in the near future.

5.1. Moving further to the infrared

There are numerous advantages in observing Cepheids further in the infrared, and these have been exploited from the ground for nearby galaxies (e.g. Jacoby et al. 1992). In particular, dust extinction is much reduced, there is less sensitivity to metallicity variations, and the spread around the mean PL relation is smaller than in the optical (e.g. McGonegal et al. 1982; Laney & Stobie 1994). As an illustration of the benefit of reduced reddening corrections, the true distance modulus determined by combining $V$- and $H$-band magnitudes is given by:

$$\mu_0 = \mu_{AH} - 0.24(\mu_{AV} - \mu_{AH})$$

(5.9)

which should be compared with equation 2.2. As shown in figure 1 this results in significantly lower sensitivity to zero-point photometry calibration uncertainties.

The installation of the NICMOS infrared camera on HST in 1997, will provide the exciting opportunity to revisit some of the HST-observed galaxies again to obtain $H$-band magnitudes for the known Cepheids. Furthermore, the small amplitudes in the $H$-band means that only one or two epochs are required for these observations, which makes them very good value in terms of telescope time!

5.2. Maximum light relations

Cepheid PL relations based on maximum as well as mean light were presented by Sandage & Tammann (1968). Recently the idea has been resurrected (Simon, Kanbur
and Mihalas, 1993), motivated by theoretical considerations, and it has been shown empirically that the PL relations at maximum light have somewhat lower dispersion than at mean light (Kanbur and Hendry, 1996). Observationally, working at maximum light removes problems of obtaining good photometry through the minimum of a Cepheid’s cycle, which is difficult close to the magnitude limit especially in crowded fields. The drawback of the method is that it requires careful accounting of the biases introduced by sparse sampling of the light curve and the effects of noise. It appears that these can be addressed by use of simulations and therefore \( PL_{\text{max}} \) relations hold considerable promise.

6. Conclusions
We have seen that the “standard” method of extinction correction for Cepheids is equivalent to fitting a PL relation to their reddening-free magnitudes. Because the reddening correction also accounts for much of the intrinsic colour dependence of Cepheid magnitudes, this distance indicator has a very small intrinsic dispersion. Thus precise results can be obtained for even small samples of Cepheids, providing the samples are “clean”. Bias effects are complicated but likely to be small, as is contamination from non-Cepheids. Crowding errors may be a more serious problem at faint magnitudes, but all of these can be addressed by simulations and alleviated if we impose a conservative lower \( \log(P) \) cutoff.

By using the reddening of the Cepheids themselves we avoid the large uncertainties associated with any other way of estimating extinction. The flip-side of this is that this distance indicator is quite sensitive to systematic calibration uncertainties. With this in mind we have produced revised, and apparently robust, new calibrations of the \( V \)-band, \( I \)-band and \( W_{\text{VI}} \) PL relations based on a careful analysis of a sample of 53 LMC Cepheids. The formal uncertainty on the zero-point of the \( W_{\text{VI}} \) relation is only 1% in distance. These new relations highlight the calibration sensitivity since a fairly small change from the old \( I \)-band relation means that the new calibration gives true distance moduli which are approximately 0.1 mag closer. The effect of the change will be less for the Sandage et al. SNIa calibration program (e.g. Sandage et al. 1996), in those cases for which the extinction to the Cepheids was assumed to be the same as that to the SN.

Finally, it is instructive to revisit the error budget which was summarized by equation 2.3. The first term, due to the intrinsic errors, will vary from sample to sample, but because larger samples of Cepheids also tend to imply more crowded fields and uncertain photometry, it is reasonable to take a representative figure of \( \sigma_{\text{internal}}/\sqrt{n} = 0.1 \) mag. From section 2, we estimate the next two terms contribute 0.11 mag whilst for the fourth term we shall assume a conservative value of \( \sigma_\mu = 0.4 \) and a typical value of \( \delta(\mu_{\text{AV}} - \mu_{\text{AI}}) \) of 0.05 mag. From section 3, we take \( \sigma_{\text{pl}} = 0.1 \) mag, which is just the adopted error on the LMC distance since the internal error on the fit to the Wesenheit magnitudes is only 0.02 mag and any photometric zero-point uncertainties on the ground-based measurements have been included in \( \sigma_\nu \) and \( \sigma_\epsilon \). In section 4 we saw that the metallicity question is still unsettled, so we assume a contribution of \( \sigma_z = 0.06 \) mag for an average HST observed galaxy, although of course this uncertainty would be removed for a galaxy which could be shown to be close to LMC metallicity. Finally, \( \sigma_{\text{systematic}} \) remains poorly determined, but we shall adopt a value of 0.05 mag, since these uncertainties will tend to be small and correlated so as not to affect the reddening correction. In total, then, we find a typical uncertainty of \( \sigma_{\text{total}} = 0.2 \) mag which amounts to \( \sim 10\% \) in distance.

We have attempted to take reasonable account here of all the significant sources of error. That such precision can be achieved at distances of 20Mpc is a testament to the exceptional capabilities of the HST. It is clear, however, that this uncertainty is
dominated by systematic errors, and the value of the whole sample of HST galaxies can be enhanced considerably if these systematics are addressed. In particular, improved calibration of WFPC2, specifically those aspects relating to the Cepheid observations; better understanding of the role of metallicity variations on Cepheid properties; more precise determination of the LMC distance; and infrared observations of known Cepheids with NICMOS are all highly desirable. With these improvements, and given the many parallel developments in secondary distance indicators, the goal of estimating $H_0$ to better than 10% looks to be achievable.

I would like to thank Dimitar Sasselov, Abi Saha, Dave Laney, John Caldwell, Wendy Freedman, Barry Madore and Mike Feast for useful discussions and communications, and a security man at Heathrow airport for helping me feel that at least someone was interested (see appendix B). Special thanks go to my collaborators on various Cepheid related projects: Tom Shanks, Martin Hendry, Shashi Kanbur, Shaun Hughes, Harry Ferguson, David Robinson and Robin Catchpole for many insightful discussions.

Appendix A. Transforming $V$ to $I$ light curves

HST Cepheid studies have all used $V$-band light curve shape information to predict the $I$-band light curve in order to reduce the number of $I$ epochs which are necessary to determine $\langle I \rangle$. The most important factor is the reduction in amplitude between $V$ and $I$. We can investigate this empirically with the large sample of well-studied galactic Cepheids analyzed by Moffett & Barnes (1985). In figure 6 we plot the ratios of the amplitudes in Johnson $R_J$- and $I_J$-bands to the amplitudes in $V$. This shows that the ratio is very nearly independent of $\log(P)$ and amp($V$). Since F814W, and Cousins $I$, are intermediate between Johnson $R_J$ and $I_J$, we recommend use of a ratio $\text{amp}(I) : \text{amp}(V) = 0.6$.

It is interesting to note that this amplitude ratio is almost identical to the reddening ratio, $A_I/A_V = 0.59$. This suggests that random phase $VI$ observations of Cepheids of known period should give precise estimates of true distance modulus after reddening correction. However, we note that the small phase shift and change of light curve shape in transforming between the bands will lead to some additional noise.

Appendix B. A funny thing happened on the way to this meeting

Traveling to this meeting the author passed through London Heathrow airport and was stopped by a security guard for a standard interview, which proceeded something like this:

SG: “What is the purpose of your trip to America?”
NRT: “I’m going to a conference in Baltimore on the age of the universe.”
SG: “That sounds interesting, do you think you’ll decide on an answer or is it something we don’t have much idea about yet?”
NRT: “Well, it’s very likely we won’t all agree with each other, but I think we’re closer to an answer than you might imagine.”
SG: “Oh really, I thought there was a problem with the ages of the globular clusters.”!!

The moral of this story, I think, is that there are people out there, even airport security guards, who are really interested in the big questions about the universe which we are trying to answer...either that or these people receive exceptionally thorough training on how to catch out unsuspecting academics! 😊
Figure 6. These two plots show how the ratio of Cepheid amplitudes in $R_j$ (crosses) and $I_j$ (circles) to the amplitude in $V$ depends on (a) log($P$) and (b) $V$-amplitude (max-to-min) itself. Apparently there is little dependence. The data are for 90 galactic Cepheids taken from the sample of Moffett & Barnes (1985), from which all type II Cepheids and those noted as having companions have been removed. The dashed lines show the means in each case.

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