The First Acoustics Peak in CMBR and Cosmic Total Density

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Abstract:

Boomerang measured the first peak in CMBR to be at location of $l_D = 196 \pm 6$, which excites our strong interest in it. A widely cited formula is $l_D \simeq 200\Omega_T^{-0.50}$ to estimate the cosmic total density. Weinberg shows it is not correct and should be $l_D \propto \Omega_T^{-1.58}$ near the interest point $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$. We show further that it should be $l_D \propto \Omega_T^{-1.43} \Omega_m^{-0.147}$ or $\Omega_T^{-1.92} \Omega_\Lambda^{0.343}$ near the same point in the more veracious sense if we consider the effect from the sound horizon. We draw a contour graph for the peak location, show that the recent data favor to a closed universe with about $\Omega_T \simeq 1.03$. If we insist on obtaining a flat universe, a point $(0.36, 0.64)$, i.e., more matter and less vacuum energy, is still possible, which has a more right-side first peak $l_D = 208$ in CMBR and a smaller acceleration parameter $-q_0 = 0.10$ for the $z = 0.4$ redshift SNIa.

I. INTRODUCTION

Recently, the Boomerang\cite{1} has observed that there exits a vivid peak structure in the power spectrum of Cosmic Microwave Background Radiation (CMBR) anisotropy. The first acoustics peak appears at location of Legendre multipole $l_D = 196 \pm 6$, then they obtained their conclusion that the universe is almost Euclidean flat, i.e., the total density is near critical, $\Omega_T = \Omega_m + \Omega_\Lambda \simeq 1.00 \pm 0.12$, which is a most important cosmological parameter concerned by us. People often uses a widely cited formula $l_D \simeq 200/\sqrt{\Omega_T}$ to estimate the cosmic total density if
one knows the first peak location\textsuperscript{[2,3]}. However, Weinberg shows that this formula is not even a crude approximation in the greatest current interest region\textsuperscript{[4]}. He says that this formula should be \( l_D \propto \Omega_T^{-1.58} \) near the favorite point \((\Omega_m, \Omega_\Lambda) = (0.3, 0.7)\). Weinberg’s work start a precedent of how to analysis the complicated phenomena of the acoustics peaks in CMBR in a simple way. It is a significant for us how to hold the physical essential by using as possible as fewer calculations. Then this puts forward an important question, i.e., we must be careful to analysis the relation between the first peak location and the cosmological parameters \( p_i \), such as the matter density \( \Omega_m \) (which includes both of the baryon density \( \Omega_b \) and the cold dark matter density \( \Omega_d \)), radiation density \( \Omega_\gamma \), the vacuum energy density (e.g., cosmological constant) \( \Omega_\Lambda \), and the redshift \( z_r \) of the recombination epoch. We also concern of how much value is the proportional coefficient in Weinberg’s formula.

In this paper we shall consider a neighbor analysis based on Efstathiou-Bond’s formula to calculate the position of the first peak. This analysis is more precious and will give a reappearance of the Weinberg’s phenomenon\textsuperscript{[4]}, i.e., the acoustics peak provides a more stringent constraint on \( \Omega_T \) than an usual expected case.

II. THE POSITION OF THE FIRST PEAK

The calculation of the first peak location is very complicated if we use the relevant sets of the pertubative evolution equations in the cosmology\textsuperscript{[5]}. Efstathiou and Bond have obtained a good experiential formula for the first peak location\textsuperscript{[6]}, we can rewrite it as the following in a more clear way with pertinent variable dependent,

\[
l_D(p_i) = C_{eb} \cdot A(p_i) \cdot |\Omega|_k^{-1/2} \text{Sinn}[|\Omega|_k^{1/2} \int_1^{2r} B(w, p_i)dw],
\]

where the function \"Sinn\", which origins from the angular diameter distance to the last scattering surface, means that if \( \Omega_k > 0 \), \( \text{Sinn}[f] = \sinh[f] \); if \( \Omega_k < 0 \), \( \text{Sinn}[f] = \sin[f] \); if \( \Omega_k \rightarrow 0 \), \( \text{Sinn}[f] \rightarrow f \). Here \( \Omega_k = 1 - \Omega_m - \Omega_\Lambda \) is the curvature term density. The function \( A(p_i) \) origins from the sound horizon of the photon-baryon fluid before the recombination,

\[
A(p_i) = \frac{3\pi}{4} \sqrt{\frac{\omega_b \Omega_m}{\omega_\gamma}} \cdot \left( \ln \frac{\sqrt{\Omega_m(4\omega_\gamma z_r + 3\omega_b)}}{\sqrt{\omega_\gamma z_r} \cdot (\sqrt{4\Omega_m} + \sqrt{3\omega_b h^{-2}})} \right)^{-1},
\]

and the function \( B(p_i) \) is relative with the cosmic conformal expansion ratio,

\[
B(w, p_i) = (\Omega_\Lambda + (1 - \Omega_m - \Omega_\Lambda)w^2 + \Omega_m w^3 + \omega_\gamma h^{-2}w^4)^{-1/2}.
\]
Hereafter we use only the parameters $\omega_\gamma = \Omega_\gamma h^2$ and $\omega_b = \Omega_b h^2$ rather than $\Omega_\gamma$ and $\Omega_b$ due to former higher accuracy. We know $\omega_\gamma = 4.31 \times 10^5$ exactly\cite{7} from the background temperature $2.73^\circ K$, and the error of $\Omega_\gamma$ comes mainly from the Hubble constant $100h$ km-sec$^{-1}$-Mpc$^{-1}$. We know $\omega_b$ more accurately than $\Omega_b$ from the Big Bang Nucleosynthesis. So that we must consider the $l_D$ error originated from an uncertainty of the Hubble constant $h$, which appears in Eqs.(2-3). An important coefficient $C_{eb}$ appears in the formula of $l_D$ expressed by us. I call it as the Efstathiou-Bond coefficient. Its origination is the projection from the three-dimensional temperature power spectrum to a two-dimensional angular power spectrum, its value is taken as $C_{eb0} = 0.746$ by Efstathiou and Bond. In order to adapt to the need of the future more accurate calculation or modification, we take it as $C_{eb} = c \cdot C_{eb0}$ and $c$ is a constant to be established. If $c$ is 1 it is corresponding to the above Efstathiou-Bond value.

III. NEIGHBOR ANALYSIS

Considering various achievements come from the different fields of the Cosmology, we choose a favorite point for various cosmological parameters $p_i$, which is $\Omega_m0 \simeq 0.3$, $\Omega_\Lambda0 \simeq 0.7$, $\omega_b0 \simeq 0.02$, $h_0 \simeq 0.7$, $z_{r0} \simeq 1100$. In the neighbor of any points, the $l_D$ can be expressed approximately as

$$l_D = l_D(p_i0) \cdot \prod (\frac{p_i}{p_i0})^{I_i},$$

(4)

the power indexes can be calculated by

$$I_i = \frac{\partial l_D}{\partial p_i}|_{p_i0} \cdot \frac{p_i0}{l_D(p_i0)}.$$  

(5)

In the neighbor of the favorite point, relation between the first peak location and the cosmological parameters is, in according to a direct numerical calculation of Eqs.(1-5) by Mathematica,

$$l_D = l_{D0}(\frac{C_{eb}}{C_{eb0}})(\frac{z_{r0}}{20})^{0.670}(\frac{h}{h_0})^{-0.487}(\frac{\omega_b}{\omega_b0})^{0.059}(\frac{\Omega_m}{\Omega_{m0}})^{-0.576}(\frac{\Omega_\Lambda}{\Omega_{\Lambda0}})^{-1.004},$$

(6)

where $l_{D0} = 213$. If we want to express it in terms of the parameter $\Omega_T$, we have

$$l_D \propto (\frac{\Omega_T}{\Omega_{T0}})^{-1.43}(\frac{\Omega_m}{\Omega_{m0}})^{-0.147} \propto (\frac{\Omega_T}{\Omega_{T0}})^{-1.92}(\frac{\Omega_\Lambda}{\Omega_{\Lambda0}})^{0.343}. $$

(7)

We see that in the point $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$, this power index is different from Weinberg’s result. The main reason is that the function $A(p_i)$ is still dependent on
Ω_m. Anyhow, the Weinberg’s conclusion, i.e., a more stringent constraint on Ω_T will be provided by the acoustics peak position, is correct, which can be seen from a large index absolute value about Ω_T. If we only want to change Ω_T alone, the first formula in Eq.(7) means to fix Ω_m and to vary Ω_Λ, the second formula means to fix Ω_Λ and to vary Ω_m.

However, the index varies quite a bit for other points, and rather complicated for all parameters. The most important parameters are Ω_m, and Ω_Λ, therefore we fix the three parameters z_r, h and ω_b at first. We choose our greatest current interest region as 0.2 <Ω_m < 0.4 and 0.6 <Ω_Λ < 0.8. In this region we can obtain a good fitting by a simple formulas, which accuracy is higher than 0.5 percent (notice that out of this region the error immediately becomes large!),

\[ l_D = c[71.4 + (1485 - 9275Ω_m + 30646Ω_m^2 - 51076Ω_m^3 + 33844Ω_m^4)(1 - 0.86Ω_Λ)] \tag{8} \]

As a comparison, the accuracy of Eq.(6) is only 1 percent in a small region of 0.25 <Ω_m < 0.35 and 0.65 <Ω_Λ < 0.75. Another important observation constraint on the cosmological parameters comes from the cosmic deceleration parameter q_0, which can be expressed[8] as \( q_0 = 0.8Ω_m - 0.6Ω_Λ = -0.2 \pm 0.1 \) for the SNIa with redshift \( z \simeq 0.4 \). A contour graph about \( l_D \) is drawn in the figure.

IV. WEINBERG’S PHENOMENON IN CONTOUR GRAPH

This figure has the following feature. Near the favorite point \( P_1 \), the \( l_D \) contour (a straight line in fact) is not parallel with the Ω_T contour, we can see a clear cross between the two lines near this point. When Ω_T approach 1.07 or \( l_D \) approach less than 190c, the both lines begin to parallel. In this case the \( l_D \) is alone dependent on \( Ω_T \). Only in this case we can express \( l_D \propto Ω_T^{l_T} \), but it has not already been a flat universe. The point \( P_2 \) is the cross point of Ω_T = 1 and \( q_0 = -0.20 \). The point \( P_3 \) is a cross point of \( l_D = 200c \) and \( q_0 = -0.20 \), which has value about \( Ω_T = 1.03 \), i.e., if the formula (1) is correct (specially for Efstathiou-Bond coefficient) to calculate the first peak position, then the recent result of the first peak squints towards a closed universe! (In this case Ω_T is nearly independent on \( q_0 \).) If we hope to obtain a flat universe, then we may choose the point \( P_4 = (0.36, 0.64) \), which has \( l_D = 208c \) and \( q_0 = -0.10 \) at edge of observation values. It is notable that such as point can exist for flat universe. The character of this point, as comparing with current favor
value, owns higher hubble constant, lower acceleration parameter, more right-side first peak (i.e., a little large \(l_D\)), higher cold dark mater density, lower vacuum energy density, which is still flat universe. From this figure we can see if the accuracy of \(l_D\) is risen, how huge progress should be made for establishment of \(\Omega_T\) value. We shall wait for the exiting precious results from the future Map and Planck.

We can see clearly the phenomenon claimed by Weinberg from this figure. The first peak position \(l_D\) determines the cosmic total density \(\Omega_T\) sensitively. If the precision about \(l_D\) of Boomerang measurement is reliable, i.e., \(190 < l_D < 202\) (see Ref.[1]), then we obtain a result \(1.03 < \Omega_T < 1.08\) from this figure. This shows that our universe may be a closed rather than flat\(^{[9]}\) ! This will bring a great challenge for the present elegant cosmological theories based on an eternal chaotic inflation\(^{[10]}\).

In another hand, if we review their conclusion \(0.88 < \Omega_T < 1.12\) (see Ref.[1] again) and suppose that all error of \(l_D\) comes from uncertainty of \(\Omega_T\), we shall get an unexpected range \(180 < l_D < 240\) from the just same figure. It is obvious that this error range of \(l_D\) is too large for current measurement. The Ref.[1] considered of course many complicated factors (a little example is the error from \(z_r, h\) and \(\omega_b\)), therefore in spirt of the accuracy of \(l_D\) is very high, we can still only obtain the \(\Omega_T\) value with very low accuracy.

V. CONCLUSION

Conforming to the Weinberg’s thought, the formula (1) and its simplification (Eqs.(6,8) and figure.1) supply for us a shortcut method to analysis error origination to determine the cosmic total density. It can help us to understand deeply the physical essential from the data of the CMBR anisotropy. Weinberg’s phenomenon, i.e., sensitivity of \(\Omega_T\) with respect to \(l_D\), is very clear in our figure. In spirt of our qualitative analysis is available widely, however our concrete numerical result depends seriously on the Efstathiou-Bond coefficient \(C_{eb}\), we hope that people can understand it deeply in the further investigation.
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FIG. 1. A contour graph of the first peak position $l_D$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{contour_graph.png}
\caption{A contour graph of the first peak position $l_D$.}
\end{figure}