Topological Inflation in Dual Superstring Models

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Abstract
We study the possibility of obtaining inflationary solutions from S-dual superstring potentials. We find, in particular, that such solutions occur at the core of domain walls separating degenerate minima whose positions differ by modular transformations.
1. Various attempts have been made at combining the ideas of superstring unification and the inflationary cosmological scenario. Although the existence of numerous scalar fields (the moduli) would seem to provide a necessary ingredient, the fact that they remain massless at all orders in string perturbation theory leads to serious difficulties in building a successful cosmological scenario. Among the many moduli, the dilaton ($\phi$) is of special interest because it controls the string coupling and variations of this field correspond to changes in masses and coupling constants which are strongly constrained by observation. This problem is usually addressed by assuming that the dilaton develops a potential, which must have its origin in non-perturbative effects, such as condensation of gauginos, so that Einstein’s gravity can be recovered after $\phi$ settles into the minimum of its potential. However, general arguments show that the dilaton cannot be stabilized in the perturbative regime of string theory leading to a runaway problem [1]. Moreover, even if the dilaton were stabilized by non-perturbative potentials, these are too steep to be suitable for inflation without fine-tuning the initial conditions [2]. Of course, once the dilaton is fixed, inflation can be achieved via other fields, e.g. chiral fields (gauge singlets) for a suitable choice of the inflationary sector of the superpotential [3, 4]. Furthermore, there are additional difficulties related with the Polonyi problem associated with scalar fields that couple only gravitationally and which may dominate the energy density of the universe at present [5, 6].

In the present letter, we show that the abovementioned difficulties can be avoided through a new mechanism introduced in [7] for fixing the dilaton. Indeed, it was shown that the dilaton potential develops a suitable minimum once the requirement of S-duality invariance (see below) is imposed. Already in Ref. [8] this mechanism was used to stabilise the dilaton while inflation was accomplished by chiral fields. Our analysis shows that inflation can be achieved via the dilaton itself provided one considers a novel way of implementing the inflationary expansion of the universe recently put forward by Linde [9] and Vilenkin [10]: topological or defect inflation. They have shown that the core of a topological defect may undergo exponential inflationary expansion provided the scale of symmetry breaking satisfies the condition

$$\eta > \mathcal{O}(M_P).$$

The inflation that ensues is eternal since the core of the defect is topologically stable and it is the restored symmetry in the core that provides the vacuum energy for inflation. We
shall see that the conditions for successful inflation are satisfied by domain walls that separate degenerate minima in S-dual superstring potentials, implying that topological domain wall inflation is, as anticipated in Ref. [6] through general arguments, an attractive cosmological scenario in string inspired models.

2. S-duality was conjectured [7] in analogy with a well-established symmetry of string compactification – T-duality. Indeed, it was shown that the effective supergravity action following from string compactification on orbifolds or even Calabi-Yau manifolds is severely constrained by an underlying string symmetry, the so-called target space modular invariance [11, 12]. The target space modular group \( \text{PSL}(2, \mathbb{Z}) \) acts on the complex scalar field \( T \) as

\[
T \rightarrow \frac{aT - ib}{icT + d}; \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1,
\]

and \( < T > \) is the background modulus associated to the overall scale of the internal six-dimensional space on which the string is compactified. Specifically, \( T = R^2 + iB \), with \( R \) being the “radius” of the internal space and \( B \) an internal axion. The target space modular transformation contains the well-known duality transformation \( R \rightarrow 1/R \) as well as discrete shifts of the axionic background \( B \) and it was shown that this symmetry remains unbroken at any order of string perturbation theory [12]. The conjectured symmetry would be a further modular invariance symmetry in string theory, where the modular group now acts on the complex scalar field \( S = \phi + i\chi \), where \( \chi \) is a pseudoscalar (axion) field. This symmetry includes a duality invariance under which the dilaton gets inverted (S-duality). S-modular invariance strongly constrains the theory since it relates the weak and strong coupling regimes as well as the “\( \chi \)-sectors” of the theory.

The form of the N=1 supergravity action including gauge and matter fields is specified by the functions \( G(\Phi, \Phi^*) = K(\Phi, \Phi^*) + \ln |W(\Phi)|^2 \) and \( f(\Phi) \); \( K(\Phi, \Phi^*) \) is the Kähler potential and \( W(\Phi) \) the superpotential, where \( \Phi \) denotes all chiral matter fields. Let us consider, to start with, only two chiral fields \( S, T \). At string tree-level, the Kähler potential for these fields looks like

\[
K = -\ln(S + S^*) - 3\ln(T + T^*).
\]

The scalar potential can be written in the following form [7]
\[ V = |h^S|^2 G_{SS}^{-1} + |h^T|^2 G_{TT}^{-1} - 3 \exp(G), \quad (4) \]

where \( h^i = \exp(\frac{1}{2} G) G^i \), \( i = S, T \) and the indices denote derivatives with respect to the indicated variable.

We shall first look at the case where there is only one modulus field, \( T \). It is known that the purely \( T \)-dependent superpotential has to vanish order by order in perturbation theory so that the VEV of \( T \) remains undetermined. However, one expects that non-perturbative effects will generate a superpotential for \( T \). The simplest expression for \( W(T) \) compatible with modular invariance is \[ W(T) \sim \eta(T)^{-6} \quad (5) \]
and the related scalar potential is given by \[ V(T) = \frac{1}{T_R^3 |\eta(T)|^{12}} \left( \frac{T_R^2}{4\pi^2} |\hat{G}_2(T)|^2 - 1 \right), \quad (6) \]
where \( T_R = 2 \text{Re} \, T \). The function \( \eta(T) = q^{1/24} \prod_n (1 - q^n) \) is the well-known Dedekind function, \( q \equiv \exp(-2\pi T) \); \( \hat{G}_2 = G_2 - 2\pi/T_R \) is the weight two Eisenstein function and \( G_2 = \frac{1}{3}\pi^2 - 8\pi^2 \sigma_1(n) \exp(-2\pi n T) \), where \( \sigma_1(n) \) is the sum of the divisors of \( n \).

If, on the other hand, one considers only the \( S \) field, the requirement of modular invariance leads to a scalar potential \[ V(S) = \frac{1}{S_R |\eta(S)|^{12}} \left( \frac{S_R^2}{4\pi^2} |\hat{G}_2(S)|^2 - 3 \right). \quad (7) \]

This potential (just like \( V(T) \)) diverges in the limit \( S \to 0, \infty \) and has minima at finite values of \( S \), close to the critical value \( S = 1 \). Indeed, the function \( \hat{G}_2(S) \) has its only zeros at \( S = 1 \), \( S = \exp(\frac{1}{6}i\pi) \) and therefore these self-dual points are necessarily extrema of the potential \( V(S) \):

\[
\frac{dV}{dS} = \frac{1}{\pi S_R |\eta(S)|^{12}} \left[ \hat{G}_2 \left( \frac{S_R^2}{4\pi^2} |\hat{G}_2(S)|^2 - 3 \right) + \frac{S_R}{\pi} \left( |\hat{G}_2(S)|^2 + \frac{S_R}{4} (\hat{G}_2^* \hat{G}_2 + \hat{G}_2^* \hat{G}_2) \right) \right] = 0; \quad (8)
\]

more precisely, the former corresponds to a local maximum and the latter to a saddle point. The potential has other extrema, namely minima, for \( \text{Re} \, S \sim 0.8, \, 1.3 \) and \( \text{Im} \, S = n, \, n \in \mathbb{Z} \);
these are minima both along the Re $S$ and Im $S$ directions. The qualitative shape of the potential is shown in Fig. 1 and the periodic structure of minima along the Im $S$ direction is shown in Fig. 2. Therefore, we see that the same way target space modular invariance fixes the value of $T_R$ thus forcing the theory to be compactified, S-modular invariance fixes the value of $S_R$ thus stabilizing the potential and avoiding the dilaton runaway problem. It is clear that the theory has to choose among an infinity of degenerate minima whose positions differ by modular transformations. Once one of them is chosen, target space modular invariance is spontaneously broken. Since duality is a discrete symmetry, if there were a phase in the evolution of the universe in which the compactification radius was already spontaneously chosen, S-duality domain walls would be created separating different vacua.

A more realistic model is obtained once all gauge singlet fields of the theory: $S, T_i, i = 1, 2, 3$ are considered. Imposing S and T-duality on the Lagrangian for $N = 1$ supergravity theory, one obtains the following potential [8]:

$$V = e^K|\eta(T_2)\eta(T_3)\eta(S)|^{-4} \left( |P|^2 \left[ \frac{S_R^2}{4\pi^2}|\hat{G}_2(S)|^2 + \frac{T_{R_i}^2}{4\pi^2}|\hat{G}_2(T_i)|^2 - 2 \right] + F_0 \right),$$

(9)

where $P = P(T_1, \psi) = \lambda(T_1)\Theta(\psi)$, $\psi$ denotes the untwisted chiral fields related to the $T_1$ sector, $\lambda$ and $\Theta$ are gauge invariant functions and $F_0 = P_m(K^{-1})^mP^n + (K_mP(K^{-1})^mP^n + h.c.)$.

Clearly, this potential is S (and T)-duality invariant since all dependence on $S$ is given in terms of duality-invariant functions $e^K|\eta(S)|^{-4}$ and $S_R^2|\hat{G}_2(S)|^2$. Again, the dual invariant points $< S >= 1$, $e^{-\pi/6}$ and $< T_i >= 1$, $e^{-\pi/6}$ are extrema (maxima and saddle points, respectively) and the minima of $V$ are nearby. In what follows we will show that the conditions for topological inflation to occur at the core of the domain walls separating degenerate minima of the above potential can be met for some range of parameters.

3. Let us now turn to the discussion of the conditions for successful topological inflation. Along a domain wall $\chi$ ranges from one minimum in one region of space to another minimum in another region. Somewhere between, $\chi$ must traverse the top of the potential, $\chi \approx \chi_0$ and we hence start expanding the potential about $\chi_0$

$$V \approx V_0 \left( 1 - \alpha^2 \left( \frac{\chi - \chi_0}{M^2} \right)^2 \right),$$

(10)

where $M = M_P/\sqrt{8\pi}$, which is the natural scale of the fields in supergravity and was set to
one in the previous section.

In flat space, the wall thickness is equal to the curvature of the effective potential, that is \( \delta^{-1} \sim \alpha (V_0/M_p^2)^{1/2} \). The Hubble parameter in the interior of the wall is given by \( H \approx (8\pi GV_0/3)^{1/2} \). If \( \delta \ll H^{-1} \), gravitational effects are negligible. However, if \( \delta > H^{-1} \), the region of false vacuum near the top of the potential, \( V \approx V_0 \), extends over a region greater than a Hubble volume. Hence, if the top of the potential satisfies the conditions for inflation, the interior of the wall inflates. Demanding that \( \delta > H^{-1} \), one obtains the following condition on \( \alpha \),

\[
\alpha^2 < \frac{8\pi}{3}.
\] (11)

It turns out that this condition is more stringent than the ones that can be derived from demanding an inflationary slow rollover regime \[6\]. However, the requirement that there are at least \( N_e \) e-folds of inflation, i.e

\[
\frac{-V''}{V} \ll \frac{6\pi}{N_e}
\] (12)

leads to the most stringent constraint on \( \alpha \) (we assume here \( N_e = 65 \)) \[6\]

\[
\alpha^2 < \frac{3\pi}{65}.
\] (13)

We have computed \( \alpha^2 \) for the purely T-dual and S-dual potentials, Eqs. (6) and (7), assuming that the real part of the \((T, S)\) field has already settled at the minimum of the potential. We envisage a scenario where inflation would take place at the core of domain walls separating different vacua, as the imaginary part of the \((T, S)\) field expands exponentially once the conditions discussed above are satisfied at the top of potential (see Fig. 2). We find \( \alpha^2 = 0.30 \) and \( \alpha^2 = 0.09 \), respectively; hence, we conclude that the conditions for successful defect inflation to occur are fulfilled in the purely S-dual case only.

For the more realistic model of Eq. (9), the value of \( \alpha^2 \) depends on the vacuum contributions of \(|P|^2\) and \(F_0\). In this case, we assume that the T-fields and the untwisted fields of the \( T_1 \) sector have already settled at the minimum of the potential and inflation takes place due to the S-field; the relevant potential can be then written as

\[
V(S) = \frac{1}{S_R |\eta(S)|^2} \left( \frac{S_R^2}{4\pi^2} |\tilde{G}_2(S)|^2 - a \right),
\] (14)
where $a$ is a constant. As for the models discussed above, we shall further assume that $S_R$ has settled at the minimum of the potential (at $<S_R> = 2$) and inflation would take place at the core of domain walls that separate different vacua, along the Im $S$ direction. We have computed $\alpha^2$ for different values of $a$ and found that the condition (13) is not fulfilled if $F_0 \geq 0$. However, if the vacuum contribution of untwisted chiral fields is such that $F_0 < 0$, actually when $a \gtrsim 2.5$, then successful topological inflation can occur in realistic models as well. Notice that we have included in our computation the effect of the non-canonical structure of the kinetic terms of $S$ (and $T$) dictated by N=1 supergravity, $(S_R)^{-2}\partial_\mu S\partial^\mu S^* ((T_R)^{-2}\partial_\mu T\partial^\mu T^*)$. Hence, we conclude that topological inflation is possible for $a \gtrsim 2.5$, thereby solving the initial condition problems in these models.

Of course, in order to have a complete cosmological scenario, it is still required that primordial energy density fluctuations are generated and a successful phase of reheating is achieved. For that, we notice that the $S$ field is coupled to radiation through $\text{Re } f(\Phi)$, i.e. via the term $\text{Re}SF_{\mu\nu}^aF^{\mu\nu a}$. Let us now see how domain wall evolution and dynamics leads, as discussed in Refs. [9, 10, 14], to a consistent cosmological scenario. At the top of the potential the field $S$ will be submitted to large fluctuations within time $\Delta t = H^{-1}$. These fluctuations look like sinusoidal waves with amplitude $H/2\pi$ and wavelength $O(H^{-1})$. During this time the original domains inflate about $e$ times and therefore the horizon, whose dimension is about $H^{-1}$, becomes divided into about $e^3$ domains. The presence of horizons for de Sitter spaces implies that the evolution of the field in each domain is independent of the others. In half of the domains the average values of the field is given by $<\chi>_T = H/2\pi$ and $\chi$ moves towards $\bar{\chi} \equiv <\text{Im } S> = n$, the VEV of Im $S$ (cf. Figure 1), while for the other half $<\chi>_T = -H/2\pi$ and $\chi$ moves towards $\bar{\chi} = n + 1$ or $\bar{\chi} = n - 1$. Since for small values of the field its dynamics is dominated by quantum fluctuations, the universe becomes divided into many thermalized regions surrounded by exponentially inflating domain walls. That is, the regime dominated by quantum fluctuations allows, as in the new inflationary model, that primordial energy density fluctuations are generated and reheating to occur. The domain walls create, on their turn, new inflating walls as, due to fluctuations, domains where for instance $\bar{\chi} = n + 1$ or $\bar{\chi} = n - 1$ can be produced inside a domain where $\bar{\chi} = n$. The latter regions will then be like islands of $\bar{\chi} = n + 1$ or $\bar{\chi} = n - 1$ in a sea of $\bar{\chi} = n$. Thus, the evolution of the walls does not destroy the original domain walls but, on the contrary, creates new domain walls on a smaller scale. These new walls will be generated in regions
where $\chi \approx \chi_0$ such that a jump of $\chi$ from one vacuum to another is not too unlikely. This implies that the new domain walls will be predominantly created close to the older ones, leading in this way to a self-similar fractal domain wall structure [10, 14]. Due to the domain wall classical dynamics, once topological inflation starts it never ends and that ensures that the $S$ field is free of any postmodern Polonyi type problem.

4. In conclusion, we have shown that topological inflation is a viable alternative to achieve inflation in string models with S-duality. Our study shows that in purely S-dual models the potential does allow topological inflation to occur due to the domain walls separating the different degenerate $\chi$-sectors of the theory. This does not happen for purely T-dual models. For models where both S and T duality are imposed, defect inflation triggered by domain walls occurs only for a limited range of the parameters associated with the vacuum contributions of untwisted fields to the potential, i.e. for $F_0 < 0$ (cf. Eq. (8)) and $a \gtrsim 2.5$ in the model of Eq. (14).
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Figure Captions

**Fig. 1** The scalar potential $V$ for the purely S-dual model, Eq. (7), as a function of $(\text{Re } S, \text{Im } S)$.

**Fig.2** The periodic structure of minima of $V$ for the purely S-dual model along the Im $S$ direction, for Re $S$ at its minimum ($\sim 1.3$).
Figure 2