Large-scale Structure with the SKA

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Abstract. A standard theoretical paradigm for the formation of large-scale structure in the distribution of galaxies has now been established, based on the gravitational instability of cold dark matter in a background cosmology dominated by vacuum energy. Significant uncertainties remain in the modelling of complex astrophysical processes involved in galaxy formation, perhaps most fundamentally in the relationship between the distributions of luminous galaxies and the underlying dark matter. I argue that the Square Kilometre Array is likely to provide information crucial to understanding this relationship and how it evolves with time.

1. Introduction

Over the last few years, cosmology has witnessed unprecedented improvements in our knowledge of the basic parameters governing the expansion of the Universe originating with observations of high-redshift supernovae (Riess et al. 1998; Perlmutter et al. 1999) and culminating in the recently-released data from the WMAP satellite (Spergel et al. 2003). As a result of these developments and parallel advances in theory, the field of large-scale structure has now entered a period of transition. Before the onset of the current data explosion, there were two basic reasons for wanting to study galaxy clustering. One was that it might furnish observational ways of pinning down cosmological parameters, and the other was that it provided the context within which to study galaxy formation and evolution. These two approaches are not mutually exclusive, of course, but one might associate the first with cosmologists of a more astrophysical persuasion, whereas the second is more likely to come from particle-cosmologists or inflationary specialists. Both points of view have stimulated the development of this field over the past twenty years. Now things are changing. Given the apparent precision with which we now know the cosmological parameters, observations of galaxy clustering will at most be seen as consistency checks on the fundamental properties of the Universe. On the other hand, accelerating improvements of observational technology have opened up the possibility of probing the very detailed and subtle properties of galaxies that are regarded as a nuisance to those interested in fundamental parameters.

In this paper, I will argue that the strongest contribution likely to be made by Square Kilometre Array, given the timescale required for its completion, is likely not to be in the pristine world of particle cosmology but in the grubby astrophysics of galaxy formation. I start by giving a very brief overview of
structure formation theory for non-specialists and then try to draw out some of the areas in which the character of the subject is changing. I will then discuss briefly the merits of 21cm galaxy surveys discussed in the science case for SKA which can be found at:

http://www.skatelescope.org/ska_science.shtml

2. Basics of Cosmological Structure Formation

2.1. Basic Framework

The Big Bang theory is built upon the Cosmological Principle, which requires the Universe on large scales to be both homogeneous and isotropic. Space-times consistent with this requirement can be described by the Robertson–Walker metric

\[ ds_{\text{FRW}}^2 = c^2 dt^2 - a^2(t) \left( \frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \tag{1} \]

where \( \kappa \) is the spatial curvature, scaled so as to take the values 0 or \( \pm 1 \). The case \( \kappa = 0 \) represents flat space sections, and the other two cases are space sections of constant positive or negative curvature, respectively. The time coordinate \( t \) is called cosmological proper time and it is singled out as a preferred time coordinate by the property of spatial homogeneity. The quantity \( a(t) \), the cosmic scale factor, describes the overall expansion of the universe as a function of time. If light emitted at time \( t_e \) is received by an observer at \( t_0 \) then the redshift \( z \) of the source is given by

\[ 1 + z = \frac{a(t_0)}{a(t_e)}. \tag{2} \]

The dynamics of an FRW universe are determined by the Einstein gravitational field equations which become

\[ 3 \left( \frac{\dot{a}}{a} \right)^2 = 8\pi G \rho - \frac{3\kappa c^2}{a^2} + \Lambda, \tag{3} \]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3\frac{p}{c^2} \right) + \frac{\Lambda}{3}, \tag{4} \]

\[ \dot{\rho} = -3\frac{\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right). \tag{5} \]

These equations determine the time evolution of the cosmic scale factor \( a(t) \) (the dots denote derivatives with respect to cosmological proper time \( t \)) and therefore describe the global expansion or contraction of the universe. The behaviour of these models can further be parametrised in terms of the Hubble parameter \( H = \dot{a}/a \) and the density parameter \( \Omega = 8\pi G \rho / 3H^2 \), a suffix 0 representing the value of these quantities at the present epoch when \( t = t_0 \). The cosmological constant is denoted \( \Lambda \) here, but it can be regarded instead as an additional energy density various forms of which have a similar effect; see Huterer & Turner (2001).
2.2. Linear Theory

In order to understand how structures form we need to consider the difficult problem of dealing with the evolution of inhomogeneities in the expanding Universe. We are helped in this task by the fact that we expect such inhomogeneities to be of very small amplitude early on so we can adopt a kind of perturbative approach, at least for the early stages of the problem. If the length scale of the perturbations is smaller than the effective cosmological horizon $d_H = c / H_0$, a Newtonian treatment of the subject is expected to be valid. If the mean free path of a particle is small, matter can be treated as an ideal fluid and the Newtonian equations governing the motion of gravitating particles in an expanding universe can be written in terms of $x = r / a$ (the comoving spatial coordinate, which is fixed for observers moving with the Hubble expansion), $v = \dot{r} - Hr = a \mathbf{x}$ (the peculiar velocity field, representing departures of the matter motion from pure Hubble expansion), $\phi(x, t)$ (the peculiar Newtonian gravitational potential, i.e. the fluctuations in potential with respect to the homogeneous background) and $\rho(x, t)$ (the matter density). Using these variables we obtain, first, the Euler equation:

$$\frac{\partial (av)}{\partial t} + (v \cdot \nabla_x) v = -\frac{1}{\rho} \nabla_x p - \nabla_x \phi.$$ \hspace{1cm} (6)

The second term on the right-hand side of equation (6) is the peculiar gravitational force, which can be written in terms of $g = -\nabla_x \phi / a$, the peculiar gravitational acceleration of the fluid element. If the velocity flow is irrotational, $v$ can be rewritten in terms of a velocity potential $\phi_v$: $v = -\nabla_x \phi_v / a$.

Next we have the continuity equation:

$$\frac{\partial \rho}{\partial t} + 3H\rho + \frac{1}{a} \nabla_x (\rho v) = 0,$$ \hspace{1cm} (7)

which expresses the conservation of matter, and finally the Poisson equation:

$$\nabla^2_x \phi = 4\pi G a^2 (\rho - \rho_0) = 4\pi G a^2 \rho_0 \delta,$$ \hspace{1cm} (8)

describing Newtonian gravity. Here $\rho_0$ is the mean background density, and

$$\delta \equiv \frac{\rho - \rho_0}{\rho_0}$$ \hspace{1cm} (9)

is the density contrast.

The next step is to linearise the Euler, continuity and Poisson equations by perturbing physical quantities defined as functions of Eulerian coordinates, i.e. relative to an unperturbed coordinate system. Expanding $\rho$, $v$ and $\phi$ perturbatively and keeping only the first-order terms in equation (7) gives the linearised continuity equation:

$$\frac{\partial \delta}{\partial t} = -\frac{1}{a} \nabla_x \cdot v,$$ \hspace{1cm} (10)

which can be inverted, with a suitable choice of boundary conditions, to yield

$$\delta = -\frac{1}{a H f} (\nabla_x \cdot v).$$ \hspace{1cm} (11)
The function \( f \simeq \Omega_0^{0.6} \); this is simply a fitting formula to the full solution. The linearised Euler and Poisson equations are

\[
\frac{\partial v}{\partial t} + \frac{\dot{a}}{a} v = -\frac{1}{\rho a} \nabla x p - \frac{1}{a} \nabla x \phi,
\]

(12)

\[
\nabla x^2 \phi = 4\pi G a^2 \rho_0 \delta;
\]

(13)

\(|v|, |\phi|, |\delta| \ll 1\) in equations (10), (12) & (13). From these equations, and if one ignores pressure forces, it is easy to obtain an equation for the evolution of \( \delta \):

\[
\ddot{\delta} + 2H \dot{\delta} - \frac{3}{2} \Omega H^2 \delta = 0.
\]

(14)

For a spatially flat universe dominated by pressureless matter, \( \rho_0(t) = 1/6\pi G t^2 \) and equation (14) admits two linearly independent power law solutions \( \delta(x, t) = D_\pm(t) \delta(x) \), where \( D_+(t) \propto a(t) \propto t^{2/3} \) is the growing mode and \( D_-(t) \propto t^{-1} \) is the decaying mode.

### 2.3. Primordial density fluctuations

The above considerations apply to the evolution of a single Fourier mode of the density field \( \delta(x, t) = D_+(t) \delta(x) \). What is more likely to be relevant, however, is the case of a superposition of waves, resulting from some kind of stochastic process in which the density field consists of a superposition of such modes with different amplitudes. A statistical description of the initial perturbations is therefore required, and any comparison between theory and observations will also have to be statistical.

The spatial Fourier transform of \( \delta(x) \) is

\[
\hat{\delta}(k) = \frac{1}{(2\pi)^3} \int d^3 x e^{-ik \cdot x} \delta(x).
\]

(15)

It is useful to specify the properties of \( \delta \) in terms of \( \hat{\delta} \). We can define the power-spectrum of the field to be (essentially) the variance of the amplitudes at a given value of \( k \):

\[
\langle \hat{\delta}(k_1) \hat{\delta}(k_2) \rangle = P(k_1) \delta^D(k_1 + k_2),
\]

(16)

where \( \delta^D \) is the Dirac delta function; this rather cumbersome definition takes account of the translation symmetry and reality requirements for \( P(k) \); isotropy is expressed by \( P(k) = P(-k) \). The analogous quantity in real space is called the two-point correlation function or, more correctly, the autocovariance function, of \( \delta(x) \):

\[
\langle \delta(x_1) \delta(x_2) \rangle = \xi(|x_1 - x_2|) = \xi(r) = \xi(r),
\]

(17)

which is itself related to the power spectrum via a Fourier transform. The power-spectrum is particularly important because it provides a complete statistical characterisation of a particular kind of stochastic process: a Gaussian random field. This class of field is the generic prediction of inflationary models, in which the density perturbations are generated by Gaussian quantum fluctuations in a scalar field during the inflationary epoch (e.g. Brandenberger 1985).
The shape of the initial fluctuation spectrum, is assumed to be imprinted on the universe at some arbitrarily early time. Many versions of the inflationary scenario for the very early universe (Guth 1981) produce a power-law form

\[ P(k) = A k^n, \]  

with a preference in some cases for the Harrison–Zel’dovich form with \( n = 1 \) (Harrison 1970; Zel’dovich 1972). Even if inflation is not the origin of density fluctuations, the form (18) is a useful phenomenological model for the fluctuation spectrum.

These considerations specify the shape of the fluctuation spectrum, but not its amplitude. The discovery of temperature fluctuations in the CMB (Smoot et al. 1992) plugged that gap.

2.4. The transfer function

We have hitherto assumed that the effects of pressure and other astrophysical processes on the gravitational evolution of perturbations are negligible. In fact, depending on the form of any dark matter, and the parameters of the background cosmology, the growth of perturbations on particular length scales can be suppressed relative to the growth laws discussed above.

We need first to specify the fluctuation mode. In cosmology, the two relevant alternatives are \textit{adiabatic} and \textit{isocurvature}. The former involve coupled fluctuations in the matter and radiation component in such a way that the entropy does not vary spatially; the latter have zero net fluctuation in the energy density and involve entropy fluctuations. Adiabatic fluctuations are the generic prediction from inflation and form the basis of most currently fashionable models, although interesting work has been done on isocurvature models (e.g. Peebles 1999).

In the classical Jeans instability, pressure inhibits the growth of structure on scales smaller than the distance traversed by an acoustic wave during the free-fall collapse time of a perturbation. If there are collisionless particles of hot dark matter, they can travel rapidly through the background and this free streaming can damp away perturbations completely. Radiation and relativistic particles may also cause kinematic suppression of growth. The imperfect coupling of photons and baryons can also cause dissipation of perturbations in the baryonic component. The net effect of these processes, for the case of statistically homogeneous initial Gaussian fluctuations, is to change the shape of the original power-spectrum in a manner described by a simple function of wave-number – the transfer function \( T(k) \) – which relates the processed power-spectrum \( P(k) \) to its primordial form \( P_0(k) \) via \( P(k) = P_0(k) \times T^2(k) \). The results of full numerical calculations of all the physical processes we have discussed can be encoded in the transfer function of a particular model (Bardeen et al. 1986). For example, fast moving or ‘hot’ dark matter particles (HDM) erase structure on small scales by the free-streaming effects mentioned above so that \( T(k) \to 0 \) exponentially for large \( k \); slow moving or ‘cold’ dark matter (CDM) does not suffer such strong dissipation, but there is a kinematic suppression of growth on small scales (to be more precise, on scales less than the horizon size at matter–radiation equality); significant small-scale power nevertheless survives in the latter case. These two alternatives thus furnish two very different scenarios for the late stages of structure formation: the ‘top–down’ picture exemplified by
Figure 1. Examples of adiabatic transfer functions for baryons, hot dark matter (HDM), cold dark matter (CDM) and mixed dark matter (MDM; also known as CHDM). Isocurvature modes are also shown. Picture courtesy of John Peacock.

HDM first produces superclusters, which subsequently fragment to form galaxies; CDM is a ‘bottom–up’ model because small-scale structures form first and then merge to form larger ones. The general picture that emerges is that, while the amplitude of each Fourier mode remains small, i.e. $\delta(k) \ll 1$, linear theory applies. In this regime, each Fourier mode evolves independently and the power-spectrum therefore just scales as

$$P(k, t) = P(k, t_1) \frac{D^2_+ (k, t)}{D^2_+ (k, t_1)} = P_0(k) T^2(k) \frac{D^2_+ (k, t)}{D^2_+ (k, t_1)} .$$

For scales larger than the Jeans length, this means that the shape of the power-spectrum is preserved during linear evolution.

2.5. Beyond linear theory

The linearised equations of motion provide an excellent description of gravitational instability at very early times when density fluctuations are still small.
(\(\delta \ll 1\)). The linear regime of gravitational instability breaks down when \(\delta\) becomes comparable to unity, marking the commencement of the quasi-linear (or weakly non-linear) regime. During this regime the density contrast may remain small (\(\delta < 1\)), but the phases of the Fourier components \(\delta_k\) become substantially different from their initial values resulting in the gradual development of a non-Gaussian distribution function if the primordial density field was Gaussian. In this regime the shape of the power-spectrum changes by virtue of a complicated cross-talk between different wave-modes. Analytic methods are available for this kind of problem, but the usual approach is to use \(N\)-body experiments for strongly non-linear analyses (Davis et al. 1985; Jenkins et al. 1999).

Further into the non-linear regime, bound structures form. The baryonic content of these objects may then become important dynamically: hydrodynamical effects (e.g. shocks), star formation and heating and cooling of gas all come into play. The spatial distribution of galaxies may therefore be very different from the distribution of the (dark) matter, even on large scales. Attempts are only just being made to model some of these processes with cosmological hydrodynamics codes, but it is some measure of the difficulty of understanding the formation of galaxies and clusters that most studies have only just begun to attempt to include modelling the detailed physics of galaxy formation. In the front rank of theoretical efforts in this area are the so-called semi-analytical models which encode simple rules for the formation of stars within a framework of merger trees that allows the hierarchical nature of gravitational instability to be explicitly taken into account (Baugh et al. 1998).

The usual approach is instead simply to assume that the point-like distribution of galaxies, galaxy clusters or whatever,

\[
n(\mathbf{r}) = \sum_i \delta_D(\mathbf{r} - \mathbf{r}_i),
\]

(20)

bears a simple functional relationship to the underlying \(\delta(\mathbf{r})\). An assumption often invoked is that relative fluctuations in the object number counts and matter density fluctuations are proportional to each other, at least within sufficiently large volumes, according to the linear biasing prescription:

\[
\frac{\delta n(\mathbf{r})}{\bar{n}} = b \frac{\delta \rho(\mathbf{r})}{\bar{\rho}},
\]

(21)

where \(b\) is what is usually called the biasing parameter. Alternatives, which are not equivalent, include the high-peak model (Kaiser 1984; Bardeen et al. 1986) and the various local bias models (Coles 1993). Non-local biases are possible, but it is rather harder to construct such models (Bower et al. 1993). If one is prepared to accept an ansatz of the form (21) then one can use linear theory on large scales to relate galaxy clustering statistics to those of the density fluctuations, e.g.

\[
P_{\text{gal}}(k) = b^2 P(k).
\]

(22)

This approach is the one most frequently adopted in practice, but the community is becoming increasingly aware of its severe limitations. A simple parametrisation of this kind simply cannot hope to describe realistically the relationship between galaxy formation and environment (Dekel & Lahav 1999).
3. Large-scale Structure: Past and Present

3.1. Modelling

Models of structure formation involve many ingredients which interact in a complicated way: (i) A background cosmology, basically a choice of $\Omega_0$, $H_0$ and $\Lambda$ if we are prepared to stick with the Robertson–Walker metric (1) and the Einstein equations (3)-(5); (ii) an initial fluctuation spectrum, usually taken to be a power-law usually with $n = 1$; (iv) a choice of fluctuation mode, usually adiabatic; (iii) a statistical distribution of fluctuations usually Gaussian; (v) a transfer function, which requires knowledge of the relevant proportions of ‘hot’, ‘cold’ and baryonic material as well as the number of relativistic particle species; (vi) a ‘machine’ for handling non-linear evolution, so that the distribution of galaxies and other structures can be predicted, usually an $N$-body code, an approximated dynamical calculation or simply, with fingers crossed, linear theory; (vii) a prescription for relating fluctuations in mass to fluctuations in light, frequently the linear bias model. I will now discuss how the attitude to these ingredients has changed in the past, and is likely to in the near future.

Historically speaking, the first model incorporating non-baryonic dark matter to be seriously considered was the hot dark matter (HDM) scenario, in which the universe is dominated by a massive neutrino with mass around $10^{-30}$ eV. This scenario has fallen into disrepute because the copious free streaming it produces smooths the matter fluctuations on small scales and means that galaxies form very late. The favoured alternative for most of the 1980s was the cold dark matter (CDM) model in which the dark matter particles undergo negligible free streaming owing to their higher mass or non-thermal behaviour. A ‘standard’ CDM model (SCDM) then emerged in which the cosmological parameters were fixed at $\Omega_0 = 1$ and $h = 0.5$, the spectrum was of the Harrison–Zel’dovich form with $n = 1$ and a significant bias, $b = 1.5$ to 2.5, was required to fit the observations (Davis et al. 1985).

The SCDM model was ruled out by a combination of the COBE-inferred amplitude of primordial density fluctuations, galaxy clustering power-spectrum estimates on large scales, cluster abundances and small-scale velocity dispersions (Peacock & Dodds 1996). It seems the standard version of this theory simply has a transfer function with the wrong shape to accommodate all the available data with an $n = 1$ initial spectrum. Nevertheless, because CDM is such a successful first approximation and seems to have gone a long way to providing an answer to the puzzle of structure formation, the response of the community has not been to abandon it entirely, but to seek ways of relaxing the constituent assumptions in order to get a better agreement with observations. Various possibilities have been suggested.

If the total density is reduced to $\Omega_0 \simeq 0.3$, which is favoured by many arguments, then the size of the horizon at matter–radiation equivalence increases compared with SCDM and much more large-scale clustering is generated. This is called the open cold dark matter model, or OCDM for short. Those unwilling to dispense with the inflationary predilection for flat spatial sections have invoked $\Omega_0 = 0.2$ and a positive cosmological constant to ensure that $k = 0$; this can be called $\Lambda$CDM and is also favoured by observations of distant supernovae. Much the same effect on the power spectrum may also be obtained
in $\Omega = 1$ CDM models if matter-radiation equivalence is delayed, such as by the addition of an additional relativistic particle species. The resulting models are usually called $\tau$CDM.

Another alternative to SCDM involves a mixture of hot and cold dark matter (CHDM), having perhaps $\Omega_{\text{hot}} = 0.3$ for the fractional density contributed by the hot particles. For a fixed large-scale normalisation, adding a hot component has the effect of suppressing the power-spectrum amplitude at small wavelengths. Another possibility is to invoke non-flat initial fluctuation spectra, while keeping everything else in SCDM fixed. The resulting ‘tilted’ models, TCDM, usually have $n < 1$ power-law spectra for extra large-scale power and, perhaps, a significant fraction of tensor perturbations. Models have also been constructed in which non-power-law behaviour is invoked to produce the required extra power: these are the broken scale-invariance (BSI) models.

### 3.2. Past Observational Developments

In 1986, the CfA survey (de Lapparent, Geller & Huchra 1986) was the ‘state-of-the-art’, but this contained redshifts of only around 2000 galaxies with a maximum recession velocity of 15 000 km s$^{-1}$. The subsequent Las Campanas survey contained around six times as many galaxies, and goes out to a velocity of 60 000 km s$^{-1}$ (Shectman et al. 1996). Quantitative measures of spatial clustering obtained from these data sets offer the simplest method of probing $P(k)$, assuming that these objects are related in some well-defined way to the mass distribution and this, through the transfer function, is one way of constraining cosmological parameters. For example, Peacock & Dodds (1996) made compilations of power-spectra of different kinds of galaxy and cluster redshift samples. Within the (considerable) observational errors, and the uncertainty introduced by modelling of the bias, all the data lie roughly on the same curve. A consistent picture thus emerged in which galaxy clustering extends over larger scales than is expected in the standard CDM scenario. It was difficult to say much in terms of testing the variations on the CDM theme I have discussed so far, however, because of the sparseness and limited scale coverage of the available data.

### 3.3. The Present: Entering the Precision Era

The next generation of redshift surveys, prominent among which are the Sloan Digital Sky Survey of about one million galaxy redshifts (Gunn & Weinberg 1995) and an Anglo-Australian collaboration using the two-degree field facility (Colless et al. 2001). The latter survey, called 2dFGRS, has now finished taking data while the Sloan Survey is still in progress. Both exploit multi-fibre methods that can obtain 400 galaxy spectra in one go, and will increase the number of redshifts by about two orders of magnitude over those previously available. The huge increase in survey depth (2dFGRS reaches redshifts $z \sim 0.3$) has allowed a much better measurement of the matter power-spectrum (Percival et al. 2001) and better statistics have allowed some progress to be made using higher-order statistical diagnostics of non-linearity and bias (Verde et al. 2002).

It is evident from Figure 2 that, although the three non-SCDM models are similar at $z = 0$, differences between them are marked at higher redshift. This suggests the possibility of using measurements of galaxy clustering at high redshift to distinguish between models and reality. This has now become possible,
with surveys of galaxies at $z \sim 3$ already being constructed (Steidel et al. 1998, 1999). Unfortunately, the interpretation of these new data is less straightforward than one might have imagined. If the galaxy distribution is biased at $z = 0$ then the bias is expected to grow with $z$ (Davis et al. 1985). If galaxies are rare peaks now, they should have been even rarer at high $z$. There are also many distinct possibilities as to how the bias might evolve with redshift (Matarrese et al. 1997; Moscardini et al. 1998; Coles et al. 1998).

But large-scale structure is not just about clustering power spectra. There are other ways in which it is possible to use information about the velocities of galaxies to constrain models (Strauss & Willick 1995). Probably the most useful information pertains to large-scale motions, as small-scale data populate the highly nonlinear regime. The basic principle is that velocities are induced by fluctuations in the total mass, not just the galaxies. Comparing measured velocities with measured fluctuations in galaxies with measured fluctuations in galaxy counts, it is possible to constrain both $\Omega$ and $b$. From equations (10) to (13) it emerges that

\[ v = -\frac{2f}{3\Omega H a} \nabla \phi + \text{const.} a(t), \]

which demonstrates that the velocity flow associated with the growing mode in the linear regime is curl-free, as it can be expressed as the gradient of a scalar potential function. Notice also that the induced velocity depends on $\Omega$. This is the basis of a method for estimating $\Omega$ which is known as POTENT. Since all matter gravitates, not just the luminous material, there is a hope that methods such as this can break the degeneracy between clustering induced by gravity and that induced statistically, by bias. See Dekel (1994) for a review. These methods are prone to error if there are errors in the velocity estimates. Perhaps a more robust approach is to use peculiar motion information indirectly, by the effect they have on the distribution of galaxies seen in redshift-space (i.e. assuming total velocity is proportional to distance). The information gained this way is statistical, but less prone to systematic error (Peacock et al. 2001) and the evolution of the effect with redshift is also a test of cosmological models (Ballinger, Peacock & Heavens 1996).

Another class of observations that can help break the degeneracy between models involves gravitational lensing. The most spectacular forms of lensing are those producing multiple images or strong distortions in the form of arcs. These require very large concentrations of mass and are therefore not so useful for mapping the structure on large scales. However, there are lensing effects that are much weaker than the formation of multiple images. In particular, distortions producing a shearing of galaxy images promise much in this regard (Kaiser & Squires 1993). With the advent of new large CCD detectors, this should soon be realised (Mellier 1999).

The combination of lensing, peculiar motions and galaxy clustering studies would be impressive enough even without the dramatic arrival of WMAP on the scene (Bennett et al. 2003). The WMAP data have really heralded the precision era, allowing direct determinations of the primordial fluctuation spectrum and the basic cosmological parameters in a manner that bypasses most of important sources of uncertainty in clustering analysis.
The bumps and wiggles shown in the transfer functions of Figure 1 do find themselves into the present-day spectrum of galaxy clustering, but they are strongly affected by non-linear evolution on the way. Moreover galaxy surveys probe the distribution of luminous matter so one can’t infer the matter spectrum directly from that of galaxies without a model for the bias. Galaxies also have peculiar motions so their redshifts do not exactly represent their proper distances. Survey determinations of $P(k)$ will inevitably be harder to interpret to those obtained from the cosmic microwave background, where none of these complications arise (Hinshaw et al. 2003). This is the reason for the tremendous precision of WMAP’s determination of cosmological parameters (Spergel et al. 2003), which can be improved still further by combining constraints from the 2dFGRS and lensing studies. Nevertheless, there are very strong possibilities that a redshift survey performed with the SKA could probe both the spectrum and the background cosmology, for example by using the ‘wiggles’ as standard rulers (e.g. Blake & Glazebrook 2003).

4. The Way Ahead: A Role for SKA Redshift Surveys?

WMAP, 2dFGRS and the other manifestations of precision cosmology have certainly made great strides towards the determination of the cosmological parameters. The standard model that has emerged (which is very similar to the ΛCDM model described above). Although it would be premature to say that no departures from this model are possible, the emphasis as far as galaxy clustering is concerned will be away from its use as a probe of the background cosmology. So what is the future? And is there a role for the SKA in cosmological studies other than consistency checks of the standard model? The answer to both questions is emphatically “yes”.

One can see evidence of a new direction already. Some of the most interesting results to have emerged from 2dFGRS concern the clustering of galaxies selected by spectral type (Madgwick et al. 2003). Preliminary results from the Sloan Digital Sky Survey reveal a complicated dependence of clustering the colours of selected galaxies (Zehavi et al. 2002). There is evidence of clustering dependence on intrinsic galaxy properties emerging also from infra-red selected galaxies (Hawkins et al. 2001). While these dependencies are simply a nuisance when it comes to determining cosmological parameters, they indicate that the large-scale distribution of galaxy clustering may hold clues to their formation process. The relative clustering strength of different populations may be complex and scale-dependent, requiring more sophisticated description that the simple bias parameters described above.

In principle observations such as these can be used to test semi-analytic models of galaxy formation of the form discussed by Baugh et al. (1998). On the other hand, all the classes of galaxy mentioned are selected by radiation coming from sources with a complex and poorly understood formation process. The Square Kilometre Array could produce a great step forward in this area, by mapping galaxy positions and redshifts in neutral hydrogen via the 21cm line. Detailed theoretical predictions are so far lacking, but two “straw man” surveys are described in the SKA science case.
4.1. The SKA Shallow Survey

The first case is a “traditional” redshift survey along the lines of 2dFGRS but using 21cm to select the galaxies. Depending on the eventual choice of instrumental sensitivity, such a survey might take 12 months, cover about 1000 square degrees of sky and be capable of detecting galaxies out to $z \sim 2$; compare the limit $z \sim 0.3$ of 2dFGRS. All in all, this means a survey of around $10^7$ galaxies in a volume of order $10^7$ Mpc. This is impressive enough in itself, but such a survey would also bring with it the possibility of HI Tully-Fisher measurements for the galaxies in it. In this respect its nearest present relative is the 6dF galaxy survey described at:

http://www.mso.anu.edu.au/6dFGS/6dF_survey_plan.html

The potential to combine redshifts and Tully-Fisher distances enables velocity field mapping on an immense scale.

4.2. The SKA Pencil Beam Survey

An alternative mode of redshift survey for SKA is to look at a smaller area for much longer. Using the same sensitivity as in the previous example of a shallow survey, a 360 hour survey covering one square degree could contain $10^5$ galaxies. A present-day $L^*$ galaxy could be detected in its HI emission out to a redshift $z \sim 3$. The limiting HI mass would be a few times $10^9 M_\odot$ at $z \sim 4$ and of order $10^8 M_\odot$ at $z \sim 1$.

The possibility of detecting objects at high redshift offers the prospect of constraining models of galaxy formation extremely strongly. In all hierarchical clustering models, the bias associated with galaxies increases dramatically with redshift. This results in a strange conspiracy: the matter correlations decrease with increase redshift while the bias increases in compensation, producing a very slow evolution of measured clustering with epoch. However, the probes we have of high-redshift clustering, such as Lyman-break galaxies (Steidel et al. 1998, 1999) and QSOs (Outram et al. 2001), suffer from low sampling density and uncertain interpretation of the host object. More importantly, the supply of cold gas plays a central role in the detailed semi-analytic models of galaxy formation and the evolution of HI mass function with redshift will be a decisive test of the basic framework. However, much theoretical work is needed to make detailed predictions for such surveys. The hierarchical nature of structure formation involves gas being distributed in less massive haloes at high redshift, but gas is also used up to form stars as time goes. The number of HI sources seen as a function of redshift may be drastically different from that expected of a non-evolving population of present-day galaxies.

5. Discussion and Conclusions

I have emphasized the importance of clustering properties and their implications for galaxy and large-scale structure formation. There will no doubt be many that disagree with this emphasis. Large-scale matter power spectrum determination will be possible using SKA and will be enormously better even that 2dFGRS or Sloan. Such studies are well-worth doing, as are the numerous possible tests
of departures from the standard model, especially with respect to the possible forms of dark energy such as quintessence (Huterer & Turner 2001). A 21cm survey would be better fitted to such a task than QSO surveys (e.g. Outram et al. 2001) because of the higher sampling density.

Interesting though the results of such studies will be, they will almost certainly turn out merely to provide consistency checks on a cosmology largely fixed by studies of the cosmic microwave background. For me, the distribution of cold gas on large-scales, how it relates to stellar populations of various kinds and how the supply of this gas has evolved with cosmic epoch offers the richest scientific possibilities. What is now needed is proper theoretical modelling of HI-selected galaxies to produce mock catalogues to drive the science case further forward. Watch this space.

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References

Ballinger, W.E., Peacock, J.A. & Heavens, A.F., 1996, MNRAS, 282, 877
Bardeen, J.M., Bond, J.R., Kaiser, N. & Szalay, A.S., 1986, ApJ, 304, 15
Baugh, C.M., Cole, S., Frenk, C.S., Lacey, C.G., 1998, ApJ, 498, 504
Blake, C. & Glazebrook, K., 2003, ApJ, 594, 665
Bower, R.G., Cole, P., Frenk, C.S., & White, S.D.M., 1993, ApJ, 405, 403
Brandenberger, R.H., 1985, Rev. Mod. Phys., 57, 1
Coles, P., 1998, MNRAS, 262, 1065
Coles, P., Lucchin, F., Matarrese, S. & Moscardini, L., 1998, MNRAS, 300, 183
Colless M.M., et al., 2001, MNRAS, 328, 1039
Davis, M., Efstathiou, G., Frenk, C.S. & White, S.D.M., 1985, ApJ, 292, 371
Dekel, A., 1994, ARA&A, 32, 371
Dekel, A. & Lahav, O., 1999, ApJ, 520, 24
de Lapparent, V., Geller, M.J., & Huchra, J.P., 1986, ApJ, 302, L1.
Gunn, J.E. & Weinberg, D.H., 1995, in: Wide Field Spectroscopy and ther Distant Universe, eds. S.J. Maddox & A. Aragón-Salamanca, World Scientific, Singapore, pp. 3-14
Guth, A.H., 1981, Phys. Rev. D., 23, 347
Harrison, E.R., 1970, Phys. Rev. D., 1, 2726
Hawkins, E., Maddox, S.J., Branchini, E. & Saunders, W., 2001, MNRAS, 325, 589
Hinshaw, G., et al., 2003, ApJS, 148, 135
Huterer, D. & Turner, M., 2001, Phys. Rev. D., 123527
Jenkins, A., et al., 1999, ApJ, 499, 20
Kaiser, N., 1984, ApJ, 284, L9
Kaiser, N. & Squires, G., 1993, ApJ, 404, 441
Matarrese, S., Coles, P., Lucchin, F. & Moscardini, L., 1997, MNRAS, 286, 115
Madgwick, D.S., 2003, MNRAS, 348, 857
Mellier, Y., 1999, ARA&A, 37, 127
Moscardini, L., Coles, P., Lucchin, F. & Matarrese, S., 1998, MNRAS, 299, 95
Outram, P.J. et al., 2001, MNRAS, 328, 174
Peacock, J.A. & Dodds, S.J., 1996, MNRAS, 280, L19
Peacock, J.A., et al., 2001, Nature, 410, 169
Peebles, P.J.E., 1999, ApJ, 510, 523
Percival, W.J., et al., 2001, MNRAS, 327, 1297
Perlmutter, S., et al., 1999, ApJ, 517, 565
Riess, A., et al., 1998, AJ, 116, 1009
Shectman, S.A., et al., 1996, ApJ, 470, 172
Smoot, G.F., et al., 1992, ApJ, 396, L1
Spergel, D.N., et al., 2003, ApJS, 148, 175
Steidel, C.C., et al., 1998, ApJ, 492, 428
Steidel, C.C., et al., 1999, ApJ, 519, 1
Strauss, M.A & Willick, J.A., 1995, Phys. Rep., 261, 271
Verde L., et al., 2002, MNRAS, 335, 432
Zehavi, I., 2002, ApJ, 571, 172
Zel'dovich, Ya.B., 1972, MNRAS, 160, 1P