Pre-collapse Evolution of Galactic Globular Clusters

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Abstract

This paper is concerned with collisionless aspects of the early evolution of model star clusters. The effects of mass loss through stellar evolution and of a steady tidal field are modelled using $N$-body simulations. Our results (which depend on the assumed initial structure and the mass spectrum) agree qualitatively with those of Chernoff & Weinberg (1990), who used a Fokker-Planck model with a spherically symmetric tidal cutoff. For those systems which are disrupted, the lifetime to disruption generally exceeds that found by Chernoff & Weinberg, sometimes by as much as an order of magnitude. Because we do not model collisional effects correctly we cannot establish the fate of the survivors. In terms of theoretical interpretation, we find that tidal disruption must be understood as a loss of equilibrium, and not a loss of stability, as is sometimes stated.

Key words: galaxies: star clusters – globular clusters: general
1. Introduction

Our understanding of the evolution of an idealised globular cluster, i.e. an isolated, bound, spherically symmetric stellar system with stars of equal mass, has increased rapidly in recent decades (Spitzer 1987, Djorgovski & Meylan 1993). Recently, the focus of interest has moved to the study of the evolution of more realistic models of globular clusters. For example, stars in a real globular cluster have different masses, and they change mass in response to their internal evolution. Moreover, most globular clusters exist within galaxies, and are influenced by the effects of the galactic tide.

These processes are linked. Each star (above the turnoff mass corresponding to the age of the system) loses a certain fraction of its mass near the end of its own evolution, and this process decreases the total mass of the cluster. Therefore the potential of the cluster weakens and the cluster expands. As a result some stars flow over the tidal boundary, and so further mass is lost by the cluster. The time scale of stellar evolution is \( \sim 10^7 \) years for a \( 10 M_\odot \) star, which is roughly comparable to the dynamical time scale (or crossing time) of the cluster. If the cluster contains a large fraction of massive stars initially, the dynamics of the cluster is greatly affected by the mass loss due to stellar evolution.

Chernoff and Weinberg (1990; hereafter CW) investigated these and other aspects of the evolution of globular clusters using Fokker-Planck models. Their study included the following realistic effects: the spectrum of stellar masses, mass loss due to stellar evolution, and a tidal cutoff to model the effect of the galactic tidal field. They performed an extensive survey of models differing with regard to the initial mass function, the central potential of the cluster, and the galactocentric distance. For example, they obtained the result that the mass loss during \( 5 \times 10^9 \) yr is sufficiently strong to disrupt weakly bound clusters with a Salpeter initial mass function.

The main purpose of the present paper is to check the results of CW using a model which should be an improvement in several respects. We use direct \( N \)-body calculations, which allow us to include processes taking place on the dynamical timescale. These are neglected by CW, who used an orbit-averaged method, and because the time scale for mass loss and the dynamical time are not well separated, it is not clear \textit{a priori} that their approximation is justified. Like CW, we also include the following “realistic” effects: a spectrum of stellar masses, mass loss due to stellar evolution, and the tidal field of the galaxy. This last feature differs from CW’s tidal cutoff, because it is not spherically symmetric, and also because it affects stars even while they remain inside the tidal radius. Like CW, we performed a survey of King models which differed in respect of the dimensionless central potential, \( W_0 \), the slope (\( \alpha \)) of the initial stellar mass function (assumed to be
a power law), and the galactocentric distance, $R_g$, of the cluster. In order to calculate the gravitational forces we used a special purpose computer for the gravitational $N$-body problem: GRAPE-3A (Okumura et al. 1993).

In general we obtained qualitatively similar results to those of CW: the less concentrated clusters ($W_0 < \sim 4$) and/or ones that contained a greater proportion of massive stars initially ($\alpha > \sim -2.5$) are disrupted before reaching the stage at which core collapse can begin. The main quantitative difference is that the lifetime to disruption (for those clusters which do not survive until core collapse) may be much longer than that found by CW.

The structure of the paper is as follows. In section 2 we describe the initial conditions and the way in which stellar mass loss and tidal effects were modelled in our calculations. (Most parameters were chosen so as to be the same as those used by CW, for purposes of comparison). In section 3 we describe briefly the hardware used for our calculations, and the results are presented in §4. In section 5 we discuss the mechanism of disruption, and §6 sums up, with discussion of some residual issues.

2. Model

2.1 Equations of Motion

We adopt the conventional model in which the cluster is assumed to move on a nearly circular orbit in a spherically symmetric galaxy potential, taken to be that of a distant point mass $M_g$. Thus the stars are affected by the steady tidal field of the galaxy, as well as by the forces from other stars in the cluster. We set the initial center of mass of the cluster at the origin ($x, y, z$) = (0, 0, 0), with axes orientated so that the position of the galactic center is ($-R_g, 0, 0$). We assume that the size of the globular cluster is much smaller than $R_g$. If the globular cluster rotates around the galactic center at an angular velocity $\Omega = (0, 0, \omega)$ (cf.§2.4), the equation of motion of an individual cluster member can be expressed as

$$\frac{d^2 \mathbf{r_i}}{dt^2} = -\nabla \Phi_{c,i} - 2\Omega \times \frac{d\mathbf{r_i}}{dt} + \omega^2(3x_i\hat{e}_x - z_i\hat{e}_z),$$

(1)

where $\mathbf{r_i}$ is the position of the $i$-th star, and $\hat{e}_x, \hat{e}_z$ are unit vectors that point along the $x, z$ axes, respectively. The first term on the right-hand side in equation (1) is the gravitational acceleration from other stars in the cluster, the second term is the Coriolis acceleration, and the third term is a combination of the centrifugal and tidal forces. In our simulations we used a softened gravitational potential, so that

$$-\nabla \Phi_{c,i} = -\sum_{j=1, j\neq i}^{N} \frac{Gm_j(t)(\mathbf{r_i} - \mathbf{r_j})}{(|\mathbf{r_i} - \mathbf{r_j}|^2 + \varepsilon^2)^{3/2}},$$

(2)
where $N$ is the number of particles, $G$ is the gravitational constant, and $\varepsilon$ is the softening parameter.

We performed numerical integrations of equation (1) using a second-order predictor-corrector scheme with shared and constant timestep. In models with $N = 16384$ the timestep, $\Delta t$, was $1/64$, and the softening parameter, $\varepsilon$, was $1/32$, in standard units (Heggie & Mathieu 1986), while for smaller values of $N$ we took $\Delta t = 1/32$ and $\varepsilon = 1/16$. These values of $\varepsilon$ are comparable with the mean interparticle distance.

### 2.2 Initial Conditions

We used King’s models (King 1966) to generate the initial conditions. Thus the distribution function is a lowered Maxwellian, given by

$$g(E) = K[e^{-\beta(E-E_t)} - 1], \quad (3)$$

for $E < E_t = \phi(r_t)$, where $E = v^2/2 + \phi(r)$, $K$ is constant, and $r_t$ is the radius of the edge of the cluster. (We use $\phi$ for the Newtonian potential to distinguish it from the softened potential $\Phi_c$.) The King model is determined by the dimensionless central potential, $W_0 = \beta(\phi(r_t) - \phi(0))$.

We assign the masses of the stars according to a power-law mass function:

$$dN(m) = m^\alpha dm, \quad (4)$$

between $0.4M_\odot$ and $15M_\odot$. We assume that there is no correlation between the position and mass; in particular there is no initial equipartition of energies between stars of different mass. Even at the present day mass segregation is difficult to observe in the dynamically evolved globular clusters of our galaxy, but our main reasons for making this assumption are (i) simplicity and (ii) consistency with the assumptions of CW.

For this paper we performed a survey of models defined by combinations of the values of the dimensionless central potential of the King model, $W_0 = 1, 3, 5, 7$, and the slope of the initial mass function, $\alpha = -1.5, -2.5, -3.5$. King models are not perfectly suited to the task, because the function $g(E)$ defined in eq.(3) is not an equilibrium distribution for eq.(1), even if $\Phi_c$ is approximated by a smooth potential, because $\phi$ excludes the tidal and centrifugal potentials, and it is an unsoftened potential. Therefore we first carried out scaling of the stellar velocities to ensure virial equilibrium, i.e. the condition

$$\ddot{I} = 0, \quad (5)$$
where
\[ I = \sum_{i=1}^{N} m_i r_i^2. \]

This condition can also be expressed as
\[
\ddot{I} = \sum_{i=1}^{N} 2m_i \ddot{r}_i^2 + \sum_{i=1}^{N} 2m_i (r_i \cdot \ddot{r}_i)
= 4T + 2V_c + V_s - 4V_g = 0,
\]

(6)

where
\[
T = \sum_{i=1}^{N} \frac{1}{2} m_i \dot{r}_i^2,
\]
\[
V_c = \sum_{i=1}^{N} m_i r_i \cdot (-\nabla \Phi_{c,i}),
\]
\[
V_s = 4\omega \sum_{i=1}^{N} L_{z,i},
\]

(7)

and
\[
V_g = \omega^2 \sum_{i=1}^{N} m_i (-\frac{3}{2} x_i^2 + \frac{1}{2} z_i^2).
\]

Here, \(T\) is the total kinetic energy, \(L_{z,i} \equiv m_i (x_i \dot{y}_i - \dot{x}_i y_i)\) is the z-component of the angular momentum of the \(i\)th star, and \(V_c\), \(V_s\), and \(V_g\) are contributions due to the gravitational force within the cluster, the Coriolis force, and the combined centrifugal and tidal forces, respectively. For an unsoftened potential \(V_c\) would be the potential energy. The external contribution to the energy of the cluster, i.e. that due to tidal and centrifugal forces, is \(V_g\). The value of \(\omega\) was determined in such a way that the tidal radius (§2.4) coincided with the initial radius of the King model.

### 2.3 Stellar evolution

We modelled the effect of stellar evolution by appropriately changing the mass of each star. At the last stage of stellar evolution stars lose a significant fraction of their mass in winds and supernova explosions. The potential well of a cluster is not sufficiently deep to retain the lost mass, since the escape velocity is only a few times 10km/s. Therefore we assume that the lost mass disappears abruptly from the cluster.
We change the mass of each particle according to

\[ m(t) = \begin{cases} 
  m_{\text{ini}} & (t < t_{se}) \\
  m_{\text{rm}} & (t \geq t_{se})
\end{cases} \]  \( (8) \)

where \( m_{\text{ini}} \) is the initial mass, \( m_{\text{rm}} \) is the mass of any remnant after mass loss, and \( t_{se} \) is the time when the star loses its mass (Table 1). We obtain values between the points listed in Table 1 by linear interpolation. The remnant mass, \( m_{\text{rm}} \), is summarized in Table 2. These tables are due to Iben and Renzini (1983), and were also those used by CW, from which these tables have been copied.

2.4 Tidal Boundary

During the course of a simulation stars escape from the cluster. The precise dynamical definition of escape is not easy if there is a tidal field, and here we adopt a simple geometric definition: escapers are defined to be those stars beyond the tidal radius. All stars within the tidal radius are taken to be members, even though the tidal field included in equation (1) is not spherically symmetric. More precisely, we use the distance between the center of the cluster (defined below) and the Lagrangian point in the direction of the galactic center as the tidal radius. If, as before, the galaxy is represented by a distant point mass, it follows that the tidal radius is given by

\[ r_t = \left( \frac{M}{3M_g} \right)^{1/3} R_g, \]  \( (9) \)

approximately, where \( M \) is the mass of the cluster, \( M_g \) is the mass of the galaxy and \( R_g \) is the distance to the galaxy. Here \( M \) is taken to be the total mass of the ‘members’, and since this depends on \( r_t \) itself, some iteration is usually required. Since the angular velocity of the cluster around the galaxy is given by \( \omega^2 = GM_g/R_g^3 \), it follows that

\[ \omega = \sqrt{\frac{GM}{3r_t^3}}. \]  \( (10) \)

We define the (density weighted) center of the cluster by

\[ \mathbf{r}_c = \sum_{i=1}^{N} \rho_i^2 \mathbf{r}_i / \sum_{i=1}^{N} \rho_i^2, \]  \( (11) \)

where \( \rho_i \) is the local density around the \( i \)-th particle. For this purpose we use the local density that was introduced by Casertano & Hut (1985) and defined by

\[ \rho_i = \frac{3M_{6,i}}{4\pi R_{6,i}^3}, \]  \( (12) \)
where $R_{6,i}$ is the distance to the sixth nearest neighbor of star $i$, and $M_6$ is the total mass lying within the distance $R_{6,i}$ from star $i$.

2.5 Scaling

We use standard units such that $M = G = -4E = 1$, where $M$ is the initial total mass and $E$ is the initial total self-energy (Heggie and Mathieu 1986), and we now consider how the results should be scaled to astrophysical units. Simple $N$-body simulations have two degrees of freedom for scaling, e.g. the units of length and time. However, we have to specify the unit of time in years in order to determine the time of mass loss by stellar evolution in the simulation. Thus two simulations in which the value of the unit of time differs will experience different evolution in general.

We wish to select scalings appropriate to a cluster of mass $M$, at galactocentric radius $R_g$, moving at circular speed $v_g$, with mean stellar mass $\langle m \rangle$, and tidal radius $r_t$, so that it may be simulated by a model composed of $N$ particles with standard units. Because of the definition of these units we select the units of mass, $U_m$, and length, $U_l$, as follows;

$$U_m = M, \quad (13)$$

$$U_l = r_t/r_t^*, \quad (14)$$

where $r_t^*$ is the tidal radius of the King model in standard $N$-body units. Since a cluster with $M/\langle m \rangle$ stars is being modelled using $N$ particles, a particle in the simulation of mass $m^* (\sum m^* = 1)$ represents a total mass $Mm^*$ which consists of $(M/\langle m \rangle)/N$ stars of individual mass $Nm^*/\langle m \rangle$.

Conversion to standard units also requires that the unit of time is given by

$$U_t = \left( \frac{U_l^3}{GU_m} \right)^{1/2} = \left( \frac{r_t^3}{GMr_t^3} \right)^{1/2} = \frac{R_g}{v_g} \left( \frac{1}{3r_t^*} \right)^{1/2}, \quad (15)$$

where we have used eqs.(9), (13) and (14). It follows that the crossing time of the $N$-body model scales correctly to that of the real globular cluster. Also, to determine when a given star loses mass, we compute $Nm^*/\langle m \rangle$, and obtain the evolution time, $t_{se}$, from Table 1. Then, the corresponding $N$-body time is $t_{se}/U_t$.

There are two characteristic time scales for the evolution of a globular cluster: the crossing time, $t_{cr}$, characterizing the orbital motions of the stars, and the relaxation time, $t_r$, characterizing two body relaxation. Ideally, we need to follow the evolution of a globular cluster with a technique which correctly models effects on both time scales. The time scale of the internal evolution of massive stars is comparable to the dynamical (crossing) time.
scale of a typical globular cluster ($\lesssim 10^7$ years). If the fraction of massive stars is not very small, the globular cluster evolves significantly within the dynamical timescale. On the other hand, relaxation is equally important in the long run. Evaporation due to two-body encounters reduces the mass of the cluster, while relaxation increases the central density, thus helping the cluster to avoid disruption by the tidal field.

The ratio of the crossing time to the relaxation time (within the half-mass radius) is approximately linearly dependent on the particle number $N$ as follows (Spitzer 1987):

$$\frac{t_r}{t_{cr}} = \frac{N}{11 \ln(0.4N)}.$$  \hfill (16)

When we model a cluster consisting of $\sim 10^5 - 10^6$ stars using a smaller number of particles (in most of our simulations $N = 8192$), the ratio is different from that of the real globular cluster system.

Under the scaling that we have adopted, the results of the simulation have little significance after the time when the system is considerably affected by two-body relaxation, since the system will become relaxed much sooner than a real globular cluster. We used a relatively large number of particles, $N$, and a large softening parameter, $\varepsilon$, comparable to the mean particle separation, in order to suppress two body relaxation effects as much as possible.

As an alternative, it would be possible to choose a scaling which ensures that the relaxation of the simulation takes place on the same time scale as that of the real cluster (Giersz & Heggie 1994; cf. Heggie 1994). On the other hand this implies that the crossing time of the simulation is much greater than it should be, and so loss of mass by stellar evolution is much more impulsive in character than in a real cluster.

3. Hardware

For the force calculation, we used a special-purpose computer for $N$-body problems, called GRAPE-3A, which is a modified version of GRAPE-3 (Okumura et al. 1993). It consists of 8 LSI chips dedicated to the calculation of gravity and has a peak performance of about 5 Gflops-equivalent per board (at 20MHz clock cycle). We also used the nearest-neighbor list produced by GRAPE-3A to compute the local density as defined in section 2.4.

GRAPE-3A is a special-purpose computer mainly for collisionless $N$-body calculations. When calculated with GRAPE-3A the force between two particles has a relative error of a few percent, because of the low accuracy of internal expressions in the GRAPE chip. However, GRAPE-3A can follow properly the evolution of self gravitating systems
unless the system contains a high-density core or its evolution is governed by the behaviour of binaries. According to Hernquist et al. (1993), the change in velocity due to the error of GRAPE-3A, denoted by $v_{err}$, may be estimated by

$$
\langle v_{err} \rangle^2 \leq e^2 \langle v_{2b} \rangle^2,
$$

where $\langle v_{2b} \rangle$ is the change in velocity due to 2-body relaxation effects. Here, $e$ is the relative error of the force between two particles. In the case of GRAPE-3A, $e$ is estimated at $2 \sim 3\%$ (Makino et al. 1990; Okumura et al. 1993). Therefore, the error due to GRAPE-3A is small compared to two-body relaxation effects, and we have already pointed out that these are required to be small if our simulations are to be valid.

4. Results

In this section we present the results of our calculations, in which we performed a survey of models differing in the slope, $\alpha$, of the initial power-law mass function, and in the dimensionless central potential of King’s model, $W_0$. In section 4.1 we summarize the results of the survey. In section 4.2 we investigate the evolution of the total mass for each set of parameters. In sections 4.3 and 4.4 we discuss the effects of varying the galactocentric distance and of two-body relaxation, respectively. In section 4.5 we present data on the anisotropy of the velocity dispersion.

4.1. Summary of Survey Results

In Table 3, the parameters of all runs are listed. The choices of $\alpha$ and $W_0$ were made for consistency with those of CW, except that we added the cases in which the dimensionless central potential $W_0 = 5$. Now the choice of $U_t$ will be explained. CW defined four families of models in terms of a combination of parameters which we denote by $F_{cw}$, and which is defined by

$$
F_{cw} = \left( \frac{M}{M_\odot} \right) \left( \frac{R_g}{Kpc} \right) \left( \frac{220 kms^{-1}}{v_g} \right) \left( \frac{1}{\ln (M/\langle m \rangle)} \right).
$$

In the case of family 1 of CW, this value was given as $F_{cw} = 5.0 \times 10^4$. For a tidally truncated cluster with a given structure and mass spectrum, $F_{cw}$ is a measure of the mean relaxation time, and different clusters within this family have different crossing times. In our $N$-body models, the evolution of systems with different crossing times will differ (in principle), and so we must choose a specific model from family 1 of CW for comparison. CW themselves gave as an example a cluster with total mass, $M$, equal to $1.49 \times 10^5 M_\odot$, and took $v_g$, the circular speed around the galaxy, to be $220 kms^{-1}$. If we specify the
slope of the IMF and the maximum and minimum stellar masses, the mean mass \( \langle m \rangle \) is determined. Then, for the stated value of \( F_{cw} \), we obtain the galactocentric distance, \( R_g \), from equation (18), and the unit of time is determined by equation (15). In this way the galactocentric distance, \( R_g \), is calculated as 3.7Kpc for \( \alpha = -1.5 \), 4.0Kpc for \( \alpha = -2.5 \), and 4.1Kpc for \( \alpha = -3.5 \). We list the corresponding unit of time \( U_t \) for each set of parameters in Table 3. Note, however, that our models equally well represent the evolution of clusters within CW’s other 3 families (cf.§4.3), for appropriate values of the various parameters, and provided always that relaxation is negligible. The particle number, \( N \), in our survey was usually \( N = 8192 \).

The lifetime of the cluster is also listed in Table 3 for each set of parameters. The first and second columns are the dimensionless central potential, \( W_0 \), and the slope of the power-law IMF, \( \alpha \). The third column is the unit of time calculated by means of equation (15). In the fourth column, the lifetime of the cluster is given in \( N \)-body units and in years (in brackets). The letter S means that the cluster survived at the end of calculation, which was set at \( T = 2000 \) in \( N \)-body units. We determined this value in relation to the initial half-mass relaxation time \( t_{rh} \sim 700 \) for \( N = 8192 \) and \( \varepsilon = 1/16 \). The fifth column is the lifetime obtained by CW. The letter C means that they found that the cluster experiences core collapse. The last column, which concerns anisotropy, is discussed in §4.5.

### 4.2. Evolution of Total Mass

In this and the following subsections we discuss several aspects of our models in some detail. Figure 1 shows the evolution of the total mass, \( M \), of the cluster. The bold curves indicate the decrease of mass of the cluster defined by those stars within the tidal boundary. The thin curves indicate the decrease of the mass of all stars due to stellar evolution only.

As can be seen from Table 3 and Fig.1, the less concentrated clusters and/or those containing more massive stars are disrupted sooner. When the initial concentration of the cluster is small and it contains relatively many massive stars, it disrupts within about 10 \( \sim \) 20 crossing times. The runs for \( (W_0, \alpha) = (1, -1.5) \) and \( (3, -1.5) \) are representative of this behaviour. In these cases, massive stars evolve immediately and the cluster loses a large fraction of its mass. Since the cluster is not at all concentrated, it is easily disrupted by the tidal field of the galaxy. Details of the evolution of the spatial structure are shown in figure 2, which displays Lagrangian radii for the case \( (W_0, \alpha) = (3, -1.5) \). The number over each curve denotes the fraction (in percent) of the initial total mass.

When the cluster initially has fewer massive stars than in the above cases, we can observe rapid disruption at the final stage of the evolution. \( (W_0, \alpha) = (1, -2.5), (1, -3.5) \) and \( (3, -2.5) \) are examples of this kind of evolution. In these cases, before this happens
the cluster gradually loses its mass by the following two processes. First, the mass loss by stellar evolution weakens the potential well of the cluster and the cluster expands. Then, the stars outside the tidal boundary escape from the cluster. Moreover, the evaporation of stars due to two body relaxation decreases the mass of the cluster, though we have taken care that this has a minor influence on our results (cf. §4.4). After the cluster has reached a certain critical point, it disrupts more-or-less immediately (cf. §5 for a detailed investigation of this final disruption process). In figure 3, the Lagrangian radii are plotted for the case of \((W_0, \alpha) = (3, -2.5)\).

When the cluster is initially concentrated, or does not contain many massive stars, it does not disrupt (within the duration of the simulation). Then the cluster may experience core collapse. In figure 4, the Lagrangian radii are plotted for the case of \((W_0, \alpha) = (3, -3.5)\), which is typical of this behaviour.

4.3. Effect of galactocentric distance

Figure 5 shows the evolution of the total cluster mass for clusters of the same initial mass at four different galactocentric distances, \(R_g\), in the case of \(W_0 = 3\) and \(\alpha = -2.5\). The number of particles is always \(N = 8192\). In the calculations, the different \(R_g\) correspond to different values of the unit of time, \(U_t\), according to equation (15). The numbers in the Figure indicate the galactocentric distance if, as before, we set the mass of the cluster to be \(M = 1.49 \times 10^5 M_\odot\). These four galactocentric distances, 4.0, 10.6, 18.0 and 47.4 kpc, correspond to the families 1, 2, 3 and 4 of CW, respectively. In table 4, the lifetimes of the clusters are summarized.

4.4. Effect of two-body relaxation

We investigated the effect of two-body relaxation by means of calculations with different particle number, \(N\). Figures 6, 7, and 8 show the evolution of the total cluster mass for several different particle numbers in the cases \((W_0, \alpha) = (3, -1.5), (3, -2.5)\) and \((1, -3.5)\), respectively. The calculations were performed with \(N = 1024, 2048, 4096\) and 8192 for all cases and \(N = 16384\) for \((W_0, \alpha) = (3, -2.5)\) and \((1, -3.5)\).

As shown in figure 6, the cluster disrupts in several crossing times in the case of \((W_0, \alpha) = (3, -1.5)\). Different values of the particle number, \(N\), do not significantly affect the result. In this case, the cluster disrupts only through processes occurring on the dynamical and stellar evolution time scales.

As shown in figure 8, the cluster disrupts after approximately 300 crossing times in the case \((W_0, \alpha) = (1, -3.5)\). The result depends greatly on the number of particles \(N\),
until this is rather large. In this case, unless \( N \) is very large the cluster loses mass due to both stellar evolution and evaporation by two-body relaxation. When the cluster loses enough mass and reaches a certain critical structure, it disrupts immediately. Clusters with smaller particle number lose more mass, because the two-body evaporation effect is stronger. Moreover, those clusters with the smallest particle numbers (\( N \leq 4096 \)) would experience core collapse well within the duration of these simulations (if the evolution were followed with an accurate, collisional code), and this would help to prevent tidal disruption. Nevertheless the similarity of the two largest simulations in Fig.8 suggests that relaxation effects are relatively unimportant for them. Assuming that mass loss from evaporation in a given time varies approximately as \( N^{-1} \) (i.e. in proportion to the reciprocal relaxation time), one may estimate that the lifetime to disruption of the model with \( N = 16384 \) differs from the result for very large \( N \) by about 10% only.

As shown in figure 7, the case \((W_0, \alpha) = (3, -2.5)\) is intermediate between the above two cases. The clusters with smallest particle number \((N \leq 2048)\) are substantially affected by two-body relaxation, and when \( N = 1024 \) it survives without disruption. The results with large particle number \((N \geq 8192)\) differ very little from each other. This shows that the decrease of mass is determined almost entirely by stellar mass loss.

4.5. Anisotropy in Velocity Dispersion

Figure 9 shows data on the anisotropy in the velocity distribution of the particles. Here the anisotropy is measured by the parameter

\[
A = 2 - \frac{\langle v^2_t \rangle}{\langle v^2_r \rangle},
\]

where \( \langle v^2_t \rangle \) and \( \langle v^2_r \rangle \) are the mean square tangential and radial velocities, respectively, of particles within the tidal radius. The approximate minimum value of \( A \) throughout the lifetime of each model is given in the last column of Table 3.

The parameter \( A \) vanishes for an isotropic distribution of velocities (as is true initially), and would be positive if there were a preponderance of radial orbits, as happens in the evolution of isolated models driven by two-body relaxation only (cf. Spitzer 1987). In all cases which we have computed, however, there is a tendency for \( A \) to decrease, which implies a deficit of stars on nearly radial orbits. We interpret this as being due to the preferential escape of stars moving on radial orbits. In those clusters which disrupt within the time scale of our simulations, however, the disruption is signalled by a rather sharp rise in \( A \). This presumably arises from the predominantly radial motions of the stars as the cluster finally dissolves.
Figure 10 shows one component of the angular velocity, $\Omega_c$, of the case $W_0 = 3$, $\alpha = -2.5$. This is defined here by

$$\Omega_c = \frac{L_z}{\sum m_i (r_{i,x}^2 + r_{i,y}^2)},$$

(20)

where $L_z$ is the z-component of angular momentum, and $r_x$ and $r_y$ are the $x$- and $y$-coordinates of the $i$th star relative to the density center. The angular velocity is so small that the rotation of the cluster is insignificant dynamically; the corresponding term $V_s$ in the virial relation, eq.(6), is almost always negligible. The initial decrease in $\Omega_c$ may be interpreted as being due to the expansion of the cluster following mass loss within the first few crossing times: the Coriolis term in eq.(1) causes $L_z$ to decrease if the radial motion is predominantly outwards.

5 Mechanism of Disruption

In this section, we investigate the mechanism of disruption of a cluster. As shown in figure 1, the final stages of this process can occur rapidly, which suggests that disruption could be caused by a loss of equilibrium. The abruptness with which disruption occurs is shown even more clearly in Figure 11, which plots the virial ratio, defined by

$$q = -4T/(2V_c + V_s - 4V_g)$$

(21) (cf. eq.(6)) for four of those clusters which exhibit disruption. This figure shows that when rapid disruption occurs the cluster loses virial balance.

When the cluster is in dynamical equilibrium, as in eq.(6) we have

$$\ddot{I} = 4T + 2V_c + V_s - 4V_g = 0.$$  

(22)

If we eliminate $T$ using the total energy of the cluster, $E$, given by $E = T + V_c + V_g$, we obtain the relation:

$$\ddot{I} = 4E - 2V_c + V_s - 8V_g.$$  

(23)

Therefore the equilibrium condition is expressed as

$$4E = 2V_c - V_s + 8V_g.$$  

(24)

To analyse this condition we reexpress $V_c$ and $V_g$ as

$$V_c = -\frac{\mu GM^2}{r_h},$$  

(25)

$$V_g = -\nu \omega^2 M r_h^2,$$

(26)
where $\mu$ and $\nu$ are dimensionless parameters, and $r_h$ is the half mass radius of the cluster. If we substitute equations (25) and (26) into equation (24), this condition becomes

$$4E + V_s = -\frac{2\mu GM^2}{r_h} - 8\nu \omega^2 Mr_h^2.$$  \hspace{1cm} (27)

Finally, by eliminating $\omega$ in favour of the tidal radius $r_t$, the equilibrium condition is expressed as

$$\frac{r_t(4E + V_s)}{2GM^2} = -\frac{\mu}{(r_h/r_t)} - \frac{4\nu}{3} \left(\frac{r_h}{r_t}\right)^2.$$ \hspace{1cm} (28)

When $r_h/r_t$ is small the right-hand side of equation (28) is dominated by the first term, which comes from the self-potential of the cluster, while when $r_h/r_t$ is large it is dominated by the second term, which is due to the potential of the galaxy and the centrifugal force. Therefore, the right-hand side of equation (28), when regarded as a function of $r_h/r_t$, has a maximum value at a particular point, if $\mu$ and $\nu$ are constant (cf. Fig.13). (In practice these parameters are indeed nearly constant, as shown in one case below.) Initially the cluster has a value of $r_h/r_t$ smaller than that at the maximum point, and the value of the function on the left side of eq.(28) is below the maximum value. As the cluster loses mass this quantity increases, as does the ratio $r_h/r_t$, and the cluster moves towards the maximum point along the curve. When the function on the left side of eq.(28) exceeds the maximum value on the stated curve there is no longer a solution on the equilibrium curve. Therefore the cluster loses equilibrium and disrupts.

It is not hard to understand why the ratio $r_h/r_t$ increases, but the reason for this behaviour is not simply the expansion resulting from mass loss. As the cluster loses mass due to stellar evolution, it does indeed expand, and the half mass radius increases. But the particles which move beyond the tidal radius cease to be members, and the tidal radius decreases in proportion to $M^{1/3}$. There is a consequent reduction in $r_h$. However, since the density around the tidal radius is smaller than that around the half mass radius, $r_h$ does not decrease so much. Therefore, the ratio $r_h/r_t$ becomes larger.

We illustrate this loss of equilibrium using numerical data for the case $W_0 = 1$, $\alpha = -3.5$ and $N = 16384$. Figure 12 confirms that the parameters $\mu$ and $\nu$ are almost constant until the cluster loses equilibrium. We plot the equilibrium curve, eq.(28), using constant values $\mu = 0.42$ and $\nu = 0.52$, which gives the bold curve in figure 13. The star-shaped symbols plotted in the figure were obtained from a simulation, for which data are plotted every 100 time units from $t = 0$ to $t = 900$, and every 10 time units after $t = 900$. As the cluster loses mass, the point representing the cluster moves upward and to the right on this curve, while the cluster maintains virial equilibrium. When the dimensionless
energy becomes larger than the maximum value on the equilibrium curve, the cluster loses virial equilibrium, and the disruption is extremely rapid.

6. Discussion

Our $N$-body calculations have been designed to investigate the survival of large star clusters (i.e. those with large $N$, such as globular star clusters) against two important disruptive processes: (i) mass loss resulting from the internal evolution of the stars, and (ii) tidal stripping due to the gravitational field of a parent galaxy. The importance of the size of $N$ stems from the fact that we have suppressed two-body relaxation effects as much as possible, and so our models are only applicable to large star clusters in which relaxation effects are negligible over timescales equivalent to the duration of our simulations. We checked that relaxation is negligible in our models by repeating runs, especially the longest ones, with much larger values of $N$, and also (though this was not discussed in our paper) by checking for mass segregation. Indeed, one way of suppressing relaxation even further in our models would have been to use models with particles of equal mass, each of which changes with time in proportion to the mass of the entire population. Conceptually it was simpler to use a distribution of particle masses and to let each particle lose mass at an appropriate time, but this did have the result of enhancing two-body effects through mass segregation.

Our model of mass loss through stellar evolution is a simplification of what is thought to happen in nature. In particular, we assume that each star abruptly loses a certain fraction of its mass at the end of its internal evolution, the time at which this occurs, and the fraction of mass, being determined uniquely by its initial mass. No attempt was made to model gentler phases of mass loss, via stellar winds, for example. Though the assumptions are a considerable idealisation, they were partly made for consistency with the work of Chernoff & Weinberg (1990), as one aim of our models was to verify their results with a different dynamical model.

As with our treatment of stellar evolution, we have modelled the galactic tide in a very simplified but conventional manner, i.e. as that due to a distant point mass, about which the cluster describes a circular orbit. In particular this neglects any time-dependence in the tide. The time-dependence of the tide is almost certainly of importance for an adequate understanding of the history of the globular clusters which survive to the present day. On the other hand a steady tide illustrates some important aspects of tidal stripping, and we have improved the treatment of Chernoff & Weinberg by including the acceleration of the tide in the equations of motion of all stars, and not simply as a cutoff which demarcates
the bound from the unbound stars. In addition our model of the tide is not spherically symmetric, unlike the treatment of Chernoff & Weinberg.

The evolution of a typical model may be summarized as follows. Mass loss due to stellar evolution decreases the total mass of the cluster. In consequence the tidal radius decreases, and the stars outside the tidal radius are stripped off. The mass loss from stellar evolution also weakens the gravitational potential. One consequence is that the cluster expands, and so some stars overflow the tidal boundary. (Also, the total mass of the cluster decreases as a result of evaporation caused by two-body encounters, but, as already explained, we have checked that this effect is relatively weak.) Though mass loss by both effects (stellar evolution and tidal stripping) always occur simultaneously, as the cluster loses mass the tidal effect becomes progressively stronger relative to the effect of stellar evolution. When the influence of the tide on the virial balance of the cluster becomes sufficiently large relative to its self-gravitation, the whole cluster loses dynamical equilibrium, and disrupts rapidly. In some of our models, however, especially those in which the rate of mass loss due to stellar evolution is small, the cluster does not reach this critical point within the duration of our simulations (which is determined by the onset of two-body relaxation effects). In these cases the ultimate fate of the cluster cannot be determined from our models, though it seems likely that such systems would eventually evolve towards core collapse.

As has been stated, one of the main purposes of our investigation was to verify the results of Chernoff & Weinberg (1990), which have proved such an important and stimulating source of information on the dynamical evolution of tidally bound clusters which lose mass by stellar evolution. Unlike us, they did include two-body relaxation effects. In other respects their models were somewhat more idealised; besides the treatment of the tide, to which we have already referred, their models treated the distribution of stellar velocities as isotropic.

In view of these differences it is interesting to find that our results do not differ qualitatively from those of Chernoff & Weinberg: the models which disrupt tidally are the same in both studies, and the models which survive until core collapse (according to CW) are those which, in our study, avoid tidal disruption to the point at which two-body relaxation may become effective. Where our results differ is in the time to disruption, for those models which disrupt tidally. Our models almost always outlast those of CW, and sometimes by a factor of ten or more. It is impossible to determine which of the differences in the models is responsible for this disagreement. In addition to the aspects discussed above, our models are able to follow processes occurring on a dynamical (crossing) time scale, and so there are several factors which could contribute to this result.
It is clear, however, that the models we have studied are not necessarily the most important for future cluster studies. In particular, the lower limit of the initial mass function is almost certainly too high, and the addition of an appropriate number of low mass stars would enhance the survivability of the clusters. In addition, non-steady tides (e.g. disk-shocking) are an important mechanism, missing in both studies, which acts in the opposite direction. Our results do, however, suggest that a Fokker-Planck model with a spherically symmetric cutoff and an isotropic distribution of velocities can be quantitatively very unreliable.

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Table 1. Stellar evolution time

| Initial Mass $m$ ($\log_{10}[m/M_\odot]$) | Main Sequence Time ($\log_{10}[t_{se}/yr]$) |
|------------------------------------------|------------------------------------------|
| 1.79                                     | 6.50                                     |
| 1.55                                     | 6.57                                     |
| 1.33                                     | 6.76                                     |
| 1.11                                     | 7.02                                     |
| 0.91                                     | 7.33                                     |
| 0.72                                     | 7.68                                     |
| 0.54                                     | 8.11                                     |
| 0.40                                     | 8.50                                     |
| 0.27                                     | 8.90                                     |
| 0.16                                     | 9.28                                     |
| 0.07                                     | 9.63                                     |
| −0.01                                    | 9.93                                     |
| −0.08                                    | 10.18                                    |

Table 2. Mass evolution

| Initial Mass ($M_\odot$) | Remnant Mass ($M_\odot$) | Comments               |
|--------------------------|--------------------------|------------------------|
| $< 4.7$                  | $0.58 + 0.22 \times (m/M_\odot - 1)$ | White dwarf           |
| [4.7,8.0]                | 0.0                      | No remnant             |
| [8.0,15.0]               | 1.4                      | Neutron star           |
Table 3. Results of Survey

| $W_0$ | $\alpha$ | $U_t$ (yr.) | Lifetime CW (yr.) | $A_{min}$ |
|-------|-----------|-------------|-------------------|-----------|
| 1     | $-1.5$    | $2.37 \times 10^6$ | $32(7.6 \times 10^7)$ | $9.2 \times 10^6$ | $-0.9$ |
| 1     | $-2.5$    | $2.56 \times 10^6$ | $108(2.9 \times 10^8)$ | $3.4 \times 10^7$ | $-0.6$ |
| 1     | $-3.5$    | $2.65 \times 10^6$ | $830(2.2 \times 10^9)$ | $2.5 \times 10^9$ | $-0.8$ |
| 3     | $-1.5$    | $1.76 \times 10^6$ | $60(1.1 \times 10^8)$ | $1.4 \times 10^7$ | $-0.5$ |
| 3     | $-2.5$    | $1.90 \times 10^6$ | $480(9.1 \times 10^8)$ | $2.8 \times 10^8$ | $-0.75$ |
| 3     | $-3.5$    | $1.96 \times 10^6$ | $S$ | $C$ | $-0.3$ |
| 5     | $-1.5$    | $1.07 \times 10^6$ | $220(2.4 \times 10^8)$ | $- - - -$ | $-0.75$ |
| 5     | $-2.5$    | $1.16 \times 10^6$ | $S$ | $- - - -$ | $-0.3$ |
| 5     | $-3.5$    | $1.19 \times 10^6$ | $S$ | $- - - -$ | $-0.15$ |
| 7     | $-1.5$    | $5.28 \times 10^5$ | $S$ | $1.0 \times 10^9$ | $-0.35$ |
| 7     | $-2.5$    | $5.71 \times 10^5$ | $S$ | $C$ | $-0.1$ |
| 7     | $-3.5$    | $5.91 \times 10^5$ | $S$ | $C$ | $-0.05$ |

Table 4. Lifetime for different galactocentric distances

| $R_g$ (Kpc) | $U_t$ (yr.) | Lifetime CW (yr.) | CW Family | CW Lifetime |
|-------------|-------------|--------------------|-----------|-------------|
| 4.0         | $1.9 \times 10^6$ | $480(9.1 \times 10^8)$ | 1         | $2.8 \times 10^8$ |
| 10.6        | $5.0 \times 10^6$  | $320(1.6 \times 10^9)$ | 2         | $2.9 \times 10^8$ |
| 18.0        | $8.5 \times 10^6$  | $250(2.1 \times 10^9)$ | 3         | $2.9 \times 10^8$ |
| 47.4        | $2.2 \times 10^7$  | $170(3.8 \times 10^9)$ | 4         | $2.9 \times 10^8$ |
Figure Captions

Fig.1 Evolution of the mass with time. The title of each graph gives the initial dimensionless central potential and the slope of the mass function. The abscissa is time, in standard $N$-body units. The thin line denotes the mass of all stars, and the thick line the mass of those within the tidal radius.

Fig.2 Evolution of Lagrangian radii for the model with initial dimensionless central potential $W_0 = 3$ and mass function slope $\alpha = -1.5$. The thin lines give the radii of spheres, centred on the density centre, containing corresponding fractions (stated as percentages) of the total initial mass of all stars. The thick curve gives the tidal radius, and the abscissa is time in $N$-body units.

Fig.3 As Fig.2 for an intermediate mass function slope.

Fig.4 As Fig.2 for a steep initial mass function.

Fig.5 Evolution of the mass of stars within the tidal radius for fixed initial parameters $W_0$, $\alpha$ and total mass, but for different galactocentric radii. The conversion of the abscissa to astrophysical time units is given in Table 4.

Fig.6 Dependence of the evolution on the number, $N$, of particles used in the simulation, for a case with a relatively flat initial mass function. The ordinate is the mass of stars within the tidal radius.

Fig.7 As Fig.6, but for an intermediate slope of the initial mass function.

Fig.8 As Fig.6, but for a steep mass function.

Fig.9 Total anisotropy of stars within the tidal radius, as a function of time in $N$-body units. The anisotropy parameter $A$ is defined in eq.(19), and the graphs are arranged as in Fig.1.

Fig.10 Spin angular velocity of one case as a function of $N$-body time. The angular velocity $\Omega_c$ is defined in terms of the component of the spin angular momentum orthogonal to the plane of motion of the cluster about the galaxy.

Fig.11 The virial ratio, defined by eq.(21), for several representative cases, plotted against $N$-body time. Each case is labelled by initial values of the parameters $W_0$ and $\alpha$.

Fig.12 Form factors $\mu$ and $\nu$ for the contributions to the potential energy from the gravitational field of the cluster and galaxy, respectively. They are defined in terms of the potential energies, mass and half-mass radius by eqs.(25) and (26).

Fig.13 Theoretical interpretation of tidal disruption. Definitions: $r_t$, tidal radius; $E$, total energy of the cluster; $V_s$, contribution of spin angular momentum to the virial balance (cf. eq.(7)); $M$, total mass within $r_t$; $r_h$, half-mass radius. The thick line plots eq.(28) (a version of the equation of virial balance) for the stated values of the form factors $\mu$ and
\( \nu \), and the thin lines sketch the two contributions (due to the cluster and the tide) to the right side of eq.(28). The stars denote the evolution of a model with stated parameters \( W_0 \) and \( \alpha \), some of which are labelled with \( N \)-body time.