Measurement-induced phase transition: A case study in the non-integrable model by density-matrix renormalization group calculations

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We study the effects of local projective measurements on the quantum quench dynamics. As a concrete example, one-dimensional Bose-Hubbard model is simulated by using matrix product state and time evolving block decimation. We map out a global phase diagram in terms of the measurement rate in spatial space and time domain, which demonstrates a volume-to-area law entanglement phase transition. When the measurement rates reach the critical values, we observe a logarithmic growth of entanglement entropy as the sub-system size or evolved time increases. This is akin to the character in the many-body localization, implying a general picture of the dynamical transitions separating quantum systems with different entanglement features. Moreover, we find that the probability distribution of the single-site entanglement entropy distinguishes the volume and area law phases just as the case of disorder-induced many-body localization. This suggests that the different type entanglement phase transitions may be understood as a nonlocalized-to-localized transition. We also investigate the scaling behavior of entanglement entropy and mutual information between two separated intervals, which is indicative of a single universality class and thus suggests a possible unified description of this transition.

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I. INTRODUCTION

Quantum entanglement is an invaluable tool to access intrinsic nature of the underlying states and their non-equilibrium properties in quantum physics \cite{1-3}. For example, typical excited eigenstates exhibit a volume-law scaling, i.e. the entanglement entropy of the reduced state of a sub-system grows as its volume. While for most of gapped ground states, an area-law of entanglement entropy emerges (entropy is proportional to the surface area of the sub-system). The scaling behavior of entanglement entropy could vary in out-of-equilibrium driving \cite{4-17}. If the thermalization is triggered by a global unitary quench with an interacting post-quench Hamiltonian, the volume-law entropy will replace the area-law behavior \cite{4, 6, 10}. But many factors may affect the non-equilibrium dynamics, for instance, randomness may lead to the many-body localization (MBL) \cite{18-32}. Due to avoiding thermalization in the MBL, the stationary state (quench steady state) exhibits area-law entropy for the short-entangled systems, and the entanglement entropy grows logarithmically in time, which is in contrast with the linear growth in thermalized case.

Except for the disorder, the presence of non-unitary operations (e.g. relaxation, dissipation, measurements) will also influence the entanglement dramatically. One notable example is, by introducing a continuous monitoring (damping) term in quench dynamics of free Fermion chain, it is found that the volume law entanglement is unstable under arbitrary weak damping if it is applied everywhere \cite{33}. This is similar to the quantum Zeno effect \cite{34}, where the total system is measured continuously and localized near a trivial product state.

Interestingly, when the measurement rate is finite, one may expect that the local measurements destroy the volume law produced by pure unitary dynamics, especially in low-dimensional systems. However, this is not the case in generic interacting systems, where more subtle entanglement structures could survive under measurements. A stable volume law phase was found in several numerics \cite{35-40}. By simulating hundreds qubits in Clifford circuits, it is found a continuous quantum dynamical phase transition from volume law to area law entanglement by continuously tuning the measurement rate \cite{36, 37}. The presence of the volume-to-area law transition is also identified in Floquet circuits \cite{35}, and can be understood by a classical percolation problem. Furthermore, it has been argued that the presence of the volume-to-area law transition can be reinterpreted in a quantum error correction point of view \cite{39}. The authors of Ref. \cite{39} considered the influence of two parameters – depth and fraction – on entanglement dynamics rather than consider only the measurement rate (density of non-unitaries in whole dynamics, which is equivalent to the fraction in case of depth is one). On the other hand, it should be noticed that the stable volume law phase does not exist in all models. Ref. \cite{41} reported that the entanglement in non-interacting Bell pair model, exhibits area law for arbitrary measurement rate. However, they also showed that in more generic models, an volume-to-area law entanglement transition can exist, i.e. the volume law exhibits under rare local measurements. They argued that the volume law entanglement is stable only if it features a subleading correction term.

Taken the above facts, another question immediately arises: What is the nature of this measurement-induced entanglement phase transition? In Ref. \cite{35}, by mapping the calculation of zeroth Rényi entropy to a classical percolation problem, a toy model describing the disentan-
lement process for unitary dynamics with measurements was provided. The scale invariance in the critical percolation system exhibits a logarithmic growth of entanglement, and also lead to power-law decay correlations. It is also found that at the critical point [37], the von Neumann entropy (the first Rényi entropy) grows logarithmically in time, and the mutual information exhibits power-law decay in space, thus an underlying conformal field theory (CFT) description was proposed. Moreover, they showed that the peak of mutual information can be an indicator of the volume-to-area law phase transition [37]. Please note that, the above discussions are mainly based on the circuits model [35, 37], and non-interacting models [37]. Besides, much less is known about the universality (if any) of the entanglement entropy of more generic models, e.g. quantum many-body lattice systems and non-integrable models, or of models mappable to them.

In this paper, we study the quantum dynamics of one-dimensional Bose-Hubbard model in presence of random projective measurements by using matrix product state (MPS) and time evolving block decimation (TEBD). The Bose-Hubbard model has been a paradigmatic non-integrable model to understand the quantum dynamics and non-equilibrium properties. We map out a global phase diagram controlled by the measurement rate in spatial space and time domain (see Fig. 1 for definition). A volume-to-area law phase transition is observed. We find that the single-site entanglement entropy can indicate the transition, this suggests that the observed entanglement transition may be understood as a nonlocalized-to-localized transition. At the critical point, we obtain a logarithmic growth of entanglement and power-law decay of correlations. The scaling behavior of entropy around the critical point appears to belong to a single universality class. Our work provides a wealth of evidences that non-unitary factors, such as projective measurements, can induce a dynamical phase transition, adding more pieces of message to the recently proposed theoretical scenario [35, 37, 39], from the microscopic view on non-integrable quantum lattice model.

II. MODEL AND METHOD

In present work, we consider a quench dynamics of the von Neumann entropy on an one-dimensional Bose-Hubbard model [42] as the post-quench Hamiltonian

$$H = -\sum_{i}(b_{i}^\dagger b_{i+1} + b_{i+1}^\dagger b_{i}) + \frac{U}{2} \sum_{i} n_{i}(n_{i} - 1)$$

(1)

where $b_{i}^\dagger, b_{i}$, and $n_{i} = b_{i}^\dagger b_{i}$ are the boson creation, annihilation, and particle number operators on site $i$ respectively. The model has a critical point $U_c \sim 3.3$ [43–52], which separates a superfluid phase in $U < U_c$ from a Mott insulator phase in $U > U_c$. Our setup for the unitary time evolution background is a quench dynamics from Mott phase into to superfluid phase. Without losing the generality, the strength of onsite interaction $U$ in post-quench Hamiltonian is chosen to be 0.14 in our simulation, and the initial state is chosen to be a trivial product state with occupation number on each site $|n_i⟩ = 1$. The maximum number of bosons per site is set to be 5.

The quench dynamics is simulated using MPS and TEBD [53–57] performing in TeNPy package [58], a network diagrammatic representation is shown in Fig. 1(a). The unitary time evolution background is built by several layers of the matrix product operator (MPO), describing the time evolution operator. By performing TEBD, each unitary layer is written in terms of two-site gates by using a second order Suzuki-Trotter decomposition with $dt = 0.02$. A bond dimension $\chi$ up to 2048 was tested to be fine for the time scale considered in present work. In our calculation, an open boundary chain with total system size $L_0 = 36$ is considered. We also test some calculations on $L_0 = 48$ size and get the very similar behavior, which gives us confidence the results shown below is free of the finite-size effect.

The local projective measurements are set to be applied randomly in space and time. The local projection operator is defined by $P = |1_x⟩⟨1_x|$, which projects the quantum state at position $x$ to $|1⟩$. In time domain, we set the number of measured unitary layers per 50 layers to be a fixed value $n_t$, and define the measurement rate in time $P_t = n_t/50$. For each measured layer, we define the probability of local projective measurements applied a single site to be the measurement rate in space.
$P_x$. After each local measurement, the total state has been renormalized, therefore the full dynamics is nonlinear. For smaller measurement rates we have simulated more random realizations, like for $P_x = 0.02$, $P_t = 1$ we simulate 900 random realizations, since the effect of single local projective measurement is larger. For larger measurement rates, the effect of single local projective measurement is much smaller, so we do not need many random realizations to obtain the smooth averaged curve with small standard error, for example, for $P_x = 0.08$, $P_t = 1$ we only simulate 40 random realizations.

We note that the two parameters $P_x$ and $P_t$ considered in our simulations are independent and not simply the same as the measurement rate $P$ considered in Ref. [35–38, 40], since we have set up that the measurements only applied after some of unitary layers. To see the difference, just need to consider the case of a finite small measurement rate in time, for example $P_t = 0.1$, where the local projective measurements applied only after 5 unitary layers in per 50 unitary layers. In this case, at least in 9 continuous unitary layers, which corresponds to the physical time $dt = 0.18$, does not exist any local projective measurement, i.e. the $P_t$ considered in our simulation controls the degree of information spreading effectively, just as the “depth” defined in Ref. [39] but with randomness.

III. RESULTS AND DISCUSSIONS

By examining the von Neumann entropy, we observe the competing tendencies between the local projective measurements and unitary time evolution background, which suggest a phase transition between entangling and disentangling phases. A two-dimensional phase diagram of von Neumann entropy consisting with the measurement rate in time $P_t$ and space $P_x$ is presented in Fig. 1(b). In the case of $P_t = 1$, $P_x$ is equivalent to the measurement rate considered in previous works [35–38, 40]. In this case, the region of volume law phase in our model is very narrow. Due to this, one may question about the stability of the strong entangling phase in combination of unitary evolution and local projective measurements. We note that our numerical results for different values of $P_t$ show that, with decreasing $P_t$ the critical value of $P_x$ exhibits a growth much faster than linear. Therefore, the competing tendencies between density of the local projective measurements applied and the degree of information scrambling suggest a stable volume law phase when $P_t$ is finite small.

The existence of two phases in the designed dynamical process is more clear in Fig. 2(a-c), where we show the time evolution of the entropy for various values of $P_x$. For the smaller measurement rate (Fig. 2(a)), the entanglement entropy increases ballistically at initial times and then saturates at a large value. The entropy in the quasi-stationary regime at long times is close to the volume-law values in the unitary evolution. For larger measurement rate (Fig. 2(c)), the entropy saturates quickly to very small values, corresponding to an area law phase or localized phase.

Besides the long-time behaviors exhibit a phase transition between volume law and area law phases, the short-time behaviors also display interesting features. We first note an interesting fact observed in our calculation. For pure unitary quench, the finite-size effect will lead to the dip in the entropy at long times [47]. In our simulation with local measurements, we find that there is no dip in the averaged entropy at long times. This can be understood in a quasi-particle picture. Consider a quasi-particle at the center of the chain moving to the boundary, where the quasi-particle will reflect back to the center. Due to this, in pure unitary dynamics, there are dips appearing at long times periodically. In the case of local measurement applied, the quasi-particles cannot propagate across the measured position, so the reflecting is destroyed and also the dips of entropy. This allows us to investigate the short-time entanglement structures. Now, let us discuss the short-time behaviors of the states. In the area law phase, the state is close to the initial trivial product state due to the known quantum Zeno effect, therefore the entropy is very small and almost unchanged in time. In volume law phase, the entanglement growth exhibits a linear dependence (with possible logarithmic correlations) with time like in the case of pure unitary evolution, and the growth velocity decreases with increasing the measurement rate. At the critical point, the linear growth is
totally destroyed by the local measurements, the entanglement exhibits a logarithmic growth in time as shown in Ref. [35] for zeroth Rényi entropy. The logarithmic scaling of the entanglement entropy with time was known to be the characteristic of MBL. This key commonality between MBL and the measurement-induced volume-to-area law phase transition in our model implies a general picture of the entanglement transition separating quantum systems with different entanglement features. One can consider that MBL system with impurities, which lead to the localization, can be mapped onto a random tensor network [59]. In our case, where the local projective measurements disentangle the local degrees of freedom, the system can be mapped onto a random tensor network where the bond dimension of each tensor is random. Therefore, it is possible to consider the general picture of two transitions can be described by the random tensor networks as in Ref. [59]. Also due to this reason, we expect an emergent CFT, and corresponding logarithmic entanglement and power-law correlations. We also note that in Ref. [59] it was shown that the percolation fixed point is unstable, so that the critical exponents may be different in different models.

The entanglement growth in space (sub-system size) also indicates a phase transition. In Fig. 3(a), the averaged von Neumann entropy of long-time steady states for different \( P_x \), with fixed \( P_t = 1 \), is presented. The growth velocity of the von Neumann entropy in space decreases as a result of increasing measurement rate. As shown in Fig. 3(b), at the critical point, a logarithmic dependence of the entropy on the sub-system size, as an universal property of entanglement phase transition, is observed. Our numerical results of the spacial distribution of the entropy suggests the scaling behavior introduced in Ref. [37], where a logarithmic correction is added onto the linear dependence in volume law phase. We note that a strong volume law entanglement is also observed in unitary dynamics with weak disorders [28]. But the general behavior of the dynamics with disorders is still not determinate, one can expect the possible logarithmic corrections of volume law entanglement in unitary time evolution with disorders in the post quench Hamiltonian, since we expect a general picture of entanglement phase transition.

We now turn to consider global scaling behavior of entanglement. In order to extract critical behavior around the critical measurement rate \( P_{x,c} \), we perform a finite-size scaling form for the von Neumann entropy [35, 37, 39, 59]

\[
S(P_x) - S(P_{x,c}) = F((P_x - P_{x,c})L^{1/\nu})
\]

where \( L \) is the sub-system size. The data collapse yields the critical value \( P_{x,c} \approx 0.060 \pm 0.004 \) and the exponent \( \nu \approx 2.00 \pm 0.15 \). The fine data collapse presented in Fig. 4(b) show the correctness of the scaling form, and also strongly support the universal phase transition. Interestingly, the obtained scaling index \( \nu \approx 2.00 \) in our calculation is close to the dynamics of random unitaries in one dimension [35]. The emergence of such a consistency implies that the randomness of local measurements and unitary gates may have similar origin.

Here we present more commons between the volume-to-area law transition and MBL, which suggest a possible generic picture of entanglement phase transition. In Fig. 5 we plot the probability distribution of the single-site von Neumann entropy for different \( P_x \) with fixed \( P_t = 1 \). We find that the single-site entanglement entropy raises signature of the transition just like in the case of

\[ P_x = \begin{align*}
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08, \\
0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08,
\end{align*} \]
the thermal-to-MBL transition [60]. In the volume law phase, as shown in Fig. 5(a), the single-site entropy distribution hosts two dominate peaks. One is located at $S = 0$ and the other one is at $S \approx 1.25$. With increasing the measurement rate, the peak centered at $S > 0$ becomes broadened, and the $S = 0$ peak becomes sharper. For the case of the measurement rate near the critical value, as the curves for $P_x = 0.05$ and 0.055 shown in Fig. 5(b), the $S > 0$ peak is very small. In the area law phase, the $S > 0$ peak totally disappears. We point out that the above discussed behavior of the single-site entanglement entropy is the consequence of a nonlocal-to-local transition. Consider a pure unitary dynamics without any impurities, which will lead the state to be localized. In this case, the state goes to thermalization, and the information spreads into the whole space, so that the single-site entanglement for each site is strong, only one peak appears in the right side. Another limit is a dynamics with strong impurities, each site is almost localized, therefore there is only a peak at $S = 0$. This suggests that the different type entanglement phase transitions, including measurement-induced volume-to-area law and disorder-induced thermal-to-MBL transitions, can be understood as a nonlocalized-to-localized phase transition. We also note that although the single-site entanglement distribution indicates a transition from non-local to local, we still do not know the volume law phase is thermalized or not (follows eigenstate thermalization hypothesis or not), since the single-site reduced density matrix does not give us such information.

In the general picture of the entanglement phase transition, one expects the scale invariance of the entanglement feature at the critical point [35, 59]. Besides the logarithmic growth entanglement as we discussed above, the scale invariance also leads to a power-law decay of correlations in space. To check the expected scale invariance, we investigate the mutual information between two distant sites $A$ and $B$

$$I_{A,B} = S_A + S_B - S_{A\cup B} \quad (3)$$

to test the power-law correlations. From the percolation model, one expects that the critical exponent of power-law decay correlation is $\Delta = 2$ [35], which leads to $I_{A,B} \propto r^{-\Delta}$. A compassion between different values of measurement rate $P_x$ is presented in Fig. 6(a). Note first that, the mutual information decays slowest at the critical point. When $P_x$ is large (after transition), the quantum Zeno effect plays a role, and the correlation decays exponentially since the long-time steady state is close to a product state. When $P_x$ is small (before transition), the unitary time evolution leads system to thermalization, where the local information vanishes, so that results a fast decay of correlations. At the critical point, the mutual information between two sites $I_{A,B}$ with long distance almost unchange with increasing the distance, and $I_{A,B}$ for critical value of $P_x$ exhibits a power-law decay. As shown in Fig. 6(b), the linear fitting in form $\ln I_{A,B} = a \ln r + b$ results a critical exponent $\Delta = -a/2 \approx 1.29$ for total system size $L_0 = 36$. On larger system size ($L_0 = 48$), it is found this value enhances to $\Delta \approx 1.56$. As discussed above, the obtained critical exponent from percolation model is $\Delta = 2$ [35]. Through this comparison, our result seems non-universal, which can be attributed to the following reasons. First, in the case of the von Neumann entropy, the problem cannot map onto a percolation model. Second, different from the circuits model, the current calculations are based on a non-integrable quantum lattice model where the correlation effects are much stronger. Third, our calculations are limit to finite sizes, and the non-universal behavior is a finite-size effect.

At last, we stress that the critical behaviors, including the power-law decay correlation and logarithmic growth entanglement, support the presence of a scale invariance around the criticality. It implies a possible unified de-
scription of the observed volume-to-area law phase transition, as proposed in Ref. [35, 37].

IV. SUMMARY AND OUTLOOKS

In this paper we have studied the entanglement dynamics of 1d Bose-Hubbard model with local projective measurements randomly appearing in space and time. A volume-to-area law entanglement phase transition was observed. By considering two parameters – measurement rate in space $P_x$ and time $P_t$, a two-dimensional phase diagram was presented. It is found that the volume law phase is robust with local measurements applied. Finite-size scaling analysis indicates the transition falls into a single universality class, despite that the critical exponent $\nu \approx 2$ is close to the random unitary circuits [35, 37].

For the critical values of measurement rate, a logarithmic growth of entanglement entropy on sub-system size and evolved time was observed. Moreover, the mutual information between two distinct sites was found to exhibit a power-law decay in space. Both the logarithmic growth entanglement entropy and the power-law decay correlations support the presence of a scale invariant quench steady state. Based on this, our results support the possibility to describe the observed volume-to-area law phase transition by a conformal field theory [35, 37].

The similar entanglement features of the measurement-induced phase transition and MBL implies a possible generic picture of entanglement phase transition. In MBL, the system cannot be thermalized. It would be interesting to explore a possible relation between random local projective measurements and thermalization. In MBL, a dephasing type damping will lead to the system thermalized [61]. A similar mechanism may works for the volume-to-area law phase transition induced by measurements. Or just consider the unitary time evolution background to be MBL, one expected the local projective measurements would “speed up” thermalization as a dephasing-type damping term.

In the case of MBL, the randomness is not necessary, a quasi-periodic potential [21] also can lead to failure of thermalization. In the case of measurement-induced entanglement transition, the role of randomness is still unclear. In Ref. [37], it was found that the randomness of the unitary gates and the local measurements is inessential by simulating Clifford circuits. But the numerics beyond Clifford circuits in Ref.[35] show that the randomness of the unitary gates leads to a different critical exponent, which means a different universality. We leave the discussion of the possible non-randomness induced entanglement transition for future work.

It would be also interesting to consider a quench dynamics from different initial conditions. For example, Gullans and Huse [62] considered a dynamical purification phase transition by replacing the initial states from pure states to mixed states. The numerics for Clifford circuits showed that the bipartite mutual information for mixed states, which reduces to entanglement entropy for pure states, grows at most logarithmically in time. This is very different from the case of initial pure states, where the entanglement entropy grows linearly in time.

Note added.—Before finalizing this manuscript, we noticed two more papers arXiv:1908.04305 and arXiv:1908.08051. Both of them provide theoretical understanding of the transition by mapping onto a percolation problem within a replica method.

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