Stochastic Geometry Modeling for Uplink Cellular V2X Communication

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Abstract—To overcome the limitations of Dedicated Short Range Communications (DSRC) with short range, non-supportability of high density networks, unreliable broadcast services, signal congestion and connectivity disruptions, vehicle-to-everything (V2X) communication networks, standardized in 3rd Generation Partnership Project (3GPP) Release 14, have been recently introduced to cover broader vehicular communication scenarios including vehicle-to-vehicle (V2V), vehicle-to-pedestrian (V2P) and vehicle-to-infrastructure/network (V2I/N). Motivated by the stringent connection reliability and coverage requirements in V2X, this paper presents the first comprehensive and tractable analytical framework for the uplink performance of cellular V2X networks, where the vehicles can deliver their information via V2N/cellular network (vehicle-to-base station, V2B communication link) or directly between vehicles in the sidelink, based on their distances, propagation environments and the bias factor. By practically modeling the vehicles on the roads using the doubly stochastic Cox process and the BSs, we derive new association probabilities of V2B and V2V links, new success probabilities of the V2B and V2V communications, and overall success probability of the V2X communication, which are validated by the simulations results. Our results reveal the benefits of V2X communication compared to V2V communication in terms of success probability.

Index Terms—V2X communication, 5G, stochastic geometry, V2V, V2I, V2N, V2P, uplink cellular networks.

I. INTRODUCTION

VEHICLE-to-everything (V2X) communications based on cellular infrastructure have been defined by the Third Generation Partnership Project (3GPP) group [1] to cover broader spectrum of communication scenarios, such as vehicle-to-vehicle (V2V), vehicle-to-pedestrian (V2P) and vehicle-to-infrastructure/networks (V2I/N). It is regarded as a promising technology to support various novel applications such as road safety, infotainment services, traffic management, traffic optimization and online services to car manufacturers. Specifically, this innovation promises to eliminate 80% of the current road accidents and help in fostering auto-mobile and telecommunication industries for a smarter and safer ground transportation system [2].

Existing V2V communication can be supported via the DSRC standard, however, it has certain limitations such as short range, unable to support high density of networks and has unreliable broadcast services [3]. The underlying carrier sensing multiple access (CSMA) medium access control (MAC) protocol also exhibits signal congestion and connectivity disruptions due to rapid changing network topology and ad-hoc vehicular networks [4]. More importantly, single DSRC technology can not support a variety of incoming vehicular oriented applications, and only V2V and V2I communication is achievable using DSRC without the provisioning of V2P, infotainment and traffic management services.

To augment DSRC communication, V2X communication was proposed to meet V2X capacity, latency and coverage requirements [1], [3], [5] and [6] via operating in the following three scenarios as shown in Fig. 1. In V2X Scenario I, V2V, V2P or V2I messages are exchanged directly between vehicles without involvement of cellular nodes. In V2X Scenario II, V2V and V2P messages are first transmitted to the cellular node (eNB or E-UTRAN denoted as CE) in uplink, and then being forwarded to multiple vehicles at a local area in the downlink by cellular node. For V2I and V2N communications, the vehicle can communicate with the E-UTRAN type RSU and application server, respectively. In V2X scenario III (A), the vehicle first transmits the V2X message to UE type RSU which is further relayed to multiple vehicles through cellular node. In V2X scenario III (B), the V2X message is received by cellular node in the uplink and then transmitted to UE type RSU on downlink for further relaying it to multiple...
vehicles. In all three scenarios, the vehicular communication can coexist with the cellular carriers called shared (S) or using dedicated carriers referred as dedicated channels (D). To model and analyze these V2X scenarios, stochastic geometry has been proposed, considering that it has been utilized as a powerful tool to model and analyze mutual interference between transceivers in the wireless networks, such as conventional cellular networks [7], [8], wireless sensor networks [9], cognitive radio networks [10], [11], and heterogeneous cellular networks [12]–[14].

The initial studies on vehicular communication have focused on modeling the V2V communication (i.e. without involvement of the cellular nodes) using stochastic geometry [15]–[22], where simple spatial models with a single road, a multi-lane road, or orthogonal roads were considered. The works in [23]–[28] accounted for the randomness of roads distributions. In [23], the nodes in the WiFi mesh networks were modeled by a Cox process on a Poisson-Line tessellation (PLT), and the nodes on each line are modeled by an inhomogeneous 1D PPP, where the probability density function of the shorted Euclidean distance between two inter-nodes was derived. Later on in [24], the Cox process on a PLT was generalized to a Poisson-Line tessellation (PLT), Poisson-Voronoi tessellation (PVT), or a Poisson-Delaunay tessellation (PDT), and the nodes on each line are modeled by a homogeneous 1D PPP. Their results have shown that PLT often gains preference over PVT and PDT in modeling road systems [1] due to its analytical tractability. In [25], the uplink coverage probability was derived for a network where the typical receiver is randomly chosen from a PPP, and the locations of transmitter mobile users alongside roads are modeled as a Cox process on a Poisson line process (PLP). In [28], the coverage probability of the V2V communication was derived, where the transmitters and receivers were modeled using independent Cox processes on the same PLP (i.e. a doubly-stochastic spatial model), and it captures the irregularity in the spatial layout of roads via the PLP model, and the distribution of vehicles on each road via the 1D PPP model.

Note that [23]–[28] are limited to the V2V communication or V2I communication. The first performance characterization of the V2X downlink communication was studied in the master thesis in [30], where the association probabilities and the coverage probabilities for the V2V and the base station to vehicle downlink communications were derived for the maximum power based association scheme and the threshold distance based association scheme. Recently, [31] has performed downlink coverage analysis of cellular network leveraging vehicles where authors have derived the distance of typical receiver vehicle at center from nearest base station or vehicle. Further, they derived downlink association and the coverage probabilities of the typical vehicle in terms of integral formulas to characterize the downlink performance of vehicular communication. However, in these works, uplink performance or association probabilities have not been studied. Moreover, authors have not suggested any mechanism during association process to control vehicular communication over cellular network to limit interference on cellular users.

The present paper can be seen as an extension to downlink communication work presented in [30], [31]. In this paper, we focus on the uplink communication of V2X networks driven by the stringent high reliability requirements for safety, traffic management and infotainment applications where the locations of vehicles are modeled as a Cox process on a Poison Line Process (PLP), and the cellular BSs are deployed as 2D PPP. A flexible V2X uplink selection scheme is proposed to control the amount of vehicular interference and the number of vehicles operating on the cellular band which has not been previously presented in the existing literature. Our contributions can be summarized as follows:

- We present a comprehensive and tractable analytical framework for analyzing the uplink communication of cellular V2X networks, where the vehicles decide to transmit to other vehicle or cellular BS via shared carriers depending on their corresponding distances, propagation environments and association bias.
- Based on the proposed V2X link selection scheme, we derive the shortest distances and the association probabilities of the vehicles using the V2V link or the V2B link via the shared cellular carriers, respectively.
- We derive the analytical expressions for the success probabilities of the V2V communication and the V2B communication, and the overall success probability of typical vehicle (i.e., V2X communication) using the uplink cellular resources, which are validated by Monte Carlo simulation.
- In V2X network with low and medium vehicle intensity on the roads, there is almost equal probability of transmitting via the V2V and the V2B link. Moreover, cellular based V2V link success probability increases at faster rate with the increase of vehicle nodes, and the success probability of the V2X communication improves with increasing the road intensity.
- Our results have shown that the success probability of the V2X communication is much higher than that of the V2V communication, as both cellular based V2V and V2B links supplement each other to achieve higher success probability of V2X communication.

The rest of the paper is organized as follows. The mathematical preliminary, system model along with assumptions, and the methodology of analysis are described in Section II and III, respectively. Section IV presents the analysis of association probabilities of V2X communication. The success probability is analyzed in Section V. Section VI presents and discusses the numerical and simulation results. The paper is concluded in Section VII. A list of the key mathematical notations used in this paper is given in Table I.

II. PRELIMINARY: POISSON LINE PROCESS

The V2X networks exhibit unique spatial characteristics due to the fact that vehicles are only driven on roadways, which are predominantly linear in nature and layout of the roads is often irregular, which makes it possible to model the road system as a realization of a line process [23]–[25], [28].

1 It has also been used in other related applications, such as in modeling the effect of blockages in localization networks [20].
Therefore, we model the roadways as a network of lines that are distributed on the plane according to a Poisson Line Process (PLP). In this section, we provide a brief introduction of PLP, the detailed information of the underlying theory can be found in [28], [30], [33].

A Poisson line process is a random collection of lines in a 2D plane. Any undirected line \( L \) in \( \mathbb{R}^2 \) can be uniquely characterized by its perpendicular distance \( y \) from the origin \( O(0,0) \) and the angle \( \theta \) subtended by the perpendicular dropped onto the line from the origin with respect to the positive x-axis in counter clockwise direction, as shown in Fig. 2. The pair of parameters \( \theta \) and \( y \) can be represented as the coordinates of a point on the cylindrical surface \( C = [0, 2\pi) \times [0, \infty) \) as illustrated in Fig. 2. Clearly, there is a one-to-one correspondence between the lines in \( \mathbb{R}^2 \) and points on the cylindrical surface \( C \). Thus, a random collection of lines can be constructed from a set of points on \( C \). In other words, the set of points generated by a PPP with certain density on \( C \) correspond to the PLP with the same density for lines on \( \mathbb{R}^2 \).

For a PLP \( \phi_R \) with the intensity \( \lambda_R \) within a circular region \( B(0, R) \), where radius \( R \in \mathbb{R} \), the corresponding points are independent and uniformly distributed in representation space \( C = [-R, R] \times [0, \pi) \) with a surface area of \( 2\pi R \). Thus, the expected number of points in the PPP that lie in \( C \) is \( 2\pi \lambda_R R^2 \), and the number of lines intersecting a disc of radius \( R \) is a Poisson distributed with mean \( 2\pi \lambda_R R^2 \). In PLP, the values of \( \theta \) and \( y \) of each line follow a uniform distribution over an appropriate range defined by \( C \). In this work, we limit ourselves to motion-invariant PLP, where the line process is invariant to the rotation of axes to the origin, for analytical simplicity [28]. The PLP is also considered to be stationary, where translated line process \( T\phi_R = \{T(L_1), T(L_2), \ldots\} \) of PLP, \( \phi_R = \{L_1, L_2, \ldots\} \) has the same distribution of lines as that of \( \phi_R \) for any translation \( T \) in the plane [28].

### III. System Model

In this work, we consider a cellular V2X networks with coexistence of V2V, V2P and V2I/N communications as per scenario I and II defined by 3GPP Release 14 and shown in Fig. 1. In this scenario, V2V, V2P and V2I/N messages are exchanged in uplink and downlink for the applications like vehicular safety. The system model are described in detail in the following subsections.

#### A. V2X Vehicular Nodes

As mentioned in Section II, we model the roads as motion-invariant PLP \( \phi_R \) with line intensity, \( \lambda_R \) as per details given in [28], [30], [31], thus the intensity of equivalent Poisson Point Process (PPP) on the representation space \( C \) is \( \lambda_R \). The V2X vehicular nodes are randomly distributed on each road as homogeneous 1D PPP with intensity \( \mu_v \). In this paper, we only distribute the V2X vehicular nodes to derive the basic analytical framework which can be further extended by including pedestrian and RSUs by distributing them according to independent PPP’s of intensities \( \mu_p \) and \( \mu_r \), receptively as discussed by [28], [30]. However, the pedestrian near roadside and RSUs will have channel conditions different from vehicular nodes due to varying antenna heights.

Assuming that each vehicle transmits independently with a probability \( p \), the locations of transmitting vehicles on each road is then given by a thinned PPP with intensity, \( \mu_t = p \mu_v \), and we denote the set of locations of the transmitting vehicles on a line \( L \) by \( W_L \). Correspondingly, the distribution of receiving vehicles on each line is also a thinned PPP with intensity, \( \mu_r = (1-p)\mu_v \). In other words, the transmitting and receiving vehicles are modeled as the doubly stochastic processes called Cox processes, \( \phi_t \) and \( \phi_r \), which are driven by the same PLP, \( \phi_R \).

For analytical simplicity, we can translate the origin \( O = (0,0) \) to the location of the typical receiver vehicle. The translated point process \( \phi_{R_0} \) can be treated as the superposition of the point process \( \phi_r \), an independent 1D PPP with intensity \( \mu_r \) on a line passing through the origin and a vehicle at the origin \( O \). This can be realized according to the following steps defined by [28]. We first add a point at the origin to the PPP in the representation space \( C \) by applying the Slivnyak’s theorem [33], thereby obtaining a PLP \( \phi_{R_0} = \phi_R \cup L_0 \) with a line \( L_0 \) passing through the origin in \( \mathbb{R}^2 \) and second, we add a point at the origin to the 1D-PPP on the line \( L_0 \) passing through the origin in \( \mathbb{R}^2 \) by applying the Slivnyak’s theorem [33]. The line, \( L_0 \) passing through the origin is referred as typical line in this paper. Since, both \( \phi_t \) and \( \phi_r \) are driven by the same line process, the translated point process \( \phi_{R_0} \) is also the superposition of \( \phi_t \) and an independent PPP with intensity \( \mu_t \).
TABLE I
NOTATIONS

| Notations | Definition |
|-----------|------------|
| $\lambda_R$ | Intensity of roads, 2D PLP |
| $\mu_v$ | Intensity of V2X nodes, 1D PPP |
| $\lambda_b$ | Intensity of base-stations, 2D PPP |
| $\phi_v$ | Poison Line Process (PLP) for roads |
| $\phi_b$ | Cox process for transmitting nodes |
| $\phi_r$ | Cox process for receiving nodes |
| $\phi_b$ | 2D PPP for cellular base-stations |
| $B$ | Association bias, 0 to $\infty$ |
| $P_b$ | Base-station transmit power |
| $P_v$ | Vehicle transmit power |
| $\alpha_v$ | Path loss exponent for V2V link |
| $\alpha_b$ | Path loss exponent for V2B link |
| $r_0$ | Perpendicular distance of road from origin |
| $\theta$ | Angle of road from x-axis |
| $BW$ | V2X communication channel bandwidth, 10 MHz |
| $V2B$ | Vehicle to base-station uplink |
| $V2V$ | Vehicle to vehicle uplink |
| $r_v$ | V2V distance |
| $r_{ub}$ | V2B uplink distance |
| $r_{vb}$ | V2V downlink distance |
| $\sigma^2$ | Thermal noise |

D. Channel Model

A general power-law path-loss model is considered in which the signal power decays at the rate, $r^{-\alpha}$, with the propagation distance $r$, where $\alpha$ is the path-loss exponent. Due to the different propagation environments experienced in the V2B and V2V links, each type of link is given its own path-loss exponent, namely, $\alpha_b$ and $\alpha_v$, respectively. The small-scale channel fading is modeled as slow-flat Rayleigh fading as used by [17], [21], [30], [31], [34], [36], where its channel gain is assumed to be exponentially distributed with unit mean. All the channel gains are assumed to be independent of each other, independent of the spatial locations, symmetric, and are identically distributed (i.i.d.). We use Rayleigh fading for V2B and V2V links for its analytical tractability and this assumption is also the most popular in the literature to get closed-form expressions [11]. However, the analytical expressions can be easily extended to interesting scenarios such as non-exponential fading [31]. For log normal shadowing, we have included the shadowing in a transparent way by using the displacement theorem given by [37] and method described by [33].

E. V2X Communication Reliability or Success Probability

The transmitting vehicle connects to a receiver (either BS or vehicle) depending on the corresponding distances, propagation environments and association bias, and thus the transmitting vehicle can operate in modes $M \in \{v2b, v2v\}$, where $v2b$ and $v2v$ modes denote the shared link communication between vehicle and cellular BS and that between vehicle and vehicle, respectively. The success probability (reliability) of the typical receiver (base-station or vehicle) conditioned on minimum distance $r$ between transmitter and receiver and mode $M$ can be defined as the probability that the SINR of receiver is greater than a SINR Threshold, $z$, which is given as

$$P_M^S(z|r, M) = \Pr[SINR > z] = \Pr\left[\frac{P\rho^{-\alpha}h}{I + \sigma^2} > z\right],$$

where $P$ is transmit power, $h$ is channel gain and $r$ is the minimum distance between transmitter and receiver.

Note that the transmitting vehicle always connects to typical receiver (BS or vehicle) at distance $r$, thus there will be no interfering vehicles present in the disk $B(0, r)$ with radius $r$ as shown in Fig. 3. The size of $B(0, r)$ affects the distribution of interfering nodes, $\Phi_I$ as well as the interference to typical receiver. We need to characterize the distribution of interferers, $\Phi_I$ as per their locations (interferers located on road passing through origin or on all other roads) to determine the Laplace transform of the distribution of interference power conditioned on the serving distance $r$ and mode $M$.

Remind that $r$ is the minimum possible distance between the receiver and transmitter, and the interfering vehicles are located outside the disk $B(0, r)$. Therefore, the interferers can be broadly divided into two categories, (a) the interfering vehicles that are located outside the disk $B(0, r)$ on the road passing through the origin and (b) the interfering vehicles located outside the disk $B(0, r)$ on all other roads. The
distribution of interferes located on all other roads can be denoted as $\Phi_v(B(0, r))$ and the disk $B(0, r)$ does not contain any interfering vehicles. Similarly, the distribution of interferes located on road passing through the origin can be denoted as $\Phi_r|e$ and there are no vehicles on the road segment from 0 to $2r$ for road passing through origin. As such, the success probability can be rewritten as

$$P_M^S(z|\sigma, M) = \Pr \left[ \frac{P_{\sigma} - \alpha}{I_\sigma + I_\alpha + \sigma^2} > z \right],$$

where $\sigma^2$ is the thermal noise power, $I_\sigma$ and $I_\alpha$ are the aggregate interference due to the vehicles located on all other roads, and on the road passing through the origin that operate in mode $v_2b$ or $v_2v$, respectively.

IV. ASSOCIATION PROBABILITIES

To facilitate the reliability analysis of proposed V2X networks, we first derive the distance distributions of the V2V link and the V2B link, and their corresponding association probability in the following.

Lemma 1 (V2V Distance): The CDF of the shortest distance $R_v$ between the vehicular transmitter and a typical vehicular receiver in the V2V link, $F_{R_v}(r_v)$ is given in [30], we still present results below for completeness

$$F_{R_v}(r_v) = 1 - \exp \left( -2\pi \lambda_R \int_{y_\alpha=0}^{r_v} \left( 1 - e^{-2\mu_v \sqrt{r_v^2 - y_\alpha^2}} \right) dy_\alpha \right) \times \exp \left( -2\pi \mu_v r_v \right).$$

Corollary 1 (The PDF of V2V Link): The Probability Density Function (PDF), $f_{R_v}(r_v)$ of the shortest distance between the vehicular transmitter and a typical vehicular receiver, $R_v$ in the V2V link is derived as

$$f_{R_v}(r_v) = \left( -2\pi \lambda_R \int_{0}^{r_v} \frac{e^{-2\mu_v \sqrt{r_v^2 - y_\alpha^2}}}{\sqrt{r_v^2 - y_\alpha^2}} dy_\alpha - 2\mu_v \right) \times \exp \left( -2\pi \lambda_R \int_{0}^{r_v} \left( 1 - e^{-2\mu_v \sqrt{r_v^2 - r_\alpha^2}} \right) dy_\alpha \right) - 2r_v \mu_v.$$

Proof 1: The Probability Density Function (PDF) of $R_v$ can be found by taking derivative of CDF given in Eq. (3), $f_{R_v}(r_v) = \frac{d}{dr_v}(F_{R_v}(r_v))$ and final result for PDF of $R_v$ is given as Eq. (4). The closed form solution of Eq. (4) is derived as

$$f_{R_v}(r_v) = \left( -2\pi^2 \lambda_R \mu_v \int_{0}^{r_v} \frac{I_0(2r_v \mu_v) - L_0(2r_v \mu_v)}{B \frac{\alpha_b}{r_v^2}} - 2\mu_v \right) \times \exp \left( -2\pi \lambda_R r_v + \pi^2 \lambda_R r_v \left( L_1(2r_v \mu_v) - I_1(2r_v \mu_v) \right) - 2r_v \mu_v. \right)$$

where $I_n(z) \ (n = 0, 1)$ denotes the modified Bessel functions of the first kind and $L_n(z) \ (n = 0, -1)$ denotes the modified Struve functions. This completes the proof.

Lemma 2 (V2B Distance): The PDF of the distance between a vehicular transmitter and the nearest BS, $R_b$ for the shared link $f_{R_b}(r_b)$ is given in [31] Eq. (2) as

$$f_{R_b}(r_b) = 2\pi \lambda_b r_b e^{-\pi \lambda_b r_b^2}.$$

According to the flexible V2X link selection scheme prosed in section [11], the vehicle selects the V2V link if $Br_v^{\alpha_v} \geq r_b^{\alpha_b}$, otherwise the vehicle selects the V2B link, where $r_v$ is the shortest V2V link distance, and $r_b$ is the distance between the transmitting vehicle and its closest cellular BS. Therefore, the vehicular transmitter can connect with the vehicular receiver or BS in the uplink with their corresponding association probabilities. In Lemma 3 and Lemma 4 we derive the association probabilities of the V2V link and the V2B link, respectively.

Lemma 3 (Association probability of the V2V Link): The probability of the vehicular transmitter selecting the V2V link is derived as

$$P_{v_2v}^A = \int_{0}^{\infty} \exp \left( -2\pi \lambda_R r_v + \pi^2 \lambda_R r_v \left( L_1(2r_v \mu_v) - I_1(2r_v \mu_v) \right) - 2r_v \mu_v \right) \times \exp \left( -2\pi^2 \lambda_R r_v \mu_v \right) \times \exp \left( -\pi \lambda_B \left( \frac{r_v}{B \frac{\alpha_b}{r_v^2}} \right)^2 \right) dr_v.$$

where $\lambda_B$ is the cellular base-station intensity, $\mu_v$ is the V2X nodes intensity, $\lambda_R$ is the road intensities, $B$ is the association bias, $I_n(z) \ (n = 0, 1)$ is the modified Bessel functions of the first kind, and $L_n(z) \ (n = 0, -1)$ is the modified Struve functions.

Proof 2: See Appendix A.

Lemma 4 (Association probability of the V2B Link): The probability of the vehicular transmitter selecting V2B link is derived as

$$P_{v_2b}^A = \int_{0}^{\infty} \exp \left( -2\pi \lambda_R \right) \times \int_{y_\alpha=0}^{r_v} \exp \left( -2\mu_v \right) \times \exp \left( -\mu_v \left( \frac{\alpha_b}{r_v^2} \right)^2 \right) \times \exp \left( -2\mu_v \left( \frac{\alpha_b}{r_v^2} \right)^2 \right) \times 2\pi \lambda_b r_b \exp \left( -\pi \lambda_b r_b \right) dr_b.$$

where $\lambda_B$ is the cellular base-station intensity, $\mu_v$ is the V2X nodes intensity, $\lambda_R$ is the road intensities, $B$ is the association bias.

Proof 3: See Appendix B.
V. V2X SUCCESS PROBABILITY

In this section, we derive the success probability of the V2X communication. To do so, we first need to characterize the interference from each type of interferer category ($I_v$, $I_r$). Thus, we calculate the general form of Laplace Transform, and we derive the expressions of Laplace Transform of interference from $I_v$ and $I_r$ in this section.

A. Laplace Transform of Interference Under Rayleigh Fading

As we know that Laplace Transform of interference, $I_X$ is $L_{I_X}(s) = E[I_x(t)e^{-st}]$. The interfering set of vehicles X for each category of interfering vehicles ($I_v$, $I_r$) based on their location can be represented as

$$I_X = \sum_{x \in X} P_v h_x D_x^{-\alpha},$$

where $X$ represent various interference sources ($I_v$, $I_r$) and $P_v$ is the transmit power of the vehicle. For a vehicle $x \in X$, we denote its distance to the typical receiver as $D_x$. Although the random variables $|D_x| \in X$, are identically distributed, they are not independent in general [7]. However, authors in [7] have shown that this dependence is weak and we will henceforth, assume each $D_x$ to be i.i.d. Due to the different propagation environments experienced in the V2B and V2V links, each type of link will have its own path-loss exponent and $\alpha$ can be replaced with $\alpha_1$ or $\alpha_2$ in (9) as per selected link. The expression for $L_{I_X}(s)$ is given as

$$L_{I_X}(s) = E[I_X] \left[ -\sum_{x \in X} sP_v h_x D_x^{-\alpha} \right].$$

(10)

By taking expectation over $h_x, D_x$, we get

$$L_{I_X}(s) = E[h_x, D_x] \left[ \prod_{x \in X} \exp \left( -sP_v h_x D_x^{-\alpha} \right) \right].$$

(11)

By assuming all $h_x$ as independent, we obtain

$$L_{I_X}(s) = E[D_x] \left[ \prod_{x \in X} E[h_x] \left[ \exp \left( -sP_v h_x D_x^{-\alpha} \right) \right] \right].$$

(12)

Based on the fact that $h \sim \exp(1)$, we obtain

$$L_{I_X}(s) = E[D_x] \left[ \prod_{x \in X} \frac{1}{1 + sP_v D_x^{-\alpha}} \right].$$

(13)

Now, we calculate the Laplace transform of interference for each category of road ($I_v$, $I_r$) in the following. Let us denote the outer circular region in which all roads exist as $B(0, R)$, and inner circular region $B(0, r)$ with radius $r$ having minimum distance between transmitter and receiver as shown in Fig. 5. Let us denote two types of road as $R_{in}$ and $R_{out}$, where $R_{in}$ are the roads intersecting the circular region $B(0, r)$, and $R_{out}$ are the roads that lie outside the circular region $B(0, r)$ and within circular region $B(0, R)$. Thus, road $R_{in}$ is located at distance, $y < r$ and in case of $R_{out}$, it is located at distance $y > r$. In this case, the interferers can be located anywhere on road $R_{out}$ as it is located outside $B(0, r)$. However, in case of $R_{in}$, the interferers will be located in regions between $(-\sqrt{r^2 - y^2}, -\sqrt{r^2 - y^2})$ and $(\sqrt{r^2 - y^2}, \sqrt{r^2 - y^2})$. In the following, we calculate the Laplace Transform for the interferences from the vehicular transmitters located in these two types of roads (i.e., $R_{in}$ and $R_{out}$).

**Corollary 2 (Laplace Transform of Interference for Single Road Located Outside Inner Circular Region, $B(0, r)$):** The conditional Laplace transform of interference at typical receiver, originating from a single road located at a distance $y$ ($y > r$), outside inner circular region, $B(0, r)$ with radius $r$ is expressed as

$$L_{I_{R_{out}}}(s|r) = \exp \left[ -2\mu_v \int_0^\infty \left[ 1 - \frac{1}{1 + sP_v(y^2 + t^2)^{-\alpha}} \right] dt \right].$$

(14)

For $\alpha = 4$, the closed form solution can be simplified as

$$L_{I_{R_{out}}}(s|r) = \exp \left[ -\frac{\pi r^2 \sin \left( \frac{1}{2} \tan^{-1} \left( \frac{y^2}{y^2 + r^2} \right) \right)}{\sqrt{\frac{1}{4} y^4 + r^4 z}} \right].$$

(15)

**Proof 4:** See Appendix C.

**Corollary 3 (Laplace Transform of Interference for Single Road Intersecting the Inner Circular Region, $B(0, r)$):** The conditional Laplace transform of interference at typical receiver, originating from a single road located at distance $y$ ($y < r$), intersecting the inner circular region, $B(0, r)$ with radius $r$ is derived as

$$L_{I_{R_{in}}}(s|r) = \exp \left[ -2\mu_v \int_0^\infty \left[ 1 - \frac{1}{1 + sP_v(y^2 + t^2)^{-\alpha}} \right] dt \right].$$

(16)

**Proof 5:** For this case, the value of $t$ varies from $t = (-\sqrt{r^2 - y^2})$ to $(\sqrt{r^2 - y^2})$ and the range of region in which the interfering nodes will be located is $(-\infty, -\sqrt{r^2 - y^2})$ and $(\sqrt{r^2 - y^2}, \infty)$. Therefore, the Laplace Transform of interference can be calculated by changing the limits of Eq. (16) and is given as Eq. (17). This completes the proof.

Based on the results in Corollary 4 and 5, we can derive the Laplace transform of the interference from the vehicles located on the road passing through the origin and that located on all other roads in Corollary 4 and 5 respectively.

**Corollary 4 (Laplace Transform of Interference from Vehicles Located on Road Passing Through Origin):** The conditional Laplace transform of interference at typical receiver, originating from road located at a distance of $y = 0$ is derived as

$$L_{I_{R_{in}}}(s|r) = \exp \left[ -\frac{2x \times 2x \times 2F_1 \left( \frac{1 - \frac{1}{\alpha}}{2 - \frac{1}{z}} \right)}{\alpha - 1} \right], \alpha > 1.$$

(17)
where $2 F_1 (a, b, c, z)$ is the Hypergeometric function and $z$ is the SINR threshold. For $\alpha = 4$, the closed form expression for the above equation is simplified as

$$L_{I_v} (s|r) = \exp \left[ - \frac{r \sqrt{z}}{\sqrt{2}} \mu_r \right] \times \left( -\tan^{-1} \left( \frac{\sqrt{z} + 1}{\sqrt{2} \sqrt{z}} \right) + \pi \right).$$

(18)

**Proof 6:** The conditional Laplace transform of interference can be calculated using Corollary 3 and $y = 0$, and the resultant equation is derived as

$$L_{I_v} (s|r) = \exp \left( -\frac{1}{2} \mu_r \right) \times \left( 1 - \frac{1}{1 + s \mu_r^{1-\alpha}} \right).$$

(19)

The closed form solution of Eq. (19) for all values of $\alpha$ is proved in (17).

**Corollary 5 (Laplace Transform of Interference from All Roads Excluding Road Passing Through Origin):** The conditional Laplace transform of the total interference at the typical receiver, originating from vehicular transmitters located on all roads except the road passing through the origin is derived as

$$L_{I_v} (s|r) = \left[ \exp \left( 2 \mu_r \lambda R \right) \int_0^r \left( 1 - L_{I_{R_{in}}} (s) \right) dy \right] \times \left[ \exp \left( 2 \mu_r \lambda R \right) \int_0^r \left( 1 - L_{I_{R_{out}}} (s) \right) dy \right].$$

(20)

where $L_{I_{R_{in}}} (s)$ and $L_{I_{R_{out}}} (s)$ are given in (16) and (14), respectively.

**Proof 7:** See Appendix D

With the help of Corollary 4 and 5 we can derive the success probabilities for the V2V link and that for the V2B link for a given distance $r$ in the following theorems.

**Theorem 1 (Success Probability of the V2V Shared Link):** The success probability of the V2V link for given minimum distance $r_v$ between transmitter and receiver is derived as

$$P_{V2V}^S (z|r_v) = \exp \left( -\frac{z \sigma^2}{P_v \times r_v^{-\alpha}} \right) \times L_{I_v} \left( \frac{z}{P_v \times r_v^{-\alpha}} | r_v \right) \times \int_0^r f_r (r) dr,$$

(21)

where $L_{I_v} (s|r_v)$ and $L_{I_v} (s|r_v)$ are given in Eqs. (17) and (20) by substituting $s = \frac{z \sigma^2}{P_v \times r_v^{-\alpha}}$.

**Proof 8:** The success probability of the V2V shared link for given minimum distance, $r_v$, between transmitter and receiver and operating mode $M = v2v$ is

$$P_{V2V}^S (z|r_v) = Pr [SINR > z].$$

(22)

Using Eq. (2), the above equation can be written as

$$P_{V2V}^S (z|r_v) = Pr \left[ \frac{P_v r_v^{-\alpha + h}}{I_v + I_r + \sigma^2} > z \right].$$

(23)

With mathematical simplification, the final equation for the success probability of the V2V link for given minimum distance, $r_v$ between transmitter and receiver is proved in Eq. (21).

**Theorem 2 (Uplink Success Probability for the V2B Link):** The uplink success probability for the V2B shared link for given minimum distance, $r_b$ between transmitter and receiver is derived as

$$P_{V2B}^S (z|r_b) = \exp \left( -\frac{z \sigma^2}{P_b \times r_b^{-\alpha}} \right) \times L_{I_v} \left( \frac{z}{P_v \times r_b^{-\alpha}} | r_b \right) \times L_{I_r} \left( \frac{z}{P_r \times r_b^{-\alpha}} | r_b \right).$$

(24)

where $L_{I_v} (s|r_b)$ and $L_{I_r} (s|r_b)$ are given in Eqs. (17) and (20) by substituting $s = \frac{z \sigma^2}{P_v \times r_b^{-\alpha}}$.

**Proof 9:** The success probability for the V2B link for given minimum distance, $r_b$ between transmitter and receiver is presented as

$$P_{V2B}^S (z|r_b) = Pr [SINR > z].$$

(25)

By using Eq. (2), the above equation can be written as

$$P_{V2B}^S (z|r_b) = Pr \left[ \frac{P_v r_b^{-\alpha + h}}{I_v + I_r + \sigma^2} > z \right].$$

(26)

With mathematical simplification, the final equation for the success probability of the V2B link for given minimum distance, $r_b$ between transmitter and receiver is proved in Eq. (24).

**Corollary 6 (Success Probability of Cellular V2X Network):** The success probability of cellular V2X network is derived as

$$P_{V2X}^S (z) = \int_0^r \exp \left( -\frac{z \sigma^2}{P_v \times r_v^{-\alpha}} \right) \times L_{I_v} \left( \frac{z}{P_v \times r_v^{-\alpha}} | r_v \right) \times L_{I_r} \left( \frac{z}{P_r \times r_v^{-\alpha}} | r_v \right) \times f_r (r) dr,$$

(27)

where $L_{I_v} (s|r_r)$ and $L_{I_r} (s|r_r)$ are given in Eqs. (17) and (20) by substituting $s = \frac{z \sigma^2}{P_v \times r_v^{-\alpha}}$ and $r = r_v$ or $r = r_b$ and $\alpha = \alpha_b$ or $\alpha_c = \alpha_v$. The $P_{V2V}^A (v2v|r_v)$ is given in Eq. (32), and $P_{V2B}^A (v2b|r_b)$ is given in Eq. (37).

**Proof 10:** By using total probability law, the success probability of V2X network is

$$P_{V2X}^S (z) = P_{V2V}^S (z|r_v) \times P_{V2V}^A (v2v|r_v) + P_{V2B}^S (z|r_b) \times P_{V2B}^A (v2b|r_b),$$

(28)

where $P_{V2V}^S (z|r_v)$ is given in Eq. (21), $P_{V2B}^S (z|r_b)$ is given in Eq. (24), $P_{V2V}^A (v2v|r_v)$ is given in Eq. (32), and $P_{V2B}^A (v2b|r_b)$ is given in Eq. (37). By removing the distance ($r_v$ and $r_b$)

$$P_{V2X}^S (z) = \int_0^r \exp \left( -\frac{z \sigma^2}{P_v \times r_v^{-\alpha_v}} \right) \times L_{I_v} \left( \frac{z}{P_v \times r_v^{-\alpha_v}} | r_v \right) \times L_{I_r} \left( \frac{z}{P_r \times r_v^{-\alpha_v}} | r_v \right) \times f_r (r) dr,$$

(27)

where $L_{I_v} (s|r_v)$ and $L_{I_r} (s|r_v)$ are given in Eqs. (17) and (20) by substituting $s = \frac{z \sigma^2}{P_v \times r_v^{-\alpha_v}}$ and $r = r_v$ or $r = r_b$ and $\alpha = \alpha_b$ or $\alpha = \alpha_v$. The $P_{V2V}^A (v2v|r_v)$ is given in Eq. (32), and $P_{V2B}^A (v2b|r_b)$ is given in Eq. (37). By removing the distance ($r_v$ and $r_b$).
condition on Eq. (28), the V2X overall success probability is proved in Eq. (27), which completes the proof.

For the purpose of comparison, we derive the success probability of vehicular communication without involvement of cellular network in the following:

**Corollary 7 (Success Probability of the V2V Communication without Cellular Network):** The success probability of V2V Communication without cellular network is derived as

\[ P_{v2v}^{\text{only}}(z) = \int_0^\infty \exp \left( -\frac{z^2}{P_v \times r_v^{\alpha_v}} \right) \times \mathcal{L}_{I_v} \left( \frac{z}{P_v \times r_v^{\alpha_v}} \right) \times f_{R_v}(r_v) \times dr_v, \tag{29} \]

where the PDF of \( r_v \) is given in Eq. (4), and \( \mathcal{L}_{I_v}(s|r_v) \) and \( \mathcal{L}_{\mu_v}(s) \) are given in Eqs. (17) and (20) by substituting \( s = \frac{P_v r_v^{\alpha_v}}{P_v r_v^{\alpha_v}} \).

**Proof 11:** The success probability of V2V Communication without cellular network can be derived by removing condition on \( r_v \) in Eq. (21) and the final expression is proved in Eq. (29).

VI. NUMERICAL RESULTS

In this section, the association probability of V2V link, association probability of V2B link, and the success probability are plotted using (4), (8), and (27), respectively. We also plot the success probability of the cellular based V2V and V2B link along with its association probability using the first part of Eq. (27), and the second part of Eq. (27), respectively. The analytical results are validated by Monte Carlo simulations as shown in each figure. In all the figures, we set the path loss at \( \alpha_v = \alpha_b = 4 \) and the thermal noise spectral density, \( \sigma^2 = -174 \text{ dBm/Hz} \) for 10 MHz bandwidth. The transmit power of vehicles are set to be 23 dBm as defined by (1). For comparison purposes, only V2V Communication without cellular networks has also been plotted using (29) to exhibit advantages of cellular V2X Communication over V2V Communication. In the figures, “analyt.” represents analytical plot, “sim” represents simulation plot, “C-V2V” represents cellular based V2V Communication, “C-V2B” represents cellular based V2B Communication, “V2X” represents cellular V2X Communication, and “V2V” represents V2V Communication without cellular. Note, cellular based V2V Communication (C-V2V) is different from V2V Communication without cellular as it involves V2V association probability depending upon V2X link selection scheme.

A. Impact of the SINR threshold

In this subsection, we examine the effect of SINR threshold, \( z \) on the success probability of the proposed model. In Fig. 4 we set \( \lambda_R = 1 \text{ Km/Km}^2 \), \( \mu_v = 10 \text{ nodes/Km} \), \( \lambda_b = 20 \text{ BSs/Km}^2 \) and \( B = 1 \). Fig. 4 plots the success probability of the V2X Communication and V2V Communication at the typical receiver versus the SINR Threshold. Following insights are observed: 1) The cellular V2X network performs better than V2V Communication in the range of -20 dB to 20 dB SINR threshold because both V2V and V2B links are contributing in achieving the better reliability of V2X Communication. However, after that, the success probability of V2X Communication matches with V2V Communication as there is no contribution by V2B link. 2) We see that the reliability advantage is more significant in V2X Communication over V2V Communication once vehicle intensities are low or medium. 3) We observe that in a highly dense V2X network, most of the vehicles connect via V2V link instead of V2B link.

B. Impact of the vehicle intensity

In this subsection, we examine the effect of vehicle intensity, \( \mu_v \), at success probability and association probabilities of V2X network. In Fig. 5 we set \( \lambda_R = 5 \text{ Km/Km}^2 \), \( \lambda_b = 20 \text{ BSs/Km}^2 \), \( z = 0 \text{ dB} \) and \( B = 1 \).

Fig. 5 plots the success probability at the typical receiver versus the intensity of vehicular nodes. The following insights can be observed: 1) The success probability of V2X Communication decreases as the intensity of vehicles on the roads increases due to decrease in V2B success probability. 2) In low and medium intensity networks, there is almost equal probability of connecting to V2V or V2B link. However, in highly dense networks, this advantage of cellular V2X Communication over V2V Communication reduces to certain extent. The success probability of V2X Communication are still better than V2V Communication. 3) We see that the cellular based V2V link success probability increases at faster rate with the increase of vehicle nodes. This is because of reduction in distance between the typical receiver and vehicle located on the lines that are closer to the origin. However, the distance between typical receiver and vehicles located on the lines that are farther away from the origin does not decrease at same rate due to effect of perpendicular distance of roads. This increases the desired signal power at a faster rate than the interference power, thus improving the SINR and hence the success probability at the typical receiver.

In Fig. 6 we plot the association probabilities of V2V and V2B links versus the intensity of vehicles. From the figure, we observed that with the increase of vehicular node intensities, more vehicles start to use the V2V link as the distance between...
C. Impact of the road intensity

In this subsection, we examine the effect of road intensity, $\Lambda_R$ at packet success probability of V2X network and V2V communication. In Fig. 7, we set $\mu_v = 5$ nodes/Km, $\lambda_b = 20$ BSs/Km$^2$, $B = 1$ and $z = 0$ dB.

Fig. 7 plots the success probability at the typical receiver versus the intensity of roads. The following insights can be observed: 1) The success probability of V2X communication in low and medium intensities are much better than V2V communication. 2) In densely populated area with roads, the gain of V2X communication over V2V communication reduces. However, the success probability of the V2V communication remains constant as variation of BS intensities has no impact on V2V communication. 4) It is expected that in current heterogeneous networks where the intensity of BS is mostly high and sufficient, the number of cellular BSs are available to provide highly reliable coverage to vehicular networks. From this analysis, we can conclude that V2X communication is going to provide better reliability performance than V2V communication in current deployment scenarios of cellular networks.

D. Impact of the base-station intensity

In this subsection, we examine the effect of BS intensity, $\Lambda_b$ at the success probability of the V2X communication and solely V2V communication. In Fig. 8, we set $\mu_v = 5$ nodes/Km, $\lambda_R = 5$ Km/Km$^2$, $B = 1$ and $z = 0$ dB.

Fig. 8 plots the success probability at the typical receiver versus the intensity of BSs. The following insights can be observed: 1) The success probability of cellular V2X network continuously increases at steady rate due to continuous rise in success probability of V2B link. 2) We observed that with the BS densification, the probability of having base-station in near vicinity to vehicular transmitter is higher than the PLP based V2V link distance, because the BSs are uniformly distributed instead of non-uniform PLP distribution of roads. 3) The success probability of the V2V communication remains constant as variation of BS intensities has no impact on V2V communication. 4) It is expected that in current heterogeneous networks where the intensity of BS is mostly high and sufficient, the number of cellular BSs are available to provide highly reliable coverage to vehicular networks. From this analysis, we can conclude that V2X communication is going to provide better reliability performance than V2V communication in current deployment scenarios of cellular networks.

E. Impact of the association bias

In this subsection, we examine the effect of association bias, $B$ at packet success probability of V2X network and V2V communication. In Fig. 9, we set $\mu_v = 5$ nodes/Km, $\lambda_R = 5$ Km/Km$^2$, $\lambda_b = 20$ BSs/Km$^2$ and $z = 0$ dB.

Fig. 9 plots the success probability at the typical receiver versus the association bias. The following insights can be observed: 1) The success probability of V2X communication is much higher than V2V communication as both cellular based V2V and V2B links supplement each other for success probability of V2X communication and there are no coverage gaps for the network. 2) We observed that there is an increase in the success probability of the V2X communication with increase in values of bias, because the success probability of
cellular networks can be controlled through association bias.

5) We can summarize that traffic loading and interference to association bias has no effect on this type of communication.

VII. CONCLUSION

In this paper, we presented a comprehensive and tractable analytical framework for the reliability performance of cellular based vehicle-to-everything (V2X) communication in which vehicular communication can be established through cellular network or directly between vehicles on shared links. A flexible V2X uplink selection scheme has been proposed for the vehicular transmitter to decide between the vehicle to vehicle (V2V) and the vehicle to base station (V2B), with a bias factor controlling the amount of vehicular interference and traffic on the cellular network. By modeling the vehicles on roads as doubly stochastic Cox process, and the BSs as 2D PPP, we derived the expressions for the success probabilities of the V2V link, the V2B link, as well as the V2X link, which are validated by the simulations. By comparing the proposed V2X communication with the solely V2V communication, we have shown the reliability enhancement brought by the shared communication via cellular networks. Future works can be extended to 1) interference mitigation techniques for cellular V2X network 2) the derivation of V2V communication analytical model for Ricean fading channel and results comparison with Rayleigh fading channels 3) modeling of vehicles on PLP as hard-core process on the line instead of PPP 4) implementation of the proposed model for vehicular communication and validation of reliability results of cellular V2X communication in comparison with V2V communication by the industry 5) correctness verification of using PLP for stochastic modeling the roads by using the Google maps and real roads.

APPENDIX A

PROOF OF LEMMA 3

The conditional association probability of the V2V communication for shared link is given by

\[
P_{v2v}(v2v|R_v) = P \left[ B \times r_v^{-\alpha_v} \geq r_b^{-\alpha_b} \right],
\]

where \( B \) is the association bias, \( r_v \) is the V2V link distance and \( r_b \) is the V2B link distance. By simplifying the above equation, we get

\[
P_{v2v}(v2v|R_v) = P \left[ r_v \geq \frac{B^{-1}}{B_0^{-1}} \right].
\]

It means that the vehicular transmitter which is located at distance \( r_v \) is connected to vehicular receiver. Therefore, the radius of disk \( b(0, \frac{B^{-1}}{B_0^{-1}}) \) does not contain any BS. By inserting CCDF of V2B link in (31), we get

\[
P_{v2v}(v2v|R_v) = \exp \left[ -\pi \lambda_B \left( \frac{B^{-1}}{B_0^{-1}} \right)^2 \right].
\]

Now, removing condition on \( R_v \), we get

\[
P_{v2v} = \int_0^{\infty} \exp \left[ -\pi \lambda_B \left( \frac{B^{-1}}{B_0^{-1}} \right)^2 \right] \times f_{R_v}(r_v) dr_v.
\]

The PDF of \( R_v \) \( f_{R_v}(r_v) \) is given in Eq. 5. The solution for the V2V association probability is proved in Eq. 7.

APPENDIX B

PROOF OF LEMMA 4

The conditional association probability of the V2B shared link is given as

\[
P_{v2b}(v2b|R_b) = P \left[ B \times r_v^{-\alpha_v} < r_b^{-\alpha_b} \right].
\]

Simplifying the above equation, we get

\[
P_{v2b}(v2b|R_b) = P \left[ r_v > B \frac{\alpha_v}{\alpha_b} r_b^{-\alpha_b} \right].
\]
It means that the disk \( b(0, B \frac{\alpha}{r_b} v) \) does not contain any vehicle, and thus
\[
P_{\nu 2b}(v 2b|R_b) = P \left[ \text{No vehicle in circle of radius, } B \frac{\alpha}{r_b} v \right].
\]

By using the CCDF of \( R_v \) given in Eq. 3, we get
\[
P_{\nu 2b}(v 2b|R_b) = \exp \left[ -2 \pi \lambda R \right.
B \frac{\alpha}{r_b} v \int_{y_n=0}^{1 - e^{-2 \mu_v \sqrt{1 - y_n^2}}} \left( \int_{y_n=0}^{1 - e^{-2 \mu_v \sqrt{1 - y_n^2}}} dy_n \right) \times \exp(-2 \mu_v B \frac{\alpha}{r_b} v).
\]

(36)

Now, by removing condition on \( R_b \) and by inserting CDF of \( R_b \) in (37), we get Eq. 8 for the unconditional V2B shared link association probability.

**APPENDIX C**

**PROOF OF COROLLARY 2**

The length of road \( R_{out} \) lying inside the circular region \( B(0, R) \) is \( 2 \sqrt{R^2 - y^2} \) and distance between two vehicles can be denoted as \( t \). By using the properties of a PPP, the probability of there being \( m \) points in this line segment can be calculated from 1D PPP. The conditional Laplace transform from these vehicles lying on this road segment to typical receiver can be calculated as follows
\[
\mathcal{L}_{I_{out}}(s|r) = E_{D_x} \left[ \prod_{x \in R_{out}} \left[ \frac{1}{1 + s P_v D_x} \right] \right].
\]

(38)

Now conditioning over number of vehicles lying on the road and then deconditioning on interference caused by each node, we get
\[
\mathcal{L}_{I_{out}}(s|r) = \sum_{m \geq 0} P[N_v = m]
\sqrt{2 \sqrt{R^2 - y^2}} \frac{f(t)dt}{1 + s P_v (y^2 + t^2)^{\frac{\alpha}{2}}} \bigg]^m.
\]

(39)

The number of points on the line segment of length \( 2 \sqrt{R^2 - y^2} \) is a Poisson random variable with mean \( 2 \mu_v \sqrt{R^2 - y^2} \) and \( t \) is uniformly distributed between \( (-\sqrt{R^2 - y^2}, \sqrt{R^2 - y^2}) \) and has a PDF \( f(t) = \frac{1}{2 \sqrt{R^2 - y^2}} \).

By inserting pdf of \( f(t) \) in the above equation, we get
\[
\mathcal{L}_{I_{out}}(s|r) = \sum_{m \geq 0} \frac{e^{-2 \mu_v \sqrt{R^2 - y^2}} (2 \mu_v \sqrt{R^2 - y^2})^m}{m!} \left[ \sqrt{2 \sqrt{R^2 - y^2}} \frac{dt \left( \frac{dt}{1 + s P_v (y^2 + t^2)^{\frac{\alpha}{2}}} \right)} \right]^m.
\]

(40)

By simplifying above equation by using the property of integral of even function and by using the Taylor series expansion, \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \), we get
\[
\mathcal{L}_{I_{out}}(s|r) = \exp \left( -2 \mu_v \sqrt{R^2 - y^2} \right)
\times \exp \left( 2 \mu_v \int_0^{\sqrt{R^2 - y^2}} \left[ \frac{dt}{1 + s P_v (y^2 + t^2)^{\frac{\alpha}{2}}} \right] \right).
\]

(41)

The final expression for Laplace transform of interference from a road located outside the inner circular region \( b(0, r) \) is given in equation 14.

**APPENDIX D**

**PROOF OF COROLLARY 3**

We condition on number of roads, \( j \) crossing the region \( B(0, r) \) to calculate the interference from vehicles which lie on roads which interest the region \( B(0, r) \). However, on these lines, the interfering vehicles are located outside the region \( B(0, r) \). This is a Poisson random variable with mean \( 2 \mu_v \lambda_R \). Similarly, we condition on number of roads, \( k \) which lie between circular regions \( B(0, r) \) and \( B(0, R) \). This is also a Poisson random variable with mean \( 2 \mu_v (R - r) \lambda_R \). Therefore, the Laplace Transform of interference of the total interference originating from Poisson Line Process can be written as
\[
\mathcal{L}_{I_{out}}(s|r) = \sum_{j \geq 0} \frac{\exp \left( -2 \mu_v d \lambda_R \right) \times (2 \mu_v r \lambda_R)^j}{j!}
\times \left[ \left( \int_{y=r}^{r} \mathcal{L}_{I_{R1}}(s) \times f_Y(y)dy \right)^j \right]
\times \sum_{k \geq 0} \frac{e^{-2 \mu_v (R-r) \lambda_R} \times (2 \mu_v (R - r) \lambda_R)^k}{k!}
\times \left[ \left( \int_{y=r}^{R} \mathcal{L}_{I_{R2}}(s) \times f_Y(y)dy \right)^k \right].
\]

(42)

By writing the above equation in the form of Taylor series, we have
\[
\mathcal{L}_{I_{out}}(s|r) = \sum_{j \geq 0} \frac{\left( \mu_v \lambda_R \int_{y=r}^{r} \mathcal{L}_{I_{R1}}(s)dy \right)^j}{j!}
\times \left[ \left( \mu_v \lambda_R \int_{y=r}^{R} \mathcal{L}_{I_{R2}}(s)dy \right)^k \right].
\]

(43)
By simplifying the above equation, we get the final result for Laplace Transform of interference of vehicles located on all roads excluding the road passing through the origin and it is proved in Eq. (20).

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