Can black holes and naked singularities be detected in accelerators?

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We study the conditions for the existence of black holes that can be produced in colliders at TeV-scale if the space-time is higher dimensional. On employing the microcanonical picture, we find that their life-times strongly depend on the details of the model. If the extra dimensions are compact (ADD model), microcanonical deviations from thermality are in general significant near the fundamental TeV mass and tiny black holes decay more slowly than predicted by the canonical expression, but still fast enough to disappear almost instantaneously. However, with one warped extra dimension (RS model), microcanonical corrections are much larger and tiny black holes appear to be (meta)stable. Further, if the total charge is not zero, we argue that naked singularities do not occur provided the electromagnetic field is strictly confined on an infinitely thin brane. However, they might be produced in colliders if the effective thickness of the brane is of the order of the fundamental length scale ($\sim$ TeV$^{-1}$).

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\section{I. INTRODUCTION}

The current interest in the possibility that there exist large extra dimensions \cite{ADD} is based on the attractive features that the hierarchy problem is by-passed by identifying the ultraviolet cutoff with the electroweak energy scale $m_{\text{ew}}$ (without ancillary assumptions to achieve radiative stability) and that, since the fundamental scale of the theory is $m_{\text{ew}}$, predictions drawn from the theory such as deviations from the $1/r^2$ law of Newtonian gravity can be experimentally tested in the near future. In the extra-dimensions scenario all of the interactions, gravity (which propagates in the whole “bulk” space-time) as well gauge interactions (which are confined on the four-dimensional brane), become unified at the electroweak scale. This means that if the model is viable, particle accelerators such as the CERN Large Hadron Collider (LHC), the Very LHC (VLHC) and the Next Linear Collider (NLC) will be able to uncover the features of quantum gravity as well as the mechanism of electroweak symmetry breaking.

The large-extra-dimension scenario also has significant implications for processes involving strong gravitational fields, such as the formation and decay of black holes. Since the fundamental scale of quantum gravity is now pulled down to order $m_{\text{ew}}$, black holes can be produced with mass of a few TeV which behave semiclassically \cite{BDD}. Since this energy scale will be available in the forthcoming generation of colliders, they might then become black hole factories \cite{ADD}. Black holes in $4+d$ extra dimensions have been studied in both compact \cite{ADD} and infinitely extended \cite{RS} extra dimensions \cite{ADD}. The basic feature of black hole production is that its cross section is essentially the horizon area of the forming black hole and grows with the center of mass energy of the colliding particles as a power which depends on the number of extra dimensions \cite{ADD}. Although the high non-linearity of the describing equations and the lack of a theory of quantum gravity hinder a fully satisfactory description of this process, there are good reasons to believe in the qualitative picture outlined above \cite{ADD}.

Once the black hole has formed (and after a possible transient, or “balding” stage \cite{ADD}), Hawking radiation \cite{ADD} is expected to set off. The phenomenology of black hole evaporation has been described within the context of the microcanonical ensemble in four space-time dimensions \cite{ADD} and in the context of compact extra dimensions \cite{ADD} in Refs. \cite{ADD}. Our starting point is the idea that black holes are (excitations of) extended objects (p-branes), a gas of which satisfies the bootstrap condition. This yields a picture in which a black hole and the emitted particles are of the same nature and an improved law of black hole decay which is consistent with unitarity (energy conservation).

The use of the microcanonical picture will lead us to the conclusion that the evaporation process in the presence of extra dimensions strongly depends on the details of the model. In particular, if the extra dimensions are compact (ADD scenario of Ref. \cite{ADD}) the luminosity of tiny black holes is in poor qualitative agreement with that predicted by the canonical picture since the occupation number density departs from thermality for masses slightly above the TeV-scale. On applying the formalism of Ref. \cite{ADD} to the cases of interest, we shall then argue that a black hole produced in a collider would be relatively longer-lived with respect to estimates in the existing literature \cite{ADD}. However, the typical life-time is short enough that black holes can be considered to decay (at least down to the fundamental mass scale) instantaneously. On the other hand, if there is one warped extra
dimension (RS scenario of Ref. [2]), the microcanonical luminosity differs significantly from the canonical expression, and the evaporation process might be frozen below the scale at which corrections to Newton’s law become effective.

It is also important to note that such tiny singularities in four dimensions, besides being beyond the realm of classical general relativity, would be black holes only provided their electric charge is zero, otherwise they are naked singularities. In the following Section we shall consider such cases. We know of no conclusive argument which completely rules out their existence.

We shall use units with $c = 1$, $\hbar = m_p l_p$ ($l_p$ is the four-dimensional Planck length) and $G_N = l_p/m_p$ denotes the four-dimensional Newton constant.

II. NAKED SINGULARITIES

The four-dimensional argument about naked singularities mentioned at the end of the Introduction easily generalizes to higher dimensions. In fact, one observes that charged (spherically symmetric) black holes must satisfy the inequality \[Q^2 \frac{m_p^2}{M^2} < \frac{(2 + d) (1 + d)}{2}, \tag{1}\]
where $Q$ is the charge in dimensionless units. The condition in Eq. (1) is obviously violated in the ADD scenario, where the effect of the brane on the space-time geometry is essentially neglected, since an object with mass of order a few TeV and charge equal to (fractions of) the electron charge has $Q (m_p/M) \sim 10^8$.

A possible way to circumvent the bound (1) is by requiring that the electromagnetic field of a point-like charge be confined to the brane, thereby, spoiling the full $(3 + d)$-dimensional spherical symmetry [10]. The system would thus appear spherically symmetric only from the four-dimensional point of view. The only known metric on the brane which might represent such a case was found in Ref. [17] in the context of the RS scenario [1]. Such a solution has the Reissner-Nordström form
\[- g_{tt} = \frac{1}{g_{rr}} = 1 - 2 \frac{M l_p}{m_p r} + Q^2 \frac{l_p^2}{r^2} - q \frac{m_p^2 l_p^2}{m^2 (5) r^2}, \tag{2}\]
and the (outer) horizon radius is given by
\[R_H = l_p \frac{M}{m_p} \left[ 1 + \sqrt{1 - Q^2 \frac{m_p^2}{M^2} + \frac{q m_p^4}{m^2 (5)}} \right], \tag{3}\]
where $m_p (5) \sim m_{ew}$ is the fundamental mass scale and $q$ represents a (dimensionless) tidal charge. The latter can be estimated on dimensional grounds as [17]
\[q \sim \left( \frac{m_p}{m_{ew}} \right)^{\alpha} \frac{M}{m_{ew}}, \tag{4}\]
and for $\alpha > -4$ the tidal term $\sim 1/r^2$ dominates over the four-dimensional potential $\sim 1/r$ (as one would expect for tiny black holes). The condition (1) is therefore replaced by
\[Q^2 \frac{m_p^2}{M^2} < 1 + \left( \frac{m_p}{m_{ew}} \right)^{3+\alpha} \frac{m_p}{M}, \tag{5}\]
which can be fairly large for $\alpha > -4$ and $M \sim m_{ew}$, in contrast to the right hand side of Eq. (1).

Which of the two conditions (1) and (5) is relevant remains an open question, since one might in fact argue that the brane cannot be infinitely thin (see, e.g., [19] and Refs. therein). Gauge fields would then extend along the extra dimension(s), roughly to a width of the order of $m^{-1}_{ew}$, and this likely yields a bound somewhere in between the expressions given in Eq. (1) and Eq. (5) for a singularity with $R_H \sim m^{-1}_{ew}$. There is thus no compelling reason to discard the possibility that the collision of charged particles produces a naked singularity, an event which would probably be indistinguishable from ordinary particle production, with the naked singularity (possibly) behaving as an intermediate, highly unstable state. The phenomenology of naked singularities is probably rather different from that of black holes, as they are generally expected to explode in a very sudden event instead of evaporating via the Hawking process (at least in an early stage; see, e.g., [20] and Refs. therein).

We should however add that the present literature does not reliably cover the case of such tiny naked singularities and their actual phenomenology is an open question. A naked singularity is basically a failure in the causality structure of space-time mathematically admitted by the field equations of general relativity. Most studies have thus focused on their realization as the (classical) end-point of the gravitational collapse of compact objects (such as dust clouds) and on their stability by employing quantum field theory on the resulting background. However, one might need more than semiclassical tools to investigate both the formation by collision of particles and the subsequent time evolution [20]. In particular, to our knowledge, no estimate of the life-time of a naked singularity of the sort of interest here is yet available.

To summarize, the following two cases might occur in a proton-proton collider such as the LHC,
\[p^+ + p^+ \rightarrow \begin{cases} \text{B. H.} + X^{++} \\ \text{N. S. or B. H.} + Y^{0,+} \end{cases}, \tag{6}\]
where $X^{++}$ denotes a set of particles whose total charge is twice the proton charge and $Y^{0,+}$ a set of particles with vanishing total charge or with one net positive charge.

* Its extension into the bulk is still under study (see, e.g., Ref. [21] for a numerical analysis).
III. BLACK HOLES

In a four-dimensional space-time, a black hole might emerge from the collision of two particles only if its center of mass energy exceeds the Planck mass \( m_p \). In fact, \( m_p \) is the minimum mass for which the Compton wavelength \( l_p = l_p(m_p/M) \) of a point-like particle of mass \( M \) equals its gravitational radius \( R_H = 2G_N M \). For energies below \( m_p \) the very (classical) concept of a black hole would lose its meaning. However, since the fundamental mass scale is shifted down to \( M/m_p \) energies below would lose its meaning. However, since the fundamental mass scale is shifted down to \( M/m_p \) can now exist as classical objects provided
\[
l_p m_p/M \ll R_H \ll L,
\]
where \( L \) is the scale at which corrections to the Newtonian potential become effective. The left hand inequality ensures that the black hole behaves semiclassically, and one does not need a full-fledged theory of quantum gravity, while the right hand inequality guarantees that the black hole is small enough that its gravitational field can depart from the Newtonian behavior without contradicting present experiments. In this Section we check that black holes with \( m_{\text{ew}} \leq M < 10 m_{\text{ew}} \) are allowed and then study their evaporation process. We shall have nothing new to report about the cross section for their production.

The luminosity of a black hole in \( D \) space-time dimensions is given by
\[
\mathcal{L}(D)(M) = A(D) \int_0^\infty \sum_{\omega} n_{(D)}(\omega) \Gamma_{(D)}^{(s)}(\omega) \omega^{D-1} d\omega
\]
where \( A(D) \) is the horizon area in \( D \) space-time dimensions, \( \Gamma_{(D)}^{(s)} \) the corresponding grey-body factor and \( S \) the number of species of particles that can be emitted. For the sake of simplicity, we shall approximate \( \sum_{\omega} \Gamma_{(D)}^{(s)}(\omega) \) as a constant (see Section II.C in [9] and below). The distribution \( n_{(D)} \) is the microcanonical number density \([13,14]\)
\[
n_{(D)}(\omega) = C([M/\omega]) \sum_{l=1}^{[M/\omega]} \exp \left[ S_{(D)}^E(M - l\omega) - S_{(D)}^E(M) \right]
\]
where \([X]\) denotes the integer part of \( X \) and \( C(\omega) \) encodes deviations from the area law \([13]\) (in the following we shall also assume \( C \) is a constant in the range of interesting values of \( M \)). The basic quantity in Eq. (11) is the Euclidean black hole action, which usually takes the form
\[
S_{(D)}^E \sim A(D) \left( \frac{M}{m_{\text{eff}}} \right)^{\beta}
\]
where \( m_{\text{eff}} \) and \( \beta \) are model-dependent quantities and \( A(D) \) (\( m_{(D)} \)) is the fundamental length (mass) in \( D \) space-time dimensions related to the fundamental Newton constant by
\[
G_{(D)} = \frac{l_p(D-3)}{m_{(D)}},
\]
We recall that for \( \beta = \beta_d = (2 + d)/(1 + d) \) and in the limit \( M/m_{\text{eff}} \to \infty \), \( n_{(D)}(\omega) \) mimics the canonical ensemble (Planckian) number density in \( 4 + d \) space-time dimensions and the luminosity becomes
\[
\mathcal{L}(4+d) \sim A(4+d) \left( T(4+d) \right)^{4+d} \sim \frac{1}{R_H^d},
\]
where \( T(4+d) \) is the Hawking temperature in \( 4 + d \) dimensions.

On using Eqs. (11) and (12) one can show that the luminosity is in general given by
\[
\mathcal{L}(4+d) = K m^\beta e^{-m^\beta} \int_0^\infty e^{x^\beta} (m - x)^{3+d} dx,
\]
where \( m \equiv M/m_{\text{eff}} \) and \( K \) is a coefficient which contains all the dimensionful parameters but does not depend on \( M \). The above integral can be performed exactly for the models under consideration.

We shall now analyze the ADD and RS scenarios separately.

A. ADD scenario

If the space-time is higher dimensional and the extra dimensions are compact and of size \( L \), the relation between the mass of a spherically symmetric black hole and its horizon radius is changed to \([15]\)
\[
R_H \simeq l_{(4+d)} \left( \frac{2 M}{m_{(4+d)}} \right)^{\frac{1}{1+\gamma}},
\]
where
\[
G_{(4+d)} \simeq L^d G_N
\]
is the fundamental gravitational constant in \( 4 + d \) dimensions. Eq. (14) holds true for black holes of size \( R_H \ll L \), or, equivalently, of mass
\[
M \ll M_c \equiv m_p L / l_p.
\]
Since \( L \) is related to \( d \) and the fundamental mass scale \( m_{(4+d)} \sim m_{\text{ew}} \sim 1 \text{ TeV} \) by \([11]\)
\[
\frac{L}{l_p} \sim \left( \frac{m_{\text{ew}}}{m_{(4+d)}} \right)^{1+\frac{d}{2}} \times 10^{\frac{d}{2}+16} \equiv \gamma^{1+\frac{d}{2}} \times 10^{\frac{d}{2}+16},
\]
Eq. (12) translates into
\[
10^{\frac{31+16+d}{2}} \times m_p \sim 10^{-16} \gamma m_p \ll M \ll M_c.
\]
(18)
The microcanonical ensemble is thus given by the occupation number density for the Hawking particles in the effective only for \( \Delta M \sim 5 \text{ TeV} \), before it reaches \( 1 \text{ m}_{\text{ew}} \), which is approximately

\[
\frac{dT}{dt} \bigg|_{M \sim \text{m}_{\text{ew}}} \approx -10^{-10} \frac{L_{(10)}}{\text{L}(10)} \sim -10^{17} \frac{\text{TeV}}{s} .
\] (22)

In the above analysis we have only considered masses for a black hole larger than the fundamental scale has been reached, a black hole continues to evaporate. However, it is also possible that the radiation simply switches off at that point (as the microcanonical luminosity suggests) and the small black hole escapes as a stable remnant. If heavy (\( \sim 10 \text{ TeV} \)) black holes are produced, they will be moving slowly and will quite likely decay (at least down to about 1 TeV) in the detector producing a “sudden burst” of light particles (electrons, positrons, neutrinos and \( \gamma \)-rays) at the collision time it would take to emit the remaining \( \Delta M \).

Although the integral in Eq. (13) can be performed exactly, its expression is very complicated and we omit it. Instead, in Table I we show the relevant quantities for \( d = 2, \ldots, 6 \) (upper bounds for the grey-body factors are estimated as in Ref. [2] for s-wave modes only, since one expects significantly smaller values for non-zero angular momentum modes [2]).

In all cases, the microcanonical luminosity becomes smaller for \( M \sim \text{m}_{\text{ew}} \) than it would be according to the canonical luminosity, which makes the life-time of the black hole somewhat longer than in the canonical picture. In particular, for \( d = 6 \) one finds

\[
T \sim \left( \frac{dT}{dt} \right)^{-1} \Delta M \sim 10^{-17} s .
\] (23)

The above relatively long time does take into account the dependence of the grey-body factor \( \Gamma_{(4+\ell)}^{(s)} \) on \( d \) but not the actual number \( S \) of particle species into which the black hole can decay. The latter would increase the luminosity by a factor \( S \sim 10 \rightarrow 100 \) but this is already taken care of by the normalizing procedure defined by Eq. (23)

\[
\text{FIG. 1. Microcanonical luminosity (solid line) for a small black hole with } d = 6 \text{ extra dimensions compared to the corresponding canonical luminosity (dashed line). Vertical units are chosen such that } L_{(10)}^{(d+1)}(\text{m}_{\text{ew}}) = 1.
\]

We then notice that the above expression differs from the four-dimensional one for which \( m_{\text{eff}} = m_{p} \gg m_{\text{ew}} \) and \( \beta = 2 > \beta_{d} \). In four dimensions one knows that microcanonical corrections to the luminosity become effective only for \( M \sim m_{p} \), therefore, for black holes with \( M \gg m_{\text{ew}} \) the luminosity \( \text{L}_{(4+\ell)} \) should reduce to the canonical result given in Eq. (23).

In order to eliminate the factor \( K \) from Eq. (13), one can therefore equate the microcanonical luminosity to the canonical expression at a given reference mass \( M_{0} \sim M_{c} \gg m_{\text{ew}} \) and then normalize the microcanonical luminosity according to

\[
\text{L}_{(4+\ell)}(M) \sim \frac{\text{L}_{(4+\ell)}(M_{0})}{\text{L}_{(4+\ell)}(M_{0})} .
\] (20)

The black hole luminosity thus obtained differs significantly from the canonical one for \( M \sim m_{\text{ew}} \), as can be clearly seen from the plot for \( d = 6 \) in Fig. [1]. For smaller values of \( d \) the picture remains qualitatively the same.

\[\text{This was also shown to be a good approximation of the luminosity as seen by an observer on the brane, since most of the emission occurs into particles confined on the brane.}\]
point. However the cross section for the production of such heavy black holes is very small [4]. If a neutral black hole is produced with a mass \( m \sim m_{ew} \) and is stable, its detection will depend upon the ability of the detectors to measure the missing transverse momentum or the missing mass accurately enough to prove the existence of a massive neutral particle. If instead it is charged and stable, it should not be difficult to track its path. Limits to the existence of stable remnants should also come, e.g., from estimates of the allowed density of primordial black holes [23].

**B. RS scenario**

In order to study this case, we shall again make use of the solution given in Ref. [17] (although new metrics were given in Ref. [24]). From Eq. (3) with \( Q = 0 \) and \( \alpha > -4 \) one obtains

\[
R_H \simeq l_p \left( \frac{m_p}{m(5)} \right)^{1+\frac{3}{2}} \sqrt{\frac{M}{m(5)}},
\]

(24)

since the tidal term \( q \) dominates for both \( M \) and \( m(5) \ll m_p \), and one must still have Eq. (7). With one warped extra dimension [3], the length \( L \) is just bounded by requiring that Newton’s law not be violated in the tested regions, since corrections to the \( 1/r^2 \) behavior are of order \( (L/r)^2 \). This roughly constrains \( l_p < L < 10^{-3} \) cm. Hence the allowed masses are, according to Eq. (6),

\[
\left( \frac{m(5)}{m_p} \right)^{\frac{3}{2}} \ll \frac{M}{m(5)} \ll \left( \frac{l_p}{l_p} \right)^2 \left( \frac{m(5)}{m_p} \right)^{2+\alpha} .
\]

(25)

In particular one notices that black holes with \( M \sim m(5) \sim m_{ew} \) could exist only if the following two conditions are simultaneously satisfied

\[
\alpha \geq 0 \quad \text{and} \quad \frac{L}{l_p} \gg \left( \frac{m_p}{m_{ew}} \right)^{\frac{3+\alpha}{2}} .
\]

(26)

The luminosity is now given by the four-dimensional expression [13] with \( D = 4 \), \( \beta = 1 \) and

\[
m_{eff} = \left( \frac{m_{ew}}{m_p} \right)^{2+\alpha} m_{ew} .
\]

(27)

The result is simple enough to display, namely

\[
\mathcal{L} \sim K m e^{-m},
\]

(28)

where the last expression follows from \( m \to M/m_{eff} \ll 1 \) since \( m_{eff} \ll m_{ew} \) for \( \alpha \geq 0 \). We again eliminate \( K \) by normalizing the luminosity to the (four-dimensional) canonical expression \( \mathcal{L}^H_{(4)}(M_0) \), where now \( M_0 \sim M_e = m_p (L/l_p) \) is the mass above which corrections to Newton’s law are negligible. For the limiting case \( \alpha = 0 \), taking into account the second condition in Eq. (26) one obtains

\[
\mathcal{L}_{(4)} < 10^{-9} \frac{M}{m_{ew}} \text{ TeV s} ,
\]

(29)

which yields an exponential decay with typical life-time \( T > 10^9 \) s.

The above result is certainly striking, since it means that microscopic black holes are (meta)stable objects and would be detected just as missing energy (if neutral) or stable heavy particles (if charged). Hence, either they escape from the detector and carry away a large amount of energy or in rare instances they give rise to an isotropic (almost steady) vanishingly faint flux of particles (a “star”) inside the detector. Black holes with lifetimes this long would have had an effect on the evolution of the early universe. The allowed density of primordial black holes [23] might thus be able to provide some evidence as to the validity of the RS scenario.

**IV. CONCLUSIONS**

We have analyzed the conditions for the existence of naked singularities and black holes with masses that can be reached in accelerators such as the LHC. We have shown that the typical life-times of tiny black holes depend strongly on the model employed, since they decay almost instantaneously in ADD and are (quasi)stable in RS. As we argued in Section II for the existence of naked singularities, it is likely that the results in RS apply only to a brane-world of zero thickness. For a brane of width of order TeV\(^{-1} \), the correct life-times would probably be in between those predicted in ADD (where the brane is totally neglected) and those in RS. We think this shows that the phenomenology of such objects is not completely settled, mainly due to the persistent lack of a sensible solution representing a black hole in a space-time with extra dimensions in which our brane-world would be embedded.

| \( \mathcal{L}^H_{(4+4)} \) | \( \Gamma_{(4+4)} \) | \( R_{(4+4)} \) | \( \mathcal{L}_{(4+4)} \) |
|---|---|---|---|
| 2 \( 10^{28} \) | \( 10^{-1} \) | \( 10^{-2} \) | \( 10^{29} \) |
| 3 \( 10^{27} \) | \( 10^{-2} \) | \( 10^{-3} \) | \( 10^{22} \) |
| 4 \( 10^{28} \) | \( 10^{-2} \) | \( 10^{-5} \) | \( 10^{21} \) |
| 5 \( 10^{27} \) | \( 10^{-3} \) | \( 10^{-6} \) | \( 10^{20} \) |
| 6 \( 10^{27} \) | \( 10^{-4} \) | \( 10^{-1} \) | \( 10^{19} \) |

**TABLE I.** Relevant quantities for the ADD scenario. \( \mathcal{L}^H_{(4+4)} \) is the canonical luminosity in TeV/s; \( \Gamma_{(4+4)} \) is an upper bound for the grey-body factor; \( R_{(4+4)} \) is the ratio defined in Eq. (21) and \( \mathcal{L}_{(4+4)} \) the microcanonical luminosity in TeV/s. All quantities are evaluated for \( M \sim m_{ew} \sim 1 \) TeV.
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