ABSTRACT

We investigate the correlation between the frequencies of the twin kilohertz quasi-periodic oscillations (kHz QPOs) and the neutron star (NS) spins in low mass X-ray binaries (LMXBs), based on the data sets of 12 sources with simultaneously detected twin kHz QPOs and NS spins, and find that the histogram of the ratio between the frequency difference of twin kHz QPOs ($\Delta \nu \equiv \nu_2 - \nu_1$) and NS spin $\nu_s$ shows a non-uniform distribution with a gap at $\Delta \nu/\nu_s \sim 0.65$. We try to classify the 12 sources into two categories according to this gap: (I) The slow rotators with $\langle \nu_s \rangle \sim 311$ Hz, XTE J1807.4-294, 4U 1915-05, IGR J17191-2821, 4U 1702-43, 4U 1728-34 and 4U 0614+09 follow a relation $\Delta \nu/\nu_s > 0.65$; (II) The fast rotators with $\langle \nu_s \rangle \sim 546$ Hz, SAX J1808.4-3658, KS 1731-260, Aql X-1, 4U 1636-53, SAX J1750.8-2900 and 4U 1608-52 satisfy the relation $\Delta \nu/\nu_s < 0.65$. However, the linear fittings of $\Delta \nu$ versus $\nu_s$ relations of group (I) and (II) are unsatisfactory to ensure any certain correlations. We suggest that this phenomenon may arise from the fact that most measured kHz QPOs and spins satisfy the conditions of $1.1 \nu_s \leq \nu_2 < 1300$ Hz and $\Delta \nu$ decreasing with $\nu_2$. Apparently, the diversified distribution of $\Delta \nu/\nu_s$ refutes the simple beat-frequency model, and the statistical correlations between the twin kHz QPOs and NS spins may arise from the magnetosphere-disk boundary environments, e.g., co-rotation radius and NS radius, that modulate the occurrences of X-ray signals. Furthermore, we also find that the distribution of the ratio of $\nu_2$ to $\nu_1$ clusters around the value of $\langle \nu_2/\nu_1 \rangle \sim 3:2$, which shows no obvious correlation with NS spins.

Key words: X-rays: binaries–binaries: close–stars: neutron – accretion: accretion discs

1 INTRODUCTION

The launch of the Rossi X-Ray Timing Explorer (RXTE) has led to the discovery of Kilohertz quasi-periodic oscillations (kHz QPOs) in neutron star low mass X-ray binaries (NS-LMXBs) (Strohmayer et al. 1996; van der Klis et al. 1996). The frequencies of these QPOs cover the range from $\sim 100$ Hz to $\sim 1300$ Hz (van der Klis 2000; 2006) and correlate with other timing and spectral features (Wijnands et al. 1997a,b; Homan et al. 2002; Kaaret et al. 1998; Ford & van der Klis 1999; Méndez et al. 1999; Ford et al. 2003; Psaltis et al. 1999; Belloni et al. 2002). These high-frequency QPOs usually appear in pairs (upper $\nu_2$ and lower $\nu_1$), with the frequencies show a nonlinear relation (Belloni et al. 2003; Zhang et al. 2006a; Belloni et al. 2007), and so far there are over twenty LMXBs have shown twin kHz QPOs in accreting millisecond X-Ray pulsars (AMXP), atoll and Z sources (Hasinger & van der Klis 1985; van der Klis 2000; 2002; Wang et al. 2014).

It is believed that twin kHz QPOs may reflect the matter motion around the inner accretion disk dozens of kilometers away from the neutron star (Kluźniak et al. 1996; van der Klis 2000), and references therein, Zhang & Wang 2013; Wang et al. 2015, 2017), which can be used to probe the strong gravitational field, the strong magnetic field...
The frequencies of the simultaneously detected twin kHz QPOs and NS spins suggest the frequency of kHz QPOs to be related to the NS spin (van der Klis 2006). In addition, the resonance model also presents the averaged peak separation of twin kHz QPOs to be close to the origin of kHz QPOs.

There is currently no consensus as to the sonic-point beat-frequency (SPBF) (Miller et al. 1998; Abramowicz et al. 2003a,b) within an accretion disk, or non-linear resonance (Klužniak & Abramowicz 1990; van der Klis 2006; Wang et al. 2013; Miller & Miller 2015). Although several theoretical interpretations are proposed, such as the relativistic precession (Stella & Vietri 1993; Stella et al. 1996; Török et al. 2016), the magnetohydrodynamic wave (Zhang et al. 2004; or non-linear resonance (Klužniak & Abramowicz 2001; Abramowicz et al. 2003a,b) within an accretion disk, the sonic-point beat-frequency (SPBF) (Miller et al. 1998; Lamb & Miller 2001), there is currently no consensus as to the origin of kHz QPOs.

The sonic-point beat-frequency model interprets the frequency separation of twin kHz QPOs to be close to the NS spin (Miller et al. 1998; Lamb & Miller 2001). However, the following observations show the averaged peak separations are found to be either close to the spin frequency or to its half (Lamb & Miller 2001; Wijnands et al. 2003; van der Klis 2003). In addition, the resonance model also suggests the frequency of kHz QPOs to be related to the NS spin (Lee et al. 2004). Thanks to the high timing revolution of RXTE, there are ∼30 NS-LMXBs observed NS spin signals (Burderi & Di Salvo 2013; van der Klis 2013), in which some sources have even been observed NS spin derivative (e.g. Burderi et al. 2006) and twelve sources have also been detected the twin kHz QPOs, making it is possible to further analyze the correlation between the frequencies of twin kHz QPOs and NS spins.

The goal of this paper is to analyze the frequency correlation between the twin kHz QPOs and NS spins, and its structure is organized as follows: In § 2, we introduce the data of simultaneously detected twin kHz QPOs and NS spins of 12 sources adopted in the analysis, and their correlations are studied and investigated in § 3. Finally, we present the discussions and conclusions in § 4.

### Table 1

| Source (12) | ν1 (Hz) | ν2 (Hz) | ν3 (Hz) | Δν1 (Hz) | Δν2 (Hz) | Δν/νa | ν2/ν1 | References |
|------------|---------|---------|---------|---------|---------|-------|-------|-----------|
| XTE J1807.4-294 | 106 ~ 370 | 337 ~ 587 | 191 (A) | 179 ~ 247 | 197 | 0.94 ~ 1.29 | 1.51 ~ 3.18 | [1, 13] |
| 4U 1915-05 | 224 ~ 707 | 514 ~ 1055 | 270 (N) | 290 ~ 353 | 299 | 1.07 ~ 1.31 | 1.49 ~ 2.30 | [2, 13] |
| IGR J17191-2821 | 681 ~ 870 | 1037 ~ 1185 | 294 (N) | 315 ~ 362 | 349 | 1.07 ~ 1.23 | 1.36 ~ 1.53 | [3, 13] |
| 4U 1702-43 | 722 | 1055 | 330 (N) | 333 | 333 | 1.00 | 1.46 | [4, 13] |
| 4U 1728-34 | 308 ~ 894 | 582 ~ 1183 | 363 (N) | 231 ~ 363 | 341 | 0.64 ~ 1.00 | 1.31 ~ 1.89 | [5, 13] |
| SAX J1808.4-3658 | 435 ~ 567 | 599 ~ 737 | 401 (AN) | 164 ~ 195 | 185 | 0.41 ~ 0.49 | 1.30 ~ 1.39 | [6, 13] |
| AX J1-0014+09 | 153 ~ 843 | 449 ~ 1162 | 415 (N) | 238 ~ 382 | 317 | 0.57 ~ 0.92 | 1.36 ~ 2.93 | [7, 14] |
| KS 1731-260 | 898 ~ 903 | 1159 ~ 1183 | 524 (N) | 260 ~ 283 | 272 | 0.50 ~ 0.54 | 1.29 ~ 1.31 | [8, 13] |
| Aquil北 | 795 ~ 803 | 1074 ~ 1083 | 550 (AN) | 278 ~ 280 | 279 | 0.50 ~ 0.54 | 1.29 ~ 1.31 | [8, 13] |
| 4U 1636-53 | 529 ~ 979 | 823 ~ 1228 | 581 (N) | 230 ~ 341 | 277 | 0.40 ~ 0.59 | 1.23 ~ 1.56 | [10, 13] |
| SAX J1750.8-2900 | 936 | 1253 | 601 (N) | 317 | 317 | 0.53 | 1.34 | [11, 13] |
| 4U 1608-52 | 473 ~ 867 | 796 ~ 1104 | 619 (N) | 225 ~ 326 | 304 | 0.36 ~ 0.53 | 1.26 ~ 1.69 | [12, 13] |

§1: The sources are listed in the order of NS spin frequency.

ν1—Frequency of the lower kHz QPO.

ν2—Frequency of the upper kHz QPO.

ν3—NS spin frequency inferred from periodic or nearly periodic X-ray oscillations. A: accretion-powered millisecond pulsar, N: nuclear-powered millisecond pulsar.

Δν: Δν ≡ ν2 − ν1.

Δν/νa: Weighted mean value of Δν calculated by equation (1).

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Figure 1. Plot of the upper kHz QPO (ν2) versus NS spin (νν/νν/ν). The upper frequency ν2 of the 12 sources range at ∼337–1253 Hz, which are constrained in the range of νν/ν < 1300 Hz by the dashed straight lines.
Correlations between ... 

Figure 2. (a) The histogram of $\Delta \nu/\nu_s$, where $\Delta \nu$ is the frequency difference between the upper and lower kHz QP Os ($\Delta \nu = \nu_2 - \nu_1$) and $\nu_s$ is the NS spin. (b) Plot of $\Delta \nu$ versus $\nu_s$, where the lines of $\Delta \nu = 0.5 \nu_s$, $\Delta \nu = (2/3) \nu_s$ and $\Delta \nu = \nu_s$ predicted by the resonance model [Lee et al. 2004] are also plotted. (c) Plot of the weighted mean values of $\Delta \nu$ ($\langle \Delta \nu \rangle$) versus $\nu_s$.

2 THE PUBLISHED SAMPLES OF THE SIMULTANEOUSLY DETECTED TWIN KHZ QPO AND NS SPIN FREQUENCIES

We focus on the 12 NS-LMXBs which have been simultaneously detected the twin kHz QPOs and NS spins, and collect the data of these QPO and spin frequencies from the published literature. The samples contain 26 pairs of twin kHz QPOs from the two accreting millisecond X-ray pulsars (XTE J1807.4-294 and SAX J1808.4-3658), and the 177 ones from ten atoll sources. The NS spin frequencies of the 12 sources are inferred from either the periodic or nearly periodic X-ray burst oscillations (Boutloukos & Lamb 2008; van der Klis 2006).

The frequency range of the twin kHz QPOs from 12 NS-LMXBs are reported in Table 1, from which it can be seen that the lower kHz QPOs show the frequency range of $\nu_1 \sim 106 - 979$ Hz, while the upper kHz QPOs show the frequency range of $\nu_2 \sim 337 - 1253$ Hz. Table 1 also shows the NS spin frequencies of the 12 sources, with the range of $\nu_s \sim 191 - 619$ Hz and the average value of $\sim 428$ Hz. It is also noticed from Table 1 that XTE J1807.4-294 shows the relative lower frequencies of twin kHz QPOs and NS spin than other sources, i.e. $\nu_1 \sim 106 - 370$ Hz, $\nu_2 \sim 337 - 587$ Hz and $\nu_s \sim 191$ Hz, respectively.

Table 2. Fitting results.

| Relation | Function | Parameter | $\chi^2$/d.o.f. |
|----------|----------|-----------|----------------|
| $f(x)$ | | $a_1 = 32 \pm 11$ | |
| $\Delta \nu/\nu_s$ | $a_1 e^{-(x-b_1)/c_1}$ & $b_1 = 0.78 \pm 0.02$ & |
| | $+ a_2 e^{-(x-b_2)/c_2}$ & $c_1 = 0.65 \pm 0.02$ & $a_2 = 32 \pm 10$ |
| | & $b_2 = 0.49 \pm 0.02$ & $c_2 = 0.08 \pm 0.03$ |
| $\Delta \nu \sim \nu_s$ | (I) $\Delta \nu = a_1 \nu_s + b_1$ & $a_1 = 0.3 \pm 0.1$ & $b_1 = 200 \pm 45$ |
| | (II) $\Delta \nu = a_2 \nu_s + b_2$ & $a_2 = 0.5 \pm 0.1$ & $b_2 = -22 \pm 65$ |
| $\nu_2/\nu_1 \sim \nu_s$ | $\nu_2/\nu_1 = a\nu_s^b$ & $a = 2.4 \pm 0.6$ & $b = -0.08 \pm 0.04$ |

van der Klis (2006).
3 FREQUENCY CORRELATION BETWEEN THE TWIN KHZ QPOS AND NS SPINS

3.1 The correlation between \( \nu_2 \) and \( \nu_s \)

Firstly, we try to probe the correlation between \( \nu_2 \) and \( \nu_s \) with the collected data of kHz QPOs and NS spins for 12 NS-LMXBs, which is shown in Fig.1 where we notice that the upper limit of \( \nu_2 \) is around \( \nu_2 < 1300 \text{ Hz} \). It is also noticed from Table 1 and Fig.1 that the NS spin frequency of each source is smaller than its upper kHz QPO frequencies, and the kHz QPO data are constrained in the region of \( \nu_2 \gg 1.1 \nu_s \). The distribution of the \( \nu_2 \) versus \( \nu_s \) relation is quite dispersive and there is no obvious concentrated phenomenon.

3.2 The correlation between \( \Delta \nu \) and \( \nu_s \)

The sonic-point beat-frequency model suggests the frequency difference between the upper and lower kHz QPOs (\( \Delta \nu \equiv \nu_2 - \nu_1 \)) may be related to the NS spin frequency \( \nu_s \) (Miller et al. 1998; Lamb & Miller 2001; Lamb 2003). The resonance model predicts that the ratio between \( \Delta \nu \) and \( \nu_s \) (\( \Delta \nu/\nu_s \)) is approximately equal to small integers, e.g. \( \Delta \nu/\nu_s \sim 0.5 \), \sim 2/3 or \sim 1, etc (Lee et al. 2004). However, there is currently no consensus as to these correlations. Here we calculate the values of \( \Delta \nu \) for all the twin kHz QPOs and show the results in Table 1. The range of \( \Delta \nu \) is \( \sim 164 - 382 \text{ Hz} \), where SAX J1808.4-3658 and XTE J1807.4-294 show the lower \( \Delta \nu \) values of \( \sim 164 - 195 \text{ Hz} \) and \( \sim 179 - 247 \text{ Hz} \), respectively. We probe the correlation between \( \Delta \nu \) and \( \nu_s \) from the following aspects:

We calculate the ratio between \( \Delta \nu \) and \( \nu_s \) and show the results in Table 1 where the range of \( \Delta \nu/\nu_s \) spans a lot of values from 0.36 to 1.31. Fig.2 (a) shows the histogram of \( \Delta \nu/\nu_s \), where the bump distribution is noticed. The Kolmogorov-Smirnov (K-S) test suggests that \( \Delta \nu/\nu_s \) distribution is not uniform at the 95\% confidence level, implying that there may exist a possible dependence of \( \Delta \nu \) on \( \nu_s \). It can be seen from Fig.2 (a) that there exists a distribution gap at \( \Delta \nu/\nu_s \sim 0.65 \), based on which we suspect that the 12 sources can be classified into two categories by this gap, and we also make a test of the double-gaussian function fitting that presents the central values of the two peaks at \( \sim 0.5 \) and \( \sim 0.8 \), respectively, as shown in Table 2. Furthermore, we find that the \( \Delta \nu/\nu_s \) distribution from 0.7 to 1.3 is quite dispersive, which is not sufficiently obvious to classify them as a group. Moreover, we also notice that the \( \Delta \nu/\nu_s \) distribution shown in Fig.2 (a) is partly similar to the expectation of resonance model (see Fig.4 of Lee et al. 2004), which predicts a peak at \( \Delta \nu/\nu_s \sim 0.5 \) and other values, however its predicted peak \( \Delta \nu/\nu_s \sim 2/3 \) is not found in Fig.2 (a).

Fig.2 (b) shows the plot of \( \Delta \nu \) versus \( \nu_s \), and a big range of \( \Delta \nu \) is found to be \( \sim 150 - 400 \text{ Hz} \). So, the distribution of \( \Delta \nu \) versus \( \nu_s \) is quite dispersive, however, the clustering phenomena separated by \( \Delta \nu/\nu_s \sim 0.65 \) can be seen, i.e. group (I) with \( \Delta \nu/\nu_s > 0.65 \): XTE J1807.4-294, 4U 1915-05, IGR J17191-2821, 4U 1702-43, 4U 1728-34 and 4U 0614+09; and group (II) with \( \Delta \nu/\nu_s < 0.65 \): SAX J1808.4-3658, KS 1731-260, Aql X-1, 4U 1636-53, SAX J1750.8-2900 and 4U 1608-52. Furthermore, we try to find the non-biased linear correlations between \( \Delta \nu \) and \( \nu_s \) of group (I) and (II) respectively, by fitting them with the linear functions. The fitting results are shown in Table 2 where the reduced \( \chi^2 \) of fitting on the two groups are \( \chi^2/\text{d.o.f.} \sim 5.3 \) and \( \chi^2/\text{d.o.f.} \sim 3.0 \) respectively, implying the unsatisfactory fittings to show any certain correlations. In addition, Fig.2 (b) also shows the relations of \( \Delta \nu = 0.5 \nu_s \), \( \Delta \nu = (2/3) \nu_s \) and \( \Delta \nu = \nu_s \) by the straight lines, as predicted by the resonance model (Lee et al. 2004), however, it can be seen that the predicted correlations seem to be partly consistent with the data, except lacking of data around the line \( \Delta \nu = (2/3) \nu_s \).

For clarity, we calculate the weighted mean value of \( \Delta \nu \), i.e. \( \langle \Delta \nu \rangle \), of each source by the following equation:

\[
\langle \Delta \nu \rangle = \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \sum_{i=1}^{N} \frac{\Delta \nu_i/\nu_s}{\sigma_i^2}
\]

where \( \sigma_i \) is the error of \( \Delta \nu_i \). The \( \langle \Delta \nu \rangle \) values are shown in Table 1 and Fig.2 (c) shows the \( \langle \Delta \nu \rangle \) versus \( \nu_s \) plot, where the clustering phenomena of the two category sources, i.e. group (I)—\( \Delta \nu/\nu_s \sim 1 \) and group (II)—\( \Delta \nu/\nu_s \sim 0.5 \), are more obvious, except for 4U 0614+09 whose \( \langle \Delta \nu \rangle \) value lie near the lines \( \langle \Delta \nu \rangle = 0.76 \nu_s \).
In order to further investigate the difference between the two category sources, we compare their $\Delta \nu$ and $\nu_s$ distributions, and show the corresponding cumulative distribution function (CDF) curves in Fig.3 (a) and Fig.3 (b), respectively. The K-S test shows that the $\Delta \nu$ data of the two category sources come from the different continuous distribution at the 95% significance level, so does the $\nu_s$ data. For group (I) and group (II), the mean values of $\Delta \nu$ are $\langle \Delta \nu \rangle \sim 302$ Hz and $\langle \Delta \nu \rangle \sim 284$ Hz, respectively, and the mean values of $\nu_s$ are $\langle \nu_s \rangle \sim 311$ Hz and $\langle \nu_s \rangle \sim 546$ Hz, respectively.

### 3.3 The correlation between $\nu_2/\nu_1$ and $\nu_s$

The non-linear resonance model by Abramowicz et al. (2003a,b, 2005) claimed that the ratio relation between the frequencies of the pair high frequency QPOs in compact X-ray binaries to be approximately peaked about 3:2, which have been observed in both stellar and intermediate black hole (BH) binaries (Pasham et al. 2014), however this 3:2 phenomenon is not so sharp in the twin kHz QPOs of NS-LMXBs (Belloni et al. 2005).

Here, we calculated the values of $\nu_2/\nu_1$ and show the results in Table 1 and find that the range of $\nu_2/\nu_1$ is $\sim 1.23 - 3.18$ with the mean value of $\nu_2/\nu_1 \sim 3 : 2$. XTE J1807.4-294 shows the relative larger values of $\nu_2/\nu_1$ ($\sim 1.51 - 3.18$) than those of other sources. Fig.4 (a) shows the histogram of $\nu_2/\nu_1$, from which a ratio clustering phenomenon around 3:2 is noticed, as noted by the non-linear resonance model (see Fig.4 of Abramowicz et al. 2003a). Fig.4 (b) shows the diagram of $\nu_2/\nu_1$ versus $\nu_s$, where the distribution is quite dispersive. We fit the relation with a power-law function $\nu_2/\nu_1 = a \nu_s^b$ and show the results in Table 2. However, the reduced $\chi^2$ value of the fitting is quite poor ($\chi^2$/d.o.f. $\sim 0.04$), which means that there is no obvious correlation between $\nu_2/\nu_1$ and $\nu_s$.

### 4 DISCUSSIONS AND CONCLUSIONS

Based on the data sets of the 12 sources with simultaneously detected twin kHz QPOs and NS spins, we investigate the correlation between the twin kHz QPOs and the NS spins, and find that there exists a gap at $\Delta \nu/\nu_s \sim 0.65$ in the $\Delta \nu/\nu_s$ distribution. The distribution of the ratio of $\nu_2$ to $\nu_1$ clusters around the mean value of $\langle \nu_2/\nu_1 \rangle \sim 3 : 2$, which shows no obvious correlation with NS spins. The details of the conclusions are discussed and summarized as follows:

1. The upper frequencies of twin kHz QPOs satisfy the conditions of $1.1 \nu_s \leq \nu_2 < 1300$ Hz (see Table 1 and Fig.1), where the maximum value of 1300 Hz may arise from the constrain by the NS stellar surface (van der Klis 2006) and the relation $\nu_2 \geq 1.1 \nu_s$ may arise from the fact that the appearance of the twin kHz QPOs needs a critical orbital Keplerian velocity of the accretion matter at the NS co-rotational radius (Wang et al. 2017). It should also be noticed that the lower kHz QPO frequency $\nu_1$ is larger or smaller than the NS spin frequency $\nu_s$ (see Table 1 and Wang et al. 2014). As the NS cannot rotate faster than Keplerian frequency at the equator, so the relation of $\nu_1 > \nu_s$ and $\nu_1 > < \nu_s$ is compatible with the fact that the upper kHz QPO should be intimately involved in the Keplerian orbital frequency, then the lower kHz QPO might not be directly given by the orbital frequency.

2. We focus on the statistical tests about how the relations proposed by various kHz QPO models compile with the detected data. Firstly, the relativistic precession model predicts no correlation of the twin kHz QPO frequency difference with NS spin (Stella & Vietri 1998; Stella et al. 1999; Török et al. 2016). The histogram of $\Delta \nu/\nu_s$ shows the bump phenomena (see Fig.2 (a)), which is not compatible with a uniform distribution by the K-S test. So, we suspect that the frequencies of twin kHz QPOs and NS spins may obey an indirect relation, which may arise from the magnetosphere-disk boundary environments, e.g., co-rotation radius and NS radius. Secondly, the sonic-point beat-frequency model interprets the frequency difference of twin kHz QPOs to be close to the NS spin frequency (Miller et al. 1998; Lamb & Miller 2001). However, the $\Delta \nu/\nu_s$ histogram in Fig.2 (a) shows a diversified distribution, which obviously rejects the idea of claiming $\Delta \nu/\nu_s \sim 1$, therefore, a simple beat-frequency model should be refused. Thirdly, the forced resonance model by Lee et al. (2004) predicts a peak at $\Delta \nu/\nu_s \sim 0.5$, $\sim 2/3$ and $\sim 1$, etc. It is noticed that the $\Delta \nu/\nu_s$ distributions of the detected data...
De-Hua Wang et al.

at 0.5 and 1 seem to be consistent with the expectation of model, however, the values around 2/3 are short of detected samples.

(3) We try to classify the 12 sources into two categories based on the gap value at \( \Delta \nu/\nu_s = 0.65 \): (I) As the slow rotators with \( \nu_s \sim 311 \text{ Hz} \), XTE J1807.4-294, 4U 1915-05, IGR J17191-2821, 4U 1702-43, 4U 1728-34 and 4U 0614+09 follow a relation \( \Delta \nu/\nu_s > 0.65 \); (II) As the fast rotators with \( \nu_s \sim 546 \text{ Hz} \), SAX J1808.4-3658, KS 1731-260, Aql X-1, 4U 1636-53, SAX J1750.8-2900 and 4U 1608-52 satisfy the relation \( \Delta \nu/\nu_s < 0.65 \) (see Fig.2 (b) and Fig.2 (c)). Because of the condition \( \nu_2 > 1.1 \nu_1 \) \cite{Wang2017} and \( \Delta \nu \) decreases with \( \nu_e \) \cite{van der Klis2006, Zhang2006}, the slow rotators \cite{Lamb2003} with the smaller \( \nu_e \) show the big \( \Delta \nu \), which causes the big \( \Delta \nu/\nu_s \) value of > 0.65. On the contrary, the fast rotators with the big spin frequencies \( \nu_e \) correspond to the small \( \Delta \nu \), which cause the smaller \( \Delta \nu/\nu_s \) value of < 0.65. However, it is not clear if the bimodal distribution of \( \Delta \nu/\nu_s \) is possible, nor what physical process can interpret the gap around \( \Delta \nu/\nu_s \sim 0.65 \). If the correlation of \( \Delta \nu \) versus \( \nu_e \) can be confirmed, it may be applied to estimate the NS spin frequencies in LMXBs.

(4) As known, various bands of QPO frequencies in BH-LMXBs and NS-LMXBs follow the tight relations \cite{Belloni2003a, Belloni2003b}, and a pair of high frequency QPOs in stellar and intermediate black hole binaries follows, approximately, a 3:2 ratio relation, which may be the particular phenomena from the innermost stable circular orbit \cite{Abramowicz2003b, Pasham2014}. In fact, there has been a suggestion in the literature for the 3:2 ratio as a parametric resonance between two particular modes of torus oscillations \cite{Bursa2004, Kluzniak2003, Kluzniak2007}. However, we find that the distribution of the ratio of \( \nu_2 \) to \( \nu_1 \) of the 12 NS-LMXBs sources cluster around the value of \( \sim 3 : 2 \) (see Fig.3 (a)), which shows no obvious correlation with NS spin. The statistical result of \( \sim 3:2 \) QPO ratio relation shares the similar conclusion from the non-linear resonance model \cite{Abramowicz2003a}, implying the \( \sim 3:2 \) relation may be the common property of the compact X-ray binaries around some particular radius. The cause of why the ratios of the pair QPOs of NS-LMXBs and BH-LMXBs show the \( \sim 3:2 \) correlations is still unclear, and it needs more efforts to uncover this secret by analyzing more QPO data in the future detections.

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