Design of Position Servo Control System Based on MATLAB

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Abstract. The position servo system is a feedback control system, and its output variable varies with the input variable, to achieve accurate tracking for the position command by the actuator. The controlled variable (output variable) of the position servo system is the linear displacement and angular displacement of the position of load space. When the given variable (input variable) of the position servo system is arbitrarily changed, the main task of the system is to make the output reproduce a given amount of change quickly and accurately. MATLAB software is widely used in academic and many practical fields. After the system is designed perfectly, we need to use MATLAB software to simulate the results. MATLAB software can well reflect the characteristics of the three-loop position servo system. The system designed in this time belongs to the analog servo system, and its selected model is a high-power three-loop position servo control system.

1. Establishment of mathematical model of three-loops servo system

1.1. The basic composition of the three-loop servo system
The system can be divided into the following nine parts:

1.1.1. Position loop
Here we only analyze the mathematical model of the position loop. The position loop can be approximated as a first-order inertia element, and the transfer function is:

\[ W_j(s) = \frac{K}{T_j s + 1} \]  (1)

1.1.2. Position sensor
The position sensor used in this design is a synchro. The synchro is a sensor of angular displacement. The amplification coefficient of the synchro used in this model is \( K_{bs} = 1.25 \) (°). The detection error of the synchro itself \( e_d = 0.5° \). Amplification coefficient \( K_{rp} = 2 \). The BST is a synchro transmitter, and the BSR is a synchro receiver. The rotor winding of transmitter BST is connected to the single-phase AC excitation power supply, and the receiver BSR rotor winding output reflects the voltage signal of angular displacement \( U_{bs} \).

\[ U_{bs} = U_{bsm} \sin(\Theta_m - \Theta_m) \]  (2)
• $U_{bms}$ - the maximum value of the sinusoidal voltage that synchro receiver output;
• $\theta'_m$ - the mechanical corner of the transmitter;
• $\theta_m$ - the mechanical corner of the receiver.

1.1.3. Voltage comparison amplifier (A)
This is an indispensable device for the position servo system. Its role is to send out control signals $U_c$, since $\Delta U$ can be positive or negative, the amplifier must have the ability to identify the polarity of the voltage, and the output control voltage $U_c$ is also reversible. Amplification coefficient is $K_a = 5$, function relationship $U_c = K_a \Delta U$.

1.1.4. Power Electronic Converter (UPE)
The power electronic converter acts as a power amplifier. This design selects the bridge-type reversible PWM converter with bipolar control. The switching frequency of the PWM converter is $f = 2000 \text{Hz}$, and the out-of-control time is $T_s = 0.5 \text{ms}$. The short out-of-control time greatly improves the rapidity of the system. The lagging link can be approximated as a first-order inertia element (in which $T_s = T_i$), and the transfer function is:

$$W_i(s) = \frac{K_i}{T_i s + 1}$$

(3)

1.1.5. Current regulator (ACR)
This design select a typical I-type system and select the PI regulator according to the engineering design method. The transfer function is:

$$W_{ACR}(s) = K_{pi} \frac{T_i s + 1}{T_i s}$$

(4)

1.1.6. Speed Regulator (ASR)
This design select a typical I-type system and select the PI regulator according to the engineering design method. The transfer function is:

$$W_{ASR}(s) = K_{pm} \frac{T_m s + 1}{T_m s}$$

(5)

1.1.7. Position adjuster (AWR)
This design select a typical II-type system and select the PID regulator according to the engineering design method and the correction of the position system. The transfer function is:

$$W_{AWR}(s) = K_{pw} \frac{T_{aw} s + 1}{T_{aw} s + 1}$$

(6)

1.1.8. Servo Motor (SM)
This design selects permanent magnet DC servo motor, the parameters of motor are: $P_n = 4kW$, $U_n = 220V$, $I_n = 22.7A$, $n_N = 1500 \text{r/min}$. The servo motor can be regarded as a second-order system, which is divided into two transfer functions, one part is the motor armature, which is approximated as a first-order inertia element, and the transfer function is:

$$K_2(s) = \frac{K_2}{T_a s + 1}$$

(7)
One part is the transmission, which is approximately the integrating element. The transfer function is:

\[ K_j(s) = \frac{K_3}{T_{ps} s} \]  \hspace{1cm} (8)

1.1.9. Load

The load is the controlled location object of the entire system, we mainly study its mathematical model. The transfer function is approximated as an integrating element:

\[ W(s) = \frac{2\pi P}{60s}. \]  \hspace{1cm} (9)

The servo system uses a low-speed DC servo motor, so the design eliminates the reducer.

1.2. Mathematical model of the three-loop servo system

The structure diagram of the three-loop servo system is shown in Figure 1.

![Figure 1 Structure diagram of the three-ring follower system](image)

As shown in Figure 1.1, ACR is a current regulator, ASR is a speed regulator, AWR is a position regulator, \( K_i \) is the current feedback coefficient, \( K_e \) is the electromotive force coefficient, \( K_t \) is the speed feedback coefficient and \( K_f \) is the position feedback coefficient.

The parameters of the current regulator, speed regulator and position regulator are designed according to engineering methods. The following are the transfer functions of the current regulator ACR, the speed regulator ACR, and the position regulator AWR with the adjusted parameters:

\[ W_{ACR}(s) = K_{pi} \frac{T_{ps} s + 1}{T_{ps} s} = \frac{2s + 1}{2s}, \quad K_{pi} = 1, T_i = 2 \]

\[ W_{ASR}(s) = K_{pm} \frac{T_{ps} s + 1}{T_{ns} s} = 200 \frac{s + 1}{30 s}, \quad K_{pm} = 200, T_n = \frac{1}{30} \]

\[ W_{AWR}(s) = K_{pw} \frac{T_{w1} s + 1}{T_{w2} s + 1} = \frac{4.73s + 118}{s + 50}, \quad K_{pw} = 2.36, T_w = 0.04, T_{w2} = 0.02 \]

2. Calculation of steady-state parameters of three-loop servo system

The calculation of steady-state parameter is to calculate the load torque at this time when the input shaft of motor rotates at the highest speed, and then determine the steady-state error of the system.
The steady state error of the system is the sum of the detection error, the given error and the disturbance error, calculated according to the I-type system. According to the system error analysis method of position servo system, the whole three-loop system is simplified as a linear system with unit negative feedback, without considering the correction device, that is, without adding the regulator. As shown in Figure 2, the static structure diagram of the system for calculating steady-state error is given.

![Figure 2 Diagram of the static structure of the position servo system](correction device are not considered)

The calculation process is as follows:

- The rated efficiency of the motor is:
  \[
  \eta_N = \frac{P_N}{U_N I_N} = \frac{4000}{220 \times 22.7} = 0.80
  \]

- The armature resistance of the motor is:
  \[
  R_a = \frac{1}{2} (1 - \eta_N) \frac{U_N}{I_N} = 0.5 \times (1 - 0.8) \times \frac{220}{22.7} = 0.97 \Omega
  \]

- The armature resistance of the motor is:
  \[
  C_e = \frac{U_N - I_N R_a}{n_N} = \frac{220 - 22.7 \times 0.97}{1500} = 0.132 \text{v} / \text{r} \cdot \text{min}^{-1}
  \]

- The torque coefficient of the motor is:
  \[
  C_m = \frac{30}{\pi} C_e = 9.55 \times 0.132 = 1.26 \text{N} \cdot \text{m} / \text{A}
  \]

- The maximum speed of the output shaft of the known motor is \( \omega_m^* = 250 \text{r} / \text{s} \), and the load torque at this time is:
  \[
  T_L = \frac{30}{\pi} \frac{P_N}{n_N} = 9.55 \times \frac{4000}{1500} = 25 \text{N} \cdot \text{m}
  \]

- The corresponding load current is:
  \[
  I_{dl} = \frac{T_L}{C_m} = \frac{25}{1.26} = 19.8 \text{A}
  \]

- As can be seen from Figure 2, the given error of the speed input is:
  \[
  e_{\omega} = \frac{\omega_m^*}{K_{bs} K_{rp} K_a K_s \frac{1}{C_e} \frac{360}{60}} = \frac{250}{1.25 \times 2 \times 5 \times 33.3 \times \frac{1}{0.132} \times 6} = 0.0132
  \]

The gain of the power electronic converter in the formula is \( K_s = K_1 = 33.3 \),

- Similarly, the disturbance error caused by the load torque \( T_L = 25 \text{N} \cdot \text{m} \) is:
\[ e_{ef} = \frac{I_{mt}R}{K_{bs}K_{sp}K_{a}K_{s}} = \frac{19.8 \times 2}{0.25 \times 2 \times 5 \times 33.3} = 0.095^5 \]

Total resistance of thyristor motor main circuit: \( R = 2 \Omega \).

- The steady state error of the system is:

  \[ e = e_d + e_m + e_{ef} = 0.5 + 0.0132 + 0.095 = 0.6082^2 \]

This is the steady-state error calculated by the I-type system. When the position PID regulator is introduced into the dynamic correction system, it becomes a typical II-type system. Then the given error and disturbance error of the system can be eliminated, the steady-state error can be significantly reduced, and the steady-state accuracy of the system can be significantly improved. Therefore, for the step response of this study, the system has higher steady-state accuracy and strong anti-interference ability. Through calculation, the system meets the design requirements in terms of steady-state accuracy and stability.

3. Analysis of MATLAB simulation results of three-loop servo system

3.1. MATLAB simulation of three-loop position servo system

The structural diagram of the three-loop servo system with given parameters is shown in Figure 3. Where is the amplification factor and time constant of the positional link, \( K_j = 1.11, T_j = 0.0132 \). The parameters in the figure are:

- \( K_e = 0.133, T_a = 0.0035s \)
- \( K_f = 0.26, K_l = 0.01 \)
- \( K_f = 2.5, T_m = 0.116s \)
- \( T_1 = 0.0005s \)
- \( K_1 = 33.3 \)
- \( K_2 = 0.5 \)
- \( K_3 = 15.05 \)

![Figure 3 Block structural diagram of the three-loop servo system](image)

The MATLAB simulation results are shown in Figure 4.

As shown in Figure 4, the step response-curve of the three-loop servo system. The ordinate represents the displacement, unit is meter. The abscissa represents time, unit is second.
As shown in Figure 4.2, the following performance indicators of the system are: overshoot is $\sigma = 12.75\%$, adjustment time is $t_s = 0.0335$, peak time $t_p = 0.0423$.

3.2. Analysis of MATLAB simulation results

The simulation results of the three-loop servo control system show that the three-loop servo control system is stable after PID correction. The anti-interference performance of the system meets the requirements, the dynamic response is fast, the sensitivity and stability are good, and the requirements of following performance indicators are met.

4. Conclusion

This paper first introduces the concept, characteristics, classifications and methods of error analysis of the position servo system. Let us have a preliminary understanding of the position servo system. Then it introduces the DC double closed loop system and engineering design regulator. Finally, the mathematical model of the three-loop servo system and the MATLAB simulation are performed. By using the function library control module of simulink, the control system can be easily modeled. Pressing the simulation button can start the simulation of the system, and the simulation process is mutual. Therefore, the simulation parameters can be changed, and the proper correction of the dynamic system by using MATLAB can enhance the performance of the system.

References

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