On the Evidence for Postacceleration Effects in Breakup Reactions with Halo Nuclei

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We study the postacceleration of charged fragments in reactions with unstable nuclear beams. For elastic breakup processes we show that the postacceleration effect can be well understood in terms of closed analytical forms derived in a quantum mechanical formulation. This gives theoretical support to the effect experimentally observed in the breakup of \textsuperscript{11}Li projectiles at intermediate energies.

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Over the last four years an extensive effort was made, both experimentally and theoretically, to understand the break-up mechanism of halo nuclei [1-9]. In particular, Refs.[1,2], reported a measurement of the parallel velocity distribution of \textsuperscript{9}Li in the elastic break-up of \textsuperscript{11}Li at 28 MeV-A, as predicted in ref. [3] by means of semiclassical arguments. This distribution showed an asymmetry with respect to the parallel velocity distributions of the neutrons and of the \textsuperscript{9}Li fragments: \textsuperscript{9}Li comes out faster than the neutrons as it splits from \textsuperscript{11}Li. This was interpreted as a postacceleration due to the Coulomb interaction in the final state. This observation was used to deduce the nature of the so-called soft dipole mode. The authors of Refs.[1,2] came to the conclusion that this mode of excitation is not a resonance since the break-up seems to have occurred in the vicinity of the charged field of the target. However, such an interpretation came under questioning in a recent publication that discussed the break-up and postacceleration of the also “exotic” deuteron [8], and also a DWBA [9] and a semiclassical [10] calculation.

It is clear that more data are required in order to better understand the postacceleration phenomenon. Simple closed form models that explicitly exhibit the effect are certainly welcome as they will supply a guide to experimentalists. The knowledge of the width of the parallel velocity distribution is particularly important since it allows the preparation of experimental setups. The purpose of this note is to present such a simple description of the postacceleration effect.

Our starting point is the prior form of the Distorted Wave Born Approximation (DWBA) description of elastic break-up of nuclear projectile. The amplitude is given by, for the \textsuperscript{11}Li \rightarrow \textsuperscript{9}Li +2n reaction,

\[ T_{ij} = \langle \chi_{2}^{(-)}(r_2)\chi_{9}^{(-)}(r_9) \mid [U_2(r_2) + U_9(r_9) - U_{11}(R)] \mid \chi_{11}^{(+)}(R) \rangle \phi_{g.s.}(r) \]  \hspace{1cm} (0.1)

where \(U_i(r_i)\) is the complex optical potential of nucleus \(i\), and \(\chi_{i}^{(\pm)}(r_i)\) is the corresponding optical wave function with outgoing (\(+\)) and incoming (\(-\)) wave boundary conditions. At intermediate energies (~50 MeV/nucleon), the Sommerfeld parameter \(\eta = Z_p Z_T e^2 / \hbar v \gg 1\), and the reaction is very forward peaked, so that we can employ the eikonal form for these wave functions, viz.,

\[ \chi_{11}^{(+)}(R) = e^{i k_{11} R} \exp \left[ -\frac{i}{\hbar \nu_{11}} \int_{-\infty}^{z_{11}} U_{11}(b_{11}, z'_{11}) d z'_{11} + i \phi_{11}^{(c)}(b_{11}, z_{11}) \right] \]  \hspace{1cm} (0.2)

\[ \chi_{9}^{(-)}(r_9) = e^{-i k_9 r_9} \exp \left[ -\frac{i}{\hbar \nu_9} \int_{z_9}^{\infty} U_9(b_9, z'_9) d z'_9 + i \phi_9^{(c)}(b_9, z_9) \right] \]  \hspace{1cm} (0.3)

\[ \chi_{2}^{(-)}(r_2) = e^{-i k_2 r_2} \exp \left[ -\frac{i}{\hbar \nu_2} \int_{z_2}^{\infty} U_2(b_2, z'_2) d z'_2 \right] \]  \hspace{1cm} (0.4)
\[ \phi_{11}^{(c)} = \eta_{11} \ln (k_{11}(r_{11} - z_{11})) ; \quad \phi_{9}^{(c)} = \eta_{9} \ln (k_{9}(r_{9} + z_{9})) ; \quad \text{with} \quad \eta_i = \frac{Z_i Z_T e^2}{\hbar v_i}, \] (0.5)

are the Coulomb phases for \( i \equiv 11 \text{Li}, \ 9\text{Li} \), respectively.

We now assume that the potentials in the exponents in Eqs.(2-4) are related \( U_{11}^{(N)} = U_{9}^{(N)} + U_{2}^{(N)} \) and ignore the difference between the quantities \( 1/\hbar v_i \). The effect of this approximation is very small for the calculation of postacceleration, since it modifies only slightly the nuclear phases. As we show next the post-acceleration effect arises from the differences between the Coulomb phases for the \( 9\text{Li} \) and the two neutrons.

For the interaction \( \Delta V \equiv U_2 + U_9 - U_{11} \simeq U_2^{(C)} + U_9^{(C)} - U_{11}^{(C)} \), we get

\[ \Delta V_C = Z_P Z_T e^2 \left[ \frac{R - r_9}{R r_9} \right]. \] (0.6)

The vectors \( \mathbf{R} \equiv r_{11}, \ r_9 \) and \( \mathbf{r}_2 \) are related through

\[ \mathbf{r}_2 = \mathbf{R} + \frac{9}{11} \mathbf{r} , \quad \mathbf{r}_9 = \mathbf{R} - \frac{2}{11} \mathbf{r} \] (0.7)

where \( \mathbf{r} \) is the relative distance between the di-neutron and \( 9\text{Li} \). With Eq. (7), \( \Delta V_C \) assumes the form, in the dipole approximation,

\[ \Delta V_C = \frac{2}{11} Z_P Z_T e^2 \left[ \frac{4\pi}{3} \right] r Y_{10}(\hat{\mathbf{r}}) \] (0.8)

Using the results above, the DWBA amplitude can be rewritten as [1]

\[ T_{fi}(Q, q) = T_{exc}(q) T_{el}(Q) \] (0.9)

where

\[ T_{exc}(q) = \frac{2}{11} \sqrt{\frac{4\pi}{3}} \int d\mathbf{r} \ e^{-i\mathbf{q} \cdot \mathbf{r} + \Delta \phi_c} \ e Y_{10}(\hat{\mathbf{r}}) \ \varphi_{g.s.}(\mathbf{r}) \] (0.10)

and

\[ T_{el}(Q) = Z_P Z_T e^2 \int d\mathbf{R} \ e^{-i\mathbf{Q} \cdot \mathbf{R}} \ S_{11}(b_{11}) \ \frac{1}{R^2} \] (0.11)

where

\[ Q = \frac{9}{11} k_2 - \frac{2}{11} k_9 , \quad Q = k_9 + k_2 - k_{11} \] (0.12)

and \( S_{11}(b) \) is the elastic S-matrix element of the projectile.

The term \( \Delta \phi_c \) in the exponential appearing in eq. (10) is the responsible for the post-acceleration effect. It is a consequence of the independent propagation of the neutrons and of the \( 9\text{Li} \) wavefunctions in the final channel (we neglect final state interactions between the two neutrons and the \( 9\text{Li} \) fragment). This term is identically zero if one considers the excitation to bound states (which are absent in the case of \(^{11}\text{Li} \) projectiles), or for the breakup of the projectile into fragments with same charge-to-mass ratios (to first order). The term \( \Delta \phi_c \) is what is left, when one incorporates the Coulomb phase for the elastic scattering, \( 2i\eta_{11} \ln(k_{11}b_{11}) \), in the S-matrix element, \( S_{11}(b_{11}) \), by using the relations (7). Specifically, it is given by

\[ \Delta \phi_c = 2i\eta_{11} \ln (k_{11}b_{11}) - i\eta_{11} \ln [k_9(r_{11} - z_{11})] - i\eta_{9} \ln [k_9(r_9 + z_9)] \simeq \frac{2\eta_{11}(\rho + z)}{11b_{11}}, \] (0.13)
where \( \rho \) is the transverse coordinate of \( r \). The approximation is valid for high energy scattering \( (z_{11} \ll b_{11}) \), and to first order in \( \rho/b_{11} \) and \( z/b_{11} \).

The factorization in the form of eq. (9) is only useful if we can separate the variables \( r \) and \( R \equiv r_{11} \) completely. This can be done by noticing that the 1/\( R^2 \) term in the integrand of eq. (11) favors small values of \( R \), as usual for Coulomb excitation processes. This is the classical equivalent of grazing collisions. Thus we set \( b_{11} \approx b_{\text{min}} \approx 1.2(A_T + A_P)^{1/3} \) fm in eq. (13).

Using a Yukawa-type function ground state wavefunction,

\[
\phi_{g.s.}(r) = \sqrt{\frac{\kappa}{2\pi}} \frac{e^{-\kappa r}}{r}, \quad \kappa^2 = \frac{2\mu E_B}{\hbar^2}, \quad \mu = \frac{18}{11} m_N,
\]

where \( E_B \) is the binding energy and \( m_N \) is the nucleon mass, the integral in eq. (10), with the approximation (13), can be done analytically, yielding

\[
T_{\text{exc}}(q) = \frac{16\pi}{11} \sqrt{\frac{2\kappa}{3}} i \frac{q_z^2}{(\kappa^2 + q_z'^2)^2},
\]

where

\[
q'_z = q + \frac{2}{11} \frac{\eta_{11}}{b_{\text{min}}} \left( \hat{z} + \hat{b} \right),
\]

and \( \hat{z} (\hat{b}) \) is the unit vector along the longitudinal (perpendicular) direction.

The elastic break-up cross-section is given by

\[
d^6\sigma = \frac{2\pi}{\hbar v_{11}} |T_{fi}|^2 \frac{d^3q d^3Q}{(2\pi)^6} \delta(E_i - E_f).
\]

Since we are interested in the momentum differences between the two neutrons and the \( ^9\text{Li} \) fragments, we integrate over the elastic scattering momentum \( Q \), i.e.,

\[
(2\pi)^3 \frac{d^6\sigma}{d^3q} = \frac{2\pi}{\hbar v_{11}} \int \frac{d^3Q}{(2\pi)^3} |T_{\text{exc}}(q)|^2 |T_{el}(Q)|^2 \delta(E_i - E_f),
\]

and we make use of the completeness of the plane waves \( e^{-iQ \cdot R} \). We find,

\[
\frac{d^2\sigma}{dq_z dq_t} = \frac{2\pi}{\hbar v_{11}} C_{cl} |T_{\text{exc}}(Q)|^2
\]

where

\[
C_{cl} = (Z_P Z_T e^2)^2 \int dR |S_{11}(b_{11})|^2 \frac{1}{R^4}.
\]

Since we are only interested in the relative velocity of the fragments, which is implicit in the momentum \( q \), we can rewrite eq. (19) as

\[
\frac{d^2\sigma}{dq_z dq_t} = C \frac{q_z'^2}{(\kappa^2 + q_z'^2 + q_t'^2)^4},
\]

where \( C \) includes all constants and factors which do not depend on \( q \).

From eq. (21) one can immediately see where the postacceleration effect resides. It shows that the longitudinal momentum distribution is shifted by \( \Delta q_z^{(0)} = 2\eta_{11}/(11b_{\text{min}}) \). For \( ^{11}\text{Li} \) projectiles incident on lead at 30 MeV/nucleon \( \Delta q_z \approx (8 \text{ fm})^{-1} \). Using the definition of \( q \) from eq. (12), we get \( v_9 - v_2 \approx 0.08 c \) which is not small compared to the beam velocity, \( v_{11} \approx 0.25 c \). This gives a clear theoretical explanation of the post-acceleration effect which was verified experimentally \[12\]. We can also give a classical interpretation of this result: the postacceleration originates from
the extra-momentum gained by the $^9$Li after the breakup, assumed to occur at the distance of closest approach. This extra-momentum is roughly given by $\Delta p = F \cdot \Delta t \simeq (Z_9 Z_T e^2 / b_{\text{min}}^2) \cdot (b_{\text{min}} / 2v) \simeq h\eta / 2b_{\text{min}}$. This also is consistent with the findings of ref. [3], where a semiclassical approach to this problem was undertaken. It is however, in contrast with ref. [9] where a DWBA calculation was also performed. The approximations used there were probably the reason for the negative result.

The width of the momentum distribution is given approximately by $\Gamma = \sqrt{\kappa^2 + (\Delta q_z)^2} / 2$. The first term inside the square root is due to the Fermi momentum of the ground state wavefunction. In figure 1 we plot $\Gamma$ and $\Delta q_z$ as a function of the bombarding energy of $^{11}$Li projectiles incident on lead. The post-acceleration effect is quite large for low energies, tending to disappear at $E/A \sim 1000$ and higher. The width of the momentum distribution is not so sensitive to the energy, since it also carries an important fraction which originates from the Fermi momentum of the ground state wavefunction. Of course, our derivation depends on the assumptions that (a) Coulomb dissociation is the major inelastic process, and that (b) the fragments are focused at forward directions. However, these assumptions are quite reasonable for systems with small binding energies, e.g., $^{11}$Li, and at intermediate energy collisions.

In conclusion, we have presented in this paper a simple analytical derivation of the parallel velocity distribution of the fragments produced in the elastic break-up of halo nuclei. We find a significant postacceleration effect, in contrast to other works [8–10]. Our results are only valid if the breakup proceeds direct to the structureless continuum, i.e. without a relevant resonance. The experimental data seems to favor the first situation.

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Figure captions

**Figure 1** The parallel momentum distribution shift for $^{11}\text{Li}$ break-up on $^{208}\text{Pb}$ as a function of the bombarding energy per nucleon. Also shown is the width of the momentum distribution, $\Gamma$, as due to the Coulomb dissociation process (solid line).
