Two-Dimensional Linear and Nonlinear Talbot Effects from Rogue Waves

Yiqi Zhang1,∗, Milivoj R. Belić2,†, Milan S. Petrović2,3, Huaibin Zheng1, Haixia Chen1, Changbiao Li1, Keqing Lu4, and Yanpeng Zhang1

1 Key Laboratory for Physical Electronics and Devices of the Ministry of Education & Shaanxi Key Lab of Information Photonic Technique, Xi'an Jiaotong University, Xi'an 710049, China
2 Science Program, Texas A&M University at Qatar, P.O. Box 23874 Doha, Qatar
3 Institute of Physics, P.O. Box 68, 11001 Belgrade, Serbia
4 School of Electronics and Information Engineering, Tianjin Polytechnic University, Tianjin 300387, China

(Dated: December 2, 2014)

We introduce two-dimensional (2D) linear and nonlinear Talbot effects (LTE and NLTE). They are produced by 2D diffraction patterns and can be visualized as 3D stacks of Talbot carpets. The NLTE originates from 2D rogue waves and forms in a bulk 3D nonlinear medium. The recurrence of an input rogue wave (with a phase shift) can only be observed at the (half) Talbot length. Different from NLTE, the LTE displays the usual fractional Talbot images as well. We also find that the smaller the period of incident rogue waves, the shorter the Talbot length. Increasing the beam intensity increases the Talbot length, but above a threshold this leads to a catastrophic self-focusing phenomenon which destroys the nonlinear effect.

PACS numbers: 42.65.Hw, 42.65.Sf

I. INTRODUCTION

Recently, optical rogue waves attracted a lot of attention, owing to their strange properties [1–4]. As a phenomenon, rogue wave originated in oceans, as an extreme localized wave that suddenly appears and disappears without a trace. However, it is now accepted that, as a kind of nonlinear phenomenon, it can be well described by the cubic nonlinear Schrödinger equation (NLSE). The cubic NLSE possesses a variety of solutions [5–7], some of which can serve as prototypical rogue waves. These include Peregrine solitons [2, 8], Kuznetsov–Ma breathers [9, 10], Akhmediev breathers (AB) [11], higher-order rogue waves [12, 13], and Fermi–Pasta–Ulam (FPU) recurrent pulses [14–16]. It should be emphasized that these solutions are not rogue waves per se but can be used to model ones. The true rogue waves are extreme wave phenomena that sporadically appear in the solutions of different NLSEs and require statistical description for evidence and confirmation.

The solutions mentioned above are by and large exact solutions to NLSE that contain solitary or trains of pulses, ride on finite backgrounds, and are prone to modulation instabilities. It is in the development of unstable wavefronts that extreme or freak waves may appear. In this Paper we restrict our attention to just one such solution that is used to describe rogue waves but also displays the nonlinear Talbot effect (NLTE) – the AB. An AB wave is periodic along the transverse coordinate and decays along the longitudinal coordinate – it is a transverse train of optical pulses. When such a train is launched into a nonlinear Kerr medium, owing to modulation instability and nonlinear interference of propagating pulses, a Talbot recurrence phenomenon is observed. Thanks to these properties, the NLTE of rogue waves has recently been reported [17]. Such TE is in stark contrast to the linear TE, in which real gratings or periodic diffracting structures are needed, it forms in linear homogeneous media, and can be generally explained by the Fresnel diffraction theory [18]. In addition, the NLTE reported in [17] not only originates from a nonlinear wave but also requires bulk nonlinear medium to form, which is different from the NLTE reported in [19].

As it is well known, TE represents a self-imaging phenomenon in the near-field diffraction of plane waves, first observed in 1836 by H. F. Talbot [20] and theoretically explained in 1881 by Lord Rayleigh [21]. Owing to its potential applications in image preprocessing and synthesis, photolithography, optical testing, optical metrology, spectrometry, and optical computing, TE has been reported in, but not confined to, atomic optics [22, 23], quantum optics [24, 25], waveguide arrays [26], photonic lattices with P T-symmetry [27], Bose-Einstein condensates [28, 29], X-ray imaging [30], and in the interferometer for C70 fullerene molecules [31]. Even though the NLTE was previously investigated [17, 19, 32], the topic is still in need of further exploration, because interdisciplinary research – in this case of rogue waves, TE, and nonlinear interference – tends to induce new ideas and applications.

TE is a spatial recurrence phenomenon: if one records directly beyond a narrow diffraction slit the beam intensity along the propagation direction, then one sees a series of periodic diminishings and revivals, coming from the interference of secondary diffracted wavelets.
where $\zeta$ is the period of the AB. The maximum of $|\nabla \psi(z, x/\sqrt{2})| = 0$ (1)

The propagation of a beam with envelope $\psi$ in a Kerr medium can be generally described by the normalized cubic NLSE

$$i \frac{\partial \psi}{\partial z} + \frac{1}{2} \nabla^2 \psi + |\psi|^2 \psi = 0$$

where $\nabla^2$ is the transverse Laplacian, in one dimension $\nabla^2 = \partial^2 / \partial x^2$ and in two dimensions $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$. One of the rogue solutions of the 1D NLSE is the AB [11]

$$\psi(z, x) = (1 - 4q) \cosh(az) + \sqrt{2q} \cos(\Omega x) + ia \sinh(az) \sqrt{2q} \cos(\Omega x)$$

which is the product of two perpendicular 1D ABs $\psi(z = 0, x)$ and $\psi(z = 0, y)$. The periods along $x$ and $y$ directions are the same, $D_x = D_y = \pi / \sqrt{1 - 2q}$. The coefficient $1 / \sqrt{M}$ in Eq. (3) makes the maximum amplitude equal to that of Eq. (2), if $C = 1$.

In the following, we study the propagation and dynamics of the 2D rogue wave with $C = 1$ in Eq. (3), in both linear and nonlinear bulk media. To guarantee high numerical precision, we utilize the fourth-order split step fast Fourier transform (FFT) method [35] in double precision. To make beams of finite energy and prevent FFT spill-over effects, we utilize an aperture with a diameter large enough to enforce fast convergence of beam intensity to zero at the transverse infinity.

In Fig. 1(a) we display the transverse intensity iso-surfaces of the propagating beam with $q = 1/4$, at different propagation distances. It is immediately seen that the linear 2D TE is there – the incident rogue wave reappears at $z \approx 6.192$. This distance defines the Talbot length $\zeta_T$. At $\zeta_T / 4$ and $\zeta_T / 2$, the fractional Talbot images also form. Similar to previous research [19, 23, 26], the fractional Talbot image at $\zeta_T / 2$ exhibits a $\pi$ phase shift in comparison with the incident, while for the $\zeta_T / 4$ fractional Talbot image, the transverse period is halved. To see the Talbot effect more clearly, we also plot the intensity carpet versus $x$ and $z$ in the plane $y = 0$, in Fig. 1(b). The Talbot images at $\zeta_T / 4$ and $\zeta_T / 2$ are quite apparent, while the one at $\zeta_T / 2$ is missing. The reason is that because of the $\pi$ phase shift, the Talbot image at $\zeta_T / 2$ is shifted for half of the period transversely and cannot be seen in the $y = 0$ plane. In Fig. 1(c), $z$ is the intensity profiles at $z = 0, z = \zeta_T / 4, z = \zeta_T / 2$, and $z = \zeta_T$ along the $x/y$ axes are displayed. In conclusion, the 2D LTE of rogue waves is easily visible and verified.

Numerical simulations demonstrate that the LTE does not change with different incident intensities, for the same value of $q$. Therefore, we can check the influence of $q$ (also of the transverse period) on the formation of LTE, even though the intensity of the incident beam changes with $q$ [17]. More numerical experiments indicate that
the Talbot length is proportional to the transverse period (nearly a linear relationship), as shown by the solid-circle line in Fig. 1(d). The same figure also displays the relationships of $D_x$ vs $q$ (dashed curve) and $D_x$ vs $M$ (dash-dotted curve).

One can only observe the secondary and primary Talbot images at $z = 0$ plane in Fig. 2(b), from which one can verify that there are no fractional Talbot images. It is evident that in the propagation range $z \in [z_T/2 \ z_T]$ a structure similar to the fractional Talbot image appears, but this is not a fractional Talbot image, because of the following reasons: (1) the transverse period is same as of the input; (2) the position is not at $3z_T/4$. Such images also exist in the range $z \in [0 \ z_T/2]$, however one cannot see them in Fig. 2(b) because the maximum is shifted from the plane $y = 0$, similar to the image at $z = z_T/2$. More clear details about the nonlinear, as well as linear evolution of Talbot images, can be seen in the movies provided in the Supplemental Material [36].

**IV. NONLINEAR TALBOT EFFECT**

We investigate the 2D NLTE, by including nonlinearity in the Schrödinger equation. The practical problems are, the self-focusing effect and the lack of superposition principle. Nonetheless, following the same numerical procedure as in the linear case, the 2D NLTE is demonstrated, with $z_T \approx 6.544$. Results are presented in Fig. 2, which follows the same setup as Fig. 1. In Fig. 2(a) we show the isosurface intensity plots which display the formation of 2D-NLTE. One striking difference with the LTE is immediately apparent: the fractional Talbot images are gone. One can only observe the secondary and primary Talbot images at $z_T/2$ and $z_T$. This is fundamentally different from the 2D LTE. We also display the intensity carpet in the $y = 0$ plane in Fig. 2(b), from which one can verify that there are no fractional Talbot images. It is evident that in the propagation range $z \in [z_T/2 \ z_T]$ a structure similar to the fractional Talbot image appears, but this is not a fractional Talbot image, because of the following reasons: (1) the transverse period is same as of the input; (2) the position is not at $3z_T/4$. Such images also exist in the range $z \in [0 \ z_T/2]$, however one cannot see them in Fig. 2(b) because the maximum is shifted from the plane $y = 0$, similar to the image at $z = z_T/2$. More clear details about the nonlinear, as well as linear evolution of Talbot images, can be seen in the movies provided in the Supplemental Material [36].

The intensity profiles of the beam at $z = 0$, $z = z_T/2$ and $z = z_T$ along the $x/y$ axes are shown in Fig. 2(c). Even though the intensity carpet in Fig. 2(b) is not entirely symmetric about $z = z_T/2$, the image at $z = z_T$ is the same as the input, and that at $z_T/2$ has a $\pi$ phase shift (and is absent from the figure). The intensity profiles in Fig. 2(c) demonstrate the formation of the 2D NLTE. Note that the modulation instability in the Kerr nonlinear medium results in the formation of the 2D NLTE that is a bit imperfect – the intensity maximum is a bit smaller than that of the incidence and small humps appear between the neighboring peaks. However, the formation of 2D NLTE of rogue waves in bulk nonlinear medium is clearly demonstrated, because the images at $z_T/2$ and $z_T$ are definitely there. Again, one should check the evolution displayed in the movie in the Supplemental Material [36], to see the formation of 2D NLTE more clearly.

Since $q$ (or the transverse period) is related to the intensity of the input beam and optical response of the nonlinear medium is sensitive to the beam intensity, the formation of 2D NLTE with different transverse periods will be much more complex than that of the 2D LTE. In Fig. 2(d) we show the changing trend of the Talbot length versus the transverse period. The relationships
between the transverse period and $q$ as well as $M$ are also displayed, which are similar to those in Fig. 1(d). As indicated by numerical simulations, the relationship between $D_x$ and $2T$ is not linear at all. Phenomenologically, the Talbot length increases more slowly with the increase in $D_x$ for $q < 0.2$, while for $q > 0.2$ the relation seems more linear. Viewed as a whole, the relation between $D_x$ and $2T$ is parabolic-like.

![Figure 3](image-url)  
**FIG. 3.** (Color online) Talbot dependence on the amplitude of the incident with fixed transverse periods.

The amplitude of the incident 2D AB in Eq. (3) can be adjusted by changing the value of $C$, so we investigate numerically the relationship between the intensity and Talbot length for the same transverse period. This is important because of the potential wave collapse in 2D. We fix $q$ (correspondingly fixing $D_x$ and $D_y$) and change the amplitude of the incident beam, to calculate the Talbot length; the results are depicted in Fig. 3. Three values of $q$ are chosen, 1/5, 1/4 and 3/10, respectively. As shown in Fig. 2(d), the bigger the $q$, the larger the Talbot length $z_T$, so that in Fig. 3, the Talbot length is the biggest when $q = 3/10$, for the same $C$. With $C$ increasing, the Talbot length also increases, and such increment becomes quite fast when $C > 1$. The reason is that higher intensity leads to stronger modulation instability, which demands a longer distance to adjust itself during propagation. Since the nonlinearity is Kerr in the NLSE, the value of $C$ cannot be chosen very high, because this would result in the well-known “catastrophic self-focusing effect” [37], i.e., the beam will collapse during propagation. Our numerics suggests the existence of a critical value of $C$ above which the collapse happens.

## V. CONCLUSIONS

We have discovered 2D NLTE for the first time theoretically, to the best of our knowledge. For the effect to be seen, not only the incident beam should be nonlinearly prepared, but the formation should also happen in the bulk Kerr nonlinear medium. We also demonstrate the 2D linear Talbot effect. Different from the 2D LTE, there are no fractional Talbot images in the 2D NLTE. Numerical experiments demonstrate that the smaller the transverse period and the smaller the amplitude of the incident beam, the shorter the Talbot length in 2D NLTE. We believe there are still many interesting questions to be answered concerning the 2D NLTE; for example: How will the effect change if the input is changed, for example to a product of doubly-periodic AB breathers? Will the effect appear if the nonlinearity in NLSE is of saturable or cubic-quintic nature? What is the threshold value of intensity before the wave collapse takes over? How long along the propagation axis the effect persists, i.e., what are the finite-size effects? We hope that our research has broadened the potential applications of Talbot effect and have deepened the understanding of rogue waves.

This work was supported by the 973 Program (2012CB921804), CPSF (2014T70923, 2012M521773), KSTIT of Shaanxi province (2014KCT-10), NSFC (11474228, 61308015, 11104214, 61108017, 11104216, 61205112), and the NPRP 6-021-1-005 project of the Qatar National Research Fund (a member of the Qatar Foundation).

[1] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, Nature **450**, 1054 (2007).
[2] B. Kibler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, N. Akhmediev, and J. M. Dudley, Nature Phys. **6**, 790 (2010).
[3] A. Montina, U. Bortolozzo, S. Residori, and F. T. Arecchi, Phys. Rev. Lett. **103**, 173901 (2009).
[4] S. Birkholz, E. T. J. Nibbering, C. Brée, S. Skupin, A. Demircan, G. Genty, and G. Steinmeyer, Phys. Rev. Lett. **111**, 243903 (2013).
[5] I. Kaminer, M. Segev, and D. N. Christodoulides, Phys. Rev. Lett. **106**, 213903 (2011).
[6] Y. Q. Zhang, M. Belić, Z. K. Wu, H. B. Zheng, K. Q. Lu, Y. Y. Li, and Y. P. Zhang, Opt. Lett. **38**, 4585 (2013).
[7] Y. Zhang, M. R. Belić, H. Zheng, H. Chen, C. Li, Y. Li, and Y. Zhang, Opt. Express **22**, 7160 (2014).
[8] D. H. Peregrine, J. Aust. Math. Soc. Ser. B **25**, 16 (1983).
[9] Y.-C. Ma, Stud. Appl. Math. **60**, 43 (1979).
[10] B. Kibler, J. Fatome, C. Finot, G. Millot, G. Genty, B. Wetzel, N. Akhmediev, F. Dias, and J. M. Dudley, Sci. Rep. **2**, 463 (2012).
[11] N. Akhmediev, V. Eleonskii, and N. Kulagin, Theor. Math. Phys. **72**, 809 (1987).
[12] M. Erkintalo, K. Hammani, B. Kibler, C. Finot, N. Akhmediev, J. M. Dudley, and G. Genty, Phys. Rev. Lett. **107**, 253901 (2011).
[13] D. J. Kedziora, A. Ankiewicz, and N. Akhmediev, Phys. Rev. E **88**, 013207 (2013).
[14] G. Van Simaeyns, P. Emplit, and M. Haelterman, Phys. Rev. Lett. **87**, 033902 (2001).
[15] A. Mussot, A. Kudlinski, M. Droques, P. Szriftgiser, and N. Akhmediev, Phys. Rev. X **4**, 011054 (2014).
[16] S. Wabnitz and B. Wetzel, Phys. Lett. A 378, 2750 (2014).
[17] Y. Q. Zhang, M. R. Belić, H. B. Zheng, H. X. Chen, C. B. Li, J. P. Song, and Y. P. Zhang, Phys. Rev. E 89, 032902 (2014).
[18] J. Wen, Y. Zhang, and M. Xiao, Adv. Opt. Photon. 5, 83 (2013).
[19] Y. Zhang, J. Wen, S. N. Zhu, and M. Xiao, Phys. Rev. Lett. 104, 183901 (2010).
[20] H. F. Talbot, Philos. Mag. 9, 401 (1836).
[21] L. Rayleigh, Philos. Mag. 11, 196 (1881).
[22] J. Wen, S. Du, H. Chen, and M. Xiao, Appl. Phys. Lett. 98, 081108 (2011).
[23] Y. Q. Zhang, X. Yao, C. Z. Yuan, P. Y. Li, J. M. Yuan, W. K. Feng, S. Q. Jia, and Y. P. Zhang, IEEE Photon. J. 4, 2057 (2012).
[24] X.-B. Song, H.-B. Wang, J. Xiong, K. Wang, X. Zhang, K.-H. Luo, and L.-A. Wu, Phys. Rev. Lett. 107, 033902 (2011).
[25] H. Jin, P. Xu, J. S. Zhao, H. Y. Leng, M. L. Zhong, and S. N. Zhu, Appl. Phys. Lett. 101, 211115 (2012).
[26] R. Iwanow, D. A. May-Arrioja, D. N. Christodoulides, G. I. Stegeman, Y. Min, and W. Sohler, Phys. Rev. Lett. 95, 053902 (2005).
[27] H. Ramezani, D. N. Christodoulides, V. Kovanis, I. Vitebskiy, and T. Kottos, Phys. Rev. Lett. 109, 033902 (2012).
[28] L. Deng, E. W. Hagley, J. Denschlag, J. E. Simsarian, M. Edwards, C. W. Clark, K. Helmerson, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. 83, 5407 (1999).
[29] C. Ryu, M. F. Andersen, A. Vaziri, M. B. d’Arcy, J. M. Grossman, K. Helmerson, and W. D. Phillips, Phys. Rev. Lett. 96, 160403 (2006).
[30] F. Pfeiffer, M. Bech, O. Bunk, P. Kraft, E. F. Eikenberry, C. Brönnimann, C. Grünzweig, and C. David, Nature Mater. 7, 134 (2008).
[31] B. Brezger, L. Hackermüller, S. Uttenthaler, J. Petschinka, M. Arndt, and A. Zeilinger, Phys. Rev. Lett. 88, 100404 (2002).
[32] J. Wen, Y. Zhang, S.-N. Zhu, and M. Xiao, J. Opt. Soc. Am. B 28, 275 (2011).
[33] N. N. Akhmediev, Nature 413, 267 (2001).
[34] G. V. Simaeys, P. Emplit, and M. Haelterman, J. Opt. Soc. Am. B 19, 477 (2002).
[35] J. Yang, Nonlinear Waves in Integrable and Non-Integrable Systems (SIAM, Philadelphia, 2010).
[36] See Supplemental Material at http://link.aps.org/supplemental/ for the whole evolution of the beams shown in Figs. 2(a) and 1(a).
[37] A. L. Gaeta, Phys. Rev. Lett. 84, 3582 (2000).