Influence of shear motion on evolution of molecular clouds in the spiral galaxy M 51

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Abstract

We have investigated the dynamics of the molecular gas and the evolution of giant molecular associations (GMAs) in the spiral galaxy M 51 with the Nobeyama Radio Observatory 45-m telescope. The velocity components of the molecular gas perpendicular and parallel to the spiral arms are derived at each spiral phase from the distribution of the line-of-sight velocity of the CO gas. In addition, the shear motion in the galactic disk is determined from the velocity vectors at each spiral phase. It is revealed that the distributions of the shear strength and of GMAs are anti-correlated. GMAs exist only in the area of the weak shear strength and further on the upstream side of the high shear strength. GMAs and most giant molecular clouds (GMCs) exist in the regions where the shear critical surface density is smaller than the gravitational critical surface density, indicating that they can stably grow by self-gravity and the collisional agglomeration of small clouds without being destroyed by shear motion. These factors indicate that the shear motion is an important factor in evolution of GMCs and GMAs.

Key words: galaxies: individual (M 51) — galaxies: ISM — galaxies: kinematics and dynamics — galaxies: spiral — radio lines: galaxies

1 Introduction

The evolution of molecular clouds is one of the keys to understanding star formation in a galaxy and hence the evolution of that galaxy. Massive stars are formed in GMCs (giant molecular clouds), and GMCs constitute GMAs (giant molecular associations), where typical sizes of a GMC and a GMA are a few 10 pc (Sanders et al. 1983) and a few 100 pc (Vogel et al. 1988), respectively. Spiral arms are a key to understanding the evolution of GMCs. Observationally the molecular arms are traced by CO lines with small offset relative to the optical arms identified by Hα (e.g., Vogel et al. 1988). A density wave (Lin & Shu 1964) generates a galactic shock in the spiral potential (Fujimoto 1968) which explains the offset of the optical arms from the molecular arms. Massive star formation could be induced by the shock and cloud–cloud collisions in the molecular arms (Roberts et al. 1990), so that the optical arms are located at the slightly downstream side of the molecular arms. GMCs can be destroyed by photodissociation owing to UV radiation from massive stars and shocks caused by supernova explosions (e.g., McKee & Ostriker 1977; Allen et al. 1986; Seta et al. 1998). The
Table 1. Parameters of M51.

| Parameter                        | Value                                      | Source                        |
|----------------------------------|--------------------------------------------|-------------------------------|
| Center position RA (J2000.0)     | 13h29m52.711                               | Turner and Ho (1994)          |
| Dec (J2000.0)                    | 47°11′42″61                               | Sandage and Tammann (1974)    |
| Distance                         | 9.6 Mpc                                    | de Vaucouleurs et al. (1991)  |
| Morphological type               | SAbc                                       | This paper                    |
| Systemic velocity (LSR)          | 469 ± 4 km s\(^{-1}\)                      | This paper                    |
| Position angle of the major axis | −9° ± 6°                                   | This paper                    |
| Inclination angle                | 22° ± 3°                                   | This paper                    |

Fig. 1. Measured spectra of \(^{12}\)CO(1–0) emission in the central region of M51, parallel to the minor \((\theta_{PA} = 80°)\) and major \((\theta_{PA} = −10°)\) axes of the galactic disk. For each spectrum the abscissa is the LSR velocity \((V_{LSR} = 200–900\ \text{km s}^{-1})\) and the ordinate is the main-beam brightness temperature \((T_{mb} = 0.5–2.0\ \text{K})\). The grid spacing between the spectra is 10″.

destroyed gas would be in the atomic phase rather than the molecular phase in the downstream and interarm regions as far as the conventional density wave and galactic shock theories hold. However, detections of GMCs and GMAs have recently been reported in the interarm regions. In the grand design spiral galaxy M51, Koda et al. (2009) showed that GMCs \((M \sim 10^5–10^6\ M_\odot)\) exist not only in the arms but also in the interarm regions. They also found GMAs \((\leq 10^7\ M_\odot)\) in the arms and the remnants of fragmented GMAs in the interarms. From these results they suggested that GMAs were fragmented into GMCs by kinematic shear caused by abrupt changes in velocity. Muraoka et al. (2009) found that molecular clouds in the interarms of M83 were not necessarily virialized, while molecular clouds in the arms were virialized, and suggested that the non-virialized molecular clouds in the interarms might be formed by the shear motion. Strong shear may generate turbulence in the clouds that suppresses star formation. However, the relation between molecular clouds and the kinetic shear motion in the clouds is still speculation, because the kinetic shear in the scale of GMCs and/or GMAs has not been directly measured in a galaxy. We have to know the strength and length of the shear observationally.

The nearby galaxy M51 at the distance of 9.6 Mpc (Sandage & Tammann 1974) has a nearly face-on disk (inclination angle \(i = 20°\); Tully 1974a) with the grand design two-fold spiral structure. Although the galaxy is interacting with its companion galaxy NGC 5195, the inner disk is less affected (see subsection 3.6). Since CO emission is fairly strong throughout the galactic disk, the
distribution and kinematics of molecular gas can be easily measured. Thus M 51 is an ideal target to obtain the shear motion in the galactic disk. Using the data of $^{12}$CO($J = 1$–0) newly mapped with the Nobeyama 45-m telescope, we determined the kinetic shear in the whole disk of the galaxy from velocity vectors of molecular gas obtained by applying the method of Kuno and Nakai (1997). In this paper, we report the results and discuss the relation between the strength of the shear and formation and destruction of GMCs and GMAs. The basic parameters of M 51 adopted in this paper are summarized in table 1. Velocities used here are in the radio definition and with respect to the local standard of rest (LSR). The LSR velocity, $V_{\text{LSR}}$, is converted from the heliocentric velocity, $V_{\text{helio}}$, by using $V_{\text{LSR}} = V_{\text{helio}} + 11.7 \text{ km s}^{-1}$ for this galaxy.

## 2 Observations

Observations of $^{12}$CO($J = 1$–0) emission were carried out between 2004 December and 2007 March using the 45-m telescope of the Nobeyama Radio Observatory (NRO). The antenna was equipped with the 5 × 5-beam SIS heterodyne receiver array (BEARS), which could measure 25 spectra of CO simultaneously (Sunada et al. 2000). The beam separation of BEARS was $41.2''$, and the full half power beamwidth (HPBW) was $16''$ at $115$ GHz, corresponding to $745$ pc at the distance of $9.6$ Mpc (Sandage & Tammann 1974). The receiver backends were the 1024-ch digital spectrometers (Sorai et al. 2000). The total bandwidth and frequency resolution of the spectrometers were $512$ MHz and $605$ kHz, which correspond to $1330$ km s$^{-1}$ and $1.57$ km s$^{-1}$, respectively. The line intensity was calibrated by the chopper wheel method, yielding an antenna temperature, $T_A^{*}$, corrected for both atmospheric and antenna ohmic losses (Ulich & Haas 1976). In this paper we use the main beam brightness temperature $T_{\text{mb}} = T_A^{*}/\eta_{\text{mb}}$, using the main beam efficiency of the antenna $\eta_{\text{mb}} = 0.32$ to 0.39. The observations were made in the position-switching mode with an integration time of $20$ s per scan and OFF-positions offset by $\pm 8''$ in the direction of the azimuth. The intensity was calibrated every 10 sequences of ON(source)–OFF(sky) positions. The telescope pointing was checked every $45$ min–1 hr by observing SiO maser emission of the late-type star R CVn. The pointing error was mostly less than $5''$, but $2''$ in the center region of the galaxy. The system noise temperatures during the observations were $600$–$1100$ K (SSB) in
resolution. Figure 1 shows the spectra in the central region of M 51. The maximum of the main beam brightness temperature is $T_{mb} = 1.6$ K. In the disk of M 51, the brightness temperatures in the spiral arms and the interarms are $T_{mb} \geq 0.5–1$ K and $\approx 0.2–0.4$ K, respectively, which are consistent with the previous values measured with the 45-m telescope (Nakai et al. 1994). The rms noise level, $\Delta T_{\text{rms}}$, is less than 0.1 K, but the velocity width is very wide ($\Delta V_{\text{FWHM}} \approx 45$ km s$^{-1}$). In more detail, the spectra show two velocity components of CO emission; a narrow component at $V_{\text{LSR}} \approx 400–450$ km s$^{-1}$ and a wide component at $V_{\text{LSR}} \approx 450–750$ km s$^{-1}$ in the vicinity of $(X, Y) = (145^\circ, 400^\prime)$). The two distinct components have been pointed out in an earlier study (Sage 1989). Figure 3 shows the distributions of the narrow and wide components separately. Since the distribution of the narrow component is connected to a spiral arm of M 51 (subsection 3.2), the component arises from the overlying spiral arm of M 51, while the wide component, which includes the systemic velocity ($V_{\text{LSR}} = 642$ km s$^{-1}$; Sage 1989) of NGC 5195, is associated with NGC 5195.

3.2 Distribution of the CO intensity

Figure 4a shows the distribution of the CO integrated intensity, $I_{\text{CO}} \equiv \int T_{mb} dv$ (K km s$^{-1}$). The distribution of $I_{\text{CO}}$ shows two spiral arms as seen in the previous maps (Rand & Kulkarni 1990; García-Burillo et al. 1993a; Nakai et al. 1994; Aalto et al. 1999; Helder et al. 2003; Schuster et al. 2007; Shetty et al. 2007; Koda et al. 2009; Vlahakis et al. 2013). One is extending to the companion NGC 5195 and another, the counterpart arm, is extending toward the southwest. The CO(1–0) distribution is in agreement with interferometric observations of CO(1–0) (Shetty et al. 2007; Koda et al. 2011) and analogous to the distribution of CO(2–1) (Schuster et al. 2007) except for the vicinity of NGC 5195. The curvature of the CO(1–0) distribution at the connection between M 51 and NGC 5195 in figure 3 and figure 4a is steeper than that of CO(2–1) in Schuster et al. (2007). The difference between those distributions can be attributed to the narrowness of the integration range in CO(2–1), $V_{\text{LSR}} = 350–600$ km s$^{-1}$, despite that the velocity components around NGC 5195 extend out to 750 km s$^{-1}$.

3 Results

3.1 CO spectra

All the measured $^{12}$CO(1–0) spectra have been smoothed to a velocity resolution of 5 km s$^{-1}$ to reduce noise. One $\sigma$ of the $I_{\text{CO}}$, i.e., $\Delta I_{\text{rms}}$, is expressed by $\Delta I_{\text{rms}} = \Delta T_{\text{rms}} \sqrt{\Delta V_{\text{FWHM}} \Delta V_c}$, where $\Delta V_c$ is the smoothed velocity at observing elevations. We observed 2760 points in the region of about $9' \times 10'$ (but no point in north-west of the map), which contained the whole optical disks of M 51 and NGC 5195, and the bridge between them. Observed grid points were located parallel to the major ($X$) and the minor ($Y$) axes of the galactic disk whose position angle was $-10^\circ$ (Tully 1974a). The transformation from the ($X$, $Y$) coordinate to ($dRA$, $dDec$) is given by $dRA = 0.9848X + 0.1736Y$, $dDec = 0.1736X + 0.9848Y$, adopting the position angle of the disk to be $\theta_{PA} = -10^\circ$. The interval of the grid points were 10$''$.

![Image](https://example.com/image.png)

Fig. 3. $^{12}$CO(1–0) intensity maps of the narrow component integrated at $V_{\text{LSR}} = 400–450$ km s$^{-1}$ (dotted contours) and of the wide component integrated at $V_{\text{LSR}} = 450–750$ km s$^{-1}$ (solid contours) overlaid on the R-band image [NASA/IPAC Extragalactic Database (NED); Hoopes et al. 2001]. The first contours and contour intervals are 5, 5, and 5 K km s$^{-1}$ for the narrow (dotted lines) and 10 K km s$^{-1}$ and 5 K km s$^{-1}$ for the wide component (solid lines), respectively. The narrow and wide components arise from an overlying spiral arms of M 51 and from molecular gas inside NGC 5195, respectively. A cross denotes the center of NGC 5195. (Color online)
Fig. 4. (a) Map of the $^{12}$CO(1–0) integrated intensity, $I_{\text{CO}}(1–0) = \int T_{mb} \, dv$ [K km s$^{-1}$], in M 51 and its companion galaxy NGC 5195 in the northeast. Contours are from 5, 10, 15, 20 to 70 K km s$^{-1}$ in steps of 10 K km s$^{-1}$. The origin of the coordinates is the center of M 51 (table 1). The image resolution is 16″. (b) The $^{12}$CO(1–0) velocity field map (solid contours) derived from intensity-weighted mean velocities overlaid with the distribution of $I_{\text{CO}}$ (colour). The velocity is in km s$^{-1}$ with respect to LSR and the radio definition.

Fig. 5. Position–velocity diagram along the major axis (Y) of M 51. The lowest contour and the contour interval of the main beam brightness temperature are 0.15 K. Other data of velocities are also plotted; H I observed with the Very Large Array (Rots et al. 1990), Hα with the Kitt Peak 4-m telescope (Goad et al. 1979) and CO(1–0) with the BIMA (Shetty et al. 2007).
In addition, the distribution of CO(3–2) (Vlahakis et al. 2013), which is similar to that of CO(2–1) rather than CO(1–0), is also explained by the limited integration range ($V_{\text{LSR}} \approx 380–580$ km s$^{-1}$). The distribution of CO(1–0) in figure 3 corresponds with the dust lane in the R-band image (Hoopes et al. 2001).

3.3 Basic parameters of M 51

3.3.1 Dynamical center and systemic velocity

Figure 5 shows the position–velocity diagram of CO (contours) along the major axis of M 51 with the position angle of $\theta_{\text{PA}} = -10^\circ$ (Tully 1974a). The dynamical center and the systemic velocity were derived from the position–velocity diagram by averaging the CO intensity-weighted velocity at $|r| < 150''$, assuming that the velocity field was symmetrical to the dynamical center. The resultant systemic velocity was $V_{\text{sys}} = 469 \pm 4$ km s$^{-1}$ which was consistent with the result of Tully (1974a), $V_{\text{sys}} = 475 \pm 3$ km s$^{-1}$ in radio definition and with respect to LSR. As shown in the next subsection, our derived position angle is $\theta_{\text{PA}} = -9^\circ$ which is slightly different to $\theta_{\text{PA}} = -10^\circ$. Even if we use $\theta_{\text{PA}} = -9^\circ$, the difference of $V_{\text{sys}}$ is less than 1 km s$^{-1}$. Therefore we adopt $V_{\text{sys}} = 469$ km s$^{-1}$ hereafter.

The dynamical center which showed the systemic velocity was $\Delta Y = +2'' \pm 2''$ away from the adopted center of the 5-GHz continuum peak (Turner & Ho 1994). Since the position is consistent with that of the continuum peak within the error and the apparent difference of $\Delta Y = +2''$ is much smaller than our angular resolution ($16''$), we use the position of the radio continuum peak, whose accuracy was $0''.1-0''.5$, as the center of M 51 for analyses in discussion.

3.3.2 Position angle

The position angle of the major axis of the M 51 disk has been determined from the kinematics of Hα and CO (Tully 1974a; Kuno & Nakai 1997; Shetty et al. 2007). We determined it in a similar way to Kuno and Nakai (1997), using our new data. The galactic disk was divided into annuli with a width of $\Delta r = 10''$ centered on the galactic center, and the observed radial velocities at observed points ($r$, $\theta_{\text{obs}}$) in each annulus ($r =$ distance from the center, $\theta_{\text{obs}} =$ position angle), $V_{\text{rad}}[= V_{\text{sys}} - V_{\text{gal,o}}\cos(\theta_{\text{obs}} - \theta_{\text{PA}})]$, were fitted with two parameters, the observed rotation velocity, $V_{\text{gal,o}}$, and the position angle of the galaxy, $\theta_{\text{PA}}$, where $V_{\text{sys}}$ is 469 km s$^{-1}$. Figure 6 shows the result of $\theta_{\text{PA}}$ derived by this method. The position angle declines with the galactic radius from $\theta_{\text{PA}} \approx 5^\circ$ at $r = 40''$ to $\approx -20^\circ$ at $r = 140''$. The mean values of $\theta_{\text{PA}}$ in the range of $r = 10''-150''$ ($r = 40''-70'', 70''-110''$, and $110''-140''$) are $-9^\circ \pm 6^\circ$ ($-3^\circ$, $-10^\circ$, and $-15^\circ$, respectively), which is consistent with the results of Tully (1974) ($\theta_{\text{PA}} = -10^\circ \pm 3^\circ$) and Kuno and Nakai (1997) ($\theta_{\text{PA}} = -8.4^\circ$).

It is to be noted that the position angle derived by this method is biased from the true value in the counter-clockwise direction due to the non-circular motion of the molecular gas influenced by the spiral potential. The spiral arms cross the major axis in the region of $r = 40''-70''$ (see figure 5), where the deviation from the rotation of the disk ($V_{\text{gal,o}} \approx 70$ km s$^{-1}$) is $\Delta V_{\text{gal,o}} \approx 10$ km s$^{-1}$. This offset induces the bias of the position angle $\theta_{\text{PA}}$ of $\Delta \theta_{\text{PA}} \approx 10^\circ$ in the counter-clockwise direction. However, the systematic variation of the position angle at $r = 10''-150''$ in figure 6 cannot be explained by the effect of only the non-circular motion but must also be due to the warped disk which may be caused by interaction with the companion NGC 5195 (e.g., Shetty et al. 2007).

3.3.3 Inclination angle

We estimated the inclination angle ($i$) of the M 51 disk by using the baryonic Tully–Fisher relation (McGaugh 2005; Shetty et al. 2007), where $i = 90^\circ$ is edge-on. McGaugh (2005) showed the relation of $M_b = 50 V_{\text{rot}}^4$ between the total baryonic mass of a galaxy, $M_b(= M_{\text{star}} + M_{\text{gal}})$, and the rotation velocity of it, $V_{\text{rot}} = V_{\text{gal,o}}/\sin i$. We used the baryonic masses in M 51 of $M_{\text{star}} = 5.2^{+1.1}_{-1.0} \times 10^{10} M_{\odot}$ (HyperLeda database, Bell et al. 2003) and $M_{\text{gas}} = 1.36(M_{\text{He}} + M_{\text{H}} + M_{\text{H}_2})$ including helium, where $M_{\text{He}} = (5.2 \pm 0.7) \times 10^9 M_{\odot}$ (subsection 3.4), $M_{\text{H}} = (2.9 \pm 0.2) \times 10^9 M_{\odot}$ (Walter et al. 2008) and $M_{\text{H}_2} \leq 1.4 \times 10^9 M_{\odot}$ (van der Hulst et al. 1988; Read & Ponman 2001). We obtained the inclination angle of $i = 22^\circ \pm 3^\circ$, using $V_{\text{rot}} = 190^{+12}_{-15}$ km s$^{-1}$ derived from the baryonic mass (see table 2) and $V_{\text{gal,o}} = 70 \pm 7$ km s$^{-1}$ in the range.
of $40'' \leq |Y| \leq 180''$ and $|X| \leq 10''$ (see figure 5). The derived inclination angle is consistent with the value of $i = 20'' \pm 5''$ derived by a morphological analysis of Hα data (Tully 1974a). The radial variation of the inclination angle of the disk ($20'' \lesssim r \lesssim 105''$) was evaluated kinematically by Shetty et al. (2007), who suggested that the disk was warped and twisted. Actually, the rotation velocities are steeply decreased in the outer disk traced by HI (figure 5). This can indicate that the outer disk is warped and more inclined than the inner disk.

### 3.4 Mass and radial distribution of molecular gas

The H$_2$ surface density is obtained from $I_{\text{CO}}$ by applying a CO(1–0)-to-H$_2$ conversion factor of $1 \times 10^{20}$ cm$^{-2}$ (K km s$^{-1}$)$^{-1}$ in M 51 (Nakai & Kuno 1995),

$$N(\text{H}_2)[\text{cm}^{-2}] = 1 \times 10^{20} I_{\text{CO}}[\text{K km s}^{-1}] \cos i,$$

which is equivalent to

$$\sigma(\text{H}_2)[M_\odot pc^{-2}] = 1.59 I_{\text{CO}}[\text{K km s}^{-1}] \cos i.$$  \hspace{1cm} (2)

Multiplied by 1.36 to include helium, the masses of molecular gas in M 51 and NGC 5195 were obtained to be $M(\text{H}_2) = 7.1 \times 10^9 M_\odot$ and $4.1 \times 10^8 M_\odot$, respectively, from figure 4a. Table 2 summarizes the masses of stars and various gasses in M 51. The molecular gas mass is about twice as much as the neutral atomic gas mass, and the total gas mass of $M(\text{H}_1) + M(\text{H}_2) + M(\text{H}_\text{II})$ is about 16% of the star mass or 14% of the total baryonic mass.

Figure 7 shows the radial distributions of $\sigma(\text{H}_1)$ and $\sigma(\text{H}_2)$, including helium, which were derived by averaging into annuli with a width of 10''. In addition to $\sigma(\text{H}_2)$ derived from CO(1–0), we also show $\sigma(\text{H}_2)$ evaluated from the data of CO(2–1) (Schuster et al. 2007) for

![Fig. 7. Radial distributions of the surface mass densities, $\sigma(\text{H}_1)$ and $\sigma(\text{H}_2)$ derived from CO(1–0) and CO(2–1), including helium, where H$_1$ data of Walter et al. (2008) and CO(2–1) data of Schuster et al. (2007) were used, and $R = \sqrt{X'^2 + Y'^2}$ and $X' = X / \cos i$. The surface density of the molecular gas decreases outward with an exponential shape fitted well by $\sigma(\text{H}_2, \text{CO}(1–0)) = 202 \exp \left[ -R/2.0 \text{ kpc} \right] M_\odot \text{pc}^{-2}$ at $1 \leq R \leq 4 \text{ kpc}$ (dotted line). The increase at $R \approx 12$ kpc shows the molecular gas in NGC 5195.](https://academic.oup.com/pasj/article-abstract/66/2/36/1524341)
reference, simply using same conversion factor of equation (1), which would be an underestimation for CO(2–1), because the intensity of CO(2–1) is usually lower than that of CO(1–0) in normal spiral galaxies (e.g., Nakai et al. 1994). The radial distribution of $\sigma$(H$_2$) from CO(1–0) shows an exponential decrease (dotted line) of $\sigma$(H$_2$) = 202 exp $[-R/2$ kpc] $M_\odot$ pc$^{-2}$ at 1 $\leq R \leq$ 4 kpc, where

$$\sigma_{\rm H_2} = 202 \exp \left[ -\frac{R}{2 \text{ kpc}} \right] M_\odot \text{ pc}^{-2}$$

Figure 8 shows the differential rotation. At $R < 1$ kpc, where figure 8 shows the rigid rotation, $\sigma$(H$_2$) shows a bump at $R \approx 6$ kpc and again decreases exponentially with the radius at 6 $< R < 10$ kpc. The local maximum at $R \approx 6$ kpc could be caused due to the kinks or fractures of the spiral arms (see subsection 3.6) by the interaction with the companion galaxy NGC 5195. At $R \geq 6$ kpc, the spiral arms bend inward then outward (see figures 4 and 9), and thus the mean surface density at $R \approx 6$ kpc becomes larger. The increase at $R \approx 12$ kpc is due to the molecular gas in NGC 5195. The above trend of the radial distribution of CO(1–0) is more conspicuous in CO(2–1), which traces warmer and denser molecular gas than CO(1–0). The molecular gas is the dominant component [$\sigma$(H$_2$) > $\sigma$(HI)] at $\sigma$(HI) + $\sigma$(H$_2$) $\geq 20 M_\odot$ pc$^{-2}$, which has been seen in many galaxies (e.g., Nishiyama et al. 2001).

3.5 Rotation curve

Figure 8 shows the rotation curve derived from the average of the northern ($Y > 0$) and southern ($Y < 0$) rotation velocities in figure 5, using the inclination angle of $i = 22^\circ$ and the position angle of $\theta_{PA} = -10^\circ$. The uncertainties of the inclination angle, $\Delta i = \pm 3^\circ$, and the position angle,
angle, $\Delta \theta_{PA} \approx \pm 10^\circ$, influence the rotation velocity by 10% and $\sim 2\%$, respectively. The velocities in H$\alpha$ with the Kitt Peak 4-m telescope (Goad et al. 1979) and in CO with the BIMA (Berkeley–Illinois–Maryland Association; Shetty et al. 2007) were used in the region of $R \leq 30'$ and $R \leq 40'$, respectively. In the central region ($R < 15'' \approx 0.7$ kpc), the rotation curve rises steeply like the rigid rotation. On the other hand, in the outer region, the velocity gradually increases from $R \approx 2.0$ kpc to $\approx 6.0$ kpc, showing the differential rotation with a dip at $R \approx 2.0$ kpc due to non-circular motion of the molecular gas influenced by the spiral potential (sub-section 3.3.2). We obtained the angular velocities, $\Omega, \Omega \pm \kappa/2$, and $\Omega \pm \kappa/4$, from the rotation curve in order to estimate the locations of resonances in the orbits of stars and gas clouds, where $\kappa = \sqrt{R(d\Omega^2/dR) + 4\Omega^2}$ is an epicyclic frequency. The locations of resonances are related closely to the structure of the spiral arms and the disk (e.g., Lin & Shu 1964; Binney and Tremaine 1987, 2008). Adopting the single pattern speed of $\Omega_p = 38$ km s$^{-1}$ kpc$^{-1}$ (Tully 1974b; Zimmer et al. 2004), the radii of the inner (IIILR) and the outer (OILR) inner Lindblad resonances, $\Omega_p = \Omega - \kappa/2$ are about $R = 0.7$ kpc (15") and 1.6 kpc (35"), respectively. We also find the co-rotation (CR: $\Omega = \Omega_p$) at 5.1 kpc (110") and the upper Lindblad resonance (OLR: $\Omega_p = \Omega + \kappa/2$) at 7.9 kpc (169") and the 4/1 resonance ($\Omega_p = \Omega - \kappa/4$) at 2.6 kpc (55"). The OILR and CR nearly correspond to the positions of Tully (1974b; $R \sim 1.9$ kpc) and Nikola (2001: $R \sim 5.6$ kpc), respectively.

Some previous studies (Elmegreen et al. 1989; Salo & Laurikainen 2000b; Meidt et al. 2008) have however suggested that the pattern speed of M 51 varies with the radial distance due to the tidal perturbation induced by an encounter with the companion NGC 5195 (e.g., Toomre & Toomre 1972). Elmegreen, Seiden, and Elmegreen (1989) proposed two pattern speeds; the inner spiral mode and the outer mode. The former mode, adopting the pattern speed ($\Omega_p = 38$ km s$^{-1}$ kpc$^{-1}$) of Tully (1974b), has an OLR at the position ($R \sim 8$ kpc) of a prominent intensity gap of the arms in M 51 (figure 4a, figure 9). On the other hand, the latter mode is the material pattern that rotates with the companion, whose ILR coincides with the CR of the inner mode. Salo and Laurikainen (2000b) also demonstrated the variation of the pattern speed with the radius by an N-body simulation. Using the outer pattern speed of $\Omega_p \approx 8$ km s$^{-1}$ kpc$^{-1}$ (e.g., Salo & Laurikainen 2000b), the ILR of the outer mode is coincident with the CR (5.1 kpc) of the inner mode ($\Omega_p = 38$ km s$^{-1}$ kpc$^{-1}$). We hereafter adopt the pattern speeds of $\Omega_p = 38$ km s$^{-1}$ kpc$^{-1}$ and $8$ km s$^{-1}$ kpc$^{-1}$ in the range of $R = 40'' - 110''$ and $110'' - 140''$, respectively.

### 3.6 Molecular spiral structure

M 51 has two very prominent spiral arms (figure 4) which are logarithmic but are broken at some radius ($R \sim 140''$), as pointed out by Nakai et al. (1994), and this feature also can be seen in the H$\alpha$ image (Tully 1974b). These kinks can be caused by the tidal interaction with NGC 5195, and the kinks were reproduced by disk simulations (Salo & Laurikainen 2000a; Dobbs et al. 2010). The angle between the tangent of a spiral arm and the azimuthal direction (i.e., pitch angle, $p$) is represented by

$$-\tan p = \frac{1}{R} \frac{dR}{d\theta},$$

where $\theta$ is the azimuthal angle of the arm measured counterclockwise from the major axis of the galaxy disk ($\theta_{PA} = -10$") and $R$ the radius corrected for the inclination angle ($i = 22''$). When $p$ is constant, equation (3) can be integrated as follows:

$$\theta_0(R) = \theta_0 - \frac{1}{\tan p} \ln \left( \frac{R}{R_0} \right),$$

where $R_0$ and $\theta_0$ are constants and $\theta_0(R)$ the position angle of a spiral arm at the radius $R$, measured from the major axis of the galactic disk. Figure 9a shows a $\ln R - \theta$ plot of the IC$_{CO}$ in figure 4a corrected for the inclination angle. The spiral arms can be fitted by equation (4) at 40" $\leq R \leq 140''$, with a pitch angle of $p = 19'' \pm 1''$, where $R_0 = 40''$ and $\theta_0 = 230''$. The pitch angle agrees with the previous values $p = 15'' \pm 4''$ (Elmegreen et al. 1989) and $p = 21'' \pm 5''$ (Nakai et al. 1994). At $R > 140''$, the two spiral arms are broken.

### 4 Discussion

#### 4.1 Derivation of velocity vectors

Figure 4b shows a contour map of the CO velocity field with the gas approaching us in the northern part of the disk of M 51 and receding in the south. The distorted isovelocity contours indicate the disturbed motion from pure circular rotation. Large non-circular motions in the spiral arms, regarded as the streaming motion of interstellar gas, have been detected in H$\alpha$ (Tully 1974b) and CO (Vogel et al. 1988; Garcia-Burillo et al. 1993a, 1993b; Rand 1993; Kuno & Nakai 1997; Aalto et al. 1999). It is difficult however to know variations of velocity vectors in the galactic disk, because we can measure only the velocity component in the line of sight. In order to investigate the motion and orbit of interstellar gas in the spiral potential, Kuno and Nakai (1997) proposed a method (the KN method) to derive the velocity vectors in the spiral phases from measured velocity data. Figure 10 shows its technique for obtaining
a velocity vector. It is regarded that the velocity vectors, $V$, located at the same spiral phase ($\Psi$; dotted line in figure 10a) in nearly the same radius (e.g., $R_a < R < R_b$) are the same. Under this assumption, we can observe different components of a velocity vector at a spiral phase in the line of sight (figure 10a), and thus the velocity vector can be determined using at least two different observed components (figure 10b).

The observed line-of-sight velocities $V_{\text{obs1}}$ and $V_{\text{obs2}}$ at two different observed positions $P_1$ and $P_2$ at $R_1$ and $R_2$ ($R_a < R_1 < R_2 < R_b$) in figure 10 are given by

\[ V_{\text{obs1}} = V_{\text{sys}} - V \cos(\theta_1 - \phi) \sin i, \tag{5} \]
\[ V_{\text{obs2}} = V_{\text{sys}} - V \cos(\theta_2 - \phi) \sin i, \tag{6} \]

where $\theta_1$ and $\theta_2$ are the position angle of the positions measured from the major axis in the galactic plane, $i$ the inclination angle of the galactic plane, and $V_{\text{sys}}$ the systemic velocity of the galaxy. $V$ and $\phi$ represent the amplitude of the velocity vector and the offset angle from the tangential direction of a circular orbit, respectively (figure 10a).

Two measured line-of-sight velocities $V_{\text{obs1}}$ and $V_{\text{obs2}}$ determine two unknown parameters $V$ and $\phi$ by solving equations (5) and (6), where $V_{\text{sys}}$ and $i$ are given in subsection 3.3. Thus we can know the velocity vector $V = (V, \phi)$ at the spiral phase $\Psi$. In other words, as shown in figure 10b, the velocity vector $V = (V, \phi)$ can be determined from two different components, $V_{\text{obs1}}$ and $V_{\text{obs2}}$, of the velocity vector at the spiral phase $\Psi$. If there are more than two line-of-sight velocities observed at the same phase $\Psi$ at $R (R_a < R < R_b)$, the velocity vector $V = (V, \phi)$ can be determined more accurately. If the velocity vector $V = (V, \phi)$ at $R_i$ ($R_a < R_i < R_b$) however is not the same even at the same spiral phase, the mean of the velocity, $\langle V \rangle$, and the offset angle, $\phi$, of the evaluated velocity $V$ at $R_a < R_i < R_b$ have large dispersions.

We applied this method to our new data of figure 4b. Although Kuno and Nakai (1997) regarded the velocity vectors in the wide range of the radius ($R_a = 40'' < R < R_b = 140''$) as constant values, we derived the velocity vectors at the radii subdivided into $R = 40''-110''$ and $110''-140''$, because the depth of the spiral potential would vary with radius and hence the motion and the orbit of gas would also vary. In the range of $R = 40''-140''$, the spiral arms can be fitted by the logarithmic spiral with the same pitch angle, $p = 19''$ (subsection 3.6), but the adopted pattern speed is different at $R < 110''$ and $R > 110''$ (subsection 3.5). Next, these regions were divided into the spiral phase width of $\Delta \Psi = 20''$ (actual angular width of $\Delta \Psi/2 = 10''$ whose spacing of $\sim 650$ pc at $R = 80''$ matches our observational angular resolution). The spiral phase $\Psi(R, \theta)$ is defined by

\[ \Psi(R, \theta) = 2 \left[ \theta(R) - \theta_0(R) \right], \tag{7} \]

where $\theta(R)$ is the position angle of the observed point at the radius $R$, measured from the major axis of the galactic disk (figure 10a). We derived $V$ and $\phi$ by a least-squares fit using the observed velocities at the various position angles in a same spiral phase, using the inclination angle of

---

**Fig. 10.** Pattern diagram of the spiral phase. (a) The phase of arm 1 starts from $\Psi = 0''$, and arm 2 from $\Psi = 360''$. The velocity vectors $V = (V, \phi)$ at the same spiral phase are regarded as same in the region between the radii $R_1$ and $R_2$, which are not greatly different from each other. We see the same velocity vector $V = (V, \phi)$ located at the spiral phase $\Psi$ in two different directions at $P_1 = (R_1, \theta_1)$ and $P_2 = (R_2, \theta_2)$ and hence can measure two different components of the velocity vector, $V_{\text{obs1}} = V \cos(\theta_1 - \phi)$ and $V_{\text{obs2}} = V \cos(\theta_2 - \phi)$. (b) The velocity vector $V = (V, \phi)$ at the spiral phase $\Psi$ can be determined from the two different velocity components $V_{\text{obs1}}$ and $V_{\text{obs2}}$ measured in the line of sight. If we measure more than two components of the velocity vector, the velocity vector can be determined statistically more accurately.
Representative velocity vectors in $R = 40''-110''$ (dark gray) and $R = 110''-140''$ (light gray) and the spiral phases $\Psi$ with $\Delta \Psi = 20''$. The dotted lines show the tangential direction of a circular orbit (light gray line). The velocity vectors change their direction from outward to inward in the arm.

Figure 11. (a) Representative velocity vectors in $R = 40''-110''$ (dark gray) and $R = 110''-140''$ (light gray) and the spiral phases $\Psi$ with $\Delta \Psi = 20''$. (b) Enlargement of the velocity vectors across the arm, $\Psi = 250''-410''$, in $R = 40''-110''$ around $|X', Y'| \sim (-50'', 60'')$. The dotted lines show the tangential direction of a circular orbit (light gray line). The velocity vectors change their direction from outward to inward in the arm.

$i = 22''$ (sub-subsection 3.3.3) and the position angles of $\theta_{PA} = -5''$ ($40'' \leq R \leq 110''$) and $-15''$ ($110'' \leq R \leq 140''$) (sub-subsection 3.3.2). The uncertainty of the inclination angle, $\Delta i = \pm 3''$, gives the uncertainty of $\Delta V/V \sim 10\%$. Figure 11a shows the resultant velocity vectors in each spiral phase in the range of $40'' \leq R \leq 110''$ and $110'' \leq R \leq 140''$. The velocity vectors in the range of $40'' \leq R \leq 110''$ change their direction with respect to a circular orbit from inward to outward in the upstream region of a spiral arm and from outward to inward in the arm (see figure 11b and $\phi$ in figure 12) as expected by the density wave theory (Levinson & Roberts 1981; Roberts & Stewart 1987; Roberts et al. 1990). At the outer region of $X' \approx 0''-100''$ and $Y' \approx -50''-120''$, however, the direction is extremely disturbed (see subsection 3.6).

In order to investigate the velocity variations with respect to the spiral potential, we divided the velocity vectors into two components parallel ($V_{\parallel}$) and perpendicular ($V_{\perp}$) to the arms; $V_{\parallel} = V \cos(\phi + p)$ and $V_{\perp} = V \sin(\phi + p)$, where $p$ is the pitch angle of $19''$ (subsection 3.6; see figure 10a). Figure 12 shows the variations of the velocity components seen in each corotating frame of the spiral arms, where we adopted the pattern speeds of $\Omega_p = 38 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $8 \text{ km s}^{-1} \text{ kpc}^{-1}$ at $R = 40''-110''$ and $110''-140''$, respectively (subsection 3.5). The azimuthal variations of the surface density of the molecular gas and the relative flux in the K band (Jarrett et al. 2003), which indicates the surface density of stars, are also displayed for comparison. In the inner region of $R = 40''-110''$, the variations of these velocity components with respect to the surface density are consistent with the results of Kuno and Nakai (1997), and are in agreement with the particle simulations (Levinson & Roberts 1981; Roberts & Stewart 1987; Roberts et al. 1990) which insist on the decline of the perpendicular velocity components $V_{\perp}$ across the arms ($-2 \leq \Delta V_{\perp}/\Delta \Psi \leq -1 \text{ [km s}^{-1} \text{ degree}^{-1}]$) caused by a type of shock due to collisions between gas clouds entering the arms, although figure 12 is not as sharp as expected in the simulations for the gas as fluid ($\Delta V_{\perp}/\Delta \Psi \ll -10 \text{ [km s}^{-1} \text{ degree}^{-1}]$; see figure 5 of Roberts 1969).

In the outer regions of $R = 110''-140''$, however, the variations of these velocity components are different from those expected in the simulations. The inclination of the outer warped disk can influence the variations of the derived velocity components as shown in the rotation curve (subsection 3.5). The motions perpendicular to the arms ($V_{\perp}$) are rapidly accelerated in the downstream of the arms ($\Psi = 490''-510''$), while the accelerated motions in the upstream of the arms decrease in the arms as well as the inner regions. Also $V_{\perp}$ in the interarms ($\Psi = 130''-150''$) shows sudden deceleration. The variations are particularly striking in the amplitude of the velocity vectors ($V$). The positions of $\Psi = 130''-150''$ and $\Psi = 490''-510''$ in the outermost regions correspond to the east side and the west side of the M 51 disk [e.g., ($X', Y'$) $\approx (-115'', -30'')$ and ($115'', 30''$) at $R = 110''-140''$], respectively. According to Salo’s simulation model (2000a), which accounts for not only the morphological feature of the M 51 system but also observational kinematics, the azimuth of the companion NGC 5195 across the M 51 disk is $\theta_{PA} = -15''$, where the azimuth is
Fig. 12. Relative flux density of the K-band, the surface density of molecular gas (H$_2$), the amplitude of the velocity vector, the offset angle from the tangential direction of a circular orbit and the velocity parallel ($V_\parallel$) and the perpendicular ($V_\perp$) to the arms in the spiral phases $\Psi_1$, from the top panel to the bottom panel. The left and right panels represent the regions in $R = 40''$–110'' and 110''–140'', respectively. The spiral phases $\Psi$ in the regions are measured counterclockwise from $\theta_{PA} = -5^\circ$ and $-15^\circ$, respectively.

counted counterclockwise from the major axis of $\theta_{PA} = 170^\circ$. They suggest that the materials in the outer regions are perturbed by multiple-encounter with NGC 5195 and have out-of-plane velocities. As suggested by Salo and Laurikainen (2000a), in the case that the outer disk is inversely tilted at $\sim 50$ to the inclination of the inner disk ($i = 22$), the effect of the difference between those inclinations on the projected velocity is about 30%. The increasing and decreasing of $V$ at $\Psi = 130^\circ$–150$^\circ$ and $\Psi = 490^\circ$–510$^\circ$ can be affected by the out-of-plane velocities. This suggests that the molecular gas in the outermost region is no longer able to move as expected from the density wave theory.
Fig. 13. Streamlines of the molecular gas derived from the velocity vectors at $R = 40''-110''$ and $110''-140''$ in figure 11a overlaid on the CO integrated intensity of figure 4. The frame corotates with the spiral pattern in each region. The direction of the rotation of the gas is counterclockwise.

This view is consistent with the contention of Elmegreen, Seiden, and Elmegreen (1989) and Tully (1974b) who suggested the existence of the tidal spiral arms in the outer disk as a consequence of the tidal interaction with NGC 5195.

4.2 Orbits of the molecular gas

The streamline of the molecular gas in the frame corotating with the spiral pattern is derived from the velocity vectors in figure 11a, using the equations (6), (7), and (8) in Kuno and Nakai (1997). Figure 13 shows the resultant orbits of the molecular gas, where the pattern speeds $\Omega_0$ are $38 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $8 \text{ km s}^{-1} \text{ kpc}^{-1}$ at $R = 40''-110''$ and $110''-140''$, respectively (subsection 3.5). At $R = 40''-110''$, the radial velocities ($V_r = V \sin \phi$) decelerate when the gas goes from the upstream side of the arms into the arms (figure 12), and therefore the orbit changes along the spiral arms after the gas enters into the arms. As a result, the orbit of the molecular gas becomes an oval as predicted by the density wave theory.

In the outer region of $R = 110''-140''$, however, the molecular gas does not move along the arms, because the radial velocities randomly have negative values even in the interarms (figure 12). Also, the radial velocities reach minima in the interarms, whereas those in the inner region of $R = 40''-110''$ minima in the arms. The orbit deviates from the shape expected from the density wave theory. The deviation is associated with the motion of the gas in the outer disk as in the material arms rather than in the density wave, caused by the tidal interaction with NGC 5195 (e.g., Tully 1974b; Elmegreen et al. 1989).

4.3 Destruction of GMAs by shear effect

The variation of the streaming motion of the molecular gas leads to the radial gradient of the tangential velocity, i.e., shear. The Oort’s $A$-constant,

$$A = \frac{1}{2} \left( \frac{V_r}{R} - \frac{dV}{dR} \right) = -\frac{1}{2} \frac{\Omega}{dR},$$

represents the rate of the shear in the disk (Elmegreen 1988) and can be derived from velocity vectors ($V_r = V \cos \phi$). Figure 14 shows the distribution of the shear strength, $A$, derived from the velocity vectors in figure 11a with the distribution of GMCs ($M \sim 10^5-10^6 \, M_\odot$) and GMAs ($M \sim 10^7-10^8 \, M_\odot$) which were identified by Koda et al. (2009). The figure shows the anticorrelation between the distribution of the shear strength and of GMAs. Namely, GMAs exist only in the area of the weak shear strength and further on the upstream side of high shear strength, while...
GMCs exist on the downstream side and in the interarms. Histograms in figure 13 show the ratio of the number of GMCs or GMAs in each range of the shear strength from A = 0 km s\(^{-1}\) kpc\(^{-1}\) to 60 km s\(^{-1}\) kpc\(^{-1}\), in intervals of \(\Delta A = 5\) km s\(^{-1}\) kpc\(^{-1}\), to the total number of GMCs or GMAs. The gray and black bars show the fractions of GMCs and GMAs, respectively. There is a cut-off of the number counts of GMAs at the shear of 40 km s\(^{-1}\) kpc\(^{-1}\), in contrast to the extended distribution of GMCs at > 40 km s\(^{-1}\) kpc\(^{-1}\). The positions of GMCs and GMAs (figure 14) and the fractions of GMCs and GMAs with respect to the shear strength (figure 15) strongly suggest that GMAs are fragmented at the downstream side of the arms by the strong shear, and then are released into the interarms as smaller clouds of GMC.

### 4.4 Probability for formation of GMAs

Gravitational instability is one of the most probable mechanisms of GMA formation. The criterion for gas disk stability can be defined by Toomre’s Q parameter,

\[
Q = \frac{\kappa v_s}{\pi G \Sigma_{\text{gas}}},
\]

where \(\kappa\) is the epicyclic frequency (see subsection 3.5), \(G\) the gravitational constant, \(v_s\) the gas velocity dispersion, \(\Sigma_{\text{gas}}\) the gas surface density (Toomre 1964; Binney & Tremaine 1987). The critical surface density is described as

\[
\Sigma_{\text{crit}} = \frac{\alpha \kappa v_s}{\pi G},
\]

where \(\alpha\) is a dimensionless constant. While the constant of \(\alpha = 1\) has been applied in a thin stellar disk, \(\alpha = 0.67\) has been derived from the observed star formation thresholds in the disk galaxies (Kennicutt 1989). When the gas surface density is larger than the critical density, the gaseous disk is gravitationally unstable to the formation of clumps such as GMAs. The Toomre stability \(Q\) in the disk of M 51 is derived by adopting \(v_s \approx 10\) km s\(^{-1}\). Figure 16a shows the distributions of \(Q\), GMCs and GMAs. The white contours show values of \(Q = 1\) and 1.5 (= 1/0.67). The region of \(Q < 1.5\) corresponds to the spiral arms where GMAs \((M \geq 10^7 M_\odot)\) and most GMCs \((M \leq 10^6 M_\odot)\) exist.

On the other hand, tidal shear motion can prevent the formation of GMAs by the gravitational instability when the shear force is stronger than the self-gravitational force. Assuming a spherical and uniform density cloud, the cloud destruction by the shear overcomes the self gravity when the gas density is less than the critical shear density, \(\rho_{\text{shear crit}}\), presented by

\[
\rho_{\text{shear crit}} = \frac{3 A (A - B)}{\pi G},
\]

where \(B\) is Oort’s B-constant defined by \(B = -\Omega + A\) (Mihalas & Routly 1968). The critical shear surface density is simply defined as \(\Sigma_{\text{crit}} = 2 h \rho_{\text{crit}}\), where \(h\) is the scale height of the gas disk. The relative importance between the gravitational instability and the shear with respect to the evolution of GMAs can be determined from the ratio of the gravitational critical surface density, \(\Sigma_{\text{crit grav}}\), to the shear
critical surface density, $\Sigma_{\text{crit}}^{\text{shear}}$. The ratio, $Q^*$, was derived by Kenney, Carlstrom, and Young, (1993) as below,

$$Q^* = \frac{\Sigma_{\text{crit}}^{\text{shear}}}{\Sigma_{\text{crit}}^{\text{grav}}} = 0.87 \left( \frac{A}{B} \right)^{1/2},$$

(12)

by adopting $b = \frac{v_z^2}{2 \pi G \Sigma_{\text{gas}}}$ as the scale height and assuming that the vertical velocity dispersion of $v_z$ is comparable to the gas velocity dispersion $v_z$ ($v_z \approx v_z$).

The distributions of $Q^*$ and of GMCs and GMAs are displayed in figure 16, where white dashed lines show a value of $Q^* = 1$. GMAs and most GMCs clearly exist in the region of $Q^* < 1$, although some GMCs in $Q^* \geq 1$. These relationships strongly suggest that GMAs can stably grow up due to the self-gravity and the accumulation of small clouds in a region such as the spiral arms, where the shear critical density is less than the gravitational critical density, because gravitational bind and instability are not interrupted by shear destruction. Some studies (e.g., Kenicutt 1989; Kenney et al. 1993; Luna et al. 2006) have examined the relation between the tidal shear and formations of molecular clouds and stars. They found that star formation is enhanced in the center region of galaxies and in the spiral arms because the shear strength in those regions is weak due to the solid-body rotation, and suggested that the tidal shear can control the star formation rate through cloud destruction.

To check the growth mechanism of molecular clouds, we compare the time scale of the GMA formation in the spiral arms and interarms by gravitational instability and by collisional agglomeration with the GMA disruption time scale by the shear. Here we representatively regard the arms ($Q^* < 1$) and the interarms ($Q^* > 1$) as the region of the spiral phase $\Psi = -30^\circ$ - $30^\circ$, $330^\circ$ - $390^\circ$ and $\Psi = 110^\circ$ - $170^\circ$, $450^\circ$ - $510^\circ$, respectively. The averaged crossing time scales in the arms and the interarms are $\tau_{\text{cross}} \sim 2 \times 10^7$ yr and $\sim 1 \times 10^7$ yr, respectively, given by

$$\tau_{\text{cross}} = \frac{R - R_0}{V_r},$$

(13)

where $V_r (= V \sin \phi)$ is the radial velocity, typically $\sim 20$ km s$^{-1}$ in the arms and $\sim 6$ km s$^{-1}$ in the interarms, and $(R - R_0)$ the radial travel distance of the molecular gas, typically $\sim 350$ pc in the arms and $\sim 50$ pc in the interarms (figure 13). The time scale of the gravitational instability can be estimated as follows (Larson 1987):

$$\tau_{\text{grav}} = \frac{v_z}{\pi G \Sigma_{\text{gas}}},$$

(14)

In the case where we use $v_z = 10$ km s$^{-1}$ as typical velocity dispersion, we obtain the averaged time scale of $\tau_{\text{grav}} \sim 2 \times 10^7$ yr in the arms ($\Sigma_{\text{gas}} \approx 40 M_\odot$ pc$^{-2}$; see figure 12) and $\sim 4 \times 10^7$ yr in the interarms ($\Sigma_{\text{gas}} \approx 20 M_\odot$ pc$^{-2}$). The shear time scale is simply derived as an inverse function of the gas speed relative to the pattern speed (Elmegreen et al. 1980):

$$\tau_{\text{shear}} = \frac{D}{R_1 \left( \Omega(R_1) - \Omega_\phi \right) - R_2 \left( \Omega(R_2) - \Omega_\phi \right)},$$

(15)
where $D$ is the diameter of the GMA, and $\Omega_p$ the pattern speed. $\Omega(R_1)$ and $\Omega(R_2)$ are the angular velocities at the distances from the galactic center $R_1$ and $R_2$ ($R_2 - R_1 = D$), respectively. The adopted $D$ is the typical value of 500 pc (e.g., Muraoka et al. 2009) and the pattern speed $\Omega_p$ is $38 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $8 \text{ km s}^{-1} \text{ kpc}^{-1}$ at $R = 40^\circ$–$110^\circ$ and $110^\circ$–$140^\circ$, respectively (subsection 3.5). The shear time scales in the arms and in the interarms are $\tau_{\text{shear}} = 4 \times 10^7 \text{ yr}$ and $1 \times 10^7 \text{ yr}$, respectively. Table 3 summarizes these time scales. In the arms, the gravitational instability time scale ($2 \times 10^7 \text{ yr}$) is comparable to the crossing time scale ($2 \times 10^7 \text{ yr}$) and shorter than the shear time scale ($4 \times 10^7 \text{ yr}$), while in the interarms, the gravitational instability time scale ($4 \times 10^7 \text{ yr}$) is longer than the crossing ($1 \times 10^7 \text{ yr}$) and the shear time scale ($1 \times 10^7 \text{ yr}$). This means that GMAs can be stably formed by gravitational instability in the spiral arms. The collisional time scale is given by

$$
\tau_{\text{coll}} = \frac{1}{n_c \sigma_{\text{coll}} v_s}, \quad (16)
$$

where $n_c$ is the number density of GMC and GMA, and $\sigma_{\text{coll}}$ is the collisional cross-section. $n_c$ is given by

$$
n_c = \frac{N}{2\pi b}, \quad (17)
$$

where $S$ is the area of the arms and the interarms, $b = v_s^2/2\pi G \Sigma_{\text{gas}}$ the scale height of the gas disk, and $N = (N_{\text{GMC}} + N_{\text{GMA}})$ the sum of the number of GMCs ($N_{\text{GMC}}$) and GMAs ($N_{\text{GMA}}$) counted by Koda et al. (2009). $N_{\text{GMC}}$ and $N_{\text{GMA}}$ are 83 and 22 in the arms, and 26 and 0 in the interarms, respectively. The used area of the arms and the interarms are $S \approx 6 \text{ kpc}^2$, for the range of the radius of $40^\circ \leq R \leq 110^\circ$ and the spiral phase width of $\Delta \Psi = 60^\circ$ [actual angular width of $\Delta \Psi/2 = 30^\circ$; see equation (7)]. Adopting $v_s = 10 \text{ km s}^{-1}$ and $\Sigma_{\text{gas}} \approx 40$ and $20 M_\odot \text{ pc}^{-2}$ in the arms and interarms, respectively, the averaged number densities are $n_c \approx 60 \text{ kpc}^{-3}$ in the arms and $\approx 3 \text{ kpc}^{-3}$ in the interarms. The effective radius $R_e = (N_{\text{GMC}} D_{\text{GMC}} + N_{\text{GMA}} D_{\text{GMA}})/N$ of the collisional cross-section ($\sigma_{\text{coll}} = \pi R_e^2$) is $144 \text{ pc}$ in the arms and $50 \text{ pc}$ in the interarms, assuming the diameters of the GMC and GMA to be $D_{\text{GMC}} = 50 \text{ pc}$ (e.g., Sanders et al. 1985) and $D_{\text{GMA}} = 500 \text{ pc}$ (e.g., Muraoka et al. 2009), respectively. The time scales of collisions between molecular clouds are $\tau_{\text{coll}} \sim 2 \times 10^7 \text{ yr}$ in the arms and $4 \times 10^7 \text{ yr}$ in the interarms for $v_s = 10 \text{ km s}^{-1}$ (table 3). Here we note that these collisional time scales are the upper limits, because we use the number counts of GMCs and GMAs in Koda et al. (2009) in which the angular resolution was $4^\circ$, corresponding to 186 pc at the distance of M 51. In order to discuss the time scale of collisions between GMCs with the size of a few 10 pc (e.g., Sanders et al. 1985) each other, observations with the higher angular resolution are required. These results however suggest that GMAs can be effectively grown by collisional agglomeration and the gravitational instability in the spiral arms, while it is difficult for GMAs to be newly formed in the interarms.

In this paper we discussed the global influence of galactic dynamics, such as the spiral arm streaming motion and the shear, on the evolution of GMCs and GMAs. However, to discuss fragmentation of GMAs to GMCs by shear and GMA formation by collisions between GMCs in more detail, observations with high enough spatial resolution to identify GMCs without missing the flux of extended gas are essential (e.g., Atacama Large Millimeter/submillimeter Array, ALMA).

## 5 Conclusions

The $^{12}\text{CO}(J = 1-0)$ line emission in the region of about $9^\prime \times 10^\prime$ ($25 \text{ kpc} \times 28 \text{ kpc}$) of the spiral galaxy M 51 was mapped with the NRO 45-m telescope. We investigated the kinematics and distribution of the molecular gas using the CO data. The main conclusions are summarized as follows:

(i) The position angle of the major axis of the galactic disk systematically varies from about $\theta_{PA} = -3^\circ$ to $-15^\circ$ with increasing radius from $R = 40^\circ$ to $140^\circ$. The systematic variation of the position angle with radius could be due to warping of the galactic disk caused by the tidal effect of the companion galaxy NGC 5195 rather than non-circular motion of the molecular gas. The systemic velocity is $V_{\text{sys(LSR)}} = 469 \pm 4 \text{ km s}^{-1}$. The inclination angle is $i = 22^\circ \pm 3^\circ$, which was derived from the baryonic Tully–Fisher relation. The outer disk traced by H I is more inclined than the inner disk.

(ii) The radial distribution of the surface density of the molecular gas shows an exponential decrease of $\sigma(H_2) = 202 \exp[-R/2 \text{ kpc}] M_\odot \text{ pc}^{-2}$ at $1 \leq R \leq 4 \text{ kpc}$ where the rotation curve shows the differential rotation, suggesting the inflow of the molecular gas.
due to viscosity of the gas. At $R < 1$ kpc of the rigid rotation, $\sigma(H_2)$ dips below the value expected from the exponential shape. At $4 < R < 10$ kpc, $\sigma(H_2)$ shows a local maximum at $R \approx 6$ kpc due to the kinks or fractures of the spiral arms, caused by an overall pattern of non-circular motions associated with the interaction with the companion galaxy NGC 5195. The increase of $\sigma(H_2)$ at $R \approx 12$ kpc is due to the molecular gas in NGC 5195.

(iii) Two molecular arms are fitted by the logarithmic spirals with a pitch angle of $\rho = 19^\circ \pm 1^\circ$ at $40^\circ \leq R \leq 140^\circ$. The location of OILR and CR is $R = 39^\circ$ and $115^\circ$, respectively, for the pattern speed of $\Omega_p = 38 \text{ km s}^{-1} \text{kpc}^{-1}$.

(iv) The velocity components parallel and perpendicular to the spiral arms were derived from the distribution of the line-of-sight velocity of the molecular gas in each spiral phase. In the inner region ($R = 40^\circ$–$110^\circ$) of the M51 disk, the variations of the velocities and the density distribution of the molecular gas with respect to the spiral potential are qualitatively in good agreement with the density wave theory in the particle-system model. The streamline of the molecular gas derived from the velocity vectors shows an oval as predicted by the density wave theory. In the outer region ($R = 110^\circ$–$140^\circ$), however, the orbit deviates from the shape expected from the theory, because the motion of the molecular gas is affected by the tidal interaction with NGC 5195.

(v) The distributions of GMAs and the shear strength due to the differential rotation show their anticorrelation with each other. GMAs exist only in the area of the weak shear strength and on the upstream side of the high shear strength. The strong shear motion displaces GMAs into the smaller clouds of GMCs which are ejected into the interarms. This indicates that shear motion is an important factor in the evolution of GMAs.

(vi) GMAs and most GMCs exist in the regions where the shear critical surface density is smaller than the gravitational critical surface density. GMAs in the arms can be stably formed by self-gravity and grow quickly by the collisional agglomeration of small clouds without being destroyed by shear motion, but GMAs cannot be newly formed in the interarms.

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