Polarized tops from new physics: signals and observables

Jessie Shelton

NHETC, Rutgers Department of Physics and Astronomy

Top quarks may be produced in large numbers in association with new physics at the LHC. The polarization of these top quarks probes the chiral structure of the new physics. We discuss several kinematic distributions which are sensitive to the polarization of single top quarks and can be used without full event reconstruction. For collimated tops we construct polarization-sensitive observables for both hadronic and leptonic decay modes and plot their distributions. We compute the observable polarization signals from top quarks produced in the on-shell cascade decay of a stop squark into a top quark and a neutralino, as well as top quarks produced in the analogous decay chain in same-spin partner models.

I. INTRODUCTION

The LHC will provide a wealth of data on the physics of the top quark [1,2]. Standard Model pair production $pp \rightarrow t\bar{t}$ alone is approximately 830 picobarns, allowing for detailed study of top properties, such as spin correlations and rare decays. In addition, any new physics which is responsible for electroweak symmetry breaking must couple strongly to the top quark, leading to many events where top quarks are produced in association with new physics.

Models which aim to stabilize the electroweak hierarchy typically predict a top quark partner, such as the stop squark in SUSY or the $T^*$ in little Higgs models. In order to suppress contributions to precision electroweak observables, this top partner is frequently taken to be odd under a discrete parity, which in turn typically leads to a new stable invisible particle. Thus a standard signature of a broad class of well-motivated models is the production of a single top quark in the cascade decay of an on-shell top partner, accompanied by additional particles in the event and possibly missing energy.

The coupling of top partners to top quarks is in general chiral, and a top quark produced in the cascade decay of a top partner will be polarized to a degree which depends on the kinematics of the decay chain as well as the couplings. The top undergoes weak decay before the process of hadronization can dilute information about its polarization, so the angular distributions of its daughter particles reflect the polarization of the parent top. Observables which measure single top polarization can therefore yield measurements which are sensitive to the chiral structure of any new physics sector.

We focus here on kinematic distributions which are can be used without fully reconstructed events. To fully reconstruct events with missing energy one must make assumptions about the event topology and the identities of particles not directly observed. This is not always desirable in events with high multiplicity final states and large missing energy. In such events hadronic top decays will likely be more useful than leptonic top decays for both event selection and top reconstruction, especially in the initial stages of analysis. Fully collimated tops allow polarization study without full event reconstruction in both leptonic and hadronic decay modes, as in the collinear limit one may construct observables which become independent of the unknown boost between the top rest frame and the lab frame.

In section II we discuss several observables whose distributions can measure single top polarization, for both collimated and uncollimated top quarks. In section III we compute the polarization of top quarks produced from the cascade decay of a top partner to a top quark and an additional particle which is not directly observed. In section IV we relate this polarization to experimentally observable signals, and in section V we conclude.

II. KINEMATIC DISTRIBUTIONS FROM STANDARD MODEL TOP DECAY

In the Standard Model the top decays almost entirely through $t \rightarrow Wb \rightarrow f_1f_2b$. The angular distributions of the final state particles depend on the spin of the top. In general the angular distribution of any one of the top’s three daughter particles can be written

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_f} = \frac{1}{2} (1 + P_t \kappa_f \cos \theta_f),$$

where $\theta_f$ is the angle between the daughter fermion $f$ and the top spin axis, as measured in the top rest frame, and $P_t$ is the polarization of the top along that axis. We use conventions where a top with spin $\pm 1/2$ has $P_t = \pm 1$. The “spin analyzing power” of the fermion $f$, $\kappa_f$, is a calculable coefficient, which depends on the identity of $f$. Unitarity restricts $|\kappa_f| \leq 1$.

The anti-fermion from the $W$ decay—the charged lepton, in leptonic $W$ decays—is maximally correlated with the top spin, $\kappa_{\bar{f}} = 1$. Therefore the charged lepton is the natural first object to consider in a study of top polarization. However, in many inclusive final states, $pp \rightarrow tX$, the leptonic decay mode may be of limited utility, as the neutrino makes top reconstruction indirect.
While it is possible to study top polarization without re-
constructing the top rest frame, for instance with highly
boosted tops, the most direct window into top polariza-
tion uses knowledge of the top rest frame. Moreover, de-
pending on the particular study, top identification may
proceed only via hadronic decays where three jets can
be combined within the appropriate invariant mass win-
dow. It is therefore interesting to consider polarization-
sensitive observables that can be constructed for hadronic
top quarks.

Hadronically decaying tops present three natural can-
didate objects whose angular distributions measure the
polarization of the parent top. One is the $b$-jet. At tree
level, the distribution of the $b$-quark is given by equation
\[ \kappa_b = \frac{m_f^2 - 2m_W^2}{m_f^2 + 2m_W^2} \simeq -0.4. \]  
(2)

One can also consider the angular distribution of the (re-
constructed) $W$ boson, which is of the form \[ \kappa_W = -\kappa_b \simeq 0.4. \]  
(3)

The spin analyzing power of the $b$-jet (or the re-
constructed $W$) is less than half of the spin analyzing power
of the anti-fermion from the decay of the $W$. Unfortunately,
in hadronic decays, the down-quark jet cannot be dis-
tinguished from the up-quark jet. However, in the top
rest frame the down quark is on average less energetic
than the up quark. Thus the less energetic of the two
light quark jets in the top rest frame constitutes another
interesting object whose angular distribution can serve
to measure the top quark polarization. At tree level, this
jet has a net spin analyzing power \[ \kappa_j \simeq 0.5, \]  
(4)

where, again, $j$ denotes the less energetic of the two light
quark jets in the top rest frame. Leading QCD corrections
to $\kappa_b$ and $\kappa_j$ have been calculated, and are of order
a few percent. In both cases the effect of the QCD
corrections is to decrease $|\kappa|$.

Depending on the method of top identification used in
a given study, the angular distributions of one or several
of the three objects mentioned here (the $b$-jet, the re-
constructed $W$, or the softer of the two light quark jets)
can form attractive observables which are sensitive to the
top quark polarization along the top quark direction of
motion in the lab frame.

A. Collimated top quarks

Suitably massive new physics may produce daughter
top quarks which are highly boosted in the lab frame.
The top decay products will then be partially or fully
collimated, necessitating novel approaches to top iden-
tification and reconstruction. When the tops
are highly collimated, the angular distributions discussed
above cannot be well measured and other kinematic vari-
ables are more appropriate.

We first consider collimated hadronic tops. When the
$b$-jet (or equivalently the $W$) can be separately identified
within the top, then the natural polarization-sensitive
observable is the fraction of the lab-frame top energy
carried by the $b$-jet,

\[ z = \frac{\mathcal{E}_b}{\mathcal{E}_t}, \]
(5)

where $\mathcal{E}_f$ denotes the energy of a particle $f$ in the
lab frame. Taking the axis of polarization to be given by the
top direction of motion, the variable $z$ is simply related to $\cos \beta$,

\[ \cos \beta = \frac{1}{\beta} \left( \frac{2m_t^2}{m_t^2 - m_W^2} z - 1 \right), \]
(6)

where $\beta$ is the boost between the top rest frame in the
lab frame (we work to tree level and in the narrow width
approximation). The distribution of $z$ is given by

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dz} = \frac{m_t^2}{m_t^2 - m_W^2} (1 - \kappa_\beta P_t) + \kappa_\beta P_t \frac{2m_t^2}{m_t^2 - m_W^2} z \]
(7)
in the collinear limit $\beta \to 1$. This distribution is plotted in Figure 1 for $P_t = \pm1$. Exactly analogous distributions
pertain for the reconstructed $W$, interchanging the roles
of positive and negative helicities. The distribution of the
total $p_T$ of the $b$-jet has also been proposed as a
polarimeter for highly boosted top quarks.

In the highly relativistic limit, leptonic tops can be
used to study polarization without any need for addi-
tional event reconstruction, as observables can be con-
structed which in the collinear limit are independent of

![FIG. 1: The distribution $1/\Gamma d\Gamma/dz$ of the fraction of visi-
table lab frame energy carried by the $b$-jet in a highly boosted
hadronic top, $z = \mathcal{E}_b/\mathcal{E}_t$, plotted as a function of $z$. The blue
line (weighted towards high values of $z$) is the distribution for
a negative-helicity top quark; the red line (weighted towards
low values of $z$) is the distribution for a positive-helicity top
quark.](image-url)
where $\theta_\ell$ is the angle between the lepton momentum and the top direction of motion, $E_\ell$ is the lepton energy in the top rest frame, $\alpha$ describes the orientation of the $b-\ell$ plane relative to the top direction of motion, and

$$\cos \zeta = \frac{m_t m_W^2 - E_\ell (m_\ell^2 + m_W^2)}{E_\ell (m_t^2 - m_W^2)}$$

is the angle between the lepton and the $b$ momenta in the $b-\ell$ plane. The (unknown) boost from the top rest frame to the lab frame is parameterized by $\beta$, which we will henceforth take to unity. In these coordinates, the matrix element for top decay is

$$|\mathcal{M}|^2 \propto (m_t E_\ell - 2E_\ell^2)(1 + \mathcal{P}_t \cos \theta_\ell),$$

where again we take the axis of spin quantization to lie along the direction of top motion. The distribution of $u$ can then be obtained from

$$\frac{1}{\Gamma} \frac{d\Gamma}{du} \propto \int dE_\ell d\cos \theta_\ell d\alpha |\mathcal{M}|^2 (\mathcal{P}_t, E_\ell, \cos \theta_\ell) \times \delta(u - u(E_\ell, \cos \theta_\ell, \alpha)).$$

We use the delta function to carry out the integral over $\alpha$, and evaluate the remaining integrals numerically. The resulting distributions for positive- and negative-helicity top quarks are plotted in Figure 2. The distributions have a shoulder at

$$u = \frac{2m_t E_{\ell, \text{min}}}{2m_t E_{\ell, \text{min}} + (m_t^2 - m_W^2)} = \frac{m_W^2}{m_t^2} \simeq 0.215,$$

where $E_{\ell, \text{min}} = m_W^2/(2m_t)$ is the minimum possible value of the lepton energy in the top rest frame. Above this value of $u$, the integrand can receive contributions from multiple angular configurations as $E_\ell$ varies, while below this value of $u$, only limited angular configurations can contribute.

**III. SINGLE TOP POLARIZATION FROM CASCADE DECAY**

Cascade decays of top partners to tops + missing energy are a prototypical example of the processes for which the above observables are well-suited. In this section we compute the polarization of top quarks produced from the decay of a top partner into a top quark and a boson partner, for both SUSY models (where the boson partner is a neutralino) and same-spin partner models (where the boson partner is a massive vector boson). We work to tree level and in the narrow width approximation, and take the top quark to have standard model decays. The polarization $P_F$ is computed along the axis determined by the direction of the top motion in the parent rest frame, which is not directly observable; in section IV we will show how to relate this to the polarization $P_D$ along the axis determined by the direction of the top motion in the laboratory frame, which yields predictions for experimentally observable distributions.
A. Production polarization in SUSY decay chains

Consider the two-body decay of an on-shell stop squark $\tilde{t}$ into a top quark and a neutralino $\chi^0$. The coupling for this process can be parameterized as

$$-\mathcal{L}_{\text{int}} = a \bar{t} t \gamma^0 + b \bar{t} t \gamma^0 + \text{H.c.},$$

where the Yukawa couplings $a$ and $b$ are in general not equal. The top quark is thus produced with some nonzero polarization, the magnitude of which depends on (1) the masses of the particles in the decay chain and (2) the mixing of the stop squarks and the neutralinos. We take the axis of spin quantization to lie among the top direction of motion in the parent stop rest frame.

The production amplitudes for positive and negative helicity top quarks depend on the following functions of the top and neutralino energy-momenta,

$$F_\pm = \frac{(E_t + m_t \pm |p_t|)(E_\chi + m_\chi \pm |p_\chi|)}{\sqrt{4(E_t + m_t)(E_\chi + m_\chi)}},$$

where all quantities are to be evaluated in the stop rest frame. For this on-shell process, the top and neutralino energy-momenta are fixed in terms of the masses of the particles in the cascade. The functions $F_\pm$ result simply from explicit evaluation of the usual spinor wavefunction operators $\sqrt{p \cdot \sigma}$, $\sqrt{p \cdot \bar{\sigma}}$ on helicity eigenspinors. They measure the separate contributions of the two helicity states in the left- or right-handed fermion wave function. The matrix element to produce a positive (negative-) helicity top quark is then the sum of the contributions from the left- and right-handed tops,

$$M_\pm = \mp i(aF_+ + bF_-).$$

For finite $m_t$ this gives, as expected, a nonvanishing amplitude for top quarks of both helicities even in the limit of a purely chiral vertex, $a \to 0$ or $b \to 0$. We find the net polarization along the production axis to be the

$$\langle P_P \rangle = \frac{(|b|^2 - |a|^2)M|p_t|}{(|a|^2 + |b|^2)(ME_t - m_t^2) + 2\text{Re}(ab)m_\chi m_t}, \quad (14)$$

where $M$ is the mass of the initial squark. The net polarization $\langle P_P \rangle$ arising from this cascade decay is plotted in the solid curves in Figure 3 as a function of the masses of the stop squark and the neutralino. For simplicity we take purely chiral couplings ($b = 1$, $a = 0$) in the figure, so the plotted polarization is an upper bound.

B. Production polarization in same-spin partner model decay chains

We now consider the analogous decay chain in the case where the top partner $\tilde{T}$ has spin 1/2 and the vector boson partner $\tilde{V}_\mu$ has spin 1. We parameterize this process by

$$-\mathcal{L}_{\text{int}} = (a \bar{t} \gamma^\mu \gamma^5 \tilde{T} + b \bar{t} \gamma^\mu \gamma^5 \tilde{T}) \tilde{V}_\mu + \text{H.c.}.$$ 

Again, the couplings $a$ and $b$ are in general unequal and depend on the mixings of the top and vector boson partners. The computation is analogous to the one in the previous subsection, except now we must also average over the spins of the top partner $\tilde{T}$ and sum over the spins of the vector boson partner $\tilde{V}$. We find the net degree of polarization along the production axis to be

$$\langle P_P \rangle = \frac{|b|^2 - |a|^2}{(|a|^2 + |b|^2)(3E_t m_{\tilde{T}}^2 + 2M p_t^2) - 6\text{Re}(ab)m_{\tilde{V}}^2 m_t}, \quad (15)$$

where all kinematic quantities ($E_t, p_t$) are to be evaluated in the parent $\tilde{T}$ rest frame. A similar computation was carried out in [10] for a benchmark model point. The net polarization $\langle P_P \rangle$ arising from this cascade decay is plotted in the dashed curves in figure 3 as a function of the masses of the top partner and the vector boson partner. The kinematic suppression of the “wrong” polarization state is less for the fermion-fermion-vector interaction than it is for the fermion-fermion-scalar interaction. That is, for a given set of masses and couplings, the top polarization predicted by a same-spin cascade is less than that predicted by an opposite-spin cascade. This is due to the additional suppression of the “wrong” helicity contributions coming from the longitudinal polarizations of the vector boson partner, but no suppression from the transverse polarizations.
IV. OBSERVABLE TOP POLARIZATION SIGNALS

In events where the top is reconstructed but the rest of the event is not, the natural axis for the top polarization is the top’s direction of motion in the lab frame, which we will refer to as the detection axis. The most natural experimental observable for hadronic tops is then the angular distribution of the direction of the bottom jet (or the less energetic light quark jet) with respect to the top quark direction of motion in the top rest frame,

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta'} = \frac{1}{2} \left( 1 + \langle P_D \rangle \kappa \cos\theta' \right), \quad (16)$$

The observed polarization is thus along the detection axis, $\langle P_D \rangle$. For highly boosted tops, the natural polarization axis is again the top direction of motion. Notice that there is no interference between the two top spin states, as the observables we consider are insensitive to rotations around the top direction of motion.

In order to relate the observed polarization to the polarization along the production axis $\langle P_P \rangle$ computed in the previous section, we need to relate the two spin bases. The detected polarization $\langle P_D \rangle$ is related to the production polarization $\langle P_P \rangle$ by

$$\langle P_D \rangle = \langle P_P \rangle \cos\omega, \quad (17)$$

where $\cos\omega$ is the angle between the production and detection axes in the top rest frame. This angle is the Wigner angle determined by the composition of boosts $\Lambda_{LL}$ from the top rest frame to the lab frame, followed by $\Lambda_{LP}$ from the lab frame to the parent rest frame, and finally $\Lambda_{Pt}$ from the parent rest frame to the top rest frame,

$$\mathcal{M}(\Lambda_{LL})\mathcal{M}(\Lambda_{LP})\mathcal{M}(\Lambda_{Pt}) = \mathcal{R}(\omega), \quad (18)$$

where $\mathcal{M}(\Lambda)$, $\mathcal{R}(\theta)$ are the representation matrices for the Lorentz transformations. From this composition of boosts we find

$$\cos\omega = \frac{E_t \beta \cos\theta + |p_t|}{\sqrt{|p_t|^2 (1 + \beta^2 \cos^2\theta) + m_t^2 \beta^2 + 2 E_t |p_t| \beta \cos\theta}}, \quad (19)$$

where $E_t, p_t$ are the energy and momentum of the top in the parent rest frame, $\theta$ is the angle of the top in the parent rest frame, and $\beta$ is the boost between the parent rest frame and the lab frame. The energy and momentum of the top are fixed by the masses of the particles in the decay. The phase space integration over $\cos\theta$ can be carried out simply as (after averaging over top partner and vector boson partner spins) the matrix element is independent of $\cos\theta$. This leaves the boost $\beta$ between the parent rest frame and the lab frame. In the limit where the top partner is produced near threshold with little velocity in the lab frame, $\beta \to 0$ and $\cos\omega \to 1$, as there is little difference between the production and detection spin axes. The Wigner angle is also lessened when the top quark is highly boosted in the top partner rest frame. Thus, as expected, the maximal observable polarization signals are realized when the top partner is heavy and the vector boson partner is light. We plot $\cos\omega$ as a function of $\beta$ in three different scenarios for the new particle masses in Figure 4 (we integrate over $\cos\theta$).

For boosts $\beta \approx 0.6$, we find that the scenarios with a moderate to sizable mass splitting have $\cos\omega \sim 0.85$ or better (the red and green curves shown in the figure), while in the case of heavy, degenerate standard model partners (the blue curve) the suppression is much larger, $\cos\omega \sim 0.5$. For smaller boosts $\beta \approx 0.25$ the situation is much better, with a suppression $\cos\omega \sim 0.9$ even for the heavy, degenerate scenario. In principle for a given model the distribution of boosts $D(\beta)$ can be computed from the production dynamics and the PDFs, and the observable polarization is then given by

$$\langle P_D \rangle = \langle P_P \rangle \frac{1}{2} \int d\cos\theta d\beta D(\beta) \cos\omega(\theta, \beta), \quad (20)$$

allowing detailed quantitative predictions. In practice, for sufficiently heavy top partners (and a sufficiently large mass splitting between the top partner and its daughter particles), the detailed boost distribution may not be necessary.

V. SUMMARY

Top quarks produced in association with new physics can serve as an important window into the chiral structure of physics beyond the Standard Model. We have computed here the expected top polarization arising from a well-motivated class of events, namely on-shell cascade
decays of a top partner into a top quark and a vector boson partner. Observation of this signal constrains both the masses of the particles in the cascade and the chirality of the couplings.

We have focused on variables which can be used in events which are not fully reconstructed. In the happy circumstance that the LHC reveals a rich spectrum of new particles with complicated cascade decays, this strategy is likely to prove the cleanest way to assemble a large sample of inclusive top + new physics events, particularly if the new physics includes a stable invisible particle. We have discussed several angular distributions relevant for hadronic top quarks, and for collimated tops have constructed variables for both hadronic and leptonic decay modes and plotted their distributions.

Acknowledgments

The author would like to thank Scott Thomas, Matt Strassler, and Michael Graesser for useful discussions. This work was supported in part by DOE grant DE-FG02-96ER40959.

[1] G. L. Bayatian et al. [CMS Collaboration], “CMS technical design report, volume II: Physics performance,” J. Phys. G 34, 995 (2007).
[2] W. Bernreuther, “Top quark physics at the LHC,” arXiv:0805.1333 [hep-ph]; T. Han, “The 'Top Priority' at the LHC,” arXiv:0804.3178 [hep-ph].
[3] M. Jezabek and J. H. Kuhn, “Lepton Spectra From Heavy Quark Decay,” Nucl. Phys. B 320, 20 (1989).
[4] M. Jezabek, “Top quark physics,” Nucl. Phys. Proc. Suppl. 37B, 197 (1994) [arXiv:hep-ph/9406411].
[5] A. Brandenburg, Z. G. Si and P. Uwer, “QCD-corrected spin analysing power of jets in decays of polarized top quarks,” Phys. Lett. B 539, 235 (2002) [arXiv:hep-ph/0205023].
[6] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, “LHC signals from warped extra dimensions,” Phys. Rev. D 77, 015003 (2008) [arXiv:hep-ph/0612015].
[7] D. E. Kaplan, K. Rehermann, M. D. Schwartz and B. Tweedie, “Top-tagging: A Method for Identifying Boosted Hadronic Tops,” arXiv:0806.0848 [hep-ph].
[8] L. G. Almeida, S. J. Lee, G. Perez, I. Sung and J. Virzi, “Top Jets at the LHC,” arXiv:0810.0954 [hep-ph].
[9] See for instance Martin, A.D. and Spearman, T.D., Elementary Particle Theory, John Wiley & Sons, 1970.
[10] M. M. Nojiri and M. Takeuchi, “Study of the top reconstruction in top-partner events at the LHC,” arXiv:0802.4142 [hep-ph].