Some Theoretical Results on $V_{ub}$ and $V_{cb}$

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I will highlight some of our theoretical results on $V_{ub}$ and $V_{cb}$ from inclusive decays of $b$ hadrons. Please refer to the original papers for details.

1 $V_{ub}$ from inclusive charmless semileptonic decays of $b$ hadrons

1.1 A new method — a model-independent determination

It was proposed recently to use the decay distribution in terms of the observable $\xi_u = (q^0 + |q|)/M_B$ in the $B$ rest frame to measure $V_{ub}$, where $q$ is the momentum transfer to the lepton pair.

This decay spectrum is unique in that the tree-level and virtual gluon processes $b \rightarrow u\ell\nu$ at the parton level generate a trivial $\xi_u$ spectrum — a discrete line at $\xi_u = m_b/M_B$, solely on kinematic grounds. Two distinct effects, gluon bremsstrahlung and hadronic bound state effects, spread out the spectrum, but most of the decay rate remains at large $\xi_u$. Consequently, about 99% of the $b \rightarrow u$ events pass the kinematic cut $\xi_u > 1 - M_D/M_B$, where no $b \rightarrow c$ transition is allowed. This discrimination between $b \rightarrow u$ signal and $b \rightarrow c$ background is even more efficient than the cut on the hadronic invariant mass.

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Because of heaviness of the decaying hadron, the light-cone expansion is applicable to inclusive $B$ decays $[2, 3, 4, 5]$. The leading nonperturbative QCD effect is attributed to the distribution function. The spectrum $d\Gamma/d\xi_u$ is directly proportional to the distribution function $[1]$. The detailed form of the distribution function is not known. However, the normalization of it is exactly known due to $b$ quantum number conservation. Using the known normalization, the dependence on the distribution function can be eliminated in the weighted integral of the decay spectrum:

$$\int_0^1 d\xi_u \frac{1}{\xi_u} \frac{d\Gamma}{d\xi_u} = \frac{G_F^2 M_B^5}{192\pi^3} |V_{ub}|^2.$$ (1)

Thus a measurement of the above weighted integral of the $\xi_u$ spectrum determines $V_{ub}$. This method is based on the light-cone expansion, which is, in principle, model independent as it is in deep inelastic scattering. Note that this method does not rely on the heavy quark effective theory.

Therefore, at least potentially, this theoretically sound, clean and experimentally efficient method allows for a model-independent determination of $V_{ub}$ with a minimum overall (experimental and theoretical) error. By this method the dominant hadronic uncertainty associated with the distribution function is avoided. The residual hadronic uncertainty due to higher-order, power-suppressed corrections of order $O(\Lambda_{QCD}^2/M_B^2)$ is expected to be at the level of 1%. The perturbative corrections are calculable. The study of these remaining theoretical uncertainties are in progress. The precision of this determination of $V_{ub}$ will mainly depend on its experimental feasibility. This method appears quite feasible by the similar techniques used to measure the inclusive charmless semileptonic branching ratio of $b$ hadrons at LEP. This method may be the best one available for the $V_{ub}$ determination.

1.2 From the inclusive charmless semileptonic branching ratio

I have calculated $[7]$ the inclusive charmless semileptonic $B$ decay width in the approach $[2, 3, 4, 5, 6]$ based on light-cone expansion and heavy quark effective theory. This approach is from first principles and the nonperturbative QCD effect can be computed in a systematic way. This allows for a largely model-independent determination of $V_{ub}$ from the inclusive charmless semileptonic branching ratio of $b$ hadrons measured at LEP.
A crucial observation is that both dynamic and kinematic effects of non-perturbative QCD must be taken into account. The latter results in the extension of phase space from the quark level to the hadron level, which obviously increases the decay width. It turns out that the net effect of non-perturbative QCD enhances the semileptonic decay width.

The heavy quark expansion approach [8] fails to take into account the kinematic effect of nonperturbative QCD and, as a result, the calculation [9] of the decay width in this approach lead to a higher value of $V_{ub}$. This failure is the consequence of the theoretical limitations in the heavy quark expansion approach: the operator product expansion breaks down for low-mass final hadronic states; the truncation of the expansion enforces the use of quark kinematics rather than physical hadron kinematics. These theoretical limitations were already indicated by the $\tau(\Lambda_b)/\tau(B_d)$ measurements. The recent CLEO analysis [10] of the hadronic mass and lepton energy moments in $B \to X_c \ell \nu$ may also hint such limitations if the experiment is correct.

Moreover, the interplay between nonperturbative and perturbative QCD effects has been accounted for in our approach, since confinement implies that free quarks are not asymptotic states of the theory.

### 1.3 From the hadronic invariant mass spectrum

I have analysed [11] the hadronic invariant mass spectrum in the QCD-based approach [2, 3, 4, 5, 6]. I found that the theoretical error on $V_{ub}$ depends strongly on the hadronic invariant mass cutoff. The higher it can be experimentally made to be, the smaller the theoretical error on $V_{ub}$.

The hadronic invariant mass spectrum has also been analysed [12] in the approach [13] based on the resummation of the heavy quark expansion. The distinct approximations are made in this approach. A comparison found [4, 6] that this approach contains less information on nonperturbative QCD in the leading approximation than the approach [2, 3, 4, 5, 6] based on light-cone expansion.

### 1.4 From the lepton energy endpoint spectrum

This determination of $V_{ub}$ has statistical power. Our analysis in the QCD-based approach showed [4] that the theoretical uncertainty on $V_{ub}$ from the lepton energy endpoint spectrum is under control.
A key step towards the improvement of the theoretical uncertainties on $V_{ub}$ from the inclusive charmless semileptonic branching ratio, the hadronic invariant mass spectrum or the lepton energy endpoint spectrum is a direct extraction of the distribution function from experiment. It was pointed out \cite{1, 6} that the nonperturbative distribution function can be directly extracted by measuring either the spectra in $\xi_f = (q^0 + \sqrt{|q|^2 + m_f^2})/M_B$ ($f = u, c$) in inclusive semileptonic $B$ decays or the photon energy spectrum in inclusive radiative $B$ decays.

2 Model-independent determinations of the ratios of the CKM matrix elements

It was found \cite{6} that the distribution function is universal in the sense that the same distribution function encodes the leading nonperturbative QCD contributions to inclusive semileptonic $B$ decays as well as inclusive radiative $B$ decays. It was proposed \cite{6} that a model-independent determination of the ratio $|V_{ub}/V_{ts}|$ can be obtained by measuring the ratio of the $\xi_u$ spectrum in $B \rightarrow X_u \ell \nu$ and the photon energy spectrum in $B \rightarrow X_s \gamma$, since the universal distribution function cancels in the ratio, $[d\Gamma(B \rightarrow X_u \ell \nu)/d\xi_u]/[d\Gamma(B \rightarrow X_s \gamma)/dE_{\gamma}]|_{E_{\gamma} = M_B \xi_u/2}$. By the similar methods one can also obtain model-independent determinations of $|V_{ub}/V_{cb}|$ \cite{3} and $|V_{cb}/V_{ts}|$ \cite{6}. These methods depend on the validity of universality of the distribution function, which can be tested experimentally.

3 $V_{cb}$ from inclusive charmed semileptonic decays of $b$ hadrons

3.1 From the inclusive semileptonic branching ratio

I have calculated \cite{5} the semileptonic decay width of the $B$ meson, which can be used to gain a largely model-independent determination of $V_{cb}$. It was also shown that it is important to include the kinematic nonperturbative QCD effect, as in the $b \rightarrow u$ case discussed above. I found \cite{6} that the semileptonic decay width is enhanced by long-distance strong interactions, in contrast
to the result of the heavy quark expansion where a reduction of the free quark decay width is claimed. The primary reason for the difference is that the heavy quark expansion approach has to use the quark-level kinematics rather than the hadron-level kinematics, as mentioned above. Consequently, compared with the light-cone approach [2, 3, 4, 5, 6], the inclusive rate calculated in the heavy quark expansion approach leads to a larger gap between the inclusive and exclusive determinations of $|V_{cb}|$.

Our prediction for the lepton energy spectrum was found [4] to be in good agreement with the experimental data. This experimental test increases our confidence in the determination of $V_{cb}$.

3.2 A new method

It was proposed [1] to use the $\xi_c$ spectrum to obtain a model-independent determination of $V_{cb}$. The idea is the same as the use of the $\xi_u$ spectrum to determine $V_{ub}$ discussed in Section 1.1. However, this way of determining $V_{cb}$ may still suffer from large theoretical systematic error. For $B \to X_c \ell \nu$, the maximum momentum transfer squared is $q^2 = (M_B - M_D)^2$. This means that $q^2$ is not large enough to neglect the higher order corrections. Actually the semileptonic $b \to c$ decay rate is dominated by a few exclusive decay modes ($D, D^* and D^{(*)}\pi$), which suggests that the light-cone picture cannot be valid point by point. The theoretical prediction in the light-cone expansion refers only to the smeared spectrum. A related problem is the uncertainty in the charm quark mass.

The method proposed for a model-independent determination of $V_{ub}$ is, on the other hand, theoretically very reliable. The light-cone expansion works much better for $B \to X_u \ell \nu$ because a much larger momentum transfer with the maximum $q^2 = M_B^2$ can occur in $B \to X_u \ell \nu$ than in $B \to X_c \ell \nu$. Many final hadronic states contribute to the $\xi_u$ spectrum above the charm threshold, without any preferential weighting towards the low-lying resonance states. Both theoretical and experimental situations in this way of determining $V_{ub}$ are so attractive that the feasibility of the experiment is worth investigating.

Note added. After this note was completed, a paper by Uraltsev [14] appeared, in which the work of Refs. [3, 7] was criticized. It has been observed in Refs. [3, 4] that the kinematic nonperturbative QCD effects are missed in the heavy quark expansion approach and must be incorporated additionally.
The author of Ref. [14] disproves that observation by criticizing the approach in Refs. [3, 7]. However, it should be stressed that that observation does not depend on any specific approach. The total decay rate receives an enhancement when the phase space is extended from the quark level determined by the $b$ quark mass to the hadron level determined by the $B$ meson mass. This is physically quite obvious and general, without the intervention of any specific theoretical approach. The heavy quark expansion approach has to use the quark-level phase space, while the $B$-meson decay rate should be calculated using the physical hadron-level phase space. As a consequence, there is rate missing in the calculation of the inclusive charmless semileptonic decay rate in [8] in the heavy quark expansion approach.

Let me next turn to the issue of the theoretical foundation of the approach in [3, 7]. The light-cone expansion has long been recognized as the theoretical foundation for the description of deep inelastic scattering processes that are dominated by light-cone singularities. The same formalism is at the basis of the approach [2, 3, 4, 5] to inclusive $B$ decays. Inclusive semileptonic $B$ decays involve large momentum transfer in most of phase space. The light-cone expansion is applicable to the inclusive decays. On the other hand, the heavy quark expansion for inclusive semileptonic $B$ decays is grounded in the operator product expansion in the case of large energy release. Making a serious scrutiny into their formulations, one can realize that as concerns the theoretical foundation the approach based on the light-cone expansion in [3, 7] is not less firm at all than the heavy quark expansion approach.

Actually these two approach tackle nonperturbative QCD effects in the different ways. The discrepancy between their results is an inevitable consequence of the difference between the underlying methods. Given the theoretical problems of the heavy quark expansion approach mentioned previously, it is not justified to regard such a discrepancy as the deficiency of the approach based on the light-cone expansion. The discrepancy could be just a reflection of the merit of the light-cone approach. As an example, Eq. (14) in Ref. [14] results from the first three terms in the moment expansion [3] of the distribution function in the light-cone approach:

$$f(\xi) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} M_n \left( \frac{m_b}{M_B} \right) \delta^{(n)}(\xi - \frac{m_b}{M_B}).$$

(2)

In fact, the truncation of this expansion is illegal in the light-cone approach. Thus Eq. (14) given in [14] is not a true result of the light-cone approach.
Nevertheless, assuming they are comparable the emerging discrepancy between Eq. (14) and the heavy quark expansion result Eq. (1) shown in [14] is conceivable and not surprising, and not in a problem with the light-cone approach itself.

In the free quark limit, the \( B \) meson and the \( b \) quark in it move together with the same velocity. Eq. (2) of Ref. [7] is the expression for the inclusive charmless semileptonic decay width of the \( B \) meson at rest, derived from the light-cone expansion. In the free quark limit, corresponding to the limit \( f(\xi) \to \delta(\xi - m_b/M_B) \), it correctly reproduces the decay width of the free \( b \) quark at rest. The claimed meaning of the heavy quark expansion correction in Eq. (1) of Ref. [14] corresponding to a free quark moving with the small velocity is self-contradictory and not a result of QCD.

A related point is that if true the results from QCD in \( 1 + 1 \) dimensions may provide some insights when we have to deal with real systems that are not simple, but cannot be regarded as the truth or proof in real QCD.

I would conclude that it is premature to dismiss the results of Refs. [3, 7] on the basis of the work in [14]. It is an indisputable fact that the kinematic nonperturbative QCD effects identified in [3, 7] are missed in the heavy quark expansion approach.

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