Big bang simulation in superfluid $^3$He-B — Vortex nucleation in neutron-irradiated superflow

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We report the observation of vortex formation upon the absorption of a thermal neutron in a rotating container of superfluid $^3$He-B. The nuclear reaction $n + ^3$He = $p + ^1$H + 0.76MeV heats a cigar shaped region of the superfluid into the normal phase. The subsequent cooling of this region back through the superfluid transition results in the nucleation of quantized vortices. Depending on the superflow velocity, sufficiently large vortex rings grow under the influence of the Magnus force and escape into the container volume where they are detected individually with nuclear magnetic resonance. The larger the superflow velocity the smaller the rings which can expand. Thus it is possible to obtain information about the morphology of the initial defect network. We suggest that the nucleation of vortices during the rapid cool-down into the superfluid phase is similar to the formation of defects during cosmological phase transitions in the early universe.

An issue of current interest is the generation of topological defects during symmetry-breaking phase transitions $^{[1]}$. In our experiment we study the density of vortex lines produced during a temperature quench of normal $^3$He liquid into its superfluid B-phase. A sufficiently rapid transition results in a dense network of vortex lines $^{[2]}$. In a large enough bias field of superfluid flow some vortices will be stabilized and survive the subsequent relaxation towards equilibrium while others contract and annihilate. Current theories of condensed matter physics generally provide the tools for understanding defect nucleation in slow phase transitions close to equilibrium. The present nucleation process, however, is far out of equilibrium and resembles the discussion of cosmological phase transitions in the early universe $^{[3]}$. Primordial defect nucleation may explain the cosmological large scale structure and also in some scenarios the net baryon asymmetry of the present universe. To test the validity of such estimates it is important to find laboratory experiments in which defect formation in a rapid quench can be studied.

Liquid $^3$He has several advantages over other systems which in the past have served as laboratory models for the cosmological defect-formation scenario, such as liquid crystals $^{[4]}$, liquid $^4$He $^{[5]}$, and superconductors $^{[6]}$. Many direct parallels and formal analogies connect superfluid $^3$He theory with field theories which are used to describe the physical vacuum, gauge fields or fermionic elementary particles $^{[7]}$. Superfluid $^3$He exhibits a variety of phase transitions and topological defects which can be detected with NMR methods with single-defect sensitivity $^{[8]}$. Of particular relevance to our rapid quench experiment is the fact that liquid $^3$He can be locally efficiently heated with thermal neutrons (Fig. 1).

The neutrons are produced with a paraffin moderated Am-Be source of 10 mCi activity. They are then incident upon the $^3$He sample container. At the minimum distance of 22 cm between source and sample, $\nu \approx 20$ neutrons/min are absorbed by the $^3$He liquid. For thermal neutrons the mean free path is 0.1 mm in liquid $^3$He and thus all absorption reactions occur close to the walls of cylindrical container which has radius $R = 2.5$ mm. Since the incident thermal neutron has low momentum, the 573 keV proton and 191 keV triton fly apart in opposite directions, producing 70 and 10$^{-5}$ cm long ionisation tracks, respectively. The subsequent charge recombination yields a heated region of the normal liquid phase which for simplicity we here assume to be of a spherically symmetric shape $^{[9]}$.

The bubble of normal fluid cools by the diffusion of quasi-particle excitations out into the surrounding superfluid with a diffusion constant $D \approx v_F l$ where $v_F$ is their Fermi velocity and $l$ the mean free path. The difference from the surrounding bulk temperature $T_0$ as a function of the radial distance $r$ from the centre of the bubble is given by

$$T(r, t) - T_0 \approx \frac{E_0}{C_v} \frac{1}{(4\pi D t)^{3/2}} \exp \left( -\frac{r^2}{4Dt} \right),$$

(1)

where $E_0$ is the energy deposited by the neutron as heat and $C_v$ the specific heat. The maximum value of the bubble radius $R_b$ with fluid in the normal phase, $T(r) > T_c$, is

$$R_b \sim \left( \frac{E_0}{C_v T_c} \right)^{1/3} (1 - T_0/T_c)^{-1/3}.$$

(2)

The bubble cools and shrinks away rapidly with the characteristic time $\tau_Q \sim R_b^2/D \sim 10^{-6}$ s. The rapid superfluid phase transition leads to the formation of a random network of vortices as was originally formulated by Kibble $^{[10]}$ and later refined by Zurek $^{[11]}$.

The fate of nucleated vorticity depends on the bias field, the velocity $v_s$ of the superflow outside the bubble...
in the bulk $^3$He-B. The superflow is created by rotating the container: the normal component is clamped to rotation with the container while the superfluid component remains stationary. This results in a relative superfluid velocity $v_s = \Omega R$ at the wall of the container, when rotated at an angular velocity $\Omega$. If the radius of the vortex loop exceeds the value $r_\sigma(v_s) = (\kappa/4\pi v_s) \ln r_0/\xi$, where $\kappa = \pi \hbar/m_s$ is the circulation quantum and $\xi = \xi_0(1 - T/T_c)^{-1/2}$ is the superfluid coherence length, the loop expands. An expanding vortex ring eventually results in a rectilinear vortex line which is pulled to the center of the container. If several vortex lines are formed they accumulate in a vortex cluster.

The number of the vortex lines is monitored with NMR. In the inset of Fig. 3, two NMR absorption records are shown, starting from the moment when the neutron source is placed in position. The absorption events, which lead to nucleation, are visible as distinct steps. The step height gives the number of vortex lines nucleated per event, if the neutron source is sufficiently far from the sample and the absorption events are well separated in time. The number of nucleated vortex lines per unit time increases rapidly with increasing $v_s$ (Fig. 3). The extrapolation to zero gives the threshold value $v_{cn}$ plotted in Fig. 3 as a function of $T$ for different pressures. This $v_{cn}$ is smaller and has a different dependence on $T$ than the critical velocity $v_c$ at which a vortex is nucleated in the absence of neutrons.

Let us estimate the critical velocity $v_{cn}$ and the number of vortices $N(v_s)$ created in one neutron absorption event [2]. The topological defects are nucleated during the superfluid transition to $^3$He-B when the order parameter begins to fall from the false ground-state into the true degenerate ground-states which form independently in regions which are not causally connected. The different regions grow in size and ultimately the order parameter fills all space but with a multitude of defects. These are laid down during the freeze out time $\sqrt{\tau_0/\tau_0}$, where $\tau_0 = \xi_0/\nu v$ characterizes the relaxation time of the order parameter in the low temperature limit. The value of the coherence length $\xi$ at that point, $\xi_0(\tau_0/\tau_0)^{1/4}$, determines the initial distance between the defects, which in this experiment is of order 1 nm.

The later evolution of the vortex network leads to a gradual increase of the intervortex distance $\xi(t)$ [3], caused almost entirely by the interaction of the defects with each other, rather than by fluctuations in the order parameter field as before. With increasing time loops with a line length $l < \xi$ are smoothed out while larger loops straighten and reconnect such that the network appears scale invariant. For a homogeneous system the number of loops $n(l)$ per unit length and unit volume with line lengths $l > \xi$ is given by [4]

$$n(l) = C \xi^{-3/2} l^{-5/2}$$

where $C \sim 0.4$. For our case of finite volume of the bubble, where all vortices form closed loops, our numerical simulation gives the same distribution law but with slightly different $C \sim 0.3$. For the average straight-line dimension $D$ of a loop we find $D = \beta(\xi)^1/2$ with $\beta \approx 1$.

The loop size distribution in terms of $D$ is scale invariant, $n(D) dD \approx 2C dD/D^4$, and does not depend on the distance between vortices. The evolution of the network leads to an increasing lower cutoff of the distribution, $D_{\min} = \alpha \xi(t)$, where $\alpha$ is of order unity. The upper cutoff is the diameter of the bubble $D_{\max} = 2R_b$.

When the average radius of curvature, determined by $\hat{\xi}$, exceeds $r_\sigma(v_s)$ the vortices start to escape from the bubble. The number of vortices $N(v_s)$, created per one neutron, is thus the number of loops with $\alpha r_\sigma(v_s) < D < 2R_b$ within the bubble volume $V_b$:

$$N(v_s) = V_b \int_{\alpha r_\sigma(v_s)}^{2R_b} dD n(D) \propto C \frac{\pi}{9} \left( \frac{2R_b}{\alpha r_\sigma(v_s)} \right)^3 - 1.$$  

(4)

The critical velocity $v_{cn}$ is determined by the requirement $N(v_{cn}) = 0$, which gives $v_{cn} = \frac{\pi \xi_0^2}{6 \alpha R_b} \ln(R_b/\xi)$. Our numerical simulation suggests $\alpha \approx 1$. Thus $v_{cn}$ is inversely proportional to $R_b$ and according to Eq. (2) has the temperature dependence $v_{cn} \propto (1 - T_0/T_c)\xi^{1/3}$, in agreement with the measurements in Fig. 3. In terms of $v_{cn}$ one obtains the universal curve

$$N(v_s/v_{cn}) = \frac{\pi C}{9} \left( \frac{v_s}{v_{cn}} \right)^3 - 1.$$  

(5)

Eq. (4) reproduces the observed cubic velocity dependence and gives the correct order of magnitude estimate of $N(v_s)$, as seen in Fig. 3. It also carries the same universality feature as the measured results in Figs. 2 and 4, and to reach a stable state in the rotating container so that they can be detected one by one. With increasing superflow velocity smaller loops, which represent an earlier stage in the evolution of the network, are extracted. This allows us to probe the morphology of the defect network at different stages of its evolution. The measured number of extracted vortices as a function of velocity is consistent with an initial loop size distribution which characterizes the random phases of the order parameter in the quench. We expect to learn more about this distribution from measuring the velocity dependence of events which lead to a given number of extracted vortex lines. More sophisticated numerical simulation is expected to reveal how the network of large loops, which exceed critical size, is modified by the bias superflow in the late stages of expansion.
FIG. 1. Principle of the experiment and of vortex nucleation during neutron irradiation: a) Superfluid $^3$He-B contained in a cylinder, which rotates at constant angular velocity $\Omega$, is exposed to neutron radiation. b) In a neutron absorption event a proton and a triton are formed which fly apart in opposite directions. Their kinetic energies are used up in ionization reactions of $^3$He atoms. c) The ionized particles recombine generating heat in the form of quasiparticle excitations which corresponds to a substantial part of the total reaction energy. A cigar-shaped hot region of liquid is formed where the temperature rises above $T_c$. d) The hot bubble cools rapidly towards the bulk liquid temperature $T_b$. While passing through the superfluid transition at $T_c$ a tangled network of vortex lines is formed. Vortex loops which exceed a critical size start to expand in the superflow created by the rotation of the container whereas smaller loops contract and annihilate. e) An expanding vortex ring ultimately intersects the walls of the container and gives rise to one rectilinear vortex line. The vortex lines accumulate in the center of the rotating container where they are counted with NMR.

FIG. 2. Nucleation of vortices during neutron irradiation. Insert: NMR absorption as a function of running time $t$ at high and low rotation velocity. The neutron source is turned on at $t = 0$. Each step corresponds to one neutron absorption event. The height of the step gives the number of nucleated vortex lines and is denoted by the adjacent number. The high velocity trace is recorded with the neutron source at a 2.8 times larger distance from the $^3$He-B sample. The bulk liquid temperature is $T_b = 0.96 T_c$. Main frame: The rate $\dot{N}$ at which vortex lines are created during neutron irradiation, plotted as a function of the normalized superflow velocity $v_s/v_{cn}$. Below the critical threshold at $v_{cn}$ the rate vanishes while above the rate follows the fitted cubic dependence $N = \gamma [(v_s/v_{cn})^3 - 1]$, with $\gamma = (1.37 \pm 0.03)$ vortex lines/min, shown by the solid line. The rate of vortex nucleation is the number of vortices per one neutron $N(v_s)$ multiplied by the neutron flux $\nu$: $\dot{N} = \nu N(v_s)$. The theoretical estimate for $N(v_s)$ from Eq. (5) with $\nu \sim 20$ neutrons/min gives $\gamma = \nu \pi C/\nu \approx 2$, which is in order of magnitude agreement with the experiment. The filled data points are measured at a bulk liquid temperature $T_b = 0.96 T_c$ and the open points at 0.90 $T_c$. Both sets of data fall on the same cubic dependence although $v_{cn}$ differs by 50% (Fig. 1). The cubic dependence on the superflow velocity is ultimately interrupted by the spontaneous critical velocity $v_c$ in Fig. 2, which is the maximum possible superflow velocity. The liquid pressure $P = 18.0$ bar and the magnetic field $H = 11.8$ mT are the same as in the insert.

FIG. 3. Critical superflow velocities as a function of the normalized bulk liquid temperature $1 - T_b/T_c$ and pressure $P$ of the liquid $^3$He-B sample in a magnetic field of $H = 11.8$ mT. Solid lines: Critical value of superflow velocity $v_{cn} \propto (1 - T/T_c)^{1/3}$ at which vortex creation starts in the presence of the neutron source. Dashed lines: Critical threshold for superflow $v_c \propto (1 - T/T_c)^{1/4}$ in the absence of the neutron source.

FIG. 4. Scale invariance of the vortex nucleation law during neutron irradiation in different ambient conditions: in Eq. (5) the dependence on temperature, pressure, and magnetic field is contained in the critical velocity $v_{cn}$. In this plot the nucleation rate $\dot{N}$ is shown as a function of $v_s^3$. The intercept with the horizontal axis gives $v_{cn}$ for each line while the common intercept with the vertical axis, $\gamma = (1.46 \pm 0.12)$ vortex lines/min, defines the numerical prefactor in Eq. (5). The experimental variables $P$, $T$, and $H$ are listed in the legend for each set of data points.