Effect of wall normal velocity on velocity distribution in unsteady flow

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Abstract. The effect of wall normal velocity on velocity distribution in unsteady flow has been investigated. We established a modified log-law model by adding the wake-term by Coles and deviation term together to depict the velocity deviation caused from classic log-low model. An empirical formula for deviation-correction factor $\alpha$ has been determined based on the present experimental data, and it yields a good agreement with other published experimental data. Experimental measurements of velocity distributions in unsteady open channel flows have been performed to prove this model. The cause of this addition was discussed and explained, and the theoretical velocity images from the improved model have been compared with the present data, they are well correlated.

1. Introduction

In fully developed open channel flow, how to accurately calculate the mean streamwise velocity distribution has challenged scientists and hydraulicians for several decades. The well-known Prandtl-von Kármán logarimthic law depicts how the mean velocity distribute in the boundary layer of plate according to mixing length hypothesis [1]. Some years later, the experimental studies about investigating velocity distribution over flat plates and in circular pipes has done by Nikuradse [2]. Since then the log law has been widely applied in rectangular and circular ducts and open channels to describe the velocity distribution. By adding a new term to the log-low, Coles [3] introduced a purely empirical correction function, which explained the error of velocity distribution caused by wake. This function is also named wake law in open channel flows. Nowadays it is widely acknowledged that the log-law can only accurately describe the velocity distribution in the inner wall part ($y/h < 0.2$) of an open channel flow [4] $y$ represents the distance to the bottom of the channel, and $h$ represents the depth of water. However, In the upper part, the velocity distribution is different from the result of log-low due to the effect of the wake. How does wake-law and the wake strength works is not fully explained despite achieved significant advances. For more than a century, scientists have found the dip-phenomenon which can be understood that if the depth ratio of the open channel is greater than a certain value, its maximum velocity is located a little bit below the surface. Thus the mean streamwise velocity changes the maximum velocity as observed by Cardoso et al [5]. Yang et al [6] presented a new law based on Reynolds-averaged Navier-Stokes (RANS) equations. There are more than one distance parameters, the first from the bed and the last from the free surface, and a deviation-correction factor, $\alpha$, which can be determined empirically. But the model is unable to fit the cases where measured velocity is locally
bigger than the prediction of log-law. So, further research is required to establish a general expression to depict the velocity in open channel flows. Yang et al [6] found that turbulent features with upward motion \((v > 0)\) and downward are totally different under the effect of a time-averaged wall-normal velocity \((v \neq 0)\) and how does the dip phenomenon works can be understood by vertical velocity in open channel flows.

As a matter of fact, we can say that the vertical velocity is not zero in steady or in uniform flows. So, it is not hard for us to take into consideration other factors, both of those can cause the deviation of the wall-normal velocity distribution in unsteady flows. Guo and Julien [7] added a cubic term to modify the log-wake law. However, this model requires that the velocity fits to the log-law in the inner bottom part and also fits to the parabolic law in the outer part. Based on the assumption of vertical velocity component, an advanced model is introduced by Lassabatere et al [8] which is suitable for rough flow and smooth flow. However, all the calculation does well in the outer and central part of a channel. As to the outer region, the uncertainties are greater. Wu et al [9] investigated how does the velocity distributes in a circular open channel with the help of the software Fluent. He added a new constant to dissipation rate formula, it summarized the effect of the secondary currents in the velocity distribution. All in all, it is valuable to analyses the linkage of velocity deviation with the time-averaged up/down motion. The purpose of the study was to determine the velocity distribution in unsteady open channel flow by developing a modified equation model. The present experimental results will be compared with the predictions obtained from the improved velocity distribution model.

2. Effect of wall-normal velocity in unsteady flow
Considering all the factors mentioned above, it is necessary to investigate the linkage between the deviation and up/down motions in unsteady flow. As we all know, wave-current flows in coastal waters are characterized by unsteadiness of the flow as shown in figure 1. In the region near the bottom of the sea, the velocity distribution of the current flows follows the log-law because the waves in the surface will not disturb the flows near the bed, as shown in figures 2 and 3. Mean velocities differs from the velocity predicted with the log-law in the upper layer, and they have the relationship with the direction of wave propagation.

![Figure 1. Waves in unsteady flows.](image)

**Figure 1.** Waves in unsteady flows.

![Figure 2. measured velocity images and results from log-law for wave opposing currents (v>0).](image)

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![Figure 3. measured velocity images and results from log-law for waves following currents (v<0).](image)

**Figure 3.** measured velocity images and results from log-law for waves following currents (v<0).
Investigation of velocity profiles has also been done by Yang [10] in combined wave. By compared experiments, he analysed how does wall-normal velocity affect streamwise velocity. In figure 4, with the distance normal to wall, the non-dimensional velocity deviations $\Delta u/u_*$ increase, which is attributable to the upward motion. The present data given in figure 5, for a negative wall-normal velocity increases along the distance normal to wall, the deviation is also increase. Significantly, the effect of the wall normal velocity on the measured value is different from the calculated value, either bigger or smaller, and this trend will continue to increase in the process of the wall normal velocity rising.

![Figure 4](image4.png)

**Figure 4.** (a) Effect of wall-normal velocity ($v>0$) and (b) Distributions of streamwise velocity deviation measured by Klopman (1997).

![Figure 5](image5.png)

**Figure 5** (a) Effect of wall-normal velocity ($v<0$) and (b) Distributions of streamwise velocity deviation measured by Klopman (1997).

### 3. Theoretical Solutions to the deviation of velocity in Unsteady flows

The Reynolds-averaged equations can be written as follows by adding the continuity equation to the x-direction momentum equation [6] to a steady turbulent flow in an open channel:

$$
\frac{\partial (uv - \frac{\tau_{xy}}{\rho})}{\partial y} + \frac{\partial (uw - \frac{\tau_{xz}}{\rho})}{\partial z} = gS
$$

(1)

In the equation, $x$ represents the streamwise direction; $y$ represents the normal distance from the bottom of the channel; $z$ represents the distance from the sidewall; $u$, $v$ and $w$ represents the mean velocities in the $x$, $y$ and $z$ directions respectively; $\tau_{xy} = \mu \frac{\partial u}{\partial y} - \rho u v$ and $\tau_{xz} = \mu \frac{\partial u}{\partial z} - \rho u w$ where $u'$, $v'$, $w'$ are turbulent fluctuating components; $\mu$ is the fluid dynamic viscosity; $g$ is the gravitational acceleration; and $\rho$ is the fluid density.

According to Yang and Lim [11] and Yang et al [12] there are three parts in the open channel. In the
near-bed region, vertical gradient \( \frac{\partial}{\partial y} \) is much larger than the spanwise direction \( \frac{\partial}{\partial \hat{i}} \); thus, \( \frac{\partial}{\partial \hat{i}} \) is negligible, see e.g. Yang et al [6] and Tracy [13]. In the main flow region, the viscous part \( \mu \frac{\partial l}{\partial \hat{j}} \) of the shear stress \( \tau_{ij} \) is smaller than the turbulent part \( -\rho u' v' \). Therefore, the viscous part in equation (1) can be neglected, and it becomes

\[
\frac{\partial(uv+u'v')}{\partial y} = gS
\]  

Integration of equation (2) with respect to \( y \) gives

\[
-\frac{u'v'}{u_*} = (1 - \frac{y}{h}) - \alpha_i \frac{y}{h} + \frac{uv}{u_*^2}
\]  

In the equation \( u = \sqrt{gRS} \) represents the bed friction velocity; \( h \) represents the depth of water; \( R \) represents the hydraulic radius; and \( \alpha_i = (gSh - u_*^2)/u_*^2 \). The third term on the right-hand side of equation (3) represents the influence of secondary currents; It is noted that a nonzero wall-normal velocity \( v \) plays an important role for the third term. Thus, the third term could be empirically modelled using a linear relationship for simplicity [6], i.e.

\[
\frac{uv}{u_*} \approx -\alpha_2 \frac{y}{h}
\]  

Where \( \alpha_2 \) is taken as a positive value in downward motion \( (v<0) \), and it is taken a negative value in upward motion \( (v>0) \).

The Reynolds shear stress in smooth open channel flows can be approximated as,

\[
-\frac{u'v'}{u_*^2} = (1 - \frac{y}{h}) - \alpha \frac{y}{h}
\]  

The effect of secondary current is accounted for by the additional term, i.e.

\[
\alpha \frac{y}{h} = (\alpha_i + \alpha_z) \frac{y}{h}
\]

On the right-hand side of equation (5), it subsequently move the maximum horizontal velocity vertically down a little bit from the surface, as is explained by Cardoso et al [14]. By applying the Boussinesq assumption

\[
-u'v' = v_i \frac{du}{dy}
\]  

Where \( v_i \) is the eddy viscosity, \( v_f \) could be modelled by empirical equation.

\[
v_i = k u_* y(1 - \frac{y}{h})
\]
In the equation $k$ is the von Karman constant ($=0.41$). Substituting equations (6) and (7) into (5),

$$\frac{du}{dy} = \frac{u_*}{k y} - \left[ \frac{\alpha u_*}{k h (1 - \frac{y}{h})} \right]$$

Integration of equation (8) with respect to $y$ and using the non-slip boundary condition i.e., $u = 0$ at $y = y_0$, gives:

$$\frac{u}{u_*} = \frac{1}{k} \ln(\frac{y}{y_0}) + \frac{\alpha}{k} \ln\left(\frac{1 - \frac{y}{h}}{1 - \frac{y_0}{h}}\right)$$

In the equation $y_0 = \nu/(c u_{(c)})$ is the $y$-distance where the velocity is almost zero; $u_{(c)}$ is the local bed friction velocity and can be determined theoretically [11].

Since $y_0/h \ll 1$, equation (9) can be written as follows:

$$\frac{u}{u_*} = \frac{1}{k} \ln(\frac{y}{y_0}) + \frac{\alpha}{k} \ln(1 - \frac{y}{h})$$

Actually, the second term, $\frac{\alpha}{k} \ln(1 - y/h)$, on right-hand side of equation (10) is of vital importance in the outer part. As $y/h \rightarrow 1, \ln(1 - y/h) \rightarrow \infty$, thus the second term of $\frac{1}{k} \ln(1 - y/h)$ must be a negative value. Hence, the coefficient $\alpha$ plays an important role in the positive and negative deviation caused by the logarithmic law. For example, considering a downward currents ($v<0$) in flow, the value of the second term is negative because $\alpha = \alpha_1 + \alpha_2 > 0$. This is why the negative deviation existed in downward motion. Thus, it is different from the downward secondary currents in the flow region, the positive deviation will be existed due to upward currents affection.

The location of the maximum velocity in an open channel can be derived analytically by differentiating equation (10) and setting $\frac{du}{dy} = 0$, that is

$$\frac{h_{\text{max}}}{h} = \frac{1}{1 + \alpha}$$

As can be seen in the equation (11) that $\alpha$ is only related to the ratio of the location of maximum velocity to the water depth, $h_{\text{max}}/h$. By combining equation (10) and the wake term by Coles [3], the complete deviation-modified velocity distribution model in a 3-D open channel flow can be written as follow:

$$\frac{u}{u_*} = \frac{1}{k} \ln(\frac{y}{y_0}) + \frac{2\Pi}{k} \sin^2 \left(\frac{\pi y}{2h_{\text{max}}} + \frac{\alpha}{k} \ln(1 - \frac{y}{h})\right)$$

The velocity profiles measured in the present study will be compared with the analytical predictions.
obtained from equation (12) in the forthcoming. Taking into account the upward motion, \( \alpha \ln(1 - \frac{y}{h}) \) should be negative, and vice versa.

The empirical relationship between \( \alpha \) and \( z/h \) is proposed with the R2 value of 0.97 [6]:

\[
\alpha = 1.5 \exp(-\frac{z}{h})
\] (13)

For the velocity profile in the center line of the channel where \( z = b/2 \), equation (13) becomes

\[
\partial = 1.5 \exp(-\frac{b}{2h})
\] (14)

4. Velocity profile fitting and results

The comparison of results from equation (13) and measured data by Klopman [15] are shown in figures 6 and 7, in which the friction velocity 0.009337m/s is calculated by using inner layer data. The curves of equation (13) are in good relationship with the data (\( \Pi = 0.6 \)) and the relative error is 8% in following currents. The relative error \( E \) in opposing is 18% by using \( \Pi = -0.1 \) larger than that of following currents. It is easy to see from the results of the experiment that the applicability of the model is good.

![Figure 6](image_url)

**Figure 6.** Results of measured and calculated velocity images for unsteady flow (data from Klopman’s paper).

![Figure 7](image_url)

**Figure 7.** Results of measured and calculated velocity images for unsteady flow (data from Klopman’s paper).

5. Why does the wake-term appear?

As can be observed from figures 6 and 7 that the model can provide much better agreement with the measured velocity, which proved that the wake-term and deviation term are needed to depict the velocity distribution in open channel flows, thus it is worthwhile to investigate the mechanism of the additional terms as shown in equation (12).
Yang [16] analyzed the wall-normal velocity $v$ in equation (3) in sediment-laden flows, he concluded that the resultant $v$, which can be expressed as following, is a joint effect of secondary currents and an upward motion induced by the sediment settlement.

$$v = v_1 + v_2$$

(15)

Where $v_1$ is the wall-normal velocity induced by the secondary currents and $v_2$ is the upward velocity due to the sediment.

Similarly, equation (15) is also valid to express the joint impact of secondary currents, since it is not difficult to imagine that every water particle is influenced by two corner vortices based on the measured secondary currents. Therefore, these two velocities represent one upward, and the other downward motion.

Previous research stressed each term in a simple condition, i.e., either upward motion or downward motion. But for an open channel flow, the upward motion co-exists with the downward motion, and they are driven by two eddies, thus it is understandable that both deviation term and the wake term are needed. In other words, the deviation term is caused by the upper secondary flow, and the wake-term in equation (12) is the result of upward motion driven the lower secondary cell. In other words, the co-existence of wake-term and the deviation-term indicates the existence of a pair of secondary currents.

6. Conclusions

A modified log-law model has proposed by adding the wake-term by Coles [3] and deviation term. Experimental measurements of velocity distributions in unsteady open channel flows, have been performed to prove this model. An empirical formula for deviation-correction factor $\alpha$ has been determined based on the present experimental data, and it yields a good agreement with other published experimental data. Both deviation-phenomenon term and the wake term are needed for an open channel flow. The predicted velocity images from the improved model have been compared with the present measured data. The measured and the predicted velocity images are in reasonably good agreement.

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