ON GRAVITATION AND QUANTA*

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Abstract

We show that a gravitating mass $M$ in thermal equilibrium behaves statistically like a system of some number $N$ of harmonic oscillators whose zero-point energy $\frac{\hbar}{2}$ depends on $N$ universally in such a way that $N\epsilon^2 \sim \frac{\hbar c^5}{G}$, where $G$ is the Newton constant, $c$ is the velocity of light in vacuum, and $\hbar$ is the Planck constant. The large number $N$ is also the number of weakly interacting gravitational atoms [2] which are the constituents of a black hole. The sum over all oscillators of the squares of zero-point energies is fixed and independent of the number of those oscillators. It is well known that the classical gravitating systems behave in the way foreign to statistical quantum mechanics. The negative specific heat of those systems and the phenomenon of a gravitational collapse are different facets of the same reality. The enigmatic Bekenstein entropy of black holes was not yet derived on the basis of microscopic theory. Our work may be considered a first step in direction of presenting such a basis [1-4]. The problem with all present approaches to this problem has been the silent assumption that the total entropy of gravitating systems (black holes) must be given by the Bekenstein formula $S_{bh} = 4k\pi M^2$, where the mass $M$ is in Planck units. The postulate of gravitational constituents (gravitational atoms) and gravitational oscillators (quanta) leads to Bekenstein formula only after a part of mass-energy fluctuations is neglected [2]. The most unusual character of the gravitational mass-energy oscillators (quanta) is that they somehow manage, via the quadratic sum rule defining the Newton constant $G$, $\sum_i \epsilon_i^2 = \frac{\hbar c^5}{G}$, to reduce their zero-point energy when the number $N$ of gravitational atoms grows [2]. This also means that a cold large gravitational mass $M \sim \mu \sqrt{N}$ consists of $N$ constituents. The formula $M^2 = \frac{\mu^2}{2\pi} N$, $\mu^2 = \frac{\hbar c}{G}$, was derived long time ago by the present author. The physical meaning of the ‘phenomenological’ entropy of Bekenstein is that it is the measure of the number $N$ of constituents making up a very ‘cold’ large body. The zero-point energy $\epsilon_i$ of gravitational quanta for a very large ‘cold’ mass is of an order of the Hawking thermal energy of quanta, $\epsilon_i \sim \left(\frac{\mu^2}{(4\pi)^2 M}\right)^{\frac{1}{2}}$. The more massive is a gravitating mass the softer are the gravitational quanta. The number $N$ of constituent gravitational atoms of a given spin-zero quantum Schwarzschild black hole determine the energy of quasi-thermal quanta and the Bekenstein-Hawking entropy, $kT_{bh} \sim \frac{\mu}{\sqrt{N}}$, $S_{bh} \sim kN$, but it is valid only in the particular limit when the interference terms are neglected [2]. Otherwise, as usual with oscillators, there are two sources of statistical fluctuations of mass-energy corresponding to the corpuscular and wave aspect of quanta [2]. We calculate the energy fluctuations

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of the system considered in [2] in terms of the mean energy $\overline{E}$, where $\overline{E} = \frac{E_0 \cosh(\frac{E}{2\hbar})}{N}$, $E_0^2 = \frac{b^2 \hbar^2 c^2 N}{4}$, and $(\Delta E)^2 = N^{-1} \overline{E}^2 - \frac{1}{4} N c^2 = N^{-1} \overline{E}^2 - \frac{1}{2} \hbar \mu^2 c^4$. Neglecting the $\frac{1}{N}$ term in this formula, when $\overline{E}$ is fixed, we obtain the expression for statistical fluctuations typical of gravitating systems: $(\Delta E)^2_{bh} = -\frac{1}{4} \hbar \mu^2 c^4$. The well known relation between the mass-energy fluctuations and the behaviour of entropy near the state of thermal equilibrium, $(\Delta E)^2 = - k \left( \frac{\partial^2 S}{\partial E^2} \right)^{-1}$, leads to the entropy of such a truncated system: $\frac{\partial^2 S_{bh}}{\partial E^2} = 4 k b^{-1} \mu^{-2} c^{-4}$ where an arbitrary integration constant is fixed to be zero by demanding that a very massive body is also very cold [2]. The entropy is given by the ‘phenomenological’ entropy formula of Bekenstein: $S_{bh} = 2 k b^{-1} \mu^{-2} \overline{E}^2 + \text{const}$. The model calculation of Hawking leads to a numerical value of the constant $b$, $b = \frac{\hbar \mu}{4 \pi}$. Quite independently of the actual value of the numerical constant $b$ the entropy $S_{bh}$ has a lower bound $S_0 = 2 k b^{-1} \mu^{-2} E_0^2$ which depends only on $N$. This follows from the fact that the total mass-energy $\overline{E}$ is bounded from below by the zero temperature value $E_0$, $\overline{E} \geq E_0$. Now, $E_0^2 = \frac{1}{2} \hbar \mu^2 c^4 N$, and, therefore, the lower bound on the entropy does not depend on $b$ at all, $S_0 = k \frac{N}{2}$. It is quite natural for the entropy to be bounded from below by the number $\frac{N}{2}$ of constituents.$^1$

We have seen the emergence of the Bekenstein formula for the black hole entropy from the hypothesis about the microscopic nature of gravitational phenomena $[1,2,4,8,9]$. We shall propose to apply the same argument to the whole Universe. It seems natural to consider $N_U \sim 10^{123}$ gravitational atoms [2] and apply to them the same physical argument. We obtain the simple estimate [2] for cosmological constant regarded as the zero-point vacuum energy density $\lambda \sim \frac{4}{c^4}$. It was shown [8] that a system of a large number of gravitational atoms described in the framework of the Atomic Theory of Gravitation (the new gravitational mechanics) by a large $N \sim N_U$ matrix of operators $[1,2,8,9]$ bears

$^1$Two examples of gravitating systems are discussed in [2] simultaneously: a spin-zero quantum black hole and the whole non-rotating Universe. The present upper bound on cosmological constant $\lambda$ in Planck units, regarded as the vacuum zero-point energy density, was used to estimate the lower bound on the total number $N_U \sim 10^{123}$ of gravitational atoms (and gravitational oscillators) in the Universe.

$^2$It is clear to this author that only the large $N$ limit ($\frac{1}{N} = \nu \to 0$, where $\nu$ plays the role analogous to the Planck constant in the new gravitational noncommutative mechanics $[1-4,8,9]$, with fixed observable quantities like $\overline{E}$), produces such an expression for entropy. If we were to impose the Boltzmann statistics on the very ‘cold’ collection of gravitational atoms this would have been so much against the spirit and the letter of quantum theory that we would have been led to the conclusion that ‘quantum gravity’ indeed requires really revolutionary ideas like the abandonment of quantum statistics. Such revolutionary step forward would make it possible to reproduce the Bekenstein-Hawking entropy by an order of magnitude estimate. On the other hand we are not prepared for such a revolutionary step forward yet.

The analogy which invites itself quite naturally is this: in the limit $\hbar \to 0$ in quantum statistical mechanics of ideal gases when we neglect the fact of identical nature of quantum particles we obtain the classical Boltzmann gas plagued with its Gibbs paradox [6] and etc. We should perhaps recall here the story of the now well understood phenomenon of superfluidity in the liquid Helium II. The moral of this very well known story, as told by R. P. Feynman [7], is this: **DO NOT QUANTIZE WHAT IS ALREADY QUANTIZED**. This basic observation [7] which was found valid for superfluidity in the past is equally valid today for quantum black holes $[2,4,8,9]$. Landau’s quantum hydrodynamics was only formally ‘quantum’ in view of quantum commutators appearing in the nonrelativistic current algebra. Feynman has discovered [7] that the real problem with quantum hydrodynamics was that hydrodynamical description of the superfluid Helium II failed to take into account the Bose statistics of Helium 4 atoms. Now, it is our opinion that we seem to be facing the same dilemma posed by the currently fashionable description of black holes. Incidentally, the cases for quantization of general relativity (GRT) and its later fermionic deformation known as ‘supergravity’ seem to follow the same pattern which was so well understood by Feynman [7] in the context of experimentally rich phenomenon of superfluidity. It seems that general relativity, ‘supergravity’, ‘superstrings’ and recently ‘supermembranes’ were also quantized in the same way the superfluid hydrodynamics was quantized with the well known results [7]. Again, the moral of the story as told by R. P. Feynman [7] when adopted to the present situation of quantum black holes seems to be that “we should give to the emperor what is emperor’s”. The Atomic Hypothesis and Quantum Statistics rule. Gravitation is the many-body phenomenon [9]. It is our responsibility now to find the physically correct Hamiltonians for systems of gravitational atoms in the framework of the new gravitational noncommutative mechanics $[1,3,4,8,9]$.  

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remarkable similarity to the thermal Universe with a nonvanishing positive cosmological constant.

If the Bekenstein entropy were the whole thing as far as the thermal properties of gravitating masses are concerned, then the World would be always in a state of the lowest thermodynamic probability. This conclusion would lead then to the statement that the behaviour of a visible Universe is determined by the condition that it is in a state of the lowest statistical weight. Considering an ensemble of such Universes, regarded as local thermal phenomena in a sense suggested in [2,4,8], we would be persuaded to conclude that our Universe is the least probable one. The Universe must be regarded as a very typical one in the statistical ensemble of Universes, which is also the statement of the maximal thermodynamic probability $W$ of Boltzmann. It should be noticed that the notion of a statistical ensemble for the observable Universe is justified only after we identify atoms whose existence is underlying the totality of phenomena. The Gibbs-Jaynes principle of the maximum of $H$ function [5] is applicable to a closed system once we postulate the general Atomic Hypothesis [1-4]. According to this hypothesis the totality of phenomena should be derived from the properties of space-time-matter atoms, which I prefer to call gravitational atoms [1-4]. The result described in this short note (Abstract), which was based on the purely phenomenological considerations, was first derived in the Spring of 1995 and published in [2]. I have applied the simple interpolation argument, originally due to Planck, to the problem of mass-energy fluctuations of a gravitating mass. I have proposed that the formalism of the new gravitational noncommutative mechanics [1-4,8,9] be applied to gravitating particles (black holes) and to the whole Universe.

The point made very clearly in [1] was that Hamilton’s optical analogy should be taken seriously for both GRT and QM. It is obvious to the present author that this analogy is quite useful once we realize that the perihelion precession of the Mercury, and the Complementarity Principle for a gravitational mass and a inertial mass aspect of a gravitating particle [1,3,8,9] allow for the transition to new gravitational mechanics in the same way the nonrelativistic Kepler problem and the spectroscopic data has made it possible for Heisenberg to propose the matrix quantum mechanics [1]. The idea of using the GRT Kepler problem and the ‘optical analogy’ of Hamilton to make a similar transition to the new gravitational noncommutative mechanics was first described by the present author [1,3] as early as in the Spring 1995 (the submittal date of [1] was May 31, 1995), and later during the USC Summer 1995 Institute [3]. The idea of what I have called the Second Heisenberg Algebra and the Second Period of Nature was described in numerous lectures and talks (CALTECH, February 1996; Tel-Aviv University, March 1996; Ecole Polytechnique, Paris, October 1996).

The picture of a gravitating particle (a black hole) which has emerged from my work is not unlike that of a giant nucleus or a baryon.[3] The calculation of specific properties of such a complex object like a quantum black hole must be carried out in a sort of large $N$ approximation in the new gravitational noncommutative mechanics proposed in [1,3,8,9], and it is under way [4].

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