Methods of Parametric Geometric Modeling in the Design and Construction of Shells of Gaussian Curvature

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Abstract. The article discusses the method of minimizing costs in the design and construction of spans (shells) with the use of 3D construction printers. The results of the developed program in AutoLISP language for optimizing the surface of a shell of positive Gaussian curvature are presented. The technique of developing a model of a complex projected object for the implementation of a new class of applied problems in the field of additive technologies in construction is presented. The fundamental possibility of the process of extracting and generating new knowledge about the projected object is shown, consisting in replacing it with the corresponding model obtained by formalizing expert information. The construction of mathematical models of complex objects is performed in a fuzzy environment using linguistic variables, fuzzy numbers and arithmetic operations on them. The solution to such problems is based on the application of traditional methods widely used in technical applications to the analysis of fuzzy systems. The mathematical basis of the traditional approach is the algebra of fuzzy numbers.

1. Introduction
In extreme conditions and natural and man-made situations, new technologies for the design and construction of fast-moving facilities are required.

Many engineering facilities for providing water supply systems, water treatment, etc. have typical architectural forms. Construction of these structures should be carried out at a high rate, with minimum cost and high reliability.

Developing additive technologies can change the existing technological chain in the construction of construction facilities and not only simple forms.

Additive 3D technologies allow to create full-size forms of small architectural constructions with using construction 3D printers [1].

Process of creation of an object consists in: the special software creates a virtual solid-state 3D model of designed structures, then information is transferred to the code, clear for the construction 3D printer, which online layer-by-layer creates construction elements by method of extrusive printing by semi-fluid material.

The selection of the type of spatial shell is carried out on the basis of a feasibility study taking into account the functional purpose of the object and architectural requirements, as well as the conditions of manufacture and construction.
The shells (thin-walled spatial surfaces of Gaussian curvature) have significant advantages, allowing to overlap large spans without intermediate supports.

The design of these shells allows to organize the operation of floors due to uniform distribution of stresses over the whole surface, without massive bearing elements.

Such structures significantly allow to reduce the thickness of the coating due to the rational operation of the material. The thickness of the shells does not exceed 50-100 mm, and when used in concrete specially selected aggregates can be less. Due to rational operation of concrete (mainly for compression), reduced material consumption of coatings is achieved [3]. Instead of concrete, engineering plastic with the addition of fiberglass for strengthening of a design (ASA) can be used.

The vaults and shells allow economic overlapping of spans up to 200 m, spending 25-40% less material than in traditional "linear" coating systems.

Shells meet the requirements to increase strength and reduce the cost of construction. The geometry of the shell is determined by the shape of its midsurface.

Parametric models of different surfaces are becoming increasingly popular. Parametric modeling is a design based on the use of feature parameters that are component of a general model, and the ratio between those parameters that define the geometric shape of the model.

The main advantage of parametric geometric modeling (PGM) is that you can change the model geometry significantly in a short time by changing a parameter.

When designing buildings and structures, varying geometric parameters can reduce internal forces in the finite elements of the general model, as well as adopt the most rational constructive scheme of the building with minimal costs for the construction of the object.

A significant difference between parametric modeling and conventional three-dimensional modeling is that the model is given by mathematical equations in the form of varying functions.

Such simulations have become available very recently because mathematical operations require powerful computational complexes. Parametric architecture objects are external geometric shapes of the model.

The surface equation is: \( z(x,y) = \varphi_1(x) \pm \varphi_2(y) \).

The surface of the shells in question (Figure 1) can be outlined as a sphere, torus, elliptical paraboloid.

Equations of named surfaces have the form:

\( a) \ x^2 + y^2 - z^2 = R_2^2, \quad b) \ x^2 + y^2 + z^2 - 2R_2(z + R_1 - R_2) = -z(R_2 - R_1)\sqrt{y^2 + (z - R_2)^2}, \quad c) \ z = f_1 \left( \frac{x^2}{l_1^2} \right) + f_2 \left( \frac{y^2}{l_2^2} \right), \)

where \( x, y, z \) – are the current coordinates; \( R_1, R_2 \) – are the radii of curvature in the \( x, y \) direction.

\[ \text{Figure 1. Diagrams of designed shells of positive Gaussian curvature and coordinate systems:} \]
\[ \text{a) transfer surface;} \]
\[ \text{b) spherical surface;} \]
\[ \text{c) torus surface;} \]
\[ \text{d) elliptical paraboloid surface.} \]
Application of parametrical modeling and new information technology of Unigraphics at design and production of shells of complex geometric shapes allows to make reverse engineering analysis and to provide the automated preparation of managing programs for production of surfaces by means of the CNC (3D-printers) on the basis of the mathematical model of a product created in CAD subsystem.

The greatest effect gives use of the parametrical geometrical models (PGM) in typical design and construction of fast-moving structures.

Calculation of any type of the arch includes:

- Selection of an optimal working scheme, i.e. a system of primary and secondary arched elements that is most appropriate to the nature of the distribution of forces and the actual significance of each element;
- Determination of dimensions of design elements;
- Collecting and division of loading;
- Checking their bearing capacity by the value of compressive stresses in masonry.

Calculations and optimization of positive Gaussian curvature shells on a rectangular plan should be carried out in a software set of AutoCAD according to the equations of these surfaces [2].

So, for the shell in the form of a barrel vault (Figure 2), the formula of the surface description is

\[ z = \left( -\frac{x^2}{a} - \frac{y^2}{b} \right) / K, \]

where \( a, b \) – shell semi-rolls; \( K \) – lifting boom index \( \frac{h}{l} = \frac{1}{2K} \).

![Figure 2. Barrel vault.](image)

As an example, consider a barrel (parabolic) vault constructed by formula (1) for a rectangular plan measuring 6x24 m:

\[ z = \left( -\frac{x^2}{3} - \frac{y^2}{12} \right) / K, \]

where \( x, y \) – coordinates of the points of the shell with the origin at the top point and in the center of the site (Figure 2); \( K \) – lifting boom index.

If \( K = 4 \), then \( \frac{h}{l} = \frac{1}{8} \), i.e. height of shell lifting \( h \) is less than span \( l \) by 8 times for both directions.

Under distributed vertical load \( q = 7 \) kN/m² and under load from dead weight, the body of the roof shell mainly works for compression in two directions.

We will make optimization of a cover of positive Gaussian curvature on the rectangular plan in this case for the volume of required material on its device at some relation of height of rise to flight of \( \frac{h}{l} \).

After substitution of the accepted numerical values, the shape of the shell, for our case, will take the form:

\[ z = \left( -\frac{x^2}{3} - \frac{y^2}{12} \right) / 4, \]

Let’s set the conditions: material – concrete B25; thickness – \( \delta = 50-100 \)mm; load - dead weight and \( q = 7 \) kN/m².

The compression force from the distributed load \( q \) in the vault lock is determined by the formula:
\[ N = \frac{q l^2}{8h}, \quad (3) \]

As the height of the roof \( h \) decreases, the force \( N \) and the stress in the compressed shell will increase, which requires an increase in its thickness to ensure the strength condition:

\[ \sigma = \frac{N}{\delta} \leq R_b, \]

where \( \delta \) – thickness of material; \( \sigma \) – compression stress due to design load; \( R_b \) – design strength of concrete.

It follows from formulae (2) and (3) that the volume of the shell will increase with decreasing boom of lifting \( h \).

On the other hand, the volume of the shell depends on its surface area:

\[ V = S\delta, \]

where \( S \) – is the surface area of the shell.

Under these conditions there is a difficult optimizing task which can be solved with application of the AutoLISP programming language and a CAD of AutoCAD at different values \( K \).

Note that concrete shells for technological reasons should be taken to be at least 50 mm thick.

The program in the AutoLISP language was developed for creation of the barrel vault with the set parameters.

The designed shell model in AutoCAD units is shown in Figure 3.

![Figure 3. Solid Shell Model in M1:1 (in AutoCAD units).](image)

The parameters of solid-state model of a cover calculated in AutoLISP are presented to M1:1 in Figure 4.

![Figure 4. Design Parameters of the Solid Shell Model in M1:1.](image)

The parameters of the barrel vault are estimated within the range of change of the boom value \( K \) from 2 to 6.
The results of the optimization are shown in Table 1, and the dependence of the volume of the building material of the shell on the boom index $K$ is shown in Figures 4, 5.

**Table 1.** The results of the optimization.

| $K$ | Thickness 50 mm | 75 mm | 100 mm |
|-----|-----------------|-------|--------|
| 2   | 35.15819643     | 52.81291218 | 70.51807643 |
| 3   | 32.32003908     | 48.53799274 | 64.79459399 |
| 4   | 30.97501741     | 46.50866031 | 61.95003482 |
| 5   | 30.26364431     | 45.43353426 | 60.52728863 |
| 6   | 29.84870729     | 44.80535855 | 59.69741458 |

**Figure 5.** Shell Construction Material Volume Dependencies on $K$.

The minimum concrete flow rate corresponds to $K$ values from 5 to 6 in the range of shell thicknesses from 50 to 100 mm.

For further research, consider the design of a separate public building (Figure 6) in a monolithic design with a smooth monolithic shell resting on rows of columns. The contour structure of the building is represented by a designed curved bar (Figure 3).

**Figure 6.** Designed separate public building.

The decision-maker (LPR) for the construction of this building raises questions about the time, price costs for design and the choice of the most acceptable model.
The LPR is primarily interested in indicators that should answer the question: "To what extent will the virtual object in question have the properties (ability) to meet the needs of the consumer?"

The criterion for correct decision-making is the effectiveness of the chosen path to achieving the goal.

Such criteria may be simple or complex, including several indicators.

The selection of criteria is determined by specific objectives at different stages of object creation. The criteria should best take into account the linkages between individual objectives and their impact on the achievement of the ultimate goal.

These tasks include not only quantitative estimates (parameters), but also qualitative indicators that cannot be clearly formalized, can be taken into account only very roughly and are resolved by involving expert assessments, scientific hypotheses, intuition of researchers.

The solution of such problems is based on the application of traditional and widely used methods in technical applications for analysis of fuzzy weakly structured complex systems.

Mathematical basis of traditional approach in such cases, as well as in our case, is the algebra of indistinct numbers.

Modeling of statuses of a virtual designed difficult project causes the necessity of creation of mathematical models in the indistinct environment with use of linguistic variables, indistinct numbers and arithmetic operations over them.

In the offered approach the fact that the expert is capable to measure and "not measured" is important, i.e. to work in "virtual space" and to build models of forecasting of a status of an object at a predesign stage when the object does not exist yet.

Thus, in conditions of considerable uncertainty, the problems of expert knowledge extraction and presentation are provided by specially developed methods.

In order to solve the problem of minimization of expertise and ensuring consistency of expert estimates, it is advisable to use the ideas of experiment planning theory and the methodology described in [4].

In a mathematical look the solution of a task is carried out in a certain order.

Survey is originally conducted, and data with a set of expressions of the expert (experts) on separate components of a difficult construction object (Figure 6) in the form of production rules with indistinctly set linguistic variables are collected:

\[ A_{j1} = \langle \text{Если} \ X_1 = \bar{U}_{j1}^{\land} X_2 = \bar{U}_{j2}^{\land} ... \land X_n = \bar{U}_{jn}, \text{то} \ Y = \bar{V}_{j1} \rangle, j = 1, ..., 2^n \]

in \( n \)-dimensional fuzzy space. At the same time values of factors belong to term sets specified in fuzzy linguistic scales

\[ \bar{U}_{j1} \in T(X_1), M_{11}: T(X_1) \rightarrow F(R^1), 1 = 1, ..., n \]
\[ \bar{V}_j \in T(Y), M_{11}: T(Y) \rightarrow F(R^1) \]

as fuzzy numbers (LR)-type

\[ M_f(\bar{U}_{j1}) = \bar{U}_{j1} = (\bar{U}_{1j}, \bar{U}_{2j}, a_{j1}, \beta_{j1}, h_{j1})_{LR}, \]
\[ M_f(\bar{V}_{j1}) = \bar{V}_{j1} = (\bar{V}_{1j}, \bar{V}_{2j}, a_{j1}, \beta_{j1}, h_{j1})_{LR}. \]

Required: find and construct an approximation function

\[ V = f(\bar{U}, \bar{B}), \]

by means of which, estimation of a status of a difficult construction object will be carried out.

At the same time this function should meet in \( n \)-dimensional space of linguistic variables the following condition:

\[ \sum_{j=1}^{2^n} \left( V_j - f(U_{j1}, ..., U_{jn}, \bar{B}) \right)^2 \rightarrow \min_{\bar{B} \in F(R^2)} \]

where \( U_{j1} \) – indistinct number; \( \bar{U}_{j1} \) – value of indistinct number; \( \bar{U}_{j1} \in T(X_1) \) – term set of linguistic variable \( X_1 \), characterizing the factor space corresponding to the component in the task of assessing
the generalized quality of the designed construction object: \( \tilde{V}_j \in T(Y) \) – term set of dependent linguistic variable \( Y \), \( \tilde{B} \) – vector of coefficients of a polynom; \( F(R^k) \) – indistinct \( k \)-dimensional space.

The analysis of expressions (4)-(8) demonstrates that the problem of quantitative estimation of a status of a designed difficult construction project in \( n \)-dimensional space of indistinct variables by formalization of expert information (4) can be solved on the basis of the offered concept of creation of the generalized indicator of the status presented by function (7) in indistinct factor space (5) and (6) at execution of a condition (8).

The basic feature of process of extraction and forming of new knowledge of a designed project consists in substitution by its corresponding model by formalization of expert information with the subsequent its research for justification of acceptance of the correct solutions.

It allows to reduce decision-making process to formal procedures, leaving the decision-maker carrying out professional interpretation of results of mathematical researches.

The presented technique of development of model of a difficult designed project is intended for implementation of a new class of applied tasks in the field of additive technologies in construction.

2. Conclusion

1. In extreme conditions and at emergency situations of natural and technogenic character are required to apply new technologies of design and construction of pre-fabricated constructions.
2. Additive 3D technologies allow to create full-size forms of small architectural constructions using construction 3D printers.
3. At design of buildings and constructions variation of geometrical parameters (PGM) is able to afford to reduce internal efforts in terminal elements of the general model and also to accept the most rational constructive scheme of the building with the minimum costs of construction of an object.

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