Accretion of Phantom energy onto a Rotating BTZ Black Hole

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Abstract. In this paper, we study the accretion of phantom energy onto a (2+1)-dimension rotating Banados-Teitelboim-Zanelli (BTZ) black hole. An interesting finding is that the rate of change of mass of the rotating BTZ black hole depends not only on the energy density and pressure of the phantom energy but also on the mass of the rotating BTZ black hole.

1. Introduction
In Astrophysics accretion is defined as [1] “a process by which matter is collected around a central object”. In binary system accretion, one star is tidally deformed and matter flows out from it to the compact companion. When one deals with an isolated object, it may accrete from the interstellar medium at a very low rate. In many of the galactic centers there is evidence of supermassive black holes. There are no companions, but matter is accreted from winds of surrounding stars. In these cases, stars may also be tidally disrupted if they come very close to the black hole and the matter would be accreted from the disrupted stars to the central black hole.

The recent observational evidence obtained from Wilkinson Microwave Anisotropy Probe (WMAP) strongly suggests that the current expansion of the Universe is accelerating [2]. This accelerated expansion of the Universe is explained by the dominance of dark energy with a negative pressure in Einstein’s theory of gravity [3]. A peculiar property of cosmological models with dark energy is the possibility of a Big Rip [4]: an infinite increase in the scale factor of the Universe in a finite time. The Big Rip scenario is realized in the case of phantom energy (for which \( \rho + p < 0 \)). In the Big Rip scenario the phantom energy tends to infinity and all
Here we consider $(\rho + p) < 0$ alone is not enough for the Big Rip scenario to be realized [5].

The accretion of dark energy onto a black hole has been studied by many authors [6] after the seminal work of Babichev et al [7] who found that

$$dM = 4\pi AM^2 (\rho + p) dt,$$

which shows that the mass of the black hole increases as it accretes the gas of particles when $(\rho + p) > 0$, but decreases as it accretes the phantom energy. In particular this implies that the black hole mass in the Universe filled with phantom energy must decrease [7].

The accretion of phantom energy onto a (2+1)-dimensional stationary BTZ black hole was studied by M. Jamil and M. Akbar [8]. In their work they showed that the change in BTZ black hole mass is

$$dM = 2\pi A_1 (\rho + p) dt.$$

Note that for phantom energy $(\rho + p) < 0$, which leads to the decrease in the mass of the BTZ black hole. In this paper we extend this work to rotating BTZ black hole.

2. Model of Accretion for Rotating BTZ Black Hole

The axial symmetric rotating solution to the (2+1)-dimensional Einstein field equations is represented by the rotating BTZ black hole metric [9]

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 \left(d\phi - \frac{J}{2r^2} dt\right)^2,$$

where $N(r) = -M + \frac{r^2}{\rho^2} + \frac{J^2}{4\rho^2}$ is the lapse function and $M$ is the dimensionless mass, while $J$ is the angular momentum of the rotating BTZ black hole. In order to discuss the accretion dynamics of phantom energy onto the rotating BTZ black hole we follow the formulism from the work of Babichev et al [7]. Considering the phantom energy to be a perfect fluid for which the energy momentum tensor is

$$T^{ab} = (\rho + p)u^a u^b + pg^{ab},$$

where $\rho$ and $p$ are the energy density and pressure of the phantom energy respectively, while $u^a = (u^0, u^1, 0)$ is the three vector velocity of the fluid with pressure $p = p(r)$ and energy density $\rho = \rho(r)$. The normalization condition of the fluid velocity implies

$$g_{ab}u^a u^b = -1.$$

Here we consider $u^1 = u$, which is the radial component of velocity of the fluid flow. Solving equation (5) for $u^0$, gives

$$u^0 = \frac{N(r) + u^2}{N(r) \left(N(r) - \frac{J^2}{4r^2}\right)}.$$

There are two important equations of conservation on which the accretion process mainly depends. One is the energy flux equation, $T^{0a}_{\;;a} = 0$, and the other is the conservation of mass flux equation, $J^a_{\;;a} = 0$, where $J$ is the current density of the fluid. Since the black hole is stationary, the energy flux conservation equation leads to

$$(\rho + p)u^3 \sqrt{\frac{N(r)(N(r) + u^2)}{N(r) - \frac{J^2}{4r^2}}} \left(\frac{\sqrt{M^2 - \frac{J^2}{r^2}} + (M - \frac{2\rho^2}{r^2})}{\sqrt{M^2 - \frac{J^2}{r^2}} - (M - \frac{2\rho^2}{r^2})}\right) \frac{M}{\sqrt{\frac{M^2 - \frac{J^2}{r^2}}{4}} - \frac{J^2}{4}} = C,$$

(7)
where $C$ is an arbitrary positive constant. In order to find the second integral of motion we use the energy momentum conservation along the three vector velocity (the energy flux equation)
\[ u_a T^{ab}_{;b} = 0. \] (8)
Considering the phantom energy to be a perfect fluid, the conservation law becomes [10]
\[ \rho_b u^b + (\rho + p) u^b_{;b} = 0. \] (9)
The solution of equation (9) is
\[ u_r \exp \left( \int_{\rho_h}^{\rho} \frac{d\rho}{\rho + p} \right) = -A, \] (10)
where $A$ is an arbitrary positive constant. Also $\rho_h$ and $\rho_\infty$ are the energy density of the phantom energy at BTZ black hole horizon and at infinity respectively. Substituting equation (10) in equation (7), we have
\[ (\rho + p) r^2 \exp \left[ - \int_{\rho_\infty}^{\rho_h} \frac{d\rho}{\rho + p} \right] \sqrt{\frac{N(r)(N(r) + u^2)}{N(r) - \frac{J^2}{4r^2}}} \left( \sqrt{\frac{M^2 - \frac{J^2}{4r^2}}{M^2 - \frac{J^2}{4r^2}} + (M - \frac{2r^2}{T})} \right)^2 \frac{M}{\sqrt{M^2 - \frac{J^2}{4r^2}}} \times \frac{1}{\sqrt{r^2 2^2(M - \frac{r^2}{T}) - \frac{J^2 2^2}{4}}} = C_1, \] (11)
where
\[ C_1 = \frac{-C}{A} = \rho_\infty + p_\infty < 0, \] (12)
$\rho_\infty$ is defined above and $p_\infty$ is the pressure at infinity. The rate of change in the mass of rotating BTZ black hole is [8]
\[ \frac{dM}{dt} = 2\pi r T^1_0. \] (13)
The value of $T^1_0$ from the stress energy tensor is obtained as
\[ T^1_0 = (\rho + p) u \sqrt{\frac{N(r) + u^2}{N(r)}}. \] (14)
Using equations (10), (11), (12) and (14) in equation (13), for general $\rho$ and $p$, we finally have
\[ dM = 2\pi r A (\rho + p) \left( \frac{M - \frac{r^2}{T}}{-N(r)} \right)^{\alpha} \left( \frac{\sqrt{M^2 - \frac{J^2}{4r^2}} + (M - \frac{2r^2}{T})}{\frac{4r^2}{T}} \right)^2 \frac{M}{2\sqrt{M^2 - \frac{J^2}{4r^2}}} \] \[ dt, \] (15)
where
\[ \alpha = \frac{\sqrt{M^2 - \frac{J^2}{4r^2}} + M}{2\sqrt{M^2 - \frac{J^2}{4r^2}}}. \] (16)
Note that from equation (15) mass of the rotating BTZ black hole decreases for $\alpha$ to be a positive even integer, $M > \frac{r^2}{T}$ and $M^2 > \frac{J^2}{4r^2}$.
Critical Accretion

At the critical point of accretion the phantom energy shows a variety of changes in its behavior close to the compact object. In order to observe these changes we determine solutions that pass through the critical points. Such solutions correspond to the material falling into the black hole with monotonically increasing speed. To evaluate the critical point of accretion we consider the equation of mass flux or the continuity equation

\[ J^a_{a} = 0, \]  

(17)

which gives

\[ p u r = C_2, \]  

(18)

where \( C_2 \) is the constant of integration. From equations (18) and (7), we have

\[ \left( \frac{\rho + p}{\rho} \right)^2 r^4 N(r) \left( N(r) + u^2 \right) \left( \sqrt{M^2 - \frac{J^2}{r^2}} + (M - \frac{2\rho}{\rho} \right) \sqrt{\frac{M}{M^2 - \frac{J^2}{r^2}} - (M - \frac{2\rho}{\rho})} \right) \frac{1}{\sqrt{r^2 l^2 \left( M - \frac{2\rho}{r} \right) - \frac{J^2}{r^2}}} = C_3, \]  

(19)

where \( C_3 = (\frac{C_2}{C_2})^2 \). Taking the differential of equations (18) and (19) and eliminating \( \frac{d\rho}{\rho} \), we get

\[ \left[ -v^2 + \frac{u^2}{N(r) + u^2} \right] \frac{du}{u} + \left[ -v^2 + \frac{\frac{J^2}{r} - \frac{J^2}{r^2}}{N(r) + u^2} - \frac{M}{M + \frac{2\rho}{r}} + \frac{M}{f(r)} \right] \frac{dr}{r} = 0, \]  

(20)

where

\[ v^2 = \frac{d \ln(\rho + p)}{d \rho} - 1. \]  

(21)

It is clear from equation (20) that if one or the other bracket factor vanishes, we get a turn-around point corresponding to double valued solution in either \( r \) or \( u \). In order to get the critical points of accretion we take both the bracket factors in equation (20) equal to zero which gives the critical points of accretion as

\[ v_c^2 = \frac{r^2 - \frac{J^2}{r^2}}{N(r_c) + u_c^2} \left( -M + \frac{2\rho}{\rho} \right) + \frac{M}{N(r_c)}, \]  

(22)

and

\[ v_c^2 = \frac{u_c^2}{N(r_c) + u_c^2}, \]  

(23)

where the subscript \( c \) refers to the critical quantities. Comparing equations (22) and (23), we have

\[ u_c^2 = \frac{r^2}{l^2} \left( \frac{(-M + \frac{2\rho}{\rho})^2 - (\frac{J^2}{r^2})}{(-M + \frac{2\rho}{\rho})^2 + \frac{J^2}{r^2}} \right). \]  

(24)

In equation (24) the quantity \( u_c \) represents the critical speed of flow at the critical point. Furthermore by taking \( J = 0 \) in equations (23) and (24), one gets the corresponding results of the non-rotating BTZ black hole [8]. For physically acceptable solution, we require \( v_c^2 > 0 \), hence we get the following restriction on speed and location of critical points

\[ u_c^2 > M - \frac{r^2}{l^2} - \frac{J^2}{4r_c^2}, \]  

(25)

and

\[ v_c^2 > \frac{l^2 M}{2r_c^2}. \]  

(26)

Conditions (25) and (26) reduce to the conditions for non-rotating BTZ black hole [8] by taking \( J = 0 \).
Conclusion
In this paper accretion of phantom energy onto a rotating BTZ spacetime is discussed. We follow the same procedure as followed for the non-rotating BTZ black hole [8]. Our analysis shows that the evolution of the rotating BTZ black hole mass depends not only on the energy density and pressure of the phantom energy but also on the mass of the rotating BTZ black hole. The mass of the rotating BTZ black hole decreases if \( \alpha = \frac{\sqrt{M^2 - \frac{J^2}{2}} + M}{2\sqrt{M^2 - \frac{J^2}{2}}} \) is a positive even integer, \( M > \frac{J^2}{2\pi} \) and \( M^2 > \frac{J^2}{2} \). The physical cause for the decrease in the black hole mass is as follows: the phantom energy falls to the black hole, but the energy flux associated with this fall is directed away from the back hole. We also discussed the critical accretion and found the critical parameters. All the results presented in [8] can be obtained as a special case of our analysis by taking \( J = 0 \).

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