Renormalizability of the center-vortex free sector of Yang-Mills theory
A Virtual Tribute to Quark Confinement and the Hadron Spectrum

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Outline

- Description of the gauge fixing procedure.

- Motivations: could be free from Gribov copies and provide a path from pure Yang-Mills to center vortex ensembles.

- Renormalizability.

- Conclusions and future perspectives.
Preliminary remarks about the gauge fixing procedure

- This gauge was proposed by L. Oxman and G. Rosa (2015), for continuum pure SU(N) Yang-Mills (YM) theory in $3 + 1$ dimensions.

- The gauge condition is imposed on auxiliary fields.

- This is in contrast with most continuum procedures\(^1\), but resembles the lattice Laplacian Center Gauge (Ph. de Forcrand and M. Pepe (2001)).

\(^1\)See H. Reinhardt and T. Tok (2001) for an exception.
The gauge fixing procedure

- Step 1: correlate $A$ with the auxiliary fields, by means of the minimization of the $SU(N)$ invariant auxiliary action

$$S_{aux}(A, \psi) = \int d^4x \left( \langle D_\mu (A) \psi_I, D_\mu (A) \psi_I \rangle + V_{aux}(\psi) \right).$$

- $\psi_I$ are a set of $N^2 - 1$ adjoint scalar fields. $V_{aux}$ is chosen such that $S_{aux}$ has a $SU(N) \rightarrow Z(N)$ SSB.

- This provides a map $A \rightarrow \psi_I(A)$. 
Step 2: extract a phase $S \in SU(N)$ from the tuple $\psi_I(A)$. This is accomplished by a generalized polar decomposition, i.e. $\psi_I(A) = S(A) q_I(A) S^{-1}(A)$.

In general, the phase $S \in SU(N)$ will contain defects associated to center vortices, e.g. $S_{1v}(\varphi) = e^{i\varphi \beta q} T_q$. $\beta$ is proportional to a fundamental weight, $T_q$ are the Cartan generators of $SU(N)$. 
In general, the phase $S \in SU(N)$ will contain defects associated to center vortices, e.g. $S_{1v}(\varphi) = e^{i\varphi \beta_q T_q}$.

**Figure:** The phase $S_{1v}(\varphi)$ is close to the identity at point $P$, and to a center element at point $Q$, satisfying $S_{1v}(2\pi) = e^{-i 2\pi/N} S_{1v}(0)$. This phase can’t be eliminated by a regular gauge transformation.
An equivalence relation between phases is naturally introduced:
\[ [S] = [S'] \iff S = US', \ U \text{ regular.} \]

It is natural to split the configuration space \( \{A\} \) into sectors \( \mathcal{V}(S_0) \), where \( A \in \mathcal{V}(S_0) \iff S(A) = US_0, \ U \text{ regular.} \)

The gauge is fixed separately in each sector \( \mathcal{V}(S_0) \) by means of the sector-dependent pure modulus condition
\[
f_{S_0}(\psi) = [S_0^{-1}\psi I(A)S_0, T_I] = 0.
\]

This is a local gauge fixing procedure.

I. M. Singer (1978): no continuous global gauge fixing is possible.
**Figure:** The connection fiber bundle \( \{A\} \). Gauge orbits are represented by green vertical lines.
Figure: The connection fiber bundle \( \{A\} \). Gauge orbits are represented by green vertical lines. Blue line: attempt at a global gauge fixing condition (global cross-section). Singer’s theorem implies that this type of gauge fixing is impossible.
Figure: The connection fiber bundle \( \{A\} \). Gauge orbits are represented by vertical green lines. The different shaded regions represent the sectors \( V(S_{1v}) \), \( V(S_{2v}) \), \( V(S_{3v}) \). The red lines correspond to a local gauge fixing procedure (local cross-sections).
After gauge fixing, the YM partition function is written as

$$Z_{YM} = \sum_{S_0} Z(S_0) \ , \quad Z(S_0) \propto \int_{\mathcal{V}(S_0)} [DA_\mu][D\phi] e^{-S_{YM}(A)} e^{-S_{gf}^{(S_0)}(A,\phi)} \ ,$$

$$\phi \equiv \{ b, c, \bar{c}, b_I, \bar{c}_I, c_I, \psi_I, \lambda_I, \xi_I \}. $$

The action $S_{gf}^{(S_0)}$ implements the sector-dependent gauge condition.
• The full action $S_{YM} + S_{gf}^{(S_0)}$ is sector dependent.

• In each sector, the action is invariant under a BRST symmetry $s$.

• $S_{gf}^{(S_0)}$ may be written as a trivial variation, i.e., all the terms associated to the gauge fixing procedure belong to the trivial part of the cohomology of $s$. 
The $s$ symmetry of the vortex-free sector ($S_0 = I$):

\[
\begin{align*}
  sA^a_\mu &= i g D^a_{\mu} c^b, \\
  s\bar{c}^a &= -b^a, \\
  s\psi^a_i &= if^{abc} \psi^b_i c^c + c^a_i, \\
  sb^a_i &= if^{abc} b^b_i c^c, \\
  s\mu^2 &= U^2, \\
  s\kappa &= \kappa, \\
  s\lambda &= \Lambda, \\
  sM^{ab}_i &= N^{ab}_i, \\
  s\bar{C}^a &= sK^a_\mu = sL^a_i = s\bar{L}^a_i = sQ^a_i = sB^a_i = 0.
\end{align*}
\]
In this framework, observables are computed as follows:

\[
\langle O \rangle_{YM} = \frac{1}{Z_{YM}} \sum_{S_0} \frac{Z(S_0)}{Z(S_0)} \int_{\mathcal{V}(S_0)} [DA_\mu][D\phi] \ O \ e^{-S_{YM}(A)} e^{-S_{gf}(A,\phi)} .
\]

What if...

\[
\frac{Z(S_0)}{Z_{YM}} = e^{-S_{eff}(S_0)} .
\]
Then,

\[ \langle O \rangle_{YM} = \sum_{S_0} e^{-S_{\text{eff}}(S_0)} \langle O \rangle_{S_0}, \]

with \( S_0 \) ranging over the set of all possible center-vortex configurations.

This is a glimpse of a path from pure YM to center-vortex ensembles in the continuum.

Phenomenological center-vortex ensembles containing tension and stiffness terms in \( S_{\text{eff}}(S_0) \) are known to reproduce the properties of the confining flux tube at asymptotic distances. For a recent review on this subject, see D. R. Junior, L. E. Oxman, G. M. Simões, Universe 7(8), 253 (2021). See also Luis Oxman’s talk tomorrow!
Are these partial contributions $e^{-S_{\text{eff}}(S_0)}$ calculable?

As a first step to answer this question, we analyzed the renormalizability of the vortex-free (perturbative) sector $S_0 = I$.

We used the algebraic method, where one characterizes $\Sigma^{c.t.}$ by using the Ward Identities of the action $\Gamma$ (O. Piguet and S. Sorella, 1995).
Ward identities: Slavnov-Taylor, ghost equation, ghost number, gauge-fixing equation, antighost equation, exact rigid symmetry, color-flavor symmetry.

Ghost equation:

\[
\frac{\delta}{\delta c^a} + (v f^{abc} f^{clm} + i f^{abn} M^m_{l}^{mn}) \frac{\delta}{\delta n^m_{l}} \Sigma = \Phi^a,
\]

\(\Phi^a\) being a linear polynomial in the fields.
The most general counterterm $\Sigma^{c.t.}$ compatible with the Ward Identities may be absorbed by a redefinition of the parameters and fields.
Conclusions

- The vortex-free sector is perturbatively renormalizable.

- Work in progress: renormalizability of a general sector $\mathcal{V}(S_0)$.

- Future perspectives: compute $S_{\text{eff}}(S_0)$. Casimir-like problem with codimension 2. See Ref. C. D. Fosco, D. R. Junior, L. E. Oxman (2020) for a much simpler, but analogous problem.
Thank you!