Combining the baryon budget with CMBR measurements

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ABSTRACT

Measurements of the Cosmic Microwave Background Radiation (CMBR) provide a powerful tool for measuring the primary cosmological parameters. However, there is a large degree of parameter degeneracy in simultaneous measurements of the matter density, \( \Omega_m \), and the Hubble parameter, \( H_0 \). In the present paper we use the presently available CMBR data together with measurements of the cosmological baryon-to-photon ratio, \( \eta \), from Big Bang nucleosynthesis, and the relative mass fraction of baryons in clusters to break the parameter degeneracy in measuring \( \Omega_m \) and \( H_0 \). We find that present data is inconsistent with the standard \( \Omega = 1 \), matter dominated model. Our analysis favours a medium density universe with a rather low Hubble parameter. This is compatible with new measurements of type Ia supernovae, and the joint estimate of the two parameters is \( \Omega_m = 0.45^{+0.07}_{-0.05} \) and \( H_0 = 39^{+13}_{-13} \) km s\(^{-1}\) Mpc\(^{-1}\). We stress that the upper bound on the Hubble parameter is likely to be much more uncertain than indicated here, because of the limited number of free parameters in our analysis.

Key words: Cosmology: cosmic background radiation, dark matter

1 INTRODUCTION

Many recent measurements seem to indicate that our universe is not a critical density, matter dominated Friedmann universe. Rather, the recent measurements of type Ia supernovae (Perlmutter et al. 1998, Perlmutter et al. 1997, Garnavich et al. 1998, Riess et al. 1998, Schmidt et al. 1998) strongly suggest that, although the universe has a flat geometry, the matter density is low (Perlmutter et al. 1998, Riess et al. 1998). The energy density is instead dominated by either a cosmological constant (Carroll, Press and Turner 1992, or by a similar type of energy with negative pressure, such as quintessence (Wang et al. 1998, Zlatey, Wang and Steinhardt 1998, Caldwell, Dave and Steinhardt 1998).

Numerous investigations have shown that the fluctuation spectrum of the cosmic microwave background radiation (CMBR) provides a potentially very powerful tool for determining cosmological parameters (Jungman et al. 1996, Bond, Efstathiou and Tegmark 1997, Zaldarriaga, Spergel and Seljak 1997, Hu, Eisenstein and Tegmark 1998). This, and one could hope that accurate measurements of these fluctuations could resolve the issue of whether or not the universe is dominated by vacuum energy. Based on present observations (Lineweaver and Barbosa 1998, Bond and Jaffe 1998, Bernardis et al. 1997, Tegmark 1998, Coble et al. 1999, Olivei-Costa et al. 1999, Coile et al. 1999), there already exist a number of estimates of the cosmological parameters (Efstathiou et al. 1999, Webster et al. 1998, Hancock et al. 1998, Lineweaver and Tegmark 1998, Bond and Jaffe 1998, Bernardis et al. 1997, Tegmark 1998, Coble et al. 1999, Olivei-Costa et al. 1999, Coile et al. 1999). The energy density is instead dominated by vacuum energy. Based on present observations (Efstathiou et al. 1999, Webster et al. 1998, Hancock et al. 1998, Lineweaver and Tegmark 1998, Bond and Jaffe 1998, Bernardis et al. 1997, Tegmark 1998, Coble et al. 1999, Olivei-Costa et al. 1999, Coile et al. 1999)

There is, however, a severe problem in that a change in one parameter can often be mimicked by suitable changes in a combination of other parameters (Hu, Eisenstein and Tegmark 1998, Eisenstein, Hu and Tegmark 1997, Eisenstein, Hu and Tegmark 1998). This is for instance very problematic when trying to simultaneously determine the cosmological matter density, \( \Omega_m \), and the Hubble parameter, \( H_0 \). It has therefore been suggested that CMBR measurements should be combined with other constraints, coming for instance from large scale structure surveys (Eisenstein, Hu and Tegmark 1998, Eisenstein, Hu and Tegmark 1998a, Eisenstein, Hu and Tegmark 1998b, Webster et al. 1998, or supernova type Ia measurements (Tegmark 1998, White 1998, Lineweaver 1998, Eisenstein, Hu and Tegmark 1998b, Eisenstein, Hu and Tegmark 1998a, Eisenstein, Hu and Tegmark 1998b, Webster et al. 1998). In this paper we explore another possibility, namely how the cosmic baryon abundance can be used together with CMBR to provide tight constraints on \( \Omega_m \) and \( H_0 \).

Galaxy clusters contain large amounts of hot, X-ray emitting gas from which the baryon mass to total mass ratio can be measured (White and Fabian 1995, David, Jones and Forman 1998, Evrard, Metzler and Navarro 1998, Evrard 1997). It turns out that measurements of this baryon cluster fraction can break the parameter degeneracy inherent in the CMBR measurements, and although most of the present CMBR data is of relatively low accuracy...
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(Hu and Tegmark 1998b; Hu, Eisenstein and Tegmark 1999), it is still sufficiently accurate to provide good constraints on the relevant cosmological models. We have used the previously available data, together with data from Big Bang nucleosynthesis and measurements of the cluster baryon fraction, and find that the observations are indeed strongly incompatible with a critical density matter dominated model. Our results are easily compared with for instance the new supernova type Ia measurements (Perlmutter et al. 1998), and are found to be completely compatible.

There still remains a quite large uncertainty on the determination of $\Omega_m$ and $H_0$ because of the low accuracy of the CMBR data. However, in the very near future, CMBR data of very high quality should become available from several different experiments. There is the balloon borne experiment BOOMERANG (Hanany 1997) which has already been flown. Also, there are two new satellite experiments, MAP and PLANCK, which will measure the fluctuation spectrum very accurately on sub-degree scales. This should provide data which is accurate enough to diminish the uncertainties by an order of magnitude.

2 BREAKING DEGENERACY

As mentioned above, parameter extraction from the CMBR data suffers from some very large parameter degeneracies (Eisenstein, Hu and Tegmark 1998a; Eisenstein, Hu and Tegmark 1998b). For instance it is not possible to constrain $\Omega_m$ and $H_0$ separately, effectively only the combination $\Omega_m h^2$ (Eisenstein, Hu and Tegmark 1998a; Eisenstein, Hu and Tegmark 1998b; Hu, Eisenstein and Tegmark 1999), where

$$h = \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}. \quad (1)$$

can be measured accurately.

However, as shown previously by several authors, it is possible to break this degeneracy by combining CMBR data with other, complementary, measurements. It was shown by Eisenstein et al. (Eisenstein, Hu and Tegmark 1998a; Eisenstein, Hu and Tegmark 1998b) that large scale structure surveys like the Sloan Digital Sky Survey (SDSS) can probe the combination $\Omega_m h^{−1/3}$. This means that the error ellipses of such surveys are almost orthogonal to those of CMBR. The joint likelihood function should show a spike-like structure at the crossing of the two ellipses instead of the very elongated structure of either of the two individual measurements (Eisenstein, Hu and Tegmark 1998a; Eisenstein, Hu and Tegmark 1998b). In fact, Webster et al. (Webster et al. 1998) have already performed a joint analysis of the present CMBR data together with data from the IRAS 1.2 Jy galaxy survey (Fisher et al. 1995). They find that the CMBR likelihood contours are indeed narrowed significantly in the ($\Omega_m$, $H_0$) plane. The above way of breaking degeneracy will work when combining CMBR with any type of measurement that does not depend on $\Omega_m$ and $H_0$ as $\Omega_m h^2$.

One such possibility is to use the cluster baryon fraction. It has long been known that measurements of the baryon cluster fraction favour a low density universe because the measured fraction is so high that it cannot support $\Omega_m = 1$ without violating BBN constraints (Bludman 1998; Steigman, Hata and Feltan 1999; White et al. 1993; Evrard 1993). A standard assumption in this game is to assume that the cluster baryon fraction is the same as the universal fraction, an assumption usually referred to as the fair sample hypothesis. Numerical simulation seem to justify this assumption. In fact recent simulations indicate that the cluster baryon fraction is slightly lower that that of the universe as a whole (see Steigman, Hata and Feltan 1999 for a discussion). One problem is that observed clusters have diverging baryon fractions, depending on their total mass. It is argued by Evrard et al. (Evrard, Metzler and Navarro 1998) that this is most likely due to errors in the estimate of the total cluster mass, and that it can be corrected by use of statistical methods. In this paper we shall use the results obtained by Evrard (Evrard 1997) for the universal cluster baryon fraction.

The method for estimating $\Omega_m$ and $H_0$ then works as follows: Big Bang nucleosynthesis (BBN) can be used to measure the baryon-to-photon ratio (Kolb and Turner 1990), $\eta$, which is related to $\Omega_b$ by

$$\Omega_b h^2 = 3.66 \times 10^7 \eta. \quad (2)$$

Thus, nucleosynthesis only measures $\Omega_b h^2$, not $\Omega_b$.

On the other hand, the cluster baryon fraction, $f_B$, is really measured as (Bludman 1998; Evrard 1997)

$$f_B h^{3/2} = \frac{\Omega_b}{\Omega_m} h^{3/2}. \quad (3)$$

By combining BBN measurements with the cluster baryon fraction we can therefore constrain the combination $\Omega_m h^{1/2}$.

This combination of $\Omega_m$ and $h$ is sufficiently different from $\Omega_m h^2$ that when combining them it becomes possible to constrain both parameters well.

3 MEASUREMENTS

3.1 CMBR measurements

In general, the fluctuations in the CMBR is measured in terms of spherical harmonics

$$T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \quad (5)$$

where the coefficients are related to the power-spectrum $C_l$ coefficients by

$$C_l \equiv \langle |a_{lm}|^2 \rangle_m. \quad (6)$$

At present there is a host of different CMBR experiments, ranging from the largest scales (COBE), down to very small scales. The data that we use are based on the compilation by Lineweaver and Barbosa (Lineweaver and Barbosa 1998).
but with the addition of the new Python V results, and the results from the QMAP experiment (Oliveira-Costa et al. 1999).

3.2 BBN measurements

The past few years have seen a very large fluctuation in the estimated baryon-to-photon ratio (see for instance Steigman 1998 for a review). It has long been known that the primordial value of deuterium provides a very sensitive probe of \( \eta \) (Kolb and Turner 1990), but the problem has been to measure the primordial deuterium abundance. In the local interstellar medium, the abundance is quite well determined (Hata et al. 1993),

\[
D/H = 1.6 \times 10^{-5},
\]

but this can only really be used to provide a strict lower limit to the primordial abundance since deuterium is only destroyed, not produced, in astrophysical environments.

Measurements of deuterium in quasar absorption systems at high redshift have provided a completely new way of measuring the primordial abundance because such systems are chemically unevolved and should therefore contain deuterium abundances close to the primordial (Steigman 1998).

There have been two conflicting estimates of the deuterium abundance in these systems, one which is much higher than the local value (Songaila et al. 1994, Carswell et al. 1994, Rugers and Hogan 1996, Webb et al. 1997, Tytler et al. 1998),

\[
D/H \simeq 2 \times 10^{-4},
\]

and one which is only a factor of two higher than the local value (Burles and Tytler 1998a, Burles and Tytler 1998b),

\[
D/H \simeq 3 \times 10^{-5}.
\]

There is growing evidence that the low deuterium value is the correct one, and as observational values we shall take the so-called Low-deuterium/High-helium data set, which is given by (Burles and Tytler 1998a, Burles and Tytler 1998b),

\[
Y_P = 0.245 \pm 0.002 \quad (10)
\]

\[
D/H = (3.4 \pm 0.3) \times 10^{-5}. \quad (11)
\]

This set of data is completely compatible with standard Big Bang nucleosynthesis for a baryon-to-photon ratio of

\[
\eta = (5.1 \pm 0.3) \times 10^{-10}, \quad (12)
\]

or, in terms of baryon density \( \Omega_b h^2 = 0.019 \pm 0.001 \).

3.3 Baryon cluster fraction

As mentioned above, Evrard (1994) has calculated the universal cluster baryon fraction based on a large number of clusters. His result is

\[
f_B = (0.060 \pm 0.003)h^{-3/2}. \quad (13)
\]

The 1σ uncertainty is perhaps somewhat underestimated in the measurement, and we shall follow Hata et al. (Steigman, Hata and Felten 1991) in assuming taking the 1σ uncertainty to be twice as large, i.e. 0.006. The above value is derived from the gas fraction alone. The fraction in hot gas relative to collapsed baryonic objects has been estimated by White et al. (White et al. 1993) to be

\[
\frac{M_{\text{gas}}}{M_{\text{gal}}} = 5.5 h^{-3/2} \quad (14)
\]

which is large enough to be insignificant.

4 LIKELIHOOD ANALYSIS

In order to estimate the underlying cosmological parameters, we have calculated a large number of synthetic models which are then compared with the different data sets. Our synthetic models range through a large parameter space and have all been calculated using the publicly available CMBFAST code (Seljak and Zaldarriaga 1996). We restrict the discussion to strictly flat models, i.e. models with \( \Omega_m + \Omega_s = 1 \). As free parameters we choose the normalisation, \( Q \), the matter density, \( \Omega_m \), the baryon density, \( \Omega_b \), the Hubble parameter, \( h \), and the spectral index, \( n \). All the above parameters are allowed to vary with the restrictions described in table 1. Altogether, 65000 independent CMB spectra have been calculated, and, apart from the fact that we do not investigate open models, this analysis is comparable to that of Lineweaver (Lineweaver 1998).

In a new analysis, Tegmark (Tegmark 1998) has calculated the likelihood function in a larger, 9-dimensional parameter space. The extra parameters are: the optical depth to reionisation, \( \tau \), and the amplitude and spectral index of tensor fluctuations. Here, it was shown that relaxing the assumption about zero curvature significantly loosens the constraint on the combination \( \Omega_m h^2 \). In fact Tegmark finds that there is no upper bound on \( \Omega_m h^2 \), even at the 68% confidence limit, contrary to our findings below. The reason for this is that the constraint relies to a certain extent on the amplitude differences in the power spectrum at different \( l \). The information stored in this difference is sensitive to changing the spatial curvature, and therefore relaxing the flatness assumption leads to a much looser upper bound on \( \Omega_m h^2 \). This being said, there are good reasons for neglecting spatial curvature. Flatness is a generic prediction of almost all inflationary models, and furthermore the new data on Supernovae of type Ia clearly favour a spatially flat universe (Perlmutter et al. 1998).

From the same prejudice that led us to consider only flat models, we have neglected possible tensor fluctuations. Almost all inflationary models predict only very small tensor fluctuations, so this is likely to be a good assumption. We have also neglected reionisation in our calculations. There is no justification for this since we know for a fact that the universe was reionised at fairly high redshift. As shown by Tegmark (Tegmark 1998), adding reionisation to the model parameters broadens the likelihood function, but does not move the maxima. Therefore we stress that our likelihood estimates should be considered optimistic and that they will broaden if reionisation is taken into account, but that they should still home in on the correct central values.

In order to compare with the different data sets we then perform a \( \chi^2 \) analysis. In general, \( \chi^2 \) is given as

\[
\chi^2 = \sum_{i=1}^N \frac{1}{\sigma_i^2} (F_{i, \text{obs}} - F_{i, \text{theory}})^2, \quad (15)
\]
Table 1. The free parameters used in our analysis

| Parameter | range      | no. grid points |
|-----------|------------|-----------------|
| $Q_2$     | 5–40 μK    | 200             |
| $\Omega_m$ | 0.2–1       | 17              |
| $\Omega_b h^2$ | 0.002–0.030 | 15              |
| $h$       | 0.30–0.75  | 16              |
| $n$       | 0.7–1.3    | 16              |

Figure 1. The likelihood function plotted in the $(\Omega_m, H_0)$ plane after maximising over all other free parameters. The contours shown are 68%, 95% and 99% confidence limits.

where $F$ for CMBR data is equal to $\Delta T$ for different values of $l$. For BBN data, $F$ is equal to either the helium or the deuterium abundance. $\sigma_i$ is the uncertainty in the given data point.

Here it has been assumed that the experimental errors are of Gaussian nature, which in reality they are not. Therefore the confidence limits that we obtain are not strict in a mathematical sense, but should rather be seen as indicative.

If the measurements are furthermore assumed to be independent then the likelihood function is then given by

$$L \propto e^{-\chi^2/2},$$

and

$$L_{\text{joint}} = L_{\text{CMBR}} L_{\text{BBN}} L_{\text{clusters}}.$$ (16)

In Fig. 1 we show the likelihood contours taking into account different measurements. The likelihood contours have been based on $\Delta \chi^2$, such that the 68%, 95%, and 99% confidence limits correspond to $\Delta \chi^2 = 2.29, 6.18$ and 9.21 respectively. The top panel shows the likelihood contours for CMBR data alone. As expected it shows a very elongated structure, roughly on the $\Omega_m \propto h^{-2}$ line. The middle panel shows how the likelihood contours are affected if one combines CMBR with the BBN estimate of $\Omega_b h^2$. Clearly, there is only a quite small effect on the determination of $\Omega_m$ and $H_0$, because $\Omega_b$ is not very degenerate with either of these two parameters. The real improvement only comes when one takes into account the cluster baryon fraction measurements.

Doing this narrows the likelihood contours to a more vertical and much tighter structure. Intriguingly, the combined likelihood function from this method favours a medium density universe with $\Omega_m$ around 0.5 and a very low value of the Hubble parameter. This is strongly incompatible with the standard $\Omega_m = 1$ CDM model, unless the Hubble parameter is much lower than 0.3. However, as mentioned above our likelihood contours are somewhat optimistic, especially the upper bound on the Hubble parameter is sensitive to adding extra parameters to the analysis.

It is also of interest to compare our results with the recent results from measurements of supernova type Ia’s. The best available data come from the supernova cosmology project, who have measured 42 high redshift supernovae (Perlmutter et al. 1998). From these they obtain stringent limits on $\Omega_m$ in a flat universe, but no constraint on the Hubble parameter. Their best fit value for $\Omega_m$ in a flat universe is

$$\Omega_m = 0.28^{+0.09}_{-0.08} \text{(statistical)} \pm 0.05 \text{(systematic).}$$ (17)

These data are compatible with our results, even at the 1σ level, but only for relatively low values of the Hubble parameter. Joint maximum likelihood for our analysis and the supernova data are shown in Fig. 2. The likelihood function has a maximum at

$$(\Omega_m, h) = (0.45, 0.39),$$ (18)

but with a fairly large uncertainty on the Hubble parameter. At the 95% confidence level the Hubble parameter is only constrained to be less than 0.64 and, again, we stress that adding more parameters to the analysis could weaken this bound further.

5 CONCLUSIONS

We have calculated the possible constraints on $\Omega_m$ and $H_0$ from combining CMBR data with data on the cosmic baryon abundance. It was found that these measurements are inconsistent with the standard flat CDM model, even for very low values of the Hubble parameter. Our favoured region of parameter space is consistent with recent measurements from type Ia supernovae at the 1σ level (Perlmutter et al. 1998). The joint likelihood function suggests that $\Omega_m$ is close to 0.4 and that a rather low value of the Hubble parameter is favoured. It is interesting to compare our results with those of Webster et al. (Webster et al. 1998), who did a joint analysis of the CMBR data and the IRAS 1.2 Jy data (Fisher et al. 1993) on large scale structure. They found a best fit for the joint likelihood of

$$(\Omega_m, h)_{\text{CMBR+IRAS}} = (0.39^{+0.14}_{-0.10}, 0.53^{+0.05}_{-0.14}),$$ (19)

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Convergence towards the true, underlying, cosmological model, using different methods. This suggests that there is a consistent estimate making it increasingly unlikely that the data are contaminated by some large, unknown, systematic error. Of course when the new high precision data become available from the BOOMERANG, MAP and PLANCK experiments, these uncertainties should be resolved.

In conclusion what we have shown in the present paper is that if the cosmic baryons are used in unison with CMBR data to constrain the cosmological parameters $\Omega_m$ and $h$, the results are very close to other recent measurements using different methods. This suggests that there is a convergence towards the true, underlying, cosmological model, which is apparently a flat, low density universe. The fact that so many completely independent methods all yield roughly consistent estimates make it increasingly unlikely that the data are contaminated by some large, unknown, systematic error. Of course when the new high precision data become available from the BOOMERANG, MAP and PLANCK experiments, these uncertainties should be resolved.

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