MESON SYNCHROTRON EMISSION FROM CENTRAL ENGINES OF GAMMA-RAY BURSTS WITH STRONG MAGNETIC FIELDS

AKIRA TOKUHISA1,2,3 AND TOSHITAKA KAJINO1,2,3,4

Received 1999 August 11; accepted 1999 September 14; published 1999 October 8

ABSTRACT

Gamma-ray bursts (GRBs) are presumed to be powered by the still unknown central engines with timescales in the range from 1 ms to approximately a few seconds. We propose that the GRB central engines would be a viable site for strong meson synchrotron emission if they were compact astrophysical objects, such as neutron stars or rotating black holes with extremely strong magnetic fields \( (H \sim 10^{12} - 10^{17} \text{ G}) \), and if protons or heavy nuclei were accelerated to ultrarelativistic energies on the order of \( \sim 10^{12} - 10^{22} \text{ eV} \). We show that the charged scalar mesons like \( \pi^\pm \) and heavy vector mesons like \( \rho \), which have several decay modes onto \( \pi^\pm \), could be emitted, with a high intensity that is a thousand times larger than photons, through strong couplings to ultrarelativistic nucleons. These meson synchrotron emission processes eventually produce a burst of very high energy cosmic neutrinos with \( 10^{12} \text{ eV} \leq E_\nu \). These neutrinos are to be detected during the early-time duration of short GRBs.

Subject headings: cosmic rays — elementary particles — gamma rays: bursts — magnetic fields — stars: neutron

1. INTRODUCTION

The accumulated observational evidence shows the existence of astrophysical objects with extremely strong magnetic fields, \( \sim 10^{13} \text{ G} \). Kouveliotou et al. (1998) have revealed that a Galactic X-ray pulsar with an estimated magnetic field of \( \sim 8 \times 10^{14} \text{ G} \) causes recurrent bursts of soft gamma rays, which are called soft gamma repeaters. Gamma-ray bursts (GRBs) with short and more intense bursts of 100 keV–1 MeV photons still remain puzzling, although it has become progressively clearer that they are likely to have a cosmological origin. Some of the theoretical models of GRBs (Usov 1992; Kluźniak & Ruderman 1998; Paczyński 1998) invoke compact objects for their central engines, such as neutron stars or rotating black holes with extremely strong magnetic fields \( (H \sim 10^{16} - 10^{17} \text{ G}) \), in order to produce an ultrarelativistic energy flow with a huge Lorentz factor of \( \Gamma \sim 10^3 \). Furthermore, neutron stars or black holes with strong magnetic fields have been proposed by many authors (Hillas 1984 and references therein) as an acceleration site of ultrahigh-energy cosmic rays (UHECRs). Milgrom & Usov (1995) suggested a possible association of the two highest energy UHECRs with strong GRBs.

An ultrarelativistic nucleus gives rise to efficient meson emission, in analogy with the canonical photon synchrotron radiation, in such a strong magnetic field because it couples strongly to meson fields as well as to an electromagnetic field. Due to its large coupling constant, the meson synchrotron emission is \( \sim 10^3 \) times stronger than the usual photon synchrotron radiation. Ginzburg & Syrovatskii (1965a, 1965b) calculated the intensity of \( \pi^0 \) emission by a proton in a given magnetic field. At that time, however, they could hardly find the astrophysical sites to which their formulae could be applied.

In the present Letter, we propose that GRB central engines could be a viable site for strong meson emission. Waxman & Bahcall (1997) proposed that high-energy neutrinos with \( \sim 10^{18} \text{ eV} \) are produced by photomeson interactions on shock-accelerated protons in the relativistic fireball (Rees & Mészáros 1992). Paczyński & Xu (1994) suggested that charged pions, which are produced in \( pp \) collisions when the kinetic energy of the fireball is dissipated through internal collisions within the ejecta, produce a burst of \( \sim 10^{10} \text{ eV} \) neutrinos. Our proposed process of meson production is different from these two, which can operate without magnetic fields. The very rapid variability timescale, \( \sim 0.1 \text{ ms} \), of many GRBs implies that each sub-burst reflects the intrinsic primary energy release from the central engine (Sari & Piran 1997). Thus, the length scale of the central engine is estimated to be \( \sim 10 \text{ km} \). The BATSE detector on board of the Compton Gamma-Ray Observatory recorded the shortest burst with a duration of 5 ms and with substructure on a scale of 0.2 ms (Bhat et al. 1992). Accumulated BATSE data (Fishman & Meegan 1995) confirmed that more than 25% of total events are short (\( \leq 2 \text{ s} \)) bursts. Therefore, if the central engines of GRBs were the compact stellar objects like neutron stars or rotating black holes associated with strong magnetic fields, relativistic protons or heavy nuclei would trigger meson synchrotron emission whose decay products could provide several observational signals even before the hidden explosion energy is transported to the radiation in the later stage of a relativistic fireball.

We extend the previous treatment of \( \pi^0 \) emission by Ginzburg & Syrovatskii (1965a, 1965b) to several kinds of neutral and charged mesons, which couple to a nucleon through scalar- or vector-type interaction, in a manner somewhat different from theirs. If the mesons produced are \( \pi^0 \)'s or heavier mesons that have main decay modes onto \( \pi^0 \)'s, then they produce high-energy photons that are immediately converted to lower energy \( \gamma \)'s or e\(^+\)e\(^-\) pairs through \( \gamma \rightarrow \gamma + \gamma \) or \( \gamma \rightarrow e^+e^- \) (Erber 1966) because of their large optical depth in strong magnetic fields. However, if they are the charged mesons like \( \pi^\pm \)'s, the decay modes are very different and result in more interesting consequences. Since the source emitters of charged nuclei are accelerated to high energy (Hillas 1984) to some extent, very high energy neutrinos are produced by \( \pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \overline{\nu}_\mu \).
\( g + \bar{\nu}_\mu + \nu_\mu \) or by its mirror conjugate process for the similar mechanism as proposed by Waxman & Bahcall (1997). These neutrinos could be a signature for strong meson emission from the GRB central engines and would be detected in the very early phase of the short GRBs. We formulate the meson synchrotron emission in the next section, and the calculated results are shown and discussed in § 3.

2. SEMICLASSICAL TREATMENT OF MESON EMISSION

We follow the established semiclassical treatment (Peskin & Schroeder 1995; Itzykson & Zuber 1980) of the synchrotron emission of the quantum fields interacting with the classical source current in strong external fields. In our present study, the meson field is second-quantized, while the nucleons are not and obey classical motion.

The energy spectrum of \( \pi^0 \) synchrotron emission by a proton was first derived by Ginzburg & Syrovatskii (1965a, 1965b), and it is given by

\[
\frac{dI_\gamma}{dE_\gamma} = \frac{g^2}{\sqrt{3\pi}} \frac{E_\gamma}{h c} \frac{1}{\gamma_\rho} \int_{\gamma_\rho}^{\infty} K_{1/3}(\eta) d\eta, \tag{1}
\]

where \( g \) is the strong coupling constant, \( g^2/hc \approx 14 \) (Sakurai 1967), \( E_\gamma \) is the energy of the emitted pion, and \( \gamma_\rho \) is the Lorentz factor, \( \gamma_\rho = E_\rho/m_\rho c^2 \), of the rotating proton in a given magnetic field. \( K_{1/3} \) is the modified Bessel function on the order of 1/3.

The function \( y(x) \) is given by

\[
y(x) = \frac{2}{3} \frac{m_\rho}{m_\pi} x \left( 1 + \frac{1}{x^2} \right)^{3/2}, \tag{2}
\]

where \( m_\rho c^2 = 938 \text{ MeV} \) and \( m_\pi c^2 = 135 \text{ MeV} \) are the rest masses of the proton and the \( \pi^0 \) meson, respectively, and the parameter \( x \), which characterizes the synchrotron emission, is determined by the proton energy and the strength of magnetic field as \( x = (H/H_0) \gamma_\rho \), with \( H_0 \equiv m_\pi c^2/eh = 1.5 \times 10^{20} \text{ G} \).

In the above equations, variable \( x \) is introduced for mathematical simplicity, \( x = (E_\gamma/E_\rho)(m_\rho/m_\pi) \). Since the available energy of the pion satisfies \( m_\rho \leq E_\rho \leq E_\gamma \), the corresponding variable range of \( x \) is \( \gamma_\rho^{-1} \leq x \leq m_\rho/m_\pi \).

Applying the same treatment to vector mesons, we obtain an energy spectrum of \( \rho \) meson synchrotron emission:

\[
\frac{dI_\rho}{dE_\rho} = \frac{g^2}{\sqrt{3\pi}} \frac{E_\rho}{h c} \frac{1}{\gamma_\rho} \int_{\gamma_\rho}^{\infty} K_{1/3}(\eta) d\eta, \tag{3}
\]

where \( K_{1/3} \) is the modified Bessel function on the order of 5/3. The function \( y(x) \) is defined by equation (2) by replacing \( m_\rho \) with a \( \rho \) meson mass of \( m_\rho c^2 = 770 \text{ MeV} \), and \( x = (E_\gamma/E_\rho)(m_\rho/m_\pi) \). Note that one can easily get the expression for photon synchrotron radiation in the limit of \( m_\rho \to 0 \) by replacing the strong coupling constant \( g \) with the electromagnetic coupling constant \( e \).

The total intensity of a scalar or vector meson as a function of \( x \) is obtained by integrating equation (1) or equation (3) over meson energies \( m_\rho \leq E_\rho \leq E_\gamma \), or, equivalently, over \( \gamma_\rho^{-1} \leq x \leq m_\rho/m_\pi \). It is useful to give an approximate formula of the total intensity in the limit of large or small \( \chi \):

\[
I_{\gamma} = \frac{g^2 m_\pi^2 c^3}{6h^2} \quad \text{for} \ \chi \gg 1, \tag{4a}
\]

\[
I_{\gamma} = \frac{g^2 m_\pi^2 c^3}{\sqrt{3}h^2} \chi \exp \left( -\frac{\sqrt{3} m_\pi}{\chi m_\rho} \right) \quad \text{for} \ \chi \ll 1, \tag{4b}
\]

and

\[
I_\rho = \frac{27(3^{1/6})}{16\pi} \Gamma \left( \frac{5}{3} \right) \frac{2g^2 m_\pi c^3}{3h^2} \chi^{2/3} \quad \text{for} \ \chi \gg 1, \tag{5a}
\]

\[
I_\rho = \frac{3}{2} \sqrt{\frac{3}{2}} \left[ \frac{m_\rho}{m_\pi} \right] \left( 1 + \frac{m_\rho}{m_\pi} \right)^{3/4} \left[ 2 \left( \frac{m_\rho}{m_\pi} \right)^2 - 1 \right]^{-1/2} \times \frac{g^2 m_\rho c^3}{h^2} \chi^{1/2} \exp \left( -\frac{2}{3\chi} \left[ 1 + \frac{m_\rho}{m_\pi} \right]^{2/3} \right) \quad \text{for} \ \chi \ll 1, \tag{5b}
\]

where we have made the approximation \( K_{1/3}(\eta) \approx 2^{-1/3} \Gamma(\rho/\eta') \) for \( \eta \ll 1 \) (\( \chi \gg 1 \)) or the approximation \( K_{1/3}(\eta) \approx (\pi/2\eta')^{1/2} \times \exp(-\eta) \) for \( \eta \gg 1 \) (\( \chi \ll 1 \)). These approximations are in reasonable agreement with exact numerical integrals within \( \pm 3\% \) for the \( \pi \) meson and \( \pm 10\% \) for the \( \rho \) meson at \( \chi \leq 0.01 \) or \( 10^7 \leq \chi \).

Let us make a short remark on our classical treatment. When the proton energy is very high or when the external magnetic field is strong (i.e., \( \chi \gg 1 \)), the quantum effects, not only in the meson field but in the source nucleon current, may not be negligible. In the case of photon synchrotron radiation, quantum effects were carefully studied (Erber 1966), and the semiclassical treatment was found to be a good approximation to the exact solution within a few percent. In our treatment, the prefactor in equation (5a) is \( (27(3^{1/6})/16\pi)\Gamma(5/3) \approx 0.583 \). Taking the limit of \( m_\rho \to 0 \) and \( g = e \), equation (5a) is applied to photon synchrotron radiation. It is shown by Erber (1966) that this factor should be 0.5563 in a fully quantum mechanical calculation. It therefore is expected to hold true for the hadron processes as well.

3. RESULTS AND DISCUSSIONS

The left panel of Figure 1 shows a comparison between the calculated spectra of scalar \( \pi^0 \) meson emission and \( \gamma \) synchrotron radiation. Since Usov (1992), Kluźniak & Ruderman (1998), and Paczyński (1998) suggested strong magnetic fields on the order of \( H \approx 10^{16} - 10^{17} \text{ G} \), which are presumed to be associated with neutron stars or black holes of the GRB central engines, here we take \( H = 1.5 \times 10^{16} \text{ G} \). The observed Lorentz factor of the fireball is \( \Gamma \approx 10^3 \), which indicates that the energy of charged particles is at least \( \sim 10^{42} \text{ eV} \). Although the acceleration mechanism in GRBs is still unknown, there are suggestions (Milgrom & Usov 1995) that the GRBs are associated with UHECRs. Six events of UHECRs beyond the Greisen-Zatsepin-Kuz’min cutoff energy, \( \sim 10^{19} \text{ eV} \) (Hill & Schramm 1985), have been detected by the AGASA group (Takeda et al. 1998). We therefore vary the proton energy from \( E_\gamma = 10^{12} \) to \( 10^{20} \text{ eV} \). All calculated spectra cut off sharply at an incident proton energy \( E_{\gamma, \gamma} = E_\gamma \). As seen in Figure 1, the
intensity of $\pi^0$ emission for $E_p = 10^{12}$, $10^{14}$, and $10^{16}$ eV, which correspond, respectively, to $\chi \approx 0.1$, 10, and 1000, is $10^3$-$10^9$ times stronger than that of the γ radiation in the high-energy parts of the spectra. This reflects the different coupling constants, $g^2/e^2 \sim 10^3$. A very sharp declination of the $\pi^0$ spectra at lower energy arises from the integral in equation (1) for finite pion mass because $K_{\gamma}(\eta) \approx (\pi/2\eta)^{1/2} \exp(-\eta)$ for $\eta \gg 1$, which is a good approximation in this energy region where $\gamma(\chi) \gg 1$.

The right panel of Figure 1 shows a comparison between the calculated spectra of ρ meson emission and γ synchrotron radiation. In this figure, both spectra look very similar to each other, except for the absolute intensity and the sharp declination of low-energy spectra due to finite ρ meson mass. This is because the interactions between ρ meson and proton and between photon and proton are of vector type. For the proton energies above $\sim 10^5$ eV, the intensity of ρ meson emission is roughly a thousand times larger than that of γ synchrotron radiation, reflecting again its stronger coupling constant.

Figure 2 displays the calculated total intensities of the synchrotron emission of $\pi^0$ and ρ mesons as a function of $\chi$. These are the integrated spectra shown in Figure 1 over available energy regions. The total intensity of γ synchrotron radiation is also shown in Figure 2. $I_\gamma$ exceeds $I_\rho$ at $3 \times 10^{-2} < \chi < 3 \times 10^4$, and $I_\rho$ exceeds $I_\gamma$ at $\chi > 0.2$. Both $I_\rho$ and $I_\gamma$ decrease exponentially with decreasing $\chi$ because of their finite masses (see the asymptotic forms at $\chi \ll 1$ in eqs. [4a]–[5b]). This sharp declination of $I_\rho$ takes place at higher $\chi$ than $I_\gamma$ because ρ meson mass is larger than pion mass. $I_\rho$ resembles $I_\gamma$ at $\chi > 1$, except that their respective intensities are a thousand times different from each other, reflecting that both the ρ meson and the photon have the same type of vector coupling to the proton but with different coupling constants, $g^2/e^2 \sim 10^3$.

A nucleon strongly couples to the $\pi^+$ as well as the $\pi^0$ field. The interaction Hamiltonian is charge independent, and $H_{\text{int}} = ig(\sqrt{2} \gamma \bar{\psi} \gamma \phi_{\rho} + \bar{\psi}_{\gamma} \gamma \phi_{\rho}) + \text{c.c.}$ (Sakurai 1967). The initial state $\bar{\psi}_{\rho}$ in both $p \rightarrow n + \pi^+$ and $p \rightarrow n + \pi^0$ processes is identical. The difference comes from the final states: the proton and $\pi^+$ couple to an external magnetic field, but the neutron and $\pi^0$ do not. (The neutron has very small magnetic moment.) When we describe these processes in quantum mechanics, we should use wave functions from the solution of the Dirac and Klein-Gordon equations for the nucleon and the pion separately. However, the final states are more or less the same for the same charge state, aside from the different masses. These might not change the reaction amplitudes by many orders at ultrarelativistic energies where the rest mass is neglected. Note that the conjugate processes $n \rightarrow p + \pi^+$ and $n \rightarrow n + \pi^0$ can also occur when a composite nucleus like $^{56}$Fe orbits in the strong magnetic fields.

Heavy mesons, including the ρ meson, have several appreciable branching ratios for the decay onto $\pi^\pm$. Let us discuss what kinds of observational signals they may make.

We assume that some fraction of $10^{51}$–$10^{52}$ ergs of the gravitational or magnetic field energy is released by some unknown mechanism operating at the GRB central engine during a very short time duration, $\sim 0.1$ ms, of the first sub-burst. We also assume that an appreciable part of this energy is deposited into the relativistic motion of the material leading to UHECRs. In a somewhat different context, Waxman & Bahcall (1997) proposed that the photons produced in the ejecta of the fireball would make a burst of $\sim 10^{53}$ eV neutrinos. Although their proposed mechanism of meson production is completely different from ours, we can apply similar results on the physical

**Fig. 1**—Left: Calculated energy spectra of scalar $\pi^0$ meson emission (solid curves) and photon synchrotron radiation (dotted curves) for various incident proton energies $E_p$ with a fixed magnetic field of $H = 1.5 \times 10^{16}$ G. From left to right, the denoted numbers are the proton energies $E_p = 10^{12}$, $10^{14}$, $10^{16}$, $10^{18}$, $10^{20}$, and $10^{22}$ eV. Right: Same as the left panel, but for vector ρ meson emission.

**Fig. 2**—Calculated total intensities of the emission of the scalar $\pi^0$ meson (solid curve), vector ρ meson (dashed curve), and photon γ (dotted curve) as a function of $\chi = (\mu H_0) \gamma_e$. 

---

*Source: Tokuhisa & Kajino, 1999 (L119)*
consequences of $\pi^+$ decay. Extending our previous discussions of $\pi^0$ in Figure 2 to $\pi^+$, we expect that an intensity of high-energy neutrinos a thousand times stronger than the photons can be produced universally at $0.1 \leq \chi$ from $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$ and its mirror conjugate process. Since neutrinos can escape from the ambient matter, these neutrinos could be a clear signature showing strong meson synchrotron emission near the central engines of GRBs associated with extremely strong magnetic fields.

The neutron emerging from $p \rightarrow n + \pi^+$ inherits almost all the proton energy and can escape from the region of strong magnetic field. If there is no dense shell surrounding the central engine, it travels $\sim 10^3 \times (\gamma_r/10^{10})$ pc before $\beta$-decay. This process may also produce a very high energy neutrino. The generic picture of GRBs (Mészáros & Rees 1993; Piran 1999) suggests that a baryon mass of $\sim 10^{-3} M_\odot$ is involved in a single explosion. If this is the case, such an amount of baryon mass is huge enough to stop almost all neutrons before running through the ambient matter.

If the central engines are neutron stars or black holes, the materials ejected from these compact objects contain heavy nuclei such as oxygen and iron because these are the products from evolved massive stars. Therefore, meson emission from a heavy nucleus as well as from a proton is worth our consideration. Let us consider a nucleus of total energy $E_{\text{tot}}$, mass number $A$, and charge $Z$ in a magnetic field of strength $H$. The energy of each nucleon is $E = E_{\text{tot}}/A$. When the strength of the effective magnetic field is $H_{\text{eff}} = (Z/A)H$, the orbital trajectory of a proton is the same as the trajectory of the nucleus in the magnetic field $H$. The intensity of $\pi^0$ emission by the nucleus should be the sum of each nucleonic contribution, provided that the synchrotron emission is incoherent. Thus, the total intensity is given by

$$I_{\pi^0}^{\text{tot}}(E_{\text{tot}}, H) \approx A I_{\pi^0}^{\text{p}}(E, H_{\text{eff}}),$$

(6)

where $I_{\pi^0}^{\text{p}}(E, H_{\text{eff}})$ is the intensity of $\pi^0$ emission by the nucleon of energy $E$ in the magnetic field $H_{\text{eff}}$. Note that both the proton and the neutron emit $\pi^0$. Figure 3 shows $I_{\pi^0}^{\text{Fe}}(E_{\text{tot}}, H)$ for $^{56}\text{Fe}

as a function of total energy $E_{\text{tot}}$ with a fixed magnetic field of $H = 1.5 \times 10^{12}$ G.

There is now more motivation to study the GRBs in association with UHECRs. It is highly desirable to proceed with a project like OWL (Orbiting Wide-angle Light collectors) in order to detect ultrahigh-energy neutrinos from GRBs so as to understand the true nature of the central engines.

This work has been supported in part by the Grant-in-Aid for Scientific Research (10640236, 10044103, and 11127220) of the Ministry of Education, Science, Sports, and Culture of Japan and also by a JSPS-NSF grant of the Japan-US Joint Research Project. We thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for its partial support during the completion of this work.

REFERENCES

Bhat, P. N., et al. 1992, Nature, 359, 217
Erber, T. 1966, Rev. Mod. Phys., 38, 626
Fishman, G. J., & Meegan, C. A. 1995, ARA&A, 33, 415
Ginzburg, V. L., & Syrovatskii, S. I. 1965a, Uspekhi Fiz. Nauk, 87, 65
Hill, C. T., & Schramm, D. N. 1985, Phys. Rev. D, 31, 564
Hillas, A. M. 1984, ARA&A, 22, 425
Itzykson, C., & Zuber, J.-B. 1980, Quantum Field Theory (New York: McGraw-Hill)
Kluźniak, W., & Ruderman, M. 1998, ApJ, 505, L113
Kouveliotou, C., et al. 1998, Nature, 393, 235
Mészáros, P., & Rees, J. M. 1993, ApJ, 405, 278
Milgrom, M., & Usov, V. 1995, ApJ, 449, L37
Paczynski, B. 1998, ApJ, 494, L45
Paczynski, B., & Xu, G. 1994, ApJ, 427, 708
Peskin, M. E., & Schroeder, D. V. 1995, An Introduction to Quantum Field Theory (Reading: Addison-Wesley)
Piran, T. 1999, Phys. Rep., 314, 575
Rees, M. J., & Mészáros, P. 1992, MNRAS, 258, 41P
Sarazin, C. L. 1987, Advanced Quantum Mechanics (Redwood City: Addison-Wesley)
Sari, R., & Piran, T. 1997, ApJ, 485, 270
Takeda, M., et al. 1998, Phys. Rev. Lett., 81, 1163
Usov, V. 1992, Nature, 357, 472
Waxman, E., & Bahcall, J. N. 1997, Phys. Rev. Lett., 78, 2292