Effects of Alfvénic Drift on Diffusive Shock Acceleration at Weak Cluster Shocks

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Abstract

Non-detection of \(\gamma\)-ray emission from galaxy clusters has challenged diffusive shock acceleration (DSA) of cosmic-ray (CR) protons at weak collisionless shocks that are expected to form in the intracluster medium. As an effort to address this problem, we here explore possible roles of Alfvén waves self-excited via resonant streaming instability during the CR acceleration at parallel shocks. The mean drift of Alfvén waves may either increase or decrease the scattering center compression ratio, depending on the postshock cross-helicity, leading to either flatter or steeper CR spectra. We first examine such effects at planar shocks, based on the transport of Alfvén waves in the small amplitude limit. For the shock parameters relevant to cluster shocks, Alfvénic drift flattens the CR spectrum slightly, resulting in a small increase of the CR acceleration efficiency, \(\eta\). We then consider two additional, physically motivated cases: (1) postshock waves are isotropized via MHD and plasma processes across the shock transition, and (2) postshock waves contain only forward waves propagating along with the flow due to a possible gradient of CR pressure behind the shock. In these cases, Alfvénic drift could reduce \(\eta\) by as much as a factor of five for weak cluster shocks. For the canonical parameters adopted here, we suggest \(\eta \sim 10^{-4} - 10^{-2}\) for shocks with sonic Mach number \(M_s \approx 2-3\). The possible reduction of \(\eta\) may help ease the tension between non-detection of \(\gamma\)-rays from galaxy clusters and DSA predictions.

Key words: acceleration of particles – cosmic rays – galaxies: clusters: general – shock waves

1. Introduction

Weak shocks with sonic Mach number typically \(M_s \lesssim 2\) a few are expected to form in the intracluster medium (ICM) during the course of hierarchical clustering of the large-scale structure of the universe (e.g., Ryu et al. 2003; Kang et al. 2007). The presence of such shocks has been established by X-ray and radio observations of many merging clusters (e.g., Markevitch & Vikhlinin 2007; Brüggen et al. 2012; Brunetti & Jones 2014). In particular, diffuse radio sources known as radio relics, located mostly in cluster outskirts, could be explained by cosmic-ray (CR) electrons (re-)accelerated via diffusive shock acceleration (DSA) at quasi-perpendicular shocks (e.g., van Weeren et al. 2010; Kang et al. 2012; Kang 2017). Although both CR electrons and protons are known to be accelerated at astrophysical shocks such as Earth’s bow shocks and supernova remnant shocks (e.g., Bell 1978; Drury 1983; Blandford & Eichler 1987), the \(\gamma\)-ray emission from galaxy clusters, which would be a unique signature of CR protons, has not been detected with high significance so far (Ackermann et al. 2014, 2016; Brunetti 2017).

In galaxy clusters, diffuse \(\gamma\)-ray emission can arise from inelastic collisions of CR protons with thermal protons, which produce neutral pions, followed by the decay of pions into \(\gamma\)-ray photons (e.g., Miniati et al. 2001; Brunetti & Jones 2014; Brunetti 2017). Using cosmological hydrodynamic simulations, the \(\gamma\)-ray emission has been estimated by modeling the production of CR protons at cluster shocks in several studies (e.g., Ensslin et al. 2007; Pizzone & Pfrommer 2010; Vazza et al. 2016). In particular, Vazza et al. (2016) tested several different prescriptions for DSA efficiency by comparing \(\gamma\)-ray flux from simulated clusters with Fermi-LAT upper limits of observed clusters. They found that non-detection of \(\gamma\)-ray emission could be understood only if the CR proton acceleration efficiency at weak cluster shocks is on average less than \(10^{-3}\) for shocks with \(M_s = 2-5\). On the other hand, recent hybrid plasma simulations demonstrated that about 5\%–15\% of the shock kinetic energy is expected to be transferred to the CR proton energy at quasi-parallel shocks with a wide range of Alfvén Mach numbers, \(M_A\), (Capirol & Spitkovsky 2014a). So there seems to exist a tension between the CR proton acceleration efficiency predicted by DSA theory and \(\gamma\)-ray observations of galaxy clusters.

It is well established that CR protons streaming along magnetic field lines upstream of parallel shock resonantly excite Alfvén waves with wavenumber \(k \sim 1/r_s\) via two-stream instability, where \(r_s\) is the proton Larmor radius (Wentz 1974; Bell 1978; Lucek & Bell 2000; Schure et al. 2012). These Alfvén waves are circularly polarized in the same sense as the proton gyromotion, i.e., left-handed circularly polarized when they propagate parallel to the background magnetic field. The waves act as scattering centers that can scatter CR particles in pitch-angle both upstream and downstream of the shock, leading to the Fermi first order (Fermi I) acceleration at parallel shocks (Bell 1978).

As CRs are scattered and isotropized in the mean wave frame, the spectral index \(\Gamma\) of the CR energy spectrum, \(N(E) \propto E^{-\Gamma}\), is determined by the convection speed of scattering centers in the shock rest frame, \(u + u_w\), instead of the gas flow speed, \(u\) (Bell 1978). Here, \(u_w\) is the mean speed of scattering centers in the local fluid frame, or the speed of so-called Alfvénic drift. The direction and amplitude of Alfvénic drift depend on the difference between the intensity of forward waves (moving parallel to the flow) and that of backward waves (moving anti-parallel to the flow), i.e., \(\langle \delta B^f \rangle^2 - \langle \delta B^b \rangle^2\) (Skilling 1975). If forward and back waves have the same intensity or if waves are completely isotropized, i.e., \(\langle \delta B^f \rangle^2 = \langle \delta B^b \rangle^2\), then \(u_w \approx 0\).

A non-resonant instability due to the electric current associated with CRs escaping upstream is also known to
toward the center of supernova explosion, i.e., the excited waves are not Alfvén waves and have a circular shock with the background magnetic field parallel to the shock normal (parallel shock). Here, the subscripts 1 and 2 are for preshock and postshock quantities, respectively. The shock faces to the right, so the preshock flow speed is $u = -u_1$. After upstream backward waves (moving anti-parallel to the flow in the flow rest frame) cross the shock, both transmitted backward waves and reflected forward waves are advected downstream. The convection speeds of waves, $W_{b1}$, $W_{b2}$, and $W_{f2}$, are given in the shock rest frame.

operate on small wavelengths (Bell 2004; Schure et al. 2012). The excited waves are not Alfvén waves and have a circular polarization opposite to the sense of the proton gyromotion, i.e., are right-handed circularly polarized when they propagate parallel to the background magnetic field. This non-resonant instability is more unstable at higher $k$’s (smaller wavelengths), and the ratio of the growth rates of non-resonant to resonant instability is roughly, $\Gamma_{\text{nonres}}/\Gamma_{\text{res}} \sim M_\Lambda/30$ (Caprioli & Spitkovsky 2014b). In cluster outskirts where the magnetic field is observed to have $B \sim 1 \mu G$ (e.g., Govoni & Feretti 2004), shocks have $M_\Lambda \lesssim 30$ (see below), so resonant instability is expected to be dominant there. As we are interested in cluster shocks here, we focus mainly on Alfvén waves excited by resonant streaming instability.

Bell (1978) noted that resonant instability would produce mostly backward waves in the preshock region, because CR protons streaming upstream excite waves that move parallel to the streaming direction (that is, travel upstream away from the shock in the upstream rest frame), and any forward waves pre-existing in the preshock flow would be damped due to the gradient of the CR distribution in the shock precursor (Wentzel 1974; Skilling 1975; Lucek & Bell 2000). Then, the Alfvénic drift speed in the preshock region may be approximated as $u_{w1} \approx +V_{A1}$, where $V_{A1} = B_0/\sqrt{4 \pi \rho}$ is the local Alfvén speed. See Figure 1 for the velocity configuration in the shock rest frame. Hereafter, the subscripts 1 and 2 refer to the quantities in the preshock and postshock regions, respectively.

Alfvénic drift in the postshock region was previously considered in studies of CR acceleration at strong supernova remnant (SNR) shocks (e.g., Zirakashvili & Ptuskin 2008, 2012; Caprioli et al. 2009; Lee et al. 2012; Kang 2013). Those studies suggested that owing to the positive gradients of the CR pressure, $P_{\text{CR}}$, forward waves (moving away from the shock toward the center of supernova explosion) could be dominant in the postshock region, then $u_{w2} \approx -V_{A2}$ (see Figure 2).

The effects of Alfvénic drift should be substantial, only if the Alfvén speed is a significant fraction of the flow speed. In SNR shocks, for instance, the Alfvén Mach number is $M_\Lambda = u_1/V_{A1} \sim 20$–200, depending on the density of the background medium, yet the Alfvénic drift effects could be appreciable (e.g., Caprioli et al. 2009; Kang 2013). For the ICM in cluster outskirts, the sound and Alfvén speeds are given as $c_s \approx 1.14 \times 10^3$ km s$^{-1}$($k_B T/5$ keV)$^{1/2}$ and $V_A \approx 184$ km s$^{-1}$($B/1 \mu G$)($n_H/10^{-4}$ cm$^{-3}$)$^{-1/2}$, respectively, so

$$\beta \equiv \left(\frac{c_A}{V_A}\right)^2 \approx 40\left(\frac{n_H}{10^{-4}}\text{ cm}^{-3}\right)\left(\frac{k_B T}{5.2\text{ keV}}\right)\left(\frac{B}{1 \mu G}\right)^2,$$  

where $k_B$ is the Boltzmann constant. For $M_\Lambda \approx 2$–3, the Alfvén Mach number of cluster shocks ranges $M_\Lambda = \sqrt{\beta} M_e \approx 13$–19, which is smaller than that of SNR shocks. Thus, we expect that the Alfvénic drift could have non-negligible effects on DSA at cluster shocks. Note that this definition of $\beta$ differs from the usual plasma beta by a factor of 1.2 for the gas adiabatic index $\gamma = 5/3$; the plasma beta of the ICM has been estimated to be $\sim 50$–100 (e.g., Ryu et al. 2008; Porter et al. 2015).

The transmission and reflection of upstream Alfvén waves at shocks can be calculated by solving conservation equations across the shock transition (e.g., Campeau & Schlickeiser 1992; Vainio & Schlickeiser 1998, 1999; Caprioli et al. 2009). Vainio & Schlickeiser (1998), for instance, used the conservation of mass, flux, transverse momentum, and tangential electric field to calculate them, in the small wave amplitude limit ($b \equiv B_2/B < 1$) in the one-dimensional (1D) plane-parallel geometry. They showed that after purely backward waves cross the shock, forward waves are also generated in the postshock region. Vainio & Schlickeiser (1999, hereafter VS99) extended the work by including the pressure and energy flux of waves across the shock. The transmission and reflection of Alfvén waves and so the ensuing CR spectrum are governed by $M_\Lambda$, $\beta$, $b$, and the properties of upstream waves. For certain shock parameters, the effective compression ratio, $r_{\text{eff}}$, which is defined as the velocity jump of scattering centers (see Section 3), can be even larger than the gas compression ratio, $r$, leading to a flatter CR energy spectrum.

In this paper, we first estimate the effects of Alfvénic drift on the DSA of protons for 1D planar shocks in high beta ($\beta \geq 1$) plasmas, with the transport of Alfvén waves across the shock transition described in VS99. We then consider two other cases, which are physically motivated: (1) postshock waves are isotropized, i.e., $u_{w2} \approx 0$, and (2) forward waves are dominant in the postshock region, i.e., $u_{w2} \approx -V_{A2}$. We examine the Alfvénic drift effects in these cases too.

In the next section, the transmission and reflection of upstream Alfvén waves at 1D planar shocks are described. In Section 3, the effects of the drift of Alfvén waves are discussed with the power-law CR proton spectrum in the test-particle limit. A brief summary including implications of our results at weak cluster shocks is given in Section 4.

2. Transmission and Reflection of Alfvén Waves at Shocks

VS99 derived necessary jump conditions for the transport of Alfvén waves across parallel shocks, whose configuration is illustrated in Figure 1. We here repeat some of them to keep this paper self-contained. The shock moves to the right, so the preshock and postshock flow speeds in the shock rest frame are $u_1 = -u_2 \hat{x}$ and $u_2 = -u_2 \hat{x}$, respectively. The background magnetic field is given as $B_0 = -B_0 \hat{x}$. CR protons streaming upstream along $B_0$ excite backward waves that travel anti-parallel to the background flow in the local fluid frame. The shock amplifies the incoming backward waves and also generates forward waves in the postshock region. The convection speed of backward waves is $W_{b1,2} = -(u_{1,2} - V_{A1,2}) < 0$ (to the left) both...
upstream and downstream of a parallel shock for the high beta plasmas with $\beta \geq 1$ considered here.

We consider nondispersive, circularly polarized Alfvén waves with small amplitudes ($b = \delta B / B \ll 1$), propagating along the mean background magnetic field, $B_0$, at 1D planar shocks. Note that the formulae below do not differentiate the handedness of wave polarization, as the conservation equations do not depend on it.

The relation for the gas compression ratio, $r$, across the shock jump can be derived from the Rankine–Hugoniot figures.
condition including the pressure and energy flux of waves and is given as the following cubic equation,
\[ b^2 M_\text{A}^2 r \{(\gamma - 1) r^2 + [M_\text{A}^2(2 - \gamma) - (\gamma + 1)] r + \gamma M_\text{A}^2 \} + (M_\text{A}^2 - r^2) [2r\beta - M_\text{A}^2(\gamma + 1 - (\gamma - 1) r)] = 0, \]
for a given set of parameters, \( M_\text{A}, \beta, \) and \( b \) (VS99). Here, \( \gamma = 5/3 \) is used for the ICM gas.

The bottom-left panel of Figure 3 shows the solution of Equation (2), \( r \), for three beta’s (\( \beta = 1, 10, \) and 80) and \( b = 0.1 \) in the Mach number range of \( M_\text{A} \leq 5 \). Because the background magnetic field is parallel to the shock flow (i.e., parallel shocks) and the transverse components of wave fields are small (\( \delta B = 0.1 B_0 \)), \( r \) is almost identical to the gas compression ratio of gas-dynamic shocks, \( r_{\text{gas}} = (\gamma + 1) M_\text{A}^2 \left( \gamma - 1 \right)^{-1} \), regardless of \( \beta \). In fact, \( r \) would deviate from \( r_{\text{gas}} \) only if \( b \) is substantially large or \( \beta \) is small. In the same panel, two such cases with \( b = 0.3 \) and \( \beta = 1 \) and \( b = 0.1 \) and \( \beta = 0.5 \) are shown for comparison, with the green and magenta lines, respectively, to illustrate such dependence.

Following VS99, the cross-helicity is defined as
\[ H_c = \frac{\delta B^b}{\delta B^f} = \frac{\delta B^b}{\delta B^f} \cdot \frac{\delta B^f}{\delta B^b}, \]
where \( \delta B^b \) and \( \delta B^f \) are the magnetic fields of backward and forward waves, respectively. In the preshock region, backward waves are expected to be dominant for CR-mediated shocks (see Introduction), so we assume \( H_{c1} \approx -1 \).

For power-law energy spectra of waves with slope \( q \), \( I(k) \propto k^{-q} \), the transmission and reflection coefficients for backward and forward waves, respectively, in the postshock region are derived from the equations for transverse momentum and tangential electric field, as follows:
\[ T = \frac{\delta B^b}{\delta B^f} = \frac{r^{1/2} + 1}{2r^{1/2}} \left( \frac{M_\text{A} + H_{c1}}{M_\text{A} + r^{1/2} H_{c1}} \right)^{q+1/2}, \]
\[ R = \frac{\delta B^f}{\delta B^b} = \frac{r^{1/2} - 1}{2r^{1/2}} \left( \frac{M_\text{A} + H_{c1}}{M_\text{A} - r^{1/2} H_{c1}} \right)^{q+1/2} \]
(Vainio & Schlickeiser 1998). Note that these coefficients are independent of the wavenumber. According to hybrid simulations of collisionless shocks by Caprioli & Spitkovsky (2014b), for shocks with \( M_\text{A} \leq 30 \) where resonant streaming instability dominantly operates, the spectrum of excited magnetic turbulence in the precursor is consistent with \( I(k) \propto k^{-1} \). So we adopt \( q = 1 \). With these coefficients, the downstream cross-helicity can be estimated as
\[ H_{c2} = H_{c1} \cdot \frac{T^2 - R^2}{T^2 + R^2}. \]

The top panels of Figure 3 show \( T, R, \) and \( H_{c2}, \) calculated with \( b = 0.1, q = 1, \) and \( H_{c1} = -1.0 \). One can see that incident backward waves are amplified across the shock with \( T > 1 \), while forward waves are generated with \( 0 < R < 1 \) (greater \( R \) for higher \( \beta \)) in the postshock region. The ensuing downstream cross-helicity ranges \(-1 < H_{c2} \leq -0.85 \) for the shocks considered here. We note that the quasi-linear treatment adopted here should break down for nonlinear waves, which are expected to develop via streaming instabilities at strong shocks.

## 3. Effects of Alfvénic Drift on DSA

### 3.1. Scattering Center Compression Ratio and CR Spectral Index

The CR transport at shocks can be described by the diffusion-convection equation,
\[ \frac{\partial f}{\partial t} + \frac{\partial (u + u_w) f}{\partial x} = \frac{1}{3} \frac{\partial u}{\partial x} \cdot \frac{\partial f}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} \left( \frac{p^2 D_{pp}}{\partial f} \right), \]
where \( f(x, p, t) \) is the pitch-angle-averaged phase space distribution function for CRs, \( u \) is the flow speed, \( u_w \) is the local speed of scattering centers, \( \kappa(x, p) \) is the spatial diffusion coefficient, and \( D_{pp} \) is the momentum diffusion coefficient (Skilling 1975; Bell 1978; Schlickeiser 1989). The effects of Alfvénic drift enter through \( u_w \), which is given here as \( u_{w1} = H_{c1} V_{A1} \) and \( u_{w2} = H_{c2} V_{A2} \) in the preshock and postshock regions, respectively.

CR particles then experience the velocity change from \( u_1 + H_{c1} V_{A1} \) to \( u_2 + H_{c2} V_{A2} \) across the shock, as they are isropized in the local wave frame. Then the compression ratio of scattering centers, defined as the velocity jump of scattering centers, is given as
\[ r_{sc} \equiv \frac{u_1 + H_{c1} V_{A1}}{u_2 + H_{c2} V_{A2}} = \frac{M_\text{A} + H_{c1}}{M_\text{A} + r^{1/2} H_{c2}}. \]
Thus, \( r_{sc} \) can be different from the gas compression ratio, \( r \), from Equation (2), depending on the cross-helicity. The bottom-middle panel of Figure 3 shows that \( r_{sc} \), calculated for 1D planar shocks (VS99), depends on \( \beta \) and can be greater than \( r_{\text{gas}} \). But for \( \beta \gg 1 \), \( r_{sc} \approx r_{\text{gas}} \), as \( M_\text{A} \gg 1 \).

At weak cluster shocks, the CR pressure is dynamically insignificant, that is, shocks are in the test-particle regime, in which the CR energy spectrum, \( N(E) \), is represented by a power-law form. Then, its power-law index, \( \Gamma \), is determined by \( r_{sc} \) as
\[ \Gamma = \frac{r_{sc} + 2}{r_{sc} - 1}. \]
(Bell 1978). The bottom-right panel of Figure 3 shows \( \Gamma \) calculated for 1D planar shocks. Flattening of \( N(E) \) due to Alfvénic drift could be substantial for \( \beta \approx 1 \) (red solid lines), for which even \( \Gamma < 2 \) is predicted. It can be seen that for \( \beta \gg 1 \) (see blue dot-dashed lines), \( \Gamma \approx \Gamma_{\text{gas}} \) as \( r_{sc} \approx r_{\text{gas}} \).

### 3.2. CR Acceleration Efficiency

In the test-particle regime, the amplitude of the CR proton spectrum can be fixed by setting it at the injection momentum, \( p_{\text{maj}} \), and then the momentum distribution function at the shock position, \( x_i \), is given as
\[ f(x_i, p) = f_N \left( \frac{p}{p_{\text{maj}}} \right)^{-(\Gamma - 2)} \exp \left[ -\left( \frac{p}{p_{\text{cut}}} \right)^2 \right], \]
where \( f_N \) is the normalization factor (Kang & Ryu 2010). The cut-off momentum, \( p_{\text{cut}} \), represents the maximum momentum of CR protons that can be accelerated within the shock age, \( t_{\text{age}} \).
and is given as \( p_{\text{cut}} \propto u_i^2 B_0 t_{\text{age}} \). As long as \( p_{\text{cut}} \gg m_p c \), the CR energy density does not depend on its exact value if \( \Gamma > 2 \).

Here, we define \( p_{\text{inj}} \) as the minimum momentum above which protons can cross the shock transition and participate in the Fermi I acceleration process, and we describe it using the injection parameter, \( Q_i \), as

\[
p_{\text{inj}} = Q_i \cdot p_{\text{th}},
\]

where \( p_{\text{th}} = \sqrt{2 m_p k_B T_2} \) is the thermal proton peak momentum of the postshock gas with temperature \( T_2 \) (Kang & Ryu 2010).

Using hybrid simulations, Caprioli et al. (2015) demonstrated that the injection momentum increases with the shock obliquity angle, \( \Theta_{\text{Bn}} \), and \( Q_i \approx 3.3-4.6 \) for quasi-parallel shocks (\( \Theta_{\text{Bn}} \lesssim 45^\circ \)) with \( \mathcal{M}_A = 5-50 \) and \( \beta \approx 1 \). The injection parameter should be affected by the strength of self-generated MHD turbulence, which in turn depends on \( \mathcal{M}_A \) and \( \beta \), in addition to \( \Theta_{\text{Bn}} \). It is also expected to increase in time as the particle spectrum extends to higher energies for strong shocks with \( p^{-4} \) momentum distribution, considering that the CR conversion efficiency cannot be greater than 100\%.

More accurate estimation of \( Q_i \) for weak cluster shocks in high \( \beta \) ICM plasmas, however, could only be made through kinetic plasma simulations; however, its value has not yet been precisely defined (see, e.g., Caprioli & Spitkovsky 2014a; Caprioli et al. 2015).

Assuming that \( f(x_i, p) \) is anchored to the postshock Maxwellian distribution at \( p_{\text{inj}} \), the normalization factor is given as

\[
f_N = n_{H2} \frac{m_p}{\pi^2 k_B^3} \exp(-Q_i^2),
\]

where \( n_{H2} \) is the postshock hydrogen number density (Kang & Ryu 2010).

Figure 4 illustrates how the test-particle spectrum in Equation (10) depends on the sonic Mach number, \( \mathcal{M}_e \). We adopt the relevant parameters for cluster shocks, \( k_B T_1 = 5.2 \) keV, \( n_{H1} = 10^{-4} \) cm\(^{-3} \), and \( B_0 = 1 \) \( \mu \)G \( (\beta = 40) \), resulting in \( \beta \approx 40 \). We set \( Q_i = 3.5 \) as a representative value, as we here model mostly parallel shocks with small obliquity angles. (Below, we also consider \( Q_i = 3.8 \) as a comparison case.) The CR spectra shown have the power-law indices, \( \Gamma \)’s, from Equations (8) and (9), which are calculated with \( H_{\text{c2}} \) estimated according to VS99 for 1D planar shocks (also with \( H_{\text{c2}} = 0 \), see Section 3.3). The cut-off momentum for \( t_{\text{age}} = 10^8 \) years is drawn for an illustrative purpose.

With the spectrum in Equation (10) and \( \Gamma > 2 \) for weak shocks, the CR injection fraction can be estimated as

\[
\xi = \frac{1}{n_{H2}} \int_{p_{\text{cut}}}^{p_{\text{min}}} 4 \pi f(x_i, p) p^2 dp 
\]

\[
\approx \frac{4}{\sqrt{\pi}} \frac{Q_i}{(\Gamma - 1)} \exp(-Q_i^2),
\]

if we take \( p_{\text{min}} = p_{\text{inj}} \) as the lower boundary of the CR momentum distribution (Kang & Ryu 2010). According to this definition, the CR injection fraction depends mainly on \( \Gamma \) and \( Q_i \), as normally \( p_{\text{cut}} \gg m_p c \).

We also define the CR acceleration efficiency as the ratio of the downstream CR energy flux to the shock kinetic energy flux, as follows,

\[
\eta \equiv \frac{f_{\text{CR}}}{f_{\text{kin}}} = \frac{u_1 E_{\text{CR}}}{(1/2) \rho_1 u_1^2},
\]

(Kang & Ryu 2013). Here, the postshock CR energy density is given as

\[
E_{\text{CR}} = 4 \pi m_p c^2 \int_{p_{\text{min}}}^{p_{\text{cut}}} (\sqrt{p^2 + 1} - 1) f(x_i, p) p^2 dp,
\]

where the particle momentum \( p \) is expressed in units of \( m_p c \). Again, we take \( p_{\text{min}} = p_{\text{inj}} \) in the calculation of \( E_{\text{CR}} \) below. Note that in general, the CR injection fraction and the DSA efficiency sensitively depend on how one specifies \( p_{\text{min}} \) because the CR number is dominated by nonrelativistic particles with \( p \sim p_{\text{inj}} \).

The left panel of Figure 5 shows the power-law slope, \( \Gamma \), estimated with \( H_{\text{c2}} \), which is calculated according to VS99. Here, \( \beta \) varies in the ranges relevant to cluster shocks, \( \beta = 10-80 \), as does the background magnetic field as \( B_0 = 1 \) \( \mu \)G \( (\beta/40)^{-1/2} \). One can see that at weak cluster shocks, \( H_{\text{c2}} \) based on VS99 could flatten the CR spectrum slightly, compared to gas-dynamic shocks without Alfvénic drift (green line). But for \( \beta \gg 1 \), the dependence of \( \Gamma \) on \( \beta \) is rather weak.
The middle and right panels of Figure 5 show the injection fraction, $\xi$, and the CR acceleration efficiency, $\eta$, respectively, calculated with the test-particle spectrum in Equation (10) with the slope $\Gamma$ shown in the left panel. Here, the adopted values of $k_{B}T_{i}$ and $n_{HII}$ are the same as in Figure 4. Both $\xi$ and $\eta$ strongly depend on $Q_{i}$ through the normalization factor $f_{N}$, due to the exponential nature of the tail in the Maxwellian distribution. While $\xi \propto Q_{i}^{2} \exp(-Q_{i}^{2})$ from Equation (13), the CR acceleration efficiency can be approximated as $\eta \propto Q_{i}^{5} \exp(-Q_{i}^{2})$ for weak shocks with power-law spectra much steeper than $p$ (dominated by nonrelativistic particles). So $\xi$ decreases by a factor of 7 as $Q_{i}$ increases from 3.5 to 3.8, while $\eta$ decreases roughly by a factor of 6 or so.

For cluster shocks with $M_{s} \lesssim 3$, $\xi \lesssim 3.2 \times 10^{-4}$ and $\eta \lesssim 2.2 \times 10^{-2}$ for $Q_{i} = 3.5$, while $\xi \lesssim 4.6 \times 10^{-5}$ and $\eta \lesssim 3.6 \times 10^{-3}$ for $Q_{i} = 3.8$. This indicates that the estimated CR injection fraction and acceleration efficiency could easily differ by an order of magnitude, depending on the adopted $Q_{i}$.

For parallel shocks with small obliquity angles (i.e., $\Theta_{BN} \lesssim 15^\circ$), however, we expect that $Q_{i}$ is unlikely to be much larger than 3.5 (Caprioli et al. 2015).

3.3. Cases with $H_{c2} \approx 0$ and $H_{c2} \approx +1$

The overall morphology of cluster shocks, induced mainly by merger-driven activities in turbulent ICMS, is expected to be quite complex and different from simple 1D planar shocks (see, e.g., Ha et al. 2018; Vazza et al. 2017). Rather, it can be characterized by portions of spherically expanding shells, composed of multiple shocks with different properties. In addition, vorticity is generated behind curved shock surfaces, leading to a turbulent cascade over a wide range of length scales and turbulent amplification of magnetic fields in the postshock flow (see, e.g., Ryu et al. 2008; Vazza et al. 2017). Then, downstream waves could be isotropized through various MHD and plasma processes in the postshock region, resulting in zero cross-helicity, $H_{c2} \approx 0$ (equal strengths of $T$ and $R$). Note that Fermi II acceleration should be operative in this case, but it is expected to be much less efficient than Fermi I acceleration.

In addition, as mentioned in the Introduction, the CR particle distribution peaks at the shock (i.e., decreases downstream) in spherical shocks or even in evolving planar shocks in which the CR pressure at the shock is increasing with time. In that case, the gradient of $P_{CR}$ is expected to damp backward waves, leaving dominantly forward waves with $H_{c2} \approx +1$ in the postshock region (Bell 1978; Zirákatashili & Ptuskin 2008; Caprioli et al. 2009). Hence, we here quantitatively examine the effects of Alfvénic drift in these physically motivated cases with $H_{c2} = 0$ and $H_{c2} = +1$, as phenomenological models.

In the panels for $r_{sc}$ and $\Gamma$ of Figure 3, the magenta and cyan lines show $H_{c2} = 0$ and $H_{c2} = +1$ cases, respectively. In fact, the scattering center compression ratio is minimized for $H_{c2} = +1$ (see Equation (8)). So this represents the case with the greatest impact of Alfvénic drift (the largest $\Gamma$). Moreover, Figure 4 compares the models with $H_{c2}$ estimated according to VS99 and the models with $H_{c2} = 0$ (isotropic waves), demonstrating how the Alfvénic drift may affect the CR spectrum.

Figure 6 shows $\Gamma$, $\xi$, and $\eta$ for the cases with $H_{c2} = 0$ (magenta lines) and $H_{c2} = +1$ (cyan lines), for the model
parameters relevant to cluster shocks and \( r_{\text{sc}} \) in Equation (8). The magenta and cyan lines are for \( H_{c,2} = 0 \) and \( H_{c,2} = +1 \), respectively, while black line shows the case with \( H_{c,2} \) calculated by following VS99. The model parameters are \( n_{\text{HII}} = 10^{-4} \, \text{cm}^{-3}, kT_1 = 5.2 \, \text{keV}, \) and \( B_0 = 1 \, \mu \text{G} (\beta = 40) \). Middle panel: CR injection fraction, \( \xi \), with \( Q_i = 3.5 \) (solid lines with circles) and 3.8 (dashed lines with triangles). Right panel: CR acceleration efficiency, \( \eta \), calculated with the test-particle spectrum given in Equation (10) with \( Q_i = 3.5 \) (solid lines with circles) and 3.8 (dashed lines with triangles). The green lines show \( \Gamma_{\text{gas}} \) and \( \eta \) for gas-dynamic shocks without Alfvénic drift.

4. Summary

We study the effects of Alfvén drift on the DSA of CR protons at weak shocks in high beta ICM plasmas. We assume that upstream Alfvén waves are self-excited by CR protons via resonant streaming instability at parallel shocks (Lucek & Bell 2000; Schure et al. 2012). Such waves are mostly backward, moving anti-parallel to the background flow (Bell 1978), so they can be characterized by the cross-helicity of \( H_{c,1} \approx -1 \) (see Equation (3) for the definition of \( H_c \)). As CR protons are scattered and isotropized in the local wave frame, the scattering center compression ratio, \( r_{\text{sc}} \), in Equation (8), which accounts for the mean drift of Alfvén waves, determines the spectral index, \( \Gamma \), of the CR spectrum in the test-particle limit.

We first consider 1D planar shocks where the transport of Alfvén waves across the shock transition is described in the small wave amplitude limit \( (b = B/B \ll 1) \) (Vainio & Schlickeiser 1998, and V99). In this limit, as noted by VS99, Alfvénic drift may increase or decrease \( r_{\text{sc}} \), depending on the shock parameters. This results in the CR spectra either flatter or steeper, compared to that for gas-dynamic shocks without Alfvénic drift. For shocks with \( M_s \lesssim 3 \) and \( \beta \equiv (M_A/M_s)^2 \sim 40-80 \), a mixture of backward and forward waves are present in the postshock region with the postshock cross-helicity estimated to \( -1 \lesssim H_{c,2} \lesssim -0.85 \), leading to only a slight decrease of \( \Gamma \) (see Figure 3). That is, for weak cluster shocks, \( r_{\text{sc}} \approx r_{\text{gas}} \) and \( \Gamma \approx \Gamma_{\text{gas}} \), and so the effects of Alfvénic drift on the DSA efficiency are only marginal (see Figure 5).

We then consider two additional, physically motivated cases: (1) downstream waves are isotropic with \( H_{c,2} \approx 0 \), and (2) they are dominantly forward with \( H_{c,2} \approx +1 \). The former could be realistic, if waves are isotropized via a variety of MHD and plasma processes including turbulence while they cross the shock transition. The latter may be relevant, if the CR pressure distribution peaks at the shock as in spherical SNR shocks or evolving planar shocks. In these two cases, Alfvénic drift causes the CR spectrum to be steeper, which results in significant reductions of the CR injection fraction, \( \xi \), and the
CR acceleration efficiency, $\eta$ (see Figure 6). In the case of $H_{c2} \approx +1$, for example, the CR proton acceleration efficiency for shocks with $M_s \gtrsim 3$ and $\beta \approx 40$ could be reduced by “a factor of up to five,” compared to that for gas-dynamic shocks. So, we conclude that the Alfvénic drift effects on the DSA efficiency could be substantial at weak cluster shocks.

We note that the CR acceleration efficiency is most sensitive to the injection momentum, or, the injection parameter, $Q_i$, defined in Equation (11). Increasing $Q_i$ from 3.5 to 3.8 (about 10%), for instance, reduces $\eta$ by a factor of five to seven. For parallel shocks with small obliquity angles, we expect that $Q_i = 3.5–3.8$ would be a reasonable range. Thus, in order to reliably estimate the CR proton acceleration efficiency at weak cluster shocks, it is important to understand the kinetic plasma processes that govern the particle injection to Fermi I acceleration at collisionless shocks at high beta plasmas.

We suggest $\eta$ could vary in a wide range of $10^{-4}–10^{-2}$ for weak cluster shocks with $M_s \approx 2–3$, depending on $H_{c2}$, $\Theta_{\delta n}$, and $\beta$. Such an estimate could be smaller by up to an order of magnitude than those adopted in previous studies, such as that of Vazza et al. (2016). Thus, this study implies that there remains room for the DSA prediction for CR proton acceleration at cluster shocks to be compatible with non-detection of $\gamma$-ray emission from galaxy clusters (Ackermann et al. 2014, 2016). Yet, we emphasize that eventually detailed quantitative studies of DSA at weak cluster shocks using kinetic plasma simulations should be crucial for solving this problem.

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