Experimental Verification of the Inverse Method of the Heat Transfer Coefficient Calculation

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Abstract: The purpose of this work is to formulate a method which can be used to solve nonlinear inverse heat conduction problems and to calculate the heat transfer coefficient distribution on the unknown boundary. The domain under consideration is divided into control volumes in polar coordinates, and heat balance equations are written. Based on temperature transients measured in selected points on the outer surface, temperature values in other points of the domain are determined. Finally, the heat transfer coefficient distribution on the inner surface with the unknown boundary condition is calculated from the presented heat balance equation. The proposed inverse method was verified experimentally using a collector that is part of a semi-industrial laboratory system. This collector is a horizontal, cylindrical thick-walled tank with flat side walls with outlets that enable oil supply and removal. Each side wall has an additional connector to ensure venting. The calculations made it possible to identify the phenomena occurring inside the collector during the experiment. The transient temperature distribution identified by the proposed inverse method was verified by a comparison of the calculated and the measured temperature transients in points inside the collector wall. Very good agreement is observed between the calculated and the measured temperature transients, which confirms the correctness of the identification. This proposed inverse method of the temperature and the heat transfer coefficient calculation is fast enough to apply in online thermal state monitoring systems. The proposed algorithm presented in this paper can easily be implemented industrially.

Keywords: inverse problems; control volume method; diagnostic system; experimental verification

1. Introduction

Inverse methods play an important role in thermal and mechanical analysis. They make it possible to solve problems even if some dimensions, properties or boundary conditions are not defined. Methods that can be applied in steady-state conditions can be found in [1,2]. Algorithms solving transient inverse problems are presented in the literature. An iterative regularization method used to solve three-dimensional ill-posed boundary inverse problems is formulated in [3]. A general method for solving multidimensional inverse heat conduction problems is presented in [4]. Identification of the transient temperature and stress distribution in an atmospheric reentry capsule is shown in [5]. Example applications of a modified Levenberg-Marquardt algorithm for the identification of the temperature-dependent conductivity of solids and heat capacity can be found in [6,7]. An experimental study of inverse identification of the unsteady heat transfer coefficient in a fin-and-tube heat exchanger assembly using an infrared thermography system is presented in [8]. A comparison between direct measurements and inverse predictions of a molten salt bank formation inside an experimental setup representing a real metallurgical reactor can be found in [9]. An inverse methodology for the heat flux estimation based on strain measurements, instead of the usual temperature data, is proposed.
This presented method is especially useful in cases where the temperature does not change in measuring points in a certain time period. A method is presented in [11] for online estimation of the time-varying surface heat flux of a nonlinear heat conduction system with a complex geometry based on transient temperature measurements.

If parameters or boundary conditions are unspecified, commercial software cannot be used directly. By transforming the task into an optimization problem, it is possible to use commercial codes iteratively [2,5–8]. However, such offline algorithms are not suitable for structure monitoring systems, which must work in the online mode. Various diagnostic methods based on direct solutions can be found in the literature [12–14]. An online monitoring system for detecting defects in a hydrogen storage vessel using the wave scattering phenomenon is presented in [12]. The signals used by a real-time structural health monitoring system can be related to those obtained for a healthy hydrogen storage vessel. Such signals carry all essential information about potential damage and can be used very effectively. The Green’s-Function Technique is used for online thermal stress determination in rotors and casings of next-generation turbines [13]. This method enables fast calculation of thermal stresses in supervised areas for any changes in the fluid temperature causing element heating or cooling [14]. A crucial effect of the variation of the heat transfer coefficient on the shape of Green’s function and on thermal stress evolution is shown on the example of a simple cylinder representing a turbine rotor heated at a constant rate. The heat transfer coefficient in a labyrinth seal is calculated using the correlation developed by Kapinos et al. [15]. In the inverse method, there is no need to assume an approximate value of the heat transfer coefficient. Non-iterative inverse methods are usually based on complex calculation algorithms, including the meshless radial point interpolation method [10], the FEM and the artificial neural network [11] or the semi-discrete control-volume method [16]. In [17], the finite-element control-volume method was applied to solve problems in components with complex shapes. Inverse problems are ill-posed due to the lack of some dimensions, properties or boundary conditions. To improve the inverse problem solution stability, many different techniques have been proposed in the literature, including regularization [10,11,16], future time steps [5] or smoothing digital filters [17]. These techniques can also lead to a loss of important information included in the measured data. For this reason, proposed algorithms should be tested not only in numerical simulations but they should also be verified experimentally. Based on the literature review, it can be seen that despite many proposed methods, there are few simple calculation techniques that can be easily applied in practice. There are also not enough works on experimental verification of inverse methods.

The purpose of this work was to formulate a simple method that can be used to solve nonlinear inverse heat conduction problems and calculate the heat transfer coefficient distribution on the unknown boundary. The method accuracy and stability will be demonstrated in numerical tests and confirmed by experimental verification.

2. Testing Stand

The experiment was carried out using the semi-industrial system presented in Figure 1. It consists of two oil tanks, horizontal and vertical, with a capacity of 3 m³ each, two pumps, pipelines and two thick-walled collectors. Four equally spaced electric heaters intended for oil heating and a blade impeller are installed in the vertical tank. The two thick-walled collectors were positioned horizontally, with an outer diameter of 219.1 mm and a wall thickness of 40 mm. The lower pipe was divided in half using a horizontally arranged 5 mm thick partition, to ensure tightness between the upper and the lower half. Using appropriate valves, oil can be directed to one of the collectors or to the bypass. This system was equipped with sensors measuring the oil flow rate, temperature and pressure.
The heat transfer coefficient was identified on the inner surface of collector 1 based on the temperature measurement on its outer surface. Details concerning the thermocouple location are presented in Figures 2 and 3. The thermocouples measuring the outer surface temperature are located on the A–A cross section. Sheathed sensors with a calibration certificate and with the diameter of 1.5 mm and tolerance of ±(0.15 + 0.002 | T |) °C were used. These thermocouples measuring the wall temperature were installed on the B–B and C–C cross sections. In-wall temperature measurements on the thick-walled pipes were performed with sheathed sensors with a diameter of 3 mm and tolerance of ±(0.15 + 0.002 | T |) °C, using a mechanism that ensures a proper downforce to the structure. The sensors measuring the temperature on the outer surface and in inner locations can be seen in Figure 4. Various methods of the thermocouples fixing were tested to achieve good contact and a low outflow of heat in the temperature measuring point on the wall thickness. Collector 1 and the partition are made of St41K steel. The steel thermal and physical properties are presented in Figure 5.
3. Inverse Method Formulation

The temperature distribution must satisfy the transient heat conduction equation:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q}$$  \hspace{1cm} (1)

where $\mathbf{q}$ is the heat flux vector defined by Fourier’s law:

$$\mathbf{q} = -k\nabla T$$  \hspace{1cm} (2)
For homogeneous and isotropic materials, all material properties ($c$—specific heat, $\rho$—density, $k$—thermal conductivity) are temperature-dependent only.

The problems occurring in a cylindrical collector are solved using the control-volume method [18]. Equation (1) is integrated over a general control volume $V$ with bounding surface $S$:

$$\int_V c(T) \rho(T) \frac{dT}{dt} dV = -\int_V \nabla \cdot \mathbf{q} dV$$  \hspace{1cm} (3)

By applying the mean value theorem for the integrals on the left and the divergence theorem on the right, the following equation is obtained:

$$V_c(T) \rho(T) \frac{dT}{dt} = -\int_S \mathbf{q} \cdot \mathbf{n} dS$$  \hspace{1cm} (4)

where the bar indicates the mean value in volume $V$.

The domain under consideration is divided into control volumes in polar coordinates $(r, \phi)$. Three of them are marked in Figure 6 using striped areas. Control volume $VA$ is related to an internal node, $VB$ to a boundary node on the outer surface with a known boundary condition and $VC$ to a boundary node on the inner surface with an unknown boundary condition.

![Figure 6. Division of the cylindrical component cross-section into control volumes.](image)

Forty-eight heat balance equations can be formulated based on Equation (4). The notation shown in Figure 6 is introduced for simplification. $N$ and $S$ are nodes with coordinate $r$ by $2\Delta r$ bigger and smaller compared to node $P$. Similarly, $W$ and $E$ are nodes with coordinate $\phi$ by $\Delta \phi$ smaller and bigger compared to node $P$; $T_P$ is the temperature in node $P$. For control volumes related to any internal node (such as $VA$), the following equation is obtained:

$$c_P \rho_P \frac{(r_P + \Delta r)^2 - (r_P - \Delta r)^2}{2} \Delta \phi \cdot \frac{dT_P}{dt} = \frac{k_x + k_E}{2} \frac{2\Delta r}{r_P \Delta \phi} (T_E - T_P) + \frac{k_y + k_P}{2} \frac{2\Delta \phi}{\Delta r} \left( \frac{k_y + k_P}{2} \frac{(r_P + \Delta r) \Delta \phi}{2\Delta r} (T_S - T_P) + \frac{k_y + k_P}{2} \frac{(r_P + \Delta r) \Delta \phi}{2\Delta r} (T_N - T_P) \right)$$  \hspace{1cm} (5)

where $c_P = c(T_P)$, $\rho_P = \rho(T_P)$, $k_i = k(T_i)$ for $i \in \{P, E, W, S, N\}$ and $P = 13...36$. 

For control volumes related to any boundary node on the outer surface (such as VB), the heat balance equation has the following form:

\[
\frac{c_P p_p (r_p^2-(rp-\Delta r)^2)}{2} \frac{dT_p}{dt} = \frac{k_s+k_p}{r_p \Delta p} (T_E - T_P) + \frac{k_w+k_p}{2 \frac{r_p \Delta p}{2}} (T_W - T_P) + \frac{q_{\text{out}} p p}{r_p \Delta p} \epsilon
\]

where \( P = 37...48 \). Term \( q_{\text{out}} p \) is the heat flux which is transferred to control volume \( P \) from the environment by convection and radiation, respectively. It can be expressed as:

\[
q_{\text{out}} p = h_{\text{out}} p (T_{\text{OUT}} - T_P) + \epsilon_{\text{out}} p \sigma \left(T_{\text{OUT}}^4 - (T_P + T_0)^4\right)\]

where \( h_{\text{out}} p \) is the convective heat transfer coefficient, \( \epsilon_{\text{out}} p \) is emissivity on the control volume outer surface, \( \sigma = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4 \) is the Stefan-Boltzmann constant and \( T_0 \) is equal to 273.15 °C. For pipes that are perfectly insulated on the outer surface, the two terms defined in Equation (7) are both equal to zero and can be omitted. For control volumes related to any boundary node on the inner surface (such as VC), the heat balance equation has the following form:

\[
\frac{c_P p_p (r_p^2-(rp+\Delta r)^2)}{2} \frac{dT_p}{dt} = \frac{k_s+k_p}{r_p \Delta p} (T_E - T_P) + \frac{k_w+k_p}{2 \frac{r_p \Delta p}{2}} (T_N - T_P) + h_{\text{in}} p (T_m - T_P) p r \Delta p
\]

where \( P = 1...12 \). Term \( h_{\text{in}} p \) is the convective heat transfer coefficient on the control volume inner surface and \( T_m \) is the medium temperature.

Transient heat conduction problems are initial-boundary problems for which appropriate initial and boundary conditions should be defined. The initial (Cauchy) condition is the value of the body temperature at the beginning of the process \( t_0 = 0 \) s.

\[
T_P|_{t_0=0} = T_0 \text{ for } P = 1...48
\]

If all dimensions, properties and boundary conditions are defined, the system of 48 ordinary differential equations based on Equations (5–8) with the initial condition as expressed in Equation (9) can be solved using the Runge-Kutta method.

If boundary conditions on the cylindrical component inner surface are unknown, the problem becomes ill-posed and additional temperature measurements \( f_P(t) \) are included in the analysis:

\[
T_P = f_P(t) \text{ for } P = 1...12
\]

Based on temperature transients measured in point \( P \), temperature derivative \( dT_P/dt \) can also be obtained. The temperature transients in nodes \( S = 25...36 \) can be calculated iteratively using Equation (6) for the control volumes associated with points \( P = 37...48 \):

\[
T_S = T_P + \frac{4 \Delta r}{(k_s+k_p)(r_p-\Delta r) \Delta p} \left[ \frac{dT_P}{dt} \frac{c_P p_p (r_p^2-(rp-\Delta r)^2)}{2} \frac{\Delta p}{r_p \Delta p} (T_E - T_P) - \frac{k_w+k_p}{2 \frac{r_p \Delta p}{2}} (T_W - T_P) - q_{\text{out}} p p \epsilon \right]
\]

The first iteration, the result is: \( T_S^{(1)} = T_P, S = 25...36, P = 37...48 \). This iterative algorithm continues until the following condition is satisfied:

\[
\left| \frac{T_S^{(n+1)} - T_S^{(n)}}{T_S^{(n+1)}} \right| \leq \varepsilon, \quad n = 0, 1, 2..., \quad S = 25...36
\]
Once the temperature transients in nodes 25...36 are obtained, an inverse solution can be run again iteratively to find temperatures in points S = 13...24, using the heat balance equation in Equation (5) for the control volumes associated with nodes P = 25...36:

\[
T_S = T_P + \frac{4\Delta r}{\mu_s + \mu_p} \left[ \frac{dT_P}{dt} \frac{r_p \rho_p c_p}{2} \left( (r_p + \Delta r)^2 - (r_p - \Delta r)^2 \right) \Delta \varphi \right] - k_p + k_p \frac{2\Delta r}{r_p \Delta \varphi} (T_E - T_P) - k_w + k_p \frac{2\Delta r}{r_p \Delta \varphi} (T_W - T_P) - k_w + k_p \frac{\Delta \varphi}{2\Delta r} (T_N - T_P) \tag{13}
\]

The same heat balance equation can be used once more to calculate temperature transients in nodes S = 1...12.

Finally, Equation (8) can be used for control volumes P = 1...12 to calculate heat transfer coefficient \( h_{inP} \) on the cylindrical component inner surface:

\[
h_{inP} = \frac{1}{(T_m - T_P) \rho_p \Delta \varphi} \left[ \frac{dT_P}{dt} \frac{r_p \rho_p c_p}{2} \left( (r_p + \Delta r)^2 - (r_p - \Delta r)^2 \right) \Delta \varphi \right] - k_p + k_p \frac{2\Delta r}{r_p \Delta \varphi} (T_E - T_P) - k_w + k_p \frac{2\Delta r}{r_p \Delta \varphi} (T_W - T_P) - k_w + k_p \frac{\Delta \varphi}{2\Delta r} (T_N - T_P) \tag{14}
\]

It is difficult to solve this ill-posed problem, as calculated temperatures or heat transfer coefficients are very sensitive to errors arising in real-time measurements. These errors can create oscillation in determined values and even make the solution unstable. Regularization techniques require the estimation of the regularization factor appropriate value, which depends on the size of the task and the value of measurement error. For this reason, to stabilize the solution, smoothing digital filters [17] were used to rectify temperature transients and calculate time derivatives.

4. Numerical Verification

The accuracy of the proposed inverse method is presented in numerical tests conducted for the collector shown in Figures 1–4 and in Figure 6. This collector is filled with oil in both parts; above and below the partition. At the beginning of the process, the temperature of the collector and of the oil inside is \( T_0 = 22 \) °C. It was assumed that during the process the ambient temperature remained constant and the heat exchange between the outer surface of the collector and the environment was due to convection and radiation. The heat transfer coefficient on the outer surface is estimated as \( h_{out} = 4.6 \) W/m²K. The emissivity of the pipe’s outer surface was assumed as \( \varepsilon = 0.8 \). Then, oil with a temperature of 175 °C was pumped into the collector. The oil flow rate in the collector upper part was higher than in the lower one, and therefore, different values of the heat transfer coefficient of \( h_{inup} = 400 \) W/m²K and \( h_{indown} = 40 \) W/m²K were adopted in the upper and in the lower part of the collector, respectively. Similar changes in heat transfer coefficients occur, for example, in a power boiler drum in which the liquid and saturated steam flow in the lower and the upper part, respectively. Naturally, there was no partition, the medium was different and so were the heat transfer coefficients. A direct solution was applied to generate temperature transients for the inverse solution. The finite element method, which is implemented in the ANSYS program [19], was used. The collector cross-section was divided into four-node quadrilateral finite elements. Fifteen elements through the wall thickness and 60 along the circumference were adopted. In order to bring the numerical tests closer to real conditions, “measured data” were produced by adding normal random errors to the temperature transients obtained from the direct solution. The generated error mean value was equal to zero and the standard deviation \( \sigma_f = 1/12 \) (error \( \pm 0.25 \) °C). These “measured” temperature transients in nodes 37–48 for the inverse solution are shown in Figure 7. The temperature distribution produced by the ANSYS code was symmetrical with respect to the vertical plane. After disturbance with measurement errors, slight differences appeared between the right and the left part, which can be seen in Figure 7.
Using the proposed inverse method, the temperature was identified with $\Delta t = 10$ s. The measured temperature transients in nodes 37–48 were smoothed using a third-degree polynomial with 11 subsequent points and the same filter was used to calculate temperature derivative $dT/dt$ [20]. After the temperature derivative was determined, Equation (11) became algebraic. Due to the small non-linearity of material properties (Figure 5), after several iterations, temperature transients in nodes 25–36 could be calculated from Equation (11). The calculated temperature transients in nodes 25–36 were smoothed and temperature derivative $dT/dt$ was calculated. Next, temperature transients in points 13–24 were obtained iteratively using Equation (13). The same equation was used to calculate temperature in nodes 1–12. The temperature transients calculated in points 1–12, located on the inner surface with an unknown boundary condition, $w$ presented in Figure 8. They were compared with the values obtained from the direct solution. Figure 8a,b show temperatures in nodes on the left and on the right side of the cross-section, respectively. Despite the introduced measuring errors, the determined transients were very close to those generated from the direct method. The identified temperature distribution was almost symmetrical with respect to the vertical plane. This is also visible in Figure 9, which shows the temperature distribution after 500 s of the heating process.

Figure 7. “Measured” temperature transients in nodes 37–48.

Figure 8. Comparison between exact temperature transients and those calculated by the inverse method on the pipe inner surface in the selected nodes: (a) left side, (b) right side.
Finally, the heat transfer coefficient $h_{inP}$ on the cylindrical component inner surface was calculated from Equation (14). The identified values were compared with those assumed in the direct solution. The values of 400 and 40 W/m$^2$K were adopted in the upper and in the lower part, respectively. For the control volume assigned to nodes 4 and 10, average values, i.e., 220 W/m$^2$K, were assumed. The oscillations in the calculated heat transfer coefficient transients were bigger compared to the identified temperatures, as can be seen in Figure 10. They were caused by the inverse problem being ill-posed on the one hand and by the introduced measuring errors on the other. The distribution of the coefficients identified on the collector entire inner surface is also shown for selected time moments in Figure 11. Additionally, the following quantitative measure of the error was proposed:

$$E_h = \sqrt{\frac{\sum_{j=1}^{N} \left( h_{inP,j}^{ex} - h_{inP,j}^{cal} \right)^2}{N - 1}}$$  (15)

where $h_{inP,j}^{ex}$ is the heat transfer coefficient calculated by the inverse method in node $P$ and in time point $j$, $h_{inP,j}^{cal}$ is the heat transfer coefficient assumed in the direct method (exact value) and $N$ is the number of compared coefficients.

![Figure 9. Calculated temperature distribution after 500 s.](image)

![Figure 10. Comparison between exact heat transfer coefficient transients and those calculated by the inverse method on the pipe inner surface in the selected nodes: (a) left side, (b) right side.](image)
The errors are calculated on the surface with the unknown boundary condition in the upper part \((P = 1, 2, 3, 11, 12)\), the lower part \((P = 5, 6, 7, 8, 9)\) and between them \((P = 4, 10)\) during the whole heating process. Their values are: \(E_{h\text{, up}} = 38.4\,[W/m^2K]\), \(RE_{h\text{, up}} = 9.6\%, E_{h\text{, down}} = 7.3\,[W/m^2K]\), \(RE_{h\text{, down}} = 18.4\%, E_{h\text{, be}} = 11.2\,[W/m^2K]\), \(RE_{h\text{, be}} = 5.1\%.\) The proposed inverse method accuracy and stability were also demonstrated in numerical tests conducted for standard deviation \(\sigma_f = 1/6\) (error \(\pm 0.5^\circ\text{C}\)). The errors of the identified heat transfer coefficient are: \(E_{h\text{, up}} = 42.6\,[W/m^2K]\), \(RE_{h\text{, up}} = 10.6\%, E_{h\text{, down}} = 9.8\,[W/m^2K]\), \(RE_{h\text{, down}} = 24.7\%, E_{h\text{, be}} = 16.7\,[W/m^2K]\), \(RE_{h\text{, be}} = 7.6\%.\)

The good agreement of the identified temperatures and heat transfer coefficient transients with the exact histories is due to the introduced time smoothing and the method of the time derivative determination. Calculations were also carried out without smoothing the temperature in time and using a central differential scheme to obtain the temperature derivative. The calculated heat transfer coefficient transients are presented in Figure 12.
5. Experimental Verification

The proposed inverse method was verified experimentally during the heating process of collector 1 in the semi-industrial system located in the Laboratory of the Institute of Thermal and Process Engineering of the Cracow University of Technology. This system and collector 1 are presented in Figures 1–4. The collector parts above and below the partition were both filled with oil. The initial temperature of the collector and of the oil inside was $T_0 = 22^\circ\text{C}$. The air temperature in the laboratory hall was constant during the process $(22^\circ\text{C})$. The heat transfer coefficient on the outer surface was estimated as $h_{\text{out}} = 4.6 \ \text{W/m}^2\text{K}$. The emissivity of the pipe outer surface $\varepsilon = 0.8$ was estimated by the FLIR i7 infrared camera with the following parameters: IR resolution $140 \times 140$ pixels, thermal sensitivity/NETD $< 0.1 \ ^\circ\text{C} (0.18 \ ^\circ\text{F})/100$ mK, field of view (FOV) $29^\circ \times 29^\circ$. Then, oil with an initial temperature of $180 \ ^\circ\text{C}$ was pumped into the collector. The partition was installed so as not to cause residual stress in the thick-walled pipe. Despite the sealing, it was not possible to ensure full tightness. Some of the hot oil pumped into the pipe upper part also flowed into the bottom part. Figure 3 shows the resulting leaks at the beginning and at the end of the partition. The temperature and pressure transients of oil flowing through the collector are shown in Figure 13. Additionally, the temperature values measured in 12 points on the collector outer surface are presented. The measured temperature distribution on the outer surface is not symmetrical with respect to the vertical surface. The temperature in node 38 is higher compared to node 48. This is similar with the temperatures in points 39 and 47. The highest asymmetry appears between nodes 40 and 46.

![Figure 13](image_url)  
**Figure 13.** Measured values of the oil pressure and temperature in the upper and the lower part, and temperatures in 12 points of the collector outer surface.

Using the proposed inverse method, the temperature was identified with $\Delta t = 10 \ \text{s}$. The measured temperature transients in nodes 37–48 were smoothed using a third-degree polynomial with 11 subsequent points and the same filter was used to calculate temperature derivative $dT_P/dt$. Similar to the numerical verification in Section 3, the temperature distribution in the collector was identified using Equations (11–13). Figure 14 presents the temperature transients calculated by the inverse method on the pipe inner surface. The highest asymmetry can be seen between nodes 4 and 10. This is probably due to a leak in the right part of the partition. Because of the leak, hot oil also flows under the partition near node 5. For this reason, the temperature identified in point 5 is higher than in the other points in the bottom part of the collector.
Finally, heat transfer coefficient $h_{in P}$ on the cylindrical component inner surface was calculated from Equation (14). Due to slight differences between the wall and oil temperatures in the bottom part of the collector, it was difficult to determine the heat transfer coefficient distribution in the lower part. However, it was possible to determine the heat transfer coefficient average value, which is shown by the dashed line in Figure 15. Additionally, the coefficients on other control volumes on the inner surface were identified and are presented in Figure 15. The sealing between the partition and the pipe hindered the heat transfer, and therefore, the heat transfer coefficients on the control volumes associated with nodes 4 and 10 are comparable with the average heat transfer coefficient in the collector lower part. The flow of hot oil under the partition near node 5, which is the effect of the leak, made $h_{in 4}$ higher compared to $h_{in 10}$. Additionally, Figure 16 presents the distribution of the coefficients identified on the collector entire inner surface for selected time moments.

Figure 14. Temperature transients calculated by the inverse method on the pipe inner surface in selected nodes: (a) left side, (b) right side.

Figure 15. Heat transfer coefficient transients calculated by the inverse method on the control volumes associated with selected nodes on the pipe inner surface: (a) left side, (b) right side.
The transient temperature distribution identified by the proposed inverse method was verified by a comparison of the calculated and the measured temperature transients in points inside the collector wall. The location of the thermocouples measuring the wall temperature is shown in Figures 2 and 3. A comparison between the calculated and measured temperature transients on the B–B and the C–C cross-section is shown in Figures 17 and 18, respectively. The biggest differences appear for temperature $f_{w,7}$, but they do not exceed 6 °C and 5% compared to the measured temperature. Very good agreement can be observed between the calculated and measured temperature transients, which confirms the correctness of the identification.

**Figure 16.** Heat transfer coefficient values obtained from the inverse method for selected times on the inner surface (node numbers as in Figure 6).

**Figure 17.** Comparison between measured temperature transients and those calculated by the inverse method on the pipe cross-section B–B.
6. Summary and Conclusions

A simple method is presented that can be used to solve inverse transient heat conduction problems. The algorithm can be used when not all boundary conditions are known. If the temperature of the medium in contact with the surface with an unknown boundary condition is known, the method can calculate the heat transfer coefficient distribution in time and space. The solution is possible thanks to temperature transients measured on the easily accessible outer surface. To improve the solution accuracy, the method is formulated in cylindrical coordinates. An iterative algorithm was proposed to take account of temperature-dependent material properties. To improve the inverse problem solution stability, the measured and calculated temperature transients were smoothed using a third-degree polynomial with 11 subsequent points. Additionally, the same filter was used to calculate the temperature derivative \( \frac{dT}{dt} \).

The proposed inverse method accuracy and stability were demonstrated in numerical tests conducted for a thick-walled collector. A direct solution was used to generate temperature transients for the inverse solution. The inverse method calculated the transient temperature distribution and the heat transfer coefficient in time and space on the surface with an unknown boundary condition. Despite the measurement data disturbance with random measuring errors, the identified temperature values and the heat transfer coefficient distribution were very close to the direct solution. Calculations were also performed without smoothing the temperature in time and using a central differential scheme to obtain the temperature derivative. The lack of temperature smoothing and less accurate determination of the temperature derivative resulted in large oscillations in the obtained waveforms.

Finally, the proposed inverse method was verified experimentally using the semi-industrial system located in the Laboratory of the Institute of Thermal and Process Engineering of the Cracow University of Technology. Based on temperature values measured in 12 points on the outer surface of a thick-walled collector with an installed partition, the temperature distribution in the collector and the heat transfer coefficient on the component inner surface were identified. The calculations made it possible to identify the phenomena occurring inside the collector during the experiment. The sealing between the partition and the pipe hindered the heat exchange locally and lowered the heat transfer coefficient near the partition. Despite the sealing, it was not possible to ensure full tightness. Some of the hot oil pumped into the pipe upper part also flowed into the bottom part. The calculated heat transfer coefficients show that the flow of hot oil under the partition occurs mainly near its right end.

The transient temperature distribution identified by the proposed inverse method was verified by a comparison of the calculated and the measured temperature transients in points inside the collector.
Very good agreement was observed between the calculated and measured temperature transients, which confirmed the correctness of the identification. The proposed inverse method of the temperature and the heat transfer coefficient calculation is fast enough to apply in online thermal state monitoring systems. The proposed algorithm presented in this paper can easily be implemented industrially.

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