Capacity Games with Supply Function Competition

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Abstract

This paper studies a setting in which multiple suppliers compete for a buyer’s procurement business. The buyer faces uncertain demand and there is a requirement to reserve capacity in advance of knowing the demand. Each supplier has costs that are two dimensional, with some costs incurred before demand is realized in order to reserve capacity and some costs incurred after demand is realized at the time of delivery. A distinctive feature of our model is that the marginal costs may not be constants, and this naturally leads us to a supply function competition framework in which each supplier offers a schedule of prices and quantities. We treat this problem as an example of a general class of capacity games and show that, when the optimal supply chain profit is submodular, in equilibrium the buyer makes a reservation choice that maximizes the overall supply chain profit, each supplier makes a profit equal to their marginal contribution to the supply chain, and the buyer takes the remaining profit. We further prove that this submodularity property holds under two commonly studied settings: (1) there are only two suppliers; and (2) in the case of more than two suppliers, the marginal two-dimension costs of each supplier are non-decreasing and constant, respectively.

Keywords: capacity game, supply function, option contract, submodularity, Nash equilibrium

1 Introduction

When demand is uncertain, the characteristics of the supply chain and the contract arrangements will determine investment decisions. In capital-intensive industries, such as petrochemical, electronics and semiconductors, manufacturers need to invest heavily in building capacity and the lead times are long [Kleindorfer and Wu, 2003], so that it remains a critical challenge to incentivize and manage capacity investment. When manufacturers are competing against each other and a buying firm may switch to a different manufacturer, the problems are intensified. If the financial risks from investment are carried by the firms who build capacity, it is difficult to find good solutions as there is

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no certainty on how much capacity will be required in the future. The result may be that capacity is only built for the part of the demand that can be more or less guaranteed: this conservative approach will reduce the buyer’s ability to meet customers’ demand. From a supply chain perspective, it is usually preferable for buyers to reserve capacity from suppliers in advance, with a payment made to the suppliers associated with this capacity reservation. Then the buyer also shares the risk that demand is low and not all the purchased capacity is required.

We model capacity reservation in a supply option framework. In the first stage, before knowing the actual demand, a buyer reserves a certain amount of capacity by paying a reservation price. In the second stage, after discovering the actual demand, the buyer asks for supply up to the lesser of the reserved amount and the observed demand. At this stage, the buyer pays an execution price only for the amount of capacity that is used. The underlying assumption of this model is that suppliers have to install capacity, or place orders with their own suppliers, before demand materializes because of the lead times involved.

A distinctive feature of our model is that marginal costs (of capacity and production) may depend on the volume, which is often the case in practical situations. For example, capacity investment often involves a one-off setup cost, there may be (dis)economies of scale in production, or in cases where a supplier manages a portfolio of facilities with heterogeneous technologies the overall cost is far from being linear (especially in the electricity industry). In our model, each supplier has a total reservation cost (incurred before demand realization), which is an arbitrary increasing function of the capacity reserved, and also a total execution cost (incurred after demand realization), which is an arbitrary increasing function of the amount actually supplied. This framework includes constant marginal costs as a special case, so our model is flexible enough to capture many practical settings in terms of cost modelling. We believe this is an important theoretical advancement as most literature on supply chain contracts assumes constant marginal costs. The linear treatment may help simplify the analysis and develop high-level intuitions especially for competitive models, but may not align with real-world practices.

With constant marginal costs, it is plausible to focus on simple contract forms, but with general cost functions, more sophisticated contract formats may be worthwhile, such as bids which specify a schedule of different prices for different quantities. This type of supply function bid often occurs in practice through the application of some form of quantity discount contract.

In this paper, we consider a supply chain with a single buyer and multiple suppliers, who supply a homogeneous item to the buyer. The suppliers compete by offering price functions for both reservation and execution. Given the suppliers’ bids, the buyer first reserves capacity before knowing the actual demand, and then decides how much capacity to use after observing the demand. The buyer may choose not to enter into a contract with a specific supplier in which case no costs are incurred by that supplier. Competition with function bids fits well with the situation where a buyer does not stipulate the specific bidding format in its Request for Quote (RFQ), so that suppliers can bid in whatever format they like.

We suppose that the buyer faces a random demand, with a known distribution. There are $n$ suppliers, and each has a total reservation cost, which is expressed as a general
increasing function of the amount reserved, as well as a total execution cost which is a general increasing function of the amount supplied. We model this game in a Stackelberg framework, where the suppliers are leaders and the buyer is a follower. Each supplier has complete information about the buyer’s demand distribution and all supplier costs, but the buyer may not know the suppliers’ costs. This assumption has been widely used in supplier competition models and is appropriate for industries where the operating environments are more transparent and/or the production technologies are more mature such as electricity, electronics and semiconductors (Wu and Kleindorfer, 2005; Martínez-de Albéniz and Simchi-Levi, 2009; Anderson et al., 2017).

The sequence of events is depicted in Figure 1. First each supplier, with the aim of maximizing their own expected profits, offers a function bid consisting of a (marginal) reservation price function and a (marginal) execution price function to the buyer. Second, prior to knowing the actual demand, the buyer determines how much to reserve from each supplier and pays the reservation price. After demand is observed, the buyer chooses what capacity to use and pays the execution price for the amount of capacity that is used. If the demand exceeds the total amount of reserved capacity, there will be lost sales. Finally, the buyer sells the product to the consumer market at the retail price $\rho$. We will call this the capacity game with function bids. This game can be considered as a special case of a general capacity game setting, in which a first stage choice by the buyer implies a constraint on the decisions to be made by the buyer at a second stage when demand information becomes known. Thus, our discussion will be carried out for the general capacity game, which in turn provides a theoretical foundation for the capacity game with function bids.

The framework we consider also applies to capacity mechanisms in electricity markets. In this context the buyer and customers are replaced by a set of consumers and a system operator who aims to maximize system welfare. Problems of this form are usually formulated with a value of lost load (VoLL). In the case where demand is stochastic but independent of price, the system operator’s problem, of minimizing the expected cost of procurement less VoLL for demand not met, becomes equivalent to maximizing VoLL times (demand met) minus the cost of procurement, which is the same as the buyer’s problem in our supply chain setting when we set the retail price to be VoLL.

It is not uncommon to have a supply chain where multiple suppliers act as Stackelberg leaders and a buyer must select amongst the available suppliers on the basis of contracts that the suppliers offer. This framework will lead to suppliers setting as high a price as they can while still being chosen. The consequence is that each supplier makes a bid that leads to the buyer being indifferent between choosing or not choosing the supplier. From
this it can be shown that the profit available to each supplier is simply its contribution to the overall supply chain profit, i.e., the difference in the supply chain profits when it is chosen and when it is not.

A major contribution of this paper is to study submodularity in this setting. We will demonstrate that a natural sufficient condition for a well-behaved equilibrium is that the supply chain profits are submodular in terms of the set of suppliers available. This property is equivalent to a requirement that the profit available to a supplier cannot increase if a new supplier is added to those available. We are therefore led to considering whether or not the supply chain profits achieved are submodular. This is not always the case, but can be established in commonly studied settings. Specifically, the property holds if either: (i) there are only two suppliers; or (ii) each supplier’s marginal reservation cost is non-decreasing and marginal execution cost is constant.

A second contribution of this paper is to study how suppliers compete with each other by using supply functions rather than scalar prices. In other words, the strategy space of each supplier is extended to a function space, and as a result, the suppliers have more flexibility in their bidding decisions. On the other hand, the buyer can determine how much to reserve from each supplier (i.e., pick any point from a supply function) rather than being restricted to a fixed quantity-payment contract. Our aim is to develop an understanding of the players’ strategic behavior in such a market setting. Previous models have focused on competition problems when suppliers are restricted to a simple strategy space (i.e., scalar decision variables, including prices, quantities, lead times, quality, or specific contract types). To the best of our knowledge, this paper is among the first to study supply function competition in a capacity reservation setting.

We find that given knowledge of the other suppliers’ bids, it is optimal for each supplier to set execution prices at execution costs and make profits only through the buyer’s reservation payment. Our findings also show that, under some reasonable assumptions, there exists a continuum of equilibria in which the suppliers charge costs only for the execution prices but add a margin to the reservation costs. Despite the multiplicity of equilibrium bidding strategies, the equilibrium outcome is essentially unique: in equilibrium the buyer makes a reservation choice that maximizes the overall supply chain profit, each supplier makes a profit that is equal to their marginal contribution to the supply chain, and the buyer takes the remaining profit.

The rest of this paper is organized as follows. After a review of the relevant literature in Section 2, we present our general capacity game model in Section 3 and study the best response and equilibrium strategies for suppliers in Sections 4. Then we show how these results apply to the more specific case we have introduced above in Section 5. We discuss in Section 6 two commonly studied settings and show that the supply chain optimal profit is submodular in a broad class of capacity games with function bids. Finally, we conclude in Section 7. All technical proofs are presented in the appendix.

2 Literature review

Supply options have been extensively studied in the operations management literature (see, e.g., Barnes-Schuster et al. [2002], Burnetas and Ritchken [2003], Wu et al. [2005b], Fu et al. [2010], Secomandi and Wang [2012]). An initial step is to investigate a buyer’s optimal
purchasing decision given a fixed set of supply options (see, e.g., Martínez-de Albéniz
and Simchi-Levi [2005]). As an extension of this, several papers examine option contract
design problems in a Stackelberg game between a buyer and a supplier with a focus on the
interaction between option markets and spot markets (see, e.g., Wu et al. [2002] Pei et al.
2011). Further extensions have been made to consider supplier competition in an option
market (Wu and Kleindorfer 2005 Martínez-de Albéniz and Simchi-Levi 2009 Anderson
et al. 2017). Wu and Kleindorfer (2005) study a commodity procurement problem where
an industrial buyer purchases supply options from a small set of suppliers, while having
access to an open spot market. No reservation cost at the suppliers is considered, and the
buyer’s downward-sloping demand (arising from utilization maximization) is determined
by the spot price and the execution prices. Such a demand modelling approach allows
the buyer to rank suppliers by using a single price index combining the reservation and
execution price of supply options, which leads to a Bertrand type of equilibrium for the
suppliers. We study a similar situation but consider suppliers having a two-dimensional
cost and a newsvendor type buyer facing uncertain demand.

The closest papers to ours within this literature are Martínez-de Albéniz and Simchi-
Levi (2009) (hereafter, “MS”) and Anderson et al. (2017) (hereafter, “ACS”). MS studies
a setting where marginal costs are constants and each supplier chooses a reservation price
and an execution price for their limitless capacity. They show that there may be efficiency
loss in equilibrium, which is up to 25% of the overall supply chain optimal profit. Our
paper differs from MS by considering general cost functions and allowing suppliers to
submit function bids. By enlarging the strategy space of suppliers, we find a relatively
clean and intuitive result as discussed earlier. In particular, when the supply chain
optimal profit is submodular, there is no efficiency loss in equilibrium and each supplier
makes a profit equal to their marginal contributions. We further show that this condition
is not restrictive as it holds under some commonly studied settings. Comparing with MS,
we also find that allowing suppliers to offer a function bid makes the buyer worse off,
thereby highlighting the significant impact of the strategy space on equilibrium outcomes.

ACS studies a similar problem where each supplier owns a block of capacity with the
same size and the buyer must reserve all of a block or none. Our general framework of
capacity games covers the model by ACS as a special case since we can recover ACS by
restricting the buyer’s capacity choice from each supplier to be either zero or the block
size. In this respect, our paper generalizes ACS by studying a situation where the buyer
can reserve any amount from each supplier and suppliers can charge different prices for
different quantities.

This paper studies a situation where suppliers compete with each other by offering a
function bid, and this resembles what is studied in the supply function equilibrium (SFE)
literature (Klemperer and Meyer 1989 Anderson and Philpott 2002 Anderson and Hu
2008 Johari and Tsitsiklis 2011). However, the game type is different. Our model is a
Stackelberg game with multiple leaders (i.e., suppliers) and one follower (i.e., buyer), and
the buyer strategically responds to the supplier offers. In contrast, the SFE literature
focuses on Nash games that aim to study the strategic interaction between multiple
firms, and this literature does not involve a profit-maximizing buyer. In other words,
we examine the buyer’s optimization problem explicitly; while in the SFE literature, the
buyer’s problem is to choose a clearing price to equate the total supply with the demand,
and each supplier’s best response is characterized by a differential equation. In addition,
our capacity games involve the buyer making a two-stage decision and each supplier submits a two-dimensional bid consisting of a reservation price function and execution price function, which is not considered in the SFE literature.

This work is also related to the multi-unit auction literature. Problems of decentralized resource allocation [Klaganam and Parkes 2004] are central to auctions. A subset of this literature examines the efficiency and profit allocation of a given type of auction, for example, share auction (Wilson, 1979), menu auction (Bernheim and Whinston, 1986), split award auction (Anton and Yao, 1989, 1992), discriminatory price auction (Menezes and Monteiro, 1995), and uniform price auction [Bresky, 2013]. Those papers generally assume that the total purchase amount of a buyer is exogenously given, while in our model it is endogenously determined by the buyer based on the supplier bids. Furthermore, we study capacity reservation games where the buyer’s capacity reservation problem is a two-stage stochastic program, which is new to this literature.

Finally, the equilibrium profit allocation obtained in this paper is of Vickrey-Clarke-Groves (VCG) style, so it is instructive to discuss the key point of departure from VCG models [Vickrey, 1961; Rothkopf et al., 1990]. Those models study profit allocation from a mechanism design perspective: the VCG profit allocation is an outcome that arises from the payment rule. In essence the VCG approach selects bids that maximize system welfare, and pays according to each bidder’s contribution (i.e., the difference between system welfare with and without that bidder). With this setup the equilibrium result is for bidders to bid truthfully and the auction result is efficient. The model we propose uses the same bid selection approach, but pays as bid. It is then an equilibrium (in the case with submodularity) for the bidders to mark up by the exact VCG amounts. We still get an efficient auction result, but without truthful bids. This is a different approach and has some advantages over the usual VCG mechanism. [Hobbs et al.] (2000) and [Rothkopf] (2007) amongst others have discussed some of the difficulties of using a VCG auction in practice. Many of these problems are resolved with pay-as-bid prices. For example, in the auction format we discuss, there is not the same incentive for the buyer to encourage bids that will not be accepted with the aim of reducing the payments to the successful bidders. Note that a submodularity condition has also been discussed in the VCG framework by [Ausubel and Milgrom] (2006), who show that this condition reduces the problems ordinarily associated with VCG. Within the mechanism design literature, several papers study multi-dimensional bidding environments in an electricity context, where the focus is on designing scoring and payment rules that incentivize bidders to truthfully reveal their costs [Bushnell and Oren, 1994; Chao and Wilson, 2002; Schummer and Vohra, 2003].

3 Model setup

The problem we study is an example of a broader class of capacity games, in which the suppliers offer bids, then a buyer selects a capacity amount to buy from each supplier, then depending on the outcome of a random variable (the demand) in the second stage, the choice of amounts to buy is made by the buyer (with amounts constrained by the capacity already bought). The revenue to the buyer is a function of demand and the amounts bought. We are not considering risk aversion, so we can use the expected payoff
as the objective for the buyer and the suppliers. Supplier \( i \) faces costs that are the sum of a capacity cost and an execution cost. We will establish our results in this general framework and then apply this to the model described in the introduction section.

The buyer makes capacity decisions \( \mathbf{t} = (t_1, \ldots, t_n) \), where capacity \( t_i \) is restricted to lie in a set \( T_i \subseteq \mathbb{R}_+ \), and hence \( \mathbf{t} \in \mathbf{T} = T_1 \times T_2 \times \cdots \times T_n \). We assume that each \( T_i \) is compact containing 0 and is specified as part of the capacity game. Typically, \( T_i \) is a closed interval \([0, d_i]\) or \( T_i = \{0, W_i\} \), which corresponds to the case where the buyer either does not use this supplier, or uses the full amount \( W_i \) of the supplier.

The capacity payment to be made by the buyer to each supplier can depend on the complete set of capacity decisions \( \mathbf{t} \), which may occur in practice, for example, when the suppliers have some exclusivity clauses. Note that, we are making our model as general as possible, although in the specific applications of our model considered in Section 5, the capacity payment made to a supplier depends only on the buyer’s choice from that supplier. In the second stage, after demand \( D \in \mathcal{D} \) becomes known, there is a set of amounts (execution quantities) \( \mathbf{s} \in \mathbf{T} \) chosen by the buyer, where \( \mathbf{s} \) depends on the observed demand \( D \) and satisfies constraints imposed by the capacity decisions of the first stage. Thus we have \( \mathbf{s} \in X(\mathbf{t}) \) for some set \( X(\mathbf{t}) \subseteq \mathbf{T} \). We require that if \( t_i = 0 \) then \( s_i = 0 \). The case that is most natural is to allow any selection at the second stage that is no more than the capacity purchase at the first stage, so that \( \mathbf{s} \leq \mathbf{t} \) and \( X(\mathbf{t}) = \{ \mathbf{s} : s_i \leq t_i, i \in N \} \). In general, however, we only assume \( X(\mathbf{t}) \) is compact.

Then supplier \( i \) decides on two payment functions \( R_i(\cdot) \) and \( P_i(\cdot) \) that the buyer will be offered, where \( R_i(\mathbf{t}) \) is the payment made to supplier \( i \) when the buyer makes a capacity reservation of \( \mathbf{t} \), and \( P_i(\mathbf{s}) \) is the payment made to supplier \( i \) given execution amounts \( \mathbf{s} \). In the formulation of the game there may be restrictions on the bids allowed, which we capture by specifying an allowed set of functions \( A_i, i \in N \), and restrict the supplier’s choice to \( R_i(\cdot) \in A_i \) and \( P_i(\cdot) \in A_i \). Since the buyer can decide not to include supplier \( i \), in which case no payment is made, the feasible bids \( A_i \) have the property that for all \( R_i(\cdot) \in A_i \), \( R_i(\mathbf{t}) = 0 \) whenever \( t_i = 0 \) (similarly for \( P_i(\cdot) \)). For maximal generality of our model, we specify only the minimal further restrictions on \( A_i \) at the end of this section.

Besides payments made by the buyer to the suppliers, we also have costs \( E_i(\mathbf{t}) \) and \( C_i(\mathbf{s}) \) incurred by supplier \( i \) that may depend on the complete set of capacities purchased and amounts executed. This occurs when competing players can partially collaborate (e.g., they share warehouse facilities or transport links). However, in the capacity game, a supplier’s costs may be independent of other suppliers’ quantities, as in our specific applications considered in Section 5. We assume that these costs are zero if supplier \( i \) is not used at either stage and hence \( E_i(\mathbf{t}) = 0 \) when \( t_i = 0 \), and \( C_i(\mathbf{s}) = 0 \) when \( s_i = 0 \).

Finally, we have the expected revenue \( V(D, \mathbf{s}) \) to the buyer from an external source, which depends on the demand \( D \) and the execution amounts \( \mathbf{s} \). Rather than include a restriction that the total execution amounts are no more than the demand (i.e., \( \sum_{i \in N} s_i \leq D \)), we will assume that this is achieved through setting appropriate values for the revenue function \( V \).

**Example 1.** We consider an example that illustrates the range of modelling applications for the general capacity game. Consider a buyer facing uncertain demand: 50 with probability 0.5 and 100 with probability 0.5. There are two suppliers with identical cost structures who produce products that are fully substitutable. We take \( T_1 = T_2 = [0, 100] \). Each supplier has costs of reservation for an amount \( t \) of capacity given by \( t^2/150 \) with
an additional fixed cost of $20 associated with building a transport link. This fixed cost is split between any suppliers who have capacity reserved, so it is $20 if just one supplier is used, and otherwise $10 each. Thus the reservation costs are

\[ E_i(t) = \begin{cases} t_i^2/150 + 20 - 10\chi(t_j), & j \neq i, \text{ if } t_i > 0; \\ 0, & \text{if } t_i = 0; \end{cases} \]

where the indicator function \( \chi(t_j) = 1 \) if \( t_j > 0 \), and 0 otherwise. Execution costs are $1 per unit so that \( C_i(s) = s_i \). Customers pay an amount $3 per unit and in addition there are fixed costs at the buyer of $20 for each product that apply only if that product is supplied. Thus, \( V(D,s) = 3\min(D,s_1 + s_2) - 20\chi(s_1) - 20\chi(s_2) \). We have the standard form for \( X(t) \) so there is a restriction that \( s_i \leq t_i \). □

Since execution amounts will depend on \( D \), we can write a policy for the buyer as \( (t,s(\cdot)) \), where \( t \in T \) and \( s(\cdot) \) is a function from the demand set \( D \) to \( T \), taking the value \( s(D) \) at \( D \). For simplicity, we will write \( s(\cdot) \) hereafter. The total expected buyer profit with this policy choice is

\[ \Pi_B(t,s) = \mathbb{E}[V(D,s(D)) - \sum_{i \in N}(P_i(s(D)) + R_i(t))] \]

given a set of supplier bids \( B = \{(P_i(\cdot), R_i(\cdot)) \in A_i : i \in N\} \). The buyer’s problem is to maximize her expected profit by choosing an optimal reservation choice \( t \) and a set of execution amounts \( s(D) \) for each possible demand \( D \in \mathcal{D} \), i.e., to solve

\[ \max \{ \Pi_B(t,s) : t \in T, s(D) \in X(t) \text{ for all } D \in \mathcal{D} \} \tag{1} \]

With these bids and buyer choice \( (t,s) \), supplier \( i \) has a profit of

\[ \pi_i(t,s) = \mathbb{E}[R_i(t) - E_i(t) + P_i(s(D)) - C_i(s(D))] \], \( i \in N \). \tag{2}

The supply chain profit is

\[ \Pi_C(t,s) = \Pi_B(t,s) + \sum_{i \in N} \pi_i(t,s) \]

\[ = \mathbb{E}[V(D,s(D)) - \sum_{i \in N}(C_i(s(D)) + E_i(t))] \], \tag{3} \]

which is independent of the bids made.

We write \( I(t) = \{i : t_i > 0\} \) for the support of a vector \( t \). We use \( \Pi_C^*(S) \), \( S \subseteq N \), to denote the optimal supply chain profit when the capacity reserved is restricted to be zero outside the set \( S \). Thus

\[ \Pi_C^*(S) = \max \{ \Pi_C(t,s) : t \in T, s(D) \in X(t) \text{ for all } D \in \mathcal{D}, I(t) \subseteq S \} \tag{4} \]

Similarly, given a set of supplier bids \( B = \{(P_i(\cdot), R_i(\cdot)) \in A_i : i \in N\} \), we use \( \Pi_B^*(S) \), \( S \subseteq N \), to denote the optimal buyer profit when the capacity reserved is restricted to be zero outside the set \( S \). Thus

\[ \Pi_B^*(S) = \max \{ \Pi_B(t,s) : t \in T, s(D) \in X(t) \text{ for all } D \in \mathcal{D}, I(t) \subseteq S \} \]

The maximizations above are taken over \( t \in T \) (a vector) and \( s(\cdot) \in X(t) \) (a function). To ensure that the maxima exist and are attained, we make the following (additional) assumptions: (a) the set-valued function \( X(t) \) is upper semi-continuous; (b) the revenue function \( V(t,s) \) is upper semi-continuous in both arguments; and (c) the set \( A_i \) of possible bids is such that \( P_i(\cdot), R_i(\cdot) \in A_i \) are lower semi-continuous.
4 Best response and equilibrium behavior

We now look at each supplier’s best response problem. As a Stackelberg leader, each supplier is able to anticipate the buyer’s reservation choice provided that the competitors’ bids are observed. Since the buyer’s optimization problem is embedded in the suppliers’ best responses, we need to specify the buyer’s choice when there are different options with the same expected value to the buyer. We say that supplier $i$ has preferred status when a solution with $t_i > 0$ is chosen by the buyer even if other options give the same value.

Theorem 1 (Best Response). For any fixed supplier $i \in N$, we have the following statements for the supplier’s best response:

(a) Given bids $\{(P_j(\cdot), R_j(\cdot)): j \in N, j \neq i\}$, the profit for supplier $i$ is no more than

$$Z_i = \Pi_{\mathcal{B}_i}(N) - \Pi_{\mathcal{B}_i}(N\{i\}),$$

where $\mathcal{B}_i = \{(P_j(\cdot), R_j(\cdot)): j \in N, j \neq i\} \cup \{(C_i(\cdot), E_i(\cdot))\}$.

(b) If $Z_i > 0$ and supplier $i$ has preferred status, then the profit $Z_i$ is achieved by the offer $(\bar{P}_i(\cdot), \bar{R}_i(\cdot))$ defined by:

$$\bar{P}_i(s) = C_i(s),$$

$$\bar{R}_i(t) = E_i(t) + Z_i \text{ when } t_i > 0, \bar{R}_i(t) = 0 \text{ when } t_i = 0.$$

(c) If $Z_i > 0$, then for any $\epsilon > 0$, an offer of $(\bar{P}_i(\cdot), \bar{R}_i^{(\epsilon)}(\cdot))$ will achieve within $\epsilon$ of the maximum possible supplier profit, where $\bar{R}_i^{(\epsilon)}(t) = \bar{R}_i(t) - \epsilon$.

Theorem 1 shows the maximum profit for a supplier $i$, which is the increase in profit available to the buyer from including the bids of supplier $i$ when these are made at cost. Moreover, when optimizing for the supplier, it is sufficient to consider supplier bids that are at cost for the execution component and make money only from the reservation payments. However, we should note that the expected profit to the supplier is unaffected by parts of the function bids that are never selected by the buyer whatever demand occurs. The consequence is that there are a continuum of other best response function offers available.

Now let us consider the equilibrium strategies for suppliers. In general, the optimal solution to the buyer’s maximization problem is not unique at an equilibrium as we shall show later. Since our focus is on supplier competition, and each supplier’s payoff depends on the buyer’s reservation choice, we need a tie-breaking rule to pin down the buyer’s optimal choice in case of multiple solutions. The suppliers have an interest in raising prices to the point where the buyer is about to drop them from consideration. Therefore, a tie-breaking assumption is critical, otherwise, we can have a difficulty in defining optimal behavior for the suppliers, as a type of $\epsilon$-optimality could occur when a supplier sets his price just below a benchmark value at which the buyer no longer wishes to select the supplier. Consequently, we make the following assumption.

Assumption 1 (Tie-Breaking Rule). In case of multiple optimal solutions, the buyer will select the one that gives the largest supply chain profit.
This assumption is in line with that for the classic Bertrand competition model, where the firm with the lowest cost wins when all the firms charge the same price. The rationale of this assumption is as follows: if the buyer selects an alternative that gives a lower supply chain profit, then keeping the other suppliers’ bids unchanged, any supplier in the supply chain optimal set (with respect to the buyer’s equally good choices) will have incentives to raise his price by an amount that is smaller than the difference between the supply chain optimal profit and the one corresponding to the buyer’s choice. This would make both the buyer and the supplier better off.

We need to rule out cases where two suppliers are identical and the optimal supply chain solution can use either one or the other, so we introduce the following assumption.

**Assumption 2 (Uniqueness of Support).** All the supply chain optimal capacity choices use the same set of suppliers.

We will write \((t^*_N, s^*_N)\) to denote an optimal solution to the supply chain optimization problem \([4]\). So Assumption 2 amounts to a statement that the support \(I(t^*_N)\) of the supply chain optimal capacity choices \(t^*_N\) is unique. Let us identify the suppliers who make a contribution to the optimal supply chain profit by defining

\[
N^*(C) = \{ j \in N : \Pi^*_C(N) > \Pi^*_C(N\{j\}) \}, \\
N^*_i(C) = \{ j \in N : \Pi^*_C(N\{i\}) > \Pi^*_C(N\{i,j\}) \}.
\]

It is easy to observe that, under Assumption 2, we have

\[
I(t^*_N) = N^*(C). \tag{5}
\]

To establish an equilibrium, we will need to make use of submodularity of the optimal supply chain profit as a set function of the suppliers available, where the submodularity property states that, for any \(i, j \in S \subseteq N\) and \(i \neq j\),

\[
\Pi^*_C(S) - \Pi^*_C(S\{i\}) \leq \Pi^*_C(S\{j\}) - \Pi^*_C(S\{i,j\}). \tag{6}
\]

Inequalities (6) are satisfied for a large class of problems of interest, as we shall discuss in detail in Section 6. It is convenient to make this property an assumption:

**Assumption 3 (Submodularity).** The supply chain optimal profit \(\Pi^*_C(S)\) is submodular as a function of the set \(S \subseteq N\) of available suppliers.

This submodularity assumption implies two properties of the set function \(\Pi^*_C(\cdot)\), which are established in the following two lemmas.

**Lemma 1.** Under the Submodularity Assumption, the following inequality holds for any \(S \subseteq N\):

\[
\Pi^*_C(N) - \Pi^*_C(S) \geq \sum_{i \in N\setminus S} (\Pi^*_C(N) - \Pi^*_C(N\{i\})). \tag{7}
\]

The following result shows that all the suppliers in \(N^*(C) \setminus \{i\}\) will still be included in the optimal supply chain choice when supplier \(i\) is unavailable.

**Lemma 2.** Under the Submodularity Assumption, for any \(i \in N\), we have \(N^*(C) \setminus \{i\} \subseteq N^*_i(C)\).
We will consider the possibility of varying the offers that a supplier makes in a way that does not change the supply chain optimal solutions. We say that a set \( \{ \Delta_i(\cdot) : i \in N \} \) of functions from \( T \) to \( \mathbb{R} \) is \textit{consistent} with the cost functions \( \{ (C_i(\cdot), E_i(\cdot)) : i \in N \} \) if adding \( \Delta_j(t) \) to the cost \( E_j(t) \) does not change the optimal solution or the optimal value, i.e., for every \( S \subseteq N \),

\[
\max \left\{ \Pi_C(t, s) + \sum_{j \in S} \Delta_j(t) : t \in T, s \in X(t) \text{ for all } D \in D, \mathcal{I}(t) \subseteq S \right\} = \Pi_C^*(S), \tag{8}
\]

and the maximization is attained at \( t^*_S \). It is easy to see that this is equivalent to the conditions of (a) \( \sum_{j \in S} \Delta_j(t^*_S) = 0 \) and (b) if \( I(t) \subseteq S \) and \( s(D) \in X(t) \) for all \( D \in D \), then

\[
\sum_{j \in S} \Delta_j(t) \leq \Pi_C^*(S) - \Pi_C(t, s). \tag{9}
\]

Note that when \( \Delta_i(t) \) is zero for any \( t \), then it is trivially consistent with the costs. Now we are ready to characterize the equilibrium that occurs when \( \Pi_C^*(S) \) is submodular as a function of \( S \).

\textbf{Theorem 2} (Nash Equilibrium). If Assumptions 1, 3 hold, then the set of bids \( \bar{B} = \{(C_i(\cdot), \bar{R}_i(\cdot)) : i \in N^*(C)\} \cup \{(C_i(\cdot), E_i(\cdot)) : i \notin N^*(C)\} \) is a Nash equilibrium, where

\[
\bar{R}_i(t) = \begin{cases} E_i(t) + \Pi_C^*(N) - \Pi_C^*(N \setminus \{i\}) - \Delta_i(t), & \text{if } t_i > 0; \\ 0, & \text{if } t_i = 0; \end{cases}
\]

and \( \{ \Delta_i(\cdot) \geq 0 : i \in N^*(C) \} \) is any set of functions consistent with the costs. In equilibrium, the buyer makes a supply chain optimal choice \( (t^*_N, s^*_N) \). The buyer makes profit \( \Pi_C^*(N) - \sum_{i=1}^n (\Pi_C^*(N) - \Pi_C^*(N \setminus \{i\})) \) and supplier \( i \) makes profit \( \Pi_C^*(N) - \Pi_C^*(N \setminus \{i\}) \).

We can see immediately from Theorem 2 that the following set of bids is a Nash equilibrium (by setting all \( \Delta_i(t) = 0 \)):

\[
\bar{R}_i(t) = E_i(t) + \Pi_C^*(N) - \Pi_C^*(N \setminus \{i\}), \text{ when } t_i > 0,
\]

and \( \bar{R}_i(t) = 0 \) when \( t_i = 0 \).

Several points about this theorem are worth mentioning. First, the profit allocation in equilibrium is a VCG result. However, this is not from a mechanism design that sets prices paid in a particular way, but arises as an equilibrium from our pay-as-bid capacity game. It is straightforward that each supplier makes a nonnegative profit. Suppliers in \( N(C)^* \) each make a strictly positive profit while the other suppliers each make zero profits. On the buyer’s side, the profit is nonnegative, which is a direct result of the submodularity property of \( \Pi_C^*(S) \) and can also be implied by the fact that the buyer will not purchase from any supplier should it make a negative profit.

The profit allocation is primarily driven by the level of competition between suppliers. In the extreme case where there is a perfectly competitive market, then each supplier’s contribution is zero, since in an individual supplier’s absence another supplier will step in with no overall reduction in supply chain profits. Thus the entire supply chain profit will go to the buyer.
Another property of the equilibrium is that the buyer’s profit will remain the same if any single supplier is dropped from the set of available suppliers. We establish this in the proof of Theorem 2 (see equation (A-3)). The intuition is that if the buyer makes a lower profit when any supplier is removed, then this supplier would have an incentive to increase its bid by a positive amount while still ensuring it is chosen by the buyer.

Finally, in equilibrium the buyer makes a choice that maximizes the supply chain profit, i.e., the supply chain is coordinated in equilibrium. It is instructive to relate our findings with those of the supply chain coordination literature (Cachon, 2003). In general, this literature focuses on designing sophisticated contracts (e.g., revenue sharing or buy back) to achieve the chain optimal profit in a dyadic supply chain. In contrast, we have a different supply chain structure (i.e., many to one), and the supply chain optimality arises as a result of supplier competition (rather than by way of design).

Example 1 (continued). Let us consider again the capacity game given in Example 1. If just one of the suppliers, say \( i \), is available, then it can be shown that the optimal reservation amount for the supply chain as a whole is \( t_i = 75 \) and the supply chain profit is $47.5. In this case, the amount supplied is less than the demand in the high demand case. On the other hand, if both suppliers are available, then it is optimal (to the supply chain) to reserve 50 from each, so \( t_1 = t_2 = 50 \). In this case no demand is lost, but the buyer finds it worthwhile to purchase from only one of the suppliers in the event that demand is low (thus saving the additional $20 cost in this case). The supply chain profit is $66.67. Thus the supply chain profits are submodular and there is an equilibrium where each supplier offers \( P_i(s) = s_i \) and \( R_i(t) = E_i(t) + 19.17 \) for \( t_i > 0 \) and \( R_i(t) = 0 \) otherwise, where \( i = 1, 2 \). □

5 Application to capacity games with function bids

We will apply our results for the general capacity game to the specific case we described in the introduction section. The buyer faces a random demand \( D \), which follows a probability distribution \( F(d) \) with density \( f(d) \) over \([d, \bar{d}]\) where \( 0 \leq d < \bar{d} < \infty \). For each unit of items sold, the buyer collects a revenue \( \rho \). Supplier \( i \) has a reservation cost, which is expressed as a general increasing function of the amount reserved, with a marginal reservation cost for an amount \( t \) given by \( e_i(t) \) (so that total reservation costs are given by \( \int_0^t e_i(s) ds \)). In addition supplier \( i \) has an execution cost which is a general increasing function of the amount supplied, with a marginal execution cost of \( c_i(t) \). We have switched to lower case letters here as an indication that these are marginal costs rather than the total costs as occur in the general capacity game. Note that \( c_i(t) \) and \( e_i(t) \) could be constants. Suppliers each maximize their own expected profits by offering a function bid consisting of a (marginal) reservation price function \( r_i(t) \) and a (marginal) execution price function \( p_i(t) \) for \( t \in [0, \tilde{t}] \). Without loss of generality, we assume \( \tilde{t} = \bar{d} \) since the buyer will never reserve more than \( \bar{d} \) units. In addition, if a supplier does not want to offer that much, he may simply set a very high price for the quantities beyond the desired amount so that the buyer will never choose to reserve more than the supplier wishes to offer.

Suppose the bids offered by suppliers are \( B = \{(p_i(\cdot), r_i(\cdot)) : i \in N\} \), where the functions \( p_i(\cdot) \) and \( r_i(\cdot) \) are defined on \([0, \bar{d}]\) and may be constants. The buyer makes
a reservation decision in the first stage and makes an execution decision in the second stage. Since the reservation payment becomes sunk at the time when the buyer makes the execution decision, with any realized demand, the buyer simply chooses the cheapest way to meet the demand based on execution prices up to the amount reserved from each supplier. If prices are continuous and less than \( \rho \), then it is not hard to see that the buyer will meet the demand \( D \) through choosing amounts \( x_i \) from supplier \( i \) where \( \sum_{i \in \mathcal{N}} x_i = D \) and the prices are all equal so that we may write \( p_i(x_i) = p(D) \), \( i \in \mathcal{N} \), except where \( x_j \) is the reserved capacity for \( j \), in which case \( p_j(x_j) \leq p(D) \). However in the general case, when prices can decrease, there may be more than one solution having these properties even with continuous prices. In this case the buyer will select the solution with the lowest total payment.

To avoid these complications and allow us to develop explicit expressions for \( \Pi_B \) and \( \Pi_C \), we make the following simplifying assumption.

**Assumption 4.** The execution price function \( p_i(\cdot) \) is non-decreasing for every \( i \in \mathcal{N} \).

In order to put this problem into the general form of a capacity game, we need to switch from marginal costs to the full costs given by their integrals. Also we note that for the capacity game with function bids, \( p_i, c_i, r_i \) and \( c_i \) only have dependence on \( t_i \) and \( s_i \) and not on the full vectors \( t \) and \( s \) (so that we need to capture the restriction that bids \( p \) and \( r \) are of this form through the specification of the sets \( A_i \)). Thus by setting \( V(D,s) = \rho \min(\sum_{i \in \mathcal{N}} s_i) \), \( P_i(s) = \int_0^{s_i} p_i(u) du \), \( C_i(s) = \int_0^{s_i} c_i(u) du \), \( R_i(t) = \int_0^{t_i} r_i(u) du \), \( E_i(t) = \int_0^{t_i} c_i(u) du \), we obtain the problem in the required form.

For the capacity game with function bids we can write the vector of second stage choices \( s \) explicitly in terms of \( t \) by recognising that the optimal choice for the buyer will use the low cost suppliers first. As we will demonstrate below, this allows an explicit expression for

\[
\Pi_B(t) := \max_{s \leq t} \Pi_B(t,s) = \max_{s \leq t} \mathbb{E}\left[ \rho \min(D, \sum_{i \in \mathcal{N}} s(D)_i) - \sum_{i \in \mathcal{N}} (P_i(s(D)_i) + R_i(t_i)) \right],
\]

where \( \mathcal{B} = \{ (P_i(\cdot), R_i(\cdot)), i \in \mathcal{N} \} \) and we write \( s(D)_i \) for the \( i \)th component of \( s(D) \). Similarly we define \( \Pi_C(t) := \max_{s \leq t} \Pi_C(t,s) \). Let

\[
\gamma_j(p) = \sup\{0 \leq y \leq t_j : p_j(y) \leq p\}. \tag{10}
\]

Then \( \gamma_j(p_i(x)) \) is the amount purchased from supplier \( j \neq i \) if an amount \( x \) is purchased from supplier \( i \). When \( p_j(t) \) is strictly increasing and continuous, then \( \gamma_j(p) = \min\{p_j^{-1}(p), t_j\} \). In the special case where marginal execution costs are constant (i.e., \( p_k(t) = p_k \) for all \( k \in \mathcal{N} \) where \( p_k \) is a constant), we have \( \gamma_j(p) = 0 \) if \( p_j > p \), and \( \gamma_j(p) = t_j \) if \( p_j \leq p \).

Let \( t_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \) denote the vector of reservation choices from suppliers other than \( i \). We now write the cumulative amount of capacity with execution prices less than \( p_i(x) \) as follows,

\[
h_i(x, t_{-i}) = x + \sum_{j \neq i} \gamma_j(p_i(x)), \quad x \in [0, t_i], \tag{11}
\]
Corollary 1. Given bids \( \{ (p_j(\cdot), r_j(\cdot)) : j \in N, j \neq i \} \) of the other suppliers, if supplier \( i \) has preferred status, then it is optimal for supplier \( i \) to set \( p_i(t) = c_i(t) \) for \( t \in [0, \hat{t}] \) and choose the function \( r_i(\cdot) \) such that

\[
\int_0^{\hat{t}_i} r_i(t)dt = \int_0^{\hat{t}_i} e_i(t)dt + \Pi_{\mathcal{B}_i}(N) - \Pi_{\mathcal{B}_i}(N\backslash \{i\}),
\]

where \( \mathcal{B}_i = \{ (p_j(\cdot), r_j(\cdot)) : j \in N, j \neq i \} \cup \{ (c_i(\cdot), e_i(\cdot)) \} \) and \( \hat{t} \in \arg \max \Pi_{\mathcal{B}}(t) \) provided \( r_i(\cdot) \) is chosen in such a way that the optimal buyer choice is \( \hat{t} \). An optimal choice for supplier \( i \) is to choose \( r_i(t) = e_i(t) \) for all \( t \in [0, \hat{d}] \) and in addition charge a fixed amount of \( \Pi_{\mathcal{B}_i}(N) - \Pi_{\mathcal{B}_i}(N\backslash \{i\}) \) for any non-zero amount reserved. In the case where supplier \( i \) does not have preferred status, then it is possible for supplier \( i \) to achieve within \( \epsilon \) of the best profit by choosing the same bid of \( p_i(\cdot) \) and a bid of \( r_i(\cdot) \) that is reduced by \( \epsilon \).

This result shows that for the capacity game with function bids there is an optimal solution with \( p_i(t) = c_i(t) \) for the supplier \( i \)'s best response problem, but there will be multiple possible solutions for \( r_i(t) \) such that the buyer’s optimal choice is \( \hat{t} \) and the supplier makes a profit of \( \Pi_{\mathcal{B}_i}(N) - \Pi_{\mathcal{B}_i}(N\backslash \{i\}) \) (which will imply that the buyer makes the same profit as \( \Pi_{\mathcal{B}_i}(N\backslash \{i\}) \)). In all of these solutions, the optimal buyer choice is \( \hat{t} \), which maximizes the buyer’s profit when supplier \( i \) charges only his costs.

For the capacity game with function bids we also have the following immediate corollary from Theorem 2.
Corollary 2 (Nash equilibrium with a lump sum payment). Suppose Assumptions \(^1, 3\) hold. Then there exists a Nash equilibrium \(\{(p^*_i(\cdot), r^*_i(\cdot), K^*_i) : i \in N\}\), where each supplier \(i\) sets prices to be costs, i.e., \(p^*_i(\cdot) = c_i(\cdot), r^*_i(\cdot) = e_i(\cdot)\), and charges a lump-sum reservation payment \(\Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})\). The buyer’s reservation choice is \(t^*_N\), the buyer makes a profit of \(\Pi^*_C(N) − \sum_{i=1}^n \Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})\), and supplier \(i\) makes a profit of \(\Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})\). □

One property of the equilibrium involving a lump sum payment is that it cannot be represented simply by defining marginal prices for capacity; to do so would require an infinite price for the first \(\varepsilon\) of capacity. However, we can use the functions \(\Delta_i(t)\) occurring in Theorem\(^2\) to construct an equilibrium solution with finite marginal costs. This is done by adjusting the capacity offer from the lump sum bid by smoothing out the beginning of the offer. We will make use of power functions to do this.

Proposition 1 (Nash equilibrium with power functions). Suppose Assumptions \(^1, 3\) hold. Further we assume that \(c_i(\cdot)\) and \(e_i(\cdot)\) are non-decreasing. Then the following strategies form a Nash equilibrium: For each \(i \in N\), \(p^*_i(\cdot) = c_i(\cdot)\) and

\[
r^*_i(t) = \begin{cases} 
  e_i(t) + \delta_i(t; \beta_i), & \text{if } 0 \leq t < \theta_i, \\
  e_i(t), & \text{if } \theta_i \leq t \leq \bar{d},
\end{cases}
\]

where

\[
\delta_i(t; \beta_i) = (\Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})) \frac{(\beta_i + 1)}{\theta_i} \left(\frac{\theta_i − t}{\theta_i}\right)^{\beta_i},
\]

\(\theta_i = \min((t^*_S)_i : S \subseteq N, S \ni i)\), and \(\beta_i > 1\) is a constant large enough. In equilibrium, the buyer’s reservation choice is \(t^*_N\). The buyer makes a profit of \(\Pi^*_C(N) − \sum_{i=1}^n (\Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})\) and supplier \(i\) makes a profit of \(\Pi^*_C(N) − \Pi^*_C(N \setminus \{i\})\).

This construction involves adding the margin \(\delta_i(t; \beta_i)\) to the reservation costs, which is a decreasing function of \(t \in (0, \theta_i)\). Choosing a high value for \(\beta_i\) will imply that \(\delta_i(t; \beta_i)\) decreases more steeply at the beginning and becomes flatter at the end of this interval.

From Proposition\(^1\) we see that there is a continuum of Nash equilibria with power functions, each associated with a possible set of \(\beta_i\) values. We can show that when \(\beta_i\) approaches positive infinity, the power function bid reduces to the lump sum bid. Thus, Corollary\(^2\) can be considered as a limiting example of Proposition\(^1\). Moreover, one may easily construct an equilibrium where some suppliers use lump sum bids (with infinitely large \(\beta\) values) while others use power function bids. Despite the multiplicity of equilibrium bidding strategies, all these equilibria lead to the same profit allocation and thus the equilibrium outcome characterized in Proposition\(^1\) is essentially unique.

Comparison with MS

As discussed earlier, existing models have focused on the case where the marginal costs are constant. As we will show in Section\(^6\) the supply chain optimal profit is submodular for this special case, and hence the equilibria characterized in Corollary\(^2\) and Proposition\(^1\) apply. It is interesting to draw a parallel between our model and the existing ones for this case. To this end, we draw on an example from MS to demonstrate the difference in equilibrium outcomes.
Example 2 (Example 1 in MS). The buyer’s demand is uniformly distributed over [0, 1], so \( F(t) = t \) for \( t \in [0, 1] \). There are two suppliers and their marginal costs are \((c_1, e_1) = (0, 60)\) and \((c_2, e_2) = (75, 5)\). The retail price is \( \rho = 100 \). We now examine the supply chain optimal problems.

If the buyer reserves from two suppliers, the supply chain problem is:

\[
\max_{t_1, t_2 \in [0, 1]} \left\{ \int_0^{t_1} ((\rho - c_1) \bar{F}(x) - e_1) \, dx + \int_0^{t_2} ((\rho - c_2) \bar{F}(x + t_1) - e_2) \, dx \right\}.
\]

The optimal solution is \( t^*_N = (4/15, 8/15) \). The supply chain optimal profit is \( \Pi^*_C(N) = 32/3 \).

If the buyer chooses supplier 1 only, the supply chain problem is:

\[
\max_{t_1 \in [0, 1]} \left\{ \int_0^{t_1} ((\rho - c_1) \bar{F}(x) - e_1) \, dx \right\}.
\]

The optimal solution is \( t^*_1 = (2/5) \) and the supply chain optimal profit is \( \Pi^*_C(\{1\}) = 8 \).

If the buyer chooses supplier 2 only, the supply chain problem is:

\[
\max_{t_2 \in [0, 1]} \left\{ \int_0^{t_2} ((\rho - c_2) \bar{F}(x) - e_2) \, dx \right\}.
\]

The optimal solution is \( t^*_2 = (4/5) \) and the supply chain optimal profit is \( \Pi^*_C(\{2\}) = 8 \).

If suppliers are restricted to each offering a pair of reservation price and execution price, in equilibrium these two suppliers bid infinitesimally close to each other. MS show that the following is a continuum of \( \epsilon \)-equilibria, which are parameterized with \( p \in [50, 75] \):

\[
(p^*_1, r^*_1) = (p^*_2, r^*_2) = \left( p, 60 - \frac{55}{75} p \right).
\]

In equilibria, the buyer’s reservation choice is

\[
t^* = \left( \frac{4}{15}, \frac{40}{3(100 - p)} \right).
\]

The profit split amongst players is given by

\[
\Pi^*_B = \frac{8(150 - p)^2}{225(100 - p)}; \quad \pi^*_1 = \frac{8p}{225}; \quad \pi^*_2 = \frac{800(75 - p)}{9(100 - p)^2}; \tag{15}
\]

and the supply chain profit is given by

\[
\Pi^*_C = \frac{32(225 - 2p)}{9(100 - p)} + \frac{800(75 - p)}{9(100 - p)^2}.
\]

Note that all the above equilibria are inefficient (i.e., not supply chain optimal) except the one with \( p = 75 \). At this efficient equilibrium, each supplier offers a bid \((75, 5)\) which is identical to the supplier 2’s cost. The supplier 2’s profit is 0, the supplier 1’s profit is \(8/3\), and the buyer’s profit equals 8.
We now demonstrate that the above strategies do not form an equilibrium if we allow suppliers to offer function bids. Suppose supplier 1 chooses the proposed bid \((p_1^*, r_1^*) = (p, 60 - (55/75)p)\), and we now examine supplier 2’s best response in choosing a function bid.

First, if supplier 1 is the sole supplier, the buyer’s reserved amount will be the sum of the two components of \(t^*\), and the buyer’s profit is equal to \(\Pi_B^*\). Therefore, we have

\[
t^*_1 = \frac{600 - 4p}{15(100 - p)} \quad \text{and} \quad \Pi_B^*(\{1\}) = \frac{8(150 - p)^2}{225(100 - p)}.
\]

Second, we show that the following strategy for supplier 2 improves his profit: setting prices to be costs and charging a lump-sum payment of \(\frac{32(75 - p)}{9(100 - p)}\). Given this offer (and the supplier 1’s offer \((p_1^*, r_1^*)\)), we can show that the solution for the buyer’s problem is

\[
\hat{t} = \left(\frac{4}{15}, \frac{8}{15}\right),
\]

and the buyer’s profit from choosing \(\hat{t}\) is equal to \(\Pi_B^*(\{1\})\). Also if the buyer purchases from only supplier 2, the buyer makes a profit of \(\Pi_B^*(\{1\})\) as well. According to the tie-breaking rule, the buyer will select \(\hat{t}\). Then supplier 2’s profit becomes

\[
\tilde{\pi}_2 = \frac{32(75 - p)}{9(100 - p)},
\]

which is strictly greater than \(\pi_2^*\) for any \(p < 75\). This shows that the equilibria in MS do not hold if we allow suppliers to offer a function bid.

Under our model setup, the problem is well behaved and there is an equilibrium with lump-sum payments (and there is also an equilibrium with power functions). The following bids form a Nash equilibrium (where \(K_1\) and \(K_2\) are lump sum payments):

\[
(p_1^*, r_1^*, K_1) = \left(0, 60, \frac{8}{3}\right) \quad \text{and} \quad (p_2^*, r_2^*, K_2) = \left(75, 5, \frac{8}{3}\right).
\]

At this equilibria, the buyer’s optimal reservation choice is \(t^* = (4/15, 8/15)\). The profit split amongst players is given by

\[
\Pi_B^* = \frac{16}{3} \quad \text{and} \quad \pi_1^* = \pi_2^* = \frac{8}{3}.
\]

In this equilibria, each supplier’s profit equals his contribution to the supply chain system and the buyer takes the remaining profit. Moreover, the reservation choice by the buyer is supply chain optimal (in distinction to the case with constant prices).

The key message of Example 2 is that imposing the restriction that each supplier submits a pair of reservation price and an execution price rather than a function bid leads to a higher buyer profit. On the other hand, the suppliers are better off if they submit function bids. This can be easily seen by comparing the profit splits at (15) and (16). \(\square\)
Comparison with ACS

In our setting, each supplier offers a function bid with a marginal reservation price function and a marginal execution price function, and the buyer has the freedom to reserve whatever amount she likes. This setting resembles the two extensions to the setting with blocks of capacity discussed in Anderson et al. (2017). The two extensions they discussed were:

(a) **Partial Reservation**, in which the buyer is not restricted to reserving a whole block. Each supplier owns a single block and the blocks can be of different sizes. Every supplier chooses an execution price and a reservation price that apply to all elements of his block.

(b) **Multiple Blocks with Common Owner**, in which each supplier owns multiple unit-blocks and can choose different prices for different unit blocks. The buyer can freely choose among the offered blocks.

In both extensions, the authors demonstrate that one of their key results in the baseline model, \( p_i = c_i \) for supplier \( i \)'s best response, does not hold. In contrast, we show that the result \( p_i = c_i \) holds in our setting as shown in Corollary 1.

The key differences are as follows. In the case (a) of partial reservation, the buyer can freely reserve any amount as in our setting. However, suppliers are restricted to offering just two scalar prices rather than general function bids. In the case (b) of multiple blocks with common owner, each supplier can choose different prices for different blocks of capacity as in our setting. However, the buyer can freely choose any of the blocks, whereas in our function bidding setting, the buyer does not have as much flexibility, because the buyer will need to accept the prices for the earlier blocks in order to enjoy the prices for the next blocks. These differences in results highlight the importance of each player’s choice flexibility on the equilibrium outcomes.

6 Discussion of submodularity assumption

In this section we examine two commonly studied settings in supply chains: (a) there are only two suppliers competing for the buyer’s procurement business, and (b) there are more than two suppliers and the marginal execution costs are constants. In each case we are interested in establishing conditions that will ensure submodularity of \( \Pi^*_C(S) \).

6.1 Case with two suppliers

When there are only two suppliers (denoted by 1 and 2) we can find conditions that are enough to ensure that submodularity occurs, and it turns out that in the special case of the capacity game with function bids submodularity will hold without any further restrictions.

**Proposition 2** (Subadditivity). If \( V(D,s) \) is subadditive in \( s \) and the cost functions are defined separately on each component, so \( C_i(s) \) is determined by the \( i \)-th component of \( s \), and \( E_i(t) \) is determined by the \( i \)-th component of \( t \), then \( \Pi^*_C(S) \) is submodular in \( S \subseteq N \). In the case of the capacity game with function bids, these conditions will hold.
6.2 Case with constant marginal execution costs

In the case of more than two suppliers, the submodularity of supply chain profit may not hold in general, as we show in Example 4 below. The following theorem identifies some sufficient conditions for the submodularity property and these conditions are satisfied for a large class of the capacity games with function bids. The notion of laminar convexity stated in the theorem is defined in the Appendix.

**Theorem 3** (Submodularity). Let $\mathbf{T} = [0, \bar{d}]^n$ and $\mathbf{X}(\mathbf{t}) = \{\mathbf{s} \in \mathbb{R}_+^n : \mathbf{s} \leq \mathbf{t}\}$ for $\mathbf{t} \in \mathbf{T}$. If $\max_{\mathbf{s} \in \mathbf{X}(\mathbf{t})} (V(D, \mathbf{s}) - \sum_{i=1}^n C_i(\mathbf{s}))$ is laminar concave in $\mathbf{t}$ and each cost function $E_i(\mathbf{t})$ ($i \in N$) is a convex function of (only) the $i$-th component of $\mathbf{t}$, then $\Pi^*_C(S)$ is submodular in $S \subseteq N$. In the case of the capacity game with function bids, these conditions are satisfied if for each supplier the marginal execution cost is constant and the marginal reservation cost is non-decreasing.

The complexity of the analysis arises from the fact that the supply chain profit function cannot be easily decomposed into components related to individual suppliers. The profit made from supplier $i$ is related to the capacity reservations from all the suppliers with lower execution prices.

Theorem 3 implies that, in settings considered by the existing studies where the marginal costs are constant, the supply chain optimal profit is submodular. We use the following example to illustrate this point.

**Example 3.** Suppose the buyer demand $D$ follows a uniform distribution over $[0, 1]$. There are three suppliers whose costs are: $c_1 = 1, e_1 = 3; c_2 = 2.5, e_2 = 2; \text{ and } c_3 = 5, e_3 = 1$. The retail price is $\rho = 10$. We know from Theorem 3 that the problem is submodular, and we can carry out the detailed calculations to find the supply chain optimal solutions for different sets of available suppliers.

Using the results in the table, we can easily check the submodularity of $\Pi^*_C(S)$. We can now construct an equilibrium set of offers where the suppliers offer at cost and in addition require a lump sum reservation payment of $\Pi^*_C(N) - \Pi^*_C(N \setminus \{i\})$, which then becomes the supplier profit. Here these amounts are 0.08333 for supplier 1, 0.0333 for...
supplier 2, and 0.0333 for supplier 3. In this equilibrium the buyer receives the remainder of the total supply chain profit: \(2.1333 - 0.15 = 1.9833\). □

The submodularity property may not hold when suppliers have decreasing marginal costs as we demonstrate with the following example.

**Example 4.** Suppose the demand is fixed with \(D = 10\) and the retail price is \(\rho = 20\). There are three suppliers with \(N = \{1, 2, 3\}\). Supplier 1 and supplier 2 have the same costs with \(c_1(t) = c_2(t) = e_1(t) = e_2(t) = 0\), for \(t \in [0, 5]\) (and an infinite cost for any larger amount). Supplier 3’s costs are \(c_3(t) = 0\) and \(e_3(t) = 10 - t\) for \(t \in [0, 10]\). So both supplier 1 and supplier 2 have the capacity of 5 and supplier 3’s capacity is 10.

We now look at the supply chain optimal problems. If all the three suppliers are available, the buyer will choose 5 units from each of supplier 1 and supplier 2. The supply chain optimal profit is \(\Pi(\{1, 2, 3\}) = 200\). If only suppliers 3 and 1 (or 2) are available, the buyer will choose 5 units from each of 3 and 1 (or 2). The supply chain optimal profit is \(\Pi(\{1, 3\}) = \Pi(\{2, 3\}) = 162.5\). If supplier 3 is the sole supplier, the buyer will choose 10 units from supplier 3 and the supply chain optimal profit is \(\Pi(\{3\}) = 150\). Therefore, we have \(\Pi(\{1, 2, 3\}) + \Pi(\{3\}) = 350 > 325 = \Pi(\{1, 3\}) + \Pi(\{2, 3\})\), which contradicts the submodularity property.

We can also show that the proposed equilibrium structure will not apply in this case. If each of suppliers 1 and 2 asks for a lump-sum payment of \(200 - 162.5 = 37.5\), and the buyer makes the supply chain optimal choice of selecting these two suppliers, then the buyer profit is \(200 - 75 = 125\). However, this is less than the profit available to the buyer from selecting supplier 3 alone, which gives the buyer 150 as profit. So this is not an equilibrium. We can check that an equilibrium exists where both suppliers 1 and 2 ask for a lump-sum payment of 25, and the buyer chooses both of these offers. □

### 7 Discussion and conclusions

In this paper, we have developed a general framework to study a broad class of capacity games. This framework allows us to examine supplier competition in an option market where suppliers’ costs may be nonlinear. In this setting, suppliers each submit a function bid consisting of a reservation price function and an execution price function. The buyer decides how much capacity to reserve from each supplier before knowing the actual demand.

When the competitors’ bids are observed, we have shown that an optimal strategy for each supplier is to set the execution price to be the execution cost and add a margin on the reservation cost. This implies that suppliers make profits only from the buyer’s reservation payments. This result does not hold in the case of bids of constant marginal costs (considered by MS).

We have also shown that, under the assumption that the supply chain optimal profit is submodular in the set of available suppliers, there is a class of equilibria in which the buyer’s reservation choice is first best, each supplier’s profit equals his marginal contribution to the supply chain and the buyer takes the remaining profit. The implication is that by allowing suppliers to compete using function bids, the supply chain is coordinated.

To demonstrate the generality of the submodularity assumption, we have shown that the supply chain optimal profit is indeed submodular when each supplier’s marginal
execution cost is constant, a setting that has been studied extensively in the existing literature, or when there are only two suppliers.

From an electricity market perspective, the profits for the buyer correspond to the overall consumer welfare. In this context the capacity mechanism we have described combines the capacity auction and the market for energy into a single pay-as-bid auction, rather than having separate uniform price auctions. This achieves an efficient outcome at equilibrium, even in the case where generators can exercise market power. In the special case of a competitive environment when no single generator has a significant impact on overall system welfare when removed, we have found that each generator bids at cost and we retain the property of an efficient set of capacity and generation choices. The result we obtain on best response is interesting from the perspective of electricity capacity mechanisms since it demonstrates that when generators are required to specify their energy bids in advance with pay-as-bid in the spot, then there is no incentive to use market power in the energy component of the bids, with profits being made entirely from the capacity payments. This result applies without an assumption of monotone prices and allows quite general dispatch mechanisms. This has relevance to the issues of uplift payments that occur in US wholesale markets because of no-load and start-up costs (see, for example, Hogan 2014).

This paper can be extended in several directions. First we assume, as in other supplier competition models, that supplier costs are known to the other suppliers. This assumption fits some settings better than others. For example, in the electricity market, generators tend to know each other’s generation technologies; thus it is prudent to assume complete cost information. However, in other settings a model that considers cost uncertainties may be more appropriate; see supply function equilibrium models with private information (Vives 2011). In addition, our model focuses on the contract market only, and incorporation of a spot market would be another interesting direction. Our equilibrium analysis builds on the submodularity condition. Even though in Example 4 we demonstrate that, when the submodularity property fails, the VCG strategies are not a Nash equilibrium, it would be interesting to investigate further what the equilibrium looks like when the supply chain optimal profit is not submodular. Finally, we should note that our assumption is that the random demand is exogenous. An important extension that we do not consider is the case where the buyer can influence demand through setting a price.

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Appendix

In this section, we prove the results derived in this paper and also give some basic definitions from Murota (2003) for Section 6.

A.1 Some definitions

The set of real numbers is denoted by $\mathbb{R}$, and $\bar{\mathbb{R}} = \mathbb{R} \cup \{ +\infty \}$ and $\underline{\mathbb{R}} = \mathbb{R} \cup \{- \infty \}$. The set of integers is denoted by $\mathbb{Z}$, and $\bar{\mathbb{Z}} = \mathbb{Z} \cup \{ +\infty \}$ and $\underline{\mathbb{Z}} = \mathbb{Z} \cup \{- \infty \}$. We use $\mathbb{D}$ to denote either $\mathbb{Z}$ or $\mathbb{R}$. Denote $[n] = \{1, \ldots, n\}$ for any positive number $n$. The characteristic vector of $S \subseteq [n]$ is denoted by $\chi_S \in \{0, 1\}^n$. For $i \in [n]$, we write $\chi_i$ for $\chi_{\{i\}}$, which is the $i$th unit vector, and $\chi_0 = \mathbf{0}$ (zero vector).

\textbf{M$^2$-convexity}

For a function $f: \mathbb{D}^n \to \bar{\mathbb{R}} \cup \{-\infty, +\infty\}$, the set 
\[ \text{dom}_f = \{ x \in \mathbb{D}^n : f(x) \in \mathbb{R} \} \]
is called the \textit{effective domain} of $f$. For a vector $z \in \mathbb{R}^n$, define the \textit{positive} and \textit{negative supports} of $z$ as 
\[ \text{supp}^+(z) = \{ i \in [n] : z_i > 0 \}, \quad \text{supp}^-(z) = \{ i \in [n] : z_i < 0 \} . \]

A function $f: \mathbb{Z}^n \to \bar{\mathbb{R}}$ is said \textit{M$^2$-convex} if for any $x, y \in \text{dom}_f$ and any $i \in \text{supp}^+(x-y)$, there exists $j \in \text{supp}^-(x-y) \cup \{0\}$ such that the following \textit{exchange property} is satisfied:
\[ f(x) + f(y) \geq f(x - \chi_i + \chi_j) + f(y + \chi_i - \chi_j) . \]

Similarly, a function $f: \mathbb{R}^n \to \bar{\mathbb{R}}$ is said \textit{M$^2$-convex} if for any $x, y \in \text{dom}_f$ and any $i \in \text{supp}^+(x-y)$, there exist $j \in \text{supp}^-(x-y) \cup \{0\}$ and $\lambda_0 > 0$ such that
\[ f(x) + f(y) \geq f(x - \lambda(\chi_i - \chi_j)) + f(y + \lambda(\chi_i - \chi_j)) \]
for all $\lambda \in \mathbb{R}^n$ with $0 \leq \lambda \leq \lambda_0$. A function $f: \mathbb{D}^n \to \bar{\mathbb{R}}$ is said \textit{M$^2$-concave} if $(-f)$ is M$^2$-convex.

\textbf{Laminar convexity}

A non-empty set $\mathcal{L} \subseteq 2^{[n]}$ is called a \textit{laminar family} if for any $A, B \in \mathcal{L}$, we have $A \cap B = \emptyset$, or $A \subseteq B$, or $B \subseteq A$. A function $f: \mathbb{D}^n \to \bar{\mathbb{R}}$ is said \textit{laminar convex} if it can be represented as 
\[ f(x) = \sum_{S \in \mathcal{L}} f_S(x(S)), \]
where $\{f_S\}$ are univariate convex functions, $\mathcal{L}$ is a laminar family, and $x(S) = \sum_{i \in S} x_i$. A function $f: \mathbb{D}^n \to \bar{\mathbb{R}}$ is said \textit{laminar concave} if $(-f)$ is laminar convex.

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A.2 Proof of Theorem 1

(a) We consider any feasible offer \((\tilde{P}_i(s), \tilde{R}_i(t))\) from supplier \(i\), giving a set of bids

\[ \tilde{B}_i = \{(P_j(s), R_j(t)) : j \in N, j \neq i\} \cup \{(\tilde{P}_i(s), \tilde{R}_i(t))\}. \]

The buyer can obtain a profit of \(\Pi^*_B(N \setminus \{i\})\) through restricting consideration to choices with \(t_i = 0\) (which also implies \(s_i = 0\)), and so we have a lower bound on buyer profit:

\[ \Pi^*_B(N) \geq \Pi^*_B(N \setminus \{i\}). \] (A-1)

Taking \((\tilde{t}, \tilde{s})\) as an optimal choice by the buyer given bids \(\tilde{B}_i\), we have

\[ \Pi^*_B(N) = \mathbb{E} \left[ V(D, \tilde{s}(D)) - \sum_{j \neq i} (P_j(\tilde{s}(D)) + R_j(\tilde{t})) - (\tilde{P}_i(\tilde{s}(D)) + \tilde{R}_i(\tilde{t})) \right]. \]

Hence

\[ \mathbb{E} \left[ \tilde{R}_i(\tilde{t}) - E_i(\tilde{t}) + \tilde{P}_i(\tilde{s}(D)) - C_i(\tilde{s}(D)) \right] = \mathbb{E} \left[ V(D, \tilde{s}(D)) - \sum_{j \neq i} (P_j(\tilde{s}(D)) + R_j(\tilde{t})) - E_i(\tilde{t}) - C_i(\tilde{s}(D)) \right] - \Pi^*_B(N) \]

Using \(\Pi^*_B(\tilde{t}, \tilde{s}) \leq \Pi^*_B(N)\) and (A-1), with a complete set of bids \(\tilde{B}_i\), we can calculate the profit for supplier \(i\) as follows:

\[ \mathbb{E} \left[ \tilde{R}_i(\tilde{t}) - E_i(\tilde{t}) + \tilde{P}_i(\tilde{s}(D)) - C_i(\tilde{s}(D)) \right] = \Pi^*_B(N) - \Pi^*_B(N \setminus \{i\}) = Z_i, \]

as required.

(b) Since \(Z_i > 0\), we know that \(\Pi^*_B(N) > \Pi^*_B(N \setminus \{i\})\), which implies that the optimal choice \((\tilde{t}, \tilde{s})\) for the buyer given bids \(\tilde{B}_i\) has \(t_i > 0\), otherwise these two would be the same. We need to show that when the offer of \((\tilde{P}_i(\cdot), \tilde{R}_i(\cdot))\) is made the buyer will choose supplier \(i\) (i.e., have \(t_i > 0\)), which will then automatically give a profit of \(Z_i\) for supplier \(i\) from (A-1). We will show that \((\tilde{t}, \tilde{s})\) is optimal for the buyer given bids

\[ \tilde{B}_i = \{(P_j(s), R_j(t)) : j \in N, j \neq i\} \cup \{(\tilde{P}_i(s), \tilde{R}_i(t))\}, \]

and this suffices since \(i\) is preferred. Now by definition

\[ \Pi^*_B(N) = \max_{t \in T} \mathbb{E} \left[ V(D, s(D)) - \sum_{j \neq i} (P_j(s(D)) + R_j(t)) - (\tilde{P}_i(s(D)) + \tilde{R}_i(t)) \right]. \]

Now

\[ \Pi^*_B(\tilde{t}, \tilde{s}) = \Pi^*_B(\tilde{t}, \tilde{s}) - Z_i = \Pi^*_B(N) - Z_i, \]
since $\bar{t}_i > 0$. Now consider an arbitrary choice for the buyer $(t, s)$ with $t_i > 0$. This has

$$\Pi_{B_i}(t, s) = \Pi_{B_i}(t, s) - Z_i \leq \Pi_{\tilde{B}_i}^*(N) - Z_i,$$

and, moreover, any choice $(t, s)$ with $t_i = 0$ has

$$\Pi_{B_i}(t, s) = \Pi_{B_i}(t, s) \leq \Pi_{\tilde{B}_i}(N\{i\}) = \Pi_{\tilde{B}_i}^*(N) - Z_i.$$

Thus we have shown the optimality we needed.

(c) In this case with the bid $(C_i(s), \bar{R}_i(t) - \varepsilon)$ the profit for supplier $i$ is $Z_i - \varepsilon$ provided $t_i > 0$. Given this offer, the buyer by choosing $(\bar{t}, \bar{s})$, defined in part (b), will achieve a profit of $\Pi_{\tilde{B}_i}^*(N) - Z_i + \varepsilon = \Pi_{\tilde{B}_i}^*(N\{i\}) + \varepsilon$, which is therefore greater than any buyer profit available when $t_i = 0$. Hence the buyer’s optimal choice must have $t_i > 0$ and this completes the proof.

A.3 Proof of Lemma 1

We prove the lemma by induction. It is trivial when $|N \setminus S| \leq 1$, so we begin with $|N \setminus S| = 2$, and we suppose that $N = S \cup \{j, k\}$. From (6), we obtain

$$\Pi_i^c(N) + \Pi_i^c(N\{j, k\}) \leq \Pi_i^c(N\{j\}) + \Pi_i^c(N\{k\}),$$

which can be rearranged to obtain the result required. Suppose (7) holds for any $S$ with $|N \setminus S| = k$. Then for $j \in S$, we have

$$\sum_{i \in N \setminus (S \cup \{j\})} (\Pi_i^c(N) - \Pi_i^c(N\{i\}))$$

$$= \sum_{i \in N \setminus S} (\Pi_i^c(N) - \Pi_i^c(N\{i\})) + \Pi_i^c(N) - \Pi_i^c(N\{j\})$$

$$\leq \Pi_i^c(N) - \Pi_i^c(S) + \Pi_i^c(N) - \Pi_i^c(N\{j\})$$

$$\leq 2\Pi_i^c(N) - (\Pi_i^c(N) + \Pi_i^c(S\{j\})) = \Pi_i^c(N) - \Pi_i^c(S\{j\}),$$

where the first inequality follows from the inductive hypothesis and the second inequality follows from the submodularity property as assumed. Hence, we establish that the result holds for $|N \setminus S| = k + 1$, which completes the proof.

A.4 Proof of Lemma 2

We prove the lemma by contradiction. Suppose otherwise and there exists $j \in N^s(C)$ and $j \neq i$ such that $j \notin N_{*i}(C)$. By definition, $\Pi_i^c(N\{i\}) = \Pi_i^c(N\{i, j\})$. Moreover, inequality (6) implies $\Pi_i^c(N\{i\}) + \Pi_i^c(N\{j\}) \geq \Pi_i^c(N\{i, j\}) + \Pi_i^c(N)$. Thus, $\Pi_i^c(N\{j\}) \geq \Pi_i^c(N)$, which contradicts the fact that $j \in N^s(C)$.

A.5 Proof of Theorem 2

We establish that with these bids the profit for supplier $i$ is $\Pi_i^c(N) - \Pi_i^c(N\{i\})$, and then that given the bids of the other players, no improvement on this is possible
for supplier $i$. We consider two cases. First suppose that $\Pi^*_C(N) - \Pi^*_C(N \setminus \{i\}) = 0$ then $i \notin N^*(C)$ and with bids $\vec{B}$ supplier $i$ makes zero profit. In the second case $i \in N^*(C)$ and $\Pi^*_C(N) > \Pi^*_C(N \setminus \{i\})$. This implies that any optimal solution for $\Pi^*_C(N)$ must have $t_i > 0$.

Next we show that the buyer has an optimal choice $(t_N^*, s_N^*)$ when facing bids $\vec{B}$. Now for any feasible $(t, s)$,

$$
\Pi_B(t, s) = E[V(D, s(D)) - \sum_{i \in N^*(C)} (P_i(s(D)) + R_i(t))
- \sum_{i \notin N^*(C)} (C_i(s(D)) + E_i(t))]
= E[V(D, s(D)) - \sum_{j \in N^*(C), t_j > 0} (\Pi^*_C(N) - \Pi^*_C(N \setminus \{j\}) - \Delta_j(t))
- \sum_{i \in N}(C_i(s(D)) + E_i(t))]
= E[V(D, s(D)) + \sum_{j \in N^*(C), t_j > 0} \Delta_j(t) - \sum_{i \in N}(C_i(s(D)) + E_i(t))]
- \sum_{j \in N}(\Pi^*_C(N) - \Pi^*_C(N \setminus \{j\}) + \Pi^*_C(N) - \Pi^*_C(I(t))],
$$

where the inequality follows from Lemma 1 and we have used $\Pi^*_C(N) - \Pi^*_C(N \setminus \{j\}) = 0$ for $j \notin N^*(C)$. Since $\Delta_j(t) \geq 0$, using (9) we obtain

$$
\sum_{j \in N^*(C), t_j > 0} \Delta_j(t) \leq \sum_{j \in I(t)} \Delta_j(t) \leq \Pi^*_C(I(t)) - \Pi_C(t, s)
= \Pi^*_C(I(t)) - E[V(D, s(D)) - \sum_{i \in N}(C_i(s(D)) + E_i(t))].
$$

Substitution into the inequality for $\Pi_B(t, s)$ leads to

$$
\Pi_B(t, s) \leq \Pi^*_C(N) - \sum_{j \in N}(\Pi^*_C(N) - \Pi^*_C(N \setminus \{j\})). \quad (A-2)
$$

We will show that with the choice $(t_N^*, s_N^*)$ the buyer can achieve this bound. Since
\[ \sum_{j \in N} \Delta_j(t^*_N) = 0, \] we have
\[ \Pi_B(t^*_N, s^*_N) = E[V(D, s^*_N(D)) - \sum_{j \in N} (\bar{P}_j(s^*_N(D)) + \bar{R}_j(t^*_N))] \]
\[ = E[V(D, s^*_N(D)) - \sum_{j \in N} (C_j(s^*_N(D)) + E_j(t^*_N))] \]
\[ - \sum_{j \in N^*(C)} (\Pi^*_C(N) - \Pi^*_C(N\{j\})) \]
\[ = \Pi^*_C(N) - \sum_{j \in N} (\Pi^*_C(N) - \Pi^*_C(N\{j\})) \].

Here we have used the fact that \( N^*(C) = I(t^*_N) \) and also that \( \Pi^*_C(N) - \Pi^*_C(N\{j\}) = 0 \) when \( j \notin N^*(C) \). Hence we have established that \( (t^*_N, s^*_N) \) is an optimal choice for the buyer. By Assumption the buyer choice \( (t^*_N, s^*_N) \) that achieves supply chain optimality is preferred and hence we have \( t_i > 0 \), from which it follows that the profit for supplier \( i \) when bids are \( \bar{B} \) is \( \Pi^*_c(N) - \Pi^*_c(N\{i\}) \). Now we evaluate \( \Pi^*_B(N\{i\}) \). For any feasible \((t, s)\), the bound still applies and hence
\[ \Pi_B(t, s) \leq \sum_{j \in N} \Pi^*_c(N\{j\}) - (|N| - 1)\Pi^*_c(N). \]

Since \( \sum_{j \in N\{i\}} \Delta_j(t^*_N) = 0 \) and each \( \Delta_j \) is non-negative we have \( \Delta_j(t^*_N) = 0 \) for each \( j \in N\{i\} \). Thus
\[ \Pi_B(t^*_N, s^*_N) = E[V(D, s^*_N(D)) - \sum_{j \in N^*(C)} (\bar{P}_j(s^*_N(D)) + \bar{R}_j(t^*_N))] \]
\[ - \sum_{j \notin N^*(C)} (C_j(s^*_N(D)) + E_j(t^*_N))] \]
\[ = E[V(D, s^*_N(D)) - \sum_{j \in N} (C_j(s^*_N(D)) + E_j(t^*_N))] \]
\[ - \sum_{j \in I(t^*_N\{i\}) \cap N^*(C)} (\Pi^*_C(N) - \Pi^*_C(N\{j\})) \]
\[ = \Pi^*_C(N\{i\}) - \sum_{j \in N^*(C) \setminus \{i\}} \Pi^*_C(N\{j\}) \],

where the last equality is based on Lemma Hence, as \( \Pi^*_C(N) = \Pi^*_C(N\{j\}) \) for \( j \notin N^*(C) \), we have
\[ \Pi_B(t^*_N, s^*_N) = \Pi^*_C(N\{i\}) - \sum_{j \in N, j \neq i} (\Pi^*_C(N) - \Pi^*_C(N\{j\})) \]
\[ = \sum_{j \in N} \Pi^*_C(N\{j\}) - (|N| - 1)\Pi^*_C(N), \]

and so \( (t^*_N, s^*_N) \) is also optimal for the buyer faced with bids \( \bar{B} \). Thus
\[ \Pi_B(N\{i\}) = \Pi^*_B(N) = \sum_{j \in N} \Pi^*_C(N\{j\}) - (|N| - 1)\Pi^*_C(N), \]

(A-3)
and there is no loss to the buyer from a restriction that one of the $t_i$ values is zero.

Now suppose that there is a different offer $(\tilde{P}_i(s), \tilde{R}_i(t))$ by one of the suppliers $i \in N^*(C)$, giving a set of bids

$$\tilde{B}_i = \{(\tilde{P}_j(s), \tilde{R}_j(t)) : i \neq j \in N^*(C)\} \cup \{(\tilde{P}_i(s), \tilde{R}_i(t))\} \cup \{(C_j(s), E_j(t)) : j \notin N^*(C)\}.$$ 

The buyer can obtain a profit of $\Pi_{\tilde{g}}(N\setminus\{i\})$ through restricting consideration to choices with $t_i = 0$. This follows because $\tilde{B}_i(t) = 0$ when $t_i = 0$. We can write this lower bound on buyer profit as

$$\Pi_{\tilde{g}}(N) \geq \Pi_{\tilde{g}}(N\setminus\{i\}).$$

Suppose that $(\hat{t}, \hat{s})$ is an optimal choice by the buyer given bids $\tilde{B}_i$, so

$$\Pi_{\tilde{g}}^*(N) = \mathbb{E}[V(D, \hat{s}(D))] - \sum_{j \notin N^*(C)\setminus\{i\}} (\tilde{P}_j(\hat{s}(D)) - \tilde{R}_j(\hat{t})) - (\tilde{P}_i(\hat{s}(D)) - \tilde{R}_i(\hat{t})) - \sum_{j \notin N^*(C)} (C_j(\hat{s}(D)) - E_j(\hat{t})).$$

Given bids $\tilde{B}_i$, the profit for supplier $i$ is

$$\mathbb{E}[(\tilde{P}_i(\hat{s}(D)) + \tilde{R}_i(\hat{t}) - (C_i(\hat{s}(D)) + E_i(\hat{t})))$$

$$= \mathbb{E}[V(D, \hat{s}(D))] - \sum_{j \notin N^*(C)\setminus\{i\}} (\tilde{P}_j(\hat{s}(D)) + \tilde{R}_j(\hat{t}))$$

$$- \sum_{j \notin N^*(C)} (C_j(\hat{s}(D)) + E_j(\hat{t})) - \Pi_{\tilde{g}}^*(N) - (C_i(\hat{s}(D)) + E_i(\hat{t}))$$

$$= \Pi_C(\hat{t}, \hat{s}) - \sum_{j \notin N^*(C)\setminus\{i\}, t_j > 0} (\Pi_C^*(N) - \Pi_C^*(N\setminus\{j\}) - \Delta_j(\hat{t})) - \Pi_{\tilde{g}}^*(N).$$

Since $\Delta_j(\hat{t}) \geq 0$ and $\Pi_{\tilde{g}}^*(N) \geq \Pi_{\tilde{g}}^*(N\setminus\{i\})$, the above quantity is at most

$$\Pi_C(\hat{t}, \hat{s}) + \sum_{j \in I(\hat{t})} \Delta_j(\hat{t}) - \sum_{j \notin N^*(C)\setminus\{i\}} \sum_{t_j > 0} (\Pi_C^*(N) - \Pi_C^*(N\setminus\{j\})) - \Pi_{\tilde{g}}^*(N\setminus\{i\})$$

$$\leq \Pi_C^*(I(\hat{t})) - \sum_{j \in N, j \neq i} (\Pi_C^*(N) - \Pi_C^*(N\setminus\{j\})) + \sum_{j \notin I(\hat{t}), j \neq i} (\Pi_C^*(N) - \Pi_C^*(N\setminus\{j\})) - \Pi_{\tilde{g}}^*(N\setminus\{i\})$$

$$\leq \Pi_C^*(I(\hat{t})) - \sum_{j \in N} (\Pi_C^*(N) - \Pi_C^*(N\setminus\{j\})) + \Pi_C^*(N) - \Pi_C^*(I(\hat{t})) - \Pi_{\tilde{g}}^*(N\setminus\{i\})$$

$$= \Pi_C^*(N) - (|N| - 1)\Pi_C^*(N) + \sum_{j \neq i} \Pi_C^*(N\setminus\{i\}) - \Pi_{\tilde{g}}^*(N\setminus\{i\}),$$

where the first inequality is from the fact that functions $\{\Delta_i(t) : i \in N\}$ are consistent with the costs, while the second inequality is from Lemma 1. We can use (A-3) and cancel terms to obtain

$$\mathbb{E}[(\tilde{P}_i(\hat{s}(D)) + \tilde{R}_i(\hat{t}) - (C_i(\hat{s}(D)) + E_i(\hat{t}))) \leq \Pi_C^*(N) - \Pi_C^*(N\setminus\{i\}),$$

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The final step is to consider an offer \((\tilde{P}_i(s), \tilde{R}_i(t))\) by one of the suppliers \(i \notin N^*(C)\). For these suppliers there is no profit under the set of bids \(\mathcal{B}\). We will show that there is no chance to make a profit with a different bid. It is clear that any choice that has \(t_i > 0\) will give the buyer less than if the same choice was used with the bids \(\mathcal{B}\), and we have seen that for \(\mathcal{B}\) an optimal choice is given by \((t^*_N, s^*_N)\). Thus this choice that has \(t_i = 0\) (and gives preference to those in the supply chain optimal choice) gives the buyer at least the same profit as any other and is therefore chosen by the buyer. Hence the profit made by supplier \(i\) with the new offer can never be greater than zero. Thus we have established that in both cases no other bid can achieve a higher profit for supplier \(i\), establishing that the bids given in the theorem are indeed a Nash equilibrium.

### A.6 Proof of Corollary 1

This is immediate, since Theorem 1 part (a) establishes that supplier \(i\) can make no more than \(\Pi^*_B(N) - \Pi^*_B(N \setminus \{i\})\), which by construction is the supplier \(i\) profit under the conditions of the Corollary. Moreover the example of an optimal choice for the supplier corresponds to part (b) of Theorem 1. Finally, the statement on \(\epsilon\)-optimality follows from part (c) of Theorem 1.

### A.7 Proof of Proposition 1

First we want to show that \(\theta_i > 0\), for \(i \in N^*(C)\). Assumption 1 is enough (from uniqueness of the support of supply chain capacity choices) to show \((t^*_N)_i > 0\), as we observed in \([6]\). Then use of submodularity and repeated application of Lemma 2 with \(N\) decreasing in size shows that \((t^*_S)_i > 0\) for \(S \subseteq N\).

It suffices to show that the execution and reservation payments induced by \(p^*_i(s)\) and \(r^*_i(t)\) satisfy the conditions given in Theorem 2. It is straightforward that the execution payment is \(\int_0^{t_i} p^*_i(s)ds = \int_0^{t_i} c_i(s)ds = C_i(s_i)\) for \(i \in N\). For \(i \notin N^*(C)\), \(\theta_i = 0\) and thus \(\int_0^{t_i} r^*_i(t)dt = E_i(t_i)\). For \(i \in N^*(C)\), we obtain the reservation payment as follows:

\[
\int_0^{t_i} r^*_i(t)dt = \int_0^{t_i} c_i(t) + \int_0^{\min(t_i, \theta_i)} \delta_i(t; \beta_i)dt
\]

\[
= E_i(t_i) + (\Pi^*_C(N) - \Pi^*_C(N \setminus \{i\})) \int_0^{\min(t_i, \theta_i)} \left(\frac{\theta_i}{\beta_i + 1}\right)^{\beta_i} dt
\]

\[
= E_i(t_i) + (\Pi^*_C(N) - \Pi^*_C(N \setminus \{i\})) \left(1 - \left(\frac{\theta_i - \min(t_i, \theta_i)}{\theta_i}\right)^{\beta_i + 1}\right).
\]

Thus,

\[
\Delta_i(t_i; \beta_i) = (\Pi^*_C(N) - \Pi^*_C(N \setminus \{i\})) \left(\frac{\theta_i - \min(t_i, \theta_i)}{\theta_i}\right)^{\beta_i + 1} \geq 0.
\]

Next we show that when all \(\beta_i\)s are large enough, the set of \(\Delta_i(t_i; \beta_i)\) derived from the power function bids is consistent with the costs. There are two requirements for consistency, first we need \(\sum_{j \in S} \Delta_j((t^*_S)_j; \beta_j) = 0\). This follows because \((t^*_S)_j \geq \theta_j\) for \(j \in S\), and hence \(\Delta_j((t^*_S)_j; \beta_j) = 0\) independent of the choice of \(\beta_j\). The second
Thus, profit from supplier $j$ from (14). Thus we can look at this inequality on a component by component basis, and $h$ is increased by taking out the capacity from supplier $i$ from the optimality of $\Pi_i^*$ where $\bar{t}$ and $\bar{t}'$th component set to zero. The first inequality here comes from submodularity and the requirement is that for any $t$ with $I(t) \subseteq S$,

$$\sum_{j \in S} \Delta_j(t) \leq \Pi_c^*(S) - \Pi_c(t).$$

The right hand side of this expression is also a sum over the non zero elements of $t$ from (14). Thus we can look at this inequality on a component by component basis, and we want to find values of $\beta$ large enough that, for every $t$

$$(\Pi_c^*(N) - \Pi_c^*(N \setminus \{i\})) \left( \frac{\theta_i - \min(t_i, \theta_i)}{\theta_i} \right)^{\beta_i + 1} \leq W(t),$$

where

$$W(t) = \int_0^{(t_s^*_j)} \left\{ [\rho - c_i(x)]F(\bar{h}_j(x, (t_s^*)_{-j})) - e_i(x) \right\} dx$$

$$- \int_0^{(t_s^*)_i} \left\{ [\rho - c_i(x)]F(\bar{h}_i(x, (t)_-i)) - e_i(x) \right\} dx,$$

(A-4)

and $t_s^*$ is optimal for $\Pi_c^*(S)$. We note that

$$\Pi_c^*(N) - \Pi_c^*(N \setminus \{i\}) \leq \Pi_c^*(S) - \Pi_c^*(S \setminus \{i\}) \leq \Pi_c^*(S) - \Pi_c(\tilde{t}_S^*)$$

where $\tilde{t}_S^* = ((t_s^*)_1, (t_s^*)_2, \ldots, (t_s^*)_{i-1}, 0, (t_s^*)_{i+1}, \ldots, (t_s^*)_n)$, i.e., it consists of $t_s^*$ with the $i$’th component set to zero. The first inequality here comes from submodularity and the second from the optimality of $\Pi_c^*(S \setminus \{i\})$. Now we observe that for $j \neq i$ the buyer’s profit from supplier $j$, which is

$$\int_0^{(t_s^*)_j} \left\{ [\rho - c_j(x)]F(\bar{h}_j(x, (t_s^*)_{-j})) - e_j(x) \right\} dx,$$

is increased by taking out the capacity from supplier $i$ (i.e., moving from $t_s^*$ to $\tilde{t}_S^*$) since $\bar{h}_j(x, (t_s^*)_{-j})$ is reduced. Hence

$$\Pi_c^*(S) \leq \sum_{j \in S, j \neq i} \int_0^{(t_s^*)_j} \left\{ [\rho - c_j(x)]F(\bar{h}_j(x, (\tilde{t}_S^*_{-i})) - e_j(x) \right\} dx$$

$$+ \int_0^{(t_s^*)_i} \left\{ [\rho - c_i(x)]F(\bar{h}_i(x, (t_s^*)_{-i})) - e_i(x) \right\} dx$$

$$= \Pi_c(\tilde{t}_S^*) + \int_0^{(t_s^*)_i} \left\{ [\rho - c_i(x)]F(\bar{h}_i(x, (t_s^*)_{-i})) - e_i(x) \right\} dx.$$ 

Thus,

$$(\Pi_c^*(N) - \Pi_c^*(N \setminus \{i\})) \left( \frac{\theta_i - \min(t_i, \theta_i)}{\theta_i} \right)^{\beta_i + 1} \leq W_{\beta_i}(t),$$

where

$$W_{\beta_i}(t) = \left( \frac{\theta_i - \min(t_i, \theta_i)}{\theta_i} \right)^{\beta_i + 1} \int_0^{(t_s^*_i)} \left\{ [\rho - c_i(x)]F(\bar{h}_i(x, (t_s^*)_{-i})) - e_i(x) \right\} dx.$$
We need to show that $W_{β_i}(t) ≤ W(t)$. Observe from (A.4) that $W_{β_i}(t) = W(t)$ if $t_i = 0$. Moreover $W_{β_i}(t)$ is zero when $t_i = θ_i$, and hence $W_{β_i}(t) ≤ W(t)$ at $t_i = θ_i$, with equality in the case that $θ_i = (t^*_S)_i$.

Since $(t^*_S)_i$ gives the optimal solution for $Π^*_C(S)$ we know that

$$\frac{∂}{∂t_i} W(t^*_S) = ((ρ - c_i((t^*_S)_i)) \bar{F}(h_i(t_i, (t^*_S)_{-i})) - e_i((t^*_S)_i)) = 0.$$  

Looking at the second derivative:

$$\frac{∂^2}{∂t_i^2} W(t) = \frac{∂}{∂t_i} ((ρ - c_i(t_i)) \bar{F}(h_i(t_i, (t^*_S)_{-i})) - e_i(t_i)) < 0$$

from our assumptions on the functions $c_i$ and $e_i$. The strict inequality here can be established from observing that $\bar{F}$ is decreasing and $h_i(t_i, (t^*_S)_{-i})$ is increasing in $t_i$. Now for $t_i ≤ θ_i$

$$\frac{∂}{∂t_i} W_{β_i}(t) = -\frac{(β_i + 1)}{θ_i} \left(\frac{θ_i - t_i}{θ_i}\right)^{β_i} \int_{0}^{(t^*_S)_i} \left\{ [ρ - c_i(x)] \bar{F}(h_i(x, (t^*_S)_{-i})) - e_i(x) \right\} dx,$$

$$\frac{∂^2}{∂t_i^2} W_{β_i}(t) = \frac{(β_i + 1)β_i}{θ_i^2} \left(\frac{θ_i - t_i}{θ_i}\right)^{β_i-1} \int_{0}^{(t^*_S)_i} \left\{ [ρ - c_i(x)] \bar{F}(h_i(x, (t^*_S)_{-i})) - e_i(x) \right\} dx,$$

which are both zero at $t_i = θ_i$. Hence a Taylor series expansion shows that for $t_i$ close enough to $θ_i$ but below it, we will have $W_{β_i}(t) < W(t)$. The multiplier $\left(\frac{θ_i - min(t_i, θ_i)}{θ_i}\right)^{β_i+1}$ decreases as $β_i$ increases and has limit zero for any $t_i > 0$. Note that $\frac{∂}{∂t_i} W_{β_i}(t)$ decreases towards $-∞$ at $t_i = 0$ as $β_i$ increases, and so is less than $\frac{∂}{∂t_i} W(t^*_S)$ for $β_i$ large enough. Hence we can set $β_i$ large enough that $W_{β_i}(t) < W(t)$ for all $t_i \in (0, θ_i)$. Therefore, we have established that the set of $Δ_i(t_i; β_i)$ is consistent with the costs.

### A.8 Proof of Proposition 2

For submodularity we simply need to show that $Π^*_C(\{1, 2\}) ≤ Π^*_C(\{1\}) + Π^*_C(\{2\})$. Let $(t^*, s^*(D))$ be the optimal supply chain choice when both suppliers are available. Then

$$Π^*_C(\{1, 2\}) = E[V(D, s^*(D)) - \sum_{i \in \{1, 2\}} (C_i(s^*(D)) + E_i(t^*))]$$

$$= E[V(D, [s^*(D)]_1 + [s^*(D)]_2)$$

$$- \sum_{i \in \{1, 2\}} (C_i([s^*(D)]_1 + [s^*(D)]_2) + E_i([t^*]_1 + [t^*]_2))],$$

where for any vector $x$ we define $[x]_1 := (x_1, 0)$ and $[x]_2 := (0, x_2)$. Using subadditivity of $V$ and the restriction on $C$ and $E$ functions we have

$$Π^*_C(\{1, 2\}) ≤ E[V(D, [s^*(D)]_1) + V(D, [s^*(D)]_2) - \sum_{i \in \{1, 2\}} (C_i([s^*(D)]_i) + E_i([t^*]_i))]$$

$$= Π_C([t^*]_1, [s^*(D)]_1) + Π_C([t^*]_2, [s^*(D)]_2) ≤ Π^*_C(\{1\}) + Π^*_C(\{2\}).$$
The final stage is simply to observe that for the capacity game with function bids, we have \( V(D, s) = \rho \min(D, s_1 + s_2) \) and this is a subadditive function, since
\[
V(D, s_A + s_B) = \min(D, s_{A1} + s_{A2} + s_{B1} + s_{B2}) \\
\leq \min(D, s_{A1} + s_{A2}) + \min(D, s_{B1} + s_{B2}).
\]

A.9 Proof of Theorem 3

Note that the supply chain optimal profit as a set function can be expressed as follows:
\[
\Pi^*(\sigma) = \max \{ \Pi_C(t) : t \leq \sigma \bar{d}, \sigma \in \{0,1\}^n \}.
\]
According to (3), with the conditions of the theorem, \( \Pi_C(t) \) is laminar concave and hence \( M^2 \)-concave in \( t \in T \) (see the Appendix for definition) according to Murota (2009, Section 4.2), which implies that \( \Pi_C(t) = \Pi_C(t\bar{d}) \) is \( M^2 \)-concave in \( t \in [0,1]^n \). For any \( \bar{\sigma} \in [0,1]^n \), let
\[
\Pi(\bar{\sigma}) = \max \{ \Pi_C(t) : 0 \leq t \leq \bar{\sigma} \}.
\]
We show that \( \Pi(\bar{\sigma}) \) is \( M^2 \)-concave over \([0,1]^n \). To this end, consider the informal convolution function
\[
\Pi(\bar{\sigma}) = \sup \{ \Pi_C(t) + \alpha(t) : t + \bar{t} = \bar{\sigma}, t, \bar{t} \in \mathbb{R}^n \}, \bar{\sigma} \in \mathbb{R}^n;
\]
where \( \Pi_C(t) = \tilde{\Pi}_C(t) - \sum_{i=1}^n \delta_{[0,1]}(t_i) \) and \( \alpha(t) = -\sum_{i=1}^n \delta_{[0,1]}(t_i) \), with \( \tilde{\Pi}_C(t) = -\infty \) if \( t \notin [0,1]^n \) and \( \delta_{[0,1]}(\cdot) \) the indicator function of set \([0,1]\) (i.e., \( \delta_{[0,1]}(x) = 0 \) if \( x \in [0,1] \) and \( \delta_{[0,1]}(x) = +\infty \) otherwise). It is clear that both \( \tilde{\Pi}_C(\cdot) \) and \( \alpha(\cdot) \) are \( M^2 \)-concave over \( \mathbb{R}^n \). According to Murota (2009, Section 4.2), \( \Pi(\bar{\sigma}) \) is \( M^2 \)-concave over \( \mathbb{R}^n \). Note that the restriction of \( \Pi(\bar{\sigma}) \) on \([0,1]^n \) is exactly \( \Pi(\bar{\sigma}) \), which therefore is \( M^2 \)-concave over \([0,1]^n \) with straightforward direct verification according to the definition of \( M^2 \)-concavity. We then conclude that \( \Pi(\cdot) \) is submodular over \([0,1]^n \) (Murota and Shioura, 2004), which implies that \( \Pi^*(\cdot) \), as the restriction of \( \Pi(\cdot) \) to \([0,1]^n \), is submodular over \([0,1]^n \), as desired.

On the second part of the theorem for the case of the capacity game with function bids, we assume that, for each supplier \( i \), the marginal execution cost \( c_i(\cdot) = \rho \) is constant and the marginal reservation cost \( e_i(\cdot) \) is non-decreasing. Let \( 0 \leq c_1 < \cdots < c_n \leq \rho \). We will show that \( \Pi_C(t) \) as defined in (14) is laminar concave in \( t \in [0, \bar{d}]^n \). With constant \( c_i(\cdot) \) in (13) we have \( \bar{h}_i(x, t_{-i}) = x + \sum_{j=1}^{i-1} t_j, x \in [0, t_i] \). Denote \( \tilde{\rho}_i = \rho - c_i \) and define
\[
\rho_i(x, t_{-i} \mid d) = \begin{cases} \tilde{\rho}_i, & \text{if } \bar{h}_i(x, t_{-i}) \leq d; \\ 0, & \text{otherwise}. \end{cases}
\]
We can write
\[
\sum_{i=1}^n \int_0^{t_i} (\rho - c_i) \bar{F}(\bar{h}_i(x, t_{-i}))dx = E_D \left[ \sum_{i=1}^n \omega_i(t \mid d) \right],
\]
where
\[
\omega_i(t \mid d) = \int_0^{t_i} \rho_i(x, t_{-i} \mid d)dx = \int_0^{b_i(t \mid d)} \tilde{\rho}_i dx = \tilde{\rho}_i b_i(t \mid d),
\]
and \( b_i(t \mid d) = \min\{d - \sum_{j=1}^{i-1} t_j, t_i\} \). Denote \( a_i = d - \sum_{j=1}^{i} t_j \) for \( i = 0, 1, \ldots, n \). Then
\[
b_i(t \mid d) = \min\{a_i^{i-1}, t_i\} = a_i^{i-1} - (a_i^{i-1} - t_i)^+ = a_i^{i-1} - a_i^+,
\]
which leads to
\[
\omega_i(t \mid d) = \bar{\rho}_i \left( a_i^{i-1} - a_i^+ \right).
\]
Therefore,
\[
\sum_{i=1}^{n} \omega_i(t \mid d) = \bar{\rho}_1 d - \sum_{i=1}^{n-1} (\bar{\rho}_i - \bar{\rho}_{i+1}) a_i^+ - \bar{\rho}_n a_n^+
\]
\[
= \bar{\rho}_1 d - \sum_{i=1}^{n-1} (c_{i+1} - c_i) \left( d - \sum_{j=1}^{i} t_j \right)^+ - \bar{\rho}_n \left( d - \sum_{j=1}^{n} t_j \right)^+.
\]
Noticing that both \((d - \tau)^+\) and \(\int_0^{\tau} e_i(x) dx\) \((1 \leq i \leq n)\) are convex in \(\tau \geq 0\), we conclude that \(\Pi_C(t)\) is laminar concave with the corresponding laminar family \(\mathcal{L} = \{\{i\} : i \in N\} \cup \{\{1, \ldots, i\} : i \in N\}\).