Numerical Study on the Influence of Angles of Repose on the Stability of Cargo in a Cargo Hold

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Abstract—In recent years, cargo instability caused by fluidization is one of the main reasons for bulk carrier capsizing. Considering the complexity and diversity of the actual cargo, the mechanism of fluidization has not been fully revealed. Taking a single cargo hold of a 57,000 DWT bulk carrier as an example, a 3D cargo hold scaled model was established, and the movement state of the cargo in the cargo hold under different angles of repose is numerically simulated. Results show that under the precondition of model parameters, angle of repose of the cargo can be further modified by the friction coefficient between particles. The smaller of the angle of repose, the stronger of the fluidity of the cargo becomes. There exists a critical angle of repose, below which the center of gravity offset of the cargo becomes more obvious and the stability of the cargo drops sharply. Simulation model and numerical method involved in this research can provide some references for predicting the stability of the marine transportation of solid bulk cargoes.

1. Introduction

Fluidization, mainly refers to the special state that a type of bulk solid cargo is prone to occur during ship navigation. This type of cargo usually consists of a certain proportion of fine particles and moisture, such as laterite nickel ore and iron concentrate. Due to the continuous dynamic load, including the ship's own swaying motion and the strong wind and waves under severe sea conditions, the cargo gradually presents a fluid state, completely losing its strength and stiffness [1]. In severe cases, the ship may even capsize due to the instability of cargo caused by fluidization [2]. For this kind of problem, Guan et al. [3] analyzed the evolution process of nickel ore fluidization in detail through a six-degree-of-freedom motion platform test loaded with a nickel ore cargo hold model. It is concluded that roll motion plays a major role in cargo fluidization. Based on the indoor small-scale shaking table model test, Zhou et al. [4,5] studied the fluidization characteristics of iron ore concentrate and laterite nickel ore. Results all show that water migration is a key factor leading to cargo fluidization. Due to the poor repeatability of model test and many external interference factors, however, numerical simulation methods such as finite element method (FEM) and discrete element method (DEM) have been adopted by scholars to further study the movement law of easily fluidized cargoes in the cabin. Based on the finite element software FLUENT, Zhang et al. [6] established the numerical model of cargo sloshing in cabin and rolling coupling motion between cabin segments by using simplified surface nonlinear method and volume of fluid (VOF) method. It is further verified that higher wave amplitude and resonance frequency are the main factors leading to ship capsizing. Ju et al. [7] established a DEM numerical model to compare with the static tilt test of Japan Classification Society,
and further verified that the friction coefficient can be equivalent to the effect of moisture between cargo particles.

Previous studies have shown that DEM is more suitable than FEM in studying the dynamic change process of multiphase, discrete and physical property dispersion of particle motion and particle flow characteristics [8]. Therefore, in view of the uniform loading of a single cargo hold of 57,000 DWT grade bulk carrier, a 3D cargo hold scale model is established based on the discrete element software PFC3D to simulate the movement of cargo particles in the cargo hold. Considering the complexity of the actual cargo properties, this study focuses on the influence of different repose angles on the movement of cargo particles in the cargo hold, in order to analyze and discuss the stability of the cargo in detail from the perspective of initial cargo properties.

2. Theoretical Background and Equations

The contact behavior of multiple particles, as the key issue of DEM, is determined by the selected contact model. In the contact model, particles are regarded as rigid bodies, and the contact between particles is a soft contact, that is, the particles are allowed to deform slightly at the contact point. When the contact behavior occurs between particles, the selected contact model would be activated. The force-displacement law provided by the contact model acts on the particles in contact to update the contact force and moment between the particles. The updated force and moment respectively act on the particles in contact, thereby updating the force of each particle and the current position change of the particle. Considering that the rolling resistance linear model has certain advantages in simulating granular materials, especially in the case of angle of repose formed by them. Therefore, the model is mainly used to complete the DEM modeling. Law of force-displacement in PFC3D are further described as follows.

In a timestep, the force-displacement law updates the relative displacement increment of the particles according to the change in the contact gap between the particles, that is, the following equations is satisfied:

\[
\Delta \delta_n := \alpha \Delta \delta_n \quad \Delta \delta_s := \alpha \Delta \delta_s \quad \alpha = \left\{ \begin{array}{ll}
\frac{g_s}{g_s - (g_s)_o}, & (g_s)_o > 0 \text{ and } g_s < 0 \\
1, & \text{otherwise}
\end{array} \right.
\]

where, \( \Delta \delta_n \) is the normal relative displacement increment, \( \Delta \delta_s \) is the shear relative displacement increment; \( g_s \) is the surface gap between particles; \((g_s)_o \) is the surface gap at the beginning of the time step. In addition, the force-displacement law updates the contact force and moment between particles as follows:

\[
F_c = F^l + F^d \\
M_c = M^r
\]

where, \( F^l \) is the linear force, \( F^d \) is the dashpot force, \( M^r \) is the rolling resistance moment. Generally, the linear force \( F^l \) can be decomposed into linear normal force \( F^l_n \) and linear shear force \( F^l_s \), which satisfies the following equations:

\[
F^l_n = \begin{cases} 
 k_n g_s, & g_s < 0 \\
0, & g_s \geq 0
\end{cases} \\
F^l_s = \begin{cases} 
 (F^l_s)_o - k_s \Delta \delta_s, & ||(F^l_s)_o - k_s \Delta \delta_s|| \leq -\mu F^l_n \\
-\mu F^l_n \left( (F^l_s)_o - k_s \Delta \delta_s \right) / ||(F^l_s)_o - k_s \Delta \delta_s||, & \text{otherwise}
\end{cases}
\]

where, \( k_n \) is the normal stiffness, \((F^l_s)_o \) is the linear shear force at the beginning of the timestep, \( k_s \) is the shear stiffness, \( \Delta \delta_s \) is the adjusted relative shear-displacement increment of Eq. (2), and \( \mu \) is the particle friction coefficient. The rolling resistance moment \( M^r \) is updated through the following steps.

First, the rolling resistance moment \( M^r \) is incremented as:

\[
M^r := M^r - k_s \bar{R}^2 \Delta \theta_b
\]
where, $\Delta \theta_{b}$ is the relative bend-rotation increment, $\vec{R}$ is the contact effective radius, defined as:

$$1/\vec{R} = 1/R^{(1)} + 1/R^{(2)}, \quad R^{(1)} \text{ and } R^{(2)} \text{ respectively correspond to the radii of the two particles in contact.}$$

The magnitude of the updated rolling resistance moment $M^r$ is then checked against a threshold limit:

$$M^r = \begin{cases} M^r, & ||M^r|| \leq \mu_r \vec{R}F_n^i \\ M^* (M^r/||M^r||), & \text{otherwise} \end{cases}$$

(9)

where, $\mu_r$ is the rolling friction coefficient.

### 3. Simulation of angle of repose

#### 3.1. Basic model parameters

Angle of repose, as an important physical parameter to characterize the bulk cargo's looseness and fluidity, its study is of great significance to the safety of bulk cargo transportation. Before using contact model to simulate the angle of repose, it is necessary to calibrate the particle parameters [9]. Previous studies have shown that the main factors affecting the angle of repose are the friction coefficient between particles, friction coefficient in contact with the ground and the types of cargoes (especially the stiffness and modulus) [10-12]. Considering the diversity of influencing factors, this numerical simulation study takes nickel ore particles as the research object, unifies the relevant variable parameters, and only changes the friction coefficient between the particles to adjust the value of the angle of repose.

In addition, reference [13] points out that when the ratio of the average particle size of the cargo to the boundary size of the model is greater than or equal to 25, the size effect of the particle size in the model can be ignored. In this numerical study, the selected particle size is 0.01 m, and the ratio of particle size to model boundary size is greater than 25. Basic parameters of the model are shown in Table 1.

#### Table 1. Specific setting of rolling resistance linear model parameters.

| Model Parameter                     | Value | Model Parameter                     | Value            |
|-------------------------------------|-------|-------------------------------------|------------------|
| Normal stiffness $k_n$ (Pa)         | 1e5   | Particle friction coefficient $\mu$ | Calibrated by simulation of different angles of repose |
| Shear stiffness $k_s$ (Pa)          | 5e4   | Rolling friction coefficient $\mu_r$ | 0.3*             |
| Normal-to-shear stiffness ratio $k^*$ | 2     | Wall friction coefficient           | 9.81             |
| Effective modulus $E^*$ (Pa)        | 1e6   | Gravity field $g$ (m/s²)            | 9.81             |
| Particle density $D$ (kg/m³)        | 2500  | Time step (s)                       | 1e-4             |

*Note: Bottom friction coefficient

#### 3.2. Simulation model

Funnel method is usually used to measure the angle of repose of soil particles. However, once the friction coefficient or the viscous force between cargo particles is large, it is easy to block the discharge port. Therefore, considering the complex characteristics of the actual easily fluidized cargoes, the inclined plane method is mainly used for numerical simulation of angle of repose.

Comparing reference [8] and reference [10], a numerical simulation model for measuring the angle of repose of the cargo is designed. Numerical simulation of angle of repose is shown in Fig. 1, and the specific steps of using the model to test the angle of repose are as follows:

- Establishment of the cabin (see Fig. 1a). The baffle is used to divide the cabin, and the left cabin is used to load cargo.
- Cargo loading (see Fig. 1b). The loading depth is set to 0.37 m, and the total mass of the cargo is about 77.40 kg.
- Remove the middle baffle and wait for the cargo to reach a state of equilibrium (see Fig. 1c).
- After the cargo reaches equilibrium (see Fig. 1d), save the current view and data for further processing.
- Modify the friction coefficient between particles and repeat the above steps to obtain different angles of repose.

3.3 Simulation results processing

From the angle of repose simulation results, it can be seen that the cargo slides freely under the condition of their own gravity, and often present an uneven surface. To further solve the effective angle of repose, the boundary drawing method and Origin software are used for further processing.

The specific steps are as follows:
- When the cargo reaches equilibrium, the cross section in x and z direction is taken, and the x and z coordinate points of boundary particles are extracted. Then, the plane rectangular coordinate system is reestablished to complete the drawing of the cargo repose angle boundary.
- Linear regression equation is used to fit the boundary line, and the corresponding curve fitting equation is obtained. Then, the effective angle of repose $\overline{\theta}$ is calculated by weighted processing according to the change interval of curve slope. Boundary fitting of angle of repose is shown in Fig. 2, and the data processing and analysis results are shown in Table 2.

From Table 2, it can be known that under the condition of calibrated model parameters, the angle of repose of the cargo can be corrected by the change of the friction coefficient between particles. At
the same time, it is further verified that the friction coefficient between particles has a significant effect on the angle of repose.

| Serial number | Particle friction coefficient $\mu$ | Rolling friction coefficient $\mu_r$ | Maximum angle of repose $\theta_{max}$ (°) | Minimum angle of repose $\theta_{min}$ (°) | Effective angle of repose $\bar{\theta}$ (°) |
|---------------|------------------------------------|-------------------------------------|-------------------------------------------|------------------------------------------|-----------------------------------------|
| 1             | 0.90                               | 0.80                                | 31.19                                     | 29.63                                    | 30.41                                   |
| 2             | 0.60                               | 0.45                                | 29.16                                     | 27.68                                    | 28.42                                   |
| 3             | 0.40                               | 0.20                                | 25.53                                     | 24.90                                    | 25.22                                   |
| 4             | 0.35                               | 0.18                                | 23.57                                     | 23.06                                    | 23.32                                   |
| 5             | 0.24                               | 0.17                                | 21.28                                     | 20.96                                    | 21.12                                   |
| 6             | 0.22                               | 0.16                                | 20.91                                     | 20.49                                    | 20.70                                   |
| 7             | 0.20                               | 0.15                                | 19.34                                     | 19.11                                    | 19.23                                   |
| 8             | 0.18                               | 0.10                                | 17.63                                     | 17.37                                    | 17.50                                   |
| 9             | 0.12                               | 0.08                                | 15.04                                     | 14.76                                    | 14.90                                   |
| 10            | 0.10                               | 0.05                                | 13.85                                     | 13.60                                    | 13.73                                   |
| 11            | 0.05                               | 0.02                                | 11.46                                     | 11.11                                    | 11.29                                   |
| 12            | 0.01                               | 0.01                                | 9.69                                      | 9.16                                     | 9.43                                    |

4. Swing Table Test Simulation

4.1. Cargo hold model
Refer to reference [5], a 3D rectangular cargo hold is established in this simulation based on DEM. The similarity ratio between the size of the model and the actual cargo hold is 1:25, which can approximately simulate the uniform loading of a single cargo hold of a 57,000 DWT grade bulk carrier. Set the origin O (0,0,0) as the swing center, and the motion form of the cargo hold corresponds to the roll motion of the actual ship. Considering the complexity of actual sea conditions, the roll motion amplitude is set to 20° and the frequency is 1 Hz. In addition, similar to the angle of repose simulation test, the total mass of the cargo is about 77.40 kg, and the actual loading height is 0.22 m. The model frame and calculation model are shown in Fig. 3.

![Fig. 3. Cargo hold model: (a) Model frame; (b) DEM calculation model.](image_url)

4.2. Simulation steps
Based on the previous angle of repose model, six groups of angles of repose parameters are selected for simulation and analysis. The simulation mainly follows the following steps:

- Establish a cargo hold, complete cargo loading, and wait for the cargo to reach a balanced state.
- Roll motion simulation. The loading of roll motion is realized by applying sinusoidal cyclic buffer velocity to the cargo bulkhead.
- Data record. The traversal function and the command of monitoring are called to record the coordinate change of real-time movement of cargo particles and the change of contact force on cargo hold.
Data export and graphic post-processing. Change the corresponding parameters and continue to simulate the next angle of repose condition.

4.3. Result analysis
The movement of the cargo with angle of repose of 14.90° in one period is shown in Fig. 4. Whenever the cargo hold returns to the equilibrium state, the overall cargo is more likely to be tilted to one side, and the center of gravity alters obviously at this time.

The center of gravity offset of the cargo under different angles of repose is shown in Fig. 5, in which the real-time loading curve of cargo hold roll motion is shown in Fig. 5a. It can be seen that with the decrease of the angle of repose of the cargo, the variation of the center of gravity offset of the cargo becomes more obvious. The variation of dynamic contact force on the cargo hold under different angles of repose is shown in Fig. 6. Among them, Fig. 6a shows the variation of the dynamic contact force on the cargo hold when the cargo angle of repose is 30.41°, while Fig. 6b shows the variation of the dynamic contact force difference of the other groups of angles of repose relative to the angle of repose of 30.41°. It can be seen that with the decrease of the angle of repose, the dynamic contact force on the cargo hold changes more and more large. The main reason is that the decrease of the angle of repose causes the shift of center of gravity of the cargo. The variation of dynamic contact force exactly corresponds to the variation of gravity center offset of the cargo. In general, the relationship between the center of gravity offset of the cargo and the angle of repose is mainly divided into the following three stages:

- When the angle of repose is 30.41°, the cargo has almost no center of gravity deviation.
- When the angle of repose is in the interval 23.32°~25.22°, the center of gravity of the cargo has a small deviation, which is specifically expressed as: the maximum center of gravity deviation in the x-direction is 0.00121 m, and the maximum center of gravity deviation in the z-direction is 0.00055 m.
- When the angle of repose is in the interval 14.90°~21.12°, the center of gravity offset of the cargo changes significantly.

Fig. 4. Movement of the cargo with angle of repose of 14.90° in one period.
Based on the above analysis, there should be a critical angle of repose in the interval of 21.12°~23.32°, which makes the cargo appear a large center of gravity deviation. Therefore, in order to ensure the stability of the cargo in the cabin, the angle of repose of the cargo should be greater than this critical value.

Fig. 5. Center of gravity offset of the cargo: (a) X-direction; (b) Z-direction.

Fig. 6. Variation of dynamic contact force on the cargo hold under different angles of repose: (a) \( \theta = 30.41° \); (b) \( \theta = 25.22°, 23.32°, 21.12°, 19.23° \) and \( 14.90° \).

5. Conclusion

Based on the rolling resistance linear model, firstly, a numerical simulation model for measuring the angle of repose of cargo is established on the basis of previous studies. Then, the movement of the cargo in the cargo hold under different angles of repose is simulated. The main conclusions are as follows:

- In the case of partial parameter calibration, the change of friction coefficient between cargo particles has a significant impact on the angle of repose. Different angle of repose can be obtained by changing the particle friction coefficient and rolling friction coefficient between particles.
- The angle of repose is proportional to the fluidity. The smaller the angle of repose is, the stronger the fluidity of cargo is, leading to the greater deviation of the center of gravity and the gradual decline of the stability of the cargo.
- Under the condition of this study, there exists a critical angle of repose: above this value, the center of gravity of the cargo is less affected and the cargo are basically stable; below this value, the center of gravity of the cargo is greatly affected, and the stability drops sharply.
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