Lower limit on the neutralino mass in the general MSSM.

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Abstract

We discuss constraints on SUSY models with non-unified gaugino masses and \(R_P\) conservation. We derive a lower bound on the neutralino mass combining the direct limits from LEP, the indirect limits from \((g-2)_\mu\), \(b \rightarrow s\gamma\), \(B_s \rightarrow \mu^+\mu^-\) and the relic density constraint from WMAP. The lightest neutralino \(m_{\tilde{\chi}_1^0} \approx 6\) GeV is found in models with a light pseudoscalar with \(M_A < 200\) GeV and a large value for \(\tan \beta\). Models with heavy pseudoscalars lead to \(m_{\tilde{\chi}_1^0} > 18(29)\) GeV for \(\tan \beta = 50(10)\). We show that even a very conservative bound from the muon anomalous magnetic moment can increase the lower bound on the neutralino mass in models with \(\mu < 0\) and/or large values of \(\tan \beta\). We then examine the potential of the Tevatron and the direct detection experiments to probe the SUSY models with the lightest neutralinos allowed in the context of light pseudoscalars with high \(\tan \beta\). We also examine the potential of an \(e^+e^-\) collider of 500 GeV to produce SUSY particles in all models with neutralinos lighter than the \(W\). In contrast to the mSUGRA models, observation of at least one sparticle is not always guaranteed.
1 Introduction

The minimal supersymmetric standard model (MSSM) is certainly one of the most attractive extensions of the standard model. Apart from providing a solution to the hierarchy problem it also fits in very well with the idea of the gauge unification and gives as a bonus a good dark matter candidate represented by its lightest supersymmetric particle, the LSP for short. One the other hand, to the question of how light this LSP can be so that one has an idea about the masses for the rest of the spectrum, the model provides no answer. The reason for this is that there is no definite model for the masses of the sparticles (and other scales) which would ultimately point out to a theory of the (soft) breaking of supersymmetry. One exception relates to the mass of the Higgs if one makes the mild requirement that the scale of supersymmetry breaking to be much above the electroweak scale. This gives one of the robust predictions of the model which is that there must be at least one light Higgs with mass below about 130GeV [1], for alternatives see [2, 3]. For practically all the other (s)particles of the model one does now have some lower bounds on their masses from extensive phenomenological studies and searches both at colliders and in astroparticle experiments. One can for example mention the impact of both LEP and the Tevatron in constraining the model parameters through the direct searches, the importance of rare decays notably \(b \rightarrow s\gamma\) or even low energy experiments like the recent measurement of the muon anomalous moment \((g - 2)_\mu\) [4]. Cosmological considerations like the thermal relic density of the LSP also put some stringent bounds on the MSSM. It should however be emphasised that many of the bounds and constraints, especially those from indirect searches, but not only, are based on theoretical assumptions some of which with strong prejudices. The aim of this paper is to critically review the lowest bound on the neutralino LSP with as little theoretical bias as possible. Instead of reviewing a host of models, we will try to delimit the parameter space that has an incidence on the LSP mass bound in a, as much as possible, model independent way [5]. We will see that one does not have to play with a large number of parameters to arrive at the lowest bound on the LSP, instead the parameter space will be naturally reduced to only a few critical parameters.

A large amount of work has been devoted to the so-called minimal supergravity model (mSUGRA) mainly because of its simplicity. Indeed, instead of dealing with about 100 parameters in the general MSSM, one works with only four parameters and a sign. With such a constrained model it becomes possible to combine the various experimental data and severely delimit the parameter space. Within this model the lowest limit on the LSP is extracted in a straightforward way from the LEP data which give \(m_{\tilde{\chi}_1^0} > 59\text{GeV}\) [6, 7]. Moreover, with the new precise determination of the relic density of cold dark matter by WMAP [8], the mSUGRA parameter space considerably shrinks to thin strips [9] suggesting a rather focused phenomenology and benchmarks for the next colliders [10]. Supersymmetric signals can be radically different in other scenarios, scenarios where one can have much more room for manoeuvre even after imposing all the latest data. In particular, the LSP bound can be much smaller than the one found in the context of mSUGRA as we will see.

Different approaches have been followed aiming at a reduction of parameters. Theoretically motivated approaches include the so-called anomaly mediated (AMSB) models [11] and generalisations [12] for example, or schemes that appeal to superstrings that are either moduli-dominated or contain a mixture of moduli and dilaton fields [13, 14, 15, 16] with varying degrees of simplification. There are even scenarios that interpolate between
mSUGRA and some of the AMSB and string inspired methodology [17]. Another approach without necessarily an underlying framework for supersymmetry breaking, lifts, at the GUT scale, the mass degeneracies of the mSUGRA model either in the sector of the gaugino \([18, 19, 20, 21, 22]\), Higgs \([23, 24]\) or the sfermions \([25, 21]\). Some analyses let loose the shackles of mSUGRA and models defined at high scale but still impose ad-hoc constraints, like equality of scalar masses at the electroweak scale \([20]\). Reviewing this situation one can still ask about the lowest stable LSP mass allowed by the present data regardless of a pre-defined scheme. This is important because not only one would want to know what the impact of the lightest LSP is for cosmology and direct detection but how the collider phenomenology can get affected when one puts aside assumptions related to the LSP mass. For example one might inquire about the sensitivities of the direct search detectors such as CDMS \([27]\), EDELWEISS \([28]\) and ZEPLIN \([29]\) for neutralino masses below those typical of the LSP of mSUGRA. The LSP, in the R-parity conserving MSSM, being the end product of the decay of any supersymmetric particle, is important for signatures at the colliders since it will always show up in any analysis. Another reason why it is interesting to study the light neutralino is that it opens the door for a sizable branching fraction of the Higgs into invisible thus reducing all the branching fractions into the usual discovery channels, \(h \to \gamma \gamma\) or \(h \to bb\) \([30, 31]\). This possibility has triggered analyses by the LHC experiments to search for the Higgs with a sizable branching fraction into invisible \([32, 33]\). This has also motivated studies to reanalyse Tevatron data in models with light neutralinos and establish whether or not supersymmetry could be discovered there in the chargino-neutralino channel leading to tri-leptons \([34]\).

The oft-quoted lower limit, \(m_{\tilde{\chi}_1^0} > 59\text{GeV}\) \([6, 7]\) which applies to mSUGRA, is basically derived from the quite robust model independent lower limit on the chargino mass \(m_{\tilde{\chi}_1^+}\). The (lightest) chargino mass is set from chargino pair production at LEP2. Deriving the lower limit on the neutralino mass from this analysis, on the other hand, tacitly assumes a model of unified gaugino masses at the GUT scale, much like what is hypothesised in mSUGRA. However in a general MSSM, the masses \(m_{\tilde{\chi}_1^0}\) and \(m_{\tilde{\chi}_1^+}\) are uncorrelated and the lower limit on the LSP neutralino mass weakens considerably if only the LEP constraint is taken into account. This occurs for a bino neutralino when \(M_1 \ll M_2\) where \(M_1(M_2)\) is the \(U(1)(SU(2))\) soft-susy breaking gaugino mass. To derive a lower bound on the LSP in these conditions one has to turn to cosmology. Indeed, the neutralino LSP cannot be too light as it would conflict with the precise measurement of the thermal relic density of cold dark matter inferred from WMAP \([8]\). One is then indirectly led to include the sleptons into the picture, in particular the right handed ones, \(\tilde{l}_R\). Indeed a light neutralino that is mostly a bino will annihilate preferably into fermions through right-handed sleptons because the latter have the largest hypercharge among all sfermions \([35, 36]\). As a rule of thumb, with all sfermions heavy but the three right sleptons, a rough approximate requirement is

\[
m_{\tilde{l}_R}^2 < 10^3 \sqrt{(\Omega_{\chi} h^2)_{\text{max}}} \times m_{\tilde{\chi}_1^0}.
\]

with all masses expressed in GeV. We identify \(\Omega_{\chi}\) with the fraction of the critical energy density provided by neutralino LSP and \(h\) is the Hubble constant in units of 100 km sec\(^{-1}\) Mpc\(^{-1}\). Therefore the lower bound on the slepton from LEP2, when used in conjunction with the upper limit from the relic density\(^1\), plays an important role in constraining a light neutralino. We will see that to accommodate the relic density constraint, a slight

\(^1\)Note that we only consider scenarios where the neutralino contributes to the cold dark matter.
Higgsino component can make the mostly bino LSP couple to the $Z$ and the Higgses. If one is not far from these resonances there will be efficient annihilation of the neutralino LSP.

To further constrain the lower bound on the LSP mass and bring in new observables in the picture, some assumptions on the sfermion masses need to be made. A well motivated mild assumption relies on so-called minimal flavour violation in order to evade bounds on flavour changing neutral currents. In the context of a generic minimal flavour violation model, sfermions with the same quantum numbers would share the same mass, with perhaps a slight breaking from Yukawa mixing. We would then treat all the sleptons on an equal footing. This then brings into the picture the constraint from the measurement of the muon anomalous magnetic moment, $\delta a_\mu$. The latter is also sensitive to the presence of light charginos/neutralinos and smuons. Both the theoretical predictions and experimental results on the muon anomalous magnetic moment have been refined on several occasions in the last year and the situation is still evolving. We will therefore be very conservative in imposing limits from $(g - 2)_\mu$ using numbers that encompass different estimates. Nevertheless we will find that the upper bound on the $(g - 2)_\mu$ does constrain some of our scenarios. In our study we will also show how results change if one does not impose the $(g - 2)_\mu$ bound.

The squarks on the other hand hardly enter the game. One needs however to assume them to be sufficiently heavy so that, especially for large $\tan \beta$, one does not conflict with the bound from $b \to s\gamma$. Moreover, squarks of the third generation should be heavy enough and or the tri-linear mixing parameter of the top be large so that, especially for low $\tan \beta$, one evades more easily the lower bound on the Higgs mass. One usually also chooses the pseudoscalar Higgs mass to be heavy enough to achieve this. With the assumption that the pseudoscalar Higgs is heavy (beyond say $\sim 300 \text{GeV}$), an absolute lower limit on the neutralino mass, $m_{\tilde{\chi}_1^0} = 18 \text{GeV} (29 \text{GeV})$ for $\tan \beta = 50 \ (10)$ results from combining cosmological and collider constraints.

Relaxing the assumption that the pseudoscalar is very heavy, after all the LEP lower bound is only about $90 \text{GeV}$, leads to an even smaller lower bound on the neutralino mass, $m_{\tilde{\chi}_1^0} > 6 \text{GeV}$. This is because, as was pointed out in [38], a new possibility for escaping the relic density constraint for very light neutralinos opens up: neutralino annihilation into $b\bar{b}$ pairs via a Higgs resonance. This channel becomes efficient in the large $\tan \beta$ regime due to the enhanced couplings to $b$ quarks and $\tau$’s. We will show that the concomitant presence of a light charged Higgs at large $\tan \beta$ means that these models are tightly constrained by $b \to s\gamma$ as well as by $B_s \to \mu^+\mu^-$.

Unstable neutralinos with masses in the MeV range have been entertained [37] and their astrophysical (supernova) implications studied.
will then analyse the constraints from the relic density and from the muon anomalous magnetic moment, \((g-2)_\mu\) and the rare decay \(b \rightarrow s\gamma\). Other constraints that we use, although in most cases without much impact, are the effect on the invisible width of the \(Z\), \(Z \rightarrow b\bar{b}\) and the rare decay \(B_s \rightarrow \mu^+\mu^-\). The latter can play a rôle for a light pseudoscalar. We will concentrate on the intermediate to large \(\tan\beta\) region and present absolute lower limits on the mass of the neutralino LSP. The case of models with a light pseudoscalar Higgs deserves a special section both as concerns the impact of the various constraints as well as the prospects at future colliders. Here the Tevatron plays an important role. The prospects for direct detection experiments will be discussed in this context. Finally, a section will be devoted to the potential of the linear collider to produce supersymmetric particles in models with light neutralinos, \(m_{\tilde{\chi}_1^+} < M_W\). We will close with a conclusion that summarises our results.

## 2 MSSM parameters

### 2.1 Physical parameters

When discussing the physics of charginos and neutralinos it is best to start by defining one’s notations and conventions. All our parameters, unless stated otherwise, are defined at the electroweak scale. The chargino mass matrix in the gaugino-Higgsino basis is defined as

\[
\begin{pmatrix}
M_2 & \sqrt{2}M_W \cos \beta \\
\sqrt{2}M_W \sin \beta & \mu
\end{pmatrix}
\]  

where \(M_2\) is the soft SUSY breaking mass term for the \(SU(2)\) gaugino while \(\mu\) is the so-called Higgsino mass parameter whereas \(\tan \beta\) is the ratio of the vacuum expectation values for the up and down Higgs fields.

Likewise the neutralino mass matrix is defined as

\[
\begin{pmatrix}
M_1 & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta \\
0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta \\
-M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\mu \\
M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\mu & 0
\end{pmatrix}
\]

where the first entry \(M_1\) (corresponding to the bino component) is the \(U(1)\) gaugino mass. The oft-used gaugino mass unification condition corresponds to the assumption

\[M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \simeq M_2/2\]  

Then constraints from the charginos alone can be easily translated into constraints on the neutralino sector. Relaxing Eq. 4, or removing any relation between \(M_1\) and \(M_2\) means that one needs further observables specific to the neutralino sector. It is a trivial observation that if \(M_1 \ll M_2, |\mu|\) one can get a very low neutralino mass independently of the chargino mass derived from Eq. 2.

In the approach we are taking, the free parameters include the ones of the gaugino sector as well as the parameters of the slepton sector. As discussed earlier, we will
assume that all the squarks are heavy. Allowing light squarks would not affect much the cosmological constraints for a neutralino that is mostly bino and in any case light squarks are strongly constrained from $b \to s\gamma$. For the slepton sector we will consider primarily a model reminiscent of mSUGRA models but without gaugino mass unification. This model features a common mass for the sleptons at the GUT scale. As we will see, for the purpose of constraining the LSP mass, the slepton mass spectrum is essentially the same had we imposed the more general requirement of minimal flavour violation by giving a common mass to the left and a common mass to right sleptons of all three generations.

Using the renormalisation group equations, the masses at the weak scale can be related to the ones at the GUT scale. For sleptons this can be done rather independently of the other MSSM parameters. Defining the parameter

$$r_{12} = \frac{M_1}{M_2},$$

which characterises the amount of non-universality in the gaugino masses, the weak scale slepton masses write

$$m_{\tilde{e}_{R}}^2 = M_0^2 + 0.88 r_{12}^2 M_2^2 - \sin^2 \theta_W D_z$$

$$m_{\tilde{e}_{L}}^2 = M_0^2 + (0.72 + 0.22 r_{12}^2) M_2^2 - (0.5 - \sin^2 \theta_W) D_z$$

$$m_{\tilde{\nu}_e}^2 = M_0^2 + (0.72 + 0.22 r_{12}^2) M_2^2 - D_z/2 \quad \text{with} \quad D_z = M_Z^2 \cos(2\beta)$$

Here the gaugino masses, $M_{1,2}$, are defined at the electroweak scale. To obtain $M_1 = r_{12} M_2$ at the electroweak scale one needs $\overline{M}_1 \approx 2 r_{12} M_2$ where $\overline{M}_i$ are defined at the GUT scale and $M_2 \approx 0.825 M_2$. In the mSUGRA models one has $\overline{M}_1 = \overline{M}_2$. The parameter $M_0$ is here defined at the GUT scale. For the sake of simplicity we have neglected the Yukawa couplings in the RGE that could affect the $\tilde{\tau}$ sector, however mixing will be taken into account through the $\mu$ term as we will see.

We will be mainly interested in models where $r_{12} << 1$, then in the RGE equation all terms in $r_{12}$ will be negligible. One obtains a natural splitting between the right-handed/left-handed sfermion masses. Indeed, $m_{\tilde{e}_{R}} \approx M_0$ and is typically much lighter than $\tilde{e}_{L}$ which receives in addition a contribution, $m_{\tilde{e}_{L}}^2 \propto M_2^2$.

The non-universality in the GUT-scale relation for gauginos which we investigate in this paper are quite plausible as many models beyond mSUGRA feature some kind of non-universal masses. For example SUGRA models with non-minimal kinetic terms [18], superstring models with moduli-dominated or a mixture of moduli and dilaton fields [13, 14, 12, 15, 17] and anomaly mediated SUSY breaking models [11] all feature non-universal masses in the gaugino and/or scalar masses.

Note in passing that Eq. (6) can be extended to squarks and if we take $\overline{M}_3 = r_{32} \overline{M}_2$ with $r_{32} > 1$ at the GUT scale one could make the squarks “naturally heavy” as we have assumed. Then the gluino mass relation $m_{\tilde{g}} \sim 4 M_2$ obtained in mSUGRA type models turns into $m_{\tilde{g}} \sim 4 r_{32} M_2$ ($r_{32} > 1$). First and second generation squark masses, neglecting the small bino contribution $\propto M_1^2$, can be approximated as

$$m_{\tilde{q}_{L,R}}^2 \sim m_{\tilde{e}_{L,R}}^2 + 0.6 m_{\tilde{g}}^2$$

We will first consider the case where the pseudoscalar is heavy, $M_A = 1$ TeV. Altogether we allow 6 free parameters at the electroweak scale and fix $M_3 = m_{\tilde{q}} = 1$ TeV:

$$\tan \beta, M_1, M_2, M_0, \mu, A_q$$

(8)
\(A_{q,l}\) are the tri-linear terms for the quark \(q\) and lepton \(l\). For scans over the parameter space, unless otherwise specified, we will consider the range

\[
5 < \tan \beta < 50, \\
M_2 < 2000 \text{ GeV}, \\
0.001 < r_{12} < 0.6, \\
|\mu| < 1000 \text{ GeV}, \\
|A_t| < 2400 \text{ GeV}, \\
M_0 < 1000 \text{ GeV}.
\]

(9)

We will fix \(A_t = 0\) for all sleptons as most of the processes we will discuss are not very sensitive to the exact value of this parameter. Although the mixing in the stau sector, which is \(\tilde{A}_\tau = A_\tau - \mu \tan \beta\), can be relevant for the calculation of the relic density, in our case we always have \(|\mu| \tan \beta > 1000\) so this term usually dominates the mixing. Finally we will consider also the case where the pseudoscalar mass is a free parameter varying it in the range

\[
92 \text{GeV} < M_A < 1 \text{TeV}.
\]

(10)

3 Limits from LEP

The direct limits from LEP2 on charginos, neutralinos as well as on sleptons are relevant for the lower bound on the lightest neutralino. As argued, the sleptons play an important role in the relic density calculation especially when the light neutralino is a bino. Here we revisit the lower limits that can be obtained by LEP2 on sfermions and gauginos/Higgsinos when one relaxes the gaugino universality assumptions.

Charginos

The lower bound on the chargino mass rests near the kinematic limit, \(m_{\tilde{\chi}^\pm} > 103.5\text{GeV}\) unless the sneutrino lies in the range \(75 < m_{\tilde{\nu}_e} < 85\text{GeV}\), then the bound drops to \(m_{\tilde{\chi}^\pm} > 73 \text{GeV}\) \[7\]. This is due to the destructive interference between the \(t\) and \(s\)-channel contributions to \(e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^-\). These bounds basically translate into bounds on \((M_2, |\mu|) > (73, 103)\text{GeV}\).

Neutralinos

The LEP experiments quote a lower limit on the neutralino mass, \(m_{\tilde{\chi}_1^0} < 59.6\text{GeV}\), while assuming unified gaugino masses at the GUT scale, \(r_{12} = 0.5\) \[6\]. This constraint on the neutralino mass is basically derived from the lower limit on the chargino mass obtained in the pair production process, \(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-\) which depends on \(M_2, \mu\). It is only through the relation between the gaugino parameters \(M_1\) and \(M_2\) that the neutralino mass limit is obtained. In a general MSSM, the charginos and neutralino masses are uncorrelated and the lower limit on the neutralino mass weakens when \(r_{12} < 0.5\) becoming roughly \(m_{\tilde{\chi}_1^0} \geq r_{12} \times m_{\tilde{\chi}_1^+}\). Taking into account the constraint from the chargino sector, the lightest neutralinos are then typically found in scenarios where \(M_1 \ll M_2, \mu\), that is the light LSP mass is set by the parameter \(M_1\) and is thus mostly a bino.

In such scenarios, the processes \(e^+e^- \rightarrow \chi_{11}^0 \chi_{12}^0, \chi_{13}^0\) can be used to somewhat constrain the parameter space. In particular, when sleptons are light, the neutralino cross-sections
depends crucially on the Higgsino content. The cross-sections are generally enhanced and regions with small $\mu$ can be more severely constrained by the neutralino production cross sections than by the chargino process \cite{30}. In our scans, we have implemented the upper limit from the L3 experiment on $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 + \tilde{\chi}_3^0 \tilde{\chi}_3^0 \rightarrow E^+l^-)$, with $l = e, \mu$, using the tables in \cite{39} that simulate both the signal and background.

The radiative processes where a photon is emitted in addition to a pair of invisible supersymmetric particles, like the lightest neutralino, will contribute to the process $e^+e^- \rightarrow \gamma + \text{invisible}$ which has been searched for by the LEP2 experiments. The neutralino radiative process $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma$, is essentially dominated by the $t$-channel selectron exchange diagrams and has a small cross-section \cite{40, 37}. We computed the double differential cross-section (energy and angular distribution of the photon) as well as the total cross-section exactly using calcHEP \cite{41} for a wide range of parameters in the MSSM. The cross-section can reach $\sigma = 50 \text{fb}$ for $m_{\tilde{e}_R} \approx 100 \text{GeV}$ but drops steadily as the scalar mass increases. While a few events are expected at LEP2, this is not enough to overcome the uncertainty in the standard model neutrino contribution to the $\gamma + \text{invisible}$ channel. Only mild improvement is obtained from fitting to the full energy-angular distribution as the distribution is somewhat similar to the background, peaked near low-energy photon. We conclude that one cannot constrain any further the neutralino mass using this process independently of the selectron mass.

In principle the upper limit on the invisible width of the $Z$, $\Gamma_{Z_{\text{inv}}} < 3 \text{MeV}$ from LEP1 can also constrain the lightest neutralino. In the particular case of a bino LSP with $M_1 \ll M_2$, the coupling to the $Z$ strongly depends on the amount of Higgsino mixing. Thus one obtains a lower bound on $\mu$, in the large $\tan \beta$ limit, $|\mu| > 110 \text{GeV}$. This constraint is in general already satisfied after taking into account the LEP2 data on chargino/neutralino production.

**Sfermions**

For selectrons, a limit of 99.5GeV can be set on both $\tilde{e}_L$ as well as $\tilde{e}_R$ in the case of a light neutralino, whereas basically model independent limits of $m_{\tilde{\mu}} > 96 \text{GeV}$ and $m_{\tilde{\tau}} > 86 \text{GeV}$ can be reached \cite{7}. It is important to point out that the lower limit on the stau is about 10GeV smaller compared to the smuon and the selectron as this will have a consequence on the relic density contribution. Note also that because of the mixing is stau sector (which in our case is induced solely through the $\mu$ term), we can with the same minimum value of $M_0$ in Eq. 6 arrive at lower values for staus than for selectrons.

**Higgs**

In models where the pseudoscalar mass is heavy, the limit on the lightest CP-even Higgs mass from LEP2, $M_h > 114.4 \text{ GeV}$, applies. We have used FeynHiggsfast \cite{42} to calculate the Higgs mass and have imposed the limit $M_h > 113 \text{ GeV}$ to allow for theoretical uncertainties.

In models with a light pseudoscalar the above LEP2 constraint is relaxed. When $M_h \approx M_A$ and $\cos(\alpha - \beta) \approx 1$, the channel $e^+e^- \rightarrow hZ$ is strongly suppressed. LEP2 can only make use of the $hA \rightarrow b\bar{b}b\bar{b}, \tau\bar{\tau}bb$ channels to put an absolute bound of $M_h, M_A > 91.6 \text{ GeV}$ \cite{43, 44, 45}. In these models, the heavy CP-even Higgs channel $e^+e^- \rightarrow HZ$ which is $\propto \sin^2(\alpha - \beta)$ is favoured. Unfortunately the mass splitting between the two scalar Higgses ($M_H > M_A$) is generally sufficient to put the heavy Higgs beyond the reach of LEP2.
4 Indirect limits : $\Omega h^2, (g - 2)_\mu, b \to s\gamma, B_s \to \mu^+\mu^-$ and $Z \to \bar{b}b$

4.1 Relic density of neutralinos

The MSSM model with a light stable neutralino must be consistent with at least the upper limit on the amount of cold dark matter. Here we take the latest bound from WMAP, $\Omega h^2 < .128$ and also compare with the old limit (of 2001) from BOOMERANG [16], MAXIMA [17] and DASI [48] with $\Omega h^2 < 0.3$. We will refer to this limit as pre-WMAP. Our calculations of the relic density is based on micrOMEGAs [49], a program that calculates the relic density in the MSSM including all possible coannihilation channels [49]. For the light neutralino masses under consideration, it is the main neutralino annihilation channels that are most relevant, in particular annihilation into a pair of light fermions. Basically two types of diagrams contribute, s-channel $Z$ (or Higgs) and $t$-channel sfermion exchange. A light neutralino that is mainly a bino couples preferentially to right-handed sleptons, the ones that have the largest hypercharge. To have a large enough annihilation rate (in order to bring down the relic density below the upper limit allowed) one needs either a light slepton or a mass close to $M_{Z,h,h,H,A}/2$. In the former case, the constraint from LEP on sleptons and in particular staus plays an important role. In the heavy slepton case, the coupling of the $Z$ should be substantial, which requires that the neutralino should have a certain Higgsino component [30, 31]. This means $\mu$ small, but still consistent with the chargino constraint.

4.2 $(g - 2)_\mu$

To derive the bound on $\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{theo}}$ we take into account the recent correction to the sign of the hadronic light by light contribution, $a_\mu^{\text{LBL}}$ [50, 51]. Considering that there are still a few issues that need to be clarified, see for instance [52], we allow a larger theoretical error and take like in [53] $a_\mu^{\text{LBL}} = (80 \pm 40) \times 10^{-11}$. For the hadronic vacuum polarisation we take an average of the new results. Jegerlehner has reported a new value that includes BES, CMD-2 with a value on the hadronic part $a_\mu^{\text{had}} = (6889 \pm 58) \times 10^{-11}$ [54]. Teubner [55] quotes two values (inclusive and exclusive, depending how the data is integrated) but uses QCD sum rules to favour the value extracted from inclusive data, $a_\mu^{\text{had}} = (6831 \pm 59 \pm 20) \times 10^{-11}$. Finally Davier [56] reported a value based on $e^+e^-$-data $a_\mu^{\text{had}} = (6847 \pm 60_{\text{exp}} \pm 36_{\text{th}}) \times 10^{-11}$ as well as another one based on $\tau^+\tau^-$ data, $a_\mu^{\text{had}} = (7090 \pm 47 \pm 12_{\text{exp}} \pm 38_{\text{SU(2)}}) \times 10^{-11}$. Since these two values are slightly inconsistent we will first consider a limit coming from an average of the hadronic estimates from $e^+e^-$ alone as well as a more conservative limit encompassing both $e^+e^-$ and $\tau$ results. 

The theoretical estimate is obtained after adding the pure QED contribution, $a_\mu^{QED}$, the weak contribution, $a_\mu^{\text{weak}}$ and the three different hadronic contributions: $a_\mu^{\text{had,LBL}}$ and the NLO hadronic contribution, $a_\mu^{\text{had/NLO}}$.

$$a_\mu^{\text{theo}} = a_\mu^{QED} + a_\mu^{\text{weak}} + a_\mu^{\text{had}} + a_\mu^{\text{LBL}} + a_\mu^{\text{had/NLO}}.$$ 

\(^2\)The new update analysis of Davier et al. [56] finds a similar $\tau$-based result and a much better agreement between the $e^+e^-$-based and $\tau$-based value. We have not used these values in our analysis as our conservative range encompasses already the newest estimates.
With the latest experimental data on the $g - 2$ measurement \[4\], bringing the world average to
\[
\alpha_{\mu}^{\text{exp}} = 11659203 \pm 8 \times 10^{-10}
\] (11)
we get, after averaging of the three different hadronic contributions from $e^+e^-$ data alone,
\[
\delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{theo}} = (33.6 \pm 8_{\text{exp}} \pm 11.5_{\text{theo}}) \times 10^{-10}
\] (12)
Adding linearly the theoretical error to a $3\sigma$ experimental error leads to the allowed range,
\[-2 < \delta a_{\mu} \times 10^{10} < 69 \] (13)

Including the $\tau$ data analysis of Davier \[56\] in the average, reduces the discrepancy between theory and experiment
\[
\delta a_{\mu} = (27.8 \pm 8_{\text{exp}} \pm 11.3_{\text{theo}}) \times 10^{-10}
\] (14)
The $3\sigma$ allowed range now reads,
\[-7.5 < \delta a_{\mu} \times 10^{10} < 63 \] (15)
As the results have changed frequently in the last months and since many issues need to be clarified in particular in the estimation of the hadronic contribution, the most conservative allowed range is found by including two allowed bands, instead of performing an average. Combining the value obtained from $e^+e^-$ data (Eq. 13) with the one from $\tau$ data alone of Davier, one gets a $3\sigma$ range
\[-25 < \delta a_{\mu} \times 10^{10} < 69 \] (16)
where the lower bound is set by the $\tau$ data and the upper bound from Eq. 13. We will refer to Eq. 16 as our conservative bound on $(g - 2)_{\mu}$.

In the MSSM, $a_{\mu}$ gets a contribution both from neutralino and chargino loops, the latter being dominant. One expects large effects for large $\tan \beta$ and light sfermions ($\tilde{\mu}/\tilde{\nu}_{\mu}$) \[57\]. Typically, the sign of $\mu$ is strongly correlated to the one of $\delta a_{\mu}$. However there are special cases where cancellations can occur between the chargino and the neutralino diagrams thus flipping the relative sign of $\delta a_{\mu}$ and $\mu$. This happens for charginos much heavier than neutralinos and the resulting value of $\delta a_{\mu}$ is small. This property will open a small corner of parameter space where models with $\mu < 0$ will be allowed even when imposing the bound in Eq. 13. When $\mu > 0$ on the other hand, one expects rather mild constraints on the parameters of the MSSM, furthermore the constraints will be mostly in the chargino/slepton sector rather than directly on the neutralino sector.

4.3 $b \to s\gamma$

Our calculation of the $b \to s\gamma$ closely follows the approach of Kagan and Neubert \[58\] (see also \[59\]) with NLO, bremsstrahlung and some non-perturbative effects taken into account.

\[\text{This last bound is similar to the one we had obtained (at } 2\sigma) \[4] \text{ before the new calculations and the more precise results came out in summer 2002.} \]
This approach makes it easier to include the effects of New Physics. Our rates refer to the so-called “faked” total rate with a cut on the photon energy corresponding to $\delta = 0.9 \, (E_\gamma > m_b/20)$. We have updated this analysis by including in a simple way the recent suggestion of Misiak and Gambino [60] of using the $\overline{MS}$ charm mass. The Wilson coefficients for the standard model and the charged Higgs are evaluated at the NLO [61], whereas we only include the remaining SUSY contributions at LO [62] with however the important inclusion of the enhanced large $\tan$ $\beta$ effects [63] (SUSY threshold corrections to the running $b$ mass). For the latter we include both the strong $\alpha_s$ contribution as well as the Yukawa contribution for the SUSY Wilson coefficients as well as the charged Higgs and the Goldstone contribution\footnote{We have corrected some typos in [63]. We thank Paolo Gambino for checking our results and agreeing with our implementation.}. Our standard model value (with scale parameters set at $m_b$), gives $Br(b \to s\gamma) = 3.72 \times 10^{-4}$ while the scale and other parameter uncertainty ($\alpha_s$, CKM matrix elements) are about 10%. To bound the SUSY contribution we take the world weighted average of the CLEO [64] BELLE [65] and ALEPH [66] measurements

$$Br(b \to s\gamma) = 3.23 \pm 0.42 \times 10^{-4}$$

(17)

We require that after allowing for the (scale) uncertainty in the theory calculation the result must be within $2\sigma$ of the experimental result, Eq. (17). Since the theory uncertainty is roughly constant over the SUSY parameter space and in order to have a faster scan we have allowed for a conservative fixed uncertainty of 10% independently of the SUSY parameters. Thus in effect we require the theory prediction to fall within the range

$$2.04 < Br(b \to s\gamma) \times 10^{-4} < 4.42$$

(18)

In the MSSM, contributions to the $b \to s\gamma$ depends mostly on the squark and gaugino/Higgsino sector as well as on the charged Higgs. The heavy squarks that we consider do not completely decouple and one can get substantial corrections to the SM branching ratio. In particular at large $\tan$ $\beta$ there is a strong $A_t$ dependence through a term $\propto A_t \tan$ $\beta$ from the mixing in the stop sector. However, the light Higgs mass is also sensitive to the mixing in the stop sector. As long as the pseudoscalar is heavy, one finds that for small values of $\tan$ $\beta$, one can find values of $A_t$, typically $A_t$ large and positive, that satisfy both the Higgs mass limit as well as the $b \to s\gamma$ bound. For light pseudoscalars on the other hand we will see that it is very difficult to satisfy the $b \to s\gamma$ bound. At large values of $\tan$ $\beta$, the Higgs bound is more easily satisfied and one can pick a set of values for $A_t$ (typically $A_t < 0$) that allows the correct amount of $b \to s\gamma$ whether or not the pseudoscalar is heavy. Note that the light squarks contribution to $b \to s\gamma$ rapidly becomes too important. This again justifies our choice of large squark masses.

In the end we find that because of the free mixing parameter in the stop sector, the $b \to s\gamma$ constraint has a mild impact on the models considered at least as concerns the mass of the neutralino. One exception is the light pseudoscalar case that will be discussed in more details in the last section.

4.4 $B_s \to \mu^+ \mu^-$

The CDF experiment at Fermilab has obtained an upper bound on the branching ratio

$B.R.(B_s \to \mu^+ \mu^- < 2.6 \times 10^{-6})$ [67] and should be able to reach $B.R.(B_s \to \mu^+ \mu^- <$...
2. \times 10^{-7}) \text{ in RunIIa. In the SM, this branching ratio is expected to be very small (} \approx 3 \times 10^{-9}). \text{ In the MSSM SUSY loop contributions due to chargino, sneutrino, stop and Higgs exchange can significantly increase this branching ratio. In particular, the amplitude for Higgs mediated decays goes as } \tan^3 \beta \text{ and orders of magnitude increase above the SM value are expected for large } \tan \beta. \text{ This process is relevant mainly in the light pseudoscalar scenario. Our calculation is based on [68] and agrees with [69]. } \Delta m_b \text{ effect relevant for high } \tan \beta \text{ are taken into account.}

4.5 \quad Z \rightarrow b \bar{b}

We have also included the constraint from } Z \rightarrow b \bar{b} \text{ although in most cases it is completely harmless. For example, the charged Higgs also contributes to } Z \rightarrow b \bar{b}. \text{ However this puts a constraint on the Higgs sector only for Higgs masses below the LEP limits. Our calculation of } Z \rightarrow b \bar{b} \text{ is along the lines of [70] but we have corrected a few typos contained in [70].}

5 \quad \text{The lower bound on } m_{\chi^0_1} \text{ when } M_A = 1 \text{ TeV}

In this section we present our results for the case of a heavy pseudoscalar after imposing all direct and indirect constraints. In particular we have imposed only the conservative limit on \((g-2)_\mu\), Eq. 13. Nevertheless it is worth discussing the impact of the much stricter bound, Eq. 16. As the allowed region in } \mu < 0 \text{ models changes drastically depending on how one evaluates the hadronic contribution, we will defer the extensive discussion of this constraint in the section devoted to the } \mu < 0 \text{ case.}

5.1 \quad \mu > 0

We first consider the case } \mu > 0. \text{ In these scenarios, the relic density of dark matter provides the main constraint on models with light neutralinos. Indeed the LEP limits on charginos and neutralinos allow very low values for neutralino masses provided } M_1 \ll M_2. \text{ The } (g-2)_\mu \text{ constraint is in general easily satisfied except for very large values of } \tan \beta. \text{ The constraint for } b \rightarrow s \gamma \text{ also affects mostly models with large values of } \tan \beta.

1) Heavy sleptons: } M_0 = 500 \text{GeV.}

In this situation all sfermions are heavy. As a consequence their contribution to the relic density is negligible. Annihilation of the LSP dark matter through s-channel } Z \text{ and Higgs exchange is on the other hand possible. Then in order to have a sufficient neutralino annihilation rate, to satisfy the upper limit on the relic density, a large enough coupling to the } Z \text{ is necessary. This requires the LSP (which is mostly a bino) to have a certain amount of Higgsino component, especially as one moves away from the } Z \text{ peak. Bino-Higgsino mixing has little } \tan \beta \text{ dependence and scales like } M_Z/\mu \text{ which calls for the smallest } |\mu| \text{ possible. It can also be enhanced somewhat if there is little splitting between } M_1 \text{ and } \mu, \text{ but this would generally not minimise the LSP mass. We scanned over the parameters } r_{12}, M_2, \mu \text{ and } \tan \beta \text{ as specified in Eq. 9. The relic density as a function of the neutralino mass clearly shows the effect of the } Z \text{ peak and the lower limit }
Figure 1: a) Relic density of the LSP vs $m_{\tilde{\chi}}$ in the MSSM with $M_0 = 500$ GeV. To guide the eye the WMAP upper bound is displayed. b) Values of $\mu$ consistent with the LEP and relic density constraints. We scan on $\tan \beta$, $M_2$, $r_{12}$ and $\mu$.

on the neutralino mass, $m_{\tilde{\chi}} \approx 32$ GeV, Fig. 1a. Note that although some points give a relic density that is much too low, models with $\Omega h^2 \approx 0.1$ can be found for any value of the neutralino mass consistent with the lower limit of $\approx 32$ GeV. In particular for the lightest neutralinos in this scenario (say within 2 GeV of the lower limit) the value of the relic density falls within the range of WMAP. Incidentally, Fig. 1 also shows that the relic density can drop quite dramatically even beyond the $Z$-peak and upon inspection even higher $\mu$ values are allowed in this range. This corresponds to the contribution to the $s$-channel lightest Higgs $h$ with $m_{\tilde{\chi}} \approx m_h/2$. Since in the scenario with $M_A = 1$ TeV, $m_h > 113$ GeV this occurs beyond the $Z$-peak. One of the reasons this second peak looks broader is because we are scanning over a wide range of $h$ masses. Another reason is that the coupling of the (almost) bino LSP is larger for the Higgs than it is for the $Z$. For the latter the coupling is in fact quadratic in the Higgsino-bino mixing. As we will discuss later, we expect the presence of light Higgses other than $h$ to reduce the value of $\Omega h^2$ and for masses of the order of the $Z$ mass to allow an even lower limit of neutralino LSP.

Fig. 1b clearly shows the preference for a significant Higgsino component, the small $\mu$ region, when the neutralino mass is below $M_Z/2$. The minimum value for the LSP mass, $m_{\tilde{\chi}} \approx 32$ GeV, occurs for $r_{12} < 0.2$, see Fig. 2. As $r_{12}$ approaches the mSUGRA value, $r_{12} = 0.5$, the limit from the chargino mass at LEP dominates and the relic density constraint has no effect. This lower bound is more or less independent of $\tan \beta$ in the range under consideration $5 < \tan \beta < 50$. Fig. 2 also shows how the improvement of the bound on the relic density from WMAP has strengthened the lower limit on $m_{\tilde{\chi}}$ by $\approx 5$ GeV when $r_{12} < 0.2$. For larger values of the non universality parameter, the improvement from WMAP is only marginal. Indeed, already with $\Omega h^2 < 0.3$, one is confined to a region ($m_{\tilde{\chi}} > 35$ GeV) close enough to the $Z$ peak so that $\Omega h^2$ drops sharply with $m_{\tilde{\chi}}$ (Fig. 1a).

The preference for the small $\mu$ region also implies that light neutralinos, say below 40 GeV, are necessarily accompanied by light charginos (below 250 GeV). However, as soon as one moves closer to $M_Z/2$, the Higgsino component does not have to be too small and
one generally has $m_{\tilde{\chi}^+} < 450$GeV. This has important consequences for colliders. For example it could be difficult for the Tevatron to find charginos in the trilepton channel ($\chi^+\chi^0_2$). Similarly a 500GeV collider would not be able to find charginos in some scenarios as will be discussed in the last section. Note that under the conditions specified here, no improvement on the lower limit on the neutralino can be set from $(g-2)_{\mu}$.

2) Light sleptons: $M_0 < 500$GeV

In addition to the effect of the $Z$ exchange one expects the contribution from $t$-channel sfermions to $\chi\chi \rightarrow f\bar{f}$ to weaken the constraint from the relic density. Here the constraints on the masses of selectrons/smuons and staus from LEP2 are important. The latter is relevant for large values of $\tan \beta$ when the $\tilde{\tau}$ can be much lighter than the selectron. As the left-right mixing in the stau sector is proportional to $A_{\tau} - \mu \tan \beta$ this is especially true in the large $\mu$ region. The (lightest) $\tilde{\tau}$ contribution to the relic density in setting the lower bound on the LSP is largest among all sleptons. Not only there is a $S$-wave contribution ($\propto m_{\tilde{\tau}}^2$) but the (dominant) $P$-wave contribution also features an additional (positive) left-right mixing. But most importantly this mixing allows for the smallest mass of the $\tilde{\tau}$ compatible with the lowest mass on the $\tilde{\tau}_1$ set by LEP which is about 10GeV lower than for the lightest smuon and selectron.

First consider the case $\tan \beta = 10$ and $r_{12} = 0.1$. With the constraint $\Omega h^2 < 0.3$, one finds that at low values of $M_0$, the parameter that governs the mass of the right-handed sfermions, the lower limit on $m_{\tilde{\chi}_1^0}$ goes down by a few GeV’s relative to the one at large values of $M_0$, Fig. 3. However the tail at low $M_0$ is cut-off to a large extent (by $\approx 10$GeV) when taking into account WMAP. When imposing the constraint $\Omega h^2 < 0.3$, we find that one does not rely exclusively on the Higgsino content of the neutralino to derive a lower bound. Large $\mu$ values are compatible with neutralinos $m_{\tilde{\chi}_1^0} \approx 18$GeV. However, with the much tighter constraint from WMAP, and after including the LEP constraint, the $t$-channel sfermion exchange alone is not sufficient to bring the relic density in the allowed range for $m_{\tilde{\chi}_1^0} < 35$GeV. Once again a non-negligible Higgsino component is required to have

![Figure 2: Lower limit on $m_{\tilde{\chi}_1^0}$ vs $r_{12}$ after imposing the relic density constraints. The limit from LEP2 alone is also displayed. Here $M_0 = 500$ GeV, $\mu > 0$ and we scan on $M_2, \tan \beta, |\mu|$. The region below the lines is excluded.](image-url)
Figure 3: Region allowed after imposing LEP and relic density constraints for $\tan \beta = 10$, $r_{12} = 0.1, \mu > 0$ and $M_A = 1\text{ TeV}$, a) in the $M_0 - m_{\tilde{\chi}_1^0}$ plane b) in the $\mu - M_0$ plane. Here we scan on $M_0, M_2, \mu$ and $A_t$. Circles (crosses) have $\Omega h^2 < 0.3(0.128)$. Half the points in the scan are generated in the region corresponding to $M_0, \mu < 150\text{ GeV}$.

sufficient coupling to the $Z$ (Fig. 3). This entails, as was the case for heavy sleptons, that light neutralinos with masses close to the lower bound are necessarily accompanied by light charginos. However, for $m_{\tilde{\chi}_1^0} \approx M_Z/2$, $m_{\tilde{\chi}_1^+}$ can now reach as much as $\approx 1\text{ TeV}$.

We show in Fig. 4 the lower limit on the neutralino mass for $\tan \beta = 10$ and $\tan \beta = 50$ as a function of $r_{12}$ after scanning over $M_2, \mu, M_0$ and $A_t$. The relic density improves the constraint on the mass of the light neutralino only after having taken into account the LEP2 constraint on selectrons (and on staus for large values of $\tan \beta$) as well as the constraint from $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0, \tilde{\chi}_1^0\tilde{\chi}_3^0$. After WMAP, the absolute lower limit moves to $m_{\tilde{\chi}_1^0} \approx 29\text{ GeV}$ for $\tan \beta = 10$. This limit is obtained in the range $r_{12} < 0.1$ and increases with $r_{12}$. In the whole region $r_{12} < 0.3$, the new WMAP data improves by up to $10\text{ GeV}$ the lower limit on the neutralino mass obtained with pre-WMAP data. As for heavy sleptons, the constraint on the chargino mass from LEP2 sets the lower limit on the neutralino mass when $0.3 < r_{12} < 0.6$. We have here also applied the upper bound on the $(g-2)_\mu$, Eq. 16. However this additional constraint does not affect the lower bound on the neutralino mass.

For $\tan \beta = 50$, a lower limit of $m_{\tilde{\chi}_1^0} > 18\text{ GeV}$ is obtained for $r_{12} < 0.06$ and increases rapidly with $r_{12}$. This lower limit lies below the one obtained for lower values of $\tan \beta$. The main reason is the contribution of the channel $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow \tau^+\tau^-$ with $\tilde{\tau}$ exchange which is enhanced because of the large $\tilde{\tau}_L-\tilde{\tau}_R$ mixing. The new WMAP data sets the lower limit on neutralino as long as $r_{12} < .4$. Although we have taken a unified scheme for generating the slepton masses along Eq. 6 with a common $M_0$ at high scale, we would have arrived at the same LSP lower limit had we varied the slepton masses independently. This is due, as explained earlier, to the fact that from LEP the lowest bound on the slepton applies to the $\tilde{s}$. On the other hand to use the bound from $(g-2)_\mu$ to further constrain the parameter space tacitly assumes universality, at least as implemented through Eq. 6. Fig. 4b) shows the impact of applying the $(g-2)_\mu$ bound alongside the relic constraint. For $\tan \beta = 10$

\footnote{In \cite{71} a lower limit of 18 GeV was obtained for any value of $\tan \beta$, using $\Omega h^2 < 0.3$, however a universal limit on $m_{\tilde{l}} > 100\text{ GeV}$ was imposed.}
there is no impact, however for \( \tan \beta = 50 \) although the lowest bound at very small \( r_{12} \) is hardly affected by \((g - 2)_\mu\), for intermediate \( r_{12} \sim 0.1 - 0.2 \) one can improve the limit on the LSP bound by about 6GeV. This is related to the fact that both the smallest values of \( M_0 \) as well as of \( M_2 \), corresponding to light smuons and charginos, give too large a contribution to \((g - 2)_\mu\) as will be discussed below.

### 5.2 \( \mu < 0 \)

It has often been claimed that models with negative values of \( \mu \) are ruled out by the \((g - 2)_\mu\). Within our conservative approach this is not the case not only because the present data and the calculation of the hadronic contribution allows \( \delta a_\mu < 0 \) but also because with nonuniversal gaugino masses the sign of \( \delta a_\mu \) and \( \mu \) are not necessarily correlated. Considering the potential relevance of the \((g - 2)_\mu\) for \( \mu < 0 \) models, we will first discuss only this constraint together with LEP before folding in the constraints from the relic density and \( b \to s\gamma \). To a certain extent we will see that these two sets of constraints work in the same direction restricting the region with light sleptons and light charginos/neutralinos.
The most important contribution to the \((g - 2)_{\mu}\) occurs for light smuons and light charginos and is enhanced at large values of \(\tan \beta\). First consider the case \(r_{12} = 0.1\) and light sleptons, \(M_0 = 150\) GeV. When \(\tan \beta = 10\), only a few models with very light

\begin{align*}
\delta a_{\mu} (10^{-9})
\end{align*}

large, one derives a lower limit on the neutralino of \(m_{\tilde{\chi}_0^1} > 50\)GeV for \(\tan \beta = 50\). It is in the small \(|\mu|\) region that one finds the largest deviation in the value of \(\delta a_{\mu}\). For \(\tan \beta = 50\) one quickly exceeds even the conservative 3σ bound. However a cancellation between the chargino and the neutralino diagrams can change the relative sign of \(\delta a_{\mu}\) and \(\mu\), Fig. 5b-c.

As the neutralino contribution is in general rather small, for this type of cancellation to occur, the chargino contribution must be somewhat suppressed (the large \(M_2\) region) furthermore a significant amount of Higgsino/gaugino mixing is required, (Fig. 5b(c)). Typically this sign flip is compatible with light neutralinos, say \(< 50\)GeV, only in the context of nonuniversal models where the appropriate hierarchy of parameters can be found, that is \(M_1 < \mu << M_2\). For heavy sleptons, the predicted values for \(\delta a_{\mu}\) are in general much smaller and all the parameter space (for \(r_{12} = 0.1\)) satisfies the experimental bound (see Fig. 5d).

Fig. 5 shows the exclusion region in the \(M_0 - M_2\) plane for three different exclusion values for \(\delta a_{\mu}\) (depending on the assumptions on the calculation of the hadronic contribution).
and for different values of the non-universality parameter. Here both $\mu$ and $A_\tau$ are kept as free parameters and the LEP limits on $m_h$ and on charginos and sleptons are imposed. The $(g-2)_{\mu}$ rules out regions where $M_0$ and $M_2(M_1)$ are small. However, no lower limit on the neutralino can be derived as light charginos/neutralinos are allowed when sleptons are very heavy. If one would impose the more restrictive limits on $\delta a_\mu$, Eq. 13, it would be possible to accommodate light sleptons and neutralinos in models where $r_{12} < 0.5$, Fig. 6. Indeed, the partial cancellation between the chargino and neutralino diagrams leads to a limit on $M_2$ that is not as strong as in the universal case. With this restrictive bound it is possible to set a lower limit on the neutralino from $(g-2)_{\mu}$ only for large values of $\tan \beta$.

![Figure 6: Contours $\delta a_\mu \times 10^{-10}$ in the $M_0 - M_2$ plane with $\mu < 0$ for $\tan \beta = 10$ (left) and $\tan \beta = 50$ (right). From top to bottom $r_{12} = 0.5, 0.1, 0.03$. The top panel $r_{12} = 0.5$ is based on the usual GUT assumption. Only LEP constraints are folded in. For $\tan \beta = 50$, the region allowed by $b \to s\gamma$ lies to the right of the dotted line and as can be seen it does not further restrict the bound set by $(g-2)_{\mu}$.](image)

Note that we have only discussed explicitly the case $A_\mu = 0$, while corrections to the $(g-2)_{\mu}$ are expected for large values of $A_\mu$ it does not strongly affect the allowed region as long as $A_\mu < 1$ TeV. Indeed when $\tan \beta$ is small there is not much constraint and when $\tan \beta$ is large we have in any case $|\mu| \tan \beta >> |A_\mu|$. 

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5.2.2 Combining all constraints

The impact of the relic density constraint coupled to direct collider limits from LEP is somewhat similar to what was discussed in the case $\mu > 0$. Here we stress the impact of adding the indirect constraints from the $(g-2)_{\mu}$.

- $\mu < 0$, $\tan \beta = 10$
  With the new WMAP data, it becomes increasingly difficult to satisfy the relic density constraint. Even with light sleptons, sufficient annihilation of neutralinos into fermion-antifermion pairs requires the neutralino to be not too far away from the $Z$ pole. As we had already found for $\mu$ positive the lower limit on the neutralino mass increases significantly in the region $r_{12} < 0.1$, from $m_{\tilde{\chi}_1^0} > 18\text{GeV}$ based on the pre-WMAP to $m_{\tilde{\chi}_1^0} > 29\text{GeV}$ with WMAP, Fig. 7a. Note that in this region, the bound on the $e^+e^- \rightarrow \tilde{\chi}_1\tilde{\chi}_1^0$ cross section from LEP has a significant impact in constraining the small $\mu$ region. As a result, the lightest neutralino allowed by LEP and WMAP alone occurs for for larger values of the non universality parameter, $r_{12} \approx 0.15$ with a bound $m_{\tilde{\chi}_1^0} > 26\text{GeV}$. It is only by imposing the $(g-2)_{\mu}$ bound that one increases the lower bound on the LSP in the region $0.1 < r_{12} < 0.4$. It is also because of the importance of $(g-2)_{\mu}$ that WMAP does not sensibly improve the LSP bound in this region compared to the pre-WMAP data.

- $\mu < 0$, $\tan \beta > 10$
  For larger values of $\tan \beta$, even our most conservative bound on $\delta a_{\mu}$ significantly restricts the LSP mass, except for the smallest $r_{12}$ values below about 0.03. Past this value the $(g-2)_{\mu}$ bound is more restrictive in setting a limit on the LSP mass, see Fig. 7b and

Figure 7: Lower bound on the neutralino mass vs $r_{12} = M_1/M_2$ for $\mu < 0$ and $a) \tan \beta = 10$, $b) \tan \beta = 50$. All direct and precision measurements constraints are combined with the relic density limits (dotted lines) while the constraint from $\delta a_{\mu}$ is removed (full lines).
very much improves on the WMAP result. As a result the lowest LSP mass $m_{\tilde{\chi}_1^0} \sim 22$ GeV is found for $r_{12} \sim 0.03$. Had we used the pre-WMAP result, this limit would have been 6GeV lower. The effect of $(g - 2)_\mu$ for $r_{12} = 0.1$ and $\tan \beta = 50$ is more clearly seen in Fig. [8]. For this set of parameters $m_{\tilde{\chi}_1^0} > 32$GeV are allowed, an increase of nearly 10 GeV had we not implemented the $(g - 2)_\mu$ bound. Note that the $(g - 2)_\mu$ bound also cuts on the region where both $M_2(M_1)$ and $M_0$ are smallest. Fig. [8] also makes it clear that the lightest LSP have mass near the $Z$ threshold which as we have argued calls for a not too large $\mu$. In turn one sees that this scenario also predicts charginos (with a large Higgsino component) that are relatively light, for instance for $m_{\tilde{\chi}_1^+} < 40$ GeV, one expects $m_{\tilde{\chi}_1^0} < 250$GeV. In the region $0.1 < r_{12} < 0.3$, we find that combining all constraints increases the lower bound on the neutralino and that the increase is more significant for very large $\tan \beta$. However, for this range of values for the non universality parameter, WMAP has much less impact. In this region the lower limit on the neutralino depends strongly on the lower allowed value for $\delta a_\mu$. Had we used the stricter bound on $(g - 2)_\mu$, Eq. [13] we would not have found neutralinos below 60GeV for the large values of $\tan \beta$.

![Figure 8](image_url)

**Figure 8:** Allowed region in the $M_0 - M_2$ plane using direct limits and WMAP constraints only (light grey circles) and when taking into account all constraints (dark crosses). Here $\tan \beta = 50$, $\mu < 0$ and $r_{12} = 0.1$.

In summary we have found that the lower limit on the neutralino mass is $m_{\tilde{\chi}_1^0} > 29$GeV for $\tan \beta = 10$ for either sign of $\mu$ and $m_{\tilde{\chi}_1^0} > 18(22)$GeV for $\tan \beta = 50$ and $\mu > 0(\mu < 0)$. The latter is allowed only for large mass splitting in the gaugino sector, $M_1 \ll M_2$. The muon anomalous magnetic moment does constrain models with $\mu < 0$ as well as the large $\tan \beta$ region except in the region where $M_1 \ll M_2$. Furthermore, in the region $r_{12} > 0.1$
and when $\mu < 0$ the lower bound on $m_{\tilde{\chi}_1^0}$ strongly depends on how much negative one allows $\delta a_\mu$ to be.

6 Lowering $M_A$

6.1 Limits on the LSP

The effect of lowering $M_A$, within the range allowed by LEP, has an impact on both the relic density constraint and the indirect limits from the $B$ sector. The relic density can be lower than the one obtained in the heavy pseudoscalar case due to the increase of the $s$-channel Higgs exchange contribution (notably the from the pseudoscalar $A$) to the neutralino annihilation cross section. This had to be expected in view of the importance of the $s$-channel pole contribution as we have already seen for the $Z$ and the lightest Higgs, $h$. Recall that in these models $M_h$ and $M_A$ can be as low as 91.6 GeV without being in conflict with the LEP2 data. In the large tan $\beta$ region, the contribution of such light Higgses can be sufficient to bring the relic density below the upper bound even if one is quite far from the $s$-channel pole due to the enhanced coupling of the Higgs to $b$ quarks and $\tau$'s. This opens up a new region of parameter space for light neutralinos ($m_{\tilde{\chi}_1^0} < 18$ GeV) as was pointed out in [72, 38].

Figure 9: $\Omega h^2$ as a function of $M_A$ for $\mu = 120$ GeV, $M_0 = 1000$ GeV, $M_1 = 0.01 M_2$ and $a)$ tan $\beta = 10$ $b)$ tan $\beta = 50$. Full lines correspond to (from top to bottom) $M_2 = 800, 1200, 1800$ GeV (left) and $M_2 = 300, 500, 800, 1200, 1800$ GeV (right). Additional dashed lines show the effect of varying $\mu$ when $M_2 = 800$ GeV (from bottom to top) $\mu =$ 160, 200, 240, 280 GeV. The dash line indicates the current upper bound from WMAP. Only direct LEP limits are imposed.

The dependence of the relic density, for small $\mu$, on $M_A$ is displayed in Fig. 9 for
$r_{12} = 0.01$ and for $\tan \beta = 10$ and $\tan \beta = 50$. The upper bound on the relic density is satisfied most easily for a light pseudoscalar, $M_A \approx 100 - 120\text{GeV}$ and for large values of $\tan \beta$. For instance $\tan \beta = 10$ is totally excluded by WMAP for the set of parameters chosen, in particular for heavy sleptons, as seen in Fig. 9a. Due to the enhanced coupling of the pseudoscalar to the heavy fermions, $\propto \tan \beta$, Fig. 9 also shows that changing $\tan \beta$ from 10 to 50 makes the relic density compatible for low values of $M_A$. The relic density decreases as one increases $M_1$ (or $M_2$ since $r_{12}$ is fixed), thus increasing the LSP mass. Once again, we confirm that in order to maximize the Higgsino-bino mixing, small values of $\mu$ are preferred. Generally, the light neutralinos (say below 16\text{GeV}) are found when $M_1 << \mu << M_2$ and we therefore also expects relatively light charginos as was the case with the heavy pseudoscalar scenario.

In models with low $M_A$ and large $\tan \beta$ one expects large effects from the $b$ observables. Most affected are $b \to s\gamma$ and $B_s \to \mu^+\mu^-$. On the other hand we have checked that the branching ratio to $Z \to b\bar{b}$ remains insensitive even in this scenario. The branching ratio for $b \to s\gamma$ receives $\tan \beta$ enhanced contributions from both the charged Higgs diagrams as well as the chargino/stop diagrams. Typically the contributions form each types of diagrams individually could be far above the allowed limit. However a partial cancellation between these two contributions occurs when $A_t$ is large and negative. For example for $\tan \beta = 50$, one needs values of $A_t < -750 \text{GeV}$. For $\tan \beta = 10$, on the other hand, even the value $A_t = -2400 \text{GeV}$, is not sufficient to induce a cancellation between the chargino sector and the large contribution from the charged Higgs to $b \to s\gamma$. This process then forbids the very low values of $M_A$ for intermediate values of $\tan \beta$, Fig. 10. One sees for

![Figure 10: Allowed values for $m_{\tilde{\chi}_1^0}$ as a function of the pseudoscalar mass for $\tan \beta = 10$. The impact of the WMAP data on the relic density is displayed: pink (light grey) circles corresponds to $\Omega h^2 < 0.3$ and green(medium grey) crosses to $\Omega h^2 < 0.128$. The black crosses correspond to the region allowed after including the $b \to s\gamma$ constraint.](image-url)

example that the lower bound for $\tan \beta = 10$, moves from $m_{\tilde{\chi}_1^0} \approx 12\text{GeV}$ to 29\text{GeV} after imposing the $b \to s\gamma$ constraint. Models with very light neutralinos that are compatible
with the $B$-sector constraints as well as with the relic density are then expected to be models with light pseudoscalars and large values of $\tan \beta$ as one can see in Fig. 11.

![Figure 11: Region of the $\tan \beta - M_A$ plane where one can find light neutralinos. Light grey (green) crosses have $10 \text{ GeV} < m_{\tilde{\chi}^0_1} < 16 \text{ GeV}$ while black crosses correspond to $m_{\tilde{\chi}^0_1} < 10 \text{ GeV}$. Here all constraints are implemented.](image)

In these models, the branching ratio for $B_s \to \mu^+\mu^-$ can be strongly enhanced at low values of $M_A$. In particular diagrams with chargino loops could give a large contribution when the chargino has a large Higgsino component. This corresponds to not too large values of $M_2$. The predicted values for $B_s \to \mu^+\mu^-$ are displayed in Fig. 12 for a variety of models that pass both LEP and WMAP constraints. One sees that the present limit from the Tevatron on the $B_s \to \mu^+\mu^-$ eliminates some models but leaves open the possibility of light pseudoscalars.

A scan over the full parameter space, Eq. 9, with $92 \text{ GeV} < M_A < 500 \text{ GeV}$ and imposing all the above mentioned constraints leads the lowest LSP mass bound of $m_{\tilde{\chi}^0_1} = 6 \text{ GeV}$. This LSP is found in models with large values of $\tan \beta > 30$ and is associated with $M_H, M_A, |\mu| < 120 \text{ GeV}$. The sleptons need not be light. The allowed region in the $M_A - m_{\tilde{\chi}^0_1}$ plane (Fig. 13) for $\tan \beta = 50$ clearly shows that when $M_A < 250 \text{ GeV}$ one can lower the bound on the neutralino compared to the case when the pseudoscalar is 1 TeV. Contrary to what we have seen for other cases, the impact of the WMAP results is marginal as concerns the lower bound on the neutralino mass $\approx 6 \text{ GeV}$; see Fig. 13. Although we found that a large number of models were ruled out by the requirement of $b \to s\gamma$ (Eq. 17), this constraint does not affect the absolute lower limit on the neutralino mass, $m_{\tilde{\chi}^0_1} = 6 \text{ GeV}$, in the large $\tan \beta$ scenario. Similarly the $B_s \to \mu^+\mu^-$ constraint does not impact on this lower bound. Moreover, for intermediate values of $\tan \beta \sim 10 - 20$, we find that lowering $M_A$ does not affect the lower limit on the neutralino mass which we derived for $M_A = 1 \text{ TeV}$. This is due to an incompatibility with the $b \to s\gamma$ constraint as shown explicitly for $\tan \beta = 10$ in Fig. 10. In fact, restricting the analysis to the region where $m_{\tilde{\chi}^0_1} < 16 \text{ GeV}$, we found that only models with $\tan \beta > 25, |\mu| <$

\footnote{This agrees with the results of \cite{38}.}
Figure 12: Branching fraction for $B_s \rightarrow \mu^+\mu^-$ vs the pseudoscalar mass for $\tan \beta = 50$, $\mu > 0$. We scan over all parameters and impose the LEP and WMAP constraints (green/light grey) as well as all constraints (dark). The preliminary 95% upper limit obtained by CDF with 113 pb$^{-1}$ (9.5 $\times$ 10$^{-7}$) and the expected reach of RunIIa (2 $\times$ 10$^{-7}$) are also displayed\cite{73}.

Figure 13: Allowed values for $m_{\tilde{\chi}_1^0}$ as a function of the pseudoscalar mass for $\tan \beta = 50$. The impact of the data on the relic density is displayed. Green stars (medium grey) correspond to $\Omega h^2 < 0.3$ while black crosses correspond to $\Omega h^2 < 0.128$. b) Chargino masses as a function of LSP mass. All constraints are implemented.
400\,\text{GeV}, M_A < 200\,\text{GeV} and a large negative stop mixing parameter A_t < -750 \,\text{GeV} were consistent with all constraints. The first three conditions are necessary for sufficient annihilation of the LSP into $\tau\tau(b\bar{b})$ pairs, while the last condition ensures cancellation between the chargino and charged Higgs contributions to $b \to s\gamma$. Obviously, as the coupling of the light Higgs scalar is enhanced with $\tan\beta$, the range of Higgs masses allowed is wider for larger values of $\tan\beta$ (see Fig. 11). With not so heavy pseudoscalar Higgs and charginos, colliders, and in particular the Tevatron, might then have good prospects to constrain the models or discover new particles in such a framework as will be discussed next.

### 6.2 Prospects at the Tevatron

Based on the existing Tevatron analyses\cite{74}, we infer that the Tevatron should be sensitive to some neutral Higgs bosons in a large fraction of the models having very light neutralinos with $m_{\tilde{\chi}_1^0} < 16 \,\text{GeV}$. As we have just discussed, the various constraints already impose that $M_A < 220 \,\text{GeV}$ and that $\tan\beta$ be large to arrive at such low values for the neutralino mass (see Fig. 11). Then the channel $p\bar{p} \to b\bar{b}\phi$, with $\phi \to b\bar{b}$ for $\phi = h, H, A$ is rather sensitive to the large $\tan\beta$ region due to the enhanced coupling to the $b$ quarks. For example, for $\tan\beta = 50$, in the $M_{h_{\max}}$ scenario, values of $M_A \approx 105\,\text{GeV}$ are excluded already with 0.1 fb$^{-1}$ of integrated luminosity and with 2 fb$^{-1}$ the region $M_A < 200\,\text{GeV}$ can be excluded at 3$\sigma$\cite{74}. For $\tan\beta = 40(30)$, $M_A < 160(130) \,\text{GeV}$ can be excluded with 2 fb$^{-1}$. For the very large values of $\tan\beta$ under consideration we have checked that the invisible mode, $h \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ carries only a small fraction of the total width so that the $b\bar{b}$ branching ratio would not be significantly suppressed. The models with light Higgses, $M_{h,H,A} \approx 100\,\text{GeV}$ can also be probed at the Tevatron in the charged Higgs channel via disappearance searches for light charged Higgses produced in the decay of top quarks $t \to H^+b$. With 2 fb$^{-1}$ of integrated luminosity in RunIIa, the region $m_{H^+} < 130(150)\,\text{GeV}$ for $\tan\beta = 30(50)$ can be excluded at 95%. However, as the charged Higgs are heavier than the pseudoscalar, this channel does not improve on the potential of the Tevatron.

Moreover the required values of $\mu$ for models leading to $m_{\tilde{\chi}_1^0} < 16\,\text{GeV}$ are such that the lightest chargino masses can also fall within the range accessible by the Tevatron in RunIIa. In particular, when $M_A$ gets close to 200 GeV and higher luminosities are needed for the Higgs channels, the predicted values for the charginos cluster near 100 $–$ 150 GeV. Although a detailed analysis of chargino searches in the trilepton mode in non-universal models has not been completed yet,\cite{34}, it is expected that the Tevatron would be sensitive to charginos within this range.

Finally, the Tevatron can also probe the large $\tan\beta$-light LSP scenario, via the $B_s \to \mu^+\mu^-$. In Fig. 12 we show the range of predicted values for $B_s \to \mu^+\mu^-$ as well as the expected sensitivity of RunIIa (2 x 10$^{-7}$). A large number of models with light neutralinos, predict a branching ratio above this sensitivity. The largest branching ratios are found for $M_A < 250\,\text{GeV}$.

To estimate the potential of RunIIa to probe models with very light neutralino LSP, we assume the expected limits, $m_A < 130(200) \,\text{GeV}, m_{H^+} < 130(150) \,\text{GeV}$ and $B_s \to \mu^+\mu^- < 2 \times 10^{-7}$, for values of $\tan\beta = 30(50)$. For the chargino channel we take as a guideline the value $m_{\tilde{\chi}_1^+} < 150 \,\text{GeV}$. Basically, we find that the additional region of parameter space where light neutralinos were allowed as one lowered $M_A$ will be probed.
entirely, at $\tan \beta = 50$, by the Tevatron, both in the pseudoscalar searches as well as in the $B_s \to \mu^+ \mu^-$. The chargino searches will also somewhat help constraint the allowed models. For $\tan \beta = 30$, due to the lower reach in the Higgs channel, we found some models with $m_{\tilde{\chi}_1^0} > 15\text{GeV}$ which could escape detection in RunIIa.

In summary, decreasing the value of $M_A$ opens up new possibilities for light neutralinos ($6 - 16\text{GeV}$). With Higgs and chargino searches, the Tevatron should be able in the near future to cover the small remaining part of parameter space where very light neutralinos ($\approx 6\text{GeV}$) can exist.

### 6.3 Direct detection with light scalars

The models we have just discussed admitting pseudoscalars (and hence the other neutral Higgses) with masses close to the lowest limit set by LEP offer interesting prospects for the direct detection experiments as well. The spin independent scalar cross section for neutralino scattering on a nucleus is sensitive to squark and Higgs exchanges. However, in the models with heavy squarks considered here, the dominant contribution arises from $t$-channel Higgs exchange. The dependence of the cross section on the mass of the Higgs scalars goes simply as $\sigma_{S.I.}^{\chi p} \propto \frac{1}{M_H^2}$. For large values of $\tan \beta$, the enhanced coupling of the Higgs to $d$-type quarks tends to further increase the cross section. Moreover because of the low $\mu$ values that these models admit, the LSP coupling to the Higgses is maximised.

![Figure 14: The spin independent neutralino-proton cross section as a function of a) $M_A$ and b) $m_{\tilde{\chi}_1^0}$ for $\tan \beta = 10$ (dark grey) and $\tan \beta = 50$ (green/light grey). Scan over the parameters specified in Eq. 9. All constraints were imposed. The reach of the direct detection experiments ZeplinI (full), Edelweiss (dot) CDMS (dot-dash) as well as for ZeplinII (dash) is displayed.](image)

We have calculated the scalar cross section for neutralino scattering off nucleons. We have included the contribution of all six flavors of quarks. Our results agree with the ones given in Ref. [75, 76]. We have used the values for the coefficients of the quark mass.
operator in the proton, \( j_{Tq}^{(p)} \) as given in Ref. [5]. We show the predictions for the spin-independent cross section for neutralino-proton scattering as a function of \( M_A \), Fig. 14a, after scanning over the parameter space as specified in Eq. 9. One witnesses the large enhancement at low values of \( M_A \). The various constraints from LEP, cosmology as well as from precision measurements have been folded in the global scan over the parameter space, hence the much smaller predictions for the cross section at intermediate values of \( \tan \beta \). There, as was discussed above, the \( b \to s\gamma \) constraint is not compatible with light pseudoscalars. The cross section for \( \tan \beta = 10 \) therefore never exceeds a few \( 10^{-8} \) pb.

For \( \tan \beta = 50 \), one reaches cross sections that are in the region of detectability of CDMS/Edelweiss/Zeplin although the largest cross sections are often found in regions where neutralinos are very light, precisely where detectors have shown less sensitivity, Fig. 14b. Nevertheless a few models, albeit not the ones with the LSP near the lowest limit \( \approx 6 \) GeV, are already ruled out by CDMS and Zeplin[29]. Furthermore, in the majority of models with \( M_A < 200 \) GeV, the spin-independent cross section exceeds \( 10^{-8} \), detectable by the next generation of direct dark matter detectors such as EdelweissII or ZeplinII.\(^7\) We emphasize that to cover all models it is crucial to have a good sensitivity in the region \( m_{\tilde{\chi}_1^0} \approx 6 - 10 \) GeV and that future detectors should extend their capabilities and searches in this mass range.

\[ \text{Figure 15: Regions of the } M_A - \mu \text{ plane where the spin independent neutralino-proton cross section exceeds } 10^{-7} \text{ pb (black), } 10^{-8} \text{ pb (green/light grey), } 10^{-9} \text{ pb (blue/medium grey). Here, } \tan \beta = 50 \text{ and we scanned over the parameters specified in Eq. 4. All constraints were imposed.} \]

\(^7\)Note that there are uncertainties in the exclusion curves written in terms of the LSP scattering cross section on proton, for a discussion of the effect of halo modeling and LSP velocity distribution see[77].
Although the spin independent cross sections are enhanced at large $\tan \beta$, cross sections above $10^{-7}$ pb are found for all values of $\tan \beta$ where light pseudoscalars are allowed ($\tan \beta > 25$). Isocurves of the scalar cross section are displayed in the $M_A - \mu$ plane in Fig. 15 emphasizing the importance of a sizable amount of gaugino-Higgsino mixing to obtain large spin-independent cross sections. Here only the upper bound on the relic density has been considered. Many of the models, near the $Z$ or Higgs resonance actually have a value for the relic density that is much too low. Usually one rescales the cross section by a factor of the relic density to some minimal value (say 0.05) in order to take into account the fact that the neutralino would not constitute the main cold dark matter component. Applying a rescaling factor of $\rho = \Omega h^2/0.05$, would strongly affect the effective cross section ($\sigma_{eff} = \rho \sigma_S^{S.I.}$) near the $Z$ or Higgs pole but not for the very light LSP where the relic density is near the allowed upper bound.

7 Prospects at $e^+e^-$ colliders

The aim of this section is to address the issue whether SUSY models with non-universal gaugino masses that have a light LSP, say $m_{\tilde{\chi}_1^0} < 70$ GeV, can always be probed at an $e^+e^-$ linear collider of centre of mass 500 GeV. In the usual mSUGRA type model, imposing such a light neutralino entails $m_{\tilde{\chi}_1^+} < 150 - 200$ GeV and therefore independently of any other sparticles, chargino discovery is guaranteed [42]. The non-universal models do not always ensure $m_{\tilde{\chi}_1^+} < 250$ GeV, moreover some models do not have light sleptons as we have seen and yet are consistent with all data (including cosmology).

The main channels for producing supersymmetric particles at a 500 GeV linear collider are the slepton pair production and chargino pair production with $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0$ being important processes as well. Of course the lightest Higgs boson can always be probed. In non-universal models, the difficult cases for the LC will be the ones where sleptons are heavy with the mass of the LSP near $M_Z/2^8$. In this case large values of $\mu$ are still allowed. This entails charginos (and $\tilde{\chi}_2^0$) too heavy to be directly pair produced. The only processes accessible would then be $\tilde{\chi}_1^0\tilde{\chi}_j^0$.

To ascertain which (s)particles can be reached at a 500 GeV, we perform scans over the parameters $M_2, \mu, M_0$ for fixed values of $\tan \beta$ and $r_{12}$, the non universality parameter. We compute the unpolarized cross sections for $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0, \tilde{\gamma}_1\tilde{\chi}_1^0, \tilde{\chi}_1^0\tilde{\chi}_2^0, \tilde{\chi}_1^0\tilde{\chi}_3^0$ as well as for $\tilde{\chi}_1^0\tilde{\chi}_1^0\tilde{\gamma}$. We have not simulated $\tilde{\chi}_L\tilde{\chi}_R$ and $\tilde{e}_L\tilde{e}_L$ production. In our scenario these would be accessible only when $\tilde{e}_R\tilde{e}_R$ is accessible. The cross sections are computed with CalcHEP [41] [42], a program for the automatic calculation of Feynman diagrams in the standard model or the MSSM. Because the LSP is very light, clean signatures for all sparticles that decay into $\tilde{\chi}_1^0$ are guaranteed. In fact, there is in general sufficient phase space for the decay into LSP to constitute the main decay mode for the selectron, $\tilde{e}_R \rightarrow e^-\tilde{\chi}_1^0$, the chargino $\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0$ as well as heavy neutralinos, $\tilde{\chi}_i \rightarrow \tilde{\chi}_1^0Z^{(*)}$. For charginos and neutralinos, decays into sleptons are also sometimes kinematically accessible. We will not perform a detailed analysis of signal and background for all these processes, in the $e^+e^-$ clean environment it should not be a problem to see a signal for $\sigma > 1$ fb.

These models should not be thought of as fine tuned, from the relic density point of view, more than the mSUGRA model is. The latter for example does require, for example, near degeneracy between the $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ masses to pass the WMAP constraint.
We also consider $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^{-0} \gamma$, the process that contribute to $\sigma_{\gamma+inv}$. Although this process has to fight a large background from the standard $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, with 95% $e^-$ polarization and after a cut on the small angle photons, it was shown that it was possible to reduce the background to $< 150fb$. [30] From this we estimate that with a luminosity of 500$fb^{-1}$, the polarized cross section for the signal should exceed $\sigma = 1.6\ fb$ in order to see a 3$\sigma$ deviation. For simplicity we compute only the unpolarised cross section and impose conservatively the same value as the pair production processes, $\sigma_{unpol} > 1fb$.

Here we concentrate on the $\mu > 0$ region. We find that in large regions of the parameter space the cross section for at least one process exceeds 1fb. We do, however, uncover regions compatible with existing limits where no sparticle is produced. To look more precisely at the potential of a 500GeV LC, we discuss again two typical values of $\tan \beta$.

- $\tan \beta = 10$

First consider the case $\tan \beta = 10$. For unified models ($r_{12} \approx .5$) and with $m_{\tilde{\chi}_1^0} < 70$ GeV, the whole parameter space is covered basically by the processes $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^{-0}$. While chargino pair production is the preferred channel especially for the large $M_0$ region, in this scenario, $\tilde{\chi}_2^0\tilde{\chi}_1^{-0}$ also exceeds 1fb in the full parameter space, Fig. [16] The process $\tilde{\chi}_3^0\tilde{\chi}_1^{-0}$ has a significant cross section only for $\mu \leq 400$GeV. When $M_0 < 200$GeV, the slepton pair production is also always accessible. As one decreases the value of the non-universality parameter, $r_{12} = 0.1$, one starts uncovering permitted regions where none of the charginos/neutralinos production processes have a large enough cross section while the sleptons are too heavy for pair production. This occurs in the large $M_0$ region and for very large $\mu$ where the coupling to the Z is not sufficient. As the chargino mass can be much higher than the LSP, the chargino process is accessible only in the small $\mu$ region while the $\tilde{\chi}_2^0\tilde{\chi}_2^{-0}$ and $\tilde{\chi}_3^0\tilde{\chi}_3^{-0}$ both extend the reach in parameter space. The process $\tilde{\chi}_1^0\tilde{\chi}_2^{-0}$ is also observable in a narrow region at small $M_0$ even for large values of $\mu$, these were the points where $m_{\tilde{\chi}_1^0} \approx M_Z/2$.

When $r_{12}$ decreases to $r_{12} = 0.03$, one observes the same features. For $M_0 < 200$GeV, observable cross sections for the slepton production processes $\tilde{e}_R\tilde{e}_R, \tilde{\mu}_R\tilde{\mu}_R$ and $\tilde{\tau}_1\tilde{\tau}_1$ are expected as for the universal case. For larger values of $M_0$, it is $\sigma_{\tilde{\chi}_1^0\tilde{\chi}_2^0}$ that eventually exceeds $\sigma_{\tilde{\chi}_1^0\tilde{\chi}_2^0}$ in the region $M_1 << \mu < M_2$. Then the LSP is bino-like and both $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ are Higgsino-like and nearly degenerate. However the $Z\tilde{\chi}_1^0\tilde{\chi}_3^0$ features the largest coupling. With the new data from WMAP, the large $M_0/\mu$ region is more severely constrained, thus reducing the allowed parameter space where no supersymmetric particles can be produced.

Note that one can also get indirect evidence of light neutralinos by measuring the invisible branching fraction of the Higgs. A large branching fraction into invisible is expected in models with non-universal gaugino masses in the region of intermediate $\tan \beta$ and when $\mu$ is small [30]. For example, for $\tan \beta = 10, r_{12} = 0.1, \mu < 300$ GeV the branching fraction $h \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^{-0}$ can reach 60%. This region of parameter space corresponds to the one where one expects many other signals of supersymmetry at the LC500, $\tilde{\chi}_1^+\tilde{\chi}_1$, $\tilde{\chi}_1^0\tilde{\chi}_j$ and sometimes $\tilde{e}_R\tilde{e}_R$, Fig. [16] Correlating the information from the Higgs and the supersymmetric sector will then help to establish the nature of the invisible decay mode of the Higgs.

- $\tan \beta = 50$

In the large $\tan \beta$ scenario, the slepton pair production process is not so useful, at least in the unified model, as most of the low $M_0$ region is ruled out. The selectron pair
Figure 16: Region of $M_0$-$\mu$ plane where the cross section for sparticles production in $e^+e^-$ exceeds 1fb, for $\tan \beta = 10$ (left) and $\tan \beta = 50$ (right) and from top to bottom $r_{12} = 0.5, 0.1, 0.03$. $r_{12} = 0.5$ is the usual GUT value. The processes are $\tilde{e}_R\tilde{e}_R$ (yellow hatch) $\tilde{\chi}_1^+\tilde{\chi}_1^-$ (red/medium grey) $\tilde{\chi}_1^0\tilde{\chi}_2^0$ (blue/dark grey) $\tilde{\chi}_1^0\tilde{\chi}_3^0$ (green/light grey). Note that there is a large overlap between the region covered by $\tilde{e}_R\tilde{e}_R$ and the ones (not featured) covered by $\tilde{\mu}_R\tilde{\mu}_R$ or $\tilde{\tau}_1\tilde{\tau}_1$. The region to the right of the full (dashed) line do not satisfy the constraint $\Omega h^2 < 0.128(0.3)$ coupled with LEP and precision measurements. In the region (white) to the left of the WMAP line no supersymmetric particle can be produced. Such regions are present only when $r_{12} = 0.03$ and $r_{12} = 0.1$. In the top figures, the whole parameter space is also covered by $\tilde{\chi}_1^0\tilde{\chi}_2^0$. In the bottom right figure, the dotted line delimits the region where $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\gamma) > 1$fb.
production process is only accessible in a corner of parameter space when \( \mu, M_0 \approx 100 \text{GeV} \). The stau pair production is also accessible when \( \mu \) is large enough to induce significant mixing in the \( \tau \) sector. However, with the \textit{WMAP} constraint there remains only a very narrow region at large \( \mu \). Both chargino pairs and \( \tilde{\chi}^0_1 \tilde{\chi}^0_2 \) cover all allowed parameter space while \( \tilde{\chi}^0_1 \tilde{\chi}^0_3 \) is also important when \( \mu < 400 \text{ GeV} \). As \( r_{12} \) decreases, the low \( M_0 \) region is allowed and observable cross sections for \( \tilde{e}_R \tilde{e}_R \) as well as \( \tilde{\tau}_1 \tilde{\tau}_1 \) are predicted. Again there is a portion of the large \( M_0 \)-large \( \mu \) region of the parameter space where supersymmetric particles are too heavy to be produced. Note that imposing the new \textit{WMAP} constraint has considerably shrunk this region. Finally, the cross section for the process \( \tilde{\chi}^0_1 \tilde{\chi}^0_1 \gamma \) can exceed 1 fb, although this occurs only in the low \( M_0 \) region when slepton pair production is also accessible.

Increasing the energy of the collider would obviously increase the parameter space where sparticles could be pair produced. Basically, the slepton pair production is large for \( M_0 < 400 \text{GeV} \) and chargino pair production for \( \mu < 450 \text{GeV} \). Thus, one nearly recovers full coverage with only these two processes, in the case of nonuniversal models with light neutralinos at a 800 GeV collider. At first sight, the more problematic models are those with \( r_{12} \approx 0.1 \) where much larger values of \( \mu \) are allowed at intermediate \( \tan \beta \). However, the process \( e^+e^- \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2 \) has a reasonable cross section for most of the large \( M_0 - \mu \) region. For example when \( \tan \beta = 10 \), \( \tilde{\chi}^0_1 \tilde{\chi}^0_2 \) is measurable everywhere except in a small region where \( M_0 > 900 \text{GeV} \). Then both the \( t \)-channel sfermion exchange and the \( s \)-channel \( Z \) exchange contributions are small and no signal for supersymmetric particles can be expected. With \textit{WMAP}, this difficult region is already ruled out when \( r_{12} < 0.1 \) (see Fig. 16 for the allowed region). One also avoids the difficult region when \( \tan \beta = 50 \) (Fig. 16). There, for any value of the non universality parameter the allowed region corresponds roughly to the one where chargino pair production is accessible.

Finally we mention that such light supersymmetric particles would probably first be discovered at the hadron colliders. For the \textit{Tevatron}, although the chargino mass considered are often within the range where discovery via the trilepton signal is expected (\( m_{\tilde{\chi}^\pm_1} \) up to 250 GeV) one needs to take a closer look at cross sections for signal and backgrounds in nonunified models to ascertain the viability of the signal [34]. In the models with heavy squarks and Higgs scalars, no other opportunity for SUSY discovery exists. The LHC on the other hand would have plenty of opportunities to discover SUSY in either the squark, Higgs or gaugino sector. We should also note that the LC through precision measurements on the \( h \) properties could also probe into SUSY especially by using information from the LHC.

8 Conclusion

We have reexamined the lowest bound on the neutralino LSP in the minimal supersymmetric model. We have worked within the context of minimal flavour violation and \( R \) parity conserving supersymmetric models. We have reduced the parameter space to only a few important parameters, those of the gaugino and slepton sector, the pseudoscalar mass and the trilinear coupling of the squarks. In particular we have relaxed the universality relation between the gaugino masses thus removing the most important constraint on the LSP mass arising from LEP. We find that the upper limit on the relic density contributed by neutralinos and as inferred from the new data by \textit{WMAP} basically sets the lower bound
on neutralinos in models where the gaugino masses are not unified at the high scale but
satisfy \( M_1 << M_2 \). In the limit of a heavy pseudoscalar mass, we found a lower bound on
the neutralino mass of 18GeV in models with \( \tan \beta = 50 \). For intermediate values of \( \tan \beta \),
the annihilation of neutralinos into light fermions is not as efficient and one can only set
a bound of 29GeV for \( \tan \beta = 10 \). It is however in models with light pseudoscalar masses
\( (M_A < 200 GeV) \) and large values of \( \tan \beta \) that one finds the lightest LSP \( m_{\tilde{\chi}_1} \approx 6 GeV \).
This is due to a new contribution to the annihilation cross-section of neutralinos, scalar
exchange mediating \( \tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \tau^+ \tau^- \), thus reducing the relic density.

The models with light pseudoscalars and neutralinos below \( \approx 16 \text{ GeV} \) can be probed
both in the next generation of direct detection experiments as well as at the Tevatron or
future colliders. The cross section for the former are much enhanced for a light Higgs with
significant coupling to the neutralino. It is however crucial that the detector be sensitive
to light neutralino masses. In addition, the pseudoscalar and/or scalar Higgses should be
within reach of the Tevatron RunIIa. Charginos or \( B_s \rightarrow \mu^+ \mu^- \) might also be detected
there. The LHC also has a potential for discovering charged Higgses in the large \( \tan \beta \) region using \( gb \rightarrow tH^\pm \) with \( H^+ \rightarrow tb, \tau^+ \nu \) or \( qq \rightarrow H^\pm \rightarrow \tau\nu \) \cite{81}. Studies of neutral
Higgses searches in the \( \tau^+ \tau^- \) channel, also show a good potential for discovery even with
a low luminosity \cite{81}. Of course other sparticles, such as charginos, heavier neutralinos
or sfermions might be discovered as well.

Finally we have shown that a linear collider with centre of mass of 500GeV has good
prospects for producing supersymmetric particles in models where neutralinos are below
the weak scale, although the parameter space cannot be completely covered especially in
models where \( M_1 << M_2 \). The gaugino universality relation can be directly tested at such
a collider but this necessitates performing a combined fit to the neutralino mass (measured
in slepton pair production process), the chargino mass as well as to the polarized cross
sections for \( e^+e^- \rightarrow \tilde{\ell}_R \tilde{\ell}_R, \tilde{\chi}_1^+ \tilde{\chi}_1^- \) \cite{82,83}. All these precision measurements can be realized
only if \( M_0(m_{\tilde{\ell}_R}), \mu < 250 \text{GeV} \). Of course models where \( M_A \) is near \( O(100) \text{ GeV} \) should
lead to several signals at the linear collider. Most noticeably in the Higgs sector where
one expects a large cross section \((10 - 100 \text{ fb})\) in both \( e^+e^- \rightarrow ZH, hA \) channels as well
as in the charged Higgs pair production channel. In most cases, and most importantly
when the Higgs sector is harder to probe, the charginos would be accessible as well at a
collider with \( \sqrt{s} = 500 \text{ GeV} \).

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