Superparticle actions from superfields

R. Marnelius and Sh. M. Shvartsman

Institute of Theoretical Physics
Chalmers University of Technology
S-412 96 Göteborg, Sweden

Abstract

Gauge invariant complex covariant actions for superparticles are derived from the field equations for the chiral superfields in a precise manner. The massive and massless cases in four dimensions are treated both free and in interaction with an external super Maxwell field. By means of a generalized BRST quantization these complex actions are related to real actions with second class constraints which are new in some cases.
1 Introduction

There is now an enormous amount of literature on the problem of how to quantize superparticles in a manifestly supersymmetric and Lorentz covariant way. This problem has attracted such an interest mainly since its solution is considered to be a crucial first step to a successful solution of the corresponding problem for superstrings. The problem is nontrivial mainly due to the appearance of second class constraints in the manifestly covariant actions. Such theories are not gauge theories and are therefore not possible to quantize in a gauge theoretical manner. The standard approach to circumvent the problem is to modify the theories by means of auxiliary variables in such a way that the second class constraints are transformed into first class ones. The modified theories are then gauge theories which may be quantized by means of standard BRST techniques. (We refrain from giving any references here since the literature is too vast.)

Now there is also another BRST approach to the covariant quantization of theories with second class constraints which does not require the introduction of additional variables and which has not been applied to superparticles before. We shall apply this approach and show that it to a large extent may be interpreted as an approach in which theories with second class constraints may be described by gauge invariant complex actions without second class constraints. Complex actions for gauge theories are not necessarily a bad feature since only the underlying physical theory is required to be implicitly described by a real action. However, it is true that for bosonic theories convergence of the path integrals in general severely restricts the ranges of the bosonic variables in the imaginary terms. On the other hand no such restriction is necessary if the imaginary terms involve fermionic variables. For the complex superparticle models to be derived here there are no such problems with the path integrals. For those who dislike the use of complex actions it is in general possible to reformulate such models in terms of a real action with second class constraints and apply the generalized BRST quantization of without using the interpretation in terms of complex actions.

In distinction to previous approaches to the quantization of superparticles our main concern in this paper is the precise connection between superfield equations and the corresponding pseudoclassical particle models. This problem has attracted rather little interest since it has no direct bearing on the quantization of the D=10 massless superparticle. However, it is relevant at a deeper level for our understanding of the relation between particle models and field theory. In this paper we shall apply the general method given which can be used as a precise method to derive particle models from quantum mechanical field equations. (In it was used as a general method to construct spinning particle models.) This method always yield gauge invariant particle actions which are not always real. However, when one obtains a complex action we shall demonstrate that there in general exists a corresponding real action usually then with second class constraints. The method is described below.

In it was shown that the corresponding particle model to a given set of field equations are described by a (pseudoclassical) Lagrangian whose phase space form is

\[ L = L_0 + \lambda_i \phi_i \]  

where \( L_0 \) determines the class of fields, \( \psi \), that is involved, and where \( \phi_i \) are constraint variables and \( \lambda_i \) Lagrange multipliers. The constraint variables \( \phi_i \) are classical expressions...
for the corresponding differential operators involved in the field equations, \( i.e. \)
\[
\hat{\phi}_i \psi = 0, \quad i = 1, \ldots
\] (1.2)

Consistency requires the operators \( \hat{\phi}_i \) to satisfy
\[
[\hat{\phi}_k, \hat{\phi}_l]_{\pm} = i \hat{F}_{klm} \hat{\phi}_m
\] (1.3)

where \( \hat{F}_{klm} \) are structure operators. Therefore, one may always find corresponding classical constraint variables, \( \hat{\phi}_i \rightarrow \phi_i \), that are of first class, \( i.e. \) satisfying the Poisson algebra
\[
\{ \phi_i, \phi_j \} = F_{ijk} \phi_k
\] (1.4)

where \( F_{ijk} \) are structure functions. In other words (1.1) always represents a gauge invariant Lagrangian. However, since \( \hat{\phi}_i \) are not always hermitian and since often not both \( \hat{\phi}_i \) and \( \hat{\phi}_i^\dagger \) are involved in the set of field equations (1.2), the Lagrangian (1.1) is in general complex since there may be complex constraint variables \( \phi_i \) for which \( \bar{\phi}_i \) is not involved in (1.1).

As will be shown in section 4 the corresponding real Lagrangian is obtained by adding the independent complex conjugate constraints to \( L \) which then in general is no longer gauge invariant.

In this paper we apply the above method to the field equations of a massive chiral scalar superfield in four dimensions both free and in interaction with an external super Maxwell field. In all cases the chiral superparticle is shown to be described by a complex covariant action which is gauge invariant. (Some of these results were presented in [5].) The corresponding propagators which are obtained by means of the natural gauge theoretic expressions in terms of the derived complex actions are then shown to agree with the results of [4]. We also perform a corresponding treatment of the massless limits of the field equations. The paper is organized as follows: In section 2 we give the correspondence between a particle Lagrangian and the class of scalar superfields. The free chiral scalar superfield case is then treated in section 3. Gauge invariant complex superparticle actions are derived. In section 4 we show that the quantization of these complex actions are related to a generalized BRST quantization of the conventional real Casalbuoni or Brink-Schwarz actions with second class constraints for free superparticles. Interaction with an external super Maxwell field by means of supersymmetric minimal coupling is considered in section 5. It is shown to be nontrivial to relate this construction to the corresponding superfield theory. In section 6 we derive gauge invariant complex actions from the superfield equations in the interaction case. The most natural corresponding real Lagrangians are shown to be equivalent to a Lagrangian obtained by minimal coupling. In section 7 we derive gauge invariant complex covariant actions from the superfield equations in the massless limit. In this case interaction is trivially consistent with minimal coupling since this is also valid in the field equations. Some final remarks are given in section 8. In an appendix we give some proofs for section 6.

2 Scalar superfields

Consider the class of superfields described by \( \Phi(x, \theta, \bar{\theta}) \) where \( x^\mu \) is the Minkowski coordinate in four dimensions (we are using spacelike metric \( \text{diag} \eta^{\mu\nu} = (-, +, +, +) \)) and where \( \theta^\alpha \) and \( \bar{\theta}^\dot{\alpha} \) are odd Grassmann variables. The indices \( \alpha \) and \( \dot{\alpha} \) are two-spinor ones.
(we are using the notation of Ref. [8]). Such superfields may be viewed as wave functions in a quantum theory obtained from the quantization of the Lagrangian

\[ L_0 = p_\mu \dot{x}^\mu - i \pi^\alpha \dot{\theta}_\alpha + i \bar{\pi}^{\dot{\alpha}} \dot{\bar{\theta}}^{\dot{\alpha}} \]  

(2.1)

where \( p_\mu \) and \( p_\alpha = i \pi_\alpha, \bar{p}_{\dot{\alpha}} = i \bar{\pi}_{\dot{\alpha}} \) are conjugate momenta to \( x^\mu \) and \( \theta^\alpha, \bar{\theta}^{\dot{\alpha}} \) respectively.

This Lagrangian is real and leads to the hermitian operators \( \hat{p}_\mu \) and \( \hat{x}^\mu \) satisfying

\[ [\hat{x}^\mu, \hat{p}_\nu] = i \delta^\mu_\nu \]  

(2.2)

and to the odd fermionic operators \( \hat{\pi}^\alpha \), \( \hat{\theta}^\alpha \), \( \hat{\bar{\pi}}^{\dot{\alpha}} \) and \( \hat{\bar{\theta}}^{\dot{\alpha}} \) satisfying

\[ \hat{\bar{\pi}}^{\dot{\alpha}} = (\hat{\pi}_\alpha)^\dagger, \quad \hat{\bar{\theta}}^{\dot{\alpha}} = (\hat{\theta}^\alpha)^\dagger \]

\[ [\hat{\theta}^\alpha, \hat{\pi}_\beta]_+ = \delta^\alpha_\beta, \quad [\hat{\bar{\theta}}^{\dot{\alpha}}, \hat{\bar{\pi}}^{\dot{\beta}}]_+ = \delta^{\dot{\alpha}}_{\dot{\beta}} \]  

(2.3)

The corresponding nondegenerate state space is spanned by eigenstates to \( \hat{x}^\mu, \hat{\theta}^\alpha \) and \( \hat{\bar{\theta}}^{\dot{\alpha}} \).

We define the eigenstates \( |x, \theta, \bar{\theta}\rangle \) by

\[ \hat{x}^\mu |x, \theta, \bar{\theta}\rangle = x^\mu |x, \theta, \bar{\theta}\rangle \]

\[ \hat{\theta}^\alpha |x, \theta, \bar{\theta}\rangle = \theta^\alpha |x, \theta, \bar{\theta}\rangle \]

\[ \hat{\bar{\theta}}^{\dot{\alpha}} |x, \theta, \bar{\theta}\rangle = \bar{\theta}^{\dot{\alpha}} |x, \theta, \bar{\theta}\rangle \]  

(2.4)

with the normalization

\[ \langle x', \theta', \bar{\theta}' | x, \theta, \bar{\theta} \rangle = \delta^4(x - x') \delta^2(\theta - \theta') \delta^2(\bar{\theta} - \bar{\theta}') \]  

(2.5)

Any state \( |\Phi\rangle \) may then be expanded in terms of these eigenstates

\[ |\Phi\rangle = \int dx d^2 \theta d^2 \bar{\theta} |x, \theta, \bar{\theta}\rangle \Phi(x, \theta, \bar{\theta}) \]  

(2.6)

where the superfield \( \Phi(x, \theta, \bar{\theta}) \) is given by

\[ \Phi(x, \theta, \bar{\theta}) \equiv \langle x, \theta, \bar{\theta}|\Phi\rangle \]  

(2.7)

### 3 Gauge invariant complex actions for free superparticles

Our starting point is the equations for a free chiral scalar superfield in four dimensions: (see e.g. eq.(9.24) in [3])

\[ \bar{D}^2 \bar{\Phi}(x, \theta, \bar{\theta}) = 4m \Phi(x, \theta, \bar{\theta}) \]

\[ D^2 \Phi(x, \theta, \bar{\theta}) = 4m \bar{\Phi}(x, \theta, \bar{\theta}) \]  

(3.1)

where

\[ D^2 \equiv D^\alpha D_\alpha, \quad \bar{D}^2 \equiv \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \]

\[ D_\alpha \equiv \partial_\alpha + i \sigma^\mu_\alpha \bar{\theta}^{\dot{\beta}} \partial_\mu \]

\[ \bar{D}_{\dot{\alpha}} \equiv -\partial_{\dot{\alpha}} - i \theta^\beta \sigma^\mu_{\beta \dot{\alpha}} \partial_\mu \]  

(3.2)
Provided \( m \neq 0 \) these equations imply

\[
\begin{align*}
D_\alpha \Phi(x, \theta, \bar{\theta}) &= 0 \\
\bar{D}_\alpha \Phi(x, \theta, \bar{\theta}) &= 0
\end{align*}
\] (3.3)

These chiral conditions are also used when (3.1) is derived from a Lagrangian density. Equ. (3.1) may be diagonalized:

\[
\begin{align*}
(D^2 \bar{D}^2 - 16m^2)\Phi(x, \theta, \bar{\theta}) &= 0 \\
(D^2 \bar{D}^2 - 16m^2)\bar{\Phi}(x, \theta, \bar{\theta}) &= 0
\end{align*}
\] (3.4)

which again imply (3.3). If we consider (3.3) as independent equations then (3.4) may be simplified to

\[
\begin{align*}
(\partial^2 - m^2)\Phi(x, \theta, \bar{\theta}) &= 0 \\
(\bar{\partial}^2 - m^2)\bar{\Phi}(x, \theta, \bar{\theta}) &= 0
\end{align*}
\] (3.5)

Thus, (3.3) and (3.5) are equivalent to (3.1). Within the operator formulation the chiral superfield equations are then given by

\[
\begin{align*}
(p^2 + m^2)|\Phi\rangle &= 0 \\
(\hat{\pi}_\alpha - \bar{\theta}^\beta (\hat{p} \cdot \sigma)_{\beta\dot{\alpha}})|\Phi\rangle &= 0
\end{align*}
\] (3.6)

Similarly the antichiral superfields equations may be written

\[
\begin{align*}
(p^2 + m^2)|\bar{\Phi}\rangle &= 0 \\
(\bar{\pi}_\alpha - (\hat{\dot{p}} \cdot \sigma)_{\alpha\beta}\bar{\theta}^\beta)|\bar{\Phi}\rangle &= 0
\end{align*}
\] (3.7)

where \( |\bar{\Phi}\rangle \) is related to \( |\Phi\rangle \) through the equation

\[
\bar{\Phi}(x, \theta, \bar{\theta}) = \langle \Phi | x, \theta, \bar{\theta} \rangle = \langle x, \theta, \bar{\theta} | \bar{\Phi} \rangle
\] (3.8)

According to the method of ref. [7] the equations (3.6) may be considered to be obtained from the pseudoclassical Lagrangian

\[
L = L_0 - \frac{e}{2} \phi + i\bar{\lambda}_\alpha d^\alpha
\] (3.9)

where

\[
\phi = p^2 + m^2 \\
\bar{d}_\alpha = \bar{\pi}_\alpha - \theta^\beta (\hat{\dot{p}} \cdot \sigma)_{\beta\dot{\alpha}}
\] (3.10)

\( L_0 \) is given by (2.1) and \( e \) and \( \bar{\lambda}^\dot{\alpha} \) are Lagrange multipliers. Similarly the equations (3.7) lead to

\[
\bar{L} = L_0 - \frac{e}{2} \bar{\phi} - i\lambda^\alpha d_\alpha
\] (3.11)

where

\[
d_\alpha = \pi_\alpha - (\hat{p} \cdot \sigma)_{\alpha\beta}\bar{\theta}^\beta
\] (3.12)
The positive feature of these Lagrangians compared to the ones in refs. [9, 10] is that they are both gauge invariant and manifestly supersymmetric and Lorentz invariant. The negative feature is that they are complex. Naively one would immediately discard complex actions. However, as we mentioned in the introduction complex actions for gauge theories is an allowed possibility and we shall also show that the above Lagrangians can be used in a consistent fashion.

Configuration space Lagrangians corresponding to (3.9) and (3.11) are easily obtained. A variation of $p^\mu, \pi_\alpha (\bar{\pi}_\dot{\alpha})$ and $\bar{\lambda}_\dot{\alpha}(\dot{\lambda}_\alpha)$ in $L (\bar{L})$ yields equations which determine these variables. We find

$$p^\mu = \frac{1}{e}(\dot{x}^\mu - i\theta^\alpha \sigma^{\alpha\beta} \dot{\lambda}_\beta), \quad \bar{\lambda}_\dot{\alpha} = -\dot{\theta}, \quad \bar{\pi}_\dot{\alpha} = \theta^\alpha (\sigma \cdot p)_{\alpha\dot{\alpha}} \quad (3.13)$$

from $L$ and

$$p^\mu = \frac{1}{e}(\dot{x}^\mu + i\lambda^\alpha \sigma^{\alpha\beta} \dot{\bar{\theta}}_\beta), \quad \lambda^\alpha = -\dot{\bar{\theta}}^\alpha, \quad \pi_\alpha = (\sigma \cdot p)_{\alpha\dot{\alpha}} \bar{\theta}_\dot{\alpha} \quad (3.14)$$

from $\bar{L}$. When these equations are inserted back into $L$ and $\bar{L}$ we get

$$L_{\text{conf}} = \frac{(\dot{x} + i\theta \sigma \dot{\bar{\theta}})^2}{2e} - \frac{1}{2}em^2 - i\pi_\alpha \dot{\bar{\theta}}^\alpha$$

$$\bar{L}_{\text{conf}} = \frac{(\dot{x} - i\bar{\bar{\theta}} \sigma \dot{\theta})^2}{2e} - \frac{1}{2}em^2 + i\bar{\pi}_\dot{\alpha} \dot{\theta}_\dot{\alpha} \quad (3.15)$$

These Lagrangians are gauge invariant under the following gauge transformations (the nonzero ones)

$$\delta e = \dot{\zeta}, \quad \delta x^\mu = \zeta p^\mu \quad (3.16)$$

where $\zeta$ is a real infinitesimal parameter function. The Lagrangian $L_{\text{conf}}$ is gauge invariant under the gauge transformations

$$\delta \bar{\theta}_\dot{\alpha} = i\kappa_{\dot{\alpha}}, \quad \delta x^\mu = \theta^\alpha \sigma^{\alpha\beta} \bar{k}_\beta, \quad \delta \pi_\alpha = -i(p \cdot \sigma)_{\alpha\dot{\alpha}} \bar{k}_\dot{\alpha} \quad (3.17)$$

where $\kappa_{\dot{\alpha}}$ is an odd complex parameter function with $p^\mu$ given by (3.13). The Lagrangian $\bar{L}_{\text{conf}}$ is invariant under (3.16) and the complex conjugation of (3.17) with $p^\mu$ given by (3.14). It is a peculiar property of the complex Lagrangians that we in the equations of motion (3.13),(3.14) and in the gauge transformations (3.17) must give up the strict reality properties of the involved dynamical variables.

The Lagrangians (3.13) should be compared with the Lagrangian given by Brink and Schwarz [10]

$$L_{BS} = \frac{(\dot{x} - i\bar{\psi} \gamma \dot{\psi})^2}{2e} - \frac{1}{2}em^2 \quad (3.18)$$

where $\psi = \left( \begin{array}{c} \theta_\alpha \\ \bar{\theta}_\dot{\alpha} \end{array} \right)$ ($\bar{\psi} = (\theta^\alpha, \bar{\theta}_\dot{\alpha})$) is a Majorana spinor and $\gamma^\mu$ the Dirac matrices in the Weyl representation ($\bar{\psi} \gamma^\mu \equiv \dot{\bar{\theta}} \sigma^\mu \theta - \theta \sigma^\mu \dot{\bar{\theta}}$). Or equivalently Casalbuoni’s Lagrangian [1]

$$L_C = m[(\dot{x} - i\bar{\psi} \gamma \dot{\psi})^2]^{\frac{1}{2}} \quad (3.19)$$
obtained after eliminating the einbein variable $e$ when $m \neq 0$ in $L_{BS}$. Eliminating $\pi_\alpha (\bar{\pi}_{\dot{\alpha}})$ in $L_{\text{conf}} (\bar{L}_{\text{conf}})$ we find the following correspondence:

\[
L \leftrightarrow L_{BS}(L_C) \quad \text{together with the constraint } \dot{\theta}^\alpha = 0
\]
\[
\bar{L} \leftrightarrow L_{BS}(L_C) \quad \text{together with the constraint } \dot{\bar{\theta}}^\dot{\alpha} = 0
\]

(These external constraints may also be implemented by means of Lagrange multipliers (like $\pi_\alpha (\bar{\pi}_{\dot{\alpha}})$ in (3.15)).) We have e.g. the formal path integral relations

\[
\int \exp(i \int_0^s d\tau L) DpDxD\theta D\bar{\theta} D\pi D\bar{\pi} D\bar{e} D\bar{\lambda} = \int \exp(i \int_0^s d\tau L_{BS}) \delta^2 (\dot{\theta}) Dx D\theta D\bar{\theta}
\]
\[
\int \exp(i \int_0^s d\tau \bar{L}) DpDxD\theta D\bar{\theta} D\pi D\bar{\pi} D\bar{e} D\lambda = \int \exp(i \int_0^s d\tau L_{BS}) \delta^2 (\dot{\bar{\theta}}) Dx D\theta D\bar{\theta}
\]

(3.21)

For a possible definition of such path integrals over both bosonic and fermionic is e.g. given in section 2.4 and 6.4 of ref.[11] where representations are given which automatically takes care of the boundary conditions for the integration variables. Propagators in the proper time method are obtained by means of a natural gauge fixing of the Lagrange multipliers $e$ and $\bar{\lambda}^\dot{\alpha}$ ($\lambda^\alpha$) in the complex actions. The propagators for chiral and antichiral superfields are then given by

\[
\langle \tau = s | \tau = 0 \rangle_{\text{chiral}} = \int \exp(i \int_0^s d\tau L) \delta(e - 1) \delta^2 (\bar{\theta}) DpDxD\theta D\theta D\bar{\theta}
\]
\[
\langle \tau = s | \tau = 0 \rangle_{\text{antichiral}} = \int \exp(i \int_0^s d\tau \bar{L}) \delta(e - 1) \delta^2 (\lambda) DpDxD\theta D\lambda D\bar{\theta}
\]

(3.22)

respectively. Now they are equal since

\[
\langle \tau = s | \tau = 0 \rangle_{\text{chiral}} = \int \exp(i \int_0^s d\tau \frac{1}{2} ((\dot{x})^2 - m^2)) \delta^2 (\dot{\theta}) Dx D\theta D\bar{\theta}
\]
\[
\langle \tau = s | \tau = 0 \rangle_{\text{antichiral}}
\]

(3.23)

which is in agreement with the result of ref.[3]. The results of this section were also given in [3].

4 Generalized BRST quantization of theories with second class constraints.

In order to give a brief description of the generalized BRST quantization of theories with second class constraints given in [1] we consider a real Lagrangian of the form (1.1)

\[
L = L_0 + \lambda^i \phi_i
\]

(4.1)
where $\lambda^i$ are Lagrange multipliers and $\phi_i$ constraint variables some of which are considered to be of second class in Dirac's classification, i.e. they satisfy PB-relations of the type

$$\{\phi_i, \phi_j\} = U_{ij}^k \phi_k + f_{ij}$$

(4.2)

where $f_{ij} \neq 0$. According to [1] (4.1) may be quantized by means of a generalized BRST quantization in which the BRST operator is not nilpotent. In the special case when $U_{ij}^k$ or $f_{ij}$ commutes with $\phi_i$ it is given by

$$Q = \hat{\phi}_i c_i - \frac{1}{2} iU_{ij}^k \mathcal{P}_k c^i c^j - \frac{1}{2} iU_{ij}^j c^i$$

(4.3)

where $c^i$ and $\mathcal{P}_i$ are the ghosts and their conjugate momenta and it satisfies

$$Q^2 = \frac{1}{2} f_{ij} c^i c^j \neq 0$$

(4.4)

Provided it is conserved it may still be used to project out physical states by means of the BRST condition

$$Q |ph\rangle = 0$$

(4.5)

In [12] it was shown that in order for this condition to project out the appropriate states there must exist a bigrading such that (For the standard nilpotent case this was also obtained in [13].)

$$Q = \delta + d$$

(4.6)

where

$$\delta^2 = d^2 = 0, \ [\delta, d]_+ = Q^2$$

(4.7)

and such that the physical states are determined by

$$\delta |ph\rangle = d |ph\rangle = 0$$

(4.8)

At this point we may connect this generalized BRST quantization with the method [7] since $\delta$ (or $d$) could be interpreted as a complex BRST charge operator coming from a complex gauge invariant Lagrangian. If we in addition have

$$d = \delta^\dagger$$

(4.9)

which is necessary when the considered state space is an inner product space, then $\delta^\dagger$ may be viewed as the BRST charge coming from the gauge invariant complex conjugate Lagrangian to the one for $\delta$ and (4.8) will then contain the solutions from both of these Lagrangians. Eq.(4.8) may then also be interpreted such that a BRST quantization of a complex Lagrangian requires the use of two BRST operators; the natural non-hermitian one and its hermitian conjugate. Below we illustrate this procedure for the free superparticle.
The free superparticle.

The Brink-Schwarz Lagrangian (3.18) (or the Casalbuoni one (3.19) in the massive case) may be written as

\[ L_{BS} = L_0 - \frac{e}{2} \phi - i \lambda^\alpha d_\alpha + i \bar{\lambda}_{\dot{\alpha}} \bar{d}^{\dot{\alpha}} \]  

(4.10)

in phase space where \( L_0 \) is given by (2.1). This is the corresponding real Lagrangian to (3.9) and (3.11) in which we have included all independent constraints. Notice that \( d_\alpha \) and \( \bar{d}^{\dot{\alpha}} \) are second class constraints (\( \{ d_\alpha, \bar{d}^{\dot{\alpha}} \} = 2i(\hat{p} \cdot \sigma)_{\alpha\dot{\alpha}} \)). According to the generalized BRST procedure above we should then have the BRST charge operator

\[ Q = \eta \hat{\phi} + c^\alpha \hat{d}_\alpha - \bar{c}_{\dot{\alpha}} \bar{\hat{d}}^{\dot{\alpha}} \]  

(4.11)

where \( \hat{\phi}, \hat{d}_\alpha, \bar{\hat{d}}^{\dot{\alpha}} \) are corresponding operators to \( \phi, d_\alpha, \bar{d}^{\dot{\alpha}} \), and where \( \eta \) is a hermitian fermionic ghost and \( c^\alpha, \bar{c}_{\dot{\alpha}} \) bosonic ghosts. \( Q \) is hermitian and assumed to be conserved. It satisfies

\[ Q = \delta + \delta^\dagger, \quad \delta^2 = (\delta^\dagger)^2 = 0, \quad [\delta, \delta^\dagger]_+ = Q^2 = -2c^\alpha \bar{c}_{\dot{\alpha}} (\hat{p} \cdot \sigma)_{\alpha\dot{\alpha}} \]  

(4.12)

where

\[ \delta = \frac{1}{2} \eta \hat{\phi} - \bar{c}_{\dot{\alpha}} \bar{\hat{d}}^{\dot{\alpha}}, \quad \delta^\dagger = \frac{1}{2} \eta \hat{\phi} + c^\alpha \hat{d}_\alpha \]  

(4.13)

where \( \delta \) may be viewed as the BRST charge from \( L \) in (3.9), and \( \delta^\dagger \) the one from \( \bar{L} \) in (3.11). Consider now the projection

\[ \delta \vert ph \rangle = \delta^\dagger \vert ph \rangle = 0 \]  

(4.14)

The chiral and antichiral superfield equations correspond then to two different sectors or more precisely to two different choices for the original state space in view of the treatment of ref. [13]. The chiral sector is obtained by means of the ghost fixing

\[ \mathcal{P} \vert ph \rangle = \bar{k}_{\dot{\alpha}} \vert ph \rangle = 0 \]  

(4.15)

where \( \mathcal{P} \) and \( \bar{k}_{\dot{\alpha}} \) are canonical conjugate momenta to the ghosts \( \eta \) and \( \bar{c}_{\dot{\alpha}} \) respectively. Consistency requires [14]

\[ [Q, \mathcal{P}]_+ \vert ph \rangle = [Q, \bar{k}_{\dot{\alpha}}]_- \vert ph \rangle = 0 \]  

(4.16)

or equivalently

\[ \hat{\phi} \vert ph \rangle = \hat{d}_{\alpha} \vert ph \rangle = 0 \]  

(4.17)

and

\[ [Q^2, \mathcal{P}]_- \vert ph \rangle = [Q^2, \bar{k}_{\dot{\alpha}}]_- \vert ph \rangle = 0 \]  

(4.18)

which in the massive case implies

\[ c^\alpha \vert ph \rangle = 0 \]  

(4.19)
The conditions (4.15) and (4.19) fix completely the ghost dependence of the physical states and (4.17) are exactly the equations (3.6) for the chiral superfield. Notice that (4.15), (4.17) and (4.19) imply (4.14). In the massless limit (4.18) does not imply (4.19) and we have ghost excitations from the point of view of ref. [13].

Similarly does

$$P|ph\rangle = k_\alpha|ph\rangle = 0$$

(4.20)

in the massive case imply

$$\bar{c}\dot{\alpha}|ph\rangle = 0$$

(4.21)

and

$$\hat{\phi}|ph\rangle = \hat{d}_\alpha|ph\rangle = 0$$

(4.22)

which are exactly the equations (3.7) for the antichiral superfield. In this case (4.20)-(4.22) imply (4.14).

The above physical states are not inner product states. The BRST quantization requires us therefore to work on bilinear forms of an original state space and its dual [13]. If we make use of an extended BRST charge involving dynamical Lagrange multipliers and antighosts then we may perform the BRST quantization on an inner product space from which we should obtain the propagators (3.23) in a precise manner (cf. [15]). However, such a precise analysis will not be performed in this paper.

5 External fields introduced by minimal coupling

A natural way to introduce interactions with an external Maxwell superfield in a manifestly supersymmetric way is by means of the replacements

$$p_\mu \rightarrow p_\mu - gA_\mu, \quad d_\alpha \rightarrow d_\alpha + gA_\alpha, \quad \bar{d}\dot{\alpha} \rightarrow \bar{d}\dot{\alpha} + g\bar{A}\dot{\bar{\alpha}}$$

(5.1)

in the constraints (3.10) and (3.12) in $L$ and $\bar{L}$ respectively. The super vector multiplet $(A_\mu, A_\alpha, \bar{A}\dot{\bar{\alpha}})$ may be represented in terms of a real scalar superfield $V = V(x, \theta, \bar{\theta})$ as follows [8]

$$A_\mu \equiv \frac{1}{8}\sigma^\dot{\alpha}_\mu [\bar{D}\dot{\alpha}, D_\alpha] V, \quad A_\alpha \equiv D_\alpha V, \quad \bar{A}\dot{\bar{\alpha}} \equiv \bar{D}\dot{\bar{\alpha}} V$$

(5.2)

where $D_\alpha$ and $\bar{D}\dot{\bar{\alpha}}$ are defined in (3.2). The interaction Lagrangians become then

$$L' = L_0 - \frac{e}{2} \phi' + i\bar{\lambda}\ddot{\alpha} \ddot{\bar{\alpha}}, \quad \bar{L}' = L_0 - \frac{e}{2} \phi' - i\lambda^\alpha \ddot{d}_\alpha$$

(5.3)

where $L_0$ is given by (2.1) and where

$$\phi' \equiv (p - gA)^2 + m^2, \quad d'_\alpha \equiv d_\alpha + gA_\alpha, \quad \ddot{d}_\dot{\alpha} \equiv \ddot{d}_\dot{\alpha} + g\bar{A}\dot{\bar{\alpha}}$$

(5.4)

According to the previous section the corresponding real Lagrangian to the complex conjugate ones in (5.3) should be

$$L'_{\text{real}} = L_0 - \frac{e}{2} \phi' + i\bar{\lambda}\ddot{\alpha} \ddot{\bar{\alpha}} - i\lambda^\alpha \ddot{d}_\alpha$$

(5.5)
which in configuration space becomes

\[ L'_{\text{real,conf}} = \frac{(\dot{x} - i\dot{\psi}\gamma^0\psi)^2}{2e} + gA_\mu(\dot{x}^\mu - i\dot{\psi}\gamma^\mu\psi) + igA^\alpha\dot{\theta}_\alpha - igA_\alpha\dot{\bar{\theta}}^\alpha - \frac{1}{2}em^2 \] (5.6)

One may notice that this is a massive D=4 version of the massless D=10 Lagrangian given in [16]. A transition to configuration space for the complex Lagrangians (5.3) yields on the other hand

\[ L'_{\text{conf}} = \frac{(\dot{x} + i\dot{\theta}\sigma\dot{\theta})^2}{2e} - \frac{1}{2}em^2 - i\pi^\alpha\dot{\theta}_\alpha + gA \cdot (\dot{x} + i\dot{\theta}\sigma\dot{\theta}) - ig\bar{A}_\alpha\dot{\bar{\theta}}^\alpha \]
\[ \bar{L}'_{\text{conf}} = \frac{(\dot{x} - i\dot{\theta}\sigma\dot{\theta})^2}{2e} - \frac{1}{2}em^2 + i\pi^\alpha\dot{\bar{\theta}}^\alpha + gA \cdot (\dot{x} - i\dot{\theta}\sigma\dot{\theta}) + ig\dot{\theta}^\alpha A_\alpha \] (5.7)

from which we obtain similar correspondences as in the free case

\[ L'_{\text{conf}} \iff L'_{\text{real,conf}} \] together with the constraint \( \dot{\theta}^\alpha = 0 \)
\[ \bar{L}'_{\text{conf}} \iff \bar{L}'_{\text{real,conf}} \] together with the constraint \( \dot{\bar{\theta}}^\alpha = 0 \) (5.8)

Unfortunately this expected natural construction does not work. The reason is that neither \((\phi', d'_\alpha)\) nor \((\phi', d'_\bar{\alpha})\) are first class constraints. We have

\[ \{d'_\alpha, d'_\beta\} = g\{A_\alpha, d_\beta\} + g\{d_\alpha, A_\beta\} = ig[D_\alpha, D_\beta] + V = 0 \]
\[ \{\phi', d'_\alpha\} = 2g(p^\mu - gA^\mu)(-iD_\alpha A_\mu - \partial_\mu A_\alpha) \] (5.9)

The only natural way to make them first class constraints is to require the last relation to be zero. In this case we would also have the same gauge invariance as in the free case. However, this requires

\[ A_\mu = i\partial_\mu V + a_\mu \] (5.10)

where \(a_\mu\) is an antichiral field \((D_\alpha a_\mu = 0)\). This severely restricts the superfield \(V\). In fact it requires \(\bar{D}^2D_\alpha V = 0\), a condition which is not allowed for the super Maxwell field (see (5.4) below).

Thus, the constraints \(d'_\alpha\) and \(\phi'\) are not first class ones when \(A^\mu\) is of the form (5.2). This means that the Lagrangians \(L'\) and \(\bar{L}'\) are not gauge invariant. In other words, it is not possible to have a gauge invariant coupling to a general super vector multiplet when the latter is introduced by minimal coupling in the free complex actions, and there are of course no corresponding superfield equations. However, in the next section we shall show that it is still possible to obtain a real Lagrangian almost of the form (5.6).

### 6 Gauge invariant introduction of external fields

In order to find the appropriate gauge invariant Lagrangians for the superparticle in interaction with an external super Maxwell field we must start from the corresponding field equations and apply the method of [7] described in the introduction. Now a real field Lagrangian for scalar chiral superfields in interactions with an external super Maxwell field seems to require at least a doublet of superfields. Define therefore

\[ \Phi \equiv \begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}, \quad \bar{\Phi} \equiv \begin{pmatrix} \bar{\Phi}_+ \\ \bar{\Phi}_- \end{pmatrix} \] (6.1)
The super field equations from this Lagrangian is (see e.g. ref.[8])

\[
\bar{D}^2 e^{g\sigma_3 V} \Phi = 4m\sigma_1 \Phi \\
D^2 e^{g\sigma_3 V} \Phi = 4m\sigma_1 \bar{\Phi}
\]  

(6.2)

where \(V\) is a real external scalar superfield, \(\sigma_i\) the Pauli matrices, and \(g\) as before a coupling constant. Even these equations imply the chiral conditions (3.3) for \(m \neq 0\), in which case (6.2) may also be diagonalized to yield

\[
\left( \bar{D}^2 e^{-g\sigma_3 V} D^2 e^{g\sigma_3 V} - 16m^2 \right) \Phi = 0 \\
\left( D^2 e^{-g\sigma_3 V} \bar{D}^2 e^{g\sigma_3 V} - 16m^2 \right) \bar{\Phi} = 0
\]

(6.3)

In terms of the components \(\Phi_{\pm}\) in (6.1) they become

\[
\left( \bar{D}^2 e^{\mp gV} D^2 e^{\pm gV} - 16m^2 \right) \Phi_{\pm} = 0 \\
\left( D^2 e^{\mp gV} \bar{D}^2 e^{\pm gV} - 16m^2 \right) \bar{\Phi}_{\pm} = 0
\]

(6.4)

Even (6.3) or (6.4) implies the chiral conditions (3.3). If we as in the free case choose (3.3) as separate independent equations then (6.4) may be simplified to

\[
\left\{ (i\partial_\mu \pm gA'_\mu) + m^2 \pm \frac{g}{4} [D_\alpha, W^\alpha] - \frac{g^2}{2} A^\alpha W_\alpha \right\} \Phi_{\pm} = 0 \\
\left\{ (i\partial_\mu \mp g\bar{A}'_\mu) + m^2 \mp \frac{g}{4} [\bar{D}_\alpha, \bar{W}^\bar{\alpha}] + \frac{g^2}{2} \bar{W}_{\bar{\alpha}} \bar{A}_{\bar{\alpha}} \right\} \bar{\Phi}_{\mp} = 0
\]

(6.5)

where \(A_\alpha\) and \(\bar{A}_{\bar{\alpha}}\) are given by (5.2), and where

\[
A'_\mu = A_\mu + \frac{i}{2} i\partial_\mu V = \frac{1}{4} \bar{\sigma}_\mu^\alpha \bar{D}_\alpha D_\alpha V, \\
\bar{A}'_\mu = A_\mu - \frac{i}{2} i\partial_\mu V = -\frac{1}{4} \sigma_\mu^\bar{\alpha} D_\alpha \bar{D}_{\bar{\alpha}} V, \\
W^\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V, \quad \bar{W}_{\bar{\alpha}} = -\frac{1}{4} \bar{D}^2 \bar{D}_{\bar{\alpha}} V
\]

(6.6)

where in turn \(A_\mu\) is the real super vector field defined in (5.2). \(W_\alpha\) and \(\bar{W}_{\bar{\alpha}}\) are the gauge invariant supersymmetric field strengths.

There is a peculiar feature of the use of chiral Lagrangians in this case: Although \(\bar{\Phi}\) is assumed to be the complex conjugate to \(\Phi\) in the Lagrangian density this is no longer true in the equations of motion. However, from (6.3) it looks like we may treat \(\bar{\Phi}_\mp\) as the complex conjugate to \(\Phi_{\pm}\) since the operators in (6.3) then are related by hermitian conjugation. From the corresponding operator expressions of (5.2) and (3.3) \((-i\partial_\mu \rightarrow p_\mu, D_\alpha \rightarrow d_\alpha\) and \(\bar{D}_\bar{\alpha} \rightarrow -\bar{d}_{\bar{\alpha}}\)) we may therefore derive the following interaction Lagrangians for the scalar fields \(\Phi_{\pm}, \bar{\Phi}_{\mp}\):

\[
L''_{\pm} = L_0 - \frac{e}{2} \Phi''_{\pm} + i\bar{\lambda}_\alpha \bar{d}_{\bar{\alpha}} \\
\bar{L}''_{\mp} = L_0 - \frac{e}{2} \bar{\Phi}''_{\mp} - i\lambda^\alpha d_\alpha
\]

(6.7)
where $L_0$ is given by (3.10) and $d^\alpha$ and $\bar{d}^\dot{\alpha}$ are defined in (3.12) respectively, and where

$$
\phi''_\pm \equiv (p + gA')^2 + m^2 \pm \frac{g}{2} d^\alpha W_\alpha + \frac{g^2}{2} A^\alpha W_\alpha
$$

$$
\bar{\phi''}_\pm \equiv (p + g\bar{A'})^2 + m^2 \pm \frac{g}{2} \bar{d}^\dot{\alpha} \bar{W}^{\dot{\alpha}} + \frac{g^2}{2} \bar{W}^{\dot{\alpha}} \bar{A}^{\dot{\alpha}}
$$

(6.8)

Notice that $L''_0$ and $L''_\pm$ only differ in the signs of the coupling constant $g$. In the appendix it is proved that $\phi''_\pm$ and $\bar{d}^\dot{\alpha}$ satisfy the same Poisson algebra as $\phi$ and $d^\alpha$ do in the free case. The Lagrangians (6.7) are therefore gauge invariant under the same abelian gauge group. $L''_\pm$ is invariant under (the nonzero transformations)

$$
\delta x^\mu = \zeta (p^\mu \mp gA'^\mu \mp \frac{d^\mu}{4} W A^\sigma \theta), \quad \delta e = \dot{\zeta}, \quad \delta \theta^\alpha = \mp \frac{\zeta \bar{g}^\alpha}{4} W^\alpha,
$$

$$
\delta \pi_\alpha = -i \frac{\zeta}{2} \partial_\alpha \phi''_\pm, \quad \delta \bar{\pi}_\dot{\alpha} = -i \frac{\zeta}{2} \partial_{\dot{\alpha}} \phi''_\pm, \quad \delta p_\mu = \frac{\zeta}{2} \partial_\mu \phi''_\pm
$$

(6.9)

and

$$
\delta x^\mu = \theta \sigma^\mu \bar{\kappa}, \quad \delta \bar{\theta}^\dot{\alpha} = i \bar{\kappa}_{\dot{\alpha}},
$$

$$
\delta \pi_\alpha = -i (p \cdot \sigma)_{\alpha \dot{\alpha}} \bar{\kappa}^{\dot{\alpha}}, \quad \delta \bar{\lambda}^{\dot{\alpha}} = -i \dot{\kappa}_{\dot{\alpha}}
$$

(6.10)

where $\zeta$ is a (real) infinitesimal parameter function in (6.9) and $\kappa^\alpha$ an odd complex parameter function in (6.10). $L''_\pm$ is invariant under the corresponding complex conjugated transformations.

The corresponding configuration space Lagrangians to (6.7) may e.g. be written as

$$
L''_{\pm (conf)} = \frac{1}{2} (\dot{x} - i \dot{\psi}\gamma \psi)^2 \pm gA' \cdot (\dot{x} - i \dot{\psi}\gamma \psi) - \frac{e}{2} m^2 \pm igA^\alpha \dot{\theta}_\alpha +
$$

$$
- \pi^\alpha (i \dot{\theta}_\alpha \mp \frac{eg}{4} W_\alpha)
$$

$$
\bar{L}''_{\pm (conf)} = \frac{1}{2} (\dot{\bar{x}} - i \dot{\bar{\psi}}\gamma \psi)^2 \pm g\bar{A}' \cdot (\dot{\bar{x}} - i \dot{\bar{\psi}}\gamma \psi) - \frac{\bar{e}}{2} m^2 \pm ig\bar{A}_{\dot{\alpha}} \dot{\bar{\theta}}^{\dot{\alpha}} +
$$

$$
+ \bar{\pi}_{\dot{\alpha}} (i \dot{\bar{\theta}}^{\dot{\alpha}} \mp \frac{\bar{e}g}{4} \bar{W}^{\dot{\alpha}})
$$

(6.11)

Imposing $\dot{\theta}^\alpha = \pm i \frac{eg}{4} W^\alpha$ for $L''_\pm$ we have effectively gauge invariance under ($\beta \equiv \zeta/e$)

$$
\delta x^\mu = \beta (\dot{x} + i \theta^\mu \dot{\bar{\theta}}), \quad \delta e = \dot{\beta} e + \dot{e} \beta, \quad \delta \theta^\alpha = \beta \dot{\theta}^\alpha, \quad \delta \bar{\theta}^{\dot{\alpha}} = 0
$$

(6.12)

as well as and under the transformations on the first line in (6.11). For $\bar{L}''_{\pm (conf)}$ we have invariance under the corresponding complex conjugated transformations. Notice that these gauge transformations are the same as those in the free case except for the last one in (3.12).

If we use the same gauge fixing as in (3.22) we find the propagators

$$
\langle \tau = s | \tau = 0 \rangle_{(\pm)} |_{\text{chiral}} =
$$

$$
= \int \exp (i \int_0^s dx L'_{\pm}) \delta (e - 1) \delta^2 (\bar{\lambda}) DpDx D\theta D\pi D\bar{\theta} D\bar{\pi} D\bar{\lambda} D\bar{\bar{\lambda}} =
$$
\[ = \int \exp(i \int_0^s \dd x L'(\pm)) \delta^2(\dot{\theta} \mp i \frac{g}{4} W) \delta^2(\dot{\bar{\theta}}) D\bar{\theta} D\theta D\bar{\theta} \]

\[ \langle \tau = s \mid \tau = 0 \rangle \mid \text{antichiral} = \]

\[ = \int \exp(i \int_0^s \dd x L'_\pm) \delta(\bar{e} \mp 1) \delta^2(\lambda) D\bar{\theta} D\theta D\bar{\theta} D\lambda D\bar{\theta} D\theta = \]

\[ = \int \exp(i \int_0^s \dd x L'_\pm) \delta^2(\dot{\theta}) \delta^2(\dot{\bar{\theta}} \pm i \frac{g}{4} W) D\bar{\theta} D\theta D\bar{\theta} \]

\[ \text{(6.13)} \]

where

\[ L'_\pm = \frac{1}{2}(\dot{\phi} - \imath \dot{\bar{\psi}} \gamma_\mu \psi)^2 - \frac{1}{2} m^2 \pm g A_\mu \cdot (\dot{\phi} - \imath \dot{\bar{\psi}} \gamma_\mu \psi) \]

\[ \pm i g A_\alpha \dot{\theta}_\alpha \mp i g A_\alpha \dot{\bar{\theta}}_\alpha \]

\[ \text{(6.14)} \]

is equal to \( L'_{\text{real,conf}} \) in (6.6) with \( e = 1 \) and \( A_\mu \) replaced by \( A'_\mu \) defined in (5.3).

The expressions (6.13) agree with those of [4] except for an interchange of the indices \( \pm \) in the antichiral case. Notice that we have used the Weyl ordering when we e.g. replaced the commutator \([D_\alpha, W^\alpha]_\)- by \( 2d^\alpha W_\alpha \) in the constraints \( \phi_\mu^\prime \). Therefore the Lagrangians (5.11) agrees up to the term \((D_\alpha W^\alpha)\) in \( L''_{\text{real,conf}} \) (and \((\bar{D}_\alpha \bar{W}^{\bar{\alpha}})\) in \( L''_{\text{antichiral}} \)) with those obtained in [4]. (There the operators in (6.3) also play a crucial role although they were obtained in a different manner.)

The BRST treatment in section 4 suggests that the corresponding real Lagrangian to the complex conjugate pair (6.7) is

\[ L''_{\pm(\text{real})} = L_0 - \frac{e}{2} \phi_\pm^\prime - \frac{\bar{e}}{2} \bar{\phi}_\pm^\prime + i \bar{\lambda}_\pm \bar{\psi} - i \lambda_\pm d_\alpha \]

\[ \text{(6.15)} \]

which in configuration space becomes

\[ L''_{\pm(\text{real,conf})} = \frac{1}{2} \left( \dot{\phi}^\prime - \imath \dot{\bar{\psi}} \gamma_\mu \psi \right)^2 - \frac{1}{2} m^2 \pm g A_\mu (\dot{\phi}^\prime - \imath \dot{\bar{\psi}} \gamma_\mu \psi) - \frac{1}{2} e_1 m^2 \]

\[ \mp g \frac{e_2}{2 e_1} \partial_\mu V (\dot{\phi}^\prime - \imath \dot{\bar{\psi}} \gamma_\mu \psi) + \frac{g^2 (e_1^2 + e_2^2)}{8 e_1} (\partial \cdot V)^2 \]

\[ - g^2 \frac{e_2}{8} e_1 (A^\alpha W_\alpha + \bar{W}_\alpha \bar{A}^\alpha) - i \frac{g^2}{8} e_2 (A^\alpha W_\alpha - W_\alpha \bar{A}^\alpha) \]

\[ \text{(6.16)} \]

where \( e_1 \equiv e + \bar{e} \) and \( e_2 \equiv i(\bar{e} - e) \).

It is rather obvious that \( \Phi \) is not the complex conjugate superfield to \( \Phi \) in (6.2). One may also notice that the free chiral superfield equations (5.3) are only obtained in the limit \( g \to 0 \) if \( \Phi_\pm \) is assumed to be complex conjugates to \( \Phi_\pm \) as we did above. However, even when this redefinition \((\sigma_1 \Phi \to \Phi)\) is inserted into the equations (6.2) we still do not obtain equations which are manifestly consistent with complex conjugation. Such equations are only obtained if we introduce the following fields

\[ \Phi' \equiv e^{\frac{\imath}{2} \sigma_3 V} \Phi, \quad \bar{\Phi}' \equiv e^{-\frac{\imath}{2} \sigma_3 V} \sigma_1 \bar{\Phi} \]

\[ \text{(6.17)} \]

since (6.2) then becomes

\[ \bar{D}^2 \Phi' = 4m \Phi', \quad D'^2 \Phi' = 4m \Phi' \]

\[ \text{(6.18)} \]
where
\[ D'_\alpha \equiv e^{-\frac{g}{2}\sigma_3 V} D_\alpha e^{\frac{g}{2}\sigma_3 V} = D_\alpha + \frac{g}{2} \sigma_3 A_\alpha, \]
\[ \bar{D}'_{\dot{\alpha}} \equiv e^{\frac{g}{2}\sigma_3 V} \bar{D}_{\dot{\alpha}} e^{-\frac{g}{2}\sigma_3 V} = \bar{D}_{\dot{\alpha}} - \frac{g}{2} \sigma_3 \bar{A}_{\dot{\alpha}} \]
(6.19)

The fields \( \Phi', (\bar{\Phi}') \) are no longer chiral ones since (6.17) transforms the chiral conditions (3.3) into
\[ D'_\alpha \Phi' = 0, \quad \bar{D}'_{\dot{\alpha}} \Phi' = 0 \]
(6.20)

Notice that (6.17) and (6.20) are manifestly consistent with complex conjugation. Equ. (6.18) may be diagonalized to
\[ \left(D^2 D'^2 - 16 m^2 \right) \Phi' = 0 \]
\[ \left(D'^2 \bar{D}'^2 - 16 m^2 \right) \bar{\Phi}' = 0 \]
(6.21)

If we like in the free case choose (6.20) as separate independent equations then (6.3) may be simplified to
\[ \left\{ (i \partial_\mu \pm g A_\mu)^2 + m^2 + \frac{g}{4} [D^{(\pm)}_\alpha, W^\alpha] \right\} \Phi'_\pm = 0 \]
\[ \left\{ (i \partial_\mu \pm g A_\mu)^2 + m^2 + \frac{g}{4} [\bar{D}^{(\pm)}_{\dot{\alpha}}, \bar{W}^{\dot{\alpha}}] \right\} \bar{\Phi}'_\pm = 0 \]
(6.22)
in terms of the components of \( \Phi' \) and \( \bar{\Phi}' \). Here \( A_\mu \) is the real vector superfield defined in (6.2) and
\[ D^{(\pm)}_\alpha \equiv D_\alpha \pm \frac{g}{2} A_\alpha, \quad \bar{D}^{(\pm)}_{\dot{\alpha}} \equiv \bar{D}_{\dot{\alpha}} \mp \frac{g}{2} \bar{A}_{\dot{\alpha}} \]
(6.23)

These relations correspond to (6.19) above. From the operator expressions in (6.20) and (6.22) (using \(-i \partial_\mu \to p_\mu, D_\alpha \to d_\alpha \) and \( \bar{D}_{\dot{\alpha}} \to - \bar{d}_{\dot{\alpha}} \)) we find now the following interaction Lagrangians for the scalar fields \( \Phi', \bar{\Phi}' \):
\[ L_m^m = L_0 - \frac{e}{2} \phi_m^m + i \bar{\lambda}_\alpha \bar{d}_{(\pm)\alpha} \]
\[ \bar{L}_m^m = L_0 - \frac{e}{2} \bar{\phi}_m^m - i \bar{\lambda}^\alpha d_{(\pm)\alpha} \]
(6.24)

where \( L_0 \) is given by (2.1) and where
\[ d_{(\pm)\alpha} \equiv d_\alpha \pm \frac{g}{2} A_\alpha, \quad \bar{d}^{(\pm)}_{\dot{\alpha}} \equiv \bar{d}^{\dot{\alpha}} \pm \frac{g}{2} \bar{A}_{\dot{\alpha}} \]
\[ \phi_m^m \equiv (p \mp g A)^2 + m^2 \mp \frac{g}{2} d_{(\pm)\alpha} W^\alpha \]
\[ \bar{\phi}_m^m \equiv (p \mp g A)^2 + m^2 \mp \frac{g}{2} \bar{d}^{(\pm)}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \]
(6.25)

In the appendix it is proved that \( \phi_m^m \) and \( \bar{d}^{(\pm)}_{\dot{\alpha}} \) satisfy the same Poisson algebra as \( \phi \) and \( \bar{d}_{\dot{\alpha}} \) in the free case. The Lagrangians (6.24) are therefore gauge invariant under the same abelian gauge group. \( L_m^m \) is invariant under (the nonzero transformations)
\[ \delta x^\mu = \zeta (p^\mu \mp g A^\mu \mp \frac{g}{4} W^{\sigma \mu} \bar{\theta}), \quad \delta e = \zeta, \quad \delta \theta^\alpha = \pm i \frac{\zeta g}{4} W^\alpha, \]
\[ \delta \pi_\alpha = -i \frac{\zeta}{2} \partial_\mu \phi_m^m, \quad \delta \bar{\pi}_{\dot{\alpha}} = -i \frac{\zeta}{2} \partial_{\dot{\mu}} \bar{\phi}_m^m, \quad \delta p_\mu = -\frac{\zeta}{2} \partial_\mu \phi_m^m \]
(6.26)
the following propagators for $\Phi'_{\bar{g}}$

$$
\delta x'^\mu = \theta \sigma'^\mu \bar{k}, \quad \delta \bar{\theta}'^\mu = i \bar{k}, \quad \delta p_\mu = \pm \frac{g}{2} \bar{\kappa}'^\mu \partial_\mu \bar{A}_\dot{\alpha} \quad \text{nn} \quad (6.27)
$$

$$
\delta \pi_{\alpha} = -i(p \cdot \sigma)_{\alpha \dot{\alpha}} \bar{k}'^\mu \pm i \frac{g}{2} \bar{\kappa}'^\mu \partial_\mu \bar{A}_{\dot{\beta}} \quad \delta \pi_{\dot{\alpha}} = \pm i \frac{g}{2} \bar{\kappa}'^\mu \partial_\mu \bar{A}_{\dot{\beta}} \quad \delta \bar{\lambda}' = -i \bar{k}' \quad (6.28)
$$

while $L''_{\pm}$ is invariant under the complex conjugated transformations.

The corresponding configuration space Lagrangians to (6.24) may e.g. be written as

$$
L''_{\pm (conf)} = \frac{1}{2e} (\dot{x} - i \dot{\psi} \gamma \psi)^2 + gA \cdot (\dot{x} - i \dot{\psi} \gamma \psi) - \frac{e}{2} m^2 \pm \frac{g}{2} \sigma'^\mu \partial_\mu \sigma'^\mu
$$

$$
L''_{\pm (conf)}(\dot{x} - i \dot{\psi} \gamma \psi)^2 + gA \cdot (\dot{x} - i \dot{\psi} \gamma \psi) - \frac{e}{2} m^2 \pm \frac{g}{2} \sigma'^\mu \partial_\mu \sigma'^\mu
$$

$$
+ \frac{\bar{\pi}_{\alpha}}{\dot{\theta}^2 + \frac{g}{4} \bar{W}^2}
$$

When $\dot{\theta}^2 = \frac{\bar{e}}{4} \bar{W}^2$ is imposed on $L''_{\pm}$ we have effectively gauge invariance under (6.12).

The corresponding real Lagrangian to the complex conjugate pair (6.29) is given by

$$
L''_{\pm (real)} = L_0 - \frac{e}{2} \dot{\theta}^2 + \frac{\bar{\pi}_{\alpha}}{\dot{\theta}^2 + \frac{g}{4} \bar{W}^2} + i \bar{\lambda} \bar{\sigma}^\alpha - i \lambda \sigma^\alpha
$$

which in configuration space becomes

$$
L''_{\pm (real, conf)} = \frac{1}{2} (\dot{x} - i \dot{\psi} \gamma \psi)^2 + gA \cdot (\dot{x} - i \dot{\psi} \gamma \psi) - \frac{e}{2} m^2
$$

$$
\pm \frac{g}{2} \sigma'^\mu \partial_\mu \sigma'^\mu + i \frac{g}{2} \bar{A}_{\dot{\alpha}} \dot{\bar{\theta}}
$$

where $e_1 = e + \bar{e}$. This is obviously a much nicer Lagrangian than (6.16) obtained before which indicates that $\Phi'$ and $\Phi''$ are the true complex conjugate pair of fields. It is very close to (5.6) obtained by minimal coupling. In fact, we get exact agreement if $g$ is replaced by $g/2$ in the last two replacements in (5.1) since the resulting modified (5.6) is (6.31).

This minimal coupling is also suggested by the relation

$$
\{d_{\alpha}^\pm, d_{\dot{\alpha}}^\pm\} = 2i(p_\mu + gA_\mu)\sigma_{\alpha \dot{\alpha}}
$$

A comparison between (5.5) and (6.30) yields that the Lagrange multiplier $\lambda^\alpha$ and $\bar{\lambda}^\dot{\alpha}$ in (5.5) are equal to $\lambda^\alpha = \frac{g}{2} \bar{W}^\alpha$ and $\bar{\lambda}^\dot{\alpha} = \frac{\bar{e}}{2} \bar{W}^\dot{\alpha}$ respectively in terms of the Lagrange multipliers in (6.30). This is an important difference since even when this modified minimal coupling is imposed on the free complex actions (3.15) it does not lead to gauge invariant complex covariant actions and corresponds to no superfield equations.

The propagators are found as before. Using the same gauge fixing as in (3.22) we find the following propagators for $\Phi'_\pm$

$$
\langle \tau = s|\tau = 0 \rangle_{(\pm)} =
$$

$$
= \int \exp(i \int_0^s d\tau L''_{\pm}|\delta(e - 1)\delta^2(\bar{\lambda})DpDxD\theta D\bar{\theta}D\pi D\bar{\pi}D\bar{\theta} \delta^2(\bar{\bar{\theta}})DxD\theta D\bar{\theta}
$$

(6.33)
and for $\Phi'_{\pm}$

$$\langle \tau = s | \tau = 0 \rangle (\pm) =$$

$$= \int \exp(i \int_0^s dt' L'_{\pm}^{(\text{real})}) \delta(\bar{e}' - 1) \delta^2(\lambda) DpDxD\pi D\bar{\pi} D\bar{e} D\bar{\lambda} =$$

$$= \int \exp(i \int_0^s dt' L'_{\pm}^{(\text{real})}) \delta^2(\bar{\theta}' \pm \frac{i g}{4} W) \delta^2(\bar{\theta}) DpDxD\bar{\pi}$$

(6.34)

where $L'_{\pm}^{(\text{real})}$ is equal to $L'_{\pm}^{(\text{real,conf})}$ in (6.33) with $e_1 = 1$. These expressions are in fact consistent with (6.13) since

$$L'_{\pm} - L'_{\pm}^{(\text{real})} = \pm \frac{1}{2} i g dV$$

(6.35)

inside the path integrals (6.13) and (6.33).

According to the generalized BRST procedure briefly described in section 4 we should have the following hermitian BRST charge operators to the Lagrangians (6.30)

$$Q_{\pm} = \eta \delta_{\pm}^{m} + \bar{\eta} \delta_{\pm}^{m} + c^\alpha \delta_{\pm}^{d(\pm)} - \bar{c}_\alpha \delta_{\pm}^{d(\pm)}$$

(6.36)

where $\eta, \bar{\eta}$ are fermionic ghosts and $c^\alpha, \bar{c}_\alpha$ bosonic spinor ghosts. $Q_{\pm}$ are assumed to be conserved. Eq.(6.36) may be written

$$Q_{\pm} = \delta_{\pm} + \delta_{\pm}^\dagger$$

(6.37)

where

$$\delta_{\pm} = \eta \delta_{\pm}^{m} - \bar{c}_\alpha \delta_{\pm}^{d(\pm)}, \quad \delta_{\pm}^\dagger = \bar{\eta} \delta_{\pm}^{m} + c^\alpha \delta_{\pm}^{d(\pm)}$$

(6.38)

which satisfy

$$\delta_{\pm}^2 = (\delta_{\pm}^\dagger)^2 = 0, \quad [\delta_{\pm}, \delta_{\pm}^\dagger] = Q_{\pm}^2 = c^\alpha \bar{c}_\alpha [\delta_{\pm}^{d(\pm)}, \delta_{\pm}^{d(\pm)}]_+$$

$$+ \eta \bar{\eta} [\delta_{\pm}^{m}, \delta_{\pm}^{m}]_- + \eta c^\alpha [\delta_{\pm}^{m}, \delta_{\pm}^{d(\pm)}]_- - \bar{c}_\alpha \delta_{\pm}^{d(\pm) \dagger}$$

(6.39)

where

$$[\delta_{\pm}^{d(\pm)}, \delta_{\pm}^{d(\pm)}]_+ \neq 0, \quad [\delta_{\pm}^{m}, \delta_{\pm}^{m}]_- \neq 0, \quad [\delta_{\pm}^{d(\pm)}, \delta_{\pm}^{d(\pm)}]_- \neq 0$$

(6.40)

$\delta_{\pm}$ and $\delta_{\pm}^\dagger$ may be viewed as the BRST charges from $L'_{\pm}$ and $\bar{L}'_{\pm}$ in (6.24).

Consider now the projections

$$\delta_{\pm} | ph \rangle = \delta_{\pm}^\dagger | ph \rangle = 0$$

(6.41)

The genuine physical sectors may then be obtained by means of auxiliary conditions [14]. The following ghost fixing is possible to impose

$$\mathcal{P} | ph \rangle = \bar{k}_{\alpha} | ph \rangle = 0$$

(6.42)
where $\mathcal{P}$ and $\bar{k}_\alpha$ are canonical conjugate momenta to the ghosts $\eta$ and $\bar{c}^{\dot{\alpha}}$ respectively. Consistency requires

$$[Q_\pm, \mathcal{P}]_+ |ph\rangle = [Q_\pm, \bar{k}_\alpha]_- |ph\rangle = 0 \quad (6.43)$$

or equivalently

$$\hat{\phi}^{m\prime}_\pm |ph\rangle = \hat{\gamma}^{(\pm)}_\alpha |ph\rangle = 0 \quad (6.44)$$

and

$$[Q_\pm^2, \mathcal{P}]_- |ph\rangle = [Q_\pm^2, \bar{k}_\alpha]_- |ph\rangle = 0 \quad (6.45)$$

which implies

$$\bar{\eta}|ph\rangle = c^\alpha |ph\rangle = 0 \quad (6.46)$$

The physical states are then completely ghost fixed and the matter part satisfies exactly the equations for the superfields $\bar{\Phi}'_\pm$ above. Notice also that (6.42), (6.44) and (6.46) imply (6.41). Notice also that (6.46) makes $Q_\pm^2$ vanish on physical states.

Similarly does

$$\bar{\mathcal{P}} |ph\rangle = k_\alpha |ph\rangle = 0 \quad (6.47)$$

imply

$$\eta |ph\rangle = \bar{c}_\alpha |ph\rangle = 0 \quad (6.48)$$

and

$$\hat{\phi}^{m\prime}_\pm |ph\rangle = \hat{\delta}^{(\pm)}_\alpha |ph\rangle = 0 \quad (6.49)$$

which are exactly the equations for the superfields $\bar{\Phi}'_\pm$ above.

## 7 The massless superparticle models.

We have demonstrated that the method of ref. [7] lead in a well defined manner to a gauge invariant although complex Lagrangian for the superparticle in four dimensions, free or in interaction with an external super Maxwell field. Although the derivation is only valid for massive superparticles the obtained Lagrangians have a well defined massless limit. However, in this case we cannot uniquely relate them to the massless limit of the corresponding real Lagrangians through the generalized BRST quantization (see remark after (4.19)). In fact, these two Lagrangians do not even have the same number of degrees of freedom in the massless limit. (The corresponding real one has less.) The reason why our derivation is not valid in the massless limit is due to the fact that the equations (3.6), (3.7) and (6.5) only follow from (3.1) and (6.2) respectively if $m \neq 0$. Below we consider the true massless superfield case.

Consider first the free massless superfield. The massless limit of (3.1) is given by

$$D^2 \Phi = 0, \quad \bar{D}^2 \bar{\Phi} = 0 \quad (7.1)$$
which is different from the massless limit of (3.6) and (3.7). Notice that $\Phi$ and $\bar{\Phi}$ are not chiral fields and that (7.1) cannot be derived from a chiral Lagrangian. The corresponding gauge invariant complex covariant Lagrangians are

$$L = L_0 + v d^2, \quad \bar{L} = L_0 + \bar{v} d^2$$

where $L_0$ is given by (2.1) and where $v$ is a complex bosonic Lagrange multiplier. In this case there is no complete configuration space Lagrangians available since

$$d^2 = \pi^\alpha \pi^\alpha - 2\pi(p \cdot \sigma)\bar{\theta} - \bar{\theta}^2 p^2, \quad \bar{d}^2 = \bar{\pi}^\dot{\alpha} \bar{\pi}^{\dot{\alpha}} - 2\bar{\theta}(p \cdot \sigma)\bar{\pi} - \bar{\theta}^2 p^2$$

which implies that the momentum $p_\mu$ cannot be eliminated in (7.2). However, $\pi^\alpha$ and $\bar{\pi}^{\dot{\alpha}}$ may be eliminated which yields

$$L = p \cdot (\dot{x} - i\dot{\theta} \sigma \bar{\theta}) + \frac{\dot{\theta}^2}{4v} + i\bar{\pi}^{\dot{\alpha}} \dot{\theta}^\alpha, \quad \bar{L} = p \cdot (\dot{x} + i\theta \sigma \bar{\theta}) + \frac{\overline{\dot{\theta}}^2}{4\bar{v}} - i\pi^\alpha \dot{\theta}_\alpha$$

$L$ yields the equations

$$\dot{p}^\mu = 0, \quad \dot{\theta}^\alpha = 0, \quad \dot{x}^\mu = i\dot{\theta} \sigma^\mu \bar{\theta}, \quad \dot{\pi}^\alpha = -\dot{\theta}^\alpha(p \cdot \sigma)_{\alpha\dot{\alpha}}, \quad \frac{d}{dt}\left(\frac{\dot{\theta}^\alpha}{v}\right) = 0$$

and $\bar{L}$ the complex conjugate ones. The massless condition $p^2 = 0$ can only be imposed as initial condition here as well as in (7.9) below. Notice that e.g. $L$ in (7.4) is gauge invariant under (the nonzero transformations)

$$\delta x^\mu = i\zeta \dot{\theta} \sigma^\mu \bar{\theta}, \quad \delta \theta^\alpha = \zeta \dot{\theta}^\alpha, \quad \delta \pi_{\alpha} = -\zeta \dot{\theta}^\alpha(p \cdot \sigma)_{\alpha\dot{\alpha}}, \quad \delta v = \frac{d(v\zeta)}{d\tau}$$

where $\zeta(\tau)$ is an arbitrary real infinitesimal parameter function.

The arguments in section 4 suggests here the existence of an associated real Lagrangian of the form

$$L_{\text{real}} = L_0 + vd^2 + \bar{v}d^2$$

which after eliminating $\pi^\alpha$ and $\bar{\pi}^{\dot{\alpha}}$ becomes

$$L_{\text{real,conf}} = p \cdot (\dot{x} - i\dot{\psi} \gamma \psi) + \frac{\dot{\theta}^2}{4v} + \frac{\overline{\dot{\theta}}^2}{4\bar{v}}$$

The equations of motion are

$$\dot{p}^\mu = 0, \quad \dot{x}^\mu = \dot{\psi}^\mu \psi = 0, \quad \dot{\theta}^2 = \bar{\theta}^2 = 0, \quad \frac{d}{dt}\left(\frac{\dot{\theta}^\alpha}{4v}\right) + i(p \cdot \sigma)_{\alpha\dot{\alpha}} \dot{\pi}^{\dot{\alpha}} = 0, \quad \frac{d}{dt}\left(\frac{\dot{\pi}^\alpha}{4\bar{v}}\right) - i\dot{\theta}^\alpha(p \cdot \sigma)_{\alpha\dot{\alpha}} = 0$$

We consider now the interaction case. The massless limit of (6.18) may be written

$$D^2_{(\pm)} \Phi^\prime_\pm = 0, \quad \bar{D}^2_{(\pm)} \bar{\Phi}^\prime_\pm = 0$$
where (for simplicity we replace \( g/2 \) by \( g \) here)

\[
D_{(\pm)\alpha} = D_\alpha \pm gA_\alpha, \quad \bar{D}_{(\pm)\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \mp g\bar{A}_{\dot{\alpha}}
\]  
(7.11)

(Notice that these equations cannot be obtained from chiral Lagrangians.) The corresponding gauge invariant complex covariant Lagrangians are

\[
L_\pm = L_0 + v\dot{d}_{(\pm)}^2, \quad \bar{L}_\pm = \bar{L}_0 + \bar{\dot{v}}\bar{d}_{(\pm)}^2
\]  
(7.12)

where

\[
d_{(\pm)\alpha} = d_\alpha \pm gA_\alpha, \quad \bar{d}_{(\pm)\dot{\alpha}} = \bar{d}_{\dot{\alpha}} \mp g\bar{A}_{\dot{\alpha}}
\]  
(7.13)

In configuration space they become

\[
L_\pm = p \cdot (\dot{x} - i\dot{\theta}\sigma\bar{\theta}) + \frac{\dot{\theta}^2}{4v} + i\bar{\pi}_{\dot{\alpha}}\dot{\theta}^{\dot{\alpha}} \pm ig\dot{\theta}^\alpha A_\alpha,
\]

\[
\bar{L}_\pm = p \cdot (\dot{x} + i\dot{\theta}\sigma\bar{\theta}) + \frac{\dot{\bar{\theta}}^2}{4\bar{v}} - i\bar{\pi}_{\dot{\alpha}}\dot{\theta}^{\dot{\alpha}} \mp ig\dot{\bar{\theta}}_{\dot{\alpha}}\bar{A}^{\dot{\alpha}}
\]  
(7.14)

where \( A_\alpha \) and \( \bar{A}_{\dot{\alpha}} \) are defined in (5.2). \( L_\pm \) is gauge invariant under the transformations (the nonzero ones)

\[
\delta x^\mu = i\zeta^\dot{\alpha}\dot{\theta}^\alpha, \quad \delta \theta^\alpha = \zeta\dot{\theta}^\alpha, \quad \delta \bar{\theta}^{\dot{\alpha}} = -\zeta\dot{\bar{\theta}}_{\dot{\alpha}} (p \cdot \sigma)_{\alpha\dot{\alpha}} \pm \zeta g\dot{\theta}^\alpha \partial_\alpha A_\alpha,
\]

\[
\delta p_\mu = \pm i\zeta g\dot{\theta}^\alpha \partial_\alpha A_\alpha, \quad \delta v = \frac{d(v\zeta)}{d\tau}
\]  
(7.15)

where \( \zeta (\tau) \) as before is an arbitrary real infinitesimal parameter function.

The corresponding real Lagrangians to (7.14) are

\[
L_{\pm(real)} = L_0 + vd_{(\pm)}^2 + \bar{v}\bar{d}_{(\pm)}^2
\]  
(7.16)

or

\[
L_{\pm(real,con.f)} = p \cdot (\dot{x} - i\dot{\psi}\gamma\bar{\psi}) + \frac{\dot{\theta}^2}{4v} + \frac{\dot{\bar{\theta}}^2}{4\bar{v}} \pm igA^\alpha \partial_\alpha \pm ig\bar{A}_{\dot{\alpha}}\dot{\theta}^{\dot{\alpha}}
\]  
(7.17)

The generalized BRST quantization of section 4 applied to the Lagrangian (7.17) leads to the hermitian BRST charge operators

\[
Q_\pm = \delta_\pm + \delta_\pm^\dagger, \quad \delta_\pm = \eta d_{(\pm)}^2, \quad \delta_\pm^\dagger = \eta\bar{d}_{(\mp)}^2
\]  
(7.18)

where \( \eta \) is a non-hermitian fermionic ghost. \( Q_\pm \) are hermitian and assumed to be conserved. We have the properties

\[
\delta_\pm^2 = (\delta_\pm^\dagger)^2 = 0, \quad [\delta_\pm, \delta_\mp^\dagger] = Q_\mp^2 = -4\eta\bar{\eta} (\delta_\mp + 2gA) \mu \sigma_{\alpha\dot{\alpha}} [\delta_{(\pm)}^\alpha, \delta_{(\mp)}^{\dot{\alpha}}] 
eq 0
\]  
(7.19)

where \( A_\mu \) is given by (5.2). \( \delta_\pm \) and \( \delta_\mp^\dagger \) may be viewed as the BRST charges from \( L_\pm \) and \( \bar{L}_\pm \) in (7.12). Consider now the projection (4.14). As in the previous cases the genuine physical states from (4.14) may be chosen to satisfy

\[
\mathcal{P}|ph\rangle = \bar{\eta}|ph\rangle = \frac{\dot{d}_{(\pm)}^2}{d_{(\pm)}^2}|ph\rangle = 0
\]  
(7.20)

where \( \mathcal{P} \) and \( \bar{\mathcal{P}} \) are canonical conjugate momenta to the ghosts \( \eta \) and \( \bar{\eta} \) respectively. Alternatively they may be chosen to satisfy

\[
\bar{\mathcal{P}}|ph\rangle = \eta|ph\rangle = \frac{\dot{\bar{d}}_{(\pm)}^2}{\bar{d}_{(\pm)}^2}|ph\rangle = 0
\]  
(7.21)

Eqs. (7.20) and (7.21) correspond exactly to the equations for the massless superfields \( \Phi_\pm \) and \( \bar{\Phi}_\pm \) in (7.10) respectively.
8 Final remarks.

In this paper we have derived complex gauge invariant actions for superparticles from some superfield equations given in the literature. We have then demonstrated that the resulting actions correctly describe the propagators by means of a natural proper time gauge fixing in the path integral expressions. In fact we get the same results as in ref. [4] both in the free as well as in the interaction case. The only peculiar feature of the use of complex actions is that one has to give up the strict reality properties of the involved dynamical variables both in the equations of motion and in the gauge transformations. A similar problem was also noted for one of the chiral Lagrangians: The supposed reality properties of the chiral superfields in the real Lagrangian describing chiral superfields in interactions with an external super Maxwell field are not valid in the equations of motion.

The obtained complex gauge invariant actions were shown to be related to real actions with second class constraints by means of the generalized BRST quantization proposed in [1, 12]. In this way we found a new real Lagrangian (6.30) for the interaction case which is only slightly different from (5.6) but which like (5.6) also can be obtained by minimal coupling. This generalized BRST procedure also tell us how the BRST quantization of the gauge invariant complex actions should be performed: The original state space should be spanned by all dynamical operators together with their hermitian conjugates. To the non-hermitian nilpotent BRST charge operator one should add its hermitian conjugate and perform the projection to the physical state space using both these BRST charges. With this procedure we have complete equivalence between the complex gauge invariant approach and the real approach with second class constraints. This is a little confusing in the massless limit of the massive chiral cases in section 3 and 6 since the real and complex actions classically do not leave the same physical degrees of freedom. Here the BRST quantization is also not completely equivalent to the corresponding superfield equations due to the presence of ghost excitations (see remark after (4.19)). What exactly happens in a proper BRST quantization on inner product spaces [13] remains to work out. The massless nonchiral cases considered in section 7 do not have this problem.
Appendix

In section 6 we stated that \( \bar{d}_\alpha = 0 \) and \( \phi''_\pm = 0 \) (as well as \( \bar{d}^{(\pm)}_\alpha = 0 \) and \( \phi''''_\pm = 0 \)) are first class constraints satisfying an abelian algebra. Here we give the details of this calculation.

**Proof of** \( \{ \bar{d}_\alpha, \phi''_\pm \} = 0 \).

Define

\[
d^{(\pm)}_\alpha \equiv d_\alpha \pm gA_\alpha
\]

then we have

\[
\{ \bar{d}_\alpha, d^{(\pm)}_\alpha \} = 2i(p \cdot \sigma)_{\alpha \bar{\alpha}} \pm ig\bar{D}_\alpha D_\alpha V
\]

and

\[
p_\mu \mp gA_\mu = \frac{i}{4} \bar{\sigma}^{\alpha \bar{\alpha}} \{ \bar{d}_\alpha, d^{(\pm)}_\alpha \}
\]

using the conventions in [8]. \( \phi''_\pm \) may therefore be written as

\[
\phi''_\pm = m^2 + \frac{1}{8} \{ \bar{d}_\alpha, d^{(\pm)}_\alpha \} \{ \bar{d}^\alpha, d^{(\pm)}_\alpha \} \mp \frac{g}{2} \phi^{(\pm)} W^\alpha
\]

Since

\[
\{ \bar{d}_\alpha, W^\alpha \} = i\bar{D}_\alpha W^\alpha = 0
\]
\[
\{ \bar{d}_\alpha, \{ \bar{d}_\beta, d^{(\pm)}_\beta \} \} = \pm ig\{ \bar{d}_\alpha, D_\beta D_\beta V \} =
\]

\[
\mp g\bar{D}_\alpha D_\beta D_\beta V = \pm 2g\epsilon_{\alpha \beta \gamma} W^\gamma
\]

we finally get

\[
\{ \bar{d}_\alpha, \phi''_\pm \} = \frac{1}{4} \{ \bar{d}^\beta, d^{(\pm)}_\beta \} (\pm 2g)\epsilon_{\alpha \beta \gamma} W^\gamma \mp \frac{g}{2} \{ \bar{d}_\alpha, d^{(\pm)}_\alpha \} W^\alpha = 0
\]

**Proof of** \( \{ \bar{d}^{(\pm)}_\alpha, \phi''''_\pm \} = 0 \).

\[
\{ \bar{d}^{(\pm)}_\alpha, \phi''''_\pm \} = \{ \bar{d}_\alpha, \phi''' + (\phi'''' - \phi''')_\pm \} \pm \frac{1}{2} g \{ \bar{A}_\alpha, \phi''''_\pm \} = 0 + \{ \bar{d}_\alpha, \frac{g^2}{4} (\partial V)^2 \pm
\]

\[
\pm ig\partial V \cdot (p \mp gA) \pm \frac{g^2}{4} A_\alpha W^\alpha \} \pm \frac{1}{2} g \{ \bar{A}_\alpha, (p \pm gA)^2 \pm \frac{g}{2} d^{(\pm)}_\alpha W^\alpha \} =
\]

\[
= i \frac{g^2}{2} \partial_\mu V \partial^\mu A_\alpha \mp g\partial \bar{A}_\alpha \cdot (p \mp gA) + g^2 \partial_\mu V \bar{D}_\alpha A_\mu + i \frac{g^2}{4} \bar{D}_\alpha A_\alpha W^\alpha -
\]

\[
- i \frac{g^2}{4} A_\alpha \bar{D}_\alpha W^\alpha \pm g\partial \bar{A}_\alpha \cdot (p \mp gA) + i \frac{g^2}{4} D_\alpha \bar{A}_\alpha W^\alpha = 0
\]

where we have used the explicit expressions of \( A^\alpha, \bar{A}^\alpha, A^\mu \) and \( W^\alpha \) in terms of \( V \) given in (5.2) and (6.6).
References

[1] R. Marnelius, *Nucl. Phys.* B294, 685 (1987)
   R. Marnelius, *Generalized BRST quantization*, Proc. Int. meeting on geometrical
   and algebraic aspects of nonlinear field theory, Amalfi, Italy, 1988, Ed S. De
   Filippo, M. Marinaro, G. Marmo and G. Vilasi North-Holland, Amsterdam, 1989.

[2] D.G. Boulware and D.J. Gross, *Nucl. Phys.* B233, 1 (1984)

[3] R. Marnelius, *Phys. Lett.* B318, 92 (1993)

[4] E. S. Fradkin and Sh. M. Shvartsman, *Mod. Phys. Lett.* A6, 1977 (1991)

[5] R. Marnelius and Sh. M. Shvartsman, *Complex covariant actions for superparti-
   cles*, ITP-Göteborg report 91-39 (1991) (unpublished)

[6] J. Grundberg, U. Lindström and H. Nordström, *Nucl. Phys.* B410, 355 (1993)

[7] R. Marnelius and U. M(??)rtensson, *Nucl. Phys.* B335, 395 (1990); *Int. J. Mod.
   Phys.* A6, 807 (1991)

[8] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University
   Press (1983)

[9] R. Casalbuoni, *Nuovo Cimento* 33A, 389 (1976)

[10] L. Brink and J. H. Schwarz, *Phys. Lett.* B100, 310 (1981)

[11] E.S. Fradkin, D.M. Gitman and Sh.M. Shvartsman, *Quantum Electrodynamics with
   Unstable Vacuum*, Springer-Verlag, Berlin (1991)

[12] R. Marnelius, *Nucl. Phys.* B370, 165 (1992)

[13] S. Hwang and R. Marnelius, *Nucl. Phys.* B315, 638 (1989); B320, 476 (1989)

[14] R. Marnelius, *Nucl. Phys.* B372, 218 (1992)

[15] R. Marnelius, *Proper BRST quantization of relativistic particles*. Nucl. Phys. B
   (in press)

[16] M. Rocek, W. Siegel, P. van Nieuwenhuizen and A. E. van de Ven, *Phys. Lett.*
   B227, 87 (1989)