Phase diagram of the ferrimagnetic mixed-spin Blume-Capel model with four-spin and next-nearest neighbor interactions

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Abstract. A finite cluster approximation based on a single cluster theory is developed for a ferrimagnetic mixed spin-1/2 and spin-1 Blume-Capel model with four-spin interaction $J_4$ and next-nearest neighbor couplings $J'$. The state equations are derived for the two-dimensional square lattice and the phase diagram is explored. We found that the system can exhibits both critical and tricritical behavior according to appropriate values of $J'$, $D$ and $J_4$.

1. Introduction

The synthesis of new stable crystalline room-temperature magnets, that are able to keep magnetization in the absence of an applied magnetic field, is a major issue for many industrial applications. This is not only because of their potential device application, including those in areas such as thermomagnetic recording, electronic and computer technologies [1], as well as biomedical materials [2] and catalysts [3], but also due to their common uses as research tools in multi-interdisciplinary areas of chemistry [4], geology, biology and medicine [5]. In recent years both chemists and physicists started to collaborate very closely on chemistry and material sciences in order to produce materials soluble in organic solvents, biocompatible, optically transparent, with spontaneous moments at room temperature [1][4][6]. Generally, the ferrimagnetic ordering seems often to play a prominent relevance in these materials. There is currently a great deal of interest in the synthesis of single-chain magnets (SCM) with ferrimagnetic arrangement, such as $\text{Co}^{II}[(\text{hfac})_2\text{(NITPhOMe)}]$ [7], $\text{Co}^{II} – \text{Cu}^{II}$ bimetallic single-chain magnet [8] and $[\text{Fe}^{III}(\text{ClO}_4)_2\{\text{Fe}^{III}(\text{bpca})_2\}\{\text{ClO}_4\}]$ [9], because they can act as elementary binary units (bits) used for information storage, providing potential applications in high-density information storage [10] or as qubits in quantum computers [11]. As purely one-dimensional systems are known to have a long-range order only at $T = 0$ K, these SCM materials promote long relaxation times and the system can behave like a magnet [12]. In the search for new and improved molecular materials, synthesis has been expanded toward 2-D and 3-D dimensional ferrimagnets, such as 2-D organometallic ferrimagnets [12]. 2-D networks of the mixed-metal material $\{\text{P(Phenyl)}4\}[\text{MnCr(oxalate)3}]n$ [13], 3-D ferrimagnets with $T_C=240K$ and $T_C=190K$ [14] and the amorphous $\text{V(TCNE)}_x\text{y(solvent)}$ with temperatures ordering as high...
as 400K [15]. For all these purposes, intensive efforts are required in the theoretical study of these materials in order to clarify their very interesting and sometimes unusual behaviors.

Mixed-spin Ising systems provide good models to study ferrimagnetism. The magnetic properties of these systems on the square lattice have been investigated by various methods. In particular, the mixed spin-1/2 and spin-1 Ising model with only nearest neighbor coupling and crystal field, has been studied by mean field theory [16] and high(low)-temperature series expansions [17]. Both approaches report tricritical point while compensation points were only detected using mean field approximation. As a result the phase diagram and the magnetic properties of the system have been reinvestigated using numerical transfer matrix techniques, supplemented by Monte Carlo simulations by Buendia and Novotny [18], renormalization-group technique [19] and also using extensive Monte Carlo simulations by Selke and Oitmaa [20]. The resulting papers found no evidence of either a compensation point or a tricritical point. On the other hand, they showed that compensation points are induced by the presence of an interaction between the next nearest neighbor of the spin-1/2. The inclusion of further-neighbor interactions would allow for a better modeling of real magnetic systems [21] and of course of all other systems that can be mapped onto the Ising models such as models of microemulsions [22].

Recently, there has been considerable interest in experimental and theoretical researches of Ising model with multispin interactions. These models are interesting because they found their theoretical explanation in the theories of super exchange interaction, the magnetoelastic effect and the spin-phonon coupling [23]. Moreover, it was pointed out that the models with the higher-order exchange interactions may exhibit rich phase diagrams and can describe phase transition in some physical systems. Additionally, they show physical behavior not detected in the usual spin systems. For example, the non-universal critical phenomena [24] and deviation from $T^{3/2}$ Block law at low temperature [25].

From the theoretical point of view, the monoatomic Ising models with multispin interactions have been investigated in detail within different methods, such as mean field approximation [26], effective field theory [27], some more accurate treatments such as series expansion [28], renormalization group methods [29], Monte Carlo simulations [30], and also exact calculations [31]. Experimentally, an interesting fact for the models with multispin interactions has been reported. Indeed, it can be used to describe various physical systems such as classical fluid [32], solid $^3$He [33], lipid bilayers [34], and rare gases [35]. Moreover for some materials it has been shown that the multispin interactions play a significant role; and they are comparable or even much important than the bilinear ones. The models with pair and quartet interaction have been applied successfully to study and explain the existence of first order phase transition in squaric acid crystal $\text{H}_2\text{C}_2\text{O}_4$ [36]. Such models have been also used to describe thermodynamical properties of hydrogen-bonded ferroelectric $\text{PbHPO}_4$, $\text{PbDPO}_4$ [37], some copolymers [38] and optical conductivity [39] observed in cuprate ladder $\text{La}_x\text{Ca}_{14-x}\text{Cu}_{24}\text{O}_{41}$. On the other hand, some experimental studies on $\text{La}_x\text{Ca}_y\text{Cu}_z\text{O}_{4+\delta}$ [40] and $\text{La}_x\text{Sr}_y\text{Cu}_z\text{O}_{4+\delta}$ [41] reveal that they could be explained by the introduction of the four-spin interaction. It is worthy to note here that this later plays an important role in the two dimensional antiferromagnet $\text{La}_x\text{CuO}_2$ [42], the parent material of high-$T_C$ superconductors.

Intense interest has been directed to study the magnetic properties of two-sublattices mixed spin Ising system. They have less translational symmetry than their single spin counterparts, and are well adapted to study a certain type of ferrimagnetism [43]. Experimentally, it has been shown that the
MnNi(EDTA)-6H$_2$O complex [44] is a good example of a mixed system. The thermodynamic properties of the ferromagnetic mixed Ising model consisting of spin-1/2 and spin-1 with four-spin and next nearest neighbor (NNN) interactions have been studied, very recently, by Monte Carlo simulations (MCS) [45] and finite cluster approximation (FCA) [46]. According to the value of the NNN couplings it has been shown that the system behaves qualitatively and quantitatively different from that obtained when the interaction between NNN is ignored. In the present paper we examine the same system using FCA in the context of the ferrimagnetism and compensation points. We will focus our attention principally on the behavior of the phase diagrams.

The ferrimagnetic mixed-spin Ising system, we are interested in, is shown in Fig.1. Such system can be described by the following Hamiltonian:

$$H = J_2 \sum_{\langle ij \rangle} \sigma_i \sigma_j - J_4 \sum_{\langle i,j,k,l \rangle} \sigma_i \sigma_k \sigma_j \sigma_l - J' \sum_{\langle ik \rangle} \sigma_i \sigma_k + D \sum_j (S_j)^2$$  \hspace{1cm} (1)

![Figure 1](image)

**Figure 1.** (a) Part of the square lattice. ⋄ and × correspond to σ and S-sublattice sites, respectively. (b) Neighbors of spin $\sigma_0$ with which directly interacts. (c) Neighbors of spin $S_0$ with which directly interacts.

The underlying lattice is composed of two interpenetrating sublattices. One occupied by spins with spin moment $\sigma=\pm 1/2$ and the other one is occupied by spins with moment $S= 0, \pm 1$ which interact with one another with ferrimagnetic parameter $J_2$ ($J_2>0$). The first summation carried out only over nearest-neighbor pair of spins. The second and the third summations represent the four-spin and NNN interactions, respectively, where the summations concern all alternate squares shaded in Fig.1. In the present work, our study is focused on ferrimagnetic state with $J'>0$. 
This paper is organized as fellows. The second section is devoted to the theoretical framework and the state equations whereas the third section focuses on the numerical result and the discussion. Finally, some conclusions are drawn in the fourth section.

2. Theoretical framework

The theoretical framework that we adopt in the study of the mixed spin-1/2 and spin-1 Ising model with four-spin and NNN interactions, described by the Hamiltonian (1), is the finite cluster approximation (FCA) [47] based on a single-site cluster theory. We have to mention that this method has been successfully applied to a number of interesting pure and disordered spin Ising systems [48]. It has also been used for transverse Ising models [49] and semi-infinite Ising systems [50]. In all these applications, it was shown that the FCA improves qualitatively and quantitatively the results obtained in the frame of the mean-field theory. In this approach, attention is focused on a cluster comprising just a single selected spin $\sigma_o (S_o)$ and its neighbour spins $\{\sigma_1, \sigma_2, S_1, S_2, S_3, S_4\}$ ($\{S_1, S_2, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$) with which it directly interacts (see Figs. 1-(a) & (b)).

We split the total Hamiltonian (1) into two parts, $H=H_o+H'$ where $H_o$ includes all parts of $H$ associated with the lattice site 0. In the present system, $H_o$ takes the form

$$H_{0\sigma} = -\sum_{i=1}^{4} S_i + J_4(S_1S_2\sigma_1 + S_3S_4\sigma_2) + J'(\sigma_1 + \sigma_2)\sigma_0$$

$$H_{0S} = -\sum_{i=1}^{4} \sigma_i + J_4(\sigma_1\sigma_2S_1 + \sigma_3\sigma_4S_2)S_0 + D(S_0)^2$$

whether the lattice site 0 belongs to $\sigma$ or $S$-sublattice, respectively.

The problem consists in evaluating the sublattice magnetizations and the quadrupolar moment. To this end, we denote by $<\sigma_o>_c$ and $<S_o^n>_c$ ($n=1, 2$), respectively the mean value of $\sigma_o$ and $S_o^n$ for a given configuration $c$ of all other spins (i.e. when all other spin $\sigma_i$ and $S_j$ ($i, j\neq 0$) are kept fixed). $<\sigma_o>_c$ and $<S_o^n>_c$ are given by

$$\langle \sigma_0 \rangle_c = \frac{\text{Tr}_{\sigma_0}\sigma_0\exp(-\beta H_{0\sigma})}{\text{Tr}\exp(-\beta H_{0\sigma})}$$

$$\langle S_0^n \rangle_c = \frac{\text{Tr}_{S_0^n}S_0^n\exp(-\beta H_{0S})}{\text{Tr}\exp(-\beta H_{0S})}$$

where Tr$_{\sigma_o}$ (or Tr$_{S_0}$) means the trace performed over $\sigma_o$ (or $S_0$) only. As usual $\beta=1/T$ where $T$ is the absolute temperature. The sublattice magnetizations $\mu, m$ and the quadrupolar moment $q$ are then given by

$$\mu \equiv \langle <\sigma_0>_c \rangle = \frac{\text{Tr}_{\sigma_0}\sigma_0\exp(-\beta H_{0\sigma})}{\text{Tr}\exp(-\beta H_{0\sigma})}$$

$$\mu \equiv \langle <\sigma_0>_c \rangle = \frac{\text{Tr}_{\sigma_0}\sigma_0\exp(-\beta H_{0\sigma})}{\text{Tr}\exp(-\beta H_{0\sigma})}$$
where \(<\cdots>\) denotes the average over all spin configurations. Performing the inner traces in (6), (7) and (8) over the states of the selected spin \(\sigma_o\) \((S_o)\), we obtain the following exact relations

\[
m \equiv \langle \langle S_0 \rangle \rangle_c = \frac{\text{Tr} S_0 \exp(-\beta H_{0S})}{\text{Tr} \exp(-\beta H_{0S})} \]

\[
q \equiv \langle \langle S_o^2 \rangle \rangle_c = \frac{\text{Tr} S_o^2 \exp(-\beta H_{0S})}{\text{Tr} \exp(-\beta H_{0S})} \]

where \(K=\beta J_2\), \(\alpha=J_4/J_2\) and \(\gamma=J'/J_2\).

It is a formidable task to calculate the average on the right-hand sides of Eqs.(9) to (11) over all spin configurations. We can easily observe that any function such as \(f(\sigma,S)\) of \(\sigma\) and \(S\) can be written as the linear superposition

\[
f(\sigma,S) = \sum_{i=1}^{6} f_i(\sigma_i,S_j)\]

which appropriate coefficients \(f_i\) \((i=1,\ldots,6)\). After applying this to all spins \(\sigma_i\) and \(S_j\) in expressions between brackets in equations (9) to (11), we average over all spin configurations. In this paper we use the simplest approximation in which we treat all spin self-correlations exactly while still neglecting correlations between quantities pertaining to different sites. This leads to the following coupled equations.

\[
\mu [2A_1 + 4(A_2 + A_3)q + (2A_4 + 8A_5 + 2A_6)q^2 + 4(A_7 + A_8)q^3 + 2A_9q^4] + \]
\[
m [4A_{10} + (4A_{11} + 8A_{12})q + (4A_{13} + 8A_{14})q^2 + 4A_{15}q^3] + \]
\[
m \mu^2 [4A_{16} + 4A_{17} + 8A_{18})q + (4A_{19} + 8A_{20})q^2 + 4A_{21}q^3] + \]
\[
\mu m^2 [2A_{22} + 8A_{23} + 2A_{24} + (4A_{25} + 8A_{26} + 4A_{27} + 8A_{28})q + (2A_{29} + 8A_{30} + 2A_{31})q^2] + \]
\[
m^3 [4A_{32} + 4A_{33}q] + \mu m^4 [2A_{34}] + m^2 \mu^3 [4A_{35} + 4A_{36}q] \]

(13)

\[
m \mu [4B_1 + 8B_2q + 4B_3q^2] + m [2B_4 + 2B_5q] + \mu^3 [4B_6 + 4B_7q + 4B_9q^2] + \]
\[
m \mu^2 [2B_8 + 2B_{10}q] + \mu m^2 [4B_{11} + \mu m^4 [2B_{12} + 2B_{13}q] + m^2 \mu^3 [4B_{14}] \]

(14)

\[
q [C_1 + 2C_2q + C_3q^2] + \mu^2 [6C_4 + 12C_5q + 6C_6q^2] + m^2 [C_7] + \mu m [4C_8 + 4C_9q] + \]
\[
\mu^4 [C_{10} + 2C_{11}q + C_{12}q^2] + m\mu^3 [4C_{13} + 4C_{14}q] + m^2 \mu^4 [C_{16}] \]

(15)
The non-zero coefficients quoted in Eq. (13) are listed in the Appendix while those quoted in Eqs. (14) and (15) can be taken from Appendix in [51], where the definition of the functions g(x) and h(x) must be changed, respectively, to

\[ g(x) \equiv \frac{2\sinh(Kx)}{\exp(\beta D) + 2\cosh(Kx)}, \quad h(x) \equiv \frac{2\cosh(Kx)}{\exp(\beta D) + 2\cosh(Kx)} \]

and take into account that the bilinear interaction \( J_2 \) is ferrimagnetic.

If we replace \( m \) and \( q \) in (13) by their expressions taken from (14) and (15), we obtain an equation for \( \mu \) of the form

\[ \mu = a\mu + b\mu^3 + \cdots \]  

(16)

In order to obtain the second-order transition temperature, we neglect higher-order terms in the magnetizations in Eqs. (13)-(15). Therefore, the critical temperature is analytically obtained through a determinantal equation, i.e.

\[ 1 + 2A_1 + 4(A_2 + A_3)q_0 + (2A_4 + 8A_5 + 2A_6)q_0^2 + 4(A_7 + A_8)q_0^3 + 2A_9q_0^4 + \]

\[ \frac{4B_1 + 8B_2q_0 + 4B_3q_0^2}{1 - (2B_4 + 2B_5q_0)} [4A_{10} + (4A_{11} + 8A_{12})q_0 + (4A_{13} + 8A_{14})q_0^2 + 4A_{15}q_0^3] \]  

(17)

where \( q_0 \) is the solution of

\[ q_0 = C_1 + 2C_2q_0 + C_3q_0^2 \]  

(18)

Equation (17) means that its right-hand side corresponds to the coefficient \( a \) in (16).

In the vicinity of the second-order transition, the sublattice magnetization \( \mu \) is given by

\[ \mu^2 = \frac{1 - a}{b} \]  

(19)

The right-hand side of (19) is positive since we are in the long-ranged ordered regime. This means that the signs of \( \mu^2 \) and \( b \) are the same.

When \( b \) changes sign (\( \mu \) keeping its sign), \( \mu^2 \) becomes negative. It means that we are not in the vicinity of a second-order transition line. So, the transition is of the first order. Therefore the point at which

\[ a(K, \alpha, \gamma) = 1 \quad \text{and} \quad b(K, \alpha, \gamma) = 0 \]  

(20)

characterizes the tricritical points.

To obtain the expression for \( b \), one has to solve (13)-(15) for small \( \mu \) and \( m \). The solution is of the form

\[ q = q_0 + q_1\mu^2 + q_2m^2 + q_3\mu m \]  

(21)

where \( q_1, q_2 \) and \( q_3 \) are given by
After some algebraic manipulations, (14) and (21) can be written in the following forms

\[ q_1 = \frac{6C_4 + 12C_5 q_0 + 6C_6 q_0^2}{1 - 2C_2 - 2C_3 q_0}, \quad q_2 = \frac{C_7}{1 - 2C_2 - 2C_3 q_0}, \quad q_3 = \frac{4C_9 + 4C_9 q_0}{1 - 2C_2 - 2C_3 q_0}. \]

By substituting \( m \) and \( q \) in Eq. (13), with their expressions taken from Eqs. (22) and (23), we obtain the Eq. (16), where \( b \) is given by

\[ b = \frac{4(A_2 + A_3)q_4 + 2(2A_4 + 8A_5 + 2A_6)q_0q_4 + 3(4A_7 + 4A_8)q_0^2q_4 + 8A_9q_0^3q_4}{A} + \frac{A}{B}[(4A_{11} + 8A_{12})q_4 + 2(4A_{13} + 8A_{14})q_0q_4 + 12A_{15}q_0^2q_4] + \frac{A}{B}[4A_{16} + 4(A_{17} + 8A_{18})q_0 + (4A_{19} + 8A_{20})q_0^2 + 4A_{21}q_0^3] + \frac{E A^2}{B^2} + \frac{F A^3}{B^3} + \frac{[8B_2q_4 + 8B_3q_0q_4 + C + AD + 2B_5q_4A + 4B_{11}A^2]}{B^2} + \frac{4B_{11}A^2}{B^3} \]

\[ [4A_{10} + (4A_{11} + 8A_{12})q_0 + (4A_{13} + 8A_{14})q_0^2 + 4A_{15}q_0^3] \]

with

\[ E = 2A_{22} + 8A_{23} + 2A_{24} + (4A_{25} + 8A_{26} + 4A_{27} + 8A_{28})q_0 + (2A_{29} + 8A_{30} + 2A_{31})q_0^2 \]

And

\[ F = 4A_{32} + 4A_{33}q_0 \]

3. Results and discussion

In this section, let us examine numerically the phase diagrams of the ferrimagnetic mixed spin-1 and spin-1/2 Blume-Capel model with four-spin interaction \( J_4 \) and next-nearest neighbor coupling \( J' \), described by the Hamiltonian (1), using the formulation of the FCA given in the section 3. Indeed, we study the effect of the crystal field \( D \), the four-spin interactions \( J_4 \) as well as the next nearest neighbor couplings on the phase diagram of the system under study.
First of all, let us take $J_4'J' = 0$ and consider the system with only bilinear interaction and crystal field. The phase diagram in the $(T_c/J_2, D/J_2)$ plane depicted in Fig. 2 shows a tricritical point at $(D/J_2)_{\text{Tri.}} = 1.976$ which can be compared with the mean-field treatment $(D/J_2)_{\text{Tri.}} = 1.860$ [55]. In order to elucidate the role of the next nearest neighbor interaction on the system, we plot in Fig. 3 the phase diagram for $J_4'/J_2 = D/J_2 = 0$. From this latter (Fig. 3), on one hand we note that no tricritical point can be found which means that the system exhibits second order transition for any positive value of $J'$. On the other hand, a positive value of $J'$ strengthens the ferrimagnetic order at low temperature. In Fig. 4, we plot the phase diagram in the $(T_c/J_2, D/J_2)$ plane for selected values of the ratio $J_4'/J_2$. From this figure, we point out that the $T$-component of the tricritical point decreases with decreasing values of the four-spin coupling. The insert plots in Fig. 4 show that the latter behavior (tricriticality) disappears, and all transitions are of second-order kind. One also notes that for $J' \leq -0.5$, the phase diagrams show bulges suggesting the occurrence of the reentrant phenomenon in which competing pair, quartet and crystal field interactions may occur. In order to have an idea on the effects of the four-spin interaction on the mixed spin Ising model with next nearest neighbor interaction, we have plotted in Fig. 5 sections of the critical surface $T_c(J', J_4)$ with a plan of fixed values of four-spin coupling. We note that negative values of $J_4$ decrease the critical temperature as $J'$ decreases. As seen from this figure, the effects become dramatic when $J_4$ approaches $-4$ and the critical temperature vanishes exactly for $J_4/J_2 = -4$ and $J'/J_2 = 0$. 

\textbf{Figure 2.} The phase diagram in $(D/J_2, T_c/J_2)$ plane for the mixed spin-$1/2$ and spin-$1$ Blume-Capel model.

\textbf{Figure 3.} The phase diagram of the mixed spin-$1/2$ and spin-$1$ Ising model with next nearest neighbor interaction. No tricritical point is detected in this case.
4. Conclusion
In this work, we have investigated the phase diagram of the ferrimagnetic mixed spin Blume-Capel model with four-spin $J_4$ and next nearest neighbor $J'$ interactions on the square lattice by the use of the finite cluster approximation within the framework of a single-site cluster theory. We have discussed the effect of $J_4$, $J'$ and $D$, normalized with the interaction strength $J_2$, on the phase diagrams. We have shown that the tricritical behavior arises when the four-spin or the crystal field interaction is included. This latter feature disappears when $J_4$ and $D$ take appropriate values and all transitions are of second-order. We have also shown that no tricritical behavior is detected when including next nearest neighbor interaction $J'$ between the sites of the $\sigma$-sublattice and the critical temperature vanishes when $J_4/J_2=-4$ and $J'/J_2=0$.

Appendix
The coefficients appearing in Eq.(13) are given by:

For abbreviation we define new function:

$$f(x) = \frac{1}{2}\tanh\left(\frac{kx}{2}\right)$$

$$A_1 = f(\gamma)$$

$$A_2 = A_3 = -f(\gamma) + \frac{1}{2}\{f(1 + \gamma) + f(-1 + \gamma)\}$$

$$A_4 = \{f(\gamma) - f(1 + \gamma) - f(-1 + \gamma)\} - \frac{1}{2}\{f(\frac{\alpha}{2}) + f(\frac{\alpha}{2} - \gamma)\} + \frac{1}{4}\{f(-2 + \frac{\alpha}{2} + \gamma) + f(-2 + \frac{\alpha}{2}) + f(2 + \frac{\alpha}{2} + \gamma) + f(2 + \frac{\alpha}{2})\}$$

$$A_5 = \frac{3}{2}f(\alpha') - \{f(1 + \gamma) + f(-1 + \gamma)\} + \frac{1}{4}\{f(2 + \gamma) + f(-2 + \gamma)\}$$

$$A_6 = \frac{1}{2}\{f(\frac{\alpha}{2}) - f(\frac{\alpha}{2} - \gamma)\} + \{f(\gamma) - f(1 + \gamma) - f(-1 + \gamma)\} + \frac{1}{4}\{f(-2 + \frac{\alpha}{2} + \gamma) - f(-2 + \frac{\alpha}{2}) + f(2 + \frac{\alpha}{2} + \gamma) - f(2 + \frac{\alpha}{2})\}$$
\[ A_7 \quad -2f(\alpha') + \frac{3}{2} \{ f(-1 + \gamma) + f(1 - \gamma) \} + \frac{1}{2} \{ f(\frac{\alpha}{2}) + f(\frac{\alpha}{2} - \gamma) - f(-2 + \gamma) - f(2 + \gamma) \} \\
\quad - \frac{1}{4} \{ f(2 + \frac{\alpha}{2}) + f(-2 + \frac{\alpha}{2}) + f(2 + \frac{\alpha}{2} + \gamma) + f(-2 + \frac{\alpha}{2} + \gamma) + f(1 + \frac{\alpha}{2} - \gamma) + f(1 + \frac{\alpha}{2} - \gamma) \} \\
\quad + \frac{1}{4} \{ f(3 + \frac{\alpha}{2}) - f(-1 + \frac{\alpha}{2}) - f(1 + \frac{\alpha}{2}) + f(1 + \frac{\alpha}{2} + \gamma) + f(-3 + \frac{\alpha}{2} + \gamma) + f(-3 + \frac{\alpha}{2}) + f(-1 + \frac{\alpha}{2} + \gamma) \} \\
\quad + f(3 + \frac{\alpha}{2}) + f(-1 + \frac{\alpha}{2} + \gamma) \] 

\[ A_8 \quad -2f(\alpha') + \frac{3}{2} \{ f(-1 + \gamma) + f(1 + \gamma) \} + \frac{1}{2} \{ -f(\frac{\alpha}{2}) + f(\frac{\alpha}{2}) - f(-2 + \gamma) - f(2 + \gamma) \} \\
\quad + \frac{1}{4} \{ f(2 + \frac{\alpha}{2}) + f(-2 + \frac{\alpha}{2}) - f(2 + \frac{\alpha}{2} + \gamma) - f(-2 + \frac{\alpha}{2} + \gamma) - f(1 + \frac{\alpha}{2} - \gamma) - f(1 + \frac{\alpha}{2} - \gamma) \} \\
\quad + \frac{1}{4} \{ f(-1 + \frac{\alpha}{2}) + f(1 + \frac{\alpha}{2}) + f(3 + \frac{\alpha}{2} + \gamma) + f(1 + \frac{\alpha}{2} + \gamma) + f(-1 + \frac{\alpha}{2} + \gamma) + f(-3 + \frac{\alpha}{2} + \gamma) \} \\
\quad - f(3 + \frac{\alpha}{2}) - f(-3 + \frac{\alpha}{2}) \] 

\[ A_9 \quad 3f(\alpha') - 2f(-1 + \gamma) - 2f(1 + \gamma) + \frac{1}{8} f(\alpha + \gamma) + \{ f(-1 + \frac{\alpha}{2}) + f(1 + \frac{\alpha}{2}) - f(\frac{\alpha}{2}) \} \\
\quad + \frac{1}{16} f(-4 + \alpha + \gamma) + f(4 + \alpha + \gamma) + \frac{1}{4} \{ 5f(-2 + \gamma) + 5f(2 + \gamma) - f(\alpha - \gamma) \} + \frac{1}{2} \{ f(2 + \frac{\alpha}{2} + \gamma) \} \\
\quad + f(-2 + \frac{\alpha}{2} + \gamma) - f(3 + \frac{\alpha}{2} + \gamma) - f(1 + \frac{\alpha}{2} + \gamma) - f(-1 + \frac{\alpha}{2} + \gamma) - f(-3 + \frac{\alpha}{2} + \gamma) \] 

\[ A_{10} \quad \frac{1}{2} f(-1) + \frac{1}{4} \{ f(-1 + \gamma) - f(1 + \gamma) \} \] 

\[ A_{11} \quad -\frac{1}{2} f(-1) + \frac{1}{4} \{ f(1 + \gamma) - f(-1 + \gamma) \} + \frac{1}{8} \{ f(-2 + \frac{\alpha}{2}) - f(2 + \frac{\alpha}{2}) + f(-2 + \frac{\alpha}{2}) + f(2 + \frac{\alpha}{2}) \} \\
\quad - f(2 + \frac{\alpha}{2} + \gamma) \] 

\[ A_{12} \quad \frac{1}{4} \{ f(-2) - f(-1) \} + \frac{1}{8} \{ f(-2 + \gamma) + 3f(1 + \gamma) - f(2 + \gamma) - 3f(-1 + \gamma) \} \] 

\[ A_{13} \quad \frac{1}{2} \{ f(-1 + \gamma) - f(1 + \gamma) - f(-2) \} + \frac{1}{4} \{ f(2 + \gamma) - f(-2 + \gamma) \} + \frac{1}{8} \{ f(-1 + \frac{\alpha}{2}) - f(1 + \frac{\alpha}{2}) - f(3 + \frac{\alpha}{2} + \gamma) + f(1 + \frac{\alpha}{2} + \gamma) - f(-1 + \frac{\alpha}{2} + \gamma) \} \\
\quad + f(-3 + \frac{\alpha}{2} + \gamma) + f(-3 + \frac{\alpha}{2}) - f(3 + \frac{\alpha}{2}) \] 

\[ A_{14} \quad \frac{1}{4} \{ f(-1) - f(-2) \} + \frac{1}{8} \{ 3f(-1 + \gamma) - 3f(1 + \gamma) + f(2 + \frac{\alpha}{2}) - f(-2 + \frac{\alpha}{2}) + f(2 + \frac{\alpha}{2} + \gamma) \} \\
\quad - f(-2 + \frac{\alpha}{2} + \gamma) - f(-2 + \gamma) + f(2 + \gamma) + \frac{1}{16} \{ f(-1 + \frac{\alpha}{2}) - f(1 + \frac{\alpha}{2}) - f(3 + \frac{\alpha}{2} + \gamma) \} \\
\quad - f(1 + \frac{\alpha}{2} + \gamma) + f(-1 + \frac{\alpha}{2} + \gamma) + f(-3 + \frac{\alpha}{2} + \gamma) - f(3 + \frac{\alpha}{2}) + f(-3 + \frac{\alpha}{2}) \] 

\[ A_{15} \quad \frac{1}{32} \{ f(-4 + \alpha + \gamma) - f(4 + \alpha + \gamma) \} + \frac{1}{2} \{ f(-2) + f(1 + \gamma) - f(-1 + \gamma) \} + \frac{1}{16} \{ f(-2 + \alpha) + f(2 + \alpha) + f(1 + \frac{\alpha}{2} + \gamma) - f(-1 + \frac{\alpha}{2} + \gamma) - 3f(-1 + \frac{\alpha}{2}) + 3f(1 + \frac{\alpha}{2}) \} \\
\quad - 3f(-3 + \frac{\alpha}{2} + \gamma) + 3f(3 + \frac{\alpha}{2}) - 3f(-3 + \frac{\alpha}{2}) + 3f(3 + \frac{\alpha}{2} + \gamma) + 5f(-2 + \gamma) - 5f(2 + \gamma) \} \\
\quad + \frac{1}{8} \{ f(1 + \frac{\alpha}{2} - \gamma) - f(-1 + \frac{\alpha}{2} - \gamma) - f(2 + \frac{\alpha}{2}) + f(-2 + \frac{\alpha}{2} + \gamma) + f(-2 + \frac{\alpha}{2} + \gamma) \} \\
\quad + f(2 + \frac{\alpha}{2}) \] 

\[ A_{16} \quad \{ f(1 + \gamma) - f(1 + \gamma) \} - 2f(-1) \] 

\[ A_{17} \quad 2f(-1) + f(1 + \gamma) - f(-1 + \gamma) + \frac{1}{2} \{ f(-2 + \frac{\alpha}{2} + \gamma) - f(2 + \frac{\alpha}{2} + \gamma) - f(-2 + \frac{\alpha}{2} + \gamma) \} \\
\quad + f(2 + \frac{\alpha}{2}) \]
\[
A_{18} \quad \{f((-1) - f(-2)) + \frac{1}{2} \{f(1 + \gamma) - f(2 + \gamma) - f(-1 + \gamma) + f(-2 + \gamma)\}\}
\]
\[
A_{19} \quad 2f(-2) + \{f(2 + \gamma) - f(-2 + \gamma)\} + \frac{1}{2} \{f\left(-1 + \frac{\alpha}{2} - \gamma\right) - f\left(1 + \frac{\alpha}{2} - \gamma\right)\} + \frac{1}{4} f\left(1 + \frac{\alpha}{2}\right)
\]
\[
- f\left(-1 + \frac{\alpha}{2}\right) + f\left(-3 + \frac{\alpha}{2} + \gamma\right) - f\left(3 + \frac{\alpha}{2} + \gamma\right) + f\left(1 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2}\right)
\]
\[
- f\left(-3 + \frac{\alpha}{2}\right)
\]
\[
A_{20} \quad \{f(-2) - f(-1)\} + \frac{1}{2} \{f(-1 + \gamma) - f(1 + \gamma) - f(-2 + \gamma) + f\left(2 + \frac{\alpha}{2}\right) - f\left(-1 + \frac{\alpha}{2}\right) - \frac{1}{4} f\left(1 + \frac{\alpha}{2}\right) - f\left(3 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2}\right) - f\left(-3 + \frac{\alpha}{2}\right)\}
\]
\[
A_{21} \quad -2f(-2) + \frac{1}{2} \{f(-2 + \frac{\alpha}{2} + \gamma) - f\left(2 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} - \gamma\right) + f\left(1 + \frac{\alpha}{2} - \gamma\right) + f\left(2 + \frac{\alpha}{2}\right)\}
\]
\[
+ \frac{1}{2} \{f(-4 + \alpha + \gamma) - f(4 + \alpha + \gamma)\} + \frac{1}{4} f\left(2 + \alpha\right) - f\left(-2 + \alpha\right) - f\left(-4\right) + f\left(1 + \frac{\alpha}{2}\right)
\]
\[
- f\left(-1 + \frac{\alpha}{2} + \gamma\right) + 3f\left(-1 + \frac{\alpha}{2}\right) - 3f\left(1 + \frac{\alpha}{2}\right) + 3f\left(3 + \frac{\alpha}{2} + \gamma\right) - 3f\left(-3 + \frac{\alpha}{2} + \gamma\right) - 3f\left(-3 + \frac{\alpha}{2}\right)
\]
\[
+ 3f\left(-3 + \frac{\alpha}{2}\right) + 5f(-2 + \gamma) - 5f(2 + \gamma)
\]
\[
A_{22} \quad \frac{1}{4} \{f(-2 + \frac{\alpha}{2}) + f\left(2 + \frac{\alpha}{2} + \gamma\right) + f\left(2 + \frac{\alpha}{2}\right) + f(-2 + \frac{\alpha}{2} + \gamma)\} + \frac{1}{4} f\left(\frac{\alpha}{2}\right) + f\left(\frac{\alpha}{2}\right)
\]
\[
A_{23} \quad - \frac{1}{2} f(\gamma) + \frac{1}{4} f(-2 + \gamma) + f(2 + \gamma)
\]
\[
A_{24} \quad \frac{1}{2} \{f\left(\frac{\alpha}{2} + \gamma\right) - f\left(\frac{\alpha}{2}\right)\} + \frac{1}{2} \{f(-2 + \frac{\alpha}{2} + \gamma) - f(-2 + \frac{\alpha}{2}) + f\left(2 + \frac{\alpha}{2} + \gamma\right) - f\left(2 + \frac{\alpha}{2}\right)\}
\]
\[
A_{25} \quad \frac{1}{2} \{f\left(\frac{\alpha}{2}\right) - f\left(\gamma\right)\} + \frac{1}{4} \{f\left(-1 + \frac{\alpha}{2} - \gamma\right) - f\left(-2 + \frac{\alpha}{2}\right) + f\left(1 + \frac{\alpha}{2} - \gamma\right) - f\left(2 + \frac{\alpha}{2} + \gamma\right) - f\left(-2 + \frac{\alpha}{2} + \gamma\right) - f\left(2 + \frac{\alpha}{2}\right)\}
\]
\[
+ \frac{1}{8} f\left(1 + \frac{\alpha}{2} + \gamma\right) + 3f\left(-1 + \frac{\alpha}{2}\right) + 3f\left(1 + \frac{\alpha}{2}\right) + 3f\left(3 + \frac{\alpha}{2} + \gamma\right) + f\left(1 + \frac{\alpha}{2} + \gamma\right) + f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2}\right) + f\left(-3 + \frac{\alpha}{2}\right)
\]
\[
A_{26} \quad \frac{1}{2} f\left(\alpha\right) - \frac{1}{4} \{f(-2 + \gamma) + f(2 + \gamma)\} + \frac{1}{8} f\left(-1 + \frac{\alpha}{2}\right) + f\left(-3 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2} + \gamma\right)
\]
\[
- f\left(1 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} + \gamma\right) - f\left(3 + \frac{\alpha}{2}\right) - f\left(-3 + \frac{\alpha}{2}\right) + f\left(1 + \frac{\alpha}{2}\right)
\]
\[
A_{27} \quad \frac{1}{2} f\left(\frac{\alpha}{2}\right) - f\left(\frac{\alpha}{2} - \gamma\right) + \frac{1}{4} f\left(2 + \frac{\alpha}{2}\right) + f\left(-2 + \frac{\alpha}{2}\right) - f\left(2 + \frac{\alpha}{2} + \gamma\right) - f\left(-2 + \frac{\alpha}{2} + \gamma\right) + f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(-3 + \frac{\alpha}{2} + \gamma\right) + f\left(-3 + \frac{\alpha}{2}\right) - f\left(1 + \frac{\alpha}{2}\right)
\]
\[
A_{28} \quad \frac{1}{2} f(\gamma) - \frac{1}{4} \{f(-2 + \gamma) + f(2 + \gamma)\} + \frac{1}{8} \{f(-1 + \frac{\alpha}{2}) + f\left(-3 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2} + \gamma\right)
\]
\[
- f\left(1 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} + \gamma\right) + f\left(3 + \frac{\alpha}{2}\right) + f\left(-3 + \frac{\alpha}{2}\right) - f\left(1 + \frac{\alpha}{2}\right)\}
\]
\[
A_{29} \quad \frac{1}{8} f(\alpha + \gamma) + \frac{1}{16} \{f(-4 + \alpha + \gamma) + f(4 + \alpha + \gamma)\} + \frac{1}{2} f\left(\frac{\alpha}{2} - \gamma\right) - f\left(-1 + \frac{\alpha}{2} - \gamma\right) - f\left(1 + \frac{\alpha}{2} - \gamma\right)
\]
\[
+ f\left(\frac{\alpha}{2}\right) + \frac{1}{4} \{f(-2 + \alpha) + f(\alpha - \gamma) + f(2 + \alpha) - f\left(3 + \frac{\alpha}{2} + \gamma\right) - f\left(1 + \frac{\alpha}{2} + \gamma\right) - f\left(-1 + \frac{\alpha}{2} + \gamma\right)\}
\]
\[-f\left(-3 + \frac{a}{2} + \gamma\right) - f\left(3 + \frac{a}{2}\right) - f\left(-3 + \frac{a}{2}\right) + f\left(-2 + \frac{a}{2}\right) + f\left(2 + \frac{a}{2} + \gamma\right) + f\left(2 + \frac{a}{2} + \gamma\right) - 3f\left(-1 + \frac{a}{2}\right) - 3f\left(1 + \frac{a}{2}\right)\]

\[\begin{align*}
A_{30} & - \frac{1}{2} f(g) - \frac{1}{8} f(\alpha + \gamma) + \frac{1}{16} \{f(4 + \alpha + \gamma) + f(-4 + \alpha + \gamma)\} + \frac{1}{4} f\left(1 + \frac{a}{2} + \gamma\right) \\
& - f\left(3 + \frac{a}{2} + \gamma\right) + f\left(-1 + \frac{a}{2} + \gamma\right) - f\left(-3 + \frac{a}{2} + \gamma\right) + f\left(-2 + \gamma\right) + f\left(2 + \gamma\right) \\
A_{31} & \frac{1}{8} f(-\alpha + \gamma) + \frac{1}{16} f(-4 + \alpha + \gamma) + f(4 + \alpha + \gamma)\} + \frac{1}{2} f\left(\frac{a}{2} - \gamma\right) - f\left(-1 + \frac{a}{2} - \gamma\right) \\
& - f\left(1 + \frac{a}{2} - \gamma\right) - f\left(\frac{a}{2}\right) + \frac{1}{4} f\left(-3 + \frac{a}{2} - \gamma\right) - f\left(-2 + \alpha\right) + f\left(\alpha - \gamma\right) - f\left(2 + \alpha\right) - f\left(2 + \frac{a}{2}\right) \\
& - f\left(-2 + \frac{a}{2}\right) + f\left(3 + \frac{a}{2} + \gamma\right) + f\left(2 + \frac{a}{2} + \gamma\right) + f\left(-2 + \frac{a}{2} + \gamma\right) - f\left(3 + \frac{a}{2} + \gamma\right) + f\left(1 + \frac{a}{2} + \gamma\right) \\
& - f\left(-1 + \frac{a}{2} + \gamma\right) - f\left(-3 + \frac{a}{2} + \gamma\right) + f\left(-1 + \frac{a}{2} + \gamma\right) + f\left(3 + \frac{a}{2} + \gamma\right) \\
A_{32} & \frac{1}{16} f\left(\frac{a}{2}\right) - 3f\left(-1 + \frac{a}{2}\right) + f\left(-3 + \frac{a}{2} + \gamma\right) - f\left(3 + \frac{a}{2} + \gamma\right) + f\left(1 + \frac{a}{2} + \gamma\right) \\
& - f\left(-1 + \frac{a}{2} + \gamma\right) - f\left(3 + \frac{a}{2} + \gamma\right) + f\left(-1 + \frac{a}{2} + \gamma\right) \\
A_{33} & \frac{1}{32} \left\{f\left(-4 + \alpha + \gamma\right) - f\left(3 + \alpha + \gamma\right)\right\} + \frac{1}{16} f\left(-4 + \alpha + \gamma\right) + f\left(3 + \alpha + \gamma\right) \\
& - f\left(-2 + \gamma\right) - f\left(-2 + \gamma\right) - f\left(-3 + \frac{a}{2} + \gamma\right) + f\left(3 + \frac{a}{2} + \gamma\right) + f\left(-3 + \frac{a}{2} + \gamma\right) - f\left(1 + \frac{a}{2} + \gamma\right) \\
& - 3f\left(1 + \frac{a}{2}\right) + f\left(-1 + \frac{a}{2} + \gamma\right) + \frac{1}{8} f\left(-1 + \frac{a}{2} + \gamma\right) - f\left(1 + \frac{a}{2} + \gamma\right) \\
A_{34} & \frac{1}{8} f\left(\alpha + \gamma\right) + \frac{1}{16} \{f\left(-4 + \alpha + \gamma\right) + f\left(4 + \alpha + \gamma\right)\} + \frac{1}{4} f\left(-2 + \gamma\right) - f\left(-2 + \gamma\right) - f\left(\alpha + \gamma\right) \\
A_{35} & \frac{1}{2} \left\{f\left(1 + \frac{a}{2} - \gamma\right) - f\left(-1 + \alpha + \gamma\right)\right\} + \frac{1}{4} \{f\left(-1 + \frac{a}{2}\right) - 3f\left(1 + \frac{a}{2}\right) - f\left(3 + \frac{a}{2} + \gamma\right) \\
+ f\left(-3 + \frac{a}{2} + \gamma\right) + f\left(1 + \frac{a}{2} + \gamma\right) - f\left(-1 + \frac{a}{2} + \gamma\right) + f\left(3 + \frac{a}{2} + \gamma\right) - f\left(-3 + \frac{a}{2} + \gamma\right)\right\} \\
A_{36} & \frac{1}{8} \left\{f\left(-4 + \alpha + \gamma\right) - f\left(4 + \alpha + \gamma\right)\right\} + \frac{1}{4} \{f\left(-2 + \alpha\right) - f\left(2 + \alpha\right) - f\left(-4\right) + f\left(3 + \frac{a}{2} + \gamma\right) \\
& - f\left(-3 + \frac{a}{2} + \gamma\right) - f\left(1 + \frac{a}{2} + \gamma\right) + f\left(-1 + \frac{a}{2} + \gamma\right) - f\left(3 + \frac{a}{2} + \gamma\right) + f\left(-3 + \frac{a}{2} + \gamma\right) - f\left(-2 + \gamma\right) \\
& - f\left(-2 + \gamma\right) - 3f\left(-1 + \frac{a}{2}\right) + 3f\left(1 + \frac{a}{2}\right) + \frac{1}{2} f\left(-1 + \frac{a}{2} - \gamma\right) - f\left(1 + \frac{a}{2} - \gamma\right)\right\} \\

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