High-$\gamma$ Beta Beams within the LAGUNA design study

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Abstract

Within the LAGUNA design study, seven candidate sites are being assessed for their feasibility to host a next-generation, very large neutrino observatory. Such a detector will be expected to feature within a future European accelerator neutrino programme (Superbeam or Beta Beam), and hence the distance from CERN is of critical importance. In this article, the focus is a $^{18}\text{Ne}$ and $^{6}\text{He}$ Beta Beam sourced at CERN and directed towards a 50 kton Liquid Argon detector located at the LAGUNA sites: Slanic ($L = 1570$ km) and Pyhäsalmi ($L = 2300$ km). To improve sensitivity to the neutrino mass ordering, these baselines are then combined with a concurrent run with the same flux directed towards a large Water Čerenkov detector located at Canfranc ($L = 650$ km). This degeneracy breaking combination is shown to provide comparable physics reach to the conservative Magic Baseline Beta Beam proposals. For $^{18}\text{Ne}$ ions boosted to $\gamma = 570$ and $^{6}\text{He}$ ions boosted to $\gamma = 350$, the correct mass ordering can be determined at Slanic for all $\delta$ when $\sin^2 2\theta_{13} > 4 \cdot 10^{-3}$ in this combination.

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1. Introduction

Results from a series of atmospheric [1, 2], solar [3–5], reactor [6–8] and long baseline accelerator [9, 10] neutrino experiments indicate that neutrinos are both massive and mix amongst themselves. A combined analysis points to two approximate 2-neutrino mixing schemes, each parameterised by a mixing angle and a mass-squared splitting. The extent to which these two schemes combine into a single 3-neutrino picture is controlled by the size of a third mixing angle, $\theta_{13}$. Defining $\Delta m_{ji}^2 = m_j^2 - m_i^2$, a combined analysis [11] of all available data returns the best fit values

$$|\Delta m_{31}|^2 = 2.47 \times 10^{-3} \text{ eV}^2 \quad \text{and} \quad \sin^2 \theta_{23} = 0.463 ;$$
$$\Delta m_{21}^2 = 7.59 \times 10^{-5} \text{ eV}^2 \quad \text{and} \quad \sin^2 \theta_{12} = 0.319 .$$

The third mixing angle, $\theta_{13}$, is constrained to be

$$\sin^2 \theta_{13} < 0.016 \ (0.053) \ \text{at} \ 1\sigma \ (3\sigma) ; \quad (1)$$

although some collaborations report a hint at low significance from a combined analysis of atmospheric, solar and long-baseline reactor neutrino data [12, 13].

The long term targets for the long baseline neutrino oscillation programme is to determine the unknown neutrino mixing angle $\theta_{13}$; whether there is CP-violation in the lepton sector, that manifests itself through a phase $\delta$; and to determine the sign of the atmospheric mass-squared splitting. The extraction of both the CP-phase and sign of the atmospheric mass-squared splitting require a three-neutrino analysis, the extent of the effect controlled by $\theta_{13}$. Therefore sensitivity, optimisation and general experimental strategy are intrinsically related to the size of $\theta_{13}$. Running and near future experiments will be the first to probe $\theta_{13}$ below the current experimental limit. If $\theta_{13}$ is beyond the reach of these experiments, then intense sources of neutrinos from next generation Superbeams [14–16], Neutrino Factories [17, 18] or Beta Beams [19] will be necessary.

A Beta Beam is a variation on the Neutrino Factory that instead sources neutrinos from the decays of boosted radioactive ions. Proposed by Zucchelli in 2001 [19], a Beta Beam distinguishes itself by sourcing beams of $\nu_e$ or $\bar{\nu}_e$ without contamination from other flavours and CP-conjugates. Through the analysis of the $\nu_\mu$ and $\bar{\nu}_\mu$ appearance channels, the Beta Beam provides a competitive physics reach on the unknown elements of neutrino mixing that have yet to be extracted from experiment. A Beta Beam will be a facility that will exploit existing, or proposed, ion production facilities and accelerator infrastructure. The major addition required is a large decay ring to store the ions once they are boosted to the desired energy. In the laboratory frame, the neutrino flux is a function of both the ion decay Q-value and the Lorentz boost of the ions. Low Q-value ions; $^{18}$Ne and $^6$He; and high Q-value ions $^8$B and $^8$Li have been identified as the best candidate ions [20]. The focus of phenomenological study has been on a Beta Beam source at CERN using either the Super Proton Synchrotron [21–25] or an enhanced 1 TeV version, as required in some LHC upgrade scenarios [26, 27] (Other facilities could be the Tevatron at Fermilab [34, 35], or a re-fitted HERA-ring [37].) In addition to specific setups, a number of optimisation studies have also been carried out [38, 40].
In this article, the physics reach of Beta Beams directed towards Liquid Argon detectors in Europe will be simulated on the assumption that a 1 TeV machine will be available [41]. The study of both low and high boost Beta Beams directed at large Water Čerenkov (WC) detectors has been studied in detail and is well understood [21–27]. Water Čerenkov detectors are best suited to short baselines since they only use the quasi-elastic events to reconstruct the signal. For longer baselines, and hence higher energies, it is necessary to use and reconstruct the energy of multi-particle final state interactions, especially deep inelastic scattering events. For this reason, one must use active scintillator, calorimeters and projection chamber technologies for intermediate and very long baselines. This study considers the GLACIER liquid argon detector [42] as a far detector for the baselines CERN-Slanic \((L = 1570 \text{ km})\) and CERN-Pyhäsalmi \((L = 2300 \text{ km})\) for \(\gamma = 350, 350\) and \(\gamma = 570, 350\) \(^{18}\text{Ne}\) and \(^{6}\text{He}\) Beta Beams. (The CERN-Boulby baseline with a non-WC detector was discussed in [28] for different exposures and so the very similar baseline for Sierozsowice \((L = 950 \text{ km})\) will not be included here.) These two baselines will then be combined with the same neutrino flux directed towards a large Water Čerenkov detector based at Canfranc \((L = 650 \text{ km})\) to examine the possible broad physics reach.

This work is part of the LAGUNA design study: an EC-funded project to assess the feasibility of underground laboratories in Europe capable of housing a future, large neutrino observatory. In addition to long baseline physics, the detectors aim to improve the bounds on proton decay half-lives up to \(\sim 10^{35}\) years to test a number of theoretical models; to detect neutrinos from astrophysical objects and Cosmological sources; continued study of solar and atmospheric neutrinos; and new sources such as from Dark Matter annihilations in the Earth’s centre or Sun’s core. Work towards understanding detector response to various neutrino sources in currently on-going. This work is one of a series of studies [16, 29, 43] examining the physics reach on long baselines experiments with the LAGUNA network of detectors based on available information. These studies will help prioritise the potential host laboratories for the future neutrino observatories.

The remainder of this article is organised as follows: in Sec. 2 the baselines available within Europe as part of the LAGUNA design study [44] will be listed and the basic phenomenological strategy for short and intermediate baselines will be summarised. In Sec. 3 a description of the simulations carried out is made followed by the results for Beta Beams directed along single baselines. In Sec. 4 the simulations from Sec. 3 are augmented with an additional Beta Beam directed towards a large Water Čerenkov based at Canfranc. The results are then discussed in relation to Magic Baseline Beta Beams in Sec. 5. Finally, in Sec. 6 the study is summarised.

2. Laguna sites hosting Beta Beam far detectors

Detector R&D is not part of the LAGUNA Design Study [44]; however the physics reach of these sites with respect to any European accelerator neutrino programmes will be critical in any decision making process. Since future European Superbeams and Beta Beams will be one of the main users for these detector technologies; the distance from CERN will be a critical factor in the final decision. However, since the size of \(\theta_{13}\) and the number of
future facilities and their scale is unknown, it is very hard to determine what is the optimal baseline and detector choice. More precisely, optimisation of a facility can only be carried out once its purpose and circumstance is defined. Since $\theta_{13}$ is unknown, and (specifically) the role in which a Beta Beam will take in a long term experimental strategy is uncertain; optimisation of the Beta Beam is not a well defined process. A typical strategy, therefore is to aim for a broad physics reach using the ability to rule out $\theta_{13} = 0^\circ$, $\delta = 0^\circ$ and $180^\circ$, and determine the correct neutrino mass ordering as indicators for a particular experimental setup; an approach also adopted here. How one achieves this is dependent, amongst other things, on the choice of baseline (or baselines) and detectors technologies.

Within Europe there are three detector options being considered: a single volume Liquid Argon time projection chamber (LAr) known as GLACIER [42]; a 500 kton Water Čerenkov (WC) known as MEMPHYS [45] suitable only for energy reconstruction at short baselines where quasi-elastic events dominate; and a non-segmented liquid scintillator detector known as LENA [46]. The 7 laboratory sites being considered within LAGUNA to house these detectors are listed in Tab. I along with their distances from CERN and corresponding first oscillation maximum energies. It clearly seen that a wide range of neutrino baselines are possible in Europe ranging from $L = 130$ km to $L = 2300$ km.

To develop a strategy, or interpret results of numerical simulations, for particular baselines it is useful to consider an analytical approximation to the oscillation probability. With the definitions $\alpha \equiv \frac{\Delta m^2_{23}}{\Delta m^2_{31}}$ and $\Delta \equiv \frac{\Delta m^2_{31} L}{4E}$; the assumption of a constant matter density profile along the baseline $A \equiv \frac{2\sqrt{2} G_F n_e E}{\Delta m^2_{31}}$; and by expanding in all the small parameters one finds [47]

$$P(\nu_e \rightarrow \nu_\mu) \simeq \sin^2 2\theta_{13} \cdot T_1 + \alpha \cdot \sin \theta_{13} \cdot (T_2 + T_3) + \alpha^2 \cdot T_4,$$

where

$$T_1 = \sin^2 \theta_{23} \cdot \frac{\sin^2 [(1 - A) \cdot \Delta]}{(1 - A)^2} ;$$

$$T_2 = \sin \delta_{CP} \cdot \sin 2\theta_{23} \cdot \sin \Delta \frac{\sin (A \Delta)}{A} \cdot \frac{\sin ((1 - A) \Delta)}{(1 - A)} ;$$

$$T_3 = \cos \delta_{CP} \cdot \sin 2\theta_{23} \cdot \cos \Delta \frac{\sin (A \Delta)}{A} \cdot \frac{\sin ((1 - A) \Delta)}{(1 - A)} ;$$

$$T_4 = \cos^2 \theta_{23} \cdot \sin^2 2\theta_{12} \frac{\sin^2 (A \Delta)}{A^2} ;$$

(3)

As is well known, and clear from this expression, the determination of $\theta_{13}$ and $\delta$ is severely affected by parameter correlations and degeneracies [48]. For a given bin at fixed baseline, up to 8 possible parameter sets can fit the data. The challenge for a future long baseline experiment is to successfully resolve these degeneracies and push for a good physics return over the sought region of parameter space. With a 1 TeV machine, it is in principle possible to source a Beta Beam from very short long-baselines ($L = 130$ km) up to very long long-baselines (Magic Baselines [33]). A Beta Beam proposal therefore has access to two types of baseline in which some of the degeneracy can be naturally suppressed:
| Location          | Distance from CERN [km] | Energy 1st Osc Max. [GeV] |
|-------------------|-------------------------|--------------------------|
| Fréjus (France)   | 130                     | 0.26                     |
| Canfranc (Spain)  | 630                     | 1.27                     |
| Umbria (Italy)    | 665                     | 1.34                     |
| Sierozsowice (Poland) | 950                   | 1.92                     |
| Boulby (UK)       | 1050                    | 2.12                     |
| Slanic (Romania)  | 1570                    | 3.18                     |
| Pyhäsalmi (Finland) | 2300                  | 4.65                     |

Table 1: Potential sites being studied with the LAGUNA design study \[44\]. The Umbria site is of interest if an upgrade to the CNGS beamline is pursued.

1. At short long-baselines \((L < 700 \text{ km say})\), the matter effect is sufficiently small so that the true \((\theta_{13}, \delta)\) and the fake solution corresponding to the incorrect mass ordering are close together. Consequently, the sensitivity to \(\theta_{13}\) and \(\delta\) is typically very good, especially with the availability of large Water Čerenkov detectors. Using the above expression, the CP-violation contribution to the probability is maximal for \(L/E = 515/\text{GeV}\). A Beta Beam flux with Lorentz boost \(\gamma = 350\), matches well the CERN-Canfranc baseline \((L = 650 \text{ km})\) \[30\].

2. At the ‘Magic Baseline’, cancellations leave only the atmospheric contribution \((T_1)\) to the appearance probability and thus there is no \(\delta\) dependence. Numerically, this is found to be at \(L = 7250 \text{ km}\) \[49\]. Use of the Magic Baseline in isolation will therefore return a reasonably clean measurement of \(\theta_{13}\) and \(\text{sign}(\Delta m^2_{31})\). The excellent sensitivity to these parameters, which is typical to proposals incorporating it, is because of the proximity to the matter resonance which, with the larger cross-sections at high energy, can compensate for the \(L^{-2}\) dependence of the un-oscillated neutrino flux.

Both these options are a staple in long baseline proposals since the effective removal of one of the unknown parameters helps greatly with degeneracy resolution. However, a extra baseline will be always be needed if one is to have a competitive reach on all three physics indicators: discovery of non-zero \(\theta_{13}\), CP non-conservation, and ability to rule out the incorrect neutrino mass ordering. For Beta Beams, the optimal choice for a second detector has not be determined rigorously, although several possibilities have been put forward for accompaniment of the Magic Baseline \[31–32\].

In this article, a Liquid Argon detector is considered at the two longest baselines within LAGUNA and therefore there is no natural suppression of the degeneracy. By itself, a single baseline Beta Beam using a 50 kton detector will not return competitive sensitivities at the longer LAGUNA baselines owing to a suppression of the neutrino flux from the baseline \((\propto L^{-2})\) and the presence of parameter degeneracies \[48\]. In isolation, binning the data to extract the oscillatory structure of the appearance probability is the required strategy to break parameter degeneracies. The relative weight of the atmospheric \((T_1)\), interference \((T_2)\),
and $T_3$) and solar ($T_4$) contributions to the probability changes with the neutrino energy for a fixed baseline. Since different contributory features of the appearance probability dominate different bins and that the location of parameter degeneracies are energy dependent, this strategy greatly aids degeneracy resolution.

The above strategy was discussed in [28] for the CERN-Boulby baseline ($L = 1050$ km) and is used in a number of other long baseline proposals, such as the Wide Band Superbeam [50] and the low energy Neutrino Factory [18]. It was argued semi-analytically that combining data from first and second oscillation maximum, even for a neutrinos (or anti-neutrinos) only, helps considerably in breaking the intrinsic energy degeneracy (or energy degeneracy [51]). Degeneracy in $\delta$, but not $\theta_{13}$ still remains, however. Incorporating data from surrounding bins typically allows determination of the true $\delta$. Resolving the sign($\Delta m^2_{31}$) degeneracy at short to intermediate baselines is far harder. The difference in the probability for normal and inverted ordering results from two effects: the $\sin \Delta$ in $T_2$ (which is present even in the vacuum for 3-neutrino mixing); and from $(1 - A)$ in $T_1$, $T_2$ and $T_3$. The discrepancy between normal and inverted orderings clearly increases with baseline and neutrino energy. For the baselines considered here, the approach is therefore to extract the sign from high energy bins. Since these bins also contain information on $\theta_{13}$ and $\delta$, degeneracy will need to be resolved. This, again, is achieved through the combination with the low energy bins where the probability splitting between mass orderings is much smaller and hence the degenerate solution is much closer to the true solution. The limit on the sensitivity to the mass ordering is then limited by the available neutrino flux.

In the following section, the effectiveness of the above strategy will be explored for a Beta Beam directed along the CERN-Slanic and CERN-Pyhäsalmi baselines. The event rates from the low energy bins are an important component of this approach, however, they are small owing the small detector mass and cross-sections. Therefore, following the initial analysis, the Liquid Argon baselines will be put in combination with a large Water Čerenkov based at the Canfranc laboratory. As indicated above, this will provide a clean measurement of $\theta_{13}$ and $\delta$ which can then be combined with the high energy bins of the Liquid Argon detectors. The reach on the correct mass ordering is expected to improve significantly as the inclusion of the extra (short) baseline is akin to enhancing the event rate in the low energy bins. The analysis carried out will be discussed in more detail in the following sections.

3. Single baseline study

The physics strategy in this study exploits $\nu_e$ and $\bar{\nu}_e$ beams sourced from the decays of boosted $^{18}$Ne and $^6$He. With production and acceleration of the ions based at CERN, the physics will be simulated for a 50 kton Liquid Argon detector located at the Slanic ($L = 1570$ km) and Pyhäsalmi ($L = 2300$ km) mines. In the first instance only these baseline will be considered for a 5 year run of $\nu_e$ and 5 years of $\bar{\nu}_e$ for the two boost pairings ($\gamma_{\nu}, \gamma_{\bar{\nu}}$) = (350,350) and (570,350). These boost assignments make the standard assumption that a 1 TeV Super Proton Synchrotron will be available for a Beta Beam based at CERN [41]. In the following section, the simulations will also include the combination
with a larger Water Čerenkov detector based at Canfranc ($L = 650$ km) exposed to the same neutrino source.

The current R&D in Liquid Argon detector technology is working towards a target mass of 100 kton [42]. However, in this article a 50 kton detector is used for the following reasons

- To bring it in line with other non-WC based Beta Beam studies in the literature.
- The sought level of $^{18}\mathrm{Ne}$ decays along the straight section of the storage ring for the $\gamma = 100, 100$ proposal [22] currently is beyond reach by a factor 20, using ISOLDE techniques [52]. A 100 kton detector is assumed but with a factor of 2 used to offset a smaller than sought decay rate.

The ion decay rates in the straight sections of the decay ring are currently unknown. R&D towards production of $^{18}\mathrm{Ne}$ and $^6\mathrm{He}$ has focussed on the use of ISOLDE techniques within the EURISOL design study [53], with target rates of $1.1 \times 10^{18}$ and $2.9 \times 10^{18}$ yearly decays for a $\gamma = 100, 100$ machine for $^{18}\mathrm{Ne}$ and $^6\mathrm{He}$ respectively. Currently, the projected rates are a factor 20 short for $^{18}\mathrm{Ne}$ and about 2 short for $^6\mathrm{He}$ [52]. This in part is because ISOL techniques, as applied by Nuclear Physicists, are optimized for more exotic nuclides that are of little interest for Beta Beams [54]. The route to the sought decay rate for $^{18}\mathrm{Ne}$ could be through ‘direct production’, for example through the $^{16}\mathrm{O}(^3\mathrm{He},n)^{18}\mathrm{Ne}$ reaction [55].

Atmospheric neutrino events are skewed below 1 GeV and cause a problem for sourcing decays for the short baselines and need to suppressed with a restrictive duty factor in the decay ring ($\sim 10^{-3}$). However this constraint can be loosened for longer baselines where neutrino energies $E_\nu > 1$ GeV are more important. This action will be necessary to reclaim the decay rate for high-$\gamma$ machines which, to first order, scales as $\gamma^{-1}$. (The impact of how much one inhibits the atmospheric neutrino background at the longer baselines has yet to studied in detail.) It may be possible to also claim some deficiency in the event rate through loosening the duty factor. To aid comparison with the latest studies, in this article the assumption that $3 \times 10^{18}$ useful ion decays per straight section of the decay ring can be sourced per year for each ion as suggested in [54], and used in [30], will be adopted.

For short and intermediate baselines, the aim is to exploit the energy dependence of the oscillatory structure of the appearance signal to help break the parameter degeneracies and push for a good physics reach. This is achieved through binning the signal to separate out the low and high energy appearance events so that the different strengths of the solar, interference and atmospheric contributions to the appearance probability can be observed. To this end, the detector has been assumed to possess a low energy threshold of 0.4 GeV and has binned up to 2.0 GeV in 0.2 GeV intervals. All bins thereafter are in 0.5 GeV intervals up to the maximum laboratory energy of the neutrinos. At present, detector response for incident Beta Beam fluxes have not been simulated for the GLACIER Liquid Argon detector. Work in this respect in ongoing, with response data soon to be available for Superbeams directed at this technology [56]. In its absence, a flat event efficiency of 80% has been taken for all channels with signal errors set at 2.5% and an energy resolution of $\sigma(E_\nu) = 0.15E_\nu^\frac{1}{3}$.

\footnote{These assignments are conservative with studies for FLARE [57] indicating that below 1 GeV, resolution could be as good as 2%, whilst above 2 GeV $\sigma(E_\nu) \simeq 0.10E_\nu$.}
All simulations in this study have been carried using the GLoBES long baseline simulation software [58]. The known oscillation parameters have been fixed to their current central values, taken from [11], and are always marginalised over. The exception is $\theta_{23}$ which is fixed to 45° so that the octant degeneracy is absent. The errors have been set to 1 % for the solar parameters and 5 % for the atmospheric parameters and matter potential. Negligible background from neutral current events is expected and has been set to 0.1 %. The 1 degree of freedom convention is adopted for all sensitivity plots in this paper.

3.1. Results

The physics reach of the two baselines considered in summarised is Fig. 1. The 3σ confidence level contours are shown for sensitivity to non-zero $\theta_{13}$ (left), CP non-conservation (middle), and to resolving the sign($\Delta m_{31}^2$) ambiguity (right). The analysis takes into account
the impact of intrinsic and the sign($\Delta m^2_{31}$) degeneracies. The top line of Fig. 1 shows the results for the CERN-Slanic baseline, whilst the bottom display the outcome of the CERN-Pyhäsalmi simulations.

The best sensitivity to non-zero $\theta_{13}$ and CP-violation is found for the CERN-Slanic baseline. This is to be expected since, with the same source, the flux is larger for this baseline. The weaker matter effect means that the sign($\Delta m^2_{31}$) degeneracy interferes less with these measurements. For the $\gamma = 570, 350$ boost pair, non-zero $\theta_{13}$ can be seen down to $\sin^2 2\theta_{13} \sim 10^{-3}$, and, for both boost pairs, at all values of $\delta$ for $\sin^2 2\theta_{13} > 10^{-2}$. However, there is a marked difference between the two boost pairings for sensitivity to CP-violation. For the $\gamma = 350, 350$ pair, the ability to rule out $\delta = 0^\circ$ or $180^\circ$ is restricted to $\sin^2 2\theta_{13} > 10^{-2}$, but increasing the boost of the $^{18}$Ne ions returns a large region of parameter space for $\delta < 0^\circ$ and centred on $\sin^2 2\theta_{13} = 10^{-3}$. There is little enhancement on the region for low boost pairing. This is suggestive that degeneracy is a problem for the $\gamma = 350, 350$ boost pairing; especially for $\sin^2 2\theta_{13} \sim 5 \cdot 10^{-3} - 1 \cdot 10^{-2}$ where there is a gap in CP-violation sensitivity. The lower event rates for the longer baselines mean that the data is insufficient to reduce the significance of some degenerate solutions. Although, the ability to rule out the incorrect mass ordering is poor relative to Neutrino Factories, for a Beta Beam it is not intrinsically bad. For the high boost run, the correct ordering can be indentified down to $\sin^2 2\theta_{13} = 2 \cdot 10^{-3}$, with determination for all values of $\delta$ for $\sin^2 2\theta_{13} > 10^{-2}$ in both cases. Although the increase in the $^{18}$Ne boost improves the reach, it does not do so significantly. Increasing the boost makes data from higher energies available without significantly altering the event rate and composition at lower energies. Since European baselines make use of these bins in combination, improving one without the improving other need not, and has not, dramatically improved the physics return. The low event rate at low energies is still insufficient to break the degeneracy for small values of $\theta_{13}$.

The physics return for the CERN-Pyhäsalmi baseline is weaker for each of the physics indicators, with little sensitivity for $\sin^2 2\theta_{13} < 10^{-2}$. Principally, this is due to the $L^{-2}$ dependence of the un-oscillated neutrino flux. In particular, the ability to determine the correct mass ordering is very poor even given the large matter effect at this baseline. The true and incorrect mass ordering solutions will be sufficiently separated in ($\theta_{13}, \delta$) space; however, the low event rate means that the solution regions at $3\sigma$ will be large and possibly merged together. When combined with a large solution region from the low energy bins (also owing to low event rates), the data is insufficient to break the degeneracy for small $\theta_{13}$.

In summary, neither of the baselines considered here have, in isolation, the right combination for degeneracy breaking ability and sufficient un-oscillated event rate to return a competitive physics reach on all physics indicators. One solution to this problem is to combine these baselines with another beam. It has been proposed in [35, 59] to use combinations of Beta Beams and Superbeams at these shorter baselines to break degeneracy and, in particular, improve the ability to determine the mass ordering. This strategy exploits the different forms of the appearance probability for the $\nu_e \rightarrow \nu_\mu$, $\bar{\nu}_e \rightarrow \bar{\nu}_\mu, \nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ channels which in turn affects the location of the degenerate solutions for different oscillation channel pairs.

In the next section, a variant on this approach will be adopted. Specifically, a second
Beta Beam baseline will be introduced in view of returning sensitivity to $\theta_{13}$ and $\delta$ to much greater precision. This is extrapolated from the strategy advocated earlier of using low and high energy bins and is motivated from the results for the Slanic baseline indicating that improvement in the high energy neutrino event rate is insufficient to significantly improve the hierarchy reach. Increasing the ion boost does not significantly improve the event rate at low energies; however, introducing a second Beta Beam at a shorter baseline, and with a large Water Čerenkov will have the desired effect. With this information the longer baseline can then rule out the incorrect solution from the sign($\Delta m^2_{31}$) degeneracy for much smaller $\sin^2\theta_{13}$. This idea will be presented in the next section.

4. In combination with a large Water Čerenkov

As discussed in Sec. 2, for a single baseline, the strategy is to use the complementary information from low and high energies to break degeneracies and push to physics reach at small $\theta_{13}$. However, for the intermediate baselines considered here, owing to smaller detector sizes and increasing sign($\Delta m^2_{31}$)-degeneracy dominance, the $\theta_{13}$ and $\delta$ reaches are relatively poor compared to the Water Čerenkov Beta Beams. This was demonstrated through the simulations of the previous section which indicated that degeneracy persisted for the intermediate baselines of Europe. A straightforward way to overcome this is to increase the exposure so that degenerate solutions lose statistical significance. However, for the Beta Beam, demands on production and ion storage impose restrictions on scaling up of this kind.

The challenge is to improve the event rate at the low energies and then use this data in combination with high energy neutrinos. This approach is distinct from the standard technique to resolve the neutrino mass ordering. It is typical to consider very long baselines where the matter effect is strongest and the closeness to the matter resonance allows the recovery of flux reduced by its $L^{-2}$ dependance. Most interest centres on the Magic Baseline where CP-violation effects vanish from the appearance probability, leaving only the atmospheric contributions. There have been a number of studies using a second Beta Beam in addition to a Magic Baseline Beta Beam to help break degeneracies. Thus far, a Magic Baseline/short baseline combination has been studied in [30, 31] and a Magic Baseline/long baseline combination in [32]. The strategy employed in this section is to deliberately place the far detector at a baseline where the degeneracy is strong. In a short baseline/intermediate baseline configuration the short baseline will provide the sensitivity to $\theta_{13}$ and $\delta$ down to the $\sin^2 2\theta_{13} \sim 10^{-4}$ level. This information can then used to rule out degenerate solution regions in ($\theta_{13}, \delta$) space for the longer baseline. (For a short baseline, the sign($\Delta m^2_{31}$)-degenerate solution will be very close to the true solution. As the matter increases, the two solutions diverge.) The combination of data sets is expected to improve the sensitivity considerably. This is demonstrated in the following simulations.
4.1. Results

In Fig. 2, the 3σ confidence levels for the three physics indicators are shown for a large Water Čerenkov, based at Canfranc, in combination with the same Liquid Argon detector as previous for the the Slanic and Pyhäsalmi laboratories. The Water Čerenkov dominates the θ_{13} and δ sensitivities and the reaches are essentially identical in both combinations. Therefore only the results for Slanic are shown here. The CP-discovery plot is both smooth and symmetric. There is no residue intrinsic degeneracy at $\sin^2 2\theta_{13} \sim 10^{-2}$ as its location is different for each baseline and therefore can be resolved. For both baseline pairs, non-zero θ_{13} and δ distinguishable from 0° and 180° can be achieved down to $\sin^2 2\theta_{13} = 5 \cdot 10^{-5}$.

The important result is that the combination of baselines does indeed improve the ability to rule out the incorrect mass ordering beyond the capability of either baseline separately. Specifically, for $\gamma = 350, 350$ and for both combinations, the correct mass ordering can be extracted for all δ down to $\sin^2 2\theta_{13} \sim 6 - 7 \cdot 10^{-3}$. For $\gamma = 570, 350$ this improves to $\sin^2 2\theta_{13} \sim 4 \cdot 10^{-3}$ which is the level reported for the conservative Magic Baseline proposal.
Even for $\gamma = 350,350$, there is substantial resolving power at the $4 \cdot 10^{-3}$ level indicating that a more minimal Beta Beam is capable of returning similar physics.

5. Discussion

The simulations presented in this article indicate that a two-baseline Beta Beam configuration using a short and intermediate baseline can obtain a similar physics reach as conservative Magic Baseline proposals, such as in [30]. Long baseline experiments incorporating the Magic Baseline aim to exploit the absence of the CP-violation at this baseline along with the nearby resonance associated with atmospheric neutrino mixing. Since such an environment provides a relatively clean measurement of the mass ordering, usually to very small $\theta_{13}$, Beta Beam studies aiming to achieve the best reach in this respect typically exploit it [30,33]. It is instructive to highlight the reason why a more minimal setup can return similar sensitivity to the conservative Magic Baseline proposals.

The crucial point to realize is that the sought features of the baseline, namely no CP-phase and proximity to the matter resonance, are features of the appearance probability. In an experiment, one measures the event rate which is a convolution of the unoscillated neutrino flux at detector, a cross-section, detector efficiencies and the probability. Having a signal clean from CP-violation is of little use if the event rate is too low to provide a competitive sensitivity. The unoscillated flux has a $L^{-2}$ dependence which greatly reduces the flux at very long baseline. This reduction can be recovered through the larger cross-sections at high energy and the matter enhancement of the appearance probability. On the assumption that the same number of ion decays is available for the very long baselines as for the short, the number of appearance events is roughly independent of the baseline at very long baselines [33]. In this instance, the ability to extract the mass ordering will peak where the signal is cleanest from $\delta$ ‘contamination’: the Magic Baseline. However, a second baseline is always necessary to have access to the CP-violation. In which case, it may be possible to use synergy between baselines to extract the sign of the mass splitting at a similar level as demonstrated in this study.

Nevertheless, the sensitivity presented here can still be beaten by most Magic Baseline Beta Beams considered. The point raised in [30] is that construction restrictions on the size of the decay ring could result in a considerable reduction of the flux at very long baseline in a realistic proposal. (Longer baselines require the decay ring to dip at larger angles to the surface so that the maximum depth of the ring could be beyond 2km if very powerful superconducting magnets are not available.) If one imposes a limit on the maximum depth of the decay ring, then there is a maximum baseline for which one can source $3 \times 10^{18}$ ion decays per year in the straight section. For all baselines longer than this, the length of the straight sections need to be curtailed. If the flux at the Magic Baseline is sufficiently reduced, there will become a point when the $\delta$ cleanliness of the signal is insufficient to achieve the best physics reach, even for a single baseline. At such a point, combinations involving synergy would then be sought to obtain a broad overall physics reach. By choosing to use $^{18}$Ne and $^{6}$He only, this study was able to expose both detectors simultaneously to the neutrino flux. In studies using all four candidate ions, one effectively cuts the event
rate in half since the low-Q and high-Q pairs need to be run separately and irradiate only the appropriate detector. The combination of the synergy between baselines and the higher event rate from using only two ions is the reason why the relatively minimal configurations discussed here have returned similar physics to conservative Magic Baseline proposals.

6. Summary

Assessing the physics reach of long baselines with far detectors based at the potential LAGUNA sites is of critical importance for strategic decisions towards accelerator and non-accelerator neutrino physics in Europe. Recently, as part of the LAGUNA design study, work studying the physics return for a Beta Beam with LENA [29] as a far detector, and a high powered Superbeam directed towards GLACIER [16] have been performed. In this article, this has been continued to include Beta Beam physics with GLACIER, but with the mass effectively reduced to 50 kton to offset any shortfall in the useful ion decay rate. R&D towards the detector response for the incident beam is currently unavailable, and so the approach was to be conservative in energy resolution and efficiency assumptions. Configuring the detector with a low energy threshold and binned in a manner to extract the oscillatory structure of the signal, the physics for single and double baselines was simulated.

The physics reach for a Beta Beam directed towards the Slanic Mine ($L = 1570$ km) and Pyhäsalmi ($L = 2300$ km) were simulated initially. The shorter baseline performed best owing to its larger event rate and weaker sign($\Delta m_{31}^2$) degeneracy. Non-zero $\theta_{13}$ could be established down to $\sin^2 2\theta_{13} \sim 10^{-3}$ for the $\gamma = 570, 350$ facility. The effect of altering the boost of the $^{18}\text{Ne}$ ion is large for the Slanic baseline, especially for CP-violation. This indicates that degenerate solutions pose a problem for the less energetic neutrinos. The effect was much less apparent for identifying the correct mass ordering. This is because the increasing the boost gives access to higher energy neutrinos without substantially increasing the flux at low energies. For Pyhäsalmi, the sensitivities were much weaker for all indicators with little return below $\sin^2 2\theta_{13} = 10^{-2}$. Although the matter effect is larger at Pyhäsalmi, the reduced flux from being more distant from source allows the degenerate solutions to remain statistically significant for small $\theta_{13}$.

These results indicated that the data from the Slanic and Pyhäsalmi Liquid Argon detectors should be combined with a concurrent run directed towards a large Water Čerenkov based at Canfranc. Such an addition provides an almost clean measurement of $\theta_{13}$ and $\delta$ down to $\sin^2 2\theta_{13} \sim 10^{-4}$. This information is then available for the longer baseline to remove the degenerate solutions and extract the mass ordering from its data set. The reach on the neutrino mass ordering is competitive with the conservative two baseline configurations incorporating the Magic Baseline. The access to $\theta_{13}$ and CP-violation was dominated by the short baseline with reach down to $\sin^2 2\theta_{13} = 5 \cdot 10^{-5}$.

In conclusion, the results of this study indicate that European Beta Beams can be provide competitive physics reach to move extensive proposals. The optimal Beta Beam configuration, therefore, has yet to be determined and should be investigated further, taking into account available knowledge on technological restrictions. More generally, these results demonstrate that the Magic Baseline is not mandatory for determining the sign of $\Delta m_{31}^2$:
the combination of data from a short baseline with data heavy in degeneracy can be an equally as powerful phenomenological tool.

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