Light scalar mesons in QCD

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I present a mini-review of the masses and couplings of the bare (unmixed) light scalar mesons: \( \bar{q}q \), \((\bar{q}q)(qq)\), \((\bar{q}q)(\bar{q}q)\), \(gg\) from QCD spectral sum rules (QSSR) and low-energy theorems (LET) which we compare with recent lattice calculations when available. Some unbiased comments on the different scenarios are given. The possibility for the \( \sigma(0.6) \) to be mostly a gluonium/glueball with a huge violation of the OZI rule in its decay is discussed. This review complements and updates the ones presented earlier [1]. Despite some progresses, the internal structure of the light scalar mesons remain puzzling, and some further efforts are required. It will be more fun at LHC if the Higgs of the Standard Model is a \( \sigma \)-like resonance.

1. Introduction

The nature of scalar mesons continues to be an intriguing problem in QCD. Experimentally, around 1 GeV, there are well established scalar mesons with isospin \( I = 1 \), the \( a_0(980) \), and with isospin \( I = 0 \), the \( f_0(980) \) [2]. Below 1 GeV, the long-standing existence of the wide \( I = 0 \) \( \sigma(600) \) is confirmed from \( \pi\pi \) scattering [3,4] (see, however [5]), while \( K\pi \) scattering indicates the presence of the \( I = 1/2 \) \( \kappa(840) \) [2,17], which is lower but larger than naively expected from \( SU(3) \) breaking expectations as a partner of the \( a_0(980) \) in a \( qq \) scheme. The real quark and/or gluon contents of these scalar states are not fully understood which effective theories based on the (non) linear realization of chiral symmetry [8,9,10,11,12], may not help for clarifying this issue. In the following, we shall focus on the tests of the \( \bar{q}q \), \((\bar{q}q)(\bar{q}q)\), \( \bar{q}q\bar{q}q \) and gluonium natures of these scalar mesons by confronting the recent experimental data with predictions on masses and couplings from QCD spectral sum rules (QSSR) [13,14] complemented with some low-energy theorems (LET) [11,13,15,16], which we compare with lattice calculations [17,20] and some other predictions when available. Previous reviews [12,11,22,23] have been already dedicated to some of these studies. The present paper will complement them and will update some recent developments in this field since 2006.

2. The \( I = 1, 1/2 \) scalar mesons

The \( a_0(980) \) and \( \kappa(840) \) masses

These channels are expected to be simpler as we do not expect to have any mixing with a gluonium. If one assumes that these states are \( \bar{q}q \) mesons, one can naturally associate them to the divergence of the vector currents:

\[
\begin{align*}
\alpha_0(980) & \rightarrow \partial_\mu V_{\mu}^d \equiv (m_u - m_d): \bar{u}(i)d: , \\
\kappa(840) & \rightarrow \partial_\mu V_{\mu}^s \equiv (m_u - m_s): \bar{u}(i)s: .
\end{align*}
\]

With the QSSR approach, the meson masses can be studied from the ratio of exponential Laplace/Borel sum rules [13,14]:

\[
\mathcal{R}_{n,n+1} = -\frac{d}{d\tau} \log \left( C_n \equiv \int_0^\infty dt \, t^n e^{-t\tau} \text{Im}\psi_{\mu\nu}(t) \right)
\]

of the corresponding two-point correlator:

\[
\psi_{\mu\nu}(q^2) = i \int d^4xe^{i\nu x} \langle 0| T \partial_\mu V_{\mu}^d(x) \partial_\nu V_{\nu}^d(0)|0 \rangle ,
\]

where \( \tau \) is the sum rule variable. The observed wrong splitting between the \( a_0(980) \) and \( \kappa(840) \) mesons can be understood from the crucial rôle of the four-quark condensates which reverses the splitting as can be read from the approximate QSSR formula from \( R_{0,1}(\tau) \) [1]:

\[
M_\kappa^2 \approx M_{a_0}^2 + 2m_s^2 - 8\pi^2m_s\langle \bar{s}s \rangle\tau_0 + \frac{3}{2} \frac{1408}{81} \pi^3 \rho \alpha_s \langle \langle \bar{s}s \rangle^2 - \langle \bar{u}u \rangle^2 \rangle \tau_0^2 - \frac{1}{3} \frac{M_{a_0}^2 \tau_0^2}{\kappa},
\]

where all different parameters including the \( a_0 \) mass are evaluated at the sum rule optimization scale \( \tau_0 \approx 1 \text{ GeV}^{-2} \); \( \rho \approx 2 - 3 \) [24] indicates the deviation from the vacuum saturation of the four-quark condensate; \( \langle \bar{s}s \rangle/\langle \bar{u}u \rangle \approx 0.8 \) measures the \( SU(3) \) breaking of the quark condensate [14]. The last term is the finite width correction of the \( \kappa \) which decreases its mass by about 20 MeV for \( \Gamma_\kappa \approx 300 \text{ MeV} \). From the previous analysis, one can deduce:

\[
M_{a_0} \approx 930 \text{ MeV} \quad \text{and} \quad M_\kappa \approx 920 \text{ MeV} ,
\]

with about 10% error, in good agreement with recent data [2]. A naive non-relativistic quark model gives analogous \( a_0 \) mass prediction but fails to reproduce the wrong splitting, which can be due to the unclear rôle of high-dimension operators in this approach.

Using similar QSSR analysis for the four-quark \((\bar{q}q)(\bar{q}q), (\bar{q}q)(\bar{q}q)\) operators for describing the scalar mesons, one obtains a value [25,26,14]:

\[
M_{\bar{q}q} \approx 1 \text{ GeV} ,
\]

while a lattice calculation in a quenched approximation obtains analogous result for \( M_{\bar{q}q} \) [20].

- QSSR can reproduce the wrong splitting of the \( a_0 \) and \( \kappa \) in the \( \bar{q}q \) scheme which is not the case of the non-relativistic quark model. This result can be checked on

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unquenched lattices. At present, the alone evaluation of the spectrum cannot select the right quark content of these mesons.

- **The decay constants**
  The decay constant $f_{a_0}$ of the $a_0$ normalized as:
  \[
  \langle 0 | \partial_\mu V^\mu_{\text{gd}} | a_0 \rangle = \sqrt{2} f_{a_0} M^2_{a_0},
  \]
  in the same way as $f_\pi = 92.4$ MeV has been estimated several times in the literature. The result from the exponential sum rule $\mathcal{L}_0(\tau)$ [1]:
  \[
  f_{a_0} \simeq (1.6 \pm 0.5) \text{ MeV},
  \]
  leads to a value of the $u$-$d$ quark mass difference consistent with recent lattice calculations. Using $SU(3)$ symmetry and the almost degeneracy of the $a_0$ and $\kappa$ masses, one obtains with a good accuracy:
  \[
  \frac{f_\kappa}{f_{a_0}} \simeq \frac{m_u - m_d}{m_u - m_d} \simeq 40,
  \]
  a ratio which is expected from the ChPT approach.

- **The hadronic couplings**
  The $a_0$ and $\kappa$ hadronic couplings have been obtained using either a vertex sum rule or $SU(3)$ symmetry rotation. The leading order vertex sum rule results are:
  \[
  g_{a_0 K^+ K^-} \simeq \frac{8\pi^2}{3 \sqrt{2}} \langle \bar{s}s \rangle \left(1 - \frac{2}{r} \right) \simeq 3 \text{ GeV},
  \]
  \[
  g_{a_0 K^+ K^-} / g_{\kappa K^+ K^-} \simeq e^{-\langle \bar{m}_K^2 - m^2 \rangle} \left(1 - \frac{2}{r} \right) \simeq 1.17,
  \]
  where we have used $\langle \bar{s}s \rangle \simeq -0.8 M^2_K f_K$, $r \equiv \langle \bar{s}s \rangle / \langle \bar{u}u \rangle \simeq 0.8$, and $\tau_0 \simeq 1 \text{ GeV}^{-2}$. We expect an accuracy of about 20% (typical for the 3-point function sum rules) for these estimates. Using the $SU(3)$ relation:
  \[
  g_{a_0 \pi K \pi} \simeq \sqrt{\frac{2}{3}} g_{a_0 K^+ K^-}
  \]
  one obtains
  \[
  \Gamma(a_0 \rightarrow \eta \pi) \simeq 84 \text{ MeV},
  \]
  in agreement with the range of data from 50 to 100 MeV given by PDG [2]. Using the previous value of the $\kappa$ coupling, one can deduce:
  \[
  \Gamma(\kappa \rightarrow K \pi) \simeq \frac{3}{2} \Gamma(\kappa \rightarrow K^+ \pi^-) \simeq 104 \text{ MeV},
  \]
  which is about a factor 4 smaller than the present data [2], but is a typical value for the width of a $\eta \pi$ state. The result for the hadronic coupling $g_{a_0 K^+ K^-}$ in the four-quark scenario depends crucially on the operators describing the $a_0$ and can range from 1.6 GeV [2] to (5–8) GeV [20]. However, the prediction for the $\eta \pi$ can agree with the data [20] depending on the size of the operator mixing parameter, while the four-quark prediction of the $\kappa$ hadronic width can lead to a half a value of the data.

- **A strong deviation from the $SU(3)$ relation** of the $a_0$ hadronic couplings does not favour the $\bar{q}q$ interpretation of the $a_0$, which seems not be the case [24]. One may question either the validity of the $\bar{q}q$ or four-quark scheme for the $\kappa$ or a better understanding of its hadronic width from the data. This experimental question requires a clean separation of the direct coupling (resonance) of the $\kappa$ and the rescattering $K \pi$ term like is the case of the $\sigma$ meson [23] discussed later on.

- **The $\gamma \gamma$ width**
  The $\gamma \gamma$ width of the $a_0$ has been evaluated using vertex sum rules within the $\bar{q}q$ and four-quark assignments of this meson, with the result [26] [1]:
  \[
  \Gamma(a_0(\bar{q}q) \rightarrow \gamma \gamma) \simeq (1.6 \pm 2.6) \text{ keV},
  \]
  and:
  \[
  \Gamma(a_0(4q) \rightarrow \gamma \gamma) \simeq (2 \sim 5) \times 10^{-4} \text{ keV},
  \]
  where the size of the ratio is of the order of $(\alpha_s / \pi)^2$, indicating that the four-quark assignment prediction is small (see also [11]).

- **None of the two schemes can explain the data** of $(0.24 \pm 0.08) \text{ keV}$ [2], which is not the case of an effective approach based on a kaon hadronic tadpole mechanism for $SU(2)$ breaking [22] or on a similar model involving kaon loops [11]. Unfortunately, a connection between these approaches and the $\bar{q}q$ or $4q$ scheme is not fully understood. A separate measure of the direct coupling and of the $KK$ rescattering terms may clarify this issue.

3. **The $I = 0$ bare scalar mesons**

The isoscalar scalar states are especially interesting in the framework of QCD since, in this anomalous $U(1)_V$ channel, their interpolating operator is the trace of the energy-momentum tensor:

\[
\theta_\mu = \frac{1}{4} \beta(\alpha_s) G_{\mu}^{\alpha} + \sum_{i = u,d,s} [1 + \gamma_m(\alpha_s)] m_i \bar{\psi}_i \psi_i,
\]

where $G_{\mu}^{\alpha}$ is the gluon field strengths, $\psi_i$ is the quark field; $\beta(\alpha_s) \equiv \beta_1 (\alpha_s / \pi) + ...$ and $\gamma_m(\alpha_s) \equiv \gamma_1 (\alpha_s / \pi) + ...$ are respectively the QCD $\beta$-function and quark mass-anomalous dimension ($\beta_1 = -1/2 (11 - 2n_f/3)$ and $\gamma_1 = 2$ for $n$ flavours). In the chiral limit $m_i = 0$, $\theta_\mu$ is dominated by its gluon component $\theta_g$, like is the case of the $\eta'$ for the $U(1)_A$ axial-anomaly, explaining why the $\eta'$-mass does not vanish for $m_i = 0$ [30], though it loves to couple to ordinary mesons. Then, it is natural to expect that these $I = 0$ scalar states are glueballs/glueonia or have at least a strong glue component in their wave function. This gluonic part of $\theta_\mu$ can be identified with...
the $U(1)_{V}$ term of the effective lagrangian based on a $U(3)_{L} \times U(3)_{R}$ linear realization of chiral symmetry (see e.g. [10,11,12]).

- **Unmixed $I = 0$ scalar $\bar{q}q$ mesons**

  We shall be concerned with the mesons $S_{2}$ and $S_{3}$ mesons associated respectively to the quark currents:

  \[
  J_{2} = m : \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) : \quad \text{and} \quad J_{3} = m_{s} : \bar{s}s : . \quad (17)
  \]

  From the good realization of the $SU(2)$ flavour symmetry ($m_{u} = m_{d}$ and ($\bar{u}u = (\bar{d}d)$), one expects a degeneracy between the two-point correlator $\psi_{\mu}(\tau)$ leading to:

  \[
  m_{S_{2}} \simeq m_{0} \simeq 930 \text{ MeV} ,
  \]

  while its hadronic coupling is $32,15$:

  \[
  g_{S_{2}\pi^{+}\pi^{-}} \simeq \frac{16\pi^{3}}{3\sqrt{3}} (\bar{u}u)_{\tau_0} e^{M_{S_{2}}^{2}/\tau_{0}/2} \approx 2.46 \text{ GeV} . \quad (19)
  \]

  leading to:

  \[
  \Gamma(S_{2} \rightarrow \pi^{+}\pi^{-}) \simeq 120 \text{ MeV} ,
  \]

  Using $SU(3)$ symmetry, one can also deduce:

  \[
  g_{S_{2}K^{+}K^{-}} \simeq \frac{1}{2} g_{S_{2}\pi^{+}\pi^{-}} \simeq 1.23 \text{ GeV} . \quad (21)
  \]

  The mass of the mesons containing a strange quark is predicted to be $15$:

  \[
  M_{S_{3}}/M_{S_{2}} \simeq 1.03 \pm 0.02 \implies M_{S_{3}} \simeq 948 \text{ MeV} , \quad (22)
  \]

  if one uses $M_{K} = 920 \text{ MeV}$ \[16\], while its coupling to $K^{+}K^{-}$ is $15$:

  \[
  g_{S_{3}K^{+}K^{-}} \simeq (2.7 \pm 0.5) \text{ GeV} . \quad (23)
  \]

- **The predictions for the hadronic widths suggest that the naive $\bar{q}q$ assignment of the $\sigma(600) \equiv S_{2}$ does not fit the data.**

- **Gluonia masses from QSSR**

  Masses of the bare unmixed scalar gluonium can be determined from the two Laplace unsubtracted (USR) and subtracted (SSR) sum rules \[37\]:

  \[
  \mathcal{L}_{0}(\tau) = \frac{1}{\pi} \int_{0}^{\infty} dt e^{-\tau t} \text{Im} \psi(t) ,
  \]

  \[
  \mathcal{L}_{-1}(\tau) = -\psi(0) + \frac{1}{\pi} \int_{0}^{\infty} \frac{dt}{t} e^{-\tau t} \text{Im} \psi(t) ,
  \]

  of the two-point correlator $\psi(q^{2})$ associated to $\theta^{\mu}_{\nu}$ defined in Eq. \[10\]. The subtraction constant $\psi(0) = -16(\beta_{s}/\pi)(\alpha_{s}G^{2})$ expressed in terms of the gluon condensate $83$ $\langle \alpha_{s}G^{2} \rangle = (0.07 \pm 0.01) \text{ GeV}^{4}$ \[39,40,14\] affects strongly the USR analysis which has lead to apparent discrepancies in the previous literature when a single resonance is introduced into the spectral function \[1\]. The SSR being sensitive to the high-energy region ($\tau \simeq 0.3 \text{ GeV}^{-2}$) predicts $15$:

  \[
  M_{G} \simeq (1.5 \sim 1.6) \text{ GeV} ,
  \]

  comparable with the quenched lattice value $18$, while the USR stabilizes at lower energy ($\tau \simeq 0.8 \text{ GeV}^{-2}$) and predicts a low-mass gluonium $16$ $6$.

  \[
  M_{\sigma_{B}} \simeq (0.9 \sim 1.1) \text{ GeV} , \quad (26)
  \]

  comparable with the unquenched lattice value $17$ but higher than the one $15$ using a strong coupling calculation of the gluon propagator \[8\]. Furthermore, the consistency of the USR and SSR can be achieved by a two-resonances ($G$ and $\sigma_{B}$) + “QCD continuum” parametrization of the spectral function $37,15$.

  - A recent QSSR analysis of the same gluonium correlator including direct instantons and using a two-resonance parametrization $76$ confirms the previous mass values, while the one using a single resonance $77,6$ gives the mean value of the two masses. The small effects of the direct instanton in the mass predictions is expected from the smallness of the extra $1/q^{2}$ term induced by a tachyonic gluon mass which mimics instanton effects in this approach \[73\], and which is necessary for solving the sum rule scale hierarchy between the gluonium and of the usual $\rho$ meson \[22\].

- **$OZI$ violation in $\sigma_{B} \rightarrow \pi\pi$**

  The $\sigma_{B}\pi\pi$ coupling can be obtained from the vertex function:

  \[
  V[q^{2}] \equiv \langle q_{1} - q_{2} \rangle \equiv \langle \sigma_{1} | [\theta^{\mu}_{\nu}] | \pi_{2} \rangle ,
  \]

  obeying a once subtracted dispersion relation:

  \[
  V(q^{2}) = V(0) + q^{2} \int_{4m_{s}^{2}}^{\infty} \frac{dt}{t} \left( \frac{1}{t - q^{2} + i\epsilon} \right) \text{Im} V(t) . \quad (28)
  \]

  with the condition: $V(0) = 0$ in the chiral limit. Using also the fact that $V'(0) = 1$, one can then derive the two sum rules:

  \[
  \sum_{S=\sigma_{B}} g_{S\pi\pi} \sqrt{2} f_{S} = 0 , \quad \frac{1}{4} \sum_{S=\sigma_{B}} g_{S\pi\pi} \sqrt{2} f_{S} / M_{S}^{2} = 1 \quad (29)
  \]

  where $f_{S}$ is the decay constant analogue to $f_{\pi}$. The $1^s t$ sum rule requires the existence of at least two resonances coupled strongly to $\pi\pi$. Considering the $\sigma_{B}$ and $\sigma'_{B}$ but neglecting the small $G$-coupling to $\pi\pi$ as indicated by GAMS $18$, one predicts in the chiral limit $37,15$:

  \[
  |g_{\sigma_{B}\pi^{+}\pi^{-}}| \simeq |g_{\sigma_{B}K^{+}K^{-}}| \simeq (4 \sim 5) \text{ GeV} , \quad (30)
  \]

  a universal coupling, which will imply a large width $6$.

  \[
  \Gamma_{\sigma_{B} \rightarrow \pi^{+}\pi^{-}} \simeq 0.7 \text{ GeV} . \quad (31)
  \]

\[\footnote{6}{\text{Notice that using a similar USR, the trigluonium mass is associated to the scalar operator $g_{f_{\sigma_{B}}G^{2}G^{2}G}$ is found to be about 3.1 GeV \[43,13\] and has a tiny mixing with the $\sigma$ resonance.}}\]

\[\footnote{7}{\text{A more direct comparison requires the evaluation of the gluonic two-point in this approach.}}\]

\[\footnote{8}{\text{In \[15\] the QCD continuum has also been modeled by a $\sigma'_{B}$ (radial excitation of the $\sigma_{B}$), which enables to fix the decay constant $f_{\sigma'_{B}}$, once the $\sigma_{B}$, $\sigma'_{B}$ masses are introduced as input.}}\]

\[\footnote{9}{\text{However, the analysis in Ref. \[15\] also indicates that $\sigma_{B}$ having a mass below 750 MeV cannot be wide ($\lesssim 200$ MeV) (see also some of Ref. \[1\]) due to the sensitivity of the coupling to $M_{G}$. Wide low-mass gluonium has also been obtained using QSSR (1st ref. in \[1\]), an effective Lagrangian \[9\] and LET \[38\].}}\]

\[\footnote{10}{\text{A higher value has been obtained because one has used as input the experimental mass $K_{0}^{*} = 1430$ MeV.}}\]
The large width into $\pi\pi$ is a typical OZI-violation due to non-perturbative effects expected to be important in the region below 1 GeV, where perturbative arguments valid in the region of the $G(1.5)$ cannot be applied. This result can be tested using lattice unquenched calculations of the width.

- **$G(1.5)$ widths**
  
  We shall not discuss the derivation of these widths here as they are done in details in text. An analogous low-energy theorem, like the one in Eq. (29), leads to strong couplings of the $G(1.5)$ in the $U(1)_A$ channels $\eta'/\eta'$ and $\eta'\gamma$, while, one expects its weaker couplings to $\pi\pi$ contrary to $\sigma_B$, which almost saturates the vertex sum rule in Eq. (29). Characteristic glueball decay widths in these channels are $\sim 10^{-11}$ MeV.

$$\Gamma(G \rightarrow \eta'\eta) \approx (5 \sim 10) \text{ MeV} , \quad \Gamma_{Gm}/\Gamma_{Gm'} \approx 0.22 . \quad (32)$$

In this approach, where the $\sigma_B$ is the lowest mass gluonium, one also expects that the $G(1.5)$ decays into $4\pi$ via $\sigma_B$ pairs with:

$$|g_{G\sigma_B}| \approx 1.3 \sim 3.7 \text{ GeV} \Rightarrow \Gamma_{G\rightarrow 4\pi} \approx 7 \sim 55 \text{ MeV} (33)$$

- **Gluonia couplings to $\gamma\gamma$**
  
  These couplings can be derived by identifying the Euler-Heisenberg effective Lagrangian for $gg \rightarrow \gamma\gamma$ via a quark constituent loop to the scalar one: $L_{S\gamma\gamma} = g_{S\gamma\gamma} S F^{(1)}_{\mu\nu} F^{(2)}_{\mu\nu}$, which leads to the sum rule $\sum_{q=u,d,s}(\pi/\beta_1) \sum_{Q_{q}} Q_{q}^{2}/M_{q}^{4}$, 

$$g_{S\gamma\gamma} \approx \frac{\alpha}{60} \sqrt{2} f_{S} M_{S}^{2} \left(\frac{\pi}{\beta_1} \sum_{Q_{q}} Q_{q}^{2}/M_{q}^{4}\right) , \quad (34)$$

where $Q_{q}$ is the quark charge in units of $e$; $M_{u,d} \approx M_{r}/2$ and $M_{s} \approx M_{s}/2$; are constituent masses; $S$ refers to gluonium ($\sigma_B$, ...). Then, one predicts the couplings:

$$g_{\sigma_B\gamma\gamma} \approx g_{\sigma_B'\gamma\gamma} \approx g_{\sigma/f} \gamma\gamma \approx (0.4 \sim 0.7) \alpha \text{ GeV}^{-1} , \quad (35)$$

and the corresponding widths $\Gamma(\sigma_B \rightarrow \gamma\gamma)$:

$$\Gamma_{\sigma_B \rightarrow \gamma\gamma} \equiv \frac{|g_{\sigma_B\gamma\gamma}|^2 16\pi}{M_{\sigma_{B}}^3} \approx (0.2 \sim 0.6) \text{ keV} . \quad (36)$$

Alternatively, one can match the $k^2$ dependence of the two sides of $\langle 0|\theta_{\mu}\gamma_\nu\gamma_\sigma|2\rangle$ in order to derive the sum rule $\sum_{Q_{q}} Q_{q}^{2}/M_{q}^{4}$:

$$\frac{1}{4} \sum_{S=\sigma_B,...} \frac{g_{S\gamma\gamma} \sqrt{2} f_{S}}{3\pi} = \frac{\alpha R}{3\pi} , \quad (37)$$

which one can use for checking the a self-consistency of the previous results ($R \approx 3 \sum_{Q_{q}} Q_{q}^{2}$) [37].

- **QCD tests of the $\sigma/f_{0}(600)$ as a gluonium?**
  
  The first question which comes in mind is: how can one compare the theoretical results from QSSR and LET obtained in the real axis with the measured value of the complex pole parameters from $\pi\pi$, $\pi K$ and $\gamma\gamma$ scatterings? Some approximate answers to this question are given in the literature, either by using a Breit-Wigner parametrization of the data [33], or by using [35] the on-shell mass obtained by imposing that the amplitude is purely imaginary at the phase $90^0$ [51] or by arguing that the mass obtained from QSSR appears in the tree level amplitude of $\pi\pi$ scattering and becomes much lower in the unitarized amplitude [35]. In all cases, these results indicate that the 1 GeV mass from QSSR can translate into the observed wide $\sigma(600)$. Using the complex pole mass, one can e.g. deduce the on-shell mass:

$$M_{\sigma}^0 \approx 0.92 \text{ GeV} \quad \text{and} \quad \Gamma_{\sigma}^0 \approx 1 \text{ GeV}, \quad (38)$$

in remarkable agreement with the previous QSSR and LET predictions. For (properly) comparing the $\gamma\gamma$ width of $\sigma_B$ with the one of $\sigma(600)$, one can use the Mennoniser model [35,34] for separating the direct resonance coupling from the $\pi\pi$ rescattering terms. Then, we can deduce the “partial” $\gamma\gamma$ widths at the complex pole:

$$\Gamma_{\sigma^{\text{dir}} \rightarrow \gamma\gamma} \approx (0.13 \pm 0.05) \text{ keV} , \quad \Gamma_{\sigma^{\text{resc}} \rightarrow \gamma\gamma} \approx (2.7 \pm 0.4) \text{ keV} , \quad (39)$$

and the total $\gamma\gamma$ width (direct $+$ rescattering):

$$\Gamma_{\sigma \rightarrow \gamma\gamma} \approx (3.9 \pm 0.6) \text{ keV} . \quad (40)$$

These values are in the range of the ones obtained in the literature [56,57,58] but a clean comparison is difficult to do as the separation between the direct and the rescattering terms is not explicit there. An (almost) similar model [59] leads to a result 2 times smaller for the total contributions than the one obtained here and a negligible direct $\gamma\gamma$ width of about 3.4 eV. A meaningful comparison can be done for the total contribution but requires the quotation of errors in the result of [59]. In a clean separation of the two contributions is proposed by measuring the $C$-odd asymmetry in $e^+e^- \rightarrow \pi^+\pi^-$, which can be feasible at KLOE [61]. Improvements of the previous estimates need more precise data below 700 MeV, and an extension of the analysis to higher energies. Translating the previous results to the on-shell mass, one obtains:

$$\Gamma_{\sigma \rightarrow \gamma\gamma} \approx (1.0 \pm 0.4) \text{ keV} , \quad (41)$$

in good agreement with the QSSR predictions for $\sigma_B$.

- **The previous “overall agreement” without any free mixing parameter can favour a large gluon component in the $\sigma/f_{0}(600)$ wave function, which is naturally expected due to the $U(1)_A$ anomaly. Its large width into $\pi\pi$, indicates a strong violation of the OZI rule, signals large non-perturbative effects in its treatment, and disfavors its $gg$ interpretation. The latter predicts a narrower hadronic width and a much larger $\gamma\gamma$ width of**

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10These features are expected for a pure gluonium where a perturbative argument like the chiral coupling to $\pi\pi$ and $KK$ can also hold [50].

11This sum rule has been used in [19] for the charm quark.

12Due to their $M^3$-dependence, the widths of the $\sigma_f$ and $G$ can be much larger: $(0.4 \sim 2) \text{ keV}$. These widths induce a tiny effect of $(3 \sim 9) \times 10^{-11}$ to the muon $(g-2)$ [22] and cannot be excluded.

13Unfortunately this procedure is expected to be only accurate for a narrow resonance.
about 5 keV which can be deduced from the one of the aq. In the same way, QSSR also predicts, for a four-quark state, having the same mass of 1 GeV [25,31], a tiny γγ width of about 0.4 eV [31]. Both qq and q̅q scenarios are disfavoured by the value of the direct coupling fitted from γγ scattering, while an explanation of the I = 0 scalar channel without the inclusion of a gluonium/glueball associated to the U(1)_V-anomaly, appears to be unlikely. Another point which can disfavour the 4q scenario is the expectation of the weak coupling of the σ to K K [11], which seems not be the case from the analysis of [57,22] where the ππ and KK couplings are almost equal as expected from the LET results in Eq. [67].

4. Higher scalar mesons

Understanding the dynamics of the spectrum of the radial excitations within QCD is still unanswered and is the subject of hot activities. For the I = 0 scalar mesons, the f_0(1.37) is a good candidate for being the radial excitation of the σ(600) where its hadronic width into ππ is found to be about (300 ~ 500) MeV [37,15] and its γγ width of the order of (10 ~ 30) eV. It can be viewed as a high mass tail of the gluonic sigma (red dragon) in [63], which starts from the σ(600) and continues to higher tail of about 1800 MeV. However, the experimental situation for the f_0(1.37) is not yet settled [64,65].

5. Meson-gluonium mixing

From the present unquenched lattice result where the glueball mass has shifted down to 1 GeV, existing meson-gluonium schemes [66] need to be revised. Hopefully, we have always considered [16,37,15,11] that the lowest glueonium mass is below 1 GeV making still valid the meson-gluonium mixing discussed in [32,11,14] where a maximal two-component mixing between the S_2(‡q) and σ_2(gg) have been proposed for explaining the wide σ(600) and narrow f_0(980). Above 1 GeV, a three-component mixing à la CKM between the G(1.5), σ_2(1.37) and the radial excitation S_2′(1.47) of the σ state have been proposed for explaining the f_0(1.37), f_0(1.5) and f_0(1.7), which can be updated.

6. Experimental tests

In addition to the new one [60] for measuring the ππ rescattering term in e^+e^- → π^+π^- discussed here and the reanalysis of γγ scattering experiments, some characteristic tests for detecting glueballs can be found in [11] and some other existing reviews.

7. General remarks

The scalar sector of QCD remains complex and fascinating, while many problems remain unanswered after a half century. It will be more fun if the Higgs of the Standard Model is a σ-like resonance.

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