THEORETICAL MODELING OF PROPAGATION OF MAGNETOACOUSTIC WAVES IN MAGNETIC REGIONS BELOW SUNSPOTS

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ABSTRACT

We use two-dimensional numerical simulations and eikonal approximation to study properties of magnetohydrodynamic (MHD) waves traveling below the solar surface through the magnetic structure of sunspots. We consider a series of magnetostatic models of sunspots of different magnetic field strengths, from 10 Mm below the photosphere to the low chromosphere. The purpose of these studies is to quantify the effect of the magnetic field on local helioseismology measurements by modeling waves excited by subphotospheric sources. Time–distance propagation diagrams and wave travel times are calculated for models of various field strengths and compared to the nonmagnetic case. The results clearly indicate that the observed time–distance helioseismology signals in sunspot regions correspond to fast MHD waves. The slow MHD waves form a distinctly different pattern in the time–distance diagram, which has not been detected in observations. The numerical results are in good agreement with the solution in the short-wavelength (eikonal) approximation, providing its validation. The frequency dependence of the travel times is in good qualitative agreement with observations.

Key words: MHD – Sun: helioseismology – Sun: magnetic fields – Sun: oscillations

Online-only material: color figures, mpeg animation

1. INTRODUCTION

Local helioseismology (time–distance helioseismology, acoustic holography, and ring-diagram analysis) provides valuable information about the physical properties of solar subsurface layers (e.g., Duvall et al. 1993; Kosovichev 1999, 2002; Kosovichev et al. 2000; Zhao & Kosovichev 2003; Braun & Lindsey 2000; Hill 1988; Haber et al. 2000). Time–distance helioseismology (Duvall et al. 1993; Kosovichev & Duvall 1997) makes use of wave travel times measured for wave packets traveling between various points on the solar surface through the interior. By inversion of these measurements, variations of the wave speed and velocities of mass flows can be recovered below the visible solar surface. Inversion results for the travel times have been obtained for quiet Sun regions as well as for magnetic active regions including sunspots. In most of these cases, the variations of the travel times were assumed to be caused by mass flows and magnetoacoustic speed fluctuations along the wave paths. It is known that sunspots possess strong magnetic fields with a complicated structure in the visible layers of the Sun where the Doppler measurements used by local helioseismology are taken (Solanki 2003). Consequently, such a magnetic field can cause important effects on helioseismic waves, beyond the perturbation theories employed for helioseismic data analysis (Kosovichev & Duvall 1997; Birch & Kosovichev 2000; Gizon & Birch 2002). The magnetic field can change the acoustic excitation rate and spectral properties of solar oscillations (see the recent work by Jacoutot et al. 2008), produce new wave modes, change wave propagation speeds, and change the wave propagation paths and reflection characteristics at the near-surface layers.

The influence of the magnetic field on the interpretation of local helioseismology measurements has not been fully explored. Theoretical efforts have been made by Crouch & Cally (2003), Cally (2005, 2006), Schunker & Cally (2006), Cally & Goossens (2008), and Schunker et al. (2008) to include mode conversion and model waves in magnetized structures by means of analytical and ray-path theories. These studies confirm the potential importance of magnetic effects. A more complete understanding of the problem can be reached by direct forward modeling of helioseismic data, by numerically solving the magnetohydrodynamics (MHD) equations in realistic magnetic configurations (e.g., Gizon et al. 2006; Khomenko & Collados 2006; Hanasoge 2008; Cameron et al. 2007; Parchevsky & Kosovichev 2009). In these papers, the MHD equations, governing the problem, are solved for magnetic field configurations resembling a sunspot or part of a sunspot. The results have demonstrated how magnetic fields change the speed, amplitude, and spectral properties of the waves, and also produce wave transformation and refraction in the sunspot atmosphere.

Observational helioseismology over active regions has repeatedly demonstrated that waves travel faster through sunspots in comparison with quiet Sun measurements (e.g., Duvall et al. 1993; Braun 1997). This fact is still the subject of present investigations and its explanation is still far from clear. Although given that sunspots possess an intense field at photospheric levels, direct or indirect influence of the magnetic field is generally accepted (e.g., Hindman et al. 1997). At subphotospheric levels, the influence of the magnetic field becomes less and less important with depth as pressure forces dominate over magnetic forces. This is why the phenomena related to the magnetic field are referred to as “surface effects.” Explanations based on wave perturbations due to scattering phase shifts (Braun 1997; Lindsey & Braun 2005a, 2005b) taking place within a few Mm below the photosphere or local wave speed variations below sunspots (Kosovichev et al. 2000) have been suggested to explain the different travel times measured in sunspots and in the quiet photosphere.

In this paper, we study wave properties and variations of travel times with the help of numerical simulations and eikonal
solutions for wave propagation in sunspot-like magnetic structures in stratified surface and subsurface layers. We perform two-dimensional numerical simulations of the waves produced by a single source for a series of model sunspots (Khomenko & Collados 2008). We study various types of MHD waves and their propagation properties numerically. In addition, for the fast magnetoacoustic mode we obtain a solution in the eikonal approximation, which is often important for correct interpretation of helioseismic results (Gough 2007). Our numerical results clearly indicate that the fast mode is the primary source of helioseismic signals. We study the frequency dependence of the wave travel times and compare the results of the eikonal approach with our numerical simulations and observational data. Our results provide insight into the properties of helioseismic waves in magnetic regions and are important for interpretation of time–distance helioseismology measurements.

2. NUMERICAL PROCEDURE

We use the numerical simulation code previously described by Khomenko & Collados (2006) and Khomenko et al. (2008). The code solves the basic nonlinear equations of the ideal MHD, written in a conservative form:

\[
\frac{\partial \rho}{\partial t} + \vec{V} \cdot (\rho \vec{V}) = 0, \quad (1)
\]

\[
\frac{\partial (\rho \vec{V})}{\partial t} + \vec{\nabla} \cdot \left[ \rho \vec{V}\vec{V} + \left( P + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\vec{B}\vec{B}}{4\pi} \right] = \rho \ddot{g}, \quad (2)
\]

\[
\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[ (E + P + \frac{B^2}{8\pi}) \vec{V} - \vec{B} \left( \frac{\vec{B} \cdot \vec{V}}{4\pi} \right) \right] = \rho \vec{V} \cdot \ddot{g} + \rho Q, \quad (3)
\]

\[
\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{B}), \quad (4)
\]

where \( \mathbf{I} \) is the identity tensor and \( E \) is the total energy:

\[
E = \frac{1}{2} \rho V^2 + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi}. \quad (5)
\]

All other symbols have their usual meaning. Gravity has been taken constant, with a value of 2.74 \times 10^4 \text{ cm s}^{-2}. The term describing radiative energy losses, \( Q \), is set to zero in the present study. A perfectly matching layer (PML) boundary condition (Berenger 1994) is applied to all boundaries of the simulation domain with the aim of minimizing wave reflection effects at the boundaries. This performs rather well in our simulations. The thickness of the PML layer in our simulations is 20 grid points (1 Mm).

The code solves the nonlinear MHD equations for perturbations that are obtained by subtracting the equations of initial magnetostatic equilibrium from Equations (1)–(4). However, for the purpose of the present study, we keep the perturbation amplitudes as low as possible, making the second-order nonlinear terms relatively unimportant. In this paper, we use the two-dimensional version of the code.

The wave source is modeled as a time-dependent term \( S(x, z, t) \) in the right-hand side of the momentum equation (Equation (2)) in the same way as in the paper by Parchevsky & Kosovichev (2009):

\[
f(x, z, \tau) = \begin{cases} 
A \left[ 1 - \frac{r^2}{R_{\text{src}}^2} \right]^2 \left( 1 - 2\pi^2 \right) e^{-\tau^2} & \text{if } r \leq R_{\text{src}} \\
0 & \text{if } r > R_{\text{src}}, \end{cases} \quad (6)
\]

where \( R_{\text{src}} \) is the source radius, taken to be 250 km, and \( r \) is the distance from the source center, \( r = \sqrt{(x - x_{\text{src}})^2 + (z - z_{\text{src}})^2} \).

The parameter \( \tau \) is given by the equation

\[
\tau = \frac{\omega (t - t_0)}{2} - \pi, \quad t_0 \leq t \leq t_0 + \frac{4\pi}{\omega}, \quad (7)
\]

where \( \omega \) is the central, dominating, frequency of the source and \( t_0 \) is the source start moment. In our model the source represents the vertical force in the momentum equation, \( \ddot{S} = (0, 0, \dot{f})^T \). The central frequency of the source is 3.3 mHz. In all the simulations described here, the center of the source is located at a depth of -200 km, below the photospheric level of the reference model S of Christensen-Dalsgaard et al. (1996). The depth location of the source in our simulations corresponds to the recent studies of the excitation of waves in magnetic regions by Jacoutot et al. (2008).

The source generates a broad spectrum of acoustic and surface gravity waves in the 5 minute range, resembling the wave excitation in the real Sun. In the case of a sunspot model, these waves are modified by the presence of the magnetic field.

3. SUNSPOT MODEL AND SIMULATION SETUP

The sunspot models are calculated using the method proposed in Khomenko & Collados (2008). Here we use three magnetostatic models of sunspots with a similar magnetic field structure but different values of the magnetic field strength. In this way, we are able to study the effects of the magnetic field strength on the propagating waves apart from the effects of the magnetic field inclination. Some characteristics of these models are displayed in Figures 1 and 2. The magnetostatic equilibrium is calculated in the cylindrical coordinates assuming azimuthal asymmetry and no twist of the magnetic field lines. At large distances from the axis, the sunspot models merge smoothly into the quiet Sun model, for which we take the model S of Christensen-Dalsgaard et al. (1996), modified to make it stable with respect to convective motions (see Parchevsky & Kosovichev 2007). The sound speed variations along the radial distance are evident from Figure 2. The sunspot models have a low-temperature zone below the surface down to, about, -2 Mm depth. Below this depth, the temperature gradient in the horizontal direction is small and no high-temperature zone is introduced in the present study. Note that the ratio of the acoustic to the Alfvén speeds squared changes by orders of magnitude with height and that the gradient of this ratio is largest in the surface layers.

For the wave simulations, the models are transformed from cylindrical coordinates \((r, \phi, z)\) to three-dimensional Cartesian coordinates. In a general case, all three components of the magnetic field vector are nonzero in the Cartesian coordinates. In the case of the two-dimensional simulations, we chose a slice where the \( B_r \) component of the field is equal to zero and calculate the wave propagation in this plane. By adopting this approach, we exclude Alfvén waves from consideration.

The simulation domain is 40 Mm in the horizontal direction and 10.5 Mm in the vertical direction. The sunspot axis is located at the center of the simulation domain, and the upper 0.5 Mm of the domain are located above \( Z = 0 \) km height. Above
Z = 0.5 Mm, we introduce an isothermal PML layer, 1 Mm thick. The grid resolution of the simulations is 50 km in the vertical and 100 km in the horizontal directions. The temporal resolution depends on the CFL numerical stability condition, limited by the rapid growth of the Alfvén speed, $v_A$, with height, changing typically between 0.01 and 0.08 s. For each of the three sunspot models, we perform two simulation runs with the wave source located at $X_0 = 6$ Mm and 12 Mm to the left of the sunspot axis.

In addition to the simulations of waves in the sunspot models, we have also performed a reference simulation for a model without a magnetic field and without horizontal variations in thermodynamic parameters. The vertical stratification of the thermodynamic parameters in this case corresponds to the modified model S.

Several recent works on helioseismology point out the presence of the so-called surface effects in the measurements in active regions (see e.g., Lindsey & Braun 2005a, 2005b; Schunker et al. 2005; Zhao & Kosovichev 2006; Rajaguru et al. 2007). Due to the Wilson depression in sunspots, the formation heights of spectral lines are altered introducing additional phase shifts to the velocity measurements, as compared to nonmagnetic regions. Our sunspot models describe the Wilson depression effect since the whole atmosphere inside the magnetic concentration had been shifted downward. In order to take these “surface effects” into account, we have computed the optical depth scale $\tau_5$ in our sunspot models. This has been done with the help of SIR code (Ruiz Cobo & del Toro Iniesta 1992). As expected, the $\tau_5$ scale varies with the radial distance from the sunspot center. This variation can be appreciated at the middle panel of Figure 1, where the contours of the constant optical depth $\log \tau_5 = -1.6$ are given. The optical depth $\log \tau_5 = -1.6$ represents the layer where typical photospheric spectral lines are formed. In the rest of the paper, we call “photospheric velocity” the velocity measured at a $\log \tau_5 = -1.6$ level. Note that the height variations in the position of this layer can be as large as 0.2–0.4 Mm, depending on the distance from the axis in the sunspot models.

In the nonmagnetic simulation, the “photospheric velocity” at $\log \tau_5 = -1.6$ always corresponds to the same height, as there is no Wilson depression.

### 4. WAVE FIELD FROM A SINGLE SOURCE

In this section, we describe the results of the simulations for the three model sunspots and the quiet Sun model for two positions of the oscillation source. In the first series of simulations, the source is located in the “umbral” part of the model at a distance of 6 Mm from the sunspot axis, where the field is relatively strong. In this region, the ratio $c_s^2/v_A^2$ is equal to 1.4, 0.46, and 0.11 in the three models with $B_{\text{phot}} = 0.9$, 1.5, and 2.4 kG, respectively. Thus, the wave source is located in the region dominated by the magnetic field in the last two
models. In the second series, the source is located in the "penumbral" part of the model, at a distance of 12 Mm from the axis, where the field is weaker. There, the parameter $c_s^2/v_A^2$ is equal to 8.2, 3.1, and 1.1 in the three models. In this case, the waves are excited in the acoustically dominated region. Placing the wave source at these two positions allows us to study the wave properties for the two different regimes. Note that through the whole paper, we choose to discuss the results in terms of the parameter $c_s^2/v_A^2$ instead of plasma $\beta$. The former parameter is more relevant regarding wave propagation. The ratio between these two parameters is equal to $(c_s^2/v_A^2)/\beta = \gamma/2 = 5/6$ (for an ideal gas). The layers $c_s^2/v_A^2 = 1$ and $\beta = 1$ are located very close in the atmosphere. The maximum height separation between them is about 30 km. The locations of the sources in both sets of simulations are marked by yellow dots in Figure 1. The duration of each simulation run is about 50 minutes.

4.1. Nonmagnetic Case

Figure 3 shows the time evolution of the simulated horizontal and vertical velocities in the case of the quiet Sun model. In the figure, the velocities are scaled with the square root of...
the background density in order to make visible variations in the deep layers. The simulation reveals that the single acoustic source excites a mixture of acoustic (\(p\)), gravity (\(g\)), and surface (\(f\)) modes, each propagating with its own speed. The \(p\)-modes are the fastest and run beyond the boundaries of the simulation domain after about 30 minutes. The slower \(g\) and \(f\)-modes are concentrated in the upper part of the simulation domain. It can be seen that the wave field is symmetric with respect to the source location since the medium is isotropic for the acoustic-gravity waves.

The power spectrum of the photospheric vertical velocity (\(k - \nu\) diagram) in the nonmagnetic simulations is given in Figure 4. The diagram demonstrates the presence of several ridges of acoustic (\(p\)-mode) oscillations. The darkest bottom ridge corresponds to the \(f\)-mode. The \(f\)-mode analytical dispersion relation is also plotted in the image and shows a reasonable agreement with the numerical results. The figure also reveals the presence of a weak \(g\)-mode signal below the \(f\)-mode ridge. These modes appear because the background model has been made convectively stable.
mode is mainly magnetic in the region where $v_{\text{ph}}$ power. The black line indicates the dispersion relation of the nonmagnetic, standard solar model simulation run. Darker colors indicate larger

Figure 4. $k-v$ power spectrum of the photospheric vertical velocity in the nonmagnetic, standard solar model simulation run. Darker colors indicate larger power. The black line indicates the dispersion relation of the $f$-mode: $\nu^2 = g k_x$, where $g$ is the value of solar gravity acceleration at the surface. Several $p$-modes can be distinguished above the $f$-mode. The $g$-modes, located below the $f$-mode, are also excited by the vertical force source. These modes are present because the solar model has been made convectively stable.

4.2. Waves in Sunspot Models

Figures 5 and 6 show the results of the simulations in the sunspot model with $B_{\text{phot}} = 1.5$ kG and the source location at $X_0 = 6$ Mm and 12 Mm, respectively. In the case of the simulations with the magnetic field, we use for illustrations the longitudinal (parallel to the local field) and transversal (perpendicular to the local field) velocity components. This provides an effective separation of the fast and slow magnetoacoustic waves in those regions of the simulation domain where the characteristic propagation speeds are clearly distinct ($v_A \ll c_S$ or $v_A \gg c_S$). Apart from a smaller propagation speed, the slow mode is mainly magnetic in the region where $v_A \ll c_S$, and has a displacement velocity that is perpendicular to the magnetic field lines. This means that it contributes mainly to the transversal velocity. However, the fast mode is mainly acoustic, that is, longitudinal, by its nature. As this can be seen in Figure 5, the longitudinal component of the fast mode vanishes approximately along the field line crossing the source. The direction of propagation of the slow magnetoacoustic waves (defined as the direction of their group velocity) is always parallel to the magnetic field direction, irrespective of their phase velocity. The direction of propagation of the fast magnetoacoustic waves is independent of the field direction, and the group velocity is parallel to the phase velocity. Given these properties, the fast and slow modes can be distinguished in the time evolution pictures of the transversal and longitudinal velocity components, displayed in Figures 5 and 6, respectively.

In all simulations, the wave source is located in the region where $c_S^2/v_A^2$ stay (roughly) between 0.1 and 10, and the field is significantly inclined. In this region, both the pressure and magnetic restoring forces are of the same order of magnitude. Thus, the source generates a set of fast and slow magnetoacoustic waves. The fast modes are an analog of the acoustic $p$-modes in the nonmagnetic simulations (see Figure 3). However, as will be shown later in the paper, their behavior is modified by the presence of the magnetic field. The slow modes did not appear in the nonmagnetic simulation.

Figure 5 shows that the fast wave propagating in this complex magnetic field configuration causes variations in both, longitudinal and transversal, components of the velocity of the same order of magnitude. The initial wave front expands propagating downward. There is no symmetry in the wave field with respect to the source location (unlike the case of the nonmagnetic simulation) since the wave propagation speed now depends on the direction. In the deep layers, the dominant velocity variations are due to the fast mode waves. In the upper photospheric layers, we mostly observe variations in the longitudinal velocity (right panels in Figure 5). The transversal velocity variations, caused by the fast magnetic waves ($c_S^2/v_A^2 \ll 1$), are negligible in these layers. The wave velocity fields developed in the simulations with the source located at $X_0 = 6$ and 12 Mm from the sunspot axis are similar. The differences are quantitative, and not qualitative in nature.

Figure 6 gives a more detailed view on the evolution of the transversal velocity component in the upper part of the simulation domain (note that the vertical axis has been expanded) and allows us to better understand the complex physics of the wave propagation in the vicinity of the $c_S = v_A$ layer. An even better view on the wave dynamics in this region can be gained by watching the movie of the simulation, provided as online material to this paper. In Figure 6, we observe the presence of the slow MHD mode generated directly by the source. This mode propagates with a visibly low speed downward along the sunspot magnetic field lines. It is localized close to the horizontal position $X = -10$ Mm.

The fast waves generated by the source experience significant refraction and reflection due to the strong increase of the Alfvén speed (see Figure 2) in the vicinity of the $c_S = v_A$ layer (snapshots corresponding to times between 8 and 14 minutes in Figure 6).

In addition to the slow MHD waves generated directly by the source, there is another wave type. It can be observed in Figure 5 between 14 and 24 minutes as a perturbation with a much smaller vertical wavelength compared to that of the fast modes in the sunspot interior. We can observe from Figure 5 that the propagation speed of this low-wavelength disturbance is comparable to that of the fast modes. Unlike the slow MHD waves, these waves propagate horizontally across the sunspot. Thus, these waves are different from either fast or slow MHD waves in several aspects: (1) they have a vertical wavelength that is much smaller than that of the fast waves, (2) they propagate with speeds similar to the fast waves (note, however, that they propagate in the region where the sound speed is slightly larger than the Alfvén speed), (3) they propagate across the field, a behavior impossible for slow waves in this region, and (4) the waves that possess similar characteristics (superficial horizontal propagation, smaller wavelength, etc.) in nonmagnetic simulations are the $f$-mode waves. From these arguments, we suggest that we see a wave type with properties that are a mixture of magnetic and $f$-mode waves. We call them “surface magnetogravity” waves. These waves were previously mentioned by, for example, Schwartz & Stein (1975), who described their properties in an isothermal atmosphere. The variations produced by the surface magnetogravity waves decrease rapidly with depth and almost completely disappear below $-3$ Mm.

The power spectra of the photospheric vertical velocity in the magnetic simulations for different $B_{\text{phot}}$ and the source location at 12 Mm from the sunspot axis are compared in Figure 7. Figure 7 shows that for the magnetic models the $k-v$ diagrams become progressively more and more different
from the nonmagnetic $k$--$
u$ diagram (Figure 4), with increasing magnetic field strength. Even in the case of the weakest field, $B_{\text{phot}} = 0.9$ kG, the inclination of the $p$-mode ridges is larger than in the pure acoustic case. The inclination of the ridges increases with increasing magnetic field strength. The dispersion relation for the $p$-mode ridges depends on the reflection conditions for waves in the upper layers. In the absence of the magnetic field, the waves with different frequencies reflect at different heights in the photosphere and below due to the rapid change of the cutoff frequency with height. When the magnetic field is introduced, the upper reflection condition changes even if the field is as weak as in the $B_{\text{phot}} = 0.9$ kG model. This
Figure 6. Time series of snapshots of the transversal velocity in the simulations with $B_{\text{phot}} = 1.5$ kG and $X_0 = 12$ Mm. Only the top part of the simulation box is shown and the vertical axis has been expanded for better presentation. The velocities are scaled with a factor of $\sqrt{\rho}$. Each panel is normalized to its own maximum.

happens because the propagation path of the high-frequency fast waves is modified strongly by the rapid increase of the Alfvén speed since these waves penetrate higher up in the atmosphere. The reflection height for such waves becomes lower compared to the nonmagnetic case. This naturally produces the change of the dispersion relation observed at the $k-\nu$ diagrams in Figure 7.

4.3. Time–Distance Diagrams and Travel-Time Estimates

In order to quantify the effects of the magnetic field on the travel-time measurements of local helioseismology, we have constructed time–distance diagrams for the vertical velocity. Such diagrams were constructed for the photospheric velocity measured at the optical depth $\log \tau_5 = -1.6$ (see Section 3) in the sunspot models. Since in our simulations the waves are excited by a single source, there is no need to compute the velocity correlations between the different surface points. Thus, the time–distance wave propagation diagrams are constructed by plotting the velocity variations as a function of time for all horizontal points at a constant $\log \tau_5$ level. Figure 8 shows the time–distance diagrams in the nonmagnetic simulations and the simulations with progressively increasing magnetic field strength.

In the time–distance diagram for the nonmagnetic case (Figure 8(a)), the two lowest systems of ridges are due to the $p$-modes. The green dashed line in Figure 8(a) gives a theoretical dispersion relation for these modes, calculated for the standard solar model S. Above them, the third system of ridges comes from the $f$-mode waves. It can be seen from the inclination of the ridges that the phase and group speeds of this mode are different from those of the $p$-modes, confirming that they belong to the $f$-mode. The theoretical time–distance curve for the $f$-mode (blue dashed line in Figure 8(a)) is in a good agreement with the numerical results. The system of ridges at the top part of the diagram above the source corresponds to the $g$-modes. These modes propagate with much lower speed than the $p$- and $f$-modes, in agreement with their appearance at the $k-\nu$ diagram in Figure 4.

The time–distance diagrams in the simulations with the magnetic field have different properties (Figures 8(b – d)). In all the cases $p$-mode-like ridges are present. As in the nonmagnetic case, we have added theoretical curves for $p$-modes and the $f$-mode in the absence of the magnetic field (green and blue dashed curves in Figures 8(b – d)). These curves help identifying the fast modes as the analog of the nonmagnetic $p$-modes propagating with a speed modified by the magnetic field. The $f$- and $g$-mode ridges do not appear in agreement with the

Figure 7. $k-\nu$ power spectra of the photospheric vertical velocity in the simulation runs for the source position $X_0 = 12$ Mm and $B_{\text{phot}} = 0.9$ (a), 1.5 (b), and 2.4 (c) kG. Darker colors indicate larger power. Note the change of the power spectra with the increasing field strength.
Figure 8. Time–distance diagrams of the photospheric vertical velocity for the simulations with (a) no magnetic field, (b) sunspot model with \( B_{\text{phot}} = 0.9 \) kG, (c) sunspot model with \( B_{\text{phot}} = 1.5 \) kG, and (d) sunspot model with \( B_{\text{phot}} = 2.4 \) kG. The source is located at \( X_0 = 12 \) Mm on the left of the sunspot axis. The red curves are the phase travel times calculated using the Gabor wavelet fit to the time–distance curves (see the text). The green dashed curves are theoretical time–distance curves for \( p \)-modes and the standard solar model \( S \) in the absence of the magnetic field. The blue dashed lines are the theoretical time–distance curves for the \( f \)-mode calculated as \( X = (1/2)(g/\omega) \) for the central frequency \( \omega = 2\pi/300 \) of the wave source.

(A color version of this figure is available in the online journal.)

Figure 9. Phase travel-time difference calculated between the travel times measured in simulations without a magnetic field and the simulations with different model sunspots (indicated in the figure) as a function of the horizontal distance. The source location at \( X_0 = 12 \) Mm left to the sunspot axis is marked by a yellow circle.

(A color version of this figure is available in the online journal.)

Instead of constant \( \log \tau_3 \) level, the results presented in Figures 8 and 9 are calculated for the photospheric vertical velocities taken at a constant \( \log \tau_3 \) level. These results do not change much if we take velocities at a constant geometrical height \( Z = 100 \) km, instead of constant log \( \tau_3 = -1.6 \). Only a minor additional phase shift is introduced in the travel-time variations in Figure 9 by making them about 5 s smaller close to the sunspot axis.

The travel-time differences discussed above were calculated for the central frequency of the wave packet. However, there has been a discussion in the recent literature regarding
Figure 10. Phase travel-time difference between the magnetic and nonmagnetic cases as a function of the horizontal distance, calculated for different frequency filtering of the simulations with the source located at 12 Mm. The different colors correspond to different wave frequencies. Calculations done for the model sunspot with $B_{\text{phot}} = 0.9$ kG (top), 1.5 kG (middle), and 2.4 kG (top).

The frequency dependence of the travel-time measurements (Couvidat et al. 2006; Couvidat & Rajaguru 2007; Rajaguru 2009; Braun & Birch 2008). It has been found that if the frequency filtering is applied to the data, in addition to the phase speed filtering, the resulting travel-time differences show a strong frequency dependence. High-frequency waves have, in general, larger travel-time differences (Rajaguru 2009). In order to investigate this effect in our simulations, we filtered the simulated photospheric vertical velocities in the five frequency ranges: 3.3, 3.5, 4.0, 4.5, and 4.8 mHz. We applied a Gaussian filter with a FWHM of 1 mHz. The resulting travel-time differences are shown in Figure 10, for all three model sunspots.

We find that the travel-time differences for each individual frequency behave similar to the travel-time differences calculated without filtering. The maximum difference is achieved for distances closer to the sunspot axis ($\pm 5$ Mm), decreasing for the waves with large skip distances. A strong frequency dependence is evident from Figure 10. Waves with higher frequencies have larger travel-time differences. It means that the propagation speed of the waves increases with frequency. The magnitude of the frequency dependence that we obtain from the simulations is in agreement with the observations of Rajaguru (2009). In his paper, travel-time differences changed from, about, $-15$ to $-30$ s between 3 and 5 mHz, similar to our results in Figure 10.

Interestingly, the travel-time differences get slightly positive for distances within first 5–6 Mm from the source location and for lower frequencies between 3.3 and 4.0 mHz. This effect needs further investigation.

5. EIKONAL APPROXIMATION SOLUTIONS

In order to understand more deeply the numerical results, we present here an approximate solution of the governing MHD equations for small perturbations in the eikonal approximation (see recent applications in, e.g., McLaughlin & Hood 2006; McLaughlin et al. 2008; Khomenko & Collados 2006). The eikonal approach assumes that the wavelength of the perturbation is much smaller than the characteristic scale of the variations of the background model (Gough 2007). In the zero-order eikonal approximation, we neglect the variation of the wave amplitude and consider only the variation of its phase, that is, we assume that all the variables in Equations (1)–(4) depend on $x, z$, and time as $U = a \cdot e^{i\phi(x, z)} \cdot e^{-i\omega t}$ (where $a$ is constant). We make use of the recent developments of Cally (2006) and Moradi & Cally (2008) that allow us to incorporate the effects of the acoustic cutoff frequency in the eikonal solution for MHD waves. In this approximation, after some mathematical simplifications, the following first-order nonlinear partial differential equation is obtained:

$$F(x, z, \phi, p, q) = \omega^4 - \omega^2 (c_S^2 + v_A^2) (p^2 + q^2) + c_S^2 (p^2 + q^2) \times (v_A p + v_A q)^2 - \omega^2 (c_S^2 q^2) + c_S^2 N^2 p^2 = 0,$$

where $p = \frac{\partial \phi}{\partial x}$ and $q = \frac{\partial \phi}{\partial z}$. Parameters $p$ and $q$ are equivalent to the horizontal and vertical wave numbers, respectively. Parameter $N$ is the Brunt–Väisälä frequency, $N^2 = g/H - g^2/c_S^2$. In the following calculations, we use an expression for the cutoff frequency for an isothermal atmosphere $\omega_c = c_S / 2H$. Equation (9) is solved by Charpit’s method of characteristics by transforming it into the following system of ordinary differential equations (ODEs):

$$\frac{dp}{ds} = -\frac{\partial F}{\partial x}$$

$$\frac{dq}{ds} = -\frac{\partial F}{\partial z}$$

$$\frac{dx}{ds} = \frac{\partial F}{\partial p}$$
Here, variable $s$ is the distance along the characteristic wave propagation path. These four ODEs are solved numerically using a fourth-order Runge–Kutta method. We investigate only fast-mode solutions of the equations. Parameters $c_S$, $v_A$, $N$, and $\omega_c$ are taken from the sunspot models. The solution of Equations (10)–(13) gives us $x(s)$, $z(s)$, $p(s)$, and $q(s)$ along the wave path $s$. The lines $x(z)$ give the trajectory of the group velocity of the solution. The phase along the wave path can be calculated as (Moradi & Cally 2008)

$$S(\vec{r}) = \int \vec{k} d\vec{r} - \omega t,$$

(14)

where $\vec{r} = (x, z)$ and $\vec{k} = (p, q)$. The phase function $S(\vec{r})$ divided by the wave frequency $\omega$ gives us an estimate of the phase travel time.

First, we study the paths through the sunspot models of fast magnetoacoustic waves of a given frequency $\omega$. Waves are launched from the same lower turning point at some depth below the surface. Figure 11 gives the examples of solutions $x(z)$ for the fast wave paths for the frequencies 6.6 (top), 4.5 (middle), and 3.0 (bottom) mHz. As an initial condition for the solution of Equations (10)–(13), we take in all the cases $x = -40$ Mm, $z = -3.5$ Mm, and $q = 0$. The initial value of $p$ is found from the approximated solution of the dispersion relation (9) at $q = 0$.

At each frequency, we calculate the wave paths for two cases. In the first case (red lines in Figure 11), we set the Alfvén speed $v_A$ to zero (i.e., eliminate the magnetic field), but keep the sound speed structure and frequencies $N$ and $\omega_c$ from the sunspot model. In the second case, we calculate the wave path in the sunspot model taking into account magnetic field terms (white lines in Figure 11). As can be observed in Figure 11, the paths of the waves in the magnetic and nonmagnetic cases are significantly different. Black dots on the curves mark the positions separated by 1 minute in order to outline the quantitative differences in phase times spent by the waves on their paths.

The frequency of the wave at the top panel of Figure 11 (6.6 mHz) is larger than the atmospheric cutoff. Thus, in the nonmagnetic case, this wave (red line) propagates to the upper atmospheric layers without reflection and goes outside the simulation domain. This does not happen in the magnetic case. The green horizontal line in Figure 11 marks the position of the layer where $c_S = v_A$. As the fast magnetoacoustic wave reaches this layer from below, its propagation properties become progressively more affected by the magnetic field. The fast magnetoacoustic wave in the region $v_A > c_S$ experiences

\[ \frac{dz}{ds} = \frac{\partial F}{\partial q}. \]

(13)
the refraction due to the rapid increase of sound speed \( c \) with height (a clear example of such behavior can be seen in the simulations presented by Khomenko & Collados (2006)). Finally, the fast magnetoacoustic wave reflects back to the deep layers at some height above \( c_S = v_A \). In the deep layers, this wave again experiences the refraction and reflection, but now due to the rapid increase of sound speed \( c_S \). Thus, in this model, the high-frequency fast waves in the magnetic case are trapped below the photosphere due to magnetic effects.

If we reduce the wave frequency to 4.5 mHz (middle panel in Figure 11), then in both the magnetic and nonmagnetic cases, the wave paths have an upper reflection point. The height where the wave frequency is equal to the atmospheric cutoff frequency is marked by the yellow lines in the figure. Since the sunspot model atmosphere has a temperature minimum, there is a region in between the two yellow lines, where the wave of frequency 4.5 mHz is evanescent. In the nonmagnetic case, the wave is reflected sharply from this region. In the magnetic case, the reflection process is more complex. One can observe that the relative positions of the layer where \( c_S = v_A \) and the cutoff (\( \omega c \)) layer change with the horizontal distance. Far from the sunspot axis, where the field is relatively weak, the cutoff height lies below the \( c_S = v_A \) layer. Thus, the waves are reflected before magnetic effects on the wave path take place. Note that the trajectories of the waves in the magnetic and nonmagnetic cases are very similar in this region. As the waves approximate the region close to the sunspot center (where the magnetic field is large), the position of the \( c_S = v_A \) layer lowers and the fast wave path becomes affected by the magnetic field. It can be seen from the figure that the red and white curves start to separate significantly in the region close to the sunspot axis. The waves in the magnetic case propagate with larger speeds and overtake the waves in the nonmagnetic case. Note that by introducing the magnetic field, we have changed the height of the upper turning point of the waves. This may explain the shift of the positions of the ridges in the magnetic simulations in Figure 7 compared with the nonmagnetic simulations in Figure 4.

Finally, the bottom panel of Figure 11 gives the propagation paths in the low-frequency case of 3 mHz. In this case, the position of the cutoff height always lies below the height of the \( c_S = v_A \) layer. It means that (at least in our sunspot model atmosphere) low-frequency waves are reflected before their path is significantly affected by the rapid increase of the Alfvén speed. However, it is important to note that even in this case, the trajectories of the waves in the magnetic and nonmagnetic models are different. The propagation speed of the fast magnetoacoustic wave depends on the magnetic field and is always greater than the sound speed.

6. QUANTITATIVE ESTIMATES OF THE TRAVEL TIMES AND SKIP DISTANCES

In order to quantify the effects of the magnetic field on the wave travel times in the eikonal approximation, we calculated the differences in travel times for the waves in the magnetic and nonmagnetic cases. This calculation is done for wave frequencies between 3.3 and 4.8 mHz. The resulting travel-time differences are presented in Figure 12 as a function of the wave skip distance. This calculation is similar to that reported by Moradi & Cally (2008).

The curves shown in Figure 12 were calculated in the following way. In each case, the waves are launched from their lower turning point; thus \( q = 0 \), and \( p \) is obtained from the dispersion relation in equation (9). The lower turning point is located at annuli distance \( x_A = 12 \) Mm from the sunspot axis, in order to make the eikonal calculations directly comparable to the travel-time estimates from simulations, as presented in Figure 10. In the vertical location of the lower turning point \( z_A \) varies in the range from \(-1.5\) to \(-10\) Mm below the photosphere. By varying \( z_A \) and frequency \( \omega \), we obtain different curves in Figure 12. The deeper \( z_A \) is, the larger is the value of the skip distance in the horizontal axis of Figure 12. Note that to facilitate the comparison between the numerical simulations and the eikonal approach, we present the skip distances in Figure 12 with respect to the position to the sunspot axis.
The frequency behavior calculated in the eikonal approximation is in good agreement with the simulations. It proves the reliability of the eikonal approximation for the full MHD approach. As in Figure 10, the travel-time differences are negative except for positions close to the source location, meaning that the waves propagate faster in the magnetic case. There is a strong frequency dependence of the travel-time differences. Figure 12 shows that depending on the sunspot model and on the skip distance, the maximum values change from, about, −50 s for the 4.8 mHz wave to −10 s for the 3.3 mHz wave. The agreement between simulations and the eikonal solution is worse in the $B_{\text{phot}} = 2.4$ kG case, where the latter gives slightly larger values of the travel-time differences and a different dependence. In all the cases, the radius dependence obtained from the simulations using the Gabor wavelet fit is more complicated than the skip-distance dependence obtained in the eikonal approximation. Note that both simulations and the eikonal approximation give values of the travel-time variations that fit reasonably well into the range of the observed values in solar active regions below sunspots (see e.g., Kosovichev et al. 2000; Zhao & Kosovichev 2003; Couvidat et al. 2006; Couvidat & Rajaguru 2007; Braun & Birch 2008). However, the frequency dependence in the eikonal approximation is slightly larger than that obtained from simulations and observations.

### 7. DISCUSSION AND CONCLUSIONS

In this paper, we present an investigation of the influence of the magnetic field on helioseismic wave propagation in active regions below sunspots. In order to better understand the physics of the waves in complex magnetic field configurations, we perform a single source experiment and study the propagation properties of the different wave modes excited by such a source, as well as the wave travel times. We have carried out our calculations in a series of sunspot models with different field strengths and for two source positions located in magnetically and acoustically dominated regions. The results of our two-dimensional simulations are compared to an asymptotic solution using the eikonal approximation. The results of both methods, numerical simulations and eikonal approximation, are found to be in a good qualitative agreement. Our approach has allowed us to investigate the effects of the magnetic field strength in sunspots on the travel-time variations obtained there from observations using the time–distance helioseismology techniques. In addition, the frequency dependence of such measurements has been studied.

The following conclusions summarize our study.

1. The wave source located immediately below the surface in a nonmagnetic standard solar model excites a mixture of $p$-, $f$-, and $g$-mode waves. When the magnetic field is present, the same source excites a mixture of fast and slow MHD waves. The fast modes represent an analog of the acoustic $p$-modes, modified by the magnetic field.

2. In addition to slow MHD waves excited directly by the source, there is another wave type with a low vertical wavelength, propagating with speed close to that of the fast waves horizontally across the sunspot. We call them surface magnetogravity waves. Several arguments listed in Section 4 indicate that these waves are different from either fast or slow waves. In our simulations, these waves propagate below the visible surface. This makes their detection complicated in spectral observations.

3. Slow MHD wave modes form an additional group of ridges in the time–distance diagram with much lower propagation speed. Their properties depend on the magnetic field strength of the model, as well as on the height in the atmosphere where the velocity is measured. The slow mode ridges on the time–distance diagrams are well isolated and do not affect the travel-time measurements in magnetic regions.

4. The helioseismic waves below sunspots are speed up by the magnetic field. The travel times of these waves are shorter by about 20–40 s compared to the waves in the quiet Sun. Changing the photospheric magnetic field strength from 0.9 to 2.4 kG produces a variation of the travel times by about 10–15 s.

5. The magnetic field produces a strong frequency dependence of the wave travel times. High-frequency waves travel faster than low-frequency waves. This happens because the propagation path of high-frequency fast waves is modified strongly by the rapid increase of the Alfvén speed in the top layers, since these waves penetrate higher up in the atmosphere. Low-frequency waves are reflected below the photosphere due to the acoustic cutoff frequency and do not reach the heights where $\omega_A > c_S$ and the magnetic fields produce strong effects. The travel-time differences change from −20 to −35 s between 3 and 5 mHz, in agreement with observations (e.g., Rajaguru 2009).

The analysis presented here is limited by the two-dimensional approximation. As a next step in our work, we plan to relax this assumption and analyze three-dimensional simulations, with wave excitation by either single or multiple sources. This will allow us to build a more realistic picture and to complete our understanding of the interaction of helioseismic waves with magnetic fields of sunspots.

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