Pion Electromagnetic Form Factor in the $K_T$ Factorization Formulae

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Received (received date)
Revised (revised date)

Based on the light-cone (LC) framework and the $k_T$ factorization formalism, the transverse momentum effects and the different helicity components’ contributions to the pion form factor $F_\pi(Q^2)$ are recalculated. In particular, the contribution to the pion form factor from the higher helicity components ($\lambda_1 + \lambda_2 = \pm 1$), which come from the spin-space Wigner rotation, are analyzed in the soft and hard energy regions respectively. Our results show that the right power behavior of the hard contribution from the higher helicity components can only be obtained by fully keeping the $k_T$ dependence in the hard amplitude, and that the $k_T$ dependence in LC wavefunction affects the hard and soft contributions substantially. A model for the twist-3 wavefunction $\psi_P(x, k_{\perp})$ of the pion has been constructed based on the moment calculation by applying the QCD sum rules, whose distribution amplitude has a better end-point behavior than that of the asymptotic one. With this model wavefunction, the twist-3 contributions including both the usual helicity components ($\lambda_1 + \lambda_2 = 0$) and the higher helicity components ($\lambda_1 + \lambda_2 = \pm 1$) to the pion form factor have been studied within the modified pQCD approach. Our results show that the twist-3 contribution drops fast and it becomes less than the twist-2 contribution at $Q^2 \sim 10 \text{GeV}^2$. The higher helicity components in the twist-3 wavefunction will give an extra suppression to the pion form factor. When all the power contributions, which include higher order in $\alpha_s$, higher helicities, higher twists in DA and etc., have been taken into account, it is expected that the hard contributions will fit the present experimental data well at the energy region where pQCD is applicable.

Keywords: higher helicity components, twist-3, $k_T$ dependence

1. Higher helicity components’ contributions to the pion form factor

In Ref.~\textsuperscript{1} we have systematically studied the transverse momentum effects and the higher helicity components’ contributions to the pion form factor based on the LC framework and the $k_T$ factorization formalism\textsuperscript{2}. The $k_T$ factorization theorem has been widely applied to various processes and it provides a scheme to take the dependence of the parton transverse momentum $k_T$ into account. We note that the $k_T$ dependence in the wavefunction can generate much larger effects than the usual Sudakov suppression to the hard scattering amplitude in the present experimental $Q^2$ region, which shows that it is substantial to take the $k_T$ dependence in the wavefunction into account.
Fig. 1. The combined results for the pion form factors $Q^2 F_\pi(Q^2)$. The solid line stands for the contribution from the hard part, the dotted line stands for the contribution from the soft part, the dashed line is the total Pion form factors and the dash-dot line is the usual asymptotic result.

The light cone (LC) formalism provides a convenient framework for the relativistic description of the hadron in terms of quark and gluon degrees of freedom, and the application of PQCD to exclusive processes has mainly been developed in this formalism. In Ref. we have given a consistent treatment of the pion form factor within the LC PQCD framework, i.e. both the wavefunction and the hard interaction kernel are treated within the framework of LC PQCD. Taking into account the spin space Wigner rotation, one may find that there are higher-helicity components ($\lambda_1 + \lambda_2 = \pm 1$) in the LC spin-space wavefunction besides the usual-helicity components ($\lambda_1 + \lambda_2 = 0$). The asymptotic behavior of the hard-scattering amplitude for the higher-helicity components including the transverse momentum in the quark propagator is of order $1/Q^4$, which is the next to leading order contribution compared with the contribution coming from the ordinary helicity component, but it can give sizable contribution to the pion form factor at the intermediate energies.

In order to compare the predictions with the experimental data, one needs to know the contribution from the soft part. As an example, we have considered the soft contribution to the pion form factor with a reasonable wavefunction in the LC framework. Our results show that the soft contribution from the higher helicity components has a quite different behavior from that of the hard scattering part and has the same order contribution as that of the usual helicity ($\lambda_1 + \lambda_2 = 0$) components in the energy region ($Q^2 \lesssim 1\text{GeV}^2$). As $Q^2 > 1\text{GeV}^2$, the higher helicity components’ contributions will decrease with the increasing $Q^2$. At about $Q^2 \sim 4\text{GeV}^2$, the higher helicity components’ contributions become negative and as a result the net soft contribution to the pion form factor will then decrease with the increasing $Q^2$, which tends to zero at about $Q^2 \sim 16\text{GeV}^2$. Although the soft contribution is purely non-perturbative and model-dependent, the calculated prediction for the pion form factor should take the $k_T$ dependence in the soft and hard parts into account beside including the higher order contributions. Therefore one needs to keep the transverse momentum in the next leading order corrections and to construct a realistic $k_T$ dependence in the hadronic wavefunction in order
to derive more exact prediction to the pion form factor.

The combined results for the pion form factors $Q^2 F_\pi(Q^2)$ are shown in Fig.1 where for comparison, the experimental data $^4$ and the well-known asymptotic behavior for the leading twist pion form factor have also been shown.

2. Twist-3 contributions to the pion form factor

In Ref.$^5$, we have constructed a new model wavefunction for $\psi_p(x, k_\perp)$ based on the moment calculation by using the QCD sum rule approach, i.e.

$$\psi_p(x, k_\perp) = (1 + B_p C_2^{1/2} (1 - 2x) + C_p C_4^{1/2} (1 - 2x)) \frac{A_p}{x(1 - x)} \exp \left( - \frac{m^2 + k_\perp^2}{8 \beta^2 x (1 - x)} \right),$$

(1)

where $C_2^{1/2}(1 - 2x)$ and $C_4^{1/2}(1 - 2x)$ are Gegenbauer polynomials and the coefficients $A_p$, $B_p$ and $C_p$ can be determined by the DA moments. It has a better end-point behavior than that of the asymptotic one and its moments are consistent with the QCD sum rule results. Although its moments are slightly different from that of the asymptotic DA, its better end-point behavior will cure the end-point singularity of the hard scattering amplitude and its contribution will not be overestimated at all.

With this model wavefunction, by keeping the $k_T$ dependence in the wavefunction and taking the Sudakov effects and the threshold effects into account, we have carefully studied the twist-3 contributions to the pion form factor within the modified pQCD approach, where the model dependence of $\psi_p(x, k_\perp)$ has also been given. It has been shown that the present model of $\psi_p(x, k_\perp)$ can give the right power behavior for the twist-3 contribution. This behavior is quite different from the previous observations, where the authors concluded that the twist-3 contribution to the pion form factor is comparable or even larger than that of the leading twist in a wide intermediate energy region up to 40GeV$^2$. The higher helicity components $(\lambda_1 + \lambda_2 = \pm 1)$ in the twist-3 wavefunction have also been considered and it will give a further suppression to the contribution from the usual helicity components $(\lambda_1 + \lambda_2 = 0)$, and at $Q^2 \sim 5GeV^2$, it will give $\sim 10\%$ suppression.

3. CONCLUSIONS

We have shown that both the higher helicity structure’ and twist-3’s contributions are $Q^2$-suppressed to the pion form factor. The $k_T$ factorization approach, where the transverse momentum dependence in the wavefunction and the hard scattering kernel have been kept, ensures these two are really power suppressed. And both two can give sizeable contributions in the intermediate energy regions.

In Fig.2 we show the combined hard contributions for pion form factor, where the higher helicity components have been included in both the twist-2 and the twist-3 wavefunctions, and the twist-3 contribution has been calculated with the present model wavefunction for $\psi_p(x, k_\perp)$ with $\langle \xi^2 \rangle = 0.350$. Together with the NLO corrections to the twist-2 contributions, which for the asymptotic DA, with the renormalization scale $\mu_R$ and the factorization scale $\mu_f$ taken to be $\mu_R^2 = \mu_f^2 = \mu_0^2$,
Fig. 2. Perturbative prediction for the pion form factor. The diamond line, the dash-dot line, the dashed line and the solid line are for LO twist-2 contribution, the approximate NLO twist-2 contribution, the twist-3 contribution and the combined total hard contribution, respectively.

$Q^2$, can roughly be taken as $Q^2 F_{\pi}^{NLO} \approx (0.903 GeV^2)\alpha_s^2(Q^2)$, one may find that the combined total hard contribution do not exceed and will reach the present experimental data.

There is still a room for other power corrections, such as the higher Fock states’ contributions, soft contributions etc.. Finally, we will conclude that there is no any problem with applying the pQCD theory including all power corrections to the exclusive processes at $Q^2 > \text{a few GeV}^2$. A 12 GeV upgrade to CEBAF will offer the possibility to measure pion form factor to good precision out to $Q^2 = 36 GeV^2$. This offers the possibility to study the transition between the dominance of ‘soft’ and ‘hard’ processes in the dynamics, and to learn where the pQCD limit may be reached, and also to check the pQCD results.

Acknowledgements

This work was supported in part by the Natural Science Foundation of China (NSFC).

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