Fractal Analysis on Damage of the Aggregate Gradation Crumb Rubber Concrete Beam with MATLAB

Hongquan Sun*, Jing Wang and Yunxue Yang

Silicon Lake College of Vocational Technology, Suzhou, Jiangsu, China

*Corresponding author

Abstract. The fractal dimensions of the surface crack of the three concrete beams with different gradations coarse aggregate crumb rubber in damaging state are calculated. The research indicates that the damage evolution process of the test beams is a process of the fractal dimension increasing. The research result reveals the relationship between damage variables and fractal dimension of the test beams. The small particle size of coarse aggregate gradation has the best mechanical characteristics with the increasing load. It provides a new method for studying the mechanical properties of crumb rubber concrete beams.

1. Introduction

Elastic rubber concrete is made by the concrete cement mortar mixing rubber powder ground from waste tires[1]. By studying the material mechanical properties of waste tire rubber particle, it is found that the addition of rubber powder can reduce the occurrence of cracks in the mortar's plastic shrinkage, and the cracks also decrease in length and width. Studies have shown that the mechanical properties of rubber concrete are greatly affected by the particle size, shape and surface roughness of rubber powder. Some papers present the fractal analysis of effect of coarse aggregate gradation on damage of rubber concrete[2].

The damage and fracture of materials have a large number of micro-damage nucleation, expansion and connection, which eventually lead to the fracture and failure of materials. In this paper, MATLAB is used to calculate the fractal dimension of the box image. A program is written to calculate the fractal dimension of damage and fracture by the pixel point covering method [3]. We use square boxes with different side lengths $\delta$ to cover the graphics, and count the number $N_{\delta}$ of boxes that cover the graphics’ pixels. The fractal dimension of surface crack development CRC test beam is calculated, and the quantitative relationship between CRC damage and fractal dimension is obtained.

2. The Definition of Box Dimension

In practical engineering, box dimension is one of the most widely used dimensions [4], which is relatively easy to calculate and estimate empirically. Let $F \subset R^n$ be any non-empty bounded subset, and let $N_{\delta}(F)$ be the number of grids intersecting the set $F$ with the coordinate network, the definitions of the lower box dimension $\dim B F$ and the upper box dimension $\overline{\dim} B F$ of the set $F$ are respectively:

$$\dim B F = \lim_{\delta \to 0} \frac{\log N_{\delta}(F)}{\log 1/\delta} \quad (2.1)$$
$$\overline{\dim} B F = \lim_{\delta \to 0} \frac{\log N_{\delta}(F)}{\log 1/\delta} \quad (2.2)$$
When these two values are equal, the common value $F$ is called the Box Dimension, denoted as $dim_{B}F$, where

$$dim_{B}F = \lim_{\delta \to 0} \frac{\log N_{\delta}(F)}{\log \delta} \quad (2.3)$$

In order to calculate the box dimension of the plane set, we can first draw the coordinate square grid on the plane and calculate the number $N_{\delta}(F)$ of intersections of $F$ with the grid square [5] (the square can be regarded as a box, hence the box dimension is named).

### 3. Test Beam Design

In this test, three rubber concrete beams with different coarse aggregate gradations were made [6], all of which were made of fine aggregate with substitution ratio 7.5% of rubber powder. The design strength grade of the crumb rubber concrete (CRC) is C30. The specimen numbers are CRC-small, CRC-mixed and CRC-large representing the rubber concrete with coarse aggregate gradations of 5 mm -16mm, 5 mm -31.5mm and 16 mm -31.5mm respectively. In the case of the same substitution rate, by changing the coarse aggregate gradation and using the fractal method, the damage effect of the rubber concrete beams with different gradations was studied. In order to form a 600mm pure bending section in the middle span of the test beam, the three-point method was adopted for test loading. The dimension and reinforcement of the test beam are shown in figure 1.

![Figure 1. Reinforcement dimension diagram of test beam (unit: mm)](image)

### 4. Programming with MATLAB

Read the pictures taken in the experiment into the computer and program with MATLAB (MATLAB 7 R14) [7]. The program with MATLAB is following.

```matlab
D = imread; % read the image in the Matlab working folder;
A = rgb2gray (D); % convert the original image to a grayscale image;
G = im2bw (A); % Gray image to binary image;
P = size (G); % to see the size of the binary image;
G = imresize; % Resize the matrix;
Function t = js(n,G); % writing statistical functions
X = []; Y = []; % sets two empty matrices to hold statistical data;
For m = 1: n; %traverse the binary image matrix
B = blockproc (G, (2^m 2^m], @ (bs) any (bs) data (:,:) = 0)); % counts the number of sub blocks that contain 0,
X = [x 2^m]; % traverse the size of the box and put it into x
Y = [y sum (B (:))]; % count the number of boxes and put it into y
The end
X = x/(2 ^ n); % unitize the original binary image matrix, and the box size is adjusted accordingly
X = - log (x); % take the logarithm of the inverse of the box size
Y = log (y); % take the logarithm of the number of boxes
T = (x, y), % outputs the statistical and processed value
Js1 = js (n, G); % the box width and number of boxes per count.
```
5. Fractal Analysis of Damage of Rubber Concrete Beams

The fractal dimension of cracks of each test beam from loading to failure is calculated by using the fractal dimension calculation method provided in this paper, and the stiffness change of the test beams under load is listed in the following table 1.

Table 1. Relationship Between Fracture Fractal Dimension of Test Beam under Different Loads and Damage Variables (D-S)

| Specimen Numbers | Loads (KN) | Fractal Dimensions (D) | Initial Stiffness | Post Damage Stiffness | Damage Variable (S) |
|------------------|------------|------------------------|-------------------|-----------------------|---------------------|
| CRC-small        | 35         | 1.3750                 | 40000.0           | 31372.6               | 0.216               |
|                  | 53         | 1.4445                 |                   | 25906.7               | 0.352               |
|                  | 73         | 1.4571                 |                   | 23411.4               | 0.415               |
|                  | 103        | 1.4932                 |                   | 18975.3               | 0.526               |
| CRC-mixed        | 35         | 1.4185                 | 41666.7           | 26890.8               | 0.355               |
|                  | 53         | 1.4441                 |                   | 23041.5               | 0.447               |
|                  | 73         | 1.4501                 |                   | 21084.3               | 0.494               |
|                  | 118        | 1.4602                 |                   | 14180                 | 0.660               |
| CRC-large        | 35         | 1.2908                 | 30769.2           | 27826.1               | 0.096               |
|                  | 53         | 1.3830                 |                   | 23041.5               | 0.251               |
|                  | 73         | 1.3851                 |                   | 20958.1               | 0.319               |
|                  | 108        | 1.4134                 |                   | 12152.8               | 0.605               |

From the table 1, we know that in the process of test of rubber concrete beam loading, the fractal dimension of the surface crack and the damage variable are increased with the increasing of the load. And the damage variable is proportional to the fractal dimension. Therefore, the fractal dimension value can be used to characterize the damage degree of rubber concrete beam.

The fractal dimension values of three test beams with different grades of coarse aggregate rubber-concrete can be calculated by using the fractal dimension calculation method provided in this paper. In the failure state, the fractal dimension value of the crack in the test beam is shown in table 2.

Table 2. Fracture fractal dimension of test beam in failure state

| Specimen Numbers | Fractal Dimensions (D) | Maximum Crack Width (mm) | Correlation Coefficient (R^2) |
|------------------|------------------------|---------------------------|------------------------------|
| CRC-small        | 1.4932                 | 0.4                       | 0.9972                       |
| CRC-mixed        | 1.4602                 | 1.2                       | 0.9976                       |
| CRC-large        | 1.4134                 | 0.6                       | 0.9948                       |

From the table 2, It can also be seen that, in the failure state, the fractal dimension value of CRC-small is the largest and the maximum crack width of CRC-small is the smallest. So CRC-small is the best in the three test beams. It can be explained as follows: as the rubber powder is mixed into the ordinary concrete, the aggregate gap is effectively filled and the mechanical properties of the ordinary concrete are changed. The grading of CRC-small is the smallest, so that the development and distribution of surface cracks is the most uniform. From loading to failure, CRC-small has excellent crack resistance, the maximum damage variable, and sufficient energy consumption.
6. Conclusions
1) The damage variables and fractal dimension values of the test beam show an increasing trend as the load increases, and the qualitative relationship between them can be fitted by MATLAB.
2) The damage evolution process of the rubber concrete beam is actually a process of increasing surface fractal dimension. The specimens at different damage stages have the corresponding fractal dimension value. Fractal dimension value can be used to characterize the damage degree of rubber concrete beam.
3) Due to the addition of rubber powder, the mechanical properties of ordinary concrete are improved. Among the three rubber concrete test beams with different coarse aggregate gradations, the CRC-small has the largest fractal dimension, the most uniform crack development and the least damage variable, so it has high crack resistance and sufficient energy consumption. Among the three test beams, CRC-small is the best one.

References
[1] Hong-quan SUN 2016 Computer Simulation on Surface Crack Growth of Recycled Concrete Beams with Fractals, 2016 6th International Conference on Mechanics, Biotechnology and Environment (ICMMBE 2016), pp 630-635.
[2] Hong-quan SUN, Wen-hui ZHOU and Chao-yi WEI 2014 Experimental study on the mechanical properties of rubber concrete. 2014 2nd International Forum on Mechanical and Material Engineering, IFMME 2014. Zhuhai, China. Advanced Materials Research. Volume: 915-916. pp 685-689.
[3] Hong-quan SUN 2011 Fractal Geometry and Fractal Interpolation [M], science press.
[4] Genady P Cherepanov 1995 Alexander S Balankin, Vera S Iavnova. Fractal Fracture Mechanics-A review [J]. Engng. Fract. Mech, (51) pp 997-1033.
[5] Hua ZHU and Cui-cui JI 2011 Fractal Theory and Its Application [M], science press.
[6] Hong-quan SUN, Jun DING, Jian GUO and Dong-liang FU 2011 Fractal Research on Cracks of Reinforced Concrete Beams with Different Aggregates Sizes [J].Advanced Building Materials (Advanced Materials Research), pp 1818-1822.
[7] Jun DING and Hong-quan SUN 2011 Optimization of Fractal Dimension Calculation Method [J], Sichuan Architectural Science Research. 37 (5), pp 54-56.