Electroweak Fits and Constraints on the Higgs Mass

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Abstract. The current status of the quantities entering into the global electroweak fits is reviewed, highlighting changes since Summer 2003. These data include the precision electroweak properties of the Z and W bosons, the top-quark mass and the value of the electromagnetic coupling constant \( \alpha(M_Z) \), at a scale \( M_Z \). Using these Z and W (high \( Q^2 \)) data, the value of the Higgs mass is extracted, within the context of the Standard Model (SM). The consistency of the data, and the overall agreement with the SM, are discussed.

1. The precision electroweak data

This report contains an update on the values of the precision electroweak properties and fits within the context of the SM, with respect to [1], where more details can be found. The \( e^+e^- \) data are from the ALEPH, DELPHI, L3 and OPAL experiments at LEP, from both the LEP1 and LEP2 phases, and also from the SLD experiment at SLAC. The \( p\bar{p} \) data come from the CDF and D0 experiments from both Run 1 (\( \sqrt{s}=1.8 \text{ TeV} \)) and Run 2 (\( \sqrt{s}=1.96 \text{ TeV} \)).

1.1. Z boson

The coupling of the Z boson to \( f \bar{f} \) is specified by the vector \( (g_{Vf}) \) and axial-vector \( (g_{Af}) \) couplings. These can be expressed in terms of \( \rho \) and the effective weak mixing angle \( \sin^2\theta^f_{\text{eff}} \) by

\[
g_M = \sqrt{\rho} f_3^f, \quad g_{Vf}/g_M = 1 - 4 \left| q_f \right| \sin^2\theta^f_{\text{eff}}
\] (1)
where $q_f$ is the charge, $T_3^f$ is the third component of weak isospin. The $Z$ partial width
\[ \Gamma_f \propto g_{Vf}^2 + g_{Af}^2, \]
and the pole forward-backward asymmetry, which has been measured for $e$, $\mu$ and $\tau$ pair final states, and also for $c$ and $b$ quarks, is
\[ A_{FB}^{0,f} = \frac{3}{4} A_e A_f, \]  
where
\[ A_f = \frac{2 g_{Vf}/g_M}{1 + (g_{Vf}/g_M)^2}. \]

The lepton couplings can be extracted from the $\tau$ polarisation (giving $A_e$, $A_\tau$), the SLAC polarised electron asymmetry $A_{LR}$ ($A_e$) and the forward-backward asymmetries for leptons ($A_\ell$, $\ell$=$e, \mu, \tau$). The results are unchanged with respect to [1] and are reasonably compatible with lepton universality, with $g_{Al}/g_{Ae} = 1.0002 \pm 0.0014$ and $1.0019 \pm 0.0015$, for $l=\mu, \tau$ respectively. The uncertainties are larger for the vector-couplings, with $g_{V\mu}/g_{Ve} = 0.962 \pm 0.063$ and $g_{V\tau}/g_{Ve} = 0.958 \pm 0.029$. Assuming lepton universality, these asymmetries give a value of $A_e = 0.1501 \pm 0.0016$. Within the context of the SM this favours a light Higgs mass. The invisible width of the $Z$ boson allows the number of light neutrinos to be extracted (assuming $\Gamma_\nu/\Gamma_l$ from the SM), and gives $N_\nu = 2.9841 \pm 0.0083$, which is 1.9 $\sigma$ below 3.

In the heavy-quark sector there are updates in the results from SLD. All the LEP and SLD results are now final, but the combination is not yet finalised. The quantities measured are $R_b = \Gamma_b/\Gamma_{had}$, $R_c = \Gamma_c/\Gamma_{had}$, $A_{FB}^{0,b}$, $A_{FB}^{0,c}$, $A_b$ and $A_c$ (which are obtained from the left-right-forward-backward asymmetries). There are additional (since Summer 2003) theoretical uncertainties, arising from the extrapolation of off-peak measurements to the peak, of 0.0002 and 0.0005 added to $A_{FB}^{0,c}$ and $A_{FB}^{0,b}$ respectively (see [2] for more details). There is good internal consistency in the determinations of $R_b$, $R_c$, $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$. The combined LEP and SLD results are given in Table 1. The largest correlation is -0.18, between $R_b$ and $R_c$. The $\chi^2/df$ for the combination is 53/(105-14), giving a probability close to 100%. If statistical errors only are used in the combination then this becomes 92/(105-14), indicating that the systematic errors appear to be overestimated.

The direct determinations of $A_e$ and $A_b$ are shown in figure 1. Also shown is the band in the $A_e$ $A_b$ plane, traced out by $A_{FB}^{0,b}$. The combined value, and the 68% cl, are also shown, as is the SM prediction. It can be seen that the joint result from these data is in poor agreement with the SM. The value of $A_{FB}^{0,b}$ favours a rather heavy Higgs mass.

Figure 2 shows the determinations of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$. The overall $\chi^2$ probability is reasonable (8.4%), but the value obtained from purely leptonic processes ($\sin^2 \theta_{\text{eff}}^{\text{lept}}=0.23113 \pm 0.00021$) is some 2.8$\sigma$ different to that obtained using heavy quarks ($\sin^2 \theta_{\text{eff}}^{\text{lept}}=0.23213 \pm 0.00029$). This comes mostly from the 2.8$\sigma$ difference in the SLD $A_{LR}$ and $A_{FB}^{0,b}$ values.
Table 1. Combination of Z heavy flavour results

| quantity | value  | error  |
|----------|--------|--------|
| $R_b$    | 0.21630| 0.00066|
| $R_c$    | 0.1723 | 0.0031 |
| $A_{FB}^{0,b}$ | 0.0998 | 0.0017 |
| $A_{FB}^{0,c}$ | 0.0706 | 0.0035 |
| $A_b$    | 0.923  | 0.020  |
| $A_c$    | 0.670  | 0.027  |

Figure 1. The couplings $A_b$ and $A_c$, both from direct measurements and from $A_{FB}^{0,b}$.

1.2. W boson

The W boson is produced singly at the Tevatron (eg $u + \bar{d} \rightarrow W^+$). The leptonic decays $W \rightarrow \ell \nu$ (with $\ell = e, \mu$) are used to determine the W mass and width, using the
transverse mass or $p_T^\ell$. From Run 1 the values $M_W = 80.433 \pm 0.079$ GeV (CDF) and $80.483 \pm 0.084$ GeV (D0) were obtained. Taking into account common systematics, the combined Run 1 values are $M_W = 80.452 \pm 0.059$ GeV and $\Gamma_W = 2.102 \pm 0.106$ GeV [3]. Run 2 analyses are currently underway.

At LEP2 the W bosons are pair-produced in $e^+e^- \rightarrow W^+W^-$. The analyses are still in progress. The statistical uncertainties from the $\ell\nu q\bar{q}'$ and $q\bar{q}' q\bar{q}'$ channels are similar. However, there is at present a large systematic uncertainty (97 MeV) in the $q\bar{q}' q\bar{q}'$ channel, due to final-state interaction effects. This is mostly from colour reconnection, with a smaller contribution from Bose Einstein correlations. This means that the $q\bar{q}' q\bar{q}'$ channel carries only 10% of the weight in the LEP2 average. The preliminary LEP2 values are $M_W = 80.412 \pm 0.042$ GeV and $\Gamma_W = 2.152 \pm 0.091$ GeV.

The combined Tevatron and LEP2 values are $M_W = 80.425 \pm 0.034$ GeV and $\Gamma_W = 2.133 \pm 0.069$ GeV. $\Gamma_W$ is compatible with the SM value of $2.097 \pm 0.003$ GeV. The world average $M_W$ value favours a low Higgs mass in the context of the SM.
2. The SM parameters

The SM parameters are taken to be $M_Z$, $G_F$, $\alpha(M_Z)$ and $\alpha_s(M_Z)$ (the electromagnetic and strong coupling constants at the scale $M_Z$), and the top-quark mass $m_t$. Through loop diagrams measurements of the precision electroweak quantities are sensitive to $m_t$ and, the ‘unknown’ in the SM, $m_{H}$. The SM computations use the programs TOPAZ0 and ZFITTER. The latter program (version 6.40) incorporates the recent fermion 2-loop corrections to $\sin^2\theta_{\text{eff}}$ and full 2-loop, and leading 3-loop, corrections to $M_W$ [4].

2.1. top-quark mass

The D0 Collaboration have recently improved their Run 1 measurement using a weighting method based on the matrix element, giving $m_t = 179.0 \pm 3.5 \text{ (stat)} \pm 3.8 \text{ (syst)}$ GeV. The CDF Run 1 value is $m_t = 176.1 \pm 4.2 \text{ (stat)} \pm 5.1 \text{ (syst)}$ GeV. Taking into account common systematic uncertainties the combined value is [5] $m_t = 178.0 \pm 4.3$ GeV, with statistical and systematic error components of 2.7 and 3.3 GeV respectively. This is to be compared to the previous value of $m_t = 174.3 \pm 5.1$ GeV.

Run 2 values have been obtained by both the CDF and D0 Collaborations, but these have not yet been included in the average.

2.2. $\alpha(M_Z)$

The value of $\alpha$ at the scale $M_Z$ requires the use of data on $e^+e^- \rightarrow \text{hadrons}$ at low energies and the use of perturbative QCD at higher energies. The various estimations of $\alpha(M_Z)$ differ in the extent to which QCD is used, as well as in the data used in the evaluation. The quantity needed is the hadronic contribution $\Delta\alpha^{(5)}_{\text{had}}$ and the value used by the LEP EWWG [1] is $\Delta\alpha^{(5)}_{\text{had}}(M_Z) = 0.02761 \pm 0.00036$. Recent data from the CMD-2 and KLOE Collaborations has been consider in [6], and the authors conclude that the value just quoted is still valid.

3. Electroweak fits

The measurements used in the global SM electroweak fits, and the fitted values, are shown in figure 3. The SM fit to these high $Q^2$ data gives

$$m_t = 178.2 \pm 3.9 \text{ GeV}$$
$$m_H = 114^{+69}_{-45} \text{ GeV}$$
$$\alpha_s(M_Z) = 0.1186 \pm 0.0027.$$

The $\chi^2$/df is 15.8/13, giving a probability of 26%. The variation of the fit $\chi^2$, compared to the minimum value, is shown in the ‘blue-band’ plot of figure 4, as a
function of \( m_H \). Also shown is the direct search limit of 114 GeV. The one-sided 95% upper limit is \( m_H \leq 260 \text{ GeV} \). This includes the theoretical uncertainty (blue-band) which is evaluated by considering the uncertainties in the new 2-loop calculations \([4]\). If the more theory driven value \( \Delta \alpha^{(5)}_{\text{had}}(M_Z) = 0.02749 \pm 0.00012 \) is used, then \( m_H \) increases to 129 GeV.

Since 2003 the main changes have been the change in \( m_t \) (\( \delta m_H \simeq +20 \text{ GeV} \)) and the new 2-loop effects (\( \delta m_H \simeq +6 \text{ GeV} \)).

The direct versus indirect values of \( m_t \) and \( M_W \) is a powerful test of the SM; see figure 5. The contours shown are for the 68\% cl. It can be seen that there is a reasonable degree of overlap and that the data prefer a light Higgs mass.

The above fits use only high \( Q^2 \) data. There are also low \( Q^2 \) data\([7]\) from Atomic Parity Violation in \(^{133}\text{Cs} \) (\( Q_W = -72.74 \pm 0.46 \)), the SLAC polarised electron Moller scattering experiment E158 (\( \sin^2 \theta_{\text{eff}}^{\text{ept}} = 0.2333 \pm 0.0016 \)) and the deep-inelastic \( \nu(\bar{\nu}) \) experiment NuTeV (\( \sin^2 \theta_W = 0.2277 \pm 0.0016 \)). The NuTeV value can be used to
extract $M_W$, and gives a value $3.1\sigma$ below that from direct measurement. Including all these low $Q^2$ data in the SM fit increases $m_H$ by 14 GeV to 128 GeV, and the $\chi^2$ probability drops to 5.4%, essentially due to the NuTeV result.

4. Conclusions

There has been steady progress on both the experimental and theoretical fronts. There are still issues with $A_{FB}^{0,b}$ and NuTeV (both $\simeq 3\sigma$ effects). It is difficult to see how $A_{FB}^{0,b}$ can be resolved in the near future, but for NuTeV, the further evaluation of QED and QCD effects, together with the NOMAD results, should help.

The SM fits favour a light Higgs mass, $m_H = 114^{+69}_{-45}$ GeV, and a 95% cl upper limit of 260 GeV. Thus the Higgs boson appears to be relatively light. Improved measurements of both $m_t$ and $M_W$ at the Tevatron, and then the LHC, will significantly improve the precision of the indirect estimation of $m_H$. 

Figure 4. Variation of $\chi^2$ versus $m_H$. 

\begin{figure}
\centering
\includegraphics[width=0.7\textwidth]{chi2_map.png}
\caption{Variation of $\chi^2$ versus $m_H$.}
\end{figure}
Figure 5. Direct versus indirect \( m_t \) and \( M_W \) measurements.

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References

[1] The LEP Collaborations and EWWG; CERN-EP/2003-091 and hep-ex/0312023 (2003).
[2] R. Hawkings, ICHEP 2004 proceedings.
[3] Combination of CDF and D0 results on W boson mass and width, hep-ex/0311039 (2003).
[4] M. Awramik et al., hep-ph/0311148, 0407317; M. Faisst et al. Nucl. Phys B665 (2003) 649.
[5] Combination of CDF and D0 results on the top-quark mass, hep-ex/0404010 (2004).
[6] B. Pietrzyk and H. Burkhardt, LAPP-EXP 2004-4, (2004).
[7] P. Langacker, ICHEP 2004 proceedings