Nucleon shape and electromagnetic form factors in the chiral constituent quark model

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Abstract. The electromagnetic form factors are the most fundamental quantities to describe the internal structure of the nucleon and the shape of a spatially extended particle is determined by its intrinsic quadrupole moment which can be related to the charge radii. We have calculated the electromagnetic form factors, nucleon charge radii and the intrinsic quadrupole moment of the nucleon in the framework of chiral constituent quark model. The results obtained are comparable to the latest experimental studies and also show improvement over some theoretical interpretations.

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ELECTROMAGNETIC FORM FACTORS

The electromagnetic form factors are fundamental quantities of theoretical and experimental interest to investigate the internal structure of nucleon. An understanding of the nucleon form factors is necessary to describe the strong interactions as they are sensitive to the pion cloud and provide a test for the QCD inspired effective field theories based on the chiral symmetry. The knowledge of internal structure of nucleon in terms of quarks and gluons degrees of freedom of QCD also provide a basis for understanding more complex, strongly interacting matter. Further, the measurements from deep inelastic scattering (DIS) experiments have revealed a significant amount of strangeness quark content in the nucleon [1] which has been further reinforced by the experiments performed in the recent past. In particular, the SAMPLE Collaboration has reported the proton strange form factor as $G_M^p = +0.37 \pm 0.20 \pm 0.26 \pm 0.07$ n.m. [2], whereas the HAPPEX Collaboration has reported $G_M^p = +0.18 \pm 0.27$ n.m. [3]. Thus, the importance of electromagnetic form factors increases further to determine the strange form factors contribution in the nucleon.

The internal structure of nucleon is determined in terms of electromagnetic Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ or equivalently in terms of the electric and magnetic Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$ [4]. Experimentally, these are determined in elastic $ep$ scattering cross section via the Rosenbluth separation [5] as well as using polarization transfer [6]. The issue of determination of the form factors has been revisited in the recent past with several new experiments measuring the form factors with precision at MAMI [7] and JLAB [8]. It has been shown that the proton form factors determined from the measurements of polarization transfer [8] were in significant disagreement with those obtained from the Rosenbluth separation [9]. This inconsistency leads to a large uncertainty in our knowledge of the proton electromagnetic form factors and urge the necessity for the new parameterizations and a new analysis [10]. The measurement of neutron form factors is even more difficult than that for the proton since the free neutron target does not exist. They are usually extracted from the measurement of electron-deuteron scattering or electron-helium scattering.

The most general form of the hadronic current for a spin $\frac{1}{2}$-nucleon with internal structure is given as

$$
\langle B | J_{\text{had}}^\mu(0) | B' \rangle = \bar{u}(p') \left( \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu}}{2M} q_\nu F_2(Q^2) \right) u(p),
$$

where $u(p)$ and $u(p')$ are the 4-spinors of the nucleon in the initial and final states respectively. The Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ are the only two form factors allowed by relativistic invariance. These form factors are normalized in such a way that at $Q^2 = 0$, they reduce to electric charge and the anomalous magnetic moment in units of the elementary charge and the nuclear magneton $\mu_N$, for example,

$$
F_1^p(0) = 1, \quad F_2^p(0) = \kappa_p = 1.793, \quad F_1^n(0) = 0, \quad F_2^n(0) = \kappa_n = -1.913.
$$

In analogy with the non-relativistic physics, we can associate the form factors with the Fourier transforms of the charge and magnetization densities. However, the charge distribution $\rho(r)$ has to be calculated by a 3-dimensional
Fourier transform of the form factor as function of $q$, whereas the form factors are generally functions of $Q^2 = q^2 - \omega^2$. It would be important to mention here that there exists a special Lorentz frame, the Breit or brick-wall frame, in which the energy of the (space-like) virtual photon vanishes. This can be realized by choosing $p_1 = -\frac{1}{2}q$ and $p_2 = +\frac{1}{2}q$ leading to $E_1 = E_2$, $\omega = 0$ and $Q^2 = q^2$. Thus, in the Breit frame, Eq. (1) takes the following form [4]

$$J_\mu = \left( G_E(Q^2), 1, \frac{\sigma \times q}{2M} G_M(Q^2) \right), \tag{3}$$

where $G_E(Q^2)$ stands for the time-like component of $J_\mu$ hence identified with the Fourier transform of the electric charge distribution, whereas $G_M(Q^2)$ is interpreted as the Fourier transform of the magnetization density. The Sachs form factors $G_E$ and $G_M$ can be related to the Dirac and Pauli form factors as

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \tag{4}$$

where $\tau = \frac{q^2}{4M^2}$ is a measure of relativistic effects.

The Fourier transform of the Sachs form factors can be expressed as

$$G_E(q^2) = \int \rho(r) e^{i q \cdot r} d^3 r = \int \rho(r) r^2 \cdot r - \frac{q^2}{6} \int \rho(r) r^2 d^3 r + \ldots, \tag{5}$$

where the first integral yields the total charge in units of $\epsilon$, i.e., 1 for the proton and 0 for the neutron, and the second integral defines the square of the electric root-mean-square (rms) radius, $\langle r^2 \rangle_E$. The density $\rho(r)$ is not an observable but only a mathematical construct in analogy with the classical charge distribution.

**CHARGE RADII AND INTRINSIC QUADRUPOLE MOMENT OF THE NUCLEON**

Elastic $ep$ scattering experiments apart from showing that the proton has a finite size [11], have also provided detailed information on the radial variation of the charge and magnetization densities. Charge radii contain fundamental information about the internal structure of the baryons. The shape of a spatially extended particle is determined by its **intrinsic** quadrupole moment [12], corresponding to the charge quadrupole form factor $G_{Q2}(q^2)$ at zero momentum transfer. For the spin $\frac{3}{2}$ baryons, the information on their intrinsic quadrupole moments can be obtained from the measurements of electric ($E2$) and Coulomb ($C2$) quadrupole transitions to excited states [13, 14]. The **intrinsic** quadrupole moment of a nucleus with respect to the body frame of axis is defined as

$$Q_0 = \int d^3 r \rho(r)(3z^2 - r^2). \tag{6}$$

If the charge density is concentrated along the $z$-direction (symmetry axis of the particle), the term proportional to $3z^2$ dominates, $Q_0$ is positive, and the particle is prolate shaped. If the charge density is concentrated in the equatorial plane perpendicular to $z$ axis, the term proportional to $r^2$ prevails, $Q_0$ is negative and the particle is oblate shaped.

In order to obtain information on these observables, we use a general parametrization (GP) method developed by the Morpurgo et al. [15]. It has been shown that it is possible to parameterize several hadronic properties using the general features of quantum chromodynamics (QCD) using the GP method. The most general form of the charge radius operator for the sum of one-, two-, and three-quark terms can be expressed as

$$r^2 = A \sum_{i=1}^{3} e_i 1 + B \sum_{i \neq j} e_i \sigma_i \cdot \sigma_j + C \sum_{i \neq j \neq k} e_k \sigma_i \cdot \sigma_j, \tag{7}$$

where $e_i$ and $\sigma_i$ are the charge and spin of the $i$-th quark. The constants $A$, $B$, and $C$ can be determined from the experimental observations on charge radius and quadrupole moments and the baryon charge radii for the octet and decuplet baryons can then be calculated by evaluating matrix elements of the operator in Eq. (7) between three-quark spin-flavor wave functions $|B\rangle$ as $\langle B|r^2|B\rangle$.

Similarly, the charge quadrupole operator composed of a two- and three-body term in spin-flavor space as

$$Q = B' \sum_{i \neq j} e_i (3\sigma_i \sigma_j - \sigma_i \cdot \sigma_j) + C' \sum_{i \neq j \neq k} e_k (3\sigma_i \sigma_j \sigma_k - \sigma_i \cdot \sigma_j \cdot \sigma_k). \tag{8}$$
Baryon decuplet quadrupole moments $Q_{B^+}$ and octet-decidual transition quadrupole moments $Q_{B^+ - B^+}$ are obtained by calculating the matrix elements of the quadrupole operator in Eq. (8) between the three-quark spin-flavor wave functions $|B\rangle$ as

$$Q_{B^+} = \langle B^+ | Q | B^+ \rangle, \quad Q_{B^+ - B^+} = \langle B^+ | Q | B^+ \rangle,$$

(9)

where $B$ denotes a spin $\frac{1}{2}$ octet baryon and $B^+$ a spin $\frac{3}{2}$ decuplet baryon.

Calculations have been carried out for the charge radii and intrinsic quadrupole moments in the quark model and pion cloud model [12], which lead to the several interesting observations. It is found that the intrinsic quadrupole moment of the proton is given by the negative of the neutron charge radius and therefore positive, whereas the intrinsic quadrupole moment of the $\Delta^+$ is negative. This corresponds to a prolate proton and an oblate $\Delta^+$ deformation.

**CHIRAL CONSTITUENT QUARK MODEL**

One of the most successful models in the non-perturbative regime of QCD is the chiral constituent quark model with configuration mixing ($\chi$CQM$_{config}$) [16]. The basic process in the $\chi$CQM is the emission of a GB by a constituent quark which further splits into a $q\bar{q}$ pair as $q_\pm \to GB^0 + q'_{\mp}$, whereas $q\bar{q} + q'$ constitute the “quark sea” [17, 18, 19]. The effective Lagrangian describing interaction between quarks and a nonet of GBs is

$$\mathcal{L} = g_{8}q\Phi q,$$

(10)

where $g_{8}$ and $\zeta$ are the coupling constants for the singlet and octet GBs. SU(3) symmetry breaking is introduced by considering $M_{\pi} > M_{\eta, d}$ as well as by considering the masses of GBs to be nondegenerate ($M_{K, \eta} > M_\pi$ and $M_{\eta'} > M_{K, \eta}$) [16, 17, 18, 19]. The parameter $a = |g_{8}|^2$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \to d(u) + \pi^+(\pm)$, whereas $\alpha^2a$, $\beta^2a$ and $\xi^2a$ respectively denote the probabilities of transitions of $u(d) \to s + K^{-}l$, $u(d, s) \to u(d, s) + \eta$, and $u(d, s) \to u(d, s) + \eta$.

The spin structure of a baryon can be defined as $\hat{B} \equiv \langle B | \mathcal{N} | B \rangle$, where $|B\rangle$ is the baryon wave function and $\mathcal{N}$ is the number operator $\mathcal{N} = n_u u_+ + n_u u_- + n_d d_+ + n_d d_+ + n_s s_+ + n_s s_-$, the coefficients of the $q_\pm$ giving the number of $q_\pm$ quarks.

The wave function for the octet baryons with spin-spin generated configuration mixing can be expressed as

$$|B\rangle \equiv \left|8, \begin{pmatrix} \frac{1}{2} \end{pmatrix}\right\rangle = \cos \phi \frac{1}{\sqrt{2}}(\chi' \varphi + \chi' \varphi') \psi'^{(0+)} + \sin \phi \frac{1}{2}((\varphi \chi'' + \varphi \chi') \psi'(0+) + (\varphi \chi' - \varphi \chi'') \psi''(0+)).$$

(11)

For the decuplet baryons, we have

$$|B^+\rangle \equiv |10, \begin{pmatrix} 3 \end{pmatrix}\rangle = \chi' \varphi \psi^{(0+)} ,$$

(12)

where $\chi$, $\varphi$ and $\psi$ are the spin, isospin and spatial wavefunctions [20].

**RESULTS AND DISCUSSION**

In the non-relativistic limit the electric and magnetic form factors of the baryons can be expressed as

$$G_E(Q^2) = \langle B | \sum_{j=1}^{3} e_j e^{-iQ_{\rho_j}} | B \rangle, \quad G_M(Q^2) = \langle B | \sum_{j=1}^{3} \mu_j \sigma_j e^{-iQ_{\rho_j}} | B \rangle.$$

(13)

For the case of proton and neutron, we have

$$G_{E}''(Q^2) = \cos^2 \phi \langle \psi'' | e^{-iQ_{\rho_3}} | \psi'' \rangle + \frac{\sin^2 \phi}{2} \left( \langle \psi' | e^{-iQ_{\rho_3}} | \psi' \rangle + \langle \psi | e^{-iQ_{\rho_3}} | \psi \rangle \right)^2 + \sqrt{2} \sin \phi \cos \phi \langle \psi'' | e^{-iQ_{\rho_3}} | \psi'' \rangle^2 ,$$

(14)
Using the GP method, the charge radii for the spin $\frac{1}{2}$ ground state baryons is expressed as

$$r^2 = (A - 3B) \sum e_i + 3(B - C) \sum e_i \sigma_i,$$

where $\sum e_i = 1$, $\sum e_i \sigma_i = 1$ for proton and $\sum e_i = 0$, $\sum e_i \sigma_i = -\frac{2}{3}$ for neutron.

In $\chi$CQM, the proton and neutron charge radii can be expressed as

$$r_p^2 = (A - 3B)(1 - a - 2a\alpha^2) + 3(B - C) \left( \cos^2\phi \left(1 - \frac{a}{3}(4 + 2\alpha^2 + \beta^2 + 2\xi^2)\right) + \sin^2\phi \left(\frac{1}{3} - \frac{a}{9}(6 + 3\beta^2 + 2\xi^2)\right) \right),$$

$$r_n^2 = (A - 3B)(1 - a\alpha^2) + 3(B - C) \left( \cos^2\phi \left(-\frac{2}{3} + \frac{a}{9}(3 + 9\alpha^2 + 2\beta^2 + 4\xi^2)\right) - \sin^2\phi \frac{a}{3}(1 - \alpha^2) \right).$$

We find that the $N \to \Delta$ quadrupole moment is related to the neutron charge radius as $Q_{p \to \Delta^+} = Q_{n \to \Delta^0} = \frac{1}{\sqrt{2}} r_n^2$, which is experimentally well satisfied and also show improvement over some theoretical interpretations. Thus, the neutron charge radius plays an important role in the hadron dynamics. It sets the scale not only for the charge radius splitting within and between flavor multiplets but also for the size of quadrupole moments and the corresponding intrinsic baryon deformation. The results can further be substantiated by a measurement of the baryon charge radius and other transition quadrupole moments.

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**REFERENCES**

1. J. Ashman et al. (European Muon Collaboration), Phys. Lett. B 206, 364 (1988); Nucl. Phys. 328, 1 (1989).
2. D.T. Spayde et al. (SAMPLE Collaboration), Phys. Lett. B, 583 (2004).
3. A. Acha et al. (HAPPEX Collaboration), Phys. Rev. Lett. 98, 032301 (2007).
4. R.G. Sachs, Phys. Rev. 126, 2256 (1962).
5. M.N. Rosenbluth Phys. Rev. 79, 615 (1950).
6. R.G. Arnold, C.E. Carlson and F. Gross, Phys. Rev. 23, 363 (1981).
7. D.I. Glazier et al., Eur. Phys. J. A 24, 101 (2005); D. Rohe, Eur. Phys. J. A 28, S1 29 (2006).
8. M.K. Jones et al. (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. 84, 1398 (2000); O. Gayou et al., Phys. Rev. C 64, 038202 (2001); O. Gayou et al., Phys. Rev. Lett. 88, 092301 (2002); V. Punjabi et al., Phys. Rev. C 71, 055202 (2005); G. Ron et al., Phys. Rev. 98, 052301 (2007).
9. I.A. Qattan et al., Phys. Rev. Lett. 94, 142301 (2005).
10. W.M. Alberico, S.M. Bilenky, C. Giunti and K.M. Graczyk, Phys. Rev. C 79, 065204 (2009).
11. R. Rosenfelder, Phys. Lett. B 479, 381 (2000).
12. A.J. Buchmann and E.M. Henley, Phys. Rev. C 63, 015202 (2001); A.J. Buchmann, Phys. Rev. Lett. 93, 212301 (2004); A.J. Buchmann, Proceedings of the Shape of Hadrons Workshop, Athens, Greece, edited by C.N. Papanicolas and A.M. Bernstein, AIP Conference Proceedings 904, 110 (2007); arXiv:0712.4270.
13. L. Tiator et al., Eur. Phys. J. A 17, 357 (2003); Blanpied et al., Phys. Rev. C 64, 025203 (2001).
14. A.M. Bernstein, Eur. Phys. J. A 17, 349 (2003); C.N. Papanicolas, Eur. Phys. J. A 18, 141 (2003).
15. G. Morpurgo, Phys. Rev. D 40, 2997 (1989).
16. H. Dahiya and M. Gupta, Phys. Rev. D 64, 014013 (2001); H. Dahiya and M. Gupta, Phys. Rev. D 66, 051501(R) (2002); Phys. Rev. D 67, 114015 (2003); H. Dahiya and M. Gupta, Phys. Rev. D 67, 074001 (2003); N. Sharma, H. Dahiya, P.K. Chatley and M. Gupta, Phys. Rev. D 79, 077503 (2009).
17. T.P. Cheng and Ling Fong Li, Phys. Rev. Lett. 74, 2872 (1995); T.P. Cheng and Ling Fong Li, Phys. Rev. D 57, 344 (1998).
18. X. Song, J.S. McCarthy and H.J. Weber, Phys. Rev. D 55, 2624 (1997); X. Song, Phys. Rev. D 57, 4114 (1998).
19. J. Linde, T. Ohlsson and H. Snellman, Phys. Rev. D 57, 452 (1998); J. Linde, T. Ohlsson and H. Snellman, 5916 (1998).
20. A. Le Yaouanc et al., Hadron Transitions in the Quark Model, Gordon and Breach, 1988.