Entropic Law of Force, Emergent Gravity and the Uncertainty Principle

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Abstract

The entropic formulation of the inertia and the gravity relies on quantum, geometrical and informational arguments. The fact that the results are completely classical is misleading. In this paper we argue that the entropic formulation provides new insights into the quantum nature of the inertia and the gravity. We use the entropic postulate to determine the quantum uncertainty in the law of inertia and in the law of gravity in the Newtonian Mechanics, the Special Relativity and in the General Relativity. These results are obtained by considering the most general quantum property of the matter represented by the Uncertainty Principle and by postulating an expression for the uncertainty of the entropy such that: i) it is the simplest quantum generalization of the postulate of the variation of the entropy and ii) it reduces to the variation of the entropy in the absence of the uncertainty.

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1 Introduction

In a recent paper, Verlinde has put forward a very interesting proposal for the origin of the inertia, the law of force, the law of gravity and the General Relativity as emergent phenomena \[1\]. According to this theory, the spacetime can be described as an information device made of holographic surfaces (screens) on which the information about the physical systems can be stored. The relevant information about the physical dynamics can be recovered by analysing the variation of the information on the screens and it is independent of the details of the particular theory used to describe the physical system. The screens behave as stretched horizons in the black hole physics and define emergent holographic directions in which the spacetime grow. The information on the screens is described by the information entropy and, as the black hole entropy, it is encoded in a number of bits proportional to the area of the screen \[3\]. The total energy of the degrees of freedom satisfy the equipartition theorem. Also, it is postulated that the information entropy is maximized by the entropic forces that act along the holographic directions and are defined by gradients of the entropy. If one associates non-inertial frames to these forces, the corresponding accelerated observers measure a redshifted information at the Unruh temperature \[4\]

\[
T_U = \frac{\hbar a}{2\pi k_B c},
\]

where \(k_B\) is the Boltzmann’s constant, \(c\) is the speed of light and \(a\) is the acceleration. For a particle of mass \(m\), it is postulated in \[1\] that the variation of the entropy along the holographic direction be linear in the separation \(\Delta x\) between the particle and the screen

\[
\Delta S = 2\pi k_B \frac{\Delta x}{l_c},
\]

where \(l_c = \hbar/mc\) is the Compton length. Thus, the Compton length defines the units of the entropy change. The variation of the entropy \(\Delta S\) corresponds to an arbitrary variation of the energy \(\Delta W\) at the thermodynamical equilibrium (with \(T = T_U\)) which, at its turn, can be interpreted as the result of a macroscopic force \(F\) acting on the particle over the distance \(\Delta x\)

\[
\Delta W = F \Delta x = T \Delta S.
\]

The above assumptions lead to the law of force for \(m\) and when applied to a spherical screen around a mass \(M\) they deliver the Newton’s law of gravity. Also, when properly generalized, the postulates allow one to obtain the Einstein’s equations \[1\].

Several features and consequences of the entropic postulate have already been explored. The compatibility between the entropic gravity and the loop quantum gravity was proved in \[9\]. Applications to cosmology, Friedmann’s equations and D-brane cosmology were developed in \[10, 11, 12, 13, 14\]. The connection between the entropic gravity and the black holes was explored in \[15, 16, 17, 18, 19\]. A modification of the entropic force to include the deviation from the equipartition theory was proposed in \[20\]. The derivation of a dark energy term from the entropic gravity was given in \[21\]. Further relations with the thermodynamics were discussed in \[22, 23, 24\]. A speculation about the possible interpretation of the Coulomb force as an entropic force was done in \[25\]. The realisation of the information entropy in terms of

\[1\]The Einstein’s equations have been obtained before from the thermodynamical considerations in \[2\]. A very interesting proposal for the gravitational entropy and its connection with the equipartition of the microscopic horizon degrees of freedom in the context of the emergent gravity was made in \[3, 4\].
light was investigated in [26]. Other geometrical, field theoretical and informational aspects of
the theory are presented in [27, 28, 29, 30, 31].

In the above construction, the holographic principle [7, 8] and the information entropy
[3] play fundamental roles in establishing the entropic nature of the inertia and of the law of
force. On the other hand, since the Planck constant cancels out in the law of force and the
gravitational force, it seems that the quantum effects are irrelevant to the inertia and gravity
despite the quantum nature of the key concepts from the relations (1) and (2). Thus, one may
ask the important question whether there are quantum corrections to the laws of Mechanics
derivable from the entropic postulate. The aim of this paper is to show that this question has a positive answer if one takes into account the quantum structure of matter in the most
general form of the Uncertainty Principle.

The paper is organized as follows. In Section 2 we briefly review the Verlinde’s postulate.
Then we use it to derive the uncertainty in the entropy of the holographic screen if the particle
is subjected to the Uncertainty Principle. We also argue that the uncertainty in the entropy
should imply an uncertainty in the adiabatic force which can be interpreted as a quantum
correction to it. In Section 3 we discuss the same situation in the Special Relativity as well as
in the General Relativity. The last section is devoted to some discussions.

2 Non-Relativistic Entropic Force and the Uncertainty Principle

Consider a system composed by a holographic screen $S$ and a quantum test particle of mass
$m$. Due to the quantum nature of the particle, the separation between $S$ and $m$ is determined
only up to the uncertainty $\delta x$ that satisfies the Heisenberg’s relation

$$\delta x \delta p \geq \frac{\hbar}{2},$$

where the uncertainty in the momentum along the transverse direction to $S$ is $\delta p$. If the
particle were classical and $\delta x$ were zero, the variation of the entropy on $S$ would be given
by (2) for any separation $\Delta x$. Since the test particle is quantum and there is an uncertainty
of its position and momentum, an uncertainty of the entropy on the screen is expected. The
uncertainty of $S$ can be related to the uncertainty in the total energy by the first law of the
thermodynamics

$$\delta W = T \delta S.$$

The above equality holds at thermodynamical equilibrium at the temperature $T$. The reason
for the uncertainty of the energy is that both the kinetic energy $K$ and the potential energy
$V$ (which is the relative energy of the particle with respect to the screen) are uncertain due
to the uncertainty of the position and momentum of the test particle. Thus, if the quantum
particle is non-relativistic, the relation (5) takes the following form

$$\frac{p}{m} \delta p + F \delta x = T \delta S. \tag{6}$$

We note that it is possible to add an extra term to the l. h. s. of the relation (6) of the form
$\delta Fx$. Such a term can be produced by the variation of $\delta p$ in time. If that is fixed then $\delta F = 0$.

The relation (6) can be used to evaluate the uncertainty of the entropy. Indeed, by in-
specting (6), one can see that the main difference between the entropy function of a classical
particle given in [1] and the quantum particle is the dependence of the entropy on $Fx$ and $K$
for the latter, i. e. \( S(U + K + Fx) \) for the quantum system. Moreover, the relation (6) suggests that \( \delta S \) depends not only on the undeterminacy in the momentum but also on its value. By taking into account all these considerations and the relation (2) for the classical particle, we are proposing the following relation for the uncertainty of the entropy

\[
\delta S = 2\pi k_B \left( \frac{\delta x}{l_c} + \frac{p\delta p}{mc^2} \right).
\]  
(7)

The denominators in the above relation result from dimensional considerations. In the classical case, the uncertainty of the momentum of particle is zero and the Heisenberg’s relation does not apply. Then by interpreting \( \delta x \) as the separation \( \Delta x \), the relation (7) reduces to the Verlinde’s ansatz (2). By using (1), (7) and the estimate \( \delta p = \hbar (2\delta x)^{-1} \) from (4) in the equation (6) one obtains the following relation

\[
F(\delta) = ma + \frac{\hbar p}{2m} \left( \frac{ha}{mc^2} - 1 \right) \delta x^{-2}.
\]  
(8)

The above equation represents the entropic force acting on the quantum particle subjected to the uncertainty principle. The classical limit is obtained by taking simultaneously \( \delta p = 0 \) and \( \hbar \to 0 \) in the above estimate, which renders the r. h. s. of the relation (6) zero. Thus, the above equation represents a generalization of the law of force to the Quantum Mechanics. If one defines the uncertainty of the force as

\[
\delta F = F(\delta) - F,
\]  
(9)

the following uncertainty relation holds

\[
\delta F \delta x^2 \geq \frac{\hbar p}{m} \left( \frac{ha}{mc^2} - 1 \right).
\]  
(10)

Note that in the relation (7) the uncertainty of \( S \) depends not only on \( \delta x \) and \( \delta p \) but also on the momentum \( p \). There is no other argument for this assumption than the form of the l. h. s. of the relation (6). In principle, one could relax this condition and assume instead that \( \delta S \) depends only on \( \delta x \) and \( \delta p \). In this case, the following relation is suitable for the definition of the uncertainty of the entropy

\[
\delta S = 2\pi k_B \left( \frac{\delta x}{l_c} + \frac{\delta p}{mc} \right).
\]  
(11)

By performing the same steps as above, one obtains the following generalization of the law of force

\[
F(\delta) = ma + \frac{\hbar}{2m} \left( \frac{ha}{c^2} - p \right) \delta x^{-2},
\]  
(12)

with the corresponding uncertainty relation

\[
\delta F \delta x^2 \geq \frac{\hbar}{2m} \left( \frac{ha}{c^2} - p \right).
\]  
(13)

The above analysis shows that the postulate of the entropic force can be used to obtain information about quantum corrections to the law of inertia and the law of force, thus establishing a connection between the classical and the quantum concepts. By particularizing these ideas, one can obtain quantum corrections to the law of gravity. To this end, we consider a
spherical screen that encloses a mass $M$. Following [1], we assume that the information on the screen is encoded in a number $N$ of bits proportional to the area of the sphere

$$N = \frac{A c^3}{G \hbar},$$

(14)

where $G$ is the Newton’s constant. This information can be realized in terms of a classical system with $N$ degrees of freedom and the energy cost $E$. Since in the absence of any other matter the information is exclusively about the mass $M$, the energy cost equals $Mc^2$ in the reference frame in which $M$ is at rest. Also, the $N$ classical degrees of freedom being identical, one can assume that the equipartition law holds at the thermodynamical equilibrium

$$E = \frac{1}{2}Nk_B T.$$  

(15)

Next, one considers a quantum particle $m$ outside the sphere. From the Heisenberg’s principle, the uncertainty in its position and momentum leads to an uncertainty in the information on the sphere as argued in the general case. Then, by applying the results from [8], one can see that

$$F_g(\delta) = G \frac{M m}{R^2} + \frac{\hbar p}{2m} \left( G \frac{\hbar M}{R^2 c^2} - p \right) \delta x^{-2}. $$

(16)

The above equation represents the quantum gravitational force derived from the entropic force postulate and the Heisenberg’s principle. In the classical limit, it reproduces the Newton’s law of gravity. If the uncertainty in the entropy does not depend on the momentum of $m$, the relation (12) should be used instead of (8) and the corresponding quantum gravitational force takes the following form

$$F_g(\delta) = G \frac{M m}{R^2} + \frac{\hbar}{2m} \left( G \frac{\hbar M}{R^2 c^2} - p \right) \delta x^{-2}. $$

(17)

Similar force-position uncertainty relations to the ones given in (10) and (13) can be derived for the gravitational force, too. It is important to note that even if the separation between $S$ and $m$ is along the normal direction to $S$, the delocalization of the particle is along $x$, $y$ and $z$. Furthermore, there is an uncertainty of the energy within the bounds of the inverse of the observation time. The generalization of the above relations to these cases is straightforward in the non-relativistic theory.

3 Relativistic Entropic Force and the Uncertainty Principle

We can try to infer the uncertainty of the entropy in the relativistic case from the same heuristics. The basic relations of the relativistic dynamics and the equation (5) lead to the following relationship among the uncertainties in $x$, $p$ and $S_r$

$$\frac{p \delta p}{m \gamma^2} + F \delta x = T \delta S_r,$$

(18)

where $p = m \gamma v$ is the relativistic momentum of the test particle and $\gamma$ is the Lorentz factor of the inertial frame of the particle relatively to the inertial frame associated to the screen. By
using the same arguments as before, one is led to postulate the following uncertainty relation in the entropy on the holographic screen

$$\delta S'_{r} = 2\pi k_B \left( \frac{\delta x}{l_c} + \frac{p \delta p}{m^2 c^2 \gamma^2} \right).$$

(19)

The above equation generalizes correctly the relation (17) as can be seen by taking the non-relativistic limit $v/c \to 0$ while keeping $c$ constant, where $v$ is the relative velocity of the two inertial frames. In this limit, the denominator of the second term from the r. h. s. of the equation (17) is recovered along with the classical quantities. Despite of that, the equation (19) does not lead to the relativistic generalization of the $F(\delta)$ from the equation (8) as one would naively expect, but instead one obtains the following equation

$$F'_{r}(\delta) = ma + \frac{hp}{2m\gamma^2} \left( \frac{h a}{mc^3} - 1 \right) \delta x^{-2}. \quad (20)$$

In order to obtain the uncertainty in the relativistic force acting on $m$ due to the uncertainty in the entropy on the screen, one should modify not only the ansatze given by the equation (19) but also the Verlinde’s postulate from (2). We propose the following relation for the relativistic uncertainty in the entropy

$$\delta S_r = 2\pi k_B \left( \frac{\gamma^3 \delta x}{l_c} + \frac{p \delta p}{m^2 c^2 \gamma^2} \right).$$

(21)

The first term in the above equation describes the variation of the entropy of the screen for the non quantum particle. We see that it gains an $\gamma^3$ factor which is hard to be foresought without comparing the entropy against the force. From (21) one can easily obtain

$$F_{r}(\delta) = m\gamma^3 a + \frac{hp}{2m\gamma^2} \left( \frac{h a}{mc^3} - 1 \right) \delta x^{-2}. \quad (22)$$

The $F_{r}(\delta)$ represents the correct generalization of the equation (8) to the relativistic case. Similarly, one can obtain the corresponding generalizations of the equations (11) and (12) which are left as an exercise.

Now let us extend these results to the General Relativity. The setting to discuss the effect of the Uncertainty Principle in the entropic general relativity is the one used to derive the entropic relativistic force from [1]. We consider a static background with a global timelike Killing vector $\xi^a$ which is necessary to define the temperature and the entropy variation and which determines the relativistic generalization of the Newton’s potential $\phi$. In this background, the particle of rest mass $m$ is accelerated with the acceleration

$$a^b = u^a \nabla_a u^b = -\nabla^b \phi, \quad (23)$$

where $u^a$ is the relativistic velocity of the particle (see [32]). Whenever the test particle is subjected to an acceleration, a relativistic force can be defined as $a^b = f^b/m$. In the entropic general relativity, the acceleration is produced by the entropy gradient along the normal direction to the holographic screen $S$ located at surfaces of constant redshift defined by $e^{\phi/c^2}$. This entropic acceleration should be related to an entropic force as mentioned above. The unit vector pointing in the normal direction to $S$ is denoted by $N^a$. As in the non-relativistic case, the information entropy on the screen can be related to the mechanical entropy of the system at local thermodynamical equilibrium with the local temperature

$$T = \frac{\hbar}{2\pi k_B c} e^{\frac{\phi}{c^2}} N^a \nabla_a \phi, \quad (24)$$
which corresponds to the $T_U$ in the non-relativistic case. The local variation of the entropy is defined along the normal direction to the screen and it is postulated to have the following value for the Compton length [1]

$$ \nabla_a S = -\frac{2\pi k_B}{l_c} N_a, \quad (25) $$

where the minus sign is due to the crossing of the holographic surface from outside to inside. As in the non-relativistic case defined by the equation (2), the above relation defines the units of the variation of the entropy.

Now let us assume that the test particle has a quantum mechanical description in some local neighbourhood of $S$ (for at least short times and small spacelike volumes). Then it is subjected to the local relativistic generalization of the Heisenberg’s principle $\delta x^a \delta p_a \geq \hbar/2$. In this case, an observer that moves with the velocity $v^a$ in the neighbourhood of $S$ and $m$ will register an uncertainty of the local information on the screen $\delta S$ related to the uncertainty of the entropy by the equation

$$ \delta W_v = -v^a \delta p_a + F_v \delta x^a = T \delta S. \quad (26) $$

Here, the index $v$ of the uncertainty of the energy means that the uncertainty is observer dependent. The relation (26) represents the relativistic generalization of (5) from the non-relativistic case. If we interpret (26) as being the definition of the uncertainty of $S$ as we did in the non-relativistic case, we conclude that the uncertainty in the entropy should be observer dependent. Then, the simplest relation that can be postulate for the uncertainty of the entropy is

$$ \delta S_v = -2\pi k_B \left( \frac{N_a \delta x^a}{l_c} + \frac{m v_a \delta p^a}{p^2} \right). \quad (27) $$

By plugging the equations (24) and (27) into (26) and using the uncertainty estimate $\delta p_a = \frac{\hbar}{2} \delta x_a^{-1}$ where $\delta x_a^{-1} = \delta x_a/\delta x^2$, one can show that the entropic relativistic force when the uncertainty is taken into account has the following form

$$ F_v^a(\delta) = -mc^2 \nabla^a \left( \frac{\phi}{c^2} \right) - \frac{\hbar}{2} \left[ 1 + \frac{\hbar}{mec^2} \nabla_b \left( \frac{\phi}{c^2} \right) \right] v^a \delta x^{-2}. \quad (28) $$

The Newtonian limit is defined as usual by taking the weak field approximation and the slow moving particle limit

$$ g_{ab} = \eta_{ab} + h_{ab}, \quad h_{ab} << 1, \quad h_{00} = 2 \frac{\phi}{c^2}, \quad (29) $$

$$ \frac{dx^0}{d\tau} \approx 1, \quad \left| \frac{dx^i}{d\tau} \right| << \left| \frac{dx^0}{d\tau} \right|. \quad (30) $$

Then for the flat space defined by the condition $\phi/c^2 << 1$ the equation (28) reproduces the equation (8). It is important to remark that the General Relativity seems to put a stronger constraint on the uncertainty of the entropy than the Newtonian mechanics. Indeed, the postulate given in the relation (27) is the simplest linear dependence of the uncertainty of the entropy on $\delta x^a$ and $\delta p^a$. The point is that it also contains the velocity of the observer $v^a$ as a consequence of the second equality from (26). Formally, one could replace $v^a$ by a different vector. But then the equality could not hold in the variable $v^a$. Thus, the simplest linear dependence on $\delta x^a$ and $\delta p^a$ forces the presence of $v^a$ in $\delta S$ as a logical and physical requirement of the theory. An interesting consequence of this is that the observer with $v^a = 0$ does not see any uncertainty in the gravitational force. The similar constraint in the classical case (the dependence of $\delta S$ on $p$), does not seem so strong and allows two different interpretations of the uncertainty of the entropy as discussed before.
4 Conclusions and Discussions

To conclude, the postulate of the entropic force represents a powerful tool for investigating the quantum nature of the gravitational phenomena. This feature stems from its quantum, geometric and informational foundations. The fact that its most impressive results, the new proposal about the nature of the inertia, the Newtonian gravity and the General Relativity obtained in [1] are all classical hides the predictive power of the entropic postulate at the quantum level. In this paper, we have shown that by considering the simplest and the most fundamental quantum property of the matter, the Uncertainty Principle, we are led to new insights in the quantum structure of the inertia and the gravity in both the non-relativistic and the relativistic theories, in the form of the quantum undeterminacy of forces. Generalizations of the results presented here and of the Uncertainty Principle can be found in [33, 34, 35, 36].

Our results have been obtained by postulating the expressions (7) and (11) for the undeterminacy of the entropy in the non-relativistic theory. With these relations given, we have argued that the correct generalization of the uncertainty of the entropy on the screen in the Special Relativity is given by the equation (21) while in the General Relativity is the equation (27) that should be considered for that purpose. The criteria used to establish these relations were the simplicity and the generalization of the classical postulate from [1] that is recovered in the classical limit of the quantum theory. The guide to establish the dependence of $\delta S$ on $\delta x$ and $\delta p$ is the dependence of $\delta W$ on $\delta x$ and $\delta p$, since $\delta W$ equates to $\delta S$ through the first law of the thermodynamics. Thus, interpreting $\delta x$ and $\delta p$ as variables strongly constraints the form of $\delta S$. (For example, terms of the form $\delta F = 0$ are not allowed.) However, the above criteria alone are not enough to discern between the relations (7) and (11).

This situation is improved in the General Relativity, where one can consider the velocity of the observer $v^a$ as an extra variable and one can require that it appear explicitly in the $\delta S$ as we have done in (27). Indeed, since the uncertainty of the energy $\delta W_v$ depends on the observer, it does not seem natural to require that $\delta S$ be observer independent as the first law makes these two quantities proportional to each other. By taking this point of view together with the previous criteria, the undeterminacy of the entropy is uniquely fixed to have the form given by the relation (27). If the above argument is ignored, then $\delta p^a$ can enter $\delta S$ through different terms from which the most simple ones have the form $p_a \delta p^a / p^2$ and $N_a \delta p^a / mc$. In these cases, $\delta F_v^a$ takes the form

$$\delta F_v^a = \frac{h}{2} \left[ v^a - \frac{\hbar p^a}{m^2 c^3} \frac{\phi}{c^2} N^b \nabla_b \left( \frac{\phi}{c^2} \right) \right] \delta x^{-2}, \quad (31)$$

$$\delta F_v^a = \frac{h}{2} \left[ v^a - \frac{\hbar}{mc^2} N^a N^b \nabla_b \left( \frac{\phi}{c^2} \right) \right] \delta x^{-2}. \quad (32)$$

Note that (31) and (32) are zero in the classical limit. However, in both cases the undeterminacy registered by the observer with $v^a = 0$ is no longer zero.

An interesting generalization of the results presented in this paper concerns the equipartition theorem. The derivation of the uncertainty of the law of gravity depends on the validity of the equipartition theorem as in [1]. If qubits are considered instead of bits, the equipartition theorem can still be applied since a qubit can be realized in terms of a physical system with one degree of freedom. However, it is well known that the equipartition law fails for many quantum systems due to frozen modes that can exist at low temperatures. Since our arguments rely on the quantum nature of the test particle, it would be interesting to discuss the alternatives to the equipartition theorem which, in general, are model dependent.

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