Outline

- Introduction to $SU(5)$ Model Building
- Towards Realistic $SU(5)$
- A Novel $SU(5)$ Proposal
- Summary
### SU(5) Model Building

#### SU$_5$ irreps

The smallest group that contains SM Minimal choice (rank 4)

| Dynkin label | Dimension | $l$ (index) | Quintality | SU$_4$ singlets | SU$_2 \times$ SU$_3$ singlets |
|--------------|-----------|-------------|------------|-----------------|-------------------------------|
| (1000)       | 5         | 1           | 1          | 1               | 0                             |
| (0100)       | 10        | 3           | 2          | 0               | 1                             |
| (2000)       | 15        |             |            |                 |                               |
| (1001)       | 24        |             |            |                 |                               |
| (0003)       | 35        |             |            |                 |                               |
| (0011)       | 40        |             |            |                 |                               |
| (0101)       | 45        |             |            |                 |                               |
| (0020)       | 50        |             |            |                 |                               |
| (2001)       | 70        |             |            |                 |                               |
| (0004)       | 70'       |             |            |                 |                               |
| (0110)       | 75        |             |            |                 |                               |
| (0012)       | 105       |             |            |                 |                               |
| (2010)       | 126       |             |            |                 |                               |
| (5000)       | 126'      |             |            |                 |                               |

**SU$_5 \supset$ SU$_2 \times$ SU$_3 \times$ U$_1$**

- $5 = (2, 1)(3) + (1, 3)(-2)$
- $10 = (1, 1)(6) + (1, 3)(-4) + (2, 3)(1)$
- $15 = (3, 1)(6) + (2, 3)(1) + (1, 6)(-4)$
- $24 = (1, 1)(0) + (3, 1)(0) + (2, 3)(-5) + (2, 3)(5) + (1, 8)(0)$
- $35 = (4, 1)(-9) + (3, 3)(-4) + (2, 6)(1) + (1, 10)(6)$
- $40 = (2, 1)(-9) + (2, 3)(1) + (1, 3)(-4) + (3, 3)(-4) + (1, 8)(6) + (2, 6)(1)$
- $45 = (2, 1)(3) + (1, 3)(-2) + (3, 3)(-2) + (1, 3)(8) + (2, 3)(-7) + (1, 6)(-2) + (2, 8)(3)$
- $50 = (1, 1)(-12) + (1, 3)(-2) + (2, 3)(-7) + (3, 6)(-2) + (1, 6)(8) + (2, 8)(3)$
- $70 = (2, 1)(3) + (4, 1)(3) + (1, 3)(-2) + (3, 3)(-2) + (3, 3)(8) + (2, 6)(-7) + (2, 8)(3) + (1, 15)(-2)$
- $70' = (5, 1)(-12) + (4, 3)(-7) + (3, 6)(-2) + (2, 10)(3) + (1, 15)(8)$
- $75 = (1, 1)(0) + (1, 3)(10) + (2, 3)(-5) + (1, 3)(-10) + (2, 3)(5) + (2, 6)(-5) + (2, 8)(5) + (1, 8)(0) + (3, 8)(0)$
Georgi-Glashow Model

- **Fermions**

\[
\bar{5}_F = \begin{pmatrix}
d_1^c \\
d_2^c \\
d_3^c \\
e \\
-\nu
\end{pmatrix}, \quad 10_F = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & u_3^c & -u_2^c & u_1 & d_1 \\
-u_2^c & 0 & u_1^c & u_2 & d_2 \\
u_2^c & -u_1^c & 0 & u_3 & d_3 \\
-u_1 & -u_2 & -u_3 & 0 & e^c \\
-d_1 & -d_2 & -d_3 & -e^c & 0
\end{pmatrix}.
\]

- ** Scalars**

\[
24_H : \quad SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y
\]

\[
5_H : \quad SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}
\]

*Georgi, Glashow 1974*
Georgi-Glashow Model

- GG model: $\bar{5}_{Fi} + 10_{Fi} + 5_H + 24_H$

- $M_d = M_e^T$ \hspace{1cm} [Georgi, Jarlskog 1979]

- $M_\nu = 0$

- Gauge coupling unification
In this talk

- Towards Realistic Models
  - Renormalizable models
  - Non-renormalizable models
Towards Realistic Models

- $\overline{5}_{Fi} + 10_{Fi} + 5_H + 24_H + 45_H$
  - Georgi, Jarlskog 1979

- Yukawa Lagrangian: Dorsner, Perez 2006

$$\mathcal{L}_Y = Y_{1,ij} \overline{5}_{Fi}^{\alpha} 10_{F\alpha\beta,j}^{\beta} 5^*_H + Y_{2,ij} \overline{5}_{Fi}^{\delta} 10_{F\alpha\beta,j} 45^*_H{}^{\alpha\beta} + \epsilon^{\alpha\beta\gamma\delta r} \left( Y_{3,ij} 10_{F\alpha\beta,i} 10_{F\gamma\delta,j} 5_H{}^r + Y_{4,ij} 10_{F\alpha\beta,i} 10_{Fm\gamma,j} 45^m_H{}^{H\delta r} \right)$$
Towards Realistic Models

- $\overline{5}_{Fi} + 10_{Fi} + 5_{H} + 24_{H} + 45_{H}$

- $M_d \neq M_e^T$

- $M_{\nu} = 0$

- Gauge coupling unification

- Proton decay (safe)
Neutrino Mass: Tree level

- **Type-I seesaw:** \( 5_{F_i} + 10_{F_i} + 5_H + 24_H + 45_H + 1_{F_i} \)

- **Type-II seesaw:** \( 5_{F_i} + 10_{F_i} + 5_H + 24_H + 45_H + 15_H \)
  
  Dorsner, Perez 2005; Dorsner, Mocioiu 2008

- **Type-I+III seesaw:** \( 5_{F_i} + 10_{F_i} + 5_H + 24_H + 45_H + 24_{F_j} \)
  
  Bajc, Senjanovic 2007; Perez 2007

![Diagrams](image-url)
Neutrino Mass: 1-loop

- Zee-mechanism: $\bar{5}_{Fi} + 10_{Fi} + 5_{H} + 24_{H} + 45_{H} + 10_{H}$

  Wolfenstein 1980; Barbieri, Nanopoulos, Wyler 1981; Perez, Murgui 2016
Neutrino Mass: 2-loop

$\bar{5}_{F_i} + 10_{F_i} + 5_H + 24_H + 45_H + 40_H + 50_H$

Saad 2019
### Minimal Renormalizable Model?

#### SU₅ irreps

| Dynkin label (name) | Dimension (index) | l (index) | Quintality | SU₄ singlets | SU₂ × SU₃ singlets |
|---------------------|-------------------|-----------|-------------|--------------|--------------------|
| (1000)              | 5                 | 1         | 1           | 1            | 0                  |
| (0100)              | 10                | 3         | 2           | 0            | 1                  |
| (2000)              | 15                | 7         | 2           | 1            | 0                  |
| (1001)              | 24                | 10        | 0           | 1*           | 1*                 |
| (0003)              | 35                | 28        | 2           | 1            | 0                  |
| (0011)              | 40                | 22        | 2           | 0            | 0                  |
| (0101)              | 45                | 24        | 1           | 0            | 0                  |
| (0020)              | 50                | 35        | 1           | 0            | 1                  |
| (2001)              | 70                | 49        | 1           | 1            | 0                  |
| (0004)              | 70'               | 84        | 1           | 1            | 0                  |
| (0110)              | 75                | 50        | 0           | 0            | 1*                 |
| (0012)              | 105               | 91        | 1           | 0            | 0                  |
| (2010)              | 126               | 105       | 0           | 0            | 0                  |
| (5000)              | 126'              | 210       | 0           | 1            | 0                  |

Slansky 1981
Minimality Criteria

- ✔ least number of parameters
- ✔ lowest dimensional representations
- ✖ non-renormalizable operators
- ✖ singlets
## A Novel $SU(5)$ Proposal

| Dynkin label | Dimension (name) | $l$ (index) | Quintality | $SU_4$ singlets | $SU_2 \times SU_3$ singlets |
|--------------|------------------|-------------|------------|-----------------|-----------------------------|
| (1000)       | 5                | 1           | 1          | 1               | 0                          |
| (0100)       | 10               | 3           | 2          | 0               | 1                          |
| (2000)       | 15               | 7           | 2          | 1               | 0                          |
| (1001)       | 24               | 10          | 0          | 1*              | 1*                         |
| (0003)       | 35               | 28          | 2          | 1               | 0                          |
| (0011)       | 40               | 22          | 2          | 0               | 0                          |
| (0101)       | 45               | 24          | 1          | 0               | 0                          |
| (0020)       | 50               | 35          | 1          | 0               | 1                          |
| (2001)       | 70               | 49          | 1          | 1               | 0                          |
| (0004)       | 70'              | 84          | 1          | 1               | 0                          |
| (0110)       | 75               | 50          | 0          | 0               | 1*                         |
| (0012)       | 105              | 91          | 1          | 0               | 0                          |
| (2010)       | 126              | 105         | 0          | 0               | 0                          |
| (5000)       | 126'             | 210         | 0          | 1               | 0                          |
A Novel $SU(5)$ Proposal

- $\overline{5}_{Fi} + 10_{Fi} + 5_H + 24_H + 35_H + 15_F + \overline{15}_F$

- Decompositions

  $5_H \equiv \Lambda = \Lambda_1(1, 2, 1/2) + \Lambda_3(3, 1, -1/3)$

  $24_H \equiv \phi = \phi_0(1, 1, 0) + \phi_1(1, 3, 0) + \phi_3(3, 2, -5/6)$

  $\phantom{24_H} + \phi_3(\overline{3}, 2, 5/6) + \phi_8(8, 1, 0)$

  $35_H \equiv \Phi = \Phi_1(1, 4, -3/2) + \Phi_3(\overline{3}, 3, -2/3) + \Phi_6(\overline{6}, 2, 1/6)$

  $\phantom{35_H} + \Phi_{10}((1\overline{0}, 1, 1)$

  $15_F \equiv \Sigma = \Sigma_1(1, 3, 1) + \Sigma_3(3, 2, 1/6) + \Sigma_6(6, 1, -2/3)$
Gauge Coupling Unification

Highly non-trivial:

- \( M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1} \)
- \( M_{\Phi_{10}}^2 = M_{\Phi_1}^2 - 3M_{\Phi_3}^2 + 3M_{\Phi_6}^2 \)
- \( M_{\Lambda_3} \geq 3 \times 10^{11} \text{ GeV} \) (proton decay)
- \( M_{\text{GUT}} \geq 5 \times 10^{15} \text{ GeV} \) (proton decay)
- \( M_k \geq 10 \text{ TeV} \) \( (k = \text{any BSM state}) \)
- \( \nu \)-mass requires specific \( M_{\Sigma_1} \) and \( M_{\Phi_1} \)  
  (rules out most of the parameter space consistent with unification)
Gauge Coupling Unification

\[ \alpha^{-1}_1, \alpha^{-1}_2, \alpha^{-1}_3 \]

\[ M_{\phi_6}, M_{\phi_3}, M_{\phi_1}, M_{\phi_8} \]

\[ M_{\Sigma_6}, M_{\Sigma_3}, M_{\Lambda_3}, M_{\Phi_{10}} \]

\[ \mu \text{ (GeV)} \]

\[ 10^2, 10^4, 10^6, 10^8, 10^{10}, 10^{12}, 10^{14}, 10^{16}, 10^{18} \]

\[ 7 \times 10^{15} \]
Neutrino Mass
Neutrino Mass

\[ \mathcal{L} \supset \chi' 5_H 5_H 35_H + Y^a_i 15_F 5_{F_i} 5_{H}^* + Y^b_i 15_F 5_{F_i} 35_{H}^* \]

- **tree-level contribution**

\[
(M^{d=7}_{\nu})_{ij} = -\chi' \frac{v_H^4}{M_{\Sigma_1} M_{\Phi_1}^2} \left( Y^a_i Y^b_j + Y^b_i Y^a_j \right)
\]

\[
\sim 10^{-26} \text{ GeV}
\]

- **one-loop contribution**

\[
(M^{d=5}_{\nu})_{ij} = \frac{\chi' v_H^2}{16\pi^2} \frac{(Y^a_i Y^b_j + Y^b_i Y^a_j) M_{\Sigma_1}}{M_{\Phi_1}^2 - M_H^2} \left( \frac{M_{\Phi_1}^2}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \log \left( \frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2} \right) - \frac{M_H^2}{M_{\Sigma_1}^2 - M_H^2} \log \left( \frac{M_{\Sigma_1}^2}{M_H^2} \right) \right)
\]

\[
\sim 4 \times 10^{-11} \text{ GeV}
\]

\[ Y^a, Y^b, \chi' = 1 \]
Charged fermion masses

\[
\mathcal{L} \supset Y_{ij}^d 10_F i \bar{5}_F j 5^*_H + Y_{ij}^u 10_F i 10_F j 5_H + Y_i^c 10_F i \bar{15}_F 24_H \\
+ Y_i^a 15_F 5_F i 5^*_H + M_\Sigma 15_F 15_F + y 15_F 15_F 24_H
\]

- Quarks, charged leptons, neutrino masses- all connected!

- \( M_u = \left( \mathbb{I}_{3 \times 3} + \delta^2 \ Y^c Y^c \dagger \right)^{-\frac{1}{2}} v_H Y^u \)

- \( M_d = \left( \mathbb{I}_{3 \times 3} + \delta^2 \ Y^c Y^c \dagger \right)^{-\frac{1}{2}} v_H (Y^d + \delta \ Y^c Y^a) \)

- \( M_e = v_H Y^d^T \)

- \( (M_\nu)_{ij} = m_0 (Y_i^a Y_j^b + Y_j^b Y_i^a) \)
Fit Parameters and Result

- $Y^d = (y_e, y_\mu, y_\tau)$
- $Y^a = (-0.0899, 0.551, 1)$
- $Y^b = (0.975, 2.381, 1)$
- $Y^c = -1.865 \times 10^{-7} (0.00137, 0.0942, 1)$
- $\lambda' = 0.239$

| Down-type quark masses | Fit value (GeV) | $\nu$ masses | Fit value (eV) | $\nu$ mixing angles | Fit value (°) |
|------------------------|----------------|--------------|----------------|---------------------|--------------|
| $m_d/10^{-3}$          | 1.14           | $m_1$        | 0              | $\theta_{12}$       | 34.57        |
| $m_s/10^{-2}$          | 2.15           | $m_2/10^{-3}$| 8.70           | $\theta_{23}$       | 47.41        |
| $m_b$                  | 0.99           | $m_3/10^{-2}$| 4.99           | $\theta_{13}$       | 8.56         |
A Novel $SU(5)$ Proposal: Summary

☑ Gauge coupling unification (fixed by $\nu$-mass)
☑ Safe from rapid proton decay
☑ Neutrino mass via 1-loop diagram ($m_1 = 0$)
☑ Correct charged fermion masses
☑ All fermion masses & mixings are correlated
☑ Only lowest dimensional representations
☑ Least number of Yukawa parameters
☑ $(1, 3, 0), (8, 1, 0), (3, 3, \frac{2}{3}), (6, 2, \frac{-1}{6}) \sim 1 - 10 \text{ TeV}$ (scalars)