A Simple, Entropy-based Dissipation Trigger for SPH

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Abstract

Smoothed particle hydrodynamics (SPH) schemes need to be enhanced by dissipation mechanisms to handle shocks. Most SPH formulations rely on artificial viscosity and, while this works well in pure shocks, attention must be paid to avoid dissipation where it is not wanted. Commonly used approaches include limiters and time-dependent dissipation parameters. The former try to distinguish shocks from other types of flows that do not require dissipation while in the latter approach the dissipation parameters are steered by some source term (“trigger”) and, if not triggered, they decay to a predefined floor value. The commonly used source terms trigger on either compression, $-\nabla \cdot v$, or its time derivative. Here we explore a novel way to trigger SPH-dissipation: since an ideal fluid conserves entropy exactly, its numerical nonconservation can be used to identify “troubled particles” that need dissipation because they either pass through a shock or become noisy for other reasons. Our new scheme is implemented into the Lagrangian hydrodynamics code MAGMA2 and is scrutinized in a number of shock and fluid instability tests. We find excellent results in shocks and only a very moderate (and desired) switch-on in instability tests. The new scheme is robust, trivial to implement into existing SPH codes, and does not add any computational overhead.

Unified Astronomy Thesaurus concepts: Hydrodynamical simulations (767)

1. Introduction

Smoothed particle hydrodynamics (SPH; Lucy 1977; Monaghan 1977) is a completely mesh-free method to solve the equations of hydrodynamics. It can be elegantly derived from a discretized Lagrangian of an ideal fluid (Gingold & Monaghan 1982; Speith 1998; Monaghan & Price 2001; Springel & Hernquist 2002; Rosswog 2015b) and thus ensures that Nature’s conservation laws are obeyed. As derived in this way, SPH is entirely dissipationless, and can therefore not handle shocks: in a shock front bulk kinetic energy is transformed by dissipation into internal energy, which goes along with an increase in entropy. Therefore, the ideal SPH equations need to be enhanced by some dissipative mechanism. In most modern Eulerian hydrodynamics schemes this is achieved by applying (exact or approximate) Riemann solvers, see, e.g., Toro (1999). This is also possible in SPH (Iwata et al. 2002; Cha & Whitworth 2003; Puri & Ramachandran 2014), but the use of artificial viscosity is more widespread. It is worth mentioning, however, that many artificial viscosity schemes bear similarities with approximate Riemann solvers (Monaghan 1997). While Riemann solvers are an elegant concept and less ad hoc than artificial viscosity, they always provide some amount of dissipation even in situations where it would actually not be needed. In SPH one has (at least in principle) the possibility to suppress/switch off dissipation completely. Historically, however, early implementations applied artificial viscosity terms with constant parameters and without limiters so that dissipation was always switched on whether or not it was needed. This lead to excessively dissipative SPH schemes and controlling the amount of dissipation has been a concern since.

Suggested cures include “limiters” (Balsara 1995; Cullen & Dehnen 2010; Read & Hayfield 2012; Wadsley et al. 2017) that are aimed at suppressing dissipation outside of shocks and tensor prescriptions (Owen 2004) that intend to restrict the effects of artificial viscosity to the shock travel directions and dissipation schemes with time dependent parameters. Time dependent dissipation parameters were introduced by Morris & Monaghan (1997) who suggested to evolve them separately for each particle according to an additional differential equation with a source and a decay term that, unless triggered, drives the parameter to a predefined floor value, see Equation (17) below. As a source term they used $-\nabla \cdot v$ which works well in many cases, but cannot distinguish between an adiabatic compression and an entropy producing shock. Cullen & Dehnen (2010) suggested instead using $d(-\nabla \cdot v)/dt$ as a dissipation trigger, so that a particle that moves into a shock (and thereby becomes increasingly more compressed) raises its dissipation parameter, which subsequently decays once the shock wave has passed.

Here we explore an alternative trigger that involves keeping track of some entropy measure at the particle level. Monitoring entropy violations has been used to steer dissipation in Eulerian Newtonian hydrodynamics (Guerrero et al. 2011, 2016) and it has also been used in relativistic hydrodynamics to steer to which amount low-order fluxes need to be added to higher-order fluxes for numerical stabilization (Guercilena et al. 2017). In this paper we use the local violation of exact entropy conservation to steer how much dissipation every SPH-particle needs. As shown in the tests below, our scheme yields excellent results, is trivial to implement, and comes without any computational overhead. We describe our methodology in Section 2, where we also briefly summarize the key ingredients of our MAGMA2 code in Section 2.1, and we discuss the entropy dissipation trigger in Section 2.2. In Section 3 we show a number of benchmark tests and we conclude in Section 4 with a concise summary.

2. Methodology

2.1. The SPH Formulation

The SPH code MAGMA2 (Rosswog 2020) profits from a number of new elements: (a) it uses high-order kernels, (b) calculates accurate gradients via matrix inversion techniques,
and (c) uses a new dissipation scheme where velocities are reconstructed via slope limiter techniques to the interparticle midpoint (Christensen 1990; Frontiere et al. 2017). The differences of these reconstructed velocities are used in the artificial viscosity tensor rather than the (“flat”) differences of the particle velocities, as is the standard practice in SPH. This approach drastically reduces unwanted dissipation and we have shown in an extensive set of test cases (Rosswog 2020) that excellent results are obtained even if the dissipation parameter $\alpha$ is kept constant at its maximum value. This finding is consistent with the results of Frontiere et al. (2017) who used an SPH formulation based on reproducing kernel interpolation (Liu et al. 1995). The main aim of this study is to apply dissipation only to “troubled particles” that are identified via entropy nonconservation between two subsequent time steps.

The equation set that we are using has been developed and tested extensively in a special relativistic context (Rosswog 2015a) and—in its Newtonian version—in the recent MAGMA2 code paper (Rosswog 2020). We use

$$\rho_a = \sum_b m_b W_{ab}(h_a),$$  

as density, momentum, and energy equations, where $\rho$, $v_a$, $u_a$, $c_{s,a}$ denote mass density, velocity, and specific internal energy, $m$ is the particle mass, $P$ is the gas pressure, $v_{ab} = v_a - v_b$, $W$ is the chosen SPH kernel function, and $h$ is its smoothing length. The gradient functions are given by

$$(G_a)^k = \sum_{d=1}^D C^{kd}(r_{a_d}, h_a)(r_{b_e} - r_a)^d W_{ab}(h_a),$$

$$(G_b)^k = \sum_{d=1}^D C^{kd}(r_{b_d}, h_b)(r_{b_e} - r_a)^d W_{ab}(h_b),$$

where $C$ is a “correction matrix” that accounts for the local particle distribution and is calculated as

$$(c_a^{kd}(h)) = \left( \sum_b \frac{m_b}{\rho_b} (r_{b_d} - r_a)^k (r_{b_e} - r_a)^d W(\eta_a - \eta_b, h) \right)^{-1}. (6)$$

Such gradients have been shown to work well (Cabezon et al. 2012; Garcia-Senz et al. 2012) and to be orders of magnitude more accurate than standard SPH-kernel gradient methods, see Figure 1 in Rosswog (2015a). Following the approach of von Neumann & Richtmyer (1950), we implement artificial viscosity by adding an “artificial pressure” $Q$ to the physical pressure $P$ wherever it occurs. We use the expression (Monaghan & Gingold 1983)

$$Q_a = \alpha \rho_b (-c_{s,a} \mu_a + 2 \mu_a^2),$$

where the velocity jump is (summation over $\delta$)

$$\mu_a = \min \left( 0, \frac{\delta v_{ab} \eta_a}{\eta_a^2 + \epsilon^2} \right).$$

The numerical parameters $\alpha$ and $\epsilon$ are usually set to 1 and 0.1, $c_{s,a}$ is the sound speed and

$$\eta_a^\delta = \frac{(r_a - r_b)^\delta}{h_a}, \quad \eta_a^2 = \eta_a^\delta \eta_a^\delta.$$
are (nondimensionalized) separations between particles. In SPH it is common practice to use \( \psi_{ab} = \psi_a^v - \psi_b^v \) in Equation (8), i.e., to apply the velocity difference between the two particles. In MAGMA2 we quadratically reconstruct the velocities of particle \( a \) and \( b \) to their midpoint at \( r_{ab} = 0.5(r_a + r_b) \), so that the velocities reconstructed from the \( a \)-side read

\[
\tilde{v}_i^a = v_i^a + \Phi_{ab} \left[ \left( \partial_i v_i^v \right) \delta^a + \frac{1}{2} \left( \partial_i \partial_m v_i^v \right) \delta^a \delta^m \right]_a,
\]

where the index at the square bracket indicates that the derivatives at the position of particle \( a \) are used and the increments from point \( a \) to the midpoint are \( (\delta^a)_b = \frac{1}{2}(r_b - r_a) \).

The reconstructed velocities from the \( b \)-side, \( \tilde{v}_i^b \), are calculated correspondingly, but with derivatives from position \( b \) and increments \( (\delta^b)_a = - (\delta^a)_b \). In Equation (8) we use the difference in the reconstructed velocities, i.e., \( \psi_{ab} = \tilde{v}_a^v - \tilde{v}_b^v \). To calculate the first and second velocity derivatives we also use matrix inversion techniques, see Rosswog (2020) for more details.

We use a modification of van Leer’s slope limiter (van Leer 1974; Frontiere et al. 2017)

\[
\Phi_{ab} = \max \left[ 0, \min \left[ 1, \frac{4A_{ab}}{(1 + A_{ab})^2} \right] \right] \times \left\{ \begin{array}{ll}
1, & \text{if } \eta_{ab} > \eta_{\text{crit}} e^{\frac{(\eta_{ab} - \eta_{\text{crit}})}{0.2}} \\
0, & \text{otherwise}
\end{array} \right.
\]

with \( N_{\text{min}} \) being the number of neighbors for the chosen kernel. We also apply a small amount of thermal conductivity

\[
\left( \frac{du_a}{dt} \right)_C = - \alpha_u \sum_b m_b \frac{v_{\text{sig},ab}}{\rho_{ab}} (u_a - u_b) |G_a + G_b|, \tag{14}
\]

where \( \alpha_u = 0.05 \) and \( v_{\text{sig},ab} \) is a signal velocity. For more details and the explicit expressions that we use we refer to the MAGMA2 code paper (Rosswog 2020). In all of the shown tests we use the Wendland C6 kernel (Wendland 1995) with 300 neighbor particles.

2.2. Using Entropy Nonconservation to Identify “Troubled Particles”

Our SPH formulation conserves mass, energy, momentum, and angular momentum exactly;\(^1\) entropy conservation, in contrast, is not actively enforced and therefore its potential nonconservation can be used to monitor the smoothness of the local flow. In smooth flows entropy should be conserved exactly while it may be physically increased in shocks. Flows can, however, also become “noisy” (i.e., nonnegligible velocity fluctuations appear) for numerical reasons (e.g., particles moving off an initially specified, nonoptimal lattice) and also in such cases (a smaller amount of) dissipation is desirable. In either case, shocks or noisy flows, one would want to apply artificial dissipation in order to keep the flow physically well-
behaved, and measuring the degree of numerical nonconservation of entropy (or some entropy function) is a natural way to identify “troubled particles” and to determine how much dissipation should be applied.

Here we suggest measuring the rate of numerical entropy generation between two subsequent time steps and a translation of this rate into a value for the dissipation parameter $\alpha$. Since MAGMA2 produces, due to the velocity reconstruction, excellent results even with a constant $\alpha = 1$, we choose parameters that are conservatively large so that $\alpha$ reaches already substantial values for small entropy violations. For SPH schemes without such velocity reconstructions the same functional relations can be used, but the optimal parameter values may have slightly different values.

We assume here a polytropic equation of state and use

$$s_a = \frac{P_a}{\rho_a^\Gamma}$$

as a measure for the entropy carried by particle $a$. Here $P_a$ is the gas pressure and $\Gamma$ is the polytropic exponent. Polytropic equations of state are used in most astrophysical gas simulations, but other entropy measures, e.g., the physical entropy of an ideal gas, could equally well be used along the same lines of reasoning. Even if other sources of entropy (e.g., nuclear reactions) were present, this approach could be used provided that one can cleanly separate out the contributions from the additional sources. But this is not the topic of our study here and we leave this for future investigations.

We use the nondimensionalized relative entropy rate of change of a particle $a$ between time step $t^{n-1}$ and time step $t^n$ ($\Delta t = t^n - t^{n-1}$)

$$\dot{\varepsilon}_a^n = \frac{|s_a^n - s_a^{n-1}|}{s_a^{n-1}} \frac{\tau_a}{\Delta t}$$

as a measure of how much dissipation is needed. Here $\tau_a = h_a/c_{s,a}$ is the particle’s dynamical timescale and $c_{s,a}$ is its sound speed. We use $l_n^n = \log(\dot{\varepsilon}_a^n)$ to steer the amount of dissipation. Similar to earlier work (Morris & Monaghan 1997; Rosswog et al. 2000; Cullen & Dehnen 2010; Rosswog 2015a;
Wadsley et al. 2017, we let the dissipation parameter \( \alpha \) decay according to
\[
\frac{d\alpha_n}{dt} = -\frac{\alpha(t) - \alpha_0}{30\tau_a},
\]
where \( \alpha_0 \) is a floor value, in other schemes often set to values around 0.1 to keep the particle distribution well-behaved (Tricco 2019). Note that we have conservatively chosen a rather long decay timescale of 30 \( \tau_a \). We compare at each time step the actual value to a “desirable dissipation parameter” and if the latter exceeds the current value, \( \alpha(t) \) is increased instantly (Cullen & Dehnen 2010). The desired value of \( \alpha \) is chosen according to the trigger \( t^u_a \)
\[
\alpha_{a,\text{des}} = \max \{ \alpha \left( t^u_a \right), \alpha_{\max} \},
\]
where \( \alpha \) is a smooth “switch-on” function for which we have chosen, see Figure 1,
\[
\mathcal{S}(x) = 6x^5 - 15x^4 + 10x^3,
\]
with
\[
x = \min \left\{ \max \left\{ \frac{t^u_a - l_0}{l_1 - l_0}, 0 \right\}, 1 \right\}.
\]
The reasoning behind this is that we consider dissipation unnecessary for acceptably small entropy violations (exact value set by \( l_0 \)) and beyond another threshold value (set by \( l_1 \)) we need the maximal dissipation parameter, \( \alpha_{\max} \). After some experimenting with both shocks and instability tests, we settled on values \( l_0 = \log(1 \times 10^{-4}) \) and \( l_1 = \log(5 \times 10^{-2}) \), so that our scheme does not switch on at all for entropy violations.
\[ \alpha_n \approx 10^{-4} \text{ and reaches } \alpha = \alpha_{\text{max}} = 1 \text{ for } \dot{\epsilon}_a \gg 5 \times 10^{-2}, \text{ see Figure 1. For aesthetic reasons we prefer to have only triggered dissipation rather than assigning a floor value } \alpha_0 \text{ by hand. We therefore use } \alpha_0 = 0 \text{ in our implementation, but note that with the chosen parameters } l_0 \text{ and } l_1 \text{ a small amount of dissipation (typically } \alpha \sim 0.01) \text{ is triggered even in smooth flows, see below. We find good results for this particular switch-on function and the chosen parameters, but other choices are certainly possible and the optimal parameter values might be slightly different for other SPH formulations.}

In the below tests, we compare also to the \( d(\nabla \cdot \mathbf{v})/dt \)-trigger suggested by Cullen & Dehnen (2010). Here the desired dissipation parameter is chosen as
\[
\alpha_{n,\text{des}} = \frac{A_a}{0.25 \left( \frac{u_{\text{sig}}}{h_a} \right)^2 + A_a},
\]
where \( A = \min(-d(\nabla \cdot \mathbf{v})/dt, 0) \) and the signal velocity is given by
\[
v_{\text{sig}} = \max_h [c_{\text{E,ab}} - \min \{0, v_{\text{ab}}, \dot{\epsilon}_{\text{ab}}\}].
\]

\footnote{Note that their kernels have a support radius of \( 1h \), while we follow the convention that the kernel is nonzero out to a radius of \( 2h \).}
Here,  \( c_{a,b} = 0.5(c_{a,a} + c_{a,b}) \),  \( \nu_{ab} = \nu_a - \nu_b \), and  \( \tilde{e}_{ab} = (r_a - r_b)/(r_a - r_b) \). Cullen & Dehnen (2010) chose a decay timescale of 20\(\nu_a/\nu_{a,\text{sig}}\) for the denominator of Equation (17) and this is the parameter we adopt in this comparison. It is worth pointing out that this is a comparison of triggers and there are differences between our approach with the  \( \nabla \cdot v \)-trigger (e.g., SPH-formulation, kernels, reconstruction, and conductivity) and the original Cullen & Dehnen approach. Nevertheless, we use the “CD” in some of the below figures as a shorthand for this  \( \nabla \cdot v \)-trigger approach.

### 3. Tests

To scrutinize the suggested scheme, we perform a number of benchmark tests. We perform shock tests to demonstrate that the dissipation robustly switches on and avoids spurious oscillations and Kelvin–Helmholtz and Rayleigh–Taylor tests to verify that no unnecessary dissipation is triggered in smooth portions of the flow.

#### 3.1. Sedov Taylor Blast

We begin by setting up a Sedov explosion test where a given number of SPH particles is distributed according to a centroidal Voronoi tessellation (Du et al. 1999) in the computational volume \([-0.5, 0.5] \times [-0.5, 0.5] \times [-0.5, 0.5]\). While this already produces very good quality initial conditions, they can be further improved by applying regularization sweeps, where each particle position is corrected according to Equation (20) of Gaburov & Nitadori (2011). This procedure ensures nearly perfectly spherically symmetric results in this test. Subsequently we assign masses so that the density is  \( \rho = 1 \). We use a polytropic exponent  \( \Gamma = 5/3 \) and spread an internal energy  \( E = 1 \) across a very small initial radius  \( R \), the specific internal energy  \( u \) of the particles outside of  \( R \) is entirely negligible (10–10 of the central  \( u \)). For the initial radius  \( R \) we choose twice the interaction radius of the innermost SPH particle. Boundaries play no role in this test (as long as the blast does not interact with them), we therefore place “frozen” particles with fixed properties around the computational volume as boundary particles. For more details we refer to Rosswog (2020).
We show in Figure 2 the dissipation parameter $\alpha$ (left) and the density $\rho$ (right) for 200$^3$ SPH particles (excluding boundary particles). This test requires large dissipation values, both to robustly handle the shock and to “calm” the particles in the post-shock region. Our scheme delivers large $\alpha$-values in this test with values of $\approx 1$ in the shock itself, and a moderate decay to values around 0.4 in the shocked, inner region. We show in Figure 3, upper row, the numerical solution of pressure, velocity, and density (black dots) as a function of radius together with the exact solution (red). Note that all particles are plotted. Keep in mind that this numerical test challenges most methods whether particle- or mesh-based and often noise and strong spurious post-shock oscillations occur, see, for example, Rosswog & Price (2007), Hu et al. (2014), Cardall et al. (2014), Hopkins (2015), Wadsley et al. (2017), or Frontiere et al. (2017). For our results, the overall agreement is very good and there is only a small velocity overshoot at the shock and—since the finite particle masses overestimate the close-to-zero central density—the central pressure is somewhat overestimated.

The lower row of Figure 3 shows the corresponding result for the $d(\nabla \cdot v)/dt$-trigger. In this test the $d(\nabla \cdot v)/dt$-trigger delivers results that are very similar to the entropy trigger. It provides somewhat less dissipation in the shocked inner region, see Figure 4, and therefore the velocity distribution there is slightly more noisy.

3.2. Circular Blast

As another benchmark we use a three-dimensional shock-tube problem suggested by Toro (1999). Similar to the Sedov test, we set up 200$^3$ particles in the computational domain $[-1, 1]^3$. We first place them according to a centroidal Voronoi tessellation (Du et al. 1999) and then perform 500 regularization sweeps. The fluid properties are assigned according to

$$ (\rho, v, P) = \begin{cases} (1.000, 0, 0, 0, 1.0) & \text{for } r < 0.5 \\ (0.125, 0, 0, 0, 0.1) & \text{else.} \end{cases} \quad (23) $$

The solution exhibits a spherical shock wave, a spherical contact surface traveling in the same direction and a spherical

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*Figure 8.* Schulz–Rinne test 2 with the suggested entropy steering (top row) and the scheme suggested by Cullen & Dehnen (2010; bottom row, “CD”). In each row the dissipation parameter $\alpha$ is shown on the right and density $\rho$ on the left.
rarefaction wave traveling toward the origin. Our solution (\(xy\)-plane) at \(t = 0.2\) is shown in Figure 5 with dissipation parameter \(\alpha\) (left), density \(\rho\), velocity \(v\), and pressure \(P\). While the “glass-like” initial particle distribution leaves some weak imprint on the \(\alpha\)-values (left panel) all the physical quantities (panels 2–4) are essentially perfectly spherically symmetric.

The comparison of the MAGMA2-result \(|y| < 0.018, |z| < 0.018\) with the reference solution obtained with the Eulerian weighted average flux method with 400\(^3\) grid cells (Toro 1999) shows a very good agreement between both, see Figure 6, upper row. In the lower row of this figure we show the \(d(\nabla \cdot v)/dt\) -trigger results. Again the agreement is very good,
but since less dissipation is triggered there are larger velocity
overshoots at the shock front.

### 3.3. Schulz–Rinne Tests

Schulz-Rinne (1993) designed a set of challenging 2D
Riemann problems in which four constant states meet at one
corner. The initial conditions are chosen so that one elementary
wave, either a shock, a rarefaction, or a contact discontinuity
emerges from each interface and the subsequent evolution leads
to geometrically complex solutions. No exact solutions are
known, but the benchmark tests are often used and the results
can be compared to other numerical solutions (Schulz-
Rinne 1993; Lax & Liu 1998; Kurganov & Tadmor 2002;
Liska & Wendroff 2003). Here we show the results for two
such tests, the initial conditions of which are given in Table 1.
Further tests of this type are shown in the
MAGMA2 code paper (Rosswog 2020).

These tests are rarely shown for SPH codes, in fact, we are
only aware of the work by Puri & Ramachandran (2014) who
show results for one such shock test in a study of Godunov-
SPH with different approximate Riemann solvers. Most of their
implementations, however, show serious artifacts in this test.
Since our code is intrinsically 3D, we simulate a slice thick
enough so that the midplane is unaffected by edge effects (we
use here 10 particle layers in the z-direction). We use
660 × 660 close-packed particles in the xy-plane between
\[ x_c = 0.5, x_c + 0.5 \times [y_c - 0.5, y_c + 0.5], (x_c, y_c) \] being
the contact point of the quadrants, and we use a polytropic
exponent \( \Gamma = 1.4. \)

Figure 7 shows the results for test SR1 with the upper two
panels showing dissipation parameter \( \alpha \) (left) and density
(right) for the entropy trigger. The corresponding quantities for
the \( d(\nabla \cdot v)/dt \)-trigger are shown in the lower two panels.
The general features of the solution are captured in both cases,
but the \( d(\nabla \cdot v)/dt \)-trigger provides substantially less
dissipation in the central vortex-like region, which results in
a noticeable lack of symmetry (lower right panel).

The results for the SR2 test are shown in Figure 8. Again, the
results for the entropy trigger are given in the upper panels,
those for the \( d(\nabla \cdot v)/dt \)-trigger in the lower ones. The
entropy scheme delivers sharp and noise-free density structures
with a high degree of symmetry and that are in very good
agreement with the results from Eulerian approaches (Schulz-
Rinne 1993; Lax & Liu 1998; Kurganov & Tadmor 2002;
Liska & Wendroff 2003). The CD scheme captures the overall
features, but again triggers less dissipation, which leads to the
central vortex structure being substantially less developed
(lower right panel).

### 3.4. Kelvin–Helmholtz

An interesting question is how much dissipation is triggered
in an overall smooth test such as a Kelvin–Helmholtz
instability. To find out we set up a test similar to McNally
et al. (2012) and Frontiere et al. (2017). The test is initialized as

\[
\rho(y) = \begin{cases}
\rho_1 - \rho_m e^{y/(0.5\Delta)} & \text{for } 0.00 < y < 0.25 \\
\rho_2 + \rho_m e^{(0.25-y)/\Delta} & \text{for } 0.25 \leq y < 0.50 \\
\rho_2 + \rho_m e^{y/(0.75\Delta)} & \text{for } 0.50 \leq y < 0.75 \\
\rho_1 - \rho_m e^{(0.75-y)/\Delta} & \text{for } 0.75 \leq y < 1.00
\end{cases}
\]
where $\rho_1 = 1, \rho_2 = 2, \rho_m = (\rho_1 - \rho_2)/2$, and $\Delta = 0.025$ and the initial velocities are

$$v_1(y) = \begin{cases} 
  v_1 - v_m e^{(y - 0.25)/\Delta} & \text{for } 0.00 < y < 0.25 \\
  v_2 + v_m e^{(0.25 - y)/\Delta} & \text{for } 0.25 \leq y < 0.50 \\
  v_2 + v_m e^{(y - 0.75)/\Delta} & \text{for } 0.50 \leq y < 0.75 \\
  v_1 - v_m e^{(0.75 - y)/\Delta} & \text{for } 0.75 \leq y < 1.00 
\end{cases} \quad (25)$$

with $v_1 = 0.5, v_2 = -0.5$, and $v_m = (v_1 - v_2)/2$. A small velocity perturbation in the $y$-direction is introduced as $v_y = 0.01 \sin(2\pi x/\lambda)$ with the perturbation wavelength $\lambda = 0.5$. The test is performed with a polytropic equation of state with exponent $\Gamma = 5/3$. The test is set up in quasi-2D with 20 slices of $512 \times 512$ particles which, for simplicity, are arranged in a simple cubic lattice. Here we focus exclusively on the topic of this paper, the dissipation trigger, and for more details of the setup and the analysis of the MAGMA2 performance we refer to Rosswog (2020). The result at $t = 1.5, 2.0$ and $2.5$ is shown in Figure 9. Our dissipation scheme triggers a floor value of $\alpha \approx 0.015$. In the shear interfaces where the particle lattices shear along one another sharply localized lines with values of up to $\alpha \approx 0.4$ are triggered. The average dissipation parameter value at $t = 2.5$ is $\bar{\alpha} = 0.09$, i.e., the $\alpha$-values are substantially lower than the standard value of unity.

We have repeated this test with the $d(\nabla \cdot v)/dt$-trigger. The density evolution is visually very similar to the one shown in Figure 9; the only noticeable difference is that the Kelvin–Helmholtz billows at late times are less symmetric than for the entropy trigger. The $\alpha$ values (at $t = 2.5$) are shown in Figure 10: the $d(\nabla \cdot v)/dt$ method hardly switches on anywhere and reaches even in the Kelvin–Helmholtz billows only values of $\alpha \sim 0.03$. We have also measured the growth rate of the instability (see Figure 11), calculated exactly as in McNally et al. (2012). As a reference solution we use a high resolution calculation ($4096^2$ cells) obtained by the PENCIL code (Brandenburg & Dobler 2002). Both our cases show a
healthy growth rate, but the entropy method is noticeably closer to the reference solution, likely because noise is more efficiently suppressed.

3.5. Rayleigh–Taylor Instability

As a last example we show the results for a commonly used Rayleigh–Taylor test (Abel 2011; Hopkins 2015; Frontiere et al. 2017). Again, the focus is on the amount of dissipation that is triggered; more details on this test can be found in the original code paper (Rosswog 2020). We adopt a quasi-2D setup using the full 3D code in a \( xy \)-domain of \([-0.25, 0.25] \times [0, 1] \) and use 10 layers of particles in the \( z \)-direction. The initial density is set up as

\[
\rho(y) = \rho_b + \frac{\rho_i - \rho_b}{1 + \exp[-(y - y_i)/\Delta]}
\]  

(26)
with \( \rho_i = 2, \rho_b = 1 \), transition width \( \Delta = 0.025 \), and transition coordinate \( y_t = 0.5 \). The interface is perturbed as

\[
v_y(x, y) = \delta v_{y,0} [1 + \cos(8\pi x)][1 + \cos(5\pi (y - y_t))] 
\]

for \( y \) in \([0.3, 0.7]\) with an initial amplitude \( \delta v_{y,0} = 0.025 \). The equilibrium pressure profile is given by

\[
P(y) = P_0 - g \rho(y)[y - y_t],
\]

where \( P_0 = \rho_i/\Gamma \) and polytropic exponent is chosen as \( \Gamma = 1.4 \). A constant acceleration \( g = -0.5 \hat{e}_y \) is applied. We show snapshots at \( t = 2.8, 3.4, \) and \( 4.0 \) in Figure 12 with density in the upper row and the corresponding dissipation parameters in the lower row. Throughout most of the computational domain the value of \( \alpha \) is very low \((\sim 0.01)\), only in the sharp edges of the rising plumes values up to \( \approx 0.7 \) are reached. Overall, our results in this test are very similar to those obtained with Lagrangian finite volume particle methods (Hopkins 2015) and SPH-methods (Frontiere et al. 2017) based on the reproducing kernel methodology (Liu et al. 1995).

We have repeated this test with the \( d(\nabla \cdot v)/dt \)-trigger. We find that it actually triggers more dissipation in this test than the new approach, see Figure 13, right panels, but the effects are benign since the evolution (at least with our approach) is not very sensitive to the exact values of \( \alpha \). Visually, the density evolution (left panels) is very similar between the two approaches. We further compare, more quantitatively, the position of the fluid interface in the central rising and the down-sinking parts of the flow. Practically, we use bisections to track the density value \((\rho_i + \rho_b)/2 = 1.5\) along the \( y \)-axis, \( y_{\text{rise}} \), and along \( x = 0.125 \) (the right down-sinking “mushroom”), \( y_{\text{fall}} \), to capture the interface positions. These interface positions are shown in Figure 14, but as expected from Figure 13, there are hardly any differences noticeable.

4. Summary

In this paper we have explored a novel way to steer dissipation in SPH simulations. Rather than triggering on the velocity divergence (Morris & Monaghan 1997) or its time derivative (Cullen & Dehnen 2010), we trigger on local violations of exact entropy conservation. Such violations can be caused by particles entering a shock front or by numerical noise. We find the additional triggering on noise very beneficial in calming down post-shock regions and in resolving complex fluid structures as they emerge, e.g., in the Schulz–Rinne test cases. Triggers on noise, in addition to a shock trigger, had been suggested in earlier work (Rosswog 2015a), but the new scheme discussed here is much simpler and the same triggering mechanism takes care of both shocks and noise. The new scheme switches on robustly in shocks, moderately and only very locally in the case of noise, and triggers hardly any dissipation \((\alpha \sim 0.01\) for our parameter choice\) in smooth regions of the flow. The suggested method is very robust, trivial to implement in existing SPH codes, and does not require any noticeable computational effort.

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Figure 14. Measure of the \( y \)-positions of the density interface in the rising (at \( x = 0, y_{\text{rise}} \)) and down-sinking part of the flow (at \( x = 0.125, y_{\text{fall}} \)) for both the entropy-(orange) and the \( d(\nabla \cdot v)/dt \)-steering (black).
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