The effects of varying the strengths of tensor and spin-orbit interactions on M1 and E2 rates in $^{12}$C: Comparison of results in $\Delta N = 0$ and $\Delta N = 0 + 2\hbar \omega$ spaces

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The energies and transition rates to $J = 1^+ T = 1$ and $J = 2^+ T = 0, 1$ states in $^{12}$C with matrix elements fitted to realistic $G$ matrix elements obtained in non-relativistic approaches are studied. Then the effects of varying the strengths of the two-body tensor and spin-orbit interactions are also considered. The calculations are done in both a small space (0p) and a large space (0p + 2×ω). In the small space the B(M1) from ground to the $J = 1^+ T = 1$ is enhanced and gets closer to experiment if the strength of the spin-orbit interaction is increased and/or if that of the tensor interaction is made weaker. In a large space the spin B(M1) gets reduced by almost a factor of two. A ‘self-weakening’ mechanism for the tensor interaction which succeeded in explaining anomalies in other nuclei does not seem to work for this case.

I. EXPERIMENT AND PREVIOUS STUDIES

Of necessity, shell model calculations must be carried in limited shell model spaces. It is therefore of interest to try to understand the impact of such restriction on the results that are obtained. In this paper we study the effects of enlarging the shell model space on M1 and E2 excitation rate and we will also study the effects of varying strengths of the spin-orbit and tensor interactions.

There are two well studied $J = 1^+$ states in $^{12}$C [1]. These are the $T = 0$ states at 12.71 MeV and the $T = 1$ state at 15.11 MeV. The B(M1) rates to these states from ground are respectively 0.040(3) $\mu_N^2$ and 2.63(8) $\mu_N^2$. The dominance of the isovector over the isoscalar B(M1) and $\beta$ factors, gave much too small a B(M1) to the $T = 1$ state at 15.11 MeV. It was noted that increasing the spin-orbit strength could cause the B(M1) to be increased. A justification for increasing the spin-orbit strength comes from the Dirac phenomenology [1]. Indeed, Zheng, Zamick and Mithier [17] showed that with $G$ matrices derived from the relativistic Bonn A interaction [18], the value of B(M1) 0$^+ T = 0 \rightarrow 1^+ T = 1$ increased with decreasing Dirac mass. For example, with $m^*(\mathrm{DIRAC}) = 938.9, 729.1$ and 630.0 MeV/$c^2$ respectively, the values of B(M1) were 0.69, 1.13 and 1.57 $\mu_N^2$ respectively. Zheng and Zamick had introduced the $(x, y)$ interaction [15]

$$V = V_c + x V_{s.o.} + y V_t$$  \hspace{1cm} (1)

where s.o. = spin orbit, t = tensor and c = everything else, especially a spin-dependent central interaction. For $x = 1, y = 1$ this interaction gave a good fit to the non-relativistic Bonn A matrix elements. On could simulate the effects of the Dirac phenomenology very well by making $x$ larger than one.

One can also get a larger B(M1) by making $y$ smaller. The justification for this is less clear. There are the universal scaling ideas of G. E. Brown and M. Rho [19] where inside the nucleus the masses of all mesons except for the pion decrease relative to their free space values. In this model not only does the spin-orbit interaction become stronger (as in Ref. [16]), but also the tensor interaction becomes weaker. Another possibility is that the effect could be due to higher shell admixtures, without having to invoke non-nuclear degrees of freedom. Concerning the latter, Fayache, Zheng and Zamick [20] noted that higher shell admixtures, i.e. 2×ω and higher excitations, could for many phenomena make the tensor interaction look weaker in the valence space. They called this the ‘self-weakening mechanism’. They applied this to the
problems of the quadrupole moment of the ground state of $^6\text{Li}$, the near-vanishing of the Gamow-Teller matrix element for the $A = 14$ system ($^{14}\text{O}$ [J = 0 T = 1] \rightarrow $^{14}\text{N}$ [J = 1 T = 0]) and the energy splitting of the $J = 0^+_1$ T = 1 and $J = 0^+_2$ T = 0 states in $^{16}\text{O}$. In all three cases the experimental result could be obtained either by weakening the tensor interaction in a valence space calculation i.e. making y less than one, or by keeping the full strength y = 1 and performing shell model calculations in which 2\hbar\omega and sometimes higher admixtures were included.

However, in a perturbation theory approach by Zheng, Zamick and M"uther to the problem of B(M1) $^{12}\text{C}$ [J = 0 T = 0 \rightarrow J = 1^+ T = 1] the self-weakening mechanism did not seem to work. The authors renormalized the two body matrix elements in the 0p shell and then used these in a 0p shell matrix diagonalization. If only Bertsch-Kuo-Brown bubble was exchanged between two nucleons, then indeed the B(M1) got bigger. However, this was offset by the two other diagrams - hole-hole and particle-particle. The latter re-"nerv" each other and cause B(M1) to become small again. The specific number for the Bonn A interaction was 0.639 \mu_N^2 for the bare interaction, 1.177 \mu_N^2 when the bubble diagram was included, 0.832 \mu_N^2 when the bubble plus hole-hole were included and 0.514 \mu_N^2 when the bubble plus hole-hole plus particle-particle diagrams were included. We go from small to large to smaller to smaller still.

It is not however clear if merely renormalizing the two body interaction is sufficient for calculating a transition matrix element. If one draws Goldstone diagrams for the M1 transition there are several diagrams not included by merely substituting a renormalized interaction into the shell model calculation. Examples of the latter include the diagrams where the M1 operator is inserted after the first interaction but before the second interaction. Therefore in this work we decided to do a complete valence plus 2\hbar\omega shell model matrix diagonalization for the relevant states and the M1 transition rates in $^{12}\text{C}$. In such an approach all the missing diagrams are implicitly generated.

Of particular interest is whether the large space calculation will or will not serve to justify the adjustment of the parameters x and y in the small space. For example, will and x = 1, y = 1 calculation in the large space yield results similar to those of an x = 1, y = 0.5 calculation in the small space (self-weakening of the tensor interaction)? Such a comparison makes sense only if we look at the low energy sector of the large space results.

Our calculations for the B(M1)’s are of additional interest because of the close relationship of the spin B(M1)’s to the beta decay Gamow-Teller matrix elements. Thus our studies of the effects of enlarging the shell-model space have a broader applicability.

### II. COMPARISON OF RESULTS OF B(M1) $J = 0^+ \rightarrow J = 1^+$ IN $^{12}\text{C}$ IN THE SMALL SPACE ($\Delta N = 0$) AND THE LARGE SPACE ($\Delta N = 2$)

#### A. The x = 1, y = 1 results

In the small space (see tables (a) with the x = 1, y = 1 interaction (which corresponds roughly to G matrix elements of a realistic interaction obtained in a non-relativistic approach) we obtain for B(M1)\uparrow to the lowest $J = 1^+ T = 1$ state a value of 0.89 \mu_N^2 much smaller than the experimental value of 2.63(8) \mu_N^2. Our calculation further indicates that this is dominantly a spin-mode excitation, since B(M1)l = 0.03 \mu_N^2 and B(M1)\sigma = 0.58 \mu_N^2.

Does the discrepancy between theory and experiment go away when we go to the large space? Quite the contrary, the B(M1) gets smaller, mainly because B(M1)\sigma decreases. Note the B(M1)\sigma is very closely related to B(GT) the Gamow-Teller transition. They will have identical quenching due to configuration mixing. From table (a) we see that relative to $\Delta N = 0$, the quenching factor for B(M1)\sigma in a $\Delta N = 0 + 2\hbar\omega$ calculation is 0.31/0.58 = 0.53 (or for the operator it is 0.73).

This is in accord with the general consensus of what the quenching of Gamow-Teller operators should be. We have here verified this result, not in perturbation theory, but rather in an explicit matrix diagonalization. It appears that matrix diagonalization results are close to those of perturbation theory.

A point that has been made before is worth repeating. In open shell nuclei, one can get B(M1)\sigma’s and B(GT)’s to be too small, despite the fact that we believe there should be an overall quenching. This is what is happening here when we use non-relativistic G matrixes derived from realistic interactions. As another example, in the SU(4) limit for N = Z nuclei B(GT) will vanish. This limit can be reached by using a spin-independent two-body interaction and turning off the one-body spin-orbit interaction, e.g. in the Elliott model where one has only a Q · Q interaction. Thus in a given calculation in an open shell nucleus one cannot readily deduce where one needs quenching or enhancement of the spin, magnetic and/or Gamow-Teller operator, unless one has complete confidence in the two-body interaction and single particle energies that are being used.

In table (a) we present the summed M1 strengths in the small space; in table (b) we have the summed strengths to the first 10 states in a large space calculation ($\Delta N = 0 + 2\hbar\omega$) and in table (c) the sum to 500 states.

Whereas for the lowest state in the small space calculation (table (a)) for the x = 1, y = 1 interaction the values of B(M1), B(M1)l and B(M1)\sigma were respectively 0.89, 0.03 and 0.58 \mu_N^2, the summed small space values are 1.42, 0.60 and 0.91 \mu_N^2. In particular there is some isovector orbital (scissors mode) strength in higher $J = 1^+ T = 1$
states.

Focusing on $B(M1)_{\sigma}$ we see that the small space sum of $0.91 \mu_N^2$ becomes $0.70 \mu_N^2$ when we consider the first 10 states in a large space calculation. There is quenching of the low lying strength. We will compare the small and large space total strengths in a later section.

**B. Varying $x$ and $y$**

In tables II and III we study also the variation of the $B(M1)$ rates with $x$ and $y$, the strengths of the two-body spin-orbit and tensor interactions respectively. We consider four sets of $(x, y) = (1,1), (1.5,1), (1.0,5)$ and $(1.5,0.5)$. Increasing the spin-orbit strength from $x = 1$ to $x = 1.5$ simulates to a large extent the use of Dirac phenomenology with a Dirac effective mass of $1/1.5 = 0.67$. If indeed the source of the increase in the spin-orbit interaction comes from the Dirac phenomenology, then one is justified in using this enhanced value of $x$ in both the small space ($\Delta N = 0$) and the large space ($\Delta N = 0 + 2\hbar\omega$).

For the tensor interaction the choice of the parameter $y$ in the large space calculation depends which of the scenarios discussed in the previous section for weakening the tensor interaction are correct. If the universal scaling ideas of G. E. Brown and M. Rho are correct then one should use a weaker tensor interaction, e.g. $y = 0.5$ in both the small and large space calculation. However in the ‘self-weakening mechanism’ which explained several phenomena for nuclei with mass numbers $A = 6, 14$ and 16, the higher shell admixtures were responsible for making the tensor interaction appear weaker in the valence space. In that case if one uses $y = 0.5$ in the small spaces one should use $y = 1$ in the large space and hope that the results for the low energy properties are about the same in the two calculations.

First, focusing on the small space calculation of table II we see that we get a marked improved in the $B(M1)$ rate $J = 0^+ \rightarrow T = 0 \rightarrow J = 1^+ \rightarrow T = 1$ when we increase the spin-orbit interaction and/or decrease the tensor interaction. From $(x, y) = (1,1)$ to $(1.5,0.5)$ the value of $B(M1)$ increase from 0.89 to 2.54 and the value of $B(M1)_{\sigma}$ increase from 0.58 to 2.57. We are getting close to the experimental value of 2.68 $\mu_N^2$.

In the large space the values of $B(M1)$ to the first $1^+ \rightarrow T = 1$ state are however in all cases smaller than in the small space. The ‘self-weakening mechanism’ for the tensor interaction does not appear to work here. We should compare the $(x, y) = (1.5, 1)$ large space $B(M1)$ result of 1.29 $\mu_N^2$ with the $(x, y) = (1.5, 0.5)$ small space result of 2.54 $\mu_N^2$. For $(x, y) = (1.5, 1)$ the gains from the small space to the large space does not increase $B(M1)$ – rather it decreases it from 1.89 $\mu_N^2$ to 1.29 $\mu_N^2$. This is mainly due to a decrease of $B(M1)_{\sigma}$ from 1.71 $\mu_N^2$ to 0.98 $\mu_N^2$ ($B(M1)_l$ changes from 0.04 to 0.02). Why the ‘self-weakening mechanism’ appears to work in some nuclei but not in others is not clear and bears further investigation.

It should be added that results we obtain here via matrix diagonalization are not so different from those by Zheng, Zamick and M"uther using a renormalized interaction in perturbation theory. Evidently, the missing diagrams in which the M1 operator acts between the two interactions are not so important.

We see from table II which deals with *summed* $B(M1)$ strengths, that in the small space increasing the spin-orbit strengths from $x = 1$ to $x = 1.5$ results in very significant changes in the summed isovector $M1$ excitation strengths. More specifically, with this change in $x$, the $B(M1)$ is increased by over 60%, $B(M1)_{\sigma}$ is increased by 127% and $B(M1)_l$ is decreased by 16%. The effect in the small space of decreasing the strength of the tensor force from $y = 1$ to $y = 0.5$ is similar but smaller; $B(M1)$ is increased by 11%, $B(M1)_{\sigma}$ is increased by 24% and $B(M1)_l$ is decreased by 1.6%. The combined effect of increasing the spin-orbit and decreasing the tensor strengths is more than the sum of the individual effects. Here $B(M1)$ is doubled, $B(M1)_{\sigma}$ is more than tripled which $B(M1)_l$ decreases by 23%.

From table II we also see that in the large space the effect on the isovector $B(M1)$ strength of increasing strength $x$ of the spin-orbit force is qualitatively similar to the effect of doing so in the small space, but is smaller in scale by a factor of about 0.6. When $x$ is increased from 1 to 1.5 (both with $y = 1$), the $B(M1)$ is increased by 36%, $B(M1)_{\sigma}$ is increased by 80% and $B(M1)_l$ is reduced by 10%. Very interestingly, in the large space, changing only the strength $y$ of the tensor force has essentially no effect on the summed isovector $M1$ strength. Indeed, when $x = 1$ and $y$ is changed for 1 to 0.5, then the $B(M1)$, $B(M1)_{\sigma}$ and $B(M1)_l$ all change by less than 3%. This result requires further study. On the other hand, in the large space, the effect on the isovector $B(M1)$’s of decreasing the tensor strength $y$ (from 1 to 0.5) after the spin-orbit strength has already been increased for $x = 1$ to $x = 1.5$ is much more substantial. It results in a further increase of 17% in $B(M1)$ and close to 30% in $B(M1)_{\sigma}$ and a further decrease of 8% in $B(M1)_l$.

The middle part of table II shows for any given $(x, y)$ combination what happens if we limit ourselves to the first ten states in the large space. Then we typically underestimate the isovector $B(M1)$, $B(M1)_{\sigma}$, $B(M1)_l$ values by 10% to 30% in comparison with the results obtained by summing over the lowest 500 states in the large space. In the small space there are only eight states with $J = 1^+ \rightarrow T = 1$. For any one $(x, y)$ combination the corresponding values of $B(M1)$ and $B(M1)_{\sigma}$ are typically 20-40% larger than for the ten lowest states in the large space, but the $B(M1)_l$’s hardly change (by less than 7% in all cases.)
C. Comparison of the total isovector M1 strength in the small and large spaces

By including \(2\hbar\omega\) excitations in our shell model diagonalization we are able to address the question of whether or not the isovector M1 strength which disappears from the low lying sector (and in particular from the lowest \(1^+\) \(T = 1\) state at 15.11 MeV) reappears in the high energy sector, i.e. in the \(2\hbar\omega\) region. To answer this we merely have to look at the results in table [1].

For the case \(x = 1, y = 1\) the total strength in the \(0p+2\hbar\omega\) space is larger than in the small space. For the other three cases \((x, y) = (1.5, 1), (1, 0.5)\) and \((1.5, 0.5)\) the opposite is true – the total isovector magnetic dipole strength is smaller when \(2\hbar\omega\) excitations are included.

For \(x = 1, y = 1\) the summed B(M1) strength in the small space is 1.42 \(\mu_N^2\) whereas in the large space it is 1.61 \(\mu_N^2\). In this case, where there is quenching in the low-lying sector this is more than compensated for by the strengths in the \(2\hbar\omega\) region.

For the other three cases, the reverse is true. For example in the case \(x = 1.5, y = 0.5\) the summed strength in the \(0p\) space is 2.58 \(\mu_N^2\), whereas in the \(0p+2\hbar\omega\) space it is 2.56 \(\mu_N^2\). If we look at B(M1), the drop in the sum is larger, from 2.82 \(\mu_N^2\) to 2.28 \(\mu_N^2\), a decrease of 19%. The orbital summed strength B(M1) on the other hand increases for this case from 0.46 \(\mu_N^2\) to 0.57 \(\mu_N^2\). Indeed the orbital summed strength increases for all four \((x, y)\) combinations considered here.

From the above we see that the M1 strength is redistributed but not conserved. There is definitely a quenching in the low energy sector but in three out of four cases considered here the missing strength does not reappear in toto in the high energy sector.

1. Isovector B(E2) – \(J = 0^+\) \(T = 0 \rightarrow J = 2^+\) \(T = 0\)

In the small space there are eight \(J = 2^+\) \(T = 0\) states. In that space (with the effective charges \(e_p = 1.5, e_n = 0.5\)), 95% (80.6 \(e^2\) fm\(^4\)) of the total calculated B(E2) strength of 82.7 \(e^2\) fm\(^4\) is concentrated in the transition to the lowest \(J = 2^+\) \(T = 0\) state at 3.80 MeV. In the larger space, the lowest \(J = 2^+\) \(T = 0\) state now at 4.79 MeV is still dominant with a B(E2) of 23.2 \(e^2\) fm\(^4\) and accounts for 55% of the total strength of 42.5 \(e^2\) fm\(^4\). In addition, another relatively strong \(J = 2^+\) \(T = 0\) state emerges among the ten lowest state, its energy is 4.40 MeV and its B(E2) is 6.12 \(e^2\) fm\(^4\). The B(E2)\(^{\text{isoscalar\uparrow}}\) sum, over the first 500 states in the large space, is slightly less than one half of the total B(E2)\(^{\text{isoscalar\uparrow}}\) sum over all the states in the small space calculation. The sum over the first ten states in the large space (up to about 45 MeV of excitation) adds up to 31.9 \(e^2\) fm\(^4\) or about 3/4 of the sum over the first 500 states.

The dominance of one single state suggest collectivity. In \(N = Z\) nuclei the isoscalar B(E2) is proportional to \((e_p + e_n)^2\). Usually the values of the effective charges \(e_p, e_n\) are introduced in \(\Delta N = 0\) calculations to reproduce the effects of neglecting higher shells. The comparison of the large space and small space results suggests that \(e_p + e_n\) should be more like 1.4 instead of 1.5 + 0.5 = 2. On this point it was noted by Abbas and Zamick [23] that the isoscalar effective charges is less than two for light nuclei and only reaches the value of two asymptotically for large \(A\).

2. Isovector B(E2) – \(J = 0^+\) \(T = 0 \rightarrow J = 2^+\) \(T = 1\)

In general this isovector B(E2) sum is smaller than the corresponding isoscalar one. In the small space, with a total of seven \(J = 2^+\) \(T = 1\) states the isovector B(E2)\(^{\uparrow}\) sum is dominated by the second \(J = 2^+\) \(T = 1\) state (here at 15.78 MeV), which contributes 2.61 \(e^2\) fm\(^4\) out of a total value of 3.39 \(e^2\) fm\(^4\) for the B(E2) sum. In the large space calculation the largest contribution to the isovector B(E2) sum from among the few lowest-lying state again comes from the second excited \(J = 2^+\) \(T = 1\) state now at 20.08 MeV above the ground state with a B(E2) of 1.56 \(e^2\) fm\(^4\). The sum over the B(E2)\(^{\uparrow}\)'s to the first ten \(J = 2\) \(T = 1\) states (which corresponds to an excitation energy of up to 53 MeV) is only 2.91 \(e^2\) fm\(^4\), but when the 500 lowest states are taken into account the B(E2) sum increases about seven-fold to 21.28 \(e^2\) fm\(^4\). It would thus seem that there are substantial isovector B(E2) contribution from very high lying states. The isovector B(E2)’s for \(N = Z\) nuclei are proportional to \((e_p - e_n)^2\); reductions in both \(e_p\) and \(e_n\) will necessarily reduce this quantity, unless such reduction are carried out in such a way that the difference \(e_p - e_n\) is also reduced. There is evidence that indeed \(e_p - e_n\) should be less than unity, from theoretical calculations [25,26].

III. COMPARISON OF RESULTS OF B(E2) IN \(^{12}\text{C}\) IN THE SMALL SPACE (\(\Delta N = 0\)) AND THE LARGE SPACE (\(\Delta N = 2\))

As a counterpoint to the high lying M1 transitions we wish to now study what happens to low lying states, e.g. the \(2^+_1\) state, when we increase the model space and when \(x\) and \(y\) are varied. In the small space (\(0p\)) we use effective charges \(e_p = 1.5, e_n = 0.5\). In the large space (\(0p+2\hbar\omega\)) we use bare charges \(e_p = 1, e_n = 0\). The \(\Delta N = 2\hbar\omega\) admixtures should in principle at least justify the use of effective charges in the small space. The results of the B(E2)’s to individual states are presented in table [II] and the summed strengths in table [IV].

A. The \(x = 1, y = 1\) results
and from phenomenological fits [27,28]. A popular value is 0.7.

B. Varying x and y

1. Isoscalar B(E2) – J = 0+ T = 0 → J = 2+ T = 0

We see from table IV the dominant B(E2)’s, the isoscalar ones, depend only weakly on the particular (x, y) combination that is used. For each of the three main blocks in table IV (small space, large space, the lowest states in the large space) the variation of the isoscalar B(E2) among the different (x, y) is less than 9%. Typically, with x = 1 decreasing the tensor force from y = 1 to y = 0.5 has a negligible decreasing effect (less than 1%). Increasing the spin-orbit from x = 1 to x = 1.5 both with y = 1 decreasing the isoscalar B(E2) by 3-7%, and simultaneously decreasing the tensor and increasing the spin-orbit decreases the isoscalar B(E2) by a total of 5 to 9%.

The isoscalar B(E2) does depend critically, however on the space in which the calculation is carried out and other effective charges used. The approximate B(E2) values in e² fm⁴ are 90- for the small space (with \( e_p = 1.5, e_n = 0.5 \)), 30 for the first ten \( 2^+ \) states in the large spaces and 40 for the large space. We thus see that the isoscalar B(E2) sum decreases as we calculate it in larger spaces (with the actual physical charges \( e_p = 1, e_n = 0 \)).

Several conclusions can be drawn from the results which were outlined in the previous paragraph. The isoscalar B(E2)’s for \( N = Z \) nuclei are proportional to \((e_p + e_n)^2\). We use effective charges \( e_p = 1.5 \) and \( e_n = 0.5 \) in the small space to compensate for the non-inclusion of higher shells. These charges lead to B(E2)’s that are twice as large in the \( \Delta N = 0 \) calculation as in the \( \Delta N = 2 \) calculation. If we use the large (\( \Delta N = 2 \)) space calculation as our standard, it would seem the effective charges \( e_p + e_n = 2 \) used in the small space are too large and that this sum should be smaller by a factor of about 1.4 (i.e. \( e_p + e_n \approx 1.4 \)) at least for \( 12^C \) and possibly more general in the 0p shell.

The use of effective charges that are too large in the small space (\( \Delta N = 0 \) calculations leads to exaggerated B(E2)’s and hence to exaggerated deformations. Such a calculation makes \( 12^C \) appear more deformed than it really is.

In the small space at least 96% of the dominant isoscalar B(E2) sum is due to one state, the lowest \( J = 9^+ T = 0 \) state which is calculated to have an excitation energy of about 3.8-4.1 MeV (for the different (x, y) combinations).

In the large space that state moves to 4.6-4.9 MeV (for the different (x, y) combinations) and its isoscalar B(E2) value increases but only 10-12%; however, now for all the (x, y) combinations there is another strong state (with strength 20-30% that of the first excited state), which lies high at 42-44 MeV, and is the eight or ninth excited \( J = 2^+ T = 0 \) state. Together, these two strong states supply about 70% of isoscalar B(E2) strength summed over the first 500 \( J = 2^+ T = 0 \) states in the large space.

2. Isovector B(E2) – J = 0+ T = 0 → J = 2+ T = 1

Here there is a marked difference in the isovector B(E2) values between small and large spaces. The small space isovector values in e² fm⁴ range from about 3-5 (depending on the (x, y) values) while in the large space the sums cluster between 21.2 and 21.4 for all of our (x, y) combinations. In the small space there are only seven states with \( J = 2^+ T = 1 \) with the third state at about 18 MeV supplying 65-80% of the total isovector B(E2) strength.

The great increase in the summed isovector B(E2) value when one goes to the large space is not due to the few lowest-lying \( J = 2^+ T = 1 \) states. In the lowest part of table IV we see that in the ten lowest \( J = 2^+ T = 1 \) states in the large space we obtain an isovector B(E2) sum that is 15-20% less than our sum in the small space and is less than 1/5 of the total summed B(E2) strength over the first 500 states in the large space. It will be interesting to see if there are one or a few very strong very high lying \( J = 2^+ T = 1 \) states or whether this increase is a cumulative effect of many states. Another possibility is that we are getting into the region of spurious states.

In the small space (and also for the ten low-lying states of the large space) the summed B(E2) increasing by 5% when the tensor force is decreased, increases by 22-33% when the spin-orbit is increased, and by 30-40% when both of the above effects take place. Very interestingly, however, in the large space, the summed isovector B(E2) is totally insensitive to the (x, y) values, varying by less than 1% among all the (x, y) combinations.

IV. ADDITIONAL REMARKS

The main problem addressed in this work is the fact that in a straightforward 0p calculation with realistic G matrix elements obtained in a non-relativistic calculation the computed value of B(M1)↑ from ground the \( J = 1^+ T = 1 \) state (0.89 \( \mu_N^2 \)) is much lower than the experimental value 2.63 \( \mu_N^2 \). This was already noted in the works of Zheng et al. [13,17]. It should be emphasized that in those works, as well as in this one, the single particle energies are calculated with the same interaction that is used between valence nucleons.

In the small space the B(M1) can be enhanced by making the spin-orbit interaction stronger [15,17] and this can be justified by the Dirac phenomenology [16]. The situation with the tensor interaction is a bit more complicated. Since the Dirac phenomenology in its original version [14] does not include pions, the tensor interaction is not affected. However, in the universal scaling mechanism of
G. E. Brown and M. Rho \( [19] \) all mesons except the pion become less massive inside the nucleus as compared with free space. This leads to a weaker tensor interaction inside the nucleus. This would be helpful for enhancing the value of \( B(M1) \) to the \( J = 1^+ T = 1 \) state.

However, for other nuclei – \( ^6\text{Li} \), \( A = 14 \) and \( ^{16}\text{O} \) – another mechanism for causing the tensor interaction to become weaker was given by Zamick, Zheng and Fayache \( [20] \). This was simply to allow \( \Delta N = 2 \) or higher admixtures into the valence space, i.e. do larger shell model calculations or include the appropriate Goldstone diagrams in perturbation theory.

Whereas the quadrupole moment of the deuterium is positive, that of the \( J = 1^+ T = 0 \) ground state of \( ^6\text{Li} \) is negative \( (-0.082 \text{ e fm}^2) \). In a \((0p)^2\) calculation without a tensor interaction, Q will be positive. With the realistic Nijm II G matrix \( [29] \) the value of Q is calculation to be \(-0.360 \text{ e fm}^2\) – too negative, indicating the tensor interaction is too strong. However, when \( 2\hbar \omega \) admixtures enter Q becomes \(-0.251 \text{ e fm}^2\), and with up to \( 4\hbar \omega \) admixtures Q is calculated to be \(-0.0085 \text{ e fm}^2\). Thus it appears that high shell admixtures cause the tensor interaction to appear weaker in the valence space.

In a similar vein, for the \( A = 14 \) beta decay \( ^{14}\text{C} \) \( J = 0^+ T = 1 \) \( \rightarrow ^{14}\text{N} \) \( J = 1^+ T = 0 \) the experimental Gamow-Teller matrix element is very close to zero. It was shown by B. Jancovici and I. Talini \( [30] \) (see also D. R. Inglis \( [31] \)) that one needs a tensor interaction in the \((0p)\) shell space in order to get a vanishing matrix element. Again with the Nijm II interaction, which has a tensor part, the \( 0\hbar \omega \) gives \( B(GT) = 3.967 \) – far from zero. With \( 2\hbar \omega \) admixtures this gets reduced to 1.795. The effective tensor interaction in the valence space is weaker. If we further enhance the spin-orbit interaction by about 50\% (as per discussion in the previous section) we can get \( B(GT) \) to be zero.

In \( ^{16}\text{O} \) the isospin splitting of the \( J = 0^+ T = 1 \) and the \( J = 0^+ T = 0 \) states is 1.845 MeV. In the absence of a tensor interaction these two states are nearly degenerate. With Nijm II the splitting becomes too large, 2.81 MeV. In zero these two states are one particle, one hole state – admixture of the configurations \( (1s_{1/2} 0p_{1/2}^{-1}) \) and \( (0d_{3/2} 0p_{3/2}^{-1}) \). When over and above these basic configurations we allow \( 2\hbar \omega \) excitations the energy splitting of \( T = 1 \) and \( T = 0 \) is reduced from 2.81 MeV to 1.93 MeV, much closer to experiment. Again it appears that higher shell admixtures cause the tensor interaction to appear weaker.

How do the higher shell admixtures affect the present problem \( ^{12}\text{C}^2 \)? Logically, if we expect that higher shell admixtures causes the tensor interaction to be weaker, we should do the large space calculation with \( y = 1 \), i.e. with the ‘bare’ tensor interaction. However, as seen in table \( 6 \), the \( y = 1 \) calculations in the large space yields smaller values of \( B(M1) \) than do those in the small space. For example for \( x = 0 \) the (small,large) results are \( 0.89 \mu_N^2 \) and \( 0.54 \mu_N^2 \) (recall that experiment is \( 2.63(8) \mu_N^2 \)). For \( x = 1.5 \) i.e. an enhanced spin-orbit interaction the corresponding values are \( 1.89 \mu_N^2 \) and \( 1.29 \mu_N^2 \).

The above results should not be totally unexpected. The isovector spin \( B(M1)_e \) is proportional via an isospin rotation to \( B(GT) \), the Gamow-Teller transition from \( (N,Z)=(6,6) \) to \( (7,5) \) or \( (5,7) \), and for GT transitions it has been noted that a quenching factor is generally needed. A popular choice is 0.5 for \( B(GT) \) or 0.7 for the matrix element \( M_{GT} \). Looking at \( B(M1)_e \) in table \( 6 \) we see that for \( x = 1, y = 1 \) it gets reduced from 0.58 \( \mu_N^2 \) to 0.31 \( \mu_N^2 \) when \( 2\hbar \omega \) admixtures are allowed. This is almost the factor of two that is often quoted.

So basically we are getting a quenching rather than the enhancement that we need. It would appear that the only way to get an enhancement towards the experimental value is to make the strength of the spin-orbit interaction \( x \) even larger and/or to invoke the universal scaling argument of G. E. Brown and M. Rho \( [19] \) to make the tensor interaction weaker. Which, or how much of these two mechanisms should be invoked will require further investigation.

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[1] F. Ajzenberg-Selove, Nucl. Phys. A 506, 1 (1990).
[2] M.S. Fayache, P. von Neumann-Cosel, A. Richter, Y.Y. Sharon and L. Zamick, preprint.
[3] E. G. Adelberger, R. E. Mars, K. A. Snover and J. E. Bussoletti, Phys. Lett. B 62, 29 (1976); Phys. Rev. C 15, 484 (1977).
[4] J. B. Flanz, R. S. Hicks, R. A. Cimdgren and G. A. Peterson, Phys. Rev. Lett. 43, 1922, (1979).
[5] H.-D. Gräf, W. Knüpfel, U. Krämer, P. von Neumann-Cosel, A. Richter and E. Spamer, Nucl. Phys. A, to be published.
[6] H. Sato and L. Zamick, Phys. Lett. 70B, 285 (1977).
[7] W. Ziegler, C. Rangacharyulu, A. Richter and C. Spiler, Phys. Rev. Lett. 65, 2515 (1990).
[8] K. Heyde and C. DeCoster, Phys. Rev. C 44, R2262 (1991).
[9] L. Zamick and D. C. Zheng, Phys. Rev. C 44, 2522 (1991); Phys. Rev. C 46, 2106 (1992).
[10] D. C. Zheng, L. Zamick and N. Auerbach, Ann. Phys. (N.Y.) 197, 343 (1990).
[11] N. Auerbach, G. F. Bertsch, B. A. Brown, and L. Zhao, Nucl. Phys. A 556, 190 (1993).
TABLE I. Comparison of the results of $\Delta N = 0$ (small space) and $\Delta N = 0 + 2\hbar\omega$ (large space) calculations of the value of $B(M1)$ from the ground state of $^{12}\text{C}$ to the lowest (and most strongly excited) $J = 1^+ T = 1$ state. The results are presented for different combinations of the spin-orbit ($x$) and tensor ($y$) strengths in the realistic interaction of Eq. (1).

|                | $x$ | $y$ | $E$(MeV) | $B(M1)^{a}$ | $B(M1)^{b}$ | $B(M1)^{c}$ |
|----------------|-----|-----|----------|--------------|--------------|--------------|
| small space    | 1   | 1   | 13.60    | 0.89         | 0.03         | 0.58         |
|                | 1.5 | 1   | 13.08    | 1.89         | 0.04         | 1.71         |
|                | 1   | 0.5 | 13.33    | 1.12         | 0.01         | 0.90         |
|                | 1.5 | 0.5 | 13.11    | 2.54         | 0.00         | 2.57         |
| large space    | 1   | 1   | 17.28    | 0.54         | 0.06         | 0.31         |
| $\Delta N = 0 + 2\hbar\omega$ | 1.5 | 1   | 16.46    | 1.29         | 0.02         | 0.98         |
|                | 1   | 0.5 | 16.84    | 0.88         | 0.026        | 0.60         |
|                | 1.5 | 0.5 | 16.28    | 1.96         | 0.026        | 1.83         |

Experiment\(^4\)  

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\(^a\) $g_{\pi} = 1, g_{\nu} = 0, g_{s\pi} = 5.586, g_{s\nu} = -3.726$.

\(^b\) $g_{\pi} = 0.5, g_{\nu} = -0.5, g_{s\pi} = 0, g_{s\nu} = 0$.

\(^c\) $g_{\pi} = 0, g_{\nu} = 0, g_{s\pi} = 0.5, g_{s\nu} = -0.5$.

\(^d\) From Ref. [5]. Also the isoscalar transition to the $J = 1^+ T = 0$ state at 12.71 MeV has a strength $B(M1) = 0.040(3) \mu^2_N$. 

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[5] G. F. Bertsch, Nucl. Phys. 74, 234 (1965).

[6] T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966).

[7] J. P. Elliott, Proc. Roy. Soc. London A 245, 128 (1958); 245, 562 (1958).

[8] A. Abbas and L. Zamick, Phys. Rev. C 21, 7381 (1980).

[9] L. Zamick and P. Federman, Phys. Rev. 177 #4, 1534 (1969).
TABLE II. Same as table I, but for the summed strengths. In the small space, the sum is over all eight states.

| Delta N = 0 |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
| small space$^a$ | $x$ | $y$ | B(M1) | B(M1)$_{l}$ | B(M1)$_{s}$ |  |
| 1 | 1 | 1.42 | 0.60 | 0.91 |  |
| 1.5 | 1 | 2.29 | 0.50 | 2.07 |  |
| 1 | 0.5 | 1.58 | 0.59 | 1.13 |  |
| 1.5 | 0.5 | 2.85 | 0.46 | 2.82 |  |

| Delta N = 0 + 2$\hbar\omega$ |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|
| large space$^b$ | $x$ | $y$ | B(M1) | B(M1)$_{l}$ | B(M1)$_{s}$ |  |
| 1 | 1 | 1.21 | 0.56 | 0.70 |  |
| 1.5 | 1 | 1.77 | 0.49 | 1.43 |  |
| 1 | 0.5 | 1.31 | 0.56 | 0.83 |  |
| 1.5 | 0.5 | 2.27 | 0.46 | 2.085 |  |

TABLE III. Comparison of experimental B(E2) values to the first 2$^+$ state in $^{12}$C with shell model calculations for different combinations of spin-orbit ($x$) and tensor ($y$) interaction strengths. Effective charges $e_p = 1.5$, $e_n = 0.5$ are used for $\Delta N = 0$ calculations. Bare charges $e_p = 1.0$, $e_n = 0.0$ are used for $\Delta N = 0 + 2\hbar\omega$ calculations.

| Experiments$^a$ | $x$ | $y$ | $E$ (MeV) | B(E2) ($e^2 fm^4$) | $E$ (MeV) | B(E2) ($e^2 fm^4$) |  |
|---|---|---|---|---|---|---|---|
| Experiment |  |  |  |  |  |  |  |
| small space | 1 | 1 | 4.44 | 39.9(2.2) | 16.11 | 2.6 |  |
| $\Delta N = 0$ | 1.5 | 1 | 3.80 | 80.6 | 15.78 | 2.6 |  |
| $e_p = 1.5, e_n = 0.5$ | 1 | 0.5 | 3.82 | 75.3 | 14.44 | 3.1 |  |
| 1.5 | 0.5 | 3.79 | 80.1 | 14.86 | 1.2 |  |
| 1 | 0.5 | 4.11 | 72.9 | 15.58 | 1.6 |  |

| large space | 1 | 1 | 4.79 | 23.2 | 20.08 | 1.6 |  |
| $\Delta N = 0 + 2\hbar\omega$ | 1.5 | 1 | 4.63 | 21.7 | 18.31 | 1.1 |  |
| $e_p = 1.0, e_n = 0$ | 1 | 0.5 | 4.69 | 23.2 | 19.49 | 1.5 |  |
| 1.5 | 0.5 | 4.87 | 21.2 | 17.94 | 2.3 |  |

$^a$sum over all eight $J = 1^+ T = 1$ states in the $\Delta N = 0$ space.
$^b$sum over all first ten $J = 1^+ T = 1$ states in the $\Delta N = 2$ space.
$^c$sum over the first 500 $J = 1^+ T = 1$ states in the $\Delta N = 2$ space.

$^a$From Ref. [4].
$^b$From Ref. [3].
TABLE IV. Comparison of the results in $\Delta N = 0$ (i.e. small) and $\Delta N = 2$ (i.e. large) spaces of shell model calculations for the sums of B(E2) transitions from the ground state of $^{12}$C to $J = 2^+ T = 0$ states. The results are presented for different combinations of spin-orbit ($x$) and tensor ($y$) strengths in the realistic interaction Eq. (1).

| $x$ | $y$ | isoscalar B(E2)$^a$ | isovector B(E2) |
|-----|-----|---------------------|-----------------|
|     |     | $J = 0^+ T = 0 \rightarrow J = 2^+ T = 0$ | $J = 0^+ T = 0 \rightarrow J = 2^+ T = 1$ |
| small space | 1 | 1 | 82.7$^a$ | 3.39$^a$ |
| $\Delta N = 0$ | 1.5 | 1 | 78.4 | 4.44 |
| total sum | 1 | 0.5 | 82.0 | 3.54 |
| $e_p = 1.5, e_n = 0.5$ | 1.5 | 0.5 | 75.3 | 4.80 |
| large space | 1 | 1 | 31.9$^a$ | 2.91$^a$ |
| $\Delta N = 0 + 2\hbar\omega$ | 1.5 | 1 | 29.8 | 3.55 |
| $e_p = 1, e_n = 0$ | 1 | 0.5 | 31.8 | 3.08 |
| | 1.5 | 0.5 | 29.6 | 3.87 |
| large space | 1 | 1 | 42.5$^a$ | 21.23$^a$ |
| $\Delta N = 0 + 2\hbar\omega$ | 1.5 | 1 | 41.1 | 21.40 |
| $e_p = 1, e_n = 0$ | 1 | 0.5 | 42.1 | 21.24 |
| | 1.5 | 0.5 | 39.9 | 21.36 |

$^a$sum over all eight $J = 2^+ T = 0$ states in the $\Delta N = 0$ space.
$^b$sum over all eight $J = 2^+ T = 1$ states in the $\Delta N = 0$ space.
$^c$sum over all first ten $J = 2^+ T = 0$ states in the $\Delta N = 2$ space.
$^d$sum over all first ten $J = 2^+ T = 1$ states in the $\Delta N = 2$ space.
$^e$sum over the first 500 $J = 2^+ T = 0$ states in the $\Delta N = 2$ space.
$^f$sum over the first 500 $J = 2^+ T = 1$ states in the $\Delta N = 2$ space.
$^g$The experimental value to the $2^+_1$ state at 4.44 MeV is B(E2)=$40 e^2 fm^4$ [13].