Microscopic optical potentials for Li isotopes

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Abstract. The microscopic optical potentials for Li isotopes (A=6,7) without free parameter are obtained by folding the microscopic optical potentials of their internal nucleons with density distributions generated from corresponding internal wave functions of Li isotopes. An isospin-dependent nucleon microscopic optical potential based on the Skyrme nucleon-nucleon effective interaction is used as the nucleon optical potential. Shell model is employed to construct the internal wave functions of Li isotopes and derive their density distributions of internal nucleons. The Li microscopic optical potentials are used to calculate the elastic-scattering angular distributions and reaction cross sections. The results reproduce experimental data well and are comparable to those calculated by phenomenological optical model potentials in many cases.

1 Introduction

The microscopic optical potential (MOP) has great significance in nuclear reaction theory and astrophysics. It is derived from nucleon-nucleon interaction theoretically and need not adjust its parameters to fit experimental data. Moreover, it can give guidance for nuclear reaction, especially in the research involving interaction systems without or lack of scattering measurement.

Nuclear reactions involving Li isotopes have been a subject of great interest for decades, not only because of their application in isotopes production and nuclear astrophysics, but also for the reaction mechanism as they are all well-known weakly bound nuclei \cite{1}. However, the scattering experimental data for them is not abundant. Therefore, microscopic optical potentials for Li isotopes will be useful to understand the reaction mechanism and analyze those nuclear reactions.

In the present work, the MOPs for Li isotopes are obtained by folding the MOPs of its constituent nucleons with their density distributions. An isospin-dependent nonrealistic nucleon MOP derived by using the Green’s function method based on the Skyrme nucleon-nucleon interaction is applied to be the MOP for internal nucleon. Shell model is used to construct the internal wave function and generated the nucleon density distributions. In order to evaluate the predictive power of the MOPs, they are used to calculate the elastic-scattering angular distributions and reaction cross sections and the calculated results are compared with those calculated by a phenomenological global optical potential (GOP) and experimental data.

This paper is organized as follow: the theoretical models and formulas of the MOPs are presented in Sec. 2; the calculated results and analysis are given in Sec. 3; the conclusions are drawn in Sec. 4 finally.

2 Theoretical model

Folding model is a common method to get the optical potential model of complex nucleus, in which the potential for the whole nucleus is considered to be the sum of its internal nucleons or clusters. Hence, the MOPs for the Li isotopes are expressed as

\[
U(\vec{R}) = \int U_n(\vec{R} + \vec{r}_n\delta) + U_p(\vec{R} + \vec{r}_p\delta) d\vec{r},
\]

where

\[
\int \rho_n(\vec{r}) d\vec{r} = N; \int \rho_p(\vec{r}) d\vec{r} = Z.
\]

\(U_n\) and \(U_p\) are the MOPs of neutron and proton from Ref. \cite{2,3} respectively. They are an isospin-dependent nonrealistic microscopic optical potential for nucleon base on Skyrme nucleon-nucleon effective interactions. Different Skyrme parameters, SGOII \cite{4} and SKC16 \cite{2}, are used to obtain the MOPs of internal nucleons for \(^6\)Li and \(^7\)Li respectively. The incident energy of each nucleon is assumed to be the total incident energy multiplied by \(1/A\). \(\vec{R}\) is the relative coordinate between the center of mass of target and projectile, \(\rho_n\) and \(\rho_p\) are the density distributions of neutron and proton of Li isotopes respectively. \(\rho_n\) and \(\rho_p\) are derived from the ground state internal wave function of Li, \(\Phi_{gr}\), directly and are expressed as

\[
\rho_{n/p}(\vec{r}) = \left| \Phi_{gr} \right| \sum_{i=1}^{A} \delta(\vec{r} - \vec{r}_i)\delta_{\tau_n/\tau_p} \left| \Phi_{gr} \right|,
\]

where \(\vec{r}_i\) is the coordinate of \(i\)th nucleon of Li relative to the center of mass of the projectile and \(\tau_i\) is its isospin. \(\tau_n\) and \(\tau_p\) are the isospins of neutron and proton respectively.
Table 1. The parameters of $\rho_n$ and $\rho_p$ derived from shell model. The value of $r_{rms}^{m}$ are taken from Ref. [1]

|        | $^6\text{Li}$ | $^7\text{Li}$ |
|--------|---------------|---------------|
| $r_{rms}^{m}$ (fm) | 2.54          | 2.50          |
| $\beta$ (fm$^{-2}$) | 0.2454        | 0.2743        |
| $a_n$ (fm$^{-3}$) | 0.0567         | 0.0921         |
| $b_n$ (fm$^{-5}$) | 0.0058         | 0.0081         |
| $a_p$ (fm$^{-3}$) | 0.0567         | 0.0621         |
| $b_p$ (fm$^{-5}$) | 0.0058         | 0.0076         |

Figure 1. The density distributions of neutron ($\rho_n$) and proton ($\rho_p$) of Li isotopes. The solid lines and dash lines represent $\rho_n$ and $\rho_p$ respectively. The black and red lines donate the results of $^6\text{Li}$ and $^7\text{Li}$ respectively.

The internal wave function of Li isotopes could be constructed in the manner of shell model. As we only concern the properties of nucleus in the ground state, a full $0\hbar\omega$ harmonic oscillator space, $(1s)^4(1p)^{A-4}$, is used to obtain $\Phi_{gs}$. Hence, it is expressed as

$$\Phi_{gs} = \mathcal{A} \left\{ \sum_{LS} \prod_{i=1}^{A} \varphi(\vec{r}_i) \zeta \right\}, \quad (4)$$

where $\mathcal{A}$ is the antisymmetric operator and $\varphi$ is harmonic oscillator wave function. $\zeta$ represents the spin and isospin part. $\sum$ represents the summation for all functions which meet the requirements of total angular momentum by $LS$ coupling method.

$\rho_n$ and $\rho_p$ derived from Eq. (3) and (4) have a unified form as

$$\rho_{n(p)}(\vec{r}) = (a_{n(p)} + b_{n(p)} \beta r^2) \exp(-A A - 1 \beta r^2). \quad (5)$$

$\beta = m\omega/\hbar$ is the harmonic oscillator constant and its value is determined by the matter root mean square radius $r_{rms}^{m}$ [1]. The parameters are listed in Table 1 and the density distributions are plotted in Fig. 1.

Furthermore, the spin-orbit coupling potential of the MOP, $U^{so}$, is calculated in a similar way with Eq. (3), while the effect of the nucleon-nucleus interaction should be taken into account, as $U^{so}$ contributes mainly in the surface region of target nucleus. The nucleon spin-orbit coupling potential, $U^{so}_n$ for neutron and $U^{so}_p$ for proton, taken from Ref. [2] should be multiplied with $m^*/m$, where $m^*$ is the nucleon effective mass inside the target nucleus. Hence, $U^{so}$ is expressed as

$$U^{so}(\vec{R}) = \int \frac{m_n}{m} U^{so}_n(\vec{R} + \vec{r}) \rho_n(\vec{r}) + \frac{m_p}{m} U^{so}_p(\vec{R} + \vec{r}) \rho_p(\vec{r}) d\vec{r}. \quad (6)$$

3 Calculated result and discussion

Based on the MOPs of Li isotopes, elastic scattering angular distributions and reaction cross sections are calculated below 100 MeV. Moreover, comparison with global optical potential [5, 6] and experimental data is made in order to evaluate their predictive power.
for $^6$Li and $^{208}$Pb reaction calculated by the MOP are about 150mb less than the experimental data at EL $\geq$ 30-40 MeV, while the MOP reproduce correct cross sections for $^6$Li + $^{27}$Al around EL $\geq$ 10 MeV. On the other hand, the reaction cross sections of $^6$Li calculated by the MOP are smaller than those calculated by the GOP at low incident energies. $\sigma_R$ is expressed as

$$\sigma_R = -\frac{2}{\hbar v} \langle \chi^+ | W | \chi^+ \rangle,$$

where $\chi^+$ represents the outgoing wave, $W$ is the imaginary part of the optical potential and $v$ is the relative velocity of target and projectile in center of mass frame. A stronger $W$ of the MOP, relative to the real part of the GOP, is needed to obtain correct $\sigma_R$ for heavy target nucleus at low incident energies. Breakup effect is expected to generate this correction, as it can provide an absorptive contribution to the imaginary part of optical potential and a repulsive contribution to the real part [28].

4 Conclusion

The microscopic optical potentials for Li isotopes without free parameter are obtained by folding model base on Skyrme nucleon-nucleon effective interaction. Shell model is used to construct the internal wave functions of Li isotopes. The elastic scattering angular distributions and reaction cross sections for targets from $^{27}$Al to $^{208}$Pb at incident energies no more than 100 MeV are calculated by the Li isotopes microscopic optical potentials. Generally, the MOP obtained can well reproduce the elastic scattering angular distributions and performs better than the GOP in some cases, especially for $^7$Li. The MOPs can both reproduce the reaction cross sections reasonably. The divergence with experimental data is expected to be solved or impaired by taking the breakup effect into account, which will be our next subject.

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