On the Coulomb interaction in chiral-invariant one-dimensional electron systems

S. Bellucci 1 and J. González 2

1Laboratori Nazionali di Frascati. INFN. P. O. Box 13. I-00044 Frascati. Italy.
2Instituto de Estructura de la Materia. Consejo Superior de Investigaciones Científicas. Serrano 123, 28006 Madrid. Spain.

We consider a one-dimensional electron system, suitable for the description of the electronic correlations in a metallic carbon nanotube. Renormalization group methods are used to study the low-energy behavior of the unscreened Coulomb interaction between currents of well-defined chirality. In the limit of a very large number $n$ of subbands we find a strong renormalization of the Fermi velocity, reminiscent of a similar phenomenon in the graphite sheet. For small $n$ or sufficiently low energy, the Luttinger liquid behavior takes over, with a strong wavefunction renormalization leading to a vanishing quasiparticle weight. Our approach is appropriate to study the crossover from two-dimensional to one-dimensional behavior in carbon nanotubes of large radius.

71.27.+a, 73.20.D, 05.30.Fk

The recent experimental availability of single-walled fullerene nanotubes has renewed the interest in the study of one-dimensional electron systems [1–4]. In one spatial dimension the Luttinger liquid concept replaces the Fermi liquid picture, and provides the paradigm of a system with strong electronic correlations [5]. The investigation of the generic electronic instabilities of metallic nanotubes has been accomplished in Refs. [6] and [7]. There have been also recent attempts to look for signatures of Luttinger liquid behavior in single-walled nanotubes that are packed in the form of ropes [8]. Another interesting instance that seems to be feasible from the experimental point of view is that of nanotubes in the absence of external screening charges or, at least, with a screening length much larger than the typical transverse dimension. The phenomenology of these systems has been studied in Refs. [9–12]. Anyhow, as long as the description of the Luttinger liquid behavior is usually made under the assumption of a short-range interaction, the carbon nanotubes with unscreened Coulomb interaction should deserve further theoretical analysis, devoted to ascertain possible deviations from the standard Luttinger liquid picture.

In the particular case of the metallic carbon nanotubes [13], the Coulomb interaction is also special in that the long-range potential does not lead to the hybridization between left- and right-moving electrons in the nanotube. At half-filling, the metallic nanotubes have two Fermi points, characterized respectively by the large momenta $K_F$ and $-K_F$, with the typical band structure shown in Fig. [1]. The underlying lattice is such that it allows to arrange the modes of the two linear branches at each Fermi point into a Dirac-like spinor, whose components stand for the respective amplitudes in the two sublattices of the honeycomb lattice [14]. The kinetic part of the hamiltonian can be approximated as $H_{\text{kin}} = -t a \delta k_x \sigma_1$, where $a$ is the lattice spacing and $t$ is the hopping parameter. This means that the right-movers have an amplitude $\Psi(x)$ that alternates sign from one sublattice to the other, while the left-movers keep the same sign on both of them. When projecting the Coulomb interaction to the longitudinal dimension of the tube, one has to sum over the points of the two transversal rings of the tube $C_x$ and $C_x'$.

$$
(\Psi_\alpha(x)\Psi_\beta(x') | V(x-x') | \Psi_\gamma(x)\Psi_\delta(x')) = \sum_{i \in C_x, j \in C_x'} \Psi_\alpha^+(r_i) \Psi_\gamma(r_i) \frac{1}{|r_i - r_j'|} \Psi_\beta^+(r_j') \Psi_\delta(r_j')
$$

It becomes clear that, for a distance $x - x'$ much greater than the lattice spacing, the matrix element is only nonvanishing when $\alpha$ and $\gamma$, as well as $\beta$ and $\delta$, have the same (left or right) chirality.

Thus, the relevant interaction in the study of the metallic carbon nanotubes turns out to be the long-range Coulomb interaction between currents of well-defined chirality. In this work we study the genuine effects due to the $1/|x|$ interaction in this kind of one-dimensional systems, focusing on the physics at each of the Fermi points. We disregard in this way backscattering processes and residual short-range interactions mixing chiralities, as they have a smaller nominal strength ($\sim 0.1 e^2/n$ in terms of the number $n$ of subbands [13,14] and they stay small down to extremely low energies ($\sim t \exp(-75)$ [14]). Our present purpose is to discern the low-energy properties of the system and, in particular, whether the long-range interaction remains unscreened to arbitrarily large distances. It is known that the bosonization approach yields divergent results for some of the observables like, for instance, the plasmon velocity at vanishing momentum. This shortcoming is remedied in practice by introducing some infrared cutoff dictated by the external environment of the one-dimensional system. Anyhow, it is worthwhile to analyze to what extent the singular interaction may be renormalized in the infrared by means of a dynamical screening effect. This problem may be also relevant when studying the properties of nanotubes of large radius, since in those systems a crossover from two-dimensional to one-dimensional behavior is to be expected when taking the infrared limit. It is known that the
Coulomb interaction is strongly renormalized on the graphite sheet \[18\], what points again at the question about the nature of the screening effects in carbon nanotubes with large number of subbands.

In this Letter we use renormalization group (RG) methods to find the low-energy effective theory of the 1/|x| interaction. To be more precise, we pose the problem of a one-dimensional model with an interaction hamiltonian

\[
H_{\text{int}} = \frac{e^2}{8\pi} \int dx dx' \left( \Psi_L^* (x) \Psi_L (x) + \Psi_R^* (x) \Psi_R (x) \right) \frac{1}{|x - x'|} \left( \Psi_L^* (x') \Psi_L (x') + \Psi_R^* (x') \Psi_R (x') \right)
\]

where \(\Psi_L (x)\) and \(\Psi_R (x)\) are the electron field operators for the left and the right branch of the linear dispersion relation, respectively. The RG method is a sensible approach to deal with this problem since the 1/|x| interaction potential (as well as the \(\delta (x)\) potential) gives rise to a marginal four-fermion interaction. The scaling dimension of the electron field \(\Psi (x)\) is \(-1/2\), in length units. This means that the interaction hamiltonian in (2) scales appropriately, with a dimensionless coupling constant \(e^2\) (in units in which \(h = c = 1\)), as the energy scale is reduced down to the Fermi level \([13]\).

However, the drawback in dealing with (2) is that it contains a highly nonlocal operator, which makes unclear the applicability of RG methods, usually devised to deal with a set of local operators. This problem can be circumvented by introducing a local auxiliary field to propagate the Coulomb interaction. The hamiltonian can be written in the form

\[
H = iv_F \int dx \left( \Psi_R^* (x) \partial_x \Psi_R (x) - \Psi_L^* (x) \partial_x \Psi_L (x) \right) + e \int dx \left( \Psi_L^* (x) \Psi_L (x) + \Psi_R^* (x) \Psi_R (x) \right) \phi (x)
\]

where the \(\phi (x)\) field propagates the interaction

\[
\delta (T \phi (x, t) \phi (x', t')) = \frac{1}{4\pi} \delta (t - t') \frac{1}{|x - x'|}.
\]

We may think of \(\phi (x)\) as the scalar potential in three-dimensional quantum electrodynamics. However, the differences with that theory in the present case are notorious since the propagation of \(\phi (x)\) is that of a genuine field in three spatial dimensions, while the electrons are confined to one dimension. In general, one may expect a better infrared behavior in the present model. The propagator of the \(\phi (x)\) field in momentum space can be read from the relativistic expression \([20]\), after sending the speed of light to infinity,

\[
i(T \phi (x, t) \phi (x', t')) = \int \frac{dq d\omega}{(2\pi)^2} \int \frac{dq_y dq_z}{(2\pi)^2} e^{iq (x - x')} e^{-i\omega (t - t')} \frac{e^{i\omega (t - t')}}{q^2 + q_y^2 + q_z^2 - i\epsilon}
\]

The usual one-dimensional propagator \(\sim \log (|q|/\Lambda)\) is recovered from (3) upon integration of the dummy variables \(q_y\) and \(q_z\). We remark that the ultraviolet cutoff \(\Lambda\) for excitations along the \(y\) and \(z\) transverse directions is needed when projecting the three-dimensional interaction down to the one-dimensional system.

The usefulness of the representation (3) can be appreciated in the renormalization of the model at the one-loop level. We study the scaling behavior of the irreducible functions as the bandwidth cutoff \(E_c\) is sent towards the Fermi level, \(E_c \rightarrow 0\). The self-energy to the one-loop order is

\[
i\Sigma(k, 0) = i e^2 \int_{-E_c}^{E_c} \frac{dp}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega_p}{2\pi} \frac{v_F (p + k)}{\omega_p^2 + v_F^2 (p + k)^2} \int \frac{d p_y d p_z}{(2\pi)^2} \frac{1}{p^2 + p_y^2 + p_z^2}
\]

The limit \(k \rightarrow 0\) has to be taken carefully in this expression, by first combining the two denominators with the use of Feynman parameters \([22]\). Finally we get

\[
i\Sigma(k, 0) = \frac{i}{4\pi v_F} \int_0^{E_c} \frac{du}{\sqrt{u}} \int_{-E_c}^{E_c} \frac{dp}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega_p}{2\pi} \frac{v_F k}{\omega_p^2 + p^2 + v_F^2 k^2 u (1 - u)}
\]

\[
\approx i e^2 \frac{v_F^2 k \log E_c}{4\pi^2}
\]

The term linear in \(k\) in \(\Sigma(k, 0)\) represents a renormalization of the Fermi velocity, which grows upon integration of the high-energy modes. Obviously, there is no correction linear in \(\omega_k\) renormalizing the electron wavefunction at the one-loop level. This is consistent with the fact that the integration of high-energy modes at \(\sim E_c\) does not renormalize the three-point vertex \(\Gamma\). We stress the difference of the logarithmic renormalization of \(v_F\) with respect to the usual
finite corrections due to a short-range interaction. The nontrivial scaling of \( v_F \) is a genuine effect of the long-range Coulomb interaction, which also takes place in higher dimensions \([15, 21]\).

Incidentally, the above computation exemplifies how the Ward identity that ensures the integrability of the Luttinger model does not hold in the present case. The Ward identity is a relation between the electron Green function \( G(p, \omega_p) \) and the three-point vertex \( \Gamma(p, \omega_p; k, \omega_k) \) at a given branch \([6]\). For the right-handed modes, for instance, it is

\[
\Gamma(p, \omega_p; k, \omega_k) = \frac{G^{-1}(p, \omega_p) - G^{-1}(p - k, \omega_p - \omega_k)}{\omega_k - v_F k}
\]  

(8)

By focusing on the singular dependences on the bandwidth cutoff \( E_c \), one can check that \([6]\) is already violated in our model to first order in perturbation theory. Actually, the dependence of the vertex \( \Gamma \) on the variables \((k, \omega_k)\) of the external interaction line is

\[
\Gamma(p, \omega_p; k, \omega_k) \approx 1 - \frac{e^2}{4\pi^2} \frac{k \log \Lambda}{\omega_k - v_F k} + \ldots
\]

(9)

We notice that the Ward identity would be satisfied if the scaling could be implemented simultaneously in the transverse ultraviolet cutoff \( \Lambda \) and the bandwidth cutoff \( E_c \), that is, by taking \( \Lambda = E_c \). However, in a real system the scaling in \( \Lambda \) gets locked by the finite cross section of the wire, while only the scaling in the longitudinal direction operated by \( E_c \) is allowed. In this respect, the gauge invariance of quantum electrodynamics is broken by the anisotropy of the electron system, as felt by the propagation of the three-dimensional electromagnetic field.

The renormalization of \( v_F \) at the one-loop level is not, in general, a sensible effect from the physical point of view, since the propagator of the electron system, as felt by the propagation of the three-dimensional electromagnetic field. The expression (11) provides a sensible approximation for the scalar propagator, as it incorporates the effect of plasmons in the model. Thus, our approach is that of using the scalar propagator (11) in the renormalization of the Fermi velocity and the electron wavefunction. We compute the electron self-energy by replacing the Coulomb potential by the dressed interaction (11)

\[
i\Sigma(k, i\omega_k) = \frac{e^2}{2\pi} \int_{-E_c}^{E_c} \frac{dp}{2\pi} \int_{-\infty}^{+\infty} \frac{d\omega_p}{2\pi} \frac{1}{i(\omega_p + i\omega_k) - v_F(p + k)} \frac{\log(|p|/\Lambda)}{1 - n \frac{e^2}{4\pi^2} \frac{v_F p^2}{\omega_p + \omega_k} \log(|p|/\Lambda)}
\]

(12)

Alternatively, one can think of the diagrammatics encoded in Eq. (12) as the leading order in a \( 1/n \) expansion, in a model with \( n \) different electron flavors. It can be shown that this approximation reproduces the exact anomalous dimension of the electron field in the Luttinger model with conventional short-range interaction \([7]\). In our case,
such approximation to the self-energy is also justified since it takes into account, at each level in perturbation theory, the most singular contribution at small momentum transfer of the interaction. Due to the cancellation of fermion loops with more than two interaction vertices which still takes place in the same way as in the Luttinger model, the representation (12) for the self-energy only misses the effects of vertex and electron self-energy corrections, which may be incorporated consistently in the RG framework by an appropriate scaling of the renormalized parameters.

The only contributions in (12) depending on the bandwidth cutoff are terms linear in \( \omega_k \) and \( k \). In this respect, it is worth mentioning that, although the usual perturbative approach gives rise to poles of the form \( k^2/(\omega_k - v_F k) \) in the self-energy [26], these do not arise in the GW approximation. There is no infrared catastrophe at \( \omega_k \approx v_F k \), because of the correction in the slope of the plasmon dispersion relation with respect to its bare value \( v_F \).

Because of the correction in the slope of the plasmon dispersion relation with respect to its bare value \( v_F \), the result that we get for the renormalized electron propagator is

\[
G^{-1}(k, \omega_k) = Z_{\Psi}^{-1}(\omega_k - v_F k) - \Sigma(k, \omega_k) \\
\approx Z_{\Psi}^{-1}(\omega_k - v_F k) + Z_{\Psi}^{-1}(\omega_k - v_F k) \frac{1}{n} \int_{E_c}^{E_F} \frac{dp}{|p|} \frac{(1 - f(p))^2}{2 \sqrt{f(p)} \left(1 + r \sqrt{f(p)}\right)^2} \\
- Z_{\Psi}^{-1} k \frac{e^2}{4\pi^2} \int_{E_c}^{E_F} \frac{dp}{|p|} \frac{f(p)^{3/2} + (4r/3 + r^3/3)f(p) + r^2 \sqrt{f(p)} + r/3}{f(p)^{3/2} \left(1 + r \sqrt{f(p)}\right)^3}
\]

(13)

where \( f(p) \equiv 1 - ne^2 \log(|p|/\Lambda)/(2\pi^2 \tilde{v}_F) \), \( r \equiv \tilde{v}_F/v_F \) and \( Z_{\Psi}^{-1/2} \) is the scale of the bare electron field compared to that of the cutoff-independent electron field

\[
\Psi_{bare}(E_c) = Z_{\Psi}^{1/2} \Psi.
\]

In the RG approach, we require the cutoff-independence of the renormalized Green function, since this object leads to observable quantities in the quantum theory. For this purpose, the quantities \( Z_{\Psi} \) and \( v_F \) have to be promoted to cutoff-dependent effective parameters, that reflect the behavior of the quantum theory as \( E_c \to 0 \) and more states are integrated out from high-energy shells of the band. Regarding the problem of self-consistency for the renormalized value \( \tilde{v}_F \) of the Fermi velocity, we find two possible solutions leading to different physical pictures:

i) Large-n limit solution. In this limit we know that the expression (14) gives the exact result for the polarization operator, since any vertex or self-energy correction makes any diagram subdominant from the point of view of the \( 1/n \) expansion. Then the correct choice for \( \tilde{v}_F \) has to be the fixed-point value of the Fermi velocity. Self-consistency is therefore attained by requiring that the solution of the scaling equation

\[
E_c \frac{d}{dE_c} v_F(E_c) = - \frac{e^2}{4\pi^2} \left(1 + r \sqrt{f(E_c)}\right)^2 \frac{1}{f(E_c)^{3/2} \left(1 + r \sqrt{f(E_c)}\right)^3}
\]

(15)

matches the fixed-point \( \tilde{v}_F \) in the limit \( E_c \to 0 \). It can be checked, however, that any finite value of \( \tilde{v}_F \) in the right-hand-side of (15) does not lead to a strong enough flow to reach \( \tilde{v}_F \) at \( E_c = 0 \). The only self-consistent solution is found for \( \tilde{v}_F = \infty \). For this value the right-hand-side of (15) has a finite limit, which produces the asymptotic scaling of the Fermi velocity \( v_F(E_c) \sim e^2/(12\pi^2) \log(E_c) \).

ii) One-band solution. If we stick to the picture in which we only pay attention to the left and right linear branches of the dispersion relation, we have to assume that (10) only provides an approximate expression for the polarization operator. The incomplete cancellation between electron self-energy and vertex corrections to that object arises from the mismatch between \( E_c \) and the transverse cutoff \( \Lambda \). As a consequence of that, the renormalized Fermi velocity in (10) gets an effective dependence on the variable \( \log(E_c/\Lambda) \) and \( \tilde{v}_F \) is to be taken as the scale dependent Fermi velocity, \( \tilde{v}_F = v_F(E_c) \). With regard to the carbon nanotubes, this is consistent with the regime in which the scaling has progressed to distances larger than the radius of the nanotube. The RG flow equations then turn out to be

\[
E_c \frac{d}{dE_c} \log Z_{\Psi}(E_c) = \frac{\left(1 - \sqrt{f(E_c)}\right)^2}{8 \sqrt{f(E_c)}}
\]

(16)

\[
E_c \frac{d}{dE_c} v_F(E_c) = - \frac{e^2}{4\pi^2} \sqrt{f(E_c)} - 4/3 + 1/ \left(3f(E_c)^{3/2}\right)
\]

(17)

Eq. (17) is now the requirement of self-consistency, with \( \tilde{v}_F = v_F(E_c) \). As mentioned before, the three-point vertex only gets the cutoff dependence given by the wavefunction renormalization in (16). This means that the electron
charge is not renormalized in low energies in our field theory framework. The behavior of the effective interaction is therefore completely encoded in Eq. (17), which can be rewritten for the effective coupling constant \( g \equiv e^2/(4\pi^2v_F) \) in the form

\[
E_c \frac{d}{dE_c} g(E_c) = \frac{1}{64(\log E_c)^2} \left( \sqrt{f(E_c)} - \frac{4}{3} + \frac{1}{3f(E_c)^{3/2}} \right)
\]  

(18)

The right-hand-side of Eq. (18) vanishes as \( e^2/(4\pi^2v_F) \to 0 \) and it could still admit a solution of the form \( g \sim -g_0/\log E_c \), but this is not realized in the present regime as the equation \( 8g_0 = \sqrt{1 + g_0} - 4/3 + 1/(3\sqrt{(1 + g_0)^3}) \) does not have any real solution. The flow of \( g(E_c) \) given by (18) quickly approaches some fixed-point value, after which it becomes little sensitive to further scaling in the infrared [28]. A plot of the flow for different values of the bare coupling constant at \( E_c = \Lambda \) is given in Fig. 1, which includes for comparison the scaling behavior of the large-\( n \) solution.

The existence of two different regimes in the model is not surprising, since the dependence on the number \( n \) of subbands is the way in which the system keeps memory of the finite transverse dimension in the carbon nanotube. A nanotube of very large radius, for instance, leads to a picture in which a large number of subbands are stacked above and below the linear branches in Fig. 1. This system falls into the description of the model with a very large \( n \) value. On the other hand, there is always a sufficiently small value of \( E_c \) for which all the subbands, but those contributing to the gapless part of the spectrum, become higher in energy than the RG cutoff. At that point, one proceeds paying attention to the linear branches crossing at the Fermi points alone, what leads to the one-band regime of the model.

From the point of view of the real space, the above transition in the number \( n \) of subbands represents the crossover from the two-dimensional regime to the effective one-dimensional description of a nanotube of large radius. Actually, in the limit of a very large \( n \) we recover the scaling behavior of the effective coupling constant in the graphite layer, which flows towards the free fixed-point. This scaling of the coupling constant is actually what makes consistent the linear quasiparticle decay rate recently measured [29] with the metallic properties of graphite [30,31]. In a carbon nanotube, the scaling appropriate for the graphite regime has to be followed up to a distance scale of the order of the diameter of the nanotube. The value of the coupling constant at that scale is what dictates the bare value for the one-dimensional regime ii). The one-dimensional effective field theory contains the explicit dependence on the ultraviolet cutoff \( \Lambda \), that is actually needed to fix the conditions at the crossover between the two regimes.

The issue of the renormalization of \( v_F \) we have discussed is important, since it implies a reduction in the strength of additional interactions in the system, whether short or long-range, as the effective couplings are all given by the couplings in the interaction hamiltonian divided by \( v_F \). We stress that this renormalization of the Fermi velocity is the relevant effect for a phenomenology at realistic energies, because the short-range interactions like umklapp or backscattering are nominally tiny and their RG flow only becomes appreciable at extremely small energies.

Incidentally, such a renormalization of \( v_F \) affecting every effective interaction in the model may also be present in small chains, contributing to explain the insulator-metal transition by the effect of the Coulomb interaction observed in the exact diagonalization of finite rings [28]. The study of low-dimensional systems carried out in Ref. [28] also indicates a reduction of the electron correlations similar to our findings in the RG framework.

The reason for the distinctive behavior of the Coulomb interaction in the fullerene tubules is the preservation of the chiral invariance in such systems. This comes from the different symmetry properties that characterize the left and the right modes at each Fermi point, leading to the selection rule that forbids their hybridization by the interaction. Thus, although it has been proposed that in a generic one-dimensional electron system the \( 2k_F \) and higher harmonics may lead to strong correlation effects in the presence of the Coulomb interaction [34], the absence of such hybridization makes the carbon nanotubes fall into the more conventional Luttinger liquid regime. As stated above, the wavefunction renormalization [14] is just the generalization for the long-range interaction of the expression giving the anomalous dimension of the electron field in the Luttinger model. The important point is the existence of a stable fixed-point for the effective coupling constant in the infrared. All the properties of the Luttinger liquid universality class are then expected to hold in the metallic carbon nanotubes, with a sensible renormalization of the effective interaction strength depending on the radius of the tubule.

We thank C. Castellani for useful discussions. This work has been partially supported by a CICYT-INFN exchange program and by the spanish Ministerio de Educación y Cultura Grant No. PB96-0875.

[1] T. W. Ebbesen, Physics Today 49(6), 26 (1996).
[2] S. J. Tans et al., Nature 386, 474 (1997).
[3] J. W. G. Wildöer et al., Nature 391, 50 (1998).
[4] T. W. Odom et al., Nature 391, 62 (1998).
[5] P. Kim et al., report cond-mat/9812408.
[6] J. Sólyom, Adv. Phys. 28, 201 (1979).
[7] F. D. M. Haldane, J. Phys. C 14, 2585 (1981).
[8] J. Vot, Rep. Prog. Phys. 58, 977 (1995).
[9] J. González et al., Quantum Electron Liquids and High-T$_c$ Superconductivity, Chap. 4 (Springer-Verlag, Berlin, 1995).
[10] L. Balents and M. P. A. Fisher, Phys. Rev. B 55, R11973 (1997).
[11] Yu. A. Krotov, D.-H. Lee and S. G. Louie, Phys. Rev. Lett. 78, 4245 (1997).
[12] M. Bockrath et al., report cond-mat/9812233.
[13] C. Kane, L. Balents and M. P. A. Fisher, Phys. Rev. Lett. 79, 5086 (1997).
[14] R. Egger and A. O. Gogolin, Phys. Rev. Lett. 79, 5082 (1997); Eur. Phys. J. B 3, 281 (1998).
[15] H. Yoshioka and A. A. Odintsov, Phys. Rev. Lett., in press.
[16] We refer to the kind of armchair nanotubes described in N. Hamada, S. Sawada and A. Oshiyama, Phys. Rev. Lett. 68, 1579 (1992); J. W. Mintmire, B. I. Dunlap and C. T. White, Phys. Rev. Lett. 68, 631 (1992).
[17] J. González, F. Guinea and M. A. H. Vozmediano, Nucl. Phys. B 406, 771 (1993).
[18] J. González, F. Guinea and M. A. H. Vozmediano, Nucl. Phys. B 424, 595 (1994).
[19] For the general discussion of the RG approach applied to interacting electron systems, see R. Shankar, Rev. Mod. Phys. 66, 129 (1994).
[20] See, for instance, J. J. Sakurai, Advanced Quantum Mechanics (Benjamin/Cummings, Menlo Park, 1984).
[21] C. Nayak and F. Wilczek, Nucl. Phys. B 417, 359 (1994).
[22] T. J. Pollehn, A. Schindlmayr and R. W. Godby, cond-mat/9711126 report.
[23] C. Castellani, C. Di Castro and W. Metzner, Phys. Rev. Lett. 72, 316 (1994).
[24] P.-A. Bares and X.-G. Wen, Phys. Rev. B 48, 8636 (1993).
[25] C. Castellani, C. Di Castro and A. Maccarone, Phys. Rev. B 55, 2676 (1997).
[26] I. E. Dzyaloshinskii and A. I. Larkin, Soviet Phys. JETP 38, 202 (1974).
[27] It can be checked that the right-hand-side of Eq. (16) matches the critical exponent of the electron propagator in the Luttinger model, after replacement of $-e^2\log(E_c/\Lambda)$ by a constant interaction $g_2 = g_4$.
[28] S. Bellucci and J. González, report cond-mat/9802011. S. Bellucci, report hep-th/9810181. to appear in the Proceedings of the VI Conference on Path Integrals (Florence, August 1998).
[29] S. Xu et al., Phys. Rev. Lett. 76, 483 (1996).
[30] J. González, F. Guinea and M. A. H. Vozmediano, Phys. Rev. Lett. 77, 3589 (1996).
[31] J. González, F. Guinea and M. A. H. Vozmediano, Phys. Rev. B 59, R2474 (1999).
[32] D. Poilblanc et al., Phys. Rev. B 56, 1645 (1997).
[33] J. van den Brink and G. A. Sawatzky, report cond-mat/9802133.
[34] H. J. Schulz, Phys. Rev. Lett. 71, 1864 (1993).
FIG. 1. Low-energy band structure of a carbon nanotube with $n = 20$ subbands. The energy is measured in units of the hopping parameter and the momentum is in units of the inverse lattice spacing.
FIG. 2. Flow of the effective coupling constant for different bare values, computed in the large-$n$ solution (thick lines) and in the model with only one band (thin lines).