A scaling relation between proton-nucleus and nucleus-nucleus collisions

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Abstract

It is recently discovered that at high multiplicity, the proton-nucleus (pA) collisions give rise to two particle correlations that are strikingly similar to those of nucleus-nucleus (AA) collisions at the same multiplicity, although the system size is smaller in pA. Using an independent cluster model and a simple conformal scaling argument, where the ratio of the mean free path to the system size stays constant at fixed multiplicity, we argue that flow in pA emerges as a collective response to the fluctuations in the position of clusters, just like in AA collisions. With several physically motivated and parameter free rescalings of the recent LHC data, we show that this simple model captures the essential physics of elliptic and triangular flow in pA collisions.

1. Introduction

As the recent measurements by the LHC [1, 2, 3] and RHIC [4] collaborations have shown, the particle production in high multiplicity proton-nucleus (pA) collisions exhibits striking long range two particle correlations which are quantitatively similar to the corresponding correlator in nucleus-nucleus (AA) events at the same multiplicity. Some features of these correlations are reproduced by the Color Glass Condensate (CGC) without reference to the fluctuating geometry [5, 6, 7, 8]. However, hydrodynamic simulations of p + A events also qualitatively predicted the correlations observed in the data [9, 10, 11], suggesting that the origin of the flow in p + A is similar to A + A. The aim of this talk is to give a brief explanation for this similarity by arguing that both in high multiplicity pA and in AA events, such long range correlations emerge from a collective response to the underlying geometry. It turns out implementation of an independent cluster model, and a simple "conformal scaling" framework [12] where the ratio of the mean free path to the system size is approximately the same for the pA and AA events are enough to capture the essential physics of such a collective response. According to this framework, at a given multiplicity, the pA event is smaller but hotter and denser, such that it develops a similar flow pattern as in AA. Below, these statements are made quantitative using both integrated and transverse momentum (pT) dependent v2[2] and v3[2] measurements of LHC. It is also worth to mention that conformal scaling framework was applied to pT dependent v4[2] and v5[2] as well, giving excellent results [7]. All these findings provide a strong evidence for the existence of collective physics in pA collisions.

2. Independent cluster model and conformal dynamics

We describe the initial state by N_{clus} independently distributed clusters such that the multiplicity N is proportional to N_{clus}. There is a single dimensionful parameter, say mean free path, l_{mfp} \propto T_i^{-1}, in our model that controls the

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response dynamics. The conformal scaling is manifested by the assumption $l_{mfp}L^{-1} = f \left( \frac{dN}{dy} \right)$, where $L$ is the system size. For instance in a saturation inspired model the dimensionful parameter would be $Q$, where $N_{clus} = \pi Q^2 L^2$ and $f = (dN/dy)^{-1/2}$. The distribution of clusters in the transverse plane, $n(x)$, is random around their mean value, $\bar{n}(x)$,

$$n(x) = \bar{n}(x) + \delta n(x) \quad \langle \delta n(x) \delta n(y) \rangle = \bar{n}(x) \delta^2(x - y)$$

(1)

where $\delta n(x)$ denotes the fluctuations around the average distribution $\bar{n}$. The flow emerges as a collective response to the geometry defined by the distribution of the clusters. We adopt linear response. The conformal scaling framework then dictates the response coefficients, $k_{2,3}$, to depend only on $l_{mfp}/L$, hence the multiplicity, i.e. $v_{2,3} = k_{2,3}(l_{mfp}/L)\epsilon_{2,3}$, where $\epsilon_2$ and $\epsilon_3$ are the eccentricity and triangularity respectively.

It is important to understand what the sources of the eccentricity and triangularity are. The $AA$ events with multiplicity comparable to $pA$ events are peripheral and the eccentricity is sourced by both the average cluster distribution and the fluctuations around it. On the other hand a high multiplicity $pA$ event is central and the eccentricity is sourced only by the fluctuations. The triangularity is sourced only by fluctuations both for $pA$ and $AA$. A gaussian distribution for the clusters, which gives a very good approximation to more complicated Glauber models, leads to the following expressions for the mean $\epsilon_2^2$ and $\epsilon_3^2$

$$\langle \epsilon_2^2 \rangle_{AA} = \epsilon^2_2 + \langle \delta \epsilon_2^2 \rangle, \quad \langle \epsilon_2^2 \rangle_{pA} = \langle \delta \epsilon_2^2 \rangle = \frac{\langle r^2 \rangle_{clus}}{N_{clus} \langle r^2 \rangle^2} \quad \langle \epsilon_3^2 \rangle_{AA} = \langle \epsilon_3^2 \rangle_{pA} = \langle \delta \epsilon_3^2 \rangle = \frac{\langle r^2 \rangle_{clus}}{N_{clus} \langle r^2 \rangle^3}$$

(2)

where $\epsilon_2$ is the average eccentricity. Note that we have assumed the same transverse distribution for $pA$ and $AA$. In the first glance it seems like a dangerous assumption, but since what enter into the formulas above are double ratios, the sensitivity to different shapes is rather small (around $\sim 15\%$ at the most) and no fine tuning is needed.

3. Elliptic and triangular flow

An immediate consequence of the discussion above is that at a given multiplicity, $v_3$ of $pA$ and $AA$ should be the same as they are both sourced by the fluctuations in the cluster distribution. As the LHC measurement shows, this is indeed true; $\langle v_3 \rangle_{pA} = \langle v_3 \rangle_{AA}$ up to a few percent. In order to compare the $v_2$s justly, one needs to “remove” the effect of the average geometry and isolate the fluctuations driven part of $\langle v_2 \rangle_{AA}$, since in $pA$ fluctuations constitute the only source. This can be done by the following rescaling, where the scaling factor projects onto $\sqrt{\langle \delta \epsilon_2^2 \rangle}$:

$$\langle v_2 \rangle_{pA,\text{rescl}} \equiv \sqrt{1 - \frac{\epsilon_2^2}{\langle \delta \epsilon_2^2 \rangle_{pA}}} \langle v_2 \rangle_{pA}.$$  

(3)

![Figure 1. The comparison between the fluctuation driven $v_2$ in AA and $pA$. Left: The actual data. Right: The fluctuation driven part isolated in AA.](image)

We have computed the rescaling factor in (3) via a Monte Carlo Glauber simulation. Once the average geometry is taken out, the conformal scaling predicts

$$\langle v_2 \rangle_{pA,\text{rescl}} \equiv k_2 \sqrt{\langle \delta \epsilon_2^2 \rangle_{pA}} \approx k_2 \sqrt{\langle \delta \epsilon_2^2 \rangle_{pA}^2} \equiv \langle v_2 \rangle_{pA}$$

(4)
The excellent agreement between the $v_2$ of $pA$ and $AA$ after the rescaling is shown in Figure 1. Note that there is no fitting parameter in the plot. Furthermore, the rescaling factor is a nontrivial function of centrality, hence multiplicity, and such a remarkable agreement is very nontrivial, indicating a strong evidence for a common origin for anisotropy in $pA$ and $AA$ which is a collective response to the geometry.

Conformal dynamics also allows one to compare the transverse momentum, $p_T$, dependence of $v_2$ and $v_3$. Since $p_T$ is a dimensionful quantity, it should enter into the expression for $v_{2,3}(p_T)$ as

$$v_2(p_T) = \varepsilon_2 f_2 (p_T / \langle p_T \rangle), \quad v_3(p_T) = \varepsilon_3 f_3 (p_T / \langle p_T \rangle)$$

where the momentum dependent response coefficients $f_{2,3}$ are universal functions and the average transverse momentum $\langle p_T \rangle \sim l_{mp}^{-1} \sim L^{-1}$ for fixed $dN/dy$. Note that such a relation is expected to hold for small $p_T \sim \langle p_T \rangle$, where we expect the collective behavior to dominate. As a consequence, in order to compare the momentum dependence of flow in $pA$ and $AA$, we should rescale the $p_T$ axis of the $AA$ to take into account the difference in $\langle p_T \rangle$ of $pA$ and $AA$. The prediction of the conformal scaling is then

$$[v_2(2)(p_T)]_{pA} = [v_2(2)(p_T/k)]_{pPb}, \quad [v_3(2)(p_T)]_{pA} = [v_3(2)(p_T/k)]_{pPb}, \quad k \equiv \langle p_T \rangle_{pA} / \langle p_T \rangle_{pPb} \approx 1.25.$$  

The measurement of $\langle p_T \rangle$ is taken from [14]. The original data for $v_2$ and $v_3$ together with this complete (and parameter free) rescaling is shown in Figure 2. From the lower panels, we see that the agreement between the dimensionless slopes in the low $p_T$ region is remarkable, and seems to affirm the conformal rescaling. At higher $p_T$, the $v_2(2)$ starts to systematically differ. This difference seems to become larger for lower multiplicities where non-flow could become significant.

Another prediction of the conformal scaling is that, the ratio of the system sizes of $pA$ and $AA$ systems is roughly $L_{AA}/L_{pA} = \langle p_T \rangle_{pA} / \langle p_T \rangle_{AA} = 1.25$. The recent Hanbury-Brown Twiss (HBT) measurement in LHC reveals that $R_{AA}/R_{pA} \approx 1.4$ [15]. Of course one expects some difference between the HBT radii and the system size as defined here, yet such an agreement is remarkable.

4. Conclusions

The presented parameter free analysis of the two particle angular correlations in $pA$ and $AA$ collisions at the LHC, with several physically motivated rescalings based on a simple conformal scaling argument, provides an explanation for the similarity in two systems. First, once the effect of average geometry is taken from AA measurement, the integrated $v_2$ in AA is the same as in $pA$ at fixed multiplicity. The integrated $v_3$ in these two colliding systems are already equal. Since the separation of $v_2$ into average and fluctuations in AA was entirely motivated by linear response and geometry, it is reasonable to conclude that both the elliptic and triangular flow in $pA$ should also be understood as a linear response to initial geometric fluctuations. Furthermore, the response coefficients of $pA$ and $AA$ are argued to be approximately equal based on the conformal scaling, which assumes a single dimensionful quantity controlling the response dynamics. The $p_T$ dependence of the $v_2$ and $v_3$ provide further support for such scaling under which both $v_2(p_T)$ and $v_3(p_T)$ curves for AA collapse onto the ones for $pA$. Consequently, phenomenologically, it seems highly unlikely that the angular correlations in $pA$ and $AA$ arise from different physics and likely that the underlying physics is collective response to fluctuation driven eccentricities.

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Scaling of \( v_2 \) for pA and AA collisions is dictated by conformal dynamics. For \( v_2 \), the y-axis is also rescaled to isolate the fluctuation driven part. The data is from [1].

Figure 2. Comparison of the momentum dependent \( v_2, v_3 \) for pA and AA. The \( p_T \) axes of AA are rescaled by the ratio of \( < p_T > \) of pA to AA as dictated by conformal dynamics. For \( v_2 \), the y-axis is also rescaled to isolate the fluctuation driven part. The data is from [1].

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