Research on Credit Strategy of Small and Medium Enterprises Based on Subjective and Objective Comprehensive Evaluation

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Abstract. In order to study the adjustment of national policies on medium, small and micro enterprises, and to determine the credit risk and credit strategy of banks to medium, small and micro enterprises. Firstly, the enterprise strength and comprehensive reputation of small and micro enterprises are analyzed, and their weight is determined by the subjective and objective comprehensive evaluation method, and the credit risk factors of each enterprise are calculated. A dual objective optimization model was established and solved by simulated annealing algorithm. The optimal profit amount and annual average interest rate of banks for different industries and different categories of credit strategy are obtained. Analytic Hierarchy process and Fuzzy Bayesian network are used to explore the influence of multiple emergent factors on banks adjustment credit strategy.

1. Introduction

With the development of economy, medium, small and micro enterprises have gradually become the main economic force of China. In order to support the development of medium, small and micro enterprises, banks have gradually included medium, small and micro enterprises into loanable enterprises. But in practice, the scale of medium, small and micro enterprises is small, and some enterprises also lack mortgage assets, so banks usually provide loans to enterprises with strong strength and stable supply and demand relationship based on credit policy, business transaction bill information and the influence of upstream and downstream enterprises. This paper first determines the evaluation weight by subjective and objective comprehensive evaluation method and calculates the credit risk factor of the enterprise. The optimal solution of each variable is obtained by simulated annealing algorithm.

2. Data sources and assumptions

Data source of this paper is from Chinese 2020 National College students Mathematical Modeling Competition C questions. For ease of handling, the following assumptions are put forward: (1) Assuming that the credit data of banks and medium, small and micro enterprises are symmetrical; (2) Assuming that the amount of loans required by each enterprise is the maximum amount that the enterprise can lend; (3) Assuming that the same enterprise has the same peak season each year; (4) Assuming that banks do not lend to enterprises with D credit ratings; (5) Assuming that the cost of rising the same ranking is the same;
3. Credit strategy model of subjective and objective comprehensive evaluation method

3.1 Research ideas
First of all, establish the credit risk evaluation two-tier index system, taking the enterprise strength, comprehensive reputation as the first class index, taking the enterprise tax amount, the stability of the off-season monthly sales item, the peak season sales item amount, the enterprise profit growth rate, the proportion of invalid invoice, the enterprise type, credit rating, whether default is the eight second-level index, determine the combination weight of credit risk evaluation through subjective and objective comprehensive evaluation method, establish the credit risk evaluation model, calculate the credit risk factor of 123 enterprises and use the large Cauchy distribution membership function to determine the relationship between the credit risk rating and credit risk factor. From 123 enterprises to determine the loan enterprises. Finally, a two-objective optimization model of loan quota and interest rate maximization is established from the perspective of bank, which is solved by simulated annealing algorithm.

3.2 Research methods

3.2.1 Fuzzy mathematical Cauchy membership function
The credit risk of medium, small and micro enterprises is rated by the internationally recognized credit risk seven rating method, and the credit risk of small and medium-sized enterprises is rated according to the credit risk of small and medium-sized enterprises, and the Cauchy distribution membership function of fuzzy mathematics is used to quantify the rating:

(1) Fuzzy set and membership degree: \( x \), any element in the study set \( U \) has a number \( A(x) \in [0,1] \) corresponding to it, \( A \) is called the fuzzy set on the \( U \), and \( A(x) \) is called the membership degree of the \( x \) to the \( A \).

(2) Membership function: if the \( x \) changes in the \( U \), the \( A(x) \) becomes a function, which is the membership function of the \( A \). When the membership degree \( A(x) \) tends to 1, the higher the degree of \( x \) belonging to the \( A \); When the membership degree \( A(x) \) tends to 0, the lower the degree of belonging to the \( A \); The establishment of a large Cauchy distribution membership function:

\[
 f(x) = \left\{ \begin{array}{ll}
 \frac{1}{\beta + \alpha(x-\beta)^{\frac{1}{2}}} & \text{if } 1 \leq x \leq 4 \\
 0 & \text{if } 4 < x < 7 
\end{array} \right. 
\]

(1)

For determining the value of a variable, assumed membership specific value: high risk membership 1, which is \( f(7)=1 \); General risk membership is 0.4, which is \( f(4)=0.4 \); No risk membership is 0.01, which is \( f(1)=0.01 \). The parameters in formula (1) are obtained:

\[
 a = 1.072 \quad b = -1.086 \quad \alpha = 17.547 \quad \beta = 0.579 
\]

(2)

Thus, the degree of membership under each rating is obtained: extremely high, a bit high, high, general, low, a bit low, risk-free rating quantification is 1,0.921,0.639,0.4,0.25,0.103,0.01.

3.2.2 Subjective and objective comprehensive evaluation method
1. Principal component analysis to reduce dimension and determine objective weight.

The principal component analysis method is used to reduce the dimension of the index and determine the objective weight of the principal component. The principal component analysis method is as follows:

I. Calculation of correlation coefficient matrix

\[
 R = \begin{bmatrix}
 r_{11} & r_{12} & \cdots & r_{1n} \\
 r_{21} & r_{22} & \cdots & r_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 r_{n1} & r_{n2} & \cdots & r_{nn}
\end{bmatrix}
\]

(3)

Among them, \( r_{ij} (i,j=1,2,\ldots,8) \) is the original indicator variable \( x_i \) and \( x_j \). Its calculation formula is:
II. Calculation of eigenvalues and eigenvectors

The eigenvalue $\lambda_i (i=1,2,...,p)$ of correlation coefficient matrix $R$ is first obtained according to the characteristic equation and sorted according to their size, which is $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p \geq 0$, and get its corresponding eigenvector $e_i (i=1,2,...,p)$, and 8 new index variables are composed of feature vectors.

III. Select $p (p \leq 8)$ principal components based on contribution rate and calculate the comprehensive evaluation value.

1. Principal component contribution and cumulative contribution of eigenvalues calculated

The cumulative contribution rate of the first five indexes was more than 85%, and the first five principal components were taken for comprehensive evaluation. According to the contribution rate of these five principal components, the weight vector obtained by principal component analysis is as follows: $K = [0.336, 0.216, 0.192, 0.138, 0.118]$;

2. Improved AHP to determine principal component weights

The analytic hierarchy process (Analytic Hierarchy Process, abbreviated as AHP) is a subjective assignment method to make decisions on more complex and fuzzy problems. Because the assignment of principal component analysis is unstable, the subjective assignment of 8 indexes is carried out by AHP and substituted into five principal components. The hierarchy process steps are as follows:

First: To construct the judgment matrix of criterion layer.

Then: The consistency index needs to be tested first $CI$, and its formula is as follows:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

Among them, $\lambda_{max}$ is the maximum characteristic root of the judgment matrix.

Third: Calculate the average random consistency index $RI$:

$$RI = \frac{\lambda'_{max} - n}{n - 1}$$

Finally: Calculate Consistency Ratio $CR$:

$$CR = \frac{CI}{RI}$$

At $CR<0.10$, the consistency of the judgment matrix is acceptable, otherwise the judgment matrix should be corrected appropriately. Calculated $CR=0.006 <0.10$, the judgment matrix is acceptable. An improved hierarchical analysis method is used to find the weight vector as follows: $Z = [0.423, 0.252, 0.164, 0.085, 0.076]$;

3. Determination of subjective and objective combination weights

Suppose there are $m$ evaluation indicators and $a$ evaluation objects. By using the improved analytic hierarchy process and principal component analysis, the subjective and objective weights of each principal component index are obtained, and the weight matrix is established $W$ as follows:

$$W = \begin{bmatrix}
    w_{11} & w_{12} & \cdots & w_{1a} \\
    w_{21} & w_{22} & \cdots & w_{2a} \\
    \vdots & \vdots & \ddots & \vdots \\
    w_{m1} & w_{m2} & \cdots & w_{ma}
\end{bmatrix}$$

where $m$ is the number of indicators; $n$ is the number of weighting methods; $w_{ik}$ uses the $k_{th}$ method to get the $i_{th}$ weight.

The subjective and objective weights of principal components are combined and modeled, as shown in formula (9):

$$\min M = \frac{\sum \sum \sum \left( (w_{ij} - c_i) \cdot x_{ij} \right)^2}{n \sum c_i = 1, c_i \geq 0}$$

\[ \text{(9)} \]
Among them, \( c_i \) is the combinatorial weight of the \( i \)th index, \( x_{ij} \) is the element of the evaluation matrix; 

The Lagrange function is constructed to solve the model as follows:

\[
L(c, \lambda) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( (w_j - c_i)^2 + \lambda (\sum_{i=1}^{m} c_i - 1) \right)
\]

(10)

And according to the existence condition of extreme value, take the first derivative of \( c_i \) and \( \lambda \), make it 0:

\[
\begin{align*}
\frac{\partial L}{\partial c_i} &= -2 \sum_{j=1}^{n} (w_j - a) x_{ij} + \lambda = 0 \\
\frac{\partial L}{\partial \lambda} &= \sum_{i=1}^{m} c_i - 1 = 0
\end{align*}
\]

(11)

By solving the above formula, the combined weight vector can be obtained as follows: \( C = [0.571, 0.219, 0.127, 0.047, 0.036] \);

\( 1-Df \) represents the total risk assessment index value of all indicators:

\[
1-Df = \sum_{i=1}^{m} c_i f_i
\]

(12)

According to the total risk evaluation index and the grade of Cauchy membership function of fuzzy mathematics, the credit risk is evaluated.

### 3.2.3 Dual-objective optimization model

Taking the maximum bank income as the objective function and the known relationship as the constraint condition, a two-objective optimization model of loan line and interest rate is established. (1) The bank shall extend a loan amount of ten thousand yuan to an enterprise determined to lend. (2) The annual interest rate is 4 to 15 percent. (3) In order to repay the loan, the enterprise must meet the one-year profit value of the enterprise can repay the loan at the beginning of the year and the annual interest rate of a year. (4) The preference ratio for the preference function is between \([0, 1]\). (5) Customer retention is equal to 1 minus customer churn. (6) Churn rate is a fitting function of the bank's annual interest rate.

The bank loan annual interest rate and customer turnover rate are drawn, and the scattered plot is fitted based on the least square method. That is, the functional relationship between the loan annual interest rate and the customer turnover rate can be obtained. Companies with different credit rating have different functional relationships. The functional relationship is shown in formula (13):

\[
\begin{align*}
t &= 640.9 a - 258.6 a^2 + 37.97 a - 1.12 & \text{A Reputation Company} \\
t &= 552.8 a - 225.0 a^2 + 34.0 a - 1.02 & \text{B Reputation Company} \\
t &= 504.7 a - 207.4 a^2 + 32.16 a - 0.97 & \text{C Reputation Company}
\end{align*}
\]

(13)

In view of the customer turnover rate, it is assumed that the potential customers corresponding to the wastage rate are 59 enterprises that can lend. The retention rate index is the probability that the bank can continue to borrow after giving the quota and interest rate. Add the retention rate as a factor to the bank's annual interest objective function.

Because the bank will give interest rate concessions to enterprises with high reputation and low credit risk. Hence, the \( m \) of preferential function \((R,G)\), \( m \in (0, 1) \), Credit rating is \( A \), \( B \), \( C \), Using cluster analysis, So that the cluster is 4, Four \( A \), \( B \), \( C \), \( D \), of lending risk Using entropy weight method to give weights, Given 12 possible interest rates, Added to the optimization model, The two-objective optimization model of loan quota and interest rate is shown in formula (14):

\[
\begin{align*}
\text{max} x &= f(m) \cdot m \cdot x \\
10 \leq x \leq 100 \\
0.04 \leq a \leq 0.15 \\
x(l + a) \leq P \\
x & \cdot 0 < f(R,G) < 1 \\
m + f(R,G) \\
l &= 1 - t \\
t &= f(a)
\end{align*}
\]

(14)
3.2.4. simulated annealing algorithm for knapsack problem

Step 1: randomly generate initial solutions. The value and weight of each item are randomly generated in the solution space; 
Step 2: set objective function: calculate the total value of all items; 
Step 3: generate a new solution: select an item \( i \), If the item \( i \) in the backpack, Take it out, And randomly put \( j \); an item If the item \( i \) not in the backpack, And \( i \) the items into the backpack, At the same time take out, item And the quality and value of the solution is poor; 
Cooling Step 4: simulated annealing; 
Step 5: determine the probability of accepting a new solution \( P \); if the total weight of the solution does not exceed the total mass of the backpack, If the new solution is superior to the original solution, Then the new solution replaces the original solution, Otherwise, set a probability to accept the new solution: after the number of iterations or the algorithm converges, Get the discrete value of each enterprise loan quota and interest rate. With the "backpack model, Optimized by a two-objective model of problem one, The simulated annealing algorithm is used to find the optimal value.

3.3 Results analysis

According to the data visualization diagram, the higher the credit risk, the better the credit risk evaluation model is. After analysis, the credit rating is higher and the risk factor is small. The total amount of the bank's best loan to each enterprise is 51.2164 million yuan, the average annual interest rate is 5.88%, and the annual profit of the bank is 2.988 million yuan.

The data of enterprise credit risk are attached, and the visual diagram of credit risk of different credit rating of enterprise is shown in Figure 1:

![Figure 1 Visualization of Credit Rating and Credit Risk](chart.png)

4. Credit Strategy Adjustment Based on Fuzzy Bayesian Network

4.1 Research ideas

Firstly, the factors affecting enterprises are natural factors, market factors and policy factors, and the influence weight matrix of different types and industries under the same unexpected factors are determined. Based on AHP and fuzzy Bayesian network model, the risk values of different unexpected factors are given.

4.2 Research methodology

4.2.1 Analytic Hierarchy Process and Fuzzy Bayesian Network

Bayesian network is a method of risk quantification. Through the definition of expert judgment statement, the probability events expressed by fuzzy judgment statement are quantified, that is, transformed into triangular fuzzy number or trapezoidal fuzzy number. The Bayesian network model is used to predict the risk probability after the fuzzy solution.

The risk node of sudden factor is divided into three levels: state, event and risk factor. The first
layer is the event layer, which directly affects the risk source; the second layer is the state layer, which covers the inducing factors of the risk event; the third layer is the risk layer, that is, the possible emergency; in the enterprise emergency fuzzy Bayesian network.

Each node has two different states. \( s_0 \) and \( s_1 \) represent the probability level of the event not occurring and the occurrence of the event. The schematic diagram of the fuzzy Bayesian network is shown in figure 2:

![Figure 2 Fuzzy Bayesian network](image)

In view of the loss caused by the burst factor, the node is assigned. Due to the lack of complete data, the AHP is established to determine the loss weight. In order to express the expert's judgment result more intuitively, the risk probability membership function of seven grades is adopted. Based on the evaluation results of all experts, the fuzzy comprehensive probability values of \( n \) experts can be obtained according to the arithmetic mean method:

\[
P = \frac{f_1 + f_2 + \cdots + f_n}{n}, \quad i = 1, 2, \ldots, m
\]

Among them, \( P_i \) represents the event fuzzy probability; \( f_j \) represents the fuzzy value of the \( j \)-th expert evaluation; \( m \) is the number of events; The probability value of the fuzzy number \( P \) is the probability value of the risk factor. The fuzzy number is processed by full integral algorithm, and the fuzzy value function of fuzzy number solution is established:

\[
I(P) = a \mu_L(P) - (1 - a) \mu_R(P)
\]

The \( I \) represents the fuzzy value and the \( a \) is the optimistic coefficient. When the optimistic coefficient \( a=0 \), corresponds to the upper bound of fuzzy number \( P \) solution fuzzification value function, and \( a=1 \) corresponds to the lower bound. This paper selects \( a=0.5 \) as the representative value of fuzzy number \( P \) fuzzy value function. \( \mu_L(P) \) and \( \mu_R(P) \) are respectively the integral values of the inverse functions of the membership functions of the left and right fuzzy numbers:

\[
\mu_L(P) = \frac{1}{2} \left( \sum_{\lambda=0}^{1} p_i^L(\lambda) \Delta \lambda + \sum_{\lambda=0}^{4.5} p_i^R(\lambda) \Delta \lambda \right)
\]

\[
\mu_R(P) = \frac{1}{2} \left( \sum_{\lambda=0}^{1} p_i^L(\lambda) \Delta \lambda + \sum_{\lambda=0}^{4.5} p_i^R(\lambda) \Delta \lambda \right)
\]

The prior probability value \( P_i \) under each node is obtained by solving the fuzzification value function with fuzzy number \( P \). An operational formula of the index risk value can be obtained: the product of the risk value of an index = the prior probability value and the index weight. The risk values of different burst factors are obtained and normalized to obtain the weight of each index.

4.2.2 The weight matrix of various burst factors under different types in the same industry.

Classify the company according to the industry and type, give the influence weight matrix of different types and industries under the same sudden factor, and give the subjective weight matrix of \( 3 \times 3 \) influence weights of different types and industries under the same sudden factor. The risk value of each sudden factor is shown in Table 1:
### Table 1 Risk Value of Emergent Factors in Different Enterprises

| Enterprise            | Labour Intensive | Science and Technology Innovation | Self-employed |
|-----------------------|------------------|-----------------------------------|----------------|
| Middle East           | 0.0475           | 0.0259                            | 0.0366         |
| Small                 | 0.0953           | 0.0519                            | 0.0735         |
| Microcosmic           | 0.1431           | 0.078                             | 0.1105         |

### 4.3 Results analysis

Analysis of variance is carried out on the credit risk $R_0$ of 302 enterprises without considering the risk of different unexpected factors and the credit risk of 302 enterprises considering different unexpected factors. According to the $p$ value less than 0.05, there was a significant difference between the two columns. It shows that the impact of sudden factors on credit risk is significant. The statistical tables of significance are shown in Table 2:

### Table 2 Significant tables

|        | SS  | df | MS   | F    | $p$ value |
|--------|-----|----|------|------|-----------|
| Column | 2.87| 1  | 2.87 | 46.3 | 0         |
| Interaction effects | 4.10 | 146 | 0.03 | 0.45 | 1         |
| Error  | 18.24 | 294 | 0.06 |      |           |
| Total  | 42.84 | 587 |      |      |           |

According to the $p$ value less than 0.05, there is a significant difference between the two columns. It shows that the impact of sudden factors on credit risk is significant. Based on this, ANOVA analysis is used to compare the impact of unexpected factors on credit risk, and the results have significant statistical significance. The empirical results show that under a variety of unexpected factors, banks can spread the loan risk by increasing loan enterprises and reducing bank interest rates, which has certain reference value and significance to the actual lending strategy of banks.

### 5. Conclusion

Using subjective and objective comprehensive weighting to give the combined weight effectively reduces the weight error. The two-objective optimization model is established, and the global optimal solution is effectively searched by simulated annealing algorithm of knapsack-like problem. The results are reliable and the model is stable. Hierarchical analysis and fuzzy Bayesian network are used to determine the risk value, which reduces the information loss caused by subjective determination weight. Finally, the feasibility of the model is increased through the data significance analysis of unexpected factors and no unexpected factors. The empirical results show that under various unexpected factors, banks can disperse loan risks by increasing loan enterprises and reducing bank interest rates. It has certain reference value and significance to the actual lending strategy of banks.

### References

[1] Ya Geng, Huisheng Wu. A Study on Backpack Problem Based on Particle Swarm-simulated Annealing Algorithm [J]. Control Engineering, 2019, 26(05): 991-996 China.
[2] Qingyi Pan. Model of creditor's rights matching for P2P lending platform based on simulated annealing algorithm [J]. Digital Technology and Applications, 2017(02): 143 China.
[3] Jing Liu, Fu Cao. Assessment of the Emergency Risk of Mutual Fund Assistance of Farmers'Professional Cooperative Based on Hierarchical Analysis Entropy Weight Method and Fuzzy Bayesian Network [J]. World Agriculture, 2020(08): 67-77+85
[4] Xiao Yuan. Discussion on the Probability Model of Customer Loss in Telecommunications Enterprises [J]. Wireless Interconnection Technology, 2015(19): 58-60 China.
[5] Zaya Zhang, Xuan Fei etc. Design [J] of Low Voltage Multi-level Multi-indicator Weight in Dist-
[6] Cenk Sakar, Burak Koseoglu, Ali C. Toz, Muge Buber. Analysing the effects of liquefaction on capsizing through integrating interpretive structural modelling (ISM) and fuzzy Bayesian networks (FBN)[J]. Ocean Engineering, 2020, 215.

[7] Mostafa Mirzaei Aliabadi, Afshin Pourhasan, Iraj Mohammadfam. Risk modelling of a hydrogen gasholder using Fuzzy Bayesian Network (FBN)[J]. International Journal of Hydrogen Energy, 2020, 45(1).