Magnetothermoelectric Effects of 2D Electron Gas in Quantum Well with Parabolic Confinement Potential in-plane Magnetic Field

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Abstract. We developed a quantitative theory of the phonon-drag thermopower and the Nernst-Ettingshausen effects for the 2D electron gas in a quantum well with parabolic confinement potential in the magnetic field in the plane of the free movement of electrons. Our theoretical results are in a good agreement with experimental data [1] in the range 1-10K. In this temperature interval the main contribution to the thermopower comes from phonon-drag. Our calculations show that the temperature dependence of both the longitudinal and transverse Nernst-Ettingshausen effects is non-monotonic. We find that phonon-drag Nernst-Ettingshausen effects change sign with the temperature.

1. Introduction
The thermomagnetic effects in quantum wells have been studied theoretically and experimentally in [1-4]. Here we have study the phonon-drag thermopower and the Nernst-Ettingshausen effects in a quantum well with parabolic confinement potential. The magnetic field is directed across the confinement direction, i.e. along the plane of a two-dimensional electron gas. Thus, two cases for the relative arrangement of the current direction and the confinement direction are possible. In the case where the current is directed along the plane of the two-dimensional electron gas it is sufficient to consider the relaxation time approximation and to use the Boltzmann equation. In the case when the current is along the direction of confinement it is necessary to use the density matrix approach [5] for calculation of the diagonal components of the conductivity tensor.

2. Theory
We consider a simple model for the quantum well, in which a two-dimensional electron gas is confined in the x-direction and a homogenous static magnetic field $B$, with the vector-potential $A(0,x,B,0)$ in the Landau gauge, is parallel to the z-axis,. The parabolic confining potential can be written as $U(x) = m\omega_p^2 x^2/2$, where $m$ is the effective mass of a conduction electron, and $\omega_p$ is the parameter of the parabolic potential. For this model, the one-particle Hamiltonian, its eigenvalues and eigenfunctions are given in [6].

Using the concept of generalized force given in [5],

$$\tilde{\Phi} = -e \tilde{E} - \frac{\tau}{T} \nabla T - A_{ph} k_B \nabla T ,$$

(1)

where the last term is related to the phonon-drag effect, we can obtain the non-diagonal, $\beta_{xy}^{ph}$, and diagonal, $\beta_{yy}^{ph}$, components of tensor $\beta$:

$$\beta_{xy}^{ph} = \frac{k_B e \omega_p}{m \omega_p^2} \sum_A f_A A_{ph} , \quad \beta_{yy}^{ph} = -e k_B \sum_A \tau_A v_A^2 \left( \frac{\partial f_A}{\partial E} \right)^{\beta_{ph}} A_{ph} .$$

(2)
Here $\varepsilon$, $\vec{v}$, $\zeta$, $\tau_e$ and $\omega_e = eB/\hbar c$ are the energy, velocity, chemical potential, relaxation time and cyclotron frequency of electrons, respectively, $\omega = (\omega_0^2 + \omega_e^2)^{1/2}$, $f_0$ is the equilibrium electron distribution function, $\alpha = (N, k, k_z)$ is a set of quantum numbers that determine electronic states in a magnetic field, $N$ is the oscillation quantum number, $\vec{k}(k_x, k_z)$ is the wave vector of electrons,

\[ A_{ph} = -\frac{s^2}{k_0} \sum q \left[ 2\hbar \left( w_{da}(q) + w_{pa}(q) \right) \exp \left( -\frac{R^2 q_x^2}{2} \right) \right] \tau_{ph}(q) \left( \frac{q \nabla T}{\nabla T} \right) \frac{dN_q^0}{dT} \delta \left( \varepsilon_{n+1} - \varepsilon_{n} \right), \]  

\[ w_{da}(q) = \frac{\pi E^2}{\varepsilon(q)} \frac{q^2}{\rho \omega_q}, \quad w_{pa}(q) = \frac{\pi e^2}{\varepsilon(q)} \frac{q^2}{\rho \omega_q}, \] 

$\vec{q}$ and $\tau_{pa}(q)$ are the phonon wave vector and the relaxation time of phonons, respectively, $N_q^0$ is the occupation number (the Planck function) for phonons with frequency $\omega_q = s q$, $s$ is the sound velocity, $\rho$ is the density of the material, and $q_{||}$ is the component of wave vector of phonon in the plane of the free movement of electrons. In (3) both the piezoelectric (pa) and the deformation (da) interaction of electrons with acoustic phonons are taken into account. Here $E_{i}$ is the deformation potential coefficient, $\beta = 0.89 e_1 \chi$, $e_{13}$ is the piezoelectric constant, $\chi$ is the static dielectric constant, and $\varepsilon(q)$ is dielectric function of the electron gas. For the 2D electron gas we have

\[ \varepsilon(q) = 1 + \frac{2m e^2}{\sqrt{\pi} \chi} \frac{1}{\sqrt{2} q_{||}} \text{erfc} \left( \frac{R q_{||}}{\sqrt{2}} \right), \] 

where $R = (\hbar / m c)^{1/2}$ is the magnetic length.

The scattering mechanisms of electrons explicitly considered in the present paper are the acoustic phonon scattering via deformation and piezoelectric couplings, and the impurity scattering arising from ionized impurities in the quantum well. For the different mechanisms of relaxation of phonons we have

\[ A_{ph}(y) = \frac{(e \beta)^2 s^2 m^2}{\pi^2 \rho \hbar (k_0 T)^2 R} \tau_{ph} \int_0^1 F_i(y x) \left[ 1 + \frac{m e^2 R e^{2y^2} \text{erfc} \left( \sqrt{2} y x \right)}{\sqrt{\pi} \chi} \right]^{-2} \frac{x^2}{\sqrt{1-x^2}} dx, \]

where $y = R k$, \[ F_i(y x) = \int_{-\infty}^{y} q' \left( \text{Sinh} \frac{\hbar s q}{2k_0 T} \right)^{-2} \left[ 1 + \left( \frac{E_{i} q}{e \beta R} \right)^2 \right] e^{\frac{r^2}{4}} dt, \quad q = \left( 4 R^2 k^2 x^2 + t^2 \right)^{1/2}. \]

For Herring’s ($\ell = 2$), Simons’s ($\ell = 1$) and boundary scattering ($\ell = 0$) mechanisms, respectively,
\[ \tau_{ph,2} = \frac{\rho \hbar^2 s^2 R^2}{(k_o T)}; \quad \tau_{ph,1} = \frac{\rho \hbar^2 s^2 R}{(k_o T)}; \quad \tau_{ph,0} = \frac{L}{s}, \]  

(8)

where \( L \) is the phonon mean free path due to boundary scattering. For the calculation of thermopower, \( \alpha_{yy} \), and the Nernst-Ettingshausen coefficient, \( Q_{yy} \), it is necessary to calculate both the diagonal and the non-diagonal components of \( \sigma_{ik} \) and \( \beta_{ik} \) \([5]\). \( \beta_{ik} \) consists of two components, the diffusion and the phonon-drag: \( \beta_{ik} = \beta_{ik}^{diff} + \beta_{ik}^{ph} \).

For the degenerated electron gas we obtain

\[
\sigma_{xy} = -\sigma_{yy} = \frac{e^2 n}{m \omega^2}; \quad \sigma_{yy} = \frac{e^2 k_F^2 \tau_F}{2\pi m}; \quad \sigma_{xx} = \frac{2\pi e^2 n^2 \omega_F^2}{m \omega^4 k_F^2 \tau_F}; \quad \sigma_{xx} \sigma_{yy} = \sigma_{xy}^2;
\]

(9)

\[
\beta_{yy}^{ph} = -\frac{k_F}{e} \sigma_{yy} A_{ph}(y_F), \quad y_F = R k_F; \quad \beta_{xy}^{ph} = -\frac{k_F}{e} \sigma_{yy} A_{ph}, \quad \bar{A}_{ph} = \frac{1}{\pi n R^2} \int_0^\infty A_{ph}(y) f_0(y) y dy,
\]

(10)

where \( k_F \) and \( n = k_F^2/2\pi \) are the Fermi’s wave number and the real density of electrons.

The diffusion component of thermopower and the Nernst-Ettingshausen effects are given in [6], so here we consider only the phonon component. In the magnetic fields satisfying conditions \( \omega_c \ll \omega_0, \omega_c \tau_F \gg 1 \) (here \( \tau_F \) is the relaxation time of electrons at the Fermi surface), for the phonon component of the change in thermoelectric power in a magnetic field, \( \Delta \alpha_{yy}^{ph}(B) = \left[ \alpha_{yy}^{ph}(B) - \alpha_{yy}^{ph}(0) \right] \), and for the Nernst-Ettingshausen coefficient we find

\[
\alpha_{yy}^{ph}(0) = -\frac{k_F}{e} A_{ph}(y_F), \quad \Delta \alpha_{yy}^{ph}(B) = -\frac{k_F}{2e} \left[ A_{ph}(y_F) - \bar{A}_{ph} \right],
\]

(11)

\[
Q_{yy}^{ph} = -\frac{k_F}{2e B} \left[ A_{ph}(y_F) - \bar{A}_{ph} \right] \left( \frac{\omega}{\omega_c} \right) \left( \omega_c \tau_F \right), \quad \bar{A}_{ph} = \frac{1}{\pi n R^2} \int_0^\infty A_{ph}(y) f_0(y) y dy.
\]

(12)

3. Results and discussion

In this section we apply our theoretical results to the 2D degenerate electron gas in GaAs/Al_Ga_As quantum wells. Numerical calculations for the thermopower have been performed for sample 1 in [1], with the electron density \( n=1.78 \times 10^{15} m^{-2} \), the mobility \( \mu=22.7 m^2/Vs \), the mean free path of phonons \( L=3 \times 10^{-4} m \), the width of the quantum well \( L_x=10^{-8} m \) [1], the mass density \( \rho=5.3 \times 10^3 kg/m^3 \), the longitudinal sound velocity \( s=5 \times 10^4 m/s \), the effective mass \( m=0.067 m_0 \) ( \( m_0 \) is the free electron mass), the acoustic deformation potential \( E_s=7.4 eV \), and the piezoelectric constant \( e_{14}=0.16 C/m^2 \) [7].

Our analysis shows that the situations considered in [1-3] satisfied the conditions of the quantum limit and the strong degeneracy of electron gas. Estimation shows that the dominant mechanism of scattering of the electrons is the scattering by ionized impurities, and for the phonons the dominant mechanism is the surface scattering.

We estimated parameter \( \omega_0=7 \times 10^{13} s^{-1} \) of the parabolic potential using condition \( R-L_c/2 \). Our theoretical results for the variation of zero-field thermopower \( \alpha_{yy} \) as a function of temperature \( T \) are shown in Fig. 1. The results are in a good agreement with experimental data in the range 1-10 K. In this temperature interval the diffusion thermopower is by 3 to 8 times smaller than the phonon-drag.
thermopower. Thus, for example, at $T=5K$, $\alpha_{yy}^{ph}=215\mu V/K$ and $\alpha_{yy}^{dif}=36\mu V/K$. At temperatures below 1.5K the main contribution to the thermopower comes from the piezoelectric interaction, and from the deformation interaction with acoustic phonons. The calculations indicate the importance of screening, so the value of the thermoelectric power in the range 1-10K, without taking into account the screening, is 1.5-2 times higher.

![Figure 1. Variation of zero-field thermopower $\alpha_{yy}$ as a function of temperature $T$: the diffusion component (1), the phonon-drag thermopower (2: due to pa-coupling, 3: due to da-coupling, 4: the total phonon-drag thermopower), and the total thermopower (5). Open and solid points are the experimental data[1].](image1)

![Figure 2. Phonon-drag magneto-thermopower $\Delta\alpha_{yy}^{ph}(B)$ as a function of temperature $T$.](image2)

Our calculations show that the temperature dependence of both the longitudinal and transverse Nernst-Ettingshausen effects is nonmonotonic (Fig. 2). Both effects change sign with temperature. Note that usually the change of sign of the Nernst-Ettingshausen effects with temperature arises due to the change in the mechanism of scattering, but in this case the mechanism of scattering remains unchanged.

It is also interesting to note that the transverse thermopower $\alpha_{xy}^{ph}=BQ_{xy}^{ph}$ exceeds the zero-field thermopower $\alpha_{xy}^{ph}$ by one order of magnitude. Thus, for example, at $T=5K$, $B=1T$, and $\omega_0=10^{13}$ sec$^{-1}$, for the transverse thermopower we have $\alpha_{xy}^{ph} \approx 3mV/K$.

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