On the structures of new scalar resonances $T_{c^{+0}}^{a}(2900)^{++}$ and $T_{c^{+0}}^{a}(2900)^{0}$

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We investigate properties of the new scalar resonances $T_{c^{+0}}^{a}(2900)^{++}$ and $T_{c^{+0}}^{a}(2900)^{0}$, which were recently reported by the LHCb collaboration. These states were observed as resonant structures in $D_{s}^{+}\pi^{-}$ and $D_{s}^{+}\pi^{+}$ invariant mass distributions in $B^{+}$ meson decays. We argue that $T_{c^{+0}}^{a}(2900)^{++}$ and $T_{c^{+0}}^{a}(2900)^{0}$ may be modeled as molecules $M^{++} = D_{s}^{+}\rho^{+}$ and $M^{0} = D_{s}^{+}\rho^{-}$ of conventional vector mesons, respectively. The mass $m$ and current coupling $f$ of the molecule $M^{++}$ are calculated using two-point sum rule method. The sum rule analysis is performed by taking into account vacuum condensates up to dimension 8. The obtained result for the mass, $m = (2917 \pm 135)$ MeV, permits us to consider the molecule $M^{++}$ as one of possible models of the resonance $T_{c^{+0}}^{a}(2900)^{++}$. Because the second structure $T_{c^{+0}}^{a}(2900)^{0}$ is isospin partner of the doubly charged state, it should have the mass close to $m$.

I. INTRODUCTION

A few months ago the LHCb collaboration reported about observation of three new exotic hadrons labeled as $P_{uds}^{a}(4338)^{0}$, $T_{c^{+0}}^{a}(2900)^{0}$ and $T_{c^{+0}}^{a}(2900)^{++}$, respectively. First of them is presumably a pentaquark discovered with a high significance in the $J/\psi\Lambda$ invariant mass distribution in the decay $B^{+} \rightarrow J/\psi\Lambda p$. Remaining two resonances are four-quark mesons $T_{c^{+0}}^{a}(2900)^{0}$ and $T_{c^{+0}}^{a}(2900)^{++}$ (hereafter $T_{c^{+0}}^{a0}$ and $T_{c^{+0}}^{a++}$, respectively) fixed in the processes $B^{0} \rightarrow D^{0}D_{s}^{+}\pi^{-}$ and $B^{+} \rightarrow D^{-}D_{s}^{+}\pi^{+}$. They are scalar particles and were seen in the mass distributions of the mesons $D_{s}^{+}\pi^{-}$ and $D_{s}^{+}\pi^{+}$. The LHCb measured the masses and widths of the exotic mesons $T_{c^{+0}}^{a0}$

$$m_{1\text{exp}} = (2892 \pm 14 \pm 15) \text{ MeV},$$

$$\Gamma_{1\text{exp}} = (119 \pm 26 \pm 13) \text{ MeV},$$

and $T_{c^{+0}}^{a++}$

$$m_{2\text{exp}} = (2923 \pm 17 \pm 20) \text{ MeV},$$

$$\Gamma_{2\text{exp}} = (137 \pm 32 \pm 17) \text{ MeV}.$$

The collaboration also provided the following data for these structures:

$$m_{\text{exp}} = (2908 \pm 11 \pm 20) \text{ MeV},$$

$$\Gamma_{\text{exp}} = (136 \pm 23 \pm 11) \text{ MeV},$$

supposing that they are isospin partners and share these parameters.

Because the new scalar resonances were fixed in the $D_{s}^{+}\pi^{-}$ and $D_{s}^{+}\pi^{+}$ mass distributions, the processes $T_{c^{+0}}^{a0} \rightarrow D_{s}^{+}\pi^{-}$ and $T_{c^{+0}}^{a++} \rightarrow D_{s}^{+}\pi^{+}$ are their main decay channels. Then, it is clear that they are built of quarks $uds\bar{u}$ and $uds\bar{d}$, provided the four-quark picture can be applied to model these states. They are also fully open flavor tetraquarks, and hence belong to a family of exotic mesons established by the structures $X_{0}(2900)$ and $X_{1}(2900)$. In fact, in Refs. [3, 4] the LHCb informed on scalar $X_{0}(2900)$ and vector $X_{1}(2900)$ resonances (in what follows $X_{0}$ and $X_{1}$) in the invariant mass distribution of $D^{-}K^{+}$ mesons in the channel $B^{+} \rightarrow D^{+}D^{-}K^{+}$. This means that the resonances $X_{0}$ and $X_{1}$ decay to mesons $D^{-}K^{+}$, and in the four-quark model are composed of four different valence quarks $uds\bar{u}$. This fact placed $X_{0}$ and $X_{1}$ to special position in $X Y Z$ family of exotic mesons, because they were first evidences for the fully open flavor tetraquarks.

The discovery of the resonances $X_{0}$ and $X_{1}$ triggered interesting theoretical investigations aimed to understand their nature, calculate their masses, and estimate widths of these states [5–8]. Naturally, authors of numerous publications made different assumptions about internal structures of $X_{0}$ and $X_{1}$, and invoked various models and calculational schemes to evaluate their parameters. For instance, in Refs. [3, 4] $X_{0}$ was treated as a scalar diquark-antidiquark state $[sc\bar{c}]$. Contrary, in Ref. [5] $X_{0}$ was assigned as the $S$-wave hadronic molecule $D^{*-}K^{+}$, whereas for $X_{1}$ the authors adopted a diquark-antidiquark model.

We studied the resonance $X_{0}$ by considering it as a molecule $D^{0}K^{*0}$ and evaluated its mass and width [26]. Obtained results allowed us to confirm the molecule nature of $X_{0}$. We explored in Ref. [27] also the resonance $X_{1}$ and interpreted it as a vector diquark-antidiquark state $X_{V} = [ud][cs]$. The discovery of $X_{0}$ and $X_{1}$ by LHCb made the diquark-antidiqurk $[ud][cs]$ objects of intensive investigations. Indeed, the vector state $X_{V}$ from this group of particles presumably was seen in experiment as the resonance $X_{1}$. The masses and full widths of the ground-state and radially excited scalar particles $X_{0}^{(i)} = [ud][cs]$ were computed in Ref. [28]. Spectroscopic parameters and widths of the axial-vector and pseudoscalar tetraquarks $X_{AV}$ and $X_{PS}$ with the same content were calculated [29], as well.
It is worth to emphasize that the fully open flavor exotic mesons were already in agenda of researchers. Thus, the scalar diquark-antidiquark state $X_c = [su]$ was investigated in Ref. [30], in which its mass and full width were computed in the framework of QCD sum rule method using the $C\gamma_5 \otimes \gamma_5 C$ and $C\gamma_\mu \otimes \gamma^\mu C$ structures and, accordingly, the scalar-scalar and axial-axial interpolating currents. The scalar, pseudoscalar and axial-vector diquark-antidiquark states $[sd][\bar{s}\bar{q}]$, which carry two units of electric charge $-2|e|$ were studied in Ref. [31]. The particles $Z^{++}$ with the content $[cu][\bar{s}\bar{q}]$ are positively charged counterparts of these states and have the same masses and decay widths. Parameters of the vector tetraquark $Z_V^{++}$ became available recently [32]. Knowledge gained during these investigations is very useful to examine the resonances $T_{c^{0}0}^{a^{0}}$. 

In our view, the resonance $T_{c^{0}0}^{a^{0}}$ is a more interesting object for exploration, because it has additional attractive feature as a first doubly charged tetraquark observed in the experiment. As is seen, the $T_{c^{0}0}^{a^{0}}$ has a content which is identical to one of the tetraquarks $Z^{++}$. The scalar tetraquark $Z_{S}^{++}$ has the mass and width [31]

$$m_{Z_c} = 2628^{+166}_{-153} \text{ MeV, } \Gamma_{Z_c} = (66.89 \pm 15.11) \text{ MeV}, \quad (4)$$

which are far from the parameters of $T_{c^{0}0}^{a^{0}}$. 

As we have noted above, the neutral scalar resonance $T_{c^{0}0}^{a^{0}}$ is composed of quarks $c\bar{d}s\bar{u}$. The quark content and spin-parity of $T_{c^{0}0}^{a^{0}}$ coincide with parameters of the scalar tetraquark $X_c = [cd][\bar{s}\bar{q}]$. The latter is an antiparticle of $X_c$ and should have the same parameters as $X_c$.

$$m_{S} = (2634 \pm 62) \text{ MeV and } \Gamma_{S} = (57.7 \pm 11.6) \text{ MeV}, \quad (5)$$

and

$$m_{A} = (2590 \pm 60) \text{ MeV and } \Gamma_{A} = (63.4 \pm 14.2) \text{ MeV}, \quad (6)$$

which were found using for $X_c$ scalar-scalar or axial-axial currents. Let us note that the prediction $(2.55 \pm 0.09)$ GeV for the mass of the $X_c$ was obtained in Ref. [33], as well. In other words, neither $T_{c^{0}0}^{a^{0}}$ nor $T_{c^{0}0}^{a^{0}}$ can be interpreted as the ground-level scalar diquark-antidiquark systems.

In this situation, we can study the $T_{c^{0}0}^{a^{0}}$ and $T_{c^{0}0}^{a^{0}}$ within a hadronic molecule model, i.e., as a bound state of conventional mesons. Let us analyze in details the resonance $T_{c^{0}0}^{a^{0}}$. Interpretation of a molecule $D_{s}^{+}\pi^{\pm}$ as the resonance $T_{c^{0}0}^{a^{0}}$ seems is difficult, because the mass of such system is considerably smaller than $m_{d^{0}}$. The hadron states $D_{s}^{+}\pi^{\pm}$ and $D^{++}K^{+}$ or their superposition provide alternative choices for $T_{c^{0}0}^{a^{0}}$. Two-particle thresholds for these molecules are equal to $2887$ MeV and $2902$ MeV, respectively. They cannot decay to $D_{s}^{+}\pi^{0}$ and $D^{++}K^{+}$ meson pairs if their masses are less than these thresholds. Otherwise, the molecules $D_{s}^{+}\pi^{+}$ and $D^{++}K^{+}$ with masses above these limits dissociate to these mesons. In both cases, decay to a pair of the pseudoscalar $D_{s}^{0}$ and $\pi^{+}$ mesons are kinematically allowed channels for the molecules $D_{s}^{0}\pi^{+}$ and $D^{++}K^{+}$.

In our paper [34], we investigated the resonance $T_{c^{0}0}^{a^{0}}$ by modeling it as a hadronic molecule $D^{++}K^{+}$. The results for the mass and width of such compound system $(2924 \pm 107)$ MeV, and $(123 \pm 25)$ MeV are consistent with parameters of $T_{c^{0}0}^{a^{0}}$ given by Eq. (2). The resonances $T_{c^{0}0}^{a^{0}}$ were investigated using different models and approaches in Refs. [35–38]. It is interesting that conclusions made about nature of these structures also differ from each other. Thus, in Ref. [35], the one-boson exchange model was used to explore the interactions in $D^{(*)}K^{(*)}$ systems. Analysis performed in this article allowed the authors to assign $T_{c^{0}0}^{a^{0}}$ to be an isovector $D^{++}K^{+}$ molecule state with the spin-parity $J^{P} = 0^{+}$ and mass $2891$ MeV. Interpretation of the new tetraquark candidate $T_{c^{0}0}^{a^{0}}$ as the resonance-like structure induced by threshold effects was suggested in Ref. [36]. Here, it was argued that the triangle singularity generated by the $c\bar{c}K^{*}D^{*}$ loop peaks around the threshold $D^{*}K^{*}$ and may simulate $T_{c^{0}0}^{a^{0}}$.

A multiquark color flux-tube model was used to investigate the resonances $T_{c^{0}0}^{a^{0}}$ in the framework of the diquark-antidiquark model [37]. The authors found that a system $[cu][\bar{s}\bar{q}]$ built of the color antitriplet diquark and triplet antidiquark with the mass $2923$ MeV is very nice candidate to the resonance $T_{c^{0}0}^{a^{0}}$. The properties of the charmed-strange tetraquarks were studied in Ref. [38] by employing a nonrelativistic potential quark model.

In present work, we explore the spectroscopic parameters of the molecule $M^{++} = D_{s}^{+}\rho^{+}$. The molecular structure $M^{0} = D_{s}^{+}\rho^{+}$ may be considered as a model for $T_{c^{0}0}^{a^{0}}$. We calculate the mass and current coupling of $M^{++}$ using QCD two-point sum rule method, and confront our predictions with the experimental data of the LHCb Collaboration.

This article is structured in the following manner: In Sec. III we derive the sum rules for the mass $m$ and current coupling $f$ of the molecule $M^{++}$ in the context of QCD sum rule method. Numerical analysis of the quantities $m$ and $f$ is performed in Sec. IV where we determine working windows for the Borel and continuum subtraction parameters, and evaluate $m$ and $f$. The section IV contains our concluding remarks. 

### II. SPECTROSCOPIC PARAMETERS OF THE HADRONIC MOLECULE $M^{++} = D_{s}^{+}\rho^{+}$

We compute the mass $m$ and current coupling $f$ of the hadronic molecule $M^{++}$ using the QCD two-point sum rule method [39, 40]. To obtain sum rules for $m$ and $f$, we start the analysis by considering the following correlation function:

$$\Pi(p) = i \int d^{4}x e^{ipx} \langle 0| T\{J(x)J(0)\}|0\rangle, \quad (7)$$
where $T$ is the time-ordering operator, and $J(x)$ stands for the interpolating current of the molecule $M^{++}$.

In the molecule model colorless four-quark structures come from the singlet-singlet $[1_c] \otimes [1_c]$ and octet-octet $[8_c] \otimes [8_c]$ terms of the color group $SU_c(3)$. In the case of $M^{++}$, we suppose that the hadronic molecule $M^{++}$ is made of two vector mesons $D_s^{++}$ and $\rho^+$, and consider the singlet-singlet type current. Then, in the $[1_c] \otimes [1_c] \otimes [1_c] \otimes [1_c]$ representation $J(x)$ has the following form

$$J(x) = (\gamma^\mu c_a(x) \mathcal{D}_a(x) \gamma^\mu u_b(x)), \quad (8)$$

with $a$ and $b$ being color indices.

It is worth to note that $J(x)$ couples not only to the molecule $M^{++}$ but also to diquark-antidiquark states. The reason is that a molecule current by means of Fierz transformation can be expressed as the sum of different diquark-antidiquark currents with some numerical factors [41]. In other words, the molecule current is a special weighted sum of diquark-antidiquark currents. Contrary, a diquark-antidiquark current can be rewritten via molecule structures Refs. 42 43. Only comparison with experimental data can justify a choice of molecule or diquark-antidiquark type structures to model the resonance $M^{++}$.

In the sum rule approach, the correlator $\Pi(p)$ has to be presented using physical parameters of $M^{++}$, and also written down in terms of different quark-gluon condensates of QCD. For the physical side of the sum rule, we get

$$\Pi^{\text{Phys}}(p) = \langle 0 | J|M^{++}(p) \rangle \langle M^{++}(p) | J | 0 \rangle \frac{m^2}{m^2 - p^2} + \cdots, \quad (9)$$

where $p$ is the four momentum of $M^{++}$. In Eq. (9) the term shown explicitly is contribution of ground-state particle $M^{++}$, whereas ellipses stand for effects of higher resonances and continuum states in the $M^{++}$ channel. To derive the physical side $\Pi^{\text{Phys}}(p)$ of the sum rule from Eq. (7), we insert a complete set of intermediate states with content and quantum numbers of the $M^{++}$ state, and carry out integration over $x$.

Introducing the physical parameters of $M^{++}$ through the matrix element

$$\langle 0 | J|M^{++}(p) \rangle = f m, \quad (10)$$

we recast $\Pi^{\text{Phys}}(p)$ into the final form

$$\Pi^{\text{Phys}}(p) = f^2 \frac{m^2}{m^2 - p^2} + \cdots. \quad (11)$$

The function $\Pi^{\text{Phys}}(p)$ includes only one Lorentz structure, namely the unit matrix $I$, and term in rhs of Eq. (11) is the invariant amplitude $\Pi^{\text{Phys}}(p^2)$ corresponding to this structure.

The QCD side of the sum rules, $\Pi^{\text{OPE}}(p)$, should be calculated in the operator product expansion (OPE) with certain accuracy. To find $\Pi^{\text{OPE}}(p)$, we use in Eq. (7) the interpolating current $J(x)$, and contract the corresponding heavy and light quark fields using the Wick’s theorem. By performing these operations for $\Pi^{\text{OPE}}(p)$, we obtain

$$\Pi^{\text{OPE}}(p) = i \int d^4x e^{ipx} \mathrm{Tr} \left[ \gamma^\mu S^{aa'}_c(x) \gamma^\mu S^a_s(x) \right] \times \mathrm{Tr} \left[ \gamma^\mu S^{bb'}_w(x) \gamma^\mu S^b_t(-x) \right], \quad (12)$$

with $S_c(x)$ and $S_w(x)$ being the quark propagators. The explicit expressions for the heavy and light quarks propagators are collected in Ref. 44.

The $\Pi^{\text{OPE}}(p)$ has also a trivial structure ~I and is characterized by an amplitude $\Pi^{\text{OPE}}(p^2)$. After evaluating the invariant amplitudes $\Pi^{\text{Phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$, one obtains the QCD sum rule equality. In order to suppress contributions of higher resonances and continuum states one has to apply the Borel transformation to both sides of this expression. We apply the continuum subtraction supported by the quark-hadron duality assumption as well. These operations generate dependence of the sum rule equality on the Borel $M^2$ and continuum threshold $s_0$ parameters. Final expression and its derivative over $d/d(-1/M^2)$ can be used to derive sum rules for the mass $m$ and coupling $f$ of the molecule $M^{++}$

$$m^2 = \frac{\Pi(M^2, s_0)}{\Pi(M^2, s_0)}, \quad (13)$$

and

$$f^2 = \frac{\epsilon m^2 / M^2}{m^2 - \Pi(M^2, s_0)}. \quad (14)$$

Here, $\Pi(M^2, s_0)$ is the invariant amplitude $\Pi^{\text{OPE}}(p^2)$ after Borel transformation and subtraction procedures, and $\Pi(M^2, s_0) = d \Pi(M^2, s_0)/d(-1/M^2)$.

The Borel transformation of the amplitude $\Pi^{\text{Phys}}(p^2)$ has the simple form

$$\frac{B \Pi^{\text{Phys}}(p^2)}{\Pi(M^2, s_0)} = f m e^{-m^2 / M^2}, \quad (15)$$

whereas the correlator $\Pi(M^2, s_0)$ is given by the expression

$$\Pi(M^2, s_0) = \int_{(m_+ + m_+)^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s / M^2} + \Pi(M^2). \quad (16)$$

The spectral density $\rho^{\text{OPE}}(s)$ is calculated as an imaginary part of the amplitude $\Pi^{\text{OPE}}(p^2)$. The term $\Pi(M^2)$ in Eq. (16) is the Borel transformations of some of terms evaluated directly from their expressions in $\Pi^{\text{OPE}}(p)$. In this article, we neglect the masses of the $u$ and $d$ quarks, but take into account terms $\sim m_a$ setting, at the same time, $m_a = 0$. Computations are performed by including into analysis the vacuum expectation values of the non-perturbative operators up to dimension 8. The higher dimensional contributions to $\Pi(M^2, s_0)$ are obtained as
products of basic vacuum condensates using the factorization procedure. First few terms in OPE do not contain such condensates. They appear at higher dimensions and are numerically small. Therefore, we neglect the ambiguities caused by the factorization and their impact on the sum rules’ results.

Analytical expressions of $\rho^{\text{OPE}}(s)$ and $\Pi(M^2)$ are rather lengthy and not presented here explicitly.

III. NUMERICAL ANALYSIS

The sum rules in Eqs. (13) and (14) contain different quark, gluon and mixed condensates. They are universal parameters, and were extracted from the analysis of numerous processes. The mass $m$ and coupling $f$ depend also on the masses of $c$ and $s$ quarks. To carry out numerical computations, one has to fix values all of these parameters. Below, we list the values of these condensates

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3, \langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle,$$

$$\langle \bar{g}g, \sigma Gq \rangle = m_2^2 \langle \bar{q}q \rangle; \langle \bar{g}g, \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle,$$

$$m_2^2 = (0.8 \pm 0.2) \text{GeV}^2,$$

$$\langle \alpha_s G^2 \rangle = (0.012 \pm 0.004) \text{GeV}^4,$$

$$\langle g_s^2 G^3 \rangle = (0.57 \pm 0.29) \text{GeV}^6,$$

$$m_s = 93^{+11}_{-5} \text{MeV}, \ m_c = 1.27 \pm 0.02 \text{GeV}. \quad (17)$$

First of them is a measure of the pole contribution and is necessary to find the upper limit $M^2_{\text{max}}$ of the Borel region. In sum rule analyses of the conventional hadrons a constraint $\text{PC} \geq 0.5$ is a standard requirement. In the case of multiquark hadrons this constraint may be satisfied, although it shrinks a region for $M^2$. The function $R(M^2)$ is employed to find the lower limit, $M^2_{\text{min}}$, of the window for the Borel parameter. Here, $\Pi^{\text{Dim8}}(M^2, s_0)$ indicates the last term in the operator product expansion of $\Pi(M^2, s_0)$. For computations performed up to dimension 8 the constraint $R(M^2_{\text{min}}, s_0) \leq 0.05$ is a reasonable requirement.

Analysis demonstrates that the regions for the parameters $M^2$ and $s_0$

$$M^2 \in [2, 3] \text{ GeV}^2, \ s_0 \in [11, 12.2] \text{ GeV}^2, \quad (20)$$

obey all the necessary constraints. Indeed, at $M^2 = 3 \text{ GeV}^2$ the pole contribution on average in $s_0$ is 0.50, whereas at $M^2 = 2 \text{ GeV}^2$ it becomes equal to 0.72. In Fig. [1] the pole contribution is depicted as a function of $M^2$ at various fixed $s_0$. Only at $s_0 = 11 \text{ GeV}^2$ in a small region, $M^2 \geq 2.8 \text{ GeV}^2$, the pole contribution is less than 0.5. But on average in $s_0$ the constraint $\text{PC} \geq 0.5$ is satisfied in the entire working region for the Borel parameter. At the minimum point, $M^2 = 2 \text{ GeV}^2$, we get $R(2 \text{ GeV}^2) \approx 0.022$ and the contribution of dimension-8 term does not exceed 2.2% of the whole result.

![FIG. 1: The pole contribution to the correlator $\Pi(M^2, s_0)$ as a function of the Borel parameter $M^2$ at different $s_0$. The horizontal black line limits the border $\text{PC} = 0.5$. The red diamond marks the point at which the mass $m$ of the molecule $M^{++} = D_s^+ \rho^+$ has effectively been extracted.](image)

We included into Eq. (17) the masses of $c$ and $s$ quarks, as well.

The working regions for the parameters $M^2$ and $s_0$ should meet the standard constraints imposed on the pole contribution (PC) and convergence of the operator product expansion. To quantify these restrictions, it is convenient to introduce the formulas

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \quad (18)$$

and

$$R(M^2) = \frac{\Pi^{\text{Dim8}}(M^2, s_0)}{\Pi(M^2, s_0)}. \quad (19)$$
Results for $m$ and $f$ are obtained by taking their mean values at different choices of the parameters $M^2$ and $s_0$

$$m = (2917 \pm 135) \text{ MeV},$$

$$f = (4.65 \pm 0.95) \times 10^{-3} \text{ GeV}^4. \quad (21)$$

The $m$ and $f$ from Eq. (21) effectively correspond to sum rules’ predictions at $M^2 = 2.4 \text{ GeV}^2$ and $s_0 = 11.8 \text{ GeV}^2$ marked in Fig. 1 by the red diamond, where the pole contribution is $PC \approx 0.64$. This fact ensures the ground-state nature of $M^{++}$ and reliability of obtained results.

The mass $m$ as functions of the parameters $M^2$ and $s_0$ is shown in Fig. 3. Here, one can see a dependence of $m$ on the parameter $M^2$, though the physical quantity should not depend on it. Nevertheless, such residual dependence of $m$ and $f$ on the Borel parameter exists and generates essential part of theoretical uncertainties shown in Eq. (21). There is also dependence on the choice of $s_0$, but this effect may be used to extract information on the mass of the first excited particle in the $D_s^+ \rho^+$ channel. Because $\sqrt{s_0}$ should be less than the mass $m^*$ of the first excited molecule $M^{++}$, we find an estimate $m^* \geq m + 500 \text{ MeV}$, which may be expected for $M^{++}$ molecule containing a $c$ quark.

As is seen, the result $m = (2917 \pm 135) \text{ MeV}$ for the mass of the molecule $M^{++} = D_s^+ \rho^+$ obtained in the current paper agrees nicely with the LHCb datum $m_{2\text{exp}}$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Left: Perturbative and nonperturbative contributions to $\Pi(M^2, s_0)$ normalized to 1 as functions of the Borel parameter $M^2$. Right: Normalized contributions of different operators to the correlator as functions of $M^2$. All curves in the figure have been calculated at $s_0 = 11.6 \text{ MeV}^2$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{The dependence of the mass $m$ of the molecule $M^{++}$ on the Borel parameter $M^2$ (left panel), and on the continuum threshold parameter $s_0$ (right panel).}
\end{figure}

\section*{IV. CONCLUDING REMARKS}

As is seen, the result $m = (2917 \pm 135) \text{ MeV}$ for the mass of the molecule $M^{++} = D_s^+ \rho^+$ obtained in the
In our article \cite{34}, we modeled the resonance $T_{ca0}^{a++}$ as the hadronic molecule $D^+K^{++}$, and calculated its mass and width. The mass ($2924 \pm 107$) MeV of $D^+K^{++}$ is also consistent with the LHCb data. Comparing these two models with each other, one sees that they lead to very close results and can be applied to describe the resonance $T_{ca0}^{a++}$. Therefore, at this level of our knowledge, we can interpret the molecules $D_s^+\rho^-$ and $D^0K^{*0}$, or their superposition as candidates to the resonance $T_{ca0}^{a++}$.

The neutral molecules $M^0 = D_s^+\rho^-$ and $D^0K^{*0}$ are possible models for the resonance $T_{ca0}$. A linear superposition of these molecules also may be used to model $T_{ca0}$. The mass of $T_{ca0}^{a++}$ as an isoscalar partner of $T_{ca0}^{a++}$ should be close to the extracted value $m$. In fact, experimentally measured mass and width differences between $T_{ca0}^{a++}$ and $T_{ca0}^{a++}$ are equal to $\Delta m \approx 28$ MeV and $\Delta \Gamma \approx 15$ MeV, respectively. But, to be accepted as a reliable model for $T_{ca0}^{a++}$ the molecule picture should be successfully confronted with the LHCb data.

As we have noted in Sec. \[\text{III}\] the $J(x)$ couples to various diquark-antidiquark states, and their specially chosen superposition gives the molecule current. The fixed ground-level diquark-antidiquark state, as it has been discussed in a detailed form in Sec. \[\text{I}\] does not describe parameters of the resonances $T_{ca0}^{a0+}$ in the width. In these circumstances, assumptions about molecule nature of $T_{ca0}^{a0+}$ seem more realistic than other four-quark models. Number of alternative molecule models for $T_{ca0}^{a0+}$ is restricted by the masses and widths of these states, as well as by the fact that the resonance $T_{ca0}^{a0+}$ carries two units of electric charge. This question requires additional detailed analysis, which is beyond the scope of the present work.

The structures $T_{ca0}^{a0+}$ were studied in the context of other models as well \cite{35,38}. At present there are no definite conclusion on the quark structure not only of resonances $T_{ca0}^{a0+}$ but also other exotic hadrons. This paper and Ref. \cite{34} are attemps to clarify a situation around of very interesting structure $T_{ca0}^{a++}$.

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