Efficient Non-linear Equalization for 1-bit Quantized Cyclic Prefix-Free Massive MIMO Systems

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\textbf{Abstract}—This paper addresses the problem of data detection for a massive \textit{Multiple-Input-Multiple-Output} (MIMO) base station which utilizes 1-bit \textit{Analog-to-Digital Converters} (ADCs) for quantizing the uplink signal. The existing literature on quantized massive MIMO systems deals with \textit{Cyclic Prefix} (CP) transmission over frequency-selective channels. In this paper, we propose a computationally efficient block processing equalizer based on the \textit{Expectation Maximization} (EM) algorithm in CP-free transmission for 1-bit quantized systems. We investigate the optimal block length and overlapping factor in relation to the \textit{Channel Impulse Response} (CIR) length based on the \textit{Bit Error-Rate} (BER) performance metric.

As EM is a non-linear algorithm, the optimal estimate is found iteratively depending on the initial starting point of the algorithm. Through numerical simulations we show that initializing the EM-algorithm with a Wiener-Filter (WF) estimate, which takes the underlying quantization into account, achieves superior BER performance compared to initialization with other starting points.

I. INTRODUCTION

Massive MIMO plays an important role for future communication systems, since the large number of antennas is capable of increasing the spectral efficiency and the amount of useable spectrum [1]. However, a simple and power-efficient analog \textit{Radio Frequency} (RF)-frontend design, together with the use of appropriate baseband-processing algorithms, becomes crucial to support a large number of antennas. Especially in high-speed millimeter-wave communication systems, the increasing complexity and power-consumption of the key components in the RF-chain, such as the high-speed ADC, can be identified as the primary bottlenecks. Whereas the power-consumption of the ADC scales roughly exponentially with the number of quantization bits [2], the use of 1-bit ADCs consumes the least amount of power and simplifies the hardware-complexity of the entire RF-frontend significantly. The lost information due to the coarse quantization can furthermore be recovered by designing data-detection algorithms, which take the effect of coarse quantization into consideration.

Equalization algorithms for narrowband systems with frequency-flat channels have been investigated in [3] and [4] for the case of 1-bit ADCs at the receive antennas. In [5], C. Struder and G. Durisi have recently proposed algorithms for quantized maximum a-posteriori (MAP) channel estimation and data detection under frequency-selective channels. In [6], low-complexity channel estimation and data detection for frequency-selective massive MIMO systems employing 1-bit ADCs was proposed based on linear combiners. A message passing algorithm for data-detection for an underlying quantized single-carrier system is proposed in [7]. To the best of our knowledge, all the mentioned contributions in the massive MIMO literature utilize CP for \textit{Orthogonal Frequency Division Multiplexing} (OFDM) and \textit{Single Carrier} (SC) transmission techniques, i.e., CP-OFDM and CP-SC.

State-of-the-art communication systems apply CP for efficient \textit{Frequency Domain Equalization} (FDE) by means of the FFT. However, the use of a CP comes at the price of a loss in spectral efficiency. It is therefore desirable to investigate computationally-efficient equalization methods without CP. In [8] we have proposed efficient linear-FDE for 1-bit quantized wide-band massive MIMO systems without CP, using an overlap-save method for equalization, while taking the quantization effect into consideration.

The EM-algorithm was originally introduced for channel estimation in wide-band \textit{Single-Input-Single-Output} SISO systems [9] and extended to flat-fading channels for massive MIMO systems in [10] and [11]. In [12] the authors propose a slightly modified EM-algorithm for mmWave frequency-flat MIMO channels, which is found to have a high computational complexity based on a matrix inversion in time-domain [13]. In [14] channel estimation based on the EM-algorithm is performed for frequency-selective massive MIMO systems, however, under the reassumption of CP. In this work, we therefore propose efficient nonlinear-FDE using the EM-algorithm for 1-bit quantized, frequency-selective, massive MIMO systems without CP.

The paper is organized as follows: An exact and mismatched quantized system model is introduced in Section II. Section III describes a probabilistic model for data-detection. In Section IV and V we derive the time-domain and frequency-domain representation of the EM-algorithm, respectively, and assess their complexity. In Section VI the performance of EM is evaluated and compared to linear equalization methods. Section VII concludes the paper.

Notation: Bold letters indicate vectors and matrices, non-bold letters express scalars. For a matrix $A$, we denote complex conjugate, transpose and Hermitian transpose by $A^\ast$, $A^T$ and $A^{\dagger}$, respectively. The operator $\text{diag}(A)$ describes a diagonal matrix containing only the diagonal elements of $A$ and $\text{vec}(A)$ denotes the vectorization operation with column-major order. The Kronecker product between matrices is given
as $A \otimes B$. The $n \times n$ identity matrix is denoted by $I_n$, while
the $n \times m$ all-zeros matrix is defined as $0_{n \times m}$.

II. System Model

The uplink of a single-cell scenario is considered where the Base-Station (BS) equipped with $M$ antennas receives the signals from $K$ single-antenna Mobile-Station (MSs). We assume a frequency-selective block fading channel between each pair of MS and BS antennas. The channel between BS $m \in \{1, 2, \ldots, M\}$ and MS $k \in \{1, 2, \ldots, K\}$ is completely characterized by an impulse response of $L + 1$ taps, denoted by $h_{mk} \in \mathbb{C}^{(L + 1) \times 1}$.

We will derive the input-output relationship based on the exact and the mismatched model in the next two subsections.

A. Exact Model (ExaMod): Block-Toeplitz Channel Matrix

The unquantized receive signal at BS $m$ is written as:

$$y_m[n] = \sum_{l=0}^{L} h_{m}^T[l]x[n - l] + \eta_m[n], \tag{1}$$

where $x[n] = [x_1[n] \ x_2[n] \ \cdots \ x_K[n]]^T \in \mathbb{C}^{K \times 1}$ is the zero-mean circularly-symmetric complex valued transmit vector with $E_x[x[n] \cdot x^H[n]] = \sigma_x^2I_K$ and $h_m[l] = [h_{m1}[l] \ h_{m2}[l] \ \cdots \ h_{mK}[l]]^T \in \mathbb{C}^{K \times 1}$ is constructed from the $l$th tap of the channel impulse response from all users on the $m$th antenna. Let the noise be drawn from the i.i.d. zero-mean circularly-symmetric complex Gaussian vector $\eta[n] = [\eta_1[n] \ \eta_2[n] \ \cdots \ \eta_M[n]]^T \in \mathbb{C}^{M \times 1}$, having the noise-covariance of $E_\eta[\eta[n] \cdot \eta[n]^H] = \sigma_n^2I_M$. We furthermore assume that the transmit and noise symbols are temporarily uncorrelated.

Using (1), the unquantized receive vector $y[n] = [y_1[n] \ y_2[n] \ \cdots \ y_M[n]]^T \in \mathbb{C}^{M \times 1}$ at time instant $n$ can be written as

$$y[n] = \sum_{l=0}^{L} H_l x[n - l] + \eta[n], \tag{2}$$

where $H_l = [h_1[l] \ h_2[l] \ \cdots \ h_K[l]]^T \in \mathbb{C}^{K \times K}$ is a channel impulse response matrix. The signal vector $y[n]$ is then quantized by a 1-bit uniform scalar quantizer to obtain

$$r[n] = Q(y[n]) = Q\left(\sum_{l=0}^{L} H_l x[n - l] + \eta[n]\right), \tag{3}$$

where $Q(\cdot)$ is applied element-wise to $y[n]$ and keeps only the sign of the real and imaginary part, i.e., $Q(z) = \text{sign}(\Re\{z\}) + j\text{sign}(\Im\{z\})$ for $z \in \mathbb{C}$ with

$$\text{sign} : \mathbb{R} \rightarrow \{-1, +1\}, \ x \mapsto \text{sign}(x) = \begin{cases} -1, & x \leq 0 \\ +1, & x > 0 \end{cases}.$$

Let us collect $N_b$ vectors, with a condition that $N_b > L$, corresponding to time instances $n, n - 1, \ldots, n - (N_b - 1)$ in order to form a space-time quantized receive matrix $R[n]$, unquantized receive matrix $Y[n]$, and noise matrix $N[n]$ as:

$$R[n] = [r[n] \ r[n-1] \ \cdots \ r[n-(N_b-1)]] \in \mathbb{C}^{M \times N_b},$$

$$Y[n] = [y[n] \ y[n-1] \ \cdots \ y[n-(N_b-1)]] \in \mathbb{C}^{M \times N_b},$$

$$N[n] = [\eta[n] \ \eta[n-1] \ \cdots \ \eta[n-(N_b-1)]] \in \mathbb{C}^{M \times N_b}.$$  

The transmit matrix $X[n] \in \mathbb{C}^{K \times (N_b+L)}$ is given by

$$X[n] = [X_c[n] \ X_m[n]], \tag{4}$$

$$X_c[n] = [x[n] \ x[n-1] \ \cdots \ x[n-(N_b-1)]] \in \mathbb{C}^{K \times N_b},$$

$$X_m[n] = [x[n-N_b] \ \cdots \ x[n-(N_b+L)]] \in \mathbb{C}^{K \times L}, \tag{5}$$

such that the space-time input-output relationship of the quantized MIMO system is given as

$$\text{vec}(Y[n]) = \tilde{H} \text{vec}(X[n]) + \text{vec}(N[n]) \in \mathbb{C}^{M \cdot N_b \times 1}, \tag{6}$$

$$\text{vec}(R[n]) = Q(\tilde{H} \text{vec}(X[n]) + \text{vec}(N[n])), \tag{7}$$

where the channel matrix $\tilde{H} \in \mathbb{C}^{M \cdot N_b \times K(N_b+L)}$ has a block-Toeplitz structure of the form

$$\tilde{H} = \begin{bmatrix} H_0 & H_1 & \cdots & H_L & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \cdots & \cdots & \cdots & H_L \\ 0 & \cdots & 0 & H_0 & H_1 & \cdots & H_L \end{bmatrix}. \tag{8}$$

Here, the matrix $0$ denotes $0_{M \times K}$ for the sake of brevity.

B. Mismatched Model (MisMod): Block-Circulant Channel Matrix Approximation

The first step in the mismatched model is to represent a block-Toeplitz channel matrix in the system model (7) as a block-circulant channel matrix with an interference term:

$$\text{vec}(Y[n]) = \tilde{H}_{\text{cir}} \text{vec}(X_c[n]) + \text{vec}(N[n]) + \tilde{\gamma}[n]. \tag{9}$$

In (10), $\hat{H}_{\text{cir}} \in \mathbb{C}^{M \cdot N_b \times K \cdot N_b}$ is a block-circulant matrix

$$\hat{H}_{\text{cir}} = \begin{bmatrix} H_0 & H_1 & \cdots & H_L & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ H_L & \ddots & \cdots & \cdots & \cdots & \cdots & H_0 \\ 0 & \cdots & 0 & H_0 & H_1 & \cdots & H_L \end{bmatrix}$$

and

$$\tilde{\gamma}[n] = \tilde{H}_{\text{in}}\left(\text{vec}(X_{\text{in}}[n])^T 0_{1 \times (N_b+L)K} - \text{vec}(X_c[n])\right). \tag{11}$$

(11) can be considered as an interference noise which corrupts the last $M \cdot L$ equations in (10), where $\tilde{H}_{\text{in}} \in \mathbb{C}^{M \cdot N_b \times K \cdot N_b}$ is given as:

$$\tilde{H}_{\text{in}} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ H_L & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & H_0 & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \cdots & \ddots & \cdots & \vdots \\ H_1 & \cdots & H_L & 0 & \cdots & \cdots & 0 \end{bmatrix}. \tag{12}$$
We can now obtain a mismatched model by ignoring the interference term, i.e.,
\[
\begin{align*}
\text{vec}(Y[n]) & \approx \hat{H}_\text{cir} \text{vec}(X_c[n]) + \text{vec}(N[n]), \\
\text{vec}(R[n]) & \approx \hat{Q}(\hat{H}_\text{cir} \text{vec}(X_c[n]) + \text{vec}(N[n])).
\end{align*}
\]
In Section IV we will show that using the mismatched model (14) will enable a computationally efficient inversion of \( \hat{H}_\text{cir} \) in the frequency domain.

III. PROBABILISTIC MODEL FOR DATA DETECTION

This section derives the joint Probability Density Function (PDF) between the transmit signal \( X[n] \), unquantized receive signal \( Y[n] \) and quantized receive signal \( R[n] \). Let us generically express the exact (ExaMod) and the mismatched (MisMod) system model as follows:
\[
\begin{align*}
\text{vec}(Y[n]) = A \text{vec}(X[n]) + \text{vec}(N[n]), \\
\text{vec}(R[n]) = Q(\text{vec}(Y[n])),
\end{align*}
\]
where \( A = \hat{H} \in \mathbb{C}^{M \times (N_0 + L)} \), \( \text{vec}(Y[n]) = [X[n] \mid N[n]] \in \mathbb{C}^{K \times (N_0 + L)} \) for the exact system model (7), and \( A = \hat{H}_\text{cir} \in \mathbb{C}^{M \times N_0 \times K \times N_0} \), \( \text{vec}(X_c[n]) \in [X_c[n] \mid N_0] \in \mathbb{C}^{K \times N_0} \) for the mismatched system model (13).

For the sake of brevity, let us represent: \( \hat{y} = \text{vec}(Y[n]), \hat{\xi} = \text{vec}(X[n]), \hat{\eta} = \text{vec}(N[n]), \hat{r} = \text{vec}(R[n]) \) and \( \hat{z} = A\hat{\xi} \). The MIMO system model can then be rewritten as
\[
\begin{align*}
\hat{y} &= A\hat{\xi} + \hat{\eta} = \hat{z} + \hat{\eta}, \\
\hat{r} &= Q(\hat{y}),
\end{align*}
\]
where \( A \in \mathbb{C}^{M \times N_0 \times P} \), \( \hat{\xi} \in \mathbb{C}^P \), \( \hat{y}, \hat{r}, \hat{\eta} \in \mathbb{C}^{M \times N_0} \), \( P = K (N_0 + L) \) for the exact and \( P = K N_0 \) for the mismatched system model. The relationship between \( \hat{\xi} \) and \( \hat{y} \) in (17) can be described by the conditional PDF:
\[
p(\hat{y}|\hat{\xi}) = \frac{1}{(\pi \sigma_\eta^2)^{M+N_0}} \exp \left(-\frac{\|\hat{y} - A\hat{\xi}\|^2}{\sigma_\eta^2}\right)
\]
as \( \hat{y} \sim \mathcal{CN}(A\hat{\xi}, \sigma_\eta^2 I_{M \times N_0}) \) [15]. Similarly, (13) can be represented as the conditional Probability Mass Function (PMF)
\[
p(\hat{r}|\hat{y}) = I_{D(\hat{r})}(\hat{y}) = p(\hat{r}|\hat{y}, \hat{\xi})
\]
of \( \hat{r} \) given \( \hat{y} \), where
\[
I_{D(\hat{r})}(\hat{y}) = \begin{cases} 
1, & \hat{r} = Q(\hat{y}) \\
0, & \text{otherwise}
\end{cases}
\]
With (19) and (20), the joint PDF of \( \hat{r}, \hat{y} \) and \( \hat{\xi} \) reads as
\[
p(\hat{r}, \hat{y}, \hat{\xi}) = I_{D(\hat{r})}(\hat{y}) \frac{1}{(\pi \sigma_\eta^2)^{M+N_0}} \exp \left(-\frac{\|\hat{y} - A\hat{\xi}\|^2}{\sigma_\eta^2}\right) p(\hat{\xi}).
\]

IV. EXPECTATION-MAXIMIZATION (EM) BASED MAXIMUM A POSTERIORI (MAP) SOLUTION

The direct maximization of \( p(\hat{r}, \hat{\xi}) \) using the MAP estimate
\[
\hat{\xi} = \text{argmax}_{\hat{\xi} \in \mathbb{C}^P} p(\hat{r}, \hat{\xi})
\]
is in general intractable [15]. The EM-algorithm computes the MAP estimate \( \hat{\xi} \) by iteratively maximizing the MAP log-likelihood function \( \ln(p(\hat{r}, \hat{\xi})) \) [10, 15].

A. EM-Algorithm

At each iteration, the following two steps are performed:

1) Expectation (E)-step: In the \( u \)-th iteration of the E-step, the expected MAP log-likelihood function is computed
\[
q(\xi, \hat{\xi}) = E_{y|\hat{r}, \xi} \left[ \ln p(\hat{r}, \hat{y}, \xi) \right].
\]
It is shown in [14] c.f. Eq. (21) that (24) reduces to
\[
\xi^{(u)} = E_{y|\hat{r}, \xi^{(u)}}[y] = E_{y|\hat{r}, \xi^{(u)}}[\hat{y}],
\]
which is an estimate of the unquantized receive signal \( \hat{y} \).

A closed form expression for this expectation is derived in Appendix A. The \( i \)-th element of \( \xi^{(u)} \), \( i \in \{1, 2, \ldots, M \cdot N_b\} \), is given by:
\[
\xi^{(u)}_i = \frac{\sigma_\eta^2}{\sqrt{2}} \left( R \{ r_i \} \varphi(w_i) \frac{\Re \{ r_i \}}{\Phi(w_i)} + j \frac{\Im \{ r_i \}}{\Phi(w_i)} \right) + z_i,
\]
where \( z_i = a_i^T \xi^{(u)} \), \( w_i = R \{ r_i \} R \{ z_i \} / \sqrt{\sigma_\eta^2/2} \), \( w_i = \Re \{ r_i \} \Re \{ z_i \} / \sqrt{\sigma_\eta^2/2} \) and \( r_i \) is the \( i \)-th element of \( \hat{r} \), \( a_i^T \) is the \( i \)-th row of \( A \).

2) Maximization (M)-step: The maximization of the expected MAP log-likelihood function \( q(\hat{\xi}, \hat{\xi}) \) with respect to \( \hat{\xi} \) in the \( u \)-th iteration is given as [14] c.f. Eq. (22):
\[
\hat{\xi}^{(u+1)} = \text{argmin}_{\xi \in \mathbb{C}^P} \left\| A\xi - \hat{\xi}^{(u)} \right\|_2^2 - \frac{\sigma_\eta^2}{2} \ln p(\xi) \]
\[
= (A^H A + \sigma_\eta^2 R_{\xi^{(u)}}^{-1})^{-1} A^H \hat{\xi}^{(u)} = G \hat{\xi}^{(u)}.
\]

A priori information about the vector \( \xi \) can be incorporated with the prior PDF \( p(\xi) \). A Gaussian prior is assumed in (a) and the matrix \( G \) represents the space-time linear equalizer. The EM-algorithm is summarized in Algorithm 1.

Algorithm 1 Expectation-Maximization (EM) Algorithm

**Input:** \( A, \hat{r}, \xi^{(0)}, \sigma_\eta^2, p(\xi) \)

**Initialize:** \( u = 0 \)

while stopping criterion not met do

**E-step:** \( \hat{\xi}^{(u)} = E_{y|\hat{r}, \xi^{(u)}}[y] \)

**M-step:** \( \xi^{(u+1)} = (A^H A + \sigma_\eta^2 R_{\xi^{(u)}})^{-1} A^H \hat{\xi}^{(u)} \)

\( u := u + 1 \)

end while

**Output:** \( \hat{\xi} \)

The EM-algorithm can be stopped after \( I_{\text{max}} \) iterations or in the case \( \|\xi^{(u)} - \xi^{(u-1)}\|_2 \leq \gamma_{\text{EM}} \|\xi^{(u)}\|_2 \) with \( \gamma_{\text{EM}} > 0 \).
B. Computational Complexity

The necessary number of complex multiplications in relation to the coherence time $T_c$ is derived in this section. It is assumed that the coherence time represented by $T_c$ symbols is divided into $B$ blocks, each consisting of $N_b$ symbols:

$$ T_c = B \cdot N_b. \quad (28) $$

In the following we distinguish between static and dynamic complexity. As $G$ in (27) needs to be computed only once during $T_c$, its computational complexity is

$$ \mathcal{T}_G = P^3 + P^2 \cdot M \cdot N_b, \quad (29) $$

The dynamic complexity is a matrix-vector product calculated from the E-step (25) and M-step (27) for the $v$th block as

$$ \mathcal{T}_E^{(v)} + \mathcal{T}_M^{(v)} = M \cdot N_b \cdot (P + 1) + P^2 \cdot M \cdot N_b, \quad (30) $$

where $v \in \{1, \ldots, B\}$. The computational complexity of the Gaussian PDF $\varphi(\cdot)$ and Cumulative Distribution Function (CDF) $\Phi(\cdot)$ are ignored in (30) as the latter can be pre-calculated offline. With (29) and (30), the total complexity of the EM-algorithm during $T_c$ is given as:

$$ \mathcal{T}_{tot} = \sum_{v=1}^{B} I_v \cdot \left[ \mathcal{T}_E^{(v)} + \mathcal{T}_M^{(v)} \right] + \mathcal{T}_G $$

$$ = \sum_{v=1}^{B} I_v \left[ M \cdot N_b \cdot (2P + 1) \right] + P^3 + P^2 \cdot M \cdot N_b, \quad (31) $$

where $I_v$ is the number EM-iterations until convergence for the $v$th block. As $P \gg N_b$ for both models, furthermore assuming a constant $I_v$, i.e. $I_v \approx I \forall v$, the computational complexity for the static and dynamic part are $O \left( N_b^3 (K^3 + K^2 M) \right)$ and $O \left( 2N_b^2 BIKM \right)$, respectively. It is shown in [18] that $N_b \ll L$. Therefore, running the EM-algorithm in time-domain using (25) and (27) becomes computationally infeasible for $N_b \gg L$.

V. THE EM-ALGORITHM IN FREQUENCY DOMAIN

This section deals with the reduction of computational complexity of the EM-algorithm by exploiting the block-circulant structure of the channel matrix $A = \hat{H}_{\text{cir}}$ using the mismatched model in (25) and (27). Therefore, $\hat{H}_{\text{cir}}$ can be diagonalized by the Discrete Fourier Transform (DFT),

$$ \hat{H}_{\text{cir}} = \left( F^H \otimes I_M \right) \mathcal{H} \left( F \otimes I_K \right), \quad (32) $$

where $F \in \mathbb{C}^{N_b \times N_b}$ is an $N_b$-point DFT-matrix and $\mathcal{H}$ is block-diagonal:

$$ \mathcal{H} = \text{diag} \{ H_{f_1} \}_{i=1}^{N_b}, \quad (33) $$

$$ H_{f_i} = \sum_{l=0}^{L-1} H_l \cdot \exp \left( -j \frac{2 \pi i}{N_b} (i-1) \right), \quad \text{for } 1 \leq i \leq N_b, $$

represents the multi-path MIMO channel in frequency domain.

A. Efficient E-Step

Applying (32) in $\hat{z} = \hat{H}_{\text{cir}} \hat{\xi}$ (c.f. Eq. (17)), the vector $\hat{z}$ is calculated in frequency domain

$$ \hat{z} = \left( F^H \otimes I_M \right) \mathcal{H} \left( F \otimes I_K \right) \hat{\xi}. \quad (34) $$

B. Efficient M-Step

Applying (32) in (27), the matrix $G$ can be efficiently calculated in frequency domain:

$$ G = \left( F^H \otimes I_K \right) G_f \left( F \otimes I_M \right), \quad (35) $$

where $G_f$ represents the frequency domain equalizer:

$$ G_f = \left( \mathcal{H}^H \mathcal{H} + \frac{\sigma_z^2}{\sigma_x^2} I_{KN_b} \right)^{-1} \mathcal{H}^H $$

$$ = \text{diag} \left\{ \left( H_{f_i}^H H_{f_i} + \frac{\sigma_z^2}{\sigma_x^2} I_K \right)^{-1} \right\}_{i=1}^{N_b}. \quad (37) $$

With (35) in (27), the efficient M-step can be implemented as:

- Frequency domain conversion of $\hat{y}_f^{(u)}$:
  $$ \hat{\gamma}_f^{(u)} = (F \otimes I_M) \hat{y}_f^{(u)}. \quad (38) $$

- Frequency domain equalization of $\hat{\gamma}_f^{(u)}$:
  $$ \hat{x}_f^{(u+1)} = G_f \hat{\gamma}_f^{(u)}. \quad (39) $$

- Time domain conversion of $\hat{\gamma}_f^{(u+1)}$:
  $$ \hat{x}_f^{(u+1)} = (F^H \otimes I_K) \hat{\gamma}_f^{(u+1)}. \quad (40) $$

Note that the term $G \tilde{\gamma}_f[n]$ in (10) is an interference distortion, which corrupts the whole estimated data-block $\hat{\xi}$. The next subsection deals with minimizing the interference distortion.

C. Interference Analysis

It was shown in [19] that the ensemble-averaged interference distortion power has a bathtub-like distribution. This behavior can be exploited to minimize the resulting error by using a $L'$ samples overlapping of data blocks, i.e., $R[n]$ contains vectors corresponding to the time instances $n, \ldots, n-(N_b-1)$ and is followed by $R[n-(N_b-L')]$ with corresponding time elements $n-(N_b-L'), \ldots, n-(2N_b-L'-1)$. In this work we shall show that $L'$ is directly related to the length of the channel memory [17], i.e., $L' \ll L$. Since $L'$ can be even or odd, we define the pre-discard- and post-discard-lengths as $L_{pre} = \lfloor L'/2 \rfloor$ and $L_{post} = \lceil L'/2 \rceil$.

D. Computational Complexity

Let $\mathcal{T}_{E_f^{(v)}}$, $\mathcal{T}_{M_f^{(v)}}$ and $\mathcal{T}_{G_f}$ denote the computational complexity of the E-step, M-step and calculation of the matrix $G$ in frequency domain. With a $L'$ samples overlapping of data-blocks, we will process $B' = T_c / (N_b - L')$ blocks per $T_c$. Using Equations (37) to (40), the total complexity during $T_c$ for performing FDE with the EM-algorithm evaluates to:

$$ \mathcal{T}_{tot} = \sum_{v=1}^{B'} I_v \cdot \left[ \mathcal{T}_{E_f^{(v)}} + \mathcal{T}_{M_f^{(v)}} \right] + \mathcal{T}_{G_f} $$

$$ = \sum_{v=1}^{B'} I_v \left[ \left( (M + K) \cdot \log_2 N_b + K \cdot M \right) \cdot N_b \right] \cdot 2 $$

$$ + \left( K \cdot \log_2 (N_b) + 2 \cdot K^2 M + K^3 \right) \cdot N_b. \quad (41) $$
For derivation of $T_{\gamma}$, we refer to [8], Eq. (31), with the slight modification that the complexity regarding the noise-covariance matrix is not accounted for in [41], as it is a scaled identity matrix [13]. The terms $T_{\gamma}(v)$ and $T_{\gamma}^{(v)}$ are equal, and describe the FFT and IFFT of the data-blocks, both needed in the E-step and the M-step.

Therefore, the static and dynamic computational complexity become log-linear in $N_b$, effectively reducing the computational complexity by orders of magnitude compared to [31].

VI. SIMULATION RESULTS AND ANALYSIS

Consider a MIMO setup having $K$ single transmit antenna users and $M$ receive antennas. The CIR between each pair of transmit and receive antenna consists of $L + 1$ taps. The transmitter is employing 16-QAM, CP is omitted at the transmit side and the receiver is using 1-bit quantizers at each of the receive antennas. The channel coherence time is assumed to be $T_c = 50 \times 10^3$ symbols. The noise is i.i.d. zero-mean circularly-symmetric AWGN with variance $\sigma_n^2 = 1$ (cf. Sec. II) and the channel is chosen based on an Extended Vehicular A model (9 nonzero taps). The results are averaged over $N_{\text{sim}} = 200$ channel realizations. Perfect CSI and synchronization between transmitter and receiver is assumed throughout the equalization process. For the EM-algorithm, the stopping criterion is set to $\gamma_{\text{EM}} = 1 \times 10^{-3}$ and the maximum number of iterations is upper bounded by $I_{\text{max}} = 1000$.

We define the bit energy to noise spectral density $E_b/N_0$ as:

$$\frac{E_b}{N_0} = \frac{P_t}{KM\sigma_n^2} \text{trace}\left(\mathbf{E} \mathbf{H} \left(\mathbf{\hat{H}} \mathbf{\hat{H}}^H\right)\right),$$

where $P_t$ is the total transmit power, $B$ is the number of bits per constellation symbol and $\mathbf{\hat{H}}$ is the MIMO channel matrix.

The proposed EM-algorithm with different settings of overlap-discard processing is assumed throughout this section. A comparison to linear equalization based on [8] is established for performance comparison. The linear WF-approach for quantized and unquantized MIMO systems from [8] is denoted as WF$_{\text{EQ}}$ and WF$_{\text{E}}$ respectively, whereas our nonlinear EM-approach for the exact and mismatched model is denoted as EM$_{\mu}$ for $\mu \in \{E,M\}$ and initialized with WF$_{\mu,Q}$, $\mu \in \{E,M\}$ respectively, if not otherwise noted.

1) Comparison between linear and nonlinear equalization methods: Fig. 1 depicts the BER of an EM-based FDE block-processor for different equalization block lengths $N_b$, discarding factors $L'$ and Initial Guesses (IG). We compare our results to the linear Wiener-Filter WF$_{\text{EQ}}$ from [8].

The BER-performance improves with increasing $N_b$ and the number of discarded samples $L'$. Performing EM-FDE with $N_b = 1024$ and $L' = 3L$ achieves almost the same BER performance as choosing $N_b = T_c$, which is running the exact EM-algorithm EM$_{\text{E}}$. The convergence of the EM-algorithm is very sensitive to a good initial guess. On the other hand, Fig. 1 shows that the performance of the EM-algorithm degrades if WF$_{\text{E}}$ is taken as an IG. On the other hand, a substantial improvement in performance compared to linear equalization [8] is achieved, if the algorithm is initialized with WF$_{\text{EQ}}$.

2) Fixed number of EM iterations: The convergence of the EM algorithm depends strongly on the number of iterations. Assuming that only a limited processing-power budget is available at the base-station, we investigate running the EM-algorithm for a fixed number of iterations. Fig. 2 shows that the performance of EM$_{\text{M}}$ improves after each iteration, taking WF$_{\text{EQ}}$ as an initial guess. Moreover, the performance of EM$_{\text{M}}$ comes quite close to EM$_{\text{E}}$ in the mid-SNR range after 8 iterations. Note that the performance of EM$_{\text{E}}$ is taken as a benchmark, as it takes $N_b = T_c$ and $I_{\text{max}} = 1000$. The results indicate that the proposed EM$_{\text{M}}$ achieves the same performance as EM$_{\text{E}}$ with substantially reduced complexity.

VII. CONCLUSION

This paper applies data detection using the EM-algorithm in 1-bit quantized massive MIMO systems without CP. We propose a computationally efficient hybrid time-frequency approach where the E-step is performed in the time domain and the M-step in the frequency domain using the mismatched model. The interference distortion due to the block-circulant channel matrix approximation is minimized by selecting block-length $N_b$ and overlapping factor $L'$ in relation to the CIR length $L$. Numerical results show that $N \approx 4 \cdot L$ and $L' \approx 2 \cdot L$ is a good choice.
The simulation results show that the initial guess of the estimated data is crucial for convergence of the EM-algorithm. It is shown that taking the WF-estimate for quantized systems as an initial guess is a better choice compared to taking the WF-estimate for unquantized systems. The results also indicate that the performance of the EM-algorithm improves after each iteration under the condition that \( N_0, L' \) and the initial guess are chosen optimally.

Future work might include the investigation of the proposed methods under non-perfect CSI.

### Appendix A

**DERIVATION OF E-STEP**

According to (17), the \( i \)th element of \( y_i \) is \( y_i = a_i^T \xi + \eta_i \), where \( \eta_i \sim \mathcal{CN}(0, \sigma_n^2) \) is the \( i \)th element of \( \eta \), and, according to (18), \( r_i = Q(y_i) \), \( i \in \{1, 2, \ldots, M \cdot N_0\} \). Hence, the \( i \)th element of \( \tilde{y}_i^{(u)} \) in the E-step (25),

\[
\tilde{y}_i^{(u)} = E \left[ y_i \left| r_i, \xi \right. \right] = E \left[ y_i \left| r_i, \xi \right. \right] = E \left[ y_i \left| r_i, a_i^T \xi \right. \right] ,
\]

is a conditional expectation of the form \( E[y|r, z] \) with \( y = z + \eta, \eta \sim \mathcal{CN}(0, \sigma_n^2) \), and \( r = Q(y) \). With \( r_R = \Re \{ r \}, r_I = \Im \{ r \}, r_R = \Re \{ y \}, y = \Re \{ y \}, z_R = \Re \{ z \}, z_I = \Im \{ z \}, \eta_R = \Re \{ \eta \} \) and \( r_I = \Im \{ \eta \} \), we arrive at

\[
y_i = z_i + \eta_i, \text{ where } \eta_i \sim \mathcal{N}(0, \sigma_n^2/2) \text{ and } r_i = \text{sign}(y_i),
\]

for \( i \in \{R, I\} \). As a consequence,

\[
E[y|r, z] = E[y|r_R, z_R] + j \cdot E[y|r_I, z_I],
\]

with

\[
E[y|r_I, z_I] = \int_{-\infty}^{+\infty} \left. y \right| \left. p(r_I, y, z_I) \right. \, dy.
\]

Since

\[
p(r_I, y, z_I) = p(r_I|y) \cdot p(y|z_I) \cdot p(z_I),
\]

where \( y|z_I \sim \mathcal{N}(z_I, \sigma_n^2/2) \) according to (43) such that

\[
p(y|z_I) = \frac{1}{\sqrt{\sigma_n^2/2}} e^{-\frac{(y-z_I)^2}{\sigma_n^2/2}},
\]

where \( \varphi(x) = \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{2}} \). The relationship between \( y_i \) and \( r_I \) in (44) is reflected by

\[
p(r_I|y) = \begin{cases} 1, & r_I = \text{sign}(y_i) \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad r_I^{\text{up}} = \begin{cases} {-\infty}, & r_I = -1 \\ 0, & r_I = +1 \end{cases}.
\]

The conditional expectation (50) can be written as

\[
E[y|r_I, z_I] = \frac{\int_{-\infty}^{0} y \varphi \left( \frac{z_i-y}{\sqrt{\sigma_n^2/2}} \right) \, dy}{\int_{-\infty}^{0} \varphi \left( \frac{z_i-y}{\sqrt{\sigma_n^2/2}} \right) \, dy}.
\]

Evaluating the integrals in (50) gives the expression

\[
E[y|r_I, z_I] = \frac{r_I}{\sqrt{2}} \frac{\varphi \left( \frac{z_i-r_I}{\sqrt{\sigma_n^2/2}} \right)}{\Phi \left( \frac{r_I}{\sqrt{\sigma_n^2/2}} \right)} + z_I,
\]

where \( \Phi(x) = \int_{-\infty}^{x} \varphi(t) \, dt \). The conditional expectation in (45) can be expressed as

\[
E[y|r, z] = \frac{\sigma_n}{\sqrt{2}} \left( \frac{r_R \varphi \left( \frac{z-R}{\sqrt{\sigma_n^2/2}} \right)}{\Phi \left( \frac{r_R}{\sqrt{\sigma_n^2/2}} \right)} + j \cdot \frac{r_I \varphi \left( \frac{z-I}{\sqrt{\sigma_n^2/2}} \right)}{\Phi \left( \frac{r_I}{\sqrt{\sigma_n^2/2}} \right)} \right) + z.
\]

Using this result in (45) with \( y_i, r_i = \Re \{ r \} + j \cdot \Im \{ r \} \) and \( a_i^T \xi \) instead of \( y, r = \Re \{ r \} + j \cdot \Re \{ r \} \) and \( z = z_R + j \cdot z_I \), respectively, results in the elementwise computation of the E-step (26).

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