Optical analogues of black-hole horizons

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Hawking radiation is unlikely to be measured from a real black hole, but can be tested in laboratory analogues. It was predicted as a consequence of quantum mechanics and general relativity, but turned out to be more universal. A refractive index perturbation produces an optical analogue of the black-hole horizon and Hawking radiation that is made of light. We discuss the central and recent experiments of the optical analogue, using hands-on physics. We stress the roles of classical fields, negative frequencies, ‘regular optics’ and dispersion. Opportunities and challenges ahead are briefly mentioned.

This article is part of a discussion meeting issue ‘The next generation of analogue gravity experiments’.

1. Introduction

Analogue systems are used to test theories of predicted phenomena that are hard to observe directly [1]. Hawking [2,3] predicted black holes to thermally radiate from the quantum vacuum. Its measurement is unlikely, having a temperature inversely related to the black-hole mass. Stellar and heavier black holes should emit Hawking radiation that is much weaker than the cosmic microwave background (CMB) fluctuations [4]. Hypothetical microscopic primordial black holes might emit significant Hawking radiation [5], but it is highly model dependent, with rates seriously bounded by the lack of its observation [6–8].

Volovik [9] and Unruh [10] have shown that a transsonic fluid flow is analogous to the space–time geometry surrounding a black hole, and should emit sound waves analogous to Hawking radiation.

Wave kinematics control the Hawking process, which appears in the presence of an effective horizon [11,12].

\[ \text{The temperature of Hawking radiation from a black hole of solar mass is about 60 nK, where CMB temperature fluctuations are about 100 } \mu \text{K.} \]
Black-hole dynamics and Einstein equations of gravity are not an essential requirement. The analogue of the event horizon is the surface separating the subsonic and supersonic flows. The surface gravity determines the strength of Hawking radiation and is analogous to the flow acceleration at the horizon. The horizon should also last long enough such that the modes of Hawking radiation will have well-defined frequencies [12].

Many analogues have been proposed [13,14] with realizations using water waves [15–20], Bose–Einstein condensates (BECs) [21–24] and optics [25–32]. The analogues gave new perspectives in both gravity and the analogue systems [13], extending Hawking’s theory to wave dynamics in moving media. The universality of the Hawking effect shows it persists in real-world scenarios, where complicated dynamics replace simplified and possibly fine-tuned theories. Hawking’s original derivation seems questionable, since the radiation originates from infinitely high frequencies, neglecting unknown high-energy physics (known as the ‘trans-Planckian problem’). The microscopic physics of the analogue systems is well known, allowing the role of high frequencies in Hawking predictions to be tested [33,34].

This paper discusses the optical analogues that use a refractive index perturbation to establish an artificial black-hole horizon (figure 1). Extensive reviews of analogue gravity exist [13,14], but they pay little attention to optical analogues and substantial progress was made after their publication. Additional useful resources include [35–38]. This paper aims to present the main experiments that transformed the optical analogues from simple ideas to established reality. It aims to do so in the light of recent developments, but in terms of hands-on physics. We believe that experiments have reached the state where their data may lead the way for new questions and discoveries in the physics of Hawking radiation.

Section 2 briefly describes early proposals for optical analogues using slow light and why they could not materialize. Transformation optics is also mentioned, where stationary dielectrics are viewed to change the spatial parts of the metric. Section 3 explains the standard theory of optical analogues discussed in this paper and the first demonstration of optical horizons. Further measurements of the frequency shift at group velocity horizons are briefly discussed, and additional related work is mentioned. Section 4 focuses on attempts in bulk optics rather than optical fibres, stressing the roles of group-velocity and phase-velocity [39] horizons. Section 5 discusses the first measurements of negative frequencies in optics, a phenomenon closely related to the Hawking effect. Its theory is also presented and shortly explained. Section 6 considers additional interpretations of the optical horizon. Theory and experiments are shown to directly relate the effect to cascaded four-wave mixing, but analysis in the time domain seems to be unavoidable. The use of temporal analogues of reflection and refraction and numerical solutions are also mentioned. Section 7 discusses the first demonstration of stimulated Hawking radiation in an optical analogue, and lessons learnt from it. Section 8 concludes the paper with a brief outlook to the future.

2. Early attempts

Fresnel’s drag [40] was an ether-based theory of light propagation in transparent media, ‘confirmed’ by Fizeau [41]. The drag effect is just a relativistic velocity addition [42], but Fresnel’s wrong theory was based on correct intuition of velocity addition. The ether was replaced by the space–time geometry and the quantum vacuum, which continued to have an intimate connection with moving media and produce puzzling phenomena [2,3,43–45]. Analogue gravity took this connection a step further, but, despite theoretical progress in the 1990s, no practical realization of analogue black holes was suggested at that time [13].

Technology drove ideas in the right directions (optics and BECs) around 2000 [46–50]. In optics, Leonhardt and Piwnicki [46–48] suggested slowing down light such that its medium could be moved in super-luminal velocities and form a horizon. These ideas used the technology of ‘slow light’ [51,52], where incredibly low group velocities are produced using electromagnetically induced transparency (EIT). However, the phase velocity of ‘slow light’ is fast, preventing the crucial formation of a phase-velocity horizon (where
the medium moves at the phase velocity of light; see below) [53]. Another problem with realizing these ideas is the narrow bandwidth of light that can be slowed [53], and severe absorption around it [39]. The inevitable conclusion was to move the medium in relativistic velocities.

Despite missing key concepts of Hawking radiation, these ideas have pushed analogue gravity forwards and beyond the scope of relativity (or relativitists). They showed that an analogue black-hole metric can be made in optics. Similarly, stationary dielectrics mimic spatial geometries. This interpretation inspired transformation optics, and the development of meta-materials technology extended the range of practical geometries [54–59].

3. Changing a reference frame

The groups of Leonhardt & König [25] started a new approach that follows a simple idea [60]: light itself travels at the speed of light. A light pulse (also denoted pump) travels in a dielectric medium with group velocity $u$. In the reference frame co-moving with the pulse (the co-moving frame), the medium seems to flow in the opposite direction with velocity magnitude $|u|$ (figure 1b).

Probe light of a different frequency differs in velocity as a result of dispersion. The nonlinear Kerr effect [61] slows down the probe upon interaction with the pulse$^2$—the refractive index changes as $n = n_0 + n_2 I$, where $n_0$ is the linear refractive index, $n_2$ is a parameter being typically $10^{-16} \text{cm}^2 \text{W}^{-1}$ [61] and $I$ is the light intensity. A group-velocity horizon forms where the probe group velocity obeys $v_g = |u|$, blocking the probe from further entering the pump. A black-hole horizon is formed in the leading end of the pump, where the flow is directed into the pulse. Its time reversal, a white-hole horizon with outward flow, is formed in the pump trailing end [25,62].

$^2$This has a similar effect to the flow acceleration in the fluid analogues, changing the probe velocity relative to the flow.
If the pump is slowly varying \([12]\), the probe co-moving (and Doppler shifted) frequency,

\[
\omega' = \gamma \left( 1 - \frac{n}{c} \right) \omega,
\]

is conserved. Here \(\gamma = (1 - u^2/c^2)^{-1/2}\) is the Lorentz factor, \(c\) is the vacuum speed of light, \(n\) is the refractive index and \(\omega\) is the probe frequency in the laboratory frame. Photon pairs of Hawking radiation are produced at each horizon: one with positive \(\omega'\), and its negative partner with \(-\omega'\) [37,63]. For positive \(\omega\), \(\omega'\) is positive only if the probe phase velocity, \(v_{\phi}\), is greater than \(|u|\). A phase-velocity horizon forms where \(v_{\phi} = |u|\), so \(\omega' = 0\). The Hawking radiation outgoing from the horizon mixes incoming radiation of positive and negative frequencies [37]. Quantum mechanically, it is described by a Bogoliubov transformation,

\[
b_\pm = a_\pm + \beta \alpha_\mp, \quad |\alpha|^2 - |\beta|^2 = 1,
\]

where the sign of the operators equals the sign of \(\omega'\). Time-dependent annihilation and creation operators for incoming modes, \(\hat{a}_\pm, \hat{a}_\mp\), mix to form annihilation operators for outgoing modes, \(\hat{b}_\pm\). This extracts metric energy (pump energy) to amplify the radiation, thus spontaneously creating outgoing radiation from incoming vacuum [32,37]. The transformation parameters, \(\alpha\) and \(\beta\), give the flux of Hawking radiation. When neglecting dispersion, the radiation effective temperature is proportional to the analogue of the surface gravity—the steepness of the pulse (giving the probe velocity gradient in the co-moving frame [12,37], which is related to the refractive index (and pulse intensity) gradient or rate of change [25,37,62]).

In [25], an optical fibre called a photonic crystal fibre (PCF) provided both the desired dispersion (figure 2) and nonlinearity. Its unique structure guided light in a very small region—the fibre’s core [64,65]. This increased the light intensity and the fibre’s nonlinear response. Its nonlinear parameter was \(\gamma(\omega_0) = \omega_0 n_2/\omega A_{\text{eff}} = 0.1 \text{ W}^{-1} \text{ m}^{-1}\) at \(\omega_0\) corresponding to 780 nm wavelength, where \(A_{\text{eff}}\) is the fibre effective mode area and \(\epsilon_0\) is the vacuum permittivity [65]. The fibre’s structure was engineered to change its dispersion relation, \(n(w)\), to have two points with matching group velocities [65]: one in a normal dispersion region, where \(n(w)\) is increasing; and another in an anomalous dispersion region, where \(n(w)\) is decreasing. The anomalous dispersion included the pump spectra, generated by a Ti:sapphire mode-locked laser. Self-phase modulation (SPM) due to the nonlinear refractive index counteracted the anomalous dispersion and formed stable solitons [65].

The solitons in [25] created horizons. A continuous wave (CW) probe was added to the fibre, being red-detuned from the frequency of matching group velocity (white diamond in figure 2). The fast probe mainly interacted with the pump trailing end, gradually slowed down, and blue-shifted (frequency up-conversion; figure 3b). This pump–probe interaction is known as cross-phase modulation (XPM), and induces chirp that depends on the pump pulse shape [65]. If \(\delta n = n_2 I\) is large enough to form a horizon, the shifting probe becomes slower than the pulse and they separate. In the co-moving frame, the probe is reflected (figure 1b) while conserving its co-moving frequency \(\omega_{\text{probe}}'\), but not \(\omega\) (figure 2). This reflection demonstrated the existence of optical horizons [25].

Philbin et al. [25] used 70 fs duration full width at half maximum (FWHM), 800 nm carrier wavelength pulses with a peak power of about 50 W. The PCF was 1.5 m long with a core diameter smaller than 2 μm. The CW probe was tunable around 1500 nm wavelength with 100–600 μW power. Pump power was kept low to maintain a stable single soliton and reduce higher order nonlinearities, such as the Raman effect [65]. Self-steepening was hoped to realize high Hawking temperatures.

This set-up achieved a mere \(10^{-3}\%\) efficiency for probe shifting. Only \(10^{-4}\) of the CW probe power was expected to interact with the pump over the 1.5 m fibre, and another order of magnitude reduction was attributed to tunnelling of the probe through the narrow pump barrier [25]. While clearly showing probe blue-shifting at the white-hole horizon, this set-up could not produce detectable negative frequency partners. It was unclear whether they should form around the phase-velocity horizon, where \(\omega' = 0\), or around the supported frequency, \(-\omega_0'\) (figure 2).
**Figure 2.** Schematic Doppler curve of the fibre-optic analogue with phase-matching conditions. Frequency $\omega'$ at the reference frame co-moving with the pump pulse is plotted against the laboratory frequency $\omega$ (equation (3.1)). The pump pulse is in a local minimum, where the dispersion is anomalous. The phase-matching condition of dispersive waves (DW) corresponds to conservation of $\omega'_{\text{pump}}$ (upper dotted line). Negative frequency resonant radiation (NRR) conserves $-\omega'_{\text{pump}}$ (lower dotted line; see also figure 3f). Probe light (black diamond) is red-shifted (white diamond) at the black-hole horizon, conserving its co-moving frequency, $\omega'_{\text{probe}}$ (upper solid line). At the white-hole horizon, the probe (now white diamond) is blue-shifted (black diamond), as seen also in figure 3b. At the local maxima, the group velocity matches that of the pump, $u$. Negative Hawking radiation (NHR) conserves $-\omega'_{\text{probe}}$ (lower solid line; see also figure 3f). The phase horizon (PH), where the phase velocity equals $u$, corresponds to $\omega' = 0$. (Online version in colour.)

Choudhary & König [66] reported probe red-shifting and studied its tunnelling. They used similar experimental parameters to [25], with tuneable 50 fs pump pulses and a visible CW probe at 532 nm (to avoid the difficulty of synchronization). Tunnelling was minimal for probe detuning up to twice the soliton bandwidth (relative to the matching frequency), but limited pump–probe interaction kept the conversion efficiency small. Tartara [67] derived both pump and probe from the same 105 fs source and propagated them in a 1.1 m fibre, demonstrating conversion efficiencies of tens of per cent. An optical parametric amplifier produced the tuneable probe.

The Raman effect causes the pump to decelerate, and was shown to only slightly change the shifted probe spectra [68]. Shifting of dispersive waves [69,70] and trapping the probe light [71–75] were also related to the Raman effect. An optical ‘black-hole laser’ that uses both the white- and black-hole horizons was predicted and analysed [76–80]. The horizon dynamics was also related to the formation of optical rogue waves and champion solitons [81–84], and the ability to form all-optical transistors [85]. Similar ‘front-induced transitions’ were studied in other areas of photonics [86], and the quest for measuring analogue Hawking radiation continued.

### 4. Horizonless emissions

Faccio’s group realized optical phase-velocity horizons, without a group-velocity horizon, in bulk optics [26,87]. They constructed pulses of super-luminal group velocities, by using their three-dimensional nature [88]. A pulsed Bessel beam is one such example. It is composed of infinitely many plane waves on a cone with a constant angle $\theta$ with the propagation axis. It can be made with an axicon lens [88]. At the apex of the cone, the plane waves interfere to form a bright spot that moves with super-luminal velocity that depends on $\theta$. Belgiorno et al. [26] used super-luminal pulses that travelled as narrow and powerful filaments inside fused silica. Looking perpendicularly to their propagation direction, they detected radiation around the phase-velocity horizon—where the radiation phase velocity matched the pulse group velocity (figure 3c). They could not measure along the propagation direction, because vast radiation was produced by the powerful pulses there (the peak intensity was as high as $10^{13}$ W cm$^{-2}$, centred at 1055 nm wavelength).
Spurious radiation was a key issue even for observation at 90°. Special care was taken to tune the radiation into spectral windows of minimal noise. Correctly identifying all the signals in such a set-up is another major challenge. Concerns were raised [89–91] and in-depth analysis...
related the radiation to a horizonless super-luminal perturbation, possibly linked to Hawking radiation [92,93]. This work stressed the importance of a blocking group-velocity horizon to the Hawking effect. Thermal Hawking radiation was predicted for super-luminal pulses in materials with linear dispersion at low energies (such as diamond) [92,93], but is yet to be measured.

Leonhardt & Rosenberg [94] related the emission in [26] to the physics of Cherenkov radiation in a surprising way. The pulse was modelled as a super-luminal light bullet [96], and found to behave like a moving magnetic dipole that is predicted to emit Cherenkov radiation with a discontinuous spectrum [97,98]. This discontinuity is exactly at the phase horizon—where the effective dipole velocity equals the phase velocity of light and \( \omega' = 0 \) (figure 2). Interferences turn the discontinuity into a peak if the dipole is an extended object, like the light bullet.

5. The role of negative frequencies

Negative frequencies are an integral part of Hawking radiation and can also appear as dispersive waves. Solitons emit dispersive waves (also known as resonant radiation) at a shifted frequency when disturbed by higher order dispersion. Non-dispersive three-dimensional light bullets emit similar radiation [88]. Both emissions can be viewed to originate from self-scattering of the pulse by its nonlinear refractive index barrier. Conservation of momentum dictates the emitted frequency through a phase-matching condition [65,99]. In the reference frame co-moving with the stable pulse, this condition becomes a conservation of energy, given by the pulse co-moving frequency, \( \omega'_{\text{pulse}} \) (figure 2). The frequency shifting at optical horizons extends this picture to the scattering of probe light. The prediction of negative Hawking modes suggested that negative dispersive waves should similarly appear. Rubino et al. [27] had measured this negative frequency resonant radiation (NRR) after it was neglected and disregarded for a long time. This required very rapid (non-adiabatic) temporal changes of the pulse envelope.

Intense pulses created NRR through steep shocks in two ways [27]: 7 fs higher order solitons [65] of about 300 pJ energy and 800 nm carrier wavelength were placed in 5 mm PCFs; and pulsed Bessel beams of 60 fs duration, about 20 \( \mu \)J energy and 800 nm carrier wavelength were sent through 2 cm of bulk calcium fluoride (CaF\(_2\)), as seen in figure 3d. Despite being seen both in optical fibres and in bulk, further work was needed to exclude other possible mechanisms for the effect—contributions due to higher order spatial modes, and possible interactions between the complex pulse and additional radiation.

Rubino et al. [100] later used a relativistic scattering potential to directly explain NRR formation, and Petev et al. [92] further related it to Hawking radiation. Conforti et al. [101] extended the theory to pulses in materials of normal dispersion (where the pulse experiences dispersive broadening) and dominating second-order nonlinearity, which effectively creates a nonlinear refractive index. The same conditions for phase matching and non-adiabatic temporal evolution were found. Analytic derivation of the NRR [102] directly related it to interactions between fields and their conjugates (or the mixing of positive and negative frequency modes). It further strengthened the connection between NRR and Hawking radiation, which originates from a Bogoliubov transformation that mixes creation and annihilation operators [37,63]. McLenaghan & König [103] studied NRR for different PCF lengths and input chirps, and inferred UV propagation loss of \( \sim 2 \) dB mm\(^{-1}\). We stress that the negative frequency (and negative norm) of the Hawking modes beyond the horizon is key to the Hawking amplification process [37], extracting energy from the background metric in accordance to a Bogoliubov transformation.

6. It is just optics

Hawking radiation is a universal geometric effect that emerges because of conversion of modes at a horizon, regardless of the microscopic physics that create the background space–time geometry [12]. The same (generalized) derivation applies for both astrophysical black holes and analogue

\[ ^3 \text{Not to be confused with dispersive waves [95].} \]
systems [12,25,104–109]. This established universality proved insightful for both gravity and optics (arguably solving the trans-Planckian problem and discovering negative frequencies in optics are two examples [13,14]). However, some researchers from both the optics and gravity (or quantum field theory on curved space–times) communities are still not comfortable with analogue Hawking radiation, claiming that the effect is ‘just optics’.\footnote{The author has personally encountered such allegations from both communities, and believes that they partially originate from not separating ‘gravity’, predicted to create the black-hole event horizon, and ‘kinematics’, which are responsible for the Hawking effect once a horizon is formed [12].} Such claims do not undermine (analogue) Hawking radiation, but complete our understanding of it. It is also possible to directly explain the effect using the underlying microscopic physics—unknown quantum gravity for real black holes, and nonlinear optics for the optical analogues.

The classical dynamics at optical horizons can be captured using numerical solutions that take into account the entire electric field (including negative frequencies), the dispersion relation and all nonlinearities [65,110–112]. These are extensions and alternatives for solving the usual generalized nonlinear Schrödinger equation, which is used extensively in standard descriptions of ultrafast nonlinear fibre optics [65,99,113].

Another approach directly relates the phenomena to discrete photonic interactions. It uses cascaded four-wave mixing of discrete spectra, and takes the continuum limit for comparison [114,115]. A CW probe and a pair of beating quasi-CWs (of 1 ns duration) positioned symmetrically around the pump central frequency were being mixed continuously (figure 3e). At each step of the cascaded process, the probe shifted by the pair’s detuning, and all three generated an equidistant frequency comb. A resonant amplification was produced at the frequency corresponding to conservation of $\omega'$ of the probe. Similar amplification appeared at the dispersive wave resonance, at $\omega'_\text{pump}$. The experiments used low cascade orders ($n = 5–7$ for the probe shift and about $n = 15$ for the dispersive waves) and were supported by numerical analysis. This picture shows how energy is being transferred from the pump to the shifting probe, by effectively absorbing pump photons of one frequency and emitting at another. The cascaded four-wave mixing also details the back-reaction mechanism, where the pump undergoes spectral recoil.

The studies [114,115] addressed the relevant phase-matching conditions, but the efficiency of the cascaded process was found only in [116], for dispersive waves. Remarkably, it showed that the known concepts from horizon physics are crucial ingredients even when the pump is made of beating quasi-CWs that form a frequency comb: the mixing was analysed using (temporal) soliton fission dynamics, being efficient only when the frequency spacing between the CWs effectively generated compressing (non-adiabatic) higher order solitons. The frequency-domain analysis was intractable for high cascade orders. The study used low cascade orders, and was supported by numerical analysis. The maximally compressed beat cycle corresponded to about 102 fs duration FWHM in the efficient regime, of high cascading orders. Conversion efficiencies of $10^{-4}$ over a 100 m fibre were reported.

The generation of NRR in media with quadratic nonlinearity was also related to a cascading process [101], which must also be the case for cubic materials. However, negative frequencies have not yet been demonstrated using a beating CW pump. It would be interesting to test how the cascading mixing behaves as it reaches negative frequencies, past the phase-velocity horizon.

Temporal analogues of reflection and refraction [117,118] are also used to explain the frequency shifts at optical horizons. They compare the frequency change due to XPM with the wave number change during refraction. The reflection at the horizon is analogous to total internal reflection. The tunnelling through the refractive index barrier is compared with frustrated total internal reflection, where an evanescent wave extends beyond the pulse and tunnels through. The optical horizon can be seen as a temporal beam splitter for probe light (acting also as an amplifier when considering negative frequencies as well).

Comparing the mode mixing and squeezing of parametric amplification [119] with the Hawking effect is also useful [37]. The origins of the two phenomena are completely different, and, even in the optical analogue, simple down-conversion obeys different phase-matching conditions that conserve total laboratory frequency [61,65] and cannot explain Hawking radiation. However, the two share intuition and the Bogoliubov transformation between
incoming and outgoing modes. In both cases, a pump creates ‘signal’ and ‘idler’ waves: the vacuum spontaneously generates quantum emissions, while stimulating the effect with classical light amplifies the effect for specific modes.

7. Observing stimulated Hawking radiation

Spontaneous Hawking radiation originates from the quantum vacuum. Classical laser fields can replace the vacuum and stimulate the effect. Drori et al. [32] observed stimulated negative Hawking radiation in an optical analogue (figure 3f). The idea of [25] was used, but with more suitable pump pulses and a pulsed probe. Analysis of the fibre dispersion (schematically seen in figure 2) and simulations of the experiment allowed the experimental parameters to be optimized. The pump was a higher order few-cycle soliton that compressed and collapsed as a result of soliton fission [65]. Its peak power reached a few hundred kilowatts, with 800 nm central wavelength. Very non-adiabatic dynamics generated NRR in the mid-UV (figure 3f), at the negative of the pump co-moving frequency, −ω′ p. This ensured the formation of steep refractive index variations, increasing the analogous surface gravity and reducing the probability of probe tunnelling. The probe was derived from the pump’s laser, and was generated through cascaded Raman scattering in a metre-long PCF. Negative prechirp was used to enhance the Raman-induced frequency shift [120], whose wavelength was continuously tuned up to 1650 nm by varying the input power. Peak powers were around 1 kW.

Efficient and broad-band probe frequency shifts at the horizon accompanied negative Hawking radiation in the mid-UV, conserving the probe co-moving frequency ω′ p and its conjugate −ω′ p, respectively. Since the fibre had strong dispersion, with different notions for phase- and group-velocity horizons, the spectrum was not Planckian [32,63,106]. Varying ω′ p both signals shifted according to theory, supporting the correct interpretation of the negative Hawking radiation. By this, also the interpretation of the NRR was supported, and its analytic theory [102] was generalized to include the Hawking process [112]. Linear relation was verified between the probe power and that of the negative Hawking signals up to a point where the signal saturated. This saturation was related to a back-reaction of the probe on the pulse, which is a prerequisite for Hawking radiation (since its energy is drawn from the pump, or the black hole that curves the metric). It also slightly reduced the magnitude of the NRR (figure 3f).

The rate of spontaneous Hawking emission was estimated based on the magnitude of the stimulated effect [32]. It was found to be too low for measurement in the system used, calling to design an improved set-up. The predicted spontaneous effect is minuscule and is overwhelmed by the noise in the system, which for the negative Hawking radiation is dominated by fluctuations of the overlapping NRR. The multi-mode nature of the PCF in the UV [65] was found to reduce the observed signal power by up to four orders of magnitude.

The experiment [32] showed how robust the Hawking effect is: it appeared despite the extreme nonlinear dynamics of the collapsing soliton. Since time in the co-moving frame is related to the propagation distance along the fibre, rapid variations in the pump profile do not violate the required slow evolution of the metric as long as they appear over a length scale much larger than the Hawking radiation wavelength [12]. As such, pump deceleration due to the Raman effect could be accounted for by simply correcting its central frequency.

8. Beyond the horizon?

Analogue gravity in optics has come a long way: from erroneous visionary ideas to careful analysis of experimental demonstrations. It drove new research directions and understandings in optics. The reality of optical horizons and negative frequencies became clear, and multiple optical interpretations made their physics more tangible. The crucial role of dispersion was stressed, determining the Hawking spectrum (figure 2). A group-velocity horizon is blocking the radiation
modes and allows their efficient conversion. A phase-velocity horizon marks the support of non-trivial negative frequency modes, which allow the Hawking amplification. The optical and other analogues, especially in water waves and BECs, provide extensive insights for gravity through concrete real-world examples. The universality of the Hawking effect separates it from gravity and high-energy physics, and emphasizes the central role of classical fields to the process. This hints to more possibilities in gravity [14] and in optics [121], even before going fully quantum.

The main challenge for observing the spontaneous (quantum) effect in optics is its low power. Noise from spurious radiation makes matters worse. Using knowledge from the stimulated effect [32] could help overcome these challenges.

The demonstrated robustness of the Hawking effect is calling for new experiments to lead the way. Our increased confidence in the interpretation of the results allows further separation from the traditional scheme of Hawking radiation, developing bolder questions and ideas. Optical technologies allow wild ideas to be realized and studied with high precision. We call for more cross-fertilization between research of the different black-hole analogues, which might open new horizons in these fields.

Data accessibility. This article has no additional data.

Competing interests. I declare I have no competing interests.

Funding. Weizmann Institute Sustainability and Energy Research Initiative; European Research Council; and the Israel Science Foundation.

Acknowledgements. I am grateful for discussions and comments from David Bermudez, Jonathan Drori and Ulf Leonhardt. I acknowledge valuable discussions with the participants of the scientific meeting ‘The next generation of analogue gravity experiments’ at the Royal Society (London, UK, December 2019), and thank its organizers: Maxime Jacquet, Silke Weinfurtner and Friedrich König.

References

1. Georgescu IM, Ashhab S, Nori F. 2014 Quantum simulation. Rev. Mod. Phys. 86, 153. (doi:10.1103/RevModPhys.86.153)
2. Hawking SW. 1974 Black hole explosions? Nature 248, 30–31. (doi:10.1038/248030a0)
3. Hawking SW. 1975 Particle creation by black holes. Commun. Math. Phys. 43, 199–220. (doi:10.1007/BF02345020)
4. Samtleben D, Staggs S, Weinstein B. 2007 The cosmic microwave background for pedestrians: a review for particle and nuclear physicists. Annu. Rev. Nucl. Part. Sci. 57, 245–283. (doi:10.1146/annurev.nucl.54.070103.181232)
5. Halzen F, Zas E, MacGibbon J, Weekes T. 1991 Gamma rays and energetic particles from primordial black holes. Nature 353, 807–815. (doi:10.1038/353807a0)
6. Alexandreas D et al. 1993 New limit on the rate-density of evaporating black holes. Phys. Rev. Lett. 71, 2524. (doi:10.1103/PhysRevLett.71.2524)
7. Fichtel C et al. 1994 Search of the energetic gamma-ray experiment telescope (egret) data for high-energy gamma-ray microsecond bursts. Astrophys. J. 434, 557–559. (doi:10.1086/174758)
8. Linton E et al. 2006 A new search for primordial black hole evaporation using the Whipple gamma-ray telescope. J. Cosmol. Astropart. Phys. 2006, 013. (doi:10.1088/1475-7516/2006/01/013)
9. Volovik GE. 2003 The universe in a helium droplet, vol. 117. Oxford, UK: Oxford University Press.
10. Unruh WG. 1981 Experimental black-hole evaporation? Phys. Rev. Lett. 46, 1351. (doi:10.1103/PhysRevLett.46.1351)
11. Visser M. 1998 Acoustic black holes: horizons, ergospheres and Hawking radiation. Class. Quantum Gravity 15, 1767–1791. (doi:10.1088/0264-9381/15/6/024)
12. Visser M. 2003 Essential and inessential features of Hawking radiation. Int. J. Mod. Phys. D 12, 649–661. (doi:10.1142/S0218271803003190)
13. Barceló C, Liberati S, Visser M. 2011 Analogue gravity. Living Rev. Relativ. 14, 3. (doi:10.12942/lrr-2011-3)
14. Barceló C. 2019 Analogue black-hole horizons. Nat. Phys 15, 210–213. (doi:10.1038/s41567-018-0367-6)
15. Rousseaux G, Mathis C, Maïssa P, Philbin TG, Leonhardt U. 2008 Observation of negative-frequency waves in a water tank: a classical analogue to the Hawking effect? *New J. Phys.* 10, 053015. (doi:10.1088/1367-2630/10/5/053015)

16. Jannes G, Piquet R, Maïssa P, Mathis C, Rousseaux G. 2011 Experimental demonstration of the supersonic-subsonic bifurcation in the circular jump: a hydrodynamic white hole. *Phys. Rev. E* 83, 056312. (doi:10.1103/PhysRevE.83.056312)

17. Weinfurtner S, Tedford EW, Penrice MC, Unruh WG, Lawrence GA. 2011 Measurement of stimulated Hawking emission in an analogue system. *Phys. Rev. Lett.* 106, 021302. (doi:10.1103/PhysRevLett.106.021302)

18. Euvé L-P, Michel F, Parentani R, Rousseaux G. 2015 Wave blocking and partial transmission in subcritical flows over an obstacle. *Phys. Rev. D* 91, 024020. (doi:10.1103/PhysRevD.91.024020)

19. Euvé L-P, Michel F, Parentani R, Philbin TG, Rousseaux G. 2016 Observation of noise correlated by the Hawking effect in a water tank. *Phys. Rev. Lett.* 117, 121301. (doi:10.1103/PhysRevLett.117.121301)

20. Torres T, Patrick S, Coutant A, Richartz M, Tedford EW, Weinfurtner S. 2017 Rotational superradiant scattering in a vortex flow. *Nat. Phys.* 13, 833–836. (doi:10.1038/nphys4151)

21. Lahav O, Itah A, Blumkin A, Gordon C, Rinott S, Zayats A, Steinhauer J. 2010 Realization of a sonic black hole analog in a Bose-Einstein condensate. *Phys. Rev. Lett.* 105, 240401. (doi:10.1103/PhysRevLett.105.240401)

22. Steinhauer J. 2014 Observation of self-amplifying Hawking radiation in an analogue black-hole laser. *Nat. Phys.* 10, 864–869. (doi:10.1038/nphys3104)

23. Steinhauer J. 2016 Observation of quantum Hawking radiation and its entanglement in an analogue black hole. *Nat. Phys.* 12, 959–965. (doi:10.1038/nphys3863)

24. de Nova JRM, Golubkov K, Kolobov VI, Steinhauer J. 2019 Observation of thermal Hawking radiation and its temperature in an analogue black hole. *Nature* 569, 688–691. (doi:10.1038/s41586-019-1241-0)

25. Philbin TG, Kuklewicz C, Robertson S, Hill S, König F, Leonhardt U. 2008 Fiber-optical analog of the event horizon. *Science* 319, 1367–1370. (doi:10.1126/science.1153625)

26. Belgiorno F, Cacciatori SL, Clerici M, Gorini V, Ortenzi G, Rizzi L, Rubino E, Sala VG, Faccio D. 2010 Hawking radiation from ultrashort laser pulse filaments. *Phys. Rev. Lett.* 105, 203901. (doi:10.1103/PhysRevLett.105.203901)

27. Rubino E et al. 2012 Negative-frequency resonant radiation. *Phys. Rev. Lett.* 108, 253901. (doi:10.1103/PhysRevLett.108.253901)

28. Elazar M, Fleurov V, Bar-Ad S. 2012 All-optical event horizon in an optical analog of a laval nozzle. *Phys. Rev. A* 86, 063821. (doi:10.1103/PhysRevA.86.063821)

29. Nguyen HS, Gerace D, Carusotto I, Sanvitto D, Galopin E, Lemaître A, Sagnes I, Bloch J, Amo A. 2015 Acoustic black hole in a stationary hydrodynamic flow of microcavity polaritons. *Phys. Rev. Lett.* 114, 036402. (doi:10.1103/PhysRevLett.114.036402)

30. Bekenstein R, Schley R, Mutzafi M, Rotschild C, Segev M. 2015 Optical simulations of gravitational effects in the Newton–Schrödinger system. *Nat. Phys.* 11, 872–878. (doi:10.1038/nphys3451)

31. Bekenstein R, Kabessa Y, Sharabi Y, Tal O, Engheta N, Eisenstein G, Agranat AJ, Segev M. 2017 Control of light by curved space in nanophotonic structures. *Nat. Photonics* 11, 664–670. (doi:10.1038/s41566-017-0008-0)

32. Drori J, Rosenberg Y, Bermudez D, Silberberg Y, Leonhardt U. 2019 Observation of stimulated Hawking radiation in an optical analogue. *Phys. Rev. Lett.* 122, 010404. (doi:10.1103/PhysRevLett.122.010404)

33. Jacobson T. 1991 Black-hole evaporation and ultrashort distances. *Phys. Rev. D* 44, 1731–1739. (doi:10.1103/PhysRevD.44.1731)

34. Unruh WG. 1995 Sonic analogue of black holes and the effects of high frequencies on black hole evaporation. *Phys. Rev. D* 51, 2827–2838. (doi:10.1103/PhysRevD.51.2827)

35. Francesco DB, Sergio LC, Daniele F. 2018 *Hawking radiation: from astrophysical black holes to analogous systems in lab*. Singapore: World Scientific.

36. Faccio D, Belgiorno F, Cacciatori S, Gorini V, Liberati S, Moschella U. 2013 *Analogue gravity phenomenology: analogue spacetimes and horizons, from theory to experiment*. Lecture Notes in Physics, vol. 870. Berlin, Germany: Springer.
37. Leonhardt U. 2010 Essential quantum optics: from quantum measurements to black holes. Cambridge, UK: Cambridge University Press.
38. Unruh W, Schützhold R. 2007 Quantum analogues: from phase transitions to black holes and cosmology. Lecture Notes in Physics, vol. 718. Berlin, Germany: Springer.
39. Milonni PW. 2004 Fast light, slow light and left-handed light. Boca Raton, FL: CRC Press.
40. Fresnel A. 1818 Lettre d’Augustin Fresnel à François Arago sur l’influence du mouvement terrestre dans quelques phénomènes d’optique. *Ann. Chim. Phys.* 9, 57–66.
41. Fizeau A. 1851 Sur les hypothèses relatives à l’éther lumineux, et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur. *C. R. Acad. Sci* 33, 349–355.
42. Rindler W. 2006 Relativity: special, general, and cosmological. Oxford, UK: Oxford University Press.
43. Fulling SA. 1973 Nonuniqueness of canonical field quantization in Riemannian space-time. *Phys. Rev. D* 7, 2850. (doi:10.1103/PhysRevD.7.2850)
44. Davies PC. 1975 Scalar production in Schwarzschild and Rindler metrics. *J. Phys. A* 8, 609–616. (doi:10.1088/0305-4470/8/4/022)
45. Unruh WG. 1976 Notes on black-hole evaporation. *Phys. Rev. D* 14, 870. (doi:10.1103/PhysRevD.14.870)
46. Leonhardt U, Piwnicki P. 1999 Optics of nonuniformly moving media. *Phys. Rev. A* 60, 4301. (doi:10.1103/PhysRevA.60.4301)
47. Leonhardt U, Piwnicki P. 2000 Relativistic effects of light in moving media with extremely low group velocity. *Phys. Rev. Lett.* 84, 822–825. (doi:10.1103/PhysRevLett.84.822)
48. Leonhardt U. 2002 A laboratory analogue of the event horizon using slow light in an atomic medium. *Nature* 415, 406–409. (doi:10.1038/415406a)
49. Garay LJ, Anglin J, Cirac JI, Zoller P. 2000 Sonic analog of gravitational black holes in Bose-Einstein condensates. *Phys. Rev. Lett.* 85, 4643–4647. (doi:10.1103/PhysRevLett.85.4643)
50. Garay LJ, Anglin J, Cirac JI, Zoller P. 2001 Sonic black holes in dilute Bose-Einstein condensates. *Phys. Rev. A* 63, 023611. (doi:10.1103/PhysRevA.63.023611)
51. Hau LV, Harris SE, Dutton Z, Behroozi CH. 1999 Light speed reduction to 17 metres per second in an ultracold atomic gas. *Nature* 397, 594–598. (doi:10.1038/17561)
52. Kash MM, Sautenkov VA, Zibrov AS, Hollberg L, Welch GR, Lukin MD, Rostovtsev Y, Fry ES, Scully MO. 1999 Ultraslow group velocity and enhanced nonlinear optical effects in a coherently driven hot atomic gas. *Phys. Rev. Lett.* 82, 5229–5232. (doi:10.1103/PhysRevLett.82.5229)
53. Unruh W, Schützhold R. 2003 On slow light as a black hole analogue. *Phys. Rev. D* 68, 024008. (doi:10.1103/PhysRevD.68.024008)
54. Leonhardt U. 2006 Optical conformal mapping. *Science* 312, 1777–1780. (doi:10.1126/science.1126493)
55. Pendry JB, Schurig D, Smith DR. 2006 Controlling electromagnetic fields. *Science* 312, 1780–1782. (doi:10.1126/science.1125907)
56. Leonhardt U, Philbin TG. 2006 General relativity in electrical engineering. *New J. Phys.* 8, 247. (doi:10.1088/1367-2630/8/10/247)
57. Chen H, Chan CT, Sheng P. 2010 Transformation optics and metamaterials. *Nat. Mater.* 9, 387–396. (doi:10.1038/nmat2743)
58. Leonhardt U, Philbin T. 2010 *Geometry and light: the science of invisibility.* Mineola, NY: Dover Publications.
59. Xu L, Chen H. 2015 Configural transformation optics. *Nat. Photonics* 9, 15–23. (doi:10.1038/nphoton.2014.307)
60. Leonhardt U, König F. 2005 Invention disclosure. Research and Enterprise services, University of St Andrews, St Andrews, UK.
61. Boyd RW. 2003 *Nonlinear optics.* Amsterdam, The Netherlands: Elsevier.
62. Belgioioso F, Cacciatori S, Ortenzi G, Rizzi L, Gorini V, Faccio D. 2011 Dielectric black holes induced by a refractive index perturbation and the Hawking effect. *Phys. Rev. D* 83, 024015. (doi:10.1103/PhysRevD.83.024015)
63. Leonhardt U, Robertson S. 2012 Analytical theory of Hawking radiation in dispersive media. *New J. Phys.* 14, 053003. (doi:10.1088/1367-2630/14/5/053003)
64. Russell P. 2003 Photonic crystal fibers. *Science* 299, 358–362. (doi:10.1126/science.1079280)
65. Agrawal GP. 2019 Nonlinear fiber optics. New York, NY: Academic Press.
66. Choudhary A, König F. 2012 Efficient frequency shifting of dispersive waves at solitons. Opt. Express 20, 5538–5546. (doi:10.1364/OE.20.005538)
67. Tartara L. 2012 Frequency shifting of femtosecond pulses by reflection at solitons. IEEE J. Quantum Electron. 48, 1439–1442. (doi:10.1109/JQE.2012.2213584)
68. Robertson S, Leonhardt U. 2010 Frequency shifting at fiber-optical event horizons: the effect of Raman deceleration. Phys. Rev. A 81, 063835. (doi:10.1103/PhysRevA.81.063835)
69. Wang S, Mussot A, Conforti M, Bendahmane A, Zeng X, Kudlinski A. 2015 Optical event horizons from the collision of a soliton and its own dispersive wave. Phys. Rev. A 92, 023837. (doi:10.1103/PhysRevA.92.023837)
70. Bendahmane A, Mussot A, Conforti M, Kudlinski A. 2015 Observation of the stepwise blue shift of a dispersive wave preceding its trapping by a soliton. Opt. Express 23, 16 595–16 601. (doi:10.1364/OE.23.016595)
71. Nishizawa N, Goto T. 2002 Pulse trapping by ultrashort soliton pulses in optical fibers across zero-dispersion wavelength. Opt. Lett. 27, 152–154. (doi:10.1364/OL.27.000152)
72. Nishizawa N, Goto T. 2002 Characteristics of pulse trapping by use of ultrashort soliton pulses in optical fibers across the zero-dispersion wavelength. Opt. Express 10, 1151–1159. (doi:10.1364/OE.10.001151)
73. Gorbach AV, Skryabin DV. 2007 Light trapping in gravity-like potentials and expansion of supercontinuum spectra in photonic-crystal fibres. Nat. Photonics 1, 653–657. (doi:10.1038/nphoton.2007.202)
74. Hill S, Kuklewicz C, Leonhardt U, König F. 2009 Evolution of light trapped by a soliton in a microstructured fiber. Opt. Express 17, 13 588–13 601. (doi:10.1364/OE.17.013588)
75. Wang W, Yang H, Tang P, Zhao C, Gao J. 2013 Soliton trapping of dispersive waves in photonic crystal fiber with two zero dispersive wavelengths. Opt. Express 21, 11 215–11 226. (doi:10.1364/OE.21.011215)
76. Corley S, Jacobson T. 1999 Black hole lasers. Phys. Rev. D 59, 124011. (doi:10.1103/PhysRevD.59.124011)
77. Leonhardt U, Philbin TG. 2007 Black hole lasers revisited. In Quantum analogues: from phase transitions to black holes and cosmology, pp. 229–245. Berlin, Germany: Springer.
78. Faccio D, Arane T, Lamperti M, Leonhardt U. 2012 Optical black hole lasers. Class. Quantum Gravity 29, 224009. (doi:10.1088/0264-9381/29/22/224009)
79. Gaona-Reyes JL, Bermudez D. 2017 The theory of optical black hole lasers. Ann. Phys. 380, 41–58. (doi:10.1016/j.aop.2017.03.005)
80. Bermudez D, Leonhardt U. 2018 Resonant Hawking radiation as an instability. Class. Quantum Gravity 36, 024001. (doi:10.1088/1361-6382/aaf435)
81. Solli DR, Ropers C, Koonath P, Jalali B. 2007 Optical rogue waves. Nature 450, 1054–1057. (doi:10.1038/nature06402)
82. Demircan A, Amiranashvili S, Brée C, Mahnke C, Mitschke F, Steinmeyer G. 2012 Rogue events in the group velocity horizon. Sci. Rep. 2, 850. (doi:10.1038/srep00850)
83. Demircan A, Amiranashvili S, Brée C, Mahnke C, Mitschke F, Steinmeyer G. 2014 Rogue wave formation by accelerated solitons at an optical event horizon. Appl. Phys. B 115, 343–354. (doi:10.1007/s00340-013-5609-9)
84. Pickartz S, Bandelow U, Amiranashvili S. 2016 Adiabatic theory of solitons fed by dispersive waves. Phys. Rev. A 94, 033811. (doi:10.1103/PhysRevA.94.033811)
85. Demircan A, Amiranashvili S, Steinmeyer G. 2011 Controlling light by light with an optical event horizon. Sci. Rep. 1, 84. (doi:10.1038/srep00850)
86. Gaafar MA, Baba T, Eich M, Petrov AY. 2019 Front-induced transitions. Nat. Photonics 13, 737–748. (doi:10.1038/s41566-019-0511-6)
87. Rubino E et al. 2011 Experimental evidence of analogue Hawking radiation from ultrashort laser pulse filaments. New J. Phys. 13, 085005. (doi:10.1088/1367-2630/13/8/085005)
88. Faccio D, Couairon A, Di Trapani P. 2007 Conical waves, filaments, and nonlinear filamentation optics. Rome, Italy: Aracne.
89. Schützhold R, Unruh WG. 2011 Comment on: Hawking radiation from ultrashort laser pulse filaments. Phys. Rev. Lett. 107, 149401. (doi:10.1103/PhysRevLett.107.149401)
90. Unruh W, Schützhold R. 2012 Hawking radiation from ‘phase horizons’ in laser filaments? Phys. Rev. D 86, 064006. (doi:10.1103/PhysRevD.86.064006)
91. Liberati S, Prain A, Visser M. 2012 Quantum vacuum radiation in optical glass. *Phys. Rev. D* **85**, 084014. (doi:10.1103/PhysRevD.85.084014)

92. Petev M, Westerberg N, Moss D, Rubino E, Rimoldi C, Cacciatori S, Belgiorno F, Faccio D. 2013 Blackbody emission from light interacting with an effective moving dispersive medium. *Phys. Rev. Lett.* **111**, 043902. (doi:10.1103/PhysRevLett.111.043902)

93. Finazzi S, Carusotto I. 2014 Spontaneous quantum emission from analog white holes in a nonlinear optical medium. *Phys. Rev. A* **89**, 053807. (doi:10.1103/PhysRevA.89.053807)

94. Leonhardt U, Rosenberg Y. 2019 Cherenkov radiation of light bullets. *Phys. Rev. A* **100**, 063802. (doi:10.1103/PhysRevA.100.063802)

95. Akhmediev N, Karlsson M. 1995 Cherenkov radiation emitted by solitons in optical fibers. *Phys. Rev. A* **51**, 2602–2607. (doi:10.1103/PhysRevA.51.2602)

96. Silberberg Y. 1990 Collapse of optical pulses. *Opt. Lett.* **15**, 1282–1284. (doi:10.1364/OL.15.001282)

97. Frank I. 1942 Doppler effect in a refractive medium. *Bull. Russ. Acad. Sci.: Phys.* **6**, 2.

98. Frank I. 1984 Vavilov-Cherenkov radiation for electric and magnetic multipoles. *Uspekhi* **27**, 772–785. (doi:10.1070/PU1984v027n10ABEH004129)

99. Dudley JM, Genty G, Coen S. 2006 Supercontinuum generation in photonic crystal fiber. *Rev. Mod. Phys.* **78**, 1135–1184. (doi:10.1103/RevModPhys.78.1135)

100. Rubino E, Lotti A, Belgiorno F, Cacciatori S, Couairon A, Leonhardt U, Faccio D. 2012 Soliton-induced relativistic-scattering and amplification. *Sci. Rep.* **2**, 932. (doi:10.1038/srep00932)

101. Conforti M, Westerberg N, Baronio F, Trillo S, Faccio D. 2013 Negative-frequency dispersive wave generation in quadratic media. *Phys. Rev. A* **88**, 013829. (doi:10.1103/PhysRevA.88.013829)

102. Conforti M, Marini A, Tran TX, Faccio D, Biancalana F. 2013 Interaction between optical fields and their conjugates in nonlinear media. *Opt. Express* **21**, 31239–31252. (doi:10.1364/OE.21.031239)

103. McLenaghan J, König F. 2014 Few-cycle fiber pulse compression and evolution of negative resonant radiation. *New J. Phys.* **16**, 063017. (doi:10.1088/1367-2630/16/6/063017)

104. Finazzi S, Carusotto I. 2013 Quantum vacuum emission in a nonlinear optical medium illuminated by a strong laser pulse. *Phys. Rev. A* **87**, 023803. (doi:10.1103/PhysRevA.87.023803)

105. Jacquet M, König F. 2015 Quantum vacuum emission from a refractive-index front. *Phys. Rev. A* **92**, 023851. (doi:10.1103/PhysRevA.92.023851)

106. Bermudez D, Leonhardt U. 2016 Hawking spectrum for a fiber-optical analog of the event horizon. *Phys. Rev. A* **93**, 053820. (doi:10.1103/PhysRevA.93.053820)

107. Linder MF, Schützhold R, Unruh WG. 2016 Derivation of Hawking radiation in dispersive dielectric media. *Phys. Rev. D* **93**, 104010. (doi:10.1103/PhysRevD.93.104010)

108. Jacquet M. 2018 Negative frequency at the horizon: theoretical study and experimental realisation of analogue gravity physics in dispersive optical media. Berlin, Germany: Springer.

109. Jacquet MJ, Koenig F. 2019 Analytical description of quantum emission in optical analogues to gravity. (http://arxiv.org/1908.02060).

110. Amiranashvili S. 2016 Hamiltonian framework for short optical pulses. In *New approaches to nonlinear waves* (ed. E Tobisch), pp. 153–196. Berlin, Germany: Springer.

111. Bermudez D. 2016 Propagation of ultra-short higher-order solitons in a photonic crystal fiber. *J. Phys.: Conf. Ser.* **698**, 012017. (doi:10.1088/1742-6596/698/1/012017)

112. Aguero R, Rosenberg Y, Leonhardt U, Bermudez D. In preparation. On the origin of the stimulated Hawking radiation in nonlinear optics.

113. Skryabin DV, Gorbach AV. 2010 Colloquium: looking at a soliton through the prism of optical supercontinuum. *Rev. Mod. Phys.* **82**, 1287–1299. (doi:10.1103/RevModPhys.82.1287)

114. Webb KE, Erkintalo M, Xu Y, Broderick NG, Dudley JM, Genty G, Murdoch SG. 2014 Nonlinear optics of fibre event horizons. *Nat. Commun.* **5**, 4969. (doi:10.1038/ncomms5969)

115. Erkintalo M, Xu Y, Murdoch S, Dudley J, Genty G. 2012 Cascaded phase matching and nonlinear symmetry breaking in fiber frequency combs. *Phys. Rev. Lett.* **109**, 223904. (doi:10.1103/PhysRevLett.109.223904)

116. Webb K, Erkintalo M, Xu Y, Genty G, Murdoch S. 2014 Efficiency of dispersive wave generation from a dual-frequency beat signal. *Opt. Lett.* **39**, 5850–5853. (doi:10.1364/OL.39.005850)
117. Plansinis B, Donaldson W, Agrawal G. 2015 What is the temporal analog of reflection and refraction of optical beams? Phys. Rev. Lett. 115, 183901. (doi:10.1103/PhysRevLett.115.183901)

118. Plansinis BW, Donaldson WR, Agrawal GP. 2018 Cross-phase-modulation-induced temporal reflection and waveguiding of optical pulses. JOSA B 35, 436–445. (doi:10.1364/JOSAB.35.000436)

119. Gerry C, Knight P, Knight PL. 2005 Introductory quantum optics. Cambridge, UK: Cambridge University Press.

120. Rosenberg Y, Drori J, Bermudez D, Leonhardt U. 2020 Boosting few-cycle soliton self-frequency shift using negative prechirp. Opt. Express 28, 3107–3115. (doi:10.1364/OE.383014)

121. Leonhardt U. 2015 On cosmology in the laboratory. Phil. Trans. R. Soc. A 373, 20140354. (doi:10.1098/rsta.2014.0354)