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A Comparison between different Optimization Techniques for CNC End Milling Process

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Abstract

Different kind of statistical optimization techniques are available for optimizing the different parameters of a CNC end milling process. In this paper a comparison is done between five different techniques such as principal components analysis, utility theory, Grey relational analysis, technique of order preference by similarity to ideal solution and their hybrid variants. The Taguchi optimization principle is common to all the methods which are presented in the paper. The experiments were carried out and the different response features such as surface roughness (Ra, Rz and Rq) and material removal rate (MRR) were measured and the different optimization techniques were applied. Three different surface roughness values are used for the analysis and they act as indices of surface quality whereas MRR acts as index of productivity. Hence the optimization is carried out such that the resulting optimized parameters will lead to a compromise between the productivity and the surface quality. The aim of the work is to carry out multi objective optimization on a single process and compare the results.

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1. Introduction

In this paper the various surface roughness measurements of the product machined by CNC end milling operation are studied experimentally and the results are interpreted analytically. Quality and productivity are two of the most important indices in any manufacturing operation.

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But it is found that quality is inversely proportional to the productivity. Hence it becomes essential to optimize both quality and productivity simultaneously. Different surface roughness parameters such as Ra, Rz and Rq are considered here. The product being machined has to have the minimum surface roughness leading to high quality which in turn affects the processing time. Hence a multi factor optimization problem is considered here. MRR is considered as the index of productivity. The experimentation is carried out in LV65 CNC Milling machine. The work piece chosen is Aluminium and the cutting tool is 10mm carbide tool. The different parameters and their levels chosen for carrying out the experiments are shown in the Table 1. The L25 Orthogonal array is chosen for carrying out the experiments with different parameter combinations. The response features measured are the surface roughness values such as Ra, Rz, Rq and Material Removal rate (MRR). The different responses measured are shown in the Table 2.

| Table 1. Parameter Levels |
|---------------------------|
| **Levels** | Depth (mm) | Speed (rpm) | Feed (mm) |
| 1          | 0.10       | 3000         | 550       |
| 2          | 0.20       | 3500         | 600       |
| 3          | 0.30       | 4000         | 650       |
| 4          | 0.40       | 4500         | 700       |
| 5          | 0.50       | 5000         | 750       |

| Table 2. Measured Responses |
|-----------------------------|
| **S. No.** | Measured responses | **Ra** | **Rz** | **Rq** | **MRR** |
| 1          | 0.53         | 3.1    | 0.66   | 7.500  |
| 2          | 0.46         | 2.75   | 0.59   | 8.333  |
| 3          | 0.6          | 3.36   | 0.74   | 8.333  |
| 4          | 0.52         | 2.8    | 0.64   | 9.375  |
| 5          | 0.58         | 3.2    | 0.73   | 9.375  |
| 6          | 0.67         | 3.83   | 0.83   | 15.000 |
| 7          | 0.62         | 2.62   | 0.7    | 16.667 |
| 8          | 0.56         | 3.2    | 0.7    | 18.750 |
| 9          | 0.64         | 3.62   | 0.8    | 21.429 |
| 10         | 0.65         | 3.41   | 0.8    | 15.000 |
| 11         | 0.48         | 2.69   | 0.58   | 25.000 |
| 12         | 0.61         | 3.47   | 0.75   | 25.000 |

2. Principal Components Analysis – Procedure Adapted for optimization

Assuming, the number of experimental runs in Taguchi’s OA design is \( m \), and the number of quality characteristics is \( n \). The Experimental results can be expressed by the following series: \( X_1, X_2, X_3, ..., X_i, ..., X_m \)

Here,
\[
X_i = \{X_1(1), X_1(2), ..., X_i(k), ..., X_i(n)\}
\]

\[
X_i = \{X_i(1), X_i(2), ..., X_i(k), ..., X_i(n)\}
\]

\[
X_m = \{X_m(1), X_m(2), ..., X_m(k), ..., X_m(n)\}
\]

Here, \( X_i \) represents the \( i \)th experimental results
Let, \( X_0 \) be the reference sequence:
Let, \( X_0 = \{X_0(1), X_0(2), ..., X_0(k), ..., X_0(n)\} \)

The value of the elements in the reference sequence means the optimal value of the corresponding quality
characteristic. $X_0$ and $X_i$ both include $n$ elements, and $X_0(k)$ and $X_i(k)$ represent the numeric value of $k$th element in the reference sequence and the comparative sequence, respectively, $k=1,2,........,n$. The following illustrates the proposed parameter optimization procedures in detail, (Su and Tong, 1997).

**Step 1: Normalization of the responses (Surface Roughness and MRR)**

Here the range of response values is very high. Such a high range may lead to biased results that’s why the original experimental data must be normalized. There are three different types of data normalization according to whether we require the LB (lower the better), the HB (higher the better) and NB (nominal the best). The normalization is taken by the following equations.

(a) LB (lower the better)

\[ X_i^*(k) = \frac{\min X_i(k)}{X_i(k)} \]  

(b) HB (higher the better)

\[ X_i^*(k) = \frac{\max X_i(k)}{X_i(k)} \]  

(c) NB (nominal the best)

\[ X_i^*(k) = \frac{\min (X_i(k), X_0(k))}{\max (X_i(k), X_0(k))} \]

Here, $i = 1, 2 ...m$; $k = 1, 2 ...n$ $X_i^*(k)$ is the normalized data of the $k$th element in the $i$th sequence. $X_{0b}(k)$ is the desired value of the $k$th quality characteristic. After data normalization, the value of $X_i^*(k)$ will be between 0 and 1.

**Step 2: Checking for correlation between two quality characteristics**

Let,

\[ Q_i = \{ X_i^*(1), X_i^*(2), X_i^*(3), \ldots, X_i^*(n) \} \]

It is the normalized series of the $i$th quality characteristic. The correlation coefficient between two quality characteristics is calculated by using the following equation:

\[ \rho_{jk} = \frac{\text{Cov}(Q_j, Q_k)}{\sigma_{Q_j} \sigma_{Q_k}} \]

\[ j = 1, 2, 3, \ldots, n \]

\[ k = 1, 2, 3, \ldots, n \]

\[ j \neq k \]

$\rho_{jk}$ is the correlation coefficient between quality characteristic $j$ and quality characteristic $k$; $\text{Cov}(Q_j, Q_k)$ is the covariance of two quality characteristics $j$ and $k$; $\sigma_{Q_j}$ and $\sigma_{Q_k}$ are the standard deviation of quality characteristic $j$ and $k$, respectively. The correlation is checked by testing the following hypothesis:

$H_0 : \rho_{jk} = 0$ (There is no correlation)

$H_0 : \rho_{jk} \neq 0$ (There is correlation)

**Step 3: Calculation of the principal component score**

(a) Calculation of the Eigen value $\lambda_k$ and the corresponding eigenvector $\beta_k$ ($k = 1, 2, \ldots, n$ ) from the correlation matrix.

(b) Calculation of the principal component scores of the normalized reference sequence and comparative sequences using the equation shown below:

\[ Y_i(k) = \sum_{j=1}^{n} X_i^*(j) \beta_{kj} \]

Where, $Y_i(k)$ is the principal component score of the $k$th element in the $i$th series. $X_i^*(j)$ is the normalized value of the $j$th element in the $i$th sequence, and $\beta_{kj}$ is the $j$th element of eigenvector $\beta_k$. 
2.1. Data Analysis

Table 3. Major Principal Components

| S.No. | Ideal sequence | ψ1   | ψ2   |
|-------|----------------|------|------|
| 1     | 0.0000         | -1.4140 |
| 2     | -0.5146        | -0.7126 |
| 3     | -0.5970        | -0.8170 |
| 4     | -0.4321        | -0.6520 |
| 5     | -0.5017        | -0.7491 |
| 6     | -0.4370        | -0.6844 |
| 7     | -0.2874        | -0.6834 |
| 8     | -0.3046        | -0.7445 |
| 9     | -0.3333        | -0.8282 |
| 10    | -0.2254        | -0.7910 |
| 11    | -0.3024        | -0.6983 |
| 12    | -0.3476        | -1.0075 |
| 13    | -0.2032        | -0.8631 |
| 14    | -0.0805        | -0.8229 |
| 15    | -0.1548        | -0.7486 |
| 16    | 0.0780         | -0.9118 |
| 17    | -0.1555        | -1.1453 |
| 18    | -0.0823        | -0.8742 |
| 19    | -0.0384        | -0.9182 |
| 20    | -0.1978        | -1.0776 |
| 21    | 0.3104         | -1.1036 |
| 22    | 0.0484         | -0.8514 |
| 23    | 0.1803         | -0.9195 |
| 24    | -0.0309        | -1.1306 |
| 25    | -0.0207        | -1.1205 |

Table 4. Quality Loss Estimates

| sl. No. | Quality loss estimates | S/n ratio |
|---------|------------------------|-----------|
| 1       | 0.5146                 | 5.7699    |
| 2       | 0.5970                 | 4.4802    |
| 3       | 0.4321                 | 7.2892    |
| 4       | 0.5017                 | 5.9912    |
| 5       | 0.4370                 | 7.1904    |
| 6       | 0.2874                 | 10.8290   |
| 7       | 0.3046                 | 10.3256   |
| 8       | 0.3333                 | 9.5433    |
| 9       | 0.2254                 | 12.9426   |
| 10      | 0.3024                 | 10.3890   |
| 11      | 0.3476                 | 9.1782    |
| 12      | 0.2032                 | 13.8409   |
| 13      | 0.0805                 | 21.8820   |
| 14      | 0.1548                 | 16.2071   |
| 15      | 0.2032                 | 13.8409   |
| 16      | 0.0780                 | 22.1635   |
| 17      | 0.1555                 | 16.1632   |
| 18      | 0.0823                 | 21.6873   |
| 19      | 0.0384                 | 28.3239   |
| 20      | 0.1978                 | 14.0766   |
| 21      | 0.3104                 | 10.1618   |
| 22      | 0.0484                 | 26.3026   |
| 23      | 0.1803                 | 14.8791   |
| 24      | 0.0309                 | 30.2118   |
| 25      | 0.0207                 | 33.6921   |

Fig.1 S/N Ratio Plot
Step 4 Calculation of Quality loss

(c) Accountability proportion (AP) values are seen and the quality characteristic with the highest value is considered and can be treated as the overall quality index; which is to be optimized finally. The quality loss $\Delta_{0,i}(k)$ of that index (compared to ideal situation) is calculated as follow:

$$\Delta_{0,i}(k) = \frac{|X_i^*(k) - X_i^*(k)|}{|Y_0(k) - Y_1(k)|} \text{ No significant correlation between quality characteristics}$$

$$\Delta_{0,i}(k) = \frac{|Y_0(k) - Y_1(k)|}{|X_i^*(k) - X_i^*(k)|} \text{ Significant correlation between quality characteristics}$$

Step 5 Optimization of Quality loss using Taguchi method

Finally the quality loss is optimized using Taguchi method. For calculating S/N ratio the higher the better criterion is selected.

3. Utility Theory – Procedure Adapted for optimization

According to the utility theory (Kumar et al 2000; Walia et al 2006), if $X_i$ is the measure of effectiveness of a quality characteristics $i$ and there are $n$ attributes evaluating the outcome space, then the joint utility function can be expressed as:

$$U(X_1, X_2, \ldots, X_n) = f(U_1(X_1), U_2(X_2), \ldots, U_n(X_n))$$

Here $U_i(X_i)$ is the utility of the $i^{th}$ attribute.

The overall utility function is the sum of individual utilities if the attributes are independent, and is given as follows:

$$U(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} U_i(X_i)$$

The attributes may be assigned weights depending upon the relative importance or priorities of the characteristics. The overall utility function after assigning weights to the attributes can be expressed as:

$$U(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} W_i \cdot U_i(X_i)$$

Here $W_i$ is the weight assigned to the attribute $i$. The sum of the weights for all the attributes must be equal to 1.

A scale is selected for the range of the utility index and that value is taken from 0 to 9 where 0 is the lowest and 9 is the highest. The preference number $P_i$ can be expressed on a logarithmic scale as follows:

$$P_i = A \cdot \log \left( \frac{X_i}{X_i^*} \right)$$

(6)

Here $X_i$ is the value of any quality characteristic or attribute $i$, $X_i^*$ is just an acceptable value of quality characteristic or attribute $i$ and $A$ is a constant. The value $A$ can be found by the condition that if $X_i = X^*$ (where $X^*$ is the optimal or the best value), then $P_i = 9$

Therefore,

$$A = \frac{9}{\log \frac{X_i}{X_i^*}}$$

(7)

The overall utility index can be expressed as follows:

$$U = \sum_{i=1}^{n} W_i \cdot P_i$$

(8)

Subject to the condition:

$$\sum_{i=1}^{n} W_i = 1$$

Since the utility function is a kind of grade and the grade always preferred is high we go by the Taguchi higher the better formula for the analysis. Here the objective function is the Utility index and hence it has to be optimized.
Table 5. Utility Values

| S. No. | Utility values | \(\psi_1\) | \(\psi_2\) | \(\psi_3\) |
|--------|----------------|------------|------------|------------|
| 1      | 3.4429         | 0.2650     | 0.1969     |
| 2      | 6.0958         | 0.0000     | 0.0000     |
| 3      | 2.0725         | 0.6180     | 0.5829     |
| 4      | 4.4324         | 0.2440     | 0.8005     |
| 5      | 2.5025         | 0.6269     | 0.7386     |
| 6      | 1.2566         | 1.8313     | 0.8680     |
| 7      | 3.8528         | 1.0430     | 5.3383     |
| 8      | 3.4810         | 1.7415     | 0.8899     |
| 9      | 2.0259         | 2.6933     | 1.2688     |
| 10     | 1.8114         | 1.5521     | 1.5010     |
| 11     | 8.2140         | 1.8987     | 1.1214     |
| 12     | 2.8305         | 3.1486     | 1.3462     |
| 13     | 1.8059         | 4.2649     | 4.4687     |
| 14     | 1.1265         | 3.2978     | 1.9951     |
| 15     | 2.9084         | 3.0875     | 1.5372     |
| 16     | 1.8396         | 8.8492     | 3.7428     |
| 17     | 7.9457         | 5.0087     | 1.7884     |
| 18     | 1.5217         | 5.6932     | 0.9817     |
| 19     | 2.8163         | 5.3836     | 6.7102     |
| 20     | 9.0000         | 3.2267     | 6.9426     |
| 21     | 2.5530         | 5.4024     | 1.1655     |
| 22     | 1.0452         | 7.7297     | 7.4847     |
| 23     | 0.0000         | 7.2738     | 1.8559     |
| 24     | 6.0943         | 7.4247     | 9.0000     |
| 25     | 5.0321         | 9.0000     | 2.5733     |

Table 6. Utility Index

| Overall Utility Index |
|-----------------------|
| S. No | Utility index | S/N ratio |
|-------|---------------|-----------|
| 1     | 1.2886        | 2.2024    |
| 2     | 2.0116        | 6.0709    |
| 3     | 1.0802        | 0.6702    |
| 4     | 1.8073        | 5.1408    |
| 5     | 1.2764        | 2.1200    |
| 6     | 1.3054        | 2.3152    |
| 7     | 3.3773        | 10.5713   |
| 8     | 2.0171        | 6.0944    |
| 9     | 1.9761        | 5.9160    |
| 10    | 1.6053        | 4.1112    |
| 11    | 3.7073        | 11.3811   |
| 12    | 2.4174        | 7.6669    |
| 13    | 3.4780        | 10.8267   |
| 14    | 2.1184        | 6.5201    |
| 15    | 2.4859        | 7.9098    |
| 16    | 4.7624        | 13.5566   |
| 17    | 4.8651        | 13.7419   |
| 18    | 2.7049        | 8.6430    |
| 19    | 4.9203        | 13.8399   |
| 20    | 6.3259        | 16.0224   |
| 21    | 3.0999        | 9.5711    |
| 22    | 5.3657        | 14.5925   |
| 23    | 3.0128        | 9.5795    |
| 24    | 7.4313        | 17.4213   |
| 25    | 5.4798        | 14.7753   |

Fig.2 S/N Ratio Plot

4. Grey Relational Analysis

In grey relational analysis, normalization is first carried out. Grey relational coefficients are calculated from the normalized values in order to represent the correlation between the response features. Then overall grey relational grade is determined by averaging the grey relational coefficient corresponding to selected responses. The overall performance characteristic of the multiple response process depends on the calculated grey
relational grade. In this approach also a multi response optimization problem is converted into a single objective optimization problem. The objective function here is represented by the Grey Relational grade.

In grey relational generation, the normalized data corresponding to Lower-the-Better (LB) criterion can be expressed as:

\[ x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)} \]

For Higher-the-Better (HB) criterion, the normalized data can be expressed as:

\[ x_i(k) = \frac{\max y_i(k) - y_i(k)}{\max y_i(k) - \min y_i(k)} \]

Where \( x_i(k) \) is the value after the grey relational generation, \( \min y_i(k) \) is the smallest value of \( y_i(k) \) for the \( k \)th response, and \( \max y_i(k) \) is the largest value of \( y_i(k) \) for the \( k \)th response. An ideal sequence is \( x^*_i(k) \) for the responses. The purpose of grey relational grade is to reveal the degrees of relation between the sequences say \([x_0(k) \text{ and } x_i(k), i = 1,2,3...9]\). The grey relational coefficient \( \xi_i(k) \) can be calculated as

\[ r_{0,i}(k) = \frac{\min \Delta_i + \xi \Delta_{\max}}{\Delta_{\min} + \xi \Delta_{\max}} \]  \hspace{1cm} (9)

Where \( \Delta_i = ||x_0(k) - x_i(k)|| \) = difference of the absolute value \( x_0(k) \) and \( x_i(k) \); \( \xi \) is the distinguishing coefficient \( 0 \leq \xi \leq 1 \);

\[ \Delta_{\max} = \left( \max_i; \max_k |x_0^*(k) - x_i^*(k)| \right) \]

\[ \Delta_{\min} = \left( \min_i; \min_k |x_0^*(k) - x_i^*(k)| \right) \]

After averaging the grey relational coefficients, the grey relational grade \( \gamma_i \) can be computed as:

\[ \gamma_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i(k) \]

where \( n \) = number of process responses. The higher value of grey relational grade corresponds to intense relational degree between the reference sequence \( x_0(k) \) and the given sequence \( x_i(k) \). The reference sequence \( x_{0(k)} \) represents the best process sequence. Therefore, higher grey relational grade means that the corresponding parameter combination is closer to the optimal.

However, Equation (11) assumes that all response features are equally important. But, in practical case, it may not be so. Therefore, different weightages have been assigned to different response features according to their relative priority. In that case, the equation for calculating overall grey relational grade (with different weightages for different responses) is modified as shown below:

\[ \gamma_i = \frac{\sum_{k=1}^{n} w_k \xi_i(k)}{\sum_{k=1}^{n} w_k} \]  \hspace{1cm} (10)

Here, \( \gamma_i \) is the overall grey relational grade for ith experiment. \( \xi_i(k) \) is the grey relational coefficient of the kth response in ith experiment and \( w_k \) is the weightage assigned to the kth response.

Table 7. Grey Relational Coefficients

| Coefficients | 12 | 0.6428 | 0.6339 | 0.5051 |
|--------------|----|--------|--------|--------|
| 13           | 0.5823 | 0.7168 | 0.8401 |
| 14           | 0.5433 | 0.6451 | 0.5869 |
| 15           | 0.6474 | 0.6293 | 0.5293 |
| 16           | 0.5843 | 0.9933 | 0.7797 |
| 17           | 0.9440 | 0.7700 | 0.5611 |
| 18           | 0.5659 | 0.8167 | 0.4590 |
| 19           | 0.6419 | 0.7959 | 0.9547 |
| 20           | 1.0000 | 0.6398 | 0.9617 |
| 21           | 0.6263 | 0.7972 | 0.4822 |
| 22           | 0.5387 | 0.9386 | 0.9754 |
| 23           | 0.4810 | 0.9138 | 0.5696 |
| 24           | 0.8386 | 0.9222 | 1.0000 |
| 25           | 0.7754 | 1.0000 | 0.6568 |
Table 8. Grey Relational Grade

| Sl.no | $\gamma_i$ | S/n Ratio |
|-------|------------|-----------|
| 1     | 0.4837     | -6.3087   |
| 2     | 0.5228     | -5.6325   |
| 3     | 0.4800     | -6.3749   |
| 4     | 0.5270     | -5.5646   |
| 5     | 0.4949     | -6.1097   |
| 6     | 0.5051     | -5.9332   |
| 7     | 0.6857     | -3.2776   |
| 8     | 0.5470     | -5.2398   |
| 9     | 0.5577     | -5.0716   |
| 10    | 0.5352     | -5.4300   |
| 11    | 0.6518     | -3.7174   |
| 12    | 0.5880     | -4.6125   |
| 13    | 0.7060     | -3.0244   |
| 14    | 0.5859     | -4.6442   |
| 15    | 0.5960     | -4.4949   |
| 16    | 0.7779     | -2.1815   |
| 17    | 0.7508     | -2.4897   |
| 18    | 0.6077     | -4.3258   |
| 19    | 0.7895     | -2.0528   |
| 20    | 0.8585     | -1.3254   |
| 21    | 0.6289     | -4.0290   |
| 22    | 0.8094     | -1.8367   |
| 23    | 0.6482     | -3.7654   |
| 24    | 0.9111     | -0.8091   |
| 25    | 0.8026     | -1.9099   |

Finally the Grey Relational Grade is optimized using Taguchi method. The S/N ratio is calculated using the higher the better criterion.

5. TOPSIS

‘TOPSIS’ is Technique of order preference by similarity to ideal solution. The procedure is given in the steps below.

**Step 1 Obtain the normalized decision matrix $r_{ij}$**

The quality loss $\Delta_{0,i}(k)$ that has been estimated by aforesaid procedure has been normalized by the following equation

$$ r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} X_{ij}^2}} $$

where, $r_{ij}$ represents the normalized performance of $\Lambda_i$ with respect to attribute $X_j$.

**Step 2. Obtain the weighted normalized decision matrix**

$$ V = w_j r_{ij} $$
\[ \sum_{j=1}^{n} w_j = 1 \]

**Step 3. Determine the ideal (best) and negative ideal (worst) solutions**

\[
A^+ = \{ (\max_v v_j | j \in J), (\min_v v_j | j \in J) | i = 1,2,\ldots,m \} \\
= \{ v_1^+, v_2^+, v_3^+ \ldots v_n^+ \} \\
A^- = \{ (\min_v v_j | j \in J), (\max_v v_j | j \in J) | i = 1,2,\ldots,m \} \\
= \{ v_1^-, v_2^-, v_3^- \ldots v_n^- \}
\]

**Step 4. Determine the distance measures**

The separation of each alternative from the ideal solution is given by n-dimensional Euclidean distance from the following equations:

\[
S_{ij}^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \\
S_{ij}^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}
\]

**Step 5. Calculate the relative closeness (closeness coefficient) to the ideal solution**

\[
C_i^+ = \frac{S_{ij}^-}{S_{ij}^+ + S_{ij}^-} \quad 0 \leq C_i^+ \leq 1 \quad (11)
\]

**Step 6. Determine the optimum process variable by optimization OPI using Taguchi method**

The optimum process parameter combination ensures highest OPI value. The closeness coefficient value is optimized using Taguchi method. For calculating S/N ratio (corresponding to the values of closeness coefficient); Higher-the-Better (HB) criterion is to be considered. As larger the value of closeness coefficient, better is the proximity to the ideal solution.

| Closeness Coefficient | S No | Ci     | S/N Ratio         |
|-----------------------|------|--------|-------------------|
| 1                     |      | 0.1225 | -18.2381          |
| 2                     |      | 0.1962 | -14.1472          |
| 3                     |      | 0.2575 | -11.7848          |
| 4                     |      | 0.0863 | -21.2794          |
| 5                     |      | 0.2088 | -13.6058          |
| 6                     |      | 0.2756 | -11.1940          |
| 7                     |      | 0.5328 | -5.4688           |
| 8                     |      | 0.3100 | -10.1737          |
| 9                     |      | 0.3126 | -10.0997          |
| 10                    |      | 0.3343 | -9.5178           |
| 11                    |      | 0.4889 | -6.2152           |
| 12                    |      | 0.3868 | -8.2496           |
| 13                    |      | 0.5323 | -5.4773           |
| 14                    |      | 0.4198 | -7.5393           |
| 15                    |      | 0.3672 | -8.7020           |
| 16                    |      | 0.6838 | -3.3012           |
| 17                    |      | 0.6011 | -4.2045           |
| 18                    |      | 0.5479 | -5.2264           |
| 19                    |      | 0.4874 | -6.2428           |
| 20                    |      | 0.4379 | -7.1730           |
| 21                    |      | 0.8200 | -1.7238           |
| 22                    |      | 0.6647 | -3.5479           |
| 23                    |      | 0.6070 | -4.3367           |
| 24                    |      | 0.4987 | -6.0435           |
| 25                    |      | 0.6047 | -4.3693           |
6. Conclusion

Hence the study is carried out and four different kinds of optimization techniques have been adapted. The results of the different techniques have been tabulated as shown in the Table 10.

Table 10. Results

| Method adapted for optimization                        | Response features | Optimum values |
|---------------------------------------------------------|-------------------|----------------|
| PCA                                                     | $R_a$, MRR        | Depth (mm) 0.1 | Speed (rpm) 3000 | Feed (mm/min) 750 |
| PCA combined with utility theory                        | $R_a$, $R_z$, $R_q$, MRR | Depth (mm) 0.5 | Speed (rpm) 3500 | Feed (mm/min) 650 |
| PCA combined with Grey Relational Analysis             | $R_a$, $R_z$, $R_q$, MRR | Depth (mm) 0.5 | Speed (rpm) 3500 | Feed (mm/min) 650 |
| PCA combined with TOPSIS                               | $R_a$, $R_z$, $R_q$, MRR | Depth (mm) 0.5 | Speed (rpm) 3500 | Feed (mm/min) 650 |

From the results it can be seen that the last three methods produced similar results. Whereas when only Ra and MRR were considered there was a variation in the results. This indicates the influence of the parameter levels when considering different responses. The above study can be carried out using other heuristic techniques and the results can be compared.

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