Method to Obtain Target Speed with Underwater Acoustic Positioning System

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Abstract. There has been considerable recent interest in the positioning of autonomous underwater vehicles (AUVs). Underwater acoustic positioning system (UAPS) are usually used to determine position and speed of the moving target in water environment. Generally, moving target speed is estimated by derivative of the target position with traditional method. However, this approach relies heavily on accuracy of positioning. Thus, this paper presents a novel method for estimation of target speed with the UAPS only, especially with long baseline (LBL) acoustic positioning system. The method is made by estimating the Doppler frequency shift of the received signals and solving the set of linear equations. It could depend less on positioning accuracy than the traditional method. What’s more, it works well even though the moving target is subjected to acceleration or jerk. With simulations by MATLAB, it is shown that the method achieves very good performance.

1. Introduction
In recent years, autonomous underwater vehicles (AUVs) technology has been developed rapidly with the strong demands by military and civil applications. The positioning system is essential for the AUV technology. Global positioning system (GPS) is used widely by aerial or ground vehicles. However, it is used rarely by AUVs due to the strong attenuation that the electromagnetic field suffers in water. An available solution is the GPS intelligent buoy system [1]. The inertial navigation system (INS) can be used by AUVs since it does not need external information and has highly autonomous character. But INS can output positioning data stably in short period, the positioning accuracy would decrease by time. Therefore, Underwater acoustic positioning system (UAPS) are usually used by AUVs because of the high positioning accuracy and long-time stability compared with other positioning technology. Based on the length of baseline, the underwater acoustic positioning system is divided three classes [2]: long baseline (LBL) acoustic positioning system, short baseline (SBL) acoustic positioning system and ultra-short baseline (USBL) acoustic positioning system. Nowadays, the researches about UAPS mostly focus on the sensor formation optimization of the UAPS [3,4], method to increase the user capacity of the UAPS [5,6], algorithm to improve positioning accuracy of the UAPS [7,8], combination of underwater acoustic positioning equipment and multi-sensors [9,10]. So far, few researchers have studied the speed estimate of movement targets used UAPS. Generally speaking, the traditional method to estimate the moving target speed is calculating the derivative of the target position. However, this method can be satisfactory when the target speed is almost constant during the selected time. Specifically, the target is not subjected to acceleration or jerk. In addition, the positioning errors influence the speed errors heavily with this method. To solve the problem...
mentioned above, a novel method for obtaining the target speed with UAPS is presented and the simulations by MATLAB is established. In Section 2 and 3, the method to obtain the target position and speed are introduced. The simulative experiments are designed to show the advantage of the method in Section 4 and the conclusion are summarized in Section 5.

2. Calculation of Target Position

LBL utilizes the concept of time of arrival (TOA) ranging to determine target position. This concept entails measuring the time it takes for ranging signal transmitted by a beacon at known location to reach the target hydrophone. The time interval, referred to as the signal propagation time, is then multiplied by the speed of the sound to obtain the beacon to target distance. The process is assumed that the clock on the beacon controls the timing of the ranging signal. All clock on each of the beacons are effectively synchronized to a system time. The target also contains a clock synchronize d to system time.

Assuming the positions of the three beacons could be attained by calibration, and the coordinate of beacon is \((x_i, y_i, z_i)\), where \(i\) ranges from 1 to 3 and references the beacons. The sound speed is \(c\) in water, \(\Delta t_i\) is the one way travel time between the \(i\)th beacon and target, then \(r_i = c \times \Delta t_i\) is the range measurement between the \(i\)th beacon and the target. In order to determine target position in three dimensions \(\mathbf{u} = (x, y, z)\), range measurements are made to at least three beacons resulting in the system of equations:

\[
\begin{align*}
\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} &= r_1 \\
\sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} &= r_2 \\
\sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} &= r_3
\end{align*}
\]

(1)

If the beacons do not all lie in a plane, through down times processing, equation (1) could be simplified as equation (2):

\[
\begin{bmatrix}
x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\
x_1 - x_3 & y_1 - y_3 & z_1 - z_3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
r_1^2 - r_2^2 + d_2^2 - d_1^2 \\
r_2^2 - r_3^2 + d_3^2 - d_1^2 \\
r_3^2 - r_1^2 + d_1^2 - d_3^2
\end{bmatrix}
\]

(2)

where \(d_i = \sqrt{x_i^2 + y_i^2 + z_i^2}, \ i = 1,2,3\). These equations can be put in matrix form by making the definitions:

\[
\mathbf{A} = \begin{bmatrix}
x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
x_3 - x_2 & y_3 - y_2 & z_3 - z_2 \\
x_1 - x_3 & y_1 - y_3 & z_1 - z_3
\end{bmatrix}
\quad \mathbf{X} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\quad \mathbf{B} = \begin{bmatrix}
r_1^2 - r_2^2 + d_2^2 - d_1^2 \\
r_2^2 - r_3^2 + d_3^2 - d_1^2 \\
r_3^2 - r_1^2 + d_1^2 - d_3^2
\end{bmatrix}
\]

(3)

One obtains, finally,

\[
\mathbf{AX} = \mathbf{B}
\]

(4)

The matrix \(\mathbf{A}\) will be invertible provided the beacons do not all lie in a plane, then equation (4) has the solution:

\[
\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}
\]

(5)

If the beacons all lie in the same plane, equation (1) could be simplified as equation (6):
The coordinates \((x, y)\) of target can be calculated by equation (6), then the solution of the coordinate \(z\) of target can be solved by equations (1).

3. Method to Obtain the Target Speed

If the target position in three dimensions \(u = (x, y, z)\) were obtained in Section 2, then the three-dimensional target speed would be defined as \(\mathbf{u} = (\hat{x}, \hat{y}, \hat{z})\). Thus, with traditional method, target speed is estimated by forming an approximate derivative of the target position:

\[
\mathbf{u} = \frac{du}{dt} = \frac{\mathbf{u}(t_2) - \mathbf{u}(t_1)}{t_2 - t_1}
\]

(7)

The accuracy of speed estimate used traditional method depends heavily on the positioning accuracy. Specifically, the errors in the positions \(\mathbf{u}(t_2)\) and \(\mathbf{u}(t_1)\) need to be small relative to \(\frac{\mathbf{u}(t_2) - \mathbf{u}(t_1)}{t_2 - t_1}\). In addition, this method could be satisfactory when the target speed is almost constant during the selected time interval, namely, the target is not subjected to acceleration or jerk.

To solve the problems mentioned above, in this paper, speed measurements are made by processing Doppler frequency shift of the received beacons. The Doppler frequency shift is produced by the relative motion of the beacon with respect to the target. The received frequency increases as the target approaches the beacons and decreases as it recedes from the beacons. For the target, the received frequency \(f_R\), can be approximated as follows:

\[
f_R = f_T \left( 1 - \frac{\mathbf{v}_R \cdot \mathbf{a}}{c} \right)
\]

(8)

where \(f_T\) is the transmitted beacon signal frequency, \(\mathbf{v}_R\) is the beacon to target relative speed vector, \(\mathbf{a}\) is the unit vector pointing along the line of sight from the target to the beacon, and \(c\) is the sound speed in water. The dot product \(\mathbf{v}_R \cdot \mathbf{a}\) represents the radial component of the relative speed vector along the line of sight from the target to the beacon. Vector \(\mathbf{v}_R\) is given as the difference:

\[
\mathbf{v}_R = \mathbf{v} - \mathbf{u}
\]

(9)

where \(\mathbf{v}\) is the speed of the beacon, for the beacon on the seafloor, \(\mathbf{v} = 0\).

For the \(i\)th beacon, substituting (9) into (8) yields:

\[
f_{RI} = f_T \left[ 1 - \frac{1}{c} (\mathbf{u} \cdot \mathbf{a}_i) \right]
\]

(10)

Expanding the dot products in terms of the vector components yields:

\[
\frac{c(f_{RI} - f_{T})}{f_T} = \frac{c \Delta f_{RI}}{f_T} = \hat{x}a_{xi} + \hat{y}a_{yi} + \hat{z}a_{zi}
\]

(11)

where \(\Delta f_{RI}\) is the Doppler frequency shift, \(\mathbf{a}_i = (a_{xi}, a_{yi}, a_{zi})\), and the components of \(\mathbf{a}_i\) are obtained as follows:
To simplify equation (11), we introduce the new variable \( d_i \), defined by:

\[
d_i = \frac{c \Delta f_{th}}{f_n}
\]

With the simplification, equation (11) can be rewritten as:

\[
d_i = x a_{u} + y a_{v} + z a_{w}
\]

We now have three unknowns \( \mathbf{u} = (x, y, z) \), which can be solved by using measurements from three beacons. We calculate the unknown quantities by solving the set of linear equations using matrix algebra. The matrix and vector scheme are defined as:

\[
\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} \\ a_{x2} & a_{y2} & a_{z2} \\ a_{x3} & a_{y3} & a_{z3} \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}
\]

In matrix notation, we can obtain:

\[
\mathbf{d} = \mathbf{Hg}
\]

and the solution for the speed are obtained as:

\[
\mathbf{g} = \mathbf{H}^{-1} \mathbf{d}
\]

If range measurements are made to more than four beacons, the least squares estimation techniques can be employed to obtain improved estimates of the unknowns. In that case, equation (17) will be rewritten as:

\[
\mathbf{g} = (\mathbf{H}^{T} \mathbf{H})^{-1} \mathbf{H}^{T} \mathbf{d}
\]

The phase measurements that lead to the Doppler frequency shift used in the speed formulation are corrupted by errors such as measurement noise and multipath. Furthermore, the computation of target speed is dependent on target positioning accuracy and correct knowledge of ocean current speed.

4. Simulation and Discussion

Consider the coordinates of the four calibrated seafloor beacons are defined as (0,0,-1000)m, (2000,0,-1000)m, (0,2000,-1000)m, (2000,2000,-1000)m, respectively. The kinematic equations of the target can be written as \( x = t \) m, \( y = 50\sin(0.05t) \) m, \( z = -200 \) m, where \( x, y \) and \( z \) denote the target position coordinates, \( t \) is the sampling time. In this simulation, the system is sampled for every 1s, and the total simulation time is 1000s. To simplify the model, during the measurement, the statistic character of the system noise and the observation noise are defined to be white Gaussian noise, and their variance are 1 and 9, respectively. The speed of ocean current is supposed to be 0.04~0.06m/s in the direction of \( x \)-axis, and, 0.02~0.04m/s in the direction of \( y \)-axis. The estimate accuracy of Doppler frequency shift is within ±20%.

The simulation results are shown from figure 1~figure 4. Figure 1 is the simulation results of real trajectory and their estimated trajectory. Figure 2 show the positioning errors in three- dimensional coordinates. As vividly shown in figure 1 and figure 2, the target realizes the estimation of the real trajectory in the presence of large positioning errors. The errors are produced by the observation noise, mainly are the range measurement errors. The positioning errors directly cause the speed errors with traditional method, shown in figure 3. Compare figure 3(a), figure 3(b) and figure 3(c), it can be seen
the speed estimation with traditional method is satisfactory when the target speed is almost constant during the selected time. Specially, the accuracy of speed estimation is varying with the target’s acceleration. Figure 4 shows the virtual speed errors with method mentioned in this paper. Compared with the traditional method, the speed errors are obviously smaller, which implies the speed accuracy with method presented in this paper depend on the positioning accuracy less than the traditional method.

5. Conclusion
This paper offered a method to obtain the speed of moving target used underwater acoustic positioning system only. The approach is made by precise estimation of the Doppler frequency shift of the received beacon signals. In comparison with traditional method to estimate speed by forming an approximate derivative of the target position, the present approach still works even though the target speed is not constant over the selected time interval. Although both methods are dependent of
positioning accuracy, the proposed method were less sensitive than the traditional method. Future work will consider the influence of time offset between the beacon clock and the target clock on the speed accuracy.

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