High Order Sliding Mode Backstepping Control for a Class of Unknown Pure Feedback Nonlinear Systems

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This work was supported by the National Natural Science Foundation of China under Grant 61703269, Grant 61703270, and Grant 61803254.

ABSTRACT This paper aims at addressing the problem of finite time exact tracking control for a class of unknown nonlinear systems in pure feedback form while guaranteeing the closed-loop stability. The uncertain nonlinear system considered in this paper is not only in completely non-affine form but also explicitly dependent on the time and therefore, can cover a general class of nonlinear systems. By incorporating high order sliding mode controller into backstepping design procedure, a new robust control scheme with novel integrators is proposed, which is able to steer the tracking error to zero. Moreover, the closed loop stability can be proved with the help of the constructed integrators in every design step. Two simulation examples are presented to illustrate the correctness and effectiveness of the proposed control scheme.

INDEX TERMS Pure feedback systems, nonlinear systems, backstepping, high order sliding mode, robustness.

I. INTRODUCTION

Control of unknown pure feedback nonlinear systems has long been a challenging issue. Due to their non-affine properties, it is difficult to find the explicit virtual controllers and the actual controller to stabilize the nonlinear pure feedback systems. Thus fewer results for pure feedback systems are available compared with those for strict feedback systems [1]–[4]. In the past decade, the works mainly focused on the time-invariant pure feedback nonlinear systems and neural networks (NN) and fuzzy logic systems (FLS) as powerful tools were applied to approximate unknown nonlinear functions. With the last one or two equations of the systems being affine, some early research dealt with the pure feedback nonlinear systems in the simple form and guaranteed the convergence of the output tracking error to a small residual set by appropriately choosing design parameters [5], [6].

Then the research was extended to the unknown nonlinear systems in the completely pure feedback form [7]–[9]. For a class of pure feedback nonlinear systems in non-affine form, [7] utilized radial basis function neural networks (RBFNN) technique and the ISS (input to state stability)-modular approach was given by combining with the backstepping design method and the small-gain theorem. Then it was proved that stability of the closed loop system was guaranteed and the output tracking error can converge to a small residual set. Furthermore, [8] and [9] took the transient performance of the tracking error into account. Adaptive dynamic surface controllers by neural network technique were proposed for unknown nonlinear pure feedback systems such that semiglobal stability of the system was ensured and the $L_\infty$ performance of the tracking errors was achieved. Recently, some output feedback control approaches were investigated in [10]–[12]. Moreover, [13]–[15] investigated the finite-time control for non-affine nonlinear systems. Reference [13] achieved dynamic containment in finite time for networked multiagent systems in nonaffine pure-feedback form by utilizing fraction dynamic surface and the containment errors between the leaders and followers converge to an adjustable neighborhood of zero. For a class of switched nonlinear systems in pure-feedback form, a backstepping controller was developed in [14] by adding a power integrator technique to solve the practical stabilization problem. When states of the systems are unavailable, [15], [16] investigated...
the output feedback control for nonlinear multiagent systems. Based on the state observers, finite-time practical tracking controllers can be obtained. Other results with respect to pure feedback systems can be found in [17]–[19] and the references contained therein for more details.

In spite of tremendous progress, we note that all the existing control schemes for unknown pure feedback nonlinear systems can not steer the tracking error to zero due to disturbances and approximate errors of unknown nonlinear functions caused by NN or FLS. Moreover, the pure feedback nonlinear systems considered in previous works are either time invariant or in partially affine form. Even if some uncertain disturbance nonlinearities considered in the systems explicitly depend on time, they were restricted by the known or unknown time invariant functions [5], [8].

On the other hand, high order sliding mode (HOSM) controller was presented for the output feedback real time control of strict feedback systems and can make the tracking error converge to zero in finite time [20]. Then, high order sliding mode control was applied to handle the unmatched perturbations and the zero tracking error after finite time can be guaranteed [21], [22]. However, the nonlinearities of the systems were required to be known and the systems were assumed to be bounded input to bounded states (BIBS) stable in [21]. Furthermore, some results [12], [23] applied HOSM differentiator as observer to develop the adaptive controllers for non-affine nonlinear systems, which can steer the tracking error to a small compact set around zero and other related research [24]–[27] can obtain asymptotically convergence for strict feedback systems.

Based on the above analysis, the main challenge is still that the control design for the unknown pure feedback nonlinear systems such that the tracking error converge to zero and the closed loop stability can be achieved. The motivation behind the research is the high precision tracking control for some general electro-mechanical systems in pure feedback form, in which tracking error can converges to zero in finite time, not to a neighborhood of zero. Specifically, in this paper, the high order sliding mode controllers are incorporated into backstepping technique to form a new recursive method.

In the design procedure, with the aid of the HOSM controllers, novel integrators are constructed to obtain the virtual control laws and the actual control law. The main contribution of this paper is summarized below.

- The novel integrators are constructed in each design step, which pave the way for solving the problem of the closed loop stability for high order sliding model control, meanwhile keeping the relative degree of the system states.

The rest of paper is organized as follows. In section 2, we state the problem formulation and preliminary knowledge. In section 3, the controller design procedure is presented, while section 4 gives the stability and tracking performance analysis. The chattering phenomenon of the controller is discussed in section 5. Finally, the correctness and the effectiveness of our proposed control approach is verified by two simulation examples in section 6.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. SYSTEM FORMULATION

We consider the following unknown pure feedback nonlinear system:

\[
\begin{align*}
\dot{x}_i &= f_i(\tilde{x}_i, x_{i+1}, t), \quad 1 \leq i \leq n - 1, \\
\dot{x}_n &= f_n(\tilde{x}_n, u, t), \\
y &= x_1,
\end{align*}
\]

where \( \tilde{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i \) and \( \tilde{x}_n = [x_1, \ldots, x_n]^T \in \mathbb{R}^n \) are the state vectors; \( f_i(\cdot) \) and \( f_n(\cdot) \) are unknown smooth and non-affine nonlinear functions including uncertain disturbance terms due to parameter variations, internal and external perturbations; \( y \in \mathbb{R} \) is the system output signal, and \( u \in \mathbb{R} \) is the system input signal. In this paper, all differential equations are understood in the Filippov sense [28], which allows the discontinuous signals to be utilized in the controller design.

B. SYSTEM TRANSFORMATION AND BASIC ASSUMPTIONS

According to the mean-value theorem, \( f_i(\cdot) \) in (1) are represented by

\[
\begin{align*}
f_i(\tilde{x}_i, x_{i+1}, t) &= f_i(\tilde{x}_i, x^0_{i+1}, t) + \frac{\partial f_i(\tilde{x}_i, x_{i+1}, t)}{\partial x_{i+1}} |_{x_{i+1} = x^0_{i+1}} (x_{i+1} - x^0_{i+1}), \\
&= f_i(\tilde{x}_i, x^0_{i+1}, t) + \frac{\partial f_i(\tilde{x}_i, x^0_{i+1}, t)}{\partial x_{i+1}} |_{x_{i+1}} (x_{i+1} - x^0_{i+1}),
\end{align*}
\]

where \( x^0_{i+1} = \partial_i x_{i+1} + (1 - \partial_i) x^0_{i+1} \) with \( 0 < \partial_i < 1 \), and \( u^0 = \partial_n u + (1 - \partial_n) u^0 \) with \( 0 < \partial_n < 1 \). When \( x^0_{i+1} = 0 \) and \( u^0 = 0 \), (2) can be written as

\[
\begin{align*}
f_i(\tilde{x}_i, x_{i+1}, t) &= f_i(\tilde{x}_i, 0, t) + \frac{\partial f_i(\tilde{x}_i, x^0_{i+1}, t)}{\partial x_{i+1}} |_{x_{i+1}} (x_{i+1} - x^0_{i+1}), \\
&= f_i(\tilde{x}_i, 0, t) + \frac{\partial f_i(\tilde{x}_i, 0, t)}{\partial x_{i+1}} |_{x_{i+1}} (x_{i+1} - x^0_{i+1}),
\end{align*}
\]

where \( x^0_{i+1} = \partial_i x_{i+1} + (1 - \partial_i) x^0_{i+1} \) with \( 0 < \partial_i < 1 \), and \( u^0 = \partial_n u + (1 - \partial_n) u^0 \) with \( 0 < \partial_n < 1 \). When \( x^0_{i+1} = 0 \) and \( u^0 = 0 \), (2) can be written as

\[
\begin{align*}
f_i(\tilde{x}_i, x_{i+1}, t) &= f_i(\tilde{x}_i, 0, t) + \frac{\partial f_i(\tilde{x}_i, x^0_{i+1}, t)}{\partial x_{i+1}} |_{x_{i+1}} (x_{i+1} - x^0_{i+1}), \\
&= f_i(\tilde{x}_i, 0, t) + \frac{\partial f_i(\tilde{x}_i, 0, t)}{\partial x_{i+1}} |_{x_{i+1}} (x_{i+1} - x^0_{i+1}),
\end{align*}
\]
For simplicity, define
\[
g_i(\cdot) = g_i(\dot{x}_i, x_{i+1}, t) = \frac{\partial f_i(\dot{x}_i, x_{i+1}, t)}{\partial x_{i+1}} \bigg|_{x_{i+1}=\dot{x}_{i+1}},
\]
i = 1, \ldots, n - 1,
\[
g_n(\cdot) = g_n(\dot{x}_n, u^0, t) = \frac{\partial f_n(\dot{x}_n, u, t)}{\partial u} \bigg|_{u=u^0}. \tag{4}
\]
Substituting (3) and (4) into (1) yields
\[
\dot{x}_i = g_i(\dot{x}_i) + f_i(\dot{x}_i, 0, t),
\]
\[
\dot{x}_n = g_n(u) + f_n(\dot{x}_n, 0, t),
\]
y = x_1, i = 1, \ldots, n - 1. \tag{5}

A.1: The sign of \(g_i(\cdot), i = 1, \ldots, n\) is known. Without loss of generality, it is assumed that \(g_i(\cdot) > 0\).

A.2: There exist constants \(g_m, g_M\) such that \(0 < g_m \leq g_i(\cdot) \leq g_M, i = 1, \ldots, n\).

A.3: The first \(n - i\) time derivatives of \(g_i(\cdot)\) and the \((n - i)\)th time derivative of \(f_i(\dot{x}_i, 0, t)\) are bounded for \(i = 1, \ldots, n\).

Remark 1: A.1 is a common assumption with respect to the control direction existing in considerable literature, such as [2], [5], [6], [8], [17], [19]. As shown in [29], A.2 and A.3 are satisfied at least locally for any smooth system and from the engineering point of view, always hold in some area of the state space containing the actual region of the system operation [18], [21], [22], [30].

Remark 2: Unlike the pure feedback nonlinear systems discussed in [5]–[19], the nonlinear functions of the system (1) explicitly depend on the time \(t\), which results in Neural Networks and Fuzzy Logic Systems invalid to approximate the unknown nonlinearities when \(t \in [0, \infty)\). Therefore, the existing methods for the common unknown pure feedback systems can not directly cope with the system (1).

Remark 3: Since the non-affine nonlinear functions of the system (1) may contain the unknown perturbation terms, the proposed control scheme ensure the robustness of the controlled system. Also, it should be noted that the nonlinear functions of (5) transformed from (1) are still completely unknown, which is more general than the systems with known nonlinearities considered in [22], [30] and [31].

C. HIGH ORDER SLIDING MODE (HOSM) CONTROLLER
Consider a class of SISO nonlinear system:
\[
\dot{x} = \xi(t, x) + \eta(t, x)u, x \in \mathbb{R}^n, u \in \mathbb{R},
\]
\[
\varepsilon = \varepsilon(t, x), \tag{6}
\]
where \(u\) is the control input to be designed, \(\varepsilon : \mathbb{R}^{n+1} \to \mathbb{R}\) is the output of the system, which can be measured, \(\xi, \eta, \varepsilon\) are unknown sufficiently smooth functions and \(n\) is the dimension of the state. The task is to obtain \(\varepsilon \equiv 0\).

We have a basic assumption that the relative degree of the system (6) is \(r\), which is a known constant. Then from [32], the control signal \(u\) explicitly appears in the \(r\)th derivative of \(\varepsilon\) for the first time, which implies that the equation
\[
\varepsilon^{(r)} = a(t, x) + b(t, x)u, b(t, x) \neq 0 \tag{7}
\]
holds, where \(a(t, x) = \varepsilon^{(r)}|_{u=0}, b(t, x) = (\partial / \partial u)\varepsilon^{(r)}\). For any Lebesgue measurable bounded input \(u(t, x)\), trajectories of the system (6) are assumed can infinitely extendible. Also, it is assumed that for some \(C_m, C_M, D > 0\), the inequalities
\[
0 < C_m \leq (\partial / \partial u)\varepsilon^{(r)} \leq C_M, \left|\varepsilon^{(r)}|_{u=0}\right| \leq D \tag{8}
\]
hold. Then the following differential inclusion can be obtained
\[

\varepsilon^{(r)} \in [D, D] + [C_m, C_M]u. \tag{9}
\]
The above problem can be solved by the following HOSM controller [20], which was constructed to ensure the \(r\)-sliding mode \(\varepsilon = \varepsilon = \ldots = \varepsilon^{(r-1)} = 0\) finite time stable.

\[
u = -\alpha \Phi_{r-1, r}(\varepsilon, \dot{\varepsilon}, \ldots, \varepsilon^{(r-1)}),
\]
\[
\varphi_{0, r} = \varepsilon, N_{0, r} = |\varepsilon|, \Phi_{0, r} = \varphi_{0, r}/N_{0, r} = \text{sign}, \Phi_{i, r} = \varepsilon^{(i)} + \beta_i N_{i-1, r}^{(r-i+1)} \Phi_{i-1, r},
\]
\[
N_{i, r} = \left|\varepsilon^{(i)}\right| + \beta_i N_{i-1, r}^{(r-i+1)}, \Phi_{i, r}/N_{i, r}, i = 1, \ldots, r - 1, \tag{10}
\]
where \(\beta_1, \ldots, \beta_{r-1}\), are positive design parameters.

Lemma 1 [20]: Let \(i = 0, \ldots, r - 1\). From (10), \(N_{i, r}\) is positive definite except the point \(\varepsilon = \dot{\varepsilon} = \ldots = \varepsilon^{(r-1)} = 0\) and define \(N_{i, r} = 0\) when \(\varepsilon = \dot{\varepsilon} = \ldots = \varepsilon^{(i)} = 0, |\Phi_{i, r}| \leq 1\) holds when \(N_{i, r} > 0\). Note that the function \(\Phi_{i, r}(\varepsilon, \dot{\varepsilon}, \ldots, \varepsilon^{(r-1)})\) is continuous except the point \(\varepsilon = \dot{\varepsilon} = \ldots = \varepsilon^{(r-1)} = 0\). Also, it can be redefined by continuity.

Lemma 2 [20]: The controller
\[
u = -\alpha \Phi_{r-1, r}(\varepsilon, \dot{\varepsilon}, \ldots, \varepsilon^{(r-1)}) \tag{11}
\]
is capable of guaranteeing the finite time stability of (9) and (11) by choosing the parameters \(\beta_1, \ldots, \beta_{r-1}, \alpha\) in (10) sufficiently large, and for the system (6), (11), the finite time stable \(r\)-sliding mode \(\varepsilon = \dot{\varepsilon} = \ldots = \varepsilon^{(r-1)} = 0\) is established.

Our control purpose is to design the control law \(u\) for system (1) such that all signals of the system are uniformly bounded and the tracking error between the output signal \(y\) and the reference signal \(y_r\) converges to zero, where \(y_r\) and its derivatives at least up to the order \(n\) are assumed to be continuous and bounded.

III. HOSM BACKSTEPPING CONTROLLER DESIGN
In this section, the high order sliding mode controllers [20] are incorporated into the backstepping technique to develop a robust exact tracking control design. The recursive design contains \(n\) steps. At each step, we construct the novel integrators to form the control signal. The virtual control signals are given in the first \(n - 1\) steps and the actual control signal \(u\) is given in the last design step.
**Step 1:** Define the system tracking error

\[ \varepsilon_1 = x_1 - y_r, \]  

Then denote \( v_{1,1} \) as the desired signal of \( x_2 \), which is designed as the virtual controller for the first step and given through \( n - 1 \) integrators as follows:

\[ \dot{v}_{1,1} = v_{1,2}, \]

\[ \vdots \]

\[ \dot{v}_{1,n-1} = -k_{1,1}v_{1,1} - \cdots - k_{1,n-1}v_{1,n-1} - \alpha_1 \Phi_{n-1,n}(\varepsilon_1, \dot{\varepsilon}_1, \ldots, \varepsilon_1^{(n-1)}), \]

where \( \alpha_1 \) is a positive design parameter, \( \Phi_{n-1,n}(\cdot) \) is given in (10) by setting \( r = n \) and \( \varepsilon = \varepsilon_1 \). Define \( k_1 = [k_{1,1}, \ldots, k_{1,n-1}]^T \), which is chosen such that \( A_1 - b_1k_1^T \) is Hurwitz with

\[ A_1 = \begin{bmatrix} 0 & I_{n-2} \\ 0 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0_{n-2}x_1 \end{bmatrix}. \]

The signals \( \varepsilon_1, \dot{\varepsilon}_1, \ldots, \varepsilon_1^{(n-1)} \) in (13) with \( \varepsilon_1^{(n-1)} \) having a Lipschitz constant \( L (> 0) \) can be calculated by the \( n \)-th order differentiator [29]:

\[ \dot{z}_{1,0} = \omega_{10}, \]

\[ \omega_{10} = -\lambda_n \Delta \left| z_{1,0} - \varepsilon_1 \right|^{\frac{n-1}{2}} \text{sign}(z_{1,0} - \varepsilon_1) + z_{1,1}, \]

\[ \dot{z}_{1,j} = \omega_{1j}, \]

\[ \omega_{1j} = -\lambda_n \Delta \left| z_{1,j} - \omega_{1(j-1)} \right|^{\frac{n-j-1}{2}} \text{sign}(z_{1,j} - \omega_{1(j-1)}) + z_{1,j+1}, \]

\( j = 1, \ldots, n - 2 \)

\[ \dot{z}_{1,n-1} = -\lambda_1 \Delta \text{sign}(z_{1,n-1} - \omega_{1(n-2)}), \]

where the parameters \( \lambda_n > \lambda_{n-1} > \cdots > \lambda_1 > 0 \) are chosen recursively large such that the estimates \( z_{1,0}, z_{1,1}, \ldots, z_{1,n-1} \) converge to the derivatives \( \varepsilon_1, \dot{\varepsilon}_1, \ldots, \varepsilon_1^{(n-1)} \) in finite time. As a matter of fact, \( \Phi_{n-1,n}(\varepsilon_1, \dot{\varepsilon}_1, \ldots, \varepsilon_1^{(n-1)}) \) in (13) is implemented by \( \Phi_{n-1,n}(z_{1,0}, \ldots, z_{1,n-1}) \).

**Step i** (\( 2 \leq i \leq n - 1 \)): Since the desired dynamic of \( x_i \) is \( v_{i-1,1} \), the \( i \)-th error is defined by

\[ \varepsilon_i = x_i - v_{i-1,1}. \]

Similar to step 1, we denote \( v_{i,1} \) as the desired signal of \( x_{i+1} \), which is designed as the virtual control signal and given by the following \( n - i \) integrators:

\[ \dot{v}_{i,1} = v_{i,2}, \]

\[ \vdots \]

\[ \dot{v}_{i,n-i} = -k_{i,1}v_{i,1} - \cdots - k_{i,n-i}v_{i,n-i} - \alpha_i \Phi_{n-i,n-i+1}(\varepsilon_i, \dot{\varepsilon}_i, \ldots, \varepsilon_i^{(n-i)}), \]

where \( \alpha_i (> 0) \) is design parameter. The vector \( k_i := [k_{i,1}, \ldots, k_{i,n-i}]^T \) is chosen such that \( A_i - b_ik_i^T \) is Hurwitz with

\[ A_i = \begin{bmatrix} 0 & I_{n-i-1} \\ 0 & 0 \end{bmatrix}, \quad b_i = \begin{bmatrix} 0_{n-i-1}x_1 \end{bmatrix}. \]

In the same fashion, \( \Phi_{n-i,n-i+1}(\varepsilon_i, \dot{\varepsilon}_i, \ldots, \varepsilon_i^{(n-i)}) \) is actually implemented by the combined function \( \Phi_{n-i,n-i+1}(z_{i,0}, z_{i,1}, \ldots, z_{i,n-i}) \), which can be obtained by considering (10) and the differential equation below.

\[ \dot{z}_{i,0} = \omega_{i0}, \]

\[ \omega_{i0} = -\lambda_{n-i+1}L \frac{1}{n-i+1} \left| z_{i,0} - \varepsilon_i \right|^{\frac{n-i-1}{n-i+1}} \text{sign}(z_{i,0} - \varepsilon_i) + z_{i,1}, \]

\[ \dot{z}_{i,j} = \omega_{ij}, \]

\[ \omega_{ij} = -\lambda_{n-i+1-j}L \frac{1}{n-i+1-j} \left| z_{i,j} - \omega_{ij-1} \right|^{\frac{n-i-j}{n-i+1-j}} \text{sign}(z_{i,j} - \omega_{ij-1}) + z_{i,j+1}, \]

\( j = 1, \ldots, n - i - 1 \)

\[ \dot{z}_{i,n-i} = -\lambda_{1} \Delta \text{sign}(z_{i,n-i} - \omega_{i(n-i)-1}), \]

where the parameters \( \lambda_{n-i+1} > \cdots > \lambda_1 > 0 \) are chosen recursively large.

**Step n**: At the final step, the error between \( x_n \) and \( v_{n-1,1} \) is

\[ \varepsilon_n = x_n - v_{n-1,1}. \]

Then, the actual control law is given by

\[ u = -\alpha_n \Phi_{0,1}(\varepsilon_n) = -\alpha_n \text{sign}(\varepsilon_n) \]

with \( \alpha_n \) being positive design parameter.

**Remark 4**: As far as we know, it is the first time that the quasi-continuous HOSM controllers are applied to address the exact tracking problem of the nonlinear system in completely pure feedback form. Further, in the recursive design procedure, the novel integrators are constructed to be BIBS with \( \alpha_i \Phi_{n-i,n-i+1}(\varepsilon_i, \dot{\varepsilon}_i, \ldots, \varepsilon_i^{(n-i)}) \) \( (i = 1, 2, \ldots, n - 1) \) as the input, whose purpose is twofold: to guarantee all signals of the integrators bounded, and to make the tracking error of every design step converge to zero in finite time. And for the step \( i \), it is the \( n - i \) integrators that are able to keep the \( n - i \) times differentiability of the \( v_{i,1} \). That is to keep the relative degree of \( x_{i+1} \) in system (1).

**IV. STABILITY AND TRACKING PERFORMANCE**

This section is devoted to both stability proof and tracking performance analysis. By constructing integrators in the recursive design steps, it is shown that our proposed algorithm can ensure the finite time exact tracking of the desired trajectory and the closed loop stability. The main results are summarized as follows.

**Theorem 1**: Consider the closed loop system consisting of the plant (1), the actual control law (21), the virtual control signals given by (13), (17) and the differentiators (15) and (19) with finite time convergence. Suppose the assumptions A.1-A.3 hold. Then by properly choosing the design parameters \( \alpha_i, l_i \) \( (i=1,2,\ldots,n) \), \( L \) and \( k_i \), \( \beta_i (i = 1, 2, \ldots, n - 1) \) given in (10), the tracking error between the system output signal and the reference signal can converge to zero in finite time meanwhile the boundedness of all the signals of the closed loop system can be achieved.
Proof:

a. Convergence of the tracking error

we shall prove that the system tracking error between the output signal and the desired signal can converge to zero in finite time.

• By considering (5), differentiating \( \varepsilon_n(= x_n - v_{n-1,1}) \)
yields

\[
\dot{\varepsilon}_n = g_n(\cdot)u + f_n(\tilde{x}_n, 0, t) - \dot{v}_{n-1,1}
\]

\[
= g_n(\cdot)u + h_n(\cdot),
\]

where \( h_n(\cdot) = f_n(\tilde{x}_n, 0, t) - \dot{v}_{n-1,1} \). The selection of \( k_{n-1,1} \) and the construction of the integrator (17), which is understood in the Filippov sense, imply that \( \dot{v}_{n-1,1} \) is bounded. Subsequently, from A.1-A.3, it is not difficult to conclude that the following inequalities hold for some positive constants \( g_m, g_M \) and \( D_n \).

\[
0 < g_m \leq g_n(\cdot) \leq g_M, \quad |h_n(\cdot)| \leq D_n
\]

(23)

hold for some positive constants \( g_m, g_M \) and \( D_n \). Following the similar line as the quasi-continuous controller design in the [20], (22) and (23) imply the following closed differential inclusion

\[
\dot{\varepsilon}_n \in [g_m, g_M]u + [-D_n, D_n].
\]

(24)

Obviously, \( \dot{\varepsilon}_n \leq \alpha_n g_M + D_n \leq L. \) According to the conclusion of [20], the controller (21) was proved to guarantee the finite time stability of (24) and 1-sliding mode \( \varepsilon_0 = 0 \) is finite time stable.

• In view of (5), (17) and (20), the time derivative \( \varepsilon_{n-1}(= x_n - v_{n-2,1}) \) is

\[
\dot{\varepsilon}_{n-1} = g_{n-1}(\cdot)(\varepsilon_n + v_{n-1,1}) + f_{n-1}(\tilde{x}_{n-1}, 0, t)
\]

\[-v_{n-2,2}.
\]

(25)

From (17) and (19), differentiating (25) gives

\[
\dot{\varepsilon}_{n-1} = g_{n-1}(\cdot)\left(-\alpha_{n-1}\Phi_{1,2}(\varepsilon_{n-1,0}, \varepsilon_{n-1,1}) \right)
\]

\[-k_{n-1,1}g_{n-1}(\cdot)v_{n-1,1} + \dot{g}_{n-1}(\cdot)v_{n-1,1} + g_{n-1}(\cdot)\varepsilon_n + f_{n-1}(\tilde{x}_{n-1}, 0, t) - \dot{v}_{n-2,2}.
\]

\[
= g_{n-1}(\cdot)v_{n-1}^\prime + h_{n-1}(\cdot).
\]

(26)

where

\[
\dot{v}_{n-1} = -\alpha_{n-1}\Phi_{1,2}(\varepsilon_{n-1,0}, \varepsilon_{n-1,1}),
\]

\[
\dot{h}_{n-1}(\cdot) = -k_{n-1,1}g_{n-1}(\cdot)v_{n-1,1} + \dot{g}_{n-1}(\cdot)v_{n-1,1} + g_{n-1}(\cdot)\varepsilon_n + f_{n-1}(\tilde{x}_{n-1}, 0, t) - \dot{v}_{n-2,2}.
\]

(27)

Then define

\[
\dot{v}_{n-1} = -\alpha_{n-1}\Phi_{1,2}(\varepsilon_{n-1,0}, \dot{\varepsilon}_{n-1}),
\]

\[
\dot{h}_{n-1}(\cdot) = \dot{h}_{n-1}(\cdot) + g_{n-1}(\dot{v}_{n-1} - v_{n-1}).
\]

(28)

(26) can be rewritten as

\[
\varepsilon_{n-1} = g_{n-1}(\cdot)v_{n-1}^\prime + g_{n-1}(\dot{v}_{n-1} - v_{n-1}).
\]

(29)

Moreover, the vector \( k_i := [k_{i,1}, \ldots, k_{i,n-1}]^T, \ i = 1, 2, \ldots, n - 1 \) are chosen such that \( A_t - b_i k_i^T \) are Hurwitz. Thus the system (17) is BIBS, which together with the boundedness of \( \Phi_{n-1-i,1}(\cdot) \), the boundedness of \( v_{n-1,1} \), \( \dot{v}_{n-2,2} \) can be inferred and obviously, \( \dot{v}_{n-1} \) and \( v_{n-1} \) are bounded. Then, based on the previous step analysis, \( \varepsilon_n \) and \( \dot{\varepsilon}_n \) are also bounded. Viewing A.1-A.3, it is readily conclude that \( h_{n-1}(\cdot) \) is bounded. Consequently, there exist positive constants \( g_m, g_M \) and \( D_n-1 \) such that

\[
0 < g_m \leq g_{n-1}(\cdot) \leq g_M, \quad |h_{n-1}(\cdot)| \leq D_n-1.
\]

and we have

\[
\bar{\varepsilon}_{n-1} \in [g_m, g_M]v_{n-1} + [-D_n-1, D_n-1].
\]

(31)

which implies that \( \bar{\varepsilon}_{n-1} \leq \alpha_n g_M + D_n-1 \leq L. \) From Lemma 2, the finite time stable 2-sliding mode \( \varepsilon_{n-1} = \dot{\varepsilon}_{n-1} = 0 \) is established. In the same fashion, step by step, it can be checked that the finite time stable \((n + 1 - i)\)-sliding mode \( \varepsilon_i = \dot{\varepsilon}_i = \cdots = \varepsilon_{(n-i+1)} = 0 \) \((i = 2, 3, \ldots, n - 2)\) is established and \( \varepsilon_{(n-i+1)} \leq \alpha_i g_M + D_i \leq L. \)

• At last, from (5), (16) and (17), the time derivative of the tracking error \( x_1 - y_r \) is expressed as

\[
\dot{\varepsilon}_1 = g_1(\cdot)(\varepsilon_2 + v_{1,1}) + f_1(\tilde{x}_1, 0, t) - \dot{y}_r.
\]

(32)

Furthermore, from (13), the nth time derivative of \( \varepsilon_i \) is computed as

\[
\varepsilon_i^{(n)} = g_i(\cdot)v_i^{(n)} + h_i(\cdot),
\]

(34)

where

\[
v_i = -\alpha_i\Phi_{1,1}(\varepsilon_1, \varepsilon_1),
\]

\[
\dot{v}_1 = -\alpha_i\Phi_{1,1}(\varepsilon_1, \dot{\varepsilon}_1),
\]

\[
h_1(\cdot) = (g_1(\cdot)e_2^{(n-1)} + g_1(\cdot)v_{1,1} - k_{1,1}v_{1,1} - \cdots - k_{1,n-1}v_{1,1} + \dot{g}_1(\cdot)v_{1,1} + \cdots + g_1(\cdot)v_{1,1} + f_1(\cdot)(\tilde{x}_1, 0, t) - \varepsilon_1^{(n-1)} + g_1(\cdot)v_i^{(n)} - \varepsilon_1^{(n-1)}).
\]

(35)

Similar to (30), for some positive constants \( g_m, g_M \) and \( D_1 \), we have

\[
0 < g_m \leq g_1(\cdot) \leq g_M, \quad |h_1(\cdot)| \leq D_1.
\]

(36)

Consequently, the following differential inclusion holds

\[
\varepsilon_i^{(n)} \in [g_m, g_M]v_i^{(n)} + [-D_1, D_1].
\]

(37)
In virtue of Lemma 2, the controller $v_i'$ in (35) can guarantee the finite time stability of (37) and provides for the finite time convergence of each trajectory to the $n$-sliding mode $e_1 = \dot{e}_1 = \cdots = e_i^{(n-1)} = 0$. Moreover, it is clear that $e_i^{(n)} \leq \alpha_1 g_{BM} + D_1 \leq L$, hence, the convergence of the tracking error to zero in finite time is verified.

b. The closed loop stability

We shall prove that all signals of the closed loop system are bounded.

From the above proof of the convergence of tracking error, it is easily checked that the errors $e_i$, $i = 1, \ldots, n$, are bounded, which follows that $x_1$ is bounded due to the boundedness of $e_1$ and $y_r$. Then in view of the BIBS systems (13) and (17), the boundedness of $v_{i-1,1}$, $i = 2, \ldots, n$ can be achieved. Therefore, $x_i(x = e_i + v_{i-1,1})$, $i = 2, \ldots, n$, are bounded and all signals of the close-loop system are uniformly bounded.

Remark 5: Different from [21] and [30], the errors $e_i, i = 2, \ldots, n$ between $x_i$ and $v_{i-1,1}$ in the transient process are considered in our proof, see (25) and (32). The errors between $\Phi_{n-1,n-i+1}(e_i, \xi_i, \ldots, e_i^{(n-i)})$ and $\Phi_{n-1,n-i+1}(\xi_0, \xi_1, \ldots, \xi_{n-1})$ are also analyzed, such as (26)-(29). Then the closed loop stability involving the HOSM controller is solved in our paper by constructing the novel integrators (13) and (17) without the BIBS assumption in [21]. In contrast to [30], it is unnecessary to employ the Lyapunov theoretical analysis and the proof of the closed loop stability is quite simple.

Remark 6: It is worth noting that the tracking error can converge to zero in finite time by utilizing our proposed scheme, while the existing methods dealing with pure feedback nonlinear systems only steer the error to a small neighborhood of zero in [5]-[9], [17]-[19] and [33], [34].

V. CHATTERING PHENOMENON

Apparently, same to the standard sliding mode control, the actual control signal designed in section 3 is discontinuous and subjected to the dangerous chattering effect. This problem is discussed in this section. Actually, we can increase the order of the high order sliding mode in the controller design to avoid discontinuities of the actual control. The specific method will be briefly introduced as follows:

By increasing the order of the system (1), the system (1) can be seen as

\[ \dot{x}_i = f_i(\dot{x}_i, x_{i+1}, t), 1 \leq i \leq n - 1, \]
\[ \dot{x}_n = f_n(x_n, u, t), \]
\[ \dot{u} = u_1, \]
\[ \dot{u}_1 = u_2, \]
\[ \vdots \]
\[ \dot{u}_{p-1} = u_p, \]
\[ y = x_1, \] (38)

where the positive integer $p$ is arbitrary. $u_p$ can be seen as control input. Then the relative degree of the system becomes $n + p$. By applying the mean-value theorem, (38) is transformed into

\[ \dot{x}_i = g_i(x_{i+1} + f_i(\dot{x}_i, 0, t), 1 \leq i \leq n - 1, \]
\[ \dot{x}_n = g_n(u + f_n(x_n, 0, t), \]
\[ \dot{u} = u_1, \]
\[ \dot{u}_1 = u_2, \]
\[ \vdots \]
\[ \dot{u}_{p-1} = u_p, \]
\[ y = x_1, \] (39)

which satisfies the assumption that the first $n - i + p$ time derivatives of $g_i(\cdot)$ and the $(n - i + p)$th time derivative of $f_i(\dot{x}_i, 0, t)$ are bounded.

Following the design procedure in section 3, the control law for the step $i (1 \leq i \leq n)$ is developed by $n - i + p$ integrators, where the order of the high order sliding mode increases to $n - i + p$. Consequently, $u$ is generated by the $p$ integrators and we can get a continuous control signal. Thus the chattering phenomenon can be effectively mitigated, which will be illustrated in the simulation results. Moreover, the stability and the tracking performance analysis is similar to that in section 4 and is omitted here.

VI. ILLUSTRATIVE EXAMPLES

In this section, two examples are presented to illustrate the effectiveness of the proposed control method.

Example 1 (Numerical Example): We consider the following third order system described by

\[ \dot{x}_1 = x_2, \]
\[ \dot{x}_2 = x_3 + 0.5 \sin x_3 + 0.2 \sin 2t, \]
\[ \dot{x}_3 = u + 0.1 \sin u + \frac{1 - e^{-x_2}}{1 + e^{-x_2}}, \]
\[ y = x_1. \] (40)

The initial condition of the system is $(x_1(0), x_2(0), x_3(0)) = (0.5, 1, 2)$. Our control objective is to design a HOSM backstepping controller such that the system output signal $y$ tracks the desired signal $y_r = (2 \sin 0.5t)$ with guaranteeing the closed loop stability. It is not difficult to check that (40) satisfies A.1-A.3. According to Section 3, the design procedure is shown below.

Step 1: The tracking error is $e_1 = y - y_r$ and the virtual control signal $v_{1,1}$ is designed by

\[ \dot{v}_{1,1} = v_{1,2}, \]
\[ \dot{v}_{1,2} = -k_{1,1} v_{1,1} - k_{1,2} v_{1,2} - \alpha_1 \Phi_{2,3}(e_1, \dot{e}_1, \ddot{e}_1). \] (41)

$k_{1,1}, k_{1,2}, \alpha_1$ are chosen as 2, 3, 6, respectively and

\[ \Phi_{2,3}(e_1, \dot{e}_1, \ddot{e}_1) \]
\[ = \frac{\ddot{e}_1 + 2(\ddot{e}_1 \dddot{e}_1 + |e_1| \dddot{e}_1 \frac{2}{3} \text{sign}(e_1))}{|\ddot{e}_1| + 2(|\dddot{e}_1| + |e_1| \dddot{e}_1 \frac{2}{3} \text{sign}(e_1))}. \] (42)
Then by (15), $\varepsilon_1, \dot{\varepsilon}_1, \ddot{\varepsilon}_1$ can be substituted by $z_{1,0}, z_{1,1}, z_{1,2}$, which are given by the following third order differentiator with finite-time convergence

$$
\dot{z}_{1,0} = \omega_{10} \\
\omega_{10} = -2L^{\frac{1}{2}} |z_{1,0} - \varepsilon_1|^{\frac{3}{2}} \text{sign}(z_{1,0} - \varepsilon_1) + z_{1,1} \\
\dot{z}_{1,1} = \omega_{11} \\
\omega_{11} = -1.5L^{\frac{1}{2}} |z_{1,1} - \omega_{10}|^{\frac{1}{2}} \text{sign}(z_{1,1} - \omega_{10}) + z_{1,2} \\
\dot{z}_{1,2} = -1.1L\text{sign}(z_{1,2} - \omega_{11}), \quad L = 400. \quad (43)
$$

Step 2: Define the second error as $\varepsilon_2 = x_2 - v_{1,1}$. The virtual control signal $v_{2,1}$ for this step is designed as

$$
\dot{v}_{2,1} = -k_{2,1}v_{2,1} - \alpha_2\Phi_{1,2}(\varepsilon_2, \dot{\varepsilon}_2). \quad (44)
$$

$k_{2,1}, \alpha_2$ are chosen as 1, 10, respectively and

$$
\Phi_{1,2}(\varepsilon_2, \dot{\varepsilon}_2) = \frac{\dot{\varepsilon}_2 + |\varepsilon_2|^{\frac{1}{2}} \text{sign}(\varepsilon_2)}{\dot{\varepsilon}_2 + |\varepsilon_2|^{\frac{1}{2}}}. \quad (45)
$$

In the same fashion, $\varepsilon_2, \dot{\varepsilon}_2$ can be substituted by $z_{2,0}, z_{2,1}, z_{2,2}$, which are obtained by

$$
\dot{z}_{2,0} = \omega_{20} \\
\omega_{20} = -1.5L^{\frac{1}{2}} |z_{2,0} - \varepsilon_2|^{\frac{1}{2}} \text{sign}(z_{2,0} - \varepsilon_2) + z_{2,1} \\
\dot{z}_{2,1} = -1.1L\text{sign}(z_{2,1} - \omega_{20}) \quad (46)
$$

with $L = 400$. Step 3: The last error is $\varepsilon_3 = x_3 - v_{2,1})$. Then the actual control law is designed as

$$
u = -\alpha_3\text{sign}(\varepsilon_3), \quad (47)$$

where $\alpha_3$ is chosen as 5.

The simulation results are shown in Figs. 1-4, from which one can see that the output of the system tracks the desired trajectory $y_r = 2\sin 0.5t$ after a finite time and all the states of the system are bounded, which accords with Theorem 1. However, the controller in Figs. 3 presents obvious chatter phenomenon.

Furthermore, based on section 5, we can make the actual control signal continuous and eliminate the chattering effect. When $p = 1$ in (38), we provide for the following design procedure from section 3.
where $L = 1000$.

Step 2: The second error is defined as $e_2 = x_2 - v_{1,1}$, and the virtual control signal $v_{2,1}$ is designed by

$$
\begin{align*}
\dot{v}_{2,1} &= v_{2,2}, \\
\dot{v}_{2,2} &= -k_{2,1}v_{2,1} - k_{2,2}v_{2,2} - \alpha_2 \Phi_{2,3}(\varepsilon_2, \dot{\varepsilon}_2), \\
\end{align*}
$$

where $k_{2,1}$, $k_{2,2}$, $\alpha_2$ are chosen as 2, 3, 35, respectively. $\Phi_{2,3}(\varepsilon_2, \dot{\varepsilon}_2)$ is obtained by considering (42) and $z_{2,0}$. $z_{2,1}$, $z_{2,2}$ can be calculated by (43) with $L = 1000$ to estimate $\varepsilon_2, \dot{\varepsilon}_2, \ddot{\varepsilon}_2$.

Step 3: The last error is $e_3 = x_3 - v_{2,1}$, and the actual control law is given by

$$
\dot{u} = -k_3u - \alpha_3 \Phi_{1,2}(\varepsilon_3, \dot{\varepsilon}_3),
$$

where $k_3$, $\alpha_3$ are chosen as 1, 50, respectively. In view of (45), $\Phi_{1,2}(\varepsilon_3, \dot{\varepsilon}_3)$ is obtained and $z_{3,0}$, $z_{3,1}$ substituting for $\varepsilon_3, \dot{\varepsilon}_3$ can be calculated by (46) with $L = 1200$.

The simulation results are shown in Figs. 5-6, from which it is clear that the chattering phenomenon of the control input can be eliminated effectively and the control signal is continuous. However, the price we paid for is that the transition time became longer though we increased some parameters. Therefore, the tradeoff between improving the tracking performance of transition process and mitigating chattering effect must be made in the engineering applications.

Example 2 (Application Example): Applied example [34], we utilize our designed controller to control the Van der Pol oscillator described by

$$
\begin{align*}
\dot{x}_1 &= x_2 + d_1(t), \\
\dot{x}_2 &= -x_1 + x_2 + u + (x_1^2 + x_2^2)(\frac{1 - e^{-u}}{1 + e^{-u}}) \\
&- x_1^2 x_2 + d_2(t),
\end{align*}
$$

where $d_1(t)(= 0.2 \sin t)$ and $d_2(t)(= 0.1 \sin t)$ are disturbance. The pure feedback system described by (53) can represent various kinds of electrical circuits, which are usually composed by resistors, a capacitor, inductance coils and a triode with two DC power sources. The control objective is to design controller proposed in this paper such that the output signal $y$ tracks the reference signal $y_r$ described by $y_r = \sin t + 0.2$ while guaranteeing the closed loop stability. It is not difficult to verify that (53) satisfies A.1-A.3 at least locally. Then, following the same procedure as that in Example 1, the control signal $u$ can be obtained. In the simulation, the system initial conditions $(x_1(0), x_2(0)) = (0.5, 2)$. The design parameters $\alpha_1$, $k_{1,1}$, $k_{1,2}$, $\alpha_2$, $k_{2,1}$ and $L$ are chosen as 6, 2, 10, 1 and 400 respectively. $\Phi_{2,3}(\cdot)$ and $\Phi_{1,2}(\cdot)$ are employed in the light of (42)-(43) and (45)-(46).

The simulation results are shown in Figs. 7-10, from which it can be seen that by using our proposed control scheme, satisfactory tracking performance can be achieved and the closed loop stability can be guaranteed. It is clear that the tracking error can converge to zero in finite time.
By comparison, for the existing schemes dealing with pure feedback nonlinear systems [5]–[9], [18], [19] and [34], the tracking error can not converge to zero and only converges to a neighborhood of zero and the advanced sliding mode controllers with chattering reduction [35], [36] can not cope with the pure feedback nonlinear systems.

VII. CONCLUSION

In this paper, a high order sliding mode backstepping control is proposed for a class of unknown pure feedback nonlinear system, which is in completely non-affine form and involves some uncertainties caused by parameter variations, internal and external disturbances. Combining with the backstepping technique and high order sliding mode controllers generated by the novel integrators, our proposed method can not only ensure the convergence of the tracking error to zero in finite time, but also guarantee the stability of the closed loop system. Simulation examples have been provided to show the correctness and effectiveness of our proposed approach.

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