Nonlocal Gate Of Quantum Network Via Cavity Quantum Electrodynamics

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We propose an experimentally feasible scheme to realize the nonlocal gate between two different quantum network nodes. With an entangled qubit (ebit) acts as a quantum channel, our scheme is resistive to actual environment noise and can get high fidelity in current cavity quantum electrodynamics (C-QED) system.

PACS numbers: 42.50.Pq, 03.67.Hk, 32.80.-t

Quantum information science aims to develop new theories and methods for processing information at the quantum level. Superposition and entanglement play the central roles in this field and make it quite different from classical case. Many interesting schemes have been proposed during the past ten years [1, 2, 3]. Most of these schemes rely on the realization of Quantum networks [4], which are considered as spatially separated nodes connected by quantum communication channels. Actually quantum communication and operation between different nodes make it possible for various of novel applications, such as distributed quantum computation [5], quantum key distribution [6], etc. As to physical implementation, the network nodes are often thought as clusters of trapped atoms and ions, which are the ideal candidates for their long-lived internal states. On the other hand, photon represents the best carriers for fast and reliable communication over long distance. Thus how to implement non-local interaction between spatially distant quantum network nodes seems an interesting question.

Earlier schemes for the realization of the interaction between two different network nodes [4] rely on the transmission of a single photon through quantum channel. Thus environment noise is unavoidably introduced. Moreover, the transmitted signals can not be amplified in quantum area because of the so-called non-cloning theorem [7]. When photon absorption or transmission error occurs, the probability of success and the fidelity will be dramatically decreased. However, if entanglement is concerned here, thing will be changed. In fact, entanglement purification [8] together with quantum repeaters [9] provides an efficient method against the environment noise.

In this paper, we consider the most simple case for future’s network quantum computation. With the help of high-quality cavities, we use a pair of entangled photons to realize the Controlled-NOT (C-NOT) operation between two spatially-separated nodes. Our scheme is probabilistic, insensitive to actual environment noise and can reach high fidelity with finite efficient detectors.

The quantum network node consists of a trapped-atom inside a high-Q cavity in our scheme. The atom has a standard “Λ” type internal states, and the relevant level diagram is shown in Fig. 1. This kind of level structure can be found in alkali atom with |0⟩ and |1⟩ denoting hyperfine manifolds of the ground state. A cavity mode $a_h$ resonantly couples the atom transition $|0⟩ \rightarrow |e⟩$ with horizontal (h) polarization. The ebits concerned here are to be polarization-entangled photons shining onto the cavity through a polarizing beam splitter (PBS) (Fig. 3). Only the $h$ component can transmit the PBS and resonantly drives the corresponding cavity mode $a_h$. The vertical (v) component is reflected by the PBS and the mirror. This kind of C-QED system has been proposed by Duan et al. [10] to realize the interaction between two single-photon pulses. Here with the help of a pair of entangled photons we use similar arrangement to implement the C-NOT operation between two distant network nodes.

Before describing the detailed model, we want to summarize the basic ideas of our scheme first. We consider two network nodes A and B with A as the control part and B as the target, and set the initial states to be $|\phi_A⟩ = \alpha |0⟩_A + \beta |1⟩_A$, $|\phi_B⟩ = a|0⟩_B + b|1⟩_B$. Generally, we assume that Alice holds system A and Bob holds system B. To implement a quantum C-NOT gate, besides one bit of classical communication in each direction, one shared ebit is also necessary [11]. Here we choose the ebit to be a polarization-entangled photon pair $|\phi⟩_{A1,B1} = \frac{1}{\sqrt{2}} (|h⟩_A |v⟩_B + |v⟩_A |h⟩_B)$. The whole scheme can be understood from the following steps.

Step (A): First, the incoming photon $A_1$ passes a local Hadamard gate and then enters into the C-QED system to realize the CPF operation between the atom and the photon. The pulse is then reflected and undergoes another Hadamard rotation. This process actually realizes the C-NOT operation with the node A as the control qubit. After

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this process the combined state of $A$, $A_1$ and $B_1$ becomes
\[
\frac{1}{\sqrt{2}}[\alpha|0\rangle_A |1\rangle_{A_1} |0\rangle_{B_1} + |\beta\rangle_{A_1} |0\rangle_A |1\rangle_{B_1} + |1\rangle_{A_1} |0\rangle_A |0\rangle_{B_1} + |0\rangle_{A_1} |1\rangle_A |1\rangle_{B_1}],
\]
where we have set $|h\rangle \rightarrow |1\rangle$ and $|v\rangle \rightarrow |0\rangle$. The photon now reaches the PBS and causes a click on one of the two
detectors $D_A$ and $D_{A_b}$. Alice then registers the corresponding result $r_a$.

Step (B): Similar to step (A), after a C-NOT operation between $B$ and $B_1$ with the photon $B_1$ as the control qubit, Bob performs another Hadamard operation on part $B_1$. For example, if $r_a = D_{A_b}$, after this step we can get the
combined state of $A$, $B$ and $B_1$ as
\[
|1\rangle_{B_1} (\alpha|0\rangle + \alpha b|00\rangle + \beta a|10\rangle + \beta b|11\rangle)_{AB} + |0\rangle_{B_1} (\alpha a|01\rangle + \alpha b|00\rangle + \beta a|10\rangle + \beta b|11\rangle)_{AB}.
\]
Bob then detects the reflected photon on $D_{B_1}$ and $D_{B_b}$ and registers the result $r_b$.

Step (C): Now Alice and Bob commute their measurement results through classical channel. Proper local rotations
(Table. 1) are then performed on nodes $A$ and $B$ respectively according to the information they have got. For example, if $r_a = D_{A_b}$, and $r_b = D_{B_b}$, after local operations $\sigma_{zA}$ and $\sigma_{xB}$, they can get the desired state $\alpha a^\dagger |0\rangle_A |0\rangle_B + \alpha b |0\rangle_A |1\rangle_B + \beta a |1\rangle_A |1\rangle_B + \beta b |1\rangle_A |0\rangle_B$.

In the ideal case, the detectors always register a click. However, perfect single-photon detectors are unreachable
with current technology. When either one of Alice or Bob doesn’t get a click, they have to reset the whole system and
repeat these steps. Fig. 2 depicts a general quantum circuit to perform the non-local gate, where we have decomposed
the C-NOT gate into a controlled-phase gate plus two Hadamard rotations. A possible experimental setup can be
found in Fig. 3.

Table. 1
Fig. 1
Fig. 2
Fig. 3

We now turn to a theoretical description [12, 13, 14, 15]. The crucial parts of the whole setup are the C-QED
systems which supply ideal platforms to realize the strong coupling between photons and atoms. Since the photon
will be detected and discarded, here we concentrate on the atom inside the cavity. The relevant Hamiltonian, including
the coupling of the external fields [16], can be expressed as ($\hbar = 1$) [17]
\[
H = \omega_0 |0\rangle\langle 0| + \omega_1 |1\rangle\langle 1| + \omega_e |e\rangle\langle e| + \omega_h a_1^\dagger a_h + g(|e\rangle\langle 0|a_h + a_1^\dagger |0\rangle \langle e|) + i \int d\omega \kappa(\omega) [b(\omega) a_h - a_1^\dagger b(\omega)] + \int d\omega \omega b(\omega) a_h(\omega),
\]
where $b(\omega)$ denotes the one-dimensional free-space mode and has a standard commutation relation $[b(\omega), b(\omega')^\dagger] = \delta(\omega - \omega')$. Since only fields with the carrier frequency close to $\omega_h$ contribute mostly to the cavity mode, we take $\kappa(\omega) = (\frac{\gamma}{\pi})^\frac{1}{2}$, where $\gamma$ is the effective cavity decay rate. To describe the effect of the spontaneous emission, we introduce the Lindblad operator $L = \sqrt{\gamma} |0\rangle \langle e|$, where $\gamma$ denotes the decay rate of the excited state $|e\rangle$ to the
ground state $|0\rangle$. We also neglect the spontaneous loss of the upper level to other states.

The Heisenberg equations of the system can be described by the following form in rotating frame
\[
\partial_t \sigma_{1} = 0, \quad (4)
\partial_t \sigma_{0} = \gamma_1 \sigma_{ee} - ig(\sigma_{1e}^\dagger \sigma_{0e} - \sigma_{eo} \sigma_{1e}), \quad (5)
\partial_t \sigma_{10} = -ig \sigma_{1e}^\dagger \sigma_{1e}, \quad (6)
\partial_t \sigma_{0e} = \frac{\gamma_2}{2} \sigma_{0e} - ig(\sigma_{0e} - \sigma_{ee}) \sigma_{1e}, \quad (7)
\partial_t \sigma_{ee} = -\gamma_2 \sigma_{ee} + ig(\sigma_{0e}^\dagger \sigma_{0e} - \sigma_{ee}), \quad (8)
\]
where we have set the detune $\Delta = \omega_h - \omega_{e0} = 0$, $\sigma_{1e} = a_h e^{i\omega_{ij} t}$, and $\sigma_{ij} = \sigma_{ij} = |j\rangle \langle i| e^{-i\omega_{ij} t}$ with $\omega_{ij} = \omega_i - \omega_j$. The influence of the external fields can be taken into account by the input-output relation
\[
\partial_t \bar{a}_h = -ig \bar{a}_{0e} - \frac{\gamma}{2} \bar{a}_h - \sqrt{\gamma} b_{in}(t), \quad (10)
\]
where $\hat{b}_{in}(t)$ is field operator corresponding the input flux of photons with $[\hat{b}_{in}(t), \hat{b}_{in}^\dagger(t')] = \delta(t-t')$. The cavity output $\hat{b}_{out}(t)$ (Fig. 3) is connected with the input by $\hat{b}_{out}(t) = \hat{b}_{in}(t) + \sqrt{\gamma}\hat{a}_{b}(t)$. In the above equations, we have also omitted the terms concerning to the Langevin noise, which has negligible contribution to the dynamics.

To see the effect of the input pulse clearly, we introduce the Fourier component of the operator as $\hat{A}(\omega) = \int \frac{d\omega}{\sqrt{2\pi}} \hat{A}(t)e^{i\omega t}$. From Eq. (10) and Eq. (8), we get

$$\hat{a}_h(\omega) = \frac{g\hat{\sigma}_{0e}(\omega) - i\sqrt{\gamma}\hat{b}_{in}(\omega)}{\omega + i\frac{\gamma}{2}}. \quad (11)$$

$$\hat{\sigma}_{0e}(\omega) = \frac{g}{\omega + i\frac{\gamma}{2}} \int \frac{d\omega'}{\sqrt{2\pi}} \hat{\sigma}_z(\omega - \omega')\hat{a}_h(\omega'), \quad (12)$$

where $\hat{\sigma}_z = \hat{\sigma}_{00} - \hat{\sigma}_{ee}$.

We have assumed that the transition between $|0\rangle$ and $|e\rangle$ is a closed optical transition, so the projector operator $\hat{P}_z = \hat{\sigma}_{00} + \hat{\sigma}_{ee}$ is independent of time. From Eq. (5) and Eq. (9), we find

$$\hat{\sigma}_z(\omega) = \frac{i\gamma_s}{\omega + i\gamma_s} \hat{P}_z|_{t=0}\delta(\omega) + \frac{2g}{\omega + i\gamma_s} \int \frac{d\omega'}{\sqrt{2\pi}} [\hat{a}_h^\dagger(\omega')\hat{\sigma}_{0e}(\omega - \omega') - h.c]. \quad (13)$$

In our scheme the input field is considered to be a single-photon pulse, so the light intensity is so low that at most one photon is inside the cavity at a time. Inserting Eq. (13) and Eq. (12) into Eq. (11), and omitting the terms involving more than one $\hat{a}_h(\omega)$ operator, we obtain

$$\hat{a}_h(\omega) = \frac{-i\sqrt{\gamma}\hat{b}_{in}(\omega)}{\omega + i\frac{\gamma}{2}} - \frac{i\gamma_s|\varphi|^2\hat{P}_z|_{t=0}}{(\omega + i\gamma_s)(\omega + i\frac{\gamma}{2})}, \quad (14)$$

$$\hat{b}_{out}(\omega) = \frac{\omega - i\frac{\gamma}{2} - \frac{i\gamma_s|\varphi|^2\hat{P}_z|_{t=0}}{(\omega + i\gamma_s)(\omega + i\frac{\gamma}{2})}}{\omega + i\frac{\gamma}{2}} \hat{b}_{in}(\omega). \quad (15)$$

Now we can see clearly how the CPF gate works. When the input photon is $v$ polarized, it is reflected by the PBS and the mirror without any change. In this situation, we get $\hat{b}_{out}(\omega) = \hat{b}_{in}(\omega)$. Otherwise it transmits the PBS and enters into the cavity. For ideal input fields, $(\gamma, \gamma_s) \gg \omega$, we get

$$\hat{b}_{out} = \frac{-1 + \frac{4\varphi^2}{\gamma_s^2} \hat{P}_z|_{t=0}}{1 + \frac{4\varphi^2}{\gamma_s^2} \hat{P}_z}|_{t=0} \hat{b}_{in}. \quad (16)$$

If the atom inside the cavity is in the state $|1\rangle$, which means $\hat{P}_z|_{t=0} = 0$, we have $\hat{b}_{out} = -\hat{b}_{in}$. However, if $\hat{P}_z|_{t=0} = 1$ and $|\varphi|^2 \gg 1$, we obtain the approximate expression as $\hat{b}_{out} = \hat{b}_{in}$.

So far we have realized the CPF gate $e^{-i\pi|1\rangle\langle 1|\otimes |h\rangle\langle h|} \hat{b}_{out}$ between a trapped-atom and a single-photon pulse. Combining this kind of two-body interaction with proper local operations, the desired non-local gate can be realized. In fact, general local operations are easy to perform for photons with the help of wave-plates and beam splitters. As to this kind of “Λ” type atoms, single-qubit rotation [18, 19] can be achieved by coupling $|1\rangle$ and $|0\rangle$ via Raman transition. Since only the two lower levels are concerned, the decoherence caused by the level $|e\rangle$ can be strongly suppressed.

The above-mentioned CPF gate can also be generalized to multiple-atom case. In this situation, $\hat{P}_z|_{t=0}$ represents the total number of the atom in the state $|0\rangle$. When $\frac{|\varphi|^2}{\gamma_s} \hat{P}_z|_{t=0} \gg 1$, we get the CPF gate between multiple-atom and the single-photon pulse.

In order to estimate the influence of the external fields on the atom inside the cavity, we consider the density matrix elements explicitly. The diagonal elements are unaffected when we get a click on the detectors (the transition $|0\rangle \rightarrow |e\rangle$ is optical closed). The off-diagonal elements, however, decay due to spontaneous emission from the upper level $|e\rangle$ to $|0\rangle$, we can roughly estimate the effect from Eq. (6) and Eq. (7).

Integrate Eq. (7) and substitute the expression into Eq. (6) by introducing the Fourier transformation of cavity mode $\hat{a}_h(t) = \int \frac{d\omega}{\sqrt{2\pi}} \hat{a}_h(\omega)e^{-i\omega t}$, we find
\[
\partial_t \tilde{\sigma}_{10} = -|g|^2 \int_{t_0}^t dt_1 \int \frac{d\omega_1 d\omega_2}{2\pi} e^{i(\omega_2 - \omega_1)t_1} \tilde{a}_h^\dagger(\omega_1)\tilde{\sigma}_{10}(t_1)\tilde{a}_h(\omega_2).
\] (17)

If we assume that the coherence between the two lower levels doesn't change much on time scale \(1/\gamma_s\), we can get the following relation (by setting \(t_0 \to -\infty\))

\[
\partial_h \tilde{\sigma}_{10} = -\int \frac{d\omega_1 d\omega_2}{2\pi} \frac{|g|^2}{\gamma_s - i\omega_2} \tilde{a}_h^\dagger(\omega_1)\tilde{\sigma}_{10}(t)e^{i(\omega_1 - \omega_2)t}\tilde{a}_h(\omega_2).
\] (18)

Integrating this equation we obtain the formal solution of \(\tilde{\sigma}_{10}\) as

\[
\tilde{\sigma}_{10}(\infty) = \tilde{\sigma}_{10}(t_0)e^{-\int \frac{d\omega}{\gamma_s} \tilde{a}_h^\dagger(\omega)\tilde{a}_h(\omega)}.
\] (19)

where \(\cdot\) denotes the normal ordering of the corresponding operators.

Eq. (19) illustrates the influence of the input pulses, and from which we can roughly estimate the quality of the non-local gate. If we assume that the bandwidth of the input fields is narrow and symmetrical around the resonant frequency \(\omega_h\), we can obtain

\[
\tilde{\sigma}_{10}(\infty) = \tilde{\sigma}_{10}(t_0)(1 - \frac{8|g|^2}{\gamma_s(1 + 4|g|^2P_\Delta(0))}).
\] (20)

The fidelity of the non-local gate can be approximately expressed as

\[
F = 1 - 2(|ab|^2 \Delta_B + |a\beta|^2 \Delta_A),
\] (21)

\[
\Delta_{A(B)} = \frac{8G_{A(B)}}{(1 + 4G_{A(B)}\Delta_B(0))}.
\] (22)

where \(G = \frac{|g|^2}{\gamma_s}\), and the subscript \(A(B)\) means that the value is obtained according to the parameters of system \(A(B)\). If \(|a|^2\) and \(|\alpha|^2\) are large enough [20] and if the two cavity systems \(A\) and \(B\) have the same parameters, we have \(1 - F \sim G^{-1} = \frac{\gamma_s}{|g|^2}\). For current technology level, the parameter \(G\) can reach 100 [21]. So we can get \(F \sim 0.99\) in this situation. Eq. (21) indicates that the fidelity of our scheme depends only on the magnitude of coupling factor \(g\), and not on the phase. So it is not necessary to confine the atom within a wavelength of the field. The factor \(g\) also suffers significant random variation in current experiment due to residual atomic motion. However, from the numerical simulation of reference [10], we expect that it is also not necessary to fix the atom inside the cavity.

The major source of noise comes from the dark counts of the single-photon detectors, which can produce false positive results. Fortunately, if there is no photon loss, the dark counts will not affect the measurement results of \(A\) and \(B\), or can be definitely distinguished when both of the two detectors get a click on one side. This effect can be considered as a shrinking factor \(1 - 2p_lp_{dc}\) connected to the fidelity of the final state, where \(p_l\) is the probability of photon loss (the inefficient of single-photon detectors can be included here as a modified photon loss rate) and \(p_{dc}\) describes the dark count rate. The input pulse may be reflected because of the mismatch between the incident field and the cavity mode or other imperfections. If we introduce \(f\) to describe these imperfections, the total fidelity will be reduced by \(\left[\frac{1 - (1-f)R}{1 + (1-f)R}\right]^2\), where we assumed that the two cavity systems have the same reflection coefficient \(R\) (for resonant case, \(R = \frac{1 - 4|g|^2P_\Delta}{1 + 4|g|^2P_\Delta}\)). All these effects cause the fidelity reduced by \(\left[\frac{1 - (1-f)R}{1 + (1-f)R}\right]^{2N}\). And the success probability can also be estimated as \((1 - p_l)(1 - p_l - 2p_{dc})\).

The whole scheme can be extended to multi-party case, where more cavity systems and entangled pairs should be contained here. If a quantum network computation contains \(N\) times such kind of C-NOT operation, the total shrinking factor which is connected to the fidelity will be \(\left[\frac{1 - (1-f)R}{1 + (1-f)R}\right]^{2N}\) (we assumed the dark counts rate is small enough), and the corresponding success probability is \((1 - p_l)^N(1 - p_l - 2p_{dc})^N\).

It should be pointed out that we have supposed the bandwidth of the entangled pulses is narrow compared with the linewidth of the cavity and the atom. Such kind of entangled source can't be obtained with spontaneous parametric
down-conversion (SPDC). However, Gheri et al. [22] have proposed an inspiring scheme to generate entangled pairs with the assistance of optical cavity. With this scheme the required bandwidth could be achieved.

In summary, we have proposed a very simple scheme for future’s network quantum computation. With the C-QED system and entanglement, we have realized the non-local gate between two different quantum network nodes. Compared with the schemes proposed before, which often use photon as flying qubit to transmit interaction between two separate nodes, our method contains entangled qubit as quantum channel, so environment noise can be suppressed. Moreover, the entanglement between the two nodes can be implemented and stored before the whole process. For example, we can store the entanglement with atomic ensemble [23], and then by a weak pulse, the stored entanglement can be released to complete the desired non-local operation.

We thank M. Paternostro and A. Beige [24] for drawing our attention to their related works. This work was funded by the National Fundamental Research Program (2001CB309300), the National Natural Science Foundation of China (10304017) and the Innovation Funds from the Chinese Academy of Sciences.

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Figure captions:
Fig. 1. Energy levels of the atom inside the cavity. An cavity mode $a_h$ resonantly drives the transition $|e\rangle \rightarrow |0\rangle$ with coupling strength $g$, and $\gamma_e$ describes the corresponding decay rate.
Fig. 2. Logical circuit to implement the non-local gate with A as the control qubit and B as the target. Classical communication is represented by the dashed line, and suitable local rotations can be found in Table. 1.
Fig. 3. Experimental setup of the whole scheme. The produced entangled photon pairs enter the CPF system and reach the detectors to cause clicks, then classical communication is performed to complete the non-local gate. The half wave plates (HWP) used here are to perform the local Hadamard rotation of photons, and local operation of the atom can be realized with the help of two classical fields.

Table captions:
Table. 1. Local operation for the corresponding measurement result. For example, $D_{A_b}$ means the photon detected by Alice is horizontal ($h$) polarization.
Fig. 1
| $r_a$ | $r_b$ | local rotation |
|------|------|----------------|
| $D_{A}$ | $D_{B}$ | $I_A \otimes \sigma_{x_B}$ |
| $D_{A}$ | $D_{B}$ | $-\sigma_{z_A} \otimes \sigma_{x_B}$ |
| $D_{A}$ | $D_{B}$ | $I_A \otimes I_B$ |
| $D_{A}$ | $D_{B}$ | $\sigma_{z_A} \otimes I_B$ |

Table 1
Fig. 2