Measurement of $B_0^d$-$\bar{B}_0^d$ mixing rate from the time evolution of dilepton events at the $\Upsilon(4S)$

The Belle Collaboration
(26-November, 2000; revised 1-March, 2001)

K. Abe\textsuperscript{8}, K. Abe\textsuperscript{36}, I. Adachi\textsuperscript{8}, Byoung Sup Ahn\textsuperscript{14}, H. Aihara\textsuperscript{27}, M. Akatsu\textsuperscript{19}, G. Alimonti\textsuperscript{17}, K. Aoki\textsuperscript{8}, K. Asai\textsuperscript{20}, M. Asai\textsuperscript{9}, Y. Asano\textsuperscript{42}, T. Aso\textsuperscript{41}, V. Aulchenko\textsuperscript{2}, T. Asuhev\textsuperscript{12}, A. M. Bakich\textsuperscript{33}, E. Banas\textsuperscript{15}, S. Behari\textsuperscript{8}, P. K. Behera\textsuperscript{43}, D. Beilune\textsuperscript{2}, A. Bondar\textsuperscript{2}, A. Bozek\textsuperscript{15}, T. E. Browder\textsuperscript{7}, B. C. K. Casey\textsuperscript{7}, P. Chang\textsuperscript{23}, Y. Chao\textsuperscript{23}, B. G. Cheon\textsuperscript{32}, S.-K. Choi\textsuperscript{6}, Y. Choi\textsuperscript{32}, Y. Doi\textsuperscript{8}, J. Dragic\textsuperscript{17}, S. Eidelman\textsuperscript{2}, Y. Enari\textsuperscript{19}, R. Enomoto\textsuperscript{8,10}, C. W. Everton\textsuperscript{17}, F. Fang\textsuperscript{7}, H. Fujii\textsuperscript{8}, Y. Fujita\textsuperscript{8}, C. Fukunaga\textsuperscript{39}, M. Fukushima\textsuperscript{10}, A. Garmash\textsuperscript{2,8}, A. Gordon\textsuperscript{17}, K. Gotow\textsuperscript{44}, H. Guler\textsuperscript{7}, R. Guo\textsuperscript{21}, J. Haba\textsuperscript{8}, T. Haji\textsuperscript{4}, H. Hamasaki\textsuperscript{8}, K. Hanagaki\textsuperscript{29}, F. Handa\textsuperscript{36}, K. Hara\textsuperscript{27}, T. Hara\textsuperscript{27}, N. C. Hastings\textsuperscript{17}, K. Hayashi\textsuperscript{8}, H. Hayashii\textsuperscript{20}, M. Hazumi\textsuperscript{27}, E. M. Heenan\textsuperscript{17}, I. Higuchi\textsuperscript{36}, T. Higuchi\textsuperscript{37}, T. Hirai\textsuperscript{38}, H. Hirano\textsuperscript{40}, T. Hojo\textsuperscript{27}, Y. Hoshi\textsuperscript{35}, W.-S. Hou\textsuperscript{23}, S.-C. Hsu\textsuperscript{23}, H.-C. Huang\textsuperscript{23}, Y.-C. Huang\textsuperscript{21}, S. Ichizawa\textsuperscript{38}, Y. Igarashi\textsuperscript{8}, T. Iijima\textsuperscript{8}, H. Ikeda\textsuperscript{8}, K. Ikeda\textsuperscript{20}, K. Inami\textsuperscript{19}, Y. Inoue\textsuperscript{26}, A. Ishikawa\textsuperscript{19}, M. Iwai\textsuperscript{8}, G. Iwai\textsuperscript{25}, H. Iwasaki\textsuperscript{8}, Y. Iwasaki\textsuperscript{8}, D. J. Jackson\textsuperscript{27}, P. Jalocha\textsuperscript{15}, H. Kakuno\textsuperscript{38}, J. Kaneko\textsuperscript{38}, J. H. Kang\textsuperscript{45}, J. S. Kang\textsuperscript{14}, P. Kapusta\textsuperscript{15}, K. Kasami\textsuperscript{8}, N. Katayama\textsuperscript{8}, H. Kawai\textsuperscript{3}, M. Kawai\textsuperscript{8}, N. Kawamura\textsuperscript{1}, T. Kawasaki\textsuperscript{25}, S. K. Kim\textsuperscript{31}, H. J. Kim\textsuperscript{45}, Hyunwoo Kim\textsuperscript{14}, S. K. Kim\textsuperscript{31}, K. Kinoshita\textsuperscript{8}, S. Kobayashi\textsuperscript{30}, S. Koike\textsuperscript{8}, S. Koishi\textsuperscript{38}, H. Konishi\textsuperscript{40}, K. Korotushenko\textsuperscript{29}, P. Krokovny\textsuperscript{2}, R. Kulasiri\textsuperscript{5}, S. Kumakura\textsuperscript{28}, T. Kuniya\textsuperscript{30}, E. Kurihara\textsuperscript{3}, A. Kuzmin\textsuperscript{2}, Y.-J. Kwon\textsuperscript{45}, M. H. Lee\textsuperscript{8}, S. H. Lee\textsuperscript{31}, C. Leonidopoulos\textsuperscript{29}, H.-B. Li\textsuperscript{11}, R.-S. Lu\textsuperscript{23}, Y. Makida\textsuperscript{8}, A. Manabe\textsuperscript{8}, D. Marlow\textsuperscript{29}, T. Matsubara\textsuperscript{37}, T. Matsuda\textsuperscript{8}, S. Matsui\textsuperscript{19}, S. Matsumoto\textsuperscript{4}, T. Matsumoto\textsuperscript{19}, K. Miyabayashi\textsuperscript{20}, H. Miyake\textsuperscript{27}, H. Miyata\textsuperscript{25}, L. C. Moffitt\textsuperscript{17}, A. Mohapatra\textsuperscript{43}, G. R. Moloney\textsuperscript{17}, G. F. Moorhead\textsuperscript{17}, S. Mori\textsuperscript{42}, T. Mori\textsuperscript{4}, A. Murakami\textsuperscript{30}, T. Nagamine\textsuperscript{36}, Y. Nagasaka\textsuperscript{18}, Y. Nagashima\textsuperscript{27}, T. Nakada\textsuperscript{47}, E. Nakano\textsuperscript{26}, M. Nakao\textsuperscript{8}, H. Nakazawa\textsuperscript{4}, J. W. Nam\textsuperscript{32}, S. Narita\textsuperscript{36}, Z. Natkaniec\textsuperscript{15}, K. Neichi\textsuperscript{35}, S. Nishida\textsuperscript{16}, O. Nitoh\textsuperscript{40}, S. Noguchi\textsuperscript{20}, T. Nozaki\textsuperscript{8}, S. Ogawa\textsuperscript{34}, T. Ohshima\textsuperscript{19},

(to appear in Phys. Rev. Lett.)
Y. Ohshima38, T. Okabe19, T. Okazaki20, S. Okuno13, S. L. Olsen7, H. Ozaki8, P. Pakhlov12, H. Palka15, C. S. Park31, C. W. Park14, H. Park14, L. S. Peak33, M. Peters7, L. E. Piilonen44, E. Prebys29, J. Raaf5, J. L. Rodriguez7, N. Root2, M. Rozanska15, K. Rybicki15, J. Ryuko27, H. Sagawa6, Y. Sakai8, H. Sakamoto16, H. Sakaue26, M. Satapathy43, N. Sato8, A. Satpathy8,5, S. Schrenk44, S. Semenov12, M. E. Sevior17, H. Shibuya34, B. Shwartz2, A. Sidorov2, V. Sidorov2, S. Stanic42, A. Sug19, A. Sugiyama19, K. Sumisawa27, T. Sumiyoshi8, J. Suzuki8, K. Suzuki19, S. Y. Suzuki8, S. K. Swain7, H. Tajima37, T. Takahashi26, F. Takasaki8, M. Takeita27, K. Tamai8, N. Tamura25, J. Tanaka37, M. Tanaka8, Y. Tanaka18, G. N. Taylor17, Y. Teramoto26, M. Tomoto19, T. Tomura37, S. N. Tovey17, K. Trabelsi7, T. Tsuboyama8, Y. Tsujita42, T. Tsukamoto8, T. Tsukamoto30, S. Uehara8, K. Ueno23, N. Ujiie8, Y. Unno3, S. Uno8, Y. Ushiroda16, Y. Usov2, S. E. Vahsen29, G. Varner7, K. E. Varvell33, C. C. Wang23, C. H. Wang22, M.-Z. Wang23, T. J. Wang11, Y. Watanabe38, E. Won21, B. D. Yabsley8, Y. Yamada8, M. Yamaga36, A. Yamaguchi36, H. Yamaguchi8, H. Yamaoka8, Y. Yamaoka8, Y. Yamashita24, M. Yamauchi8, S. Yanaka30, M. Yokoyama37, K. Yoshida19, Y. Yusa36, H. Yuta1, C. C. Zhang11, H. W. Zhao8, Y. Zheng7, V. Zhilich2, and D. Zontar42

1 Aomori University, Aomori
2 Budker Institute of Nuclear Physics, Novosibirsk
3 Chiba University, Chiba
4 Chuo University, Tokyo
5 University of Cincinnati, Cincinnati, OH
6 Gyeongsang National University, Chinju
7 University of Hawaii, Honolulu HI
8 High Energy Accelerator Research Organization (KEK), Tsukuba
9 Hiroshima Institute of Technology, Hiroshima
10 Institute for Cosmic Ray Research, University of Tokyo, Tokyo
11 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing
12 Institute for Theoretical and Experimental Physics, Moscow
13 Kanagawa University, Yokohama
14 Korea University, Seoul
15 H. Niewodniczanski Institute of Nuclear Physics, Krakow
16 Kyoto University, Kyoto
17 University of Melbourne, Victoria
18 Nagasaki Institute of Applied Science, Nagasaki
19 Nagoya University, Nagoya
20 Nara Women's University, Nara
21 National Kaohsiung Normal University, Kaohsiung
22 National Lien-Ho Institute of Technology, Miao Li
23 National Taiwan University, Taipei
24 Nihon Dental College, Niigata
25 Niigata University, Niigata
26 Osaka City University, Osaka
27 Osaka University, Osaka
28 Panjab University, Chandigarh
29 Princeton University, Princeton NJ
Abstract

We report a determination of the $B_d^0 \bar{B}_d^0$ mixing parameter $\Delta m_d$ based on the time evolution of dilepton yields in $\Upsilon(4S)$ decays. The measurement is based on a 5.9 fb$^{-1}$ data sample collected by the Belle detector at KEKB. The proper-time difference distributions for same-sign and opposite-sign dilepton events are simultaneously fitted to an expression containing $\Delta m_d$ as a free parameter. Using both muons and electrons, we obtain $\Delta m_d = 0.463 \pm 0.008$ (stat.) $\pm 0.016$ (sys.) ps$^{-1}$. This is the first determination of $\Delta m_d$ from time evolution measurements at the $\Upsilon(4S)$. We also place limits on possible CPT violations.

PACS numbers: 13.20.H
The frequency of $B^0_d\bar{B}^0_d$ mixing is proportional to the mass difference between the two mass eigenstates of the neutral $B$ meson, $\Delta m_d$, and is a fundamental parameter of the $B$ system. Measurements of $\Delta m_d$ derived from the time evolution of $B^0_d$ decays have been reported by CDF, SLD, and the LEP experiments [1]; ARGUS and CLEO have measured it using the integrated fraction of same-flavor $B$ pair decays in $\Upsilon(4S)$ events [2,3]. We report here the first determination of $\Delta m_d$ based on the time evolution of $B^0_d$ decays in $\Upsilon(4S)$ events produced in asymmetric $e^+e^-$ collisions, using data collected by the Belle detector [4] at the KEKB storage ring [5].

At the $\Upsilon(4S)$, the asymmetry in time evolution between same-flavor ($B^0_dB^0_d$, $\bar{B}^0_d\bar{B}^0_d$) and opposite-flavor ($B^0_d\bar{B}^0_d$) decay pairs exhibits an oscillation as a function of the proper time difference between the two $B$-meson decays, $\Delta t$, with a frequency that is proportional to $\Delta m_d$. In KEKB, collisions between 8.0 GeV electrons and 3.5 GeV positrons have a center of mass (CM) motion along the electron beam direction (z direction) with a Lorentz boost of $\gamma\beta = 0.425$. Since each of the two $B$’s is produced nearly at rest in the CM, the separation of their decay vertices in the lab frame is proportional to $\Delta t$ and has an average magnitude of 200$\mu$m. High-momentum leptons can be used both for tagging the $B$ flavor and for determining the decay vertex with good accuracy. The $\Delta t$ in dilepton events can thus be used to measure the time evolution of $B$ decays. The same analysis can be used to test CPT conservation by the inclusion of the complex parameter $\cos \theta$ in the fit [6].

The analysis presented here is based on integrated luminosities of 5.9 fb$^{-1}$ at the $\Upsilon(4S)$ resonance and 0.6 fb$^{-1}$ at an energy that is 60 MeV below the peak.

The Belle detector consists of a silicon vertex detector (SVD), a central drift chamber (CDC), an array of 1188 aerogel Cerenkov counters (ACC), 128 time-of-flight (TOF) scintillation counters, and an electromagnetic calorimeter containing 8736 CsI(Tl) crystals (ECL), all located inside the 3.4-m-diameter superconducting solenoid that generates a 1.5 T magnetic field. An iron return yoke, outside the solenoid, is segmented into 14 layers of 4.7-cm-thick iron plates alternating with a system of resistive plate counters that is used to identify muons and $K_L$ mesons (KLM).

Hadronic events are required to have at least five tracks, an event vertex with radial and $z$ coordinates respectively within 1.5 cm and 3.5 cm of the origin, a total reconstructed CM energy greater than 0.5$W$ ($W$ is the $\Upsilon(4S)$ CM energy), a $z$-component of the net reconstructed CM momentum less than 0.3$W/c$, a total CM calorimeter energy between 0.025$W$ and 0.90$W$, and a ratio $R_2$ of the second and zeroth Fox-Wolfram moments that is less than 0.7. While the $R_2$ cut suppresses events of non-$\Upsilon(4S)$ origin, all other cuts are intended to remove the beam-related background and QED events.

For electron identification, we use position, cluster energy, and shower shape in the ECL, $dE/dx$ in the CDC, and hit information in the ACC. This is $\sim 90\%$ efficient for electrons and has a $\sim 0.3\%$ misidentification probability for charged hadrons with momenta above 1 GeV/$c$. Electrons from $\gamma$ conversions are removed.

Muon selection is based on KLM hits associated with charged tracks. The range of the tracks and the matching quality of the hits are used. The efficiency is $\sim 85\%$ for muons with momentum above 1 GeV/$c$ and the misidentification probability is $\sim 2\%$.

Events containing leptons from $J/\psi$ decays are rejected. In addition, lepton candidates are required to satisfy $30^\circ < \theta < 135^\circ$; $1.1$ GeV/$c < p^* < 2.3$ GeV/$c$; $|d_{z\text{IP}}| < 0.05$ cm; $|dz\text{IP}| < 2.0$ cm; and have at least one (two) associated SVD hit(s) in the $r$-$\phi$ ($r$-$z$) view,
where $\theta$ is the laboratory polar angle, $p^*$ is the CM momentum, and $d^t_{IP}$ and $dz_{IP}$ are the distances of closest approach to the run-dependent interaction point. To reduce secondary leptons and fakes from the same $B$ and from the continuum, which tend to be back-to-back, the opening angle $\theta_{\ell\ell}$ between the leptons in the CM frame is required to satisfy $-0.8 < \cos\theta_{\ell\ell} < 0.95$. The application of the above-listed criteria yields 8573 same-sign (SS) and 40981 opposite-sign (OS) dilepton events on the $\Upsilon(4S)$, and 40 SS and 198 OS dilepton events below the resonance.

The $z$-vertex of leptons is determined from the intersection of the lepton tracks with the profile of $B_d^0$ decay vertices, which is estimated from the profile of the beam interaction point (IP) convolved with the average $B$ flight length ($\sim 20\mu m$ in the $\Upsilon(4S)$ rest frame). The mean position and the width ($\sigma_{xIP}, \sigma_{yIP}, \sigma_{zIP}$) of the IP are determined on a run-by-run basis using hadronic events. We find $\sigma_{xIP} = 100-120\mu m$, $\sigma_{yIP} \sim 5\mu m$ and $\sigma_{zIP} = 2-3\mu m$. The proper-time difference is calculated from the $z$ positions of the two lepton vertices using the relation $\Delta t = \Delta z/c3\gamma$, where $\Delta z = z_1 - z_2$ is the difference between the two $z$ vertices. For OS events, the positively charged lepton is taken as the first lepton ($z_1$). For SS events the absolute value of $\Delta z$ is used.

The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.

Each dilepton is identified with one of the event types listed in Table I. Each event type is categorized as either signal (S), correctly tagged background (C), or incorrectly tagged background (W). For each, we parameterize the proper-time distribution as the product of distributions. The observed SS and OS dilepton proper-time distributions have contributions from “signal,” defined as events where both leptons are primary leptons from semileptonic decay of $B_d^0$ or $B^+$, and “background,” where at least one lepton is secondary or fake, or the event is from the non-$\Upsilon(4S)$ continuum. The value of $\Delta m_d$ was extracted by simultaneously fitting the two distributions to the respective sums of contributions from all known signal and background sources.
primary lepton paired with a secondary lepton from a c-quark. The shape as well as the normalization of the background from neutral B events depends on $\Delta m_d$. To account for this, we generated two samples of generic neutral B events, one with $\Delta m_d = 0.464 \text{ ps}^{-1}$ and one with $\Delta m_d = 0.423 \text{ ps}^{-1}$. Background distributions for arbitrary $\Delta m_d$ are determined by linear interpolation.

We use the $\Delta z$ distribution of dileptons from $J/\psi$ decays in the data for $g$; for these events the root distribution is a delta function and the lepton momentum spectra are in the same region as those of primary leptons from B decays. The $\Delta z$ distribution for $J/\psi$ events, which has $\sigma = 112 \mu m$, agrees with the MC distribution if it is convolved with a Gaussian of $\sigma = 50 \pm 18 \mu m$. This is due to an imperfect detector simulation, and we correct this effect by applying a convolution with $\sigma = 50 \mu m$ to each MC-determined background distribution.

To extract $\Delta m_d$, a binned maximum likelihood fit is performed simultaneously to the $\Delta z$ distributions of the SS and OS dileptons. Each fitting function is a sum of signal and background distributions. In order to properly take into account the tails of the $\Delta z$ distributions, the signal response function and the background distribution are given in the form of a lookup table rather than an analytic function. We fix the parameters $\tau_{B_d^0} = 1.548 \text{ ps}$, $f_+/f_0 = 1.05$, and $\tau_{B^+}/\tau_{B_d^0} = 1.06$, and limit the fit region to $|\Delta z| < 1.85 \text{ mm}$. The constraint $b_+/b_0 = \tau_{B^+}/\tau_{B_d^0}$ is imposed. The continuum contribution is fixed to that of off-resonance data, scaled to account for luminosity and energy differences. The relative selection efficiencies for the event types (mixed $B^0_d$, unmixed $B^0_d$, and charged B) within each tag type (S, C, and W) are fixed, resulting in two free parameters (efficiency ratios in C/S and W/S) in addition to $\Delta m_d$ and the overall normalization. The fit result is $\Delta m_d = 0.463 \pm 0.008 \text{ ps}^{-1}$ with $\chi^2/DOF = 333/376$. The efficiency ratios in C/S and W/S are $(9.66 \pm 1.39) \times 10^{-3}$ and $(6.98 \pm 0.25) \times 10^{-3}$, respectively, which give signal fractions to be 32.1%(SS) and 77.5%(OS). Figure 4 shows the $\Delta z$ distributions for the data together with the fitted curves. Figure 5 shows the OS and SS asymmetry, $(N_{OS} - N_{SS})/(N_{OS} + N_{SS})$, for data together with the result of the fit.

As a cross check, we also measured $\Delta m_d$ using a fitting method that differs from the one described above in the following aspects [10]: a) an unbinned rather than binned maximum likelihood fit; b) response function is the sum of three Gaussians, with parameters determined from the dileptons from $J/\psi$ decays; c) backgrounds separated into SS, OS, rather than C, W; d) background distributions were analytic functions, with parameters determined by fitting to MC. We find $\Delta m_d$ in the range 0.460 to 0.483 ps$^{-1}$ depending on the choice of analytic forms for the backgrounds, which is consistent with the primary result.

The systematic errors were estimated by repeating the fits for different input parameters. The main contribution originates from uncertainties on input parameters and from determination of the response functions. Contributions of $f_+/f_0$, $\tau_{B_d^0}$ and $\tau_{B^+}/\tau_{B_d^0}$ are estimated by adjusting each in turn by the amount of its uncertainty. Contribution of the response function arises from the possibility that it differs from the true dilepton response function, from the statistical uncertainties of the determination, and from the fact that the calculation of proper time $\Delta t = \Delta z/c\beta\gamma$ is not exact due to the motion of the $B$’s in the CM and the energy spread of the beams. To estimate the first possibility, we used the MC dilepton response function convolved with a Gaussian of $\sigma = 50 \mu m$. For the second, we varied the number of entries on a bin-by-bin basis by the amount of the statistical errors. For the third, we compared two fits, one using a response function obtained for the true $\Delta t$ difference and
a second obtained for $\Delta z$.

We also consider the uncertainty from the background simulation. We assigned a 35% error for the fake rate and adjusted the fake rate by this amount. We varied the branching ratios of $B$ decaying to $D^0$ and $D^+$ in the MC in accord with the experimental uncertainties [11]. We varied the width of the Gaussian used to correct for an imperfect detector simulation by $\pm 18 \mu m$.

It is assumed in the fit that $\Delta \Gamma$, the difference between the decay widths of the neutral $B$ mass eigenstates, is zero. Although no significant experimental constraint exists [3], it is predicted based on solid theoretical grounds to be very small ($\Delta \Gamma / \Gamma < 1\%$) [8]. We repeated the fit including the effects of $\Delta \Gamma / \Gamma = 1\%$ and found the shift in the result to be negligible ($< 0.001 \text{ ps}^{-1}$).

Contributions to the systematic error from the above sources are summarized in Table II. The total systematic error is obtained by summing all errors in quadrature:

$$\Delta m_d = 0.463 \pm 0.008 \ (\text{stat}) \pm 0.016 \ (\text{sys}) \ \text{ps}^{-1}.$$ 

When the constraint of CPT conservation is removed in $B^0_d - \bar{B}^0_d$ mixing, the theoretical functions (2) and (3) are modified and become

$$F(\Delta t) = (| \sin \theta |^2 / 4 \tau_{B^0_d}) e^{-|\Delta t|/\tau_{B^0_d}} [1 - \cos(\Delta m_d \Delta t)]$$

$$F(\Delta t) = (1 / 4 \tau_{B^0_d}) e^{-|\Delta t|/\tau_{B^0_d}} [1 + | \cos \theta |^2$$

$$+ (1 - | \cos \theta |^2) \cos(\Delta m_d \Delta t)$$

$$- 2 Im(\cos \theta) \sin(\Delta m_d \Delta t)] \quad (6)$$

and $\chi_d$ becomes $| \sin \theta |^2 x_d^2 / | \sin \theta |^2 x_d^2 + (2 + x_d^2 + x_d^2 | \cos \theta |^2)$. A non-zero value of the complex parameter $\cos \theta$ would be an indication of CPT violation. The result of the fit is [12]

$$Im(\cos \theta) = 0.035 \pm 0.029 \ (\text{stat}) \pm 0.051 \ (\text{sys})$$

$$Re(\cos \theta) = 0.00 \pm 0.15 \ (\text{stat}) \pm 0.06 \ (\text{sys})$$

and $\Delta m_d = 0.461 \ \text{ps}^{-1}$. These results imply [3] the upper limits $|m_{B^0} - m_{\bar{B}^0}|/m_{B^0} < 1.6 \times 10^{-14} \text{ and } |\Gamma_{B^0} - \Gamma_{\bar{B}^0}|/\Gamma_{B^0} < 0.161 \text{ at the 90\% C.L.}$

In summary, we report the first determination of $\Delta m_d$ using the time evolution of $B^0_d$-mesons produced in $\Upsilon(4S)$ decays. We obtain $\Delta m_d = 0.463 \pm 0.008 \ (\text{stat}) \pm 0.016 \ (\text{sys}) \ \text{ps}^{-1}$, which is consistent with the world average value $\Delta m_d = 0.472 \pm 0.017 \ \text{ps}^{-1}$ [3]. We have also examined CPT violation and obtain the first limit on $(m_{B^0} - m_{\bar{B}^0})/m_{B^0}$ and a limit on $(\Gamma_{B^0} - \Gamma_{\bar{B}^0})/\Gamma_{B^0}$ that is compatible with the previous measurement [13].

We wish to thank the KEKB accelerator group for the excellent operation. We acknowledge support from the Ministry of Education, Science, Sports and Culture of Japan and the Japan Society for the Promotion of Science; the Australian Research Council and the Australian Department of Industry, Science and Resources; the Department of Science and Technology of India; the BK21 program of the Ministry of Education of Korea and the Basic Science program of the Korea Science and Engineering Foundation; the Polish State Committee for Scientific Research under contract No.2P03B 17017; the Ministry of Science and
Technology of Russian Federation; the National Science Council and the Ministry of Education of Taiwan; the Japan-Taiwan Cooperative Program of the Interchange Association; and the U.S. Department of Energy.
REFERENCES

[1] LEP B Oscillations Working Group, see
http://www.cern.ch/LEPBOSC/ and references therein.

[2] ARGUS Collaboration, H. Albrecht et al., Phys. Lett. B324, 249 (1994).

[3] CLEO Collaboration, B. H. Behrens et al., Phys. Lett. B490, 36 (2000).

[4] BELLE Collaboration, KEK Report 2000-4 (2001), to appear in Nucl. Instrum. Methods.

[5] KEKB accelerator group, KEKB B Factory Design Report, KEK Report 95-7, 1995; Y. Funakoshi et al., Proc. 2000 European Particle Accelerator Conference, Vienna (2000).

[6] A. Mohapatra et al., Phys. Rev. D58, 036003 (1998); V. Alan Kostelecký and R. Van Kooten, Phys. Rev. D54, 5585 (1996); M. Kobayashi and A. I. Sanda, Phys. Rev. Lett. 69, 3139 (1992); T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965).

[7] G. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).

[8] D. E. Groom et al., Eur. Phys. J. C15, 1 (2000).

[9] CLEO Collaboration, J. P. Alexander et al., CLNS 00-1670, CLEO 00-7, submitted to Phys. Rev. Lett.

[10] M. Tomoto, PhD Thesis, Nagoya University (2000).
http://belle.kek.jp/bdocs/theses.html.

[11] CLEO Collaboration, L. Gibbons et al., Phys. Rev. D56, 3783 (1997).

[12] C. Leonidopoulos, PhD Thesis, Princeton University (2000).
http://belle.kek.jp/bdocs/theses.html.

[13] OPAL Collaboration, K. Ackerstaff et al., Z. Phys. C76, 417 (1997).
| $\ell\ell$ event type | $N$ | tag type |
|----------------------|-----|----------|
| SS signal            | $B_d^0$, mixed | $N_{4S}f_0\chi_d b_0^2$ | S |
| SS background        | $B_d^0$, mixed | $N_{4S}f_0\chi_d$ | C |
|                      | $B_d^0$, unmixed | $N_{4S}f_0(1 - \chi_d)$ | W |
|                      | $B^+B^-$ | $N_{4S}f_+$ | W |
|                      | continuum | $N_{\text{cont}}$ | |
| OS signal            | $B_d^0$, unmixed | $N_{4S}f_0(1 - \chi_d)b_0^2$ | S |
| OS background        | $B_d^0$, unmixed | $N_{4S}f_0\chi_d$ | W |
|                      | $B_d^0$, unmixed | $N_{4S}f_0(1 - \chi_d)$ | C |
|                      | $B^+B^-$ | $N_{4S}f_+$ | C |
|                      | continuum | $N_{\text{cont}}$ | |

| Source (uncertainty) | $\Delta m_d$ |
|----------------------|--------------|
| $f_+/f_0$ (± 0.08)   | ±0.009       |
| $B_d^0$ lifetime (± 0.032 ps) | ±0.004       |
| $\tau_{B^+}/\tau_{B_d^0}$ (± 0.03) | ±0.009       |
| response function     | ±0.005       |
| background fake rate (±35%) | ±0.004 |
| $B(B \to DX)$ ($D^0$: ±4.6%, $D^+$: ±14.3%) | ±0.002       |
| continuum (SS: ±16%, OS: ±7%) | ±0.002       |
| detector resolution, BG (±18 µm) | ±0.001       |
| Monte Carlo statistics | ±0.004       |
| total                | ±0.016       |
FIG. 1. $\Delta z$ distribution of dileptons for data together with fit result. The upper plot shows the distributions for same-sign, and the lower plot for opposite-sign dileptons. Signal and background dileptons obtained from the fit are also shown.
FIG. 2. Opposite and same-sign dilepton asymmetry vs $\Delta z$. The asymmetry is defined as $A(\Delta z) = (N_{OS} - N_{SS})/(N_{OS} + N_{SS})$. The points are the data. The curve is the result of the fit.