Topological Bose-Mott insulators in one-dimensional non-Hermitian superlattices

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We study topological properties of Bose-Mott insulators in one-dimensional non-Hermitian superlattices, which may serve as effective Hamiltonians for cold atomic optical systems with either two-body loss or one-body loss. We find that in strongly repulsive limit, the Mott insulator states of the Bose-Hubbard model with a finite two-body loss under integer fillings are topological insulators characterized by a finite charge gap, nonzero integer Chern numbers and nontrivial edge modes in low-energy excitation spectrum under open boundary condition. The two-body loss suppressed by the strong repulsion results in a stable topological Bose-Mott insulator which behaves similar properties as the Hermitian case. However, for the non-Hermitian model related to the one-body loss, we find the non-Hermitian topological Mott insulators are unstable with a finite imaginary excitation gap.

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Introduction.- Non-Hermitian topological systems have attracted much attention in the past years with the aid of the fast development of topological photonics \cite{1}--\cite{8}. Since the non-Hermitian systems posses more fundamental nonspatial symmetries, their topological classification goes beyond the standard ten classes of the corresponding Hermitian systems \cite{9}--\cite{16}. The non-Hermitian systems have been shown to exhibit many exotic properties without Hermitian counterparts \cite{17}--\cite{33}, including half-integer topological invariants \cite{24}--\cite{29}, non-Hermitian skin effect (NHSE) and breakdown of bulk-boundary correspondence in some non-reciprocal system \cite{30}--\cite{50}. Recently, exploring topological phases in interacting non-Hermitian systems has also been addressed in several works \cite{51}--\cite{52}.

Due to their highly controllability, one-dimensional (1D) optical superlattices, which can be realized by superimposing two 1D optical lattices with commensurate wavelengths, have provided an ideal playground for exploring topologically nontrivial phases \cite{53}--\cite{64}. The interplay between many-body interaction and single-particle band topology can lead to intriguing correlated states exhibiting nontrivial topological properties, e.g., fractional topological states \cite{65}--\cite{68} and topological Mott insulators (TMIs) \cite{59}--\cite{61} in the interacting superlattice systems. Recent experimental progress demonstrate that the optical lattice systems may be a controllable candidate for studying the quantum open systems by introducing a dissipation process \cite{65}, which can be effectively described by an effective non-Hermitian Hamiltonian under certain condition. So far, different kinds of literatures on the effect of the dissipation have been reported due to the particle loss or photon scattering \cite{66}--\cite{79}. Especially, the two-body loss on quantum many-body systems is realized in the form of inelastic collisions which can be widely controlled \cite{80}--\cite{84}. One can tune the inelastic two-body scattering via a photo-association resonance in a bosonic or fermionic system, where a delay of the melting of the Mott insulating state was detected \cite{83}--\cite{84}.

Motivated by these progresses, in this work we study the realization of topological Mott phase in 1D non-Hermitian superlattice systems. For an interacting bosonic gas trapped in a superlattice with an integer band filling factor, a TMI is in the formation of the Mott phase characterized by a nontrivial topological invariant \cite{59}--\cite{60}. When we consider a non-Hermitian system, its eigenenergies are generally complex and natural questions arise here are whether the Mott phase still exists and how to characterize the non-Hermitian TMI?

Model and method.- To address these problems, we first consider interacting bosons trapped in a 1D two-periodic optical lattice with the Hamiltonian described by $H_{BH} = H_0 + H_I$ with

\begin{equation}
H_0 = - \sum_j J(j, \theta)(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.}) + \sum_j \frac{V(j, \theta)}{2} \hat{n}_j,
\end{equation}

and

\begin{equation}
H_I = \frac{U}{2} \sum_j \hat{n}_j(\hat{n}_j - 1),
\end{equation}

Here, $\hat{b}_j$ is the annihilation operator of bosons at site $j$, $\hat{n}_j = \hat{b}_j^\dagger \hat{b}_j$ is the number operator of bosons, and the alternating hopping strengths are given by $J(j, \theta) = J[1 + \delta \sin(\pi j + \theta)]$ with the dimerization strength $\delta$ and
$J$ being set as the energy unit ($J = 1$). The on-site potential is given by $V(j, \theta) = V \cos(\pi j + \theta)$ with $V \cos \theta$ denoting the energy offset between neighboring sites for real phase $\theta$ of the potential. The interaction strength $U = U_r$ is always real and can be experimentally controlled by the Feshbach resonance. Complex-valued interactions can emerge in some effective Hamiltonians of ultracold atomic systems induced by considering the inelastic processes between different orbitals which give rise to two-particle loss.

When atoms undergo inelastic collisions, the scattered atoms are lost from the system. The atom losses are described by a quantum master equation:

$$\partial_t \rho(t) = -i[H, \rho(t)] - \frac{1}{2} \gamma \sum_j (L_j \rho(t) + \rho(t)L_j^\dagger),$$

where $\rho(t)$ is the density matrix of the atomic gas, $L_j$ is a Lindblad operator at site $j$ which describes a loss with the rate $\gamma > 0$. The process of two-particle loss can be described by setting $L_j \to b_j b_j^\dagger$. Considering the short time evolution, the quantum-jump term can be negligible and the dynamical evolution is described by

$$\partial_t \rho(t) = -i[H^{\text{eff}}(\theta), \rho(t)] + \rho(t)H^{\text{eff}}(\theta),$$

where $H^{\text{eff}} = H_0 + H_I$ is an effective Hamiltonian with the interaction amplitude $U$ taking a complex value

$$U = U_r - i\gamma.$$}

Here $U_r \geq 0$ represents the repulsive interaction and $\gamma \geq 0$ the rate of loss.

When $U = 0$, the Hamiltonian reduces to the topologically nontrivial Rice-Mele model $[55]$. The energy spectrum in momentum space with momentum $k$ is $E_{\pm} = \pm \sqrt{2 + V^2 \cos^2 \theta/4 + 2\delta^2 \sin^2 \theta + 2(1 - \delta^2 \sin^2 \theta) \cos k}$ and the energy gap is $\Delta_b = 2 \sqrt{V^2 \cos^2 \theta/4 + 4\delta^2 \sin^2 \theta}$. Fig. 1(a) shows the single-particle energy spectrum of Rice-Mele model as the function of $\theta$ under OBC. (b) The charge gap $\Delta_c$ as the function of $U_r$ for the Hermitian case with $L = 14$ and $\nu = 1$ under periodic boundary condition (PBC). In the small $U_r$ limit, the charge gap $\Delta_c \to 0$ and the charge gap $\Delta_c$ grows with the increase of $U_r$. In the strongly repulsive limit, $\Delta_c$ tends to $\Delta_b/2$, as predicted by applying the Bose-Fermi mapping $[59]$. Our results indicate that a Mott-insulator phase emerges in the large $U_r$ case.

The topological feature of the Mott-insulator state can be characterized by the many-body Chern number in the two-dimensional (2D) parameter space $(\varphi, \theta)$ $[57, 59, 60]$. Here, we introduce the twist boundary condition which is corresponding to a shift momentum $k = (2m + \varphi)/N_{\text{cell}}$ in Brillouin zone with $\varphi$ being a generalized boundary phase and $m = 0, 1, \ldots, N_{\text{cell}} - 1$. The many-body Chern number for the Hermitian case is defined as

$$C = \frac{1}{2\pi} \int d\varphi d\theta B(\varphi, \theta)$$

with the Berry curvature

$$B(\varphi, \theta) = \text{Im} \left( \left\langle \frac{\partial \tilde{\psi}}{\partial \varphi} | \frac{\partial \tilde{\psi}}{\partial \theta} \right\rangle - \left\langle \frac{\partial \tilde{\psi}}{\partial \theta} | \frac{\partial \tilde{\psi}}{\partial \varphi} \right\rangle \right).$$

When the Mott insulator is formed for $\nu = 1$, the corresponding state has the Chern number $C = 1$, indicating that the Mott insulator is topologically nontrivial, i.e., the formation of TMI. As shown in Fig. 1(c), even for a small $U_r$, the topological number $C$ already approaches to 1. Our numerical calculations demonstrate that the TMI emerges with the increase of $U_r$, which is consistent with previous studies $[59, 60]$.

**Non-Hermitian TMI.-** Now we study the non-Hermitian effect induced by the imaginary part of $U$. As a concrete example, we consider the system described by Hamiltonian $H^{\text{eff}}(\theta)$ with $\nu = 1$, $\delta = 0.6$, $V = 2$, and $\theta = \pi/4$.
θ = π/4 and γ = 1, and calculate the charge gap Δ_c defined by Eq. (6). Here, due to the complex energy spectrum for a non-Hermitian system, we define E_0(N) as the ground state energy for the N-boson system with the minimum value of the real-part. Correspondingly, the real part of the charge gap for non-Hermitian case, is defined as Δ_c^R = Re(Δ_c) and the imaginary part of the charge gap Δ_c^I = Im(Δ_c) is also calculated. Fig. 2 shows Δ_c^R and Δ_c^I as the function of U_r under PBC. For U_r = 0, the real part of the charge gap Δ_c^R → 0 and Δ_c^I takes a finite value. The real part of the charge gap Δ_c^R shows a monotonic increase with the increasing of U_r and tends to Δ_e/2 in the strongly repulsive limit. However, with the increase of U_r, the absolute value of Δ_c^I firstly increases and then decreases when U_r > 3.76. In the large U_r limit, Δ_c^I → 0 and two-body collisions are strongly suppressed which leads to the decreasing of the atomic loss, indicating that a stable Mott insulator emerges in the strongly interacting limit.

To characterize the topological feature, we now construct the many-body Chern number for the non-Hermitian case. In 2D parameter space of (ϕ, θ), the Chern number of the ground state for a non-Hermitian system is an integral invariant, which can be defined as

\[ C^{αβ} = \frac{1}{2\pi} \int dϕ dθ B^{αβ}(ϕ, θ), \]

where the Berry curvatures \( B^{αβ}(ϕ, θ) = \) Im \((\frac{∂ψ^n_α}{∂ϕ} \frac{∂ψ^n_β}{∂θ} - \frac{∂ψ^n_α}{∂θ} \frac{∂ψ^n_β}{∂ϕ})\) with \( α, β = R/L \). These definitions are a direct generalization of non-Hermitian Chern numbers [20] to the many-body systems. Here, the right eigenstates \( |ψ^R⟩ \) and the left eigenstates \( |ψ^L⟩ \) can be respectively defined as \( H|ψ^R⟩ = E|ψ^R⟩ \) and \( H^†|ψ^L⟩ = E^†|ψ^L⟩ \) with the normalization condition \( ⟨ψ^n_α|ψ^n_β⟩ = 1 \). We numerically calculate four different Chern numbers \( C^{LL}, C^{LR}, C^{RL} \) and \( C^{RR} \) of the Hamiltonian \( H^{(1)}_{\text{eff}} \) with \( ν = 1, γ = 1 \) and different \( U_r \). We find \( C^{LL} = C^{LR} = C^{RL} \) and \( C^{RR} = 1 \) for this non-Hermitian interacting boson system with finite repulsive \( U_r \). In the large \( U_r \) limit, the stable Mott insulators are formed for \( ν = 1 \), corresponding to the states with nontrivial Chern numbers, which show similar features as the Hermitian case and the stable Mott insulators are topologically nontrivial.

According to the bulk-edge correspondence, one may expect that the non-Hermitian TMI also exhibit nontrivial edge states under OBC. In Fig. 3 we show the real part (left column) and the imaginary part (right column) of the low-energy spectrum \( E_n - E_0 \) versus θ with different \( U_r \), \( L = 12, ν = 1, δ = 0.6, V = 2 \) and \( γ = 1 \). (a) \( U_r = 10 \) under PBC; (b) \( U_r = 10 \) under OBC; (c) \( U_r = 100 \) under PBC; (d) \( U_r = 100 \) under OBC.
excited state for systems with $U_r = 10$ and $U_r = 100$ under PBC, respectively. Correspondingly, the edge states emerge in the real gap regimes of the low-energy excitation spectrum under OBC. As the phase shift $\theta$ varies from $0$ to $2\pi$, the edge states connect the ground state to the excited band, as shown in Figs. 3(b) and 3(d) for systems with $U_r = 10$ and $U_r = 100$ under OBC, respectively. The real part of low-energy excitation spectrum for system with $U_r = 100$ of $H_{\text{eff}}^{(1)}$ exhibits almost the same behaviors as its corresponding Hermitian case. The imaginary part of low-energy excitation spectrum gradually converges to zero with the growth of $U_r$, indicating that the effect of the finite two-body loss is almost completely suppressed and a stable TMI exists in strongly repulsive limit.

**TMI in the non-Hermitian Rice-Mele model.** Next we also consider another non-Hermitian extension of the Hamiltonian by taking a complex phase $\theta$, i.e., $H_{\text{eff}}^{(2)} = H_0^{NH} + H_I$, where $H_0^{NH}$ is obtained by replacing $\theta \rightarrow \theta + i\beta$ in $H_0$ with $\beta$ being an imaginary phase shift. The non-Hermitian Hamiltonian $H_0^{NH}$ may serve as an effective Hamiltonian related to some one-body loss processes [80,87], and similar models have been studied in Ref. [88-90]. We take $\beta = 0.1\pi$ as an example for exhibiting our calculation results. In the absence of $U_r$, the energy gap in the complex-energy plane as defined in Ref. [20] is emergent for rolling $\theta \in [0, 2\pi]$ which is shown in Fig. 4(a) with $\delta = 0.6$ and $V = 2$ under PBC. When OBC is considered, the edge states emerge between the two bulk bands [see Fig. 4(b)], with the corresponding bulk state characterized by a nonzero band Chern number [20].

In the presence of the interaction, we first calculate the charge gap $\Delta_c$ versus $U_r$ with $\beta = 0.1\pi$, $\delta = 0.6$, $V = 2$, $\theta = \pi/4$ and $L = 14$ under PBC, as shown in Fig. 5(a). With the increase of repulsion, both the real and imaginary parts of the charge gap $\Delta_c^{R}$ increase and in the strong repulsion, $\Delta_c^R \rightarrow \text{Re}(\Delta_c)/2 = 1.1065$ and $\Delta_c^I \rightarrow \text{Im}(\Delta_c)/2 = 0.0667$. A finite imaginary-valued gap indicates that the Mott phase will collapse in the long-time evolution. To characterize the topological property of the unstable Mott insulators, we numerically calculate the Chern number Eq. (7) defined in non-Hermitian case. When $U_r > 0$, the four Chern numbers are equal and $C^{LL} = C^{LR} = C^{RL} = C^{RR} = 1$. The existence of the nonzero Chern number implies the repulsively interacting system with an integer filling factor having nontrivial topological properties. The topological phase would exhibit nontrivial edge modes in the gap regions under OBC. We calculate the low-energy excitation spectra of $H_{\text{eff}}^{(2)}$ with $L = 14$, $\nu = 1$, $\delta = 0.6$, $V = 2$ and $\beta = 0.1\pi$.
by rolling $\theta \in [0, 2\pi]$ shown in Figs. 5(b)-(e). As shown in Figs. 5(b) and 5(d), an obvious energy gap is shown between the ground state and the low-energy excitation states for $U_r = 10$ and 100 under PBC, respectively. Under OBC, as the phase shift $\theta$ varies from 0 to $2\pi$, the low-energy spectra with $U_r = 10$ and 100 are shown in Figs. 5(e) and 5(e), respectively. The edge states connecting the ground states and low-energy part emerge in the gap and the position of the edge states continuously varies with the rolling of $\theta$ in the complex-energy plane. Specifically, the energies of the ground states and the edge modes exhibit a finite imaginary part which implies that the non-Hermitian TMIs with the one-body loss are formed, but due to the finite imaginary parts of the ground energies, the TMIs are unstable and shall be broken down during the time evolution.

Conclusions.- In summary, we have discussed topological Bose-Mott insulators in 1D non-Hermitian superlattices which are characterized by a finite charge gap, nonzero integer Chern numbers defined in non-Hermitian case and nontrivial edge modes in low-energy excitation spectrum under OBC. We found that for the non-Hermitian effect induced by a finite two-body loss, the nontrivial TMIs are stable in strong repulsion limit. However, for a non-Hermitian TMI associated with one-body loss, the low-energy excitation spectrum with a finite imaginary part suggests the TMI is unstable and shall collapse with time.

Note added: While main results of this manuscript were completed, we became aware of the similar works of [91, 92], which discuss the non-Hermitian topological Mott insulators in the interacting non-Hermitian Aubry-André-Harper models with different focus issues.

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