A Travel Decision Making Model Based on Nested Logit Model Considering Bounded Rationality

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ABSTRACT In this study, a Boundedly Rational Nested Logit (BRNL) model is proposed by introducing the concept of indifference threshold into Nested Logit (NL) model. In BRNL model, the traveler is assumed to choose randomly or in accordance with his preference if the expected cost difference between two alternatives is within the indifference band. Otherwise, the traveler will choose the alternative with the minimum expected cost. A nested method of successive average is developed to solve the proposed problem. Finally, the rationality of the model and effectiveness of the algorithm are proved by using a numerical example with three-mode transportation network. The results indicate that, different from NL model, the choices of the travelers in BRNL model do not depend on the costs always. The value of the indifference threshold will affect the choice probability. The larger the indifference threshold is, the less sensitive travelers are to the change of cost. In particular, if the indifference threshold is large enough, the travelers’ choices will always depend on the preference.

INDEX TERMS Travel decision, bounded rationality, nested logit, indifference threshold, preference.

I. INTRODUCTION

Discrete choice model is widely applied in an analysis of traveler behavior. The model assumes that the traveler is expected utility maximizer, namely traveler chooses an alternative within utility maximization always, even if the difference in utilities is negligible [1]–[4].

However, this assumption is not actual sometime. Relative to “utility maximization”, Simon [5] considered that people are bounded rationality, who tend to seek a satisfying solution among those feasible solutions rather than to find an optimal one. Quantd [6] pointed out that there is an indifference band about utility level came from a commodity. Georgescu-Roegen [7] brought the concept of thresholds into the consumer choice theory, proposing that the consumers will not change their choice unless the quantities exceed some “necessary minimum”. In 1979, Kahneman and Tversky [8] put forward the prospect theory based on Simon’s theory of bounded rationality and, which is used to describe people’s decision-making behaviors under uncertain conditions. In 1992, Tversky and Kahneman [9] proposed the cumulative prospect theory on the basis of the theory of hierarchical dependent expected utility.

There are some researches refer to “indifference threshold” in the study of traveler’s route/mode choice behaviors. For example, Mahmassani and Chang [10], [11] and Mahmassani and Jou [12] conducted a series of experimental studies in the 1990s and proposed an indifference curve of departure time and route choice and established a Boundedly Rational User Equilibrium model. They indicated that the travelers would not change their choice as long as the result of departure time and route choice is in the indifference band. Ridwan [13] introduced a choice function based on fuzzy preference relations for individual choice in traffic assignment considering the travelers with non-maximizing behavior. Lou et al. [14] established Boundedly Rational User Equilibrium model and defined the concept of indifference band. Li et al. [15] developed a named Bounded Rational Binary Logit (BRBL) model based on the “bounded rationality” hypothesis. It assumed that all the travelers are homogeneous which means that the indifference thresholds of all the travelers are same. Di et al. [16] modeled travelers’ route switching choice behavior after a new route was introduced by assuming the travelers will choose the new bridge
only if the time saving efficient by taking the new bridge than an indifference band. Furthermore, they assumed the travelers are heterogeneous and the indifference band follows lognormal distribution. Carrion and Levinson [17] found that the threshold exists in the traveler’s route choice behavior by the analysis of GPS data.

Incorporating “indifference threshold” into the research of travelers’ mode choice behavior, Krishnan [18] hypothesized that the “satisfier” in Simon’s theory would again become a utility maximizer if one alternative draws sufficient attraction than the other. He defined indifference threshold as “minimum perceivable differences” between the utilities of two alternatives and proposed a minimum perceivable difference (MPD) model. Lioukas [19] extended Krishnan’s binary theory to multiple-alternative situations using multinomial logit model. Cantillo et al. [20] designed a state choice survey that the subjects are faced with two alternatives, but they are allowed to choose an indifferent potion, such as “both are the same” and established a discrete choice model to inspect the individuals’ choice behaviors with indifference threshold. Wei et al. [21] and Wang et al. [22] proposed the notion of indifference thresholds-based bi-modal equilibrium (ITBE) with the assumption of indifference thresholds-based travelers’ mode choice behavior. They indicated that the solution of ITBE might not be unique by assuming travelers have different indifferent threshold for different modes. Unlike aforementioned approaches, the research of Obermeyer et al. [23] concentrates on attribute thresholds through estimating thresholds concerning travel time differences of discrete choice models. Zhang et al. [24] defines the “indifference threshold” as the traveler’s sensitivity to changes in travel utilities. In the framework of the theory of planned behavior (TPB), a structural equation model (SEM) considering the indifference threshold is established to analyze a traveler’s mode choice behavior.

The aforementioned researches extended modified the Binary Logit (BL) and Multinomial Logit (MNL) models by introducing the concept of indifference threshold. However, there were no studies refer to NL model considered indifference threshold.

Some researches pointed that, the application of MNL models is limited for its Independence from Irrelevant Alternatives (IIA) property. The IIA property indicates that the choice decision between a pair of alternatives is irrelevant to any other alternatives. This assumption may lead to prediction error of choice probabilities [3]–[4]. There are different models directed at reducing or eliminating the effects of the IIA property. One of them is the Nested Logit (NL) model which is the most simply and widely used [25]–[29]. For example, Koppelman and Bhat [30] presented a classic example with IIA property, so called “red bus/blue bus paradox”, and indicated that NL model prevents significant deviations from the IIA property but keeps the computational advantages of the MNL model. Yao et al. [31] established a MNL model and a NL model to research the traveler’s mode choice behaviors, respectively. The estimating results of the two models showed that the prediction accuracy of NL model is significantly higher than that of the MNL model, which indicated NL model is more suitable for describing the mode choice behaviors of the traveler. Kitthamkesorn et al. [32] chosen NL model for mode choice to handle the mode similarity. Liu et al. [33] established a multi-mode stochastic user equilibrium allocation model based on nested Logit by considering the traveler’s route choice and mode choice behavior.

In this article, a Boundedly Rational Nested Logit (BRNL) model was established. The model considering the indifference threshold into NL model. Travelers are assumed boundedly rational, namely they are indifference to the choices that have a random utility difference within the indifference band. Otherwise, they behave according to a logit model when the random utility difference is outside the indifference band. Two new parameters are introduced, one for the threshold that defines the indifference band, and another one for the preference of alternatives. Different from NL model, the choices of travelers in BRNL model does not depend on the utilities but their preferences if the random utility difference between choices is within the indifference band. Moreover, compared with the model established by Lioukas [19], BRNL model overcomes the significant deviations from the IIA property.

The remainder of the paper is organized as follows: Next Section presents the assumptions and the formulation of BRNL model. Section 3 represents a nested method of successive average algorithm to solve BRNL model. A numerical example is conducted in Section 4 to demonstrate BRNL model and solution algorithm, and also compare it with the NL model. Finally, conclusions and further researches are presented in Section 5.

![FIGURE 1. Three-modal transport network.](image-url)
part for the excepted cost of car, respectively. The excepted costs for bus and subway each include a distinct deterministic part, \(V_{bus}\) and \(V_{sub}\), and a common deterministic part, \(\varepsilon_{PT}\), for public transit; they also include a common random part, \(\varepsilon_{bus}\) and a distinct random parts, \(\varepsilon_{bus}\) and \(\varepsilon_{sub}\). The common error part creates a covariance between the total errors for bus, \(\varepsilon_{bus} + \varepsilon_{PT}\) and subway, \(\varepsilon_{sub} + \varepsilon_{PT}\).

The total error for each of the three alternatives is assumed to be distributed Gumbel with scale parameter, \(\theta_1\). The variance of these distributions is

\[
Var(\varepsilon_{car}) = Var(\varepsilon_{bus} + \varepsilon_{PT}) = Var(\varepsilon_{sub} + \varepsilon_{PT}) = \frac{\pi^2}{6\theta_1^2} \tag{4}
\]

The distinct error components, \(\varepsilon_{bus}\) and \(\varepsilon_{sub}\), also are assumed to be distributed Gumbel with scale parameter, \(\theta_2\). The variance of these distributions is

\[
Var(\varepsilon_{bus}) = Var(\varepsilon_{sub}) = \frac{\pi^2}{6\theta_2^2}, \quad 0 < \theta_2 < \theta_1 \tag{5}
\]

III. THE ASSUMPYIONS OF THE MODEL

As described in [33], indifference threshold refers to the perceived utility difference between modes that enables the traveler to move from one travel mode to another. In this study we assume that the travelers cannot tell the difference between the except costs of two modes and will choose travel mode randomly or in keeping with his preference if the absolute value of the cost difference is less than the indifferent threshold. Otherwise, travelers will choose the travel mode with minimum excepted cost. The model hypothesis can be expressed as equations (6) and (7).

\[
p(bus|PT) = \begin{cases} 
1, & U_{bus} - U_{sub} < -\Delta_1 \\
\tau_1, & -\Delta_1 \leq U_{bus} - U_{sub} \leq \Delta_1 \\
0, & U_{bus} - U_{sub} > \Delta_1 
\end{cases} \tag{6}
\]

\[
p(car) = \begin{cases} 
1, & U_{car} - U_{PT} < -\Delta_2 \\
\tau_2, & -\Delta_2 \leq U_{car} - U_{PT} \leq \Delta_2 \\
0, & U_{car} - U_{PT} > \Delta_2 
\end{cases} \tag{7}
\]

where, \(p(bus|PT)\) represents the conditional choice probability for the bus alternative under the condition of choosing public transit alternative. \(p(car)\) is the conditional choice probability of the car alternative. \(U_{bus}, U_{sub}, U_{car}\) and \(U_{PT}\) denote the excepted costs of bus, subway, car and public transit. \(\tau_1\) and \(\tau_2\) are parameters reflecting a traveler’s preference for the lower and upper level nested alternatives, respectively, \(0 \leq \tau_1 \leq 1, 0 \leq \tau_2 \leq 1\). \(\Delta_1\) and \(\Delta_2\) are the indifference thresholds of lower and upper level nests. The indifference thresholds reflect the level of rationality of a traveler. It is assumed that all travelers have the same level of rationality, that is, they have the same indifference thresholds, \(\Delta_1 \geq 0, \Delta_2 \geq 0\).

Equation (6) and (7) mean that:

a. When the excepted cost difference between bus (car) and subway (public transit) is greater than \(\Delta_1(\Delta_2)\), the traveler will definitely not choose the bus (car).

b. When the excepted cost difference between bus (car) and subway (public transit) is less than \(-\Delta_1(-\Delta_2)\), the traveler must choose the bus (car).

c. When the excepted cost difference between bus (car) and subway (public transit) is greater than \(-\Delta_1(-\Delta_2)\) and less than \(\Delta_1(\Delta_2)\), the traveler will make the choice randomly or according to his preference. \(\tau_1 = 0.5(\tau_2 = 0.5)\) represent that the traveler does not prefer any modes, and will choose randomly. The larger \(\tau_1(\tau_2)\) is, the more likely the traveler is to choose bus (car). \(\tau_1 = 1(\tau_2 = 1)\) if the traveler completely prefers bus (car) and \(\tau_1 = 0(\tau_2 = 0)\) if the traveler completely prefers subway (public transit).

Thus, the choice probabilities for the bus and subway under the condition of public transit can be expressed as equations (8) and (9):

\[
P(bus|PT) = \text{Prob}(U_{bus} - U_{sub} < -\Delta_1) \cdot 1 + \text{Prob}(-\Delta_1 \leq U_{bus} - U_{sub} \leq \Delta_1) \cdot \tau_1 \tag{8}
\]

\[
P(sub|PT) = 1 - P(bus|PT) \tag{9}
\]

And, the choice probabilities for car and public transit can be expressed as equations (10) and (11):

\[
P(car) = \text{Prob}(U_{car} - U_{PT} < -\Delta_2) \cdot 1 + \text{Prob}(-\Delta_2 \leq U_{car} - U_{PT} \leq \Delta_2) \cdot \tau_2 \tag{10}
\]

\[
P(PT) = 1 - P(car) \tag{11}
\]

IV. THE FORMULATION OF BRNL MODEL

The choice probabilities for the lower level nested alternatives (bus and subway) can be expressed as

\[
P(bus) = P(PT) \cdot P(bus|PT) \tag{12}
\]

\[
P(sub) = P(PT) \cdot P(sub|PT) \tag{13}
\]

Substitute equations (2) and (3) into equations (8) and (9)

\[
P(bus|PT) = \text{Prob}(U_{bus} - U_{sub} < -\Delta_1) \tag{14}
\]

According to the properties of the Gumbel distribution,

\[
P(bus|PT) = \frac{1}{1 + \exp(\theta_1(V_{bus} - V_{sub} + \Delta_1))}
\]

\[
P(bus) = P(PT) \cdot \text{Prob}(U_{bus} - U_{sub} < -\Delta_1)
\]

\[
P(sub) = P(PT) \cdot \text{Prob}(-\Delta_1 \leq U_{bus} - U_{sub} \leq \Delta_1)
\]

\[
P(car) = \text{Prob}(U_{car} - U_{PT} < -\Delta_2) \cdot 1 + \text{Prob}(-\Delta_2 \leq U_{car} - U_{PT} \leq \Delta_2) \cdot \tau_2
\]

\[
P(PT) = 1 - P(car)
\]
According to the properties of the Gumbel distribution,

\[
P_{\text{car}}(\theta) = \frac{1}{1 + \exp(\theta_2(V_{\text{car}} - V_{\text{PT}} + \Delta_2))} \cdot \frac{1 + \exp(\theta_1(V_{\text{bus}} - V_{\text{sub}}))}{\tau_2} \]

where, \( V_{\text{PT}}^* \) is the minimum expected travel cost for public transit

\[
V_{\text{PT}}^* = -\frac{1}{\theta_1} \cdot \ln[\exp(-\theta_1 V_{\text{bus}}) + \exp(-\theta_1 V_{\text{sub}})]
\]

Equations (12), (13), (15), (16) and (18) - (20) are BRNL model for two-level nest structure with two alternatives in one lower nest. In particular, when \( \Delta_1 = \Delta_2 = 0 \), BRNL model will degrade to the regular NL model.

The following is the situation of two-level nest structure with more than two alternatives in one lower nest as depicted in Figure 3.

The choice probability for the lower nested alternatives \( B_i (i = 1, 2, \ldots, m) \) can be expressed as

\[
P(B_i) = P(B_i|A_k) \cdot P(A_k)
\]

where, \( P(B_i|A_k) \) presents the choice probability for \( B_i \) under the condition of choosing the upper nested alternative \( A_k \) and \( P(A_k) \) is the choice probability for the alternative \( A_k \).

Suppose that \( U_{Ak} \) and \( U_{Bi} \) are the expected utilities for \( A_k \) and \( B_i \) and consist of two parts

\[
U_{Ak} = V_{Ak} + \varepsilon_{Ak}
\]

\[
U_{Bi} = V_{Bi} + \varepsilon_{Bi}
\]

\( V_{Ak} \) and \( V_{Bi} \) are the deterministic parts of the utilities and \( \varepsilon_{Ak} \) and \( \varepsilon_{Bi} \) are the random parts assumed to be distributed Gumbel with scale parameters, \( \theta_2 \) and \( \theta_1 \), respectively.

In lower nest, the traveler faces the following situations: First, consider the probability of choosing a single alternative \( B_i \). This would occur when: \( U_{Bi} > U_{Bj} + \Delta_1, j = 1, 2, \ldots, m; j \neq i, \Delta_1 > 0 \). \( \Delta_1 \) is the indifference threshold of lower nest. The probability \( P_1(\{B_i|A_k\}) \) can be expressed as

\[
P_1(\{B_i|A_k\}) = \frac{\exp(\theta_1 V_{Bi})}{\exp(\theta_1 V_{Bi}) + \sum_j \exp(\theta_1 (V_{Bi} + \Delta_1))}
\]

\( j = 1, 2, \ldots, m; j \neq i; \Delta_1 > 0; \theta_1 > 0 \)

Second, consider the case where the traveler prefers two alternatives, say \( B_i \) and \( B_j \), to all others but is indifferent between these two alternatives. This would occur when: \( U_{Bi} < U_{Bj} \leq U_{Bj} + \Delta_1, U_{Bi} > U_{Bk} + \Delta_1 \) or \( U_{Bi} < U_{Bj} \leq U_{Bj} + \Delta_1, U_{Bi} > U_{Bk} + \Delta_1 \). In this circumstance, the choice probability \( P_2(\{B_i,B_j|A_k\}) \) can be described as

\[
P_2(\{B_i,B_j|A_k\}) = \tau_{ij} \cdot P(B_i|A_k)
\]

\[
+ \tau_{ij} \cdot \exp(\theta_1 V_{Bj}) + \frac{\exp(\theta_1 (V_{Bj} + \Delta_1))}{\exp(\theta_1 V_{Bi}) + \exp(\theta_1 (V_{Bj} + \Delta_1))}
\]

where, \( P(B_i|A_k) \) refers to the probability of choosing \( B_i \) and \( B_j \) simultaneously. \( \tau_{ij} \) represents the level of preference of the traveler for choosing \( B_i \) to \( B_j \).

Further, it can be deduced that as (26), as shown at the bottom of the next page, Third, the traveler may prefer any of three alternatives, say \( B_i, B_j \) and \( B_k \), but with all these alternatives being equally preferred. This can occur under six possible orderings of costs:

1) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)
2) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)
3) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)
4) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)
5) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)
6) \( U_{Bj} > U_{Bk} > U_{Bj} > U_{Bk} \)

Using \( U_{Bj} > U_{Bj} > U_{Bk} \) as an example, indifference between \( B_i \) and \( B_k \) occurs when:

\( U_{Bj} - U_{Bj} < \Delta_1, U_{Bj} - U_{Bk} < \Delta_1 \) and \( U_{Bj} - U_{Bk} < \Delta_1 \) or \( U_{Bj} - U_{Bj} < \Delta_1, U_{Bj} - U_{Bk} < \Delta_1 \) and \( U_{Bj} - U_{Bk} < \Delta_1 \)
The probability that the traveler chooses three alternatives at the same time can be expressed as (27), as shown at the bottom of the page, where as (28), as shown at the bottom of the page, thus,

\[ P_3(B_i | A_k) = \tau_{i-jk} \cdot \sum P(B_{ijk} | A_k) \]  

(29)

where, the sum extends over the six orderings for \( B_i, B_j \) and \( B_k \), \( \tau_{i-jk} \) denotes the level of preference of the traveler for choosing \( B_i \) to \( B_j \) and \( B_k \).

Similarly, the aggregation process can be extended to four or more alternatives. The choice probability of alternative \( B_i \) is equal to the sum of the probabilities under all the conditions.

The probability that the traveler chooses \( A_k \) can be deduced in the same way.

\[
P_2(B_i | A_k) = \tau_{ij} \cdot \frac{\exp (\theta_1 V_{B_i})}{\exp (\theta_1 (V_{B_i} - \Delta_1)) + \exp (\theta_1 V_{B_i}) + \sum_k \exp (\theta_1 (V_{B_i} + \Delta_1))} \\ - \frac{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_k \exp (\theta_1 (V_{B_i} + \Delta_1))}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} \\ + \frac{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_k \exp (\theta_1 (V_{B_i} + \Delta_1))}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} \\ - \exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_k \exp (\theta_1 (V_{B_i} + \Delta_1)) \\
\]

(26)

First, \( P_1(A_k) \) is the probability of choosing a single alternative \( A_k \).

\[
P_1(A_k) = \exp (\theta_2 (V_{A_k} + V_{A_k}^*)) / \exp (\theta_2 (V_{A_k} + V_{A_k}^* + \Delta_2)) + \sum_l \exp (\theta_2 (V_{A_l} + V_{A_l}^* + \Delta_2)) \\
\]

(30)

Second, \( P_2(A_k) \) refers to the probability of choosing \( A_k \) and \( A_p \) simultaneously. (32), as shown at the bottom of the next page, where, \( \omega_{kp} \) represents the level of preference of the

\[
P(B_{ijk} | A_k) = \text{Prob} \left( U_{B_i} < U_{B_i} \leq U_{B_i} + \Delta_1 \cap U_{B_i} < U_{B_i} \leq U_{B_k} + U_{B_k} + \Delta_1 \cap U_{B_k} + \Delta_1 \cap U_{B_k} > U_{B_p} + \Delta_1 \right) \\ + \text{Prob} \left( U_{B_i} < U_{B_i} \leq U_{B_i} + \Delta_1 \cap U_{B_i} < U_{B_i} \leq U_{B_k} + U_{B_k} + \Delta_1 \cap U_{B_k} + \Delta_1 \cap U_{B_k} > U_{B_p} + \Delta_1 \right) \\
= \frac{Q_{BP} \exp (\theta_1 (V_{B_i} + \Delta_1))}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} + \frac{Q_{BP} \exp (\theta_1 V_{B_i})}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} \\
\]

(27)

\[
Q_{BP} = \frac{\exp (\theta_1 V_{B_i})}{\exp (\theta_1 (V_{B_i} - \Delta_1)) + \exp (\theta_1 V_{B_i}) + \sum_{\rho} \exp (\theta_1 (V_{B_{\rho}} + \Delta_1))} \\ - \frac{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_{\rho} \exp (\theta_1 (V_{B_{\rho}} + \Delta_1))}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} \\ + \frac{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_{\rho} \exp (\theta_1 (V_{B_{\rho}} + \Delta_1))}{\exp (\theta_1 V_{B_i}) + \exp (\theta_1 (V_{B_i} + \Delta_1))} \\ - \exp (\theta_1 V_{B_i}) + \exp (\theta_1 V_{B_i}) + \sum_{\rho} \exp (\theta_1 (V_{B_{\rho}} + \Delta_1)) \\
\]

(28)
traveler for choosing $A_k$ to $A_p$.

Third, The probability that the traveler chooses three alternatives $A_k$, $A_p$ and $A_q$ at the same time can be expressed as

$$P(A_{kpq}) = \frac{Q_{A3} \exp (\theta_2 (V_{Ak} + \Delta_2))}{\exp (\theta_2 V_{Ak}) + \exp (\theta_1 (V_{Ap} + \Delta_2))}$$

$$+ \frac{Q_{A3} \exp (\theta_2 V_{Ap})}{\exp (\theta_2 V_{Ak}) + \exp (\theta_1 (V_{Aq} + \Delta_2))}$$

$$+ \frac{Q_{A3} \exp (\theta_2 V_{Aq})}{\exp (\theta_2 V_{Ak}) + \exp (\theta_1 (V_{Aq} + \Delta_2))}$$

where as (34), as shown at the bottom of the page, thus,

$$P_3 (A_k) = \omega_{k-pq} \cdot \sum P (A_{kpq})$$

where, the sum extends over the six orderings for $A_k$, $A_p$ and $A_q$. $\omega_{k-pq}$ denotes the level of preference of the traveler for choosing $A_k$ to $A_p$ and $A_q$.

Similarly, the aggregation process can be extended to four or more alternatives. The choice probability of alternative $A_k$ is equal to the sum of the probabilities under all the conditions.

Taking the two-level nest structure with three alternatives in one lower nest as an example, the choice probability of any alternative is

$$P (B_1) = P (A_2) \cdot P (B_1 | A_2)$$

$$P (B_2) = P (A_2) \cdot P (B_2 | A_2)$$

$$P (B_3) = P (A_2) \cdot P (B_3 | A_2)$$

$$P (B_1 | A_2) = P_1 (B_1 | A_2) + \tau_{12} \cdot P_2 (B_2 | A_2)$$

$$+ \tau_{13} \cdot P_2 (B_3 | A_2) + \tau_{1-3} \cdot P_3 (B_3 | A_2)$$

$$P (B_2 | A_2) = P_1 (B_2 | A_2) + \tau_{23} \cdot P_2 (B_2 | A_2)$$

$$+ \tau_{21} \cdot P_2 (B_1 | A_2) + \tau_{2-3} \cdot P_3 (B_3 | A_2)$$

$$P (B_3 | A_2) = P_1 (B_3 | A_2) + \tau_{32} \cdot P_2 (B_3 | A_2)$$

$$+ \tau_{31} \cdot P_2 (B_1 | A_2) + \tau_{3-12} \cdot P_3 (B_3 | A_2)$$

$$P (B_1 | A_2) + P (B_2 | A_2) + P (B_3 | A_2) = 1$$

$$\tau_{12} + \tau_{13} = 1; \tau_{13} + \tau_{31} = 1;$$

$$\tau_{23} + \tau_{32} = 1; \tau_{1-23} + \tau_{2-3} + \tau_{3-12} = 1$$

$$P (A_1) = p (A_1) + \omega_{12} \cdot p (A_12)$$

$$+ \omega_{13} \cdot p (A_31) + \omega_{1-23} \cdot p (A_123)$$

$$P (A_2) = p (A_2) + \omega_{23} \cdot p (A_23)$$

$$+ \omega_{21} \cdot p (A_12) + \tau_{2-13} \cdot p (A_123)$$

$$P (A_3) = p (A_3) + \omega_{32} \cdot p (A_23)$$

$$+ \omega_{31} \cdot p (A_31) + \omega_{3-12} \cdot p (A_123)$$

$$P_2 (A_k) = \omega_{kp} \cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$\cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$\cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$q = 1, 2, \ldots, n; q \neq k \text{ and } q \neq p; \Delta_2 > 0$$

$$Q_{A3} = \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q (\theta_2 (V_{Aq} + \Delta_2))}$$

$$- \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q (\theta_2 (V_{Aq} + \Delta_2))}$$

$$+ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q (\theta_2 (V_{Aq} + \Delta_2))}$$

$$- \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q (\theta_2 (V_{Aq} + \Delta_2))}$$

$$\cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$\cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$\cdot \left[ \frac{\exp (\theta_2 V_{Ap})}{\exp (\theta_2 (V_{Ak} + \Delta_2)) + \exp (\theta_2 V_{Ap}) + \sum_q \exp (\theta_2 (V_{Aq} + \Delta_2))} \right]$$

$$q = 1, 2, \ldots, n; q \neq k \text{ and } q \neq p; \Delta_2 > 0$$
\[ P(A_1) + P(A_2) + P(A_3) = 1 \] (47)
\[ \omega_{12} + \omega_{21} = 1; \omega_{13} + \omega_{31} = 1; \omega_{23} + \omega_{32} = 1; \omega_{12-3} + \omega_{2-13} + \omega_{3-12} = 1 \] (48)

V. SOLUTION ALGORITHM

In this section, the nested method of successive average (NMSA) algorithm is developed to solve the aforementioned problem. The followings are the steps of the algorithm.

Step 1. Initialization. Let the iteration counter \( n_1 = n_2 = 0 \) and the convergence tolerance \( \varepsilon > 0 \). Randomly load the flow of upper nest meeting the total demand \( N \), that is, \( f_{0}^{\text{car}} + f_{P}^{\text{PT}} \).

Step 2. Cost calculation. Calculate the cost of each mode, and then determine the flow of upper nest according to (18)-(20). Randomly load the flow of lower nest that satisfying \( P(PT) \cdot N \), and then calculate the flow of the lower nest given by (15)-(16).

Step 3. Flow update. Generate the auxiliary flow pattern \( \mu \) and the outer loop updates the flow of each mode.
\[ f_{n+1}^{\text{car}} = f_{n}^{\text{car}} + 1/n_1 (y_{n}^{(n)} (3) - f_{n}^{(n)}) \]
\[ f_{n+1}^{\text{PT}} = f_{n}^{\text{PT}} + 1/n_1 (y_{n}^{(n)} (3) - f_{n}^{(n)}) \] (49)

The nested inner loop updates the flow of each mode.
\[ f_{n+1}^{\text{bus}} = f_{n}^{\text{bus}} + 1/n_2 (y_{n}^{(n)} (3) - f_{n}^{(n)}) \]
\[ f_{n+1}^{\text{sub}} = f_{n}^{\text{sub}} + 1/n_2 (y_{n}^{(n)} (3) - f_{n}^{(n)}) \] (50)

Step 4. Checking the convergence. If (51), (52) is satisfied, the algorithm stops and the present \( f^{(n)} \), \( f^{(n)} \) are the equilibrium results.
\[ \frac{\sqrt{\sum (f_{n+1}^{(n)} - f_{n}^{(n)})^2}}{\sum f_{n}^{(n)}} \leq \varepsilon \] (51)
\[ \frac{\sqrt{\sum (f_{n+1}^{(n)} - f_{n}^{(n)})^2}}{\sum f_{n}^{(n)}} \leq \varepsilon \] (52)

Otherwise, make \( n_1 = n_1 + 1, n_2 = n_2 + 1 \) and return to Step 2.

VI. NUMERICAL EXAMPLE

We demonstrate and validate BRNL model with a three-mode transportation network shown as Figure 1 in this section. Moreover, the sensitivity analysis is performed to depict the impacts of indifference threshold and preference on a traveler’s choice behavior.

A. GENERALIZED TRAVEL COST

The followings are the generalized travel costs of different transport modes, namely the fixed terms in the excepted travel costs.

The generalized travel cost of a car user is
\[ V_{\text{car}} = \alpha \cdot T(f_{\text{car}}) + F + \pi_1 \] (53)
where, \( \alpha \) presents the unit cost of travel time, \( F \) is the operating cost of choosing the car, and \( \pi_1 \) is the parking fee. \( T(f_{\text{car}}) \) denotes the travel time by car and throughout the paper, the travel time is expressed as follow
\[ T(f_{\text{car}}) = t_{0}^{\beta} \left[ 1 + \beta \left( \frac{f_{\text{car}}}{b} \right)^n \right] \] (54)
where, \( t_{0}^{\beta} \) represents travel time of car under the free-flow condition. Let \( f_{\text{car}} \) denote the numbers of car travelers and \( b \) be the capacity of link. \( \beta \) and \( n \) are the parameters of equation (13).

The generalized travel cost of a bus user is
\[ V_{\text{bus}} = \alpha \cdot T_{\text{bus}} + \mu_1 \cdot g(f_{\text{bus}}) + \pi_2 \] (55)
where, \( T_{\text{bus}} \) represents the travel time of bus. \( \mu_1 \) is the unit cost of discomfort for bus, \( \pi_2 \) is the bus fare, and \( f_{\text{bus}} \) is the numbers of bus travelers. An increasing function of the number of car users is adopted to describe the body congestion discomfort of the traveler. The function is expressed as follow
\[ g(f_{\text{bus}}) = 0.05f_{\text{bus}}^2 + 0.25f_{\text{bus}} \] (56)

The generalized travel cost of a subway user is
\[ V_{\text{sub}} = \alpha \cdot T_{\text{sub}} + \mu_2 \cdot g(f_{\text{sub}}) + \pi_3 \] (57)
where, \( T_{\text{sub}} \) is the travel time of subway and \( \mu_2 \) is the unit cost of discomfort for subway. \( \pi_3 \) represents the subway fare. \( f_{\text{sub}} \) denotes the numbers of subway travelers. The body congestion discomfort experienced by a subway user can be described through an increasing function of the number of users selecting this mode. The function takes the form as follow
\[ g(f_{\text{sub}}) = 0.05f_{\text{sub}}^2 + 0.25f_{\text{sub}} \] (58)

The parameters of the generalized travel costs of different transport modes in equation (53)-(55) and (57) are \( \alpha = 30 \text{Y} / \text{h}, T_{\text{sub}} = 1.5h, T_{\text{bus}} = 2h, t_{0}^{\beta} = 1h, \mu_1 = 0.01 \text{Y} , \mu_2 = 0.005 \text{Y}, \pi_1 = 20 \text{Y}, \pi_2 = 4 \text{Y}, \pi_3 = 8 \text{Y}, F = 10 \text{Y}, = b 800 \text{pcu/h}, \beta = 1.5, n = 4 \). The total demand is fixed at 2000 pcu/h. The dispersion parameters \( \theta_1 \) and \( \theta_2 \) are supposed to be 1 and 0.5.

B. SENSITIVITY ANALYSIS

First, the effect of monetary cost on the probability of choosing car for travelers when \( \Delta_1 = 0, \tau_1 = 0.5, \tau_2 = 0.5 \) are shown in Figure 4. According to equation (53), the generalized travel cost of choosing the car includes time cost and monetary cost. \( I \) represents monetary cost, which consists of operating cost and parking fee. We can see that the probability of choosing car decreases as monetary cost increases in NL model(\( \Delta_2 = 0 \)), namely the choices of travelers depend on the costs.

Because of the different decision-making mechanisms, BRNL model gives different results relative to NL model. From Figure 4 we can see that, when \( \Delta_2 = 20 \), the probabilities of choosing a car are both 50% when the value of \( I \) is 40 to 60, which means that the choices of travelers do not depend on the cost but are random if the cost difference
Choosing car decreases as $\Delta_1$ increases. This is because the minimum expected cost of public transit decreases as $\Delta_1$ increase. It further presents that the value of the indifference threshold for lower nest will affect the choice probability of upper nest.

The choice probabilities of bus and subway increase as $\Delta_2$ increase in Figure 6, which means that the value of the indifference threshold for upper nest will affect the choice probability of lower nest. The probability of choosing bus is going to be equal to the probability of choosing subway as $\Delta_1$ increase. This means that the traveler tends to choose randomly with the increase of indifference threshold when he does not prefer any mode.

Then, Figure 7 depicts the joint impacts of parameters $\tau_2$ and $\Delta_2$ on traveler’s choice behavior. The other parameters are $\tau_1 = 0.5$ and $\Delta_1 = 0$. The model degrades into the regular NL model if $\Delta_1 = \Delta_2 = 0$. In NL model, the probability of choosing car does not change with the change of $\tau_2$, which indicates that the traveler’s choice are not affected by the preference without considering the bounded rationality of the traveler. In BRNL model, the probability of choosing car varies with the change of the preference parameters. It illustrates that, if the traveler is boundly rational, the probability of choosing the car increases with the increase of preference parameter when the cost difference between car and public transit falls within an indifference band. Moreover, the higher the indifferent threshold is, the greater the effect of the traveler’s preference on the choice. Especially, if $\Delta_2$ is large enough, which indicates that the cost difference cannot be rationally distinguished by traveler, all the choices the traveler made are according to his preferences. At this case, the choice probability of car is constantly equal to $\tau_2$.

VII. CONCLUSION

In this article, BRNL model is developed by introducing the idea of the indifference threshold. In particular, BRNL model will degrade to the regular NL model when the indifference threshold is equal to zero. For the purpose of demonstrating the proposed model and comparing it with regular NL model, an example is used to represent the equilibrium results and the sensitivity analysis is implemented.
It can be deduced that, different from NL model, the choices of the travelers in BRNL model does not depend on the costs always. The indifference threshold and preferences will affect the choice probability if the cost difference between choices is within the indifference band. In addition, the value of the indifference threshold of lower (upper) nest will affect the choice probability of upper (lower) nest. The mode choice probability of the traveler will increase with the increase of preference parameter. Moreover, the larger the indifference threshold is, the greater the effect of the traveler’s preference on the mode choice. In particular, if the indifference threshold is large enough, the travelers’ choices will always depend on the preference.

The values of the parameters in this article are given and the further research is to estimate these parameters by conducting a stated preference survey to improve the accuracy of the model. It is also meaningful to further extend BRNL model to day-to-day choice behavior.

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REFERENCES

[1] Y. Shao, W. Wang, and L. Cheng, “Study on the individual travel mode choice based on the discrete choice theory,” J. Highway Transp. Res. Devolop., vol. 23, no. 8, pp. 110–115, Aug. 2006.

[2] Q. Sun, L. Zhu, and B. Chen, “A dynamic generalized cost based Logit model for passenger corridors,” J. Transp. Syst. Eng. Inf. Technol., vol. 13, no. 4, pp. 15–22, 2013.

[3] H. Z. Guan, Disaggregate Model-A Tool of Traffic Behavior Analysis. Beijing, China: Communication Press 2004.

[4] M. Ben-Akiva and S. R. Lerman, Discrete Choice Analysis: Theory and Application to Travel Demand. Cambridge, MA, USA: MIT Press, 1985.

[5] H. A. Simon, “A behavioral model of rational choice,” Quart. J. Econ., vol. 69, no. 1, pp. 99–118, 1955.

[6] R. E. Quandt, “A probabilistic theory of consumer behavior,” Quart. J. Econ., vol. 70, no. 4, pp. 507–536, 1956.

[7] N. Georgescu-Roegen, “Threshold in choice and the theory of demand,” Econometrika, vol. 26, no. 1, p. 157, Jan. 1958.

[8] D. Kahneman and A. Tversky, “Prospect theory: An analysis of decision under risk,” Econ. J. Econ. Soc., vol. 47, no. 2, pp. 263–291, 1979.

[9] A. Tversky and D. Kahneman, “Advances in prospect theory: Cumulative representation of uncertainty,” J. Risk Uncertainty, vol. 5, no. 4, pp. 297–323, Oct. 1992.

[10] H. S. Mahmassani and G.-L. Chang, “On boundedly rational user equilibrium in transportation systems,” Transp. Sci., vol. 21, no. 2, pp. 89–99, May 1987.

[11] H. S. Mahmassani and G.-L. Chang, “Experiments with departure time choice dynamics of urban commuters,” Transp. Res. B, Methodol., vol. 20, no. 4, pp. 297–320, Aug. 1986.

[12] H. S. Mahmassani and R.-C. Jou, “Transferring insights into commuter behavior dynamics from laboratory experiments to field surveys,” Transp. Res. A, Policy Pract., vol. 34, no. 4, pp. 243–260, May 2000.

[13] M. Ridwan, “Fuzzy preference based traffic assignment problem,” Transp. Res. C, Emerg. Technol., vol. 12, nos. 3–4, pp. 209–233, Jun. 2004.

[14] Y. Lou, Y. Yin, and S. Lawphongpanich, “Robust congestion pricing under bounded rational user equilibrium,” Transp. Res. B, Methodol., vol. 44, no. 1, pp. 15–28, Jan. 2010.

[15] T. Li, H. Z. Guan, and K. K. Liang, “Day-to-day dynamical evolution of network traffic flow under bounded rational view,” Acta Phys. Sinica, vol. 65, no. 15, pp. 17–27, 2016.

[16] X. Di, H. Xu, L. Liu, S. Zhu, and D. M. Levinson, “Indifference bands for boundedly rational route switching,” Transportation, vol. 44, no. 5, pp. 1169–1194, Sep. 2017.