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Mathematical model and interpretation of crowding effects on SARS-CoV-2 using Atangana-Baleanu fractional operator

Rupakshi Mishra Pandeya, Ankita Chandola\textsuperscript{a}, and Ritu Agarwal\textsuperscript{b}

\textsuperscript{a}AMITY INSTITUTE OF APPLIED SCIENCES, AMITY UNIVERSITY UTTAR PRADESH, NOIDA, INDIA
\textsuperscript{b}DEPARTMENT OF MATHEMATICS, MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY, JAIPUR, RAJASTHAN, INDIA

1 Introduction

1.1 About COVID-19

Coronaviruses are large, enveloped, positive-stranded RNA viruses, that are responsible for infecting large number of mammalian and avian species\cite{1}. These viruses have spike-like projections of glycoproteins on their surface (see Fig. 1), which appears like a crown when viewed under the electron microscope; hence, the name coronavirus.

The first case of the ongoing pandemic of COVID-19 was identified in Wuhan, China in December 2019. It is a contagious disease that is caused by Severe Acute respiratory Syndrome Coronavirus 2 (SARS-CoV-2) and has spread worldwide across 210 nations. Coronavirus causes infections that resemble the common cold-type symptoms but became the most disastrous virus, taking immense toll in terms of human, economic, and social impact resulting in the highest death tolls. The numerical data and the graphical interpretation of the total cases, new cases, and deaths as of February 2021 taken from various sources\cite{2–4} are shown in Figs. 2–5.

Currently, more than 50 COVID-19 vaccines are under various phases of trials. In some countries, vaccination drives have also begun. Even then, until the vaccine is given to every person, preventive measures such as social distancing, sanitization, and masks are necessary as the COVID-19 cases are rising on a daily basis. Various strains of COVID-19 have been found in various countries in the world. A new variant found in late December in
the United Kingdom, named as “VUI 202012/01” is one of the deadly mutants of the virus, which has a 70% transmission rate. Crowding may worsen the current situation.

2 Spread of new SARS-CoV-2 variant in India

After the first case was recorded on January 31, 2020, the number of COVID-19 cases in India rose at a comparatively slow pace. Around September 15, daily cases peaked at about 98,000, then gradually declined for the next 5 months. A month into 2021, it seemed that
FIG. 3 Graphical interpretation of COVID-19 cases worldwide as of February 2021. Data collected on February 4, 2021 and April 22, 2021 from https://ourworldindata.org/coronavirus-data-explorer?yScale=log&zoomToSelection=true&country=OWID_WRLAfricaEuropeNorthAmericaAsiaSouthAmericaOceania&region=World&casesMetric=true&interval=smoothed&hideControls=true&smoothing=7&pickerMetric=location&pickerSort=asc.

FIG. 4 Graphical interpretation of new COVID-19 cases per day in India as of February 2021. Data collected on February 4, 2021 and April 22, 2021 from https://www.google.com/search?client=firefox-b-d&q=covid-19#wptab=s:H4sIAAAAAAAAANgVuLV9t9cNMwv5y18zJecTozSv9Y5mnWtoXmO04eKzsgv80ry5ypFNljYoOyVLgLpVB1ajBIBXOhCvHsYuLz2SE3MKckILkksKV7EKpicX55f1iWWVrFAMEgMAoubRkiEAAAA.
India’s history will be different from that of the United States and Brazil, all of which experienced several waves of the disease and recorded immense amount of deaths last year.

The daily infections began to rise in the mid of February 2021 and since then, the numbers of daily cases have risen immensely as compared to the previous year. Though Maharashtra currently has the highest number of cases, the number of infected across India continues to rise inexorably, following the classic trend of a second wave.

Viruses are increasingly changing due to replication, and changes in the SARS-CoV-2 virus have been found around the world as a result of evolution and adaptation processes. While the majority of new mutations would have no effect on the virus’s propagation, certain mutations or combinations of mutations may give the virus a selective advantage, such as increased transmissibility or the ability to evade the host immune response.

Variants are most likely the reasons for such a drastic increase in the number of COVID-19 cases and deaths. Various variants of SARS-CoV-2 virus have been found in India: B.1.36 that contains specific mutation N440K and B.1.617 that contains two specific mutations E484Q and L452R. The mutation of SARS-CoV-2 virus in India has an increased transmissibility rate. According to the researchers and doctors, this mutated virus spreads in less than 1 min as compared to the 10 min spreading time of the virus last year.

India has experienced a drastic change in the number of cases and deaths from February 2021 to April 2021.

FIG. 5 Graphical interpretation of deaths per day due to COVID-19 in India as of February 2021. Data collected on February 4, 2021 and April 22, 2021 from https://www.google.com/search?client=firefox-b-d&q=covid-19#wptab=s:H4sIAAAAAAAAAONgVuLV7c3Nwly5k6OL8zJecT0zS3w8sc9YSmnSWtOXm004eIKzsgvd80ry5ypFNlJYoOyVlgEpVB1ajBi8XOhCvHuL25EMKckIlOksKV7EKpicX55fl1iWlWRarFAMEgMAoubRkIEAAAA.
The statistics of COVID-19 cases in India is dangerous and scary. The increase in the number of cases and the number of deaths from February 2021 to April 2021 shown in Figs. 4–8 represent how bad the situation is and that various strict measures need to be taken and followed.

Overcrowding should be avoided at any cost. Nonessential travel should be avoided to delay the importation and dissemination of the current SARS-CoV-2 variants of concern. In addition to warnings against unnecessary travel and bans on travel for those who have been infected, travel precautions such as testing and quarantining of travelers should be maintained, particularly for those coming from places where the virus is prevalent. Earlier, vaccination drives focused on protecting those most at risk from severe disease. It was important to use the available vaccines to provide protection for those who were most vulnerable and for key workers against the virus. But as per the current situation, even the young generation is at risk and hence the decision by the Government of India for vaccination to all above 18 years. Vaccination done till date in India is represented in Figs. 9–11.

India is experiencing more than 2 lakh cases per day since April 14, 2021 and considering the population of India and the fact that transmissibility rate of this virus is faster, it will only take 2–3 weeks for the cases to reach more than 5 lakh statistics and the availability of resources for the treatment will reduce.

![Graphical interpretation of COVID-19 cases in India as of April 2021.](https://ourworldindata.org/coronavirus-data-explorer?yScale=log&zoomToSelection=true&country=OWID_WRLAfricaEuropeNorthAmericaAsiaSouthAmericaOceania&region=World&casesMetric=true&interval=smoothed&hideControls=true&smoothing=7&pickerMetric=location&pickerSort=asc)
FIG. 7 Graphical interpretation of new COVID-19 cases per day in India as of April 2021. Data collected on February 4, 2021 and April 22, 2021 from https://www.google.com/search?client=firefox-b-d&q=covid-19#wptab=s:H4siAAAAAQAONgVuLVTrc3Nw5k6OL8zJecTozS3w8sc9YSmnSWtOXmO04eliKzsgvd80rySypFNlJYoOyVLgEpVB1ajBI8XOhCvHyL2SE3MKckILkksKV7EKpcX5sf1iWWVrFAMEgMAoubRkJEAAAA.

FIG. 8 Graphical interpretation of deaths per day due to COVID-19 in India as of April 2021. Data collected on February 4, 2021 and April 22, 2021 from https://www.google.com/search?client=firefox-b-d&q=covid-19#wptab=s:H4siAAAAAQAONgVuLVTrc3Nw5k6OL8zJecTozS3w8sc9YSmnSWtOXmO04eliKzsgvd80rySypFNlJYoOyVLgEpVB1ajBI8XOhCvHyL2SE3MKckILkksKV7EKpcX5sf1iWWVrFAMEgMAoubRkJEAAAA.
**FIG. 9** Graphical interpretation of vaccination done for COVID-19 in India as of April 2021. Data collected on April 22, 2021 from https://www.google.com/search?client=firefox-b-d&q=covid+19+and+vaccination.

**FIG. 10** COVID-19 cases across India as of February 4, 2021. Data collected on February 4, 2021 and April 22, 2021 from https://www.covid19india.org/ and https://www.worldometers.info/world-population/india-population/.
To avoid such a situation, precautionary measures have to be taken, not just by the government but by every individual.

Different methods of mathematical modeling involving fractional calculus exist in the literature (see, e.g., Refs. [5, 6]) and using these methods, mathematical models for various diseases and phenomena have been studied by various authors (see, e.g., Refs. [7–14]). Recently, various models have been created to study the spread of the COVID-19 virus, its mutations, effect of lockdown, and other precautionary measures (see, e.g., Refs. [15–31]).

In this chapter, we will create a model for crowding effects on COVID-19 using Atangana-Baleanu (AB) fractional operator. We will discuss the existence and uniqueness of the system of equations using fixed-point method. We will also interpret the model explaining its significance.

3 Model for crowding effects on COVID-19

We will use the Caputo AB-derivative to define a model for COVID-19 showing crowding effects. The Caputo AB-derivative is defined as [32]

\[
(0^{ABC}_{t} D_{t}^\xi x)(t) = \frac{B(\xi)}{1-\xi} \int_{0}^{t} x'(u) E_{\xi} \left[ -\xi \frac{(t-u)^{\xi}}{1-\xi} \right] du,
\]

where \( \xi \in [0, 1] \), \( x' \in H(a, b) \), \( a \leq b \), and \( B(\xi) \) is a normalizing positive function satisfying \( B(0) = B(1) = 1 \) and \( E_{\alpha}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n + 1)} \) [33].
We propose a model with recruitment rate $\omega N$ for the susceptible individual and non-linear incidence rate $\frac{\alpha_2 SI}{1 + \alpha_1 I}$. Consider the following system of equations:

$$
\begin{align*}
_0^{ABC}D^\xi_t(S(t)) &= \omega N - \frac{\alpha_2 SI}{1 + \alpha_1 I} - \omega S, \\
_0^{ABC}D^\xi_t(I(t)) &= \frac{\alpha_2 SI}{1 + \alpha_1 I} - (\gamma + \omega)I, \\
_0^{ABC}D^\xi_t(R(t)) &= \gamma I - \omega R,
\end{align*}
$$

(1)

where

- $\alpha_2 I$ = infection force of the disease,
- $\frac{1}{1 + \alpha_1 I}$ = crowding effect,
- $S(t)$ = susceptible population at time $t$,
- $I(t)$ = infectious population at time $t$,
- $R(t)$ = recovered population at time $t$,
- $\omega$ = death rate,
- $\gamma$ = recovery rate,
- $\alpha_2$ = transmission coefficient,
- $N(t)$ = total constant population,

$$N(t) = S(t) + I(t) + R(t)$$

(2)

with

$$N(0) = S(0) + I(0) + R(0)$$

(3)

and

$$S(0) = S_0 \geq 0, \ I(0) = I_0 \geq 0, \ R(0) = R_0 \geq 0.$$  

(4)

### 3.1 Solution of the system of equations

In this section, we will need the following results to find the solution of the system of Eqs. (1).

1. The left Riemann-Liouville integral is given by [34, 35]

$$[_{0}J_t^\xi](x) = \frac{1}{\Gamma(\xi)} \int_0^t (t-u)^{\xi-1} x(u) du, \quad \xi > 0.$$

2. The $AB$-fractional integral is given by [36]

$$[_{0}^{AB}J_t^\xi](x) = \frac{1 - \xi}{B(\xi)} x(t) + \frac{\xi}{B(\xi)} [_{0}J_t^\xi](x).$$

3. For $0 < \xi < 1$ [37, 38],

$$
_{0}^{ABC}J_t^\xi[_{0}^{ABC}D_t^{\xi}(x)](t) = x(t) - x(0) E_\xi(\lambda t^\xi) - \frac{\xi}{1 - \xi} E_{\xi, \xi+1}(\lambda t^\xi).
$$

$$= x(t) - x(0).$$
We will now consider system of Eqs. (1), and using $AB$-fractional integral on both side of the system of Eqs. (1), we get

$$\frac{\alpha_{2}SI}{1 + \alpha_{1}I} - \omega S]$$

Using the result for $\frac{\alpha_{2}SI}{1 + \alpha_{1}I} - (\gamma + \omega)I$, we substitute the initial conditions (4), the earlier system of Eqs. (6) become

$$\frac{\alpha_{2}SI}{1 + \alpha_{1}I} - (\gamma + \omega)I]$$

Substituting the initial conditions (4), the earlier system of Eqs. (6) become

$$\frac{\alpha_{2}SI}{1 + \alpha_{1}I} - (\gamma + \omega)I]$$

For convenience, we replace

$$H_{1}(t, S) = \frac{\alpha_{2}SI}{1 + \alpha_{1}I} - \omega S,$$

$$H_{2}(t, I) = \frac{\alpha_{2}SI}{1 + \alpha_{1}I} - (\gamma + \omega)I,$$

$$H_{3}(t, R) = \gamma I - \omega R.$$

Using system of Eqs. (8) in Eq. (7)

$$S(t) - S_{0} = \frac{\alpha_{2}SI}{1 + \alpha_{1}I} - \omega S,$$

$$I(t) - I_{0} = \frac{\alpha_{2}SI}{1 + \alpha_{1}I} - (\gamma + \omega)I,$$

$$R(t) - R_{0} = \frac{\alpha_{2}SI}{1 + \alpha_{1}I} - \omega S.$$

Using the result for $\frac{\alpha_{2}SI}{1 + \alpha_{1}I} - \omega S]$ in system of Eqs. (9)

$$S(t) - S_{0} = 1 - \frac{\alpha_{2}SI}{B(\xi)} H_{1}(t, S) + \frac{\alpha_{2}SI}{B(\xi)} 0^{\xi} H_{1}(t, S),$$

$$I(t) - I_{0} = 1 - \frac{\alpha_{2}SI}{B(\xi)} H_{2}(t, I) + \frac{\alpha_{2}SI}{B(\xi)} 0^{\xi} H_{2}(t, I),$$

$$R(t) - R_{0} = 1 - \frac{\alpha_{2}SI}{B(\xi)} H_{3}(t, R) + \frac{\alpha_{2}SI}{B(\xi)} 0^{\xi} H_{3}(t, R).$$
The system of Eqs. (10) is the solution of the system of Eqs. (1).

Rewriting the system of Eqs. (10) as

\[
T[S(t)] = \frac{1 - \xi}{B(\xi)} H_1(t, S) + \frac{\xi}{B(\xi)} 0^3 H_1(t, S),
\]

\[
T[I(t)] = \frac{1 - \xi}{B(\xi)} H_2(t, I) + \frac{\xi}{B(\xi)} 0^3 H_2(t, I),
\]

\[
T[R(t)] = \frac{1 - \xi}{B(\xi)} H_3(t, R) + \frac{\xi}{B(\xi)} 0^3 H_3(t, R).
\]

### 3.2 Existence of solution

We will discuss the existence of the solution of the system of Eqs. (1) by using the fixed-point method.

**Theorem 1.** The kernels \( H_1, H_2, \) and \( H_3 \) satisfy the Lipschitz condition.

**Proof.** Consider

\[
\| H_1(t, S) - H_1(t, S_1) \| = \left\| \left( \omega N - \frac{\alpha_2 S I}{1 + \alpha_1 I} - \omega S \right) - \left( \omega N - \frac{\alpha_2 S_1 I}{1 + \alpha_1 I} - \omega S_1 \right) \right\|
\]

\[
\leq \left\| \omega N - \frac{\alpha_2 S I}{1 + \alpha_1 I} - \frac{\alpha_2 S_1 I}{1 + \alpha_1 I} + \omega S + \omega S_1 \right\| + \| \omega N - \omega N \|
\]

\[
= \left\| \left( \frac{\alpha_2 I}{1 + \alpha_1 I} + \omega \right) (S - S_1) \right\|
\]

\[
= \frac{\alpha_2 I}{1 + \alpha_1 I} + \omega \| (S - S_1) \|
\]

where \( g_1 > 0 \) is a Lipschitz constant. Hence, the Lipschitz condition is satisfied for \( H_1 \). Similarly, the other kernels \( H_2 \) and \( H_3 \) satisfy the Lipschitz condition as

\[
\| H_2(t, I) - H_2(t, I_1) \| \leq g_2 \| (I - I_1) \|,
\]

\[
\| H_3(t, R) - H_3(t, R_1) \| \leq g_3 \| (R - R_1) \|,
\]

where \( g_2, g_3 > 0 \) are Lipschitz constant.

**Corollary 1.** The kernels \( H_1, H_2, \) and \( H_3 \) are contraction maps if the following conditions are satisfied:

\[
0 \leq g_1 < 1, \quad 0 \leq g_2 < 1, \quad \text{and} \quad 0 \leq g_3 < 1.
\]

We will use “Arzela-Ascoli theorem” to prove the next result.

**Lemma 1.** Arzela-Ascoli theorem. Let \( \Omega \) be a compact Hausdorff metric space. Then, \( \bar{N} \in C(\Omega) \) is relatively compact iff \( \bar{N} \) is uniformly bounded and uniformly continuous.
**Theorem 2.** Let \( \tilde{N} \subset P \) be bounded and

\[
\begin{align*}
\| S(t_2) - S(t_1) \| &\leq D_1 \| t_2 - t_1 \|, \\
\| I(t_2) - I(t_1) \| &\leq D_2 \| t_2 - t_1 \|, \\
\| R(t_2) - R(t_1) \| &\leq D_3 \| t_2 - t_1 \|, 
\end{align*}
\]

(16)

where \( S, I, R \in \tilde{N} \). Then, \( T(\tilde{N}) \) is compact.

**Proof.** We assume there exists constants \( l, m, n > 0 \) such that

\[
\| S(t) \| < l, \quad \| I(t) \| < m, \quad \| R(t) \| < n.
\]

(17)

Let

\[
X_1 = \max_{0 \leq t \leq 1, \ 0 \leq S < l} H_1(t, S(t)), \\
X_2 = \max_{0 \leq t \leq 1, \ 0 \leq I < m} H_2(t, I(t)), \\
X_3 = \max_{0 \leq t \leq 1, \ 0 \leq R < n} H_3(t, R(t)).
\]

(18)

For all \( S, I, R \in \tilde{N} \), we have

\[
\begin{align*}
\| T[S(t)] \| &= \left\| \frac{1 - \xi}{B(\xi)} H_1(t, S) + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) H_1(t, S) \right\|, \\
\| T[I(t)] \| &= \left\| \frac{1 - \xi}{B(\xi)} H_2(t, I) + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) H_2(t, I) \right\|, \\
\| T[R(t)] \| &= \left\| \frac{1 - \xi}{B(\xi)} H_3(t, R) + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) H_3(t, R) \right\|.
\end{align*}
\]

(19)

Consider

\[
\begin{align*}
\| T[S(t)] \| &= \left\| \frac{1 - \xi}{B(\xi)} H_1(t, S) + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) H_1(t, S) \right\| \\
&\leq \frac{1 - \xi}{B(\xi)} \| H_1(t, S) \| + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) \int_0^t (t-u)^{\xi-1} H_1(u, S) du \| \\
&\leq \frac{1 - \xi}{B(\xi)} \| H_1(t, S) \| + \frac{\xi}{B(\xi)} \xi \Gamma(\xi) \| H_1(t, S) \||
\end{align*}
\]

(20)

\[
\begin{align*}
&\leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t-0)^{\xi}}{\xi \Gamma(\xi)} \right) X_1,
\end{align*}
\]

Similarly,

\[
\begin{align*}
\| T[I(t)] \| &\leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t-0)^{\xi}}{\xi \Gamma(\xi)} \right) X_2,
\end{align*}
\]

(21)
\[ \| T[R(t)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t - 0)^\xi}{B(\xi)\Gamma(\xi)} \right) X_3. \] (22)

Consequently \( T(\tilde{N}) \) is bounded.

Now, consider

\[
\| T[S(t_2)] - T[S(t_1)] \| \\
= \left\| \left( \frac{1 - \xi}{B(\xi)} H_1(t_2, S(t_2)) + \frac{\xi}{B(\xi)} t^{\xi} H_1(t_2, S(t_2)) \right) - \left( \frac{1 - \xi}{B(\xi)} H_1(t_1, S(t_1)) + \frac{\xi}{B(\xi)} t^{\xi} H_1(t_1, S(t_1)) \right) \right\| \\
\leq \frac{1 - \xi}{B(\xi)} \| H_1(t_2, S(t_2)) - H_1(t_1, S(t_1)) \| \\
+ \frac{\xi}{B(\xi)\Gamma(\xi)} \left| \int_0^{t_2} (t_2 - u)^{\xi-1} H_1(u, S(u)) du - \int_0^{t_1} (t_1 - u)^{\xi-1} H_1(u, S(u)) du \right| \\
= \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) \| H_1(t_2, S(t_2)) - H_1(t_1, S(t_1)) \|. \] (23)

Similarly,

\[
\| T[I(t_2)] - T[I(t_1)] \| = \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) \| H_2(t_2, I(t_2)) - H_2(t_1, I(t_1)) \|, \] (24)

\[
\| T[R(t_2)] - T[R(t_1)] \| = \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) \| H_3(t_2, R(t_2)) - H_3(t_1, R(t_1)) \|. \] (25)

Further, we consider

\[
\| H_3(t_2, R(t_2)) - H_3(t_1, R(t_1)) \| = \| (\gamma I - \omega R(t_2)) - (\gamma I - \omega R(t_1)) \| \\
\leq \omega \| R(t_2) - R(t_1) \| \\
\leq D_3 \| t_2 - t_1 \|. \] (26)

Similarly,

\[
\| H_1(t_2, S(t_2)) - H_1(t_1, S(t_1)) \| \leq D_1 \| t_2 - t_1 \|, \] (27)

\[
\| H_2(t_2, I(t_2)) - H_2(t_1, I(t_1)) \| \leq D_2 \| t_2 - t_1 \|. \] (28)

Substituting Eqs. (26)–(28) in Eqs. (23)–(25), we get

\[
\| T[S(t_2)] - T[S(t_1)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) D_1 \| t_2 - t_1 \|, \] (29)
\[\| T[I(t_2)] - T[I(t_1)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)^{\xi}}{B(\xi)\Gamma(\xi)} \right) D_2 \| t_2 - t_1 \| , \] (30)

\[\| T[R(t_2)] - T[R(t_1)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t_2 - t_1)^{\xi}}{B(\xi)\Gamma(\xi)} \right) D_3 \| t_2 - t_1 \|. \] (31)

This implies

\[\| T[S(t_2)] - T[S(t_1)] \| \to 0 \text{ as } t_2 \to t_1,\]
\[\| T[I(t_2)] - T[I(t_1)] \| \to 0 \text{ as } t_2 \to t_1,\]
\[\| T[R(t_2)] - T[R(t_1)] \| \to 0 \text{ as } t_2 \to t_1.\]

Thus, \( T(\bar{N}) \) is equicontinuous.

According to Arzela-Ascoli theorem, \( T(\bar{N}) \) is compact.

### 3.3 Uniqueness of solution

We will now discuss the uniqueness of the system of Eqs. (11).

Consider

\[\| T[S_1(t)] - T[S_2(t)] \| = \left\| \frac{1 - \xi}{B(\xi)} [H_1(t,S_1(t)) - H_1(t,S_2(t))] + \frac{\xi}{B(\xi)} \frac{\partial}{\partial t}[H_1(t,S_1(t)) - H_1(t,S_2(t))] \right\| \]
\[\leq \frac{1 - \xi}{B(\xi)} \| H_1(t,S_1(t)) - H_1(t,S_2(t)) \| + \frac{\xi}{B(\xi)} \| H_1(t,S_1(t)) - H_1(t,S_2(t)) \| \frac{(t - 0)^{\xi}}{\xi\Gamma(\xi)} \]
\[\leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t - 0)^{\xi}}{B(\xi)\Gamma(\xi)} \right) g_1 \| S_1(t) - S_2(t) \|. \] (32)

Similarly,

\[\| T[I_1(t)] - T[I_2(t)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t - 0)^{\xi}}{B(\xi)\Gamma(\xi)} \right) g_2 \| I_1(t) - I_2(t) \| , \] (33)

\[\| T[R_1(t)] - T[R_2(t)] \| \leq \left( \frac{1 - \xi}{B(\xi)} + \frac{(t - 0)^{\xi}}{B(\xi)\Gamma(\xi)} \right) g_3 \| R_1(t) - R_2(t) \|. \] (34)

If \( T \) is a contraction, then the system of Eqs. (1) has a unique solution (“Banach fixed-point theorem”).
$T$ is a contraction if the following conditions are satisfied:

$$\left( \frac{(1-\xi) + (t-0)^{\xi}}{B(\xi) + B(\xi)\Gamma(\xi)} \right) g_1 < 1,$$
$$\left( \frac{(1-\xi) + (t-0)^{\xi}}{B(\xi) + B(\xi)\Gamma(\xi)} \right) g_2 < 1,$$
$$\left( \frac{(1-\xi) + (t-0)^{\xi}}{B(\xi) + B(\xi)\Gamma(\xi)} \right) g_3 < 1.$$

Hence, we have proved the existence and uniqueness of the solution to the system of Eqs. (1) that represents the spreading of SARS-CoV-2 and the crowding effect with $AB$-derivative using the fixed-point method.

### 3.4 Interpretation of the model

Spatial heterogeneity in communicable disease transmission is often influenced by native variations in population or human movements, in a way that high native population densities may speed up the spread of new pathogens because of higher contact rates with prone individuals. The model shows that the degree of spread of COVID-19 is strongly shaped by population aggregation. It shows that the pandemic in crowded places is spread faster over time and total attack rates or the total infection rates are larger in the crowded places than in the less populated places. Hence, when the contact range is small, the spread of COVID-19 may be controlled.

Infected population and recovered population are inversely related: decrease in infectious population yields an increase in recovered population. Reducing the infection risks in the most affected populations ultimately serves the entire population. In order for the infection to decline, the daily infected as a proportion of the total infected and the daily deceased as a proportion of the total deceased should decrease over time. In contrast, the daily recovery as a proportion of the overall recovery should improve over time.

Also, since $N(t) = S(t) + I(t) + R(t)$, increase in susceptibility implies reduction in the COVID-19 cases or we can say “infection goes to extinction.”

We used Python to represent the number of infected, active, recovered, deceased, and susceptible cases as per February 4, 2021 and as per April 22, 2021 graphically using Bar graph, where the data are taken from various sources [39, 40].

### 4 Conclusions

The epidemic of COVID-19 is a powerful reminder of the continuing threat of evolving and reemerging infectious diseases and the need for continuous monitoring, timely detection, and rigorous study to recognize and establish successful countermeasures in order to
understand the fundamental biology of new species and our vulnerability to them. This work presented the crowding effects by the nonlinear incidence rate in the mathematical model using AB fractional operator for the current pandemic of COVID-19. We derived the solution, discussed the existence results and the uniqueness of the solution using the fixed-point method. In future, we can apply numerical simulation techniques in the defined model to further predict the COVID-19 scenario in different situations in different places. This information can help guide the transmission of public health information and the nature and extent of the government’s response to mandatory public health practices or regulated business operations to limit transmission. Timely prediction of the number of cases is essential for planning medical resources and ensuring that populations facing the uncertainty of emerging infectious diseases during the pandemic response have access to available care and the best possible outcome.

Availability of data and materials
The data and other materials such as pictures and graphs used have been properly cited and referred to in the chapter.

Competing interests
The authors declare that they have no competing interests.

Authors’ contributions
The authors contributed equally and significantly in writing this chapter. All authors read and approved the final manuscript.

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