A study on the numerical simulation of thermal stress during the solidification of shaped castings

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Received 14 June 1999

Abstract

In order to study the development of thermal stress and to predict the hot tearing and residual stress of shaped casting, two models were used to carry out the stress analysis of the two stages of solidification. The rheological model \([H]—[H[N]]—[N[S]]\) was used for the quasi-solid zone while the thermo-elastic-plastic model was used for the period after solidification. Coupling the thermal analysis based on the finite difference method with the stress analysis based on the finite element method, a FDM/FEM integrated system of thermal stresses analysis during the solidification process was developed. After experimental verification, the system was put into practical application. The analysis results during the quasi-solid zone show that the visco-plastic strain is an important factor for the occurrence of hot tearing. The hot tearing of a case steel casting and the residual stresses and deformation of a hydro-turbine blade steel casting were analyzed and predicted using the system. The simulation and the practical results were basically in agreement. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Numerical simulation; Solidification process; Shaped casting; Thermal stress; Rheological model; Thermo-elastic-plastic model

1. Introduction

The numerical simulation of thermal stress during solidification is of considerable significance in understanding the development of thermal stress and the prediction of hot tearing, cold cracks, residual stress and deformation and even the thermal boundary conditions between the casting and the mold. Great attention has been paid to this area in the past decades. The thermal stresses of continuous casting and ingots were analyzed in some works [1—4] and simplified as two-dimensional problems.

However, the development of the thermal stresses of shaped casting during the solidification process is very complicated due to the complex mechanical behavior of the liquid, the liquid—solid coexistence zone and the solid zone at high temperatures. Therefore, little work has been done to model the stress development of the mushy zone and the shaped casting also [5,6].

Recently, the rheological properties of cast alloys such as aluminium alloys, copper alloys and cast steels have been investigated [7—9]. It was found that nearly all of these alloys comply with the \([H]—[H[N]]—[N[S]]\) model in the quasi-solid zone. Therefore, in this paper, the \([H]—[H[N]]—[N[S]]\) model and thermo-elastic-plastic model were used to carry out the stress analysis of the quasi-solid zone and of the period after the solidification of the shaped casting, respectively. In addition, an integrated FDM/FEM analysis system of thermal stresses was established, verified and put into practical application.

2. Constitutive equations

The rheological model and the thermo-elastic-plastic model describe the different mechanical behavior of the quasi-solid and the period after solidification, respectively. They are depicted as follows.

2.1. Rheological model \([H]—[H[N]]—[N[S]]\)

The mechanical model of \([H]—[H[N]]—[N[S]]\) is shown in Fig. 1, where \([H]\), \([N]\) and \([S]\) are Hooke, Newton and St. Venant elements, respectively, \([-]\) denotes series and \([\|]\) denotes parallel. The Hooke element is elastic; the \([H[N]\) so-called Kelvin element is visco-elastic and the \([H]—[N[S]\) so-called Bingham element is visco-plastic.

For the Hooke and Kelvin models, the relationships of stress \([\sigma]\) and elastic strain \([\varepsilon]\) and visco-elastic strain \([\varepsilon]\) can be expressed by the following equations, respectively:

\[
[\sigma] = [D_h][\varepsilon],
\]

(1)
\[ \sigma = [D_h][\varepsilon] + [\lambda_b][\dot{\varepsilon}], \]  

(2)

where \([D_h]\) and \([D_k]\) are the elastic moduli of Hooke and Kelvin elements, respectively, and \(\lambda_b\) is the viscosity of the Kelvin element.

In all the following equations the subscripts \(h\), \(k\), \(b\) denote the variables of the Hooke, Kelvin, and Bingham elements, respectively, while the dots above the symbols denote the derivative of time.

In the three-dimensional situation, the visco-plastic strain rate of the Bingham element is given as follows [10]:

\[ \dot{\varepsilon}_b = \frac{1}{3\lambda_b} \langle \Phi(F) \rangle \frac{\partial Q}{\partial \sigma} \],  

(3)

where \(\lambda_b\) is the viscosity of the Bingham element, \(F\) the yield function which can be denoted by \(\tilde{\sigma} = \sigma - \sigma_e\), for Von Mises yield criterion, \(Q\) the visco-plasticity potential, and \(\langle \Phi(F) \rangle\) is a switch on/switch off operator.

\[ \langle \Phi(F) \rangle = 0 \quad \text{if } F < 0, \]

\[ \langle \Phi(F) \rangle = \Phi(F) \quad \text{if } F > 0. \]

As to the associative situation \(Q = F\). Taking \(\langle \Phi(F) \rangle = F\), then

\[ \dot{\varepsilon}_b = \frac{1}{2\lambda_b} \left(1 - \frac{\sigma_e}{\tilde{\sigma}}\right)\{\sigma'\}, \]

(4)

where \(\sigma_e\) is the yield stress, \(\tilde{\sigma}\) the equivalent stress, and \(\{\sigma'\}\) is the deviatoric stress vector. In the solidification zone there is no strain hardening, and therefore the yield stress is only a function of temperature.

The total strain increment can be divided into elastic, visco-elastic, thermal part and visco-plastic after yielding. Therefore, the constitutive equations before yielding and after yielding are Eqs. (5) and (6), respectively

\[ \{\sigma\} = [D_h][\varepsilon] + [\lambda_b][\dot{\varepsilon}], \quad \{\sigma\} = [D_k][\varepsilon] + [\lambda_k][\dot{\varepsilon}]. \]

(5)

\[ \{\varepsilon\} = \{\varepsilon_h\} + \{\varepsilon_k\} + \{\varepsilon_{th}\}, \quad \{\varepsilon\} = \{\varepsilon_h\} + \{\varepsilon_k\} + \{\varepsilon_{th}\}, \]

\[ \{\varepsilon\} = \{\varepsilon_h\} + \{\varepsilon_k\} + \{\varepsilon_{th}\} + \{\varepsilon_{th}\}. \]

\[ \{\dot{\varepsilon}\} = \{\varepsilon\} + \{\dot{\varepsilon_h}\} + \{\dot{\varepsilon_k}\} + \{\dot{\varepsilon_{th}}\}. \]

A three-dimensional algorithm based on the rheological model for FEM was then developed.

2.2. Thermo-elasto-plastic model

The stress increment \(\{d\sigma\}\) and elastic strain increment \(\{d\varepsilon\}\) satisfy

\[ \{d\sigma\} = [D][d\varepsilon], \]  

(7)

where \([D]\) is the elasticity matrix.

The plastic model complies with the flow rule and hardening rule. The flow rule determines the direction of plastic straining and is given as follows:

\[ \{d\varepsilon_{pl}\} = \lambda \left\{ \frac{\partial Q}{\partial \sigma} \right\}, \]

(8)

where \(\lambda\) is a plastic multiplier which determines the amount of plastic straining, and \(Q\) is the plastic potential which determines the direction of plastic straining and is also a function of stress. For the associate flow

\[ Q = F. \]  

(9)

To calculate the plastic strain increment it is necessary to determine \(\lambda\) by the following deduction.

The hardening rule states that the yield criterion changes with work hardening, so that

\[ F(\{\sigma\}, \kappa, \{\alpha\}) = 0, \]  

(10)

where \(\kappa\) is the plastic work, and \(\{\alpha\}\) is the transition of the yield surface. \(\kappa\) and \(\{\alpha\}\) are internal variables. Especially, the plastic work is the sum of the plastic work done over the history of loading

\[ \kappa = \int \{\sigma\}^T[\{d\varepsilon_{pl}\}], \]  

(11)

and the transition of the yield surface is also history dependent and is given as

\[ \{\alpha\} = \int C[\{d\varepsilon_{pl}\}], \]  

(12)

where \(C\) is a material parameter and \(\{\alpha\}\) represents the location of the center of the yield surface and moves in the direction of plastic straining.

Eq. (10) is differentiated so that

\[ dF = \left\{ \frac{\partial F}{\partial \sigma} \right\}^T[d\sigma] + \frac{\partial F}{\partial \kappa} d\kappa + \left\{ \frac{\partial F}{\partial \alpha} \right\}^T[d\alpha] = 0, \]  

(13)

which is the plastic consistency condition. Noting from Eqs. (11) and (12) there are

\[ d\kappa = \{\sigma\}^T[\{d\varepsilon_{pl}\}], \]  

(14)

\[ [d\alpha] = C[d\varepsilon_{pl}]. \]  

(15)

Since the total strain increment can be divided into an elastic, plastic and thermal part

\[ \{d\varepsilon\} = \{d\varepsilon\} - \{d\varepsilon_{pl}\} - \{d\varepsilon_{th}\}. \]  

(16)
Combining Eqs. (7) and (16) and substituting Eqs. (8), (14) and (15) into Eq. (13), then

$$\lambda = \frac{-\frac{\partial F}{\partial \sigma} + C\left(\frac{\partial Q}{\partial \sigma}\right)}{-\frac{\partial F}{\partial \sigma} + C\left(\frac{\partial Q}{\partial \sigma}\right)} \frac{\partial Q}{\partial \sigma} - C\left(\frac{\partial Q}{\partial \sigma}\right) + \left(\frac{\partial F}{\partial \sigma}\right)^T [D] \left(\frac{\partial Q}{\partial \sigma}\right).$$

(17)

Finally, substituting Eq. (17) into Eq. (8), the plastic strain can be calculated. Therefore, the plastic strain is related to the total increment in strain, the current stress state, and the specific forms of the yield and potential surfaces.

In addition, the thermo-elasto-plastic model is provided by ANSYS software, in which the Von Mises yield criteria and the initial stress method are employed.

3. The FDM/FEM integrated analysis system of thermal stresses

The development of thermal stress is closely related to the cooling of the casting, and therefore the numerical simulation of thermal stress is based on the thermal analysis. The finite difference method (FDM) based thermal analysis software FT-STAR was applied to perform the thermal analysis because of the simplicity of FDM, while the finite element method (FEM) was applied for the stress analysis. Coupling the thermal analysis based on FDM with stress analysis based on FEM, a FDM/FEM integrated system of thermal stress analysis during the solidification process was developed, its schematic diagram being shown in Fig. 2. The coupling is through the transfer of the temperature fields of FDM into the thermal loads needed in the FEM program by the temperature fields of FDM into the thermal loads needed in the FEM program by the temperature transferring module [6]. The rheological model is used to model the stress of the quasi-solid zone with the aim of predicting hot tearing. At the same time, the thermo-elasto-plastic model provided in the finite element analysis software ANSYS is used to model the stress during the period after solidification and to predict the residual stress and deformation. In the pre-processing module the first step is to construct the three-dimensional solid geometry model, which provides files for both the finite difference and finite element mesh generation. In this paper, three-dimensional solid geometry models were constructed by using the commercial software CADDS5. Further, the pre-processing

![Fig. 2. Schematic diagram of the FDM/FEM integrated system for the thermal stress analysis of shaped castings.](image-url)
and post-processing for FEM were performed in the ANSYS system (Fig. 2).

4. Experimental verification

The system was verified by the experimental and empirical results of two different sample castings. A sample steel casting for investigating hot tearing was used for the rheological model while a bow-shaped cast iron sample casting was used for the thermo-elasto-plastic model.

4.1. Thermal stress analysis based on a rheological model

A steel sample casting of 0.25% carbon with a hot spot, as shown in Fig. 3, was used to verify the thermal stress analysis based on a rheological model. Two castings with and without constraints of the middle part of the specimen were cast and simulated by the system. The rheological parameters of the cast steel used in the numerical simulation were from Ref. [8]. The castings were poured at 1590°C into a sodium silicate bonded sand mold. The middle part of the sample casting was considered to be in free contraction due to the good collapsibility of the mold sand at high temperatures. The casting obtained under no constraints was sound. However, hot tearing occurred at the hot spot under constraints when a rigid steel pipe was put between the two sides of the casting, as shown in Fig. 4.

The characteristics of elastic $\varepsilon_u$, visco-elastic $\varepsilon_k$ and visco-plastic strain $\varepsilon_b$ are different, as shown in Fig. 5. The elastic strain reach the maximum value immediately with stress; the visco-elastic strain increased gradually under constant stress and reaches the maximum value $\varepsilon_{kmax}$ after a certain period of time; and the visco-plastic strain increases continuously without limitation. Therefore, the visco-plastic strain was further investigated for the prediction of hot tearing. The distribution of visco-plastic strain $\varepsilon_b$ with and without constraints at the end of solidification is shown in Fig. 6. It is obvious that the visco-plastic strain $\varepsilon_b$ concentrates at the hot spot with constraints and on the other hand there is no obvious concentration of strains under no constraints. These simulated results are in agreement with the experimental results.

Therefore, the visco-plastic strain is a main factor determining the hot tearing tendency and the criteria for hot tearing can be expressed as $\varepsilon_b/\varepsilon_{bcrit}$, which means that if the visco-plastic strain surpasses the critical value $\varepsilon_{bcrit}$, hot tearing occurs.

4.2. Thermal stress analysis based on the thermo-elasto-plastic model

The bow-shaped cast iron sample casting, shown in Fig. 7, was used to verify the stress analysis based on thermo-elasto-plastic model provided by ANSYS5.3. The mechanical properties of cast iron are from Ref. [5]. The constraints are applied at the symmetrical faces. The calculated and the

![Fig. 3. Geometry of the steel sample casting (mm).](image)

![Fig. 5. Characteristics of $\varepsilon_u$, $\varepsilon_k$ and $\varepsilon_b$.](image)

![Fig. 4. Hot tearing occurring under constraints: (a) the constraints; (b) hot tearing, shown by the arrow.](image)
Fig. 6. Distribution of visco-plastic strain $\dot{\varepsilon}_p$: (a) under no constraints; (b) under constraints.

Fig. 7. Geometry of the bow-shaped sample casting (mm).

Fig. 8. Comparison of the calculated and the measured cooling curves.

measured cooling curves of two specific points 1 and 2 as shown in Fig. 7 are compared in Fig. 8. They are basically in agreement with the maximum deviation being 10%. The difference between cooling of the thinner straight and thicker semicircular bars determines their difference in contraction, which will further result in the stress and deformation development of the sample casting.

The stresses at 30 and 100 min after solidification as shown in Fig. 9, represent the two periods of solidification, respectively. At the beginning the contraction of the straight bar and the outer side of the semicircular bar is larger than that of the inner side of the semicircular bar, which results that the former is in tension and the later is in compression. During further cooling the contraction of the inner side of the semi-circular bar comes to be larger than that of the straight bar and the outer side of the semicircular bar. Consequently, the inner side of the semicircular bar is in tension and the straight bar and the outer side of the semicircular bar are in compression. The calculated residual stress of point 1 is $-9.858$ MPa, and the measured value $-12.0$ MPa. In addition, the deformation development of the sample casting is shown in Fig. 10. Therefore, the calculated results of the bow-shaped sample casting comply with the theoretical analysis and the measured results.

5. Engineering application

Two steel castings from the casting plants were analyzed by using the system, one for the prediction of hot tearing with the other for residual stress and deformation.

5.1. Prediction of hot tearing of a case steel casting

Hot tearing occurred at the joints of the bars and the body of a case steel casting in the foundry, as shown in Fig. 11. When the two bars were changed into one thick bar in the middle of the body and two chills placed at the joints between the bar, as shown in Fig. 12, the hot tearing defects were basically eliminated.

Then, stress analysis during the quasi-solid zone of the case casting under the original and improved technologies were carried out and the hot tearing tendency was predicted. The calculated distribution of hot tearing tendency or the distribution of $\dot{\varepsilon}_p/\varepsilon_{\text{crit}}$ at the end of solidification with the original and the improved technologies are illustrated in Fig. 13, where $\varepsilon_{\text{crit}}$ is taken as 0.5% [11]. It is obvious that there is great hot tearing tendency at the joints of the original technology because $\dot{\varepsilon}_p/\varepsilon_{\text{crit}}$ is larger than 1.0. On the other hand, the hot tearing tendency is greatly decreased
with $\varepsilon_\text{f}/\varepsilon_{\text{crit}}$ smaller than 1.0 by using the modified technology. The predicted results are in agreement with the practical results.

5.2. Analysis of residual stress and deformation of a hydro-turbine blade casting

The hydro-turbine blade stainless steel casting is a very important component in the hydro-turbine and has a very complicated shape, as shown in Fig. 14. There was great deforming tendency during solidification in production. A finite element model with tetrahedral elements was used for the convex part and hexahedral elements were used for the body, as shown in Fig. 15. The total mesh elements are 2884 and the nodes are 3030. The simulated residual deformation is shown in Fig. 16. It is obvious that the main deformation locates at the left side of the dashed line shown in Fig. 16. It is obvious that the main deformation resides after shake-out. The simulated residual deformation at point A is 60.9 mm as shown in Fig. 16, while the actual deformation at point A is about 33 mm. The reason for this discrepancy may be: (1) for stress analysis the action of the mold is neglected; (2) the deformation of the blade could recover to some extent during later heat treatment. Further, the maximum simulated equivalent stress is located in the middle part of the blade, as shown in Fig. 17.

Fig. 9. The stress distribution in the x-direction during solidification.

Fig. 10. The deformation during solidification (magnification x 10): (a) 30.0 min; (b) 100.0 min.

Fig. 11. Illustrating: (a) the original technology; (b) hot tearing (shown by the arrow).
6. Conclusions

1. A three-dimensional constitutive equations of rheological model [H]–[H][N]–[N][S] was established.

2. A FDM/FEM integrated analysis system of thermal stresses was developed, and the rheological and thermo-elasto-plastic models were used to simulate the thermal stresses in the quasi-solid zone and the stage after solidification, respectively.

3. The system was verified by using two sample castings. The results show that the visco-plastic strain is an important factor for the occurrence of hot tearing, and a hot tearing criterion was then established.

4. For engineering application, the hot tearing tendency of a case steel casting and the residual stress and deformation of a hydro-turbine blade stainless steel casting were simulated and the results were discussed.
Acknowledgements

The study was supported financially by the State Significant Fundamental Research Project, the R and D fund of Machinery Building and the Significant Project of NSFC.

References

[1] R. Aboutalebi, M. Hasan, R.L.L. Guthrie, Thermal modeling and stress analysis in the continuous casting of arbitrary sections, Steel Res. 65 (6) (1994) 225–233.
[2] B.G. Thomas, A. Moitra, H. Zhu, in: M. Cross, J. Campbell (Eds.), Coupled Thermo-Mechanical Model of Solidifying Steel Shell Applied to Depression Defects in Continuous-Cast Slabs, Proceedings of Modeling of Casting, Welding and Advanced Solidification Processes VII, TMS, London, 1995, pp. 241–248.
[3] G. Funk, J.R. Boehmer, F.N. Fett, Coupled FDM/FEM model for the continuous casting process, Int. J. Comput. Appl. Technol. 7 (3–6) (1994) 214–228.
[4] T.C. Tseng, S. Kobayashi, Stress analysis in solidification process application to continuous casting, Int. J. Mach. Tools Manufact. 29 (1) (1989) 121–140.
[5] Liu Baicheng, Zhu Riming, Residual stress computation and analysis of machine tool bed casting, in: Proceedings of the Fifth International Symposium of the Physical Metallurgical of Cast Iron (SCI-5), Nancy, France, 1994.
[6] Y. Chen, J.W. Kang, B.C. Liu, in: B.G. Thomas, C. Beckermann (Eds.), Study on Residual Stress of Cylinder Block Casting by Using an Integrated FDM/FEM System, Proceedings of Modeling of Casting, Welding and Advanced Solidification Processes VIII, TMS, New York, 1998, pp. 771–778.
[7] L. Qingchun, C. Kuiying, L. Chi, Z. Songyan, Rheological behavior in solid–liquid coexistence zone and simulation of stress–strain and hot tearing of Al–Cu alloy during solidification, AFS Trans. (1991) 245–253.
[8] Cheng Jun, Cheng Mei, Dang Jingzhi, Numerical simulation of thermal stress in Al–Si alloy castings during solidification process, Special Foundry Nonferrous Alloys (5) (1995) 1–4 (in Chinese).
[9] Li Defu, Lin Bainian, Yan Yonggui, An Geying, Experimental modeling for rheological properties of cast steel, Acta Metall. Sinica 31 (9) (1995) B389–B393 (in Chinese).
[10] T. Inoue, D.Y. Ju, Analysis of solidification and visco-plastic stresses incorporating a moving boundary: an application to simulation of the centrifugal casting process, J. Thermal. 15 (1992) 109–128.
[11] K. Nakayama, M. Kinefuchi, K. Tsutsui, in: B.G. Thomas, C. Beckermann (Eds.), A Measurement of Critical Strain for Internal Crack in Continuously Cast Slabs, Modeling of Casting, Welding and Advanced Solidification Processes VIII, TMS, New York, 1998, pp. 915–922.