Pre-inflationary partition of extra dimensions

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We consider a mechanism when extra dimensions become dynamically discriminated during an anisotropic evolution caused by a special isotropic energy-momentum tensor under initial conditions equivalent for all spatial directions.

\section*{I. INTRODUCTION}

Trying to create a consistent quantum gravity reveals a problem of extra spatial dimensions\cite{1,2} since we do not see those dimensions in any experimental phenomena. If so, we have to point to rigorous dynamical reasons or conditions for such the decoupling of extra dimensions outside our three dimensional space (3D). The most known approaches in this way are the following: i) extra dimensions are compact in contrast to our ordinary infinite space, ii) extra dimensions are large but almost empty, since the matter propagates only in three separate dimensions\cite{3}. Both statements imply the situations when properties of extra dimensions specifically differ from that of our 3D-space. In this paper we consider a mechanism providing such the difference between extra and 3D spaces. Namely, we show that initially isotropic homogeneous sources of evolution in general relativity with extra dimensions can result in an anisotropic solution when some spatial dimensions become exponentially long in comparison with others\textsuperscript{1}. We also speculate about conditions providing the case with the number of such the exponentially long dimensions equal to three.

\section*{II. EVOLUTION}

In the isotropic homogeneous case the Einstein tensor, composed of Ricci tensor $R_{\mu\nu}$, metrics $g_{\mu\nu}$ and scalar curvature $\mathcal{R} = g^{\mu\nu} R_{\mu\nu}$,

\[ G^\mu_\nu = R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu \mathcal{R}, \]

should have the spatial components in the form of

\[ G^\alpha_\beta = -p \delta^\alpha_\beta, \quad \alpha, \beta \neq 0, \]

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\textsuperscript{1}In the similiar aspect, sources of gravity with non-equivalent dimensions were considered in\textsuperscript{4}, solutions with a cosmological constant and non-equivalent initial data were investigated in\textsuperscript{5}, a non-liner sigma model was analysed in\textsuperscript{6}, a generic case of expanding and shifting dimensions was discussed recently in\textsuperscript{7}.
with $\alpha, \beta$ running from 1 to $d$ being the dimension of space and $p$ denoting the pressure of matter with the energy-momentum tensor $T_{\mu\nu}$ in the Einstein equations

$$R_{\nu}^{\mu} - \frac{1}{2} \delta_{\nu}^{\mu} R = 8\pi G T_{\nu}^{\mu},$$

where $G$ is the Newton constant.

With the stationary tensor of matter background we expect that the pressure remains constant in time, that could be easily reached if the metrics has the form of

$$ds^2 = dt^2 - a_+^2(t) dr_+^2 - a_-^2(t) dr_-^2,$$

where the scale factors have the exponential dependence on the time

$$a_\pm(t) = e^{\pm H\pm t},$$

while the spatial dimensions of “+” and “−” components are equal to $d_+$ and $d_−$, respectively, at $d = d_+ + d_-$. Thus, we study the possibility of spontaneously broken isotropy and homogeneity during the evolution with the isotropic and homogeneous initial conditions and energy-momentum tensor. Indeed, the spatial components of Einstein tensor with metrics (2) are equal to

$$G_+^+ = -d_+ H_+^2 + d_- H_- H_+ - \frac{1}{2} R, \quad G_-^- = -d_- H_-^2 + d_+ H_+ H_- - \frac{1}{2} R,$$

where the scalar curvature equals

$$R = -(H_- d_- - H_+ d_+)^2 - H_+^2 d_- - H_-^2 d_+.$$

The temporal component is given by the formula

$$G_0^0 = -d_+ H_+^2 - d_- H_-^2 - \frac{1}{2} R.$$

The isotropic condition $G_+^+ = G_-^-$ requires

$$d_+ H_+^2 + (d_- - d_+) H_+ H_- - d_- H_-^2 = 0,$$

that has 2 solutions: the first is $H_+ = -H_-^2$ representing de Sitter space-time being the isotropic solution with $T_{\mu}^{\nu} = \rho_\Lambda \delta_{\mu}^{\nu}$ of vacuum with the energy density $\rho_\Lambda = d(d - 1) H_+^2 / (16\pi G)$ setting the curvature $R = -d(d + 1) H_+^2$, while the second solution gives $d_+ H_+ = d_- H_-$ corresponding to the spontaneously broken isotropy of space-time, that is of our interest.

So, the anisotropic evolution takes place at

$$G_0^0 = -G_+^+ = -G_-^- = \frac{1}{2} R = -\frac{1}{2} d H_+ H_-.$$

Note, that $H_+$ and $H_-$ have the same sign. Therefore, the solution corresponds to the stiff matter with $\rho = p < 0$. This could take place in an effective field theory of scalar $\phi(t)$ depending exceptionally on the time if one has the lagrangian

$$\mathcal{L} = \frac{1}{2} \left\{1 - F(\phi)\right\} \dot{\phi}^2, \quad \dot{\phi} = \partial_0 \phi,$$



2 The same relation between the dimensions and Hubble rates was obtained in [6] under the condition of constant physical volume at $d_+ = 3$ (see Section III.)
where $F$ is a function of kinetic self-interaction, that has the limit of large fields $F(\infty) = 2$ and the free field limit of $F(0) = 0$, say,

$$F(\phi) = 2 \frac{\phi^2}{M^2 + \phi^2} = 2 \sum_{n=0}^{\infty} (-1)^n \left( \frac{\phi^2}{M^2} \right)^{n+1}.$$ 

At $\phi^2 \gg M^2$ we arrive to the stiff matter with the negative density of energy in the strong field limit.

Then, we get the following pattern: in the case of a restricted potential energy at large fields (a plateau of potential) the kinetic term can dominate and give the stiff matter providing the evolution with the broken isotropy of space-time; after the field reaches values with the comparable kinetic and potential energy density the regime of evolution switches to the inflation, i.e. a slow roll of field from the flat plateau to the minimum of potential.

### III. EFFECTS

The anisotropic solution for the evolution with the isotropic energy-momentum tensor reveals two sub-spaces: the large expanding space of $d_+$ dimension and small contracting space of $d_-$ dimension. A co-moving volume $V_0$ corresponds to the physical volume

$$V(t) = a_+^{d_+}(t) a_-^{d_-}(t) V_0 = V_0 e^{(d_+ H_+ - d_- H_-) t} = V_0,$$

hence, the physical volume remains constant but its “$+$” and “$-$” dimensions do scale in different and opposite rates compensating each other.

Let us consider properties of external matter evolution at fixed metrics. So, we neglect by any back reaction of external matter to the metrics, that means a contribution of matter to the total energy balance to be essentially suppressed. For the sake of simplicity we focus on massless particles, i.e. the radiation.

#### A. Soft and hard modes

The mass shell condition of radiation with comoving momentum $k$, $g^{\mu \nu} k_\mu k_\nu = 0$ transforms to

$$k_0^2 = k_+^2 e^{-2H_+ t} + k_-^2 e^{2H_- t}. \quad (9)$$

Modes moving exclusively along the “$+$” direction, i.e. at $k_- \equiv 0$, are exponentially ultra-soft,

$$k_0^2 = k_+^2 e^{-2H_+ t} \to 0,$$

while modes propagating in both the contracting “$-$” sub-space and expanding “$+$” sub-space, are exponentially ultra-hard,

$$k_0^2 - k_-^2 e^{-2H_+ t} = k_-^2 e^{2H_- t} \to \infty.$$ 

Therefore, an observer living in the large dimensions, considers the ultra-hard modes as super-heavy alike super-Planckian massive particles with masses rising during such the evolution.

We see that the energy of soft modes scales as

$$k_0^+ \sim \frac{1}{a_+} \sim e^{-H_+ t}, \quad (10)$$

...
while the energy of hard modes scales as
\[ k_0^- \sim \frac{1}{a_-} \sim e^{H_- t}. \]  
(11)

At the constant physical volume \( V(t) \) we expect that for the energy densities of soft and hard modes we get
\[ \rho_\pm \sim \frac{1}{a_\pm} \sim e^{\mp H_\pm t}, \]  
(12)

that is valid. Indeed, the energy-momentum conservation \( \nabla_\mu T^{\mu}_\nu = 0 \) for the temporal component, i.e. at \( \nu = 0 \), gives
\[ \dot{\rho}_+ + \rho_+ + (d_+ H_+ - d_- H_-)(\rho_+ + \rho_-) + d_+ H_+ p_+ - d_- H_- p_- = 0, \]  
(13)

where \( p_\pm \) denote pressures of soft and hard modes. Since the sub-spaces are separately isotropic, but the overall isotropy is spontaneously broken, we put the relations for radiation in the sub-spaces
\[ p_\pm = \frac{1}{d_\pm} \rho_\pm, \]
that results in
\[ (\dot{\rho}_+ + H_+ \rho_+) + (\dot{\rho}_- - H_- \rho_-) = 0, \]
which means that the evolution of soft and hard modes can be separated and it gives relation of (12).

Then, free isotropic soft and hard modes compose two cold and hot matter components with different temperatures and pressures. However, interactions could mix these components.

Collisions of soft modes with soft modes cannot produce hard modes. i.e. the extra contracting space is decoupled from the soft \( d_+ \) dimensional space. Collisions of hard modes with hard and soft modes can produce both hard and soft modes, that can cause the cooling of contracted sub-space and warming of expanding sub-space. The balance depends on the rates of evolution and scattering, that determines the scenario of future. For instance, the high density of hard modes can change a geometry of extra sub-space, say, causing its compactification before the cooling, that means further complete decoupling of extra dimensions.

**B. Hubble horizons and optimal dimensions**

Hubble constants restrict the wave lengths since only the wave lengths less than the Hubble horizons evolve. In this respect we can evaluate the contribution of curvature to the lagrangian inside the Hubble volumes of sub-spaces, that gives
\[ L_H \sim R H_+^{-d_+} H_-^{-d_-} \sim d H_+^{1-d_+} H_-^{1-d_-}. \]

At fixed contraction rate \( H_- \) and space dimension \( d \) we get the following dependence on the dimension \( d_+ \) of expanding sub-space:
\[ L \sim L_d = \left( \frac{d_-}{d_+} \right)^{1-d_+} \sim \left( \frac{d-d_+}{d_+} \right)^{1-d_+}. \]  
(14)
It is the easy numerical task to calculate the discrete spectrum versus $d_+$. For instance, in 10 dimensional space-time ($d = 9$) the absolute value gets the minimum of $|L_d| = 1/4$ at $d_+ = 3$. One could refer to such the extremal condition as the optimal dimension of large sub-space. We find that $d_+ = 3$ is optimal only at $d = 9, 10, 11, 12$. So, at $d < 9$ the 2 dimensional expansion optimal, while at $d > 12$ the expanding sub-space has got extra soft dimensions. It is notable that the superstring theory points to the 10-dimensional space-time and a 11-dimensional supergravity, that stands in consistence with our consideration of optimal condition giving the 3 dimensional expanding Universe.

IV. CONCLUSION

We have described the new anisotropic evolution of space with extra dimensions under the isotropy of homogeneous energy-momentum tensor. This anisotropy discriminates 3 expanding spatial dimensions and extra contracted dimensions at the total dimension of space $d$ in the range from 9 to 12. Modes propagating in sub-spaces are differentiated in the following way: soft modes live exclusively in the expanding sub-space, while other modes look like ultra-hard and heavy for observers in the expanding world. Such the effects are provided by the specific limit of energy-momentum tensor that can be reached if one introduces the appropriate field possessing the kinetic self-interaction of definite kind. One could call this field ‘pre-inflaton’.

A knowledge on a further fate of such the evolution itself requires to take into account an influence of matter to the metrics. In addition, the pre-inflationary regime described, needs studies of an effective field model with deviations of energy-momentum tensor from the leading approximation given by values constant in time.

To the moment the properties of space partition during the pre-inflationary evolution allow us to conjecture that extra dimensions do decouple from the expanding sub-space of 3 dimensions.

Acknowledgments

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