Scaling and superconductivity in heavy electron materials

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Understanding the origin of the unconventional superconductivity found in heavy electron materials, the cuprates, the pnictides, and the organics, continues to be a central problem in condensed matter physics. In the present communication we focus on understanding this connection for one important member of this family, the heavy electron materials, where, as may be seen in Fig. 1, finding such a connection appears at first sight highly problematic because Curro scaling covers a range of temperatures in which both hybridized localized spins and the itinerant heavy electron Kondo liquid contribute to the spin-lattice relaxation rate. We show that a connection becomes possible at the optimal pressure, $p_L$, provided both $T_c$ and the total spin-lattice relaxation rate scale with the coherence temperature, $T^*$, at which heavy electrons first emerge. We show that the first condition is easily met, and that the second scaling relation requires that the intrinsic spin-lattice relaxation rate of the heavy electron Kondo liquid be independent of temperature over much of the relevant temperature region, an expected signature of magnetic quantum critical spin fluctuations. We present a simple way to test whether the second conditions is met, and find that is satisfied for a number of heavy electron superconductors.

The phenomenological two-fluid model, which explains so many other aspects of heavy electron behavior, turns out to be key to understanding the scaling of their remarkable dynamic behavior with both $T_c$ and $T^*$. In the two-fluid model, below $T^*$, the collective hybridization of the Kondo lattice of $f$-electron local moments with the background conduction electrons gives rise to an itinerant heavy electron Kondo liquid (KL) of strength, $f$, that coexists with the hybridized local moment spin liquid, of strength $1 - f$, with

$$f(T, p) = f_0(p) \left(1 - \frac{T}{T^*}\right)^{3/2}. \quad (1)$$

Here $f_0(p)$ provides a quantitative measure of hybridization effectiveness; it is unity at the delocalization quantum critical point (QCP) that marks a zero temperature transition from partially localized to fully itinerant behavior. For $f_0 > 1$, the model predicts the pressure and temperature dependence of the delocalization line $T_L$, shown in Fig. 2, that begins at the delocalization QCP and marks, at finite temperatures, the end of the collective hybridization process at $f(T_L, p) = 1$; according to

![FIG. 1: (Color online) The scaling of the spin-lattice relaxation rate with temperature normalized by the superconducting transition temperature for a number of heavy electron superconductors at pressures at or near that at which $T_c$ is maximum. The analysis is extended to higher temperatures in the insert, where the arrows indicate $T^*$.](image-url)
FIG. 2: (Color online) A schematic phase diagram for heavy electron superconductors that is based on the pressure-induced changes in the behavior of CeCoIn$_5$ and CeRhIn$_5$ [12, 13]. It shows the coherence temperature, $T^*$ ($>30T_c$), at which itinerant heavy electrons emerge to form a Kondo liquid, and the delocalization line that is the boundary between fully itinerant and partially localized heavy electron quasiparticles. $T_c$ is maximum at the pressure, $p_L$, at which $T_c = T_L$. At $p_L$, between $T^*$ and $T_c$, both the hybridized local spin liquid and the itinerant Kondo liquid contribute to the spin lattice relaxation rate.

Eq. (1), it takes the form.

$$T_L(p) = T^*(p) \left[ 1 - f_0(p)^{-2/3} \right].$$  \hspace{1cm} (2)

Since localization competes with superconductivity, it is physically appealing to assume that the maximum in the superconducting transition temperature, $T_c^{\text{max}}$, is found at the pressure $p_L$, at which the delocalization line, $T_L$, intersects $T_c$; experiments on CeCoIn$_5$ and CeRhIn$_5$ [12, 13] show that this is the case. For pressures less than $p_L$, quasiparticle localization reduces the number of heavy electrons able to become superconducting and so suppresses $T_c$, while for pressures greater than $p_L$, the spin-fluctuation induced interaction between heavy electron quasiparticles becomes increasingly less effective [14]. Since $f_0(p_L)$ is not far from its value at the quantum critical point, $f_0(p_{QC}) = 1$, we then obtain a simple relation between $T_c^{\text{max}}$ and $T^*(p_L)$,

$$\frac{T_c^{\text{max}}}{T^*(p_L)} = 1 - f_0(p_L)^{-2/3} \approx \frac{2}{3}[f_0(p_L) - 1],$$  \hspace{1cm} (3)

that produces $T_c/T^*$ scaling for materials with comparable values of $f_0(p_L)$. This is roughly the case for the materials [1, 15] 20] shown in Table I in which the value of $f_0$ is estimated from other experiments for CeCoIn$_5$, CeRhIn$_5$ and URu$_2$Si$_2$ and assumed to be near unity at the optimal pressure $p_L$ for PuCoGa$_5$ and UPt$_3$. For CeRhIn$_5$, experiment shows a maximal $T_c$ at 2.0 GPa [21].

FIG. 3: (Color online) Exploration of quantum critical scaling behavior by plotting the spin-lattice relaxation rate against $(1 - T/T^*)^{3/2}$. The arrows indicate the temperature where the scaling fails.

The value of its $f_0$ at ambient pressure is unknown; we assume it is $\sim 0.9$ in order to compare, at a qualitative level, CeIn$_5$ with its sister materials.

We turn next to the spin-lattice relaxation rate, noting that to the extent it exhibits $T/T^*$ scaling at $p_L$, $T/T_c$ scaling will be present for materials with comparable values of $f_0$. To see how this might come about, we consider the two-fluid expression for the spin-lattice relaxation rate \[3\],

$$\frac{1}{T_1} = \frac{1 - f(T)}{T_1^{SL}} + f(T)T_1^{KL},$$  \hspace{1cm} (4)

where $T_1^{SL}$ and $T_1^{KL}$ are the intrinsic spin-lattice relaxation times of the hybridized local moment spin liquid and the itinerant Kondo liquid. Eq. (1) tells us that $T/T^*$ scaling will be found to the extent that both intrinsic spin-lattice relaxation rates are independent of temperature. For the spin-liquid, experiment suggests that $1/T_1^{SL}$ becomes almost temperature independent as one approaches $T^*$ for materials that are at or near the quantum critical or localization pressure [cf the insert in Fig. 1] and it is reasonable to assume that this behavior continues below $T^*$. Importantly, the intrinsic Kondo
TABLE I: Results of an analysis of the two-fluid parameters and the spin-lattice relaxation rate in several heavy electron superconductors at indicated pressures. The unit for all the spin-lattice relaxation rates is sec⁻¹. \( T_x \) is the cut-off temperature below which the scaling breaks down and \( t_0 = T_1(0.T^{*}\ast)/T_1(T^{*}) \).

| \( p \) (GPa) | \( T_l \) (K) | \( T^{*} \) (K) | \( T_1/T^{*} \) | \( f_0(p) \) | \( 1/T_1^{SL} \) | \( 1/T_1^{SL} - 1/T_1^{KL} \) | \( f_0 \) | \( 1/T_1^{KL} \) | \( T_1^{KL}/T_1^{SL} \) | \( T_x \) | \( t_0 \) | ref. |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| PuCoGa\(_5\) | 0           | 18.5        | 430         | 0.043       | 1.068       | 746         | 654         | 134         | 5.6         | \( \sim T_c \) | 2.68        | [1, 2]      |
| UPt\(_3\)   | 0           | 0.5         | 20          | 0.025       | 1.039       | 17.0        | 14.8        | 3.5         | 4.9         | \( \sim T_N \) | 2.65        | [7, 17]     |
| CeCoIn\(_5\) | 1.2         | 2.6         | 85          | 0.031       | 1.023       | 300         | 268         | 38          | 7.9         | \( \sim T_{g9} \) | 2.77        | [11, 12, 15]|
| CeCoIn\(_5\) | 2.2         | 56          | 0.041       | 0.87        | 450         | 202         | 218         | 2.1         | \( \sim T_c \) | 1.48        | [11, 12, 15]|
| CeRhIn\(_5\) | 2.7         | 42          | 0.048       | 1.24        | 516         | 358         | 228         | 2.3         | \( \sim T_L \) | 1.99        | [13, 15]    |
| CeIrIn\(_5\) | 0           | 4.0         | 40          | 0.01        | 0.9         | 475         | 359         | 76          | 6.3         | \( \sim 5 \text{ K} \) | 2.18        | [7, 18]     |
| URu\(_2\)Si\(_2\) | 0.5         | 1.5         | 75          | 0.02        | 1.6         | 2.91        | 3.28        | 0.86        | 3.4         | \( \sim T_{HO} \) | -           | [16]        |
| CeCu\(_2\)Si\(_2\) | 0.6         | 70          | 0.009       | -           | 148         | 0           | 148         | 1.0         | \( \sim T_{SDW} \) | -           | [7, 19]     |
| YbRh\(_2\)Si\(_2\) | 0           | 50          | -           | 89         | 0           | 89         | 1.0         | \( \sim T_L \) | -           | [11, 20]    |

The liquid spin-lattice rate will be nearly independent of temperature over a wide range of temperatures provided the spin fluctuations responsible for its spin-lattice relaxation rate, \( 1/T_1^{KL} \), exhibit the magnetic quantum critical behavior \( \frac{1}{T_1^{KL}} \) that goes to zero at \( T_1^{KL} \sim (T + T_0) \) with \( T_0 \) being small and going to zero at the QCP.

For any material for which \( T^* \) is known, it is straightforward to test whether these conditions are met, by plotting \( 1/T_1 \) vs \( (1 - T/T^*)^{3/2} \) for temperatures below \( T^* \).

On rewriting Eq. (4) as

\[
\frac{1}{T_1} = \frac{1}{T_1^{KL}} + \left( \frac{1}{T_1^{KL} - \frac{1}{T_1^{SL}}} \right) f_0(p) \left( 1 - \frac{T}{T^*} \right)^{3/2},
\]

we see that to the extent that one finds linear behavior, one can determine directly both \( 1/T_1^{KL} \) and the product \( f_0(p) \left( 1/T_1^{KL} - \frac{1}{T_1^{SL}} \right) \) from such a scaling plot. Our results for a number of heavy electron materials are given in Fig. 3, where the hoped-for linear scaling is found between \( T^* \) and a cut-off temperature, \( T_x \), for each material shown there, while the corresponding results for the intrinsic spin lattice relaxation rates are given in Table I.

To the extent that the ratio, \( T_1^{KL}/T_1^{SL} \), is similar for different superconductors, one can obtain a simple scaling formula for the spin-lattice relaxation rate normalized at a fixed temperature, say \( 0.2T^* \):

\[
\frac{T_1(0.2T^*)}{T_1(T)} \approx t_0 + 1.4(1 - t_0) \left( 1 - \frac{T}{T^*} \right)^{3/2},
\]

where \( t_0 = T_1(0.2T^*)/T_1(T^*) \). As shown in Table I, \( t_0 \) varies from 2.0 to 2.8, for the materials examined there. Since the hyperfine coupling constants and spin liquid contributions may differ from material to material and not all of these are at the pressure \( p_L \) at which \( T_c \) is maximum, such a slight variation is somewhat unexpected; the solid lines in Fig. 4(a) shows that \( t_0 = 2.7 \) provides a good fit to the data.

We now look into the details of the scaling behavior for individual materials. In Fig. 4(a), one sees that the \( T^* \) scaling continues down to the superconducting transition in CeCoIn\(_5\) (\( p = 1.2 \) GPa) and PuCoGa\(_5\), and the lowest measured temperature in UPt\(_3\). This proves that at \( p_L \) the spin fluctuations responsible for the intrinsic Kondo liquid \( T_1 \) in these materials exhibit quantum critical magnetic behavior. Since UPt\(_3\) exhibits \( T_1 \) scaling behavior identical to that seen for CeCoIn\(_5\) and PuCoGa\(_5\), there can be little question that a similar quasiparticle interaction is the physical origin of its superconductivity; its different pairing state reflects its quite different Fermi surface. For these three materials, \( t_0 \) clusters around 2.7.

Deviations from scaling behavior are found at a temperature \( T_x \), that is higher than \( T_c \) for CeRhIn\(_5\) at \( p = 2.7 \) GPa and CeIrIn\(_5\) and CeCoIn\(_5\) at ambient pressure. In CeRhIn\(_5\), we find \( T_x \sim 6 \) K. Using \( T^* = 42 \) K and \( f_0(2.7) = 1.24 \) estimated from other experiments \( \text{[14]} \), we find using Eq. 2 that \( T_x \) is close to the calculated delocalization temperature \( T_{KL} = 5.7 \) K, so that the loss of scaling below \( T_x \) must be attributed to the Kondo liquid. As may be seen in Fig. 4, a reasonable fit to the experimental data may be obtained if we take \( T_1^{KL}T \sim (T + T_0) \) with \( T_0 \sim 3 \) K providing a measure of the distance away from the quantum critical pressure. Although CeIrIn\(_5\) at ambient pressure is below the optimal pressure at \( \sim 2 \) GPa and possesses a quite large scaling cut-off temperature \( T_x \approx 5 \) K compared to its \( T_c = 0.4 \) K, we find that taking \( T_1^{KL}T \sim (T + T_0) \) with \( T_0 \sim 2 \) K explains the departure from quantum critical scaling and provides a good fit to the experimental data, as may be seen in Fig. 4(b). In CeIn\(_3\) at 2.43 GPa, a similar deviation from scaling behavior is seen at \( \sim 15 \) K, far above the antiferromagnetic and superconducting ordering temperatures \( \text{[23, 24]} \). PuCoGa\(_5\) exhibits complex behavior in that a scaling plot of its \( T_1 \) exhibits several changes of slope with lowering temperature whose physical origin remains to be determined \( \text{[25]} \). For CeCoIn\(_3\) at ambient pressure, we find deviations from the scaling behavior below \( \sim 7 \) K, far above the superconducting transi-
The behavior of the spin-lattice relaxation rate of both CeCu$_2$Si$_2$ and YbRh$_2$Si$_2$ at ambient pressure is anomalous. Although a number of experiments have suggested a $T^* \sim 70 \text{ K}$ for CeCu$_2$Si$_2$ [26], we see in Fig. 3(b) that its spin-lattice relaxation rate $1/T_1$ continues to be temperature-independent down to $\sim 8 \text{ K}$, below which the onset of the spin density wave fluctuations is observed in neutron scattering experiments [27]. We encounter a similar anomaly in YbRh$_2$Si$_2$ for which $T^* \sim 50 \text{ K}$ [28]. Assuming the two-fluid model is applicable, this requires a non-accidental cancellation ($T_{1\text{KL}}^* \approx T_{1\text{SL}}^*$) of the independent spin liquid and Kondo liquid contributions to $1/T_1$ over a substantial temperature range. This might happen if both contributions are dominated by valence fluctuations or some other physical phenomenon other than magnetic quantum critical behavior.

In summary, we have used the two-fluid model in this paper to explore the origin of $T/T_1$ scaling in the spin-lattice relaxation rate of heavy electron materials at the pressure at which $T_c$ is maximum. We have shown that because there is a simple relation between $T^*$ and $T_c$ at this pressure, $T/T_1$ scaling is to be expected in the spin-lattice rate of heavy electron materials to the extent that $T_1$ exhibits quantum critical $T^*/T_1$ scaling, and have proposed a simple experimental test for its existence.

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**FIG. 4:** (Color online) Comparison of theory and experiment for the spin-lattice relaxation rate in several heavy electron superconductors. (a) The scaling of $T_1$ as a function of $T/T^*$. The solid line is the proposed scaling formula with $t_0 = 2.7$. (b) The total $1/T_1$ and the Kondo liquid contribution for CeRhIn$_5$ at 2.7 GPa and CeIrIn$_5$ and CeCoIn$_5$ at ambient pressure. The inset shows the intrinsic Kondo liquid term $1/T_1^{KL}$ derived after subtracting the spin liquid contribution. The (solid and dashed) lines are fit to experiment using $T_1^{KL} \sim (T + T_0)$ with $T_0 = 3 \text{ K}$ for CeRhIn$_5$, 2 K for CeIrIn$_5$, and 2.2 K for CeCoIn$_5$ at ambient pressure.

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