TESTS OF CONVECTIVE FREQUENCY EFFECTS WITH SOI/MDI HIGH-DEGREE DATA

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ABSTRACT

Advances in hydrodynamical simulations have provided new insights into the effects of convection on the frequencies of solar oscillations. As more accurate observations become available, this may lead to an improved understanding of the dynamics of convection and the interaction between convection and pulsation (Rosenthal et al. 1999). Recent high-resolution observations from the SOI/MDI instrument on the SOHO spacecraft have provided the so-far most-detailed observations of high-degree modes of solar oscillations, which are particularly sensitive to the near-surface properties of the Sun. Here we present preliminary results of a comparison between these observations and frequencies computed for models based on realistic simulations of near-surface convection. Such comparisons may be expected to help in identifying the causes of the remaining differences between the observed frequencies and those of solar models.

Key words: solar oscillations; convection.

1. INTRODUCTION

In stellar-structure theory, the standard Mixing-Length Model of convection (MLT) continues to be the industry standard, despite its many limitations. For example MLT cannot account for the broadening of photospheric spectral lines, presumably due to overshooting, nor can it model the effect of turbulent pressure on the mean structure in a consistent way. More subtly perhaps, the reduction of the true 3-D radiation hydrodynamics to 1-D hydrostatic stratification eliminates some significant contributing effects including the role of correlations, which result in a mean structure which does not satisfy the locally-valid equation of state, and radiative effects due to opacity inhomogeneities, which result in hotter surface layers for a given effective temperature (see Stein & Nordlund 1998a).

Helioseismology has considerable potential as a tool for testing the accuracy of numerical models of these surface regions through comparison of predicted and measured frequencies. A major contribution is the effect of turbulent pressure on the mode frequencies (Kosovichev 1995). In such a comparison, it is vital to distinguish between model effects on frequencies, resulting from errors or inaccuracies in the mean structure, and modal effects resulting from uncertainties in the physics of wave propagation in these turbulent layers. We use numerical simulations of the surface layers to handle model effects while studying various physically-motivated assumptions about the modal physics.

2. DATA ANALYSIS

Our data analysis procedure is described in more detail by Rhodes et al. (1997, 1998). A 60.75 day time series of MDI Full-Disk Dopplergrams, beginning May 23, 1996, was used. After limited gap-filling, the final duty cycle was 97.3 %. The so-called “averaged-spectrum” method is used to obtain frequency estimates, with results from low- and intermediate-degree power spectral peaks being used to obtain a correction due to asymmetry which is then applied to the frequencies obtained by ridge-fitting at higher degrees. Results are obtained for degrees up to $\ell = 1000$. 
3. THE STANDARD SOLAR MODEL

The standard solar model we use is Model S of Christensen-Dalsgaard et al. (1996). It uses the OPAL equation of state (Rogers et al. 1996), OPAL opacities (Iglesias et al. 1992) at high temperature and Kurucz (1993) opacities in the atmosphere. The atmosphere is determined by a T − τ relation (where T is temperature and τ is optical depth) fitted to the HSRA reference atmosphere (Soderblom et al. 1971) and standard MLT (Bohm-Vitense 1958) is used for the convection zone. The effects of turbulent pressure are not included in Model S.

Figure 1 shows a comparison between adiabatic eigenfrequencies of Model S and the data. The frequency residuals have been scaled with the quantities Qnl which represent the ratio of the mode inertia of a particular mode to that of a radial mode of the same frequency. Mode inertias were normalised at the photosphere, defined as the location where the temperature is equal to the effective temperature. It is evident that, except for modes of low order, the scaled residuals are essentially a function of frequency alone, an indicator that the predominant effect responsible for the residuals lies close to the solar surface (Christensen-Dalsgaard & Berthomieu 1991).

4. MODEL CONTRIBUTIONS AND NUMERICAL SIMULATION

We treat the model contributions to the residuals by using radiation-hydrodynamic simulations of the surface layers to construct new mean models, by patching temporal and spatial averages of the simulations to convective envelope models constructed with standard physics. The simulations are described in detail by Stein & Nordlund (1989, 1998).

Our philosophy is to construct numerical simulations and standard MLT envelope models each using the same physics, thus allowing us to isolate the effects of 3-D convective dynamics. In this case we used simulations calculated with the MHD equation of state (Hummer & Mihalas 1988, Mihalas et al. 1988, Dappen et al. 1988, Mihalas et al. 1990) and Kurucz opacities. The effective temperature of the simulation was 5777K.

A standard envelope model was constructed with the same physics, and using standard MLT with the addition of a parameterised turbulent pressure of the form \( p_t = \beta p v^2 \), where \( \rho \) is density and \( v \) is the convective velocity, obtained from the mixing-length formulation. The extra parameter \( \beta \) and the usual mixing length parameter were then adjusted to allow a continuous matching of density, pressure and temperature. Because the matching point is at a depth where the turbulent pressure is already very small (of order 1% of the total pressure) we do not consider this matching process to be a useful parameterisation of the relation between mixing-length velocity and turbulent pressure in general. Strikingly, the resulting matched model has a convection zone depth of 0.286\( R_\odot \), very close to the helioseismic value of (0.287 ± 0.003)\( R_\odot \) (Christensen-Dalsgaard et al. 1991, Kosovichev & Fedorova 1991) or (0.287 ± 0.001)\( R_\odot \) (Basu & Antia 1997). This is a remarkable, parameter-free validation of the simulations.

5. MODAL CONTRIBUTIONS

The process described above yields a hydrostatic pressure and density stratification for a patched model based on the simulation. We consider only adiabatic oscillations of this patched model. Under the assumption of adiabaticity, the modal effects are essentially confined to the relation between the Lagrangian perturbations \( \delta p \) and \( \delta \rho \) in pressure and density,

\[
\frac{\delta p}{p} = \Gamma_1 \frac{\delta \rho}{\rho}.
\]

When convection is ignored, \( \Gamma_1 = (\partial \ln p/\partial \ln \rho)_s \), the derivative being at constant specific entropy \( s \), is simply the thermodynamic adiabatic exponent. Here we incorporate modal effects by the use of different recipes for \( \Gamma_1 \); in particular, they define the adiabatic sound speed \( c = (\Gamma_1 p/\rho)^{1/2} \). We consider two possibilities: the Gas Gamma Model (GGM) and the Reduced Gamma Model (RGM).

In the GGM we simply use the horizontal and temporal mean value of the thermodynamic \( \Gamma_1 \) from the simulations. In the RGM, this is replaced by the so-called “reduced gamma”:

\[
\Gamma_1^{(r)} = \frac{(\Gamma_1 p_\text{g})}{p},
\]

where \( p_\text{g} \) is the gas pressure and \( p \) is the total pressure. The reduced gamma was originally introduced by Rosenthal et al. (1995) who argued that if the Lagrangian perturbation to the turbulent pressure,
\[ \delta \tilde{\rho}, \text{ were zero then} \]
\[ \frac{\delta \tilde{p}}{\tilde{p}} = \frac{\delta \tilde{\rho}}{\tilde{\rho}} = \Gamma^*_1 \frac{\delta \tilde{\rho}}{\tilde{\rho}}. \]  

Rosenthal et al. (1999) show that the definition (2) is the correct one to use if \( \delta \tilde{\rho} = 0 \) provided certain other conditions are also met, principally that the time variation of the convective energy fluxes can be neglected.

We also construct a Standard Envelope Model (SEM) which uses the same physics as the simulations, has no turbulent pressure, and is calibrated to have the same convection-zone depth as the patched models. The SEM is constructed using the Rosseland Mean of the opacities used in the simulations and a \( T - \tau \) relation determined from the averaged simulation itself. Thus the differences between the models should isolate truly convective effects.

**6. MODEL DIFFERENCES**

Christensen-Dalsgaard & Thompson (1997) have shown that when model differences are expressed in Lagrangian terms (i.e., on a fixed mass scale) the frequency change due to near-surface perturbations can be written approximately as

\[ \frac{\delta \omega_{nl}}{\omega_{nl}} \simeq \int_0^R K^{nl}_{\nu,c}(r) \frac{\delta m \nu}{\nu} dr, \]

where \( \delta m \nu/\nu \) is the Lagrangian perturbation to the quantity \( \nu = \Gamma^*_1/c \), and where the kernels \( K^{nl}_{\nu,c}(r) \) can be calculated from the structure and eigenfunctions of the reference model. For an isothermal layer, \( \nu = 2\omega_c/g \), where \( \omega_c \) is the acoustic cutoff frequency. Hence changes in \( \nu \) correspond physically to changes in the size of the acoustic cavity.

Figure 2 shows Lagrangian differences in \( \nu \) between the various models. The GGM-SEM differences largely reflect changes in density due to the elevation of the photospheric regions by turbulent pressure. In addition, the RGM-SEM differences show the effect of the turbulent reduction of \( \Gamma_1 \). Comparing (GGM-SEM) with (GGM-Model S) we can see that the different physics between SEM and Model S (principally OPAL \( \nu \), MHD equation of state and a different \( T - \tau \) relation) produces further small changes confined extremely close to the surface.

**7. OSCILLATION FREQUENCIES**

We restrict ourselves to modes trapped within the convection zone. This allows us to include direct comparisons also with Model S. Figure 3 shows that the frequency differences between GGM and SEM are similar in magnitude and shape to those between the data and Model S (Figure 1), whereas corresponding differences between RGM and SEM are much larger than required. GGM-Model S differences are broadly similar to GGM-SEM differences.

**8. INTERPRETATION AND CONCLUSIONS**

Our statement that the remaining discrepancy in frequency must be due to modal, rather than model, effects is based on the belief that the simulations are fundamentally correct. Why do we have such faith in the simulations? The principal model effect on the frequencies is the elevation of the photospheric layers by the combined effect of turbulent pressure and an effect of the 3-D dynamics, whereby hotter regions, which have higher opacity, contribute less to the emitted radiation. The higher mean temperature of the 3-D model is thus “hidden” from view, but is reflected in the pressure stratification.

We have carried out a number of convergence tests on the simulations involving runs at various resolutions. The thermal stratification is rather insensitive to numerical resolution (cf. Stein & Nordlund 1998b, Fig. 26). The variation of turbulent pressure with resolution is shown in Figure 5.

The highest-resolution runs already produce the correct turbulent broadening, asymmetries, and blueshift of spectral lines and so the maximum turbulent pressure cannot increase much beyond the largest values found so far (Nordlund & Dravins 1990b, Dravins & Nordlund 1990a, Dravins & Nordlund 1990b). In fact, despite the apparent steady increase of turbu-
Figure 3. Scaled differences between frequencies of patched (RGM and GGM) and comparison (SEM) envelope models and between the GGM envelope and the standard solar model (Model S).

Figure 4. Measured frequency residuals in the sense (observations) – (model), scaled by $Q_{nl}$. The model frequencies are for the GGM, with the (unreduced) gas $\Gamma_1$.

Figure 5. Mean turbulent pressure divided by total (gas + turbulent) pressure as a function of depth for four different runs, with different numerical resolutions: $253^2 \times 163$, $125^2 \times 82$, $63^2 \times 63$, and $32^2 \times 41$. (Note that the total duration of the $253^2 \times 163$ run is only 10 minutes, while we estimate that a minimum duration of 30–60 minutes is required for statistically stable results.)

What then are the nature of the modal effects which account for the remaining frequency differences? One clue is provided by Stein & Nordlund (1991), who show that the time variation of the turbulent pressure is related to the compression in a much more complicated fashion than that implied by either the GGM or the RGM. In practice the turbulent component of gamma, $\partial \ln p_t / \partial \ln \rho$, can be negative, and is a strong function of height. Evidently, we need to use such information from the simulations themselves to determine a better model for $\Gamma_1$ than the two simple cases considered here.

However, the limitations even of this approach are evident from consideration of the f modes. These are almost entirely insensitive to the mean solar structure and so no treatment based on variations in either $\Gamma_1$ or the hydrostatic structure will ever improve the agreement between measurement and theory in their case. The principal contribution to the f-mode residuals probably comes from the advection of wavefronts by turbulent motions (Murawski & Roberts 1993a, Murawski & Roberts 1993b, Gruzinov 1998), an effect which has proved extremely difficult to model realistically. However, Murawski et al. (1998) have derived a dispersion relation for the f mode in a simple model containing a random velocity field, and shown that the frequency shift and linewidth of the f mode for this model are consistent with the properties obtained from the high-resolution MDI data. It is likely that turbulent advection results in frequency shift of the p modes in addition to the turbulent pressure effect. It may be, once again, that we will have to turn to the simulations to provide us with clues as to how to describe these processes.

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