SECULAR DYNAMICS IN HIERARCHICAL THREE-BODY SYSTEMS WITH MASS LOSS AND MASS TRANSFER

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ABSTRACT

Recent studies have shown that secular evolution of triple systems can play a major role in the evolution and interaction of their inner binaries. Very few studies explored the stellar evolution of triple systems, and in particular the mass-loss phase of the evolving stellar components. Here we study the dynamical secular evolution of hierarchical triple systems undergoing mass loss. We use the secular evolution equations and include the effects of mass loss and mass transfer, as well as general relativistic effects. We present various evolutionary channels taking place in such evolving triples, and discuss both the effects of mass loss and mass transfer in the inner binary system, as well as the effects of mass loss/transfer from an outer third companion. We discuss several distinct types/ regimes of triple secular evolution, where the specific behavior of a triple system can sensitively depend on its hierarchy and the relative importance of classical and general relativistic effects. We show that the orbital changes due to mass-loss and/or mass-transfer processes can effectively transfer a triple system from one dynamical regime to another. In particular, mass loss/transfer can both induce and quench high-amplitude (Lidov–Kozai) variations in the eccentricity and inclination of the inner binaries of evolving triples. They can also change the system dynamics from an orderly periodic behavior to a chaotic one, and vice versa. 

Key words: binaries: close – stars: evolution – stars: mass-loss – stars: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION

Triple systems are some of the most frequent astrophysical phenomena, manifesting themselves in almost any given scale, where triple stars, planets in binaries, and even our own Sun–Earth–moon system serve as a few obvious examples. About 15% of all stars reside in triples (Raghavan et al. 2010; and possibly >50 (40)% for more massive O/B stars; Tokovinin 1997; Eggleton et al. 2007; Remage Evans 2011; S. De-Mink 2014, private communication). Some aspects of the rich and complex gravitational dynamics of such systems have been studied extensively (as evidenced by the well known three-body problem; Valtonen & Karttunen 2006). However, study of the realistic evolution of such systems, including the coupling of their dynamics with non-gravitational processes and/or the realistic treatment of the physical properties such as mass loss and mass transfer, is still in its infancy. Given the mounting evidence for the importance of such systems and their ubiquity in stellar and planetary systems, our poor understanding of these systems is quite disconcerting. Here we explore the secular dynamics of evolving triples, and in particular the effects of mass loss and mass transfer on the dynamical evolution of triples.

Stable triple systems are hierarchical, namely, they consist of an inner binary and an outer binary orbit, i.e., the tertiary (third object). The secular evolution of such systems is the change of orbital elements on timescales on timescale much larger than the dynamical timescale of the system. Triple stellar evolution processes, i.e., mass loss, mass transfer, tidal friction, gravitational wave emission, etc., influence the orbital elements through changes in energy and angular momentum. Hence, in order to understand triple stellar evolution, one should couple the secular dynamics and the evolutionary processes to get a complete and more comprehensive picture.

Several astrophysical phenomena are likely to be produced via triple interaction and evolution. A key dynamical long term effect is the Lidov–Kozai mechanism (Kozai 1962; Lidov 1962), in which perturbations by the third outer companion lead to periodic/quasi-periodic (and sometimes chaotic; Ford et al. 2000; Blaes et al. 2002; Katz & Dong 2012; Naoz et al. 2013b) large amplitude oscillations (sometimes called Kozai-cycles) of the mutual inclination of the inner and outer binaries in the triple, as well as the inner binary eccentricity. This secular effect, first introduced in the context of solar system bodies, has since been suggested to play an important role over a wide range of systems and scales, from the dynamics of super-massive black holes and stars to exoplanets, moons, and planetesimals (e.g., Kozai 1962; Lidov 1962; Blaes et al. 2002; Nesvorný et al. 2003; Eggleton & Kisseleva-Eggleton 2006; Wu & Murray 2003; Fabrycky & Tremaine 2007; Perets & Naoz 2009; Thompson 2011; Antonini & Perets 2012; Katz & Dong 2012; Naoz et al. 2012, 2013a). The basic effect of the Kozai cycle is that on timescale much larger than the orbit periods, the inner orbit eccentricity and relative inclination fluctuate due to mutual torque between the inner and outer binaries. As a result, orbits exchange angular momentum (but not energy) and become more eccentric, possibly to the point at which physical collisions/mergers, tidal friction, gravitational wave emission, and/or other short range effects become important. In the following, we consider all the system components to be point-like mass particles; the coupling of tidal effects and/or mergers/mergers where the physical size plays a role will be explored elsewhere.

The analysis of secular processes make use of the mass-averaging technique; the masses of the triple components are averaged out over the inner and outer binaries’ orbital periods, and the dynamics then follow the mutual torques of these mass-averaged rings, using a perturbative expansion. For a very low-mass secondary in the inner binary (the test-particle regime), using expansion terms up to the quadruple level is sufficient to describe its evolution (e.g., for the cases of
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a star–planet–comet or star–planet–moon systems; Naoz et al. 2013a). The quadruple expansion is also sufficient in symmetric cases, where the primary and secondary in the inner binary have equal masses and the octupole expansion term becomes negligible (see Section 2.1 for a quantitative description). The octupole expansion treatment exhibits features lacking in the quadrupole analysis, in particular, a system with no inner binary symmetry in mass, i.e., \( m_0 \neq m_1 \), where \( m_0 \) and \( m_1 \) are the masses of the inner binary (Mazeh & Shaham 1979; Ford et al. 2000; Blaes et al. 2002; Naoz et al. 2013a). Octupole level dynamical treatment covers a wider range of systems, but it becomes inaccurate once the triple hierarchy is weaker, i.e., the timescales for the third body perturbation during periastron passage become comparable to the dynamical timescale of the inner binary, and the secular averaging approach is no longer valid (Antognini & Perets 2012; Katz & Dong 2012; Seto 2013; Antognini et al. 2014; Antonini et al. 2014; Prodan et al. 2014). Each one of these approaches is also characterized by a different dynamical behavior, from a periodical behavior in the quadrupole regime, to a quasi-periodic and chaotic behavior at the octupole and the non-secular regime.

In the following, we use the secular approach to explore mass-transfer and mass-loss processes in triple systems. This approach is valid as long as the mass-loss and mass-transfer processes occur on long enough timescales, such that the changes in angular momentum/energy on dynamical timescales are small; prompt mass-loss processes such as supernovae explosions are therefore not studied here. This is done by adding the appropriate terms for mass loss and mass transfer to the secular equations of motion at the octupole level, as well as using direct N-body integration to validate our secular analysis. As we show, such processes can change the dynamical regime characterizing the system, and thereby lead to significant changes in the system behavior. We focus only on mass-loss-induced transitions in the secular regimes (see also an N-body study of such transitions by Shappee & Thompson 2013); discussions of mass-loss-induced transitions from the secular regime to non-secular or even unstable triple regime are discussed elsewhere (e.g., Kratter & Perets 2012; Perets & Kratter 2012; Veras & Tout 2012). For related studies of mass loss in multiple planets (orbiting single stars), see Voyatzis et al. (2013).

Our paper is organized as follows. In Section 2, we briefly review the standard octupole order expansion and describe the equations of motion; readers familiar with this approach may skip to Section 3, where we present the additional secular mass loss/transfer terms to the equation of motion. In Section 4, we provide detailed examples for the evolution of triple systems, and compare the results with direct N-body simulations. We then (Section 5) discuss novel triple evolutionary channels accessible due to the coupling of secular dynamics with mass loss/mass transfer.

2. OCTUPOLE ORDER OF SECULAR EVOLUTION

In the following, we first briefly present the analytic treatment for the long-term, secular evolution of the hierarchical triple systems following previous studies (Harrington 1968; Ford et al. 2000; Naoz et al. 2013a; Mazeh & Shaham 1979). These provide the basis for the secular evolution approach, which we then extended to include mass-loss and mass-transfer processes. We use time-independent Hamiltonian perturbation theory and expand it up to the third, octupole order.

One can treat a hierarchical triple as two weakly interacting Kepler orbits, an inner orbit and an outer orbit (see Figure 1), together with a weak interaction term. We than exploit the fact that an hierarchical system, by definition, satisfies the following condition

\[ \alpha \equiv \frac{a_1}{a_2} \ll 1, \]

where \( a_1 \) is the inner binary semi-major axis (SMA) and \( a_2 \) is the outer binary SMA. The interaction term can then be expanded by powers of \( \alpha \), as we show in the following.

The Hamiltonian of a stable triple system is given by

\[ H = \frac{G m_0 m_1}{2 r} + \frac{G m_0 (m_0 + m_1) m_2}{2 R} + V(r, r_{12}, r_{23}), \]

where \( r \) represents the position of mass \( m_1 \) relative to mass \( m_0 \), \( r_{12} \) represents the position of mass \( m_2 \) relative to mass \( m_0 \), and \( r_{12} \) represents the position of mass \( m_1 \) relative to mass \( m_2 \). The first term is the kinetic energy of the inner orbit and the second term is the kinetic energy of the outer orbit. The last term is the potential energy of the triple system,

\[ V(r, r_{12}, r_{23}) = -\frac{G m_0 m_1}{r} - \frac{G m_0 m_2}{r_{12}} - \frac{G m_1 m_2}{r_{12}}. \]

The complete Hamiltonian, expanded in powers of \( \alpha \), is then

\[ H = -\frac{G m_0 m_1}{2 a_1} - \frac{G (m_0 + m_1) m_2}{2 a_2} - \frac{G}{a_2} \sum_{j=2}^{\infty} \alpha^j M_j \left( \frac{r}{a_1} \right)^j \left( \frac{a_2}{R} \right)^{j+1} P_j(\cos \psi), \]

where \( \psi \) is the angle between \( r \) and \( R \) (see Figure 1) and \( P_j \) is the \( j \)th Legendre polynomial and

\[ M_j \equiv m_0 m_1 m_2 \frac{m_1^{j-1} - (-1)^{j+1} m_1^{j+1}}{(m_0 + m_1)^j}. \]

The first and second terms are the total energy of the inner and outer binaries, respectively, and the third term is the expanded series corresponding to the interaction between the two orbits. It is convenient to write the Hamiltonian (4) in the angle-action

Figure 1. Hierarchical three-body system. Two weakly interacting binaries: the inner Keplerian orbit (binary) consists of mass \( m_0 \) and \( m_1 \) and an outer Keplerian orbit (binary) of \( m_2 \) and the center of mass of the inner binary, denoted in a gray circle. \( r \) is the separation vector of the inner binary; \( R \) is the separation vector of the outer binary; \( i \) is the inclination between the two orbiting planes; and \( \psi \) is the angle between the separation vectors \( r \) and \( R \).
variables called Delaunay’s elements (Valtonen & Karttunen 2006), which provide a convenient dynamical description of the three-body system. The coordinates are chosen to be the mean anomalies, $l_1$ and $l_2$, and their conjugate momenta

$$L_1 = \frac{m_0 m_1}{m_0 + m_1} \sqrt{G(m_0 + m_1)a_1},$$

$$L_2 = \frac{m_2(m_0 + m_1)}{m_0 + m_1 + m_2} \sqrt{G(m_0 + m_1 + m_2)a_2}, \quad (5)$$

as well as the arguments of the periastron, $g_1$ and $g_2$, and their conjugate momenta (the orbital angular momenta of the orbits),

$$G_1 = L_1 \sqrt{1 - e_1^2}, \quad G_2 = L_2 \sqrt{1 - e_2^2}, \quad (6)$$

where $e_1$ and $e_2$ are the inner and outer orbit eccentricities, respectively. We also make use of the longitudes of ascending nodes, $h_1$ and $h_2$, and their conjugate momenta (these are the $z$-components of the orbital angular momenta of the orbits)

$$H_1 = G_1 \cos l_1, \quad H_2 = G_2 \cos l_2 \quad (7)$$

and

$$H \equiv H_{\text{tot}} = G_1 \cos l_1 + G_2 \cos l_2, \quad (8)$$

where subscripts 1 and 2 denote the inner and outer orbits, respectively and $H$ is the total angular momentum. For geometric intuition, see Figure 2.

Using this coordinate system, the Hamiltonian can be written in third order in $\alpha$ (octupole approximation) Naoz et al. (2013a); Blaes et al. (2002).

$$\mathcal{H} = -\frac{\beta_0}{2L_1^2} - \frac{\beta_1}{2L_2^2} - 4\beta_2 \left( \frac{L_1^4}{L_2^2} \right) \left( \frac{r_1}{a_1} \right)^2 \left( \frac{a_2}{r_2} \right)^3 (3 \cos 2\psi + 1)$$

$$- 2\beta_3 \left( \frac{L_1^6}{L_2^2} \right) \left( \frac{r_1}{a_1} \right)^3 \left( \frac{a_2}{r_2} \right)^4 (5 \cos^3 \psi - 3 \cos \psi), \quad (9)$$

where

$$\beta_0 = Gm_0 m_1 \frac{L_1^2}{a_1}, \quad (10)$$

$$\beta_1 = Gm_2 (m_0 + m_1) \frac{L_2^2}{a_2}, \quad (11)$$

and

$$\beta_2 = \frac{G^2}{16} \frac{(m_0 + m_1)^2 m_2^2}{(m_0 + m_1 + m_2)^3} \quad (12)$$

$$\beta_3 = \frac{G^2}{4} \frac{(m_0 + m_1)^3 m_2^2}{(m_0 + m_1 + m_2)^3} \quad (13)$$

In order to obtain the equations of motion, one should use the Hamilton equation with respect to the relevant Hamiltonian. However, in the case of hierarchical systems, a separation of timescales approach can be employed. The short timescale is the inner and outer orbit period, and the long timescale is the timescale for the change in the orbital elements. The timescale difference enables us to average over the short timescales and obtain a much simpler Hamiltonian, and hence simpler set of the equations of motion. This is explained in the following subsection.

2.1. Secular Dynamics

The Hamiltonian (9) contains information about the long and short timescale behavior of the system. Because of the separation of timescales, one can average over the short time evolution. This is done by averaging over rapidly varying $l_1$ and $l_2$ in the Hamiltonian, so-called double averaging, and using the Von Zeipel method, i.e., transforming the coordinates into different angle-action variables that align the total angular momentum vector to the $z$-axis (see Naoz et al. 2011). Using this method, we eliminate the nodes by the relation $h_1 - h_2 = \pi$. Next, we define

$$\theta \equiv \cos i = \frac{H^2 - G_1^2}{2G_1G_2}, \quad (14)$$

where $i = l_1 + l_2$ is the total inclination (see Figure 2). One then obtains the equations of motion through the canonical equations (Ford et al. 2000; Blaes et al. 2002; Naoz et al. 2011):

$$\frac{d\theta_1}{dt} = 6C_2 \left\{ \frac{1}{G_1} \left[ 4\theta^2 + (5 \cos 2g_1 - 1)(1 - e_1^2 - \theta^2) \right] \right\}$$

$$+ 6C_2 \left\{ \frac{\theta}{G_2} \left[ 2 + e_1^2(3 - 5 \cos 2g_1) \right] \right\}$$

$$+ C_3 \epsilon_2 c_1 \left( \frac{1}{G_2} \right) \left( \frac{\theta}{G_1} \right) \left\{ \sin g_1 \sin g_2 \left[ A + 10(3\theta^2 - 1)(1 - e_1^2) \right] \right\}$$

$$- C_3 \epsilon_2 c_1 \left( \frac{1}{G_2} \right) \left( \frac{\theta}{G_1} \right) \left\{ 5\theta \cos \psi \right\}$$

$$- C_3 \epsilon_2 \frac{1 - e_1^2}{\epsilon_1 G_1} \left[ 10\theta(1 - \theta^2)(1 - 3e_1^2) \sin g_1 \sin g_2 \right]$$

$$- C_3 \epsilon_2 \frac{1 - e_1^2}{\epsilon_1 G_1} \left[ \cos \phi(3A - 10\theta^2 + 2) \right] \quad (15)$$

$$\frac{d\epsilon_1}{dt} = 30C_2 \epsilon_1 \frac{1 - e_1^2}{G_1} \left[ (1 - \theta^2) \sin 2g_1 - C_3 \epsilon_1 \frac{1 - e_1^2}{G_1} \right]$$

$$\times \left[ 35 \cos \psi (1 - \theta^2)e_1^2 \sin 2g_1 \right] - C_3 \epsilon_1 \frac{1 - e_1^2}{G_1}$$

$$\times \left[ - 10\theta(1 - e_1^2)(1 - \theta^2) \cos g_1 \sin g_2 \right] - C_3 \epsilon_1 \frac{1 - e_1^2}{G_1}$$

$$\times \left[ \left. - A \sin g_1 \cos g_2 - \theta \cos g_1 \sin g_2 \right] \right\} \quad (16)$$
\[
\frac{dg_2}{dt} = 3C_2 \left\{ \frac{2\theta}{G_1} \left[ 2 + e_1^2(3 - 5\cos 2g_1) \right] \right\} \\
+ 3C_2 \left\{ \frac{1}{G_2} \left[ 4 + 6e_1^2 + (5\theta^2 - 3) \right] \times (2 + 3e_1^2 - 5e_1^2 \cos 2g_1) \right\} \\
- C_3 e_1 \sin g_1 \sin g_2 \left\{ \frac{4e_2^2 + 1}{e_2 G_2} \right\} \times \left[ A + 10(3\theta^2 - 1)(1 - e_1^2) \right] \\
- C_3 e_1 \sin g_1 \sin g_2 \left\{ \frac{1}{G_1 + \theta G_2} \right\} \\
+ C_3 e_1 \cos \psi \left\{ 5B\theta e_2 \left( \frac{1}{G_1 + \theta G_2} \right) + \frac{4e_2^2 + 1}{e_2 G_2} A \right\} 
\]

\[
\frac{de_2}{dt} = C_3 e_1 \frac{1 - e_2^2}{G_2} \left\{ 10\theta(1 - \theta^2)(1 - e_1^2) \sin g_1 \cos g_2 \right\} \\
+ C_3 e_1 \frac{1 - e_2^2}{G_2} \left\{ A(\cos g_1 \sin g_2 - \theta \sin g_1 \cos g_2) \right\} 
\]

\[
\frac{dG_1}{dt} = - C_3 e_1 e_2 \left\{ - 35e_1^2(1 - \theta^2) \sin g_1 \cos \psi \right\} \\
- C_3 e_1 e_2 (A(\sin g_1 \cos g_2 - \theta \sin g_1 \sin g_2)) \\
- C_3 e_1 e_2 \left\{ 10\theta(1 - \theta^2)(1 - e_1^2) \cos g_1 \sin g_2 \right\} 
\]

\[
\frac{dG_2}{dt} = - C_3 e_1 e_2 \cdot A(\cos g_1 \sin g_2 - \theta \sin g_1 \cos g_2) \\
- C_3 e_1 e_2 \left\{ 10\theta(1 - \theta^2)(1 - e_1^2) \sin g_1 \cos g_2 \right\} 
\]

from Equation (2), the law of sines and the geometrical relations

\[
H_1 = \frac{H^2 + G_1^2 - G_2^2}{2H} 
\]

and

\[
H_2 = \frac{H^2 + G_2^2 - G_1^2}{2H}, 
\]

one can get the \( z \)-component angular momentum equation of motion

\[
\frac{dH_1}{dt} = \frac{\sin i_2}{\sin i_{tot}} \frac{G_1 - \sin i_1}{\sin i_{tot}} \frac{G_2}{\sin i_{tot}} 
\]

\[
\frac{dH_2}{dt} = \frac{\sin i_1}{\sin i_{tot}} \frac{G_2}{G_1} - \frac{\sin i_2}{\sin i_{tot}} \frac{G_1}{G_1} 
\]

Now, using the geometrical relations \( \cos i_1 = \frac{(H^2 + G_1^2 + G_2^2)}{2HG_1} \) and \( \cos i_2 = \frac{(H^2 + G_1^2 + G_2^2)}{2HG_2} \), we obtain

\[
\frac{d\cos i_1}{dt} = \frac{H_1}{G_1} - \frac{G_1 \cos i_1}{G_1} 
\]

where

\[
C_2 = \frac{G m_0 m_1 m_2}{16 (m_0 + m_1) a_2 (1 - e_2^2)^{3/2}} \left( \frac{a_1}{a_2} \right)^2 
\]

and

\[
C_3 = \frac{15G m_0 m_1 m_2 (m_2 - m_1)}{64 (m_0 + m_1)^2 a_2 (1 - e_2^2)^{5/2}} \left( \frac{a_1}{a_2} \right)^3 
\]

are the quadrupole and octupole coefficients, respectively. The quantities \( A \) and \( B \) in these terms are given by

\[
A = 4 + 3e_1^2 - \frac{5}{2}(1 - \theta^2)B 
\]

and

\[
B = 2 + 5e_1^2 - 7e_1^2 \cos 2g_1 
\]

and \( \psi \) is the angle between the periastron directions,

\[
\cos \psi = - \cos g_1 \cos g_2 - \theta \sin g_1 \sin g_2 
\]

\( C_3 \) then makes it clear that the octupole terms vanish when \( m_0 = m_1 \).

Let us now define

\[
\epsilon_3 = \left( \frac{m_0 - m_1}{m_0 + m_1} \right) \left( \frac{a_1}{a_2} \right) \frac{e_2}{1 - e_2^2}, 
\]

which measures the importance of the octupole terms, \( C_3/C_2 \). Note that in this approximation, the energies for the inner and outer orbits are conserved, respectively, namely, \( \dot{a}_1 = 0 \) and \( \dot{a}_2 = 0 \), and similarly the total angular momentum \( H = 0 \). Hence, the interaction only involves exchanging angular momentum between the two orbits.

We obtain the equations of motion of the orbital elements. As can be seen, the secular evolution, as described by the quadrupole level equations, gives rise to an oscillatory behavior, where both the eccentricity and the inclination of the inner binary periodically change with a potentially large amplitude; these are called Kozai–Lidov oscillations/cycles (Kozai 1962; Lidov 1962), which have been discussed extensively in recent years. The characteristic timescale of a full Kozai cycle is given by (Kozai 1962; Valtonen & Karttunen 2006)

\[
P_{\text{Kozai}} \approx \left( \frac{a_2}{a_1} \right)^3 \frac{m_0 + m_1}{m_2} P_1 = \frac{P_2}{P_1} \left( \frac{m_0 + m_1 + m_2}{m_2} \right), 
\]

where

\[
P_1 = 2\pi \sqrt{a_1^3/G (m_0 + m_1)} 
\]

is the time period of the inner binary, and

\[
P_2 = 2\pi \sqrt{a_2^3/G (m_2 + m_1 + m_0)} 
\]

is the time period of the outer binary. The maximum inner eccentricity during a cycle is given by Valtonen & Karttunen (2006)

\[
e_{\text{max}} = \sqrt{1 - \frac{5}{3} \cos^2 i_0}, 
\]
where $i_0$ is the initial inclination between the inner and outer binaries in the triple (assuming an initial circular orbit).

In the octupole regime, an additional, long-term, low-frequency modulation can be seen on a different timescale (see Section 2.3). In particular, in this regime, the system evolution is much more complex, and it is not periodic on the long term. Moreover, the amplitude of the changes in the inclinations and eccentricities could be much larger. In this regime, transitions between prograde and retrograde orbits due to extremely high eccentricities can also occur. The latter effect becomes highly important when discussing realistic non-point particles, where close encounters between the inner triple components can lead to significant orbital changes through the coupling of short-range dissipative processes.

Making use of the secular formulation for the evolution of classical point-like particles described above, we are now ready to take the next step and couple the triple dynamics with dissipative and non-classical processes. The secular classical point-particle dynamics of the triple systems described above conserve energy, and therefore the SMAs of the system remain constant throughout their evolution, $a_1 = \text{const}$, $a_2 = \text{const}$. In the following subsections, general relativistic (GR) interaction, mass-loss, and mass-transfer processes are described. In a realistic triple system, these effects are important and influence the equation of motion. In Section 2.2, we consider the general relativistic effects on the equation of motion, and in Section 3, we added our novel treatment for the mass-loss and mass-transfer processes.

### 2.2. Post-Newtonian Terms

General relativity plays a key role in close compact systems. These post-Newtonian effects come in two separate manifestations, and both are taken into account only for the inner binary. GR effects on the outer binary are typically insignificant in comparison besides the cases of weakly hierarchical systems, where the secular approach becomes invalid; in the following, GR effects on the external binary are typically insignificant in comparison besides the cases of weakly hierarchical systems, where the secular approach becomes invalid; in the following, GR effects on the external binary are neglected (for the treatment of such systems, see Naoz et al. 2013b; Will 2014).

The first post-Newtonian effect included is the precession of periastron (not a dissipative effect, but rather an additional dynamical process), and the second is gravitational wave (GW) radiation (which is a dissipative process). For the former, Hamiltonian analytic treatment can be used (Blaes et al. 2002) by adding the averaged post-Newtonian Hamiltonian to the averaged Hamiltonian described in Section 2.1

$$
\mathcal{H} = C_2 \left[ (2 + e_1^2)(1 - 3\theta^2) - 15e_1^2(1 - \theta^2)\cos 2g_1 \right] + C_3 e_1 e_2 [A \cos \phi + 109(1 - \theta^2)(1 - e_1^2) \sin g_1 \sin g_2] + \frac{G^2 m_0 m_1}{c^2 a_1^3} \times \left[ \frac{15m_1^2 + 15m_0^2 + 29m_0 m_1}{8(m_0 + m_1)} - \frac{3(m_0 + m_1)}{(1 - e_1^2)^{3/2}} \right].
$$

From this Hamiltonian, one obtains the additional term for the precession of the inner binary

$$
\frac{dg_1}{dt} = \frac{3}{c^2 a_1(1 - e_1^2)} \left( \frac{G(m_0 + m_1)}{a_1} \right)^{3/2}.
$$

For the gravitational wave treatment, one can follow Peters (1964) and compute the loss of energy, angular momentum, and the change in eccentricity, averaged per orbit, and add the appropriate terms to the Hamiltonian equations. The GR terms are as follows; for the inner binary SMA,

$$
\frac{da_1}{dt} = -\frac{64G^3 m_0 m_1 (m_0 + m_1)}{5c^5 a_1^3 (1 - e_1^2)^{7/2}} \left( 1 + \frac{73}{24} e_1^2 + \frac{37}{96} e_1^4 \right);
$$

for the inner orbit eccentricity,

$$
\frac{de_1}{dt} = -\frac{304G^3 m_0 m_1 (m_0 + m_1) e_1}{15c^5 a_1^3 (1 - e_1^2)^{5/2}} \left( 1 + \frac{121}{304} e_1^2 \right);
$$

and for the total loss of angular momentum due to GW radiation from the inner binary,

$$
\frac{dH}{dt} = -\frac{32G^3 m_0^2 m_1^2}{5c^5 a_1^3 (1 - e_1^2)^{3/2}} \left[ \frac{G(m_0 + m_1)}{a_1} \right]^{1/2} \times \left( 1 + \frac{7}{8} e_1^2 \right) \frac{G_1 + G_2 \theta}{H}.
$$

Note that the inner binary radiates GWs and changes the magnitude of $G_1$ and $H$, while the magnitude of $G_2$ remains unchanged.

### 2.3. Timescales

In this subsection, we present the different timescales of the dynamical problem at hand. The short timescales of the system are the inner and outer binary periods, $P_1$ and $P_2$, respectively. Using the double-averaging method, we effectively assume no variation occurs on these (or shorter) timescales. The next timescale corresponds to the quadrupole term, which gives rise to the “standard” Kozai cycling. Its characteristic timescale is $t_2 \sim G_1/G_2$. More specifically, the timescale is (Naoz et al. 2013b)

$$
P_{\text{Kozai}} = t_2 \sim 2\pi \frac{a_1^3 (1 - e_1^2)^{3/2} (m_0 + m_1)^{1/2}}{G^{1/2} m_2 a_1^{1/2}}.
$$

The timescale corresponding to the octupole level perturbations has the form of $t_3 \sim e_1^{3/2} t_2$ and the explicit term is

$$
t_3 \sim 2\pi \frac{a_1^4 (1 - e_1^2)^{5/2} (1 - e_1^2)^{1/2} (m_0 + m_1)^{3/2}}{G^{1/2} m_2 |m_0 - m_1| e_2 a_1^{5/2}}.
$$

The long-term modulation of the standard Kozai cycles occurs on this timescale.

An additional important timescale is that arising from GR effects, the timescales corresponding to the different post-Newtonian terms, which has two manifestations. The first is the precession timescale of the inner binary, and the second it the GW radiation timescale from the inner binary. For the GR precession timescale,

$$
t_{1,PN} \sim 2\pi \frac{a_1^{5/2} c^2 (1 - e_1^2)}{3 G^{3/2} (m_0 + m_1)^{3/2}}
$$

which corresponds to the time the ellipse makes a complete revolution due to GR precession (1 PN represents the first post-Newtonian term). The GW timescale is

$$
t_{2.5,PN} \sim \frac{5}{256} \frac{c^5 a_1^4 (1 - e_1^2)^{7/2}}{G^3 m_0 m_1 (m_0 + m_1)}.
$$
where $2.5\,\text{PN}$ represents the 2.5 post-Newtonian term. The physical meaning of this post-Newtonian term is the timescale for orbital energy loss due to GW emission, which could eventually lead to the inner binary merger.

Equipped with all the necessary equations arising from the gravitational dynamics, we now continue to add relevant terms arising from the evolutionary processes of mass loss and mass transfer to the equations of motion.

### 3. SECULAR EVOLUTION WITH MASS LOSS AND MASS TRANSFER

In this section, we treat the secular evolution equations of motion with mass loss. For a single star, mass-loss rates vary over a wide range, from $M \sim 10^{-14}M_\odot\text{yr}^{-1}$ to $M \sim 1M_\odot\text{yr}^{-1}$, depending on stellar mass, evolutionary stage, and the mass-loss mechanism, e.g., a supernova (SN) explosion causes prompt mass loss, and a common envelope (CM) stage in a binary system expel up to a few solar mass per year. Mass loss is therefore an important evolutionary effect for single, binary, and triple stellar evolution. Here we treat only spherical symmetric secular mass-loss/mass-transfer processes, which are defined as slow changes in mass, with respect to the orbital period, namely,

$$P_1 \cdot \dot{M}_b \ll M_b,$$

where $M_b$ is the total mass of a given mass losing binary system. For simplicity, we assume secular mass loss of mass $m_0$ in the inner binary of the form

$$\frac{d}{dt}m_0 = -\alpha,$$

where $\alpha$ is an arbitrary function satisfying Equation (46). The corrected equations of motion are presented for this case; a complete treatment for mass loss from both the inner and outer binaries is presented at the end of this section. For secular mass loss in a binary, the SMA and the eccentricity evolve according to the following equations of motion

$$\dot{a}_{\text{ML}} = -\frac{\alpha}{(m_0 + m_1)}\left(m_0 + m_1\right)$$

and a similar change in $\dot{a}_{1\text{ML}}$, which was originally a constant of motion, in the absence of mass-loss processes

$$\dot{a}_{2\text{ML}} = \frac{\alpha}{m_2 + (m_0 + m_1)^2}.$$

Figure 3. Evolution of a triple system with mass loss from the inner binary primary component, showing a MIEK-like behavior. The initial setup of the system is given by $m_0 = 7.0M_\odot$, $m_1 = 6.5M_\odot$, $m_2 = 6M_\odot$, $a_1 = 10\text{AU}$, $a_2 = 250\text{AU}$, $e_1 = 0.1$, $e_2 = 0.7$, $g_1 = 0^\circ$, $g_2 = 0^\circ$ and the mutual inclination is $i = 60^\circ$, similar to the case studied by Shappee & Thompson (2013). Constant mass loss is introduced for $m_0$ after $t = 3\text{Myr}$ for $\Delta t_{\text{ML}} = 1\text{Myr}$ until $m_0(t = 4\text{Myr}) = 1.15M_\odot$. No mass transfer is considered. Top left: the evolution of the inner binary eccentricity with time. Top right: the evolution of the inclination with time. Bottom left: the evolution of the inner angular momentum, $G_1(t)$, in the relevant units. Bottom right: the evolution of the inner angular momentum, $G_1(t)$, in the relevant units. After mass loss, from $t = 4\text{Myr}$, the system enters the octupole regime and displays an octupole level of evolution, i.e., high eccentric inner binary and even retrograde orbits.

(A color version of this figure is available in the online journal.)
Figure 4. Similar evolution as shown in Figure 3, comparing the secular method approach and $N$-body simulation. The secular equation of motion is shown in blue, and the direct $N$-body integrations are shown in red. Top left: inner eccentricity in log$(1 - e_1)$ scale. Top right: inclination vs. time. Bottom left: inner binary angular momentum, $G_1$, in the relevant units. Bottom right: total angular momentum vs. time, $H(t)$, in the relevant units. Excellent agreement can be seen between the direct $N$-body simulation and the secular code. (A color version of this figure is available in the online journal.)

The rate of change of the orbital angular momentum $G_1$ with mass loss is given by differentiating Equation (6) with respect to time

$$\dot{G}_{1,ML} = -\frac{\alpha m_1}{m_0(m_0 + m_1)} G_1$$

and the outer angular momentum $G_2$ follows

$$\dot{G}_{2,ML} = -\frac{\alpha m_2}{(m_0 + m_1)(m_0 + m_1 + m_2)} G_2.$$  

These additional terms change only in magnitude and not in direction. Note the angular momentum per unit mass is conserved; hence at the test particle limit, the angular momentum itself would also be conserved. The total angular momentum change within one orbital period time, due to the mass loss in the inner binary, is given by differentiating Equation (8)

$$\dot{H}_{ML} = \dot{G}_1 \cos i_1 + \dot{G}_2 \cos i_2 + \dot{G}_1 \cos i_{1,ML} + \dot{G}_2 \cos i_{2,ML}$$

$$= \dot{G}_1 \cos i_1 + \dot{G}_2 \cos i_2$$

Note that the last equality is obtain under the assumption of a spherical symmetric mass loss. Spherical mass loss cannot change the direction of the binary angular momentum. The change in the total angular momentum $H$ effects (23) and (24). Therefore, one needs to recalculate the rate of change of $H_1$ and $H_2$. Taking the time derivatives of Equations (21) and (22), we get

$$\dot{H}_{1/2} = \frac{H \dot{H} + G_{1/2} \dot{G}_{1/2} - G_{2/1} \dot{G}_{2/1}}{H} - \frac{H \dot{H}_{1/2}}{H}$$

for $H_1$ or $H_2$, respectively, adds an additional term proportional to $\dot{H}$

$$\dot{H}_{1,ML} = \dot{H} \left( 1 - \frac{H_1}{H} \right)$$

$$\dot{H}_{2,ML} = \dot{H} \left( 1 - \frac{H_2}{H} \right).$$

In the general case where all three objects lose mass,

$$\frac{d}{dt} m_2 = -\beta,$$

$$\frac{d}{dt} m_1 = -\gamma,$$

where $\beta$ and $\gamma$ are arbitrary functions satisfying Equation (46), the additional terms added to the inner and outer angular momenta, (19) and (53), are

$$\dot{G}_{1,ML} = \frac{-\alpha m_1}{m_0(m_0 + m_1)} G_1 + \frac{-\gamma m_0}{m_1(m_0 + m_1)} G_1$$

$$\dot{G}_{2,ML} = \left( \frac{-\alpha - \gamma}{m_0 + m_1} \right) m_2 + \frac{-\beta (m_0 + m_1)}{m_3(m_0 + m_1 + m_2)} G_2,$$
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The functions $\alpha$, $\beta$, and $\gamma$ are usually obtained from stellar evolution codes or analytic approximations.

In order to account for mass transfer between the triple components, we also need to introduce efficiency parameters $\psi_{0,1}$, $\psi_{1,0}$ and $\psi_{2,01}$. These correspond to the efficiency of mass transfer in a given binary sub-system in the triple: $\psi_{0,1}$ is the efficiency of the mass transfer from $m_0$ to $m_1$, $\psi_{1,0}$ is the efficiency of the mass transfer from $m_1$ to $m_0$, and $\psi_{2,01}$ is the efficiency of the mass transfer from $m_2$ to the inner binary. It is important to note that throughout our modeling of the mass transfer we assume no angular momentum exchange, besides that arising directly from the mass changes; i.e., we assume mass “disappears” from one object and “reappears” in another. We can then obtain the following general set of equations which can also account for mass transfer

$$
\dot{m}_0 = -\alpha + \psi_{1,0} \cdot \gamma + \psi_{2,01} \cdot \beta \cdot \frac{m_0}{m_0 + m_1}
$$

$$
\dot{m}_1 = -\gamma + \psi_{0,1} \cdot \alpha + \psi_{2,01} \cdot \beta \cdot \frac{m_1}{m_0 + m_1}.
$$

Note that the last term in Equations (63) and (64) is obtained from mass transfer from the tertiary companion, with the assumption that the mass accretion is higher into the more massive object in the inner binary (following the results of de Vries et al. 2014).

Figure 5. Evolution of the $e_3$ parameter (Equation (32)) as a function of time for the same system, as shown in Equation (32). A dramatic growth of the $e_3$ coefficient is noticeable after the mass-loss epoch at $t = 4$ Myr by more than an order of magnitude. The oscillatory behavior of $e_3$ after mass loss is due to small oscillatory evolution of $e_2$, as one can expect from octupole level of evolution. (A color version of this figure is available in the online journal.)

$$
\dot{a}_{1,ML} = \frac{\alpha + \gamma}{m_0 + m_1} a_1,
$$

$$
\dot{a}_2 = \frac{\alpha + \gamma + \beta}{m_2 + (m_0 + m_1)} a_2.
$$

Figure 6. Evolution of a triple system with mass loss from both the inner binary primary component and a consecutive mass loss from the tertiary companion, showing SEFO behavior. The system parameters are $m_0 = 7.0 M_\odot$, $m_1 = 6 M_\odot$, $m_2 = 6.5 M_\odot$, $a_1 = 10$ [AU], $a_2 = 250$ [AU], $e_1 = 0.1$, $e_2 = 0.7$, $g_1 = 0^\circ$, $g_2 = 0^\circ$, and the mutual inclination is $i = 60^\circ$. The first constant mass loss from $m_0$ is introduced after $t = 1$ Myr for $\Delta t_{ML} = 1$ Myr until $m_0 (t = 2$ Myr) $= 1.15 M_\odot$. A second constant mass loss from $m_2$ is then introduced starting at $t = 3$ Myr for $\Delta t_{ML} = 1$ Myr until $m_2 (t = 4$ Myr) $= 1.15 M_\odot$. Top left: inner eccentricity as a function of time in log $(1 - e_1)$ scale. Top right: mutual inclination of the system. Bottom left: $P_{Kozai}$ is plotted against time. Bottom right: $e_3$ is plotted against time. The SEFO evolution can be seen in the transition occurring during the second mass-loss epoch, where $e_3$ becomes less (by a factor of approximately two) and $P_{Kozai}$ grows by an order of magnitude.

(A color version of this figure is available in the online journal.)
4. EVOLUTIONARY CHANNELS

In Sections 2.3 and 2.1, we presented the relevant timescales and the parameter determining the importance of the octupole term in the secular dynamics equations of motion. These timescales are dependent on the masses and SMAs of the triple components. By varying the mass, we change the relevant timescale and the dynamics, as well as the relative importance of the octupole term. Mass loss/mass transfer can therefore change the dynamical behavior of a triple system from one regime of evolution to another. In this section, we provide examples of several triple systems showing such changes by fully integrating the equations of motions for these systems. In addition, we use direct N-body integration (based on Hut 1981)
and compare them to the results from our secular evolution method. In Section 4.1, we present a case of mass loss in the inner binary, followed by the case of mass loss from the third outer companion in Section 4.2.

4.1. Mass Loss from the Primary Components in the Inner Binary, and the Mass-loss-induced Eccentric Kozai (MIEK) Process

Shappee & Thompson (2013) studied the case of mass loss from a component in the inner binary, which leads to a transition from a more regular Kozai–Lidov secular behavior to the regime where octupole level perturbations become significant, and the amplitude of eccentricity changes become significant; they called this behavior mass-loss-induced eccentric Kozai (MIEK). They used a full $N$-body integration, which provides an excellent test case for comparison with our secular evolution approach.

We integrated the system using both the secular method as well as full $N$-body integration. The evolution of the systems is shown in Figure 3. We introduce a constant mass loss from $m_0$ starting after $t = 3$ Myr for a period of $\Delta t_{ml} = 1$ Myr, until $m_0 (t = 4$ Myr) = 1.15 $M_\odot$. No mass transfer was considered in this case, i.e., $\psi_{1,0} = \psi_{0,1} = \psi_{2,01} = 0$, and GR effects are negligible for this system. We can see very good agreement between our secular method results and the direct $N$-body simulations (which are themselves consistent with those obtained by Shappee & Thompson 2013), as can be seen in Figure 4.

As discussed in Shappee & Thompson (2013, Figure 2), before mass loss, the system is in the standard quadrupole Kozai mechanism regime, showing an oscillatory behavior of the inclination and the inner eccentricity. At this stage $e_3 \approx 0.002$, and the system is in the quadrupole regime, while after mass loss, $e_3$ becomes significantly larger ($e_3 \approx 0.045$; see Figure 5). This increase by more than an order of magnitude drives the system to an octupole-dominated regime, where the system can be driven into much higher eccentricities; hence the term mass-loss-induced eccentric Kozai (MIEK).

4.2. Mass Loss/Transfer from the Third Component and the Secular Evolution Freeze Out (SEFO) Process

We now consider the case of mass loss from the inner binary system of a triple (similar to the previous case), but now also consider an additional later mass-loss epoch from the third companion in the system. No mass transfer is considered, $\psi_{1,0} = \psi_{0,1} = \psi_{2,01} = 0$ and GR effects are negligible. Figure 6 shows the system evolution (see the figure caption for initial conditions).

The first mass-loss epoch drives the system through a MIEK process to the octupole regime, namely, $e_3$ significantly grows and the system evolves to be in the octupole level regime, where the inner binary goes through a very eccentric orbit, and the inclination evolves into a retrograde orbit. The second mass-loss epoch drives the system away from the octupole regime and closer into the quadrupole regime. The system then seems to freeze in its current state, and the inclination is kept on a retrograde orbit for longer than the $t_3$ timescale (not shown in plots). This occurs due the mass loss, which changes both $P_{Kozai}$ and $e_3$ in such a way that the octupole level of evolution does not significantly affect the system evolution on these long timescales, i.e., this process leads to what we call a SEFO.

Figure 9. Same as Figure 8, but now showing the long-term evolution. Top left: mutual inclination as a function of time. Top right: inner binary eccentricity in log $(1 - e_1)$ scale. Bottom left: octupole timescale, $t_3$, as a function of time. Bottom right: ratio of Kozai period and the octupole timescale as a function of time, $P_{Kozai}/t_3$.

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We now consider the effects of mass transfer for the same system. We integrated a triple with the same initial condition, but now we allowed for mass transfer in the inner binary, namely $\psi_{0,1} = \kappa$, for $\kappa < 1$. For a wide range of $\kappa$, we got two distinct outcomes: for $\kappa \lesssim 0.45$, the end result was a freeze-out retrograde system, while for $\kappa \gtrsim 0.45$, the system did not evolve into a retrograde orbit during its evolution. A representative case for this mass-transfer evolution is shown in Figure 7.

4.3. Mass Transfer from a Third Companion to the Inner Binary and the Formation of a Short Period Inner Binary

We now consider a case of mass transfer from the third companion into the inner binary. This mass-transfer epoch increases the masses of the inner binary components and leads to the compactification of the inner binary. We show an example of such evolution in the following, where we also demonstrate the effects of GR precession on the processes by showing the evolution of the system both with and without including GR effects. In this case, a fraction of $\psi_{2,01} = 0.2$ of the mass lost from $m_2$ is gained by the inner binary; the mass is divided between the binary components according to Equations (63) and (64). Figures 8 and 9 show the evolution of the system on both short and long timescales, respectively, for the case without GR effects (see the system parameters in the figure caption). As can be seen, after the mass transfer to the inner binary the inner SMA shrinks by a factor of approximately two, and the period of the inner binary reaches approximately three days.

Figure 9 shows the evolution of the mutual inclination and the inner binary eccentricity on a longer timescale compared with Figure 8. On this longer timescale, the modulation from the octupole perturbation regime can also be seen.

GR effects become important when the timescales of $t_{1,PN}$ and $t_{2,5PN}$ become comparable to or smaller than the other dynamical timescales. This occurs when the inner binary becomes highly eccentric and/or the inner binary SMA, $a_1$, becomes small. Once the timescale for $t_{1,PN}$ becomes comparable to the $P_{\text{Kozai}}$, GR precession perturbs the coherent evolution of the Kozai–Lidov evolution and the amplitude of the oscillatory behavior of the Kozai mechanism (Naoz et al. 2013b) is quenched. In Figure 10, we show the evolution of the same system, where

Figure 10. Similar to Figure 8, but including GR effects. The system parameters: $m_0 = 0.5 M_\odot$, $m_1 = 0.6 M_\odot$, $m_2 = 7 M_\odot$, $a_1 = 0.1$ AU, $a_2 = 20$ [AU], $\epsilon_1 = 0.01$, $\epsilon_2 = 0.6$, $g_1 = 0^\circ$, $g_2 = 0^\circ$, and $\iota = 60^\circ$. Constant secular mass loss on $m_2$ after $t = 0.5$ Myr for $\Delta t_{\text{ML}} = 10^5$ yr until $t_2 = 1.15 M_\odot$, $\psi_{2,01} = 0.2$. GR effects are included. Top left: mutual inclination as a function of time. Top right: inner binary eccentricity in log(1 − $e_1$) scale. Bottom left: $P_{\text{Kozai}}$ as a function of time. Bottom right: $t_{\text{Kozai}}/t_{1,\text{PN}}$ as a function of time. After mass transfer $t = 0.6$ Myr, the precession timescale is much shorter than the Kozai period, resulting in quenching the Kozai effect.

(A color version of this figure is available in the online journal.)
we now include GR effects; the quenching of the oscillatory behavior by the GR effects is clearly seen and can be well understood, since after the mass-transfer the 1PN timescale becomes smaller than the Kozai period. Indeed, the Kozai period increases by more than an order of magnitude up to 10 Myr, and we find no variation of the orbital elements occurring on this (or longer) timescale (not shown).

5. DISCUSSION AND SUMMARY

Realistic stellar systems undergo different evolutionary scenarios where secular mass loss and mass transfer play an important role. These include winds from evolved stars, Roche lobe overflow (RLOF), wind RLOF, etc. Incidents of mass transfer or mass loss may also be relevant to other systems, whether planets, moons, or asteroidal systems (e.g., through atmospheric evaporation). Such processes have been studied extensively and their effects on binary systems have been explored. Here we extended the study of these effects and model the effects of secular mass loss transfer on the long-term dynamical behavior of triple systems; the rich dynamics of triples becomes even more complex when coupled to such variation in the mass of the components.

Here we have shown for the first time that the dynamics of a triple hierarchical system, including mass-loss and mass-transfer, can be well modeled via double averaged Hamiltonian expanded up to an octupole level (Harrington 1968; Ford et al. 2000; Blaes et al. 2002). This Hamiltonian presents several relevant timescales, which are significantly dependent on the mass and separation of the system components. Mass loss/mass transfer therefore affect these relevant timescales and can thereby transfer a triple system from one type of dynamical behavior regime to another, where the dynamics are dominated by a different type of perturbations/physical processes.

We used our model to study the evolution of various types of triple systems and demonstrated several of such evolutionary channels. These include the mass-loss induced eccentric Kozai (MIEK) process (also studied by Shappee & Thompson 2013 using $N$-body integration) and its reversed inverse-MIEK process, the secular evolution freeze-out (SEFO), and mass transfer from a third companion to an inner binary (see also the related study by SPH and $N$-body integration by de Vries et al. 2014). The MIEK process transfers a triple from the quadrupole (stable oscillatory behavior) to the octupole regime (quasi-periodic and even chaotic behavior leading to extremely large changes in eccentricities and inclinations), where an inverse-MIEK, transferring a system from the octupole to the quadrupole regime, can also occur when mass is lost from the third outer companion. An additional similar process, SEFO, can transfer a system from the quadrupole regimes to a state where secular evolution is either quenched or operates on excessively long timescales. Mass transfer from a third companion can both induce the formation of short-period binaries, as well as lead to quenched secular evolution, at the point where the systems become more susceptible to GR effects, which quench any further oscillatory behavior.

Finally, the model developed here shows excellent agreement with full $N$-body integration schemes, but has the important advantage of providing much faster (orders of magnitude) calculations. It provides a useful tool for the study of triple systems, and in particular for the exploration of a large phase space of initial conditions. It could therefore also be integrated into studies of larger-scale systems (such as stellar clusters, where triples can play an important role) and/or population synthesis studies where understanding the evolution of many triple systems becomes important.

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