Early Supernova Emission: Logarithmic Corrections to the Planar Phase

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Abstract

When the shock wave generated in a supernova explosion breaks out of the stellar envelope, the first photons, typically in the X-ray-to-UV range, escape to the observer. Following this breakout emission, radiation from deeper shells diffuses out of the envelope as the supernova ejecta expands. Previous studies have shown that the radiation throughout the planar phase (i.e., before the expanding envelope has doubled its radius) originates in the same mass coordinate, called the “breakout shell.” We derive a self-similar solution for the radiation inside the envelope and show that this claim is incorrect and that the radiation propagates logarithmically into the envelope (in a Lagrangian sense), rather than remaining at a fixed mass coordinate. The logarithmic correction implies that the luminosity originates in regions where the density is \( \sim 10 \) times higher than previously thought, where the photon production rate is increased and helps thermalization. We show that this result has significant implications for the observed temperature. In our model, the radiation emitted from blue supergiant and Wolf–Rayet explosions is still expected to be out of thermal equilibrium during the entire planar phase, but the observed temperature will decrease by 2 orders of magnitude, contrary to previous estimates. Considering the conditions at the end of the planar phase, we also find how the temperature and luminosity transition into the spherical phase.

Key words: radiation; dynamics – shock waves – supernovae: general

1. Introduction

The first emission from a supernova (SN) explosion is called the “shock breakout” and marks the emergence of a shock wave from the stellar envelope. During the explosion, a radiation-dominated shock wave accelerates in the decreasing density profile of the progenitor star. It deposits its energy into the envelope, leaving behind it an expanding radiation-dominated gas. Once the width of the shock becomes comparable to the distance from the edge of the star, the photons that power the shock wave diffuse ahead of it, and the shock “breaks out” of the envelope (Colgate 1974; Falk 1978; Klein & Chevalier 1978; Imshennik et al. 1981; Ensmann & Burrows 1992; Matzner & McKee 1999). This breakout emission typically lasts for a few seconds and is followed by a long phase of photon diffusion, which produces emission that decays slowly (Nakar & Sari 2010; Piro et al. 2010; Rabinin & Waxman 2011).

Immediately after breakout, the evolution of the ejecta is described by planar expansion, during which the radius of the envelope remains roughly constant. The duration of the planar phase depends on the radius of the progenitor and on the shock velocity and typically lasts from a few minutes in compact progenitors to hours for more extended stars. After the radius of the SN has doubled its size, the envelope enters the spherical phase, in which the ejecta expands homologously. In this paper, we focus on the planar phase evolution. This phase was investigated in very few works, including Nakar & Sari (2010), who examined the post-breakout emission for various types of progenitors, and Piro et al. (2010), who looked at the breakout from Type Ia SNe. Both of these works assumed that the diffusion wave stays at a fixed mass, implying that the same material is the source of radiation throughout the entire planar phase. They estimated that the breakout emission peaks somewhere between the UV and soft \( \gamma \) rays, depending on the type of the progenitor. As the SN ejecta expands and cools adiabatically, the radiation is shifted to lower frequencies. Early shock breakout emission might have been observed as an X-ray outburst in the cases of SN 2006aj and SN 2008D (Campana et al. 2006; Soderberg et al. 2008). Later optical emission suggested that both were Ib/c SNe, whose progenitors are compact Wolf–Rayet (WR) stars (Maeda et al. 2007; Malesani et al. 2009; Mazzali et al. 2006, 2008; Modjaz et al. 2006; Pian et al. 2006).

In this work, we find self-similar solutions for the radiation in the SN envelope. We first solve for the hydrodynamic profiles of the SN envelope after shock passage and during the planar expansion, according to Sakurai (1960). We then use these profiles as initial conditions for the following stage of photon diffusion. This derivation takes into account the density and shock velocity profiles (given that they can be described as polytropes) and is therefore expected to produce more accurate solutions than the approximate order-of-magnitude models. A self-similar solution for the radiation of Type II SNe was also found by Chevalier (1992). However, they treated only the spherical phase and did not provide a solution for the earlier planar phase.

The most important result of this paper is that through the similarity solution, we were able to find a logarithmic correction to the dynamics of the planar phase, compared to what was thought so far. We find the mass from which radiation escapes to the observer at the end of the planar phase to be an order of magnitude larger than estimated before. Correspondingly, the emission peaks at lower frequencies than would have been inferred without the logarithmic correction.

In Section 2 we describe the pre-explosion structure of the progenitor, and in Section 3 we find the planar hydrodynamic evolution of the envelope. We treat the breakout phase and find a self-similar solution for the energy in the envelope including diffusion in Section 4. We then provide the resulting luminosity in Section 5 and observed temperature in Section 6. We discuss light travel time effects in Section 7 and apply the results of this paper to various progenitors in Section 8.
2. Pre-expansion Hydrodynamics

The radiation-dominated shock wave that forms during the explosion propagates through the ejecta and accelerates in the decreasing density profile of the envelope. The pre-explosion density profile of the progenitor star near its surface can be approximated as a power law of the distance from the edge of the star, \( R - r \), namely,

\[
\rho \propto (R - r)^n,
\]

where \( R \) is the stellar radius, \( r \) is the distance from the center of the star, and \( n = 3/2 \) for radiative envelopes and 3 for convective envelopes. This density profile is a simplification that describes well only the outer regions of the stellar envelope. Since in this work, we focus on the first seconds to hours of the SN evolution, the above description for the density profile serves as a good approximation.

The hydrodynamics of shock passage through a stellar envelope in the form of Equation (1) were treated in Sakurai (1960); hereafter S60. As in S60, we assume that the shock wave is a discontinuity that reaches all the way to the edge of the star, i.e., \( R - r = 0 \). In reality, the shock has a certain width, and it breaks out of the envelope when its width is comparable to the distance to the edge of the star. However, an accurate description of the dynamics at breakout lies beyond the scope of this paper. We therefore adopt the simplistic view of S60 but note that our analysis may not well describe material lying ahead of the breakout location.

Throughout this work, we follow the notations of S60 for the pre-explosion profile of the star. The density is parameterized as

\[
\rho_i = \kappa_1(R - r)^\mu,
\]

and the shock velocity is

\[
U_{\text{shock}} = \kappa_2(R - r)^{-\mu n},
\]

where \( \mu \) is determined by requiring continuity at the sonic point, while \( \kappa_1 \) and \( \kappa_2 \) are constants that depend on the progenitor mass \( M \), radius \( R \), and explosion energy \( E \) of the progenitor star. We choose the following representation for \( \kappa_1 \) and \( \kappa_2 \):

\[
\kappa_1 = C_1 \frac{M}{R^{n+3}}, \quad \kappa_2 = C_2 \left( \frac{E}{M} \right)^{1/2} R^m.
\]

The constants \( C_1 \) and \( C_2 \) are dimensionless constants that depend on the inner structure of the progenitor star. In Section 8 we estimate their values for different progenitors.

Using Equations (2) and (3), we find self-similar solutions for the hydrodynamic equations (Equations (2)–(4) in S60). We do not provide the full calculation here but note that the only difference from S60’s work is that we do not consider an external gravitational force, since the energy of the explosion is typically much larger than the gravitational energy, i.e., \( GM^2/R \ll E \). For given values of \( n \) and the adiabatic index \( \gamma \), there is a single value of \( \mu \) that satisfies the continuity condition at the sonic point. Since the value of \( \mu \) is insensitive to \( n \), we use the canonical value of \( \mu = 0.19 \). For further details on the solution, we refer the reader to S60. After the passage of the shock, the ejecta gains kinetic energy and starts to expand while decreasing in density and internal energy. The following hydrodynamic evolution can be divided into two temporal regimes: a planar phase, during which the radius of the envelope is nearly constant \( (r \approx R) \), and a spherical phase that starts after the envelope has roughly doubled its size, where the radius evolves homologously and satisfies \( r = v \times t \). In this paper, we treat only the planar phase and the short transition into the spherical phase while taking into account both adiabatic cooling and photon diffusion.

3. Planar Hydrodynamics

The stage of expansion into vacuum was first treated in S60. We repeat the procedure and solve the hydrodynamic equations (Equations (16)–(19) in S60) using the solution of the preceding stage when the shock is at \( r = R \) as initial conditions. Here, instead of the spatial Lagrangian coordinate “\( a \)” used in S60, we write the expressions in terms of the Lagrangian mass coordinate \( m \), which is related to \( a \) by

\[
m = \int 4\pi R^2 \rho_0 da,
\]

where \( \rho_0 \) is the density profile in the envelope when the shock reaches the stellar edge. We find the self-similar solutions for the fluid velocity \( \nu \), density \( \rho \), pressure \( P \), and distance from the pre-expansion stellar radius \( x \) during the planar expansion phase,

\[
x(m, \xi) = \bar{x} \left( \frac{m}{M} \right)^{1/(n+1)} r(\xi),
\]

\[
\nu(m, \xi) = \bar{\nu} \left( \frac{m}{M} \right)^{-\mu n/(n+1)} F(\xi),
\]

\[
\rho(m, \xi) = \bar{\rho} \left( \frac{m}{M} \right)^{n/(n+1)} H(\xi),
\]

\[
P(m, \xi) = \bar{P} \left( \frac{m}{M} \right)^{(n-2\mu n)/(n+1)} G(\xi),
\]

where we define the following normalizations:

\[
\bar{x} \equiv (4\pi R^2 h(0) \kappa_1 M^{-1} n^{-1/(n+1)}) = R [4\pi h(0) C_1]^{1/\gamma},
\]

\[
\bar{\nu} \equiv f(0) \kappa_2 \bar{x}^{-\mu} = \left( \frac{E}{M} \right)^{1/2} f(0) C_2 [4\pi h(0) C_1]^{\mu/\gamma},
\]

\[
\bar{\rho} \equiv h(0) \kappa_1 \bar{x}^n = \frac{M}{R^3} [4\pi]^{\gamma} h(0) C_1^{1/\gamma},
\]

\[
\bar{P} \equiv \frac{g(0)}{f(0)^2 h(0)} \bar{\rho} \bar{\nu}^2 = \frac{E}{R^3} [4\pi h(0)]^{\mu/(n+1)} C_2^2 C_1^{2\mu n/(n+1)},
\]

and \( F(\xi), H(\xi), G(\xi), \) and \( r(\xi) \) are numerical functions of the self-similar variable \( \xi \), which is defined as

\[
\xi = \left[ \frac{m(n+1)}{4\pi R^2 h(0) \kappa_1} \right]^{(n+1)/(n-1)} 2 \kappa_2 t.
\]

The parameters \( h(0), g(0), f(0) \) are constants that depend on \( n \) and are found by solving the equations of the previous stage of shock passage numerically (see S60). Their values for \( n = 3 \) and 3/2 are given in Table 1.

The functions \( F(\xi), H(\xi), G(\xi), \) and \( r(\xi) \) do not have an analytic solution on the whole range of \( \xi \). However, they approach a single power-law profile at \( \xi \gg 1 \) (for a given \( t \), this would refer to the outer layers of the envelope that have already
expanded). Power-law fitting to the numerical solutions of \( F(\xi), G(\xi), H(\xi), \) and \( \eta(\xi) \) in the limit of \( \xi \gg 1 \) gives

\[
F(\xi) = \alpha_F, \quad (9a)
\]

\[
H(\xi) = \alpha_H \xi^{-1}, \quad (9b)
\]

\[
G(\xi) = \alpha_G \xi^{-\gamma}, \quad (9c)
\]

\[
\eta(\xi) = \alpha_\eta \xi, \quad (9d)
\]

where \( \alpha_F, \alpha_H, \alpha_G, \) and \( \alpha_\eta \) are numerical parameters of the fit whose values are given in Table 2. Using equation sets (6) and (9) and the expression for \( \xi \) in Equation (8), we write the expressions for \( x, v, \rho, \) and \( P \) as functions of \( m \) and \( t \):

\[
x(m, t) = \alpha_r \kappa_2 \bar{X}^{-\mu} P(m) \left( \frac{m}{M} \right)^{\frac{\mu}{\gamma-1}} t, \quad (10a)
\]

\[
v(m) = \alpha_F \sqrt{P(m)} \left( \frac{m}{M} \right)^{\frac{\mu}{\gamma-1}} t, \quad (10b)
\]

\[
\rho(m, t) = \frac{\alpha_H \kappa_2}{\kappa_2} \bar{X}^{\mu+1} \left( \frac{m}{M} \right)^{1+\frac{\mu}{\gamma-1}} t^{-\gamma}, \quad (10c)
\]

\[
P(m, t) = \frac{\alpha_G \kappa_2}{\kappa_2} \bar{X}^{\gamma} \left( \frac{m}{M} \right)^{\frac{\mu}{\gamma-1}} t^{-\gamma}. \quad (10d)
\]

Comparing Equation 10(b) with Equation 6(b), we reproduce the known result by S60, showing that the velocity reaches an asymptotic value, which is roughly twice the initial velocity and a function only of \( m \). Furthermore, since the radius remains approximately constant during the planar phase and \( x \propto t \), the density at a fixed mass coordinate evolves as \( \rho \propto t^{-1} \). The evolution described in equation set (10) does not start at \( t = 0 \) for every \( m \), since the width of a shell is effectively constant as long as its size has not doubled. This earlier stage of the evolution is not described by the asymptotic solution presented in equation set (10).

We use the solutions for \( P \) and \( \rho \) in equation set (10) to find the evolution of the internal energy,

\[
u(m, t) = \frac{1}{\gamma-1} \frac{P}{\rho} = 3 \frac{\alpha_G \kappa_2^{-1/3} P \bar{X}^{\gamma+1} (\mu+1) \left( \frac{m}{M} \right)^{1-3\mu} \left( \frac{m}{M} \right)^{\frac{\mu}{\gamma-1}} t^{-1/3}, \quad (11)
\]

where we substituted \( \gamma = 4/3 \) for a radiation-dominated gas. In the internal regions, diffusion is negligible, and the internal energy changes only due to adiabatic cooling.

### 4. Diffusion during the Planar Phase

For the purpose of describing the diffusion of radiation inside the SN ejecta, it is useful to treat the envelope as a series of successive shells, where inside each shell, the hydrodynamic properties, such as density, velocity, and radius, do not vary a lot. Each shell is assigned a mass \( m \) corresponding to that defined in Equation (5), which is roughly the integrated mass measured from the edge of the stellar envelope down to the internal radius of the shell. According to this definition, the more massive the shell, the more internal its position inside the envelope. Each shell has a characteristic width \( d = d_0 + vt \), where \( d_0 \) is its pre-explosion width. The planar self-similar dynamics for each shell starts once its width has doubled, such that \( d \sim v t \gg d_0 \).

In the previous section, we found an expression for the internal energy when photon diffusion is negligible. In this section, we find the energy profile that forms due to diffusion of photons through the ejecta and use that to compute the observed bolometric luminosity. We begin with an order-of-magnitude derivation of the luminosity shell, which is the source of the escaping photons, and then solve the equation that describes the energy evolution inside the envelope to find an accurate self-similar solution for the energy and the postbreakout shock cooling emission.

#### 4.1. The Breakout and Luminosity Shells

In the early stages after shock passage, the density and temperatures inside the envelope are very high, and the matter is opaque to radiation. Since the temperatures are higher than \( \sim 1 \text{ eV} \), the gas is fully ionized, and the dominant opacity source is Thomson scattering. The diffusion time through the envelope therefore depends only on the mass of the shell and its width \( d \).

\[
t_{\text{diff}} = 3 \kappa_T \frac{m}{4 \pi R^2 c} \frac{d}{c}, \quad (12)
\]

where \( \kappa_T \) is the Thomson opacity and \( m \) is the mass of the shell. A photon originating in a shell of mass \( m \) will be able to escape the envelope when the diffusion time from that shell is equal to its dynamical time, namely,

\[
t_{\text{dyn}} = t_{\text{diff}}, \quad (13)
\]

where \( t_{\text{dyn}} = t \). Equation (13) is equivalent to the condition \( \tau = c / v \), where \( \tau \) is the optical depth and \( c \) is the speed of light. The shell that satisfies this condition is called the “breakout shell,” and the time it takes photons to diffuse out of the bottom of that shell before expansion is the breakout time, \( t_{\text{bo}} \). The properties of the breakout shell are denoted by the subscript \( \text{bo} \).

Using Equation (12) and the solutions for \( \rho \) and \( x \) in equation set (10) (and setting \( d \sim x \)), we find the expression for the mass of the breakout shell:

\[
m_{\text{bo}} = M \left( \frac{3 \kappa M}{4 \pi R^2 c} \alpha_r \kappa_2 \bar{X}^{-\mu} \right)^{\frac{\mu}{\gamma-1}}. \quad (14)
\]
The breakout time is evaluated as the initial dynamical time of the breakout shell, namely, \( t_{bo} = \frac{d_0}{v_{bo}} \), where

\[
d_0(m) = M \left( \frac{m}{M} \right)^{\frac{1}{1+\mu_0}}
\]

is the initial width of a shell of mass \( m \) before expansion and \( v_{bo} \) is the velocity of the breakout shell before the planar acceleration. Using Equations (10b), (14), and (15), we find the expression for the breakout time:

\[
t_{bo} = \frac{c}{1+\mu_0} \left( \frac{3\pi M}{4\pi R_c^3} \right)^{\frac{1}{1+\mu_0}} \left( \frac{M}{m} \right)^{\frac{\mu_0}{1+\mu_0}}
\]

The properties of the breakout shell at \( t_{bo} \) can be found by substituting the mass of the breakout shell in Equation (14) into equation set (10). The typical breakout properties for red supergiant (RSG), blue supergiant (BSG), and WR explosions are given in Appendix A.

At later times, the shell out of which the observed photons originate is called the “luminosity shell,” and its properties are denoted by the subscript \( L \). The luminosity of an SN at time \( t \) is dominated by the energy stored in the luminosity shell. External to the luminosity shell, at \( m < m_{bo} \), the luminosity is constant. Equations (12) and (13) imply that the optical depth of each shell is constant in time during the planar phase. Therefore, there is only one shell that satisfies \( \tau = c/v \), and naively, the breakout shell should serve as the luminosity shell throughout the entire planar phase. Although this is the picture that was accepted until now (Nakar & Sari 2010, hereafter NS10), it can be contradicted with simple arguments. Given that the duration of the planar phase is much longer than the dynamical time of the breakout shell at breakout, the time will double itself many times until the beginning of the spherical phase, such that at some point, the breakout shell will have lost roughly all of its energy, and photons from shells that are further in will be able to diffuse out of the envelope. The luminosity shell will thus recede, though slowly, and expose deeper regions of the outflow. To estimate the correction to the mass of the luminosity shell, consider a shell of mass \( m \) that is internal to \( m_{bo} \) such that \( t_{bo}(m) < t_{diff}(m) \). A fraction of the energy in that shell is able to escape, and the energy that remains in it at time \( t \) can be approximated as

\[
E(t) \approx E_0 \left( 1 - \frac{t_{dyn}(m)}{t_{diff}(m)} \right)^{\log \left( \frac{\rho_{in} / \rho_{in}}{\rho_{out} / \rho_{out}} \right)}
\]

where \( E_0 \) is the initial internal energy in the shell and \( \log \left( \frac{\rho_{in} / \rho_{out}}{\rho_{out} / \rho_{in}} \right) \) counts the number of dynamical times that have passed for that shell since the beginning of expansion. The shell out of which photons effectively diffuse out, namely, the luminosity shell, can be defined as the shell that satisfies \( E(t)/E_0 \sim 0.5 \). Solving Equation (17) for this condition with \( t_{dyn}/t_{diff} \ll 1 \), we find an approximate expression for the mass of the luminosity shell:

\[
m_{LS} \approx m_{bo} \left[ 2 \cdot \log \left( \frac{t}{d_0(m) / v(m)} \right) \right]^{\frac{(n+1)}{(n+1-\mu)}}
\]

The luminosity shell satisfies \( t < t_{diff} \) but is nevertheless the shell out of which photons diffuse out to the observer.

In the next section, we derive a more rigorous and accurate expression for \( m_{LS} \) that agrees with the scaling in Equation (18).

4.2. Self-similar Solution for the Radiation

The equation that describes the evolution of the specific energy is

\[
\frac{\partial u}{\partial t} = \frac{1}{\partial m} (4\pi R_c^3 c^3 - u)^{\frac{1}{\alpha}}
\]

where

\[
F = D \frac{\partial (u \cdot \rho)}{\partial x}
\]

is the flux and \( D = c/3\kappa \) is the diffusion coefficient. Equation (19) can be solved with a self-similar solution. We want to transform \( u \) and \( m \) into new variables, such that the dependence on \( t \) is eliminated from the equation and it becomes an ordinary differential equation (ODE). In this problem, the luminosity shell serves as the self-similar coordinate, since it is the location from which photons effectively diffuse out, and the specific energy profile changes from an inner adiabatic profile to an outer diffusive profile. Equation (18) is an estimate of the mass of the luminosity shell. A more formal definition is obtained by determining the correct transformation of the variable \( m \). We represent the luminosity shell as an unknown function of the time, \( m_{LS} = F(t) \), and define new variables by normalizing \( m \) and \( u \) to the mass and the adiabatic energy of the luminosity shell, respectively:

\[
\tilde{m} \equiv \frac{m}{F(t)} \quad \tilde{u} \equiv \frac{u}{u_{bo}[F(t)/m_{bo}]^{1-\mu_o}(t/t_{bo})^{-1/3}}
\]

Plugging the new variables into Equation (19), it can be shown that in order to transform Equation (19) into an ODE, \( F(t) \) needs to satisfy

\[
F'(t) \cdot [F(t)]^{n/(n+1)} \propto t^{-1}
\]

with the initial condition \( F(t = t_{bo}) = m_{bo} \). The two conditions above give the solution for \( F(t) \):

\[
m_{LS}(t) = F(t) = m_{bo} \left[ 1 + \log(\frac{t}{t_{bo}}) \right]^{(n+1)/(n+1-\mu)}.
\]

We note that although radiation leaks out of deeper shells, only the breakout shell satisfies \( t = t_{diff} \).

In the problem we are solving, the pre-diffusion profiles are represented by power laws as evaluated in equation set (10), which implies unphysical infinite values at \( x = 0 \). In this representation, according to Equation (23), the first shell out of which energy leaks is located at \( m = 0 \), which is satisfied when \( t = t_{bo}/c \). Therefore, between the time the first photons started leaking out of the envelope and \( t = t_{bo} \), the time has only roughly doubled itself. The considered power-law profiles are obviously not realistic close to \( x = 0 \) and serve as a good description of the real SN envelope only at small values of \( m \) or large \( t \) (i.e., \( \xi \gg 1 \)).
With the corrected expression for \( m_{\text{bo}} \), we perform the following variable transformation:

\[
\tilde{m} \equiv \frac{m}{m_{\text{bo}}[1 + \log(t/t_{\text{bo}})]^{\frac{n+1}{n-1+\mu n}}},
\]

\[
\tilde{u} \equiv \frac{u}{u_{\text{bo}}[1 + \log(t/t_{\text{bo}})]^{\frac{1-\mu n}{n-1+\mu n}}}^{1/3},
\]

and obtain the self-similar form of Equation (19):

\[
\frac{d^2\tilde{u}}{d\tilde{m}^2} \tilde{m}^{1+\mu n/(n+1)} \frac{n+1}{\mu n} \\
+ \frac{d\tilde{u}}{d\tilde{m}} \left[ 2 \frac{n+1}{\mu n} \left( 1 + \frac{\mu n}{n+1} \right) \tilde{m}^{\mu n/(n+1)} + \tilde{m}^{-\mu n/(n+1)} \frac{n+1}{n+1-\mu n} \right] \\
+ \tilde{u} \left[ 1 + \frac{\mu n}{n+1} \right] \tilde{m}^{\mu n/(n+1)-1} \frac{1 - 5\mu n}{3(n+1-\mu n)} = 0.
\]

This equation can be solved analytically in the limits of \( \tilde{m} \gg 1 \) and \( \tilde{m} \ll 1 \) by substituting a general power-law solution of the form \( \tilde{u} = A\tilde{m}^\beta \). We find

\[
\tilde{u} = \begin{cases} 
    \tilde{m}^{\beta(1-5\mu n)/3(n+1)}, & \tilde{m} \gg 1 \\
    A\tilde{m}^{-\mu n/(n+1)}, & \tilde{m} \ll 1
\end{cases},
\]

where \( A \) is a numerical constant that is found by solving Equation (25) numerically. The solution for \( u \) in the limit \( m \gg m_{\text{um}} \) is accurately known from Equation (11) since it is purely adiabatic, and we use it as a boundary condition at \( \tilde{m} \gg 1 \). Fitting the numerical solution for \( A\tilde{m}^\beta \) in the limits \( \tilde{m} \gg 1 \) and \( \tilde{m} \ll 1 \), we find that \( A = 1.17 \) for \( n = 3/2 \) and \( A = 0.97 \) for \( n = 3 \). In Figure 1, we plot the numerical solution for \( n = 3/2 \) and the analytic approximations at \( \tilde{m} \ll 1 \) and \( \tilde{m} \gg 1 \).

Using Equations (24) and (26), we write the solution for \( u \) in the external parts of the ejecta:

\[
u(m \ll m_{\text{bo}}) = \mathcal{A}u_{\text{bo}} \left( \frac{m}{m_{\text{bo}}} \right)^{\frac{n}{n-1+\mu n}} [1 + \log(t/t_{\text{bo}})]^{\frac{1-2\mu n}{n+1+\mu n}} \\
\times \left( \frac{t}{t_{\text{bo}}} \right)^{1/3}.
\]

The solution at \( \tilde{m} \ll 1 \) is then used to compute the bolometric luminosity of the SN.

### 5. The Bolometric Luminosity

#### 5.1. Planar Phase Luminosity

The bolometric luminosity during the planar phase is computed according to

\[
L = 4\pi R^2 \mathcal{F},
\]

where \( \mathcal{F} \) was defined in Equation (20). Using the self-similar solution for \( u(m \ll m_{\text{bo}}) \) in Equation (27), together with the expression for \( \rho \) in Equation 10(c), we can express the luminosity as

\[
L = \mathcal{A}u_{\text{bo}} m_{\text{bo}} \left( \frac{n+1}{\mu n} \right) [1 + \log(t/t_{\text{bo}})]^{\frac{1-2\mu n}{n+1+\mu n}} \\
\times \left( \frac{t}{t_{\text{bo}}} \right)^{-4/3} \left( 1 + \log(t/t_{\text{bo}}) \right)^{0.06}, \quad n = 3/2 \\
= \left( r^{-4/3} \left[ 1 + \log(t/t_{\text{bo}}) \right]^{0.01} \right)^3, \quad n = 3.
\]

The factor \( [1 + \log(t/t_{\text{bo}})]^{\frac{1-2\mu n}{n+1+\mu n}} \) can thus either slow down or accelerate the decrease of the luminosity, depending on the value of \( n \). However, the power of the logarithmic correction is very small and does not have a noticeable effect on the observed luminosity of typical SN events. The reason for the weak response of the luminosity to the significant increase in \( m_{\text{bo}} \) is that the flat profile of the internal luminosity in the adiabatic part of the envelope (denoted \( L_{\text{int}}(m) \), the energy per unit time that crosses a shell inside the SN envelope). Using the expression for the adiabatic evolution of \( u \) in Equation (11), we can write the profile of the internal luminosity in shells deeper than the luminosity shell:

\[
L_{\text{int}}(m) \approx \frac{u(m_{\text{bo}} < m) \cdot m}{t_{\text{diff}}} \propto r^{-4/3} m^{\frac{1-2\mu n}{n+1+\mu n}}.
\]

For \( n = 3/2 \), the power of \( m \) is \( \sim 0.06 \), and for \( n = 3 \), it is \( \sim 0.01 \), implying that the spatial profile of the luminosity is rather flat. Therefore, the increase in \( m_{\text{bo}} \) can have only a scarce effect on the luminosity.

#### 5.2. Planar-spherical Transition

The planar phase ends when the ejecta radius has roughly doubled itself. The dynamics then transition into the spherical phase, where the SN radius is no longer constant but evolves homologically as \( r \sim v \times t \). Since the ejecta velocity is a function only of the mass (Equation 10(b)), lower-mass shells enter the spherical phase earlier than more massive, internal
spherical phase. At the end of the planar phase, enters the spherical phase.

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luminosity. Afterward, the luminosity shell is located where

spherical phase, it

remains so until it

τ

= c/v, since it is the source of the observed luminosity. Afterward, the luminosity shell is located where

τ

= c/v. As a result, before the radiation enters the “nominal” spherical phase, it first goes through a transition phase, during which the luminosity is governed by the energy that diffused out of the luminosity shell at t


.In Figure 2, we compare the luminosity shell defined in this work to the previously perceived “naive” luminosity shell, which is located at m (τ = c/v) throughout the evolution.

In order to find the bolometric luminosity during the transition phase, we first find the expressions for the relevant hydrodynamic properties during their spherical evolution.

The radii of the shells within the spherical phase follow a homologous expansion and have the following dependence on m and t:

\[ r(m, t) \approx v(m) \cdot t = v_{bo} \cdot t \left( \frac{m}{m_{bo}} \right)^{-\nu m/(n+1)} . \]  

(33)

The evolution of r, together with conservation of mass, implies that the density of a shell drops as \( \rho \propto r^{-3} \) starting at \( R/v(m) \):

\[
\rho(m, t = R/v(m)) = \rho_{bo} \left( \frac{m}{m_{bo}} \right)^{1 + 3\nu m/(n+1)} \left( \frac{t}{t_{bo}} \right)^{-3}.
\]

(34)

With the above expressions for \( \rho \) and r, we can estimate the mass of the shell that satisfies \( \tau = c/v \) during the spherical phase by requiring that \( t_{diff} = t_{dyn} \). The diffusion time during the spherical phase can be written as

\[ t_{diff} \approx 3K_T m_{pl} (4\pi\rho c)^{1/2} , \]

and therefore,

\[ m(\tau = c/v) = m_{bo} \left( \frac{t}{t_{bo}} \right) \left( \frac{t_{bo}}{m_{bo}} \right)^{2/(n+1)} . \]

(36)

The specific energy density at \( m_{pl} < m \) can be found using the solutions for \( u \) and \( \rho \) at \( t = R/v(m) \) in Equations (10(c) and (11) and the expression for \( \rho \) in the spherical phase in Equation (34):

\[ u(m_{pl} < m) = u(m_{pl} < m, t = R/v(m)) \left( \frac{\rho(m, t)}{\rho(m, t = R/v(m))} \right)^{\gamma - 1} \]

\[ = u_{bo}(t_{bo}) \left( \frac{m}{m_{pl}} \right)^{1 - 3\nu m/(n+1)} \left( \frac{t}{t_{bo}} \right)^{-1} \left[ 1 + \log(t_{bo}/t_{bo}) \right]^{1 - 3\nu m/(n+1)} . \]

(37)

The specific energy profile at \( m < m_{pl} \) was formed by diffusion at the end of the planar phase. Using Equation (27), we find

\[ u(m < m_{pl}) = u(m < m_{pl}, t = R/v(m)) \left( \frac{\rho(m, t)}{\rho(m, t = R/v(m))} \right)^{\gamma - 1} \]

\[ = u_{bo}(t_{bo}) \left( \frac{m}{m_{bo}} \right)^{1 - \frac{\nu m}{n+1}} \left( \frac{t}{t_{bo}} \right)^{-1} \left[ 1 + \log(t_{bo}/t_{bo}) \right]^{1 - 2\nu m/(n+1)} . \]

(38)
The luminosity in the transition phase can be approximated using Equations (36) and (38):

\[ \frac{L(t)}{t} = \frac{u(\tau = c/v) \cdot m(\tau = c/v)}{t_b} \]

\[ = \frac{u_{bb}}{t_s} \left( \frac{m}{m_{bo}} \right)^{1/3} \left( \frac{t}{t_s} \right)^{1 - \frac{k m}{\rho m (v_{mm} + m)}}. \] (39)

At \( \tau < c/v \), the specific energy profile is formed by photon diffusion and is given by \( u = L \tau / 4\pi r^2 c \rho \), where \( \tau \approx 3kTm/4\pi r^2 \). Using Equations 10(c), 33, and 39, we have

\[ u(\tau < c/v) = u_{bo}(t_s) \left( \frac{m}{m_{bo}} \right)^{1/3} \left( \frac{t}{t_s} \right)^{1 - \frac{k m}{\rho m (v_{mm} + m)}}. \] (40)

The transition to the spherical phase starts at \( t_s \) and ends when \( m(\tau = c/v) = m_{pl} \). This time is denoted as \( t_r \) and estimated using Equations 32 and 36:

\[ t_r = t_s \left[ 1 + \log (t_s/t_{bo}) \right]^{1/1 - \frac{k m}{\rho m (v_{mm} + m)}} \approx 3t_s. \] (41)

Past \( t_r \), the dynamics of the radiation are fully spherical and have been treated in several previous works (NS10; Rabinak & Waxman 2011).

6. The Observed Temperature

As implied from Equation (29), the luminosity is only mildly affected by the logarithmic correction to \( m_{bo} \) during the planar phase. Nevertheless, the increase in \( m_{bo} \) means that the radiation originates in regions of higher optical depth and density, where the ability of the radiation to thermalize with the gas is significantly better than in the breakout shell. This will have a nonnegligible effect on the observed temperature.

Throughout this section, we use the method and terminology of NS10 to estimate the observed temperature of the SN. Instead of assuming thermal equilibrium, they took into account the thermal coupling between the radiation and gas. The photon–electron coupling is achieved via several physical processes: free–free, bound–bound, and bound–free emission and absorption and Compton and inverse Compton scattering.

At the high-temperature conditions that prevail during the planar phase, most of the hydrogen atoms are ionized (\( T \gg 1 \) eV), such that bound–bound and bound–free absorption opacities are negligible compared to that of free–free. We thus take free–free absorption and emission as the dominant photon production processes, which can be complemented by Compton upscattering of lower-energy photons. During the thermalization process, radiation with an initial temperature of \( T > T_{BB} \) (where \( T_{BB} \) is its blackbody temperature) cools by sharing its energy with other photons via emission and absorption from electrons. In order for a shell to achieve thermal equilibrium, it needs to produce the following number of photons of energy \( 3kT_{BB} \):

\[ n_{BB} \approx \frac{\alpha T_{BB}^4}{3kT_{BB}}, \] (42)

where \( k \) is Boltzmann’s constant, \( \alpha \) is the radiation constant, and the blackbody temperature can be expressed as

\[ T_{BB} = \left( \frac{\mu \cdot \rho}{a} \right)^{1/4}. \] (43)

In order to determine whether a certain shell is capable of achieving thermal equilibrium, we use the following definition for the thermal coupling coefficient:

\[ \eta \equiv \frac{n_{BB}}{\min \{t, \, t_{diff} \} \dot{n}(T_{BB})}, \] (44)

where \( \dot{n}(T_{BB}) \) is the production rate of photons of energy \( 3kT_{BB} \), and \( \min \{t, \, t_{diff} \} \) is the time a photon spends in the shell and is equal to \( t_{diff} \) for shells satisfying \( m < m_{bo} \) and \( t \) for shells with \( m > m_{bo} \) during the planar phase. The dominant process for photon production is considered to be free–free emission, for which \( \dot{n}(T_{BB}) \) in Equation (44) is the free–free photon production rate,

\[ \dot{n}_{ph, ff} = 3.5 \times 10^{36} \text{s}^{-1} \text{cm}^{-3} \rho^2 T_{BB}^{-1/2}, \] (45)

for which Equation (44) becomes

\[ \eta_{ff} = \frac{7 \times 10^2}{\min \{t, \, t_{diff} \} \left( \frac{\rho}{10^{-10} \text{g cm}^{-3}} \right) \left( \frac{kT_{BB}}{100 \text{eV}} \right)^{2/3} \left( \frac{\mu \cdot \rho}{a} \right)^{1/4}}. \] (46)

If photon production is the only process that thermalizes the radiation, \( \eta < 1 \) is the condition for thermal equilibrium in a shell. However, the definition of \( \eta \) in Equation (44) does not take into account the effect of inverse Compton scattering. Photons that were created with an energy \( h\nu < 3kT \) cannot contribute to thermalization if they are Compton upscattered to \( h\nu \sim 3kT \), where \( h \) is Planck’s constant. The lowest-energy photon that can contribute to the spectrum at \( 3kT \) is limited by two main processes: free–free self-absorption and photon diffusion out of the shell. Free–free self-absorption causes the number of available photons for Comptonization to decrease by a factor of \( \nu^2 \). The self-absorption frequency, \( \nu_{sa} \), is found by equating the Rayleigh–Jeans (RJ) energy density to the energy density produced by free–free emission (which depends on the time a photon spends in a shell) and is equal to

\[ h\nu_{sa} \approx 10 \text{ eV} g_{ff}^{1/2} \left( \frac{n}{10^{15} \text{ cm}^{-3}} \right)^2 \left( \frac{T}{100 \text{eV}} \right)^{-3/4} \times \left( \frac{\min \{t, \, t_{diff} \}}{1 \text{ s}} \right)^{1/2}, \] (47)

where we used the fact that at the RJ tail, \( h\nu \ll kT \). On the other hand, another limit comes from the fact that scattered photons must interact with the electrons a sufficient amount of times to reach an energy of \( 3kT \) before escaping the shell by diffusion. The minimal frequency that satisfies this requirement is

\[ h\nu_{diff} = kT \exp \left\{ -\frac{kT}{m_e c^2 \nu} \right\}. \] (48)
The lowest energy that can be Comptonized, $\nu_{\min}$, is then determined by

$$\nu_{\min} = \max \{ \nu_{\text{diff}}, \nu_{\text{sa}} \}. \quad (49)$$

The Comptonization parameter, $\xi(T)$, is the ratio of photons at energy $3kT$ created by Comptonization relative to free–free processes,

$$\xi(T) \approx \max \{ 1, 1 + 0.5 \cdot \log(\gamma_{\max})(1.5 + \log(\gamma_{\max})) \}, \quad (50)$$

where

$$\gamma_{\max} = \frac{kT}{\nu_{\min}}. \quad (51)$$

Note that $\xi$ is an implicit function of $T$. Furthermore, the logarithmic dependence of $\xi$ on $\gamma_{\max}$ resolves the exponential sensitivity of $\nu_{\text{diff}}$.

A shell in which $\eta/\xi < 1$ implies that the radiation has had enough time to thermalize with the gas during the available interaction time and has achieved a temperature of $T_{\text{BB}}$. However, when $\eta/\xi > 1$, the shell did not manage to generate enough photons with an energy of $3kT$ to reach thermal equilibrium. It can be shown that $\eta$ is an increasing function of $r$; therefore, if a certain shell is out of thermal equilibrium, so are all of the shells external to it. This implies that the outermost shell that is able to achieve thermal equilibrium satisfies $\eta/\xi = 1$. This shell is termed the “color shell,” and its location is more commonly known as the “thermalization depth.” External to the color shell, the gas can no longer modify the energy of the radiation, and from that point, the observed temperature is constant and satisfies $T_{\text{cl}} = T_{\text{BB}}(\eta/\xi = 1)$.

If the luminosity shell is out of thermal equilibrium, it is the only shell that affects the observed temperature, since the energy of the photons cannot be changed as they propagate outward. The observed temperature will thus be the typical energy of the photons in the luminosity shell:

$$T_{\text{obs}} = \begin{cases} \frac{\nu_{\text{ls}}^2}{\xi_{\text{ls}}}T_{\text{BB,ls}}, & \eta_{\text{ls}}/\xi_{\text{ls}} > 1 \\ T_{\text{BB}}(\eta = 1), & \eta_{\text{ls}}/\xi_{\text{ls}} < 1. \end{cases} \quad (52)$$

The observed emission from an SN can be in or out of thermal equilibrium, depending on the progenitor properties. High-energy explosions in compact stars tend to be out of thermal equilibrium. In the following subsections, we find the evolution of $T_{\text{obs}}$ as a function of time in the planar and transition phases for both cases.

### 6.1. $T_{\text{obs}}$ during the Planar Phase

The observed temperature during the planar phase is determined by the ability of the radiation to thermalize at $m \leq m_{\text{ls}}$. We assume here that thermalization occurs at $m_{\text{bo}} < m \leq m_{\text{ls}}$, such that the available time for thermalization in a shell is $t$. The dependence of $\eta$ on $m$ and $t$ is found using Equations 10(c), 27, 43, and 46:

$$\eta(m_{\text{bo}} < m < m_{\text{ls}}) = \eta_{\text{bo}}(t_{\text{bo}})^{-1/6} \times \left( \frac{m}{m_{\text{bo}}} \right)^{16\mu + 9n + 9/88t^{-1}} \left[ 1 + \log(t/t_{\text{bo}}) \right]^{1/2(2\mu - 1)}. \quad (53)$$

Whether the observed radiation is thermalized depends on the values of $\eta$ and $\xi$ in the luminosity shell. The expression for $\eta_{\text{ls}}$ is found by substituting the solution for $m_{\text{ls}}$ from Equation (23) into Equation (53):

$$\eta_{\text{ls}}(t) = \eta_{\text{bo}}(t_{\text{bo}})^{-1/6} \left[ 1 + \log(t/t_{\text{bo}}) \right]^{1/2(2\mu - 1)} \times \left\{ \begin{array}{ll} t^{-1/6} & n = 3/2 \\ t^{-1/6} & n = 3. \end{array} \right. \quad (54)$$

The thermal coupling of the luminosity shell thus significantly improves owing to the logarithmic increase of $m_{\text{ls}}$. The luminosity shell probes regions of higher density (by the end of the planar phase, the density of the luminosity shell is $\sim 10$ times higher than in the breakout shell), where the photon production rate increases accordingly as $n \propto \rho^2$ (Equation 45).

Since $\xi$ is an implicit function of $T$, the value of $\xi_{\text{ls}}$ ought to be computed numerically. Using Equations (50) and (54), we can determine whether the observed radiation is in thermal equilibrium or not, which will define the nature of the solution according to Equation (52).

#### 6.1.1. $T_{\text{obs}}$ during the Planar Phase for $\eta_{\text{ls}}/\xi_{\text{ls}} > 1$

If $\eta_{\text{ls}}/\xi_{\text{ls}} > 1$, the luminosity shell and all shells external to it are out of thermal equilibrium. According to Equations (52) and (54), when the luminosity shell is out of thermal equilibrium, $T_{\text{obs}}$ behaves as

$$T_{\text{obs}}(\eta_{\text{ls}}/\xi_{\text{ls}} > 1) = T_{\text{BB,bo}}(t_{\text{bo}})^{\xi_{\text{ls}}/\xi_{\text{bo}}} \left( \frac{t}{t_{\text{bo}}} \right)^{-2/3} \left[ 1 + \log(t/t_{\text{bo}}) \right]^{16\mu + 9n + 4/88t^{-1}} \times \left\{ \begin{array}{ll} \xi_{\text{ls}}(T)^{-2} & n = 3/2 \\ \xi_{\text{ls}}(T)^{-2} & n = 3. \end{array} \right. \quad (55)$$

where $T_{\text{bo}}$ is the observed temperature at breakout. The solution of $T_{\text{obs}}$ is obtained by solving the implicit Equation (55) using the expression for $\xi(T)$ in Equation (50). Contrary to $T_{\text{obs}}$, the dependence of $\eta_{\text{ls}}/\xi_{\text{ls}}$ on the logarithmic factor is not negligible, and the radiation approaches thermal equilibrium faster than previously thought. Equilibrium is achieved once $T_{\text{obs}} = T_{\text{BB,ls}}$.

The expression for $T_{\text{obs}}$ derived in Equation (55) is valid only while $T < 50$ KeV. Above that temperature, relativistic effects such as pair production start being important (Weaver 1976; Svensson 1984). In that case, pair production will lead to a lower temperature than what Equation (52) predicts. Therefore, if Equation (55) results in temperatures higher than 50 KeV, it is not correct, and the temperature should be computed using relativistic shock breakout models (Budnik et al. 2010; Nakar & Sari 2012).
6.1.2. \( T_{\text{obs}} \) during the Planar Phase for \( \eta_0/\xi_0 < 1 \)

When the luminosity shell is in thermal equilibrium, i.e., \( \eta_0/\xi_0 < 1 \), the observed temperature is determined in the color shell, which satisfies \( \eta = 1 \) (throughout this paper, we assume that the color temperature is low enough such that Compton scattering is not important in the color shell, and \( \xi_0 \sim 1 \)). Assuming that the color shell is external to the luminosity shell but internal to the breakout shell, \( t < t_{\text{faii}} \) in the color shell, and \( t \) is the available time for the radiation to thermalize. Using Equation (53), we find the mass of the color shell by solving \( \eta = 1 \),

\[
m_{\text{cl}} = m_{\text{bo}} \left[ \frac{\eta_{\text{bo}}(h_{\text{bo}})}{\eta_{\text{bo}}(b_{\text{bo}})} \right]^{\frac{4(n+1)}{3(n+1)+5n+4}} \left( \frac{t}{t_{\text{bo}}} \right)^{\frac{4n+1}{3(n+1)+5n+4}} \times \left[ 1 + \log(t/t_{\text{bo}}) \right]^a
\]

\[
\propto \begin{cases} t^{-0.12}[1 + \log(t/t_{\text{bo}})]^{0.04}, & n = 3/2 \\ t^{-0.12}[1 + \log(t/t_{\text{bo}})]^{-0.01}, & n = 3 \end{cases} \tag{56}
\]

where

\[
\alpha = \frac{7(n+1)(2\mu n - 1)}{3(\mu n - n - 1)(16\mu n + 9n + 9)}. \tag{57}
\]

The color temperature is the blackbody temperature of the color shell. We thus use Equations (10(c), 27, 43), and (56) to find \( T_{\text{cl}} \),

\[
T_{\text{cl}} = T_{\text{BB}}(m_{\text{bo}}, h_{\text{bo}}) \left( \frac{t}{t_{\text{bo}}} \right)^{-2(2\mu n + 9n + 9) \left[ 1 + \log(t/t_{\text{bo}}) \right]^\beta} \propto \begin{cases} t^{-0.36}[1 + \log(t/t_{\text{bo}})]^{0.03}, & n = 3/2 \\ t^{-0.36}[1 + \log(t/t_{\text{bo}})]^{-0.01}, & n = 3 \end{cases} \tag{58}
\]

where

\[
\beta = \frac{4(\mu n + n + 1)(2\mu n - 1)}{3(\mu n - n - 1)(16\mu n + 9n + 9)}. \tag{59}
\]

When the radiation is in thermal equilibrium, the logarithmic correction is negligible. The reason for that is that in equilibrium, \( T_{\text{cl}} \) can be expressed as

\[
T_{\text{cl}} = \left( \frac{\tau_{\text{cl}} L}{4\pi r_{\text{cl}}^2 ac} \right)^{1/4}. \tag{60}
\]

As we saw, \( L \) and \( m_{\text{cl}} \) depend very weakly on the logarithmic correction, and therefore so do \( \tau_{\text{cl}}, T_{\text{cl}} \) and \( T_{\text{cl}} \).

6.2. \( T_{\text{obs}} \) during the Transition to the Spherical Phase

Once the breakout shell enters the spherical phase at \( t_s = R/\nu_{\text{bo}} \), the energy in the envelope has an inner adiabatic profile \( (m > m_{\text{pl}}) \) and an outer diffusive profile \( (m < m_{\text{pl}}) \). At \( t_s < t \), the shell that satisfies \( \tau = c/v \), which is initially located at \( m_{\text{bo}} \), starts propagating through the external diffusive profile and increases in mass.

As explained in Section 5.2, the transition to the spherical phase starts at \( t_s \) and ends when \( \tau = c/v \) reaches \( m_{\text{pl}} \). At the end of the transition phase, the ejecta has no recollection of what happened in the planar phase, in the sense that the increase of \( m_{\text{pl}} \) during the planar phase does not affect the temperature at \( t_s < t \). Nevertheless, the logarithmic correction to \( m_{\text{pl}} \) in the planar phase has an effect on the temperature of the transition phase, both when the radiation was thermalized at \( t_s \) and when it was not. We treat these two scenarios in the following subsections.

6.2.1. \( T_{\text{obs}} \) during the Transition Phase for \( \eta_0(t_s)/\xi_0(t_s) > 1 \)

We first consider the case in which the luminosity shell is out of thermal equilibrium at \( t_s \). As demonstrated by Equation (53), during the planar phase, the thermal coupling increases with time, as the value of \( \eta \) decreases for a given \( m \). During the spherical phase, however, the thermal coupling of a shell becomes weaker, as we show next.

The evolution of \( \eta \) is found using Equations (34), (37), (38), (43), and (46):

\[
\eta(m) = \begin{cases} \eta_{\text{bo}}(h_{\text{bo}}) \left( \frac{t_s}{t_{\text{bo}}} \right)^{-1/6} \left( \frac{m}{m_{\text{bo}}} \right)^{3/2} & m(t = c/v) < m < m_{\text{pl}} \\ \eta(m_{\text{pl}}, t_{s,\text{pl}}) \left( \frac{t_s}{t_{s,\text{pl}}} \right)^{3/2} \left( \frac{m}{m_{\text{pl}}} \right)^{-2 - \frac{4(3\mu n + 9n + 9)}{4(\mu n + n + 1)(2\mu n - 1)}} & m_{\text{pl}} < m \end{cases} \tag{61}
\]

where \( t_{s,\text{pl}} = R/\nu(m_{\text{pl}}) \), and for simplicity, we neglected the weak dependence on the logarithmic factor.

Equation (61) shows that \( \eta \) increases with time and therefore receives its minimal value at \( t = R/\nu(m) \), which is the time a shell of mass \( m \) enters the spherical phase. Consequently, shells that were out of thermal equilibrium at the end of the planar phase remain so during the spherical phase.

The number of photons in a shell in which \( \eta > 1 \) during the spherical phase is determined by the minimal value of \( \eta \) in Equation (53), namely,

\[
\eta_{\text{min}}(m) = \eta(m, t = R/\nu(m)) = \eta_{\text{bo}}(h_{\text{bo}}) \left( \frac{t_s}{t_{\text{bo}}} \right)^{-1/6} \left( \frac{m}{m_{\text{bo}}} \right)^{\frac{52\mu n + 27n + 27}{24(\mu n + 1)}}, \tag{62}
\]

again neglecting the weak dependence on the logarithmic correction for simplicity. In the diffusive energy profile at \( m < m_{\text{pl}} \), the radiation in each coordinate \( m \) has the same characteristic energy (only reduced by adiabatic cooling) and is the typical photon temperature that resides in the shell of mass \( m_{\text{pl}} \) at \( t_s \). As the breakout shell enters the spherical phase, the shell that now satisfies \( \tau = c/v \) starts propagating inward in the external energy profile. All shells external to \( m_{\text{pl}} \) are also out of thermal equilibrium, since they were so at the end of the planar phase, and now their thermal coupling only becomes weaker. Therefore, the shell in which \( \tau = c/v \) is out of thermal equilibrium and does not generate its own photons. It merely releases the energy that had reached there by diffusion from the
inner shells during the planar phase. The temperature of each shell is thus the temperature of the luminosity shell at $t_s$, which has evolved adiabatically:

$$T(m < m_{pl}, t_s < t < t_{tr}) = T_{obs}(t_s) \left( \frac{R/v(m)}{t_s} \right)^{1/3} \left( \frac{t}{R/v(m)} \right)^{-1}$$

$$= T_{BB,ls}(t_s) \left( \frac{\eta_h(t_s)^2}{\xi_h(T_s)^2} \right) \left( \frac{m}{m_{bo}} \right)^{\frac{2\eta_m}{\eta_m + 1}} \left( \frac{t}{t_s} \right)^{-1},$$

where $T_s = T_{obs}(t_s)$. The observed temperature is determined by the photons that diffuse out of $\tau = c/v$. Substituting Equation (36) into Equation (63), we find

$$T_{obs}(t_s < t < t_{tr}) = T_{BB,ls}(t_s) \eta_h(t_s)^2 \xi_h(T_s)^2 \left( \frac{m}{m_{bo}} \right)^{\frac{2\eta_m}{\eta_m + 1}} \left( \frac{t}{t_s} \right)^{-1}.$$  

(64)

At $t_{tr}$, the luminosity shell starts probing shells in which the temperature profile is not constant as before. The luminosity shell might still be out of thermal equilibrium if $\eta_{max}(m_{pl})/\xi (m_{pl}) > 1$. In that case, the observed temperature will drop quickly until the luminosity shell reaches the shell that satisfies $\eta_{min} = 1$, which was in thermal equilibrium when it entered the spherical phase. The time when this occurs is denoted by $t_1$:

$$t_1 = t_s \left[ \frac{\eta_{bo}(t_{bo})}{\eta_{bo}(t_s)} \right]^{1/6} \left( \frac{m_{bo}}{m_{bo}} \right)^{\frac{12\eta_{bo}(t_{bo})}{\eta_{bo}(t_s)}}.$$  

(65)

At $t_{tr} < t < t_1$, the temperature evolves according to

$$T_{obs}(t_{tr} < t < t_1) = T_{BB,ls}(t_s) \eta_{min,ls}^2 \xi_{ls}^2$$

$$= T_{BB,ls}(t_s) \eta_{min,ls}^2 \xi_{ls}^2 \left( \frac{t}{t_{tr}} \right)^{\frac{11 + 15\eta_s + 3\eta_m}{\eta_m + 1}},$$

(66)

where $T_s = T_{obs}(t_{tr})$. Since thermal coupling is kept through adiabatic cooling when the energy is radiation-dominated, the observed radiation will go into thermal equilibrium at $t_1$. The $\xi_{ls}$ decreases with temperature, and therefore $T_{obs}$ will not behave as a simple power law; also, in this phase, the evolution of $T_{obs}$ is computed numerically.

At $t_1 < t$, the observed radiation is in thermal equilibrium, and $T_{obs}$ is simply the blackbody temperature of the luminosity shell, computed using Equations (34), (36), (37), and (43):

$$T_{obs}(t_1 < t < t_2) = T_{BB,ls}(t_1) \left( \frac{t}{t_1} \right)^{\frac{2\eta_m}{6\eta_m + 1}}.$$  

(67)

where $t_2$ is the time when $\eta_{ls} = 1$ and the observed temperature from that point is determined that satisfies $\eta = 1$. It was found by NS10 that the color temperature evolves roughly as $T_{cl} \propto t^{-0.6}$ at $t_2 < t$, where the exact value depends on the choice of $n$. The $\eta_{ls}$ is found by substituting Equation (36) into Equation (61) for $m_{pl} < m$, and, requiring that $\eta_{ls} = 1$, we find an expression for $t_2$:

$$t_2 = t_{tr} \eta(m_{pl}, t_{tr}) \left( \frac{m_{bo}}{m_{bo}} \right)^{\frac{12\eta_{bo}(t_{bo})}{6\eta_{bo}(t_s)}}.$$  

(68)

6.2.2. $T_{obs}$ during the Transition Phase for $\eta_{cl}(t_s) \eta_{cl}(t_s) < 1$

If the luminosity shell is in thermal equilibrium at the end of the planar phase, then all shells with $m < m_{cl}(t_s)$ have the same temperature at $t_s$, namely, the color temperature. As the shell in which $\tau = c/v$ propagates in $m < m_{cl}(t_s)$, the observed temperature is the adiabatically cooling color temperature at $t_s$. It is important to note that although the luminosity shell is in thermal equilibrium, the energy release is controlled by the shell that satisfies $\tau = c/v$, which is not necessarily in thermal equilibrium. Therefore, if $m_{bo} < m_{cl} < m_{pl}$, initially, the radiation is out of thermal equilibrium.

Internal to $m_{cl}(t_s)$, all shells are in thermal equilibrium at $t = t_s$, and their temperature is their blackbody temperature, $T_{BB}(m)$. Since thermal equilibrium is kept through adiabatic cooling, the temperature of these shells continues to be their blackbody temperature. We denote the time when $m(\tau = c/v) = m_{cl}(t_s)$ by $t_c$:

$$t_c = t_s \left( \frac{m_{cl}(t_s)}{m_{bo}} \right)^{\frac{\eta_{cl} - 1}{6\eta_{cl} + 1}}.$$  

(69)

The typical temperature of a shell is

$$T(m) = \begin{cases} \frac{T_{cl}(t_s)}{t_s} \left( \frac{R/v(m)}{t_s} \right)^{1/3} \left( \frac{t}{R/v(m)} \right)^{-1}, & m < m_{cl}(t_s) \\ \frac{T_{BB}(t_s)}{t_s} \left( \frac{R/v(m)}{t_s} \right)^{1/3} \left( \frac{t}{R/v(m)} \right)^{-1}, & m_{cl}(t_s) < m. \end{cases}$$  

(70)

As before, the observed temperature is determined by location of $\tau = c/v$:

$$T_{obs} = \begin{cases} T_{cl}(t_s) \left( \frac{t}{t_s} \right)^{1+\frac{\eta_{cl}}{6\eta_{cl} + 1}}, & t_s < t < t_c \\ T_{BB}(t_s) \left( \frac{t}{t_c} \right)^{1+\frac{\eta_{cl} - 1}{6\eta_{cl} + 1}}, & t_c < t < t_{2,d}. \end{cases}$$  

(71)

The temporal evolution for $t_s < t < t_c$ is the same as that in Equation (64), since it is simply an adiabatic evolution of a single temperature.

For some progenitor properties, the shell where $\tau = c/v$ can reach $\eta = 1$ while still in the transition phase, i.e., while $m_{bo} < m(\tau = c/v) < m_{pl}$. We call this time $t_{2,d}$, to differentiate it from $t_2$ in Equation (68) (which occurs if the luminosity shell reaches $\eta = 1$ during the spherical phase, while propagating at $m_{pl} < m$). Starting at $t_{2,d}$, the luminosity shell generates enough photons to reach thermal equilibrium, and the observed temperature is $T_{cl} = T_{BB}(n = 1)$. The time $t_{2,d}$ is found by substituting $m(\tau = c/v)$ (Equation (36)) into Equation (61) for $m(\tau = c/v) < m < m_{pl}$ to find $\eta(\tau = c/v)$ and requiring $\eta(\tau = c/v) = 1$:

$$t_{2,d} = \left[ \frac{\eta_{bo}(t_{bo})}{\eta_{bo}(t_s)} \right]^{1/6} \left( \frac{m_{bo}}{m_{bo}} \right)^{\frac{12\eta_{bo}(t_{bo})}{6\eta_{bo}(t_s)}}.$$  

(72)
Using the specific energy profile at \( \tau < c/\nu \) given by Equation (40), we find the evolution of \( \eta \) at \( m < m(\tau = c/\nu) \):

\[
\eta(m < m(\tau = c/\nu)) = \eta_\text{bo}(t_\text{bo}) \left( \frac{t}{t_\text{bo}} \right)^{-1/6} \left( \frac{m}{m_\text{bo}} \right)^{-12 - \frac{7\nu_m}{\nu_\text{bo}} \left( \frac{t}{t_\text{bo}} \right)^2 \frac{\gamma_\text{nu}}{\gamma_\text{nu} + 3} \right) \tag{73}
\]

(again, neglecting the weak dependence on the logarithmic correction). The mass of the color shell is found by requiring \( \eta = 1 \) and using the solution in Equation (73):

\[
m_{\text{cl}} = m_\text{bo} \left[ \eta_\text{bo}(t_\text{bo}) \left( \frac{t}{t_\text{bo}} \right)^{-1/6} \right]^{28(n+1)(\mu+1)/(30n+3)} \times \left( \frac{t}{t_\text{bo}} \right)^{32(n+1)(\mu+1)/(30n+3)} \tag{74}
\]

The color temperature during the transition phase is therefore

\[
T_{\text{cl}} = T_{\text{obs}}(t_2, \alpha) \left( \frac{t}{t_2, \alpha} \right)^\chi,
\]

where

\[
\chi = -\frac{2(15 + 30n + 39\mu n) + n^2(15 + 39\mu + 56\mu^2)}{3(\mu n + n + 1)(28\mu n + 17n + 17)}
\]

\[
\approx -0.60
\]

for both \( \mu = 0.19 \), \( n = 3/2 \) and \( \mu = 0.19 \), \( n = 3 \).

### 7. Light Travel Time Effects

In a spherical explosion, radiation emitted from different parts of the progenitor surface arrives at the observer at different times. Due to this effect, the observed radiation at time \( t \) will be a mix of light emitted at different times and thus at different intensities. This effect is important if \( t_\text{bo} < R/c \) and affects the observed radiation at \( t < R/c \). The apparent luminosity, \( L_{\text{at}} \), is calculated using the following integral:

\[
L_{\text{at}}(t) = \int_0^r L(t') \frac{r}{R} dr
\]

\[
= \int_{\max[0,R/c]}^{t} \frac{L(t')}{R/c} \left[ 1 - \frac{c}{R} (t - t') \right] dt', \tag{77}
\]

where \( r \) is the projected radius and \( t' = t - \Delta t \) is the time at the emitting source, where \( \Delta t = (R - \sqrt{R^2 - r^2})/c \) is the travel time of photons emitted from an apparent radius \( r \) to the plane tangent to the front of the sphere. The luminosity is thus smeared over a timescale of \( \sim R/c \), and for \( t \ll R/c \), it is roughly constant and equal to \( L_{\text{bo}} t_{\text{bo}} c R \). At \( R/c < t \), the light travel time ceases to affect the observed luminosity, and

\[
L \propto t^{-4/3} \text{ as in Equation (29)}.
\]

The observed spectrum is also affected by the different arrival times of the emitted radiation. At \( t < t_\text{bo} \), the spectrum is dominated by radiation escaping from the shock front, and the typical observed photon energy will be \( T_{\text{bo}} \). While \( t_\text{bo} < t < R/c \), the observed spectrum will be a combination of radiation arriving from different apparent radii, each with a different typical energy \( T \), and thus will not be a simple blackbody (or diluted blackbody) spectrum. The observed spectrum receives a simple form if the intrinsic temperature and luminosity evolve as a power laws. This is a very good approximation for the luminosity, for which the logarithmic correction has no significant effect, but does not hold for the temperature, as is evident from Equation (35). Nevertheless, the early temperature evolution, including the logarithmic correction, can be fit relatively well by a power law. Taking \( L(t) = L_{\text{bo}} (t/t_{\text{bo}})^{-4/3} \) and \( T(t) = T_{\text{bo}} (t/t_{\text{bo}})^{-\alpha} \), the spectrum broadens in time to form a power law:

\[
F_\nu \propto \frac{dL}{d\nu} \propto \nu^{\frac{1}{\alpha} - 1}.
\]

We will show in the next section that typically, \( 1/(3\alpha) > 0 \), so \( \nu F_\nu \) is an increasing function of \( \nu \). The upper end of this spectrum corresponds to the initial breakout temperature, \( T_{\text{obs}}(t_{\text{bo}}) \), and the lowest observed frequency corresponds to the nondelayed temperature, \( T_{\text{obs}}(t) \). Here \( \nu F_\nu \) is a slowly increasing function of \( \nu \), and therefore no typical energy can be defined during this stage. The observed spectrum will thus be broadband until \( t = R/c \).

### 8. Applications to Various Progenitors

In this section, we apply our results to different progenitors —RSG, BSG, and WR—and discuss the effect of the planar logarithmic correction on the observed properties of each progenitor.

The expressions derived in the previous sections for \( T_{\text{cl}}, T_{\text{obs}} \), and \( L \) are functions of the progenitor density structure and shock velocity, which depend on the progenitor properties \( E, M, R, \) and \( \kappa \); the dimensionless constants \( C_1 \) and \( C_2 \), defined in Equation (4); and the density power-law index \( n \). Throughout the paper, we assume a single power-law density profile. This assumption may not accurately describe the profile of a realistic progenitor in the entire radius range relevant for this analysis (e.g., Morozova et al. 2016). However, the model is not very
sensitive to the exact power-law index, but rather depends more on the normalization of the profiles.

The constants $C_1$ and $C_2$ determine the normalization of the envelope structure and velocity when the density is described by a polytrope. We use Matzner & McKee (1999) to estimate these constants, which depend on whether the envelope is radiative or convective. For each of the progenitors discussed here, we provide the appropriate values of $C_1$ and $C_2$.

We use the following notations to specify the progenitor properties: $M_\text{e} = x M_\odot$, $R_\text{e} = x R_\odot$, $E_\text{e} = 10^6$ erg, and $\kappa_\text{e} = x \text{ cm}^2 \text{ g}^{-1}$.

### 8.1. RSG

The RSGs are the progenitors of most Type II explosions. They have a typical radius of 500 $R_\odot$, and since their envelopes are hydrogen-rich, their typical Thomson opacity is $\sim 0.34 \text{ cm}^2 \text{ g}^{-1}$. The envelopes of RSGs are convective, and therefore similar properties, evolved using MESA by a polytrope. We use Matzner & McKee (1999) to estimate these constants, which now depend on the detailed envelope structure. Taking typical values for the total, envelope mass of the RSGs is 15 $M_\odot$, and since their envelopes are convective, and therefore similar properties: $M_\text{e} = 15$, $M_\text{env} = 10 M_\odot$, respectively, we find $C_1 \sim 0.5$ and $C_2 \sim 0.9$. The values of $C_1$ and $C_2$ fit well the numerical profiles of a progenitor with similar properties, evolved using MESA (Paxton et al. 2011).

For the above typical RSG properties, the breakout time according to Equation (16) is

$$ t_\text{bo}^{\text{RSG}} = 92 \sqrt{M_\odot^{2.16} R_\odot^{3.20} E_\text{e}^{-0.79} \kappa_\text{e}^{-0.58}}, $$

(79)

The breakout time enters the spherical phase at $t_s = 7.2 \text{ hr} M_\odot^{-0.44} R_\odot^{1.26} E_\text{e}^{-0.56} \kappa_\text{e}^{-0.13}$,

(80)

according to Equation (31). During the planar phase, the luminosity evolves according to Equation (29), whereas during the transition from the planar to the spherical phase, we estimate the luminosity using Equation (39):

$$ L^{\text{RSG}}(t) = \begin{cases} 
5.5 \times 10^{45} \text{ erg s}^{-1} M_\odot^{0.37} R_\odot^{2.46} E_\text{e}^{0.30} \kappa_\text{e}^{-0.10} \\
\times [1 + \log(t/92 \text{ s})]^{0.06} \left( \frac{t}{90 \text{ s}} \right)^{4/3}, \quad t_\text{bo} < t < t_s \\
5.8 \times 10^{41} \text{ erg s}^{-1} M_\odot^{1.03} R_\odot^{5.13} E_\text{e}^{0.90} \kappa_\text{e}^{-0.03}, \quad t_s < t < t_\text{fr}
\end{cases} $$

(81)

where $t_\text{fr} = 24 \text{ hr} \sqrt{M_\odot^{2.16} R_\odot^{3.20} E_\text{e}^{-0.79} \kappa_\text{e}^{-0.58}}$ according to Equation (41) and the estimates for $t_\text{bo}$ and $t_s$ obtained in Equations (79) and (80).

In Figure 3, we show the luminosity calculated in this work (excluding the transition phase) with and without the logarithmic correction, together with those of Shussman et al. (2016) for $n = 3/2$, for the typical RSG properties. Shussman et al. (2016) performed radiative transfer calculations on a set of analytic progenitors with a power-law density profile and calibrated the numerical factors of the analytic model using the simulated results. The luminosity of NS10’s model is a factor of $\sim 2$ higher than the luminosity calculated in this work, while the numerically calibrated luminosity of Shussman et al. (2016) is $\sim 1.6$ times lower. The behavior of the luminosity during the spherical phase is different in Shussman et al.’s (2016) model, since they took into account the transition in the density profile in the inner regions probed during the spherical phase. From Figure 3, it is evident that the logarithmic correction does not play a role in shaping the observed luminosity, in agreement with Equation (29).

The behavior of the temperature during the planar phase depends upon whether the luminosity shell is in thermal equilibrium at $t_\text{bo}$, which is determined by Equations (50) and (54). For an RSG using Equation (54), we find the thermal coupling coefficient during the planar phase:

$$ \eta_\text{ls}(t) = 1.3 M_\odot^{-1.68} R_\odot^{0.40} E_\text{e}^{0.23} \left( \frac{t}{92 \text{ s}} \right)^{-1/6} \times [1 + \log(t/92 \text{ s})]^{-1.47}. $$(82)

Hence, the luminosity shell is marginally in thermal equilibrium at breakout time. In RSGs, Comptonization does not play an important role in thermalizing the radiation, since $kT_\text{obs}$ is not much higher than $kT_\text{eq}$. Indeed, we find that at breakout time, $T_{\text{ls}}(t_\text{bo}) \sim 1$, while $T_{\text{ls}}(t_\text{fr}) \sim 1.3$. This implies that $\frac{\eta_\text{ls}}{\eta_\text{eq}} > 1$, and initially, the radiation is out of thermal equilibrium. However, as the luminosity shell retreats into regions of higher density, the observed radiation becomes thermalized, and $T_{\text{obs}}$ quickly reaches $T_{\text{BB,ls}}$. We denote this time as $t_\text{eq}$. After reaching thermal equilibrium, the color temperature is determined in the shell where $\eta = 1$, i.e., $T_{\text{obs}} = T_{\text{cl}} = T_{\text{BB}}(\eta = 1)$, and $T_{\text{BB,ls}} > T_{\text{cl}}$. At $t_\text{fr}$, the SN enters into the transition phase, and the temperature behaves as described in Section 6.2.2. At the end of the transition phase, the envelope enters the spherical phase, and none of the observed properties are affected by the planar phase logarithmic correction.

We use Equations (55), (58), and (71) to obtain

$$ T_{\text{obs}}^{\text{RSG}} = \begin{cases} 
5.3 \times 10^5 K M_\odot^{-3.35} R_\odot^{-0.62} E_\text{e}^{0.98} \kappa_\text{e}^{-0.34} \left( \frac{t}{100 \text{ s}} \right)^{-2/5} \\
\times [1 + \log(t/92 \text{ s})]^{2.64}, \quad t_\text{bo} < t < t_\text{eq} \\
4.1 \times 10^5 K M_\odot^{-0.29} R_\odot^{0.11} E_\text{e}^{0.31} \kappa_\text{e}^{-0.03} \left( \frac{t}{100 \text{ s}} \right)^{-0.36} \\
\times [1 + \log(t/92 \text{ s})]^{0.03}, \quad t_\text{eq} < t < t_s \\
1.8 \times 10^5 K M_\odot^{-0.12} R_\odot^{0.42} E_\text{e}^{0.09} \kappa_\text{e}^{-0.12} \left( \frac{t}{1 \text{ hr}} \right)^{-0.60}, \quad t_s < t < t_\text{fr}
\end{cases} $$

(83)
where the expression for \(t_{bo} < t < t_{eq}\) is for the nonthermal radiation. For the above progenitor properties, the breakout shell itself (and therefore also the observed radiation) is in thermal equilibrium at \(t_e\). The time \(t_e\) is not defined for this specific progenitor, since the color shell at \(t_c\) is external to the breakout shell.

We plot the evolution of \(T_{obs}\) according to Equation (83) in Figure 4, together with the observed temperature derived without the logarithmic correction for comparison and the blackbody temperature of the luminosity shell. We find that without the logarithmic correction, the radiation remains out of thermal equilibrium for a few minutes but thermalizes while still in the planar phase. From that point, \(T_{obs} < T_{BB,ls}\) since the radiation thermalizes in \(m_{ls} < m_{ls}^*\), where \(T_{BB}\) is lower than that of the luminosity shell. This result is only in mild contradiction with previous works (e.g., NS10).

\[ T_{obs}(t) = T_{BB}(t) \times \left(1 + \log\left(\frac{l}{l_{tt}}\right)\right)^{-0.07} \]

\[ T_{BB}(t) = \frac{E}{4\pi R^2} \left(1 + \log\left(\frac{l}{l_{tt}}\right)\right)^{-0.07} \]

**Figure 4.** Solid black line: observed temperature of an RSG with \(M_{15} = 1, R_{500} = 1, E_{51} = 1\), and \(\kappa_{0.34} = 1\). The dashed–dotted blue line is the blackbody temperature of the luminosity shell, and the dashed red line is the observed temperature that would have been obtained without the logarithmic correction to the luminosity shell. In this plot, we show the typical radiation temperature in the progenitor frame (not taking into account light travel time effects). Initially, the radiation is out of thermal equilibrium, as \(T_{obs} > T_{BB}\). Due to the increase of \(m_{ls}\), the radiation quickly reaches thermal equilibrium at \(t = t_{eq}\), while the uncorrected \(T_{obs}\) remains out of thermal equilibrium for a few more minutes before reaching thermalization. While \(t < R/c \sim 1200\) s, light travel time variations create a broadband spectrum that follows \(F_\nu \propto \nu^{-0.07}\), such that no typical temperature can be defined during that period in the observer frame. At \(t_{eq} < t < t_e\), the observed temperature is determined in the shell where \(\eta = 1\). The luminosity enters the transition phase at \(t_t\), and decreases gradually (see text for details). At \(t = t_s\), the transition phase ends, and the observed temperature is no longer affected by the logarithmic correction.

**Figure 5.** Same as Figure 4 but for an RSG with \(M_{10} = 1, R_{500} = 1, E_{51} = 1.5\), and \(\kappa_{0.34} = 1\). In this case, the spectrum in the observer frame at \(t < R/c \sim 1200\) s will follow \(F_\nu \propto \nu^{-0.87}\), where the peak of \(sF_\nu\) is at \(T_{obs}(t_{bo})\).
that predicted that an RSG will be in thermal equilibrium at breakout and throughout the planar phase while considering a constant luminosity shell. For the progenitor properties considered, the radiation peaks in the near-UV during the planar phase.

During $t < R/c \sim 1200$ s, the observed spectrum is affected by light travel time variations. Since the radiation is in thermal equilibrium throughout most of this phase, the effective power law that the temperature follows is $T \propto t^{-0.36}$. Therefore, according to Equation (78), the observed spectrum will follow $F_{\nu} \propto \nu^{-0.07}$ below the peak of $\nu F_{\nu}$ at $T_{\text{obs}}(t_{\text{bo}})$.

For a less massive and more energetic progenitor of, e.g., $M = 10 M_\odot$ and $E = 1.5 \times 10^{51}$ erg, thermal coupling becomes weaker and $t_{\text{bo}}(t_{\text{bo}}) = 6$, in accordance with Equation (82). Without the logarithmic correction, the radiation would have remained out of thermal equilibrium throughout all of the planar phase (see Figure 5). However, with the logarithmic correction, the radiation reaches thermal equilibrium after $\sim 400$ s. The breakout and spherical times for these progenitor properties are

$$t_{\text{bo}} = 60 \, s \, M_1^{0.21} R_5^{2.16} \nu^{-0.79} \kappa_{0.34}^{0.58}$$

and

$$t_s = 4.8 \, hr \, M_1^{0.44} R_5^{1.26} \nu^{-0.56} \kappa_{0.34}^{-0.13}.$$  

Using Equations (55), (64), (66), and (67), we find the temperature evolution for this case:

$$t_{\text{obs}}^{\text{RSG}} = \begin{cases} 
1.9 \times 10^7 \, K \, M_1^{-3.35} R_5^{-0.62} \nu^{3.98} E_5^{1.26} \kappa_{0.34}^{0.58} \left( \frac{t}{60 \, s} \right)^{-2/3} [1 + \log(t/60 \, s)]^{-2.64} , & \text{if } t_{\text{bo}} < t < t_{\text{eq}} \\
2.8 \times 10^5 \, K \, M_1^{-0.29} R_5^{0.11} E_5^{1.26} \kappa_{0.34}^{-0.31} \left( \frac{t}{10 \, m} \right)^{-0.36} [1 + \log(t/60 \, s)]^{0.03} , & \text{if } t_{\text{eq}} < t < t_s \\
3.3 \times 10^5 \, K \, M_1^{-0.08} R_5^{0.73} E_5^{0.03} \kappa_{0.34}^{-0.31} \left( \frac{t}{1 \, hr} \right)^{-0.36} , & \text{if } t_s < t < t_c \\
5.2 \times 10^4 \, K \, M_1^{0.05} R_5^{0.28} E_5^{1.11} \kappa_{0.34}^{-0.31} \left( \frac{t}{10 \, hr} \right)^{-0.41} , & \text{if } t_c < t < t_{2,d} \\
5.4 \times 10^4 \, K \, M_1^{-0.12} R_5^{0.42} E_5^{0.09} \kappa_{0.34}^{-0.31} \left( \frac{t}{10 \, hr} \right)^{-0.60} , & \text{if } t_{2,d} < t < t_{\text{fr}} 
\end{cases}$$

where

$$t_c = 7 \, hr \, M_1^{-0.28} R_5^{1.00} E_5^{0.31} \kappa_{0.34}^{0.49}$$

and

$$t_{2,d} = 11 \, hr \, M_1^{0.95} R_5^{0.79} E_5^{1.10} \kappa_{0.34}^{0.05}.$$  

and $t_{\text{fr}} \sim 16$ hr. This temperature evolution is plotted in Figure 5.

For these progenitor parameters, the radiation first peaks in the X-ray range but shifts to the UV during the planar phase owing to the logarithmic correction.

A fit to the temperature at the early phases shows that it decreases approximately as $t^{-2.5}$ while it is out of thermal equilibrium. As a result, while the observed spectrum is affected by light travel time variations (at $t < R/c$), it will follow $F_{\nu} \propto \nu^{-0.87}$.

8.2. BSG

Explosions of BSGs also produce Type II SNe (e.g., SN 1987A) and are therefore hydrogen-rich with $\kappa = 0.34 \, \text{cm}^2 \, \text{g}^{-1}$. Their typical radius is $R \sim 50 \, R_\odot$, and they have a mass similar to an RSG of $M \sim 15 \, M_\odot$. However, their envelopes are radiative, and therefore $n = 3$. For radiative envelopes, we find $C_1 \sim 0.35$ and $C_2 \sim 0.97$. These values agree with the BSG pre-explosion density profiles and the post-explosion velocity profiles in Dessart & Hillier (2018). We use these values to describe a typical BSG.

The breakout time of a BSG is significantly shorter compared to an RSG, since the material is ejected at higher velocities due to the smaller radius and steeper density profile:

$$t_{\text{bo}}^{\text{BSG}} = 7.5 \, s \, M_1^{0.27} R_5^{1.92} E_5^{1.73} \kappa_{0.34}^{0.46}.$$
For the assumed BSG properties, the planar phase of the breakout shell ends at

$$t_{BSG} = 15 \text{ m} M_{15}^{0.12} R_{50}^{1.33} E_{51}^{-0.58} \eta_{0.17}^{-0.17}.$$  \hfill (90)

Therefore, the planar phase evolution of a BSG is not likely to be observed, as the cadence of current surveys is days to hours. The luminosity of a typical BSG in the planar and transition phases is obtained using Equations (29) and (39),

$$L_{BSG}(t) = \begin{cases} 7.7 \times 10^{44} \text{ erg s}^{-1} M_{15}^{-0.33} R_{50}^{2.31} E_{51}^{0.34} \eta_{0.99}^{-0.99} \\ \times \left[1 + \log(t/7 \text{ s})\right]^{-0.01} \left(\frac{t}{10^8}\right)^{-4/3}, \quad t < t_s \\ 1.8 \times 10^{41} \text{ erg s}^{-1} M_{15}^{-0.74} R_{50}^{0.97} E_{51}^{0.92} \eta_{0.82}^{-0.82} \\ \times \left(\frac{t}{1 \text{ hr}}\right)^{-0.33}, \quad t_s < t < t_{tr} \end{cases}$$

$$t_{tr} \approx 47 \text{ m}.$$  \hfill (91)

In Figure 6, we plot the luminosity derived in this work along with that obtained by NS10. The result of NS10 is ~3 higher than what we obtain in Equation (91). Radiation from a BSG will be much fainter than that of an RSG (Equation (81)). The low luminosity is expected for compact progenitors that lose most of their energy to adiabatic expansion and is expressed by the high power of $R$ in Equation (91). A good example is SN 1987A, whose light curve is powered mostly by $^{56}$Ni and has only a small and early contribution from shock cooling radiation (e.g., Woosley et al. 1988).

A BSG in the planar phase will initially be out of thermal equilibrium with the following value of $\eta_{1\nu}$:

$$\eta_{1\nu}(t) = 70 M_{15}^{1.59} R_{50}^{-0.77} E_{51}^{2.12} \xi_{0.34}^{1.66} \left(\frac{t}{7 \text{ s}}\right)^{-1/6} \times \left[1 + \log(t/7 \text{ s})\right]^{-1.66}.$$  \hfill (92)

Nevertheless, in the case of BSG explosions, upscattering of lower-energy photons by Compton interactions is important due to the high gas temperature and therefore contributes greatly to thermalization. At breakout, we find that $\xi_{1\nu} \sim 3$, such that $\eta_{1\nu}/\xi_{1\nu} \sim 20$ and the observed radiation is still out of thermal equilibrium. The dependence of $T_{\text{obs}}$ on time differs from a power law, since $\xi_{1\nu}$ depends logarithmically on $T_{\text{obs}}$, which itself decreases nonnegligibly. We find that at the end of the planar phase, $\eta_{1\nu} \sim 1.7$ and $\xi_{1\nu} \sim 1$, and the radiation is almost in thermal equilibrium.

We use Equations (55), (64), (66), and (67) to obtain

$$t_1 = 57 \text{ m} M_{15}^{0.24} R_{50}^{0.94} E_{51}^{0.29} \eta_{0.34}^{0.50}$$

and

$$t_2 = 2.9 \text{ hr} M_{15}^{0.77} R_{50}^{0.62} E_{51}^{1.00} \eta_{0.34}^{1.03}.$$  \hfill (94)

At $t_{tr} < t$, $\xi_{1\nu} \sim 1$, and Comptonization no longer affects the observed temperature. In Figure 7, we plot the temperature evolution represented in Equation (93). We also overplot the blackbody temperature of the luminosity shell and the observed temperature that would have been obtained without the logarithmic correction found in this work. The drop in the observed temperature during the planar phase is significant compared to the uncorrected temperature, and at $t_s$, $T_{\text{obs}}$ is an order of magnitude lower than the temperature calculated without the logarithmic correction. As a result, the decrease in temperature toward thermal equilibrium in the end of the planar phase is much more gradual than previously thought.

One can notice that the corrected and uncorrected evolution becomes identical at $t_{tr} \lesssim t$, once the luminosity shell reaches $m_{pl}$, since the observed radiation now originates in the same shell.

Without the logarithmic correction, the typical energy of the BSG radiation would have exceeded 10 KeV throughout the planar phase, emitting most of the energy in hard X-rays. Nevertheless, the fast decrease in $T_{\text{obs}}$ brings the typical energy to the extreme UV ($\sim$100 eV) at the end of the planar phase. Comparing our results to work that does not assume thermal equilibrium (e.g., Enssn & Burrows 1992 and NS10), we find higher observed temperatures that exceed their results by about an order of magnitude. The discrepancy of our results compared to those of NS10 originates mainly from the difference in the normalization of $\eta_{1\nu}$ and the weaker role of Comptonization imposed by the higher value of $h\nu_{\text{min}}$, which causes our values of $\xi_{1\nu}$ to be much lower and result in higher temperatures.

For this BSG progenitor, $R/c \sim 120$ s. With an effective power-law temperature evolution of $T \propto t^{-1.2}$ at $t < R/c$, light travel time variations will create a spectrum of $F_{\nu} \propto \nu^{-0.52}$ during the first $\sim 2$ minutes.
8.3. WR

The WR stars are believed to be the progenitors of Ibc SNe. They lost all of their H envelope (and some of their He) prior to the explosion and therefore have an electron scattering opacity of 0.2 cm$^2$ g$^{-1}$. We use the following properties to describe a typical WR star: $M = 15M_{\odot}$, $R = 5R_{\odot}$, and $E = 10^{51}$ erg. These objects also have radiative envelopes for which $n = 3$, and therefore $C_1 = 0.35$ and $C_2 = 0.97$ if the whole envelope is a single polytrope. For these typical parameters, the breakout time of a WR star would be

$$t_{\text{bo}}^{\text{WR}} = 0.12 \, s \, M_{15}^{0.27} R_{5}^{1.92} E_{51}^{-0.73} \kappa_{0.2}^{0.46},$$

and the planar phase ends at

$$t_{\text{s}}^{\text{WR}} = 45 \, s \, M_{15}^{0.42} R_{5}^{1.33} E_{51}^{-0.58} \kappa_{0.2}^{0.17},$$

which, similar to a BSG, means that the planar phase evolution of a WR explosion cannot be resolved in observations. The luminosity of a typical BSG in the planar and transition phases is found using Equations (29) and (39),

$$L^{\text{WR}}(t) = \begin{cases} 
2.5 \times 10^{45} \, \text{erg s}^{-1} M_{15}^{-0.33} R_{5}^{2.21} E_{51}^{0.34} \kappa_{0.2}^{0.99} \\
\times [1 + \log(t/0.1 \, \text{s})]^{-0.01} \left( \frac{t}{0.1 \, \text{s}} \right)^{-4/3}, & t < t_{\text{s}} \\
1.1 \times 10^{44} \, \text{erg s}^{-1} M_{15}^{-0.74} R_{5}^{0.97} E_{51}^{0.92} \kappa_{0.2}^{-0.82} \\
\times \left( \frac{t}{1 \, \text{m}} \right)^{-0.33}, & t_{s} < t < t_{\text{u}} \\
\end{cases},$$

where $t_{\text{u}} = 2.7$ minutes. The small radius of a WR star leads to very high velocities at breakout (almost $10^5$ km s$^{-1}$) resulting in very high temperatures, which makes it hard for the radiation to thermalize. Accordingly, the value of $\eta_{\text{h}}$ is very high,

$$\eta_{\text{h}}^{\text{WR}}(t) = 343 M_{15}^{1.59} R_{5}^{-0.77} E_{51}^{2.12} \kappa_{0.2}^{1.60} \left[ 1 + \log(t/0.1 \, \text{s}) \right]^{-1/6},$$

while $\xi_{\text{h}}(t_{\text{bo}}) \sim 12$. This means that $\eta_{\text{h}}/\xi_{\text{h}} \sim 30$, and the radiation is out of thermal equilibrium at breakout. As in the case of BSG explosions, in WR explosions, the radiation remains out of thermal equilibrium throughout the planar phase where $\eta_{\text{h}}/\xi_{\text{h}} \sim 5$ at $t_{\text{s}}$.

The observed temperature of the nonthermal radiation from a WR explosion, according to Equations (55), (66), and (67), is

$$T_{\text{obs}}^{\text{WR}}(\eta_{\text{h}} > 1) = \begin{cases} 
2.0 \times 10^9 \, K \, M_{15}^{3.02} R_{5}^{-1.38} E_{51}^{4.18} \kappa_{0.2}^{-3.02} \\
\times \left( \frac{\xi_{\text{h}}}{12} \right)^{-2} [1 + \log(t/0.1 \, \text{s})]^{-3.02}, & t_{\text{bo}} < t < t_{\text{s}} \\
1.1 \times 10^7 \, K \, M_{15}^{3.08} R_{5}^{-1.16} E_{51}^{4.08} \kappa_{0.2}^{3.0} \left( \frac{t}{1 \, \text{m}} \right)^{-0.83}, & t_{s} < t < t_{\text{u}} \\
2.5 \times 10^6 \, K \, M_{15}^{1.16} R_{5}^{5.0} E_{51}^{1.39} \kappa_{0.2}^{2.23} \left( \frac{t}{3 \, \text{m}} \right)^{-5.46}, & t_{\text{u}} < t < t_{\text{b}} \\
1.5 \times 10^5 \, K \, M_{15}^{0.05} R_{5}^{0.24} E_{51}^{0.11} \kappa_{0.2}^{0.31} \left( \frac{t}{5 \, \text{m}} \right)^{-0.40}, & t_{\text{b}} < t < t_{2} \\
\end{cases},$$

where

$$t_{1} = 5 \, m \, M_{15}^{-0.24} R_{5}^{0.94} E_{51}^{-0.29} \kappa_{0.2}^{-0.50},$$

and

$$t_{2} = 24 \, m \, M_{15}^{-0.77} R_{5}^{0.62} E_{51}^{1.00} \kappa_{0.2}^{1.03}.\) (102)

We plot this temperature evolution in Figure 8.

The breakout temperature of a WR explosion derived using our model exceeds the energy in which pair production starts regulating the temperature (∼50 keV), and our treatment for the temperature evolution might therefore not be valid in the very early stages of a WR SN. By the end of the planar phase, the typical energy will have reached ∼1 keV, which lies in the soft X-ray range.

X-rays were associated with the early detection of the Ib/c SN 2008D (Soderberg et al. 2008; Modjaz et al. 2009), which was discovered during breakout in NGC 2270. However, it is not clear whether the X-ray transient originated in the shock breakout (Chevalier & Fransson 2008; Soderberg et al. 2008) or whether its source was a mildly relativistic jet that penetrated through the envelope of the star (e.g., Mazzali et al. 2008; Xu et al. 2008; Modjaz et al. 2009).

An effective power-law fit to the temperature at early times gives $T \propto t^{-0.72}$. Therefore, the spectrum at $t < R/c \sim 12$ s will follow $F_{\nu} \propto \nu^{-0.52}$.

9. Summary

We derive a self-similar solution for the energy inside an SN envelope when the envelope is fully ionized ($T > 1$ eV) and the SN is in the planar phase ($t < R/v$). We rely on the exact density and energy profiles of the envelope after shock passage, which serve as initial conditions to the subsequent expansion phase (S60). Our solution is applied to various types of progenitors, and for each, we use the appropriate normalization of the density and velocity profiles. Our results for the luminosity and observed temperature are therefore more accurate than previous analytic solutions that use order-of-magnitude normalizations. Using a self-similar solution, we find a logarithmic correction to the radiation diffusion in the planar phase, whose strongest effect is on the observed temperature of the SN. We list here our main conclusions.

1. Since the time doubles itself many times during the planar phase, photons from more internal shells relative to the breakout shell are able to diffuse out through the envelope. The luminosity shell is thus not constant, as previously thought, but penetrates farther into the star logarithmically with time. It therefore no longer satisfies [Equation]...
The logarithmic correction to \( m \) = \( R_1 \), and factor of \( \sim 5 \) relative to the breakout shell. The observed radiation now originates in shells that satisfy \( t < t_{\text{diff}} \). By the end of the planar phase, the mass of the luminosity shell will have increased by a factor of \( \sim 10 \) relative to the breakout shell.

2. The logarithmic correction to \( m_{ls} \) highly affects the observed radiation if the luminosity shell is out of thermal equilibrium during the planar phase. This is more often the situation in compact progenitors with a steep density profile, such as BSG and WR stars, where the velocities of the ejecta are very high, and the radiation does not manage to thermalize. The logarithmic correction to the luminosity shell causes the radiation to originate in much denser regions, roughly 10 times denser than without the correction. This increases the photon production rate by free–free emission by a factor of \( \sim 10^2 \) and drives the radiation more quickly toward thermal equilibrium.

3. As the luminosity shell recedes into the envelope, the thermal coupling increases with time, causing the observed temperature to decrease more than adiabatically (when the radiation is nonthermal). We reach similar conclusions to previous works that found the emission from BSG and WR explosions to be nonthermal. However, we also show that the typical photon energy is expected to decrease much faster than previously thought due to the logarithmic correction, which would otherwise cool only adiabatically. The radiation peak is thus directed toward lower frequencies; a BSG explosion will first peak in the X-rays, but the radiation will reach the soft far-UV after \( \sim 10 \) minutes. A WR star is expected to initially radiate in the hard X-ray–to–gamma-ray regime and cool down to the soft X-ray by the end of the planar phase. However, if the model described in this paper yields typical energies that exceed \( \sim 50 \) KeV, the calculation is no longer valid, since the temperature is now regulated by pair production.

4. Using our accurate self-similar solutions, we find that the radiation from an RSG explosion is expected to be mildly out of thermal equilibrium at breakout but quickly reach thermalization owing to the logarithmic increase of the luminosity shell. Nevertheless, due to light travel time variations, the observed spectrum will not be that of a blackbody until \( t < R/c \sim 12 \) s. Previous works predicted that the emission from BSG and WR explosions to be nonthermal. However, we also show that the typical photon energy is expected to decrease much faster than previously thought due to the logarithmic correction, which would otherwise cool only adiabatically. The radiation peak is thus directed toward lower frequencies; a BSG explosion will first peak in the X-rays, but the radiation will reach the soft far-UV after \( \sim 10 \) minutes. A WR star is expected to initially radiate in the hard X-ray–to–gamma-ray regime and cool down to the soft X-ray by the end of the planar phase. However, if the model described in this paper yields typical energies that exceed \( \sim 50 \) KeV, the calculation is no longer valid, since the temperature is now regulated by pair production.

5. Due to the weak dependence of the luminosity on the Lagrangian mass coordinate \( m \), the logarithmic correction...
is not expected to have an observed effect on the bolometric luminosity. For the same reason, when the radiation is in thermal equilibrium, the temperature will also not be affected by the logarithmic correction.

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Appendix A
The Properties of the Breakout Shell at Breakout

\[
\nu_{bo}^{\perp} = \begin{cases} 
7.1 \times 10^3 \text{ km s}^{-1} M_{15}^{-0.44} R_{500}^{0.26} E_5^{0.56} \rho_5^{0.13} \kappa_{0.34} (\text{RSG}) \\
2.3 \times 10^4 \text{ km s}^{-1} M_{15}^{-0.42} R_{50}^{-0.33} E_5^{0.58} \rho_5^{0.17} \kappa_{0.34} (\text{BSG}) \\
4.6 \times 10^4 \text{ km s}^{-1} M_{15}^{-0.42} R_{5}^{-0.33} E_5^{0.58} \rho_5^{0.17} \kappa_{0.34} (\text{WR}) 
\end{cases} 
\]

(103a)

\[
\rho_{bo} = \begin{cases} 
1.2 \times 10^{-9} \text{ g cm}^{-3} M_{15}^{0.87} R_{500}^{0.51} E_5^{-1.13} \rho_5^{-1.26} \kappa_{0.34} (\text{RSG}) \\
1.6 \times 10^{-10} \text{ g cm}^{-3} M_{15}^{0.83} R_{50}^{0.66} E_5^{-1.17} \rho_5^{-1.33} \kappa_{0.34} (\text{BSG}) \\
4.5 \times 10^{-10} \text{ g cm}^{-3} M_{15}^{0.83} R_{5}^{0.66} E_5^{-1.17} \rho_5^{-1.33} \kappa_{0.34} (\text{WR}) 
\end{cases} 
\]

(103b)

\[
\mu_{bo} = \begin{cases} 
1.6 \times 10^{17} \text{ erg g}^{-1} M_{15}^{0.80} R_{500}^{0.21} E_5^{0.87} \rho_5^{0.06} \kappa_{0.34} (\text{RSG}) \\
1.2 \times 10^{18} \text{ erg g}^{-1} M_{15}^{0.74} R_{50}^{0.63} E_5^{0.92} \rho_5^{0.18} \kappa_{0.34} (\text{BSG}) \\
4.5 \times 10^{19} \text{ erg g}^{-1} M_{15}^{0.74} R_{5}^{0.63} E_5^{0.92} \rho_5^{0.18} \kappa_{0.34} (\text{WR}) 
\end{cases} 
\]

(103c)

\[
d_{0,bo} = \begin{cases} 
6.5 \times 10^{10} \text{ cm} M_{15}^{0.23} R_{500}^{1.90} E_5^{0.23} \rho_5^{-0.45} \kappa_{0.34} (\text{RSG}) \\
1.7 \times 10^{10} \text{ cm} M_{15}^{-0.15} R_{50}^{1.58} E_5^{-0.15} \rho_5^{-0.29} \kappa_{0.34} (\text{BSG}) \\
5.3 \times 10^{8} \text{ cm} M_{15}^{-0.15} R_{5}^{1.58} E_5^{-0.15} \rho_5^{-0.29} \kappa_{0.34} (\text{WR}) 
\end{cases} 
\]

(103d)

\[
m_{bo} = \begin{cases} 
2.5 \times 10^{29} \text{ g} M_{15}^{0.44} R_{500}^{0.26} E_5^{0.56} \rho_5^{-1.13} \kappa_{0.34} (\text{RSG}) \\
8.7 \times 10^{26} \text{ g} M_{15}^{0.42} R_{50}^{0.58} E_5^{-1.17} \rho_5^{-1.17} \kappa_{0.34} (\text{BSG}) \\
7.5 \times 10^{24} \text{ g} M_{15}^{0.42} R_{5}^{0.58} E_5^{-1.17} \rho_5^{-1.17} \kappa_{0.34} (\text{WR}) 
\end{cases} 
\]

(103e)

Normalized to the breakout velocity before acceleration.

Appendix B
Glossary of Main Symbols and Notations

1. \( t \): time.
2. \( r \): radius.
3. \( v \): velocity.
4. \( m \): integrated mass measured from the stellar radius inward.
5. \( \rho \): density.
6. \( d \): shell width.
7. \( x \): distance from the fixed position of the stellar edge at \( t = 0 \). This is also the width of a shell during the planar phase evolution.
8. \( u \): energy per unit mass.
9. \( R_d \): progenitor radius.
10. \( R_s \): stellar radius.
11. \( M \): progenitor mass.
12. \( E \): explosion energy.
13. \( P \): pressure.
14. \( L \): observed luminosity.
15. \( T_{obs} \): observed temperature, defined as the typical photon energy.
16. \( \kappa \): opacity.
17. \( \tau \): optical depth.
18. \( T_{BB} \): blackbody temperature, thermal equilibrium temperature for a given energy density, defined by Equation (43).
19. Breakout shell: shell from which the shock breaks out; satisfies \( \tau = c/v \) during the planar phase.
20. Luminosity shell: shell that is the source of the observed luminosity.
21. Color shell: shell where the observed temperature is determined. Coincides with the shell satisfying \( \eta = 1 \) when the radiation is in thermal equilibrium and with the luminosity shell when the radiation is out of thermal equilibrium.
22. For any quantity \( x \), we use the following subscripts:
   \( x_{bo} \): value at the breakout shell at breakout.
   \( x_{cl} \): value at the luminosity shell.
   \( x_{cs} \): value at the color shell.
23. \( m_{bo} \): mass of the breakout shell.
24. \( m_{pl} \): mass of the luminosity shell at the end of the planar phase (at \( t = t_f \)).
25. \( t_{bo} \): time of breakout.
26. \( t_{diff} \): diffusion time.
27. \( t_s \): time of breakout shell transition to the spherical phase.
28. \( t_{sph} (m) \): time a shell of mass \( m \) enters the spherical phase, defined in Equation (31).
29. \( t_{eq} \): time when the radiation reaches thermal equilibrium during the planar phase, in case it was not in thermal equilibrium at breakout.
30. \( t_{en} \): time when the transition phase ends.
31. \( t_1 \): first time the shell that satisfies \( \tau = c/v \) during the transition phase reaches a shell in which the radiation is in thermal equilibrium. Thermal equilibrium in that shell was achieved by photons produced at the end of the planar phase.
32. \( t_{2a} \): first time the shell that satisfies \( \tau = c/v \) coincides with the shell satisfying \( \eta = 1 \) during the transition phase.
33. \( t_1 \): first time in which the observed radiation is in thermal equilibrium, in case it was not in thermal equilibrium at the end of the transition phase. Thermal equilibrium was achieved by photons produced at an earlier time.
34. \( t_2 \): first time in which the radiation is in thermal equilibrium achieved by photons produced at \( t_2 \), in case it was not in thermal equilibrium at the end of the transition phase.
35. \( \eta \): thermal coupling coefficient, defined in Equation (44).
36. \( \zeta \): logarithmic Comptonization factor, defined in Equation (50).
37. \( n \): power-law index describing the pre-explosion stellar density profile; \( n \) is related to the adiabatic index \( \gamma \) by \( \gamma = 1 + 1/n \).
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