Anomalies in the differential cross sections at 13 TeV.

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Abstract

Analysis of differential cross sections of the TOTEM Collaboration data, carried out without model assumptions, shows the existence of new effects in the behavior of the hadron scattering amplitude at a small momentum transfer at a high confidence level. The quantitative description of the data in the framework of the high energy generalized structure (HEGS) model supports such a phenomenon that can be associated with the specific properties of the hadron potential at large distances. It is shown that the value of $\rho(s, t)$ at $\sqrt{s} = 13$ TeV and small $t$ exceeded 0.1.

1. Introduction

The unique experiment carried out by the TOTEM Collaboration at LHC at 13 TeV gave excellent experimental data on the elastic proton-proton scattering in a wide region of transfer momenta [1, 2]. It is especially necessary to note the experimental data obtained at small momentum transfer in the Coulomb-hadron interference region. The experiment reaches very small $t = 8 \times 10^{-4}$ GeV$^2$ with small $\Delta t$, which gave a large number of experimental points in a sufficiently small region of momentum transfer. This allows one to carry out careful analysis of the experimental data to explore some properties of the hadron elastic scattering.

There are two sets of data - at small momentum transfer [1] and at middle and large momentum transfer [2]. They overlap in some region of the momentum transfer which supply practically the same normalization of both sets of the differential cross section of elastic proton-proton scattering.

A research of the structure of the elastic hadron scattering amplitude at superhigh energies and small momentum transfer - $t$ can give a connection between the experimental knowledge and the basic asymptotic theorems based on first principles [3, 4, 5]. It gives information about the hadron interaction at large distances where the perturbative QCD does not work [6, 7], and a new theory as, for example, instanton or string theories must be developed.

There is a very important characteristic of the elastic scattering amplitude such as the ratio of the real part to imaginary part of the scattering amplitude - $\rho(s, t)$. It is tightly connected with the integral and differential dispersion relations. Of course, especially after different results obtained by the UA4 and UA4/2 Collaborations at SPPS physicists understand that $\rho(s, t = 0)$ is not simple experimental value but heavily dependent on theoretical assumptions about
the momentum dependence of the elastic scattering amplitude. Our analysis of both experimental data obtained by the UA4 and UA4/2 Collaborations, shows a small difference value of $\rho(s, t = 0)$ between both the experiments if the non-linear behaviour of the elastic scattering amplitude is taken into account [8]. Hence, this is not a experimental problem but a theoretical one [9].

Usually, a small region of $t$ is taken into account for extraction of the sizes of $\sigma_{\text{tot}}$ and $\rho(t = 0)$ (for example [1, 10]). However, a form of the scattering amplitude assumed for small $t$ and satisfying the existing experimental data at small $t$, can essentially be different from experimental data at large momentum transfer. One should take into account the analysis of the differential cross section at 13 TeV where the diffraction minimum impacts the form of $d\sigma/dt$ already at $t = -0.35$ GeV$^2$.

2. Thin effects in the differential cross sections at $\sqrt{s} = 13$ TeV

The extraction of values of the basic parameters of the elastic hadron interaction requires some model that can describe all experimental data at the quantitative level with minimum free parameters. Now many groups of researchers have presented some physical models satisfying more or less these requirements. This is especially related with the HEGS (High Energy Generalized Structure) model [11, 12]. As it takes into account two form factors (electromagnetic and gravitomagnetic), which are calculated from the GPDs function of nucleons, it has a minimum of free parameters and gives a quantitative description of the exiting experimental data in a wide energy region and momentum transfer. One of the specific properties of our analysis is that in the fitting procedure we take into account only statistical errors. The systematic errors are taken as an additional coefficient which changes the normalization of one set of experimental data. In this case, the space for theoretical functions decreases essentially but can lead to an increase in the whole $\chi^2$ [13].

However, let us carried out analysis of $d\sigma/dt$ of the TOTEM Collaboration data, without model assumptions to catch out some possible thin effects like some periodical structure in the behavior of the hadron scattering amplitude at a small momentum transfer [14].

For model free analysis, let us use the method of comparison of two statistically independent choices, for example [15]. Such method does not require knowledge of the form of the additional periodic part of the scattering amplitude. If we have two statistically independent choices $x'_{n_1}$ and $x''_{n_2}$ of values of the quantity $X$ distributed around a definite value of $A$ with the standard error equal to 1, We can find the difference between $x'_{n_1}$ and $x''_{n_2}$. For that, we can compare the arithmetic mean of these choices

$$\Delta X = (x'_1 + x'_2 + \ldots + x'_{n_1})/n_1 - (x''_1 + x''_2 + \ldots + x''_{n_2})/n_2 = \overline{x'_{n_1}} - \overline{x''_{n_2}}.$$ 

The standard deviation for this case will be $\delta_x = [1/n_1 + 1/n_2]^{1/2}$. And if $\Delta X/\delta_x$ is larger than 3, we can say that the difference between these two choices has the 99% probability.
The deviations $\Delta R_i$ of experimental data from these theoretical cross sections we will be measured in units of experimental error for an appropriate point

$$\Delta R_i = \left[ \frac{(d\sigma/dt)_i^{exp} - (d\sigma/dt)_i^{th}}{\delta_{i}^{exp}} \right],$$

where $\delta_{i}^{exp}$ is an experimental error. To take this effect into account, we break the whole studied interval of momentum transfer into $k$ equal pieces $k\delta t = (t_{max} - t_{min})$, and then sum $\Delta R_i$ separately over even and odd pieces. Thus, we receive two sums $S^{up}$ and $S^{dn}$ for $n_1$ even and $n_2$ odd intervals. At this $n_1 + n_2 = k$ and $|n_1 - n_2| = 0$ or 1

$$S^{up} = \sum_{j=1}^{n_1} \left( \sum_{i=1}^{N} \Delta R_i |_{\delta q(2j-1) < q_i \leq \delta q(2j)} \right), \quad S^{dn} = \sum_{j=1}^{n_2} \left( \sum_{i=1}^{N} \Delta R_i |_{\delta q(2j) < q_i \leq \delta q(2j+1)} \right).$$

In the case of some difference of experimental data from the theoretical behavior, expected by us, or incorrectly determined parameters, these two sums will deviate from zero; but their sizes should coincide within experimental errors. However, this will be so in the case if experimental data have no any periodic structure or a sharp effect coincides with one interval. We assume that such a periodic structure is available and its period coincides with the chosen interval $2\delta t$. In this case, the sum $S^{up}$ will contain, say, all positive half-cycles; and the sum $S^{dn}$, all negative half-cycles. The difference between $S^{up}$ and $S^{dn}$ will show the magnitude of an additional effect summed over the whole researched domain.

The method does not require exact representation of the periodical part of the scattering amplitude, and now let us apply it to new LHC data of the TOTEM Collaboration at 13 TeV. The region of momentum transfer examined up to $-t < 0.4$ GeV$^2$ includes the Coulomb-hadron interference range. Of course, it is necessary to choose the true interval $\delta t$ to obtain the maximum of the difference between the sums $S^{up}$ and $S^{dn}$. To evaluate the size of a possible effect, one should examine the difference of the arithmetic mean values $\Delta S$ and the corresponding dispersion - $\delta S$

$$\Delta S = S^{up} - S^{dn}, \quad \delta S = \left( \frac{1}{1/n_1 + 1/n_2} \right)^{1/2}/N.$$

Let us calculate the sum of $\Delta S$ and its arithmetic mean chosen in the most appropriate interval $\delta t$

$$\Delta S = 285/325 = 0.877 \pm 0.028.$$

Obviously, this is shows the existence of some periodical structure at a high confidence level.

Now let us try to find the form of such an additional periodical contribution to the basic elastic scattering amplitude. As a basis, take our high energy generalized structure (HEGS) model which quantitatively describes, with only a few parameters, the differential cross section of $pp$ and $p\bar{p}$ from
\[ s = 9 \text{ GeV} \text{ up to } 13 \text{ TeV}, \text{ includes the Coulomb-hadron interference region and the high-}|t| \text{ region up to } |t| = 15 \text{ GeV}^2 \text{ and quantitatively well describes the energy dependence of the form of the diffraction minimum} \] 16. However, to avoid possible problems connected with the low-energy region, we consider here only the asymptotic variant of the model 17.

The total elastic amplitude in general receives five helicity contributions, but at high energy it is enough to write it as \( F(s, t) = F^h(s, t) + F^{em}(s, t) e^{\varphi(s, t)} \), where \( F^h(s, t) \) comes from the strong interactions, \( F^{em}(s, t) \) from the electromagnetic interactions and \( \varphi(s, t) \) is the interference phase factor between the electromagnetic and strong interactions 18. The Born term of the elastic hadron amplitude at large energy can be written as a sum of two pomeron and odderon contributions. All terms are supposed to have the same intercept \( \alpha_0 = 1 + \epsilon_0 = 1.11 \), and the pomeron slope is fixed at \( \alpha' = 0.24 \text{ GeV}^{-2} \).

The model takes into account two hadron form factors \( F_1(t) \) and \( A(t) \), which correspond to the charge and matter distributions 19. Both form factors are calculated as the first and second moments of the same Generalized Parton Distributions (GPDs).

As a probe for the oscillatory function take \[ f_{osc}(t) = h_{osc}(i + \rho_{osc}) J_1(\tau))/\tau; \tau = \pi (\phi_0 - t)/t_0, \quad (5) \]

here \( J_1(\tau) \) is the Bessel function of the first order. This form has only a few additional fitting parameters and allows one to represent a wide range of possible oscillation functions.

After the fitting procedure, we obtain \( \chi^2/dof = 1.24 \) (remember that we used only statistical errors). One should note that the last points of the second set above \( -t = 2.8 \text{ GeV}^2 \) show an essentially different slope, and we removed them. The total number of experimental points of both sets of the TOTEM Collaboration equals 415. If we remove the oscillatory function, then \( \chi^2/dof = 2.7 \), so an increase is more than two times. If we make a new fit without \( f_{osc} \), then \( \chi^2/dof = 2.5 \) decreases but remains large.

To see the oscillations in the differential cross sections, let us determine two values - one is pure by theoretical and the other with the experimental data

\[
\begin{align*}
R_{\Delta_{th}} &= \frac{[\sigma/dt_{th0} + osc - \sigma/dt_{th0}]/\sigma/dt_{th0}}{\sigma/dt_{th0}}, \\
R_{\Delta_{exp}} &= \frac{[\sigma/dt_{exp} - \sigma/dt_{th0}]/\sigma/dt_{th0}}{\sigma/dt_{th0}}. \quad (6)
\end{align*}
\]

The corresponding values calculated from the fit of two sets of the TOTEM data at 13 TeV are presented in Fig.1.

However, the additional normalization coefficient reaches a sufficiently large value, about 13%. It can be in a large momentum transfer region but is very unusual for a small momentum transfer. However, both sets of experimental data (small and large region of \( t \)) overlap in some region and, hence, affect each other’s normalization. It is to be noted, that the size of the normalization coefficient does not impact the size and properties of the oscillation term. We have examined many different variants of our model (including large and unity
normalization coefficient), but the parameters of the oscillation term have small variations.

In the work [20], the analysis of both sets of the TOTEM data at 13 TeV is carried out with additional normalization equal to unity and taking into account only statistical errors in experimental data. The Born scattering amplitude has four free parameters (the constants $C_i$) at high energy: two for the two pomeron amplitudes and two for the odderon. The real part of the hadronic elastic scattering amplitude is determined through the complexification $\hat{s} = s \exp(-i\pi/2)$ to satisfy the dispersion relations.

Now let us put the additional normalization coefficient to unity and continue to take into account in our fitting procedure only statistical errors. Of course, we obtain an enormously huge $\sum \chi^2$. The new fit changes the basic parameters of the Pomeron and Odderon Born terms but does not lead to a reasonable size of $\chi^2$. We find that the main part of $\sum \chi^2$ comes from the region of a very small momentum transfer. It requires the introduction of a new term which can help to describe the CNI region of $t$. This kind of term can be taken in different forms. In the present paper, we examined

$$F_d(t) = h_d(i + \rho_d)e^{-B_d|t|^\gamma \log \hat{s}},$$

where $G^2_\text{el}$ is the squared electromagnetic form factor of the proton. For simplicity, in a further fitting procedure the constant $\rho_{osc}$ and the phase $\phi_0$ of the oscillatory term are taken equal to zero. Hence, the oscillatory term depends only on two parameters - $h_{osc}$ and $t_0$ period of oscillation. Also, to reduce the number of fitting parameters the correction to the main slope, determined by $\pi$-meson loop [23], is taken in a simple form, we obtain the slope as

$$B(t) = \alpha' \log \hat{s}(1 - te^{B_\text{ad}t}).$$
The differential cross sections are calculated in the framework of the HEGS model with fixed additional normalization by 1.0 and with additional term a) [left] the magnification of the region of the small momentum transfer of a); b) [right] the magnification of the region of the diffraction minimum.

The fit of both sets of the TOTEM data simultaneously with taking into account only statistical errors, with additional normalization equal to unity and with the additional term, eq.(7), gives a very reasonable $\chi^2 = 551/425 = 1.29$. The results are presented for zoom of the region of small $t$ in Fig.2a, and zoom of the region of the diffraction minimum in Fig.2b.

The parameters of the additional term are well defined $h_d = 1.7 \pm 0.01; \quad \rho_d = -0.45 \pm 0.06; \quad B_d = 0.616 \pm 0.026; \quad \kappa = 1.119 \pm 0.024$.

To check up the impact of the form of the CNI phase - $\varphi(t)$, we made our calculations with the original Bethe phase $\varphi = -(\ln(B_{sl}/2. \ t) + 0.577)$ as well. We found that $\sum \chi^2$ changes by less than 0.2% and practically does not impact the parameters $F_d(t)$. Hence, our model calculations show two possibilities in the quantitative description of the two sets of the TOTEM data. One - takes into account an additional normalization coefficient, which has a minimum size of about 13% ; the other - the introduction of a new anomalous term of the scattering amplitude, which has a very large slope and gives the main contributions to the Coulomb-nuclear interference region.

Of course, there are some other ways to obtain good descriptions of the new experimental data of the TOTEM Collaboration. One is to use a model with an essentially increasing number of the fitting parameters and many different parts of the scattering amplitude. Another is to use a polynomial model with many free parameters. In both cases, the physical value of such a description is doubtful.

3. Size of $\rho(\sqrt{s} = 13 \ TeV, t)$

There is a large discussion about the value of the $\rho(s, t = 0)$ - the ratio of the real to imaginary part of the elastic scattering amplitude at $\sqrt{s} = 13$ TeV. If the TOTEM Collaboration gives the size of $\rho(s, t = 0) = 0.09 \pm 0.01$
using own simple phenomenological analysis, other researchers obtained the value \( \rho(s, t = 0) \) some above that value using the model description of the differential cross sections in a wide region of momentum transfer. For example in [24], it is noted that "... the value of \( \rho \) would be higher than the TOTEM value for \( \rho \) found under the hypothesis that the real part of the elastic nuclear amplitude is devoid of such a zero in the CNI region."

There is some simple method to obtain the value of \( \rho(s, t = t_{CN}) \) at one point \( t_{CN} \) without any assumptions of the momentum transfer behavior of the real part of the elastic scattering amplitude and check up some model assumptions. Let us consider from this point of view experimental data on nucleon-nucleon elastic scattering, being available in the range of small transfers of a pulse.

The differential cross sections measured experimentally are described by the squared scattering amplitude

\[
d\sigma/dt = \pi \left( F_C^2(t) + (1 + \rho^2(s, t)) \, \text{Im} F_N^2(s, t) \right)
\]

\[ 
\pm 2(\rho(s, t) + \alpha \phi) \, F_C(t) \text{Im} F_N(s, t) 
\]

\[ 
\pm 2(\rho(s, t) + \alpha \phi) \, F_C(t) \text{Im} F_N(s, t). 
\]

(9)

where \( F_C = \pm 2\alpha G^2/|t| \) is the Coulomb amplitude; \( \alpha \) is the fine-structure constant and \( G^2(t) \) is the proton electromagnetic form factor squared; \( \text{Re} \, F_N(s, t) \) and \( \text{Im} \, F_N(s, t) \) are the real and imaginary parts of the nuclear amplitude; \( \rho(s, t) = \text{Re} \, F(s, t)/\text{Im} \, F(s, t) \). Just this formula is used to fit experimental data determined by the Coulomb and hadron amplitudes and the Coulomb-hadron phase to obtain the value of \( \rho(s, t) \).

From equation (9) one can obtain an equation for the real (or imaginary) part of the scattering amplitude or for \( \rho \) for every experimental point - if we take the ordinary exponential form for the imaginary (or real) part of the scattering amplitude [25].

\[
\text{Re} F_N(s, t) = (\text{Re} F_c(s, t) + \text{Re} F_h(s, t)) \sqrt{\frac{1}{\pi} \frac{d\sigma}{dt} - (\text{Im} F_c + \text{Im} F_N)^2}. 
\]

(10)

As the imaginary part of the scattering amplitude is defined by

\[
\text{Im} F_N(s, t) = \frac{\sigma_{tot}}{4\pi} \exp(Bt/2),
\]

(11)

it is evident from (10) that the real part depends on \( n, \sigma_{tot}, B \). It is clear that if the differential cross sections have a special dependence, this will manifest itself most strongly in the calculated real part of the hadron scattering amplitude.

Let us determine the value \( \Delta_R \) in two ways. One gives purely theoretical \( \Delta^{th}_R(s, t) \) that is dependent on the size of the real part of the scattering amplitude

\[
\Delta^{th}_R(s, t) = (\text{Re} F_C(t) + \text{Re} F_h(s, t))^2. 
\]

(12)

Obviously, it gives the minimum at one point of \( t_{min} \) where the Coulomb amplitude equals by module the real part of the scattering amplitude.

Other determination gives the \( \Delta^{exp}_R(s, t_i) \) that is dependent on the experimental data of the differential cross sections and the size of the imaginary part of the scattering amplitude.

\[
\Delta^{exp}_R(s, t_i) = (\text{Re} F_C(t_i) + \text{Re} F_h(s, t_i))^2. 
\]

(13)
Figure 3: The value $\Delta_{R}^{th}$ and experimental $\Delta_{R}^{exp}$ are calculated with the parameters determined by TOTEM Collaboration [1] with $\rho(t = 0) = 0.09$.

Figure 4: The value $\Delta_{R}^{th}$ and experimental $\Delta_{R}^{exp}$ are calculated with $\rho(t = 0) = 0.12$. 
of the scattering amplitude.

\[
\Delta_{exp}^R(s, t_i) = \left[ \frac{d\sigma}{dt_i}_{exp} - k\pi \star (ImF_c(t_i) + ImF_h(t_i))^2 \right]/(k\pi).
\] (13)

If the real and imaginary parts of the scattering amplitude are indeed determined, then both above values have to be the same.

Now let us calculate these values using the parameters obtained by the TOTEM Collaboration through the fitting procedure of the experimental data at \(\sqrt{s} = 13\) TeV. The results are shown in Fig.3. It can be seen that most parts of the values of \(\Delta_{exp}^R(s, t_i)\) have the negative sign. It shows the wrong determination of the imaginary part of the scattering amplitude or the wrong determination of the normalization of the experimental data.

A different situation is presented in Fig.4. In this case, the value \(\rho(s, t = 0) = 0.12\) and slightly changes the slope of the scattering amplitude. The dependence of \(\Delta_{exp}^R(s, t_i)\) and \(\Lambda_{exp}^R(s, t)\) on momentum transfer is related with eqs.(12,13).

Such different results, presented in Fig.3 and Fig.4 show that the real part of the scattering amplitude has to be more than 0.1.

4. Conclusion

Thus, the study of the behavior of the differential cross sections in the range of small momentum transfer can give essential information on the behavior of the interaction potential at large distances.

Using only statistical errors and fixing additional normalization of differential cross sections equal to unity, we have limited the possible forms of the theoretical representation of the scattering amplitude. The phenomenological model, the HEGSh model, was used for examining the whole region of the momentum transfer of two sets of experimental data obtained by the TOTEM Collaboration at 13 TeV. The simple exponential form of the scattering amplitude was used to examine only a small region of momentum transfer. In both cases, an additional fast decreasing term of the scattering amplitude was required for a quantitative description of the new experimental data. The large slope of this term can be connected with a large radius of the hadronic interaction and, hence, can be determined by the interaction potential at large distances. It can be some part of the hadronic potential responsible for the oscillation behavior of the elastic scattering amplitude [26].

The discovery of such anomaly in the behaviour of the differential cross section at very small momentum transfer in LHC experiments will give us important information about the behavior of the hadron interaction potential at large distances. It may be tightly connected with the problem of confinement. We have shown the existence of such anomaly at the statistical level and that some other models also revealed such unusual behaviour of the scattering amplitude. Very likely, such effects exist also in experimental data at essentially smaller energies [26, 27]. However, the results of the TOTEM Collaboration have a unique unprecedentedly small statistical error and reach minimally small
angles of scattering with the largest number of experimental points in this small region of the momentum transfer. The new effects can impact the determination of the sizes of the total cross sections, the ratio of the elastic to the total cross sections and the size of the $\rho(s,t)$, the ratio of the real to imaginary part of the elastic scattering amplitude. It is to be noted that the detected new phenomena can impact the determination of the size of the $\rho(s,t)$, the ratio of the real to imaginary part of the elastic scattering amplitude and the sizes of the total cross sections, the ratio of the elastic to the total cross sections.

The comparison of the sizes of the total cross sections and $\rho(t=0)$ obtained for the case with additional coefficient normalization $k$ and the cases with an additional fast decreasing term and $k = 1.0$, show the large difference. If in the first case we obtain $\sigma_{\text{tot}}(\text{TOTEM}) = 106.2 \pm 0.2$ mb which is less than the value extracted by the TOTEM Collaboration - $\sigma_{\text{tot}}(\text{TOTEM}) = 110.6 \pm 3.4$ mb in the analysis of only small momentum transfer region. In this case the size of $\rho(t=0) = 0.142 \pm 0.004$. However, in the case with the $k = 1.0$, which require an additional term with a large slope, the value of $\sigma_{\text{tot}} = 112.6 \pm 0.11$ mb which exceeds the $\sigma_{\text{tot}}(\text{TOTEM})$, and $\rho(t=0)$ practically coincides with the predictions of the COMPETE Collaboration [28]. These results show the necessity the complete analysis of all the sets of the LHC data from $\sqrt{s} = 7$ TeV up to $\sqrt{s} = 13$ TeV including the results of both the Collaborations (TOTEM and ATLAS).

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