A new formulation of perturbation theory for a description of the Dirac and scalar fields (the Yukawa model) is suggested. For a main approximation, the self-consistent field model that considers to a certain degree forces caused by interaction of fields is chosen. This approximation leads to a normally ordered form of the interaction Hamiltonian. Generation of fermion mass in the process of interaction with exchange by a scalar boson is investigated. It is demonstrated that for zero bare mass, the fermion can acquire mass only if its coupling constant exceeds the critical value determined by the boson mass. In this regard, the problem of the neutrino mass is discussed.

Keywords: Yukawa model, fermion, boson, self-consistent field, perturbation theory, neutrino, mass generation.

1. The theory of interacting Dirac and scalar fields was suggested by Yukawa for a description of nucleon and pion interactions [1]. In modern particle theory, the standard model contains the term with the Yukawa interaction which relates the Higgs scalar field to quarks and leptons so that the majority of free parameters of this standard model are the Yukawa coupling constants [2]. The Yukawa theory can also be used as a simplified model of quantum electrodynamics. However, since massive scalar bosons rather than massless vector bosons are examined, this model presents no difficulties which can arise in the process of electromagnetic field quantization [3].

In the present work, a modified perturbation theory is suggested for the Yukawa model where the self-consistent field model is chosen as a main approximation. For nonrelativistic many-particle Fermi and Bose systems, an analogous approach was developed in [4, 5]. In [6], the idea of this approach was realized on the example of solving the quantum-mechanical problem of an anharmonic oscillator.

The Lagrange density function for the Yukawa model has three terms

\[ L = L_D + L_S + L_I, \]  \hspace{1cm} (1)

where

\[ L_D = -\left( \bar{\psi} \gamma^\mu \partial_\mu \psi + m \bar{\psi} \psi \right) \]  \hspace{1cm} (2)

is the Dirac field Lagrangian,
The scalar field Lagrangian, and

\[ L_I = -g \phi \psi \overline{\psi} \]  

is the interaction Lagrangian of the scalar and Dirac fields \( \phi \) and \( \psi \). Here \( m \) and \( \kappa_0 \) are the bare masses of the Dirac and scalar fields, \( g \) is the dimensionless coupling constant, \( \partial_\mu = \partial / \partial x_\mu \), \( x_\mu = (x, x_0) \), and \( \gamma_\mu \) are the Dirac matrices. The metric \( ab = ab - a_0 b_0 \) and the system of units \( h = c = 1 \) are used. Field operators entering into initial Lagrangian (1) are written down in the Heisenberg representation and obey the standard commutation conditions for coinciding times.

2. The Yukawa model, like the majority of field theories, can be studied within the framework of perturbation theory. As a rule, the free field Lagrangian \( L_0 = L_D + L_S \) is used as a main approximation, and interaction (4) is considered as a perturbation. The validity of the approach is the more rigorously substantiated the smaller the coupling constant. Meanwhile, in the models with the Yukawa interaction, the perturbation theory in its conventional form can be inapplicable because this interaction is strong. However, natural separation of the Lagrangian into the main component and perturbation is not unambiguous and necessary. Indeed, instead of \( L = L_1 + L_2 \), we can use \( L = L'_1 + L'_2 \), where \( L'_1 = L_1 + \Delta L \), \( L'_2 = L_2 - \Delta L \), and \( \Delta L \) is any arbitrary operator addition. Assigning a method of constructing the Lagrangian which describes one-particle states, we thereby define the term noninteracting particle within the framework of nonlinear theory.

We now consider the influence of perturbation (4) on the free Dirac and scalar fields. The structure of perturbation (4) is such that it has the form of the mass term (with the operator coefficient) involved in the Lagrangian \( D_L \). Therefore, we assume that this interaction will lead to a change in the particle mass described by the field \( \psi \). To consider this effect, we introduce the Lagrangian

\[ L'_D = -\left( \overline{\psi} \gamma_\mu \partial_\mu \psi + M \psi \overline{\psi} \right), \]  

where \( M \) is the mass different from the bare particle mass \( m \) of Lagrangian (2). We note also that interaction (4) is linear in the scalar field and can break the symmetry of Lagrangian \( L_S \) with respect to changes in the scalar field sign: \( \phi \rightarrow -\phi \). To consider this effect, we introduce the Lagrangian

\[ L'_S = -\frac{1}{2} \left( \partial_\mu \phi \right)^2 + \kappa^2 \phi^2 - b \phi, \]  

where \( b \) is a real coefficient. The mass \( \kappa \) has been introduced instead of \( \kappa_0 \) in Eq. (6), assuming that the interaction can change the scalar particle mass as well. However, as demonstrated below, the structure of interaction (4) is such that this change does not occur. Thus, Lagrangians (5) and (6) consider to a certain degree the forces caused by interaction of the fields. For the main approximation, we choose the Lagrangian which is the sum of Lagrangians (5) and (6):

\[ L_0 = -\left( \overline{\psi} \gamma_\mu \partial_\mu \psi + M \psi \overline{\psi} \right) - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - b \phi - V. \]  

Here \( V \) is the numerical \( c \)-constant which is important for constructing the perturbation theory in this approach. In this main approximation, the perturbation Lagrangian is \( L_C = L - L_0 \), so that

\[ L_C = -g \phi \overline{\psi} \psi + (M - m) \psi \overline{\psi} - \frac{1}{2} \left( \kappa_0^2 - \kappa^2 \right) \phi^2 + b \phi + V. \]