Relativistic mean-field approximation with density-dependent screening meson masses in nuclear matter

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Abstract

The Debye screening masses of the $\sigma$, $\omega$ and neutral $\rho$ mesons and the photon are calculated in the relativistic mean-field approximation. As the density of the nucleon increases, all the screening masses of mesons increase. It shows a different result with Brown-Rho scaling, which implies a reduction in the mass of all the mesons in the nuclear matter except the pion. Replacing the masses of the mesons with their corresponding screening masses in Walecka-1 model, five saturation properties of the nuclear matter are fixed reasonably, and then a density-dependent relativistic mean-field model is proposed without introducing the non-linear self-coupling terms of mesons.

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I. INTRODUCTION

In spite of the great success of the relativistic quantum field theory (RQFT) in many areas of modern physics, some difficulties have been found in applying this theory to nuclear systems. In nuclear physics, the coupling constant of strong interaction between nucleons is far larger than the fine structure constant in quantum electrodynamics (QED). The ground state of the nuclear matter or the finite nuclei is often defined as "vacuum", where the Fermi sea is filled with interacting nucleons, and no anti-nucleons and "holes" exist.

In 1970’s, in order to solve the nuclear many-body problems with RQFT, Walecka et al. developed the quantum hadrodynamics theory (QHD)[1, 2]. It is an important progress in nuclear physics. Since then, the QHD theory has widely been used in nuclear physics and nuclear astrophysics[3, 4]. However, in the earliest QHD theory, the resultant compression modulus is almost 550MeV[1], which is far from the experimental data range of 200 − 300MeV. To solve this problem, the nonlinear self-coupling terms among $\sigma$ mesons were introduced in addition to the mass term $\frac{1}{2}m_\sigma^2\sigma^2$[5],

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \quad (1)$$

Moreover, a self-coupling term of the vector meson

$$\frac{1}{4}c_3(\omega^\mu\omega_\mu)^2$$

is added to produce the proper equation of state of nuclear matter[6, 7]. No doubt, additional parameters would give more freedoms to fit the saturation curve of nuclear matter. Zimanyi and Moszkowki developed the derivative scalar coupling model yielding a compression modulus of 225MeV without any additional parameter[8]. Because all of these models include the nonlinear density-dependent terms in the Lagrangian in substance, the proper value of the compression modulus can be obtained.

In the relativistic quantum field theory, the nonlinear density-dependent terms in the Lagrangian represent higher-order corrections. In this paper, we would calculate the self-energies of virtual mesons with zero time-space momentum in the framework of the relativistic mean-field approximation, and then the Debye screening masses of the mesons would be discussed. Finally, we would replace the masses of the mesons with their corresponding screening masses in the Walecka-1 model, and develop a density-dependent relativistic
mean-field theory without additional parameters, which is similar to the density-dependent quark-meson coupling model, i.e. quark-meson coupling model-2(QMC-2)\cite{9}.

II. THE SCREENING MESON MASSES IN THE RELATIVISTIC MEAN-FIELD APPROXIMATION

the Lagrangian density in nuclear system can be written as

\[ \mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\text{Int}}. \]  

(2)

In this expression, the Lagrangian density for the free nucleon field can be described by:

\[ \mathcal{L}_N = \bar{\psi} (i \gamma_\mu \partial^\mu - M_N) \psi, \]

(3)

where \( \psi \) is the field of the nucleon and \( M_N \) is the bare mass of the nucleon. The free Lagrangian densities for the \( \sigma, \omega, \rho \) meson fields and the photon field can be expressed by:

\[ \mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2, \]

(4)

\[ \mathcal{L}_\omega = -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu, \]

(5)

\[ \mathcal{L}_\rho = -\frac{1}{4} \vec{R}_{\mu\nu} \cdot \vec{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu, \]

(6)

\[ \mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]

(7)

where

\[ \omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \]

(8)

\[ \vec{R}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu, \]

(9)

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

(10)

are corresponding field tensors, respectively. The interactive Lagrangian density can be written as

\[ \mathcal{L}_{\text{Int}} = -g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi \]

\[ -g_\rho \bar{\psi} \gamma^\mu \frac{\vec{r}}{2} \cdot \vec{\rho}_\mu \psi - e \bar{\psi} \gamma_\mu \frac{1 + \gamma_3}{2} A^\mu \psi \]

(11)

with \( \vec{r} \) being the Pauli matrix.
In the framework of relativistic mean-field approximation, the meson fields operators and the electromagnetic field operator can be replaced by their expectation values in the nuclear matter [2]:

\[ \sigma \rightarrow \langle \sigma \rangle = \sigma_0, \tag{12} \]
\[ \omega_\mu \rightarrow \langle \omega_\mu \rangle = \omega_0 \delta_\mu^0, \tag{13} \]
\[ \vec{\rho}_\mu \rightarrow \langle \vec{\rho}_\mu \rangle = \rho_0 \delta_\mu^0, \tag{14} \]
\[ A_\mu \rightarrow \langle A_\mu \rangle = A_0 \delta_\mu^0, \tag{15} \]

where \( \sigma_0 \) is the expectation value of the scalar meson field operator, \( \omega_0 \) and \( \rho_0 \) are the expectation values of the time-like components of \( \omega \) meson and neutral \( \rho \) meson field operators, respectively, and \( A_0 \) denotes the scalar potential of the electromagnetic field in the nuclear system. The Lagrangian density for nuclear matter in the relativistic mean-field approximation then reads

\[
\mathcal{L}_{RMF} = \bar{\psi} (i \gamma_\mu \partial^\mu - M_N) \psi - g_\sigma \bar{\psi} \psi \sigma_0 - g_\omega \bar{\psi} \gamma_\mu \omega_\mu \psi - g_\rho \bar{\psi} \gamma_\mu \vec{\tau} \cdot \vec{\rho}_\mu \psi - \frac{1}{2} m^2 \sigma^2_\sigma - \frac{1}{2} m^2 \omega^2_\omega - \frac{1}{2} m^2 \rho^2_\rho. \tag{16} \]

In the nuclear matter, because of the long-range Coulomb repulsive interaction between protons, the nuclear matter would not be bound. As a model of nuclear matter, the electromagnetic interaction usually be ignored. However, in the finite nuclei, the electromagnetic interaction should be considered in the structure calculation.

The screening mass of the meson \( m^*_\alpha \) is defined from the static infrared limit \( k^0 = 0 \) and then \( k \to 0 \) as Ref. [10]

\[ m^*_\alpha = \sqrt{m^2_\alpha + \Sigma_\alpha (k = 0, \rho_p, \rho_n)}, \quad (\alpha = \sigma, \omega, \rho_0). \tag{17} \]

It represents the inverse Debye screening length and implies the long-distance correlations. In the following, we will calculate the self-energy of virtual mesons with \( k = 0 \) in the framework of relativistic mean-field approximation, and then the screening masses of the mesons and the photon are obtained.

The interactive perturbation Hamiltonian of the model can be expressed as

\[
\mathcal{H}_I = g_\sigma \bar{\psi} \sigma \psi + g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi + g_\rho \bar{\psi} \gamma_\mu \vec{\tau} \cdot \vec{\rho}_\mu \psi + e \bar{\psi} \gamma_\mu \frac{1}{2} A^\mu \psi, \tag{18} \]
and the S-matrix can be written as
\[ \hat{S} = \hat{S}_0 + \hat{S}_1 + \hat{S}_2 + \ldots, \] (19)
where
\[ \hat{S}_n = \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n T [\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_n)]. \] (20)

In the 2nd-order approximation, only
\[ \hat{S}_2 = \frac{(-i)^2}{2!} \int d^4x_1 \int d^4x_2 T [\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)]. \] (21)
should be calculated.

The nucleon field operator \( \psi(x) \) and its conjugate operator \( \bar{\psi}(x) \) can be expanded in terms of a complete set of solutions of the Dirac equation:
\[
\begin{align*}
\psi(x, t) &= \sum_{\eta=1,2} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \sqrt{\frac{M_N^*}{E^*(p)}} \\
&\quad \left( A_{p\lambda} U_{\eta}(p, \lambda) \exp \left( i\vec{p} \cdot \vec{x} - i\varepsilon^{(+)}(p) t \right) + B^\dagger_{p\lambda} V_{\eta}(p, \lambda) \exp \left( -i\vec{p} \cdot \vec{x} - i\varepsilon^{(-)}(p) t \right) \right),
\end{align*}
\] (22)
\[
\begin{align*}
\bar{\psi}(x, t) &= \sum_{\eta=1,2} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3/2} \sqrt{\frac{M_N^*}{E^*(p)}} \\
&\quad \left( A^\dagger_{p\lambda} \bar{U}_{\eta}(p, \lambda) \exp \left( -i\vec{p} \cdot \vec{x} + i\varepsilon^{(+)}(p) t \right) + B_{p\lambda} \bar{V}_{\eta}(p, \lambda) \exp \left( i\vec{p} \cdot \vec{x} + i\varepsilon^{(-)}(p) t \right) \right),
\end{align*}
\] (23)
where \( U_{\eta}(p, \lambda) \) and \( V_{\eta}(p, \lambda) \) are Dirac spinors for the positive and negative energies, respectively, and
\[
\begin{align*}
\sum_{\lambda=1,2} U_{\eta}(p, \lambda) \bar{U}'_{\eta'}(p, \lambda) &= \left( \frac{\hat{p} + M_N^*}{2M_N^*} \right) \delta_{\eta\eta'},
\end{align*}
\] (24)
\[
\begin{align*}
\sum_{\lambda=1,2} V_{\eta}(p, \lambda) \bar{V}'_{\eta'}(p, \lambda) &= \left( \frac{\hat{p} - M_N^*}{2M_N^*} \right) \delta_{\eta\eta'},
\end{align*}
\] (25)
with \( \lambda \) and \( \eta \) denoting the spin and isospin for the nucleon, and \( M_N^* = M_N + g_\sigma \sigma_0 \) is the effective mass of the nucleon.
\[
\varepsilon^{(\pm)}(p) = \pm E^*(p) + g_\omega \omega_0 + g_\rho \frac{\tau_3}{2} \rho_0 + c \frac{1 + \tau_3}{2} A_0
\] (26)
with \( E^*(p) = \sqrt{p^2 + M_N^*} \), are the positive and negative energy eigenvalues for the Dirac equation of the nucleon in the nuclear matter, respectively. Assuming there are no antinucleons in the nuclear matter or finite nuclei, only positive-energy components are considered in Eqs. (22) and (23).

When a scalar meson of momentum \( k \) is considered in nuclear matter, the scalar meson field operator \( \sigma(k, x) \) can be expressed as

\[
\sigma(k, x) = a(k) \exp(-ik \cdot x) + a^\dagger(k) \exp(ik \cdot x). \tag{27}
\]

If the coupling constants \( g_\sigma, g_\omega \) and \( g_\rho \), the proton charge \( e \), and the masses of mesons are supposed have already been renormalized, the contribution of a single nucleon loop is not necessarily calculated\([11]\), only these contributions from the Feynman diagrams in Fig.1 should be considered.

The expectation value of \( \hat{S}_2 \) can be written as

\[
\langle k_2 | \hat{S}_2 | k_1 \rangle = -ig_\sigma^2(2\pi)^4 \delta^4(p_1 + k_1 - p_2 - k_2) \sum_{\eta=1,2} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) U_\eta(p, \lambda) \left( \frac{1}{\rho - \vec{K} - M_N^*} + \frac{1}{\rho + \vec{K} - M_N^*} \right) U_\eta(p, \lambda), \tag{28}
\]

where \( k_1 = k_2 = k \), and \( p_1 = p_2 = p = (E^*(p), \vec{p}) \), and \( \theta(x) \) is the step function.

Considering the diagrams in Fig.1, the scalar meson propagator \( G(k) \) in nuclear matter can be derived as

\[
G(k) = \frac{1}{(2\pi)^4 k^2 - m_\sigma^2 + i\varepsilon} + \frac{1}{(2\pi)^4 k^2 - m_\omega^2 + i\varepsilon} \sum_{\eta=1,2} \sum_{\lambda=1,2} (-ig_\sigma^2(2\pi)^4 \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) U_\eta(p, \lambda) \left( \frac{1}{\rho - \vec{K} - M_N^*} + \frac{1}{\rho + \vec{K} - M_N^*} \right) U_\eta(p, \lambda) \frac{1}{(2\pi)^4 k^2 - m_\sigma^2 + i\varepsilon} + \frac{1}{(2\pi)^4 k^2 - m_\omega^2 + i\varepsilon} \sum_{\eta=1,2} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) U_\eta(p, \lambda) \left( \frac{1}{\rho - \vec{K} - M_N^*} + \frac{1}{\rho + \vec{K} - M_N^*} \right) U_\eta(p, \lambda) \frac{1}{k^2 - m_\omega^2 + i\varepsilon}. \tag{29}
\]

According to the Dyson equation

\[
\frac{i}{k^2 - m_\sigma^2 - \Sigma_\sigma + i\varepsilon} = \frac{i}{k^2 - m_\sigma^2 + i\varepsilon} + \frac{i}{k^2 - m_\omega^2 + i\varepsilon} \Sigma_\sigma \frac{1}{k^2 - m_\sigma^2 + i\varepsilon}. \tag{30}
\]
we obtain the self-energy of the scalar meson in the nuclear matter

\[ \Sigma_{\sigma} = g^2_{\sigma} \sum_{\eta=1,2} \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) \]

\[ \bar{U}_\eta(p, \lambda) \left( \frac{1}{\bar{p} - \bar{k} - M_N^*} + \frac{1}{\bar{p} + \bar{k} - M_N^*} \right) U_\eta(p, \lambda), \quad (31) \]

where

\[ \sum_{\lambda=1,2} \bar{U}_\eta(p, \lambda) \left( \frac{1}{\bar{p} - \bar{k} - M_N^*} + \frac{1}{\bar{p} + \bar{k} - M_N^*} \right) U_\eta(p, \lambda) \]

\[ = \sum_{\lambda=1,2} Tr \left[ \left( \frac{1}{\bar{p} - \bar{k} - M_N^*} + \frac{1}{\bar{p} + \bar{k} - M_N^*} \right) U_\eta(p, \lambda) \bar{U}_\eta(p, \lambda) \right] \]

\[ = Tr \left[ \left( \frac{\bar{p} - \bar{k} + M_N^*}{(p-k)^2 - M_N^*} + \frac{\bar{p} + \bar{k} + M_N^*}{(p+k)^2 - M_N^*} \right) \frac{\bar{p} + M_N^*}{2M_N^*} \right]. \quad (32) \]

In the above deduction, Eq. (24) is used.

In order to obtain the Debye screening mass of the scalar meson, we calculate the self-energy of the scalar meson in the nuclear matter in the static infrared limit \( k^0 = 0 \) and then \( \vec{k} \to 0 \). Because the time-space momentum of virtual scalar meson is zero, \( k = 0 \), we have \( k^2 = 0 \). Considering \( p^2 = M_N^* \), the virtual scalar meson self-energy

\[ \Sigma_{\sigma} = g^2_{\sigma} \sum_{\eta=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) \]

\[ Tr \left[ \left( \frac{\bar{p} - \bar{k} + M_N^*}{-2p \cdot k} + \frac{\bar{p} + \bar{k} + M_N^*}{2p \cdot k} \right) \frac{\bar{p} + M_N^*}{2M_N^*} \right] \]

\[ = g^2_{\sigma} \sum_{\eta=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) Tr \left[ \frac{k}{p \cdot k} \cdot \frac{\bar{p} + M_N^*}{2M_N^*} \right] \]

\[ = \frac{g^2_{\sigma}}{M_N^*} \sum_{\eta=1,2} \frac{2}{(2\pi)^3} \int d^3p \frac{M_N^*}{E^*(p)} \theta(p_F - |\vec{p}|) \]

\[ = \frac{g^2_{\sigma}}{M_N^*} \left( \rho^p_S + \rho^n_S \right), \quad (33) \]

where \( \rho^p_S \) and \( \rho^n_S \) are the scalar densities of protons and neutrons with

\[ \rho^p_S = 2 \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{(p^2 + M_N^* \frac{1}{4})}, \quad (34) \]

\[ \rho^n_S = 2 \int \frac{d^3p}{(2\pi)^3} \frac{M_N^*}{(\vec{p}^2 + M_N^* \frac{1}{4})}. \quad (35) \]
Similarly, the self-energies of the $\omega$, neutral $\rho$ and $\gamma$ with $k = 0$ may be obtained:

\[
\Sigma_\omega = \frac{g_\omega^2 (\rho_\rho^p + \rho_\rho^n)}{2M_N^*},
\]

(36)

\[
\Sigma_\rho = \frac{g_\rho^2 (\rho_\rho^p + \rho_\rho^n)}{8M_N^*},
\]

(37)

\[
\Sigma_A = \frac{e^2 \rho_S^p}{2M_N^*}.
\]

(38)

According to Eq. (17), the Debye screening masses of the $\sigma$, $\omega$ neutral $\rho$ and the photon can be written as

\[
m^*_\sigma(\rho_N, f_p) = \sqrt{m^2_\sigma + \Sigma_\sigma},
\]

(39)

\[
m^*_\omega(\rho_N, f_p) = \sqrt{m^2_\omega + \Sigma_\omega},
\]

(40)

\[
m^*_\rho(\rho_N, f_p) = \sqrt{m^2_\rho + \Sigma_\rho},
\]

(41)

\[
m^*_A(\rho_N, f_p) = \sqrt{\Sigma_A} = \sqrt{\frac{e^2 \rho_S^p}{2M_N^*}}.
\]

(42)

They are also the functions of the density of nuclear matter $\rho_N$ and the proton fraction $f_p = Z/A$.

In the relativistic mean-field approximation,

\[
m^2_\sigma \sigma_0 = - g_\sigma (\rho_\rho^p + \rho_\rho^n),
\]

(43)

so the effective mass of the nucleon can be written as

\[
M^*_N = M_N + g_\sigma \sigma_0
\]

\[
= M_N - \frac{g_\sigma^2 (\rho_\rho^p + \rho_\rho^n)}{m^2_\sigma}.
\]

(44)

From Eq. (33), the screening mass of the scalar meson can be expanded as

\[
m^*_\sigma = m_\sigma \left( 1 + \frac{1}{2} \cdot \frac{g_\sigma^2 (\rho_\rho^p + \rho_\rho^n)}{m^2_\sigma} \frac{M^*_N}{M_N^*} + \ldots \right).
\]

(45)

To the first order of the scalar density of the nucleon,

\[
\frac{\delta m_\sigma}{m_\sigma} / \frac{\delta M_N}{M_N} = - \frac{1}{2} \cdot \frac{M_N}{M_N^*},
\]

(46)

where $\delta m_\sigma = m^*_\sigma - m_\sigma$, and $\delta M_N = M^*_N - M_N$. At the low density of the nuclear matter, $M^*_N \approx M_N$, and

\[
\frac{\delta m_\sigma}{m_\sigma} / \frac{\delta M_N}{M_N} \approx - \frac{1}{2}.
\]

(47)
Obviously, the fractional change of the mass of the scalar meson is $-1/2$ times that of the nucleon in the nuclear matter, and this simple value of the ratio is similar to the scaling relation in Ref. [3].

The ratio of the effective mass to the bare mass for the nucleon and the ratios of screening mass to the bare mass for mesons as the functions of the nucleon density for the symmetric nuclear matter are shown in Fig.2. It is seen that when the number density of nucleon increases, the effective mass of the nucleon decreases, and the screening masses of mesons increase. The increase of screening masses of the mesons means that as the density of the nuclear matter become dense, the range of nuclear forces decreases, and the Debye screening effect enhances in the denser nuclear matter. Our results on the screening masses of the mesons are different from the Brown-Rho scaling[12], in which the masses of the mesons are reduced in the nuclear matter, similarly to the reduction of the effective mass of the nucleon, as the density of the nuclear matter increases.

III. THE EQUATION OF STATE FOR NUCLEAR MATTER

The relativistic mean-field results may be derived by summing the tadpole diagrams self-consistently in nuclear matter, retaining only the contributions from nucleons in the filled Fermi sea in the evaluation of the self-energy and energy density[4], so the relativistic mean-field approximation is consistent to the relativistic Hartree approximation in the calculation of nuclear matter. Because the time-space momentum $k$ of virtual mesons in the tadpole diagrams is zero approximately, if the Debye screening effect is considered, the masses of mesons in the relativistic mean-field approximation should be replaced by their screening masses, respectively. Therefore, the total energy density and the pressure of nuclear matter can be deduced to

$$\varepsilon = \frac{1}{2} m_{\sigma}^2 \sigma_0^2 - \frac{1}{2} m_{\omega}^2 \omega_0^2 - \frac{1}{2} m_{\rho}^2 \rho_0^2 + \sum_{B=p,n} \varepsilon_B, \quad (48)$$

and

$$p = -\frac{1}{3} \sum_{B=p,n} \left(-\varepsilon_B + M_N^\pi \rho_S^B + g_\omega \omega_0 \rho_V^B + g_\rho \frac{\tau_3}{2} \rho_0 \rho_V^B\right)$$

$$-\frac{1}{2} m_{\sigma}^2 \sigma_0^2 + \frac{1}{2} m_{\omega} \omega_0^2 + \frac{1}{2} m_{\rho} \rho_0^2, \quad (49)$$
with
\[ \varepsilon_B = \frac{2}{(2\pi)^3} \int_0^{p_F(B)} dp \left( \left( \vec{p}^2 + M_N^2 \right)^{\frac{3}{2}} + \left( g_\omega \omega_0 + g_\rho \frac{\tau_3}{2} \rho_0 \right) \right), \] (50)

and the vector densities of protons and neutrons being
\[ \rho_V^p = 2 \int \frac{d^3p}{(2\pi)^3}, \] (51)
\[ \rho_V^n = 2 \int \frac{d^3p}{(2\pi)^3}, \] (52)

respectively.

By fitting the saturation properties of nuclear matter, the parameters of the relativistic mean-field approximation in which the mesons have density-dependent screening masses can be fixed
\[ \frac{g_\sigma^2}{m_\sigma^2} = 8.297 \text{fm}^2, \quad \frac{g_{\omega}^2}{m_{\omega}^2} = 3.683 \text{fm}^2, \quad \frac{g_{\rho}^2}{m_{\rho}^2} = 5.187 \text{fm}^2. \] (53)

With these parameters we obtain a saturation density of 0.149fm\(^{-3}\), a binding energy of 16.669 MeV, a compression modulus of 280.1 MeV, a symmetry energy coefficient of 32.8 MeV and an effective nucleon mass of 0.808M\(_N\) for the symmetric nuclear matter.

The saturation curves for the nuclear matter with different parameter sets are plotted in Fig.3. Comparing these results with each other, we see that when the density of the nucleon becomes larger than the saturation density of nuclear matter, the average energy per nucleon with our model parameter set increases more slowly than those with NL3\([13]\), NLSH\([14]\) and TM1\([7]\). It manifests that the equation of state for nuclear matter in our model is softer than those in the other models, because the mesons have density-dependent screening masses.

Due to the approximate chiral symmetry restoration in nuclear matter, the effective nucleon mass is reduced. Brown-Rho scaling would imply a similar reduction in the mass of all the mesons except the pion. However, this scaling would not lead to reasonable equation of state for the nuclear matter. Our model gives different results with Brown-Rho scaling, and can fit the correct saturation properties of nuclear matter in the framework of relativistic mean-field approximation.

In the nuclear matter, The screening mass of photon is only related to the scalar density of the proton, but not the momentum of the photon. In a symmetric nuclear matter, if one takes a nucleon number density of 0.16fm\(^{-3}\), and the bare nucleon mass, the resultant
screening mass of photon is about $5.42\,\text{MeV}$ \textsuperscript{13}. Although the photon gains an screening mass in the nuclear matter, the range of Coulomb repulsive force is still large enough, and the nuclear matter would not be bound as the electromagnetic interaction is considered. In the case of finite nuclei, the contribution from the photon mass term should be included in the relativistic Hartree approximation or relativistic mean-field approximation.

In nuclear matter, the screening masses of the mesons increase with the density of the nuclear matter. This result is equivalent to the statement that the coupling constants decrease with the increasing number density of nucleon when the masses of mesons retain constant. At this point, our model is consistent to the model in Ref. \textsuperscript{16}.

**IV. SUMMARY**

In summary, we calculate the self-energies of virtual mesons with zero time-space momentum in the relativistic mean-field approximation, and the Debye screening masses of the mesons are obtained, which are the functions of the scalar densities of protons and neutrons. The screening masses of the mesons increase with the density of the nucleon. It shows different results with Brown-Rho scaling. Replacing the masses of mesons with corresponding screening masses in the relativistic mean-field approximation, we obtain a relativistic density-dependent nuclear model. In this model, the nonlinear self-coupling terms of mesons are not needed and only three model parameters are required. With this model, five saturation properties of symmetric nuclear matter, the saturation density, the binding energy, the effective nucleon mass, the compression modulus and the asymmetric energy, are calculated. In denser nuclear matter, the equation of state with this model is softer than those of previous models. This implies that the dense matter in the core of neutron stars or the center of the relativistic heavy ion collision might be described correctly.

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\textsuperscript{[1]} J. D. Walecka, Ann. Phys.(N.Y.)83 (1974)491
[2] B. D. Serot, J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1.

[3] P. Ring, Prog. Part. Nucl. Phys. 37 (1996) 193.

[4] N. K. Glendenning, Compact Star: Nuclear Physics, Particle Physics, and General Relativity, (Springer-Verlag, New York, 1997)

[5] J. Boguta, A. R. Bodmer, Nucl. Phys. A292 (1977) 413

[6] A. R. Bodmer, Nucl. Phys. A526 (1991) 703

[7] Y. Sugahara, H. Toki, Prog. Theor. Phys., 92 (1994) 803

[8] J. Zimanyi, S. A. Moszkowski, Phys. Rev. C 42 (1990) 1416

[9] K. Saito, K. Tsushima and A. W. Thomas, Phys. Rev. C 55 (1997) 2637.

[10] S. Gao, Y. J. Zhang, R. K. Su, Phys. Rev. C 52 (1995) 380

[11] J. D. Bjorken, S. D. Drell, Relativistic Quantum Mechanics, (Mc Graw-Hill Book Company, 1964).

[12] G. E. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720.

[13] G. A. Lalazissis, J. Koenig, P. Ring, Phys. Rev. C 55 (1997) 540.

[14] M. M. Sharma, M. A. Nagarajan, P. Ring, Phys. Lett. B 312 (1993) 377.

[15] B. X. Sun et al., nucl-th/0204013.

[16] R. Brockmann, H. Toki, Phys. Rev. Lett. 68 (1992) 3408; H. Shen, Y. Sugahara, H. Toki, Phys. Rev. C 55, (1997) 1211.
FIG. 1: Feynman diagrams for the meson or the photon self-energy in nuclear matter, while 1 and 2 denote particles of the initial state, 3 and 4 denote particles of the final state.
FIG. 2: The ratios of the effective or screening mass to the bare mass $M^*/M$ for the nucleon and the mesons as a function of the nucleon density $\rho_N$ in the symmetric nuclear matter in this model.

FIG. 3: Average energy per nucleon $\varepsilon/\rho_N - M_N$ as a function of nucleon density $\rho_N$ with the different parameter sets. The solid line denotes the curve obtained in this model, the dash-dot line is for the NL3 parameters[13], the dash line is for the NLSH parameters[14], and the dot line is for the TM1 parameters[7].