A new approach to quarkonium polarization studies

Pietro Faccioli\textsuperscript{1)}, Carlos Lourenço\textsuperscript{2)}, and João Seixas\textsuperscript{1,3)}

Abstract

Significant progress in understanding quarkonium production requires improved polarization measurements, fully considering the intrinsic multidimensionality of the problem. We propose a frame-invariant formalism which minimizes the dependence of the measured result on the experimental acceptance, facilitates the comparison with theoretical calculations, and provides a much needed control over systematic effects due to detector limitations and analysis biases. This formalism is a direct and generic consequence of the rotational invariance of the dilepton decay distribution and is independent of any assumptions specific to particular models of quarkonium production.

\textsuperscript{1)} Laboratório de Instrumentação e Física Experimental de Partículas (LIP), Lisbon, Portugal
\textsuperscript{2)} CERN, Geneva, Switzerland
\textsuperscript{3)} Physics Department, Instituto Superior Técnico (IST), Lisbon, Portugal
Quarkonium polarization measurements should provide key information for the understanding of quantum chromodynamics (QCD) [1], with the competing mechanisms dominating in the different theoretical approaches to quarkonium production leading to very different predictions. In particular, non-relativistic QCD calculations [2], dominated by colour-octet components, predict that, at Tevatron energies and asymptotically high $p_T$, the directly produced $J/\psi$ and $\psi'$ mesons should be almost fully transversely polarized (angular momentum component $J_z = \pm 1$) with respect to their own momentum direction (helicity frame), while NLO calculations of colour-singlet quarkonium production [3] indicate a strong longitudinal ($J_z = 0$) polarization component.

The present experimental knowledge on quarkonium polarization is also contradictory and puzzling. The slightly longitudinal prompt-$J/\psi$ polarization measured by CDF [4] along the helicity axis ($J/\psi$ momentum direction in the center-of-mass of the collision system) is in clear disagreement with the expectations of all existing models. The fact that these recent measurements disagree with the results previously published by the same experiment [5] adds further confusion to the picture. Equally disturbing is the lack of continuity between fixed-target and collider results [6]. Bottomonium polarization should be easier to interpret theoretically, given that the heavier bottom quark mass should satisfy the non-relativistic approximation much better than the $c\bar{c}$ case. However, the $\Upsilon(1S)$ Tevatron measurements [7], available in the helicity frame and extending to $p_T$ values around 15 GeV/c, are contradictory: CDF indicates unpolarized production; D0 indicates longitudinal polarization. The discrepancy between the two results cannot be justified by their different rapidity windows. At lower energy and $p_T$, E866 [8] has shown yet a different polarization pattern: the $\Upsilon(2S)$ and $\Upsilon(3S)$ states have maximal transverse polarization with respect to the direction of motion of the colliding hadrons (Collins-Soper frame [9]), with no significant dependence on transverse or longitudinal momentum, while the $\Upsilon(1S)$ is only weakly polarized, indicating a dominant role of feed-down contributions.

To clarify this intricate situation, improved measurements are needed. So far, most experiments have presented results based on a fraction of the physical information derivable from the data: only one polarization frame is used and only the polar projection of the decay angular distribution is studied. These incomplete results prevent model-independent physical conclusions. Moreover, such partial descriptions of the observed physical processes reduce the chances of detecting possible biases induced by insufficiently controlled systematic effects. The forthcoming LHC measurements, in particular, would benefit from an improved formalism. In Ref. [6] we emphasized the importance of approaching the polarization measurement as a multidimensional problem, determining the full angular distribution in more than one frame. In this letter we show the existence of a frame-independent relation among the observable parameters of the dilepton decay angular distribution. The determination of invariant quantities facilitates the comparison between measurements, and with theory, reducing the kinematic dependence of the results to their intrinsic (and physically relevant) component. Invariant relations can also be used to perform self-consistency checks which can expose unaccounted systematic effects or eventual biases in the experimental analyses, a crucial advantage given the challenging nature
of quarkonium polarization measurements due, in particular, to the difficult subtraction of the spurious kinematic correlations induced by the detector acceptance.

We begin with preliminary considerations on the kinematics of the dilepton decay of inclusively produced vector mesons. The most general observable distribution is

\[ W(\cos \vartheta, \varphi) \propto \frac{1}{(3 + \lambda_\vartheta)} (1 + \lambda_\vartheta \cos^2 \vartheta + \lambda_\varphi \sin^2 \vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos \varphi), \]

where \( \vartheta \) and \( \varphi \) are the (polar and azimuthal) angles formed by the positive lepton with, respectively, the polarization axis \( z \) and the production plane \( xz \) (containing the directions of the colliding particles and of the decaying meson). All experimentally definable polarization axes belong to the production plane. A transformation from one observation frame (A) to another (B) is a rotation around the \( y \) axis by a \( \delta \) angle, the parameters changing as:

\[ \lambda_\vartheta^{(B)} = \frac{\lambda_\vartheta^{(A)} - 3\Lambda}{1 + \Lambda}, \quad \lambda_\varphi^{(B)} = \frac{\lambda_\varphi^{(A)} + \Lambda}{1 + \Lambda}, \]

\[ \lambda_{\vartheta\varphi}^{(B)} = \frac{\lambda_{\vartheta\varphi}^{(A)} \cos 2\delta - \frac{1}{2} (\lambda_\vartheta^{(A)} - \lambda_\varphi^{(A)}) \sin 2\delta}{1 + \Lambda}, \]

with \( \Lambda = \frac{1}{2} (\lambda_\vartheta^{(A)} - \lambda_\varphi^{(A)}) \sin^2 \delta - \frac{1}{2} \lambda_{\vartheta\varphi}^{(A)} \sin 2\delta. \)

Since the magnitude of the “polar anisotropy”, \( \lambda_\vartheta \), never exceeds 1 in any frame, we deduce the frame-independent inequalities

\[ |\lambda_\varphi| \leq \frac{1}{2} (1 + \lambda_\vartheta), \quad |\lambda_{\vartheta\varphi}| \leq \frac{1}{2} (1 - \lambda_\vartheta), \]

which imply the bounds \( |\lambda_\varphi| \leq 1 \) and \( |\lambda_{\vartheta\varphi}| \leq 1 \). More interestingly, we can see that \( |\lambda_\varphi| \leq 0.5 \) when \( \lambda_\vartheta = 0 \) and must vanish when \( \lambda_\vartheta \to -1 \).

In general, the transformation of the polarization parameters explicitly depends on the production kinematics. Considering, for example, the Collins-Soper (CS) and helicity (HX) frames, the angular terms in Eq. 2 are

\[ \sin^2 \delta_{HX \to CS} = \sin^2 \delta_{CS \to HX} = \frac{p_T^2 E^2}{p^2(m^2 + p_T^2)}, \]

\[ \sin 2\delta_{HX \to CS} = -\sin 2\delta_{CS \to HX} = \frac{2 m p_T p_L E}{p^2(m^2 + p_T^2)}, \]

where \( m, E, p, p_T \) and \( p_L \) are, respectively, the mass, the energy and the total, transverse and longitudinal momenta of the meson in the center-of-mass of the collision. As a result, the observed quarkonium polarization has, in general, an “extrinsic”, frame-related, kinematic dependence, superimposed on the “intrinsic” physical dependence due to the properties of the production processes and their varying mixture.

Such an extrinsic dependence can introduce artificial differences between the polarization results obtained by experiments probing different acceptance windows. Figure [1] shows how natural \( \Upsilon \) polarizations \( \lambda_\vartheta = +1 \) and \( -1 \) in the CS frame (with
Figure 1: Kinematic dependence of the $\Upsilon(1S)$ decay angular distribution seen in the HX frame, for natural polarizations $\lambda_\varphi = +1$ (a-b-c) and $\lambda_\varphi = -1$ (d-e-f) in the CS frame. The curves correspond to different rapidity intervals; from the solid line: $|y| < 0.6$ (CDF), $|y| < 0.9$ (ALICE), $|y| < 1.8$ (D0), $|y| < 2.5$ (ATLAS and CMS), $2 < |y| < 5$ (LHCb). For simplicity the event populations were generated flat in rapidity. The sign of $\lambda_\varphi \varphi$ depends on the definition of the $y$ axis of the polarization frame, here taken as sign($p_L$)($\vec{P}_1 \times \vec{P}_2$)/|$\vec{P}_1 \times \vec{P}_2$|, where $\vec{P}_1,2$ are the momenta of the colliding particles in the meson’s rest frame.

$\lambda_\varphi = \lambda_\varphi \varphi = 0$ and no intrinsic kinematic dependence) translate into different $p_T$-dependent polarizations measured in the HX frame in different rapidity acceptances. In this simple case, a common choice of the “natural” frame (CS) by all experiments would avoid such a misleading differentiation of results. In general, however, it may be impossible to find one frame providing a simple representation of the quarkonium polarization scenario. This is shown in Fig. 2 where we consider, for illustration, that 60% of the $\Upsilon$ events have natural polarization $\lambda_\varphi = +1$ in the CS frame and the remaining fraction has $\lambda_\varphi = +1$ in the HX frame. While the polarizations of the two event subsamples are intrinsically independent of the production kinematics, in neither frame will measurements performed in different transverse and longitudinal momenta windows find identical results.
Figure 2: Kinematic dependence of the $\Upsilon(1S)$ decay angular distribution seen in the HX (a-b-c) and CS (d-e-f) frames, when 60% (40%) of the events have full transverse polarization in the CS (HX) frame. The curves represent measurements in different acceptance ranges, as detailed in Fig. 1.

Also the comparison between experimental data and theory must consider the experimental acceptance and efficiency. Experiments measure the net polarization of the specific cocktail of quarkonium events accepted by the detector, trigger and analysis cuts. If the polarization depends on the kinematics, the measured angular parameters depend on the effective population of collected events in the probed phase space window. Two experiments covering the same kinematic interval may find different average polarizations if they have different acceptance shapes in that range. The problem can be solved by presenting the results in narrow intervals of the probed phase space. Similarly, theoretical calculations of the average polarization in a certain experiment should consider how the momentum distribution is distorted by its acceptance. Alternatively, the prediction should avoid as much as possible kinematic integrations or provide event-level information to be embedded into Monte Carlo simulations of the experiments.

The general frame-transformation relations in Eq. 2 imply the existence of an invariant quantity, definable in terms of $\lambda_\theta$, $\lambda_\varphi$ and $\lambda_{\theta\varphi}$, in one of the following
equivalent forms:
\[
\mathcal{F}_{\{c_i\}} = \frac{(3 + \lambda_\theta) + c_1(1 - \lambda_\phi)}{c_2(3 + \lambda_\theta) + c_3(1 - \lambda_\phi)}.
\] (5)

An account of the fundamental meaning of the frame-invariance of these quantities can be found in Ref. [10]. We will consider here, specifically, the form

\[
\tilde{\lambda} \equiv \mathcal{F}_{\{-3,0,1\}} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}.
\] (6)

In the special case when the observed distribution is the superposition of \(n\) “elementary” distributions of the kind \(1 + \lambda_\theta^{(i)}\cos^2 \vartheta\), with event weights \(f^{(i)}\), with respect to \(n\) different polarization axes, \(\tilde{\lambda}\) represents a weighted average of the \(n\) polarizations, made \textit{irrespectively} of the orientations of the corresponding axes:

\[
\tilde{\lambda} = \frac{\sum_{i=1}^{n} f^{(i)} \lambda_\theta^{(i)}}{\sum_{i=1}^{n} f^{(i)}}.
\] (7)

The determination of an invariant quantity is immune to “extrinsic” kinematic dependencies induced by the observation perspective and is, therefore, less acceptance-dependent than the anisotropy parameters \(\lambda_\theta, \lambda_\phi, \lambda_\theta\phi\). For instance, in the case illustrated in Fig. 2 as well as in the simpler case of Fig. 1(a-b-c), any arbitrary choice of the experimental observation frame will always yield the value \(\tilde{\lambda} = 1\), independently of kinematics. This particular case, where all contributing processes are transversely polarized, is formally equivalent to the Lam-Tung relation [11], as discussed in Ref. [10]. Analogously, the example represented in Fig. 1(d-e-f), or any other case where all polarizations are longitudinal, yields \(\tilde{\lambda} = -1\).

The existence of frame-invariant parameters also provides a useful instrument for experimental analyses. Checking, for example, that the same value of an invariant quantity (Eq. 5) is obtained (within systematic uncertainties) in two distinct polarization frames is a non-trivial verification of the absence of unaccounted systematic effects. In fact, detector geometry and/or data selection constraints strongly polarize the reconstructed dilepton events. Background processes also affect the measured polarization, if not well subtracted. The spurious anisotropies induced by detector effects and background do not obey the frame transformation rules characteristic of a physical \(J = 1\) state. If not well corrected and subtracted, these effects will distort the shape of the measured decay distribution differently in different polarization frames. In particular, they will violate the frame-independent relations between the angular parameters. Any two physical polarization axes (defined in the rest frame of the meson and belonging to the production plane) may be chosen to perform these tests. The HX and CS frames are ideal choices at high \(p_T\), where they tend to be orthogonal to each other (in Eq. 4, \(\sin^2 \delta \to 1\) for \(p_T \gg m\)). At low \(p_T\), where the difference between the two frames vanishes, any of the two and its exact orthogonal may be used to maximize the significance of the test. Given that \(\tilde{\lambda}\) is “homogeneous” to the anisotropy parameters, the difference \(\tilde{\lambda}^{(B)} - \tilde{\lambda}^{(A)}\) between the results obtained in two frames provides a direct evaluation of the level of systematic errors not accounted in the analysis.

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We conclude with a summary of our messages. Choosing a given polarization axis in an experimental analysis or theoretical calculation has a radical effect on the magnitudes and signs of the polar and azimuthal anisotropies observed in the dilepton decay distribution: all terms of the distribution must be determined to provide unambiguously interpretable physical information. Rotational invariance imposes frame-invariant constraints on the polar and azimuthal anisotropy parameters and, for any mixture of production mechanisms in a given kinematic condition, there exists an invariant relation depending on one frame-independent parameter. Measurements of the anisotropy parameters are necessarily affected by the fact that the transformation from one frame to another is an explicit function of the production kinematics. These effects may result in a misleading interpretation of the measurements. Reporting polarization results in terms of invariant quantities facilitates the comparison between different measurements, and with theory, reducing the kinematic dependencies to their intrinsic (and physically most relevant) component. The invariant relation can also be used in the data analyses to perform self-consistency checks which can expose unaccounted detector effects or eventual biases.

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