Structural Transformation of Bilayer Ferro-foams Caused by Homogeneous Static Magnetic Field

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Abstract
Structural transformation of honeycomb structure bilayer ferro-foams caused by homogeneous magnetic field is investigated in this paper. Previous researchers have proved that monodisperse bilayer foams with honeycomb structure can transform reversibly by changing foam liquid fraction. However, we have observed that, when applying a homogeneous magnetic field, bilayer foams can also perform this transformation at a constant liquid fraction. This self-organizational behavior of foam structure is believed to be based on energy minimization principle, the magnetic interaction of Ferro-foams Plateau borders change the total energy of the whole system, which trigger the structure transformation process.

Keywords: Honeycomb Structure; Magnetic interactive potential energy; Ferro-Foams.

Introduction
Monodisperse bilayer foams are composed of series of identical units. There are two kinds of these units. One is honeycomb unit, which is the basic cell constituting honeycombs in nature, and it is once believed to have minimal surface area with given volume [1, 2]. The other is discovered by L.Fejes Tóth[3] in 1964. It is proved to be 0.35% less in surface area than that of honeycomb unit, and now named Tóth unit, as shown in Fig. 1. We call the monodisperse bilayer foams structure composed of honeycomb units honeycomb structure and Tóth units Tóth structure below. But no counterpart in nature can be found of the Tóth structure. Until 1994, Weaire and Phelan[4] find Tóth structure in dry bilayer soap foams for the first time, and also realize that Tóth structure transform to honeycomb structure by adding liquid to the dry foams. Researchers believe this structural transformation is a conduct of foam self-organization for minimizing system energy. Afterwards, Reinhard Höhler et al[5] analyze the energy of ordered monodisperse foam systems, and conclude an energy($E$)-liquid fraction($\Phi$) relation. It shows that, foams prefer Kelvin structure with low liquid fraction and fcc structure with high liquid fraction. There exit a critical transformation liquid fraction point between these two structures. It is believed that foam structures are chose by minimizing their energy. So, is it possible to realize structure transforms by exerting a kind of energy into the foam system while keep the liquid fraction constant?

Fig. 1: (a) Tóth unit, vertical distribution and a view of such units projected on a plane; (b) honeycomb unit, vertical distribution and a view of such units projected on a plane.

In this paper, we prepare monodisperse bilayer ferro-fluid foams at certain specific liquid fractions in Hele-Shaw cells. Then a vertical homogeneous magnetic field is imposed to observe the foam structural transformation behavior. Results show that foam unit cell changes from Tóth unit to honeycomb unit at a specific liquid fraction. And when removing the
field, foam restores to its initial Tóth unit. A possible explanation based on foams system energy analysis was given. This study opens possible applications in the field of metal foam preparation, but also as a model experimental system to study the structure controlling in the foam physics.

**Method**

The homogeneous magnetic field is generated by Helmholtz double coils\(^6\), as shown in Fig. 2. The liquid is ferro-fluid and the parameters concerned are shown in Table 1 below. Surfactant (Sodium dodecyl benzene sulfonate, SDBS) concentration is set at 0.4g/L to maintain foams stabilization.

| Carrier fluid | Color | Saturation magnetic susceptibility (A/m) | Density (kg/m\(^3\)) | Initial density magnetic susceptibility (A/m) | Surface tension (N/cm) |
|---------------|-------|------------------------------------------|----------------------|---------------------------------------------|-----------------------|
| water         | black | 15915                                    | 1180                 | 0.6                                         | 2.6×10\(^{-4}\)       |

The bilayer foams are prepared in a Hele-Shaw cell (30mm wide × 60mm high × 3mm thick). As illustrated in Fig. 2, a flat syringe needle is used to control bubble sizes. And it is connected to a gas flowmeter. In this experiment, inner diameter of the needle is 0.41mm, and gas flow is set at 5ml/min. In this case, the generated monodisperse bubbles\(^7\) will fill the Hele-Shaw cell in two layers. Then, the cell is placed horizontally on the symmetry plane of Helmholtz double coils which provides a homogeneous magnetic field. Experimental phenomena are recorded from the top by a digital camera.

![Fig. 2: Helmholtz double coils and foam preparation device system.](image-url)

![Table.1 Physical parameters of magnetic fluid](image-url)
Results & Discussions

In this experiment, bilayer foams at different liquid fractions are prepared by foam preparation device. Foam transformation critical value is found at about 0.28. The value of the liquid fraction is obtained from measurements of the volume of Hele-Shaw cell and the volume of containing ferro-fluid. This estimate may not be entirely accurate, but can at least serve as a nominal value for future research. Afterwards, we have observed foam structures at certain specific liquid fractions before or after applying a homogeneous magnetic field. And the results show that, when liquid fraction is below 0.19 (including 0.19), bilayer foams structure take the form of Tóth unit whether applying a homogeneous magnetic field or not. When liquid fraction comes up to 0.20, foams still remain as Tóth structures before applying a magnetic field. But, after applying the field, foams transform from Tóth structures to honeycomb structures. When removing the field, foams restore to their initial Tóth structures, which are shown in Fig. 3.

The total free energy is supposed to determine foam structures. So when applying homogeneous magnetic field to foam systems its effect should be taken into considerations on the transformation process. When magnetic substances are put in a magnetic field, they will be magnetized to generate induced magnetic fields [8-10]. After that, these induced fields will interact leading to system energy change, which can be considered as magnetic moment interaction. In our experiment, the magnetic substance is ferro-fluid, which is mainly existed in plateau borders. So, plateau borders can be viewed to be composed of dozens of moment units.

As to bilayer ferro-foams, they can be divided into three layers in vertical direction, which are upper plateau borders, lower plateau borders and middle interfacial plateau borders in Fig. 1. We just take the upper and lower parts into consideration when figuring out magnetic interactive potential energy, because the middle interfacial plateau borders occupy a low percentage in height. The total magnetic interactive potential energy consists of interactive energy of plateau borders in same layers and different layers. As to plateau borders in same layer, the interactive energy of these two types of foams are equal. However, when considering the energy generated by plateau borders in different layers, border spacing of Tóth foams is different from that of honeycomb, as shown in Fig. 4, which leads to the difference of magnetic interactive potential energy. And this is the basic reason that causes the difference of foam system energy.

Fig. 4: (a) top view of a Tóth unit and its longitudinal section; (b) top view of a honeycomb unit and its longitudinal section

Given to small interfacial part, we views it as a plane when analyzing average plateau border spacing in the longitudinal section of bilayer ferro-foams, as shown in Fig. 4. Average plateau border spacing is defined as the average distance \( d \) of two adjacent plateau borders in a regular hexagon prism bottom \( S \), which can be written as:

\[
\frac{n\pi}{2} \left( \frac{d}{2} \right)^2 = S
\]

Sides of foam units are obtained by image analysis. In our experiment, the side of a regular hexagon prism bottom is 0.5mm. Then, the average plateau border spacing \( d_{av} \) of Tóth foams is 0.45mm, and \( d_{av} \) is 0.53mm.

For any magnetic mass point \( dL_2 \) in plateau border \( L_2 \) and \( dL_1 \) in plateau border \( L_1 \). Supposing that, there is a magnetic moment \( \vec{m} \) at \( dL_2 \) and another magnetic moment \( \vec{m} \) at \( dL_1 \). These two magnetic moments will interact to generate magnetic interactive potential energy. Then the total magnetic interactive potential energy \( W_{L_1-L_2} \) of plateau border \( L_1 \) and \( L_2 \) can be solved. For Tóth structure foams, the energy between two plateau borders is

\[
W_{L_1-L_2} = -\frac{\mu_{m_1} m_2}{4\pi} \times 1.07
\]

And for honeycomb structure foams, the energy between two plateau borders is

\[
W_{L_1-L_2} = -\frac{\mu_{m_1} m_2}{4\pi} \times 0.81
\]

Let us note that among equation (2) and (3), the negative sign represents attractive energy. Obviously, the absorbed energy of Tóth structure foams is bigger than that of honeycomb structure foams in two plateau borders model. This is the same in the whole bilayer foam systems for plateau borders of different layers. Since the interactive energy of these two types of foams are equal in same layers, we can conclude Tóth structure foams absorb more magnetic interactive potential.
energy than honeycomb structure foams in whole bilayer foam systems, denoted $W_{TH} > W_{HC}$.

To stack bilayer foams in form of Tóth structure or honeycomb structures a question of energy. As shown in Fig. 5, the black curves are energy trends of these two structures as foam liquid fraction changes without magnetic field\[5\]. Since

Fig. 5: A predicted relation between system energy and bilayer foams liquid fraction after applying a homogeneous magnetic field (red curves) and a known relation between them without magnetic field (black curves).

foam system inclines to stay in a minimal energy, bilayer foams present different structures at different liquid fraction range. According to the black curves, bilayer foams will change their structures when foam liquid fraction above 0.28. However, when applying a homogeneous magnetic field, the foam at liquid fraction 0.20. Which should take the form of Tóth structure without magnetic field, transform into honeycomb structure. This phenomenon is novel but can be explained by foam system energy absorption. As introduced above, after applying a homogeneous magnetic field, the absorbed energy of bilayer Tóth structure foams is bigger than that of honeycomb structure foams. As shown in Fig. 5, the red curves present the foam system energy after applying a homogeneous magnetic field. Due to $W_{TH} > W_{HC}$, the energy curve of Tóth structure foams lifts higher than that of honeycomb structure foams, which results that the critical transformation liquid fraction shift left, as $\Phi'$ shown in Fig. 5. In our experiment, after applying a homogeneous magnetic field, bilayer foams at liquid fraction 0.19 do not change their structures, while bilayer foams at liquid fraction 0.20 change. We can conclude that, after applying a homogeneous magnetic field, the critical transformation liquid fraction is in the range of 0.19 to 0.20. Comparing to the critical transformation liquid fraction ($\Phi=0.28$) without the field, the transformation liquid fraction is 8-9% ahead.

**Conclusion**

Introducing a homogeneous magnetic field will lead to a structural transformation at a certain specific liquid fraction below the critical points, and this transformation can be explained by magnetic interactive potential energy generated by magnetic moments.

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