A process fault estimation strategy for non-linear dynamic systems

Marcin Pazera, Józef Korbicz
Institute of Control and Computation Engineering, University of Zielona Góra, Podgórna 50, 65-246 Zielona Góra, Poland
E-mail: {M.Pazera,J.Korbicz}@issi.uz.zgora.pl

Abstract. The paper deals with the problem of simultaneous state and process fault estimation for non-linear dynamic systems. Instead of estimating the fault directly, its product with state and the state itself are estimated. To derive the fault from the product, a simple algebraic approach is proposed. The estimation strategy is based on the quadratic boundedness approach. The final part of the paper presents an illustrative example concerning a laboratory multi-tank system. The real data experiments clearly exhibit the performance of the proposed approach.

1. Introduction
In the last decades, the problem of Fault Diagnosis (FD) [3, 17, 20, 11, 14, 27, 26] takes important part in the correct functioning of the industrial systems. It also plays a significant role in conditions monitoring of the plants. The FD issue can be divided into three tasks, i.e., detection, isolation and identification [11, 5]. Industrial systems consist of process dynamics, sensors and actuators. Thus, it leads to fault classes [3, 8, 28] such as process faults (or component faults), sensor faults and actuator faults. The fault estimation is also basis for the implementation of so-called Fault-Tolerant Control (FTC) [8, 19, 21, 16, 25, 29, 23, 22] which can be divided into passive and active one. The active FTC needs the fault estimation to calculate current control signal, and hence, to compensate the influence of occurred fault. In contrast to active one, the passive FTC is designed to be robust for predefined faults. The faults are treated as disturbances.

Among Fault Detection and Isolation (FDI) methodologies available in the literature, strategies such as minimum variance input and state estimator [9], sliding mode high-gain observers [24, 2], Kalman filter [13], adaptive estimation [32] and also $H_\infty$ approach [28, 18, 12, 4, 30, 15] draws considerable attention in the field of control engineering. However, there are no many FDI solutions dedicated to the problem of process fault estimation, especially for non-linear systems. Most of them are concentrated to the problem of sensor or actuator faults. The paper aims at providing a novel process fault estimation strategy for non-linear dynamic systems. The proposed strategy is based on Quadratic Boundedness (QB) [1, 6, 7] approach allowing the convergence analysis.

The paper is organized as follows: section 2 presents the problem formulation, subsequently in section 3 methodology proceeding due to design the observer is shown. Final part of the paper shows an illustrative example (section 4) wherein the algorithm has been implemented to the multi-tank system, and section 5 concludes the paper.
2. Preliminaries

Let us consider a non-linear discrete-time system:

\[ x_{k+1} = Ax_k + \sum_{i=1}^{n_f} A_{f,i} f_{i,k} x_k + Bu_k + g(x_k) + W_1 w_k, \] (1)
\[ y_k = C x_k + W_2 w_k, \] (2)

where \( x_k \in \mathbb{X} \subseteq \mathbb{R}^n \), \( y_k \in \mathbb{R}^m \), \( u_k \in \mathbb{R}^r \), \( f_k \in \mathbb{R}^{n_f} \) are the state, output, control input and process fault vectors, respectively and \( g(x_k) \) is the non-linear function which describes the behaviour of the system with respect to state. Note that \( A_{f,i} \) denote the distribution matrix of \( i \)-th process fault \( f_i \), i.e., it describes the way in which the fault influences the system matrix.

Moreover, \( W_1 \) and \( W_2 \) denote the noise distribution matrices and \( w_k \) expresses the exogenous disturbance vector. Furthermore, \( w_k \) can be split as follows \( w_k = [w_{1,k}^T, w_{2,k}^T]^T \) where \( w_{1,k} \) and \( w_{2,k} \) represent the process and measurement uncertainties, respectively.

The problem is to design an observer which makes it possible to simultaneous estimate the state and process fault. Throughout the paper, the following substitution is applied:

\[ z_{i,k} = f_i x_k, \] (3)

which leads to the new representation of the system:

\[ x_{k+1} = Ax_k + \sum_{i=1}^{n_f} A_{f,i} z_{i,k} + Bu_k + g(x_k) + W_1 w_k, \] (4)
\[ y_k = C x_k + W_2 w_k. \] (5)

Thus, the problem is to design an estimator that will be able to estimate \( z_{i,k} \) and \( x_k \) simultaneously. While the idea to estimate the fault is based on recovering it from \( z_{i,k} \) and \( x_k \). For the aim of further deliberations, let us consider the following assumptions:

**Assumption 1:**

\[ \varepsilon_{i,k} = z_{i,k+1} - z_{i,k}, \]
\[ \varepsilon_{i,k} \in \mathcal{E}_{\varepsilon,i}, \quad \mathcal{E}_{\varepsilon,i} = \{ \varepsilon : \varepsilon^T Q_{\varepsilon,i} \varepsilon \leq 1 \}, Q_{\varepsilon,i} > 0. \] (6)

**Assumption 2:**

\[ w_k \in \mathcal{E}_w, \quad \mathcal{E}_w = \{ w : w^T Q_w w \leq 1 \}, Q_w > 0. \] (7)

These assumptions are required for the subsequent fault estimation algorithm. First of them (Assumption 1) concerns the system behaviour in case of fault, which means that all real faults and states are bounded. Consequently, all \( z_{i,k} \) are bounded as well. The second one, i.e., Assumption 2, similarly as Assumption 1, states that the external disturbances are unknown, but bounded.

To handle the aforementioned problem of simultaneous estimation of the state \( x_k \) and \( z_{i,k} \), it is proposed to use the Process Fault Estimator (PFE) of the following structure:

\[ \hat{x}_{k+1} = A\hat{x}_k + \sum_{i=1}^{n_f} A_{f,i} \hat{z}_{i,k} + Bu_k + g(\hat{x}_k) + K(y_k - C\hat{x}_k), \] (8)
\[ \hat{z}_{i,k+1} = \hat{z}_{i,k} + L_i (y_k - C\hat{x}_k), \quad i = 1 \ldots s. \] (9)
Note that $i$th fault estimate obeys

$$\hat{z}_{j,k} = f^j_{i,k} \hat{x}_{j,k}, \quad i = 1, \ldots, s, \quad j = 1, \ldots, n. \quad (10)$$

Indeed, there is no single estimate $\hat{f}_{i,k}$ satisfying $n$ equations of the above form (10). Thus, rather than having one single estimate, a set of $n$ estimates will be obtained $\hat{f}^j_{i,k}, j = 1, \ldots, n$. Thus, the final step is to calculate the Mean Value (MV) of these estimates

$$\hat{f}_{i,k} = \frac{1}{n} \sum_{j=1}^{n} \hat{f}^j_{i,k}. \quad (11)$$

A complete scheme of the proposed estimation strategy is depicted in figure 1.

![Diagram of proposed approach](image)

**Figure 1.** Diagram of proposed approach

### 3. Observer design

The main goal of this section is to present the design strategy of the process fault estimator. The estimator should be robust to the process and measurement uncertainties. From (4), (5) and (8), the state estimation error can be described by

$$e_{k+1} = x_{k+1} - \hat{x}_{k+1}$$

$$= [A - KC] e_k + \sum_{i=1}^{n_f} A_f e_{z,i,k} + g(x_k) - g(\hat{x}_k) + [W_1 - KW_2] w_k, \quad (12)$$

where $e_{z,i,k} = z_{i,k} - \hat{z}_{i,k}$.

Subsequently, following the Differential Mean Value Theorem (DMVT) [31], it can be shown that

$$g(x_k) - g(\hat{x}_k) = M_x (x_k - \hat{x}_k), \quad (13)$$
Thus, equation (21) can be rewritten in an equivalent compact form expressed by
\[ M_x = \begin{bmatrix} \frac{\partial g_1(x)}{\partial x}(c_1) \\ \vdots \\ \frac{\partial g_n(x)}{\partial x}(c_n) \end{bmatrix}, \tag{14} \]

where \( c_1, \ldots, c_n \in \text{Co}(x_k, \hat{x}_k) \), \( c_i \neq x_k, c_i \neq \hat{x}_k \), \( i = 1, \ldots, n \). Following that all states are bounded in real system, \( x_k \in \mathbb{X} \) let
\[ \hat{x}_{i,j} \geq \frac{\partial g_i(x)}{\partial x_j} \geq x_{i,j}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n, \tag{15} \]

it is clear that there exist \( M_x \in \mathbb{M} \) such that
\[ M_x = \left\{ M \in \mathbb{R}^{n \times n} | \hat{x}_{i,j} \geq m_{x,i,j} \geq \underline{x}_{i,j}, \quad i, j = 1, \ldots, n \right\}. \tag{16} \]

Thus, the equation (12) can be rewritten as
\[ e_{k+1} = x_{k+1} - \hat{x}_{k+1} = [A - KC + M_x] e_k + \sum_{i=1}^{nf} A_f e_{z,i,k} + [W_1 - KW_2] w_k. \tag{17} \]

Moreover, the dynamics of the estimation error of the product of the state and the fault can be expressed by
\[ e_{z,i,k+1} = z_{i,k+1} + z_{i,k} - \hat{z}_{i,k+1} = \varepsilon_{i,k} + z_{i,k} - \hat{z}_{i,k+1} = \varepsilon_{i,k} + z_{i,k} - L_i C x_k - L_i W_2 w_k + L_i C \hat{x}_k = \varepsilon_{i,k} + e_{z,i,k} - L_i C e_k - L_i W_2 w_k, \tag{18} \]

where \( \varepsilon_{i,k} = z_{i,k+1} - z_{i,k}. \)

By constructing the following extended vectors:
\[ \bar{e}_{k+1} = \begin{bmatrix} e^T_{k+1} \\ e^T_{z,1,k+1} \\ \vdots \\ e^T_{z,nf,k+1} \end{bmatrix}, \tag{19} \]
\[ v_k = \begin{bmatrix} w^T_k \\ e^T_{1,k} \\ \vdots \\ e^T_{nf,k} \end{bmatrix}, \tag{20} \]

the state and fault estimation error dynamics can be formulated as
\[ \bar{e}_{k+1} = \begin{bmatrix} A - KC + M_x & A_{f,1} & A_{f,2} & \cdots & A_{f,n_f} \\ -L_1 C & I & 0 & \cdots & 0 \\ -L_2 C & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ L_{nf} C & 0 & 0 & 0 & I \end{bmatrix} \bar{e}_k + \begin{bmatrix} W_1 & 0 & 0 & \cdots & 0 \\ -L_1 W_2 & I & 0 & \cdots & 0 \\ -L_2 W_2 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -L_{nf} W_2 & 0 & 0 & \cdots & I \end{bmatrix} v_k. \tag{21} \]

Thus, equation (21) can be rewritten in an equivalent compact form
\[ \bar{e}_{k+1} = X \bar{e}_k + Z v_k, \tag{22} \]
Therefore, the ellipsoidal set bounding $v_k$ can be described by:

$$E_v = \{ v : v^T Q_v v \leq 1 \},$$

with

$$Q_v = \frac{1}{2 + n_f} \text{diag}(Q_w, Q_{e,1}, \ldots, Q_{e,n}, Q_w).$$

For the purpose of further deliberations, the so-called Quadratic Boundedness (QB) [1, 6, 7] approach can be use. The Lyapunov function can be formulated as follows

$$V_k = \bar{e}_k^T P \bar{e}_k,$$

with $P > 0$.

To introduce the QB approach, the following definitions are recalled [1]:

**Definition 1:** System (22) is strictly quadratically bounded for all allowable $v_k \in E_v$, if $\bar{e}_k^T P \bar{e}_k > 1$ implies $\bar{e}_{k+1}^T P \bar{e}_{k+1} < \bar{e}_k^T P \bar{e}_k$ for any $v_k \in E_v$.

It should be highlighted that the strict quadratic boundedness of (22) ensures that $V_{k+1} < V_k$ for any $v_k \in E_v$ when $V_k > 1$.

**Definition 2:** A set $E$ is a positively invariant set for (22) for all $v_k \in E_v$ if $\bar{e}_k \in E$ implies $\bar{e}_{k+1} \in E$ for any $v_k \in E_v$.

The results in [1] and the above definitions determine the following lemma for (22):

**Lemma 1:** The following statements are equivalent:

(i) The system (22) is strictly quadratically bounded for all $v_k \in E_v$ and $M_x \in \mathbb{M}$.

(ii) The ellipsoid

$$E = \{ e_k : e_k^T P e_k \leq 1 \},$$

is an invariant set for (22) for any $v_k \in E_v$.

(iii) There exists a scalar $\alpha \in (0,1)$ such that:

$$\begin{bmatrix}
X^T P X_x - P + \alpha P & X^T P Z \\
Z^T P X_x & Z^T P Z - \alpha Q_v
\end{bmatrix} \preceq 0.
$$

Note that the above lemma was initially developed for linear systems [1], and hence, it is straightforwardly extended to the class of systems being considered. As a consequence, the following theorem is proposed:
**Theorem 1**: The system (22) is strictly quadratically bounded for all \( v_k \in E_v \) and \( M_x \in M \) if there exist matrices \( P > 0, U \) and a scalar \( \alpha \in (0, 1) \), such that the following inequality is satisfied:

\[
\begin{bmatrix}
-P + \alpha P & 0 & A_x^T P - \bar{C}^T U^T \\
0 & -\alpha Q_v & 0 \\
P \bar{A}_x - U \bar{C} & PW - UV & -P
\end{bmatrix} \preceq 0.
\] (30)

**Proof**: Inequality (29) can be rewritten to the following form:

\[
\begin{bmatrix}
X^T P X & Z^T P Z \\
0 & -\alpha Q_v Z^T P
\end{bmatrix} \preceq 0.
\] (31)

Then, using the Schur complement and multiplying left and right side by \( \text{diag}(I, I, P) \) give

\[
\begin{bmatrix}
-P + \alpha P & 0 & A_x^T P - \bar{C}^T U^T \\
0 & -\alpha Q_v & 0 \\
PX_x & PZ & -P
\end{bmatrix} \preceq 0.
\] (32)

Introducing

\[
P X_x = P (\bar{A}_x - \bar{K} \bar{C})
\]

\[= P \bar{A}_x - P \bar{K} \bar{C} = P \bar{A}_x - U \bar{C},\] (33)

\[
P Z = PW - P \bar{K} - \bar{V} = PW - UV,
\] (34)

and substituting (33) and (34) into (32) completes the proof.

Finally, the design procedure brings down to solve (30) and then calculate

\[
\bar{K} = \begin{bmatrix}
K \\
L_1 \\
L_2 \\
\vdots \\
L_{nf}
\end{bmatrix} = P^{-1} U.
\] (35)

Solving (30) is equivalent to solve (for \( i = 1, \ldots, N \))

\[
\begin{bmatrix}
-P + \alpha P & 0 & A_x^T P - \bar{C}^T U^T \\
0 & -\alpha Q_v & 0 \\
P \bar{A}_x,i - U \bar{C} & PW - UV & -P
\end{bmatrix} \preceq 0,
\] (36)

and then determine

\[
\bar{K} = \begin{bmatrix}
K \\
L_1 \\
L_2 \\
\vdots \\
L_{nf}
\end{bmatrix} = P^{-1} U.
\] (37)

A complete design procedure of the Process fault estimator can be presented as follows:

(i) Select

\[Q_v = \frac{1}{2 + n_f} \text{diag} \left( Q_w, Q_{e,1}, \ldots, Q_{e,n_f}, Q_w \right),\]
(ii) Obtain \( U, P \) by solving (for \( i = 1, \ldots, N \))

\[
\begin{bmatrix}
-\bar{P} + \alpha P & 0 & \bar{A}_{x,i}^T P - \bar{C}^T U^T \\
0 & -\alpha Q_v W - \bar{V}^T U^T & -P
\end{bmatrix} \leq 0,
\]

\( P > 0, \ 0 < \alpha < 1 \)

(iii) Calculate

\[
\bar{K} = \begin{bmatrix}
K \\
L_1 \\
L_2 \\
\vdots \\
L_{n_f}
\end{bmatrix}
\]

4. Illustrative example
To verify proposed approach for the process fault estimation task, it was implemented for the multi-tank system which is depicted in figure 2. Such a system is designed for simulating the real industrial multi-tank system in the laboratory conditions. It can be regularly used to practically examine both, linear and non-linear control, identification and diagnostics methods. The considered system consists of three separate tanks which are placed each above other and equipped with drain valves and level sensors based on a hydraulic pressure measurement. Those tanks are differently shaped to reflect the nonlinearities of the system. The lower bottom tank is a water reservoir for the system. The multi-tank system is fitted with a DC water pump which is used to fill the upper tank. The water outflows from the tanks due to gravity. The multi-tank system exchanges data with the level sensors, it also communicates with valves and a pump with a PC-based digital controller through the dedicated I/O board and the power interface. Real time software is controlled by the I/O board with MATLAB/SIMULINK environment. For more information the reader is referred to [10]. The non-linear model of the multi-tank system can be presented as:

\[
\begin{align*}
\frac{dH_1}{dt} &= \frac{1}{\beta_1(H_1)} q - \frac{1}{\beta_1(H_1)} C_1 H_1^{\alpha_1}, \\
\frac{dH_2}{dt} &= \frac{1}{\beta_2(H_2)} C_1 H_1^{\alpha_1} - \frac{1}{\beta_2(H_2)} C_2 H_2^{\alpha_2}, \\
\frac{dH_3}{dt} &= \frac{1}{\beta_3(H_3)} C_2 H_2^{\alpha_2} - \frac{1}{\beta_3(H_3)} C_3 H_3^{\alpha_3},
\end{align*}
\]

where \( H_i, \beta_i(H_i), \alpha_i \) and \( C_i \) is the liquid level in \( i \)th tank, the surface area of the \( i \)th tank with the liquid level equal to \( H_i \), flow coefficient of the \( i \)th tank and outflow resistance of the \( i \)th tank, respectively. The parameters were obtained by identification with using real data as follows: \( C_1 = 1.0057 \cdot 10^{-4}, \ C_2 = 1.1963 \cdot 10^{-4}, \ C_3 = 9.8008 \cdot 10^{-5}, \alpha_1 = \alpha_2 = \alpha_3 = 0.5 \). The above mentioned equations which are describing the system were discretized using Euler method with sampling time \( T_s = 0.01[s] \). Furthermore, the output equation is characterised by

\[
C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},
\]

which means that liquid level in the second tank is unavailable during the experiment.
According to the methodology of the PFE design described in section 3, by solving the inequality (36) the following matrices were obtained:

\[
K = \begin{bmatrix}
1.0031 & -0.0000 \\
-0.0720 & 0.0000 \\
-0.0000 & 0.0679 \\
0.0000 & -0.0000 \\
0.0000 & 0.0003 \\
\end{bmatrix},
\]

(42)

\[
L = \begin{bmatrix}
0.002 & -0.0000 \\
-0.0000 & 0.0003 \\
0.0000 & 0.9739 \\
\end{bmatrix},
\]

(43)

for \( \alpha = 0.4 \). The parameter \( \alpha \) is chosen for the greatest value of the trace of matrix \( P \) as the best value. In figure 3 is presented the evolution of the trace.

Let us assume that the initial state for the system and observer are \( x_0 = [0.001, 0.001, 0.001] \), \( \hat{x}_0 = [0.01, 0.01, 0.01] \), respectively while the input is \( u_k = 5 \cdot 10^{-5} \).

Moreover, let us consider the following fault scenario

\[
f_k = \begin{cases}
-3 \cdot 10^{-4}, & 5000 \leq k \leq 10000, \\
0, & \text{otherwise},
\end{cases}
\]

(44)

with the fault distribution matrix

\[
A_f = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix},
\]

(45)

which can be perceived as a temporary leakage in the third tank.
All experimental results were performed exclusively with the real system. Figures 4–6 present the state for the first, second and third tank, respectively and their estimates. It can be seen that all states were estimated very well. The estimates are following the real states with high accuracy even in case of leakage from the third tank. The unmeasurable liquid level (second tank) was also estimated quite well. Taking into account that the level varies from 0 to 0.35m, the state estimation error (figure 5 on the right) which varies from 0.5 to 1.5cm should be viewed as highly satisfactory. Figure 7 depicts the product of the real state $x_3$ and the fault as well as its estimate. Figure 8 show the real fault and its estimate (left figure) and the fault estimation error (right figure). The obtained results clearly exhibit that the proposed approach can be successfully applied to the non-linear systems.

5. Conclusions
The main objectives of this paper was to propose the strategy to handle with the problem of simultaneous state and process fault estimation for a class of non-linear systems. At first, description of the systems which includes the process fault and adequate observer structure is...
Figure 5. State $x_2$ and its estimate $\hat{x}_2$ (left figure) and state estimation error (right figure) – second tank

Figure 6. State $x_3$ and its estimate $\hat{x}_3$ – third tank

Figure 7. Product of the state and fault $z_3$ and its estimate $\hat{z}_3$
Figure 8. Process fault $f$ and its estimate $\hat{f}$ (left figure) and fault estimation error (right figure)

presented. Subsequently, the design strategy of the robust observer, which takes into account uncertainties influences onto the system is shown. The proposed approach is based on Quadratic Boundedness strategy which allows the convergence analysis of the system. The design strategy is quite simple and at last it brings down to solve a set of linear matrix inequalities. The final part of the paper shows an illustrative example with an application to multi-tank system. The results were obtained with real system, exclusively. They show huge quality of the received estimates and confirm the correctness of the proposed approach.

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