Implementing General Gauge Mediation

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Abstract

Recently there has been much progress in building models of gauge mediation, often with predictions different than those of minimal gauge mediation. Meade, Seiberg, and Shih have characterized the most general spectrum which can arise in gauge mediated models. We discuss some of the challenges of building models of General Gauge Mediation, especially the problem of messenger parity and issues connected with R symmetry breaking and CP violation. We build a variety of viable, weakly coupled models which exhibit some or all of the possible low energy parameters.
INTRODUCTION

If evidence for low energy supersymmetry is discovered at the Tevatron or LHC, the most urgent task will be elucidating the superpartner spectrum. Gauge mediation is a leading candidate for the messenger of supersymmetry breaking. Gauge mediation has a number of virtues: it is flavor blind, and so accounts for the absence of flavor-changing processes at very low energies; it fits well with ideas about dynamical supersymmetry breaking; and, at least in its simplest forms, it is a highly predictive framework.

Gauge mediation is a well-studied subject, but two developments over the last few years have lead to a renewal of interest. First has been the recognition that theories in which supersymmetry is broken in metastable ground states, are common, even generic. This has greatly enlarged – and simplified – the possibilities for model building. Second has been the growing appreciation that supersymmetric models generally – and gauge mediated models in particular – must be tuned if they are to yield electroweak symmetry breaking with Higgs particles and superpartners consistent with experimental constraints.

Early models of gauge mediation did not invoke dynamical supersymmetry breaking. In their simplest version, so-called minimal gauge mediation (MGM), there was a singlet field, $X$, coupled to a set of messengers, filling out a 5 and $\bar{5}$ representation of $SU(5)$. The vacuum expectation value of the field $X$ had the form

$$\langle X \rangle = x + \theta^2 F_x$$

(1)

with couplings to messengers:

$$W = X \left( \lambda q \bar{q} + \lambda \ell \bar{\ell} \right).$$

(2)

The resulting sfermion spectrum is easily calculated:

$$\tilde{m}^2 = 2\Lambda^2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right],$$

(3)

where $\Lambda = F_x/x$. $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets. This formula predicts definite ratios of squark, slepton and gaugino masses. In the early models, O’Raifeartaigh-like couplings of $X$ to various fields were responsible for $F_x$. $R$ symmetries were typically broken explicitly, and so the models were not natural in the strictest sense.
The first well-studied models of dynamical supersymmetry breaking (DSB) possessed stable, non-zero energy vacua, before coupling to messengers. It was possible to construct models of minimal gauge mediation with such DSB, but the resulting theories were quite baroque. The usual strategy was to invoke one (or more!) hidden sectors, whose couplings to singlets like $X$ eventually yielded the desired structure. Typically, once messengers were included, the desired vacuum state was metastable. But only with the observation of of Intriligator, Shih and Seiberg that metastable, dynamical supersymmetry breaking is a generic phenomenon, was it appreciated that one should consider hidden sectors in which (even before coupling to messengers or MSSM fields) the supersymmetry-breaking vacuum is metastable. This has greatly expanded the possibilities for model building.

Even in this broader framework, there are model-building challenges. For models with stable vacua, there is a theorem due to Nelson and Seiberg that the underlying theory, if generic, must possess an R symmetry. In the metastable models, this requirement is relaxed to the requirement of an approximate R symmetry. This would seem an advantage for model building, since one does not have to account for spontaneous breaking of the symmetry. In the simplest ISS model, however, the low energy theory has an accidental, unbroken R symmetry. In the “retrofitted” models of, the simplest models also have such a symmetry at the level of the low energy effective theory. Shih has formulated a general theorem which suggests that this problem is challenging: in a weakly coupled theory without gauge interactions and with a global continuous R symmetry, the $R$ symmetry is never spontaneously broken if the fields all have R charge 0 or 2 (as in the simplest O’Raifeartaigh models). This result does not hold in the presence of gauge interactions, but the simplest models which exploit this loophole are not particularly attractive and are sometimes fine-tuned. This theorem can be circumvented, as shown in, when a hierarchy between masses can lead to two-loop effects that dominate the one-loop Coleman-Weinberg potential. Shih has exhibited models with fields with more exotic R charges (and without gauge interactions) in which the R symmetry is spontaneously broken.

But, as we will comment further below, these models often introduce new challenges. The authors of constructed retrofitted models without low energy R symmetries altogether. But these models required small couplings which, while technically natural, added additional complexity, and also raised the value of the underlying supersymmetry breaking scale.

Another challenge for gauge-mediated models involves the MGM spectrum itself. Eqn.
predicts definite ratios of squark, slepton and gaugino masses. Coupled with the current
limits on the lightest sleptons (approximately 100 GeV), it implies that slepton doublets
have masses greater than 215 GeV, while squark masses are larger than 715 GeV. If one
considers the top/stop contribution to $m_{H_U}^2$ (the coefficient of $|H_U|^2$ in the lagrangian),
this is logarithmically divergent. The cutoff, $M$, in gauge-mediated models, is of order
the messenger scale. Assuming a value of this scale of order the GUT scale, as in many
models[12, 13],

$$\delta m_{H_U}^2/M_Z^2 \approx 130;$$  

if $M = 10^2$ TeV, 130 is reduced to 21, still suggestive of a 5% fine tuning.

A number of models have been proposed recently in which the predictions of MGM do
not hold[11, 14, 15]. Meade, Shih and Seiberg[16] have provided a general framework for
considering gauge mediated models. First, they give a definition: a model is gauge mediated
if the couplings between the hidden and the visible sector vanish as the gauge couplings
tend to zero. As they stress, framing this definition raises interesting questions, especially
since, in a theory which is truly gauge-mediated in this sense, the $\mu$ term cannot arise from
supersymmetry-breaking dynamics. In the course of this paper, we will return to this and
other issues raised by this definition. Meade et al go on to characterize the most general
spectrum of gauginos and squarks and sleptons which can arise in this framework (“General
Gauge Mediation” or GGM). Setting aside a possible Fayet-Iliopoulos D term, there are, in
general, six parameters which characterize the low energy superparticle spectrum. We will
discuss existing models, some of their problems, and their parameter counting. We will then
extend these constructions, describing simple, weakly coupled theories with up to the full
set of six parameters.

Very often, in the metastable models with DSB, the low energy theory can be described
by a renormalizable theory without gauge interactions. Motivated by this, and by a desire for
simplicity and explicitness, Shih and collaborators have pursued a program of model building
with weakly coupled fields, without gauge interactions. The discussion above suggests the
following rules:

1. The model should possess an R symmetry.
2. There should be fields in the model with R charge not equal to zero or two.
3. There should be a rich enough structure that the predictions of minimal gauge mediation do not hold.

Within this framework, the authors of [15] generate a class of models with interesting features: an MGM spectrum for gauginos, but not for squarks and sleptons. But the models suffer from certain difficulties. Most important, unless certain parameters are tuned, there are one loop corrections to squark and slepton masses (squared) proportional to hypercharge. These are problematic parametrically, since gaugino masses are generated at one loop, and because some masses will be tachyonic (in the absence of tuning). One of the goals of the present paper is to explore these issues. We will consider ways to “fix” the models of [15] so as to obtain, automatically, an approximate symmetry (first noted in [4]) which can suppress these one loop contributions. This symmetry has been dubbed “messenger parity” in [17]. We will also introduce simpler classes of models which exhibit this feature automatically.

Another interesting feature of these constructions is the frequent appearance of “runaway” directions, directions in field space where, classically, the potential tends to zero for large fields. In the models of [15], these directions are separated from the state of interest by a barrier, but it is interesting to ask why these directions arise, and whether they can be useful for model building. We will see that the existence of runaway is common, and related to symmetries [15]. The argument will immediately indicate how such behavior can be avoided. In the models of [15], classically, there is a (pseudo) moduli space, separated, as we indicated, by a barrier from the runaway directions. On the moduli space, there is a point of unbroken R symmetry, and a Coleman-Weinberg calculation is required to determine whether or not the symmetry is broken. In our modified construction, there are branches of the moduli space on which the R symmetry is everywhere broken. A Coleman-Weinberg calculation is still required to determine the decay constant of the R axion and the precise spectrum of the model.

In the next section, we survey a number of existing approaches to model building. We review the results of [8] concerning spontaneous breaking of R symmetries in models with only gauge singlets, and introduce the problem of messenger parity. We explain, following [11], that a model with a single 5, 5 pair of messengers and several singlets with scalar and F term vev’s provides an example of GGM with two parameters describing both the gaugino and sfermion spectrum, with messenger parity automatic. We show that a model with the structure 10 + 10 gives a richer parameter set, with the possibility of significant
compression of the sfermion spectrum. We explain why, without imposing extra symmetries, one can obtain at most five parameters with this set of constructions, and describe simple models (with symmetries) with the full complement of GGM parameters. All of these models automatically possess an approximate messenger parity. In section we discuss the spectra of these models from a phenomenological viewpoint. In section , we discuss issues connected with breaking the R symmetry. We present simple models with supersymmetry and R symmetry broken by multiple singlets, needed for the GGM models described in section . We explain why runaway directions are typical of models with fields with R charge different than zero or two. We provide a simple example of a model where the R symmetry is everywhere broken on a branch of the moduli space.

In section we consider other approaches to model building. We discuss the model of “Extraordinary Gauge Mediation” of [15]. In this model, the presence of (multiple) messengers plays a crucial role in supersymmetry breaking. We explain why, without additional fields, one cannot obtain messenger parity as an accident in these theories. We exhibit the minimal additional field content required, and discuss some of the challenges to building a working model. We then consider the class of models in which the R symmetry is broken classically, without runaway behavior in the hidden sector. Coupling these to messengers allows realizations of GGM with CP conservation, before coupling to the MSSM fields. As a result, EdM’s are highly suppressed. In the conclusions, we discuss the possible phenomenological implications of these observations. The difficulties of building models with the full parameter set of GGM suggest that gauge mediation may make robust predictions beyond those of [16]. We also remark on some questions of definition: we critique the definition of gauge mediation in [16], as well as a definition of direct mediation given in [11].

IMPLEMENTING GGM

For our purposes, it will be convenient to parameterize the GGM spectrum in terms of the gaugino masses, $m_{\lambda}$, $m_w$ and $m_b$ and three parameters contributing to sfermion masses, $\Lambda_c^2$, $\Lambda_w^2$ and $\Lambda_Y^2$. In terms of these latter numbers, the sfermion masses are given by:

$$\tilde{m}^2 = 2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right) ^2 \Lambda_c^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right) ^2 \Lambda_w^2 + \frac{5}{3} \left( \frac{Y}{2} \right) ^2 \left( \frac{\alpha_1}{4\pi} \right) ^2 \Lambda_Y^2 \right]$$

(5)
In the MGM, there is a simple relation between the various parameters; our goal is to construct models in which these are independent.

**MGM and Messenger Parity**

The simple model of gauge mediation, eqn. [2], has one particularly attractive feature. Without tuning of parameters, it automatically has an approximate messenger parity symmetry, under which

$$q \leftrightarrow \bar{q} \quad \ell \leftrightarrow \bar{\ell} \quad V \rightarrow -V \quad (6)$$

This symmetry is necessarily violated by couplings of the MSSM fields, but the symmetry is good enough to ensure that an expectation value for $\langle D_Y \rangle$ is only generated at high loop order, so the usual two-loop contributions of eqn. [3] give the dominant contribution to scalar masses. This cancellation is discussed in the appendix.

This model is hardly complete. One needs to add some additional structure to account for the vev of the superfield $X$ (and, of course, the $\mu$ term). With our requirement that the model possess an R symmetry, in light of Shih’s theorem, one needs to add fields with unconventional R charges. Before doing this, we consider generalizations with more fields and messengers.

**Multiple Singlets: Simple models of GGM**

A very simple generalization, mentioned in [3], contains several singlets and a single set of messengers:

$$W = X_i \left( \lambda_i^i q \bar{q} + \lambda_i^i \ell \bar{\ell} \right) + F_i X^i. \quad (7)$$

This class of models, again, automatically exhibits a messenger-parity symmetry. It yields a spectrum of gauginos, squarks and sleptons, however, which is already different than that of minimal gauge mediation. There are now two parameters which describe the full spectrum (here $\langle X_i \rangle = x_i + \theta^2 F_i$):

$$\Lambda_q = \frac{\lambda_i^i F_i}{\lambda_q^i x_j} \quad \Lambda_\ell = \frac{\lambda_i^i F_i}{\lambda_\ell^j x_j} \quad (8)$$
The masses of the gluinos are given by
\[
m_\lambda = \frac{\alpha_3}{4\pi} \Lambda_q, \quad m_w = \frac{\alpha_2}{4\pi} \Lambda_\ell, \quad m_b = \frac{\alpha_1}{4\pi} \left[\frac{2}{3} \Lambda_q + \Lambda_\ell\right].
\]
Similarly, for the squark and slepton masses we have:
\[
\Lambda_c^2 = \Lambda_q^2; \quad \Lambda_w^2 = \Lambda_\ell^2; \quad \Lambda_Y^2 = \left(\frac{2}{3} \Lambda_q^2 + \Lambda_\ell^2\right).
\]
Note that in these models, the unified prediction for the gaugino mass is lost. There is, in general, a relative phase between \( \Lambda_q \) and \( \Lambda_\ell \), which leads, potentially, to edm’s for quarks and leptons at one loop. The details depend on the origin of \( \mu \). This problem will be common to many, but not all, of the models we discuss.

While more general than MGM, this model is described by two independent parameters, rather than the six (not allowing for a Fayet-Iliopoulos term) permitted by the analysis of [16]. There are thus four mass relations, which hold at the messenger scale:
\[
\left(\frac{\alpha_3}{4\pi}\right)^2 \Lambda_c^2 = m_\lambda^2
\]
\[
\left(\frac{\alpha_2}{4\pi}\right)^2 \Lambda_w^2 = m_w^2
\]
\[
\frac{2}{3} \alpha_1 \alpha_2 m_\lambda + \alpha_1 \alpha_3 m_w - \alpha_2 \alpha_3 m_b = 0
\]
and
\[
\frac{2}{3} \Lambda_c^2 + \Lambda_w^2 - \Lambda_Y^2 = 0.
\]
Note that the range of the low energy parameters (gaugino masses, \( \Lambda_c^2 \), etc.) depend on the details of the microscopic model. For example, it is easy to see that if there are \( N \) singlets, the ratio of \( m_\lambda^2 \) to \( \Lambda_c^2 \) is at most \( N \).

Additional parameters arise if we slightly complicate the messenger sector. If, for example, we replace the 5 and \( \bar{5} \) by a 10 and \( \bar{10} \) \((Q, \bar{Q}, U, \bar{U}, E, \bar{E})\), then there are three independent parameters which describe the low energy spectrum. The resulting model:
\[
W = X_i \left( y_i \bar{Q} Q + r_i \bar{U} U + s_i \bar{E} E \right)
\]
still, automatically, respects a messenger parity symmetry. Here it is natural to define the three parameters:
\[
\Lambda_Q = \frac{y_i F_i}{y_j x_j} \quad \Lambda_U = \frac{r_i F_i}{r_j x_j} \quad \Lambda_E = \frac{s_i F_i}{s_j x_j}
\]
In terms of these, the low energy GGM parameters are:

\[ m_\lambda = \frac{\alpha_3}{4\pi} (2\Lambda_Q + \Lambda_U) \quad m_w = \frac{\alpha_2}{4\pi} 3\Lambda_Q \quad m_b = \frac{\alpha_1}{4\pi} \left( \frac{4}{3} \Lambda_Q + 2\Lambda_E + \frac{8}{3} \Lambda_U \right) \]  

(17)

and

\[ \Lambda_c^2 = 2\Lambda_Q^2 + \Lambda_U^2 \quad \Lambda_w^2 = 3\Lambda_Q^2 \quad \Lambda_Y^2 = \frac{4}{3} \Lambda_Q^2 + 2\Lambda_E^2 + \frac{8}{3} \Lambda_U^2 \]  

(18)

We will discuss the low energy spectrum of the model in section , but note that the presence of \( \bar{E}E \) means that the masses of the lightest sleptons are not correlated with the masses of doublets or triplets.

Combining the models with 10 and \( \overline{10} \) and 5 and \( \bar{5} \) (specifically adding the superpotentials of \( 7 \) and \([15]\)) yields a theory which still possess a messenger parity and which now exhibits five of the GGM parameters. There are now five parameters which characterize the full sparticle spectrum.

\[ m_\lambda = \frac{\alpha_3}{4\pi} (\Lambda_q + 2\Lambda_Q + \Lambda_U) \, , \quad m_w = \frac{\alpha_2}{4\pi} (\Lambda_l + 3\Lambda_Q) \, , \]

\[ m_b = \frac{\alpha_1}{4\pi} \left( \frac{2}{3} \Lambda_q + \Lambda_l + \frac{4}{3} \Lambda_Q + 2\Lambda_E + \frac{8}{3} \Lambda_U \right) \]  

(19)

and

\[ \Lambda_c^2 = \Lambda_q^2 + 2\Lambda_Q^2 + \Lambda_U^2 \quad \Lambda_w^2 = \Lambda_l^2 + 3\Lambda_Q^2 \quad \Lambda_Y^2 = \Lambda_q^2 + \Lambda_l^2 + \frac{4}{3} \Lambda_Q^2 + 2\Lambda_E^2 + \frac{8}{3} \Lambda_U^2 \]  

(20)

Again, in this case, since the six low energy parameters are described by five microscopic ones, there is a sum rule. In this case, the rule is more complicated. It turns out to be eighth order in the masses, with 294 terms.

Five is the largest number of parameters one can obtain with the requirements:

1. Automatic messenger parity (in all of the above models, none of the fields, \( q, \bar{q}, \ell, \bar{\ell}, Q, \bar{Q}, U, \bar{U}, E, \bar{E}, \) can mix, due to the gauge quantum numbers).

2. Complete \( SU(5) \) multiplets.

3. Perturbative unification: multiplets such as the adjoint permit the full set of six parameters, but unification is problematic due to the large, negative beta functions.

4. No additional symmetries, beyond gauge symmetries, which distinguish the messengers.
Relaxing the last requirement permits construction of models with the full set of six parameters. For example, consider a theory with two 5 and \(\bar{5}\)'s and a single 10 and \(\bar{10}\), coupled to at least three singlets, where there are discrete symmetries which permit only the couplings:

\[
\lambda_{\ell}^{ai} X_a \ell_i \ell_i + \lambda_{q}^{ai} X_a \bar{q}_i q_i + \lambda_{Q} Q Q + \lambda_{U} \bar{U} U + \lambda_{E} \bar{E} E
\]  

(21)

Now there are four parameters associated with the two 5's,

\[
\Lambda_{q}^i = \frac{\lambda_{q}^{ai} F_a}{\lambda_{q}^{ai} x_a}, \quad \Lambda_{\ell}^i = \frac{\lambda_{\ell}^{ai} F_a}{\lambda_{\ell}^{ai} x_a}.
\]

(22)

while \(\lambda_{Q}, \Lambda_{U}\) and \(\Lambda_{E}\), are as in equation [16]. The gaugino and sfermion masses are now as in eqns. [19,20], but with \(\Lambda_{q} \to \sum \Lambda_{q}^i\) in the gaugino formulas, and \(\Lambda_{q}^2 \to \sum (\Lambda_{q}^i)^2\) in the sfermion mass formulas. The three gaugino masses and the three \(\Lambda^2\) combinations multiplying the different \(\alpha_i\) in the sfermion masses are all independent for a total of six parameters.

Because of the large, non-asymptotically free beta functions for \(SU(3) \times SU(2) \times U(1)\), unification in these theories is a delicate matter. If the messenger scale is 10’s of TeV, then the couplings become strong before the unification scale. For higher messenger scale, the theory can remain perturbative.

The increasing complexity associated with theories with more parameters raises the possibility that, if gauge mediation is realized in nature, the underlying theory generates only a subset of the full set of GGM parameters, leading to additional predictions. We will describe shortly how one can construct models with multiple singlets with suitable vev's. First, however, we consider some other possible model building strategies.

Models with one singlet and multiple messengers do not have difficulties with messenger parity, but they have the MGM spectrum. Taking the messengers to be 5, and \(\bar{5}\), by separate unitary transformations of the fields one can write:

\[
W = X \left( \lambda_{a}^{q} \bar{q}_a q_a + \lambda_{a}^{\ell} \bar{\ell}_a \ell_a \right)
\]

(23)

This class of models still respects an approximate messenger parity symmetry. However, the spectrum is that of MGM. This is also true if the messengers are in the \(\bar{10}\) and 10 representations.
With multiple singlets and multiple messengers, with symmetries, we saw that one can readily construct models with automatic messenger parity. But without symmetries, the situation is more complicated. We cannot, in general, bring the superpotential to a simple, diagonal form, but instead, have:

\[ W = X_i (\lambda_{ab} \tilde{q}_a q_b + \ldots) \].

(24)

We can also allow mass terms (as in [15]), \( m_{ab} \tilde{q}_a q_b + \ldots \). Here we encounter a serious difficulty: there is no messenger parity symmetry, unless the \( \lambda^i \)’s are each diagonal. As a result, there is a one loop contribution to \( \langle D_Y \rangle \) (this is illustrated in a simple case in the Appendix), unless there are additional symmetries (we will see examples shortly).

**SQUASHING THE SPECTRUM**

The five parameter model has interesting phenomenological features. For plausible values of the parameters, it exhibits a significant “squashing” or “compression” of the spectrum. In other words, the masses of the squarks, and the \( SU(2) \) singlet and doublet sleptons, can be quite close. This can appreciably ameliorate the fine tuning problems of gauge mediation, especially if the scale of the messenger masses is low. To illustrate in a simple limit, take two of the five parameters to vanish:

\[ \Lambda_Q = \Lambda_U = 0 \]

(25)

This leaves us with what we will call the “three parameter model.” (\( \Lambda_E, \Lambda_\ell, \) and \( \Lambda_q \)). The sparticle spectrum can be squashed in two different ways; both exhibit interesting phenomenology. First, one can take \( \Lambda_\ell \sim \Lambda_E > \Lambda_q \). In this case we can raise the slepton masses so for example: \( \tilde{m}_{\tilde{\ell}} \sim \tilde{m}_{s\ell} \sim 1 \text{TeV} \). In [15] it was pointed out that this region of parameter space can allow a tuned cancelation between the soft Higgs mass generated by integrating out the messengers and the radiative corrections from the heavy stop mass. This allows a small \( \mu \)-term. The second way to squash the spectrum is by taking \( \Lambda_\ell \sim \Lambda_E > \Lambda_q \) in such a way that \( \tilde{m}_{s\ell} \sim 300 \text{GeV} \) and \( \tilde{m}_{\tilde{\ell}} \sim 100 \text{GeV} \). Such a limit ameliorates the “little hierarchy” problem mentioned in the introduction which arises because heavy squarks typically renormalize the Higgs soft mass to a large, negative value. However, even in this “light” region of parameter space the requirement that the lightest chargino mass be greater than 100GeV
implies that \( m_{H_u} \) gets an \( SU(2) \) charged messenger contribution: \( m_{H_u}^2(SU(2)) \geq (150\text{GeV})^2 \).
The squarks then renormalize \( m_{H_u} \) to give: \( m_{H_u}^2(sq) \leq -(170\text{GeV})^2 \). This still implies a modest amount of fine tuning.

**BREAKING THE R SYMMETRY**

Shih\[8\] provided a simple model whose Coleman-Weinberg potential leads to breaking of R symmetry. The model has fields with R charges 1, -1, 3 and 2, \( \phi_1, \phi_{-1}, \phi_3, X \):

\[
W = -FX + \lambda X \phi_1 \phi_{-1} + m_1 \phi_1^2 + m_2 \phi_{-1} \phi_3.
\] (26)
The theory has a pseudomoduli space with \( \phi_i = 0 \) and \( X \) undetermined. The Coleman-Weinberg analysis leads to a non-zero value of \( X \) at one loop.

We have seen that perhaps the simplest way to obtain a more general gauge mediated spectrum is with multiple singlets with scalar and F term vev's. A simple example of such a model is provided by taking several copies of the model of eqn. [26]:

\[
W = -FX^a + \lambda X^a \phi_1^a \phi_{-1}^a + m_1 \phi_1^a \phi_1^a + m_2 \phi_{-1}^a \phi_3^a.
\] (27)
Obviously, this is not the most general model consistent with symmetries; one should allow couplings among the fields with different labels, \( a \). But given that this model has broken symmetries at a (meta)stable local minimum, one sees that at least for small values of these additional parameters, one can obtain the desired structure: multiple singlets, broken supersymmetry, and broken \( R \) symmetry. The singlets can then be coupled to messengers, allowing the full GGM spectrum.

While an existence proof, the existence of so many fields (a minimum of nine in the above construction) is perhaps unappealing. A model with fewer fields exploits the couplings to messengers to fix the \( R \)-symmetry breaking vev’s. Consider

\[
W = -F_\alpha X_\alpha + \lambda_\alpha X_\alpha \phi_1 \phi_{-1} + m_1 \phi_1^2 + m_2 \phi_{-1} \phi_3 + y_\alpha X_\alpha \tilde{5} 5.
\] (28)
Here there are two \( X \) type fields, but only a single set of \( \phi \) fields. For simplicity, we have indicated only a single set of 5, \( \tilde{5} \) messengers; the generalization with 10, \( \overline{10} \) and/or multiple 5’s and \( \tilde{5} \)’s is immediate. In this model, one linear combination of \( X \)’s, call it \( X_2 \), decouples from the \( \phi \)’s; the other, \( X_1 \), obtains a vev from one loop diagrams containing \( \phi \) fields, as in
the previous model. \( X_2 \) then receives an (in general non-zero) vev from loops of messengers. For a suitable range of parameters, the messenger masses are non-tachyonic.

We should note that a simpler version of eqn.\[26\] omits the field \( \phi_{-3} \) and the mass parameter \( m_2 \):

\[
W = -FX + \lambda X \phi_1 \phi_{-1} + m\phi_1^2.
\]

This model has a runaway direction starting from the origin in field space. Still, the Coleman-Weinberg analysis yields a local minimum of the potential for \( X \), with \( \phi_1 = \phi_{-1} = 0 \). For small \( \lambda \), this minimum is highly metastable. While simpler, however, the range of parameters over which the \( R \) symmetry is broken is somewhat tuned. There is a metastable minimum for \( \frac{\lambda F}{m^2} \sim 3 \cdot 10^{-3} \). Approximating the potential with those considered in \[18\] we find that this minimum is long lived in the limit \( \lambda (\frac{F}{m^2})^\frac{1}{2} \rightarrow 0 \).

**Breaking R Symmetry at Tree Level: The Problem of Runaway**

In the model of eqn.\[26\], in addition to the pseudomoduli space, one can obtain vanishing of the energy as a limiting process. If

\[
\lambda \phi_{-1} \phi_1 = F
\]

while, at the same time, \( \phi_1 \rightarrow 0, \phi_X \rightarrow 0 \), then the potential tends to zero. More precisely we can take:

\[
\phi_1 = e^\alpha \sqrt{\frac{F}{\lambda}}, \quad \phi_{-1} = e^{-\alpha} \sqrt{\frac{F}{\lambda}}, \quad X = -\frac{2m_1}{\lambda} e^{2\alpha}, \quad \phi_3 = \frac{2m_1}{m_2} \sqrt{\frac{F}{\lambda}} e^{3\alpha}
\]

we see that

\[
F_X = 0; \ F_{\phi_1} = 0; \ F_{\phi_{-1}} = 0
\]

while

\[
F_{\phi_3} = m_2 \sqrt{\frac{F}{\lambda}} e^{-\alpha}
\]

so the potential tends to zero as \( \alpha \rightarrow \infty \). It is easy to understand why this happens. In general, the manifold of classical solutions of the supersymmetry equations is larger than implied by the symmetry group; it is described by the *complexification* of the group. Here we
have solved a subset of the equations, those for which the $F$ terms have charge 0 or smaller. The manifold of solutions of this subset of equations is still described by the complexified group. Since all of the non-vanishing $F_i$’s have R charge greater than zero, they vanish as we take the parameter of the complexified group transformation to $\infty$.

Note that these field configurations, in which the R symmetry is everywhere broken, are distinct from the pseudomoduli space. As shown in [8], the pseudomoduli space is stabilized. Decay to the runaway directions must proceed by tunneling. Note that, because the runaway is classical, quantum effects, at weak coupling, will not give rise to metastable minima in these directions.

This argument indicates that in theories with fields with non-standard R charges, the appearance of runaway behavior is common [15]. All that is required is that one be able to solve the equations for the vanishing $F$ terms for all fields with R charge greater than or equal to two, or less than or equal to two. On the other hand, this discussion also makes clear how one can avoid the runaway: it must not be possible to satisfy these equations. In this case, one has a branch of the pseudomoduli space on which the R symmetry is everywhere broken. In the following subsection, we give an example of such a model.

**Breaking R Symmetry at Tree Level: No Runaway**

Based on our discussion above, the existence of runaway directions in models with fields with $R$ charge both greater than and less than two would seem to be a generic feature. Divide the fields of the model into a set with R charge greater than two, $Y_i$, a set with $R$ charge less than two, $Z_i$, and a set with $R$ charge equal to two, $X_i$. (Note that with this division, the R charges of F components of chiral superfields with $R < 2$ are negative.) Suppose one can solve the equations $\partial W/\partial \phi_i = 0$ for $X_i$ and $Y_i$. Then, even if we can’t solve the equations for $Z_i$, since all of the remaining $F$’s have the $R$ charge of the same sign, we can make the corresponding $F_i$’s arbitrarily small (while keeping the rest zero) by a complexified R transformation. Clearly the same is true if we switch the role of $Y_i$ and $Z_i$. Roughly speaking, to determine if such solutions exist, we have to count equations and unknowns. Essentially we need, both for $R \geq 2$ and $R \leq 2$, an overdetermined set of equations.

As an existence proof, consider a model with superpotential (the R charges of the fields
are indicated by the subscripts):

\[ W = X \left( \gamma \phi_{2/3} \phi_{-2/3} - \mu^2 \right) + \frac{\delta}{3} \phi_{2/3}^3 + m_1 \phi_{2/3} Y_{4/3} + m_2 \phi_{-2/3} Y_{8/3}. \]  

(34)

All of the parameters in the model can be taken to be real.

There is a branch of the (pseudo)moduli space with all fields 0 except for \( X \) where the potential is equal to \( V = \mu^4 \). However, there is a second branch on which

\[ \phi_\frac{2}{3} = m_2 \sigma e^{i\theta}; \quad \phi_{-\frac{2}{3}} = m_1 \sigma e^{-i\theta}. \]  

(35)

where \( \sigma \) is defined such that \( \mu^2 = \frac{m_1 m_2}{\gamma}(1 + \sigma^2 \gamma^2) \). This branch exists for \( \sigma \in \mathbb{R} \) and on it the value of the potential is

\[ V = \frac{m_1^2 m_2^2}{\gamma^2} \left( 1 + 2 \sigma^2 \gamma^2 \right) \leq \mu^4. \]

Therefore it is lower than the previous one and stable.

Unlike the runaways we have discussed above, one cannot now allow, say, \( \phi_{3/2} \) to be large and \( \phi_{-3/2} \) small at the same time. The equations for \( \frac{\partial W}{\partial Y_i} \) and \( \frac{\partial W}{\partial X} \) are incompatible. The (pseudo)moduli space is of real dimension 3. Again, one should note that on this branch, there is no point at which the \( R \) symmetry is restored. A one loop Coleman-Weinberg calculation is required to determine the values of the \( X \) and \( Y_i \) fields.

This model, however, has a serious deficiency. The symmetries allow an additional coupling

\[ \delta W = h Y_{4/3}^2 \phi_{-2/3}. \]  

(36)

Adding this coupling reintroduces the problem of runaway. One can avoid this difficulty by adding an additional field, \( \chi_{2/3} \), and a \( Z_2 \) symmetry, under which all fields but \( X \) and \( \chi_{2/3} \) are odd. For the superpotential we take:

\[ W = X \left( \gamma \phi_{2/3} \phi_{-2/3} - \mu^2 \right) + \frac{\delta}{3} \phi_{2/3}^2 \chi_{2/3} + m_1 \phi_{2/3} Y_{4/3} + m_2 \phi_{-2/3} Y_{8/3} + \lambda \chi_{2/3}^3. \]  

(37)

These are all of the couplings permitted by the symmetries. The features of the model are similar to those we encountered above. Supersymmetry is broken, and there is a branch of the moduli space on which \( R \) symmetry is everywhere broken. Notice also that all the parameters appearing in (37) can be made real by field redefinitions.
OTHER APPROACHES TO MODEL BUILDING

The models with multiple singlets we have described in section, while providing a realization of GGM, hardly exhaust the possibilities for model building. In this section, we consider some other approaches.

Extraordinary Gauge Mediation

Ref. \[15\] presented models in which the messengers coupled at tree level to the goldstino, and in which the dynamics of the messenger fields figures crucially in determining the pattern of R symmetry breaking. We will take the messenger fields $\phi_i, \tilde{\phi}_i$, to fill out a 5 and $\bar{5}$ representation of $SU(5)$. Writing the lagrangian in a schematic, $SU(5)$ invariant form:

$$W = \lambda_{ij} \phi_i \tilde{\phi}_j + m_{ij} \tilde{\phi}_i \phi_j + X F$$

where $\lambda$ and $m$ are such that the theory has a continuous R symmetry. For example, if $i = 1, 2$,

$$W = X \left( \lambda_1 \phi_1 \tilde{\phi}_1 + \lambda_2 \phi_2 \tilde{\phi}_2 \right) + m \phi_1 \tilde{\phi}_2.$$

To avoid one loop D terms, it is necessary that $\lambda_1 = \lambda_2$ to a high degree of approximation. If we make this assumption, then one has a spectrum for scalars with two of the three parameters allowed by GGM, but, perhaps surprisingly, with “unification” relations for the gaugino masses\[15\].

Imposing that $\lambda_1 = \lambda_2$, one has an approximate messenger parity, insuring that the D term is generated only at high orders. We might try to understand this equality of couplings as arising from a symmetry which interchanges the labels 1 and 2. The symmetry is violated only by the mass term, and this violation is soft (e.g. corrections to the kinetic terms for $\phi_1$ and $\phi_2$ are finite, and arise first at three loops). This suggests that one can understand the model as arising from an exact $1 \leftrightarrow 2$ symmetry which is spontaneously broken.

We can illustrate this possibility by introducing two fields, $\rho_+$ and $\rho_-$, with couplings

$$X \left( \phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2 \right) + X \mu^2 + \rho_+ \phi_1 \tilde{\phi}_2 + \rho_- \tilde{\phi}_1 \phi_2.$$

If $\rho_+$ has an expectation value, and $\rho_- = 0$, then the low energy theory has the structure of Shih’s model. The $Z_2$ symmetry interchanges $\rho_+$ and $\rho_-$, as well as interchanging the labels “1” and “2”.

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In order that the vanishing of $\rho_-$ be natural, we would like the model to possess a symmetry under which $\rho_-$ transforms and $\rho_+$ is neutral. Because $\rho_+$ and $\rho_-$ carry different $R$ charges, this means that it is possible to define two different $R$ symmetries, or alternatively, one $R$ symmetry and an ordinary, global $U(1)$. Since these symmetries are spontaneously broken, there are additional Goldstone bosons, and, unless $\phi_\pm$ couple to $X$ at tree level, additional pseudomoduli. To try and fix the values of $\rho_\pm$, one might try to write, for example, couplings:

$$Z (\rho_+ \chi_- + \rho_- \chi_+ - \mu^2) + Y \rho_+ \rho_-$$

in the hopes of obtaining a pattern of vev’s with the desired structure. In this model, however, coupling of $Z$’s to $X$ cannot be forbidden, nor certain additional dangerous couplings of $Y$. Ignoring this, we encounter the runaway issues which we have encountered before.

It is easy to achieve the desired structure in a theory with a classical moduli space, if we are willing to introduce more fields and more symmetries. E.g. add

$$Y_1 (\rho_+ \chi_-) + Y_2 (\rho_- \chi_+)$$

This model has an additional (ordinary) $U(1)$ symmetry under which the fields $Y_i$ transform. As a result, there are not additional dangerous couplings of $Y$ to $X$ (or $\chi$). The model, as expected, has a moduli space with several branches. There is a branch of the desired type, with $\rho_+ = a, \chi_+ = b$, and all other fields vanishing. On this branch, the only extra light fields are $\rho_+$ and $\chi_+$. Achieving a model where classically one does not have additional moduli of this type is difficult. Whether in a complete model, the Coleman-Weinberg calculation can yield a sensible metastable minimum along one of these branches is a question which requires investigation.

**Broken R Symmetry at Tree Level, Multiple Messengers**

We have outlined above how to obtain a pseudomoduli space on which $R$ symmetry is broken everywhere. We can easily write a model of this type, coupled to messengers. Start with the model of equation [37]. As explained previously, this is a model where one cannot set all of the $F$ terms with $R$ charge greater than or less than two to zero. As a result, supersymmetry is broken, and there is a branch of the classical moduli space in which the $R$ symmetry is everywhere broken.
Introduce couplings of the $Y$ fields to messengers:

$$W_m = a Y^4 (M_1 \bar{M}_1) + b Y^8 (M_2 \bar{M}_2)$$

(43)

These couplings are schematic. $M_i$’s can be 5 or 10’s of $SU(5)$, and the Yukawa couplings need not be unified. This is at most a two parameter model, in which messenger parity holds automatically. This model has the virtue that CP is automatically conserved. More complexity is required, however, to obtain more than two parameters.

The messenger pair $M_1 \bar{M}_1$ has R charge $2/3$ while the pair $M_2 \bar{M}_2$ has R charge $-2/3$. One needs to check, then, that the local minimum obtained from the Coleman-Weinberg calculation without messengers does not yield tachyonic masses for messengers. This can be shown to hold for a range of parameters.

CONCLUSIONS

We have presented entire classes of models with GGM spectra, and with messenger parity as an automatic feature. We have explored qualitatively different possibilities for R symmetry breaking, in which there are branches of the pseudomoduli space on which the R symmetry is everywhere broken.

The discussion of this paper shows that it is easy to construct models with more parameters than that of the MGM, and that even in weakly coupled theories, one can obtain the full set of GGM parameters. Models with more parameters are progressively more complicated. We have seen that one must be careful to insure the existence of a messenger parity symmetry. The fact that obtaining the full set of GGM parameters requires extra symmetry structure suggests that models with few parameters might be more likely. We have indicated simple ways in which to compress the spectrum, obtaining models without the severe fine tuning of MGM. We have also seen that problems with CP violation are typical of models in which the MGM predictions are modified, but we have also seen exceptions. The exceptions arose in cases where there are several singlets, with distinct quantum numbers under some symmetry. An alternative solution to the problem of edm’s is that CP violation is spontaneous, as discussed in [11, 19].

One might wonder about our focus (and that of [15]) on renormalizable theories with continuous R symmetries. After all, we do not expect fundamental theories to exhibit global,
continuous R symmetries. But the work of [11] on retrofitted model building suggests that it is challenging to build models without an approximate R symmetry at the level of the low energy lagrangian. This symmetry must then be spontaneously broken, and the criteria discussed by Shih then must be satisfied. In [11], low energy models without continuous symmetries were constructed, but they required the existence of certain quite small Yukawa couplings. These, it was argued, could arise by a Frogatt-Nielsen mechanism. One can debate whether these models are more or less complicated (or plausible) than those presented in this paper.

These studies have lead us to rethink certain questions of definition. We are not entirely satisfied with the definition of gauge mediation presented in [16]. In particular, it can be applied to certain proposals, like anomaly mediation, in which gravitational effects (or more generally, very high scale effects) are important. In the case of anomaly mediation, for example, if one tunes the Kahler potential to have the sequestered form (or perhaps provides a higher dimensional, dynamical explanation), then there are no contributions to scalar and gaugino masses in the limit that the gauge couplings vanished. Of course, the spectrum one obtains is problematic, and the solutions to this problem might involve non-gauge interactions. Still, it is not clear whether one really wants to call even the unrealistic version “gauge mediation”.

Another question of definition has to do with the term “direct mediation”. Loosely, what is usually meant by this is that the messengers couple directly (as opposed to through loop effects) to the fields responsible for the underlying breaking of supersymmetry. In [11], a definition was offered, that mediation is direct if in the limit that the couplings to messengers are turned off, supersymmetry is restored. This definition is appealing. Models which fit in this framework include strongly coupled models with dynamical supersymmetry breaking, in which some of the fields of the strongly coupled sector act as messengers. But this definition is readily seen, after a moment’s thought, to be too restrictive. Any O’Raifeartaigh model where the messengers couple to the field $X$ with a non-vanishing F component, such as that of eqn. 39 would not fit this definition. A more precise definition would be: mediation is indirect if, decoupling the messengers, the features of the hidden sector (spectra and patterns of R symmetry breaking) are not appreciably affected.

While recent developments in dynamical supersymmetry breaking and gauge-mediated model building may not make discovery of gauge mediation seem a certainty, they do make
the possibility much more plausible. The phenomenology is likely to be much richer than that of MGM, yet still restricted. The detailed examination of the possible parameter space is worthy of further exploration.

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Appendix

To illustrate the problem of one loop D-terms, consider a model with a pair of messengers $\tilde{q}_i$ and $q_i$ ($i = 1, 2$). If the messengers both carry $U(1)_Y$ charge then the component lagrangian will contain quadratic interactions between the Messengers and the squarks and sleptons of the form

$$L_{\text{int}} \sim g_1^2 \left( \frac{Y_{\tilde{q}}}{2} \right) \left( \frac{Y_Q}{2} \right) q_i^\dagger q_i Q^\dagger Q.$$ (44)

This can lead to the graph shown in Fig. 1. In order for these contributions to be zero, it is sufficient that there exist a symmetry in the Messenger sector where $\tilde{q}_i \leftrightarrow q_j$ and $V \rightarrow -V$. If such a symmetry exists, then it necessarily implies that no linear term in $V$ (or any component of $V$) can be generated at one loop in the Lagrangian. The "D-term" is an auxiliary component of $V$. A linear term in $D$ is forbidden by this Messenger parity
symmetry. The absence of a linear term in D implies that the one loop graph for the squark mass must be zero. This is easily seen by not integrating out the D-term as is typically done. In this case, the one loop graph in Fig. 1 is replaced by the graph in Fig. 2. So it is clear that if a linear term in D is forbidden then the one-loop squark mass is zero.

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In many earlier models, both weakly and strongly interacting, the inclusion of messengers lead to the existence of vacua in which supersymmetry was restored, but the possibility that the hidden sector, by itself, was described by a metastable, supersymmetry breaking state, had not received great attention.