\[ \mathcal{O}(N_f \alpha^2) \] Corrections in Low-Energy Electroweak Processes

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Abstract

We provide a compendium of techniques that can be used to compute \[ \mathcal{O}(N_f \alpha^2) \] corrections to low-energy electroweak processes. Specifically, these are the 2-loop electroweak corrections containing a light fermion loop. It is shown that the vast majority of such corrections can be reduced to expressions involving a universal master integral for which an exact analytic form in dimensional regularization is given. The only exceptions are certain photon vacuum polarization diagrams. Examples are presented for diagrams that occur in a variety of processes of practical interest.
1 Introduction

Among the most precisely measured electroweak observables the electromagnetic coupling constant, $\alpha$, the muon decay constant, $G_\mu$, the anomalous magnetic moment $g - 2$, and the ratio of charged to neutral current cross-sections in semi-leptonic neutrino scattering, feature prominently. Because of their high experimental precision $\alpha$ and $G_\mu$ are commonly used as input in calculations of electroweak radiative corrections. Until relatively recently the experimental errors on $\alpha$ and $G_\mu$ were entirely negligible compared to those on the third input quantity, $M_Z$. Now, however, both $G_\mu$ and $M_Z$ are both measured to 2 parts in $10^{5}$. Any further improvement on the accuracy of $M_Z$ would mean that the measurement of the muon lifetime in principle limits the accuracy with which electroweak theory can be tested.

In this arena, theoretical calculations currently lag experiment. Calculations are not available that could exploit the $2 \times 10^{-5}$ accuracy available on $G_\mu$ and $M_Z$. It is therefore of technical interest and practical necessity to develop electroweak calculations that approach the $2 \times 10^{-5}$ accuracy of experiment. While a full 2-loop calculation would be ideal, it is likely that the analytic form of such results would be too complicated to usefully write down. One would therefore probably wish to rely on computer algebra and integration to obtain numerical results [1]. However it is still true that certain dominant subclasses of 2-loop diagrams can be calculated and written down in compact analytic form [2–5]. Such results are, of course, always preferable to numerical ones.

The set of 2-loop Feynman diagrams containing one fermion loop are such a class. These we will refer to as $\mathcal{O}(N_f \alpha^2)$ corrections where $N_f$ is the number of fermions and $N_g = N_f/8$ is the number of complete generations. In these corrections we will assume that fermions are massless which is an excellent approximation for all but the third generation. The $\mathcal{O}(N_f \alpha^2)$ results can be used either with $N_g = 3$ and mass corrections computed for the third generation or $N_g = 2$ and corrections for the third generation computed separately.

Because $N_f$ is quite large — up to 24 — the $\mathcal{O}(N_f \alpha^2)$ corrections are expected to be a dominant subset of 2-loop graphs and since $N_f$ provides a unique tag, the complete set of $\mathcal{O}(N_f \alpha^2)$ corrections contributing to a particular physical process will form a gauge-invariant set. Diagrams of this type have been considered in connection with the muon anomalous magnetic moment [4].

A priori the $\mathcal{O}(N_f \alpha^2)$ corrections can be expected to contribute at the level of $1.5 \times 10^{-4}$. Without their calculation and inclusion, theoretical predictions are uncertain at this level.

To test the Standard Model, one needs at least four well-measured electroweak observables. Three of these are used as input to the model and the fourth is then predicted and can be compared with experimental determinations. In addition, relating electroweak parameters, such as $M_Z$, extracted at high energies with those, such as $\alpha$ and $G_\mu$, extracted at low energies inevitably involves an uncertainty aris-
ing from the hadronic contribution to the photon vacuum polarization, $\Pi'_\gamma(0)$. This hadronic uncertainty can be reduced by improved measurements of the cross-section $\sigma(e^+e^- \rightarrow \text{hadrons})$ and the use of dispersion relations [7–9] or eliminated completely by sacrificing one high precision observable [10].

At present the fourth well-measured electroweak observable, needed to test the Standard Model at very high precision, is missing but it is not unlikely that quantities, such as the polarization asymmetry, $A_{LR}$, or $g−2$ of the muon, could fulfill this role in the foreseeable future. In any event the calculation of the $\mathcal{O}(N_f\alpha^2)$ electroweak radiative corrections are a necessary prerequisite to be able exploit the extremely accurate experimental measurement that are the legacy of LEP.

2 The Master Integral

In what follows the general coupling of a fermion to a vector boson will be denoted

$$\gamma_{\mu} \equiv i\gamma_{\mu}(\beta_{L}\gamma_L + \beta_{R}\gamma_R)$$

where $\gamma_L$ and $\gamma_R$ are the usual left- and right-handed helicity projection operators and $\beta_L$ and $\beta_R$ are the corresponding coupling constants. We assume throughout an anticommuting $\gamma_5$. A Euclidean metric with the square of time-like momenta being negative will be used and all calculations will be done in $R_{\xi=1}$ gauge. The sine and cosine of the weak mixing angle $\theta_W$ will be denoted $s_\theta$ and $c_\theta$ respectively.

We will be interested in the $\mathcal{O}(N_f\alpha^2)$ corrections for massless fermions. In corresponding Feynman diagrams the fermions couple only to vector bosons and never to Goldstones or the physical Higgs particle. Many, but by no means all, such diagrams are obtained simply by inserting a fermion loop into the boson propagator of a one-loop diagram, see Fig.1a. Because the original one-loop diagram is logarithmically divergent, the fermion loop insertion will need to be calculated to $\mathcal{O}(n−4)$ in dimensional regularization, where $n$ is the dimension of space-time. It turns out, however, that for all $\mathcal{O}(N_f\alpha^2)$ diagrams for low-energy processes, it is possible to obtain expressions that are exact in $n$.

For the massless fermion loop insertion, it may be shown that

$$\gamma_{\mu} \equiv i\gamma_{\mu}(\beta_{L}\gamma_L + \beta_{R}\gamma_R)$$

where $\beta_L$ and $\beta_R$ are the couplings of the attached vector bosons.
A number of special cases of low energy 2-loop integrals have appeared in the literature \[11\]–\[13\]. By using the projection operator techniques described in the following sections, most $\mathcal{O}(N_f\alpha^2)$ diagrams occurring in low-energy electroweak processes, including box diagrams, can be reduced to expressions involving a general master integral of the form

$$I(j, k, l, m, n, M^2) = \int \frac{d^n p}{i\pi^2} \frac{1}{[p^2][p^2 + M^2]^k} \int \frac{d^n q}{i\pi^2} \frac{1}{[q^2][(q + p)^2]^m}$$

(3)

provided the diagram contains at least one massive boson. The only exceptions to this are certain contributions to the photon vacuum polarization. In that case some diagrams must be calculated retaining the fermion mass, $m_f$, and then applying the asymptotic expansion of ref. [13] before taking the limit $m_f \to 0$. Other $\mathcal{O}(N_f\alpha^2)$ diagrams that cannot be expressed in terms of the integral (3) are of the pure QED type containing only internal photons and fermions. The relevant master integrals for this case can be found in ref. [12].

In eq.(3), the integration over $q$ can be performed using standard Feynman parameter techniques and yields

$$\int \frac{d^n q}{i\pi^2} \frac{1}{[q^2][(q + p)^2]^m} = \frac{\pi^{n-2}}{[p^2][p^2 + m - \frac{n}{2}]^k} \frac{\Gamma \left( l + m - \frac{n}{2} \right) \Gamma \left( \frac{n}{2} - j \right) \Gamma \left( \frac{n}{2} - l \right) \Gamma \left( n - l - m \right)}{\Gamma(l)\Gamma(m)\Gamma(n - l - m)}. \quad (4)$$

The resulting integral with respect to $p$ in eq.(3) is independent of angle and hence

$$\int \frac{d^n p}{i\pi^2} \frac{1}{[p^2][p^2 + M^2]^k} = \frac{2\pi^{n-2}}{\Gamma \left( \frac{n}{2} \right)} \int_0^\infty dp \frac{p^{2n-2j-2l-2m-1}}{[p^2 + M^2]^k}$$

$$\quad = \frac{\pi^{n-2}}{(M^2)^{k+j+l+m-n}} \frac{\Gamma(n - j - l - m)\Gamma(k + j + l + m - n)}{\Gamma \left( \frac{n}{2} \right) \Gamma(k)} \quad (5)$$

from which it follows

$$I(j, k, l, m, n, M^2) = \frac{\pi^{n-4}}{(M^2)^{k+j+l+m-n}}$$

$$\times \frac{\Gamma(n - j - l - m)\Gamma(k + j + l + m - n)\Gamma \left( l + m - \frac{n}{2} \right) \Gamma \left( \frac{n}{2} - l \right) \Gamma \left( \frac{n}{2} - l \right) \Gamma \left( \frac{n}{2} - l \right)}{\Gamma \left( \frac{n}{2} \right) \Gamma(k)\Gamma(l)\Gamma(m)\Gamma(n - l - m)}$$

(6)

### 3 Self-energy Diagrams

For electroweak processes, such as muon decay, in which the external fermions can be considered massless, only the self-energy diagrams of vector bosons need to be
considered. General techniques are available to reduce the tensor integrals that occur to standard form factors \([14]\) for which scalar integral representations are known. For the present case of zero external momentum, \(q = 0\), the vector boson self-energies, \(\Pi_{\mu\nu}(q^2)\), can only take the form

\[
\Pi_{\mu\nu}(0) = \delta_{\mu\nu} F
\]

where \(F\) is a function of the internal masses only and may be obtained from the tensor integral representation of \(\Pi_{\mu\nu}(0)\) by means of the projection operator, \(\delta_{\mu\nu}/n\). Thus

\[
F = \left(\frac{\delta_{\mu\nu}}{n}\right) \Pi_{\mu\nu}(0).
\]

(7)

The resulting scalar integral can always written in terms the master integral, \(I(j, k, l, m, n, M^2)\), of eq.(6) and results obtained that are exact for all \(n\).

### 4 Vertex Diagrams

Again for processes with massless external fermions the only relevant vertex corrections are those involving vector bosons and these will necessarily be purely vector and axial-vector in character. A general vertex correction can then be represented as

\[
\equiv V_\mu = i \gamma_\mu (V_L \gamma_L + V_R \gamma_R)
\]

where \(V_L\) and \(V_R\) are functions only of the internal masses. The tensor integral representation of \(V_\mu\) can easily be obtained by standard techniques and from it the scalar integral representations of \(V_L\) and \(V_R\) follow by means of projection operators. Thus

\[
V_{L,R} = -\frac{i}{2n} \text{Tr}\{V_{\mu} \gamma_\mu \gamma_{R,L}\}.
\]

(8)

where \(\text{Tr}\{\gamma_\mu \gamma_\nu\} = 4n\) is assumed. This method for directly obtaining the scalar integral representation of the vertex form factors is particularly convenient when computer algebra is being employed. Once again the resulting scalar integrals can be written in terms of the master integral, \(I(j, k, l, m, n, M^2)\).

#### 4.1 Example

The diagram shown in Fig.4(a) occurs in the calculation of the \(\mathcal{O}(N_f \alpha^2)\) electromagnetic charge renormalization. It will be assumed that the external fermion has \(t_3 = +1/2\) in which case the fermion to which the photon couples has \(t_3 = -1/2\).
associated with the external fermion currents \( J \)

where the square brackets \([ \quad ]\)

intended use at one-loop. For general diagrams appears in ref. \[15\]. They are, however, valid only for

develop logarithmic divergences. A useful set of identities for calculating one-loop box diagrams are finite but at

4-fermion processes one-loop box diagrams are finite but at

\( \mathcal{O}(N_f \alpha^2) \) they develop logarithmic divergences. A useful set of identities for calculating one-loop box diagrams appears in ref. \[15\]. They are, however, valid only for \( n = 4 \) because of their intended use at one-loop. For general \( n \) it may be shown that these relations become

\[
\begin{align*}
[\gamma^\rho \gamma^\mu \gamma^\sigma \gamma^L, [\gamma^\nu \gamma^\rho \gamma^\sigma \gamma^R, 2] & = 4 \delta_{\mu\nu} [\gamma^\alpha \gamma^L, [\gamma^\alpha \gamma^L, 2] + (n - 4) [\gamma^\mu \gamma^L, 2] [\gamma^\nu \gamma^L, 2] \quad (9) \\
[\gamma^\rho \gamma^\mu \gamma^\sigma \gamma^L, [\gamma^\rho \gamma^\nu \gamma^\sigma \gamma^L, 2] & = 4 [\gamma^\nu \gamma^L, 2] [\gamma^\rho \gamma^L, 2] + (n - 4) [\gamma^\mu \gamma^L, 2] [\gamma^\nu \gamma^L, 2] \quad (10) \\
[\gamma^\rho \gamma^\mu \gamma^\sigma \gamma^L, 2] [\gamma^\rho \gamma^\nu \gamma^\sigma \gamma^L, 2] & = 4 [\gamma^\gamma^L, 2] [\gamma^\rho \gamma^L, 2] + (n - 4) [\gamma^\rho \gamma^L, 2] [\gamma^\sigma \gamma^L, 2] \quad (11) \\
[\gamma^\rho \gamma^\mu \gamma^\sigma \gamma^L, 2] [\gamma^\sigma \gamma^\nu \gamma^\sigma \gamma^R, 2] & = 4 \delta_{\mu\nu} [\gamma^\alpha \gamma^L, 2] [\gamma^\rho \gamma^L, 2] + (n - 4) [\gamma^\rho \gamma^L, 2] [\gamma^\nu \gamma^L, 2] \quad (12)
\end{align*}
\]

where the square brackets \([ \quad ]\) and \([ \quad ]\) indicate that the enclosed \( \gamma \)-matrices are associated with the external fermion currents \( J_1 \) and \( J_2 \) respectively.

The general \( \mathcal{O}(N_f \alpha^2) \) box diagram for massless external fermions at \( q = 0 \) therefore takes the form

\[
B_{\mu\nu} J_{\mu \alpha} J_{2\nu} = B \cdot J_{1\alpha} J_{2\alpha}.
\]

(13)
Here $B_{\mu\nu}$ is a tensor integral and the product of $J_{1,\alpha}J_{2,\alpha}$ can be constructed from one or a combination of the $\gamma$-matrices appearing in eq. (9)–(12). As for the self-energy contributions, the tensor integral $B_{\mu\nu}$, can only be proportional to $\delta_{\mu\nu}$ and hence, using the projection operator method of section 2, the scalar integral representation for the form factor $B$ is seen to be $B = (\delta_{\mu\nu}/n)B_{\mu\nu} = B_{\mu\mu}/n$. In this case the identities (9)–(12) simplify to become

$$\frac{\delta_{\mu\nu}}{n}[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_{L,R}]_1[\gamma_\rho\gamma_\nu\gamma_\sigma\gamma_{L,R}]_2 = \frac{(5n-4)}{n}[\gamma_\alpha\gamma_{L,R}]_1[\gamma_\alpha\gamma_{L,R}]_2 \quad (14)$$

$$\frac{\delta_{\mu\nu}}{n}[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_{L,R}]_1[\gamma_\rho\gamma_\nu\gamma_\rho\gamma_{L,R}]_2 = [\gamma_\alpha\gamma_{L,R}]_1[\gamma_\alpha\gamma_{R,L}]_2 \quad (15)$$

$$\frac{\delta_{\mu\nu}}{n}[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_{L,R}]_1[\gamma_\sigma\gamma_\nu\gamma_\rho\gamma_{L,R}]_2 = [\gamma_\alpha\gamma_{L,R}]_1[\gamma_\alpha\gamma_{L,R}]_2 \quad (16)$$

$$\frac{\delta_{\mu\nu}}{n}[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_{L,R}]_1[\gamma_\sigma\gamma_\nu\gamma_\rho\gamma_{R,L}]_2 = \frac{(5n-4)}{n}[\gamma_\alpha\gamma_{L,R}]_1[\gamma_\alpha\gamma_{R,L}]_2 \quad (17)$$

### 5.1 Example

The diagram shown in Fig. 1(b) occurs in the $\mathcal{O}(N_f\alpha^2)$ corrections to muon decay. Using (1) and (13) the resulting the Feynman diagram can be written as

$$\frac{g^2}{16c_\theta^2} \left( \frac{g^2}{16\pi^2} \right)^2 [\gamma_\alpha\gamma_{L,\mu}[\gamma_\alpha\gamma_{L,\nu}] \frac{n-2}{n-1} \int \frac{d^n p}{i\pi^2} \int \frac{d^n q}{i\pi^2} \frac{1}{[p^2 + M_Z^2][q^2 + M_W^2] q^2 (q + p)^2}$$

$$= \frac{g^2}{16c_\theta^2} \left( \frac{g^2}{16\pi^2} \right)^2 [\gamma_\alpha\gamma_{L,\mu}[\gamma_\alpha\gamma_{L,\nu}] \frac{n-2}{n-1} \right]$$

$$\times \left\{ I(0,1,1,1,n,M_Z^2) - I(0,1,1,1,n,M_W^2) \right\}$$

$$= \frac{g^2}{24M_W^2 s_\theta^2} \left( \frac{g^2}{16\pi^2} \right) [\gamma_\alpha\gamma_{L,\mu}[\gamma_\alpha\gamma_{L,\nu}] (\pi M_W^2)^{-2\epsilon} \Gamma(\epsilon)$$

$$\times \left\{ (s_\theta^2 + \ln c_\theta^2) \left( 1 + \epsilon \left( \frac{8}{3} - \gamma \right) \right) + \epsilon \ln^2 c_\theta^2 \right\}$$

up to $\mathcal{O}(1)$ in $\epsilon = 2 - n/2$.

### 6 Derivatives of Self-energy Diagrams

The derivatives of self-energies with respect to the square of the external momentum, $k^2$, are required for the calculation physical processes. They are often thought of as wavefunction renormalization factors although they occur even when wavefunction renormalization has not been explicitly performed [14]. As we are concerned with
low-energy processes and massless fermions, these derivatives will only be required at \( k^2 = 0 \). In that case the scalar integral representation may be obtained by the following method.

For a vector boson the self-energy may be written in the form
\[
\Pi_{\mu\nu}(k^2) = \left( \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) \Pi_T(k^2) + \left( \frac{k_{\mu}k_{\nu}}{k^2} \right) \Pi_L(k^2)
\]
(18)

with \( \Pi_T \) and \( \Pi_L \) being the transverse and longitudinal form-factors respectively.

\[
\Pi_T(k^2) = \frac{1}{n-1} \left( \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) \Pi_{\mu\nu}(k^2)
\]
(19)

\[
\Pi_L(k^2) = \left( \frac{k_{\mu}k_{\nu}}{k^2} \right) \Pi_{\mu\nu}(k^2).
\]
(20)

For massless fermions self-energy diagrams take the form
\[
\Sigma(k) = i \gamma \left( F_L(k^2) \gamma_L + F_R(k^2) \gamma_R \right)
\]
(21)

and
\[
F_{L,R}(k^2) = -\frac{i}{2k^2} \text{Tr} \{ \Sigma(k) \gamma_{R,L} \}
\]
(22)

In either case given expressions for the self-energies in the form of tensor integrals, the scalar integral representations for the corresponding form factors \( \Pi_L, \Pi_T \) or \( F_L, F_R \) are easily obtained. In general, then, these scalar integral representations may be written
\[
F(k^2) = \int \frac{d^n p}{i\pi^2} \int \frac{d^n q}{i\pi^2} \frac{F(k^2, p^2, q^2, p \cdot k, k \cdot p, q \cdot p)}{[(k + p)^2 + M_1^2]^N_1 [(k + q)^2 + M_2^2]^N_2 [(k + p + q)^2 + M_3^2]^N_3}
\]
(23)

where \( N_1, N_2 \) and \( N_3 \) are integer powers.

In the region of small \( k^2 \) we may expand the propagators as a power series in \( (k \cdot p) \) and \( (k \cdot q) \). Thus, for example,
\[
\frac{1}{[(k + p)^2 + M_1^2]^N_2} = \frac{1}{[k^2 + p^2 + M_1^2]^N_2} \left( 1 - 2N_2 \frac{(k \cdot p)}{k^2 + p^2 + M_1^2} + \ldots \right)
\]

\[
\frac{1}{[(k + p + q)^2 + M_3^2]^N_3} = \frac{1}{[k^2 + (p + q)^2 + M_3^2]^N_3} \left( 1 - 2N_3 \frac{(k \cdot p) + (k \cdot q)}{k^2 + (p + q)^2 + M_3^2} + \ldots \right)
\]

with the result that the integral in eq. (23) becomes
\[
F(k^2) = \int \frac{d^n p}{i\pi^2} \int \frac{d^n q}{i\pi^2} \sum_{i,j} (k \cdot p)^i (k \cdot q)^j F_{ij}(k^2, p^2, q^2).
\]
(24)
Upon integration terms in which \( i + j \) is odd vanish. Terms in the integrand with \( i + j = 2 \) are \((k \cdot v_1)(k \cdot v_2)F(k^2, q^2, p^2)\) in which \( v_1 \) and \( v_2 \) are either \( p \) or \( q \). Because \( F \) is a function only of squared momenta the integration of \( p_\mu q_\nu F(k^2, q^2, p^2) \) can only yield a result proportional to \( \delta_{\mu\nu} \). Its coefficient may be extracted using the projection operator \( \delta_{\mu\nu}/n \) as in section 2. It follows then that the terms \((k \cdot v_1)(k \cdot v_2)F(k^2, q^2, p^2)\) in the integrand may be replaced by \( k^2(v_1 \cdot v_2)/n F(k^2, q^2, p^2) \) without changing the integral.

In a similar way for \( i + j = 4 \) one can make the replacement
\[
(k \cdot v_1)(k \cdot v_2)(k \cdot v_3)(k \cdot v_4)F(k^2, q^2, p^2) \rightarrow \frac{k^4(v_1 \cdot v_2)(v_3 \cdot v_4) + (v_1 \cdot v_3)(v_2 \cdot v_4) + (v_1 \cdot v_4)(v_2 \cdot v_3)F(k^2, q^2, p^2)}{n(n + 2)}
\]
where again the \( v_i \) are \( p \) or \( q \).

The integral is thus transformed into a form whose only \( k \) dependence is enters only through \( k^2 \).

Differentiation of the integrand with respect to \( k^2 \) can now be performed and setting \( k^2 = 0 \) yields a result expressible in terms of the master integral (3).

For most practical purposes \( i + j = 4 \) is as high as one needs to go in the series expansion. The general term in this series can be found in [13] and requires repeated application of the momentum-space d’Alembertian operator.

### 6.1 Example

A self-energy diagram for a massless fermion is shown in Fig.1c and may be written
\[
\Sigma(k) = i k A_L(k^2) \gamma_L.
\]

After obtaining the scalar integral representation for \( A_L(k^2) \) and expanding the propagator as in (24) up to \((k \cdot p)^2\) one obtains
\[
A_L(k^2) = \left. \frac{1}{4k^2} \left( \frac{g^2}{16\pi^2} \right)^2 \frac{(n-2)}{(n-1)} \int \frac{d^n p}{i\pi^2} \int \frac{d^n q}{i\pi^2} \frac{1}{[p^2 + M_W^2]^2 q^2 (q + p)^2} \times \left\{ \frac{(3-n)k^2}{k^2 + p^2} + \frac{2(p^2 - k^2)(k^2 + (n-2)p^2)}{p^2(k^2 + p^2)^3} (k \cdot p) \right\} \right|_{k^2=0}
\]

After the replacement \((k \cdot p)^2 \rightarrow k^2 p^2/n\) the derivative with respect to \( k^2 \) at \( k^2 = 0 \) is found to be
\[
\left. \frac{\partial A_L(k^2)}{\partial k^2} \right|_{k^2=0} = \left. \left( \frac{g^2}{16\pi^2} \right)^2 \frac{(n-2)(n-4)}{4n} \int \frac{d^n p}{i\pi^2} \int \frac{d^n q}{i\pi^2} \frac{1}{[p^2 + M_W^2]^2 q^2 (q + p)^2} \right|_{k^2=0}
\]
\[
= \left. \left( \frac{g^2}{16\pi^2} \right)^2 \frac{(n-2)(n-4)}{4n} I(0, 2, 1, 1, n, M_W^2) \right|_{k^2=0}
\]
\[
= -\left( \frac{g^2}{16\pi^2} \right)^2 (\pi M_W^2)^{n-3} \frac{1}{2n} \Gamma(5-n) \Gamma \left( 2 - \frac{n}{2} \right) \Gamma \left( \frac{n}{2} - 1 \right)
\]
exactly. This result is related to vertex diagrams by Ward identities that serve as a useful check.

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