The decay of the omega meson at finite temperature

Indrajit Mitra and Abhee K. Dutt-Mazumder

Saha Institute of Nuclear Physics
1/AF, Bidhan Nagar
Calcutta - 700 064, India

Abstract

The decay width of the $\omega$ meson at finite temperature is calculated using the Gell-Mann Sharp Wagner model of $\rho$ pole dominance. Effective masses of the $\rho$ and $\omega$ are determined within the framework of real-time formalism of finite temperature field theory. It is shown that even though the mass of the $\omega$-meson decreases with temperature, its decay width increases because of an interesting interplay between the phase space factors, the transition matrix element and Bose enhancement.

\footnote{email: abhee@tnp.saha.ernet.in}
Properties of nuclear matter at finite temperature have recently acquired particular interest as in the laboratory such exotic conditions can now be created by colliding heavy nuclei. In this context, therefore, the behaviour of hadrons at high temperature and/or high density have been investigated by several authors [1, 2, 3]. In particular, efforts have been directed towards the unraveling of the properties of light vector mesons at finite temperature because they show up in the dilepton spectra observed experimentally. Li, Ko and Brown have shown that the dropping vector meson masses can account for the enhanced yield of the dileptons in the low mass region as observed in SPS, CERN [4].

Issues such as whether the vector meson masses go up or down with temperature or how their decay widths change with increasing T, have been a source of intense debate in recent years [3, 5]. In fact models used to examine the thermal properties of these vector mesons cover a wide range, starting from hadronic models to calculations involving quarks as fundamental constituent of mesons. If the phase transition to a QGP phase is primarily one of deconfinement, argues Pisarski, then this may be modeled by an effective bag constant which decreases with temperature and the effective mass of the $\rho$ meson, like all hadronic bound states, should then decrease with temperature. A gauged sigma model on the contrary predicts that masses of $\rho$, $\omega$ and $\phi$ increase monotonically with T, while an alternate scenario has been suggested in Ref.[6]. Recently, Gale and Kapusta, using an effective model involving pseudoscalar and vector meson, have showed that the effective masses of vector meson increase with temperature at leading order of $T^2$ [2], while in QCD sum rules, it has been demonstrated that with increasing temperature vector and axial vector masses decrease with the exception of

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the $\omega$-meson mass which is independent of temperature \[7, 8\]. The variation of their decay width with temperature, has also been pointed out in Ref.\[3\].

In the present letter we focus in particular on the properties of the $\omega$ meson, the shift in its mass and the corresponding change in its decay width in a thermal bath. We also calculate the effective $\rho$ meson mass at finite temperature as we use the Gell-Mann Sharp Wagner (GSW) model of $\rho$ pole dominance to calculate the matrix element involved in the decay rate for $\omega \rightarrow 3\pi$. In fact, this has a significant consequence on the width of $\omega$ meson as shall be explained later.

Vector meson masses, in the present context are mainly modified by the vacuum excitation of a nucleon-antinucleon pair at finite temperature. The $\omega$ meson mass, (as also that of the $\rho$) decreases with increasing temperature basically due to the nucleon loop and the fact that the effective nucleon mass (generated by the $\sigma$ meson mean field) is reduced. Although because of the reduction in mass of the $\omega$ meson the phase space is suppressed, the downward shift in the $\rho$ pole position at finite $T$ results in an increase of the matrix element appearing in the $\omega$ decay width at finite $T$. Furthermore, the stimulated emission into a pion gas, commonly referred to as the Bose Enhancement (BE) would give rise to a further increase as in the present context the decay is assumed to take place in a thermal bath and the $\omega$ meson dominantly decays into three pions. These factors in fact finally lead to a broader width for $\omega$ meson in a heat bath compared to what is observed at zero temperature. This particular feature, which has not been remarked upon earlier, is in contrast to what has been assumed in Ref.\[4\] that the $\omega$ meson decay width is proportional to the $\omega$ meson mass (which decreases with temperature).
To calculate the decay width of the ω meson, as has already been mentioned, we use the GSW model of ρ pole dominance, i.e. \( \omega \rightarrow \rho \pi \rightarrow 3\pi \) \[9, 10\]. The \( \omega \rho \pi \) interaction Lagrangian can be taken to be

\[ \mathcal{L} = g_{\omega \rho \pi} \varepsilon_{\mu \nu \alpha \beta} \partial^\mu \omega^\nu \partial^\alpha \rho^\beta \pi \]  

(1)

The matrix element, therefore, can be calculated considering the three channels depending upon the charged state of the intermediate ρ meson.

\[ \mathcal{M} = 2g_{\omega \rho \pi}g_{\rho \pi \pi}m_\omega^* \left[ \frac{\varepsilon^\cdot(\vec{k}_+ \times \vec{k}_-)}{(P-k_0)^2-m_\rho^2} - \frac{\varepsilon^\cdot(\vec{k}_- \times \vec{k}_0)}{(P-k_-)^2-m_\rho^2} + \frac{\varepsilon^\cdot(\vec{k}_- \times \vec{k}_0)}{(P-k_+)^2-m_\rho^2} \right] \]  

(2)

where the asterisk reminds us that instead of the free meson masses the thermal masses have to be used which are determined from the poles of the dressed propagator at finite temperature as discussed below. Here P represents the momentum of the ω meson, the matrix element is written in its rest frame. i.e. \( P = (m_\omega^*, 0, 0, 0) \), and \( k_\alpha \) with \( \alpha = +, 0, - \) denoting the momenta of \( \pi^+, \pi^0, \pi^- \). The \( \omega \rho \pi \) coupling constant can be determined by using the relation \( g_{\omega \rho \pi}^2 = \frac{g_{\rho \pi \pi}^2}{8\pi f_\pi^2} \) \[11\] which, in fact, is consistent with phenomenological studies, while the \( g_{\rho \pi \pi} \) coupling constant is determined by fitting the \( \rho \rightarrow \pi \pi \) decay width. For the coupling constants as usual we do not make any finite temperature correction. The square of the matrix element averaged over the polarization states of the ω meson is given by

\[ |\mathcal{M}|^2 = \frac{4}{3}g_{\omega \rho \pi}^2g_{\rho \pi \pi}m_\omega^* m_{\rho}^2 (\vec{k}_+ \times \vec{k}_-)^2 \sum_{\alpha=0,+,-} \left[ \frac{1}{(P-k_\alpha)^2-m_\rho^2} \right]^2 \]  

(3)

where the three momentum conservation condition has been used.

The decay width in the hot pion gas may be written as

\[ \Gamma_\omega(T) = \frac{1}{8m_\omega^*(2\pi)^4} \int |\mathcal{M}|^2 f_{BE}(E_+, E_-) dE_+ dE_- \]  

(4)
The factor \( f_{BE}(E_+, E_-) = \Pi(1 + n(E_\alpha)) \) accounts for the BE due to the induced emission of pions where \( n(E_\alpha) \) is the pion distribution function with \( E_0 = m_\omega^* - E_+ - E_- \).

Now, to calculate the decay width integrations have to be performed over \( E_+, E_- \) in the range \( E_+(\text{max}) = \frac{m^2 - 3m_\rho^2}{2m_\rho^*} \) and \( E_+(\text{min}) = m_\pi \), where the limits of \( E_- \) are \( \frac{m_\rho^* - E_+}{2} \pm \frac{1}{2} \left\{ \sqrt{\left(E_+^2 - m_\rho^2\right)(m_\rho^2 - 2E_+m_\rho^* - 3m_\pi^2)} \right\} \).

In the present study the effective thermal masses of the vector mesons are generated, as has already been mentioned, by the nucleon-antinucleon excitation at finite temperature. We need to find the effective masses of both the \( \rho \) and \( \omega \) meson. The vector meson-nucleon interaction Lagrangian may be written as

\[
\mathcal{L}_{\text{int}} = g_\alpha [\bar{N}\gamma_\mu \tau^\alpha N - \frac{\kappa_\alpha}{2M}\bar{N}\sigma_{\mu\nu}\tau^\alpha N\partial^\nu]V_\mu^{\alpha} \tag{5}
\]

where \( V_\alpha = \{\omega, \rho\} \), \( \alpha \) running from 0 to 3, indexes quantities relevant for \( \omega \) when \( \alpha = 0 \), while \( \alpha = 1 \) to 3 refers to the \( \rho \) meson; \( \tau^0 = 1 \) and \( \tau^i \) are the isospin Pauli matrices. The coupling constants \( g_\rho \) and \( g_\omega \) as also the "anomalous" or tensor-coupling parameters \( \kappa_\rho \) and \( \kappa_\omega \) may be estimated \([12]\) from the Vector Meson Dominance (VMD) model for nucleon form-factors or from the fitting of the nucleon-nucleon interaction data as done by the Bonn group \([13]\). In view of the relatively small value of the iso-scalar magnetic moment of the nucleon as compared to the iso-vector part, the tensor coupling is more important for the \( \rho \) than it is for the \( \omega \) meson.

The effective vector meson masses, as has already been mentioned, are determined from the poles of the dressed propagator or the zeros of the inverse propagator

\[
D_{\mu\nu}^{-1} = D_{0\mu\nu}^{-1} + \Pi_{\mu\nu} \tag{6}
\]
where the polarization tensor is given by

$$\Pi_{\mu \nu}^{\alpha \beta} = -i \int \frac{d^4k}{(2\pi)^4} \text{Tr}[i\Gamma_\mu^\alpha iS_F(k + q)i\Gamma_\nu^\beta iS_F(k)]$$  \hspace{1cm} (7)

here \( \Gamma \) represents the appropriate vertex factors and \( \alpha, \beta \) are the isospin indices.

The nucleon propagator in the real time formalism at finite temperature is represented by

$$S_F(k) = \frac{k + M^*}{k^2 - M^{*2} + i\epsilon} + \frac{k + M^*}{e^{\beta|u.k|} + 1}2\pi i\delta(k^2 - M^{*2})$$ \hspace{1cm} (8)

\( u_\mu = (1, 0, 0, 0) \) here defines the thermal bath frame. \( M^* \) is the effective nucleon mass generated by the sigma meson exchange tadpole at finite temperature. The separation of the nucleon propagator [eq.(8)] into the T-dependent \( (S^T_F) \) and the zero temperature contribution \( (S^0_F) \) is particularly useful.

Now clearly, in the polarization tensor there are terms arising from \( S^0_F S^0_F, S^T_F S^0_F + S^0_F S^T_F \) and \( S^T_F S^T_F \). The first combination corresponds to the zero temperature part of the self-energy represented by \( Q_{\mu \nu} \Pi^0_F \), where \( Q_{\mu \nu} = (-g_{\mu \nu} + \frac{u_\mu u_\nu}{q^2}) \) is the relevant projection tensor. The \( \Pi^0 \) integral is divergent and is same as that at zero temperature except that now it involves instead of the free nucleon mass the thermal nucleon mass. We regularize it using dimensional regularization and use the renormalization scheme that the on-shell vacuum contribution to \( \Pi_{\mu \nu} \) vanishes at zero temperature. Expressed mathematically, we impose the condition \( \partial^n \Pi^0(q^2)/\partial(q^2)^n|_{M^* \rightarrow M, q^2 = m^2} = 0 \ (n = 0, 1, 2..., \infty) \) similar to what one adopts at finite density \( (T = 0) \) calculations here and henceforth \( \Pi^0(q^2) \) denoted the renormalized quantity. It may be mentioned that as we are presently dealing with the fermion loop there is no additional source of divergence in the present case at finite temperature. Therefore the polarization tensor can be split up into \( T=0 \) and
\( T \neq 0 \) part in the following way

\[
\Pi_{\mu\nu}^0 = Q_{\mu\nu} \Pi_{\mu\nu}^0(q^2) + \Pi_{\mu\nu}^{(T)}
\]  

(9)

Explicit expressions are given by

\[
\Pi_{\mu\nu}^{v(T)} = \frac{2g_v^2}{\pi^3} \int \frac{d^3k}{E^*(k)} \frac{1}{e^{\beta E^*(k)} + 1} \frac{K_{\mu\nu} q^2 - Q_{\mu\nu} (k \cdot q)^2}{q^4 - 4(k \cdot q)^2}
\]  

(10)

\[
\Pi_{\mu\nu}^{t(T) + tv(T)} = \frac{2g_v^2}{\pi^3} \left( \frac{\kappa M^*}{4M} \right)^2 2q^4 Q_{\mu\nu} \int \frac{d^3k}{E^*(k)} \frac{1}{e^{\beta E^*(k)} + 1} \frac{1}{q^4 - 4(k \cdot q)^2}
\]

(11)

\[
\Pi_{\mu\nu}^{u(T)} = -\frac{2g_v^2}{\pi^3} \left( \frac{\kappa}{4M} \right)^2 (4q^4) \int \frac{d^3k}{E^*(k)} \frac{1}{e^{\beta E^*(k)} + 1} \frac{K_{\mu\nu} + Q_{\mu\nu} M^*}{q^4 - 4(k \cdot q)^2}
\]

(12)

where \( K_{\mu\nu} = (k_{\mu} - \frac{k \cdot q}{q^2} q_{\mu})(k_{\nu} - \frac{k \cdot q}{q^2} q_{\nu}) \) and the superscripts v & t represent the vector and tensor coupling contributions.

The effective masses can now be determined from zeroes of the inverse propagator eq.(6) for \( \rho \) and \( \omega \) meson which then can be used to determine the \( \omega \) meson decay width in a hot pion gas. Here we take only the nucleon loop as others like \( \rho - \pi \) or \( \omega - \pi \) loops for the \( \omega \) and \( \rho \) respectively do not contribute much to the mass shift as compared to the shift brought about by the one considered here. Actually in the present context the vector meson mass shift is inextricably related, as mentioned before, to the nucleon mass shift at finite T because of the \( \sigma \) meson mean field. Masses of these hadrons are reduced mainly because the nucleon mass decreases. This is depicted in Fig.(1)
Fig. 1 The variation of the effective nucleon and meson masses with temperature. Solid, dashed and dotted lines represent effective nucleon, rho and omega meson masses respectively.

As has already been mentioned we have calculated the vector meson effective masses at finite temperature using real-time formalism. For the $\rho$ meson effective mass our findings are consistent with what has been obtained using the imaginary-time formalism in Ref[1].

We observe from Fig(2) that although the mass of the $\omega$ meson goes down with temperature thereby suppressing the phase space, the decay width shows opposite trend because of the reduction of the $\rho$ meson mass. This is a new observation which indicates that we cannot take the $\omega$ meson decay width to be proportional to its mass in contrast to what has been used in Ref[4]. Another important observation is that the Bose enhancement causes an appreciable increase of the decay width at finite temperature in a pionic medium.
**Fig. 2** The variation of the decay width of omega meson with temperature with and without the Bose enhancement factor

In conclusion we observe that $\omega$ meson decay rate in a thermal bath increases because of an interesting interplay between the phase space factor (which reduces because of the decrease in $\omega$ meson mass), the transition matrix element (which increases because of the downward shift of the $\rho$ meson pole) and Bose enhancement (caused by the stimulated emission of pions at finite temperature). All these factors are likely to have significant bearings on the dilepton spectra observed in experiments involving heavy ions. Such investigations are in progress.

We sincerely acknowledge useful discussion with B. Dutta-Roy, B. Sinha, and J. Alam.
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1email: abhee@tnp.saha.ernet.in
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Issues such as whether the vector meson masses go up or down with temperature or how their decay widths change with increasing T, have been a source of intense debate in recent years [3, 5]. In fact models used to examine the thermal properties of these vector mesons cover a wide range, starting from hadronic models to calculations involving quarks as fundamental constituent of mesons. If the phase transition to a QGP phase is primarily one of deconfinement, argues Pisarski, then this may be modeled by an effective bag constant which decreases with temperature and the effective mass of the $\rho$ meson, like all hadronic bound states, should then decrease with temperature. A gauged sigma model on the contrary predicts that masses of $\rho$, $\omega$ and $\phi$ increase monotonically with T, while an alternate scenario has been suggested in Ref.[6]. Recently, Gale and Kapusta, using an effective model involving pseudoscalar and vector meson, have showed that the effective masses of vector meson increase with temperature at leading order of $T^2$ [2], while in QCD sum rules, it has been demonstrated that with increasing temperature vector and axial vector masses decrease with the exception of
the \( \omega \)-meson mass which is independent of temperature \([7, 8]\). The variation of their decay width with temperature, has also been pointed out in Ref.\([3]\).

In the present letter we focus in particular on the properties of the \( \omega \) meson, the shift in its mass and the corresponding change in its decay width in a thermal bath. We also calculate the effective \( \rho \) meson mass at finite temperature as we use the Gell-Mann Sharp Wagner (GSW) model of \( \rho \) pole dominance to calculate the matrix element involved in the decay rate for \( \omega \rightarrow 3\pi \). In fact, this has a significant consequence on the width of \( \omega \) meson as shall be explained later.

Vector meson masses, in the present context are mainly modified by the vacuum excitation of a nucleon-antinucleon pair at finite temperature. The \( \omega \) meson mass, (as also that of the \( \rho \)) decreases with increasing temperature basically due to the nucleon loop and the fact that the effective nucleon mass (generated by the \( \sigma \) meson mean field) is reduced. Although because of the reduction in mass of the \( \omega \) meson the phase space is suppressed, the downward shift in the \( \rho \) pole position at finite T results in an increase of the matrix element appearing in the \( \omega \) decay width at finite T. Furthermore, the stimulated emission into a pion gas, commonly referred to as the Bose Enhancement (BE) would give rise to a further increase as in the present context the decay is assumed to take place in a thermal bath and the \( \omega \) meson dominantly decays into three pions. These factors in fact finally lead to a broader width for \( \omega \) meson in a heat bath compared to what is observed at zero temperature. This particular feature, which has not been remarked upon earlier, is in contrast to what has been assumed in Ref.\([4]\) that the \( \omega \) meson decay width is proportional to the \( \omega \) meson mass (which decreases with temperature).
To calculate the decay width of the $\omega$ meson, as has already been mentioned, we use the GSW model of $\rho$ pole dominance, i.e. $\omega \to \rho \pi \to 3\pi$ [9, 10]. The $\omega \rho \pi$ interaction Lagrangian can be taken to be

$$\mathcal{L} = g_{\omega \rho \pi} \epsilon_{\mu
u\alpha\beta} \partial^\mu \omega^\nu \partial^\alpha \rho^\beta \pi$$

The matrix element, therefore, can be calculated considering the three channels depending upon the charged state of the intermediate $\rho$ meson.

$$\mathcal{M} = 2g_{\omega \rho \pi} g_{\rho \pi \pi} m_\omega^* \left[ \frac{\vec{\epsilon} \cdot (\vec{k}_+ \times \vec{k}_-)}{(P - k_0)^2 - m_\rho^2} - \frac{\vec{\epsilon} \cdot (\vec{k}_+ \times \vec{k}_0)}{(P - k_-)^2 - m_\rho^2} + \frac{\vec{\epsilon} \cdot (\vec{k}_- \times \vec{k}_0)}{(P - k_+)^2 - m_\rho^2} \right]$$

where the asterisk reminds us that instead of the free meson masses the thermal masses have to be used which are determined from the poles of the dressed propagator at finite temperature as discussed below. Here $P$ represents the momentum of the $\omega$ meson, the matrix element is written in its rest frame. i.e. $P = (m_\omega^*, 0, 0, 0)$, and $k_\alpha$ with $\alpha = +, 0, -$ denoting the momenta of $\pi^+, \pi^0, \pi^-$. The $\omega \rho \pi$ coupling constant can be determined by using the relation $g_{\omega \rho \pi}^2 = \frac{9g_{\rho \pi \pi}^2}{8\pi f_\pi^2}$ [1] which, in fact, is consistent with phenomenological studies, while the $g_{\rho \pi \pi}$ coupling constant is determined by fitting the $\rho \to \pi\pi$ decay width. For the coupling constants as usual we do not make any finite temperature correction. The square of the matrix element averaged over the polarization states of the $\omega$ meson is given by

$$|\tilde{\mathcal{M}}|^2 = \frac{4}{3} g_{\omega \rho \pi}^2 g_{\rho \pi \pi}^2 m_\omega^* (\vec{k}_+ \times \vec{k}_-)^2 \sum_{\alpha=0,+,,-} \frac{1}{(P - k_\alpha)^2 - m_\rho^2}$$

where the three-momentum conservation condition has been used.

The decay width in the hot pion gas may be written as

$$\Gamma_\omega(T) = \frac{1}{8m_\omega^*(2\pi)^4} \int |\tilde{\mathcal{M}}|^2 f_{BE}(E_+, E_-) dE_+ dE_-$$
The factor \( f_{BE}(E_+, E_-) = \Pi_k (1 + n(E_\alpha)) \) accounts for the BE due to the induced emission of pions where \( n(E_\alpha) \) is the pion distribution function with \( E_0 = m_\omega^* - E_+ - E_- \).

Now, to calculate the decay width integrations have to be performed over \( E_+, E_- \) in the range \( E_{+(\text{max})} = \frac{m_\pi^2 - 3m_\pi^2}{2m_\pi^2} \) and \( E_{+(\text{min})} = m_\pi \), where the limits of \( E_- \) are \( \frac{(m_\pi^*-E_+)}{2} \pm \frac{1}{2} \sqrt{\frac{(E_+^2-m_\pi^2)}{(m_\pi^2+m_\pi^*-2E_+m_\pi^*)}} \).

In the present study the effective thermal masses of the vector mesons are generated, as has already been mentioned, by the nucleon-antinucleon excitation at finite temperature. We need to find the effective masses of both the \( \rho \) and \( \omega \) meson. The vector meson-nucleon interaction Lagrangian may be written as

\[
\mathcal{L}_{\text{int}} = g_\alpha [\bar{N} \gamma_\mu \tau^\alpha N - \frac{\kappa_\alpha}{2M} \bar{N} \sigma_{\mu\nu} \tau^\alpha N \partial^\nu] V^\mu_{\alpha} \tag{5}
\]

where \( V_{\alpha} = \{\omega, \rho\} \), \( \alpha \) running from 0 to 3, indexes quantities relevant for \( \omega \) when \( \alpha = 0 \), while \( \alpha = 1 \) to 3 refers to the \( \rho \) meson; \( \tau^0 = 1 \) and \( \tau^i \) are the isospin Pauli matrices. The coupling constants \( g_\rho \) and \( g_\omega \) as also the “anomalous” or tensor-coupling parameters \( \kappa_\rho \) and \( \kappa_\omega \) may be estimated [12] from the Vector Meson Dominance (VMD) model for nucleon form-factors or from the fitting of the nucleon-nucleon interaction data as done by the Bonn group [13]. In view of the relatively small value of the iso-scalar magnetic moment of the nucleon as compared to the iso-vector part, the tensor coupling is more important for the \( \rho \) than it is for the \( \omega \) meson.

The effective vector meson masses, as has already been mentioned, are determined from the poles of the dressed propagator or the zeros of the inverse propagator

\[
D_{\mu\nu}^{-1} = D_{0\mu\nu}^{-1} + \Pi_{\mu\nu} \tag{6}
\]
where the polarization tensor is given by

$$\Pi_{\mu\nu}^{\alpha\beta} = -i \int \frac{d^4k}{(2\pi)^4} \text{Tr}[i\Gamma_\mu^\alpha iS_F(k + q)i\bar{\Gamma}_\nu^\beta iS_F(k)]$$

(7)

here $\Gamma$, $\bar{\Gamma}$ represent the appropriate vertex factors and $\alpha, \beta$ are the isospin indices.

The nucleon propagator in the real time formalism at finite temperature is represented by

$$S_F(k) = \frac{\slashed{k} + M^*}{k^2 - M^*^2 + i\epsilon} + \frac{\slashed{k} + M^*}{e^{\beta |u.k|} + 1} 2\pi i \delta(k^2 - M^*^2)$$

(8)

$u_\mu = (1, 0, 0, 0)$ here defines the thermal bath frame. $M^*$ is the effective nucleon mass generated by the sigma meson exchange tadpole at finite temperature. The separation of the nucleon propagator [eq.(8)] into the $T$-dependent ($S^T_F$) and the zero temperature contribution ($S^0_F$) is particularly useful.

Now clearly, in the polarization tensor there are terms arising from $S^0_FS^0_F$, $S^T_FS^0_F + S^0_FS^T_F$ and $S^T_FS^T_F$. The first combination corresponds to the zero temperature part of the self-energy represented by $Q_{\mu\nu}\Pi^0$, where $Q_{\mu\nu} = (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2})$ is the relevant projection tensor. The $\Pi^0$ integral is divergent and is same as that at zero temperature except that now it involves instead of the free nucleon mass the thermal nucleon mass. We regularize it using dimensional regularization and use the renormalization scheme that the on-shell vacuum contribution to $\Pi_{\mu\nu}$ vanishes at zero temperature. Expressed mathematically, we impose the condition $\partial^n\Pi^0(q^2)/\partial(q^2)^n|_{M^* \rightarrow M, q^2 \rightarrow m^2} = 0$ ($n = 0, 1, 2..., \infty$) similar to what one adopts at finite density ($T = 0$) calculations [12] (here and henceforth $\Pi^0(q^2)$ denotes the renormalized quantity). It may be mentioned that as we are presently dealing with the fermion loop there is no additional source of divergence in the present case at finite temperature. Therefore the polarization tensor can be split up into $T=0$ and
$T \neq 0$ part in the following way

$$\Pi^0_{\mu\nu} = Q_{\mu\nu}\Pi^0(q^2) + \Pi^{(T)}_{\mu\nu}$$

(9)

Explicit expressions are given by

$$\Pi^{v(\mu)}_{\mu\nu} = \frac{2g^2_v}{\pi^3} \int \frac{d^3k}{E^*(k)e^{\beta E^*(k)}} \frac{1}{q^4 - 4(k \cdot q)^2}$$

(10)

$$\Pi^{t(\mu)}_{\mu\nu} = \frac{2g^2_v}{\pi^3} \left(\frac{\kappa M^*}{4M}\right)^2 q^4 Q_{\mu\nu} \int \frac{d^3k}{E^*(k)e^{\beta E^*(k)}} \frac{1}{q^4 - 4(k \cdot q)^2}$$

(11)

$$\Pi^{u(\mu)}_{\mu\nu} = -\frac{2g^2_v}{\pi^3} \left(\frac{\kappa}{4M}\right)^2 (4q^4) \int \frac{d^3k}{E^*(k)e^{\beta E^*(k)}} \frac{1}{q^4 - 4(k \cdot q)^2}$$

(12)

where $\mathcal{K}_{\mu\nu} = (k_{\mu} - \frac{k \cdot q}{q^2} q_{\mu})(k_{\nu} - \frac{k \cdot q}{q^2} q_{\nu})$ and the superscripts v & t represent the vector and tensor coupling contributions.

The effective masses can now be determined from zeroes of the inverse propagator eq.(6) for $\rho$ and $\omega$ meson which then can be used to determine the $\omega$ meson decay width in a hot pion gas. Here we take only the nucleon loop as others like $\rho - \pi$ or $\omega - \pi$ loops for the $\omega$ and $\rho$ respectively do not contribute much to the mass shift as compared to the shift brought about by the one considered here. Actually in the present context the vector meson mass shift is inextricably related, as mentioned before, to the nucleon mass shift at finite $T$ because of the $\sigma$ meson mean field. Masses of these hadrons are reduced mainly because the nucleon mass decreases. This is depicted in Fig.(1)

As has already been mentioned we have calculated the vector meson effective masses at finite temperature using real-time formalism. For the $\rho$ meson effective mass our findings are consistent with what has been obtained using the imaginary-time formalism in Ref[1].
We observe from Fig(2) that although the mass of the $\omega$ meson goes down with temperature thereby suppressing the phase space, the decay width shows opposite trend because of the reduction of the $\rho$ meson mass. This is a new observation which indicates that we cannot take the $\omega$ meson decay width to be proportional to its mass in contrast to what has been used in Ref[4]. Another important observation is that the Bose enhancement causes an appreciable increase of the decay width at finite temperature in a pionic medium.

In conclusion we observe that $\omega$ meson decay rate in a thermal bath increases because of an interesting interplay between the phase space factor (which reduces because of the decrease in $\omega$ meson mass), the transition matrix element (which increases because of the downward shift of the $\rho$ meson pole) and Bose enhancement (caused by the stimulated emission of pions at finite temperature). All these factors are likely to have significant bearings on the dilepton spectra observed in experiments involving heavy ions. Such investigations are in progress.

We sincerely acknowledge useful discussion with B. Dutta-Roy, B. Sinha, and J. Alam.
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Figure captions

1. Fig. 1: The variation of the effective nucleon and meson masses with temperature. Solid, dashed and dotted lines represent effective nucleon, rho meson and omega meson masses respectively.

2. Fig. 2: The variation of the decay width of omega meson with temperature with and without the Bose enhancement factor.