A novel method of prescribed constraint control without initial condition of nonlinear systems

Hui Liu, Xiaohua Li, Xiaoping Liu

Abstract—A novel constraint control strategy without initial condition of constrained variables is investigated based on backstepping technique for nonlinear systems. In this paper, the novel constraint control strategy is presented for a class of strict-feedback nonlinear systems with actuator saturation and external disturbances by using a nonlinear mapping and a novel performance constraint function. In this control strategy, there are two prescribed constraint functions, the design of these functions is not related to the initial conditions of the constrained variables. Unlike the existing constraint control method without initial condition, the proposed method gives a new solution. It can guarantee that the constraint variable gets into a prescribed constraint region from any initial value no later than a setting time. And the setting time is a design parameter, it can be set arbitrarily. A prescribed performance constraint tracking controller is designed in this paper. It can make that the tracking error of the nonlinear system is constrained to a given region no later than the given setting time, and the transient and steady state performance of the system are ensured. Finally, the proposed method is compared with the existing method, the effective and superiority of the proposed method are demonstrated by two practical examples.

Index Terms—Nonlinear systems; prescribed constraint control; nonlinear mapping; performance constraint function; without initial condition.

I. INTRODUCTION

Because the prescribed performance control can guarantee the transient performance of systems, including overshoot, convergence rate and convergence accuracy, it has attracted a great deal of attentions in control field. In fact, the prescribed performance control can be regarded as a kind of constraint control. At present, the existing constraint control methods include prescribed performance control, funnel control, output constraint control, and so on. And the constraint control is usually designed by barrier Lyapunov function (BLF) method. The types of constraint boundary can be summarized as the constant value constraint (CVC) and the time-varying function constraint (TVFC).

In CVC control design approaches, there are mainly four categories at present. The first is the logarithm-type BLF method [1]–[7]. The logarithm-type BLF was originally investigated for a class of systems with the Brunovsky normal form in [1]. Inspired by [1], [2]–[7] discussed the control problem on tracking error constraints or state constraints based on the logarithm-type BLFs. The second is the tan-type BLF method [8]–[11]. The third is the integral-type BLF method, see [12]–[15]. They were presented to constrain the tracking errors of systems or the full states of systems. The fourth is the nonlinear mapping method, see [16]–[19]. In this method, a one-to-one nonlinear mapping is introduced in order to achieve the constraint performance of systems.

TVFC control is usually called the prescribed performance control. In this control method, the system tracking error is constrained by two time-varying boundaries. The control idea was firstly proposed in [20]. Generally, in this kind of control strategy, an error transformation function is adopted firstly to transform the original inequality constraint for the tracking error into an equality constraint, then the prescribed performance controller is designed according to the transformed error so that the prescribed performance of the system is guaranteed, see [21] and [22]. At present, the prescribed performance control strategy has been widely used in many researches, such as [21]–[24]. Specially, it should be noted that the funnel control method is actually a prescribed performance control method without the error transformation before control design, see [25], [26].

Although the aforementioned constraint control schemes have satisfactory control performance, they have still serious flaws, that is, the choice of prescribed performance function depends on the initial condition of the constrained variable. In other words, the initial condition of the constrained variable must be known before the system controller is designed. But it is difficult to obtain the initial condition in practical systems. Therefore, the developed prescribed performance control design methods will be difficult to be applied in practice. How to removes the effect of initial condition is an extremely challenging and meaningful topic in constraint control design. Therefore, in [28]–[31], a class of improved prescribed performance control design schemes, which are independent of the initial condition, were proposed. The control schemes in [28]–[31] circumvented the initial condition of the constrained variables by the similar error transformations. The transformations employed a shifting function or a tuning function to make the transformed error being zero at initial time. Compared with the existed methods, the proposed method in this paper presents a new transformation idea. A nonlinear mapping and a new prescribed performance function are adopted, and the constrained variable is transformed to a prescribed scope at
initial time rather than to zero. At present, the similar work is not still found. However, actuator saturation is not considered in the existing works [28]–[31].

Actuator saturation is a widespread phenomenon in control systems [32]. It should be mentioned that the actuator saturation in practical systems may limit or degrade the system control performance, even destabilize the system, see [33] and [34]. Therefore, the control research for the systems with actuator saturation is meaningful. In this paper, the actuator saturation problem is considered simultaneously while the novel prescribed constraint control scheme is proposed.

Motivated by the above discussions, this paper proposes a low-complexity and novel constraint control design scheme for nonlinear systems with actuator saturation. The main contributions of this paper are summarized as follows:

1) Compared with the traditional constraint control works [5]–[7], [9]–[11] and [22], a novel prescribed constraint control scheme is presented for a class of nonlinear systems with actuator saturation, which circumvents completely the problem that the traditional performance constraint control schemes depend on the initial conditions of the constrained variables. 2) Unlike the existing works [28]–[31], this paper presents a new constraint idea. A nonlinear mapping and a new prescribed performance function are adopted, and the constrained variable is transformed to a defined scope rather than to zero at initial time. 3) The setting time, at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter and can be set arbitrarily.

The rest of this paper is organized as follows. Section II presents the problem formulation. Section III formulates a novel constraint control idea. The design process for adaptive neural network prescribed performance constraint controller is shown in Section IV. The simulation examples are given in Section V. Section VI gives the conclusion of the paper.

II. PROBLEM FORMULATION

Consider the tracking control problem of a class of strict-feedback systems with actuator saturation and unknown nonlinear functions, and external disturbances. The mathematical model of the system is described by the following differential equation.

\[
\begin{align*}
\dot{x}_i &= g_i(\bar{x}, x_{i+1}) + f_i(\bar{x}) + d_i(t), \\
\dot{y} &= x_1,
\end{align*}
\]

where \(x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n\) is the state vector of the system, \(\bar{x} = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i\). And \(y \in \mathbb{R}\) is the output of the system. \(v\) is the designed control input, \(u(v)\) denotes the plant input with saturation type nonlinearity. \(g_i(\cdot)\) and \(f_i(\cdot)\), \(i = 1, 2, \ldots, n\) are unknown, continuous and smooth nonlinear functions. \(d_i(t)\), \(i = 1, 2, \ldots, n\) are the bounded external disturbances. The tracking error of the system (1) is defined as \(e_1 = x_1 - y_r\), where \(y_r\) is a reference signal. Specially, the system (1) satisfies the following assumptions:

**Assumption 1 [32]** The reference signal \(y_r(t)\) and its derivatives \(\dot{y}_r(t)\), \(i = 1, 2, \ldots, n\) are known and bounded.

**Assumption 2 [35]** The sign of \(g_i(\bar{x})\) is known and unchanged. Without loss of generality, it is further assumed as \(g_i(\bar{x}) > 0\) and there exists an unknown constant \(b_m\) such that

\[0 < b_m < g_i(\bar{x}), \forall \bar{x} \in \mathbb{R}^i\]

for \(i = 1, 2, \ldots, n\).

The saturation model in (1) is asymmetric, it is described as [36]

\[u(v) = u_g \times \text{erf}\left(\frac{\sqrt{2} \lambda v}{2 u_g}\right),\]

where \(u_g = (u_+ + u_-) + (u_+ - u_-) \text{sign}(v)/2\); \(\text{erf}(\cdot)\) denotes a Gaussian error function (GEF) which can be defined as

\[\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.\]

To facilitate the control design later, we define

\[\Delta(v) = u(v) - \lambda v,\]

where \(\lambda\) is an unknown positive constant, see [36], [37]. Hence, the asymmetric smooth model (2) can be written as

\[u(v) = \lambda v + \Delta(v).\]

**Assumption 3 [37]** There exist positive constants \(\Delta_{\max}\) and \(\Delta_{\min}\) such that \(\Delta_{\min} \leq \Delta(v) \leq \Delta_{\max}\).

Substituting (5) into (1) gives

\[\begin{align*}
\dot{x}_i &= g_i(\bar{x}_i) x_{i+1} + f_i(\bar{x}_i) + d_i(t), \\
\dot{x}_n &= g_n(x) u(v) + f_n(x) + d_n(t), \\
y &= x_1.
\end{align*}\]

In this paper, the following RBFNN is used to approximate a continuous function \(\tilde{f}_m(Z) : \mathbb{R}^l \rightarrow \mathbb{R}\)

\[\tilde{f}_m(Z) = W^T S(Z),\]

where \(Z \in \Omega_Z \subset \mathbb{R}^l\) is the input vector, \(W = [w_1, w_2, \ldots, w_l]^T \in \mathbb{R}^l\) is the weight vector, \(l > 1\) is the node number of the neural network, \(S(Z) = [s_1(Z), s_2(Z), \ldots, s_l(Z)]^T \in \mathbb{R}^l\) means the basis function vector with \(s_i(Z)\) being chosen commonly as Gaussian function defined by the following form.

\[s_i(Z) = \exp\left[-\frac{(Z - \mu_i)^T(Z - \mu_i)}{\eta^2}\right], i = 1, 2, \ldots, l,\]

where \(\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{il}]^T\) and \(\eta\) stand for the center of the receptive field and the width of the Gaussian function, respectively.

Just as [34], [38], the RBFNN (7) can be used to approximate a smooth nonlinear function \(\tilde{f}(Z)\) over the compact set \(\Omega_Z\) to arbitrary accuracy as

\[\tilde{f}(Z) = W^T S(Z) + \delta(Z), \quad |\delta(Z)| \leq \varepsilon,\]

where \(W^* = [w_1, w_2, \ldots, w_l]^T \in \mathbb{R}^l\) is the ideal constant weight vector. \(W^*\) is defined as the value of \(W\) that minimizes \(\delta(Z)\) for all \(Z\), i.e.,

\[W^* \triangleq \arg\min_{W \in \mathbb{R}^l} \left\{\sup_{Z \in \Omega_Z} |\tilde{f}(Z) - W^T S(Z)|\right\},\]

\(\delta(Z)\) is the approximation error, \(\varepsilon\) is an unknown bound constant.
Lemma 1 [40]: Let function $V(t) \geq 0$ be a continuous function defined $\forall t \in R^+$ and $V(0)$ bounded, and $\rho(t) \in L_{\infty}$ be real-valued function. If the following inequality holds

\[ \dot{V}(t) \leq -a_0 V(t) + b_0 \rho(t), \]

where $a_0 > 0$, $b_0$ are constants, then $V(t)$ is bounded.

The control objective of this paper is summarized as follows.

Design an adaptive neural prescribed constraint tracking controller by using a nonlinear mapping and a novel performance constraint function which make the prescribed performance constraint function be independent of the initial condition of the constrained variable. The designed controller can guarantee that all the signals in the closed-loop system (1) are bounded, and the tracking error $\varepsilon_1(t)$ is constrained by a prescribed performance constraint function and can converge to a given compact set no later than the given setting time. And the setting time can be set arbitrarily. Specially, the pre-specified constraint performance of the system can’t be affected by external disturbances.

III. A Novel Constraint Control Idea

In the traditional prescribed performance control method, the tracking errors of systems need to satisfy the following constraint:

\[ -\mu(t) < \varepsilon_1(t) < \mu(t), \quad t \geq 0, \]

where $\mu(t) = (\mu_0 - \mu_{\infty})e^{-\tau t} + \mu_{\infty}$ is a prescribed performance function with $\mu_0 > \mu_{\infty} > 0$, $\tau > 0$. In control design, the achievement of the prescribed performance control requires the initial condition of the controlled variable $\varepsilon_1(t)$ to satisfy $-\mu(0) < \varepsilon_1(0) < \mu(0)$. However, the initial conditions of practical systems maybe difficult to be obtained, therefore, for this case, the traditional prescribed performance control method is limited in application.

In order to avoid the defect, a novel prescribed constraint control method is given in this paper. The method is independent of the initial condition of the constrained variable. The novel idea of the method is described as follows.

According to the control objective, the tracking error $\varepsilon_1(t)$ needs to satisfy $-\rho_1(t) < \varepsilon_1(t) < \rho_1(t)$ when $t \geq T > 0$ without the limit of the tracking error $\varepsilon_1(0)$. Here, $T$ is a designable time parameter and it is called the setting time. The constraint function $\rho_1(t)$ is defined as

\[ \rho_1(t) = (\rho_0 - \rho_{\infty})e^{-\lambda_1(t-T)} + \rho_{\infty}, \]

where $\rho_1(t)$ is a direct constraint function, $\rho_0 > \rho_{\infty} > 0$, $\lambda_1 > 0$ and $T > 0$ are the design parameters.

The statement for the novel constraint control idea is clearly illustrated by the graphical representation in Fig. 1.

In order to make the control effect of the tracking error be independent of the initial condition, a new nonlinear mapping of $\varepsilon_1(t)$ is proposed. The error $\varepsilon_1(t)$ is mapped to $z_1$ by the following hyperbolic tangent mapping.

\[ z_1 = \tanh(\varepsilon_1). \]

Remark 1: From (12), it can be observed that $z_1(t)$ and $\varepsilon_1(t)$ is approximately linear around the origin.

Remark 2: $\dot{z}(t)$ is a smooth function, which has been proven in [41]. In addition, it is easy to prove that $\rho_2(t)$ is a smooth function.

Remark 3: In order to ensure the prescribed performance of $\varepsilon_1(t)$, $\varepsilon_1(0)$ must meet the condition $|\varepsilon_1(0)| < \rho_2(0)$. From (12), it can be seen that $-1 < \varepsilon_1(t) < 1$ holds for any value of $\varepsilon_1$. Because $-1 < \varepsilon_1(t) < 1$, the condition can be achieved by choosing $\rho_2(0) \geq 1$. And $\rho_2(0) \geq 1$ can be ensured by $M \geq 1$. Therefore, the proposed method doesn’t need the initial value of the constrained variable $\varepsilon_1(t)$.

Theorem 1: If the inequality (13) holds, then $-\rho_1(t) < \varepsilon_1(t) < \rho_1(t)$ holds for $t \geq T$.

Proof: From (12) and (13), it follows that

\[ -\rho_2(t) < \varepsilon_1(t) < \rho_2(t). \]

If $t \geq T$, according to (14), the following mathematical relationships can be obtained.

\[ 0 < \rho_2(\varepsilon_1(t)) < \tanh(\rho_1(t)). \]

From (16) and (17), we have $-\rho_1(t) < \varepsilon_1(t) < \rho_1(t)$, $t \geq T$, so Theorem 1 is proved.

The control objective of the tracking error $\varepsilon_1(t)$ can be reduced to constrain $z_1(t)$ according to (13). Next, the constraint control design method for $z_1(t)$ will be presented.
IV. ADAPTIVE NEURAL NETWORK PREScribed PERFORMANCE CONSTRAINT CONTROLLER

Based on the proposed constraint control idea in the above section, an adaptive neural network constraint controller will be designed by adopting backstepping technique. The following coordinate transformation is chosen firstly.

\[ \zeta_1 = \tan \left( \frac{\pi x_1}{2 \rho_1} \right). \]  

(18)

\[ \eta_i = x_i - \alpha_{i-1}(x_i, \hat{\beta}, \hat{y}_{r}^{(i-1)}, \hat{\rho}_2^{(i-1)}), \quad i = 2, 3, \ldots, n, \]  

(19)

where \( \alpha_{i-1} \) is the virtual control signal, \( \hat{y}_{r}^{(i-1)} = [y_r, y_r^{(i)}, \ldots, y_r^{(i-1)}] \), \( \hat{\rho}_2^{(i-1)} = [\rho_2, \rho_2^{(i)}, \ldots, \rho_2^{(i-1)}] \), \( \hat{\beta} \) denotes the estimated value of the unknown constant \( \beta \), which is defined as \( \hat{\beta} = \max \{ \| W_i^T \| ; \ i = 1, 2, \ldots, n \} \), and the estimation error \( \tilde{\beta} = \beta - \hat{\beta} \).

**Step 1:** Choose a Lyapunov function candidate as

\[ V_1 = \frac{1}{2} b_m^2 \zeta_1^2 + \frac{1}{2 k_1^2} b_m^2 \tilde{\beta}^2, \]  

(20)

where \( k_1 \) is a positive design parameter. The derivative of \( V_1 \) is obtained as

\[ \dot{V}_1 = b_m \zeta_1 \dot{\zeta}_1 - \frac{1}{k_1^2} b_m^2 \tilde{\beta} \dot{\beta}, \]  

where \( Q = \frac{\pi}{2 \rho_1} \) and \( \cos^2 \left( \frac{\pi x_1}{2 \rho_1} \right) \). The following inequalities can be obtained by invoking Young’s inequality.

\[ \frac{b_m Q}{\rho_2} \left( \frac{4}{(e^{x_1} + e^{y_1})^2} g_1 \eta_2 \right) \leq \frac{b_m Q^2}{\rho_2} \left( \frac{16}{(e^{x_1} + e^{y_1})^2} g_1 + b_m g_1 \eta_1^2 \right). \]  

(22)

\[ \frac{b_m Q}{\rho_2} \left( \frac{4}{(e^{x_1} + e^{y_1})^2} d_1 \right) \leq \frac{b_m^2 Q^2}{\rho_2} \left( \frac{16}{(e^{x_1} + e^{y_1})^2} d_1 \right) \]  

(23)

Substituting (22) and (23) into (21) gives

\[ V_1 \leq \frac{b_m Q}{\rho_2} \left( \frac{4}{(e^{x_1} + e^{y_1})^2} g_1 \alpha_1 + \frac{Q}{\rho_2} \left( \frac{16}{(e^{x_1} + e^{y_1})^2} g_1 \right) \right) + \frac{4}{(e^{x_1} + e^{y_1})^2} (f_1 - y_r) - \frac{z_1 \dot{\rho}_2}{\rho_2} + \frac{b_m Q}{\rho_2} \left( \frac{16}{(e^{x_1} + e^{y_1})^2} \right) - \frac{1}{k_1} b_m^2 \tilde{\beta} \dot{\beta} + b_m g_1 \eta_1^2 + d_1^2. \]  

(24)

**Step i (2 \leq i \leq n - 1):** Consider the following Lyapunov function

\[ V_i = \frac{1}{2} b_m \eta_i^2. \]  

(25)

The derivative of \( V_i \) is

\[ \dot{V}_i = b_m \eta_i \left( g_i(\eta_{i+1} + \alpha_i) + f_i + d_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + f_j + d_j) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \beta} \right) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right). \]  

(26)

Applying Young’s inequality yields

\[ b_m \eta_i \eta_{i+1} \leq b_m g_1 \eta_1^2 + b_m g_1 \eta_1^2. \]  

(27)

\[ b_m \eta_i d_i \leq b_m^2 \eta_1^2 + d_1^2. \]  

(28)

\[ b_m \eta_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} d_j \leq b_m^2 \eta_1^2 \sum_{j=1}^{i-1} \left( \frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \sum_{j=1}^{i-1} d_j^2. \]  

(29)

With the help of (27)~(29), (26) can be rewritten as

\[ \dot{V}_i \leq b_m \eta_i \left( g_i(\alpha_i + \eta_i f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (g_j x_{j+1} + f_j) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{i-1}}{\partial \beta} \right) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} + b_m \eta_i + b_m \eta_i \sum_{j=1}^{i-1} \left( \frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \sum_{j=1}^{i-1} d_j^2 + b_m g_1 \eta_1^2. \]  

(30)

**Step n:** Similarly to Step i, we take a Lyapunov functional as:

\[ V_n = \frac{1}{2} b_m \eta_n^2. \]  

(31)

Differentiating (31) with (1) and (19) yields

\[ \dot{V}_n = \frac{1}{A} b_m \eta_n \left( g_n (\Delta + \Delta(\nu)) + f_n + d_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j + d_j) \right) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{n-1}}{\partial \beta} \right) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right). \]  

(32)

By Young’s inequality, the following formulas hold.

\[ \frac{1}{A} b_m \eta_n \Delta(\nu) \leq \frac{1}{A^2} b_m^2 \eta_n^2 + \Delta^2, \]  

(33)

\[ \frac{1}{A} b_m \eta_n d_n \leq \frac{1}{A^2} b_m^2 \eta_n^2 + d_n, \]  

(34)

\[ \frac{1}{A} b_m \eta_n \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} d_j \leq \frac{1}{A^2} b_m^2 \eta_n^2 \sum_{j=1}^{n-1} \left( \frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 + \sum_{j=1}^{n-1} d_j^2. \]  

(35)

Substituting (33)~(35) into (32) yields

\[ \dot{V}_n \leq \frac{1}{A} b_m \eta_n \left( \lambda g_n^2 \nu + \frac{1}{A} b_m \eta_n \Delta(\nu) + f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j) \right) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \frac{\partial \alpha_{n-1}}{\partial \beta} \right) - \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_2^{(j)}} \rho_2^{(j+1)} \right) + \frac{1}{A} b_m \eta_n \sum_{j=1}^{n-1} \left( \frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 + \Delta^2 + \sum_{j=1}^{n} d_j^2. \]  

(36)

Then, according to (20), (25) and (31), Lyapunov functional candidate for the system (1) can be defined as

\[ V = \sum_{i=1}^{n} V_i. \]  

(37)
From (24), (30), (36) and (37), the following result holds:

\[
V \leq b_m \eta_1 \left( \frac{4}{(e^{e^1} + e^{-e^1})^2} g_1 \alpha_1 + \tilde{f}_1(Z_1) \right) - \frac{b_m \eta_1^2}{4} - \frac{1}{k_1} b_m^2 \tilde{\beta}^2
\]

\[+ \sum_{i=2}^{n-1} \left( b_m \eta_i (g_i \alpha_i + \tilde{f}_i(Z_i)) - \frac{b_m \eta_i^2}{4} \right) + \frac{1}{\lambda} b_m \eta_0 (\lambda g_n v)
\]

\[+ \tilde{f}_n(Z_n) - \frac{b_m^2 \eta_n^2}{4\lambda^2} + \Delta^2 + \sum_{k=1}^{n} d_j^2 + \Lambda.
\]  

(38)

where \(\eta_1 = Q / \rho_2\) and

\[
\tilde{f}_i(Z_i) = \frac{16 g_1 \eta_1}{(e^{e^1} + e^{-e^1})^2} + \frac{4}{(e^{e^1} + e^{-e^1})^2} (f_1 - \tilde{y}_r)
\]

\[- \frac{3 \rho_2}{\rho_2} + \frac{16 b_m \eta_1}{(e^{e^1} + e^{-e^1})^2} + \frac{b_m \eta_1}{4},
\]

\[
\tilde{f}_i(Z_i) = g_i \eta_1 + f_i - \sum_{j=1}^{i-1} \frac{\partial g_i}{\partial x_j} (g_j x_j + f_j) - \sum_{j=0}^{i-1} \frac{\partial g_i}{\partial y_j} \hat{y}_r
\]

\[- \sum_{j=0}^{i-1} \frac{\partial g_i}{\partial y_j} \hat{y}_r
\]

\[+ g_i \eta_1 - \xi_i (Z_i) + \frac{b_m \eta_i}{4},
\]

\[
\tilde{f}_n(Z_n) \leq \frac{1}{4} b_m \eta_n g_n^2 + f_n - \sum_{j=1}^{n-1} \frac{\partial g_j}{\partial x_j} (g_j x_j + f_j) - \sum_{j=0}^{n-1} \frac{\partial g_j}{\partial y_j} \hat{y}_r
\]

\[- \sum_{j=0}^{n-1} \frac{\partial g_j}{\partial y_j} \hat{y}_r
\]

\[+ \lambda g_{n-1} \eta_0 = \lambda \xi_n (Z_n) + \frac{b_m \eta_n}{4\lambda}.
\]

with \(\Lambda = b_m \sum_{i=2}^{n-1} \eta_i \xi_i (Z_i) - \frac{\partial \xi_i}{\partial y_j} \hat{y}_r + b_m \eta_{n-1} (\lambda \xi_n (Z_n) - \frac{\partial \xi_n}{\partial y_j} \hat{y}_r), Z_i = [x_i, \tilde{y}_r, \tilde{p}_i]^T \in \Omega_i \subset R^5, Z_i = [x_i, \tilde{\beta}, \tilde{\beta}, \hat{y}_r, \hat{y}_r, \hat{y}_r] \in \Omega_i \subset R^{3(n+1)}, Z_n = [x_n, \tilde{\beta}, \tilde{\beta}, \hat{y}_r, \hat{y}_r, \hat{y}_r] \in \Omega_n \subset R^{3(n+1)}.
\]

\(\xi_i (Z_i) (2 \leq i \leq n)\) are the auxiliary functions to compensate the term \(\frac{\partial \xi_i}{\partial y_j} \hat{y}_r\) in (38), they will be given in Appendix I. Here, each unknown function \(\tilde{f}_i(Z_i)\) is approximated by a RBFNN in this control design, namely, \(\tilde{f}_i(Z_i) = W_i^T S_i(Z_i) + \delta_i (Z_i), (1 \leq i \leq n)\) and \(|\delta_i (Z_i)| \leq |e_i|, \hat{e}_i\) is an unknown positive constant.

With the aid of Young’s inequality, it follows that

\[
\frac{1}{4} b_m \eta_n \tilde{f}_n(Z_n) = \frac{1}{4} b_m \eta_n (W_n^T S_n(Z_n) + \delta_n)
\]

\[\leq \frac{b_m^2 \eta_n^2}{2b_1} W_n^T W_n S_n^T (Z_n) S_n (Z_n) + \frac{b_1}{2} + \frac{b_m^2 \eta_n^2}{4} + \delta_n^2
\]

\[\leq \frac{b_m^2 \eta_n^2}{2b_1} \beta S_n^T (Z_n) S_n (Z_n) + \frac{b_1}{2} + \frac{b_m^2 \eta_n^2}{4} + \delta_n^2,
\]

(39)

\[
\frac{1}{4} b_m \eta_1 \tilde{f}_1(Z_1) = \frac{1}{4} b_m \eta_1 (W_1^T S_1 (Z_1) + \delta_1)
\]

\[\leq \frac{b_m^2 \eta_1^2}{2} W_1^T W_1 S_1^T (Z_1) S_1 (Z_1) + \frac{b_1}{2} + \frac{b_m^2 \eta_1^2}{4} + \delta_1^2,
\]

(40)

where \(b_1 > 0 \ (1 \leq i \leq n)\) are design parameters. Based on (38)~(40), it can be obtained that

\[
V \leq b_m \eta_1 \left( \frac{4}{(e^{e^1} + e^{-e^1})^2} g_1 \alpha_1 + \frac{b_m \eta_1}{2b_1} \beta S_1^T (Z_1) S_1 (Z_1) \right)
\]

\[- \frac{1}{k_1} b_m^2 \tilde{\beta}^2 + \sum_{i=2}^{n-1} \left( b_m \eta_i (g_i \alpha_i + \tilde{f}_i(Z_i)) - \frac{b_m \eta_i^2}{4} \right) + \frac{1}{\lambda} b_m \eta_0 (\lambda g_n v)
\]

\[+ \tilde{f}_n(Z_n) - \frac{b_m^2 \eta_n^2}{4\lambda^2} + \Delta^2 + \sum_{k=1}^{n} d_j^2 + \Lambda.
\]

(41)

According to (41), we can design the virtual control laws

\[
\alpha_1 = \frac{(e^{e^1} + e^{-e^1})^2}{4} (-c_1 \xi_1 \cos^2 \left( \frac{\pi c_1}{2\rho_2} \right) \rho_2
\]

\[- \frac{\eta_1}{2b_1} \beta S_1^T (Z_1) S_1 (Z_1),
\]

(42)

\[
\alpha_i = -c_i \eta_i - \frac{\eta_1}{2b_1} \beta S_i^T (Z_i) S_i (Z_i), 2 \leq i \leq n - 1,
\]

(43)

the real control law

\[
v = -c_n \eta_n - \frac{\eta_1}{2b_n} \beta S_n^T (Z_n) S_n (Z_n),
\]

(44)

and the adaptive law

\[
\dot{\hat{\beta}} = \sum_{i=1}^{n} \frac{k_1}{b_1} \eta_i \xi_i^T (Z_i) \xi_i (Z_i) - d_0 \hat{\beta},
\]

(45)

where \(c_i (i = 1, 2, \ldots, n)\) and \(d_0\) are positive constants.

Based on the above deduction, we give the following theorem.

**Theorem 2:** Consider the nonlinear system (1) satisfying Assumptions 1-3, if the direct constraint function \(\rho_1)\) and indirect constraint function \(\rho_2)\) are chosen respectively in (11) and (14), the virtual control laws, actual control law and adaptive laws of the system are designed according to (42)~(45), then the tracking error of the controlled system (1) can be driven into the prescribed scope given by \(\rho_1\) and \(-\rho_1\) when \(t \geq T\).

**Proof:** 1) On stability for the closed-loop system (1).

By substituting (42)~(45) into (41), the following inequalities can be obtained

\[
\dot{V} \leq - \frac{\pi c_1 b_1^2 \eta_1^2}{4} - \sum_{i=2}^{n} c_i b_m \eta_i^2 - \frac{d_0}{k_1} \eta_1^2 \tilde{\beta}^2 + \sum_{i=1}^{n} \epsilon_i^2
\]

\[+ \sum_{i=1}^{n-1} b_i + \frac{b_n}{2\lambda^2} + \Delta^2 + \sum_{k=1}^{n} d_j^2 + \Lambda.
\]

(46)

**Remark 4:** According to (45), \(\hat{\beta}(t) \geq 0\) always holds for all \(t \geq 0\) if \(\hat{\beta}(0) \geq 0\). \(\hat{\beta}(t) \geq 0\) has been used in the deduction of (46).
In (46), it can be seen that \( V \) is bounded if \( \lambda \leq 0 \) holds. In fact, \( \lambda \leq 0 \) can be proven. In order to clarify the derivation, the proof of \( \lambda \leq 0 \) is placed in Appendix I. Therefore, (46) can be rewritten as

\[
\dot{V} \leq -\frac{\pi}{2} c_1 b_1^2 \epsilon_1^2 - \sum_{i=2}^{n} c_i b_i^2 \epsilon_i^2 + \frac{d_0}{k_1} b_1^2 \beta \beta + \sum_{i=1}^{n} e_i^2 \\
+ \sum_{i=1}^{n-1} b_i + \frac{b_n}{2 \lambda^2} + \Delta_{\text{max}} \sum_{i=1}^{n} d_i.
\]

(47)

For the term \( \frac{d_0}{k_1} b_1^2 \beta \beta \) in (47), the following inequality is verified easily by using Young’s inequality.

\[
\frac{d_0}{k_1} b_1^2 \beta \beta \leq - \frac{d_0}{2k_1} b_1^2 \beta \beta + \frac{d_0}{k_1} b_1^2 \beta \beta.
\]

(48)

Then, substituting (48) into (47) yields

\[
\dot{V} \leq -\frac{\pi}{2} c_1 b_1^2 \epsilon_1^2 - \sum_{i=2}^{n} c_i b_i^2 \epsilon_i^2 + \frac{d_0}{2k_1} b_1^2 \beta \beta + \sum_{i=1}^{n} e_i^2 \\
+ \sum_{i=1}^{n-1} b_i + \frac{b_n}{2 \lambda^2} + \Delta_{\text{max}} \sum_{i=1}^{n} d_i.
\]



where \( a = \min\{\pi c_1 b_1, 2c_1 b_1, 2\lambda c_1 b_1, d_0; i = 2, \ldots, n-1\}, b = 1, \rho(t) = \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n-1} b_i + \frac{b_n}{2 \lambda^2} + \Delta_{\text{max}} \sum_{i=1}^{n} d_i.

From the formula (49) and Lemma 1, we can know that \( V \) is bounded, which means that \( x_1 \), \( \eta_i \), \( i = 2, \ldots, n \) and \( \beta \) are bounded. From (43) and (44), we also know \( c_1 \), \( i = 2, \ldots, n-1 \) and \( \nu \) are bounded, so \( x_3, x_4, \ldots, x_n \) are bounded due to the coordinate transformation (19). However, the boundedness of \( x_1 \) and \( x_2 \) cannot be obtained from the above analysis. The proof of the boundedness of them will be given in the part 3).

2) On constraint of \( z_1 \)

Considering \( z_1(t) = \tan(\frac{\pi \eta}{2\eta}) \) is bounded, \( \tan(\pm \frac{\pi}{2}) = \infty \) and \( z_1(0) < \rho_2(\tilde{v}(0)) \), then the following inequality always holds that

\[
-\rho_2(\tilde{v}(t)) < z_1(t) < \rho_2(\tilde{v}(t)).
\]

(50)

3) On constraint performance and boundedness of the tracking error \( e_1(t) \)

From (50) and Theorem 1, it can be known that the control object has been achieved, that is, \( -\rho_1(t) < e_1(t) < \rho_1(t) \) when \( t \geq T \). With the help of the continuity of the tracking error \( e_1(t) \), it is easy to be known that \( e_1(t) \) is bounded in whole time domain by mathematical logic. Thus, \( x_1 \) is bounded. Furthermore, \( x_1 \) can be proved to be bounded easily, so \( x_2 \) is also bounded. Therefore, the control objective of this paper is achieved.

The proof of Theorem 2 has been completed.

Remark 5: There are the design parameters \( \rho_0, \rho_\infty, \lambda_1 \) and \( T \) in the performance function \( \rho_1(t) \). From (11), it can be found that increasing the design parameter \( \lambda_1 \) and decreasing the design parameters \( \rho_0, \rho_\infty, T \) can bring better tracking performance. The smaller the parameter \( \rho_\infty \), the smaller the steady state error. Increasing \( \lambda_1 \) can accelerate the convergence rate of the tracking error. However, the shorter setting time \( T \), the smaller \( \rho_\infty \) and the larger \( \lambda_1 \) can induce the larger control input. Therefore, the design parameter \( T \) and the function \( \rho_1(t) \) should be chosen depending on the physical limitations and performance requirements of the output in practical applications. For the design parameters \( \xi, M, \nu_0 \) and \( \nu_1 \) in \( \rho_2(t) \), we find that increasing the value of \( \xi \) can achieve a better tracking performance, but the design parameters \( M, \nu_0 \) and \( \nu_1 \) are only need to satisfy the design condition given in this paper. The larger the parameter \( c_i \), the larger the control input. The smaller the parameter \( b_i \), the larger the control input. The design parameters \( k_1 \) and \( d_0 \) slightly affect the control performance.

V. SIMULATION STUDIES

In order to illustrate the effectiveness and superiority of the proposed control method, the developed adaptive neural prescripted constraint controller is applied to two practical examples.

Example 1: Consider a rigid robot manipulator described in [42], and input saturation and external disturbance are added to the system, then the dynamic model of the system is

\[
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = f_2 + g_2 u(v) + d_2, \\
y = x_1,
\end{cases}
\]

(51)

where \( g_2 = \frac{1}{J_0}, f_2 = -\frac{m_g}{J_0} l_0 \cos(x_1), x_1, x_2, m_r, g_r, \) and \( l_r \) are the angular position of manipulator, the relative angular velocity, the load mass, the gravity and the length of manipulator, respectively. And \( J_0 = 4 m_r l_0^2 / 3 \) is the inertia coefficient, \( m_r = 5 \text{mg}, g_r = 9.8 \text{ms}^2/\text{ms} \) and \( l_r = 0.25 \text{m} \).

The simulations is conducted with the system initial condition \( [x_1(0), x_2(0)]^T = [0, 0]^T \), and \( \beta(0) = 2 \). The reference signal is \( y_r = \cos(t) \). The parameters of the asymmetric smooth saturation nonlinearity model are \( u_0 = 20 \) and \( u_{-} = -10 \).

The neural network \( W_{1}^T S_{1}(Z_1) \) contains \( l_1 = 7^5 \) nodes with centers \( \mu(l = 1, \ldots, l_1) \) evenly spaced in \([-3, 3], \ldots, [-3, 3]\), and width \( \eta_l = 2 \) \( (l = 1, \ldots, l_1) \). The neural network \( W_{2}^T S_{2}(Z_1) \) contains \( l_2 = 7^9 \) nodes with centers \( \mu(l = 1, \ldots, l_2) \) evenly spaced in \([-3, 3], \ldots, [-3, 3]\), and width \( \eta_l = 2 \) \( (l = 1, \ldots, l_2) \). The design parameters are chosen as \( c_1 = 4, c_2 = 4, d_0 = 4, b_1 = 1, b_2 = 1, k_1 = 0.4, \rho_0 = 0.1, \rho_\infty = 0.01, \lambda_1 = 0.7, \nu_0 = 5, \nu_1 = 3, M = 10, \xi = 20 \) and the setting time \( T = 3 \text{s} \).

The simulation of the rigid robot manipulator is carried out after the controller is designed according to Theorem 2. The simulation results are given in Figs. 2-7.

In order to compare with the method in [30], according to the constraint control idea in [30], an adaptive constraint tracking controller is designed for the rigid robot manipulator system. The designed controller (70)-(72) can be founded in Appendix II. For the purpose of comparison, the same constraint function is chosen as the same as the prescribed performance function in this paper for the tracking error \( e_1 \), that is, \( F_1 = F_2 = 0.07 e^{-0.6(t-5)} + 0.01 \). The other design parameters and neural networks of the method in [30] are as the same as the proposed method in this paper. The designed
controller is applied to the rigid robot manipulator, and the simulation results are also given simultaneously in Figs. 2-7.

Fig. 2 shows the tracking effect of the output $y$. Fig. 3 gives the control effect of the tracking error $e_1$. Fig. 4 illustrates the actual controller input $v$ and the asymmetric saturation output $u$. The boundedness of the system state $x_2$ is shown in Fig. 5. Fig. 6 shows the boundedness of the adaptive parameter $\hat{\beta}$. Fig. 7 gives the external disturbance $d_2$.

From Figs. 2-3, we can observe that the tracking performance of the system output $y$ is satisfactory, $e_1$ is constrained by the constraint function $\rho_1$ no later than the given setting time $T = 3\text{s}$ when $e_1(0)$ isn’t between $\rho_1(0)$ and $-\rho_1(0)$ in the two design methods. Figs 2-7 demonstrate clearly that the proposed control method is effective. From Fig. 3, it can be observed that the tracking performance of our method is better than the method in [30].

**Example 2:** To further illustrate the effectiveness of the proposed constraint control scheme, consider a inverted pendulum system taken from [4], and actuator saturation and external disturbance are added to the inverted pendulum system. The
dynamic model of the system is
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= g \sin(x_1) - \frac{ml_1^2 x_1 \cos(x_1) \sin(x_1)}{M_0 + m} \\
&\quad + \frac{\cos(x_1)}{M_0 + m} \left( \frac{4}{3} - \frac{m \cos^2(x_1)}{M_0 + m} \right) u(v) + d_2, \\
y &= x_1,
\end{align*}
\]
where \( g = 9.8 \text{m/s}^2 \), \( M_0 = 1 \text{kg} \), \( m = 0.1 \text{kg} \), \( l_0 = 0.5 \text{m} \) are the acceleration of gravity, mass of cart, the mass of pole and the half length of pole, respectively.

After the controller is designed according to Theorem 2, the simulation is conducted with the initial condition \([x_1(0), x_2(0), \hat{\beta}(0)]^T = [1, 0.3, 0.1]^T\). The reference signal is \( y_r(t) = \sin(t) \). The parameters of the asymmetric smooth saturation nonlinearity model are \( u_+ = 20 \) and \( u_- = -30 \). The design parameters are chosen as \( c_1 = 2 \), \( c_2 = 15 \), \( d_0 = 0.4 \), \( b_1 = 1 \), \( b_2 = 1 \), \( k_1 = 0.4 \), \( \rho_0 = 0.12 \), \( \rho_1 = 0.01 \), \( \lambda_1 = 0.6 \), \( \bar{v}_0 = 5 \), \( \bar{v}_1 = 2 \), \( M = 3 \), \( \tau = 3 \) and the setting time \( T = 1 \text{s} \). The neural networks are the same as in this paper.

The simulation results for the inverted pendulum system are shown in Figs. 8-13. Fig. 8 shows the tracking effect of the output \( y \). Fig. 9 exhibits the control effect of the tracking error \( e_1 \), and \( e_1(t) \) converges to the prescribed constraint region no later than the given setting time \( T = 1 \text{s} \) when \( e_1(0) \) isn’t between \( \rho_1(0) \) and \( -\rho_1(0) \). Fig. 10 gives the actual controller input \( v \) and the asymmetric saturation output \( u \). The boundedness of the system state \( x_2 \) and the adaptive parameter \( \hat{\beta} \) is depicted in Fig. 11 and 12, respectively. Fig. 13 presents the external disturbance \( d_2 \). These figures verify the effectiveness and feasibility of the proposed method.

VI. Conclusion

In this paper, a novel adaptive neural network prescribed constraint control design scheme is presented based on backstepping technique, which is independent of the initial condition of the constrained variable. Be different from the existing constraint control method, in this paper, the purpose of constraint control is achieved by using a nonlinear mapping and a novel performance constraint function. In this method,
the setting time $T$, at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter. The designed controller not only guarantees the system stability but also achieves the desired constraint performance no later than the setting time for any initial condition even if the system is affected by external disturbance. The effectiveness of the proposed control method has been demonstrated via the simulations of two practical systems. Therefore, the proposed constraint control strategy is more practical in application. Our future work will focus on the application research of the proposed method to stochastic systems, switched systems, large-scale systems and multi-agent systems.

\section*{Appendix I}

\textbf{Proof of $\Lambda \leq 0$}

\textit{Proof:} As we know, it always holds that

\begin{equation}
\Lambda = b_m \sum_{i=2}^{n+1} \eta_i (\dot{\xi}_i(Z_k) - \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}) + b_m \eta_n \frac{1}{\lambda} (\dot{\xi}_n(Z_n) - \frac{\partial \alpha_{n-1}}{\partial \beta} \dot{\beta}) = b_m \sum_{i=2}^{n+1} \eta_i \dot{\xi}_i(Z_k) - b_m \sum_{i=2}^{n+1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} = b_m \left[ \sum_{i=2}^{n+1} \eta_i \dot{\xi}_i(Z_k) - b_m \frac{1}{\lambda} \sum_{i=2}^{n+1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} \right] = b_m \left[ \sum_{i=2}^{n+1} \eta_i \dot{\xi}_i(Z_k) - b_m \frac{1}{\lambda} \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} \right] = b_m \Lambda (\lambda - 1) \sum_{i=2}^{n-1} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}. \quad (52)
\end{equation}

To clarify the deducing procedure of the proof, the following two inequalities are given firstly by using the formula (45) and Lemma 1 in [39].

The first inequality is

\begin{equation}
\begin{aligned}
- b_m \sum_{i=2}^{n} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} &= - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}. \quad (53)
\end{aligned}
\end{equation}

where

\begin{equation}
\begin{aligned}
\Xi_i &= \frac{1}{\lambda} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\quad - k_1 \eta^2 \sum_{k=2}^{i} \eta_k \frac{\partial \alpha_{k-1}}{\partial \beta} \bigg| S_k(Z_k), i = 2, \ldots, n.
\end{aligned}
\end{equation}

The second inequality is

\begin{equation}
\begin{aligned}
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\quad + k_1 \eta^2 \sum_{k=2}^{i} \eta_k \frac{\partial \alpha_{k-1}}{\partial \beta} \bigg| S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\quad + k_1 \eta^2 \sum_{k=2}^{i} \eta_k \frac{\partial \alpha_{k-1}}{\partial \beta} \bigg| S_k(Z_k).
\end{aligned}
\end{equation}

To clarify the deducing procedure of the proof, the following two inequalities are given firstly by using the formula (45) and Lemma 1 in [39].

The first inequality is

\begin{equation}
\begin{aligned}
- b_m \sum_{i=2}^{n} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} &= - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \left( -d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right) \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k) \\
&\leq - b_m \sum_{i=2}^{n} \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta}. \quad (54)
\end{aligned}
\end{equation}

where

\begin{equation}
\begin{aligned}
\Xi_i &= - d_0 \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \dot{\beta} + (\lambda - 1) \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \sum_{k=1}^{i-1} k_1 \eta^2 \xi^T(Z_k) S_k(Z_k).
\end{aligned}
\end{equation}
\[
- \frac{1}{\lambda} |\lambda - 1| \frac{k_1}{2b_1} \eta_s^2 \sum_{i=2}^{n} \left| \eta_i \frac{\partial \alpha_{i-1}}{\partial \beta} \right|, \quad i = 2, \ldots, n - 1,
\]

\[\tilde{Z}_m = - \frac{1}{\lambda} |\lambda - 1| \frac{k_1}{2b_1} \eta_n \sum_{i=2}^{n} \eta_i \frac{\partial \alpha_{n-1}}{\partial \beta} .\]

Here, \( \sigma \) is the an upper bound of \(|\sigma(z, \lambda)|\).

Substituting (53) and (54) into (52), one has

\[\Lambda \leq b_m \sum_{i=2}^{n} \eta_i \xi_i (Z_i) - b_m \sum_{i=2}^{n} \eta_i \xi_i - b_m \sum_{i=2}^{n} \eta_i \xi_i \]

\[= b_m \sum_{i=2}^{n} \eta_i (\xi_i (Z_i) - \tilde{Z}_i - \tilde{Z}_i) .\]

If the auxiliary functions \( \xi_i (Z_i), i = 2, \ldots, n \) are chosen as \( \xi_i (Z_i) = \Xi_i + \tilde{Z}_i \), then

\[\Lambda \leq 0 .\]

So far, the proof has been completed.

**Appendix II**

**The controller design based on the method in [30]**

According to the design idea in [30], the design process is presented as follows:

According to (4), the rigid robot manipulator in (51) can be rewritten as

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= g_2 (\alpha v + \Delta v) + f_2 + d_2, \\
y &= x_1 ,
\end{align*}
\]

(55)

Define the coordinate transformation

\[e_1 = x_1 - y_r, \quad e_2 = x_2 - \alpha_1,\]

where \( e_1 \) is the tracking error, \( y_r \) is the desired signal, \( \alpha_1 \) is the virtual control.

Adopting the idea of [30], the shifting function is chosen as

\[
\varphi(t) = \begin{cases} 
1 - \left(\frac{T - t}{T}\right)^4, & 0 \leq t < T \\
1, & t \geq T
\end{cases}
\]

(56)

then

\[\xi(t) = \varphi(t) e_1(t) .\]

(57)

**Step 1:** Choose a barrier Lyapunov function candidate as

\[V_1 = \frac{b_m e_2^2}{(F_1 + \xi(t))F_2 - \xi_1} + \frac{1}{2k_1} b_m \dot{\beta}^2 ,\]

(58)

where \( b_m, k_1 \) and \( \dot{\beta} \) are the same as the definition in this paper, \( F_1 \) and \( F_2 \) are the prescribed constraint function of \( \xi_1 \).

Then, the time derivative of \( V_1 \) is

\[
\dot{V}_1 = b_m \dot{M}_1 \xi_1 \left( \dot{\varphi} e_1 + \varphi \alpha_1 + \varphi e_2 \right)
- \varphi \varphi \dot{y}_r + \varphi \dot{y}_1 \frac{\alpha_1}{k_1} b_m \dot{\beta} .
\]

(59)

where

\[
\dot{M}_1 = \frac{2F_1 F_2 + \xi_1 F_2 - F_1 \xi_1}{(F_1(t) + \xi_1(t))^2 (F_2(t) - \xi_1(t))^2} ,
\]

Then, the following inequality can be derived by using Young’s inequality.

\[-b_m \dot{M}_1 \xi_1 \varphi e_1 \leq \frac{1}{4} k_1 b_m \dot{M}_1^2 \varphi^2 \xi_1^2 e_1^2 \frac{1}{k_1} + \frac{1}{k_1} \]

\[= \frac{1}{4} k_1 b_m \dot{M}_1^2 \varphi^2 \xi_1^2 e_1^2 + \frac{1}{k_1} .\]

(60)

Substituting (59) into (58) generates

\[\dot{V}_1 \leq b_m \dot{M}_1 \xi_1 \varphi (\alpha_1 + \dot{f}_1 (Z_1)) + b_m \dot{M}_1 \xi_1 e_2
- \frac{1}{4} \left( b_m \dot{M}_1 \xi_1 \varphi \right)^2 + \frac{1}{k_1} ,\]

(61)

where

\[Z_1 = [x_1, y_r, \dot{y}_r, F_1, F_1, F_2, F_2]^T ,\]

\[\dot{f}_1 (Z_1) = \frac{1}{4} k_1 b_m \dot{M}_1 \varphi^2 \xi_1^2 - \dot{\xi}_1 + e_1 \eta_1 + \frac{1}{4} b_m \dot{M}_1 \xi_1 \varphi .\]

**Step 2:** Consider a Lyapunov function candidate as

\[V_2 = \frac{1}{2\lambda} b_m e_2^2 ,\]

(62)

then

\[\dot{V}_2 = \frac{1}{\lambda} b_m e_2 (g_2 \alpha v + \Delta v + f_2 + d_2 - \alpha_1) \]

\[= \frac{1}{\lambda} b_m e_2 \left( g_2 \alpha v + \Delta v + f_2 + d_2 - \xi_1 - \frac{\partial \alpha_1}{\partial \beta} \dot{\beta} \right) ,\]

(63)

where

\[\xi_1 = \frac{\partial \alpha_1}{\partial \beta} x_2 + \sum_{j=0}^{i-1} \frac{\partial \alpha_1}{\partial \beta} (j) y_r^{(j+1)} + \sum_{j=0}^{i-1} \frac{\partial \alpha_1}{\partial \beta} \varphi^{(j+1)}
+ \sum_{k=1}^{2} \sum_{j=0}^{1} \frac{\partial \alpha_1}{\partial \beta} F_k^{(j+1)} .\]

Applying Young’s inequality, the following inequalities can be obtained.

\[-\frac{1}{\lambda} b_m e_2 d_2 \leq \frac{1}{4\lambda} b_m e_2^2 + d_2 .\]

(64)

\[-\frac{1}{\lambda} b_m e_2 \Delta v \leq \frac{1}{4\lambda} b_m e_2^2 + \Delta v_{\text{max}} .\]

Now, substituting (63)–(64) into (62), it follows that

\[\dot{V}_2 \leq \frac{1}{\lambda} b_m e_2 \left( g_2 \alpha v + \bar{f}_2 (Z_2) \right) + \Delta_2 - b_m \dot{M}_1 \xi_1 e_2
- \frac{1}{4} \left( \frac{1}{\lambda} b_m e_2 \right)^2 + \frac{1}{\lambda} b_m e_2 \left( \xi_2 (Z_2) - \frac{\partial \alpha_1}{\partial \beta} \dot{\beta} \right) ,\]

(65)

where

\[Z_2 = [x_1, \dot{x}_2, \dot{y}_r, \dot{y}_y, \dot{y}_r, F_1, \dot{F}_1, \dot{F}_1, \dot{F}_2, \dot{F}_2]^T ,\]

\[\bar{f}_2 (Z_2) = f_2 - \xi_1 + \frac{1}{2\lambda} b_m e_2 + \lambda M_1 \xi_1 \varphi + \frac{1}{4\lambda} b_m e_2 - \xi_2 (Z_2) ,\]

\[\xi_2 (Z_2) = \frac{\partial \alpha_1}{\partial \beta} \frac{k_1}{2b_1} M_1^2 \varphi^2 \xi_1^2 S_1^T (Z_1) S_1 (Z_1)
- \frac{\partial \alpha_1}{\partial \beta} \frac{k_1}{2b_2} E_2^2 \frac{d_2}{\partial \beta} d_2 \dot{\beta} .\]
The unknown function $\tilde{f}_i(Z_i)$ can be approximated by a neural network, that is, $\tilde{f}_i(Z_i) = W_i^T S_i(Z_i) + \delta_i(Z_i)$, $i = 1, 2$. By applying the Young’s inequality, it produces

$$b_m M_i \xi_i \tilde{f}_i(Z_1) = b_m M_i \xi_i \tilde{f}_i(W_i^T S_1(Z_1) + \delta_1(Z_1))$$

$$= b_m M_i \xi_i \tilde{f}_i W_i^T S_1(Z_1) + b_m b_1 M_i \xi_i \delta_1(Z_1)$$

$$\leq \frac{1}{2b_1} \left( b_m M_i \xi_i \tilde{f}_i^2 W_i^T W_i S_1^T(Z_1) + b_1^2 \right) + \frac{1}{4} \left( b_m M_i \xi_i \tilde{f}_i^2 \right) + \varepsilon_1^2$$

$$\leq \frac{1}{2b_1} \left( b_m M_i \xi_i \tilde{f}_i^2 b_2 S_1^T(Z_1) + b_1^2 \right) + \frac{1}{4} \left( b_m M_i \xi_i \tilde{f}_i^2 \right) + \varepsilon_1^2,$$

where $b_1$ and $b_2$ are positive design parameters.

The total Lyapunov function of the system (55) is

$$V = V_1 + V_2.$$

With the aid of (60), (65), (67) and (68), one has

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

$$\leq b_m M_i \xi_i \varphi \left( \alpha_1 + \tilde{f}_i(Z_1) \right) + \frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) + \frac{1}{k_1} \right) + d_1^2 + d_1^2 + \Delta_{\text{max}}$$

$$+ \frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \hat{\beta} \right) + \frac{1}{k_1} b_m \varepsilon_2 \beta.$$

According to (69), design the virtual control

$$\alpha_1 = -c_1 \varepsilon_1 - \frac{\xi_1}{2b_1} M_i \xi_i \tilde{f}_i S_1^T(Z_1) S_1(Z_1),$$

the control input

$$v = -c_2 \varepsilon_2 - \frac{\varepsilon_2}{2b_2} b_2 S_2^T(Z_2) S_2(Z_2),$$

and the adaptive law

$$\dot{\beta} = \frac{k_1}{2b_2} M_i \xi_i S_1^T(Z_1) S_1(Z_1) + \frac{k_1}{2b_2} \varepsilon_2 S_2^T(Z_2) S_2(Z_2) - d_0 \hat{\beta},$$

where $c_1$, $c_2$, $k_1$ and $d_0$ are positive design parameters.

By means of (66) and (72), one has

$$\frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \hat{\beta} \right)$$

$$= \frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \left( M_i \xi_i \tilde{f}_i \right) S_1^T(Z_1) S_1(Z_1) - d_0 \hat{\beta} \right)$$

$$\leq \frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \left( M_i \xi_i \tilde{f}_i \right) S_1^T(Z_1) S_1(Z_1) \right)$$

$$- \frac{\partial \alpha_1}{\partial b} 2b_2 \varepsilon_2 S_2^T(Z_2) S_2(Z_2) + d_0 \hat{\beta}$$

$$\leq \frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \left( M_i \xi_i \tilde{f}_i \right) S_1^T(Z_1) S_1(Z_1) \right)$$

$$- \frac{\partial \alpha_1}{\partial b} 2b_2 \varepsilon_2 S_2^T(Z_2) S_2(Z_2) + d_0 \hat{\beta}$$

$$= 0,$$

that is,

$$\frac{1}{\lambda} b_m \varepsilon_2 \left( \xi_2(Z_2) - \frac{\partial \alpha_1}{\partial b} \hat{\beta} \right) \leq 0. \tag{73}$$

By substituting (70)-(73) into (69), it is derived that

$$\dot{V} \leq -c_1 b_m \xi_1 - \xi_2 b_2 S_2^T(Z_2) S_2(Z_2) + b_2 \frac{1}{2} + \frac{1}{2} \left( \frac{b_m}{k_1} \xi_2 \right)^2 + \varepsilon_2^2$$

$$\leq \frac{b_2}{2b_2} \xi_2 b_2 S_2^T(Z_2) S_2(Z_2) + b_2 \frac{1}{2} + \frac{1}{2} \left( \frac{b_m}{k_1} \xi_2 \right)^2 + \varepsilon_2^2,$$

where $b_1$ and $b_2$ are positive design parameters.

According to Lemma 1, $V$ is bounded, furthermore,

$$\dot{F}_1(t) < \xi_1(t) < \xi_2(t). \tag{75}$$

When $t \geq T$, it follows from (56), (57) and (75) that

$$\dot{F}_1(t) < \xi_1(t) < \xi_2(t),$$

which means that $F_1$ and $F_2$ will become the prescribed performance function of the tracking error $\varepsilon_1$.

So far, the design process is completed.

**Declarations**

**Funding** This work is supported by the Natural Science Foundation of Liaoning Province, under Grant 20180550319, and the Education Foundation of Liaoning Province, under Grant 2019LNJC09, in part by Doctoral Start-up Foundation of Liaoning Province under Grant 2019-BS-126.

**Conflict of interests** The authors declare that there is no conflict of interests regarding the publication of this paper.

**Availability of data and material** Not applicable.

**Code availability** Not applicable.

**Authors contributions** 1) Compared with the traditional constraint control work [5]–[7], [9]–[11] and [22], a novel prescribed constraint control scheme is presented for a class of nonlinear systems with actuator saturation, which circumvents completely the problem that the traditional performance constraint control schemes depend on the initial conditions of the constrained variables. 2) Unlike the existing works [28]–[31], this paper presents a new constraint idea. A nonlinear mapping
and a new prescribed performance function are adopted, and the constrained variable is transformed to a defined scope rather than to zero at initial time. 3) The setting time, at which the constrained variable of the system gets into the prescribed constraint region, is a design parameter and can be set arbitrarily.

References

[1] Ngo, K.B., Mahony, R., Jiang, Z.P.: Integrator backstepping using barrier functions for systems with multiple state constraints. In: Proceedings 44th IEEE Conference on Decision and Control, pp. 8306-8312. Seville, December (2005)

[2] Zhou, Q., Wang, L.J., Wu, C.W., Li, H.Y., Du, H.P.: Adaptive fuzzy control for nonstrict-feedback systems with input saturation and output constraint. IEEE Trans. Syst. Man Cybern. Syst. 47(1), 1-12 (2017)

[3] Zhao, S.Y., Liang, H.J., Du, P.H., Qi, S.W.: Adaptive NN finite-time tracking control of output constrained nonlinear system with input saturation. Nonlinear Dyn. 92(4), 1845-1856 (2018)

[4] Li, H.Y., Zhao, S.Y., He, W., Lu, R.Q.: Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone. Automatica 100, 95-107 (2019)

[5] Liu, Y.J., Gong, M.Z., Tong, S.C., Chen, C.L.P., Li, D.J.: Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints. IEEE Trans. Fuzzy Syst. 26(5), 2607-2617 (2018)

[6] Chen, L., Wang, Q.: Prescribed performance-barrier Lyapunov function for the adaptive control of unknown pure-feedback systems with full-state constraints. Nonlinear Dyn. 95(3): 2443-2459 (2019)

[7] Xu, X., Cao, J.D., Hu, X., Liu, Q.: Extended state observer-based adaptive prescribed performance control for a class of nonlinear systems with full-state constraints and uncertainties. Nonlinear Dyn. https://10.1007/s11071-021-06564-3

[8] He, W., Dong, Y.T.: Adaptive fuzzy neural network control for a constrained robot using impedance learning. IEEE Trans. Neural Netw. Learn. Syst. 29(11), 1174-1186 (2018)

[9] Sun, W., Su, S., Wu, Y., Xia, J., Nguyen, V.: Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints. IEEE Trans. Cybern. 50(8), 3424-3432 (2020)

[10] Xia, J.W., Zhang, J., Sun, W., Zhang, B.Y., Wang, Z.: Finite-time adaptive fuzzy control for nonlinear systems with full state constraints. IEEE Trans. Syst. Man, Cybern. Syst. 49(7), 1541-1548 (2019)

[11] Wang, C.X., Wu, Y.Q., Wang, F.H., Zhao, Y.: TABLF-based adaptive control for uncertain nonlinear systems with time-varying asymmetric full state constraints. Int. J. Control. 94(5), 1238-1246 (2021)

[12] Kim, B.S., Yoo, S.J.: Adaptive control of nonlinear pure-feedback systems with output constraints: Integral barrier Lyapunov functional approach. Int. J. Control Autom. Syst. 13(1), 249-256 (2015)

[13] Liu, Y.J., Tong S.C., Chen, C.L.P., Li, D.J.J.: Adaptive NN control using integral barrier Lyapunov functionals for uncertain nonlinear block-triangular constraint systems. IEEE Trans. Cybern. 47(11), 3747-3757 (2017)

[14] Li, D., Liu, L., Liu, Y.J., Tong, S.C., Chen, C.L.P.: Fuzzy approximation-based adaptive control of nonlinear uncertain state constrained systems with time-varying delays. IEEE Trans. Fuzzy Syst. 28(8), 1620-1630 (2020)

[15] Sun, T., Pan, Y.: Robust adaptive control for prescribed performance tracking of constrained uncertain nonlinear systems. J. Franklin Inst. 356(1), 18-30 (2019)

[16] Liu, B., Wang, D., Li, H., Xie, X., Alostaib, N.D.: A novel neural-network-based adaptive control scheme for output-constrained stochastic switched nonlinear systems. IEEE Trans. Syst. Man, Cybern. Syst. 49(2), 418-432 (2019)

[17] Yin, S., Yu, H., Shahnaizi, R., Adel, H.: Fuzzy adaptive tracking control of constrained nonlinear switched stochastic pure-feedback systems. IEEE Trans. Cybern. 47(3), 579-588 (2017)

[18] Zhang, T., Wang, N., Wang, Q., Yi, Y.: Adaptive neural control of constrained strict-feedback nonlinear systems with input unmodeled dynamics. Neurocomputing 272, 596-605 (2018)

[19] Wang, X., Wu, Q., Yin, X.: Command filter based adaptive control of asymmetric output-constrained switched stochastic nonlinear systems. ISA Trans. 91, 112-124 (2019)

[20] Becchelli, C.P., Rovithakis, G.A.: Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance. IEEE Trans. Autom. Control 53(9), 2090-2099 (2008)

[21] Wang, M., Wang, C., Shi, P., Liu, X.: Dynamic learning from neural control for strict-feedback systems with guaranteed predefined performance. IEEE Trans. Neural Netw. Learn. Syst. 27(12), 2564-2576 (2016)

[22] Qiu, J., Sun, K., Wang, T., Gao, H.: Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance. IEEE Trans. Fuzzy Syst. 27(11), 2152-2162 (2019)

[23] Zhao, X., Tong, S.: Adaptive prescribed performance decentralized control for stochastic nonlinear large-scale systems. Int. J. Adapt. Control Signal Process. 32(12), 1782-1800 (2018)

[24] Meng, W., Yang, Q., Sun, Y.: Adaptive neural control of nonlinear mimo systems with time-varying output constraints. IEEE Trans. Neural Netw. Learn. Syst. 26(5), 1074-1085 (2015)

[25] Wang, S., Ren, X., Na, J.: Extended-state-observer-based funnel control for nonlinear servomechanisms with prescribed tracking performance. IEEE Trans. Autom. Sci. Eng. 14(1), 98-108 (2017)

[26] Han, S.L., Lee, J.M.: Fuzzy echo state neural networks and funnel dynamic surface control for prescribed performance of a nonlinear dynamic system. IEEE Trans. Ind. Electron. 61(2), 1099-1112 (2014)

[27] Wang, Y., Hu, J., Zheng, Y.: Improved decentralized prescribed performance control for non-affine large-scale systems with uncertain actuator nonlinearity. J. Franklin Inst. 356(13), 7091-7111 (2019)

[28] Zhang, J.X., Yang, G.H.: Robust adaptive fault-tolerant control for a class of unknown nonlinear systems. IEEE Trans. Ind. Electron. 64(1), 55-59 (2016)

[29] Zhang, J.X., Yang, G.H.: Fuzzy adaptive output feedback control of uncertain nonlinear systems with prescribed performance. IEEE Trans. Cybern. 48(5), 1342-1354 (2017)

[30] Song, Y.D., Zhou, S.: Tracking control of uncertain nonlinear systems with deferred asymmetric time-varying full state constraints. Automatica 98, 314-322 (2018)

[31] Wang, A., Liu, L., Qiu, J., Feng, G.: Event-triggered adaptive fuzzy output-feedback control for nonstrict-feedback nonlinear systems with asymmetric output constraint. IEEE Trans. Cybern. (2020). https://doi.org/10.1109/TCYB.2020.2974775

[32] Wen, C., Zhou, J., Liu, Z., Su, H.: Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance. IEEE Trans. Autom. Control 56(7), 1672-1678 (2011)

[33] Xu, Q., Wang, Z., Zhen, Z.: Adaptive neural network finite time control for quadrotor UAV with unknown input saturation. Nonlinear Dyn. 98(3), 1973-1998 (2019)

[34] He, W., Sun, Y., Yan, Z., et al: Disturbance observer-based neural network control of cooperative multiple manipulators with input saturation. IEEE Trans. Neural Netw. Learn. Syst. 31(5), 1735-1746 (2020)

[35] Wang, H., Shi, P., Li, H., Zhou, Q.: Adaptive neural tracking control for a class of nonlinear systems with dynamic uncertainties. IEEE Trans. Cybern. 47(10), 3075-3087 (2017)

[36] Ma, J., Ge, S.S., Zheng, Z., Hu, D.W.: Adaptive NN control of a class of nonlinear systems with asymmetric saturation actuators. IEEE Trans. Neural Netw. Learn. Syst. 26(7), 1532-1538 (2015)

[37] Zhang, Q., Liu, Y., Wen, G.: Adaptive neural network control for time-varying state constrained nonlinear stochastic systems with input saturation. Inf. Sci. 527, 191-209 (2020)

[38] Ma, L., Hua, X., Zhao, X., Zong, G.: Observer-based adaptive neural tracking control for output-constrained switched MIMO nonstrict-feedback nonlinear systems with unknown dead zone. Nonlinear Dyn. 99(2), 1019-1036 (2020)

[39] Wang, C., Hill, D.J., Ge, S.S., Chen, G.R.: An ISS-modular approach for adaptive neural control of pure-feedback systems. Automatica 42(5), 723-731 (2006)

[40] Ge, S.S., Wang, C.: Adaptive neural control of uncertain MIMO nonlinear systems. Trans. Neur. Netw. 15(3), 674-692 (2004)

[41] Liu, Y., Liu, X., Jing, Y.: Adaptive neural networks finite-time tracking control for non-strict feedback systems via prescribed performance. Inf. Sci. 468, 29-46 (2018)

[42] Liu, X., Wang, H., Gao, C., Chen, M.: Adaptive fuzzy funnel control for a class of strict feedback nonlinear systems. Neurocomputing 241, 71-80 (2017)