Simulation of partial discharge in helium filled elliptic cavity in dielectric

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Abstract. Partial discharge features in cavities in a condensed dielectric was studied. Two models for description of the electric charge transfer during partial discharge were used that are model of constant conductivity of a cavity and a diffusion-drift model. The numerical methods for both models were realized for parallel computations with GPU that accelerates the calculations significantly. The three-dimensional calculations of apparent and true charge in cavities of the shapes of different size spheres and prolate spheroid were conducted. The simulation of the initial stage of streamer development in elliptic helium filled cavity was performed.

1. Introduction
One of the effective methods of testing the insulation in high-voltage equipment is the registration of partial discharges (PD). The equipment is subject to malfunctions because of the appearance of gas bubbles in liquid insulation as well as cracks and cavities in solid insulation which appear over time. The application of a high voltage results in the formation of electronic avalanches that leads to the ionization of gas in cavities. The gas is ionized, and the gas cavity becomes conductive for a short period of time equal to ~10 ns. Thus, the understanding of the charges relaxation mechanisms in gas bubbles elucidates the characteristics of PD in insulation.

The works in the field of gas discharge devoted to simulations of avalanches and streamers are known for a long time [1-5]. They studied the conditions of avalanche development, their transition to streamer and the role of diffusion in streamer evolution [2], the structure of the streamer head and role of the photo-ionization [3], etc. There are many works devoted to the streamer discharge simulation for technical applications, for example [4, 5]. Nevertheless, the simulations of streamer development in gas cavities and its applications to describe PD in liquid and solid dielectrics began to appear just recently [6]. In these works, the calculations of gas discharges in spherical cavities were conducted in the net of cavities regularly distributed in dielectric. Though the electrical characteristics of PD were not studied in details in these works.

In the present work we calculated the characteristics of PD in spherical and elliptical cavities of different sizes within the model of constant conductivity of plasma in the cavity. Also the model of streamer development in helium filled elliptic cavity was made within the diffusion-drift approximation. All the models were realized as parallel algorithms for calculations with graphic processing units (GPU).
2. The models of charge relaxation in a cavity
The electric field and the charge distribution in the gap between electrodes were calculated at each time step by solving the system of equations listed below. In the region occupied by dielectric with a dielectric permittivity \( \varepsilon \)

\[
\text{div}(\varepsilon \mathbf{E}) = 0, \quad \text{where } \mathbf{E} = -\nabla \varphi.
\]

In the gas cavity where the electron avalanches can develop

\[
\text{div}(\varepsilon_0 \mathbf{E}) = \rho,
\]

where \( \rho \) is the charge density.

We used two models to describe the transport of positive and negative charge to the walls of the cavity. In the first model, we consider that plasma in a cavity occurs instantly and then the conduction of a cavity is constant during PD. The charge is transferred in accordance with the continuity equation

\[
\frac{\partial \rho}{\partial t} + \text{div}(\mathbf{j}) = 0,
\]

where the current density is

\[
\mathbf{j} = \sigma \mathbf{E},
\]

with \( \sigma = 0 \) in dielectric and \( \sigma = \text{const} \) in the cavity. In the second model, we solved the transport equations in the form [7, 8]

\[
\frac{\partial n_e}{\partial t} - \text{div}(\mu_e n_e \mathbf{E} + D_e \nabla n_e) = S_i - S_r + S_{ph},
\]

\[
\frac{\partial n_+}{\partial t} + \text{div}(\mu_+ n_+ \mathbf{E} - D_+ \nabla n_+) = S_i - S_r + S_{ph},
\]

where \( n_e \) and \( n_+ \) are the concentrations of the electrons and positive ions, \( \mu_e \) and \( \mu_+ \) are the mobilities of the electrons and ions, \( D_e \) and \( D_+ \) are the diffusion coefficients. The ionization term is \( S_i = (\mu_e n_e \mathbf{E} + D_e \nabla n_e) \alpha(\mathbf{E}) \). Here \( \alpha(\mathbf{E}) \) is the first Townsend coefficient.

The recombination was taken into account as \( S_i = \beta n_e n_+ \), where \( \beta \) is effective recombination coefficient. The charge density in (1) is \( \rho = e(n_+ - n_e) \) where \( e \) is the elementary charge.

This model was applied for simulating the initial stage of a streamer growth in a cavity.

3. Calculations with Graphic Processing Units
It is well known that computations of electric potential distribution take the most time when simulating electro physical problems despite the fact that many numerical solution methods of the equation (1) were proposed. We used NVIDIA Graphic Processing Units (GPU) which allowed processing many nodes of the lattice simultaneously that accelerates the computations significantly.

The conservative finite-difference numerical method was realized for the GPU for calculating the currents and the electric potential from the system (1) – (3).

The CUDA programming technology with C language was used to implement the algorithm in GPU. The graphic card with 2880 processor cores was used. Each lattice node was processed in its own thread. The blocks of 32 threads provided the maximum computing performance. The use of GPU accelerated the calculations of the electric field by about 100 times. All the data were allocated in the fast global memory of GPU.

At each time step, the distribution of the potential was calculated by simple iterations. The accuracy of the reproduction of the shape of PD current, especially the peak value depends strongly on the accuracy of field calculation. For the constant conductivity the PD pulse is described by exponential decay function. The simulations showed that the magnitude of this pulse is calculated with the relative error less than 1 percent if the relative error of the potential calculation is less than \( 10^{-10} \).

The numerical method for the solution of the diffusion-drift model equations (4) – (5) were also realized for GPU. The calculations were performed for the lattices of sizes up to \( 2^{24} \approx 16 \times 10^6 \) nodes.
4. The simulations of apparent and true charges in spherical and prolate bubble

Three-dimensional simulations of PD in cavities of various sizes and shapes were performed. We used the Maxwellian relaxation time \( \tau = \varepsilon / 4 \pi \sigma \) as the scale of time. The applied voltage \( V \) and the gap length \( d \) were used as the units of the voltage and the distances. Thus, the unit of charge is \( Q_0 = V \cdot d \).

![Figure 1](image)

**Figure 1.** The dependence of true (a) and apparent (b) charge on the cavity radius of PD (b).

The calculations of the apparent and the true charge at PD in a spherical cavity were performed. The influence of the cavity size on the values of these charges was studied. It was considered that a spherical cavity was placed at the center of the inter-electrode gap. The gap was filled with a dielectric with the dielectric permittivity \( \varepsilon = 2.2 \) corresponding to the dielectric permittivity of a transformer oil. The cavity had the dielectric permittivity \( \varepsilon = 1 \).

The dependences of the true and the apparent charges on the spherical cavity radius are shown in figure 1, a and b, respectively. The best approximation of the curve for the true charge is close to the power dependence with the exponent 2 and can be written as \( q_t = 1.8 \cdot q_0 \cdot (R/d)^{2.058} \). For the apparent charge \( q_{ap} \), we found that \( q_{ap} = 2.92 \cdot q_0 \cdot (R/d)^{3.059} \) that is close to the power dependence with the exponent 3. These results show that the relation \( q_{ap}/q_t \) increases approximately linearly with the increase of the size of spherical cavity in a dielectric at least if the cavity is in the center of the gap. This dependence of \( q_{ap}/q_t \) on \( R \) is in a good agreement with the results of Ref. [9]. For spherical helium bubble of the diameter 1.86 mm and electrode spacing of 6.8 mm similar that of the work [10] we obtained the apparent charge value \( q_{ap} = 123.5 \) pC at the applied voltage \( V = 22.4 \) kV that is in a satisfactory agreement with the experiments.

It is well known that the gas bubble in the dielectric liquid is deformed under the action of a strong electric field due to the electrostrictive forces. The shape of the bubble is usually described by the model of the prolate spheroid with the larger semi-axis directed along the electric field in this case. The apparent charge of PD for deformed cavities was calculated. The cavity had the shape of the prolate spheroid with the larger semi-axis \( b \) and the shorter semi-axis \( a \). To describe the shape of the spheroid we will use the deformation coefficient in the form of \( k = b/a \). As it was shown earlier the value of the apparent charge measured during PD depends strongly on the cavity size.

The calculation performed in this work showed that the deformation of the cavity significantly affects the value of the apparent charge of PD. Nevertheless, the apparent charge value reduced to the volume of the cavity \( q_{red} = Q_{app} / \pi a^2 b \) depends on just the deformation coefficient provided that the other parameters are the same. The results of this calculations are shown in figure 1 (dots) and Table 1.
The dependence of \( q_{\text{red}} \) on \( k \) is well approximated by the polynomial 
\[
q_{\text{red}} = -37.25 + 114.57 \cdot k - 59.43 \cdot k^2 + 12.89 \cdot k^3
\]
that is shown in figure 1 (solid line).

![Graph showing the dependence of reduced apparent charge on deformation coefficient.](image)

**Figure 2.** The dependence of the reduced apparent charge in a circuit on the deformation \( k \) of the elliptic cavity (dots). The approximation by a polynomial is also shown (solid line).

**Table 1.** The deformation coefficient of the cavity dependence \( k = b/a \) and the corresponding «apparent» charge reduced to the cavity volume \( Q_{\text{app}} \).

| \( k \)  | 1.0 | 1.067 | 1.138 | 1.214 | 1.286 | 1.538 | 1.769 | 1.917 | 2.0 |
|--------|-----|-------|-------|-------|-------|-------|-------|-------|-----|
| \( Q_{\text{app}}, \text{fCl/m}^3 \) | 30  | 34    | 35    | 37    | 39    | 45    | 51    | 55    | 57  |

5. **Simulation of the electron avalanche development in helium bubble**

The dependence of the electron mobility on the electric field is taken from [11, 12]. The data were reduced to the gas density at atmospheric pressure. The approximation of these experimental data were obtained in the form of
\[
\alpha(E) = \exp(b_0 + b_1 (E + b_2)),
\]
where \( b_0 = 12.5741, b_1 = -5.8983, b_2 = 6.246 - 10^5 \) for the electric field range from 3 kV/cm to 100 kV/cm, and \( b_0 = 13.3426, b_1 = -2.5558 - 10^7, \) and \( b_2 = 5.8672 - 10^6 \) for the values of \( E \) from 100 kV/cm to 2 MV/cm.

The diffusion coefficient and the mobility for ions were chosen to be constant for the simplicity since the diffusion and the mobility of ions are significantly smaller than these values of the electrons, and change not so strongly with \( E \). The mobility and the diffusion coefficient for the helium ions in
pure helium were taken from [11]. The diffusion coefficient was \( D_+ = 0.27 \text{ cm}^2/\text{s} \) and the mobility was \( \mu_+ = 10.4 \text{ cm}^2/\text{V s} \). The recombination coefficient was taken from [15] for low density and low temperature range. The average electric field \( E_0 = V/d \), the gap distance \( d \), and the diffusion coefficient for ions \( D_+ \) were taken as the basic units. In this case the time step in figure 3 is measured in units of \( \tau_0 = h^2/D_+ \), where \( h \) is the lattice step.

![Image](image-url)

**Figure 3.** Distribution of the vertical component of the electric field stress in central cross-section of the gap along the axis of electrode gap (a). The shape of the streamer at the timepoint t4 (b). Profiles of the electric field (c) and the electron density (d) along the axis of electrode gap at different timepoints. \( t_1 = \tau_0 \), \( t_2 = 1.86 \tau_0 \), \( t_3 = 1.96 \tau_0 \), \( t_4 = 2.12 \tau_0 \).

The simulations of the initial stage of streamer development in elliptical cavity filled with helium were performed (figure 3). The deformation coefficient for the cavity was \( k = b/a = 1.5 \). The ratio of the large axis of the bubble to the gap length was \( 2b/d = 0.487 \). The cavity located in the center of the gap. The dielectric had the permittivity \( \varepsilon = 2.2 \) while gas inside the cavity - \( \varepsilon = 1 \).

### 6. Conclusion

The methods of computation of PD in dielectrics with the gas cavities inside were developed. For accelerating computations, the methods were realized for graphic processing units. The simulations of PD in gas cavities inside condensed dielectric were performed using GPU with the number of...
computing cores up to 2880 on very large lattices. The dependences of apparent and true charge on the size of the spherical cavity and the deformation of elliptical cavity were obtained. It was shown that the true charge increases as the square of the radius while the apparent charge is proportional to the cubic radius of the sphere. The simulation of the electron avalanche development in the elliptic bubble filled with helium was performed using GPU. The concentrations of the electrons and the ions as well as the field distribution in the inter-electrode gap were obtained.

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