Nuclear halo and the coherent nuclear interaction

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(Dated: November 8, 2018)

Abstract

The unusual structure of $^{11}\text{Li}$, the first halo nucleus found, is analyzed by the Preparata model of nuclear structure. By applying Coherent Nucleus Theory, we obtain an interaction potential for the halo-neutrons that rightly reproduces the fundamental state of the system.

PACS numbers: 21.30.Fe, 21.60.-n,21.45.+v

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I. INTRODUCTION

The understanding of halo nuclei is one of the issues of nuclear research still awaiting a satisfactory explanation. The nucleus $^{11}\text{Li}$ is the first observed and the most interesting case of a two neutrons halo ($^9\text{Li} + n + n$) and different kinds of experiment have been performed to investigate its structure. The generally accepted picture of a halo nucleus describes it by a core nucleus surrounded by loosely bound valence neutron which tunnel with significant probability into a region outside the core potential. Many theoretical efforts have been carried out to describe this Borromean system where the three-body system is bound and its all binary subsystem are unbound, but these attempts with the nature of the $^9\text{Li} + n$ interaction that is not exactly known.

In this paper we wish to show that the unusual properties of halo nuclei can be explained in the framework of the Coherent Nucleus Theory proposed more the ten years ago by Giuliano Preparata. This theory which lays at the foundation of the nuclear Shell Model has been applied to several problems of nuclear physics, namely: EMC nuclear effect, Coulomb sum rule in quasi-elastic electron-nucleus scattering, deep-inelastic scattering at low $x$, hypernuclear interactions and decays, low-energy photoabsorption in nuclei, neutron stars. According to this approach, inside a nucleus the nucleons are involved in a laser-like process whose two levels, strongly coupled to the pion field, are the $\text{N}(940)$ and the $\Delta (1232)$. The solutions of the coherence equations for this $N\Delta\pi$ coupled system, at the resonant $\pi$-mode for which $\omega_\pi \equiv \omega_0 = m_\Delta - m_N = 292$ MeV and $q \equiv |\vec{q}| = 256$ MeV, are characterized by time-independent amplitudes and phases that vary linearly with time. In this way, the laser process produces a coherent $\pi$-condensate, characterized by its well-defined phase relation with the $N-\Delta$ system, which is spread out throughout the spatial region where the collective ”$N-\Delta$ current” is localized, i.e. within the nucleus.

The most stable configuration is reached in a spatial region of radius $R_{CD} \simeq 4.2$ fm called Coherence Domain (CD) and within a single CD the resulting $p$-wave $\pi$-field is given by ($i = 1, 2, 3$ is the isospin index)

$$\phi_i(\vec{x}, t) = 8\pi \sqrt{\frac{\rho}{2\omega_0}} (\hat{x} \cdot \vec{\alpha}_i) j_1(qr) \sin(\omega_r t) \quad (r < R_{CD})$$

where $\rho$ is the nuclear density, $\vec{\alpha}_i$ is the $\pi$-amplitude

$$\sum_i |\vec{\alpha}_i|^2 \simeq (0.3)^2$$
and

\[ \omega_r = \omega_o (1 - \phi) \simeq 100 \text{ MeV} \quad (3) \]

is the "renormalized frequency" (\( \dot{\phi} \) is the pion phase velocity) inside the nuclear medium. We should emphasize that this pion condensation, that is generated via long-range coherent hadronic forces, is totally unrelated to the static incoherent pion condensate proposed by Migdal \[12\] and predicted at densities far from normal nuclear density \[13\]. As a fundamental result, the total energy of the coherent state is lowered and the average energy gain per particle, at nuclear matter density \( \rho_o \simeq 0.166 \text{ fm}^{-3} \), is about 60 MeV, which well represents the depth of the self-consistent nuclear potential.

A coherent evolution of the nuclear dynamics is done also for light nuclei \[10\] if it is possible to match Eq. (1) with the solution of the free-field equation (\( \hbar = c = 1 \))

\[ (\Box + m^2)\phi_i(\vec{x},t) = 0 \quad (4) \]

valid outside the nucleus \( (r > R_A \simeq r_o A^{1/3}) \), whose \( p \)-wave solution is given by

\[ \phi_i(\vec{x},t) = (\vec{x} \cdot \vec{A}_i) k_1(\lambda r) \sin(\omega_r t) \quad (r > R_A) \quad (5) \]

where

\[ \vec{A}_i = 8\pi \sqrt{\rho} \frac{j_1(q R_A)}{2\omega_o k_1(\lambda R A)} \vec{\alpha}_i \quad (6) \]

and \( \lambda \) determined by joining Eq. (5) together with its first radial derivative to the inner \( \pi \)-field of Eq. (1). A simple calculation provides for a critical radius \( R_c \simeq 2.42 \text{ fm} \) below which there can be no exponentially decaying solution of (4). The radius of \( ^9\text{Li} \) is approximately \( R_A \simeq 2.5 \text{ fm} \) and we obtain \( \lambda \simeq 62 \text{ MeV} \).

We possess now all the ingredients needed to analyze a possible mechanism for explaining the basic properties of the halo nuclei in terms of interaction between the extra neutrons and the evanescent tail of the pion condensate \[5\].

**II. MODEL AND METHOD OF CALCULATION**

In our approach to determine the ground state of the halo nucleus we assume that the extra neutrons interact with the core-nucleus through their coupling to the evanescent tail of its coherent pion field \( \pi_c \). The virtual dispersive interaction potential for the basic process
\[ \pi_c + n \rightarrow \pi_c + n \] is calculated by applying second order perturbation theory:

\[ V_n(\vec{x}) = -\frac{i}{4m_n} \int_{-\infty}^{+\infty} dt \int d^3\xi \langle n|T \left[ H_I \left( \vec{x} + \frac{\vec{\xi}}{2}, \frac{t}{2} \right) H_I \left( \vec{x} - \frac{\vec{\xi}}{2}, -\frac{t}{2} \right) \right]|n\rangle \] (7)

where \(|n\rangle\) is the ground state we search for and the interaction hamiltonian \(H_I\) will be given explicitly later.

Inserting into Eq.(7) a complete sum over intermediate state and re-arranging the time-ordered product we obtain:

\[ V_n(\vec{x}) = -\frac{i}{2m_n} \int_{0}^{+\infty} dt \int d^3\xi \sum_N \langle n|H_I \left( \vec{x} + \frac{\vec{\xi}}{2}, t \right) |N\rangle \langle N|H_I \left( \vec{x} - \frac{\vec{\xi}}{2}, -t \right) |n\rangle \] (8)

For our low-energy calculation the intermediate state are the nucleons on themselves and the interaction Hamiltonian that we assume is the usual non relativistic reduction of the pseudoscalar \(\pi NN\) coupling:

\[ H_{\pi NN} = ig\bar{N}\gamma_5 \tau \cdot \vec{\pi}N \] (9)

with \(g^2/4\pi \simeq 14.3\).

In non-relativistic limit the matrix element is:

\[ \langle n|H_I(\vec{x}, t)|N\rangle = g(\vec{\sigma} \cdot \vec{k})[\vec{\tau} \cdot \vec{\phi}(\vec{x})]e^{i\vec{k} \cdot \vec{x}}e^{-i(E_N^N - m_n t) \sin (\omega_r t)} \] (10)

where \(\vec{\phi}(\vec{x})\) is the \(\pi\)-field of Eq.(5), \(\vec{k}\) is the momentum of the intermediate nucleon, \(\vec{\sigma}\) and \(\vec{\tau}\) are the spin and the isospin operators respectively. Performing the necessary algebra and integrating over the time, we have:

\[ V_n(\vec{x}) = \frac{g^2}{4m_n} \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{k^2} - \frac{1}{2} \left( \frac{1}{k^2 + 2m_N \omega_r} \right) + \left( \frac{1}{k^2 - 2m_N \omega_r} \right) \right] \times \sum_i \int d^3\xi \phi_i \left( \vec{x} + \frac{\vec{\xi}}{2} \right) \phi_i \left( \vec{x} - \frac{\vec{\xi}}{2} \right) e^{i\vec{k} \cdot \vec{\xi}} \] (11)

Inserting the explicit expression for the pion field and performing the integration over the variable \(\vec{\xi}\) we obtain, taking the limit \(|\vec{\xi}| \ll |\vec{x}|\), the following result

\[ V_n(\vec{x}) \simeq \frac{g^2}{4m_n} \left( \frac{4\pi r}{\lambda} \right)^{\frac{3}{2}} e^{-2\lambda r} \left( 1 + \frac{1}{\lambda r} \right)^2 \sum_i (\hat{x} \cdot \vec{A}_i)^2 \times \int \frac{d^3k}{(2\pi)^3} e^{-\frac{k^2}{2}} \left[ 1 - \frac{k^2}{2} \left( \frac{1}{k^2 + 2m_N \omega_r} + \frac{1}{k^2 - 2m_N \omega_r} \right) \right] \] (12)

where \(r = |\vec{x}|\). The integration over \(|\vec{k}|\) is extended to all those values for which the exponential in the above expression doesn’t make the integrand function vanishing, therefore the
significant region for the integration is such that $R_{\text{halo}}k^2/\lambda < 1$. For this set of values we have $k \ll \sqrt{2m_N\omega_r}$ and

$$\int d^3k e^{-\frac{\vec{k}^2}{\lambda}} \left( \frac{1}{\vec{k}^2 + 2m_N\omega_r} + \frac{1}{\vec{k}^2 - 2m_N\omega_r} \right) \approx 0$$

(13)

that allows us to rewrite the potential as:

$$V_n(\vec{x}) \approx -\frac{g^2}{4m_n} \frac{e^{-2\lambda r}}{\lambda^2 r^2} \left( 1 + \frac{1}{\lambda r} \right)^2 \sum_i (\hat{\vec{x}} \cdot \vec{A}_i)^2, \quad r > R_A$$

(14)

The process $\pi_c + [nn] \rightarrow \pi_c + [nn]$, actually the one involving the neutron pair in $^{11}\text{Li}$, is a few-body problem whose solution requires some approximations. The interaction Hamiltonian is

$$H_I = i \frac{g}{2m_n} \sum_{\lambda=1}^2 \vec{r}_\lambda (\vec{\sigma}_\lambda \cdot \vec{k}_\lambda) \vec{\phi}(\vec{x}_\lambda)$$

(15)

where $\vec{x}_\lambda$ and $\vec{k}_\lambda$ stand for position and momentum of the $\lambda^{th}$ neutron ($\lambda = 1, 2$) respectively.

A special case of interest that will be explored is that for which the interaction Hamiltonian reduces to

$$H_I = i \frac{g}{2m_n} [\tau_1^k (\vec{\sigma}_1 \cdot \vec{k}) + \tau_2^k (\vec{\sigma}_2 \cdot \vec{k})] \phi_k(\vec{x})$$

(16)

where

$$\vec{k} = \vec{k}_1 + \vec{k}_2, \quad \vec{x} = \frac{\vec{x}_1 + \vec{x}_2}{2}$$

(17)

At this point, with the adopted approximation, we can proceed as the one extra neutron case. An intermediate result for the potential is given by

$$V_{2n}(\vec{x}) \approx -\frac{g^2}{2m_n} \left( \frac{4\pi r}{\lambda} \right)^3 \frac{e^{-2\lambda r}}{\lambda^2 r^2} \left( 1 + \frac{1}{\lambda r} \right) \sum_i (\hat{\vec{x}} \cdot \vec{A}_i)^2 + (\vec{\sigma}_1 \cdot \hat{\vec{k}})(\vec{\sigma}_2 \cdot \hat{\vec{k}}) \tau_1^i \tau_2^j (\hat{\vec{x}} \cdot \vec{A}_i)(\hat{\vec{x}} \cdot \vec{A}_j)$$

(18)

but, with respect to the case of one neutron, additional considerations have to be done for the spin-isospin terms in the above expression. When we consider the $n-n$ system in a singlet spin state $S = 0$ and in a triplet isotopic spin state $I = 1$ we have

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3$$

(19)

and

$$\chi_{I=1}^\dagger \tau_1^i \tau_2^j \chi_{I=1} = (\chi_1^+)^\dagger (\chi_2^-)^\dagger \tau_1^i \tau_2^j \chi_1^+ \chi_2^- = \delta^{i3} \delta^{j3}$$

(20)
\[ r_1^i r_2^j (\hat{x} \cdot \vec{A}_i)(\hat{x} \cdot \vec{A}_j) = (\hat{x} \cdot \vec{A}_3)^2 = \frac{1}{3} \sum_i (\hat{x} \cdot \vec{A}_i)^2 \]  

the last from symmetry conditions. The interaction potential is finally given by

\[ V_{2n}(\vec{x}) \simeq -\frac{g^2}{3m_n} \left( \frac{R_A}{r} \right)^4 \left( \frac{1 + \lambda r}{1 + \lambda R_A} \right)^2 e^{-2\lambda(r-R_A)} \sum_i (\hat{x} \cdot \vec{A}_i)^2, \quad r > R_A \]  

III. RESULTS

The effective Schrödinger equation for \(^9\)Li + 2\(n\) system is given by

\[ \left[ -\nabla^2 + V_{2n}(\vec{x}) \right] \psi(\vec{x}) = E_{2n} \psi(\vec{x}), \quad r = |\vec{x}| > R_A \]  

This equation can be solved for the state with \(l = 0\) observing that

\[ V_{2n}(\vec{x})_{l=0} \equiv V_{2n}(r) = -V_0 \left( \frac{R_A}{r} \right)^4 \left( \frac{1 + \lambda r}{1 + \lambda R_A} \right)^2 e^{-2\lambda(r-R_A)}, \quad r > R_A \]  

where

\[ V_0 = \frac{g^2}{3m_n} \frac{\alpha^2}{3} \eta^2 \simeq 61.3 \text{ MeV} \]  

with

\[ \eta = 8\pi \sqrt{\frac{\rho}{2\omega_0}} j_1(qR_A) \]  

Writing for \(l = 0\)

\[ \psi(r) = \frac{1}{r} \chi(r) \]  

and assuming for the interaction potential the following approximate expression

\[ V_{ap}(r) = -V_0 e^{-\mu(r-R_A)}, \quad r > R_A \]  

with \(\mu = 1.8 \text{ fm}^{-1}\), calculated by interpolation of the potential \[28\] with the exact potential \[24\], we can introduce the new variable

\[ y = e^{-\frac{\mu}{2}(r-R_A)} \]  

and rewrite the radial part of the Schrödinger equation as

\[ \frac{d^2 \chi}{dy^2} + \frac{1}{y} \frac{d\chi}{dy} + \left( c^2 - \frac{q^2}{y^2} \right) \chi = 0 \]
with the abbreviations
\[ c^2 = 16 \frac{m_n V_0}{\mu^2}, \quad q^2 = -16 \frac{m_n E_{2n}}{\mu^2} \] (31)

The equation (30) is a differential Bessel equation, whose general solution is given by

\[ \chi(y) = C_1 J_q(cy) + C_2 J_{-q}(cy). \] (32)

By Eq. (29) we have that for \( y = 0, r \to \infty \) and \( \chi \) must vanish. Therefore \( C_2 = 0 \) and the reduced wave function becomes

\[ \chi(r) = C_1 J_q \left[ ce^{-\mu^2(r-R_A)} \right] \] (33)

Due to the Pauli principle between the extra and the core nucleons, we require \( \chi(R_A) = 0 \) i.e.

\[ J_q(c) = 0 \] (34)

stipulating that the surface of the nucleus acts as a infinite potential barrier.

Using Eq. (31) and (34) and the above numerical values for \( \mu \) and \( V_0 \), we can calculate the energy eigenvalue of the bound state and we obtain

\[ E_{2n} \simeq 300 \text{ keV} \] (35)

to be compared with the experimental value \( E = 294 \pm 30 \text{ keV} \) [1]. By means of the derived wave function the root mean square radius of the halo is given by

\[ r_h = \sqrt{\int_{R_A}^{+\infty} r^2 \chi^2(r) dr} \simeq 7.0 \text{ fm} \] (36)

so that the root mean square radius of the total system is

\[ r_{RMS} = \sqrt{\frac{A_c r_c^2}{A} + \frac{2}{A} r_h^2} \simeq 3.6 \text{ fm} \] (37)

where \( r_c \) is \( rms \) radius of core nucleus. The experimental value of the above quantity is \( 3.55 \pm 0.10 \text{ fm} \) [14].

Let us now compare the potential obtained for one neutron and the above one just calculated for the system of the two neutrons. We have

\[ \frac{(V_0)_n}{(V_0)_{2n}} \propto \frac{(\tau \cdot \sigma)^2}{(\tau_1 \cdot \sigma_1 + \tau_2 \cdot \sigma_2)^2} = \frac{3}{4} \] (38)
The limiting value of $V_0$ is obtained from (34) in the limit of vanishing energy $q = 0$ and is given by

$$(V_0)_{\text{min}} = \frac{(2.4\mu)^2}{16m_n} \simeq 48.5 \text{ MeV} \quad (39)$$

From (38) we obtain a smaller value

$$(V_0)_n \simeq 46.5 \text{ MeV} \quad (40)$$

This fact points out that the $^{10}\text{Li}$ is unbound as expected.

IV. CONCLUSIONS

The qualitative difference between the standard approaches and our calculation is clear and can be easily understood. In ordinary potential models the neutrons are loosely bound to an inert core and occupy the (possible) vacant state of the nuclear potential and the pairing between the neutrons in two-neutron halo plays a crucial role in their stability. In our approach the extra neutrons are localized outside the core through the virtual interaction with the evanescent $\pi$-wave. On the other hand, if $\pi$-condensation does occur, like it happens in our model, an interaction between two neutrons and the evanescent $\pi$-wave become possible, leading to a halo nucleus. In this way, we are able to reproduce the experimental results for the two-neutron separation energy and root mean square radius of $^{11}\text{Li}$ and the fact that the $^{10}\text{Li}$ is unbound.

With this work we have taken only the first step in a research program aimed at analyzing the consequences of the Coherent Nucleus Theory on the structure of halo nuclei and leave for a future publication a more detailed investigation of the correlations in two neutron halos.

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