The spin content of the proton in quenched QCD.

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We present preliminary results on the proton spin structure function at zero momentum, in the quenched approximation. Our calculation makes use of a nonperturbative means of determining the multiplicative renormalization of the topological charge density.
I. INTRODUCTION

Many years after its experimental observation, the proton spin crisis remains a puzzle for QCD to address. The problem may be stated as follows: Consider the singlet axial current

\[ j^5_\mu = \sum_{f=1}^{N_f} \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f \]  

Its on-shell nucleon matrix element has the form

\[ \langle \vec{p}, e | j^5_\mu | \vec{p}', e' \rangle = \bar{u}(\vec{p}, e) \left[ G_1(k^2) \gamma_\mu \gamma_5 - G_2(k^2) k_\mu \gamma_5 \right] u(\vec{p}', e') \]  

where \( e, e' \) label the helicity states and \( k \) is the momentum transfer. In a naive wave function picture \( G_1(0) \) can be interpreted as the fraction of the nucleon spin carried by the quarks. Experimental determinations lead to an unexpectedly small value of \( G_1(0) \), calling for a theoretical explanation in the context of QCD.

In the chiral limit, \( j^5_\mu \) is not conserved; its anomalous divergence is proportional to the topological charge density,

\[ q(x) = \frac{g^2}{32 \pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}(x) \]  

One has

\[ \langle \vec{p}, e | \partial^\mu j^5_\mu | \vec{p}', e' \rangle = -\langle \vec{p}, e | 2N_f q | \vec{p}', e' \rangle = -2MA(k^2) \bar{u}(\vec{p}, e) i\gamma_5 u(\vec{p}', e') \]  

where \( A(k^2) = G_1(k^2) + G_2(k^2) k^2 / M \) and, therefore, \( A(0) = G_1(0) \). Finding \( G_1(0) \) thus reduces to a calculation of the proton matrix elements of \( q(x) \).

For a determination of this nonperturbative matrix element, we turn to the lattice. To proceed, we need a (lattice-)regularized version of \( q(x) \), for example

\[ q_L(x) = -\frac{1}{24 \times 32 \pi^2} \sum_{\mu\nu=\pm 1}^{\pm 4} \epsilon_{\mu\nu\rho\sigma} \text{Tr} [\Pi_{\mu\nu} \Pi_{\rho\sigma}] \]  

where \( \Pi_{\mu\nu} \) is the product of link variables around a plaquette. As with any regularized operator, the correct approach to the continuum requires a proper renormalization. A consideration of the quantum numbers of \( q_L(x) \) leads to the following simple form

\[ q_L(x) \xrightarrow{a \to 0} a^4 Z_q(\beta) q(x) + O(a^6) \]  

A precise determination of \( Z_q \) is a prerequisite for the calculation of the hadronic matrix element.

Using the numerical (heating) method described in Refs. [6, 7], we evaluate \( Z_q(\beta) \) without any recourse to perturbation theory. Subsequently, an estimate of \( G_1(0) \) is obtained by measuring the matrix element \( \langle \vec{p}, e | q_L | \vec{p}', e' \rangle \) on the lattice, using Wilson fermions in the quenched approximation.

We should also mention two earlier investigations of this issue: Ref. [9] estimates \( G_1(0) \) in a quenched calculation; the result quoted there must be corrected by renormalization effects of the lattice topological charge density. In Ref. [10] a simulation of unquenched staggered fermions is performed, using the “geometrical” topological charge density.
II. DETERMINATION OF $Z_Q$

We determine $Z_q$ nonperturbatively by a method described in Refs. [7,8]. The method relies on the fact that $Z_q(\beta)$ is produced by short ranged quantum fluctuations, and therefore should not suffer from critical slowing down, unlike “global” quantities, such as the topological charge. We start from a configuration $C_0$ which is an approximate minimum of the lattice action and carries a definite topological charge $Q_{L,0}$ (typically $Q_{L,0} \approx 1$). We construct ensembles $C_n$ made out of independent configurations obtained by heating (i.e. updating by a local algorithm) the starting configuration $C_0$, at a given $\beta$, for the same number $n$ of updating steps, and average the topological charge over $C_n$ at fixed $n$. Plotting $Q_L = \sum_x q_L(x)$ averaged over $C_n$ as a function of $n$, we should see first a decrease of the signal, originated by the onset of $Z_q(\beta)$ during thermalization of the short-ranged modes, followed by a plateau. The average of $Q_L$ over plateau configurations should be equal to $Z_q(\beta)Q_{L,0}$.

On a $16^4$ lattice we constructed an approximate instanton configuration whose charge is $Q_{L,0} \approx 0.95$. Heating was performed using a 10-hit Metropolis algorithm (tuned to 50% acceptance) at $\beta = 6.0$, and collecting about 4500 heating trajectories. In Fig. 1 we plot $Q_L(C_n)/Q_{L,0}$. We see clearly a plateau starting from $n \approx 15$. The value of $Q_L/Q_{L,0}$ at the plateau gives an estimate of $Z_q(\beta)$:

$$Z_q(\beta = 6.0) = 0.18(1)$$

In order to check the stability of the background topological structure of the initial configuration, after the heating procedure we cool the configurations, by locally minimizing the action; we find $Q_L \approx Q_{L,0}$ after a few cooling steps.

III. SPIN CONTENT OF THE PROTON

In order to calculate $G_1(0)$ we performed Monte Carlo simulations in the quenched approximation using Wilson fermions. In terms of the proton annihilation operator $a_\alpha = \epsilon^{abc}[u^b_\gamma \gamma_3 d^c]u^a_\alpha$ and its Fourier transform $\tilde{a}(\vec{p}, t)$, we should have

$$\langle \tilde{a}_\alpha(\vec{0}, t)\tilde{\bar{a}}_\beta(\vec{0}, 0) \rangle \simeq Z_a \left( \frac{1 + \gamma_0}{2} \right)_{\alpha\beta} e^{-Mt} \quad t \gg 1$$

and

$$\langle \tilde{a}_\alpha(\vec{0}, t)\tilde{q}_L(\vec{k}, t_q)\tilde{\bar{a}}_\beta(\vec{0}, 0)P_{\alpha\beta} \rangle \simeq i (-)^P V Z_a Z_q \frac{M}{2E(k)} \frac{A(-k^2)}{N_f} |\vec{k}| e^{-Mt} e^{-\left(\frac{1}{2}\right)(E(k)-M)t_q} \quad t, t_q \gg 1$$

where $E(k) = \sqrt{M^2 + k^2}$; $P_{\alpha\beta}$ and $P$ are the helicity projection operator and its eigenvalue. Given that, with the Wilson fermion discretization, the state of opposite parity propagates backward in time, we must keep $t \leq T/2$, where $T$ is the temporal size of the lattice, in order to isolate the proton state. Then $G_1(0) = A(0)$ can be obtained extrapolating to $\vec{k} = 0$ the following relationship.
\[
\frac{A(-k^2)}{N_f} = -i (-)^p \frac{2E(k)}{M|\vec{k}|} e^{M_e(E(k)-M)t_q} \frac{1}{Z_\alpha Z_q} V (\bar{a}_\alpha (\vec{0}, t) \bar{q}_L (\vec{k}, t_q) \bar{a}_\beta (\vec{0}, 0) P_{\beta\alpha})
\] (10)

when \( t, t_q \gg 1 \). In practice, since in a finite lattice we cannot reach \( \vec{k} = 0 \), the smallest available nonzero momentum \( \vec{k} = (2\pi/L, 0, 0) \) can be used to estimate \( A(0) \).

Our measurements were performed on a sample of about 100 configurations generated on a \( 16^3 \times 32 \) lattice at \( \beta = 6.0 \). We computed the Wilson fermion propagator using the smearing technique described in Ref. [11]: we fixed the Coulomb gauge and used a delta function localized on a \( 6^3 \) cube in the Dirac equation. We considered the following values of the hopping parameter: \( k = 0.153, 0.154, 0.155 \).

We found the quantity \( (10) \) to be zero within errors, so we can only conclude giving a bound on \( G_1(0) \) (within one standard deviation)

\[
G_1(0) \lesssim 0.025 \frac{N_f}{Z_q} \approx 0.4
\]

(11)

While the above bound is not stringent enough for a comparison with experimental data, the situation is actually quite promising: A higher statistics calculation is called for, and we are currently undertaking it on an APE/Quadrics dedicated machine. Also, an improved lattice topological charge density operator (less noisy or with \( Z_q \) closer to one) would be welcome. Finally, an obvious further question to address is the effect of unquenched fermions. We will be reporting on these issues in future publications.
REFERENCES

[1] R. D. Carlitz, Proceedings, XXVI Int. Conf. on High Energy Physics, Dallas (1992), ed. J. R. Sanford, and references therein.
[2] J. Ellis and M. Karliner, Phys. Lett. B313 (1993) 131, and references therein.
[3] G. Veneziano, Mod. Phys. Lett. A4 (1989) 1605; Shore and Veneziano, Mod. Phys. Lett. A8 (1993) 373.
[4] P. Di Vecchia, K. Fabricius, G.C. Rossi and G. Veneziano, Nucl. Phys. B192 (1981) 392.
[5] M. Campostrini, A. Di Giacomo and H. Panagopoulos, Phys. Lett. B212 (1988) 206.
[6] M. Teper, Phys. Lett. B232 (1989) 227.
[7] A. Di Giacomo and E. Vicari, Phys. Lett. B 275 (1992) 429.
[8] B. Allés, M. Campostrini, A. Di Giacomo, Y. Gündüz and E. Vicari, Phys. Rev. D48 (1993) 2284.
[9] J. E. Mandula, Phys. Rev. Lett. 65 (1990) 1403; Nucl. Phys. B (Proc. Suppl.) 26 (1992) 356.
[10] R. Altmeyer, M. Göckeler, R. Horsley, E. Laermann, G. Schierholz, Proceedings, “Lattice 92”, Amsterdam (The Netherlands). Nucl. Phys. B (Proc. Suppl.) 30 (1993) 483.
[11] APE Collab., P. Bacilieri et al, Nucl. Phys. B317 (1989) 509.
FIGURES

FIG. 1. $Q_L/Q_{L,0}$ versus the number of updatings when heating an instanton configuration at $\beta = 6$. A dashed line indicates the value of $Z_q$ estimated by averaging data on the plateau.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9402019v1