Comprehensive Heading Error Processing Technique Using Image Denoising and Tilt-Induced Error Compensation for Polarization Compass

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\textbf{ABSTRACT} Bionic polarization navigation has a broad variety of application in diverse fields for high reliability and strong robustness to interference, fundamental to which is the use of a polarization compass based on polarized light cues. Nevertheless, dramatical reduction of the orientation accuracy resulted from the noise in a measured angle of polarization (AoP) and the tilted angles of a polarization compass during operation gives imperative influence on navigation precision. Herein, we investigate how to improve the navigation accuracy effectively by the proposed comprehensive heading error processing technique for a polarization compass, where a novel denoising scheme is designed to eliminate the noise in AoP images directly by integrating the strength of iterative variance-stabilizing transformation (IVST) and adaptive soft interval thresholding (SIT) so as to compensate the following tilt-induced error accurately. Subsequently, a promising compensation approach inspired by efficient extreme learning machine (EELM) is introduced to correct the tilt-induced error caused by realistic execution. The AoP image denoising advance and the tilt-induced error modeling advance combine to produce remarkable performance gains on the heading error. Experimental results and comparisons with prior arts reveal that the proposed comprehensive heading error processing technique is highly appealing in terms of improving the orientation accuracy for a polarization compass with superiority to state-of-the-art alternatives.

\textbf{INDEX TERMS} Polarization compass, error processing, de-noising, error modeling.

\section{I. INTRODUCTION}

Studies of certain insects and migratory birds provide rich evidence that they extract navigation cues from regular atmosphere polarization pattern [1]–[3], which gives stable and crucial reference information for bionic polarization navigation in complex natural environment [4], [5]. To ensure the high accuracy of autonomous polarization navigation, an effective and reliable orientation approach is urgently required for a polarization compass. The polarization compass is a key component in a polarized light navigation system, which is an autonomous navigation device depending on the atmosphere polarization mode of the polarized light in sky. The measurement orientation accuracy by the compass
determines the performance of the whole polarized light navigation system directly. Consequently, how to improve the accuracy of a polarization compass has always been the research focus in the field of polarized light navigation. Currently, some research on improving the orientation performance of a polarization compass mainly focuses on the point of view optimizing the algorithm of heading angles, such as an orientation method via Pulse Coupled Neural Network algorithm is addressed for the highly accurate and robust compass information calculation from the polarized skylight imaging [6], a calculation algorithm of heading angles for an imaging polarization navigation sensor based on a machine-vision algorithm is proposed [7], a computational model can directly estimate the solar azimuth and also infer the confidence of estimating the potential accuracy of polarized light compass information both in absolute terms and in the context of path integration [8]. Some methods to raise orientation accuracy are in terms of calibration which is based on the intrinsic characteristics of polarization navigation sensors. For instance, a calibration model by taking full advantages of both reference angle of polarization and constant degree of polarization is designed to determine calibration parameters for a polarization navigation sensor [9], the central-symmetry calibration method optimized the nonconstant calibration voltage deviations is presented to increase the accuracy of a bioinspired polarization sensor with lenses for navigation [10], a calibration method based on variable substitution and non-linear curve fitting is exploited to gain a relatively high degree of accuracy in polarization measurement [11], and so on. Other schemes are investigated from the view of integrating different navigation technologies such as inertial navigation system (INS) [12] and global navigation satellite system (GNSS) [13] in polarization navigation.

Nevertheless, it is worthwhile to highlight that the latest ordinary methods applied to improve the orientation performance for a polarization compass are lack of considering both the influence of noisy AoP images acquired from a polarization camera that will directly generate noisy data for the compass, and the influence of the tilted angles of a polarization compass during actual navigation that will also produce a great heading error simultaneously. To date, there are some denoising algorithms are reported to attenuate the noise in color images [14]–[16] and only very few denoising algorithms for the division of focal plane (DoFP) images [17]–[19]. Denoising approaches investigated for AoP images have not been previously advanced as far as the author knows. Furthermore, from the previous analysis, it can be seen the existing methods of improving the orientation accuracy for a polarization compass are through optimizing the algorithm of heading angles and integrated navigation. The complex nonlinear relationship between heading error and the tilted angles is still not solved. Consequently, not only can the orientation accuracy be improved but also the distribution law of heading error can be clearly described by modeling the complex nonlinear relationship after denoising AoP images.

Furthermore, neural network model such as backpropagation (BP) neural network, Elman neural network (Elman NN), and radical basis function (RBF) neural network have been employed to model nonlinear relationship between various data due to their flexible function approximators. However, they perform poor abilities in improving the orientation accuracy for a polarization compass. Accordingly, a promising tilt-induced error modeling and compensation approach based on efficient extreme learning machine (EELM) [20], [21] is introduced to establish the relationship between tilted angles and heading error since the EELM neural network exploits a reduced complete orthogonal decomposition to calculate the model output matrix instead of the commonly used singular value decomposition, which can achieve a much faster learning speed and effectiveness than other neural networks.

In this study, in order to improve the heading accuracy effectively for a polarization compass, a novel denoising scheme using IVST-SIT is designed to eliminate the noise in the AoP images directly by integrating the strength of iterative variance-stabilizing transformation (IVST) [22], [23] and adaptive soft interval thresholding (SIT) [24]. Subsequently, the tilt-induced error model based EELM is employed to obtain the outstanding orientation performance gains since it is highly appealing for decreasing the error originated from tilted angles of a polarization compass. The main contributions in this article are summarized as follows:

(1) A novel IVST-SIT denoising scheme is presented to remove the noise in AoP images, in which an IVST via the combination of noisy AoP images with the denoised estimate by an adaptive SIT filter from the last iteration is executed overall the filter procedure, while an exact unbiased inverse on the processed image is introduced to reconstruct properly the final estimation.

(2) An effective modeling and compensation method inspired by EELM algorithm using reduced complete orthogonal decomposition (RCOD) is designed for the tilt-induced error of heading angle for a polarization compass. The trained EELM is able to establish the complex nonlinear relationship between the input acted as pitch, roll angles and the heading error output to increase the orientation accuracy.

The remainder of this article is organized as follows: Section II depicts the calculation method of the heading angle and existing problems. Section III elucidates the heading error processing technique on the basis of IVST-SIT and EELM algorithms for the polarization compass in detail. The proposed technique for improving the navigation precision through experiments and comparisons with various image denoising and heading error modeling methods is demonstrated in Section IV. Section V concludes the proposed heading error processing technique.

II. HEADING ANGLE CALCULATION AND PROBLEM FORMULATION

Polarized light navigation imitates the mechanism of insects such as desert ants utilizing the polarization properties of the
skylight to navigate straight home after a long and random foraging [25]. Accordingly, the calculation of heading angle \( \phi \) is realized in real-time according to the solar azimuth \( \alpha_s \) together with the angle \( \alpha_c \) of a carrier forward direction and the solar meridian (SM), which is the symmetry axis of atmospheric polarization mode regarded as reference datum, as shown in Fig.1 and Eq.1.

where the North by East direction is defined as positive and the reference coordinate system (RCS) coincides with the incident light coordinate system (ICS) completely at the zenith. \( \varphi_E \) is the AoP whose range is \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\) such that is represented as (2). The final heading angle \( \phi \) can be obtained by combining (1) and (2). Due to \( \alpha_s \) being determined by astronomical calendar, the heading angle is directly affected by the measured AoP with a polarization camera. Meanwhile, the measured AoP is extremely susceptible to the influence of the noise in camera circuit and incident light photon fluctuation [26].

\[
\begin{align*}
\phi &= \alpha_s - \alpha_c \quad (1) \\
\alpha_c &= \frac{\pi}{2} \pm \varphi_E \quad (2)
\end{align*}
\]

In addition, a polarization compass is not always in the horizontal state during actual navigation process, which is described by the tilted angles, namely the pitch angle \( \theta \) and the roll angle \( \delta \). Rotational experiments shown as Fig.2 by varying the roll angle from \(-30^\circ\) to \(30^\circ\) at an interval of \(3^\circ\) and leaving the pitch angle \( \theta \) \(0^\circ\) demonstrate the great influence of the tilted angles and the noisy AoP images acquired with a polarization camera on the accuracy of the heading angle output by a polarization compass.

**III. PROPOSED HEADING ERROR PROCESSING METHOD**

In this section, we present the comprehensive heading error processing scheme shown as Fig.3 for a polarization compass, in which a promising IVST-SIT denoising strategy is carried out to remove the noise in AoP images while the heading error originated from the tilted angles of a polarization compass will be modeled and compensated by EELM algorithm subsequently.

**A. AoP IMAGE DENOISING ALGORITHM**

AoP images acquired with polarization cameras are easy to be corrupted by the noise described in Section 2 that can be expressed as stochastic Mixed-Poisson-Gaussian (MPG) noise [27]. Herein, a novel IVST-SIT denoising algorithm is exploited to remove the noise in AoP images by integrating the strength of an IVST and adaptive SIT, where the VST is exploited to convert the MPG noise into the near Gaussian noise with asymptotic constant variance which will be removed by the SIT filter and the whole ergodic process is implemented iteratively. Toward the more effective goal of AoP image denoising, an iterative convex combination and an exact unbiased inverse will be utilized in the VST and an adaptive thresholding operator is given to the SIT.

**FIGURE 1.** Schematic diagram of heading angle calculation.

**FIGURE 2.** Generated process of heading error.

**FIGURE 3.** Block diagram of the proposed heading error processing method.
filter. Fig. 4 illustrates the specific procedure of the proposed IVST-SIT denoising algorithm, the steps of which are shown as follows:

Step 1: Initialization. Setting the first estimation \( \hat{p}_0 \) is equal to the original noisy AoP image \( r \) comprised of pixels \( r(x) \) modeled as the Gaussian process of the parameter \( p(x) \), namely, \( \hat{p}_0 = r, \hat{p} \) is the estimation of the parameter \( p(x) \) \( \geq 0 \) and \( x \in \Omega \subset N^2 \).

Step 2: Combination. An iterative convex combination \( r_i \) by the raw noisy AoP image \( r \) with the estimate \( \hat{p}_{i-1} \) of a previously denoised image is calculated in the entire ergodic, namely

\[
r_i = c_i r + (1 - c_i) \hat{p}_{i-1} \tag{3}
\]

where, \( 0 < c_i \leq 1 \) indicates the convex combination coefficient.

Step 3: Transformation. In order to improve the performance of the universal iteration, a binning operator \( \beta_{hi} \) is introduced to first act on \( r_i \), especially when \( \hat{p}_{i-1} \) performs a poor estimation of \( p \), and then the \( f_i \) of VST is used to obtain a new noisy AoP image, i.e.,

\[
\tilde{r}_i = f_i (\beta_{hi} (r_i)) \tag{4}
\]

Step 4: Denoising. The iterative SIT filter with an adaptive thresholding operator works straightforward for the new noisy AoP image \( \tilde{r}_i \), leading to a denoised image \( D_i = F_i (\tilde{r}_i) \).

Step 5: Inversion. An unbiased inverse \( I_{fi} \) of \( f_i \) is applied to the \( D_i \), which is represented as

\[
I_{fi} (D_i) = c_{i}^{2} I_{fi} (D_i + \frac{c_i - 1}{c_i} D_i^{-1}) \tag{5}
\]

where \( \alpha = 2\sqrt{r + \frac{3}{8}} \) denotes Anscombe forward transformation and is a Poisson process.

Step 6: Returning. A final denoised AoP image is achieved via an inverse binning operator \( \hat{p}_{i-1}^{-1} \).

\[
\hat{p}_i = \beta_{hi}^{-1} \left[ I_{fi} \left( F \left[ f_i \left( \beta_{hi} \left( c_i r + (1 - c_i) \hat{p}_{i-1} \right) \right) \right] \right) \right] \tag{6}
\]

Step 7: Output. Provided each iteration is less than the total number of iterations \( K \), i.e., \( i < K \), \( \hat{p}_i \) is regarded as the next iteration, or else output as final estimate \( \hat{p}_i \).

The entire IVST-SIT denoising algorithm for AoP images in four directions is shown as follows:

| Algorithm 1 IVST-SIT |
|----------------------|
| **Requirement:** four polarimetric images |
| **Transformation Domain** |
| 1: VST transform, the polarimetric images in four directions are transformed into the VST domain |
| **Denoising** |
| 2: for \( i = 1 \) to \( 4 \) do |
| 3: The transformed polarimetric images are denoised by SIT |
| 4: VST domain inverse transformation |
| 5: end for |
| **Calculation final AoP** |
| 6: Stokes vector method is used to solve the four processed polarimetric images |
| 7: Obtain the AoP |

### B. OTHER RECOMMENDATIONS

In order to obtain more accurate and more reliable navigation performance, an efficient and robust learning algorithm based on EELM neural network, which is able to establish the complex nonlinear relationship between heading error and tilted angles by utilizing the pitch and roll angles as the input of the model and heading angle error as the output of the model, is employed to model and compensate the heading error of a polarization compass in tilted status during the real operation and given as the following steps. Training the error model is from Step 1 to Step 4 and testing is Step 5:

Step 1: Determination of input and output parameters of the heading error model. The pitch angle \( \theta \) and the roll angle \( \delta \) are employed as the input vectors, and the output vector is heading error \( e \), which is mathematically modeled as:

\[
e_j = F_{i,j} E_i, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, N \tag{7}
\]

\[
F_{i,j} = \begin{pmatrix}
F(\alpha_1, t_1, \theta_1, \delta_1) & \ldots & F(\alpha_1, t_1, \theta_1, \delta_1)
\end{pmatrix}
\]

\[
E_i = \begin{pmatrix}
E_{1}^{T} \\
\vdots \\
E_{m}^{T}
\end{pmatrix}
\]

\[
e_j = \begin{pmatrix}
e_{1}^{T} \\
\vdots \\
e_{n}^{T}
\end{pmatrix}
\]

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where $F_{i,j}$ and $E_i$ indicate the output vector of the hidden layer and the weight vector connecting the $i$th hidden node with the output nodes, respectively. The number of hidden nodes is $n$ and the dimension of the vectors is $m$. $a_i$ and $t_i$ represent the weight vector and the threshold of the $i$th hidden node, respectively.

Step 2: The hidden node parameters, i.e., $n$, $m$, $a_i$, $t_i$ are yielded randomly.

Step 3: The hidden layer output matrix $F_{i,j}$ and the model output matrix $e_j$ are calculated as Eqs. (8) and (10).

Step 4: The weight vector $E_i$ connecting the $i$th hidden node with the output nodes is achieved using RCOD the matrix $F_{i,j}$ and expressed as

$$RCOD(F_{i,j}) = ON^{-1}U$$
$$E_i = ON^{-1}U^te_j$$

where $O \in R^{n \times r}$ and $U \in R^{N \times r}$ are orthogonal matrices, while $N \in R^{r \times r}$ is nonsingular and $r$ is the rank of the matrix $F$.

Step 5: The heading error $e_j$ is calculated according to (7).

The whole heading error modeling procedures by an EELM neural network are described as Fig. 5 and Algorithm 2.

**IV. EXPERIMENT AND VERIFICATION**

**A. EXPERIMENTAL SETUP**

The polarization compass was consisted of a PHX050S-P polarization camera with Sony IMX250MZR CMOS sensor and a Nvidia Jetson TX2 embedded processing module. The whole polarization data measurement system was connected to a TBR100 three-axis turntable with a built-in original SUNX PM-L25 sensor and the repeat positioning accuracy of less than $\pm 0.005^\circ$. A high-resolution multimedia interface monitor was attached to the system for information feedback, and a SC3003P-XB controller was used to vary the pitch and roll angles of the turntable. The computer was programmed with a real-time user interface to control the system while viewing the live heading angle data. The experimental equipment of this work and the characteristics are shown as Fig.6 and Table 1.

![Architecture of the tilt-induced error model.](image)

**Algorithm 2 EELM**

**Input:** pitch angle $\theta_j$, roll angle $\delta_j$

**Output:** heading error $e_j$

**Training**

1: Construct input space $I_j$ with $\theta_j$ and $\delta_j$

2: Establish the input space and output space of the heading error model as follows:

$$M = \{ (I_j, e_j) | j = 1, 2, \ldots, N \}$$

where $I_j \in R^m$ and $e_j \in R$ are the input and output, respectively.

3: Generate $a_i$, $t_i$, $m$, and $n$ randomly

4: Calculate $F_{i,j}$ and $e_j$ as Eqs. (8) and (10)

5: Calculate $E_i$ as Eq. (12)

**Testing**

6: Calculate $e_j$ by Eq. (7)

7: end

**B. AOI IMAGE DENOISING VERIFICATION**

The proposed comprehensive heading error processing technique using image denoising and tilt-induced error compensation for a polarization compass is verified by a two-phase process. During the first phase, the IVST-SIT denoising scheme described as Section 3.1 was performed for the AOI images acquired by the polarization camera. Fig. 7 and Table 2 illustrate the results and evaluation index of the AOI images with four various polarization directions (PD) before and after denoising by static experiments. Larger definition and entropy values indicate clearer images, while smaller standard deviation (Std) suggests more stable and less discrete polarization information contained in the AOI images.

Though the limited denoising effect of each direction AOI image using the proposed algorithm is visualized from Fig. 7 and Table 2, the overall significant denoising result compared with original AOI images and different denoising methods is shown as Fig. 8 and Table 3. That is because the calculation method of AOI needs the light intensity information in four various polarization directions incorporated in
AoP images. Fig. 8 shows the experimental results of heading angle error from original AoP images and different image denoising methods including traditional soft threshold (ST), two-dimensional empirical mode decomposition (BEMD), and a block-matching and 3-D (BM3D) filtering algorithm, which further verifies the obvious effectiveness of the proposed IVST-SIT denoising scheme on AoP images.

The root mean square error (RMSE) and Std of the heading angle error of static test before and after different image denoising algorithms are illustrated in Table 3. The smallest Std and RMSE of the heading error are 0.0551 and 0.0548 of the proposed IVST-SIT denoising algorithm, respectively. The second effect is the use of BEMD denoising method. Compared to the original noisy AoP image and BEMD, the Std improvement of heading error with the proposed image denoising algorithm is 90.6% and 81.7%, and the RMSE improvement is 90.6% and 81.6%, which implies the proposed IVST-SIT method is able to eliminate the noise in AoP images effectively and improve the heading accuracy significantly.

### C. VERIFICATION OF HEADING ERROR MODELING AND COMPENSATION

The second phase of the proposed comprehensive heading error processing technique used EELM neural network.
detailed in Section 3.2 to model and compensate the tilt-induced error of the heading angle to improve the orientation accuracy for a polarization compass. Rotational experiment depicted Section 4.1 was implemented in the campus of North University of China (NUC) in Taiyuan City Shanxi Province on 9th May 2020 at sunset (from 16:00 to 17:30). The tested data was acquired by varying the pitch angle $\theta$ from $-30^\circ$ to $30^\circ$ at an interval of $5^\circ$, and the roll angle varied from $-15^\circ$ to $15^\circ$ at an the same interval of $5^\circ$ in each fixed pitch angle. The experimental and modeling results with EELM of the tilt-induced error was plotted in Fig.9 (a) and (b), the difference of which was shown in Fig.9 (c). It can be seen clearly the difference of the heading error between the actual measurement and EELM modeling is smaller enough to verify the strong fitting ability of EELM in establishing the complex relationship between the heading error and the tilted angles.

Furthermore, Fig.10 presented the contrast results of tilt-induced error compensation algorithm based on EELM before and after image denoising together with state-of-the-art models such as BPNN (back-propagation neural network), Elman NN, and RBFNN (radial basis function neural network) only after image denoising for avoiding complex graphics by rotational experiments. The mean, Std, and RMSE of the compared compensation results of the tilt-induced error are listed in Table 4 and Table 5 further to verify the performance of the proposed model. The RMSE of the EELM compensation after image denoising is 0.0303 and improves by 84.2% and 93.7% compared with no image denoising and RBFNN which is inferior to EELM error.
compensation effect after image denoising, respectively. The results indicate the proposed tilt-induced error compensation method based on EELM achieves excellent orientation performance.

V. CONCLUSION
In this article, we present a comprehensive heading error processing technique using image denoising and tilt-induced error compensation method for a polarization compass. The heading error processing scheme is implemented in two phases: (1) it utilizes a novel IVST-SIT denoising method combining the strength of IVST and adaptive SIT so as to model the following heading error accurately, and (2) a promising tilt-induced error approach based on EELM is exploited to model and compensate the heading error for the polarization compass during actual operation. Finally, the effectiveness of the proposed method is verified by various real experiments. By monolithically combining these two advances, we have created a comprehensive tilt-induced error compensation method that has an improvement of the RMSE by 86% compared with prior arts. The departure from traditional processing methods of the orientation error, which accurately establishes the relationship between the tilted angles and heading error by nonlinear fitting, has enabled us to create an effective heading error compensation method with a RMSE of 0.0303 for a polarization compass. The orientation performance can be improved significantly by the proposed heading error processing technique, which makes polarization orientation become a more promising navigation method.

In future, greater emphasis can be placed on the research into the exploitation of digital signal processing (DSP) techniques to devise new orientation algorithms of a lower computational complexity represent another possible area for further investigation.

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