A quantum critical superconducting phase transition in quasi-two-dimensional systems with Dirac electrons

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Abstract

We present a theory describing the superconducting (SC) interaction of Dirac electrons in a quasi-two-dimensional system consisting of a stack of $N$ planes. The occurrence of a SC phase is investigated both at $T = 0$ and $T \neq 0$. At $T = 0$, we find a quantum phase transition connecting the normal and SC phases. Our theory qualitatively reproduces the SC phase transition occurring in the underdoped regime of the high-Tc cuprates. This fact points to the possible relevance of Dirac electrons in the mechanism of high-Tc superconductivity.

Key words: Dirac electrons, superconductivity, quantum criticality

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1 Introduction

There are many condensed matter systems in one and two spatial dimensions containing electrons that may be described by a relativistic, Dirac-type lagrangian, namely Dirac electrons. Among these we may list the high-Tc cuprates, graphene sheets and dichalchogenides [1]. Even though these are evidently non-relativistic systems these materials have special points in the Brillouin zone where two bands touch in a single point around which the electron dispersion relation behaves as $\epsilon(\vec{k}) = v_F|\vec{k}|$. The elementary excitations around such a point are Dirac electrons. They are, after all, a result of the electron-lattice interaction.

We present here, a theory describing the superconducting interaction of Dirac electrons associated to two distinct Dirac points [2]. We show that, at $T = 0$, the system presents a quantum critical point separating the normal and superconducting phases and determine the superconducting gap as a function of the coupling constant. The quantum phase transition occurring in our model and the behavior of $T_c$ around the quantum critical point qualitatively reproduce very well the superconducting transition in the high-Tc cuprates in the underdoped region. This suggests that Dirac electrons may play an important role in the mechanism of high-Tc superconductivity.

We consider a quasi-two-dimensional electronic system consisting of a stack of planes containing two Dirac points. In addition, we introduce an internal index $a = 1, ..., N$, supposed to characterize the different planes to which the electrons may belong. The electron creation operator, therefore, is given by $\psi^\dagger_{iaa}$, where $i = 1, 2$ are the Dirac indices, corresponding to the two Fermi
points, \( \sigma = \uparrow, \downarrow \), specifies the z-component of the electron spin and \( a = 1, \ldots, N \) labels the electron plane. The complete lagrangian we will consider is given by

\[
\mathcal{L} = i \overline{\psi}_{\sigma a} \not\partial \psi_{\sigma a} + \frac{\lambda}{N} \left( \psi_{1\uparrow a}^{\dagger} \psi_{2\downarrow a}^{\dagger} + \psi_{2\uparrow a}^{\dagger} \psi_{1\downarrow a}^{\dagger} \right) \times \left( \psi_{2\downarrow b} \psi_{1\uparrow b} + \psi_{1\downarrow b} \psi_{2\uparrow b} \right),
\]

(1)

where \( \lambda > 0 \) is a constant that may depend on some external control parameter, such as the pressure or the concentration of some dopant.

We now introduce a Hubbard-Stratonovitch complex scalar field \( \sigma \), in terms of which the lagrangian becomes

\[
\mathcal{L}[\Psi, \sigma] = i \overline{\psi}_{\sigma a} \not\partial \psi_{\sigma a} - \frac{N}{\lambda} \sigma^* \lambda + \sigma \left( \psi_{1\uparrow a}^{\dagger} \psi_{2\downarrow a}^{\dagger} + \psi_{2\uparrow a}^{\dagger} \psi_{1\downarrow a}^{\dagger} \right). \]

(2)

From this we obtain the field equation for the auxiliary field: \( \sigma = -\frac{1}{N} \left( \psi_{2\downarrow a} \psi_{1\uparrow a} + \psi_{1\downarrow a} \psi_{2\uparrow a} \right) \). The vacuum expectation value of \( \sigma \) is an order parameter for the superconducting phase.

Integrating on the fermion fields, we obtain the effective action

\[
S_{\text{eff}}[\sigma] = \int d^3x \left( -\frac{N}{\lambda} |\sigma|^2 \right) - i2N \text{Tr} \ln \left[ 1 + \frac{|\sigma|^2}{\Box} \right]
\]

(3)

Let us consider firstly \( T = 0 \). In this case, we get the renormalized effective potential per plane corresponding to (3):

\[
V_{\text{eff},R}(|\sigma|) = \frac{|\sigma|^2}{\lambda_R} - \frac{3\sigma_0}{2\alpha} |\sigma|^2 + \frac{2}{3\alpha} |\sigma|^3,
\]

(4)

where \( \lambda_R \) is the (physical) renormalized coupling and \( \sigma_0 \) is an arbitrary finite scale, the renormalization point.
Studying the minima of the previous expression, we can infer that the ground state of the system will be

\[
\Delta_0 = \begin{cases} 
0 & \lambda_R < \lambda_c \\
\alpha \left( \frac{1}{\lambda_c} - \frac{1}{\lambda_R} \right) & \lambda_R > \lambda_c 
\end{cases}
\]

(5)

where \( \Delta = |\sigma| \). Expression (5) implies that the system undergoes a continuous quantum phase transition at the quantum critical point \( \lambda_c = \frac{4\pi v_F^2}{3\sigma_0} \), separating a normal from a superconducting phase.

We turn now to finite temperature effects. Using a large \( N \) expansion and evaluating (3) at \( T \neq 0 \), we find the effective potential, whose minima provide a general expression for the superconducting gap as a function of the temperature, namely

\[
\Delta(T) = 2T \cosh^{-1} \left[ \frac{\Delta_0}{e^{\frac{\pi T}{2}}} \right],
\]

(6)

where \( \Delta_0 \) is given by (5). From (6) we can verify that indeed \( \Delta(T = 0) = \Delta_0 \). Also from the above equation, we may determine the critical temperature \( T_c \) for which the superconducting gap vanishes. Using the fact that \( \Delta(T_c) = 0 \), we readily find from (6)

\[
T_c = \frac{\Delta_0}{2 \ln 2}.
\]

(7)

In Fig. 1, using (5) and (7), we display \( T_c \) as a function of the coupling constant. This qualitatively reproduces the superconducting phase transition of the high-Tc cuprates in the underdoped region. Since our theory describes the generic superconducting interaction of two-dimensional Dirac electrons, we may see this result as an indication of the possible relevance of this type of electrons in the high-Tc mechanism.
Fig. 1. The superconducting critical temperature $T_c$ as a function of the renormalized coupling $\lambda_R$.

In terms of the critical temperature, we may also express the gap as

$$\Delta(T) = 2 T \cosh^{-1} \left[ 2 \left( \frac{T}{T_c} - 1 \right) \right].$$

Near $T_c$, this yields

$$\Delta(T) \overset{T \ll T_c}{=} 2 \sqrt{2 \ln 2} \ T_c \left( 1 - \frac{T}{T_c} \right)^{\frac{1}{2}},$$

which presents the typical mean field critical exponent $1/2$.

Finally, we would like to make two remarks. Firstly, both the gap $\Delta(T)$ (and hence the critical temperature) and the renormalized effective potential do not depend on the arbitrary renormalization point $\sigma_0$. This can be seen by
a renormalization group analysis [2]. The theory does not predict the value of $\lambda_c$, it has to be determined experimentally. Second, we can show that the results, obtained in mean field, are robust against quantum fluctuations [2].

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