Ginzburg-Landau functional for nearly antiferromagnetic perfect and disordered Kondo lattices

M. Kiselev1, K. Kikoin2 and R. Oppermann1

1 Institut für Theoretische Physik, Universität Würzburg, D-97074, Germany,
2 Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

Interplay between Kondo effect and trends to antiferromagnetic and spin glass ordering in perfect and disordered bipartite Kondo lattices is considered. Ginzburg-Landau equation is derived from the microscopic effective action written in three mode representation (Kondo screening, antiferromagnetic correlations and spin liquid correlations). The problem of local constraint is resolved by means of Popov-Fedotov representation for localized spin operators. It is shown that the Kondo screening enhances the trend to a spin liquid crossover and suppresses antiferromagnetic ordering in perfect Kondo lattices and spin glass ordering in doped Kondo lattices. The modified Doniach’s diagram is constructed, and possibilities of going beyond the mean field approximation are discussed.

PACS:71.27+a, 75.20.Hr, 75.10.Nr, 75.30.Mb

I. INTRODUCTION

The Kondo lattice (KL) systems are famous with their unusual electronic and magnetic properties, including giant effective masses observed in thermodynamic and de Haas-van Alphen measurements. Unconventional superconductivity and fascinating variety of magnetic properties. The overwhelming majority of metallic KL systems demonstrate antiferromagnetic (AFM) correlations, and one can meet all types of AFM order: localized spins in U2Zn17, UCD11, CeIn3, quadrupole ordering in CeB2; interplay between localized and itinerant excitations in several U- and Ce-based compounds. Puzzling magnetic order of the moments in UPt3, URu2Si2, UNi2Al3 that, however, rapidly transform in usual large localized moments with doping; quantum phase transition in CeCu6−xAx; fluctuation-type dynamical ordering in U(Pt1−xPdx); short-range magnetic correlations without long range order but with astonishingly wide temperature interval of critical behavior in CeCu6 and CeRu2Si2. The list is by no means exhaustive. The superconducting state in most cases also coexists with antiferromagnetism, and, apparently, Cooper pairing in KL is mediated by magnetic fluctuations. The dominant contribution to the low-temperature thermodynamic is also given by spin degrees of freedom.

On the other hand, low-temperature characteristics of KL are scaled by a Kondo temperature $T_K$. These characteristics are Fermi-like, but the energy scale of "fermion" spectrum is compressed with a factor $T_K/\varepsilon_F$ relative to a conventional electron Fermi liquid. Apparently, the AFM correlations due to Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction $I$ partially suppressed by intra-site Kondo effect should be treated as a background for all unusual properties of Kondo lattices, so the main theoretical challenge is to find a scenario of crossover from a high temperature regime of weak interaction (scattering) between localized spins and conduction electron Fermi liquid to a low-temperature strong coupling regime where the spins lose their localized nature and are confined into unconventional quantum liquid involving spin degrees of freedom of conduction electrons.

On the phase diagram of disordered KL more exotic possibilities arise, such as non Fermi liquid regime observed, e.g. near the $T = 0$ quantum critical point in $Y_{1-x}U_xPd_3$ (see, e.g., Ref. 13). In this family of ternary alloys the spin glass (SG) behavior was discovered in a U concentration range $0.3 < x < 0.5$ with a freezing temperature $T$ growing monotonically with $x$. Among other U-based heavy fermion compounds with SG behavior URh2Ge2, U2Rh3Si14, U2PdSi12 should be mentioned. The effects of "Kondo disorder" were reported for UCu5−xPdx in Ref. 18. Later on the competition between RKKY and Kondo exchange for disordered Ce alloys was discovered experimentally (see Refs. 19-22). The magnetic phase diagram of CeNi1−xCux exhibits change of magnetic ordering from AFM to FM at $x = 0.8$, whereas for $0.2 < x < 0.8$ the SG state appears only at high temperatures above the FM order. Apparently, the Kondo interaction could be considered as the mechanism leading to reduction of magnetic moments because increasing Ni contents effectively reduces the strength of indirect exchange interaction, and then, a larger temperature stability range of the SG phase appears (see Refs. 19-22).

The competition between the one-site Kondo type correlations and the indirect inter-site exchange is visualized in a Doniach’s diagram where all possible phase transition and crossover energies are plotted as a function of a "bare" coupling parameter $\alpha = J/\varepsilon_F$ that characterizes exchange interaction between the spin and electron subsystem in KL. Only Kondo screening and RKKY coupling were competing in original Doniach’s diagram. Later on it was...
noticed that the trend to spin liquid (SL) ordering is the third type of correlation which modifies essentially the magnetic phase diagram of KL in a critical region $T_K \sim I$ of the Doniach’s diagram.  

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type "condensate" within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is

In this paper we present a high temperature mean-field description of transitions from a paramagnetic state to correlated spin states in KL, which does not violate the local constraint for spin-fermion operators. We use the Popov-Fedotov representation of spin operator to construct the effective action for KL. In this representation the local constraint is satisfied automatically. We consider the mutual influence of various order parameters (Kondo, AFM, SL and SG correlation functions) and derive a Ginzburg Landau functional (Section II). On the basis of this functional we construct generalized Doniach’s diagrams that take into account all interplays. The Doniach’s diagram for a perfect KL is presented in Section III and the influence of Kondo screening on the SG diagram in disordered KL is considered in Section IV.

All existing theories appeal to mean-field approximations that violate the local gauge invariance both in Kondo and SL channel. As a result, fictitious second order phase transitions from a free spin (paramagnetic) state to a confined spin (Kondo singlet or resonating valence bond SL) state arise in spite of the fact that neither symmetry is violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” within a framework of mean field approach. To eliminate fictitious phase transition to a SL state one should refuse violated by these transformations. New approach allows us to get rid of assumption about Kondo-type “condensate” confined spin (Kondo singlet or resonating valence bond SL) state rise in spite of the fact that neither symmetry is
Here all energies are measured in $\varepsilon_F = 1$ units, and an infinitesimal staggered magnetic field is introduced that respects the symmetry of magnetic bipartite lattice in AFM case ($\varepsilon_F$ is restored in further calculations wherever it is necessary).

To calculate the spin part of free energy $F_s(T) = -T \ln Z_s$ we represent the partition function $Z$ in terms of path integral. The spin subsystem is described by means of Popov-Fedotov trick.

$$Z_s = \text{Tr} \, e^{-\beta H} = iN \text{Tr} \, e^{-\beta (H+i\pi N^f/(2\beta))},$$

(5)

Here $\beta = T^{-1}$, $N$ is the number of unit cells, $N^f = \sum_j N_j^f$, and the spin $S = 1/2$ operators are represented as bilinear combinations of fermion operators.

$$S_j^z = (f_j^\dagger f_j^\uparrow - f_j^\dagger f_j^\downarrow)/2, \quad S_j^+ = f_j^\dagger f_j^\downarrow, \quad S_j^- = f_j^\dagger f_j^\uparrow.$$  

(6)

These operators obey the constraint

$$N_j^f = \sum_\sigma f_j^\dagger f_j^\sigma = 1.$$  

(7)

In accordance with Ref. [23], the Lagrange term with a fixed imaginary chemical potential $-i\pi T/2$ is added to the Hamiltonian ([3]). We use the path integral representation for the partition function,

$$Z \over Z_0 = \int \mathcal{D}c \mathcal{D}f \mathcal{D}f \exp \mathcal{A}$$

(8)

Then the Euclidean action for the KL is given by

$$\mathcal{A} = \mathcal{A}_0 - \int_0^\beta d\tau \mathcal{H}_{int}(\tau)$$

$$\mathcal{A}_0 = \mathcal{A}_0[c, f] = \int_0^\beta d\tau \sum_{j,\sigma} \left\{ \bar{c}_{j}\sigma(\tau) \left[ \partial_\tau - \varepsilon(-i\nabla) + \mu \right] c_{j}\sigma(\tau) + \bar{f}_{j}\sigma(\tau) \left[ \partial_\tau - i\pi T/2 \right] f_{j}\sigma(\tau) \right\}$$

(9)

Following the Popov-Fedotov procedure, the imaginary chemical potential is included in discrete Matsubara frequencies for semi-fermion operators $f_{j}\sigma$. As a result the Matsubara frequencies are determined as $\omega_m = 2\pi T(n+1/4)$ for spin semi-fermions and $\varepsilon_n = 2\pi T(n+1/2)$ for conduction electrons. In terms of temperature Green’s function the Euclidian action has the form

$$\mathcal{A} = \mathcal{A}_0 + \mathcal{A}_{int} = \sum_{k,\sigma} \bar{e}_{k}\sigma G_0^{-1}(k)c_{k}\sigma + \sum_{j,\sigma} \bar{f}_{j}\sigma(\omega_n) D^{-1}_{0}\sigma(\omega_n)f_{j}\sigma(\omega_n)$$

$$+ \frac{\bar{J}}{2} \sum_{j,\sigma\sigma',\varepsilon,\omega_n} \bar{c}_{j}\sigma(\varepsilon_1) f_{j}\gamma(\omega_2) \bar{f}_{j}\sigma'(\omega_1) c_{j}\sigma'(\varepsilon_2) \delta_{\varepsilon_1 - \varepsilon_2, \omega_1 - \omega_2}$$

$$+ \int \sum_{j,\sigma\gamma,\omega_n} \bar{f}_{j}\sigma(\omega_1) \bar{\hat{\sigma}}_{\sigma\gamma} f_{j}\sigma'(\omega_2) \bar{\hat{f}}_{j}\gamma(\omega_3) \hat{\hat{\sigma}}_{\gamma\gamma'} f_{l}\gamma'(\omega_4) \delta_{\omega_1 - \omega_2 - \omega_3 - \omega_4}.$$  

(10)

Here the Green’s functions (GF) for bare quasiparticles are

$$G_0(k, i\varepsilon_n) = \frac{1}{i\varepsilon_n - \varepsilon_k + \mu}, \quad D_{0}\sigma(\omega_m) = \frac{1}{\imath\omega_m - \sigma \varepsilon_n / 2}$$

(11)

(11)

$(\nu$ is the index of magnetic sublattice that defines the direction of staggered magnetic field).

The first interaction term in this equation is responsible for low-energy Kondo correlations, and we will treat it in conventional manner. In the RKKY term two modes should be considered, i.e. the local mode of AFM fluctuations and the nonlocal spin liquid correlations. We adopt for these modes the Neel type antiferromagnetism and resonating valence bond (RVB) type spin liquid state, respectively. In accordance with the general path integral approach to KL, we first integrate over fast (electron) degrees of freedom. Then in the sf-exchange contribution to the action we are left with the auxiliary field $\phi$ with statistics complementary to that of semi-fermions. The spin correlations in the inter-site RKKY term are treated in terms of vector Bose fields $Y$ (AFM mode) and scalar field $W$ (spin liquid RVB mode). As a result, $\mathcal{A}_{int}$ is represented by a following expression
\[ A_{\text{int}} = -\frac{2}{J} \text{Tr} |\phi|^2 - \text{Tr} \frac{1}{l_q} Y_q Y_{-q} - \text{Tr} \frac{1}{l_{q_1-q_2}} W_{p_1} W_{p_2} - \text{Tr} \tilde{f}_{\sigma} \phi_j G_0(r) \tilde{\phi} f_{\sigma} . \]  

(12)

When making a Fourier transformation for non-local spin liquid correlations (the third term in the r.h.s.) we introduced the coordinates \( R = (R_1 + R_2) / 2 \) and \( r = R_1 - R_2 \) for RVB field, so that \( P, q \) are the corresponding momenta. Below we assume \( P = 0 \) and omit it in notations for the SL mode, \( W_{0q} \equiv W_q \).

Consequent mean-field approach demands introduction of three "condensates", i.e. three time independent c-fields for Kondo coupling, AFM coupling and SL coupling, respectively, that arise as a consequence of a saddle-point approximation for all three modes. For example, the mean-field description of the interplay between the Kondo and RVB couplings was presented in Ref. 12. The undesirable consequence of this approximation is violation of the electromagnetic \( U(1) \) gauge invariance, when the electrical charge is ascribed to initially neutral spin fermion field \( f \) (see, e.g. Ref. 12). According to a scenario offered in Ref. 23 there is no necessity in introducing the mean-field saddle point for Kondo coupling because the transition to a correlated spin state occurs at \( T > T_K \). In this case the one site Kondo correlations suppress the Neel phase transition (reduce \( T_N^0 \to T_N \)) in favor of spin liquid state with a characteristic crossover temperature \( T^* > T_N \). So we refrain from using the saddle point approximation for the field \( \phi \) but still use it for the fields \( Y \) and \( W \).

To compactify the equation for the action \( A \) we introduce a spinor representation for semi-fermions

\[ \bar{F}_F = (f_{p+} f_{p-} f_{p+Q} f_{p+Q+}) , \]

and the following definition of the Fourier transform of inverse semi-fermionic Green’s function

\[ D^{-1}_m(P, Q) = \begin{pmatrix} i\omega_m - W_P & 0 & Y^+_Q & Y^+_Q \\ 0 & i\omega_m - W_P & Y^-_Q & -Y^-_Q \\ Y^+_Q & Y^-_Q & i\omega_m - W_{P+Q} & 0 \\ Y^-_Q & -Y^+_Q & 0 & i\omega_m - W_{P+Q} \end{pmatrix} . \]

(13)

The same function in a lattice representation is presented in Appendix I. This operator arises as a result of Hubbard-Stratonovich transformation decoupling the magnetic modes \( Y \) and the spin-liquid mode \( W \). Then the effective action \( A_s \) acquires the following form

\[ A_s = \text{Tr} \bar{F} D^{-1}_m F + A_{\text{int}} . \]

(14)

Now we integrate over semi-fermionic fields and obtain the effective action for a KL model

\[ A_s = \text{Tr} \ln (D^{-1}_m(Y, W) + \phi_j G_0(r) \tilde{\phi}_j) - \frac{2}{J} \text{Tr} |\phi|^2 - \text{Tr} \frac{1}{l_q} Y_q Y_{-q} - \text{Tr} \frac{1}{l_{q_1-q_2}} W_{p_1} W_{p_2} . \]

(15)

Here the argument \( |\phi|^2 \) appeared in the Green’s function \( D_m \) as a result of integration of the last term in eq. (12) over the semi-fermionic fields.

In a mean-field approximation for two independent modes (neglecting renormalization due to Kondo scattering) eq. (14) results in a free energy with two local minima reflecting two possible instabilities of high-temperature paramagnetic state relative to Neel and SL states. To describe these instabilities one should pick out the classic part of Neel field

\[ Y = (\beta N)^{1/2} \frac{J}{2} \mathcal{N} \delta_q \delta_{\omega,0} \epsilon_z + \bar{Y}_q \]

(16)

and use the eikonal approximation for the SL field

\[ W_{R,r} = J \Delta(r) \exp(i\theta) . \]

(17)

Here \( \mathcal{N} = (Y^2) \) is the staggered magnetization, \( \bar{Y}_q \) are the fluctuations around the mean-field magnetization, \( Q \) is the AFM vector for a given bipartite lattice, \( \epsilon_z \) is the unit vector along the magnetization axis, \( \Delta = \mathcal{D}(r) \) is the modulus of RVB field, and \( \theta = (r \cdot A(R)) \) is the phase of this field.

As is known, in Heisenberg lattices for dimension \( d > 1 \), \( T_N \) is higher than the temperature \( T_{sl} \) of crossover to the SL state, so that the ordered magnetic phase is the Neel phase. Due to Kondo fluctuations that screen dynamically local magnetic correlations and slightly enhance the inter-site semi-fermionic correlations, the balance between two modes is shifted towards spin liquid phase in a critical region of Doniach’s diagram, \( T_K \sim J \). To show this we include in the free energy the corresponding corrections induced by the last term in Eq. (12). As was mentioned above we
refrain from using the mean-field approach to Kondo field, so that the interplay between the Kondo mode and two other modes is taken into account by including the Neel mean filed corrections to the semi-fermionic Green’s function. Then instead of (13) one has the following equation for $D^{-1}$:

$$D^{-1}_m(N, \Delta) = \begin{pmatrix}
i\omega_m - \Delta I_q & 0 & 0 \\
0 & N I_q / 2 & 0 \\
0 & 0 & -N I_q / 2 \\
i\omega_m - \Delta I_q & 0 & 0 \\
0 & N I_q / 2 & 0 \\
0 & 0 & -N I_q / 2 \\
\end{pmatrix}.$$  \hspace{1cm} (18)

The next steps, i.e. calculations of fluctuation corrections to the stationary point mean field solutions, can be performed by introducing the auxiliary self energies

$$M(\tilde{Y}, \theta) = D^{-1}_m(Y, W) - D^{-1}_m(N, \Delta)$$

$$K_\phi(\omega_{n_1}, \omega_{n_2}) = -T \sum_{\Omega} \phi_j(\omega_{n_1} - \Omega) G_0(r, \Omega) \tilde{\phi}_l(\omega_{n_2} - \Omega)$$ \hspace{1cm} (19)

Then the effective action is approximated by the polynomial expansion

$$\text{Tr} \ln (D^{-1}_m(Y, W) + K_\phi) = \text{Tr} \ln D^{-1}_m(N, \Delta) + \text{Tr} \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} \left\{ D_m(N, \Delta) [M(\tilde{Y}, \theta) + K_\phi] \right\}^n$$ \hspace{1cm} (20)

(the Fourier transform of diagonal part of the Green’s function $K_\phi$ is calculated in Appendix I).

Neglecting all fluctuations, i.e. retaining only the first term in the r.h.s. of Eq. (20) together with quadratic terms for AFM and SL modes (15), one obtains the following expression for the free energy per lattice cell:

$$\beta F(N, \Delta) = \frac{\beta z |\Delta|^2}{4} - \ln |2 \cosh(\beta z |\Delta|/2)| + \frac{\beta z |\Delta|^2}{2} - \sum_q \ln |2 \cosh(\beta I_q |\Delta|)|$$ \hspace{1cm} (21)

($I_\phi = -I$). The standard self-consistent mean-field equations for order parameters are obtained by differentiating eq. (21) from condition of minima of the free energy. These are

$$N = \tanh \left( \frac{I_{\phi} N}{2T} \right)$$ \hspace{1cm} (22)

for the Neel parameter and

$$\Delta = -\sum_q \nu(q) \tanh \left( \frac{I_{\phi} \Delta}{T} \right)$$ \hspace{1cm} (23)

for the real part of the RVB order parameter. Here $\nu(q) = I_q / I_\phi$. The latter equation was first derived in Ref. [34].

Then making the high temperature expansion of Eq. (21), one obtains a Ginzburg-Landau (GL) equation in the approximation of two independent modes:

$$\beta F(N, \Delta) = \frac{\beta |I| z N^2}{4} \tau_N + c_N N^4 + \frac{\beta |I| z \Delta^2}{2} \tau_{sl} + c_{sl} \Delta^4$$ \hspace{1cm} (24)

where $\tau_N = 1 - T_N/T$ and $\tau_{sl} = 1 - T_{sl}/T$. The temperatures of two magnetic instabilities are determined as the temperatures of sign inversion in the coefficients in quadratic terms of GL expansion $T_N = z |I| / 2$ and $T_{sl} = |I|$. The forth order GL coefficients $c_N$ and $c_{sl}$ are positive and depend only on temperature. Up to this point the theory is formulated for arbitrary dimension $d$. In fact, the dimensionality enters the RKKY coupling parameter (see below) and determines the number of nearest neighbors $z$. We consider $z I$ as a universal parameter in further calculations.

III. DONIACH’S DIAGRAM REVISITED

To describe the contribution of Kondo scattering to magnetic part of the Doniach’s diagram one should integrate $A$ over auxiliary field $\phi$ and thus find the Kondo corrections both to Neel and RVB instability points. One should consider two cases: (i) $T_N > T_{sl}$ (Kondo corrections screen AFM magnetic moments), and (ii) $T_{sl} > T_N$ (Kondo corrections enhance nonlocal RVB correlations).

(i) Kondo screening of AFM order. In this case one takes $\Delta = 0$ in the Green’s function [18]. Then adding the last term of eq. (12) to the effective action and integrating over the semi-fermionic fields one gets correction to the effective action in a form of polarization operators given by the first diagram in Fig. 1a.
Here the external wavy lines stand for "semi-bosonic" field $\phi$ describing Kondo correlations (see $^{33}$). These "semi-bosonic" fields are still bosons from the point of view of the permutation relations, but unlike true bosonic fields they do not satisfy symmetric boundary conditions, and cannot condense in a state with zero frequency and momentum. So the Popov-Fedotov formalism gives an adequate description of the fact that there is no broken symmetry corresponding to Kondo temperature $^{35}$. The polarization loop is formed by conduction electron propagator $G_0$ (solid line) and local semi-fermionic Green’s function $D_m$ given by Eq. (18) (see Appendix I for the explicit form of these Green’s functions).

The logarithmic renormalization of the coupling constant is already taken into account in $\tilde{J}$. Therefore the dimensionless integral $\delta\Pi$ includes only contribution due to nonzero magnetic molecular field $^{36}$,

$$
\delta\Pi(N) = \frac{\pi}{2} \left( \frac{1}{\cosh(\beta N)} - 1 \right) + O \left( \frac{N^2}{T \epsilon_F} \right).
$$

(26)

(see Appendix II for detailed calculations). This correlation correction should be incorporated in the equation for the free energy, so that

$$
\beta F(N) = \beta F(N,0) + \ln \left[ \frac{1}{J} - \delta\Pi(N) \right].
$$

(27)

Then differentiation of Eq. (27) over the Neel order parameter $N$, give the following self-consistent equation in the vicinity of renormalized transition point,

$$
N = \tanh \left( \frac{\beta Q N}{2} \right) \left[ 1 - \frac{a_N}{\ln (T/T_K)} \frac{\cosh^2(\beta Q N/2)}{\cosh^2(\beta Q N)} \right]
$$

(28)

instead of Eq. (22). Here the Kondo temperature, $T_K$, is defined as a temperature where the coefficient in front of $|\phi_{n=0}|^2$ in Eq. (25), i.e. the function $J^{-1} - \delta\Pi(N)$, turns into zero. It is seen that the screening corrections near the Neel transition point are negative, $\delta\Pi(N \rightarrow 0) = -a_N(\beta N)^2 < 0$, so that Kondo screening effectively increases the magnetic free energy, and eventually the logarithmic local field corrections reduce the Neel temperature. The results of numerical solution of Eq. (28) are shown by a bold line in Fig. 2. Inset (a) illustrates the reduction of $T_N$ in comparison with the bare mean field Neel temperature $T_N^0 = z \epsilon_F \alpha^2 / 2$, where $\alpha = J/\epsilon_F$ is dimensionless coupling constant for the Doniach’s diagram.

(ii) Kondo enhancement of SL transition. Now we assume $N = 0$ in Eq.(21) and subsequent equations. Following the same lines as in preceding subsection, one obtains the modified effective action

$$
A_{\phi} = 2 \sum_{\mathbf{q},n} \left[ \frac{1}{J} - \delta\Pi(I_{\mathbf{q}} \Delta) \right] |\phi_n(\mathbf{q})|^2
$$

(29)
instead of (25), and the polarization integral with the use of the diagram (b) from Fig.1 is given by

\[
\delta \Pi(Iq\Delta) = \sum_k \left[ \frac{1}{\cosh(\beta I_k\Delta) - 1 + I_k\Delta \tanh(\beta I_k\Delta)} \right] \frac{1}{\xi_{k+q}^2 + (\pi/2\beta)^2}.
\] (30)

instead of (26) (see Appendix II). Here \(\xi_p = \varepsilon_p - \varepsilon_F\) is the dispersion law for conduction electrons near the Fermi surface. Inserting the corresponding corrections to the free energy,

\[
\beta F(\Delta) = \beta F(0, \Delta) + \text{Tr} \ln \left[ 1 - \delta \Pi(Iq\Delta) \right] .
\] (31)

one obtains the corrected self-consistent equation for \(\Delta\). When deriving this equation, the spinon dispersion can be neglected since \(\Delta \to 0\) in a critical point. Then one has

\[
\Delta = -\sum_q \nu(q) \left[ \tanh \left( \frac{I(q)\Delta}{T} \right) + a_{sl} \frac{I_q\Delta}{T \ln(T/T_K)} \right]
\] (32)

Here \(a_{sl} \sim 1\) is a numerical coefficient. It is seen from (32) that unlike the case of local magnetic order, Kondo scattering favors transition to a SL state, because this scattering means in fact involvement of itinerant electron spin degrees of freedom into spinon dynamics. Mathematically, enhancement arises because \(\delta \Pi(Iq\Delta \to 0) = a_{sl}(\beta I_q\Delta)^2 > 0\), so that Kondo “anti-screening” effectively decreases the magnetic free energy. The results of numerical solution of Eqs. (31) and (32) are presented by circles in Fig. 2.

FIG. 2. Doniach diagram reconstructed due to Kondo screening (see text for explanation).

Here filled circles correspond to the region where the AFM order overcomes the SL phase, and the light circles show unphysical ”suppressed” AFM solutions obtained beyond the region of validity of the mean-field equation (28). Two other characteristic temperatures, renormalized \(T_K\) and \(T_{sl}\), are shown by dashed and solid line, respectively. The effects of suppression of \(T_N\) (thin and thick solid lines for bare and renormalized temperatures) and \(T_K\) (thin and thick dotted lines) are illustrated by the upper and lower inset, respectively. As is seen from the modified Doniach’s diagram, the interplay between three modes becomes significant in a critical region where the exchange coupling constant is close to the point \(\alpha_c = 0.13\) where \(I = \Delta_K\) in the conventional Doniach’s diagram. If the Kondo screening is not taken into account, then \(T_N^{(0)}(\alpha) < T_N^{(0)}(\alpha)\) (thin solid and dotted lines in the lower inset). The Kondo screening changes this picture radically, and as a result wide enough interval of the values of parameter \(\alpha\) just to the right of the
critical value $\alpha_c$ arises, where the enhanced transition temperature $T_d$ exceeds both the reduced Neel temperature $T_N$ and the Kondo temperature $T_K$. The calculations of $T_d$ presented in Fig. 2 are performed for $d = 2$. The similar picture exists for $d = 3$, although the domain of stable SL state is more narrow (for a given value of $zI$). This means that in this region the stable magnetic phase is, in fact, the spin liquid phase. If one descends from high temperatures in a hatched region of Doniach’s diagram where $T_K \sim T_N$, the Kondo scattering suppresses the AFM correlations, but the SL correlations quench Kondo processes at some temperature $T_d > T_K$. As a result the Kondo-type saddle point is not realized in the free energy functional in agreement with the assumption used above in our derivation of Ginzburg-Landau expansion. The preliminary version of this scenario was presented in \cite{41}. More refined mean field approach described here confirms and enhances this scenario, however the SL liquid phase is still described in the mean field approximation. Although the local constraint for spin operators is not violated in Popov-Fedotov formalism, the gauge phase is still fixed\cite{41}, so the next task is consideration of fluctuation backflows described by the higher order terms of Ginzburg-Landau expansion.

IV. ISING SPIN GLASSES IN DONIACH’S DIAGRAM

In this section we consider the interplay between Kondo scattering and magnetic correlations in case of random RKKY interaction \cite{3}, where the randomness results in formation of a spin glass phase. We consider disorder induced by paramagnetic impurities in KL. As was shown in \cite{42}, elastic scattering results in appearance of a random phase $\delta(r)$ in RKKY indirect exchange parameter,

$$I_{ij} \equiv I(r) \simeq -\left(\frac{J^2}{\varepsilon_F}\right) \frac{\cos[2pFr - \frac{\pi}{2}(d+1) + \delta(r)]}{(2pFr)^d}$$

(33)

where $r = |R_i - R_j|$, $d$ is the dimensionality of KL. This form of random exchange predetermines two possible scenarios of SG ordering.

(i) Fluctuations take place around a node of RKKY interaction \cite{3}. This asymptotic is derived from the general equation for RKKY exchange parameter \cite{3}:

$$I_{ij} = -\frac{J^2}{\varepsilon_F} \frac{\pi}{d-1} \left(\frac{pFr^2}{2}\right)^d (pFr)^2 \left[ J_{d/2-1}(pFr)Y_{d/2-1}(pFr) + J_{d/2}(pFr)Y_{d/2}(pFr) \right].$$

($a_0$ is the lattice spacing, $J_\nu(x)$ and $Y_\nu(x)$ are the Bessel functions of 1st and 2nd kind). In this case FM and AFM bonds enter the partition function on equal footing, and quenched independent random variables $I_{ij}$ can be described by Gaussian distribution $P(I_{ij}) \sim \exp(-I_{ij}^2 N/2I^2)$ \cite{3}. The magnetic ordering effects also can be included in our approach by introducing nonzero standard deviation $\Delta I \neq 0$ into the distribution $P(I_{ij})$ that, in turn, results in additional competition between SG and AFM (or, in some cases, FM) states. Recently, the competition between AFM and SG regimes was considered in \cite{43}.

(ii) RKKY exchange fluctuates around some negative value in AFM domain of exchange parameters. In this case there is a competition between SG, SL and AFM phases. The third possibility, i.e. fluctuations in FM domain is somewhat trivial because in this case Kondo fluctuations cannot significantly change freezing scenario.

We start with the case (i). To understand the situation qualitatively we make following simplifying approximations. First, we consider only Ising-like exchange in the Hamiltonian \cite{3}:

$$H' = -\sum_{\langle ij \rangle} I_{ij} S_i^z S_j^z$$

(34)

This is usual approximation in the theory of spin glasses that allows one to forget about quantum dynamics of spin variables \cite{3}. In the original paper \cite{3} the simplifying assumptions ($d = \infty$, separate electron bath for each localized spin) were made. Thus the form of spin-spin correlator was predetermined, and these assumptions allowed the authors to obtain exact solution in a framework of dynamical mean field theory approach. We refrain from using these approximations. Second, we confine ourselves with the mean field (replica symmetrical) solution of the Edward-Anderson (EA) model \cite{3}. This means that only a pairwise interaction of nearest neighbors is taken into account. The number $z$ of neighbors should be big enough ($z^{-1} \ll 1$) to verify the mean field approximation. We consider the interplay between SG and Kondo-type correlations by means of the replica method. We use the approach developed in Ref. \cite{3} for the Sherrington-Kirkpatrick model \cite{3}. Both electron and semi-fermion variables are replicated ($c \rightarrow c^a, f \rightarrow f^a$, where $a = 1, \ldots, n$), and the number of replicas is tended to zero, so that the free energy per cell is given by the limit $F = \beta^{-1} \lim_{n \to 0}(1 - \langle Z^n \rangle_{av})/(nN)$. Here the replicated partition function is

\[ \]
The Hubbard-Stratonovich trick one comes to the following equation:

\[
(Z^n)_{av} = \prod \int dI_{ij} P(I_{ij}) \prod D\{c^{a\sigma}_{i,\sigma}, f^{a\sigma}_{i,\sigma}\} \exp\left( A_0[c^a, f^a] - \int_0^\beta d\tau H_{int}(\tau) \right)
\]  

(35)

where \(A_0\) corresponds to noninteracting fermions.

Averaging over disorder and integrating out high-energy electronic states with the help of replica-dependent Hubbard-Stratonovich trick one comes to the following equation:

\[
(Z^n)_{av} = \prod \int D\{c^a, f^a, \phi^a\} \exp\left( A_0 + \frac{zI^2}{4N} \text{Tr}[X^2] + \int_0^\beta d\tau \text{Tr}\left\{ \phi^a c^a f^a + \bar{\phi}^a \bar{f}^a c^a - \frac{2}{J} |\phi^a|^2 \right\} \right)
\]

(36)

with

\[
X^{ab}(\tau, \tau') = \sum_i \sum_{\sigma, \sigma'} f^a_{i,\sigma}(\tau) f^b_{i,\sigma}(\tau') \phi^a_{i,\sigma}(\tau') \phi^b_{i,\sigma,\sigma'}(\tau).
\]

Then following the standard pattern of replica theory for spin glasses one fixes the saddle point in spin space related to EA order parameter \(Q\). At this stage the initial problem is mapped onto a set of independent Kondo scatterers for low energy conduction electrons in external replica dependent effective magnetic field:

\[
(Z^n)_{av} = \exp\left( -\frac{1}{4} z(\beta I)^2 N(nq^2 + n(n-1)q^2) + \sum_i \ln \left[ \prod \int D\{f^a, \phi^a\} \int_x^G \int_{y^a}^G \exp(A\{f^a, \phi^a, y^a, x\}) \right] \right)
\]

(37)

where \(f^G(x)\) denotes \(\int_\infty^G dx / \sqrt{2\pi} \exp(-x^2/2) f(x)\),

\[
A\{f^a, \phi^a, y^a, x\} = \sum_{a,\sigma} f^a_{a,\sigma} \left[ (D_m^{(a)})^{-1} - \sigma h(y^a, x) \right] f^a_{a,\sigma} - \frac{2}{J} \sum_n |\phi^a_n|^2
\]

(38)

and \(h(y^a, x) = I \sqrt{2qx} + I \sqrt{z(\bar{q} - q)y^a}\) is effective local magnetic field, which depends on diagonal, and off-diagonal elements of Parisi matrix, \(\bar{q} = \langle S_0^a(0)S_0^a(t \to \infty) \rangle\) and \(q = \langle S_0^a(0)S_0^b(t \to \infty) \rangle\) \((a \neq b)\), respectively. The latter one is the EA order parameter \(q_{EA} = q\). Neglecting all fluctuations and retaining only first two terms in the exponent in Eq. (37), one comes to EA mean field equation for the free energy,

\[
\beta F = \frac{z(\beta I)^2}{4} \left[ (1 - \bar{q})^2 - (1 - q)^2 \right] - \int_x^G \ln(2 \cosh(\beta I x \sqrt{2q})).
\]

(39)

(see[2]). Then making the high temperature expansion, one obtains the Ginzburg-Landau equation in the vicinity of SG transition

\[
\beta F_{sg} = \frac{z(\beta I)^2}{4} q^2 \tau_{sg} - c_{sg} q^3 + d_{sg} q^4
\]

(40)

where \(\tau_{sg} = 1 - \tau_f / T\) and \(\tau_f = \sqrt{z} I\) is a spin glass freezing temperature.

Like in the previous case of ordered KL we incorporate the static replica dependent magnetic field \(h\) in semi-fermionic Green’s functions. As a result, the modified effective action for Kondo fields arises like in Eqs. (23) and (24)

\[
A[y^a, x] = \ln \left( 2 \cosh(\beta h(y^a, x)) \right) - \sum_n \left[ \frac{1}{J} - \delta \Pi(h(y^a, x)) \right] |\phi^a_n|^2
\]

(41)

Here similarly to Eq. (24)

\[
\delta \Pi(h) = \left[ \frac{\pi}{2} \left( \frac{1}{\cosh(\beta h)} - 1 \right) + O \left( \frac{h^2}{T \epsilon_F} \right) \right]
\]

(42)

Finally, performing Gaussian averaging over \(\phi\) fields and taking the limit \(n \to 0\) one obtains the free energy

\[
\beta F(\bar{q}, q) = \frac{1}{4} \frac{z(\beta I)^2}{2} (\bar{q}^2 - q^2) - \int_x^G \ln \left( \int_y^G 2 \cosh(\beta h(y, x)) \right) \left[ 1 - J \Pi(0, h(y, x)) \right] \right). \]

(43)
Corrected equations for $q$ and $\tilde{q}$ are determined from conditions $\partial F(\tilde{q}, q) / \partial \tilde{q} = 0$, $\partial F(\tilde{q}, q) / \partial q = 0$. These are

$$\frac{1}{2} z(\beta I)^2 \tilde{q} = \int_x^G \frac{\partial \ln C}{\partial \tilde{q}} \, , \quad \frac{1}{2} z(\beta I)^2 q = -\int_x^G \frac{\partial \ln C}{\partial q} \, .$$

$$C = \int_y^G 2 \cosh(\beta h(y, x)) / [1 - J \Pi(0, h(y, x))] \, . \quad (44)$$

Under the condition $h(y, x) \leq 1$ a useful approximate equation for $C$ is obtained

$$\ln (C C(x, \tilde{q}, q)) = -\frac{1}{2} \ln (1 + \gamma u^2(\tilde{q} - q)) + \frac{u^2(\tilde{q} - q)(1 + \gamma x^2)}{1 + \gamma u^2(\tilde{q} - q)} + \ln \left[ \cosh \left( \frac{ux \sqrt{q}}{1 + \gamma u^2(\tilde{q} - q)} \right) \right] \quad (45)$$

Here the following short-hand notations are used: $u = \beta I \sqrt{z}$, $C = J / \epsilon_F \ln(T/T_K)$ and $\gamma = 2c / \ln(T/T_K)$ with $c = (\pi/4 + 2/\pi^2) \sim 1$. We note again that when $J = 0$, which corresponds to the absence of Kondo interaction, $C(x, \tilde{q}, q) = 2 \exp \left( \frac{1}{2} z(\beta I)^2(\tilde{q} - q) \right) \cosh(\beta l x \sqrt{zq})$, and standard EA equation takes place, providing, e.g. an exact identity $\tilde{q} = 1$.

In the vicinity of a freezing point Eq. (44) acquires the form

$$\tilde{q} = 1 - \frac{2c}{\ln(T/T_K)} - O \left( \frac{1}{\ln^2(T/T_K)} \right) , \quad (46)$$

$$q = \int_x^G \tanh^2 \left( \frac{\beta l x \sqrt{zq}}{1 + 2cz(\beta I)^2(\tilde{q} - q) / \ln(T/T_K)} \right) + O \left( \frac{q}{\ln^2(T/T_K)} \right) \, .$$

As a result of numerical solution of Eqs. (46) we obtain the analog of Doniach’s diagram for a disordered KL where the spin glass freezing temperatures without and with Kondo screening contributions are shown ($T_f^{(0)}$ and $T_f$, respectively).

![Doniach's diagram for spin glass transition in disordered KL](image)

FIG. 3. Doniach’s diagram for spin glass transition in disordered KL (see text for explanation).

Here $T_f^{(0)}$ is obtained from GL equation (40) neglecting Kondo screening effect, and $T_f$ was defined from Eqs. (46) under additional condition $\partial^2 F_{sg} / \partial q^2 = 0$. The influence of Kondo screening on the diagonal element of Parisi matrix $\tilde{q}$ is illustrated by the inset (bare value of $\tilde{q} = 1$ is shown by the dashed line). Like in the case of perfect KL, the screening effect is noticeable when $T_f^{(0)} \sim T_K^{(0)}$. 

The influence of SG transition on a Kondo temperature for a KL with SG freezing was studied recently in Ref. 49. Although the Kondo effect in this paper is considered in a mean field approximation (i.e. Kondo screening is treated as a true phase transition) and a static ansatz was applied for SG, the authors obtained strong reduction of Kondo temperature in the same region $T_K \sim T_f^{(0)}$.

V. CORRELATIONS IN KONDO LATTICE BEYOND THE MEAN FIELD

The mean-field Doniach’s diagram even in its improved form oversimplifies enormously the real picture of interplay between three competing modes in effective action (12). First of all, the proximity of three characteristic temperatures, $T_K$, $T_{sl}$ and $T_N$ means that even when one of them is dominant, i.e. determines the local minimum of the free energy, two other define the fluctuations around the saddle point. Second, it is clear physically that only Neel temperature, $T_N$, is a temperature of a real phase transition, whereas $T_{sl}$ and $T_K$ are merely characteristic crossover temperatures. The main shortcoming of the mean field approximation is that this approach treats all three modes on equal footing. The method described in preceding section allows one to get rid of artificial phase transition at $T = T_K$, however the problems with description of SL phase still exist. Meanwhile, it is known that the mean-field approximation for SL state violates the local gauge invariance and fixes the phase $\theta$ of SL mode $W$ (17). Second-order phase transition from paramagnetic to SL state is an undesirable corollary of this crude approximation, and fluctuation corrections to the mean-field solution cannot improve this defect of the theory.

In this section we consider several scenarios of mode-mode correlations in a system described by the general equation (1), (12) for effective action $\mathcal{A}$. First, we offer the description of crossover to a SL state, which allows one to bypass the mean field saddle point (23). It will be demonstrated that the interplay between fluctuations of the fields $\phi$ and $Y_Q$ can trigger the transformation of localized critical relaxation AFM modes into SL type correlations without loss of criticality. The main idea of our scenarios is that the heavy fermion state of KL is, in fact, unconventional AFM state with spin excitations changing their character from Bose-like spin fluctuations or spin waves to Fermi-like spinon modes. Next, we consider the behavior of Kondo mode below $T_{sl}$ and describe the quenching of Kondo scattering by SL fluctuations in a hatched part of Doniach’s diagram (Fig. 2) where the static molecular field is absent.

We demonstrated above that the Kondo screening enhances SL correlations on a level of the mean field approximation. Similar effect should exist on a more refined level of interacting fluctuation modes. To find the corresponding mechanism we refuse from bilocal representation of spin mode. Instead of introducing the mode $W$ associated with gauge non-invariant $U(1)$ field described by the phase $\theta$ in Eq. (17), we consider the effect of interference of Kondo screening modes associated with spins located on different sites of KL. In fact we consider the high-temperature precursors of orthogonality catastrophe mentioned by Nozieres in his formulation of ”exhaustion problem” (51). In a revised scheme we start with the action determined by the Hamiltonian (1). Starting with integration over ”fast” electronic variables (with energies $\sim \varepsilon_F$), we obtain $\mathcal{A}_{int}$ in a form

$$
\mathcal{A}_{int} = -\frac{2}{\tilde{J}} \text{Tr} |\phi|^2 - \text{Tr} \frac{1}{I_q} Y_q Y_{-q} - \text{Tr} \bar{f}_j \phi_j G_0(r) \bar{\phi}_l f_l \\
- \text{Tr} \bar{\phi}_j \phi_l \Pi_4 \bar{\phi}_l \phi_j - \text{Tr} Y_j \bar{\phi}_j \phi_l \Pi_6 \bar{\phi}_l \phi_j Y_l . \tag{47}
$$

![FIG. 4. Diagrams for fourth and sixth order polarization operators $\Pi_4$ (a) and $\Pi_6$ (b) in effective action responsible for mode-mode coupling.](image-url)
Here instead of introducing the scalar mode $W$ we retained higher order terms in Kondo screening fields. These terms are illustrated by the diagrams of Fig. 4.

The diagram of Fig. 4a describes interference of Kondo clouds around the sites $R_j$ and $R_l$. Zig-zag lines stand for AFM vector mode. Like all screening diagrams in Fermi systems it contains Friedel-like oscillating factor. To estimate the polarization operator we use the asymptotic form of the electron Green's function in d-dimension at large distances:

$$G(r, \Omega) \sim \frac{1}{(p_{Fr})^{d-2}} \exp \left[ \frac{|\Omega|}{2\varepsilon_F} p_{Fr} + i \left( p_{Fr} - \pi \frac{d+1}{4} \right) \text{sgn} \Omega \right]$$

Inserting this function in a four-tail diagram of Fig. 4a, one comes to

$$\Pi_4 \sim - \frac{1}{T \varepsilon_F} \frac{\cos(2p_{Fr} - (d+1)\pi/2)}{(2p_{Fr})^{d-1}} + O \left( \frac{1}{\varepsilon_F} \ln \left( \frac{T}{\varepsilon_F} \right) \right)$$

Therefore we expect that this interference correlates with RKKY-type magnetic order, and the interaction between the corresponding modes represented by the diagram (b) in Fig. 4 influences the magnetic response in a "critical" region of the Doniach’s diagram. This response is determined by the fluctuation corrections to Neel effective action,

$$\delta A_{eff} = \frac{1}{4} \sum_{\mathbf{q}, \alpha, \omega_n} Y^\alpha(\mathbf{q}, \omega_n) \left[ I^{-1}(\mathbf{q}) + \chi_0 \delta n, 0 \right] Y^\alpha(\mathbf{q}, \omega_n)$$

Here $\alpha$ are Cartesian coordinates, $\chi_0 = \beta/4$ is a static Curie susceptibility of isolated spin 1/2 (Fig. 5a). The term in square brackets is, in fact, inverse Ornstein-Zernicke correlator $\sim a^2_0 (Q - \mathbf{q})^2 + \tau_N$ at $T \gtrsim T_N$ and $Q - \mathbf{q} \rightarrow 0$. First non-vanishing correction to $\chi_0$ is given by Fig. 5b. In this diagram the spins $S_j$ and $S_j$ are screened independently, (the wavy lines represent all parquet vertex insertions). In the mean field approach the similar effects are described by Eq. (28).

\[FIG. 5.\] Diagrams describing local (Curie-type) magnetic susceptibility $\chi_0$ (a) and nonlocal correction taking into account Kondo screening of vertices (b).

Indeed, each vertex correction $\Gamma_{i<j,l}(\omega, \epsilon) \sim \langle \phi(\epsilon) \delta(\epsilon) \rangle$ gives the contribution $\sim 1/\ln(\epsilon/T_K)$, and integration over internal frequency $\epsilon$ results in $1/\ln(T_K/\tau_N)$ correction in Eq. (28).

The effects essentially beyond the mean field are described by the diagrams that cannot be cut along a pair of electron propagators (solid lines) (see Fig. 6. and 7a).

\[FIG. 6.\] Leading diagrams describing interference of Kondo clouds in magnetic susceptibility (see text for details).
The first of these diagrams (Fig. 6a) can be treated as a nonlocal correction to one site spin susceptibility (Fig. 5a) induced by interfering flow and counterflow of two Kondo clouds. As a result, the spin-fermion propagator becomes nonlocal without introducing the mean field order parameter \((17)\). The next diagram (Fig. 6b) is a kind of “exchange” by these clouds in the course of two-spinon propagation. Up to now we exploited the “proximity” effects \(T \sim T_K\). A critical AFM mode given by the Fourier transform of the diagram of Fig. 5a with the wave vector \(q \approx Q\) also exists in this temperature interval, and, moreover, this mode is dominant in spin susceptibility at \(T \sim T_N\). This means that the nonlocal contributions of Fig. 6 should be taken also at these \(q\). Due to nonlocality, the temperature dependence of spin polarization loop will be slower than the Curie law \(1/T\), and the inverse static susceptibility given by these diagrams is

\[
\chi^{-1}_Q(T) = \chi_0^{-1}(T) + \chi_{sl}^{-1}(T) + \tilde{I}_Q
\]

(51)

This deviation from Curie law results in delay of Neel phase transition or, in other words, in extension of critical regime to temperatures well below \(T_N\) in accordance with scenario described in Ref. 28. Magnetic instabilities that can emerge at \(T \ll T_N\) will be the instabilities of spin liquid phase. These instabilities have much in common with itinerant fluctuational magnetism considered, e.g. in Refs. 52,53.

FIG. 7. (a) Next to parquet approximation for Kondo correction to the magnetic susceptibility; (b) magnetic fluctuation correction to single site Kondo scattering.

The diagram of Fig. 7a with bare spinon propagators gives only local correction to the susceptibility, however at \(T \ll T_N^0\) where the spinon lines are dressed by the self energies shown in Fig. 6a, this diagrams also becomes nonlocal and, therefore contributes in nonlocal term in the r.h.s. of Eq. (51). The processes taken into account in the diagram of Fig. 7b describe the feedback influence of spin fluctuations on the Kondo screening. This diagram together with higher order terms of the same type results in dynamical suppression of \(T_K\) as a result of appearance of spin fluctuation energy \(\omega_{sf} \sim \xi^{-2}\) in the Kondo logarithm. \(\ln(\varepsilon_F/\max\{T, \omega_{sf}\})\). This mechanism is effective not too close to real \(T_N\) where the magnetic correlation length \(\xi\) determining the short-range magnetic order is still comparable with the lattice spacing (here \(z\) is dynamical critical exponent).

This schematic description is only scenario of the theory of critical phenomena in KL. We leave discussion of fluctuations around SG transitions beyond the scope of this paper. Some details of a new modulated replica symmetry breaking schemes, which combine tree and wave-like structures in AFM SG may be found in Ref. 41. More detailed calculation of critical magnetic and spin glass fluctuations in spin liquid will be published separately.

VI. CONCLUDING REMARKS

We derived in this paper the phase diagram for the Kondo lattice model, starting with a high temperature expansion of effective action. As a first step, we succeeded in getting rid of one of fictitious saddle points, i.e. we avoided the introduction of ”Kondo-condensate” averages \(\langle c_{\sigma f}^\dagger f_{\sigma i} \rangle\) used in previous revisions of the Doniach’s diagram\(23,25\). In our modified Doniach’s diagram (Fig. 3) the renormalized \(T_K\) is the lowest of all characteristic temperatures for all reasonable values of coupling constant \(\alpha\) where one can neglect valence fluctuations. In fact, the mean-field calculations of Ref. 23 give similar picture. The feedback of this result is that the strong Kondo regime is unachievable in a critical region of Doniach’s diagram, and the real role of Kondo screening for small \(\alpha\) where \(T_N > T_{sl} > T_K\) is to reduce localized magnetic moments and enhance the electronic density of states around \(\varepsilon_F\). Thus the moderately heavy fermion systems with relatively big magnetic moments ordered antiferromagnetically arise (CeIn\(_3\), CeAl\(_2\)) are possible examples\(54\).
In a critical region of Doniach’s diagram Kondo screening changes radically the behavior of KL. According to our mean field results the conventional AFM order is suppressed at \( T \sim T_{sl} \geq T_N \). The SL phase that arises instead is, nevertheless, close to magnetic instability, and one can expect that spin subsystem eventually orders magnetically. If new transition temperature, \( T_N \), is finite, the singlet spinon coupling is incomplete, so that RVBs have residual magnetic moments, and these moments are ordered at \( T = T_N \) (we emphasize once more that \( T_N \) marked by light circles in a hatched region of the phase diagram of Fig.3 is not a real transition temperature. It rather designates the temperature region where critical AFM fluctuations arise). Of course, the magnitude of these moments is extremely small, and one can qualify this type of magnetic order is intermediate between localized and itinerant AFM. In the temperature interval \( T_N < T < T_N \) the critical AFM relaxation mode characterizes the magnetic response of the system. When \( T_N = 0 \), one deals with quantum phase transition, and the case \( T_N < 0 \), apparently, corresponds to short-range correlations existing in a wide temperature interval \( 0 < T < T_N \). This picture describes in gross features the magnetic properties of magnetic KL, but any kind of quantitative description will be possible only after realization of scenarios for the critical behavior of spin liquid briefly sketched in Section V.

Now we turn to discussion of conclusions that could be derived from our theory concerning the nature of heavy fermion state. The most important one is that the separation of charge and spin degrees of freedom existing in KL at high temperatures take place also in a strong coupling regime at \( T \ll T_K \). Indeed, at high \( T \) exceeding all characteristic temperatures in KL the spin excitation spectrum is simple structureless peak of the width \( T \) around zero energy. This peak is manifested as Curie-type magnetic susceptibility and trivial high-temperature corrections \( \sim 1/T^\alpha \) to all thermodynamic quantities due to weak paramagnetic spin scattering of conduction electrons, whose Fermi liquid continuum exists as independent charge branch of elementary excitations. Since all transformations of spin subsystem occur at \( T > T_K \) (at least in a region of \( \alpha < 0.2 \) where the valence fluctuations are still negligible), this central peak still exist in a strong coupling regime. Below \( T_{sl} \sim T_K \) this peak is formed by spin liquid excitations. The character of these excitations reminds relaxation modes in a picture of fluctuation itinerant magnetism,\(^{22a,b}\) in a wide temperature interval down to \( T_{coh} \) where the coherent spin liquid regime of Fermi type is established. The interaction between SL mode and conduction electrons is the same exchange-type scattering as at high temperatures. This coupling constant \( \tilde{J} \) is, however, enhanced by the Kondo effect (see Eq. 25). The electrons in a layer of the width \( T_K \) around Fermi level interact non-adiabatically with spin fermions at low \( T \). As a result the giant Migdal effect arises,\(^{52a}\) which results in strong electron mass enhancement. So, the heavy fermion state in accordance with this picture is a two-component Fermi liquid where the characteristic energies of charge subsystem (slow electrons with \( \epsilon < T_K \)) and spin subsystem (spinons with \( \omega \sim T_{sl} \)) are nearly the same).

Exponentially narrow low-energy peak of predominantly spin origin appears practically in all theories of strongly correlated electron systems. In the archetypal Hubbard model this peak arises on a dielectric side of Mott-Hubbard transition, and still exists on metallic side, where the charge and spin degrees of freedom are already coupled. This is the point where the links between Hubbard and Anderson models arise at least on a level of dynamical mean field theory (DMFT) valid at \( d \rightarrow \infty \). On the other hand the mean field solution that results in merged charge and spin degrees of freedom in a central peak becomes exact in the large-\( N \) theories for \( N = \infty \) saddle point.\(^{22c}\) Recent achievements in this direction are connected with confirmation of Nozieres’s prediction of second scale in Kondo lattice\(^{22a,b}\) in the limit of exhaustion regime of small electron concentration. At this temperature the “bachelor” spins form a coherent Fermi liquid and lose their localized nature. This anticipation was confirmed by recent calculation within mean field slave boson approximation of \( N \rightarrow \infty \) theory.\(^{57} \) In our approach the regime of bachelor spins does not arise, because the Kondo coupling remains weak even at \( T < T_K \) (see above), but the spin degrees of freedom become coherent at \( T \sim T_{coh} \), so that two coherence scales is the intrinsic property of the model.

Another aspect of large \( N \) theories is the possibility of supersymmetric description that allows combined description of spin degrees of freedom in a mixed fermion-boson \( SU(N) \) representation.\(^{58} \) This approach allowed the authors to retain inter-site RKKY interaction in the limit of \( N \rightarrow \infty \) in spite of \( 1/N^2 \) effect of suppression of all inter-site magnetic correlations in a standard large \( N \) approach. The use of Popov-Fedotov representation allows treatment of different magnetic modes described by these operators as ”semi-fermions” or ”semi-bosons” in different physical situations.\(^{59} \) In this paper we appealed to \( SU(2) \) symmetry. The general recipe of generation of modes with intermediate statistics between Fermi and Bose limiting cases for \( SU(N) \) algebra is offered in.\(^{59} \) In fact the eventual transformation of the states with intermediate statistics into true fermions (bosons) occurs only at \( T \rightarrow 0 \). so this approach may be extremely useful for adequate description of quantum phase transitions.

In principle other collective modes can modify the scenario of AFM phase transition in KL. In particular, the low-lying crystal field excitations may intervene the magnetic phase transition in the same fashion as Kondo clouds in our theory. Probably the CeNiSn family of semimetallic Kondo lattices is an example of such intervention.
There are normal and anomalous components. The components \( D \) transform of the Green’s function \( T \). The support of the Alexander von Humboldt Foundation, K.K. is grateful to Israeli-USA BSF-1999354 for partial support and to the University of Würzburg for hospitality.

**APPENDIX I**

To evaluate the contribution of Kondo mode in the expansion \( \mathcal{L} \) for effective action, one needs the Fourier transform of the Green’s function \( K_0 \). This is

\[
\begin{pmatrix}
\phi_n(k)G_0(q)\phi_n(k) & 0 & \phi_n(k)G_0(q)\phi_n(k + Q) & 0 \\
0 & \phi_n(k)G_0(q)\phi_n(k) & 0 & \phi_n(k)G_0(q)\phi_n(k + Q) \\
\phi_n(k + Q)G_0(q)\phi_n(k) & 0 & \phi_n(k + Q)G_0(q)\phi_n(k + Q) & 0 \\
0 & \phi_n(k + Q)G_0(q)\phi_n(k) & 0 & \phi_n(k + Q)G_0(q)\phi_n(k)
\end{pmatrix} \tag{AI.1}
\]

The components \( D_{m\sigma}(q) \) of the semi-fermionic Green’s function \( D \) in (20) are determined by inverting the matrix (18). There are normal and anomalous components,

\[
-\int_0^\beta d\tau e^{i\omega_m\tau}\langle T_\tau f_\sigma(q,\tau)\bar{f}_\sigma(q,0) \rangle = \frac{i\omega_m - W_q}{(i\omega_m - W_q)^2 - Y^2} \tag{AI.2}
\]

and

\[
-\int_0^\beta d\tau e^{i\omega_m\tau}\langle T_\tau f_\sigma(q,\tau)\bar{f}_\sigma(q + Q,0) \rangle = \frac{Y_{\sigma\sigma}^2}{(i\omega_m - W_q)^2 - Y^2}, \tag{AI.3}
\]

respectively. Here \( Y = N I_{Q}/2 \) and \( W_q = I_q \Delta \).

To perform calculations in real space, one should know the inverse Green’s function \( K_0^{-1} \) in coordinate representation:

\[
D_{m\sigma}^{-1}(W,Y) = \begin{pmatrix}
  i\omega_m + Y_{j}^z & Y_j^+ & W_{jl} & 0 \\
  Y_j^- & i\omega_m - Y_j^z & 0 & W_{jl} \\
  W_{ij} & 0 & i\omega_m - Y_i^z & Y_i^+ \\
  0 & W_{ij} & Y_i^- & i\omega_m + Y_i^z
\end{pmatrix}. \tag{AI.4}
\]

It should be noted that the nonlocal term \( W_{jl} \) in (AI.4) responsible for SL correlations transforms into diagonal term \( W_q \) in momentum representation (AI.3), whereas the local staggered field \( Y_i \) has non-diagonal matrix elements in momentum space corresponding to AFM correlations at \( q = Q \).

**APPENDIX II**

The sum of polarization integrals presented in Fig. 1 is given by the following equation

\[
\Pi_n(Y,W_q) = -T \sum_{m\sigma,p} D_{m\sigma}(p)G_{m+n}(p + q) \tag{AII.1}
\]

Only normal component (AI.2) survives in this equation as a result of spin summation. The Neel loop (Fig.1a) after performing frequency summation acquires the form

\[
\Pi(Y,0) = \sum_p \left\{ \tanh\left(\frac{\xi_p}{2T}\right) \left[ \frac{\xi_p - Y}{(\xi_p - Y)^2 + \lambda^2} + \frac{\xi_p + Y}{(\xi_p + Y)^2 + \lambda^2} \right] \right. \\
\left. + \frac{\lambda}{\cosh(Y/T)} \left[ \frac{1}{(\xi_p - Y)^2 + \lambda^2} + \frac{1}{(\xi_p + Y)^2 + \lambda^2} \right] \\
- \tanh\left(\frac{Y}{T}\right) \left[ \frac{\xi_p - Y}{(\xi_p - Y)^2 + \lambda^2} - \frac{\xi_p + Y}{(\xi_p + Y)^2 + \lambda^2} \right] \right\}. \tag{AII.2}
\]
Here $\xi_p = \varepsilon_p - \varepsilon_F$, $\lambda = \pi T/2$. This integral is an even function of the order parameter, $\Pi(Y) = \Pi(-Y)$. Using the inequality $Y \ll \varepsilon_F$, two last terms can be simplified, and introducing the integral over the electron band with constant density of states $\rho_0$, one has

$$
\Pi(Y, 0) = \frac{1}{4} \rho_0 \int_{-\varepsilon_F}^{\varepsilon_F} d\xi \left\{ \tanh \left( \frac{\xi_p}{2T} \right) \left[ \frac{\xi_p - Y}{(\xi_p - Y)^2 + \lambda^2} + \frac{\xi_p + Y}{(\xi_p + Y)^2 + \lambda^2} \right] \right. \\
+ \left. \frac{\pi \rho_0}{2 \cosh(Y/2T)} + \frac{\rho_0 Y}{\varepsilon_F} \tanh \left( \frac{Y}{2T} \right) \right\}.
$$

(AII.3)

Incorporating $\rho_0$ in dimensionless variables, one has in the vicinity of Neel point where $Y \ll T$

$$
\Pi(Y, 0) = \frac{1}{2} \pi N \left( \frac{Y}{T} \right)^2 + O \left( \frac{Y^2}{\varepsilon_F^2} \right).
$$

(AII.4)

The logarithmic term is, in fact, included in the renormalized coupling constant $\tilde{J}$ in Eq. (23) for effective action, and the remaining terms give Eq. (26) for $\delta \Pi$. Deeper in magnetic phase where $Y \gg T$, the Kondo effect is quenched by molecular field, so that

$$
\Pi = \ln \left( \frac{\varepsilon_F}{Y} \right) + b_N \left( \frac{T}{Y} \right)^2 + O \left( \frac{T^2}{\varepsilon_F^2} \right)
$$

(AII.5)

Numerical coefficients $a_N, b_N$ arising from approximate estimates of the integrals in Eq. (AII.3) are of the order of unity.

The SL loop (Fig. 1b) can be estimated for $q = 0$. After frequency summation it is presented by the following integral

$$
\Pi(0, \Delta) = \frac{1}{2} \sum_p \xi_p \tanh \left( \frac{\xi_p}{2T} \right) + I_p \Delta \tanh \left( \frac{I_p \Delta}{T} \right) + \frac{\lambda}{2 \cosh(I_p \Delta/T)}
$$

(AII.6)

This function is also even, $\Pi(\Delta) = \Pi(-\Delta)$. Extracting from (AII.6) the logarithmic term $\ln(\varepsilon_F/2T)$, one comes to Eq. (30) for $\delta \Pi$. In a critical region of Doniach’s diagram where $\Delta \ll T$, one has

$$
\delta \Pi(0, \Delta) = a_{sl} \frac{\Delta^2}{2T^2} \sum_p \frac{\nu_p^2}{\xi_p^2 + \lambda^2}, \quad a_{sl} \sim 1.
$$

(AII.7)

\begin{enumerate}
\item G.G. Lonzarich, J. Magn. Magn. Mat. 76-77, 1 (1988); M. Springford, Physica B 171, 151 (1990); Y. Onuki and A. Hasegawa, J. Magn. Magn. Mat. 108, 19 (1992).
\item M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991); R.H. Heffner and M.R. Norman, Comments Cond. Mat. Phys. 17, 361 (1996).
\item Rauchschwalbe U (87) Physica B 147, 1 (1987); H.R. Ott, in Progress in Low Temperature Physics. Vol. XI, ed. D.F. Brewer (Elsevier, Amsterdam, 1987), p.215; N. Grewe and F. Steglich, in Handbook on the Physics and Chemistry of Rare Earths, Vol. 14, eds. K.A. Gschneider, Jr, and L. Eyring (Elsevier, Amsterdam, 1991), p. 343
\item J.M. Effantin, J. Rossat-Mignod, P. Burlet, H. Bartholin, S. Kunii, and T. Kasuya, J. Magn. Magn. Mater. 47-48, 145 (1985).
\item A. Loidl, A. Krimmel, K. Knorr, G. Sparn, M. Lang, C. Geibel, S. Horn, A. Grauel, F. Steglich, B. Welslau, N. Grewe, H. Nakotte, F. de Boer, and A.P. Murani, Ann. Physik, 1, 78 (1992); A. Bernasconi, M. Mombelli, Z. Fisk, and H.R.Ott, Z. Phys. B 94, 423 (1994).
\item A. de Visser, J.J.M. France, J. Magn. Magn. Mat. 100, 204 (1991).
\item H. von Löhneysen, A. Neubert, T. Pietrus, A. Schröder, O. Stockert, U. Tutsch, M. Löwenhaupt, A. Rosch, and P. Wölfle. Eur. J. Phys. B 5, 447 (1998).
\end{enumerate}
1. A. de Visser, M.J. Graf, P. Estrela, A. Amato, C. Baines, D. Andreica, F.N. Gygax, and A. Schenk, Phys. Rev. Lett. 85, 3005 (2000).
2. J. Rossat-Mignod, L.P. Regnault, J.L. Jacoud, C. Vettier, P. Lejot, J. Flouquet, E. Walker, D. Jaccard, and A. Amato, J. Magn. Magn. Mat., 76&77, 376 (1988).
3. N.D. Mathur, F.M. Groschke, S.R. Julian, J.R. Walker, D.M. Freye, R.K.W. Haselwimmer, and G.G. Lonzarich, Nature 394, 39 (1998).
4. C.M. Varma in Theory of Heavy Fermions and Valence Fluctuations, Springer Series in Solid State Sciences, T. Kasuya and T. Saso (eds), Vol.62 (Springer: Berlin, Heidelberg), p.277
5. Yu. Kagan, K.A. Kikoin, and N.V. Proko‘ev, Physica B 182, 201 (1992).
6. M.B. Maple, M.C. de Andrade, J. Herrmann, Y. Dalichaouch, D.A. Gajewski, C.L. Seaman, R. Chau, R. Movshovich, M.C. Aronson, and R. Osborn, J. Low Temp. Phys., 99 314 (1995).
7. L.Z. Liu, J.W. Allen, C.L. Seaman, M.B. Maple, Y. Dalichaouch, J.S. Kang, M.S. Torikachvili, and M.A. Lopez de la Torre, Phys. Rev. Lett. 68, 1034 (1992).
8. S. Süllov, G.J. Nieuwenhuys, A.A. Menovsky, J.A. Mydosh, S.A.M. Mentink, T.E. Mason, W.J.L. Buyers, Phys. Rev. Lett. 78, 354 (1997).
9. S. Becker, S. Ramakrishnan, A.A. Menovsky, G.J. Nieuwenhuys, and J.A. Mydosh, Phys. Rev. Lett. 78, 1347 (1997).
10. D.X. Li, Y. Shiokawa, Y. Homma, A. Uesawa, A. Doniach, T. Suzuki, Y. Haga, E. Yamamoto, T. Homma, Y. Ônuki, Phys. Rev. B 57, 7434 (1998).
11. O.O. Bernal, D.E. MacLaughlin, H.G. Lukefahr and A. Andraka. Phys.Rev.Lett. 75, 2023 (1995).
12. J.C. Gomez Sal, J. Garcia Soldevilla, J.A. Blanco, J.I. Espeso, J. Rodriguez Fernandez, F. Luis, F. Bartolome, and J. Bartolome. Phys. Rev B56, 11747 (1997).
13. J. Garcia Soldevilla, J.C. Gomez Sal, J.A. Blanco, J.I. Espeso, J. Rodriguez Fernandez, Phys. Rev. B61, 6821 (2000).
14. D. Eom, M. Ishikawa, Y. Homma, A. Doniach, T. Suzuki, Y. Haga, E. Yamamoto, T. Homma, Y. Ônuki. Phys. Rev. B 57, 7434 (1998).
15. K. A. Kikoin, M.N. Kiselev and A.S. Mishchenko, Physica B 195, 231 (1977) ; C. Lacroix and M. Cyrot, Phys. Rev. B 20, 1969 (1979).
16. P. Coleman, N. Andrei, J. Phys.: Cond. Mat. 1, 4057 (1989).
17. K. A. Kikoin, M.N. Kiselev and A.S. Mishchenko, Physica B 60, 600 (1994).
18. J.R. Iglesias, C. Lacroix, and B. Coqblin, Phys. Rev. B56, 11820 (1997); B.H. Bernhard, C. Lacroix, J.R. Iglesias, and B. Coqblin, Phys. Rev. B61, 441 (2000).
19. V.N. Popov and S.A. Fedotov, Sov. Phys. – JETP 67, 535 (1988).
20. L.B. Ioffe and A.I. Larkin, Phys. Rev. B 39, 8988 (1989); P.A. Lee and N. Nagaosa, Phys. Rev. B 46, 5621 (1992).
21. K. A. Kikoin, M.N. Kiselev and A.S. Mishchenko, Physica B 230-232, 490 (1997).
22. N. Read and D.M. Newns, J. Phys. C: Solid State 16, 3273 (1983).
23. F. Bouis and M.N. Kiselev, Physica B 259-261, 195 (1999).
24. K. A. Kikoin, M.N. Kiselev and A.S. Mishchenko, Physica B 230-232, 490 (1997).
25. K. A. Kikoin, M.N. Kiselev and A.S. Mishchenko, Sov. Phys. – JETP 85, 490 (1997).
26. The specific feature of $S=1/2$ is that the set $2\pi T(m + 1/4)$ can be interpreted both as "semi-fermionic" and "semi-bosonic" field. The complementary field $\phi$ which appears in decoupling procedure for bi-Grassmann products in representation \[ \phi \] will be referred as "semi-bosonic" field for the physical reasons, which are discussed below.
27. G. Baskaran, Z. Zou and P.W. Anderson, Solid State Comm., 63, 973 (1987); A. Ruckenstein, P. Hirschfeld and J. Appel, Phys. Rev. B 36, 857 (1987).
28. A.M. Tsvelik and P. Wiegmann. Adv. Phys. 32, 483 (1983).
29. A.A. Abrikosov and A.A. Migdal. Journ. Low Temp. Phys. 3, 519 (1970)
30. A. Yu. Zvyuzin and B.V. Spivak, JETP Lett. 43, 234 (1986); L.N. Bulayevskii and S.V. Panyukov, ibid, 240.
31. D.N. Aristov, Phys. Rev. B 55, 8064 (1996).
32. J. Stein, Eur. Phys. Journ. B 12, 5 (1999).
33. S. Sachdev, N. Read, and R. Oppermann, Phys. Rev. B 52, 10286 (1995).
34. R. Oppermann, D. Sherrington, and M. Kiselev, cond-mat/0106066.
35. A mean-field solution of fully connected quantum Heisenberg SG model in a large $N$ limit that involves the quantum dynamics of spin variables was presented in A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. Lett. 85, 840 (2000).
36. A. Sengupta and A. Georges, Phys. Rev. B 52, 10295 (1995).
37. S.F. Edwards and P.W. Anderson, J. Phys. F: Metal Phys. 5, 965 (1975).
38. M.N. Kiselev and R. Oppermann, JETP Lett. 71 (2000) 250.
39. D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. 35, 1972 (1975).
40. R. Oppermann and A. Muller-Groeling, Nucl. Phys. B 401, 507 (1993).
41. K. Binder and A.P. Young, Rev. Mod. Phys. 58, 801 (1986).
42. A. Theumann, B. Coqblin, S.G. Magalhães, and A.A. Schmidt, Phys. Rev. B 63, 054409 (2001).
43. I. Affleck, Z. Zou, T. Hsu and P.W. Anderson, Phys. Rev. B 38, 745 (1988).
44. P. Nozières, Ann. Phys. (Paris) 10, 19 (1985); Eur. Phys. J. B6, 447 (1998).
45. T. Moriya, Spin Fluctuations in Itinerant Electron Magnetism (Springer, Berlin, Heidelberg, 1985)
46. Y. Okuno and K. Miyake, J. Phys. Soc. Jpn, 67, 3342 (1998).
Metallic compounds containing U, apparently, should be described by the Anderson-lattice Hamiltonian because of partially itinerant nature of the $5f$ electrons, so we refrain here from direct application of our model to U-based heavy fermion systems.

55. K. Kikoin, J. Phys.: Cond Mat. 8, 3601 (1996).
56. A. Georges, G. Kotliar, W. Krauth, and M. Rozenberg, Rev. Mod. Phys. 68, 13 (1996).
57. P. Coleman, Phys. Rev. B 35, 5072 (1987).
58. S. Burdin, A. Georges, and D.R. Grempel, Phys. Rev. Lett. 85, 1048 (2000).
59. P. Coleman, C. Pepin, and A.M. Tsvelik, Phys. Rev. B 62, 3852 (2000).
60. M.N. Kiselev, H. Fedmann, and R. Oppermann, Eur. Phys. Journ. (in press).
61. A.M. Tsvelik, private communication.
62. Yu. M. Kagan, K.A. Kikoin and A.S. Mishchenko, Phys. Rev. B 55, 12348 (1997); K.A. Kikoin, M.N. Kiselev, A.S. Mishchenko, and A. de Visser, Phys. Rev. B 59, 15070 (1999).