Transverse “resistance overshoot” in a Si/SiGe two-dimensional electron gas in the quantum Hall effect regime

I. Shlimak\textsuperscript{1}, V. Ginodman\textsuperscript{1}, A. B. Gerber\textsuperscript{2}, A. Milner\textsuperscript{2}, K.-J. Friedland\textsuperscript{3}, and D. J. Paul\textsuperscript{4}

\textsuperscript{1}Minerva Center and Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics, Bar-Ilan University, 52900 Ramat-Gan, Israel
\textsuperscript{2}School of Physics \& Astronomy, Raymond \& Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, 69976 Tel Aviv, Israel
\textsuperscript{3}Paul-Drude-Institut für Festkörperphysik, Hausvogteiplatz 5-7, 10117, Berlin, Germany
\textsuperscript{4}Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, U.K.

We investigate the peculiarities of the “overshoot” phenomena in the transverse Hall resistance $R_{xy}$ in Si/SiGe. Near the low magnetic field end of the quantum Hall effect plateaus, when the filling factor $\nu$ approaches an integer $i$, $R_{xy}$ overshoots the normal plateau value $h/e^2$. However, if magnetic field $B$ increases further, $R_{xy}$ decreases to its normal value. It is shown that in the investigated sample n-Si/Si$_{0.7}$Ge$_{0.3}$, overshoots exist for almost all $\nu$. Existence of overshoot in $R_{xy}$ was observed in different materials and for different $\nu$, where splitting of the adjacent Landau bands has different character, hints at the common origin of this effect. Comparison of the experimental curves $R_{xy}(\nu)$ for $\nu = 3$ and $\nu = 5$ with and without overshoot showed that this effect exist in the whole interval between plateaus, not only in the region where $R_{xy}$ exceeds the normal plateau value.

Observations have been reported of anomalous peaks in the Hall resistance $R_{xy}$ in high mobility GaAs/AlGaAs heterojunctions measured in the quantum Hall effect regime. In incremental magnetic fields $B$, when the filling factor $\nu$ approaches an odd integer $i$, $R_{xy}$ overshoots the normal plateau value $h/e^2$. However, if $B$ increases further, $R_{xy}$ decreases to its normal value. It was mentioned that in GaAs/AlGaAs heterostructures, the overshots occur near the low magnetic field end of the spin resolved odd plateaus. The explanatory model was based on the assumption that these anomalies are due to the decoupling of the two edge states of the topmost spin-split Landau bands (LB), which occurs with an increase of the magnetic field because of the enhancement of the $g$-factor and the corresponding spin splitting due to the electron-electron interaction. Subsequently, this effect was also observed in n- and p-Si/SiGe heterostructures, where the above explanation is questionable. Indeed, for Si-based heterostructures, the $g$-factor is initially large. Moreover, in $n$-type structures, the overshoot was observed near $i = 3$, where spin splitting is not relevant, because at odd $i$, adjacent LBs in $n$-type Si/SiGe are valley-split. This hints at a more universal character of the overshoot.

In this work we present the results of experimental investigations of the overshoot in $n$-type Si/Si$_{1-x}$Ge$_x$ heterostructure. The sample investigated was Hall-bar patterned $n$-type Si/Si$_{0.7}$Ge$_{0.3}$ double heterostructure, 7 nm i-Si quantum well was situated between 1 $\mu$m i-Si$_{0.7}$Ge$_{0.3}$ layer and 67 nm Si$_{0.7}$Ge$_{0.3}$ layer with 17 nm spacer followed by 50 nm Si$_{0.7}$Ge$_{0.3}$ heavily doped with As. A 4 nm silicon cap layer protects the surface. The electron concentration $n$ and mobility $\mu$ at 1.5 K were $n_0 = 9 \cdot 10^{15}$ m$^{-2}$, $\mu_0 = 8$ m$^2$/V$\cdot$s. The sample resistance was measured using a standard lock-in-technique, with the measuring current being 20 nA at a frequency of 10.6 Hz. The results of measurements weakly depend on the choice of contacts of similar geometry. The results of investigation of longitudinal conductivity in this sample were published in Ref.\textsuperscript{2},\textsuperscript{3}.

Figure 1 shows the transversal Hall resistivity $\rho_{xy}$ measured in units of $h/e^2 = 25.8$ k$\Omega$ at different temperatures 0.2, 0.6 and 1.2 K as a function of filling factor $\nu = n_0 h/eB$. The enhanced view of the overshoots near the vicinity of $\nu = 3$ is shown in the inserts. The maximal amplitude and sharpness of the overshoot occur at lowest temperatures (0.05–0.2 K), while with increasing $T$ above 0.2 K the overshoot is smeared gradually. Overshoot is observed at almost all $\nu$, with the exception of $\nu = 2$ and 4, where overshoot is not observed at all temperatures; for $\nu = 5$ and 7, it is feebly marked only at intermediate temperatures.

In Ref.\textsuperscript{\textsuperscript{2}}, it was reported that in a Si/Si$_{1-x}$Ge$_x$ in tilted magnetic fields, the overshoot at odd filling factor $\nu = 3$ increases significantly at a tilting angle of around 69°. This was interpreted as being a consequence of the

![Graph showing transverse Hall resistivity $\rho_{xy}$ in units of $h/e^2$ as a function of filling factor $\nu$ measured at $T = 0.2$, 0.6 and 1.2 K. The insert shows the enhanced view of the overshoot for $\nu = 3$.](image-url)
exchange enhancement of spin- and valley-splitting attributed to band crossing of the cyclotron and valley-split LB (proportional to the perpendicular component $B_\perp$) and spin-split LB (proportional to the total field $B$).

Our measurements show that an increase of overshoot in tilted magnetic fields can also be observed for even filling factors $\nu = 8$ and $10$ (Fig. 2), where the origin of the adjacent LBs splitting differs in principle from the case $\nu = 3$.

Insert in Figure 2 shows the overshoot near $\nu = 3$ at $T = 0.33$ K measured for opposite directions of the perpendicular magnetic field (marked as $B^+$ and $B^-$) and in the case when $B$ is tuned up and down (solid and open circles). The coexistence of the curves as the magnetic field is tuned up and down provides an evidence for the steady-state phenomenon. Changing the direction of $B$ shows that the overshoot is non-erasable after averaging and, therefore, cannot be explained by the admixture of $\sigma_{xx}$ caused by geometrical asymmetry in the sample.

In Ref. [5], it has been shown that in two-dimensional SiGe hole gas, overshoots in $\rho_{xy}$ may occur as a result of oscillations described by semi-classical theory and can be related to the oscillations of longitudinal conductivity $\Delta \sigma_{xx}$ around the classical Drude conductivity: $\sigma_{xx} = n_0 e \mu_0/(1 + \mu_0^2 B^2)$. Accordingly, the values of $\Delta \sigma_{xy}$ are obtained from oscillations around the classical curve $\sigma_{xy} = \mu_0 B \sigma_{xx}$. In accordance with this model, the amplitude of the resistivity oscillations should be given by $\Delta \rho_{xy} = -\Delta \sigma_{xx}/2\mu_0 B$. Our experimental data in $n$-type SiGe are in disagreement with this relation: calculated $\Delta \rho_{xy}$ are too small in comparison with the measured values which was mentioned also in [5]. This means that the above model cannot explain overshoot in $n$-type SiGe.

Let us compare the experimental curves with and without overshoot. Fig. 3a shows the dependence of the normalized Hall resistivity $\rho^{(4-3)} = R_{xy}(\nu)/(h/e^2)$ for filling factor $\nu$ between $i = 4$ and $3$ where overshoot is clearly visible. This curve is labeled as $\rho^{(3)}$. As an experimental curve without overshoot, we use the dependence of $\rho^{(6-5)}$ between $i = 6$ and $5$. For comparison of these two curves, we replot $\rho^{(6-5)}$ in the interval $\nu$ between $i = 4$ and $3$ using the following scaling procedure: $\rho^{(6-5)} = [(\rho^{(6-5)} - 1/6)/(1/3 - 1/4)/(1/5 - 1/6)] + 1/4$. This curve is labeled as $\rho^{(5)}$. The experimental points of the ratio $\rho^{(i)} = \rho^{(3)}/\rho^{(5)}$ are plotted in Fig. 3a. One can see that the overshoot effect exists in the almost whole interval between plateaus, not only in the region where $\rho_{xy}$ exceeds $h/e^2$, moreover, the maximal value of $A$ is achieved approximately in the middle point between $\nu$ and $\nu + 1$.

Observation of the overshoot for different materials and for different $\nu$, where splitting of the adjacent LBs has different character, hints at the common origin of this effect.

We assume that the overshoot could be caused by existence of the second sort of electrons with lower mobility which is determined at zero and weak magnetic fields. In 3D classical conductivity, parallel contribution of two sorts of carriers with different mobility leads to the overshoot in the measured value of the Hall volt-
energy interval between the delocalized states in the central part of LB and strongly localized states in the tails of the band.

It turns out that the above assumption is useful for understanding of some peculiarities of the overshoot phenomenon. For example, smearing of the overshoot with increase of temperature (Fig. 1) can be explained by temperature-induced expansion of the delocalized states and corresponding narrowing of the energy interval for barely localized states. Broadening of LBs leads to the expansion of the barely localized states and therefore is favorable for the overshoot observation. The broadening of LBs may be caused, for example, by increase of overlapping of the adjacent LBs in tilted magnetic fields (Fig. 2) or by increase of disorder. This may explain the reversible defect-induced appearance of the overshoot after proton irradiation observed in a GaAs/AlGaAs sample. In Fig. 4 the transversal Hall resistivity $\rho_{xy}$ in the investigated n-Si/SiGe heterostructure is compared with a Si-MOSFET sample with close electron concentration $n_0 = 8.6 \cdot 10^{15}$ m$^{-2}$, but lower mobility $\mu_0 = 1$ m$^2$/V·s. One can see that in the case of disordered 2D Si-MOSFET, the overshoot is strongly pronounced.

The next example of the influence of disorder is shown in Fig. 5. In this experiment, p-type Si/SiGe sample was investigated. Concentration of holes and mobility measured at 1.5 K were $p_0 = 1.6 \cdot 10^{15}$ m$^{-2}$ and $\mu_0 = 0.7$ m$^2$/V·s. The transversal resistance $R_{xy}$ of this sample was measured in the dark and after illumination by a red LED (LED current was 100 $\mu$A, duration 5 s). As a result of illumination, additional concentration of carriers was "frozen" and residual hole concentration measured up to $1.8 \cdot 10^{15}$ m$^{-2}$. It is known that increase of concentration of "frozen" carriers leads to the more effective screening of the random potential relief and therefore decreases disorder. Fig. 5 shows $R_{xy}(\nu)$ in the vicinity of $\nu = 3$ in dark (solid line) and after illumination (dashed line) measured at $T = 300$ mK. It is seen that decrease of disorder results in decrease of the overshoot.

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