Research Article

Application of Vibration Signals for the Quantitative Analysis of the Optimal Threshold of Bearing Failure

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This study established the prognostics and health management system for bearing failure. The vibration signals measured during the bearing operation were used for prognostics. First, the time-domain signal of vibration was calculated through generalized fractal dimensions, and the relationship diagram of generalized fractal dimensions and time was obtained. Then, the trend of bearing failure was compared by the GFDal results. However, the results can only be used for qualitative feature extraction. The bearing failure at the beginning cannot be determined by qualitative methods. Therefore, this study further converted the calculation results of GFDs into a Gauss distribution curve based on the statistical method under normal operation of the bearing. The Gauss distribution curve of the bearing under normal operation and at different time was overlapped. The overlap rate of the bearing area under different times was calculated. The minimum value was taken as the diagnostic standard, which was the optimal threshold of bearing failure defined in this study and was used as the quantitative basis for bearing failure. Therefore, the comparison of the area overlap rate under the Gauss distribution curve between the normal bearing and the bearing under test could provide diagnosis to the bearing failure. Moreover, the time point of the initial failure of the bearing could also be estimated based on the optimal failure threshold.

1. Introduction

In industrial machinery, the rotational behavior is a frequent motion. The rotation may induce aging, wear even cracking of parts and materials due to vibration or fatigue, thereby influencing the service life and production scheduling of machines. Moreover, it is likely to form a part failure, damaging multiple mechanisms in succession. Therefore, providing preventive maintenance or timely renewal of parts, such as bearings, can avoid faults or long-term negligence causing heavy losses of the production units.

There are four types of maintenance methods for general mechanical equipment after a period of operation [1]. One of the methods is predictive maintenance with prognostic diagnostic tools, as shown in Figure 1. Four quadrants are established on the parts of a machine, where the repair time is the X-axis, the frequency of failure is the Y-axis, and each quadrant represents a different processing mode, as shown in Figure 2. As the bearing assembly used in this study requires a long maintenance time (including assembly and disassembly time of other components), as well as arrival time of parts, but the frequency of failure is not very high, it is suitable to adopt the prognostic maintenance mode.

In recent years, universal and multiple smart solutions have been developed for fault diagnosis [2–4], such as the numerical analysis [5] and machine learning [6, 7], which have been discussed and analyzed in many studies. However, these detection methods have some defects. First, the experiment data are unbalanced, and the measured data excessively concentrate on normal operation of bearing. As a result, it is unlikely to obtain the data from normal operation to stop due to failure. Even if the long-term monitoring data can be obtained, the key is to determine the time point of initial failure of bearing. In terms of model prediction of
many machine learning algorithms [8–10], the bivariate analysis of normal or failure of calculation samples must be specified before constructing an effective model. Based on the abovementioned reasons, this study proposed to obtain the classification criteria for judging normal and damaged bearings and determine the optimal (best) classification threshold to construct a reliable bearing prediction model.

The present diagnostic methods for bearing failure include conventional signal analysis method and machine supervised learning algorithm. The method used in this paper and prior references are discussed and compared below. In 2018, Li et al. [11] used short-time Fourier transform (STFT) to determine the time-frequency signal of simulation tests and divided the data into training and validation data. Finally, the training data were imported into convolutional neural network (CNN) for learning and the parameter model of CNN was updated to judge the validation data. However, their method did not use real measured signals for training. In 2020, Pichler et al. [12] used vibration signals to accelerate bearing failures through the accelerated lifetime (ALT) test platform. The performance of bearing was degraded to diagnose faults, but this was different from the long running data of bearing. In 2018, Huang et al. [13] used the optimization algorithm to extract instantaneous fault characteristic frequency (IFCF) from time-frequency representation (TFR) and to extract multiple T-F curves from TFR. The bearing fault could be detected and diagnosed at unknown time-dependent speed without resampling. In 2019, Glowacz [14] used acoustic measurement for electric coffee machine bearing fault diagnosis, diagnosed normal, minor damage, and moderate damage conditions. He used the method of selection of amplitudes of frequency ratio of 24% multiexpanded filter 8 Hz and k-means clustering for detection, but the background noise was not further solved. In 2019, Glowacz et al. [15] used vibration signals for signal processing, inspected the rotor of motor, and used the amplitude of vibration signal frequency for feature selection. The classification methods were nearest neighbor (NN), linear discriminant analysis (LDA), and linear support vector machine (LSVM). The analysis result was in the range of 97.61%–100%. Although results were satisfactory, there was no long-term monitoring data for analysis, which was different from the condition of practical application to some extent. In 2018, Yuan et al. [16] proposed an adaptive high-order local projection denoising method, which could reduce noise effectively. The high-order polynomial was used to estimate the centroid of neighboring region for diagnosis. The numerical simulation was used with the vibration signal model of faulted rolling bearing. The method could extract the characteristic frequency of simulated signal, and it was validated by experiment data. In 2018, Yuan et al. [17] used HHT time-frequency analysis and CNN for diagnosing the type and severity of bearing fault. They used HHT to transform the time signal of vibration into time-frequency analysis and then used CNN to establish fault sensitive features in these time-frequency charts and to perform fault classification. The bearing experiment validated the effectiveness and efficiency of the method. To sum up, these methods could diagnose the faults in rolling bearing, but these methods have some defects. First, the major problem in noise signal processing is the influence of background noise, and the differences in measurement environment will induce difficulties and uncertainties in feature extraction. Second, it is difficult to obtain long-term monitoring signals of bearing, especially the data from normal condition to failed operation, so that the signal analysis is likely to be incomplete. In some studies, the data for building diagnostic models were predicted according to the measured data of normal or severe damage, and the data of initial damage were not provided, leading to incorrect identification of initial damage to bearing.

Another kind of diagnosis is based on supervised machine learning algorithm. It characterizes and labels the data in the training process, and then the data are classified. The binary classification is used in general, for example, the damage condition is “yes/no” or the sample inspection result is “positive/negative.” In 2018, Yan and Jia [18] used a novel optimized SVM with multidomain feature multidomain classification algorithm for diagnosis and used three
methods, including statistical analysis, FFT, and VMD, to extract the fault features. The fault feature information was extracted from time domain, frequency domain, and time-frequency domain. Finally, the identification of rolling bearing multiple fault state was validated by the particle swarm optimization-based support vector machine (PSO-SVM), different from conventional signal analysis method. In 2020, Zhao et al. [19] used a normalized convolutional neural network to diagnose different fault severities as a few measured failure data resulted in unbalanced diagnosis and proposed a special model structure based on unbalanced information data, which was combined with exponential moving average technology to optimize the overall performance of the proposed model. This method uses the data under normal operating condition for diagnosis. Although it improved the precision, the measured data of the slightly damaged bearing were not provided. In 2019 [20], Xu et al. proposed the theoretical method of deep learning, which was combined with deep convolutional neural network (CNN) and random forest ensemble learning (RT). First, the time-domain vibration signal was transformed by continuous wavelet transform (CWT) into time-frequency gray-scale 2D image. Then, the CNN model was used to extract the multilevel feature of fault detection from the image. The multilevel feature can diagnose bearing fault through multiple RF classifiers. This method is better than traditional method and standard deep learning method, but it cannot specify the diagnostic standard of initial damage in the experiment data. In 2019 [21], Dai et al. suggested that the vibration signals contained a lot of unauthentic diagnostic features; thus, they used bandpass filter to process the original signals and used demodulation technology to obtain the fault feature. Their proposed method for diagnosing bearing fault used the kurtosis of vibration signal as the failure information index. In 2018, Hoang and Kang [22] proposed the convolutional neural network, in which the measured vibration signal data were substituted in calculation to perform automatic fault diagnosis without extracting features. However, the process is too simple, and it is very important to select the hyperparameters of the CNN model, so selecting appropriate hyperparameters is relatively difficult for diagnosis. In 2020, Sun et al. [23] proposed a rapid diagnostic model of Lévy moth-flame optimization (LMFO) and Naive Bayes algorithm, which was combined with the recognition method of conventional model. Their model could overcome the difficulty in selecting classifier parameters in small sample classification. The feature extraction of ensemble empirical mode decomposition (EEMD) can effectively reduce the complexity of background noise. Moreover, the feature selection approach of LMFO can effectively reduce the calculation dimensions, as well as improve the accuracy of classifier, enhancing the practicability of fault diagnosis.

The machine learning method has remedied many defects in traditional analysis tools, and many specific feasible programs are proposed for implementing bearing intelligentization. However, if the supervised learning algorithm is used for modeling, the foremost meaning is to confirm the label of sample. In other words, the normal or damaged class of sample must be confirmed, and the samples of the two classes shall be provided, so as to enhance the usability of model. Therefore, the feature extraction is attained by the calculation of GFDs in this paper, and the qualitative analysis of bearing with relatively severe damage can be obtained from the calculation results of GFDs. The result of GFDs is statistically transformed into Gauss distribution curve. The normal bearing signal of initial operation is transformed into Gauss distribution curve as base, and the Gauss distribution curves at different times are obtained continuously, overlapping the base to obtain the overlap rate of area. All the area overlap rates of normal bearing were determined and compared to obtain the minimum value of area overlap as the optimal diagnostic threshold. This process is the calculation procedure for optimal threshold of bearing failure. This threshold is used as the diagnostic standard of bearing failure. The continuous signals from normal operation to failure are diagnosed to determine the time of initial bearing failure, so as to classify and mark the failure of all signals. The data of initial failure are added in the calculated samples to meet the requirements of two classes, reducing imbalanced data problem. The diagnostic method proposed in this paper has important significance and practicability for equipment maintenance. The production line shutdown maintenance requires certain scheduling, and it does not influence the operation of production line. Therefore, the intelligent health management system for diagnosing bearing failure developed in this study is of value and can provide health prognostics of important components of mechanical bearing. It is the key technology for operation and maintenance of equipment manufacturers and users in the future.

2. Introduction to the Theory

2.1. Generalized Fractal Dimensions. In this study, the fractal dimensions were calculated based on the fractal theory in order to determine the abnormal fractal dimensions rule and normal fractal dimensions rule. The results were compared for further diagnosis. The fractal dimensions computing procedures are defined as follows:

The generalized fractal dimensions (GFDs) of the multifractal theory can be used to solve many problems in a nonlinear system. The process from normal condition to damage and failure due to serious damage of bearing measurement signals is nonlinear signal processing. First, the fractal data are put in a plane lattice with different divisions, and different grids generate different weighted factors. The average probability in each grid is taken as a weight. This weighted probability is used to calculate the two-dimensional GFDs, which is known as the box dimension. Dimension $\alpha$ is called the holder exponent [24, 25].

If the abovementioned $\alpha$ (holder exponent) is regarded as a nonmean object, i.e., the same object but doped with different proportions of materials, there should be different holder exponents for calculation. Thus, function $F(q, \alpha)$ is required to describe different materials, which is expressed as follows:
\[
F(q, \varepsilon) = \sum_{i=1}^{n} p_i^q = \varepsilon^{\alpha(q)},
\]
where \(\alpha(q)\) can be expressed as the following equation:

\[
\alpha(q) = \lim_{\varepsilon \to \infty} \frac{\log \sum_{i=1}^{n} (P_i)^q}{\log \varepsilon}.
\]

The generalization of \(\alpha(q)\) and the dimension of scale index \(q\), as defined by Renyi’s entropy in 1959 [26], are used to replace the numerator of \(\alpha(q)\) in the following equation:

\[
I(q) = \frac{-1}{q-1} \log \sum_{i=1}^{n} (P_i)^q.
\]

If the signal set is covered with boxes sized \(\varepsilon\) and the number of these small boxes is \(n\), then \(P_i(\varepsilon)\) represents the probability of a time-domain pressure signal occurring in the \(i\)-th box, and the holder exponent \(\alpha(q)\) is not a constant value, but varies with the \(q\) value equation (2) which can be changed to the following equation:

\[
D(q) = -\lim_{\varepsilon \to \infty} \frac{\log n}{\log \varepsilon} - \lim_{\varepsilon \to \infty} \frac{\log \sum_{i=1}^{n} P_i}{\log \varepsilon}.
\]

\[
D(0) = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i^q = -\lim_{\varepsilon \to \infty} \frac{\log \sum_{i=1}^{n} P_i^0}{\log \varepsilon}.
\]

As equation (5) is \(P_i\) to the power of zero, all the values of \(P_i\) to the power of zero are 1, and only self-similarity is considered without weights or different probabilities. Thus,

\[
D(1) = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i^q - 1 = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i^{q-1} = \lim_{\varepsilon \to \infty} \log \sum_{i=1}^{n} P_i \log P_i.
\]

When \(q = 2\), equation (4) can be changed to

\[
D(2) = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i^q = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i = \lim_{\varepsilon \to \infty} \frac{1}{q-1} \log \sum_{i=1}^{n} P_i^2.
\]

According to equation (7), \(P_i^2 = P_i \times P_i\), calculates the correlation between two weights, which is mainly for describing the natural fractal of an irregular discrete body and can analyze the closeness relation between components, known as the correlation dimension [27, 28].

While different GFDs can be obtained from equation (4), such as \(D(3), D(4), D(5), \ldots\), the amount of calculation increases with the \(q\) value, and when the \(q\) value reaches a certain magnitude, the benefit ratio may not be very high.

In the computing process of this value, \(P_i\) is the calculation result of the probability \(P_{uv}\). First, the signal is converted into time-dependent function \(X(t)\), where \(T\) is the sampling time of measurement, \(\Delta t\) is the sampling time, and \(y\) is the sampling point, where \(y = T/\Delta t\). Thus, the signal can be expressed as \(X(i)\), \(i = 1, 2, 3, \ldots, y\). As shown in Figure 3, \(\varepsilon\) is the width of mesh segmentation, and the subscript of \(\varepsilon, j\) denotes the width of the position. Thus, the quantity of the column and row is \(C_j = T/\varepsilon, j = 1, 2, 3, \ldots, J\), where \(u\) is

\[
\text{the } u\text{-th column}, \nu \text{ is the } \nu\text{-th row}, \text{ and } J \text{ is the type of mesh segmentation. Figure 4 shows the mesh division of a mesh coordinate, where } E_{uv} \text{ denotes the set of all } P_i. \text{ Thus, the calculation of the probability } P_{uv} \text{ of the mesh segmentation is defined as follows [29]:}
\]

\[
P_{uv}(\varepsilon_i) = \frac{E_{uv}}{\sum_{i=1}^{J} \sum_{j=1}^{C_j} E_{uv} - \sum_{j=1}^{C_j} E_{uv}/y}.
\]

2.2. Gaussian Distribution Curve. The GFDs used in this study can be used for qualitative analysis of damage diagnosis, thus identifying the trend of damage. However, the critical point of bearing failure cannot be defined. Thus, an optimal failure criterion, meaning the threshold of damage, should be determined to reach quantitative criteria. If it is higher than this value, the bearing is in normal range, and vice versa. Therefore, the calculated GFDs are transformed into Gauss distribution, also known as normal distribution, to determine the standard value. Two important factors determining Gauss distribution are the mean value and the standard deviation, where the mean value determines the center of the curve, and the standard deviation determines
The foremost physical significance of the Gauss curve is its laterally symmetric shape, meaning the curve is bell shaped, and such a distribution curve is a perfect distribution curve. While the experimental data or calculation results may not belong to this perfect theoretical model, it quite approaches this model, and the data are assumed to be of normal distribution in the course of analysis. This paper uses the theoretical characteristics of a normal curve for diagnostic analysis. The mean value of the normal curve determines the intermediate point, and the standard deviation determines the amplitude of distribution. The coordinate graph is drawn using the horizontal axis as the variable value, and the vertical axis is the probability of occurrence (times) [31] in the Gauss distribution curve. The mean value determines the symmetrical position of Gauss distribution, and the standard deviation determines the type of amplitude. The discrete normal distribution can be expressed as follows:

\[ f(x_i) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}, \quad -\infty < x_i < \infty, \quad (11) \]

where \( \pi = 3.14159, \) \( e = 2.17828, \) \( \bar{x} = \) sample mean value of variable \( x_i, \) and \( s = \) sample standard deviation of variable \( x_i. \)

### 3. Research Process and Experimental Case

#### 3.1. Research Structure

This paper builds a vibration characteristic estimation model for bearings in operation and uses this model to estimate the service behavior of the bearings. The vibration signal data of the bearings are derived from the actual experiment, and the analysis process is shown in Figure 5:

The detailed calculation steps of bearing failure diagnosis are as follows:

Step 1. The GFDs of vibration signal measured are calculated, and the relationship between GFDs and time is plotted.

Step 2. GFDs of different time are compared for qualitative analysis.

Step 3. The GFDs calculation results of bearings no. 2, no. 3, and no. 4 at different time are transformed into the Gauss distribution curve.

Step 4. The Gauss distribution curve calculated in Step 3 for each bearing is taken to overlap each other to calculate the area overlap rate. The area overlap rate of each bearing at different time can thus be obtained.

Step 5. The area overlap rate of each bearing is compared, and the minimum value of the area overlap rate is taken as the optimal threshold of bearing failure and the standard for bearing failure diagnosis.

Step 6. Based on the tested bearing signals, the area overlap rate is calculated by the calculation method from Step 1 to Step 4.

Step 7. The results of Step 6 are compared with the diagnostic criteria of Step 5.
3.2. Experimental Structure and Data. The test case of bearing failure is taken from the experiment data, as obtained by the NSFI/UCR intelligent maintenance system center [32] of the Rexnord company in Milwaukee, Wisconsin. The test layout for this bearing is shown in Figure 6. As seen, four double groove ball bearings of the Rexnord model ZA-2115 are installed on the shaft, which are given forced lubrication under load and during rotation. The speed is fixed at 2000 RPM, and an axial load of 6000 lbs is applied to the spring mechanism above the bearings and shaft.

A set of uniaxial accelerometers is affixed to each bearing and stuck on the bearing shell. The accelerometer is a PCB 353B33 high sensitivity-ICP sensor, and the data are collected by NI_6062E data acquisition card. The sampling frequency of data acquisition is 20 kHz. The test continues until a bearing fails, and the vibration pressure signal acquisition stops.

The recorded time for calculation is 10:32:39 February 12 to 06:22:39 February 19. The data are captured every 10 minutes and recorded one second each time. Each data has 20480 points, and there are 984 recorded files. Each data has four columns of data, where the first column of data represents the accelerometer measurement data of bearing no. 1, and so forth. The four accelerometers corresponding to the four bearings are shown in Table 1. This experiment stops when the outer ring of bearing 1 fails. Bearings no. 2, no. 3, and no. 4 are not damaged.

4. Experimental Comparison and Analysis

This section is divided into qualitative and quantitative parts for analysis and comparison. The GFDs can be used to distinguish normal bearings from severely damaged bearings, but the bearings with no serious damage cannot be distinguished. Thus, qualitative analysis can only be conducted on the damaging trend. In order to obtain a more accurate diagnosis, this paper further uses the Gauss distribution curve area overlap rate to find out the reference value for quantitative analysis and accurately defines the criteria for damage. First, the GFDs are calculated after taking out the normal bearing for a period of time, and then the calculated results are transformed into the Gauss distribution curve. If the Gauss distribution curves are overlapped at different time and in normal operation, the area overlap rate of the curves will be very close, because there is a high similarity between them in normal operation. Therefore, in this paper, the area overlap rate under the Gauss distribution curve is calculated based on the normal bearing signals of nos. 2, 3, and 4, and the size of the area overlap rate is compared. The minimum value is taken as the optimal threshold of bearing failure and the quantitative judgment standard for bearing failure. Finally, bearing no. 1 is taken out for comparison and the rationality of the optimal threshold was observed.

4.1. Experimental Qualitative Analysis Results. The vibration signal measurement results are shown in Figures 7 and 8. Figure 7 shows the normal bearing, and Figure 8 shows the bearing vibration signals when damage occurs. As it is difficult to identify bearing failure according to the vibration measurement signals, this study performs 1 kHz filter signal processing for the bearing vibration signals, which are converted into dB values (reference value is 1\times10^{-6} Pa), and the GFDs of all bearings are obtained. The X-axis denotes time, the Y-axis denotes the magnitude of GFD, and the results are shown in Figure 9. The calculated value of the GFDs of bearing no. 1 was extracted. The results of three different intervals are also extracted, as shown in Figure 10. As seen, when the bearing works in normal condition, as shown by blue line (no symbol) and red line (circle symbol) in the figure, the GFDs are obviously overlapped, rather than separated apart. In addition, according to the blue line (no symbol) and green line (square symbol) in Figure 10, the normal bearing is apparently different from the abnormal bearing, and the two curves are obviously separated from each other. While it is observed that the bearing has been damaged, this result is only available for general qualitative analysis, meaning whether the trends are obviously separated can be observed. However, there is no specific definition standard. Thus, it is likely to form an ambiguous zone, and the errors cannot be distinguished or judged. Therefore, this paper further defines the optimal damage threshold, namely, the definition standard, as the basis of quantitative analysis.
4.2. *Experimental Quantitative Analysis Results*. This paper uses the Gauss distribution curve to calculate the optimal damage threshold. First, the calculation results of the GFDs of bearing no. 1 are extracted and the GFDs of front-end are in normal mode. Every 60 data are considered one unit, which are transformed into the Gauss distribution curve. The two sections of signals are stacked, and the Gauss distribution overlap of three sections is calculated, as shown in Figure 11. The measured signals of bearing no. 1 start from a normal bearing to severe damage, as shown in

**Figure 6**: Experimental platform and accelerometer arrangement plan [33].

**Table 1**: Bearing number and accelerometer number.

| Measured data channel | Bearing number | Accelerometer number |
|-----------------------|----------------|---------------------|
| Channel 1             | Bearing 1     | No. 1               |
| Channel 2             | Bearing 2     | No. 2               |
| Channel 3             | Bearing 3     | No. 3               |
| Channel 4             | Bearing 4     | No. 4               |

**Figure 7**: Vibration signals of normal bearing.

**Figure 8**: Vibration signals of abnormal bearing.

**Figure 9**: Temporal distribution of GFDs of different bearings.
Figures 11(a)–11(c). The overlap of the Gauss distribution curve decreases from high overlap to low overlap, and eventually slight overlapping occur. This indicates that the normal bearing has very similar GFDs in operation. When the bearing is damaged, the Gauss distribution curve is apparently different. According to the mean value of Gauss distribution, when the bearing failure is worse, the mean value is larger, which shifts right. In other words, the value of GFDs is increased when the bearing is damaged, and the condition of the bearing has become more complicated. Therefore, based on this principle, the signals of the undamaged bearings no. 2, no. 3, and no. 4 are used to make the time-dependent graphs of the Gauss distribution overlap of GFDs, as shown in Figure 12, respectively. When bearing no. 1 is damaged, the measurement is stopped to avoid the serious failure of bearing no. 1 indirectly influencing the measurement signals of normal bearings no. 2, no. 3, and no. 4. However, bearings no. 2, no. 3, and no. 4 may have been damaged slightly, leading to inaccuracy in estimating the optimal overlap. Thus, the overlap is calculated up to the 140th hour, and the value in the last period is disregarded to avoid misrecognition. The overlap values in Figure 12 are compiled into Table 2. The minimum overlap is 77.86%, which is the minimum Gauss distribution overlap converted from different bearings in normal bearing operation. The optimal damage threshold is defined as 77.68%. When the overlap of the bearing is lower than this threshold, the bearing has been preliminarily damaged; if the overlap is much lower than this threshold, the bearing failure is considered serious; when the overlap is higher than this threshold, the bearing state is considered normal.

Finally, according to the result of bearing no. 1, the overlaps of the time-dependent distribution of bearings no. 2, no. 3, and no. 4 are overlapped, as shown in Figure 13. The minimum five overlaps of bearings no. 2 to no. 4 are compiled in Table 2. As seen, the minimum threshold is 77.86%, and this value is called the optimal damage threshold, which is the standard for judging whether the bearing is damaged or not. The obtained optimal damage threshold result is validated by using bearing no. 1, as shown in Figure 13. First, the front-end time of bearing no. 1 in the overall measurement process, namely, the normal part, is completely higher than the calculated optimal damage threshold. The bearing is shown normal. The damaged part of bearing no. 1 in the back-end is completely lower than the calculated optimal damage threshold. This predicted damage threshold is shown applicable, and the feasibility and accuracy of this quantitative method are validated. The time point of the initial bearing failure can be predicted in Figure 13. When the overlap is exactly lower than 77.86%, at the 100th hour in operation, the time point of the initial bearing failure can be estimated using this optimal damage threshold. This result can provide a reference for manufacturers to evaluate the time point to change the bearing, as well as for arranging a shutdown overhaul of production lines.

5. Discussion

This study used time-domain fractal dimensions and the Gauss distribution curve overlap to build a feasible bearing failure estimation model. The feasibility of this diagnostic method of feature extraction was proved, and the actual measurement cases of the bearings were used for diagnostic evaluation.

The present research findings are discussed as follows:

(1) Based on the vibration time signal obtained in the experiment, the results were calculated by the GFDs. In the GFDs diagram, the trend of normal and damaged bearings can be clearly distinguished, which can be used as the basis for judging the bearing with serious damage. However, it cannot provide accurate judgment for the bearing at the beginning of
damage. Therefore, only qualitative analysis can be performed, and quantitative diagnostic criteria cannot be provided.

(2) Therefore, to obtain accurate criteria, this study converted the GFD values calculated for bearings no. 2, no. 3, and no. 4 at different time into Gauss distribution curves and compared the area overlap rates of different bearings at different time. At the same time, the area overlap rate of different bearings at different time was compared. The minimum area overlap rate was taken as the optimal threshold for bearing diagnosis and the judgment benchmark for bearing failure, so as to establish the estimated model of bearing failure.

(3) When the area overlap rate of the bearing under test is smaller than the optimal threshold of bearing failure and the difference is larger, the bearing failure is more serious.

(4) When the area overlap rate is higher than the optimal threshold of bearing failure, it is similar to the normal operation of the bearing, so the bearing is in the normal operation. On the contrary, when the area overlap rate is lower than the optimal threshold, it shows a significant difference from the normal bearing operation, indicating that the bearing has been damaged.

(5) Based on the area overlap rate of bearings no. 2, no. 3, and no. 4 at different time, the optimal threshold of bearing failure was obtained when the area overlap rate is 77.86%. This threshold can be used to estimate the occurrence of slight damage of bearing no. 1 about 100 hours after the start of the operation. This
result is of great significance to the estimation of the residual life of bearings.

6. Conclusion

In this study, a reliable diagnostic method for the damage to double channel ball bearing was proposed. The vibration signals were measured in actual operation of bearing to calculate the GFDs and time variation. The apparently damaged bearing was identified, and accurate qualitative analysis of relatively severe damage was obtained, but the initial damage to bearing cannot be diagnosed specifically. Therefore, this study used normal bearing and abnormal bearing to determine the Gauss curve area overlap and the standard threshold of damage for quantized improvement and as the criteria of initial damage. The very moment of initial
damage could be found by using the obtained the standard threshold of damage. The method proposed in this paper can propose a reasonable judgment base specifically, the data of initial damage can be increased when building the diagnostic model, ensuring the feasibility of diagnostic model and solving the imbalanced data problem. In addition, the users can give an early warning at the initial stage of bearing failure and arrange the schedule of breakdown maintenance, so as to attain the goal for bearing intelligentization. The method proposed in this paper is the model for double channel ball bearing, and different standard models can be rebuilt for different bearing types.

**Data Availability**

The data used to support the findings of this study are available in [32].

**Conflicts of Interest**

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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