Decays of $Z_b^+$ and $Z'_b^+$ as hadronic molecules

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Abstract. The two newly observed hidden-bottom mesons $Z_b^+(10610)$ and $Z'_b^+(10650)$ with quantum numbers $J^P = 1^+$ are considered as hadronic molecules composed of $B\bar{B}^*$ and $B^*\bar{B}$, respectively. We give predictions for the widths of the strong two-body decays $Z_b^+ \to \Upsilon(nS) + \pi^+$ and $Z'_b^+ \to \Upsilon(nS) + \pi^+$ in a phenomenological Lagrangian approach.

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1. Introduction

Recently, two hidden-bottom charged meson resonances were observed by the Belle Collaboration [1] as two narrow resonance structures in the $\pi^+\Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^+h_b(mP)$ ($m = 1, 2$) mass spectra. They are produced in association with a single charged pion in $\Upsilon(5S)$ decays with the following values of mass and width:

- $M[Z_b^+(1061)] = 10608.4 \pm 2.0$ MeV, $\Gamma[Z_b^+(10610)] = 15.6 \pm 2.5$ MeV, $M[Z_b'^+(10650)] = 10653.2 \pm 1.5$ MeV, $\Gamma[Z_b'^+(10650)] = 14.4 \pm 3.2$ MeV. Analyses of the charged pion angular distributions favor the $I^G(J^P) = 1^+(1^+)$ quantum numbers of the $Z$-states [1].

Theoretical structure assignments for these hidden-bottom meson resonances were proposed immediately after their observation [2]–[10], mainly based on molecular [2] and tetra-quark interpretations [9, 10] using the analogy to the charm sector. Also, in [5] the new resonances were identified as a hadro-quarkonium system based on the channel coupling of light and heavy quarkonia to intermediate open-flavor heavy-light mesons.

In this paper we analyze the two-body strong decays $\Upsilon(nS)\pi^+$ of $Z_b^+$ and $Z_b'^+$ using a phenomenological Lagrangian approach developed in Refs. [11]–[16] which is based on the compositeness condition [17, 18]. In particular, in [11]–[16] recently observed unusual hadron states (like $D_{s0}^*(2317)$, $D_{s1}(2460)$, $X(3872)$, $Y(3940)$, $Y(4140)$, $Z(4430)$, $\Lambda_c(2940)$, $\Sigma_c(2800)$) were analyzed within the structure assumption as hadronic molecules. The compositeness condition implies that the renormalization constant of the hadron wave function is set equal to zero or that the hadron exists as a bound state of its constituents. It was originally applied to the study of the deuteron as a bound state of proton and neutron [17] (see also Ref. [12] for a further application of this approach to the case of the deuteron). Then it was extensively used in low–energy hadron phenomenology as the master equation for the treatment of mesons and baryons as bound states of light and heavy constituent quarks (see e.g. Refs. [18, 19, 20]).

By constructing a phenomenological Lagrangian including the couplings of the bound state to its constituents and the constituents to other final state particles, we evaluated meson–loop diagrams which describe the different decay modes of the molecular states (see details in [11]).

In the present report we proceed as follows. In Sec. II we briefly review the basic ideas of our approach. We now consider the two new resonances $Z_b^+$ and $Z_b'^+$ as the two molecular states of $\bar{B}B^*$ and $\bar{B}^*B^*$. Then we proceed to estimate their strong two-body decays $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$ where $n = 1, 2, 3$ based on an phenomenological Lagrangian approach. In Sec. III we present our numerical results and a short summary is given in Sec. IV.

2. Phenomenological Lagrangian approach

Here we briefly discuss the formalism for the study of the composite (molecular) structure of the $Z_b^+$ and $Z_b'^+$ resonances. In the following calculation we adopt the spin and parity quantum numbers $J^P = 1^+$ for the two resonances $Z_b^+$ and $Z_b'^+$. We
consider these two new charged hidden-bottom meson resonances as a superposition of 

molecular states of $\bar{B}B^*$ and $\bar{B}^*B^*$ as

$$|Z_b^+(10610)\rangle = \frac{1}{\sqrt{2}} \left| B^{*+}\bar{B}^0 + \bar{B}^{*0}B^+ \rightangle,$$

$$|Z_b^+(10650)\rangle = |B^{*+}B^{*0}\rangle.$$  \hfill (1)

Our approach is based on an interaction Lagrangian describing the coupling of the $Z_b^+$ 
(or $Z_b^+$) to its constituents. The simplest forms of such Lagrangians read

$$\mathcal{L}_{Z_b}(x) = \frac{g_{Z_b}}{\sqrt{2}} M_{Z_b} Z_b^\mu(x) \int d^4y \Phi_{Z_b}(y^2) \left( B(x + y/2) \bar{B}^*_{\mu}(x - y/2) ight),$$

$$\mathcal{L}_{Z_b'}(x) = \frac{g_{Z_b'}}{\sqrt{2}} \epsilon_{\mu\nu\alpha\beta} \bar{B}^*(x - y/2) \bar{B}^*_{\mu\nu}(x + y/2) \bar{B}^*_{\alpha\beta}(x - y/2),$$ \hfill (2)

where $y$ is a relative Jacobi coordinate, $g_{Z_b}$ and $g_{Z_b'}$ are the dimensionless 

coupling constants of $Z_b^+$ and $Z_b^+$ to the molecular $\bar{B}B^*$ and $\bar{B}^*B^*$ components, respectively. 

Here $\Phi_{Z_b}(y^2)$ and $\Phi_{Z_b'}(y^2)$ are correlation functions, which describe the distributions 

of the constituent mesons in the bound states. A basic requirement for the choice of 

an explicit form of the correlation function $\Phi_H(y^2)$ ($H = Z_b, Z_b'$) is that its Fourier 

transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render 

the Feynman diagrams ultraviolet finite. We adopt a Gaussian form for the correlation 

function. The Fourier transform of this vertex function is given by

$$\tilde{\Phi}_H(p^2/\Lambda^2) \equiv \exp(-p^2/\Lambda^2),$$ \hfill (4)

where $p_E$ is the Euclidean Jacobi momentum. $\Lambda$ is a size parameter characterizing the 

distribution of the two constituent mesons in the $Z_b^+$ and $Z_b^+$ systems, which also leads to 

a regularization of the ultraviolet divergences in the Feynman diagrams. From our 

previous analyses of the strong two-body decays of the $X, Y, Z$ meson resonances and of the 

$\Lambda_b(2940)$ and $\Sigma_b(2880)$ baryon states we deduced a value of $\Lambda \sim 1$ GeV \hfill \cite{[15]}. \hfill \cite{[14]}

For a very loosely bound system like the $X(3872)$ a size parameter of $\Lambda \sim 0.5$ GeV \hfill \cite{[14]}

is more suitable. The coupling constants $g_{Z_b}$ and $g_{Z_b'}$ are then determined by the compositeness 

condition \hfill \cite{[17] \cite{18} \cite{19} \cite{15} \cite{11}}. \hfill \cite{[14]}

It implies that the renormalization constant of the hadron 

wave function is set equal to zero with:

$$Z_H = 1 - \Sigma_H^2(M_H^2) = 0.$$ \hfill (5)

Here, $\Sigma_H'$ is the derivative of the transverse part of the mass operator $\Sigma_H^{\mu\nu}$ of the 

molecular states (see Fig.1), which is defined as

$$\Sigma_H^{\mu\nu}(p) = g_{H}^{\mu\nu} \Sigma_H(p) + \frac{p^\mu p^\nu}{p^2} \Sigma_H^L(p), \quad g_{H}^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}.$$ \hfill (6)

The compositeness condition \hfill \cite{[15]} \hfill \cite{[14]} gives a constraint on the choice of the free parameter $\Lambda$. 

Analytical expressions for the couplings $g_{Z_b}$ and $g_{Z_b'}$ are given in Appendix A. In the 

calculation the masses of $Z_b$ and $Z_b'$ are expressed in terms of the constituent masses
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\begin{align}
M_{Z_b} &= M_B + M_{B^*} - \epsilon, \quad M_{Z_b'} = 2M_{B^*} - \epsilon. \quad (7)
\end{align}

Here we assume bound states for the $Z_b$ and $Z_b'$. In the calculation of the two-body decays $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$ we include the direct four-point interactions for the $BB^*\Upsilon\pi$ and $B^*B^*\Upsilon\pi$ vertices. The respective phenomenological Lagrangians take the form

\begin{align}
\mathcal{L}_{BB^*\Upsilon\pi}(x) &= g_{BB^*\Upsilon\pi} \Upsilon\mu(x) \bar{B}^\alpha(x) \bar{B}(x) + H.c., \\
\mathcal{L}_{B^*B^*\Upsilon\pi}(x) &= i\epsilon_{\mu\alpha\beta} \left( g_{B^*B^*\Upsilon\pi} \Upsilon\mu(x) \bar{B}^\beta(x) \partial^\nu \bar{\pi}(x) \cdot \bar{\tau} B^\alpha(x) \\
&\quad + f_{B^*B^*\Upsilon\pi} \partial^\nu \Upsilon\mu(x) \bar{B}^\beta(x) \bar{\pi}(x) \cdot \bar{\tau} B^\alpha(x) \right). \quad (9)
\end{align}

The four-particle coupling constants defined in Eqs. (8) and (9) are effective, which also include off-shell effects. Such couplings are obviously different from the strong couplings of molecular states to their constituents which model their composite structure via vertex function distributions. We use effective Lagrangians (using both SU(4) and SU(5) classification schemes) developed by Ko and collaborators [21] which worked phenomenologically quite successfully. Corresponding diagrams contributing to the $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$ processes are shown in Fig.2.

Among the three couplings $g_{BB^*\Upsilon\pi}$, $g_{B^*B^*\Upsilon\pi}$ and $f_{B^*B^*\Upsilon\pi}$ we have the relations [21]

\begin{align}
g_{BB^*\Upsilon\pi} &= \frac{g_{TB\bar{B}} g_{TB\bar{B}^*}}{2\sqrt{2}}, \quad g_{B^*B^*\Upsilon\pi} = f_{B^*B^*\Upsilon\pi} = \frac{g_{BB^*\Upsilon\pi}}{2\sqrt{M_B M_{B^*}}}. \quad (10)
\end{align}

The hadronic couplings $g_{TB\bar{B}}$ and $g_{BB^*\pi}$ are defined as [13]

\begin{align}
\mathcal{L}_{B^*B\pi}(x) &= \frac{g_{B^*B\pi}}{\sqrt{2}} \bar{B}_\mu(x)i\partial^\mu \bar{\pi}(x) \bar{\tau} B(x) + H.c., \\
\mathcal{L}_{TB\bar{B}}(x) &= g_{TB\bar{B}} \Upsilon\mu(x) \bar{B}(x)i\partial^\mu B(x) + H.c. \quad (11)
\end{align}
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The coupling constant $g_{\Upsilon(nS)BB}$ is given by

$$g_{\Upsilon(nS)BB} = \frac{M_{\Upsilon(nS)}}{f_{\Upsilon(nS)}}$$

where $f_{\Upsilon(nS)}$ is determined from the leptonic decays of the $\Upsilon(nS)$ states as

$$\Gamma(\Upsilon(nS) \rightarrow e^+e^-) = \frac{4\pi\alpha_{\text{EM}}^2 f_{\Upsilon(nS)}^2}{27 M_{\Upsilon(nS)}}$$

where $\alpha_{\text{EM}} = 1/137.036$ is the fine-structure constant. The relation (12) is analogue of the $\rho$-meson universality

$$g_{\rho \pi \pi} = \frac{M_{\rho} f_{\rho}^2}{2g_{\rho \gamma}}$$

extended to the bottom sector in Ref. [21], where $g_{\rho \gamma}$ is the $\rho \rightarrow \gamma$ transition coupling. Here For the last couplings we get $f_{\Upsilon(1S)} = 715.2$ MeV, $f_{\Upsilon(2S)} = 497.5$ MeV, $f_{\Upsilon(3S)} = 430.2$ MeV, where we used the mass values $M_{\Upsilon(1S,2S,3S)} = 9460.30 \pm 0.26$ MeV, 10023.26 $\pm$ 0.31 MeV and 10355.2 $\pm$ 0.5 MeV as well as the results for the leptonic decay widths of the $\Upsilon(nS)$ states

$$\Gamma(\Upsilon(1S) \rightarrow e^+e^-) = 1.340 \pm 0.018 \text{ keV},$$
$$\Gamma(\Upsilon(2S) \rightarrow e^+e^-) = 0.612 \pm 0.011 \text{ keV},$$
$$\Gamma(\Upsilon(3S) \rightarrow e^+e^-) = 0.443 \pm 0.008 \text{ keV}.$$ (15)

Note that we explicitly take into account the $M_{\Upsilon(nS)}$ dependence of the $f_{\Upsilon(nS)}$ and $g_{\Upsilon BB}$ couplings.

The coupling $g_{B^*B^\pi}$ can be related to the effective coupling constant $\hat{g} = 0.44 \pm 0.03^{+0.01}_{-0.00}$ determined in a lattice calculation [22]. The relation is

$$g_{B^*B^\pi} = \frac{2\hat{g}}{f_{\pi}} \sqrt{M_B M_{B^*}} \simeq 35.34,$$ (16)

where $f_{\pi} \simeq 132$ MeV is the pion decay constant. We should stress that the phenomenological Lagrangians developed in Refs. [21] were successfully applied to different aspects of heavy flavor physics.

3. Numerical results

With the phenomenological Lagrangians introduced and discussed in Sec. II one can proceed to determine the widths of the two-body decays $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b^{'+} \rightarrow \Upsilon(nS) + \pi^+$ with $n = 1, 2, 3$ (see corresponding diagrams in Fig.2). The corresponding decay widths are given by:

$$\Gamma_{Z_b^+ \rightarrow \Upsilon(nS)\pi^+} \simeq \frac{g_{Z_b^+ \Upsilon(nS)\pi}^2}{16\pi M_{Z_b^+}} \lambda^{1/2}(M_{Z_b^+}^2, M_{\Upsilon(nS)}^2, M_{\pi}^2),$$

$$\Gamma_{Z_b^{'+} \rightarrow \Upsilon(nS)\pi^+} \simeq \frac{g_{Z_b^{'+} \Upsilon(nS)\pi}^2}{16\pi M_{Z_b^{'+}}} \lambda^{1/2}(M_{Z_b^{'+}}^2, M_{\Upsilon(nS)}^2, M_{\pi}^2),$$ (17)
where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen function. The decay coupling constants $g_{Z_b \Upsilon(nS)\pi}$ and $g_{Z_b' \Upsilon(nS)\pi}$ involve the products $g_{Z_b \Upsilon(nS)\pi} = g_{Z_b} g_{BB^* \Upsilon(nS)\pi} J_1$ and $g_{Z_b' \Upsilon(nS)\pi} = g_{Z_b'} g_{B^*B^* \Upsilon(nS)\pi} M_{Z_b'} J_2$ where the loop integrals $J_1$ and $J_2$ are given in Appendix A.

For our numerical evaluation hadron masses are taken from the compilation of the PDG [23]. The only free parameter of our calculation is the dimensional parameter $\Lambda$ entering in the correlation function of Eq. (4). As mentioned before, the parameter $\Lambda$ describes the distributions of $BB^*$ and $B^*B^*$ in the $Z_b^+$ and $Z_b'^+$ bound state systems, respectively. Tables I and II contain our estimates for the decay widths of $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$. We also indicate the values for the couplings $g_{Z_b}$ and $g_{Z_b'}$ as determined from the compositeness condition. We find that the data on the strong $\Upsilon(nS)\pi$ decays of $Z_b$ and $Z_b'$ can be qualitatively described for $\Lambda \simeq 0.5$ GeV. This value is close to the one used for the molecular system of the $X(3872)$ [14]. This also means that the $Z_b$ and $Z_b'$ states are considered as extended molecular states. For transparency we present the results for several choices of the size parameter $\Lambda = 0.4, 0.45, 0.5, 0.55$ GeV and the binding energy $\epsilon$. Note that an increase of $\Lambda$ leads to a larger decay width. Although the dependence of the decay widths on the binding energy is rather moderate, a quantitative prediction for these decays strongly depends on the size of the system. The decay pattern of $Z_b$ and $Z_b'$ to $\Upsilon(nS) + \pi^+$, that is the relative importance of the $\Upsilon(nS) + \pi^+$ decay channels for $n = 1, 2, 3$, can be reproduced and could give further support for the molecular interpretation of these states. One can see from Tables I and II that the rates are increased by a factor 2-2.5 which means that the amplitudes are roughly increased by a factor 1.4-1.6. Latter value is consistent with a growth of the cutoff parameter of 0.55/0.4 = 1.375. In this respect the calculation is consistent and we do not see any disagreement. It is also clear that any model based on cutoff regularization has cutoff-dependent result. Here the cutoff is related to the size of hadronic molecular compound. As done in our previous analyses of other heavy hadron molecules data help to do a fine tuning of the cutoff parameter which is specific for a particular molecular state. There is no a universal cutoff for the whole tower of possible hadronic molecular states. Our previous analyses of molecular states consisting of two heavy mesons (like the $X(3872)$ state) indicate that the cutoff parameter in the vertex functions of $Z_b$ and $Z_b'$ states should be around 0.5 GeV. This is the reason why we do predictions for the value of $\Lambda$ close to 0.5 GeV and are also waiting for more precise data with smaller error bars. Please also note that the ratio of rates is less dependent on the cutoff and hence a stable prediction of the model. Since data are given for the absolute rates we also choose to use this presentation for our results and not the relative rates.
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Table I. $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ decay properties.

| $\epsilon$(MeV) | $g_{Z_b}$ | $\Gamma_{1S}$(MeV) | $\Gamma_{2S}$(MeV) | $\Gamma_{3S}$(MeV) |
|------------------|------------|--------------------|--------------------|--------------------|
| 1                | 3.3, 3.4, 3.6, 3.7 | 11.6, 16.4, 22.3, 29.5 | 13.7, 19.4, 26.4, 34.9 | 7.2, 10.2, 13.9, 18.4 |
| 5                | 4.0, 4.0, 4.1, 4.2 | 11.0, 16.5, 22.4, 29.5 | 13.0, 19.4, 26.3, 34.7 | 6.7, 10.1, 13.6, 18.0 |
| 10               | 5.0, 4.9, 4.8, 4.8 | 11.7, 16.9, 22.7, 29.7 | 13.7, 19.8, 26.6, 34.8 | 7.0, 10.1, 13.5, 17.6 |
| 20               | 7.2, 6.6, 6.3, 6.0 | 13.3, 18.3, 24.0, 30.8 | 15.4, 21.2, 27.8, 35.7 | 7.4, 10.2, 13.4, 17.3 |
| 30               | 9.4, 8.5, 7.9, 7.4 | 14.5, 19.6, 25.4, 32.3 | 16.7, 22.5, 29.2, 37.1 | 7.7, 10.3, 13.4, 17.0 |
| 40               | 11.7, 10.4, 9.5, 8.8 | 15.4, 20.4, 26.7, 33.7 | 17.5, 23.2, 30.3, 38.3 | 7.5, 10.0, 13.1, 16.5 |
| 50               | 13.9, 12.3, 11.1, 10.2 | 15.9, 21.2, 27.6, 34.9 | 17.8, 23.9, 31.1, 39.2 | 7.1, 9.6, 12.5, 15.7 |
| Exp.             |             | 22.9 ± 7.3 ± 2 | 21.1 ± 4.3 ± 2 | 12.2 ± 1.7 ± 4 |

Table II. $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$ decay properties.

| $\epsilon$(MeV) | $g_{Z_b'}$ | $\Gamma_{1S}$(MeV) | $\Gamma_{2S}$(MeV) | $\Gamma_{3S}$(MeV) |
|------------------|------------|--------------------|--------------------|--------------------|
| 1                | 3.2, 3.4, 3.6, 3.7 | 12.0, 16.9, 23.0, 30.4 | 14.7, 20.8, 28.3, 37.4 | 9.0, 12.8, 17.4, 23.0 |
| 5                | 3.9, 4.0, 4.1, 4.2 | 12.1, 17.0, 23.0, 30.3 | 14.9, 20.8, 28.2, 37.2 | 9.0, 12.6, 17.1, 22.6 |
| 10               | 5.0, 4.8, 4.8, 4.7 | 12.7, 17.4, 23.4, 30.6 | 15.4, 21.3, 28.5, 37.3 | 9.2, 12.7, 17.1, 22.3 |
| 20               | 7.2, 6.6, 6.3, 6.0 | 14.0, 18.8, 24.6, 31.7 | 16.9, 22.7, 29.8, 39.3 | 9.8, 13.2, 17.3, 22.3 |
| 30               | 9.4, 8.5, 7.8, 7.3 | 15.1, 20.1, 26.1, 33.1 | 18.1, 24.1, 31.3, 39.7 | 10.2, 13.5, 17.6, 22.3 |
| 40               | 11.7, 10.4, 9.4, 8.7 | 15.8, 21.1, 27.3, 34.5 | 18.9, 25.1, 32.4, 41.0 | 10.2, 13.6, 17.6, 22.2 |
| 50               | 13.9, 12.3, 11.0, 10.1 | 16.2, 21.7, 28.2, 35.6 | 19.1, 25.5, 33.2, 41.9 | 9.9, 13.3, 17.3, 21.8 |
| Exp.             |             | 12 ± 10 ± 3 | 16.4 ± 3.6 ± 4 | 10.9 ± 2.6 ± 4 |

4. Summary

To summarize, we have pursued a hadronic molecular interpretation for the two recently observed hidden-bottom charged mesons $Z_b^+$ and $Z_b'^+$. In our calculation we have used the spin-parity assignment $J^P = 1^+$ for the two resonances, which is currently favored by the experimental decay distributions. We have studied the consequences for their two-body decays $Z_b^+ \rightarrow \Upsilon(nS) + \pi^+$ and $Z_b'^+ \rightarrow \Upsilon(nS) + \pi^+$ within a phenomenological Lagrangian approach. The calculated results for the decay widths (see Tables I and II) are of the order of MeV and for the most part qualitatively consistent with the numbers deduced by the Belle collaboration. Especially the experimental results for $Z_b^+ (Z_b'^+) \rightarrow \Upsilon(nS) + \pi^+$ can be reproduced taking a value for the free size parameter $\Lambda$ near 0.5 GeV.

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Appendix A. Coupling constants and structure integrals

The expressions for the coupling constants $g_{Z_b^+ b}$, $g_{Z'_b^+ b}$ and structure integrals $J_1$, $J_2$ are

\[ g_{Z_b^+ b}^{-2} = \frac{M_{Z_b}^2}{32\pi^2\Lambda^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1^3} \left( \alpha_{12} + 2\alpha_1 \alpha_2 \right) \left( 1 + \frac{\Lambda^2}{2M_{B^*}^2\Delta_1} \right) \times \exp \left\{ -\frac{M_{B^*}^2\alpha_1 + M_{B^*}^2\alpha_2 + M_{Z_b}^2\alpha_{12} + 2\alpha_1 \alpha_2}{\Delta_1} \right\}, \]  

(A.1)

\[ g_{Z'_b^+ b}^{-2} = \frac{M_{Z'_b}^2}{16\pi^2\Lambda^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1^2} \left( \Lambda^2 + \alpha_{12} + 2\alpha_1 \alpha_2 \right) \left( 1 + \frac{\Lambda^2}{M_{B^*}^2\Delta_1} \right) \times \exp \left\{ -\frac{M_{B^*}^2\alpha_1 + M_{B^*}^2\alpha_2 + M_{Z'_b}^2\alpha_{12} + 4\alpha_1 \alpha_2}{\Delta_1} \right\}, \]  

(A.2)

\[ J_1 = \frac{1}{8\pi^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2^2} \left( 1 + \frac{\Lambda^2}{2M_{B^*}^2\Delta_2} \right) \times \exp \left\{ -\frac{M_{B^*}^2\alpha_1 + M_{B^*}^2\alpha_2 + M_{Z_b}^2\alpha_{12} + 4\alpha_1 \alpha_2}{\Delta_2} \right\}, \]  

(A.3)

\[ J_2 = \frac{1}{8\pi^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2^2} \left( 1 + \frac{\Lambda^2}{M_{B^*}^2\Delta_2} \right) \times \exp \left\{ -\frac{M_{B^*}^2\alpha_1 + M_{B^*}^2\alpha_2 + M_{Z'_b}^2\alpha_{12} + 4\alpha_1 \alpha_2}{\Delta_2} \right\}, \]  

(A.4)

where

\[ \Delta_1 = 2 + \alpha_{12}, \quad \Delta_2 = 1 + \alpha_{12}, \quad \alpha_{12} = \alpha_1 + \alpha_2. \]  

(A.5)

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