SELECTION OF OPTIMAL METHOD OF CORRELATED COLOUR TEMPERATURE CALCULATION

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Abstract

Correlated color temperature (CCT) is a value that characterizes the chromaticity of radiation and it is also used for calculation of color rendering according to the metrics CRI and CQS and also in the theoretical researches. And if in the first case the specific accuracy is not required during its estimation because there are chromatic thresholds within the limits of which CCT is accepted to be the same, in the second and the third cases the accuracy of the methods cannot be neglected. Also, due to the fact that there are several methods of the CCT calculation, all of them possess different degree of complexity and accuracy and the researchers face the problem what method and in what case they shall prefer. The above mentioned determines the urgency of the studied problem. The objective of the article is to determine the distribution of the CCT absolute error in the field of its determination for the most well-known methods: Robertson’s method, Yoshi Ohno’s method, Javier Hernandez-Andres’ method, Mc Camy’s method. The leading approach to the research of this problem is to use the coordinates of chromaticity located on the lines of the constant correlated color temperature with the further evaluation of the absolute error as the initial data for the CCT calculation. As a result of the research it was revealed that the numerical methods of Robertson and Yoshi Ohno are significantly more precise than the analytical methods of Hernandez-Andres and Mc Camy in the whole CCT definition domain. On the base of the obtained distributions of the absolute error the recommendation can be given to use

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the different methods of calculation for different cases. The work compares the “classical” variant of Robertson’s method using the 31 isotherm and the variants with a bigger number of isotherms. It is shown that when the step between the isotherms is reducing the error is decreasing too. The developed method of estimation of the CCT calculation is universal and can be applied to other methods apart the methods considered in the article.

Keywords: Correlated color temperature, Planckian locus, blackbody locus, line of constant correlated temperature, chromaticity coordinates, absolute error.

I. Introduction

The first methods of the CCT calculation appeared at the end of 1960s. Later about ten such methods were offered. The authors of the methods mentioned the errors, certainly, but they did not always give the methodology of its evaluation. The analytical method of the CCT calculation is offered in and the absolute errors of the CCT calculation method are provided but the evaluation method is not given. Thus, there is no single approach to the evaluation of errors of the CCT calculation by all existing methods. Therefore, we developed a test for evaluation of the absolute errors of the CCT calculation method. The test was written in the python language using the libraries numpy, scipy and matplotlib.

But before starting to describe the method we need to remind that according to, correlated color temperature is a blackbody temperature with chromaticity that is the closest one on the CIE 1960 uniform-chromaticity-scale diagram to the chromaticity of the given spectral distribution. The notion of correlated color temperature cannot be changed if the distance between the chromaticity of the tested source and the Planckian locus is more than

\[ \Delta C = \left[ (u'_p - u'_t)^2 + 4/9(v'_p - v'_t)^2 \right]^{1/2} = 5 \cdot 10^{-2} , \]

where \((u'_t, v'_t)\) and \((u'_p, v'_p)\) are coordinates of chromaticity of the tested source and blackbody correspondingly on the CIE 1974 uniform-chromaticity-scale diagram. Isotherm is a line all points of which correspond to one and the same CCT. Therefore, when we do not know the temperature to which the isotherm corresponds, taking the point on it and calculating the CCT for it we can find the error (absolute and relative) of the method in the set point. It means we shall take the temperature of the isotherm as the actual value of CCT to evaluate the error. Besides, basing upon the definition of CCT the error of method shall be determined not in one point but in a band along the Planckian locus, width 0.10 (Fig.1).
II. Methodological Procedures

The essence of the error evaluation method of the CCT calculation is the following:

– the table is formed on the first stage that contains the coordinates of the isotherms' chromaticity. The following equation is used (1):

\[
\begin{align*}
    u_t &= u_0 \pm \frac{lu'}{\sqrt{u'^2+v'^2}}, \\
    v_t &= v_0 \pm \frac{lu'}{\sqrt{u'^2+v'^2}},
\end{align*}
\]

(1)

Where \( u, v \) are unknown coordinates of chromaticity, distant from the Planckianlocus at the distance \( l \); \( u_0, v_0 \) are coordinates of the Planckianlocus chromaticity; \( l \) is the distance between the Planckianlocus and itsparallel curve; \( u' = du/dt, v' = dv/dt \) are partial derivative coordinates of the Planckianlocus chromaticity. It should be noted that we used the numerical differentiationto find \( u' \) and \( v' \). The procedure of isotherm calculation is described in detail in;

–later the second table is formed the cells of which contain the results of the CCT calculation for chromaticities from the first table;

– at the third stage the table is formed the cells of which contain the absolute error. The temperature of isotherms is taken as the actual value for the error calculation.

Thus, the essence of the method is to use the coordinates of isotherm chromaticity as the initial data and to determine the absolute error as the difference between the CCT calculated value and the isotherm temperature. The use of reciprocal mega kelvin MK\(^{-1}\) as the unit of temperature measurement is related to the fact that on the uniform-chromaticity-scale diagram the approximately equal number of chromatic thresholds will correspond to the selected temperature step (for example, \( \Delta T = 20 \) MK\(^{-1}\)), that means the distance between the isotherms will be equal.

Fig. 1: Planckianlocusandisothermson CIE 1964 uniform-chromaticity-scale diagram
III. Results

Robertson’s method for the CCT calculation was offered in 1968. He found the wide practical application and it is still relevant. For example, this method is often used in the software of the modern spectral radiometers; it is based upon the linear interpolation of isotherms.

The original method is based upon the calculation table describing the properties of 31 isotherms (from $10 \text{ MK}^{-1}$ to $660 \text{ MK}^{-1}$, step of $10 \text{ MK}^{-1}$). We decreased the step between the isotherms. As a result the tables were calculated for the step of $5 \text{ MK}^{-1}$, $2.5 \text{ MK}^{-1}$ and $1 \text{ MK}^{-1}$.

The results of error evaluation of the CCT calculation using this method are shown in the Table 1. Analyzing them we can conclude that the reduction of the step decreases the error. For the step of $1 \text{ MK}^{-1}$, for the practically important interval of temperatures from $1667 \text{ K}$ to $10000 \text{ K}$ the method error approaches 0.

Table 1: ErrorsofRobertson'smethod

| Errors in CCT (K) | $10 \text{ MK}^{-1}$ | $5 \text{ MK}^{-1}$ | $2.5 \text{ MK}^{-1}$ | $1 \text{ MK}^{-1}$ |
|------------------|----------------------|---------------------|----------------------|---------------------|
|                  | 12.4                 | 0.6                 | 0.16                 | 0.025               |

Figure 2 describes the distribution of the absolute error in the range of $1667 \text{ K} — 10000 \text{ K}$ for the table offered by G. Wyszecki. In this figure the temperatures are shown in the axis of abscissa (X), and the distance from Planckian locus is shown in the axis of ordinates (Y), and the absolute error is shown in the applicate axis (Z).

Fig. 2: Graph of absolute error distribution of CCT calculation according to Robertson’s method for G. Wyszeck’s table

As the graph shows (Fig. 2) the increase of temperature increases the error.
Mc Camy’s method was offered in 1992. It is offered as the third order polynomial for the CCT calculation. The polynomial was obtained basing on the assumption that all isotherms cross in the certain point on the chromaticity diagram XYZ.

The error evaluation of the CCT calculation using this method showed (Fig. 3) that it is not so acute in comparison with Robertson’s method. The maximal error was 285.4 K in the range of 1700-10000K.

![Fig. 3: Graph of absolute error distribution of CCT calculation according to Mc Camy's method in the range of 1700 — 10000 K](image)

Javier Hernandez-Andres’ method was offered in 1999. Like McCamy’s method is it an analytical solution. The evaluation of the CCT calculation error using this method allowed concluding (Fig. 4) that it is exceeded significantly by the accuracy of the Robertson’s method. The maximal error was 429.4 K in the range of 1700 — 10000 K.

In 2013 Yoshi Ohno offered three methods of the CCT calculation: triangular method, method of parabolas and combined method. All of them are based on the same calculation table.

![Fig. 4: Graph of absolute error distribution of CCT calculation according to Javier Hernandez-Andres’ method in the range of 1700 — 10000 K](image)
The absolute errors of the CCT calculation using Yoshi Ohno’s method are shown in the Table 3. The graphs of the absolute error distribution for the corresponding CCT calculation methods are shown in the Figures 5-7.

**Table 3: Absolute errors of CCT calculation according to Y. Ohno's method**

| Method          | Error in CCT (K) |
|-----------------|------------------|
| Triangular method | 19.1             |
| Method of parabolas | 8.5              |
| Combined method | 0.9              |

**Fig. 5:** Graph of absolute error distribution of CCT calculation according to triangular method in the range of 1700K – 10000 K

**Fig. 6:** Graph of absolute error distribution of CCT calculation according to method of parabolas in the range of 1700K – 10000 K
The Result of the Performed Work Shows

– the error of the CCT calculation for Robertson’s method increases together with the temperature increase. For the step of 1 μ of the calculation table in the range of temperatures from 1700 to 10000 K the error of the method approaches 0;

– for Yoshi Ohno’s triangular method in the range of temperatures from 1667 K to 10000 K the absolute error does not exceed 19.1 K. The method of parabolas in the same range is more acute and the absolute error does not exceed 8.5 K. The combined method is the most acute. In the range of 1667 K – 10000 K the absolute error is 0.9;

– for Mc Camy’s method in the range of 1700 – 10000 K the error reaches 250K.

– for Javier Hernandez-Andres’ method the error reaches 429.4 K in the range of 1700 – 10000 K.

IV. Discussion

In his article Yoshi Ohno represents the graphs for his method characterizing the behavior of the CCT error depending upon the distance of chromaticity from the Planckian locus and from the value of CCT. Looking at these graphs and Fig. 5-7 we can notice that the error behavior determined by Yoshi Ohno is the same as the one obtained in our research. Regarding these values of the absolute error, the work does not describe the certain values in the form of numbers but it provides the graphs. Thus, when looking at the Fig.5 in, we can observe, for example, that at the temperature 10000 K the maximal error for the triangular method is approximately equal to 11.2 K. Our methodology for the same method and the temperature show 11.5 K. For the method of parabolas and the temperature 9500 K the maximal error is 8 according to the Yoshi Ohno’s graph. According to our method it is also 8 K. Such correlation of values means the correctness of the offered methodology. However, we shall mention that its accuracy depends upon the accuracy of the coordinates determination $u_l, v_l$ (see...
formula 1). The partial derivatives \( u' \) and \( v' \) are used to find them, and as the Planckian locus is set discretely we determine them by means of the numerical differentiation that, as it is known gives only approximate value of function increment. The work specifies the errors of the different methods but the methodology of their determination is not specified and the character of the error distribution is not described. This does not allow estimating the conditions when method works properly.

Summarizing the results of the researches we can make the following conclusions. The accuracy of the CCT calculation of Javier Hernandez-Andres’ method or Mc Camy’s method depends heavily upon the position of the studied chromaticity regarding the Planckian locus. Such methods are popular because of the simplicity of their implementation. Despite such methods are suitable for the domestic white-light sources, but when the universality and accuracy are required Robertson’s method or Yoshi Ohno’s combined method shall be used.

V. Conclusion

The article offers the universal method of error valuation of the various methods of the correlated color temperature calculation for the whole range of determination. The distribution of the absolute error of the correlated color temperature was determined for Robertson’s method (1968), Yoshi Ohno’s method (2013), Javier Hernandez-Andres’ method (1999), Mc Camy’s method (1992). On the base of the obtained distributions the recommendations to apply the various methods of calculation were given. The developed method of error valuation of the correlated color temperature calculation can be applied not only to the methods considered in the article.

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