Naïve Dimensional Analysis
and Supersymmetry

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Abstract

In strongly-coupled theories with no small parameters, there are factors of $4\pi$ that appear when the couplings of the low-energy effective lagrangian are written in units of the effective cutoff $\Lambda$. These numerical factors can be explained using “naïve dimensional analysis.” We extend these ideas to supersymmetric theories, and show how to systematically include small parameters and couplings to weakly-interacting fields. The basic principle is that if the fundamental theory is strongly coupled, then the effective theory must also be strongly coupled at the scale $\Lambda$. We use our results to analyze several examples where strong supersymmetric dynamics may be relevant for phenomenology. For models that break supersymmetry through strong dynamics with no small parameters, we show that the Goldstino decay constant $F$ is given by $F \sim \Lambda^2/(4\pi)$. We also consider theories with standard-model gauge bosons coupled directly to strong supersymmetry-breaking dynamics near the weak scale; smoothly-confining theories; and a model that breaks supersymmetry through the mechanism of a deformed moduli space.

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1 Introduction

If nature is supersymmetric at short distances, then dynamical supersymmetry breaking provides an attractive possible explanation for the origin of the supersymmetry breaking scale. Many interesting models of dynamical supersymmetry breaking involve non-calculable strong dynamics, and one would like to construct the low-energy effective theory for such models and estimate the couplings involved. If there are no small parameters in the theory and the only scale is the energy where the dynamics becomes strong, then one might expect all couplings in the low-energy theory to be order one in units of the scale of strong dynamics. However, our experience with QCD suggests that the situation is not so simple. In QCD, there are differences by factors of order 10 in quantities that are formally of order $\Lambda_{QCD}$. For example, the pion decay constant is $f \sim 100$ MeV, while the scale that controls the convergence of chiral perturbation theory is of order 1 GeV.

These numerical factors can be counted using “naïve dimensional analysis” (NDA). The starting point of NDA is to assume that there is a single mass scale $\Lambda$ in the fundamental theory that sets the energy scale for the states that are integrated out to obtain the effective theory. One then assumes that subleading corrections will be suppressed compared to leading terms by powers of $E/\Lambda$, where $E$ is the energy scale of the observable of interest. The factors of $4\pi$ that appear can then be counted using the renormalization properties of the effective theory and naturalness arguments. For example, if the strong dynamics breaks a global symmetry, the decay constant of the Nambu–Goldstone bosons is given by $f \sim \Lambda/(4\pi)$. This counting accounts for some striking facts about QCD: for example, the existence of an approximate $SU(3)$ flavor symmetry despite the fact that (numerically) $m_s \sim 100$ MeV $\sim f$.

In this paper, we first apply NDA to theories that spontaneously break supersymmetry through strong dynamics at a scale $\Lambda$. (Examples of such theories are $SU(5)$ gauge theory with a 10 and a $\bar{5}$, or $SO(10)$ gauge theory with a single spinor.) We argue that the Goldstino decay constant $F$ (with dimension mass-squared) is given by $F \sim \Lambda^2/(4\pi)$. We then extend this counting to include weakly-coupled fields and small parameters, and also consider theories with confining dynamics.

In all cases, the principle we use is that if the fundamental theory is strongly coupled at the scale $\Lambda$, then the effective theory must also be strongly coupled at this scale. We assume that strong coupling means that all loops are equally important (“loop democracy”). Even if we cannot perform the matching calculation at the scale $\Lambda$, we can count the factors of $4\pi$ that must appear in the effective theory so that the dynamics is strong at the matching scale.
This paper is organized as follows. In Section 2, we briefly review the arguments for NDA in nonsupersymmetric theories with spontaneously broken symmetries. In Section 3, we extend these arguments to spontaneous breaking of supersymmetry. In Section 4, we show how to include small parameters and couplings to weakly-interacting fields. In Section 5, we analyze some examples where the $4\pi$ counting discussed above may be relevant for phenomenology and model-building. We first consider theories with standard-model gauge bosons coupled directly to strong supersymmetry-breaking dynamics near the TeV scale. We then consider models with confining dynamics $^{[5]}$, and analyze a model that breaks supersymmetry through the mechanism of a deformed moduli space $^{[6]}$. Section 6 contains our conclusions.

2 Power Counting for Nambu–Goldstone Bosons

2.1 Linear Sigma Model

We begin our review of NDA for global symmetry breaking with the linear sigma model. We consider a theory of a complex scalar field $\Phi$ with lagrangian

$$L = \partial^\mu \Phi^\dagger \partial_\mu \Phi - \frac{1}{2} \lambda (\Phi^\dagger \Phi - v^2)^2.$$  \hspace{1cm} (2.1)

If $v^2 > 0$, then $\langle \Phi \rangle = v$ and the $U(1)$ global symmetry is spontaneously broken. The theory contains a massless Nambu–Goldstone boson with decay constant $f = v$, and a massive scalar ("$\sigma$") with mass $m_\sigma^2 = \lambda v^2$. We therefore identify $\Lambda \sim m_\sigma$ as the scale of “new physics” in the low-energy effective lagrangian.

It is straightforward to integrate out the heavy scalar field at tree level by making the field redefinition

$$\Phi = v \hat{\sigma} e^{i\hat{\Pi}}$$ \hspace{1cm} (2.2)

and using the $\hat{\sigma}$ equations of motion. To order $\partial^4$, we obtain the effective lagrangian

$$L_{\text{eff}} = v^2 \hat{L}_{\text{eff}}, \quad \hat{L}_{\text{eff}} = \partial^\mu \hat{\Pi} \partial_\mu \hat{\Pi} + \frac{1}{2m_\sigma^2} (\partial^\mu \hat{\Pi} \partial_\mu \hat{\Pi})^2 + O(\partial^6).$$  \hspace{1cm} (2.3)

To this order, $v^2$ appears as an overall factor in the effective lagrangian, and all couplings in $\hat{L}_{\text{eff}}$ are order 1 in units where $m_\sigma = 1$.

In fact, the scaling in Eq. (2.3) holds to all orders in the derivative expansion. This is because the fundamental lagrangian Eq. (2.1) can be written

$$L = v^2 \hat{L}, \quad \hat{L} = \partial^\mu \hat{\sigma} \partial_\mu \hat{\sigma} + \hat{\sigma}^2 \partial^\mu \hat{\Pi} \partial_\mu \hat{\Pi} - \frac{m_\sigma^2}{2} (\hat{\sigma}^2 - 1)^2.$$  \hspace{1cm} (2.4)
Again, \( v^2 \) appears as an overall factor and all couplings in \( \hat{L}_{\text{eff}} \) are order 1 in units where \( m_\sigma = 1 \). Since the overall factor does not affect the classical equations of motion, it is easy to see that integrating out the field \( \hat{\sigma} \) at tree level results in an effective lagrangian of the form \( \text{Eq. (2.3)} \) to all orders in the derivative expansion. This shows that the effective lagrangian has the form

\[
\hat{L}_{\text{eff}} \sim v^2 \sum_{p,q} \left( \frac{\partial \mu}{m_\sigma} \right)^p \left( \frac{\Pi}{v} \right)^q,
\]

where we have written the result in terms of the canonically normalized field \( \Pi \equiv v \hat{\Pi} \). Eq. (2.5) is the NDA scaling for the effective lagrangian (with the identifications \( f \sim v \), \( \Lambda \sim m_\sigma \)).

The reason that the effective lagrangian exhibits the scaling Eq. (2.5) is that there are only two scales in the full theory. In a more complicated sigma model, we could have several widely different scales, invalidating the power-counting formula Eq. (2.5). However, if the parameters of the model are adjusted so that all nonzero tree masses are of the same order, then arguments such as these show that the NDA scaling is also valid in these more general theories. (We will give the analogous arguments in detail for the case of supersymmetry breaking below.) We therefore expect the NDA scaling to hold in all effective theories with a single scale of “new physics.”

### 2.2 Strong Coupling

We can try to get some insight into strongly-coupled theories from the linear sigma model above by allowing the coupling \( \lambda \) to become large. When \( \lambda \) (defined at a suitable subtraction scale) becomes sufficiently large, the perturbative Landau pole of the theory is near the physical mass of the \( \sigma \) particle. It is unlikely that this theory can be made sensible as an effective theory at the scale \( m_\sigma \). Nonetheless, we can make sense out of this theory by imposing a cutoff of order \( m_\sigma \) on the theory, and study the dynamics below the scale \( m_\sigma \). This procedure is certainly far from rigorous, but in this way we can hope to get some qualitative insight into the low-energy effective theory resulting from strong dynamics.

Because \( v^2 \) appears only as an overall factor in Eq. (2.4), \( 1/v^2 \) acts as a loop counting parameter analogous to \( \hbar \). The remaining dimensions are made up by \( m_\sigma \) and powers of momentum, so loop effects in the fundamental theory at the scale \( m_\sigma \) are suppressed by powers of

\[
\ell = \frac{1}{16\pi^2} \frac{m_\sigma^2}{v^2} \sim \frac{\lambda}{16\pi^2} \tag{2.6}
\]
compared to tree effects. This theory becomes strongly coupled when all loop effects are equally important ("loop democracy"), which means $\ell \sim 1$. Note that in this limit, we have $m_\sigma \sim 4\pi v$, which corresponds to the usual NDA estimate $\Lambda \sim 4\pi f$ for strong theories with no small parameters.

Even though the dynamics at the scale $\Lambda$ becomes non-calculable for $\Lambda \sim 4\pi f$, the effective theory given by Eq. (2.5) is still predictive at scales small compared to $\Lambda$. The reason is that in the effective theory the Nambu–Goldstone bosons are derivatively coupled and $m_\sigma$ does not appear as a kinematic scale in the loop diagrams. Loops in the effective theory are therefore suppressed by powers of

$$\ell_{\text{eff}} = \frac{p^2}{16\pi^2 v^2},$$

where $p$ is an external momentum. Loop effects are small for $p \ll \Lambda$, and we have a predictive low-energy expansion. This reasoning embodies an appealing picture of the origin of the factors of $4\pi$: they are put in the effective lagrangian so that the effective description is strongly coupled at the scale $\Lambda$ where it matches onto the fundamental theory. The effective theory is weakly coupled at energies below $\Lambda$ because the Nambu–Goldstone bosons are derivatively coupled.

One might wonder whether it is possible to have $\Lambda \gg 4\pi f$ in the effective theory. This is unnatural because the loop corrections logarithmically renormalize higher-derivative coefficients in the effective lagrangian. Therefore the condition $\Lambda \gg 4\pi f$ could hold only at a specific value of the renormalization scale, which is clearly unphysical [1].

The loop-counting parameter in the effective theory may be smaller than the estimates above in a strongly-coupled theory if there is a small parameter in the theory. For example, in QCD for a large number of colors $N$, the loop counting parameter in the low-energy theory is $1/N \ll 1$. In this case, the relation between the decay constant and the scale of new physics is $\Lambda \sim 4\pi f/\sqrt{N}$ [8].

3 Power Counting for Goldstinos

3.1 A Simple O’Raifeartaigh Model

We begin our discussion of supersymmetry breaking with a model containing chiral superfields $\Phi$, $S$, and $\Sigma$, with superpotential

$$W = \kappa S + \frac{1}{2} \lambda S \Phi^2 + M\Sigma \Phi.$$  \hspace{1cm} (3.1)

We choose all couplings to be real and positive by rescaling the fields.
Supersymmetry is spontaneously broken by \( \langle F_S \rangle = \kappa \). \( \langle S \rangle \) is undetermined at tree level, corresponding to the tree-level flat direction that exists in all O’Raifeartaigh models. The global minimum of the tree potential is at \( \langle \Phi \rangle = \langle \Sigma \rangle = 0 \) provided that \( M^2 \geq \lambda \kappa \). This is the parameter range we will consider.

The spectrum of this theory is as follows. The fermionic components of \( \Phi \) and \( \Sigma \) get a tree-level Dirac mass \( M \), and the \( S \) fermion is exactly massless (it is the Goldstino). The complex \( \Sigma \) scalar gets a tree-level mass \( M \), and the two real \( \Phi \) scalars get tree-level mass-squared \( M^2 \pm \lambda \kappa \). (The splitting \( \lambda \kappa \) in the scalar masses breaks supersymmetry in the tree-level spectrum.) The \( S \) scalar is massless at tree level, and gets a positive mass-squared at one loop of order

\[
m_S^2 \sim \frac{\lambda^2}{16\pi^2} \kappa.
\]

(The global minimum is at \( \langle S \rangle = 0 \).) For weak coupling, \( m_S \) is much smaller than the other masses in the problem, and we do not obtain a simple scaling analogous to the NDA result for global symmetry breaking. However, we will argue that a simple scaling emerges in a strong-coupling limit where all heavy particles have comparable mass.

We first choose the parameters so that all the non-zero tree-level masses are of the same order. This means

\[
M^2 \sim \lambda \kappa.
\]

We then define rescaled component fields and couplings

\[
\hat{\phi} \equiv \frac{M}{\kappa} \phi, \quad \hat{\psi} \equiv \frac{M^{1/2}}{\kappa} \psi, \quad \hat{\lambda} \equiv \frac{\kappa}{M^2} \lambda,
\]

and write the lagrangian as

\[
\mathcal{L} = \kappa^2 \left\{ \frac{i}{M} \hat{\psi}^\dagger \sigma^\mu \partial_\mu \hat{\phi} + \frac{1}{M^2} \partial^\mu \hat{\phi}^\dagger \partial_\mu \hat{\phi} + \cdots \right.
\]

\[
- (\hat{\psi}_\Phi \hat{\psi}_\Sigma + \text{h.c.}) - \hat{\lambda} \left( \frac{1}{2} \hat{\phi}_S \hat{\psi}_\Phi + \hat{\phi}_\Phi \hat{\psi}_S \right) + \text{h.c.}
\]

\[
- \left| \frac{1}{2} \hat{\lambda} \hat{\psi}_\Phi - 1 \right|^2 - \left| \hat{\phi}_\Phi \right|^2 - \left| \hat{\lambda} \hat{\phi}_S \hat{\phi}_\Phi + \hat{\phi}_\Sigma \right|^2 \right\}.
\]

The condition that the tree masses are of the same order means that \( \hat{\lambda} \sim 1 \) (see Eq. (1.3)), so the lagrangian has the form

\[
\mathcal{L} = \kappa^2 \hat{\mathcal{L}},
\]
where all the couplings in $\hat{L}$ are order 1 in units where $M = 1$.

We see that the parameter $1/\kappa^2$ acts as a loop counting parameter if we work in units where $M = 1$, so loops are suppressed by powers of

$$\ell = \frac{1}{16\pi^2} \frac{M^4}{\kappa^2} \sim \frac{\lambda^2}{16\pi^2}.$$  \hspace{1cm} (3.7)

We now consider the na"ive strong-coupling limit $\ell \sim 1$. We then find that the $S$ mass is of order $M$ (see Eq. (3.2)), the same as the other heavy masses. Integrating out the heavy particles gives rise to an effective lagrangian for the Goldstino of the form

$$\mathcal{L}_{\text{eff}} = \kappa^2 \hat{\mathcal{L}}_{\text{eff}},$$ \hspace{1cm} (3.8)

where all couplings in $\hat{\mathcal{L}}_{\text{eff}}$ are order 1 in units of $M$. This means

$$\hat{\mathcal{L}}_{\text{eff}} \sim \sum_{p,q} \left( \frac{\partial_\mu}{M} \right)^p (\hat{\chi})^q,$$ \hspace{1cm} (3.9)

where $\hat{\chi}$ is the (rescaled) Goldstino field. Defining a canonically-normalized Goldstino field $\chi \equiv \kappa \hat{\chi}/M^{1/2}$ and identifying $F \sim \kappa$ and $\Lambda \sim M$, the effective lagrangian can be written

$$\mathcal{L}_{\text{eff}} \sim F^2 \sum_{p,q} \left( \frac{\partial_\mu}{\Lambda} \right)^p \left( \frac{\Lambda^{1/2} \chi}{F} \right)^q,$$ \hspace{1cm} (3.10)

with $\Lambda^2 \sim 4\pi F$. This is the NDA form of the Goldstino lagrangian. It is easy to see that this scaling is compatible with the constraints on the couplings coming from the non-linear realization of supersymmetry \[9\].

### 3.2 General O’Raifeartaigh Model

We now extend this analysis to arbitrary O’Raifeartaigh models. We assume the theory has a canonical kinetic term\[\footnote{The argument is easily extended to theories with non-trivial Kähler potentials.}]

$$\mathcal{L}_D = \int d^2 \theta d^2 \bar{\theta} \Phi_+^a \Phi^a,$$ \hspace{1cm} (3.11)

and F terms

$$\mathcal{L}_F = \int d^2 \theta \sum_{n=1}^\infty \frac{1}{n!} \chi_{a_1 \cdots a_n}^{(n)} \Phi^{a_1} \cdots \Phi^{a_n} + \text{h.c.}$$ \hspace{1cm} (3.12)
We will show that the couplings can be chosen so that all tree-level masses are of the same order $\Lambda$, and derive the scaling of the low-energy theory. We will work in units where $\Lambda = 1$. We begin by defining rescaled superfields

$$\Phi \equiv F \hat{\Phi},$$

(3.13)

in terms of which the kinetic term is

$$L_D = F^2 \int d^2 \theta d^2 \bar{\theta} \hat{\Phi}_a^\dagger \hat{\Phi}^a.$$  

(3.14)

In order to make all physical masses of order 1, we write the superpotential couplings

$$\lambda^{(n)}_{a_1 \ldots a_n} = F^{2-n} \hat{\lambda}^{(n)}_{a_1 \ldots a_n},$$

(3.15)

and choose values

$$\hat{\lambda}^{(n)} \sim 1.$$  

(3.16)

The full lagrangian then has the form

$$L_D + L_F = F^2 \hat{\mathcal{L}},$$

(3.17)

where all couplings in $\hat{\mathcal{L}}$ are of order 1. Since $F$ scales out of the classical equations of motion, the tree-level masses are independent of $F$, and hence the non-zero tree vacuum expectation values and masses are all of order 1. With the choice of parameters Eq. (3.16), the supersymmetry breaking order parameter is

$$\langle \frac{\partial W}{\partial \hat{\Phi}^a} \rangle = F \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \hat{\lambda}^{(n)}_{a_{b_2} \ldots a_n} \langle \hat{\Phi}^{b_2} \ldots \hat{\Phi}^{b_n} \rangle \sim F.$$  

(3.18)

The loop-counting parameter is

$$\ell \sim \frac{1}{16 \pi^2 F^2}.$$  

(3.19)

More precisely, what is meant here is that all nonzero couplings are of order 1. If an O’Raifeartaigh model breaks supersymmetry, it will break it for all values the parameters except for isolated critical values where the limiting behavior of the potential for large field values changes. We can therefore always find a supersymmetry-breaking vacuum where the non-zero couplings $\lambda^{(n)}$ are order 1.

In “inverse hierarchy” models, there are mass hierarchies of order $e^{-16 \pi^2/\lambda^2}$, where $\lambda$ is a superpotential coupling. These hierarchies disappear in the strong-coupling limit considered below, and we expect our final results to hold even in this class of models.
For $\ell \ll 1$, the theory has a light complex scalar with mass of order $\ell$, corresponding to the tree-level flat direction that exists in all O’Raifeartaigh models. To get a theory where all massive particles have masses of the same order, we take the naïve strong coupling limit $\ell \sim 1$. In this case, the effective lagrangian below the scale $\Lambda$ contains only the Goldstino field, with the scaling

$$\mathcal{L}_{\text{eff}} \sim F^2 \mathcal{L}_{\text{eff}}.$$  

This leads to the NDA form Eq. (3.10) for the effective lagrangian.

### 3.3 Global Symmetry Breaking

In the above analysis, note that the vacuum expectation values of the fields $\hat{\Phi}$ were order 1. This means that

$$\langle \Phi \rangle \sim \frac{F}{\Lambda} \sim \frac{\Lambda}{4\pi}.$$  

Identifying $f \sim \langle \Phi \rangle$, this agrees with the estimate $\Lambda \sim 4\pi f$, so the power-counting above is consistent with the NDA arguments for global symmetry breaking.

### 4 Spurion Analysis

We now extend these power-counting arguments to include small coupling constants and couplings to weakly-interacting light fields. We begin with the general O’Raifeartaigh model discussed in the previous section. We now add to the fundamental theory couplings of the form

$$\delta \mathcal{L} = \int d^2 \theta d^2 \bar{\theta} \mathcal{J}_{D,d} \mathcal{O}_{D,d} + \left[ \int d^2 \theta \mathcal{J}_{F,d} \mathcal{O}_{F,d} + \text{h.c.} \right],$$  

where $\mathcal{O}_{D,d}, \mathcal{O}_{F,d} \sim \Phi^d$ are dimension-$d$ operators composed of strongly-interacting chiral superfields. $\mathcal{J}_{D,d}$ and $\mathcal{J}_{F,d}$ may be coupling constants or products of coupling constants and weakly-coupled superfields. If $\mathcal{J}_{D,d}$ or $\mathcal{J}_{F,d}$ involve weakly-coupled superfields, the leading contributions to the effective lagrangian arise from diagrams with no loops involving weakly-interacting superfields. In either case, $\mathcal{J}_{D,d}$ ($\mathcal{J}_{F,d}$) may be viewed as a source (or “spurion”) coupled to the operator $\mathcal{O}_{D,d}$ ($\mathcal{O}_{F,d}$).

We now write the operators $\mathcal{O}_{D,d}$ and $\mathcal{O}_{F,d}$ in terms of the rescaled fields $\hat{\Phi}$ defined in Eq. (3.13)

$$\hat{\mathcal{O}}_{D,d} \equiv F^{-d} \mathcal{O}_{D,d} \sim \hat{\Phi}^d, \quad \hat{\mathcal{O}}_{F,d} \equiv F^{-d} \mathcal{O}_{F,d} \sim \hat{\Phi}^d,$$  

where

$$\hat{\mathcal{O}}_{D,d} \equiv F^{-d} \mathcal{O}_{D,d} \sim \hat{\Phi}^d, \quad \hat{\mathcal{O}}_{F,d} \equiv F^{-d} \mathcal{O}_{F,d} \sim \hat{\Phi}^d,$$  

(4.2)
and rescale $J_{D,d}$ and $J_{F,d}$ according to

$$
\hat{J}_{D,d} \equiv F^{d-2} J_{D,d}, \quad \hat{J}_{F,d} \equiv F^{d-2} J_{F,d}.
$$

(4.3)

In terms of these, the coupling to the spurions can be written

$$
\delta \mathcal{L} = F^2 \left[ \int d^2 \theta d^2 \bar{\theta} \hat{J}_{D,d} \hat{O}_{D,d} + \int d^2 \theta \hat{J}_{F,d} \hat{O}_{F,d} + \text{h.c.} \right].
$$

(4.4)

The point of this rescaling is that $\hat{J}_{D,d}$ and $\hat{J}_{F,d}$ now appear in the fundamental lagrangian in the same way as a rescaled strongly-coupled field $\hat{\Phi}$. We therefore have (in units where $\Lambda = 1$)

$$
\langle 0 | T \hat{O}_{d_1} \cdots \hat{O}_{d_n} | 0 \rangle \sim F^2 + \text{loop corrections}
$$

(4.5)

for any operators with dimensions $d_1, \ldots, d_n$. This counting is reproduced in the effective lagrangian by

$$
\mathcal{L}_{\text{eff}} \sim F^2 \sum_{p, \ldots, t} \left( \frac{\partial}{\Lambda} \right)^p \left( \frac{\Lambda^{1/2} \chi}{F} \right)^q \left( \frac{1}{\Lambda^{1/2} \partial \bar{\theta}} \right)^r \left( \frac{F^{d-2}}{\Lambda^{d-1}} J_{D,d} \right)^s \left( \frac{F^{d-2}}{\Lambda^{d-1}} J_{F,d} \right)^t,
$$

(4.6)

where we have put back factors of $\Lambda$ by dimensional analysis. The derivatives with respect to the superspace coordinate $\theta$ are used to project out the component fields of the spurions $J_{D,d}$ and $J_{F,d}$.

It is worth emphasizing that the arguments above are simply a compact way of taking the naïve strong coupling limit of O’Raifeartaigh models. The scaling above holds only if the spurion $J$ in the fundamental theory is coupled to operator $O$ that is a simple product of strong chiral superfields. It is straightforward to analyze general operators in the same way. The result (in units where $\Lambda = 1$) is that $J$ appears in the effective lagrangian with coefficient $F^{w-2}$, where $w$ is the sum of “weights” of the strong fields appearing in $O$. Strong chiral fields and factors of the strong gauge field strength $W_\alpha$ have weight 1, and gauge-covariant derivatives (superspace or spacetime) have weight 0.

A counting of $4\pi$’s was given (without derivation) for dynamically-generated $D$ terms in Ref. [12], for dimensionless couplings to weakly-coupled fields ($J_{F,d}$ with $d = 2$ in our notation). We agree with the results of Ref. [12] in this case.

5 Examples
5.1 Supersymmetry Breaking in the Observable Sector

An interesting application of these ideas is to models of gauge-mediated supersymmetry breaking \[13\]. In these models, supersymmetry breaking is communicated to superpartners of observed fields by strong and electroweak gauge interactions. Explicit models of this type usually involve a weakly-coupled “messenger” sector that separates the observable sector from the supersymmetry breaking dynamics. These models have the virtue of avoiding many phenomenological problems (although they may have problems with color breaking \[14\]), but they are rather complicated. We will consider the possibility that the standard model gauge fields couple directly to strong supersymmetry-breaking dynamics near the weak scale.\[^4\] This would require supersymmetry-breaking gauge theory with an unbroken symmetry that is large enough to contain the standard-model gauge group. On the other hand, the model should not have extra symmetries that give rise to unwanted massless particles. If one is interested in maintaining grand unification, the models must not contain too many fields carrying standard-model quantum numbers. It may be difficult to construct explicit models satisfying all these constraints, but we can easily analyze the low-energy theory resulting from such a model using the ideas of this paper. We will find that the phenomenology of these models is very similar to that of weakly-coupled messenger models, with \(\Lambda \sim 10\) TeV.

\[^4\]There are models in the literature in which the electroweak gauge bosons are coupled directly to a supersymmetry breaking sector with a large scale hierarchy \[15\]. In these models, the scale of strong dynamics \(\Lambda\) is far above the weak scale.

\[\text{Fig. 1. Typical diagrams contributing to (a) gaugino masses and (b) scalar masses in theories where fields are coupled weakly to the strongly interacting fields. The shaded blob denotes strong interactions.}\]
to the strongly-interacting fields is

$$\delta L = \int d^2 \theta d^2 \bar{\theta} \Phi^\dagger g V_A T_A^{(r)} \Phi,$$

(5.1)

where \( \Phi \) is a strongly-interacting field in the representation \( r \) of the standard-model gauge group, \( V_A \) is the weakly-interacting gauge superfield, and \( T_A^{(r)} \) is the gauge generator. The gaugino mass arises from the diagram of Fig. 1a. The operator \( g V_A T_A^{(r)} \) acts as a source for a dimension-2 operator, so we have

$$m_\lambda \sim g^2 I_2(r) \frac{F^2}{\Lambda^3} \sim \frac{g^2 I_2(r)}{16\pi^2} \Lambda,$$

(5.2)

where \( I_2(r) \) is the index of the representation \( r \). Requiring that the weak gaugino masses be of order 100 GeV gives \( \Lambda \sim 10 \text{ TeV} \).

The scalar masses arise through the diagrams such as the one in Fig. 1b. This effect can be estimated as a 1-loop perturbative diagram with the insertion of non-perturbative gaugino mass and wavefunction renormalization. This gives rise to a scalar mass-squared of order

$$m_\phi^2 \sim I_2(r) C_2(r) \left( \frac{g^2}{16\pi^2} \right)^2 \Lambda^2,$$

(5.3)

where \( C_2(r) \) is the quadratic Casimir invariant of the representation \( r \).

These estimates are the same order of magnitude as the minimal messenger sector with singlet \( S \) coupled to messenger “quarks” \( q \) and \( \bar{q} \) via a superpotential term \( \lambda Sqq \), provided we make the identifications

$$F \sim \langle F_S \rangle, \quad \Lambda^2 \sim m_\lambda^2 \sim \lambda^2 \langle S \rangle^2 \sim \lambda \langle F_S \rangle, \quad \lambda \sim 4\pi.$$

(5.4)

The main difference between the strongly-coupled models considered here and perturbative messenger models is that we expect a rich spectrum of strong resonances near the scale \( \Lambda \sim 10 \text{ TeV} \) in the strong models.

5.2 Confining Theories

Another interesting class of supersymmetric theories are those that confine and have light composite chiral superfields \[\boxed{\text{}}\]. Models have been constructed where the composite particles have the quantum numbers of quarks, leptons, and Higgs fields, and

\[\text{It may appear surprising that the gaugino mass is proportional to } F^2 \text{ rather than } F, \text{ since the gaugino mass can be written as an } F \text{-term with a supersymmetry-breaking spurion } \theta \theta. \text{ However, the powers of } F \text{ and } \Lambda \text{ in our formulas serve only to count powers of } 4\pi. \text{ Also, our results are (by construction) consistent with the strong-coupling limit of weakly-coupled models.}\]
attempts at model-building have been made \cite{16, 17}. Confining models can break supersymmetry through strong dynamics \cite{6}, or they may be coupled to a separate supersymmetry breaking sector in some way.

We begin by considering confining theories in which supersymmetry is not broken. The prototype model of this kind is supersymmetric QCD with gauge group $SU(2)$ and 6 fields $Q$ in the fundamental representation. Seiberg has argued convincingly that the low-energy effective theory near the origin of moduli space of this model has a confined description in terms of composite superfields with the quantum numbers of the operators $M^{jk} \sim \epsilon_{ab} Q^a Q^b$, where $\epsilon_{ab}$ is the $SU(2)$ metric and $j, k = 1, \ldots, 6$.

The effective theory has a dynamically-generated superpotential

$$W_{\text{eff}} = \text{Pf}(M) \propto \epsilon_{j_1 \cdots j_6} M^{j_1 j_2} M^{j_3 j_4} M^{j_5 j_6}. \quad (5.5)$$

This theory is controlled by strong dynamics with no small parameters, so the arguments above lead us to expect that the effective lagrangian should have the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{16\pi^2} \hat{\mathcal{L}}_{\text{eff}}, \quad (5.6)$$

where all couplings in $\hat{\mathcal{L}}_{\text{eff}}$ are order 1 in units where $\Lambda = 1$. In terms of canonically normalized composite fields $M$, the lagrangian has the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{16\pi^2} \left[ \int d^2\theta d^2\bar{\theta} K_{\text{eff}} \left( \frac{4\pi M}{\Lambda}, \frac{D_\alpha}{\Lambda^{1/2}}, \ldots \right) + \int d^2\theta W_{\text{eff}} \left( \frac{4\pi M}{\Lambda}, \ldots \right) \right]. \quad (5.7)$$

Couplings to other fields can be included using the results of the previous section.

In any confining model, we therefore expect that dynamically generated cubic couplings in the superpotential such as the one in Eq. (5.3) is of order $4\pi$ (not 1) when expressed in terms of canonically normalized fields. This means that confined description of this theory is not weakly coupled at the scale $\Lambda$. However, we expect the cubic coupling to renormalize logarithmically to zero in the infrared, so this theory is still weakly coupled at energies far below $\Lambda$. If we want to identify a cubic term in a dynamically-generated superpotential with the top-quark Yukawa coupling, the compositeness scale must be large enough that the top-quark Yukawa coupling can become perturbative at the weak scale. The top quark mass is then controlled by an approximate infrared fixed point, as in Refs. \cite{18}. (These points have also been noted in Ref. \cite{17}, which appeared as this paper was nearing completion.)

\footnote{There are $SU(N)$ and $Sp(2N)$ generalizations of this theory, and if $N \gg 1$ these theories have an additional small parameter. We will discuss this elsewhere.}

\footnote{The counting for the Kähler potential was given (without derivation) in Ref. \cite{12}.}
The confining theory described here is related to other theories by adding mass terms, and we must check that our estimates are consistent with these relations. In fact, by adding mass terms for all but 2 $Q$’s, one obtains a model whose non-perturbative dynamics far from the origin in moduli space can be analyzed by a direct instanton calculation \[19\]. However, to use this solution to find the physical couplings in the confining theory near the origin of moduli space, it is necessary to know the Kähler potential close to the origin to determine the normalization of the fields. A similar ambiguity will affect any attempt to relate the physical superpotential couplings to a weak-coupling calculation, and we therefore believe that there is no conflict between our arguments and the well-established relations between theories with different numbers of colors and flavors.

5.3 Dynamical Supersymmetry Breaking on a Deformed Moduli Space

We now consider the model of Refs. [6] as an example where the factors of $4\pi$ are rather non-trivial. The model has gauge group $SU(2)$ with 4 fields $Q$ in the fundamental representation. There are also gauge singlet fields $S_{jk} = -S_{kj}$ ($j = 1, \ldots, 4$), coupled to the $Q$’s via the superpotential

$$W = \lambda S_{jk} \epsilon_{ab} Q^{aj} Q^{bk}.$$  \hspace{1cm} (5.8)

If we set $\lambda = 0$, Seiberg has argued that this theory has a confined description in terms of “meson” fields $M^{jk}$ satisfying a quantum constraint $\text{Pf}(M) = \text{constant}$. This constraint is incompatible with the condition that $\partial W/\partial S_{jk} = 0$, and supersymmetry is spontaneously broken \[6\].

We will write the effective lagrangian assuming that $\langle S \rangle$ is small enough that the confined description is valid. We use fields $\hat{M}$ in terms of which the effective lagrangian has the form of Eq. (5.6). We have argued above that the strong dynamics gives rise to vacuum expectation values for such fields of order 1 (in units where $\Lambda = 1$). Therefore, the fields $\hat{M}$ satisfy

$$\text{Pf} \hat{M} = c \sim 1.$$ \hspace{1cm} (5.9)

The effective lagrangian written in terms of these fields is

$$\mathcal{L}_{\text{eff}} = \int d^2\theta d^2\bar{\theta} \left[ S^\dagger S + \frac{1}{16\pi^2} K_{\text{dyn}}(\hat{M}, \lambda S) \right]$$

$$+ \int d^2\theta \frac{a\lambda}{16\pi^2} S_{jk} \hat{M}^{jk} + \text{h.c.},$$ \hspace{1cm} (5.10)

where $K_{\text{dyn}}$ is the dynamically generated Kähler potential\[8\]. The coupling $a$ in the\[^8\] is also contain terms depending on supersymmetry covariant derivatives of $\hat{M}$ and $S$.\[^8\]
effective superpotential is an unknown strong-interaction parameter of order 1.

To understand the physics of this lagrangian, we write
\[ \hat{M} = \langle \hat{M} \rangle + \hat{M}', \quad S = S_0 + S', \]
where \( S' \) is defined by the constraint
\[ \langle \hat{M}^{jk} \rangle S'_{jk} = 0. \]
(There is a nonlinear constraint on \( \hat{M}' \) arising from Eq. (5.9).) With these definitions, we see that the effective superpotential gives \( \hat{M}' \) and \( S' \) a supersymmetry-invariant Dirac mass. Taking into account that the kinetic term for \( \hat{M}' \) has a coefficient of order \( 1/(16\pi^2) \), we see that this mass is of order
\[ m \sim \frac{\lambda}{4\pi} \Lambda. \]

We would like to integrate out \( \hat{M} \) and \( S' \) and write the effective theory below the scale \( m \). The only light degree of freedom in this case is the field \( S_0 \). It is not hard to see that the leading contributions come from 1-loop diagrams where all vertices have the form \( (S_0)^p (\hat{M}')^q \). The resulting effective lagrangian has the form (putting back factors of \( \Lambda \))
\[ L_{\text{eff}} = \int d^2 \theta d^2 \bar{\theta} \left[ S_0^\dagger S_0 + \left( \frac{\lambda \Lambda}{16\pi^2} \right)^2 \hat{K}_{\text{dyn}}(\lambda S_0/\Lambda) \right] + \int d^2 \theta \frac{a \lambda \Lambda^2}{16\pi^2} S_0 + \text{h.c.}, \]
(5.14)
In this description, the model breaks supersymmetry due to the combination of the linear term in the superpotential and the non-trivial Kähler potential. The supersymmetry-breaking order parameter is
\[ \langle F_{S_0} \rangle \sim \frac{\lambda \Lambda^2}{16\pi^2}. \]
(5.15)
The fermionic component of \( S_0 \) is an exactly massless Goldstino. The sign of the \( S_0 \) scalar mass term at the origin is determined by the signs of unknown coefficients in the effective Kähler potential, so we cannot determine whether \( \langle S_0 \rangle \) is nonzero. As long as \( \langle S_0 \rangle \) is small enough for the confined description to be valid, the scalar components of \( S \) have masses of order
\[ m_0 \sim \left( \frac{\lambda}{4\pi} \right)^4 \Lambda. \]
(5.16)
The factors of \( 4\pi \) reduce the mass by a factor of \( 10^{-4} \) in this model!
6 Conclusions

All of our results for strongly-interacting theories with a single mass scale can be summarized by the formula

\[ \mathcal{L}_{\text{eff}} = \frac{1}{16\pi^2} \hat{\mathcal{L}}_{\text{eff}}, \]

where all couplings in \( \mathcal{L}_{\text{eff}} \) are of order 1 in units of the mass scale \( \Lambda \) that sets the scale for the “new physics” associated with the strong dynamics. Eq. (6.1) ensures that both the fundamental and the effective theory are strongly coupled at the scale \( \Lambda \), in the sense that all loops are equally important (“loop democracy”). Below the scale \( \Lambda \), the effective theory may be weakly coupled, either because the symmetries of the effective theory do not allow renormalizable interactions (as in the case of Nambu–Goldstone bosons), or because the renormalizable couplings that appear renormalize logarithmically to zero in the infrared (as in confining theories). The arguments presented here in support of this counting are at best heuristic, but the low-energy behavior of QCD seems to support them. As illustrated in this paper, the simple scaling embodied in Eq. (6.1) has interesting implications for dimensional analysis in strongly-interacting supersymmetry breaking theories.

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\footnote{Note that the parameter \( 1/(16\pi^2) \) can be viewed as being small because the dimension of spacetime is large. It is hard to see how to construct a systematic argument based on this observation, since all interactions become irrelevant for large spacetime dimension.}
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