Uncertainty Analysis Applied to Predicting Water Inflow in Confined Water

Chengcheng Xu¹, Chuiyu Lu²*, and Jianhua Wang ²

¹School of Earth and Environment, Anhui University of Science and Technology, Huainan, China
²State Key Laboratory of Water Cycle Simulation and Regulation, China Institute of Water Resources and Hydropower Research, Beijing, China

*Email: cylu@iwhr.com

Abstract. In order to predict the amount of water inflow accurately, define the corresponding water influx response quantitatively in the parameter variation interval, this paper used the interval uncertainty analysis method, combined with the non-probability set theory and derived the predictive formula of confined water inflow under two conditions, the formula comes from the initial deterministic simple large-well method calculation formula to the consideration of parameter variation uncertainty fusion large-well method calculation formula transformation. Using the mining data, comparing the upper and lower limits of the actual calculated boundary obtained by the Monte Carlo method and of the derived formula, the relative error of the calculation result of the confined water inflow expression under the two conditions is calculated 5% and 10% or less, corresponding to the rate of change of the parameter variables, the results show that: according to the first formula calculated by the empirical formula, the relative error of the water inflow are 5% and 10%, the corresponding rate of change of the parameter variable are 0.109 and 0.14; according to the second formula calculated by the actual survey calculation, when the relative error of the water inflow are 5% and 10%, the change rates of the corresponding parameter variables are 0.274 and 0.131 respectively. Under the identical error condition, the maximum value change rates of the parameter are greater than the change rates of the parameter when the minimum value is calculated, which is beneficial to the upper limit of the calculation of mine water inflow.

1. Introduction

Water damage is one of the problems that must be paid attention to during mining [1,2]. Large-well method is a commonly used method for predicting mine water inflow [3,4]. However, during the process of calculation, the calculation results are uncertain subject to the constraints of hydrogeological conditions, which affects the accuracy of the final calculated value [5-7]. Facing the problem of water inrush in mining of current days, it is of great significance to rapidly analyse the impact of uncertainty on water inflow prediction for safe production [8,9]. The inaccuracy of water inflow prediction caused by the complexity of hydrogeological conditions in actual mining areas has become one of the key issues of groundwater science [10-13]. In the study of uncertain problems, random, grey, fuzzy mathematics, etc. are commonly used mathematical methods [14,15]. In the actual calculation process, due to the incompleteness of the field data, it is difficult to obtain the probability density function using random mathematical methods, which causes large errors in the calculation of water inflow, and that is unfavourable for mine exploitation[16,17]. The use of deterministic large-well
method calculations also does not rule out the effects of changes in hydrogeological conditions. Starting from the uncertain boundary, combined with the non-probability set theory convex model method [18,19], the formula for calculating the water inflow with upper and lower limits is derived. From the initial deterministic simple large-well method formula to the consideration of parameter variation uncertainty fusion large-well method formula change, the uncertain influence factors in the actual calculation process are expressed in the form of interval change, and integrated into the calculation formula. Using the interval uncertainty formula, the data requirements are more in line with the measured data of mining area. The calculation of mine water inflow provides a new way to predict water inflow in complicated conditions, which has certain reference significance for mining.

2. Non-Probability Set Theory Convex Model Method

Let $\beta^0=(\beta_1^0, \beta_2^0, ..., \beta_n^0)^T$ be the statistical mean or center number of the parameter $\beta=(\beta_1, \beta_2, ..., \beta_n)^T$ of the analytical model. The parameter can be expressed as:

$$\beta = \beta^0 + \varepsilon$$

According to the mathematical theory of the set theory convex model, the statistical mean of the function in the bounded uncertain parameter $\beta=(\beta_1, \beta_2, ..., \beta_n)^T$ is $\beta^0=(\beta_1^0, \beta_2^0, ..., \beta_n^0)^T$ is carried out near Taylor and retains one item, which gives:

$$\varphi(\beta) = \varphi(\beta^0 + \varepsilon) = \varphi(\beta^0) + \sum_{i=1}^{n} \frac{\partial \varphi(\beta_0)}{\partial (\beta_i)} \varepsilon_i = \varphi_0 + f^T \varepsilon$$

It is assumed that the uncertainty or unascertained quantity $\varepsilon$ of the bounded uncertain parameter that varies near the statistical mean varies within the bounded convex set of equation (3), namely:

$$C(\varepsilon, \delta) = \{ \varepsilon: \varepsilon^T M \varepsilon \leq \delta^2 \}$$

where: M-positive definite matrix;

$\delta$-positive real number.

When the bounded uncertain parameter $\varepsilon=(\varepsilon_1, \varepsilon_2, ..., \varepsilon_n)^T$ varies within the bounded convex set of equation (3), the upper bound of the bounded uncertain response of the function can be determined. The Taylor's first-order approximate solution $\varphi=\varphi_{\text{max}}$ and the lower bound of the response Taylor's first-order approximate solution $\varphi=\varphi_{\text{min}}$, namely:

$$\varphi=\varphi_{\text{max}} = \max_{\varepsilon \in C(\varepsilon, \delta)} \{ \varphi_0 + f^T \varepsilon \}$$

$$\varphi=\varphi_{\text{min}} = \min_{\varepsilon \in C(\varepsilon, \delta)} \{ \varphi_0 + f^T \varepsilon \}$$

Mathematical optimization theory has confirmed that the extreme values of equations (4) and (5) will be reached at the boundary of the ellipsoid region represented by equation (3). Let the Lagrangian function be:

$$L=\varphi_0 + g^T \varepsilon + w(\varepsilon^T M \varepsilon - \delta^2)$$

where: w-Lagrange multiplier. The necessary conditions for taking the extreme value are:

$$\frac{\partial L}{\partial \varepsilon} = f + 2wM\varepsilon = 0$$

Finished up:

$$w^2 = \frac{1}{4\delta^2} f^T M^{-1} f$$

$$w = \pm \frac{1}{2\delta} \sqrt{f^T M^{-1} f}$$

Bringing equation (9) into equations (4) and (5) yields [13]:

...
\[
\phi = \phi_{\text{max}} = \phi_0 + \delta \sqrt{\sum_{j=1}^{n} \left( \frac{n}{2} \Delta \beta_j \frac{\partial \phi^0}{\partial \beta_j} \right)^2}
\] (10)

\[
\phi = \phi_{\text{min}} = \phi_0 + \delta \sqrt{\sum_{j=1}^{n} \left( \frac{n}{2} \Delta \beta_j \frac{\partial \phi^0}{\partial \beta_j} \right)^2}
\] (11)

3. Prediction Formula for Interval Water Inflow

The formula of confined water is one of the basic formulas in groundwater dynamics, and it is also the most commonly used mathematical formula for calculating mine water inflow. The expression is often used to calculate the amount of water inflow:

\[
Q = 2.73 \frac{KMS}{\eta \left( \frac{R_0}{r_0} \right)}
\] (12)

\[
R = 105 \sqrt{K}
\] (13)

\[
r_0 = \eta \left( \frac{a+b}{4} \right)
\] (14)

\[
R_0 = r_0 + R
\] (15)

where:
- \(Q\) - water inflow, \(m^3/d\);
- \(K\) - hydraulic conductivity (m/d);
- \(a, b\) - working face length, width, m;
- \(\eta\) - calculation factor, see Table 1;
- \(r_0\) - reference radius;
- \(R\) - influence radius;
- \(R_0\) - large well reference radius;
- \(M\) - thickness of the aquifer;
- \(S\) - drawdown of the water table.

| \(\frac{b}{a}\) | 0   | 0.20 | 0.40 | 0.60 | 0.8 | 1.00 |
|----------------|-----|------|------|------|-----|------|
| \(\eta\)       | 1.00| 1.12 | 1.14 | 1.16 | 1.18| 1.18 |

According to the formula (12), the aquifer is close to the horizontal, the distance between the top and bottom plates is relatively uniform and the water medium is relatively uniform, avoiding the fluctuation of water flow in the aquifer, and there is a certain range in the pumping well with a circular long radius head boundary. In the actual mining area, during the mining process, due to the possible existence of faults, geological anomalies, etc., the formation permeability coefficient of the study area is locally increased or decreased, and the aquifer is therefore characterized by non-uniformity, it is difficult to achieve a circular or a rule head boundary.

However, in the current prediction of mine water inflow, with a series of measures such as drainage in the mining area, the groundwater level changes gradually toward a gentle process. At this time, the groundwater flow is similar to the steady flow, and the heterogeneous aquifer is directly regarded as stable. The flow is calculated using the traditional large-well method. Equation (12) is performed under the condition that the calculation object is approximated to a steady flow in the calculation process. In equation (12), \(Q\) is nonlinear with \(M, S, K,\) and \(r_0\). In mathematical processing, according to the law of the implicit function:

\[
X = \frac{R + r_0}{r_0}
\] (16)
\[ Y = \log (1 + \frac{R}{r_0}) \]  

(17)

Obtain the derivative of the flow \( Q \) versus the variables \( K, M, S, \) and \( r_0 \).

\[ \frac{\partial Q}{\partial K} = \frac{Q}{K} X \frac{Y}{\beta} \]  

(18)

\[ \frac{\partial Q}{\partial r_0} = \frac{Q}{r_0} X \frac{Y}{\beta} \]  

(19)

\[ \frac{\partial Q}{\partial M} = \frac{2.73KS}{Y} \]  

(20)

\[ \frac{\partial Q}{\partial S} = \frac{2.73KS-QX}{YS} \]  

(21)

In actual mining, some mines can directly determine the magnitude of the impact radius based on the previous exploration data, instead of using equation (13). In this case, there will be 5 variables as follows:

\[ \frac{\partial Q}{\partial K} = Q \]  

(22)

\[ \frac{\partial Q}{\partial r_0} = \frac{Q}{r_0} X \frac{Y}{\beta} \]  

(23)

\[ \frac{\partial Q}{\partial M} = \frac{2.73KS}{Y} \]  

(24)

\[ \frac{\partial Q}{\partial S} = \frac{2.73KS-QX}{YS} \]  

(25)

\[ \frac{\partial Q}{\partial R} = \frac{Q}{YR} \]  

(26)

By substituting equations (18) - (21) and equations (22) - (26) into equations (10) and (11), an interval calculation formula (Table 2) can be obtained according to both the empirical formula and the actual survey.

**Table 2. Interval formula of water inflow.**

| Ways to obtain R          | Formula 1 | Formula 2 |
|---------------------------|-----------|-----------|
| Empirical formula         | \[ Q_0 \pm \frac{\pi}{2} \sqrt{[7.45K^3M^3S^3\beta^2_\text{M}^2 + Q_0^2\beta^2_{\text{R}} + Q_0^2\beta^2_{\text{S}} + (2.73KS-QX)^2\beta^2_S]} \] |
| Actual survey             | \[ Q_0 \pm \frac{\pi}{2} \sqrt{[\beta^2_{\text{K}}(Y-0.5X)^2\beta^2_{\text{K}}^2 + Q_0^2\beta^2_{\text{M}}^2 + Q_0^2\beta^2_{\text{R}}^2 + Q_0^2\beta^2_{\text{S}}^2 + (2.73KS-QX)^2\beta^2_S]} \] |

Remark: \( Q_0 \) is the corresponding result when the corresponding variable takes the center of the interval. \( \beta_{\text{K}} = \Delta K/\Delta K_0, \beta_{\text{M}} = \Delta M/\Delta M_0, \beta_{r_0} = \Delta r_0/\Delta r_0, \beta_{R} = \Delta R/\Delta R_0, \beta_{S} = \Delta S/\Delta S_0, \beta_{\text{K}}, \beta_{\text{M}}, \beta_{r_0}, \beta_{R}, \beta_{S} \) are the rates of change of the corresponding variables. The '+' in '±' corresponds to the upper limit (maximum value) of the water influx change interval, and the '-' corresponds to the lower limit (minimum value) of the water influx change interval.

The rate of change of the five variables \( S, M, K, R \) and \( r_0 \) comprehensively represents the variation interval of the variable. It is convenient to use the formula to calculate the amount of water inflow after considering the influence of the parameter change.

**4. Effectiveness of Interval Water Forecasting Formula**

Equations (10) and (11) are based on the first-order Taylor series and optimization theory. The rate of change of the formula in Table 2 obtained on this basis must not be infinite. The actual upper and lower limit of the response interval are obtained by the Monte Carlo method to analyze the validity of the formula and the limit of the rate of change in Table 2. The calculation results are shown in Table 3.
The "absolute value of the maximum relative error" in "the allowable rate of change of the absolute value of the maximum relative error is smaller than the value of the variable" is the relative error between upper limit value $Q^*$ of the water inflow according to the formula in table 2 and actual calculation value by using Monte Carlo method.

The absolute value of the maximum relative error $= \left| \frac{Q-Q^*}{Q} \right|$ (25)

In the calculation process (show in table 2), the rate of change starts from 0 to 0.5 with an increases at a pitch of 0.01.

For the calculation of formula 2, for each set of data, multiply the influence radius value calculated by equation (13) by 4 as $R$, and the water inflow amount is calculated by equation (12), and other parameters are unchanged. Table 3 shows the maximum rate of change of the corresponding variables of the five sets of test data for the two interval water inflow prediction formulas at the error levels of 0.05 and 0.1. For example, for data 1, when formula 1 is used to calculate the water inflow, if required the absolute value of the relative error of the calculated maximum value does not exceed 0.05, then the rate of change of the four parameters in equation (12) cannot exceed 0.11; if the absolute value of the relative error required to calculate the maximum value does not exceed 0.1, then the rate of change of the four parameters in (12) cannot exceed 0.15.

It can be seen from table 3 that under the same error requirement, the upper limit is greater than the lower limit by using equations 1 and 2 corresponds. That is, when the rate of change of the variable is relatively large, the reliability of the calculated maximum water inflow is higher than the minimum water inflow.

**Table 3. Calculation rate of water inflow under different error conditions.**

| Formula | Error boundary | Parameter | Data1 | Data2 | Data3 | Data4 | Data5 |
|---------|----------------|-----------|-------|-------|-------|-------|-------|
| 1       | The allowable rate when the absolute value of the maximum relative error is less than $\beta$ | $Q_0$ (m$^3$/d) | 1200  | 2400  | 4000  | 7000  | 9000  |
|         | $\beta=5\%$  | $M_0$ (m)  | 40    | 80    | 120   | 150   | 180   |
|         | $\beta=10\%$ | $S$ (m)    | 10    | 25    | 40    | 60    | 80    |
|         | $\beta=5\%$  | $K$ (m/d)  | 0.0005| 0.005 | 0.008 | 0.01  | 0.05  |
| 2       | The allowable rate when the absolute value of the minimum relative error is less than $\beta$ | $\beta=5\%$  | 0.109 | 0.114 | 0.158 | 0.167 | 0.180 |
|         | $\beta=10\%$ | 0.146     | 0.237 | 0.257 | 0.266 | 0.304 |
|         | $\beta=5\%$  | 0.165     | 0.172 | 0.257 | 0.281 | 0.319 |
|         | $\beta=10\%$ | 0.130     | 0.178 | 0.212 | 0.272 | 0.338 |
|         | $\beta=5\%$  | 0.274     | 0.285 | 0.355 | 0.366 | 0.392 |
|         | $\beta=10\%$ | 0.131     | 0.198 | 0.226 | 0.268 | 0.297 |
|         | $\beta=5\%$  | 0.178     | 0.199 | 0.210 | 0.231 | 0.275 |
|         | $\beta=10\%$ | 0.162     | 0.193 | 0.276 | 0.278 | 0.343 |

When it is necessary to obtain the upper and lower limits of the water inflow within the larger change interval and meet certain accuracy requirements, the large interval can be divided into cells, and then the formula in table 2 can be used between the cells.

5. Application
The aquifer in the Jurassic of one mine is mainly composed of coarse sandstone. The average elevation of the aquifer is 838.18m. According to the drilling data of the working face, the average thickness of the Jurassic system is 108m. The aquifer has a maximum permeability coefficient of...
0.0654 m/d, a minimum of 0.00043 m/d, an average of 0.02265 m/d, a single-hole water inflow of 0.05-3.85 L/s, and a unit water inflow of 0.0015-0.1171 L/s·m. The layer is water-invariant and is an indirect water-filled aquifer mined by the 3-1 coal seam and is also the main aquifer. There are 14 normal faults, 1 reverse fault, 1 fault with a drop of more than 10m; 2 faults gap from 10m to 5m; 2 faults gap from 5m to 3m; the remaining 9 faults are less than 3m. After the working face of the mine is drained, the pressure outside the funnel boundary is confined water, while the inside of the funnel is unconfined water. The drawdown of the groundwater table is 351.45m, which comes from the result of confined water level (1189.63m) minus water floor level (838.18m). Using the equation 1 to calculate the corresponding water inflow change interval under the variation of each variable interval, considering the incompleteness of the existing mine data and the uncontrollability of the actual hydrogeological conditions, $\beta_s=0.05$, $\beta_M=\beta_k=\beta_r=0.2$, that is, the variation range of the water level depth is [7.64, 84.29], the variation range of aquifer thickness is [38.3, 188.04], the variation range of permeability coefficient is [0.00043, 0.0654], and the variation range of equivalent radius (m) is [972.67, 2059.4]. After calculation, the variation ranges of water inflow (m³/d) is [9.48, 5127.56]. The amount of water inflow calculated by using the average of various variables is:

There is a certain difference in the amount of water inflow calculated by using the maximum, minimum and average values of various variables, which indicates that the variable interval has certain practical significance for the prediction of water inflow.

In order to further explore the relationship between the calculated results of equation 1 and the rate of change of each variable, a correlation analysis is made between the maximum and actual maximum values calculated by the equation 1, as shown in Figure 1.

![Figure 1](image_url)  

**Figure 1.** Relative error diagrams of the maximum water inflow.

It can be concluded from Figure 1 that there is a nonlinear relationship between the relative error which is calculated by the empirical formula and the rate of change of the variable.

In Figure 1, when the rate of change of the variable is 0.3, the error of the maximum value is substantially less than 20%; when the rate of change is 0.2, the error of the minimum value is substantially less than 10%.

The actual mine case reveals that the interval water inflow calculated by the empirical formula is adaptive to each other within a certain range of variable changes.

6. Conclusion

(1) From the perspective of uncertainty analysis of interval, combined with the theory of non-probability set, the prediction formula of confined water inflow under two conditions is derived, which considered the traditional formula of large-well method, and as the parameters change, the amount of water influx also changes in a certain interval.
(2) The allowable rate of change of the variables is 0.109 and 0.146 when the maximum error of the maximum (minimum) water inflow are 5% and 10% by using the formula 1; The allowable rate of change of the variables is 0.274 and 0.131 when the maximum error of the maximum (minimum) water inflow are 5% and 10% by using the formula 2; under the same error condition, the rate of change of the parameter when the two formulas calculate the maximum value is greater than the rate of change of the parameter when the minimum value is calculated, which is beneficial to The upper limit of the mine water inflow calculation.

(3) The mathematical formula derivation and the calculation of the mining area example reveal that the calculation formula of interval water inflow can be more convenient and accurate to predict the magnitude of water inflow, and provide some reference for the calculation of water inflow in mining area.

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