KN SIGMA TERMS, STRANGENESS IN THE NUCLEON, 
AND DAΦNE†

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1. INTRODUCTION.

This talk will consist of three parts: in the first I shall stress the importance of the $KN$ sigma terms in fixing the composition of the baryonic ground states, and particularly the scalar, strange–quark density in the nucleon. I shall also briefly cover some of the reasons, besides QCD, why it is important to know with some accuracy such strong–interaction parameters, and delve on why they can not simply be inferred from the (already rather well known) $\pi N$ one.

In the second chapter, I shall review the situation of their extraction from data, using as a guideline the clearer $\pi N$ case, and discuss merits and shortcomings of three different methods. I shall try to cover the last two decades of studies in the field, i.e. those starting with A.D. Martin’s analysis of low-energy $KN$ systems\(^1\), and show that, though there is still no consensus, the most reliable methods indicate a large value for the isoscalar parts of the $KN$ sigma terms, and can only put rather generous bounds on their isovector parts.

A third, final section will be dedicated to experimental outlooks on the future of low-energy $KN$ physics, focussing on the possibilities that are opening up at $\phi$–factories: due to their high design luminosities and to the kaon production mechanisms, these will be almost monochromatic sources of extremely–low–background, low–momentum kaons. The challenge is how to exploit these kaons for scattering experiments in a geometry radically different from those we have been accustomed up to now. Not being a rugged experimentalist, I shall limit myself to a conceptual sketch of a dedicated detector and to some “back–of–the–envelope” calculations, which I hope will show that high–statistics measurements should indeed be feasible with advanced, currently available technologies. I shall also briefly list the relevant measurements already possible at the existing three DAΦNE experiments DEAR, FINUDA and KLOE.
2. \(\sigma\)-TERMS AND "MEASUREMENTS" OF THE STRANGE–QUARK SCALAR DENSITY OF THE NUCLEON.

The problems, posed by the \(\pi N\) \(\sigma\)-term being larger than expected on the basis of the simplest quark–model pictures of the nucleon, have been with us for quite a while\(^{2}\), before Donoghue and Nappi\(^{3}\) pointed to this fact as to an indicator of a large \(\bar{s}s\) component in the nucleon sea, foreign to the then standard quark–model pictures, but not unexpected in a Skyrmion picture of the nucleon\(^{4}\).

However, only a few authors have stressed\(^{5}\) that the \(\pi N\) \(\sigma\)-term is not the best indicator of a scalar strange–quark density in the nucleon sea, but just the quantity sensitive to the latter that, as of today, we know the best.

Indeed, the evidence it provides is quite indirect, resting on two other assumptions about the precise values of both the quark–mass ratio \(2m_s/(m_u + m_d)\) and the \(SU(3)_f\)-breaking terms in the octet–baryon masses. What is "measured" is indeed the proton expectation value of the operator

\[
\sigma_{\pi^+\pi^-}(x) = \frac{1}{2} (m_u + m_d) \left[ \bar{u}(x)u(x) + \bar{d}(x)d(x) \right],
\]

which clearly does not gauge directly the strange–quark density \(\bar{s}(x)s(x)\). The above operator is a pure isoscalar: the corresponding operator for neutral pions

\[
\sigma_{\pi^0\pi^0}(x) = m_u \bar{u}(x)u(x) + m_d \bar{d}(x)d(x) \tag{1'}
\]

has a small, additional isovector part coming from the \(SU(2)_f\)-violating part of the Hamiltonian, and related via \(SU(2)_f\) to the soft–pion limit of the charge–exchange, crossing–even amplitude. This part is expected to be suppressed by at least one order of magnitude with respect to the former, even when exerting the caution suggested by the recent observation of sizeable departures from the Gottfried sum rule in deep inelastic scattering\(^6\).

For sake of brevity I shall neglect here these \(SU(2)_f\)-violating effects, being their expected contributions well below present experimental (and theoretical) capabilities. To gauge directly the scalar density \(\bar{s}(x)s(x)\) one should turn instead to operators like

\[
\sigma_{K^+K^-}(x) = \frac{1}{2} (m_s + m_u) \left[ \bar{u}(x)u(x) + \bar{s}(x)s(x) \right] \tag{2}
\]

(the analogous operator for \(K^0\)'s can be obtained replacing \(u\)'s by \(d\)'s in Eqn. 2), and to the corresponding one for the octet component of the \(\eta\)-meson

\[
\sigma_{\eta_s\eta_s}(x) = \frac{1}{3} \sigma_{\pi^0\pi^0}(x) + \frac{4}{3} m_s \bar{s}(x)s(x), \tag{3}
\]
which, even if not directly measurable, still plays an important role in meson–condensation phenomena in dense nuclear matter\(^7\) and is directly related to the chiral–symmetry breaking part of the standard–model Hamiltonian

\[
H_{SB}(x) = \frac{3}{4} \left[ \sigma_{\eta s \eta s}(x) + \sigma_{\pi^0 \pi^0}(x) \right],
\]  

(4)

responsible for the shift of the nucleon mass from its chiral–symmetry value (at lowest–order in the symmetry breaking),

\[
\Delta M_N^{(0)} \simeq \langle N | H_{SB}(0)_{I=0} | N \rangle = \frac{3}{4} \left[ \Sigma_{\pi^\pm N} + \Sigma_{\eta s N} \right].
\]  

(5)

Last but not least, we also wish to mention here the dominantly isovector operator \(\sigma_{\pi^0 \eta s}(x)\), of some relevance in the study of the \(I = 1 \bar{K}N t\)–channel amplitudes, given as

\[
\sigma_{\pi^0 \eta s}(x) = \frac{1}{\sqrt{3}} \left( m_u \bar{u}(x) u(x) - m_d \bar{d}(x) d(x) \right).
\]  

(5’)

Separating \(H_{SB}(x)\) into its singlet and both isoscalar and isovector octet components respectively as

\[
H_{SB}(x) = H_{SB}^{(0)}(x) + H_{SB}^{(8)}(x) + H_{SB}^{(3)}(x),
\]  

(6)

where

\[
H_{SB}^{(0)} = m_0 S_0 = \frac{1}{3} \left( m_u + m_d + m_s \right) (\bar{u}u + \bar{d}d + \bar{s}s),
\]  

(6’)

\[
H_{SB}^{(8)} = m_8 S_8 = \frac{1}{6} \left( m_u + m_d - 2m_s \right) (\bar{u}u + \bar{d}d - 2\bar{s}s),
\]  

(6’’)

and

\[
H_{SB}^{(3)} = m_3 S_3 = \frac{1}{2} \left( m_u - m_d \right) (\bar{u}u - \bar{d}d),
\]  

(6’’’)

we can express the meson–nucleon \(\sigma\)–terms (considering only “elastic” channels, and charged pions and kaons) as

\[
\Sigma_{\pi^\pm N} = \frac{m}{m_s - m} M_8 \frac{1}{1 - y},
\]  

(7)

\[
\Sigma_{K^\pm N}^{(0)} = \frac{m_s + m_u}{m_s - m} \frac{1}{4} M_8 \frac{1 + y}{1 - y},
\]  

(8)

and

\[
\Sigma_{\eta s N} = \frac{m_s}{m_s - m} \frac{2}{3} M_8 \frac{y + m/(2m_s)}{1 - y},
\]  

(9)
where we have neglected small, $SU(2)_f$–violating terms from the nucleon wave function, introduced the “nuclear isospin” notation $\Sigma^{(I=0,1)} = 1/2 \left[ \Sigma_p + (-1)^I \Sigma_n \right]$ (useful to work in nuclear matter) plus Gasser’s notation $y = 2\langle N|\bar{s}s|N\rangle/\langle N|(\bar{u}u + \bar{d}d)|N\rangle$, and defined

\[
M_8 = -\frac{1}{2} \int d^3 \vec{x} \left[ \langle p|\{3 H^{(8)}_{SB}(\vec{x},0)\}|p\rangle + (p \to n) \right].
\]  

(10)

The scale of “isovector” combinations (and $SU(2)_f$–violating terms) is set instead by

\[
M_3 = -\frac{1}{2} \int d^3 \vec{x} \left[ \langle p|\{\sqrt{3} \frac{m_8}{m_3} H^{(3)}_{SB}(\vec{x},0)\}|p\rangle - (p \to n) \right],
\]  

(11)

which we shall not use extensively, but which has however to be kept under close scrutiny to ensure the absence of “large” $SU(2)_f$–violating terms in the soft–meson limit. These “isovector” parts can be easily written down using the above notation\(^9\), but will not be considered here as they are independent on $y$; they are not negligible, at least for kaons, and influence detailed analyses: for instance their neglect in $K$–condensation calculations\(^7\) has masked till now an interesting consequence for supernovae\(^10\) (probably already seen in the IMB and Kamiokande neutrino signals from Shelton’s supernova, SN 1987A\(^11\)), i.e. the possible presence of an “energetic”, pure $\nu_\mu$ signal a short time after the “thermal” neutrino burst from the gravitational collapse.

Putting together eqs. (5), (7) and (9), one obtains

\[
\Delta M_{SB}^{(0)}(0) \simeq \frac{m_s}{m_s - m} M_8 \frac{m/m_s + y/2}{1 - y} = \Sigma_{\pi \pm N} \left( 1 + \frac{m_s}{2m} y \right),
\]  

(12)

so that even a value of $y$ as small as 0.2 can make $\Delta M_{SB}$ quite larger than $\Sigma_{\pi \pm N}$, the traditional, quark–model expectation for the chiral–symmetry–breaking shift in the nucleon mass.

The two mass scales (10) and (11) were traditionally calculated from octet–baryon masses at lowest order in the symmetry breaking: they can however receive non–negligible corrections from higher–order terms. Already Gasser\(^8\) found a sizeable correction to $M_8$, working at one loop in chiral perturbation theory: we expect that going to higher orders, or higher number of loops, could increase $M_8$ even further (see the recent re–evaluation of the “scalar” pion form factor\(^12\), yielding a very “soft” result, in line with our dispersive estimate of eighteen years ago\(^13\)).

Note that higher orders in the symmetry breaking (with $m_u \neq m_d$) break also the isospin invariance of the nucleon wave function, so that $\langle p|H^{(8)}_{SB}|p\rangle \neq \langle n|H^{(8)}_{SB}|n\rangle$ and
\[ \langle p | H_{SB}^{(3)} | p \rangle \neq -\langle n | H_{SB}^{(3)} | n \rangle : \text{however the size of the discrepancy is of } O(m_3/m_8 \simeq 2 \cdot 10^{-2}) \]

with respect to the $SU(2)_f$-symmetric values, and thus not as important as the rest of the contribution.

To try and estimate these higher-order effects, independently of either the Bern group approach, or Skyrmion phenomenologies of all, different kinds, I have taken the rather naive approach of working in a Hamiltonian formalism, and used second-order Raleigh–Schrödinger perturbation theory. Restricting the mixing of the baryon octet to just one representation for each non-exotic multiplicity, I have found for the two mass-breaking scales

\[ M_8 = [626 \, \text{MeV}] + [(200 \pm 20) \, \text{MeV} + 8 \cdot \Sigma] \]

and

\[ M_3 = [132 \, \text{MeV}] - [(35 \pm 6) \, \text{MeV}] \]

where in each expression the first and second square bracket represents, respectively, the first- and second-order flavour-symmetry-breaking contribution, and $\Sigma \geq 0$ is the unitary–singlet–admixture term in the mass of the $\Lambda$–hyperon. The latter cannot vanish if we are to reproduce, in the same formalism, flavour–symmetry–breaking effects in the axial–vector couplings\textsuperscript{14,15}, and is better to be strongly limited from above if the octet has to stay lighter than the decuplet in the symmetry limit: one can thus estimate $M_8$ to lie between a minimum of about 850 MeV and a maximum which cannot exceed 1,150 MeV, or $M_8 \simeq (1,000 \pm 150) \, \text{MeV}$, somewhat above Gasser’s one-loop estimate\textsuperscript{8}, which can be translated in our language into the value $M_8 = (840 \pm 120) \, \text{MeV}$.

Note that to extract $y$ from eq. (7) one would also have to know the strange–to–non–strange quark mass ratio $2m_s/(m_u + m_d)$, for which Gasser used (consistently) the one–loop result $m_s/m \simeq 25$. However, QCD sum rules\textsuperscript{16} give a wider range of values for this ratio, so as to make its precise value questionable: a careful assessment of all the uncertainties makes a value of $y \simeq 0$ not incompatible with the value $\Sigma_{\pi \pm N} \simeq 50 \, \text{MeV}$, on which consensus seem to have been finally reached among the different methods\textsuperscript{17,18}, if all theoretical uncertainties both on $M_8$ and on $m_s/m$ are pushed toward their upper limits.

By inspecting eq. (8) one can see that: i) $\Sigma_{K \pm N}^{(0)}$ is very little dependent on $m_s/m$ for not too small values of the ratio, and ii) much more dependent on $y$ than $\Sigma_{\pi \pm N}$. The sad note is that, despite all efforts including mine, we are far from reaching consensus but for its order of magnitude, expected to be of several hundred MeV’s. Since this lack of consensus is due in part to the theoretical difficulties inherent to an extrapolation over
much larger four-momentum intervals than in the $\pi N$ case, and in part to the poorer information coming from experiments on low-energy $\bar{K}N$ systems, we shall devote the following two sections first to a review of the extrapolation methods, and then to an outlook on possibilities opening up at the DAΦNE $\phi$-factory.

3. METHODS OF EXTRAPOLATION TO $q^2 = t = \omega^2 = 0$: A SUBJECTIVE REVIEW.

At the first presentation of such a summary, in 1991 at Bad Honnef, I updated the (unpublished) report presented in 1982 at the Black Forest Meeting in Todtnaueberg, touching only passingly results and methods where no improvements had been registered, and concentrating instead on those which had been improved upon after that date. Here the main improvement over Bad Honnef will be a revision of the “scalar form factors” following their more recent theoretical re-evaluations and the re-analyses of low-energy $\pi\pi$ data prompted by the activities of the DAΦNE Theory Group.

The matrix elements of the operators discussed in the previous section, generally known as the $\sigma$-terms, are better to be seen (from $su(3) \times su(3)$ current algebra and PCAC) as the zero-energy, zero-momentum-transfer values of the scattering amplitudes for massless mesons, shorn of their eventual pseudovector-coupling Born terms. For a process $a + B \rightarrow b + B'$ (where the mesons $a$ and $b$ are to be taken off their mass shells), the kinematics are defined by the variables $\vec{P} = \{q_a^2, q_b^2, \omega, t\}$, where $\omega = (s-u)/2(M+M')$, since energy-momentum conservation fixes $s+u = M^2 + M'^2 + q_a^2 + q_b^2 - t$, and the $\sigma$-term is thus defined as

$$\Sigma_{aB \rightarrow bB'} = \langle B' | \sigma_{ab}(0) | B \rangle = -\lim_{\vec{P} \rightarrow \vec{O}} \frac{f_a f_b}{2} \left[ A_{aB \rightarrow bB'}(\vec{P}) - A_{aB \rightarrow bB'}^{\text{Born}(pv)}(\vec{P}) \right],$$

where $\vec{O} = \{0, 0, 0, 0\}$.

At least three methods have been widely used in the literature (I choose deliberately not to mention those less recommendable or of dubious validity): i) the "improved" Altarelli–Cabibbo–Maiani technique, ii) "modified" Fubini–Furlan sum rules applied to $K^-$–nucleus scattering lengths, and iii) a "unitarized" version of the Cheng–Dashen relation.

3.1. THE "IMPROVED" ALTARELLI–CABIBBO–MAIANI METHOD.

The method originally devised by Altarelli, Cabibbo and Maiani to extract the $\pi N$ $\sigma$-term, and subsequently extended to the $KN$ ones by Reya and by Violini and
coworkers$^{22}$, consists essentially in continuing, from the threshold to $q^2 = t = \omega^2 = 0$, the elastic, crossing–even amplitudes from which all low–mass, pseudovector–coupling pole terms have been explicitly subtracted, considering such a difference to be adequately described by a truncated power series of the above invariants. The original, essential shortcomings of the seminal papers$^{21,26,27}$ were soon corrected using, instead of the amplitudes at threshold (of course a singular point), the zero–energy amplitudes derived from fixed–$t$ dispersion relations$^{22,28}$.

The quality of such an approach for the $KN$ systems can be gauged by the estimates $\Sigma_{K^\pm p} = 175 \pm 890 \text{ (sic)}$ MeV and $\Sigma_{K^\pm n} = 718 \pm 460$ MeV reported by G. Violini and his coworkers$^{22}$: they also give, for the “isoscalar” part $\Sigma_{K^\pm N}^{(0)}$ the value $599 \pm 374$ MeV: error estimates are thus of the same order or even larger than the $\sigma$–terms themselves. The same authors point out that the method requires large cancellations between terms each one of which, though correlated to the others, carries a large uncertainty, mainly due to the poor quality of what where (and still are) the best available low–energy $\bar{K}N$ data.

Note that the same method applied to $\pi N$ amplitudes gave$^{27,28}$ the result (rounding figures and giving a personal re–evaluation of the original errors) $\Sigma_{\pi^\pm N} \simeq 50 \pm 10$ MeV, not far from modern estimates coming from different methods$^{17,18}$: in this case the difference, apart from the higher quality of the data, reached already in the late seventies, is to be attributed mostly to the analytic structure of the low–energy $\bar{K}N$ unphysical region, responsible of the huge cancellations present in the $KN$ case, and totally absent in the $\pi N$ one.

3.2. THE FUBINI–FURLAN ANALYSIS OF MESON-NUCLEUS SCATTERING LENGTHS.

The second method to be briefly reviewed here has been introduced by this author a couple of decades ago$^{29}$, and later re-considered$^{17,24}$ for application to the world set of mesonic–atom data. It was originally motivated by the observations that current–algebra sum rules, derived in the collinear frame by Fubini and coworkers$^{30}$ (relating the $\sigma$–terms to the amplitudes at threshold), take a much simpler form if one can send the target mass to infinity, and that the extreme non–smoothness of the low–energy, meson–nucleus scattering amplitudes, due to nuclear excitations, can be easily eliminated, summing these excitations with standard nuclear sum–rule techniques$^{29}$. Furthermore, divergences (appearing in QCD from integrations up to infinite energy and virtuality) can be avoided by
using a finite–contour version of the sum rules, owing to the large mass gap present in the pseudoscalar–meson mass spectra.

For $\Sigma_{\pi \pm N}$ such a method produces an estimate of about $48 \pm 9$ MeV, reproducing nicely all detailed features of the data\(^{17}\) (down to typically nuclear, shell–structure effects), available for separated isotopes up to $^{27}$Al, plus an extrapolation to threshold from a phase–shift analysis of $\pi^{\pm} - ^{40}$Ca elastic scattering.

For kaonic atoms one can use data up to uranium (due to the dominantly $S$–wave nature of the interaction), but generally these are available for natural isotopic mixtures only, so that both isotopic–spin dependence and shell-structure effects can not be separated out. Depending on assumptions on the renormalization of the hyperon axial couplings in nuclear matter, one estimates $\Sigma_{K^{\pm}N}^{(0)}$ to range from 480 to 650 MeV, with purely statistical errors from the fits\(^{23}\) of the order of 20 to 30 MeV.

The comparison with the $\pi N$ case shows clearly that this analysis is limited by its systematics, which can not be resolved (as done in the $\pi N$ case) as long as we can not use data from isotopically separated atomic species; thus the different effects are lumped together in a global fit to the mass–number dependence, which gives too large a weight to the heaviest–atom data, precisely those for which the optical–potential model used to extract the kaon–nucleus scattering lengths is more open to questioning\(^{31}\).

3.3. THE “UNITARIZED” VERSION OF THE CHENG–DASHEN THEOREM.

The third approach is an improvement over the rather oversimplified, linear expansion originally employed by Cheng and Dashen for the $\pi N$ amplitude\(^{32}\), and improperly extended to $KN$ ones by some authors\(^{33}\); however, the original idea can be correctly rephrased by stating that all pseudovector Born terms of the spin–averaged scattering amplitudes vanish exactly along the line $\Gamma_{CD}$, defined by $q_1^2/m_1^2 = q_2^2/m_2^2$, $\omega^2 = 0$, $t = q_1^2 + q_2^2$, so that this line can be used as an extrapolation path to go from the current algebra point $q_1^2 = q_2^2 = \omega^2 = t = 0$ to the mass–shell point $\omega^2 = 0$, $t = m_1^2 + m_2^2$.

The major contributions to the amplitude curvature along this line are of course expected from the low–mass portion of the $t$–channel cuts\(^{13}\), while minor contributions are also expected from the discontinuities in the mass variables $q_i^2$. The improvement to the extrapolation comes from the further observation that, if the discontinuities are dominated by the $S$–waves and if one can write these latter in an $N/D$ decomposition,
Watson’s theorem holds both on and off the mass shell, and one has\textsuperscript{13,25}

\begin{equation}
A^+_{\pi\pm N}(2m^2_\pi) \simeq \frac{2\Sigma_{\pi\pm N}}{f_\pi(0)^2} \Phi_{\pi\pi}(2m^2_\pi) \left(\frac{m^2_{\pi I}}{m^2_{\pi I} - m^2_\pi}\right)^2,
\end{equation}

and

\begin{equation}
A^+_{K\pm N}(2m^2_K) \simeq \frac{2\Sigma^{(I)}_{K\pm N}}{f_K(0)^2} \Phi^{(I)}_{KK}(2m^2_K) \left(\frac{m^2_{K I}}{m^2_{K I} - m^2_K}\right)^2,
\end{equation}

where the Omnès function in the second case is related to the first one by the $N/D$ decomposition as

\begin{equation}
\Phi_{KK}^{(I=0)}(t) = 1 + R \cdot [\Phi_{\pi\pi}(t) - 1]
\end{equation}

(and $\Phi_{KK}^{(I=1)}(t) \simeq \Phi_{\pi\eta}(t)$, since $R \simeq 1$ in the latter case), where

\begin{equation}
R = \frac{m^2_K}{\sqrt{6}m^2_\pi} \cdot \frac{\Sigma_{\pi\pm N}}{\Sigma_{K\pm N}^{(0)}},
\end{equation}

and the same Omnès functions can be used on and off the mass shell. These Omnès functions (a.k.a. “the pion scalar form factor” in the $\pi N$ case\textsuperscript{18}) have obviously in the variable $t$ all the analytical properties of a form factor on the line $\Gamma_{CD}$: the criticism raised by Coon and Scadron\textsuperscript{34} is just a semantic misunderstanding.

It might appear, from the last relation, that $\Sigma_{K\pm N}^{(0)}$ can not be extracted from the on–shell amplitudes, since $\Phi_{KK}^{(0)}$ depends on it: but one can, using $R$ as a parameter, derive $\Sigma_{K\pm N}^{(0)}$ from a consistency condition, since its dependences on $R$ coming from the two relations (15) and (17) are remarkably different\textsuperscript{25}. Of course, one has to rely on a simultaneous determination of $\Sigma_{\pi\pm N}$, possibly within the same method for internal consistency. Using the values calculated by Oades\textsuperscript{35} for the zero–energy, pole–term–subtracted, non–flip amplitudes $D^+_{K\pm p}$ and $D^+_{K\pm n}$ at different values of $t \leq 0$, we reconstructed, adding the proper hyperon poles, the two zero–energy, spin–averaged amplitudes $A^+_I(t)$ for $I = 0, \ 1$. At this point these latter were divided by the Omnès functions $\Phi_{KK}^{(I)}(t)$ and, subtracting again the pseudovector hyperon Born terms (which vanish at the Cheng–Dashen point), we obtained two functions\textsuperscript{25} which extrapolated smoothly to $t = 2m^2_K$, provided we used a conformal mapping of the complex $t$–plane to ensure stability, taking there the values $\left[2\Sigma^{(I)}_{K\pm N}/f_K(0)^2 \cdot [m^2_{KI}/(m^2_{KI} - m^2_K)]^2\right]$.

The method, using as inputs the $\pi\pi$ S–waves\textsuperscript{36} and the pseudoscalar excitations’ masses from recent compilations\textsuperscript{37}, gives $\Sigma_{\pi\pm N} \simeq 50$ MeV and $\Sigma_{K\pm N}^{(0)} \simeq 460$ MeV, with the “physical” meson decay constants $f_\pi = 132$ MeV and $f_K = 154$ MeV. Errors are
difficult to assess in this case: for the $\pi N$ system such a unitarization method should do no worse than chiral perturbation theory (and has indeed been successfully checked against Gasser's scalar form factor$^{12,18}$); for the $KN$ systems even the dispersion relation results for the zero–energy amplitudes supplied by Oades$^{35}$ did not carry any error estimate to start with: varying the coupling constants between the extremes used in the dispersion relations gives the extrapolation a purely systematic uncertainty of the order of 95 MeV, already larger than that associated to the uncertainties in the scalar form factor evaluation.

For the $I = 1$ channel, describing $\pi\eta$ and $KK$ couplings to the scalar $a_0$–meson by a $K$–matrix with the observed mass and width of the resonance$^{37}$, one can extract by the same technique $\Sigma^{(1)}_{K^\pm N} \simeq 78^{+36}_{-56}$ MeV (where again the errors are only measures of the dependence of the extrapolation on the coupling constants), to be compared with an expectation (to second order in $SU(3)_f$–breaking) of $\simeq 25$ MeV.

The method is stable, at least with respect to the kaon–nucleon–hyperon couplings: the above uncertainty cover also the cases in which, e.g. $g^2_{K\Sigma N}$ was put equal to zero; the same can not be said for the first of the three methods, derived from the Altarelli–Cabibbo–Maiani technique$^{22}$. It is also non–perturbative, and general enough to be tested in the $\pi N$ system as well, where the perturbative techniques seem not in contradiction with its results$^{12,18}$. We are therefore waiting only too eagerly for new $KN$ data to put it to even more stringent tests$^{38}$.

A further comment is in order on the previous presentation: all methods resting on experimental information from the $KN$ amplitudes are presently suffering from the extremely poor quality of our knowledge of the $S = -1$ meson–baryon systems at low energy, even on its most fundamental parameters such as the PBB coupling constants. All information on the $KY N$ couplings comes indeed from subtracted, forward dispersion relations for the spin–averaged amplitudes $D = A + \omega B$ only, at variance with the $\pi N$ case, where one can use both these and the unsubtracted ones for the pure $B$ amplitude as a cross–check, and therefore it suffers from a strong correlation to the parameters of the $S$ waves at and below threshold. The simpler analytic structure of the $\pi N$ elastic scattering amplitudes allows even the use of partial–wave dispersion relations, at variance with the $KN$ case, where in some channels even the Born term singularities fall on the right–hand cuts$^{39}$. It is the very poor information on the $P$ waves which prevents use of the $B$ amplitudes (dominated by these latter at low and intermediate energies): indeed a recent dispersive analysis has shown that even the “best” low–energy phase–shift analyses
available present severe inconsistencies with simple tests coming from fixed-\(t\) analyticity\(^{39}\). New data and new analyses of these and of the older ones are therefore urgently needed.

4. CAPABILITIES FOR A \(\bar{K}N\text{-SCATTERING EXPERIMENT AT DA}\Phi NE\).

DA\(\Phi NE\) is the \(\phi\)-factory (the acronym stands for “Double Annular \(\phi\)-Factory for Nice Experiments”), which has replaced the Adone colliding-beam machine in the same experimental hall of the Laboratori Nazionali dell’I.N.F. N. in Frascati. From its expected luminosity\(^{40}\) of \(2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}\), and an annihilation cross section at the \(\phi\)-resonance peak of about \(4.40 \mu \text{b}\), one can see that its two interaction regions will be the sources of \(\simeq 436 \ K^{\pm} \text{ s}^{-1}\), at a central momentum of \(126.9 \text{ MeV/c}\), with the momentum resolution of \(\simeq 1.1 \times 10^{-2}\) due to the small energy spreads \((\Delta E/E \simeq 10^{-3})\) in the beams, as well as \(\simeq 303 \ K_L \text{ s}^{-1}\), at a central momentum of \(110.1 \text{ MeV/c}\), with the slightly worse resolution of \(\simeq 1.5 \times 10^{-2}\).

Both \(\pi^{\pm}\)’s and leptons coming out the two sources are backgrounds rather easy to control: the first because the \(\pi^{\pm}\)’s, though produced at a rate comparable to that of \(K^{\pm}\)’s (about \(341 \ \pi^{\pm} \text{ s}^{-1}\)), come almost all from events with three or more final particles, and can be greatly suppressed by momentum and acollinearity cuts; the second, as well as collinear pions from \(\phi \rightarrow \pi^{+}\pi^{-}\), produced at much lower rates, of order \(2.5 \times 10^{-1} \text{ s}^{-1}\) (the leptons) or \(3.5 \times 10^{-2} \text{ s}^{-1}\) (the pions), are completely eliminated by a momentum cut, having momenta about four times those of the \(K^{\pm}\)’s.

The interaction regions are therefore small-sized sources of low-momentum, tagged \(K^{\pm}\)’s and \(K_L\)’s, with negligible contaminations (after suitable cuts on angles and momenta on the outgoing particles are applied event by event), in an environment of very low background radioactivity: this situation is simply unattainable with conventional technologies at fixed-target machines\(^{41}\), where the impossibility of placing experiments too close to the production target limits from below the charged-kaon momenta, kaon decays in flight contaminate strongly the beams, and low-momentum experiments are thus possible only with the use of “moderators”, with a subsequent huge beam contamination at the target, as well as a large final-momentum spread due to straggling phenomena.

It is therefore of the highest interest to consider the feasibility of low-energy, \(K^{\pm}N\) and \(K_LN\) experiments at DA\(\Phi NE\), with respect to equivalent projects at machines such as, e.g., KAON studied for TRIUMF\(^{41}\) (and too hastily aborted by the Canadian government), or to ideas advanced for the equally sadly aborted European Hadron Factory project\(^{42}\).
I shall, in this final part, try and give an evaluation of the rates to be expected in a very simple, dedicated apparatus at DAΦNE. I shall assume cylindrical symmetry, with a toroidal target fiducial volume, limited by radii \( a \) and \( a + d \) and of length \( L \) (inside and outside of which you can imagine a tracking system, surrounded by a photon detecting system (e.g. lead–Sci–Fi sandwiches) and a solenoidal coil to provide the field for momentum measurements), filled, for simplicity, with a gas at moderate pressure. Such a detector immediately recalls the architecture of KLOE\textsuperscript{43}, and could be thought of as a much smaller (and cheaper) brother of the latter.

One must convert the usual, fixed-target expression for reaction rates to a spherical geometry and also include kaon decays in flight, getting (for simplicity this formula considers only the cases of either neutral kaons or zero magnetic field, but can easily be extended to the more general case)

\[
dN_r = \frac{1}{\rho^2} \left( \frac{3}{8\pi} \right) (L\sigma_\phi B_\phi) \sin^2\theta e^{-\rho/\lambda} \sigma_r \rho_t (\rho^2 d\rho d\Omega) ,
\]

(18)

with \( \rho, \theta \) and \( \phi \) spherical coordinates, \( L \) the machine luminosity, \( \sigma_\phi \) the annihilation cross section at the \( \phi \)–resonance peak, \( B_\phi \) the \( \phi \) branching ratio into the desired mode (either \( K^+K^- \) or \( K_L K_S \)), \( \sigma_r \) the reaction cross section for the process considered, \( \rho_t \) the target nuclear density, and \( \lambda = p_K \tau_K / m_K \) the decay length (0.954 m for \( K^\pm \)'s and 3.429 m for \( K_L \)'s, at the \( \phi \)–resonance momenta). The small ratio \( \lambda_+ / \lambda_L \) gives immediately a reduction in radius with respect to KLOE of a factor from 4 to 6, larger radii for the fiducial volume being useless, since most of the charged kaons would have already decayed.

The reaction rate over the fiducial volume can be cast into the simple form (valid also in the more general case)

\[
N_r = \frac{3\pi}{4} rd(L\sigma_\phi B_\phi) \rho_t \sigma_r ,
\]

(19)

with geometrical acceptance, magnetic–field effects and kaon decay in flight all thrown into the reduction factor \( r \), which we have estimated to take the values 0.50 for \( K^\pm \)'s and 0.72 for \( K_L \)'s for a fiducial volume defined by \( a = 10 \) cm, \( d = 50 \) cm and \( L = 1 \) m, to represent a person–sized detector, fitting in DAΦNE’s second interaction region.

This gives, for a target volume filled by a diatomic, nearly ideal gas, the rates for \( K^\pm \) initiated processes

\[
N_r = 10,410 \times p(\text{atm}) \times \sigma_r(\text{mb}) \text{ events/y} ,
\]

(20)
for a “physicist’s year” of $10^7$ s (for $K_L$’s the initial figure in the above equation is only slightly reduced by the interplay of $r$ and $B_\phi$ to 10,350), or, with rough estimates of the partial $K^-p$ cross sections at the $\phi$–decay momenta, to about $3 \times 10^6$ $K^-p$–initiated two–body events per year, of which about one third elastic scattering events, and the remaining two thirds more or less evenly divided between the five dominant inelastic channels ($\pi^+\Sigma^-$, $\pi^0\Sigma^0$, $\pi^0\Lambda$, $\bar{K}^0n$, and $\pi^-\Sigma^+$, more or less in order of decreasing importance). One could also expect about $1.5 \times 10^6$ $K_Lp$–initiated events, plus from 5 to 10 thousand radiative–capture events from both initial states, which should allow a good measurement on these processes as well $^{44}$.

These rates could be improved dramatically using liquid targets: the small range ($1 - 2$ cm at the $\phi$–factory momenta) of kaons in liquid hydrogen makes the target–detector complex much smaller, but suitable only for measurement of inelastic or radiative–capture rates at threshold. One has also to weigh the reduction in cost implied by the smaller dimensions against the added cost of cryogeny: mentioning costs, we wish to point out that DAΦNE, though giving the experimenters a very small momentum range, could save them the cost of the tagging system needed to reject the contaminations of a conventional low–energy, fixed–target experiment $^{45}$.

The above estimates for $K^-$ rates do not include energy losses in the beam–pipe wall and in the internal tracking system, which were assumed sufficiently thin (e. g. of a few hundred $\mu$m of low–Z material, such as carbon fibers or Mylar). I have indeed checked that, due to the shape of the angular distribution of the kaons produced, particle losses are rather contained and momentum losses flat around $\theta = \pi/2$: even for a thickness of the above–mentioned materials up to about 1 mm, kaon momenta do not decrease appreciably below 100 MeV/c and losses do not grow beyond a few percents. Rather, one could exploit such a thickness as a “moderator”, to span the interesting region of the charge–exchange threshold, measurement which would add additional constraints on low–energy amplitude analyses $^{1,39,46}$.

We have presented the above, oversimplified estimates to show that acceptable rates can be achieved, orders of magnitude above those of existing data at about the same momentum, i.e. to the lowest–energy points of the British–Polish Track–Sensitive Target Collaboration, taken in the late seventies at the (R.I.P.) NIMROD accelerator at the then Rutherford Laboratory $^{47}$.

The statistics derived above should indeed allow a determination not only of the
integrated cross sections for the dominant two–body channels (and, with a $\gamma$–detection efficiency equal to that of KLOE$^{43}$, a clear separation of final–state $\Sigma^0$’s from $\Lambda$’s), but also of those of the rarer three–body ones, plus that of the two–body angular distributions: the TST Collaboration$^{47}$ was able to measure $L_1$ for the $\pi^\pm \Sigma^\mp$ channels only, but with results consistent with zero within $2\sigma$, and therefore never used in the coupled–channel analyses. The same statistics, exploiting the self–analysing powers of $\Lambda$ and $\Sigma^+$ non–leptonic decays, should allow the determination of the polarization of the final baryons in some channels ($K^-p \to \pi^0 \Lambda, \pi^- \Sigma^+$ and $K_Lp \to \pi^+ \Lambda, \pi^0 \Sigma^+$) as well$^{46}$.

Since losses do not affect $K_L$’s, a detector of the kind sketched above, much smaller in size than but similar in geometry to KLOE, could be used without any problem to study low–energy $K_L \to K_S$ regeneration and charge–exchange in gaseous targets, providing essential information for this kind of phenomena.

I shall conclude my presentation remarking that DAΦNE (and $\phi$–factories in general) will present the opportunity of low–energy kaon experiments not feasible (with conventional technologies) at fixed–target kaon beams. Many, interesting experiments will however be already possible with existing detectors: KLOE$^{43}$ will surely be able to register all interactions of both $K^\pm$ and $K_L^0$ with the $^4$He filling its wire chamber, interactions never observed before at such low laboratory momenta, DEAR$^{48}$ will measure the K lines of kaonic hydrogen (and deuterium) giving independent information on the $\bar{K}N$ S–wave scattering lengths (and, with CCDs covering much lower $\gamma$–ray energies, they could also investigate the P waves through the study of the L lines), and FINUDA$^{49}$, though starting with a much narrower scope than KLOE, will anyway be able to make some high quality measurements, in particular of the charge–exchange processes taking place in the hydrogen of the plastic scintillators of its inner detector TOFINO.

5. “ENVOI”.

We hope this last section has helped in building in the audience the feeling that DAΦNE is an unique opportunity, too unique to be missed, for bringing the quality of our information on the low–energy, $S = -1$ meson–baryon systems as close as possible to the one we already have on the $S = 0$ one. To miss such an occasion would only be the sadder replay of another event not too far in our past, when an $e^+e^-$ machine of c.m. energy from 10 to 15 GeV, proposed to replace Adone (who remembers Super–Adone?), was killed in her crib as “not very interesting physically” (and beauty was just waiting us
around the corner...): let us then hope that people and organisations footing the bills for our community (at least on this side of the Atlantic Ocean...) have learnt something from the misjudgements made in the past.

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