Electroweak phase transition and entropy release in the early universe

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Abstract. It is shown that the vacuum-like energy of the Higgs potential at non-zero temperatures leads, in the course of the cosmological expansion, to a small but non-negligible rise of the entropy density in the comoving volume. This increase is calculated in the frameworks of the minimal standard model. The result can have a noticeable effect on the outcome of baryo-through-leptogenesis.
1 Introduction

As it is well known, the entropy density in the primeval plasma is conserved in the course of the cosmological expansion, if the plasma is in thermal equilibrium state with negligibly small chemical potentials of all particle species, see e.g. books [1, 2]. In other words

$$s = \rho + P a^3 = \text{const},$$  \hspace{1cm} (1.1)

where $T(t)$ is the plasma temperature, $a(t)$ is the cosmological scale factor, and $\rho$ and $P$ are respectively the energy density and the pressure of the plasma.

Normally the state of matter in the early universe is quite close to the equilibrium because the reaction rate, $\Gamma \sim \alpha n T$, is much faster than the cosmological expansion rate, $H = \dot{a}/a \sim T^2/m_{Pl}$. Here $n = 1, 2$ represents decays and two-body reactions respectively, $m_{Pl} = 1.2 \cdot 10^{19}$ GeV is the Planck mass and $\alpha$ is the coupling constant of the particle interactions. Typically $\alpha \sim 10^{-2}$. The above estimate for $\Gamma$ is presented for high temperatures, higher than the masses of the participating particles. The condition of equilibrium, $\Gamma > H$, is satisfied at the temperatures $T < \alpha n m_{Pl}$ up to a constant factor of order unity. Since $m_{Pl}$ is huge, thermal equilibrium existed during most of the universe history, if the reaction rate is not anomalously weak, i.e. $\alpha \ll 1$.

In thermal equilibrium, the occupation number (or what is the same, the distribution function) of any particle species is determined by two parameters only, the chemical potential, $\mu_j$, of each type of particles and the common temperature of all species. An exception is the Bose condensed state, when the chemical potential reaches the maximum value $\mu = m_B$, where $m_B$ is the boson mass. But still even in this case the system state is also determined by two parameters: the amplitude of the condensate and the temperature of the particles above the condensate. For a system of that kind, entropy, $s$, surely rises.

In equilibrium the distribution functions with $\mu_j = 0$ have the usual Bose-Einstein or Fermi-Dirac form:

$$f(E) = \frac{1}{\exp(E/T) \pm 1}, \hspace{1cm} (1.2)$$

where $E = \sqrt{q^2 + m^2}$ is the particle energy, $q$ is its momentum, and $m$ is the particle mass.

Using this form of the distributions and the expressions:

$$\rho = \int \frac{d^3 q}{(2\pi)^3} E f(E), \quad P = \int \frac{d^3 q}{(2\pi)^3} \frac{q^2}{3E} f(E),$$  \hspace{1cm} (1.3)

$$ \frac{1}{\exp(E/T) \pm 1},$$
one can easily prove the validity of the conservation law (1.1). Moreover, the conservation law (1.1) remains true for any form of $f(E/T)$ with an arbitrary function $T(t)$.

Still there might be several realistic regimes during the universe history when $s$ was not conserved. For example, if the universe was at some epoch dominated by primordial black holes with small masses [3], the entropy release could be very high, so that it might essentially delete all preexisting baryon asymmetry.

A large amount of entropy could be produced if the primeval plasma underwent the first order phase transition at some early period of the cosmological evolution. Unfortunately, we do not know for sure if such phase transition(s) indeed took place. A large entropy production might happen, in particular, during the QCD phase transition at $T \sim 100–200$ MeV. However due to strong technical problems the order of the QCD phase transition in cosmology is not known. For a review see e.g. ref. [4].

Some, realistic but most probably very weak entropy production, took place during the freeze-out of dark matter (DM) particles. However, usually the fraction of DM density was quite low at the freezing and the effect is tiny.

Possibly the largest entropy release in the standard model took place in the process of the electroweak transition from symmetric to asymmetric electroweak phase in the course of the cosmological cooling down. In principle, the transition could be either first order or second order, even very smooth crossover. Theoretical calculations say that in the minimal standard model with one Higgs field the transition is the mild crossover. However, in an extended theory with several higgses, the transition could be even first order with significant supercooling [5–10].

According to the electroweak (EW) theory at the temperatures higher than a critical one, $T > T_c$, the expectation value of the Higgs field, $\phi$, in the plasma is zero and the EW-symmetry is unbroken [11]. When the temperature drops below $T_c$, a non-zero $\langle \phi \rangle$ is created, which gradually rises, with decreasing temperature, up to the vacuum expectation value $\langle \phi \rangle = \eta$, see below. Such a state does not satisfy the conditions necessary for the entropy conservation and an entropy production is expected.

Here we calculate the entropy release in the course of the transition from the phase with unbroken electroweak symmetry to the symmetry broken phase. Presumably in the minimal EW-theory the mild cross-over regime is realized, so we make the calculations under this assumption.

### 2 Theoretical Framework

We take the Lagrangian of theory in the following slightly simplified form:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) + \sum_j i \left[ g^{\mu\nu} \partial_\mu \chi_j^\dagger \partial_\nu \chi_j - U_j(\chi_j) \right] + \mathcal{L}_{int} \quad (2.1)$$

where the Lagrangian of the Higgs boson interactions with fields, $\chi_j$, can be taken as:

$$\mathcal{L}_{int} = \phi \sum_j g_j \chi_j^\dagger \chi_j. \quad (2.2)$$

The summation is made over all relevant fields $\chi_j$. 
The equations of motion for the homogeneous classical field $\phi(t)$ and for the operators of the quantum fields $\chi_j(x, t)$ in FRW cosmological backgrounds have the form:

$$
\ddot{\phi} + 3H \dot{\phi} + U_\phi'(\phi) - \sum_x g_j \chi_j^\dagger \chi_j = 0, \quad (2.3)
$$

$$
\ddot{\chi}_j + 3H \dot{\chi}_j - \frac{1}{a^2} \Delta \chi_j - U_{\chi}'(\chi_j) - g_j \phi \chi_j = 0, \quad (2.4)
$$

where $U_\phi' = dU_\phi/d\phi$, $U_{\chi}'(\chi_j) = dU_{\chi}/d\chi_j$, $a(t)$ is the cosmological scale factor, and $H = \dot{a}/a$ is the Hubble parameter.

The equation of motion for the classical field $\phi$ (2.5) is often taken as:

$$
\ddot{\phi} + (3H + \Gamma) \dot{\phi} + U_\phi'(\phi) = 0, \quad (2.5)
$$

where $\Gamma$ is the decay width of the $\phi$-boson. Such equation is obtained by thermal averaging of the interaction term (2.2) and, strictly speaking, is valid only for quadratic potential $U_\phi \sim \phi^2$. Generally the equation is more complicated and may be even non-local in time [12]. Still the above equation is sufficiently accurate for an order of magnitude estimates.

The self-potential of $\phi$ with the temperature corrections can be written as:

$$
U_\phi(\phi) = \frac{\lambda}{4} (\phi^2 - \eta^2)^2 + \frac{T^2 \phi^2}{2} \sum_j h_j \left( \frac{m_j(T)}{T} \right), \quad (2.6)
$$

where according to experiment the vacuum expectation value of $\phi$ is equal to $\eta = 246$ GeV and the quartic self-coupling of $\phi$ is $\lambda = 0.13$ [13]. Here $T$ is the plasma temperature and $m_j(T)$ is the mass of the $\chi_j$-particle at temperature $T$, see below, eqs. (2.10) and (2.8).

The last temperature dependent term in eq. (2.6) appears as a result of thermal averaging of the interaction (2.2). It includes the contributions of $\phi$ itself and of all particles $\chi_j$. The summation over $j$ means the summation over all these particles. The functions $h_j(m_j/T)$ are positive and proportional to $g_j^2$. At high temperatures, $T > m_j(T)$, it is multiplied by a constant factor. At low temperatures, $T < m_j(T)$, the function $h_j(m_j/T)$ is exponentially suppressed, $\sim \exp[-m_j(T)/T]$.

In what follows we will be mostly interested in the contribution of fermions. Their Yukawa coupling constants to the Higgs field are determined by their masses at zero temperature, $m_f = g_f \eta$. According to the results presented e.g. in ref. [10] the fermionic loop with single fermion species gives $h_f(0) = m_f^2/(6\eta^2)$. For quarks this number should be multiplied by three due to the quark colors. So e.g. for $t$-quark $h_t(0) = m_t^2/(2\eta^2) = 0.25$. The masses of all particles depend on the temperature, $m_j = m_j(T)$, because the masses are proportional to the expectation value of the Higgs field and the latter is proportional to the temperature dependent value of $\phi$ at the minimum of the potential (2.6):

$$
\phi_{min}^2(T) = \eta^2 - (T^2/\lambda) \sum_j h_j \left( \frac{m_j(T)}{T} \right), \quad (2.7)
$$

and correspondingly

$$
m_j^2(T) = g_j^2 \phi_{min}^2(T) = g_j^2 \left[ \eta^2 - (T^2/\lambda) \sum_j h_j \left( \frac{m_j(T)}{T} \right) \right]. \quad (2.8)
$$
Here $j = f$ is the index of $\chi_f$-particle which acquires mass through a non-zero expectation value of $\phi$. The summation in the r.h.s. of this equation is made over all particles, $\chi_j$ and $\phi$. So for an accurate determination of all particle masses at non-zero $T < T_c$ we need to solve the whole system of equations for all values of $f$. However, because of large mass differences among the fermions in the standard model only the term with largest $g_f$ (or the highest mass fermion) contributes to the sum.

As we have mentioned above, at the temperatures higher than the critical value $T_c$ the expectation value of the Higgs field vanishes, while at $T < T_c$ the expectation value $\langle \phi \rangle$ becomes non-zero and all particles acquire non-zero, temperature dependent masses $m_f(T)$. As we have already mentioned, a particle gives noticeable contribution to thermal mass of the Higgs field if $m_f(T) \lesssim T$. According to eq. (2.8), the critical temperature is determined by the equation:

$$T_c^2 = \frac{\lambda \eta^2}{\sum h_j(0)}.$$  \hspace{1cm} (2.9)

In terms of $T_c$ the value of $\phi$ at the minimum of the potential or, what is the same, the expectation value of $\phi$ in the plasma can be written as

$$\phi^2_{\text{min}} = \eta^2 \left[ 1 - \frac{T^2}{T_c^2} \frac{\sum h_j(m_j/T)}{\sum h_j(0)} \right] = \eta^2 \left[ 1 - \frac{T^2}{T_c^2} \frac{h_{\text{tot}}(m/T)}{h_{\text{tot}}(0)} \right],$$  \hspace{1cm} (2.10)

where to simplify the equations we introduced the notations

$$\sum h_j(m_j/T) \equiv h_{\text{tot}}(m), \quad \sum h_j(0) \equiv h_{\text{tot}}(0).$$  \hspace{1cm} (2.11)

To describe the behavior of $h_{\text{tot}}(m)$ we need to define for each particle (fermion) the temperature at which its temperature-dependent mass becomes equal to the temperature, $m_f(T) = T$. Above this temperature the quark contribution to $h_q(m_q(T)/T)$ is equal to $h_q = g_q^2/2 = m^2_q(0)/(2\eta^2)$ (later on we use the notation $m_f(0) \equiv m_f$). The lepton contribution is $h_l = m_l^2/(6\eta^2)$. Below this temperature, $h_f$ is exponentially suppressed and can be neglected. Since masses of the quarks and leptons are very much different (except for $u$ and $d$ quarks) we may approximate $h_{\text{tot}}(m)$ as a succession of theta-functions dominated by a single fermion $f$ in the temperature range $T_{\text{min}}(f) \leq T \leq T_{\text{min}}^{(f)}$, where $f'$ is the heavier fermion nearest by mass to $f$.

According to eq. (2.8) and (2.9), $m_f(T)$ would remain smaller than $T$ for the temperatures higher than

$$(T_{\text{min}}^f)^2 = \frac{g_f^2 \eta^2}{1 + \left(g_f^2 \eta^2 / T_c^2\right) [h_{\text{tot}}(m)/h_{\text{tot}}(0)]}.$$  \hspace{1cm} (2.12)

As we have mentioned above, $h_{\text{tot}}(m)$ is dominated by the single contribution of the fermion $f$ at the temperatures near $T_{\text{min}}^f$. So eq. (2.12) is reduced to

$$(T_{\text{min}}^f)^2 \approx \frac{g_f^2 \eta^2}{1 + (N_f/3) \left(g_f^2 \eta^2 / g_t^2 T_c^2\right)},$$  \hspace{1cm} (2.13)

where $N_f = 3$ for quarks and $N_f = 1$ for leptons. Note, that only for $t$-quarks two terms in the denominator are comparable, while for lighter quarks and leptons $T_{\text{min}}^f \approx m_f$. At the
temperatures in the interval $T_{\text{min}}^t \leq T \leq T_c$ t-quark dominates, while at $T_{\text{min}}^b < T < T_{\text{min}}^t$, b-quark gives the dominant contribution, and so on.

The potential $U_{\phi}(\phi)$ is chosen in such a way that it vanishes when $\phi$ takes its vacuum expectation value, $\phi = \eta$. It ensures zero vacuum energy of the classical field $\phi$. For nonzero temperature, $\phi < \eta$ and $\dot{U}_{\phi}(\phi_{\text{min}}) \neq 0$:

$$U_{\phi}(\phi_{\text{min}}) = \frac{h_{\text{tot}}(m) T^2 \eta^2}{2} \left[ 1 - \frac{T^2 h_{\text{tot}}(m)}{2\lambda \eta^2} \right] = \frac{h_{\text{tot}}(m) T^2 \eta^2}{2} \left[ 1 - \frac{T^2}{2T_c^2} \frac{h_{\text{tot}}(m)}{h_{\text{tot}}(0)} \right]. \quad (2.14)$$

Let us note that the equations presented above are true in the broken phase, when there are one real Higgs field and massive three component intermediate W and Z bosons.

To calculate the entropy density we need the expression for the energy-momentum tensor:

$$T_{\mu\nu} = \partial_{\mu} \phi \partial_{\nu} \phi - g_{\mu\nu} \left( g^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \phi - U_{\phi}(\phi) \right) + \sum_j \left[ \partial_{\mu} \chi_j^{\dag} \partial_{\nu} \chi_j + \partial_{\nu} \chi_j^{\dag} \partial_{\mu} \chi_j - g_{\mu\nu} \left( g^{\alpha\beta} \partial_{\alpha} \chi_j^{\dag} \partial_{\beta} \chi_j - U_j(\chi_j) + 2\mathcal{L}_{\text{int}} \right) \right],$$

where $\mathcal{L}_{\text{int}}$ is given by eq. (2.2).

The operators of the energy density and pressure density for homogeneous classical field $\phi$ and all other fields $\chi_j$ (quanta of $\phi$ should be included there) in the Friedmann-Robertson-Walker background have the form:

$$\rho = \frac{\dot{\phi}^2}{2} + U_{\phi}(\phi) + \sum_j \left[ \chi_j^{\dag} \chi_j + \partial_i \chi_j^{\dag} \partial_i \chi_j / a^2 + U_j(\chi_j) \right] - \mathcal{L}_{\text{int}}; \quad (2.16)$$
$$P = \frac{\dot{\phi}^2}{2} - U_{\phi}(\phi) + \sum_j \left[ \chi_j^{\dag} \chi_j - (1/3) \partial_i \chi_j^{\dag} \partial_i \chi_j / a^2 - U_j(\chi_j) \right] + \mathcal{L}_{\text{int}}, \quad (2.17)$$

where $\partial_i \chi$ is the space derivative.

The sum $(\rho + P)$ enters into the expression for the entropy density and into the equation governing the evolution of $\rho$, see below eq. (2.19). It is equal to:

$$\rho + P = \frac{\dot{\phi}^2}{2} + \sum_j \left[ \chi_j^{\dag} \chi_j + \frac{2}{3a^2} \partial_i \chi_j^{\dag} \partial_i \chi_j \right]. \quad (2.18)$$

It can be verified that $\rho(t)$ indeed satisfies the covariant conservation law:

$$\dot{\rho} = -3H(\rho + P), \quad (2.19)$$

if we use the equations of motion (2.5, 2.4) and neglect the terms containing total spatial divergence.

Let us calculate now the variation of entropy in the course of the cosmological expansion, using the definition (1.1), expressions (2.16, 2.18), and the equations of motion (2.5, 2.4).

The calculations will be greatly simplified if we assume that the energy density consists of two parts, the energy density of the field $\phi(t)$ sitting at the minimum of the potential and of relativistic matter, so the expression for $\rho$ becomes:

$$\rho \approx U_{\phi}(\phi_{\text{min}}) + \frac{\dot{\phi}^2}{2} + \frac{\pi^2 g^{*}}{30} T^4, \quad (2.20)$$
and

$$\rho + P \approx \dot{\phi}^2 + \frac{4}{3} \frac{\pi^2 g_* T^4}{30},$$  \hspace{1cm} (2.21)

where $g_* \sim 10^2$ is the effective number of particle species at or near the electroweak phase transition. It is a function of temperature, decreasing in the course of the cosmological cooling down. Equation (2.20) is valid in the limit of instant thermalization.

The oscillations of $\phi$ around $\phi_{\text{min}}$ are quickly damped, so we take $\dot{\phi} = \dot{\phi}_{\text{min}}$ and neglect $\dot{\phi}^2$ in what follows, because the evolution of $\phi_{\text{min}}$ is induced by the universe expansion which is quite slow. In this approximation we obtain the single differential equation governing the temperature evolution with time, or what is more convenient, with the scale factor. Under these assumptions equation (2.19) can be rewritten as

$$\frac{\dot{a}}{a} = -\frac{\dot{T}}{T} \left[ \frac{\kappa \eta^2 T^2}{4 \pi^2 g_*} \left( 1 - \frac{T^2 h_{\text{tot}}(m)}{T^2_{\text{c}}} h_{\text{tot}}(0) \right) + \frac{4 \pi^2 g_* T^4}{30} \right] = -4H \frac{\pi^2 g_* T^4}{30}. \hspace{1cm} (2.22)$$

Here equations (2.14), (2.20), and (2.21) are used and the time derivative of $h_{\text{tot}}(m)$ is neglected, because $h_{\text{tot}}$ is supposed to be the succession of the step functions.

Let us note that the equation (2.22) does not take into account the modification of the temperature evolution due to annihilation of non-relativistic species, as e.g., the well known heating of plasma by $e^+ e^-$-annihilation, which takes place at $T$ below $m_e$. We disregard this effect because, if the annihilating particles are in thermal equilibrium state with vanishing chemical potential, the entropy density in this process is conserved.

It is convenient to introduce the parameters:

$$\kappa = \frac{30 h_{\text{tot}}(m)}{4 \pi^2 g_*}, \quad \nu = \frac{\kappa \eta^2}{2 T^2_{\text{c}}},$$  \hspace{1cm} (2.23)

so equation (2.22) turns into

$$\frac{\dot{a}}{a} = -\frac{\dot{T}}{T} \left[ \frac{\kappa \eta^2}{T^2} \left( 1 - \frac{T^2 h_{\text{tot}}(m)}{T^2_{\text{c}}} h_{\text{tot}}(0) \right) + 1 \right]. \hspace{1cm} (2.24)$$

In the case when the heaviest particle mass, that of $t$-quark, is lower than the temperature, we can take $h(m) = h(0)$ and equation (2.24) can be easily integrated resulting in:

$$\frac{a(T)}{a_{\text{c}} T_{\text{c}}} = x^{2 \nu} \exp \left[ \nu \left( \frac{1}{x^2} - 1 \right) \right], \hspace{1cm} (2.25)$$

where $x = T/T_{\text{c}}$ and the cosmological scale factor $a_{\text{c}}$ is taken at $T = T_{\text{c}}$.

Taking $T_{\text{c}}$ from eq. (2.9), we find for $t$-quark:

$$(x_{\text{min}}')^2 \equiv \left( \frac{T_{\text{min}}^4}{T_{\text{c}}^4} \right)^2 = \frac{g^2 (\eta/T_{\text{c}})^2}{1 + g^2 (\eta/T_{\text{c}})^2} = \frac{m_t^2/T_{\text{c}}^4}{1 + m_t^2/T_{\text{c}}^4}. \hspace{1cm} (2.26)$$

Since $t$-quark is the heaviest among the standard model particles, its contribution to the entropy release is the largest at high temperatures. But it quickly disappears when temperature drops below the running mass of $t$-quark, i.e. at $T < m_t(T)$. On the other hand, lighter particles become efficient at lower $T$. Due to that, their contribution remain more or less the same as that of $t$-quark. Moreover, as one can see from eqs. (2.23), the effect is inversely proportional to the number of the particle species $g_*$. It drops down from $g_* = 106.75$ at the electroweak phase transition to 10.75 at the temperatures below the muon mass. So, as we see in the next section, it considerably amplify the contribution of light leptons into the entropy increase in the course of the cosmological expansion.
3 Calculations and Results

We start with the calculations of the contribution to the entropy from the heaviest particles. To this end we need the numerical values of their coupling constants with the Higgs boson. According to the experimental data, they are $g_t = 0.25$, $m_t = 173$ GeV, $\lambda = 0.13$, $g_W = 0.13$, $g_Z = 0.1$. The Yukawa coupling constants of lighter fermions scale as the ratio of the masses, $g_f = g_t(m_f/m_t)$.

With the account of $t$-quark only the critical temperature, according to eq. (2.9) is

$$T_{c}^2/\eta^2 = \frac{2\lambda\eta^2}{m_t^2} \approx 0.53.$$  

The contribution of other heavy particle makes this ratio twice smaller:

$$T_{c}^2/\eta^2 = \frac{2\lambda\eta^2}{m_t^2 + m_w^2 + m_z^2 + m_H^2} \approx 0.25 \quad (3.1)$$

Let us estimate now the values of parameters $\kappa$ and $\nu$ of eq. (2.23). As we mentioned above, $h_{tot}(m)$ is a collection of theta-functions dominated by single contribution of a fermion with mass specified below eq. (2.12). Correspondingly at $T > T_{f\text{,min}}$ the contribution of a fermion to $h_{tot}(m)$ is equal to $h_f = N_f m_f^2/(6\eta^2)$, where $N_q = 3$ and $N_l = 1$. Hence the contribution of all fermions lighter than $t$-quark is

$$\kappa = \frac{5}{4\pi^2} \sum_f \left[ \frac{N_f m_f^2}{g_s(T_{f\text{,min}}^f)\eta^2} \right], \quad (3.2)$$

where $g_s(T_{f\text{,min}}^f)$ is the number of relativistic particle species present in the plasma at $T \sim T_{f\text{,min}}^f$. So $\kappa$ and $\nu \approx \kappa$ are both small numbers, $\nu \approx 0.007$ for $g_s = 106.76$ and $\nu \sim 0.07$ for $g_s = 10.75$. One should keep in mind, however, that these small numbers are multipleid by $(T_c/T_{min}^f)^2 \sim T_c^2/m_f^2$, which can be very large. It is interesting that the product $\nu(T_c/m_f)^2$ essentially does not depend upon the mass and the effect is the larger for smaller masses due to a decrease of $g_s$.

Using equation (2.25), we find that the relative increase of the entropy is

$$\frac{\delta s}{s} = \sum_f x_{f\text{,min}} \exp \left[ 3\nu_f \left( \frac{1}{x_{f\text{,min}}^2} - 1 \right) \right] - 1, \quad (3.3)$$

where $\nu_f$ includes only contribution from single fermion $f$ and, according to eq. (2.13),

$$x_{f\text{,min}} = \left( \frac{T_{f\text{,min}}^f}{T_c} \right)^2 \approx \frac{g_f^2\eta^2/T_c^2}{1 + (N_f/3) \left( g_f^2\eta^2/g_t^2T_c^2 \right)}, \quad (3.4)$$

With an exception for $t$-quark this expression is reduced to a very simple one

$$x_{f\text{,min}} = (m_f/T_c)^2, \quad (3.5)$$

and correspondingly

$$\frac{3\nu_f}{x_{f\text{,min}}^2} = \frac{15N_f}{4\pi^2g_s} \left( \frac{T_c}{\eta} \right)^2 \quad (3.6)$$

For example, the electron contribution to the relative rise of the entropy is $(\delta s/s)_e = 1.8\%$. At temperatures below the muon mass, $g_s = 14.25$ and thus the muon contribution is
\( (\delta s/s)_\mu = 1.3\% \). The contribution of \( \tau \)-lepton is \((\delta s/s)_\tau = 0.25\%\), because at \( T = 180 \text{ GeV} \), \( g_* = 75.75 \). The contribution of three quark families in this temperature range is 12 times larger and brings about 3\%. The contribution from \( t \)-quark is \((\delta s/s)_t \approx 1\%\). The contribution from \( b \)-quark is almost the same but it is to be noted that the contribution came from a bigger range of temperature \((T_{\text{min}} \text{ for } b \text{-quark is } \approx 4 \text{ GeV})\). Moreover, the contributions of the lighter \( s, u, \) and \( d \) quarks are slightly enhanced because they remain alive down to the QCD phase transition at about 150 MeV, when \( g_* \) is 72.25. The contribution of Higgs boson \((\delta s/s)_H = 1\%\) and that of Gauge bosons is \((\delta s/s)_{W,Z} \approx 2\%\).

We need to take into account that neutrinos are decoupled from the electromagnetic component of the cosmic plasma at \( T_e \approx 1.9 \text{ MeV} \) for \( \nu_e \) and at \( T_{\mu,\tau} \approx 3.1 \text{ MeV} \) for \( \nu_\mu \) and \( \nu_\tau \), see e.g. [14]. This effect would lead to the decrease of the effective number of species from 10.75 to \( g_* = 5.5 \) and the rise of the electron contribution up to \((\delta s/s)_e = 3.6\%\).

As an illustration, the entropy production from \( t \)-quark as a function of temperature is presented in Fig. 1. Contributions to the entropy release comes till \( T_{\text{min}} \) of every particles.

![Figure 1](image_url)

Similar calculations have been done for other particles and the entropy release has been calculated in the huge range of temperature, starting from the temperature of the Electroweak phase transition down to the mass of electron (511 KeV). So, in the range from \( GeV \) to \( keV \) scale, we find that the total amount of entropy is increased by about 13\%.

4 Discussion and Conclusion

It is shown that the total entropy release in the course of the electroweak symmetry breaking is quite noticeable even in the frameworks of minimal Standard model of particle physics. We have assumed here that Electroweak phase transition is second order (or smooth/mild crossover). It is to be noted that \( g_* \) decreases as the temperature falls down. But as we go to very low temperature scale, the minimum temperature \((T_{\text{min}})\) takes the value of the particle
mass and hence we find that the contribution of lighter particles in the process of entropy release is nearly similar to that of the heavy particles, like $t$-quark.

In extended versions of the electroweak theory (e.g., with several Higgs fields) the entropy release may be considerably larger.

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