BRST invariant $CP^1$ model through improved Dirac quantization

Soon-Tae Hong$^{1,2,a}$, Young-Jai Park$^{1,b}$, Kuniharu Kubodera$^{2,c}$
and Fred Myhrer$^{2,d}$

$^1$Department of Physics and Basic Science Research Institute,
Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

$^2$Department of Physics and Astronomy,
University of South Carolina, Columbia, SC 29208, USA

May 10, 2001

ABSTRACT

The Batalin-Fradkin-Tyutin (BFT) scheme, which is an improved version of Dirac quantization, is applied to the $CP^1$ model, and the compact form of a nontrivial first-class Hamiltonian is directly obtained by introducing the BFT physical fields. We also derive a BRST-invariant gauge fixed Lagrangian through the standard path-integral procedure. Furthermore, performing collective coordinate quantization we obtain energy spectrum of rigid rotator in the $CP^1$ model. Exploiting the Hopf bundle, we also show that the $CP^1$ model is exactly equivalent to the O(3) nonlinear sigma model at the canonical level.

PACS: 11.10.-z, 11.10.Ef, 11.30.-j, 14.20.-c
Keywords: $CP^1$ model, Dirac quantization, BFT scheme, BRST symmetry

$^asthong, ^byjpark@ccs.sogang.ac.kr; ^ckubodera, ^dmyhrer@sc.edu
1 Introduction

Since the (2+1) dimensional O(3) nonlinear sigma model (NLSM) was first discussed by Polyakov and Belavin [1], there have been lots of attempts to improve this soliton model associated with the homotopy group \( \pi_2(S^2) = \mathbb{Z} \). The SU(N) invariant NLSM or \( CP^{N-1} \) model was later introduced \( \cite{2, 3, 4} \) in terms of \( N \) complex fields \( Z_\alpha (\alpha = 1, ..., N) \) satisfying a constraint \( Z_\alpha^* Z_\alpha - 1 = 0 \). In addition, one can impose a local U(1) invariance

\[
Z_\alpha (x) \rightarrow e^{i\alpha(x)} Z_\alpha (x)
\]

for arbitrary space-time dependent \( \alpha(x) \) \( \cite{3} \). The \( CP^{N-1} \) model for \( N = 2 \) was shown \( \cite{3} \) to be equivalent to the O(3) NLSM where Polyakov and Belavin found instantons \( \cite{1} \). Moreover, it is well known that the topological charge in the O(3) NLSM is equivalent to that in the SU(2) invariant NLSM \( \cite{3} \).

On the other hand, the Dirac method \( \cite{5} \) is a well known formalism to quantize physical systems with constraints. In this method, the Poisson brackets in a second-class constraint system are converted into Dirac brackets to attain self-consistency. The Dirac brackets, however, are generically field-dependent, nonlocal and contain problems related to ordering of field operators. These features are unfavorable for finding canonically conjugate pairs. However, if a first-class constraint system can be constructed, one can avoid introducing the Dirac brackets and can instead use Poisson brackets to arrive at the corresponding quantum commutators.

To overcome the above problems, Batalin, Fradkin, and Tyutin (BFT) \( \cite{6} \) developed a method which converts the second-class constraints into first-class ones by introducing auxiliary fields. Recently, this BFT formalism has been applied to several models of current interest \( \cite{7, 8, 9} \). In particular, the relation between the Dirac and BFT schemes, which had been obscure and unsettled, was clarified in the SU(2) Skyrmion model \( \cite{10} \). Very recently, in Ref. \( \cite{11} \) the BFT Hamiltonian method was also applied to the SU(2) Skyrmion model to directly obtain the compact form of a first-class Hamiltonian, via the construction of the BFT physical fields. Meanwhile, this BFT approach was applied to \( CP^1 \) model \( \cite{12, 13} \), but these studies fell short of obtaining the desired compact form of a first-class Hamiltonian and as a result further developments have been deterred.

The motivation of this paper is to systematically apply the standard Dirac quantization method, the BFT scheme \( \cite{8} \), the Batalin, Fradkin and Vilkovisky (BFV) method \( \cite{14, 15, 16} \) and the Becci-Rouet-Stora-Tyutin (BRST)
method [7] to the $CP^1$ model [12, 13]. In section 2 we convert the second-class constraints into first-class ones according to the BFT method to construct first-class BFT physical fields and directly derive the compact expression of a first-class Hamiltonian in terms of these fields. The existing approach, used in Ref. [18] to derive a first-class Hamiltonian in the SU(2) Skyrmion model, involves an infinite iteration procedure. Our approach avoids this. We then investigate some properties of the Poisson brackets of these BFT physical fields to obtain the Dirac brackets in the limit of vanishing auxiliary fields. We construct in section 3 a BRST-invariant gauge fixed Lagrangian in the BFV scheme through the standard path-integral procedure. Exploiting collective coordinates, in section 4 we perform a semi-classical quantization and in section 5 we explicitly show the equivalence between the $CP^1$ model and O(3) nonlinear sigma model (NLSM) [19] at the canonical level, by using the Hopf bundle [3, 4]. In section 6 we show that the energy spectrum of rigid rotator in the $CP^1$ model obtained by the standard Dirac method with the suggestion of generalized momenta is consistent with that of the BFT scheme.

2 First-class constraints and Hamiltonian

In this section we apply the BFT scheme to the $CP^1$ model, which is a second-class constraint system. We start with the $CP^1$ model Lagrangian of the form

$$L = \int d^2x \left[ \partial_\mu Z_\alpha \partial^\mu Z_\alpha - (Z_\alpha^* \partial_\mu Z_\alpha)(Z_\beta \partial^\mu Z_\beta^*) \right]$$

(2.1)

where $Z_\alpha = (Z_1, Z_2)$ is a multiplet of complex scalar fields with a constraint

$$\Omega_1 = Z_\alpha^* Z_\alpha - 1 \approx 0.$$  

(2.2)

One notes here that, as discussed before, this model is invariant under a local U(1) gauge symmetry transformation ([11]). By performing the Legendre transformation, one can obtain the canonical Hamiltonian

$$H_c = \int d^2x \left[ \Pi_\alpha^* \Pi_\alpha + \partial_i Z_\alpha^* \partial_i Z_\alpha - (Z_\alpha^* \partial_i Z_\alpha)(Z_\beta \partial_i Z_\beta^*) \right]$$

(2.3)

where $\Pi_\alpha$ and $\Pi_\alpha^*$ are the canonical momenta conjugate to the complex scalar fields $Z_\alpha$ and $Z_\alpha^*$, respectively, given by

$$\Pi_\alpha = \dot{Z}_\alpha^* - Z_\alpha^* Z_\beta \dot{Z}_\beta^*.$$
\[ \Pi^*_\alpha = \dot{Z}_\alpha - Z_\alpha Z^*_\beta \dot{Z}_\beta. \]  \hspace{1cm} (2.4)

The time evolution of the constraint \( \Omega_1 \) yields an additional secondary constraint
\[ \Omega_2 = Z^*_\alpha \Pi^*_\alpha + Z_\alpha \Pi_\alpha \approx 0 \]  \hspace{1cm} (2.5)
and \( \Omega_1 \) and \( \Omega_2 \) form a second-class constraint algebra
\[ \Delta_{kk'}(x, y) = \{ \Omega_k(x), \Omega_{k'}(y) \} = 2 \epsilon^{kk'} Z^*_\alpha Z_\alpha \delta(x - y) \]  \hspace{1cm} (2.6)
with \( \epsilon^{12} = - \epsilon^{21} = 1. \)

Following the BFT formalism \([3, 4, 8]\) which systematically converts the second class constraints into first class ones, we introduce two auxiliary fields \( \Phi^i \) according to the number of second class constraints \( \Omega_i \) with the Poisson brackets
\[ \{ \Phi^i(x), \Phi^j(y) \} = \epsilon^{ij} \delta(x - y). \]  \hspace{1cm} (2.7)

The first class constraints \( \tilde{\Omega}_i \) fulfilling the simplest closed algebra
\[ \{ \tilde{\Omega}_i(x), \tilde{\Omega}_j(y) \} = 0 \]  \hspace{1cm} (2.8)
are then constructed as follows
\[ \tilde{\Omega}_i(x) = \Omega_i(x) + \int d^2 y X_{ij}(x, y) \Phi^j(y) \]  \hspace{1cm} (2.9)
where the matrix \( X_{ij} \) satisfies the relation
\[ \Delta_{ij}(x, y) + \int d^2 z X_{ik}(x, z) \epsilon^{kl} X_{jl}(z, y) = 0. \]  \hspace{1cm} (2.10)

The solution of Eq. (2.10) is for instance given as
\[ X_{ij}(x, y) = \begin{pmatrix} 2 & 0 \\ 0 & -Z^*_\alpha Z_\alpha \end{pmatrix} \delta(x - y) \]  \hspace{1cm} (2.11)
to yield the first class constraints with the redefinition of the two auxiliary fields \( \Phi^i = (\theta, \pi_\theta) \)
\[ \tilde{\Omega}_1 = \Omega_1 + 2 \theta = Z^*_\alpha Z_\alpha - 1 + 2 \theta, \]
\[ \tilde{\Omega}_2 = \Omega_2 - Z^*_\alpha Z_\alpha \pi_\theta = Z^*_\alpha \Pi^*_\alpha + Z_\alpha \Pi_\alpha - Z^*_\alpha Z_\alpha \pi_\theta. \]  \hspace{1cm} (2.12)
Here one notes that the physical fields $Z_\alpha$ are geometrically constrained to reside on the $S^3$ hypersphere with the modified norm $Z_\alpha^*Z_\alpha = 1 - 2\theta(x)$.

Now, we consider the uniqueness of the first class constraints. In fact, according to the Dirac terminology \[5\], the first class constraints $\tilde{\Omega}_i$ are defined to satisfy the following Lie algebra \[2.13\]
\[
\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = C^k_{ij} \tilde{\Omega}_k.
\]
Since the first class constraints $\tilde{\Omega}_i$ are "strongly zero" $\tilde{\Omega}_i = 0$ to yield $\{\tilde{\Omega}_i(x), \tilde{\Omega}_j(y)\}_\text{phy} = 0$ from Eq. \[2.13\], one does not have any difficulties in construction of the quantum commutators and in quantization of the given physical system. In that sense, one has degrees of freedom in taking a set of the first class constraints, without any criterion. For instance our set of the first class constraints \[2.12\] is a specific choice to satisfy the minimal Lie algebra \[2.3\] with $C^k_{ij} = 0$. Moreover even in this minimal case, we have an equivalent family of the first class constraints governed by SO(2) rotation group, under which the matrix $X_{ij}$ transforms as
\[
X \to X' = RXR^T \quad (2.14)
\]
where $R$ is an orthogonal $2 \times 2$ matrix satisfying the condition $RR^T = 1$. Since the matrix $\epsilon$ is invariant under the SO(2) rotation, namely, $ReR^T = \epsilon$, one can easily check that the above rotated $X'$ also satisfy the relation \[2.10\] to yield the equivalent family of first class constraints $\tilde{\Omega}'_i$ given by inserting the $X'$ into Eq. \[2.9\]. For the more important case of the uniqueness of the first class Hamiltonian, we will discuss later in details.

Next, we construct the first class BFT physical fields $\tilde{\mathcal{F}} = (\tilde{Z}_\alpha, \tilde{\Pi}_\alpha)$ corresponding to the original fields $\mathcal{F} = (Z_\alpha, \Pi_\alpha)$. The $\tilde{\mathcal{F}}$'s, which reside in the extended phase space, are obtained as a power series in the auxiliary fields $(\theta, \pi_\theta)$ by demanding that they are strongly involutive: $\{\tilde{\Omega}_i, \tilde{\mathcal{F}}\} = 0$. After some algebra, we obtain the first class physical fields as
\[
\tilde{Z}_\alpha = Z_\alpha \left( \frac{Z_\beta^*Z_\beta + 2\theta}{Z_\beta^*Z_\beta} \right)^{1/2},
\]
\[
\tilde{\Pi}_\alpha = \left( \Pi_\alpha - \frac{1}{2} Z_\alpha^*\pi_\theta \right) \left( \frac{Z_\beta^*Z_\beta}{Z_\beta^*Z_\beta + 2\theta} \right)^{1/2} \quad (2.15)
\]
In the case of the nonvanishing $C^k_{ij}$, Eq. \[2.11\] is modified as $\Delta_{ij}(x, y) + \int d^2z X_{ik}(x, z) e^{kl} X_{jl}(z, y) = C^k_{ij}(x, y)\tilde{\Omega}_k(y)$.  

---

\[1\]In the case of the nonvanishing $C^k_{ij}$, Eq. \[2.11\] is modified as $\Delta_{ij}(x, y) + \int d^2z X_{ik}(x, z) e^{kl} X_{jl}(z, y) = C^k_{ij}(x, y)\tilde{\Omega}_k(y)$. 

---

4
As discussed in Ref. [8], any functional \( K(\tilde{F}) \) of the first class fields \( \tilde{F} \) is also first class, namely, \( K(F; \Phi) = K(\tilde{F}) \). Using the property, we construct a first-class Hamiltonian in terms of the above BFT physical variables. The result is

\[
\tilde{H} = \int d^2 x \left[ \tilde{\Pi}^*_\alpha \tilde{\Pi}_\alpha + (\partial_i \tilde{Z}^*_\alpha)(\partial_i \tilde{Z}_\alpha) - (\tilde{Z}^*_\alpha \partial_i \tilde{Z}_\alpha)(\tilde{Z}_\beta \partial_i \tilde{Z}^*_\beta) \right]. \tag{2.16}
\]

We then directly rewrite this Hamiltonian in terms of the original as well as auxiliary fields \( \tilde{z}^x \) to obtain

\[
\tilde{H} = \int d^2 x \left[ \left( \Pi^*_\alpha - \frac{1}{2} Z_\alpha \pi_\theta \right) \left( \Pi_\alpha - \frac{1}{2} Z^*_\alpha \pi_\theta \right) \frac{Z^*_\beta Z_\beta + 2\theta}{Z^*_\beta Z_\beta + 2\theta} \right.
\]

\[
+ (\partial_i Z^*_\alpha)(\partial_i Z_\alpha) \frac{Z^*_\beta Z_\beta + 2\theta}{Z^*_\beta Z_\beta} - (Z^*_\alpha \partial_i Z_\alpha)(Z_\beta \partial_i Z^*_\beta) \left( \frac{Z^*_\gamma Z_\gamma + 2\theta}{Z^*_\gamma Z_\gamma} \right)^2 \left. \right] . \tag{2.17}
\]

We observe that the forms of the first two terms in this Hamiltonian are exactly the same as those of the O(3) NLSM [22].

Here \( \tilde{H} \) is strongly involutive with the first class constraints \( \{ \tilde{\Omega}_1, \tilde{H} \} = 0 \). A problem with \( \tilde{H} \) in Eq. (2.17) is that it does not naturally generate the first-class Gauss law constraint from the time evolution of the constraint \( \tilde{\Omega}_1 \). Therefore, by introducing an additional term proportional to the first class constraints \( \tilde{\Omega}_2 \) into \( \tilde{H} \), we obtain an equivalent first class Hamiltonian

\[
\tilde{H}' = \tilde{H} + \frac{1}{2} \int d^2 x \pi_\theta \tilde{\Omega}_2 \tag{2.18}
\]

which naturally generates the Gauss law constraint

\[
\{ \tilde{\Omega}_1, \tilde{H}' \} = \tilde{\Omega}_2, \quad \{ \tilde{\Omega}_2, \tilde{H}' \} = 0. \tag{2.19}
\]

One notes here that \( \tilde{H} \) and \( \tilde{H}' \) act in the same way on physical states, which are annihilated by the first-class constraints. Similarly, the equations of motion for observables remain unaffected by the additional term in \( \tilde{H}' \). Furthermore, in the limit \( (\theta, \pi_\theta) \to 0 \), our first class system is exactly reduced to the original second class one.

\[\text{In deriving the first class Hamiltonian } \tilde{H} \text{ of Eq. (2.17), we have used the conformal map condition, } Z^*_\alpha \partial_i Z_\alpha + Z_\alpha \partial_i Z^*_\alpha = 0. \]
Now it is appropriate to comment on the uniqueness in construction of the first class Hamiltonians. Similar to the above discussions on the uniqueness of the first class constraints, one can have the degrees of freedom in construction of the first class Hamiltonian \( \tilde{H} \) or \( \tilde{H}' \) where \( \tilde{H}' \) is equivalent to \( \tilde{H} \) up to the additional term \( \tilde{\Omega}_2 \) which does not affect the vacuum structure as discussed above. However, imposing the condition that in the limit \( (\theta, \pi_\theta) \to 0 \) the first class system is exactly reduced to the original second class one, one can exploit the degrees of freedom to uniquely fix the specific form of first class Hamiltonian \( \tilde{H}' \) in Eq. (2.18) which fulfills the Gauss law constraint.

Next, we consider the Poisson brackets of the fields in the extended phase space \( \tilde{F} \) and identify the Dirac brackets by taking the vanishing limit of auxiliary fields. After some algebraic manipulation starting from Eq. (2.13), one can obtain the commutators

\[
\{ \tilde{Z}_\alpha(x), \tilde{Z}_\beta(y) \} = \{ \tilde{Z}^*_\alpha(x), \tilde{Z}^*_\beta(y) \} = 0, \\
\{ \tilde{Z}_\alpha(x), \tilde{\Pi}_\beta(y) \} = (\delta_{\alpha\beta} - \frac{\tilde{Z}_\alpha \tilde{Z}^*_\beta}{2Z^*_\gamma Z^*_\gamma}) \delta(x-y), \\
\{ \tilde{Z}_\alpha(x), \tilde{\Pi}^*_\beta(y) \} = -\frac{\tilde{Z}_\alpha \tilde{Z}^*_\beta}{2Z^*_\gamma Z^*_\gamma} \delta(x-y), \\
\{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}_\beta(y) \} = \frac{1}{2Z^*_\gamma Z^*_\gamma} (\tilde{\Pi}_\alpha \tilde{Z}^*_\beta - \tilde{Z}^*_\alpha \tilde{\Pi}_\beta) \delta(x-y), \\
\{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}^*_\beta(y) \} = \frac{1}{2Z^*_\gamma Z^*_\gamma} (\tilde{\Pi}_\alpha \tilde{Z}^*_\beta - \tilde{Z}^*_\alpha \tilde{\Pi}_\beta) \delta(x-y).
\]

In the vanishing auxiliary field limit, the above Poisson brackets in the extended phase space exactly reproduce the corresponding Dirac brackets in the previous works [12, 13]

\[
\{ \tilde{Z}_\alpha(x), \tilde{Z}^*_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{Z}^*_\alpha(x), \tilde{Z}_\beta(y) \}_{D}, \\
\{ \tilde{Z}^*_\alpha(x), \tilde{Z}_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{Z}^*_\alpha(x), \tilde{Z}_\beta(y) \}_{D}, \\
\{ \tilde{Z}_\alpha(x), \tilde{\Pi}_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{Z}_\alpha(x), \tilde{\Pi}_\beta(y) \}_{D}, \\
\{ \tilde{Z}_\alpha(x), \tilde{\Pi}^*_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{Z}^*_\alpha(x), \tilde{\Pi}_\beta(y) \}_{D}, \\
\{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}_\beta(y) \}_{D}, \\
\{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}^*_\beta(y) \}_{(\theta, \pi_\theta)=0} = \{ \tilde{\Pi}_\alpha(x), \tilde{\Pi}^*_\beta(y) \}_{D}.
\]

(2.21)
where

\[ \{ A(x), B(y) \}_D = \{ A(x), B(y) \} - \int d^2 z d^2 z' \{ A(x), \Omega_k(z) \} \Delta^{k \ell} \{ \Omega_{k'}(z'), B(y) \} \]

(2.22)

with \( \Delta^{k \ell} \) being the inverse of \( \Delta_{k \ell} \) in Eq. (2.6). It is also noteworthy that the Poisson brackets of \( \tilde{F} \)’s in Eq. (2.21) have exactly the same form as those of the Dirac brackets of the field \( F \). In other words, the functional \( \tilde{\mathcal{K}} \) in \( \mathcal{K}(\mathcal{F}; \Phi) = \mathcal{K}(\tilde{\mathcal{F}}) \) corresponds to the Dirac brackets \( \{ A, B \}_D \) and hence \( \tilde{\mathcal{K}} \) corresponding to \( \{ \tilde{A}, \tilde{B} \} \) becomes

\[ \{ \tilde{A}, \tilde{B} \} = \{ A, B \}_D |_{A \to \tilde{A}, B \to \tilde{B}}. \]  

(2.23)

This kind of situation happens again when one considers the first-class constraints (2.12). More precisely, these first-class constraints in the extended phase space can be rewritten as

\[
\begin{align*}
\tilde{\Omega}_1 &= \tilde{Z}_\alpha \tilde{Z}_\alpha - 1, \\
\tilde{\Omega}_2 &= \tilde{Z}_\alpha \tilde{\Pi}_\alpha + \tilde{\Pi}_\alpha \tilde{Z}_\alpha,
\end{align*}
\]

(2.24)

which are form-invariant with respect to the second-class constraints (2.2) and (2.5).

### 3 BRST symmetries

In this section we introduce two canonical sets of ghosts and anti-ghosts together with auxiliary fields in the framework of the BFV formalism [14, 15, 16], which is applicable to theories with the first-class constraints:

\((C^i, \bar{P}_i), \ (P^i, \bar{C}_i), \ (N^i, B_i), \ (i = 1, 2)\)

which satisfy the super-Poisson algebra

\[ \{ C^i(x), \bar{P}_j(y) \} = \{ P^i(x), \bar{C}_j(y) \} = \{ N^i(x), B_j(y) \} = \delta^i_j \delta(x - y). \]

Here the super-Poisson bracket is defined as

\[ \{ A, B \} = \frac{\delta A}{\delta q} \left| \frac{\delta B}{\delta p} \right|_t - (-1)^{\eta_A \eta_B} \left. \frac{\delta B}{\delta q} \right|_t \left. \frac{\delta A}{\delta p} \right|_t, \]
where $\eta_A$ denotes the number of fermions, called the ghost number, in $A$ and the subscript $r$ and $l$ denote right and left derivatives, respectively.

In the $CP^1$ model, the nilpotent BRST charge $Q$ and the BRST invariant minimal Hamiltonian $H_m$ are given by

$$
Q = \int d^2 x \left( C^i \tilde{\Omega}_i + P_i B_i \right),
$$
$$
H_m = \tilde{H}' - \int d^2 x C^1 \tilde{P}_2,
$$

(3.1)

which satisfy the relations

$$
\{ Q, H_m \} = 0, \quad Q^2 = \{ Q, Q \} = 0.
$$

(3.2)

Our next task is to fix the gauge, which is crucial to identify the BFT auxiliary field $\theta$ with the Stueckelberg field. The desired identification follows if one chooses the fermionic gauge fixing function $\Psi$ as

$$
\Psi = \int d^2 x \left( \bar{C}_i \chi^i + \bar{P}_i N^i \right),
$$

(3.3)

with the unitary gauge

$$
\chi^1 = \Omega_1, \quad \chi^2 = \Omega_2.
$$

(3.4)

Here note that the $\Psi$ satisfies the following identity

$$
\{ \{ \Psi, Q \}, Q \} = 0.
$$

(3.5)

The effective quantum Lagrangian is then described as

$$
L_{eff} = \int d^2 x \left( \Pi_\alpha \dot{Z}_\alpha^* + \Pi_\alpha \dot{Z}_\alpha + \pi_\theta \dot{\theta} + B_2 \dot{N}^2 + \bar{P}_i \dot{\bar{C}}_i + \bar{C}_2 \dot{\bar{P}}^2 \right) - H_{tot}
$$

(3.6)

where $H_{tot} = H_m - \{ Q, \Psi \}$ and the terms $\int d^2 x \left( B_1 \dot{N}^1 + \bar{C}_1 \dot{\bar{P}}^1 \right) = \{ Q, \int d^2 x \bar{C}_1 \dot{\bar{N}}^1 \}$ have been suppressed by replacing $\chi^1$ with $\chi^1 + \dot{N}^1$.

Now we perform path integration over the fields $B_1, N^1, \bar{C}_1, \bar{P}_1, \bar{P}$ and $C^1$, by using the equations of motion. This leads to the effective Lagrangian of the form

$$
L_{eff} = \int d^2 x \left[ \Pi_\alpha \dot{Z}_\alpha^* + \Pi_\alpha \dot{Z}_\alpha + \pi_\theta \dot{\theta} + B \dot{N} + \bar{P} \dot{\bar{C}} + \bar{C} \dot{\bar{P}}
\right. \\
- \left. (\Pi_\alpha - \frac{1}{2} Z_\alpha \pi_\theta)(\Pi_\alpha - \frac{1}{2} Z_\alpha \pi_\theta) \frac{Z_\gamma Z_\gamma}{Z_\gamma Z_\gamma + 2 \theta}
\right]
$$

8
\[-(\partial_i Z^*_\alpha)(\partial_i Z_\alpha) \frac{Z^*_\gamma Z_\gamma + 2\theta}{Z^*_\gamma Z_\gamma} + (Z^*_\alpha \partial_i Z_\alpha)(Z_\beta \partial_i Z^*_\beta) \left( \frac{Z^*_\gamma Z_\gamma + 2\theta}{Z^*_\gamma Z_\gamma} \right)^2\]
\[-\frac{1}{2} \pi \theta \tilde{\Omega}_2 + 2Z^*_\alpha Z_\alpha \pi \theta \tilde{C} \tilde{C} + \tilde{\Omega}_2 N + B\Omega_2 + \tilde{P}\tilde{P} \] 

(3.7)

with the redefinitions: \(N \equiv N^2, B \equiv B_2, \tilde{C} \equiv \tilde{C}_2, C \equiv C^2, \tilde{P} \equiv \tilde{P}_2, \mathcal{P} \equiv \mathcal{P}_2.\)

After performing the routine variation procedure and identifying \(N = -B + \frac{\theta}{(1-2\theta)},\) we arrive at the effective Lagrangian of the form

\[
L_{\text{eff}} = \int d^2x \left[ \frac{1}{(1-2\theta)}(\partial_\mu Z^*_\alpha)(\partial^\mu Z_\alpha) - (1-2\theta)^2 (B + 2\tilde{C})^2 \right.
\left. - \frac{1}{(1-2\theta)^2}(Z^*_\alpha \partial_\mu Z_\alpha)(Z_\beta \partial^\mu Z^*_\beta) - \frac{1}{1-2\theta} \partial_\mu \theta \partial^\mu B + \partial_\mu \tilde{C} \partial^\mu C \right]
\]

(3.8)

which is invariant under the BRST-transformation

\[
\delta_B Z_\alpha = \lambda Z_\alpha C, \quad \delta_B \theta = -\lambda(1-2\theta)C, \\
\delta_B \tilde{C} = -\lambda B, \quad \delta_B C = \delta_B B = 0.
\]

(3.9)

4 Collective coordinate quantization

In this section, we perform a semi-classical quantization of the unit topological charge \(Q = 1\) sector of the \(CP^1\) model by exploiting the collective coordinates to consider physical aspects of the theory.

As a first approximation to the quantum ground state we could quantize zero modes responsible for classical degeneracy by introducing collective coordinates as follows

\[
Z_1 = e^{-i(\alpha + \phi)/2} \cos \frac{F(r)}{2}, \\
Z_2 = e^{i(\alpha + \phi)/2} \sin \frac{F(r)}{2},
\]

(4.1)

where \((r, \phi)\) are the polar coordinates and \(\alpha(t)\) is the collective coordinates. Here, in order to ensure the case of \(Q = 1\), we have used the fact the profile function \(F(r)\) satisfies the boundary conditions: \(\lim_{r \to \infty} F(r) = \pi\) and \(F(0) = 0.\)
Using the above soliton configuration, we obtain the unconstrained Lagrangian of the form

\[ L = -E + \frac{1}{2}I\dot{\alpha}^2, \]  

where the soliton static mass and the moment of inertia are given by

\[ E = \frac{\pi}{2} \int_0^\infty dr \left[ \left( \frac{dF}{dr} \right)^2 + \frac{\sin^2 F}{r^2} \right], \]

\[ I = \pi \int_0^\infty dr r \sin^2 F. \]

Introducing the canonical momentum conjugate to the collective coordinate \( \alpha \)

\[ p_\alpha = I\dot{\alpha}, \]

we then have the canonical Hamiltonian

\[ H = E + \frac{1}{2I}p_\alpha^2. \]

At this stage, one can associate the Hamiltonian (4.5) with the previous one (2.3), which was given by the canonical momenta \( \pi^a \). Given the soliton configuration (4.1) one can obtain the relation between \( \pi^a \) and \( p_\alpha \) as follows

\[ \Pi^*_\alpha \Pi_\alpha = \frac{\sin^2 F}{4I^2} p_\alpha^2 \]

to yield the integral

\[ \int d^2x \Pi^*_\alpha \Pi_\alpha = \frac{1}{2I}p_\alpha^2. \]

Since the spatial derivative term in (2.3) yields nothing but the soliton energy \( E \), one can easily see, together with the relation (4.7), that the canonical Hamiltonian (2.3) is equivalent to the other one (1.5), as expected.

Now, let us define the angular momentum operator \( J \) as follows

\[ J = \int d^2x \epsilon_{ij}x^i T^{\alpha j}, \]

where the symmetric energy-momentum tensor is given by

\[ T^{\mu \nu} = \partial^\mu Z^*_\alpha \partial^\nu Z_\alpha + \partial^\mu Z_\alpha \partial^\nu Z^*_\alpha - (Z_\alpha \partial^\mu Z^*_\alpha)(Z^*_\beta \partial^\nu Z_\beta) \\
- (Z^*_\alpha \partial^\mu Z_\alpha)(Z_\beta \partial^\nu Z^*_\beta) - g^{\mu \nu}(\partial_\alpha Z^*_\alpha)(\partial^\rho Z_\alpha) + g^{\mu \nu}(Z^*_\alpha \partial_\rho Z_\alpha)(Z_\beta \partial^\beta Z^*_\beta). \]
Then, substituting the configuration (4.1) into Eq. (4.9), we obtain the angular momentum operator of the form

\[ J = -\mathcal{L}\dot{\alpha} = -p_\alpha = i\frac{\partial}{\partial \alpha} \]

(4.10)
to yield the Hamiltonian of the form

\[ H = E + \frac{1}{2I} J^2. \]

(4.11)

Here one notes that the above Hamiltonian can be interpreted as mass spectrum of a rigid rotator in the \( \mathbb{C}P^1 \) model.

Next, let us consider the zero modes in the extended phase space by introducing the soliton configuration

\[ Z_1 = (1 - 2\theta)^{1/2} e^{-i(\alpha + \phi)/2} \cos \frac{F(r)}{2}, \]

\[ Z_2 = (1 - 2\theta)^{1/2} e^{i(\alpha + \phi)/2} \sin \frac{F(r)}{2}, \]

(4.12)

which satisfy the first class constraint \( Z_\alpha^* Z_\alpha = 1 - 2\theta \) of Eq. (2.12). In this configuration from Eqs. (2.1) and (5.11) we then obtain

\[ L_{\text{eff}} = -E + \frac{1}{2} L\dot{\alpha}^2, \]

(4.13)

which is remarkably the Lagrangian (4.2) given in the original phase space. Consequently the quantization of zero modes in the extended phase space reproduces the same energy spectrum (4.11). This phenomenon originates from the fact that the collective coordinates \( \alpha \) in the Lagrangian (4.2) are not affected by the constraints (2.2) and (2.12) for the complex scalar fields \( Z_\alpha \). Here one notes that in the SU(2) Skyrmion model the collective coordinates themselves are constrained to yield the modified energy spectrum\(^3\) in contrast to the case of the \( \mathbb{C}P^1 \) model.

\(^3\)Here one can easily see that the first class physical fields \( \tilde{Z}_\alpha \) of Eq. (2.15) satisfy the corresponding first class constraint \( \tilde{Z}_\alpha^* \tilde{Z}_\alpha = 1 \) of Eq. (2.24).
5 Connection to O(3) nonlinear sigma model

In this section, we will demonstrate the equivalence of the \(CP^1\) model and O(3) NLSM [19, 22] at the canonical level. In the O(3) NLSM, the dynamical physical fields \(n^a\) are mappings from the spacetime manifold, which is assumed to be the direct product of a compact two-dimensional Riemann surface \(M^2\) and the time dimension \(R^1\), to the two-sphere \(S^2\), namely \(n^a : M^2 \otimes R^1 \to S^2\). On the other hand, the dynamical physical fields of the \(CP^1\) model are \(Z_\alpha\) which map the spacetime manifold \(M^2 \otimes R^1\) into \(S^3\), namely \(Z_\alpha : M^2 \otimes R^1 \to S^3\). Here one notes that \(S^3\) is homeomorphic to SU(2) group manifold.

Since the \(CP^1\) model is invariant under a local U(1) gauge symmetry, which consists of a redefinition of the phase of \(Z_\alpha\) as in Eq. (1.1), the physical configuration space of the \(CP^1\) model are the gauge orbits which form the coset \(S^3/S^1 = S^2 = CP^1\). In order to associate the physical fields of the \(CP^1\) model with those of the O(3) NLSM, we exploit the projection from \(S^3\) to \(S^2\), namely the Hopf bundle [3, 4]

\[
n^a = Z_\alpha^* \sigma^a Z_\alpha \tag{5.1}
\]

with the Pauli matrices \(\sigma^a\), so that we can see that the \(CP^1\) model Lagrangian (2.1) is equivalent to the O(3) NLSM [19]

\[
L = \int d^2x \left[ \frac{1}{4} (\partial^a n^b)(\partial^b n^a) \right] \tag{5.2}
\]

where \(n^a\) (a=1,2,3) is a multiplet of three real scalar field with a constraint

\[
\Omega_1 = n^a n^a - 1 \approx 0. \tag{5.3}
\]

Here note that the topological charge \(Q = Z\) sector of the O(3) NLSM is guaranteed by the homotopy group \(\pi_2(S^2) = Z\).

Moreover the collective coordinates (4.1) of the \(CP^1\) model can be consistently obtained via the Hopf bundle (5.1) from those of the well known O(3) NLSM

\[
\begin{align*}
n^1 &= \cos(\alpha(t) + \phi) \sin F(r), \\
n^2 &= \sin(\alpha(t) + \phi) \sin F(r), \\
n^3 &= \cos F(r),
\end{align*} \tag{5.4}
\]

\(^4\)Here one notes that in order to eliminate all the unphysical degrees of freedom one can also supply a gauge fixing condition such as the Coulomb gauge: \(Z_\alpha \partial_\alpha Z_\alpha - Z_\alpha \partial_\alpha Z_\alpha = 0\).
to yield the rigid rotator energy spectrum (4.11), which is exactly the same as that of the $O(3)$ NLSM [22] with the same soliton static mass $E$ and moment of inertia $I$ defined in Eq. (4.3).

Now one can introduce the other bundle for the canonical momenta [13]

$$\pi^a = \frac{1}{2}(\Pi_\alpha \sigma^a Z_\alpha + Z_\alpha^* \sigma^a \Pi_\alpha^*)$$

(5.5)

to reproduce the following secondary constraint from the corresponding $CP^1$ model one (2.5)

$$\Omega_2 = n^a \pi^a \approx 0.$$  (5.6)

Exploiting the above bundles (5.1) and (5.5) one can easily show that the canonical Hamiltonian (2.3) of the $CP^1$ model is reduced to that of the $O(3)$ NLSM

$$H_c = \int d^2 x \left( \pi^a \pi^a + \frac{1}{4} \partial_i n^a \partial_i n^a \right).$$  (5.7)

Here one notes that in contrast to the Banerjee case [13], where the reduced Hamiltonian has an additional term proportional to a first class constraint, we have obtained the exactly same Hamiltonian as shown in (5.7).

Similarly, introducing the bundles for the first class physical fields $\tilde{n}^a$ and $\tilde{\pi}^a$

$$\tilde{n}^a = \tilde{Z}_\alpha^* \sigma^a \tilde{Z}_\alpha$$
$$\tilde{\pi}^a = \frac{1}{2}(\tilde{\Pi}_\alpha \sigma^a \tilde{Z}_\alpha + \tilde{Z}_\alpha^* \sigma^a \tilde{\Pi}_\alpha^*)$$ (5.8)

one can also find the equivalence between the $CP^1$ model and the $O(3)$ NLSM in the extended phase space at the classical level.

## 6 Connection to consistent Dirac quantization

Now we consider consistent connection of the quantization of $CP^1$ model in the improved Dirac scheme to that of the standard Dirac one, where one can obtain the quantum commutators via Eqs. (2.20) and (2.21)

$$[Z_\alpha(x), Z_\beta(y)] = [Z_\alpha^*(x), Z_\beta(y)] = 0.$$
\[ [Z_\alpha(x), \Pi_\beta(y)] = i \left( \delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\gamma^* Z_\gamma} \right) \delta(x-y), \]
\[ [Z_\alpha(x), \Pi_\beta^*(y)] = -\frac{i}{2Z_\gamma^* Z_\gamma} Z_\alpha Z_\beta \delta(x-y), \]
\[ [\Pi_\alpha(x), \Pi_\beta(y)] = \frac{i}{2Z_\gamma^* Z_\gamma} \left( \Pi_\alpha Z_\beta^* - Z_\alpha^* \Pi_\beta \right) \delta(x-y), \]
\[ [P_\alpha(x), \Pi_\beta(y)] = \frac{i}{2Z_\gamma^* Z_\gamma} \left( \Pi_\alpha Z_\beta - Z_\alpha^* \Pi_\beta^* \right) \delta(x-y), \]

where the quantum operator for the canonical momenta are given as

\[ \Pi_\alpha = -i(\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta \]
\[ \Pi_\alpha^* = -i(\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta^* \]

(6.1)

with the short hands \( \partial_\alpha = \frac{\partial}{\partial Z_\alpha} \) and \( \partial_\alpha^* = \frac{\partial}{\partial Z_\alpha^*} \).

Now we observe that without loss of generality the generalized momenta \( \Pi_\alpha \) fulfilling the structure of the commutators (6.1) is of the form

\[ \Pi_\alpha^c = -i(\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta - \frac{icZ_\alpha^*}{2Z_\beta^* Z_\beta} \]
\[ \Pi_\alpha^{*c} = -i(\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta^* - \frac{icZ_\alpha}{2Z_\beta^* Z_\beta} \]

(6.3)

with an arbitrary parameter \( c \) to be fixed later.

On the other hand, the energy spectrum of the rigid rotators in the \( CP^1 \) model can be obtained in the Weyl ordering scheme [23] where the Hamiltonian (2.3) is modified into the symmetric form

\[ H_N = E + \int d^2 x \Pi_\alpha^N \Pi_\alpha^* \]

(6.4)

where

\[ \Pi_\alpha^N = \frac{i}{2} \left[ (\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta + \partial_\beta (\delta_{\alpha\beta} - \frac{Z_\alpha^* Z_\beta}{2Z_\beta^* Z_\beta}) + \frac{c Z_\alpha^*}{Z_\beta^* Z_\beta} \right] \]
\[ \Pi_\alpha^{N*} = \frac{i}{2} \left[ (\delta_{\alpha\beta} - \frac{Z_\alpha Z_\beta^*}{2Z_\beta^* Z_\beta}) \partial_\beta^* + \partial_\beta^* (\delta_{\alpha\beta} - \frac{Z_\alpha^* Z_\beta}{2Z_\beta^* Z_\beta}) + \frac{c Z_\alpha}{Z_\beta^* Z_\beta} \right] . \]

(6.5)
After some algebra, one can obtain the Weyl ordered $\Pi^N_\alpha^* \Pi^N_\alpha$ as follows

$$\Pi^N_\alpha^* \Pi^N_\alpha = -\partial^* \alpha \partial^\alpha + \frac{3Z^*_\alpha Z^\beta \partial^* \alpha \partial^\beta + \frac{3}{4Z^*_\gamma Z^\gamma} Z^\alpha \partial^\alpha - \frac{1}{16Z^*_\gamma Z^\gamma} (2c - 1)(2c + 3)}$$

to yield the modified quantum energy spectrum of the rigid rotators

$$\langle H_N \rangle = E + \frac{1}{2T} J^2 - \int d^2 x \frac{(2c + 3)(2c - 1)}{16}.$$

(6.6)

Now, in order for the Dirac bracket scheme to be consistent with the BFT one, the adjustable parameter $c$ in Eq. (6.6) should be fixed with the values

$$c = \frac{1}{2} - \frac{3}{2}.$$

(6.7)

Here one notes that these values for the parameter $c$ relate the Dirac bracket scheme with the BFT one to yield the desired quantization in the $CP^1$ model so that one can achieve the unification of these two formalisms.

## 7 Conclusions and Discussions

In summary, we have constructed first-class BFT physical fields and, in terms of them we have obtained a first-class Hamiltonian, consistent with the Hamiltonian with the original fields and auxiliary fields. The Poisson brackets of the BFT physical fields are also constructed and these Poisson brackets are shown to reproduce the corresponding Dirac brackets in the limit of vanishing auxiliary fields. Subsequently, we have obtained, in the Batalin, Fradkin and Vilkovisky (BFV) scheme[14, 15, 16], a BRST-invariant gauge fixed Lagrangian including the (anti)ghost fields, and BRST transformation rules under which the effective Lagrangian is invariant. On the other hand, by performing the semiclassical quantization with the collective coordinates, we have obtained the spectrum of rigid rotator, which is shown to be consistent with that obtained by the standard Dirac method with the introduction of generalized momenta. Next, using the Hopf bundle[20], we have shown that the $CP^1$ model is exactly equivalent to the O(3) NLSM. Through further investigation it will be interesting to include the Chern-Simons or Hopf term in the $CP^1$ model since there are still subtle ambiguities in the literatures[24].
Now, it is appropriate to comment on the dynamical aspects of the $CP^{N-1}$ model which possesses a local U(1) gauge invariance. As discussed in Eq. (1.1), one has traded in two degrees of freedom (three components of $n^a$ minus one constraint (5.3)) for the three degrees of freedom (four components of $Z$ minus one constraint (2.2)). In fact, the $Z_\alpha$ field possesses only two degrees of freedom since an overall phase transformation (1.1) does not change $n^a$ and hence the Lagrangian (2.1). In order to check that the Lagrangian (2.1) is invariant under the U(1) local gauge transformation, one can rewrite the Lagrangian as 

$$L = \int d^2 x \left[ \frac{2}{g^2} (D_\mu Z_\alpha)^* (D^\mu Z_\alpha) + \lambda (Z_\alpha^* Z_\alpha - 1) \right]$$

(7.8)

with the covariant derivative $D_\mu = \partial_\mu - i A_\mu$ and the auxiliary gauge field $A_\mu = -i Z_\alpha^* \partial_\mu Z_\alpha$. Here we have explicitly included the primary constraint $\Omega_1$ in Eq. (5.3) together with the Lagrangian multiplier field $\lambda$, and the coupling constant $g^2$ [3]. Under the U(1) local gauge transformation (1.1), one can have

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha$$

(7.9)

under which the Lagrangian (7.8) is invariant. At the classical level, the gauge field $A_\mu$ associated with the U(1) symmetry is a redundant one which can be eliminated by using the equations of motion [3].

On the other hand, it was shown that in a stationary phase approximation the expectation value $\langle \lambda \rangle$ does not vanish for $g^2$ large enough [3]. In this case, one can replace the field $\lambda(x)$ in the Lagrangian (7.8) with a constant $\lambda = \langle \lambda \rangle$ to yield the effective Lagrangian of the disordered phase where $Z_\alpha$ is an effective field no longer subjected to a constraint on its magnitude [3, 25]. Thus, at the quantum level, the $Z_\alpha$ field acquires a mass and the spin-spin correlation function becomes short ranged. Here we have effectively traded in the constraint for a mass. On the other hand, the gauge field $A_\mu$ may also acquire the kinetic term to become dynamical at the quantum level [3]. Moreover, if the coupling constant $g^2$ becomes larger than some critical values, the symmetric phase appears with massless vector boson pole. Since the O(3) NLSM has no local U(1) symmetry and may become singular due to a divergence in a composite vector boson channel, the above equivalence between the $CP^1$ model and O(3) NLSM breaks down at the quantum level where one needs to take into account properly the dynamical gauge boson effects.
In the BFT scheme, at the quantum level, a similar situation happens to yield the quantum effects and the corresponding breakdown of the equivalence between the $CP^1$ model and the $O(3)$ NLSM. Moreover, through further investigation, it will be interesting to study a new term $\lambda \theta$ in $\lambda \tilde{\Omega}_1$ associated with the first class constraint $\tilde{\Omega}_1$ in Eq. (2.12), which may play a role in quantum level phenomenology.

STH would like to thank the University of South Carolina for the warm hospitality during his visit. STH and YJP would like to thank B.H. Lee for helpful discussions and acknowledge financial support in part from the Korean Ministry of Education, BK21 Project No. D-1099 and Grant No. 2000-2-11100-002-5 from the Basic Research Program of the Korea Science and Engineering Foundation. The work of KK and FM are supported in part by NSF (USA), Grant No. 9900756 and No. INT-9730847.

References

[1] A.A. Belavin and A.M. Polyakov, JETP Lett. 22, 245 (1975).
[2] H. Eichenherr, Nucl. Phys. B146, 215 (1978).
[3] E. Witten, Nucl. Phys. B149, 285 (1979).
[4] A. D’Adda, P. DiVecchia and M. Lüscher, Nucl. Phys. B152, 125 (1979).
[5] P.A.M. Dirac, Lectures in Quantum Mechanics (Yeshiva University, New York, 1964).
[6] I.A. Batalin and E.S. Fradkin, Phys. Lett. B180 (1986) 157; Nucl. Phys. B279, 514 (1987); I.A. Batalin and I.V. Tyutin, Int. J. Mod. Phys. A6, 3255 (1991).
[7] R. Banerjee, Phys. Rev. D48, R5467 (1993); W.T. Kim and Y.-J. Park, Phys. Lett. B336, 376 (1994).
[8] Y.-W. Kim, Y.-J. Park, and K.D. Rothe, J. Phys. G24, 953 (1998); Y.-W. Kim and K. D. Rothe, Nucl. Phys. B510, 511 (1998); M.-I. Park
and Y.-J. Park, Int. J. Mod. Phys. A13, 2179 (1998); Y.-W. Kim and Y.-J. Park, Mod. Phys. Lett. A13, 1201 (1998); S.-T. Hong, W.T. Kim and Y.-J. Park, Mod. Phys. Lett. A15, 1915 (2000).

[9] S. Ghosh, Phys. Rev. D49, 2990 (1994); J.-G. Zhou, Y.-G. Miao, and Y.-Y. Liu, Mod. Phys. Lett. A9, 1273 (1994); R. Amorim and J. Barcelos-Neto, Phys. Rev. D53, 7129 (1996); M. Fleck and H.O. Girotti, Int. J. Mod. Phys. A14, 4287 (1999); C.P. Nativiade and H. Boschi-Filho, Phys. Rev. D62, 025016 (2000).

[10] S.-T. Hong, Y.-W. Kim, and Y.-J. Park, Phys. Rev. D59, 114026 (1999).

[11] S.-T. Hong, Y.-W. Kim, and Y.-J. Park, Mod. Phys. Lett. A15, 55 (2000); S.-T. Hong and Y.-J. Park, Mod. Phys. Lett. A15, 913 (2000).

[12] N. Banerjee, S. Ghosh, and R. Banerjee, Phys. Rev. D49, 1996 (1994).

[13] R. Banerjee, Phys. Rev. D49, 2133 (1994).

[14] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. B55, 224 (1975); M. Henneaux, Phys. Rep. 126, 1 (1985).

[15] T. Fujiwara, Y. Igarashi, and J. Kubo, Nucl. Phys. B341, 695 (1990); Y.-W. Kim, S.-K. Kim, W.T. Kim, Y.-J. Park, K.Y. Kim, and Y. Kim, Phys. Rev. D46, 4574 (1992); S. Hamamoto, Prog. Theor. Phys. 96, 639 (1996).

[16] C. Bizdadea and S.O. Saliu, Nucl. Phys. B456, 473 (1995).

[17] C. Becci, A. Rouet, and R. Stora, Ann. Phys. 98, 287 (1976); I.V. Tyutin, Lebedev Preprint 39 (1975).

[18] W. Oliveira and J.A. Neto, Int. J. Mod. Phys. A12, 4895 (1997).

[19] M. Bowick, D. Karabali, and L.C.R. Wijewardhana, Nucl. Phys. B271, 417 (1986).

[20] M. Bergeron, G. Semenoff, and R.R. Douglas, Int. J. Mod. Phys. A7, 2417 (1992).
[21] B. Chakraborty and A.S. Majumdar, Phys. Rev. D58, 125024 (1998); ibid, Int. J. Mod. Phys. A14, 1561 (1999).

[22] S.-T. Hong, W.T. Kim, and Y.-J. Park, Phys. Rev. D60, 125005 (1999).

[23] T.D. Lee, Particle Physics and Introduction to Field Theory (Harwood, New York, 1981).

[24] P.K. Panigrahi, S. Roy, and W. Scherer, Phys. Rev. Lett. 61, 2827 (1988); A. Kovner, Phys. Lett. B224, 299 (1989).

[25] A. Zee, Physics in (2+1)-Dimension (World Scientific, Singapore, 1991) Ed. Y.M. Cho.