How precisely neutrino emission from supernova remnants can be constrained by gamma ray observations?

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Abstract

We propose a conceptually and computationally simple method to evaluate the neutrinos emitted by supernova remnants using the observed $\gamma$-ray spectrum. The proposed method does not require any preliminary parametrization of the gamma ray flux; the gamma ray data can be used as an input. In this way, we are able to propagate easily the observational errors and to understand how well the neutrino flux and the signal in neutrino telescopes can be constrained by $\gamma$-ray data. We discuss the various possible sources of theoretical and systematical uncertainties (\textit{e.g.}, neutrino oscillation parameters, hadronic modeling, \textit{etc.}), obtaining an estimate of the accuracy of our calculation. Furthermore, we apply our approach to the supernova remnant RX J1713.7-3946, showing that neutrino emission is very-well constrained by the H.E.S.S. $\gamma$-ray data: indeed, the accuracy of our prediction is limited by theoretical uncertainties. Neutrinos from RX J1713.7-3946 can be detected with an exposure of the order $\text{km}^2 \times \text{year}$, provided that the detection threshold in future neutrino telescopes will be equal to about 1 TeV.
1 Introduction

Under-water and under-ice neutrino telescopes are instruments aiming to discoveries. They could reveal an effective acceleration of cosmic rays (CR) in galactic sources (such a supernova remnants [1] and micro-quasars [2]) and/or in extragalactic sources (such as AGN and gamma ray bursts [3]). However, at present it is possible to obtain reliable expectations only for few of these sources, such as the supernova remnants (SNR) discussed in this paper.

The idea that CR could originate in supernovae has been put forward already in 1934 but the first quantitative formulation of the conjecture that the young SNRs refurbish the Milky Way of cosmic rays, compensating the energy losses, is due to Ginzburg & Syrovatskii [1]. In fact, the turbulent gas of SNRs is a large reservoir of kinetic energy and this environment can support diffusive shock waves acceleration [4]. The theory of CR acceleration in SNRs is still in evolution, but the generic expectations are stable. The CR flux in SNRs is expected to have a power law spectrum with spectral index $\Gamma = 2.0 - 2.4$ at low energies with a cutoff at an energy $E_c$ which depends on the details of the acceleration mechanism and on the age of the system. In specific implementations the cutoff energy $E_c$ can be as large as several PeV and, thus, is consistent with the “knee” in the CR spectrum at $E \sim 3 \times 10^{15}$ eV, which is believed to mark the transition from galactic to extra-galactic origin of CR [5].

In recent times, great progress has been made in the observation of SNRs. In particular, the High Energy Stereoscopic System (H.E.S.S.) [6] has determined quite precisely the gamma ray spectra of few SNRs showing that they extend above 10 TeV. In the context of Ginzburg & Syrovatskii hypothesis, it is natural to postulate that the observed gammas are produced by the decay of $\pi^0$ (and $\eta$) resulting from the collision of accelerated hadrons with the ambient medium. New and crucial observations are being collected and the hadronic origin seems to be favored for certain SNRs, such as Vela Jr [7] and RX J1713.7-3946 [8, 9]. It is not yet possible, however, to exclude that (part of) the observed radiation is produced by electromagnetic processes. The definitive proof that SNRs effectively accelerate CR could be obtained by the observation of high energy neutrinos in neutrino telescopes presently in operation, in construction or in project [11].

As well known, there is a strict connection between photon and neutrino fluxes produced by hadronic processes in transparent sources (see, e.g., [12]), which results from the fact that the same amount of energy is roughly given to $\pi^0$, $\pi^+$ and $\pi^-$ in hadronic collisions. On this basis, one can estimate the neutrino fluxes expected from a sources with known $\gamma$-ray spectra, trying to identify detectable sources and/or to optimize the detection strategies. The gamma-neutrino connection has been described in various recent papers [13, 14, 15, 16, 18, 19, 20, 21] at a different level of accuracy, relying on different assumptions on the primary cosmic ray spectrum and/or hadronic interaction model.

In this particular moment, when high energy gamma ray astronomy is flourishing and the neutrino telescopes are finally becoming a reality, it is clearly important to have solid and transparent predictions. We continue, thus, the work started in [10, 21, 22], proposing a method to calculate neutrinos fluxes which is, at the same time, simple, accurate and model-independent. Our results are in essence a straightforward applications of standard techniques [23], but we believe that that they will be useful since they improve the existing calculations in various respects. In particular, we provide simple analytic expressions for the neutrino fluxes which have a general validity and can be applied directly to gamma ray data since they do not require any parametrization of the photon spectrum. This allows us to propagate easily the observational errors in the gamma ray flux and, thus, to understand how well

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1 See also [10] for a recent analysis leading to a different conclusion.
the neutrino flux and the signal in neutrino telescopes can be constrained by γ-ray data. We also
discuss the various possible sources of theoretical and systematical uncertainties, obtaining an estimate
of the accuracy of our method. This kind of analysis is relevant in the present situation, since the
number of events expected in neutrino telescopes from SNRs is quite low (see, e.g., [16]) and even
small (downward) revisions of the expected signals may be important and/or require different detection
strategies. It is thus important to understand the relevance of different assumptions in the calculations
and the origin of (apparently) contrasting results appeared in the literature.

The plan of the paper is as follow. In Sect. 2 we review the method presented in our previous
paper [16]. The relations obtained in this work – Eqs. (8) – are physically equivalent to those presented
in [16]. However, they are more compact and more convenient for numerical computations since we
have been able to recast them in such a way that they require only one numerical integration. In
Sect. 3 we discuss the effect of neutrinos oscillations (see [24] for a review) and we discuss the relevance
of uncertainty in neutrino mixing parameters for the predicted neutrino flux. In Sect. 4 we present
our main results, i.e., Eqs. (30) and (32) which relate the (oscillated) muon neutrino and antineutrino
fluxes to the γ-ray flux. As it is explained, the only necessary information to predict neutrino fluxes are
the relative production rates of the various mesons in hadronic processes, which are robust predictions
of hadronic interaction models. In this paper, we adopt the results from Pythia [25], estimated by
using the parametrization of hadronic cross sections presented in [26]. A comparison with SYBILL [27]
and DPMJET-III [28] is performed (when possible) by using the parametrization and tabulations of
hadronic cross sections presented by [29] and [30]. In Sect. 5 and 6 we propose a procedure to predict
neutrino fluxes and event rate in ν-telescopes directly from γ-ray observational data, and we apply it to
RX J1713.7-3946 which is presently the best studied SNR. In Sect. 7 we summarize our main results.

2 The photon-neutrino relation

The interactions of cosmic ray protons with a hydrogen ambient cloud result in the production of mesons
which subsequently decay producing gamma ray and neutrinos. Both gamma rays and neutrinos depend
linearly on the flux of the primary cosmic ray. We, thus, expect that a linear relation also exists between
 photon and neutrino fluxes, which can formally be expressed as:

$$\Phi_{\nu}[E_{\nu}] = \int \frac{dE_{\gamma}}{E_{\gamma}} K_{\nu}[E_{\nu}, E_{\gamma}] \Phi_{\gamma}[E_{\gamma}].$$

as will be precised just below in this Section.

In order to determine the kernels $K_{\nu}[E_{\nu}, E_{\gamma}]$, we only need to know the relative number of pions,
kaons and η produced by cosmic rays at any given energy, as it is explained in [16] and further discussed
in the following. Let us indicate with $R_i[E]$ the number of $i-$particles produced per unit time and unit
energy in the cloud. Here, for simplicity, we assume that the momentum distributions of the various
particles are approximately isotropic, so that the differential flux produced in a detector at a distance
are produced for each $\pi$ and $\eta$, according to:

$$\Phi_\gamma[E_\gamma] = \frac{2}{4\pi D^2} \int_{E_\gamma}^{\infty} \frac{dE_\gamma}{E_\gamma} \left( R_{\pi^0}[E] + b_{\gamma\gamma} R_\eta[E] \right)$$

(2)

where $b_{\gamma\gamma} = 0.394$ is the $\eta \to \gamma\gamma$ branching ratio and the factor 2 takes into account that two photons are produced for each $\pi^0$ and $\eta$. Neutrinos are, instead, mainly produced by charged pions and charged kaons. We can formally write (neglecting neutrino oscillations):

$$\Phi_\nu[E_\nu] = \frac{1}{4\pi D^2} \sum_i \int_{E_\nu}^\infty \frac{dE_\nu}{E_\nu} R_i[E] \omega_{i\nu} \frac{E_\nu}{E_\nu}$$

(3)

where $i = \pi^+, \pi^-, K^+, K^-$ and $\nu = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. The quantity $\omega_{i\nu}[x]$ with $x = E_\nu/E_\nu$, represents the spectrum of neutrinos $\nu$ produced in the decay chain the $i$-meson.

If we assume that the ratios between the production rates of the various mesons are approximately constant (see next section), the above relations can be combined in order to obtain the neutrino fluxes as a function of the photon flux. Rel. (2) can be, in fact, inverted obtaining:

$$R_{\pi^0}[E] = \frac{4\pi D^2}{1 + b_{\gamma\gamma} f_\eta} \frac{E}{2} \frac{d\Phi_\gamma[E]}{dE}$$

(4)

where $f_\eta = (R_\eta/R_{\pi^0})$. This expression can then be used in Eq. (3) to obtain:

$$\Phi_\nu[E_\nu] = \int_{E_\nu}^\infty \frac{dE_\gamma}{E_\gamma} K_{i\nu}[E_\nu/E_\gamma] \omega_{i\nu} \frac{E_\nu}{E_\nu}$$

(5)

where:

$$K_{i\nu}[x] = \frac{1}{2(1 + b_{\gamma\gamma} f_\eta)} \sum_i f_i \frac{d\omega_{i\nu}[x]}{d\ln x}$$

(6)

and

$$f_i = R_i/R_{\pi^0} \quad \text{with} \quad i = \pi^+, \pi^-, K^+, K^-, \eta.$$  

(7)

In explicit terms, we can write:

$$K_{\nu_e}[x] = \frac{1}{1 + f_\eta^e} \left( f_{\pi^+ \nu_e} + f_{K^+ \nu_e} + g_{K\nu_e} \right)$$

$$K_{\overline{\nu}_e}[x] = \frac{1}{1 + f_\eta^e} \left( f_{\pi^- \overline{\nu}_e} + f_{K^- \overline{\nu}_e} + g_{K\overline{\nu}_e} \right)$$

$$K_{\nu_\mu}[x] = \frac{1}{1 + f_\eta^\mu} \left( f_{\pi^+ \nu_\mu} + f_{K^+ \nu_\mu} + h_{K} + f_{K^- \nu_\mu} \right)$$

$$K_{\overline{\nu}_\mu}[x] = \frac{1}{1 + f_\eta^\mu} \left( f_{\pi^- \overline{\nu}_\mu} + f_{K^- \overline{\nu}_\mu} + h_{K^-} + f_{K^+ \overline{\nu}_\mu} \right)$$

(8)

2 If this assumption is removed, one has to replace here and in the following:

$$\frac{1}{4\pi} R_i[E] \rightarrow \frac{dR_i[E, n]}{d\Omega}$$

where $dR_i/d\Omega$ is the rate of $i$-particles produced per unit energy and unit solid angle, $n$ is the unit vector in the direction connecting the SNR to the detector and we have taken into account that the produced particles are almost collinear.
where we have defined
\[ f'_{\eta} = b_{\eta\gamma} f_{\eta} \quad f'_{K^\pm} = b_{K\nu} f_{K^\pm} \] (9)
and \( b_{K\nu} = 0.634 \) is the branching ratio for \( K^\pm \) semi-leptonic decay. The functions \( h_i[x] \) account for neutrinos produced in \( \pi^+ \to \mu^+ + \nu_\mu \) and \( K^+ \to \mu^+ + \nu_\mu \) (and charge conjugated processes) and are given by
\[ h_i[x] = \frac{1}{2} \delta[x - (1 - r_i)] \] (10)
where \( r_i = (m_\mu/m_i)^2 \) with \( i = \pi, K \). The functions \( g_{\nu}[x] \) account, instead, for neutrinos produced by muons. They also encode the information on the energy distribution and polarization of muons produced by pions and kaons decay and can be expressed as:
\[ g_{\nu}[x] = \frac{g_{\nu}[x]}{2(1 - r_i)} + \frac{r_i g''_{\nu}[x]}{(1 - r_i)^2} - \left( \frac{g'''_{\nu}[x/r_i]}{2(1 - r_i)} + \frac{g'''_{\nu}[x/r_i]}{(1 - r_i)^2} \right) \theta[r_i - x] \] (11)
where the relevant polynomials are:
\[ g'_{\nu}[x] = 2(1 - x)^2(1 + 2x) \]
\[ g''_{\nu}[x] = 4(1 - x^3)/3 \]
\[ g'''_{\nu}[x] = (1 - x)^2(1 + 2x)/3 \] (12)
and
\[ g'_{\nu}[x] = 12(1 - x)^2x \]
\[ g''_{\nu}[x] = 4(1 - x)^3 \]
\[ g'''_{\nu}[x] = -2(1 - x)^3 \] (13)

Some details of the derivations and some checks of these formulae, in the limit of “isospin invariance”, are given in the appendix A of [21].

We note that the same kind of approach can be used to calculate the flux of electrons and positrons produced by hadronic processes in the cloud, which may be relevant to calculate radio synchrotron emission. By considering that the spectral distribution of electrons produced in \( \mu \to e + \bar{\nu}_e + \nu_\mu \) is identical to that of \( \nu_\mu \), we immediately obtain:
\[ \Phi_{e\pm}[E] = \int_{E^{-}}^{\infty} \frac{dE_\gamma}{E_\gamma} K_{e\pm}[E/E_\gamma] \Phi_\gamma[E_\gamma] \] (14)
where
\[ K_{e^-}[x] = \frac{1}{1 + f'_{\eta}} \left[ f_\pi - g_{\pi\nu_\mu}[x] + f'_{K^-} - g_{K\nu}[x] \right] \] (15)
\[ K_{e^+}[x] = \frac{1}{1 + f'_{\eta}} \left[ f_\pi + g_{\pi\nu_\mu}[x] + f'_{K^+} + g_{K\nu}[x] \right] \] (16)

3 The effect of neutrino oscillations

Neutrino telescopes are sensitive to muon neutrino and muon antineutrino fluxes at earth that differ from the fluxes produced in the supernova remnants due to the effect of neutrino oscillations. Denoting
EγKoscνµ[ν/γ] Φγ[γ] (17)

which is given by:

Koscνµ[x] = PµµKνµ[x] + PeµKνe[x].

This can be expressed as the sum of several contributions, each arising from a different mechanism for neutrino production:

Koscνµ[x] = A1hπ[x] + A2gπνµ[x] + A3gπνe[x] + B1hK[x] + B2gKνµ[x] + B3gKνe[x]

with the coefficients Ai and Bi given by:

A1 = Pµµfπ+/1 + fη'; A2 = Pµµfπ−/1 + fη'; A3 = Peµfπ+/1 + fη';

B1 = PµµfK+/1 + fη'; B2 = PµµfK−/1 + fη'; B3 = PeµfK+/1 + fη'.

A similar expression holds for the muon antineutrino flux. Namely, the kernel Koscνµ[x] is obtained from Eq. (19) with the replacements fπ+ → fπ− and fK+ → fK− (and vice versa) in Rel. (20)

The oscillation probabilities Pµµ and Peµ can be evaluated by assuming: θ12 = 35° ± 1°, θ23 = 42° ± 4° and θ13 = 5° ± 4°, close to the range found in a recent global analysis of the world data [32]. We obtain the central values:

Pµµ = 0.36       Peµ = 0.26

that will be used in the following calculations. The uncertainties in Pµµ and Peµ are at the level of about ∼ 10% and are almost completely anti-correlated, being mostly due to the spread of θ23 around

Figure 1: Suppression of cosmic νµ (or ¯νµ) flux as a function of the electron/muon neutrino (or antineutrino) flux ratio. The dashed (continuous) lines enclose the 1σ (2σ) region consistent with the measurements. The dotted line identify the most probable value for the suppression factor. The grey region, instead, is forbidden.
maximal mixing. It is, thus, not immediate to understand how they propagate to the final muon neutrino flux. Writing:

\[ \frac{\Phi_{\nu_\mu}}{\Phi_{\nu_\mu}^{\text{tot}}} = \frac{1}{1 + \psi} \left( P_{\mu\mu} + P_{e\mu} \psi \right) \]

where

\[ \begin{cases} \psi = \Phi_{\nu_e}^0 / \Phi_{\nu_\mu}^0 \\ \Phi_{\nu_\mu}^{\text{tot}} = \Phi_{\nu_\mu} + \Phi_{\bar{\nu}_\mu}^0 \end{cases} \] (22)

we see the effect of oscillations depends on the neutrino flavor ratio at production \( \psi \). We also understand that we expect a partial cancellation between the contributions of \( P_{\mu\mu} \) and \( P_{e\mu} \) to the total error budget.

In Fig. 1, we show the l.h.s. of the above equation as a function of \( \psi \). The dashed (continuous) lines in the figure enclose the 1\( \sigma \) (2\( \sigma \)) region consistent with the measurements. The dotted line identifies the most probable value for the muon neutrino flux suppression factor. Several remarks on this figure are in order.

(i) When we consider the values \( \psi \sim 0.5 \) characteristic of neutrinos from pion decay, we find that the most probable value is \( \Phi_{\nu_\mu} / \Phi_{\nu_\mu}^{\text{tot}} \approx 0.33 \). This is a well known result, first derived in [33] and often used in the literature (see, e.g., [18, 34]). For the sake of precision, we note that \( \psi \) is the electron/muon neutrino (or antineutrino) flux ratio at a fixed energy which may differ from 0.5 even for pion decay, due the different energy distribution of the three neutrinos produced by pions. If we assume that the neutrino spectra in the SNR is described by a power law with spectral index \( \alpha = 2 \) (\( \alpha = 3 \)), we obtain \( \psi = 0.34 \) (\( \psi = 0.61 \)). In our approach, however, we do not need to decide \textit{a priori} the neutrino flavor ratio \( \psi \) or the value of \( \Phi_{\nu_\mu} / \Phi_{\nu_\mu}^{\text{tot}} \): the energy distributions of the produced neutrinos and the oscillation effects are automatically implemented by Eq. (19).

(ii) We note that the possible suppression of the muon neutrino flux is bounded from below, as emphasized by the forbidden (grey) region in the figure. In the extreme case \( \psi = 0 \) we have \( \Phi_{\nu_\mu} / \Phi_{\nu_\mu}^{\text{tot}} > 1/3 \) that can be understood by considering that the (averaged) survival probability \( P_{\mu\mu} \) cannot be smaller than 1/3; if we assume \( \psi = 1 \) we obtain, instead \( \Phi_{\nu_\mu} / \Phi_{\nu_\mu}^{\text{tot}} = (P_{\mu\mu} + P_{e\mu})/2 = (1 - P_{\mu\tau})/2 > 1/4 \), which follows from the fact that the (averaged) oscillation probability \( P_{\mu\tau} \) cannot be larger than 1/2. The oscillation effect that is realized in Nature happens to be very close to the maximum possible effect, namely, to be very close to the forbidden region in Fig. 1.

(iii) The uncertainty due to the imprecise knowledge of the oscillation parameters is at the level of 2\% and diminishes with increasing \( \psi \), as a result of the partial cancellation of the much larger (anti-correlated) contribution of \( P_{\mu\mu} \) and \( P_{e\mu} \) to the total error budget. It should be noted that uncertainty works mostly in the direction to \textit{increase} the expected muon neutrino (or antineutrino) flux. For \( \psi \sim 0.5 \), we find, in fact, the region

\[ \frac{\Phi_{\nu_\mu}}{\Phi_{\nu_\mu}^{\text{tot}}} = (0.33 - 0.35) \text{ at } 2\sigma \] (23)

which is extremely asymmetric with respect to the most probable value \( \sim 0.33 \).

4 The meson production rates (the \( f_i \) factors)

The last information that we need to predict the neutrino fluxes are the factors \( f_i \) which give the production rates of the various mesons rescaled to that of the neutral pions (see Eq. (7)) and can be calculated from hadronic interaction models.

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3 The neutrino flavor ratio \( \psi \) for \( K^\pm \) decay can largely depart from 0.5. Assuming, again, that the neutrino spectra is described by a power law with spectral index \( \alpha = 2 \) (\( \alpha = 3 \)), we obtain \( \psi = 0.32 \) (\( \psi = 0.19 \)). In a more realistic case, in which we choose \( f_{\pi^+} = f_{\pi^-} = 1 \) and \( f_{K^+} = f_{K^-} = 0.1 \), we obtain \( \psi = 0.52 \) (\( \psi = 0.56 \)).
The rate of production of $i$--mesons at an energy $E$ is given by:

$$ R_i[E] = N \int_{E}^{\infty} \frac{dE_p}{E_p} J_p[E_p] \sigma[E_p] F_i \left[ \frac{E}{E_p}, E_p \right] $$

(24)

where $N$ is the total amount of target hydrogen in the cloud, $J_p[E_p]$ is the CR energy spectrum (averaged over the SNR volume, see Eq. (7) in [22]), and we employed the usual definition of the adimensional distribution function $F_i[x, E_p]$:

$$ \frac{d\sigma_i}{dE} = \frac{\sigma[E_p]}{E_p} F_i \left[ \frac{E}{E_p}, E_p \right] $$

(25)

where $\sigma$ is the total inelastic p-p cross section, $E_p$ is the proton energy, $d\sigma_i/dE$ is the inclusive cross section for $i$--particles production and $E$ represents the energy of the produced particle. Let us assume that the cosmic ray spectrum is roughly described by a power law in energy. We can write $J[E_p] \sigma[E_p] \propto E_p^{-\alpha}$, and using the “quasi-scaling” approximation\footnote{The “quasi-scaling” approximation is accurate at the few per cent level in the energy range of interest, see, e.g., [22]. We also remark that the total cross-section $\sigma[E_p]$ has a very weak dependence on the proton energy, see, e.g., [29]).} for hadronic cross sections, i.e., taking

$$ F_i[x, E_p] \to F_i[x] \equiv F_i[x, E_p^0] $$

(26)

where $E_p^0$ is a fixed reference values for the proton energy, we finally obtain:

$$ f_i \equiv \frac{R_i}{R_{\pi^0}} = \frac{Z_i[\alpha]}{Z_{\pi^0}[\alpha]} $$

(27)

where:

$$ Z_i[\alpha] = \int_{0}^{1} dx x^{\alpha-1} F_i[x] $$

(28)

The above relation shows that the ratios between the number of produced particles are essentially independent on energy and are determined by the $(\alpha - 1)$--momenta of the adimensional distribution function $F_i$.

Presently there are various well developed codes - such as Pythia [25], SYBILL [27], QGSJET [35], DPMJET-III [28] - which implement different hadronic interaction models to study particle production in nucleon-nucleon or nucleon-nucleus collisions. It would be interesting to make a systematic comparison between the various codes, but this is beyond the scope of this work. In Ref. [26], suitable parametrizations of the pion and kaon differential cross section in $pp$ collisions from the Pythia code are given. Moreover, parametrizations of $\pi^0$ and $\eta$ productions in $pp$ collisions, according SYBILL and QGSJET code, are presented in Ref. [29]. In the left panel of Fig. 2 we use these parametrizations to calculate the relevant $f_i$ factors for $\alpha = 2 - 4$. We see that the quantities $f_i$ are marginally dependent on the assumed spectral index and we take the values obtained for $\alpha = 2$:

$$ f_{\pi^+} = 1.08 \quad f_{\pi^-} = 0.79 \quad f_\eta = 0.48 $$
$$ f_{K^+} = 0.13 \quad f_{K^-} = 0.09 $$

(29)

as reference values in our calculations. We remark that, using these values, we improve on our previous calculation [16] where we assumed “isospin invariance” (i.e., $f_{\pi^+} = f_{\pi^-} = 1$) and we overestimated the rate of $K^\pm$ production ($f_{K^+} = f_{K^-} = 0.2$).
In order to estimate uncertainties in our method, we should compare the $f_i$ factors in Eq. (29) with those obtained by using different hadronic interaction models. For assumed spectral index $\alpha = 2$, the results of [30], which describes the various possible contribution to $\gamma$-production in p-ISM collisions, can be used to estimate the $f_i$ factors predicted by DPMJET-III. In this case, in fact, the factor $Z_i = \langle E_i \rangle / E_p$ coincides with the average fraction of the parent particle energy carried by particles of the type $i$. This quantity can be related to the energy fraction emitted in photons, by taking into account the branching ratios and the kinematic of the decay processes. By using this approach, we estimate (when possible) the values presented in Tab. 1, where we also show the predictions obtained from Pythia and SYBILL parametrizations. We remark the Pythia and SYBILL results are obtained by assuming $pp$ collision, while the DMPJET-III results refer to the collision of protons with ISM, which is assumed to be composed by 90% protons, 10% helium nuclei, 0.02% carbon and 0.04% oxygen. One sees from Tab. 1 that there is a reasonable agreement between the prediction of the different codes. In particular, the $f_i$ factors are practically unchanged in the various cases. This means that the neutrino-photon ratio is a rather solid prediction, which should not suffer from large uncertainties in hadronic models and/or in the modelization of the CR flux and the ambient medium in the SNR.

Finally, we discuss the other assumptions in our approach. Our calculation neglects the contribution of $K_S^0$ and $K_L^0$ (and more rare production channels) to photon and neutrino production. According to Tab. 1, these processes accounts for several percents increase of the photon and neutrino production rates with compensating effects in the neutrino-photon ratio, which is affected at the few percents level. Moreover, we assumed that the ratio between the production rates of the various mesons does not depend on energy, nor on the CR spectral shape. This assumption is motivated by the “quasi-scaling” behavior of hadronic cross sections and by the results presented in Fig. 2 and it is valid with few percents accuracy. In conclusion, by taking into account the approximations implicit in our method (e.g., constant $f_i$ factors, neglected production channels, etc.), the uncertainties in hadronic modeling and the uncertainty in neutrino oscillation parameters, we can safely estimate that the neutrino fluxes predicted by our approach are accurate at the level of about 20%.

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5 As recalled in the Introduction, a possible limit of our (and other) calculations is the possible presence of a relevant leptonic contamination (component) of the gamma radiation, that requires efforts in theoretical modeling of the sources and multi-wavelength observations to be precisely assessed. Such a possibility implies that, conservatively, we should
Table 1: Prediction from different interaction models. See text for details.

| Model                | $Z_i$ | $f_i$ | $\pi^0$ | $\eta$ | $\pi^+ | K^+ | K^- | K^0_L | K^0_S |
|----------------------|-------|-------|---------|--------|--------|--------|--------|--------|--------|
| Koers et al. [26]    |       |       | 0.12    | 0.13   | 0.095  | 0.016  | 0.011  | 0.013  | 0.013  |
| pp - Pythia          |       | 1     |         |        |        |        |        |        |        |
| Huang et al. [30]    |       |       | 0.16    | 0.055  | 0.019  | 0.014  | 0.016  | 0.017  |        |
| p-ISM - DPMJET-III   |       | 1     | 0.34†   |        | 0.12   | 0.09   | 0.10   | 0.11   |        |
| Kelner et al. [29]   |       |       | 0.13    | 0.062  |        |        |        |        |        |
| pp - SYBILL          |       | 1     | 0.48    |        |        |        |        |        |        |

† Estimated by assuming that “direct $\gamma$ production” in [30] is due to $\eta$ decays.

5 The neutrino flux

By using the $f_i$ factors given in Eq. (29) and the oscillation probabilities given in Eq. (21), one can calculate the photon-neutrino kernels $K_{\nu^0}[x]$ and $K_{\nu^+}[x]$ according to Eqs. (19, 20). Considering the explicit form of the the various contributions (see Eqs. (10, 11)), one obtains the following simple analytic expression:

$$\Phi_{\nu^0}[E] = 0.380 \Phi_\gamma[E/(1 - r_\pi)] + 0.0130 \Phi_\gamma[E/(1 - r_K)] + \int_0^1 \frac{dx}{x} k_{\nu^0}[x] \Phi_\gamma[E/x]$$

(30)

where the first two terms describe neutrinos produced in pions (first term) and kaons (second) decays. The kernel $k_{\nu^0}[x]$, which takes into account neutrinos produced by muon decay, is shown in the right panel of Fig. 2 and it is given by:

$$k_{\nu^0}[x] = \begin{cases} x^2(15.34 - 28.93 x) & x \leq r_K = 0.0458 \\ 0.0165 + 0.1193x + 3.747x^2 - 3.981x^3 & r_K < x < r_\pi \\ (1 - x)^2 (-0.6698 + 6.588x) & x \geq r_\pi = 0.573 \end{cases}$$

(31)

A similar relation holds for muon antineutrino flux:

$$\Phi_{\bar{\nu}_\mu}[E] = 0.278 \Phi_\gamma[E/(1 - r_\pi)] + 0.0090 \Phi_\gamma[E/(1 - r_K)] + \int_0^1 \frac{dx}{x} k_{\bar{\nu}_\mu}[x] \Phi_\gamma[E/x]$$

(32)

where

$$k_{\bar{\nu}_\mu}[x] = \begin{cases} x^2(18.48 - 25.33 x) & x \leq r_K = 0.0458 \\ 0.0251 + 0.0826x + 3.697x^2 - 3.548x^3 & r_K < x < r_\pi \\ (1 - x)^2 (0.0351 + 5.864x) & x \geq r_\pi = 0.573 \end{cases}$$

(33)

Eqs. (30) and (32) have a general validity and do not require any specific parametrization of the photon spectrum. They will be used in the following to derive the neutrino flux from the SNR RXJ1713.7-3946 directly from the observational $\gamma$-ray data. It is, anyhow, interesting to discuss the application of these relations for the functional forms most commonly used to parametrize the photon spectrum.

speak of an upper bound on the neutrino signal within the present information.
5.1 Using parametrized fluxes

If the photon spectrum is approximated with a power law $\Phi_{\gamma} \propto E^{-\Gamma}$, one immediately sees that:

$$\Phi_{\nu}[E] = Z_{\nu}[\Gamma] \cdot \Phi_{\gamma}[E]$$

(34)

where:

$$Z_{\nu_{\mu}}[\Gamma] = 0.380 \, x_{\pi}^\Gamma + 0.0130 \, x_{K}^\Gamma + \int_0^1 dx \, k_{\nu_{\mu}}[x] \, x^{\Gamma-1}$$

$$Z_{\nu_{\pi}}[\Gamma] = 0.278 \, x_{\pi}^\Gamma + 0.0090 \, x_{K}^\Gamma + \int_0^1 dx \, k_{\nu_{\pi}}[x] \, x^{\Gamma-1}$$

(35)

with $x_{\pi} = 1 - r_{\pi} = 0.427$ and $x_{K} = 1 - r_{K} = 0.954$. The functions $Z_{\nu}[\Gamma]$ are the $(\Gamma - 1)$-momenta of the photon-neutrino kernels and can be calculated analytically by using Eqs. (5,6) of [18] where the relationship between $\gamma$-ray and neutrino spectra produced by simply parametrized primary proton spectra was studied. One sees that our calculation predicts $\sim 10\%$ larger neutrino fluxes. This is not surprising considering that the calculation of [18] (which is based on [20]) do not include kaons (and assume equal production of $\pi^+$, $\pi^-$ and $\pi^0$).6

The photon spectrum is expected to have a cutoff at high energies. Thus, we can write $\Phi_{\nu} = N \xi_{\nu} \Phi_{\gamma} E^{-\Gamma}$, where $N$ is a normalization factor and the adimensional function $\xi_{\nu} \Phi_{\gamma}$ which modulates the photon spectrum is normalized to 1 at low energies. If $\gamma$-rays originate from hadronic processes, the cutoff cannot be too sharp, since the spectral features of the parent CR spectrum are diluted in hadronic cascades, as was pointed out in [22]. If the function $\xi_{\nu} \Phi_{\gamma}$ is sufficiently smooth, we can extract it from the integrals in Eqs. (31) and (32), obtaining:

$$\Phi_{\nu}[E] = N \, Z_{\nu}[\Gamma] \, \xi_{\nu}[E] \, E^{-\Gamma}$$

(36)

with

$$\xi_{\nu_{\mu}}[E] \simeq \frac{1}{Z_{\nu_{\mu}}[\Gamma]} \left( 0.380 \, x_{\pi}^\Gamma \, \xi_{\gamma} \frac{E}{x_{\pi}} + 0.013 \, x_{K}^\Gamma \, \xi_{\gamma} \frac{E}{x_{K}} + \xi_{\gamma} \frac{E}{x_{m}} \int_0^1 dx \, k_{\nu_{\mu}}[x] \, x^{\Gamma-1} \right)$$

$$\xi_{\nu_{\pi}}[E] \simeq \frac{1}{Z_{\nu_{\pi}}[\Gamma]} \left( 0.278 \, x_{\pi}^\Gamma \, \xi_{\gamma} \frac{E}{x_{\pi}} + 0.009 \, x_{K}^\Gamma \, \xi_{\gamma} \frac{E}{x_{K}} + \xi_{\gamma} \frac{E}{x_{m}} \int_0^1 dx \, k_{\nu_{\pi}}[x] \, x^{\Gamma-1} \right)$$

(37)

where $x_{m} = 0.59$ approximately coincides with the maximum of the function $k_{\nu_{\mu}}[x] \, x^{\Gamma-1}$ when $\Gamma \simeq 2-3$. The above relations shows that, besides being suppressed by a factor $Z_{\nu}[\Gamma]$ with respect to the photon flux, the neutrino fluxes are also shifted to lower energies. For $\Gamma = 2$ the numerical factors of the various terms in the r.h.s. of Eqs. (37) become 0.069, 0.012, 0.130 and 0.051, 0.008, 0.137 respectively, showing that the dominant contribution is provided by neutrino produced in muon decays (i.e., last terms in the r.h.s. of Eqs. (37)). As a consequence, the features in the photon spectrum, such as the presence of a cutoff at an energy $E_{\gamma,c}$, are essentially reproduced in the neutrino spectrum at an energy lower by a factor $E_{\nu,c}/E_{\gamma,c} = x_{m} \simeq 0.59$, as also noticed by [18].

6 We do not compare with the results of the recent publication [31] because they do not provide sufficient details to reproduce their results. In particular, we are unable to reproduce the line labeled “Vissani, 2006” in their Fig. 2 which, in the intention of the authors of [31], should amount to an application of [18].
Figure 3: The function $Z_\nu[\Gamma]$ for $\nu_\mu$ (dotted), $\bar{\nu}_\mu$ (dashed) and $\nu_\mu + \bar{\nu}_\mu$ (solid). The red line is the approximate expression $Z_\nu + \bar{\nu}[\Gamma] = 0.71 - 0.16(\Gamma + 0.1)$ obtained by Eqs. (5,6) of [15].

5.2 Using raw data

RXJ1713.7-3946 is presently the best studied SNR. It has been observed by H.E.S.S. during three years from 2003 to 2005 [8]. The data expands over three decades, exploring the energy interval $E_\gamma = 0.3 - 300$ TeV. The energy resolution of the experiment is equal to about 20% and the photon spectrum is sampled in 25 bins $\delta E_\gamma/E_\gamma \simeq 0.2$ plus three larger bins at high energy. In a previous paper [22], we have discussed the information that the observational data provide on the parent CR spectrum in the SNR. Here, we use the observational data to calculate the $\nu_\mu$ and $\bar{\nu}_\mu$ fluxes emitted by this object and to estimate the event rate expected in neutrino telescopes. The proposed procedure, which does not require any parametrization of the photon flux, is general and can be applied to any other $\gamma$-transparent source.

As a first step, we “rescale” the photon and neutrino fluxes according to:

$$\varphi_\gamma[E] \equiv \Phi_\gamma[E] \cdot E^\alpha$$

$$\varphi_\nu[E] \equiv \Phi_\nu[E] \cdot E^\alpha$$

where $\nu = \nu_\mu, \bar{\nu}_\mu$. For a proper choice of the parameter $\alpha$, the “rescaled” fluxes are expected to vary slowly with energy. In the following, we adopt:

$$\alpha = 2.5$$

which is particularly appropriate for RX J1713.7-3946 (see, e.g., Fig. 3 of [22] and related discussion).

We indicate with $\varphi_j \pm \Delta \varphi_j$ the (rescaled) photon flux measured in the $j$–th energy bin, centered at a photon energy $E_j$ and covering the energy range $(E_{j,\text{inf}}, E_{j,\text{sup}})$. We can approximate the photon flux by:

$$\varphi_\gamma[E_\gamma] = \sum_j \varphi_j W_j[E_\gamma]$$

where $W_j[E_\gamma]$ are rectangular functions which describes the various energy bins (i.e., $W_j[E_\gamma] \equiv 1$ for $E_{j,\text{inf}} \leq E_\gamma \leq E_{j,\text{sup}}$ and zero elsewhere). We immediately obtain from Eqs. (30,32) the relation:

$$\varphi_\nu[E] = \sum_j \varphi_j w_j[E]$$
Figure 4: The $\nu_\mu$ (solid) and $\overline{\nu}_\mu$ (dotted) flux expected from the SNR RX J1713.7-3946 according to the H.E.S.S. $\gamma$-ray data. The shaded area are obtained by propagating the observational uncertainties in the $\gamma$-ray data. The red dotted lines correspond to the atmospheric neutrino flux in the vertical direction integrated over an angular window $0.5^\circ$ (lower line) and $1.0^\circ$ (upper line).

$$\varphi_{\nu_\mu}[E] = \sum_j \varphi_j \overline{w}_j[E]$$

where:

$$w_j[E] = 0.0453 W_j[E/(1-r_h)] + 0.0116 W_j[E/(1-r_K)] + \int_0^1 \frac{dx}{x} k_{\nu_\mu}[x] x^\alpha W_j[E/x]$$

$$\overline{w}_j[E] = 0.0331 W_j[E/(1-r_h)] + 0.0080 W_j[E/(1-r_K)] + \int_0^1 \frac{dx}{x} k_{\nu_\mu}[x] x^\alpha W_j[E/x]$$

Rels. (42) give the neutrino fluxes as a linear combination of the observational values $\varphi_j$ of the photon flux. The functions $w_j[E]$ and $\overline{w}_j[E]$ describe the contribution that each data point gives to the reconstructed neutrino flux at the energy $E$. The uncertainty in the neutrino fluxes can be evaluated by propagating linearly the observational errors $\Delta \varphi_j$. We obtain:

$$\frac{\Delta \varphi_{\nu_\mu}[E]}{\varphi_{\nu_\mu}[E]} = \sqrt{\sum_j \Delta \varphi_j^2 w_j[E]^2} \overline{\sum_j \varphi_j w_j[E]}$$

Similarly, the correlation between the values of the $\nu_\mu$ flux at two different energies can be calculated by:

$$\vartheta_{\nu_\mu}[E, E'] = \frac{\sum_k \Delta \varphi_k^2 w_k[E] w_k[E']}{\sqrt{\sum_j \Delta \varphi_j^2 w_j[E]^2} \overline{\sum_l \Delta \varphi_l^2 w_l[E]^2}}$$

Analogous expressions can be obtained for $\overline{\nu}_\mu$ flux by replacing $w_j[E] \rightarrow \overline{w}_j[E]$ in the above expressions.

The results of the proposed method are displayed in Fig. 4 where we show with solid (dashed) line the $\nu_\mu$ flux ($\overline{\nu}_\mu$ flux) expected from RX J1713.7-3946 according to the H.E.S.S. observational $\gamma$-ray data. The H.E.S.S. data cover the energy range $E_\gamma = 0.3 - 300$ TeV. In this interval we have described the photon flux according to Eq. (41) and we have not relied on theoretical assumptions. It is, however, unnatural to assume that the photon spectrum vanishes outside the probed region. For this reason,
we have continued the photon spectrum at low energy ($E_\gamma \leq 0.3$ TeV) assuming $\Phi_\gamma = I(E/1\text{TeV})^{-\beta}$ where $\beta = 2$ and $I = 2.47 \cdot 10^{-11}/(\text{cm}^2 \text{s TeV})$. The assumed low-energy photon spectrum smoothly connects with the low energy bins observed by H.E.S.S experiment (see, e.g., [5] [22]). In order to make minimal assumptions we have not extrapolated the photon spectrum at high energies. The relevance of the assumed low and high energy behavior can be studied by considering different extrapolations for the gamma ray flux outside the region probed by H.E.S.S. experiment. In this way, we have been able to verify the spectral region directly constrained by the data is $E_\nu \simeq 0.3 - 100$ TeV.

We can see that the neutrino and antineutrino spectra are well described by a power law with spectral index $\gamma \simeq 2$ at low energies and a cutoff/transition region at $E_\nu \sim 3 - 5$ TeV. The ratio $\Phi_\nu/\Phi_\mu$ is nearly constant and equal to about 0.93. The shaded areas describe the observational errors in the neutrino fluxes and are obtained by propagating the uncertainties in the gamma ray data. The neutrino fluxes are well constrained at low energies where the relative uncertainty is at few per cents level. The information degrades at high energy. We have $\Delta \Phi_\nu/\Phi_\nu \sim 30\%$ at $E_\nu = 30$ TeV and much worse at larger energies. We remind, for completeness, that one has to consider also a systematic error in our calculation equal to about 20\% due to uncertainties in hadronic cross section, neutrino oscillation parameters and the approximations implicit in our method, as conservatively estimated in the previous section.

The red dotted lines in Fig. 4 show the atmospheric neutrino flux which provides a diffuse background for SNR neutrino detection. The relevance of this background is, clearly, reduced if one is able to observe the source with a good pointing accuracy. The lower (upper) red dotted line is obtained by integrating the muon neutrino flux in the vertical direction given by (30) over an angular window $0.5^\circ$ ($1.0^\circ$), which correspond to the angular resolution of neutrino telescopes at energies equal to about 1 TeV. We remind that, below $E_\nu \sim 1$ TeV, the angular response of the detector is mainly determined by kinematic of the detection process and, thus, cannot be improved. We see that the spectral region in which the signal is expected to be larger than the background is above $\sim 1$ TeV quite close to cutoff in the $\nu$ spectrum. The red lines in Fig. 4 should be intended as lower limits of the atmospheric $\nu$ background. The atmospheric neutrino flux, in fact, depends on the zenith angle and increase by about one order of magnitude in the horizontal direction. Moreover, the angular dimension of the galactic SNRs can be comparable with the detector angular resolution. The SNR RX J1713.7-3946, as an example, subtends an angle in the sky equal to about $1.0^\circ$.

In the next section, we will calculate the event rate produced by RX J1713.7-3946 in a telescope located in the Mediterranean sea. We conclude this section by discussing whether we can expect neutrino fluxes much larger than that from RX J1713.7-3946. In this respect, it was noted in [22] that the $\gamma$-ray spectrum of RX J1713.7-3946, corresponds to a flux of CR protons with a energy equal to about $0.05 \times 10^{51}$ erg (consistent with Ginzburg & Syrovatskii hypothesis) colliding with a molecular cloud of approximately 300 solar masses, which represents a quite favorable situation. In other words, it difficult to imagine CR fluxes in SNRs much more energetic than this or a much larger target mass. SNRs can be closer than RX J1713.7-3946 which has a distance from us equal to about $D \sim 1$ kpc. Keeping the same parameters, this would increase the $\gamma$ and $\nu$ fluxes as $D^{-2}$ and, correspondingly, the expected signal in $\nu$ telescopes. However, there will be no gain with respect to the atmospheric $\nu$ background, if the linear dimension $L$ of the sources is the same as RX J1713.7-3946, since the

\[ \frac{\delta E_\nu}{E_\nu} \simeq 0.03 \] and does not erase any significant features. The smoothing scale is, in fact, much smaller than the H.E.S.S. energy resolution (equal to about 20\%) and than the size of the energy bins.

\[ \text{The wiggy behavior of the neutrino spectra is not physically significant. It reflects the statistical fluctuations of the photon data in the simple interpolation scheme proposed in Eq. (41). To reduce the effect, we have applied a Gaussian smearing to the neutrino fluxes predicted by Eq. (22) on scales smaller than $\delta E_\nu/E_\nu \simeq 0.03$. This is equivalent to smooth the photon flux on scales $\delta E_\nu/E_\nu \leq 0.03$ and does not erase any significant features. The smoothing scale is, in fact, much smaller than the H.E.S.S. energy resolution (equal to about 20\%) and than the size of the energy bins.} \]
angular dimension $\theta = L/D$ increase with decreasing distances. For this reason, the best candidates are close and young SNRs which had no time to expand to large radii and which are also expected to produce spectra which extend to larger energies, having the most energetic proton less time to escape. A promising candidate could be the young and close SNR named Vela Jr (angular size $2^\circ$), as suggested by the extrapolations of the first H.E.S.S. observations \cite{17, 22, 21}.

6 Events in neutrino telescopes

Charged-current interactions of $\nu_\mu$ and $\overline{\nu}_\mu$ produce muons and antimuons that can be observed in neutrino telescopes. The number of muons and antimuons reaching an area $A$ and in a time of observation $T$ is:

$$N_{\mu+\overline{\mu}} = f_{\text{liv}} \cdot A \cdot T \cdot \int_{E_{\text{th}}}^{\infty} dE \, \Phi_{\nu_\mu}[E] \times Y_{\mu}[E, E_{\text{th}}](1 - \overline{\nu}_\mu[E]) + (\nu_\mu \to \overline{\nu}_\mu)$$

where $E$ is the energy of the neutrino at the point of interaction and $E_{\text{th}}$ is the energy threshold for muon detection. Following \cite{14}: (1) we assume $A = 1 \text{km}^2$ and $T = 1 \text{ solar year}$; (2) we consider that the SNR RX J1713.7-3946 is visible for $f_{\text{liv}} = 78\%$ of the sidereal year, as in Antares location; (3) the neutrino absorption coefficient $\overline{\nu}_\mu(E)$, averaged over the daily location of the source, is calculated for standard rock; (4) the probability $Y_{\mu}(E, E_{\text{th}})$ that a neutrino of energy $E$ produce a muon that reach the detector with an energy larger than $E_{\text{th}}$ is calculated for water.

The above equation can be rewritten in the form:

$$N_{\mu+\overline{\mu}} = \int_{E_{\text{th}}}^{\infty} dE_{\nu} \frac{dE_{\nu}}{E_{\nu}} n_{\nu_\mu}[E_{\nu}, E_{\text{th}}] \Phi_{\nu_\mu}[E_{\nu}] + (\nu_\mu \to \overline{\nu}_\mu)$$

where the function:

$$n_{\nu_\mu}[E_{\nu}, E_{\text{th}}] = f_{\text{liv}} \cdot A \cdot T \cdot Y_{\mu}[E_{\nu}, E_{\text{th}}] \cdot (1 - \overline{\nu}_\mu[E_{\nu}]) \cdot E_{\nu}$$

gives the relative contribution of neutrinos with energy $E_{\nu}$ to the total event rate above a detection threshold $E_{\text{th}}$. In the left panel of Fig. 5 we show the function $n_{\nu_\mu}[E_{\nu}, E_{\text{th}}]E_{\nu}^{-2.5}$ as a function of $E_{\nu}$ for
the selected values $E_{\text{th}} = 0.05, \ 0.2, \ 1, \ 5, \ 20$ TeV. One sees that the bulk of the signal is expected to arise from the spectral region $E_{\nu} \simeq 0.3 - 100$ TeV which is directly constrained by photon observational data, provided that the detection threshold is below few TeV.

The previous point can be much better appreciated by rewriting the expected event rate in $\nu$-telescopes in terms of the observed photon flux. By using the results of the previous sections, we obtain:

$$N_{\mu+\bar{\mu}} = \int_{E_{\text{th}}}^{\infty} \frac{dE_{\nu}}{E_{\nu}} n[E_{\gamma}, E_{\text{th}}] \Phi_{\gamma}[E_{\gamma}]$$

(49)

where the function $n[E_{\gamma}, E_{\text{th}}]$ is given by:

$$n[E_{\gamma}, E_{\text{th}}] = \int_{E_{\text{th}}}^{E_{\gamma}} \frac{dE_{\nu}}{E_{\nu}} K_{\nu \mu}^{\text{osc}}[E_{\nu}/E_{\gamma}] n_{\nu \mu}[E_{\nu}, E_{\text{th}}] + (\nu_{\mu} \rightarrow \bar{\nu}_{\mu})$$

(50)

and $K_{\nu \mu}^{\text{osc}}[x]$ is the photon-neutrino kernel given in Eq. (19). The function $n[E_{\gamma}, E_{\text{th}}]E_{\gamma}^{-2.5}$ is shown as a function of $E_{\gamma}$ in the right panel of Fig. 5. We see immediately that the energy region probed by H.E.S.S. data (i.e., $E_{\gamma} = 0.3 - 300$ TeV) is the most relevant energy region to derive expectations for future neutrino telescopes when the detection threshold is $E_{\text{th}} \leq 1$ TeV. In this case, neutrinos that effectively contributes to the $\nu$-telescopes signals are connected to photons which are observed with Čerenkov $\gamma$-ray telescopes. If the threshold is much larger than this, on the contrary, the function $n[E_{\gamma}, E_{\text{th}}]E_{\gamma}^{-2.5}$ is peaked outside the probed region and one is forced to rely on theoretical extrapolations.

By following a procedure analogous to that adopted in the previous section, we can calculate the expected signal from RX J1713.7-3946 directly from photon observational data. If the photon flux is approximated as in Eq. (41), we obtain:

$$N_{\mu+\bar{\mu}} = \sum_{j} \varphi_{j} \pi_{j}[E_{\text{th}}]$$

(51)

where $\varphi_{j} = \Phi_{j} \cdot E_{j}^{\alpha}$ is the observational value for the (rescaled) photon flux in the $j$-th energy bin and the factors:

$$\pi_{j}[E_{\text{th}}] = \int \frac{dE_{\gamma}}{E_{\gamma}} n[E_{\gamma}, E_{\text{th}}] E_{\gamma}^{-\alpha} W_{j}[E_{\gamma}]$$

(52)

"weights" the contribution of each energy bin to the signal above the threshold $E_{\text{th}}$ ($W_{j}[E_{\gamma}] \equiv 1$ for $E_{j,\text{inf}} \leq E_{\gamma} \leq E_{j,\text{sup}}$ and zero elsewhere). The uncertainty in the predicted neutrino signal can be evaluated by propagating linearly the observational errors $\Delta \varphi_{j}$, obtaining:

$$\Delta N_{\mu+\bar{\mu}} = \sqrt{\sum_{j} \Delta \varphi_{j}^{2} \pi_{j}[E_{\text{th}}]^{2}}$$

(53)

The number of neutrino events from SNR RX J1713.7-3946 in Antares location after one year of data taking is reported in Tab. 2. We see that a $\text{km}^{2}$ class neutrino telescope will be able to collect few events per year, if the threshold will be lower than about 1 TeV. The observational uncertainty is less than 10%, indicating that H.E.S.S. data very well constrains the expected neutrino signal. The above estimates do not include possible contributions from the low ($E_{\gamma} \leq 0.3$ TeV) and high energy ($E_{\gamma} \geq 300$ TeV) tails of the photon flux, which are not constrained by observations. However, these contributions are expected to be negligible (at the few per cents level), if the threshold is lower than few TeV. We remind that our calculation is affected by $\sim 20\%$ systematic error due to uncertainties in
Table 2: The number of neutrino events, $N_{\mu+\pi}$, from SNR RX J1713.7-3946 expected in Antares location per km$^2$ per year for various energy thresholds, according to H.E.S.S $\gamma$-ray data. The uncertainty of the predicted signal, $\Delta N_{\mu+\pi}$, is obtained by propagating H.E.S.S. observational errors. The atmospheric neutrino background, $N_{\text{Atmo}}^{\mu+\pi}$, is estimated from [36] (see text for details).

| $E_{\text{th}}$ (TeV) | $N_{\mu+\pi}$ | $\Delta N_{\mu+\pi}$ | $\frac{\Delta N_{\mu+\pi}}{N_{\mu+\pi}}$ | $N_{\text{Atmo}}^{\mu+\pi}$ |
|----------------------|----------------|----------------------|---------------------------------|-----------------------------|
| 0.05                 | 5.65           | 0.35                 | 0.06                            | 20.5                        |
| 0.2                  | 4.67           | 0.33                 | 0.07                            | 6.6                         |
| 1                    | 2.44           | 0.28                 | 0.11                            | 1.1                         |
| 5                    | 0.57           | 0.17                 | 0.30                            | 0.1                         |
| 20                   | 0.08           | 0.07                 | 0.95                            | 0.007                       |

hadronic cross sections, neutrino oscillation parameters and the approximations implicit in our method, as conservatively estimated in the previous sections.

The above results show that the detection threshold will be a crucial parameter. Clearly, it cannot be much larger than 1 TeV, otherwise the event rate would be negligible. However, it cannot be much lower than this, due the presence of the atmospheric neutrino background. At present, the atmospheric neutrino fluxes have a quite large uncertainty due to imprecise knowledge of primary CR spectra at earth, of hadronic cross sections and the poor understanding of charmed particles production (see [36]). However, in the next future, they will be measured by neutrino telescopes.

In order have feeling of the background levels, we approximate the atmospheric neutrino fluxes according to $\Phi_{\nu}^{\text{Atmo}}(E_{\nu}) = C \cdot (E_{\nu}/1\text{TeV})^{-3.6}$, as it is appropriate in the energy range of our interest. The normalization constant $C = 1.8 \times 10^{-11}\text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}$ has been obtained by fitting the vertical neutrino flux of [36] (which underestimates the average atmospheric neutrino flux in the direction of the SNR) and integrating over an angular window $\theta = 1^\circ$. By using this parametrization, we obtain the number of events given in the last column of Tab. 2. Clearly, these values are purely indicative. They allow us, however, to conclude that there is no real gain to lower the threshold below the TeV level, in agreement with the finding of [37]. In conclusion, the best choice seems $E_{\text{th}} \sim 1 \text{ TeV}$, since it allows us to probe neutrino emission: i) in an energy region very-well constrained by $\gamma$-ray data; ii) collecting few events per year; iii) with a signal-to-background ratio order one or larger.

7 Summary

Future observations in neutrino telescopes have the potential to test an important theoretical paradigm, that the young SNRs are the main site of acceleration of the galactic cosmic rays. At present, it is useful to use the existing observations of very high energy gamma rays from SNRs to derive quantitative predictions for neutrinos from these sources. The main results obtained in this paper are the following:

i) We have discussed a conceptually and computationally simple method to extract precise predictions for neutrinos from supernova remnants and from other hypothetical sources, transparent to their gamma rays. This method (that is based and that elaborates on our previous work on the subject [16, 14, 22, 21]) is superior to other ones present in the literature, in that it does not need a preliminary parametrization of the gamma ray observations and, as we demonstrated, permits one to
use directly the gamma ray data as an input.

ii) Our method allows us to propagate easily the $\gamma$-ray observational errors and, thus, to understand how well neutrino fluxes and the signal in future neutrino telescopes can be constrained by $\gamma$-ray observations. Our analysis, and specifically the application to the best studied SNR (RX J1713.7-3946) shows that the present, successful program of observations of very high energy gamma rays from certain supernova remnants (in particular, with imaging arrays of Čerenkov telescopes) covers the right energy region to derive expectations for the forthcoming neutrino telescopes. As an example, the signal produced in $\nu$-telescopes by RX J1713.7-3946 is predicted with an observational uncertainty equal to about $\sim 10\%$.

iii) We have discussed the sources of theoretical and systematic uncertainties in our calculation. In particular, we have checked that the error arising from unknown neutrino oscillations parameters is very small (at the level of $\sim 2\%$), as a result of the partial cancellation of the (much larger) anti-correlated contributions of electron and muon neutrino oscillation probabilities to the total error budget\(^8\). The main source of uncertainty in the predictions is due to the modeling of the hadronic interactions and was conservatively estimated at $\sim 20\%$ level. This error could be more precisely assessed after a systematic comparison of existing numerical codes and overview of the available experimental data.

iv) It seems possible, at least for the best observed SNR (RX J1713.7-3946), to succeed and detect a neutrino signal with exposures of the order of year $\times$ km$^2$, provided that the detection threshold in future neutrino telescopes will be equal to about $\sim 1$ TeV. Another promising and perhaps better young SNR is Vela Jr, whose higher part of gamma ray spectrum (above 20 TeV) is still to be studied. Due to the presence of the atmospheric neutrino background, it does not seem really much useful to lower the threshold for neutrino observation below the TeV region. This is in agreement with the finding of\(^{[37]}\).

In summary, we have shown that it is possible to obtain precise and reliable predictions of the neutrino signals expected in neutrino telescopes from SNRs. The observation of neutrinos from these objects would amount to a proof of the existence of the expected over-density of CR and, thus, to a confirmation of the hypothesis that young SNRs are the main site of acceleration of galactic CR.

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\(^8\)This is also due the vicinity of a physical boundary, almost saturated by oscillations, as can be seen from in Fig.\(^2\).
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