1 Introduction

The crust of a neutron star is essentially determined by the low-density region ($\rho < \rho_0 \approx 0.15 - 0.16 \text{fm}^{-3}$) of the equation of state. At the bottom of the inner crust, where the density is $\rho \lesssim 0.1 \rho_0$, the formation of light clusters in nuclear matter will be energetically favorable at finite temperature. At very low densities and moderate temperatures, the few body correlations are expected to become important and light nuclei like deuterons ($d \equiv ^2\text{H}$), tritons ($t \equiv ^3\text{H}$), helions ($h \equiv ^3\text{He}$) and $\alpha$-particles ($^4\text{He}$) will form. Due to Pauli blocking, these clusters will dissolve at higher densities $\rho \gtrsim 0.1 \rho_0$. The presence of these clusters influences the cooling process and quantities, such as the neutrino emissivity and gravitational waves emission. The dissolution density of these light clusters, treated as point-like particles, will be studied within the Relativistic Mean Field approximation. In particular, the dependence of the dissolution density on the clusters-meson couplings is studied [1].

2 The model

The Lagrangian density of a system of nucleons and light clusters ($d$, $h$, $t$, and $\alpha$ particles) is given by

$$\mathcal{L} = \sum_{j=p,n,t,h} \mathcal{L}_j + \mathcal{L}_\alpha + \mathcal{L}_d + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\omega \rho},$$

where the Lagrangian density $\mathcal{L}_j$ is

$$\mathcal{L}_j = \bar{\psi}_j \left[ \gamma_\mu i D_\mu - M^*_j \right] \psi_j,$$

(1)
the \( \alpha \) fields are given by
\[ iD_j^\mu = i\partial^\mu - g_s^j \omega^{\mu} - \frac{g^j_\rho}{2} \tau \cdot b^\mu, \]
and
\[ M^*_j = M_j - g_s^j \sigma, \quad j = p, n, t, h \]
where \( M_{p,n} = M = 938.918695 \text{ MeV} \) is the nucleon mass and \( M_{t,h} = A_{t,h}M - B_{t,h} \) are the triton and helion masses. The binding energies of the clusters \( B_i \) are in \([1]\). The couplings between the cluster \( i \) and the meson fields \( \omega, \sigma \) and \( \rho \) are given by \( g_s^i \), \( g_s^j \) and \( g_\rho^j \), respectively. The \( \alpha \) particles and the deuterons are described as in \([2]\), with their respective meson populations in a state of zero momentum, therefore
\[ (\bar{L}_i^\alpha)^* \rho_\alpha \]
and
\[ (\bar{L}_d) \]
where \( iD_j^\mu = i\partial^\mu - g_s^j \omega^{\mu} \) and \( M^*_j = M_j - g_s^j \sigma \) with \( j = \alpha, d \).

The meson Lagrangian densities are
\[ \mathcal{L}_\alpha = \frac{1}{2} (iD_\mu^\alpha \phi_\alpha)^* (iD_{\mu\alpha} \phi_\alpha) - \frac{1}{2} \phi_\alpha^* (M^*_\alpha)^2 \phi_\alpha, \]
and
\[ \mathcal{L}_d = \frac{1}{4} (iD_\mu^\rho \phi_\rho - iD_\mu^\sigma \phi_\sigma) (iD_{\mu\alpha} \phi_\alpha - iD_{\mu\alpha} \phi_\sigma) - \frac{1}{2} \phi_\rho^* (M^*_d)^2 \phi_\mu, \]
where \( iD_\mu^\mu = i\partial^\mu - g_s^\mu \omega^{\mu} \) and \( M^*_j = M_j - g_s^j \sigma \) with \( j = \alpha, d \).

The meson Lagrangian densities are
\[ \mathcal{L}_\sigma = \frac{1}{2} \left( \partial_{\mu} \sigma \partial^\mu \sigma - m_s^2 \sigma^2 - \frac{1}{3} \kappa g_s^3 \sigma^3 - \frac{1}{12} \lambda g_s^4 \sigma^4 \right) \]
and
\[ \mathcal{L}_\omega = \frac{1}{2} \left( \frac{1}{2} \Omega_{\mu\nu} \Omega^{\mu\nu} + m_\omega^2 \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{12} \xi g_\omega^4 (\omega_{\mu\nu} \omega^{\mu\nu})^2 \right) \]
\[ \mathcal{L}_\rho = \frac{1}{2} \left( -\frac{1}{2} B_{\mu\nu} \cdot B^{\mu\nu} + m_\rho^2 b_{\mu} \cdot b^{\mu} \right) \]
\[ \mathcal{L}_{\omega\rho} = A_v g_v^2 g_\rho^2 \omega_{\mu} \omega^{\mu} b_{\mu} \cdot b^{\mu} \]
where \( \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \ B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu - g_\rho (b_\mu \times b_\nu) \).

The equations of motion in the relativistic mean field approximation for the meson fields are given by
\[ m_s^2 \sigma + \frac{\kappa}{2} g_s^3 \sigma^2 + \frac{\lambda}{6} g_s^4 \sigma^3 = \sum_{i=p,n,t,h} g_s^i \rho^i_s + \sum_{i=d,\alpha} g_s^i \rho_i \]
\[ m_\omega^2 \omega^0 + \frac{1}{6} \xi g_\omega^4 \left( \omega^0 \right)^3 + 2 A_v g_v^2 g_\rho^2 \omega^0 \left( b_3^0 \right)^2 = \sum_{i=p,n,t,h,d,\alpha} g_s^i \rho_i \]
\[ m_\rho^2 b_3^0 + 2 A_v g_v^2 g_\rho^2 \left( \omega^0 \right)^2 b_3^0 = \sum_{i=p,n,t,h} g_\rho^i I_3^i \rho_i \]
where \( \rho_i^s = \frac{2S_i+1}{2\pi^2} \int_0^{\lambda_i} k^3 d\lambda \frac{M_i - g_s^i \sigma}{\sqrt{k^2 + (M_i - g_s^i \sigma)^2}} \) is the scalar density, \( \rho_i \) is the vector density, and \( I_3^i (S_i) \) is the isospin (spin) of the specie \( i \) for \( i = n, p, t, h \). At zero temperature all the \( \alpha \) and deuteron populations will condense in a state of zero momentum, therefore \( \rho_\alpha \) and \( \rho_d \) correspond to the condensate density of each specie.
Figure 1: The dissolution density $\rho_d$ of each light cluster is studied separately. We assume the coupling constants as $g_i^j = \eta_j g_v$, $g_i^l = \beta_l g_s$, $g_p^l = \delta_l g_p$ and $g^l_p = 0$ with $j = t, h, d, \alpha$, $i = t, h$ and $l = d, \alpha$ respectively. Then, we determine the dissolution density $\rho_d$ dependence on the parameters $\eta_j$, $\beta_l$ and $\delta_l$ for different global proton fractions $Y_p$.

The results for triton at $T = 0$ are shown in Fig. 1 (the results for all clusters can be seen in [1]).

The cluster formation and dissolution are insensitive to the $\rho$-meson-cluster coupling constants when compared with $g_i^j$ and $g_i^l$ [1]. Therefore, we fix the $g_p^l$ for triton and helion as $g_p$. The $\rho_d$ decreases with an increase of the $\omega$-meson-cluster coupling constant and increases with an increase of the $\sigma$-meson-cluster coupling constant. For some value of $\beta_l$ below the mass number $A_i$, $\rho_d$ increases abruptly and the clusters will not dissolve.

To fix $g_i^j$ and $g_i^l$ for each cluster two constraints are required. We consider the $\rho_d$ for symmetric nuclear matter ($Y_p = 0.5$) at $T = 0$ MeV [3], the in-medium binding energies $B_i = A_i M^* - M^*$ [2], and recent experimentally Mott points at $T \approx 5$ MeV [4]. We contract several sets with $g_i^j/g_s = 0$, $A_i/2$, $3A_i/4$ and $4A_i/5$, and calculate the $g_i^j/g_v$ value of each set that reproduces $\rho_d$ for $Y_p = 0.5$ at $T = 0$ MeV [3]. After, we compare the behavior of the different sets at $T = 5$ MeV with the in-medium binding energies [2] and the Mott points [4]. The results for the best parameter set [4] are shown in Figure 2. To test the parametrization chosen, we calculate the particle fraction at finite temperature and in chemical equilibrium, $\mu_i = Z_i \mu_p + (A_i - Z_i) \mu_n$ with $i = \alpha, h, t, d$ [5, 6]. The effect of $g_s^l$ at $T = 5$ MeV is shown in Fig. 3 (a). In Fig. 3 (b) – (d), we always set $g_s^l = 3A_i/4 g_s$. In Fig. 3 (b) we compare two models, NL3 (dashed lines) and FSU (full lines). The effect of temperature and isospin asymmetry is shown in Fig. 3 (c) and (d). In Fig. 3 (c) particle fractions in symmetric matter are compared for $T = 10$ MeV (full lines) and $T = 5$ MeV (dashed lines).
Figure 2: The in-medium binding energy in our model is compared in (a), for all the clusters (dashed lines), with the corresponding results of Typel 2010 [2] at $T = 5$ MeV (full lines). In (b) we compare our Mott points with the ones from Typel 2010 [2] (dashed lines) and the experimental prediction by Hagel 2012 [4] (full dots with errorbars). The FSU parametrization was considered.

For a larger coupling $g_{\sigma}^s/g_s$, both the fraction of particles and the dissolution density are larger. For symmetric nuclear matter at $T = 10$ MeV, the deuteron fraction is already the largest fraction and the $\alpha$ fraction the smallest. In neutron rich matter, the deuteron fraction is the largest and the tritons come in second both at $T = 5$ and 10 MeV. The results do not depend much on the nuclear parametrization chosen.

4 Conclusion

Light clusters have been included in the EOS of nuclear matter as point-like particles with constant coupling constants within relativistic mean field approach. Results demonstrate that the dissolution of clusters is mainly determined by the isoscalar part of the EOS. Recent experimental results for the in-medium binding energy of light clusters [4] at $T \sim 5$ MeV and results from a quantum statistical approach [2] were used to constraint the clusters couplings constants. It was shown that a larger $\sigma$-cluster coupling gives rise to larger dissolution densities and larger particle fractions in chemical equilibrium at $T = 5$ MeV and $T = 10$ MeV for symmetric and asymmetric nuclear matter with light clusters.

The in-medium binding energies proposed in [2] may be reproduced within our model with a temperature dependent meson-cluster coupling constants. More experimental Mott points at more temperatures would allow the determination of the temperature dependence.
Figure 3: Fraction of light clusters in equilibrium with nuclear matter: (a) for FSU with $Y_p = 0.5$ and $T = 5$ MeV, and $g_s^i = 0$ (dashed lines) and $g_s^i = 3 A_i/4 g_s$ (full lines); (b) for $g_s^i = 3 A_i/4 g_s$ with $Y_p = 0.5$ and $T = 5$ MeV and FSU (full lines) and NL3 (dashed lines); (c) for FSU with $g_s^i = 3 A_i/4 g_s$ and $Y_p = 0.5$, and $T = 10$ MeV (full lines) and $T = 5$ MeV (dashed lines); (d) for FSU with $g_s^i = 3 A_i/4 g_s$ and $Y_p = 0.3$, and $T = 10$ MeV (full lines) and $T = 5$ MeV (dashed lines).

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