Jang, Il-Seung
A combinatorial realization of Kirillov-Reshetikhin crystals for type $E$ arising from translations. (English) Zbl 1493.17014 J. Algebra 595, 660-694 (2022).

In the paper under review, the author provides a new combinatorial realization of Kirillov-Reshetikhin crystals $B^{r,s}_{\ell}$ for type $E^{(1)}_{\ell}$, where $r$ is a minuscule node and $s$ is a positive integer. Let $w_0$ be the longest element of the Weyl group $W$ and let $w_J$ be the longest element of the parabolic subgroup of $W$ determined by $r$. We set $w^J := w_0 w_J$. The author first interprets the infinite crystal $B(\infty)$ of type $E^{(1)}_{\ell}$ from the viewpoint of PBW bases and find a tensor product decomposition

$$B(\infty) \simeq B^J \otimes B_J,$$

where $B^J$ is a crystal of the quantum unipotent subalgebra $U_q(w^J)$. Following the the author’s previous work with J.-H. Kwon, J. Comb. Theory, Ser. A 168, 219–254 (2019; Zbl 1447.17013), the author then defines an affine crystal structure on $B_J$. Using the fact that the crystal $B^J$ can be understood as a limit of the crystals $B(s\varpi_r)$ as $s \to \infty$, the author describes a subcrystal of $B^J$ isomorphic to the Kirillov-Reshetikhin crystal $B^{r,s}$ in a combinatorial way.

Reviewer: Euiyong Park (Seoul)

MSC:
17B37 Quantum groups (quantized enveloping algebras) and related deformations
22E46 Semisimple Lie groups and their representations
05E10 Combinatorial aspects of representation theory

Keywords: quantum groups; Kirillov-Reshetikhin crystals; quantum nilpotent algebras; PBW crystals; type $E$

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References:
[1] Biswal, R.; Scrimshaw, T., Kirillov-Reshetikhin crystals $(B\setminus\{7, s\})$ for type $(E_7, \{1\})$, Commun. Algebra, 1-16 (2021)
[2] Berenstein, A.; Zelevinsky, A., Tensor product multiplicities, canonical bases and totally positive varieties, Invent. Math., 143, 77-128 (2001) - Zbl 1061.17006
[3] Burge, W. H., Four correspondences between graphs and generalized Young tableaux, J. Comb. Theory, Ser. A, 17, 12-30 (1974) - Zbl 0286.05115
[4] Bourbaki, N., Lie Groups and Lie Algebras, Chapter 4-6 (2002), Springer-Verlag Berlin
[5] Bourbaki, N., Lie Groups and Lie Algebras, Chapters 7-9 (2005), Springer-Verlag: Springer-Verlag Berlin
[6] Chari, V., On the fermionic formula and the Kirillov-Reshetikhin conjecture, Int. Math. Res. Not., 629-654 (2001) - Zbl 0982.17004
[7] Chari, V.; Pressley, A., Quantum affine algebras and their representations, (Representations of Groups. Representations of Groups, CMS Conf. Proc., vol. 16 (1995), Amer. Math. Soc.: Amer. Math. Soc. Providence, RI), 59-78 - Zbl 0855.17009
[8] Chari, V.; Hernandez, D., Beyond Kirillov-Reshetikhin modules, Contemp. Math., 506, 49-81 (2010) - Zbl 1277.17009
[9] Fourier, G.; Okado, M.; Schilling, A., Kirillov-Reshetikhin crystals for nonexceptional types, Adv. Math., 222, 1080-1116 (2009) - Zbl 1239.17007
[10] Fulton, W., Young Tableaux, with Application to Representation Theory and Geometry (1997), Cambridge Univ. Press - Zbl 0878.14034
[11] Greene, C., An extension of Schensted’s theorem, Adv. Math., 14, 254-265 (1974) - Zbl 0293.05006
[12] Hatayama, G.; Kuniba, A.; Okado, M.; Takagi, T.; Yamada, Y., Remarks on fermionic formula, Contemp. Math., 248, 243-291 (1999) - Zbl 1032.81015
[13] Hiroshima, T., Perfectness of Kirillov-Reshetikhin crystals $(B\setminus\{r, s\})$ for types $(E_6, \{1\})$ and $(E_7, \{1\})$ with

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a minuscule node r (2021), preprint

[14] Hong, J.; Kang, S.-J., Introduction to Quantum Groups and Crystal Bases, Graduate Studies in Mathematics, vol. 42 (2002), Amer. Math. Soc. · Zbl 1134.17007

[15] Humphreys, J. E., Reflection Groups and Coxeter Groups (1990), Cambridge University Press: Cambridge University Press Cambridge · Zbl 0725.20028

[16] Jang, L.-S.; Kwon, J.-H., Quantum nilpotent subalgebra of classical quantum groups and affine crystals, J. Comb. Theory, Ser. A, 168, 219-254 (2019) · Zbl 1447.17013

[17] Jang, L.-S.; Kwon, J.-H., Lusztig data of Kashiwara-Nakashima tableaux in type D, Algebr. Represent. Theory, 24, 4, 959-989 (2021) · Zbl 1482.17033

[18] Jones, B.; Schilling, A., Affine structures and a tableau model for \( (E_6) \) crystals, J. Algebra, 324, 2512-2542 (2010) · Zbl 1206.81062

[19] Kac, V., Infinite-Dimensional Lie Algebras (1990), Cambridge Univ. Press · Zbl 0716.17022

[20] Kashiwara, M., On crystal bases of the q-analogue of universal enveloping algebras, Duke Math. J., 63, 465-516 (1991) · Zbl 0739.17005

[21] Kashiwara, M., Crystal Bases, Representations of Groups, CMS Conf. Proc., vol. 16, 155-197 (1995), Amer. Math. Soc.: Amer. Math. Soc. Providence, RI · Zbl 0851.17014

[22] Kang, S.-J.; Kashiwara, M.; Misra, K. C.; Miwa, T.; Nakashima, T.; Nakayashiki, A., Affine crystals and vertex models, (Infinite Analysis, Part A, B, Infinite Analysis, Part A, B, Kyoto, 1991, Infinite Analysis, Part A, B, Infinite Analysis, Part A, B, Kyoto, 1991, Adv. Ser. Math. Phys., vol. 16 (1992), World Sci. Publ.: World Sci. Publ. River Edge, NJ), 449-484 · Zbl 0792.17005

[23] Kang, S.-J.; Kashiwara, M.; Misra, K. C.; Miwa, T.; Nakashima, T.; Nakayashiki, A., Perfect crystals of quantum affine Lie algebras, Duke Math. J., 68, 499-602 (1992) · Zbl 0774.17017

[24] Kimura, Y., Quantum unipotent subgroup and dual canonical basis, Kyoto J. Math., 52, 277-331 (2012) · Zbl 1282.17017

[25] Knuth, D. E., Permutations, matrices and generalized Young tableaux, Pac. J. Math., 34, 709-729 (1970) · Zbl 0199.31901

[26] Kwon, J.-H., RSK correspondence and classically irreducible Kirillov-Reshetikhin crystals, J. Comb. Theory, Ser. A, 120, 433-452 (2013) · Zbl 1292.05270

[27] Kwon, J.-H., A crystal embedding into Lusztig data of type A, J. Comb. Theory, Ser. A, 154, 422-443 (2018) · Zbl 1373.05208

[28] Lenart, C.; Naito, S.; Sagaki, D.; Schilling, A.; Shimozono, M., A uniform model for Kirillov-Reshetikhin crystals II. Alcove model, path model, and \( (P = X) \), Int. Math. Res. Not., 14, 4259-4319 (2017) · Zbl 1405.05194

[29] Lenart, C.; Lubovsky, A., A generalization of the alcove model and its applications, J. Algebraic Comb., 41, 3, 751-783 (2015) · Zbl 1314.05224

[30] Lenart, C.; Scrimshaw, T., On higher level Kirillov-Reshetikhin crystals, Demazure crystals, and related uniform models, J. Algebra, 539, 285-304 (2019) · Zbl 1454.17008

[31] Littelmann, P., Cones, crystals, and patterns, Transform. Groups, 3, 2, 145-179 (1998) · Zbl 0908.17010

[32] Lusztig, G., Canonical bases arising from quantized enveloping algebras, J. Am. Math. Soc., 3, 447-498 (1990) · Zbl 0739.17005

[33] Lusztig, G., Canonical bases arising from quantized enveloping algebras II, Prog. Theor. Phys. Suppl., 102, 175-201 (1990) · Zbl 0774.17017

[34] Lusztig, G., Introduction to Quantum Groups, Progr. Math., vol. 110 (2010), Birkhäuser · Zbl 1246.17018

[35] Nakashima, T.; Zelevinsky, A., Polyhedral realizations of crystal bases for quantized Kac-Moody algebras, Adv. Math., 131, 1, 253-278 (1997) · Zbl 0897.17014

[36] Naoi, K., Existence of Kirillov-Reshetikhin crystals of type \( (G_2) \), J. Algebra, 512, 47-65 (2018) · Zbl 1472.17060

[37] Naoi, K.; Scrimshaw, T., Existence of Kirillov-Reshetikhin crystals for near adjoint nodes in exceptional types, J. Pure Appl. Algebra, 225, 5, Article 106593 pp. (2021) · Zbl 1480.17019

[38] Okado, M.; Schilling, A., Existence of Kirillov-Reshetikhin crystals for nonexceptional types, Represent. Theory, 12, 186-207 (2008) · Zbl 1243.17009

[39] Saito, Y., PBW basis of quantized universal enveloping algebras, Publ. Res. Inst. Math. Sci., 30, 209-232 (1994) · Zbl 0812.17013

[40] Salisbury, B.; Schulze, A.; Tingley, P., Combinatorial descriptions of the crystal structure on certain PBW bases, Transform. Groups, 23, 501-525 (2018) · Zbl 1441.17016

[41] Shimozono, M., Affine type A crystal structure on tensor products of rectangles, Demazure characters, and nilpotent varieties, J. Algebraic Comb., 15, 151-187 (2002) · Zbl 1106.17035

[42] Stembridge, J., On the fully commutative elements of Coxeter groups, J. Algebraic Comb., 5, 4, 353-385 (1996) · Zbl 0864.20025

[43] Papi, P., A characterization of a special ordering in a root system, Proc. Am. Math. Soc., 120, 661-665 (1994) · Zbl 0799.20037

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