Numerical study on dynamics-dependent synchronization in mutually-coupled lasers with asymmetric feedback

Shoma Ohara¹a), Kazutaka Kanno¹, and Atsushi Uchida¹b)

¹ Department of Information and Computer Sciences, Saitama University,
255 Shimo-Okubo Sakura-ku, Saitama City, Saitama 338-8570, Japan

a) s.ohara.836@ms.saitama-u.ac.jp
b) auchida@mail.saitama-u.ac.jp

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Abstract: We numerically investigate the change in synchronization property in different frequency components associated with laser dynamics, which is termed dynamics-dependent synchronization, in two mutually-coupled semiconductor lasers. We introduce an optical amplifier to implement asymmetric feedback, and we change the feedback strength of one of the two coupled lasers to observe dynamics-dependent synchronization. In-phase synchronization is observed for the original signals, while anti-phase synchronization is found for the low-pass-filtered signals, in the presence of low-frequency fluctuation dropouts. We analyze dynamics-dependent synchronization by observing temporal changes in the short-term cross-correlation and the local optical frequency detuning.

Key Words: synchronization, chaos, nonlinear dynamics, semiconductor lasers

1. Introduction

Coupled nonlinear systems including chaotic oscillators exhibit a variety of nonlinear dynamics. There have been many reports on nonlinear dynamical phenomena in a wide range of scientific fields, such as population dynamics, electronics, neurons, and photonics [1, 2]. Recently, chaos synchronization in coupled semiconductor lasers has been studied intensively for understanding fundamental physics and for information security applications [3]. Coupled laser systems have been attracted as novel information processing sources such as secure key distribution [4] and neuromorphic information processing [5].

Semiconductor lasers with optical self-feedback show low-frequency fluctuations (LFF) dynamics [1, 6, 7]. LFF dynamics consist of high-frequency chaotic oscillations and low-frequency sudden-intensity dropouts. Furthermore, chaotic itinerancy occurs as an important feature of LFF dynamics. When lasers are mutually coupled, different types of synchronization phenomena of LFF dynamics have been reported. For example, the leader-laggard relationship of lag synchronization has been demonstrated [8–10], where the lasers can be synchronized with the time lag depending on the coupling distance between the lasers. This phenomenon can be observed even though the lasers are coupled...
symmetrically by spontaneous symmetry breaking. In addition, anti-phase synchronization has been observed [11]. Episodic synchronization has been observed when the optical frequency detuning is changed temporally [12, 13]. The influence of parameter mismatch between the lasers on the degree of synchronization has been reported [14, 15].

As monolithically integrated optical devices, Photonic Integrated Circuits (PICs) have been fabricated recently [16], which is suitable for the applications of random number generation [17–21] and secure key distribution [22]. The external cavity length plays an important role in laser dynamics with time-delayed optical feedback [23, 24]. The fundamental dynamical behaviors have been investigated for the PICs with short external cavity lengths at millimeters [25–28]. In addition, a PIC with mutually-coupled semiconductor lasers with asymmetric feedback has been proposed [29]. This PIC has been experimentally investigated to understand the relationship between the synchronization states and the laser dynamics, which is termed dynamics-dependent synchronization [29]. In this phenomenon, synchronization is dependent on different frequency ranges. For example, in-phase synchronization is observed for the original signals, and, anti-phase synchronization is observed for the low-pass filtered signals. The synchronization states also depends on the change in the laser dynamics from chaos to LFFs. However, it has not been clarified that the reason why the synchronization states are changed when the laser dynamics are changed. In addition, the role of asymmetric feedback on the dynamics-dependent synchronization in the PIC has not been well understood yet.

In this study, we numerically investigate the change in the synchronization states in two mutually-coupled semiconductor lasers. We introduce asymmetric feedback for the two lasers in our numerical model. We focus on the relationship between synchronization states and laser dynamics under the asymmetric feedback configuration when the feedback strength for one of the lasers and the coupling strength are changed simultaneously. In this scheme, we observe dynamics-dependent synchronization in which the evolution of the cross-correlation value and the bifurcation of laser dynamics coincide to each other when one of the laser parameters is changed. We investigate the temporal changes in the cross-correlation value and the optical frequency detuning to analyze the dynamics-dependent synchronization.

2. Numerical model
Our numerical model for two mutually-coupled semiconductor lasers is presented in Fig. 1 [30]. This model is based on our previous experimental system of the PIC, which consists of two semiconductor lasers with asymmetric optical feedback from a common external mirror [29]. The model consists of two semiconductor lasers (referred to as laser 1 and laser 2, respectively) that are mutually coupled via a common external mirror with partial transparency. The model has an asymmetric feedback configuration by inserting a semiconductor optical amplifier (SOA) between the laser 2 and the external mirror. Both the feedback strength for the laser 2 and the coupling strength are changed by using the SOA, while the feedback strength of the laser 1 is fixed. The amplification coefficient of the SOA is denoted as \( r_{SOA} \), where the transmitted light is amplified for \( r_{SOA} > 1 \). The feedback light for the laser 2 propagates through the SOA twice, while the coupling light between the laser 1 and 2 propagates through the SOA once. The amplification rate of the feedback strength for the laser 2 can be larger than that of the coupling strength. The feedback-induced dynamics becomes dominant in...
the laser 2 for $r_{SOA} > 1$. On the contrary, the feedback strength for the laser 2 is attenuated by the SOA for $r_{SOA} < 1$. The dynamics induced by the coupling between the two lasers becomes dominant when $r_{SOA}$ is small.

We execute numerical simulations using the rate equations known as the Lang-Kobayashi equations [31] to investigate dynamics-dependent synchronization in our model. The Lang-Kobayashi equations are written as follows:

**Laser 1:**

$$
\frac{dE_1(t)}{dt} = \frac{1 + i\alpha}{2} \left[ G_N(N_1(t) - N_0) \left( \frac{1}{1 + \epsilon|E_1(t)|^2} - \frac{1}{\tau_p} \right) E_1(t) \right. \\
+ \kappa_1 E_1(t - \tau_1) \exp(-i\omega_1 \tau_1) \\
+ \left. r_{SOA} \kappa_{inj}(E_2(t - \tau_{inj}) \exp[i(-\Delta \omega t - \omega \tau_{inj})] + \xi_1(t) \right) 
$$

\hspace{1cm} (1)

$$
\frac{dN_1(t)}{dt} = J_1 - \frac{N_1(t)}{\tau_s} - \frac{G_N(N_1(t) - N_0)}{1 + \epsilon|E_1(t)|^2} |E_1(t)|^2 
$$

\hspace{1cm} (2)

**Laser 2:**

$$
\frac{dE_2(t)}{dt} = \frac{1 + i\alpha}{2} \left[ G_N(N_2(t) - N_0) \left( \frac{1}{1 + \epsilon|E_2(t)|^2} - \frac{1}{\tau_p} \right) E_2(t) \right. \\
+ \kappa_2 E_2(t - \tau_2) \exp(-i\omega_2 \tau_2) \\
+ \left. r_{SOA} \kappa_{inj}(E_1(t - \tau_{inj}) \exp[i(-\Delta \omega t - \omega \tau_{inj})] + \xi_2(t) \right) 
$$

\hspace{1cm} (3)

$$
\frac{dN_2(t)}{dt} = J_2 - \frac{N_2(t)}{\tau_s} - \frac{G_N(N_2(t) - N_0)}{1 + \epsilon|E_2(t)|^2} |E_2(t)|^2 
$$

\hspace{1cm} (4)

where $E$ and $N$ are the complex electric field and the carrier density, respectively. $\tau_{1,2}$ and $\kappa_{1,2}$ represent the feedback delay times and the feedback strengths. $\tau_{inj}$ and $\kappa_{inj}$ represent the coupling delay time and the coupling strength. $\alpha$ is the linewidth enhancement factor, $J$ is the laser injection current. $J_{th}$ is the lasing threshold current. $J = kJ_{th}$ and $k$ is the injection current normalized by the lasing threshold $J_{th} = N_{th}/\tau_s$. $G_N$ is the gain coefficient. $N_0$ is the carrier density at transparency. $\tau_p$ and $\tau_s$ are the photon and carrier lifetimes. $\epsilon$ is the gain saturation coefficient. $\Delta \omega \equiv 2\pi(f_1 - f_2)$ is the initial detuning of the optical angular frequencies between the two lasers, where $\Delta f$ is set to 3.0 GHz. $\xi_{1,2}(t)$ is the normalized white Gaussian noise with the properties $\langle \xi_{1,2}(t) \rangle = 0$ and $\langle \xi_{1,2}(t)\xi_{1,2}(s) \rangle = \delta(t - s)$, where $\langle \cdot \rangle$ denotes the ensemble average and $\delta$ is the Dirac’s delta function. The fourth terms $\xi_{1,2}(t)$ in the right hand side of Eqs. (1) and (3) represent spontaneous emission noise. The standard deviation of the noise is set to $10^{-2}$ in our numerical simulation. We confirmed that the main result of this manuscript is not changed even without the noise.

The parameter values are set as shown in Table I. These values correspond to the lengths of the PIC used in our previous experiment [29]. For example, the external cavity lengths of the lasers are set to $L_1 = 11.0$ mm and $L_2 = 10.3$ mm for laser 1 and 2, respectively, and the coupling length is set to $L_{inj} = 21.3$ mm. The round-trip feedback delay times for laser 1 and 2 are $2.9 \times 10^{-10}$ and $2.7 \times 10^{-10}$ s, respectively, obtained from $\tau_{1,2} = (2nL_{1,2})/c$, where $n$ is the reflective index of the laser and $c$ is the speed of light. The one-way coupling delay time is $2.8 \times 10^{-10}$ s, obtained from $\tau_{inj} = (nL_{inj})/c$. The values of the feedback delay times are asymmetric, because the parameter values are determined from our previous experiment [29]. In this experiment, the feedback delay times (i.e., the external cavity lengths) are slightly different between the two lasers due to the Y-branch configuration in the photonic integrated circuit.

In the model equations, the feedback strengths for the laser 1 and 2 are represented as $\kappa_1$ and $r_{SOA}^2\kappa_2$, respectively. The coupling strength is represented as $r_{SOA}\kappa_{inj}$. Therefore, the feedback strengths for the laser 2 and the coupling strength are simultaneously changed by changing $r_{SOA}$ with different factors, while the feedback strength for the laser 1 is not changed.

We calculate the cross-correlation value between the temporal waveforms of the laser 1 and 2 to quantitatively evaluate the synchronization quality. The cross-correlation function $C(\tau)$ is defined as
for both the original and low-pass filtered signals at the cut-off frequency of the LFF dropouts. We calculate the cross-correlation value to evaluate the synchronization quality we apply a low-pass filter to the laser output signals to distinguish between the entire dynamics and LFF dynamics consist of high-frequency chaotic oscillations and low-frequency intensity dropouts [6].

3. Observation of dynamics-dependent synchronization

follows.
\[
C(\tau) = \frac{\langle (I_1(t-\tau) - \bar{I}_1)(I_2(t) - \bar{I}_2) \rangle}{\sigma_1 \cdot \sigma_2}
\]

where, \(I_1(t)\) and \(I_2(t)\) are the output intensities of the laser 1 and 2, respectively. \(\bar{I}_1\) and \(\bar{I}_2\) are the mean values of \(I_1(t)\) and \(I_2(t)\). \(\sigma_1\) and \(\sigma_2\) are the standard deviations of \(I_1(t)\) and \(I_2(t)\). The bracket \(<\rangle\) represents time averaging. We calculate the cross-correlation value \(C\) by changing the delay time \(\tau\) continuously and obtain the cross-correlation function \(C(\tau)\).

### Table I. Parameter values used in numerical simulations.

| Symbol   | Parameter                          | Value                  |
|----------|------------------------------------|------------------------|
| \(G_N\) | Gain coefficient                   | \(8.4 \times 10^{-13} \text{ m}^4\text{s}^{-1}\) |
| \(N_0\) | Carrier density at transparency    | \(1.4 \times 10^{24} \text{ m}^{-3}\) |
| \(N_{th}\) | Carrier density at threshold      | \(2.018 \times 10^{24} \text{ m}^{-3}\) |
| \(\epsilon\) | Gain saturation coefficient       | \(4.5 \times 10^{-23}\) |
| \(\tau_p\) | Photon lifetime                    | \(1.927 \times 10^{-12} \text{ s}\) |
| \(\tau_s\) | Carrier lifetime                   | \(2.04 \times 10^{-9} \text{ s}\) |
| \(\alpha\) | Linewidth enhancement factor      | 6.0        |
| \(c\)    | Speed of light                     | \(2.998 \times 10^8 \text{ ms}^{-1}\) |
| \(\kappa_1\) | Feedback strength for laser 1     | 7.77 ns\(^{-1}\) |
| \(\kappa_2\) | Feedback strength for laser 2     | 7.77 ns\(^{-1}\) |
| \(\kappa_{inj}\) | Coupling strength between laser 1 and 2 | 7.77 ns\(^{-1}\) |
| \(r_{SOA}\) | Amplification coefficient of SOA   | Variable              |
| \(J_{th}\) | Injection current at threshold     | \(9.892 \times 10^{12} \text{ m}^{-3}\text{s}^{-1}\) |
| \(J_{th}^1/J_{th}^2\) | Normalized injection current for laser 1 | 1.02 |
| \(J_{th}^2/J_{th}^1\) | Normalized injection current for laser 2 | 1.10 |
| \(L_1\) | External cavity length of laser 1  | 11.0 mm                |
| \(L_2\) | External cavity length of laser 2  | 10.3 mm                |
| \(\tau_1\) | Feedback delay time for laser 1   | \(2.9 \times 10^{-10} \text{ s}\) |
| \(\tau_2\) | Feedback delay time for laser 2   | \(2.7 \times 10^{-10} \text{ s}\) |
| \(L_{inj}\) | Distance between laser 1 and 2    | 21.3 mm                |
| \(\tau_{inj}\) | Coupling delay times between laser 1 and 2 (one-way) | \(2.8 \times 10^{-10} \text{ s}\) |
| \(\Delta f\) | Initial optical frequency detuning between laser 1 and 2 | \(3.0 \times 10^9 \text{ Hz}\) |
| \(\lambda_1\) | Optical wavelength for laser 1    | \(1.537 \times 10^{-6} \text{ m}\) |
| \(\omega_1\) | Optical angular frequency for laser 1 | \(1.226 \times 10^{15} \text{ s}^{-1}\) |

3. Observation of dynamics-dependent synchronization

LFF dynamics consist of high-frequency chaotic oscillations and low-frequency intensity dropouts [6]. We apply a low-pass filter to the laser output signals to distinguish between the entire dynamics and the LFF dropouts. We calculate the cross-correlation value to evaluate the synchronization quality for both the original and low-pass filtered signals at the cut-off frequency of \(f_c = 1.0 \text{ GHz}\), where the average frequency of the LFF dropouts ranges from tens to hundreds of MHz.

Figure 2 shows the numerical results of synchronization between the two lasers for the original and low-pass-filtered signals for \(r_{SOA} = 1.3\). The temporal waveform of the laser 2 output is shifted from that of the laser 1 output at a lag time to obtain the maximum absolute value of the cross-correlation function. In Fig. 2(a), the original temporal waveforms of the two lasers show fast chaotic oscillations, and in-phase synchronization is observed. The correlation plot between the two lasers shows good synchronization with the correlation value of 0.880 in Fig. 2(b). The dynamics of these temporal waveforms are determined as chaos (coherence collapse in [1, 2]), since the spectral peak of the chaotic oscillations (\(\sim 3 \text{ GHz}\)) is higher than the peak of the low-frequency components in the fast Fourier transform (FFT) of the temporal waveform, as shown in Fig. 2(c). The spectral peak around 3.0
Fig. 2. Numerical results of the dynamics and synchronization state for $r_{SOA} = 1.3$. (a) Temporal waveforms of the two laser outputs for the original signals, (b) correlation plot of (a), (c) fast Fourier transform (FFT) of (a), (d) temporal waveforms of the two laser outputs for the low-pass-filtered signals, (e) correlation plot of (d), and (f) cross-correlation function for the original signals (the red solid line) and the low-pass-filtered signals (the blue dashed line). The lag times at which the absolute value of the cross-correlation function in (f) is maximal are (a), (b) $-0.36$ ns and (d), (e) $-0.38$ ns, respectively.

GHz in Fig. 2(c) corresponds to the relaxation oscillation frequency of the semiconductor laser. In contrast, the temporal waveforms of the low-pass-filtered signals are shown in Fig. 2(d). In-phase synchronization is also observed for the low-pass-filtered signals, as shown in the correlation plot of Fig. 2(e) with the correlation value of 0.908. Therefore, in-phase synchronization is observed for both the original and filtered signals of chaotic dynamics. Figure 2(f) shows the cross-correlation function of the original and low-pass-filtered signals. The maximum peaks of the absolute values are found in the positive correlation values, indicating that in-phase synchronization is observed for both the original and filtered signals.

Figure 3 shows the numerical results of synchronization between the two lasers for the original and low-pass-filtered signals for $r_{SOA} = 2.6$. For the original signals, the dynamics of chaotic pulsations is observed and synchronization is degraded, as shown in Figs. 3(a) and 3(b), where the correlation value is 0.444. The RF spectra of Fig. 3(c) shows the LFF dynamics, where the peak of the low-frequency components less than 1.0 GHz is comparable to the peak of the chaotic oscillation at $\sim$3 GHz. The temporal waveforms for the filtered signals are shown in Fig. 3(d), and the correlation plots shows the anti-phase synchronization with the correlation value of $-0.698$ in Fig. 3(e). Figure 3(f) shows the cross-correlation function of the two laser outputs for the original and filtered signals. Note that the maximum peak of the absolute correlation value is negative for the filtered signals, whose temporal waveforms and correlation plots are shown in Figs. 3(d) and 3(e). Therefore, in-phase synchronization is observed for the original signals, while anti-phase synchronization is observed for the filtered signals in the case of the LFF dynamics. We term this phenomenon “dynamics-dependent synchronization.”
where in-phase or anti-phase synchronization states depend on the dynamics (chaos or LFF) of the temporal waveforms [29].

4. Synchronization states for different amplification coefficients

We investigate the relationship between the synchronization state for the different frequency components and the laser dynamics by changing the amplification coefficient of SOA ($r_{SOA}$) continuously. Figure 4(a) shows the maximum of the absolute value of the cross-correlation function for both the original and filtered signals as $r_{SOA}$ is changed. The delay time $\tau$ of the cross-correlation function is changed when $r_{SOA}$ is changed to obtain the maximum of the absolute value of the cross-correlation. In the region of $0.6 \leq r_{SOA} \leq 1.5$, positive values of the cross-correlation are obtained for both the original and filtered signals, indicating that in-phase synchronization is observed. On the contrary, in the regions of $2.1 \leq r_{SOA} \leq 2.7$ and $3.2 \leq r_{SOA} \leq 3.7$, positive and negative values of the cross-correlation are obtained for the original and filtered signals, respectively. Therefore, in-phase and anti-phase synchronization are observed for the original and filtered signals in these regions.

Figure 4(b) shows the bifurcation diagram of the FFT as $r_{SOA}$ is changed. The color indicates the heights of the power spectra of the FFT for different $r_{SOA}$. In the region of $0.3 \leq r_{SOA} \leq 1.5$, broad continuous spectral components are observed and chaotic oscillations (coherence collapse) are obtained. As $r_{SOA}$ is increased, the broad spectra are separated and some regions of continuous spectral components are obtained for $r_{SOA} > 2.1$, except the periodic windows near $r_{SOA} \sim 3.0$. In addition, the power of the low frequency components ($\leq 1$ GHz) are observed, indicating the
appearance of the LFF dynamics. Compared Fig. 4(b) with Fig. 4(a), the change in the dynamics corresponds to the change in the synchronization states, i.e., the anti-phase synchronization for the filtered signals is only observed for the appearance of the LFF dynamics, but not the chaotic dynamics. We thus found dynamics-dependent synchronization by changing $r_{SOA}$ in Figs. 4(a) and 4(b).

5. Local change in dynamics-dependent synchronization

To understand the physical origin of dynamics-dependent synchronization, we investigate the short-term change in the cross-correlation value and the local optical frequency detuning. We define the time window $\tau_W$ to calculate the short-term cross-correlation between the two laser outputs. We set different $\tau_W$ for original and low-pass filtered signals so that $\tau_W$ corresponds to roughly twice the inverse of the main frequency of laser dynamics, which are set to 0.68 ns and 2.00 ns for the original and filtered signals, respectively. We calculate the short-term cross-correlation value $C(t)$ at time $t$ using the two laser intensities from $I(t - \tau_W/2)$ to $I(t + \tau_W/2)$. We also calculate the local optical frequency detuning between the original signals of the two laser outputs. The local optical frequency detuning $\Delta f_{local}$ is defined as follows,

$$\Delta f_{local} = \Delta f + \frac{1}{2\pi} \left[ \frac{d\Phi_1(t)}{dt} - \frac{d\Phi_2(t)}{dt} \right],$$

where $\Phi(t) = \arctan(E_{im}(t)/E_{re}(t))$ is the instantaneous phase of the complex electric-field amplitude $E(t) = E_{re}(t) + iE_{im}(t)$.

We calculate the short-term cross-correlation for the original signal, filtered signal, and the local optical frequency detuning of the original signals between the two lasers. Figure 5 shows the temporal waveforms, the short-term correlations, and the local optical frequency detuning of the original and filtered signals for $r_{SOA} = 1.3$. We set the delay time of $\tau = -0.36$ ns to adjust the time shift between the two temporal waveforms. In Fig. 5(a), two temporal waveforms are well matched between the two lasers for both the original and filtered signals. In Fig. 5(b), the short-term cross-correlations indicate positive values for most of the time, and in-phase synchronization is achieved for both the original and filtered signals. In Fig. 5(c), the local optical frequency is fixed near zero and injection locking is achieved. These results indicate that the optical frequencies are locked between the two lasers and in-phase synchronization is observed for both the original and filtered signals.
We increase \( r_{\text{SOA}} \) to 2.6 and investigate the temporal waveforms, the short-term correlations, and the local optical frequency detuning of the original and filtered signals, as shown in Fig. 6. We set the delay time of \( \tau = -0.28 \text{ ns} \) to adjust the time shift between the two temporal waveforms for \( r_{\text{SOA}} = 2.6 \). In Fig. 6(a), two temporal waveforms look differently for the original signals. Long pulsations are obtained for the laser 2, while the pulsations do not continue for the laser 1. The difference is clearly observed for the filtered signals, where large amplitude of the laser 2 outputs continues while the output of the laser 1 decays rapidly. These different temporal waveforms result from the asymmetry of the feedback strengths between the two lasers when \( r_{\text{SOA}} \) is increased. In Fig. 6(b), the short-term cross-correlation changes rapidly. For the original signals, positive correlation values are obtained during the pulsation of the laser outputs, and the correlation decreases rapidly when the LFF dropouts occur (the laser outputs become zero). For the filtered signals, the short-term cross-correlation fluctuates chaotically, and negative correlations are obtained, since the two temporal waveforms of the filtered signals are very different. In Fig. 6(c), the local optical frequency detuning increases when the LFF dropouts occurs. This result indicates that the optical frequencies are unlocked when the LFF dropouts are observed, due to the lack of the coupling power.

We interpret that dynamics-dependent synchronization results from the asymmetry of the feedback strengths between the two lasers. For small \( r_{\text{SOA}} \), the chaotic dynamics induced by the coupling is dominant, and in-phase synchronization is observed under injection locking. In contrast, the feedback strength for the laser 2 increases and LFF dynamics appear by increasing \( r_{\text{SOA}} \), while the feedback strength for the laser 1 is fixed. The LFF dynamics of the laser 2 is dominant and the dynamics of the laser 1 follows that of the laser 2. However, synchronization is not complete due to the lack of injection locking by the LFF dropouts. This asymmetric change in the feedback strength results in the change in the dynamics and synchronization states for different frequency components.
6. Local changes at $r_{SOA} = 2.6$.

(a) Temporal waveforms, (b) short-term correlations and (c) optical frequency detuning. The upper part indicates the original signals and the lower part indicates the filtered signals in (a) and (b). (c) is calculated by the original signals. Each figure is calculated with the time lag which is shifted by $-0.28$ ns. The Greek numbers correspond to the positions on the chaotic attractors of Fig. 8.

![Fig. 6. Local changes at $r_{SOA} = 2.6$. (a) Temporal waveforms, (b) short-term correlations and (c) optical frequency detuning. The upper part indicates the original signals and the lower part indicates the filtered signals in (a) and (b). (c) is calculated by the original signals. Each figure is calculated with the time lag which is shifted by $-0.28$ ns. The Greek numbers correspond to the positions on the chaotic attractors of Fig. 8.]

6. Local change in synchronization properties on chaotic attractors

We plot the short-term cross-correlation values and the local optical frequency detuning on the chaotic attractors of the original signals to investigate the local relationship between the synchronization state and the laser dynamics, as shown in Fig. 7 for $r_{SOA} = 1.3$. The chaotic attractors are plotted on the coordinates of the optical frequency shift and the carrier density [10, 26, 27]. In Figs. 7(a) and 7(b), large positive values of the short-term cross-correlation are plotted on the whole regions of the attractors of the laser 1 and 2 (red color), and in-phase synchronization is achieved for wide regions of the attractors. The short-term cross-correlation values for the filtered signals are also plotted on the chaotic attractors of the original signals, as shown in Figs. 7(c) and 7(d). Large positive values of cross-correlation are found over the whole regions of the attractors, which indicates that in-phase synchronization is achieved. In Figs. 7(e) and 7(f), the local optical frequency detuning is plotted on the attractors, and injection locking is achieved (black color) for the wide regions of the attractors. These results show that in-phase synchronization is achieved under injection locking in the whole regions of the attractor for both of the original and filtered signals.

We found more interesting results for $r_{SOA} = 2.6$. Figure 8 shows the short-term cross-correlation values and the local optical frequency detuning plotted on the chaotic attractors of the original signals for $r_{SOA} = 2.6$. In Figs. 8(a) and 8(b), the short-term cross-correlation values between the original signals of the laser 1 and 2 are plotted on the attractors, and the Greek numbers on the attractors correspond to the positions on the temporal waveforms of Fig. 6(a). Pulsation outputs appear in both lasers at the position I and continue over II (also see Fig. 6(a)), which corresponds to the decrease of the carrier density (the vertical axis). The pulsations disappear (increase of the carrier density) for the laser 1 at III, while the laser 2 still produces pulsation outputs at III (decrease of the carrier density). In fact, the chaotic trajectory of the laser 1 cannot follow that of the laser 2, because the
Fig. 7. Local changes on the chaotic attractors of the original signals for $r_{\text{SOA}} = 1.3$. (a), (b) short-term correlation for the original signals, (c), (d) short-term correlation for the filtered signals, and (e), (f) local optical frequency detuning for the original signals. (a), (c), (e) laser 1 and (b), (d), (f) laser 2.

A decrease in the optical frequency of the laser 1 stops at about $-35$ GHz (the horizontal axis), while the frequency of the laser 2 keeps decreasing to $-50$ GHz. A LFF dropout occurs for the laser 1 first at III, then a dropout is observed for the laser 2 at IV. Finally, the trajectory goes back to the first position at V. Large correlation values (red color) are obtained only in the region around III, but not other regions, because the times at which the LFF dropouts occur are different between the laser 1 and 2.

The short-term cross-correlation values for the filtered signals are also plotted on the attractors of the original signals, as shown in Figs. 8(c) and 8(d). Large positive values of cross-correlation (red color) are observed in the region around II, while negative values (blue color) are obtained in the regions from III to V. In the regions of III and V, the shape of the two chaotic attractors are different between the laser 1 and 2 due to the different dynamics of the occurrence of the LFF dropouts, and this mismatch of the shape of the attractors results in the anti-phase synchronization for the filtered signals. Note that the distribution of the cross-correlation values on the attractors are completely different between the original and filtered signals (e.g. Figs. 8(b) and 8(d)).

In Figs. 8(e) and 8(f), the local optical frequency detuning are plotted on the attractors. The local
optical frequency detuning shows almost zero at the lower end of the attractors (the regions of II), where the optical frequencies are matched between the two lasers under the pulsation outputs. The optical frequencies are unlocked in the region of IV, due to the occurrence of the LFF dropouts. The switching between the optical frequency locking and unlocking is observed on the attractors, and these frequency changes affect different synchronization states for different frequency components.

From these local analysis, we found that different chaotic attractors are generated due to the asymmetric feedback by the SOA. Different dynamics of the occurrence of LFF dropouts results in the local change in the synchronization state for the original and filtered signals on the chaotic attractors. Therefore, we found that dynamics-dependent synchronization is observed locally on the chaotic attractors for the original and filtered signals under asymmetric feedback strengths.

7. Conclusions
We numerically investigated dynamics-dependent synchronization in two mutually-coupled semiconductor lasers with asymmetric feedback. We introduced a semiconductor optical amplifier (SOA) to
implement asymmetric feedback strengths. We investigated synchronization property for different frequency ranges of the laser dynamics by applying a low-pass filter to the laser output signals. We observed in-phase and anti-phase synchronization states, depending on the frequency components, and these synchronization states can be changed by the transition of the laser dynamics from chaos to LFF. This phenomenon results from the asymmetry of the feedback strengths between the two lasers. To analyze more details of dynamics-dependent synchronization, we calculated the short-term correlation and the local optical frequency detuning. Short-term correlation and the local optical frequency detuning fluctuate when anti-phase synchronization is observed due to the occurrence of LFF dynamics under the asymmetric feedback, while the short-term correlation is almost constant when both lasers show chaotic dynamics. The short-term correlation values are plotted on the chaotic attractors, and local change in the cross-correlation is observed, showing that the positive and negative correlation values are observed in the different regions of the attractors for the original and filtered signals. This result suggests that different dynamics of LFF dropouts results in the local change in the synchronization states. The revealed mechanism of dynamics-dependent synchronization is indicated to be important in many other coupled dynamical systems.

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