Photon-neutrino interaction in $\theta$-exact covariant noncommutative field theory

R. Horvat,1 D. Kekez,1 P. Schupp,2 J. Trampečić,1,3 and J. You1,4

1Institute Rudjer Bošković, Bijenička 54 10000 Zagreb, Croatia
2Jacobs University Bremen, Center for Mathematics, Modeling and Computing, Campus Ring 1, 28759 Bremen, Germany
3Max-Planck-Institut für Physik, (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany
4Mathematisches Institute Göttingen, Bunsenstr. 3-5, 37073 Göttingen, Germany

Photon-neutrino interactions arise quite naturally in noncommutative field theories. Such couplings are absent in ordinary field theory and imply experimental lower bounds on the energy scale $\Lambda_{\text{NC}} \sim |\theta|^{-2}$ of noncommutativity. Using non-perturbative methods and a Seiberg-Witten map based covariant approach to noncommutative gauge theory, we obtain $\theta$-exact expressions for the interactions, thereby eliminating previous restrictions to low-energy phenomena. We discuss implications for plasmon decay, neutrino charge radii, big bang nucleosynthesis and ultrahigh energy cosmic rays. Our results behave reasonably throughout all interaction energy scales, thus facilitating further phenomenological applications.

PACS numbers: 11.10.Nx, 13.15.+g, 26.35.+c, 98.70.Sa

II. INTRODUCTION

Neutrinos, being neutral particles, cannot couple directly to photons. They can couple indirectly via weak interaction and other effects, potentially leading to significant observational effects in astrophysics. The photon-neutrino scattering cross section predicted by the Standard Model is, however, exceedingly small and likely of little practical importance in astrophysics, see e.g. [3]. Field theories on noncommutative (NC) spaces offer a different interaction channel for neutrinos and photons: Such theories have been introduced as effective models for the quantum geometric structure of spacetime that is generically expected in any reasonable quantum theory of gravity (including string theory). The presence of gauge fields influences such noncommutative structures and neutrinos propagating in the modified background feel the effect. The non-zero star commutator

$$[A_\mu \star \Psi] = A_\mu \star \Psi - \Psi \star A_\mu.$$  \hspace{1cm} (1)

is a striking illustration of this effect in noncommutative $U_q(1)$ theory: Fermions that are neutral or even sterile in the commutative limit can nevertheless directly couple to a $U_q(1)$ gauge field. In this article we shall investigate such noncommutativity induced interactions using methods that are non-perturbative in the noncommutativity parameters $\theta$.

Studies in noncommutative particle phenomenology, aiming at predicting possible experimental signatures and estimating bounds on space-time noncommutativity from existing experimental data, started around the time that string theory indicated that noncommutative gauge theory could be one of its low-energy effective theories. Early attempts with simple models based on star products (with expressions like the one given above) soon ran into several serious difficulties: (i) Such theories have no local gauge invariant quantities, (ii) fields like $A_\mu$ do not transform covariantly under coordinate changes and (iii) there are unphysical restrictions on representations and charges. These shortcomings are overcome in an approach based on Seiberg-Witten (SW) maps, which enables one to deform commutative gauge theories with essentially arbitrary gauge group and representation. Since this approach also fixes problems of non-covariance under coordinate changes, we shall refer to this class of theories as covariant noncommutative field theories. After some initial enthusiasm, renormalizability of these theories turned out to be a delicate issue. In a Dirac fermion 4-vertex was identified as one of the major culprits in this issue. This vertex is absent in theories with chiral fermions. Particularly well behaved are theories where the gauge fields couple via a star commutator to fermions, as is the case in our model: Cancelations between fermion and boson loops lead to softened UV/IR coupling. Regardless of the question of renormalizability, SW-map based models serve as effective theories for some of the quantum geometric effects expected in more fundamental theories such as string theory and quantum gravity. In the Seiberg-Witten map approach, noncommutative fields $A_\mu$, $\Psi$, ... and gauge transformation parameters $\Lambda$ are interpreted as non-local, enveloping algebra-valued functions of their commutative counterparts $a_\mu$, $\psi$, $\lambda$ and of the noncommutative parameters $\theta_{\mu \nu}$, in such a way that ordinary gauge transformations of the commutative fields induce noncommutative gauge transforma-

1 These models should be understood as effective theories and are not necessarily renormalizable.

2 All these models are consistent, contrary to slightly misleading statements in a recently published "no-go theorem", where a mal-constructed model is shown to lead to contradictions.
tions of the noncommutative fields. This procedure allows the construction of noncommutative extensions of important particle physics models like the NC Standard Model (NCSM) and GUT models \cite{21,22}, as well as various follow-on studies with NC modifications of particle physics \cite{27,34}. In more recent development, it has been found that SW expanded models at first order in \( \theta \) are well-behaved regarding anomalies and renormalizability. For example the NCSM \cite{21} at \( \theta \)-order appears to be anomaly free \cite{35,50}, has remarkably well-behaved one-loop quantum corrections \cite{57} and breaks Lorentz symmetry; see also \cite{29,58,41}.

To avoid strong backgrounds from known processes, considerable efforts in noncommutative phenomenology has been directed at interactions which are suppressed in Standard Model settings. One important candidate or this type is the aforementioned tree-level coupling of neutrinos with photons or, more precisely, plasmons. Such interactions have already been studied in the framework of noncommutative gauge theories defined by Seiberg-Witten maps \cite{42,43}. There, like in almost all other studies of covariant NC field theory, an expansion and cut-off in powers of the noncommutativity parameters \( \theta^{\mu \nu} \) was used for computational simplicity. Such an expansion corresponds to an expansion in momenta (derivatives) and restrict the range of validity to energies well below the noncommutativity scale \( \Lambda_{\text{NC}} \). This is usually no problem for experimental predictions because the noncommutativity parameters \( \theta^{ij} = \frac{c^{ij}}{\Lambda_{\text{NC}}^2} \) are in general considered to be small. There exists, however, exotic processes like ultra high energy cosmic rays \cite{44} in which the interacting energy scale runs higher than the current experimental bound on the noncommutative scale \( \Lambda_{\text{NC}} \). Here the previously available approximate results are inapplicable. To overcome the \( \theta \)-expansion and cut-off approximation, we are using in this article \( \theta \)-exact expressions and expand in powers of the coupling constant as in ordinary gauge theory.

The \( \theta \)-exact approach has been inspired by exact formulas for the Seiberg-Witten map \cite{12,43,44}. For arbitrary non-Abelian gauge theories the \( \theta \)-exact approach is still a challenging problem, in particular in loop computations and at higher orders in the coupling constant. Since perturbative renormalization and UV/IR mixing are still fairly poorly understood, it is not clear how to interpret the quantum corrections and to relate them to observations \cite{48,50}. Another interesting venue for applications of \( \theta \)-exact methods is the investigation of quantum corrections in covariant noncommutative quantum field theories. Recently it was suggested \cite{50} that noncommutative QED might be renormalizable by adding proper counter-terms. A covariant \( \theta \)-exact version of this theory could be a very interesting object for future studies. First \( \theta \)-exact results have been published in the investigation of UV/IR mixing in covariant NC gauge theory \cite{4} and later in the context of NC photon-neutrino phenomenology, namely scattering of ultra high energy cosmic ray neutrinos on nuclei \cite{44}. Those topics were off-limits in the old \( \theta \)-expansion method.

### II. MODEL

In this section we recall some basic facts about covariant noncommutative gauge theory based on Seiberg-Witten maps. We then review the derivation of \( \theta \)-exact interaction terms and apply the method to the computation of neutrino-photon tri-particle vertices. We close with a comment on an alternative covariant vertex. Throughout the article we shall concentrate on Abelian noncommutative gauge theory.

It is straight-forward to formulate field theories on noncommutative spaces by inserting a star product \( \ast \) between all fields in the action. This introduces ordering ambiguities and it breaks ordinary gauge invariance (because local gauge transformations do not commute with star products). In analogy to the introduction of covariant derivatives in gauge theory, the star product can be promoted to a gauge-field dependent covariant star product \( \ast' \). Together with a gauge-field dependent covariant integral measure this leads to a noncommutative gauge theory. Alternatively and in fact equivalently \cite{11,12}, it is possible to retain the original star product and instead promote all fields to noncommutative fields and the gauge transformations to noncommutative gauge transformations. In this construction the “noncommutative fields” are obtained via Seiberg-Witten maps \cite{4} and their generalizations from the original “commutative fields”.

With some field-ordering “fine-tuning”, it is possible to obtain noncommutative models, were neutrinos and other neutral fermion fields do not couple to photons – the minimal NC Standard Model is an example of this type. More generally, however, electrically neutral matter fields will be promoted via (hybrid) Seiberg-Witten maps to noncommutative fields that couple via star commutator to photons and transform in the adjoint representation of \( U \ast (1) \) – this is the case for phenomenologically promising NC GUTs. The inclusion of all gauge covariant coupling terms is furthermore a prerequisite for reasonable UV behavior. Taking all this into account we eventually arrive at the following model of a Seiberg-Witten type noncommutative \( U \ast (1) \) gauge theory \footnote{In this section we set the coupling constant \( e = 1 \), to restore the coupling constant one simply substitute \( a_\mu \) by \( ea_\mu \), then divide the gauge-field Lagrangian by \( e^2 \).} (for more details, see the discussion at the end of the section)

\[
S = \int \left( -\frac{1}{4} F_{\mu \nu} F_{\mu \nu} - i \bar{\Psi} \left[ D^\mu - m_\nu \right] \Psi \right) d^4x \quad (2)
\]

with \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu] \) and

\[
D_\mu \Psi = \partial_\mu \Psi - i [A_\mu, \Psi] \quad . \quad (3)
\]
All the fields in this action are images under (hybrid) Seiberg-Witten maps of the corresponding commutative fields $a_\mu$ and $\psi$. In the original work of Seiberg and Witten and in virtually all subsequent applications, these maps are understood as (formal) series in powers of the noncommutativity parameter $\theta^{\mu\nu}$. Physically, this corresponds to an expansion in momenta and is valid only for low-energy phenomena. Here we shall not subscribe to this point of view and instead interpret the noncommutative fields as the primary fields in the theory up to lowest nontrivial order in $\theta^{\mu\nu}$ from the recursion and consistency relations and direct recursive computations using consistency conditions. For the lowest nontrivial order a direct deduction from the recursion and consistency relations

$$\delta_\Lambda A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$$

$$\delta_\Lambda \Psi = i[\Lambda, \Psi]$$

with the ansatz

$$\Lambda = \Lambda_1 a_\mu \lambda = (1 + \hat{\Lambda}^1 a_\mu + \hat{\Lambda}^2 a_\mu + O(a^3))\lambda$$

$$\Psi = \Psi[a_\mu] \psi = (1 + \hat{\Psi}^1 a_\mu + \hat{\Psi}^2 a_\mu + O(a^3))\psi$$

is already sufficient. Here $\hat{\Psi}[a_\mu]$ and $\hat{\Lambda}[a_\mu]$ are gauge-field dependent differential operators that we shall now determine: Starting with the fermion field $\Psi$, at lowest order we have

$$i[\lambda, \psi] = \hat{\Psi}[\partial \lambda] \psi$$ (9)

Writing the star commutator explicitly as

$$[f \star g] = f(x)(e^{-\theta^{\mu\nu} \partial_\mu \partial_\nu} - e^{\theta^{\mu\nu} \partial_\mu \partial_\nu})g(y) \bigg|_{x = y}$$

$$= 2if(x) \sin \left(\frac{\partial_\mu \partial_\nu}{2}\right)g(y) \bigg|_{x = y}$$ (10)

we observe that

$$\hat{\Psi}[a_\mu] = -\theta^{ij} a_i \star_2 \partial_j$$ (11)

will fulfill the consistency relation. The generalized star product $\star_2$ that appears here is defined as follows

$$f \star_2 g = f(x) \sin \left(\frac{\partial_\mu \partial_\nu}{2}\right)g(y) \bigg|_{x = y}$$ (12)

The gauge transformation $\Lambda$ can be worked out similarly, namely

$$0 = [\lambda_1 \star [\lambda_2 - \lambda_1], a_\mu] + i\hat{\Lambda} \partial \lambda_1] a_\mu - i\hat{\Lambda} \partial \lambda_2] a_\mu$$

$$= \frac{1}{2}([\lambda_1 \star [\lambda_2 - \lambda_1], a_\mu] + i\hat{\Lambda} \partial \lambda_1] a_\mu - i\hat{\Lambda} \partial \lambda_2] a_\mu]$$

and hence

$$\hat{\Lambda} = -\frac{1}{2} \theta^{ij} a_i \star_2 \partial_j$$ (14)

The gauge field $a_\mu$ requires slightly more work. The lowest order terms in its consistency relation are

$$-\partial_\mu \frac{1}{2} \theta^{ij} a_i \star_2 \partial_j \lambda - i[\lambda, a_\mu] = A_\mu^2 [a_\mu + \partial_\mu \lambda] - A_\mu^2 [a_\mu]$$ (15)

where $A^2$ is the $a^2$ order term in the expansion of $A$ as power series of $a$. The left hand side can be rewritten as $-\frac{1}{2} \theta^{ij} \partial_\mu a_i \star_2 \partial_j \lambda - \theta^{ij} a_i \star_2 \partial_j a_\mu \star_2 \partial_j a_\mu$, where the first term comes from $-\frac{1}{2} \theta^{ij} \partial_\mu a_i \star_2 a_j$, while the third one comes from $-\theta^{ij} a_i \star_2 \partial_j a_\mu$. After a gauge transformation, the sum of the first and third terms equals the second term. Ultimately, we obtain

$$A_\mu = a_\mu - \frac{1}{2} \theta^{ij} a_i \star_2 (\partial_\mu a_\mu + f_{\mu\nu}) + O(a^3)$$ (16)

$$\Psi = \psi - \theta^{ij} a_i \star_2 \partial_j \psi + O(a^3)\psi$$ (17)

$$\Lambda = \lambda - \frac{1}{2} \theta^{ij} a_i \star_2 \partial_j \lambda + O(a^3)\lambda$$ (18)

with $f_{\mu\nu}$ being the commutative field strength $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$. 

---

4 Notation: Capital letters denote noncommutative objects, small letters denote commutative objects, hatted capital letters denote differential operator maps from the latter to the former.
Expanding the action (2) in the terms of the commutative fields, one gets the following \( \theta \)-exact cubic terms up to first order in \( a_\mu \):

\[
\mathcal{L} = \bar{\psi} \gamma^\mu [a_\mu, \psi] - (\theta^{ij} a_i \ast_2 \partial_j \bar{\psi})(i\partial - m_\nu)\psi
- \bar{\psi}(i\partial - m_\nu)(\theta^{ij} a_i \ast_2 \partial_j \psi) + \bar{\psi} \mathcal{O}(a^2)\psi.
\]

(19)

Here \( \psi \) is a Dirac-type massive or massless (i.e. Weyl) neutrino field. To extract Feynman rules in an appropriate form, we use the arithmetic property \( i\partial [f \ast_2 g] = [f \ast_2 g] \) to obtain the effective neutrino-photon Lagrangian density

\[
\mathcal{L} = -(\theta^{ij} a_i \ast_2 \partial_j \bar{\psi})(i\partial - m_\nu)\psi
- \bar{\psi}(i\partial - m_\nu)(\theta^{ij} a_i \ast_2 \partial_j \psi)
+ i\bar{\psi} \gamma^{\mu} (\theta^{ij} a_i \ast_2 \partial_j \psi) + \mathcal{O}(a^2)\psi.
\]

(20)

The rest of the derivation resembles that of NCQED without a Seiberg-Witten map. Just like the Moyal-Weyl star product turns into an exponential function in momentum space, the generalized star product \( \ast_2 \) turns into a function

\[
F(q, k) = \frac{\sin \frac{q_\mu k_\mu}{2}}{\frac{q_\mu k_\mu}{2}},
\]

(21)

where \( q \) and \( k \) are the momenta of the fields involved in the product. We notice that for tri-field interaction, 4-momentum conservation \( q = k-k' \) renders the function \( F \) independent of the order of momenta involved: \( F(q, k) = F(k, q) = F(k, q') = F(k', k) \). We can hence pull out \( F \) as an universal factor. In the end we obtain the following \( \theta \)-exact Feynman rule for the neutrino-photon tri-particle vertex:

\[
\Gamma^\mu = iF(q, k) \left[ (\tilde{k} - m_\nu)\bar{\psi}(q_\mu k^\mu - q\tilde{k}) + (q\tilde{k})\gamma^\mu - \tilde{k}\gamma^\mu \right],
\]

(22)

with the shorthand notations \( q\tilde{k} \equiv q_i \theta^{ij} k_j \) and \( \tilde{k}^\mu = \theta^{\mu j} k_j \).

If we compare this with to the first order in \( \theta \) vertex that was used in previous work, we see that only the factor \( F(q, k) \) is new. Consequently, it is this factor that leads to modifications in \( \theta \)-exact computations.

Interestingly, the first term in (22),

\[
\Gamma^\mu_{alt} = iF(q, k)(\tilde{k} - m_\nu)\bar{\psi}(q_\mu k^\mu),
\]

(23)

is already consistent with gauge invariance on its own. This simplified vertex defines an alternative NC theory of neutrino-photon interaction that is attractive for computations beyond tree level. In this article we will use the full vertex which is more natural from the point of view of NC gauge theory as it is derived from a covariant derivative.

The two choices of NC vertices is ultimately related to a choice of generalized SW map in the construction of the NC theory. In the remainder of this section we shall explore this construction in more detail. For simplicity of presentation we shall set \( e = 1 \) and focus on the massless case.

We start with the action for a neutral massless free fermion field

\[
S = \int \bar{\psi} \gamma^\mu \partial_\mu \psi \, d^4x = \int \bar{\psi} \gamma^\mu \partial_\mu \psi \, d^4x,
\]

(24)

where, as indicated, a Moyal-Weyl type star product can be inserted or removed by partial integration. Following the method of constructing a covariant NC gauge theory outlined at the beginning of this section, we lift the factors in the action via (generalized) Seiberg-Witten maps \( \hat{\Psi}[a_\mu], \hat{\Phi}[a_\mu] \) to noncommutative status as follows:

\[
S = \int \hat{\Psi}(\bar{\psi}) \gamma^\mu \hat{\Phi}(\partial_\mu \psi) \, d^4x = \int \hat{\Psi}(\bar{\psi}) \gamma^\mu \hat{\Phi}(\partial_\mu \psi) \, d^4x.
\]

(25)

Now if the SW maps \( \hat{\Psi}, \hat{\Phi} \) and a corresponding map \( \hat{\Lambda} \) for the gauge parameter \( \lambda \) satisfy

\[
\delta_{\lambda}(\hat{\Psi}(\bar{\psi})) = i[\hat{\Lambda}(\lambda) \ast_2 \hat{\Psi}(\bar{\psi})],
\]

(26)

\[
\delta_{\lambda}(\hat{\Phi}(\partial_\mu \psi)) = i[\hat{\Lambda}(\lambda) \ast_2 \hat{\Phi}(\partial_\mu \psi)],
\]

we will have a noncommutative action that is gauge invariant under infinitesimal commutative gauge transformations \( \delta_{\chi} \) and reduces to the free fermion action in the commutative limit \( \theta \to 0 \).

The appropriate map \( \hat{\Psi} \) is the one [14] that we have already derived:

\[
\hat{\Psi}(\bar{\psi}) = \psi - \theta^{ij} a_i \ast_2 \partial_j \psi + \mathcal{O}(a^2)\psi.
\]

(27)

Recalling that we are dealing with neutral fields, i.e. \( \delta_{\chi} = 0 \) and \( \delta(\partial_\mu \psi) = 0 \), we notice that we can in principle use the same map also for \( \hat{\Phi} \):

\[
\hat{\Phi}(\partial_\mu \psi) = \hat{\Psi}(\partial_\mu \psi) = \partial_\mu \psi - \theta^{ij} a_i \ast_2 (\partial_j \partial_\mu \psi) + \mathcal{O}(a^2)\psi.
\]

(28)

This construction is quite unusual from the point of gauge theory, as it yields a covariant derivative term without introducing a covariant derivative. In any case the resulting action

\[
S_{alt} = \int \left( i\bar{\psi} \gamma^\mu \partial_\mu \psi - i(\theta^{ij} \partial_i \bar{\psi} \ast_2 a_i) \gamma^\mu \partial_\mu \psi
+ i\bar{\psi} \gamma^\mu (\theta^{ij} a_i \ast_2 \partial_j \partial_\mu \psi) \right) d^4x + \mathcal{O}(a^2)
\]

(29)

is consistent and gauge invariant. The corresponding photon-fermion interaction vertex

\[
\Gamma^\mu_{alt} = iF(q, k)\bar{\psi}(q_\mu k^\mu)
\]

(30)

is surprisingly simple and therefore quite attractive for loop-level computations. The vertex satisfies

\[
q_\mu \Gamma^\mu_{alt} = (q \cdot \tilde{k})iF(q, k)\tilde{k} = 0.
\]

(31)

There is, however, a second choice for \( \hat{\Phi} \):

\[
\hat{\Phi}(\partial_\mu \psi) = D^\mu_\mu \hat{\Psi}(\psi) = \partial_\mu \hat{\Psi}(\psi) - i[a_\mu, \hat{\Psi}(\psi)]
= \partial_\mu \psi - \theta^{ij} a_i \ast_2 \partial_j \partial_\mu \psi + \theta^{ij} \bar{f}_{ij} \ast_2 \partial_j \partial_\mu \psi + \mathcal{O}(a^2)\psi,
\]

(32)
based on the well-known NC QED-type covariant derivative. This second choice of SW map differs from the first one by the gauge invariant term \( \theta^a f_{\mu \nu} \gamma^2 \partial_\nu \psi \), indicating a freedom in the choice of Seiberg-Witten map. The second choice leads to the vertex
\[
\Gamma^\mu = iF(q, k) \left[ \bar{\psi} \gamma^\mu + (q \theta k) \gamma^\mu - \theta k^\mu \right].
\]
(33)

In general one can chose any superposition of the two SW maps \( \Phi_{\text{alt}} \) and \( \Phi \), but in this article we shall focus on the second choice as it is more natural from the point of view of gauge theory.

### III. APPLICATIONS

#### A. Plasmon decay into \( \bar{\nu} \nu \) pairs

Our first phenomenological application of the new neutrino-photon vertex \(^{22}\) will include a decay of transverse plasmon modes into neutrino pairs, which we then compare with the result obtained with perturbative methods (to first order in \( \theta \)) in \(^{42}\). Starting from the tree-level vector-like coupling to photons \(^{22}\), standard \( \gamma \)-matrix techniques yield the amplitude squared for the process \( \gamma_{pl} \rightarrow \bar{\nu} \nu \) summed over polarizations:

\[
|M_{\text{NC}}(\gamma_{pl} \rightarrow \bar{\nu} \nu)|^2 = 4 e^2 |F(q, k)|^2 (q \theta k)^2 (q^2 + 2 m_e^2).
\]

In the plasmon rest frame the total rate of decay into massless neutrinos involves the following phase space integral over the outgoing neutrino momenta:

\[
\Gamma_{\text{NC}}(\gamma_{pl} \rightarrow \bar{\nu}(l) \nu(l)) = \frac{\alpha \omega_{pl}}{4 \pi} \sin \theta \sin^2 \frac{q \theta k}{2}
\]

\[
= \frac{1}{4} \alpha \omega_{pl} \int \frac{dx}{-1} \left[ 1 - (\cos A x) J_0(B \sqrt{1 - x^2}) \right],
\]

where \( J_0 \) is the zeroth order Bessel function of the first kind, and

\[
A = \frac{c_{03} \omega_{pl}^2}{2 \Lambda_{\text{NC}}^2}, \quad B = \frac{\omega_{pl}^2}{2 \Lambda_{\text{NC}}^2} \sqrt{c_{01}^2 + c_{02}^2}.
\]

(36)

The plasma frequency \( \omega_{pl} \) is defined as the frequency of plasmons at \( |q| = 0 \). In the regime where the motion of background electrons is irrelevant, i.e. \( q_0 > 2 m_e \) and \( |q| > m_e \), the dispersion relation for transverse and longitudinal waves can be calculated analytically, giving (see e.g. \(^{32}\))

\[
\omega_{pl}^2 = R e \Pi_T(q_0, |q| = 0) = \frac{e^2 T^2}{9},
\]

(37)

where \( R e \Pi_T \) is the transverse part of the one-loop contribution to the photon self-energy at finite temperature/density and \( T \) is the temperature. The integral \(^{35}\) can be solved analytically (see Appendix):

\[
\Gamma_{\text{NC}}(\gamma_{pl} \rightarrow \bar{\nu}(l) \nu(l)) \rightarrow \frac{\alpha}{2} \omega_{pl} \left( 1 - \frac{\sin \xi}{\xi} \right),
\]

(38)

where \( \xi = \omega_{pl}^2/(2 \Lambda_{\text{NC}}^2) \) and \( \alpha \) is the fine structure constant.

The Standard Model (SM) neutrino-penguin-loop decay rate for transverse plasmons (of energy \( E_\nu \)) into neutrinos is proportional to \( \omega_{pl}^2/E_\nu \). \(^{33}\) Comparing the SM rate to our NC rate \(^{33}\) for a plasmon at rest, and taking into account that our NC photon-neutrino interaction \(^{22}\) has equal strength for both neutrino chiralities we obtain the ratio

\[
R = \frac{\sum_{\text{flavors}} \Gamma_{\text{NC}}(\gamma_{pl} \rightarrow \bar{\nu}L \nu_L + \bar{\nu}R \nu_R)}{\sum_{\text{flavors}} \Gamma_{\text{SM}}(\gamma_{pl} \rightarrow \bar{\nu}L \nu_L)}
\]

\[
= \frac{3 \cdot 48 \pi^2 \alpha^2}{(c_{\nu_e}^2 + c_{\nu_e}^2 + c_{\nu_e}^2) G_F \omega_{pl}^2} \left( 1 - \frac{\sin \xi}{\xi} \right),
\]

(39)

see Figure 1. For \( \nu_e \), we have \( c_{\nu_e} = \frac{1}{2} + 2 \sin^2 \Theta_W \), while for \( \nu_\mu \) and \( \nu_\tau \) we have \( c_{\nu_\mu} = -\frac{1}{2} + 2 \sin^2 \Theta_W \). At small \( \omega_{pl} \) and reasonably low NC scale \( \Lambda_{\text{NC}} \) the NC rate clearly dominates, while for large \( \omega_{pl} \) the Standard Model rate dominates regardless of the value of \( \Lambda_{\text{NC}} \).

Comparing to the previous result \(^{42}\), we note an overall suppression factor \( 1 - \frac{\sin \xi}{\xi} \) here, which, depending on the parameters, may assume any value between 0 (for \( \xi = 0 \)) and 1.22 (for \( \xi \approx 4.5 \)). The previous result was perturbative in \( \theta \), corresponding to an expansion of the suppression factor as a power series of \( \xi \)

\[
1 - \frac{\sin \xi}{\xi} = \frac{1}{6} \xi^2 - \frac{1}{120} \xi^4 + O(\xi^6).
\]

(40)

\(^5\) A few parenthetical remarks are in order here. In a different model \(^{51}\), it is claimed that Dirac masses for neutrinos are not consistent with NC gauge invariance of the Yukawa terms. Our model features a vector-like interaction, since both \( \nu_L \) and \( \nu_R \) are singlets under the residual \( U(1)_Q \) and Dirac mass terms for neutrinos are allowed. The tree-level coupling to photons is experienced by both neutrino chiralities even above the electroweak symmetry breaking scale. Note also that the vertex \(^{22}\) is zero for Majorana neutrinos, unless transition electromagnetic moments are invoked. In all applications we will work with neutrinos with definite chiralities, i.e. Weyl neutrinos, as the neutrino mass can be neglected. The interaction \(^{22}\) is generation-independent, and hereafter the coupling constant is restored.

\(^6\) Here the dimensionless normalized matrix elements \( e^{\theta i} \) are defined by \( \theta^{\theta i} = e^{\theta i}/\Lambda_{\text{NC}}^2 \), with \( \sum_{i=1}^{3} |e^{\theta i}|^2 = 1 \).
The perturbative approach clearly fails, when $\xi$ is considerably larger than one, while the new $\theta$-exact results remain valid. Using Eqs. (38) and (40) for small $\xi$, we recover the old result (42), thus showing consistency of the new computation.

Keeping just the first term in (40), the ratio $R$ becomes independent of the plasma frequency $\omega_{pl}$. This feature is reflected in the approximately constant $R = 1$ contour for small $\omega_{pl}$ shown in Fig. 1 and gives a lower bound $\Lambda_{NC} \gtrsim 70$ GeV, in agreement with (42), where a bound on $\Lambda_{NC}$ was derived from the requirement that NC contributions to plasmon decay in stars should not go beyond the Standard Model predictions. In practice, the plasmon frequency in stars, $\omega_{pl} \approx 10$ keV, is too low to see the effect of the modified interaction (19). In the following, we shall explore other examples.

**B. Neutrino charge radius**

We make use of the full $\theta$-exact expression for the plasmon decay rate to recompute also NC neutrino charge radii for both chiralities. Noting (43)

$$\Gamma(\gamma_{pl} \rightarrow \bar{\nu}_L \nu_L) = \frac{\alpha}{144 E_\gamma} |\langle r_\nu^2 \rangle|^2$$

and using (40) we find that the $\theta$-exact NC induced neutrino charge radius is given by

$$|\langle r_\nu^2 \rangle| = \lim_{\omega_{pl} \rightarrow 0} \frac{6\sqrt{2}}{\omega_{pl}^2} \sqrt{1 - \frac{\sin \xi}{\xi}}.$$  (42)

The limit $\omega_{pl} \rightarrow 0$ implies, through the dispersion relation $q^2 = \omega_{pl}^2$, the familiar $q^2 \rightarrow 0$ limit entering the definition of the charge radius. It is interesting to note that the expansion in the plasma frequency coincides with an expansion in $\theta$, because the effect of the full $\theta$-exact interaction enters only through the parameter $\xi$. Therefore, the limit $\omega_{pl} \rightarrow 0$ picks up only the first term in (40) and that corresponds to the first-order-in-$\theta$ result, as stated before. This lucky coincidence implies that there are no $\theta$-exact corrections to the first-order-in-$\theta$ charge radius that was obtained earlier (43):

$$|\langle r_\nu^2 \rangle| = \frac{\sqrt{3}}{\Lambda_{NC}^2}.$$  (43)

Note that the $\theta$-parameter – when interpreted as the length scale of the fuzziness of spacetime which arises as a consequence of space-space uncertainty relations – directly runs up a charge radius for a neutrino (by giving spatial extent to a point particle), as expected.

With (43) at hand, one can immediately place a constraint on $\Lambda_{NC}$ by employing a very stringent bound on $|\langle r_\nu^2 \rangle|$ based on SN1987A (54). With $|\langle r_\nu^2 \rangle| \approx 2 \times 10^{-33} \text{cm}^2$, and using (43) one obtains $\Lambda_{NC} \gtrsim 0.6$ TeV.

**C. Big Bang Nucleosynthesis (BBN)**

Over the past decades, BBN has established itself as one of the most powerful available probes of physics beyond the Standard Model, giving many interesting constraints on particle properties (an extensive summary is available, for instance, in (55)). One uses it to parametrize the energy density of new relativistic particles at the time of BBN in terms of the effective number of additional neutrino species, $\Delta N_\nu$, whose determination involves both a lower limit on the barion-to-photon ratio ($\eta \equiv n_b/n_\gamma$) as well as an upper bound on the primordial mass fraction of $^4\text{He}$, $Y_p$ (54). The energy density of three light right-handed (RH) neutrinos produced by plasmon decay during BBN is equivalent to the effective number $\Delta N_\nu$ of additional doublet neutrinos

$$\Delta N_\nu = 3 \frac{(T_{\nu_L}/T_{\nu_R})}{4},$$

where $T_{\nu_L}$ is the temperature of the SM neutrinos, being the same as that of photons down to $T \sim 1 \text{ MeV}$. A better limit on $\Delta N_\nu$ leads to a smaller value of $T_{\nu_R}$ and consequently a higher decoupling temperature of the RH neutrinos. For $\Delta N_\nu = 1$, one finds $T_{\nu_R} > T_C$, where $T_C$ is the critical temperature for the deconfinement restoration phase transition, $T_C \sim 200 \text{ MeV}$. If $\Delta N_\nu \approx 0.2$, then $T_{\nu_R}$ would be close to a critical temperature of the electroweak phase transition, $T_{dec} \lesssim 300 \text{ GeV}$. Unfortunately, with the WMAP value for $\eta$ (55), $Y_p$ was predicted to increase (55), having a tendency to loosen the tight bounds on $\Delta N_\nu$ that existed before.
The RH neutrino is commonly considered to decouple at the temperature $T_{\text{dec}}$ when the condition
\[ \Gamma(\nu_l \rightarrow \bar{\nu}_R \nu_R) \simeq H(T_{\text{dec}}) \] (45)
is satisfied. The plasma frequency in this case is given by
\[ \omega_{\text{pl}} = \frac{e T_{\text{dec}}}{3} g^*_{\text{ch}}, \] (46)
while the Hubble expansion rate satisfies
\[ H(T) \simeq 1.66 g_* \frac{T_{\text{dec}}^2}{M_{\text{Pl}}}, \] (47)
where $g_*$ and $g^*_{\text{ch}}$ are the degrees of freedom specifying the entropy of the interacting species for all and charged species, respectively; $M_{\text{Pl}}$ is the Planck mass.

Computing the decoupling temperature $T_{\text{dec}}$ based on the assumption that the decay rate (45) is solely due to noncommutative effects and comparing with lower bounds on $T_{\text{dec}}$ that can be inferred from observational data, we can determine lower bounds on the scale of noncommutativity $\Lambda_{\text{NC}}$, assuming that \( \xi \) is fully due to noncommutative contributions:
\[ T_{\text{dec}} \simeq \frac{M_{\text{Pl}} e^3 g^*_{\text{ch}}}{39.84 \pi g_*} \left(1 - \sin \frac{\xi}{\xi}\right), \quad \xi = \frac{e^2 (g^*_{\text{ch}})^2 T_{\text{dec}}^3}{18 \Lambda_{\text{NC}}^2}. \] (48)

Let us consider the pre-factor
\[ \frac{M_{\text{Pl}} e^3 g^*_{\text{ch}}}{39.84 \pi g_*} = \frac{\pi^2 \alpha^2 g^*_{\text{ch}}}{4.98 g_*} M_{\text{Pl}}, \] (49)
and taking into account $\alpha \simeq 137^{-1}$, $g^*_{\text{ch}}/g_* \simeq 1$, we have
\[ \frac{\pi^2 \alpha^2 g^*_{\text{ch}}}{4.98 g_*} M_{\text{Pl}} \simeq 2.22 \times 10^{-4} M_{\text{Pl}}. \] (50)

Since this factor is amplified by the Planck mass, we can test the full interaction \( \text{(19)} \) only for $\Delta N_\nu < 1$. Unfortunately, such a precision in observational data is not expected to be reached anytime soon. Thus, to account for decoupling temperatures associated with cosmological phase transitions (200 MeV/300 GeV), we have to require $\left(1 - \sin \frac{\xi}{\xi}\right) \ll 1$. This only occurs when $\xi \to 0$, so that within this regime we can use the leading order term in the Taylor expansion in $\xi$. This gives
\[ T_{\text{dec}} \simeq \frac{\pi^2 \alpha^2 g^*_{\text{ch}}}{4.98 g_*} M_{\text{Pl}} = \frac{4 \pi^2 \alpha^2 g^*_{\text{ch}}}{4.98 g_*} \frac{M_{\text{Pl}}}{108}, \] (51)
so that
\[ \Lambda_{\text{NC}} \simeq \left(\frac{\pi^2 \alpha^2 (g^*_{\text{ch}})^3 M_{\text{Pl}} T_{\text{dec}}^3}{4.98 \cdot 27 g_*} \right)^{\frac{1}{4}}. \] (52)

Now setting further $g_* \sim g^*_{\text{ch}} \sim 100$, $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV and $T_{\text{dec}} > 200$ MeV (quark-hadron phase transition), a lower bound on $\Lambda_{\text{NC}}$ can be obtained as
\[ \Lambda_{\text{NC}} > 3.68 \text{ TeV}. \] (53)

For $T_{\text{dec}} > 300$ GeV (eletroweak phase transition), we have
\[ \Lambda_{\text{NC}} > 887 \text{ TeV}, \] (54)
confirming the previous results obtained by making use of scattering processes of $\nu_{\text{NS}}$ \[59\].

For the sake of demonstration, we have also studied \( \text{(15)} \) numerically, to investigate how the plasmon rate obtained in the full theory affects determination of $\Lambda_{\text{NC}}$ when $T_{\text{dec}} \simeq 10^4 M_{\text{Pl}}$. The relation \( \text{(48)} \) is depicted in Fig. 2, in which the approximate result \( \text{(52)} \) is also superimposed for comparison.

Looking at the asymptotic behavior of the approximate \( \text{(52)} \) and exact \( \text{(48)} \) relations between $\Lambda_{\text{NC}}$ and $T_{\text{dec}}$, one notes that Fig. 2 reveals quite a different behavior for them: While the solution of \( \text{(15)} \) obtained by employing the leading order term in $\xi$, shows no restriction on $T_{\text{dec}}$ all the way up till the beginning of the radiation era, the solution of \( \text{(52)} \) reveals a maximal decoupling temperature in the said epoch. (Of course, if inflation occurred...
well below $T_{\text{max}}^{\text{dec}} \simeq 3 \times 10^{-5} M_{\odot}$, then any distinction between the two solutions would practically disappear.) This feature is accompanied by a maximal scale of noncommutativity, above which the RH neutrinos can no longer retain thermal contact with the rest of the universe. This is in contrast with the exact relation \cite{18}, where thermal equilibrium is maintained for much larger $\Lambda_{NC}$’s (if inflation occurred well above $T_{\text{max}}^{\text{dec}}$), and stops when $T_{\text{dec}}$ hits the reheating temperature. This means that the effect of the full interaction \cite{22} is to bring about the maximal upper limit $\Lambda_{NC}^{\text{max}} \simeq 9 \times 10^{-4} M_{\odot}$, occurring at a decoupling temperature slightly below $T_{\text{dec}}^{\text{max}}$, which can be extracted by our method. Note that for decoupling temperatures lying in a region where the oscillatory patterns inherent in \cite{22} becomes manifest, it may seem troublesome to infer a bound on the scale of noncommutativity since several (many) $\Lambda_{NC}$’s correspond to the same $T_{\text{dec}}$. In these cases one chooses the highest $\Lambda_{NC}$, otherwise $\Lambda_{NC}$’s obtained with a much smaller decoupling temperature (where the oscillatory term is shut down) would give much better lower limits. Again, this characteristic is missing in the perturbative solution, where better and better limits on the NC scale are always accompanied with progressively increasing decoupling temperatures.

Note that these ranges of $T_{\text{dec}}$ are of course tremendously above the bounds that can be inferred from current observational data.

D. Ultrahigh Energy (UHE) Cosmic Rays

The non-observation of UHE neutrino induced events in neutrino observatories implies a strong model-independent constraint on the inelastic neutrino-nucleon cross section \cite{63}, which consequently gives a constraint on the scale of noncommutativity for a NC gauge-field theory in which neutrinos couple directly to photons \cite{44}. It was observed \cite{14} that at energies as high as $10^{11}$ GeV, the usual expansion in $\theta$ is no longer meaningful. In order to fix the breakdown in the perturbative expansion, a resummation in the neutrino-photon vertex was undertaken \cite{14}. Although devoid of a firm theoretical background and with only the zeroth order in the Seiberg-Witten map employed, this ad hoc approach did produce the correct sin term that we have now obtained, redoing the computation using the $\theta$-exact vertex \cite{22}.

Curiously enough, \cite{22} can be put in a much simpler form if the NC vertex connects external (on-shell) neutrino lines. Indeed, with the aid of the Dirac equation for free fields, the first term in \cite{22} vanishes if one line is on-shell, while the third term in \cite{22} vanishes if both lines are on-shell. Thus, for tree-level processes with no internal neutrino lines, \cite{22} reduces to

$$\Gamma^\mu = 2i e \gamma^\mu \sin \left( \frac{q \theta k}{2} \right), \quad (55)$$

This exactly coincides with the result \cite{44} obtained with the ad-hoc method. This way, the powerful bounds on $\Lambda_{NC}$ obtained there, in the range 200-900 TeV (depending on a model for the cosmogenic neutrino flux), get further credence as far as the underlying theoretical background is concerned. Note, though, that \cite{55} should not be used to calculate tree-level processes with internal neutrino lines, nor in calculations involving loops. Hence, if one is, for instance, to study the UV/IR mixing in the neutrino sector, then the complete expression as given by \cite{22} should be used.

IV. CONCLUSION

In summary, we showed that the tree-level tri-particle decay $\gamma pl \to \nu \bar{\nu}$ in the covariant noncommutative quantum gauge theory based on Seiberg-Witten maps can be computed without an expansion over the noncommutative parameter $\theta$. As an application, we focus on plasmon decay into neutrinos, reconsidering previous computations that were done with less sophisticated tools and deriving new bounds on the scale of noncommutativity.

Comparing to previous results, the total decay rate is modified by a factor which remains finite throughout all energy scales. Thus the new results behave much better than the $\theta$-expansion method when ultra high energy processes are considered. We expect that similar control on the high energy behavior can be extended to $\theta$-exact perturbation theory involving more than three external fields in the near future. This would provide a considerably improved theoretical basis for research work in the field of noncommutative particle phenomenology.

Acknowledgments

J.T. would like to acknowledge support of W. Hollik, and MPI Munich for hospitality. The work of R.H., D.K and J.T. are supported by the Croatian Ministry of Science, Education and Sports under Contracts Nos. 0098-0982930-2872 and 0098-0982930-2900, respectively. The work of J.Y. was supported by the Croatian NSF and the IRB Zagreb, and by the German Research Foundation (Deutsche Forschungsgemeinschaft (DFG)) through the Institutional Strategy of the University of Göttingen.

Appendix A: Integral

We evaluate the integral in \cite{63}

$$I = \int_{1}^{0} dx (\cos Ax) J_0(B \sqrt{1 - x^2})$$

$$= \int_{1}^{0} dx \cos(A \sqrt{1 - x^2}) \frac{J_0(Bx)}{\sqrt{1 - x^2}} \quad (A1)$$
analytically by Taylor expansion of both \( \cos x \) and \( J_0(x) \)

\[
\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \quad J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x^2/2)^{2k}}{(2k)!}.
\]  

(\text{A2})

We consider the integral over the product of the \( m \)-th order term in the Taylor expansion of \( \cos(A\sqrt{1-x^2}) \) and the \( n \)-th order term in the Taylor expansion of \( J_0(Bx) \)

\[
\mathcal{I}(m,n) = \frac{1}{0} dx \frac{(-m)(A\sqrt{1-x^2})^{2m}}{(2m)!} \frac{(-n)(Bx)^{2n}}{(2n)!} \int_0^1 dx (1-x) \frac{2m-1}{2} x^n.
\]

(\text{A3})

The integrals over \( x \) can now be done by successive integration by parts, yielding

\[
\int_0^1 dx (1-x)^{2m-2} x^n = \frac{n!l^{2n+1}}{(2m)(2m+3)...(2m+2n+1)}
\]

\[
= \frac{n!l^{2n+1}}{(2m+n+1)!l!}
\]

(\text{A4})

and

\[
\mathcal{I}(m,n) = \frac{(-)^{m+n}}{(2m+n+1)!} \frac{(n+m)!}{n!m!} \frac{(A^2)(B^2)^n}{(2m+n+1)!}.
\]

(\text{A5})

Re-summng

\[
\sum_{n+m=l} \mathcal{I}(m,n) = \frac{(-)^{l}}{(2l+1)!} \left( \sqrt{A^2 + B^2} \right)^{2l}
\]

(\text{A6})

and noting

\[
\sin x = \sum_{l=0}^\infty \frac{(-)^l x^{2l+1}}{(2l+1)!} = x \sum_{l=0}^\infty \frac{(-)^l x^{2l}}{(2l+1)!}
\]

(\text{A7})

we finally obtain the surprisingly simple result

\[
\mathcal{I} = \frac{\sin \sqrt{A^2 + B^2}}{\sqrt{A^2 + B^2}} = \frac{\sin \xi}{\xi}.
\]

(\text{A8})

[1] J. B. Adams, M. A. Ruderman, and C.-H. Woo, Phys. Rev. 129, 1383 (1963).
[2] P. Bandyopadhyay, Phys. Rev. 173, 1481 (1968).
[3] D. A. Diesch and W. W. Repko, Phys. Rev. D 48, 5106 (1993), [arXiv:hep-ph/9305284].
[4] P. Schupp and J. You, JHEP 0808, 107 (2008), [arXiv:0807.4886].
[5] I. Hinchliffe, N. Kersting, and Y. L. Ma, Int. J. Mod. Phys. A19, 179 (2004), [arXiv:hep-ph/0205040].
[6] N. Seiberg and E. Witten, JHEP 9909, 032 (1999), [arXiv:hep-th/9908142].
[7] R. Jackiw and S. Y. Pi, Phys. Rev. Lett. 88, 111603 (2002), [arXiv:hep-th/0111122].
[8] J. Madore, S. Schranil, P. Schupp, and J. Wess, Eur. Phys. J. C16, 161 (2000), [arXiv:hep-th/0001203].
[9] B. Jurco, S. Schranil, P. Schupp, and J. Wess, Eur. Phys. J. C17, 521 (2000), [arXiv:hep-th/0006246].
[10] A. A. Bichl, J. M. Grimstrup, L. Popp, M. Schweda, and R. Wulkenhaar, [arXiv:hep-th/0102103].
[11] B. Jurco and P. Schupp, Eur. Phys. J. C14, 367 (2000), [arXiv:hep-th/0001052].
[12] B. Jurco, P. Schupp, and J. Wess, Nucl. Phys. B604, 148 (2001), [arXiv:hep-th/0001052].
[13] B. Jurco, L. Möller, S. Schranil, P. Schupp, and J. Wess, Eur. Phys. J. C21, 383 (2001), [arXiv:hep-th/0104155].
[14] M. Chaichian, P. Presnajder, M. M. Sheikh-Jabbari, and A. Tureanu, Phys. Lett. B683, 55 (2010), [arXiv:0907.2646 [hep-th]].
[15] R. Wulkenhaar, JHEP 0203, 024 (2002), [arXiv:hep-th/0112248].
[16] J. M. Grimstrup and R. Wulkenhaar, Eur. Phys. J. C26, 139 (2002), [arXiv:hep-th/0205133].
[17] M. Burić, D. Lalas, V. Radovanović, and J. Trampetić, Phys. Rev. D 77, 045031 (2008), [arXiv:0711.0887 [hep-th]].
[18] C. P. Martin and C. Tamarit, Phys. Rev. D 80, 065023 (2009), [arXiv:0907.2464 [hep-th]].
[19] A. Matusis, L. Susskind, and N. Toumbas, JHEP 0012, 002 (2000), [arXiv:hep-th/0002075].
[20] A. Bichl, J. Grinstrup, H. Grosse, L. Popp, M. Schweda, and R. Wulkenhaar, JHEP 0106, 013 (2001), [arXiv:hep-th/0104097].
[21] X. Calmet, B. Jurco, P. Schupp, J. Wess, and M. Wohlgenannt, Eur. Phys. J. C23, 363 (2002), [arXiv:hep-ph/0111115].
[22] W. Behr, N. Deshpande, G. Dunlapčić, P. Schupp, J. Trampetić, and J. Wess, Eur. Phys. J. C29, 441 (2003), [arXiv:hep-ph/0202121].
[23] P. Aschieri, B. Jurco, P. Schupp, and J. Wess, Nucl. Phys. B651, 45 (2003), [arXiv:hep-th/0205214].
[24] B. Melić, K. Passek-Kumerički, J. Trampetić, P. Schupp, and M. Wohlgenannt, Eur. Phys. J. C42, 483 (2005), [arXiv:hep-th/0502249].
[25] B. Melić, K. Passek-Kumerički, J. Trampetić, P. Schupp, and M. Wohlgenannt, Eur. Phys. J. C42, 499 (2005), [arXiv:hep-th/0503064].
[26] C. P. Martin, PoS CNCQG2010, 026 (2010), [arXiv:1011.4783 [hep-th]].
[27] T. Ohl and J. Reuter, Phys. Rev. D 70, 076007 (2004), [arXiv:hep-th/0406098].
[28] T. Ohl and J. Reuter, arXiv:hep-ph/0407337.
[29] B. Melić, K. Passek-Kumerički, and J. Trampetić, Phys. Rev. D 72, 057502 (2005), [arXiv:hep-th/0507231].
[30] A. Alboteanu, T. Ohl, and R. Rückl, PoS HEP2005,
322 (2006), [arXiv:hep-ph/0511188].
[31] A. Alboteanu, T. Ohl, and R. Ruckl, Phys. Rev. D 74, 096004 (2006), [arXiv:hep-ph/0608155].
[32] A. Alboteanu, T. Ohl, and R. Ruckl, Phys. Rev. D 76, 105018 (2007), [arXiv:0707.3595].
[33] A. Alboteanu, T. Ohl, and R. Ruckl, Acta Phys. Polon. B 38, 3647 (2007), [arXiv:0709.2359].
[34] M. Burić, D. Latas, V. Radovanović, and J. Trampetić, Phys. Rev. D 74, 096004 (2006), [arXiv:hep-ph/0608155].
[35] A. Alboteanu, T. Ohl, and R. Ruckl, Phys. Rev. D 76, 105018 (2007), [arXiv:0707.3595].
[36] A. Alboteanu, T. Ohl, and R. Ruckl, Acta Phys. Polon. B 38, 3647 (2007), [arXiv:0709.2359].
[37] M. Burić, D. Latas, V. Radovanović, and J. Trampetić, Phys. Rev. D 76, 105018 (2007), [arXiv:0707.3595].
[38] M. Burić, V. Radovanović, and J. Trampetić, JHEP 0703, 030 (2007), [arXiv:hep-th/0609073].
[39] C. P. Martin and C. Tamarit, JHEP 0912, 042 (2009), [arXiv:0910.2677 [hep-th]].
[40] C. Tamarit, Phys. Rev. D 81, 025006 (2010), [arXiv:0910.5195 [hep-th]].
[41] M. Burić, D. Latas, V. Radovanović, and J. Trampetić, Phys. Rev. D 83, 045023 (2011), [arXiv:1009.4603 [hep-th]].
[42] P. Schupp, J. Trampetić, J. Wess, and G. Raffelt, Eur. Phys. J. C36, 405 (2004), [arXiv:hep-ph/0212292].
[43] P. Minkowski, P. Schupp, and J. Trampetić, Eur. Phys. J. C37, 123 (2004), [arXiv:hep-th/0302175].
[44] R. Horvat, D. Kekez, and J. Trampetić, Phys. Rev. D 83, 065013 (2011), [arXiv:1005.3209 [hep-ph]].
[45] T. Mehen and M. B. Wise, JHEP 0012, 008 (2000), [arXiv:hep-th/0002094].
[46] H. Liu, Nucl. Phys. B614, 305 (2001), [arXiv:hep-th/0011125].
[47] Y. Okawa and H. Ooguri, Phys. Rev. D 64, 046009 (2001), [arXiv:hep-th/0104036].
[48] S. A. Abel, J. Jaeckel, V. V. Khoze, and A. Ringwald, JHEP 0609, 074 (2006), [arXiv:hep-ph/0607188].