Holographic formation of photon structures by Bessel-like and Gaussian light fields in photopolymer materials taking into account two-beam interaction and small contrasts

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Abstract. This paper presents a theoretical model of the holographic formation of photon structures (PS) in a photopolymer material (PPM) by Bessel-like and Gaussian light beams, taking into account the self-diffraction of recording light beams in areas of low contrast of the interference pattern. It was demonstrated by numerical simulation and experiment that the influence of the self-diffraction effect leads to an increase in the level of the side maxima of the diffracted light beam.

1. Introduction
Nowadays, the task of the formation of quasi-diffraction-free light beams seems to be really relevant. In particular, Bessel beams [1-2] belong to such ones. But real Bessel beams cannot exist because of the impossibility of transferring infinite optical power; therefore, Bessel-like beams with spatial distributions of intensity close to the theoretical are more often considered. Thus, the search for available and technologically advanced methods for the formation of such beams seems relevant. More often than others, diffraction optical elements (DOEs), which convert incident light beams into quasi-diffraction-free ones, are used [3-5].

One of the most effective methods for DOEs formation is holographic one using photosensitive media. The effectiveness of the method is determined by the fact that the distribution of the refractive index of the photosensitive medium completely depends on the holographic recording scheme and the physicochemical processes that take place in the medium. Therefore, it becomes possible to form arbitrarily complex photon structures (PS) that allow to transform the spatial profiles of the incident light beams. In this paper, the process of the holographic formation of the PS by the Bessel-like and Gaussian light fields in the PPM with two-beam interaction and small contrasts of recording light beams is investigated.

2. Theoretical part
Let two monochromatic light beams with amplitude profiles $E_0(r)$ (Gaussian distribution) and $E_1(r)$ (Bessel-like distribution), wave vectors $k_0$ and $k_1$ fall onto a flat sample containing an absorbing photopolymer layer at angles $\theta_0$ and $\theta_1$ (figure 1.a). Vectors $k_0$ and $k_1$ lie in the XOY plane, and the width of the beams is much larger than the sample’s thickness $d$. Then the optical field at the entrance to the sample can be represented as [6-9]:
\[ E(t,r) = \sum_{j=0,1} e_j \cdot E_j \cdot \exp \left[ i(\omega \cdot t - k' \cdot r) \right], \quad (1) \]

where \( e_j \) – polarization vectors; \( r \) – radius-vector; \( k' = k \cdot N_j \), \( k = n \cdot c/\omega \) – wave number and \( N_j \) – normal to the wavefront, \( n \) – refraction index.

**Figure 1.** (a) Spatial recording geometry; (b) local contrast of interference pattern.

**Under the action of diffraction of the recording light beams during recording process inside the sample with a PPM, the intensity distribution of the recording field \( I(t,r) \) changes. The formation of the PS at each time point continues in accordance with the changed distribution of the light field, and in areas of low contrast, under the action of self-diffraction, an additional structure is formed. It should be noted that, the effect of self-diffraction is most observed in areas of low contrast (<0.1, figure 1.b).**

The process of holographic formation of a PS is described in general form by the kinetic equations of monomer concentration \( M \) and refractive index \( n \) [6-9]:

\[ \frac{\partial M}{\partial t} = \text{div} \left[ D(M) \text{grad}(M) \right] - h \left[ I(t,r) \right]^k M, \quad (2) \]

\[ \frac{\partial n}{\partial t} = \delta n_p, h \left[ I(t,r) \right]^k \left( M/M_n \right) + \delta n_d \text{div} \left[ D(M) \text{grad}(M/M_n) \right], \quad (3) \]

where \( M = M(t,x,y) \), \( n = n(t,x,y) \), \( I(t,r) \) – intensity distribution of the recording field inside the PPM, \( \delta n_p \) and \( \delta n_d \) – model parameters characterizing the \( n \) change due to polymerization and diffusion of material components respectively; \( M_n \) – initial concentration of the monomer; \( h \) – coefficient depending on the material parameters; \( k \) – degree of non-linearity of the photopolymerization process.

The solution of the system of coupled waves equations, describing the interaction of light waves with the PS being formed, is obtained in the approximation of a given field [10, 11]. The change in the light wave \( E_i \) in the interaction area is written as:

\[ \delta E_i(t,x,y) = E_i(t,x,y) - E_i(x) = -iGE_0(x) \int_0^y n_1(t,x,y') \, dy', \quad (4) \]

where \( n_1 \) – first harmonic of the refractive index; \( G = \pi / \left[ \lambda \cos(\theta_0) \right] \), where \( \lambda, \theta_0 \) – wavelength of light and the recording angle of the wave \( E_0 \) in the material.

The intensity distribution of the light field for expressions (2) and (3) can be written as [9]:

\[ I(t,r) = I^0(x) \left[ 1 + m(x) \cos(Kr) + \frac{E_0(x)\delta E_i(t,x,y)}{I^0(x)} e^{-ikr} + \text{c.c.} \right], \quad (5) \]
where \( I^0(x) = [I_0(x) + I_1(x)] \), \( I_j(x) = |E_j(x)|^2 \), \( j = 0, 1 \), \( m(x) = 2\sqrt{I_0(x)I_1(x)(e_1 \cdot e_2)} / [I_0(x) + I_1(x)] \) — local contrast of interference pattern.

The solution of the system of kinetic equations for the amplitude of the first harmonic of the refractive index is the expression [9]:

\[
\eta_1(\tau, x, y) = \delta n_p F_2(x) \sqrt{m_0(x)} \int_0^\tau R(\tau', x) H_0(\tau', \tau, x, y) d\tau',
\]

where \( \tau = t / T_m \) — relative time, \( T_m = 1 / \left(K_1^2 D_m\right) \) — diffusion time, \( K_1 = |K_1| \) — wave number of the first harmonic, \( D_m \) — the initial value of the diffusion coefficient, \( F_2(x) = 2k^2 \sqrt{b_1} (1 + m_0(x)) \), \( b_1 = b(x) = T_p(x) / T_m \),

\( T_p(x) = h^{-1} \left[I^0(x)\right]^2 \) — local polymerization time, \( R(\tau, x) = \frac{M_0(\tau)}{M_n} \left(\frac{2\tau}{b_1} - C_n\right) \int_0^\tau M_0(\tau') e^{-\int_0^\tau \tau' e(\tau') d\tau'} d\tau' \),

\( M_0(\tau) \) — monomer concentration for zero harmonic, \( M_n \) — initial monomer concentration,

\( C_n = \delta n_p / \delta n_p \), \( H_0(\tau, \tau, x, y) = 1 + \frac{i \cdot F_2(x) \cdot \Gamma}{y / d} \int_0^\tau \int_0^\tau R(\tau') d\tau' \cdot J_1 \left[2\sqrt{\int_0^\tau \int_0^\tau R(\tau') d\tau'}\right], \)

Bessel function, \( F_1(\tau) = 2^2 / b_1 b_2 (\tau, x) \), \( \Gamma = \delta n_p \cdot G \cdot d = \omega \cdot d \cdot \delta n_p / 2 \cdot \omega \cdot \cos(\varphi) \) — normalized coupling coefficient characterizing the efficiency of the interaction of light waves with a structure.

The resulting expression (6) determines the temporal dynamics of the spatial distribution of the amplitude of the PS, taking into account self-diffraction of the recording light beams.

As a result of recording the PS by light beams with different amplitude and phase distribution, the amplitude and phase profile of the structure becomes non-uniform. The inhomogeneity of the amplitude-phase profile of the structure leads to the rotation of the effective vector of the grating, and as a result, the shift of the Bragg angle when reading, as well as to the exchange of energy between the beams.

3. Numerical simulation

For the numerical simulation, non-uniform profiles of recording light beams were taken, corresponding to the Gaussian and Bessel-like field distributions (\( m_n(x) = I_1(x) / I_0(x) = \text{var} \)).

According to expression (6), the amplitude profile \( \eta_1(\tau, x, y) \) and the corresponding intensity distribution for the diffracted light beam (figure 2) were calculated with and without two-beam interactions.

![Figure 2. The normalized intensity distribution profile of the diffracted light beam along the x coordinate with and without two-beam interaction.](image-url)
From figure 2 it can be seen that for the diffracted light beam, when two-beam interaction is taken into account, the side maxima are enhanced in level, and the gain is more pronounced in areas of low contrast. This phenomenon is due to the effect of self-diffraction of recording light beams.

4. Experimental part

Figure 3 shows the experimental setup for the holographic formation and reading of photon structures in a sample with PPM [12-14]. A helium-neon (He-Ne) laser with a radiation wavelength of 633 nm forms a reference light beam with a Gaussian light distribution of 2 mm in diameter and a power of 2 mW. After reflection from the mirror (M), the beam is divided into two using a beam-splitting cube (BSC). Further, the signal beam with a Gaussian intensity distribution through an amplitude transparency (AT) is transformed into a one-dimensional Bessel-like [15]. AT has a slit width of 200 microns and a distance between slits of 900 microns. The distance from the AT to the lens (L) and from the lens to the sample corresponded 16 cm. The angle of incidence of the reference and signal beam is 4 degrees. The reference beam after the mirror (M) was broadened using a collimator (C) to an aperture value of 4 mm. In the bulk of the PPM sample, the reference and signal beams interfere. Further, following to the holographic principle, a phase transmission hologram is formed in the sample.

Photopolymer films “GFPM633.5” produced by LLC Polymer Holograms-Novosibirsk with a layer thickness of 45 ± 5 μm on a glass substrate with a thickness of 1 ± 0.1 mm were used as PPM. The laser beam analyzer (A) captures the intensity distribution of the transmitted signal and reference beam. To read the obtained hologram, the signal light beam was blocked by a shutter (B). At the output of a formed structure, the analyzer recorded the intensity distribution of the diffracted light field.

Figure 3. (a) Experimental setup for PS recording and (b) experimental setup for PS reading.

Figure 4 shows experimentally obtained two-dimensional intensity profiles of the Gaussian, signal, and diffracted beam from the distance between the lens and beam analyzer. The measurements were carried out using setup scheme shown in Fig. 3 by displacing the beam analyzer along the direction of propagation of light beams from 9 to 30 cm relative to the lens. From a qualitative comparison of changes in the intensity profiles from 9 to 30 cm, it can be seen that the width of signal beam along the X coordinate remains almost constant, whereas width of the Gaussian beam varies several times over a short distance. It is worth noting that for a one-dimensional Bessel-like beam, the nature of change is valid along the X axis, along the Z axis signal beam behaves like a Gaussian. The diffracted beam repeats the nature of the signal beam.
Figure 4. Two-dimensional intensity profiles of the Gaussian, signal and diffracted beam from the distance between the lens and the beam analyzer.

For a quantitative estimate of the change in width of Gaussian beam and width of the central maximum of signal and diffracted beams along the transverse coordinate X at an intensity level of 0.5, figure 5 shows a normalized graph of the light constrictions’ width with a longitudinal displacement of the beam analyzer. From Figure 5 it can be seen that at a distance of 18 to 30 cm the width of the diffracted and signal beams practically coincide, then to the width of Gaussian beam increases by 6 times, which confirms the properties of Bessel-like beams (signal and diffracted beams) to compensate the diffraction along the length of focal segment by lateral supply of radiation energy.

Figure 5. The result of measuring the width of beams at longitudinal displacement from the lens to the beam analyzer.

To observe another characteristic property of a Bessel-like beam (profile recovery), an obstacle was located at a distance of 2 cm from the DOE along the propagation of the diffracted beam. The obstacle was a thin wire, the size of which was 260 μm is comparable to the width of the central maximum of the diffracted beam along the X-axis. Figure 6 illustrates two-dimensional profiles of the diffracted beam at a distance of 2 cm from the DOE (Figure 6.a), with the introduction of the obstacle (Figure 6.b), and the recovered beam. The qualitative restoration of the central region of the Bessel-like light field was observed at a distance of 10 cm from the DOE.

For a quantitative assessment with the presented model, figure 7 presents the results of comparing the normalized intensity distribution profiles along the X axis of the diffracted light beam, obtained experimentally and by the method of numerical simulation with allowance for two-beam interactions.
Figure 7. Theoretical and experimental normalized intensity distribution profiles of diffracted light beams along the X coordinate.

The estimated standard deviation of theoretical results from the experiment was 1.9%. The theoretical and experimental research confirms the need to take into account the effect of self-diffraction on the diffraction characteristics of holographically formed DOE.

5. Conclusion
In this paper, we developed a theoretical model of the formation of a PS for the conversion of light fields into one-dimensional Bessel-like, taking into account two-beam interaction and small contrasts. An experimental study of the holographic formation of the DOE by Bessel-like and Gaussian light beams in PPM.

The ability of signal and diffracted beam to compensate diffraction by lateral radiation energy was experimentally demonstrated. The unique property of restoring the Bessel-like beam’s profile during the passage of an obstacle is also demonstrated.

It has been shown by numerical simulation and experiment that, due to the effect of self-diffraction in areas of low contrast (corresponding to the regions of the side maxima of the diffracted light beam), the amplitude of the diffracted field is higher relative to the recording one.

Thus, to determine the spatial distribution of the refractive index of the PPM in the process of PS holographic formation, it is necessary to take into account the two-beam interaction at low contrasts of the recording light beams, which will make it possible to more accurately determine the diffraction characteristics of the formed optical elements.

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