Tracking Rydberg atoms with Bose-Einstein Condensates

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We propose to track position and velocity of mobile Rydberg excited impurity atoms through the phase they imprint on their host condensate by elastic interactions with the Rydberg electron. This signal is then naturally converted to features in the condensate density or momentum distribution. The condensate thus acts analogous to the cloud or bubble chambers in the early days of elementary particle physics. The technique will be useful for exploring Rydberg-Rydberg scattering, rare inelastic processes involving the Rydberg impurities, coherence in Rydberg motion and forces exerted by the condensate on the impurities. Our simulations show that resolvable tracks can be generated within the immersed Rydberg life time and condensate heating is under control.

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Introduction: Tracking particle motion has helped to advance physics for centuries, in developing Newtonian mechanics, understanding Brownian motion and, more recently, unravelling the standard model of elementary particle physics. Tracks allowed the deduction of fundamental theories through studying the deflection of trajectories by conservative and frictional forces. They further indicate decay products and give clues on particle lifetimes via track lengths. Early examples of tracking devices were the bubble chamber [1] and cloud chamber [2], in which an energetic particle leaves an optically visible mark of its passing through interaction with the chamber medium. These devices were later replaced by wire chambers [3] and recently silicon detectors [4].

In ultra-cold atomic physics, about 25 orders of magnitude below the particle physics energy scales, experiments can dope Bose-Einstein condensates (BECs) with impurities such as ions [5] or Rydberg atoms [6, 8]. We show that the phase coherence of the BEC allows its use as a tracking instrument for Rydberg impurities, reminiscent of a bubble chamber in the early days of particle physics. Through elastic collisions of the Rydberg electrons with the condensate atoms, a dynamical quantum phase is imprinted onto the wave function of the latter. The phase records Rydberg trajectories and velocity, and could be read out by interference with a reference condensate [9, 13]. Alternatively, after a short time, the impurity track will be converted into in-situ detectable [14, 10] density depressions.

Tracking can first verify dipole-dipole and van-der-Waals (vdW) interactions [17, 19] of Rydberg atoms, and then explore less well studied inelastic reactions that occur when ground-state atoms interact with Rydberg impurities in the ultracold regime [0, 7, 20]. While such reactions may somewhat limit BEC in tracking other Rydberg dynamics, they also provide the opportunity to study the timing and evolution during decay processes through the length and features of imprinted tracks.

To demonstrate the feasibility of our ideas, we model up to five Rydberg excited atoms mutually interacting via van-der-Waals forces while embedded in a BEC. Rydberg atoms move as in Newton’s equations, while the BEC evolves according to the Gross-Pitaevskii equation including the effect of the impurity potentials [21, 23]. We show that for typical parameters, resolvable phase tracks are obtained within the effective Rydberg life-time [7]. We show that the classical electron probability distribution in a Rydberg state is a helpful tool for modelling tracking, and find that condensate heating during the tracking process is limited.

Interactions between Rydberg impurities and BEC: Consider a Bose-Einstein condensed gas of N Rb87 atoms with mass m mostly in their ground state, among which Nimp impurity atoms are excited to a Rydberg state |Ψ⟩ = |νs⟩, with principal quantum number ν ≫ 10 and l = 0 angular momentum. This is sketched for Nimp = 2 in Fig. 5(a). We denote the location of impurities with xμ. As discussed in [6, 22, 25], we can then

\begin{align*}
\text{FIG. 1: Sketch showing a one-dimensional cut through a Bose-Einstein condensate that is tracking two repelling Rydberg impurities (black ●). (a) Condensate density } \rho \text{ before (black) and after (green) Rydberg motion, with corresponding phase signal (red) and sketched Rydberg electron wavefunctions (blue bottom). (b) Interaction potential } V_{\text{R}(R)} \text{ (black) between condensate atoms and a single Rydberg impurity at } x = 0, \text{ for principal quantum number } \nu = 20. \text{ The classical approximation } V_c \text{ (red) averages over fast oscillations. We also employ the indicated cut-off } V_{\text{max}} \text{ (dotted blue).}
\end{align*}
model the BEC in the presence of Rydberg impurities with the Gross-Pitaevskii equation (GPE):
\[
\frac{i\hbar}{\partial t} \phi(R) = \left( -\frac{\hbar^2}{2m} \nabla^2 + W(R) + g_{2D} |\phi(R)|^2 \right) \phi(R) + \sum_{n} V_{0}\Psi(R-x_n)^2 \phi(R),
\]
(1)

where \(\phi(R)\) is the condensate wave function and \(W(R) = \frac{m\omega_r^2}{2}(x^2 + y^2)/2\) describes a two-dimensional (2D) harmonic trap, \(R = [x, y]^T\). The third dimension is frozen through tight trapping \(\omega_z^2 \gg \omega_r\), see e.g. [20, 27], hence \(g_{2D} = g_{3D}/(\sqrt{2\pi}\sigma_z)\) describes the effective strength of atomic collisions, where \(g_{3D} = 4\pi\hbar^2a_s/m\), with atom-atom s-wave scattering length \(a_s\) and \(\sigma_z = \sqrt{\hbar/m\omega_z}\).

The last line in [3] represents interactions between the ground state condensate atoms and impurities due to elastic collisions of the Rydberg electron and condensate atoms [28]. The potential for a single impurity \(V_{\text{R},n}(R) \equiv V_0|\Psi(R-x_n)|^2\), is sketched in Fig. 3(b). Its strength is set by \(V_0 = \pi \hbar^2 a_e/m\) containing the electron-atom scattering length \(a_e = -16.04a_0\), where \(a_0\) is the Bohr radius [29]. The potential shape is set by the Rydberg electron density |\(\Psi(R)|^2\), where \(\Psi(R) = \langle R|\Psi \rangle\).

To describe mobile Rydberg impurities, we couple Eq. (3) to Newton’s equations governing their motion
\[
m \frac{\partial^2}{\partial t^2} x_n = -\nabla_x \left[ V_{\text{RR}}(X) + \tilde{V}(x_n) \right],
\]
(2)

where \(V_{\text{RR}}(X) = \sum_{n>m} C_6(\nu)/|x_n - x_m|^6\) is the vdW interaction between Rydberg impurities, with dispersion coefficient \(C_6\) taken from [30], and \(X = [x_1, \ldots, x_{N_{\text{imp}}}]^T\) grouping all impurity positions. In addition, the \(n^{th}\) impurity feels an effective potential \(\tilde{V}(x_n) = \int d^2 R V_0|\Psi(R-x_n)|^2|\phi(R)|^2\) from the backaction of the condensate [21, 23, 24]. See [27] for the 2D reduction.

Let us split the BEC wave function \(\phi(R) = \sqrt{\rho(R)} e^{i\varphi(R)}\) into a real density \(\rho\) and phase \(\varphi\). Then, the initial effect of each impurity is to imprint a phase \(\varphi \sim -V_{\text{R},n}(R) \Delta t/\hbar\), within a short time \(\Delta t\). When the impurity moves, a phase pattern is “painted” over the condensate. As we will show, this allows impurity tracking with the added practical benefit that oscillatory quantum features in Fig. 3(b) are averaged, so that we can replace \(V_{\text{R},n}(R)\) with a smoother classical approximation \(V_{\text{c},n}(R)\) as discussed in the SI [27].

**Phase imprinting versus density modulation:** Using XMDS [32, 33], and Newton’s equations (2) for a comparatively small 2D BEC cloud, with 3D peak density at the centre of \(p_0 = 3.6 \times 10^{19} \text{ m}^{-3}\). Five atoms are excited to \(\nu = 80\) Rydberg states and placed at \(t = 0\) initially on the inner starting points of the tracks evident in Fig. 5(a). The panel shows the condensate phase \(\varphi(R)\).

The final position of each atom after an imprinting time \(\tau_{\text{imp}} = \tau_{\mu s}\) is shown by colored circles with size matching the Rydberg electron orbital radius \(r_{\text{orb}} = 3a_0\nu^2/2\).

Atoms have repelled as in [8], on time-scales very short compared to those typical for BECs. While tracks are clearly visible in the condensate phase, the effect on the density in panel (b) is almost negligible for times as short as \(\tau_{\text{imp}}\), corresponding to the Raman-Nath regime [34]. The imprinted phase also carries velocity information, since it depends on how long a given location is visited by the impurity, see SI [27]. This is shown in the inset of panel (a) for the initial atomic acceleration.

Interferometry can deduce a condensate phase pattern [9,13], but is not commonly available. Fortunately, the phase tracks are converted into density tracks through motion of the ground-state atoms. These have received an initial impulse from the passing Rydberg impurity, causing motion on larger time scales \(\tau_{\text{mov}} \approx 0.8\text{ ms}\) in panels (c,d). Hence density depressions with 41% contrast appear on either side of the Rydberg track. These can be read out through in-situ density measurements [14,16]. As third resultant signal in dilute condensates, the post time-of-flight image may allow reconstructing the early stage phase [22].

Tracking information is dominated by imprinting via the wide tails of the Rydberg electron density, between \(x = 10\) and 50 nm in Fig. 3(b). We thus cut the large central peak off at \(V_{\text{max}}\) as shown in blue, to significantly
ease simulations. Tracks were largely independent of this technical step.

**Classical electron distribution:** For Fig. 2 we replaced the potential based on the Rydberg electron wave function $V_{\text{eh,atom}}(R)$ by the corresponding classical probability distribution $V_{\text{c,atom}}(R)$, shown in Fig. 3 (b). This is possible since the passage of the mobile impurities through the BEC spatially averages over the electron distribution along the direction of motion. According to the correspondence principle, the averaging results in the classical probability distribution, see 24, 35.

To verify this technique, we show what amounts to a zoom onto a Rydberg track in Fig. 3. It is created by a single impurity with velocity $v = 0.7$ m/s traversing a homogeneous 2D condensate ($\rho_{\text{3D}} = 3.6 \times 10^{19}$ m$^{-3}$, $\omega_z/(2\pi) = 1$ kHz). Interactions are modelled with both, $V_{\text{eh,R}}(R)$, black line in Fig. 3 (b), and $V_{\text{c,R}}(R)$, red line in Fig. 3 (b). The difference between imprinted phases is small, see Fig. 3 (b). Besides technical utility, this agreement indicates a further use of Rydberg tracking for explorations of the correspondence principle.

![FIG. 3: (a) Zoom onto the condensate phase imprinted by a mobile Rydberg impurity in a homogeneous background. (b) The difference between the imprinting effect caused by the real potential $V_{\text{eh,R}}(R)$ and its classical approximation $V_{\text{c,R}}(R)$ is small. See also supplementary movie. (c) Modifying the potential cut-off only affects the less relevant central part of the imprinting track. We show the difference between phases resulting from $V_{\text{max}} = 2$ MHz and 6 MHz. (d) Number of un-condensed atoms $N_{\text{unc}}$, due to Rydberg impurity determined from the TWA for different $V_{\text{max}}$. Dashed lines match the linear rate equation $dN_{\text{unc}}/dt = \Gamma$, with $\Gamma \approx 280 - 300$ atoms/µs. In contrast, for Fig. 2 heating remains negligible.](image)

**Condensate heating:** Ref. 26 reported atom-loss and heating when sequentially exciting a large number of Rydberg impurities in a BEC. This could occur in our scenario in addition to mechanical effects of Rydberg-BEC interactions explored so far. Instead, for the substantially smaller number of Rydberg impurities discussed here, and shorter times, we show that heating is limited.

For this, we augment simulations of Eq. 3 beyond the mean-field using the truncated Wigner approximation (TWA) 39, 42. In brief, this adds quantum fluctuations to the initial state of 3 by specific addition of random noise. Averaging over an ensemble of solutions, we can then extract the number of uncondensed atoms $N_{\text{unc}}$ from the noise statistics as discussed in the SI. We expect the TWA to give reliable results for the short times $\tau_{\text{imp}}$ considered here 43.

Under conditions of Fig. 2 we find that while Rydberg atoms exist, up to $\tau_{\text{imp}} = 7$ µs, they cause only 30 atoms to become uncondensed. Heating would thus only become significant much later. For different conditions we further explored heating in Fig. 3. We find that final heating rates $\Gamma$ shown in the figure scale with the principal quantum number as $\Gamma \sim \nu^{-1}$. Increasing the cut-off led to a linear increase in uncondensed atom number $N_{\text{unc}} \propto V_{\text{max}}$ until this saturates, with mostly unchanged final heating rates. See SI for more details.

Experiments on heating by mobile Rydberg impurities may unravel incoherent versus coherent aspects of impurity-superfluid interactions, involving critical velocities 44, frictional forces 45 or Cherenkov radiation 46, all due to the creation of elementary excitations 47.

**Detection of condensate backaction:** We mainly focus on the phase and density tracks imparted by Rydberg impurities on the BEC, however 3 also includes a backaction of the BEC onto the Rydberg motion: the effective potential in Eq. 3 set by BEC density. This was negligible in Fig. 2 and hence disabled, but can become crucial for denser and smaller condensates (now $N = 28000$ atoms with $\omega_r/(2\pi) = 96$ Hz), shown in Fig. 4. There, just two impurities with $\nu = 80$ are initially separated by $d = 5.57$ µm, symmetrically placed on either side of the centre of the condensate, which has a Thomas-Fermi radius $R_{\text{TF}} = 8.21$ µm. Rydberg atoms initially accelerate quickly to $v \approx 0.4$ m/s due to vdW repulsion. Once they leave the high density region of the condensate cloud, the attraction to ground-state atoms provided by the Rydberg electron results in an effective potential well $V(x_n)$, significantly slowing down the Rydberg impurities as shown in panel (a). Had we not included this force in the simulation, the total energy would not be conserved as shown in panel (b).

The backaction effect in Fig. 4 should be observable in experiment, using tracking or conventional techniques.

**Outlook and utility:** Our simulation in Fig. 2 demonstrates that kinematic data of mobile Rydberg impurities can be viably extracted through their interaction with a host BEC, mimicking particle physics tracking techniques. Importantly, the required imprinting stage $\tau_{\text{imp}} = 7$ µs is shorter than the life-time $\tau \approx 45$ µs/$N_{\text{imp}} \approx 9$ µs that we expect based on 7 for all five atoms even
within the condensate. While tracks due to simple vdW interactions among equal state Rydberg $l = 0$ excitations are not expected to hold too many surprises, our technique opens a window on more intriguing ultracold quantum dynamical processes sketched in Fig. 5: (i) When different Rydberg states are involved, potentials may contain an attractive component leading to collisional ionisation [48, 51]. In an $E \approx 1 \text{ V/cm}$ extraction field along $z$, the ionized electron would leave the BEC faster than 1 ns. Thus ionized tracks instantly terminate and allow a measurement of the ionization distance $d_{\text{ion}}$ as sketched in Fig. 3 (a). (ii) While our theory assumes the Rydberg electron to remain in the same quantum state during acceleration [52], there will be some probability of inelastic processes, dominated by angular momentum $l$-changing collisions [7]. Since the imprinting signature depends on $l$ [23], tracking may provide a time sequence of $l$ changes. (iii) Multiple dipole-dipole coupled Rydberg impurities move according to aggregate Born-Oppenheimer surfaces [53], providing directed energy transport [54, 55] or conical intersections [56, 58]. Tracking enables detection of such effects, and will introduce a well defined decoherence channel akin to the discussion of [59].

At present none of the phenomena (i)-(iii) in a BEC has established theory, the creation of which can benefit from Rydberg tracking experiments such as proposed here. Conclusions: Mobile Rydberg atoms can be tracked through the phase they imprint while passing through a BEC. Modelling this is greatly facilitated by approximating the Rydberg-BEC potential using the classical electron position distribution. On the short time scales required for the tracking, heating and inelastic decay are under control. BEC based Rydberg tracking can now advance our understanding of a variety of ultracold quantum dynamical processes, such as ionisation, state changing collisions and motional decoherence. Other effects explorable may include phonon mediated Rydberg-Rydberg interactions [60] or motion damping [61].

All phenomena discussed here should remain qualitatively unchanged if the direct Rydberg-electron-BEC interaction is replaced with dressed impurity-BEC long range interactions discussed in [22]. This turns the range of the imprinting potential and thus the width of tracks into a tuneable parameter.

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Supplemental information: Tracking Rydberg atoms with Bose-Einstein Condensates

Dimensionality reduction for Gross-Pitaevskii equation: The main article employs a GPE in two spatial dimensions, which eases simulations and in an experiment would ease detection. Let us briefly discuss the approximations that allow the reduction of the 3D GPE to a 2D GPE. The evolution of BEC in the presence of a Rydberg impurity in 3D is governed by:

$$i\hbar \frac{\partial}{\partial t} \phi(\mathbf{R}) = \left( -\frac{\hbar^2}{2m} \nabla_{\mathbf{R}}^2 + V(\mathbf{R}) + g_{3D}|\phi(\mathbf{R})|^2 + \sum_{n} N_{\text{imp}} V_{0}(\mathbf{R} - \mathbf{x}_{n})^2 \right) \phi(\mathbf{R}),$$

where $V(\mathbf{R}) = m(\omega_{x}^2(x^2 + y^2) + \omega_{z}^2 z^2)/2$ is the 3D harmonic potential and $\mathbf{R} = [x, y, z]^T$ is the 3D coordinate vector. We take $\omega_{z} > \omega_{x}$ and assume the wavefunction $\phi(\mathbf{R})$ factors into a part for the in plane coordinate ($\mathbf{R} = [x, y]^T$) and a part for the z-direction as, $\phi(\mathbf{R}, t) = \phi(\mathbf{R}, t)|\phi(z)$. Importantly $|\phi(z)|$ is frozen in the harmonic oscillator ground state along $z$, normalized to unity.
After multiplying by \( \phi^*(z) \) and integrating here along the \( z \)-direction we obtain Eq. (1) of the main article, which effectively describes a tightly trapped pancake BEC. We consider parameters for which \( \sigma_z = 0.3 \) \( \mu \text{m} \) \( < r_{\text{orb}} = 0.6 \) \( \mu \text{m} \). We can thus assume the Rydberg wave-function does not vary significantly over the range of \( z \) with non-vanishing BEC, see Fig. [6]. Then, during the 3D\( \rightarrow \)2D reduction, we can approximate:

\[
\int dz \, V_0 |\Psi(\mathbf{R} - \mathbf{x}_n)|^2 |\phi(z)|^2 = V_0 |\Psi(\mathbf{R} - \mathbf{x}_n)|^2 \int dz |\phi(z)|^2 = V_0 |\Psi(\mathbf{R} - \mathbf{x}_n)|^2.
\]

Effectively, in Eq. (4), we thus only use a 2D cut at \( z = 0 \) through the effective potential \( V_{\text{eff},n}(\mathbf{R}) \), as shown in Fig. [8].

**Velocity dependence of phase imprinting:** Ignoring all energy contributions except BEC-impurity interaction, for a single impurity with trajectory \( \mathbf{x}(t) = \mathbf{v} t \) the solution of Eq. (3) of the main article at time \( t \) is given by \( \phi(\mathbf{R}, t) = \phi(\mathbf{R}, 0) \exp(i \varphi(\mathbf{R}, t)) \), where

\[
\varphi(\mathbf{R}, t) = -\int dt \, V_0 |\Psi(\mathbf{R} - \mathbf{v} t)|^2 / \hbar
\]

is the phase imprinted by the moving Rydberg impurities at a location \( \mathbf{R} \) in the BEC. Using the definition of a line-integral along the curve \( \mathbf{R}' = \mathbf{R} - \mathbf{v} t \), we can re-write this as

\[
\varphi(\mathbf{R}, t) = -\int_C d\ell |\Psi(\mathbf{R}'(\ell))|^2 / (|\mathbf{v}| \hbar),
\]

where the curve \( C \) traces the trajectory of the location \( \mathbf{R} \) as it moves through the Rydberg electron orbit \( \Psi \) in the rest frame of the Rydberg atom. We show Eq. (6) to demonstrate two features: (i) the accumulated phase is a spatial average over \( |\Psi|^2 \) in the direction of motion. (ii) The phase is proportional to \( |\mathbf{v}|^{-1} \) for uniform motion, and thus contains velocity information.

**Quantum versus classical Rydberg electron probability distributions:** The Rydberg-condensate interaction potential \( V_{\text{GR}} \) contains a 2D cut of the quantum probability density (QPD), \( \rho^Q(\mathbf{R}) = |\Psi(\mathbf{R})|^2 \) (for a single impurity at \( \mathbf{x} = 0 \)). Modelling the ensuing BEC dynamics encounters two major computational challenges: (i) Resolving the highly oscillatory Rydberg wave function in Fig. [6](b) on nm scales, while spanning the whole host BEC of radius \( \sim 20 \) \( \mu \text{m} \) necessitates a large number of discretized spatial points. (ii) At the same time, very short time steps are forced by large interaction energies \( \mathcal{O}(10 \text{ GHz}) \) of ground-state and Rydberg atoms near the Rydberg core only. We tackle the former point (i) by replacing the QPD in Eq. (3) by the classical probability density (CPD) \( \rho^{\text{CPD}} \)

\[
\rho^{\text{CPD}}(\mathbf{R}) = \frac{1}{8\pi^2 R} \frac{1}{\sqrt{\epsilon^2 b^2 - (R - b)^2}},
\]

where \( R = |\mathbf{R}|, b = -k/2E \) is the semi-major axis for the elliptical electron orbit in a Coulomb field \( U(\mathbf{R}) = -k/R \) with \( E \) the energy of the \( \nu \)th level and \( \epsilon = \sqrt{1 + 2E \ell^2 / \hbar^2} \) the eccentricity. Here \( L \) is the angular momentum of the Rydberg state.

The CPD becomes ill defined after the outer classical turning point \( R_{\text{ct}} \), where \( R_{\text{ct}} = b(1 + \epsilon) \). Therefore, for \( R > R_{\text{ct}} \) we revert from \( \rho^{\text{CPD}} \) back to the tail of the QPD for a smooth and well approximated distribution.

Overall we thus employ the classical approximation to the Rydberg-BEC interaction potential given by:

\[
V_{\text{c,n}}(\mathbf{R}) = V_0 \begin{cases} 
\rho^{\text{CPD}}(\mathbf{R}) & R < R_{\text{ct}}, \\
\rho^2(\mathbf{R}) & R \geq R_{\text{ct}},
\end{cases}
\]

which is sketched in Fig. 1 (b) of the main article as a red line.

The replacement of \( V_{\text{GR,n}}(\mathbf{R}) \) by \( V_{\text{c,n}}(\mathbf{R}) \) is validated in Fig. 3 of the main article, where we find good agreement between both methods. It works due to the spatial averaging caused by Rydberg motion as seen in [9]. The average QPD yields the CPD as known from the correspondence principle.

We solve problem (ii) by using a high-energy cutoff at \( |V(\mathbf{R})| = V_{\text{max}} \). Due to the minor spatial extent of the potential region affected by the cut-off, its impact on our results is small. This is seen by comparing Fig. [7] here with Fig. 2 of the main article.

**Energy conservation:** In this section, we discuss the relevant components of the BEC energy and Rydberg impurity energy, used in the main article to explore energy conservation in the coupled system of effective GPE and Newton’s equation with back action.

Since Eq. (1) in the main article contains the Rydberg-BEC interaction potential \( V_{\text{GR}} \), we also should include this as a back-action of the BEC into the equation of motion for the Rydberg impurities. The total energy of
We thus use the initial stochastic field depletion or thermal fluctuations beyond the mean field.

The total energy of the BEC is given by the Gross-Pitaevskii energy functional,

\[
\begin{align*}
E_{\text{BEC}} &= \int d^2R \left( -\frac{\hbar^2}{2m} \nabla^2 + V(R) \right) \phi^*(R) \\
&\quad + g_{2D} |\phi(R)|^2 \phi(R).
\end{align*}
\]

The interaction energy due to Rydberg-BEC interaction potential \(V_{RR}\) is

\[
E_{\text{int}} = \sum_n \int d^2R \left| \Psi(R) - x_n \right|^2 |\phi(R)|^2.
\]

The total energy of the complete system (Ry whole + BEC) \(E = E_{\text{Ryd}} + E_{\text{BEC}} + E_{\text{int}}\) is constant throughout the dynamics.

**Truncated Wigner approximation:** After its introduction to BEC \[60\] \[62\] the TWA in a BEC context is described in many articles including the review \[62\]. The central ingredient of the method is adding random noise to the initial state of the GPE, Eq. (1) of the main article, in order to provide an estimate for the effects of quantum depletion or thermal fluctuations beyond the mean field. We thus use the initial stochastic field

\[
\alpha(R, 0) = \phi_0 + \sum_k [\eta_k u_k(R) - \eta_k^* v_k(R)]/\sqrt{2}.
\]

with random complex Gaussian noises \(\eta_k\) fulfilling \(\eta_k\overline{\eta_k} = 0\) and \(\eta_k \overline{\eta_k} = \delta_{kl}\), where \(\rightarrow\) is a stochastic average. \(u_k(R)\) and \(v_k(R)\) are the usual (2D) Bogoliubov mode functions in a homogenous BEC with density \(\rho = |\phi_0|^2\) \[43\].

A different symbol \(\alpha_\delta(R)\) has been chosen for the stochastic field compared to the mean field, to emphasise the difference in physical interpretation due to the presence of noise: The stochastic field now allows the approximate extraction of quantum correlations using the prescription

\[
\frac{1}{2} \left( \langle \hat{\Psi}^\dagger(R') \hat{\Psi}(R) \rangle + \langle \hat{\Psi}(R) \hat{\Psi}^\dagger(R') \rangle \right) \rightarrow \alpha^*(R') \alpha(R),
\]

Using restricted basis commutators \(\delta_c\) \[63\] \[64\], we can then extract the total atom density

\[
n_{\text{tot}}(R) = |\alpha(R)|^2 - \frac{\delta_c}{2},
\]

condensate density \(n_{\text{cond}}(R) = |\alpha(R)|^2\) and from these both the uncondensed density \(n_{\text{unc}}(R) = n_{\text{tot}}(R) - n_{\text{cond}}(R)\), see also \[39\] \[41\]. Uncondensed atom numbers as a measure of non-equilibrium “heating” referred to in the main article are finally \(N_{\text{unc}} = \int d^2R n_{\text{unc}}(R)\).

**Condensate Heating:** In addition to the mean field dynamics which is the main focus of this article, the presence of the Rydberg impurities inside the BEC will cause incoherent excitations due to Rydberg-condensate interaction. Using the TWA discussed above, we calculate
heating via the number of uncondensed atoms $N_{\text{unc}}(t)$ discussed above. This turned out negligible for Fig. 2 of the main article. Thus in Fig. 3 we rather show the heating for a single impurity with $v = 0.03m/s$ traversing a homogeneous 2D condensate ($\rho_D = 8.5 \times 10^{21} m^{-3}$, $\omega_\perp/(2\pi) = 1kHz$).

The uncondensed atom number, $N_{\text{unc}}(t)$, increases with time and then typically shows a linear trend after $t \approx 25\mu s$, as shown in Fig. 3a for different potential cut-offs $V_{\text{max}}$ at $\nu = 80$. We also show in Fig. 3b that the number of uncondensed atoms at a fixed time, $N_{\text{unc}}(t_0)$, increases linearly with the potential cut-off $V_{\text{max}}$ and finally reaches saturation.

The linear increase of $N_{\text{unc}}(t)$ at late times in (a) allows the definition of a (final) heating rate $\Gamma$ via

$$\frac{dN_{\text{unc}}}{dt} = \Gamma.$$  \hspace{1cm} (15)

as fitted by dashed lines in panel (c). The resultant $\Gamma$ are shown in panel (d).

For our 2D TWA calculations we employed $256 \times 256$ spatial grid-points, $129 \times 129$ Bogoliubov modes with noise and averaged over 30000 trajectories.

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The kinetic energies of accelerated Rydberg atoms is at most \( E_{\text{kin}} \approx 703 \text{ MHz} \), that of thermal uncondensed atoms \( E_{\text{kin}} \approx 13 \text{ KHz} \) at a temperature of \( T = 100 \text{ nK} \), none of which exceed the energy gap of \( \Delta E \approx 1.66 \text{ GHz} \), between \( |\nu = 80, s\rangle \) and the nearest state \( |\nu = 77, d\rangle \), suppressing the most basic state changing processes.