Parity Splitting and Polarized-Illumination Selection of Plasmonic Higher-Order Topological States

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Topological states, originated from interactions between internal degree of freedoms (like spin and orbital) in each site and crystalline symmetries, offer a new paradigm to manipulate electrons and classical waves. The accessibility of spin degree of freedom has motivated much attention on spin-related topological physics. However, intriguing topological physics related to atomic-orbital parity, another binary degree of freedom, have not been exploited since accessing approaches on atomic orbitals are not well developed. Here, we theoretically discover spectral splitting of atomic-orbital-parity-dependent second-order topological states on a designer-plasmonic Kagome metasurface, and experimentally demonstrate it by exploiting the easy controllability of metaatoms. Unlike previous demonstrations on Hermitian higher-order topological insulators, radiative non-Hermicity of the metasurface enables far-field access into metaatomic-orbital-parity-dependent topological states with polarized illuminations. The atomic-orbital parity degree of freedom may generate more intriguing topological physics by interacting with different crystalline symmetries, and promise applications in polarization-multiplexing topological lasing and quantum emitters.

Conventional topological states, one-dimension lower than that of topological insulators, have attracted much attention from condensed matter\textsuperscript{1,2} to photonic\textsuperscript{3-5}, acoustic\textsuperscript{6}, and mechanic\textsuperscript{6,7} communities. The disorder-immune feature of these states drives much effort in developing various robust devices\textsuperscript{3-7}. Recently, a novel class of
topological systems, higher-order topological insulators (HOTIs), have been discovered\textsuperscript{8-30}. Such emerging topological phases, hosting topological states two or more dimensions lower than that of HOTIs, offer a new paradigm to localize/transport electrons or classical waves in extended dimensionalities. Due to the easy controllability, metamaterials are intensively exploited to demonstrate the intriguing topological states in HOTIs\textsuperscript{11,12,14-19,21-30}.

Fundamentally, topological states originate from the interactions between internal degree of freedoms (DoF) in each site and crystalline symmetries. As accessing approaches on spin DoF in electronic crystals have been well established\textsuperscript{31}, this accessibility has motivated much attentions on spin-related topological states\textsuperscript{1,2}, subsequently extended to photonics\textsuperscript{3-5} and acoustics\textsuperscript{6,7}. Recently, the extensions to spin-related physics are also occurring in higher-order topological phases\textsuperscript{32-36}. In contrast, topological physics related to atomic-orbital parity, another binary DoF universally existing in crystals, has not been exploited. The fundamental reason is that atomic orbitals, being probability distribution of electrons around the nucleus\textsuperscript{37}, are challenging to access. The orbital-related topological edge-state has been demonstrated in a two-dimensional nanomechanical topological insulator\textsuperscript{38}, while without considering the parity DoF of orbitals. In addition, the parity DoF has not been connected with higher-order topology yet.

Here, we realize a second-order topological insulator by constructing a breathing Kagome metasurface (shown in Fig. 1a) with designer-plasmonic metaatoms\textsuperscript{39-41}, whose metaatomic orbitals (MAOs) are easily accessible. The metaatom is a textured
metal disk, whose electromagnetic properties are analogous to a plasmonic resonator in nanooptics (see Supplementary Information I). We focus on the middle resonance peak ($\omega_f = 6.069$ GHz) of a single metaatom illustrated in Fig. 1b, which is hexapole mode (as shown in Fig. 1c) commensurate with the $C_3$ symmetry of the Kagome lattice. The three resonance modes can emulate the atomic orbitals shown in Fig. 1c, therefore we term the hexapole mode as $f$-MAO in the following. Three crucial features of the $f$-MAO make the designer-plasmonic second-order topological insulator distinct from the previous demonstrations \cite{8,30,32-36,38}. Firstly, the $f$-MAO has two degenerate modes with even and odd parities, as illustrated in Fig. 1c, and thus gives rise to two copies of parity-dependent second-order topological corner states (SOTCS). The parity is short for the parity of MAOs. In contrast, both the $s$- and the $p_z$-MAOs in previous demonstrations \cite{9,13,14,16-20,23-28} are isotropic in the plane, thus inducing parity-less SOTCS. It is noteworthy that the parity DoF is governed by intrinsic chiral symmetry, different from the spin DoF governed by time/pseudo-time reversal symmetry\cite{32-36}. Secondly, the $f$-MAO spectrally overlaps with its neighbouring $d$-MAO and $g$-MAO as shown in Figs. 1b and 1c. The spectral overlapping neglected in previous works induces effective parity-dependent in-line long-range couplings (LRC) in the Kagome lattice. This in-line LRC can result in parity splitting of SOTCS, and is essentially different from the dipole-dipole LRC\cite{24}, which gives rise to the type-II corner states. Finally, the plasmonic MAOs exhibit radiative non-Hermicity (shown in Fig. 1d), fundamentally distinct from the energy-conserved natural atomic orbitals. Moreover, the radiative non-Hermicity ($\gamma_f = 0.01225$ GHz of $f$-MAO) links the far-field polarizations to the near-
field parities, and enables polarization selection of parity-dependent SOTCS. This phenomenon neither exists in their condensed matter counterparts nor has been observed in previous demonstrations\textsuperscript{8,30,32-36,38} whose systems are closed.

We first show the intrinsic chiral symmetry of parity DoF by investigating the couplings of \(f\)-MAOs in a dimer system. A plasmonic meta-dimer is shown in the inset of Fig. 2a. It can be generally described with the effective Hamiltonian

\[
\hat{H}_{\text{dimer}} = \omega_f + \begin{bmatrix} -\Delta \omega_{\text{eff}} & \kappa_f \omega_f \\ \kappa_f \omega_f & -\Delta \omega_{\text{eff}} \end{bmatrix},
\]

where \(\omega_f\) is the frequency of \(f\)-MAOs in each metaatom, \(\kappa_f\) represents the coupling coefficient between \(f\)-MAOs, and \(\Delta \omega_{\text{eff}}\) is effective on-site detuning (EOD). The dimer system exhibits two supermodes \(\varphi_{\pm} = (1/\sqrt{2})[1, \pm 1]^T\), corresponding to eigenfrequencies \(\omega_+ = \omega_f - \Delta \omega_{\text{eff}} + \kappa_f \omega_f\), \(\omega_- = \omega_f - \Delta \omega_{\text{eff}} - \kappa_f \omega_f\), respectively. The sign +/- denotes in-phase/out-of-phase, respectively. When the frequency \(\omega_+\) of the in-phase supermode \(\varphi_+\) is higher (lower) than that of the out-of-phase one \(\varphi_-\), the dimer system exhibits a positive (negative) coupling \(\kappa_f\). The EOD \(\Delta \omega_{\text{eff}} = -(\overline{\omega} - \omega_f)\) measures the shift of the average frequency \(\overline{\omega} = (\omega_+ + \omega_-)/2\), with respect to the \(\omega_f\).

With the analysis above, we show the parity-dependence of both \(\Delta \omega_{\text{eff}}\) and \(\kappa_f\). In the plasmonic meta-dimer, the even (odd) \(f\)-MAO is selectively excited by end (side) excitations (shown in the inset of Fig. 2a), and the corresponding spectrum is shown as the red (blue) line in Fig. 2a. Regarding the even (odd) \(f\)-MAO, the dimer system exhibits an in-phase supermode at a frequency lower (higher) than that of out-of-phase one (shown in Fig. 2b). It means that parity-dependent couplings are \(\kappa_{f,e} < 0\) and \(\kappa_{f,o} > 0\), where the subscripts ‘e’ and ‘o’ denote the even and odd modes respectively. The
average frequency $\overline{\omega}_{c,e}(\overline{\omega}_{c,o})$ of the even (odd) supermodes blueshifts (redshifts) with respect to $\omega_f$. It means the EOD $\Delta \omega_{\text{eff}}$ exhibits the same parity-dependence as $\kappa_f$. These sign-reversed interactions imply that the parity DoF obeys an intrinsic chiral symmetry, fundamentally different from the spin/pseudospin DoF governed by (pseudo) time-reversal symmetry$^{32-36}$. Consequently, the dimer Hamiltonian should be completely expressed as $\hat{H}_\text{dimer} = \omega_f + \begin{bmatrix} -\Delta \omega_{\text{eff}} & \kappa_f \omega_f \\ \kappa_f \omega_f & -\Delta \omega_{\text{eff}} \end{bmatrix} \otimes \sigma_z$, where $\sigma_z$ represents the Pauli matrix for the parity DoF. We theoretically find that the origin of EOD can be attributed to the interorbital coupling between $f$-MAO and $d$-MAO (or $g$-MAO) (details in Supplementary Information II), due to their spectral overlapping.

To directly display the interorbital couplings, we take a straight plasmonic trimer for illustration (as shown in the inset of Fig. 2c), typically existing in the Kagome lattice. We consider first the trimer model without interorbital couplings, thus only couplings between $f$-MAO ($i.e.$, $f$-$f$) exist. The trimer Hamiltonian is expressed as $\hat{H}_\text{trimer,0} = \begin{bmatrix} 0 & \kappa_f \omega_f & 0 \\ \kappa_f \omega_f & 0 & \kappa_f \omega_f \\ 0 & \kappa_f \omega_f & 0 \end{bmatrix}$, without considering the parity DoF. Three eigenfrequencies are obtained as $\omega_{1,3} = \omega_f (1 \mp \sqrt{2} \kappa_f)$, $\omega_2 = \omega_f$. Interestingly, the middle eigenfrequency $\omega_2$ does not depend on $\kappa_f$. It means that for both even ($\kappa_f < 0$) and odd ($\kappa_f > 0$) modes, the middle eigenfrequencies $\omega_{2,e}$ and $\omega_{2,o}$ are degenerate. Both $\omega_{2,e,o}$ correspond to the supermode $[A_L,A_M,A_R]_2^T = \frac{\sqrt{2}}{2}[1,0,1]^T$, where $A_{L,M,R}$ denote lattice modes on the left, middle and right metaatoms, respectively. Different from the vanished $A_M$ in the $f$-$f$ model, the simulated field patterns (Fig. 2d) show nonvanishing fields. By mode decomposing, the simulated $A_M$ is a superposition of $d$-MAO and $g$-MAO. It implies that the interorbital couplings ($f$-$d$-$f$ and $f$-$g$-$f$) do exist.
The interorbital coupling, effectively expressed as $\hat{H}_{\text{trimer, LOC}} =$

\[
\begin{bmatrix}
-\Delta \omega & 0 & \kappa_3 \omega_f \\
0 & -2\Delta \omega & 0 \\
\kappa_3 \omega_f & 0 & -\Delta \omega
\end{bmatrix}
\]

results in a modified trimer Hamiltonian $\hat{H}_{\text{trimer}} = \hat{H}_{\text{trimer, 0}} + \hat{H}_{\text{trimer, LOC}}$, where $\kappa_3$ and $\Delta \omega$ represent the effective in-line LRC (Fig. 2c) and EOD, respectively (see details in Supplementary Information III). Solving the modified Hamiltonian, we find that the middle eigenfrequency $\omega_2 = \omega_f - (\Delta \omega + \kappa_3 \cdot \omega_f)$ shows a spectral shift with respect to $\omega_f$. According to the spectral results in Fig. 2c, the middle resonance peaks of even ($\omega_{2e} = 6.079$ GHz) and odd ($\omega_{2o} = 6.060$ GHz) show blue- and red-shifts with respect to $\omega_f$. That means the interorbital couplings are also parity dependent, consistent with the theoretical analysis (Supplementary Information III).

Therefore, the trimer Hamiltonian should be completely expressed as $\hat{H}_{\text{trimer}} = \omega_f +$

\[
\begin{bmatrix}
-\Delta \omega & \kappa_f \omega_f & \kappa_3 \omega_f \\
\kappa_f \omega_f & -2\Delta \omega & \kappa_f \omega_f \\
\kappa_3 \omega_f & \kappa_f \omega_f & -\Delta \omega
\end{bmatrix} \otimes \sigma_z
\]

The effective off-line LRC also exists when the trimer is bended and has been discussed in Supplementary Information III. These effective LRCs originate from spectral overlapping of neighbouring MAOs, essentially different from the direct LRC induced by dipole-dipole interaction\textsuperscript{33}. It is also noteworthy that the EOD is induced by the interorbital couplings, different from that by changing metaatoms\textsuperscript{29}.

Based on the microscopic analysis above, we can abstract the designer plasmonic metasurface in Fig. 1a as the model shown in Fig. 3a, where $\kappa_1$, $\kappa_2$, and $\kappa_3$ represent the intracell, intercell coupling, and the effective in-line LRC, respectively. Its Hamiltonian is expressed as

\[
\hat{H}_1 = (\hat{H}_0 + \hat{H}_{\text{LOC}}) \otimes \sigma_z\]

(1)
where $\hat{H}_0$ is the Hamiltonian of conventional Kagome model, and $\hat{H}_{\text{IOC}}$ represents the interorbital coupling in the Kagome lattice. And the interorbital coupling $\hat{H}_{\text{IOC}}$ is expressed as $\hat{H}_{\text{IOC}} = 2\kappa_3 \omega_f \star \text{diag} \left\{ 2\cos \left( \frac{d}{2} k_x \right) \cos \left( \frac{\sqrt{3}}{2} d k_y \right), \cos (d k_x) + \cos \left( \frac{d}{2} k_x + \frac{\sqrt{3} d}{2} k_y \right), \cos (d k_x) + \cos \left( \frac{d}{2} k_x + \frac{\sqrt{3} d}{2} k_y \right) \right\} - \Delta \omega$. The Hamiltonian can reduce to the conventional Kagome model with parity multiplexing $\hat{H}_0 \otimes \sigma_x$ by switching off the interorbital coupling $\hat{H}_{\text{IOC}} (\kappa_3 = 0, \Delta \omega = 0)$ first. The reduced Hamiltonian gives rise to six bands (See Supplementary Information IV). When $|\kappa_1| > |\kappa_2|$, the parity-dependent bands are topologically nontrivial, and their bulk topologies can be characterized with parity-dependent Wannier centres $(P_x, P_y)_{e,o}$,

$$P_j = -\frac{1}{S} \int_{\text{BZ}} A_j d^2 k$$

where $j = x, y$, $A_j = -i (\langle u | \partial k_j | u \rangle)$ is the Berry connection of the bands below the gap. $u$ represents the periodic part of Bloch functions, and $S$ is the area of the first Brillouin zone. When $\kappa_1/\kappa_2 > 2$, the Wannier centre lies at $(0,0)$ and implies a topologically trivial system. When $0 < \kappa_1/\kappa_2 < 1/2$, the Wannier centre lies at $\left( -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right)$, and implies a topologically nontrivial system. Taking $\kappa_{e,o} = +0.0098$, and $\kappa_{e,o} = +0.0215$ extracted from simulation, the calculated eigen spectra of a triangular-shaped lattice are shown in Fig. 3b, where two copies of parity-dependent SOTCS degenerately emerge at $\omega_f$.

We examine the effect of the interorbital coupling $\hat{H}_{\text{IOC}}$ on the SOTCS, by taking nonzero effective in-line LRC ($\kappa_{e,o} = +0.0029$) and EOD ($\Delta \omega_{e,o} / \omega_f = +0.0029$) in the Hamiltonian. Since both $|\kappa_3|$ and $|\Delta \omega / \omega_f|$ are much smaller than the min$\{ |\kappa_1|, |\kappa_2| \}$, the bulk bands are slightly altered but without closing the parity-dependent gaps (See Supplementary Information V). As such, the parity-dependent bulk topologies do not
change. Regarding the triangular-shaped lattice, the SOTCS show parity splitting (in Fig. 3c), i.e., the even- and odd-SOTCS at $\omega_{\text{corner},e} = 6.110$ GHz and $\omega_{\text{corner},o} = 6.028$ GHz, respectively. This parity-dependent frequency shifting of SOTCS is fundamentally different from that induced by changing metaatoms$^{29}$ and has not been observed in previous demonstrations$^{8,30,32-36,37}$. The off-line LRC, in the same order as $\kappa_3$, imposes no effect on the parity-dependent SOTCS (See Supplementary Information V) but would give rise to type-II corner states$^{24}$, if strong enough.

The parity splitting of SOTCS is further experimentally demonstrated. The local densities of states are detected at the A & B points of the corner resonator, respectively (see Methods and Supplementary Information VIII). The spectra show dominated peaks at 6.025GHz (blue) and 6.134GHz (red) for Point B and A, respectively. Near-field imaging results further confirm the parity-dependent topological corner states. In Fig. 3e, the corner resonator shows brighter odd $f$-MAO at 6.025GHz than those on other resonators. Some lattices at the edges show bright $f$-MAOs, but they belong to the even parity. This is consistent with the theoretical results in Fig 3c, where odd-parity corner states are close to the even-parity edge states in spectra. The even-parity SOTCS at 6.134GHz is also shown in Fig. 3f. These SOTCS are robust to structural disorders as shown in Supplementary Information VI.

Besides the near-field observations, the parity-dependent SOTCS can also be probed with far-field polarized illuminations. The fundamental reason is that the designer plasmonic metasurface exhibits radiative non-Hermicity, which imposes no effect on the bulk topologies, but opens radiation channels to observe the far-field
response of SOTCS. The radiative non-Hermicity can be described as $\hat{H}_{\text{NH}} \otimes \sigma_0$ for the breathing Kagome lattice, where $\hat{H}_{\text{NH}} = \begin{bmatrix} i\gamma_f & 0 & 0 \\ 0 & i\gamma_f & 0 \\ 0 & 0 & i\gamma_f \end{bmatrix}$, and $\sigma_0$ represents the identity matrix. Since a diagonal and uniform Hamiltonian cannot change the bulk topologies, the SOTCS still exist even with the radiative non-Hermicity. On the other hand, the radiative non-Hermicity links the near-field MAO parities to far-field polarizations (details in Supplementary Information IX). This unique feature of the SOTCS is experimentally verified through the setup in Fig. 4a (details in Supplementary Information IX). $E_x$ ($E_y$) polarized plane waves are normally incident onto the metasurface, and a near-field probe is placed vertically at Point A (B) of the corner resonator to detect $E_z$ component. The detected spectrum at A (B) shows a resonance peak at 6.1 (6.0) GHz, and implies the excitation of even (odd) SOTCS. For comparison, the background shows the vanished $E_z$ component, when the metasurface is removed (see Fig. 4b). The near-field imaging result shown in Fig. 4c further verifies the even (odd) SOTCS at 6.1 (6.0) GHz. These frequencies deviate a little from the near-field results, due to different supports for the metasurfaces. This polarized illumination selection on parity-dependent SOTCS is consistent with both theoretical and near-field experimental results.

In conclusion, we experimentally observed parity splitting and realized polarization selection of parity-dependent SOTCS on a designer-plasmonic metasurface. In contrast to pseudospin DoF relying on $C_6$ or $C_3$ symmetries\textsuperscript{35,36}, the MAO parity is an intrinsic DoF and does not depend on any crystalline symmetries.
Hence, intriguing topological physics could be anticipated by incorporating MAO parities into other lattices. Moreover, the parity DoF could be manipulated by incorporating interorbital couplings, originated from spectral overlapping between neighbouring MAOs. If interfacing with gain media, our results may promise applications in polarization-dependent topological lasing\textsuperscript{42,43} and quantum emitters\textsuperscript{44}.

Methods

**Calculation and simulation.** The transmission spectrum of a single metaatom is fitted with the Lorentz formula

\[
\text{Transmission} = \left| \frac{A_d}{\omega - \omega_d + i \gamma_d} + \frac{A_f}{\omega - \omega_f + i \gamma_f} + \frac{A_g}{\omega - \omega_g + i \gamma_g} \right|,
\]

where \(d, f, g\) are short for \(d-, f-, g\)-MAOs respectively, and \(\omega, \gamma, A\) denote centre frequencies, losses and normalized amplitudes of \(i\)-MAO. The numerical simulations on the plasmonic dimer and trimer systems are conducted with CST Microwave Studio. The band structures and the eigen spectra of the Kagome metasurface are obtained with the coupled mode theory.

**Sample fabrication.** The metasurface is fabricated with standard printed circuit board technology. The designer plasmonic resonators consist of 0.018-mm-thick copper standing on a 0.254 mm-thick RT/duriod 5880 laminate with relative permittivity of 2.2+0.0009i.

**Experimental setup.** All the measurements are carried out with a Keysight E5071 Vector Network Analyzer in a microwave absorber chamber. A 2-mm-long monopole is mounted on an automatic near-field scanning system to capture the field distributions. The \(E_z\) components of near-field profiles are detected by suspending the monopole vertically 1 mm above the sample. In far-field experiments, a high-gain reflector antenna HD-10180RA1000NZJ is employed to generate collimated linearly polarized beams.

Acknowledgments

The work at Zhejiang University was sponsored by the National Natural Science Foundation of China (NNSFC) under Grants No. 62171406, No. 61801426, No.11961141010, the ZJNSF under Grant No. Z20F010018, the National Key Laboratory Foundation No. 614220520040, and the Fundamental Research Funds for the Central Universities No. 2020XZZX002-15. S. X., Q.-D. C., H.-B. S., Q. C., and H. S. acknowledge NSFC grant nos. 61805097, 61935015, 61590930, and 61825502; and the National Key R&D Program of China grant 2017YFB1104300.

Author contributions

Y. L., H. X., X. X. and Y. Y. performed theoretical analysis. Y. L., S. X., Z. Z. and W. Y. performed the measurements. Y. L., and Z. W. conducted simulation. All authors analysed data and wrote the manuscript. F. G. and H. C. supervised the project.
Data Availability

The data that support the plots within this paper are available from the corresponding authors upon reasonable request. Source data of Fig. 3d and Fig. 4b are provided with this paper.

Code Availability

We use the commercial software COMSOL MULTIPHYSICS and CST Microwave Studio to perform the electromagnetic simulations. Request for computation details can be addressed to the corresponding authors.
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Figure 1 | A Breathing Kagome metasurface consisting of designer-plasmonic metaatoms. a. The illustration of designer-plasmonic Kagome metasurface, with $d_1 = 29$ mm, and $d_2 = 26$ mm. The inset shows the photograph of a single metaatom, where the inner and outer radius are $r = 3$mm, $R = 12$mm respectively. The groove number is $N = 60$, and the filling ratio is $FR = 50\%$. b. The near-field transmission spectrum of a single metaatom. The points ‘S’ and ‘P’ denote the locations of source and probe, respectively. The circle and dashed lines are simulated and fitted results, respectively. The red, green, and blue denote quadrupole, hexapole, and octupole modes, respectively. c. The correspondence between resonance modes in (b) and meta-atomic orbitals (MAOs). The red, green, and blue colours denote $d$, $f$, and $g$-MAOs, respectively. The inset shows the field profiles of even- and odd-parity $f$-MAOs. d. The far-field radiation pattern of an individual metaatom excited by a near-field monopole. $S_i$ and $S_o$ represent the in-plane and out-of-plane radiation of the $f$-MAO, respectively.
**Figure 2 | The MAO parity-dependent couplings.**

**a.** The transmission spectra of a plasmonic meta-dimer with a centre-to-centre distance of 30 mm. The red and blue lines are measured with the end and side excitations, respectively. The red ($\omega_e$) and blue ($\omega_o$) dashed lines indicate the average frequencies of two split resonance peaks. The black dashed line represents the frequency $\omega_f$ of $f$-MAO. The inset shows the schematic of the meta-dimer. 

**b.** The illustration of parity-dependent couplings in the meta-dimer. The even and odd parities exhibit negative and positive couplings, respectively. 

**c.** The transmission spectra of a plasmonic meta-trimer. The inset shows the schematic of the meta-trimer. 

**d.** The field patterns for odd and even parities at $\omega_{2,o}$ and $\omega_{2,e}$, respectively. The patterns on the middle metaatom are the superpositions of $d$- and $g$-MAOs. 

**e.** The illustration of the effective LRC induced by the interorbital coupling. a. u., arbitrary units.
Figure 3 | The mechanism of parity splitting and near-field imaging of parity-dependent SOTCS. 

- a. The abstract model for the plasmonic Kagome metasurface.
- b-c. The calculated eigen spectrums of different models with $\kappa_3 = 0$ (b), and $\kappa_3 = 0.0029$ (c), respectively. The red and blue colours represent even and odd parity, respectively. The solid, semi-transparent solid and empty circles represent the corner, edge and bulk states, respectively.
- d. The experimentally measured local density of states at points A and B in the inset of (a), respectively.
- e-f. The experimentally captured near-field patterns of SOTCS. The odd-parity SOTCS at 6.025 GHz is shown in (e), and even at 6.134 GHz in (f).
Figure 4 | The far-field demonstration of polarization-selection on parity-dependent SOTCS. a. The schematic of far-field experiment setup. The HOTI metasurface is normally illuminated with linear-polarized collimated beams. b. The measured spectra at the top corner resonator in (a). The blue and red spectra are detected at points A and B in (a), respectively. The grey line represents the background spectrum without the HOTI. c-d. Patterns of SOTCS on the top corner resonator. (c) shows odd-parity SOTCS excited by $y$-polarized incidence, and (d) for the even parity by $x$-polarized incidence.