Einstein-Brans-Dicke Cosmology

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Abstract

We studied the Einstein-Brans-Dicke cosmology in detail. The difference of the evolution of the universe is significant between Einstein-Brans-Dicke cosmology and standard big-bang model during the radiation-dominated era. The power-law evolution of the scale factor is fast enough to solve the cosmological puzzles and slow enough to avoid the graceful exit problem. However, the constraints from the satisfactory bubble distribution ($\beta^2 > 0.25$) and the solar system observations ($\beta^2 < 0.002$) are mutually exclusive. This suggests that this kind of inflationary model is ruled out. We also clarify the distinction between Einstein frame and Jordan frame in Brans-Dicke theory.

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In Brans-Dicke (BD) gravity, the effective gravitational constant $G_{\text{eff}} = \phi^{-2}$, varies as the BD field $\phi$ evolves. The evolution of the scalar field slows down the expansion rate of the universe during inflation, and allows nucleation of bubbles to end the inflationary era. But it was soon found that the bubbles could lead to unacceptable distortions of the microwave background [1]. A large number of inflationary models were proposed in the framework of multi-scalar tensor gravity to solve the problem [2]-[7]. For instance, the introduction of a potential for the scalar field $\phi$ and a scalar field dependent coupling constant $\omega(\phi)$ solved some problems. In [4], D. La considered the BD cosmology in Einstein frame, but he didn’t analyze the constraints from cosmological models in Einstein frame [4]. Instead he used the constraints from the original Jordan-Brans-Dicke (JBD) inflation. In the literature, some people considered the inflationary models in Einstein frame in order to solve equations easily, but they analyzed their final results in Jordan frame because most people insist that the Jordan frame is the physical frame to keep the equivalence principle. In fact, the equivalence principle can be kept in Einstein frame if we use Einstein frame as the physical frame. Sometimes people just mixed up Jordan frame and Einstein frame. As showed by Cho and Damour etc., Pauli metric represented the spin-two massless graviton [3][4]. They also showed that the two frames were not conformal invariant for the case used in inflationary models. For arguments in favor of Einstein frame as the physical frame, see [8][9]. In Kaluza-Klein unification, one must identify the physical 4-dimensional metric as Pauli metric $g_{\mu\nu}$ in order to describe Einstein gravity [11]-[13]. Apart from the higher dimensional Kaluza-Klein theory, the Einstein-Brans-Dicke (EBD) like theory may also derived from the induced gravity and $R^2$ gravitational theories. In this paper, we choose Einstein frame as the physical one and refer the cosmology based on the EBD gravity as EBD cosmology. We analyze the detailed evolutions of the universe during the radiation-dominated (RD), matter-dominated (MD) and the inflationary epochs. The physical differences from these two frames were discussed in [3][8][11][14]. The distinctive features of the EBD gravity are: (1) a massive test particle deviates from geodesic motion, and a photon follows geodesic motion. Cho and Magnano and Sokolowski derived that a photon remained geodesic motion by using the equation of motion for the BD scalar field [10][12]. The derivation is wrong because the use of the equation of motion for the BD scalar field means that the test particle is the source of spin-0 gravitational field. In fact, we should consider the motion of a test particle in some known background, not the interaction between the test particle and the background space-time. The geodesic motion of a photon is the consequence of $ds^2 = 0$ and the conformal invariance of $ds^2 = 0$. In [15], we got the wrong results about the deflection angle and the time delay of radar echo. They should be different from those in general relativity by a factor $1 - \beta^2/2$. Therefore, the constraint on $\beta^2$ from both experiments is $\beta^2 \leq 0.002$. (2) The coupling constant $\omega$ in Brans-Dicke theory must be positive, but in EBD theory, Pauli metric can be defined even when $\omega$ is negative but larger than $-3/2$. (3) the dilaton $\sigma$ does appear in the matter Lagrangian. (4) the dilaton field no longer plays the role of time-varying gravitational coupling constant.

\footnote{Einstein frame is also called Pauli frame in the literature}
The JBD Lagrangian is given by
\[ \mathcal{L}_{BD} = -\sqrt{-\gamma} \left[ \phi \tilde{\mathcal{R}} + \omega \gamma^{\mu\nu} \frac{\partial_\mu \phi \partial_\nu \phi}{\phi} \right] - \mathcal{L}_m(\psi, \gamma_{\mu\nu}). \] (1)

The above Lagrangian (1) is conformal invariant under the conformal transformations,
\[ g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}, \quad \Omega = \phi^\lambda, \quad (\lambda \neq \frac{1}{2}), \quad \sigma = \phi^{1-2\lambda}, \quad \bar{\omega} = \frac{\omega - 6\lambda(\lambda - 1)}{(2\lambda - 1)^2}. \]

For the case \( \lambda = 1/2 \), we make the following transformations
\[ g_{\mu\nu} = e^{a\sigma} \gamma_{\mu\nu}, \quad (2a) \]
\[ \phi = \frac{1}{2\kappa^2} e^{a\sigma}, \quad (2b) \]
where \( \kappa^2 = 8\pi G, \quad a = \beta \kappa, \) and \( \beta^2 = 2/(2\omega + 3) \). Remember that the JBD Lagrangian is not invariant under the above transformations (2a) and (2b). After the conformal transformations (2a) and (2b), we get the EBD Lagrangian
\[ \mathcal{L} = \sqrt{-g} \left[ -\frac{1}{2\kappa^2} \tilde{\mathcal{R}} - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right] - \mathcal{L}_m(\psi, e^{-a\sigma} g_{\mu\nu}). \] (3)

In this frame, \( h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \) represents the spin-2 massless graviton. That’s one of the reasons why we use the Einstein frame as the physical frame. In general, we may use the matter Lagrangian \( \mathcal{L}_m(\psi, g_{\mu\nu}) \) to keep the equivalence principle if we identify the metric \( g_{\mu\nu} \) as the physical one. However, we will lose the interactions between the dilaton field \( \sigma \) and the matter fields \( \psi \). In order to avoid this and see the differences between the two frames, I use the Lagrangian (3) as the basis of EBD cosmology. In this paper, I consider the simplest case, i.e., a constant coupling constant without potential for the dilaton field \( \sigma \). The generalization to a more complicated multi-scalar tensor gravity in Jordan frame can be found in (5). For the cosmological models in the context of general scalar-tensor theory in Jordan frame, see (6).

Based upon the homogeneous and isotropic Friedman-Robertson-Walker spacetime
\[ ds^2 = -dt^2 + R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right], \] (4)
and the perfect fluid \( T^\mu_\nu_m = e^{-2a\sigma}[(\rho + p) U^\mu U^\nu + p g^{\mu\nu}] \) as the matter source, we can get the evolution equations of the universe from the action (3)
\[ H^2 + \frac{k}{R^2} = \frac{\kappa^2}{3} \left( \frac{1}{2} \dot{\sigma}^2 + e^{-2a\sigma} \rho \right), \]
\[ \ddot{\sigma} + 3H \dot{\sigma} = \frac{1}{2} a e^{-2a\sigma} (\rho - 3p), \] (6)
where \( \rho \) is the mass-energy density and \( p \) is the pressure. The motion of the matter field satisfying the covariant conservation law
\[ \nabla_\nu [T^\nu_{\sigma} + T^\nu_m] = 0 \]
with

\[ \nabla_\nu T^\mu_\sigma = g^\mu_\nu \partial_\nu \sigma \Box \sigma, \]

becomes

\[ \dot{\rho} + 3H(\rho + p) = \frac{3}{2}a\dot{\sigma}(\rho + p). \] (7)

If we are given a state equation for the matter \( p = \gamma \rho \), then the above equation gives us a first integral,

\[ \rho R^{3(\gamma + 1)} e^{-3a(\gamma + 1)\sigma/2} = C_2. \] (8)

Combining Eqs. (5), (6) and (7), we get another first integral for the flat universe \( k = 0 \),

\[ Re^{-a(1-\gamma)\sigma/\beta^2(1-3\gamma)} = C, \] (9)

where the above equation is valid for \(-1 \leq \gamma < 1 - 2/(3 + \sqrt{6}/\beta) \) and \( \gamma \neq 1/3 \).

From Eq. (5), we have

\[ k = \frac{8\pi G}{3H^2} \left( \frac{1}{2} \dot{\sigma}^2 + e^{-2a\sigma} \rho \right) - 1 \equiv \Omega - 1, \] (10)

where \( \Omega \equiv 8\pi G(\frac{1}{2} \dot{\sigma}^2 + e^{-2a\sigma} \rho)/3H^2 = (\frac{1}{2} \dot{\sigma}^2 + e^{-2a\sigma} \rho)/\rho_c \). From the above expressions, we see how the \( \sigma \) field contributes to the matter source. The EBD cosmology was discussed in [17], here we give more detailed analyses and make some corrections.

## 1 Matter-Dominated Epoch

In MD epoch, we have the state equation \( p = 0 \). The solutions to Eqs. (3)-(7) for flat universe \( k = 0 \) are

\[ \rho(t) = \rho_p \left[ 1 + \frac{6 + \beta^2}{4} H_p(t - t_p) \right]^{-6(2-\beta^2)/(6+\beta^2)}, \] (11)

\[ R(t) = R_p \left[ 1 + \frac{6 + \beta^2}{4} H_p(t - t_p) \right]^{4/(6+\beta^2)}, \] (12)

\[ e^a\sigma = (16\pi) \left[ 1 + \frac{6 + \beta^2}{4} H_p(t - t_p) \right]^{4\beta^2/(6+\beta^2)}, \] (13)

\[ H(t) = \frac{H_p}{1 + \frac{6 + \beta^2}{4} H_p(t - t_p)}. \] (14)

where \( R_p, H_p \) and \( t_p \) are the present radius of the universe, the present Hubble constant and the present age of the universe, respectively, and \( e^a\sigma_p = 16\pi \). If we let \( t_p = 4H_p^{-1}/(6 + \beta^2) \), then we have

\[ \rho(t) = \rho_p \left( \frac{t}{t_p} \right)^{-6(2-\beta^2)/(6+\beta^2)} \], \( R(t) = R_p \left( \frac{t}{t_p} \right)^{4/(6+\beta^2)} \), \( e^a\sigma = (16\pi) \left( \frac{t}{t_p} \right)^{4\beta^2/(6+\beta^2)} \). (15)
It is obvious that the present age of the universe given by EBD theory is a little less than that given by the standard big-bang model. Because the smallness of the value of $\beta^2$ determined by the present experiments, we obtain the approximate solutions

$$\rho \approx \rho_p \left[1 + \frac{3}{2} H_p (t - t_p) \right]^{-2} = \rho_p \left( \frac{t}{t_p} \right)^{-2},$$  \hspace{1cm} (16)

$$R \approx R_p \left[1 + \frac{3}{2} H_p (t - t_p) \right]^{2/3} = R_p \left( \frac{t}{t_p} \right)^{2/3},$$  \hspace{1cm} (17)

$$\sigma \approx \ln \left( \frac{16\pi}{\beta \kappa} \right) + \frac{2\beta}{3\kappa} \ln \left[1 + \frac{3}{2} H_p (t - t_p) \right] \approx \frac{\ln (16\pi)}{a}.$$  \hspace{1cm} (18)

From the above results, we see that the contribution of dilaton field is negligible and the evolution of the universe is almost indistinguishable from the usual hot-big-bang model based on Einstein gravity during MD epoch.

### 2 Radiation-Dominated Epoch

For the RD epoch, the equation of state for radiation is $\rho = 3p$. After using the equation of state, we get solutions to Eqs. (3) and (7) for $k = 0$

$$R(t)^3 \sigma(t) = C_1,$$  \hspace{1cm} (19)

$$\rho(t) R^4(t) e^{-2a\sigma} = C_2,$$  \hspace{1cm} (20)

where $C_1$ and $C_2 > 0$ are the integration constants. $C_1$ can be positive, negative or zero determined by the initial condition of the universe. If we choose $C_1 = 0$, then we find that $\sigma$ is a constant, so the solutions are the same as those of the standard big-bang model during RD era. Combining Eqs. (19), (20) and (4), we get

$$\left( \frac{\dot{R}}{R} \right)^2 = \frac{\kappa^2}{3} \left( \frac{C_1^2}{2 R^6} + \frac{C_2}{R^4} \right).$$  \hspace{1cm} (21)

It is easy to solve the above equation if we use the cosmic time defined by $dt = R(\eta) d\eta$. The solutions to the above equations (19)-(21) in terms of the cosmic time $\eta$ are

$$R(\eta) = \left[ \frac{C_2}{3\kappa^2 \eta^2} + \frac{2}{\sqrt{6} |C_1| \kappa \eta} \right]^{1/2},$$  \hspace{1cm} (22)

$$\sigma = \sigma_0 \pm \frac{3}{\sqrt{6} \kappa} \ln \frac{\eta + \sqrt{6|C_1|}}{C_2 \kappa},$$  \hspace{1cm} (23)

$$\rho(\eta) = \frac{C_2 e^{2a\sigma}}{R^4(\eta)},$$  \hspace{1cm} (24)
\[ t = \frac{1}{2C_2} \sqrt{\frac{3}{C_2 \kappa^2}} \left[ \left( \frac{1}{\sqrt{6}} C_2 \kappa \eta + \frac{|C_1|}{2} \right) \sqrt{\frac{2}{3} C_2^2 \kappa^2 \eta^2 + \frac{4}{\sqrt{6}} |C_1| C_2 \kappa \eta} \right. \]

\[ - \frac{C_1^2}{2} \ln \left( \frac{2}{\sqrt{6}} C_2 \kappa \eta + |C_1| + \sqrt{\frac{2}{3} C_2^2 \kappa^2 \eta^2 + \frac{4}{\sqrt{6}} |C_1| C_2 \kappa \eta} \right) + \frac{C_1^2}{4C_2} \sqrt{\frac{3}{C_2 \kappa^2}} \ln |C_1|, \]

\[ \text{(25)} \]

\[ R(t) \sqrt{R^2(t) + \alpha^2 - \alpha^2 \ln(R(t) + \sqrt{R^2(t) + \alpha^2}) + \frac{\alpha^2}{2} \ln \alpha^2 = 2 \sqrt{\frac{C_2}{3} \kappa} t = 2 \bar{t}} \quad \text{(26)} \]

where \( \bar{t} = \sqrt{C_2 \kappa t/\sqrt{3}} \), \( \alpha^2 = C_1^2/(2C_2) \), \( \sigma_0 \) is another integration constant determined by the initial condition of the dilaton field \( \sigma \) and the sign in the equation (23) is the same as the sign of \( C_1 \). From Eq. (26), it is clear that (see Fig. 1)

\[ 2 \bar{t} \approx \begin{cases} 
\frac{R^3(t)}{2\alpha} & \text{if } R(t) \ll \alpha, \\
R^2(t) & \text{if } R(t) \gg \alpha.
\end{cases} \]

In the early times, (here we suppose \( C_1 \) is not very large so that \( R(t) \) will exceed \( \alpha \)) in a very short time, the universe in EBD cosmology evolves much faster than that in standard cosmology. After some time (\( R(t) \gg \alpha \)), the solution will evolve to the solution with \( C_1 = 0 \) asymptotically, i.e., we have the same evolution as that given by the standard model. From Eq. (21) and (26), we can get the proper distance to the horizon measured at time \( t \)

\[ d_H(R) = R(t) \int_0^t \frac{dt'}{R(t')} = \frac{\sqrt{3}}{\sqrt{C_2 \kappa}} R(\sqrt{R^2 + \alpha^2} - \alpha). \quad \text{(27)} \]

Let \( \bar{d}_H(R) = \sqrt{C_2 \kappa} d_H(R)/\sqrt{3} \), we have \( \bar{d}_H(R) \approx R^3/2\alpha \ll R^2 \) if \( R \ll \alpha \). Therefore, the horizon distance in the early times is much smaller than that in standard model, we will need more e-foldings to solve the horizon problem. If \( R \gg \alpha \), \( \bar{d}_H(R) \approx R^2 \) (see Fig. 2). From the above analyses, we know that during the early times, the expansion of the universe is faster than that in standard big bang model. This means that at the same temperature or same size, the EBD universe is younger.
Figure 2: The horizon $d_H$ as the function of the scale factor $R$. Curve 1 refers to $\alpha = 10$, curve 2 refers to $\alpha = 0$ and the third curve in the left figure is $R^3/2\alpha$ with $\alpha = 10$.

3 Inflationary Epoch

As the universe enters the inflationary epoch, the energy density approaches the false-vacuum energy density $\rho_f = -p_f = \text{const}$. The solutions to Eqs. (5)-(7) for flat universe are

$$e^{\alpha \sigma} = 2\kappa^2 \tilde{m}_{Pl}^2 (1 + 2\beta^2 H_B t),$$

or

$$\sigma = \sigma_B + \frac{1}{\beta \kappa} \ln (1 + 2\beta^2 H_B t),$$

$$R(t) = R(B)(1 + 2\beta^2 H_B t)^{\frac{1}{2\beta^2}},$$

where $H_B = \sqrt{\rho_f/(2\kappa \tilde{m}_{Pl}^2 \sqrt{3 - 2\beta^2})}$ is the Hubble parameter at the beginning of inflation, $t = 0$ (here I set the beginning of the inflation to be time scale zero), $\tilde{m}_{Pl}$ (greater than the scale of phase transition) is an arbitrary integration constant corresponding to the effective Planck mass at the beginning of inflation, and $\sigma_B = \ln (2\kappa^2 \tilde{m}_{Pl}^2)/a$ is the value of dilaton field at the beginning of inflation. Because the variation of the dilaton field $\sigma$ is very small during the MD era and the late times of the RD epoch, we may suppose that at the end of inflation, the dilaton field becomes $\sigma(t_e) \approx \ln(16\pi)/a$. If $C_1$ is large enough that the universe will stay at the range $R(t) < \alpha$ during most of the times of RD era, then the above assumption is not true.

For short times, $t < t_c \equiv 1/2\beta^2 H_B$, $R(t) \sim \exp (H_B t)$, which is the Einstein-de-Sitter inflation. However, for $t > t_c$, $R(t)$ crosses over to power-law expansion, $R(t) \sim (t/t_c)^{2/\beta^2}$. The cross-over from exponential to power-law expansion changes the rate at which bubble nucleation converts the universe from false- to true-vacuum phase. The phase transition will be completed when the nucleation rate per Hubble volume per Hubble time

$$\epsilon(t) = \frac{\lambda_0}{H(t)^4} \sim \lambda_0 (2\beta^2)^4 t^4 \sim 1.$$
Let us suppose that the phase transition ends at time (here we follow Weinberg’s method \[1\])

\[ t_e = q \lambda_0^{-1/4} (2\beta^2)^{-1}, \] (31)

where \( q \) is an order of unity constant and \( \lambda_0 \) is the nucleation rate per unit volume per unit time. The consistency condition by the dilaton field at the end of inflation gives

\[ \sigma(t_e) = \sigma_B + \frac{1}{a} \ln(1 + 2\beta^2 H_B t_e) \approx \frac{1}{a} \ln(16\pi), \] (32)

\[ 2\beta^2 H_B t_e \approx \frac{1}{G \bar{m}_{Pl}^2} \gg 1. \] (33)

Combining Eqs. (31), (33) and the definition of \( H_B \), we get

\[ \lambda_0 = \left( \frac{1}{96\pi} \right)^2 q^4 \rho_f^2 G^2. \] (34)

During the phase transition, the scale factor will increase by a factor

\[ \frac{R(t_e)}{R(0)} = (1 + 2\beta^2 H_B t_e)^{1/2}\beta^2 \approx (2\beta^2 H_B t_e)^{1/2}\beta^2. \] (35)

The requirement that the scale factor increases at least by a factor of 65-e foldings gives us the constraint

\[ \beta^2 < \frac{1}{65} \ln \left( \frac{T_{Pl}}{T_c} \right) = 0.14, \] (36)

where \( T_{Pl} = 10^{19} \) Gev is the Planck energy and \( T_c = 10^{15} \) Gev is the energy scale for GUT phase transition. The probability of a point remaining in the false-vacuum phase during a bubble nucleation process beginning at time \( t_B \) is

\[ p(t) = \exp \left[ - \int_{t_B}^{t} dt' \lambda(t') R^3(t') \frac{4\pi}{3} \left[ \int_{t'}^{t} \frac{dt''}{R(t'')} \right]^3 \right], \] (37)

where \( \lambda(t) \) is the nucleation rate per unit time per unit volume, approximately constant (\( \sim \lambda_0 \)) during the inflationary phase. Combining Eqs. (29), (37) and (34), we get

\[ p(t) = \exp \left[ - \frac{\pi}{3} \delta \left( y^4 g(\beta) - 1 + O((1 + 2\beta^2 H_B t_e)^{-1/2}\beta^2) \right) \right], \] (38)

where \( y = 1 + 2\beta^2 H_B t, \delta = (q/(2\beta^2 H_B t_e))^4 \), and

\[ g(\beta) = 1 - \frac{24\beta^2}{6\beta^2 + 1} + \frac{12\beta^2}{1 + \beta^2} - \frac{8\beta^2}{3 + 2\beta^2}. \]
If $2\beta^2/H_B t$ is large and $\beta$ is small, we have

$$p(t) \approx \exp \left[ -\frac{\pi}{3} \frac{q^4}{2\beta^2} \left( \frac{t}{t_c} \right)^4 \right].$$

From the bounds on the anisotropy of the microwave background, we will get a constraint if we require that no more than $10^{-5}$ of space was still undergoing thermalization at the recombination $T \approx 4000$ K,

$$\beta^2 > \frac{1}{2 + \frac{2}{5} \log_{10}(T_c/T)} \approx 0.025 \quad \text{or} \quad \omega < \frac{1}{2} + \frac{8}{5} \log_{10}(T_c/T).$$

Remember that the solar system observation requires $\beta^2 < 0.002$, so the EBD inflation can’t avoid the big-bubble distribution problem either.

It is true that we can get the Eqs. (39)-(41) and all the solutions in this paper from the corresponding equations and solutions in the original JBD cosmology by the transformations

$$dt = e^{\sigma/2} d\tilde{t}, \quad R(t) = e^{\sigma/2} \tilde{R}[\tilde{t}(t)],$$

and the transformation (2b), where $\tilde{R}(\tilde{t})$ is the scale factor in JBD cosmology. That is, our solutions in terms of $t$ can be derived from the solutions in terms of $\tilde{t}$ in original JBD cosmology by the above transformations. This is easily understood. Note that in this paper we use the Robertson-Walker metric (4) and our Lagrangian is related to the JBD Lagrangian by the transformations (2a) and (2b). Under the transformations (2a) and (2b), the Robertson-Walker metric becomes

$$ds^2 = e^{\sigma} ds^2 = e^{\sigma} \left\{ -dt^2 + \tilde{R}^2(\tilde{t}) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right] \right\}.$$ 

Therefore, if we make the transformations (2b) and (2c), we can get the solutions to EBD cosmology from the corresponding solutions to the JBD cosmology. So the two cosmological solutions are related by the transformations (2b) and (2c). Although we have the relationships (2b) and (2c) between the two cosmological models, the actual physical contents are different if we use different identifications of the physical metrics. This point is obvious from the results in this paper. The comoving coordinates are $(\tilde{t}, r, \theta, \phi)$ in JBD cosmology and the comoving coordinates are $(t, r, \theta, \phi)$ in EBD cosmology. Note that the coordinate symmetry is broken in cosmology. As showed in this paper, the problem arising from bubble distribution is also unavoidable in EBD inflation.

In EBD cosmology, I find that the expansion of the universe is faster than that given by standard cosmology during the early times of RD era. This has an important dynamical effect upon the early universe. The total entropy in EBD cosmology defined as $S = e^{-2\sigma} (\rho + p) R^3 / T$ is conserved. Since we have $e^{-2\sigma} \rho \sim T^4$, so $R(t) \sim T^{-1}(t)$. The faster expansion makes $T(t)$ decrease, i.e., a given temperature will occur at an earlier epoch.

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