Curvature Method to Detect Location and Depth of a Plastic Zone in Frame Members during an Earthquake

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Abstract:

This paper presents a new method for detecting beam and frame damage caused by an earthquake. Despite comprehensive investigations on structural damage detection, which mostly have focused on damage detection of elastic structures, the present study deals with nonlinear damage detection in structures. Furthermore, in the proposed method, only measurements of the damaged structures during an earthquake are required and the measurement of undamaged structure is redundant. The proposed method is based on the use of curvature of beams in order to detect plastic zones. To evaluate the efficiency of this method, the beams with different boundary conditions and mass distributions as well as a one-bay single-story moment frame were modeled in OpenSees software and exposed to the accelerations of Cape, Chichi, and El Centro earthquakes. The curvature vectors calculated using data from acceleration recording points were utilized to detect the place and severity of the damage. Furthermore, in an attempt to reduce costs of actual damage detection, the number of accelerometers were reduced using the cubic spline method of interpolation. Finally, an experimental study was operated to show the effectiveness of the proposed method to determine the length and depth of plastic zones with reasonable accuracy.

1. Introduction

Damage in a structural system is defined as an undesirable change in its behavioral characteristics (Glisic and Inaudi 2008). Indeed, damage can be considered as the changes in a system which will unfavorably affect the present or future operations. The damage to structural elements is either linear or nonlinear. After the occurrence of linear damage, the behavior of the structure will continue to be linear, whereas, in the case of nonlinear damage, the structure will tend to be nonlinear. Linear damage is usually modeled as a crack before loading, which is normally in the form of an impulse (Wei Fan and Pizhang Qiao 2011). However, nonlinear damage occurs in the form of a plastic zone in the structure.

One of the oldest methods is based on the modal parameters of a structure (Doebling SW et al. 1996, Turner and Pretlove 1998). Modal parameters are frequency and structural mode shape, which produces relatively acceptable results. There are more accurate methods of damage detection which are based on curvature mode shape and modal strain energy (Qiao et al. 2007, Ciambella et al. 2011, and Jeong Kim et al. 2013). For instance, Cao et al. (2014) detected multiple cracks in a cantilever beam using curvature mode shape. However, there are a few drawbacks in this method; first, it requires a relatively large number of accelerometers; so, the cost of experiments increases (Chuanshuang Hu and Afzal 2006). Secondly, the natural excitations from ambient sources have a propensity to only excite the lower frequency modes that are mainly insensitive to local damage. Further, many structures experience a wide-ranging operational environment. Varying operational and environmental conditions can create changes in the modal properties of the structure that can be mistakenly identified as being caused by damage (Sohn, H. et al. 2002).
A commonly used mathematical tool to detect damage-induced changes in the structural response is the wavelet transform. The difference between wavelet transform coefficients before and after damage (Amaravadi et al. 2001, Castro et al. 2006, and Garcia-Hernandez et al. 2007) or the ratio of these coefficients prior to and following damage (Zhu XQ and Law SS 2006) is often used as a criterion of damage. Ovanesova and Suarez (2004) detected cracks in a 2D frame using discrete wavelet transform (DWT). They detected the cracks in the lower fourth of the column and also in the middle section of the beam by computing the deformation of the frame and using DWT. Mario Solis et al. (2013) applied cracks in the form of cuts in some parts of the flange and web section of a simple steel beam. They determined the damage in this beam using the mode shape difference between the damaged and undamaged structure and performing Continuous Wavelet Transform (CWT). Nikos Pnevmatikos et al. (2016) detected damage in a steel frame using the residual wavelet coefficient of the healthy and damaged specimen. In the experimental cases, the damage has been defined as losing two out of three bolts at one of the frame connections.

However, despite comprehensive investigations on structural damage detection, most researchers have focused on damage detection of elastic structures and researches on nonlinear structures are infrequent in the reports. An important point to consider is that the damage caused by an earthquake to a structure is mostly nonlinear. Thus, it is necessary to detect plastic zones after or during an earthquake in order to improve the structure. Jann Yang et al. (2011) proposed a new method to detect time-varying characteristics of a nonlinear structure with hysteresis behavior. Li Qin and Yun Niu (2017) introduced a steel frame model with two adjustable rotational dampers, to simulate the plastic hinge effect as a benchmark model for future nonlinear damage detection studies. Eric Hernandez et al. (2018) proposed a methodology for element-by-element estimation of demand-to-capacity ratios in instrumented steel moment-resisting frames subject to earthquakes.

The main part of the existing nonlinear damage detection techniques use a nonlinear finite element model updating method which is a new approach that can detect nonlinear structural damage. Eunjong Yu and Chung (2012) presented the application of a finite element model updating method to seismic damage detection of a small-scale reinforced concrete building structure. Song et al. (2012) via the nonlinear model updating method, detected the location and properties of damage in a reinforced concrete shear wall. Also, Asgarieh et al. (2014) obtained the modal characteristics of a frame using the nonlinear finite element model updating method and used these characteristics to detect the structural damage caused by the seismic load. Unfortunately, complicated calculation imposed a limitation on this method, especially when the material properties were not the same in all of the structural elements.

The present research was an attempt to detect the location and scope of plastic zones during an earthquake. This method is based on the use of acceleration records in order to find the curvature of the beam. The curvature vectors were then utilized to detect plastic zones. In addition to different beam and frame structures under various mass and loading conditions, an experimental study was used to verify the effectiveness of the proposed method. Results show that the proposed method is quite effective in determining the length and depth of plastic zones with reasonable accuracy.

2. Crack detection by obtaining the difference between slope vectors

In this study, as the proposed method, curvature and slope vector is calculated from measured accelerations, it is necessary to check the ability of these deformation parameters in locating the damage.

Mario Solis et al. (2013) modeled a simple beam and applied a crack in different sections of the beam as different scenarios. Then, they calculated the differences between the mode shapes of the damaged beam and those of the undamaged beam and obtained the CWT coefficients of these differences. Subsequently, they detected the crack by obtaining the mode shapes of the beam and performing CWT.

Following Mario Solis et al. (2013), the authors of the present study modeled a simple steel beam with a length of 1.28 m, a height of 100 mm, a flange width of 50 mm, a flange thickness of 6.8 mm, a web thickness of 4.5 mm, and a weight per unit length of 8.1 kg. A crack was applied to the middle section of the beam with a depth of 30 mm. The mass of the beam was distributed in a uniform manner among 64 points along the beam at 20 mm intervals. The beam was exposed to a net bending and was statically analyzed. After beam loading, displacement vectors and their slope vectors throughout the beam were computed. Then, the difference between slope vectors \(y'_d\) and \(y'_ad\), in the damaged and undamaged state of the structure, respectively, was calculated using Eq. (1) below.

\[
[S'] = [y'_d - y'_ad]
\]

It should be noted that we need the derivative of parameters because the only available data in the proposed method, is acceleration measurement.

A CWT of \([S']\) is performed to obtain information about the damage state. Fig. 1 compares the results obtained in the present study (1a) with those reported by Mario Solis et al. (2013) (1b). A sudden increase in CWT coefficients in both figures indicates the crack location.

As presented in Fig. 1a, slope vector can be used for damage detection, and cracks can be detected only by calculating the slope vectors of the beam, indicating that it is unnecessary to compute the mode shapes of the beam. This naturally depends on the accuracy of accelerometers, which may lead to poor precision in numerical computing.

3. Plastic zone detection due to monotonic loading

By using the same beam as in the previous section, plastic zones were detected in this section due to the monotonic loading. The location and scope of plastic zones were affected by load patterns. More specifically, the load applied to the beam had either a uniform or concentrated pattern, while the gravity load pattern had only a uniform distribution.
As mentioned before, the derivative of parameters is essential because the only available data in the proposed method is acceleration measurement. For damage detection, the difference between curvature vectors of the damaged and undamaged beam \( S^d \) is considered as the damage index and is calculated using Eq. (2).

\[
\{S^d\} = \{y''_d\} - \{y''_{ud}\}
\]

(2)

Where \( \{y''_d\} \) and \( \{y''_{ud}\} \) are the curvature vectors of the undamaged and damaged beams, respectively. The CWT coefficients were calculated for \( \{S^d\} \). The mother wavelet used in this study is db2, which is widely used in solving a broad range of problems.

Fig. 2 is the force-displacement diagram of the middle section of the beam subjected to a uniform load pattern. As can be seen, once the middle section of the beam was pushed by 4.3 mm, this section entered the nonlinear zone, and the more the push value increased, the more the nonlinear zone expanded. Fig. 3 shows the CWT coefficients for the \( \{S^d\} \) of a simply-supported beam exposed to monotonic loads under different conditions.

As shown in Fig. 3a, the middle of the beam was pushed by 3 mm under a uniform load. In this case, the beam remained in the elastic region. The damage index was calculated as the difference between curvature vectors of the last two steps of loading. There was no sudden increase in the CWT coefficients of the \( \{S^d\} \) vector because, at both push steps, no part of the beam became plastic as indicated in Fig. 3a.

Fig. 3b shows that when the middle of the beam was pushed by 4.5 mm under the previous load, the CWT coefficients of the \( \{S^d\} \) vector pertinent to this push step suddenly increased as shown by the white lines appearing in the middle of the beam. This means that a plastic zone was created in this region.

It can be seen in Fig. 3c that the \( \{S^d\} \) vector was obtained when the middle of the beam was pushed by 3 mm (the undamaged case), and 4.5 mm (the damaged case) under a concentrated load. The white line seen in the middle of the beam indicates a sudden increase in the CWT coefficients of the \( \{S^d\} \) vector pertinent to this push step. This denotes the creation of a plastic zone. If the push load changes from uniform to a concentrated pattern, a non-derivative point will be created in the diagram for the curvature vector of the beam at the point of load application.

This will cause a sudden increase in the CWT coefficients of the \( \{S^d\} \) vector at just one point and can give the erroneous impression that there is structural damage. The fact that the sudden increase in the CWT coefficient is distributed or concentrated can be used as an exact method for distinguishing a plastic zone from a non-plastic one. Fig. 3c clearly shows that the increase in the wavelet coefficients was not limited to just one point, but it also affected the areas around the middle of the beam.

Finally, as for Fig. 3d and 3e, the push load was applied at two points along the beam: the first one-fourth and the middle.
Fig. 3: The CWT coefficients for the \{S''\} of a simply-supported beam exposed to push loads under different conditions: a) elastic beam / uniform load, b) plastic zone in the middle of the beam / uniform load, c) plastic zone in the middle of the beam / concentrated load, d, e) plastic zone in the middle and quarter of the beam / two concentrated loads.

As can be seen in Fig. 3d, when the middle of the beam was pushed by 2 mm and 7 mm, the CWT coefficients of the \{S''\} vector related to these two push steps underwent a sudden increase and the areas around the middle of the beam were also affected. This indicates the creation of a plastic zone. Furthermore, it is clear in Fig. 3e that when the middle of the beam was pushed by 2 mm and 15 mm, the CWT coefficients of the \{S''\} vector suddenly increased at the interval between the concentrated loads. Before bringing this section of the paper to an end, it can be concluded from the previous discussion that using the curvature vectors of the beam at different push steps is a good way of detecting the plastic zones formed in a beam.

4. Plastic zone detection under earthquake loading

It is worth noting that in reality, the output from accelerometers is usually the only available data in health monitoring. In the previous sections, the curvature and displacement measures were directly obtained from the numerical model of the study only to indicate the efficiency...
and effectiveness of the curvature method. In this section, the vertical acceleration response of the beam obtained from the installed accelerometers will be used for calculating the displacement and curvature vectors.

4.1. Curvature calculation

In the time-history analysis, to calculate the curvature vector pertinent to each time step, it is necessary to calculate the displacement vector of the beam. A displacement vector on the other side can be calculated through double numerical integration of acceleration response with respect to time. Now, the curvature vector of the beam for each time step can be obtained through the double derivation of the displacement response with respect to distance. To eliminate distortions caused by errors in numerical calculations, a curve fitting technique should be employed. For this purpose, the `mslowess` command in MATLAB software was used at each derivation step. Smoothing `mslowess` procedure is usually applied in filtering out the noise for accurate results.

4.2. Reduction of acceleration recording points

The accuracy with which the curvature vectors of the beam are calculated is a function of two key factors: (1) the number of time steps and (2) the number of points along the beam for recording the acceleration vectors. Increasing the number of time steps includes in numerical integration and the number of points to record acceleration along the beam will increase the accuracy. However, for the real measurement conditions, it is not cost-effective to increase the number of accelerometers. In this regard, in the first section of this study, the number of accelerometers is reduced for analytical study. Then, the same method is used for the experimental case.

Through interpolation using the cubic spline method, it was found that the data obtained from 9 points on the beam are not significantly different from the data obtained from 64 points (as used in Mario Solis et al. 2013). Thus, it could be claimed that 9 rather than 64 accelerometers could be sufficient in cost reduction.

4.3. Critical time step

Plastic zones can be formed at an unknown time step, which is introduced here as a critical time step. Given this fact, the situation of the beam at every time step has to be checked for the purpose of determining the critical time step. Since this is obviously impractical, it is necessary to develop a procedure to identify the critical time step. The following procedure is introduced here for this purpose:

1. A matrix denoted as “curvature matrix” is formed, whose rows and columns correspond to the number of time steps and to the number of recording points, respectively. Therefore, the curvature matrix is a matrix whose row \( i \) is the curvature vector corresponding to the \( i \)th time step.

2. The first critical time step could be considered at the time step in which the sum of absolute values of the vector entries is maximum in comparison with the corresponding value calculated for all the other time steps (comparison between rows).

3. The second critical time step could be considered at the time step where the curvature entry with the maximum absolute value in comparison with all the other entries in the curvature matrix belongs to it (comparison between columns).

It is noteworthy to explain that the critical time steps identified by the above-mentioned criteria are often near the time step at which peak ground acceleration (PGA) is applied to the beam. CWT is calculated for both critical time steps and the best CWT diagram was selected for damage detection.

4.4. Detection of the plastic zone by CWT

In this section, a simply-supported beam was exposed to Chichi and Cape earthquakes. Then, the acceleration response vector was obtained for each critical time step, and the curvature vector was calculated, accordingly. Finally, the CWT coefficients for the curvature vector of the beam at the critical time steps were obtained (Fig. 4). The white areas in Fig. 4 demonstrate the location of plastic zones in each case. Figs. 4a and b represent the beam with 64 acceleration recording points. Also, Figs. 4c and d are related to the beam with only 9 acceleration recording points. By comparing Figs. 4a and 4c and also Figs. 4b and 4d, it can be concluded that almost the same results were obtained when 9 or 64 acceleration recording points were used.

5. Using curvature vectors as damage detection tools

In this section, the curvature vectors of the beam under the Chichi earthquake were applied as damage detection tools. Fig. 5 shows the curvatures corresponding to two different time steps. The curvature of a section, whether the section exhibits linear or nonlinear behavior, is a function of the strain of the fibers in that section.

If the yielding stress of the beam material was 2400 kg/cm², and height \( (h) \) of the beam section was 0.1 m, the yielding curvature \( (\phi_y) \) of the beam section (Aschheim M. 2002) can be calculated to be 0.023 using Eq. (3).

\[
\phi_y = \frac{2 \times \varepsilon_y}{h}
\]  

(3)

In which, \( \varepsilon_y \) is the yielding strain of the fibers in the section.
Fig. 4: The CWT coefficients for the curvature vector at the critical time step: a) the beam exposed to the Cape earthquake with 64 acceleration recording points, b) the beam exposed to the Chichi earthquake with 64 acceleration recording points, c) the beam exposed to the Cape earthquake with 9 acceleration recording points, d) the beam exposed to the Chichi earthquake with 9 acceleration recording points.

Fig. 5: The curvature of the beam under the Chichi earthquake with the middle section being elastic or plastic.

As can be seen in Fig. 5, when a plastic zone was formed, the value of beam curvature at the middle suddenly increased, and a non-derivative point appeared in the curve. When the curvature in the middle section of the beam exceeded the yielding curvature, a plastic zone started to form in the areas around the middle section of the beam. The length and depth of the plastic zone can be easily seen in Fig. 5. The advantage of this method of damage detection is that it prevents us from wrongly considering non-derivative points as plastic zones, as is, in the case with the CWT method.

6. Plastic zone detection in a fixed-end beam with uniformly-distributed masses

The beam modeled in this section is the same as the one in previous sections, with the only difference being that the two ends of the beam are fixed in this case. The masses were uniformly distributed along the beam, and the beam was exposed to the Chichi earthquake acceleration.

Once the acceleration response vectors of the beam were recorded at each time step, the curvature vectors of the beam were computed on the basis of the data obtained from 9 and 64 acceleration recording points. In the present circumstances, since fixed-end beams are sensitive to the formation of plastic zones, it was necessary for time-history analysis to set the time steps as small as 0.0001 s. This was performed with the purpose of increasing the accuracy of finite difference calculations. The curvature vectors of the beam at critical time steps are depicted in Fig. 6.
7. Plastic zone detection in a simply-supported beam with a non-uniform mass

It should be noted at this juncture that in all the sections discussed so far, the masses were distributed uniformly along the beam, and the earthquake acceleration was applied uniformly to all masses. These two conditions caused the middle section of the beam to be the first nonlinear section at the critical time step. However, in this section of the paper, the masses of the beam were distributed non-uniformly along the beam; thus, the beam was exposed to asymmetric loading. The mass of the first 20% of the beam span from the left was increased (Fig. 7), the Chichi earthquake acceleration was applied, and the curvature vector of the beam at the critical time step was calculated (Fig. 8). Under these latter conditions, the plastic zone was not accepted to appear in the middle section of the beam.

As shown in Fig. 8, as the distribution of the masses along the beam changed from uniform to non-uniform pattern, the curvature vector of the beam became asymmetric, and its maximum curvature tended to appear on the left rather than appearing in the middle section.
This phenomenon is completely rational for the reason that, as the mass of the first 20% of the beam span was increased, the load applied to that section also increased. Another observation was that the plastic zone increased in depth but decreased in length. It means that a more concentrated and steeper plastic zone was detected here.

8. Plastic zone detection in a simple frame

A one-bay single-story moment frame was modeled in this section as displayed in Fig. 9. The columns were 2 m high, and the frame span was 1.28 m. The frame was designed under combinations of dead and live loads, and all the beam and column sections were assigned the specifications of IPE 120. Data was obtained from 34 acceleration recording points located uniformly along the columns and the beam.

To increase the accuracy of finite difference calculations, the time steps in the time-history analysis were set at 0.0001 s. and the frame was exposed to the El Centro earthquake. In order to compute the curvature vectors of the beam and the columns, the local coordinate vector of each frame member was used. The acceleration response vectors of the columns and the beam were recorded at each time step, and their curvature vectors were calculated, as seen in Fig. 10, and Fig. 11.

As illustrated in Fig. 10a, the plastic zone appeared at the base and top of the column at the critical time step, in which the plastic zone at the base is far steeper. Also, Fig. 10b shows that the non-derivative points appeared in the curvature diagram of the beam near the beam-column connection, indicating the formation of a plastic zone in the beam in this location.

Figs. 11a, 11b, and 11c show the time histories of the curvature values in the location where plastic zones appeared at critical time steps. As can be seen, near the beam-column connection, plastic zones appeared on the beam and also on top of the column at the same time step and the principle of strong column/weak beam was not met. This is quite rational because the specifications of beam and column sections were identical.
9. Experimental Study

As a practical example, the curvature vector calculation of a simply supported beam has been performed experimentally.

9.1 Test Specimen

The experimental setup is shown in Fig. 12 and 13. The test setup consist of an aluminum beam (with density of ρ=2700 kg/m³ and Young’s modulus of E=50 GPA, and Yield Stress of Fy=70 MPa) which is a box section, with dimensions a×b×t=0.02×0.02×0.001 m, so that cross-sectional area and second moment of area with respect to z-axis are A=8 ×10⁻⁵ m² and I_{zz}=4.6 ×10⁻⁹ m⁴, respectively. The length of the beam is 1.1 m.

The concentrated mass was added at the middle of the beam to reduce the natural frequency of the beam. This work was performed due to the limitation of the linear actuator. The linear actuator (Sclater N. et al. 2001) is a device that develops a force and a motion through a straight line. A stepper motor-based linear actuator uses a stepping motor as the source of rotary power. As the rotor turns, linear motion is achieved directly through the screw. Stepper motors have been used in a wide array of applications for many years. With trends towards miniaturization, computer control, and cost reduction, stepper motor actuators are being used in an ever-increasing range of applications. Also, The Micro-Electro-Mechanical Systems (MEMS) accelerometers type MPU6050 are used to measure the vibration of the beam. MEMS accelerometers have proven to offer a suitable solution for SHM in civil engineering applications. Such devices are typically characterized by high portability and durability, as well as limited cost, hence resulting in ideal tools for applications in buildings and infrastructure (Bedon C. et al. 2018).

Fig.11: a) The curvature of the column base section, b) the column top section, c) the beam section that is near the connection to the columns, at every time step

Fig.12: Photograph of the experimental setup
As shown in Fig. 13, a displacement time history of a sine sweep signal from 1-4 Hz was applied to the right support of the beam, and the acceleration responses were measured at the seven positions. Fig. 15 depicts acceleration at the middle of the beam.

In this method, only acceleration data was used to assess damage location and its severity. This data will be used for calculating the displacement and curvature vectors. As mentioned before, the displacement vector can be calculated through double numerical integration of acceleration response with respect to time, and the curvature vector of the beam for each time step can be obtained through the double derivation of the displacement response with respect to distance. Fig. 16 shows the curvature vector of the beam under support excitation.

The specimen yielding curvature of 0.14 can be calculated using Eq. (3). As can be seen in Fig. 16, the curvature value...
of the sections of the beam increased, and the curvature values exceed the yielding curvature, so the plastic region occurred in this section. Fig. 17 shows the plastic zone of the beam after the experiment.

Fig. 17: Damage in the test specimen

9. Conclusion

The present study proposed a new method for detecting structural damage. Several beam and frame structures subjected to earthquake accelerations were studied for damage detection with the proposed method. Also, an experimental study was carried out for the purpose of evaluation of the method in detecting the plastic hinge formation of a simple aluminum beam under support excitation. In this study, the curvature method was used to detect the development of plastic zones in beam or frame sections.

The advantage of using the curvature method is that, it prevents from erroneously regarding non-derivative points as plastic zones. It can be concluded from the results of this study that almost the same result was obtained when 7 or 64 acceleration recording points were used in curvature method, indicating the ability of this method to decrease the need for numerous expensive accelerometers. It should be noted that it is necessary to use this method knowing that the yielding stress of members can be easily obtained.. Using the proposed method leads to the sufficiently-accurate determination of the length and depth of plastic zones as well as their location. The experimental study showed that the nonlinear damaged position could be accurately determined with the proposed approach.

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