Soft gamma-ray repeaters and anomalous X-ray pulsars as highly magnetized white dwarfs

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Abstract. We explore the possibility that soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are powered by highly magnetized white dwarfs (B-WDs). We take a sample of SGRs and AXPs and provide the possible parameter space in mass, radius, and surface magnetic field based on their observed properties (period and its derivative) and the assumption that these sources obey the mass-radius relation derived for the B-WDs. The radius and magnetic field of B-WDs are adequate to explain energies in SGRs/AXPs as the rotationally powered energy. In addition, B-WDs also adequately explain the perplexing radio transient GCRT J1745-3009 as a white dwarf pulsar. Note that the radius and magnetic fields of B-WDs are neither extreme (unlike of highly magnetized neutron stars) nor ordinary (unlike of magnetized white dwarfs, yet following the Chandrasekhar’s mass-radius relation (C-WDs)). In order to explain SGRs/AXPs, while the highly magnetized neutron stars require an extra, observationally not well established yet, source of energy, the C-WDs predict large ultra-violet luminosity which is observationally constrained from a strict upper limit. Finally, we provide a set of basic differences between the magnetar and B-WD hypotheses for SGRs/AXPs.

Keywords: magnetic fields, white and brown dwarfs, X-ray pulsar, neutron stars

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1 Introduction

Soft gamma-ray repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are astronomical objects which exhibit pulsations and their properties are different from the rotation powered radio pulsars and accretion powered X-ray pulsars. SGRs/AXPs are, as of now, most popularly hypothesized to be isolated, spinning down, highly magnetized neutron stars (NSs) (magnetar model) [1]. According to this model, emission is powered by the energy stored in strong magnetic fields of NSs with the surface field \( B_s \sim 10^{14} - 10^{15} \) G. Such strong magnetic fields, however, have not been detected from observed data yet and alternate models like fast rotating white dwarfs (WDs) have been proposed [2–4].

AXPs are distinguished from X-ray binaries by their narrow period distribution, soft X-ray spectrum, faint optical counterparts, and long term spin-down. SGRs, on the other hand, are observed with their bright and short bursts and hence they are considered as a subclass of gamma-ray bursts. Based on their persistent X-ray counterparts, SGRs were found to be very similar to AXPs and hence they are often classified together, as is in the present work. The rotational period of AXPs/SGRs, as of now, lies in a narrow range (2–12 s) compared to that for the ordinary pulsars. Moreover, their generally large spin-down rates, strong outburst energies and giant flares make them different from ordinary pulsars.

Typical rotationally powered energy release rate in AXPs/SGRs is \( \sim 10^{32} - 10^{34} \) ergs s\(^{-1}\) while their X-ray luminosity \( \gtrsim 10^{34} \) ergs s\(^{-1}\), which rules them out to be considered as rotationally powered NSs. A NS of radius 10km with \( B_s \sim 10^{14} \) G and central field \( B_c \sim 10^{16} \) G can have magnetic energy \( \sim 10^{48} \) ergs, which could produce the luminosity \( \sim 10^{36} \) ergs s\(^{-1}\), in its typical age (if they are associated with supernova remnants or young clusters.
of massive stars). Such a high field, based on the propagation time of magnetic instability in the NS surface with Alfvén speed, explains the short duration of initial spike in giant flares. Furthermore, the strong field explains the confinement of the hot plasma required for the subsequent tail with a softer spectrum pulsating at the NS rotation period. Other phenomena being explained due to the existence of large magnetic field include short bursts in almost all AXPs/SGRs with peak luminosity exceeding by a few orders of magnitude the Eddington limit for a NS and high frequency quasi-periodic oscillations (QPOs).

While the above description generally makes the foundation of magnetar model concrete, there are certain shortcomings in it. First of all, as of now, there is no evidence for the strongly magnetized NS — as strong as required for the magnetar model. The inferred/measured strongest possible field, in some occasions, has been only $\sim 10^{12}$ G [5–7]. Second, recent Fermi observations are inconsistent with the predicted high energy gamma-ray emissions in the magnetars [8]. Third, inferred upper limit of $B_s$ for some magnetars, e.g. SGR 0418+5729, is quite smaller than the field required to explain observed X-ray luminosity. Fourth, it has been shown (e.g. [9–11]) that the attempt to relate magnetars to the energies of the supernova remnants or the formation of black holes is not viable. Among other inconsistencies, there are radio pulsars discovered with inferred $B_s$ overlapping with those of AXPs/SGRs, but without any signature of magnetically powered emissions like bursts/flares. Nevertheless, by arguing all of them to be rotating high-field NSs with respective magnetic axes having different orientations with respect to their rotation axes, they were attempted to be unified [12]. In the presence of strong magnetic fields, the high-field pulsars can have active inner accelerators while the AXPs cannot, revealing different observed emission characteristics among themselves. However, recently discovered NSs with relatively low inferred dipole field exhibit outburst properties similar to those in AXPs/SGRs. These observations imply that a high magnetic dipole moment is not a mandatory condition for a magnetar.

In order to remove these shortcomings, at least a part, AXPs/SGRs have been again re-argued to be magnetized WDs [13], following the idea originally proposed more than two decades back [2, 3] (see also [14]). Due to their larger radius ($\sim 10^4$ km for a typical WD), the rotationally powered energy for WDs could be quite larger than that for NSs. Hence, these authors attempted to explain the energy released in AXPs/SGRs through the occurrence of glitch and subsequent loss of the rotational energy. Indeed, the possibility of glitch in WDs, as starquakes, with mass around the Chandrasekhar limit [15] was shown earlier [3] which explained the mean spin-down rate observed for 1E 2259+586. However, this WD based model (hereinafter C-WD) is challenged by the observed short spin periods (e.g. [5]). In addition, due to larger radius, they should exhibit larger ultra-violet (UV) luminosities, which, however, suffer from a deep upper limits on the optical counterparts (e.g. [5, 16]) of some AXPs/SGRs, e.g. SGR 0418+5729 (see, however, [17]).

Recently, Mukhopadhyay and his collaborators, in a series of papers, have proposed for the existence of highly (as well as very highly) magnetized WDs [18–23] with mass significantly super-Chandrasekhar. $B_s$ of such WDs (hereinafter B-WDs) could be as high as $10^{12}$ G and $B_c$ could be 2–3 orders of magnitude higher (see [24]).

Any new idea, when proposed, generally is tested with a simplistic model first. Once, the results based on a simplistic model show promise to explain observations and/or experiments, then more realistic self-consistent models, introducing more sophisticated physics, are introduced in order to fine-tune the original model. Without being an exception, Mukhopadhyay and collaborators have also followed the same tactics to develop their super-Chandrasekhar WD model (see [25], which discusses the evolution of this topic so far).
These authors have, so far, approached towards this mission through the following steps. First, they considered most simplistic, spherically symmetric, very highly magnetized WDs in the Newtonian framework, assuming the magnetic field to be constant or almost constant throughout (or modeling, as if, the inner region of WDs) [19]. To assure stability of such WDs, the authors assumed that the large magnetic field in them is *tangled/fluctuating* in a length scale larger than the quantum length scale such that the average field and hence corresponding magnetic pressure is much smaller than the matter pressure modified by the Landau effects [21, 26].

However, it has been speculated in those work itself that with very high fields the self-consistent consideration of deformation of WDs would reveal a similar super-Chandrasekhar mass at lower fields. In the same model framework, they also showed that magnetized WDs altogether have a new mass-limit, 80% larger than the Chandrasekhar-limit [21], in the same spirit as the Chandrasekhar-limit was obtained [15].

Afterwards, the authors removed both the assumptions: the Newtonian description and spherical symmetry (e.g. [27, 28]). Note that magnetized WDs could be significantly smaller in size compared to their conventional counter-parts [19, 21] and, hence, general relativistic effects therein may not be negligible. Thus, based on a full scale general relativistic magnetohydrodynamic (GRMHD) description [29], they explored more self-consistent WDs which are ellipsoid and revealed similar stable masses, as obtained in the simpler framework, but at smaller fields [28, 30], as speculated earlier [19, 26]. In fact, in the later work [28, 30], the authors were able to show that depending on the field profiles (which were chosen self-consistently in accordance with other conditions/equations) and hence magnetic pressure gradient and magnetic density, maximum mass of a B-WD could be even slightly higher than that proposed earlier [21], but at high fields (not *very high* fields).

In order to understand, how to acquire the strong magnetic fields in B-WDs, the present authors argued [22] for a possible evolutionary scenario by which super-Chandrasekhar WDs with high magnetic fields could be formed by accretion on to a commonly observed magnetized WD, invoking the phenomenon of flux freezing. Based on a varying accretion rate scenario, they showed that a highly super-Chandrasekhar B-WD could be formed in the time scale of \( \sim 2 \times 10^7 \) years from a 0.2\( M_\odot \) WD with surface field \( \sim 10^9 \) G. The idea is, as WD accretes matter, its magnetic field amplifies as a consequence of the increase in (central) density, hence gravitational force, due to the contraction in size (via flux freezing theorem) of WD [31]. Nevertheless, other ways (including possible dynamo based mechanisms) of generating such strong fields, at least partially, cannot be ruled out (however note that depending in field profiles, a very strong field is not necessarily needed to form a super-Chandrasekhar B-WDs). The authors also showed that the estimated number of super-Chandrasekhar B-WDs, governed from cataclysmic variables (CVs), is consistent with the observed rate of peculiar type Ia supernovae, which could be their ultimate fate (at least one of the possible fates).

Such WDs are already shown to help in explaining peculiar, over-luminous type Ia supernovae whose progenitor masses are necessarily super-Chandrasekhar. Nevertheless, these B-WDs are smaller or significantly smaller in size, depending on the field geometry, compared to their ordinary counterparts (e.g. polar with \( B_s \sim 10^9 \) G). Typically their radius can just be an order or two magnitude(s) higher than that of a NS. As the surface temperatures of WDs with the change of magnetic fields are not expected to be significantly higher [32] (in fact could be lower [33]), smaller the radius, smaller the luminosity of the WD is. Therefore, B-WDs should be consistent with the UV-luminosity \( (L_{UV}) \) cut-off in AXPs/SGRs. Moreover, their typical \( B_s \) is consistent with observation, but adequate to explain AXP/SGR energies as rotationally/spin-down powered energy, unlike magnetars.
Furthermore, B-WDs could be adequate candidates to explain certain peculiar radio pulsars/transients as white dwarf pulsars (WDPs), e.g. GCRT J1745-3009 [34], which is otherwise thought to be the prototype of a hitherto unknown class of transient radio sources.

Hence, in the present paper, we explore AXPs/SGRs as B-WDs. Note that the idea to explore B-WDs as AXPs/SGRs has already been presented by independent groups [35–37]. Although the evolution of B-WDs could be by accretion, they may appear as AXPs/SGRs at the exhaustion of mass supply after significant evolution. Such WDs’ $B_s$ and $R$ combination can easily explain AXPs/SGRs as rotationally powered WDs. All the machineries implemented in the magnetar model can be applicable for B-WDs as well, however, with a smaller $B_s$ which is physically more viable. As the ranges of radii and magnetic fields of B-WDs lie in between those of highly magnetized NSs and C-WDs — neither very extreme nor ordinary, we propose these ranges to be explored extensively in understanding related observations (not restricted to AXPs/SGRs and WDPs only), in particular the ones which remain unresolved yet. Therefore, the present work aims at initiating this venture.

The paper is organized as follows. In the next section, we outline the model to explain magnetized WDs as rotating dipoles. Subsequently, in sections 3 and 4, we explore a few sources of SGR/AXP and GCRT J1745-3009 as B-WDs respectively. In section 5, we present the basic differences between the magnetar and B-WD models. Finally, we end in section 6 with summary and implications.

2 Modelling magnetized white dwarfs as rotating dipoles

Following standard electrodynamics [38], the rate of energy loss from a rotating, magnetized compact star, assuming it to be a rotating dipole, is [39]

$$\dot{E}_{\text{rot}} = - \frac{4\Omega^4 \sin^2 \alpha}{5c^3} |m|^2,$$

(2.1)

when the variation of dipole moment $m$ arises due to the inclination of magnetic axis with respect to the rotational axis of the star by the angle $\alpha$, $\Omega$ is assumed to be the angular frequency of the star at the surface, $c$ the light speed. Now the dipole nature of the magnetic field can be expressed as

$$B = \frac{2|m|}{R^3},$$

(2.2)

when $R$ is the radius (or average radius if it is spheroid) of the star. However, the above energy loss rate can be defined as the rate of change of rotational kinetic energy of the star with moment of inertia $I$ so that

$$I\dot{\Omega} = \dot{E}_{\text{rot}},$$

(2.3)

which leads to

$$B_s = \sqrt{\frac{5c^3 IP \dot{P}}{4\pi^2 R^6 \sin^2 \alpha}} G,$$

(2.4)

when $P$ is the rotational period and $\dot{P}$ the period derivative. This is the upper limit of $B_s$. As $P$ and $\dot{P}$ for AXPs/SGRs (and WDPs) are known from observation, $B_s$ can be computed from a given mass-radius ($M - R$) relation for rotating B-WDs when $\alpha$ is a parameter. Note that only that $M$ and $R$ (or equatorial radius $R_e$) are the appropriate set from the $M - R$ relation whose surface angular velocity ($\Omega_{\text{eq}}$) corresponds to the observed $P$. Once $B_s$ is estimated for an observation, the rotational/dipole energy $E_{\text{rot}}$ stored in the star can be
computed. This furthermore quantifies the maximum energy stored in it, if there is no other source as adopted in the magnetar model.

We explore the possibility to explain the origin of high energy phenomena in AXPs/SGRs and WDPs by $\dot{E}_{\text{rot}}$ and $B_s$ of B-WDs — there is no need to invoke extraordinary, yet observationally unconfirmed, sources of energy. This is possible because B-WDs have larger $I$ (due to larger $R$) than NSs, revealing larger $m$, which is however small enough to produce UV-luminosity.

The $M - R$ relations of B-WDs depend on the magnetic field [19, 21, 28, 30]. It has been self-consistently found by GRMHD simulations [28, 30] that field decreases from the central region to the surface at least in 2–3 orders of magnitude. As $B_c$ plays the major role in holding the mass [19], an $M - R$ relation corresponds to the strict value(s) of $B_c$ too. However, $B_s$ is weakly constrained. A range of $B_s$ corresponds to very similar $M$ and $R$ for a given $B_c$, as long as $B_c/B_s \gtrsim 10^3$. Nevertheless, from the solution of stellar structure, a given $M$ and $R$ corresponds to a given $B_c$ and $B_s$ [19, 21], as well as given $\Omega_{\text{eq}}$ [30] for rotating stars. On the other hand, for a given observation ($P$ and $\dot{P}$), a particular set of $M$, $R$ and $\Omega_{\text{eq}}$ corresponds to a particular $B_s$ from equation (2.4), which has to be same/similar to the value of $B_s$ corresponding to the magnetic field profile giving rise to chosen $M$ and $\Omega_{\text{eq}}$ in the first place. This helps in removing the apparent degeneracy in $B_s$, are useful for explaining AXPs/SGRs and WDPs. This outlines the rule followed here.

3 Explaining AXPs/SGRs

We consider nine AXPs/SGRs to estimate the range of possible parameters of B-WDs. We consider two cases separately. First, we consider models with highly magnetized (with $B_c \lesssim 5 \times 10^{14}$ G) B-WDs considering anisotropic effects of magnetic field self-consistently formulated in GRMHD simulations [28, 30]. Subsequently, we assume the B-WDs to be very highly magnetized (with $B_c \gtrsim 10^{15}$ G) and follow the respective approximate models [19, 21].

3.1 B-WDs with high magnetic fields

We consider a typical $M - R_e$ relation, as shown in figure 20 of [30], described for poloidal magnetic field profiles with $B_c = 3.1 \times 10^{14}$ G, $R_e$ ranging in 1534-1586 km and corresponding $R_p/R_e$ in 0.82–0.55 ($R_p$ being the polar radius). If the surface temperature is assumed to be $\sim 10^4$ K, then $L_{\text{UV}} \sim 10^{29}$ ergs/s. However, a more self-consistent computation reveals $L_{\text{UV}}$ to be much smaller in the presence of field considered here [33]. The observed values of $P$, $\dot{P}$ and $L_x$ for nine SGRs/AXPs are given in table 1. Figure 1 shows that $\dot{E}_{\text{rot}}$ computed based on our model, with a fixed $\alpha = 15^\circ$, in the range $1 \leq P/\text{sec} \leq 20$ is several orders of magnitude larger than observed $L_x$ for each source. Note that only one point in each curve (corresponding to fixed $\dot{P}$) in figure 1a corresponds to the respective source with observed $P$, when different $M$ in the $M - R_e$ relation corresponds to different $\Omega_{\text{eq}}$ and, hence, different $P$. From figure 1b we can retrieve $M$ and, hence $R_e$, corresponding to the respective sources. It can be mentioned that the sin $\alpha$ factor in equation (2.1) (and hence other equations) does not have much importance, because a wind component can also spin down a pulsar. In fact, the detailed $\alpha$-dependence can be different in different models (see, e.g., [40, 41]). Hence, we do not intend to constrain $\alpha$-dependence in our model and the computations are done for a particular, fixed value of $\alpha$, unless stated otherwise.
Table 1. Various observational and theoretical parameters of AXPs/SGRs: $P$, $\dot{P}$, $L_x$ are observed values and inputs and $\alpha$, minimum of $L_{UV}$ are outputs of our very highly magnetized B-WD model, discussed in section 3.2. See, http://www.physics.mcgill.ca/~pulsar/magnetar/main.html.

| AXPs/SGRs          | $P$  | $\dot{P}$  | $L_x$     | $\alpha$ | $L_{UV \text{min}}$ B-WD | $L_{UV \text{min}}$ C-WD |
|--------------------|------|------------|-----------|----------|----------------------------|--------------------------|
| 1E 1547-54         | 2.07 | 2.32       | 0.031     | 5 – 15   | $5.7 \times 10^{25}$      | $4.8 \times 10^{29}$     |
| 1E 1048-59         | 6.45 | 2.7        | 0.054     | 5 – 15   | $3.5 \times 10^{26}$      | $9.2 \times 10^{29}$     |
| 1E 1841-045        | 11.78| 4.15       | 2.2       | 15       | $1.6 \times 10^{28}$      | $1.7 \times 10^{30}$     |
| 1E 2259+586        | 6.98 | 0.048      | 0.19      | 2 – 3    | $3.4 \times 10^{26}$      | $1.5 \times 10^{29}$     |
| SGR 1806-20        | 7.56 | 54.9       | 1.5       | 15       | $3.4 \times 10^{26}$      | $3.5 \times 10^{29}$     |
| SGR 1900+14        | 5.17 | 7.78       | 1.8       | 15       | $8.6 \times 10^{28}$      | $1.3 \times 10^{30}$     |
| SGR 0526-66        | 8.05 | 6.5        | 2.1       | 15       | $6.4 \times 10^{27}$      | $1.7 \times 10^{29}$     |
| SGR 0418+5729      | 9.08 | $5 \times 10^{-4}$ | $6.2 \times 10^{-4}$ | 1 – 5 | $3 \times 10^{25}$      | $1.8 \times 10^{29}$     |
| SGR 1822-1606      | 8.44 | $9.1 \times 10^{-3}$ | $4 \times 10^{-3}$ | 1 – 5 | $3.4 \times 10^{26}$ | $8 \times 10^{28}$ |

Figure 1. The ratio of rate of rotational energy released to X-ray luminosity as a function of (a) spin period, and (b) mass, for B-WDs when from the top to bottom various curves correspond to 1E 1547-54, 1E 1048-59, SGR 1806-20, SGR 1900+14, SGR 0526-66, SGR 1822-1606, 1E 1841-045, SGR 0418+5729 and 1E 2259+586. For other details, see table 1.

Figure 2 shows $B_s$ for the respective sources. As above, only one point in each curve corresponds to the respective sources with known $P$. The values of $B_s$, along with their $B_c$, are confirming these B-WDs to be excellent storage of magnetic energy. The computed $B_s$ values turn out to be much higher compared to those in C-WDs (see figure 3b, which will be discussed furthermore in section 3.2). Hence, the $M - R_e$ relation adequately explains all nine AXPs/SGRs without requiring an extra-ordinary source of magnetic energy.

3.2 B-WDs with very high magnetic fields

Some of the properties of chosen AXPs/SGRs, particularly useful for the present modelling, are listed in table 1. Like in section 3.1, we primarily consider $\alpha = 15^\circ$ which is not observationally much constrained either. However, in some sources, $\alpha$ must be smaller (if no other source of spin down is considered) in order to explain observed data better according
to our theory. Indeed, $\alpha$ for SGR 0418+5729 has been argued to be very small [42]. For such sources, a range of $\alpha$ is tabulated, when smaller the $\alpha$, larger the allowed mass range of the WD is. However, a larger $\alpha$ reveals a smaller $L_{UV}$ which argues the WDs to be more difficult to observe. We also assume the surface temperature of WDs to be $T_{UV} \sim 10^4$ K and set their radius of gyration assuming the WDs to be semi-solid sphere/ellipsoid.

Figure 3a shows the $M-R$ combinations for B-WDs, which exhibit $100 \lesssim \dot{E}_{rot}/L_x \lesssim 10^7$ from equation (2.1) for the AXPs/SGRs listed in table 1, explaining them well as rotational powered pulsars. The corresponding $L_{UV}$ appears to be as small as $\sim 10^{26}$ ergs s$^{-1}$ (see table 1). Figure 3b along with table 1, however, shows that the $M-R$ combinations for the C-WDs exhibit 1–4 orders of magnitude higher $L_{UV}$.

Figures 4a and 4b show that $B_s$ of the B-WDs is quite stronger compared to that of the C-WDs. This reveals that B-WDs are better storage of rotational/spin-down/magnetic
energy. They naturally explain AXPs/SGRs without requiring an extra-ordinary source of magnetic energy. Generally higher the \( B_s \) and \( B_c \), higher the \( M \) is, which corresponds to a lower \( R \) and hence a lower \( L_{UV} \). However, as shown earlier \cite{19}, depending on the central density, a B-WD of lower \( R \) with lower \( M \) (\( \sim 1.5M_\odot \)) can be formed. Therefore, for a given \( M \), \( L_{UV} \) for B-WDs could be smaller than that for C-WDs. Figure 5 shows that a B-WD of \( M \sim 1.45M_\odot \) can have \( L_{UV} \sim 10^{27} \text{ergs s}^{-1} \) which is significantly smaller than the smallest possible \( L_{UV} \) for a C-WD.

Generally (see table 1), the expected \( L_{UV} \)-s of B-WDs are two orders of magnitude lower than that of C-WDs, except a few cases where they are just a few factors lower. It should be interesting to concentrate on these sources and measure their UV flux and spectrum, simultaneously with the X-ray spectrum, so that contributions from putative companions could be subtracted and the real blackbody emission from the compact objects could be measured and radius be estimated.

**Figure 4.** Surface magnetic field as a function of mass for (a) B-WDs when from the top to bottom various curves correspond to SGR 1806-20, 1E 1048-59, SGR 0526-66, 1E 1841-045, 1E 1547-54, SGR 1900+14, 1E 2259+586, SGR 1822-1606, SGR 0418+5729, (b) C-WDs when from the top to bottom various curves correspond to SGR 1806-20, SGR 0526-66, 1E 1841-045, SGR 1900+14, 1E 1048-59, 1E 1547-54, 1E 2259+586, SGR 1822-1606, SGR 0418+5729.

**Figure 5.** UV-luminosity (solid line) and radius (dashed line) as functions of mass for a particular central magnetic field \( 7 \times 10^{15} \text{G} \).
4 Explaining GCRT J1745-3009

GCRT J1745-3009 is a transient radio source in the direction of the galactic center, which exhibited five peculiar consecutive outbursts at 0.33 GHz with a period of 77.13 minutes. Zhang and Gil [34] argued it to be a WDP with a period of 77.13 minutes within 0.8 kpc. Later on, by color-magnitude analysis, Kaplan et al. [43] showed that WDs with typical temperatures (5000-20,000 K) and radii (5000 km) could not be this much close to us and would be at least at around 2 kpc. This challenged the idea of the source to be a WDP. Now in the framework of very slowly rotating B-WDs (see figures 17 and 18 of ref. [30]) with \( B_s \sim 3.3 \times 10^{11} - 2 \times 10^{12} \) G, corresponding \( R \sim 1580 - 500 \) km (depending on whether the magnetic field is high or very high) and central density \( \sim 10^{10} \) gm cm\(^{-3}\), we revisit all the calculations, e.g. radius of polar cap and unipolar potential drop therein, done by previous authors [34], and find them to be consistent with WDP idea when the unipolar potential drop is at least one order of magnitude smaller than that in radio pulsars. However, the emission altitude turns out to be in accordance with radio pulsars for \( \alpha \sim 30 \) and \( \sim 15 \) for high and very high fields respectively. Furthermore, the sum of mean free paths for electrons to produce inverse-Compton gamma-ray photons and for the photons to attenuate turns out to be larger than the radius of B-WDs, which rules out the possible pair productions and explains why GCRT J1745-3009 is dormant before and after the bursting cycles. The maximum gamma-ray/X-ray flux appears to be only a factor of 4 larger than that obtained by Zhang and Gil [34] for the same parameters for very highly magnetized B-WDs, which is still quite small to detect. While for highly magnetized cases it appears to be \( \sim 100 \) times larger than that obtained by previous authors, it is very consistent with the X-ray flux upper limit \( \sim 5 \times 10^{-10} \) ergs s\(^{-1}\) cm\(^{-2}\) [44]. Moreover, the solid angle of radio emission is quite unknown and is a free parameter in order to reduce gamma-ray/X-ray flux furthermore. Interestingly, \( \dot{P} \sim 10^{-14} \) for such B-WDs which is difficult to measure.

Finally, from the condition of radio luminosity not exceeding the spin-down luminosity reveals the distance of the source to be \( \lesssim 8.5 \) kpc (for the same solid angle chosen by the previous authors), for high magnetic fields, which is much larger than that predicted by Zhang and Gil [34] and in accordance with the lower limit predicted by Kaplan et al. [43]. For very high magnetic fields, the distance turns out to be smaller, but still at least \( \sim 1.6 \) kpc which is larger than that predicted by Zhang and Gil [34]. As shown in section 3, such B-WDs are significantly cooler and hence being further away than that predicted by Zhang and Gil [34], its optical flux will be dimmer to evade detection. This strongly supports the source to be a WDP.

5 Magnetar versus B-WDs

The magnetar hypothesis is around for quite some time and several observational features of SGRs/AXPs are worked out under the premise of this hypothesis. While attempting to explain new observations, sometimes new factors in the magnetar hypothesis are invoked, which, although not proven, are quite plausible. For example, the period derivatives in magnetars are often found to vary, sometimes related to other parameters, like, luminosity and temperature, and these are explained by invoking the torque exerted by the magnetosphere (see, e.g., [45]). Many of these explanations can possibly be tailored to suit the B-WD hypothesis, presented in this work.

Instead of embarking on such an adventure, we enumerate here the most fundamental and basic differences between the magnetar model prevailing in the literature and the B-WD hypothesis presented in this paper, to explain the properties of SGRs/AXPs. Future
observations can be tuned to pin down these differences which will help in either identifying the exciting new objects called B-WD or it will strengthen the magnetar hypothesis.

5.1 Narrow period range and rarity of the objects

The narrow period range and the rarity of the SGRs/AXPs is explained in the magnetar hypothesis as due to the peculiar conditions required to produce the extremely high magnetic field. They form presumably with a very high spin, and, due to the extremely high magnetic field, spin down very fast to the observed range of periods. At longer periods, they are too weak to be detected. The B-WDs, on the other hand, are rare objects and their formation may require still rarer constraint of being able to accrete matter from a low mass companion. The lower period range would be too fast for a WD, whereas at longer periods they could be undetectable.

The observational consequence of these would be as follows. In the magnetar hypothesis, there is a rare possibility of detecting SGRs/AXPs with fast rotation (significantly less than a second), whereas such objects would not be possible in the B-WD hypothesis. Furthermore, in the magnetar hypothesis, there should be a large number of high magnetic field long period NSs with quite weak emission. In the B-WD hypothesis, however, slowly rotating high magnetic WDs could be detected as the progenitors of peculiar Type Ia Supernovae.

5.2 Origin and age

Magnetars necessarily have to be young and should be associated with a young supernova remnant (SNR). If one measures the space velocities of SGRs/AXPs, from the measured age, one should be able to identify the recent SNR for each and every SGR/AXP. The B-WDs would be preferentially seen in dense stellar regions (so that the peculiar high magnetic field WDs capture a low mass companion and increase their core magnetic field by accretion), but they need not be associated with SNR.

5.3 Companion and relic accretion disks

In the B-WD hypothesis, one of the methods to generate higher $B_c$ is by accretion and the resultant flux freezing in WDs. Hence if B-WDs are the central sources in SGRs/AXPs, some of these objects may retain a low mass companion and/or relic accretion disks. Being a recently born NS in a supernova, no such companion or relic accretion disk should be found in SGRs/AXPs, in the magnetar hypothesis.

5.4 Explaining giant flares

One of the motivations to introduce the magnetar concept was to explain the huge amount of energy released in repeated bursts, in particular giant flares, in SGRs. In the B-WD model, the SGR/AXP systems are perhaps old systems, mildly powered by relic accretion and mostly powered by rotation. Hence, a continuous decay of magnetic field as well as increase in mass (due to accretion) is possible (and, hence, decrease in radius due to stronger gravity and increase in magnetic field due to flux freezing) in them. This is expected to result in frequent re-adjustment of the high magnetic field and its gradient within the star, as well as equation of state (for very high magnetic field cases). This could furthermore result in outbursts in SGR/AXP systems in the B-WD hypothesis.
5.5 Size

One of the most fundamental differences in the two models is the size of the compact object. In the magnetar hypothesis, they are NSs with a size of 8–10 km. In the B-WD hypothesis, they can have a wide range of size ranging from a few tens of km to a few thousands of km. A detailed modelling and understanding of the X-ray spectral components should be able to resolve the source size. Indeed, detailed X-ray spectroscopy of AXPs has revealed the presence of thermal components in the X-ray spectrum (see, e.g., [46]) and it is shown that the inferred sizes of the emission region are sometimes lower than a km, demonstrating small hot spots in the emission, rather than emission from the full surface of the compact object. The detection of UV/optical emission of sources, on the other hand, points towards the emission from the full surface of the compact object. Hulleman et al. [47] already pointed out that the optical data of AXP 4U 0142+61 are consistent with a hot WD.

Hence, the multi-wavelength data of AXPs need to be re-looked in the perspective of optical/UV emission coming from the surface of a B-WD and X-rays coming from hot spots, to derive sensible source parameters.

5.6 Non-detection of gamma-rays and radio emissions

As per the B-WD model, the spin energy of the WD is used to power the X-ray emission with a mechanism similar to that seen in rotation powered pulsars (RPPs), with, however, different size and magnetic fields. Hence, many of our understanding of RPPs must be applicable to AXPs/SGRs, as per the B-WD model. RPPs are normally seen in radio wavelengths and among the thousands of RPPs discovered in radio wavelengths, only a few dozen are seen in X-rays and fewer still in gamma-rays. It is found that $L_x$ scales as $B^2/P^4$ (when $B$ is average magnetic fields) and the gamma-ray luminosity, $L_\gamma$, scales as $B/P^2$ (e.g. [48]). Hence, only the young RPPs with lower periods and higher magnetic fields are seen in X-rays and gamma-rays.

Now B-WDs could have larger values of $B$ than ordinary NS RPPs, which also implies their narrower polar caps. The source XTE J1810-197, however, was seen in radio wavelengths [49] and the authors pointed out that employing RPP typed mechanism cannot be excluded for the handful of known magnetars, because of their long periods implying small active polar caps and narrow beams which may miss the observers for random orientations. Hence the rare detection of radio pulsations in AXPs/SGRs, as compared to NS RPPs, is consistent with the high magnetic field assumed for B-WDs.

The non-detection of AXPs/SGRs in gamma-ray energies is already in conflict with the outer gap model in the magnetar scenario [50]. Since gamma-ray luminosity scales as $B$, the lower-than magnetar field for B-WDs explains the lower $L_\gamma$ for AXPs/SGRs in the B-WD model.

To summarize, in the B-WD scenario, the magnetic field could be higher than NS RPPs to have narrow beams and field could be lower than magnetar model to have lower $L_\gamma$. A deeper search, however, for gamma-ray emission in AXPs/SGRs and a detailed modelling using the B-WD model parameters will certainly help in distinguishing/refining the B-WD model.

5.7 Capability of measuring magnetic fields by latest experiments

Recently launched ASTROSAT and Hitomi satellites are capable of detecting hard X-rays of energy upto $\sim 100$ keV. Hence, they are expected to observe more sources with the capabilities of wide band spectroscopy. Now the electron Cyclotron resonance energy of
absorption spectrum is to be \( E_c = 11.6 \left( B_s / 10^{12} \right) \text{ keV} \). Therefore, a compact object with \( B_s \sim 10^{12} - 10^{13} \text{ G} \) is possible to be observed by ASTROSAT/LAXPC and could be identified as a B-WD. However, for a NS based magnetar with \( B_s \gtrsim 10^{14} \text{ G} \), \( E_c \gtrsim 1000 \text{ keV} \), which is quite beyond the scope of these satellites.

6 Summary and implications

We have demonstrated important applications of recently formulated B-WDs [19–21, 23]. The present work indicates a possibility of wide application of B-WDs in modern astrophysics. The idea that AXPs/SGRs need sources of energy other than accretion is certainly inevitable, but the hypothesis that they are highly magnetic NSs, although attractive, did not neatly fit in with furthermore observations (unlike other ideas in Astrophysics, like, spinning NSs as radio pulsars and accreting compact objects as X-ray binaries, which quickly established themselves as paradigms). In other words, while there is a more standard model as of now, namely magnetar, to explain AXPs/SGRs, alternate models must be explored keeping shortcomings in magnetar model in mind and here we have explored one such model developed recently, namely B-WDs.

Apart from the limitations of the magnetar model discussed earlier, the model does not have natural explanation of several observed features, like, the narrow period range (however, within the framework of the magnetic field decay models, it was attempted it explain [51]) and several orders of magnitude range in period derivative, lack of large proper motion, lack of supernova identification for all AXPs/SGRs etc. Hence, it is very important that other possible explanations for the AXP/SGR phenomena, at least a part of them, need to be seriously explored. The B-WD concept is an extremely attractive alternate for AXPs/SGRs. This is because of its range of mass and radius to satisfy different observations.

Recently, the source 1E 2259+586 has been reported to exhibit an anti-glitch [52]. While the presence of glitch has been explained in the theory of NS, an anti-glitch is difficult to argue under the same framework. We, however, can speculatively describe it in B-WDs as follows. As we argued earlier, an equilibrium B-WD will have a larger interior field which decays (starting from the core-crust boundary) down away from the center [22]. However, B-WDs are expected to encounter continuous mass-loss, along with the decay of its angular velocity and magnetic field exhibiting rotational power. Now, when the surface field decays below a critical value, the stiffness of the field profile increases significantly enough to produce an outward force arising from the additional gradient of magnetic pressure, say \( \sim 2 \times 10^{11} \text{ ergs cm}^{-3} \) at a radius \( \sim 600 \text{ km} \) with a density \( \sim 10^9 \text{ gm cm}^{-3} \). This in turn leads to the increase of the stellar radius in equilibrium and, from the conservation of angular momentum, a decrease of the frequency with an anti-glitch \( \sim -4.5 \times 10^{-8} \text{ Hz} \) as observed. Furthermore, above a critical time scale, the density will decrease appreciably to decrease the inner magnetic field due to continuous mass-loss, when the outer field has been already smaller. This in turn decreases the stiffness of the field profile and hence the gradient of magnetic pressure. As a result, this leads to the gravitational power to be suddenly stronger to exhibit glitch. This cycle may continue depending on the competitive powers between mass-loss and rotational energy extraction rates. The glitch, however, can also be occurred due to the loss of angular velocity alone and subsequent star-quake, in the same model platform, as is argued in the standard NS picture.

Another important application of B-WDs has been explaining the peculiar radio transient GCRT J1745-3009 as a WDP. While earlier authors indeed argued it to be a WDP,
subsequently it was ruled out based on a more accurate prediction of its distance in the framework of a C-WD. However, considering it to be a B-WD furthermore opens up its possibility of a WDP, due B-WDs’ larger magnetic field and smaller radius which could predict it to be low luminous and further away and hence dimmer to evade detection.

Now, in order to verify the usefulness of B-WDs, it is important to refine X-ray spectroscopic tools to measure the mass and radius of the compact objects in AXPs/SGRs. In the calculations described in this work, we could only derive a range of possible magnetic field related to the corresponding mass and radius for a given source based on the observed $P$ and $\dot{P}$. If, observationally, some parameters could be tied to either mass or radius (even magnetic field, e.g. by electron Cyclotron absorption line, which would be in the X-ray regime, unlike the gamma-ray regime of NS based model), then unique solutions and hence definite predictions can be made based on the B-WD model. In the larger astrophysical context, it is important to examine various observations in the light of the existence of B-WDs and their creation by the phenomena of flux-freezing. For example, the fact that in the SDSS survey, the magnetic WDs are found to have, on an average, larger mass compared to the non-magnetic WDs [32] can be explained naturally if we assume that the magnetic WDs are generated by accretion (and the resultant contraction in radius and flux freezing). Similarly, magnetic WDs in CVs have higher field strength below their period gap (which are evolved systems), again, comes naturally from the fact that evolved WDs, due to accretion, should have larger mass, lower radius, and higher magnetic field. Finally, a precise mass measurement of evolved CVs should identify a few of them as B-WDs.

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