Interplay between electron spin and orbital pseudospin in double quantum dots

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We investigate theoretically spin and orbital pseudospin magnetic properties of a molecular orbital in parabolic and elliptic double quantum dots (DQDs). In our many body calculation we include intra- and inter-dot electron-electron interactions, in addition to the intradot exchange interaction of ‘p’ orbitals. We find for parabolic DQDs that, except for the half or completely filled molecular orbital, spins in different dots are ferromagnetically coupled while orbital pseudospins are antiferromagnetically coupled. For elliptic DQDs spins and pseudospins are either ferromagnetically or antiferromagnetically coupled, depending on the number of electrons in the molecular orbital. We have determined orbital pseudospin quantum numbers for the groundstates of elliptic DQDs. An experiment is suggested to test the interplay between orbital pseudospin and spin magnetism.

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I. INTRODUCTION

Nanometer-scale quantum dots have potential for many technological applications[1]. A useful model of these systems is a parabolic quantum dot. In this model two-dimensional electrons are confined in a parabolic potential. The eigenenergies of a parabolic quantum dot are given by

\[ E_n = \hbar \omega (n + 1), \quad n = 0, 1, \cdots, \] (1)

where \( \omega \) is the characteristic frequency of the parabolic potential. This dot is rotationally invariant about the symmetry axis of the parabolic potential. (Hereafter this axis will be called the \( z \)-axis). The energy states for \( n = 0, 1, 2, \cdots \) are labeled as ‘s’, ‘p’, ‘d’ ... orbitals, as in the real atoms. Each of these states is \((n+1)-fold\) degenerate. The ‘s’ orbital is non degenerate with \( z \)-component of the angular momentum quantum number \( \alpha = 0 \) while the ‘p’ orbital for each dot is doubly degenerate with \( \alpha = \pm 1 \). Parabolic dots form basis for understanding more realistic dots, for example, elliptic dots[2].

When two of these dots are coupled an artificial diatomic molecule can be formed[3]. These DQDs have potential application in quantum computing since they may provide a basic building block of solid state realization of a quantum computer[4, 5, 6, 7, 8, 9]. Drawing the analogy with the transition metal compounds[10], we expect to find interesting physics in the molecular orbital originating from the degenerate levels of a quantum dot, such as the p state. The orbital degeneracy of a shell makes the physics rich in the transition metal compounds. To illustrate the physics we consider two sites each with a two-fold degenerate orbital. The degenerate orbitals are denoted by \( a \) and \( b \). These states can be labeled with a pseudospin value: the state \( a \) with pseudospin 1/2 and the state \( b \) with pseudospin -1/2. In such a model, there are on-site repulsion energies \( U_a, U_b \) and \( U_{ab} \), intra-atomic exchange energy \( J \), and the tunneling energies \( t_a \) and \( t_b \). When \( t_a = t_b = t, U_a = U_b = U \) and \( U_{ab} = U - J \), the effective model Hamiltonian for \( t \ll U \) can be written[10] as

\[ H_{eff} = -\frac{4t^2}{U - 2J} \left( \frac{3}{4} \vec{S}_1 \cdot \vec{S}_2 \right) \left( \frac{1}{4} - \vec{\tau}_1 \cdot \vec{\tau}_2 \right) -\frac{4t^2}{U} \left( \frac{1}{4} - \vec{\tau}_1 \cdot \vec{\tau}_2 \right) \left( \frac{3}{4} + \vec{\tau}_1 \cdot \vec{\tau}_2 \right). \] (2)

Here, spin and pseudospin of the electrons on the \( j \)-th atom are denoted by \( \vec{S}_j \) and \( \vec{\tau}_j \), respectively. This particular Hamiltonian is rotationally invariant both in spin and pseudospin spaces, i.e., it has the symmetry of \( SU(2) \times SU(2) \)[10]. In the groundstate of this Hamiltonian spins in different atoms are ferromagnetically coupled while their pseudospins are antiferromagnetically coupled. The ground-state energy is \(-4t^2/(U - 2J)\). This state has the total spin and pseudo spin quantum numbers \( S = 1 \) and \( \tau = 0 \), and is 3-fold degenerate. The first excited state with energy \(-4t^2/U\) has \( S = 0 \) and \( \tau = 1 \), and is also 3-fold degenerate. The second excited state has zero energy with \( S = \tau = 0 \) or \( S = \tau = 1 \) (10-fold degenerate).

Magnetic and excited state properties of DQDs are also of experimental interest. Recently an experimental investigation of orbital magnetism of DQDs containing about 50 electrons was carried out[12]. The single-electron tunneling spectroscopy in a finite source drain voltage[13] allows one to explore the excited states of DQDs[14]. This technique is used to detect the singlet-triplet transition in DQDs[15], which plays an important role in two-qubit quantum gates of quantum computing[3].

The physics of the molecular orbital made of the doubly degenerate ‘p’ states of DQDs is expected to be more complicated than that of the effective Hamiltonian described in Eq. (2). In DQDs, ‘p’ states can be labeled with a pseudospin value: the state with a plus value of the \( z \)-component of the angular momentum has pseudospin...
1/2 and the state with a minus value has pseudospin -1/2. An electron has thus a pseudospin 1/2 on top of ordinary spin 1/2. Unlike transition metal compounds, since \( U_{ab} = U \) rotational symmetry of \( f \) is broken, i.e., the \( SU(2) \times SU(2) \) symmetry is broken in DQDs. However the system is invariant under pseudospin rotations about the \( z \)-axis. It will be interesting to investigate how pseudospins and real spins of two dots couple in the molecular orbital made of doubly degenerate ‘p’ states of DQDs. The other issue is the orbital magnetism of this Hamiltonian. The proper basis is suggested to test the interplay between orbital and pseudospin and spin magnetism.

II. MODEL HAMILTONIAN FOR DOUBLE QUANTUM DOTS

We describe our model Hamiltonian for double quantum dots. If the interdot exchange energy is ignored, Coulomb interactions for the parabolic dots can be decomposed as follows:

\[
U_{\alpha,\alpha'} = \int \int d^2x \, d^2x' |\varphi_\alpha(x)|^2 |\varphi_{\alpha'}(x')|^2 v(x - x')
\]

\[
V_{\alpha,\alpha'} = \int \int d^2x \, d^2x' |\varphi_\alpha(x)|^2 |\varphi_{\alpha'}(x + d)|^2 v(x - x' - d)
\]

\[
J_{1,-1} = \int \int d^2x \, d^2x' \varphi_1^* (x) \varphi_{-1} (x) \varphi_{-1}^* (x') \varphi_1 (x') \times v(x - x'),
\]

where \( \varphi_\alpha \) is the wavefunction of an electron with the \( z \)-component of the angular momentum \( \alpha \) and \( v \) is the Coulomb potential. \( U_{\alpha,\alpha} \) is the on-dot direct repulsion energy between two electrons, one with the \( z \)-component of the angular momentum \( \alpha \) and the other with \( \alpha' \). \( V_{\alpha,\alpha'} \) is the same as \( U_{\alpha,\alpha} \) except that it is the interdot direct interaction. \( J_{1,-1} = J \) is the orbital exchange energy between two electrons in different orbitals of the same dot. Since the direct interactions are independent of \( \alpha \) and \( \alpha' \), i.e., \( U_{\alpha,\alpha} = U \) and \( V_{\alpha,\alpha'} = V \), we have the Hamiltonian for parabolic DQDs with no hopping between dots

\[
\mathcal{H}_0 = U \left( \sum_{j\sigma} n_{j\alpha 1} n_{j\alpha, -1} + \sum_{j\sigma\sigma'} n_{j\alpha\sigma} n_{j, -1, \sigma'} \right) + J \sum_{j\sigma} c_{j1\sigma}^\dagger c_{j,-1,\sigma'}^\dagger c_{j1\sigma'} c_{j,-1,\sigma} + V \sum_{\sigma\sigma'} n_{1\alpha\sigma} n_{2\alpha'\sigma'} + E_1 \sum_{j\alpha \sigma} n_{j\alpha \sigma}
\]

where \( n_{j\alpha \sigma} = c_{j\alpha \sigma}^\dagger c_{j\alpha \sigma} \) and the fermion operator \( c_{j\alpha \sigma} \) annihilates (creates) the electron with the spin \( \sigma/2 \) in the state of the \( z \)-component of the angular momentum \( \alpha \) on the \( j \)-th dot. The summations are performed for \( j = 1, 2, \alpha = \pm 1 \) and \( \sigma = \pm 1 (\uparrow, \downarrow) \).

The hopping Hamiltonian between double dots is given by

\[
V_{\text{hop}} = -t \sum_{\alpha \sigma} \left( c_{1\alpha \sigma}^\dagger c_{2\alpha \sigma} + \text{h.c.} \right).
\]

where \( t \) denotes the tunneling energy of an electron between the opposite dots. It is assumed that the hopping of an electron between dots changes neither the angular momentum nor the spin. Furthermore, we assume that the hopping energy \( t \) is not dependent of the angular momentum of the electron.

The confinement potential of each quantum dot can be assumed to be parabolic with a distortion. The distortion an electron experiences has an elliptic form and is
represented by
\[ V_{\text{elliptic}}(\vec{r}) = 2 \left( \frac{m^* \omega}{\hbar} \right) \epsilon xy, \quad (6) \]
where \( \epsilon \) is a positive parameter describing the deviation from parabolicity. The distortion breaks the \( U(1) \) symmetry and lifts the degeneracy of a parabolic dot. It splits the ‘\( p \)’ orbitals of \( E_1 = 2\hbar \omega \) with the angular momenta \( \alpha = \pm 1 \) into two states
\[
| -1 \rangle' = \frac{1}{\sqrt{2}} (| 1 \rangle - i | -1 \rangle), \text{ for } E'_1 = (2\hbar \omega - \epsilon),
\]
\[
| 1 \rangle' = \frac{1}{\sqrt{2}} (| 1 \rangle + i | -1 \rangle), \text{ for } E'_1 = (2\hbar \omega + \epsilon),
\]
where \( | \alpha \rangle \) denotes the state with the angular momentum \( \alpha = \pm 1 \) and \( E'_1 \) is the single particle energy of the elliptic dot. The total Hamiltonian for the DQD with the elliptic potential can be written as
\[ H_{\text{tot}} = H + V_{\text{elliptic}}, \quad (8) \]
where \( H = H_0 + V_{\text{hop}} \) is the total Hamiltonian for the DQD with a parabolic potential and \( V_{\text{elliptic}} \) represents the Hamiltonian of the elliptic potential. The elliptic potential in the second quantized form is
\[ V_{\text{elliptic}} = (2\hbar \omega - \epsilon) \eta_{-1} + (2\hbar \omega + \epsilon) \eta_1, \quad (9) \]
where \( \eta_\beta \) is the number operator of particles in the states \( | \beta \rangle = \pm 1 \rangle' \). It can be written as
\[
\eta_\beta = \frac{1}{2} \sum_j (c_{j+\sigma} - i \beta c_{j-\sigma}) (c_{j-\sigma} - i \beta c_{j+\sigma})
\]
\[ = \frac{1}{2} \left( \sum_{j,\alpha} c_{j+\alpha}^+ c_{j-\alpha} + i \beta \sum_{j,\alpha} \alpha c_{j-\alpha}^+ c_{j+\alpha} \right). \quad (10) \]

In terms of pseudospin operators the elliptic potential can be written as
\[ V_{\text{elliptic}} = 2\epsilon \tau_y + E_1 N_p, \quad (11) \]
where
\[ \tau_y = -\frac{i}{2} \sum_j \left( c_{j+1,\sigma}^+ c_{j-1,\sigma} - c_{j-1,\sigma}^+ c_{j+1,\sigma} \right). \quad (12) \]

In general, the intradot interaction is greater than the interdot interaction since two dots are separated with a distance \( d \) each other. One can assume that \( U > J > V \gg t \). We consider elliptic DQDs that are significantly distorted from parabolic DQDs, so we assume that \( \epsilon \gg t \). Both the elliptic potential and the hopping terms will be treated as perturbations. The number of ‘\( p \)’-orbital electrons \( N_p \) that can be injected in the system is from 0 to 8. For these numbers, the low energy states will be obtained in Sec. V for parabolic dots and in Sec. VI for elliptic dots. Before these, we will explain the symmetry operators of our Hamiltonian and the basis states for \( H_0 \).

### III. SYMMETRY OPERATORS

**Orbital pseudospin operators** Let \( \tilde{\tau} \) be the total orbital pseudospin operator defined by
\[ \tau^2 = \frac{1}{2}(\tau_+ \tau_- + \tau_- \tau_+) + \tau_z^2 \quad (13) \]
with
\[ \tau_+ = \tau_x + i\tau_y = \sum_j \tau_{j+} = \sum_j \left( \sum_{\sigma} c_{j+\sigma}^+ c_{j-\sigma} \right) \]
\[ \tau_- = \tau_x - i\tau_y = \sum_j \tau_{j-} = \sum_j \left( \sum_{\sigma} c_{j-\sigma}^+ c_{j+\sigma} \right) \]
\[ \tau_z = \sum_j \tau_{jz} = \sum_j \left( \sum_{\sigma} (c_{j+\sigma}^+ c_{j-\sigma} - c_{j-\sigma}^+ c_{j+\sigma}) \right). \quad (14) \]

These operators have the following properties. When a single dot contains one electron then \( \tau_{j+} \uparrow \downarrow = | \uparrow \downarrow > = | \uparrow \downarrow \rangle \) and \( \tau_{j-} \downarrow \uparrow = 0 \), where \( | \uparrow \downarrow \rangle \) and \( | \downarrow \uparrow \rangle \) denote the states of the \( j \)-th dot with the up and down pseudospin of the electron, respectively. If a single dot contains two electrons with opposite spins then \( \tau_{j-} | \uparrow \downarrow \rangle = | \downarrow \uparrow \rangle \).

In general, neither parabolic dot nor elliptic dot has the orbital pseudospin number \( \tau \) as a good quantum number since \( [H_0, \tau^2] \neq 0 \) holds. This can be understood as follows. Let us define the total orbital pseudospin of \( j \)-th dot \( \tau_j \) as \( \tau_j^2 = (\tau_{j+} + \tau_{j-})/2 + \tau_{jz}^2 \). The ferromagnetic exchange term of \( H_0 \) in Eq. (11) can be written as \( H_{\text{ex}} \equiv -J_{\text{ex}} \sum_j \left( \tau_{j+} + \tau_{j-} + \tau_{j+} \tau_{j-} \right) \). It can be shown that this term and \( \tau^2 \) do not commute yielding
\[ [H_{\text{ex}}, \tau^2] = 2J \sum_{j=1}^2 (-1)^{j-1}\left| \tau_{j+} \right| \tau_{j+} + \tau_{j+} \tau_{j-} \tau_{j+} = 0 \quad (15) \]
where \( \tau_{3,\pm} \equiv \tau_{1,\pm} \).

We will show nonetheless that for some states
\[ [H_{\text{ex}}, \tau^2]| \psi > = 0. \quad (16) \]
If \( | \psi > \) is a one electron state of \( j \)-th dot then this equation holds since \( H_{\text{ex}} | \psi > = 0 \) and \( H_{\text{ex}} \tau^2 | \psi > = 0 \) (\( \tau^2 | \psi > \) is also a one electron state). Also if \( | \psi > \) has two electrons in a singlet orbital-pseudospin state of the \( j \)-th dot \( | \tau_j = 0, \tau_{jz} = 0 > = \frac{1}{\sqrt{2}}(| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle) \) then the commutator operating to this eigenstate vanishes. It is because the commutator in Eq. (15) has \( \tau_{j+}, \tau_{j-} \) or \( \tau_{jz} \) in
every term and those numbers for the singlet state are all zero as follows: $\tau_+ = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) = \frac{1}{\sqrt{2}}(\uparrow\uparrow - \downarrow\downarrow)$, and $\tau_- = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) = \frac{1}{\sqrt{2}}(\uparrow\uparrow - \downarrow\downarrow) = 0$, and $\tau_z = \frac{1}{\sqrt{2}}(\uparrow\uparrow - \downarrow\downarrow) = 0$. Hence, if any dot of double dots has a singlet orbital-pseudospin states with either $\tau_1 = 0$ or $\tau_2 = 0$, the number of the total orbital pseudospin $\tau$ becomes a good quantum number.

Meanwhile, the $z$ component of the total orbital pseudospin commutes for parabolic dots, $[\mathcal{H}, \tau_z] = 0$, so that $\tau_z$ becomes a good quantum number. For elliptic dots, however, it is not a good quantum number since $[\mathcal{V}_{\text{elliptic}}, \tau_z] \neq 0$.

Spin operators Let $\hat{S}$ be the total spin operator. Both parabolic and elliptic dots have spin rotational symmetry, i.e., $[\mathcal{H}_{\text{tot}}, \hat{S}] = 0$. This symmetry can be verified using the spin operators

$$S_+ = S_x + iS_y = \sum_{\alpha} c_{\alpha 1}^\dagger c_{\alpha -1},$$
$$S_- = S_x - iS_y = \sum_{\alpha} c_{\alpha -1}^\dagger c_{\alpha 1},$$
$$S_z = \frac{1}{2} \sum_{\alpha} (c_{\alpha 1}^\dagger c_{\alpha -1} - c_{\alpha -1}^\dagger c_{\alpha 1}).$$

(17)

Dot pseudospin operators One can consider another pseudospin depending on which dot an electron belongs to, 1 or 2. If the electron resides on the 1st dot, the dot pseudospin of it is given by $\frac{1}{2}$. With the similar definitions to the spin and the orbital pseudospin operators, we have

$$\Omega^2 = \frac{1}{2} (\Omega_+ \Omega_- + \Omega_+ \Omega_+) + \Omega_z^2$$

(18)

with

$$\Omega_+ = \Omega_x + i\Omega_y = \sum_{\alpha} c_{\alpha 1}^\dagger c_{2\alpha},$$
$$\Omega_- = \Omega_x - i\Omega_y = \sum_{\alpha} c_{2\alpha}^\dagger c_{\alpha},$$
$$\Omega_z = \sum_{\alpha} \left( c_{\alpha 1}^\dagger c_{\alpha 1} - c_{2\alpha}^\dagger c_{2\alpha} \right).$$

(19)

In general, the number $\Omega$ is not a good quantum number, i.e., $[\mathcal{H}_0, \Omega^2] \neq 0$ since the term $\mathcal{H}_{\text{ex}}$ does not commute with $\Omega^2$. From the operational calculations, however, we find the number $\Omega$ is well-defined for some eigenstates. If $|\psi\rangle$ is a one-electron state of the $j$-th dot, then $\mathcal{H}_{\text{ex}}|\psi\rangle = 0$. And $\mathcal{H}_{\text{ex}} \Omega^2 |\psi\rangle = 0$ since $\Omega^2 |\psi\rangle$ is also a one-electron state. Hence, $[\mathcal{H}_0, \Omega^2] |\psi\rangle = 0$.

(20)

We also see that $[\mathcal{H}_0, \Omega_z] = 0$ but $[\mathcal{V}_{\text{hop}}, \Omega_z] \neq 0$, and hence $\Omega_z$ does not commute with the total Hamiltonian.

Particle-hole transformation Both parabolic and elliptic dots have particle-hole symmetry. For the number of electrons in ‘p’ orbital $N_p > 4$, one can straightforwardly obtain the energy spectra and the corresponding states from the results for the systems of $N_p \leq 4$ via a particle-hole transformation without detailed calculations. Here we will prove this for the parabolic case. Let us consider first the vacuum state in the hole space. The maximum number of electrons that we can insert on the double dots is 8. In this case there exists one possible state, i.e. all orbitals are filled with particles. This fully occupied state is defined as a hole vacuum $|0\rangle_h$ which is related to the particle vacuum $|0\rangle$ as

$$|0\rangle_h \equiv c_{11}^\dagger c_{11-1}^\dagger c_{11-1,1}^\dagger c_{11-1,1}^\dagger c_{211}^\dagger c_{211-1}^\dagger c_{211-1,1}^\dagger c_{211-1,1}^\dagger |0\rangle.$$  

(21)

The hole operators are defined as $h_{j\alpha\sigma} = c_{j\alpha\sigma}^\dagger (h_{j\alpha\sigma} = c_{j\alpha\sigma})$. They, of course, fulfill the anticommutation relations

$$\{h_{j\alpha\sigma}, h_{j'\alpha'\sigma'}^\dagger\} = \delta_{jj'}\delta_{\alpha\alpha'}\delta_{\sigma\sigma'},$$
$$\{h_{j\alpha\sigma}, h_{j'\alpha'\sigma'}\} = 0,$$

(22, 23)

and operating the hole annihilation operators on $|0\rangle_h$ gives zero, i.e. $h_{j\alpha\sigma}|0\rangle_h = 0$. The total number of holes is given by

$$N_h = \sum_{j\alpha\sigma} h_{j\alpha\sigma}^\dagger h_{j\alpha\sigma} = 8 - N_p.$$  

(24)

By replacing $c_{j\alpha\sigma}$ ($c_{j\alpha\sigma}^\dagger$) with $h_{j\alpha\sigma}^\dagger$ ($h_{j\alpha\sigma}$) and rearranging the operators, our Hamiltonian $\mathcal{H}$ can be easily transformed as

$$\mathcal{H}(U, J, V; t; N_p) =$$

$$\mathcal{H}_h(U, J, V, -t; N_h) + 8E_1 + (3U - J + 4V)(4 - N_h).$$  

(25)

In Eq. 25, the hole Hamiltonian $H_h$ is defined by

$$H_h = H_0^h + V_{\text{hop}}^h.$$  

(26)

with

$$H_0^h = U \left( \sum_{ja} n_{ja1}^h n_{ja-1}^h + \sum_{ja} n_{ja1}^h n_{ja-1,1}^h \right) + J \sum_{ja} h_{ja1}^\dagger h_{ja-1,1}^\dagger h_{ja1} h_{ja-1,1} + V \sum_{a\alpha\sigma} n_{a\alpha\sigma}^h n_{a\alpha\sigma'}^h + E_1 \sum_{ja\alpha} n_{ja\alpha}^h$$

(27)

where $n_{ja\alpha}^h = h_{ja\alpha}^\dagger h_{ja\alpha}$ and $E_1 = -E_1$, and

$$V_{\text{hop}}^h = -t \sum_{a\alpha} \left( h_{1\alpha}^\dagger h_{2\alpha} + \text{h.c.} \right).$$  

(28)

The energy eigenvalues for $H_h$ with $N_h$ holes is nothing more than the spectra for $H$ with the same number of
FIG. 1: (a) Basis set $B^1(N_p = 1)$. $\uparrow \downarrow (\downarrow \uparrow)$ is the electron with up (down) spin. Upper (lower) half circle represents a state in the ‘$p$’ orbital with $z$-component of angular momentum $\alpha = 1(-1)$. Not all the basis states are shown. Other states can be obtained by flipping all spins, by altering the signs of all angular momenta, or by doing both. (b) One of the eigenstates for parabolic dots is shown as an example. This state has the quantum numbers $\sigma = 1$ and $\alpha = 1$ and is given by a linear combination of the basis states in the set $B^1(1)$.

| $\alpha = 1$ | $\sigma = 1$ | $\sigma = 0$ |
|-------------|-------------|-------------|
| $\downarrow \downarrow$ | $\uparrow \uparrow$ | $\downarrow \uparrow$ |

FIG. 2: Some basis states in the set $B^1(N_p = 2)$. States with two electrons in a single dot are not included in this basis set since we are interested only in the low energy states.

| $\alpha = 2$ | $\sigma = 0$ | $\sigma = 1$ |
|-------------|-------------|-------------|
| $\uparrow \uparrow$ | $\uparrow \downarrow$ | $\downarrow \uparrow$ |

FIG. 3: Some basis states in the three sets from $B^1(N_p = 3)$ to $B^3(N_p = 3)$.

| $\alpha = 3$ | $\sigma = 0$ | $\sigma = 1$ |
|-------------|-------------|-------------|
| $\uparrow \uparrow \uparrow$ | $\uparrow \uparrow \downarrow$ | $\uparrow \downarrow \uparrow$ |

| $\alpha = 4$ | $\sigma = 0$ | $\sigma = 1$ |
|-------------|-------------|-------------|
| $\uparrow \uparrow \uparrow \uparrow$ | $\uparrow \uparrow \uparrow \downarrow$ | $\uparrow \uparrow \downarrow \uparrow$ |

FIG. 4: Some basis states in the five sets from $B^1(N_p = 4)$ to $B^5(N_p = 4)$.

| $\alpha = 5$ | $\sigma = 0$ | $\sigma = 1$ |
|-------------|-------------|-------------|
| $\uparrow \uparrow \uparrow \uparrow \uparrow$ |

IV. BASIS STATES AND DEGENERATE PERTURBATIONAL CALCULATION

The total Hamiltonian of our system is $\mathcal{H}_0 + V_{\text{hop}} + V_{\text{elliptic}}$. In the parabolic DQD $V_{\text{elliptic}}$ is absent and we will treat $V_{\text{hop}}$ in degenerate perturbation theory using the eigenstates of $\mathcal{H}_0$ as basis states. In the elliptic DQD we will use the eigenstates of $\mathcal{H}_0$ to find eigenstates of $\mathcal{H}_0 + V_{\text{elliptic}}$. Then we will treat $V_{\text{hop}}$ in degenerate perturbation theory using eigenstates of $\mathcal{H}_0 + V_{\text{elliptic}}$ as basis states.

In both cases we need to find the the eigenstates of $\mathcal{H}_0$. We consider only the states with the energies of the lowest order of $U$ and comment on the number of the degeneracies, $g$, of each eigenenergy. For $N_p = 1$ the eigenenergy and the degeneracy are $(E^{(0)}, g) = (0, 8)$. These basis states $B^1(N_p = 1)$ are shown in Fig. 4 for $N_p = 2$ we find $(E^{(0)}, g) = (V, 16)$. These basis states $B^1(N_p = 2)$ are shown in Fig. 3. For $N_p = 3$ there are three groups of basis states $B^1(N_p = 3)$, $B^2(N_p = 3)$
and $B^3(N_p = 3)$. They have $(E^{(0)}_p, g) = (U + 2V - J, 24)$, $(U + 2V, 16)$ and $(U + 2V + J, 8)$, respectively. For $N_p = 4$ there are five groups of basis states from $B^4(N_p = 4)$ to $B^5(N_p = 4)$. They have $(E^{(0)}_p, g) = (2U + 4V - 2J, 9)$, $(2U + 4V - J, 12), (2U + 4V, 10), (2U + 4V + J, 4)$, and $(2U + 4V + 2J, 1)$, respectively.

Consider basis states $|\phi_j\rangle$ that belong to a group $B^l(N_p)$. Let us call this group $l$ and its energy $E^{(0)}_l$. In the first order degenerate perturbation theory we diagonalize the matrix

$$V^{(1)}_{ij} = <\phi_i|V_{\text{pert}}|\phi_j>, \quad (31)$$

and find the eigenstates and energies. If this matrix is zero we apply the 2nd order degenerate perturbation theory by diagonalizing the matrix

$$V^{(2)}_{ij} = <\phi_i|V_{\text{pert}}|\phi_j>, \quad (32)$$

Note that the state $V_{\text{pert}}|\phi_j\rangle$ may not belong to the group $l$; it may belong to another group $l'$ with degenerate energy $E_{l'}$. The resulting eigenvectors are the linear combinations of the basis states $|\phi_j\rangle$ with appropriate expansion coefficients $c_j$: $|\psi_k\rangle = \sum_j c_j|\phi_j\rangle$. The perturbed eigenergies are $<\psi_k|V_{\text{pert}}|\psi_k> = \frac{1}{E_{l'} - H_{\text{unpert}}} V_{\text{pert}}|\psi_k\rangle$.

In the parabolic DQDs when the number of electrons is odd, the degeneracy of the low energy states for $H_0$ is lifted in the 1st order perturbation of $V_{\text{hop}}$. For a even number of electrons, on the other hand, we need to use the 2nd order energy. It indicates that the hoppings of electrons are more important for the odd-numbered system than for the even-numbered one. All the matrix elements $V_{ij}$ have common proportionality factor $\pm t$, and consequently the expansion coefficients $c_i$ in the eigenvectors are all independent of these factors. For an elliptic potential degenerate perturbation theory is applied twice. First we apply it to $V_{\text{elliptic}}$, then and apply it to $V_{\text{hop}}$ using the eigenstates of $H_0$ and $V_{\text{elliptic}}$.

We have tested our degenerate perturbation theory for the following special case of $t_\alpha = t_\beta = t$, $U_{ab} = U_{ab} = U$ and $U_{ab} = U - J$. We have calculated the eigenstates, eigenenergies, degeneracies, and quantum numbers. These results are consistent with those for the Hamiltonian of the transition metal oxides in Eq. 2.

V. LOW LYING ENERGY STATES FOR PARABOLIC QUANTUM DOTS

The parabolic DQD is described by the Hamiltonian $H_0 + V_{\text{hop}}$. Here the low lying energies and the eigenstates of our DQD model are investigated by using the degenerate perturbation theory for $N_p \leq 4$ and by transforming electrons to holes for $N_p \geq 5$. From now on, in order to avoid the long expression, we drop the constant term $E_1 N_p$ in the Hamiltonian $H_0$ when writing the energy eigenvalues $E^{(0)}_p$ for $H_0$ and $E$ for $H$. We denote the $N_p$-particle eigenfunction for the parabolic Hamiltonian $H$ as $|\psi^{(k)}_{\alpha \tau, \sigma \tau}(N_p)\rangle$ where $\alpha \tau$ and $\sigma \tau$ indicate the $z$-component of the total angular momentum and the twice of the total spin, respectively. In order to distinguish the states with the same values of $\alpha \tau$ and $\sigma \tau$, we do number by assigning $k$. The state with the lower $k$ value has either the same or the lower energy than the higher $k$-valued one. Since the hopping term can be written as $V_{\text{hop}} = -2t\Omega_x$ we need to diagonalize the first order matrix elements $(\Omega_x)_{ij}^{(1)}$. If these first order matrix elements are all zero we need to diagonalize the second order matrix elements $(\Omega_x)_{ij}^{(2)}$ (the definitions of the superscript 1 and 2 are given in Eqs. 31 and 32).

$N_p = 1$: The energy spectrum is depicted in Fig. 5(a). Since $|V_{\text{hop}}, \Omega_x\rangle \neq 0$ leads to the broken dot pseudospin rotational symmetry; the energy is split into two levels. Explicit calculations show that all eigenstates have $\tau = 1/2$, $S = 1/2$ and $\Omega = 1/2$. Each split level is 4-fold degenerate with $\tau_z = \pm 1/2$ ($\alpha = \pm 1$) and $S_z = \pm 1/2$ ($\sigma = \pm 1$). For the ground state $|\psi^{(1)}_{\alpha \sigma}\rangle$ we have $\Omega_x = 1/2$. One of the ground states, which has $\tau_z = 1/2$ and $S_z = 1/2$, is shown, as an example, in Fig. 5(b). Other ground states can be obtained by flipping the orbital-pseudospin or the spin, or doing both in the example state. For the excited states $|\psi^{(2)}_{\alpha \sigma}\rangle$, we have $\Omega_x = -1/2$. For all these eigenstates $[H_0, \tau^2]|\psi^{(k)}_{\alpha \tau, \sigma \tau}(N_p)\rangle = 0$ although the operational identity is as $[H_0, \tau^2]\neq 0$. This is because the eigenstates are one electron states. See Eq. 10 and the explanation following it. Consequently, $\tau$ is a good quantum number. For the same reason $\Omega$ is a good quantum number (see Eq. 20). The spectrum for $N_p = 7$ is exactly the same as that of $N_p = 1$ except that the eigenstates corresponding to $|\psi^{(1)}_{\alpha \sigma}\rangle$ have the
energy $E = 9U - 3J + 12V + t$ while $|\psi^{(2)}_0> \text{ have } E = 9U - 3J + 12V - t$. This is because $t$ is replaced by $-t$ in the particle-hole transformation.

$N_p = 2$: In the basis states we include only states with one electron in each dot. With these basis states the Hamiltonian matrix of $V_{\text{hop}}$ vanishes since the hopping term produces states with two electrons on the same dot. Therefore we apply the 2nd order degenerate perturbational calculation to find eigenstates. The results are shown in Fig. 6(a). For the ground states, we find $S_z = 1$ and $J = 1$. It is 3-fold degenerate with $S_z = \pm 1$ and $S_z = 0$. Fig. 6(b) shows the graphical representations for the ground states with $S_z = 1$ ($\sigma_T = 2$) and $S_z = 0$ ($\sigma_T = 0$). The states with $S_z = -1$ can be obtained by flipping all spins of the state with $S_z = 1$. Note that for all eigenstates each dot contains only one electron. Therefore from Eqs. (16) and (20) it follows that the numbers $\tau$ and $\Omega$ are good quantum numbers of the eigenstates. There are competitions between ferromagnetic and superexchange interactions in our model Hamiltonian. Two-electron parabolic DQDs are stable when the ferromagnetic interactions win, making the spins of electrons parallel. On the other hand, the orbital pseudospins are antiparallel so that $\tau = 0$.

$N_p = 3$: Fig. 7(a) shows how the low energy spectrum for $N_p = 3$ is changed by the perturbation $V_{\text{hop}}$. Note that in the groundstates each dot contains either one or two electrons and the dot with two electrons has the orbital pseudospin singlet state of $\tau = 0$. Therefore from Eq. (16) it follows that $\tau$ is a good quantum number of the groundstates. $\Omega$ also happens to be a good quantum number for the groundstates. The ground-state energy corresponds to the states with $\tau = \frac{1}{2}$, $S_z = \frac{3}{2}$ and $\Omega = \frac{1}{2}$. They have $S_z = \pm \frac{1}{2}$ or $S_z = \pm \frac{3}{2}$ and $\tau_z = \pm \frac{1}{2}$ respectively, yielding the ground state 8-fold degenerate. In Fig. 7(b), we show two ground states of them, one with $\tau_z = \frac{1}{2}$ and $S_z = \frac{3}{2}$ and the other with $\tau_z = \frac{1}{2}$ and $S_z = \frac{1}{2}$. The others can be obtained by flipping all spins or all orbital pseudospins or both. From the ground states, one can notice that our three-electron system favors the total spin of electrons to be maximized, showing the spin ferromagnetism. On the other hand, the orbital and dot pseudospins are minimized. Not all the eigenenergy states are eigenstates of $\Omega^2$, for example, $|\psi^{(2,3)}_0>$. But the numbers $S$, $S_z$ and $\tau_z$ are good quantum numbers of all the eigenenergy states.

$N_p = 4$: After the 2nd-order perturbation calculations, we obtain the energy spectra in Fig. 8(a). The bases $B^1(4)$ with the lowest unperturbed energy $E^{(0)} = 2U - 2J + 4V$ are recombined to be the eigenstates with the ground-state energy $E = 2U - 2J + 4V - 6t^2/(U + J - V)$, and the others with $E = 2U - 2J + 4V - 4t^2/(U + J - V)$ and $E = 2U - 2J + 4V$. The ground state $|\psi^{(1)}_{0,0}(4)>$ is unique and has $\tau_z = 0$, $S = 0$ and $S_z = 0$. It is also the eigenstate for $\tau = 0$. Note that in the groundstates each dot contains two electrons and both dot have
the singlet states of the orbital pseudospins $\tau_1 = \tau_2 = 0$. Therefore from Eq. (10) it follows that $\tau$ is a good quantum number of the groundstates. These results suggest that the system with $N_p = 4$ is stable when the spins and the orbital pseudospins of electrons are antiparallel each other. The reason is as follows. For $N_p = 4$, the electrons occupy four different quantum states of double dots. Unlike the case for $N_p = 1, 2$ and 3, whenever an electron of one dot hops onto the other dot it must face another electron with the opposite spins due to the Pauli exclusion principle. Hence, the superexchange interaction dominates the ferromagnetic interaction in order to lower energy by hopping. It is worth noting that, in the ground state, $S_1 = S_2 = 1$. Within a dot, the spins are ferromagnetic. However, $\vec{S}_1$ and $\vec{S}_2$ are aligned in opposite directions, leading $S = 0$. Furthermore, the total orbital pseudospin $\tau$ is naturally zero. In the excited states, $\tau_\perp$, $S_\perp$ and $S_\parallel$ are good quantum numbers.

5 ≤ $N_p$ ≤ 8: The spectra for $N_p = 5, 6$ and 7 are obtained from those for $N_p = 3, 2$ and 1, respectively, by the particle-hole transformation as we mentioned in Sec. III. The procedure is as below. First, we obtain the energy spectra of $\mathcal{H}^b(U, J, V, t; N_h = 8 - N_p)$, which is nothing but the spectra of $\mathcal{H}(U, J, V, t; N_p = N_h)$. And each eigenstate of $\mathcal{H}^b$ is given by simply writing $h_{j_\alpha\sigma}$ ($h_{j_\alpha\sigma}$) instead of $c_{j_\alpha\sigma}$ ($c_{j_\alpha\sigma}$) and $|0>_{h}$ instead of $|0>_{h}$ in the eigenstate of $\mathcal{H}$. Then, we change the hopping energy from $t - t$ in order to obtain the energy spectrum for $\mathcal{H}^b(U, J, V, t; N_h)$. Finally we add $(3U - J + 4V)(4 - N_h)$ to the energy and transform all states by replacing $h_{j_\alpha\sigma}$ ($h_{j_\alpha\sigma}$) with $c_{j_\alpha\sigma}$ ($c_{j_\alpha\sigma}$) and $|0>_{h}$ with $c_{1111}c_{11-1,11-1,11-1,1}c_{1211}c_{2121,11-1,1,2,1-1,1-1,1}$ for $N_p = 8$. All of the quantum states on the double dots are filled up and the energy of the system is $E = 12U - 4J + 16V$. The ground states for $N_p > 4$ have the same magnetic properties for the states with $8 - N_p$. This results from the fact that $(2N_p - 8)$ particles among $N_p$ electrons always remain nonmagnetic, each two bound as a pair of electrons with up and down spins in the same orbitals on the same dots. For $N_p = 8$, the system is naturally nonmagnetic.

VI. LOW-LYING ENERGY STATES OF ELLIPTIC POTENTIAL

In the following we will investigate the energy spectra for elliptic dots from $N_p = 1$ to $N_p = 4$. We obtain the total energies and the eigenstates up to the vanishing leading terms using two successive degenerate perturbation calculations, the first, with respect to $V_{\text{elliptic}}$ and the second, to $V_{\text{hop}}$, as we mentioned in Sec. IV. In these calculations, the term $E_1N_p$ in the potential $V_{\text{elliptic}}$ as well as in the Hamiltonian $\mathcal{H}_0$ are ignored.

We will denote the final eigenstates for $\mathcal{H}_{\text{tot}}$ as $\psi^{(k)}(N_p)$ > where $\sigma_T$ and $k$ are equivalently defined as in Sec. VI. Note that $\sigma_T$ is dropped this time because $V_{\text{elliptic}}$ breaks the orbital pseudospin’s up-down symmetry so that the $z$-component of the total angular momentum is not conserved anymore.

In the first order perturbation calculation with respect to $V_{\text{elliptic}}$ the matrix $(V_{\text{elliptic}})^{(1)}_{i,j}$ of $(\tau_\parallel)^{(1)}_{i,j}$ has to be diagonalized (see Eq. (11)). All matrix elements of $(V_{\text{elliptic}})^{(1)}_{i,j}$ vanish, the second order matrix elements $(V_{\text{elliptic}})^{(2)}_{i,j}$ is calculated (see Eq. (12)).

$N_p = 1$: The resulting spectra for $N_p = 1$ are shown in Fig. 8 (a). For $N_p = 1$, the Hamiltonian $\mathcal{H}_{\text{tot}}$ is exactly diagonalized. Unlike the Hamiltonian for parabolic dots $\mathcal{H}_0$, the total Hamiltonian does not commute with the $z$-component of the total orbital pseudospin, i.e., $[\mathcal{H}_{\text{tot}}, \tau_\parallel] \neq 0$. This is due to the elliptic potential $V_{\text{elliptic}}$. Hence, after the perturbation $V_{\text{elliptic}}$ is introduced, the energy is split into two levels. Each level is split again into two levels by the hopping term $V_{\text{hop}}$, which is responsible for the fact that $\mathcal{H}_{\text{tot}}$ does not commute with $\Omega_\perp$, i.e., $[\mathcal{H}_{\text{tot}}, \Omega_\perp] \neq 0$ as we mentioned in the previous
The ground state is depicted in Fig. 9 (b). The other ground state can be obtained by flipping the electron's spin. For these ground states $|\psi_{0}^{(1)}\rangle$, we have $(\tau, \tau_y) = (\frac{1}{2}, -\frac{1}{2})$, $(S, S_z) = (\frac{1}{2}, \frac{1}{2})$ and $(\Omega, \Omega_z) = (\frac{1}{2}, \frac{1}{2})$. Note that in all eigenstates each dot contains either one or zero electrons. Therefore from Eqs. (13) and (20) it follows that $\tau$ and $\Omega$ are good quantum numbers. For the excited states, all eigenstates have $\tau = \frac{1}{2}$, $S = \frac{1}{2}$ and $\Omega = \frac{1}{2}$. Each energy is doubly degenerate with $S_z = \pm \frac{1}{2}$. $N_p = 2$: The energy spectra of $\mathcal{H}_{tot}$ for $N_p = 2$ are shown in Fig. 10 (a). Note that similar to the case of two-electron parabolic DQDs, the numbers $\tau$ and $\Omega$ are good quantum numbers. The ground state $|\psi_{0}^{(1)}\rangle$ is unique and is drawn in Fig. 10 (b). It has $(\tau, \tau_y) = (1, -1)$, $(S = 0, S_z) = (0, 0)$ and $\Omega = 1$. One can notice that the eigenvalues of the total orbital pseudospin and the total spin are different from those of the ground state of a parabolic DQD with $\tau = 0$ and $S = 1$. Excited states $|\psi_{2\sigma}^{(2)}\rangle$ and $|\psi_{0}^{(3)}\rangle$ of elliptic DQDs have the same form as the degenerate groundstates $|\psi_{0,2\sigma}^{(1)}\rangle$ and $|\psi_{0,0}^{(3)}\rangle$ of parabolic DQDs. In these ground states of parabolic DQDs, the total orbital pseudospin of two electrons forms a singlet state, $(|\uparrow\downarrow> - |\downarrow\uparrow>)/\sqrt{2}$ with $\tau = 0$. Since $\psi_{elliptic}$ flips all pseudospins in turn, this singlet state vanishes as $2e\tau y/\sqrt{2}(|\uparrow\uparrow> - |\downarrow\downarrow>) = -ie\sqrt{2}(|\uparrow\downarrow> + |\downarrow\uparrow>) = 0$. Hence, the antisymmetric property of the orbital pseudospin singlet leads to the unchanged total energy. In the groundstate of a two-electron elliptic DQD, the total spin is minimized while the total orbital pseudospin is maximized, showing orbital pseudospin ferromagnetism.

$N_p = 3$: The results of the elliptic DQD for $N_p = 3$
TABLE I: Ground-state magnetic properties of parabolic DQDs for different values of \( N_p \). A hyphen means that the relevant quantum number is not a good quantum number.

| \( N_p \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| \( S \) | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 |
| \( \tau \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| \( \Omega \) | 1/2 | 0 | 1/2 | 0 | 1/2 | 0 | 1/2 | 0 |
| \( \Omega_z \) | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 | 1/2 |

are shown in Fig. 11 (a). Each unperturbed energy is split into two levels by \( V_{\text{elliptic}} \): the lower energy state has \( \tau_y = -\frac{1}{2} \) and the higher energy state \( \tau_y = +\frac{1}{2} \). They are split again when the hopping perturbation \( V_{\text{hop}} \) is introduced, as shown in the spectra in the right side. The ground states are four-fold degenerate with \( S_z = \pm \frac{1}{2} \) and \( S_z = \pm \frac{3}{2} \). Two of them are depicted in Fig. 11 (b), and others can be obtained by flipping all spins in the depicted states. In the ground states, we have \((\tau, \tau_y) = \left( \frac{1}{2}, -\frac{1}{2} \right)\), \( S = \frac{3}{2} \) and \((\Omega, \Omega_x) = \left( \frac{1}{2}, \frac{1}{2} \right)\). For the same reason as in the parabolic DQD for \( N_p = 3 \), \( \tau \) and \( \Omega \) are good quantum numbers in the ground states. Similar to the parabolic DQD, the ground states show the spin ferromagnetism while the orbital and the dot pseudospins are minimized. This can be understood as follows: the ground states for elliptic dots are the linear combinations of the parabolic-dot ground states with the same spin quantum number \( S \), i.e., \(|\psi_{S\sigma}^{(1)}(3)\rangle = \frac{1}{\sqrt{2}}(|\psi_{1,3\sigma}^{(1)}(3)\rangle - i|\psi_{-1,3\sigma}^{(1)}(3)\rangle\) and \(|\psi_{S\sigma}^{(1)}(3)\rangle = \frac{1}{\sqrt{2}}(|\psi_{1,3\sigma}^{(1)}(3)\rangle + i|\psi_{-1,3\sigma}^{(1)}(3)\rangle\). Hence, it is natural that they show the same magnetic properties of the ground states of parabolic dots. Note that the numbers \( S \) and \( S_z \) are good quantum numbers in the excited states.

\( N_p = 4 \): Fig. 12 (a) shows the energy spectra for \( N_p = 4 \). The energy of \( E^{(0)} = 2U + 4V - 2J \) and the corresponding basis states \( B^1(4) \) are not changed after the perturbation \( V_{\text{elliptic}} \). The reason is as follows. Each dot in the basis states has \( S_j = 1, \ j = 1, 2 \) (triplet states). The spins of two electrons within a dot are parallel which does not permit the change of the orbital pseudospins of the electrons due to Pauli exclusion principle. Hence, the elliptic potential does not change the basis states \( B^1(4) \). Since only the hopping term \( V_{\text{hop}} \) has an effect on those states, the ground state is the same as that for the parabolic dots. We depict it in Fig. 12 (b). Of course, \( S = 0 \) and \( \tau = 0 \) in the ground state. In the excited states, \( S \) and \( S_z \) are, in general, good quantum numbers.

\( 5 \leq N_p \leq 8 \): The elliptic potential is unchanged under the particle-hole transformation. The energy spectra and the states for \( N_p \geq 5 \) can thus be obtained from those for \( N_p \leq 4 \) via the particle-hole transformation.

VII. CONCLUSION AND DISCUSSION

We have investigated theoretically low excitation states in the ‘p’ molecular orbital of parabolic and elliptic DQDs. In our many body calculation we include intra- and inter-dot electron-electron interactions, in addition to the intradot exchange interaction of ‘p’ orbitals. Wave-functions, energy splittings, and degeneracies of states of the molecular orbitals are determined. The low lying excited states may be probed experimentally by measuring current when a finite source drain voltage is present. Our result shows how many current peaks should be observed in such an experiment.

We have also determined the properties of orbital mag-
| $N_p$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------|---|---|---|---|---|---|---|---|
| $S$  | 1/2 | 0 | 3/2 | 0 | 3/2 | 0 | 1/2 | 0 |
| $\tau$ | 1/2 | 1 | 1/2 | 0 | 1/2 | 1 | 1/2 | 0 |
| $\tau_0$ | -1/2 | -1 | -1/2 | 0 | -1/2 | -1 | -1/2 | 0 |
| $\Omega$ | 1/2 | 1 | 1/2 | -1/2 | 1 | 1/2 | 0 | 1/2 |
| $\Omega_0$ | 1/2 | 1/2 | -1/2 | -1/2 | 1/2 | -1/2 | 0 |

TABLE II: Groundstate magnetic properties of elliptic DQDs for different values of $N_p$.

netism of groundstates. Since the magnetization is directly related to the $z$-component of the total angular momentum our results may be used to determine the value of the magnetization of DQDs. Note that the $z$-component of the total angular momentum is in fact equal to the value of the $z$-component of the total pseudospin. In elliptic DQDs the $z$-component of the total angular momentum is not a good quantum number.

The orbital pseudospin and spin properties for parabolic and elliptic DQDs are summarized in Tables I and II, respectively. We see for parabolic DQDs that the total spin of electrons takes the maximum possible value except for $N_p = 4$ and $N_p = 8$. For these values of $N_p$ the total spin is minimum. The orbital pseudospin is zero for $N_p = 2, 4, 6, 8$. Note that for both $N_p = 2$ and $6$ the spins and pseudospins are ferromagnetic and antiferromagnetic, respectively, i.e., $S = 1$ and $\tau = 0$. However, for elliptic DQDs the opposite is true. In this case the groundstate energy is minimized when the total spin takes the value $S = 0$. This difference in the interplay between orbital pseudospin and spin magnetism in parabolic and elliptic DQDs can be tested experimentally. When a weak magnetic field is applied the groundstate energy of the parabolic DQD should split into three while that of the elliptic dot remains unsplit. A sufficient deformation from the parabolicity is required since the strength of the ellipticity assumed to be stronger than the magnitude of the tunneling energy in our results. In our calculations we have assumed that the exchange interaction between dots is negligible. In some cases the interdot exchange interaction can be important, and can lead to canted phases. Further investigations including the interdot exchange are needed to test the stability of canted states in our DQDs.

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