Double parton interaction: the values of $\sigma_{\text{eff}}$

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(Dated: September 11, 2019)

In this letter we show that the two parton showers mechanism for $J/\Psi$ production, that has been discussed in Ref.[28], leads to small values of $\sigma_{\text{eff}}$ for the production of a pair of $J/\Psi$. We develop a simple two channel approach to estimate the values of $\sigma_{\text{eff}}$, which produces values that are in accord with the experimental data.

PACS numbers: 12.38.Cy, 12.38.g, 24.85.+p, 25.30.Hm

A. Introduction

The double parton interaction has been under close scrutiny over the past three decades both by theoreticians (see Ref.[1] and references therein) and by the experimentalists[2–26]. The sizable cross sections for the double parton interaction at high energy support the assumption, that a dense system of partons is produced in the proton-proton collisions at high energy. Such dense system of partons appears naturally in the CGC/saturation approach to high energy QCD[27], which permits us to consider hadron-hadron, hadron-nucleus and nucleus-nucleus interactions from a unique point of view. As a quantitative measure of the strong double parton interaction, the value of $\sigma_{\text{eff}}$ is used, this was introduced by considering the double inclusive cross sections of two pairs of back-to-back jets with momenta $p_{T,1}$ and $p_{T,2}$, measured with rapidities of two pairs ($y_1$ and $y_2$) which are close to each other ($y_1 \approx y_2$, see Fig. 1). These pairs can only be produced from two different parton showers. The data were parameterized in the form

$$\frac{d\sigma}{dy_1 d^2p_{T,1} dy_2 d^2p_{T,2}} = \frac{m}{2\sigma_{\text{eff}}} \frac{d\sigma}{dy_1 d^2p_{T,1}} \frac{d\sigma}{dy_2 d^2p_{T,2}}$$

where $m = 2$ for different pairs of jet and $m = 1$ for identical pairs. The values of $\sigma_{\text{eff}}$ are shown in Fig. 2 for different final states.

FIG. 1: Two parton showers contribution to two pairs of back-to-back gluon jet production in hadron-hadron collisions (Fig. 1-a). Helical lines denote the gluons. The red helical lines depict the jets. The wavy lines describe the BFKL Pomeron. Fig. 1-b shows the Mueller diagram[29] for the double inclusive cross section while Fig. 1-c is the Mueller diagram for the inclusive cross section.

One can see from this figure, that in spite of large errors, pairs $J/\Psi + J/\Psi$ and $J/\Psi + \Upsilon$ have a small value of $\sigma_{\text{eff}}$ of about 5 mb, while other final states lead to $\sigma_{\text{eff}} \approx 15$ mb.

In this letter we wish to show that the two parton shower mechanism for $J/\Psi$ production, that has been discussed in Ref.[28], leads to small values of $\sigma_{\text{eff}}$ and to develop a simple two channel approach to estimate the values of $\sigma_{\text{eff}}$.

B. $\sigma_{\text{eff}}$ in the BFKL Pomeron calculus

The double inclusive cross section for two pairs of the jets shown in Fig. 1 can be written in the form (see Fig. 1-b)

$$\frac{d\sigma}{dy_1 d^2p_{T,1} dy_2 d^2p_{T,2}} = \int \frac{d^2Q_T}{4\pi^2} \frac{N^2(Q_T)}{g^4(Q_T = 0)} \frac{d\sigma}{dy_1 d^2p_{T,1}}(Q_T) \frac{d\sigma}{dy_2 d^2p_{T,2}}(Q_T)$$

(2)
Experiment (energy, final state, year)

| Experiment     | Energy (GeV) | Final State | Year |
|----------------|--------------|-------------|------|
| ATLAS          | 7            | W + 2 jets  | 2013 |
| CMS            | 8            | W±W±        | 2018 |
| ATLAS          | 8            | 4 leptons   | 2018 |
| LHCb           | 7            | J/ψ + J/ψ   | 2017 |
| LHCb           | 7            | J/ψ + J/ψ   | 2017 |
| CMS            | 8            | W + W±      | 2018 |
| ATLAS          | 8            | J/ψ + J/ψ   | 2017 |
| LHCb           | 7            | 4 jets      | 2016 |
| CMS            | 7            | J + J/ψ     | 2015 |
| ATLAS          | 7            | 4 jets      | 2016 |
| LHCb           | 7            | J + J/ψ     | 2016 |
| DØ             | 1.796        | γ + b + c + 2 jets | 2014 |
| ATLAS          | 7            | W + 2 jets  | 2013 |
| CMS            | 8            | W±W±        | 2018 |
| ATLAS          | 7            | J/ψ + J/ψ   | 2017 |
| LHCb           | 7            | J/ψ + J/ψ   | 2017 |
| CMS            | 8            | W + W±      | 2018 |
| ATLAS          | 7            | J/ψ + J/ψ   | 2017 |
| LHCb           | 7            | J + J/ψ     | 2016 |
| DØ             | 1.796        | γ + b + c + 2 jets | 2014 |

FIG. 2: Summary of measurements and limits on the effective cross section, determined in different experiments[2–26]. The measurements that were made by different experiments are denoted by different symbols and colours. The figures is taken from Ref. [21].

FIG. 3: The two parton shower mechanism for J/Ψ production (see Fig. 3a). Fig. 3b shows the Mueller diagram[29] for the inclusive J/Ψ production.

In Eq. (2) \( \frac{d\sigma}{dy_1 d^2 p_{T,1}} (Q_T) \) describes the production of two back-to-back jets from the BFKL Pomeron[30] and can be written in the form:

\[
\frac{d\sigma}{dy_1 d^2 p_{T,1}} (Q_T) = G_{BFKL}^B (Y - y_1, k_T, Q_T) \otimes \sigma_{hard} \otimes G_{BFKL}^B (y_2, p_{1,T} + p_{2,T} - k_T; Q_T) \tag{3}
\]

where \( \otimes \) stands for all necessary integrations. The amplitude of the BFKL Pomeron \( G_{BFKL}^B (Y - y_1, k_T, Q_T) \) can be simplified if we take into account that the \( Q_T \) dependence of the BFKL Pomeron is determined by the size of the
largest of the interacting dipoles *

\[ G_{BFKL}^Y (Y - y_1, k_T, Q_T) \approx G_{BFKL}^Y (Y - y_1, k_T, Q_T = 0) \ g(Q_T) \]  

Plugging Eq. (5) into Eq. (2) we obtain that

\[ \frac{1}{\sigma_{\text{eff}}} = \int \frac{d^2 Q_T}{4\pi^2} \ \frac{N^2 (Q_T)}{g^4 (Q_T = 0)} \]  

\( N (Q_T) \) is the scattering amplitude of the BFKL Pomeron with the hadron integrated over all produced mass, which has non-perturbative origin and should be taken from non-perturbative QCD or from high energy phenomenology with current embryonic stage of non-perturbative approach. Eq. (2) is discussed in more detail in Ref. [32]. The amplitude \( N(Q_T) \) has a complex structure and can be viewed as the sum of the elastic contribution and the inelastic one which should be summed over entire range of the produced mass (see Fig. 5).

In the two channel approximation we replace the rich structure of the produced states, by a single state with the wave function \( \psi_D \). The observed physical hadronic and diffractive states are written in the form

\[ \psi_h = \alpha \Psi_1 + \beta \Psi_2; \quad \psi_D = -\beta \Psi_1 + \alpha \Psi_2; \quad \text{where} \quad \alpha^2 + \beta^2 = 1; \]  

Functions \( \psi_1 \) and \( \psi_2 \) form a complete set of orthogonal functions \( \{ \psi_i \} \) which diagonalize the interaction matrix \( T \)

\[ A_{i,k}^{r,r'} = \langle \psi_i \ | \ T \ | \psi_{k'} \rangle = A_{i,k} \delta_{i,i'} \delta_{k,k'} . \]  

Bearing Eq. (8) in mind, we can write the following expression for the amplitude \( N \) (see Fig. 5a):

\[ N (Q_T) = \alpha^2 g_1^2 (Q_T) + \beta^2 g_2^2 (Q_T) \]  

* The fact that the \( Q_T \) dependence is determined by the size of the largest dipole stem from the general features of the BFKL Pomeron. Indeed, the eigenfunction of the BFKL Pomeron in coordinate space is equal to [31]

\[ N (r, r'; b) = \left( \frac{r^2 \ r'^2}{(b - \frac{1}{2} (r - r'))^2 (b + \frac{1}{2} (r - r'))^2} \right)^{\gamma} \]  

where \( b \) is the conjugate variable to \( Q_T \). From Eq. (4) one can see that the typical value of \( b \) is of the order of the largest of \( r \) and \( r' \). In our process \( r' \) is of the order of \( R_h \), where \( R_h \) denotes the radius of the nucleon. The value of \( 1/r \) is of the order of the mass of the heavy quark \( m_c \), or the saturation scale \( Q_s \) and, therefore, turns out to be much larger than \( 1/R_h \), and can be neglected. The dependence on \( Q_T \approx 1/R_h \) is described by \( g(Q_T) \) in Eq. (5), which has a non-perturbative origin and, in practice, has to be taken from the experiment.
FIG. 5: Diffractive production of the $J/\Psi$ meson in deep inelastic scattering at low $x$. Wavy lines denote the BFKL Pomerons.

$g_i(Q_T)$ are phenomenological functions while $\alpha$ is a constant whose value we determine by fitting to the experimental data. Fortunately, we can determine the values of $g_i(Q_T)$ and $\alpha$ by considering the diffractive production of $J/\Psi$ in DIS. These data indicate that the diffractive production of $J/\Psi$ has different slopes in $Q_T$ for elastic (see Fig. 5-a) and for inelastic (see Fig. 5-b) production. For the reaction $\gamma^* + p \rightarrow J/\Psi + X$ the slope turns out to be much smaller $d\sigma_{\text{inel. diff.}}/dQ_T^2 \propto \exp(-B_{\text{in}}Q_T^2)$ with $B_{\text{in}} \approx 1.69 GeV^{-2}$. The second conclusion from the data is that the cross sections of elastic and inelastic $J/\Psi$ diffractive production are the same. We can implement the described features of the diffractive production of $J/\Psi$ by introducing $g_1(Q_T) = g_1 e^{-\frac{1}{2}B_1 Q_T^2}$ and $g_2(Q_T) = g_1 e^{-\frac{1}{2}B_2 Q_T^2}$ and imposing the following restrictions on the parameters

$$B_1 = B_{\text{el}}; \quad B_2 = B_{\text{in}}; \quad \alpha^2 \frac{g_1^2}{B_1} = \beta^2 \frac{g_2^2}{B_2}$$

(10)

Plugging Eq. (5) into Eq. (2) we have

$$\frac{1}{\sigma_{\text{eff}}} = \int \frac{d^2 Q_T}{4\pi^2} \frac{(\alpha^2 g_1^2(Q_T) + \beta g_2^2(Q_T))^2}{(\alpha^2 g_1(0) + \beta^2 g_2(0))^4}$$

(11)

In Fig. 6 we plot the value of $\sigma_{\text{eff}}$ for a pair of back-to-back jets.

FIG. 6: $\sigma_{\text{eff}}$ for a pair of back-to-back jets (see Fig. 6-a and Eq. (2)) and for pair of $J/\Psi$ (Fig. 6-b). The red lines show $\sigma_{\text{eff}} = 15 \text{ mb}$ in Fig. 6-a and $\sigma_{\text{eff}} = 5 \text{ mb}$ in Fig. 6-b.

We reach the average experimental value of $\sigma_{\text{eff}} = 15 \text{ mb}$ at $\alpha = 0.8$. It should be noted that the information that we obtain from the diffractive production of $J/\Psi$ is enough to claim that $\sigma_{\text{eff}} \leq 20 \text{ mb}$ in a two channel model.
C. \( \sigma_{\text{eff}} \) for the production \( J/\Psi \) and \( \Upsilon \) pairs

As one can see from Fig. 4 the single inclusive production of \( J/\Psi \) is proportional to

\[
\frac{d\sigma}{dy} \propto g(Q_T = 0) \int \frac{d^2 Q_T}{4\pi^2} N(Q_T) = (\alpha^2 g_1(Q_T = 0) + \beta^2 g_2(Q_T = 0)) \int \frac{d^2 Q_T}{4\pi^2} \left( \alpha^2 g^2_1(Q_T = 0) + \beta^2 g^2_2(Q_T) \right) 
\]

\[
= \frac{1}{4\pi} (\alpha^2 g_1 + \beta^2 g_2) \left( \alpha^2 g^2_1 + \beta^2 g^2_2 \right) = \frac{1}{4\pi} \left( \alpha^2 g_1 + \beta^2 g_2 \right) \left( \alpha^2 g^2_1 \right) \tag{12}
\]

The Mueller diagrams for the double inclusive cross section of \( J/\Psi \) pair production are shown in Fig. 7. From this figure we see that the double inclusive cross section can be estimated using the amplitudes of the interaction with three and four Pomerons, which are shown in Fig. 5-b and Fig. 5-c. Using these amplitudes, we obtain

\[
\frac{d^2\sigma}{dy_1 dy_2} \propto \int \frac{d^2 Q_T}{4\pi^2} \frac{d^2 Q'_T}{4\pi^2} \frac{d^2 Q''_T}{4\pi^2} \left( \alpha^2 g^2_1(Q_T) + \beta^2 g^2_2(Q_T) \right) 
\]

\[
\times \left( \alpha^2 g_1 \left( Q'_T + \frac{1}{2} Q_T \right) g_1 \left( -Q'_T + \frac{1}{2} Q_T \right) g_1 \left( Q''_T + \frac{1}{2} Q_T \right) g_1 \left( -Q''_T + \frac{1}{2} Q_T \right) 
\]

\[
+ \beta^2 g_2 \left( Q'_T + \frac{1}{2} Q_T \right) g_2 \left( -Q'_T + \frac{1}{2} Q_T \right) g_2 \left( Q''_T + \frac{1}{2} Q_T \right) g_2 \left( -Q''_T + \frac{1}{2} Q_T \right) \right) 
\]

\[
+ \left( \alpha^2 g_1 \left( Q'_T + \frac{1}{2} Q_T \right) g_1 \left( -Q'_T + \frac{1}{2} Q_T \right) g_1 (Q_T) + \beta^2 g_2 \left( Q'_T + \frac{1}{2} Q_T \right) g_2 \left( -Q'_T + \frac{1}{2} Q_T \right) g_2 (Q_T) \right) 
\]

\[
\times \left( \alpha^2 g_1 \left( Q''_T + \frac{1}{2} Q_T \right) g_1 \left( -Q''_T + \frac{1}{2} Q_T \right) g_1 (Q_T) + \beta^2 g_2 \left( Q''_T + \frac{1}{2} Q_T \right) g_2 \left( -Q''_T + \frac{1}{2} Q_T \right) g_2 (Q_T) \right) \right) \tag{13}
\]

All momenta are shown in Fig. 7. From Eq. 12 and Eq. 13 we estimate the values of \( \sigma_{\text{eff}} \) using Eq. 1. In Fig. 6-b we plot the results of these estimates. Comparing Fig. 6-a and Fig. 6-b one can see that the values of \( \sigma_{\text{eff}} \) for \( J/\Psi \) pair production, turns out to be smaller than 5 mb, which in accord with the experimental data of Fig. 2.

![Mueller diagrams](image)

**FIG. 7:** Mueller diagrams for the double inclusive cross section of pair \( J/\Psi \) production.

D. Conclusions

In this letter we demonstrated that the two parton showers mechanism leads to much smaller values of \( \sigma_{\text{eff}} \leq 5 \text{ mb} \) for the double parton interaction. This result is in qualitative agreement with the data, and we consider it as a support for the idea that the production of two parton showers is responsible for \( J/\Psi \) inclusive cross section. The second result of this letter is the claim, that a simple two channel model with restrictions that stem from the results...
of the experiments on diffractive production in DIS, is able to describe the size of the double parton interaction at high energies leading to the values of $\sigma_{int} \lesssim 20 \text{mb}$ for 4 jets production, in accord with the experimental data.

Our numerical estimates are based on Eq. (10) and on the numerical values for the slopes $B_{\ell}$ and $B_{in}$. We checked that the smallness of $\sigma_{int}$ for $J/\Psi$ pair production has only a mild dependence on the value of $B_{in}$, which was measured with large errors. The two channel model has been used for describing the soft high energy data. It should be noted, that for the BFKL Pomeron, whose intercept is larger than 1, the integral over the produced mass in diffraction is convergent, and the Good-Walker mechanism is able to describe the diffractive production of both small and large masses. Therefore, we believe it reasonable to use this model to obtain the first estimates of the value of $\sigma_{int}$.

Acknowledgements.

We thank all participants of Low-x 2019 WS and especially L. Motyka for encouraging discussions. This research was supported by CONICYT PIA/BASAL FB0821(Chile) and Fondecyt (Chile) grant 1180118.
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