Investigation of the Effect of the Mortar Thickness in Masonry Minarets by Using Anisotropic Model

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Abstract. As is known, the masonry structures, together with the main structural materials, such as stone or brick, consist of mortar material which serves as the adhesive between these materials. In general, the mechanical properties of brick, stone and mortar materials are different. However, they have been used throughout history and have been the unchanging materials used in masonry structures. The structural walls are formed by the heterogeneous joining of two materials with different mechanical properties. However, these heterogeneous structural elements are generally considered homogeneous for ease of calculation. In this study, the effects of different joint thicknesses on masonry structures on minarets structure were investigated. Changes in structure behaviour were investigated by using different material properties to a masonry minaret structure. By examining these differences, the effect of isotropic or anisotropic modelling on the result, the effect of mortar thickness on the result was compared and the results were interpreted.

1. Introduction
While examining masonry structures, there are some differences between them. Vertical loads in masonry structures are generally less than the vertical bearing capacity of the bearing wall. Therefore, damages in masonry structures generally consist of horizontal loads. In addition, cracks due to ground settlements, loading and movement of the wall outside the axis can be shown among the reasons.

Although an idea can be made about cracks that may occur in bearing walls under horizontal loads in masonry structures, it is not known exactly whether the crack will form in joint or brick or stone. This uncertainty can have many different causes. Many variables such as joint material used, the thickness of the joint, the quality of the joint work, the size of the stone material used, the mechanical properties of the brick, can affect the formation of the crack.

2. Micro model in composite material
The main purpose of modelling masonry structures is to create a model that acts as close to the real structure as possible. It is a composite material consisting of masonry structure, brick and mortar. Micro-modelling becomes more realistic, considering its components one by one, but not practical for a large build. To overcome this problem, anisotropic continuity is obtained by using the material properties of the macro-model components and the composite material behaviour in the micro-model. The presence of horizontal and vertical grout joints in the wall construction shown in Figure 1 causes
the wall to be anisotropic. It increases the need to consider both the mortar thickness and the anisotropy of different brick wall structures.

![Figure 1. Typical masonry wall model](image1)

In the study conducted by Laurenço [1], the spring constants that can be used in the joints between the bricks are obtained from the

\[ k_n = \frac{E_u \times E_m}{t_m \times (E_u - E_m)}, \quad k_s = \frac{G_u \times G_m}{t_m \times (G_u - G_m)} \] (1)

equations and solutions are made. In addition, the elasticity module used in some studies and experiments to switch to the macro model is given with the equation

\[ E_k = \frac{(t_m + t_u) \times \rho_k}{(t_m/E_m) + (t_u/E_u)} \] (2)

where \( E_k \) is the equivalent modulus of elasticity for the composite material, \( t_m \) is mortar thickness, \( t_u \) is brick thickness, \( E_m \) is mortar elasticity module, \( E_u \) is a brick modulus of elasticity. \( \rho_k \) is a coefficient ranging from 0 to 1 for adherence between brick and mortar.

Therefore, as shown in Figure 2, the change in mortar thickness was examined. Linear elastic assumptions were made in brick and mortar, and elasticity constants were obtained depending on mortar thickness / brick thickness ratios.

![Figure 2. (a) horizontal and vertical mortar thickness fixed, (b) horizontal mortar thickness increased (c) macro model](image2)

An example of such a wall structure, the photograph taken during the restoration of a minaret in Istanbul, where the mortar / brick thickness ratio is close to 1, is shown in Figure 3.
2.1. Stress-strain relations
Stress-strain relations in elastic material: There are 36 independent material constants in general. These material constants fall to 21 in anisotropic material, 13 in monocline material, 9 in orthotropic material, 5 in transverse isotropic material and 2 in isotropic material.

Stress-strain relations For the plane stress condition, there are 4 independent constants in the transversely isotropic material. \((E_1, E_2, \nu_{12}, G_{12})\) Stress-strain relations in transversely isotropic material:

\[
\varepsilon_1 = \frac{1}{E_1} \sigma_1 - \frac{\nu_{12}}{E_1} \sigma_2, \quad \varepsilon_2 = \frac{1}{E_2} \sigma_2 - \frac{\nu_{12}}{E_1} \sigma_1, \quad \gamma_{12} = \frac{\tau_{12}}{G_{12}},
\]

where \(E_1, E_2, \nu_{12}, G_{12}\) are elastic constants of composite material.

2.2. Obtaining \(E_1, E_2, \nu_{12}, G_{12}\) elastic constants of composite material
In the \(\sigma_1\) loading in direction 1, the equations of strain (elongation ratios oranları = \(\Delta L / L\)) are obtained from the condition \(\varepsilon_1 = \varepsilon_u = \varepsilon_m\) (geometric conformity condition). Total force in equilibrium equations \(F_1; F_u; F_m\): the force the brick carries; \(F_m\): the force carried by the mortar; \(A_1\): total cross-sectional area; \(A_u\): cross-sectional area of the brick; \(A_m\): the cross-sectional area of the mortar; \(V_u\): ratio of brick area to total area; \(V_m\): ratio of mortar area to total area; \(\sigma_1\): Stress in 1 axis; \(\sigma_u\): stress in brick; \(\sigma_m\): It is the stress in the mortar [2], [3].

Equilibrium equation \(F_1=F_m+F_u\); stress strain relations: using \(\sigma_i=E_i\varepsilon_i\) and stress: \(F_i=\sigma_iA_i\), the equilibrium equation
\[ E_1 \varepsilon_1 A_1 = E_u \varepsilon_u A_u + E_m \varepsilon_m A_m, \quad E_1 = E_u V_u + E_m V_m \] (4)

For the calculation of the \( E_2 \) material constant, taking into account the continuity condition of the stresses in the materials loading \( \sigma_2 \) (Figure 4b) in the direction of 2 \( \sigma_2 = \sigma_u = \sigma_m \) and the sum of displacements \( \Delta_2 = \Delta_u + \Delta_m \)

\[ \varepsilon_2 W = \varepsilon_u W V_u + \varepsilon_m W V_m, \quad \frac{1}{\varepsilon_2} = \frac{V_u}{\varepsilon_u} + \frac{V_m}{\varepsilon_m} \] (5)

Considering the ratio of deformations in the loading of \( \sigma_1 \) in direction 1 (figure 4a), the sum of \( \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} \) and vertical displacements are considered as

\[ \varepsilon_1 \nu_{12} W = \varepsilon_u W V_u \nu_u + \varepsilon_m W V_m \nu_m \quad , \quad \nu_{12} = \nu_u \nu_u + \nu_m \nu_m \] (6)

is written as the total amount of expansion. Using stress strain equation, Poisson's ratio is calculated.

Due to the shear stress in the composite material (figure 4c), the total expansion amount is written as \( \gamma = \frac{M}{W} \quad \Delta_1 = \Delta_u + \Delta_m \) from the sum of the deformations. \( G_{12} \) shear modulus is obtained by using the stress strain equation:

\[ \gamma W = W V_u \gamma_u + W V_m \gamma_m \quad \frac{1}{G_{12}} = \frac{V_u}{\gamma_u} + \frac{V_m}{\gamma_m} \] (7)

3. The transition from Micro Model to Macro Model

In this study, using the equations 4, 5, 6, 7 obtained from micro models, calculations were made by taking mortar elasticity module 2000MPa, brick elasticity module 20,000 MPa, mortar Poisson ratio 0.25 and brick Poisson ratio 0.15. Anisotropic material constants were obtained according to 6 different mortar / brick thickness ratios. Material constants, \( E_1, E_2, \nu_{12} \) and \( G_{12} \) values containing material data of macro models are given in Table 1 (Figure 5).

Isotropic walls are constructed by taking the \( E_2 \) value into consideration and used in this study to compare with the corresponding anisotropic wall and minaret models. In the isotropic model, \( E_1 = E_2 \) is taken for composite materials based on the idea of idealizing the brick and mortar as two linear springs connected in series.

**Table 1.** Material coefficients according to the ratio of mortar thickness to a total thickness

| Material constants | 1  | 2  | 3  | 4  | 5  | 6  |
|--------------------|----|----|----|----|----|----|
| \( V_u \)          | 1.0| 0.8| 0.6| 0.4| 0.2| 0.0|
| \( V_m \)          | 0.0| 0.2| 0.4| 0.6| 0.8| 1.0|
| \( E_1 \) (MPa)    | 20000| 16400| 12800| 9200| 5600| 2000|
| \( E_2 \) (MPa)    | 20000| 7143 | 4348 | 3125| 2439| 2000|
| \( \nu_{12} \)     | 0.15| 0.17| 0.19| 0.21| 0.23| 0.25|
| \( G_{12} \) (MPa) | 8696| 2924| 1757| 1256| 978 | 800|
Figure 5. Value of elasticity modules according to the brick thickness / mortar thickness change.

Typical walls are considered for isotropic-anisotropic comparisons. For this, typical walls of 4.0m width and 3.0m height and 0.25m thickness are considered. Isotropic and anisotropic solutions were made with SAP2000 program [4]. 1.0MPa normal stress, 1.0MPa shear stress was applied to the top surface of the wall as loading and the results of isotropic anisotropic solutions were compared in some parts of the wall [5], [6].

Isotropic walls are modelled considering the E2 value and used to compare anisotropic wall models (Figure 6). Three different types of walls are considered according to the wall space / total area ratio. These typical walls are; (a) wall void ratio = 0, (b) wall void ratio = 0.185, (c) wall void ratio = 0.615.

The values of $\sigma_{\text{max}}$, $\sigma_{\text{min}}$ formed on the wall base were calculated according to the wall total area/void area ratio. As the brick thickness / mortar thickness ratio, calculations are made according to $V_m/V_u = 0.6/0.4$ values and $\sigma_{\text{max}}$ diagram under the effect of the horizontal load is given in Figure 7 in the anisotropic environment. In addition, the maximum normal stress $\sigma_{\text{max}}$, horizontal displacement $u_1$ and vertical displacement $u_2$ values obtained from isotropic and anisotropic solutions are given in Table 2.

Table 2. Stress at the bottom and displacements at the top surface in typical walls loaded with a horizontal load

| Horizontal load | Wall (a) | Wall (b) | Wall (c) |
|-----------------|---------|---------|---------|
| $\sigma_{\text{max}}$ (MPa) | Anisotropy | 7.792 | 12.990 | 19.310 |
| | Isotropy | 6.691 | 11.060 | 18.120 |
| | Anisotropy/ Isotropy | 1.165 | 1.175 | 1.066 |
| $u_1$ (mm) | Anisotropy | 5.078 | 8.164 | 14.230 |
| | Isotropy | 5.182 | 9.012 | 16.480 |
| | Anisotropy/ Isotropy | 0.98 | 0.910 | 0.860 |
| $u_2$ (mm) | Anisotropy | 2.549 | 2.821 | 3.039 |
| | Isotropy | 2.675 | 3.191 | 3.695 |
| | Anisotropy/ Isotropy | 0.950 | 0.880 | 0.820 |
4. Analysis of Minaret with Anisotropic Model

To highlight the objectives of this study, typical minaret examples shown in Figure 8 are discussed. The features of this minaret are 1.74m in diameter at the base, the average diameter of 1.07m in the highest area, 20.83m in Type 1 and 17.56m in Type 2.

Material constants for static, dynamic and modal analysis are taken from Table 1 and the unit volume weight of the wall structure is taken as 20kN/m$^3$. The minaret is modelled with SAP2000 program and displacement and stresses are examined under the effect of horizontal load and vertical load. Only the weight of its own weight was taken as the vertical load. Thin shell elements are used in this model. In the horizontal load calculation, spectrum characteristic periods $T_A = 0.15$ sec, $T_B = 0.40$ sec, building behaviour coefficient $R = 2$, building importance coefficient $I = 1.5$ and damping ratio 0.05. It was selected.
In the minaret, bricks thickness/mortar thickness ratio was selected as **material 1**: 0.8/0.2 and **material 2**: 0.4/0.6 and calculations were made according to two different situations. The maximum $\sigma_{11}$, $\sigma_{22}$, $\sigma_{12}$ stresses at the base obtained from isotropy and anisotropy solutions in the minaret under the effect of horizontal load and vertical load and the horizontal displacement $u_2$ values at the top point of the minaret are given in Table 3, 4, 5. In addition, anisotropy/isotropy result rates are given in the last line of the tables. When it is examined in terms of displacements, the isotropic model and anisotropic models are close to each other.

However, when examined in terms of stresses, the stresses change in the material with the change of the mortar thickness. Therefore, modelling with an anisotropic model will be more realistic.

**Table 3.** Type 1 minaret (minaret at a height of 20.83m)

|            | Material 1 | Material 2 |
|------------|------------|------------|
| Vertical load | $\sigma_{11}$ | $\sigma_{12}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{12}$ |
| Anisotropy  | 1.238      | 0.531      | 1.131        | 0.484        |
| Isotropy   | 1.156      | 0.468      | 0.941        | 0.477        |
| Anisotropy/Isotropy | 1.071      | 1.135      | 1.202        | 1.179        | 1.015        |

|            | Material 1 | Material 2 |
|------------|------------|------------|
| Horizontal load | $\sigma_{11}$ | $\sigma_{12}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{12}$ |
| Anisotropy  | 3.050      | 0.597      | 3.862        | 0.820        |
| Isotropy   | 1.065      | 0.450      | 0.835        | 0.540        |
| Anisotropy/Isotropy | 2.864      | 1.327      | 4.625        | 1.197        | 1.519        |

**Table 4.** Type 2 minaret (minaret at a height of 17.56m)

|            | Material 1 | Material 2 |
|------------|------------|------------|
| Vertical load | $\sigma_{11}$ | $\sigma_{12}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{12}$ |
| Anisotropy  | 1.219      | 0.516      | 1.093        | 0.480        |
| Isotropy   | 1.146      | 0.471      | 0.928        | 0.481        |
| Anisotropy/Isotropy | 1.064      | 1.096      | 1.178        | 0.998        |

|            | Material 1 | Material 2 |
|------------|------------|------------|
| Horizontal load | $\sigma_{11}$ | $\sigma_{12}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{12}$ |
| Anisotropy  | 2.868      | 0.457      | 3.618        | 0.573        |
| Isotropy   | 1.001      | 0.438      | 0.865        | 0.443        |
| Anisotropy/Isotropy | 2.865      | 1.043      | 4.183        | 1.293        |

**Table 5.** Comparison of long minaret/short minaret (Type 1/Type 2)

|            | Material 1 | Material 2 |
|------------|------------|------------|
| Horizontal load | $\sigma_{11}$ | $\sigma_{12}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{12}$ |
| Anisotropy  | 1.063      | 1.306      | 1.067        | 1.431        |
| Isotropy   | 1.064      | 1.027      | 0.965        | 1.219        |

**5. Conclusions**

In this study, macro model anisotropic models were obtained from micro model walls using the values given in Table 1. In the anisotropic wall models, it was observed that the $E_1$ modulus of elasticity in the horizontal direction was higher. Therefore, it can be concluded that the anisotropic model is always higher than the isotropic stiffness in the horizontal direction.
Calculations were made for the three wall configurations (a), (b), (c) using the anisotropic and isotropic wall features in case of horizontal loading. The maximum stress at the wall base and the displacements at the top of the wall were calculated. It was observed that the mortar thickness in the filled wall (type a) was effective in terms of stress, and in the type c (void ratio 0.165), the isotropic model was 0.82 times the anisotropic model.

In this study, 2 types of minarets were modelled as isotropies and anisotropies according to their height. Calculations were made such that the ratio of mortar thickness to total thickness is 0.6 and 0.2. It has been observed that the effect of the anisotropic solution increases with increasing mortar thickness. It is understood that the effect of $\sigma_{11}$ stresses in the horizontal direction is more effective than other stress components. Since only the vertical elasticity module is taken in the effect of mortar/brick thickness in the spring resemblance or isotropic model, the horizontal elasticity module is not considered. The period and horizontal displacement of the minaret, isotropic model and anisotropic models are very close to each other.

It is important to investigate the wall features in the horizontal direction, since the damage in masonry structures mostly occurs due to horizontal loading. In the anisotropic solutions, the maximum $\sigma_{11}$ stress that occurs in the long minaret and the vertical load at the base is 1.202 times of the isotropic solution; stretching $\sigma_{11}$ at horizontal load 4.625 times the isotropic solution; stretching the $\sigma_{11}$ at vertical load in the short minaret 1.178 times the isotropic solution; $\sigma_{11}$ stress at horizontal load was obtained as 2.865 times of the isotropic solution.

In the comparison of long minaret and short minaret (table 5), in the anisotropic solutions, by increasing the mortar thickness in the long minaret ($V_u/V_m = 0.4/0.6$), the maximum $\sigma_{12}$ stretching at the base was obtained as 1.431 times of the short minaret solution.

As a result, the effect of the mortar thickness in the masonry structures is recommended to obtain not only the modulus of elasticity in the vertical direction, but also the effect of the modulus of elasticity in the horizontal direction as anisotropic.

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