The Colour-Magnitude Relation as a Constraint on the Formation of Rich Cluster Galaxies

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ABSTRACT

The colours and magnitudes of early-type galaxies in galaxy clusters are strongly correlated. The existence of such a correlation has been used to infer that early-type galaxies must be old passively evolving systems. Given the dominance of early-type galaxies in the cores of rich clusters, this view sits uncomfortably with the increasing fraction of blue galaxies found in clusters at intermediate redshifts, and with the late formation of galaxies favoured by CDM-type cosmologies. In this paper, we make a detailed investigation of these issues and examine the role that the colour-magnitude relation can play in constraining the formation history of galaxies currently found in the cores of rich clusters. We start by considering the colour evolution of galaxies after star formation ceases. We show that the scatter of the colour-magnitude relation places a strong constraint on the spread in age that is allowed for the bulk of the stellar population. In the extreme case that the stars are formed in a single event, the spread in age cannot be more than 4 Gyr. Although the bulk of stars must be formed in a short period, continuing formation of stars in a fraction of the galaxies is not so strongly constrained. We examine a model in which star formation occurs over an extended period of time in most galaxies with star formation being truncated randomly. This model is consistent with the formation of stars in a few systems until look-back times of \( \sim 5 \) Gyr. An extension of this type of star formation history allows us to reconcile the small present-day scatter of the colour-magnitude relation with the observed blue galaxy fractions of intermediate redshift galaxy clusters.

In addition to setting a limit on the variations in luminosity-weighted age between the stellar populations of cluster galaxies, the colour-magnitude relation can also be used to constrain the degree of merging between pre-existing stellar systems. This test relies on the slope of the colour-magnitude relation: mergers between galaxies of unequal mass tend to reduce the slope of the relation and to increase its scatter. We show that random mergers between galaxies very rapidly removes any well-defined colour-magnitude correlation. This model is not physically motivated, however, and we prefer to examine the merger process using a self-consistent merger tree. In such a model there are two effects. Firstly, massive galaxies preferentially merge with systems of similar mass. Secondly, the rate of mass growth is considerably smaller than for the random merger case. As a result of both of these effects, the colour-magnitude correlation persists through a larger number of merger steps.

The passive evolution of galaxy colours and their averaging in dissipationless mergers provide opposing constraints on the formation of cluster galaxies in a hierarchical model. At the level of current constraints, a compromise solution appears possible. The bulk of the stellar population must have formed before \( z = 1 \), but cannot have formed in mass units much less than about half the mass of a present-day \( L^* \) galaxy. In this case, the galaxies are on average old enough that stellar population evolution is weak, yet formed recently enough that mass growth due to mergers is small.

Key words: galaxies: formation – galaxies: evolution – galaxies: stellar content
1 INTRODUCTION

The existence of a strong correlation between the colour and luminosity of early-type galaxies in rich clusters is well established (e.g., Sandage & Visvanathan, 1978, Larson, Tinsley & Caldwell, 1980). This colour-magnitude relation (CMR) implies a physical link between the stellar populations of galaxies and their total stellar mass. The best explanation appears to be a correlation between the metal abundance of the stellar population and total galaxy mass (e.g., Arimoto & Yoshii, 1987, Bower, Lucey & Ellis, 1992b, BLE), although explanations based on the age of galaxies and on their stellar initial mass function (IMF) have also been proposed (e.g., Trager et al., 1993, Tantalo et al., 1997). Metal abundance variations are widely accepted as the more likely explanation because observations of the CMR in high redshift clusters show a uniform shift towards bluer colours for galaxies of all magnitudes while maintaining small scatter about the mean relation (Ellis et al., 1997, Kodama & Arimoto 1997, Stanford, Eisenhardt & Dickinson 1997, Kodama et al. 1998). As we discuss below, the tightness of the CMR leaves little room for additional variations in galaxy age.

In this paper, we explore the constraints that can be placed on the formation of galaxies by the existence of the CMR. Through out this paper, we will assume that the metal abundance and age of a galaxy are uncorrelated — this is the case for both the super-novae wind models of Arimoto & Yoshii (1987) and for self-regulated hierarchical galaxy formation models (White & Frenk, 1991, Kauffmann et al., 1993, Cole et al., 1994, and subsequent papers). Ferras et al. (1998) provide an extended discussion of the formal limits on correlated variations. We focus on the scatter of galaxies about the mean correlation and on the slope of the correlation relative to the observed scatter. For galaxies in clusters the rms residuals are remarkably small, less than 0.05 mag rms in $U - V$ for E and S0 galaxies in the Coma cluster. This implies a marked homogeneity in the galaxies' formation that provides an important constraint for theoretical models. For example, if the general trend of increasingly blue colours at fainter magnitudes were due to age, the galaxies would be required to range in ‘age’ (i.e., the look-back time to the epoch at which the bulk of the stars were formed) by $\sim 7$ Gyr across the first 3 mag of the relation (with the larger galaxies being formed at earlier times), while being coordinated so that galaxies of the same magnitude ‘formed’ with an rms dispersion substantially less than this. Other models for the origin of the relation are similarly constrained. Under the assumption that the CMR is driven by age, BLE used this argument to suggest that most galaxies must have formed the bulk of their stars at redshifts greater than unity. The CMR data also limit the possible contribution of bursts of star formation after the formation of the main stellar component and the degree to which present-day galaxies can have been built-up by the merger of stellar (as opposed to gaseous) systems.

Taken in isolation, the tightness of the colour-magnitude relation suggests that most galaxies in clusters are old systems, but this picture appears to conflict with observations of intermediate redshift clusters. Photometric studies (e.g., Butcher & Oemler, 1978, 1984, Couch & Newell, 1984) have suggested very significant changes to the clusters’ mix of galaxy types over look-back times of 3-5 Gyr. Prob-ably, this change is associated with the transformation of Spiral galaxies into S0 types (Dressler et al., 1997, Couch et al., 1997). This picture is reinforced by spectroscopic observations showing that a large fraction of the galaxies in these clusters have only just completed their star formation cycle (e.g., Dressler & Gunn, 1983, Couch & Sharples, 1987, Barger et al., 1996). Can this evidence of star formation activity at relatively modest look-back times be compatible with the homogeneity of the present-day galaxy populations? To answer this question, we must carefully examine the scatter that would be introduced by a population of galaxies that were forming stars in the recent past. In BLE’s data for the Coma cluster, for example, a small number of galaxies do not fit on the ridge-line of the CMR. It is important that we correctly account for the treatment of out-lying galaxies when comparing the present-day and intermediate redshift observations, and that we allow for the full range of morphological types seen in present-day clusters.

The well-defined slope of the CMR implies that the star formation process is linked to the final mass of the system: if most of the star formation were to occur in systems of low mass, a physical link between the initial systems and their final mass would be required. In the absence of such a process, the CMR sets a strong constraint on the number of pieces that can be combined to form the final system.

The approach outlined by BLE does not rely on a specific model for galaxy formation in order to derive constraints from the observational data. The short-coming of this approach is that it takes no account of the inevitable gravitational evolution of the dark matter haloes in which the galaxies are contained. For example, although the early-type galaxies on which this data is based currently reside in rich clusters, at $z = 2$ this cluster scale over-density would not yet have collapsed; the typical dark matter halo been have had a mass close to that of a small galaxy group, and the finally galaxy would probably not be recognisable as a single unit. White & Frenk (1991), Kauffmann et al. (1993), Cole et al. (1994) and subsequent papers, have therefore pursued an alternative approach. They take as a starting point the extended Press-Schechter theory (Press & Schechter, 1974, Bower, 1991, Bond et al., 1991, Lacey & Cole 1993). This allows Monte Carlo simulation of the halo merger tree corresponding to a present-day halo of a given mass. Simple prescriptions for the cooling of gas, formation of stars and the feedback of the energy released in supernovae, and the merging and accretion of galaxies can then be fed into the realistic gravitational framework. The rudimentary parameters controlling these processes can then be adjusted until a realistic galaxy population is created. Using these procedures, Kauffmann (1996) and Kauffmann & Charlot (1997, KC97) have argued that an elliptical galaxy colour-magnitude relation can be produced that accurately mimics the observed one, both in its slope and in scatter about the mean relation. This occurs despite the fact that star formation continues until relatively late epochs in such hierarchical models (eg., 50% of stars form at $z < 1$ in the models of Baugh et al., 1997 [BCFL]), and that considerable merging occurs between stellar components.

This paper re-examines the constraints which can be derived from the CMR in the light of these models. Following BLE, we limit the galaxy formation process in a model independent manner, while at the same time allowing for
realistic growth of the gravitational potential. Our aim is to synthesize the model independent approach of BLE with the all-inclusive modeling of KC97 and BCFL. We consider a simplified scheme in which the bulk of star formation occurs at relatively high redshift giving rise to a well defined CMR for proto-galactic sub-units. Merging of these increases the mass of the galaxy causing the initial relationship to evolve increasing in scatter and decreasing in slope. Random merging of the sub-lumps creates a relationship that is very flat and has considerable scatter. The key issue we address is the extent to which merging in a hierarchical universe leads to a predominance of equal mass mergers and thus to a CMR that maintains it slope and keeps an acceptably small scatter. To do this we use the galaxy merging trees from BCFL but avoid using a detailed model for the galaxy formation process in order to isolate the effect of galaxy mergers. We do not attempt to assign morphological classifications to the model galaxies. Instead, we compare the sample with a morphologically complete sample of galaxies in the core of the Coma cluster (Terlevich et al., 1998).

In outline, the structure of this paper is as follows. The first part of the paper is concerned with the conclusions that can be drawn regarding the stellar populations of cluster galaxies. In §2, we revisit the scatter analysis of BLE. We consider a range of possible star formation histories ranging from continuous almost uniform star formation to star formation that declines strongly through the lifetime of the galaxy. We also consider the effects of the clipping algorithm used to determine the scatter from the observational data. §3 investigates whether the range of star formation histories considered allow a consistent paradigm that explains both the narrowness of the CMR at low redshift and the evidence of star formation activity in intermediate redshift clusters. In §4 we investigate the effects of mergers on the evolution of the CMR. In particular, we determine whether there is a conflict between the old ages implied for the bulk of the stars and the degree of merging implied by hierarchical gravitational evolution.

Finally, in §5, we present a discussion of the results of our study. We show that the small scatter of galaxies in the cores of rich clusters remains a challenging constraint for realistic galaxy formation models. However, by weighting the rate at which star formation is truncated in the galaxies towards large look-back times, and incorporation of the effects of the colour-magnitude fitting process, it is possible to explain both the increasing blue galaxy populations of intermediate clusters and the uniformity of the present-day galaxy populations. Furthermore, we show that while random merging quickly removes any initial slope of the colour-magnitude relation, inclusion of the dynamical biases inherent in a hierarchical model sustains the ratio of the scatter and slope of the colour-magnitude relation at a value close to that observed. A summary of our investigation is given in §6.

2 CONSTRAINTING THE HISTORY OF STAR FORMATION

2.1 Single-burst models

In this section, we consider the constraints that the tightness of the colour-magnitude relation places on the star formation histories of early-type galaxies. We base our investigation on the $U-V$ colour as this colour straddles the 4000Å break and has high sensitivity to evolution of the stellar population. We will adopt the Coma cluster as the proto-type of rich present-day galaxy clusters. High precision photometry in the $U$ and $V$ bands is available from BLE and Terlevich et al., 1998. The arguments we put forward are generally applicable to all broad-band measures of stellar populations.

We first consider the case in which early-type galaxy formation occurs in a single short-lived burst. This type of formation history might be suitable for the formation of elliptical galaxies (eg., Mathews & Baker, 1977, Arimoto & Yoshi, 1987), although such models over produce the numbers of very red galaxies seen in deep number-count surveys (eg., Silk & Zepf, 1996, Zepf, 1997) and over-predict the numbers of morphologically classified E and S0 galaxies with $z > 1$ in the Hubble Deep Field (Franceschini et al., 1998). These problems can be avoided if an extensive component of dust is associated with the burst event. With an extensive dusty component, the proto-elliptical galaxies would be luminous at infra-red wavelengths, and are therefore open to detection by ISO (eg., Taniguchi et al. 1997) and sub-millimeter arrays such as SCUBA (cf., Smail et al., 1997). Such models are appealing, however, since a strong correlation between the metal abundance of a galaxy and its mass arises naturally as a consequence of the more massive galaxies greater binding energy.

The constraints on such burst models were considered extensively in BLE. Figure 1 shows the rate of colour change as a function of the time since the star burst. This figure is central to the analysis of BLE. The curve plotted uses the stellar population synthesis code of Kodama and Arimoto (1997) with varying metal abundance. The figure also shows the population synthesis code of Bruzual (1983): the rate of colour change has altered little despite the substantial improvements that have been made in the range of stellar populations that are included in these codes. This reinforces the inherent simplicity of using this colour scheme to study star formation history: changes in the $U-V$ colour are driven primarily by the evolution of the main sequence turn-off, and are thus relatively simple to model correctly.

This figure can be used to set limits to the spread in the formation epochs of the early-type galaxies, and thus to infer the approximate redshift range of galaxy formation. The rate of change of $U-V$ colour gives the spread in colour of two galaxies formed a $\Delta t$ apart in age: $\Delta(U-V) \approx \Delta t d(U-V)/dt$. Knowing $\Delta(U-V)$ from our observational data and $d(U-V)/dt$ from our models we can estimate $\Delta t$. Thus the spread of galaxies about the mean colour-magnitude relation can be converted into a spread in formation times. If we assume that galaxies form over a fraction $\beta$ of the time that is available, this can in turn be converted into an estimate of the time before which most galaxies were formed.

One simple approach is to assume that galaxy formation occurs with a uniform random distribution in look-back time between an early epoch, $t_0 = 13$ Gyr, and a time $t_{stop}$ - a parameter that we wish to estimate. The value of $t_0$ has been chosen to match the observed ages of the oldest stars in our galaxy (eg., Pont et al., 1998, Gratton et al., 1998). In this case, the rms dispersion in galaxy colours is approximately

$$\beta \left( \frac{t_0 - t_{stop}}{3.5} \right) \frac{d(U-V)}{dt}$$
would be weakened if the galaxies are formed over a smaller time spread than is actual available (ie., \( \beta < 1 \)), but current evidence points to the formation of the earliest stars and galaxies at high redshifts, certainly \( z > 3.5 \) (eg., Madau et al., 1996). Since these early objects would occur in regions destined to be incorporated into rich clusters (eg., BCFL, Governato et al., 1998) there seems little scope for weakening the constraints by delaying the onset of galaxy formation.

In principle, observations of a narrow colour-magnitude relation at higher redshift could constrain the galaxy formation epoch even more tightly (eg., Ellis et al., 1997). At \( z \sim 0.5 \), the increase in the power of the test is modest, since the predicted rate of colour evolution is still small. Moreover, the galaxy populations of intermediate redshift clusters are considerably more heterogeneous than those of their present-day counterparts, and additional caution must be exercised when deciding exactly which should be included when the properties of the CMR are calculated. We therefore consider the properties of intermediate clusters separately in §3.

### 2.2 Continuous Star Formation

An alternative model for the star formation history of cluster galaxies allows the star formation to take place over a protracted period of time. This is conveniently modelled by an exponentially declining star formation rate. Although the conventional view is that this star formation occurs in a well-defined single event, this need not be the case: the exponential decay is simply intended to characterise the change of the overall rate of star formation even if it is split between several sub-units. We model the effect of the cluster environment by truncating the star formation in the galaxy at the time when it is accreted into the cluster environment. This is intended to mimic the hostile effects of the dense environment, such as ram-pressure stripping by the dense intracluster medium (Gunn & Gott, 1972, Kent, 1981, Couch et al., 1998) or high speed galaxy-galaxy encounters (Moore et al., 1996). We have avoided incorporating a more detailed model for these effects since our over-riding aim is to produce a simple scheme that can be compared with the observations in order to investigate the coordination of the truncation event.

Our models assume that star formation can be parametrised by

\[
SFR \propto e^{-t/t_{\text{start}}} \quad \text{for } t_{\text{stop}} < t < t_{\text{start}}
\]

\[
= 0 \quad \text{otherwise}
\]

Star formation starts at a time \( t_{\text{start}} \), and declines at a rate determined by \( \tau \) until it is truncated at time \( t_{\text{stop}} \). We fix the metal abundance of the system at the solar value.

Initially we consider the case in which star formation starts in all galaxies at a similar high-redshift epoch and so set \( t_{\text{start}} = 13 \) Gyr. The rate of change of colour is shown as the thin solid line in Figure 2 for the values \( \tau = 2, 5, 10 \) Gyr. As expected, the rate of change of colour declines much more rapidly than in the case considered in the previous section.

50 km/s/Mpc. This gives an age of the universe of 13 Gyr. Note, however, that the arguments we present in §2 are primarily dependent on the cosmic look-back timescale and not on the redshift scale given by the assumed cosmology.
The Colour-Magnitude Relation

Figure 2. (a) $U-V$ colour and (b) rate of change of $U-V$ colour as a function of the ‘age’ of the composite stellar population. Solid lines are based on the models of Kodama & Arimoto, with abundance held fixed at the solar value. Top thick line: a single burst model in which all stars are formed in a short burst at $t_{\text{stop}}$. This model was used as the basis of BLE’s diagram. Lower thin lines: stars are formed at a decaying rate determined by the parameter $\tau$ until it is truncated at $t_{\text{stop}}$. Three values are shown, $\tau = 10, 5, 2$ Gyr, from top to bottom. Star formation is in each case initiated at $t_{\text{start}} = 13$ Gyr. Although the colours change very rapidly immediately after star formation finishes, the rate of change becomes small after several Gyr have elapsed, and the movement of the galaxies in the colour-magnitude diagram becomes negligible. The dot-dashed line shows effect of adding a 10% burst of star formation to the $\tau = 5$ Gyr model at $t_{\text{stop}}$. The dashed line shows the rate of change of colour (with respect to $t_{\text{start}}$) when $t_{\text{stop}}$ is held fixed at 5 Gyr, while the epoch at which star formation begins ($t_{\text{start}}$, plotted on abscissa) is varied. Dotted lines in the diagram illustrate the degree of coordination implied by the observed scatter in the colour-magnitude relation. The lines show $\beta = 0.1, 0.3, 1.0$ from top bottom. Further details are given in the text.

This is particularly true for the lowest $\tau$ models. Here very few stars are being formed at low look-back times. When star formation is truncated, very little blue flux is produced once the most massive stars have evolved off the main sequence. The presence of the remaining young stars has little effect on the overall colour. Even in a galaxy with a long star formation decay timescale, the rate of change of colour is lower than the single-burst formation scenario discussed in the previous section.

Figure 3. Simulated histograms of the distribution of galaxies across the colour-magnitude relation. (a) Star formation occurs in a single burst. The burst events are distributed uniformly between $t = 0$ and 13 Gyr. (b) Star formation begins in all galaxies at $t = 13$ Gyr, and declines at a rate $\tau = 10$ Gyr until it is truncated at $t_{\text{stop}}$. For different galaxies, $t_{\text{stop}}$ is distributed uniformly between $t = 0$ and 13 Gyr. (c) as b, but $\tau = 5$ Gyr. (d) as b, but $\tau = 2$ Gyr. In each case, the vertical solid and dashed lines show the bi-weight average and scatter.

It is clear from this experiment, that the colour-magnitude relation sets relatively weak constraints on the last epoch of star formation in these systems. However, the approach of comparing the observed scatter with the rate of colour change is a poor approximation if star formation occurs over a prolonged period. Furthermore, the clipping procedure used to measure the observed scatter is not taken into account. A more accurate method of assessing the scatter is to simulate the colour distribution across the CMR and to then reproduce the observational measurement.

The transverse cross-section of the model CMR is illustrated in Figure 3. These histograms have been created by drawing 100 galaxies with random values of $t_{\text{stop}}$ in the range 0 to 13 Gyr. Each of the histograms shows the results for different star formation models: (a) all the stars form in a single burst; (b) star formation begins at 13 Gyr, and decays with e-folding timescale $\tau = 10$ Gyr until it is truncated at $t_{\text{stop}}$; (c) and (d) show similar models but with $\tau = 5$ and 2 Gyr respectively. The simulated galaxies have the same metal abundance and thus this diagram neglects the effects of the slope of the CMR: in practice the scatter we derive from our models may be a slight underestimate due to the increased brightness of the bluer galaxies. The effect is negligible, however, since the slope of the CMR is shallow relative to the changes in brightness. For example, in the case of the $\tau = 5$ Gyr model, the increase in dispersion due to this effect is 0.01 mag at most.

In order to make a quantitative comparison with ob-
Figure 4. The bi-weight scatter in the simulated scatter in the colour magnitude relation as a function of \(t_{\text{stop,min}}\), the look-back time to the epoch at which star formation in all galaxies. Two models for the evolution of star formation are shown, \(\tau = 5\) Gyr (solid line) and \(\tau = 10\) Gyr (dashed line). Further details of the star formation histories consider are explained in the text. Error bars shown on the figure illustrate the 1\(\sigma\) variation in the scatter measured for different Monte Carlo realisations of a sample of 100 galaxies. A thin horizontal line illustrates the observed CMR scatter.

For comparison, Figure 5 shows the histogram for the colour distribution of galaxies in the core of the Coma cluster. (a) Bright elliptical galaxies in the cluster core with morphological classification from Andreon et al., 1996. (b) The CMR for all bright galaxies regardless of morphological type. Galaxies brighter than \(V < 17\) and within the central 20’ dia. of the cluster are plotted. The bi-weight scatter quoted here has not been corrected for observational uncertainties.

Table 2. Observed bi-weight scatter for galaxies in the Coma cluster from Terlevich et al., 1998, as a function of radius and morphological type. The sample is limited to galaxies with \(V < 17\) within a 13’ diameter aperture. The values given have been corrected for the contribution of measurement uncertainties (0.025 mag).

| Region         | Morphology | \(N_{\text{gal}}\) | Bi-weight scatter |
|----------------|------------|---------------------|-------------------|
| core (\(r < 20'\)) | E only     | 29                  | 0.036             |
|                | E + S0     | 80                  | 0.046             |
| all galaxies   | all galaxies | 98                  | 0.049             |
| \(r < 50'\)    | all galaxies | 165                 | 0.061             |

Figure 4 shows how the scatter varies as a function of the minimum value of \(t_{\text{stop}}\) for star formation rates declining as \(\tau = 5\) and 10 Gyr. For a given observational scatter, this can be used to estimate the lowest look-back time at which star formation can have occurred in a portion of the galaxies. A model which considers the effect of varying both the start of star formation and its truncation is considered below.

For comparison, Figure 5 shows the histogram for the colour distribution of galaxies in the core of the Coma cluster. This data is taken from a wide-area \(U-V\) Mosaic of the cluster (Terlevich et al. 1998, summarised in Table 2), with colours calculated within an 8.7 Kpc dia. aperture. Here we focus on the data taken from the core of the cluster, where accurate morphological types are available from Andreon et al. (1996), although the full photometric data-set, extending to 50’ from NGC 4874, has similar properties. Comparable results are obtained if we use data from BLE, although the galaxy selection is limited to early-types. By using Terlevich et al.’s data, it is not necessary to define a morphological classification system for the simulated galaxies since we may consider colour properties of the galaxies independently of their morphological appearance. In the core of the cluster, early-type galaxies are dominant and the measured scatter...
Table 1. Scatter in simulated colour distributions. Star formation in individual galaxies occurs between look-back times $t_{\text{start}}$ and $t_{\text{stop}}$ with a declining rate parameterised by $\tau$. 100 galaxies are simulated with $t_{\text{start}}$ and $t_{\text{stop}}$ randomly distributed as shown in the table. The columns give: (1) the type of star formation model; (2) the lowest value used for the $t_{\text{stop}}$ parameter; (3) the value or upper limit to the $t_{\text{start}}$ parameter; (4) The scatter of simulated colours calculated using the bi-weight statistic; (5) The 1σ sample to sample variation in the scatter we measure for a sample of 100 galaxies.

| Model       | $t_{\text{stop, min}}$ | $t_{\text{start}}$ | Bi-weight scatter |
|-------------|------------------------|---------------------|------------------|
| single-burst| $0$                    | $t_{\text{stop}}$  | $0.242 \pm 0.034$|
|             | $5$                    |                     | $0.103 \pm 0.004$|
|             | $10$                   |                     | $0.029 \pm 0.001$|
| $\tau = 10$| $0$ fixed at $13$      | $0.109 \pm 0.013$  |
|             | $5$                    | $0.048 \pm 0.002$  |
|             | $10$                   | $0.014 \pm 0.001$  |
| $\tau = 5$ | $0$ fixed at $13$      | $0.086 \pm 0.009$  |
|             | $5$                    | $0.041 \pm 0.002$  |
|             | $10$                   | $0.013 \pm 0.001$  |
| $\tau = 2$ | $0$ fixed at $13$      | $0.031 \pm 0.002$  |
|             | $5$                    | $0.023 \pm 0.001$  |
|             | $10$                   | $0.011 \pm 0.001$  |
| $\tau = 5$ | $0 - t_{\text{start}}$| $0 - 13$            | $0.316 \pm 0.050$|
|             | $0 - t_{\text{start}}$| $5 - 13$            | $0.138 \pm 0.015$|
|             | $5 - t_{\text{start}}$| $5 - 13$            | $0.084 \pm 0.005$|

is not overly dependent on the method used to define the ridge-line of the CMR.

The first panel of Figure 5 shows the scatter in colour about the mean relation for elliptical galaxies only. For each galaxy, its deviation from the mean relation was calculated and then used to compile the histogram shown in the figure. Selection of galaxies by morphological type does not accurately reflect the synthetic histograms that we have created, however. For example, galaxies with more recent star formation activity have predominantly disc morphologies. Because of the complete spatial coverage of Terlevich et al.’s data, we can also fit to the CMR without regard to galaxy morphology. This gives a similar result with just slightly larger scatter (Table 2). The difference is not surprising since most galaxies in the core of the cluster are early-types, and it is widely established (eg., BLE) that there is little difference in the properties of E and S0 galaxies in the cores of rich clusters.

Comparison with Table 1 shows that while allowing star formation to continue in some galaxies to the present-day produces too much scatter, truncating star formation before a look-back time of 5 Gyr reduces the scatter to be acceptably small. In contrast to the single burst models that we considered in §2.1, strong coordination of the last epoch of star formation is not necessary in order to create a narrow colour-magnitude relation. Although the star formation rate decays slowly, the bulk of stars are still formed at large lookback times. A quantitative estimate of the epoch at which the last stars were formed can be obtained from Figure 4.

The observational estimate of the scatter in the CMR given in Table 2 (all galaxies, $r < 20'$) of 0.049 suggests that star formation may continue in a fraction of galaxies until a look-back time of 5 Gyr for $\tau = 10$ Gyr, or 3 Gyr for $\tau = 5$ Gyr.

These values allow considerably more star formation activity to have occurred in the intermediate redshift universe than was suggested by consideration of the single burst models: because the complete galaxy population started to form stars at a similar epoch, we have tended to ensure that the bulk of the stars have very similar ages and colours. As a result, low scatter is measured when star formation is truncated early. To show the importance of the epoch at which star formation is started, we fix $t_{\text{stop}}$ at 5 Gyr, and allow $t_{\text{start}}$ to vary. Dot-dashed lines in Figure 2 show the rate of colour change (with respect to $t_{\text{start}}$) as a function of $t_{\text{start}}$. As expected, these lines lie close to the burst formation model confirming that the colour-magnitude relation still places significant constraint on the epoch of the formation of the bulk of the stars (or its degree of coordination). A model in which both $t_{\text{start}}$ and $t_{\text{stop}}$ are allowed to vary cannot simply be represented in the Figure 2. It is possible, however, to make Monte-Carlo simulations of this case by at first selecting $t_{\text{start}}$ with a uniform distribution and then selecting $t_{\text{stop}}$ to lie between this and some minimum value. As expected, these models produce larger variations in the CMR than when $t_{\text{start}}$ is held fixed. In this case even when no star formation occurs at look-back times less than 5 Gyr, the variation in colours is still greater than the observed relation. The key result of this section is that the scatter of the C-M relation places a strong constraint on the formation epoch of the bulk of the stars, but a relatively weak constraint on the last epoch of star formation in the cluster galaxies. Thus the conclusion reached by BLE that the bulk of star formation occurs at redshifts greater than 1 (an average redshift of 2) appears robust; however, recent truncation of residual star formation is also compatible with the tight relation.
3 COMPARISON WITH THE BUTCHER-OEMLER EFFECT IN INTERMEDIATE REDSHIFT CLUSTERS

In this section we contrast the homogeneity of the stellar populations of galaxies in nearby clusters with the properties of galaxies in clusters at intermediate redshift. Our aim is to examine the broad compatibility of these observations. A detailed comparison of morphologically selected subsamples of galaxies will be made in a subsequent paper.

The observations of intermediate redshift clusters reveal significant evolution in the galaxy populations of rich clusters. This is seen in two distinct ways. Firstly, through the changing morphological content of clusters, originally observed as an increase in the fraction of blue galaxies (Butcher & Oemler, 1978), and now (post-HST) identified as an increase in the fraction of galaxies with spiral and irregular morphology perhaps associated with a decline in the S0 galaxy content of distant clusters (Smail et al., 1997, Dressler et al., 1997). We will refer to this as the morphological Butcher-Oemler effect. Secondly, evolution is apparent in the fraction of galaxies with spectra showing abnormally strong Balmer absorption lines (e.g., Dressler & Gunn, 1983, Couch & Sharples, 1987, Couch et al., 1997, Dressler et al., 1997). The most extreme Hδ line strengths can only be reproduced if star formation is truncated after a strong burst of star formation (although the star burst may have a strongly skewed stellar IMF). We will refer to this phenomenon as the spectroscopic Butcher-Oemler effect. It is not clear whether the two versions of the Butcher-Oemler effect reflect the same physical process (e.g., Charlot & Silk, 1994, Couch et al., 1997): for example, the star burst fraction may be driven by interactions with gas rich galaxies while the general decline in the blue fraction might result from ram pressure stripping.

At first sight, the evolution seen in the distant clusters appears to conflict with the homogeneity of the colour-magnitude relation in systems nearby. Because galaxies cannot escape from these clusters, star formation histories of the type discussed above must be typical of many of the galaxies seen in our Coma cluster sample. Even if these galaxies do not have regular E or S0 morphological types, the data of Terlevich et al (Table 2) shows that similar scatter is obtained for the present-day CMR even if morphological information is ignored. The comparison can only be avoided if the galaxies responsible for the Butcher-Oemler effect are systematically destroyed (for example by 'harassment', Moore et al., 1996) to form the diffuse intra-cluster light, or are confined to the outer-parts of present-day clusters.

We will assume that the first process is not efficient in what follows. The second restriction is difficult to quantify because it is dependent on the efficiency with which these galaxies are mixed into the central parts of the cluster. However, Allington-Smith et al. (1993) concluded that there was no evidence for a radial gradient in the frequency of starburst/post-starburst galaxies. Furthermore, Terlevich et al.’s data for the Coma cluster extends out 2 Mpc (60% of the cluster’s virial radius) if the full area is considered, and incorporates the infalling NGC 4839 group in the South West of the cluster. This area is comparable with the region over which the Butcher-Oemler effect has been established at intermediate redshift (e.g., Dressler et al., 1997, Couch et al., 1997, but see Abraham et al., 1996, van Dokkum et al., 1998 for studies with wider coverage). It therefore seems unlikely that the contrast in star formation histories can be explained by a simple radial gradient in galaxy properties. Finally, we note that the comparison requires us to make a further assumption: that the clusters identified at intermediate redshift will, by the present-day, evolve into systems with properties similar to those of the Coma cluster. This comparison appears secure since the intermediate redshift clusters can only grow in richness, and the tightness of the CMR appears a generic property of all present-day rich clusters (BLE, Garilli et al., 1998).

As we have shown, the scatter about the CMR constrains the formation of the bulk of the stars much more strongly than it limits the formation of the last stars. Is it possible then that the CMR scatter and the direct evidence for evolution are compatible? To answer this question, we use the analysis of the star formation cycle in distant clusters from Barger et al. (1996). They show that the relative numbers of star burst, post-star burst and red Hδ-strong galaxies can be described by the addition of a 10% (by mass) burst of star formation to an underlying spiral or elliptical-type (it is very difficult to estimate the star formation history of the galaxy prior to the star burst, Charlot & Silk, 1994). After the burst, all star formation ceases. A simple single cycle need not apply to all galaxies (for instance, Barger et al. take no account of the relative magnitudes of the different types of object). Nevertheless, several Gyr after star formation ceases, the strength of the star burst has a relatively small effect on the colour evolution (Figure 2) and so this simple parameterisation is adequate. Below, we assume that all galaxies pass through this burst phase immediately after the truncation of the normal mode of star formation.

We investigate the compatibility between the cluster population at $z = 0.5$ and the scatter in the present-day population by developing the Monte-Carlo realisation technique introduced in §2. We initially based the progenitor galaxy population on the $\tau = 5$ Gyr model so as to mimic...
Figure 6. The evolution with look-back time of the infall rate for a rich cluster. The extended Press-Schechter method has been used to calculate the rate at which mass is incorporated into units more massive than a galaxy group (B91). These curves are used to suggest a parametric form for the rate at which star formation is truncated in the cluster galaxies. Solid line: star formation is truncated when a galaxy is accreted into a group of 5 or more galaxies \( (M_\star = M_\star, \text{ in the notation of B91}) \); dashed line, on infall into a group three times more massive \( (M_\star = 3M_\star) \).

Figure 7. Simulated colour-magnitude histograms for a model of galaxy evolution that is consistent with both the narrow scatter in the CMR in local clusters and the observation of the Butcher-Oemler effect in high redshift systems. The model mixes a 50% population of galaxies undergoing a random truncation of their star formation between \( t = 0 \) and 13 Gyr with a 50% population of galaxies that cease star formation between 10 and 13 Gyr. The two panels distinguish assumption that progenitors have star formation declining as \( \tau = 5 \) and \( \tau = 10 \) Gyr.

by the cluster. Using Eq. 18 of B91, and setting \( M_\star = 1.0 \), gives the rate at which mass is accreted into objects more massive than galaxy groups. This formula accounts for the total accretion rate even if the final cluster consists of several fragments at earlier times. We adopt the parameters \( M' = 100, n = -1.5 \) from B91 in order to provide a description of the growth of a rich cluster, and assume that the infall rate, \( R_{\text{infall}}(z) \), reflects the rate at which star formation is truncated in the cluster galaxies (note that this approach differs from B91). The evolution of the infall rate is plotted in Figure 6. The evolution of the rate of infall is modest, increasing by a factor of 3 between the present and a look-back time of 9 Gyr and then rapidly declining at larger look-back times. The function \( R_{\text{infall}}(z) \) is normalised and used to randomly select truncation times for Monte-Carlo simulation of the present-day CMR. This parameterisation assumes that star formation is truncated when a galaxy is accreted into a system more massive than a group containing ~ 5 galaxies.

We generate two models analogous to Models 1 & 2. Model 3 assumes that all galaxies are formed by truncation of exponentially decaying star formation. This overestimates rate of increase in the blue galaxy fraction. In Model 4, we add a 40% population of intrinsically old galaxies so that the blue fraction increases more slowly with redshift, reaching 25% at \( z = 0.5 \).

Table 3 shows the present-day scatter that we measure for each of the models. These models differ from the values given in Table 1 because of the inclusion of the 10% starburst at the final epoch. We first consider Model 1 in which the truncation rate is constant and the actively forming galaxies in clusters rises rapidly to a fraction of 50% at \( z = 0.5 \). This model produces too many galaxies that are still moving towards the main colour-magnitude relation. As a result, the predicted scatter exceeds that observed in present-day clusters. Replacing the constant truncation rate by the rate derived from B91 (Model 3) slightly reduces both the blue fraction at intermediate redshift (40%) and predicted present-day CMR scatter, although the latter is still too large to be compatible with the observations of Terlevich.
et al. (1998). Altering the parameterisation of the truncation rate by increasing, by a factor of three, the mass-scale at which star formation is switched off, creates a distribution that is flatter and tends to increase the predicted scatter. By contrast, adding a population of intrinsically old galaxies can dramatically reduce the predicted scatter. We add the maximum population that is allowed before the blue galaxy fraction at \( z = 0.5 \) drops below 25%. With the constant truncation rate, adding a mixture of 50% intrinsically old galaxies reduces the rate at which galaxies are supplied to the CMR sufficiently that the predicted scatter provides a reasonable match to that observed in the Coma cluster. The combination of the high rate at which the colours of these galaxies redden, and the way in which the mean relation is trimmed to remove outlying points weakens their influence on the bulk of the galaxies’ properties. Similar results are obtained by adding a 40% population of ‘old’ galaxies to Model 3. The predicted present-day scatter is again within the observational limits. In these cases, evolution of the cluster blue galaxy fraction from a few percent at the present day to \( \sim 25\% \) at \( z = 0.5 \) does not conflict with the observation of a narrow observed CMR in local clusters.

4 CONSTRAINT ON THE MERGER HISTORIES OF CLUSTER GALAXIES

In addition to placing a constraint on the star formation histories of galaxies, the colour-magnitude relation can also be used to set a limit on the amount of galaxy merging that occurs after the formation of the dominant component of the stellar population. We have shown in the previous section that the dominant component of the stellar population was formed at relatively high redshifts. In this section, we address the question of whether these stars must already have been bound into a single object, or whether they could have been formed in small units that were subsequently merged together. It is important here that we distinguish between mergers in which substantial star formation takes place and those that take place between systems consisting only of stars. The approach we adopt aims to illuminate the general constraints that can be derived regardless of the specific star formation model. As a result we concentrate only on the second case, termed dissipationless merging by Bender et al. (1992a). The case in which mergers promote star formation cannot be considered in general terms and a specific model for star formation in galaxy mergers, and this conversion between hot and cold gas phases is required (eg., KC97, BCFL).

The key ingredient to using the colour-magnitude as a constraint on galaxy merger histories is its slope. Even if the CMR is initially formed without scatter in a coeval galaxy population, dissipationless mergers between systems of differing colour tend to average the colours of galaxies towards a single value. This reduces the slope of the relation and increases scatter as galaxies undergo different numbers of mergers between galaxies covering different ranges of colours. Thus, without knowing the initial slope of the relation, we can use the ratio between the CMR scatter and its slope to place an upper-limit on the importance of dissipationless mergers.

We model this process by allocating galaxies to an initial colour magnitude diagram. We assume that the initial relation is exact and that there is no differential evolution of galaxy luminosities and colours. This is clearly an over simplification, but it distinctly separates the effects of the evolution of the stellar population (§2.3) from those due to the merging of galaxies. Under these conditions, the slope of the initial relation is the only parameter connecting the galaxy mass with its colour. We emphasise that this scaling is arbitrary, and that choice of a steeper initial slope simply rescales the final slope. The key parameter that must be matched is the ratio between the scatter about the average CMR and its slope. This ratio is independent of the initial slope in the model.

Experiment with random mergers between these objects shows that the initial relation is quickly weakened as galaxies are merged; however, this approach does not accurately mimic the gravitational evolution of the galaxies’ dark matter haloes. As KC97 have shown, there are significant correlations in galaxy merger histories meaning that large galaxies form from systematically larger progenitors. Therefore, we have refined our treatment to include the merger histories as defined by the simulations of BCFL. This has the advantage that it allows us to calibrate the merger process on to a (model dependent) redshift scale. We have included the results of the equivalent random merger process as a means of determining the importance of the correlations present in the hierarchical model.

4.1 Hierarchical clustering

In order to generate galaxy merging histories, we use the formalism described in BCFL. This first generates a tree of merging haloes using the Monte-Carlo prescription outlined in Cole & Lacey, 1993 (see also Summersville & Kolatt, 1998). In order to follow the merging history of galaxies, we must add an additional ingredient: the merging of galaxies within a common halo. This process is driven by dynamical friction, and we use the dynamical friction timescales as parameterised by Cole et al., 1994. Galaxies that have dynamical friction timescales less than the merger timescale of the halo are assumed to merge with the dominant galaxy. Since the dynamical friction timescale is shorter for more massive galaxies, this naturally generates a tendency for large galaxies to form from mergers of massive galaxies. This approach is generic to all hierarchical galaxy formation codes, and we use the code of BCFL to produce a list of galaxy fragments and their evolution as a function of redshift. We emphasise that this is only used to produce the initial CMR and to select galaxies to be merged; BCFL’s parameterisation of the star formation process is ignored in what follows.

It is important that we only select merger histories corresponding to galaxies that are found in rich clusters at the final time. We incorporate this requirement by only constructing merger trees for objects bound into a dark matter halo with circular velocity greater than 1000 km/s at the final time. In order to reduce computational overhead, also require that the final galaxy has absolute V-magnitude brighter than −19 in BCFL’s simulations. The final results are unaffected by the cut off since we determine the slope and scatter over only the brightest 4 magnitudes of the simulated CMR (see §4.3).

Once the galaxy merger tree has been constructed, our
phenomenological approach diverges from the prescriptive modelling of BCFL. We start by selecting an initial epoch ($z_{\text{form}}$). The galaxy fragments present in the tree $z = z_{\text{form}}$ are given a magnitude scaled to the log of their total baryonic mass, and a colour according to an initial ‘perfect’ CMR (i.e., colour = slope $\times$ magnitude + offset). As we have already emphasised, the ‘colour’ we calculate is purely illustrative and serves only to show the effect of mixing the stellar populations. We make no attempt to justify the initial zero-point or slope that is applied. The evolution can then be followed by simply combining the magnitudes and colours of the fragments as they merge to form the final galaxies at $z = 0$. At some points in the tree the proto-galaxies merge with ‘new’ fragments (fragments with progenitors that are below the mass resolution of the merger code); in order to be consistent with our phenomenological model, we do not incorporate these objects into the merger scheme. The treatment of these objects is only important if the CMR is set at high redshifts $z > 2.5$; at later times the fragments are very much larger than the mass resolution limit. Each tree leads to the formation of a single galaxy at the present epoch. In order to match the numbers of galaxies in our observational sample the process is repeated in order to generate a realisation of the CMR.

Panels a, b & c of Figure 8 show examples of the present-day CMRs that we derive by this approach. If the relation is established at relatively low redshift, the rate of merging is low enough that little evolution of the initial slope occurs. By contrast, a relation that is established at higher redshift is considerably weakened due to the large number of mergers that have taken place. Despite this, a discernible CMR exists at the present-day even if the initial relation is established at $z > 2.5$. We fitted the CMR of the brightest four magnitudes of the relation by minimising the bi-weight scatter. This is shown by a solid line in each of the panels. The degree of robustness is surprising. As we show below it is due to the slow rate of mass growth in the hierarchical merger tree at late times.

4.2 Random mergers

In order to test the importance of the merger correlations in the hierarchical model, we repeated the Monte Carlo process using randomised merger trees. In order to be able to compare our results directly, we have based the simulation on the same galaxy merger trees to imprint the initial CMR at $z = z_{\text{form}}$. However, rather than merging the fragments as ordered by the hierarchical tree, random fragment pairs (possibly from different trees) are selected at each branch of the original tree. Fragments without progenitors in the hierarchical tree also have no progenitor in the random tree. This approach preserves both the initial mass distribution and number weighted merger rates.

As can be seen from Panels d, e & f of Figure 8, the effect of the random mergers is to more rapidly reduce the slope of the initial tree and to rapidly increase the scatter of the CMR. Although a best fit line can be defined for these models, the slope varies considerably between different realisations. The data used below have been averaged over many realisations of the random merger trees.

4.3 Comparison with Observational Data

We use the scatter in the present day CMR to constrain its formation history, however the scatter that we measure in the simulated CMRs can be made arbitrarily small by reducing the slope of the relation. For example, initially allocating all galaxies the same colour will (assuming a coeval population) lead to a final CMR with zero scatter but also zero slope. The key requirement is therefore that the merging process is able to maintain the ratio between the scatter and slope ($R$) at or below the observed ratio. This is independent of our initial CMR, and allows us to compare both the models to each other and to real data.

Figure 9 summarises how the ratio $R$ varies with $z_{\text{form}}$. In both random and hierarchical models $R$ increases monotonically, however the hierarchical clustering model has a much slower increase of $R$ with $z_{\text{form}}$. We compare these results with our data from the Coma cluster. The observed $U - V$ scatter is 0.049 (all galaxies, Table 2) and the observed CMR slope, measured using $U - V$ colours in a metric (13") aperture and total $V$ magnitudes, is $-0.087$ (BLE). This gives a ratio of 0.6. It is important to use the slope to a total magnitude rather than an aperture measurement so that this quantity is conserved during the mergers as in the model calculation. There is, however, a further correction to this slope. The colours that are available for Coma are measured in a metric aperture that takes no account of the size of the galaxy. Thus larger galaxies have their colours measured within a smaller relative diameter. This will naturally tend to introduce a slope into the CMR since galaxies become bluer at large radii. To estimate the size of this effect, we use the elliptical galaxy colour gradients measured by Peletier et al. (1990). Their mean colour gradient is measured in $U - R$ and $U - B$, but it can be converted to a gradient in $U - V$ using the late type stellar colours from Gunn & Stryker, 1983. We find $D_\Delta (\mu_U - \mu_V) / D_\log r = -0.178$ mag/dex. This can be converted to a change in aperture colour as a function of radius by integrating over the de Vaucouleurs $r^{1/4}$ profile, giving a change in colour of $-0.134$ mag/dex. In order to assess the effect of this colour gradient, it is simplest to compare with the observed slope of the colour-magnitude relation plotted in $U - V$ vs. $D_V$ coordinates ($D_V$ measures the size of the galaxy within which the mean surface brightness is 19.80 mag arcsec$^{-2}$, Lucey et al., 1991). The observed slope of in this parameter space is 0.46 mag/dex (BLE), which should be compared with the effect expected due to colour gradients of 0.134 (the aperture used to measure the colour remains fixed and therefore the relative size of the aperture is inversely proportional to $D_V$). Thus the colour gradient effect accounts for about 30% of the CMR slope. Including this correction raises the ratio of observed scatter to slope to 0.8.

We can compare this limit with the simulated models. For the random model, the ratio $R$ is already too large for $z_{\text{form}} \geq 0.9$. For the hierarchical model, $R$ increases less rapidly, so that the constraint on $z_{\text{form}}$ is correspondingly weakened, and the observed limit on $R$ only conflicts with the model strongly for $z_{\text{form}} \geq 2$ (although there is little room for additional sources scatter once $z_{\text{form}} \geq 1.4$). This limit is roughly consistent with the formation of the bulk of the stellar population at large look-back times as discussed.
in §2. Quoting these results in terms of redshift is, however, a little unsatisfactory as the redshift evolution of the objects mass is tied to the specific model for dynamical friction and the merger timescale adopted by BCFL in generating the galaxy merger tree. A more useful classification is the factor by which the masses of the galaxies have increased over these epochs. We use the ratio of the mass-weighted mean mass (MWMM) between the final CMR and the epoch at which the CMR is set as a measure of the amount of mass growth. This is analogous to \( M_* \) of the Press-Schechter mass function, and provides a measure of the ‘typical’ stellar mass of the model galaxy population. By assuming a particular mass-to-light ratio for the stellar population, the MWMM can be converted into an equivalent luminosity, however, this is not required for the comparison we wish to make. The change in the MWMM ratio provides a simple measure of the mass a typical galaxy has gained in the merger process. In the hierarchical tree, this is not equivalent to the number of mergers: as we will see, the mergers at late times (low redshifts) are dominated by the accretion of small objects that make little change to the MWMM.

Figure 10 shows how the MWMM evolves as a function of redshift for the hierarchical and random models. We first consider the hierarchical tree. Initially, the MWMM ratio evolves only slowly: at these redshifts most mergers involve the accretion of a small companion by a large galaxy. By \( z_{\text{form}} = 1 \), the characteristic mass has grown by only 20%, and the rate of growth remains low until \( z_{\text{form}} > 2 \). By contrast, the evolution of the MWMM of the random model is more sensitive to the choice of \( z_{\text{form}} \). This is a result of the way in which the random model is constructed from the same galaxy fragments as present in the hierarchical model at the initial epoch. It is important to note that Figure 10 does not show the evolution of the MWMM for a single realisation, but rather the effect of different starting points for the random merger tree. At low redshifts, the initial mass distribution is dominated by high mass objects: in the random model merging of these components has high probability, and there is considerable evolution of the characteristic mass. At higher values of \( z_{\text{form}} \), a greater fraction of the initial mass is dominated by the smaller mass objects, and the behaviour becomes more similar to that seen in the hierarchical tree. To quantify the mass growth factor, both datasets have been interpolated by a smooth relation shown by the thin lines in Figure 10. We use this conversion to compare the evolution of the CMR as a function of mass growth. This is shown in Figure 11. Despite the large differences in the rate of mass growth, the evolution of the slope to scatter ratio, \( R \), is similar for both the random and hierarchical merger trees. Both curves show steps in the \( R \) that result from the non-linear behaviour of the bi-weight scatter estimator. Although the overall impression is one of
The Colour-Magnitude Relation

Figure 9. The effect of mergers in a hierarchical clustering scenario. The solid line shows how the ratio between present-day CMR scatter and slope ($R$) depends on the redshift $z_{\text{form}}$ at which test galaxies are assigned colours according to an exact correlation between colour and magnitude at redshift. The effect of mergers in an equivalent random merging model is shown by the dashed line.

Figure 10. The evolution of the MWMM, expressed relative to the $z = 0$ value, as a function of the formation redshift $z_{\text{form}}$ for a hierarchical galaxy model (solid line) and a model with random galaxy mergers (dashed line). Details of the models are given in the text.

Figure 11. The evolution of the ratio between CMR scatter and slope ($R$) for a hierarchical galaxy model (solid line) and a model with random galaxy mergers (dashed line). Details of the models are given in the text. The observed value of $R$ is shown as a thin horizontal line.

similarity, the correlations inherent in the hierarchical merging tree result in the scatter from this model exceeding the random model at low mass growth factors and under shooting it at high values. Nevertheless, when compared to the observed value of $R$, both models set similar limits on the mass growth factor. Although the CMR can be imprinted at higher redshift in the hierarchical tree, this is compensated by the slower growth of mass in this model.

It is interesting to compare these results with KC97. While our approach differs from their global modelling of the galaxy formation process, we find similar conclusions for the factor by which galaxy mass has increased since the formation of the bulk of the stars. The key point is that this factor is sufficiently small (between 2 and 3) that the CMR is adequately able to retain a memory of its initial slope. The particular model by which galaxies are selected for merging seems to have only a weak effect on the mass growth factor, although it has significant implications for rate of redshift evolution.

The existence of a strong colour-magnitude correlation rules out the possibility that typical cluster galaxies have grown in mass by a large factor since the formation of the bulk of their stars. Furthermore, the factor of 2-3 that we estimate probably represents an upper limit to the role of dissipationless mergers in the formation of (the majority of) present-day luminous cluster galaxies. For example, it is extremely unlikely that the initial CMR will be completely perfect. Furthermore, we have not allowed for the differential evolution of galaxy colours as discussed in §2. These effects will tend to make the limit on the degree of merging still more stringent. The constraint can only be weakened if the mergers are not purely dissipationless and are associated with significant star formation and consequent metal enrichment. This then conflicts with our initial parameterisation of the CMR. We discuss the role of these effects further, and examine possible explanations for the origin of the CMR in the following section.
5 DISCUSSION

At first sight, the early formation of the bulk of stars in cluster galaxies presents a problem for the hierarchical scenarios of galaxy formation. However, while star formation generally occurs late in these models, clusters of galaxies are special parts of the universe in which structure formation is more advanced. Thus the formation of the bulk of stars at redshifts greater than unity does not present a particular problem since our inferences apply to only rich clusters. The difference between the formation of average structure and that leading to present-day rich clusters can be seen directly from the extended Press-Schechter formalism described by Bower (1991) and Bond et al. (1991). This allows us to determine, as a function of present-day halo mass, the epoch at which 30% of the final mass has become bound into objects of a certain scale size. For example, Moore et al.’s (1990) analysis of the CfA redshift survey suggests that the present-day mass distribution has a characteristic mass scale ($M_\star$) equivalent to a group containing 5 $L_\star$ galaxies and an effective power spectrum index, $n = -1.5$. With these parameters, 30% of the universe becomes bound into galactic size objects at $z \approx 0.5$. This is a very late epoch. By contrast, the matter that is destined to be incorporated into present-day rich clusters ($M \sim 10M_\star$) reaches the same threshold at $z \approx 1.4$. This simple calculation is reinforced by the more detailed calculations that can be made using full galaxy formation codes (eg., Kauffmann, 1996).

It is quite natural therefore that although the majority of stars are formed at relatively recent epochs, the stellar populations of galaxies found in rich clusters are biased to form at high redshifts. This is in broad agreement with the small scatter we observe in the colours. From this argument alone, however, it is not clear whether these stars are formed in sub-galactic fragments and only later combined into the cluster galaxies that we observe. We have addressed this issue by investigating the effect of mergers on the slope and scatter of the CMR. Extensive dissipationless merging flattens the CMR and introduces additional scatter. This happens rapidly if the merger process selects galaxy fragments at random. Random merging is not a good description of the process expected in a hierarchical model, however, since most mergers involve the galaxy at the centre of the halo and its satellites. Encounters between the satellites themselves have high relative velocity leading to ‘harassment’ (Moore et al., 1996) rather than merging. The dynamical friction timescale is strongly sensitive to the ratio of the mass of the satellite to that of the halo as a whole (eg., Binney & Tremaine, 1987). As a result, galaxy mergers of the most massive galaxies occur shortly after the formation of a new halo, while galaxies of lower mass tend to remain in orbit about the dominant object for a number of dynamical times. This introduces strong correlations into the merger tree so that mergers at low redshifts tend to involve lower mass satellites. These contribute little to the mass of the brighter galaxies and there is little flattening of the CMR. Because of these effects, it may be possible to construct a coherent model in which the star formation occurs at relatively early epochs and yet the growth of galaxies by mergers does not overly weaken the initial CMR. One approach is to treat separately the constraints on the degree of mass growth and on the last epoch of star formation. Alternatively, the two limits may be combined using the mass growth factor given by the hierarchical merger tree. For example, consider a model in which the star formation ceases over extended periods and is truncated randomly so that star formation ceases in all systems before $z \sim 1$ (ie., $t_{stop,min} \sim 8.5\,\text{Gyr}$ in the terminology of §2.2). If the CMR has negligible scatter at $t_{stop,min}$, we can estimate the combined effects of the differential reddening and merging. Figure 9 suggests that merging will result in a present-day $R$ value of 0.6, corresponding to a scatter of 0.04 mag about the observed slope. Differential colour evolution contributes a scatter of 0.02 mag (Figure 4). Assuming that these contributions can be added in quadrature, the model would be well matched to the observed scatter. However, this estimate is clearly conservative. If the star formation rate declines as $\tau = 10\,\text{Gyr}$, the $V$-band luminosity weighted age of the stellar population is 10.5 Gyr (look-back time, $z = 2$) in even the youngest galaxies. As a result, our estimate of the contribution to the scatter from galaxy merging has probably been underestimated. If the merger contribution is calculated from a look-back time of 10.5 Gyr, the combined scatter exceeds the observed value. Unfortunately, it is not possible to perform this calculation more rigorously without a detailed model connecting star formation, merging and chemical evolution.

A model in which all star formation ceases completely at early epochs in cluster galaxies cannot, in any case, be reconciled with the increasing fraction of blue galaxies seen in intermediate redshift clusters. This is a fundamental observational problem: if we accept that the small scatter seen in the Coma cluster is representative of local clusters (cf., BLE, Garilli et al., 1996), and that we have observed a sufficient range of cluster richnesses at high redshift to encompass its likely progenitors (cf., Stanford et al., 1997, Smail et al., 1997), then there must be a solution to this paradox. We would like to avoid extreme solutions that side-step the problem: for example, the blue colours of the intermediate redshift galaxies might arise from a top heavy IMF, or the blue light might be concentrated into stellar discs which are subsequently stripped in the cluster environment (Moore et al., 1996). Our modelling of the process by which the scatter is measured shows that such conclusions are avoidable. A model in which star formation in the cluster population is truncated uniformly over the available cosmic time indeed produces an unacceptably large scatter in the present-day CMR. However, mixing this population with an equal population of older galaxies results in a predicted CMR scatter that is compatible with the observed values and accounts for the galaxy populations observed in intermediate redshift clusters.

So far, we have considered the scatter across the CMR without discussing the origin of its slope. In models in which star formation is completed over relatively short timescales, the slope of the CMR has a natural explanation (eg., Arimoto & Yoshii, 1987). Higher mass systems are better able to contain the star forming gas against the winds generated by the on-going super-novae. They thus reach a higher mean metal abundance before the energy released by the supernovae is able to drive the gas out of the galaxy and halt the formation of further stars. If we now propose that star formation in early-type galaxies occurs over an extended period of time, the connection between galaxy mass and the efficiency with which gas is converted into stars (or equiv-
alently the metal abundance of the system) is much less evident. It is reasonable to ask whether there should be any correlation between stellar mass and colour at all. There are two alternatives: the star formation process must either be self-regulated by dynamic interchange of the gas expelled by supernova driven winds and the cooling gas from a halo reservoir (White & Frenk, 1991, Kauffmann et al., 1993, Cole et al., 1994), or by episodic bursts of star formation driven by galaxy mergers (Larson & Tinsley, 1974, Ballard et al., 1997). Both of these models are capable of explaining the slope of the correlation, but the origin is very different in each. By itself, a closed box model of chemical evolution tends asymptotically to a metal abundance (the ‘yield’, eg., Tinsley 1980) that depends only on the initial mass function of the stars formed. However, if (1) star formation is allowed only to occur in discrete bursts when galactic units merge and (2) the initial gaseous building blocks are all of similar size, the rate at which a system progresses towards the asymptotic abundance is linked to the number of mergers and hence to the total mass. Nevertheless, the model’s success is dependent on the ad hoc assumption of a limited size range for the proto-galactic units (otherwise galaxies of the same final mass may be built from widely differing numbers of mergers) and the careful balancing of the rate at which gas is consumed in mergers so that only the most massive galaxies approach the yield.

By contrast, a model in which the outflow of enriched gas is balanced by inflow of less enriched material naturally produces a relationship between galaxy mass and abundance since the rate at which the enriched material is driven from the galaxy depends on its binding energy. The result is an effective yield that varies with galaxy mass. In low mass systems, most of the metals produced in supernova ejecta escape from the galaxy; those that remain are diluted by newly accreted material (White & Frenk 1991). In high mass systems, most of the metals are retained in the galaxy and the inflowing material is quickly enriched. The result is a strong correlation between mass and metal abundance with a slope determined by the relative ease with which supernovae ejecta escape the galaxy. Such feedback is required in hierarchical models in order to flatten the faint end of the galaxy luminosity function (eg., White & Frenk, 1991, Cole et al., 1993, BCFL). A consequence of this model is that the correlation should not be restricted to early-type galaxies in clusters, but should also be observable in general field spirals. The existence of such a correlation is still controversial for the stellar disc (eg., Tully et al., 1982, Mobasher et al., 1986, Peletier & de Grijs, 1998), although a correlation between HII region metal abundance and total brightness is well established (Roberts & Haynes, 1994, Zaritsky et al., 1994). A further consequence of this model is that it is reasonable to treat separately the effects of passive aging of stellar populations and their metal abundances. The metal abundance of the stars in a galaxy is set by a combination of the yield of the stellar population, the outflow of enriched material and the inflow of unenriched gas. These combine to set an effective yield that is a function of the mass of the galaxy’s dark matter halo. Unlike the scenario of Arimoto & Yoshii, the metal abundance does not depend critically on the duration for which star formation continues; thus two White & Frenk type galaxies that continue their star formation for different lengths of time will both have similar metallicities. Impor-tantly, the longer timescale of star formation in one galaxy cannot compensate for its bluer colours (due to younger stellar population age) through a higher metal abundance. In this model at least, there is no possibility of a conspiracy between metal abundance and age that conceals large variations in star formation history.

6 CONCLUSIONS

The narrowness of the colour–magnitude relation in cluster galaxies imposes a definite degree of homogeneity on their stellar populations. BLE applied this argument in the context of single burst models to conclude that the majority of star formation occurred at look-back times greater than $\sim 10$ Gyr. In this paper, we have explored a greater variety of possible star formation histories. We find that the bulk of stars must still have old ages but that star formation continuing until relatively recent epochs can be accommodated within the observed CMR scatter in local clusters. This conclusion agrees with that of van Dokkum et al., 1998, who used analytical models for the evolution of galaxies luminosities to explore a wider range of possible star formation histories. As a result, the rising fraction of blue galaxies in clusters out to $z \sim 0.5$, as observed in the B"{u}thcher-Oemler effect, does not conflict with the homogeneity of the majority of cluster galaxies. Importantly, these constraints on the star formation histories of cluster galaxies can be naturally reconciled with hierarchical models of galaxy formation since the cluster galaxy population is assembled at substantially higher redshifts than galaxies drawn at random from the universe as a whole.

The CMR can also be used to limit the degree of merging that can occur between galaxies after the stellar population has been formed. Random mergers quickly flatten the CMR leading to a high ratio between the scatter and slope. With random merging, this point is reached quickly, so that the bulk of stars in these galaxies must be formed at redshifts less than 1. However, merging between galaxies is not well modelled by a random process since the dynamical friction time scale is strongly mass dependent. We have incorporated the effect of correlated mergers using galaxy merger trees taken from BCFL. This substantially lengthens the survival time of a recognisable CMR. Even at $z = 2$, the ratio of the scatter to slope is kept small, lying just within acceptable bounds. As a result, it is possible to construct a model for the formation of cluster galaxies in which the bulk of the stars are formed at high redshifts (resulting in little differential colour evolution at the present-day) and yet undergo sufficiently little merging that the colour-magnitude relation is still well-defined at the present-day. Nevertheless, the factor by which the mass of a typical galaxy grows between the formation of the bulk of the stars and its present-day mass is small, less than a factor of 2–3 regardless of the merger tree considered.

This conclusion agrees with the study of KC97, who found that in their specific galaxy formation model the CMR of early-type galaxies was well defined and showed scatter comparable to the observational measurements. Our approach has been complementary: a tightly defined colour-magnitude relation at low redshift is not a property related to a single specific model, but will be met by any model in
which most star formation occurs at early epochs \((z > 1)\) in cluster galaxies and in which subsequent mass growth is not too large. We have avoided distinguishing galaxies on the basis of their morphology by using robust algorithm to measure the scatter of the mean relation with all morphological types present. This results in a conclusion that is as model independent as possible.

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