A Theoretical Construction of Thin Shell Wormhole from Tidal Charged Black hole

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Abstract

Recently, Dadhich et al [Phys.Lett.B 487, 1 (2000)] have discovered a black hole solution localized on a three brane in five dimensional gravity in the Randall-Sundrum scenario. In this article, we develop a new class of thin shell wormhole by surgically grafting above two black hole spacetimes. Various aspects of this thin wormhole are also analyzed.

Introduction:

In recent years, several scientists around the world have given their attention to the brane world gravity. Within the framework of brane world, one can explain one of the hierarchy problems in the current standard models of high energy physics \cite{1}. Also, brane world framework may give clue how to solve the greatest challenging problem in theoretical physics namely the unification of all fundamental forces in nature \cite{2}. Actually, the string theory was the inspiration behind the idea of brane world scenario. The brane world idea is that the matter fields are located on a three dimensional subspace, called brane embedded in 1 + 3 + d dimensions in which the gravity can propagate in the d-extra dimensions. Here, the d-extra dimensions need not all be small or even compact. Most of the recent studies consider a simple version of the brane world scenario where all matters (except gravity) are confined to a 3-brane embedded in a five dimensional spacetime (bulk) while gravity can propagate in the bulk.

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As a consequence, the gravity on the brane can be described by the modified 4-dimensional Einstein equations which contains (i) $S^\nu_\mu$, which is quadratic in the stress energy tensor of matter confined on the brane (ii) the trace less tensor $E^\nu_\mu$, originating from the 5D Weyl tensor and describing tidal effects on the brane from the bulk geometry. Recently, Dadhich et al [3] have presented a spherically symmetric solution which describes a black hole localized on a three brane in five dimensional gravity in the brane world scenario. This black hole (without electric charge) is termed as tidal charged black hole. In this case tidal charge is arising via gravitational effects from the fifth dimension.

Motivated by Morris and Thorne’s work [4], the study of traversable wormhole have been a focus of interest in recent years. These are the solutions of Einstein’s equations that have two regions connected by a throat. To get a wormhole solution, one has to tolerate the violation of null energy condition. In other words, the presence of exotic matter (i.e. the matter which violates the null energy condition) is unavoidable to get a wormhole solution. As it is difficult to deal with exotic matter, it is useful to minimize the usage of exotic matter. In recent past, Visser [5] has proposed a way, which is known as ‘Cut and Paste’ technique, of minimizing the usage of exotic matter to construct a wormhole in which the exotic matter is concentrated at the wormhole throat. In ‘Cut and Paste’ technique, the wormholes are theoretically constructed by cutting and pasting two manifolds to obtain geodesically complete new manifold with a throat placed in the joining shell. Using Darmois-Israel [6] formalism, one can determine the surface stresses of the exotic matter (located in thin shell placed at the joining surface). Recently, several physicists are interested to develop thin shell wormholes. Visser and Poisson have analyzed the stability of thin shell wormhole constructed by joining the two Schwarzschild geometries [7]. The stability of transparent spherical symmetric thin-shell wormholes was examined by Ishak and Lake [8]. Eiroa and Romero [9] have studied the linear stability of charged thin shell wormholes constructed by joining the two Reissner-Nordström spacetimes under spherically symmetric perturbations. Lobo and Crawford [10] have extended the linear stability analysis to the thin shell wormholes with Cosmological Constant. Eiroa and Simeone [11] have studied cylindrically symmetric thin shell wormhole geometries associated to gauge cosmic strings. Also, the same authors have constructed a charged thin shell wormhole in dilaton gravity and they have shown that the reduction of the total amount of exotic matter is dependent on the Dilaton-Maxwell coupling parameter [12]. The five dimensional thin shell wormholes in Einstein-Maxwell theory with a Gauss Bonnet term has been studied by Thibault et al [13]. They have made a linearized stability analysis under radial perturbations. Recently, the present authors have studied thin shell wormholes in higher dimensional Einstein-Maxwell theory which is constructed by Cutting and Pasting two metrics corresponding to a higher dimensional Reissner-Nordström black hole [14]. Also, the same authors have constructed thin shell wormhole in heterotic string theory [15]. In this article, we study thin shell wormhole in Brane worlds. We develop the model by cutting and pasting two metrics corresponding to a tidal charged black hole. According to Dadhich et al [3] tidal force arising via gravitational effects from the fifth dimension i.e. it is arising from the projection onto the brane of free gravitational field effects in the bulk.
The tidal charge ‘q’ of Dadhich et al.’s brane world black hole contains the information of the extra dimension and does affect on the geodesics as well as on the gravitational potential. So, it is of great interest to investigate how the tidal force affects on the thin shell wormhole. Various aspects of these thin shell wormhole, namely, temporal evolution of the throat, stability, total amount of exotic matter will be discussed. We have shown the tidal charge affects significantly on stability of this thin shell wormhole. It is also shown that the total amount of exotic matter is reduced by increasing of the tidal charge.

The layout of the paper as follows:

In section 2, the reader is reminded about tidal charged black hole obtained by Dadhich et al. In section 3, thin shell wormhole has been constructed by means of the Cut and Paste techniques. In section 4, the time evolution of the radius of the throat is considered whereas linearized stability analysis is studied in section 5. Section 6 is devoted to a brief summary and discussion including the calculation of the total amount of exotic matter needed.

2. Brane world Black holes:

The gravitational field equations induced on the brane is described by modified Einstein equations from 5-dimensional gravity with the aid of Gauss-Codazzi equations to the effective 4-dimensional field containing the new terms carrying bulk effects onto the brane as [1-2]

\[ G_{\mu\nu} = \Lambda g_{\mu\nu} + k_4^2 T_{\mu\nu} + k_5^4 S_{\mu\nu} - E_{\mu\nu} \]  

Here \( \Lambda \) is the 4D cosmological constant expressed in terms of the 5D, \( \Lambda_5 \) and brane tension \( \lambda \):

\[ \Lambda = \frac{1}{2}(\Lambda_5 + \frac{1}{6}k_5^4 \lambda) \ ; \]

\[ k_4^2 = 8\pi G = \frac{k_4^4 \lambda}{6\pi} \]

where \( k_4 \) is the 4D gravitational constant and \( G \) is the Newton’s constant of gravity, \( T_{\mu\nu} \) is the usual energy momentum tensor of matter on the brane and the local bulk effects on the matter, \( S_{\mu\nu} \) consists of squares of \( T_{\mu\nu} \) (it is a local higher energy correction term). \( E_{\mu\nu} \) consists of the projection of the bulk Weyl tensor onto the brane (it is non local from the brane point of view). By construction, \( E_{\mu}^\nu \) is trace less, \( E_{\mu}^\mu = 0 \) and on the brane, in the vacuum case, it satisfies conservation equation \( \nabla^\mu E_{\mu}^\nu = 0 \). Inspite of the absence of matter, \( E_{\mu}^\nu \) may not be zero.
Considering the static spherically symmetric spacetime,
\[ ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2d\Omega^2, \tag{2} \]
and using the vacuum brane field equation (with \( \Lambda \) set equal to zero)
\[ G_\mu^\nu = -E_\mu^\nu. \]

Dadhich et al [3], have obtained a new black hole solution as
\[ ds^2 = (1 - \frac{2m}{r} + \frac{q}{r^2})dt^2 - \frac{dr^2}{(1 - \frac{2m}{r} + \frac{q}{r^2})} - r^2d\Omega^2, \tag{3} \]

These black holes are characterized by two parameters: their mass and dimension less tidal charge, \( q \). The tidal charge parameter \( q \) comes from the projection on the brane of free gravitational field effects in the bulk. Here the tidal charge \( q \) can take both positive and negative values. When the tidal charge takes positive values, then the metric (3) is analogous to Reissner-Nordström Black hole solution. For \( q < m^2 \), it describes tidal charge black hole with two horizons at \( r_h = m \pm \sqrt{m^2 - q} \), which are below the Schwarzschild radius. Now for \( q < 0 \), the above tidal charge black hole has only one horizon at \( r_h = m + \sqrt{m^2 + |q|} \), above the Schwarzschild radius. The gravitational field of this black hole is increased due the presence of the tidal charge.

### 3. Cut and Paste Tactics:

Dadhich et al [3] have given exact localized black hole solution in which the brane is located at \( \chi = 0 \) (\( \chi \) is the fifth coordinate). This black hole formed from collapsed matter confined on brane. The induced four dimensional metric on the brane is given in equation (3). We work with (3 + 1) dimensional brane, which is contained in a (4 + 1) dimensional bulk and consider the 4-dimensional horizon structure of the brane world black hole because like Dadhich et al [3], we are not interested to take the effect of the brane world black hole on the bulk geometry. And for that we do not consider the bulk metric. Thus, to construct thin shell wormhole in brane world, we cut two copies of region from the tidal charged black hole geometry described by \( \Omega^\pm = (x \mid r \leq a) \), where \( a \geq r_h \) (position of event horizon). In this study, we assume the case, where tidal charge is negative i.e. horizon of this black hole is greater than the Schwarzschild horizon. Now taking the two copies of the remaining regions, \( M^\pm = (x \mid r \geq a) \), we paste the two pieces together at the hypersurface \( \Sigma = \Sigma^\pm = (x \mid r = a) \). This surgical grafting produces a geodesically complete manifold \( M = M^+ \cup M^- \) with a matter shell at the surface \( r = a \), where the throat of the wormhole is located. Thus a single manifold \( M \) is obtained which connects two asymptotically flat regions at their boundaries \( \Sigma \) and the throat is placed at \( \Sigma \) (here \( \Sigma \) is a synchronous time like hypersurface). Since \( M \) is a piece wise tidal charged black hole spacetime, the stress energy tensor is either everywhere zero or obeys all energy conditions except at the throat itself.
At $\Sigma$, one expects that the stress energy tensor to be proportional to a delta function. Following Darmois-Israel formalism, we shall determine the surface stresses at the junction boundary. The intrinsic coordinates in $\Sigma$ are taken as $\xi^i = (\tau, \theta, \phi)$ with $\tau$ is the proper time on the shell.

To understand the dynamics of the wormhole, we assume the radius of the throat be a function of the proper time $a = a(\tau)$. The parametric equation for $\Sigma$ is defined by

$$\Sigma : F(r, \tau) = r - a(\tau) \quad (4)$$

The extrinsic curvature associated with the two sides of the shell are

$$K_{ij}^\pm = -n_\nu^\pm \left[ \frac{\partial^2 X_\nu}{\partial \xi^i \partial \xi^j} + \Gamma^\nu_{\alpha \beta} \frac{\partial X^\alpha}{\partial \xi^i} \frac{\partial X^\beta}{\partial \xi^j} \right] |_{\Sigma} \quad (5)$$

where $n_\nu^\pm$ are the unit normals to $\Sigma$,

$$n_\nu^\pm = \pm |g^{\alpha \beta} \frac{\partial F}{\partial X^\alpha} \frac{\partial F}{\partial X^\beta}|^{-\frac{1}{2}} \frac{\partial F}{\partial X^\nu} \quad (6)$$

with $n^\mu n_\mu = 1$.

The intrinsic metric on $\Sigma$ is given by

$$ds^2 = -d\tau^2 + a(\tau)^2 d\Omega_2^2 \quad (7)$$

From Lanczos equation, one can obtain the surface stress energy tensor $S^i_j = \text{diag}(\sigma, -v_\theta, -v_\phi)$ (where $\sigma$ is the surface energy density and $v_{\theta,\phi}$, the surface tensions) as

$$\sigma = -\frac{1}{2\pi a} \sqrt{1 - \frac{2m}{a} - \frac{Q}{a^2} + \dot{a}^2} \quad (8)$$

$$-v_\theta = -v_\phi = v = \frac{1}{4\pi a} \frac{1 - \frac{m}{a} + \dot{a}^2 + a\ddot{a}}{\sqrt{1 - \frac{2m}{a} - \frac{Q}{a^2} + \dot{a}^2}} \quad (9)$$

where over dot means the derivative with respect to $\tau$ and $q = -Q, Q > 0$.

Negative surface energy density in (8) implies the existence of exotic matter at the shell. The negative signs of the tensions mean that they are indeed pressures.
4. **Time evolution of radius of the throat:**

The static equations are obtained with $\dot{a} = 0$ and $\ddot{a} = 0$ in equations (8) and (9):

\[
\sigma_0 = -\frac{1}{2\pi a} \sqrt{1 - \frac{2m}{a} - \frac{Q}{a^2}}
\]

\[
v_0 = -\frac{1}{4\pi a} \frac{1 - \frac{m}{a}}{\sqrt{1 - \frac{2m}{a} - \frac{Q}{a^2}}}
\]

[ suffix 0 indicates the static situation ]

Now, one can write the equations (10) and (11) in the form

\[
v_0 = w(a)\sigma_0
\]

where

\[
w(a) = \frac{1}{2} \frac{1 - \frac{m}{a}}{(1 - \frac{2m}{a} - \frac{Q}{a^2})}
\]

Following Eiroa et al and C Bejarano et al [12], we assume that the equation of state does not depend on the derivative of $a(\tau)$ i.e. it is the same form as in the static one. Now putting $\sigma$, $v$ in place of $\sigma_0$, $v_0$ from (8) and (9) in (12), we get the following expression as

\[
\ddot{a}(a - 2m - \frac{Q}{a}) - \dot{a}^2 \left(\frac{m}{a} + \frac{Q}{a^2}\right) = 0
\]

This implies,

\[
\dot{a}(\tau) = \dot{a}(\tau_0) \left[1 - \frac{2m}{a(\tau_0)} - \frac{Q}{a^2(\tau_0)}\right]^{\frac{1}{2}}
\]

Here, $\tau_0$ is arbitrary fixed time.

Thus we get,

\[
\int_{a(\tau_0)}^{a(\tau)} \frac{da}{\left[1 - \frac{2m}{a(\tau)} - \frac{Q}{a^2(\tau)}\right]^{\frac{1}{2}}} = \dot{a}(\tau_0)(\tau - \tau_0) \left[1 - \frac{2m}{a(\tau_0)} - \frac{Q}{a^2(\tau_0)}\right]^{-\frac{1}{2}}
\]

This gives,

\[
[a^2(\tau) - 2ma(\tau) - Q]^{\frac{1}{2}} + m \ln[2(a^2(\tau) - 2ma(\tau) - Q)]^{\frac{1}{2}} + 2a(\tau) - 2m = \frac{\dot{a}(\tau_0)(\tau - \tau_0)}{(1 - \frac{2m}{a(\tau_0)} \frac{Q}{a^2(\tau_0)})^{\frac{1}{2}}}
\]

The above implicit expression gives the time evolution of the radius of the throat.
The velocity and acceleration of the throat are

\[ \dot{a}(\tau) = \dot{a}(\tau_0) \left[ 1 - \frac{2m}{a(\tau)} - \frac{Q}{a^2(\tau)} \right]^{\frac{1}{2}} \]  

and

\[ \ddot{a}(\tau) = \dot{a}^2(\tau_0) \left[ \frac{m}{a(\tau)} + \frac{Q}{a^3(\tau)} \right] \left[ 1 - \frac{2m}{a(\tau_0)} - \frac{Q}{a^2(\tau_0)} \right]^{-\frac{1}{2}} \]  

From the above two expressions, one can easily see that the sign of the velocity is given by the sign of the initial velocity and the acceleration is always positive. It is immaterial whether the initial velocity is positive or negative, the throat expands forever. This would imply that the equilibrium position is always unstable. However, if the initial velocity is zero, the velocity and acceleration of the throat would be zero i.e. throat be in static equilibrium position. Now, we shall study the stability of the configuration under small perturbations around static solution situated at \( a_0 \) (initial velocity will be assumed to be zero).

5. Linearized Stability Analysis:

Rearranging equation (8), we obtain the thin shell’s equation of motion

\[ \dot{a}^2 + V(a) = 0 \]  

Here the potential is defined as

\[ V(a) = 1 - \frac{2m}{a} - \frac{Q}{a^2} - 4\pi^2a^2\sigma^2(a) \]  

Linearizing around a static solution situated at \( a_0 \), one can expand \( V(a) \) around \( a_0 \) to yield

\[ V = V(a_0) + V'(a_0)(a - a_0) + \frac{1}{2}V''(a_0)(a - a_0)^2 + 0[(a - a_0)^3] \]  

where prime denotes derivative with respect to \( a \).

Since we are linearizing around a static solution at \( a = a_0 \), we have \( V(a_0) = 0 \) and \( V'(a_0) = 0 \). The stable equilibrium configurations correspond to the condition \( V''(a_0) > 0 \). Now we define a parameter \( \beta \), which is interpreted as the speed of sound, by the relation [7]

\[ \beta^2(\sigma) = \frac{\partial p}{\partial \sigma} |_{\sigma} \]  

Here,

\[ V''(a) = -\frac{4m}{a^3} - \frac{6Q}{a^4} - 8\pi^2\sigma^2 - 32\pi^2a\sigma\sigma' - 8\pi^2a^2(\sigma')^2 - 8\pi^2a^2\sigma\sigma'' \]  

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Since the negative tension is equivalent to pressure, we take \(-v = -v_\theta = -v_\phi = p\). From equations (8) and (9), one can write energy conservation equation as

\[ \dot{\sigma} + 2 \frac{\dot{a}}{a} (p + \sigma) = 0 \]  

(24)

or

\[ \frac{d}{d\tau} (4\pi \sigma a^2) + p \frac{d}{d\tau} (4\pi a^2) = 0 \]  

(25)

From equation (24) (by using (22)), we obtain,

\[ \sigma'' + \frac{2}{a} \sigma' (1 + \beta^2) - \frac{2}{a^2} (p + \sigma) = 0 \]  

(26)

Now, the second derivative of the potential is taken the following form as

\[ V''(a_0) = -\frac{2}{a_0^3} [a_0 \frac{(a_0 - m)^3 + m(m^2 + Q)}{a_0^2 - 2ma_0 - Q} + 2(a_0^2 - 3ma_0 - 2Q)\beta_0^2] \]  

(27)

The wormhole solution is stable if \(V''(a_0) > 0\) i.e. if

\[ (a_0^2 - 3ma_0 - 2Q)\beta_0^2 < -a_0 \left[ \frac{(a_0 - m)^3 + m(m^2 + Q)}{2(a_0^2 - 2ma_0 - Q)} \right] \]  

(28)

The right hand side of this inequality is negative because, \(a_0 > m(1 + \sqrt{1 + \frac{Q}{m^2}}) \equiv r_h\).

The left hand side flips sign at \(a_0 = \frac{3}{2} m[1 + \sqrt{1 + \frac{8Q}{9m^2}}] \equiv r_+ > r_h\). Therefore, if one treats \(a_0, m\) and \(Q\) are specified quantities, then the stability of the configuration requires the following restrictions on the parameter \(\beta_0\).

\[ \beta_0^2 < -a_0 \left[ \frac{(a_0 - m)^3 + m(m^2 + Q)}{2(a_0^2 - 2ma_0 - Q)} \right] \text{ if, } a_0 > r_+ \]

\[ \beta_0^2 > -a_0 \left[ \frac{(a_0 - m)^3 + m(m^2 + Q)}{2(a_0^2 - 2ma_0 - Q)} \right] \text{ if, } r_h < a_0 < r_+ \]

This means there exists some part of the parameter space where the throat location is stable. For a lot of useful information, we show the stability regions graphically. For normal situation i.e. in case of real matter, \(\beta_0\) represents the velocity of sound and it lies within the interval, \(\beta_0^2 \in (0, 1]\). However, in the presence of exotic matter (as it happens to be in the throat), \(\beta_0\) is not velocity of sound. So far, exotic matter this range may be relaxed. One can see ref.[7] for an extensive discussion on the respective physical interpretation of \(\beta_0\) in the presence of exotic matter.
Figure 1: Here we plot $z = \beta \frac{a_0}{m}$ $Vs. x = \frac{a_0}{m}$ for $a_0 > r_+ (\frac{Q}{m} = .1)$. The stability region is situated below the curve.

Figure 2: Here we plot $z = \beta \frac{a_0}{r_{(a=a_0)}}$ $Vs. x = \frac{m}{a_0}$ for $r_h < a_0 < r_+ (\frac{Q}{m} = .1)$. The stability region is situated above the curve.
6. Summary and Discussions:

In this article, we have studied thin shell wormhole in brane world scenario by surgically grafting two tidal charged black hole spacetimes. Though the tidal charge parameter 'q' of this black hole can take both positive and negative values, but in this study, we consider the case for \( q < 0 \) [ For \( q > 0 \), the solution is analogous to the Reissner-Nordström Black hole solution ]. In this case, the black hole has only one horizon, which lies outside the Schwarzschild horizon. We have considered an equation of state that relates the tension with the surface energy density of the exotic matter at the throat. We have obtained the time evolution of the radius of throat. One could see that whether the initial velocity is positive or negative, the throat expands indefinitely. But, when initial velocity is zero, then the radius of the throat remains constant i.e. the throat be in a static equilibrium position. We have analyzed the dynamical stability of the thin shell, considering linearized radial perturbations around static solution. To analyze this, we define a parameter \( \beta^2 = \frac{\mathcal{P}}{\sigma^2} \) as a parametrization of the stability of equilibrium. We have obtained a restriction on \( \beta^2 \) to get stable equilibrium of the thin wormhole (see eq. (28)).

The total amount of exotic matter for the thin wormhole can be quantified by the integral (In this case, radial pressure, \( p_r = 0 \) and we have \( \rho < 0, \rho + p_r < 0 \) i.e. both energy conditions are violated. The transverse pressure is \( p_t = p_\theta = p_\phi = -\nu \) and one can see from (10) and (11) that the sign of \( \rho + p_t \) is not fixed but depends on the value of the parameters)

\[
\Omega = \int [\rho + p_r] \sqrt{-g} d^3 x .
\]

Following Eiroa and Simone [11], we introduce a new radial coordinate \( R = \pm (r - a) \) in \( M (\pm \text{ for } M^\pm \text{ respectively}) \) so that

\[
\Omega = \int_0^{2\pi} \int_0^\pi \int_{-\infty}^{\infty} [\rho + p_r] \sqrt{-g} Rd\theta d\phi
\]

Since the shell does not exert radial pressure and the energy density is located on a thin shell surface, so that \( \rho = \delta(R) \sigma_0 \), then we have

\[
\Omega = \int_0^{2\pi} \int_0^\pi \left[ \rho \sqrt{-g} \right]_{r=a_0} d\theta d\phi = 4\pi a_0^2 \sigma(a_0)
\]

Thus one gets, \( \Omega = -2a_0 \sqrt{1 - \frac{2m}{a_0} - \frac{Q}{a_0^2}} .\)

One could see that the tidal charge and mass of the black hole affect the total amount of exotic matter needed. The variation of the total amount of exotic matter with respect to tidal charge and mass of the black hole is shown in the figure 3.
Figure 3: Here we plot $z = \frac{\Omega}{a_0} V_s$, $x = \frac{m}{a_0}$ and $y = \sqrt{\frac{Q}{a_0}}$. 
Now we are interested to the fact that under what conditions the total amount of exotic matter could be reduced. If mass of the black hole remains fixed, then the total amount of exotic matter is reduced by increasing the tidal charge. Thus tidal charge contains the information of extra dimension plays significant role to reducing total amount of exotic matter needed.

Also if tidal charge of the black hole is kept fixed, then the total amount of exotic matter is reduced by increasing the mass of the black hole. Thus one can see that less exotic matter is needed when tidal charge and mass of the black hole are increased. These are depicted in figure 4 and 5.

![Figure 4](image1.png)

Figure 4: Here we plot $z = \frac{\Omega}{a_0}$ Vs. $y = \sqrt{\frac{Q}{a_0}}$ for $\frac{m}{a_0} = \frac{1}{10}$.

![Figure 5](image2.png)

Figure 5: Here we plot $z = \frac{\Omega}{a_0}$ Vs. $x = \frac{m}{a_0}$ for $\sqrt{\frac{Q}{a_0}} = .35$.

Further, from the above expression, one can see that $\Omega$ approaches to zero when wormhole radius tends to the event horizon (i.e. when $a_0 \rightarrow r_h$). So one can get vanishing amount of exotic matter by taking $a_0$ near $r_h$. Thus, one can note that the total amount of exotic matter needed to support traversable wormhole can be made infinitesimal small by taking wormhole radius near the event horizon of the tidal charged black hole. This is depicted in the figure-6.
Figure 6: We choose $m = 2$ and $Q = .5$. The variation of total amount of exotic matter on the shell with respect to $a_0$ is shown in the figure.

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