Gap flow between two circular plates with temperature-controlled wall: application of thermohydrodynamic lubrication theory and comparison with an iso-viscous model

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Abstract
Gaskets are widely used as static seals in industry, machinery, and living ware. Generally, leakage is reduced or eliminated by clamping seal components and blocking flow passages. However, strong clamping sometimes leads to surface damage. Surface roughness and waviness form partial paths and excitation and vibration loosen clamping bolts. Leakage is directly proportional to the cube of gap height and inversely proportional to viscosity. Moreover, the viscosity of fluids, particularly oil, strongly depends on temperature, as lower temperatures correspond to higher viscosities. In other words, oil leakage can be reduced by decreasing its temperature. Therefore, it is possible to control leakage by changing the gap temperatures. In this paper, a flange-type gasket with a gap is modeled using two circular plates with a central recess. The thermohydrodynamic lubrication (THL) theory is applied to the gap flow. The effects of wall temperature, gap height, and recess pressure on the leakage flow rate are numerically solved. The basic equations comprise the generalized Reynolds equation, the energy equation, and the heat conduction equation and the THL solutions are compared with a simple model based on the iso-viscous theory. In conclusion, the oil temperature in the gap can be controlled by the wall temperature. If the wall temperature is decreased, the oil temperature falls. Subsequently, viscosity increases, helping to decrease leakage in a wide range of operating conditions. The leakage can be estimated by the iso-viscous model with the viscosity at the wall temperature.

Keywords: Seal, Gasket, Viscosity, Oil, Leakage, Tribology, Machine element

1. Introduction
Gaskets (JIS, 2015) and gasketed joints (ASTM, 1985) are sealing devices and static seals. They are essential machine elements and widely used in the industry, machinery, and living ware. The characteristics of gaskets mainly depend on their material, geometry, fluid properties, sealing pressure, clamping pressure, and flange surface. By squeezing the sealing faces and cutting the flow passages, leakage is reduced and stopped.

Sealing elements are often inserted into the mating surfaces of the gasket devices. The materials are non-metals (e.g., rubber and resin) and metals (e.g., copper alloy and stainless steel). Many researchers have studied gaskets in terms of their surface geometry, clamping pressure, element material, morphology damage, and environmental use. Based on the percolation and contact mechanics theories, Persson presents a theory of leak rate of seals. These have recently been developed and are applicable to both static and dynamic seals (Persson & Yang, 2008), including plastic deformation (Persson, 2016).

Gaskets are always used when surfaces are contacted firmly by clamping (Kazamaki, 1969). Clamping increases the real contact area (Nitta et al., 2005) and reduces leakage. At the same time, however, it damages surfaces. Additionally, because surface roughness and waviness yield partial channels (Tsukizoe, 1968), clamping is not a panacea. Meanwhile,
fastener loosening and mechanical vibration cause temporary gaps.

Although there are several types of gaskets and gasketed joints; the flange type would be the simplest configuration and frequently used. Fessler et al. (1988) examined flat-flanged joints without gaskets and, using the fluid flow theory, finite element deflections, and initial gap measurements, experimentally collected air leakage data. They discovered that gap height can be reduced with load and that the divergent initial gap causes excessive leakage. As for gas pipe flange connections, using the finite element simulation, including the nonlinear stress–strain relationship of the gasket and the leakage test using nitrogen gas, Takaki et al. (2005) studied the sealing performance of the pipe flange connection. They discussed the effects of the flange rotation and contact stress on the leak rate. Using the material spring effect, Saeed et al. (2008) developed an all-metal gasket with circumferential annular lips and examined its effect on sealing. Pérez-Ráfols et al. (2016) presented a model of spiral groove waviness and surface roughness, matching the experimental data in the literature. They stated that the contact area’s local distribution is more strongly connected to leakage than the real percentage of the contact area. Moreover, since gaskets are used in various environments and temperatures, Marie et al. (2003) proposed an integrated approach to characterize liquid leakage through a metal-to-metal contact seal. They examined different deformation models and compared their experimental results. Interfacial leakage of air in rubber under a low-temperature condition poses a serious problem. Akulichev et al. (2018) revealed that the main cause for failure is the detachment of the elastomer seals from their mating sealing parts due to elastomer thermal contraction.

In this study, we consider an additional method for improving the sealing effect of gaskets. We focus on that oil viscosity is a strong function of the temperature, meaning that the lower the temperature, the higher the viscosity (Kazama et al., 2016). Leakage is proportional to gap height cubed and inversely proportional to viscosity. Therefore, leakage can be reduced as the oil temperature diminishes (Kazama, 2017) as well as the gap shrinks. The phenomena between the gap flow and wall temperature and the relationship between cooling the gasket and reducing the leakage are examined theoretically. A gasket joint with a gap is modeled as two circular plates with a center recess. For a theoretical approach of gap flow, the thermohydrodynamic lubrication (THL) theory (Dowson, 1962; Khonsari, 1986; Tanaka, 1993) has been well known and widely used mainly for hydrodynamic journal bearings (Mitsui and Yamada, 1979; Hatakenaka, 2015) and thrust bearings (Taniguchi and Ettes, 1975; Kim et al., 1986). The THL theory is enable to estimate precisely the temperature distributions and bearing characteristics; however, the model is complicated and the calculation is time consuming. In this report, we apply the THL theory to the gap flow, with the effects of wall temperature, gap height and sealing pressure on the leakage flow rate being solved numerically and show the effects of cooling the gap on reducing the leakage under a wide range of conditions. Additionally, we propose a simple model based on the iso-viscous lubrication theory and confirm the applicability of the model to estimation of the leakage.

2. Nomenclature

\[ a \] : radius ratio, \( = R_1/R_2 \)

\[ B \] : disk thickness

\[ Bi \] : \( = Bh_T/\Lambda \)

\[ c_p \] : specific heat at constant pressure

\[ Ec \] : \( = (\omega_0 R_2)^2/(c_p T_0) \)

\[ Eu \] : \( = 6\mu_0/(\rho \omega_0 H^2) \)

\[ \tilde{h} \] : clearance, \( = h/H \)

\[ H \] : representative clearance

\[ h_c \] : center clearance

\[ h_{\text{min}} \] : minimum clearance

\[ h_T \] : heat transfer coefficient

\[ Nu \] : \( = H h_T/\Lambda \)

\[ \bar{p} \] : pressure, \( = p/[6\mu_0 \omega_0/(R_2/H)^2] \)

\[ Pe' \] : \( = Pr Re' \)

\[ Pr \] : \( = \mu_0 c_p/\Lambda \)

\[ p_a \] : atmospheric pressure

\[ p_r \] : recess pressure

\[ p_s \] : sealing oil pressure
3. Theory

The leakage of a flange-type gasket (gasketed joint) is modeled as a gap flow between two circular plates (disks), as shown in Fig. 1. The THL theory (Kazama, 2010) is applied to the non-parallel gap flow between stationary disks under the steady-state condition. The thermal distortion is neglected.

The basic equations comprise the generalized Reynolds equation, Eq. (1), the energy equation, Eq. (2), the heat conduction equation, Eq. (3) and equations of physical properties of the oil.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial r} \right) + \frac{1}{r^2 \Omega} \frac{\partial}{\partial \Omega} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} \right) = 0 \tag{1}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial r} \right) + \frac{1}{r^2 \Omega} \frac{\partial}{\partial \Omega} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} \right) = \frac{1}{Pe' \Omega} \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Ec \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Eu \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Re' \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} \tag{2}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial r} \right) + \frac{1}{r^2 \Omega} \frac{\partial}{\partial \Omega} \left( \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} \right) = \frac{1}{Pe' \Omega} \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Ec \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Eu \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} + Re' \frac{r^2 \Omega^3}{\Omega} \frac{\partial \Omega}{\partial \Omega} \tag{3}
\]

where \( Ec \), \( Eu \), \( Pe' \) and \( Re' \) are comparable with the Eckert, Euler, Péclet and Reynolds numbers, respectively. Moreover, functions \( F_i \) (\( i = 0, 1, 2, 3, 4 \)), \( m_r \) and \( m_{\theta} \), and dissipation function \( \phi \) in Eq. (2) are defined thus:
Fig. 1 Schematic diagram of the model

\[
\begin{align*}
F_0 &= \int_0^1 \frac{d\tilde{z}}{\tilde{\mu}} \\
F_1 &= \int_0^1 \tilde{z}d\tilde{Z} \\
F_2 &= -6 \left( \int_0^1 \frac{\tilde{z} d\tilde{Z}}{\tilde{\mu}} - F_1 \frac{F_3}{F_0} \right) \\
F_3 &= \int_0^1 \frac{\tilde{z} d\tilde{Z}}{\tilde{\mu}} \\
\end{align*}
\]

(4)

\[
\begin{align*}
m_r &= \frac{\tilde{z} \partial \tilde{h}}{\tilde{h} \partial \tilde{r}} \\
m_\theta &= \frac{\tilde{z} \partial \tilde{h}}{\tilde{h} \partial \tilde{\theta}} \\
\phi &= \frac{1}{\tilde{h}^2} \left[ \left( \frac{\partial \tilde{u}}{\partial \tilde{z}} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial \tilde{z}} \right)^2 \right]
\end{align*}
\]

(5)

(6)

The boundary conditions of Eqs. (1)–(3) are as follows:
where, \( \hat{p} \geq 0 \) (\( \hat{p}_a = 0 \)), \( 1 \geq \hat{r} \geq a \), \( \hat{T} > 0 \), \( \hat{T} > 0 \), \( 1 \geq \hat{z} \geq 0 \) and \( 1 \geq \hat{z}' \geq 0 \). It is assumed that the sealing oil temperature is kept constant and the model is placed in atmosphere, so that the oil temperatures at the inlet of the gap and inner radius of the disks can be set at the sealing oil temperature and the heat at the upper and side surfaces of the disk is transferred by the air natural convection.

Thicknss \( h \), the flow velocity in radial direction \( \hat{u} \), the flow rate through gap \( \hat{Q} \) and mean temperature of leaked oil \( \hat{T}_{out} \) are derived, as shown below.

\[
\hat{h} = \hat{h}_c + \alpha \hat{r} \cos(\theta - \phi) \quad (9)
\]

\[
\hat{u} = 6\hat{h}^2 \frac{\hat{p}}{\hat{r}} \left[ \int_0^{\hat{r}} \frac{\hat{Z}}{\hat{\mu}} d\hat{Z} - \frac{\hat{F}_1}{\hat{F}_0} \int_0^{\hat{r}} \frac{\hat{Z} d\hat{Z}}{\hat{\mu}} \right] \quad (10)
\]

\[
\hat{Q}(\hat{r}) = \frac{H}{R_2} \int_0^{2\pi} \hat{h} \int_0^1 \hat{u} \hat{z} d\hat{z} d\theta \quad (11)
\]

\[
\hat{T}_{out} = \int_0^{2\pi} \hat{h} \int_0^1 \hat{T}_{in} d\hat{z} d\theta \quad (12)
\]

Viscosity \( \hat{\mu} \) is considered a function of temperature, as per the following exponential function:

\[
\hat{\mu} = e^{-\beta(\hat{T}_{in} - \hat{T}_0)} \quad (13)
\]

The effect of pressure on viscosity is ignored because of relatively low pressure. Although other physical properties such as density, specific heat at constant pressure, thermal conductivity and thermal expansivity are also functions of temperature and pressure, they are kept constant. The equations are discretized using the finite difference method and are solved numerically employing the iterative method.

The THL theory is applied to lubrication analyses, especially of high-speed journal bearings and large-scale thrust bearings, where the effect of the viscous dissipation based on the Couette flow is dominant. The theory enables to estimate accurately the pressure and temperature distributions in the fluid film and tribological characteristics of the lubricating parts such as load-carrying capacity, friction and flow rate. However, the basic equations including the Reynolds equation, the energy equation, the heat conduction equation and formulae of lubricant physical properties must be solved simultaneously, so that the model is fairly intricate and the simulation becomes large scale. Therefore, a simple model is also proposed on the basis of the iso-viscous theory. The leakage flow rate, \( \hat{Q} \), of the parallel disks can be derived as:

\[
\hat{Q} = -\frac{\pi H \hat{h}^3 \hat{p}_r - \hat{p}_a}{\hat{r} \log a} \quad (14)
\]

where, \( \hat{\mu} \) is corresponding to an effective viscosity. In the following discussion, the \( \hat{\mu} \) in Eq.(14) is calculated by
Eq.(13) at \( \tilde{T} = \tilde{T}_w \) and the solutions of Eq. (14) are compared with the results based on the THL theory.

The effects of some of the operating conditions on temperature changes and leakage are shown below. Since there are many kinds of gaskets, the shape and size cannot be specified. In this paper, the size of the gasketed joint model is derived from SOP-100A-10K in JIS B2220 (JIS, 2012), namely \( R_x = 210 \text{ mm}, R_y = 115 \text{ mm and } B = 18 \text{ mm} \). Representative parameters are as follows: \( \rho = 844 \text{ kg/m}^3; \mu_0 = 37.6 \text{ mPa-s (TP}^0_{sh} = 40\degree \text{C}); \beta = 0.02 \text{ K}^{-1}; c_p = 1968 \text{ J/(kg-K)}; \lambda = 0.134 \text{ W/(K-m)}; \) \( \Lambda = 16 \text{ W/(K-m)}; \) and \( h_w = 50 \text{ W/(m}^2\text{-K)} \). The operating parameters are as follows: \( h_c = 1\text{–}100 \text{ µm}; p_s = 1\text{–}100 \text{ kPa}; \) and \( T_w = 10\text{–}50\degree \text{C} \). In Fig. 1, \( p_w \) is atmospheric pressure, \( H = h_c, p_r = p_s, R_{10} = R_1, R_{2U} = R_2, T_r = T_{riv} = T_s = 40\degree \text{C} \) and \( T_0 = 20\degree \text{C} \). The reference temperature \( T_0 \) is set at \( T_s \) and \( \omega_0 = p_s/\mu_0 \).

Under these conditions, negative pressure is not generated.

4. Results and discussion

Figure 2 illustrates the temperature contour maps of the cross section of the parallel gap (\( \bar{a} = 0 \)). The left-hand side figure is the temperature map in the case of thin gap (\( h_c = 1 \text{ µm} \)) and the right-hand side figure is the map of thick gap (\( h_c = 100 \text{ µm} \)). The upper and lower half parts correspond to the disk and the gap, respectively. The base line (\( \bar{a} = 0 \)) is the temperature-controlled wall (\( \tilde{T}_w = 0.5 \)). Although the temperature in the case of the thicker gap is somewhat non-uniform, the oil temperature in the gap almost follows the controlled wall temperature. One can see that, in other words, the oil temperature is easy to control for thinner gap and the leakage \( Q \) reduces as wall temperature decreases and the viscosity increases. This is because the disks are stationary and the gap is small, and thus the heat generation is small and the heat conduction is large (The distributions of temperature and changes in leakage are influenced by the thermal properties and working conditions).

Figure 3 and Fig. 4 present the effects of wall temperature \( T_w \) on the mean temperature of leaked oil \( T_{out} \) and the leakage flow rate ratio \( Q^* \), which is normalized by the reference leakage flow rate \( Q_0 (= 2.312 \times 10^{-10} \text{ m}^3/\text{s}) \) under the conditions of \( h_c = 0.01 \text{ mm} \) and \( T_w = 40\degree \text{C} \), in terms of gap height \( h_c \) at the center under the conditions of sealing oil pressure \( p_s = 10 \text{ kPa} \) and the gap in parallel (\( \bar{a} = 0 \)). As wall temperature \( T_w \) decreases, leaked oil temperature \( T_{out} \) nearly decreases at \( T_w \). It is evident that gap \( h_c \) is large, and \( T_{out} \) diverges slightly from \( T_w \) because of low heat conduction. Additionally, leakage \( Q^* \) also decreases as wall temperature \( T_w \) decreases. This tendency is independent of \( h_c \). Furthermore, under the conditions of \( h_c = 0.1\text{–}0.001 \text{ mm} \) and \( T_w = 50\text{–}10\degree \text{C} \), the effect of \( h_c \) on \( Q^* \) is obviously larger than that of \( T_w \). However, from large to small \( h_c, Q^* \) decreases as \( T_w \) becomes low.

Figure 5 shows the temperature difference \( \Delta T \) between the leaked oil temperature \( T_{out} \) and the control wall temperature \( T_w \); Fig. 6 shows the ratio of the leakage flow rate \( Q_{cal} \) based on the THL calculation to the flow rate \( Q_{th} \) of Eq. (14). In Fig. 5 the absolute values of \( \Delta T \) becomes large at larger \( h_c \) and lower and higher \( T_w \). However, \( \Delta T \) is less than 3\degree C under these conditions. In Fig. 6 the ratio \( Q_{cal}/Q_{th} \) is also changed approximately up to one percent.
Fig. 3 Mean temperature of leaked oil $T_{out}$ vs. wall temperature $T_w$ in terms of gap $h_c$ ($p_s = 10$ kPa, $\alpha = 0$)

Fig. 4 Effect of wall temperature $T_w$ on leakage flow rate ratio $Q^*$ in terms of gap $h_c$ ($p_s = 10$ kPa, $\alpha = 0$)

Fig. 5 Temperature difference $\Delta T$ between leaked oil and wall temperature in terms of gap $h_c$ ($p_s = 10$ kPa, $\alpha = 0$)
Table 1 presents the effects of sealing oil pressure $p_s$ on leakage flow rate ratio $Q_{cal}/Q_{th}$. Under the conditions of low-to-high $p_s$ and $T_w$, $Q_{cal}/Q_{th}$ is independent of $p_s$ and the change is less than 0.01. Therefore, regardless of the sealing pressure, one can see that the leakage flow rate can be controlled by the wall temperature and estimated by the simple model in Eq.(14).

Table 1 Effect of wall temperature $T_w$ on leakage flow rate ratio $Q_{cal}/Q_{th}$ in terms of sealing pressure $p_s$ ($h_c = 0.01$ mm, $\bar{\alpha} = 0$)

| $T_w$ [°C] | $p_s = 1$ kPa | 10 kPa | 100 kPa |
|------------|---------------|---------|---------|
| 10         | 1.002         | 1.002   | 1.002   |
| 20         | 1.002         | 1.002   | 1.002   |
| 30         | 1.001         | 1.001   | 1.001   |
| 40         | 1.000         | 1.000   | 1.000   |
| 50         | 0.995         | 0.995   | 0.995   |

Figure 7 illustrates the contour map of the interface temperature between the gap and the upper disk, $T_{\theta=\frac{\pi}{2}} (= T_{w=0})$, under the non-parallel gap ($\bar{\alpha} = 0.9$). The normalized sealing oil temperature $\bar{T}_{s}$ and the wall temperature $\bar{T}_{w}$ are 1.0 and 0.5, respectively and the contour interval is 0.001. The temperature variations are quite small and the temperatures almost coincide with the wall temperature, while the temperature near the minimum gap ($h_{min}$) is lowest and the temperatures have the distributions.

Figure 8 shows the ratio $Q^{*}$ of leakage flow rate in terms of inclination $\bar{\alpha}$ under the conditions of $h_c = 10$ μm and $p_s = 10$ kPa. Although the leakage flow rate increases as the inclination increases, the flow rate can be reduced as the wall temperature decreases for both parallel and non-parallel gaps.
5. Conclusions

Using the changes in viscosity as a strong function of temperature, a method for reducing leakage in a flange-type gasket was proposed and the models were examined theoretically. The THL theory was applied to the gap flow of the seal parts and the THL solutions were compared with a simple model using the wall temperature. The leakage flow rate and oil temperature were mainly discussed, with the wall temperature, gap height and sealing pressure selected as the parameters. When the wall temperature was controlled, the oil temperature in the gap followed and changed. The oil temperature nearly equated to the wall temperature in various conditions. Thus, leakage decreased as the wall temperature decreased as per a wide range of height of the gap and pressure of the oil. The simple model with the viscosity at the wall temperature is also effective to estimate the leakage. Moreover, the wall temperature may be controlled by using thermoelectric conversion elements and heat pumps. The present model will be confirmed and discussed by the experiment in the near future. This method can be superimposed onto clamping and squeezing the gap to reduce and stop the leakage.
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