Geographical Variability of the Deformation Radius of the First Surface Mode

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Research Article

Keywords: surface mode, deformation radius, sea surface height

DOI: https://doi.org/10.21203/rs.3.rs-370587/v1

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Geographical Variability of the Deformation Radius of the First Surface Mode

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Abstract

In areas with rough bathymetry, the vertical structure of ocean eddies can be decomposed into “surface modes,” which are surface intensified, and exhibit a velocity of nearly zero at the bottom. Furthermore, ocean surface modes are ubiquitous. Atlases of the first surface mode (SM1) deformation radius were computed on a global 0.25° × 0.25° grid using WOA2013 and the data from Generalized Digital Environment Model (GDEM). Monthly and seasonal changes were also analyzed. The annual average SM1 deformation radius was approximately 1.5 times larger than the Rossby radius of deformation; the main difference occurred in areas with rough bathymetry, including continental margins and mid-ocean ridges. The seasonal and monthly average SM1 deformation radius shows an evident annual cycle.

Plain Language Summary

The vertical structure of ocean eddies can be decomposed into “surface modes” with rough bathymetry. We used two sets of data to calculate the global distribution of deformation radius of the first surface mode (SM1) and its seasonal and monthly changes. The annual average SM1 deformation radius was approximately 1.5 times
larger than the Rossby radius of deformation, and the main difference occurs in areas with rough bathymetry. The seasonal and monthly average SM1 deformation radius exhibits an annual cycle.

**Keywords:** surface mode, deformation radius, sea surface height

1. **Introduction**

Oceanography has developed rapidly since the appearance of satellites, which provide a means to observe the global ocean surface. Gridded sea surface height (SSH) data are now available with a spatial resolution of 0.25° and a daily temporal scale. Sea surface height and surface temperature are observed remotely by satellites, and reflect the change of the entire ocean water column. However, a lack of understanding of the sea surface field is a significant problem.

Traditionally, the vertical structure of an ocean current is decomposed into different “modes.” Wunsch (1997) studied the vertical structure of subinertial currents from globally distributed current meters and projected subsurface velocities onto “baroclinic modes.” By assuming a flat bottom, these modes, which are orthogonal basis functions, can be derived using climatological density. Wunsch found that most regions are dominated by barotropic and first baroclinic modes. In an earlier theoretical work, Fu and Flierl (1980) suggested that nonlinear interactions cause energy to accumulate in these modes, which is consistent with this observation (Wunsch, 1997). Because of the near-surface intensification of baroclinic modes, Wunsch (1997) inferred that SSH anomalies probably reflect the first baroclinic mode. Stammer (1997) found that SSH anomalies have lateral scales proportional to the Rossby radius of deformation, which is also associated with the first baroclinic mode. Generally, the empirical orthogonal function (EOF) analysis method is used to produce efficient depictions of the vertical structure of current meter data (Inoue 1985; Mercier and Colin de Verdiere 1985; Müller and Siedler 1992; Wunsch, 1997; de La
Lama et al., 2016). In most regions, the first empirical orthogonal function (EOF1) monotonically decay with depth to a value near zero at the bottom and without changing sign. EOF1 is inconsistent with standard barotropic and baroclinic modes but a combination of them. EOF1 also captures a substantial fraction of the subsurface variance, which typically exceeds 80% in many locations. LaCase (2017) defined these modes as “surface modes.”

With a flat bottom, we can obtain traditional baroclinic modes; however, with steep or rough bathymetry, the solution will be another set of orthogonal basis functions “surface modes.” These modes are surface intensified with a velocity of nearly zero at the bottom. Surface modes should ideally exhibit with a rough bathymetry, including those of continental margins (Garrett, 1978; Pickart, 1995) or mid-ocean ridges (Fu et al., 1982). However, LaCase (2017) demonstrated that with realistic bathymetry and/or bottom friction, surface modes are ubiquitous in the ocean. This would explain the EOF1 of Wunsch (1997) and de La Lama et al. (2016) in many extratropical regions.

Since surface modes are ubiquitous in the ocean, it is unclear how to explain the sea surface field. The altimeter data of SSH predominantly reflects the first baroclinic mode (Wunsch, 1997), and SSH anomalies propagate faster than the baroclinic Rossby waves (Chelton and Schlax, 1996; Tailleux and McWilliams, 2001). This may be because SSH mainly reflects information about surface modes. The phase speed of long Rossby waves is proportional to the square of the deformation radius. Furthermore, since the surface mode has a larger deformation radius, it propagates faster (LaCase, 2017). Similar to the traditional baroclinic mode, each surface mode corresponds to a deformation radius. It is unclear what the distribution of the deformation radius corresponding to surface modes is in this case. It is also unclear what the difference is between this radius and the Rossby deformation radius. In this paper, the deformation radius of the first surface mode is calculated using real topographic slopes. Its global distribution is then given and compared with that of the Rossby deformation radius.
2. Method

With exponential stratification, \( N = N_0 \exp(z/d) \), and the solution of surface modes satisfies the following (LaCase, 2017):

\[
Y_0(\gamma) \left[ J_0(\gamma e^{-\alpha d}) + \zeta J_1(\gamma e^{-\alpha d}) \right] = J_0(\gamma) \left[ \zeta Y_1(\gamma e^{-\alpha d}) + Y_0(\gamma e^{-\alpha d}) \right]
\]

(2.1)

where \( \gamma = N_0 d \lambda / f_0 \), \( \zeta = \frac{N_0 \alpha k^2 + I^2}{\beta \lambda} \), \( Y_n \) and \( J_n \) are Bessel functions. \( N \) is the buoyancy frequency, \( f_0 \) is the Coriolis parameter and \( \beta \) is its derivative with latitude, \( H_0 \) is the mean depth, \( h = \alpha y \) represents a linear slope. It can be solved numerically using Newton’s method.

The surface modes problem can be analytically solved with constant stratification or exponential stratification. With exponential stratification and when the bottom slope exceeds 10e-4, the depth-varying modes become more surface intensified, which forms surface modes (LaCase, 2017). The bottom slope calculated from Etop1 data exceeds 10e-4 nearly everywhere (LaCase, 2017, Fig. 2), which indicates that the surface modes are ubiquitous in the ocean.

To obtain a more realistic representation of the real ocean, \( N \) is set as the exponential form and the bottom topographic slopes are calculated from the Etop1 data. Equation (2.1) is then numerically solved, and the corresponding deformation radius is obtained.

3. Geographical variability of the deformation radius of the surface mode

3.1 Exponential fitting of \( N \)

To solve equation (2.1), the buoyancy frequency \( N \) should be first calculated. Here, \( N \) is estimated using a gridded annual average temperature and salinity from WOA2013. Generally, the buoyancy frequency \( N \) of the real ocean is close to the exponential function distribution. Therefore, we first calculated the value of \( N \)
using the annual average WOA2013 data and then fit it with the exponential form
\[ N = N_0 \exp(z / d) \]. However, some areas are not suitable for exponential function
fitting. First, when the water depth is less than 50 m, the distribution of \( N \) is
extremely complex because of the lateral and bottom boundary. Second, when the
fitted \( d \) exceeds the local water depth, this indicates that the fitting is not correct.
Therefore, \( N \) in these regions is set to NAN.

As shown from the expression of \( N \), a larger \( N_0 \) and \( d \) indicates a larger \( N \),
which represents strong stratification. The distribution of the fitted \( N_0 \) and \( d \) is
shown in Fig. 1. The larger values of \( N_0 \) appear at the edge of the continental shelf,
and a smaller value appears in the ACC region. In contrast, the distribution of \( d \) is
the opposite in these areas. Thus, the stratification at the edge of the continental shelf
is relatively strong, while stratification is relatively weak in the ACC region.
3.2 Geographical variability of the deformation radius of the first surface mode

In section 3.1, the buoyancy frequency $N$ of the real ocean was fitted using the exponential function. Then, equation (2.1) can be solved numerically to obtain the eigenvalues. By taking the reciprocal of the eigenvalues, the deformation radius corresponding to the surface modes can be obtained. The above process of calculating the deformation radius of the surface modes is consistent with that of traditional baroclinic modes. Thus, the deformation radius of first baroclinic mode, the Rossby deformation radius, plays a fundamentally important role in extratropical large-scale ocean circulation theory. If SSH mainly reflects the information about surface modes,
a reliable global climatology deformation radius of the surface modes is clearly of great value. If unspecified, SM1 indicates the first surface mode in the remainder of this manuscript.

![Map of the SM1 deformation radius computed from annual average WOA2013 data.](image)

Fig. 2 Map of the SM1 deformation radius computed from annual average WOA2013 data.

A map of the SM1 deformation radius computed from annual average WOA2013 data using (2.1) is shown in Fig. 2. Here, the annual average SM1 deformation radius is denoted as $D_{SM1}$. Note that since the value of $f$ is very small near the equator, we avoided regions extremely close to the equator, $5^\circ S - 5^\circ N$, in this study. Corresponding to the result of fitted $N$, $D_{SM1}$ has some sporadic blank values in the ACC region. The distribution of $D_{SM1}$ shows high latitude dependence, which is consistent with Fig. 8 of LaCasce and Groeskamp (2020) and the Rossby deformation radius ($D_{flat}$, Chelton et al., 1998). We received the Rossby deformation radius data from Chelton. The $D_{SM1}$ decreased from approximately 300 km in the near-equatorial band to less than 10 km at latitudes higher than approximately $60^\circ$. Tulloch et al. (2009) provided the global distribution of long-wave phase speed calculated from satellite altimeter data. By comparing Fig. 3 to Fig. 2a of Tulloch et al. (2009), it was apparent that the spatial distribution of the two is very similar, except for the ACC region. This also confirms that the phase speed of long waves is proportional to the square of the
deformation radius. To visualize the difference between $D_{SM1}$ and $D_{flat}$, the relative change of $D_{SM1}$ with $D_{flat}$ was calculated as $R_{rcy} = (D_{SM1} - D_{flat}) / D_{flat}$ (Fig. 3a). Since SM1 deformation radius is small at high latitude (exceeding 60°), $R_{rcy}$ can be large. For convenience, only the results between 60°S -60°N are given. From the distribution of ratio $R_{rcy}$, we can see that $D_{SM1}$ is approximately 1.5 times larger than $D_{flat}$. In the deep sea, the difference can be ignored due to the relatively small topographic change. However, at the edge of the continental shelf and due to the large topographic gradient, the difference is clearly apparent, especially eastern regions of the equatorial Pacific Ocean and Atlantic Ocean, and the Labrador Sea, where the $D_{SM1}$ is approximately twice that of $D_{flat}$. In Fig. 9 of LaCasce and Groeskamp (2020), $D_{SM1}$ is only 1.5 times as large as $D_{flat}$, which may be due to the fitting of an exponential function.

The SM1 deformation radius is related to the Coriolis parameter, buoyancy frequency, water depth, and topographic slope, as apparent in equation (2.1). To understand the effect of exponential function fitting, the deformation radius ($D_{flat.N.fit}$) with the flat bottom was also calculated using the same exponential fitting. The results (not given) demonstrated that $D_{flat.N.fit}$ was almost half of $D_{flat}$. However, in the eastern equatorial Pacific Ocean and Atlantic Ocean, $D_{flat.N.fit}$ was approximately 60%-70% of $D_{flat}$, while in the ACC region and the Labrador Sea, they were comparable. This means exponential function fitting reduces the deformation radius. However, in Fig. 3a, $D_{SM1}$ and $D_{flat}$ are roughly the same in the deep sea, which means even small topographic slope would favor surface modes. This is consistent with the result of LaCasce and Groeskamp (2020).

A larger $N_0$ or $d$ will lead to a larger $D_{SM1}$ when the topographic slope is small (cf Fig. 3a with Fig. 2). Furthermore, equatorial instability waves prevail in the
equatorial Pacific and Atlantic Oceans. They also affect the vertical structure of the stratification, and consequently change the distribution of $D_{SM1}$. Similarly, a relatively larger value of $d$ can been seen in the Labrador Sea. In addition, since the SM1 deformation radius in Labrador Sea was relatively small (approximately 10 km), a small change will also cause a large $R_{rcy}$. Accordingly, when Fig. 2a of Tulloch et al. (2009) was compared with Fig. 6 of Chelton et al. (1998), it was apparent that the long-wave phase speed calculated from satellite altimeter data was also larger than the speed of traditional first baroclinic Rossby waves in these seas. This explains why the SSH anomalies propagate faster than the speed of traditional first baroclinic Rossby waves (Chelton and Schlax, 1996; Tailleux and McWilliams, 2001; Aoki et al. 2009; Hunt et al. 2012; Wortham and Wunsch 2014; de La Lama et al. 2016). The zonally averaged $D_{SM1}$, $D_{flat}$ and $D_{flat \_ N \_ fit}$ were also calculated (Fig. 3b). Zonally averaged $D_{SM1}$ were slightly larger than $D_{flat}$ with a mean difference of approximately 15 km; while zonally averaged $D_{flat \_ N \_ fit}$ were approximately half of zonally averaged $D_{flat}$. 

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3.3 Change of the SM1 deformation radius with seasons and months

In section 3.2, $N$ was calculated using annual average WOA2013 data, and the global distribution of $D_{SM1}$ was provided. To diagnose the change of the influence of $N$ on the calculation of the SM1 deformation radius, we calculated the SM1 deformation radius based on the seasonal average and monthly average data. Since the deepest depth of the monthly average WOA 2013 data was only 3000 m, it cannot satisfy the needs of the calculation. Thus, another set of data, the Generalized Digital Environment Model - Version 3.0 (GDEM-V3.0), was used to replace it. The GDEM-V3.0 monthly climatology has a horizontal resolution of $0.25^\circ \times 0.25^\circ$ and 78 standard depths from the surface to 6600 m, with a vertical resolution varying from 2
m at the surface, to 200 m below 1600 m. The GDEM-V3.0 was derived from the temperature and salinity profiles extracted from the Mater Oceanographic Observational Data Set edited at the Naval Research Laboratory (NRL). The NRL manually examined all profiles within groups covering small geographic regions and short seasonal or monthly time periods to remove erroneous profiles. Before calculating the deformation radius, two kinds of data, WOA2013 and GDEM-V3.0, were first compared. These data were in good agreement at depths of less than 3000 m. Therefore, the SM1 deformation radius calculated from the monthly average GDEM-V3.0 data was considered reliable.

The seasonal and monthly average SM1 deformation radius, $D_{\text{season}}$ and $D_{\text{month}}$, was calculated using WOA2013 and GDEM V3.0, respectively. The relative change of the seasonal average and monthly average with annual average deformation radii,

$$ R_{\text{rcs}} = \frac{(D_{\text{season}} - D_{\text{SM1}})}{D_{\text{SM1}}}, \quad \text{and} \quad R_{\text{rcm}} = \frac{(D_{\text{month}} - D_{\text{SM1}})}{D_{\text{SM1}}}, $$

are shown in Figs. 4-7. These ratios can be used to judge the seasonal and monthly change of SM1 deformation radius. In the seasonal average WOA2013 data, for north hemisphere oceans, January-March is winter, April-June is spring and so on.
Fig. 4 Relative change of seasonal average with the annual average deformation radius.

It can be seen that in winter (Jan-Mar), $D_{season}$ was generally smaller than $D_{SM1}$ in the northern hemisphere and was approximately 30%-50% smaller than the annual average between 30° and 50°N. In the southern hemisphere, $D_{season}$ was essentially the same as $D_{SM1}$. However, in summer (Jul-Sep), the situation is the opposite. In the southern hemisphere, $D_{season}$ was generally smaller and was negative 50% between 30-50°S. In the northern hemisphere, $D_{season}$ was essentially consistent with the annual average value. Spring and autumn represent transitional periods. In summer, the SM1 deformation radius of the northern hemisphere increased and that of the southern hemisphere decreased, which is the opposite in winter and thus, an annual cycle is apparent. This phenomenon was more pronounced from the monthly average results.

Fig. 5 Same as Fig. 4 but for monthly average (Jan-Apr)
The results of monthly average data exhibited an evident annual cycle of the SM1
deformation radius (Figs. 5-7). In the northern hemisphere, $D_{\text{month}}$ was the smallest in February and approximately 30% - 50% smaller than the annual average. In particular, in the Kuroshio extension and Gulf stream area, $D_{\text{month}}$ in January was only half of the annual average. As the year progressed from January through to December, $D_{\text{month}}$ gradually increased, consistent with the $D_{\text{SM1}}$ in June. In summer (July-September), $D_{\text{month}}$ began to exceed the annual average and was maximal in August and was, approximately 10% larger than $D_{\text{SM1}}$. In autumn, the deformation radius decreased as part of the annual cycle. In the southern hemisphere, although $D_{\text{month}}$ was not as regular as in the northern hemisphere, an annual cycle remained apparent. In January, $D_{\text{month}}$ in most regions in the southern hemisphere was equivalent to the annual average, which was almost 10% larger. As the year progressed, $D_{\text{month}}$ gradually decreased and attained its minimum value in August, which was approximately 40% smaller than $D_{\text{SM1}}$. Subsequently, it began to increase and attained almost the same level as the annual average (only 10% smaller), thereby exhibiting an annual cycle.

4. Discussion and conclusions

Atlases of the SM1 deformation radius were computed on a global 0.25×0.25° grid from WOA2013 and GDEM data, and seasonal and monthly changes were also analyzed. The $D_{\text{SM1}}$ was approximately 1.5 times larger than $D_{\text{flat}}$ and the main difference occurred in areas with rough bathymetry, including the continental margins or the mid-ocean ridges, consistent with previous studies (Garrett, 1978; Fu et al., 1982; Pickart, 1995). The zonally averaged $D_{\text{SM1}}$ was approximately 15 km larger than $D_{\text{flat}}$. As indicated by LaCase (2017), ocean surface modes are ubiquitous. Because $D_{\text{SM1}}$ was larger than $D_{\text{flat}}$ and the phase speed of long Rossby waves is proportional to the square of the deformation radius, this explains why SSH anomalies
propagate faster to the west (Chelton and Schlax, 1996; Tailleux and McWilliams, 2001; Aoki et al. 2009; Hunt et al. 2012; Wortham and Wunsch 2014; de La Lama et al. 2016). The seasonal and monthly average SM1 deformation radius, $D_{\text{season}}$ and $D_{\text{month}}$, exhibited a clear annual cycle. $D_{\text{season}}$ was the smallest in spring and the largest in autumn in the northern hemisphere, and the opposite pertains in the southern hemisphere. Summer and winter represent transitional periods, and $D_{\text{season}}$ was consistent with $D_{\text{SM1}}$ globally. Analogously, in the northern hemisphere, $D_{\text{month}}$ was the smallest in February, especially in the Kuroshio extension and the Gulfstream area. $D_{\text{month}}$ the gradually increased, consistent with $D_{\text{SM1}}$ in June and attained a maximum in August, approximately 10% larger than $D_{\text{SM1}}$. $D_{\text{month}}$ then decreased, reflective of the annual cycle. However, in the southern hemisphere, $D_{\text{month}}$ was less consistent.

In energy cascade theory, the Rossby deformation radius is an important scale. According to classical baroclinic instability theory (Charney, 1947; Eady, 1949; Phillips, 1954; Smith, 2007; Tulloch et al., 2011), ocean eddies generate near the Rossby deformation radius. Ocean eddies subsequently grow through “triad interactions” until the Rhines scale, where the inverse cascade is arrested, and nonlinear turbulence will be replaced by the excitation of linear Rossby waves. Wang et al. (2015) provided the distribution of various scales related to the inverse energy cascade, including the energy injection scale and the arrest-start scale. They demonstrated that eddy scales predicted by local linear baroclinic instability agree well with the energy injection scale in the mid- and high-latitude, and eddy scales predicted from the altimeter observation were reasonably close to the energy arrest-start scale. However, the relationships between these scales and the SM1 deformation radius are unclear. These questions require further research and analysis.

**Acknowledgements**

The study is funded by the National key research and development program of China.
(No. 2016YFC0301203), the Scientific Instrument Developing Project of the Chinese Academy of Sciences (No. YJKYYQ20190047), Science Foundation of Hebei province (Grant No. D2019407046)
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