Displacement and velocity estimation of the earthquake response signals measured with accelerometers

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Abstract. We propose in this paper a method to calculate the antiderivative of signals that have the integral close to null, as the signals measured on structures during earthquakes are. The method implies performing a series of numerical integration considering the initial value being zero. Afterward, the average value for the primitive function is calculated and considered as initial value: this solves the initial value problem with acceptable precision. Because of the minor errors, the second antiderivative that is the displacement in our case will gain a continuously slight increase. We overcome this problem by finding the trendline and extracting it from the signal representing the second antiderivative. In this way, we obtain accurate instantaneous values for the displacement as well. The algorithm, nominated as PySEMO, is implemented in the Python programming language and used to demonstrate the accuracy of the method. At the end of the paper, we make recommendations for the acquisition strategy to guarantee to find precise velocities and displacements. The algorithm can be used for other signals alternating around zero, e.g. those measured on rotating machines, as well.

1. Introduction
The typical sensor for measuring the dynamic response of structures is the accelerometer. When interpreting technical standards and norms, estimation of velocities and displacements is most often required [1]. The velocity is the antiderivative of the acceleration, while the displacement is the antiderivative of the velocity [2]. Numerical methods allow obtaining the anti-derivative (or primitive integral) as a discrete function by integration. This implies calculating integrals for the original function for all intervals limited by two consecutive samples. The most challenging problem in calculating the antiderivative is finding the initial value, which implies finding the integration constant [3]. As far as we know, there is no method to calculate the antiderivative as a discrete function from an original discrete function.

We herein present a numerical method to calculate the antiderivative of alternating signals which integral has an insignificant value. As examples, we indicate the vibration of rotating machinery [4-6] or the response of structures to impulsive excitation [7]. The behavior of structures during earthquakes is a particular case that is the focus of our work. To prevent harmful effects, the structures are isolated by inserting between the ground and the superstructure devices that diminish the effect of inertial forces. Two main types of devices are used: that based on the dissipation of energy by friction [8-10] and that based on elastic supports which change the period of the system [11-13]. The latter is manufactured from various materials and can achieve different configuration [14]. The idea is to design the mentioned devices based on the history of the place, therefore the information stored in earthquake registrations is essential.
Repositories containing earthquake accelerograms in digital form exist [15-17]. In this paper, we present an algorithm to find the velocity and afterward the displacement by repetitively calculating the antiderivative of accelerograms. Two aspects are of importance in this calculus: (*) finding the initial value, and (**) extraction of the zero-frequency component from the first antiderivative (i.e. the velocity). We implemented the algorithm in Python to perform fast simulation and prove it works well for signals with one or more components and in the absence or presence of damping.

2. The algorithm used for integration
The exemplification of how we developed the algorithm to repetitively calculate antiderivatives for a digital signal is made for a sinusoid. Afterward, we show the algorithm works for signals with more harmonic components as well.

Let us consider the $i$-th harmonic component $a_i$ of an acceleration signal $a$, expressed as:

$$a^i = \bar{a}^i \sin \left(2\pi f^i t + \varphi^i\right)$$  \hspace{1cm} (1)

where: $\bar{a}^i$ is the amplitude; $f^i$ is the frequency, $t$ is the time and $\varphi^i$ is the initial phase. For the digital signal, the $k$-th sample is displayed at time:

$$t_k = (k - 1)\Delta t$$  \hspace{1cm} (2)

Hence, the signal with more components can be expressed:

$$a = a^1 + ... + a^i + ...$$  \hspace{1cm} (3)

For the acceleration represented as a simple harmonic signal (simplicity we do not use the index $i$ here), the velocity is:

$$v_{q+1} = v_q + \frac{a_k + a_{k+1}}{2} \Delta t$$  \hspace{1cm} (4)

where

$$t_q = t_k + \frac{\Delta t}{2}$$  \hspace{1cm} (5)

The problem is finding the initial value of the velocity $v_0$, in fact, the constant of integration. We propose finding it by calculating the average of the velocity signal that starts from the origin, i.e. the initial value equals zero. Or, in other words, we subtract the zero-frequency component from the signal resulted by integration involving Eq. (4) for the case $v_0 = 0$. The process is illustrated in Fig. 1, which shows the velocity signal obtained for the initial condition $v_0 = 0$ and after subtracting the average.

![Figure 1](image-url)

**Figure 1.** The velocity for the initial condition set to zero and after subtracting the average.
In a similar way, by performing again the calculus of the antiderivative, we calculate the displacement with the mathematical relation:

\[
\Delta d_{n+1} = d_n + \frac{v_n + a_{n+1}}{2} \Delta t
\]  

(6)

where

\[
t_w = t_q + \frac{\Delta t}{2}
\]  

(7)

Because the velocity curve does usually not contain an integer number of cycles, the average can differ smoothly from the real zero-frequency component, thus slight increase or decrease of the next antiderivative is expected. To find the real displacement, we extract the trendline of the displacement curve. This has as result the subtraction of the average along with the rotation to get the curve to ensure it a horizontal axis. Observe that the trendline for the displacement calculated for \(d_0 = 0\), which has the equation indicated in the figure 2, is not perfectly parallel with the abscissa and is translated in the positive direction of the ordinate. Dissimilar, the trendline after correction (subtraction of the previously calculated trendline) fit the abscissa, which means the displacement is now correctly calculated and displayed.

![Figure 2](image2.png)

**Figure 2.** The displacement for the initial condition set to zero and after subtracting the trendline.

The algorithm is implemented in the Python programming language resulting in the PySEMO (Python Seismic Motion) application. The interface of PySEMO to control the input data is presented in figure 3. It permits importing a digital real-world signal acquired with an acquisition system or generating one (especially for demonstration or didactic reasons). It is possible to mention the type of the signal, acceleration, velocity or displacement, and the application calculated the other two signals by calculating the antiderivative after the proposed algorithm or the derivative. It is possible to display any signal and to save these as images or as Excel files.

![Figure 3](image3.png)

**Figure 3.** The interface of the PySEMO application to control the input data.
We developed this application to calculate the velocities and displacements from accelerograms and use it here to demonstrate the accuracy of the proposed method. However, the application can be used for any signal alternating around zero, for instance, those measured on rotating machines. The algorithm on which the application is based is comprehensively described in figure 4.

In addition to performing two integrations, PySEMO can calculate the derivatives. Thus, we can also introduce velocities and displacements to calculate the other two curves. For demonstrating the accuracy of the proposed method, in this paper, after calculating the velocity and the displacement we reconstruct the acceleration and compare it with the original signal.

3. Numerical examples
For comparison and understanding the necessary conditions for an accurate calculus, we made simulations with the PySEMO application for 3 sets of signals. The signals, which represent measured accelerations, are generated with a sampling frequency $F_R=1000$ Hz and all signals or components have the amplitude $a=1$.

The first set consists of a short signal ($t=1.5$ s) and a long signal ($t=5.5$ s), both having the frequency $f=1$ Hz. It was desired to find out how the signal length affects the accuracy of the calculated curves. From the accelerations, velocities, and displacement represented in figure 5, we can deduce the achieved results are accurate for the long signal, while for the short signal the velocities (calculated as an antiderivative and reconstructed) are not perfectly overlapped. The reason is that the displacement curve, even after rotation, has not the trendline aligned with the abscissa.

Note that the short signal has an initial phase and the method still works.

**Figure 4.** The algorithm to find the velocity and displacement curves form the accelerograms.
The second set of signals consist in a short signal with the frequency $f=5$ Hz, undamped and damped with the damping ratio 0.5 respectively. Figure 6 represents the acceleration signal and its antiderivatives; in addition, the reconstructed velocities and accelerations are represented for comparison. One can observe the quality of the calculated velocities both for the undamped as well as for the damped signal. The curves representing the displacements are not smooth, so we conclude the sampling rate should be increased to achieve better results.
Next approach is to demonstrate the algorithm works for a signal with more harmonic components. We tested it first on an undamped signal with the time length \( t = 2.5 \) s that has three harmonic components with the frequencies: \( f_1 = 2 \) Hz, \( f_2 = 5 \) Hz and \( f_3 = 12 \) Hz. Secondly, we take the same signal, but with the damping ratio 0.5. We represent the signals in figure 7.

Figure 6. Diagrams for the generated accelerations and calculated velocities and displacements (a) undamped signal; (b) damped signal.
Figure 7. Diagrams for the acceleration signal generated with three harmonic components and the calculated velocities and displacements (a) undamped signal; (b) damped signal.

From figure 7b we can conclude that the damping does not affect the method’s accuracy, both the reconstructed velocity and acceleration fit the original one. Comparing the undamped and damped signals in figures 6 and 7, we can observe that the same sampling strategy leads to similar precision in calculating the antiderivatives.

4. Conclusions
Analyzing figures 5 to 7 we conclude that the algorithm implemented in the PySEMO application is precise and can be used for calculating antiderivatives without knowing the initial conditions of the analyzed system if some conditions are fulfilled. First, the ratio between the frequencies of the signal components and the time length should be as big as possible, to ensure a sufficiently big number of cycles in the signal. From our experience, the signal must contain at least five cycles of the fundamental frequency. A second condition concerns the time resolution, which depends on the sampling frequency. Here, we determined that each cycle of the highest frequency in the acceleration signal must include 200 samples to ensure a smooth displacement curve. Regarding the initial phase and the damping ratio, we conclude these do not affect the accuracy of the results obtained with the PySEMO application.

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