What You Expect is NOT What You Get! Questioning Reconstruction/Classification Correlation of Stacked Convolutional Auto-Encoder Features

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**Abstract**—In this paper we thoroughly investigate the quality of features produced by deep neural network architectures obtained by stacking and convolving Auto-Encoders. In particular, we are interested into the relation of their reconstruction score with their performance on document layout analysis. When using Auto-Encoders, intuitively one could assume that features which are good for reconstruction will also lead to high classification accuracies. However, we prove that this is not always the case. We examine the reconstruction score, training error and the results obtained if we were to use the same features for both input reconstruction and a classification task. We show that the reconstruction score is not a good metric because it is biased by the decoder quality. Furthermore, experimental results suggest that there is no correlation between the reconstruction score and the quality of features for a classification task and that given the network size and configuration it is not possible to make assumptions on its training error magnitude. Therefore we conclude that both, reconstruction score and training error should not be jointly used to evaluate the quality of the features produced by a Stacked Convolutional Auto-Encoders for a classification task. Consequently one should independently investigate the network classification abilities directly.

**Keywords**—Auto-Encoder, reconstruction score, features quality.

I. INTRODUCTION

With their recent success in Computer Vision, the research area of deep learning becomes more and more popular. A widely known technique for fast learning of very deep networks is layer-wise training, where the networks learns representations layer-wise. This technique has been introduced by Ballard [1] in 1987 and became popular after its successful application in the last decade by many different authors. Among them, there are Hinton et al. [2] who used it to train deep belief networks in 2006, Bengio et al. [3] who in 2007 showed how deep architectures are more efficient than shallow ones for difficult problems and Lee et al. [4] who introduced convolution and probabilistic max-pooling in deep belief networks in 2009. These innovations contributed to the development of a new architecture paradigm which is obtained by literally stacking and convolving Auto-Encoders (AEs). The concept of stacking AEs became popular with Vincent et al. [5] in 2010. One year later Masci et al. [6] put convolution into play and introduced the Stacked Convolutional Auto-Encoder (SCAE) architecture. In the more recent years several authors made used of this paradigm in very different contexts. For example Tan et al. [7] used it for steganalysis of digital images and Leng et al. [8] for 3D object retrieval.

All these work share the common practice to use AEs as feature extractors for performing classification tasks. However, by doing this, it is inherently assumed that a good AE will lead to a good classification. In this paper we question the correlation between reconstruction and classification abilities of automatically/deeply learned features.

The main contribution is to analyze the relationship between SCAEs reconstruction score and the quality of their features for classification tasks, i.e., layout analysis at pixel level. Additionally, we examine the predictability of their training error magnitude based on their architecture depth and layer configuration.

**Outline**

This paper is structured as follows: Section II contains a brief introduction on the SCAEs architecture with an explanation of the different layer types and training methods used in our experiments. In Section III we explore the issues related to using the reconstruction score as feature quality evaluation metric. Follows Section IV with the results of the experiments, where after introducing the dataset and architecture used, we present the data obtained measuring the training error of SCAEs and their classification performances.

II. STACKED CONVOLUTIONAL AUTO-ENCODERS

In this Section the concept of SCAEs is briefly explained. Additionally, the theoretical foundations of the different layer types and training methods used are presented.

The idea behind SCAEs is to literally stack and convolve AEs in order to get different levels of features which can then be used for machine learning purposes. Figure 1 shows an example where a two layered network processes an input image of size 9 × 9. In this case the architecture is the following:

| layer | 1   | 2   |
|-------|-----|-----|
| patch size | 3 × 3 | 2 × 2 |
| offset | -   | -   |
| hidden neurons | 4   | 5   |

The authors published an exhaustive theoretical foundation of SCAEs in [9]
The patch size of the second layer determines how many times the first AE must be applied on the input image. The positioning of the total input patch on the input image is typically random. In the context of this work we used the \textit{N-light-N} framework \cite{9} which allows for building such complex architectures easily while maintaining a high degree of customization.

![SCAE Architecture](image)

Fig. 1. Example of SCAE architecture. In this case there are only two layers and a 2 \times 2 convolution is performed.

\textbf{A. Neural Layer}

A Neural Layer (NN) is composed of perceptrons \cite{10} and trained with error gradient descent. It uses a soft sign activation function \cite{11, 12} as shown below:

\begin{align*}
f(x) &= \frac{x}{1 + |x|} \quad f(x)' = \frac{1}{(1 + |x|)^2}
\end{align*}

\textbf{B. Linear Layer}

This is a linear associator layer (LL). The activation function is the identity:

\begin{align*}
f(x) &= x \quad f(x)' = 1
\end{align*}

The output is given simply by \( w \cdot x \) and it is not scaled. This layer can be trained with back-propagation like usual. Because of the nature of the activation function, the weights of this layer might grow out of control saturating the whole network, or die out and never recover \cite{10}. To prevent this, if the size of a weight becomes larger than a threshold, all weights in the layer get normalized \cite{13}:

\begin{align*}
W_{i,t+1} &= \frac{W_{i,t}}{||W||}
\end{align*}

\textbf{C. Oja’s Layer}

This is exactly like a Linear Layer, but instead of upgrading the weights with back propagation this layer is designed to learn with the Oja’s rule algorithm \cite{10}. In the following equation is presented the weight update rule:

\begin{align*}
W_{i,t+1} &= W_{i,t} + \gamma (\phi x - \phi^2 W_{i,t})
\end{align*}

where \( x \) is selected from the input dataset \( X \), \( \phi \) is computed as \( \phi = w \cdot W_{i,t} \) and \( \gamma \) is the learning constant such that \( 0 < \gamma \leq 1 \). The parameter \( \gamma \) must be small enough such that the weights do not diverge as in the naive implementation of a linear associator. It is a modified version of the Hebbian learning rule \cite{14}. This algorithm makes the weights of the layer to converge towards a value that computes the principal components of the input \cite{15, 16}.

\textbf{D. Training method}

In our experiment we initialize the SCAEs in two different ways. The naming convention chosen in our work is to call the two initialization modes SCAE and PCA. The former is the Xavier random initialization \cite{12}, which ensures that the variance of the input and the output are the same:

\begin{align*}
1 - 2 \cdot \frac{r}{\sqrt{|x|}} \quad \text{where} \quad r \in [0, 1]
\end{align*}

The latter is a novel way to initialise AES which makes use of the transformation matrix \( W \) obtained via Principal Component Analysis (PCA) \cite{17}. Afterward, the training is performed layer-wise in a bottom-up fashion with standard back propagation.

\textbf{III. RECONSTRUCTION SCORE}

In this Section the reliability of reconstruction score used as metric to determine the quality of features produced by SCAEs is analysed.

The first thought, as the nature of a SCAE is to be composed of AES is to use the reconstruction score for judging its features quality. Recall that the purpose of an AE is being able to reconstruct the input from a dimensionally reduced version of it as in Equation\cite{18}

\begin{align*}
\hat{\vec{y}} &= E(\vec{x}) \ , \ \vec{x}' = D(\hat{\vec{y}}) \ | \ \vec{x} \approx \vec{x}' \quad \text{where} \quad |\vec{y}| \leq |\vec{x}| \quad (1)
\end{align*}

where \( E \) and \( D \) are the encoding/decoding functions, \( \vec{x} \) is the input vector and \( \hat{\vec{y}} \) is its encoded representation. The reconstruction score \( s \) is computed looking at the \( L_2 \) distance\footnote{This is not an official standard, but the most popular method. Other distances could be used instead.} between the input image and the reconstructed image as shown in Equation\cite{19}

\begin{align*}
s = ||\vec{x}' - \vec{x}||_2 \quad (2)
\end{align*}

Even tough this metric is often used for evaluating the quality of a SCAE in general, in our specific case it is not fitting for two main reasons:

1) The goal of this experiment is to discover a robust and consistent metric to evaluate the quality of the features extracted from the SCAE given an input image. The reconstruction score is evaluating how good the SCAE is reconstructing such input image.
and gives no reliable estimation on the quality of the features vector $\tilde{y}$. In fact, an extreme example would be when the identity function is learned from the AutoEncoder. In this case, the reconstruction score is close to 0 but the quality of the features vector $\tilde{y}$ is poor as there is no advantage from feeding them rather than the raw input to a classifier.

2) From a mathematical point of view, after substituting $\tilde{x}$ in $E$, we obtain that the reconstruction score $s$ is computed as:

$$ s = ||D(E(\tilde{x})) - \tilde{x}||_2 $$

That is, the decoding function $D(\tilde{y})$ is taken into account and affecting the reconstruction score $s$. This means that a bad reconstruction function can shadow a high quality features vector $\tilde{y}$. Furthermore, when using the SCAE as features extractor, the decoding function is not even used as the encoded array $\tilde{y}$ (and not $\tilde{x}$) gets propagated forward in the layers.

In conclusion, this metric is not really giving us an insight on the quality of the features vector $\tilde{y}$ and its reliability is jeopardized by the quality of the decoding function $D(\tilde{y})$.

**IV. EXPERIMENTS / RESULTS**

In this section are presented the dataset and the architecture used for the experiments, followed by the results obtained while evaluating the training error and the performances of a classifier that uses the extracted features.

A. Dataset

In the context of this work, we present the results obtained on the Parzival dataset$^{19}$ which contain 47 pages of a Gothic handwritten historical manuscript from the 13th century, written in Medieval German from three writers on parchment.

![Parzival dataset training set sample, showing the graphical style of the document.](image)

B. Architecture

The features extractor used is a SCAE. In this kind of architecture the choice of parameters is critical. There is no trivial way to determine the optimal parameters$^{20}$$^{21}$

and often the approach is finding them by trial and error. In this work we are not interested into finding the best performing network topology as we are comparing the results with different settings on the same architecture. Therefore we took the parameters from a previous work$^{22}$ where a well performing set of them have been found trough an extensive grid search. Those parameters define the SCAE architecture for what concerns the number of layers, size of the input patches with their respective off-sets and number of hidden layers. They are presented in the following table:

| Layer | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| patch size | $5 \times 5$ | $3 \times 3$ | $3 \times 3$ | $3 \times 3$ |
| offset | $5 \times 5$ | $3 \times 3$ | $3 \times 3$ | - |
| hidden neurons | 10 | 18 | 18 | 35 |

C. Training Error

In addition to the reconstruction score, we also measured the training error. Similarly to the reconstruction score$^{23}$ it is computed as the distance from the reconstructed input and the input itself, but it is measured on a single patch rather than on the whole input image and thus corresponds to the sum of the back propagated error from the decoder to the encoder. The idea is to consider the evolution of training error over time and not only the mere final result. This gives an insight of the behavior of the AE, on whether it is learning something or not. Additionally, it is possible to analyze the training error per layer during the training of the whole SCAE. Figure$^{5}$ shows the result of such analysis. The four plots represent the training error at each layer.

In Figure$^{5}$ three groups of performance can be easily distinguished. The lowest training error is achieved by configurations employing linear associators$^{6}$ instead of perceptrons, with the exception of SCAE-OjasLayer (orange line) that has the highest error, well above all others. Despite this suggests that such configuration is not performing well, it is not a surprising result. In fact, in contrast with other layers types the error decreases in time than to back propagation, OjasLayer trains and updates its weights to perform a better PCA operation and not to better reconstruct the input. This also applies to the decoder, which obviously will not learn how to decode the encoded vector as it also updates its weights with Ojas rule. This proves the aforementioned point that the decoding function $D$ do influences the reconstruction score in a possibly negative way. Additionally, this explains why the PCA initialised configuration with the OjasLayer (cyan line) is the one with the lowest training error: because the weights of both the encoder and the decoder are not learned from scratch, thus are near-optimal at this stage.

In Fig. 3b is presented the training error of the same configurations once another layer is added to the SCAE architecture. Interestingly, in this case the lowest training error is achieved by configurations based on perceptrons.

Finally, observing the four plots of Figure 3 in a sequence reveals that by adding more layers to the SCAE architecture.

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1Note that here is meant exactly the reconstruction score numeric value, and not in general the capability of an AutoEncoder to reconstruct the input.

2The technical reference to this datasets is St. Gallen, Stiftsbibliothek, Codex 857. The ground truth has been generated by Chen et al. 18.

3Due to their similar nature, this metric also suffers from the aforementioned problems (see Section III).

4See Section II for details about the different types of layers used in the experiment.
the training error of PCAE-OjasLayer increases and that of SCAE-OjasLayer decreases.

As a conclusion we derive that by looking only at the SCAE architecture it is not possible to determine which configuration will have the lowest training error.

D. Classification results

In previous Sections we proved the unreliability of the reconstruction score as metric to evaluate the quality of the features of a SCAE. We then analysed the training error, observing different performance levels for configurations with different initialization method and layer type.

The experiments of this Section measure empirically the performance of all configurations in a classification task for layout analysis at pixel level. The purpose is to evaluate the relationship between this value and the training error. We do this under the assumption that the classifier’s performance will reflect the features vector quality. Clearly, all different configurations will make use of the very same classifier for fairness purposes.

In order to put a classifier on top of our SCAEs we used again the N-light-N framework which allows to quickly create a Feed Forward Convolution Neural Network (FFCNN) from a SCAE. This classifier architecture is literally built out of a SCAE, whose AEs are cloned and then used as feature extractor layers. On top of them, two classification layers have been added, where the first layer has 100 hidden neurons and the second one has as many as the number of classes present in the dataset used (in this case 6). As the goal is to evaluate the features produced by the SCAE we did not perform fine tuning.

Basing ourselves on previous Section’s results, we decided to run the experiments twice, with two different classifiers: one who uses linear layers and the other with neural ones (see Section II).

In Table I are presented precision, recall, F1 score and accuracy of all different SCAE configurations, in combination with the two different classifier configurations FFCNN–NN respectively FFCNN–LL. These results are the average of repeating the training-evaluation processes for 10 times, where each time the classifier has been trained for 100'000 samples and then tested against the whole test dataset.

From Table I we can derive that the classifier with neural layer is slightly but consistently better than the linear counterpart. However, the behaviour of different configurations seems to be unaffected by this setting.

We it came to relate the training error measured previously with the obtained F1 score, our first approach was to look at the extreme values for both cases. Let’s recall that the final training error of PCAE-OJAS was very high whereas SCAE-OJAS had the lowest (see Figure 3d). The following table show in a compact way these values:

| Configuration | Precision | Recall | F1 score | Accuracy |
|---------------|-----------|--------|----------|----------|
| FFCNN with neural layer (NN) |
| PCAE-LL | 0.51 | 0.74 | 0.52 | 0.75 |
| PCAE-NN | 0.53 | 0.65 | 0.49 | 0.70 |
| PCAE-OJAS | 0.51 | 0.81 | 0.53 | 0.77 |
| SCAE-LL | 0.52 | 0.73 | 0.52 | 0.76 |
| SCAE-NN | 0.53 | 0.67 | 0.50 | 0.71 |
| SCAE-OJAS | 0.35 | 0.28 | 0.24 | 0.47 |

| FFCNN with linear layer (LL) |
| PCAE-LL | 0.49 | 0.51 | 0.41 | 0.57 |
| PCAE-NN | 0.48 | 0.58 | 0.44 | 0.63 |
| PCAE-OJAS | 0.49 | 0.54 | 0.41 | 0.59 |
| SCAE-LL | 0.50 | 0.55 | 0.42 | 0.62 |
| SCAE-NN | 0.48 | 0.60 | 0.46 | 0.65 |
| SCAE-OJAS | 0.45 | 0.36 | 0.28 | 0.48 |

TABLE I. RESULTS OF EACH ARCHITECTURE CONFIGURATION COMBINED WITH A CLASSIFIER AFTER THE TRAINING IS COMPLETED. PRECISION AND RECALL ARE COMPUTED ON A MULTI-CLASS EVALUATION WHEREAS THE ACCURACY ON A BINARY CLASS EVALUATION. BOLD VALUES INDICATE HIGHEST VALUE OF THAT SECTION.

Surprisingly, the data suggest not only that, contrary to what one might think, lower training error does not necessarily correspond to higher features quality, but also that the converse is not true. Hence, a high training error does not necessarily correspond to lower features quality.

Being this only the extreme cases evaluation, we measured the Pearson correlation coefficient over all configurations obtaining a coefficient of determination $r^2$ of 0.043. Although technically a positive correlation, the relationship between the reconstruction score and accuracy is weak.

V. CONCLUSION AND OUTLOOK

In this paper we questioned the correlation between reconstruction and classification abilities of automatically/deeply learned features of SCAEs.

The analysis of the reconstruction score shows that its value is not necessarily giving an insight on the quality of the feature vector $\vec{y}$ and that its reliability is jeopardized by the quality of the decoding function $D(\hat{y})$. Additionally, we observed that given the SCAE architecture and configuration it is not possible to safely estimate its training error magnitude.

A current limitation is that this study has been conducted in a limited domain. While this is already interesting and shows that a general correspondence is not present, the generality of the aforementioned findings have be investigated and verified on other tasks. In particular, the task performed in this paper is layout analysis at pixel level on the Parzival dataset. To support our findings, we performed preliminary experiments on other datasets (George Washington and Saint Gall). On both datasets we observed a similar behaviour. Even on different tasks, e.g., object recognition (CIFAR-10) and digit recognition (MNIST), our findings have been verified in initial tests. A detailed analysis with broader experimentation is subject of our future work.

\(^{7}\)An attentive reader might see that the F1 score values shown table are not corresponding to the harmonic mean between precision and recall of the same row. This is because the value shown is averaged only after being computed class-wise. This also applied to precision, recall and accuracy.\(^{8}\)Precision, recall and accuracy could be used as well.

\(^{9}\)The nearer the value is to zero, the weaker the relationship
This work contributes towards advancing knowledge on the SCAE architecture paradigm. While this idea is getting more and more attention from the researchers in the field of machine learning and deep learning, we strive better understanding the internals, the found representations, and the classification potential. Our findings are also very interesting for the general field of unsupervised feature learning.

Finally, we conclude that one should not rely on reconstruction score and training error to evaluate the quality of the features produced by a SCAE for a classification task, but rather aim for an alternative and independent investigation of the network classification abilities.

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