Low-frequency gravitational radiation from coalescing massive black holes

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Abstract
We compute the expected low-frequency gravitational wave signal from coalescing massive black-hole (MBH) binaries at the centres of galaxies. We follow the merging history of halos and associated holes via cosmological Monte Carlo realizations of the merger hierarchy from early times to the present in a \(\Lambda CDM\) cosmology. MBHs get incorporated through a series of mergers into larger and larger halos, sink to the centre owing to dynamical friction, accrete a fraction of the gas in the merger remnant to become more massive, and form a binary system. Stellar dynamical processes dominate the orbital evolution of the binary at large separations, while gravitational wave emission takes over at small radii, causing the final coalescence of the system. We discuss the observability of inspiralling MBH binaries by a low-frequency gravitational wave experiment such as the planned Laser Interferometer Space Antenna (LISA), discriminating between resolvable sources and unresolved confusion noise. Over a three-year observing period LISA should resolve this GWB into discrete sources, detecting \(\approx90\) individual events above a \(S/N = 5\) confidence level, while expected confusion noise is well below planned LISA capabilities.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Studies of gravitational wave (GW) emission and of its detectability are becoming increasingly topical in astrophysics. Technological developments of bars and interferometers bring the promise of a future direct observation of gravitational radiation, allowing us to test one of the most fascinating predictions of general relativity, and, at the same time, providing a
new powerful tool in the astronomical investigation of highly relativistic catastrophic events, such as the merging of compact binary systems and the collapse of massive stellar cores. Massive black-hole binaries (MBHBs) are among the primary candidate sources of GWs at mHz frequencies [1–3], the range to be probed by the space-based Laser Interferometer Space Antenna (LISA). Today, massive black holes (MBHs) are ubiquitous in the nuclei of nearby galaxies [4]. If MBHs were also common in the past (as implied by the notion that many distant galaxies harbour active nuclei for a short period of their life), and if their host galaxies experience multiple mergers during their lifetime, as dictated by popular cold dark matter (CDM) hierarchical cosmologies, then MBHBs will inevitably form in large numbers during cosmic history. MBHBs that are able to coalesce in less than a Hubble time will give origin to the loudest GW events in the universe.

2. Dynamical evolution of MBHBs

Here we describe the expected GW signal from inspiralling binaries in a hierarchical structure formation scenario in which seed holes of intermediate mass form far up in the dark halo ‘merger tree’. The model has been discussed in detail in [5]. Seed holes with \( m_{\text{seed}} = 150M_\odot \) are placed within rare high-density peaks (minihalos) above the cosmological Jeans and cooling masses at redshift 20. Their evolution and growth is followed through Monte Carlo realizations of the merger hierarchy, combined with semi-analytical prescriptions for the main processes involved, such as the dynamical friction against the dark matter background, the hardening of MBHBs via three-body interactions, the MBHB coalescence due to the emission...
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Figure 2. Characteristic strain spectrum $h_c$ against frequency: from top to bottom, the three sets of curves refer to systems with $\log(m_1/M_\odot) = 7, 6, 5$ respectively, and the solid, long-dashed, and short-dashed lines assume the binary at $z = 1, 3, 5$, respectively. A mass ratio $m_2/m_1 = 0.1$ is assumed. The lowest curve is for an equal mass binary $m_1 = m_2 = 10^5 M_\odot$ at $z = 7$. The small diamonds on each curve mark, from left to right, the frequency 1 year, 1 month and 1 day before coalescence. Thick solid curve: LISA sensitivity threshold approximately accounting for detection with $S/N > 5$ including galactic [9] and extragalactic [10] white dwarf binary (WD–WD) confusion noises (added in quadrature). A three-year observation is considered.

of gravitational waves, triple MBH interactions, and the ‘gravitational rocket’ effect. Quasar activity is triggered during major mergers. Figure 1 shows the number of MBHB coalescences per unit redshift per unit observed year predicted by our model. We expect a few tenths of events per year, the vast majority involving quite light binaries ($m_{\text{BH}} = m_1 + m_2 \lesssim 10^5 M_\odot$, where $m_1$ is the mass of the heavier BH in the binary).

The model reproduces fairly well the observed luminosity function of optically selected quasars in the redshift range $1 < z < 5$, and provides a quantitative explanation of the stellar density profiles observed in the cores of bright ellipticals [6].

3. Gravitational wave signal

We compute the GW emission from each binary in the quadrupole approximation, then, to evaluate the total GWB, we integrate the contributions from all binaries [7]. The characteristic amplitude $h_c$ of a source at comoving coordinate distance $r(z)$ is

$$h_c \approx h \sqrt{n}.$$  (1)

The strain amplitude $h$ (sky-and-polarization averaged) is given by

$$h = \frac{8\pi^{2/3} G^{5/3} M^{5/3}}{10^{1/2} c^4 r(z)} f_r^{2/3},$$  (2)

where the rest-frame frequency $f_r$ is related to the observed frequency $f$ as $f = f_r/(1 + z)$, and $M = m_1^{1/5} m_2^{1/5} (m_1 + m_2)^{1/5}$ is the chirp mass of the system. Here $n$ is the number of
cycles a source spends at frequency $f_r$, i.e., $n = f_r^2 / \dot{f}_r$ [8]. Note that for a finite observation time $\tau$, the number of cycles at a given frequency $f_r$ cannot exceed $f_r \tau$. The behaviour of $h_c$ versus $f$ is shown, for different binaries, in figure 2.

Given the strain amplitude and the orbital evolution of the system, the energy spectrum integrated over the entire radiating lifetime $dE_{gw}/d\ln f_r$ can be computed [7]. Then the total GWB due to the cosmological MBH binaries is calculated as

$$c^2 \rho_{gw}(f) = \frac{\pi c^2}{4 G} f^2 h_c^2(f) = \int_0^\infty dz \frac{N(z)}{1+z} dE_{gw} / d\ln f_r,$$

where $N(z) dz$ is the comoving number density of events in the redshift interval $z, z + dz$, and the factor $1/(1 + z)$ accounts for the redshifting of gravitons [11].

The nature (stochastic or resolved) of the GWB can be assessed by counting the number of events per resolution frequency bin whose observed signal is above a given sensitivity threshold $h_{c, \text{min}}$. The frequency resolution is simply $\Delta f = 1/\tau$, where $\tau$ is the observation time, so the longer the observation, the smaller the noise. The GW signal due to a large population of sources is unresolved if there is, on average, at least one source per eight frequency resolution bins [12].

4. Results and discussion: resolvable sources versus confusion noise

The results for GWB are presented in figure 3. The left panel shows the total background in terms of $h_c^2$ as a function of the observed frequency, while in the right panel the contribution of binaries in different mass ranges is plotted. An extensive discussion is given in [7].

Figure 3. Left panel: total GWB from inspiralling MBHBs. Thin solid line: square of the characteristic strain versus wave frequency. Thick solid line: LISA sensitivity curve. Short-dashed line: expected strain from extragalactic WD–WD [10]. Long-dashed line: expected strain from unresolved galactic WD–WD [9]. Thick dash at $f \sim 10^{-9}$: current limits from pulsar timing experiments [13]. Right panel: integrated GWB from inspiralling MBHBs in different mass ranges. From top to bottom (indicated by labels), the curves show the signal produced by events with $9 < \log(m_1/M_\odot) < 10$, $8 < \log(m_1/M_\odot) < 9$, ..., $2 < \log(m_1/M_\odot) < 3$. 
Figure 4. Upper left panel: confusion noise due to MBHBs as a function of frequency (thick solid line). Other lines have the same meaning as in figure 3, left panel. Upper right panel: differential redshift distribution of MBHBs resolved with $S/N > 5$ by LISA in a three-year mission (solid lines). The separate counts for MBs (short-dashed line) and IBs (long-dashed line) are also shown. Lower left panel: mass distribution of the more massive members of MBHBs; line style as in the right upper panel. Lower right panel: mass ratio distribution of MBHBs; line style as in the right upper panel.

LISA threshold is estimated by combining the LISA single-arm Michelson sensitivity curve (taken from [14]) with the recent analysis of the LISA instrumental noise below $10^{-4}$ Hz [15].

Figure 4, upper left panel, shows the level of unresolved confusion noise due to MBHBs, compared to the LISA instrumental noise curve and to the noise due to unresolved galactic and extragalactic WD–WD binaries. While MBHBs would be the most important sources of astrophysical confusion noise below $10^{-4}$ Hz, the level is more than an order of magnitude below the LISA instrumental noise. Then, LISA will resolve the GWB due to MBHBs into discrete sources.

The histogram in figure 4, upper right panel, shows the number of resolved sources per unit redshift above $S/N > 5$, assuming a three-year LISA observation. The total number of detectable systems is divided into ‘merging’ binaries (MBs, i.e., events whose signal emitted at the last stable orbit lies above the LISA sensitivity, see figure 2) and ‘in-spiral’ binaries (IBs, i.e., events with integrated $S/N > 5$, but with emission amplitude at the last stable orbit well below the LISA sensitivity). LISA will resolve about 90 events in a three-year mission ($\approx 35$ MBs and $\approx 55$ IBs) up to $z \approx 12$. Finally, lower left and right panels show the number of sources as a function of $m_1$ and of $m_2/m_1$ respectively. While we expect observable MBs to involve systems of about $10^5$–$10^6 M_\odot$ with mass ratios of the order of 0.1–0.2, IBs have lighter masses and mass ratios near unity.
5. Conclusions

We have computed the GW signal (in terms of the characteristic strain spectrum) from the cosmological population of inspiralling MBHBs predicted to form at the centre of galaxies in a hierarchical structure formation scenario. We note here that, while any hierarchical clustering model in ΛCDM cosmology gives similar halo merging rates (e.g., [17]), the coalescence time of MBHBs is a matter of (more uncertain) estimates of the dynamical friction and hardening time scales. On the other hand, as long as the coalescence time is short compared to the Hubble time, we expect that our results are only weakly dependent on the details of MBH dynamics.

In the LISA window (10^{-5.5} \leq f \leq 0.1 \, \text{Hz}) , the main sources of GWs are MBHBs in the mass range 10^3 \leq m_1 \leq 10^7 M_\odot . With a plausible lifetime of \approx 3 \, \text{years}, LISA will resolve the GWB into \approx 90 discrete sources above S/N = 5 confidence level. Among these, \approx 35 are MBs, the others are IBs. Most of the observable MBs are at 3 \leq z \leq 7 , while IBs can be detected up to \approx 12 . Once resolvable sources are subtracted from the total GWB, the remaining confusion noise level is expected to be well below the LISA sensitivity threshold [16].

While LISA will make it possible to probe the coalescence of early black-hole binaries in the universe, it may not be able to observe the formation epochs of first MBHs. We conclude by remarking that the bulk of detections involves binaries in the range 10^3-10^5 M_\odot , a range where black holes have never been observed.

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