Matrix model and Chern-Simons theory

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ABSTRACT: In this short note we would like to present a simple topological matrix model which has close relation with the noncommutative Chern-Simons theory.

KEYWORDS: Matrix models.
1. Introduction

In recent years there was a great interest in the area of the noncommutative theory and its relation to string theory. In particular, it was shown in the seminal paper \[1\] that the noncommutative theory can be naturally embedded into the string theory. It was also shown in the recent paper \[2\] that there is a remarkable connection between noncommutative gauge theories and matrix theory. For that reason it is natural to ask whether we can push this correspondence further. In particular, we would like to ask whether other gauge theories, for example Chern-Simons theory, can be also generalised to the case of noncommutative ones. It was recently shown \[3\] that this can be done in a relatively straightforward way in the case of Chern-Simons theory. It is then natural to ask whether, in analogy with \[2\], there is a relation between topological matrix models \[4\] and Chern-Simons noncommutative theory.

It was suggested in many papers \[6, 7, 11\] that Chern-Simons theory could play profound role in the nonperturbative formulation of the string theory, M-theory. On the other hand, one of the most successful (up to date) formulation of M-theory is the Matrix theory \[5\], for review, see \[12, 13, 14, 15, 16\]. We can ask the question whether there could be some connection between Matrix models and Chern-Simons theory. This question has been addressed in interesting papers \[6, 7\], where some very intriguing ideas have been suggested.

Noncommutative Chern-Simons theory could also play an important role in the description of the Quantum Hall Effect in the framework of string theory \[17, 18\]. All these works suggest plausible possibility to describe some configurations in the physics of the condense systems in terms of D-branes, which can be very promising area of research. On the other hand, some ideas of physics of the condense systems could be useful in the nonperturbative formulation of the string theory. In summary,
on all these examples we see that it is worth to study the basic questions regarding to the noncommutative Chern-Simons theory and its relation to the matrix theory and consequently to the string theory.

In this paper we will not address these exciting ideas. We will rather ask the question whether some from of the topological matrix model can lead to the noncommutative Chern-Simons theory. Such a model has been suggested in [8] and further elaborated in [4]. We will show that the simple topological matrix model [4] cannot lead (As far as we know.) to the noncommutative Chern-Simons theory. For that reason we propose a simple modification of this model when we include additional term containing the information about the background structure of the theory. Without including this term in the action we would not be able to obtain noncommutative version of the Chern-Simons theory. It is remarkable fact that this term naturally arises in D-brane physics from the generalised Chern-Simons term in D-brane action in the presence of the background Ramond-Ramond fields [19]. For that reason we believe that our proposal could really be embedded in the string theory and also could have some relation with M-theory.

2. Brief review of Chern-Simons theory

In this section we would like to review the basic facts about Chern-Simons actions and in particular their extensions to noncommutative manifolds. We will mainly follow [3].

The Chern-Simons action is the integral of the $2n+1$ form $C_{2n+1}$ over space-time manifold $1$ which satisfies

$$dC_{2n+1} = \text{Tr}F^{n+1},$$

where the wedge operation $\wedge$ between forms $F$ is understood. The action is defined as

$$\frac{\delta S_{2n+1}}{\delta A} = \frac{\delta}{\delta A} \int C_{2n+1} = (n+1)F^n,$$

with the conventions

$$A = A_\mu dx^\mu, \quad F = dA - iA \wedge A = \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]) dx^\mu \wedge dx^\nu.$$
Chern-Simons action in the operator formalism in the next section where this action naturally arises from the modified topological matrix model [4].

3. Matrix model of Chern-Simons theory

It was argued in [4, 8] that we can formulate the topological matrix model which has many properties as the Chern-Simons theory [8]. The action for this model was proposed in the form

\[ S = \epsilon_{\mu_1 \ldots \mu_D} \text{Tr} X^{\mu_1} \ldots X^{\mu_D}. \]  

It is easy to see that this model can be defined in the odd dimensions only:

\[ S = (-1)^{D-1} \epsilon_{\mu_D \mu_1 \ldots \mu_{D-1}} \text{Tr} X^{\mu_D} X^{\mu_1} \ldots X^{\mu_{D-1}} = (-1)^{D-1} S, \]  

so we have \( D - 1 = 2n \Rightarrow D = 2n + 1 \). The equations of motion obtained from (3.1) are

\[ \frac{\delta S}{\delta X^\mu} = \epsilon_{\mu \mu_1 \ldots \mu_2n} X^{\mu_1} \ldots X^{\mu_2n} = 0, \mu = 0, \ldots, 2n. \]  

It was argued in [4] that there are solutions corresponding to D-branes. However, it is difficult to see whether these solutions corresponding to some physical objects since we do not know how to study the fluctuations around these solutions. For example, for \( D = 3 \) we obtain from the equation of motion for \( \mu = 0, 1, 2 \)

\[ [X^1, X^2] = 0, \ [X^0, X^1] = 0, \ [X^2, X^0] = 0. \]  

We see that the only possible solutions correspond to separate objects where the matrices \( X \) are diagonal or solution \( X^1 = 0 = X^2 \) with any \( X^0 \). We do not know any physical meaning of the second solution. For that reason we propose the modification of the topological matrix model which, as we will see, has a close relation with the noncommutative Chern-Simons theory [3]. We propose the action in the form

\[ S = (2\pi)^n \epsilon_{\mu_1 \ldots \mu_D} \text{Tr} \left( (-1)^{n/2} \frac{D + 1}{2D} X^{\mu_1} \ldots X^{\mu_D} + (-1)^{(n-1)/2} \frac{D + 1}{4(D-2)} \theta_{\mu_1 \mu_2} X^{\mu_3} \ldots X^{\mu_D} \right), \]  

where \( D = 2n + 1 \) and the numerical factors \( (-1)^{n/2}, (-1)^{(n-1)/2} \) arise from the requirement of the reality of the action. The other factors \( (D + 1)/(2D), (D + 1)/(4(D-2)) \) were introduced to have a contact with the work [3]. In the previous expression (3.5) we have also introduced the matrix \( \theta_{\mu \nu} \) which characterises given configuration. The equations of motion have a form

\[ (-1)^{n/2}(n+1) \epsilon_{\mu_1 \ldots \mu_{2n}} X^{\mu_1} X^{\mu_2} \ldots X^{\mu_{2n}} + (-1)^{(n-1)/2} \frac{n + 1}{2} \epsilon_{\mu_1 \mu_2 \ldots \mu_{2n}} \theta_{\mu_1 \mu_2} X^{\mu_3} \ldots X^{\mu_{2n}} = 0. \]  

(3.6)
We would like to find solution corresponding to the noncommutative Chern-Simons action. From the fact that we have odd number of dimensions we see that one dimension should correspond to the commutative one. In order to obtain Chern-Simons action in the noncommutative space-time we will follow \[10\] and compactify the commutative direction \(X^0\). For that reason we write any matrix as

\[
X_{IJ}^\mu = (X_{ij}^\mu)_{mn}, \quad I = m \times M + i, \quad J = n \times M + j,
\]

(3.7)

where \(X_{ij}\) is \(M \times M\) matrix with \(M \to \infty\) and also \(m, n\) go from \(-N/2\) to \(N/2\) and we again take the limit \(N \to \infty\). In other words, the previous expression corresponds to the direct product of the matrices

\[
X = A \otimes B \Rightarrow X_{xy} = A_{ij}B_{kl}, \quad x = i \times M + k, \quad y = j \times M + l,
\]

(3.8)

with \(M \times M\) matrix \(B\). We impose the following constraints on the various matrices \[10\]

\[
(X_{ij}^i)_{mn} = (X_{ij}^i)_{m-1,n-1}, \quad a = 1, \ldots, 2n,
\]

\[
(X_{ij}^0)_{mn} = (X_{ij}^0)_{m-1,n-1}, \quad m \neq n,
\]

\[
(X_{ij}^0)_{mn} = 2\pi R \delta_{ij} + (X_{ij}^0)_{n-1,n-1},
\]

(3.9)

where \(R\) is a radius of compact dimension. These constraints (3.9) can be solved as \[10\]

\[
(X_{ij}^i)_{mn} = (X_{ij}^i)_{0,m-n} = (X_{ij}^i)_{m-n},
\]

\[
(X_{ij}^0)_{mn} = 2\pi R m \delta_{mn} \otimes \delta_{ij} + (X_{ij}^0)_{m-n}.
\]

(3.10)

We then immediately obtain

\[
([X^0, X^i]_{ij})_{np} = 2\pi R m \delta_{mn}(X_{ij}^i)_{np} - (X_{ij}^i)_{mn}2\pi R m \delta_{np} +
\]

\[
+ (X_{ik}^0)_{mn}(X_{kj}^i)_{np} - (X_{ik}^0)_{mn}(X_{kj}^i)_{np} =
\]

\[
= 2\pi R (m - p)(X_{ij}^i)_{m-p} + ([X^0, X^i]_{ij})_{m-p},
\]

(3.11)

where \((X_{ij}^\mu)_{0m} = (X_{ij}^\mu)_m\). We see that the commutator \(X^0\) with any \(X^i\) has a form of the covariant derivative \[10\] where the first term correspond to the ordinary derivative \(-i \partial_0\) with respect to the dual coordinate \(\tilde{x}_0\) which is identified as \(\tilde{x}_0 \sim x_0 + 2\pi/R\). The second term is the commutator of the gauge field \(X^0 = A_0\) with any matrix. We could then proceed as in \[10\] and rewrite the action in the form of the dual theory.

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defined on the dual torus with the radius $\tilde{R} = 1/R$, but for simplicity we will use the original variables. Using this result we will write $X^0 = K C_0$ as

$$(C_{0,ij})_{mn} = p_{0,mn} \otimes 1_{M \times M} + (A_{0,ij})_{mn}$$

where the acting of $p_0$ on various matrices is defined in (3.11) and where the numerical factor $K$ will be determined for letter convenience.

For illustration of the main idea, let us consider matrix model defined in $D = 2n + 1 = 3$ dimensions. Let us consider the matrix $\theta_{\mu\nu}$ in the form

$$\theta_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \theta_{1N \times N} \\ 0 & -\theta_{1N \times N} & 0 \end{pmatrix},$$

(3.13)

where $1_{N \times N}$ is a unit matrix with $N$ going to infinity. Then the equations of motion, which arise from (3.5), are

$$i\epsilon_{012} X^1 X^2 + i\epsilon_{021} X^2 X^1 + \frac{1}{2} \left( \epsilon_{012} \theta_{12} + \epsilon_{021} \theta_{21} \right) = 0,$$

$$i\epsilon_{102} X^0 X^2 + i\epsilon_{120} X^2 X^0 = 0,$$

$$i\epsilon_{201} X^0 X^1 + i\epsilon_{210} X^1 X^0 = 0,$$

(3.14)

The second and the third equation gives the condition $[X^0, X^i] = 0$ which leads to the solution $A_0 = 0$ and $[p_0, X^i] = 0$. These equations, together with the first one, can be solved as

$$X^i = \delta_{mn} \otimes x^{i}_{jk}, [x^i, x^j] = i\theta^{ij}.$$

(3.15)

Thanks to the presence of the unit matrix $\delta_{mn}$, $X^i$ commutes with $p_0 \otimes 1_{M \times M}$ and so is the solution of the equation of motion (3.14). Following [2], we can study the fluctuations around this solution with using the ansatz

$$X^0 = \omega_{12} C_0 = \omega_{12} (p_0 \otimes 1_{M \times M} + (A_{0,ij})_{mn}),$$

$$X^i = \theta^{ij} C_j, C_i = 1_{N \times N} \otimes p_i + (A_{i,ij})_{mn}, p_i = \omega_{ij} x^j, i = 1, 2, \omega_{ij} = (\theta^{-1})_{ij},$$

(3.16)

It is easy to see that this configuration corresponds to the noncommutative Chern-Simons action in $D = 3$ dimensions [3]. More precisely, let us introduce formal parameters

$$dx^\mu, \mu = 0, \ldots, 2, dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu, \epsilon^{\mu_1 \ldots \mu_D} = dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D}$$

(3.17)

and the matrix valued one form

$$C = C_\mu dx^\mu = d + A,$$

(3.18)
where $C_\mu$ is given in (3.16). Then the action describing the fluctuations around the classical solution (3.15) has a form

$$S = 2\pi \sqrt{\det \theta} \text{Tr} \left( -\frac{2}{3} C \wedge C \wedge C + 2\omega \wedge C \right). \quad (3.19)$$

We rewrite this action in the form which has a closer contact with the commutative Chern-Simons theory. Firstly we prove the cyclic symmetry of the trace of the forms

$$\text{Tr} \left( A^1 \wedge \ldots A^D \right) = \text{Tr} \left( A^1_{\mu_1} A^2_{\mu_2} \ldots A^D_{\mu_D} \right) dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} =$$

$$= \text{Tr} \left( A^1_{\mu_1} A_2 A^D A_{\mu_D} \right) dx^{\mu_2} \wedge dx^{\mu_D} \wedge dx^{\mu_1} = \text{Tr} \left( A^2 \wedge \ldots \wedge A^D \wedge A^1 \right), \quad (3.20)$$

where we have used the fact that $D$ is odd number so that $dx^{\mu_1}$ commutes with even numbers of $dx$. Then the expression (3.19) is equal to

$$S = -2\pi \sqrt{\det \theta} \text{Tr} \left( iA \wedge (d \wedge A + A \wedge d) + \frac{i2}{3} A \wedge A \wedge A \right), \quad (3.21)$$

where we have used

$$d \wedge d = p_\mu p_\nu dx^\mu dx^\nu = \frac{1}{2} [p_\mu, p_\nu] dx^\mu \wedge dx^\nu = -i\omega,$$

$$\omega = \frac{1}{2} \omega_{\mu\nu} dx^\mu \wedge dx^\nu, \quad \omega_{ij} = (\theta^{-1})_{ij}, \quad \omega_{0i} = 0. \quad (3.22)$$

Now it is easy to see that (3.19) is a correct action for the fluctuation fields $A$. The equations of motion arising from (3.19) are

$$-2i(d + A) \wedge (d + A) + 2\omega = 0. \quad (3.23)$$

Looking at (3.22) it is easy to see that the configuration $A = 0$ is a solution of equation of motion as it should be for the fluctuating field. With using

$$d \wedge A + A \wedge d = [p_\mu, A_\nu] dx^\mu \wedge dx^\nu = i\partial_\mu A_\nu dx^\mu \wedge dx^\nu = i d \cdot A, \quad (3.24)$$

we obtain the derivative $d \cdot$ that is an analogue of the exterior derivative in the ordinary commutative geometry. In this case the action has a form

$$S = 2\pi \sqrt{\det \theta} \text{Tr} \left( A \wedge d \cdot A - \frac{2i}{3} A \wedge A \wedge A \right), \quad (3.25)$$

which is the standard Chern-Simons action in three dimensions.

We observe that this action differs from the action given in [3] since there is no the term $\omega \wedge A$ in our action. This is a consequence of the presence of the second term in (3.3) that is needed for the emergence of noncommutative structure in the
Chern-Simons action. On the other hand, from the fact that similar matrix structure arises in the study of Quantum Hall Effect in D-brane physics [18] we believe that our proposal of topological action could have relation to the string theory and M theory. As usual, this action can be rewritten using in terms of the integral over space-time with ordinary multiplication replaced with star product [9].

Generalisation to the higher dimensions is straightforward. The equations of motion (3.6) give

\[ i\epsilon_{\mu\nu}X^\mu X^\nu + \frac{1}{2}\epsilon_{\mu\nu}\theta^{\mu\nu} = 0 \Rightarrow [X^\mu, X^\nu] = i\theta^{\mu\nu}, \mu, \nu = 1, \ldots, 2n. \quad (3.26) \]

We restrict ourselves to the case of \( \theta \) of the maximal rank. For simplicity, we consider \( \theta \) in the form

\[
\theta^{\mu\nu} = \begin{pmatrix}
0 & \ldots & \ldots & \ldots & 0 \\
0 & 0 & \theta_1 & 0 & \ldots & 0 \\
0 & -\theta_1 & 0 & \ldots & \ldots & 0 \\
\ldots & \ldots & 0 & \ldots & \ldots & \ldots \\
0 & \ldots & 0 & \theta_n & \ldots & \ldots \\
0 & \ldots & 0 & -\theta_n & 0 & \ldots \\
\end{pmatrix}.
\quad (3.27)
\]

As in 3 dimensional case we introduce the matrix \( \omega \) defined as follows

\[
\omega_{ij} = (\theta^{-1})_{ij}, \; i, j = 1, \ldots, 2n, \; \omega_{i0} = \omega_{0i} = 0.
\quad (3.28)
\]

and we define \( C = C_\mu dx^\mu = d + A \), \( \mu = 0, \ldots, 2n \) where the dimension \( x^0 \) is compactified as above. And finally various \( C_\mu \) are defined as

\[
X^i = \theta^{ij} C_j, \; X^0 = \left( \prod_{i=1}^n \omega_i \right) C_0, \; \omega_i = -\theta_i^{-1}.
\quad (3.29)
\]

with as \( C_\mu \) same as in (3.16). Then the action has a form

\[
S_{2n+1} = (2\pi)^n \sqrt{\det \theta} \text{Tr} \left( (-1)^n (-1)^{n/2} \frac{n+1}{2n+1} C^{2n+1} + \
+ (-1)^{n-1} (-1)^{(n-1)/2} \frac{n+1}{2n-1} \omega \wedge C^{2n-1} \right).
\quad (3.30)
\]

In order to obtain more detailed description of the action we will follow [3]. Since \( \frac{\delta}{\delta C} = \frac{\delta}{\delta A} \), we can write \(^2\)

\[
\frac{\delta S_{2n+1}}{\delta C} = (2\pi)^n \sqrt{\det \theta} \left( (-1)^n (-1)^{n/2} (n + 1) C^{2n} + \
+ (-1)^{n-1} (-1)^{(n-1)/2} (n + 1) \omega \wedge C^{2n-2} \right) = 
= (2\pi)^n \sqrt{\det \theta} \left( (n + 1)(F - \omega)^n + (n + 1) \omega \wedge (F - \omega)^{n-1} \right),
\quad (3.31)
\]

\(^2\)We will write \( C^n \) instead of \( C \wedge \ldots \wedge C \).
where we have used the fact that $C^2 = -i\omega + i(-idA - iAd - iA^2) = -i\omega + iF$. Since $\omega$ and $F$ are both two forms and $\omega$ is a pure number from the point of view of the noncommutative geometry we immediately see that $F$ and $\omega$ commute so that we can write

$$(F - \omega)^n = \sum_{k=0}^{n} \binom{n}{k} (-\omega)^{n-k} F^k .$$

(3.32)

Following [3] we introduce the other form of the Lagrangian

$$\delta \tilde{L}_{2k+1} = (k + 1) F^k .$$

(3.33)

Then we can rewrite (3.31) as

$$
\frac{\delta}{\delta C} \left\{ S_{2n+1} - (2\pi)^n \sqrt{\det \theta} \left( \sum_{k=0}^{n} \binom{n + 1}{k + 1} (-\omega)^{n-k} \tilde{L}_{2k+1}
\right.ight.
\left. - \sum_{k=0}^{n-1} (-1)^{n-k-1} \frac{1}{n} \binom{n + 1}{k + 1} \omega^{n-k} \land \tilde{L}_{2k+1} \right) \right\} = 0 \Rightarrow
$$

$$
\Rightarrow S_{2n+1} = (2\pi)^n \sqrt{\det \theta} \text{Tr} \left( \sum_{k=0}^{n} (-1)^{n-k} \binom{n + 1}{k + 1} \omega^{n-k} \tilde{L}_{2k+1} + 
\sum_{k=0}^{n-1} (-1)^{n-k-1} \frac{1}{n} \binom{n + 1}{k + 1} \omega^{n-k} \land \tilde{L}_{2k+1} \right) .$$

(3.34)

As a check, for $n = 1$ we obtain from (3.34)

$$S_3 = (2\pi) \sqrt{\det \theta} \text{Tr} \left( -2\omega \land \tilde{L}_1 + \tilde{L}_3 + 2\omega \land \tilde{L}_1 \right) = 2\pi \sqrt{\det \theta} \text{Tr} \tilde{L}_3 ,$$

(3.35)

and using

$$\frac{\delta \tilde{L}_3}{\delta A} = 2F = -2i(dA + Ad + A^2) \Rightarrow \tilde{L}_3 = -iA \land d \land A - i(2A \land A \land A - iA \land A \land A) ,$$

(3.36)

we obtain

$$S_3 = 2\pi \sqrt{\det \theta} \text{Tr} \left( A \land d \cdot A - i\frac{2}{3} A \land A \land A \right) ,$$

(3.37)

which, as we have seen above, is a correct form of the noncommutative Chern-Simons action in three dimensions.

4. Conclusion

In this short note we have shown that simple modification of the topological matrix model [4] could lead to the emergence of the noncommutative Chern-Simons action [3]. In order to obtain this action we had to introduce the antisymmetric matrix
$\theta$ expressing the noncommutative nature of the space-time. It is crucial fact that we must introduce this term into the action explicitly which differs from the case of the standard matrix theory \[2\], where different configurations with any values of the noncommutative parameters arise as particular solutions of the matrix theory.

It is also clear that we can find much more configurations than we have shown above. The form of these configurations depend on $\omega$. It is possible to find such a $\theta$ which leads to the emergence of lower dimensional Chern-Simons actions and also which leads to the emergence of point-like degrees of freedom in the Chern-Simons theory. For example, we can consider $\theta$ in the form

$$
\theta^{\mu\nu} = 1_{mn} \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A \\ 0 & -A & 0 \end{pmatrix}; A = \begin{pmatrix} 0_{k \times k} & 0 \\ 0 & \theta_{1_{N \times N}} \end{pmatrix}.
$$

(4.1)

This corresponds to the configuration describing Chern-Simons action with the presence of $k$ point-like degrees of freedom - ”partons”. We could analyse the interaction between these partons and gauge fields in the same way as in matrix theory (For more details see \[13, 15\] and reference therein.) It is possible that this simple model could have some relation to the holographic model of M-theory \[11\]. In particular, we see that the partons arise naturally in our approach. On the other hand, the similar analysis as in \[11\] could determine $\theta$, i.e. Requirements of the consistency of the theory could choose $\theta$ in some particular form. In short, we hope that the approach given in this paper could shine some light on the relation between the matrix models and Chern-Simons theory.

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