Introduction. Understanding the out-of-equilibrium dynamics of quantum many-body systems is a central issue of modern condensed matter physics from both a fundamental and an applicative point of view. Theoretical interest on these problems traces back to the studies of irreversibility in non-equilibrium thermodynamics. In quantum systems the interplay between phase coherence, strong interactions, and low dimensionality may result in surprising dynamical behaviors. Remarkably, this kind of issues can be explored experimentally at the quantum level by realizing highly controllable quantum many-particle systems. In this sense, cold atoms in optical lattices are the paradigmatic example of an interacting system where the interaction strength and the geometrical settings can be fine tuned. The engineered Hamiltonians can mimic condensed matter systems and also provide feasible tools to investigate many interesting issues in non-equilibrium statistical mechanics. Similarly, arrays of coupled microcavities have been shown to have the potential to act as simulators of quantum many-body dynamics, with characteristics complementary to those of optical lattices.

In this context it is desirable to consider simple but illustrative enough situations. An important problem that has been studied is the response of a system to a sudden change of the bath temperature. The specific example of the XY model in a transverse magnetic field whose spins are locally coupled to a set of bosonic baths is considered. The peculiar nature of the dynamics is encoded in the correlations developing out of equilibrium. By means of a kinetic equation we analyze the spin-spin correlations and block correlations. We identify some universal features in the out-of-equilibrium dynamics. Two distinct regimes, characterized by different time and length scales, emerge. During the initial transient the dynamics is characterized by the same critical exponents as those of the equilibrium quantum phase transition and resembles the dynamics of thermal phase transitions. At long times equilibrium is reached through the propagation along the chain of a thermal front in a manner similar to the classical Glauber dynamics.

We investigate the dissipative dynamics of a quantum critical system in contact with a thermal bath. In analogy with the standard protocol employed to analyze aging, we study the response of a system to a sudden change of the bath temperature. The specific example of the XY model in a transverse magnetic field whose spins are locally coupled to a set of bosonic baths is considered. The peculiar nature of the dynamics is encoded in the correlations developing out of equilibrium. By means of a kinetic equation we analyze the spin-spin correlations and block correlations. We identify some universal features in the out-of-equilibrium dynamics. Two distinct regimes, characterized by different time and length scales, emerge. During the initial transient the dynamics is characterized by the same critical exponents as those of the equilibrium quantum phase transition and resembles the dynamics of thermal phase transitions. At long times equilibrium is reached through the propagation along the chain of a thermal front in a manner similar to the classical Glauber dynamics.
magnetic field:

\[ H_S = -\frac{J}{2} \sum_j \left( \frac{1+\gamma}{2} \sigma_j^x \sigma_{j+1}^x + \frac{1-\gamma}{2} \sigma_j^y \sigma_{j+1}^y + h \sigma_j^z \right) \]

where \( \sigma^x,y,z \) are Pauli matrices. We fix the energy scale \( J = 1 \) and consider \( h > 0 \). In the anisotropic case \( 0 < \gamma \leq 1 \), the magnetic field \( h \) induces a phase transition at \( h_c = 1 \) that separates a paramagnetic phase at \( h > 1 \) from a ferromagnetic ordered phase with \( \langle \sigma^z \rangle \neq 0 \); such a phase transition belongs to the Ising universality class with critical indexes \( \nu = z = 1 \). The Hamiltonian can be diagonalized in momentum space, in terms of Jordan-Wigner fermions \( c_k \), as \( \sum_{k>0} \Psi_k \hat{H}_k \Psi_k \), where \( \Psi_k = (c_k^+, c_{-k}) \), \( \hat{H}_k = -\langle \cos k + h \rangle \hat{\tau}_x + \sin k \hat{\tau}_y \), where \( \hat{\tau}_x, y \) are Pauli matrices. The \( T = 0 \) quantum phase transition leaves an imprint at low temperatures, leading, close to the quantum critical point, to a crossover at temperatures \( T \sim \Delta \) with \( \Delta \equiv |h - h_c| \) the energy gap. In particular for \( T \ll \Delta \) the spin-spin correlation function is factorized into a quantum and a thermal term that can be described semiclassically in terms of quasiparticle excitations, while in the quantum critical region \( (T \gg \Delta) \) quasiparticle excitations no longer exist.

To model a thermal reservoir we consider a set of bosonic baths coupled locally to each spin \( \hat{X}_j \), such that the global Hamiltonian reads

\[ H = H_S + \sum_j N \hat{X}_j \hat{\sigma}_j^z + H_B. \]

where \( X_j = \sum_{\beta=\gamma} \lambda_j \beta \hat{b}_j^+ \hat{b}_{j+1}^\beta + \hat{b}_j \hat{b}_{j+1}^\beta \) and \( H_B = \sum_{j,\beta,\gamma} \omega_{j,\beta} \beta \hat{b}_j^\beta \hat{b}_{j+1}^\gamma \). The system-bath coupling is chosen to have power law spectral densities \( \hat{X}_j = \sum_{\beta=\gamma} \lambda_j \beta \hat{b}_j^\beta \). The system-bath coupling we are considering, Eq. (2), breaks the integrability of the model, inducing transitions between all energy levels, and thus complete relaxation.

The quantum quench dynamics for the closed XY model was studied in [16]. It is customary to consider the physical system initially uncorrelated, e.g. by applying a strong magnetic field \( h \). After a quench of the magnetic field, correlations between parts of the system will start to develop because of the dynamics induced by the new Hamiltonian. Analogously, in the case of thermal quenches, we consider the system to be initially prepared in equilibrium with the bath at a very high temperature, again with no correlations because the density matrix of the system is proportional to the identity. After a quench of the temperature of the bath at \( t = 0 \), the system is forced out of equilibrium and eventually reaches a new stationary thermal state. In the following we investigate how such correlations develop and how thermal equilibrium is eventually approached. All the results shown in the figures below refer to the Ising model (\( \gamma = 1 \)) coupled to Ohmic baths (\( s = 1 \)). However, the results stated in the text refer to the general case \( 0 < \gamma \leq 1, s > 0 \). We discuss the spin-spin correlation function and later we consider the quantum mutual information.

**Spin-spin correlations.** We consider the equal-time connected correlation function

\[ C_{zz}(t, R) = \langle \sigma_j^z(t) \sigma_{j+R}^z(t) \rangle - \langle \sigma_j^z(t) \rangle \langle \sigma_{j+R}^z(t) \rangle. \]  

In the case of thermal quenches the dynamics is purely dissipative. Since for weak coupling \( \alpha \ll 1 \) the dynamics of populations and coherences decouple, if the system starts in a mixed state no coherences will develop after the quench (this is consistent with the so called “secular approximation” [18]). Therefore in this limit at each time the system is approximately in a statistical mixture of the Hamiltonian eigenvectors, i.e. a gaussian state. Hence, by exploiting this the correlation function can be expressed as

\[ C_{zz} = \frac{1}{4} \left( \sum_{k>0} \cos(kR) \right)^2 - \left( \frac{1}{4} \sum_{k>0} \cos(kR) \right). \]

In order to evaluate the two point fermionic correlators we use the kinetic equation derived in [13] within the weak coupling and Markov approximations. From the analysis of our results (Fig. II) two regimes can be outlined: right after the quench, for \( t \ll \alpha \), correlations increase as \( C_{zz} \propto t^2 \), while in the opposite limit, at times \( t \gg \alpha \), the system is close to thermal equilibrium. During the initial transient, \( C_{zz} \) reaches for far distant spins \( (R \gg 1) \) values greater than those of thermal equilibrium, \( C_{zz}(R) > C_{zz}^0(R) \propto \exp(-R/\xi) \), with \( \xi \) the thermal correlation length [22]. Thus, the crossover to the long-time regime displays a non-monotonous behavior as a function of time, so that \( C_{zz} \) increases up to a maximum value and then relaxes to the thermal equilibrium value (Fig. II).

Let us analyze first the initial transient. We observe that, for noncritical values of the magnetic field \( (h \neq 1) \),
C_{zz} changes its sign at a certain distance $\xi_t(h)$ such that $C_{zz} \lesssim 0$ for $R \gtrsim \xi_t$ (see Fig. 2). That distance marks the crossover between two power-law behaviors with different exponent, respectively $R^{-4}$ and $R^{-2}$, and close to the critical point it diverges as

$$\xi_t \propto |h - h_c|^{-1}.$$  

(4)

Collecting all the above results, we find that the long $R$ behavior close to the critical point is described by

$$C_{zz} \propto t^2 \begin{cases} R^{-2} & R \ll \xi_t \\ R^{-4} & R \gg \xi_t \end{cases}$$  

(5)

At equilibrium, close to the phase transition, the correlation length $\xi$ marks the crossover between the exponential decay for $R \gg \xi$ and the critical power-law for $R \ll \xi$. Similarly, in the present non-equilibrium case $\xi_t$ can be interpreted as an effective crossover scale between two power-law regimes. Eqs. (4) and (5) are independent of the specific value of the final temperature at which the system is quenched and of the specific exponent $s$ of the bath spectral function. Moreover, they are robust within the range $0 < \gamma \leq 1$ in which the system belongs to the Ising universality class. Remarkably, although in this regime the system is far from equilibrium, Eqs. (4) and (5) are characterized by the equilibrium critical indexes: $\xi \propto |h - h_c|^{-1}$ and $C_{zz} \propto R^{-2}$, $R \ll \xi$.

We now focus on the long time regime. The analysis of Fig. 3 indicates that, at a given time, the correlation function is thermalized up to a distance $R_{th}$. This thermal front exists because the long-distance correlations are dominated by the slowly relaxing low-energy modes.

Figure 2: Initial transient: snapshots of $C_{zz}$ (left) and $\partial_t I$ (right) at a fixed $t/\alpha \ll 1$ after a quench from $T = \infty$ to $T = 0.1$ and for $h = 0.8$, 0.9, 0.95, 0.975, 1 (from bottom to top). The spikes relative to $C_{zz}$ in the left panel mark the distances $\xi_t(h)$ at which $C_{zz}$ changes sign. Dashed lines are plotted for comparison. Inset: $\xi_t$ as a function of $|h - h_c|$; for $\partial_t I$, $\xi_t$ is calculated as the maximum of $\partial_t^2 I$ which marks the crossover between $L^{-1}$ and $L^{-4}$ scaling.

Figure 3: Thermalization of the spin-spin correlation function $C_{zz}$ after a quench from $T = \infty$ to $T = 0.1$ at $h = 1$. Left: snapshots of $C_{zz}(R)$ at $t/\alpha = 10$, 20, 30, 40 from top to bottom; thick red line is the (exponential) thermal equilibrium $C_{zz}^{th}$. At a given time after the quench $C_{zz}$ is thermalized up to a distance $R_{th}$ that increases with time. Right: corresponding time dependence of $R_{th}$; the linear fit gives $v_{th} \approx 0.32$.

Figure 4: Thermal front velocity $v_{th}$ as a function of $1/T$. From top to bottom $h = 1$, 0.9, 1.2 (so that $\Delta = 0$, 0.1, 0.2). Lines are the fit $T\alpha(1 + b\frac{\Delta}{h})e^{-\Delta/T}$ with $a = 3.3$, $b = 0.9$ for the specific case $\gamma = 1$, $s = 1$.

The front is found to propagate ballistically with a speed $v_{th}$, that is a function of $T$ and $h$. In particular, as shown in Fig. 4, the velocity scales as

$$v_{th} \propto \begin{cases} T^s & T \gg \Delta \\ e^{-\Delta/T} & T \ll \Delta \end{cases}$$  

(6)

where $\Delta = |h - 1|$ is the energy gap of the XY model.

Block correlations. We now analyse how correlations between a block of spins and the rest of the chain develop after a thermal quench. In order to quantify such a correlation, we use a tool originally developed in the context of quantum information theory. For a certain bipartition of the system into two blocks of $L$ and $N - L$ spins, the mutual information is defined as

$$\mathcal{I}(L) = S(\rho_L) + S(\rho_{N-L}) - S(\rho_N),$$  

(7)

where $S(\rho) = -\text{Tr} (\rho \log \rho)$ and $\rho_N$ is the density matrix of the entire system. The mutual information measures the correlations between the two blocks of $L$ and $N - L$.
spins \(20\). For the XY model \(I(L)\) is known to diverge logarithmically as a function of \(L\) at the critical point, while it saturates for noncritical values. In the following we concentrate on the derivative \(\partial_L I\), which measures the sensitivity of the correlations to the block size. It is useful to study \(\partial_L I\) because, at equilibrium, it shows features similar to the spin-spin correlation function: it scales as \(\partial_L I \propto L^{-1}\) at the critical point, while it decays exponentially for noncritical values.

Equation (7) can be computed in terms of two-point fermionic correlators (obtained by solving the kinetic equation) using the results of Ref. \(21\). We study for the mutual information the same setting of thermal quenches investigated above for the spin-spin correlation function. Remarkably the scenario emerging is very similar to that depicted in Fig. 1. There are two regimes: an initial transient governed by

\[
\partial_L I \propto t^2 \begin{cases} 
L^{-1} & L \ll \xi_t \\
L^{-4} & L \gg \xi_t 
\end{cases}
\]

with the same correlation length \(\xi_t\) (see Fig. 2). In the quasi-equilibrium regime at long times, \(\partial_L I\) exhibits a thermal front propagation similar to that shown in Fig. 3 and with the same velocity found for \(C_{zz}\) (see Fig. 4).

**Conclusions.** We have analyzed the dynamics of spin-spin and block correlation functions following a sudden cooling of the bath coupled to a quantum system. For both quantities we find that the dynamics displays two regimes: at short times the correlations develop according to (5) and (8), while at long times a well defined thermal front propagates along the chain with velocity \(v_t\), the latter being sensitive to the critical properties of the system. We remark that the system does not exhibit aging because it is quenched away from the critical point. Nevertheless in its early stages relaxation does show critical features analogously to those of thermal phase transitions. In particular, for systems quenched at the critical temperature, it is known that equal-time two-point correlation function scales, during the initial transient, as a power law both in time \(t^n\) (here \(a = 2\)) and in space \(R^{-d+2-\eta}\) (in our case \(R^{-2}\)) \(15\). Besides, we point out that the scaling of the thermalization velocity \(v_t \propto \exp(-\Delta/T)\), which we find in the semiclassical regions \(T \ll \Delta\), holds also in the classical Ising model within the Glauber dynamics \(24\). This similarity can be ascribed to the fact that the system-bath coupling generates incoherent relaxation without conserving the order parameter, as happens in the phenomenological Glauber model for the dynamics of the classical spins.

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