Asymmetric tunneling, Andreev reflection and dynamic conductance spectra in strongly correlated metals

V.R. Shaginyan\textsuperscript{1} and K.G. Popov\textsuperscript{2}

\textsuperscript{1}Petersburg Nuclear Physics Institute, RAS, Gatchina, 188300, Russia
\textsuperscript{2}Komi Science Center, Ural Division, RAS, Syktyvkar, 167982, Russia

Landau Fermi liquid theory predicts that the differential conductivity between metallic point and metal is a symmetric function of voltage bias $V$. This symmetry holds if the particle-hole symmetry is preserved. We show that the situation can be different when one of the two metals is a strongly correlated one whose electronic system can be represented by a heavy fermion liquid. When the heavy fermion liquid undergoes fermion condensation quantum phase transition, the particle-hole symmetry is violated making both the differential tunneling conductivity and dynamic conductance asymmetric as a function of applied voltage. This asymmetry can be observed when the strongly correlated metal is either normal or superconducting. We show that at small values of $V$ the asymmetric part of the dynamic conductance is a linear function of $V$ and inversely proportional to the maximum value of the gap and does not depend on temperature provided that metal is superconducting, when it becomes normal the asymmetric part diminishes at elevated temperatures.

PACS numbers: 71.27.+a, 74.20.Fg, 74.25.Jb

Key Words: quantum phase transitions, heavy fermions, quasiparticles, asymmetric conductivity

INTRODUCTION

The unusual properties of strongly correlated electron liquid recently observed in high-$T_c$ superconductors (HTSC) and heavy-fermion (HF) metals are due to quantum phase transitions taking place at their critical points. Therefore, direct experimental studies of these transitions and critical points are of crucial importance for understanding the physics of strongly correlated metals. In case of HTSC such direct experimental studies are absent since the corresponding critical points are "shadowed" by the superconductivity. Recent experimental data on the behavior of HF metals pour a light on the nature of these critical points and corresponding phase transitions. Thus, it is extremely important to study simultaneously both the high-$T_c$ superconductivity and the anomalous behavior of HF metals. A quantum phase transition is driven by control parameters such as composition, pressure, density $x$ of electrons (holes), magnetic field $H$, etc, and occurs at temperature $T = 0$. QCP separates an ordered phase generated by quantum phase transition from previously existing disordered phase. It is expected that the universal behavior is only observable if the heavy electron liquid in question is very near QCP, for example, when the correlation length is much larger than the microscopic length scales. Quantum phase transitions of this type are quite common, and we shall label them as conventional quantum phase transitions (CQPT). In this case, the approach known as the Moria-Hertz-Millis theory is dealing with some kind of magnetically ordered state of a substance and focuses on the long wavelength low energy thermal and quantum fluctuations on this state background. The corresponding critical state near QCP is characterized by the complete absence of quasiparticles. It is commonly believed that the absence of quasiparticle-like excitations is the main cause of the non-Fermi liquid (NFL) behavior and other types of the critical behavior in the quantum critical region. However, the predictions of theories based on CQPT fail to explain the strong NFL behavior observed in such HF metals as CeCu$_6-y$Au$_y$, CeCoIn$_5$ and YbRh$_2$Si$_2$ near the magnetic ordering transition or in such HF metal as CeRu$_2$Si$_2$ showing neither evidence of the magnetic ordering and/or superconductivity down to the lowest temperatures nor the LFL behavior. Therefore, we conclude that the magnetic ordering does not play the key role in the NFL behavior and can be considered as a side effect adding specific features to the corresponding NFL behavior.

The Landau Fermi liquid (LFL) theory rests on both the notion of quasiparticles representing elementary excitations of a Fermi liquid and the concept of the order parameter which characterizes the ordered phase. Quasiparticles are appropriate excitations to describe its low temperature thermodynamic properties. The inability of the LFL theory to explain the experimental observations which point to the dependence of quasiparticle effective mass $M^*$ on temperature $T$ and applied magnetic field $H$ may lead to the conclusion that Landau paradigm related to the quasiparticles and order parameter fail to explain the experimental facts on the NFL behavior of strongly correlated electrons. However, other experimental facts show that quasiparticles do exist near QCP while $M^*$ depends strongly on magnetic field and temperature. As a result, we can safely conclude that the fluctuations are not responsible for observed NFL behavior, the quasiparticles are not suppressed by them and Landau paradigm survives also when dealing with strongly correlated electrons. Therefore the problem of the NFL behavior can be resolved within the LFL theory providing that quasiparticles form so-called
fermion-condensate (FC) state emerging behind the fermion condensation quantum phase transition (FCQPT). It is possible to explain the "whole bunch" of observed thermodynamic properties of HF metals on the basis of FCQPT allowing for existence of Landau quasiparticles down to the lowest temperatures.

The experiments on HF metals and HTSC explore mainly their thermodynamic properties. It is highly desirable to probe the other properties of the heavy electron liquid like quasiparticle occupation numbers, which are not directly linked to the density of states or to the behavior of the effective mass $M^*$. Both scanning tunneling microscopy (STM) and point contact spectroscopy (PCS) based on Andreev reflection (AR) being sensitive to both the density of states and quasiparticle occupation numbers are ideal techniques to study the effects of particle-hole symmetry violation, making the differential tunneling conductivity and dynamic conductance to be asymmetric function of applied voltage. This asymmetry can be observed when HF metals and HTSC are either normal or superconducting. We note that this asymmetry is unusual in conventional metals, especially at low temperatures. In the case of LFL the particle-hole symmetry conserves and both the differential tunneling conductivity and dynamic conductance are symmetric functions of voltage bias. Thus, STM and PCS provide a new direction in the experimental studies of the NFL behavior of HTSC and HF metals.

In this letter we show that a particle-hole symmetry is violated when the heavy electron liquid undergoes FCQPT. As a result, both the differential tunneling conductivity $\sigma_d(V)$ and the dynamic conductance $\sigma_c(V)$ become asymmetric as a function of voltage $V$. We demonstrate that the asymmetric part is defined by a temperature independent part of the entropy characterizing the normal state of the electronic system of a strongly correlated metal, and the study of the thermodynamic properties is possible by measuring the transport characteristics of the metal. While the application of magnetic field destroys the NFL behavior of the heavy electron liquid and restores the above symmetry. Therefore the measurements of asymmetric part of the conductance can be viewed as a powerful tool to investigate the NFL behavior of strongly correlated Fermi systems.

**SYMmetric Tunneling and FCQPT**

The tunneling current $I$ through the point contact between two ordinary metals is proportional to the driving voltage $V$ and to the squared modulus of the quantum mechanical transition amplitude $t$ multiplied by the difference $N_1(0)N_2(0)(n_1(p, T) - n_2(p, T))$. Here $n(p, T)$ is the quasiparticle distribution function (occupation number) and $N(0)$ is the density of states of the corresponding metal. On the other hand, the wave function calculated in WKB approximation is proportional to $(N_1(0)N_2(0))^{-1/2}$. As a result, the density of states cancels down and the tunneling current becomes independent of $N_1(0)N_2(0)$.

Now we turn to a consideration of the tunneling current at low temperatures which in the case of ordinary metals is given by

$$I(V) = 2|t|^2 \int [n_F(\varepsilon - V) - n_F(\varepsilon)] d\varepsilon. \quad (1)$$

Here $n_F(\varepsilon)$ is the distribution function of ordinary metal. We use an atomic system of units: $e = m = \hbar = 1$, where $e$ and $m$ are electron charge and mass, respectively. Since temperature is low, we approximate $n_F(\varepsilon)$ by the step function $\theta(\varepsilon - \mu)$, $\mu$ is the chemical potential. It follows from Eq. (1) that quasiparticles with the energy $\varepsilon, \mu - V \leq \varepsilon \leq \mu$, contribute to the current. One obtains from Eq. (1) that $I(V) = a_1 V$ and $\sigma_d(V) = dI/dV = a_1$, $a_1 = const$. Thus, within the LFL theory the differential tunneling conductivity $\sigma_d(V)$ is a symmetric function of the voltage $V$. In fact, the symmetry of $\sigma_d(V)$ holds if the particle-hole symmetry is conserved as it is the case in the LFL theory. Therefore, the existence of the $\sigma_d(V)$ symmetry is quite obvious and common in the case of metal-to-metal contacts when those metals are ordinary in their normal or superconducting states.

Now we briefly describe the heavy electron liquid with FC. When the density $x$ of the liquid approaches some threshold value $x_{FC}$ the effective mass diverges. Beyond $x_{FC}$ the effective mass becomes negative. To avoid physically meaningless states with $M^* < 0$, the system undergoes FCQPT with FC formation behind the critical point $x = x_{FC}$. Therefore, behind the critical point $x_{FC}$, the step-wise quasiparticle distribution function $n(p, T \to 0) = \theta(p - p_F)$ does not deliver the minimum to the Landau functional $E[n(p)]$. As a result, at $x < x_{FC}$ and $T = 0$ the quasiparticle distribution is determined by the standard equation for the minimum of a functional

$$\frac{\delta E[n(p)]}{\delta n(p, T = 0)} = \varepsilon(p) = \mu; \ p_i \leq p \leq p_f. \quad (2)$$

Equation (2) determines the quasiparticle distribution function $n_0(p)$ minimizing the ground state energy $E \equiv E[n(p)]$. Being determined by Eq. (2), the function $n_0(p)$ does not coincide with the step function $\theta(p - p_f)$ in the region $(p_f - p_i)$, so that $0 < n_0(p) < 1$, while outside the region it coincides with $\theta(p - p_f)$. It follows from Eq. (2) that the single particle spectrum is completely flat in this region. Such a state was called the state with FC since the quasiparticles in the range $p_f - p_i$ of momentum space are confined to the chemical potential $\mu$. The possible solution $n_0(p)$ of Eq. (2) and the corresponding single particle spectrum $\varepsilon(p)$ are shown.
in Fig. 1. It is seen from Fig. 1 that the particle-hole symmetry is not supported by FC since \( n_0(p) \) does not evolve from the Fermi-Dirac distribution function and we can expect that the conductivity possesses an asymmetric part. One can show that the relevant order parameter

\[ n_0(p) = \sqrt{n_0(p)(1-n_0(p))} \]

is the solution of Eq. (3) and it implies \( n_0(p < p_i) = 1, n_0(p_i < p < p_f) < 1 \) and \( n_0(p > p_f) = 0 \), while \( \varepsilon(p_i < p < p_f) = \mu \). The Fermi momentum \( p_F \) obeys the condition \( p_i < p_F < p_f \).

\[ \kappa(p) = \sqrt{n_0(p)(1-n_0(p))} \]

is the order parameter of the superconducting state with the infinitely small superconducting gap \( \Delta \) and with the entropy \( S = 0 \). Hence this state cannot exist at any finite temperatures and is driven by the parameter \( x \): at \( x > x_{FC} \) the system is on the disordered side of FCQPT; at \( x = x_{FC} \), Eq. (2) possesses the non-trivial solutions \( n_0(p) \) with \( p_i = p_F = p_f \); at \( x < x_{FC} \), the system is on the ordered side. At \( T > 0 \), \( n(p,T) \) is given by the Fermi-Dirac distribution function

\[ n(p,T) = \left( 1 + \exp \left[ \frac{(\varepsilon(p,T) - \mu)}{T} \right] \right)^{-1}, \]

where \( \varepsilon(p,T) \) is the single-particle spectrum, determined by Eq. (2) and \( \mu \) is a chemical potential. Equation (3) can be recast as

\[ \varepsilon(p,T) - \mu(T) = T \ln \left( \frac{1 - n(p,T)}{n(p,T)} \right). \]

As \( T \to 0 \), the logarithm on the right hand side of Eq. (4) is finite when \( p \in (p_f - p_i) \) and \( n(p,T) \approx n_0(p) \) so that \( T \ln(\ldots) \to 0 \), and we again arrive at Eq. (2). Near the Fermi level the single particle spectrum can be approximated as

\[ \varepsilon(p \approx p_F, T) - \mu \approx \frac{p_F(p - p_F)}{M^*(T)}. \]

It follows from Eq. (3) that \( n(p,T \to 0) \to \theta(p - p_F) \) if \( M^* \) is finite at \( T \to 0 \). At low temperatures, as it is seen from Eq. (5), the effective mass diverges as

\[ M^*(T) \simeq p_F \frac{p_f - p_i}{4T}. \]

At \( T \ll T_f \), Eq. (5) is valid and describes the quasiparticles with the energy \( \varepsilon \) and the distribution function \( n_0(p) \). Here \( T_f \) is the characteristic temperature where the influence of FCQPT becomes negligible. The energy \( \varepsilon \) belongs to the interval

\[ \mu - 2T \leq \varepsilon \leq \mu + 2T. \]

When the heavy electron liquid becomes superconducting the effective mass is temperature independent at \( T \leq T_c^* \) and given by the equation

\[ M^*(T) \simeq p_F \frac{p_f - p_i}{2\Delta_1}, \]

where \( T_c^* \) is the transition temperature at which the gap vanishes and \( \Delta_1 \) is the maximum value of the superconducting gap at \( T = 0 \).

**ASYMMETRIC CONDUCTANCE IN HF METALS AND HTSC**

In the case of the heavy electron liquid with FC, the tunneling current is of the form

\[ I(V) = \int [n(\varepsilon - V,T) - n_F(\varepsilon,T)]d\varepsilon. \]

Here we have replaced the distribution function of ordinary metal by \( n(\varepsilon,T) \) so that \( n(\varepsilon(p), T \to 0) \to n_0(\varepsilon(p)) \) where \( n_0(\varepsilon(p)) \) is the solution of Eq. (2) and also normalized the transition amplitude \( |t|^2 = 1 \). The differential tunneling conductivity, \( \sigma_d(V) = \partial I/\partial V \), is given by

\[ \sigma_d = \frac{1}{T} \int n(\varepsilon(z) - V,T)(1 - n(\varepsilon(z) - V,T)) \frac{\partial \varepsilon(z,T)}{\partial z} dz, \]

where \( z = p/p_F \). We take dimensionless momentum \( z \), instead of energy \( \varepsilon \), as an independent variable, since the distribution function \( n \) is a continuous function of \( z \) rather than of \( \varepsilon \) as it seen from Fig. 1. Indeed, the energy \( \varepsilon \) is constant in the range \( p_i - p_f \) while the distribution function varies in this range. It follows from Eq. (10) that the asymmetric part \( \Delta \sigma_d(V) = (\sigma_d(V) - \sigma_d(-V))/2 \) of the differential conductivity is of the form

\[ \Delta \sigma_d(V) = \frac{1}{2} \int \frac{\alpha(1 - \alpha^2)}{[\alpha n(z,T) + (1 - n(z,T))]^2} \left[ \frac{1 - 2n(z,T)}{\alpha n(z,T) + (1 - n(z,T))]^2} \right] dz, \]

where \( \alpha = \frac{p_f - p_i}{t}. \)
with $\alpha = \exp(-V/T)$. It is worth noting that according to Eq. (11) we have $\Delta \sigma_d(V) = 0$ if the considered HF metal is replaced by an ordinary metal. Indeed, the effective mass is finite at $T \to 0$ so that the integrand becomes an odd function of $x = z - 1$, while the limits of integration can be taken $-\infty, \infty$ since the integrand behaves like $\exp(-|x|)$ at large $|x|$. On the other hand, the integrand is no longer an odd function if the particle-hole symmetry is violated. As it is seen from Fig. 1, there are no reasons to expect that a Fermi liquid with FC conserves this symmetry. Thus, we conclude that the differential conductivity becomes an asymmetric function of the applied voltage for HF liquid with FC.

To estimate $\Delta \sigma_d(V)$, we observe that it is zero when $V = 0$ as it should be and follows from Eq. (11) as well. It is seen from Eq. (11) that at low voltage $V$ the asymmetric part behaves as $\Delta \sigma_d(V) \propto V$. Then, the natural scale to measure the voltage is $2T$, as it is seen from Eq. (4). In fact, the asymmetric part is proportional to $(p_f - p_i)/p_F$. As a result, we obtain

$$\Delta \sigma_d(V) \approx c \left(\frac{V}{2T}\right) \frac{p_f - p_i}{p_F} \propto \frac{V}{2T} \frac{S_0}{x}. \quad (12)$$

Here, $x = p_F^2/(3\pi^2)$ is a density of the heavy electron liquid, $c$ is a constant of the order of unity, and $S_0$ is the temperature independent part of an entropy $S[n(p, T)]$ which is given by the familiar expression

$$S[n(p, T)] = -2 \left[ n(p, T) \ln n(p, T) + (1 - n(p, T)) \right] \times \ln(1 - n(p, T)) \frac{dp}{(2\pi)^3}. \quad (13)$$

Inserting the solutions $n_0(p)$ into Eq. (13) we obtain that the entropy contains a temperature independent part $S_0/x \sim (p_f - p_i)/p_F$. At $T < T_f$, it can be approximated as

$$S(T) \approx S_0 + a \sqrt{\frac{T}{T_f}}, \quad (14)$$

Here $a$ is a constant. Thus, the ordered state existing at $T = 0$ is separated from the disordered state by first order phase transition. Due to this first order phase transition, both at the FCQPT point and behind it there are no critical fluctuations accompanying second order phase transitions and suppressing the quasiparticles. As a result, the quasiparticles survive and define the thermodynamic and transport properties of HF systems. This is in agreement with recent facts obtained in measurements on CeCoIn$_5$ [6]. This means that asymmetric conductivity measurements can provide a valuable information about the entropy $S_0$ which determines the divergence of the Grüneisen ratio $\Gamma(T)$ as well since, as it is seen from Eq. (13), $\Gamma(T) \propto S_0/\sqrt{T}$ [18]. So, we may conclude that STM and PCS techniques can give an experimental evidence about the NFL behavior of strongly correlated fermion systems. The constant $c$ can be evaluated using the analytically solvable models. For example, calculations of $c$ within a simple model, when Landau functional $E[n(p)]$ is of the form [19]

$$E[n(p)] = \int \frac{p^2 dp}{2m (2\pi)^3} + \frac{1}{2} \int V(p_1 - p_2)n(p_1)n(p_2) \times \frac{dp_1 dp_2}{(2\pi)^6}, \quad (15)$$

with inter-particle interaction

$$V(p) = g_0 \frac{\exp(-|p|)}{|p|}, \quad (16)$$

gives $c \approx 1/2$. It follows from Eq. (12), that when $V \approx 2T$ and FC occupies a noticeable part of Fermi volume, $(p_f - p_i)/p_F \approx 1$, so that the asymmetric part becomes comparable with differential conductivity, $\Delta \sigma_d(V) \sim \sigma_d(0) \sim \sigma_c(V)$. The asymmetric behavior of the conductivity can be observed in measurements on both high-$T_c$ metals in their normal state and the heavy fermion metals, for example, such as CeCoIn$_5$ and YbRh$_2$(Si$_{0.95}$Ge$_{0.05}$)$_2$ which are expected to have undergone FCQPT. Indeed, the measurements on these metals have shown that the Grüneisen ratio diverges [2, 20, 21]. In that case, the electronic systems of these metals are to undergo FCQPT so that the temperature independent part $S_0$ of the entropy becomes finite [18]. As a result, the asymmetric part of the conductance $\Delta \sigma_d(V)$ becomes finite [17]. We note that at sufficiently low temperatures, the application of magnetic field restores the LFL behavior making the asymmetry of the tunneling conductivity vanish [17]. Therefore, the measurements have to be carried out when the corresponding HF metal demonstrates the NFL behavior.

Point contact spectroscopy has recently been used to investigate the HF metal CeCoIn$_5$. The dynamic conductance spectra shown in Fig. 2 have been obtained and the asymmetric character of conductance has been observed in CeCoIn$_5$ both in its superconducting and NFL normal states [22]. Fig. 2 shows the conductance $dI/dV$ as a function of voltage $V$. The finite and even enhanced subgap conductance seen in Fig. 2 as well as that below the critical temperature $T_c$ of superconducting phase transition arises from Andreev reflection, see e.g. [14].

It is seen from Fig. 2 that the asymmetric conductivity develops at about 45 K, that is well above the critical temperature $T_c = 2.3$ K of superconducting phase transition. The asymmetry becomes more pronounced with decreasing temperature down to $T_c$. Then it remains almost the same at least down to 0.4 K [22].

In Fig. 3, we plot the results of our calculations (based on the functional [15]) of the asymmetric part $\Delta \sigma_d(V)$ of the conductance $\sigma_c(V)$ as a function of voltage when the heavy electron liquid is in its normal state.
Curves $\sigma_c(v)$ are shifted vertically by 0.05 for clarity and normalized by the conductance at $-2$ mV. The asymmetry is seen to develop starting at 45 K and becomes stronger with decreasing temperature.

The asymmetric conductance $\Delta \sigma_d(V)$ can also be observed when both HTSC and HF metals in question go from normal to superconducting phase. The reason now is that $n_0(p)$ is again responsible for the asymmetric part of the differential conductivity measured by both STM and PSC. As it was shown in Ref. [7], the function $n_0(p)$ is not appreciably disturbed by the superconductive pairing interaction which is relatively weak as compared to Landau interaction forming the distribution function $n_0(p)$. Therefore, the asymmetric conductance remains approximately the same below $T_c$. This result is in good agreement with experimental facts as it is seen from the lower inset of Fig. 3. We also conclude that Andreev reflection can be considered as a useful effect when studying the asymmetric conductance and the NFL behavior. When calculating the tunneling conductance measured by scanning tunneling microscopy, we have to take into account that

$$N_s(E) = N(\varepsilon - \mu) \frac{E}{\sqrt{E^2 - \Delta^2}}$$

comes now into the play since the density of states $N_s(E)$ of the superconducting metal is zero in the gap i.e. when $E \leq |\Delta|$. Here, $\Delta$ is the superconducting gap at $T = 0$, $E$ is the quasiparticle energy in the superconducting state, related to the normal state quasiparticle energy as $\varepsilon - \mu = \sqrt{E^2 - \Delta^2}$. It follows from Eq. (17) that the tunneling conductance can be asymmetric if the density of states $N(\varepsilon - \mu)$ is asymmetric with respect to the Fermi level as in the case of Fermi systems with FC. In Fig. 4, the results of our calculations (based on the functional [15]) with the same parameters when calculating the asymmetrical conductance shown in Fig. 3) of the density of states $N(\xi, T)$ as a function of normalized binding energy $\xi = (\varepsilon - \mu)/\mu$ are shown. It is seen from Fig. 4 that $N(\xi, T)$ is strongly asymmetric with respect to the Fermi level, or $\mu$. As a result, at the Fermi level the first derivative of $N(\xi, T)$ with respect to $\xi$ is not zero, and at small values of $\xi$ $N(\xi, T)$ can be approximated as $N(\xi, T) \approx a_0 + a_1 \xi$. The coefficient $a_0$ does not contribute to the asymmetric conductance which is obviously defined by $a_1$. The coefficient $a_1$ has to be proportional to the effective mass related to $T$ by Eq. (8). Now the system in its superconducting state and the effective mass is given by Eq. (8) so that the

$$g = 8$$
density of states is determined by $\Delta_1$ while the values of the normalized temperature shown in the upper right corner of Fig. 4 are defined as $2T \simeq \Delta_1/\xi_F$. So we have $\Delta\sigma_d(V) \sim V S_0/x \Delta_1$ since $(p_f - p_i)/p_F \simeq S_0/x$, while $E = V$ and $\xi = \sqrt{E^2 - \Delta^2}$. It is instructive to adjust Eq. (12) for the case of superconducting HF metal, multiplying the right hand side of this expression by $N_s(E)$ and replacing the quasiparticle energy $\varepsilon - \mu$ by $\sqrt{E^2 - \Delta^2}$ with $E$ being substituted by the voltage $V$. As a result, Eq. (12) can be again presented in the form

$$\Delta\sigma_d(V) \simeq \frac{V \sqrt{V^2 - \Delta^2} p_f - p_i}{\Delta \sqrt{V^2 - \Delta^2}} \frac{p_f - p_i}{p_F} \sim \frac{V S_0}{\Delta_1 x}. \quad (18)$$

We note that the entropy $S_0$ on right hand side of Eq. (18) is a temperature independent part of the normal (rather then superconducting) state entropy. Thus, as it follows from Eq. (18), the entropy $S_0$ characterizing the normal state can be evaluated measuring the asymmetric part of tunneling conductance of the superconducting state. We note that the scale $2T$ entering Eq. (12) is replaced by the scale $\Delta_1$ in Eq. (18). In the same way, as Eq. (12) is valid up to $V \sim 2T$, Eq. (18) is valid up to $V \sim \Delta_1$ and up to temperatures $T \leq T_c$. It is seen from Eq. (18) that the asymmetric part of the differential tunneling conductivity becomes as large as the entire differential tunneling conductivity at $V \sim \Delta_1$ under the condition that FC occupies a large part of the Fermi volume, $(p_f - p_i)/p_F \simeq 1$. In the case of a $d$-wave (or other unconventional) pairing, the right hand side of Eq. (18) has to be additionally integrated over the corresponding gap function. In our case this procedure is trivial, and $\Delta\sigma_d(V)$ becomes finite even at $V \leq \Delta_1$. In the case of PCS due to Andreev reflection the asymmetrical conduc-

tance can be observed even at $V \to 0$ and in the case of $s$-wave pairing. The tunneling differential conductance $\Delta\sigma_d(V)$ depending on the local density of states (LDOS) was obtained in measurements on $\text{Bi}_2\text{Sr}_2\text{Ca}_x\text{Cu}_{2-x}\text{O}_{8+y}$ at low temperatures [24] and is shown in Fig. 5. The presence of an electronic inhomogeneity in $\text{Bi}_2\text{Sr}_2\text{Ca}_x\text{Cu}_{2-y}\text{O}_{7+y}$ has recently been discovered in the scanning tunneling microscopy and spectroscopy experiments. This inhomogeneity is manifested as spatial variation in LDOS spectrum, in the low-energy spectral weight, and in the magnitude of the superconducting energy gap. The inhomogeneity observed in the integrated LDOS is not due to impurities, but rather is intrinsic characteristic of a substance. The observations allow to relate the magnitude of the integrated LDOS to the local oxygen concentration. The curves shown in Fig. 5 can be viewed as those corresponding to HTSC with different oxygen concentrations or corresponding to HTSC with different values of $\Delta_1$ measured at the same $T$ thus permitting to study the asymmetric conductance as a function of $\Delta_1$. It is seen from Fig. 5 that the tunneling differential conductivity is strongly asymmetric in superconducting state of the $\text{Bi}_2\text{Sr}_2\text{Ca}_x\text{Cu}_{2-y}\text{O}_{7+y}$ compound. Fig. 6 presents the asymmetric parts $\Delta\sigma_d(V)$ of the tunneling differential conductance spectra extracted from data displayed.

Figure 4: The density of states $N(\xi, T)$ as a function of $\xi = (\varepsilon - \mu)/\mu$ calculated for the three values of the normalized temperature $T$. The values of the normalized temperature are shown in the upper right corner.

Figure 5: Spatial variation of the tunneling differential conductance $\Delta\sigma_d(V)$ spectra measured in $\text{Bi}_2\text{Sr}_2\text{Ca}_x\text{Cu}_{2-y}\text{O}_{8+y}$. Curves 1 and 2 are taken at the positions where the integrated LDOS is very small. The low differential conductance and the absence of a superconducting gap are indicative for insulating behavior. Curve 3 is for a large gap 65 meV with small coherence peaks. The integrated value of the LDOS at the position for curve 3 is small but larger than those in curves 1 and 2. Curve 4 is for a gap of 40 meV, which is close to the mean value of the gap function. Curve 5, taken at the position with the highest integrated LDOS, is for the smallest gap of 25 meV with two very sharp coherence peaks [24].
in Fig. 5. It is seen that at small values of the bias voltage $V$ and in accord with Eq. (13), $\Delta\sigma_d(V)$ is a linear function of $V$ and the coefficient $a_1$ and the slope of the lines displayed in Fig. 6 are inversely proportional to the gap $\Delta_1$, see Fig. 5. Temperature dependence of the asymmetric parts $\Delta\sigma_d(V)$ of point contact spectra on YBa$_2$O$_{7-x}$/La$_{0.7}$Ca$_{0.3}$MnO$_3$ bilayers with $T_c \approx 30$ K shows that the asymmetric conductivity remains constant up to temperatures of about $T_c$ and persists up to temperatures well above $T_c$, see Fig. 7. It is also seen from Fig. 7 that at small values of the voltage $V$ the asymmetric part is a linear function of $V$ and starts to diminish at $T \geq T_c$. This behavior permits to study the asymmetric conductance as a function of temperature and is in good agreement with the behavior described by Eqs. (12) and (18). Therefore, we can conclude that the asymmetric part of the conductances described by Eqs. (12) and (18) are in good qualitative agreement with the experimental facts displayed in Figures 2, 3, 5, 6 and 7, while the asymmetry starts from FCQPT yielding the FC state with particle-hole symmetry violation.

**CONCLUSIONS**

We have shown that scanning tunneling microscopy and point contact spectroscopy being sensitive to both the density of states and the quasiparticle occupation numbers are ideal techniques for studying the effects related to a violation of the particle-hole symmetry. Above effects make the differential tunneling conductivity and dynamic conductance to be asymmetric function of applied voltage. The asymmetry appears as soon as FCQPT emerges and is closely related to the violation of a particle-hole symmetry. We have demonstrated that the asymmetric part of the conductance can be observed when HF metals and high-$T_c$ superconductors are normal and/or superconducting. We have shown that at small values of the voltage bias the asymmetric part is a linear function of the voltage and inversely proportional to the maximum value of the gap and does not depend on temperature provided that metal is superconducting, when it becomes normal the asymmetric part diminishes at elevated temperatures. Our theoretical results are in good agreement with available experimental data while it proved out to be very useful to explore the asymmetric part of conductivity rather then the conductivity. Since in pure LFL case the particle-hole symmetry is conserved and both the differential tunneling conductivity and dynamic conductance are symmetric functions of voltage bias, the measurements of asymmetric part of the conductance can be viewed as a powerful tool to investigate the NFL behavior of strongly correlated Fermi systems. We have derived that the asymmetric part is defined by the temperature independent part of the entropy characterizing the normal state of the electronic system of a strongly correlated metal. As a result, it became possible to study the thermodynamic properties measuring the transport characteristics of a substance. Thus, STM and PCS provide a new and very useful direction in the experimental studies of the NFL behavior of HTSC and HF metals.

Finally, our consideration of the conductance asymmetry observed in both HF metals and high-$T_c$ superconduc-
tors and described within the same framework suggests that FCQPT is intrinsic to strongly correlated substances and can be viewed as the universal cause of the non-Fermi liquid behavior.

This work was supported in part by RFBR, project No. 05-02-16085.

* Electronic address: vrshag@thd.pnpi.spb.ru
[1] D. Takahashi, et al., Phys. Rev. B 67 (2003) 180407.
[2] J. Custers, et al., Nature 424 (2003) 524.
[3] T. Senthil, M. Vojta, S. Sachdev, Phys. Rev. B 69 (2004) 035111.
[4] S. Fujimori, et al., cond-mat/0602296
[5] F. Ronning, et al., Phys. Rev. Lett. 97 (2006) 067005.
[6] J. Paglione, et al., cond-mat/0605124
[7] V.R. Shaginyan, M.Ya. Amusia, A.Z. Msezane, Phys. Lett. A 338 (2005) 393.
[8] V.R. Shaginyan, et al., cond-mat/0602602
[9] V.A. Khodel, V.R. Shaginyan, JETP Lett. 51 (1990) 553.
[10] G. E. Volovik, JETP Lett. 53 (1991) 222.
[11] M.Ya. Amusia, V.R. Shaginyan, Phys. Rev. B 63 (2001) 224507.
[12] A.F. Andreev, Sov. Phys. JETP 19 (1964) 1228.
[13] W.A. Harrison, Phys. Rev. 123 (1961) 85.
[14] G. Deutscher, Rev. Mod. Phys. 77 (2005) 109.
[15] V.R. Shaginyan, JETP Lett. 77 (2003) 99; V.R. Shaginyan, JETP Lett. 77 (2003) 178.
[16] M.Ya. Amusia, V.R. Shaginyan, Phys. Lett. A 298 (2002) 193.
[17] V.R. Shaginyan, JETP Lett. 81 (2005) 222.
[18] M.Ya. Amusia, A.Z. Msezane, V.R. Shaginyan, Phys. Lett. A 320 (2004) 459.
[19] V.A. Khodel, V.R. Shaginyan, V.V. Khodel, Phys. Rep. 249 (1994) 1.
[20] R. Küchler, et al., Phys. Rev. Lett. 91 (2003) 066405.
[21] N. Oeschler, et al., Phys. Rev. Lett. 91 (2003) 076402.
[22] W. K. Park, L. H. Greene, J. L. Sarrao, J. D. Thompson, Phys. Rev. B 72 (2005) 052509.
[23] P.W. Anderson, N.P. Ong, cond-mat/0405518; M. Randeria, R. Sensarma, N. Trivedi, F. Zhang, cond-mat/0412096
[24] S.H. Pan, et al., Nature 413 (2001) 282.
[25] S. Piano, et al., J. of Physics: Conference Series 43 (2006) 1123.