MASSIVE QUANTUM MEMORIES BY PERIODICALLY INVERTED DYNAMIC EVOLUTIONS

S. M. Giampaolo, F. Illuminati, A. Di Lisi, and G. Mazzarella
Dipartimento di Fisica “E. R. Caianiello”, Università di Salerno;
CNR-Coherentia, Gruppo di Salerno; and INFN Sezione di Napoli,
Gruppo Collegato di Salerno, Via S. Allende, 84081 Baronissi (SA), Italy.
giampaolo@sa.infn.it; illuminati@sa.infn.it; dilisi@sa.infn.it; mazzarella@sa.infn.it

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We introduce a general scheme to realize perfect quantum state reconstruction and storage in systems of interacting qubits. This novel approach is based on the idea of controlling the residual interactions by suitable external controls that, acting on the inter-qubit couplings, yield time-periodic inversions in the dynamical evolution, thus cancelling exactly the effects of quantum state diffusion. We illustrate the method for spin systems on closed rings with $XY$ residual interactions, showing that it enables the massive storage of arbitrarily large numbers of local states, and we demonstrate its robustness against several realistic sources of noise and imperfections.

Keywords: Quantum Information; Quantum Control; Spin Systems.

1. Introduction

Along the pathway that should eventually lead to the realization of scalable schemes of quantum computation, much theoretical work has been recently devoted to develop physical devices for the efficient processing and the coherent transfer of quantum data\textsuperscript{1–5}. Besides these two fundamental aspects, a further crucial requirement needed for the realization of a quantum computer is the possibility to store quantum information on a time scale at least comparable to the one needed to perform computational tasks. In particular, it would be very important to introduce systems acting as stable and robust quantum memories that recover and conserve large sets of quantum states that would be otherwise usually lost in very short times, due to quantum diffusion and decoherence\textsuperscript{6}.

An ideal quantum register is formed by a set of non interacting identical two–level systems (qubits), in the sense that either the Hamiltonian describing the dynamics of the internal degrees of freedom of the qubits, as well as the Hamiltonian describing the inter-qubit interactions, can be changed at will to realize the desired quantum-state manipulations. However, when we move from the ideal case to the arena of possible concrete realizations, the qubits cannot in general be considered identical due to the unavoidable presence of local imperfections, and moreover,
residual inter-qubit couplings are always present. The existence of these two sources of noise causes, as a consequence, the loss of information in the quantum register.

To ensure the ability to efficiently store quantum data, many different noise-evading schemes have been proposed\textsuperscript{7-17} All these works can be classified in two different groups: either the proposed schemes are based on some error correction technique; or they rely on some intrinsic mechanism, holding at least for a specific subset of quantum states, that leaves the information unaffected during the time-evolution of the quantum register (decoherence-free subspace schemes). The second approach is the most desirable one as it provides in principle a radical solution to the storing problem but, unfortunately, it is very sensitive to almost any source of imperfection. On the contrary, the schemes based on quantum error correction techniques are characterized by a dynamics that allows at any time the unambiguous reconstruction of the initially stored state, at least for a subset of states. The main trouble with error correction techniques lies in the fact that usually only very few states can be effectively accessed to store information and, hence, relatively large arrays of qubits are needed to store relatively small amounts of information.

In the present work, starting from Section 2, we discuss and generalize a new approach to quantum state storage that has been recently introduced\textsuperscript{18}. The basic idea of this novel scheme for the realization of massive quantum memories is to exploit time-modulated periodic dynamics that allow a perfect, periodic reconstruction of a generic initial state. In some sense, this new method can be seen as a time-periodic generalization of the decoherence-free subspace approach. We will show that our scheme is very robust against several sources of noise and imperfections.

An explicit realization of this approach is illustrated in Section 3, in which a topological linked magnetic flux with periodic step-time modulation is used to cancel the effects of the residual $XY$ interactions with site-dependent couplings induced by local imperfections. Finally, in Section 4 we summarize our results and discuss future perspectives.

2. Storing scheme: overview and prescriptions

As we have anticipated, a quantum register can be seen as an ensemble of qubits each one of them described by its own internal Hamiltonian. In the ideal case the internal interactions of the qubits are all identical and residual interactions between them can be switch on and off at will. However, realistic realizations of arrays of qubits strongly deviate from the ideal case: internal interactions are different due to imperfections, and residual inter-qubits interactions are always present. For this reason, realistic quantum processors and registers are in general properly described by generic Hamiltonians of the form\textsuperscript{19-21}

$$
\hat{H} = \hat{H}_0 + \hat{H}_{\text{err}},$$

where $\hat{H}_0$ stands for the Hamiltonian of ideal model register and $\hat{H}_{\text{err}}$ for the undesired noise terms. Due to the presence of the noise terms, immediately after having
stored a quantum state in the register (in the following we always refer to it as the initial state $|\psi_0\rangle$), such state starts to evolve, and hence the fidelity of the quantum register decreases, progressively corrupting the storing process. To overcome this problem we introduce a scheme that is based on the idea of modulating the evolution generated by the noise terms, rather than trying to completely eliminate it. In fact, we suggest to engineer the quantum register, and the corresponding quantum information storing process, in such a way that the overall dynamics yields a perfect (or almost perfect) periodic reconstruction of the initially stored state, with an a priori determined period $T$.

The idea to obtain a periodic state reconstruction, naturally leads to consider time-dependent, periodic Hamiltonians. Our approach aims in fact at modulating one or both of the two contributions in Eq. (1) to obtain the time-dependent periodic Hamiltonian that realizes the desired state reconstruction. As it is well known, in the presence of a generic time-dependent Hamiltonian, the quantum state evolution can be tracked down by resorting to the Dyson series representation. Obviously, the structural complexity of the Dyson series does not allow to identify all the possible Hamiltonian dynamical evolutions yielding perfect time-periodic state reconstruction. However, the Dyson series can be easily resummed when the Hamiltonian enjoys the property to commute at different times: $[\hat{H}(t), \hat{H}(t')] = 0$ for all $t$ and $t' \neq t$. Thus, our first prescription is to conceive a quantum register described by a time-periodic modulated Hamiltonian, commuting at all times. In the case in which this property holds only in a subspace of the whole Hilbert space of the quantum register, our approach is still valid if the state to be stored is chosen in this subspace.

Under the time-commutation hypothesis, we can determine a complete set of states $\{|\alpha\rangle\}$ that are eigenstates of the time-dependent Hamiltonian at all times even if the corresponding energy eigenvalues are time-dependent functions $\varepsilon^\alpha(t)$. Because the energy eigenstates form a complete basis set, the initial state $|\psi_0\rangle$ can be written as a linear combination: $|\psi_0\rangle = \sum_\alpha c_\alpha |\alpha\rangle$, with $c_\alpha = \langle \alpha | \psi_0 \rangle$, and the sum runs over the complete set of eigenstates. It is then simple to write the evolution of the initial state after a time $t > 0$

$$|\psi(t)\rangle = \sum_\alpha c_\alpha \exp \left( -i \int_0^t \varepsilon^\alpha(\tau) d\tau \right) |\alpha\rangle .$$  \hspace{1cm} (2)

From Eq. (2) it is immediate to see that if at a certain time $T > 0$ all the integrals in the sum are equal or differ from each other by integer multiples of $2\pi$, then the initial state $|\psi_0\rangle$ is perfectly reconstructed, but for an irrelevant global phase factor. This is then the second requirement that one needs to impose on the time-modulated dynamics in order to realize perfect time-periodic quantum state storage.

In the following of the paper we will show a simple but interesting implementation of this storing scheme based on the Aharonov-Bohm effect in systems of charged particles on a lattice.
3. Quantum register based on the Aharonov-Bohm effect

In this section we illustrate a specific realization of the general approach discussed above, that enables to eliminate the effect of the residual interactions between qubits described by \( XY \) Hamiltonians that, as we will show in subsection 3.1 rapidly destroys the storing capacity of a quantum register. As it is well known, spin Hamiltonians can be used as an effective description of systems of hopping and/or interacting quantum particles on a lattice, in the presence of an energy gap such that only two local states on each lattice site can be considered\(^{22}\). In general, in any such situation, an effective two-level system can always be mapped in a formal spin-1/2 one\(^{23}\). If the particles are charged, in the presence of a linked magnetic flux the real-valued hopping amplitude between particles, or, in spin language, the nearest-neighbor coupling, is transformed in a complex-valued quantity\(^{-22}\)\(^{26}\). In the presence of a simple geometry such as a tiny solenoid placed in the center of a circular ring of regularly spaced sites, the phase \( \theta \) of the nearest-neighbor interaction amplitude is proportional to the magnetic flux: \( \phi \propto \phi/N \) where \( N \) is the total number of qubits in the ring\(^{24}\)\(^{26}\). As shown in subsection 3.2, the magnetic flux can be suitably modulated in time to perfectly eliminate the dephasing effects due to the undesired residual interaction terms. Moreover, in subsection 3.3 we show that the storing scheme is robust with respect to other possible sources of noise that can arise in practical realizations.

3.1. The unmodulated quantum register

We begin by considering a quantum register in which \( N \) identical qubits are placed on the sites of a closed ring, and, due to the presence of local imperfections, are coupled by nearest-neighbor \( XY \) interactions with site-dependent amplitudes.

\[
\hat{H} = -\sum_i (\lambda + \chi_i) \left( \sigma_i^+ \sigma_{i+1}^- + H.c. \right) + B \sum_i \sigma_i^z,
\]

where \( B \) is the half-energy gap between the two levels of the qubits, \( \lambda \) is the global, averaged nearest-neighbor coupling amplitude, and \( \chi_i \) is a site-dependent random variable of vanishing mean, representing the local imperfection in the coupling at site \( i \). The presence of the coupling, even if \( \lambda \ll B \) and \( \chi_i = 0 \ \forall \ i \), rapidly destroys the storing capacity of the register. To illustrate this effect, let us introduce the following set of states \( |\Psi_d\rangle = |\uparrow_d\rangle \prod_{i\neq d} |\downarrow_i\rangle \), where the product involves all the qubits of the register except the one at site \( d \), and let us take the state with \( d = 0 \) \( (|\psi_0\rangle = |\Psi_0\rangle) \) as the initial one. Following the work of Amico et al.\(^{27}\), let us consider the behavior of the overlap \( \mathcal{F}_d(t) = |\langle \Psi_d | \Psi_0(t) \rangle|^2 \), where \( |\Psi_0(t)\rangle \) is the time-evolution of the initial state under the action of Hamiltonian Eq. (3). By looking at Fig. (1), we see that the quantum state diffusion grows indefinitely and the initial state is never recovered. Similar numerical results can be obtained in the limit \( \lambda \rightarrow 0 \) and nonvanishing local imperfections\(^{19,20}\).

From Fig. (1) one must conclude that either it is possible to perform all the
computational processes in times much shorter than $1/\lambda$ (time scale in which the fidelity of the state is close to 1), or, otherwise the quantum register described by Eq. (3) is not able to store quantum information for times long enough to process the desired quantum algorithms.

3.2. The step-phase modulated quantum register

To improve the storing capacity of the quantum register described in the previous subsection, we implement the scheme of quantum control that we have briefly outlined in the Introduction. The idea is to associate to the coupling amplitude a time dependent phase factor so that the modified Hamiltonian of the register reads

$$\hat{H}(t) = -\sum_i (\lambda + \chi_i) \left(e^{i\theta(t)} \sigma_i^+ \sigma_{i+1}^- + H.c. \right) + B \sum_i \sigma_i^z. \quad (4)$$

As already mentioned, complex coupling amplitudes can be realized in systems of charged hopping particles, such as electrons or charged ions hopping along a string of quantum dots or Cooper pairs tunnelling on Josephson junctions arrays\(^{22}\), when they are subject to external electromagnetic vector potentials\(^{24,26}\). To obtain a spatially constant phase in the intersite hopping amplitudes, one can consider, for instance, a thin solenoid placed at the center of a closed, circular ring. Then, in the associated spin formalism, the phase amplitude $\theta$ of the spin-spin couplings is equal for any pair of neighboring sites and is proportional to the ratio of the linked magnetic flux $\phi$ and the total number $N$ of sites: $\theta \propto \phi/N^{25}$. Clearly, not any time modulation of the phase factor can realize the desired perfect state storage. As we have already mentioned, two requirements must be
satisfied. The first prescription is that the time modulated Hamiltonian must obey
the commutation property at different times: $[\hat{H}(t), \hat{H}(t')] = 0$. In the case of the
modulated Hamiltonian Eq. (4), this property is verified if and only if $\theta(t) - \theta(t') = k\pi$, with $k$ integer. This implies that we must modulate the phase in such a way
that, during the entire evolution, it regularly jumps on and off between the two
constant values $\theta_0$ and $\theta_0 + \pi$, namely, we must realize a step-phase modulation.

The second property that must be verified in order to obtain a perfect, time-
periodic state reconstruction is that, at a certain given time $T$, all the time integrals
appearing in the Dyson series Eq. (2) for the quantum state evolut
ion must be


equal or differ by a trivial phase factor integer multiple of $2\pi$. Let us observe
that, independently of the values of $\theta$, the local term $B \sum_i \sigma_i^z$ commutes with the
$XY$ residual interaction terms in the Hamiltonian Eq. (4). Hence, there exists a
complete set of eigenstates of the total Hamiltonian that are as well simultaneous
eigenstates of both the local and the interaction terms. But then, for all eigenvalues
$\varepsilon^{\alpha}(\theta_0)$ associated to the eigenstates $|\alpha\rangle$, we have that: $\varepsilon^{\alpha}(\theta_0) = \varepsilon^{\alpha}_c(\theta_0) + \varepsilon^{\alpha}_l$, where $\varepsilon^{\alpha}_c(\theta_0)$ and $\varepsilon^{\alpha}_l$ are, respectively, the interaction and the local contributions to the
energy. On the other hand, when $\theta$ passes from the value $\theta_0$ to $\theta_0 + \pi$, the coupling
contribution to the energy changes sign while the local one remains unchanged, so
that $\varepsilon^{\alpha}(\theta_0 + \pi) = -\varepsilon^{\alpha}_c(\theta) + \varepsilon^{\alpha}_l$. Therefore any two energy eigenvalues $\varepsilon^{\alpha}(\theta)$ and $\varepsilon^{\alpha}(\theta + \pi)$ corresponding to the same eigenstate $|\alpha\rangle$, differ only in the sign of the
interaction component.

Collecting these facts together, if we select a regular step time modulation of
the phase of the form$^{18}$

$$
\theta(t) = \begin{cases} 
\theta & 0 \leq t < T/2 \\
\theta + \pi & T/2 \leq t < T 
\end{cases} 
$$

periodically repeated for any $t \geq T$, we obtain that the contribution of the residual
interaction Hamiltonian to the quantum state time evolution vanishes at any time
t = mT with $m$ arbitrary integer. Consequently, it turns out that by a proper
time modulation of the external electromagnetic potential and of the corresponding
Aharonov-Bohm phase, it is possible to whipe out the effects of the undesired $XY$
couplings in a quantum register.

Regarding the role of the local term, we must discriminate between two different
situations. The good case is when we can control and choose the period $T$ in such a
way that $BT = 2l\pi$ with $l$ integer. Under this hypothesis any initial quantum state is
reconstructed exactly at all times $t$ integer multiples of $T$. The bad instance is when
the parameters are fixed such that $BT \neq 2l\pi$. If we store an arbitrary initial state
linear combination of states with different numbers of up and down spins, i.e. states
different of magnetizations, at all times integer multiples of $T$ each of these states
is reconstructed, but, unfortunately, with extra, geometric phase factors between
them, and perfect quantum state reconstruction becomes impossible. However, even
in this case, perfect storage is still achieved in the subspace of states that can be
expressed as linear combinations of local states all with the same, fixed value of the
Fig. 2. Step-periodic time-modulation of the phase, Eq. (5): two-dimensional contour plot showing the evolution of the overlap $F_d(t)$, for the same initial state considered in Fig. (1), as a function of the distance $d$ from site 0 ($y$-axis) and of the dimensionless time $\lambda t$ in atomic units $\hbar = 1$ ($x$-axis). Here $\lambda T = \pi$ and $\theta = \pi/2$. The value of the overlap increases from white ($F_d(t) = 0$) to black ($F_d(t) = 1$).

magnetization.

To compare the unmodulated and the step-phase modulated quantum registers, in Fig. (2) we again plot the time evolution of the overlap $F_d(t)$, for the same initial state considered in Fig. (1). At striking variance with the unmodulated case reported in Fig. (1), we see that the step-phase modulated register realizes exact, time-periodic coherent revivals of the initial state. Moreover, we see from Fig. (2) that the overall spatial diffusion of the state is confined in a well defined and narrow region of the ring. this result can be generalized to any initial state $|\Psi_0\rangle$ with an arbitrary number of flipped spins (many magnons). In particular, one finds the same coherent time-periodic revival of the state as in the one-magnon case, while the spatial spread becomes a function of the size of the region along which the initial state is extended, but remains in any case finite and limited.

3.3. Possible sources of noise

Tackling the issue of practical implementations for the quantum state storage scheme that we have introduced, it is very important to verify that it does does not depend critically on a perfect realization of the step-periodic time modulation of the phase and on other possible sources of noise and imperfections. We begin by investigating whether the step phase modulated register can be efficiently implemented when considering real step function generators of finite precision. In Fig. (3) we show the evolution of the fidelity $F_0(t) = |\langle \Psi_0 | \psi(t) \rangle|^2$ for the same initial one-magnon state $|\Psi_0\rangle$ as before, as a function of the number of periods, when the step-periodic, time-modulated phase $\theta(t)$ Eq. (5) is approximated by its
Fig. 3. Fidelity $F_0(t)$ on site $i = 0$ as a function of the number of periods $\lambda T$ for the initial one-magnon state $|\Psi_0\rangle$ when we replace the step-periodic time-modulated phase $\theta(t)$ with its finite-harmonic Fourier approximations, in increasing order. Dashed line: first 5 harmonics; dot-dashed line: first 13 harmonics; dot-dot-dashed line: first 25 harmonics; dotted line: first 50 harmonics; solid line: first 100 harmonics.

Fourier decompositions, truncated at various finite orders. Remarkably, we see that even when considering only the first 100 harmonics, the fidelity remains close to the ideal limit $F_0(t) = 1$ for very long times. This demonstrates the stability of the finite-harmonic approximation and, as a consequence, the robustness of our storing protocol against imperfections in the external control of the phase.

Another important source of noise that modifies the quantum register described by Eq. (4) is the presence of local, site-dependent random imperfections $\eta - i$ in the half-energy gap, so that the Hamiltonian Eq. (4) is modified and becomes

$$\hat{H}(t) = -\sum_i (\lambda + \chi_i) \left( e^{i \theta(t)} \sigma_i^+ \sigma_{i+1}^- + H.c. \right) + \sum_i (B + \eta_i) \sigma_i^z \quad (6)$$

where $\eta_i$ are random variables of vanishing mean and finite variance $\sigma^2_\eta$. Because of the presence of the local noise term $\hat{H}_{err} = \sum_i \eta_i \sigma_i^z$, it is no longer possible to satisfy the commutation property for any value of the phase $\theta$, and hence state reconstruction will not be perfectly achieved, in general. However it is possible, and important, to determine to what extent the presence of the local noise affects the efficiency of the storing scheme. To this aim, we first note that both the local noise and the interaction terms commute with the local term $\hat{H}_l = B \sum_i \sigma_i^z$, even if they do not commute with each other. It is then possible to introduce the complete set of states $\{|\gamma\rangle\}$ that are simultaneous eigenstates of the local term, with eigenvalue $\varepsilon^\gamma_i$, and of $\hat{H}_1 = -\hat{H}_l + \hat{H}_{err}$, with eigenvalue $\varepsilon^\gamma_i$, where $\hat{H}_i$ is the XY interaction in the Hamiltonian Eq. (6). After a generic period $T$, perfect reconstruction of the initial state, written as a linear combination of states $\{|\gamma\rangle\}$, is assured if the fidelity $\langle \gamma | \gamma(T) \rangle = 1, \forall |\gamma\rangle$. We can then quantify the effect of the local noise term by studying the difference of the fidelities $\langle \gamma | \gamma(T) \rangle$ with and without the local noise term (in the latter case $\langle \gamma | \gamma(T) \rangle = 1$ trivially). With the notations introduced
above, and assuming a step time-modulation of the phase $\theta$, the application of the total Hamiltonian Eq. (6) to a generic state $|\gamma\rangle$ yields

$$
\hat{H}(t)|\gamma\rangle = \begin{cases} 
(\varepsilon_1^\gamma - \varepsilon_1^\gamma)|\gamma\rangle + 2\hat{H}_{\text{err}}|\gamma\rangle & 0 \leq t < T/2, \\
(\varepsilon_1^\gamma + \varepsilon_1^\gamma)|\gamma\rangle & T/2 \leq t < T.
\end{cases} 
$$

(7)

Because the states $|\gamma\rangle$ are not eigenstates of $\hat{H}_{\text{err}}$, we have that $\langle\gamma'|\hat{H}_{\text{err}}|\gamma\rangle \neq 0$ for any pair $(\gamma, \gamma')$. On the other hand, in the absence of the phase modulation (unmodulated register with noise), one has

$$
\hat{H}(t)|\gamma\rangle = (\varepsilon_1^\gamma - \varepsilon_1^\gamma)|\gamma\rangle + 2\hat{H}_{\text{err}}|\gamma\rangle \forall t.
$$

(8)

We may now use the expressions in Eq. (7) and Eq. (8) to evaluate the fidelity in the two different situations by the Dyson series. Assuming for instance a Gaussian distribution for the local noise $\eta_i$, we obtain, for the periodic step time modulation and for the case of constant phase, respectively,

$$
\langle\gamma|\gamma(T)\rangle = \exp\{-i\varepsilon_1^\gamma T\}A(\sigma_\eta, \varepsilon_1^\gamma, \sigma_1),
$$

(9)

and

$$
\langle\gamma|\gamma(T)\rangle = \exp\{-i(\varepsilon_1^\gamma + \varepsilon_1^\gamma)T\}A'(\sigma_\eta, \varepsilon_1^\gamma, \sigma_1).
$$

Looking at the exponential terms we immediately note that the periodic step time modulation of the phase is still effective in suppressing the undesired action of the residual $XY$ interaction terms. Concerning the attenuations terms $A$ and $A'$ in Eqs. (9), both functions depend on the Gaussian width $\sigma_\eta$ of the local noise term, on the eigenvalue $\varepsilon_1^\gamma$ of the Hamiltonian $H_1$, associated to the $|\gamma\rangle$, and on the spread $\sigma_1$ of the density of states of $H_1$ as a function of the energy. Both functions must reduce to unity in the limit of vanishing $\sigma_\eta$ but, in spite of this information, it is difficult to obtain an analytic expression for any one of them. However, close to the limit of vanishing $\sigma_\eta$ it is possible to compare their series expansions, finding that $A$ remains always closer to one than $A'$. This implies that the the scheme based on the phase step modulation Eq. (5) of the off-diagonal Hamiltonian terms has an effect on the diagonal (local) part as well, allowing for a better quantum state storage compared to the unmodulated register.

4. Summary and outlook

We have presented a novel scheme for the storage of quantum information in a quantum register based on periodic, perfect state reconstruction. As an example of application of this storing scheme we have discussed a quantum register based on time modulation of a Aharonov-Bohm phase. This kind of quantum register is able to fully cancel the effects of residual interactions of the $XY$ type. Moreover, the scheme appears to be robust even in the presence of other sources of static noise, such as phase modulations of finite precision and local noise on the internal Hamiltonian. The study of the effects of dynamic imperfections is currently under way, and we hope to soon report on it.
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