Mesoscopic Physics and the Fundamentals of Quantum Mechanics

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(Received January 11, 2022)

We start by reviewing some interesting results in mesoscopic physics illustrating nontrivial insights on Quantum Mechanics. We then review the general principles of dephasing (sometimes called ”decoherence”) of Quantum-Mechanical interference by coupling to the environment degrees of freedom. A particular recent example of dephasing by a current-carrying (nonequilibrium) system is then discussed in some detail. This system is itself a manifestly Quantum Mechanical one and this is another illustration of detection without the need for ”classical observers” etc. We conclude by describing briefly a recent problem having to do with the orbital magnetic response of conduction electrons (another manifestly Quantum Mechanical property): The magnetic response of a normal layer (N) coating a superconducting cylinder (S). Some recent very intriguing experimental results on a giant paramagnetic component of this response are explained using special states in the normal layer. It is hoped that these discussions illustrate not only the vitality and interest of mesoscopic physics but also its extreme relevance to fundamental issues in Quantum Mechanics.
I. INTRODUCTION

Mesoscopic Physics [1] deals with the realm which is in-between the microscopic (atomic and molecular) scale and the macroscopic one. As such, it can give us very fundamental information on the crossover between the microscopic, purely quantum, regime and the macroscopic regime. Macroscopic-type electrical measurements can be sensitive to nontrivial quantum phenomena in mesoscopic samples at low temperatures. The latter are needed to preserve the phase coherence of the electrons, as will be discussed later.

Following the discovery of the quantum Hall effect [2] it became clear that electrical quantities such as conductances can display discrete ”quantized” values. The basic unit being $e^2/h$, the product of the fine-structure constant and a classical conductance related to the well-known impedance of free space. In fact, from the deep and very useful Landauer [3] relationship between conductance and transmission, one can obtain [4] nontrivial predictions on the conductance of very small orifices connecting large electron reservoirs. This conductance, in the above units, is limited by the ”number of quantum channels” (or transverse states below the Fermi energy). In special cases (ballistic and tapered shapes of the orifice) the integer values can be approached. This has been found experimentaly [5] for carefully prepared ”Quantum Point Contacts” (QPC) in the 2D electron gas at low temperatures. For a 2D electron gas, the Quantum Point Contact is created by depositing, using lithographic techniques, two rather narrow metallic ”gates’ that almost touch each other at their tips. A negative bias depletes the electron gas beneath the gates, leaving a small opening, which serves as the orifice and is controlled by the gate voltage. We show in fig. [6] a recent example [6] depicting the conductance of such a Quantum Point Contact as function of the gate voltage. When the latter becomes less negative the orifice opens up and its conductance exhibits a tendency for plateaus at integer multiples of $e^2/h$ as shown in the figure.
FIG. 1. The conductance of a QPC in a 2D electron gas as a function of gate voltage, from ref. [6]. The RHS scale is for the lower curves giving the noise power for transport voltages of 1, 2 and 3 mV.

We present this particular experiment although flatter plateaus have been obtained before, because of a new feature: It obtained the spectral density of the noise power due to the current through the device (as shown). This nonequilibrium "shot noise" is due to the discreteness of the electronic charge. It has been predicted [7] to peak around the transitions among the plateaus, as in fact demonstrated by the cited results. We shall later use this result in section III. We remark that the shot noise is an interesting manifestation of the particle nature of the electron, although it obviously appears also in situations in which the wave-like properties are dominant, such as tunneling through a barrier. The sensitivity of the shot noise to the statistics and correlations of the particles makes it a very interesting subject for further study.
More recently, this effect was discovered to be of relevance in various naturally-forming atomic scale contacts at room temperatures.

The Landauer formulation enables one to consider the conductance of a mesoscopic ring as function of an Aharonov-Bohm (AB) flux through its opening \[8\]. The conductance is found to depend periodically on the flux with a period \(\Phi_0 = \frac{hc}{e}\), in agreement with general theorems (\(\Phi\) is equivalent to a phase change of \(2\pi\Phi/\Phi_0\)) and with experiment \[9\]. It is interesting to note that the conductance of macroscopically long cylinders of mesoscopic radii was predicted theoretically \[10\] and found experimentally \[11\] to have a period of \(\Phi_0/2\). This difference of a factor of two was found to be that between a "sample specific" and (impurity-) ensemble-averaged behavior. The latter implies averaging over many samples all having the same macroscopic parameters but differing in the detailed specific placement of the impurities and defects. The long cylinder consists of many such samples whose resistances are added incoherently, hence the total resistance displays the ensemble-averaged behavior in which the fundamental \(\Phi_0\) period is averaged out. This insight opened the way to a consideration of "mesoscopic" (sample-dependent) fluctuations in the conductance (and possibly other physical properties). We cannot discuss this here due to length limitations. The universal sizes \[12\] of the above fluctuations is nevertheless a fascinating fundamental property of these systems.

When the mesoscopic ring is isolated from its leads, one may still enquire what happens as a function of the AB flux through it at equilibrium or under adiabatic conditions. For free electrons, it is straightforward to see that the energy levels will depend on the flux. Therefore so will the free energy. Since the thermodynamic equilibrium current is the derivative of the free energy with respect to the flux, it follows that an equilibrium circulating current can flow in the ring and it will be periodic in the flux, with a period \(\Phi_0\), as above. Obviously, for this to happen, the electrons must stay coherent while going at least once around the ring. It has been believed that scattering of the electrons by defects, imperfect sample edges, etc. will cause this current to dissipate and therefore the equilibrium current should vanish in realistic systems. One of the fundamental quantum mechanical notions that mesoscopic
physics helped to clarify is, in fact, that elastic scattering by defects does \textit{not eliminate such coherence effects}. The impurities present just a (random) static potential to the electron. Well-defined coherent wavefunctions will prevail, and energy levels will be sharp and flux-dependent, as long as the disorder is not strong enough to cause localization. The resulting "persistent currents" will not decay \cite{13}. This initially surprising fact was proven experimentally \cite{14}. The magnitude of these currents can even be \textit{larger} than what noninteracting electrons will yield. Many-body theory is necessary to explain these magnitudes and this is still unsolved in some instances. The fact that static impurity scattering does not destroy phase coherence is obviously what enables the AB oscillations in the conductance to exist. Thus \textit{elastic scattering does not cause phase incoherence}. It takes \textit{inelastic} scattering to do that\footnote{As will become clear, here the term "inelastic" implies just changing the quantum state of the environment. It is irrelevant how much energy is transferred in this process. This includes zero energy transfer – flipping the environment to a degenerate state.}. This will be discussed in some detail in the following two sections. We remark that since the strength of the inelastic scattering decreases at low temperatures, one may reach the regime where electrons stay coherent over the whole sample. This is an important condition for the flux sensitivity and persistent currents to exist. For many experimental configurations this means that sizes in the micron range are useful at temperatures of a few degrees K. In the nanometer size range, quantum effects should be observable at room temperature.

Following the brief review above of some highlights from mesoscopic physics, we discuss in section \textsection{} II the general principles of dephasing (sometimes called "decoherence") of Quantum-Mechanical interference by coupling to the degrees of freedom which do not directly participate in the interference. Such degrees of freedom are often referred to as the "environment". Several ways to discuss this dephasing are mentioned and their equivalence proven. A particular recent example of dephasing by a current-carrying (nonequilibrium) system is then discussed in some detail in section \textsection{} III. This example also illustrates some of
the principles discussed in section II. This system is itself a manifestly Quantum Mechanical one and this is a good example of detection without the need for "classical observers" etc. We describe, in section IV, a recent surprising experimental result on the orbital magnetic response of conduction electrons (another manifestly Quantum-Mechanical property): The magnetic response of a normal layer (N) coating a superconducting cylinder (S) exhibits a very intriguing giant paramagnetic component at very low temperatures. This is explained in terms of an unusually large mesoscopic effect (on scales of a fraction of a millimeter), using special states in the normal layer. Some concluding remarks are made in the final section.

**II. GENERAL THEORY OF QUANTUM-MECHANICAL DEPHASING**

Many of the interesting effects in mesoscopic systems are due to quantum interference. We already saw in section I that elastic scattering by a static potential does not destroy quantum interference. It only modifies and perhaps complicates it. It takes inelastic scattering due to the coupling of the interfering particle to its environment, to eliminate the interference. The way such a coupling modifies quantum phenomena has been studied for a long time, both theoretically ([15,16]) and experimentally. A beautiful description of the underlying physics may be found in the Feynman Lectures on Physics [17]. The effect of the coupling to the environment may be characterized by the “phase-breaking” time, $\tau_\phi$, which is the characteristic time for the interfering particle to stay phase coherent, as explained below.

Stern et al. [18] studied the way a coupling of an interfering particle affects a two-wave interference experiment. Our discussion will be based on their work. Two descriptions have been used of the way the interaction of a quantum system with its environment might suppress quantum interference. The first regards the environment as measuring the path (sometimes referred to as "Which Path" detection) of the interfering particle. When the environment has the information on that path, no interference is obtained. The second description answers the question naturally raised by the first: How does the interfering
particle “know”, when the interference is examined, that the environment has identified its path? This question is answered by the observation that the interaction of a partial wave with its environment can induce an uncertainty in this wave’s phase (what counts physically is the uncertainty of the relative phases of the paths). This may be described as averaging out the interference pattern by turning it into a sum of many patterns, shifted relative to one another. These two descriptions were proven to be equivalent, and this has been applied to the dephasing by electromagnetic fluctuations in metals, and by photon modes in thermal and coherent states. Here we will review the two descriptions, and briefly summarize the dephasing by the electron-electron interaction in conducting matter.

As a guiding example, we consider an Aharonov-Bohm (A-B) interference experiment on a ring as described in the previous section. This experiment starts by a construction of two electronic wave packets, $\ell(x)$ and $r(x)$ ($\ell, r$ stand for left, right), crossing the ring along its two opposite sides. We assume that the two wave packets follow well defined classical paths, $x_\ell(t), x_r(t)$ along the arms of the ring. The interference is examined after each of the two wave packets had traversed half of the ring’s circumference. Therefore, the initial wavefunction of the electron (whose coordinate is $x$) and the environment (whose wavefunction and set of coordinates are respectively denoted by $\chi$ and $\eta$) is:

$$\psi(t = 0) = [\ell(x) + r(x)] \otimes \chi_0(\eta)$$  \hspace{1cm} (1)

This kind of interference gives rise to $h/e$ oscillations of the conductance. At time $\tau_0$, when the interference is examined, the wave function is, in general,

$$\psi(\tau_0) = l(x, \tau_0) \otimes \chi_\ell(\eta, \tau_0) + r(x, \tau_0) \otimes \chi_r(\eta, \tau_0)$$  \hspace{1cm} (2)

and the interference term is $2 \text{Re} \left[ \ell^*(x, \tau_0) r(x, \tau_0) \int d\eta \chi^*_\ell(\eta, \tau_0) \chi_r(\eta, \tau_0) \right]$ Had there been no environment present in the experiment, the interference term would have been just $2\text{Re}[\ell^*(x, \tau_0) r(x, \tau_0)]$. So, the effect of the interaction is to multiply the interference term by

$$\int d\eta \chi^*_\ell(\eta) \chi_r(\eta)$$  \hspace{1cm} (3)
at \( \tau_0 \). This is so since the environment is not observed in the interference experiment, its coordinate is therefore integrated upon, i.e., the scalar product of the two environmental states at \( \tau_0 \) is taken. The first way to understand the dephasing is seen directly from this expression, which is the scalar product of the two environment states, coupled to the two partial waves, at \( \tau_0 \). At \( t=0 \) these two states are identical. During the time of the experiment, each partial wave has its own interaction with the environment, and therefore the two states evolving in time become different. When the two states of the environment become orthogonal, the final state of the environment identifies the path the electron took. Quantum interference, which is the result of an uncertainty in this path, is then lost. Thus, the phase breaking time, \( \tau_\phi \), is the time in which the two interfering partial waves shift the environment into states orthogonal to each other, i.e., when the environment has the information on the path the electron took.\(^2\)

The second explanation for the loss of quantum interference regards it from the point of view of how the environment affects the partial waves, rather than how the waves affect the environment. It is well known that when a static potential \( V(x) \) is exerted on one of the partial waves, this wave accumulates a phase

\[
\phi = -\int V(x(t))dt/\hbar
\]

and the interference term is multiplied by \( e^{i\phi} \). “A static potential” here is a potential which is a function of the particle’s coordinate and momentum only, and does not involve any other degrees of freedom. For a given particle’s path, the value of a static potential is well defined. When \( V \) is not static, but created by environment degree(s) of freedom, \( V \) becomes an operator. Thus its value is not well defined any more. The uncertainty in this value results from the quantum uncertainty in the state of the environment. Therefore, \( \phi \) is not definite as well. In fact, \( \phi \) becomes a statistical variable, described by a distribution function \( P(\phi) \). (For

\(^2\)The question of whether somebody comes in to observe the change of state of the environment, does \textit{not} arise. The discussions of the effect of that further observation are irrelevant.
the details of this description see ref [18]). The effect of the environment on the interference is then to multiply the interference term by the average value of $e^{i\phi}$, i.e., $\langle e^{i\phi} \rangle = \int P(\phi) e^{i\phi} d\phi$

The averaging is done on the interference “screen”. Since $e^{i\phi}$ is periodic in $\phi$, $\langle e^{i\phi} \rangle$ tends to zero when $P(\phi)$ is slowly varying over a region much larger than one period, of $2\pi$. When this happens, one may say that the interference screen shows a superposition of many interference patterns, mutually cancelling each other. Hence, the phase-breaking time is also the time in which the uncertainty in the phase becomes of the order of the interference periodicity. This is the second explanation for the loss of quantum interference.

The statement of equivalence between the two explanations is given by the equation,

$$\langle e^{i\phi} \rangle = \int d\eta \chi_0^*(\eta) \chi_0(\eta) \quad (5)$$

When the environment measures the path taken by the particle (by $\chi_0$ becoming orthogonal to $\chi_r$), it induces a phase shift whose uncertainty is of the order of $2\pi$. The equivalence embodied in eq. (5) is proven as follows:

We start considering dephasing of the right-hand path $\chi_r$ only. The generalization to two paths will be seen later. The Hamiltonian of the environment will be denoted by $H_{env}(\eta, p_\eta)$, while the interaction term is $V(\chi_r(t), \eta)$ (the left partial wave does not interact with the environment). Starting with the initial wave function (eq. [1]) the wave function at time $\tau_0$, using the potential in the interaction picture $V_I(t) \equiv e^{iH_{env} t} V(\chi_r(t), \eta) e^{-iH_{env} t}$, is

$$\psi(\tau_0) \equiv \ell(\tau_0) \otimes e^{-iH_{env} \tau_0/\hbar} \chi_0(\eta) + r(\tau_0) \otimes e^{-iH_{env} \tau_0/\hbar} \hat{T}$$

$$\times \exp \left[ -i \int_0^{\tau_0} \frac{dt}{\hbar} V_I(x_r(t), t) \right] \chi_0(\eta). \quad (6)$$

where $\hat{T}$ is the time-ordering operator. Hence the interference term is multiplied by $\langle \chi_0 | \hat{T} \exp \left[ -\frac{i}{\hbar} \int_0^{\tau_0} dt V_I(x_r(t), t) \right] | \chi_0(\eta) \rangle$ The interpretation of this expression in terms of a scalar product of two environment states at time $\tau_0$ is obvious. The interpretation in terms of phase uncertainty emerges from the observation that the last expression is the expectation
value of a unitary operator. As all unitary operators, this operator can be expressed as the exponential of an Hermitian operator $\phi$, i.e.,

$$\langle \chi_0 | \hat{T} \exp \left[ -\frac{i}{\hbar} \int_0^{\tau_0} dt V_I(x_r(t), t) \right] | \chi_0 \rangle = \langle \chi_0 | e^{i\phi} | \chi_0 \rangle.$$  \(7\)

Hence the effect of the interaction with the environment is to multiply the interference term by $\langle e^{i\phi} \rangle$, where the averaging is done with respect to the phase probability distribution, as determined by the environmental state $\chi_0$.

The concept of the phase operator $\phi$ would seem to need more clarification. In fact, this is only a way of describing what happens. The whole physics is contained in the "phasor" unitary operator $exp(i\phi)$. The decrease of the absolute value of its average from unity determines the deterioration of the interference. Nevertheless a physical interpretation of the phase operator can be obtained in the case where the potentials exerted by the environment at different points along the particle’s path commute, i.e.

$$[V_I(x_r(t), t), V_I(x_r(t'), t')] = 0 \quad (8)$$

Then, $\phi = -\frac{1}{\hbar} \int_0^{\tau_0} dt V_I(x_r(t), t)$. In this case $\dot{\phi}$, the rate of accumulation of the phase, is just the local potential acting on the interfering particle, independent of earlier interactions of the particle with the environment. One should distinguish here between two limits: for $\langle \delta\phi^2 \rangle \ll 1$, the environment’s potential can be approximated by a single-particle (possibly time-dependent) potential $V_I(x_r(t), t) = \langle \chi_0 | V_I(x_r(t), t) | \chi_0 \rangle$. For $\langle \delta\phi^2 \rangle \gg 1$, on the other hand, the interference term tends to zero. The crossover between the two regimes occurs at $\langle \delta\phi^2 \rangle \sim 1$.

The condition in eq. (8) is typically valid when the excitation created by the electron at one time can not be absorbed at a later time. This is the case for the example of an electron interacting with a free electromagnetic field. Also, in large many-body environments the potential exerted by the environment on the interfering particle is usually practically

$^{3}exp(i\phi)$, in distinction to the phase itself, has the important feature of a built in $2\pi$ periodicity.
independent of the particle’s history since the environment’s memory time is very short. Therefore eq. 8 can be assumed to hold.

We thus see that the loss of interference due to an interaction with a dynamical environment can be understood in the two ways discussed. The interference is destroyed either when the state of the environment coupled to the right wave is orthogonal to that coupled to left wave, or, alternatively, when the width of the phase distribution function exceeds a magnitude of order unity. The interaction with the dynamical environment turns the phase into a statistical variable, and this, together with the fact that the phase is defined only over a range of $2\pi$, determines the conditions for the phase to become completely uncertain. If the potential exerted by the environment on the interfering particle at a given point along its path is assumed to be independent of the path, the phase uncertainty is given by,

$$\langle \delta \phi^2 \rangle = \frac{\tau_0}{\hbar} \int_0^{\tau_0} \int_0^{\tau_0} \frac{dt}{\hbar} \left[ \langle V_I(x_r(t), t)V_I(x_r(t'), t') \rangle ight.$$

$$- \langle V_I(x_r(t), t) \rangle \times \langle V_I(x_r(t'), t') \rangle \left]. \right.$$

(9)

This relationship will be used in the next section.

The exact behaviour of the interference term for $\langle \delta \phi^2 \rangle \gg 1$, i.e., the value of $\langle e^{i\phi} \rangle$ for broad distribution functions, depends on the phase distribution, $P(\phi)$. However, the description of the phase as a statistical variable enables us, under appropriate conditions, to apply the central limit theorem, and conclude that $P(\phi)$ is a normal distribution. The central limit theorem is applicable, for example, when the phase is accumulated in a series of uncorrelated events (e.g., by a series of scattering events by different, non-interacting, scatterers), or, more generally, whenever the potential-potential correlation function decays to zero with a characteristic decay time much shorter than the duration of the experiment.

In particular, the central limit theorem is usually applicable for coupling to a heat-bath. For a normal distribution,

$$\langle e^{i\phi} \rangle = e^{i\langle \phi \rangle - (1/2)\langle \delta \phi^2 \rangle}$$

(10)

This expression is exact for the model of an environment composed of harmonic oscillators.
with a linear coupling to the interfering waves. This model was proven in recent years to be very useful in the investigation of the effect of the environment on quantum phenomena (e.g. refs. [15,16]).

It is seen from the above discussion that the phase uncertainty remains constant when the interfering wave does not interact with the environment. Thus, if a trace is left by a partial wave on its environment, this trace cannot be wiped out after the interaction is over. Neither internal interactions of the environment, nor a deliberate application of a classical force on it, can reduce back the phase uncertainty after the interaction with the environment is over. This statement can be proven also from the point of view of the change the interfering wave induces in its environment. This proof follows simply from unitarity. The scalar product of two states that evolve in time under the same Hamiltonian does not change in time. Therefore, if the state of the system (electron plus environment) after the electron-environment interaction took place is $|r(t)\rangle \otimes |\chi_{env}^{(1)}(t)\rangle + |\ell(t)\rangle \otimes |\chi_{env}^{(2)}(t)\rangle$, then the scalar product $\langle \chi_{env}^{(1)}(t)|\chi_{env}^{(2)}(t)\rangle$ does not change with time. The only way to change it is by another interaction of the electron with the same environment. Such an interaction keeps the product $\langle \chi_{env}^{(1)}(t)|\chi_{env}^{(2)}(t)\rangle \otimes \langle r(t)|\ell(t)\rangle$ constant, but changes $\langle \chi_{env}^{(1)}(t)|\chi_{env}^{(2)}(t)\rangle$. The interference will be retrieved only if the orthogonality is transferred from the environment wave function to the electronic wave functions which are not traced on in the experiment. The above discussion will be quite relevant for one of the aspects of the dephasing problem treated in the next section.

So far we were concerned with the phase $\phi = \phi_r$, accumulated by the right hand path only. The left hand path accumulates similarly a phase $\phi_\ell$ from the interaction with the environment. The interference pattern is governed by the relative phase $\phi_r - \phi_\ell$, and it is the uncertainty in that phase which determines the loss of quantum interference. This uncertainty is always smaller than, or equal to, the sum of uncertainties in the two partial waves’ phases. The case of noncommuting phases will not be discussed here.

Often the same environment interacts with the two interfering waves. An interesting case is when both waves emit the same excitation of the medium. This radiation makes each of
the partial waves’ phases uncertain, but does not alter the relative phase. A well-known example is that of "coherent inelastic neutron scattering" in crystals (see e.g. ref. [19]). This process follows from the coherent addition of the amplitudes for the processes in which the neutron exchanges the same phonon with all scatterers in the crystal.

The last example demonstrates that an exchange of energy is not a sufficient condition for dephasing. It is also not a necessary condition for dephasing. What is important is that the two partial waves flip the environment to orthogonal states. It does not matter in principle that these states are degenerate. Simple examples were given by ref. [18]. Thus, it must be emphasized that, for example, long-wave excitations (phonons, photons) usually do not dephase the interference. But that is not because of their low energy but rather because they do not influence the relative phase of the paths. An equivalent way to state this (see refs. [17,18] is that, as in the Heisenberg microscope, the radiation with wavelength $\lambda$ can not resolve the two paths if their separation is smaller than $\lambda$. An example where that latter effect is crucial will be mentioned below.

We emphasize that dephasing may occur by coupling to a discrete or a continuous environment. In the former case the interfering particle is more likely to “reabsorb” the excitation and “reset” the phase. In the latter case, the excitation may move away to infinity and the loss of phase can usually be regarded as, practically speaking, irreversible. The latter case is that of an effective “bath” and there are no subtleties with the definition of $\phi$ since eq. 8 may be assumed. We point out that in special cases it is possible, even in the continuum case, to have a finite probability to reabsorb the created excitation and thus retain coherence. This happens, for example, in a quantum interference model due to Holstein for the Hall effect in insulators.

Our discussion shows that the dephasing physics is fully understood with all its subtleties and there is really no room for further semiphilosophical discussions. The work in mesoscopic physics has provided quantitative examinations of this. In many typical solid-state situations one is interested in the dephasing of, for example, electrons, just above the Fermi energy, performing diffusive motion due to defects, and interacting strongly via the Coulomb
interaction with all the other electrons. This electron-electron interaction provides in most cases the dominant dephasing mechanism. It is easy then to obtain the dephasing rate from the strength of the inelastic scattering of the considered electron by the electron sea, i.e.

using the trace left in the environment. A straightforward semiclassical calculation gives for a three dimensional diffusive sample [18]:

$$\tau^{-1}_{ee} = \frac{2\pi}{\hbar \Delta} \int_0^{\omega_{max}} d\omega \int \frac{d^3q}{(2\pi)^3} V_{Coul} \text{Im} \left( \frac{1}{\epsilon(q, \omega)} \right) \langle \rho^2_q \rangle \omega \left[ \coth \frac{\omega}{2T} - \tanh \frac{\omega - E}{2T} \right]$$  \hspace{1cm} (11)

In this equation, $V_{Coul} \equiv \frac{4\pi e^2}{q^2}$ is the Coulomb potential, $\epsilon(q, \omega)$ is the dielectric constant, $\langle \rho^2_q \rangle \omega$ is the matrix element squared of the density operator $\rho_q$ between two states with an energy difference $\omega$, averaged over disorder configurations (it is approximately given by $|\langle m | e^{iq \cdot r} | n \rangle|_{av}^2 = \frac{1}{\pi \hbar N(0)} \text{Re} \left[ \frac{1}{\omega + Dq^2} \right]$). Here $N(0)$ is the density of states at the Fermi energy and $E$ is the energy of the electron, measured relative to the Fermi energy. For a system in the linear response regime, the interesting case is that of $E \approx T$. Eq. (11) can be viewed as a first-order perturbation theory contribution to the imaginary part of the electron’s self energy, where the perturbation is the complex potential $\frac{4\pi e^2}{q^2 \epsilon(q, \omega)}$. For conductors, $\text{Im} \left( \frac{1}{\epsilon(q, \omega)} \right) \approx \frac{\omega}{4\pi \sigma}$. Eq. (11) was first obtained in ref [20] by using the effect of the electromagnetic fluctuations due to the electron gas on the considered electron. The equivalence of these two points of view is guaranteed by the fluctuation-dissipation theorem.

For low dimensions (thin films and wires) the integrations over the appropriate components of $q$ are replaced by summations and it is found that the remaining integrations are infrared (small $q$)-divergent. A careful evaluation of the phase difference of two paths shows that this divergence is cured by a cutoff whose physical meaning is exactly that low $q$ excitation can not distinguish paths that are separated in space by less than $1/q$. This is in agreement with the ”Heisenberg microscope”-type argument [17,18] mentioned above. Once that is done, the results are in a quantitative agreement with experiments.

Actually, the above is easily generalizable for the dephasing by the interaction with any system whose response function $\text{Im} \left( \frac{1}{\epsilon(q, \omega)} \right)$ is known. In that sense, the physics of dephasing is understood in principle. The case where the ”environment” is not at equilibrium is of
fundamental interest since the fluctuations are not given by their well-known equilibrium values and the fluctuation-dissipation theorem is applicable only for the linear transport. An example is provided in the next section.

III. DEPHASING BY A CURRENT-CARRYING QUANTUM DETECTOR

The quantum point contact, alluded to in section I, can be a sensitive detector when biased to be on the transition between two quantized plateaus. It is then rather sensitive to small changes in parameters, for example in the electrostatic field nearby. This sensitivity may be used to detect the presence of an electron in one of the arms of an interferometer, provided the two arms are placed asymmetrically with respect to the QPC. Following discussions by Gurvitz, Buks et al. performed measurements confirming "which path" detection by the QPC. The AB oscillations were measured in a ring. On one of its arms the transmission was limited by a "quantum dot" where the electron wave would resonate for a relatively long and controllable (to a degree) dwell time $\tau_d$. A QPC was placed near that arm and the degree of dephasing due to it (determined by $\tau_\phi/\tau_d$) could be inferred from the strength of the AB conductance oscillations.
FIG. 2. (right) A schematic view of the device used in ref. [23]: Electrons in the 2D gas (B) pass from the emitter (E) to the collector (C), constrained by the reflectors (bordered empty regions). The set of gates (dark regions) deplete the electrons below them and define the AB ring and the quantum dot on its right arm. Near and on the right of the latter, the conductance ($I_D/\mu_D$) of the QPC is measured by the circuit shown.

(left) An SEM micrograph of the device. The gray areas are the gates and reflectors. An "air bridge" biases the central gate which controls the hole of the ring.

Clearly, a necessary and sufficient condition for dephasing is that $\tau_\phi \ll \tau_d$. The results were in good agreement with the theory developed by the authors [23] and by Levinson [24] and, independently, by Aleiner et al [25]. We review below the main features of the theory for this dephasing, emphasizing the aspects that have general basic relevance. The new interesting feature of this nonequilibrium dephasing is that a finite current is flowing in the detector and, with increasing time, each electron transmitted there adds its contribution to the decrease of the overlap, eq.3. As discussed in the previous section, the reduction in overlap is conserved when further thermalization of the transferred electrons in the downstream reservoir occurs. This is ensured by unitarity, as long as no further interaction with the
an interfering electron takes place. The alternative picture is that nonequilibrium (shot-noise) fluctuations of the current in the QPC create a phase uncertainty for the electron in the quantum dot. While the equivalence of these two pictures is guaranteed by the discussion of the previous section (ref. [18]) it is interesting and nontrivial to see how it emerges in detail, as we shall demonstrate. This is all the more interesting, since the former point of view by which the current fluctuations in the QPC cause dephasing, seems superficially to contradict the idea behind eq. [12]. According to the latter, dephasing appears to have to overcome the shot-noise fluctuations, which therefore may be thought to oppose dephasing.

In the model considered, the existence of an electron in the quantum dot is taken to change the transmission and reflection amplitudes of the QPC from $t$ and $r$ to $t + \Delta t$ and $r + \Delta r$. This changes the transmission coefficient from $T$ to $T + \Delta T$ and the conductance by $e^2/\pi \hbar \Delta T$. For simplicity the QPC is taken as single-channel and symmetric and we consider only zero temperature (which means practically that the temperature is much smaller than the voltage $V$ on the QPC). $\tau_\phi$ can be physically estimated from the condition that the change in the number of electrons, $\langle N \rangle = (I/e)\tau_\phi$, streaming across the QPC within $\tau_\phi$, $\langle \Delta N \rangle = e \pi \hbar V \Delta T \tau_\phi$ be larger than the rms fluctuations of $N$ during the same time. For the latter one has the quantum shot-noise result [7] according to which the mean-square fluctuation $\langle (\Delta N)^2 \rangle$ is given by $(I/e)\tau_\phi(1 - T)$ (see fig.[8], where the proportionality to $T(1 - T)$ is what generates the peaks in the noise power). Thus:

$$\frac{1}{\tau_\phi} \sim \frac{e}{\pi \hbar} \frac{(\Delta T)^2 V}{T(1 - T)}$$  \hspace{1cm} (12)

Below we describe three derivations, whose equivalence is guaranteed by the discussion of section [11] of the precise result corresponding to the estimate in eq.[12].

The first two derivations consider the overlap of the states of the environment which are influenced by the partial waves going through the two arms of the ring, as in eq.[3]. Ref [23] evaluated $\int d\eta \chi_\ell^*(\eta)\chi_\nu(\eta)$ where $\chi_\ell$ and $\chi_\nu$ are the states of the QPC with the electron partial wave inside or outside the quantum dot (i.e. with and without the changes $\Delta t$ and $\Delta r$ respectively). The relevant single-electron states of the QPC are the $N(0)eV$ "scattering"
states generated by the LHS reservoir. For each such outgoing state the above overlap is easily seen to be given by \( O_1 = r^*(r + \Delta r) + t^*(t + \Delta t) \). For a symmetric "barrier" at the QPC, one may write the coefficients as:

\[
\begin{align*}
    r &= isin\theta e^{i\phi}; \\
    t &= cos\theta e^{i\phi}.
\end{align*}
\] (13)

where \( \theta \) and \( \phi \) are real. \( \Delta r \) and \( \Delta t \) are then parametrized in terms of \( \Delta \theta \) and \( \Delta \phi \) and the above overlap is given by:

\[
O_1 = cos(\Delta \theta) \cong 1.
\] (14)

In a unit time and for a length \( L/2 \) of the QPC arm, there are \( 2v_F/L \) attempts to go through the QPC in each of the above states. Thus the total rate of attempts, \( \nu \), remembering that \( N(0) = \frac{L}{2\pi \hbar v_F} \), is given by:

\[
\nu = \frac{eV}{\pi \hbar}.
\] (15)

Thus the rate of decrease of the total overlap, i.e. the dephasing rate \( 1/\tau_\phi \), is \( \nu \) times the single-state overlap \( O_1 \). Since, from eq.13, \((\Delta T)^2 = 4(1 - T)T(\Delta \theta)^2 \), one obtains:

\[
\frac{1}{\tau_\phi} = \frac{eV(\Delta T)^2}{8\pi \hbar \bar{v}T(1 - T)},
\] (16)

in agreement with the estimate of eq.12.

The above derivation [23] used the overlap of the populated states in the QPC as modified by the interaction. A direct way to obtain \( \frac{1}{\tau_\phi} \) is to calculate the rate for real transitions induced in the QPC by the same interaction. This is what was essentially done in ref. [25]. Each of the \( N(0)eV \) states populated by the higher bias (LHS) reservoir can decay into the unpopulated states emanating from the lower bias (RHS) reservoir. In the simplest model, the conductance of the single-channel QPC, of total length \( L \), is determined by a \( \delta \)-function potential \( v_\delta(x) \). The transmission amplitude of this potential is well-known to be \( e^{-z} \). The dimensionless parameter \( z \) is defined by \( z = \frac{v}{\hbar v_F} \). The existence of the electron in the QD changes \( v \) to \( v + \delta v \). This changes the transmission coefficient of the QPC by:

\[
\delta T = -\frac{2}{\hbar v_F} T^{3/2} \sqrt{1 - T} \delta v.
\]
It is easy to see that the matrix element of the perturbation $\delta v \delta(x)$ for such a transition is
\[ \frac{\delta v}{2L} (t + t^*) = \frac{T \delta v}{L}. \]

Evaluating this transition rate by the golden rule (but remembering that the spin in unflipped in the transition) and multiplying by $N(0)eV$, the number of independently decaying states, we arrive at the same result for the dephasing rate, as in eq.16. For the quantitative comparison, it has to be kept in mind that the rate for real transitions is twice the rate of decrease of the overlap with the initial state.

The reason that the above two derivations give the same results (as was explicitly checked above) is that they evaluate the same overlap. One by using the states as modified by the interaction, the other by evaluating the "real" transition rate within the unpertubed states. As discussed in the previous section, once the electron wave is out of the quantum dot and continues to diffuse along the interferometer arm with a much weaker interaction with the QPC, the dephasing can not be reset. This is guaranteed by unitarity, unless further interaction of the electron wave with the detector will undo the change of state of the latter. Before discussing below the third derivation [24], which uses the second point of view of section II, we emphasize that we considered here only the nonequilibrium part of the dephasing (due to the finite V). When the detector is in equilibrium at finite temperatures, the usual dephasing may occur. But at zero temperature and for an infinitesimal energy of the electron wave above the Fermi energy, no equilibrium dephasing takes place in usual circumstances. Ref [24] pointed out that the polarization of the electron gas in the detector by the electron in the quantum dot causes a reduced overlap as in eq.3. This is related to the "orthogonalization catastrophe"-type many-body effect. According to ref [24] this reduction of the overlap can also exist in the present case at zero temperature and voltage, due to the "Coulomb blockade" situation: The transmission through the dot is maximized by a gate voltage which compensates for the polarization energy of the electron gas.

The derivation of ref [24] utilizes the fluctuations of the phase of the electron wave in the quantum dot due to the interaction with the QPC. Eq.9 above expresses this phase
uncertainty as an integral of the correlation function of this interaction. $V_I(t)$ equals the lowest-order modulation $W(t)$ of the energy of the electron in the quantum dot by the interaction with the QPC. Thus, the phase fluctuations increases with time like the integrated correlator, $K(t)$, of $W(t)$ (as in eq.[9]). It was further noted in ref. [24], that if $[W(t),W(t')] \neq 0$, then $K(t)$ should be taken as the symmetrized (anticommutator) correlator. When the correlation time of $K(t)$, $\tau_c$, is short (as is the case here, for further consequences, see below), one finds:

\[
\frac{1}{\tau_\phi} = \frac{1}{2} \int_{-\infty}^{\infty} K(t) dt = \pi S(0),
\]

where $S(\omega)$ is the power spectrum of the fluctuations of $W(t)$, i.e. the Fourier transform of $K(t)$, $S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} K(t) \exp(i\omega t) dt$.

A fundamentally interesting result of ref. [24] is that $\tau_\phi$ also plays the role of the decay time of the average $\hat{c}(t)$, where $\hat{c}$ is the annihilation operator of an electron on the quantum dot. This decay was shown not to be related to that of the energy or the electron density of the dot. It really reflects the decay of the coherence of the state on the dot.

Since the electron on the quantum dot couples to the QPC by changing its $r$ and $t$, it turns out that $K(t)$ is proportional to the combination $|r^* \Delta t + t^* \Delta r|^2$ (which is approximately equal to $(\Delta \theta)^2$ by virtue of eq.[13]). A straightforward calculation yields for the nonequilibrium dephasing ($eV \gg k_B T$):

\[
\frac{1}{\tau_\phi} = \frac{(\Delta \theta)^2}{4\pi \hbar^2} 2eV.
\]

Using $(\Delta T)^2 = 4T(1-T)(\Delta \theta)^2$, which follows from eq.[13], this is seen to be the same as eq.[16].

The correlation time of $K(t)$ is of the order of $\hbar/(eV) \ll \tau_\phi$. This justifies the assumption made in order to get eq.[17]. The physical reason for the dephasing being much slower than the correlation time of the fluctuations is precisely [24] the motional narrowing obtained self-consistently from the same inequality.

Of course, for the QPC to dephase the interference, the dwell time $\tau_d$ in the quantum dot has to be comparable to or larger than $\tau_\phi$. The experiments of ref. [23] agree better than
qualitatively with the above picture. For QPC voltages larger than thermal, the visibility of the AB interference contribution to the conductance of the ring decreased roughly linearly in $V$ and the coefficient was in reasonable agreement with the above. The parameter $\Delta T$ was directly measured and the dependence on $T$ was qualitatively observed as well.

**IV. PARAMAGNETIC ORBITAL RESPONSE OF ELECTRONS IN PROXIMITY TO A SUPERCONDUCTOR**

The orbital magnetic response of conduction electrons is one of the oldest problems in condensed-matter and statistical physics. The Bohr-van Leeuwen theorem ensures that this is a purely quantum phenomenon. The persistent currents discussed in section I and the Landau diamagnetism are two examples of this response in normal conductors. This response is ordinarily very small, due to the almost complete cancellation between the diamagnetic and paramagnetic contributions (which must cancel exactly in the classical limit). In superconductors, where the electrons condense into a single macroscopic quantum state, the diamagnetic orbital response has the largest magnitude possible (larger by at least several orders of magnitude than normal persistent currents). The intermediate situations between "normal" and "super" are therefore of interest.

The magnetic response of a normal layer (N) coating a superconducting cylinder (S) is a good example in this connection. The diamagnetic response of the normal layer (proximity effect) is related to the formation of Andreev levels. It turns out that usually these levels, in which the electron is retroreflected from the superconductor at low energies as a time-reversed hole, do not have any paramagnetic contribution. This is the origin of the induced diamagnetism in the proximity layer. At low energies, the density of such states goes linearly to zero, which enhances the various scattering mean-free paths in the Born approximation. In particular, the low-energy glancing states (see fig.3), which can skip along the outer boundary without hitting the superconductor, can have relatively large magnetic moments that lead to a significant low-temperature paramagnetic correction to the Meissner result.
V. CONCLUDING REMARKS

It is hoped that the topics discussed in this paper demonstrate the value of considering concrete examples where basic quantum issues can be addressed by specific equations and the results eventually compared with experiment. Mesoscopic Physics offers an ideal arena for this type of demystification. In particular, no philosophical discussions of dephasing and "which path" detection are necessary. Detection happens due to well-defined processes in the detector (which is quantum mechanical by itself). There is absolutely no need, at least in this example, to resort to classical observers who will look at the data and further influence the results thereby. It is worth pointing out that in the solid state experiments considered here, as well as in many photon experiments, quantities averaged over many interfering entities are considered. The probabilistic aspects are included. What happens in an experiment with a single electron does not have to be considered in this type of discussion. Even for quantities such as the shot noise (which is the fluctuations around the average) one considers, possibly time-dependent, correlation functions, which again are averages in the above sense.

In section IV a new and interesting situation for orbital magnetism is reviewed. In
a system consisting of superconducting and normal components, strong correlation exist between electrons and holes due to Andreev reflection. These lead to the proximity-effect diamagnetism. The same correlations decrease the low-energy density of states of these Andreev states. This, in turn, stabilizes the special ”whispering gallery” modes, which at low temperatures give a surprisingly large paramagnetic orbital response.

The correlation (alias entanglement) due to interactions and statistics among particles is a ubiquitous and a very important element in condensed matter (as well as in atomic and molecular) systems. Its study is very nontrivial and extremely relevant for Mesoscopic Physics. This is one of the issues remaining in the study of the crossover between microscopic and macroscopic behavior.

Acknowledgements

The research reported here was supported by grants from the German-Israel Foundation (GIF) and the Israel Science Foundation, Jerusalem. The author would like to thank Y. Aharonov, C. Bruder and A. Stern for collaborations on these problems. Ana-Celia Mota and Eyal Buks are thanked for instructive discussions on the experiments of Refs. [27] and [23]. I. L. Aleiner, A. Altland, N. Argaman, C. W. J. Beenakker, W. Belzig, M. Berry, E. Buks, A. Fauchère, Y. Gefen, B. I. Halperin, D.E. Khmelnitskii, A. Krichevsky, R. Landauer, Y. Levinson, Y. Meir, M. Schechter, G. Schön, T.D. Schultz, A. Stern, C. Urbina, and A. Zaikin are thanked for discussions on aspects of the theory. E. Buks and M. Reznikov are thanked for permission to use their figures.

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