Performance of 2-Link Robot by utilizing Adaptive Sliding Mode Controller

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ABSTRACT

The Sliding Mode Control (SMC) has been among powerful control techniques increasingly. Much attention is paid to both theoretical and practical aspects of disciplines due to their distinctive characteristics such as insensitivity to bounded matched uncertainties, reduction of the order of sliding equations of motion, decoupling mechanical systems design. In the current study, two-link robot performance in the Classical SMC is enhanced via Adaptive Sliding Mode Controller (ASMC) despite uncertainty, external disturbance, and coulomb friction. The key idea is abstracted as follows: switching gains are depressed to the low allowable values, resulting in decreased chattering motion and control's efforts of the two-link robot system. Un-known uncertainty bounded and reducing switching gains can be considered major advantages of ASMC leading to outperform ASMC upon CSMC. Simulink MATLAB 2019a was used to obtain the simulation outcomes. The outcomes have shown that both methodologies had good tracking performance to the desired position and made the system asymptotically stable through the steady-state errors investigate approaching zero. ASMC is better than CSMC illustrated by minimizing gains values, control efforts, and chattering for each link.

Keywords: Classical Sliding Mode Controller, Adaptive Sliding Mode Controller, signum function, saturation function, chattering.
1. INTRODUCTION

Robotics is a relatively new field in digital technology, breaking the limits in conventional engineering. Understanding the function of robots and their implementations include knowledge of electrical engineering, mechanical engineering, manufacturing or industrial engineering, informatics, economy, and math. New engineering fields, such as systems engineering, application engineering, and software engineering, have arisen to address the complexity of the robotics sector (Badoniya and George, 2018). The robotic arm is widely used in the industry. Man-handling in so many field applications where analytical services are required is either dangerous or not. Two or maybe more arm-manipulators are commonly used in such circumstances. Some robots are used for inspecting and removing dangerous areas and/or destroying explosive devices. These robots will be used to make some passageways via the mined battlefields, intact ammunition manipulation and neutralization, vehicle inspection, trains, aircraft, and buildings (Patic and Gorghiu, 2009). The robot manipulator can be considered a very high non-linearity system besides a perturbation term that includes parameter uncertainty, disturbances, and friction. There are several techniques for controlling the robot manipulators performance. The set of specific controllers extended from linear to nonlinear, to other non-classical, nonlinear, adaptive non-classical, and nonlinear controllers (Pilan and Sulaiman, 2012).

Control of the sliding mode has proved to be a reliable and efficient control approach for nonlinear systems. However, this strategy usually requires high gains to reach the sliding surface in a relatively "short" time for any trajectory. Researchers have been trying to redefine the sliding surfaces optimally to reduce high energy consumption and/or attain faster reach time (Nikkhah et al., 2006). (Jamshed and Dad, 2017), designed sliding mode on a 2-link robotic manipulator. A robust Sliding Mode Control (SMC) is used to track the desired trajectory only without regard to the specified coulomb type and the control action. (Sareena and Mathew, 2019), proposed to compare the performance of a two-link robot by utilizing SMC and PID. The two link robot equations are being used without disturbances and uncertainty of parameters in the study. Comparative research is completed, and the results show that the control of sliding mode exceeds the control of PID with a minimal tracking error. (Al-Samarraie and Salih, 2017), proposed to design ASMC for a two-link robot with unknown dead-zone and LuGre friction. The proposed method reduces the switching gain and control effort to low possible value to give good performance tracking for the desired position. (Ibrahim and Sharkawy, 2018), proposed to build SMC and Adaptive PID controller for a two-link robot manipulator. APID comprises two PID controllers. The first PID forces tie is to follow the desired trajectory, and the second PID is to compensate for the error and variable uncertainty. In the APID contrast, SMC outperforms in reducing error. (Wang and Zhang, 2018) proposed to design SMC with adaptive fuzzy for 2-link manipulator. The stability and consistency of the entire closed-loop system are assured by an
adaptive weight value modified. This study is constructed to good tracking between position and desired position, so the chattering is still appearing in the simulation results caused by signum function in SMC. (Baek and Kwon, 2020) proposed to built SS-ASMC for a two-link robot to reduce the error in tracking the desired position in the past study (Baek et al., 2016). SS-ASMC method utilizing a collection from TDE and PPM. This method is centered on the switching gain value to reduce the error to very little value. The obstacle of this way is very high gain value in the presence of TDE, and the chattering isn't solved in this study.

The current work discusses applying full phases of the sliding mode of CSMC and ASMC for a high nonlinear two-link robot with the presence of disturbance and coulomb friction and uncertainty of parameters. So robust performance, good tracking for the desired position, and attenuate the switching gains that reduce the oscillation motion in the torque actions are achieved. In Section 2, the mathematical model of the two link robot arm is presented. Then, the sliding mode controllers' techniques are illustrated in Sections 3. Simulation results and discussion is presented in Section 4. Finally, in Section and conclusions are presented.

2. MATHEMATICAL MODEL OF 2-LINKS ROBOTIC MANIPULATOR

2.1 The 2-Link Robot Description

Robotics is concerned with the analysis of those devices that can replace humans in implementing a job, both in terms of physical activity and in terms of determination. A list of the most common mechanical systems is classified such as robot manipulators and mobile robots (Siciliano et al., 2009), (Kelly et al., 2005), and (Craig, 2005).

In robotics, the torque (τ) force is needed to give the position (θ) and velocity (θ̇) and make tracking of actual position to the desired position. The torque is considered input and velocity and position are considered output as Fig. (1).

![Figure 1. Representation of a robot at input-output (Kelly et al., 2005).](image)

The 2-link robot manipulator is presented in Fig. (2) with the following descriptions in Oxy coordinates.
2.2 Dynamic

The dynamic formula can be expressed as below for a 2-link robotic manipulator (Shafeek, 2007) and (Sareena and Rikesh, 2019):

\[ M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) = \tau \]  

(1)

\( \theta, \dot{\theta}, \ddot{\theta} \) are defined as 2×1 vectors of joint angular position, velocity, and acceleration. \( \tau \) is presented as a 2×1 vector of torque. \( M(\theta) \) presented a 2×2 matrices of inertia. \( C(\theta, \dot{\theta}) \) is a 2×2 matrix of Coriolis and centrifugal forces. \( G(\theta) \) is a gravity vector and it is a 2×1 matrix.

The Parameter Descriptions in Eq. (1) are as below:

1: \( M(\theta) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \)

Where,

\[ M_{11} = (m_1 + m_2) L_1^2 + m_2 L_2^2 + 2m_2 L_1 L_2 \cos(\theta_2) \]
\[ M_{12} = M_{21} = m_2 L_2^2 + 2m_2 L_1 L_2 \cos(\theta_2), \text{and} \]
\[ M_{22} = m_2 L_2^2 \]

2: \[ C(\theta, \dot{\theta}) \dot{\theta} = \begin{bmatrix} m_2 L_1 L_2 \sin(\theta_2) \dot{\theta}_2^2 - 2m_2 L_1 L_2 \sin(\theta_2) \dot{\theta}_1 \dot{\theta}_2 \\ m_2 L_1 L_2 \sin(\theta_2) \dot{\theta}_2^2 \end{bmatrix} \]

3: \[ G(\theta) = \begin{bmatrix} m_2 L_2 \cos(\theta_1 + \theta_2) + (\theta_1 + \theta_2) L_1 \cos(\theta_1) \\ m_2 L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \]

4: \[ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \]
This study presumes that the following statement for the actual position is:

\[
\begin{align*}
\theta_1 &= x_1 + \theta_{1d} \\
\theta_2 &= x_2 + \theta_{2d}
\end{align*}
\]  

(2)

For joint-1 and joint-2, respectively, the optimal angles are \(\theta_{1d}\) and \(\theta_{2d}\).

The Model Robot can be rewritten as follows:

\[
\begin{align*}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= -M(\theta)^{-1}(C(\theta, \dot{\theta})\dot{\theta} + G(\theta) + \tau + \delta(x, u))
\end{align*}
\]  

(3)

Eq. (3) should be rephrased to.

\[
\begin{align*}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= F + w u + \delta
\end{align*}
\]  

(4)

The symbols Eq. (4) can be identified as the following equations

\[
\begin{align*}
X1 &= \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, & X2 &= \begin{bmatrix} X_3 \\ X_4 \end{bmatrix}, \\
F &= \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -M(\theta)^{-1}(C(\theta, \dot{\theta})\dot{\theta} + G(\theta)) \\
u &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = M(\theta)^{-1}\tau \\
\delta &= \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \Delta F + \Delta w u + Fc + D(t)
\end{align*}
\]  

(5)  (6)  (7)  (8)

\(\Delta F = 20\%\) \(F\), and \(\Delta w = 10\%\) \(w\) are uncertain parameters for variables \(F\) and \(w\).

In both joints, Coulomb friction is \(Fc=\begin{bmatrix} Fc1 \\ Fc2 \end{bmatrix}\) and \(D(t) = \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}\) indicates the external disturbances.

The set of nonlinear equations which distinguish the system's action are, therefore:

\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= F_1 + u_1 + \delta_1 \\
\dot{x}_4 &= F_2 + u_2 + \delta_2
\end{align*}
\]  

(9)

Where, \(\delta_1\) and \(\delta_2\) are the terminology for Joint 1 and 2 perturbations, respectively.

3. THE CONFIGURATION OF THE SLIDING MODE CONTROL

The study history of Sliding Mode control theory is quite long and distinguishes, which stretches back to the structure and equilibrium analyzes of the nineteenth century. It started up in the late 1950s as an engineering discipline. Nyquist, Bode, Evan, and Wiener were the leaders for dynamic analysis and controller synthesis operating in the frequency domain. The early design of the control system techniques served the cause of automation. However, reliability, technological complexity as dynamics became complex, and many other associated drawbacks were still open research problems for the control system world (Khan, 2016).
SMC’s basic concept is to apply sliding modes in a predetermined variety. They were recognized as sliding manifold, sliding surface, hyperplane, or switching line, in state space of a given system, with a discontinuous function (switching) Controller (Khan, 2016). SMC happens in two stages. The duration when trajectories of the state of the system are pushed from an initial position to a pre-specified sliding manifold is known as the reaching phase. A special form of device motion/trajectories, known as the sliding phase, follows the reaching phase's completion. During this step, the system's states' trajectories are limited to remaining on the sliding manifold. The equilibrium (the origin) can be slid along the surface, as shown in Fig. (3).

SMC architecture involves two main phases, i.e., the layout of the sliding surface in compliance with the optimal output of the closed-loop and design of acceptable control law (Utkin, et al., 2009), (Dereje, 2018) and (Do, 2014).

3.1 CSMC design

Conventional SMC architecture is composed of the following (Salih, 2016), (AL-Samarraie, 2011) and (Do, 2014):

- Construct of an appropriate switching surface, which gives the plant what it wants Achievement.
- Implementation of a discontinuous control rule that forces on the switching surface the plant's trajectories and holds them there provided that sliding mode exists for all time or overtime.

So, the question of control in seeking a sliding surface and a rule of control is abstracted.

In practical sliding mode control applications, engineers may undergo the unwanted occurrence of finite-frequency and amplitude oscillations, which is named 'chattering' as Fig. (4). The chattering is a significant obstacle to its application in the first stage of developing the sliding mode control theory. Chattering is dangerous since it results in low control precision, high wear of mechanical moving parts, and large thermal losses in electrical devices (Utkin, 2006) and (Khan, 2018).

![Figure 3. The perfect two-phase sliding mode (Brandstädter, 2009).](image-url)
The boundary layer (saturation function) is solved this undesired motion in the control action or sliding variable, as in Fig. (5).

\[
\begin{align*}
  u &= u_{dis} = -k(x) \text{ sign}(s) \\
  s &= \lambda e + \dot{e} = 0
\end{align*}
\]

Let \( \lambda = 10 \), \( x_1 = e \) and \( x_2 = \dot{e} \), so the sliding surface can be rephrased as

\[
S = 10 \ x_1 + x_2 = 0
\]

Where, \( \text{sign}(s) = \begin{cases} 
1 & \text{if } s > 0 \\
-1 & \text{if } s < 0 \\
\epsilon[-1.1] & \text{if } s = 0
\end{cases} \)  

This equation is adjusted by using the boundary layer to suppress chattering in the control operation instead of the signum function.

\[
u = u_{dis} = -k(x) \text{ sat}(s)
\]
Sat (s) function is given as Eq. (15) (Utkin, et al., 2009) and (Hamoudi, 2014):

\[
sat(s, \varphi) = \begin{cases} 
    \text{sign}(s) & \text{if } |s| > \varphi \\
    \frac{s}{\varphi} & \text{if } |s| \leq \varphi
\end{cases}
\] (15)

where \( \varphi > 0 \), is identified as the diameter of the boundary layer.

The surface to slide can be described as follows:

\[ s_1 = \lambda x_1 + x_3 \] (16)
\[ s_2 = \lambda x_2 + x_4 \] (17)

Where, \( x_1 \) and \( x_2 \) are link 1 and link 2 angular location errors and \( x_3 \) and \( x_4 \) are link 1 and link 2 angular velocity errors, respectively.

Let \( \lambda = 10 \), then Eq. (15) and (16) is going to be rewritten as below:

\[ s_1 = 10 x_1 + x_3 \] (18)
\[ s_2 = 10 x_2 + x_4 \] (19)

The gain \( k(x) \) is calculated by utilizing the way as explained in Appendix A, in a general way.

\[ \dot{s} < 0 \] (20)

Where, \( s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \)

By utilizing Eq. (12) in Eq. (20).

\[ 10 \dot{x}_1 + \dot{x}_2 < 0 \]

By utilizing Eq. (4) and (8).

\[ w k(x) > |\delta(x, u)| \]
\[ k(x) = k_0 + \frac{\Delta F + D}{(w - \Delta w)} \] (21)

Where, \( k_0 > 0 \)

\[ k(x) = \begin{bmatrix} k_1(x) \\ k_2(x) \end{bmatrix} \] (22)

\( K_1(x) \) is a control action gain for link 1, and \( K_2(x) \) is a control action gain for link 2.

If the gain values of each link are found and substituted in Eq. (14), then, as below, torques are:

\[ \tau_1 = M_{11} u_1 + M_{12} u_2 \] (23)
\[ \tau_2 = M_{21} u_1 + M_{22} u_2 \] (24)
3.2 ASMC design

The biggest challenges to applying Sliding Mode Control are two interrelated phenomena: chatter and high control action activity. The chattering amplitude is well known to be proportional to the magnitude of a discontinuous total control. These two problems can be dealt with at the same time if the magnitude is decreased to a minimum permissible level represented by the conditions for the existing Sliding Mode (Mandal, 2013), (Larguech, et al., 2013) and (Utkin and Poznyak, 2013). In the current study, ASMC is proposed to deal with the two issues mentioned above. ASMC has suppressed the switching gain to low gains. When the gains are decreased, control actions are minimized, which causes attenuating the chattering leading to good tracking performance, which decreases thermal and mechanical losses of systems.

The SMC with the presence of the disturbance term shall be given by (Salih, 2016) and (AL-Samarraei, and Salih, 2017):

\[ u = -k(t) \text{sign}(s) \]  \hspace{1cm} (25)

Where, \( u \ (s, x) \) is the control action to be configured, and \( k(t) \) is the adaptive controller gain that is described as the following:

\[ k(t) = \begin{bmatrix} k_1(t) \\ k_2(t) \end{bmatrix} \]  \hspace{1cm} (26)

Where, \( s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \) is the sliding variable, and \( \text{sign}(s) \) is the signum function which is described in Eq. (13).

Quantify the adaptive controller gain can as (Bandyopadhyay, et al., 2013):

\[ \dot{\mu} = \{ \rho \ |s(x,t)| \text{sign}(|s(x,t)| - \epsilon) \]  \hspace{1cm} (27)

Where, \( \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \), \( \rho > 0 \) and \( \epsilon > 0 \)

and \( k(t) \) is chosen depending on the next rules.

\[ k = \begin{cases} \mu & \text{if } K_{\text{min}} < \mu < K_{\text{max}} \\ K_{\text{min}} & \text{if } \mu \leq K_{\text{min}} \\ K_{\text{max}} & \text{if } \mu \geq K_{\text{max}} \end{cases} \]  \hspace{1cm} (28)

Where, \( K_{\text{min}} \) is the lowest limit possible value of \( k(t) \), \( K_{\text{max}} \) its upper limit possible value of \( k(t) \), and \( \mu \ (0) \) is the initial point of the gain \( k(t) \).

To give more details and illustrations about adaptive control law and apply conditions of Eq. (28) in the flowchart below.

Finally, the controller law will be updated with saturation function as opposed to signum function, as described:
\[ u = -k(t) \text{sat}(s, \varphi) \]  \hspace{1cm} (29)

As suggested previously, the value of Eq. (29) or Eq. (25) is substituted in Eq. (23) and Eq. (24) to calculate the torque for each link.

![Diagram]

**Figure 6.** The scheme shows ASMC.

### 4. THE SIMULATION RESULTS and DISCUSSION

The tow-link robot is employed to investigate the robustness of the proposed control methods. In the current simulation results, there is a comparison between CSMC, ASMC, and ASMC (Baek and Kwon, 2020), Appendix B to show the effectiveness of the proposed adaptive sliding mode controller upon another two techniques by solving the chattering issues concerned with high control gains and high control efforts.

The system is simplified into four states, which have initials conditions as follow: \( x_1(0) = \frac{\pi}{4} \) (rad), \( x_2(0) = \frac{\pi}{4} \) (rad), \( x_3(0) = 0 \) (rad/sec), and \( x_4(0) = 0 \) (rad/sec) with parameter values are presented in Table 1.
To get the outcomes from ASMC (Baek and Kwon, 2020), Appendix B. The values of other parameters in Eq. (B.14) and Eq. (B.15) are given. \( \sigma_{\text{max}(1,2)} = 2 \times 10^3, \sigma_{\text{min}(1,2)} = 6 \times 10^2, p_{1,2} = 10^4, \mu_{1,2} = 10^4, \alpha_{1,2} = 5 \times 10^{-3}, \epsilon_{1,2} = 2 \times 10^{-7}, \lambda_{1,2} = 10, \delta_{1,2} = 12 \) and \( 9, \Phi_{1,2} = 10^{-1} \) and \( M_{1,2} = 10^{-1} \).

Fig. 7 illustrates the state's trajectory from the initial point to the origin point (desired position) for both links. By applying SMC's reaching condition, these trajectories reach approximately zero and make the system asymptotic stability. Fig. 8 shows the error (rad.) dose not exceed \( 294 \times 10^{-6} \) controlled by proposed ASMC, while \( 583 \times 10^{-6} \) is controlled by ASMC (Baek and Kwon, 2020) for both links simultaneously. Fig. 9 and Fig. 10 show the torques action (N.m) and sliding variables for both links. As clearly, when utilizing CSMC and ASMC (Baek and Kwon, 2020), the torques and sliding variables are still suffering from the chattering despite employing the boundary layers with the same width for three controller methods. Fig. 11 shows the switching gains very high values when utilizing ASMC (Baek and Kwon, 2020) because of the present time delay estimation in this method. CSMC also has a high gain value compared with proposed ASMC because of the known bounded parameter uncertainties. The unsuitable high gain values for CSMC and ASMC (Baek and Kwon, 2020) explain the reasons for the chattering in the torque action Fig. 9 and the sliding variables Fig. 10. Fig. 12 shows that the tracking activity between actual and desired positions does not exceed 0.55 (sec.) for three methods.

Table 2. gives more important details about the comparison between the three controller algorithms; the comparison was built about the steady-state error, chattering magnitude, and gains values.

| parameter | Description                                      | Value (unit) |
|-----------|--------------------------------------------------|--------------|
| L1        | the length of link 1.                            | 0.12 (m)     |
| L2        | the length of link 2.                            | 0.08 (m)     |
| m1        | the mass of link 1                               | 0.01996 (kg) |
| m2        | the mass of link 2                               | 0.0076 (kg)  |
| \( \theta_1 \) desired | Theta desired of link 1                        | \( \frac{\pi}{2} [1 - e(-5t)(5t + 1)] \) (rad.) |
| \( \theta_2 \) desired | Theta desired of link 2                        | \( \frac{\pi}{2} [1 - e(-5t)(5t + 1)] \) (rad.) |
| d1        | Disturbance of link 1                            | 10\times\sin(t) (N.m.) |
| d2        | Disturbance of link 2                            | 10\times\sin(t) (N.m.) |
| \( F_{c1} \) | Coulomb frictions of link 1                     | 0.03 (N.m.)  |
| Fc2       | Coulomb frictions of link 2 | 0.05 (N.m.) |
|-----------|-----------------------------|-------------|
| Φ1, Φ2    | Width of boundary layer     | 0.01, 0.01  |

**Figure 7.**

(a) The phase trajectory between x1 and x3 for link-1.

(b) The phase trajectory between x2 and x4 for link-2.
Figure 8. (a) The error $x_1$(rad.) for link-1, (b) The error $x_2$(rad.) for link-2.
Figure 9. a) The torque action (N.m) for link-1, b) The torque action (N.m) for link-2.
Figure 10. a) The sliding variable for link-1, b) The sliding variable for link-2.
Figure 11. a) The switching gain for link-1, b) The switching gain for link-2.
Figure 12. a) The performance of tracking between both the existing position and desired link-1 position. b) The performance of tracking between both the existing position and desired link-2 position.
Table 2. The Performance of the CSMC and ASMC.

| Controller | Chattering magnitude (N. m) | Gain values $k(t)$ and $k(x)$ | Steady-state error (rad.) |
|------------|-----------------------------|-------------------------------|--------------------------|
|            | Link1 | Link2 | Link1 | Link2 | Link-1 | Link-2 |
| CSMC       | $26 \times 10^{-5}$ | $27 \times 10^{-5}$ | 800   | 802   | $276 \times 10^{-6}$ | $278 \times 10^{-6}$ |
| ASMC       | $\approx 0$   | $\approx 0$   | 750   | 780   | $279 \times 10^{-6}$ | $294 \times 10^{-6}$ |
| ASMC (Baek and Kwon, 2020) | $29 \times 10^{-5}$ | $53 \times 10^{-5}$ | 1818  | 1312  | $583 \times 10^{-6}$ | $407 \times 10^{-6}$ |

5 CONCLUSIONS
An adaptive gain sliding mode controller for the accurate stabilization of a nonlinear two-link robot has been developed in this paper. Firstly, a classical control algorithm for sliding mode is being developed. The corresponding control parameters are then obtained through a dynamic adjustment of the switching gain to achieve rapid system convergence to its optimum level. Outcomes from the simulation are presented to illustrate the effectiveness of the technique proposed.

From Table 2., compared to the other sliding mode control methodologies, the ASMC method performs better in terms of stabilization precision, system control efforts, minimizing chattering, and lower-level gain values than the classic SMC method and ASMC (Baek and Kwon, 2020).

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APPENDIX (A)
The general nonlinear system can be proposed as below:
\[ \begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= f(x) + g(x)u + \delta(x,u)
\end{align*} \]  \hspace{1cm} (A.1)

And the sliding surface equation is written as in Eq. (A.2).
\[ s = \lambda x_1 + x_2, \] \hspace{1cm} (A.2)

The control action for the full SMC is written as.
\[ u = u_{eq} + u_{dis}. \] \hspace{1cm} (A.3)

where \( u_{eq} = \frac{1}{g(x)} (f(x) + \delta(x,u)), u_{dis} = -k(x)\text{sign}(s). \)

The candidate lyapunov function is:
\[ \nu = |s| > 0 \] \hspace{1cm} (A.4)

Where, \( |||s|| = s \text{ sign}(s), \) for \( s \neq 0. \)

Therefore, \( \nu \) can be written by substitute Eq. (A.2) as:
\[ \nu = s \text{ sign}(s) > 0 \] \hspace{1cm} (A.5)
\[ \nu' = s \text{ sign}(s) < 0 \] \hspace{1cm} (A.6)
\[ v = (\lambda x_1 + x_2) \cdot \text{sign}(s) < 0 \]

\[ v = (\lambda x_2 + f(x) + g(x)u + \delta(x, u)) \cdot \text{sign}(s) < 0 \]

\[ v = (\lambda x_2 + f(x) + g(x)(u_{eq} + u_{dis}) + \delta(x, u)) \cdot \text{sign}(s) < 0 \]

\[ v = (\lambda x_2 + f(x) - \lambda x_2 - f(x) - g(x)k(x)\text{sign}(s) + \delta(x, u)) \text{sign}(s) < 0 \]

\[ v = (-g(x)k(x)\text{sign}(s) + \delta(x, u)) \cdot \text{sign}(s) < 0 \]

\[ = -g(x)k(x) + |\delta(x, u)| < 0 \]

\[
K(x) = |\delta(x, u)| / g(x) \quad (A.7)
\]

\[
k(x) = k^0 + \frac{|\delta(x, u)|}{g(x)} \quad (A.8)
\]

Where \( k^0 > 0 \).

**APPENDIX (B)**

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_c = \tau + d \quad (B.1)
\]

Where \( \ddot{q}, \dot{q}, q \) are acceleration, velocity, and position of the robot manipulator. \( \tau \) is a 2\times1 vector of torque, \( M(\theta) \) are a 2\times2 matrices of inertia and \( C(\theta, \dot{\theta}) \) is a 2\times2 matrices of Carioles and centrifugal forces respectively, and finally, \( G(\theta) \) is a gravity vector and it is a 2\times1 matrix. \( F_c \) is a Coulomb friction (2\times1) vector. \( d \) are disturbance (2\times1) vector.

\[
\dot{q} = -M(\theta)^{-1}(C(q, \dot{q})\dot{q} + G(q) + F_c - \tau - d) \quad (B.2)
\]

\[
\dot{q} = -M(q)^{-1}(F + F_c - d) + (-M(q)^{-1} - \bar{M}^{-1})\tau + \bar{M}^{-1} \tau \quad (B.3)
\]

\[
\dot{q} = L + \bar{M}^{-1} \tau \quad (B.4)
\]

Where \( L = -M(q)^{-1}(F + F_c - d) + (-M(q)^{-1} - \bar{M}^{-1})\tau \)

\( \bar{M} = \text{diag}(M_1, M_2, \ldots, M_n) \) is a constant value which is known as TDE gains.

The error \( e = (e_1, e_2, \ldots, e_n) \), \( q = (q_1, q_2, \ldots, q_n) \) and \( qd = (qd_1, qd_2, \ldots, qd_n) \)

\[
e = qd - q . \quad (B.5)
\]

The sliding variable

\[
s = \lambda e + \dot{e} \quad (B.6)
\]

\[
s = (s_1, s_2, \ldots, s_n) \text{ and } \lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \text{ where } \lambda \text{ is sliding gain.}
\]

To find the control action must derive the sliding variable and \( s = 0 \).

\[
\dot{s} = \lambda \dot{e} + \ddot{e} \quad (B.7)
\]

\[
0 = \lambda \dot{e} + (\ddot{q}d - \dddot{q})
\]

By substitute (B.4)

\[
0 = \lambda \dot{e} + (\ddot{q}d - (L + \bar{M}^{-1} \tau))
\]

\[
L = \dddot{q} - \bar{M}^{-1} \tau \quad (B.8)
\]

\[
0 = \lambda \dot{e} + (\ddot{q}d - (\dddot{q} - \bar{M}^{-1} \tau + \bar{M}^{-1} \tau))
\]

\[
\tau = \bar{M}(\lambda \dot{e} + (\dddot{q}d - (\dddot{q} - \bar{M}^{-1} \tau)) + kg s) \quad (B.9)
\]
By substitute (B.6)
\[ \tau = M(\lambda \dot{e} + (\ddot{q} - (\dot{q} - M^{-1} \tau)) + kg(\lambda \dot{e} + \dot{e})) \] (B.10)
\[ \tau = M((\lambda + kg) \dot{e} + (\ddot{q} - (q\ddot{t} - M^{-1} \tau_{t-1}))) + kg \lambda \dot{e} ) \] (B.11)

The total torque action
\[ \tau = \bar{M}((\lambda + kg) \dot{e} + (\ddot{q} - (q\ddot{t} - M^{-1} \tau_{t-1}))) + kg \lambda \dot{e} + \bar{M} K \text{sign}(s) \] (B.12)

To suppress the chattering problem, utilize the boundary layer instead of the signum function.
\[ \tau = \bar{M}((\lambda + kg) \dot{e} + (\ddot{q} - (q\ddot{t} - M^{-1} \tau_{t-1}))) + kg \lambda \dot{e} + \bar{M} K \text{sat}(\frac{s}{\phi}) \] (B.13)

Where \( \phi \) is the width of the boundary layer.
K is adaptive gain is needed to suppress the affecting of TDE.

Parent adaptive law
\[ \dot{k} = \begin{cases} \delta \sigma \frac{|s|}{s} \text{sign}(|s| - \epsilon) & \text{if } k > 0 \text{ and } s \neq 0 \\ \delta \sigma |s| & \text{if } k = 0 \text{ or } s = 0 \end{cases} \] (B.14)

Where \( \sigma \) is the child adaptive law.
\[ \dot{\sigma} = \begin{cases} -\rho |s| & \text{if } |\dot{k}| < \mu \text{ and } \sigma = \sigma_{\text{max}} \\ -\rho |s|a \text{sign}(|s| - \epsilon) & \text{if } |\dot{k}| < \mu \text{ and } \sigma_{\text{min}} < \sigma < \sigma_{\text{max}} \\ \rho |s| & \text{if } |\dot{k}| < \mu \text{ and } \sigma = \sigma_{\text{min}} \\ 0 & \text{if } |\dot{k}| = \mu \end{cases} \] (B.15)