Naturally Large Tan $\beta$ *

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Abstract

We show that if there are only two Higgs doublets in the supersymmetric standard model, large $\tan \beta$ requires a fine tuning in the parameters of the Lagrangian of order $(1/\tan \beta)$, which cannot be explained by any approximate symmetry. With an extended Higgs sector, large $\tan \beta$ can be natural. We give an explicit example with four doublets in which it is possible to achieve large $\tan \beta$ as a result of an approximate symmetry, without any light superpartners. The approximate symmetry can be extended to explain all the hierarchies in the quark mass matrix.

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1. Introduction

The large ratio between the top and bottom quark masses is one of many puzzling hierarchies in the standard model. One possible explanation for this hierarchy arises in supersymmetric theories, which require at least two Higgs doublets, one which couples to up type quarks and one to down quarks. In these theories, the ratio of masses could conceivably arise entirely from a large value of $\tan \beta$, the ratio of the vacuum expectation values of the up and down type Higgs fields [1,2].

However, a large value of $\tan \beta$ is also puzzling, as one would expect all vacuum expectation values to be roughly equal and set by the weak scale. We argue that any such model with a minimal Higgs sector requires at least one fine tuning of the parameters of the potential of order $1/\tan \beta$ in order to achieve electroweak symmetry breaking at the observed scale while avoiding overly light charginos. In a recent paper, Hall, Rattazzi and Sarid (HRS) [2] explain this large ratio entirely through approximate symmetries. Consistent with our claim, the approximate symmetries they introduce reduce the number of fine tunings but do not entirely eliminate them.

The fine tuning can be eliminated at the expense of introducing additional fields. We give an illustrative example of a model with four Higgs doublets which naturally has large $\tan \beta$ without any light fields, indicating that the predictions of large $\tan \beta$ scenarios are very model dependent.

2. Large $\tan \beta$ with a minimal Higgs sector

In this section, we show that given the constraints on charginos and higgsinos, large $\tan \beta$ necessarily implies a large hierarchy (of order $\tan \beta$) between the various mass squared parameters of the potential. Approximate symmetries [2] can enforce the relatively small size of some parameters, but not all of them. There are several different philosophies as to what constitutes an acceptable hierarchy between the various parameters. Our naturalness criterion is that unless constrained by additional approximate symmetries, all mass parameters are about the same size, and all dimensionless numbers are of order one. We then find that $\tan \beta$ should also be of order one in a phenomenologically acceptable supersymmetric model with only two Higgs doublets.

Let us assume the standard scalar potential for the two Higgs fields, $H_u$ and $H_d$.

$$m_u^2 H_u^2 + m_d^2 H_d^2 + B\mu (H_u H_d + \text{h.c.}) + \frac{g^2 + g'^2}{8} (|H_u|^2 - |H_d|^2)^2,$$  \hspace{1cm} (2.1)
where $\mu$ is the higgsino mass parameter in the superpotential. If the electroweak scale is to arise naturally, one would expect the parameters $m_U^2$ and $m_D^2$ to be of order $m_Z^2$. Tan$\beta$ can be computed from this potential to be

$$\frac{1}{2} \sin 2\beta = -\frac{\mu B}{m_U^2 + m_D^2} \approx \frac{1}{\tan \beta}.$$  \hfill (2.2)

Thus large tan $\beta$ requires the parameter $\mu B$ to be much smaller than $m_U^2 + m_D^2$. Naturalness requires that a parameter can only be small if a symmetry is restored in the limit that the parameter goes to zero. As pointed out by HRS, $B$ can naturally be made small by an approximate R symmetry, while $\mu$ could be small because of an approximate Peccei-Quinn (PQ) symmetry, which is taken to commute with supersymmetry. However in a two Higgs doublet model, either of these symmetries results in a light chargino. Let the parameter which suppresses $R$ symmetry breaking terms be $\epsilon_R$. The PQ symmetry is only broken by the parameter $\mu$. Then the chargino mass matrix (in a basis where the first entry refers to a Higgsino and the second entry to a chargino) takes the form

$$\begin{pmatrix} \mu & \frac{q}{\sqrt{2}} \langle H_U \rangle \\ \frac{q}{\sqrt{2}} \langle H_D \rangle & \frac{q}{\epsilon_R M_S} \end{pmatrix},$$  \hfill (2.3)

where $M_S$ is of order the supersymmetry breaking scale. Now because $\langle H_D \rangle$ is assumed to be small, there will be a light eigenvalue unless both $\mu$ and $\epsilon_R M_S$ are large. However a light chargino is unacceptable. Because the lightest eigenstate must exceed 45 GeV, if tan $\beta$ is much larger than one we are forced to take both $\mu > 85$ GeV and $\epsilon_R M_S > 85$ GeV. Now since $B$ is of order $\epsilon_R M_S$, while $m_U^2 + m_D^2$ is naturally of order $M_S^2$, eq. (2.2) requires that

$$M_S^2 > (85 \text{ GeV})^2 \tan \beta.$$  \hfill (2.4)

Thus if there is an approximate symmetry forcing tan $\beta$ to be large, the supersymmetry scale is also forced to be much larger than the weak scale, contrary to what would be true in a natural scenario.

The reason the minimal model is so constrained is readily understood on the basis of chiral charge (or alternatively anomaly) assignments. If both a PQ symmetry and $R$ symmetry are maintained (notice the charges may be chosen so the vacuum expectation value of $H_U$ breaks neither), the chiral charge assignments require the presence of an additional massless charged fermion in addition to the down quarks.
Let us our compare our result to the parameters in the paper of HRS. In order to achieve a \( \tan \beta \) as large as 50, they take \( m_D^2 \) to be about \((700 \text{ GeV})^2\) while both \( B\mu \) and \( m_U^2 \) are of order \(-m_Z^2\), a factor of 50 smaller. While the relatively small size of \( B\mu \) is enforced by a combination of PQ and \( R \) symmetries, there is no symmetry which can suppress \( m_U^2 \), which is nonetheless required to be a small negative number since it sets the scale of the \( W \) and \( Z \) masses. Thus there is a hierarchy in the Higgs potential of order \( \tan \beta \) which is not enforced by any symmetry, although it can be arranged in a radiative symmetry breaking scenario by tuning the top mass. Our aim is to generate a model with large \( \tan \beta \) which is devoid of fine tuning, which we have shown is not possible in the context of the minimal Higgs sector, given the light chargino constraint. In the next section, we show that with an enlarged Higgs sector, large \( \tan \beta \) can be naturally arranged.

3. A model with naturally large \( \tan \beta \)

The minimal extension of the Higgs sector is the addition of a gauge singlet \( S \). However, one can easily see that this does not help make large \( \tan \beta \) more natural. With a gauge singlet one can add to the superpotential a term \( \lambda S H_U H_D \), which contributes to the Higgsino mass if \( S \) gets a vev. One then should also add to the potential (2.1) a term \( B'\lambda S H_U H_D \), where \( B' \) is of order \( M_S \) unless one imposes an approximate \( R \) symmetry, in which case \( B' \sim \epsilon_R M_S \). Now \( \tan \beta \) is of order \( (m_U^2 + m_D^2)/(B'\lambda \langle S \rangle + B\mu) \), which is naturally large if either \( \epsilon_R \) is small or \( \lambda \langle S \rangle \) and \( \mu \) are small. With only two Higgs doublets, the chargino mass matrix is now

\[
\begin{pmatrix}
\mu + \lambda \langle S \rangle & \frac{g}{\sqrt{2}} \langle H_U \rangle \\
\frac{g}{\sqrt{2}} \langle H_D \rangle & M_S \epsilon_R
\end{pmatrix}
\]

which will have a light eigenvalue if \( \tan \beta \) is naturally large, and \( m_U^2 \) and \( m_D^2 \) are both of order \( M_S^2 \). To avoid the light chargino would require a model in which the parameters providing Higgsino masses are not suppressed and in which there is no approximate \( R \) symmetry. But then the model is no more natural than the minimal model. This result can be easily understood by looking at the charged fermion charge assignments under the approximate chiral symmetries, and noting that with only two Higgs doublets any symmetry which protects \( \tan \beta \) also requires additional light charged particles.

We therefore are led to the addition of another Higgs doublet. Anomaly constraints require two new Higgs doublets. (This would change the prediction of \( \sin^2 \theta_W \) in a GUT
scenario unless we also add other particles such as a vector-like color triplet). We add two additional Higgs particles, which we call $H_U'$ and $H_D'$. We assume $\tan \beta$ is large; that is, there is a large vacuum expectation value for $H_U$, and a small one for $H_D$. To suppress the term $\mu_{ud} H_U H_D$, which would induce a large vev for $H_D$, we impose an approximate PQ symmetry under which $H_D$ has charge 1, the left handed down antiquarks have charge -1, and all other quarks and $H_U$ have charge 0. (We do not wish to impose an approximate R symmetry because this would lead to a light chargino mass.) Now the chargino mass matrix is
\[
\begin{pmatrix}
\mu_{ud} & \mu_{ud'} & \frac{g}{\sqrt{2}} \langle H_U \rangle \\
\mu_{u'd} & \mu_{u'd'} & \frac{g}{\sqrt{2}} \langle H_U' \rangle \\
\frac{g}{\sqrt{2}} \langle H_D \rangle & \frac{g}{\sqrt{2}} \langle H_D' \rangle & \mathcal{O}(M_S)
\end{pmatrix},
\]
which, since $\mu_{ud}$ and $\langle H_D \rangle$ are assumed to be small, will have a light eigenvalue unless $\mu_{u'd}$ is reasonably large. Thus $H_U'$ should have PQ charge -1, and $\langle H_U' \rangle$ should be small, or $\mu_{u'd}$ will induce a large vev for $H_D$. Now we need to determine the PQ charge of $H_D'$. This can be chosen such that the PQ charge assignments of the charginos are vectorlike, so that no chargino masses are suppressed by the approximate symmetry. Therefore we take $H_D'$ to have a PQ charge of 0.\footnote{For nonzero choice of the charge of $H_D'$, avoiding a light chargino would require spontaneous breakdown of the PQ symmetry at the weak scale. Then one would be forced to complicate the model further in order to achieve a stable vacuum with spontaneously broken PQ symmetry, and there would also be a light pseudo Goldstone boson.} Then in the limit that the PQ symmetry is exact, if only $H_U$ has negative mass squared, only $H_U$ and $H_D'$ receive vevs, and the chargino mass matrix has the form
\[
\begin{pmatrix}
0 & \mu_{ud'} & \frac{g}{\sqrt{2}} \langle H_U \rangle \\
\mu_{u'd} & 0 & 0 \\
0 & \frac{g}{\sqrt{2}} \langle H_D' \rangle & \mathcal{O}(M_S)
\end{pmatrix},
\]
which naturally has all eigenvalues of order the weak scale. In this limit there is also no Goldstone boson, since the PQ symmetry is not spontaneously broken, however the charge $-1/3$ quarks are massless. Allowing small explicit violation of the PQ symmetry by an amount $\epsilon_{PQ}$ will give $H_D$ and $H_U'$ small vevs and the down type quarks small masses. Note that the small explicit PQ violation does not imply that there must be a light pseudo Goldstone boson, even though it induces small vevs for the scalars carrying nonzero PQ charges, since the size of the vevs is no larger than the explicit symmetry breaking terms.

Thus by making a vectorial assignment of PQ charge on the charginos, and by avoiding an R symmetry, we have naturally large $\tan \beta$ without an additional light state in the theory.
A potential problem with this particular model is that with extra Higgs doublets it is possible to have more than one doublet couple to quarks with the same charge, leading to the possibility of large flavor changing neutral currents (FCNC). This does not happen in the limit that the PQ symmetry is exact, however explicit PQ violation would allow FCNC. For instance the $H'_D$ coupling to down type quarks could be suppressed by $\epsilon_{PQ}$. This does not provide sufficient suppression of FCNC. The vev of $H_D$ is also suppressed relative to $\langle H'_D \rangle$ by an amount $\epsilon_{PQ}$, and so $H_D$ and $H'_D$ would make comparable contributions to the down quark masses. In models with $\tan \beta$ of order one where all Yukawa couplings to the quarks have the same approximate “texture” enforced by approximate chiral symmetries, it is possible to have more than one Higgs couple to each type of quarks without inducing unacceptably large FCNC \[3\]. In our large $\tan \beta$ case however, the FCNC in the down quark sector are enhanced relative to the $\tan \beta \sim 1$ case by a factor of $\tan \beta$, and would generally be too large.

Of course supersymmetric theories already have potential FCNC which can arise from squark exchanges \[4\]. We will assume that the problem of FCNC from the soft supersymmetry breaking terms has been solved by some means \[3,3\], but we still need to avoid FCNC from Higgs exchange by finding some way of further suppressing the coupling of $H'_D$ to the down quark sector.

One simple possibility utilizes the same mechanism that Nir and Seiberg \[6\] used to obtain zeros in quark mass matrices. The small parameter $\epsilon_{PQ}$ could be a spurion which arises as a consequence of spontaneous symmetry breaking at high energy by a field $\phi$, which is communicated to the low energy theory by particles of mass $M$. Thus we postulate that $\epsilon_{PQ} = \langle \phi \rangle / M$. Now in the effective theory supersymmetric couplings in the superpotential can only depend on $\phi$ and not $\phi^*$. If $\phi$ carries a PQ charge of -1, then the coupling $(\langle \phi \rangle / M) \mu' H_U H_D$ is allowed, where we assume $\mu'$ is a parameter of order the supersymmetry breaking scale. Couplings such as $(\langle \phi \rangle^*/M) H'_D \lambda_{ij}^* q_i d_j$ are however not allowed by supersymmetry. The coupling $(\langle \phi \rangle^*/M) H'_U \lambda''_{ij}^* q_i u_j$ is also not allowed. There could be some additional FCNC induced by $H'_D$ in the soft supersymmetry breaking terms, but these should be acceptably small in any model which has managed to solve the supersymmetric flavor problem \[3,4\].

Large $\tan \beta$ is interesting in the context of flavor models, which explain the structure of the quark mass matrices by approximate symmetries, because it changes the charge assignments required in order to generate the correct texture for the mass matrices. For example, we could assume that there is another approximate U(1), which is explicitly
broken by a spurion of size $\alpha \sim 0.2$ carrying charge -1. Then for example we could assign charges $(3,2,0)$ to the left handed quarks, $(1,0,0)$ to the right handed down type quarks, and $(4,1,0)$ to the right handed up type quarks. Now the quark mass matrices naturally have the form

$$M_{\text{up}} = 175 \text{ GeV} \begin{pmatrix} \alpha^7 & \alpha^4 & \alpha^3 \\ \alpha^6 & \alpha^3 & \alpha^2 \\ \alpha^4 & \alpha & 1 \end{pmatrix}$$

$$M_{\text{down}} = \frac{175 \text{ GeV}}{\tan \beta} \begin{pmatrix} \alpha^4 & \alpha^3 & \alpha^3 \\ \alpha^3 & \alpha^2 & \alpha^2 \\ \alpha & 1 & 1 \end{pmatrix},$$

which gives the correct pattern of masses and mixing angles for the quarks, up to factors of two.

4. Conclusions

Our approach is orthogonal to that of previous authors. Rather than confining our attention to a two doublet model, we considered the question of whether it is possible to naturally generate a large value for $\tan \beta$, and if so, what if any are the generic predictions of such a model. We have shown that it is simple to make such a model at the expense of complicating the Higgs sector; we have shown furthermore that it is impossible to do so without additional charged fields. From this model, we see that although the minimal Higgs sector model with large $\tan \beta$ requires the presence of additional light fields, there are not necessarily additional light fields present in a nonminimal model; the minimal model with large $\tan \beta$ can be made natural only at the expense of requiring light charginos. Of course, the minimal model, which contains an unnatural fine tuning of order $1/\tan \beta$ in order to be phenomenologically acceptable, is very predictive. In nonminimal models, which can be natural, none of these predictions necessarily apply.

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