Field Reconstruction Method for Linear Tubular Permanent Magnet Motor

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I. INTRODUCTION

In the automotive industry, the current and the future trend are becoming more electrified vehicles. The hybrid electric vehicle (HEV) combines an electric motor with a combustion engine in order to increase efficiency. In the electric vehicle (EV), the combustion engine has been replaced by an electric motor.

Recently, many studies have been conducted to improve the driving quality in HEV and EV [1], [2]. The active suspension system can improve performance without conflicting demands between road condition, load carrying, and passenger comfort [3]. Therefore, the traditional passive suspension system will gradually be replaced by the active suspension system in the near future.

For the active suspension system using the electric machine (EM), both rotating and linear machines can be applied. The linear machine is more efficient than the rotating machine because the electromagnetic energy can be directly transferred without conversion from rotational motion to linear motion.

The EMs are becoming increasingly diverse and require proper mathematical modeling. The accuracy of modeling is an important factor in analysis and design of the EM.

The magnetic equivalent circuit or the analytical method was widely used as an analysis tool offering a good compromise between accuracy and computational cost [4]. These methods could be applied to various types of EMs. However, it has a limitation in considering the detail structure inside the EM. For this problem, the finite element (FE) analysis is used.

The two-dimensional (2-D) FE analysis is much faster than the three-dimensional (3-D) FE analysis, however, it does not reflect the electromagnetic effects caused by the 3-D structure, making it difficult to guarantee accuracy when evaluating models with complex shapes. On the other hand, 3-D FE analysis is computationally inefficient for the optimal design process.

To address this problem, the field-based approach using the FE analysis can be used. The field reconstruction method (FRM) has been used to reduce the computational burden in the FE analysis [5]–[8]. In the FRM, the air-gap magnetic flux density is presented as the sum of each magnetic flux distribution generated from permanent magnet (PM) and armature winding. Each magnetic flux distribution is reconstructed by the basis-function sweeping the reference magnetic flux distribution. By using the basis-function, it is possible to calculate the air-gap magnetic flux distribution at the combination of any armature current or mover position without an additional FE analysis [7], [8].
Therefore, the FRM can reduce the computational burden of the FE analysis.

The previous research work on the FRM has focused on the applications, such as axial flux PM motors [6]–[8], switched reluctance machines [9], PM synchronous machines [10]–[13], and induction machines [14], [15]. Park et al. [6] and Ajily et al. [7] presented the performance of axial flux PM motors using the 3-D FRM. Lin et al. [9] proposed a way to apply the FRM to a switched reluctance machine to consider the double-saliency and magnetic saturation. For efficient modeling, Torregrossa et al. [10], [11] and Dezheng et al. [14] used the FRM for the synchronous PM motors and induction machines, respectively. However, few papers have reported the FRM in linear machines. The focus is always on rotating machines, not linear-type machines.

In this paper, we adopted FRM for the analysis and design of the LTPMM. The LTPMM is structurally accompanied by a longitudinal end effect, which distorts the magnetic flux distribution in the air-gap [16], [17]. This effect can be further increased because the electromagnetic active suspension has a shorter stroke. Therefore, accurate analysis of this effect is important for designing the LTPMM. First, we evaluated the distortion of air-gap magnetic flux distribution due to the longitudinal end effect of the LTPMM. Then, we proposed the enhanced FRM with a new basis-function.

**II. ELECTROMAGNETIC ACTIVE SUSPENSION**

**A. LINEAR TUBULAR PERMANENT MAGNET MOTOR**

The linear machine can be classified as the linear planner permanent magnet motor (LPPMM) and the linear tubular permanent magnet motor (LTPMM) [18], [19]. The LTPMM has an advantage that the force density is higher than that of the LPPMM because the attraction force is zero [20].

The LTPMM can be sorted into interior PM type and exterior PM type according to the relative position of the PM. Fig. 1 show the LTPMM with interior PM type and exterior PM type, respectively. Magnetization can be categorized into axial, radial and halbach pattern according to the direction of the magnetization as shown in Fig. 2.

In an electric machine with the halbach magnetized PM, the air-gap magnetic flux distribution and the back-EMF waveform are more sinusoidal and the detent force is low. In addition, the halbach array does not require the back-iron for the PM, resulting in higher power density than other magnetization patterns under the same volume condition [21], [22].

Because the active suspension system locates outside of the vehicle, it would be exposed to the external pollution such as dust and sand. Therefore, the interior PM type is weaker in the external environment than the exterior PM type. As a result, we adopted the LTPMM with exterior PM type and the halbach array. The specification of the analysis model and its structure are shown in Table 1 and in Fig. 3, respectively.

**B. MAGNETIC FLUX DISTRIBUTION IN LTPMM**

In this paper, the LTPMM includes coupled modules. A single module of the PM and stator consists of 4-poles and 3-slots, respectively. In Fig. 1 (b), since the LTPMM has 4-modules of the stator and 2-modules of the mover, the air-gap magnetic flux distribution is repeated every slot pitch.

For the electromagnetic active suspension, the LTPMM has a short mover and short stator due to spatial constraints. Because of the different lengths of the mover and stator, the LTPMM has a longitudinal end effect as shown in Fig. 4. The effect distorts the periodicity of the air-gap magnetic flux distribution, affecting the thrust force. This will result in unpredictable characteristics.

Fig. 6 shows the r-component of the air-gap magnetic flux distribution under no-load conditions. The results were
obtained when the mover moved 0, 3τ, and 6τ from the left end of the stator in Fig. 5. In Fig. 6 (b), the results represents the longitudinal end effect at both ends of the PM and τend is the distance affected by the longitudinal end effect. In Fig. 6 (a) and (b), however, when the PM reaches the end of the stator, the reluctance paths at both ends of the PM gradually change, eventually making the three waveforms different.

III. CONVENTIONAL FRM

In the conventional FRM, an air-gap magnetic flux density is obtained as follows [23], [24]:

\[
B_r(t) = B_{r,m}(t) + B_{r,s}(t) \\
B_z(t) = B_{z,m}(t) + B_{z,s}(t)
\]

where \(B_r(t)\) and \(B_z(t)\) are the air-gap magnetic flux density in the \(r\)-, and \(z\)-direction in cylindrical coordinate system, and \(B_m(t)\) and \(B_s(t)\) are the air-gap magnetic flux density generated by PM and armature reaction, respectively.

The \(B_m(t)\) and \(B_s(t)\) are reconstructed by using the following basis-functions [6]:

\[
B_m(t) = f_m(t) \\
B_s(t) = \sum_{a=1}^{N} B_{s,a}(t) \\
B_{s,a}(t) = f_s \cdot i_{s,a}(t)
\]
where $f_m$ and $f_s$ are the basis-functions of the mover and the stator, $i_{s,a}(t)$ is the excitation current in the $a$th slot, $B_{s,a}(t)$ is the air-gap magnetic flux density when the $a$th slot is only excited, and $N$ is the number of slots.

In order to reconstruct the entire air-gap magnetic flux density, the basis-function sweeps the reference magnetic flux distribution. The reference magnetic flux distribution is obtained by the FE static analysis. The basis-function predicts the distribution of the air-gap field over the entire position of the mover by sweeping.

The conventional FRM assumes that the air-gap magnetic flux distribution is periodic and the air-gap is continuous. Under this condition, the reference magnetic flux distribution can be selected at any mover position. Unlike a linear machine, the air-gap of a rotating machine has continuity that is repeated for every pole pitch.

However, in the case of the linear motor, the field distribution varies depending on the position of the mover as shown in Fig. 6. Therefore, when the FRM is applied to a rotating machine, the reference magnetic flux distribution can be selected at any position of the mover. However, in the case of the LTPMM with the finite air-gap length, the air-gap is not continuous and the air-gap magnetic flux is distributed without a periodicity.

In Fig. 6, the air-gap magnetic flux distributions in each case are different in $\tau_{end}$. If one of three waveforms is selected as the reference magnetic flux distribution, the basis-function will sweep this difference at all mover positions. It will distort the overall air-gap magnetic flux density and eventually results in unexpected characteristics. Therefore, an accurate evaluation of the end effect is required in the FRM. For this reason, the conventional FRM should be modified in order to be applied to the LTPMM.

IV. ENHANCED FRM

A. BASIS FUNCTION: STATOR

In the proposed FRM, the air-gap magnetic flux density generated by the armature reaction is expressed as follows:

$$B_s(t) = F_s \cdot I_s(t)$$

$$I_s(t) = \left[ i_{s,1}(t + \theta_1) \quad i_{s,2}(t + \theta_2) \quad \cdots \quad i_{s,N}(t + \theta_N) \right]^T$$

$$F_s = \begin{bmatrix}
    f_s(L_{s,1}^{1}(t_0)) & f_s(L_{s,2}^{1}(t_0)) & \cdots & f_s(L_{s,h}^{1}(t_0)) \\
    f_s(L_{s,1}^{2}(t_0)) & f_s(L_{s,2}^{2}(t_0)) & \cdots & f_s(L_{s,h}^{2}(t_0)) \\
    \vdots & \vdots & \ddots & \vdots \\
    f_s(L_{s,1}^{k,N}(t_0)) & f_s(L_{s,2}^{k,N}(t_0)) & \cdots & f_s(L_{s,h}^{k,N}(t_0)) \\
\end{bmatrix}$$

where $f_s$ is the basis-function of the stator, $F_s$ is the basis-matrix of the stator, $\theta_N$ is the phase angle of $N$th slot, $t_0$ is the initial time, and $L_{s,h}^{k,N}$ represents the reference magnetic flux distribution. It can be obtained as follows:

$$L_{s,h}^{k,N}(t_0) = \frac{B_{s,N}(t_0, I_0)}{I_0}$$

where $I_0$ is the initial current. As shown in Fig. 7, $L_{s,h}^{k,N}$ is the magnetic flux density at $k$th position in the air-gap. It is generated by the armature reaction in $N$th slot.

In this paper, because the LTPMM adopted the halbach magnetization, the back-iron of the mover is not needed. In addition, since the stator has the same length as the air-gap, there is no distortion of the magnetic flux distribution due to the longitudinal end effect when the armature winding is excited and the PM is excluded. Therefore, for the basis-function of the stator $f_s$, $B_{s,N}$ requires only a static FE analysis.

Fig. 8 shows the results of the conventional FRM, enhanced FRM, and finite element method (FEM). Fig. 8 presents the $r$- and $z$-component of the air-gap flux distribution at the left end of the PM. Using the FEM, the air-gap flux density was obtained when the mover moved $3\tau_s$ from the initial position. The results of the FRMs were generated by sweeping the reference magnetic flux distribution at initial position. The difference between the results of the FRM and FEM is acceptable below 2.8%.

B. BASIS FUNCTION: MOVER

First, as shown in Fig. 6 (b), when the mover is located more than $\tau_{end}$ from the end of the stator, the air-gap magnetic flux density can be obtained as follows:

$$B_m(t) = \sum_{h=1}^{p} F_{m,mid}(t), \quad \left( \frac{\tau_{end}}{m} \right) \leq t < \left( \frac{x - \tau_{end}}{m} \right) \quad (10)$$

$$F_{m,mid}(t) = \left[ f_m\left(L_{m,h}^{1}(t)\right) \quad f_m\left(L_{m,h}^{2}(t)\right) \quad \cdots \quad f_m\left(L_{m,h}^{k}(t)\right) \right]^T \quad (11)$$

$F_{m,mid}(t)$

$F_{m,mid}(t)$
where $F_{m,mid}$ is the basis-matrix of the mover, $P$ is the number of poles, $m$ is the velocity, $x$ is the stoke, $f_m$ is the basis-function of the mover, $L^k_m$ is the reference magnetic flux distribution at $k$th position in air-gap. The basis-function of the mover requires a FE analysis when the armature winding is not excited and the PM is included. As shown in Fig. 6, due to the back-iron in the stator, the longitudinal end effects occur at both ends of the PM.

$B_m$ can be presented as the linear sum of the air-gap magnetic flux distribution produced by the individual PMs. $F_{m,mid}$ is repeated every slot pitch and the sweeping can be possible in $\tau_{end}/m \leq t < (x-\tau_{end})/m$. After the sweep, since the air-gap of the LTPMM is finite, it is necessary to shift the position of the air-gap flux distribution corresponding to the mover movement. The sweeping and shifting are performed simultaneously in (12)-(13).

However, in Fig. 6 (a) and (c), when the mover reaches the end of the stator and the distance between the end of PM and stator is less than $\tau_{end}$, the air-gap flux distribution is no longer predictable. Therefore, the additional basis-matrix of the mover is needed as follows:

$$B_m(t) = F_{m,end}(t)$$

$$0 \leq t < \left(\frac{\tau_{end}}{m}\right), \quad \left(\frac{x - \tau_{end}}{m}\right) \leq t < \left(\frac{x}{m}\right)$$

V. FORCE CALCULATION
The force is calculated using the Maxwell stress tensor (MST) as follows:

$$F = \int_S \mathbf{T} \cdot \hat{n} \, dS$$

where $F_{m,end}$ is the basis-matrix when the end of the PM is located within $\tau_{end}$ from the end of the stator.

Fig. 9 shows comparison results. In the results of the conventional FRM, the reference magnetic flux distribution is obtained when the mover is located at the initial time. The difference between the waveforms will cause a large error because it will be repeated several times for each position of the mover by the basis-function.

In Fig. 10, the flowchart of the proposed FRM is presented. At the initial time, the mover is fully retracted to the left end of the LTPMM. First, the static FE analysis is performed to obtain a reference magnetic flux distribution for the basis-function of mover and stator. Then, in Fig. 10, it follows the blue path until the end of the PM is $\tau_{end}$ away from the end of the stator. When the PM is located more than $\tau_{end}$ from the end of the stator, the FRM flows along the red path. This flow is repeated until the mover enters the other end of the LTPMM. Finally, the thrust of the LTPMM is calculated using $B_m$ and $B_s$ reconstructed from each basis-function.

FIGURE 8. Air-gap magnetic flux distribution generated by armature reaction. (a) $r$-component. (b) $z$-component.

FIGURE 9. Air-gap magnetic flux distribution generated by PM. (a) $r$-component. (b) $z$-component.
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FIGURE 10. Flowchart of the enhanced FRM.

In the cylindrical coordinate system, \( \vec{T} \) is

\[
\vec{T} = \begin{bmatrix}
T_{rr} & T_{r\theta} & T_{rz} \\
T_{\theta r} & T_{\theta\theta} & T_{\theta z} \\
T_{rz} & T_{\theta z} & T_{zz}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
B_r^2 - B_\theta^2 - B_z^2 & B_r B_\theta & B_r B_z \\
2\mu_0 B_r B_\theta & \mu_0 & \frac{\mu_0}{B_r B_z} \\
\mu_0 & \frac{\mu_0}{B_r B_z} & B_z^2 - B_r^2 - B_\theta^2
\end{bmatrix}
\]

assuming \( B_\theta = 0 \) to analyze an axisymmetric model, \( \vec{T} \) can be simplified as follows:

\[
\vec{T} = \begin{bmatrix}
B_r^2 - B_\theta^2 - B_z^2 & B_r B_z \\
2\mu_0 B_r & \mu_0 & \frac{\mu_0}{B_r B_z} \\
\mu_0 & \frac{\mu_0}{B_r B_z} & B_z^2 - B_r^2 - B_\theta^2
\end{bmatrix}
\]

In the LTPMM, the force is classified into the normal force and end force [25]. The normal force is related to the relationship between the stator teeth and the magnet. The end force is generated by the magnetic flux density at the end section of the stator.

Fig. 11 shows the cross section of the LTPMM. In Fig. 11, the \( \hat{n} \) of the normal force and end force are +r and ±z respectively. Using (15) and (17), the thrust force of the LTPMM can be calculated as (18).

\[
F_z = F_{z,\text{normal}} + F_{z,\text{left\_end}} + F_{z,\text{right\_end}}
\]

\[
F_{z,\text{normal}} = \frac{1}{\mu_0} \oint_{E1E5} B_r B_z d\theta dz
\]

\[
F_{z,\text{left\_end}} = \frac{1}{2\mu_0} \oint_{E1E2} (B_r^2 - B_\theta^2) d\theta dr + \frac{1}{2\mu_0} \oint_{E2E3} (B_r^2 - B_\theta^2) d\theta dr + \frac{1}{2\mu_0} \oint_{E3E4} (B_r^2 - B_\theta^2) d\theta dr
\]

\[
F_{z,\text{right\_end}} = \frac{1}{2\mu_0} \oint_{E5E6} (B_r^2 - B_\theta^2) d\theta dr + \frac{1}{2\mu_0} \oint_{E6E7} (B_r^2 - B_\theta^2) d\theta dr + \frac{1}{2\mu_0} \oint_{E7E8} (B_r^2 - B_\theta^2) d\theta dr
\]

where E1 to E8 are positions at both ends of the stator back-iron to distinguish the influence of the leakage flux.

VI. RESULT

Under no-load conditions, the total air-gap magnetic flux density includes only the \( B_m \) produced by the PMs. Therefore, the detent force calculation of the LTPMM can be performed using the basis-function of the mover. In Fig. 12, the detent force of the FRM and FEM are compared. It is noted that the basic-function of the conventional FRM provides the error of the reconstructed field by sweeping the distorted field at the end of the stator. Compared with the results of the FEM, it can
be seen that the error of the enhanced FRM is less than 5%, while the error of the conventional FRM is up to 21.2%.

On the other hand, for the calculation of thrust, both the basic-functions of mover and stator are used. First, the reference magnetic flux distribution for sweeping of each basis-function is obtained using FE static analysis. The basis-matrix is calculated in (8) and (11), and the thrust is calculated in (18) using the reconstructed entire field.

Fig. 13 illustrates comparison results of the thrust force in LTPMM. In Fig. 13, the Point A is when the mover is positioned at the end of the stator, and Point B and C are when the mover is more than \( \tau_{\text{end}} \) from the end of the stator.

Since the basis-function of the conventional FRM selects the reference magnetic flux distribution when the mover is located at the initial time, at Point A, the error between the FRMs and the FEM is 3.45%. At the Point B, the difference between the conventional FRM and FEM was 21.1%, while the difference between the enhanced FRM and FEM was 5.5%.

The Fig. 14 shows the distribution of the magnetic flux density inside the back-iron of the stator. Four cases were adopted to verify the feasibility of the FRM according to the saturation effect. The measurements of the \( B \) were performed on the same element of each case. In Table 2, the results of the FRM and FE analysis are compared. For Case 1, 2 and Case 3, the results are under soft magnetic saturations and the resulting difference between the FE analysis and FRM is less than 1.8%. In Case 5, 6 and Case 7, the difference is still acceptable at less than 7.4%, even under hard saturation conditions.

Under the same simulation conditions (CPU Intel core i7-6700K 4.00GHz, RAM 16GB), for 3-D model, the FE analysis required 38h 2m, while the FRM took 4h 13m. The model has 270 105 elements, and the simulation was performed for full stroke of the LTPMM. The FRM provides an 88.9% reduction in time costs compared to FE analysis. During the optimization process, the computational time difference between these two methods becomes larger in situations where the dimensions of the model should change. Given that the mover and stator fields are independent of each other in the FRM, this is advantageous for analyzing the characteristics of pole and coil combinations. If various magnet shapes should be reflected in a specific coil structure, there is no need for additional armature reaction field analysis, and when considering changes in the winding configuration in a specific magnet shape, additional FE calculations to obtain the no-load field are not required. In addition, when analyzing with changes in excitation current, the FE analysis should be recalculated, whereas in the case of the FRM, it is possible to predict the result with acceptable accuracy by using the basis function.

VII. EXPERIMENTAL VERIFICATION

To validate the enhanced FRM, prototype LTPMM presented in this paper has been manufactured and tested. The specification of the prototype LTPMM is listed in Table 1. The outer diameter and overall length were determined taking into account the space occupied by the suspension in the vehicle. Table 3 shows the test condition. The test setup is...
shown in Fig. 15. For measuring thrust, the specially designed load cell was attached on the housing. Four load-motors actuate the prototype motor. The speed and thrust load are controlled by inverter for load motors and the thrust force is generated by inverter for prototype motor. The comparison of calculated and measured thrust characteristics is shown in Fig. 16. The difference between the enhanced FRM and experimental result was 3.7% on average.

VIII. CONCLUSION

The contribution of this paper is to present a method for fast evaluation of the force calculation and magnetic flux distribution of linear motors using the FRM. This paper considers the LTPMM for the electromagnetic active suspension of vehicles. We adopted FRM to reduce computational burden for modeling LTPMM. Since the road condition changes every moment, detailed operation is necessary for the comfort of the driver. For this reason, accurate thrust force calculations are an important factor in modeling the LTPMM. However, because the LTPMM has short mover and short stator, the longitudinal end effect is significant. The \( r \)- and \( z \)-components of the air-gap flux were analyzed to investigate the longitudinal end effect. Finally, we proposed an enhanced FRM taking consideration into this end effect of the LTPMM. We found that using the proposed FRM compared to the FE analysis resulted in a 88.9% reduction in time cost. Furthermore, it can be used not only for the LTPMM, but also for exterior/interior magnet type, radial/axial/halbach magnetization patterns, and even flat-type linear motors.

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