Research on 3d meso-structure modeling method and penetration simulation of concrete target

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Abstract: This paper presents a new three-dimensional(3D) meso-structure modelling method, the anti-penetration performance of concrete target with aggregate is studied. Firstly, the shape, size and characteristics of random distribution of concrete aggregates are considered, a new meso-model generation algorithm is proposed, which is in accordance with the actual gradation and actual volume ratio of aggregates. Then, the finite element model was generated using a mapping algorithm. Finally, the influence of aggregate volume ratio on the anti-penetration performance is analyzed by simulating different concrete target models. The results show that, under the premise that the aggregate gradation of the concrete is continuous gradation and randomly distributed, when the volume ratio of aggregates increases, it will have a significant impact on the penetration speed of the projectile.

1. Introduction

As one of the most common man-made engineering materials, concrete has good mechanical properties and is widely used in the field of military protection engineering [1]. At present, in the research of the anti-penetration performance of concrete, researchers usually assume that the concrete material is homogeneous and isotropic based on the macro-scale model. In fact, concrete is a heterogeneous material, mainly composed of aggregates, mortar, interfacial transition layers, and various defects (such as cracks, holes, etc.). The mechanical properties of various components of concrete are quite different, so the macro-scale model cannot accurately reflect the anti-penetration performance of concrete [2-4].

Studies have shown that the macroscopic physical properties of concrete mainly depend on its components and complex meso-structure, such as the shape, size, and volume ratio of aggregates. The meso-structure of concrete can be obtained by CT scanning technology, but there are many limitations in this technology, such as: the scanning size cannot be too large, the cost is high, and it is difficult to use the control variable method. In recent years, with the development of high-speed computer, numerical simulation based on meso level has become possible. Researchers have proposed many computer simulation methods, among which the earlier-proposed and easiest to implement is the take-and-place method. However, the take-and-place method needs a lot of calculation time to realize the aggregate volume ratio of more than 30%, and its aggregate size distribution will be different from the actual situation [5]. Based on the method of 3D Voronoi and scaling, although the high aggregate volume ratio can be generated quickly, there are still deficiencies in the control of aggregate gradation [6].
In this paper, a new aggregate generation algorithm is proposed, which combines the 3D Voronoi method. The algorithm is programmed in MATLAB language. Firstly, a standard concrete specimen model is generated by using this algorithm. Then, the mechanical properties of concrete specimen under compression load are studied, and the validity of the model is verified. Finally, a 3D model of concrete target with aggregate is established, and the influence of aggregate volume ratio on the anti-penetration performance of concrete target is studied by using LS-DYNA software.

2. Model algorithm

2.1. Introduction of 3D Voronoi graphics

The 3D Voronoi diagram is a group of continuous polyhedrons formed by a vertical bisecting planes of straight lines between two neighboring points. This polyhedron is also called cell. Suppose there are \( n \) seed points in the space, denoted as \( P = \{ p_1, p_2, \ldots, p_n \} \). A cell contains only one seed point. If any point in the cell is written as \( q \), then the Voronoi graphic cell meets the following equation:

\[
v(p_i) = \bigcap_{i \neq j} \{ p \mid d(q,p_i) < d(q,p_j) \} \quad (i = 1,2,\ldots,n; j = 1,2,\ldots,n)
\]

(1)

Where \( v(p_i) \) is the distance from any point \( q \) in cell to each seed point. \( d(q,p_i) \) is the distance between \( q \) point and \( p_i \) seed point. \( d(q,p_j) \) is the distance between \( q \) point and \( p_j \) seed point.

Figure 1 shows a 3D Voronoi diagram.

![Figure 1. 3D Voronoi diagram.](image)

2.2. Size and shape control of Voronoi cell

In Voronoi diagram, the positional relationship between the seed points will affect the cell shape. When the seed points are uniformly distributed in the spatial region, the Voronoi diagram and its polyhedrons are also regular. When the position of the seed point has a certain randomness, the shape and size of the polyhedron in the Voronoi diagram are also random. Suppose there is a 3D space \( (x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, y_{\text{max}}, z_{\text{min}}, z_{\text{max}}) \), and it can be divided into \( m \) cubes. The side length of the cube is \( d \), the center of mass of each cube is \( M \), its coordinate is \( (x_{\text{init}}, y_{\text{init}}, z_{\text{init}}) \), the coordinate of seed point \( p \) is \( (x, y, z) \), and \( k \) is the random variable on \([-k_e, k_e]\). The position of the seed point satisfies the following equation:

\[
\begin{align*}
x_i &= x_{\text{init}} + N(-k_e,k_e); \\
y_i &= y_{\text{init}} + N(-k_e,k_e); \\
z_i &= z_{\text{init}} + N(-k_e,k_e); \\
\end{align*}
\]

(2)

Where the value of \( k_e \) is less than \( 0.5d \), so the seed point must still be in the original cell. The value of \( k_e \) has an effect on the shape and size of the polyhedron. When the value of \( k_e \) increases, the difference between the polyhedrons formed by Voronoi cells will increases, that is, the randomness will increases.

In a cube model with a side length of 160 mm, a 3D Voronoi partition map is generated, \( d = 20 \) mm. figure 2 is a 3D Voronoi partition map when \( k_e = 0, 0.15d, 0.30d, \) and \( 0.45d \) respectively.
The minimum sieving size of aggregate determines the division of aggregate gradation. In order to calculate the minimum size of cell, it is necessary to solve the minimum bounding box of cell. Figure 3 shows the cell with its minimum bounding box. The cells need to be reduced in the model for simulating the gap between the cells. The scaling center is the body center of the minimum bounding box of the cell, and the reduction factor is $k_s$ ($0 < k_s < 1$). Different reduction factors will change the overall cell distribution. It should be noted that the size of the reduced cell must be within its range of gradation. Figure 4 shows the scaled cell model.

The shape of the cells can be divided into polyhedral, spherical, columnar and flaky. In this paper, the shape of the cells is controlled by controlling the aspect ratio, flatness ratio, sphericity and shape factor of the minimum bounding box.

**Figure 2.** 3D Voronoi partition map.

**Figure 3.** A cell with its minimum bounding box.

**Figure 4.** Scaled cell model.
According to the method described above, the size parameters of the minimum bounding box of each cell are calculated, and each cell is screened in turn. The gradation parameters of the model are shown in the figure 5 (when $ke = 0.15d$). It can be seen from the figure 5 that the size distribution of the cell roughly conforms to the normal distribution, so the calculation method described in this paper has good applicability in obtaining the parameters of the mixed aggregate gradation.

![Figure 5](image1.png)  
**Figure 5.** The gradation parameters of the model.

![Figure 6](image2.png)  
**Figure 6.** Gradation curve of model.

2.3. Gradation control

In order to simulate the actual mechanical properties of concrete, the aggregate parameters in the model must be quantitatively constrained, such as aggregate size, aggregate quantity, and reduction factor, so as to strictly control the volume ratio of aggregate and gradation curve. The aggregate gradation follows Fuller's 3D aggregate gradation curve distribution, which is as follows:

$$P(D) = 100\left(\frac{D}{D_{\text{max}}}\right)^n$$

(3)

Where $P(D)$ is the cumulative percentage of aggregate passing the sieve with mesh diameter $D$, $D_{\text{max}}$ is the maximum diameter of aggregate, and $n$ is the index of the equation. In this paper, $n = 0.5$.

In this study, the aggregate gradation of concrete was taken as continuous gradation according to the common working conditions in practice, and the maximum aggregate particle size was about 80 mm. The overall size of the concrete target is $500 \text{ mm} \times 500 \text{ mm} \times 500 \text{ mm}$. The concrete is regarded as a two-phase heterogeneous composite material composed of aggregate and cement mortar. The aggregate is randomly distributed in its space, the rest of the space is mortar, regardless of the adhesive bands between aggregate and mortar. The aggregate volume ratio of the model is 30%, and the gradation curve of model is shown in figure 6.

2.4. Algorithm flow

According to the above analysis, this paper uses Matlab software to write the algorithm to generate a 3D meso model. The multi-gradation aggregate model is generated by merging cell models. Firstly, a Voronoi cell model corresponding to the single gradation is generated. Then the cells are randomly selected in the cell model, and the cell models between different gradations are combined by judgment.

The algorithm flow is as follows:

- Step 1. Determine the aggregate volume ratio, gradation and shape parameters required for the concrete target model;
- Step 2. Determine that $n$ coordinate points are uniformly dropped, according to the gradation of aggregate;
- Step 3. Generate randomly the seed point coordinates by using the Monte Carlo method;
- Step 4. Divide Voronoi cells based on seed points by the 3D Voronoi algorithm;
- Step 5. Scaling each cell equally;
Step 6. Select randomly the cells to form cell model, according to the single gradation requirements; Step 7. Judge whether there is a next gradation. If "yes", execute step 2-6. If "no", execute step 8; Step 8. Merge the cell model generated; Step 9. Generated concrete aggregate by spatial mapping method.

After the random aggregate is generated and placed, a 3D hexahedral concrete background grid is established. The background grid is mapped to the aggregate grid based on the positional relationship between the background grid and the geometric outline of the aggregate. Figure 7 shows the finite element model of the concrete meso-structure.

![Figure 7. Finite element model of concrete meso-structure.](image)

3. Numerical simulation

3.1. Material model

In this paper, RHT material model is used in the constitutive model of aggregate and mortar. In addition to the characteristics of pressure dependence, strain rate sensitivity and compression damage softening, the influence of the third invariant on partial stress tensor on the shape of failure surface is introduced. In the RHT model, the elastic limit surface, failure surface and residual strength surface are introduced as three control failure surfaces to describe the initial yield strength, failure strength and residual strength of concrete materials [7-8].

The failure equation of RHT model can be described as:

\[ Y_{\text{fail}}(P^*, \theta, \dot{\varepsilon}) = Y_c(P^*)R_3(\theta)F_{\text{rate}}(\dot{\varepsilon}) \tag{4} \]

Where \( R_3(\theta) \) is the ratio of the meridian radius of any stress angle to the compressed meridian radius; \( F_{\text{rate}}(\dot{\varepsilon}) \) is the strain rate strengthening factor; \( Y_c(P^*) \) is the compression meridian strength; \( P^* \) is the pressure normalized; \( \theta \) is the Lode angle; \( \dot{\varepsilon} \) is the strain rate.

\[ Y_c(P^*) = f_c \left\{ A \left[ P^* - P^*_{\text{spall}} \right] F_{\text{rate}}(\dot{\varepsilon}) \right\}^{N/3} \tag{5} \]

Where \( f_c \) is the tensile strength; \( P^*_{\text{spall}} \) is the normalized spall strength; \( P^*_{\text{spall}} = P_{\text{spall}}/f_c \); \( A \) is the constant of failure surface; \( N \) is the index of failure surface.

The damage of RHT constitutive model can be described as:

\[ D = \sum \left( \Delta \varepsilon_p / \varepsilon_{\text{failure}} \right) \tag{6} \]

\[ \varepsilon_{\text{failure}} = D_1 \left( P^* - P^*_{\text{spall}} \right) D_2 \geq \varepsilon_p^{\text{min}} \tag{7} \]

Where \( D \) is the damage parameter, \( D_1 \) and \( D_2 \) are the damage constants; \( \varepsilon_{\text{failure}} \) is the failure strain; \( \Delta \varepsilon_p \) is the plastic strain increment; \( \varepsilon_p^{\text{min}} \) is the minimum failure strain.

The specific parameters of mortar and aggregates in concrete target are shown in table 1 and table 2. The material parameters of mortar are selected from the standard 35 MPa concrete [9], and the aggregates parameters are selected from the granite material [10].

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Table 1. Parameters of the RHT material for mortar.

| $\rho_0$ (g/cm$^3$) | 2.314 | $D_1$ | 0.04 |
|----------------------|-------|-------|------|
| $A_1$/GPa            | 35.27 | $D_2$ | 1    |
| $G$/GPa              | 16.7  | $A$   | 1.6  |
| $f_c$/GPa            | 0.035 | $N$   | 3.0  |
| $f$/$f_c$            | 0.1   | $\alpha_0$ | 1.1884 |
| $f$/$f_c$            | 0.18  | $\varepsilon_p$ | 0.01 |

Table 2. Parameters of the RHT material for aggregates.

| $\rho_0$ (g/cm$^3$) | 2.7   | $D_1$ | 0.042 |
|----------------------|-------|-------|-------|
| $A_1$/GPa            | 86.71 | $D_2$ | 1     |
| $G$/GPa              | 24.17 | $A$   | 1.6   |
| $f_c$/GPa            | 0.119 | $N$   | 4.0   |
| $f$/$f_c$            | 0.1   | $\alpha_0$ | 1.1   |
| $f$/$f_c$            | 0.38  | $\varepsilon_p$ | 0.012 |

3.2. Numerical model of rigid projectile penetrating concrete

The rigid model is adopted for the projectile, and the geometry of rigid projectile is taken from the projectile used in Hanchak test [11]. $L=101.6$ mm, $d=25.4$ mm, $H=42.1$ mm, and warhead radius ratio is 3. The geometry of projectile is shown in figure 8.

Figure 8. Geometry of projectile.

The size of concrete target is 500 mm $\times$ 500 mm $\times$ 500 mm, the volume ratio of aggregate is 30%, the maximum aggregate size is about 80 mm, and the aggregate grading is continuous gradation. In order to observe the deflection attitude of the projectile when the rigid projectile penetrates concrete target, and reduce the modeling and calculation time, the 1/2 symmetric model is used in the calculation. The calculation model is shown in figure 9.
Figure 9. Calculation model of rigid projectile and concrete target.

3.3. Finite element calculation of concrete under uniaxial compression

In order to verify the stability of the model, three models are randomly generated according to the same parameters. The uniaxial compression simulation is carried out with LS-DYNA software to obtain the compressive stress-strain curve of concrete. In simulation, the 150 mm cube specimen is designed and placed on a based-plate whose boundary is fully fixed. A uniform pressure is applied vertically to the top-plate [12]. Figure 10 shows the axial stress and strain relationship obtained by three simulations. From the simulation results, it can be concluded that the modeling method has good stability and reliability. The maximum peak compressive strength of concrete is about 37 MPa, and the main mode of model failure is oblique shear failure.

Figure 10. Uniaxial compression results of three simulations.

3.4. Numerical results and analysis of penetration

The influence of concrete aggregate on the penetration performance of rigid projectile is studied. The rigid projectile penetrates the concrete target with the aggregate volume ratio of 30%, at the speed of 600 m/s. Penetration process of projectile is shown in figure 11. In the process of rigid projectile penetrating concrete target, part of kinetic energy will be converted into internal energy, and the remaining kinetic energy will be used to maintain the forward penetration ability of projectile. Due to the large kinetic energy of the projectile at the initial stage of penetration, the unbalanced force produced by the aggregate and mortar near the warhead is not enough to affect the deflection of the rigid projectile. However, with the decrease of the kinetic energy of the rigid projectile, the influence of the unbalanced force on the ballistic deflection begins to gradually increase, and the projectile appears obvious deflection, and the deflection position appears at the moment when the warhead penetrates the aggregate.
In the following analysis, the author focuses on the influence of aggregate volume ratio on penetration velocity of projectile, under the premise that the aggregate grading of concrete target is continuous gradation and random distribution. The aggregate volume ratio is set at 25% - 40%, and the impact velocity of rigid projectile is 600 m/s.

**Figure 11.** Penetration process of projectile.

**Figure 12.** Curve of penetration velocity and acceleration of projectile with displacement.

**Figure 13.** Effects of aggregate volume ratio on residual velocity.
Figure 12 shows the velocity and acceleration of projectile with penetration displacement in the process of projectile penetrating concrete target. The effect of aggregate volume ratio on residual velocity is shown in figure 13. It can be seen that the penetration velocity of the projectile decreases with the increase of penetration displacement. The residual velocity of projectile decreases with the increase of aggregate volume ratio, and the acceleration increases with the increase of aggregate volume ratio. When the volume ratio of aggregate increases from 25% to 30%, the residual velocity decreases by 19%. When the aggregate volume ratio increases from 30% to 40%, The change range of residual velocity and acceleration of projectile is small. There are several peaks in the acceleration curve, which are respectively at the time of penetration into the pit and impact of the projectile on the aggregate. The head of the projectile will deflect to the side with small resistance, that is, to the mortar side.

4. Conclusion
In this paper, a new aggregate generation algorithm is proposed, which combines the 3D Voronoi method, and the influence of aggregate volume ratio on the anti-penetration performance of concrete targets is analyzed. Under the premise that the aggregate gradation of the concrete is continuous gradation and randomly distributed, the penetration velocity of the projectile decreases with the increase of penetration displacement, the residual velocity and acceleration of projectile decreases with the increase of aggregate volume ratio. When the aggregate volume ratio increases from 25% to 30%, the residual velocity of the projectile decreases the most, with 19%. The analysis results can provide reference for the design of concrete protective structure.

References
[1] Bin L, Qihua W and Yonggang LU. 2014 Concrete (6) 16-9
[2] TEYMEN A and KARAHAN O. 2008 Cement and Concrete Composites (30) 290-6
[3] Forrestal M J and Tzou D Y. 1997 International Journal of Solids and Structure 34(31-32) 4127-46
[4] María López de Murphy. Characterization, 2006 Modeling and size effect of concrete-epoxy interfaces (The Pennsylvania State University. 300 North Zeeb Road)
[5] Wang Z M, Kwan A K H and Chan H C. 1999 Computers & Structures 70(5) 533-44
[6] Le X U, Huiwei Y and Jianxing HU et al. 2015 Journal of Building Structures
[7] Ridel W, Thoma K and Hiermaiser S. 1999 Penetration of reinforced concrete by BETA−B−500 numerical analysis using a new macroscopic concrete model or hydrocodes 9th International Symposium, Interaction of the Effects of Munitions with Structures
[8] Johnson G R and Holmquist T J. 2008 An improved computational constitutive model for brittle materials AIP Conference Proceedings. American Institute of Physics
[9] Hongxin H, Huisuo Z, Shanshan M and Fan Z. 2019 Ordnance Industry Automation 38(9) 60-3
[10] Hongchao L. 2016 The study of the rock RHT model and to determine the values of main parameters (China University of Mining & Technology, Beijing)
[11] Hanchak SJ and Forrestal MJ. 1992 Int J Impact Engng 12(1) 1-7
[12] Zhenhai G, Xiuqin Z and Dacheng Z et al. 1982 Journal of Building Structures 3(1) 1-12.