Damage constitutive model of pure copper at different annealing temperatures

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Abstract. Aiming at the problem of damage evolution of pure copper during the plastic deformation, the normalized shape factor is introduced based on the RO model (Ramberg-Osgood model). The mesoscopic damage constitutive model of pure copper at different annealing temperatures is established and the tensile deformation of industrial pure copper at different annealing temperatures is analyzed. The results show that the error between the calculated value and the experimental value of the damage constitutive model, based on normalized shape factor, at different annealing temperatures, is less than 10%. The model can effectively reveal the tensile damage evolution behavior of industrial pure copper and accurately predict the plastic tensile flow stress of industrial pure copper at different annealing temperatures. The hardening coefficient and hardening exponent in the model are closely related to the annealing temperature of the material. The annealing temperature has little effect on the hardening exponent and has a significant effect on the hardening coefficient and the hardening coefficient decreases with the increase in annealing temperature.

1. Introduction
Meso scale damage evolution law of materials can be obtained by the meso scale study of microstructure of materials. Constitutive relation of materials is another important way to analyze the meso damage and deformation mechanism of materials [1]. At present, research on damage constitutive relation of metal materials based on microvoid is relatively few [2-3]. However, quantitative description on pore size and pore of materials area focuses on laboratories. In addition, the damage description by micro hole is mostly concentrated in the later stage evolution behavior of materials. With development of damage, theory Weckman tried to describe the damage constitutive relation of materials through grain boundary as a damage variable. The method would become a new attempt [4].

Messner et al. developed a multi-scale grain boundary damage model for simulating grain boundary delamination [5], which can be used to analyze the failure of specific grain boundaries with common normal direction. The main characteristics of the model were physical consistency and grid independence. The damage of multiple grain boundaries on the microscopic scale and the macro scale was described in the whole failure process of the materials on the plane. Chen et al. studied the viscosity of grain boundary participating in plastic deformation titanium alloy [6]. Based on ABAQUS platform, elasto viscoplastic constitutive model of the grain boundary deformation damage was established, which can effectively reflect the grain boundary slip behavior of industrial pure titanium during uniaxial tension at low strain rate.
At present, most of research on grain boundary damage constitutive relation was analyzed by computer simulation, grain boundary damage constitutive models and the influence of annealing temperature relation between [7-8] JC (Johnson-Cook model) constitutive model and GTN (Gurson-Tvergaard-Needleman model) constitutive model, the RO model were also commonly used constitutive models. The GTN model needs nine parameters, which are difficult to solve and have great limitations. Compared to the RO model, RO model has a simple form and less parameters to be determined, which is suitable for materials with obvious nonlinear characteristics. The stress-strain behavior of materials was simulated in plastic deformation stage was studied based on the conventional mechanical properties the damage constitutive relation of industrial pure copper [9-10]. By the normalized shape factor, damage constitutive model of material was established based on the RO model. The error of the model was analyzed by comparing the calculated value to the experimental value. Finally, the influence of annealing temperature on the damage constitutive parameters was analyzed by varying the parameters of the model under different annealing temperatures.

2. Experimental materials and methods
The test material is T2 pure copper, and the main components of the material are shown in Table 1.

| Cu  | Sb | Pb | S  | Bi |
|-----|----|----|----|----|
| 99.9| 0.2| 0.4| 0.5| 0.2|

The T2 pure copper was cut into a tensile specimen by wire cutting. The thickness of the specimen was 2mm and the specimen size was shown in Figure 1. Room temperature was 20℃. The samples were annealed in the annealing furnace (OTF-1200X). The specimens were heated to 300℃ for 60 min, 500℃ for 60 min, and 600℃ for 60 min. After heat treatment, the samples were polished with a corrosive agent (5 g FeCl₃+ 10 ml HCl + 100 ml H₂O) to observe the microstructure of the surfaces. The tensile test was carried out through the MTS (MTS Landmark) and the loading rate was 0.2mm/min by displacement. The laser extensometer (MTS Laser Extensometer LX500) was used for displacement measurement.

3. The RO damage constitutive model
The effective stress can be defined as the ratio of the external loading F to the effective carrying area \( \hat{A} \), and the effective stress \( \hat{\sigma} \) was expressed as:

\[
\hat{\sigma} = \frac{F}{\hat{A}} = \frac{p}{S-S_D} \tag{2-1}
\]

According to literature [1], effective stress \( \hat{\sigma} \) and damage factor \( \omega \). The relationship can be expressed as:

\[
\hat{\sigma} = \frac{\sigma}{1-\omega} \tag{2-2}
\]
For the non-destructive materials, damage factor $\omega = 0$; When the material is damaged, the damage factor $0 < \omega < 1$; Loss factor when material fails $\omega = 1$. Damage factor $\omega$ the physical meaning of [11] is similar to that of normalized shape factor $D$, and it varies from 0 to 1. When damage factor are 0, they are all nondestructive. When damage factor are 1, the material fails or breaks, so the relationship between effective stress $\tilde{\sigma}$ and normalized shape factor $D$ can be expressed as:

$$\tilde{\sigma} = \frac{\sigma}{1-D}$$  \hspace{1cm} (2-3)

The normalized shape factor $D$ can change the grain boundary evolution from 0 to 1, which reflects the damage degree of yielding, strengthening, necking, weakening and breaking stage during the plastic deformation. When $D=0$, the material is in a state of non-destruction. When $0<D<1$, the material occurs to damage. When $D=1$ is applied, the material occurs failure. The formula derivation and evolution characterization of normalized shape factor $D$ are described in detail [11].

The industrial pure copper belong to T2 pure copper, elastic modulus of the material was 113.5 GPa. The RO model was chosen to consider strain hardening. Normalized shape factor $D$ was introduced to form the constitutive relation of pure copper with damage:

$$\varepsilon = \left(\frac{\tilde{\sigma}}{K}\right)^M = \left(\frac{\sigma}{(1-D)K}\right)^M$$  \hspace{1cm} (2-4)

where:
- $K$——Hardening coefficient of materials
- $M$——Hardening exponent of materials

The formula (2-4) can also be written as follows:

$$\sigma = (1-D)Ke^M$$  \hspace{1cm} (2-5)

The RO model was a power hardening model, which was in better agreement with the stress-strain relationship of the material in plastic deformation. Combined with the stress-strain relationship of the elastic deformation part of the material, the elastoplastic constitutive model of the material can be expressed as:

$$\sigma = \begin{cases} 
E\varepsilon & (\varepsilon \leq \frac{\sigma}{K}) \\
(1-D)Ke^M & (\varepsilon > \frac{\sigma}{K}) 
\end{cases}$$  \hspace{1cm} (2-6)

4. Results and Analysis.

In order to verify the validity of the new constitutive model with damage, it was necessary to analyze the relationship between the effective stress and the strain and the normalized shape factor. By analyzing the variation rule of effective stress of pure copper, the effective stress and the strain curves of industrial pure copper at different annealing temperatures were established, as shown in Figure 2. In the elastic stage, plastic deformation did not occur, the normalized shape factor was 0, the effective stress of industrial pure copper was the same as that of nominal stress. When the material entered the plastic deformation stage, the effective stress increased with the increase of plastic strain. The greater the plastic strain, the more effective stress increased. With the increase of annealing temperature, the effective stress of pure copper increased slowly. It can be seen that the effective stress of pure copper at different annealing temperatures had a similar change rule with the increase of strain. The annealing temperature had a great influence on the effective stress of plastic deformation stage.
Figure 2. Effective stress versus strain

Figure 3 should show the relationship curve between the effective stress and normalized shape factor of industrial pure copper at different annealing temperatures. The diagram showed that the effective stress and normalized shape factor curve of the material were similar to figure 2, with the increase of normalized shape factor. The curves of effective stress of industrial pure copper at different annealing temperatures with normalized shape factor were similar. It can be seen from reference [11] that the normalized shape factor of industrial pure copper increased, the damage degree of the material increased. For high annealing temperature, the internal grains tended to be round, the internal dislocation hindrance was small. The elongation degree of the grains was increased and the bearing area of the materials was decreased greatly. When the annealing temperature continued to increase from 500 °C to 600 °C, due to the rolling structure has disappeared [11], the reduction of dislocation slip resistance became slower. The deformation driving force and the bearing area of the material slowly decreased, the reduction of effective stress was small.

Figure 3. Effective stress versus normalized shape factor

Based on the relationship between the effective stress and plastic strain of pure copper in Figure 2, the normalized shape factors of industrial pure copper at different annealing temperatures were substituted into the power hardening constitutive model, and the effective stress-strain curve was fitted by the RO model, as shown in Figure 4. The hardening coefficient $K$ and hardening index $M$ were obtained by the fitting equation, as shown in Table 2. The data in Table 2 were substituted into formula (1-6), The damage constitutive relation of industrial pure copper at different annealing temperatures can be obtained. According to the data in Table 2, with the increase of annealing temperature the hardening coefficient $K$ of the pure copper damage constitutive model decreased the effect of annealing temperature was more significant. The hardening exponent $M$ increased first and then decreased with the increase of annealing temperature. The results showed that the hardening
parameters of the T2 pure copper were closely related to the annealing temperature and the material parameters change regularly with the increase of annealing temperature.

Figure 4. Fitting curve of effective stress versus strain

Table 2. Model parameters with different annealing temperatures

| Annealing temperature (℃) | K (MPa) | M   |
|---------------------------|---------|-----|
| 20                        | 723.2   | 0.3 |
| 300                       | 640.1   | 0.31|
| 500                       | 488.6   | 0.49|
| 600                       | 278.8   | 0.37|

To investigate the fitting accuracy of the power hardening constitutive model of industrial pure copper at different annealing temperatures, the experimental curve was compared, as shown in Figure 5. In the elastic deformation stage, the constitutive model of pure copper under different annealing temperatures followed Hooke's law. The experimental values were in agreement with the calculated values. In plastic deformation stage, the calculated values of industrial pure copper deviated from the theoretical value, and the overall deviation was small. By comparing the calculated stress values to the experimental values in the damage constitutive equation, it was found that the calculated values of stress in the power hardening constitutive model can accurately fit the experimental values at the yield stage and then higher than the experimental values. The maximum error was 11.7%, when the annealing temperature was 20 ℃; when annealing temperature was 300 ℃, the average error was 7.9%; when annealing temperature was 500 ℃, the average error was 5.6%; when annealing temperature was 600 ℃, the maximum error was 16.1%, the average error was 5.8%. The results showed that the average error of industrial pure copper at different annealing temperatures was less than 10%, which indicated that the damage constitutive equation can accurately predict the flow stress [12] of material with plastic deformation. The damage constitutive model, based on normalized shape factor, can reflect the stress-strain relationship of industrial pure copper at different annealing temperatures more accurately.
Figure 5. Comparison curve between calculated value and experimental value of constitutive model with different annealing temperature.

Figure 6 shows the relationship between the annealing temperature and the average error of the model. It can be seen that with the increase of annealing temperature, the average error between the calculated stress and the experimental value of the new damage constitutive model gradually decreased. This was because the annealing temperature increased, the grain boundary of industrial pure copper became clearer, and the statistical error of normalized shape factor was reduced, thus reducing the stress calculation error of the damage constitutive model.

Figure 6. Average error versus annealing temperature.
5. Conclusions
In this paper, the normalized shape factor was introduced into the RO model to study the damage constitutive relation of pure copper under different annealing temperatures.

(1) The average error of the damage constitutive model, based on normalized shape factor, is less than 10% of the experimental value. The damage constitutive model can reflect the stress-strain relationship of industrial pure copper more accurately and provide the reference for study of the constitutive relationship of ductile metal sheet with different annealing temperatures.

(2) The hardening parameter of the pure copper damage constitutive model was closely related to the annealing temperature of the material. The annealing temperature had few effect on the hardening exponent, but had a significant effect on the hardening coefficient. The hardening coefficient decreased with the increase of annealing temperature.

(3) With the increase of annealing temperature, the statistical error of normalized shape factor decreased. The model was suitable for metal materials with higher annealing temperature.

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