Temporal double-slit interferences of a quantum fluxon in quantum Josephson nanocircuits

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Abstract. We investigate temporal Young’s double-slit interferences of a quantum fluxon in quantum Josephson nanocircuits, in order to explore the nonlocality of time at macroscopic scales. We employ Gaussian wave-packet approaches to describe the time evolution of the quantum fluxon. We obtained an analytic formula for the probability density of finding the quantum fluxon at a detector’s position as a function of time, and found interference fringes in the time domain. This shows nonlocal nature of time at macroscopic scales and also provides potential applications for a new type of building blocks of quantum information in solid state devices.

1. Introduction
Quantum mechanics led us to a counterintuitive concept, i.e. wave-particle duality. Young’s double slit experiment which revealed the wave nature of the light also greatly contributed to clarify this concept from the viewpoint of wave nature of the matter by running the same setup except a special feature of a source that outputs just one photon at a time. Needless to say, Heisenberg’s uncertainty principle lies behind this concept. It has been predominantly studied on quantum nonlocality based on the uncertainty relation between position and momentum. On the other hand, it is possible to study quantum nonlocality based on another type of uncertainty relation, i.e. uncertainty of energy and time. In this case, the nonlocality is related to time. Thus, there has been considerable interest in temporal version of double-slit experiments [1-4] and has been successfully confirmed in experiments at microscopic scales.

In contrast, there has been discussed as another fundamental problem in quantum mechanics. It is a question of whether quantum mechanics is applicable to macroscopic systems. A concrete example is a superposition of quantum states at macroscopic scales, i.e. the Schrödinger cat as well as double-slit interferences. This issue has been tested in various systems. It is noteworthy that interference fringes were demonstrated in a double slit experiment of a fullerene molecule regarded as a macroscopic object [5]. However, double-slit interference in the time domain has not yet been demonstrated so far.

In this paper, we propose a simple scheme for investigating temporal Young’s double-slit interferences to explore quantum nonlocality in time at macroscopic scales through a quantum fluxon that allows delicate control with state-of-the-art technology instead of fullerene C$_{60}$ used in the previous studies.
2. Temporal Young’s double-slit interferences of a quantum fluxon

2.1. Quantum fluxons
A Josephson junction is a superconducting device made up of two superconducting materials separated by thin insulating layer capable of electron pair (Cooper pair) tunneling. The tunnel current across the junction can flow depending on the phase difference of the order parameters between superconductors without an applied voltage. The phase difference is governed by sine-Gordon equation based on the Maxwell equation in a long Josephson junction (see Figure 1).

The sine-Gordon equation is a nonlinear wave equation involving a unique solution called kink (antikink). The kink solution expresses a topological nonlinear solitary wave travelling in the nonlinear dispersive system without changing its shape and exhibits particle nature even if it is a basically wave. The physical background of this solution is circulating currents across the junction that produces magnetic field locally. Since it possesses a quantum unit of magnetic flux $\Phi_0 = \hbar/2e$, it was named fluxon. Thus, the fluxon is a nonlinear elementary excitation within a long Josephson junction and behaves like a Newtonian particle moving along the Josephson circuits. Note that it is regarded as a macroscopic object since the fluxon is created by a huge number of circulating electrons (supercurrents) across the junction.

The fluxon has been considerably studied in applications to information transfers or controls within the framework of classical physics so far [6]. Later, quantum nature of the fluxon has been demonstrated experimentally in quantum dissociation of a bound fluxon pair in annual Josephson circuits [7]. The quantum-mechanical interference of fluxon has been also considered theoretically in connection of the superposition of spatially distinct positions of a fluxon [8], but it has not yet observed directly due to mainly technical difficulty for making spatial fluxon interferometer in current fabrication technology. In contrast, it seems that a temporal interferometer we discuss below is feasible to achieve at existing technologies.

![Figure 1](image1.png)

**Figure 1.** A schematic diagram of a fluxon created by a circulating current in a long Josephson junction composed of an insulating thin layer sandwiched by two superconductors.

2.2. Temporal Young’s double-slit interferometers
In a temporal double-slit experiment, only one slit with a shutter is placed in front of the screen (or detector) in contrast to a spatial double-slit experiment. The shutter that can be opened and closed forms double slits in different times.

Suppose that a fluxon is initially pinned by a microshort of a long Josephson junction. The fluxon is then irradiated by a coherent superposition of two temporally separated superposition states for a single photon, sometimes called a time-bin photon, which is produced by a Mach-Zender interferometer. The pinned fluxon can then move due to collision with the photon. Since the photon is in temporally superposition states, the moving fluxon is also in temporally superposition states. In other words, the superposition of macroscopically distinct states in time is produced by the media conversion from a time-bin photon to a time-bin fluxon. Figure 2 shows the schematic diagram of a setup for temporal Young’s double slit experiments in a Josephson system we described above.
Figure 2. A proposed setup of temporal Young’s double-slit experiment of a quantum fluxon. BS and M stand for a half beam splitter and a mirror for a photon, respectively.

The trajectories of a fluxon in spacetime for this interferometer are shown in Figure 3. A fluxon is initially prepared at a microshort. At $t = 0$, the fluxon starts to move with a 50% chance due to the collision with an advanced photon of a time-bin photon. The remaining 50% portion of the fluxon remains intact until a retarded photon comes. After the duration time $T_1$, the remaining fluxon will start to move at the same velocity of the advanced portion of the fluxon. We assume that fluxon will eventually move by collision with superposed photons. The interference terms are ignored to avoid complication in Figure 3.

Figure 3. Trajectories of a fluxon after collisions with a single photon in a temporally superposition states.

3. Time evolution of temporally superposition states of a quantum fluxon

3.1. Gaussian wave-packet approach

Now let us calculate time evolution of the wavefunction in this system. The time evolution of the wavefunction of a free quantum fluxon with mass $m$ is given by

$$\psi(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{i p^2}{2m} t} e^{-i\frac{i}{\hbar}p(x-x')} \psi(x', 0) dx' dp$$  (1)
where $p$ is a fluxon momentum. We employ Gaussian wave packet as an initial state of a stationary fluxon given as

$$
\psi(x, 0) = \frac{1}{(2\pi\sigma_0^2)^{1/4}} e^{-\frac{x^2}{4\sigma_0^2}}
$$

(2)

where $\sigma_0$ is a parameter for a Gaussian width. The wave function of the fluxon which start to move by collision with a photon of momentum $p_0$ can be represented as

$$
\psi'(x, 0) = \psi(x, 0) e^{i\frac{p_0}{\hbar}x}.
$$

(3)

Using this equation, the wavefunction of the moving fluxon in the first part of the superposition state at time $t$ is given by

$$
\psi_1(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{p^2}{2\hbar m} + i\frac{p}{\hbar}(x-x')} \psi'(x', 0) dx' dp
$$

$$
= \left[\frac{1}{\sqrt{2\pi\sigma_0(1+i\frac{t}{\tau})}}\right]^{1/2} \exp\left\{ -\frac{(x-v_0)^2}{4\sigma_0^2\left[1+i(\frac{t}{\tau})\right]} \right\} \left(1 - i\frac{t}{\tau}\right) + \frac{i}{\hbar} \left[p_0 x - \frac{p_0^2}{2m} t\right] \right\}. \tag{4}
$$

where $v_0$ is a velocity converted from momentum $p_0$. On the other hand, the rest part of the superposition state of the fluxon remains intact until the subsequent partner photon in the Mach-Zender interferometer comes at time $T_s$, i.e. the time slit interval. This expression is obtained by inserting equation (2) into equation (1) as

$$
\psi_2(x, T_s) = \left[\frac{1}{\sqrt{2\pi\sigma_0(1+i\frac{t}{\tau})}}\right]^{1/2} \exp\left\{ -\frac{x_0^2}{4\sigma_0^2\left[1+i(\frac{T_s}{\tau})\right]} \right\} \left(1 - i\frac{T_s}{\tau}\right). \tag{5}
$$

The expression of the wavefunction for the moving fluxon by collision with the partner photon of the same momentum $p_0$ at time $t$ is given by

$$
\psi_2(x, t) = \left[\frac{m}{2\pi\hbar(1-v_0)}\right]^{1/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{mict(x-x')^2}{2\hbar(1-v_0)}} \psi_1(x', T_s) e^{i\frac{p_0}{\hbar}x'} dx'
$$

$$
= \left[\frac{1}{\sqrt{2\pi\sigma_0(1+i\frac{t}{\tau})}}\right]^{1/2} \exp\left\{ -\frac{(x-v_0)^2}{4\sigma_0^2\left[1+i(\frac{T_s}{\tau})\right]} \right\} \left(1 - i\frac{T_s}{\tau}\right) + \frac{i}{\hbar} \left[p_0 x - \frac{p_0^2}{2m} (t - T_s)\right] \right\}. \tag{6}
$$

Here we introduce a comoving frame by setting the spatial slit interval as $v_0T_s$, in order to simplify the situation. Then the wavefunctions can be reexpressed as

$$
\psi_1'(x, t) = \left[\frac{1}{\sqrt{2\pi\sigma_0(1+i\frac{t+T_s}{\tau})}}\right]^{1/2} \exp\left\{ -\frac{(x-v_0(t+T_s))^2}{4\sigma_0^2\left[1+i(\frac{t+T_s}{\tau})\right]} \right\} \left(1 - i\frac{t+T_s}{\tau}\right) + \frac{i}{\hbar} \left[p_0 x - \frac{p_0^2}{2m} (t + T_s)\right] \right\}. \tag{7}
$$

$$
\psi_2'(x, t) = \left[\frac{1}{\sqrt{2\pi\sigma_0(1+i\frac{t+T_s}{\tau})}}\right]^{1/2} \exp\left\{ -\frac{(x-v_0(t-T_s))^2}{4\sigma_0^2\left[1+i(\frac{t+T_s}{\tau})\right]} \right\} \left(1 - i\frac{t+T_s}{\tau}\right) + \frac{i}{\hbar} \left[p_0 x - \frac{p_0^2}{2m} (t - T_s)\right] \right\}. \tag{8}
$$

The whole wavefunction is represented by the superimposition of these,

$$
\psi(x, t) = N[\psi_1'(x, t) + \psi_2'(x, t)]
$$

(9)

where the normalization constant $N$ can be represented as
\[ N = \left( 2 \left\{ 1 + \exp \left[ -\frac{(v_0 T_s)^2}{4 \sigma_0^2} \right] \cos \left( \frac{p_0^2}{m \hbar T_s} \right) \right\} \right)^{-1/2}. \]  

Therefore, the probability density is then given as
\[
|\psi(x, t)|^2 = \frac{2N^2}{\sqrt{2\pi \sigma_0} \left[ 1 + \left( \frac{t+T_s}{\tau} \right)^2 \right]^{1/2}} \exp \left\{ \frac{-2[(x-v_0 t)^2+(v_0 T_s)^2]}{4 \sigma_0^2 \left[ 1 + \left( \frac{t+T_s}{\tau} \right)^2 \right]} \right\} \times \left( \cosh \left\{ \frac{4(x-v_0 t)v_0 T_s}{4 \sigma_0^2 \left[ 1 + \left( \frac{t+T_s}{\tau} \right)^2 \right]} \right\} + \cos \left\{ \frac{4(x-v_0 t)v_0 T_s}{4 \sigma_0^2 \left[ 1 + \left( \frac{t+T_s}{\tau} \right)^2 \right]} \left( \frac{t+T_s}{\tau} + \frac{p_0^2}{m \hbar T_s} \right) \right\} \right). \]  

The hyperbolic cosine term in the parenthesis shows the classical behavior of two Gaussian wave packets, while the cosine term represents interference between them. It is noteworthy that the frequency of interference depends on time. We will discuss this feature concretely in the following.

### 3.2. Interference fringes

Now we display the time evolution of the wave packet based on the obtained expression. Here we assume that the mass of fluxon is about 1/100 of the electron \([9]\) and the fluxon speed is 1/100 of the speed of light \([10]\). The initial spread of the Gaussian function \(\sigma_0\) is assumed to be the size of fluxon \(\lambda_f = 1 \times 10^{-6} \text{m}\). In the following discussion, optimization for the observation is not done. However, experimental observation can be feasible since the time scale is located in picoseconds.

Figure 4 shows the probability density of finding a fluxon as a function of time at various detector positions. Over time, a second wave packet is observed after the first wave packet. In addition, as a feature of the Gaussian wave packet, it can be seen that the amplitude decreases with the wave packet propagation and the wave packet also spreads due to wave number dispersion. The spread of the wave packet leads to the superposition of the two wave packets, and the interference pattern is manifested at the observation point away from the slit where the overlap becomes large.

**Figure 4.** Probability density of a fluxon as a function of time at various detector positions, (a) for \(10^2 T_s\), (b) for \(2 \times 10^2 T_s\), (c) for \(3 \times 10^2 T_s\) and (d) for \(4 \times 10^2 T_s\), respectively. The characteristics of temporal interference using the Gaussian wave packets appear in the oscillation period. Unlike the oscillation patterns in a space double-slit experiment, the oscillation periods are not
same at different detector positions. The oscillation frequencies of interference fringes in $|\psi(x, t)|^2$ is given as

$$\omega(t) = \frac{v_0^2 T_s (1 + T_s)}{\sigma_0^2 \left[ 1 + \left( \frac{T_s x}{\sigma^2} \right)^2 \right] \tau t^2}$$

and is shown as a function of time in Figure 5. The frequency monotonically decreases with time. Therefore, the period of the interference pattern increases as the position of the detector moves away from the microshort at which fluxon starts to move. This feature is a strong evidence for temporal interference.

![Figure 5. Time dependent oscillation frequencies of interference fringes.](image)

4. Concluding remarks
The temporal Young’s double-slit interferences of a quantum fluxon have been investigated for exploring quantum nonlocality of time at macroscopic scales. We employ Gaussian wave-packet approaches to describe the time evolution of the quantum fluxon. We have obtained an analytic formula for the probability density of finding the quantum fluxon at a detector’s position as a function of time and found interference fringes in the time domain with a frequency dependent on the detector position. This showed nonlocal nature of time at a macroscopic scale. This temporal interference also provides potential applications for a new type of building blocks of quantum information in solid state devices.

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6. References
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