A universal assortativity measure for network analysis

Guo-Qing Zhang\textsuperscript{1,∗} Su-Qi Cheng\textsuperscript{1,3,†} and Guo-Qiang Zhang\textsuperscript{1,‡}

\textsuperscript{1}Institute of Computing Technology, Chinese Academy of Sciences
\textsuperscript{2}School of Computer Science and Technology, Nanjing Normal University and
\textsuperscript{3}Graduate University of Chinese Academy of Sciences

Characterizing the connectivity tendency of a network is a fundamental problem in network science. The traditional and well-known assortativity coefficient is calculated on a per-network basis, which is of little use to partial connection tendency of a network. This paper proposes a universal assortativity coefficient (UAC), which is based on the unambiguous definition of each individual edge’s contribution to the global assortativity coefficient (GAC). It is able to reveal the connection tendency of microscopic, mesoscopic, macroscopic structures and any given part of a network. Applying UAC to real world networks, we find that, contrary to the popular expectation, most networks (notably the AS-level Internet topology) have markedly more assortative edges/nodes than dissortative ones despite their global dissortativity. Consequently, networks can be categorized along two dimensions—single global assortativity and local assortativity statistics. Detailed anatomy of the AS-level Internet topology further illustrates how UAC can be used to decipher the hidden patterns of connection tendencies on different scales.

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I. INTRODUCTION

Network has become a useful and proliferative tool in a wide spectrum of research areas, ranging from traditional communication and transportation networks to more recently emerging networks as complex as online social networks and brain networks \textsuperscript{[1–15]}.. Assortativity coefficient is a basic metric that characterizes the connectivity tendency of a network, i.e., globally, whether nodes of similar (or dissimilar) degrees are more likely to be connected \textsuperscript{[10]}. However, this metric is a macroscopical property, which becomes useless when microscale or mesoscale level analysis is required. In other words, one can not tell the exact intra-group or inter-group connection tendencies from the per-network assortativity coefficient.

Experimental studies have shown that various forms of groups are hidden in real networks. These groups can take the form of community, motif, clique, etc \textsuperscript{[19–21]}. Multi-scale, especially mesoscale analysis is very important to understand the roles and dynamics of these groups \textsuperscript{[17,18]}. However, previous studies typically focus on the uncovering of these groups within a network, and treat isomorphic modular components to be identical. In other words, the component is solely studied as a subgraph extracted out of the whole network, totally neglecting the links connecting this subgraph to other parts of the graph. Obviously, this traditional method inevitably fails to capture the functional difference between isomorphic modular components. Indeed, functional roles or dynamics of a group can only be comprehensively understood when it is put in the global context. An important distinguishable property is whether the group under consideration is assortatively mixed or dis assortatively mixed within its local surroundings, which can have quite different influence on the dynamics, e.g., information diffusion/disease spreading \textsuperscript{[22]}, resilience against attacks \textsuperscript{[23]}. Fig. 1 gives an illustrative example. In this figure, two triangles $A$ and $B$ are located in different surroundings. Triangle $A$ is surrounded by high-degree nodes, i.e., dissortatively mixed with the outside world, whereas triangle $B$ is surrounded by low-degree nodes, i.e., assortatively mixed with the outside world. This causes $A$ and $B$ to behave quite differently in the process of information or disease diffusion. In this simple example, suppose SIR model is used to model a disease spreading process and the infectious probability $p$ is set to 0.5. If $A$ serves as the source of the spreading process, the expected number of infected nodes accounts for about 23% of all the nodes, in contrast, if $B$ serves as the source, then only less than 4% of the nodes are expected to be infected. This drastic discrepancy apparently comes from the difference in the connectivity tendency between the two triangles and their respective outside worlds.

Hence, in order to exactly analyze and explore network structure, which is beneficial to better understand the dynamics of complex systems, it is of critical significance to perform intra-group or inter-group connection tendency measurement in the global context. In this paper, we propose a universal assortativity coefficient that is based on the unambiguous definition of each individual edge’s contribution to the global assortativity coefficient. This metric allows assortativity analysis on any part of a network and reveals some hidden network connectivity patterns.
II. UNIVERSAL ASSORTATIVITY COEFFICIENT

In order to measure the assortativity of the network on different scales, we proposed a uniform metric called universal assortativity coefficient that measures the assortativity of any subset of connections. Simply put, it is the summation of each individual edge’s contribution to the global assortativity coefficient. Hence, we begin with our definition of each individual edge’s contribution to the global assortativity coefficient.

Before the formal definition, it is necessary to review some related concepts discussed by Newman [16]. For simplicity, all the concepts we discuss are based on undirected networks. With minor or moderate adjustments, these concepts can also be applied to directed networks. Degree distribution \( p(k) \) refers to the probability that a randomly chosen node is of degree \( k \). The remaining degree distribution \( q(k) \) refers to the probability that following a randomly chosen edge, the remaining degree of the reached node is \( k \). Here, the remaining degree is the number of edges leaving this node other than the one we arrived along. This number is one less than the total degree of this node. The normalized distribution \( q(k) \) of the remaining degree is:

\[
q(k) = \frac{(k + 1)p(k + 1)}{\Sigma jp_j}
\]  

(1)

Joint probability distribution of the remaining degrees of two endpoints at either end of a randomly chosen edge \( e_{ij} \) is the probability that the remaining degrees of two endpoints of a randomly chosen edge are \( i \) and \( j \).

Following these definitions, the assortativity coefficient \( r \) is defined as:

\[
r = \frac{1}{\sigma_q^2} \Sigma [\Sigma jk (e_{jk} - q(j)q(k))] \]  

(2)

where \( \sigma_q \) is the standard deviation of the remaining degree distribution \( q(k) \).

For uncorrelated network, \( r = 0 \); when the network is assortatively mixed, i.e., nodes of similar degrees are more likely to get connected, \( r \) is positive; when the network is dissortatively mixed, i.e., nodes of dissimilar degrees tend to connect to each other, \( r \) is negative.

Now considering each individual edge’s contribution to the network assortativity coefficient \( r \). Denote \( U_q = \Sigma jq(j) \) to be the expected value of remaining degree, then \( r \) can be rewritten as:

\[
r = \frac{1}{\sigma_q^2} \left[ \Sigma jk (e_{jk} - q(j)q(k)) \right] 
= \frac{\sigma_{jk} jk e_{jk} - U_q^2}{\sigma_q^2} 
= \frac{\sigma_{jk} jk e_{jk} - \Sigma j q(j) - kq(k) + U_q^2}{\sigma_q^2} 
= \frac{\sigma_{jk} jk e_{jk} - \Sigma j \Sigma k e_{jk} - kq(k) + U_q^2}{\sigma_q^2} 
= \frac{\Sigma jk jk e_{jk} - \Sigma j \Sigma k + U_q^2 e_{jk}}{\sigma_q^2} 
= \frac{\Sigma jk (j + 1) U_q + U_q^2 e_{jk}}{\sigma_q^2} 
= \frac{\Sigma jk (j - U_q)(k - U_q)e_{jk}}{\sigma_q^2} 
= \frac{E(J - U_q)(K - U_q)}{\sigma_q^2}
\]

where \( J \) and \( K \) are variables of the remaining degree, which have the same expected value \( U_q \). Following the above equation, we see that each edge’s contribution to \( r \) is:

\[
\rho_e = \frac{(j - U_q)(k - U_q)}{M \sigma_q^2}
\]

(3)

where \( M \) is the number of edges, and \( j, k \) are the remaining degrees of the two endpoints of edge \( e \). It is easy to see that \( r = \Sigma_{i=1}^M \rho_e \).

When the network is completely homogeneous, i.e., all nodes have the same degree, then \( \sigma_q = 0 \). In this case \( \rho_e \) becomes undefinable. Since in this case, each edge has the same contribution to \( r \), we define \( \rho_e \) to be \( \frac{1}{M} \).

If \( \rho_e > 0 \), then \( e \) is called an assortative edge; otherwise if \( \rho_e < 0 \), it is called a dissortative edge. In this definition, if both the endpoints’ remaining degrees are greater(or less) than the global expected remaining degree \( U_q \), then the edge is assortative, and the more the two endpoints’ remaining degrees deviate from \( U_q \), more assortative the edge is. Otherwise, the edge is dissortative. In other words, the edge assortativeness is a scaled difference between the two endpoints’ remaining degrees and the global expected remaining degree. The absolute value of the contribution \( |\rho_e| \) is termed as the assortative/dissortative strength of the corresponding edge. We define \( S_{ae} \) to be the average strength of assortative edges,
and $S_{de}$ to be the average strength of dissortative edges. The ratio of assortative edges is denoted by $P(\rho_v > 0)$.

Finally, the universal assortativity coefficient for a targeted edge set $E_{\text{target}}$ is defined as:

$$\rho = \sum_{e \in E_{\text{target}}} \rho_e = \sum_{e \in E_{\text{target}}} \frac{(j - U_q)(k - U_q)}{M\sigma_q^2}$$

Based on this metric, it is easy to measure the assortativity on different scales. For example, to measure the connectivity tendency of a single node, denoted as $\rho_v$, simply set $E_{\text{target}}$ to be the edges emanating from the node. If $\rho_v > 0$, then we call $v$ to be an assortative node, otherwise if $\rho_v < 0$, we call $v$ to be a dissortative node.

To measure the connectivity tendency within a group, $E_{\text{target}}$ is set to the edges within this group. If we set $E_{\text{target}}$ to be the whole edge set $E$, then we arrive at Newman’s global assortativity coefficient [16]. In order to measure the connectivity tendency between groups, simply set $E_{\text{target}}$ to be the edges between the groups. In this sense, this metric can be used to measure connection tendencies on different scales, thus, it deserves the name uniform assortativity coefficient (UAC).

Back to our example in Fig. 1, the global assortative coefficient $\rho$ is -0.804, indicating strong dissortativity. However, this global knowledge is of little use to understand the functional roles of local components, such as $A$ and $B$. Based on $UAC$, we can quantitatively measure the connection tendency between $A$ and the remaining graph, as well as between $B$ and the remaining graph. It turns out that the inter-group assortative coefficient between $A$ and the remaining graph is -0.033, whereas the inter-group assortative coefficient between $B$ and the remaining graph is 0.025. As a consequence, although $A$ and $B$ are isomorphic when they are extracted out of the graph, their different connectivity tendencies to the other part of the graph result in drastic discrepancy in the disease spreading process. This example clearly tells us the significance of partial connection tendency for network analysis.

III. REAL NETWORK ANALYSIS

We apply the UAC analysis to various real-world networks. Table. 1 reports $r$, $P(\rho_v > 0)$, $S_{ar}$, $S_{de}$ and $P(\rho_v > 0)$ for different kinds of networks. These networks can be roughly categorized as five kinds: technical networks, biological networks, social networks, online social networks, and synthesized networks.

From this table, we see:

1. For a majority of real networks considered in this paper, e.g., AS, Router, Email-Enron, despite their impressive global dissortativity, we surprisingly find that the number of assortative edges/nodes exceeds dissortative edges/nodes. Whereas for the synthesized ER network, the number of assortative edges almost equals that of dissortative edges, and their average strengths are indistinguishable as well. Hence, the network as a whole has no mixing pattern.

2. The global network assortativity is determined by both the ratio of assortative edges and the strength of these edges. For instance, in SCN, both the ratio of assortative edges and the average strength of assortative edges are greater than dissortative edges, hence it exhibits strong assortativeness as a whole. In comparison, though the number of assortative edges in the AS network also exceeds dissortative ones, the average strength of assortative edges is much weaker than dissortative ones. Hence, the dissortativity of this network comes from the relatively stronger strength of smaller number of dissortative edges. This is true for quite a number of other dissortative networks.

3. Here we reconfirm the fact that online social networks are dissortatively mixed, whereas real-world social networks are assortatively mixed [36]. We observe that the ratio of assortative edges in online social networks are comparatively lower than that of real-world social networks, although the total number of assortative edges still exceeds dissortative ones. However, the average strength of dissortative edges is greater than that of assortative edges in online social networks, in contrast, in real-world social networks, the situation is just the opposite. This reflects the fact that online social networks can to some extent eliminate social barrier between people of different social positions, making it much easier for people at the bottom of society to setup links to people at the top of society.

4. According to global assortativity and local edge assortativity statistics, networks can be categorized to four kinds: globally assortative with leading number of assortative edges, globally assortative but with leading number of dissortative edges, globally dissortative with leading number of dissortative edges, globally dissortative but with leading number of assortative edges. Table II categorizes the networks along the two dimensions. Yet, it still remains an open question whether there is a real network that exhibits global assortativity but primarily consists of dissortative edges.

In the following, we use the AS-level Internet topology as an example to illustrate how the universal assortativity coefficient can be used to calculate the connectivity tendency of intra-group or inter-group connections. In the AS-level topology, a natural group partition of clear and explicit meaning is to partition the ASes according to their geographical regions. Today, five regional Internet registries (RIR) are managing the allocation and registration of Internet number resources (including AS numbers) within a particular region of the world. The five RIRs are: AfriNIC for Africa, ARIN for the United...
TABLE I. Connection tendencies for different categories of networks.

| Category            | Name                  | $r$  | $P(r_{e} > 0)$ | $S_{ae}$  | $S_{de}$  | $P(r_{e} > 0)$ |
|---------------------|-----------------------|------|---------------|-----------|-----------|---------------|
| Technical Network   | AS-2011-6 [24]        | -0.184 | 60.4%  | 1.96 × 10^{-6} | 8.71 × 10^{-6} | 58.3%  |
|                     | Router [10]           | -0.138 | 51.3%  | 5.80 × 10^{-6} | 1.07 × 10^{-4} | 57.5%  |
|                     | USAir [25, 26]        | -0.208 | 42.1%  | 2.76 × 10^{-4} | 3.70 × 10^{-4} | 37.3%  |
| Biological Network  | PPI [27]              | -0.102 | 50.5%  | 6.92 × 10^{-5} | 1.02 × 10^{-4} | 52.6%  |
|                     | celegansneural [28, 29]| -0.163 | 56.7%  | 1.11 × 10^{-5} | 3.21 × 10^{-4} | 42.1%  |
|                     | foodweb_Florida [26, 30]| -0.112 | 49.6%  | 1.90 × 10^{-5} | 2.94 × 10^{-4} | 34.4%  |
| Social Network      | SCN [11, 12]         | 0.161 | 61.4%  | 1.33 × 10^{-5} | 1.01 × 10^{-5} | 69.7%  |
|                     | CA-HepTh [31]        | 0.268 | 63.1%  | 2.78 × 10^{-5} | 9.66 × 10^{-5} | 72.8%  |
|                     | CA-GrOc [31]         | 0.659 | 80.6%  | 6.32 × 10^{-5} | 2.78 × 10^{-5} | 88.9%  |
| Online Social Network| soc-Epinions1 [32]  | -0.041 | 58.8%  | 5.81 × 10^{-6} | 1.07 × 10^{-6} | 71.6%  |
|                     | Email-Enron [33, 34]  | -0.111 | 58.9%  | 1.21 × 10^{-6} | 3.19 × 10^{-6} | 51.5%  |
| Synthesized Network | ER [35]              | -0.001 | 50.7%  | 1.24 × 10^{-6} | 1.27 × 10^{-9} | 50.4%  |

* The result of ER network is an average over 10 times. We treat Soc-Epinions1, celegansneural and foodweb_Florida, originally directed networks, as undirected networks by treating each directed edge as an undirected one and eliminating duplicated edges.

TABLE II. Categorization of networks by global assortativity and edge assortativity statistics.

| $r > 0$ | $r < 0$ |
|---------|---------|
| $P(r_{e} > 0) > 50\%$ | SCN, CA-HepTh, CA-GrOc | AS, Router, soc-Epinions, Email-Enron |
| $P(r_{e} > 0) < 50\%$ | - | USAir, foodweb_Florida |

States, Canada, several parts of the Caribbean region and Antarctica, APNIC for Asia, Australia, New Zealand, and neighboring countries, LACNIC for Latin America and parts of the Caribbean region, and RIPECC for Europe, the Middle East and Central Asia (see Fig. 2 for a graphical representation of the five RIRs’ responsible regions). This gives us a coarse partition of the ASes according to the five regions. A more fine-grained partition is to further divide each region according to countries and regions. Hence, we have a two-level partitioning. The first-level groups consist ASes adhering to the same regional Internet registries, and the second-level groups consist ASes belonging to the same country and region, following the ISO 3166-1 standard.

Table III reports both the intra-RIR and inter-RIR assortativity coefficients. We observe that except for ARIN, other RIRs all show assortativity internally. For inter-RIR connections, we observe that connections between ARIN and all other RIRs show dissortativity. RIPECC exhibits similar phenomenon with ARIN except that its connections with AfriNIC exhibits some sort of assortativity. Connections among AfricNIC, APNIC, and LACNIC, all show assortativity. This connectivity tendency reflects the fact that broadly, the regions covered by ARIN and RIPECC are the core of the Internet. However, RIPECC differs from ARIN in the sense that RIPECC itself is assortative whereas ARIN is dissortative. This could be more appropriately explained by the more fine-grained country and region connection tendencies. Fig. 3 reports the intra- and inter-country and region assortativity coefficients for those countries and regions whose observed ASN numbers are greater than 80 (we choose 80 as a threshold because we want to ensure that each RIR has at least one country or region in this map). In this figure, vacant grid means there is no observed AS connections between the two countries/regions. Different colors are used to discretize the strength of assortativity/dissortativity within and between countries/regions. Several clear patterns can be observed from this plot. Firstly, except for US, all other countries/regions are internally assortatively mixed, as illustrated by the diagonal of the plot. Secondly, there are a few countries/regions, namely, US, CA, GB, EU, DE, that primarily show dissortative connectivity tendencies to other countries/regions. Finally, inter-connections between other countries/regions are mostly assortative.

Statistically, on the RIR scale, we found that about 67.3% intra-RIR edges are assortative, whereas only 32.3% inter-RIR edges are assortative. And on the country/region scale, 69.7% intra-country/region edges are assortative, whereas only 44.9% inter-country/region edges are assortative. Considering the fact that globally an average of 60.4% edges are assortative, it is then apparent that on both scales, edges within the same regional area are more likely to be assortative than the average ratio 60.4%, whereas, edges linking different regional areas are far less likely to be assortative than the average ratio. This locality-driven difference in connec-
TABLE III. Intra-RIR and inter-RIR assortativity coefficients.

| size | AfrinIC | APNIC     | LACNIC | RIPE-NCC | ARIN     |
|------|---------|-----------|--------|----------|----------|
| 557  | 9.35 × 10⁻⁵ | 3.86 × 10⁻⁵ | 1.13 × 10⁻⁴ | 1.24 × 10⁻⁴ | -0.002   |
| 371  | 9.86 × 10⁻⁵ | 0.014     | 1.48 × 10⁻⁴ | -2.93 × 10⁻⁴ | -0.01    |
| 1299 | 3.13 × 10⁻⁴ | 1.48 × 10⁻⁴ | 0.004  | -4.61 × 10⁻⁴ | -0.007   |
| 13401| 1.24 × 10⁻⁴ | -2.93 × 10⁻⁴ | -4.61 × 10⁻⁴ | 0.021    | -0.083   |
| 11172| -0.002   | -0.01     | -0.007 | -0.083   | -0.121   |

activity patterns is a characteristic feature of AS-level Internet topology, which however, cannot be revealed by the global assortativity coefficient.

IV. DISCUSSION AND CONCLUSION

Prior to our definition, local assortativity coefficient is proposed as a local metric [37, 38] that measures the individual node’s connection tendency, which is defined by calculating the contribution of each node to the global assortativity coefficient. However, the calculation is arguable because there is no precise and unique way to deterministically quantify each node’s contribution to a combined term $U_q^2$ collectively calculated from the edge set. For example, supposing the remaining degree of a node $v$ is $j$, there may be many forms of the contribution of $v$, such as $(j / \sum_{v \in V} j) \cdot U_q^2$, $(j^2 / \sum_{v \in V} j^2) \cdot U_q^2$ and so on. None of these forms can justify itself. This is because calculation of $U_q^2$ is a unified process, which is closely related to the complex correlation of the network structure, so we could not decompose this term into each node’s contribution as if nodes were independent of each other. In contrast, our definition is more straightforward in that it calculates each edge’s contribution to the global assortativity coefficient, rather than each node’s contribution to a term in the formula. As a result, our definition completely avoids the bias issue in that definition [38]. More
TABLE IV. Country and region codes for corresponding IDs in Fig. 3 and the number of ASes owned by these countries and regions.

| RIR name | ID | country and region code | number of ASes |
|----------|----|-------------------------|----------------|
| AfriNIC  | 0  | ZA                      | 95             |
|          | 1  | AU                      | 586            |
|          | 2  | KR                      | 539            |
|          | 3  | JP                      | 483            |
|          | 4  | ID                      | 339            |
|          | 5  | IN                      | 306            |
|          | 6  | HK                      | 194            |
|          | 7  | CN                      | 166            |
|          | 8  | TH                      | 162            |
|          | 9  | NZ                      | 151            |
|          | 10 | SG                      | 123            |
|          | 11 | PH                      | 118            |
|          | 12 | TW                      | 100            |
|          | 13 | BD                      | 85             |
|          |    |                         |                |
|          | 14 | US                      | 10406          |
|          | 15 | CA                      | 674            |
| APNIC    | 16 | BR                      | 574            |
|          | 17 | AR                      | 141            |
|          | 18 | MX                      | 133            |
|          |    |                         |                |
|          | 19 | RU                      | 2544           |
|          | 20 | UA                      | 1146           |
|          | 21 | GB                      | 1059           |
|          | 22 | EU                      | 1016           |
|          | 23 | DE                      | 904            |
|          | 24 | PL                      | 882            |
|          | 25 | CZ                      | 505            |
|          | 26 | FR                      | 431            |
|          | 27 | IT                      | 423            |
|          | 28 | BG                      | 349            |
|          | 29 | NL                      | 348            |
|          | 30 | CH                      | 324            |
|          | 31 | SE                      | 306            |
|          | 32 | AT                      | 268            |
|          | 33 | RO                      | 243            |
|          | 34 | ES                      | 213            |
|          | 35 | TR                      | 170            |
|          | 36 | LV                      | 150            |
|          | 37 | IL                      | 149            |
|          | 38 | DK                      | 141            |
|          | 39 | SI                      | 125            |
|          | 40 | HU                      | 123            |
|          | 41 | IR                      | 117            |
|          | 42 | FI                      | 111            |
|          | 43 | BE                      | 110            |
|          | 44 | NO                      | 105            |

To summarize, we present a universal assortativity coefficient (UAC) which can be used to calculate connection tendencies on any part of a network, such as communities, groups in multiple network scales. Indeed, given that the target edge set is set to all edges, UAC is exactly the global assortativity coefficient (GAC). In this sense, GAC is a special case of UAC. Moreover, this definition is deterministic, completely avoiding the bias issue accompanied with the node-based local assortativity coefficient definition. UAC helps to uncover individual, partial, and global assortativity patterns in various networks. Applying UAC to real world networks, we find that contrary to the popular expectation, most globally dissortative networks are still dominated by assortative edges, though with weak strength. This observation also motivates us to classify networks along two dimensions into four categories, characterized by their global assortativity coefficient and local assortativity statistics. It is expected that this measure can be widely applied to various networks such as popular online social networks, ubiquitous modern communication networks and transportation networks, help people uncover more hidden patterns in networks, and finally allow deep understanding of network dynamics caused by the structural difference discerned by the UAC.

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