Adaptive Differential Thrust Methodology for Lateral/Directional Stability of an Aircraft with a Damaged Vertical Stabilizer

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This paper investigates the utilization of differential thrust to help a commercial aircraft with a damaged vertical stabilizer regain its lateral/directional stability. In the event of an aircraft losing its entire vertical stabilizer, the consequential loss of the lateral/directional stability and control is likely to cause a fatal crash. In this paper, an aircraft with a damaged vertical stabilizer is investigated, and a novel differential thrust based adaptive control approach is proposed to achieve a stable flight envelope. The propulsion dynamics of the aircraft is modeled as a system of differential equations with engine time constant and time delay terms to study the engine response time with respect to a differential thrust input. The novel differential thrust control module is then presented to map rudder input to differential thrust input. Model reference adaptive control based on the Lyapunov stability approach is implemented to test the ability of the damaged aircraft to track the undamaged aircraft’s (reference) response in an extreme scenario. Investigation results demonstrate successful application of such differential thrust approach to regain lateral/directional stability of a damaged aircraft with no vertical stabilizer. Finally, the robustness and sensitivity analysis results conclude that the stability and performance of the damaged aircraft in the presence of uncertainties remain within desirable limits, and demonstrate a safe flight mission through the proposed adaptive control methodology.

Nomenclature

\( b \) aircraft wing span
\( C_{Lr} \) dimensionless derivative of rolling moment coefficient with respect to yaw rate
\( C_{L\alpha v} \) lift curve slope of the vertical stabilizer
\( C_{N r} \) dimensionless derivative of yawing moment coefficient with respect to yaw rate
\( C_{N \beta} \) dimensionless derivative of yawing moment coefficient with respect to side slip angle
\( C_{N \delta r} \) dimensionless derivative of yawing moment coefficient with respect to rudder deflection
\( C_{Y \beta} \) dimensionless derivative of side force coefficient with respect to side slip angle
\( C_{Y r} \) dimensionless derivative of side force coefficient with respect to yaw rate
\( \frac{d \delta}{d \beta} \) change in side wash angle with respect to change in side slip angle
\( g \) gravitational acceleration
\( T_{xx} \) normalized mass moment of inertia about the x axis
\( T_{xz} \) normalized product of inertia about the xz axis
\( T_{zz} \) normalized mass moment of inertia about the z axis
\( L_p \) dimensional derivative of rolling moment coefficient with respect to roll rate
\( L_r \) dimensional derivative of rolling moment coefficient with respect to yaw rate
\( L_{\beta} \) dimensional derivative of rolling moment coefficient with respect to side slip angle
\( L_{\delta a} \) dimensional derivative of rolling moment coefficient with respect to aileron deflection
\( L_{\delta r} \) dimensional derivative of rolling moment coefficient with respect to rudder deflection

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I. Introduction

The vertical stabilizer is the key aerodynamic surface that provides an aircraft with its directional stability characteristic while the ailerons and rudder are the primary control surfaces that give the pilots the control authority of the yawing and banking maneuvers. In the event of an aircraft losing its vertical stabilizer, the loss of lateral/directional stability and control is likely to cause a fatal crash. Notable examples of such a scenario are the crash of the American Airline 587 in 2001 when the Airbus A300-600 lost its vertical stabilizer.
stabilizer in wake turbulence, killing all passengers and crew members\textsuperscript{10} and the crash of Japan Airlines Flight 123 in 1985 when a Boeing 747-SR100 lost its vertical stabilizer leading to an uncontrollable aircraft, resulting in 520 casualties\textsuperscript{11}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{The recovered vertical stabilizer in the American Airlines 587 crash\textsuperscript{10}}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{The Boeing 747-SR100 without its vertical stabilizer in the Japan Airlines Flight 123 crash\textsuperscript{11}}
\end{figure}

However, not all situations of losing the vertical stabilizer resulted in a total disaster. When an aircraft lost its vertical stabilizer, differential thrust was proved to be able to make the aircraft controllable in the case of the United Airlines Flight 232 in 1989\textsuperscript{14}. Another remarkable endeavor is the landing of the Boeing 52-H even though the aircraft lost most of its vertical stabilizer in 1964\textsuperscript{3}.

Research on this topic has been conducted with two main goals: to understand the response characteristics of the damaged aircraft such as the work of Bacon and Gregory\textsuperscript{2}, Nguyen and Stepanyan\textsuperscript{18} and Shal\textsuperscript{20} as well as to come up with an automatic control algorithm to save the aircraft from disasters such as the work of Burcham et al.\textsuperscript{7}, Guo et al.\textsuperscript{15}, Liu et al.\textsuperscript{15}, Tao and Ioanou\textsuperscript{21} and Urnes and Nielsen\textsuperscript{22}.

Notable research on the topic of a damaged transport aircraft includes the work of Shah\textsuperscript{20} where a wind tunnel study was performed to evaluate the aerodynamic effects of damages to lifting and stability/control surfaces of a commercial transport aircraft. In his work, Shah\textsuperscript{20} studied this phenomenon in the form of partial or total loss of the wing, horizontal, or vertical stabilizers for the development of flight control systems to recover the damaged aircraft from adverse events. The work of Nguyen and Stepanyan\textsuperscript{18} investigates the effect of the engine response time requirements of a generic transport aircraft in severe damage situations associated with the vertical stabilizer. They carried out a study which concentrates on assessing the requirements for engine design for fast response in an emergency situation. In addition, the use of differential
thrust as a propulsion command for the control of directional stability of a damaged transport aircraft was studied by Urnes and Nielsen\textsuperscript{22} to identify the effect of the change in aircraft performance due to the loss of the vertical stabilizer and to make an improvement in stability utilizing engine thrust as an emergency yaw control mode with feedback from the aircraft motion sensors.

Existing valuable research in literature provides very unique insight regarding the dynamics of such an extreme scenario, where in this paper, we provide a unique extension to the existing works, and provide an adaptive control methodology to aid such a damaged aircraft to land safely. Inspired by the intriguing endeavor of the Boeing B52 pilot\textsuperscript{3} this research is motivated to improve air travel safety by incorporating the utilization of differential thrust to regain lateral/directional stability for a commercial aircraft (in this case, a Boeing 747-100) with a damaged vertical stabilizer. For this purpose, we construct the nominal and damaged aircraft models in Section-I. In Section-III we revisit the modeling the engine dynamics of the jet aircraft as a system of differential equations with corresponding time constants and time delay terms to study the engine response characteristic with respect to a differential thrust input. In Section-IV we develop a novel differential thrust control module to map rudder input to differential thrust input. In Section-V the aircraft’s open loop system response is investigated. In Section-VI the model reference adaptive control based on the Lyapunov stability approach is implemented to test the ability of the damaged aircraft to mimic the undamaged aircraft’s (reference) response and achieve safe (and stable) operation conditions. In Section-VII the robustness and sensitivity analysis is conducted to test the stability and validate the overall performance of the system in the presence of uncertainties. In the Conclusion section, the paper is finalized, and the future work is discussed.

II. The Aircraft Models

In this research, the flight scenario is chosen to be a steady, level cruise flight for the Boeing 747-100 at Mach 0.65 and 20,000 feet. We assume that at one point during the flight, as a result of external disturbance, the vertical stabilizer is completely damaged, and in the followings we investigate the means to control such aircraft in such an extreme case scenario. For this purpose, here, we develop nominal (undamaged) and damaged aircraft models for analysis.
A. Flight Conditions

The flight conditions for both the damaged and undamaged aircraft models in this research are summarized in Table 1.

| Parameter           | Value     |
|---------------------|-----------|
| Altitude (ft)       | 20,000    |
| Air Density (slugs/ft³) | 0.001268 |
| Airspeed (ft/s)     | 673       |

B. The Undamaged Aircraft Model

The Boeing 747-100 was chosen for this research due to its widely available technical specification, aerodynamics, and stability derivative data. The data for the Boeing 747-100 is summarized in Table 2, where it represents the nominal (undamaged) aircraft.

| Parameter                                      | Value                  |
|------------------------------------------------|------------------------|
| Center of Gravity (xcg)                        | 0.25                   |
| Wing Area (ft²)                                | 5500                   |
| Wing Span (ft)                                 | 196                    |
| Wing Mean Geometric Chord (ft)                 | 27.3                   |
| Distance from Outer-Most Engine to Center of Mass (ft) | 69.83                 |
| Weight (lbs)                                   | 6.3663x10⁵             |
| Ixx (slugft²)                                  | 18.2x10⁶               |
| Iyy (slugft²)                                  | 33.1x10⁶               |
| Izz (slugft²)                                  | 49.7x10⁶               |
| Ixz (slugft²)                                  | 0.97x10⁶               |
| (CL)β                                          | -0.160                 |
| (CL)p                                          | -0.340                 |
| (CL)r                                          | 0.130                  |
| (CL)δa                                         | 0.013                  |
| (CL)δr                                         | 0.008                  |
| (CN)β                                          | 0.160                  |
| (CN)p                                          | -0.026                 |
| (CN)r                                          | -0.28                  |
| (CN)δa                                         | 0.0018                 |
| (CY)β                                          | -1.00                  |
| (CY)p                                          | -0.90                  |
| (CY)r                                          | 0                      |
| (CY)δa                                         | 0                      |
| (CY)δr                                         | 0.120                  |

Taken from Nguyen and Stepanyan [12], the lateral/directional linear equations of motion of the nominal (undamaged) aircraft, with the intact ailerons and rudder as control inputs, are presented as
surface responsible for the weathercock stability, then we also have to change, where the values that reflect such a scenario (for the damaged aircraft) are listed in Table 3.

For the modeling of the damaged aircraft, in case of the loss of the vertical stabilizer, lateral/directional stability derivatives need to be reexamined and recalculated. Since the whole aerodynamic structure is affected, the new corresponding stability derivatives have to be calculated and studied. The lateral/directional stability derivatives need to be reexamined and recalculated. Since the whole aerodynamic structure is affected, the new corresponding stability derivatives have to be calculated and studied. The lateral/directional dimensionless derivatives that depend on the vertical stabilizer include:

\[ C_{Y\beta} = -\eta \frac{S_w}{S} C_{L\alpha v} \left( 1 + \frac{d\sigma}{d\beta} \right) \]  
\[ C_{Yr} = -2 \left( \frac{l_v}{b} \right) C_{Y\beta\text{tail}} \]  
\[ C_{N\beta} = C_{N\beta\text{wf}} + \eta_v V_v C_{L\alpha v} \left( 1 + \frac{d\sigma}{d\beta} \right) \]  
\[ C_{Nr} = -2\eta_v V_v \left( \frac{l_v}{b} \right) C_{L\alpha v} \]  

Due to the loss of the vertical stabilizer, the vertical tail area, volume, and efficiency factor will all be zero; therefore, \( C_{Y\beta} = C_{Yr} = C_{Nr} = 0 \). If the vertical stabilizer is assumed to be the primary aerodynamic surface responsible for the weathercock stability, then we also have \( C_{N\beta} = 0 \). Finally, \( C_{Lr} = \frac{C_L}{I} \).

In addition, without the vertical stabilizer, the mass and inertia data of the damaged aircraft are going to change, where the values that reflect such a scenario (for the damaged aircraft) are listed in Table 3.

**Table 3. The damaged aircraft data**

| Parameter          | Value                     |
|--------------------|---------------------------|
| Weight (lbs)       | 6.2954x10³                |
| \( I_{xx} \) (slug – ft²) | 17.893x10⁶               |
| \( I_{yy} \) (slug – ft²) | 30.925x10⁶               |
| \( I_{zz} \) (slug – ft²) | 47.352x10⁶               |
| \( I_{xz} \) (slug – ft²) | 0.3736x10⁶               |

In this study, during an event of the loss of the vertical stabilizer, we propose the differential thrust component of aircraft dynamics to be utilized as an alternate control input replacing the rudder control to regain stability and control of lateral/directional flight dynamics. This requires a specific structure of open loop damaged plant dynamics. Next, we present the lateral-directional linear equations of motion of the damaged aircraft, with the ailerons (\( \delta a \)), differential thrust (\( \delta T \)), and collective thrust (\( \Delta T \)) as control inputs, as:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\beta} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & \bar{\beta} \\
0 & L_p & L_\beta & L_r \\
0 & \frac{g}{V} & \frac{g_y + g_x}{V} & \frac{g_y}{V} - 1 \\
0 & N_p & N_\beta & N_r
\end{bmatrix}
\begin{bmatrix}
\phi \\
p \\
\beta \\
r
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
L_{\delta a} & \frac{I_{xx} y_u}{I_{xx} l_x - l_{xx}} & 0 \\
\frac{V_y}{V} & 0 & \frac{-\bar{\beta}}{mV} \\
N_{\delta a} & \frac{I_{xx} y_u}{l_{xx} l_x - l_{xx}} & 0
\end{bmatrix}
\begin{bmatrix}
\delta a \\
\delta T \\
\Delta T
\end{bmatrix}
\]  

(7)
In this case, if the initial trim side-slip angle is zero, then \( \Delta T \) does not have any significance in the control effectiveness for a small perturbation around the trim condition\(^5\) which means that the above equations of motion can be reduced to the final form of governing equations of motion for damaged aircraft as:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\rho}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & \theta \\
0 & L_{dp} & L_{\beta} & L_r
\end{bmatrix}
\begin{bmatrix}
\phi \\
\rho
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & T_{\delta a} & \frac{T_{\delta a}}{I_{xx}I_{zz} - I_{xz}^2}
\end{bmatrix}
\begin{bmatrix}
\delta a \\
\delta T
\end{bmatrix}
\tag{8}
\]

III. Propulsion Dynamics

With emerging advancements in manufacturing processes, structures, and materials, it is a well known fact that aircraft engines have become highly complex systems and include numerous nonlinear processes, which effect the overall performance (and stability) of the aircraft. From the force-balance point of view, this is usually due to the existing coupled and complex dynamics between engine components and their relationships in generating thrust. However, in order to utilize the differential thrust generated by the jet engines as a control input for lateral/directional stability, the dynamics of the engine need to be modeled in order to gain an insight into the response characteristics of the engines.

Engine response, generally speaking, depends on its time constant and time delay characteristics. Time constant dictates how fast the thrust is generated by the engine, while time delay (which is inversely proportional to the initial thrust level) is due to the lag in engine fluid transport and the inertias of the mechanical systems such as rotors and turbo-machinery blades\(^13\).

It is also suggested\(^13\) that the non-linear engine dynamics model can be simplified as a time-delayed second-order linear model as

\[
\dot{T} + 2\zeta \omega \dot{T} + \omega^2 T = \omega^2 T_c (t - t_d)
\tag{9}
\]

where \( \zeta \) and \( \omega \) are the damping ratio and bandwidth frequency of the closed-loop engine dynamics, respectively; \( t_d \) is the time delay factor, and \( T_c \) is the thrust command prescribed by the engine throttle resolver angle.

With the time constant defined as the inverse of the bandwidth frequency \( (\tau = \frac{1}{\omega}) \), and \( \zeta \) chosen to be 1 representing a critically damped engine response (to be comparable to existing studies), the engine dynamics can be represented as

\[
\begin{bmatrix}
\dot{T} \\
\ddot{T}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
\frac{-1}{\tau} & \frac{-2}{\tau}
\end{bmatrix}
\begin{bmatrix}
T \\
\dot{T}
\end{bmatrix} +
\begin{bmatrix}
0 & T_c (t - t_d)
\end{bmatrix}
\tag{10}
\]

For this study, the Pratt and Whitney JT9D-7A engine is chosen for the application in the Boeing 747-100 example, where the engine itself produces a maximum thrust of 46,500 lbf\(^3\) at Mach 0.65 and 20,000 feet flight conditions, the engine time constant is 1.25 seconds, and the time delay is 0.4 second\(^13\). The engine thrust response curve at Mach 0.65 and 20,000 feet is, therefore, obtained as shown in Fig\(^4\) which provides a useful insight into how the time constant and time delay factors affect the generation of thrust for the JT9D-7A jet engine. At Mach 0.65 and 20,000 feet, with the engine time constant of 1.25 seconds, and the time delay of 0.4 second, it takes approximately ten seconds for the engine to reach steady state and generate its maximum thrust capacity at 46,500 lbf from the trim thrust of 3221 lbf. The increase in thrust generation follows a relatively linear fashion with the engine response characteristic of approximately 12,726 lbf/s during the first three seconds, and then the thrust curve becomes nonlinear until it reaches its steady state at maximum thrust capacity after about ten seconds. This represents one major difference between the rudder and differential thrust as a control input. Due to the lag in engine fluid transport and turbo-machinery inertias, differential thrust (as a control input) cannot act as instantaneous as the rudder, which has to be taken into account very seriously in control system design.
IV. Differential Thrust as a Control Mechanism

A. Thrust Dynamics and Configuration

In order to utilize differential thrust as a control input for the four-engined Boeing 747-100 aircraft, a differential thrust control module must be developed. Here, differential thrust input is defined as the difference between the thrust generated by engine number 1 and engine number 4 while the amounts of thrust generated by engine number 2 and 3 are kept equal to each other as shown in Eqs. (11-12). This concept is illustrated in further details in Fig. 5.

\[ \delta T = T_1 - T_4 \]  
\[ T_2 = T_3 \]  

Engine number 1 and 4 are employed to generate the differential thrust due to the longer moment arm \( (y_e) \), which makes the differential thrust more effective as a control for yawing moment. This brings into the picture the need of developing a logic that maps rudder input to differential thrust input, which is further explained in the following section.
B. Rudder Input to Differential Thrust Input Mapping Logics

When the vertical stabilizer of the aircraft is intact (i.e., with nominal plant dynamics), the pilot has the ailerons and rudder as major control inputs. However, when the vertical stabilizer is damaged, most probably, the pilot will keep on demanding control effort from the rudder until it is clear that there is no response from the rudder. To eliminate this mishap, but to still be able to use the rudder demand, a differential thrust control module is introduced in the control logic, as shown in Fig.6 and Fig.7, respectively. This differential thrust control module maps corresponding input/output dynamics from the rudder pedals to the aircraft response, so that when the rudder is lost, the rudder input (from the pilot) will still be utilized but be switched to differential thrust input, which will act as a rudder input for lateral/directional controls. This logic constitutes one of the novel approaches introduced in this paper.

As it can be also seen from Fig. 6 and Fig. 7, the differential thrust control module’s function is to convert the rudder pedal input from the pilot to the differential thrust input. In order to achieve that, the rudder pedal input is converted to the differential thrust input (in pounds-force) which is then provided into the engine dynamics, as discussed previously, in Section III. With this modification, the engine dynamics will dictate how differential thrust is generated, which is then provided as a “virtual rudder” input into the aircraft dynamics. The radian to pound-force conversion is derived in the next section.

C. Radian to Pound-Force Conversion Factor

Using Fig. 5 and with the steady, level flight assumption at the altitude of 20,000 feet, we get the following relationship:

\[ N_{\delta r} = N_{\delta T} \]  \hspace{1cm} (13)

\[ qSbC_{N_{\delta r}}\delta r = (\delta T)\gamma_e \] \hspace{1cm} (14)

which means the yawing moment by deflecting the rudder and by using differential thrust have to be the same. Therefore, the relationship between the differential thrust control input \((\delta T)\) and the rudder control input \((\delta r)\) can be obtained as

\[ \delta T = \left( \frac{qSbC_{N_{\delta r}}}{\gamma_e} \right) \delta r \] \hspace{1cm} (15)

Based on the flight conditions at Mach 0.65 and 20,000 feet, and the data for the Boeing 747-100 summarized in Table 1 and Table 2, the conversion factor for the rudder control input to the differential thrust input is calculated to be
\[
\frac{\delta T}{\delta r} = -4.43 \times 10^5 \frac{lb}{rad}
\]

Due to the sign convention of rudder deflection and the free body diagram in Fig. 5, \(\delta r\) here is negative. Therefore, for the Boeing 747-100, in this study, the conversion factor for the mapping of rudder input to differential thrust input is found to be

\[
\frac{\delta T}{\delta r} = 4.43 \times 10^5 \frac{lb}{rad}
\]

D. Commanded vs. Available Differential Thrust

At this point, the worst case scenario is considered, and it is assumed that the aircraft has lost its vertical stabilizer so that the rudder input is converted to differential thrust input according to the logics discussed previously in this section.

Unlike the rudder, due to delayed engine dynamics with time constant, there is a major difference in the commanded differential thrust and the available differential thrust as shown in Fig. 8.

![Figure 8. Commanded vs. available differential thrust](image)

It can be seen from Fig. 8 that compared to the commanded differential thrust, the available differential thrust is equal in amount but longer in the time delivery. For one degree of step input on the rudder, the corresponding equivalent commanded and available differential thrust are 7737 lbf, which are deliverable in ten second duration. Unlike the instantaneous control of the rudder input, there is a lag associated with the use of differential thrust as a control input. This is due to the lag in engine fluid transport and the inertias of the mechanical systems such as rotors and turbo-machinery blades. This is a major design consideration and will be taken into account during the adaptive control system design phase in the following sections.

V. Open Loop System Response Analysis

In this section, the open loop system response to a one degree step input simulating an extreme scenario of the damaged aircraft is investigated.
A. Plant Dynamics

With data taken from Table 1 and Table 2, the A, B, C, D state space matrices for the lateral/directional dynamics of the nominal (undamaged) Boeing 747-100 can be obtained as

\[
A_{latm} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -0.8566 & -2.7681 & 0.3275 \\
0.0478 & 0 & -0.1079 & -1 \\
0 & -0.0248 & 1.0460 & -0.2665
\end{bmatrix}
\]

\[
B_{latR} = \begin{bmatrix}
0 & 0 \\
0.2249 & 0.1384 \\
0 & 0.0144 \\
0.0118 & -0.6537
\end{bmatrix}
\]

\[
C_{lat} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad D_{lat} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

Based on the data for the lateral/directional stability derivatives of the aircraft without its vertical stabilizer, given in Table 1 and Table 3, the A, B, C, D matrices for the damaged aircraft can be achieved as

\[
A_{lat} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -0.8566 & -2.7681 & 0.1008 \\
0.0478 & 0 & 0 & -1 \\
0 & -0.0248 & 0 & 0
\end{bmatrix}
\]

\[
B_{latT} = \begin{bmatrix}
0 & 0 \\
0.2249 & 0.0142 \\
0 & 0 \\
0.0118 & 0.6784
\end{bmatrix}
\]

\[
C_{lat} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad D_{lat} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

Here, \(A_{latm}\) defines the state matrix of the model, undamaged aircraft whereas \(A_{lat}\) represents the state matrix of the damaged aircraft. Furthermore, \(B_{latR}\) represents the input matrix when the ailerons (\(\delta a\)) and rudder (\(\delta T\)) are control inputs of the undamaged (reference) aircraft whereas \(B_{latT}\) stands for the input matrix of the scenario where the ailerons (\(\delta a\)) and differential thrust (\(\delta T\)) are control inputs of the damaged aircraft. In addition, the damping characteristics of the damaged aircraft are also found to be

| Mode    | Pole Location | Damping | Frequency (1/s) | Period (s) |
|---------|---------------|---------|----------------|------------|
| Dutch Roll | 0.0917 ± 0.43 | -0.209  | 0.439          | 14.2969    |
| Spiral  | 6.32 * 10^{-18} | -1      | 6.32 * 10^{-18} | 9.9486 * 10^{17} |
| Roll    | -1.04         | 1       | 1.04           | 6.0422     |

which clearly indicates the unstable nature of the damaged aircraft in the Dutch roll mode by the Right Half Plane (RHP) pole locations. Furthermore, the pole of the spiral mode lies at the origin, which represents very slow (also unstable) dynamics. The only stable mode of the damaged aircraft is the roll mode by the Left Half Plane (LHP) pole location.
B. Open Loop System Response of the Damaged Aircraft

Following to this, the open loop response characteristics of the aircraft with a damaged vertical stabilizer to one degree of step input from the ailerons and differential thrust are presented in Fig.9, where it can be clearly seen that when the aircraft is majorly damaged and the vertical stabilizer is lost, the aircraft response to the pilot’s inputs is completely unstable in all four states (as it was also obvious from the pole locations). This means the pilot will not have much chance to stabilize (i.e. save) the aircraft in time, which calls for a novel approach. This is another point where the second novel contribution of the paper is introduced: automatic control strategy to stabilize the aircraft, which allows safe (i.e. intact) landing of the aircraft.

![Graphs showing the open loop system response of the aircraft with a damaged vertical stabilizer.](image)

Figure 9. Open loop system response of the aircraft with a damaged vertical stabilizer.
VI. Adaptive Control System Design

To control an aircraft with a fully damaged vertical stabilizer and no rudder is, understandably, a very stressful and laborious task for the pilots. This task also requires skills and experience which in extremely stressful moments, is hard to possess. In such instances, pilots usually have seconds to react, and as having been witnessed beforehand, coupling between the pilot and unstable aircraft dynamics, usually led to a catastrophic crash. Therefore, for the safety of the overall flight, it is crucial for an automatic control system to be developed, tested, and implemented for the aircraft to mitigate accidents and to improve safety, stability, and robustness. As an answer to such need, here, we introduce a novel, Lyapunov stability based adaptive control system design.

A. Background Theory

In conventional model reference adaptive control theory, two celebrated and widely used methods are the MIT Rule and the Lyapunov Stability approaches. Because of the Multi-Input-Multi-Output (MIMO) structure of the lateral/directional dynamics, the MIT rule will not be utilized due to its weak controllability characteristics in higher order and complex systems and the adaptive control system design in this paper will be based on the celebrated Lyapunov Stability approach.

In adaptive control theory, generally speaking, the Lyapunov stability approach is based on the characteristic of a decreasing kinetic energy function of state dynamics. Because of the reason that kinetic energy of a system is descending, the system is considered approaching its asymptotic stability (equilibrium) point. However, it is a relatively cumbersome task to derive a kinetic energy function for a complex system, but if a characteristic of a decreasing kinetic energy function of state dynamics, then it can be concluded that the solution of the governing differential equations \( \frac{dy}{dt} = f(x) \) will be stable. The function \( V(x) \) is then called Lyapunov function.

For this study, the candidate Lyapunov function is suggested as

\[
V(x) = e^T Pe + \text{Tr}[(A - BL - A_m)\,N\,(A - BL - A_m)]
\]  

(16)

where \( N \) is the weighting matrix and \( \text{Tr} \) is the "Trace" of a matrix.

Under perfect matching conditions, it is assumed and calculated that there exists an adjustment gain \( L^* \) so that it will lead the system dynamics to \( A - BL^* \rightarrow A_m \), where \( A_m \) is the model plant dynamics. The changing behavior with respect to time of the error function \( \dot{e} \) is obtained as

\[
e = y - y_m \rightarrow \dot{e} = \dot{y} - \dot{y}_m = (Ay + Bu) - (A_m y_m + B_m u_c)
\]  

(17)

With \( u = u_c - Ly \),

\[
\dot{e} = Ay + B(u_c - Ly) - A_m y_m - B_m u_c
\]  

(18)

and with \( y_m = y - e \), after some algebraic manipulations,

\[
\dot{e} = A_m e + (A - BL - A_m)y + (B - B_m)u_c
\]  

(19)

but \( B = B_m \),

\[
\dot{e} = A_m e + (A - BL - A_m)y
\]  

(20)

With \( L = L^* + \Delta L \) and \( A - A_m = BL^* \)

\[
\dot{e} = A_m e + (A - A_m - B(L^* + \Delta L))y = A_m e + (BL^* - BL^* - B\Delta L)y
\]  

(21)

Therefore,

\[
\dot{e} = A_m e - B\Delta Ly
\]  

(22)

The derivative of the Lyapunov function from Eq. [16] can be obtained as

\[
\dot{V}(x) = -e^T Q e + 2\text{Tr}[-\Delta L^T B^T P \dot{e} y^T + \Delta L^T T \Delta L]
\]  

(23)

where \( A_m^T P + PA_m = -Q \). To make sure that \( \dot{V}(x) \) is always negative, which guarantees stability, the condition \( \Delta L^T(-B^T P \dot{e} y^T + T \Delta L) = 0 \) must be satisfied. Dimensional analysis to determine the size of
matrix $L$ must be next conducted. For the model of the lateral/directional dynamics system in this paper, the matrix has four states, which suggests a 4x4 system. In addition, the control matrix ($B$ matrix) has 4 states as well ($B$ is a 4x2 matrix). Therefore, the parameter adjustment feedback gain matrix $L$ must be a 2x4 matrix, which is depicted as

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \end{bmatrix}$$

Now the condition to guarantee that the first derivative of the Lyapunov function is negative, which is $\Delta L^T(-B^T Pey^T + T\Delta \dot{L}) = 0$, must be solved.

$$\Delta L^T(-B^T Pey^T + T\Delta \dot{L}) = 0 \rightarrow -B^T Pey^T + T\Delta \dot{L} = 0 \rightarrow T\Delta \dot{L} = B^T Pey^T$$

Because $L = L^* + L$, so $\dot{L} = \Delta \dot{L}$,

$$\dot{L} = \Delta \dot{L} = T^{-1}B^T Pey^T = (B^T NB)^{-1}B^T Pey^T$$

Therefore, $\dot{L}$ can be found from $B$, $N$, $P$, $e$, and $y$ according to Eq. (24). $\dot{L}$ can be integrated to get $L$, which will lead the nominal system dynamics to match the model plant dynamics.

B. Simulation Results

The representative block diagram architecture for the suggested adaptive control system design (based on the Lyapunov stability approach) is illustrated in Fig. 10. The ultimate goal of the proposed adaptive control system design is to investigate whether the aircraft with a damaged vertical stabilizer is going to be able to mimic nominal (undamaged) aircraft dynamics and track the response of the undamaged (model) aircraft or not, by utilizing differential thrust as a control input for lateral/directional dynamics. It is also desired to compare damaged dynamics with the behavior of the undamaged (model) aircraft with an intact vertical stabilizer. The control inputs for both plants are one degree step inputs for both the ailerons and rudder/differential thrust. It is worth noting that this is an extreme scenario test to see whether the damaged aircraft utilizing differential thrust can hold itself in a continuous yawing and banking maneuver, without becoming unstable and losing control.

As it can also be seen from Fig. 10 for the nominal scenario (where the rudder is intact) the input signals of one degree step functions are routed directly to the model (reference) aircraft, whereas for the damaged
aircraft, the input signals for the ailerons and rudder are mapped through the input control module, where the rudder input signal is routed through the differential thrust control module and then converted to the differential thrust input following the transformation logics discussed in Section IV of this paper.

Following to that, the simulation results of the adaptive control system model are presented in Fig. 11. As shown in Fig. 11, after only 13 seconds, all four states of the aircraft's lateral/directional dynamics reach steady state values. It can also be clearly seen that after a time interval of 13 seconds the damaged aircraft plant can mimic the model, undamaged aircraft plant where the errors are minimized as shown in Fig. 12. This demonstrates the functionality of the Lyapunov based adaptive control system design in such an extreme scenario.

From Fig. 12, it can be observed that the error signals for all four lateral/directional states are diminished after 13 seconds. Therefore, it can be concluded that the damaged aircraft plant can track and mimic an undamaged aircraft in a remarkable fashion. However, this comes at the cost of the control efforts as shown in Fig. 13, which are still within control limits, and without any saturation of the actuators.

In order to have a feasible control strategy in real-life situation, limiting factors are imposed on the aileron and differential thrust control efforts. The aileron deflection is limited at ±26 degrees. For differential thrust, a differential thrust saturation is set at 43,729 lbf, which is the difference of the maximum thrust and trimmed thrust values of the JT9D-7A engine. In addition, a rate limiter is also imposed on the thrust response characteristic at 12,726 lbf/s.

The aileron control effort, as indicated by Fig. 13, calls for the maximum deflection of about ±26 degrees and reaches steady state at approximately 1.2 degrees of deflection after 10 seconds responding to a one degree step input. This aileron control effort is very reasonable and achievable if the ailerons are assumed to have instantaneous response characteristics by neglecting the lag from actuators or hydraulic systems. The differential thrust control effort demands a maximum differential thrust of -7120 lbf, which is within...
Figure 12. Error signals

Figure 13. Control efforts
the thrust capability of the JT9D-7A engine, and the differential thrust control effort reaches steady state at around -155 lbf (negative differential thrust means $T_4 > T_1$) after 14 seconds. Therefore, it can be concluded that the adaptive control system design with the utilization of differential thrust as a control input is proven to save the damaged aircraft by making it behave like the undamaged (model) aircraft, but the feasibility of the adaptive control method depends heavily on the thrust response characteristics of the aircraft jet engines.

VII. Robustness and Sensitivity Analysis

The robustness and sensitivity analysis is conducted by introducing 20% of full-block, additive uncertainties into the plant dynamics of the damaged aircraft to test the stability and performance of the damaged aircraft in the presence of uncertainties, as shown in Fig. 15.

A. Frequency Response in the Presence of Uncertainties

In order to study the frequency response of the lateral/directional states due to the control inputs, we investigate the damaged plant dynamics with associated additive uncertainties. The response is presented in Fig. 14, where it is obvious that even in the presence of uncertainties, high frequency dynamics of the aircraft remains (relatively) unchanged, while the low frequency content will remain within a ball of uncertainty. Moreover, for the aileron control input, lower frequencies experience more excitations than higher frequencies for roll angle and roll rate. However, for sideslip angle and yaw rate, the plant experiences excitations in both low and high frequency.

B. Robustness Analysis of Adaptive Control System Design

The robustness of the adaptive system design presented in this paper is investigated by the introduction of 20% of uncertainties into the plant dynamics of the damaged aircraft, to test its ability to track the reference response of the undamaged (model) aircraft. Fig. 15 shows the logic behind the adaptive control system design in the presence of uncertainties.

A Monte Carlo simulation was conducted to test the robustness of the damaged plant in the presence of uncertainties. Figure 16 shows the outputs of states in the presence of 20% of uncertainties. It can be clearly seen that the adaptive control system design is able to perform well under given uncertain conditions and the damaged aircraft can follow/mimic the response of the nominal (undamaged) aircraft. In that sense, the uncertain plant dynamics are well within the expected bounds of the nominal plant.

The robustness of the adaptive control system design can be further illustrated in Fig. 17 that all the error signals reach steady state and converge to zero only after 10 seconds. However, these favorable characteristics come at the expense of the control effort from the ailerons and differential thrust as shown in Fig. 18.

According to Fig. 18 when there are uncertainties, the aileron control input demands the maximum deflection of approximately -23 degrees and reaches steady state at around 1.2 degrees after 10 seconds. The aileron input demand is reasonable and feasible due to the limiting factor of ± 26 degrees of the aileron deflection and the assumption that ailerons have instantaneous response characteristics by neglecting the lag from actuators or hydraulic systems. In contrast, the differential thrust control input demands at maximum approximately -4800 lbf, which is within the capability of the JT9D-7A engine, and the differential thrust control effort reaches steady state at around -155 lbf (negative differential thrust means $T_4 > T_1$) after 14 seconds. Again, due to the differential thrust saturation set at 43,729 lbf and the thrust response limiter set at 12,726 lbf/s, this control effort of differential thrust in the presence of uncertainties is achievable in real life situation.
Figure 14. Bode diagrams of damaged and uncertain plants
Figure 15. Simulink diagram for the adaptive control system in the presence of uncertainties

Figure 16. Adaptive control outputs in the presence of uncertainties
Figure 17. Error signals in the presence of uncertainties

Figure 18. Control efforts in the presence of uncertainties
VIII. Conclusion

This paper studied the utilization of differential thrust as a control input to help a Boeing 747-100 aircraft with a damaged vertical stabilizer to regain its lateral/directional stability. The motivation of this research study is to improve the safety of air travel in the event of losing the vertical stabilizer by providing control means to safely control and/or land the aircraft.

Throughout this paper, the necessary nominal and damaged aircraft models are constructed, where lateral/directional equations of motion have been revisited to incorporate differential thrust as a control input for the damaged aircraft. The engine dynamics of the jet aircraft is modeled as a system of differential equations with engine time constant and time delay terms to study the engine response time with respect to a commanded thrust input. The ability of the damaged aircraft to track and mimic the behavior of the model undamaged aircraft in an extreme scenario is illustrated through the adaptive control system design based on the Lyapunov stability approach. Demonstrated results show that open-loop unstable damaged plant dynamics can be stabilized using adaptive control based differential thrust methodology. Further analysis on robustness show that uncertain plant dynamics can mimic nominal plant dynamics well, in presence of 20% additive uncertainty, associated with damaged aircraft dynamics.

Through listed analyses above, the ability to save the damaged aircraft by adaptive control strategy and the utilization of differential thrust has been demonstrated in this paper. This framework provides an automatic control methodology to save the damaged aircraft and avoid the dangerous coupling of the aircraft and pilots, which have led to crashes in all commercial airline incidents. Furthermore, it has also been concluded that due to the heavy dependence of the differential thrust generation on the engine response, in order to better incorporate the differential thrust as an effective control input in a life-saving scenario, major developments in the engine response is also going to better assist such algorithm.

Future studies include robust control strategy based on H infinity loop shaping approach and the incorporation of the utilization of differential thrust for lateral/directional stability into a small-scaled unmanned aerial vehicle (UAV) for real-life testing.

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