Collinear true ternary fission as the consequence of the collective nuclear model

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Abstract

The concept of collinear spontaneous true ternary fission of $^{252}$Cf is subject to critical check by means of the trajectory calculations. The conclusion is that the collinear flight of the fragments turns out to be a natural and most probable mode. The collinearity is in the model on the prescission stage, and it survives at the post-scission stage of spreading of the fragments. The angular distribution of the fragments is calculated, aimed at studying the effect of the spins of the fragments arising at the moment of scission, in spite of the zero spin of the fissile nucleus. The results prove the experimental data available.
1 Introduction

The question of fission into three comparable fragments has a long, challenging and fascinating story. As distinct from traditional ternary fission, where emission of two massive fragments is accompanied with a ternary light particle, like an $\alpha$ particle, sometimes it is called true ternary fission (TTF). Strutinsky et al. were the first who proposed search for this process [1]. Attempts of creating a theory of TTF were undertaken by many theorists. Proceeding from typical initial conditions on the top of usual fission barrier, within the framework of the liquid drop model, Nix [2] demonstrated appearance of a third very light fragment which arose between two other massive fragments in the case of very heavy fissioning systems with $A \gtrsim 300$. Family of shapes leading to fission into three massive fragments was deduced in Ref. [3, 4]. To this end, Legendre-polynomial expansion up to tenth order and more was exploited. In some papers, though, the role of hexadecapole deformation was underlined. The idea of generic mechanism of TTF was expressed in Refs. [5, 6, 7], in contrast to the consecutive one. It is suggested that TTF develops along a special dynamical path to which the nucleus enters at the very beginning of fission. The predetermining role belongs to the hexadecapole deformation, like the quadrupole deformation plays the leading role in habitual binary fission.

The first experimental searches for the effect of TTF were undertaken
in fission of actinide nuclei by thermal neutrons \cite{8} and α particles \cite{9}. The measurements were also conducted in heavy-ion collisions \cite{10}, and spontaneous fission of $^{252}$Cf \cite{11}. However, only upper limits of the probability of the processes were established at the level of $10^{-4} - 10^{-8}$. It is worthy of noting that there was a tacit contradiction between theory and experimental search. From the theoretical point of view, the linear form of fissile nuclei is more favourable than a clover-leaf shape (e. g. \cite{5} and refs. cited therein). However, experimental efforts were mainly aimed at detecting fragments at approximately similar angles, i.e., $\sim 120^\circ$. Based on general considerations, the experimenters likely believed that the mutual electrostatic repulsion could align the spreading angles.

Solyakin et al. proposed the collinear mode of tripartition \cite{12}, when searching for TTF of $^{238}$U by 1-GeV protons. This concept was most successfully realized in JINR experiments on FOBOS and mini-FOBOS setups \cite{13, 14, 15}. Use of the missing-mass method in combination with net detectors led to conclusion that the collinear mode of TTF of $^{235}$U and spontaneous fission of $^{252}$Cf may be at the level of up to $10^{-4} - 10^{-5}$. At first sight, this mode is in contrast with ordinary ternary fission \cite{16}, where α particles or protons are emitted approximately perpendicularly to the fission axis. In Ref. \cite{17} the authors doubted a possibility of a “perfectly” collinear flight in the case of three massive fragments. At the same time, there is a small fraction $\sim 10$ percent of polar alpha particles, emitted
along the fission axis (e.g. [18] and refs. cited therein). Furthermore, direct estimations of possible scenarios of the fragment spread made in Refs. [19, 20, 21] show that the collinear trajectory only can be expected if all the three fragments are born in one line. Minor displacement of the middle fragment from the axis 0.5 fm, as well as the presence of a non-zero transverse component of its velocity break the collinearity. Therefore, the question of angular distribution of the fragments is of primordial interest.

As we will see, the collinear character of spreading of the fragments follows the principles of the A. Bohr’s collective model of nuclear motion. As a result, the description turns out to be completely different in the cases of traditional ternary fission, accompanied by emission of an alpha particle, and TTF. In the former case, the two nascent massive fragments form an axially-symmetric core, in the field of which the alpha particle is formed and emitted. In the case of TTF, all the three fragments still form an axially symmetric nucleus. This means that they move coaxially until separation.

The next question, however, arises whether this pre-scission collinearity may survive in the course of post-scission propagation of the fragments. They keep memory of events, making a complex three-body motion under the action of the mutual Coulomb repulsion, at the same time continuing the rotation of the fission axis. As it is shown in the next section, devoted to consideration of the qualitative factors, combination of these factors really
destroys a collinear picture to a some extent. In section 3, formulas used for calculation are derived. The results of calculation for representative fragments of TTF are reported in section 4. The results obtained are discussed in the Conclusion.

2 Qualitative premises

As is known, $K$ is a good quantum number. In quantum mechanics, the body cannot rotate around the axis of symmetry, and thus the rotational momentum is perpendicular to the axis of symmetry, and its projection $K$ onto the axis of symmetry is zero. At the same time, a nonzero value of $K$ can also be observed in an axially symmetric system due to quasiparticle excitations unrelated to rotation. If the system is a bit non-axially symmetric, then basically it will also rotate around an axis perpendicular to the axis of symmetry, and it will only twist around the axis of symmetry slightly. In quantum mechanics, this is reflected in the fact that the wave function of the fissile nucleus has the following form [22]:

$$\Psi^I_M(r_i) = \sum_K a_K D^I_{MK}(\theta, \phi, \vartheta) \chi_K(r'_i).$$  \hspace{1cm} (1)

$I$ in Eq. (1) is the nuclear spin, $M$ — its projection in the laboratory frame. Wigner’s $D$ functions from the Euler angles $\theta, \phi, \vartheta$ define the orientation of the nucleus in space and determine the angular distribution of the fragments. $r_i$ and $r'_i$ are the nuclear variables (e. g., nucleon coordinates)
in the laboratory system and intrinsic coordinate system, related to the
nucleus, respectively.

The \( z' \) axis thus coincides with the fission axis. And let us direct the
\( x' \) axis in the plane of symmetry of the fissile nucleus (that is in the plane
of the triangular configuration). In terms of the Euler angles, the rotation
from the laboratory system \( r \) to the intrinsic system \( r' \) can be performed in
three steps. First two of them, the rotations by \( \theta \) about the \( z \) axis and by
\( \phi \) about the new axis \( x' \), respectively, impose the \( z \) and \( z' \) axes with each
other \[22\]. After which, it remains the third rotation by \( \vartheta \) about the new
\( z' \) axis, in order to impose the \( x \) axis at the \( x' \) one. In the case of a axially-
symmetric nucleus, this third rotation evidently might not be needed, as
all the azimuthal angles are equivalent. The only way to combine this
picture with Eq. (1) is to put \( K = 0 \). And \textit{vice versa}: in the case of
asymmetric — triangular configuration of the fissile nucleus, rotation by
the angle \( \vartheta \) is essential. Correspondingly, this excludes values of \( K = 0 \) in
Eq. (1), leaving only values \( K > 0 \) as adoptable.

Actually, this consideration is founded on the same arguments as the A.
Bohr’s hypothesis \[22, 23\] about the predominance of a certain channel in
photofission of \(^{238}\text{U}\). As \( I \geq K \), non-zero \( K \) values are only possible if fissile
nucleus has a non-zero spin. This is not the case if spontaneous fission of
\(^{252}\text{Cf}\) is considered. Therefore, it is only the symmetric configuration “three
in line” which survives fission.
Furthermore, even such a “co-axial” initial configuration is not enough yet for the final collinearity. It can be destroyed during spreading of the fragments, as a result of interplay of the accelerating Coulomb force between them and initial velocity conditions. Most essential is perpendicular to the fission axis component of the initial velocity, which arises at the moment of scission due to big spins and large relative angular momentum of the fragments. In the case of binary fission of actinide nuclei, the mean value of the fragment spin is about $7 - 8$ \cite{24, 25}. In papers \cite{26, 24}, appearance of the spins in the fragments at scission is explained by excitation of the collective modes of wriggling and bending vibrations. In the first case, the fragments are formed with spins parallel to each other, and perpendicular to the fission axis. Arising total spin is compensated by the orbital angular momentum $L$ of the relative motion of the fragments. The latter is thus in the opposite direction and also perpendicular to the fission axis. This orbital momentum gives evidence of the initial velocities of the fragments which are perpendicular to the fission axis, as shown in Fig. 1. The observed value of the mean spin of the fragments can be explained in this way (e.g. \cite{27}).

One concludes from the above consideration that in the case of TTF, appearance of wriggling vibrations in all three fragments can lead to the total spin of the fragments, and, respectively, to their total orbital momentum as much as $L \sim 20$. And the larger the $L$ value, the greater the final
angle of divergence between the fragments. Baring this in mind, we varied possible $L$ values within $0 \leq L \leq 20$. In the case of bending vibrations, the fragments are formed with spins antiparallel to one another. Therefore, the relative orbital momentum, together with the related destruction of the collinearity, is expected to be even smaller.

Allowance for the initial transverse velocity results in the final divergence of the spreading fragments. If the fragments could move completely freely after scission, then with reasonable initial conditions, all the fragments would stay collinear on a rotating fission axis. The necessary condition for the fragments to remain on the axis is that both the transverse and longitudinal velocity components remain proportional to the distance from c. m. When the Coulomb force is switched on, it changes only the longitudinal component. This violates the proportionality: the middle (ternary) fragment is pushed to the c. m. by the both outer fragments. This reflects, specifically, in its small final kinetic energy [19, 20, 21]. In turn, the middle fragment itself pushes both the side fragments out, which also works as to violate the proportionality. As a result, as soon as the middle fragment descends from the axis, this immediately triggers the transverse component in the Coulomb repulsion between the fragments, which enhances the further destruction of collinearity.

Note that the mechanism described has much in common with the ROT effect, which arises in fission of nuclei with spins, different from zero, by
polarized neutrons \cite{28, 29}. The ROT effect is the triple angular correlation between neutron spin and the momenta of fragments and ternary particles. The ROT effect can be explained as due to rotation of the fission axis before scission, which is transferred to the fragments at scission as the transverse initial velocity. Naturally, this mechanism would also give a contribution to destruction of collinearity in TTF, if the fissile nucleus had a spin. There is, however, a big difference in the mechanism of ROT effect and that discussed above. First, the mechanism, destroying the collinearity in TTF, appears to be much stronger, as it is related with much higher momenta \( L \lesssim 20 \). Second, it does not contribute to the ROT effect because of angular averaging: there would be no correlation of the \( L \) direction with the spin of the fissile nucleus, even if the spin were different from zero. Contrary, in the case of TTF, where each event is detected independently of the others, relative momentum \( L \) could easily manifest itself through violation of the collinearity.

To sum it up, we will assume that the total relative momentum of the fragments may reach as high as \( L \approx 20 \) in the case of wriggling vibrations. This value is compensated by the total spin of the fragments. With smaller or even zero values of the total spin of the fragments, as might be in the case of bending vibrations, and smaller angular momentum of their relative motion, the trajectories of the fragments would be all the more collinear. Let us turn to the numerical calculations.
3 Calculation formulas

3.1 Equations of motion

Numerical simulation of trajectories of representative fragments is a classical method. Its applicability follows a known fact that the wavelength related with the fragment translation is small as compared to its size. Such calculations were found to work well in description of the spectra and angular distributions of α particles, emitted in ternary fission (e.g. [30]). Specifically, they were applied to the description of the ROT effect [31].

After scission, further trajectories of the fragments are determined by the repulsive Coulomb forces between them. Representative trajectories are simulated in the next section by solving the Newton equations of motion for each of the fragments:

\[
\frac{d^2 \mathbf{r}_i}{dt^2} = \mathbf{F}_i/M_i, \quad i = 1, 2, 3, \tag{2}
\]

where \(M_i\) is mass of the \(i\)-th fragment, and \(F_i\) is the resulting force acting on it from the two other fragments. For simplicity, the latter is calculated under natural assumption of spherical fragments.

3.2 Initial conditions for the positions of the fragments

The system of coupled equations (2) has to be solved numerically with the proper initial conditions of position and velocity for each of the fragments at scission. I consider the generic mechanism of the TTF, when the both
scissions occur nearly simultaneously. The choice of the initial conditions is illustrated in Fig. 1. Let the fission axis coincide with the quantization axis \( z \) at the moment of scission. Denote the side fragments with indices 1 and 2, and the middle fragment as No. 3. In view of the axial symmetry of the problem, let \( x \) be the transverse direction axis. And let us specify

the atomic and mass numbers of the fragments as \( Z_i \) and \( A_i \), respectively, with the distances \( r_{12} \), \( r_{23} \) and \( r_{13} \) between the fragments. The positions of the fragments must be defined, baring in mind their asymptotic total kinetic energy (TKE), which must not exceed reaction heat \( Q \). For the parameterization purposes, the total Coulomb energy of the fragments is minimized, based on the position of the ternary fragment at fixed distance

Figure 1: Initial conditions for the trajectory simulations. \( V_1 \), \( V_2 \) and \( V_3 \) are the transverse velocities to the fission axis of the fragments 1 – 3, which comprise the total relative angular momentum of the collective rotation of the fragments (directed towards us). \( D \) is the distance between extreme fragments.
\[ D = r_{12} \text{ between the side fragments:} \]
\[ r_{23} = D \frac{\sqrt{Z_2}}{\sqrt{Z_1} + \sqrt{Z_2}}. \quad (3) \]

In this way, the initial positions of all three fragments are fixed by the single parameter \( D \), which in turn is defined by the TKE value \( T \), \( T \leq Q \):
\[ T = \left( \frac{Z_1 Z_2}{r_{12}} + \frac{Z_1 Z_3}{r_{13}} + \frac{Z_2 Z_3}{r_{23}} \right) e^2. \quad (4) \]

### 3.3 Initial conditions for the transverse velocities of the fragments

In the laboratory system, the most general motion of the fragments can be represented as a superposition of a linear translation and rotation around their centre of mass. The former gives linear velocity of the fragment, the latter is nothing more than the spin of the fragment. A small initial velocity of the fragments in the direction of fission is not important for the present purposes. In order to calculate the initial transverse velocity of the fragments, let us designate the masses of the fragments and their positions on the axis of fission as \( M_1, z_1, M_2, z_2 \) and \( M_3, z_3 \), respectively. The center of gravity of the fragments, determined during fission, is set as
\[ \zeta = \frac{(M_1 z_1 + M_2 z_2 + M_3 z_3)}{M}, \quad (5) \]
where the total mass \( M = M_1 + M_2 + M_3 \). The total angular momentum of the fragments \( L \) is defined as follows:
\[ \omega \left[ M_1(z_1 - \zeta)^2 + M_2(z_2 - \zeta)^2 + M_3(z_3 - \zeta)^2 \right] = L\hbar, \quad (6) \]
and the initial transverse velocity of fragment $i$ is

$$V_i = \omega(z_i - \zeta). \quad (7)$$

4 Results of the calculation

A landscape of the potential deformation energy was calculated in Ref. [33] for the case of TTF of $^{252}\text{Cf}$. It suggests the following mode as a likely candidate:

$$^{252}\text{Cf} \rightarrow ^{132}\text{Sn} + ^{48}\text{Ca} + ^{72}\text{Ni}, \quad Q = 251 \text{MeV}, \quad (8)$$

with the light and heavy side fragments of Ni and Sn, and the ternary fragment of Ca in the middle. The $Q$ value in fission (8) was calculated, using AME2012 atomic mass evaluation [32]. The presence of two magic or semimagic fragments in the final state provide a great released energy $Q$. The situation is like in three-partition of the atomic clusters of $^{27}\text{Na}^{+++} \rightarrow 3 \, ^{9}\text{Na}^{+}$ into three magic clusters of $^{9}\text{Na}^{+}$ [5]. The final TKE values depend on the scission configuration: position of the fragments, thickness of the necks. Deformation of the fragments takes a part of energy from the $Q$ value, diminishing TKE of the fragments. For this reason, I consider various representative TKE values and total angular momenta $L$.

In the landscape of the potential energy [33], pronounced valleys favorable for ternary fission were found. One of them, which may be related
with channel (8), lies after a saddle point at \( r_{12} \approx 3R_0 = 22 \text{ fm} \), where \( R_0 \) is the radius of the mother nucleus. At this distance, formation of the future fragments starts. This distance is close to the scission range in the case of binary fission, where the scission distance is approximately twice as large as the total radius of the both fragments. The valley presents a good opportunity for scission and separation of all three fragments somewhere at \( r_{12} \gtrsim 30 \text{ fm} \). Indeed, the TKE value \( T = Q \) would be achieved if scission occurred at \( r_{12} = 25.56 \text{ fm} \). In practice, part of the released energy is stored in the deformation energy of the fragments, while scission occurs at a larger distance. Baring this in mind, we varied the parameter \( D = r_{12} \) in the range up to \( D = 40 \text{ fm} \). Experimental results [15] confirm such an expectation.

Equations of motion (2) with initial conditions (3) and (7) were solved by means of the Runge—Kutta—Nyström method. The results of the trajectory simulations are presented in Fig. 2 and Table 1. The calculated kinetic energies of each of the fragments, together with their TKE, are presented in Fig. 2 versus the distance between the side fragments \( D \) at scission. All the energies smoothly decrease with increasing \( D \), while the TKE changes from \( T = Q = 251 \text{ MeV} \) for \( D = 25.6 \text{ fm} \) down to \( T = 160 \text{ MeV} \) for \( D = 40 \text{ fm} \). As well as in ordinary binary fission, the heavy fragments are produced with lower kinetic energies. We note a characteristic feature of TTF, which follows Fig. 2: the ternary fragments,
which are formed between the heavy and light ones, turn out to be very slow, with the kinetic energies of approximately 5 MeV. This is 15 – 20 times as small as the energies of the main fragments. This is in accordance with ref. [34], Qualitative reasons for such low energies are discussed in Introduction.

Results concerning the angular distribution of the fragments are presented in Table 1. As a consequence of the rotation of the fission axis, none of the fragment trajectories remains on the z axis after scission, if
\( L \neq 0 \). For the configuration presented in Fig. 1, where the momentum \( \mathbf{L} \) is aimed at the reader, the light Ni fragment goes down from the \( z \) axis. In turn, the heavy Te fragment comes up. As a result, the two main fragments scatter in the opposite directions, with the angle \( \Theta \) between them remaining close to 180°. The calculated \( \Theta \) values are displayed in the Table for various momenta \( L \). As one can see from the Table, the difference from 180° does not exceed 0.4°. This deflection is a consequence of the transverse force, discussed in Introduction.

The ternary fragment always flies in the same direction as the light one. The angle of divergence \( \Phi \) between them is also presented in the Table against the total angular momentum \( L \). The latter was varied in a wide range \( 0 \leq L \leq 20 \), as explained previously. All three fragments remain in the same plane. Projections of the velocities of the light and ternary fragments on the axis, perpendicular to the direction of the heavy fragment, have opposite signs. Scattering of the ternary fragment into the upper and lower half-planes in Fig. 1 is equally probable. The results presented clearly show that the Ni and Ca fragments, moving in the same direction, diverge within one-two degrees at most, for all the considered \( L \) values. Such a divergence can be prettily neglected in the first approximation under the conditions of the experiments on FOBOS and mini-FOBOS [13, 14, 15]. In this way, the performed trajectory calculations confirm the observed picture of the collinear TTF.
Table 1: Calculated angular distributions of the fragments of true ternary fission of $^{252}$Cf $^\text{\textbullet}$ versus the scission distance $D$ and the relative angular momentum $L$. $\Theta$ is the angle between the directions of the heavy and light fragments, $\Phi$ — the divergence angle between the light $^{72}$Ni and ternary $^{48}$Ca fragments, moving in the same direction.

| $D, \text{ fm}$ | $L$ | $\Theta^\circ$ | $\Phi^\circ$ |
|-----------------|-----|----------------|-------------|
| 25.6            | 5   | 179.9          | 0.7         |
|                 | 10  | 179.8          | 1.4         |
|                 | 15  | 179.7          | 2.1         |
|                 | 20  | 179.6          | 2.8         |
| 30              | 5   | 179.9          | 0.6         |
|                 | 15  | 179.7          | 1.9         |
| 35              | 5   | 179.9          | 0.6         |
|                 | 15  | 179.7          | 1.8         |
|                 | 20  | 179.6          | 2.4         |

5 Conclusion

It follows from the considered model that the approximately collinear picture of TTF is most probable. This is due to combination of the two circumstances. The first circumstance is axial symmetry of the fissile nucleus on its path towards scission. Because of the axial symmetry, there is a sole way of formation of the fragments, when they remain co-axial till scission. This is a remarkable illustration of the collective A. Bohr’s model. Furthermore, it is in spirit of the A. Bohr’s hypothesis which transfers the principles of symmetry from the collective model into fission. As is known, the A. Bohr’s hypothesis also works in the case of fission of $^{235}$U by thermal
neutrons, in which case the compound nucleus is characterized by strong mixing of the states with different $K$ values, so as all the possible values from 0 to 4 may become nearly equally probable due to the Coriolis mixing \[35\]. As a result, it was suggested that the channel $J^\pi K = 4^-0$ should also be considered for the resonances with $J^\pi = 4^-$. Also in this case, most probable channels with a certain $K$ reply to minimal energy over the fission barrier. As an example of application of the symmetry principles in fission, we also note paper \[36\] where, in addition to the $J^\pi K$ characteristics of fission barriers, the signature quantum numbers $s$ and $r$ related to the symmetry of the first and second fission barriers were introduced. One can say that the barrier works as the filter, which selects the channels with minimum energy over the barrier. However, the angular distribution in the case of fission of nuclei with spins becomes more complicated because of the strong mixing over $K$ values on the stage of the compound nucleus \[35\]. A consideration of such cases can be performed elsewhere, in a similar approach.

The second circumstance is good conservation of the initial pre-scission collinearity during the post-scission spreading of the fragments under the action of the Coulomb repulsive force. The repulsion leads to descent of the fragments from the axis, after which appearance of the transverse component of the Coulomb force forms the final value of the angles of divergence between the fragments. The final angle between the light and
ternary fragments remains at the level of two degrees, not resolvable in experiments [13, 14, 15]. This allows one to speak about collinear mode of TTF.

We did not take into account the strong interaction of the fragments, as in [19]. At first glance, this may seem to be a shortcoming. However, at deeper insight, necessity of taking the strong interaction into account is due to unrealistic initial configuration of three touching spheres in [19], without attention to the dynamics of fission. The distance between the side fragments $R_{12} = 20$ fm rather corresponds to binary fission. There it takes in to account that before scission, a long neck is formed. After its rupture, the remnants of the neck are snap back [37]. And only after this, the fragments can be considered as approximately spherical, being at a considerable distance from one another where short-range strong interaction really does not play a noticeable role. Choice of the initial configuration in the present work: $R_{12} = 27 – 35$ fm is in agreement with the above picture and microscopic dynamical calculation [33]. Moreover, expects that the configuration [19] will rather lead to merging of the three sheres into one mother nucleus, then their further separation.

Another difference is in treating the initial velocity. The authors in [19] do not write, how to combine availability of the transverse initial velocity $v_{3y}$ on the middle fragment with the momentum conservation, if the other velocities are zero: $v_{1x} = v_{2x} = v_{3x} = v_{1y} = v_{2y} = 0$. Our present model
harmonically satisfies the momentum conservation. Dwelling on these and other shortcomings of [19] is not among my present purposes, although. The main difference is that the main question remains unanswered in [19]. I want to show the deeper physical reason, which make collinear TTF most probable. It is remarkable that the final collinearity of the fragments gives a strong evidence of an ideal axial symmetry of the fissile system in its evolution up to scission. The final collinearity of the fragments would be already broken by the very minimum shift of the middle fragment from the fission axis by \( \approx 0.1 \text{ fm} \) [20]. This comprises as little as 0.1 percent of the distance between the side fragments. This makes TTF a unique process showing manifestation of the axial and other symmetries which underlie the A. Bohr’s model. Although the collective model was designed for description of other phenomena, such as spectra and intensities of gamma quanta, etc., in none of these examples does the description achieve such an accuracy, may be, \( \sim 10 \) percent at most. Therefore, TTF remains a process where the merits and the underlying symmetries of the collective model manifest themselves in full shine.

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