SEARCHING FOR THE MSW ENHANCEMENT

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Abstract

We point out that the length scale associated with the MSW effect is the radius of the Earth. Therefore to verify matter enhancement of neutrino oscillations, it will be necessary to study neutrinos passing through the Earth. For the parameters of MSW solutions to the solar neutrino problem, the only detectable effects occur in a narrow band of energies from 5 to 10 MeV. We propose that serious consideration be given to mounting an experiment at a location within 9.5 degrees of the equator.
Because the small angle MSW solution [1–4] gives the best fit to existing solar neutrino data [5], it could be the ultimate solution to the solar neutrino problem [6], and independent means of verifying it become important. The scale of the MSW effect [7] is set by the radius of the Earth (see below), and so the day-night effect [9] is probably the only means of demonstrating unambiguously that the passage of neutrinos through matter can enhance neutrino oscillations with small in vacuo mixing angles. It is unfortunate that the total rate for the Kamiokande experiment [8] shows no clear evidence for a nighttime signal greater than the daytime one, and that the preferred fit to the total rates of all four solar neutrino experiments lies outside the region of parameter space where such an effect is expected [1,9]. Here we want to argue that the best place to look for the day-night effect is in a band of neutrino energies from about 5 to 10 MeV and at a new location, close to the Equator.

When we rewrite the famous MSW enhancement condition [7] \(\sqrt{2}G_{\nu}N_e = (\Delta m^2/2E) \cos 2\theta\), so that \(E/\Delta m^2\) is expressed as a length in meters, we find that the controlling scale happens to be the radius of the Earth \(R_E\):

\[
\frac{E}{\Delta m^2} = \frac{6.7 \times 10^6}{\rho_E(Z/A)} \cos 2\theta = \frac{1.05R_E}{\rho_E(Z/A) \cos 2\theta}. \tag{1}
\]

The density of the Earth, \(\rho_E\), varies from about 3 gm/cc at the surface to 13 gm/cc in the core [10]; and the ratio \((Z/A)\) is of order 0.5 throughout. Therefore, in order for a neutrino oscillation to be enhanced in passage through the Earth, its in vacuo oscillation length must be at least a significant fraction of \(R_E\).

To gain some insight into possible matter oscillations, it is instructive to apply the parameters of typical MSW fits to a model of the Earth. The small angle MSW solution is typified by [1,3]:

\[
\Delta m^2 = 6 \times 10^{-6} \text{ eV}^2, \quad \sin^2 2\theta = 0.007. \tag{2}
\]

The density \(\rho_E\) [10], as a function of the distance \(xR_E, (0 \leq x \leq 1)\) from the center of the Earth, decreases monotonically from 13 gm/cc at \(x = 0\) to 10 gm/cc at \(x = 0.55\) in the core region, and after a sharp discontinuity, decreases monotonically again in the mantle from 6 gm/cc at \(x = 0.55\) to 3 gm/cc at \(x = 1\).

Neutrinos traveling through the Earth will satisfy the enhancement condition in the core when their energies fall within the narrow band [15] of 6.5 MeV \(\leq E \leq 8\) MeV, and in the mantle when they fall into a wider, higher energy band of 13.5 MeV \(\leq E \leq 27\) MeV.

The size of the enhancement region is [2] \(\Delta R = h_0 \tan 2\theta\), where \(h_0\) is the reciprocal of the logarithmic derivative of the density; for the core, it is

\[
\Delta R_{\text{core}} \approx 2R_E \tan 2\theta \approx 0.17R_E, \tag{3}
\]

and for the mantle

\[
\Delta R_{\text{mantle}} \approx 0.75R_E \tan 2\theta \approx 0.06R_E. \tag{4}
\]

Both regions are considerably smaller than the corresponding oscillation lengths at the points of enhancement, \(L_m \approx (5.3 \cot 2\theta/\rho_0)R_E\), and so the enhancements are nonadiabatic.
Since the energy band $6.5 \text{ MeV} \leq E \leq 8 \text{ MeV}$ overlaps the $^{8}\text{B}$ solar neutrino spectrum, we shall concentrate our attention on it from now on.

In passing, we note that the large angle MSW solution, typified by the oscillation parameters $\Delta m^2 = 3 \times 10^{-5} \text{ eV}^2$, $\sin^2 2\theta = 0.7$, predicts enhancement at energies well outside the solar spectrum (except perhaps for the hep branch). They are, in units of MeV: $18.5 \leq E \leq 24$, and $40 \leq E \leq 80$, for the core and mantle of the Earth respectively.

For the enhancement mechanism to have a significant effect on the $\nu_e$ survival probability of neutrinos in the $6.5 \text{ MeV} \leq E \leq 8 \text{ MeV}$ energy band, the neutrinos must travel a distance $\Delta R_c$ or more through the vicinity of the enhancement density. The discontinuity in density between the core and mantle occurs at $x = 0.55$ and subtends a half-angle of $33^\circ$ at the surface of the Earth. Therefore paths must be within $33^\circ$ of the vertical to produce significant effects. To illustrate the point, imagine these neutrinos traveling through the Earth on a path which makes an angle $\phi$ with the vertical at the site of the detector. When $\phi$ is less than $29.88^\circ$, the path length in the core will be greater than $\Delta R_{\text{core}} \approx 0.17 R_E$ as given above, and there will be a significant effect on oscillation probabilities. It falls off dramatically as $\phi$ reaches the $30^\circ$ mark.

Another subtle effect is that the neutrino energy corresponding to the largest change in oscillation probability will increase with $\phi$. Enhancement energies (see Eqs. 1 and 2) vary inversely with density, and the maximum density along any path decreases as $\phi$ increases. Since interaction cross sections increase with energy, this effect may be important in choosing a path to maximize neutrino interaction rates.

To study the magnitude of these effects, we have performed a series of numerical calculations which follow $^{8}\text{B}$ solar neutrinos from the point of origin in the Sun through different chords of the Earth. The $\nu_e$ survival probability is very sensitive to the relative phase between the neutrino mass eigenstate components when they arrive at Earth. Since the neutrinos are produced over a region of the solar core which is about $10 R_E$, and since their in vacuo oscillation length is only a fraction of $R_E$, we average over the relative phase to calculate the $\nu_e$ survival probability [16].

Our results are shown in Fig. 1. There is a significant increase in survival probability for neutrinos between 5 and 10 MeV for all angles less than $33^\circ$. The probability increases by almost a factor of 2, and the energy at which the largest increase occurs tends to move up with $\phi$. By about 10 MeV there is no longer a difference between day and night. A slight increase in the neighborhood of 12 MeV corresponds to a small enhancement in the mantle of the Earth.

To carry out the search for Earth enhancement, it will be best to detect neutrinos through their charged-current interactions with as heavy a target as possible: the heavier the target, the more closely the recoil electron energy follows the energy of the incident neutrino. In neutrino–electron scattering, for example, the recoil electron energies are almost equally distributed amongst the values kinematically allowed by the incident neutrino energy, and they are therefore spread over a considerable range [11]. In neutrino–deuteron charged-current interactions, the electron energy is sharply peaked at its maximum value, but does
have a tail extending to lower energies \[12\]. This implies that Earth enhancement can noticeably change the shape of the recoil spectrum in neutrino–deuteron interactions, but not in $\nu$–$e$ scattering. Experiments like SNO \[13\] may therefore be better suited to this measurement than ones like Super Kamiokande \[14\], although the latter has great statistical power. Heavier targets such as $^{12}$C and $^{16}$O may be even more suitable, especially if the recoil electron can be detected and its energy measured in real time.

We can demonstrate these points by calculating the differential cross sections for neutrino interactions as a function of the angle $\phi$ through the Earth and comparing them with daytime cross sections (see Fig. 2). At $\phi = 0^\circ$, the deuterium differential cross section increases significantly for recoil electrons with kinetic energies between 4 and 6 MeV, and at $\phi = 20^\circ$, the increase shifts to higher energies between 5.5 and 7.5 MeV. By contrast, the elastic scattering cross section shows a small, essentially uniform increase for all recoil energies below 7 MeV, and the increase does not vary much with $\phi$.

We can compare nighttime rates, $N$, as a function of $\phi$ and express the difference, $N - D$, from the daytime rate, $D$, as a percentage of $D$. To take shape effects into account, we use three different energy ranges for the recoil kinetic energy of the electron: (i) $0$–$T_{\text{max}}$, the maximum recoil kinetic energy; (ii) $5$ MeV–$T_{\text{max}}$; and (iii) $5$–$10$ MeV. Our results for deuterium interactions and elastic scattering are shown in Fig. 3. For elastic scattering, there is roughly a 15% increase for angles up to $27^\circ$ and, as expected, this behavior is much the same for all three energy ranges. For deuterium, however, the changing shape of the recoil spectrum with angle $\phi$ shows up when we cut out recoil energies below 5 MeV, and the behavior in ranges (ii) and (iii) is different from that in (i).

Location is an important consideration maximizing enhancement effects. There are two aspects: the coverage time in any one year that solar neutrinos must pass through the core of the Earth to reach the detector; and the minimum angle $\phi_{\text{min}}$ of the chords they travel. To study these questions we need to calculate the hours during which rays from the Sun will pass through the core of the Earth ($\phi \leq 33^\circ$) on any day of the year for a latitude $\ell$. Let $\Omega$ be the orbital phase of the Earth around the Sun ($\Omega = 0^\circ$ during the Summer Solstice, $90^\circ$ for the Autumnal Equinox, and so on). The declination of the Sun is $\delta = 23.5^\circ \cos \Omega$, and hence the Sun is located at an angle of $(90^\circ - \ell + \delta \sin \ell \cos \delta)$ from the southern horizon at high noon. If we define the hour angle $h$ to run from $(-12)$ to $(+12)$, with $h = 0$ as high noon, we can determine the critical hour $h_c$ when the Sun passes through the critical angle $\phi_c = 33^\circ$ and the duration $D_h$, in hours, that the Sun is blocked by the core:

$$h_c = \frac{24}{2\pi} \cos^{-1} \left( -\frac{\cos \theta_c + \sin \delta \sin \ell}{\cos \delta \cos \ell} \right);$$

$$D_h = 2 \left( 12 - h_c \right).$$  \hspace{1cm} (6)

It is simple to show that for the lowest latitude for which the rays from the Sun will never pass through the core of the Earth is $\ell_n = \phi_c + \delta_{\text{max}} = 33^\circ + 23.5^\circ = 56.5^\circ$; it can also be shown that the highest latitude for which solar neutrinos will pass through the core for some time during each day of the year is $\ell_h = \phi_c - \delta_{\text{max}} = 9.5^\circ$.

Figure 4 shows the coverage time per day as a function of the seasons of the year. At $0^\circ$ latitude, the coverage time oscillates between a maximum of almost 4.5 hours during the Vernal and Autumnal Equinoxes, and a minimum of about 3.25 hours at the Summer and
Winter Solstices; altogether the coverage time amounts to 14.3% of the year. As the latitude increases, this percentage decreases and, once \( \ell \geq 9.5^\circ \), the part of the year for which there is no coverage grows steadily. By 25°, coverage occurs only in the Fall and Winter and at the Winter Solstice, the daily coverage time is almost 5 hours. This has the interesting effect of doubling the coverage during the relevant part of the year. For example, at 30°, the total coverage time is 7.5% of the full year, but 15% of the Fall and Winter; thus comparing night and day during the Fall and Winter alone enhances the overall signal.

Higher latitudes also cover a smaller range of angles \( \phi \). At 12 midnight the angle \( \phi \) at latitude \( \ell \) is \( (\delta + \ell) \), its smallest value for the night. For a given latitude less than 23.5°, the angle \((\delta + \ell)\) will vanish at some time in the winter and \( \phi \) will run the full range from 0° to 33°. For \( \ell \geq 23.5^\circ \), the minimum angle \( \phi_{min} \) never vanishes and increases with latitude. By 45°, we are running out of the most effective enhancement region of the core (see Fig. 3).

These factors of coverage time and angular range make it clear that latitudes close to the Equator are the most favorable for maximal day-night effects. Present experiments are all at latitudes greater than 30° and coverage is restricted to the Fall and Winter: Super Kamiokande, at a latitude of 36.4°, is covered by the core 7% of the year and has \( \phi_{min} \) of 13°; SNO is at 46.5° and is covered 3.5% with \( \phi_{min} \) of 23°; and Borexino, at about 42°, is covered 5% of the year with \( \phi_{min} \) of 18.5°. In all three cases, coverage is restricted to the Fall and Winter. We therefore propose that a search be made for a new site, preferably within 9.5° of the Equatorial region with sufficient overburden, 3000 mwe or more, and infrastructure to mount a major experiment.

To illustrate the experimental requirements necessary for the observation of a significant effect, we consider a charged-current deuterium experiment located at the Equator. During the 14.3% of the coverage time in any one year (“nighttime”), the event rate will be greater by roughly 20% than that during the corresponding 14.3% that the detector is diametrically opposite the covered locations (“daytime”). Therefore, if \( X \) events are seen in the “daytime”, then 1.2\( X \) will be seen in the “nighttime”. If we assume that systematic errors are equal to statistical ones and ask for a 3\( \sigma \) effect, then \( X = 900 \). In other words we require about 6300 events for the full year, or 17 events per (24 hour) day. This is roughly twice the SNO rate and 40 times the rate of the Davis \( 37 \mathrm{Cl} \) experiment. Thus we would require 2 kilotons of heavy water, or 24 kilotons of cleaning fluid (assuming that \( 37 \mathrm{Cl} \) events can be seen in real time). A 5\( \sigma \) effect would require almost three times these quantities, or three times the duration, or some combination of increased size and duration.

In conclusion, we have outlined the case and the scope for a new solar neutrino experiment dedicated to determining whether the matter enhancement of neutrino oscillations does indeed occur. We believe that this experiment is a necessary sequel to the experiments presently operating, or coming online. If the small angle MSW should continue to give the best fit to existing and forthcoming solar neutrino data, the observation of a night-day effect will be the conclusive evidence that it is indeed the true solution to the solar neutrino problem.

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FIG. 1. Survival probability $P(\nu_e \rightarrow \nu_e)$ for $^{8}$B neutrinos before (day) and after passing through the Earth at various angles (night).
FIG. 2. Differential cross sections averaged over the $^8$B spectrum, before and after passing through the Earth.
FIG. 3. Fractional excess of event rates at night as a function of the neutrino path.
FIG. 4. Number of hours per night that solar neutrinos pass through the Earth’s core: annual variations for various latitudes.