Diffraction of light by a nanowire

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Abstract

A general scattering problem of a plane electromagnetic wave on an infinite cylindrical rod is formulated and solved in a form of Bessel functions series expansion. The conductivity account via Ohm law directly in Maxwell equation leads to complex wavenumber and hence the complex arguments of Bessel functions inside the cylinder. The general formula for averaged by period Pointing vector is derived. For numerical calculations asymptotics of Bessel functions are used. Dependence of scattered wave intensity as function of angle and frequency is presented for different values of the rod radius.

Keywords: electromagnetic waves, scattering problem; conducting cylinder, nanowires; state; PACS ;

1 Introduction

There is a long history related to problem of light scattering on dielectric and conducting bodies, see e.g. [1]. Recent investigation apply the theoretical results to a problems of nanowires parameters determination from optic measurements [2]. The case of dielectric rod is studied in [3].

In this manuscript we develop the famous Mie theory to the case of infinite conducting rod of radius $r_0$ in somewhat manner similar to the cited [3], but widen the problem statement and its solution. The conductivity is introduced via direct account of Ohm law in Maxwell-Ampere equation, while the dielectric dielectric permittivity supposed to be a real function of coordinates. We state and solve the problem in cylindrical coordinates. The conductivity and permittivity supposed to be a function of radial variable to describe eventual transitional domain in a vicinity of the rod. Magnetic properties are trivial: the magnetic permeability is considered as a constant, equal to 1.

We apply the theory to case related to [2], using appropriate asymptotics of cylindrical functions, namely we consider scattering by thin ($\lambda >> r_0$) conducting rod (having in mind for example scattering of light on semiconductor nanowires).
2 Electromagnetic waves in conducting medium

2.1 Electromagnetic wave equation in cylindrically stratified medium

The Maxwell equations for medium without space charges in the Lorentz-Heaviside’s unit system (c - the velocity of light in vacuum)

\[
\nabla \cdot \mathbf{D} = 0, \tag{1a}
\]

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \tag{1b}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{1c}
\]

\[
\nabla \times \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c} \mathbf{J}, \tag{1d}
\]

where the standard set of electric \( \mathbf{D}, \mathbf{E} \) and magnetic \( \mathbf{H}, \mathbf{B} \) fields is used. The material equations (of state) are assumed as for isotropic medium. We also restrict ourselves by fixed frequency of incident and, hence, scattered wave:

\[
\mathbf{D} = \epsilon \mathbf{E}, \tag{2a}
\]

\[
\mathbf{H} = \frac{1}{\mu} \mathbf{B}, \tag{2b}
\]

where the dielectric permittivity supposed to be a function of coordinates, while magnetic permeability - constant,

\[
\epsilon = \epsilon(\vec{r}), \tag{3}
\]

\[
\mu = \text{const.} \tag{4}
\]

The current density for this isotropic case is given by the simple version of Ohm’s law:

\[
\mathbf{J} = \sigma \mathbf{E}, \tag{5}
\]

where

\[
\sigma = \sigma(\vec{r}). \tag{6}
\]

Introduce the fields complex amplitudes:

\[
\mathbf{E} = \frac{1}{2} \left[ \mathbf{E}(\vec{r}) \exp(i\omega t) + \mathbf{E}^*(\vec{r}) \exp(-i\omega t) \right], \tag{7a}
\]

\[
\mathbf{D} = \frac{1}{2} \left[ \mathbf{D}(\vec{r}) \exp(i\omega t) + \mathbf{D}^*(\vec{r}) \exp(-i\omega t) \right], \tag{7b}
\]

\[
\mathbf{B} = \frac{1}{2} \left[ \mathbf{B}(\vec{r}) \exp(i\omega t) + \mathbf{B}^*(\vec{r}) \exp(-i\omega t) \right], \tag{7c}
\]

\[
\mathbf{H} = \frac{1}{2} \left[ \mathbf{H}(\vec{r}) \exp(i\omega t) + \mathbf{H}^*(\vec{r}) \exp(-i\omega t) \right]. \tag{7d}
\]
Insert conditions \([7], [5]\) and \([2]\) into the Maxwell’s equations \([1]\), then the following set of equations for amplitudes is obtained:

\[
\begin{align*}
\nabla \cdot \epsilon \mathbf{E} &= 0, \quad (8a) \\
\nabla \times \mathbf{E} &= -\frac{i\omega}{c} \mathbf{B}, \quad (8b) \\
\nabla \cdot \mathbf{B} &= 0, \quad (8c) \\
\nabla \times \mathbf{B} &= \left(\frac{i\omega\mu\epsilon}{c} + \frac{4\pi\mu}{c} \sigma\right) \mathbf{E}, \quad (8d)
\end{align*}
\]

as well as correspondent conjugated one.

Let us derive wave equation from the set of equation \([8]\), differentiating as

\[
\nabla \times \nabla \times \mathbf{E} = \left(\frac{\omega^2 \mu \epsilon}{c^2} - i \frac{4\pi \omega \mu}{c^2} \sigma\right) \mathbf{E}. \quad (9)
\]

The conjugate set of equations to \([8]\) yields just a conjugation to \([9]\).

Denote \(\frac{\omega^2 \mu \epsilon}{c^2} - i \frac{4\pi \omega \mu}{c^2} \sigma\) as \(k^2(\vec{r})\). The left side of equation \([9]\) is equal to

\[
\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E}. \quad (10)
\]

From the equation \([8a]\)

\[
\nabla \cdot \epsilon \mathbf{E} = \epsilon \nabla \cdot \mathbf{E} + \mathbf{E} \cdot \nabla \epsilon = 0,
\]

so

\[
\begin{align*}
\nabla \cdot \mathbf{E} &= -\frac{\mathbf{E} \cdot \nabla \epsilon}{\epsilon} = -\frac{\mathbf{E} \cdot \nabla \epsilon}{\epsilon}, \quad (11) \\
- \nabla \left(\frac{\mathbf{E} \cdot \nabla \epsilon}{\epsilon}\right) - \Delta \mathbf{E} &= k^2(\vec{r}) \mathbf{E}. \quad (12)
\end{align*}
\]

A statement of standard scattering problem in such case includes boundary conditions at infinity as prescribed asymptotics.

### 2.2 Energy density flux

The averaged flux density throughout the surface outside the rod is expressed via Pointing vector:

\[
\mathcal{S}_{\text{scat}} = \frac{c}{4\pi} \int_0^T dt \mathbf{E}_{\text{scat}} \times \mathbf{B}_{\text{scat}}. \quad (13)
\]

The flux density in terms of the introduced complex amplitudes \([7]\):

\[
\mathbf{E} \times \mathbf{B} = \frac{1}{4} \left\{ \mathbf{E} \times \mathbf{B} \exp(2i\omega t) + \mathbf{E}^* \times \mathbf{B}^* \exp(-2i\omega t) + 2\Re[\mathbf{E} \times \mathbf{B}^*]\right\}.
\]

Integrals of exponent function over the time period gives 0. Hence:

\[
\mathcal{S} = \frac{c}{16\pi} \Re[\mathbf{E} \times \mathbf{B}^*]. \quad (14)
\]
3 Maxwell equations in cylindrical coordinates

3.1 Description of phenomena and its geometry

The topic of further considerations is diffraction of the light by an infinitely long rod, hence all differential operators has a cylindrical symmetry and cylindrical variables will be used in cylindrical coordinates \( r, \varphi, z \). In the context we restrict ourselves by the case of \( \epsilon(\vec{r}) = \epsilon(r), \sigma(\vec{r}) = \sigma(r) \). Moreover incidental wave is a plane wave coming from infinity to 0 along \( x \) axis what causes that in general the phenomena have a symmetry with respect to \( XZ \) plane, thus all derivatives over \( z \) vanish \( \frac{\partial}{\partial z} \equiv 0 \). It means also, that all solutions have to satisfy following condition \( f(\varphi) = f(-\varphi) \).

3.2 Cylindrical coordinates

For the clarity all neccessary formulas, which will be used during calculations, are introduced in present section in terms of standard notations and unit vectors \( \hat{r}, \hat{\varphi}, \hat{z} \). Gradient in cylindrical coordinates:

\[
\nabla = \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \varphi} \hat{\varphi} + \frac{\partial}{\partial z} \hat{z},
\]

Divergence in cylindrical coordinates:

\[
\nabla \cdot \mathbf{E} = \frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{1}{r} \frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z},
\]

The curl operator:

\[
\nabla \times \mathbf{E} = \left( \frac{1}{r} \frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} \right) \hat{r} + \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \hat{\varphi} + \frac{1}{r} \left( \frac{\partial}{\partial r} (r E_\varphi) - \frac{\partial E_r}{\partial \varphi} \right) \hat{z},
\]

Laplacian (of a scalar field) in cylindrical coordinates:

\[
\Delta = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}.
\]

Laplacian of a vector field \( \mathbf{E} \) in cylindrical coordinates:

\[
\Delta \mathbf{E} = \left( \Delta E_r - \frac{E_r}{r^2} - \frac{2}{r^2} \frac{\partial E_\varphi}{\partial \varphi} \right) \hat{r} + \left( \Delta E_\varphi - \frac{E_\varphi}{r^2} + \frac{2}{r^2} \frac{\partial E_r}{\partial \varphi} \right) \hat{\varphi} + \Delta E_z \hat{z}.
\]
3.3 Generalized Helmholtz equation and division of variables

Using formulas (18) and (19) leads to the following equations for each component of the electric field amplitude:

\[
-\left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 E_r}{\partial \varphi^2} - \frac{2}{r^2} \frac{\partial E_r}{\partial \varphi} + \frac{\partial}{\partial r} \left( E_r \frac{\partial \ln \epsilon(r)}{\partial r} \right) \right) \hat{r} = k^2(r) E_r \hat{r} \\
-\left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_\varphi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 E_\varphi}{\partial \varphi^2} - \frac{2}{r^2} \frac{\partial E_\varphi}{\partial \varphi} + \frac{2}{r} \frac{\partial E_r}{\partial \varphi} - \frac{\partial}{\partial r} \left( r \frac{\partial \ln \epsilon(r)}{\partial r} \right) \right) \hat{\varphi} = k^2(r) E_\varphi \hat{\varphi} \\
-\left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \varphi^2} \right) \hat{z} = k^2(r) E_z \hat{z}
\]

The last equation for \( \hat{z} \) coordinate and \( E_z \) component (20c) is independent of any other components, that allows to use it as basic for further considerations. The variables are separated by substituting \( E_z(r, \varphi) = R(r) \Phi(\varphi) \), hence

\[
\frac{1}{R} \frac{\partial R}{\partial r} + \frac{1}{R^2} r \frac{\partial^2 R}{\partial \varphi^2} + k^2(r) r^2 = -\nu^2 \frac{\partial^2 \Phi}{\partial \varphi^2} = \nu^2,
\]

where \( \nu \) is a constant of separation. Consider equation for \( \Phi \)

\[
\frac{\partial^2 \Phi}{\partial \varphi^2} = -\nu^2 \Phi.
\]

General solution of it is \( \Phi(\varphi) = A_1 \exp(i\nu \varphi) + A_2 \exp(-i\nu \varphi) \), however the symmetry via XZ plane reduces this solution, to

\[
\Phi(\varphi) = A \exp(i\nu \varphi).
\]

Function \( \Phi(\varphi) \) must be continuous, and it has to satisfy the condition of periodicity \( \Phi(\varphi) = \Phi(\varphi + 2\pi) \). It means that the parameter \( \nu \) is an integer. Next, for each \( \nu \)

\[
r^2 \frac{\partial^2 R}{\partial \varphi^2} + r \frac{\partial R}{\partial r} + (k^2(r) r^2 - \nu^2) R(r) = 0.
\]

Despite deceptive similarity to the Bessel equation of integer order, the equation above is different because of a parameter’s \( k(r) \) dependence on radius coordinate. The solution of (24) involves unknown constants, so the constant \( A \) can be omitted. Then the basic expansion is defined by correspondent special functions \( R_\nu(r) \)

\[
E_z = \sum_{\nu=-\infty}^{\infty} R_\nu(r) \exp(i\nu \varphi).
\]

Let us expand the rest components in Fourier series in \( \exp(i\nu \varphi) \):

\[
E_r = \sum_{\nu=-\infty}^{\infty} S_\nu(r) \exp(i\nu \varphi).
\]
\[ E_\phi = \sum_{\nu = -\infty}^{\infty} T_\nu(r) \exp(i\nu \phi), \]  
(27)

because of the cyclic symmetry of the field. Plugging it into (20a) and (20b) yields:

\[
\sum_{\nu} \left[ \frac{1}{r} \frac{\partial S_\nu}{\partial r} + \frac{\partial^2 S_\nu}{\partial r^2} - \frac{\nu^2}{r^2} S_\nu - \frac{S_\nu}{r^2} \right] \frac{i2\nu}{r^2} + \nu \frac{\partial \ln \epsilon(r)}{\partial r} + S_\nu \frac{\partial^2 \ln \epsilon(r)}{\partial r^2} + k^2(r) S_\nu \right] \exp(i\nu \phi) = 0,
\]
(28a)

\[
\sum_{\nu} \left[ \frac{1}{r} \frac{\partial T_\nu}{\partial r} + \frac{\partial^2 T_\nu}{\partial r^2} - \frac{\nu^2}{r^2} T_\nu - \frac{T_\nu}{r^2} + \frac{i2\nu}{r^2} S_\nu + \frac{i\nu}{r} S_\nu \frac{\partial \ln \epsilon(r)}{\partial r} + k^2(r) T_\nu \right] \exp(i\nu \phi) = 0.
\]
(28b)

Taking the linear independence of the exponents into account, and rearranging it, one have

\[
\frac{\partial^2 S_\nu}{\partial r^2} + \left( \frac{1}{r} + \frac{\partial \ln \epsilon(r)}{\partial r} \right) \frac{\partial S_\nu}{\partial r} + \left( \frac{\partial^2 \ln \epsilon(r)}{\partial r^2} - \frac{\nu^2}{r^2} - \frac{1}{r^2} + k^2(r) \right) S_\nu - \frac{i2\nu}{r^2} T_\nu = 0,
\]
(29a)

\[
\frac{\partial^2 T_\nu}{\partial r^2} + \left( \frac{1}{r} + \frac{\partial \ln \epsilon(r)}{\partial r} \right) \frac{\partial T_\nu}{\partial r} + \left( -\frac{\nu^2}{r^2} - \frac{1}{r^2} + k^2(r) \right) T_\nu + \left( \frac{i2\nu}{r^2} + \frac{i\nu}{r} \frac{\partial \ln \epsilon(r)}{\partial r} \right) S_\nu = 0.
\]
(29b)

Next, let us use the Gauss law from Maxwell equations (8a) and divergence (16) to obtain the relationship between functions \(S_\nu(r)\) and \(T_\nu(r)\):

\[
\frac{1}{r} \frac{\partial E_r}{\partial r} + E_r \frac{\partial \ln \epsilon(r)}{\partial r} + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} = 0.
\]
(30)

Combining the results, we arrive at

\[
T_\nu = \frac{i}{\nu} \left[ \frac{1}{r} \frac{\partial S_\nu}{\partial r} + \left( 1 + \frac{\partial \ln \epsilon(r)}{\partial r} \right) S_\nu \right]
\]
(31)

Substitute formula for \(T_\nu\) into (29a)

\[
\frac{\partial^2 S_\nu}{\partial r^2} + \left( \frac{3}{r} + \frac{\partial \ln \epsilon(r)}{\partial r} \right) \frac{\partial S_\nu}{\partial r} + \left( \frac{\partial^2 \ln \epsilon(r)}{\partial r^2} + \frac{2}{r} \frac{\partial \ln \epsilon(r)}{\partial r} - \frac{\nu^2}{r^2} + \frac{1}{r^2} + k^2(r) \right) S_\nu = 0
\]
(32)

Finally we have a representation for general solution of the problem via \(S_\nu\). Let us pick up formulas expressing all components of the electromagnetic \(E, B\) field.

Using the Maxwell equation (8b) expresses the magnetic field by

\[
B_z = \frac{ic}{\omega r} \left( \frac{\partial}{\partial r} \left( r E_\phi \right) - \frac{\partial E_r}{\partial \phi} \right),
\]
(33a)

\[
B_r = \frac{ic}{\omega r} \frac{\partial E_z}{\partial \phi},
\]
(33b)

\[
B_\phi = -\frac{ic}{\omega} \frac{\partial E_z}{\partial r}
\]
(33c)
After substitution of expansions, we obtain

\[ B_z = \frac{ic}{\omega} \sum_{\nu = -\infty}^{\infty} \left( \frac{\partial T_\nu(r)}{\partial r} + \frac{1}{r} T_\nu(r) - \frac{i\nu}{r} S_\nu(r) \right) \exp(i\nu\varphi), \quad (34a) \]

\[ B_r = -\frac{c}{\omega} \sum_{\nu = -\infty}^{\infty} \nu R_\nu(r) \exp(i\nu\varphi), \quad (34b) \]

\[ B_\varphi = -\frac{ic}{\omega} \sum_{\nu = -\infty}^{\infty} \frac{\partial R_\nu(r)}{\partial r} \exp(i\nu\varphi). \quad (34c) \]

It is possible to check that plugging (2) and expressions \( E_r, E_\varphi, E_z \) into (8d) with assistance of relation \( T \) of \( S \) (31), leads to the differential equations (29) and (24), which were derived from Maxwell equations set.

4 Formulation and solution of scattering problem

4.1 Plane wave in cylindrical coordinates

Let outside the semiconductive rod vacuum. The medium is excited by a plane wave that is coming from infinity to zero along \( x \) axis:

\[ \exp(ik_{\text{out}}r \cos \varphi) = \sum_{\nu = -\infty}^{\infty} i^\nu J_\nu(k_{\text{out}}r) \exp(i\nu\varphi), \quad (35) \]

which is identified with the incident polarised wave:

\[ E = E_z \hat{z} = \sum_{\nu} i^\nu J_\nu(k_{\text{out}}r) \exp(i\nu\varphi). \quad (36) \]

The correlated perpendicular magnetic field is given by

\[ B = B_y \hat{y} = \sum_{\nu} i^\nu J_\nu(k_{\text{out}}r) \exp(i\nu\varphi). \quad (37) \]

Find projections of \( B \) on the rod surface

\[ B_r = \hat{r} \cdot B = \hat{r} \cdot \hat{y} B_y = (\cos(\varphi)\hat{x} + \sin(\varphi)\hat{y}) \cdot \hat{y} B_y = \sin(\varphi) B_y \quad (38a) \]

\[ B_\varphi = \hat{\varphi} \cdot B = \hat{\varphi} \cdot \hat{y} B_y = (-\sin(\varphi)\hat{x} + \cos(\varphi)\hat{y}) \cdot \hat{y} B_y = \cos(\varphi) B_y. \quad (38b) \]

Use Eulers formula; because \( \sum_{\nu = -\infty}^{\infty} \) - summation index can be easily switched \( \nu \equiv \nu + 1 \) or \( \nu \equiv \nu - 1 \)

\[ B_r = \frac{\exp(i\varphi) - \exp(-i\varphi)}{2i} \sum_{\nu} i^\nu J_\nu(k_{\text{out}}r) \exp(i\nu\varphi) = -\sum_{\nu} \frac{i^\nu}{k_{\text{out}}r} J_\nu(k_{\text{out}}r) \exp(i\nu\varphi), \]

Similarly

\[ B_\varphi = -\sum_{\nu} \frac{i^{\nu+1}}{k_{\text{out}}} \frac{\partial J_\nu(k_{\text{out}}r)}{\partial r} \exp(i\nu\varphi). \]
4.2 Permitivity index as a step function

Working in polar coordinates, we divide the half axis \([0, \infty)\) to two domains \([0, a)\) - points inside a rod and \([a, \infty)\) with constant parameters \(\epsilon\) and \(\sigma\).

4.2.1 Complete solution outside rod

Solution outside the rod consists of two fields: incidental wave denoted as \(\mathbf{E}_{\text{inc}}\) and scattered one \(\mathbf{E}_{\text{scat}}\).

\[
\mathbf{E}_{\text{out}} = \mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{scat}}, \quad (39)
\]

\[
\mathbf{B}_{\text{out}} = \mathbf{B}_{\text{inc}} + \mathbf{B}_{\text{scat}}, \quad (40)
\]

Incidental wave which has been already considered in the previous chapter, is well known polarised plane wave. Now, general form of the refracted wave will be derived by solving appropriate differential equations. We use the following choice for simplicity:

\[
\epsilon(r) = 1, \quad \mu = 1, \quad \sigma(r) = 0, \quad (41a)
\]

hence

\[
k^2(r) = k^2_{\text{out}}, \quad (42)
\]

all quantities do not depend on \(r\) and are purely real. Taking above conditions into account the equations are simplified. Equation (24) is a Bessel equation of integer order, whose solutions are Bessel functions of the first and the second kind.

\[
R_\nu = A_1 J_\nu(k_{\text{out}}r) + A_2 Y_\nu(k_{\text{out}}r). \quad (43)
\]

The equation (32) reduces to the form:

\[
\frac{\partial^2 S_\nu}{\partial r^2} + \frac{3}{r} \frac{\partial S_\nu}{\partial r} + \left( k^2_{\text{out}} - \frac{\nu^2}{r^2} + \frac{1}{r^2} \right) S_\nu = 0, \quad (44)
\]

Substitution \(S_\nu = \frac{1}{r} s_\nu(r)\) leads again to the well known Bessel equation

\[
r^2 \frac{\partial^2 s_\nu}{\partial r^2} + r \frac{\partial s_\nu}{\partial r} + (k^2_{\text{out}} r^2 - \nu^2) s_\nu = 0, \quad (45)
\]

so the solution is:

\[
S_\nu = \frac{1}{r} \left( A_3 J_\nu(k_{\text{out}}r) + A_4 Y_\nu(k_{\text{out}}r) \right). \quad (46)
\]

Substituting it into (31) we get formula for:

\[
T_\nu = \frac{i}{\nu} \left( A_3 \frac{\partial J_\nu(k_{\text{out}}r)}{\partial r} + A_4 \frac{\partial Y_\nu(k_{\text{out}}r)}{\partial r} \right). \quad (47)
\]

Both of \(J_\nu\) and \(Y_\nu\) are bounded for large arguments, hence general solutions of electric contributions at infinity are:

\[
E_z = \sum_\nu \left( A_1 J_\nu(k_{\text{out}}r) + A_2 Y_\nu(k_{\text{out}}r) \right) \exp(i \nu \varphi), \quad (48a)
\]

\[
E_r = \sum_\nu \frac{1}{r} \left( A_3 J_\nu(k_{\text{out}}r) + A_4 Y_\nu(k_{\text{out}}r) \right) \exp(i \nu \varphi), \quad (48b)
\]

\[
E_\varphi = \sum_\nu \frac{i}{\nu} \left( A_3 \frac{\partial J_\nu(k_{\text{out}}r)}{\partial r} + A_4 \frac{\partial Y_\nu(k_{\text{out}}r)}{\partial r} \right) \exp(i \nu \varphi). \quad (48c)
\]
Simplifying we got all components of magnetic field:

\[ B_z = -\frac{ic}{\omega} \sum_\nu \left[ A_3 \left( \frac{ik^2_{\text{out}}}{\nu} J_\nu(k_{\text{out}}r) \right) + A_4 \left( \frac{ik^2_{\text{out}}}{\nu} Y_\nu(k_{\text{out}}r) \right) \right] \exp(i\nu\phi), \quad (49a) \]

\[ B_r = -\frac{c}{\omega r} \sum_\nu \nu \left( A_1 J_\nu(k_{\text{out}}r) + A_2 Y_\nu(k_{\text{out}}r) \right) \exp(i\nu\phi), \quad (49b) \]

\[ B_\phi = -i\frac{c}{\omega} \sum_\nu \left( A_1 \frac{\partial J_\nu(k_{\text{out}}r)}{\partial r} + A_2 \frac{\partial Y_\nu(k_{\text{out}}r)}{\partial r} \right) \exp(i\nu\phi). \quad (49c) \]

Now it is possible to write down explicit forms of polarized electric and magnetic field outside the cylinder.

### 4.2.2 Solution inside

Analogously to the previous section we put

\[ \epsilon(r) = \epsilon_{\text{in}}, \mu = 1, \sigma(r) = \sigma_{\text{in}}. \quad (50a) \]

\[ k^2 = \left( \frac{\omega^2 \epsilon_{\text{in}}}{c^2} - i \frac{4\pi\omega}{c^2} \sigma_{\text{in}} \right) = k_{\text{in}}^2 \quad (51) \]

In the case of stepfunctions, \( k_{\text{in}} \) does not depend on radial coordinate but this time it is a complex number. It does not change the form of the solutions, the only difference is that the argument of the solutions is complex. Upper bounds of solutions have to be limited, so Neumann function and its derivatives contributions that blows up at zero must be suppressed. It gives

\[ E_z = \sum_\nu A_5 J_\nu(k_{\text{in}}r) \exp(i\nu\phi), \quad (52a) \]

\[ E_r = \sum_\nu \frac{1}{\nu} A_6 J_\nu(k_{\text{in}}r) \exp(i\nu\phi), \quad (52b) \]

\[ E_\phi = \sum_\nu i\frac{1}{\nu} A_6 \frac{\partial J_\nu(k_{\text{in}}r)}{\partial r} \exp(i\nu\phi). \quad (52c) \]

\[ B_z = -\frac{ic}{\omega} \sum_\nu A_6 \frac{ik^2_{\text{in}}}{\nu} J_\nu(k_{\text{in}}r) \exp(i\nu\phi), \quad (53a) \]

\[ B_r = -\frac{c}{\omega r} \sum_\nu \nu A_5 J_\nu(k_{\text{in}}r) \exp(i\nu\phi), \quad (53b) \]

\[ B_\phi = -i\frac{c}{\omega} \sum_\nu A_5 \frac{\partial J_\nu(k_{\text{in}}r)}{\partial r} \exp(i\nu\phi). \quad (53c) \]

### 4.2.3 Polarization towards \( z \)

Explicit forms of polarised electric and magnetic field outside the cylinder yields:

\[ 2\mathbf{e}_{\text{out}} = \left[ \hat{z} \sum_\nu i^\nu J_\nu(k_{\text{out}}r) \exp(i\nu\phi) + \hat{z} \sum_\nu \left( A_1 J_\nu(k_{\text{out}}r) + A_2 Y_\nu(k_{\text{out}}r) \right) \exp(i\nu\phi) \right] \exp(i\omega t) + \]

\[ + \left[ \sum_\nu (-i)^\nu J_\nu(k_{\text{out}}r) \exp(-i\nu\phi) + \sum_\nu \left( A_1^* J_\nu(k_{\text{out}}r) + A_2^* Y_\nu(k_{\text{out}}r) \right) \exp(-i\nu\phi) \right] \exp(-i\omega t) \quad (54) \]
\[ 2 \mathbf{B}_{\text{out}} = \left[ -\hat{r} \sum_{\nu} \frac{e^{i \nu}}{r} \frac{1}{k_{\text{out}}} J_\nu(k_{\text{out}} r) \exp(i\nu \varphi) + \hat{\varphi} \sum_{\nu} \frac{i e^{i \nu}}{r} \frac{\partial J_\nu(k_{\text{out}} r)}{\partial r} \exp(i\nu \varphi) + \right. \]
\[ -\hat{r} \sum_{\nu} \frac{e^{i \nu}}{r^2} \left( A_1 J_\nu(k_{\text{out}} r) + A_2 Y_\nu(k_{\text{out}} r) \right) \exp(i\nu \varphi) + \]
\[ - \hat{\varphi} \sum_{\nu} \frac{i e^{i \nu}}{r^2} \left( A_1 \frac{\partial J_\nu(k_{\text{out}} r)}{\partial r} + A_2 \frac{\partial Y_\nu(k_{\text{out}} r)}{\partial r} \right) \exp(-i\nu \varphi) + \]
\[ + \hat{r} \sum_{\nu} \frac{e^{-i \nu}}{r^2} \left( A_1 J_\nu(k_{\text{out}} r) + A_2 Y_\nu(k_{\text{out}} r) \right) \exp(-i\nu \varphi) + \]
\[ \left. + \hat{\varphi} \sum_{\nu} \frac{i e^{-i \nu}}{r^2} \left( A_1 J_\nu(k_{\text{out}} r) + A_2 Y_\nu(k_{\text{out}} r) \right) \exp(-i\nu \varphi) \right] \exp(-i\omega t) + \]
\[ 2 \mathbf{E}_{\text{in}} = \left[ \hat{z} \sum_{\nu} A_5 J_\nu(k_{\text{in}} r) \exp(i\nu \varphi) \exp(i\omega t) + \hat{r} \sum_{\nu} \frac{i e^{i \nu}}{\omega} A_5 \frac{\partial J_\nu(k_{\text{in}} r)}{\partial r} \exp(-i\nu \varphi) \exp(-i\omega t) + \right. \]
\[ \left. \hat{\varphi} \sum_{\nu} \frac{i e^{i \nu}}{\omega} A_5 \frac{\partial J_\nu(k_{\text{in}} r)}{\partial r} \exp(i\nu \varphi) \exp(-i\omega t) + \hat{r} \sum_{\nu} \frac{e^{i \nu}}{\omega} A_5 J_\nu(k_{\text{in}} r) \exp(-i\nu \varphi) \exp(i\omega t) \right. \]
\[ + \left. \hat{\varphi} \sum_{\nu} \frac{e^{i \nu}}{\omega} A_5 J_\nu(k_{\text{in}} r) \exp(i\nu \varphi) \exp(-i\omega t) \right] \exp(-i\omega t) \]
\[ 2 \mathbf{B}_{\text{in}} = \left[ -\hat{r} \sum_{\nu} \frac{e^{i \nu}}{\omega} A_5 J_\nu(k_{\text{in}} r) \exp(i\nu \varphi) + \hat{\varphi} \sum_{\nu} \frac{i e^{i \nu}}{\omega} A_5 \frac{\partial J_\nu(k_{\text{in}} r)}{\partial r} \exp(-i\nu \varphi) \exp(-i\omega t) + \right. \]
\[ + \left. \hat{r} \sum_{\nu} \frac{e^{-i \nu}}{\omega} A_5 J_\nu(k_{\text{in}} r) \exp(-i\nu \varphi) + \hat{\varphi} \sum_{\nu} \frac{i e^{-i \nu}}{\omega} A_5 \frac{\partial J_\nu(k_{\text{in}} r)}{\partial r} \exp(-i\nu \varphi) \exp(-i\omega t) \right] \exp(-i\omega t) \]

4.3 Boundary conditions on a rod surface for polarizations along \( z \),

Tangential components of electric and perpendicular contribution of magnetic field must be continuous on a surface of the rod. Components toward \( z \) and \( \varphi \) axes are always tangential to the cylinder surface, what is universal, whatever light polarisation is. Assume the wave polarisation is directed along \( z \), then:

\[ (\mathbf{E}_{\text{out}})_z|_{r_0} - (\mathbf{E}_{\text{in}})_z|_{r_0} = 0, \]  \[ (\mathbf{B}_{\text{out}})_z|_{r_0} - (\mathbf{B}_{\text{in}})_z|_{r_0} = 0, \]
\[ (\mathbf{B}_{\text{out}})_\varphi|_{r_0} - (\mathbf{B}_{\text{in}})_\varphi|_{r_0} = 0. \]
Because $\exp(i\omega t)$ and $\exp(-i\omega t)$ are linearly independent, above conditions can be separated:

\[
\begin{align*}
&\left( E_{\text{out}} \right)_{z|r_0} - \left( E_{\text{in}} \right)_{z|r_0} = 0, \quad (59a) \\
&\left( E_{\text{out}}^* \right)_{z|r_0} - \left( E_{\text{in}}^* \right)_{z|r_0} = 0, \quad (59b) \\
&\left( B_{\text{out}} \right)_{\phi|r_0} - \left( B_{\text{in}} \right)_{\phi|r_0} = 0, \quad (59c) \\
&\left( B_{\text{out}}^* \right)_{\phi|r_0} - \left( B_{\text{in}}^* \right)_{\phi|r_0} = 0, \quad (59d) \\
&\left( B_{\text{out}} \right)_{r|\nu} - \left( B_{\text{in}} \right)_{r|\nu} = 0, \quad (59e) \\
&\left( B_{\text{out}}^* \right)_{r|\nu} - \left( B_{\text{in}}^* \right)_{r|\nu} = 0. \quad (59f)
\end{align*}
\]

Explicit form after substitution of the field components leads to the set of two equations, which contain three unknown constants.

\[
A_5 J_\nu(k_{\text{in}}r_0) = (i\nu + A_1)J_\nu(k_{\text{out}}r_0) + A_2 Y_\nu(k_{\text{out}}r_0), \quad (60a)
\]

\[
A_5 \frac{\partial J_\nu(k_{\text{in}}r)}{\partial r} \bigg|_{r_0} = (i\nu + A_1) \frac{\partial J_\nu(k_{\text{out}}r)}{\partial r} \bigg|_{r_0} + A_2 \frac{\partial Y_\nu(k_{\text{out}}r)}{\partial r} \bigg|_{r_0}. \quad (60b)
\]

It gives us formula for amplitude inside rod

\[
A_5 = \frac{i\nu J_\nu(k_{\text{in}}r_0) + A_1 J_\nu(k_{\text{out}}r_0) + A_2 Y_\nu(k_{\text{out}}r_0)}{J_\nu(k_{\text{in}}r_0)}, \quad (61)
\]

and the relation between constants $A_1$ and $A_2$.

\[
A_2 = -(A_1 + i\nu) \frac{\partial}{\partial r} \left. \left( \frac{J_\nu(k_{\text{out}}r)}{J_\nu(k_{\text{in}}r)} \right) \right|_{r_0}. \quad (62)
\]

As it is shown in the formula (62) there is a freedom in choosing explicit relation of $A_1$ and $A_2$. By direct physical intuition, we choose constants such that

\[
A_2 = -iA_1. \quad (63)
\]

Hence refracted wave would be described by a Hankel function of the first kind $H^{(1)}_\nu(k_{\text{out}}r) = J_\nu(k_{\text{out}}r) + iY_\nu(k_{\text{out}}r)$. Hankel function tends to be a cylindrical wave at infinity, so it is a good choice. Finally,

\[
A_5 = \frac{i\nu J_\nu(k_{\text{out}}r_0) + A_1 H^{(2)}_\nu(k_{\text{out}}r_0)}{J_\nu(k_{\text{in}}r_0)}, \quad (64)
\]

\[
A_1 = -i\nu \frac{k_{\text{out}}(J_{\nu-1}(k_{\text{out}}r_0) - J_{\nu+1}(k_{\text{out}}r_0)) J_\nu(k_{\text{in}}r_0) - k_{\text{in}}J_\nu(k_{\text{out}}r_0) (J_{\nu-1}(k_{\text{in}}r_0) - J_{\nu+1}(k_{\text{in}}r_0))}{k_{\text{out}}(H^{(2)}_{\nu-1}(k_{\text{out}}r_0) - H^{(2)}_{\nu+1}(k_{\text{out}}r_0)) J_\nu(k_{\text{in}}r_0) - k_{\text{in}}H^{(2)}_\nu(k_{\text{out}}r_0) (J_{\nu-1}(k_{\text{in}}r) - J_{\nu+1}(k_{\text{in}}r_0))}. \quad (65)
\]

### 4.4 Properties and approximations of sum over $A_1(\nu)$

Assume $r_0 < 100\AA$. Taking into account the physical values of introduced parameters, it is possible to approximate estimated amplitude $A_1$. While the
argument is small enough \((k_{\text{out}}r_0 = \frac{2\pi r_0}{\lambda} \approx \frac{6.1004 A}{6000 A} = 0.1)\), Bessel functions tend to be a monomials.

\[
J_\nu(z) \approx \frac{\Gamma(\nu+1)}{\Gamma(1/2)} \left(\frac{z}{2}\right)^\nu, \quad \nu \neq -1, -2, \ldots 
\]

Next,

\[
\|k_{\text{in}}r_0\| = \frac{r_0 \omega}{c} \sqrt{\epsilon^2 + 16\pi^2 \left(\frac{\sigma}{\omega}\right)^2}.
\]

There is one more restrictions imposed by formula (66), that is \(\nu\) must be nonnegative. Remember that the solution \(E_z\) requires sum over all integer values of \(\nu\), from minus infinity to plus infinity. However, it can be easily check, that \(A_1(\nu)\) is even or odd function for even or odd \(\nu\) respectively:

\[
A_1(\nu) = (-)^\nu A_1(-\nu),
\]

so the constant can be approximated as follows:

\[
A_1 \approx -i^\nu \frac{i\pi}{2} \frac{(\frac{1}{2} k_{\text{out}}r_0)^{2\nu+2}}{\frac{\omega}{\nu} (\nu+1)} \left(\frac{k_{\text{in}}}{k_{\text{out}}}\right)^2 - 1,
\]

Formula above should be used very carefully, because it is no more even or odd via \(\nu\).

### 4.5 Energy density flux evaluation

The Pointing vector

\[
\mathbf{S} = \frac{c}{16\pi} \mathbb{R}[\mathbf{E} \times \mathbf{B}^*] = \frac{c}{16\pi} \left(-2\mathbb{R}[E_z B_\phi^*] r + 2\mathbb{R}[E_z B_\phi^*] \hat{r}\right),
\]

defines the instant energy density flux. We are interested in contribution toward radial direction \(\hat{r}\) for big \(z\).

\[
(\mathbf{S}_{\text{scat}})_{\hat{r}} = -\frac{c}{8\pi} \mathbb{R}[E_{\text{scat},z} B_{\text{scat},\phi}^*],
\]

Asymptotics at large \(z\) is

\[
H^{(2)}_\nu(z) \approx \sqrt{\frac{2}{\pi z}} \exp\left(-iz + i\frac{\nu \pi}{2} + i\frac{\pi}{4}\right),
\]

hence

\[
E_{\text{scat},z} = \sum_\nu A_1(\nu) H^{(2)}_\nu(k_{\text{out}}r) \exp(i\nu\phi) \approx \sqrt{\frac{2}{\pi k_{\text{out}}r}} \exp\left(-ik_{\text{out}}r + i\frac{\pi}{4}\right) \sum_\nu A_1(\nu) \exp\left(i\frac{\pi}{4}\right) \exp(i\nu\phi),
\]
Denote $A = \nu \omega$, then

$$B_{\text{scat}, \varphi} = - \sum_{\nu} \frac{i c}{\omega} A_1(\nu) \frac{\partial H^{(2)}_{\nu}(k_{\text{out} r})}{\partial r} \exp (i \nu \varphi) =$$

$$= - \sum_{\nu} \frac{i c}{\omega} A_1(\nu) k_{\text{out} r} \frac{1}{2} \left[ H^{(2)}_{\nu - 1}(k_{\text{out} r}) - H^{(2)}_{\nu + 1}(k_{\text{out} r}) \right] \exp (i \nu \varphi) \approx$$

$$\approx - \sqrt{\frac{2}{\pi k_{\text{out} r}}} \exp \left( -i k_{\text{out} r} + i \frac{\pi}{4} \right) \sum_{\nu} i A_1(\nu) \exp \left( \frac{i \nu \pi}{2} \right) \left[ \exp \left( -i \frac{\pi}{2} \right) - \exp \left( i \frac{\pi}{2} \right) \right] \exp (i \nu \varphi) =$$

$$= - \sqrt{\frac{2}{\pi k_{\text{out} r}}} \exp \left( -i k_{\text{out} r} + i \frac{\pi}{4} \right) \sum_{\nu} A_1(\nu) \exp \left( \frac{i \nu \pi}{2} \right) \exp (i \nu \varphi)$$

Therefore

$$(\mathcal{S}_{\text{scat}})_r = \frac{c}{4 \pi^2 k_{\text{out} r}} \sum_{\nu, \nu'} (-1)^{\nu - \nu'} A_1(\nu) A_1^*(\nu') \exp (i (\nu - \nu') \varphi). \quad (74)$$

### 4.6 Polarization toward $y$, the list of formulas

#### 4.6.1 Fields

$$2 \mathbf{E}_{\text{out}} = \left[ -\hat{r} \sum_{\nu} \nu^2 \frac{1}{r} J_\nu(k_{\text{out} r}) \exp (i \nu \varphi) + \right.$$

$$+ \hat{\varphi} \sum_{\nu} \nu^2 \frac{1}{k_{\text{out} r}} \frac{\partial J_\nu(k_{\text{out} r})}{\partial r} \exp (i \nu \varphi) +$$

$$\left. + \hat{r} \sum_{\nu} \frac{1}{r} \left( A_3 J_\nu(k_{\text{out} r}) + A_4 Y_\nu(k_{\text{out} r}) \right) \exp (i \nu \varphi) \right]$$

$$+ \varphi \sum_{\nu} \nu^2 \left( A_3 \frac{\partial J_\nu(k_{\text{out} r})}{\partial r} + A_4 \frac{\partial Y_\nu(k_{\text{out} r})}{\partial r} \right) \exp (i \nu \varphi) \exp (i \omega t) + c.c. \quad (75)$$

$$2 \mathbf{B}_{\text{out}} = \left[ -\hat{z} \sum_{\nu} \nu^2 J_\nu(k_{\text{out} r}) \exp (i \nu \varphi) + \hat{z} \sum_{\nu} \nu^2 \frac{k_{\text{out} r}^2}{\nu} \left[ A_3 J_\nu(k_{\text{out} r}) + A_4 Y_\nu(k_{\text{out} r}) \right] \exp (i \nu \varphi) \right] \exp (i \omega t) + c.c. \quad (76)$$

$$2 \mathbf{E}_{\text{in}} = \left[ \hat{r} \sum_{\nu} \frac{1}{r} A_6 J_\nu(k_{\text{in} r}) \exp (i \nu \varphi) + \hat{\varphi} \sum_{\nu} \nu A_6 \frac{\partial J_\nu(k_{\text{in} r})}{\partial r} \exp (i \nu \varphi) \right] \exp (i \omega t) + c.c. \quad (77)$$

$$2 \mathbf{B}_{\text{in}} = \left[ \hat{z} \sum_{\nu} \nu A_6 \frac{k_{\text{in} r}^2}{\nu} J_\nu(k_{\text{in} r}) \exp (i \nu \varphi) \right] \exp (i \omega t) + c.c. \quad (78)$$
4.6.2 Boundary conditions on a rod surface for the polarization

\[
\begin{align*}
(B_{\text{out}})|_{r_0} - (B_{\text{in}})|_{r_0} &= 0, \quad (79a) \\
\epsilon_{\text{out}} (E_{\text{out}})|_{r_0} - \epsilon_{\text{in}} (E_{\text{in}})|_{r_0} &= 0, \quad (79b) \\
(E_{\text{out}})|_{r_0} - (E_{\text{in}})|_{r_0} &= 0. \quad (79c)
\end{align*}
\]

\[
A_0 \frac{\partial J_r(k_{\text{in}} r)}{\partial r} |_{r_0} = (A_3 - \frac{i\nu}{k_{\text{out}}}) J_r(k_{\text{out}} r) - J_r(k_{\text{in}} r) \begin{pmatrix} \frac{\partial}{\partial r} J_r(k_{\text{out}} r) \end{pmatrix}_{r_0} + A_4 \frac{\partial Y_r(k_{\text{out}} r)}{\partial r} |_{r_0} \quad \text{similar to the previous case,}
A_3 = -i A_4.
\]

\[
A_3 = \frac{i\nu}{k_{\text{out}}} \begin{pmatrix} \frac{k^2_{\text{in}}}{k_{\text{out}}} \frac{\partial}{\partial r} J_r(k_{\text{out}} r) \end{pmatrix} J_r(k_{\text{in}} r) + J_r(k_{\text{out}} r) \begin{pmatrix} \frac{\partial}{\partial r} J_r(k_{\text{out}} r) \end{pmatrix}_{r_0} \]  \quad \text{(80)}
\]

4.6.3 Energy density flux

\[
\mathcal{G} = \frac{c}{16\pi} \Re[\mathbf{E} \times \mathbf{B}^*] = \frac{c}{16\pi} \left( \frac{1}{r} \Im[\mathbf{E}_\varphi B^*_z] - \frac{2}{r} \Im[\mathbf{E}_z B^*_\varphi] \right). \quad (82)
\]

We are interested again in contribution towards radial direction \(\hat{r}\):

\[
(G_{\text{scat}})_{\hat{r}} = \frac{c}{8\pi r} \Re[B_{\text{scat}, \varphi} B^*_{\text{scat}, z}], \quad (83)
\]

\[
H^{(2)}_\nu(z) \approx \sqrt{\frac{2}{\pi z}} \exp \left( -iz + \frac{i\nu\pi}{2} + i\frac{\pi}{4} \right), \quad (84)
\]

\[
E_{\text{scat}, \varphi} = \sum_{\nu} \frac{i}{\nu} k_{\text{out}} A_3(\nu) \frac{\partial H^{(2)}_\nu(k_{\text{out}} r)}{\partial r} \exp \left( i\nu\varphi \right) = \sum_{\nu} \frac{i}{\nu} k_{\text{out}} A_3(\nu) \frac{1}{2} \left[ H^{(2)}_{\nu-1}(k_{\text{out}} r) - H^{(2)}_{\nu+1}(k_{\text{out}} r) \right] \exp \left( i\nu\varphi \right) \approx
\]

\[
= \sqrt{\frac{2}{\pi k_{\text{out}} r}} \exp \left( -ik_{\text{out}} r + i\frac{\pi}{4} \right) \sum_{\nu} \frac{i}{\nu} k_{\text{out}} A_3(\nu) \exp \left( \frac{i\nu\pi}{2} \right) \left[ \exp \left( -i\frac{\pi}{2} \right) - \exp \left( i\frac{\pi}{2} \right) \right] \exp \left( i\nu\varphi \right) = \sqrt{\frac{2}{\pi k_{\text{out}} r}} \exp \left( -ik_{\text{out}} r + i\frac{\pi}{4} \right) \sum_{\nu} \frac{i}{\nu} k_{\text{out}} A_3(\nu) \exp \left( \frac{i\nu\pi}{2} \right) \exp \left( i\nu\varphi \right)
\]

\[
B_{\text{scat}, z} = -\frac{ic}{\omega} \sum_{\nu} \frac{1}{\nu} k_{\text{out}} \frac{i}{\nu} k_{\text{out}} H^{(2)}_\nu(k_{\text{out}} r) \exp \left( i\nu\varphi \right) \approx \sqrt{\frac{2}{\pi k_{\text{out}} r}} \exp \left( -ik_{\text{out}} r + i\frac{\pi}{4} \right) \sum_{\nu} \frac{k_{\text{out}}}{\nu} A_3(\nu) \exp \left( \frac{i\nu\pi}{2} \right)
A_3 = i\nu \frac{1}{k_{\text{out}}} a_3(\nu),
\]

(85)
\[
\sum \frac{1}{k_{\text{out}}} A_3(\nu) \exp \left( i \frac{\nu \pi}{2} \right) \exp (i \nu \varphi) = \\
= \sum \frac{1}{k_{\text{out}}} i \nu \frac{1}{k_{\text{out}}} A_3(\nu) \exp \left( i \frac{\nu \pi}{2} \right) \exp (i \nu \varphi) = \\
= \sum A_3(\nu) \exp (i \nu \pi) \exp (i \nu \varphi),
\]

\[
E_{\text{scat},\varphi} B^*_{\text{scat},z} = \frac{2}{\pi k_{\text{out}}} \left\| \sum \frac{1}{k_{\text{out}}} A_3(\nu) \exp (i \nu \pi) \exp (i \nu \varphi) \right\|^2, \quad (86)
\]

\[
(\mathfrak{S}_{\text{scat}})_\varphi = \frac{c}{4 \pi^2} \frac{1}{k_{\text{out}}} \left\| \sum \frac{1}{k_{\text{out}}} A_3(\nu) \exp (i \nu \pi) \exp (i \nu \varphi) \right\|^2. \quad (87)
\]

5 Results discussion

The derived formulas for the z-polarization coincide with ones from [2] if one take into account the relation

\[
k_{\text{in}}/k_{\text{out}} = \tilde{n}, \quad (88)
\]

and change notations as

\[
a_3(\nu) = a_\nu, \quad (89a)
\]

\[
a_1(\nu) = b_\nu \quad (89b)
\]

\[
Q_{TM} = \frac{1}{k_{\text{out}}} \left\| \sum b_\nu \exp (i \nu \pi) \exp (i \nu \varphi) \right\|^2 = \frac{1}{k_{\text{out}}} \sum_{\nu, \nu'} (-1)^{(\nu - \nu')} b_\nu b^*_\nu \exp (i(\nu - \nu') \varphi)
\]

(90)

The main target of this paper is to derive and investigate dependence of scattered wave on the material parameters and dimension of the rod. For this purpose a code and program were elaborated. Below we illustrate the results of calculations by plots (Figure 1). Authors use formulas for scattered light integrated over the angle. But the observed scattered light only for 180°. The dependence on frequency is rather strong for nanowires. This property was used in [2] to propose a method which allows to determine precisely the radius of a wire. Our formalism gives a link to such physical constants as \( \sigma, \epsilon \) determination by optical measurements.

References

[1] Craig F. Bohren, Donald R. Huffman Absorption and Scattering of Light by Small Particles. Wiley, 1983.

[2] G Bröstrup et al A precise optical determination of nanoscale diameters of semiconductor nanowires. 2011 Nanotechnology 38 22 385201 doi:10.1088/0957-4484/22/38/385201

[3] R. B. Keam. Plane wave excitation of an infinite dielectric rod. IEEE MICROWAVE AND GUIDED WAVE LETTERS, 4:326–328, 1994.
Figure 1: Dependence of scattering amplitude on frequency for two polarizations