The Spin Structure of the $qq$ Interaction and the Mass Spectra of Bound $q\bar{q}$ Systems: Different Versions of the 3-Dimensional Reductions of the Bethe-Salpeter Equation

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Abstract

Bound $q\bar{q}$-systems are considered in the framework of three different versions of the 3-dimensional reduction of the Bethe-Salpeter equation, all having the correct one-body limit when one of the constituent quark masses tends to infinity, and in the framework of the Salpeter equation. The spin structure of confining $qq$ interaction potential is taken in the form $x\gamma^0_1\gamma^0_2 + (1-x)I_1I_2$, with $0 \leq x \leq 1$. The problem of existence (nonexistence) of stable solutions of 3-dimensional relativistic equations for bound $q\bar{q}$ systems is studied for different values of $x$ from this interval. Some other aspects of this problem are discussed.

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As it is well-known, at present the spin (Lorentz) structure of the \(qq\) interaction is not established theoretically in QCD, a fundamental theory of strong interactions. Consequently, it is interesting to consider different possible choices for the spin structure, as it was done in Refs. [1]-[4] where the bound \(q\bar{q}\) systems in the framework of the Salpeter equation were investigated. Further, it is well known that the Salpeter equation is the simplest version of the 3-dimensional (3D) reduction of the Bethe-Salpeter (BS) equation, the latter being believed to provide a natural basis to study bound \(q\bar{q}\) systems in the framework of the Constituent Quark Model. Namely, the Salpeter equation is obtained from the BS equation in a straightforward way, when the kernel of the latter is assumed to have the instantaneous (static) form. However, in the instantaneous approximation there exist other possible versions [5, 6] of the 3D reduction of the BS equation which, unlike the Salpeter equation, have a correct one-body limit when the mass of one of the constituents tends to infinity. These versions, which can be derived by choosing appropriate effective 3D-Green’s function for two noninteracting fermions, will hereafter be referred to as the MW and CJ versions, respectively. Moreover, a new version of the effective propagator for two free scalar particles, guaranteeing the existence of the correct one-body limit for 3D-equations, was suggested in Ref. [7]. The effective 3D-Green’s function for two noninteracting fermions can be constructed along the lines similar to Ref. [7], in a standard manner (see below).

Taking into account the fact that the relativistic effects are important for \(q\bar{q}\) systems containing two light quarks, as well as for heavy-light systems, it seems interesting to carry out the investigation of this sort of systems in the framework of the above mentioned 3D relativistic equations and study the dependence of the properties of the bound \(q\bar{q}\) systems on the spin (Lorentz) structure of the confining part of \(qq\)-interaction. In the present report, we deal with this problem as concerned to the \(q\bar{q}\) mass spectrum.

The relativistic 3D equations for the wave function of bound \(q\bar{q}\) systems, corresponding to the instantaneous (static) BS kernel \((K(P;p,p') \rightarrow K_{st}(\vec{p},\vec{p}'))\), for all versions considered below can be written in a common form (in the cm frame)

\[
\tilde{\Phi}_M(\vec{p}) = \tilde{G}_{0,eff}(M,\vec{p}) \int \frac{d\vec{p}'}{(2\pi)^3} [iK_{st}(\vec{p},\vec{p}')] \equiv \tilde{V}(\vec{p},\vec{p}') \tilde{\Phi}_M(\vec{p}')
\]  

(1)

where \(M\) is the bound system mass. The equal-time wave function \(\tilde{\Phi}_M(\vec{p})\) is related to the BS amplitude \(\Phi_P(p)\) by

\[
\tilde{\Phi}_M(\vec{p}) = \int \frac{dp_0}{2\pi} \Phi_{P=(M,\vec{0})}(p)
\]  

(2)
and the effective 3D Green’s function of the two noninteracting quark system \( \tilde{G}_{0e\!ff} \) is defined as

\[
\tilde{G}_{0e\!ff}(M, \vec{p}) = \int \frac{dp_0}{2\pi i} [G_{0e\!ff}(M, p) = g_{0e\!ff}(M, p)(\not{p}_1 + m_1)(\not{p}_2 + m_2)] \tag{3}
\]

Here, \( G_{0e\!ff} \) is the effective two fermion free propagator and \( g_{0e\!ff} \) is the effective propagator of two scalar particles. The operator \( \tilde{G}_{0e\!ff} \) can be given in the form

\[
\tilde{G}_{0e\!ff}(M, \vec{p}) = \sum_{\alpha_1 = \pm} \sum_{\alpha_2 = \pm} \frac{D^{(\alpha_1\alpha_2)}(M, p)}{d(M, p)} \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}_1, -\vec{p}_2) \gamma_1 \gamma_2, \quad p \equiv |\vec{p}| \tag{4}
\]

where the projection operators \( \Lambda_{12}^{(\alpha_1\alpha_2)} \) are defined as

\[
\Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}_1, \vec{p}_2) = \Lambda_1^{(\alpha_1)}(\vec{p}_1) \otimes \Lambda_2^{(\alpha_2)}(\vec{p}_2), \quad \Lambda_1^{(\alpha_1)}(\vec{p}_1) = \frac{\omega_1 \gamma_1 + \hat{h}_i(\vec{p}_1)}{2\omega_i}, \quad \hat{h}_i(\vec{p}_1) = \vec{\alpha}_i \vec{p}_1 + m_i \gamma_0^0, \quad \omega_i = (m_i^2 + \vec{p}_i^2)^{1/2} \tag{5}
\]

and the functions \( D^{(\alpha_1\alpha_2)}(M, p) \) and \( d(M, p) \) are given by

\[
D^{(\alpha_1\alpha_2)}(M, p) = \frac{(-1)^{\alpha_1 + \alpha_2}}{\omega_1 + \omega_2 - (\alpha_1 E_1 + \alpha_2 E_2)}, \quad d(M, p) = 1
\]

\[
E_1 + E_2 = M, \quad E_1 - E_2 = \frac{m_1^2 - m_2^2}{M} \equiv b_0 \quad (\text{MW version}) \tag{6}
\]

\[
d(M, p) = 2(\omega_1 + \omega_2), \quad a = E_i^2 - \omega_i^2 = [M^2 + b_0^2 + 2(\omega_1^2 + \omega_2^2)]/4 \quad (\text{CJ version}) \tag{7}
\]

(see Ref. [8] where the mass spectrum of the bound \( q\bar{q} \) systems was investigated in the framework of the MW and CJ versions of the relativistic 3D equations in the configuration space).

Note that expression (7) was derived from Eq. (3) by using that for \( g_{0e\!ff}(M, p) \) determined from the dispersion relation. The same relation is satisfied by the expression of \( g_{0e\!ff}(M, p) \) suggested in Ref. [7] (see formula (10) therein). According to the prescription given in Ref. [7], particles in the intermediate states are allowed to go off shell proportionally to their mass, so that when one of the particles becomes infinitely massive, it automatically is kept on mass shell and the corresponding equation can be reduced to the one-body equation. Using this expression for \( g_{0e\!ff}(M, p) \) from Eq. (3) we obtain the expression for \( \tilde{G}_{0e\!ff} \) given by Eq. (4). (Note that formula (11) in Ref. [7] is not correct as it does not follow from formula (10)). So we obtain

\[
D^{(\alpha_1\alpha_2)}(M, p) = (E_1 + \alpha_1 \omega_1)(E_2 + \alpha_2 \omega_2) - \frac{R - b}{2y} \left[ \frac{R - b}{2y} + (E_1 + \alpha_1 \omega_1) - (E_2 + \alpha_2 \omega_2) \right]
\]
\[ d(M, p) = 2RB, \quad R = (b^2 - 4y^2a)^{1/2}, \quad B = \frac{R - b}{2y} \left[ \frac{R - b}{2y} + b \right] + a, \]
\[ b = M + b_0y, \quad y = \frac{m_1 - m_2}{m_1 + m_2} \quad (8) \]

This version will below be referred to as the MNK version.

Using the properties of the projection operators \( \Lambda_{12}^{(\alpha_1\alpha_2)} \) and formulae (4-8), the following system of equations can be derived from Eq. (1)

\[ [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) = \]
\[ = A^{(\alpha_1\alpha_2)}(M, p) \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}, -\vec{p}) \int \frac{d\vec{p}'}{2(2\pi)^3} \gamma_1^0 \gamma_2^0 \hat{V}(\vec{p}, \vec{p}') \sum_{\alpha_1' = \pm} \sum_{\alpha_2' = \pm} \tilde{\Phi}_M^{(\alpha_1'\alpha_2')}(\vec{p}') \quad (9) \]

where \( \tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) = \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}, -\vec{p}) \tilde{\Phi}_M(\vec{p}) \) and the functions \( A^{(\alpha_1\alpha_2)} \) are given by

\[ A^{(\pm\pm)} = \pm 1, \quad A^{(\pm\mp)} = \frac{M}{\omega_1 + \omega_2} \quad (\text{MW version}) \quad (10) \]

\[ A^{(\alpha_1\alpha_2)} = \frac{1}{2RB} \left\{ a[M + (\alpha_1\omega_1 + \alpha_2\omega_2)] - 
- [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\frac{R - b}{2y} \left[ \frac{R - b}{2y} + (E_1 + \alpha_1\omega_1) - (E_2 + \alpha_2\omega_2) \right] \right\} \quad (\text{MNK version}) \quad (12) \]

As to the Salpeter equation, it can be obtained from the MW version by putting \( A^{(\pm\mp)} = 0 \) and \( \tilde{\Phi}_M^{(\pm\mp)} = 0 \).

Further, we write the unknown function \( \tilde{\Phi}_M^{(\alpha_1'\alpha_2')} \) in Eq. (9) in the form analogous to that used in Ref. [9], where the bound \( qq \) systems were studied in the framework of the Salpeter equation

\[ \tilde{\Phi}_M^{(\alpha_1\alpha_2)}(\vec{p}) = N_{12}^{(\alpha_1\alpha_2)}(p) \left( \frac{1}{\alpha_1\vec{\sigma}_1\vec{p}/(\omega_1 + \alpha_1 m_1)} \otimes \frac{1}{-\alpha_2\vec{\sigma}_2\vec{p}/(\omega_2 + \alpha_2 m_2)} \right) \chi_M^{(\alpha_1\alpha_2)}(\vec{p}) \quad (13) \]

where

\[ N_{12}^{(\alpha_1\alpha_2)}(p) = \left( \frac{\omega_1 + \alpha_1 m_1}{2\omega_1} \right)^{1/2} \left( \frac{\omega_2 + \alpha_2 m_2}{2\omega_2} \right)^{1/2} \equiv N_1^{(\alpha_1)}(p) N_2^{(\alpha_2)}(p) \quad (14) \]

Then, if the \( qq \) interaction potential operator \( \hat{V}(\vec{p}, \vec{p}') \) is taken in the form [3]

\[ \hat{V}(\vec{p}, \vec{p}') = \gamma_1^0 \gamma_2^0 \hat{V}_{\text{eg}}(\vec{p} - \vec{p}') + [x \gamma_1^0 \gamma_2^0 + (1 - x)I_1 I_2] \hat{V}_{\text{c}}(\vec{p} - \vec{p}'), \quad (0 \leq x \leq 1), \quad (15) \]

the following system of equations for the Pauli 2 \( \otimes \) 2 wave functions \( \chi_M^{(\alpha_1\alpha_2)} \) can be derived

\[ [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\chi_M^{(\alpha_1\alpha_2)}(\vec{p}) = \]
\[
\begin{align*}
A^{(\alpha_1 \alpha_2)}(M; p) &= \sum_{\alpha_1' = \pm} \sum_{\alpha_2' = \pm} \int \frac{d\vec{p}'}{(2\pi)^3} \tilde{V}_{e_{\text{eff}}}^{(\alpha_1 \alpha_2, \alpha_1' \alpha_2')}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \chi_{M}^{(\alpha_1' \alpha_2')}(\vec{p}') \\
&= \sum_{\alpha_1' = \pm} \sum_{\alpha_2' = \pm} \int \frac{d\vec{p}'}{(2\pi)^3} \tilde{V}_{e_{\text{eff}}}^{(\alpha_1 \alpha_2, \alpha_1' \alpha_2')}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \chi_{M}^{(\alpha_1' \alpha_2')}(\vec{p}')
\end{align*}
\]

where the effective \( gg \) interaction operator \( \tilde{V}_{e_{\text{eff}}}^{(\alpha_1 \alpha_2, \alpha_1' \alpha_2')}(\vec{p}, \vec{p}', \vec{\sigma}_1, \vec{\sigma}_2) \) is expressed via the potentials \( V_{\text{og}} \) and \( V_c \) and some functions, taking account of relativistic kinematics. On using the partial-wave expansion

\[
\chi_{M}^{(\alpha_1 \alpha_2)}(\vec{p}) = \sum_{LSJM} < \vec{n}|LSJM \rangle \chi_{M}^{(\alpha_1 \alpha_2, \alpha_1' \alpha_2')}(\vec{p})
\]

From Eq. (16) the system of integral equations for the radial wave functions \( R_{LSJ}^{(\alpha_1 \alpha_2)}(p) \) can be obtained.

At the first stage our investigation is aimed at comparative analysis of the different versions of the 3D relativistic equations as concerns the existence of stable solutions for different values of the vector-scalar mixing parameter \( x \) in Eq. (15). For this reason, in Eq. (15) we neglect the one-gluon exchange potential and take the confining potential \( V_c(r) \) in the oscillator form used in Ref. [10], which is a simplified though justified version (for the light and light-heavy sectors) of a more general form used in Ref. [9]. Namely, we take

\[
V_c(r) = \frac{4}{3} \alpha_s(m_{12}^2) \left( \frac{\mu_2^2}{2} r^2 - V_0 \right)
\]

\[
\mu_{12} = \frac{m_1 m_2}{m_{12}}, \quad m_{12} = m_1 + m_2, \quad \alpha_s(Q^2) = \frac{12\pi}{33 - 2n_f} \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-1}
\]

In the momentum space, the system of integral equations for the radial functions \( R_{LSJ}^{(\alpha_1 \alpha_2)}(p) \) with the above potential is reduced to the system of second-order differential equations. The solution of this equation is written in the form similar to that given in Refs. [8, 10]

\[
R_{LSJ}^{(\alpha_1 \alpha_2)}(p) = \sum_{n=0}^{\infty} C_{LSJn}^{(\alpha_1 \alpha_2)} R_{nL}^{(\alpha_1 \alpha_2)}(p)
\]

where \( R_{nL}(p) \) are the well-known oscillator wave functions. Then, the system of equations for \( R_{LSJ}^{(\alpha_1 \alpha_2)}(p) \) is reduced to the system of linear algebraic equations for the coefficients \( C_{LSJn}^{(\alpha_1 \alpha_2)} \).

\[
M C_{LSJn}^{(\alpha_1 \alpha_2)} = \sum_{\alpha_1', \alpha_2', L', S', n'} H_{LSJn, L'S'L'n'}^{(\alpha_1 \alpha_2, \alpha_1' \alpha_2', \alpha_1'' \alpha_2'')} \chi_{M}^{(\alpha_1' \alpha_2')} C_{L'S'L'n'}^{(\alpha_1'' \alpha_2'')}
\]

Here it is necessary to stress that the matrix \( H \) explicitly depends (except the Salpeter version) on the meson mass \( M \) we are looking for. Consequently, the system of equations (20) is nonlinear in \( M \).
By truncating the sum in (19) at some fixed value \( N_{\text{max}} \), the eigenvalues \( M \) and the corresponding coefficients \( C^{(1)\alpha_2}_{LSJ_{\mu}} \) can be determined from the system of algebraic equations with the dimension \( 4(N_{\text{max}} + 1) \) (for the Salpeter case we have \( 2(N_{\text{max}} + 1) \) equations), provided the procedure converges with the increase of \( N_{\text{max}} \). If the procedure does not converge, we interpret this as absence of stable solutions to the initial equations. As it has been mentioned above, the system of equations (20) is nonlinear in \( M \) for the MW, CJ and MNK versions of the 3D equations. For the solution we use the iteration procedure: at the first step, in the r.h.s. of Eq. (20) determining \( M \), we substitute the solution of the Salpeter equation and then find \( M \) by the procedure explained above. At the second step, we substitute the obtained solution into the r.h.s. of Eq. (20) and the iterations are continued until the result converges.

In the present paper, we calculate the masses for the following \( q \bar{q} \) systems with nonequal mass quarks: \( d \bar{s} \left( ^{1}S_{0}, \ 3^{1}S_{1}, \ ^{1}P_{1}, \ ^{3}P_{0}, \ ^{3}P_{1}, \ ^{3}P_{2}, \ ^{1}D_{2}, \ ^{3}D_{1}, \ ^{3}D_{3} \right) \), \( c \bar{u} \) and \( c \bar{s} \left( ^{1}S_{0}, \ 3^{1}S_{1}, \ ^{1}P_{1}, \ ^{3}P_{2} \right) \), for which the values of the meson masses are known. In the calculations the following values of the parameters of the \( qq \)-interaction potential (18) were used \[9, 10\]: \( \omega_0 = 710 \text{ MeV} \), \( V_0 = 525 \text{ MeV} \), \( \Lambda = 120 \text{ MeV} \), \( m_u = m_d = 280 \text{ MeV} \), \( m_s = 400 \text{ MeV} \), \( m_c = 1470 \text{ MeV} \).

On the basis of calculation of mass spectra of the above \( q \bar{q} \) systems we have arrived at the following conclusions:

The stable solutions of the MW, CJ and MNK versions of the 3D relativistic equations always exist for \( x = 0 \). For \( x = 1 \) these solutions do not exist for the majority of states under consideration. For the Salpeter equation the situation is just opposite: for \( x = 1 \) stable solutions always exist whereas for \( x = 0 \) the solutions do not exist for the majority of the states studied. This agrees with the results obtained earlier in Refs. \[1\]-\[4\] for the \( q \bar{q} \) systems of equal mass quarks from the light quark sector (u,d,s). Moreover, for the CJ and MNK versions stable solutions always exist for \( (0 \leq x \leq 0.5) \). As to the MW version, this sort of solutions exist only for \( q \bar{q} \) systems with one heavy quark, whereas in order to provide the existence of stable solutions in the same interval of \( x \) for light \( q \bar{q} \) systems it is necessary to accept a much smaller value for the confining potential strength parameter \( \omega_0 \) (e.g. 450 MeV instead of 710 MeV). As to the interval \( (0.5 \leq x \leq 1) \), the existence (nonexistence) of stable solutions in the MW, CJ and MNK versions depends on the quark sector (light or heavy), quantum numbers of the \( q \bar{q} \) system and on the value of the parameter \( \omega_0 \). For the case of the Salpeter equation, the situation again is opposite - stable solutions always exist in the interval \( (0.5 \leq x \leq 1) \) for \( q \bar{q} \) systems with both quarks from light-quark sector \[2\]-\[4\], in the whole interval \( (0 \leq x \leq 1) \) for \( q \bar{q} \)
systems with both quarks from heavy-quark sector ($c\bar{c}$) [9], or from heavy-light sector (present result). The existence of stable solutions of the relativistic equations under consideration is mainly related to the presence of the mixed ($+−$ and $−+$) energy components of the wave functions in the equation for the $q\bar{q}$ bound state.

To illustrate these conclusions, in Tables 1. a,b,c we give the results of numerical solution of the system of equations (20) for $d\bar{s}$, $c\bar{u}$ and $c\bar{s}$ bound systems for the states $^1S_0$, $^3S_1$ and $^1P_1$.

Note that in order to obtain stable solutions of the 3D relativistic equations, it is sufficient to take $N_{max} = 4−7$ in the series (19). This property is common for all 3D versions and meson states under consideration.

A more detailed analysis (and the comparison with experiment) of the results in the framework of the above considered versions of 3D equations with the confining potential (18) (including the regularization problem of the wave function normalization condition in the CJ and MNK versions), as well as the description of decay properties of pseudoscalar and vector mesons ($P \rightarrow \mu \bar{\nu}$, $V \rightarrow e^+e^−$), will be published separately.

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Table 1.a The mass spectrum (in GeV) for \((d\bar{s})\) system

| States | Versions | \(\alpha_1\alpha_2\) | \(x=0.0\) | \(x=0.1\) | \(x=0.3\) | \(x=0.5\) | \(x=0.7\) | \(x=0.9\) | \(x=1.0\) |
|--------|----------|------------------|--------|--------|--------|--------|--------|--------|--------|
|        | MW\(^+\) | ++               | 0.826  | 0.836  | 0.854  | 0.870  | 0.884  | 0.895  | 0.900  |
|        |          | all              | 0.849  | 0.855  | 0.888  | 0.877  | 0.889  | *      | *      |
|        | CJ       | ++               | 0.966  | 0.997  | 1.055  | 1.107  | 1.155  | 1.197  | 1.216  |
|        |          | all              | 1.025  | 1.042  | 1.078  | 1.117  | 1.160  | *      | *      |
|        | MNK      | ++               | 0.938  | 0.966  | 1.016  | 1.060  | 1.098  | 1.131  | 1.146  |
|        |          | all              | 0.990  | 1.005  | 1.036  | 1.069  | *      | *      | *      |
|        | Sal      | ++               | 0.951  | 0.981  | 1.035  | 1.082  | 1.124  | 1.161  | 1.178  |
|        |          | all              | *      | *      | 1.026  | 1.079  | 1.121  | 1.160  | 1.172  |
|        |          |                   |        |        |        |        |        |        |        |
|        | MW\(^+\) | ++               | 0.833  | 0.843  | 0.861  | 0.877  | 0.891  | 0.902  | 0.907  |
|        |          | all              | 0.855  | 0.861  | 0.873  | 0.877  | 0.903  | *      | *      |
|        | CJ       | ++               | 0.985  | 1.017  | 1.076  | 1.129  | 1.177  | 1.220  | 1.239  |
|        |          | all              | 1.036  | 1.057  | 1.095  | 1.140  | 1.192  | *      | *      |
|        | MNK      | ++               | 0.958  | 0.986  | 1.035  | 1.079  | 1.117  | 1.150  | 1.165  |
|        |          | all              | 1.004  | 1.020  | 1.054  | 1.090  | 1.130  | *      | *      |
|        | Sal      | ++               | 0.971  | 1.001  | 1.054  | 1.102  | 1.143  | 1.181  | 1.198  |
|        |          | all              | *      | *      | 1.037  | 1.095  | 1.141  | 1.181  | 1.198  |
|        |          |                   |        |        |        |        |        |        |        |
|        | MW\(^+\) | ++               | 1.045  | 1.059  | 1.085  | 1.106  | 1.124  | 1.139  | 1.145  |
|        |          | all              | 1.082  | 1.090  | 1.104  | 1.118  | *      | 1.140  | *      |
|        | CJ       | ++               | 1.258  | 1.302  | 1.380  | 1.448  | 1.505  | 1.155  | 1.577  |
|        |          | all              | 1.342  | 1.366  | 1.415  | 1.464  | *      | *      | *      |
|        | MNK      | ++               | 1.216  | 1.251  | 1.314  | 1.367  | 1.414  | 1.455  | 1.473  |
|        |          | all              | 1.285  | 1.304  | 1.342  | 1.381  | 1.419  | *      | *      |
|        | Sal      | ++               | 1.235  | 1.274  | 1.343  | 1.403  | 1.454  | 1.499  | 1.519  |
|        |          | all              | *      | *      | 1.326  | 1.398  | 1.453  | 1.499  | 1.517  |

MW\(^+\) - \(\omega_0 = 450\) \(MeV\)
* - absence of stable solution
Table 1.b The mass spectrum (in GeV) for \((u\bar{c})\) system

| States | Versions | \(\alpha_1\alpha_2\) | x=0.0 | x=0.1 | x=0.3 | x=0.5 | x=0.7 | x=0.9 | x=1.0 |
|--------|----------|----------------------|-------|-------|-------|-------|-------|-------|-------|
| MW     | ++       | 2.012 | 2.031 | 2.065 | 2.097 | 2.127 | 2.155 | 2.168 |
|        | all      | 2.062 | 2.069 | 2.085 | 2.104 | 2.131 | *      | *     |
| CJ     | ++       | 2.018 | 2.037 | 2.073 | 2.107 | 2.138 | 2.167 | 2.181 |
|        | all      | 2.062 | 2.070 | 2.089 | 2.112 | 2.140 | *      | *     |
| MNK    | ++       | 2.011 | 2.029 | 2.063 | 2.095 | 2.124 | 2.151 | 2.164 |
|        | all      | 2.058 | 2.065 | 2.082 | 2.102 | 2.127 | *      | *     |
| Sal    | ++       | 2.012 | 2.031 | 2.065 | 2.097 | 2.127 | 2.155 | 2.168 |
|        | all      | 2.011 | 2.030 | 2.065 | 2.097 | 2.127 | 2.154 | 2.167 |
| MW     | ++       | 2.015 | 2.803 | 2.860 | 2.100 | 2.129 | 2.157 | 2.170 |
|        | all      | 2.063 | 2.070 | 2.087 | 2.107 | 2.134 | *      | *     |
| CJ     | ++       | 2.020 | 2.039 | 2.075 | 2.109 | 2.140 | 2.170 | 2.183 |
|        | all      | 2.063 | 2.072 | 2.091 | 2.114 | 2.143 | *      | *     |
| MNK    | ++       | 2.013 | 2.031 | 2.065 | 2.097 | 2.126 | 2.153 | 2.166 |
|        | all      | 2.059 | 2.067 | 2.084 | 2.104 | 2.130 | *      | *     |
| Sal    | ++       | 2.015 | 2.033 | 2.068 | 2.100 | 2.129 | 2.157 | 2.170 |
|        | all      | 2.012 | 2.031 | 2.067 | 2.099 | 2.129 | 2.157 | 2.170 |
| MW     | ++       | 2.210 | 2.244 | 2.309 | 2.369 | 2.423 | 2.435 | 2.451 |
|        | all      | 2.311 | 2.323 | 2.351 | 2.384 | 2.426 | 2.437 | 2.457 |
| CJ     | ++       | 2.265 | 2.291 | 2.338 | 2.382 | 2.422 | 2.458 | 2.457 |
|        | all      | 2.328 | 2.340 | 2.366 | 2.394 | 2.425 | 2.459 | 2.479 |
| MNK    | ++       | 2.251 | 2.274 | 2.318 | 2.358 | 2.394 | 2.427 | 2.443 |
|        | all      | 2.318 | 2.328 | 2.348 | 2.371 | 2.397 | 2.429 | 2.448 |
| Sal    | ++       | 2.254 | 2.278 | 2.323 | 2.363 | 2.401 | 2.435 | 2.451 |
|        | all      | 2.250 | 2.276 | 2.322 | 2.363 | 2.401 | 2.435 | 2.451 |

* - absence of stable solution
Table 1.c The mass spectrum (in GeV) for \((c\bar{s})\) system

| States | Versions | \(\alpha_1\alpha_2\) | \(x=0.0\) | \(x=0.1\) | \(x=0.3\) | \(x=0.5\) | \(x=0.7\) | \(x=0.9\) | \(x=1.0\) |
|--------|-----------|---------------------|--------|--------|--------|--------|--------|--------|--------|
|        | MW        | ++                  | 2.174  | 2.188  | 2.216  | 2.242  | 2.267  | 2.291  | 2.302  |
|        |           | all                 | 2.209  | 2.215  | 2.231  | 2.248  | 2.270  |        |        |
|        | CJ        | ++                  | 2.181  | 2.196  | 2.225  | 2.253  | 2.279  | 2.305  | 2.317  |
|        |           | all                 | 2.210  | 2.219  | 2.236  | 2.257  | 2.281  |        |        |
|        | MNK       | ++                  | 2.171  | 2.185  | 2.212  | 2.238  | 2.262  | 2.285  | 2.296  |
|        |           | all                 | 2.202  | 2.209  | 2.225  | 2.243  | 2.265  |        |        |
|        | Sal       | ++                  | 2.174  | 2.188  | 2.216  | 2.242  | 2.268  | 2.291  | 2.302  |
|        |           | all                 | 2.172  | 2.187  | 2.216  | 2.242  | 2.267  | 2.291  | 2.301  |
|        | MW        | ++                  | 2.176  | 2.191  | 2.218  | 2.245  | 2.270  | 2.294  | 2.305  |
|        |           | all                 | 2.210  | 2.218  | 2.232  | 2.250  | 2.273  |        |        |
|        | CJ        | ++                  | 2.183  | 2.198  | 2.227  | 2.255  | 2.282  | 2.307  | 2.319  |
|        |           | all                 | 2.212  | 2.220  | 2.239  | 2.259  | 2.284  |        |        |
|        | MNK       | ++                  | 2.173  | 2.187  | 2.216  | 2.240  | 2.265  | 2.288  | 2.299  |
|        |           | all                 | 2.204  | 2.211  | 2.227  | 2.245  | 2.268  |        |        |
|        | Sal       | ++                  | 2.176  | 2.191  | 2.218  | 2.245  | 2.270  | 2.294  | 2.305  |
|        |           | all                 | 2.174  | 2.189  | 2.217  | 2.244  | 2.270  | 2.293  | 2.305  |
|        | MW        | ++                  | 2.438  | 2.459  | 2.497  | 2.533  | 2.567  | 2.599  | 2.615  |
|        |           | all                 | 2.497  | 2.506  | 2.525  | 2.547  | 2.578  | 2.598  | 2.614  |
|        | CJ        | ++                  | 2.452  | 2.474  | 2.516  | 2.555  | 2.592  | 2.626  | 2.643  |
|        |           | all                 | 2.499  | 2.511  | 2.539  | 2.564  | 2.595  | 2.627  | 2.645  |
|        | MNK       | ++                  | 2.432  | 2.452  | 2.489  | 2.524  | 2.557  | 2.587  | 2.602  |
|        |           | all                 | 2.481  | 2.491  | 2.511  | 2.534  | 2.561  | 2.587  | 2.605  |
|        | Sal       | ++                  | 2.438  | 2.459  | 2.497  | 2.533  | 2.567  | 2.599  | 2.614  |
|        |           | all                 | 2.434  | 2.459  | 2.496  | 2.533  | 2.567  | 2.599  | 2.614  |

* - absence of stable solution