Research on Node localization Algorithm in WSN Based on TDOA

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Abstract. Many applications of wireless sensor networks (WSN) require nodes to know their location information. Monitoring data without node location information is meaningless in many situations. Since the node positioning efficiency of WSN directly affects the performance of the network, designing an efficient positioning algorithm is the focus of WSN research. Based on the principle of TDOA positioning in WSN, it studies two main positioning algorithms: Chan and Taylor algorithm. Meanwhile the least square (LS) method is used to provide the initial position for Taylor algorithm and the first calculation result for Chan algorithm. Simulation results under different conditions show that the Chan algorithm has a good positioning accuracy and less computation than that of Taylor algorithm in the line of sight (LOS) environment. However, under the actual channel, due to the influence of No Line of Sight (NLOS), the positioning accuracy of Taylor algorithm is better than that of Chan algorithm, but its computation is large.

Keywords: Wireless Sensor Network (WSN), Time Difference of Arrival (TDOA), Chan Algorithm, Taylor Algorithm.

1. Introduction
Node positioning in WSN is a new technology of target information location acquisition, which has potential development prospects in network security, positioning and navigation[1-3]. With the continuous development of science and technology, people's demand for wireless sensor networks has also increased. All aspects of life are inseparable from the convenience of wireless sensor network technology[4-7]. Therefore, it is widely used in military, environmental, medical and other aspects[8]. In many application scenarios, the position of sensor nodes is needed. The most common positioning method is GPS positioning system, which is not suitable for node localization in wireless sensor networks. Firstly, the energy consumption of GPS receiver is relatively high, and the energy consumption of nodes in WSN is limited. If each node is equipped with a GPS receiver, the lifetime of the network will be greatly shortened. Secondly, the cost of GPS receiver is relatively high. If each node is equipped with a GPS receiver, it will increase the extra cost, especially in large-scale wireless sensor network[9-10]. In conclusion, we need to study a cheap and efficient node positioning method in wireless sensor networks.

2. Principles of TDOA
The basic principle of TDOA algorithm is to measure the time difference of arrival that the moving target to be tested, also called the target signal (label signal), arrivals to each known base station first, then multiplying the propagation speed of electromagnetic wave to get the distance difference, and use the distance difference to construct hyperbolic equation.

Assume that \( t_i (i = 1, 2, \cdots, n) \) is the arriving time from the base station \( i \) to the label signal, and \( \Delta t_i \) is the equivalent clock error from the \( i^{th} \) base station to the label signal, then the pseudo-range can be calculated from the base station to the label signal. If \((x_i, y_i)\) is the actual position coordinates of base station\( i \), and \((x, y)\) is the location coordinate of the label, then the TOA equation is:

\[
d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} = c(t_i - \Delta t_i), i = 1, 2, \ldots, n
\]  

(1)

Let the base station 1 as the reference base station, the TDOA equations of \( N \) base stations are obtained according to the time difference of arrival.

\[
\begin{align*}
\sqrt{(x_1-x)^2 + (y_1-y)^2} &- \sqrt{(x-x_1)^2 + (y-y_1)^2} = d_2 - d_1 = d_{21} \\
\sqrt{(x_2-x)^2 + (y_2-y)^2} &- \sqrt{(x-x_2)^2 + (y-y_2)^2} = d_3 - d_1 = d_{31} \\
\vdots
\sqrt{(x_n-x)^2 + (y_n-y)^2} &- \sqrt{(x-x_n)^2 + (y-y_n)^2} = d_n - d_1 = d_{n1}
\end{align*}
\]  

(2)

Each TDOA determines a hyperbola, so when all base stations keep the clock in sync, that is, \( \Delta t_i - \Delta t_i \neq 0 (i \neq j \text{ & } i, j = 1, 2, \cdots, n) \). On the basis of formula (1), we can get a set of hyperbolic (2). As shown in Fig.1, the intersection point of hyperbola is the position of the label, so the position coordinates of the label can be obtained by solving (2).

But it is very difficult to solve the hyperbolic equation directly, and the amount of computation is very large. In general, the hyperbolic equation is linearized, and the position calculation is done by using the algorithm of least squares, Chan and Taylor.

3. Localization algorithms based on TDOA

3.1. Taylor Algorithm

The Taylor expansion is carried out at the initial position of the selected nodes to be measured, and the parts of above the second order are neglected for recursion calculation. According to the added convergence conditions of the actual network environment, the location result is judged, and the nodes with too much error between the positioning node and the actual node are discard, and the measured values are re-collected to solve the problem iteratively. Make the estimated position of the nodes to be measured be in a certain area centered on the actual position to improve the estimation accuracy. The algorithm flow can be described as:

**Step1:** assume that the initial position of the node to be measured is \((\hat{x}, \hat{y})\), and the real position error is \(\delta_x, \delta_y\). There are:

\[
x = \hat{x} + \delta_x, y = \hat{y} + \delta_y
\]  

(3)

**Step2:** expand the expression \( d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \) with Taylor series at \((\hat{x}, \hat{y})\), neglecting the terms of quadratic above.

\[
d_i = d_i|_{\hat{x}, \hat{y}} + \frac{\partial d_i}{\partial x}|_{\hat{x}, \hat{y}} \delta_x + \frac{\partial d_i}{\partial y}|_{\hat{x}, \hat{y}} \delta_y + \epsilon_i
\]  

(4)

where, the partial derivative is:
\[
\begin{align*}
\frac{\partial d_i}{\partial x} |_{x_i,y_i} &= \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} |_{x_i,y_i} = \frac{\hat{x}-x_i}{d} \\
\frac{\partial d_i}{\partial y} |_{x_i,y_i} &= \frac{y-y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} |_{x_i,y_i} = \frac{\hat{y}-y_i}{d}
\end{align*}
\]

\( e_i \) is:

\[
e_i = (d_i - \hat{d}_i)(\frac{\hat{x}-x_i}{d} \delta + \frac{\hat{y}-y_i}{d} \delta)
\]

Let \( \delta = \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} \), \( h_i = \begin{bmatrix} d_1 - \hat{d}_1 \\ d_2 - \hat{d}_2 \\ \vdots \\ d_n - \hat{d}_n \end{bmatrix} \), \( G_i = \begin{bmatrix} \frac{\hat{x}-x_1}{d_1} & \frac{\hat{y}-y_1}{d_1} \\ \frac{\hat{x}-x_2}{d_2} & \frac{\hat{y}-y_2}{d_2} \\ \vdots & \vdots \\ \frac{\hat{x}-x_n}{d_n} & \frac{\hat{y}-y_n}{d_n} \end{bmatrix} \), then the measurement error is:

\[
\varphi = h_i - G_i \delta
\]

**Step3:** let error matrix \( \varphi = 0 \), then the weighted least squares solution can be obtained.

\[
\delta = (G_i^T Q^{-1} G_i)^{-1} G_i^T Q^{-1} h_i
\]

where, \( Q \) is the covariance matrix of the measurement error. Let

\[
e_i = d_i - \sqrt{(x-x_i)^2 + (y-y_i)^2}
\]

Get the absolute minimum \((e_i)_{\text{min}}\), then \( Q \) can be written as: \( Q = \text{diag}((e_i)_{\text{min}},\ldots,(e_i)_{\text{min}}) \).

**Step4:** judge whether \( \delta = \sqrt{\delta_x^2 + \delta_y^2} \) is less than the given threshold, if it is less than the threshold, \((\hat{x}, \hat{y})\) is the position of label; otherwise, go on the Step5;

**Step5:** let \( x = x + \delta_x, y = y + \delta_y \) and return to Step2.

### 3.2. Chan Algorithm

By using the known relevant data of TDOA, the initial nonlinear equations are transformed into linear equations, and the initial solution is obtained by least square method. Then, using the estimated position coordinates and additional variables of the first step, the second weighted least squares method is used to estimate the estimated position of the nodes to be measured.

Set the node position to be measured is \((x, y)\), and the location of the \( i^{th} \) reference base station is \((x_i, y_i)\), then the distance between the measured node and the reference base station is:

\[
d_i^2 = (x_i - x)^2 + (y_i - y)^2 = K_i x_i x - 2y_i y + x^2 + y^2
\]

where, \( K_i = x_i^2 + y_i^2 \), \( d_{ij} \) is used to indicate the distance difference between the label to the \( i^{th} \) base station and to the first base station.

When \( i = 3 \), two measurements can be obtained, and two equations of two variables can be obtained through the transformation of equations, and then the estimated position of the target to be measured is solved.

When \( i \geq 4 \), let \( R = x^2 + y^2 \), the true location \( Z_o = (x_o, y_o, R_o) \) and then the error vector \( \varphi \) for variables can be established.

\[
\varphi = H - G_o Z_o
\]
The measurement error of each reference node be \( \delta \), then:

\[
\varphi_i = d_i^2 - (d_i^0)^2 = (d_i^0 + \delta)^2 - (d_i^0)^2 = 2d_i^0\delta_i + \delta_i^2
\]  

where, \( d_i^0 \) represents the true value of the node to the reference node. The first estimate of \( Z_a \) is obtained by weighted least squares (WLS) method.

\[
\hat{Z}_a = \left( G_a^T \phi^{-1} G_a \right)^{-1} G_a^T \phi^{-1} H
\]  

Since \( Q \) is unknown at the first estimate, the identity matrix can be used instead.

\[
\hat{Z}_a = \left( G_a^T G_a \right)^{-1} G_a^T H
\]

Now the relationship between the estimated value and the real value can be expressed as:

\[
\begin{align*}
\hat{Z}_{a1} &= x_0 + e_1 \\
\hat{Z}_{a2} &= y_0 + e_2 \\
\hat{Z}_{a3} &= R_0 + e_3
\end{align*}
\]

where \( e_1, e_2, e_3 \) are estimation errors.

Let \( \phi_1 = 2x_0 + e_1 \equiv 2x_0, \phi_2 = 2y_0 + e_2 \equiv 2y_0, \phi_3 = e_1 \), then (12) can be rewritten as:

\[
\varphi^T = H^T G_{a}^T Z_p
\]

where, \( H^T = \begin{pmatrix} \hat{Z}_{a1} \\ \hat{Z}_{a2} \\ \hat{Z}_{a3} \end{pmatrix} \), \( G_a^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \), \( Z_p = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} \). \( \varphi = [\phi_1, \phi_2, \phi_3]^T \) is the error vector of \( Z_p \).

The estimated value of \( Z_p \) is

\[
Z_p = \left( G_a^T \phi^{-1} G_a \right)^{-1} G_a^T \phi^{-1} H
\]

The location result obtained by the two WLS calculation is:

\[
Z = \pm \sqrt{Z_p}
\]

According to the known conditions, the position coordinates of the nodes to be measured can be determined and the estimated values can be obtained.

4. Simulation analysis

4.1. Influence of Number of Base Stations
In the Gauss environment, the number of base stations is 3,4,5,6,7 respectively. The positioning performance of Chan algorithm and Taylor algorithm are simulated and analyzed respectively. The variation of the root mean square error (RMSE) with TDOA standard deviation is shown in Fig. 1.
From Fig.1 we can see that with the increase of the TDOA standard deviation, the RMSE of the Chan algorithm and the Taylor algorithm in the five cases are all increased. When the base station number is 3, the positioning accuracy is the worst, and the RMSE increases rapidly with the increase of TDOA deviation. When the base station number is bigger than or equal to 5, the RMSE curve increases slowly. Under the same conditions, the Taylor algorithm is superior to Chan algorithm.

4.2. Performance Comparison
The COST259 channel model is a MIMO channel model based on geometric statistical characteristics. With the COST259 channel model, the positioning performance of the three algorithms under fixed standard deviation and different cell radius is shown in Fig.2.
From Fig.2 we can see that in the actual COST259 channel, due to the influence of NLOS propagation, the best positioning error is about 60m. Compared with Fang algorithm, Taylor algorithm and Chan algorithm have better a positioning performance, while Taylor algorithm is superior to Chan algorithm. This is because the NLOS environment will affect the positioning accuracy of the Chan algorithm. Compared with the Taylor algorithm, the Chan algorithm can get a definite solution under all circumstances, while the Taylor algorithm may diverge in the process of calculation, and can not get a definite solution. In practical application, the location algorithm should be chosen according to the different environment.

5. Conclusion
Wireless sensor network node localization is a new target information location acquisition technology. It has potential development prospects in network security and location navigation. Based on the TDOA positioning principle of wireless sensor network, this paper studies two positioning algorithms of Chan algorithm and Taylor algorithm, and uses LS method to provide initial position for Taylor algorithm. It provides the first calculation results for the Chan algorithm. In the MATLAB environment, the algorithm is simulated in three different cases. The Chan algorithm is able to make full use of the measurements through two LS estimation. It’s positioning accuracy is good, and the amount of computation is smaller than that of Taylor algorithm. However, due to the influence of NLOS error in the actual channel, the positioning accuracy is slightly lower than that of the Taylor algorithm, which usually recursively calculates coordinates based on the initial estimation position. It has a better positioning accuracy, but needs a large amount of computation. In practical applications, appropriate location algorithm should be selected according to different requirements, which provides a theoretical basis for indoor and outdoor positioning and navigation.

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