Molecular structure of highly-excited resonant states in $^{24}\text{Mg}$ and the corresponding $^{8}\text{Be}+^{16}\text{O}$ and $^{12}\text{C}+^{12}\text{C}$ decays

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(Dated: June 8, 2010)

Abstract

Exotic $^{8}\text{Be}$ and $^{12}\text{C}$ decays from high-lying resonances in $^{24}\text{Mg}$ are analyzed in terms of a cluster model. The calculated quantities agree well with the corresponding experimental data. It is found that the calculated decay widths are very sensitive to the angular momentum carried by the outgoing cluster. It is shown that this property makes cluster decay a powerful tool to determine the spin as well as the molecular structures of the resonances.

PACS numbers: 21.10.Tg, 21.60.Gx, 23.70.+j, 27.30.+t
I. INTRODUCTION

One of the most fundamental problems in theoretical nuclear physics has been the determination of the degrees of freedom that govern the behavior of the many-body system that is the nucleus. In the early 1930’s it was suspected that the nucleus was like a polyatomic molecule or a liquid [1]. In this background the liquid drop model, with its collective degrees of freedom, was introduced [2]. Also very early the alpha-particle model of the nucleus was formulated [3, 4]. This model was successful to explain the structure of light $N = Z$ nuclei, e.g., $^{24}$Mg [5]. Even in heavy nuclei a model where the nucleus is described as an alpha particle moving outside a frozen core [6] has been applied successfully to explain nuclear properties (see, e.g., Ref. [7] and references therein). This model was generalized to include exotic clusters as degrees of freedom and it could thus explain well cluster radioactivity [8]. But even before this, molecular-like states have been used as degrees of freedom since they were first suggested [1]. One expects that the cluster structures would be most noticeable in light $N = Z$ nuclei at excitation energies near the decay thresholds [9, 10].

In this paper we will analyze cluster degrees of freedom in $^{24}$Mg. It has been found that this nucleus shows a rich variety of cluster structures. Already as early as 1960 resonant states observed in the $^{12}$C+$^{12}$C elastic scattering reaction were found to correspond to quasimolecular structures in the compound nucleus $^{24}$Mg [11, 12]. Thenceforth, many reaction experiments were performed where different cluster-decay channels were observed, i.e., $\alpha + ^{20}$Ne* [13, 14], $^8$Be+${}^{16}$O [15–21], $^{12}$C+$^{12}$C [19, 21–26]. Also $\alpha$-chain structures have been suggested [27].

The question of whether the resonances observed in the $^{12}$C+$^{12}$C scattering represent true cluster states in the $^{24}$Mg compound system or whether they simply reflect scattering states in the ion-ion potential is still unsolved [28]. Further, if correspondences indeed exist between the resonances and structures in $^{24}$Mg, one may ask which configuration would be responsible for the resonances. It was suggested that the resonant structures are associated with a strongly deformed secondary minimum in the potential energy surface of $^{24}$Mg predicted by Nilsson-Strutinsky [29, 30] and Hartree-Fock [31] calculations. The cranked cluster model also predicted several quasi-stable deformed cluster configurations at high deformations [5]. The energy-spin plot of these resonant states indicates a large rotational moment of inertia and hence a large deformation [32]. This was considered as an
evidence for molecular structures.

Based on a $^{12}\text{C} + ^{12}\text{C}$ cluster model, Buck et al. has performed coupled-channel calculations for $^{24}\text{Mg}$ excited states in excitation energy $\leq 20$ MeV, providing a good descriptions of spectroscopic properties including excitation energies, electromagnetic transitions and electron scattering [33]. Similar spectroscopic studies but using the microscopic generator-coordinate method with $\alpha+$core configurations have also been carried out by Descouvemont and Baye [34, 35]. Adopting the coupled-channel orthogonality condition model, Kato et al. performed semi-microscopic calculations for the band structures of the $^{12}\text{C} + ^{12}\text{C}$ and $^{\alpha} + 2\alpha$ molecular states [36–38]. However, a reliable calculation of charged-particle decay widths corresponding to resonant molecular states is still lacking. Yet, such calculation is necessary to get a further understanding of the observed resonant decays in $^{24}\text{Mg}$.

The width corresponding to the decay of a cluster from a mother nucleus is given, in principle, by the classical Thomas expression [39], i.e.,

$$\Gamma_l(R) = 2P_l(R)\frac{\hbar^2}{2\mu R}|F_l(R)|^2,$$

(1)

where $l$ is the angular momentum carried by the outgoing cluster, $P$ is the penetration probability and $\mu$ is the cluster-daughter reduced mass. $R$ is a radius outside the surface of the daughter nucleus, where the cluster is assumed to have been formed. At this point the nuclear interaction acting upon the daughter nucleus and the cluster is negligible and the outgoing channel consists of two bodies, i.e., the daughter nucleus and the cluster, moving under the influence of the Coulomb and centrifugal forces only. Inside this radius the nucleons forming the mother nucleus do not necessarily form any cluster. The corresponding many-body wave function should describe the clusterization as the relative distance $R$ approaches the nuclear surface. At this point the wave function of the cluster already formed in the internal region is matched with the corresponding outgoing two-body wave function of the external region. The amplitude of the wave function in the internal region at $R$ is the formation amplitude, i.e.,

$$\mathcal{F}_l(R) = \int dR\xi_d\xi_c[\Psi(\xi_d)\phi(\xi_c)Y_l(\hat{R})]J_{m,M_m}\Psi_m(\xi_d, \xi_c, \hat{R}),$$

(2)

where $d$, $c$ and $m$ label the daughter, cluster and mother nuclei, respectively. $\Psi$ are the intrinsic wave functions and $\xi$ the corresponding intrinsic coordinates. $\phi(\xi_c)$ is the intrinsic wave function of the cluster. The rest of the notation is standard. Notice that since at $R$
the internal and external wave functions coincide one has $R_P(R) \propto 1/\mathcal{F}_l(R)$ and therefore the width is independent upon $R$ (for this and a simple derivation of Eq. (1) see [40]). Notice also that the penetrability is determined by the Coulomb and centrifugal forces only. Instead, the nuclear interaction is fundamental in the formation of the cluster.

Among the observed exit channels of the $^{24}\text{Mg}$ resonant states, decays into the $^{8}\text{Be}_{gs}+^{16}\text{O}_{gs}$ and $^{12}\text{C}_{gs}+^{12}\text{C}_{gs}$ ones, with all decay products in their ground states, are of particular interest. Resonant states sampled through these two channels have been expected to display quasimolecular structures [11, 12, 17, 19, 21, 28]. Experiments have shown that the $^{8}\text{Be}$ and $^{12}\text{C}$ decay channels emerge mainly in the $^{24}\text{Mg}$ excitation energy range of $\approx 23 - 34$ MeV [15, 17, 19, 21, 24]. The resonant widths of some of the states have been observed via the $^{12}\text{C}(^{12}\text{C},^{8}\text{Be}_{gs})^{16}\text{O}_{gs}$ reaction [15, 16]. An interesting question is if the same resonant states are sampled in the two different decay channels [19].

Besides the excitation energy and decay width, the spin is another important quantum property of a resonant state. Usually, the spin is determined experimentally by analyzing angular correlation data. For most of the resonant states in $^{24}\text{Mg}$, however, spin assignments remain ambiguous due to statistical limitations of the data.

We will here analyze the $^{8}\text{Be}$- and $^{12}\text{C}$-decays from the $^{24}\text{Mg}$ resonant states. Our aim is to understand the processes leading to the formation of the decaying resonances as well as to explore the structure of the resonances in terms of molecular degrees of freedom. In Section III is the formalism to be used in the applications presented in Section III. A summary and conclusions are in Section IV.

II. THE MODEL

A proper (microscopic) evaluation of the formation amplitude of the cluster is an extremely difficult undertaking. It is for this reason that many effective models have been proposed. These models take somehow into account the formation probability through a number of free parameters, which are adjusted to fit experimental data. With the parameters thus determined, the effective models usually provide a good description of the decay. Therefore effective models are very useful and extensively applied, e.g., in Ref. [41]. Among these models we will choose for our calculations the one described in Ref. [6], which has shown to provide the correct values of cluster decay widths [7, 8]. This is an extreme cluster
model, in which the mother nucleus is assumed to consist of the cluster moving around the
daughter nucleus (i.e., the core). In addition, it assumes that the decaying state is quasi-
bound and, therefore, that the mean-field potential generated by the core can be viewed as
a Woods-Saxon potential in which harmonic oscillator conditions can be applied. Thus, a
global principal quantum number $G = 2n + l$ is introduced, where $n$ is the number of nodes
and $l$ the orbital angular momentum carried by the wave function. Applying the Wilder-
muth condition (which is the harmonic oscillator condition for the conservation of energy)
the Pauli principle is partly taken into account. As a result $G$ depends upon the number of
nucleons in the daughter nucleus.

In this paper we will adopt the Woods-Saxon potential given by

$$v(r) = \frac{V_0}{1 + e^{\frac{r}{\kappa}}},$$  \hspace{1cm} (3)

where

$$V_0 = -V_{00}(1 \pm \kappa \frac{N_d - Z_d}{N_d + Z_d}),$$ \hspace{1cm} (4)

and the $+$ $(-)$ sign corresponds to proton (neutron) potentials. The index $d$ indicates
daughter quantities. The Woods-Saxon parameters are from the Chepurnov parametrization
of Ref. [42], i.e.,

$$V_{00} = 53.3 \text{ MeV},$$

$$\kappa = 0.63,$$

$$a = 0.63 \text{ fm},$$

$$R = r_0 A_d^{1/3} \text{ fm},$$ \hspace{1cm} (5)

where $A_d = N_d + Z_d$, and $r_0 = 1.24 \text{ fm}$ (however, the parameter $r_0$ will be adjusted to give
a reasonable Coulomb barrier, see the discussion later).

We use a folded mean-field-type nuclear potential for the cluster, which is constructed as
follows [43],

$$V_N(r) = \lambda [N_c v_n(r) + Z_c v_p(r)],$$ \hspace{1cm} (6)

where $\lambda$ is the folding factor; $N_c$ and $Z_c$ are the neutron and proton numbers of the cluster,
respectively; $v_n(r)$ and $v_p(r)$ are the single neutron and proton potentials (excluding the
Coulomb potential), respectively, generated by the core, i.e., the Woods-Saxon potential
given by Eq. (3). Then the cluster potential is written as

$$V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2} l(l + 1),$$ \hspace{1cm} (7)
where the Coulomb potential \( V_C(r) \) takes the usual form given in Ref. \[33\] (taking the same radius for the Coulomb and nuclear potentials).

We use the Bohr-Sommerfeld quantization condition \[6\] in the determination of the parameter values, which is written as

\[
\int_0^{r_2} dr \sqrt{\frac{2\mu}{\hbar^2}} |Q_0 - V(r)| = (2n + 1)\frac{\pi}{2} = (G - l + 1)\frac{\pi}{2},
\]

(8)

where \( Q_0 \) is the decay \( Q \) value for a given channel with the mother, daughter and cluster all in their ground states; \( r_2 \) is the turning point obtained by \( V(r) = Q_0 \). In the \( Q_0 \) case, we have \( l = 0 \). For the \( ^{12}\text{C} + ^{12}\text{C} \) system, the Coulomb barrier has been well measured experimentally \[44\]. In the present calculations, we adjust the radius parameter (i.e., \( r_0 \) in Eq. (5)) to fit the experimental Coulomb barrier by minimizing \( (V^{\text{expt}}_B - V^{\text{cal}}_B)^2 + (R^{\text{expt}}_B - R^{\text{cal}}_B)^2 \) (here \( V_B \) and \( R_B \) are the height and location of the Coulomb barrier, respectively). Therefore, the folding factor \( \lambda \) and the radius parameter \( r_0 \) can be determined by using the Bohr-Sommerfeld condition and fitting the Coulomb barrier.

As seen, the Bohr-Sommerfeld condition involves the global quantum number \( G \). In our case the Wildermuth rule can be written as \[45\] \( G = \sum_{i=1}^{A_c} g_i \), where \( A_c \) is the nucleon number of the cluster and \( g_i \) is the oscillator quantum number corresponding to the nucleons in the cluster. For the \( ^8\text{Be}_{gs} + ^{16}\text{O}_{gs} \) structure in \( ^{24}\text{Mg} \), the \( ^8\text{Be} \) cluster nucleons occupy the \( 1d_5/2 \) orbits above the \( N, Z = 8 \) closed shells (i.e., the Fermi levels of the \( ^{16}\text{O} \) core). These orbits have an oscillator quantum number \( g_i = 2 \), leading to \( G = 16 \) for the \( ^8\text{Be}_{gs} + ^{16}\text{O}_{gs} \) configuration. For the \( ^{12}\text{C}_{gs} + ^{12}\text{C}_{gs} \) structure, two protons and two neutrons of the \( ^{12}\text{C} \) cluster occupy the \( 1p_{1/2} \) orbits with \( g_i = 1 \), and the other eight cluster nucleons fill the \( 1d_{5/2} \) orbits with \( g_i = 2 \), which gives \( G = 20 \). Notice that it is by considering the system as a Fermi gas that the Wildermuth rule takes into account the Pauli principle. However, smaller \( G \) values are possible if one considers the inner quantum of the cluster. For example, Buck et al. take mainly \( G = 16 \) as the starting number in the \( ^{12}\text{C} + ^{12}\text{C} \) coupled-channel calculations \[33\]. We will discuss calculations at different \( G \) values later.

The partial decay width is calculated by using the expression \[6\]

\[
\Gamma = P \times \Gamma_p,
\]

(9)

and

\[
\Gamma_p = F \frac{\hbar^2}{4\mu} \exp \left[ -2 \int_{r_2}^{r_3} k(r)dr \right],
\]

(10)
where \( P \) is the preformation probability of the cluster being formed in a state of the mother nucleus, and \( \Gamma_p \) is the width that corresponds to the penetration probability of the cluster through the potential barrier. The normalization factor \( F \) is determined by

\[
F \int_{r_1}^{r_2} \frac{dr}{2k(r)} = 1, \tag{11}
\]

where \( k(r) \) is the standard local wave number, i.e.,

\[
k(r) = \sqrt{\frac{2\mu}{\hbar^2}} |Q^*_i - V(r)|. \tag{12}
\]

For the cluster decay from an excited state of the mother nucleus into the ground states of the decay products, we have \( Q^*_l = Q_0 + E^*_J \) where \( E^*_J \) is the excitation energy of the mother with the spin \( J = l \) for decay into spin-zero final states. \( r_1, r_2 \) and \( r_3 \) are the turning points obtained by \( V(r) = Q^*_l \). The quantity \( P \) is the probability that within the model the cluster can be found in a given model configuration. It is the equivalent of the spectroscopic factor in one-particle transfer reactions. Thus, if there is only one configuration which is relevant, then that configuration defines the decay channel and \( P = 1 \). This was the value used in, e.g., Refs. [6, 8, 43]. This quantity has no relation with the cluster wave function on the nuclear surface, except for the case of proton-decay, for which \( P \) and the square of the formation amplitude \( \mathcal{F} \) are related. This is because the proton is indeed a “cluster” in the sense of the cluster model. But for real cluster (including the \( \alpha \)-particle) the preformation factor \( P \) is unrelated to the formation amplitude \( \mathcal{F} \). Actually the values of the amplitude \( \mathcal{F} \) have been evaluated for alpha and heavier clusters by fitting experimental decay widths. One thus obtained that \( \mathcal{F} \) is of order unity for protons, as \( P \) is, but of order \( 10^{-2} \) for alpha particles and of order \( 10^{-5} \) for \( ^8\text{Be} \) clusters [46]. The formation probability is the square of this number.

### III. CALCULATIONS AND DISCUSSIONS

We take \( G = 20 \) and 16 (i.e., ignoring the inner quantum number of the cluster) for the calculations of the \( ^{12}\text{C}_{gs} + ^{12}\text{C}_{gs} \) and \( ^8\text{Be}_{gs} + ^{16}\text{O}_{gs} \) decays, respectively, as discussed above. Using the Bohr-Sommerfeld condition and fitting the Coulomb barrier, with the experimental \( Q \)-value \( Q_0 = -13.93 \text{ MeV} \) for the \( ^{12}\text{C}_{gs} + ^{12}\text{C}_{gs} \) channel, we obtained a folding factor of \( \lambda = 0.500 \) and an adjusted radius parameter of \( r_0 = 1.44 \text{ fm} \) for the \( ^{12}\text{C} \)-cluster potential.
This radius parameter is slightly larger than the one of \( r_0 = 1.24 \) fm in the Chepurnov parametrization, which would indicate an equivalent inclusion of the effect from the cluster size. For the \(^8\text{Be}_{gs} + ^{16}\text{O}_{gs}\) channel, no experimental Coulomb barrier is available. We take the same radius parameter for this channel, which should be reasonable. With \( Q_0^{\text{expt}} = -14.14 \) MeV, we obtained \( \lambda = 0.509 \) for the \(^8\text{Be}\)-cluster potential.

With all the quantities required by the formalism thus obtained we proceeded to analyze experimental data corresponding to resonances in \(^{24}\text{Mg}\) that have been measured at the high excitation energy-range of \( \approx 23 - 34 \) MeV. These resonances might be quasimolecular states since the emissions of \(^8\text{Be}\)- and \(^{12}\text{C}\)-clusters from the resonances have been observed in many experiments \([11, 15-25]\). The experimental widths of the resonances have been obtained via the measurement of the \(^{12}\text{C}(^{12}\text{C}, ^8\text{Be}_{gs})^{16}\text{O}_{gs}\) channel \([13, 16]\). The high resolution of \( \approx 100 \) keV in energy indicates that the observed widths would be the natural widths of the resonances \([15, 16]\). These can be considered narrow resonances and, therefore, quasibound states. One of the main assumptions of the cluster model is thus satisfied. To study the possibility that the resonances are indeed states of \(^{24}\text{Mg}\) we will evaluate the \(^8\text{Be}\)- and \(^{12}\text{C}\)-decay widths emitted from all the observed resonances. Those channels that provide values of the widths that agree with experiment will give us a clue on the molecular structure of the resonances as well as the corresponding spins. Regarding this last point it is to be noticed that the decay channels \(^8\text{Be}_{gs} + ^{16}\text{O}_{gs}\) and \(^{12}\text{C}_{gs} + ^{12}\text{C}_{gs}\) involve only spin-zero final states and therefore \( l = J \), where \( l \) is the angular momentum of the outgoing cluster and \( J \) is the spin of the resonant state. Moreover, in the \(^{12}\text{C}\) decay channel only even angular momentum are carried by the outgoing \(^{12}\text{C}\) cluster. This is due to the symmetric character of the \(^{12}\text{C} + ^{12}\text{C}\) partition in the entrance channel and that in the exit channel all states have zero spin.

With the folded \(^8\text{Be}\)-cluster potential obtained above, we calculated the corresponding penetration width \( \Gamma_p \) (Eq. (10)) for different spins. Since the centrifugal barrier (and therefore the penetrability) depends rather strongly upon \( l \), this calculation may be able to determine accurately the spins of the observed resonant states. This feature is even more marked for the high-lying states, with energies which may be above the Coulomb barrier.
TABLE I: Calculated penetration widths $\Gamma_p^{\text{cal}}$ (Eq. (10)) corresponding to the $^8$Be- and $^{12}$C-cluster decays from $^{24}$Mg. The total width $\Gamma^{\text{cal}}$ is from Eq. (9). The experimental widths ($\Gamma^{\text{expt}}$) are taken from Ref. [16]. The experimental spin assignments are taken from Refs. [19, 21, 22]. The theoretical spins corresponding to the $^{12}$C channel must be even (see the text).

| $E^*_J$ (expt) (MeV) | $\Gamma^{\text{expt}}$ (keV) | $J^{\text{expt}}$ | $J^{\text{theo}}$ | $\Gamma_p^{\text{cal}}(^8\text{Be})$ (keV) | $\Gamma_p^{\text{cal}}(^{12}\text{C})$ (keV) | $\Gamma^{\text{cal}}$ (keV) |
|---------------------|-----------------|-----------------|-----------------|--------------------------------|--------------------------------|-----------------|
| 23.9                | 200             | (8)             | 8               | 161                          | 110                          | 136             |
| 24.2                | (200)           | (8)             | 8               | 239                          | 170                          | 205             |
| 24.4                | (400)           | (8, 9)          | 8               | 305                          | 192                          | 249             |
|                     |                 |                 | 9               | 49                           |                              | 49              |
| 24.6                | 300             | (8)             | 8               | 386                          | 284                          | 335             |
|                     |                 |                 | 9               | 65                           |                              | 65              |
| 24.9                | 300             | (8, 9)          | 8               | 529                          | 418                          | 474             |
|                     |                 |                 | 9               | 96                           |                              | 96              |
| 25.1                | < 450           | (8)             | 8               | 646                          | 529                          | 588             |
|                     |                 |                 | 9               | 124                          |                              | 124             |
| 25.3                | 200             | (8)             | 8               | 759                          | 628                          | 694             |
|                     |                 |                 | 9               | 159                          |                              | 159             |
| 25.8                | 500             | (9, 10)         | 9               | 283                          |                              | 283             |
|                     |                 |                 | 10              | 40                           | 37                           | 39              |
| 26.3                | 300             | (10)            | 9               | 474                          |                              | 474             |
|                     |                 |                 | 10              | 74                           | 68                           | 71              |
| 26.9                | 340             | (10)            | 10              | 149                          | 154                          | 152             |
| 27.3                | 300             | (10)            | 10              | 230                          | 234                          | 232             |
| 27.8                | 240             | (10)            | 10              | 381                          | 402                          | 392             |
| 28.3                | 340             | (10)            | 10              | 600                          | 654                          | 627             |
|                     |                 |                 | 11              | 88                           |                              | 88              |
| 29.3                | $\sim$ 700     | (10,12)         | 11              | 243                          |                              | 243             |
|                     |                 |                 | 12              | 28                           | 38                           | 33              |

Continued...
Proceeding in the same fashion, we also calculated the width corresponding to $^{12}$C-decay. The results of our calculations, corresponding to both clusters, are presented in Table I. One notices that the penetrability width $\Gamma_p$ is approximately the same for both channels for a given value of the angular momentum. This can be understood because at the high excitation energies of the resonances the centrifugal barrier is dominant.

An interesting question is whether the different choices of the $G$ number influence the calculated width. The global quantum number $G$ appears in the Bohr-Sommerfeld condition (i.e., Eq. (8)) which is used mainly in the determination of the folding factor. Considering the inner quantum of the cluster, however, smaller $G$ values are possible. The $^8$Be cluster has an inner quantum number of 4, and $^{12}$C itself has a number of 8, which gives a starting $G$ number of 12 for the two channels. For highly-excited cluster states, larger $G$ numbers are also possible. As an example, Table II lists calculated widths for the 23.9 MeV ($J = 8$) state (i.e., the first state of the Table I) at different $G$ numbers. Note that the $G = 20$ situation in the $^{12}$C channel corresponds to the $G = 16$ case in the $^8$Be channel, i.e., both ignore the inner quanta. We see that the calculated widths as well as the obtained Coulomb barriers keep quite stable at different $G$ values. The experimental height and location of the $^{12}$C+$^{12}$C Coulomb barrier are $V_B = 5.8 \pm 0.3$ MeV and $R_B = 6.5 \pm 0.4$ fm [44], respectively, while Buck’s model gave $V_B \approx 6.3$ MeV and $R_B \approx 7.2$ fm [33]. Once the potential parameters
are determined, the width calculation is independent on the $G$ number because it does not appear in the formula of the width (see Eq. (10)). Different $G$ numbers lead to the different bands of spectra [33–38], which would be discussed in our future work with the possible improvement of the model. In the present work, we focus on the decay property of the resonances.

To obtain the decay width it we have still to determine the values of the preformation factor $P$. It was already mentioned that if there is only one channel which is open we assume $P \approx 1$. This assumption has been well tested within effective models for the cluster decays of the ground states in even-even heavy nuclei [6, 47]. For highly-excited resonant states, other configurations with excited cluster and/or excited core would exist, which decreases the preformation factor of the interested channel, and consequently reduces the corresponding decay width. For the resonant states in the energy range of $\approx 23 – 34$ MeV, however, all the experiments [15–26] address that only the decay channels with all the decay products in their ground states have been detected. This indicates that the $^{12}\text{C}_{gs} + ^{12}\text{C}_{gs}$ and $^{8}\text{Be}_{gs} + ^{16}\text{O}_{gs}$ configurations should be dominant in the resonances of this energy range.

We can further analyze the effect on the width from other possible channels. The $^{12}\text{C}_{gs} + ^{12}\text{C}(2^+_1)$ channel with single $^{12}\text{C}$ excitation to the $2^+_1$ (at 4.44 MeV) state should be
next important compared with the ground-state channel. This channel has a decay threshold 4.44 MeV higher than the one of the $^{12}\text{C}_{gs} + ^{12}\text{C}_{gs}$ channel, but can lead to a possible spin decrease of 2 units for the cluster motion and then reduces the centrifugal barrier. As an example, we have calculated the width of the $^{12}\text{C}_{gs} + ^{12}\text{C}(2^+_1)$ channel for the 23.9 MeV ($J=8$) state (correspondingly $l = 6$), giving a width of only 0.3 keV for this excited channel. Configurations with single and mutual $^{12}\text{C}$ excitations to the $0^+_2$ state (i.e., the Hoyle state at 7.65 MeV) are interesting. The microscopic coupled-channel calculation predicted that these configurations are of importance for resonances with excitation energy higher than 40 MeV [48]. However, for the resonances in the energy of 20 − 34 MeV, our calculations show that decay widths for the channel with single $^{12}\text{C}$ excitation to the Hoyle state are less than 1 keV, and much less for the one with double $^{12}\text{C}$ excitations to the Hoyle state. Therefore, for width calculations in the energy range studied, contributions from excited channels should be negligible.

In cases where $^{24}\text{Mg}$ decays by emitting $^8\text{Be}$ as well as $^{12}\text{C}$ one can assume $P(^8\text{Be}) + P(^{12}\text{C}) \approx 1$. But we have seen that the penetrability is about the same for both cases. Therefore the system will be trapped within the barrier approximately the same time, irrespective of whether it is in the configuration $^8\text{Be}_{gs} + ^{16}\text{O}_{gs}$ or in $^{12}\text{C}_{gs} + ^{12}\text{C}_{gs}$. In addition, experimental excitation functions indicate that the decay into both channels proceeds through approximately the same probability [19, 21, 32]. Therefore one may assume that the preformation factor is $P \approx 0.5$ for $^8\text{Be}$ as well as for $^{12}\text{C}$. As a result, the resonant widths acquire the form $\Gamma \approx 0.5[\Gamma_p(^8\text{Be}) + \Gamma_p(^{12}\text{C})]$.

One sees in Table II that, for each resonance, there is a value of $\Gamma_{\text{cal}}$ that agrees with the corresponding experimental data within a factor of three. The quality of these numbers can best be judged by noticing that in the simpler case of $\alpha$-decay such agreement would be considered excellent [49]. But the most important feature of Table II is that it shows that cluster decay from highly excited resonances is a powerful tool to investigate the structure of the decaying resonances. This is because at high energies (above the Coulomb barrier) the decay width is practically only dependent upon the centrifugal barrier. As a result, the calculated widths are very sensitive to the angular momenta carried by the decaying clusters.

Often the experimental spin assignment of the states in Table II is uncertain due to the statistical limitations of angular correlation data [19, 21]. Our calculation confirms the
TABLE III. Experimental and calculated decay widths for the $^{12}$C+$^{12}$C resonances observed in the energy range of 20 – 23 MeV [26].

| $E^*_j$ (expt) (MeV) | $J^\pi_{\text{expt}}$ | $\Gamma_{\text{expt}}$ (keV) | $\Gamma_{\text{cal}}$ (keV) | $J^\pi = 4^+$ | $J^\pi = 6^+$ |
|----------------------|------------------------|-------------------------------|-------------------------------|-------------|-------------|
| 20.77 ± 0.02         |                        | 175 ± 108                     | 159                          | 11          |             |
| 21.18 ± 0.02         | $4^+$                  | 219 ± 69                      | 330                          | 27          |             |
| 21.62 ± 0.01         | $4^+$                  | 99 ± 36                       | 589                          | 64          |             |
| 21.82 ± 0.01         | $4^+$                  | 97 ± 29                       | 584                          | 92          |             |
| 22.01 ± 0.01         | $4^+$                  | 120 ± 31                      | 128                          |             |             |
| 22.26 ± 0.02         | $4^+$                  | 123 ± 53                      | 195                          |             |             |
| 22.43 ± 0.03         | $4^+$                  | 216 ± 70                      | 253                          |             |             |
| 22.99 ± 0.03         | ($6^+$)                | 267 ± 90                      | 535                          |             |             |

experimental assignments when doubtful, as e.g., for the state at $E^*_j = 23.9$ MeV. In this context it is important to point out that our calculation definitely excludes a possible $J = 6$ value of the spin of this state, as was suggested in a previous experiment [16]. This is a general feature. The calculation helps to decide which spin is correct when there are several experimental possibilities. An example of this is the state at $E^*_j = 25.3$ MeV, where for $J=9$ the calculated decay width agrees with the corresponding experimental value, while for $J=8$ the difference between theory and experiment is more than a factor of three.

As a further test of the present model, we have also investigated the resonances in the energy range of 20 – 23 MeV which were observed to have the $^{12}$C+$^{12}$C decay [26]. The experiment [26] was performed with a high resolution of 87 keV in energy, which is important to ensure that the natural widths of resonances were measured. Table III presents the comparison between the calculated and experimental widths of the resonances. The experiment assigned the $J^\pi = 4^+$ for the resonant states, but commented that $J^\pi = 6^+$ remains tentative [26]. A later experiment pointed out that the $J^\pi = 6^+$ is more possible for these resonances [19]. See Table III it is found that observed widths increase in general with increasing energies, but a remarkable drop happens at 21.62 MeV. This would indicate a spin increase at this resonant state. From the present calculations, we should be able to conclude that the first two resonances of Table III have $J^\pi = 4^+$ and all others have $J^\pi = 6^+$. 

13
One sees that for all states in Table I the spins are relatively large. This can be understood considering that at excitation energies above the Coulomb barrier only the centrifugal force can trap the cluster inside the mother nucleus. In the framework of the shell model, when the cluster nucleons fill the $sd$ shell the maximum possible spin is 12 for the $^8\text{Be}+^{16}\text{O}$ configuration. This is an indication that for the highest-lying states in Table I with spin $J = 13$, the cluster nucleons occupy the $fp$ shell.

Another important feature is that the calculation allows one to get information about the molecular structure of the resonances. This is otherwise a very difficult undertaking. For instance, the resonance lying at 23.9 MeV can be interpreted as a mixing of the molecular configurations $^8\text{Be}+^{16}\text{O}$ and $^{12}\text{C}+^{12}\text{C}$. Instead, the state at 25.3 MeV is predicted to have spin $J = 9$ and consists entirely of the configuration $^8\text{Be}+^{16}\text{O}$. The same procedure can be applied to all states in Table I.

Finally, it is interesting to notice that since for the $^{12}\text{C}$ decay channel only even spins are possible, the number of molecular configurations is limited.

IV. SUMMARY AND CONCLUSION

In this paper we have analyzed the decay of $^8\text{Be}$ and $^{12}\text{C}$ clusters from $^{24}\text{Mg}$. The observed decaying resonances lie high in the spectrum, at the energy range of $\approx 20 - 34$ MeV. Our aim was to probe whether the clusters could be considered as elementary degrees of freedom. This is reasonable since at such high energies the nuclear density is low and therefore the Pauli principle is not very effective in hindering the cluster formation. To evaluate the cluster-decay widths we applied the cluster model of Ref. [6]. As a central field we chose the Woods-Saxon potential. The results of the calculation are in agreement with the rather large amount of available experimental data within a factor of three, as seen in Tables II and III. Considering that these are complicated decay processes, such agreement can be considered very good.

In many instances our calculation shows that the decay through the $^8\text{Be}$ channel is as probable as the one proceeding through the $^{12}\text{C}$ channel. We could thus conclude that, in terms of the cluster model, the decaying resonance corresponds to a mixing of the molecular states $|^8\text{Be}\otimes^{16}\text{O}\rangle$ and $|^{12}\text{C}\otimes^{12}\text{C}\rangle$. This also indicates that both decays occur from the same resonance and simultaneously, a point that may be doubtful to an experimental observer.
At the very high energies of the decaying resonances the centrifugal barrier is practically the only one able to trap the cluster nucleons within the mother nucleus. Therefore the penetrabilities, and the resulting decay widths, are very sensitive to the angular momentum carried by the outgoing cluster. This peculiar feature allowed us to assign precisely the spins of the molecular states in $^{24}$Mg. This is specially important in cases where there are several experimental possibilities for the spin of a given resonance. We have thus found, e.g., that the calculated width of a state lying at 33.4 MeV, which experimentally may have spin $J = 12$ or 13, agrees with the experimental value only if $J=13$.

In conclusion we have shown in this paper that the analysis of cluster decay widths from high lying resonances in light nuclei is a powerful tool to determine the spins of the states as well as their structures in terms of molecular degrees of freedom.

V. ACKNOWLEDGMENTS

One of the authors (C.X.) would like to express his thanks to Royal Institute of Technology (Alba Nova University Center) for the hospitality extended to him during his stay there. This work has been supported by the NSFC research grant (J0730316) for undergraduates, the NSFC grants under Nos. 10735010 and 10975006, the Chinese Major State Basic Research Development Program under Grant 2007CB815000, and the Swedish Research Council (VR).

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