Measuring Coherent Motions in the Universe

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We present new measurements of the coherent motion of galaxies based on observations of the large-scale redshift–space distortions seen in the two-dimensional two-point correlation function of Luminous Red Galaxies in Data Release Seven of the Sloan Digital Sky Survey. We have developed a new methodology for estimating these coherent motions, which is less dependent on the details of galaxy bias and of the cosmological model to explain the late–time acceleration of the expansion of the Universe. We measure a one-dimensional velocity dispersion of galaxies on large–scales of $\sigma_v = 3.01^{+0.45}_{-0.46} \, h^{-1} \, \text{Mpc}$ and $\sigma_v = 3.69^{+0.47}_{-0.47} \, h^{-1} \, \text{Mpc}$ at a mean redshift of $z = 0.25$ and 0.38 respectively. These values are fully consistent with predictions for a WMAP7–normalised $\Lambda$CDM Universe and inconsistent at confidence of 3.8$\sigma$ with a Dvali-Gabadadze-Porrati (DGP) model for the Universe. We can convert the units of these $\sigma_v$ measurements to $270^{+40}_{-41} \, \text{km/s}$ and $320^{+41}_{-41} \, \text{km/s}$ respectively (assuming a $\Lambda$CDM universe), which are lower than expected based on recent low redshift ($z < 0.2$) measurements of the peculiar velocity field (or “bulk flows”). It is difficult to directly compare these measurements as they cover different redshift ranges and different areas of the sky. However, one possible cosmological explanation for this discrepancy is that our Galaxy is located in unusually over, or under, dense region of the Universe.

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I. INTRODUCTION

The last decade has seen a revolution in cosmology with the emergence of a standard model for the Universe dominated by a strange substance known as “dark energy”, with an effective negative pressure. Since the first evidence of dark energy in 1998\textsuperscript{1,2}, there has been substantial observational and theoretical research aimed at understanding the true nature of this phenomenon. In recent years, many authors have started exploring the possibility that dark energy, and the observed acceleration of the expansion of the Universe, could be the consequence of an incomplete theory of gravity on cosmological scales and may require modifications to Einstein’s theory of General Relativity.

One of the most direct methods of testing our assumed theory of gravity is to perform consistency checks between the geometrical expansion history of the Universe, as measured by cosmological probes like Type Ia Supernovae and the Baryon Acoustic Oscillations, and the evolution of density inhomogeneities in the Universe\textsuperscript{3-7}. The growth of structures in the Universe can be measured using a variety of techniques, but a popular and direct method involves measuring the coherent peculiar velocities of galaxies on large scales (i.e., the motion of galaxies after the cosmological expansion has been removed) caused by their infall into large scale overdensities like clusters and superclusters of galaxies\textsuperscript{8, 9}. Traditionally, the local peculiar velocity field, or “bulk flow” of galaxies, has been estimated using samples of galaxies where the peculiar velocity of each galaxy has been determined using secondary distance indicators (e.g. the Tully–Fisher relationship\textsuperscript{10, 12} or Fundamental Plane\textsuperscript{13, 14}). Alternatively, the coherent motion, or flow, of galaxies can be statistically estimated from their effect on the clustering of galaxies out to at least 800 Mpc\textsuperscript{21, 22}, which leads to the intriguing situation that these local measurements appear to be significantly greater in amplitude, and on larger scales, than expected for the local bulk flow in the Universe (within 100 $h^{-1}$ Mpc). This new analysis is also consistent with new measurements of the local bulk flow using the kinetic Sunyaev-Zeldovich effect\textsuperscript{20} of massive clusters of galaxies out to at least 800 Mpc\textsuperscript{21, 22}, which leads to the intriguing situation that these local measurements appear to be significantly greater in amplitude, and on larger scales, than expected for the local bulk flow in the Universe (within 100 $h^{-1}$ Mpc). This new analysis is also consistent with new measurements of the local bulk flow using the kinetic Sunyaev-Zeldovich effect\textsuperscript{20} of massive clusters of galaxies out to at least 800 Mpc\textsuperscript{21, 22}, which leads to the intriguing situation that these local measurements appear to be significantly greater in amplitude, and on larger scales, than expected for the local bulk flow in the Universe (within 100 $h^{-1}$ Mpc).

Given the importance of these local large–scale bulk flow measurements, we investigated in a previous paper\textsuperscript{24} the likelihood of such large coherent flows using an alternative methodology based upon measurements of the redshift–space distortions seen in the clustering of a sample of galaxy clusters selected from the Sloan Digital Sky Survey (SDSS)\textsuperscript{25}. We detected no statistical evidence for the coherent flows of the same size, and scale, as dis-
discussed above, but due to the sample size the statistical errors were large (see [24] for details). Therefore, in this paper, we re-visit our earlier work using a larger cosmological data–set and recent theoretical improvements in the modelling of redshift–space distortions. The goal of this paper is to statistically study the likelihood of the local bulk flow measurements within a large cosmological volume of the Universe, well beyond these nearby measurements.

In section II of this paper, we outline in greater detail the fitting techniques used in [24] to formulate the line–of–sight smearing effect due to the coherent bulk flow of galaxies. In section III, we explain our data analysis of a sample of SDSS Luminous Red Galaxies (LRGs; [29]) and our measurement of their two–dimensional redshift–space correlation function. In section IV, we present our results from comparing our LRG measurements to our model for redshift–space distortions, and compare our results to cosmological predictions and previous observations. We conclude in section V.

II. MODELLING REDSHIFT–SPACE DISTORTIONS

The aim of this work is to statistically measure the coherent motion of galaxies on large scales. That is, the linear velocity of galaxies excluding both the cosmological Hubble expansion and any smaller scale non–linear components.

The coherent motions of galaxies introduce redshift–space distortions – an anisotropic feature – into the measured clustering statistics. As originally proposed in [30], a distant observer should expect a multiplicative enhancement of the overdensity field along the line–of–sight, compared to the transverse direction, due to such coherent peculiar motion of galaxies. This “Kaiser effect”, as it is now known, can be seen as a “squashing” or flattening of the two–dimensional two–point correlation function (ρ, ρ), where the correlation function is decomposed into two vectors; one parallel to the line–of–sight (ρ) and the other perpendicular to the line–of–sight (σ). Information about the coherent velocities of galaxies can then be extracted from the two–dimensional correlation function via careful theoretical modelling of these redshift–space distortion effects.

In this paper, we utilise several theoretical improvements to the original redshift–space distortion work given in [30]. Throughout this paper, we refer to the original work of [30] as the “Kaiser limit”, which formulated the coherent motions of galaxies as an additional Dopper shift to the cosmological redshift. The “Kaiser limit” is suitable to test theoretical models in the linear region of galaxy perturbations (on large scales), but over the last decade there have been significant improvements to the modelling of these effects. Below, we repeat some of the formalism presented in our first paper [24] to aid the reader and provide continuity with additional work presented in following subsections.

A. Representation of the correlation function in the Kaiser limit

The observed power spectrum of density fluctuations in redshift–space, \( P_{ab}(k, \mu) \), can be written as,

\[
P_{ab}(k, \mu) = \left[ P_{\delta \delta}(k, \mu) + 2\mu^2 P_{\delta \varpi}(k, \mu) + \mu^4 P_{\varpi \varpi}(k, \mu) \right] \times G(k, \mu, \sigma_v),
\]

where \( \delta \) and \( \Theta \) denote galaxy density and velocity fields respectively (where \( \Theta = \theta/aH \) and \( \theta \) is the divergence of the velocity fields). In Eq. (1) the function \( G \) denotes the additional suppression effect due to coherent motions which affects the galaxy–galaxy, velocity–velocity and galaxy–velocity power spectra equally, and is a function of separation \( (k) \), angle between the galaxies (denoted by \( \mu \)) and has a given smoothing scale, or velocity dispersion \( (\sigma_v) \), which is the parameter at the heart of our analysis and this paper.

As discussed in our previous paper, we can decompose these power spectra into a scale–dependent \((D)\) and scale–independent part \((g)\) given by,

\[
P_{\Phi \Phi}(k, a) = D_D(k)g_D^2(a),
\]

\[
P_{\delta \delta}(k, a) = D_m(k)g_m^2(a),
\]

\[
P_{\varpi \varpi}(k, a) = D_m(k)g_m^2(a),
\]

where the subscript \( \Phi \) denotes the curvature perturbation in the Newtonian gauge,

\[
ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2.
\]

The growth function \( g_b \) is defined as \( g_b = b g_m \), where \( b \) is the standard linear bias parameter between the density of galaxies and the underlying dark matter, \( \delta_m \). As we follow the positive sign conversion, \( g_b \) is the growth function of \(-\Theta\).

The shape factor of the perturbed metric power spectra \( D_{\Phi}(k) \) is defined as

\[
D_{\Phi}(k) = \frac{2\pi^2}{k^3} \frac{9}{25} \Delta^2(\omega_0) T_{\Phi}^2(k),
\]

which is a dimensionless metric power spectra at \( a_{eq} \) (the matter–radiation equilibrium epoch), and \( \Delta^2(\omega_0) \) is the initial fluctuations in the comoving gauge and \( T_{\Phi}(k) \) is the transfer function normalised at \( T_{\Phi}(k \rightarrow 0) = 1 \). The primordial shape \( \Delta^2_0(k) \) depends on \( n_S \) (the slope of the primordial power spectrum), as \( \Delta^2_0(k) = A^2_S(k/k_p)^{n_S-1} \), where \( A_S \) is the amplitude of the initial comoving fluctuations at the pivot scale, \( k_p = 0.002 \, \text{Mpc}^{-1} \). The intermediate shape factor \( T_{\Phi}(k) \) depends on \( \omega_m \) (\( \omega_m = \Omega_m h^2 \)). The shape factor for the matter fluctuations, \( D_m(k) \), which is important for both the galaxy–galaxy and
velocity–velocity power spectra in Eq. 2 above, is given by the conversion from $D_p(k)$ of,

$$D_m(k) \equiv \frac{4}{9H_0^2\Omega_m^2}D_p(k),$$

where, assuming $c = 1$, we can write $H_0 \equiv 1/2907 \text{h Mpc}^{-1}$.

In the Kaiser limit, we would thus assume $P(k) \equiv P_{\text{lin}}(k)$, where $P_{\text{lin}}(k)$ is the linear power spectrum, and no smearing of the large scale power spectra due to the random motions of galaxies within individual dark matter halos, or the “Finger–of–God” (FoG) effect seen on small–scales in redshift surveys. Therefore, the observed compression of $\xi_s(\sigma, \pi)$ along the line–of–sight due to coherent infall of galaxies around large–scale structures can be written in configuration space as,

$$\xi_s(\sigma, \pi)(a) = \left( g_b^2 + \frac{2}{3}g_\omega g_\Theta + \frac{1}{5}g_\Theta^2 \right) \xi_0(r)P_0(\mu)$$

$$- \left( \frac{4}{3}g_b g_\omega + \frac{4}{7}g_\Theta \right) \xi_2(r)P_2(\mu)$$

$$+ \frac{8}{35}g_\Theta^2 \xi_4(r)P_4(\mu),$$

where $P_l(\mu)$ is the Legendre polynomial and the spherical harmonic moment $\xi_l(r)$ is given by,

$$\xi_l(r) = \int \frac{k^2dk}{2\pi^2} D_m(k)j_l(kr),$$

where $j_l$ is a spherical Bessel function.

### B. CMB priors

The coherent evolution parts in Eq. 2 are not generally parameterised by known standard cosmological parameters. We thus normalise these growth factors at $a_{eq}$ such that,

$$g_b(a_{eq}) = 1,$$

$$g_m(a_{eq}) = a_{eq}g_b(a_{eq}),$$

$$g_\Theta(a_{eq}) = \frac{dg_m(a_{eq})}{da}.$$

Then, we treat $g_b$ and $g_\Theta$ as free parameters to be measured by redshift–space distortions with CMB priors used to determine the shape factor $D_m(k)$ at the last scattering surface.

The shape factor $D_m(k)$ depends mainly on the following set of cosmological parameters, $\omega_m$, $n_S$, $A_S$ and $\Omega_m$. While the first two parameters ($\omega_m$ and $n_S$) are well constrained by the CMB at the last scattering surface (regardless of theoretical model used to describe the late–time acceleration of the expansion of the Universe), the last two parameters ($A_S$ and $\Omega_m$) can not be determined by the CMB alone. The primordial amplitude $A_S$ has been measured to an accuracy of less than 5% from WMAP7 alone, however this constraint becomes weaker due to the unknown reionization history of the Universe (tighter constraints are achieved by assuming a stepwise parameterisation of the reionization history). For the matter content of the Universe, the CMB measures $\omega_m$, not $\Omega_m$, which depends on the Hubble Constant.

Therefore, given our CMB priors, $D_m(k)$ can be expressed as,

$$D_m(k) = \left( \frac{A_S^*}{\Omega_m^*} \right)^2 D_m^*(k : A_S^*, \Omega_m^*),$$

where $A_S^*$ and $\Omega_m^*$ are specific reference values. We use the best fit cosmological parameters of the WMAP7–normalised $\Lambda$CDM model in this paper, $A_S^* = \sqrt{2.41 \times 10^{-9}}$ and $\Omega_m^* = 0.264$. Then the power spectra can be re–written as,

$$P_{b\delta_s}(k, a) = \left( g_b^* A_S^* \Omega_m^* \right)^2 D_m^* \left( k : A_S^*, \Omega_m^* \right),$$

$$P_{\Theta\Theta}(k, a) = \left( g_\Theta^* A_S^* \Omega_m^* \right)^2 D_m^* \left( k : A_S^*, \Omega_m^* \right).$$

Here, we define new growth functions as,

$$g_b^*(a) \equiv g_b(a) \frac{A_S^* \Omega_m^*}{\Omega_m A_S^*},$$

$$g_\Theta^*(a) \equiv g_\Theta(a) \frac{A_S^* \Omega_m^*}{\Omega_m A_S^*}.$$  

Then the power spectra become

$$P_{b\delta_s}(k, a) = g_b^* D_m^* \left( k : A_S^*, \Omega_m^* \right),$$

$$P_{\Theta\Theta}(k, a) = g_\Theta^* D_m^* \left( k : A_S^*, \Omega_m^* \right).$$

in which the power spectra are split into two parts, i.e., $g_b^*$ and $g_\Theta^*$ are determined through our redshift–space distortion measurements, and $D_m^*$ is given by the CMB priors and the reference values of $A_S^*$ and $\Omega_m^*$.

Thus, we re-write the correlation function in Eq. 6 in terms of our new fitting parameters $g_b^*$ and $g_\Theta^*$,

$$\xi_s(\sigma, \pi)(a) = \left( g_b^{*2} + \frac{2}{3}g_b^* g_\omega^* + \frac{1}{5}g_\Theta^* \right) \xi_0(r)P_0(\mu)$$

$$- \left( \frac{4}{3}g_b^* g_\omega^* + \frac{4}{7}g_\Theta^* \right) \xi_2(r)P_2(\mu)$$

$$+ \frac{8}{35}g_\Theta^{*2} \xi_4(r)P_4(\mu),$$

where the spherical harmonic moment $\xi_l^*(r)$ is given by,

$$\xi_l^*(r) = \int \frac{k^2dk}{2\pi^2} D_m^*(k : A_S^*, \Omega_m^*) j_l(kr).$$

We measure $g_b^*$ with a normalised $D_m^*(k : A_S^*, \Omega_m^*)$, not $g_\Theta^*$ paired with $D_m(k)$. However, as $P_{\Theta\Theta} = g_\Theta^* D_m(k) = g_\Theta^* D_m^*(k : A_S^*, \Omega_m^*)$, $P_{\Theta\Theta}$ does not depend on any normalisation, i.e., $P_{\Theta\Theta}$ is invariant under the transformation between the * (asterisk) variables and those without above. Thus, we present measured coherent motions
not in terms of \( g^*_\Theta \), but in terms of velocity dispersion derived from \( P_{\Theta \Theta} \). Hereafter, we drop ‘∗’ symbol, i.e. ‘unasteriked’ quantities imply ‘asteriked’ quantities from now.

C. Modifying the Kaiser effect

It has been pointed out in \cite{31} that the description of \( \xi_s(\sigma, \pi) \) on large scales (in linear theory) will be modified due to dispersion effects in the \( \pi \) direction. This effect was recently investigated using N-body simulations and was found to be important to include \cite{32}. Therefore, in this paper, we have modified the Kaiser limit using a Gaussian velocity dispersion term for \( G \) in Eq. 1, which we will refer to as the “modified Kaiser effect” throughout. This modified model has been shown to work reasonable well on mock catalogues of a \( \Lambda \)CDM Universe (see\cite{32,33}.

In detail, we can write \( G \) as

\[
G(k, \mu, \sigma_v) = e^{-k^2 \mu^2 \sigma_v^2},
\]

(15)

where \( \sigma_v \) is the 1-D velocity dispersion on large scales, and is the parameter of interest in this paper. We can write \( \sigma_v \) as,

\[
\sigma_v^2 = \frac{1}{6\pi^2} \int P_{\Theta \Theta}^{\text{lin}}(k, z) dk,
\]

(16)

where we see that \( \sigma_v \) has units of \( h \text{Mpc}^{-1} \) as \( P_{\Theta \Theta}^{\text{lin}} \) and \( k \) have units of \( (h^{-1}\text{Mpc})^3 \) and \( h \text{Mpc}^{-1} \) respectively. This is somewhat confusing given \( \sigma_v \) is called a “velocity dispersion” in previous literature, but we retain this terminology here to remain consistent with this literature. However, we note that we can convert the units of \( \sigma_v \) to km/s using \( aH(a)\sigma_v \), but this requires a precise measurement of the expansion history of the Universe (at the redshifts of interest), in addition to the determined \( P_{\Theta \Theta}^{\text{lin}} \) from our method.

We therefore modify the Kaiser effect using \( G \) defined in Eq. 15 but keeping \( P(k) = P_{\Theta \Theta}^{\text{lin}}(k) \). Then the integration of \( \xi_l \) in Eq. 6 should be re-expressed, as \( G \) changes with varying \( g^*_\Theta \) altering \( \sigma_v \) in Eq. 16 to,

\[
\xi_l(r) = \int \frac{k^2 dk d\mu_k}{(2\pi)^2} D^s_m(k)e^{-(k\mu_k\sigma_v)^2} \cos (kr\mu_k)P_l(\mu_k\|^\pi) \]

Here \( \mu_k \) is the cosine of angle between \( \vec{k} \) and the pairwise orientation.

For illustrative purposes, in the left-hand panel of Fig. 1 we present our modelling of \( \xi_s(\sigma, \pi) \) using the standard Kaiser limit (unfilled black contours) compared to the modified Kaiser effect (filled blue contours) discussed above. In these examples, we have used a galaxy bias of \( b = 1.86 \) and \( 1.88 \) at \( z = 0.25 \) (upper panel) and \( 0.38 \) (lower panel) respectively (These values of \( b \) were chosen to correspond to our estimated bias values from Table I to illustrate our point). The additional suppression in the modified Kaiser effect can be clearly seen on large-scales (in the linear regime) at approximately the 10 to 20% level.
the conversion of redshifts and angles to distances could introduce an additional anisotropic signal in the correlation function i.e. the “Alcock–Paczynski” effect [40]. The size of this effect is smaller than the Kaiser effect on the scales of interest herein, e.g., if we vary our background cosmology, within reasonable limits allowed by the WMAP analysis, we see no significant difference in our correlation functions. However, we note that a more self-consistent approach will be required in the future (see [11,43]).

In Fig. 2 we show the co-moving number density of LRGs in our whole sample (compared to the random catalogue) and we split this sample into two redshift bins as indicated (Bin 1: 0.16 < z < 0.32 and Bin 2: 0.32 < z < 0.47). We do not split the sample further as we wish to keep the thickness of the redshift shells (δz ∼ 0.15) larger than the scales of interest in our analysis, thus preventing any aliasing in the redshift direction.

We estimate the correlation function using the “Landy-Szalay” estimator [44],

\[
\xi(\sigma, \pi) = \frac{DD - 2DR + RR}{RR},
\]

where DD is the number of galaxy–galaxy pairs, DR the number of galaxy-random pairs, and RR is the number of random–random pairs, all separated by a distance σ ± Δσ and π ± Δπ. All pairs are weighted using the minimum variance weighting scheme of [45]. Each galaxy is assigned a weight according to,

\[
w_i = \frac{1}{1 + n_i(z) P_w},
\]

where \(n_i(z)\) is the comoving space density at redshift \(z\) and \(P_w = 4 \times 10^4 h^{-3} \text{Mpc}^3\), as in [46]. The pairs are also corrected for fiber collisions, where two or more galaxies separated by less than 55 arcseconds cannot be observed simultaneously. This is achieved using a secondary galaxy weight, proportional to the number of unobserved galaxies within the fiber radius. The pair counting was achieved using the parallel kd-tree code, NTROPY [47].

In the right-hand panel of Fig. 1 we present our measured \(\xi_s(\sigma, \pi)\). The perpendicular component, \(\sigma\), is binned logarithmically into ten bins from 1 h^{-1} Mpc to 60 h^{-1} Mpc, and the parallel component, \(\pi\), is also binned logarithmically into ten bins, but from 1 h^{-1} Mpc to 30 h^{-1} Mpc.

We use the jack–knife method [48] to estimate statistical errors on \(\xi_s(\sigma, \pi)\), which involved dividing the survey into \(N\) sub-sections with approximate equal area (and thus volume) and then computing the mean and variance of \(\xi_s(\sigma, \pi)\) from these \(N\) measurements of the correlation function with the \(i^{th}\) region removed each time (where \(i = 1...N\)). In our analysis, we use \(N = 100\) jack-knifed samples which were created using an adaption of the HEALPix, equal–area, pixelisation code [49].

The jack–knife measurements provide an estimate of the covariance matrix for all bins in our correlation function. However, this matrix can be noisy, given the number of jack–knife samples used, although previous work
This shape dependence is determined by the ratio between matter and radiation energy densities and sets the location of the matter-radiation equality in the time coordinate. Therefore, the shape-dependent part of $\xi_{\ell n}(r)$ is well determined, in a model independent way, when both $n_S$ and $\omega_m$ are determined at the last scattering surface. We thus marginalise our constraints over the growth parameters with $(n_S, \omega_m)$ using WMAP7 priors on $n_S$ and $\omega_m$, $n_S = 0.963 \pm 0.014$ and $\omega_m = 0.1334^{+0.0056}_{-0.0055}$.

### C. Non-linear suppression cut-off

On small scales, there are non-linear effects that are not easily modelled via analytical methods, e.g., the internal velocities of galaxies within clusters which cause the famous “Fingers-of-God” effect seen in galaxy redshift surveys [52]. We investigate here the effect of such non-linearities and outline our strategy for mitigating them in our analysis.

First, the contribution of velocity–velocity correlation is decomposed from the observed correlation function using the anisotropic feature of redshift distorted galaxy maps of which information is most extractable at the quadrupole moment. Since the leading order of the quadrupole moment is the density–velocity correlation, what we measure is the cross-correlation between the density and velocity fields not the velocity–velocity component. The velocity–velocity correlation is the leading order at the hexadecapole moment, which is too weak to provide tight constraints on the coherent velocity flows. However, both fields (density and velocity) are perfectly correlated in the linear regime at least at scales $k < 0.1 \, h \, \text{Mpc}^{-1}$.

Deviations from a perfect cross-correlation coefficient $\epsilon = 1$ ($\epsilon \equiv P_{gg}/\sqrt{P_{\Theta\Theta}P_{gg}}$) are not caused by cosmological effects but rather by non-linear physics. If we analyse our data only in the linear regime (where $\epsilon = 1$), then $g_0$ and $g_0\Theta$ can be simultaneously determined from just the density–density and density–velocity components. Therefore, we impose a cut-off scale on $r = \sqrt{\sigma^2 + \pi^2}$ at the scale where $\epsilon$ does not significantly depart from unity. This corresponds to $k < 0.1 \, h \, \text{Mpc}^{-1}$, or $r_{\text{cut}} = 15 \, h^{-1} \, \text{Mpc}$, as seen in N-body simulations [53]. We use $k \sim 1/r$, instead of $k \sim 2\pi/r$, given by the peak contribution of convolution in Eq. [4] due to the Bessel function in the integrand.

Secondly, we also discard more bins along the $\pi$ direction as they are more contaminated by the “Finger-of-God” effect. In our model, we use a linear power spectrum instead of a non-linear power spectrum as in [31, 32, 33]. This should be fine for the accuracy required in this paper as the most important correction is due to the dispersion effect, which we have included here, and also the effect of non-linearities in the power spectra is only a few percent at $k < 0.2 \, h \, \text{Mpc}^{-1}$ [32, 54]. To estimate the impact of the non-linearity on the two-point correction function in redshift space, we use the HALOFIT model [53] for the non-linear power spectrum.
with a CDM transfer function [56]. We apply the same non–linear power spectrum for the velocity power spectrum, although this obviously overestimates the velocity power (see [34] for a non–linear fitting function of the velocity power spectrum). However, we use this non–linear model just for discarding small–scale data which may be significantly affected by non–linearities, and our rejection will be conservative. We find that the effect of non-linearity is over 20% at the scale of $\sigma < 5 h^{-1}$ Mpc in the redshift range of our data, and discard these measurements from our analysis. Shown in the right panel of Fig. 1, measured $\xi_s(\sigma, \pi)$ at bins of $\sigma > 5 h^{-1}$ Mpc is consistent with theoretical predictions (compared with the left panel of Fig. 1), but $\xi_s(\sigma, \pi)$ at bins of $\sigma < 5 h^{-1}$ Mpc does not match well with the theoretical predictions from the modified Kaiser effect. In this paper, we thus impose a cut–off of $\sigma > 5 h^{-1}$ Mpc.

In summary, we impose two cut-offs in scale to control the contamination of small-scale non-linearities on our larger scale correlation function measurements. We achieve this by not including bins with scales $r_{\text{cut}} < 15 h^{-1}$ Mpc and $\sigma < 5 h^{-1}$ Mpc in our fitting procedure.

IV. MEASURED COHERENT MOTIONS

A. Consequences of the correction to the Kaiser limit

Using the data and method outlined in Sections II and III, we measure $g_b$ and $\sigma_v$ simultaneously from the SDSS DR7 LRG correlation functions assuming the appropriate WMAP7 priors as discussed above. As illustrated in Fig. 1 there is excellent visual consistency on large scales between our predicted $\xi_s(\sigma, \pi)$ and the observed functions. In detail, we obtain a reduced $\chi^2$ of 0.89 and 0.83 for the $z = 0.25$ and 0.38 samples for 12 and 9 degrees of freedom. We have 16 and 13 eigenmodes after truncation, minus four fitting parameter respectively. In Fig. 4 we present constraints on $g_b$ and $\sigma_v$ ($\sigma_v$ is converted from measured $g_b$ using Eq. 16). In the Kaiser limit, variations of the coherent growth function of galaxy density fields amplifies mainly the monopole moment of $\xi_s(\sigma, \pi)$, and variations of the coherent motions affects mostly the anisotropy of $\xi_s(\sigma, \pi)$. Measured $g_b$ and $\sigma_v$ using the Kaiser limit are presented as unfilled black contours of Fig. 4.

The velocity dispersion in the correlation function $\xi_s(\sigma, \pi)$ of the modified Kaiser effect (Eq. 17) induces an additional suppression to the $\xi_s(\sigma, \pi)$ of the conventional Kaiser limit (Eq. 6). The variation of coherent motion leads to monopole suppression as well as anisotropic amplification. As coherent motions increase, the corresponding $g_b$ becomes smaller due to the increasing suppression by the velocity dispersion effect. Shown as filled blue contours of Fig. 4 the best fit value of $g_b$ is shifted to lower values and the contours are elongated to smaller $g_b$ at higher $\sigma_v$. While the measured difference of $\sigma_v$ between the Kaiser limit and modified Kaiser effect is less than 4%, the monopole shift of $g_b$ due to velocity dispersion with the modified Kaiser effect causes measured differences of $g_b$ from 10% to 15% presented in Table II.
TABLE I. The measured values of several common cosmological parameters, including $g_0$, $g_b$, and $\sigma_v$ which are discussed extensively throughout this paper. The quoted errors on the best fit parameters are one sigma, after marginalizing over all other parameters. Parameter values in parenthesis are theoretical predictions based on a WMAP7–normalized $\Lambda$CDM model for the Universe.

| Parameter         | $z = 0.25$ (ACDM) | $z = 0.38$ (ACDM) |
|-------------------|-------------------|-------------------|
| $g_0$             | (0.411)           | (0.422)           |
| $\sigma_v$ in Mpc/h | 3.04$^{+0.46}_{-0.45}$ | 3.82$^{+0.49}_{-0.48}$ |
| $aH\sigma_v$ in km/s | 279$^{+41}_{-20}$ | 331$^{+42}_{-32}$ |
| $g_b$             | 1.33$^{+0.045}_{-0.043}$ | 1.32$^{+0.046}_{-0.044}$ |
| $\beta$           | 0.29$^{+0.044}_{-0.043}$ | 0.36$^{+0.045}_{-0.044}$ |
| $b_{ACDM}$        | 1.98$^{+0.066}_{-0.065}$ | 2.09$^{+0.073}_{-0.072}$ |

B. Cosmological constraints

We present herein a measurement of the coherent motions using $\sigma_v$, converted from our measured value of $g_0$ (shown in Table I) and shape factor $D_m$ given by our WMAP7 priors. At $z = 0.25$, $\sigma_v$ is measured to be $3.01^{+0.46}_{-0.45}$ Mpc (one sigma errors), and at $z = 0.38$, we find $\sigma_v = 3.69^{+0.47}_{-0.46}$ Mpc. These measurements are less dependent on the cosmological model for the late-time acceleration of the expansion of the Universe (e.g. dark energy) as our CMB priors are fixed at a much earlier epoch in the Universe, e.g. we use value of $A_s$ and $\Omega_m$ constrained at the surface of last scattering. Our measurements are also independent of galaxy bias which is a major advantage compared to other methods to parameterise the motions of galaxies from redshift–space distortions. It is true that our measured $\sigma_v$ correlates with $g_b$, but we do not need to know how to separate $b$ and $g_{m0}$ for determining the coherent motions.

If we assume $H(a)$ is well–measured, then we can convert the units of $\sigma_v$ using $\alpha H a_v$ (assuming $\Lambda$CDM) to give $270^{+40}_{-20}$ km/s and $320^{+41}_{-32}$ km/s at the two redshifts listed in Table I. Although we would not recommend using $\alpha H a_v$ for cosmological constraints (without a reliable $H$ measurement to an accuracy of a few percent), this conversion is more intuitive when discussing coherent motions and allows us to compare our values with other peculiar velocity measurements, e.g., the “bulk flow” measurements discussed in Section I.

Traditionally, coherent motions are estimated using the $\beta$ parameter $[57, 63]$,

$$P_{ob}(k, \mu) = (1 + \beta \mu^2) P_{bg_{ob}(k)}.$$  

This parameter is equivalent to $g_0/g_b$ in our notation, and measured to be $\beta = 0.30^{+0.04}_{-0.03}$ and $0.39^{+0.056}_{-0.056}$ at $z = 0.25$ and 0.38 respectively. As we have measured, $g_b$ is significantly sensitive to the different redshift–space distortion models, and $\beta$ is found to be $0.29^{+0.044}_{-0.043}$ and $0.36^{+0.048}_{-0.048}$ for the Kaiser limit. Thus, when measuring $\beta$ from the SDSS DR7 LRG samples for use as cosmological constraints, it should be stated clearly which model for redshift-space distortions is assumed.

If we assume a $\Lambda$CDM model, then the galaxy bias can be estimated from our measured $g_b$ values. We present values for $b$ in Table I, which are $b_{ACDM} = 1.86^{+0.089}_{-0.093}$ and $1.88^{+0.11}_{-0.09}$ at $z = 0.25$ and 0.38 respectively. It is interesting to compare our measurements with $b_{ACDM} = 1.86 \pm 0.07$ as measured by [62] using the higher–order correlation function of LRGs from the SDSS. This consistency supports our modified Kaiser formulation, as the estimated $b_{ACDM}$ using the original Kaiser limit is different by more than one sigma.

C. Tracing the history of coherent motions of galaxies

In Fig. 5, we present our measurements of $\sigma_v$, the coherent motions of galaxies in redshift space, versus redshift and compared to theoretical predictions. As can be seen, our measurements are fully consistent with the WMAP7–normalised $\Lambda$CDM model. This represents a “clean” test of these cosmological models free from contamination by non–linear physics on small–scales and uncertainties of the galaxy bias determination. These data probe the growth history of fluctuations in the Universe and are thus complementary to the geometrical probes of the Universe.

In Fig 6, we also present predicted curves for the coherent motion of galaxies for a variety of dark energy models with varying constant equation of state from $w = -1.4$ to $-0.6$ (dotted curves). All other cosmological parameters were fixed to be the same as the WMAP7 best fit $\Lambda$CDM values, except $w$. Using these curves, we can approximately estimate that we have a constraint of $\sigma(w) \sim 0.2$, around a mean value of $w \approx -1$, from our $\sigma_v$ measurements. This is consistent and complementary to similar constraints from geometrical observations of the Universe [63].

Finally, we also provide in Fig 5 a prediction for the Dvali-Gabadadze-Porrati (DGP) self-accelerating braneworld scenario [64]. As can be seen, this cosmological model provides a poor description of our observations (excluded at confidence of 3.8$\sigma$ for both redshift bins), which support other observational constraints on this model from the cosmic microwave background (CMB) anisotropy, supernovae and Hubble constant data [65].
velocity units (as opposed to lengths presented above) and

\[ \sigma_v = 3.01^{+0.45}_{-0.46} h^{-1} \text{Mpc} \]

at a mean redshift of \( z = 0.25 \) and \( \sigma_v = 3.69^{+0.47}_{-0.45} h^{-1} \text{Mpc} \) at \( z = 0.38 \). These values for \( \sigma_v \) are fully consistent with a WMAP7-normalized ΛCDM model with \( w \simeq -1 \pm 0.2 \) as illustrated in Fig. 4. Our observations are however, inconsistent with a DGP model for the Universe to high statistical significance (> 5\( \sigma \)). Our results provide a competitive, and complementary, constraint on these cosmological models compared to the usual geometric probes of the Universe.

We have converted our measured values of \( \sigma_v \) into velocity units (as opposed to lengths presented above) and find \( 270_{-41}^{+10} \text{km/s} \) and \( 320_{-41}^{+10} \text{km/s} \) at a mean redshift of \( z = 0.25 \) and 0.38 respectively, assuming a ΛCDM Universe. As expected, these coherent motions (or velocity dispersions) are fully consistent with expectations from a ΛCDM Universe. These estimates are however, inconsistent with local measurements of the peculiar velocity field (or “bulk flows”) which have recently been measured to be greater than these velocities and expectations from ΛCDM [17, 18]. If the amplitude of these local, observed bulk flows were converted (using \( aH\sigma_v \) and assuming a flat ΛCDM model) to the redshift range studied here (0.16 \( \leq z \leq 0.47 \)), then we might expect to see larger coherent motions.

It is difficult to perform a direct comparison of these different velocity measurements because of the different methods and redshift intervals used. For example, the lower redshift measurements of [17, 18] or [21, 22] only probe the velocity field out to 150\( h^{-1} \text{Mpc} \) using their COMPOSITE sample, while most of the data is within a sphere of radius of \( \approx 60h^{-1} \text{Mpc} \) depending on the weighting scheme used (see Fig. 2 of [18]). In contrast, our statistical estimate of coherent motions on the scales up to 60\( h^{-1} \text{Mpc} \) are derived from within a large volume of the Universe at higher redshift, e.g., 0.5\( h^{-3} \text{Gpc}^3 \) (0.16 \( \leq z \leq 0.32 \)) and 1.1\( h^{-3} \text{Gpc}^3 \) (0.32 \( < z < 0.47 \)) respectively, assuming a flat WMAP7 cosmology. Therefore, our measurements have averaged over many hundreds of subregions of approximately the same size as the volume used by [17, 18] (assuming a subregion of radius of 60\( h^{-1} \text{Mpc} \)).

Furthermore, there are potential differences in the possible directions of the velocity measurements being compared. The direction of the local velocity measurements of [17, 18] point towards \( \approx 158, -51 \) degrees of Right Ascension (RA) and Declination respectively on the sky, while our SDSS DR7 data is centred at \( \approx 180, +30 \) (Equatorial coordinates) and concentrated in the northern hemisphere, i.e. our patch is not along the direction relevant to the patches used in [17, 18] or [21, 22]. A direct comparison of the directions of the various velocity measurements is hard as again our SDSS measurement is a statistical average over many subregions of space and thus has no directional information. That said, our statistical measurement of coherent velocities within such sized volumes of the universe is smaller, to high statistical significance, than that measured locally around our Galaxy.

A more interesting comparison would be with the measurements of [21, 22] who find a “bulk flow” of X-ray clusters, with respect to the CMB, of \( \approx 1000 \text{km/s} \) out to \( \approx 800h^{-1} \text{Mpc} \) (or 560\( h^{-1} \text{Mpc} \) for comparison herein) in approximately the same direction as discussed above for the local measurements [17, 18]. Again, it is hard to make a direction comparison in terms of the bulk velocity amplitude and direction as our statistical measurements are derived from higher redshift [22], they note that their bulk flow velocities peak at \( z \leq 0.16 \) with the possibility of higher redshift clusters providing little to their dipole
measurements. This is therefore, below our low redshift LRG sample data, and averaged over a larger volume, than used by 21, 22.

One possible explanation for the differences we are seeing is that our Galaxy is located in an unusual part of the Universe, e.g., in a highly over, or underdense region of the Universe. Again, our measurements are obtained at $z > 0.16$, beyond the redshift limits of all these local measurements 17, 18, 21, 22. Moreover, we can not exclude the presence of a large constant large-scale “dark flow” across the volume surveyed by our DR7 data i.e., a velocity dipole with no variation across $\simeq 1 h^{-1}$Gpc of the Universe. Such a constant flow would be undetectable by our method as the correlation function in redshift space is distorted by the divergence of the peculiar velocity field, while measuring the bulk flows via the SZ effect can include a possible global flow. However, this explanation would require the “dark flow” of 22 to extend out beyond the SDSS LRG sample ($z \simeq 0.5$) to leave it undetectable, i.e., over a $1000 h^{-1}$ Mpcs in scale. This can be tested using the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS), which will provide redshift-space distortion measurements both at higher redshift and over larger volumes of the Universe 67.

In the future we plan to extend our measurements to much higher redshifts, in order to extend the history of coherent motions (e.g. Fig. 5). As shown in our paper, these motions can be measured in an independent way, free of some of the problems associated with other measures of the growth history of the Universe, and the assumed cosmological model for the late–time Universe, e.g., we are able to test not only the conventional dark energy model, but also “interacting” and “clustered” dark energy models, not to mention the general class of modified gravity theories. Unlike other approaches, our measurements are free from any possible violation of the consistent equations.

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