Few comments on preprint cond-mat/0007430 by Aleiner et al.: It has been suggested in Ref. [1] that conduction electron generated virtual hopping to the excited states of a two-level system (TLS) could increase the amplitude of assisted tunneling and thus result in a measurable Kondo temperature. In a recent preprint, Aleiner et al. revisited the role of excited states [2] and pointed out that in the calculations of Ref. [1] only the first few excited states have been taken into account and inclusion of all excited states induces a large decrease of $T_K$ instead of an increase.

While we think that the work of Aleiner et al. addresses an important issue, there are several points that we have to comment on:

(a) The reduction of $T_K$ heavily relies on the assumption of electron-hole symmetry. In fact, using the model of Ref. [1] and assuming a contact TLS-electron interaction of strength $U$, the assisted tunneling term generated by the reduction of the cutoff is proportional to:

$$
\delta V_{\alpha\beta} = \sum_{\epsilon_j} \sum_{\nu} g(D)V_{\epsilon\alpha\beta}V_{\epsilon\nu\beta} - g(-D)V_{\epsilon\beta\alpha}V_{\epsilon\nu\beta},
$$

where $D$ denotes the bandwidth cutoff, $\epsilon_j$ is the energy of the $j$th excited state of the TLS, and the matrix element between the TLS wave functions $\varphi_i$, $\varphi_j$ in the "tower" model [2] and conduction electron states $j_i(z)$ $j\nu(z)$ is given by:

$$
V_{\alpha\beta}^{ij} \sim U \int dz \varphi_i(z)\varphi_j(z)j_i(z)j\nu(z).
$$

For large enough $D$ the identity $\sum\varphi_j(z)\varphi_j(z') \approx \delta(z-z')$ can be used and assuming $g(D) = g(-D)$ the two terms in Eq. [1] cancel. However, for electronic bands usually $g(D) \neq g(-D)$ and the cancellation does not occur. In the Figure we show $T_K$ obtained from the complete solution of the leading logarithmic scaling equations. We assumed a density of states of the form

$$
g(\epsilon) = g(0)(1 + \alpha \epsilon/D_0),
$$

where $\alpha$ is usually of the order of $1$. As one can see, a small electron-hole asymmetry can increase $T_K$ by many orders of magnitude, bringing it into the physically measurable range. Note that while for $\alpha = 0$ the inclusion of all excited states reduces $T_K$ by three orders of magnitude in agreement with Aleiner et al., for $\alpha = 0.3 - 0.4$ there is very little reduction compared to the truncated model. [The $\alpha = 0.4$ case is special: Here five heavy particle states are still active at $T_K$.]

(b) It is obvious from the figure that it is extremely difficult to give a "first principles" estimation for $T_K$, which is very sensitive to the exact model parameters such as the distance between the two wells, the mass of a heavy particle, or the value and structure of the heavy particle-conduction electron interaction. Within the Thomas-Fermi interaction approximation, e.g., $T_K$ changes many orders of magnitudes if one changes the effective charge of the heavy particle by a factor of 2 (Hall effect measurements indicate, e.g., that the valence of crystalline Cu is about 1.5. This value may considerably differ from the valence of the tunneling atoms that are most probably sitting in a more disordered region.)

In reality, one does not even know what tunneling centers look like. The simplistic model of Ref. [1] assumes a very specific TLS structure, which may be very far from that of real tunneling systems, possibly due to dislocation jogs, e.g. [3].

(c) Our calculations clearly demonstrate that $T_K$ can be in the measurable range. It is a more delicate question whether the splitting of the TLS can be small enough to allow for the development of a two-channel Kondo behavior. In our model the bare splitting is $\Delta_0 \sim 0.4K$, which is clearly smaller than $T_K$ for $\alpha > 0.2$. However, the presence of electron-hole symmetry breaking increases $\Delta_0$, which is, on the other hand considerably reduced under scaling. To decide which of these processes wins and whether the renormalized splitting can be less than $T_K$, a much more detailed study of the next leading logarithmic equations is needed.

Another possibility could be to have tunneling centers, where the degeneracy of the lowest-lying states is guaranteed by some local symmetry (rotational or possibly time reversal). A. Zawadowski\textsuperscript{1} and G. Zaránd\textsuperscript{1,2}

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[2] I. L. Aleiner, B. L. Altshuler, Y. M. Galperin, T. A. Shutenko (preprint cond-mat/0007430).
[3] Tejs Vegge et al., cond-mat/0003138.

FIG. 1. $T_K$ as a function of the excited states kept in the scaling procedure. The calculations were done for assuming a heavy particle of mass $M \sim 50 \times m_{\text{proton}}$, a barrier of height $\sim 300K$ and of width 0.5Å, and the width of the potential wells was taken to be 0.1Å.