Comparison of hadronic rescattering calculations of elliptic flow and HBT with measurements from RHIC

T. J. Humanic
Department of Physics, The Ohio State University, Columbus, OH 43210
(Dated: March 30, 2022)

Results from the data obtained in the first physics run of the Relativistic Heavy Ion Collider (RHIC) for Au+Au collisions at $\sqrt{s} = 130$ GeV have shown surprisingly large pion elliptic flow and surprisingly small HBT radii. Attempts to explain both results in a consistent picture have so far been unsuccessful. The present work shows that a simple thermal-like initial state model coupled to a hadronic rescattering calculation can explain reasonably well both elliptic flow and HBT results from RHIC. The calculation suggests a very early hadronization time of about 1 fm/$c$ after the initial collision of the nuclei.

PACS numbers: 25.75.Gz, 25.75.Ld

I. INTRODUCTION

Results of the Year-1 running of the Relativistic Heavy Ion Collider (RHIC) for Au+Au collisions at $\sqrt{s} = 130$ GeV have shown suprisingly large pion elliptic flow and suprisingly small HBT radii. Attempts to explain both results in a consistant picture have so far been unsuccessful. Hydrodynamical models agree with the large elliptic flow seen in the RHIC data but significantly disagree with the experimental HBT radii. On the other hand, relativistic quantum molecular dynamics calculations which include hadronic rescattering, for example RQMD v2.4, significantly underpredict the elliptic flow seen in the RHIC data but predict pion HBT radii comparable to the data. A calculation has recently been made to extract HBT radii with a hydrodynamical model coupled with a hadronic rescattering afterburner with the result that the HBT radii are significantly larger than measurements. This lack of a single model to explain both results has been our first big mystery from RHIC. It has been suggested that we should call into question our current understanding of what information pion HBT measurements give us.

In an effort to address this mystery, the present work explores a somewhat different picture of the nuclear collision than those presented above. In this picture, hadronization into a simple thermal-like state occurs soon (about 1 fm/$c$) after the initial collision of the nuclei followed by hadronic rescattering until freezeout. The goal will thus be to test whether hadronic rescattering alone can generate enough general flow in the system to explain both the elliptic flow and HBT results from RHIC starting from a simple non-flowing initial state. Note that this approach, while similar to the relativistic quantum molecular dynamics calculations mentioned above, differs from them in the choice of the model of the initial state of the system before hadronic rescattering commences (e.g. the other models use a color string picture for the initial state). The simple initial state model used here is meant to be a construction to parameterize the true initial state with a few parameters which can be adjusted to help the rescattering calculation agree with the RHIC results. If it is possible to obtain reasonable agreement with the data, then one might ask how early in the calculation one can go and still maintain a physically motivated model. The discussion of this important point is deferred until later.

II. CALCULATIONAL METHOD

A brief description of the rescattering model calculational method is given below. The method used is similar to that used in previous calculations for lower CERN Super Proton Synchrotron (SPS) energies. Rescattering is simulated with a semi-classical Monte Carlo calculation which assumes strong binary collisions between hadrons. The Monte Carlo calculation is carried out in three stages: 1) initialization and hadronization, 2) rescattering and freeze out, and 3) calculation of experimental observables. Relativistic kinematics is used throughout. All calculations are made to simulate RHIC-energy Au+Au collisions in order to compare with the results of the Year-1 RHIC data.

The hadronization model employs simple parameterizations to describe the initial momenta and space-time of the hadrons similar to that used by Herrmann and Bertsch. The initial momenta are assumed to follow a thermal transverse (perpendicular to the beam direction) momentum distribution for all particles,

$$(1/m_T)dN/dm_T = C m_T / [\exp (m_T/T) \pm 1]$$

where $m_T = \sqrt{p_T^2 + m_0^2}$ is the transverse mass, $p_T$ is the transverse momentum, $m_0$ is the particle rest mass, $C$ is a normalization constant, and $T$ is the initial “temperature” of the system, and a gaussian rapidity distribution for mesons,

$$dN/dy = D \exp[-(y - y_0)^2/(2\sigma_y^2)]$$
where \( y = 0.5 \ln \left( \frac{E + p_z}{E - p_z} \right) \) is the rapidity, \( E \) is the particle energy, \( p_z \) is the longitudinal (along the beam direction) momentum, \( D \) is a normalization constant, \( y_0 \) is the central rapidity value (mid-rapidity), and \( \sigma_y \) is the rapidity width. Two rapidity distributions for baryons have been tried: 1) flat and then falling off near beam rapidity and 2) peaked at central rapidity and falling off until beam rapidity. Both baryon distributions give about the same results. The initial space-time of the hadrons for \( b = 0 \) fm (i.e. zero impact parameter or central collisions) is parameterized as having cylindrical symmetry with respect to the beam axis. The transverse particle density dependence is assumed to be that of a projected uniform sphere of radius equal to the projectile radius, \( R = r_0 A^{1/3} \), where \( r_0 = 1.12 \) fm and \( A \) is the atomic mass number of the projectile. For \( b > 0 \) (non-central collisions) the transverse particle density is that of overlapping projected spheres whose centers are separated by a distance \( b \). The longitudinal particle hadronization position \( z_{\text{had}} \) and time \( t_{\text{had}} \) are determined by the relativistic equations \( z_{\text{had}} = \tau_{\text{had}} \sinh y; t_{\text{had}} = \tau_{\text{had}} \cosh y \)  

\begin{equation}
(3)
\end{equation}

where \( y \) is the particle rapidity and \( \tau_{\text{had}} \) is the hadronization proper time. Thus, apart from particle multiplicities, the hadronization model has three free parameters to extract from experiment: \( \sigma_y, T \) and \( \tau_{\text{had}} \). The hadrons included in the calculation are pions, kaons, nucleons and lambdas (\( \pi, K, N, \text{and } \Lambda \)), and the \( \rho, \omega, \eta, \eta', \phi, \Delta, \) and \( K^* \) resonances. For simplicity, the calculation is isospin averaged (e.g. no distinction is made among a \( \pi^+ \), \( \pi^0 \), and \( \pi^- \)). Resonances are present at hadronization and also can be produced as a result of rescattering. Initial resonance multiplicity fractions are taken from Herrmann and Bertsch \cite{12}, who extracted results from the HELIOS experiment \cite{11}. The initial resonance fractions used in the present calculations are: \( \eta/\pi = 0.05, \rho/\pi = 0.1, \rho/\omega = 3, \phi/(\rho + \omega) = 0.12, \eta'/\eta = K^*/\omega = 1 \) and, for simplicity, \( \Delta/N = 0 \).

The second stage in the calculation is rescattering which finishes with the freeze out and decay of all particles. Starting from the initial stage \( t = 0 \) fm/c, the positions of all particles are allowed to evolve in time in small time steps \( (dt = 0.1 \) fm/c) according to their initial momenta. At each time step each particle is checked to see a) if it decays, and b) if it is sufficiently close to another particle to scatter with it. Isospin-averaged s-wave and p-wave cross sections for meson scattering are obtained from Prakash et al. \cite{15}. The calculation is carried out to 100 fm/c, although most of the rescattering finishes by about 30 fm/c. The rescattering calculation is described in more detail elsewhere \cite{11}.

Calculations are carried out assuming initial parameter values and particle multiplicities for each type of particle. In the last stage of the calculation, the freeze-out and decay momenta and space-times are used to produce observables such as pion, kaon, and nucleon multiplicities and transverse momentum and rapidity distributions. The values of the initial parameters of the calculation and multiplicities are constrained to give observables which agree with available measured hadronic observables. As a cross-check on this, the total kinetic energy from the calculation is determined and compared with the RHIC center of mass energy of \( \sqrt{s} = 130 \) GeV to see that they are in reasonable agreement. Particle multiplicities were estimated from the charged hadron multiplicity measurements of the RHIC PHOBOS experiment \cite{14}. Calculations were carried out using isospin-summed events containing at freezeout about 5000 pions, 500 kaons, and 650 nucleons (\( \Lambda \)'s were decayed). The hadronization model parameters used were \( T = 300 \) MeV, \( \sigma_y=2.4 \), and \( \tau_{\text{had}}=1 \) fm/c. It is interesting to note that the same value of \( \tau_{\text{had}} \) was required in a previous rescattering calculation to successfully describe results from SPS Pb+Pb collisions \cite{17}.

Figure 1 shows normalized pseudorapidity distributions summed over pions, kaons, and nucleons from rescattering calculations for \( b = 0, 5, \) and \( 8 \) fm. The widths of the distributions and the flattening near \( \eta = 0 \) is similar to data from PHOBOS \cite{17}. Note that if the initial pion rapidity distribution in the calculation was slightly flattened at \( y = 0 \) from the simple Gaussian in Equation 2, the small dip seen in the data at \( \eta = 0 \) could be reproduced.

Figure 2 shows \( m_T \) distributions for pions, kaons, and nucleons from the rescattering calculation for \( b = 0 \) fm near midrapidity \((-1 < y < 1)\) fitted to exponentials of the form \( \exp \left( -m_T/B \right) \), where \( B \) is the slope parameter. The extracted slope parameters shown in Figure 2 are close in value to preliminary measurements from the STAR experiment for the \( \pi^- \), \( K^- \), and anti-proton of 190 \( \pm \) 10, 300 \( \pm \) 30, and 565 \( \pm \) 50 MeV, respectively \cite{18}. Thus, we see that if all hadrons begin at a common temperature-parameter value of 300 MeV, the hadronic rescattering alone is able to generate enough radial flow.
FIG. 2: Transverse mass distributions from the rescattering model. The lines are exponential fits to the distributions and the slope parameters are shown.

FIG. 3: Transverse momentum distributions for pions, kaons, and nucleons from a $b = 8$ fm rescattering calculation.

to account for the differences in slope among the pion, kaon, and nucleon $m_T$ distributions.

Figure 3 shows transverse momentum distributions for pions, kaons, and nucleons from a $b = 8$ fm rescattering calculation which extend to high-$p_T$, i.e. 6 GeV/c. As observed in PHENIX data [19], the pion and nucleon distributions merge for $p_T > 2$ GeV/c and the kaon distribution crosses that for nucleons at around 1 GeV/c.

The elliptic flow and two-pion HBT observables are also calculated from the freeze-out momenta and space-time positions of the particles at the end of the rescattering stage. The elliptic flow variable, $v_2$, is defined as

$$v_2 = \langle \cos(2\phi) \rangle; \phi = \arctan(p_y/p_x) \quad (4)$$

where $p_x$ and $p_y$ are the $x$ and $y$ components of the particle momentum, and $x$ is in the impact parameter direction and $y$ is in the “out of plane” direction (i.e. $x-z$ is the reaction plane and $z$ is the beam direction). The HBT pion source parameters are extracted from the rescattering calculation using the same method as was applied for previous SPS-energy rescattering calculations [20]. The Pratt-Bertsch “out-side-long” radius parameterization is used [21, 22] yielding the four parameters $R_{T \text{side}}$, $R_{\text{out}}$, $R_{\text{Long}}$, and $\lambda$, which represent two mutually perpendicular transverse (to the beam direction) radius parameters, a radius parameter along the beam direction, and a parameter related to the “strength” of the two-pion correlations, respectively.

III. ELLIPTIC FLOW RESULTS

Figure 4 shows the $p_T$ dependence of $v_2$ for pions and nucleons extracted from the $b = 8$ fm rescattering calculation compared with the trends of the STAR measurements for $\pi^+ + \pi^-$ and $p + p - \text{bar}$ at 11 – 45% centrality [1], which roughly corresponds to this impact parameter. Figure 5 compares the $p_T$ dependence of $v_2$ for kaons from the $b = 8$ fm rescattering calculation with the STAR measurements for $K^0_s$ at 11 – 45% centrality [23]. As seen, the rescattering calculation values are in reasonable agreement with the STAR measurements. The flattening out of the pion and nucleon $v_2$ distributions for $p_T > 2$ GeV/c is consistent with that seen in STAR and PHENIX results for minimum-bias hadrons [19, 24] (the kaon $v_2$ calculation does not extend higher than 2 GeV/c in Figure 5 due to limited statistics). Thus, the same rescattering mechanism that can account for the radial flow seen in Figures 2 and 3 is also seen to account for the magnitude and $p_T$ dependence of the elliptic flow for pions, kaons, and nucleons.

IV. HBT RESULTS

The pion source parameters extracted from HBT analyses of rescattering calculations for three different impact parameters, $b = 0$, 5, and 8 fm, are compared with STAR $\pi^-$ measurements at three centrality bins [2] in Figure 6. Note that the PHENIX HBT results [3] are in basic agreement with the STAR results. The STAR centrality bins labeled “3”, “2”, and “1” in the figure correspond to 12% of central, the next 20%, and the next 40%, respectively. These bins are roughly approximated by the impact parameters used in the rescattering calculations, i.e. the average impact parameters of the STAR centrality bins are estimated to be within ±2 fm of the rescattering calculation impact parameters used to compare with them. In the left panel, the centrality dependence of the HBT parameters is plotted for a $p_T$ bin of 0.125 – 0.225 GeV/c. In the right panel, the $m_T$ dependence of the HBT parameters is plotted for centrality bin 3, for the STAR measurements, or $b = 0$ fm, for the rescattering calculations. Although there are differences in some of the details, the trends of the STAR HBT measurements
are seen to be described rather well by the rescattering calculation.

V. DISCUSSION

As shown above, the radial and elliptic flow as well as the features of the HBT measurements at RHIC can be adequately described by the rescattering model with the hadronization model parameters given earlier. The results of the calculations are found to be sensitive to the value of \( \tau_{\text{had}} \) used, as was studied in detail for SPS rescattering calculations [11]. For calculations with \( \tau_{\text{had}} > 1 \text{ fm/c} \) the initial hadron density is smaller, fewer collisions occur, and the rescattering-generated flow is reduced, reducing in magnitude the radial and elliptic flow and most of the HBT observables. Only the HBT parameter \( R_{\text{Long}} \) increases for larger \( \tau_{\text{had}} \) reflecting the increased longitudinal size of the initial hadron source, as seen in Equation 3. One can compensate for this reduced flow in the other observables by introducing an ad hoc initial “flow velocity parameter”, but the increased \( R_{\text{Long}} \) cannot be compensated by this new parameter. In this sense, the initial hadron model used in the present calculations with \( \tau_{\text{had}} \sim 1 \text{ fm/c} \) and no initial flow is uniquely determined with the help of \( R_{\text{Long}} \).

At this point, one can consider the physical significance of the present rescattering model in two different ways. The first way is to accept that it is physically valid in the time range where hadronic rescattering should be valid, e.g. for times later than when the particle density reaches about 1 \( \text{fm}^{-3} \), and to take the initial state hadronization model as merely a parameterization useful to fit the data. Considering the calculation this way, one can at least expect to gain an insight into the phase-space configuration of the system relatively early in the collision (in the calculation 1 \( \text{fm}^{-3} \) occurs at a time 4 fm/c after hadronization). The second way to consider the calculation is to see if it is possible to also physically motivate the initial state hadronization model. An attempt to do this is given below.

In order to consider the present initial state model as a physical picture, one must assume: 1) hadronization occurs very rapidly after the nuclei have passed through each other, i.e. \( \tau_{\text{had}} = 1 \text{ fm/c} \), 2) hadrons or at least hadron-like objects can exist in the early stage of the collision where the maximum value of \( \rho \) approaches 8 GeV/fm\(^3\), and 3) the initial kinetic energies of hadrons can be large enough to be described by \( T = 300 \text{ MeV} \) in Equation 1.

Addressing assumption 2) first, in the calculation the maximum number density of hadrons at mid-rapidity at \( t = 0 \text{ fm/c} \) is 6.8 \( \text{fm}^{-3} \), rapidly dropping to about 1 \( \text{fm}^{-3} \) at \( t = 4 \text{ fm/c} \). Since most of these hadrons are pions, it is useful as a comparison to estimate the effective volume of a pion in the context of the \( \pi - \pi \) scattering cross section, which is about 0.8 \( \text{fm}^2 \) for s-waves [13]. The “radius” of a pion is found to be 0.25 fm and the effective pion volume is 0.065 \( \text{fm}^3 \), the reciprocal of which is about 15 \( \text{fm}^{-3} \). From this it is seen that at the maximum hadron number density in the calculation, the particle occupancy of space is estimated to be less than 50%, falling rapidly with time. One could speculate that this may be enough spacial separation to allow individual hadrons or hadron-like objects to keep their identities and not melt into quark matter, resulting in a “super-heated” semi-classical gas of hadrons at very early times, as assumed in the present calculation.

Since the calculation takes the point of view of being purely hadronic, it is instructive to consider assumptions 1) and 3) in the context of the Hagedorn thermodynamic model of hadronic collisions [24]. According to Hage-
dorn, the mass spectrum of hadrons of mass m increases proportional to \( \exp (m/T_0) \) in hadronic collisions, where \( T_0 = 160 \text{ MeV} \) is the limiting temperature of the system. This seems to contradict the value \( T = 300 \text{ MeV} \) needed in the present case in Equation 1 to describe the data. The Hagedorn model assumes that a) the system comes to equilibrium and b) the details of particle production via direct processes and through resonance decay average out. Neither of these assumptions is necessarily guaranteed at very early times in the collision. The use of the thermal functional form, Equation 1, to set up the initial transverse momenta of the hadrons in the present calculations is convenient but not required. For example, the exponential form \( \exp (-m_T/T_e) \) (where \( T_e \) is a slope parameter) which does not describe thermal equilibrium, could have equally well been used. This exponential form of the transverse mass distribution was successfully used previously in rescattering calculations to describe SPS data [1].

Assumption 1) can also be motivated by the Color Glass Condensate model [26, 27]. In the usual version of this picture, after the collision takes place the Color Glass melts into quarks and gluons in a timescale of about 0.3 fm/c at RHIC energy, and then the matter expands and thermalizes into quark matter by about 1 fm/c. In the context of the present rescattering calculations, it is tempting to modify the collision scenario such that instead of the Color Glass melting into quarks and gluons just after the collision, the sudden impact of the collision “shatters” it directly into hadronic fragments on the same timescale as in the parton scenario due to the hadronic strong interactions.

VI. SUMMARY

A simple thermal-like initial state model coupled with a hadronic rescattering calculation is able to adequately describe the large elliptic flow and small HBT radii recently measured at RHIC. A feature of this picture is a very early hadronization time of about 1 fm/c after the initial collision of the nuclei.

Acknowledgments

It is a pleasure to acknowledge Ulrich Heinz, Mike Lisa and Rainer Renfordt for their helpful suggestions regarding this work, and Larry McLerran for illuminating discussions on the Color Glass Condensate model. This work was supported by the U.S. National Science Foundation under grant PHY-0099476.

[1] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 87, 182301 (2001).
[2] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 87, 082301 (2001).
[3] K. Adcox et al. [PHENIX Collaboration], arXiv:nucl-th/0210008.
[4] P. P. Kolb, J. Sollfrank and U. W. Heinz, Phys. Lett. B 459, 667 (1999).
[5] D. H. Rischke and M. Gyulassy, Nucl. Phys. A 608, 479 (1996).
[6] H. Sorge, H. Stocker and W. Greiner, Annals Phys. 192, 266 (1989).
[7] H. Sorge, Phys. Rev. C 52, 3291 (1995).
[8] D. Hardtke and S. A. Voloshin, Phys. Rev. C 61, 024905 (2000).
[9] S. Soff, S. A. Bass and A. Dumitru, Phys. Rev. Lett. 86,
3981 (2001).
[10] M. Gyulassy, arXiv:nucl-th/0106072.
[11] T. J. Humanic, Phys. Rev. C 57, 866 (1998).
[12] M. Herrmann and G. F. Bertsch, Phys. Rev. C 51, 328 (1995).
[13] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
[14] U. Goerlach et al. [HELIOS Collaboration], Nucl. Phys. A 544, 109C (1992).
[15] M. Prakash, M. Prakash, R. Venugopalan and G. Welke, Phys. Rept. 227, 321 (1993).
[16] B. B. Back et al. [PHOBOS Collaboration], Phys. Rev. Lett. 85, 3100 (2000).
[17] B. B. Back et al. [PHOBOS Collaboration], Nucl. Phys. A 698, 88 (2002).
[18] C. Adler et al. [STAR Collaboration], Nucl. Phys. A 698, 64c (2002).
[19] W. A. Zajc et al. [PHENIX Collaboration], Nucl. Phys. A 698, 30 (2002).
[20] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 58, 1671 (1998).
[21] S. Pratt, T. Csörgő and J. Zimanyi, Phys. Rev. C 42, 2646 (1990).
[22] G. F. Bertsch, Nucl. Phys. A 498, 173C (1989).
[23] C. Adler et al. [STAR Collaboration], preprint (submitted to Phys. Rev. Lett).
[24] R. J. Snellings [STAR Collaboration], Nucl. Phys. A 698, 193 (2002).
[25] R. Hagedorn, Nucl. Phys. B 24, 93 (1970).
[26] L. D. McLerran, arXiv:hep-ph/0202025.
[27] A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D 52, 6231 (1995).