Tidal evolution of exo-planetary systems: WASP-50, GJ 1214 and CoRoT-7

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We perform numerical simulations to investigate tidal evolution of two single-planet systems, that is, WASP-50 and GJ 1214 and a two-planet system CoRoT-7. The results of orbital evolution show that tidal decay and circularization may play a significant role in shaping their final orbits, which is related to the initial orbital data in the simulations. For GJ 1214 system, different cases of initial eccentricity are also considered as only an upper limit of its eccentricity (0.27) is shown, and the outcome suggests a possible maximum initial eccentricity (0.4) in the adopted dynamical model. Moreover, additional runs with alternative values of dissipation factor Q′ may be carried out to explore tidal evolution for GJ 1214b, and these results further indicate that the real Q′ of GJ 1214b may be much larger than its typical value, which may reasonably suggest that GJ 1214b bears a present-day larger eccentricity, undergoing tidal circularization at a slow rate. For the CoRoT-7 system, tidal forces make two planets migrating towards their host star as well as producing tidal circularization, and in this process tidal effects and mutual gravitational interactions are coupled with each other. Various scenarios of the initial eccentricity of the outer planet have also been done to investigate final planetary configuration. Tidal decay arising from stellar tides may still work for each system as the eccentricity decreases to zero, and this is in association with the remaining lifetime of each planet used to predict its future.

extrasolar planets, tidal decay, planetary formation, numerical simulations

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There are 758 extrasolar planets detected up to 6 February 2012, a large portion of which are Jupiter-like planets, as well as a population of super-earths with masses ranging from 1 to 10 Earth’s mass (M⊕) [1]. Figure 1 shows the distribution of semi-major axes a and eccentricities e of all observed extrasolar planets by Doppler radial velocity. The eccentricities of the planets tend to be close to zero for very close-in orbits (a < 0.2), however, the eccentricities of planets are distributed uniformly from 0 to 0.9 when a > 0.2, which may be explained as a result of tidal decay and circularization in the exo-planetary systems. As well known, tidal dissipation is quite common in the planetary systems. For example, in our solar system, tidal effects existing in the Moon-Earth system may make the Moon gradually move away from the Earth, and at the same time the rotation of the Earth becomes a bit slower. In addition, tidal heating of Io (one of Galilean moons of Jupiter) changes the satellite’s interior. Moreover, some planets and most satellites have been in synchronous rotation [2].

In exo-planetary systems, the distribution of semi-major axes has a cut-off (0.01 AU) beyond which no planet is unveiled, because very close-in planets have been destroyed by tidal effects. The distribution of the planetary ages and the semi-major axes further shows that less younger planets have a much smaller semi-major axes because most of them have been engulfed by the host star due to the stronger tides [3].

Herein Jupiter-like planets (super-earths) with semi-major axes a < 0.1 AU, are called hot Jupiter-like planets (hot super-earths) [4]. However, it is difficult to elucidate how these
planets have formed in such close-in orbits as observed according to current planetary formation models [5–9]. However, a possible dynamical evolution picture may be described as follows: originally, the planets form in the distant protoplanetary gaseous disk (several AU), then move into closer orbits due to the migration mechanisms such as disk migration [10,11], the secular stellar perturbation or Kozai mechanism [12] and planetary scattering (planets will be usually excited to high eccentricities) [13]. Once they move to a much closer orbit especially within 0.1 AU, tidal effects arising from their host stars then play a vital role in the secular evolution of the planets, to shape their final orbits [6]. Finally, it may come into being the observed planetary configuration as tidal decay and circularization [14].

For multiple-planet systems, consisting of at least two planets, it is believed that gravitational interaction between planets is coupled with tidal process, so that this may cause the orbital evolution to be more complex than that of single-planet systems. The previous works are not well studied for such systems [15,16]. Therefore, we not only investigate tidal evolution of two single-planet systems (WASP-50 and GJ 1214), but also one two-planet system (CoRoT-7). When a planet has a nonzero eccentricity, planetary tide (arising from the central stellar acting on a planet) will decrease its semi-major axis and eccentricity until the eccentricity drops to zero, including current observed orbits in this process. The stellar tide (arising from the planet acting on the central star) continues to cause the planet’s orbit to decay even after the eccentricity decreases to zero, which is associated with the remaining lifetime of each planet to predict its future.

1 Tidal theory

According to the classical tidal theory, the mean variations of the semi-major axis and eccentricity in a single-planet system are given by [16,17]

\[ \frac{da}{dt} = -\frac{4}{3}na^{-\frac{3}{2}}\langle 1 + 2e^2 \rangle + 7e^2D \]  

(1)

and

\[ \frac{de}{dt} = -\frac{2}{3}ne a^{-\frac{3}{2}}\langle 9 + 7D \rangle, \]  

(2)

where \( D = \frac{\hat{p}}{2a} \), and

\[ \hat{p} = \frac{9}{4} \frac{k_0 m_1}{Q_0 m_0} R_0^3, \quad \hat{p} = \frac{9}{2} \frac{k_1 m_0}{Q_1 m_1} R_1^3, \]  

(3)

represent the stellar and planetary tides, respectively. In the above equations, \( m_i, R_i \) (\( i = 0, 1 \)) are the mass and radius (in this paper, the subscripts 0 and 1 mean stellar and planetary parameters, for two-planet system, the subscripts 1 and 2 relate to the inner and outer planet, respectively), \( a, e, n \) are semi-major axis, eccentricity and mean orbital motion, and \( k_i, Q_i \) are Love number and tidal dissipation factor, where \( k_i \) is related to the inner tidal-effective rigidity, the distribution of radial density and other parameters of a planet. \( Q_i \) is a characterization of the tidal strength, that is larger \( Q_i \) corresponding to more weaker tidal dissipation, otherwise a more stronger tidal interaction.

The above tidal theory shows that \( a \) and \( e \) are nonlinearly coupled and cannot be treated independently as a simple function of time. Furthermore, when the ratio of \( m_1/m_0 \) is smaller or \( Q_0 \gg Q_1 \), the stellar tide can be ignored. Hence, the stellar tide is not taken into account in the planetary tidal evolution until the planetary eccentricity drops to zero in this work. This indicates that the stellar tide predicts the remaining lifetime of the planetary system.

2 Numerical simulations of tidal evolution

2.1 Dynamical model

The reference framework used here is centered at the star and the motion of the planet is coplanar with respect to the reference plane. General relativity (GR) is considered for all planets here due to the planetary close-in orbits. Then the dynamical equations for a two-planet system are given as the following form by adding GR on the outer planet [16]:

\[ r_1 = -\frac{G(m_0 + m_1)}{r_1^3} r_1 + Gm_0 \left( \frac{r_2 - r_1}{|r_2 - r_1|^3} \right) \frac{r_2}{r_2} + \frac{(m_0 + m_1) f_{g_1} + f_{g_2}}{m_0}, \]

(4)

\[ r_2 = -\frac{G(m_0 + m_2)}{r_2^3} r_2 + Gm_2 \left( \frac{r_1 - r_2}{|r_1 - r_2|^3} \right) \frac{r_1}{r_1} + \frac{(m_0 + m_2) f_{g_1} + f_{g_2}}{m_0}, \]

(5)

where \( f_{g_1} \) and \( f_{g_2} \) are forces acting on the inner and outer planets generated from GR given by [16,18]

\[ f_{g_1} = \frac{Gm_0 m_1}{c^2 r_1^3} \left( \frac{4}{r_1} \frac{Gm_0}{r_1} - v_1^2 \right) r_1 + 4(r_1 \cdot v_1) v_1, \]

(6)

\[ f_{g_2} = \frac{Gm_0 m_2}{c^2 r_2^3} \left( \frac{4}{r_2} \frac{Gm_0}{r_2} - v_2^2 \right) r_2 + 4(r_2 \cdot v_2) v_2. \]

(7)
where \( v_1 = \mathbf{r}_1 \) and \( v_2 = \mathbf{r}_2 \), are velocities of the inner and outer planets respectively, and \( c \) is the speed of light.

In the above equations, \( f_{t1} \) and \( f_{t2} \) are tidal forces acting on the inner and outer planet excited by the host star with the expressions of \([16,19,20]\)

\[
f_{t1} = -\frac{9Gm^2R^5}{8Q'n_1r_1^5}\left[2r_1(r_1 \cdot v_1) + r_1^2(r_1 \times \Omega_1 + v_1)\right],
\]

\[
f_{t2} = -\frac{9Gm^2R^5}{8Q'n_2r_2^5}\left[2r_2(r_2 \cdot v_2) + r_2^2(r_2 \times \Omega_2 + v_2)\right],
\]

where \( G \) is gravitational constant, \( \Omega_i \) is the spin velocity, and \( Q'_i \) denotes a modified tidal dissipation factor defined as \( Q'_i \equiv 3Q_i/2k_i \), which is related to the lag time of the deformation due to tidal interaction. In general, \( Q'_i = 100 \) is adopted for Earth-like planets, e.g. CoRoT-7b, but \( 10^5 - 10^6 \) for Jupiter-like planets, for example, WASP-50b.

In this work, we adopt a modified MERCURY6 package to simulate the orbital evolution by adding GR and tidal effect in the original codes \([21]\), which is a general-purpose software package for N-body integrations and is designed to explore the dynamical evolution of objects moving in the gravitational field of a massive central body, as well to consider non-gravitational (e.g., cometary jet) process. Bulirsch-Stoer algorithm \([22]\) is utilized in the numerical simulations, by nature being slow but more accurate in the runs. The time step adopted here is \( 1/50-1/100 \) times the period of the innermost planet, with an accuracy of \( 10^{-12}-10^{-16} \).

### 2.2 WASP-50 system

WASP-50b is a hot Jupiter discovered by transit photometry in 2011 residing at a close-in orbit (0.0295 AU) around a G9 dwarf ( \( v = 11.6, 0.892 \, M_\odot, 0.843 \, R_\odot \) ) (Table 1) \([23]\). The current stellar activity and rotational period indicate that the age of the host star is about 0.8 Gyr, which is discrepant with the estimated age about 7 Gyr from stellar evolution. It may be relevant to tidal dissipation \([23]\). WASP-50b might be in the final process of undergoing tidal decay and circularization according to its current extremely tiny eccentricity (0.009). Considering the angular momentum of the system invariable throughout tidal evolution, and the initial region before migration due to tidal decay being more distant than the current location (the same situation for other systems), then the initial orbital elements in the numerical simulations are assumed as \( a_{ini} = 0.0305 \, AU, e_{ini} = 0.18 \). Herein a typical tidal dissipation factor \( Q'_i \) is taken as \( 10^5 \).

Figure 2 shows that the variations of the semi-major axis and eccentricity for WASP-50b. As expected, the semi-major axis and eccentricity decrease over secular evolution, and finally reach present location at 0.0295 AU in a nearly-circular orbit within \( \sim 50 \) Myr.

A short-period eccentric planet may possess a circular orbit if its circularization timescale is shorter than the stellar age due to tidal evolution. The circularization timescale is a vital factor to reproduce the process of tidal dissipation, which can be described by (in a single-planet when simply considering the planetary tide and assuming \( a \) as constant) \([24]\)

\[
\tau_{circ} = -\frac{e}{\dot{e}} = \frac{4}{63}Q'_i\left(\frac{a^3}{Gm_0}\right)^{1/2}\frac{m_1}{m_0}\left(\frac{a_1}{R_1}\right)^5.
\]

For \( Q'_i \approx 10^5 \) (where \( Q_1 \approx 26666.7 \) and \( k_1 \approx 0.4 \)), the circularization timescale of WASP-50 system is \( \sim 6.1 \) Myr, which is consistent with the numerical simulations of 9.4 Myr.

The semi-major axis continues to suffer from tidal decay even its eccentricity is reduced to be about zero owing to the stellar tide (Figure 3). Subsequently, WASP-50b may disintegrate by the tides near the Roche limit at \( \sim 0.01 \, AU \) in 0.4, 4 and 40 Gyr when taking the values of \( Q'_0 \) as \( 10^0, 10^5 \) and \( 10^6 \) respectively. According to the theory, the tidal inspiral time of a planet is written as \([25]\):

\[
\tau_a \approx \frac{1}{48}\frac{Q'_0}{n_1}\left(\frac{a_1}{R_0}\right)^5\frac{m_0}{m_1},
\]

which represents the remaining lifetime of the planet. For example, the very time from the current location to its Roche limit. As known, \( Q'_0 \) plays a very important part in tidal evolution, however, \( Q'_0 \) is still unknown. Hence, values of \( Q'_0 \)

Table 1 Orbital and physical parameters of WASP-50 system [23]

| Body     | Mass  | Radius | Semi-major axis | Eccentricity |
|----------|-------|--------|----------------|--------------|
| WASP-50  | 0.892 M_\odot | 0.843 R_\odot | –              | –            |
| WASP-50b | 1.468 M_J   | 1.153 R_J   | 0.0295 AU      | <0.009       |
in the investigated exo-planetary systems are always referred to those of solar system. For Jupiter-like system as WASP-50, when $Q_0 = 10^6, 10^7, 10^8$, $r_a = 0.29, 2.9, 29$ Gyr, which is in good agreement with those numerical results.

2.3 GJ 1214 system

GJ 1214b is a super-earth detected by transit method in 2009 with a mass and radius of 6.55 $M_\oplus$ and 2.678 $R_\oplus$ respectively, indicating that its size is between the Earth and ice giant planets of the solar system. As for planetary structure, GJ 1214b resembles a planet with a composition of water surrounded by a hydrogen-helium envelope with only 0.05% of the total planetary mass [26]. Present-day observations show that GJ 1214b is now orbiting its host star (0.153 $M_\odot$, 0.210 $R_\odot$) at 0.014 AU (the smallest semi-major axis in the exo-planetary system thus far, see Table 2) with an upper limit of eccentricity (0.27) [26]. Carter et al. (2011) [27] further constrained the eccentricity of GJ 1214b as 0.138 on the bases of follow-up light curves. However, the planetary eccentricity should have been deduced to zero in a very close-in orbit due to tidal interaction, however this inferring is incompatible with a moderate eccentricity of GJ 1214b as mentioned. One possible explanation is that there are so many unconfirmed factors during the process of tidal dissipation that current observations cannot provide a reasonable constraint on GJ 1214b’s eccentricity. In this sense, we then extensively investigate tidal evolution for GJ 1214b by adopting a variety of eccentricities, in attempt to understand a possible constraint on its initial eccentricity. Furthermore, we again perform additional simulations to explore the tidal evolution for this system, using an alternative $Q'_i$ since the factor is not well determined, to examine a possible range of $Q'_i$ during the evolution.

First, to simulate the tidal evolution of various eccentricities of GJ 1214b, we assume the initial orbital elements as follows: $a_1$ ini = 0.0168 AU, $e_1$ ini = 0.3, 0.35, 0.4, 0.45. Taking $Q'_i = 100$ (a typical value for terrestrial planets), Figure 4 shows variations of the semi-major axis and eccentricity of GJ 1214b for the variety of the initial eccentricities, considering both tide and GR raised by the host star. The outcomes indicate that final orbits are somewhat dependent on $e_1$ ini, where the smaller the final semi-major axis (the shorter the circularization timescale), the larger $e_1$ ini. At the time GJ 1214b finally located at 0.014 AU after experienced tidal decay, the corresponding eccentricities are deduced to be 0.104, 0.195, 0.267 and 0.330 (marked by dots in Figure 4) respectively, of which the former three cases can meet the upper limit of observed eccentricity (0.27). Hence, the possible largest initial eccentricity is 0.4 if the initial semi-major axis is 0.0168 AU.

The circularization timescale is estimated to be ~ 8702.6 yrs according to equation (10), which is in accord with our numerical result (~ 10,000 yrs) where $Q'_i = 100$ (with $Q_i = 20$, $k_i = 0.3$) in the third case ($a_1$ ini = 0.0168 AU, $e_1$ ini = 0.4). Results of this run show that the final semi-major axis moves about 0.0136 AU on a circular orbit over ~ 5 x 10^5 yrs. If it takes GJ 1214b so short time to circularize its orbit, the statistical results may imply that most exoplanets have circular orbits, but in fact many eccentric planets have been discovered so far. Therefore, the circularization timescale of GJ 1214b does not seem to be so short possibly because the

| Body   | Mass      | Radius   | Semi-major axis | Eccentricity |
|--------|-----------|----------|-----------------|--------------|
| GJ 1214| 0.153 $M_\odot$ | 0.210 $R_\odot$ | ~              | ~            |
| GJ 1214b| 6.55 $M_\oplus$ | 2.678 $R_\oplus$ | 0.014 AU       | <0.27        |

Figure 3 The stellar evolution of WASP-50b from the current location with various $Q'_i$ (The dash line means the Roche limit at ~ 0.01 AU).

Figure 4 (Color online) Variations of the semi-major axis and eccentricity of GJ 1214b in tidal evolution. The red dots show the current locations and its corresponding possible eccentricities.
adopted $Q'_1 = 100$ is not reasonable, denoting that the real value of $Q'_1$ would be likely to be much larger than 100. To make sure a potential range of $Q'_1$, two additional runs (where $Q'_1 = 1000$, $10000$) are further investigated in the study (Figure 5). As seen, $Q'_1$ simply plays an important role in circularization timescale (where a longer circularization timescale is associated with a larger $Q'_1$), but has little influence on the present-day eccentricity and final locations. Substantially speaking, it takes GJ 1214b about 0.14 Myr and 1.4 Myr to evolve into a circular orbit from one of possible current cases ($a_1 = 0.014$ AU, $e_1 = 0.267$) when $Q'_1 = 1000$ and $10000$, respectively. Therefore, it seems to be reasonable for GJ 1214b that $Q'_1$ is much larger than 100, indicating that this planet may own an alternate structure differing from that of a typical terrestrial planet of the solar system. In this sense, GJ 1214b is likely to be in a process of undergoing tidal circularization at a slow rate, and its circularization timescale is proportional to $Q'_1$. Hence, from the simulations, we may infer that a majority of close-in eccentric exoplanets observed now have larger tidal dissipation factors than those of the planets in the solar system, leading to a fact that they may still bear apparent eccentricities far from zero.

Herein we take $Q'_1 = 100$ to estimate the remaining lifetime of GJ 1214b, in this case its final orbit locates at 0.0118 AU on a circular orbit. Choosing $Q'_0 = 10^6$, $10^7$, the calculated timescale of remaining lifetime is about 28.8 and 288.0 Gyr respectively, according to equation (11), and this is consistent with numerical results (Figure 6). The remaining lifetime of $Q_0 = 10000'$, are 100 Gyr by taking $Q'_0 = 10^6$, GJ 1214b will eventually be affected by stellar tide, however, it will survive throughout the stellar lifetime for $Q'_0 = 10^7$.

### 2.4 CoRoT-7 system

CoRoT-7b is the first super-earth discovered with a determined radius and mass ($8.0 \ M_\oplus$, $1.58 \ R_\oplus$) around its host star at a close-in circular orbit (0.0172 AU), companied with a second Earth-like planet CoRoT-7c ($13.6 \ M_\oplus$, $2.39 \ R_\oplus$) at 0.046 AU (Table 3) [28,29]. Hence, the CoRoT-7 system, hosting two terrestrial planets, motivates a great many of researchers to explore its formation and dynamical evolution.

Considering the total angular momentum as invariable, the initial orbital elements are assumed as $e_1 = 0$, $i_1 = 0.1$, $a_1 = 0.0188$ AU, $a_2 = 0.0466$ AU. Figures 7 and 8 show the tidal evolution of two planets under consideration of both tide and GR ($Q'_1 = Q'_2 = 100$). The semi-major axes and eccentricities of two planets are both decreasing due to tidal effects. The initial eccentricity of CoRoT-7b is pumped up to a moderate value of 0.1 due to the excitation of the outer eccentric companion, then it is relaxed within a short time of ~ 0.5 Myr, next approaches a quasi-equilibrium state. The excitation of eccentricity is essential for the inner planet to undergo tidal decay and circularization as the tidal dissipation requires a nonzero eccentricity according to eqs. (1)–(3). After the relaxation progress, the orbit of the inner planet becomes smooth and continues to suffer from tidal decay and circularization. As a result that it stops near the current location at 0.0175 AU in a circular orbit. Concurrently, the outer

![Figure 6](image)

**Figure 6** The stellar tidal evolution of GJ 1214b with various $Q'_0$.

GJ 1214b is longer than that of WASP-50b because of a relatively low mass of GJ 1214. If the main-sequence lifetimes of stars such as the type of GJ 1214, with a mass of only 0.15 $M_\odot$, are 100 Gyr by taking $Q'_0 = 10^6$, GJ 1214b will eventually be affected by stellar tide, however, it will survive throughout the stellar lifetime for $Q'_0 = 10^7$.

| Table 3 Orbital and physical data of CoRoT-7 system [28–30] |
|-----------------|----------|-----------|-----------------|-----------------|
| Body            | Mass     | Radius    | Semi-major axis | Eccentricity    |
| CoRoT-7         | 0.93 $M_\odot$ | 0.87 $R_\odot$ | –               | –               |
| CoRoT-7b        | 8.0 $M_\oplus$  | 1.58 $R_\oplus$ | 0.017 AU       | 0               |
| CoRoT-7c        | 13.6 $M_\oplus$ | 2.39 $R_\oplus$ | 0.046 AU       | 0               |
planet of CoRoT-7c still migrates towards the star at a much slower rate comparing to that of the inner planet owning to the farther distance from its host star. Although both of the system run towards the star at various migration rate, resulting in a much wider separation between them, a pair of circular orbits have finally formed at about 2 Myr.

In addition, we carry out additional runs to investigate the tidal evolution of only a single outer planet to understand the outer planet’s orbital decay and circularization. In this case, orbital shrinkage is much quicker but circularization is much slower than that of the two-planet case (Figure 9). As a consequence, the decreasing of $e_2$ is mainly produced by the tidal decay and circulation of the inner planet, as a result of coupled tidal and gravitational interactions rather the contributions from its own tidal dissipation. However, to conserve the total angular momentum in the planet-star system, the tidal decay of the outer planet is damped in the presence of the inner planet. In theory, Mardling (2007) [31] studied the secular evolution of the outer planet in a two-planet system, expressing the variation of $e_2$ as:

$$\dot{e}_2 = -\frac{\lambda}{\tau_{\text{circ}}} \frac{e_2}{F(e_2)}, \quad (12)$$

where $\tau_{\text{circ}} = e_1/\dot{e}_1$ is the circularization timescale of the inner planet in a single-planet system, and $\dot{e}_1$ is given by eq. (2), $\lambda \equiv (25/16)(m_1/m_2)(a_1/a_2)^{5/2}$, and $F(e_2) \equiv e_2^3(1 - \alpha e_2^{-1} + \gamma e_2^3/2), \quad \alpha \equiv (m_1/m_2)(a_1/a_2)^{1/2}$.

Qualitatively, our results agree with those of Rodríguez et al. (2011) [16], but the resulting orbits we obtained differ from theirs because of the adoption of the initial orbital elements, as the initial parameters have an important part in its final orbit. In this sense, three runs of simply changing $e_2$ init = 0.1, 0.15, 0.2, respectively, are again performed to explore the situation of orbital evolution for a two-planet system. Figures 10 and 11 show that both semi-major axes and eccentricities are influenced by $e_2$ init. As a result, the final locations of the inner and the outer planets are at 0.0175, 0.0166, 0.0140 AU and 0.04656, 0.04651, 0.04647 AU, respectively, and the changes in final semi-major axis of the outer planet is smaller as it is far away from the star. The final semi-major axes are
smaller with larger $e_{2\text{ ini}}$, and the changes of their eccentricities can be ignored since all of them are deduced to zero within $\sim 2$ Myr, implying that their final locations are mainly dominated by $e_{2\text{ ini}}$. Furthermore it provides concrete evidence that tidal effects and mutual gravitational interactions are coupled with each other during the secular evolution.

Figure 12 shows the stellar tidal evolution from the current orbit for $Q'_0 = 10^5, 10^6, 10^7$, respectively. For example, if taking $Q'_0 = 10^6$, CoRoT-7b will be demolished in less than 1 Gyr, which is in concordance with the evaluated tidal inspiral timescale $\tau_{i1} = 0.4$ Gyr. However, CoRoT-7c will still remain survival in the stellar lifetime due to its extremely far proximity from the host star by taking the above values of $Q'_0$.

3 Summary and discussion

In this work, we numerically simulate two single-planet systems including one hot Jupiter-like planetary system (WASP-50) and hot super-earth system (GJ 1214), and a two-planet Earth-like system (CoRoT-7). We may summarize the main results below:

WASP-50b may arrive at its final orbit with a zero eccentricity in about 50 Myr by suffering from tidal decay and circularization. For GJ 1214 system, alternative initial eccentricities in simulations are investigated to make clear the tidal evolution for the super-earth GJ 1214b. As a result that the final place is more closer to the star and the eccentricity of the current location is much larger with a larger initial eccentricity in tidal process, and an upper limit of initial eccentricity (0.4) is given at the initial location (0.0168 AU). In addition, $Q'_1$ may have a direct influence on the circularization timescale of GJ 1214b, implying that $Q'_1$ of GJ 1214b could be probably much larger than that of terrestrial planets (100), so as to explain why GJ 1214b still bears an apparent eccentricity in such a close-in orbit. For two-planet system CoRoT-7, tidal evolution induces two planets to approximately reach their current locations with zero eccentricities respectively, although both of them suffer from mutual influences due to coupled affects of tidal and gravitational interaction, which can be explained by the conservation of angular momentum.
of the planet-star system. When the eccentricity goes down to zero, the stellar tide then begins to work and causes the orbital decay of the planet until it enters the region of Roche limit. Considering \( Q_1^* = 10^6 \), the remaining lifetimes of WASP-50b, GJ 1214b and CoRoT-7b are about a few billion years in comparison with the stellar ages, however, CoRoT-7c may survive steadily owing to the farther distance from its host star.

In this paper, GR is not analyzed although it is numerically considered in tidal evolution, because GR has no effect on secular variations of semi-major axis and eccentricity [32]. Herein our numerical simulations are based on several assumptions:

Firstly, tidal dissipation factor \( Q_1^* \) (or \( Q_0^* \)) is adopted from an equilibrium tidal theory, which is independent of tidal frequency and amplitude; secondly, \( Q_1^* \) is referred to those values of the solar system. For instance, taking \( 10^5 \) and 100 for Jupiter-like and terrestrial planets, respectively; thirdly, the stellar tide arising from the planet exerting on the host star is ignored during tidal decay and circularization;

Finally, the synchronization is assumed since the spin period of the planet is unknown and generally the synchronization timescale is much shorter than a secular tidal evolution. Thus, all assumptions may produce a deviation from an authentic dynamical evolution for the investigated systems. Circularization timescales of some planetary systems may be unreasonable when compared to the ages of planets due to the improper \( Q_1^* \), as well as the planetary ages are inconsistent with their stellar ages caused by the uncertain \( Q_0^* \). Therefore, new constraints of \( Q_1^* \) and \( Q_0^* \) are expected from observations so as to understand tidal dissipation well. In addition, the future new models of planetary formation and migration are of assistance to restrain the starting setup of the reasonable initial configurations adopted in numerical simulations that the final orbits depend on. Hence, until all above issues are well resolved, one complete scenario from planet formation, migration and tidal evolution to generation of the final planetary configuration will help us to understand how hot Jupiter-like and Earth-like planets are formed.

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1. http://exoplanet.eu
2. Murray C D, Dermott S F. Solar System Dynamics. New York: Cambridge Univ Press, 1999
3. Jackson B, Barnes R, Greenberg R. Observation evidence for tidal destruction of exoplanets. Astrophys J, 2009, 698: 1357–1366
4. Lanza A F. Hot Jupiters and the evolution of stellar angular momentum. Astron Astrophys, 2010, 512: 77–91
5. Lissauer J J. Planet formation. Ann Rev Astron Astrophys, 1993, 31: 129–174
6. Rasio F A, Ford E B. Dynamical instabilities and the formation of extra-solar planetary systems. Science, 1996, 274: 954–956
7. Zhou J L, Aarseth S J, Lin D N C, et al. Origin and ubiquity of short-period earth-like planets: evidence for the sequential accretion theory of planet of formation. Astrophys J, 2005, 631: L85–L88
8. Ji J H, Jin S, Timney C G. Forming close-in earth-like planets via a collision-merger mechanism in late-stage planet formation. Astrophys J, 2011, 727: L5–L8
9. Jin S, Ji J H. Terrestrial planet formation in inclined systems: application to the GOLE-2006-BLG-109L system. Mon Not Roy Astron Soc, 2011, 418: 1335–1345
10. Lin D, Bodenheimer P, Richardson D. Orbital migration of the planetary companion of 51 Pegasi to its present location. Nature, 1996, 380: 606–607
11. Goldreich P. Disk-satellite interactions. Astrophys J, 1980, 241: 425–441
12. Fabrycky D, Tremaine S. Shrinking Binary and Planetary Orbits by Kozai Cycles with Tidal Friction. Astrophys J, 2007, 669: 1298–1315
13. Ford E B, Rasio F A. on the relation between hot Jupiters and the Roche limit. Astrophys J, 2006, 638: L45–L48
14. Rasio F A, Tout C A, Lubow S H, et al. Tidal decay of close planetary orbits. Astrophys J, 1996, 470: 1187–1191
15. Zhou J L, Lin D N C. Migration and Final Location of Hot Super Earths in the Presence of Gas Giants. In: IAU Symp. 249, Exoplanets: Detection, Formation and Dynamics. ed. Sun Y S, Ferraz-Mello S, Zhou J L, eds. Suzhou, China, 2008, 319, 285–289
16. Rodríguez A, Ferraz-Mello S, Michilchenko T A, et al. Tidal decay and orbital circularization in close-in two-planet systems. Mon Not Roy Astron Soc, 2011, 415: 2349–2358
17. Dobbis-Dixon I, Lin D N C, Mandling R A. Spin-Orient Evolution of Short-Period Planets. Astrophys J, 2004, 610: 464–476
18. Beutler G. Methods of Celestial Mechanics. Vol I: Physical, mathematical, and numerical principles. Berlin: Springer, 2005
19. Mignard F. The evolution of the lunar orbit revisited. I. Earth Moon Planets, 1979, 20: 301–315
20. Mandling R A, Lin D N C. Calculating the tidal, spin, and dynamical evolution of extrasolar planetary systems. Astrophys J, 2002, 573: 829–844
21. Chambers J E. A hybrid symplectic integrator that permits close encounters between massive bodies. Mon Not Roy Astron Soc, 1999, 304: 793–799
22. Steor J, Bulirsch R. Introduction to Numerical Analysis. New York: Springer Verlag, 1980
23. Gillon M, Doyle A P, Lendl M, et al. WASP-50b: a hot Jupiter transiting a moderately active solar-type star. Astron Astrophys, 2011, 533: 88–95
24. Goldreich P, Soter S. \( Q \) in the Solar System. Icarus, 1966, 5: 375–389
25. Levrard B, Winsidoreff C, Chabrier G. Falling transiting extrasolar giant planets. Astrophys J, 2009, 692: L9–L13
26. Charbonneau D, Berta Z K, Irwin J. A super-Earth transiting a nearby low-mass star. Nature, 2009, 462: 891–894
27. Carter J A, Winn J N, Holman M J, et al. The transit light curve project, XIII. six transits of the super-earth GJ 1214b. Astrophys J, 2011, 730: 82–91
28. Léger A, Rouan D, Schneider J, et al. Transiting exoplanets from the CoRoT space mission VIII. CoRoT-7b: the first super-Earth with measured radius. Astron Astrophys, 2009, 506: 287–302
29. Queloz D, Bouchy F, Moutou C, et al. The CoRoT-7 planetary system: two orbiting super-Earths. Astron Astrophys, 2009, 506: 303–319
30. Ferraz-Mello S, Taiore D S M, Beaugé C, et al. On the mass determination of super-Earths orbiting active stars: the CoRoT-7 system. Astron Astrophys, 2011, 531: 161–171
31. Mandling R A. Long-term tidal evolution of short-period planets with companions. Mon Not Roy Astron Soc, 2007, 382: 1768–1790
32. Huang C, Liu L. Analytical solutions to the four post-Newtonian effects in a near-earth satellite orbit. Celest Mech Dyn Astr, 1992, 53: 293–307