The Discussion about the Spin States, the Helicity States and the Chirality States

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Abstract
The differences and the relations among the spin states, the helicity states and the chirality states of fermions in Relativistic Quantum Mechanics are discussed. It is pointed out emphatically that they are entirely different: a spin state is helicity degenerate; a helicity state can be expanded as linear combination of the chirality states; the polarization of fermions in flight must be described by the helicity states. The investigation of these differences can not only clarify many conceptual confusions, but also can predict a new left-right polarization asymmetry phenomenon—the lifetime asymmetry.

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1 Introduction

“The polarization asymmetry will play a central role in precise tests of the standard model.” [1] The SLD Collaboration [2, 3] and the SLAC E158 Collaboration [4, 5] have precisely measured the polarization asymmetry in neutral weak currents processes mediated by \( Z^0 \) exchange and got a definite proof. Their results show that the integrated cross section of left-handed (LH) polarized electrons is greater than that of right-handed (RH) ones in polarized electron-positron collisions and polarized electron-electron Møller scattering. But so far in charged weak current processes mediated by \( W^\pm \) exchange, the measurement of the polarization asymmetry, the lifetime asymmetry, has not yet been found in literature. This is because it has not yet been understood theoretically.

In Relativistic Quantum Mechanics and the Standard Model of particle physics, there exist three kinds of spinor wave functions, i.e., the spin states, the helicity states and the chirality states. They are entirely different. Formerly, however, the difference among them would often be neglected. Physicists too believed in the covariant form of four-component spinor which enjoys the invariance under the Lorentz transformation and so misunderstood the meaning of so-called the spin states. Meanwhile, the importance of helicity state and its difference from the spin state and the chirality state were often confused or overlooked, so that the lifetime asymmetry [6, 7, 8], the lifetime of RH polarized fermions is always greater than that of LH ones with the same speed in flight, used to be disregarded.

In this paper the differences and the relations among three kinds of spinors are discussed in detail. The paper is organized as follows. In Sec.2, the spin states are discussed concisely. In Sec.3, the helicity states are discussed and its differences from the spin states are carefully distinguished. In Sec.4, the chirality states and the relations among the spin states, the helicity states and the chirality states are discussed. In Sec.5 the polarization asymmetry in charged weak current processes is discussed by analyzing and calculating the lifetimes of polarized muons in flight. Finally, we briefly summarize the discussion above in Sec.6.

2 The spin states

The spin states are the plane wave solutions of Dirac equation, and in momentum representation for a given 4-momenta \( p \) and mass \( m \), the positive and the negative energy solution are respectively

\[
\psi_s(p) = \sqrt{\frac{E_p + m}{2E_p}} \left( \frac{\varphi_s}{E_p + m \varphi_s} \right),
\]
\[ v_s(p) = \sqrt{\frac{E_p + m}{2E_p}} \left( \frac{\sigma \cdot p}{E_p + m} \varphi_s \right), \]  

(2)

where \( E_p > 0 \), \( s = 1, 2 \) and \( \varphi_s \) are Pauli spin wave functions. The spin states are eigenstates of operator \( \frac{\omega(p) \cdot e}{m} \) with eigenvalues \( \pm 1 \), namely

\[ \frac{\omega(p) \cdot e}{m} u_s(p) = \begin{cases} u_s(p), & (s = 1) \\ -u_s(p), & (s = 2) \end{cases} \]  

(3)

Where \( \frac{\omega(p)}{m} \) is the Pauli-Lubanski covariant spin vector and \( e \) is the four-polarization vector in the form

\[ e_\alpha = \begin{cases} e^0 + \frac{p}{m(E_p + m)} (p \cdot e^0), & (\alpha = 1, 2, 3) \\ i \frac{p \cdot e^0}{m}, & (\alpha = 4) \end{cases} \]  

(4)

which is normalized \((e^2 = 1)\), orthogonal to \( p \) \((e \cdot p = 0)\). \( e^0 \) is the space component of the four-polarization vector \( e \) with respect to the rest frame and is equal to the unit vector of \( z \) axis, i.e., \( e^0 = (e^0, 0) = (0, 0, 1, 0) \).

Because the vector \( \frac{\omega(p)}{m} \) and \( e \) are both Lorentz covariant, their scalar product \( \frac{\omega(p) \cdot e}{m} \), formed by the projection of the Pauli-Lubanski covariant spin vector \( \frac{\omega(p)}{m} \) on the four-polarization vector \( e \), is a Lorentz scalar operator and the eigenvalue of the spin state is Lorentz invariant as indicated from Eq. (3).

In the rest frame, the Pauli-Lubanski covariant spin vector \( \frac{\omega(p)}{m} \) reduces to \((\Sigma, 0)\), where \( \Sigma \) is the \( 4 \times 4 \) matrix representation of Pauli spin matrices, the four-polarization vector \( e \) to \((e^0, 0)\), the operator \( \frac{\omega(p) \cdot e}{m} \) to \((\Sigma_3, 0)\) and the spin states \( u_s(p) \) reduce to

\[ u_1^0 = \begin{pmatrix} \varphi_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_2^0 = \begin{pmatrix} \varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]  

(5)

Physically these states correspond to particles at rest with spin up \((s = 1)\) or down \((s = 2)\) along the \( z \)-axis.

In the motion frame (i.e. in the laboratory frame), however, the spin states, vector \( \frac{\omega(p)}{m} \) and \( e \) have no intuitively and definitely physical significance. For example, in the spin states the polarization of fermions is described by the four-polarization vector \( e \) which is the relativistic generalization of three-polarization vector \( P \), whereas its space component \( e \) is completely different from vector \( P \). The vector \( P \) is the ensemble average of spin vector of fermion beam, \( P = \langle \sigma \rangle \), and it does not change its sign under the space inversion and so is called a pseudovector or an axial vector, like angular momentum. The polarization degree \(|P|\) is the length of polarization vector \( P \), and it can take any value between 0 and 1. The value of \(|P|\) can not be greater than 1 and remains constant in different frame. On the other hand, when \( p = p_z \) vector \( e \) reads

\[ e = \frac{E}{m} e^0. \]  

(6)

Differing from vector \( P \), obviously, the direction of vector \( e \) is always pointing to \( z \) axis and its value can be greater than one, i.e., \(|e| \geq 1\). It is inconsistent with the property, \(|P| \leq 1\), of vector \( P \). Only if in the rest frame, \( E = m \), \(|e| \) value is equal to 1. Strictly speaking, therefore, only in the rest frame can the four-polarization vector \( e \) be most unambiguously defined [9].

In application it is frequently necessary to evaluate spin sums in the form

\[ P_1(p) = \rho_+ \Lambda_+(p), \quad P_2(p) = \rho_- \Lambda_+(p), \quad P_3(p) = \rho_+ \Lambda_-(p), \quad P_4(p) = \rho_- \Lambda_-(p). \]  

(7)

The \( \Lambda_+(p) \) and \( \Lambda_-(p) \) are the positive and the negative energy projection operator,

\[ \Lambda_+(p) = \frac{-i \gamma \cdot p + m}{2E_p}, \quad \Lambda_-(p) = \frac{i \gamma \cdot p + m}{2E_p}, \]  

(8)

respectively, and \( \rho_\pm \) are spin projection operators:

\[ \rho_\pm = \frac{1}{2} (1 \pm i \gamma_5 \gamma \cdot e). \]  

(9)

The plus sign refers to \( s = 1 \) and the minus sign to \( s = 2 \). We have eigen equation

\[ \rho_+ u_1(p) = u_1(p), \quad \rho_- u_2(p) = u_2(p), \quad \rho_+ u_2(p) = \rho_- u_1(p) = 0. \]  

(10)
One sees that the operator $P_1(p)$ project out the positive energy states with spin up and $P_2(p)$ the positive energy states with spin down, whereas the operator $P_3(p)$ the negative energy states with spin down and $P_4(p)$ the negative energy states with spin up.

3 The helicity states

A helicity state satisfies the ordinary Dirac equation [10]. If spinor $\varphi$ is taken as the eigenstate of the helicity operator,

$$\frac{\sigma \cdot p}{|p|} \varphi_h = h \varphi_h, \quad h = \pm 1$$

then the helicity states read

$$u_h(p) = \sqrt{\frac{E_p + m}{2E_p}} \left( \frac{\varphi_h}{E_p + m} \varphi_h \right),$$

and

$$\varphi_{h=+1} = \left( \begin{array}{c} \cos \frac{\theta}{2} e^{-i\phi} \\
\sin \frac{\theta}{2} e^{i\phi} \end{array} \right), \varphi_{h=-1} = \left( \begin{array}{c} -\sin \frac{\theta}{2} e^{-i\phi} \\
\cos \frac{\theta}{2} e^{i\phi} \end{array} \right),$$

where $\theta$ and $\phi$ are the polar angles of momentum $p$ in polar-coordinates. The state with $h = +1$ is the RH helicity state while the state with $h = -1$ is the LH one. The spin projection operators of the helicity states are [11]

$$\rho_h = \frac{1}{2} \left( 1 + h \frac{\Sigma \cdot p}{|p|} \right).$$

We can see that the helicity state is different from the spin state. The helicity operator $\frac{\sigma \cdot p}{|p|}$ is the projection of the spin along the direction of the motion of polarized fermions. A fermion's helicity $h$ can change its sign under a Lorentz transformation or a space inversion, and so it is called a pseudoscalar. In other words, the helicity of a massive fermion is not a Lorentz invariant observable quantity and it cannot be considered as a fundamental property of particle.

The operator $\frac{\sigma \cdot p}{m}$ is a 4 × 4 matrix, and it should have 4 eigenvalues and 4 corresponding eigenfunctions. From Eq. (3), however, we can find out that it does not. Therefore, a spin state with the same $s$ but different values of $h$ is helicity degenerate. Similarly, the spin projection operator of the spin state is independent of $h$, so that it is also helicity degenerate. The spin states can not uniquely describe the helicity of fermions and its spin projection operator can only project out the states which have spin $s = 1$ and 2 in its rest frame, respectively.

The helicity $h$ and the polarization degree $|\mathbf{P}|$ are two different concepts. The helicity describes the mutual relation between the spin and the momentum vector, and it can only take the value of either +1 or −1, which correspond respectively to spin vector $\mathbf{\sigma}$ parallel and antiparallel to $\mathbf{p}$. Without the momentum, there would be no the mutual relation. In the rest frame of fermions, therefore, the helicity is meaningless and the polarization of fermions in flight must be described by the helicity states which are closely related to directly observable quantity experimentally.

On the other hand, taking the simplest case of $\mathbf{p} : \mathbf{p} = p_z$, which does not lose the universality of problem, we have the LH helicity state $u_{Lh}$ and the RH helicity state $u_{Rh}$, respectively

$$u_{Lh}(p) = \sqrt{\frac{E_p + m}{2E_p}} \left( \frac{\varphi_2}{E_p + m} \right),$$

$$u_{Rh}(p) = \sqrt{\frac{E_p + m}{2E_p}} \left( \frac{\varphi_1}{E_p + m} \right).$$

Even so comparing Eq. (9) with Eq. (14) one can also see that the spin projection operator of the spin state is different from that of the helicity state though the spin and the helicity state are formally identical when $\mathbf{p} = p_z$. The $\rho_{\pm}$ is Lorentz covariant [12], while $\rho_h$ is not and it is essentially a two-component operator. Hence, one can not absolutely regard the state [13] or [10] as a spin state.

In Quantum Mechanics, the variables of wave functions are separable and then a wave function can be written as $\Psi(x, y, z, \mathbf{\sigma}, t) = \psi(x, y, z, t)\chi(\mathbf{\sigma})$, where the wave function $\psi(x, y, z, t)$ is only related to the space-time variables and the wave function $\chi(\mathbf{\sigma})$ to a single spin operator $\mathbf{\sigma}$. The spin is the intrinsic property of particles and then the eigenvalue of spin wave function is Lorentz invariant. A particle which does not possess determinate spin value is not able to be accepted physically. On the other hand, operator $\mathbf{\sigma}$ has three spatial components, $\sigma_x$, $\sigma_y$ and $\sigma_z$. The third
spin operator $\sigma_z$ commutes with Hamilton operator $H$, and so it is conventionally designated as the variable of the spin wave function. In this way, a wave function can be rewritten as $\Psi(x, y, z, \sigma_z, t) = \psi(x, y, z, t)\chi(\sigma_z)$. Therefore, the solution of Schrödinger equation is simultaneously an eigenfunction of $H$ and of $\sigma_z$.

In Relativistic Quantum Mechanics, Hamilton operator $H$ still commute with $\sigma_z$ in the rest frame. In the motion frame, however, they do not commute anymore and so $\sigma_z$ can not serve as the spin variable. We need to construct new spin operator and its eigenfunctions. Based on discussion above, obviously, the spin wave function should satisfy the two conditions as follows: (1) it reduces to the eigenfunction of $\Sigma$ in the rest frame; (2) its eigenvalue is Lorentz invariant. In order to satisfy the two conditions, we must refer to the Pauli-Lubanski covariant spin vector $\frac{\omega(p)}{m}$ and the four-polarization vector $e$, as shown in Eqs. (3) and (4). The spin states can not only satisfy the two conditions but also are simultaneously an eigenfunction of $H$, of momentum $p$ and of scalar operator $\frac{\omega(p)}{m}$. Contrarily, the helicity states do not satisfy the two conditions because they are meaningless in the rest frame and their eigenvalues are not Lorentz invariant, either. Strictly speaking, therefore, the spin states are the plane wave solutions of Dirac equation, while the helicity states are only the mathematical solutions of Dirac equation, not the physical solutions.

4 The chirality states

The chirality states are the eigenstates of chirality operator $\gamma_5$. The LH and the RH chirality state are defined as, respectively

$$ u_{Lh}(p) = \frac{1}{2}(1 + \gamma_5)u_+(p), \quad u_{Rh}(p) = \frac{1}{2}(1 - \gamma_5)u_+(p). $$

(17)

Obviously, a chirality state is directly correlative with a spin state and its eigenvalue is Lorentz invariant.

In general, the chirality states are different from helicity states. Only if $m = 0$ (for example neutrinos) or $E \gg m$ (in the ultrarelativistic limit) the fermions satisfy Weyl equation [1], the spinor $\varphi_s$ must then be taken to be eigenstates of helicity operator and the polarization is always in the direction of motion [13]. In other words, when $m = 0$ or the velocity of fermion approaches light speed, the helicity states, the chirality states and the spin states are identical, i.e.

$$ u_{Lh}^W(p) = u_{Lh2}^W(p) = u_{Lh3}^W(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_2 \\ -\varphi_2 \end{pmatrix}, $$

(18)

$$ u_{Rh}^W(p) = u_{Rh1}^W(p) = u_{Rh3}^W(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 \\ \varphi_1 \end{pmatrix}. $$

(19)

The superscript $W$ refers to it being a solution of Weyl equation.

The helicity states $u_{Lh}$ and $u_{Rh}$ in Eqs. (15) and (16) can be expanded as linear combination of the chirality states, respectively

$$ u_{Lh}(p) = \frac{1}{2}(1 + \gamma_5)u_{Lh}(p) + \frac{1}{2}(1 - \gamma_5)u_{Rh}(p) $$

$$ = C_{LL}u_{Lh}^0 + C_{LR}u_{Rh}^0, $$

(20)

$$ u_{Rh}(p) = C_{RL}u_{Lh}^0 + C_{RR}u_{Rh}^0, $$

(21)

where $u_{Lh}^0$ and $u_{Rh}^0$ are the chirality states in the rest frame,

$$ u_{Lh}^0 = \frac{1}{2} \begin{pmatrix} \varphi_s \\ -\varphi_s \end{pmatrix}, \quad u_{Rh}^0 = \frac{1}{2} \begin{pmatrix} \varphi_s \\ \varphi_s \end{pmatrix}. $$

(22)

The coefficients $C_{LL}$, $C_{LR}$, $C_{RL}$ and $C_{RR}$ are given by

$$ C_{LL} = C_{RR} = \frac{1}{\sqrt{2E_p}}(\sqrt{E_p + m} + \sqrt{E_p - m}) = \sqrt{1 + \beta}, $$

(23)

$$ C_{RL} = C_{LR} = \frac{1}{\sqrt{2E_p}}(\sqrt{E_p + m} - \sqrt{E_p - m}) = \sqrt{1 - \beta}, $$

(24)

where $\beta$ is the velocity of fermions. It is obvious from Eqs. (20) and (21) that in a LH helicity state $u_{Lh}(p)$ the coefficient $C_{LL}$ is the amplitude of LH chirality state $u_{Lh}^0$, and the $C_{LR}$ that of RH chirality state $u_{Rh}^0$; while in a RH helicity state $u_{Rh}(p)$ the $C_{RL}$ that of LH chirality state $u_{Lh}^0$ and the $C_{RR}$ that of RH chirality state $u_{Rh}^0$ in the rest frame. For a LH helicity state the hidden amplitude of RH chirality state decreases with the increase of $\beta$ until it approximate zero when $\beta \to 1$, showing that a high-energy fermion can be LH polarized without hidden RH chirality state; for a RH helicity state the hidden amplitude of LH chirality state decreases with the increase of $\beta$. When $\beta = 0$, the hidden amplitude of LH chirality state is equal to that of RH one and one even can not discriminate a rest fermion being either LH or RH polarized [14]. It indicates again that the helicity states are meaningless in the rest frame.
5 the lifetime of polarized muons in flight

The Standard Model of particle physics is a chiral gauge theory and all of fundamental fermions are divided into two classes, LH and RH chirality state, and they have different gauge transformations. Especially, RH chirality states have zero weak isospin and are only present in neutral weak currents. In charged weak currents, all fermions are in the LH chirality states while all antifermions are in the RH ones. For example, the negative muon decay can be written as

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_{\mu},$$  \hspace{1cm} (25)

where subscript $L(R)$ denotes the LH (RH) chirality state.

If the muons are unpolarized and if we do not observe the polarization of final-state fermions, then the transition matrix element of muon decay is given by averaging over the muon spins and summing over all final fermion spins:

$$M^2 \sim \frac{1}{2} \sum_{s,s',r,r'} [ \bar{\pi}_{s'}(q) \gamma_{\lambda} (1 + \gamma_5) \nu_{r'}(k') ]^2 [ \bar{\pi}_{r}(k) \gamma_{\lambda} (1 + \gamma_5) u_s(p) ]^2. \hspace{1cm} (26)$$

where $p$, $q$, $k$ and $k'$ are 4-momenta, while $s$, $s'$, $r$ and $r'$ are spin indices for $\mu$, $e$, $\nu_\mu$ and $\bar{\nu}_e$, respectively. For the convenience of discussion below, in Eq. (26) we set

$$I = \frac{1}{2} \sum_{s=1}^{2} \sum_{r=1}^{2} [ \bar{\pi}_r(k) \gamma_{\lambda} (1 + \gamma_5) u_s(p) ]^2, \hspace{1cm} (27)$$

which is related to the muons. By means of Eq. (4), the evaluations of spin sums are reduced to the calculation of projection operators:

$$\sum_{s=1}^{2} u_s(p)\pi_s(p) = \sum_{n=1}^{2} P_n(p) = \Lambda_+(p), \hspace{1cm} (28)$$

$$\sum_{s=1}^{2} v_s(p)\pi_s(p) = -\sum_{n=1}^{4} P_n(p) = -\Lambda_-(p). \hspace{1cm} (29)$$

One sees that the explicit evaluations of spin projection operators disappear. Applying Eqs. (28), (29) and (8) as well as the trace theorems, the muon lifetime $\tau$ in the motion frame is given by

$$\tau = \frac{\tau_0}{\sqrt{1 - \beta^2}}, \hspace{1cm} (30)$$

where $\tau_0$ is the muon lifetime in its rest frame. We see that the lifetime increases considerably with the increase of $\beta$ as required by the special theory of relativity and so that the lifetime is not a Lorentz scalar.

If the muons are polarized and the polarized state of fermions is described by the spin state (1) or (2), as mentioned in most literatures and textbooks [15, 16, 17], then the muon spins should not be summed and averaged, and instead of Eqs. (27), we have

$$I_s = \sum_{r=1}^{2} [ \bar{\pi}_r(k) \gamma_{\lambda} (1 + \gamma_5) u_s(p) ]^2, \hspace{1cm} (31)$$

and

$$u_s(p)\pi_s(p) = P_s(p) = \frac{1}{2} (1 \pm i\gamma_5 \gamma \cdot e) \frac{(-i\gamma \cdot p + m_\mu)}{2E_p}, \hspace{1cm} (32)$$

respectively. In a similar way to get Eq. (30), we obtain

$$\tau_s = \tau, \hspace{1cm} (33)$$

where $\tau_s$ is the lifetime of polarized muons in the spin state. It is easy to see that the lifetime is independent of the polarization of muons.

If the polarized states of fermions are described by the helicity states, then substituting the spin states in Eq. (31) with the helicity states, we have

$$I_{Lh} = \sum_h [ \bar{\pi}_h(k)\gamma_{\lambda}(1 + \gamma_5)u_{Lh}(p) ]^2 \hspace{1cm} (34)$$
From Eqs. (20), (23) and (17) we easily find

\[(1 + \gamma_5)u_L(p) = 2\sqrt{1 + \beta}u_L^0 = \sqrt{1 + \beta}(1 + \gamma_5)u_L^0, \tag{35}\]

where \(u^0_L\) is the spin state in the rest frame. One can see that the chirality-state projection operator \((1 + \gamma_5)\) picks out LH chirality state \(u^0_L\) in a LH helicity state, which is factorized into two parts in the second equation, one is the spin state \(u^0_L\) and another is a factor \(\sqrt{1 + \beta}\) which is relevant to muon’s polarization. Substituting Eq. (35) into Eq. (34) we have

\[I_{Lh} = (1 + \beta) \sum_h \left[\bar{\Psi}_h(k)\gamma_\lambda(1 + \gamma_5)u^0_L\right]^2, \tag{36}\]

Though the spin projection operators of the spin state is different from that of helicity state, from Eq. (8) we can see out that the energy projection operator is independent of \(s\) or \(h\) and so that it is not difficult to prove that their energy projection operators are identical, namely

\[\sum_h u_h(k)\bar{\Psi}_h(k) = \sum_{r=1}^2 u_r(k)\bar{\Psi}_r(k) = \sum_{n=1}^2 P_n(k) = \Lambda_+(k). \tag{37}\]

On the other hand, according to Eq. (32) in the rest frame we have

\[u^0_L u^0_L = P_2(p^0) = \frac{1}{2} \left[1 - i\gamma \cdot \rho \cdot e^0 \left(\frac{-i\gamma \cdot p^0 + m_\mu}{2m_\mu}\right)\right]. \tag{38}\]

The operator \((\gamma \cdot p)\) and \(\rho_-\) are both Lorentz covariant, therefore, under a Lorentz transformation from the rest frame to the motion frame \(u^0_L u^0_L\) will go over into \(u_2(p)\bar{\Psi}_2(p)\), namely

\[u^0_L u^0_L \rightarrow u_2(p)\bar{\Psi}_2(p). \tag{39}\]

Substituting Eq. (37) into Eq. (36) and considering Eq. (39), Eq. (36) may be rewritten as

\[I_{Lh} = (1 + \beta) \sum_{r=1}^2 \left[\bar{\Psi}_r(k)\gamma_\lambda(1 + \gamma_5)u_2(p)\right]^2. \tag{40}\]

Comparing Eq. (40) with Eq. (31), we obtain the lifetime of LH polarized muons

\[\tau_{Lh} = \frac{\tau}{1 + \beta}. \tag{41}\]

Similarly, for RH polarized muons we obtain

\[I_{Rh} = (1 - \beta) \sum_{r=1}^2 \left[\bar{\Psi}_r(k)\gamma_\lambda(1 + \gamma_5)u_2(p)\right]^2. \tag{42}\]

Then the lifetime of RH polarized muons is given by

\[\tau_{Rh} = \frac{\tau}{1 - \beta}. \tag{43}\]

It is shown that for polarized fermions in flight, if we use the spin state to performing calculation, it will result in Eq. (33) which does not reveal any polarization asymmetry; if we use helicity state, it will result in Eqs. (41) and (43) in which the lifetime is the left-right polarization asymmetry, namely lifetime asymmetry.

Furthermore, this conclusion is also valid for all fermions in the decays under weak interactions. It means that the lifetime of RH polarized fermions is always greater than that of LH ones in any one of inertial systems in which fermions are in flight with a same speed. Due to the time dilation effect predicted by the special theory of relativity, the lifetime is originally not a Lorentz scalar. Now, due to the parity violation, the lifetime is left-right polarization asymmetry. Hence the lifetime is neither a four-dimensional scalar, nor a scalar under the three-dimensional space inversion.
6 Conclusion

We have discussed in detail the helicity states, the spin states and the chirality states. For massive free fermions the three kinds of spinor wave functions are entirely different. The eigenvalue of the spin state is Lorentz invariant, while that of helicity state is not. The spin projection operator of the spin state is Lorentz covariant, while that of the helicity state is not. A spin state is helicity degenerate, so that it can not uniquely describe the helicity of fermions. The helicity state is also different from the chirality state and it can be expanded as linear combination of chirality states. In the rest frame the spin states are defined and possess definitely physical significance, but in the motion frame they do not. In the motion frame, contrarily, the helicity states are defined and possess definitely physical significance, but in the rest frame they do not. In order to research the processes of weak interactions, therefore, in the rest frame one must employ the spin states, while in the motion frame the polarization of fermions in flight must be described by using helicity states.

In order to clarify the differences, Dr. Strangerep very clearly said: “there’s an important distinction between helicity and chirality.” “When textbooks say that only left-handed fermions participate in the weak interaction, they mean left-handed chirality, not left-handed helicity.” “whenever textbooks speak of handedness in connection with weak interactions, they mean chirality, not helicity. And whenever people talk about boosting into another frame to change the handedness, they’re talking about helicity.” [15] I would like to supplement and modify some words in his statements below. There are some important differences among the spin states, the helicity states and the chirality states. Whenever people talk about boosting into another frame to change the handedness, they’re discussing about the polarized state, in the rest frame they should be discussing about the spin state, while in the motion frame about the helicity state. Whenever textbooks speak of handedness in connection with weak interactions, for example, the Standard Model is a chiral gauge theory, or neutral Z particles and charged W+ and W- particles transmit the weak force and do so with different strengths for LH and RH particles; the W+ and W- interact only with LH particles, while the Z interacts with both but with different strengths, they mean chirality states, not helicity states or spin states. When textbooks say that in the beta or muon decays, all electrons are completely LH particles while all positrons completely RH ones, they mean chirality states, not helicity states or spin states. However, when textbooks say that in the beta or muon decays, all electrons are predominantly LH particles while all positrons predominantly RH ones, they mean helicity states, not chirality states or spin states. And when textbooks say that neutrinos are LH particles while antineutrinos are RH ones, they mean any of spin states, helicity states and chirality states because they are identical when \( m = 0 \).

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