Research Article

One-Way Substitution Newsboy Problem under Retailer’s Budget Constraint

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One-way substitution means that when low-end brand goods are sold out, high-end brand goods can be offered to consumers as substitute goods, but not the opposite. In realistic economic activity, “shortage of funds” is a common practical problem for the retailer in making order decision. This paper proposes a nonlinear optimization model with the retailer’s budget to study the optimal order quantities and substitution discount for two one-way substitution products under a stochastic demand scenario, and the objective is to maximize the retailer’s revenue. We solve the model mainly according to the Karush–Kuhn–Tucker (KKT) theorem and present the conditions of optimal decisions. Finally, through the numerical study, we analyze the influence of the budget constraint and other parameters on the optimal solutions.

1. Introduction

Since functions or services have become increasingly more sophisticated and diverse, products are always divided into two categories, high-end brand products and low-end brand products, according to different qualities and service levels. When low-end products are out of stock, retailers may sell high-end products at a discounted price to the customers who originally wanted to buy the low-end products in order to retain them as customers and reduce losses. This phenomenon of using high-end products to substitute for low-end products is called one-way substitution. We cannot match supply and demand exactly since the market demand is stochastic and fluctuating. Therefore, if you want to reduce losses, you have to persuade customers to accept the better product as a substitute. This is why one-way substitution is common in many industries. When seats in the economy class are sold out and seats in the first class are remained, the airlines may upgrade some economy class passengers to the first class for free so as to sell a few more economy class tickets. If standard rooms are sold out, hotels are likely to offer vacant deluxe rooms at a higher discount to tourists who are reluctant to pay a premium. Under the situation of Assemble-to-Order, one-way substitution appears more frequently because of the fluctuation of demands. If special power adapters which are only used in a certain region are in the shortage, Hewlett-Packard, one of the world’s leading printer producers, will assemble a power adapter for printers which can be used around the world, so as to ensure that the order for products can be completed on time. In addition, in computer network equipment manufacturing industry, devices with high capacity are often used to replace devices with low capacity, such as memory chips and USB flash disk. There are also many one-way substitution products in iron and steel industry. For example, steel products with high hardness are sometimes used to replace similar steel products with low hardness.

Chand et al. [1] constructed a one-way substitution model of parts selection and proposed an effective dynamic programming algorithm to find the optimal solution. Hsu and Bassok [2] proposed a single-period, multiproduct substitution model and obtained the optimal solution. Smith and Agrawal [3] established a probabilistic demand model for a class of commodities with substitutes and gave a method to select the inventory level of products in order to maximize the total expected profits. Mahajan and Van Ryzin
We assume that the retailer can only order two types of products from one supplier and that the products have the same basic functionality, but their additional features are at designed a hybrid intelligent algorithm based on the genetic algorithm (GA) and the fuzzy simulation to solve the model. Yang et al. [27] studied how to use the genetic algorithm to select suppliers and purchase products when the seller has a return policy and faces budget constraints. Zhang et al. [28] developed a solution algorithm for the constrained newsboy problem. Huang et al. [29] studied a class of multiproduct newsvendor problem with a budget constraint when considering strategic customer behaviour. Zhang and Hua [30] presented a portfolio contract for the multiproduct newsboy problem with budget constraint and showed the advantage of the portfolio model. Zhang [31] considered the multi-constraint multiproduct newsvendor problem with a budget constraint. Bajwa et al. [32] considered a dynamic problem of joint pricing and production decisions for multiple products with a capacity constraint. Zhang et al. [33] considered the budget constraint into a substitute product newsboy problem: the newsvendor sells two substitute products and determines the optimal order quantity and retail price for each product. As far as we know, few studies take budget constraints into account in the research of one-way substitution newsboy problem.

We present a nonlinear programming model in which we consider two one-way substitution products and the retailer’s budget constraint. This paper aims at answering the following critical questions: under the certain budget constraint, how does the retailer make decisions about the substitute discount and order quantity of each product to maximize revenues? How do the important parameters (e.g., the budget constraint, the retail price, the wholesale cost, the salvage value, and the understocking cost) affect the optimal substitute discount and order quantity of each product?

Our work contributes to the literature in the following ways:

1. As far as we know, this paper is the first work to consider the retailer’s budget constraint into the one-way substitution newsboy problem, which extends the research of the newsboy problem.
2. In this paper, we use the analytical approach to address this type of newsboy extension problem and obtain the conditions for the existence of the optimal solutions.
3. With the increase in the budget, the optimal order quantities increase and the optimal substitute discount decreases. The optimal substitute discount is proportional to the price and the wholesale cost of the high-end product and the salvage value of the low-end product, respectively. The conclusions are of great practical significance and can answer the above two critical questions to provide guidance of the ordering and substitution decisions to the retailer.

2. Problem Description and Related Assumptions

We assume that the retailer can only order two types of products from one supplier and that the products have the same basic functionality, but their additional features are at
different levels. We refer to high-end products as product A and low-end products as product B, and both products have a fixed wholesale cost. The total costs of ordering the two types of products cannot exceed the retailer’s budget. The market demand for each type of product is independent. When product B is out of stock and product A has some surplus, the retailer will be willing to sell product A to consumers who order product B at a discounted price. As a result, a portion of the unmet consumers for product B may shift to product A at an acceptable price. The unsold products have little salvage value at the end of a single sale cycle. If products are out of stock and cannot be substituted, the retailer will encounter understocking costs.

2.1. Notations. $x$ and $y$ represent the market demands of product A and product B, respectively. As continuous stochastic variables, the corresponding probability density functions are $f_1(x)$ and $f_2(y)$, respectively, and the cumulative functions are $F_1(x)$ and $F_2(y)$, respectively. $r$ represents the discount that is provided by the retailer when substituting product B with product A. That is, the price of product A that is sold to customers who ordered product B but cannot be satisfied is $r p_1$. $\beta$ represents the one-way substitution rate. That is, when product B is out of stock, the probability that consumers who ordered product B will be willing to buy product A instead. The parametric symbols and meanings used in this paper are shown in the following Table 1.

2.2. Model Assumptions

Assumption 1: $p_i \geq c_i \geq v_i > 0$, $i = 1, 2$. It shows that the sales of the two products are profitable, and the excess inventory has low salvage value.

Assumption 2: $p_1 > p_2$, $c_1 > c_2$, $v_1 > v_2$, and $s_1 > s_2$. It means that the retailer will give priority to satisfy customers according to their actual needs. The retailer will adopt the substitute strategy only when product B is out of stock, and product A has surplus inventory.

Assumption 3: $r p_1 + s_2 \geq v_1$. It shows that when product B is out of stock, the retailer chooses to substitute product B with product A, which brings more profits than the salvage value of product A. If this assumption is satisfied, the retailer will be willing to adopt the one-way substitution strategy.

Assumption 4: $r p_1 \geq p_2$. It shows that the substitute price should be greater than or equal to the price of low-end products, which is a requirement for maintaining market stability. $r p_1 \leq p_1$. The substitute price should be less than or equal to the price of high-end products, and then the demand for high-end products caused by the shortage of low-end products may be increased.

Assumption 5: $\beta = p_1 (1 - r)/(p_1 - p_2)$. $\beta(r)$ is a monotonic subtractive function of $r$. The lower the $r$ is, the greater the number of consumers who ordered product B and would be willing to purchase product A when product B is out of stock.

Assumption 6: the supplier has enough products for the seller to order.

3. The Model

Based on the hypotheses above, we construct a newsboy model with a one-way substitution relationship between two products. The model’s objective is to maximize the expected revenues by jointly optimizing the order quantity and the substitute discount. In this model, the retailer’s revenues are mainly divided into the following five situations:

(i) Product A is not out of stock, and product B is not out of stock ($x \leq q_1, y \leq q_2$):

$$\pi_1 = p_1 x - c_1 q_1 + v_1 (q_1 - x) + p_2 y - c_2 q_2 + v_2 (q_2 - y).$$

(ii) Product A is out of stock, and product B is not out of stock ($x > q_1, y \leq q_2$):

$$\pi_2 = p_1 q_1 - v_1 q_1 - s_1 (x - q_1) + p_2 y - c_2 q_2 + v_2 (q_2 - y).$$

(iii) Product A is out of stock, and product B is out of stock ($x > q_1, y > q_2$):

$$\pi_3 = p_1 q_1 - c_1 q_1 - s_1 (x - q_1) + p_2 q_2 - c_2 q_2 - s_2 (y - q_2).$$

(iv) Product A is not out of stock, product B is out of stock, and the remaining quantity of product A is more than or equal to the quantity that needs to be substituted ($x \leq q_1, y < q_2$):

$$\pi_4 = p_1 x - c_1 q_1 + p_2 q_2 - c_2 q_2 + r p_1 \beta(y - q_2) + v_1 (q_1 - x - \beta(y - q_2)) - s_2 (1 - \beta)(y - q_2).$$

(v) Product A is not out of stock, product B is out of stock, and the remaining quantity of product A is

---

**Table 1: Notations.**

| Parameter | Meaning |
|-----------|---------|
| $x$       | Market demand for product A |
| $y$       | Market demand for product B |
| $q_1$     | Order quantity of product A |
| $q_2$     | Order quantity of product B |
| $p_1$     | Unit retail price of product A |
| $p_2$     | Unit retail price of product B |
| $c_1$     | Unit wholesale cost of product A |
| $c_2$     | Unit wholesale cost of product B |
| $v_1$     | Unit salvage value of product A |
| $v_2$     | Unit salvage value of product B |
| $s_1$     | Unit understocking cost of product A |
| $s_2$     | Unit understocking cost of product B |
| $r$       | Price discount for substitute products |
| $\beta$   | One-way substitution rate |
| $C$       | Budget constraint |
| $\pi$     | Total profits |
| $E(\pi)$  | Expected profits |
lower than the quantity that needs to be substituted
\( q_1 - \beta (y - q_2) < x \leq q_1, y > q_2 \):

\[
\pi_5 = p_1 x - c_1q_1 + p_2 q_2 - c_2 q_2 + r p_1 (q_1 - x) - s_2 (x + y - q_1 - q_2).
\] (5)

Therefore, the total expected profits for the two products are as follows:

\[
E(\pi) = E(\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5) = \int_0^{q_1} \int_0^{\pi} \pi_1 f_1(x) f_2(y) dx dy + \int_0^{\pi} \pi_2 f_1(x) f_2(y) dx dy + \int_0^{\pi} \pi_3 f_1(x) f_2(y) dx dy + \int_0^{\pi} \pi_4 f_1(x) f_2(y) dx dy + \int_0^{\pi} \pi_5 f_1(x) f_2(y) dx dy.
\] (6)

Then, the model under the stochastic demand of the retailer can be described as follows:

\[
R = \max E(\pi),
\]

s.t.

\[
\begin{align*}
& c_1 q_1 + c_2 q_2 \leq C \\
& \max \left( \frac{p_2}{p_1} v_1 - s_2 \right) \leq \beta, \leq 1, \\
& q_1 \geq 0.
\end{align*}
\] (7)

The first-order and second-order derivatives of \( E(\pi) \) with respect to \( r \) are as follows:

\[
\frac{\partial E(\pi)}{\partial r} = \int_0^{q_1} \int_0^{\pi} p_1 (q_1 - x) f_1(x) f_2(y) dx dy + \int_0^{\pi} \left( p_1 \beta + r p_1 \beta y + s_2 \beta y - \nu_1 \beta y \right) f_1(x) f_2(y) dx dy,
\] (8)

\[
\frac{\partial^2 E(\pi)}{\partial r^2} = \left[ \beta \left( y - q_2 \right) \right] \left( r p_1 + s_2 - \nu_1 \right) f_1(q_1 - \beta y + \beta q_2) + \int_0^{\pi} \left[ f_2(y) dy + 2 p_1 \beta y \left( y - q_2 \right) \right] \left( q_1 - \beta y + \beta q_2 \right) f_1(x) f_2(y) dx dy.
\] (9)

Because \( \beta' = -p_1 / (p_1 - p_2) < 0, \partial E^2(\pi)/\partial r^2 < 0 \). When \( q_1 \) and \( q_2 \) are given, the expected profit function \( E(\pi) \) is a strictly concave function with respect to \( r \). That is, when the order quantities of two products are known, an optimal discount can be obtained to maximize the retailer’s revenue. According to the optimal condition of the first-order derivative, the optimal discount \( r^* \) satisfies \( \partial E(\pi)/\partial r^* = 0 \).

The first-order and second-order partial derivatives of \( E(\pi) \) with respect to \( q_1 \) and \( q_2 \) are as follows:

\[
\frac{\partial E(\pi)}{\partial q_1} = \left( p_1 + s_1 - c_1 \right) \left( p_1 + s_1 - v_1 \right) - \int_0^{q_1} \int_0^{\pi} r p_1 + s_2 - \nu_1 \right) f_1(x) f_2(y) dx dy,
\] (10)

\[
\frac{\partial E(\pi)}{\partial q_2} = \left( p_2 + s_2 - c_2 \right) \left( p_2 + s_2 - v_2 \right) - \int_0^{q_2} \int_0^{\pi} r p_1 + s_2 - \nu_1 \right) f_1(x) f_2(y) dx dy,
\] (11)

\[
\frac{\partial^2 E(\pi)}{\partial q_1^2} = f_1(q_1) \int_0^{\pi} \left( r p_1 + s_2 - v_1 \right) f_2(y) dy - f_1(q_1) \left( p_1 + s_1 - v_1 \right) - f_1(q_1 - \beta y + \beta q_2) \int_0^{\pi} \left( r p_1 + s_2 - v_1 \right) f_2(y) dy,
\] (12)

From the first-order and second-order partial derivatives of \( q_1 \) and \( q_2 \), we know that \( a < 0, ab - cd > 0 \), and so the Hessian matrix is negative definite, and \( E(\pi) \) is a concave function. Therefore, the objective function has a pair of optimal solutions to get the maximal value. Under the premise of giving a substitute discount, the optimal order quantities for the two products are \( q_1^* \) and \( q_2^* \). For the sake of simplicity in the following text, we make the following annotations:
\begin{align}
H_1(q_1, q_2) &= \int_{q_1-\beta}^{q_1} f_1(x) f_2(y) dx dy, \\
H_2(q_1, q_2) &= \int_{q_1-\beta}^{q_1} f_1(x) f_2(y) dx dy, \\
H_3(q_1, q_2) &= \int_{q_1-\beta}^{q_1} x f_1(x) f_2(y) dx dy, \\
H_4(q_1, q_2) &= \int_{q_1-\beta}^{q_1} y f_1(x) f_2(y) dx dy.
\end{align}

(13)

Let equation (8) = 0, equation (10) = 0, and equation (11) = 0. Then, it can be seen that there is a certain functional relationship between \( r^* \), \( q_1^* \), and \( q_2^* \). The above three formulas are combined into the following equations. The solution of the equations is the optimal solution of the model, which only considers the maximization of the expected revenues and does not consider a budget constraint:

\[
\begin{bmatrix}
F_1(q_1) - H_1(q_1, q_2) & \frac{r P_1 + s_2 - v_1}{p_1 + s_1 - v_1} = \frac{p_1 + s_1 - c_1}{p_1 + s_1 - v_1} \\
F_2(q_2) + H_2(q_1, q_2) & \frac{r P_1 + s_2 - v_1}{p_2 + s_2} = \frac{p_2 + s_2 - c_2}{p_2 + s_2 - v_2}
\end{bmatrix}
\]

\[
= \frac{p_1}{p_1 + r P_1 + s_2 - v_1} H_3(q_1, q_2) + \frac{q_2 - H_2(q_1, q_2)}{q_2 - v_1}
\]

(14)

We discuss the optimization problem in the following two cases.

Case 1. When \( \lambda^* = 0 \), condition 1 and condition 3 are set up. From condition 4, we have \( C - c_1 q_1 - c_2 q_2 \geq 0 \), which means that the budget constraints are loose and large, and the budget has no limiting effect on procurement, and condition 4 is also set up. \( \lambda^* = 0 \) can be approximated as having no budget constraint, and the retailer’s optimal decision in this case corresponds to the solution \((q_1^*, q_2^*, r^*)\) of equation (14).

Case 2. When \( \lambda^* > 0 \), condition 1 is set up. From condition 3 and condition 4, we have \( C - c_1 q_1 - c_2 q_2 = 0 \). \( \lambda^* > 0 \) means that the budget constraints are tight and small and that procurement is limited by the budget. The condition \( C - c_1 q_1 - c_2 q_2 = 0 \) is added to equation (16) to form the following equation (17). At this point, the optimal decision corresponds to the solution \((q_1^*, q_2^*, r^*, \lambda^*)\) of the following equation:

\[
\begin{bmatrix}
F_1(q_1^*) - H_1(q_1^*, q_2^*) & \frac{r^* P_1 + s_2 - v_1}{p_1 + s_1 - v_1} = \frac{p_1 + s_1 - c_1}{p_1 + s_1 - v_1} \\
F_2(q_2^*) + H_2(q_1^*, q_2^*) & \frac{r^* P_1 + s_2 - v_1}{p_2 + s_2 - v_2} = \frac{p_2 + s_2 - c_2}{p_2 + s_2 - v_2}
\end{bmatrix}
\]

\[
= \frac{p_1}{p_1 + r^* P_1 + s_2 - v_1} H_3(q_1^*, q_2^*) + \frac{q_2 - H_2(q_1^*, q_2^*)}{q_2 - v_1}
\]

(17)

4. Numerical Study

In this section, we assumed that a manufacturer of memory products sells two USB flash disks with different capacities, product A has the high capacity, and product B has the low capacity. We use subscripts 1 and 2 for products A and B, respectively. The stochastic demand of two products follows an exponential distribution with parameters \( \lambda_1 \) and \( \lambda_2 \), respectively. The probability density functions are \( f_1(x) = \begin{cases} \lambda_1 e^{-\lambda_1 x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \) and \( f_2(y) = \begin{cases} \lambda_2 e^{-\lambda_2 y}, & y \geq 0 \\ 0, & y < 0 \end{cases} \). In this
paper, we use MATLAB to solve the equations and analyze the influence of each parameter on the optimal order quantity and the optimal substitute discount. While analyzing the impacts of the budget constraints on the optimal decisions, the values of other parameters are $p_1 = 40, p_2 = 25, c_1 = 23, c_2 = 13, s_1 = 2, s_2 = 1, v_1 = 20,$ and $v_2 = 12$. $D_1$ and $D_2$ follow an exponential distribution with parameters $\lambda_1 = 0.05$ and $\lambda_2 = 0.02$, respectively. The effects of different budgets on the optimal order quantity and the optimal substitute discount are shown in the figures.

As shown in Figure 1 and Figure 2, with the increase in the budget, the optimal order quantities of product 1 and product 2 both show an upward trend, and the optimal substitution discount shows a downward trend. In this example, when analyzing the effects of the unit retail price, unit wholesale cost, unit understocking cost, and unit salvage value on the optimal quantity and the substitute discount, we set the budget as 700. That is, the procurement of two products is subject to tight budget constraints. Under certain budget constraints, the effects of the unit price, unit wholesale cost, unit understocking cost, and unit salvage value on the optimal order quantity and the substitute discount are shown in the following tables.

The data in Table 2 are $(q_1^*, q_2^*, r^*)$ under different price combinations, the values of the other parameters are $C = 700, c_1 = 23, c_2 = 13, s_1 = 2, s_2 = 1, v_1 = 20,$ and $v_2 = 12$, and $D_1$ and $D_2$ follow an exponential distribution with parameters $\lambda_1 = 0.05$ and $\lambda_2 = 0.02$, respectively. As shown in Table 2, when the price of product 2 is fixed under certain budget constraints, with the increase in the price of product 1, both the order quantity of product 1 and the substitute discount increase, while the order quantity of product 2 decreases. When the price of product 1 is fixed at a lower value under certain budget constraints, with the increase in the price of product 2, both the order quantity of product 1 and the substitute discount decrease, while the order quantity of product 2 increases. When it is fixed at a large value, with the increase in the price of product 2, the order quantity of product 1 increases at first and then decreases, the order quantity of product 2 decreases at first and then increases, and the substitute discount decreases.

The data in Table 3 are $(q_1^*, q_2^*, r^*)$ under different cost combinations, the values of other parameters are $C = 700, p_1 = 40, p_2 = 25, s_1 = 2, s_2 = 1, v_1 = 20,$ and $v_2 = 12$, and $D_1$ and $D_2$ follow an exponential distribution with parameters $\lambda_1 = 0.05$ and $\lambda_2 = 0.02$, respectively. As shown in Table 3, when the wholesale cost of product 2 is fixed under certain budget constraints, with the increase in the wholesale cost of product 1, both the order quantity of product 2 and the substitute discount increase, while the order quantity of product 1 decreases. When the wholesale cost of product 1 is fixed under certain budget constraints, with the increase in the cost of product 2, both the order quantity of product 2 and the substitute discount decrease, and the order quantity of product 1 increases. This shows that the wholesale cost of products is inversely proportional to its own order quantity and proportional to the order quantity of another product.

| $p_1$ | $p_2$ | $C$ |
|-------|-------|-----|
| 35    | 20    | (12.36, 31.98, 0.956) |
| 40    | 25    | (12.29, 32.11, 0.949) |
| 45    | 30    | (12.11, 32.42, 0.944) |

The data in Table 4 are $(q_1^*, q_2^*, r^*)$ under different understocking cost combinations, the values of other parameters are $C = 700, p_1 = 40, p_2 = 25, c_1 = 23, c_2 = 13, v_1 = 20,$ and $v_2 = 12$, and $D_1$ and $D_2$ follow an exponential distribution with parameters $\lambda_1 = 0.05$ and $\lambda_2 = 0.02$, respectively. As shown in Table 4, when the understocking cost of product 2 is fixed under certain budget constraints, with the increase in the understocking cost of product 1, both the order quantity of product 2 and the substitute discount
Table 3: The effects of the unit wholesale cost on the optimal order quantity and the substitute discount.

| $c_1$ | $c_2$ |
|-------|-------|
| 11    |       |
| 13    |       |
| 15    |       |
| (13.92, 37.06, 0.957) | (14.15, 33.57, 0.956) |
| (12.26, 38.01, 0.962) | (12.48, 34.41, 0.961) |
| (10.77, 39.16, 0.966) | (10.98, 35.46, 0.965) |

Table 4: The effects of the unit understocking cost on the optimal order quantity and the substitute discount.

| $s_1$ | $s_2$ |
|-------|-------|
| 1     |       |
| 2     |       |
| 3     |       |
| (12.72, 31.33, 0.9604) | (12.78, 31.23, 0.9602) |
| (12.75, 31.30, 0.96035) | (12.80, 31.21, 0.9601) |
| (12.76, 31.27, 0.9603) | (12.83, 31.14, 0.960) |

Table 5: The effects of the unit salvage value on the optimal order quantity and the substitute discount.

| $v_1$ | $v_2$ |
|-------|-------|
| 8     |       |
| 12    |       |
| 16    |       |
| (12.68, 31.41, 0.960) | (10.95, 34.48, 0.965) |
| (14.58, 28.04, 0.956) | (12.73, 31.33, 0.960) |
| (16.96, 23.93, 0.951) | (15.05, 27.22, 0.955) |

5. Conclusion

In this paper, we mainly consider a budget-constrained retailer who sells two kinds of products, where high-end products can substitute for low-end products when the low-end products are out of stock. In addition, the retailer may sell high-end products at a discounted price to customers who ordered low-end products. We construct a one-way substitution newsboy model with budget constraints and study how retailers determine the optimal order quantity and the substitute discount. Through the optimization of the model, we obtain the necessary conditions for the existence of the optimal solution. Finally, we use MATLAB to carry out a numerical study and find the optimal solutions under different conditions. We also analyze the effects of budget constraints, the unit price, unit wholesale cost, unit understocking cost, and unit salvage value on the optimal order quantity and the substitute discount. We have the following conclusions: first, as the budget increases, the budget constraints imposed on the retailer are increasingly more relaxed, the optimal order quantities of two products gradually increase, and the optimal substitute discount gradually decreases. Second, when the budget constraints are fixed, the variation trend in the optimal order quantity of two products about the important parameters is opposite. That is, the order quantity of one product increases, and the order quantity of the other product decreases. Finally, the substitute discount is proportional to the price and the wholesale cost of the high-end brand product and is inversely proportional to its salvage value and understocking cost. In addition, it is proportional to the salvage value of the low-end brand product and inversely proportional to its price, understocking cost, and wholesale cost. In real life, many retailers have "budget shortage" problems. To maximize their revenues, they must use the limited funds to make the optimal decisions. This paper has a certain guiding significance for the decision-making of retailers who have limited funds and simultaneously sell two products with one-way substitution relationship.

This paper studies the single-period ordering model of two products with a one-way substitution relationship under budget constraints. In real life, the retailer may face the multiple-cycle ordering decision and multiple products with one-way substitution relationship. In future studies, we can extend the model to a multiple-cycle ordering model with multiple constraints and consider a variety of one-to-one substitute products. In the case when low-end products are out of stock, the substitute rate of high-end products to low-end products is only considered as the price concession rate provided by the retailer in this paper. In fact, the relationship between the substitute price and the substitute rate needs further exploration.

Data Availability

The code to solve the problem in the numerical study section is available from the corresponding author upon request.
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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