Renormalization group approach of itinerant electron systems near the Lifshitz point

C.P. Moca, I. Tifrea and M. Crisan

Department of Theoretical Physics
University of Cluj, 3400 Cluj, Romania

Abstract

Using the renormalization group approach proposed by Millis for the itinerant electron systems we calculated the specific heat coefficient $\gamma(T)$ for the magnetic fluctuations with susceptibility $\chi^{-1} \sim |\delta + \omega|^\alpha + f(q)$ near the Lifshitz point. The constant value obtained for $\alpha = 4/5$ and the logarithmic temperature dependence, specific for the non-Fermi behavior, have been obtained in agreement with the experimental data.
The occurrence of the non-Fermi behavior in the systems of fermions coupled to a critical fluctuations mode has been suggested in connection with the neutron experiments\textsuperscript{1,2} and studied in the framework of many body theory\textsuperscript{3,4} in the case of two-dimensional (2D) and three dimensional (3D) models.

Recent experiments on the heavy fermions systems showed also a non-Fermi behavior of these materials at low temperatures and it was associated with the proximity of quantum critical point (QCP). The most studied example\textsuperscript{5} is CeCu$_{6-x}$Au$_x$ where at the QCP $x = 0.1$ the resistivity increases linearly with temperature $T$ over a wide range of $T$ and the specific heat $C(T)$ is proportional to $T \ln T$. This behavior has been explained\textsuperscript{6} by the coupling of 3D fermionic excitations to the 2D critical ferromagnetic fluctuations near the QCP.

The inelastic neutron scattering measurements performed on this materials\textsuperscript{7,8} showed the following new points in the behavior of this material

1. The inelastic neutron scattering data can be fitted with a susceptibility of the form
$$\chi^{-1} = C^{-1}[f(q) + (aT - i\omega)^\alpha]$$
where $\alpha = 4/5$ and not 1 as is predicted by the mean field approximation.

2. The quadratic stiffness vanishes, fact which shows that we are dealing with a quantum Lifshitz point (QLP).

3. The peaks for $x = 0.2$ and $x = 0.3$ can be considered as 2D precursor of 3D order.

4. The scaling analysis showed\textsuperscript{7} that $\gamma(T) = C(T)/T$ has the form
$$\gamma(T) \sim T^{(D-1/2)\alpha/2-1}$$

which for $D = 3$ and $\alpha = 4/5$ gives a temperature independent value.

This analysis has been performed taking as the most important contribution to $\chi$ the form containing $\omega^\alpha$ and the $q$-dependence of the form $f(q) = Dq^2 + Cq^4$ where $\alpha = 2/z$, $z$ being the critical exponent from the dynamical critical phenomena\textsuperscript{9}.

In this paper we will show that using the Hertz\textsuperscript{10} renormalization group method (RNG) extended for $T \neq 0$ by Millis\textsuperscript{11} we can obtain the $\ln T_0/T$ term as a quantum correction to the classical results expressed by Eq. (1). We start from an interacting Fermi system...
and by introducing the Bose field $\Phi(q)$ (associated with the magnetic fluctuations) via the Hubbard-Stratanovich transformations one can integrate out the fermions and expand the effective action up to forth order in $\Phi$ as

$$
S_{eff}[\Phi] = \int_0^1 \frac{d\omega}{2\pi} \int_0^1 \frac{d^2q_{\parallel}}{(2\pi)^2} \int_0^1 \frac{dq_{\perp}}{2\pi} \Phi \left[ |\delta + |\omega||^\alpha + q_{\parallel}^2 + Dq_{\perp}^2 + q_{\perp}^4 \right] \Phi 
+ u \int_0^1 \frac{d\omega}{2\pi} \int_0^1 \frac{d^2q_{\parallel}}{(2\pi)^2} \int_0^1 \frac{dq_{\perp}}{2\pi} (\Phi \Phi)^2
$$

where $u > 0$ is the coupling constant. The scaling variables of the models are $\delta^\alpha$ ($\delta$ is the deviation of the control parameter $x$ from its critical value $x_c$) the stiffness $D$, the temperature and the coupling constant $u$. Using the transforms

$$
\omega' = b^{2/\alpha} \omega \quad q_{\parallel}' = b q_{\parallel} \quad q_{\perp}' = q_{\perp} \sqrt{b} 
\delta' = b^{2/\alpha} \delta \quad D' = bD
$$

and interacting out on the shell $\Lambda \geq q \geq \Lambda/b$ ($b > 1$) near the cut-off $\Lambda$ we obtain the RNG equations. In order to calculate the specific heat we will use the scaling procedure to obtain an equation for the free energy, defined for the free fluctuations as

$$
F = \int_0^1 \frac{dz}{2\pi} \int_0^1 \frac{d^2q_{\parallel}}{(2\pi)^2} \int_0^1 \frac{dq_{\perp}}{2\pi} \coth \frac{z}{2T} \arctan \frac{A \sin \theta}{A \cos \theta + q_{\parallel}^2 + q_{\perp}^2}
$$

where $\theta = \alpha \tan^{-1}(z/\delta)$ and $A^{-\alpha} = (\delta^2 + z^2)^{1/2}$. Following the same procedure as in Ref. 11 we obtain the equations

$$
d \frac{T(b)}{d \ln b} = \frac{2}{\alpha} T(b)
$$

$$
\frac{d u(b)}{d \ln b} = \left( \frac{3}{2} - \frac{2}{\alpha} \right) u(b) - u^2(n + \delta)f_2
$$

$$
\frac{d \delta^\alpha(b)}{d \ln b} = 2\delta^\alpha(b) + 2u(b)(n + 2)f_1
$$

$$
\frac{d D(b)}{d \ln b} = D(b)
$$

$$
\frac{d F(b)}{d \ln b} = \left( \frac{2}{\alpha} + \frac{5}{2} \right) F(b) + f_3
$$
where $n$ is the number of the field components, $f_1 = f_1[T(b), \delta^\alpha(b), D(b)]$, $f_2 = f_2[T(b), \delta^\alpha(b), D(b)]$ and $f_3 = f_3[T(b)]$ are complicated functions but will be approximated as presenting a weak dependence of $\delta^\alpha(b)$ and $D(b)$ for $\delta^\alpha(b), D(b) \ll 1$. The renormalization procedure is stopped at

$$\delta^\alpha(b) = 1$$

(10)

and from Eqs. (5)-(7) we get

$$T(b) = T b^{2/\alpha}$$

(11)

$$u(b) = u b^{3/2 - 2/\alpha}$$

(12)

$$\delta^\alpha(b) = e^{2x} \left[ \delta^\alpha + 2u(n + 2) \int_0^{lnb} dxe^{-x(1/2 + 2/\alpha)} f_1 \left( T e^{2x/\alpha} \right) \right]$$

(13)

These equations will be analyzed in two regimes. First regime defined by

$$T(b) \ll 1$$

(14)

will be called ”quantum regime” and the second defined by

$$T(b) \gg 1$$

(15)

will be called ”classical regime”. Eqs. (5)-(8) have been solved following Ref. 11 in the quantum and classical regimes and the renormalization coupling constant has been obtained as

$$u(b) = u T^{1-3\alpha/4} \frac{T^{3\alpha/4}}{\left[ \bar{\delta}^\alpha + (A - 4B)(n + 2)uT^{1+\alpha/4} \right]^3 / 2}$$

(16)

where $\bar{\delta}^\alpha = \delta^\alpha(\bar{b})$ and $\bar{b}$ is defined by $T(\bar{b}) = 1$ and is $T^{-\alpha/2}$. The coupling constant have to satisfy the condition $u(b) \ll 1$ and this condition is not satisfied if

$$\bar{\delta}^\alpha + (A - 4B)(n + 2)uT^{1+\alpha/4} = 0$$

(17)

If we define the coherence length by $\xi^{-2} \sim \delta$ we get $\xi^{-2} \sim T^{\alpha+1/4}$. 

4
In order to calculate the specific heat and $\gamma = C(T)/T$ we will use Eq. (9) for the free energy $F$. Neglecting in the lower approximation the second term we obtain

$$ F(T) = F(b)b^{-2/\alpha-5/2} \quad (18) $$

The exact solution of Eq. (3) has the form

$$ F(b) = b^{2/\alpha+5/2} \int_{\ln b}^{\ln b}\! dx e^{-2(\alpha^{1/2}+\alpha^{3/4})/T} f_3(T e^{2x/\alpha}) \quad (19) $$

and in order to perform the integral in this expression we take the variable $x$ in the domains

$$ 0 < x < \frac{\alpha}{2} \ln \frac{1}{T} \quad (20) $$

$$ \frac{\alpha}{2} \ln \frac{1}{T} < x < \ln b^* \quad (21) $$

where $b^*$ is defined as in Ref. [11], by $\delta(b^*) = 1$. In the first domain $f_3(T) \simeq C_3 T^2$ and in the second $f_3(T) \approx DT$. Using these approximations we obtain from Eq. (19)

$$ b^{-2/\alpha-5/2} F(b) = \frac{\alpha}{2} T^{1+5\alpha/4} \left[ C_3 \int_T^{T_1} dT_1 T_1^{-5\alpha/4} + D \int_1^{T b^* 2/\alpha} dT_1 T_1^{-1-5\alpha/4} \right] \quad (22) $$

where $T_1 = T \exp [2x/\alpha]$. If we take $\alpha = 4/5$ from Eq. (22) we calculate

$$ F(T) = \frac{2}{5} C_3 T^2 \ln \frac{1}{T} + \frac{2}{5} D T^2 - \frac{2}{5} D T b^*^{-5/2} \quad (23) $$

and

$$ \gamma(T) \simeq \gamma_0 + \bar{\gamma} \ln \frac{1}{T} + O \left( \frac{1}{T^2} \right) \quad (24) $$

a result which shows that the constant value from Ref. [7] is reobtained but the RNG equations give also the second term specific to the non-Fermi behavior.

Recently, Ramazashvili[12] used the same method studying QLP for such a model with $\alpha = 1$. Our results are consistent with the results obtained in Ref. [12] but the specific heat coefficient obtained has a $T^{1/4}$ dependence which is given by the simple gaussian approximation and the value $\alpha = 1$. 

5
REFERENCES

1 P. Benard et al., Phys. Rev. B 47, 15217 (1993)

2 L. Chen and P. Benard, Phys. Rev. B 52, 1152 (1995)

3 M. Crisan and L. Tataru, J. Supercond. 8, 341 (1995); Phys. Rev. B 54, 3597 (1996)

4 Y. M. Vilk and A.-M. S. Tremblay, Phys. Rev. B 49, 13267 (1994); Europhys. Lett. 33, 159 (1996); J. Phys. I (France), 13, 1369 (1997)

5 H. van Loheysen, J. Phys. Cond. Matt. 8, 9689 (1996)

6 A. Rosch et al., Phys. Rev. Lett. 79, 159 (1997)

7 A. Schroder et al., Phys. Rev. Lett. 80, 5623 (1998)

8 O. Stockert et al., Phys. Rev. Lett. 80, 5627 (1998)

9 P. Coleman, cond-mat 9809436. This paper gives an excellent physical picture of the non-Fermi behavior in the weak coupling approach where $z = 2$ and in the strong coupling approach where $\alpha = 4/5$.

10 J. A. Hertz, Phys. Rev. B 14, 1165 (1976)

11 A. J. Millis, Phys. Rev. B 48, 7183 (1993)

12 R. Ramazashvili, cond-mat 9901191