Coulomb drag in phase-coherent mesoscopic structures

— Numerical study of disordered 1D–wires

Niels Asger Mortensen¹ ², Karsten Flensberg², Antti-Pekka Jauho¹

¹ Mikroelektronik Centret, Technical University of Denmark, Ørsteds Plads bld. 345 east, DK-2800 Kgs. Lyngby, Denmark
² Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

Abstract We study Coulomb drag between two parallel disordered mesoscopic 1D–wires. By numerical ensemble averaging we calculate the statistical properties of the transconductance G_{21} including its distribution. For wires with mutually uncorrelated disorder potentials we find that the mean value is finite, but with comparable fluctuations so that sign-reversal is possible. For identical disorder potentials the mean value and the fluctuations are enhanced compared to the case of uncorrelated disorder.

1 Introduction

Current flow in a conductor can through a Coulomb mediated drag-force accelerate charge-carriers in a nearby conductor, thus inducing a drag-current. The effect is active whenever the distance between the two conductors is of the same order as the distance between the charge-carriers – otherwise it is suppressed by screening. In the past years Coulomb drag in extended 2D-systems has been studied extensively and very recently the Thouless energy. We study how drag between disordered mesoscopic 1D–wires give rise to these new interesting phenomena such as large fluctuations and sign reversal of the drag current.

2 Formalism

Consider two 1D–wires of length L (shorter than the phase-breaking length) parallel to each other with a separation d, see Fig. 1. Writing the Laplacian with the help of finite differences the Hamiltonian of the uncoupled mesoscopic 1D–wires is mapped onto a tight-binding model

\[ H \{ n_n' \} = [2t + U_i(n)] \delta_{n_n'} - t \delta_{n_n' \pm 1}, \quad t = \frac{\hbar^2}{2ma^2}, \quad (1) \]

where i = 1, 2 label the two wires and n, n’ = 1, 2, 3, …, N the lattice points. The conducting properties can be obtained from the retarded Green functions of the isolated wires which can be written as N × N matrices:

\[ G_i = (\epsilon_F - H_i - \Sigma_L^i - \Sigma_R^i)^{-1}, \quad \text{where} \quad \Sigma_p^i \quad \text{is the retarded self-energy describing coupling to the lead} \quad p = L, R. \]

Using Kubo formalism we calculate the Coulomb drag to second order in the interaction \( U_{12} \) between the mesoscopic 1D–wires which we assume to be otherwise non-interacting. For kT ≪ \( \epsilon_F \) the dc transconductance \( G_{21} = \partial I_2 / \partial V_1 \) becomes

\[ G_{21} = \frac{e^2}{\hbar} (kT)^2 \frac{\ell^2}{3} \text{Tr} [U_{12} M_1 U_{12} M_2], \quad (2a) \]

where \( U_{12} \) is an N × N coupling matrix representing the interwire Coulomb interaction and \( M_1 \) is an N × N matrix

\[ M_i = \text{Re} \{ A_i^T \otimes [A_i A_i] \} , \quad A_i = i \{ G_i - \Sigma_i \}. \quad (2b) \]

Here, \( A_n n' = \pm \delta_{n_n' \pm 1}/(N - 1) \) and \( \{ X \otimes Y \}_{n_n' } = X_{n'n'} Y_{nn'} \). The Landauer conductance \( G_{ii} = \partial I_i / \partial V_i \) of the individual wires can be expressed in a similar form

\[ G_{ii} = \frac{2e^2}{\hbar} \text{Tr} [\Gamma_L^i \Gamma_R^i \Sigma_i \Sigma_i^i] , \quad \Gamma_p^i = i \{ \Sigma_p^i - \{ \Sigma_p^i \}^\dagger \}. \quad (3) \]

3 Ensemble averaging

The statistical properties of drag can be analyzed by generating an ensemble of different disorder configurations and using Eq. (2) to calculate the drag. For the disorder we use the Anderson model with diagonal disorder where the transport mean free path \( \ell \) can be related to the disorder strength \( W \) by \( \ell = a12(4\ell \epsilon_F - \epsilon_F^2)/W^2 \). We consider two cases: i) both wires being disordered, but with \( U_1 \) and \( U_2 \) fully uncorrelated and ii) both wires being disordered and fully correlated, i.e. \( U_1 = U_2 \). For weak disorder a diagrammatic perturbation expansion for the fluctuations \( G_{21} = G_{21} - \langle G_{21} \rangle \) gives \( \langle [G_{21}]^2 \rangle_\ell \propto 1/k_F \ell \) and it can also be argued that \( \langle [G_{21}]^2 \rangle_\ell = 2 \times \langle [G_{21}]^2 \rangle_{uc} \). Both predictions are valid to lowest order in \( 1/k_F \ell \).

4 Results

We consider quarter-filled bands (\( \epsilon_F = t \)) and wires with \( N = 100 \) lattice points so that \( k_F L = (\pi/3) \times 100 \). The separation is \( k_F d = 1 \) and for simplicity we assume an unscreened coupling of the form \( \{ U_{12} \}_{nn'} = e^2 / (4\pi \epsilon_0 \epsilon_F [(n - n')^2 a^2 + d^2]^{1/2}) \).
We study the de-localized regime $\ell \gg L$ where we as expected [10,11] find that the fluctuations $\delta G_{ii}$ of the Landauer conductance $G_{ii}$ are vanishing and $\langle G_{ii} \rangle \simeq 2e^2/h$. However, for the transconductance $G_{21}$ even weak disorder can have a large effect. The lower left panel of Fig. 2 shows a typical histogram of $G_{21}(\ell)/G_{21}(\infty)$ (where $G_{21}(\infty)$ is the result in the ballistic regime, $U_1 = U_2 = 0$) for $\ell = 36L$. Depending on the disorder configuration $G_{21}(\ell)$ can be either higher or lower than in the ballistic regime. The enhancement occurring for certain disorder configurations can be understood physically as follows. The lack of translational invariance allows forward scattering (transferred momentum $q \simeq 0$), which normally has little effect, to cause transitions between scattering states with opposite directions, thus contributing to the drag. The variance is of the same order as the mean value so that sign reversal for some disorder realizations is possible. The latter is represented by the negative tail in the histogram.

For the same system parameters but now with identical (correlated) disorder potentials we get a very different distribution as seen in the upper left panel of Fig. 2. As predicted we find the mean value is enhanced compared to uncorrelated disorder and also the fluctuations are enhanced. In the right panel of Fig. 2 we show the dependence of the fluctuations on the mean free path $k_F\ell$ which has the expected $1/k_F\ell$ behavior. Comparing the two disorder situations we find numerical support for the predicted relative strength of $\sqrt{2}$.

5 Conclusion
We have numerically studied drag of disordered mesoscopic 1D-wires in the de-localized regime $\ell \gg L$. Our results illustrate how the statistics of the transconductance depend strongly on disorder and we find that even weak disorder can give rise to fluctuations of the same order of magnitude as the transconductance for the ballistic case. This implies that the direction of drag depends on the disorder configuration and that for a given system the sign of the drag current will be arbitrary. Our results also confirm the for 2D extended systems recently predicted enhancement of the mean value for correlated disorder compared to uncorrelated disorder. In addition we have also found a corresponding enhancement of the fluctuations by a factor of $\sqrt{2}$ compared to uncorrelated disorder.

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