Wind induced vibration of the tower of offshore wind turbine

Saba Rahman¹, A K Jain¹, S D Bharti² and T K Datta²

¹Department of Civil Engineering, Indian Institute of Technology, Hauz Khas 110016, Delhi, India
²Department of Civil Engineering, Malaviya National Institute of Technology, Malaviya Nagar, Jaipur 302017, Rajasthan, India

Abstract. The floating offshore wind turbine (FOWT) may have significant displacement and acceleration of the hub caused due to the wind-wave interaction. In this paper, a simplified cascaded analysis of the FOWT is presented in order to obtain a preliminary estimate of the responses of the hub using an iterative frequency domain technique. In order to carry out the analysis, the masses of the nacelle and rotor are lumped at the top of the hub, which is discretized into elements. At the base of the hub, two interface degrees of freedom consisting of a rotation and a translation is considered, which connect the two subsystems, namely, the hub and the floating body (spar). Both the wave and fluctuating components of the wind are considered as a random process represented by their power spectral density functions. The two subsystems are analyzed separately, and the interaction effect is incorporated through an iterative process. Using the above method of analysis, a FOWT having a 90m tall tower is analyzed under two random sea states, namely, 12m-18s and 5m-10s. Two mean wind velocities, namely, 12m/s and 20m/s are taken at the reference height of 10m. The results of the study indicate that significant absolute peak displacement and acceleration at the top of the hub may take place, requiring due attention to the safety of the design of FOWT.

1. Introduction

Wind turbines are used to extract renewable energy from the wind. The turbines were mostly installed on the land in far off open spaces where unobstructed wind flow was expected. However, the rapid increase of the land-based wind turbines encountered several obstacles, such as the substantial environmental impact on people living in the vicinity of wind turbines, the limitation of being high-capacity and making large wind farms. Therefore, the attention turned on to the offshore sites, a less restrictive installation place. The wind turbines, installed in offshore sites, earned the name offshore wind turbines and are classified mostly into two categories, fixed and floating type according to how the wind turbine tower is supported. In recent years, a floating offshore wind turbine has been increasingly regarded as an attractive option to produce wind energy because of the available high wind in deep seas. However, the installation of floating wind turbines in deep waters is faced with several difficulties like the economic viability, higher maintenance, carrying the electricity over a long distance. Being able to deploy wind turbines in deep water will be crucial to determine the scale of the industry within regions where the maritime continental shelf is steep [1-3]. Different from most conventional offshore floating structures, floating wind turbines are relatively small bodies which can exhibit stronger non-linearities in their dynamic behavior associated with strong coupling between aerodynamics and hydrodynamics. The floating-type wind turbine is under the concept design stage?
because there are several issues related to core technology are not yet fully resolved [4]. In particular, the maintenance of dynamic stability against the irregular wind, wave and current loads is a challenging problem [5-7]. In order to secure the dynamic stability of floating type offshore wind turbines, the rotational oscillation control is critical [8-10]. In the offshore practice, it can be accomplished by the substructure, mooring lines, or tension legs and anchors. The common types of generating the righting moment or draft control are submerged TLP (tension-leg platform), spar, and barge-type. In the case of a spar-type floating wind turbine, the buoyancy force produced by the substructure supports the whole offshore wind turbine and the tension of mooring lines offers the station keeping. The pitch and roll stability are primarily maintained by the pitch and roll stiffness of the spar-type floating substructure [11,12] and the relative distance between the centers of gravity and buoyancy [13]. In addition, it could be improved further by passive control and active control devices using water ballast [14-16]. Wind and wave excitations induce both rigid-body degrees of freedom of floating substructure and tower displacement [17].

A complete analysis of the dynamics of the offshore wind turbine is computationally intensive. It requires the 3D modeling of the substructure, mooring lines/tension leg, anchor and the tower (hub). Usually, the rotor and nacelle are not modeled, but their masses are taken into consideration. The wind and wave interaction effects are included in the analysis, along with the different kinds of non-linearities involved in the problem. Further, the time domain solution using the simulation procedure to consider the effect of both the fluctuating component of the wind and the hydrodynamics arising due to water particle kinematics involves extensive numerical computations. In order to assess a preliminary estimate of the performance of the wind turbine, simpler modeling may be adopted, which might give an insight into the interaction effects, and the contribution of the fluctuating component of the wind on the responses of the hub.

With this background in view, the present study is undertaken. A simplified 2D model of an offshore wind turbine is analyzed here, with less computational effort, in order to estimate the spar and hub responses. The turbine is mounted on a spar platform in a water depth of 80m. A cascaded analysis, in which the hub and the spar are analyzed separately, is adopted with the interaction effect between the two being included iteratively. The analysis is performed in the frequency domain using the wind and wave forces to be represented by the power spectral density functions. In order to carry out the frequency domain spectral analysis, suitable linearization in the drag and restoring forces are made. An offshore wind turbine having a hub height of 90m and a rotor diameter of 60m under the action of two wind-driven sea states is taken as an illustrative problem.

2. Theory
The 2D model of the offshore wind turbine is shown in figure 1. The mathematical model used for the analysis is shown in figure 2. The lumped mass stick model represents the hub. The rotor and nacelle masses are lumped at the top node. At the base node of the hub, both rotational and translational degrees of freedom are considered while other nodes of the hub have only a transitional degree of freedom. The spar is modeled as a two degree of freedom model with pitch and surge defined at the center of the mass of the spar. The degrees of freedom shown at the base of the hub is the interface degree of freedom between the hub and spar. The effects at the two degrees of freedom at the center of the mass of spar are transformed into the interface degree of freedom for incorporating the interaction effects between the two systems. The two systems are analyzed separately, and the interaction effect is considered through the iterative process. The equation of the motion for the two systems is written as (figure 2).

\[
M \ddot{u} + C \dot{u} + K u = F(t); \text{ (For the hub)} \tag{1}
\]

In which M, C and K are the mass, damping and stiffness matrix of size 11x11; \( \dot{u} \) is the corresponding vector of the displacement of the hub and \( F(t) \) is the fluctuating component of the along-wind force. The right-hand side of equation (1) is linear as the second-order term of the fluctuating component of the wind is neglected. The equation is given as
\[ M \ddot{z} + C(t) + R(t) = P(t); \quad \text{(For the spar)} \]  

(2)

In which \( M \) is 2x2 matrix; \( z \) and \( P(t) \) are vectors of size 2. \( R(t) \) and \( C(t) \) are displacement and velocity-dependent restoring and damping forces of size 2.

**Figure 1.** Schematic diagram of the offshore wind turbine.

In a time-domain solution, the non-linearities arising due to dependency of the of \( R(t) \) on the surge and pitch motion, as shown in figure 3, can be handled using the incremental dynamic equation of motion. Similarly, drag non-linearity can be introduced in the incremental dynamic equation of motion. Since a frequency domain, spectral analysis is performed here, both drag and restoring forces are linearized. The non-linear drag force in Morrison’s equation is linearization by using the following assumption.

\[ C(t) = C(\dot{V} - \dot{Z})|\dot{V} - \dot{Z}| \approx C\dot{V}|\dot{V}| \approx \bar{C}\dot{V} \]  

(3)

In which, \( \bar{C} = C\sigma \dot{V} \); \( \dot{V} \) is the water particle velocity vector; \( \dot{Z} \) is the structural velocity vector. The non-linear restoring force is linearized by considering the initial stiffness of the force-displacement relationship shown in figure 3. Thus, the linearized version of equation (2) is given by

\[ \bar{M}\ddot{z} + \bar{C}\dot{z} + \bar{R}_0(z) = \bar{P}(t) \]  

(4)

In which \( \bar{C} \) and \( \bar{R}_0 \) are 2x2 matrices. Further, it is assumed that pitch and surge are uncoupled. Resulting in equation (4) to be a set of uncoupled equations of motion. In order to reduce the computational effort, only the fundamental mode is assumed to take part in the wind induced vibration of the hub. With \( \phi \) as the fundamental mode shape of the fixed base hub, equation (1) is transformed into

\[ \bar{m}\ddot{z} + \bar{c}\dot{z} + \bar{k}z = \bar{F} \]  

(5)
In which $\bar{m}$, $\bar{c}$ and $\bar{k}$ are 3x3 matrices; $\bar{F}$ and $\bar{z}$ are vectors of 3x1 with $z^T = [q_1 \quad \bar{r}_s \quad \bar{r}_\theta]^T$; $q_1$ is the first generalized displacement of the fixed base hub; $\bar{r}_s$ and $\bar{r}_\theta$ are translational and rotational motions at the interface node. They are transformed degrees of freedom from the surge $(r_s)$ and pitch $(r_\theta)$ degrees of freedom at the center of the mass of the spar.

The solution process starts by assuming the interaction forces to be zero to find the PSDFs of the responses corresponding to the hub degrees of freedom as

$$S_{zz} = H_z(\omega)S_{FF}H_z^T(\omega)$$  \hspace{1cm} (6)

In which $H_z(\omega)$ is the frequency response function matrix and $S_{FF}$ is the PSDF matrix of wind forces with $S_{zz}$ known, $S_{rs}$ and $S_{r\theta}$ are also known. Next is the spar motions $z^T = [r_s \quad r_\theta]^T$ are solved from

$$S_{zz} = H_z(\omega)S_{PP}H_z^T(\omega)$$  \hspace{1cm} (7)

Since there is no coupling between $r_s$ and $r_\theta$, $S_{rs}$ and $S_{r\theta}$ are determined separately as

$$S_{rs} = (\text{TRF}_s)^2S_w + \left|H_{rs}(\omega)\right|^2S_{r_sT}$$  \hspace{1cm} (8)

$$S_{r\theta} = (\text{TRF}_\theta)^2S_w + \left|H_{r\theta}(\omega)\right|^2S_{r_\theta T}$$  \hspace{1cm} (9)

In which $S_{rs}$ and $S_{r\theta}$ are the PSDFs of the motion of the spar in the $s$ and $\theta$ degree of freedom; TRF$_s$ and TRF$_\theta$ are pseudo transfer function of surge and pitch for the wave forces; $H_{rs}$ and $H_{r\theta}$ are the frequency response function of the spar corresponding to $r_s$ and $r_\theta$ degree of freedom; $S_{r_sT}$ and $S_{r_\theta T}$ are the transformed $S_{rs}$ and $S_{r\theta}$ to the $r_s$ and $r_\theta$ coordinates; and $S_w$ is the P-M spectrum.

Once the $S_{rs}$ and $S_{r\theta}$ are known from equations (8 and 9). They are transformed into $S_{rsT}$ and $S_{r_\theta T}$ and equation (6) is solved to find $S_{zz}$. The interaction is continued until the convergence is achieved in the mean square sense.

![Figure 3](image)

**Figure 3.** Variation of restoring forces with the motion of the spar (a) surge, (b) pitch.

### 3. Numerical example

As an illustrative example problem, the 2D model of a spar platform forming the base for a hub of an offshore turbine in a water depth of 80m is considered (figure 1). The properties of the different components used for the analysis are shown in table 1. The wind-driven P-M spectrum is used to represent the random wave. Similarly, Kaimal’s spectrum is used to represent the fluctuating component of the along-wind velocity modeled as a random process. Expressions for both spectra are given below. The logarithmic law is used to define the variation of the mean wind speed along with the height of the hub. The expression for P-M spectrum [18] is given as:
\[ S_w = \alpha g^2 \omega^{-5} \exp \left[ -\beta \left( \frac{g}{\omega U_{19.5}} \right)^4 \right] \]  

(10)

In which \( \alpha \) and \( \beta \) are two constants, \( \beta \) is taken as 0.74. The value of \( \alpha \) is taken such that it is consistent with the significant wave height and predominant wave period. The following empirical equation is used to obtain the value of \( \alpha \)

\[ \alpha = 5 \left( \frac{\omega_p^2 \sigma_c}{g} \right) \]  

(11)

In which \( \sigma_c^2 \) is the variance of the surface wave. \( \sigma_c \) is related to the significant wave height as \( p \sigma_c \); in which \( p \) is the peak factor given by Davenport [19]. By substituting \( \frac{\omega_p U_{19.5}}{g} = 0.879 \) in equation (10), the following result is obtained

\[ S_w = \alpha g^2 \omega^{-5} \exp \left[ -\frac{5}{4} \left( \frac{\omega}{\omega_p} \right)^{-4} \right] \]  

(12)

The zero-moment of the equation (12) gives the variance of the surface wave elevation as

\[ \sigma_c^2 = \int_0^\infty S_w d\omega = \frac{2.74 \times 10^{-3} U_{19.5}^4}{g^2} \]  

(13)

With the significant wave height of a sea state known, \( \sigma_c \) can be calculated as described above. Equation (13) is then used to find \( U_{19.5} \) corresponding to the significant wave height of the sea state and hence, \( U_{10} \) can be calculated. The Kaimal’s normalized power spectral density [20] for wind in non-dimensional form is given as

\[ f \frac{S_k(f)}{\sigma_k^2} = \frac{4fL_k/U_{10}}{(1 + 6fL_k/U_{10})^{5/3}} \]  

(14)

In which, \( S_k \) is the velocity component spectrum; \( \sigma_k \) is the standard deviation; \( L_k \) is the velocity component integral scale parameter. \( L_k \) and \( \sigma_k \) depend on the turbulence above the seawater level. Two sea states, namely, 5m-12s and 12m-18s, are considered in the study. The corresponding mean wind speeds at the reference height of 10m are calculated as 12m/s and 20m/s, respectively. The spectra of the wind and wave are shown in figures 4 and 5. The pseudo transfer function of the surge and pitch are determined at the center of mass of the spar. The two degrees of freedom at the center of mass of the spar are transformed to the two degrees of freedoms at the interface node, as shown in figure 2. The natural frequencies of the surge and pitch of spar and the hub are shown in table 2.

| Table 1. Structural properties of the offshore wind turbine. |
|-------------------------------------------------------------|
| Effective diameter of the spar | 12m |
| Mass of the spar | 2250 \times 10^3 \text{kg} |
| Mass moment of inertia of pitch | 1.6 \times 10^9 \text{kgm}^2 |
| Diameter of mooring lines | 0.15m |
| Unstretched length of the mooring | 450m |
| Design draft | 15m |
| Hub height | 90m |
| Base diameter of hub and thickness | 6m, 0.036m |
Top diameter of hub 3m, 0.023m
Rotor diameter 60m
Rotor mass $41 \times 10^3$ kg
Nacelle mass $63 \times 10^3$ kg
Total mass (Rotor + Hub + Nacelle) $430 \times 10^3$ kg
Mass moment of inertia of pitch $2.5 \times 10^9$ kgm$^2$
Elevation of the tower base 25m
Wind drag coefficient 1

Table 2. Natural frequencies of the system.

|       | Surge | Pitch |
|-------|-------|-------|
| Surge | 0.205 rad/s | 0.235 rad/s |
| Hub   | 3.15 rad/s   |       |

The pseudo transfer functions of the pitch and surge are shown in figure 6. It is seen from the figure that the dominant peaks of the transfer function occur at the respective frequencies of the surge and pitch frequencies. The power spectral density function (PSDFs) of the surge and pitch for the two sea states are shown in figures 7 and 8. It is seen from the figures that the dominant peaks of the PSDFs correspond to the predominant wave frequencies. This is the case because surge, pitch and fundamental hub frequencies are away from the wave frequencies of the two sea states. However, significant wind energies exist below 0.6 rad/s are noticed in all PSDFs because of the interaction effect between the wind and waves for the case of the PSDFs for the 12m-18s wave, it is observed that they are categorized by two peaks, one near the natural frequencies of pitch and surge and the other near predominant wave frequencies. However, the peaks at the natural frequencies are much higher as compared to those at the predominant wave frequencies. This is the case because the 18s wave has significant energies between 0.2 rad/s to 0.4 rad/s, as can be seen from figure 9.

Table 3. Maximum values of responses.

| Responses                                      | 5m-10s | 12m-18s |
|------------------------------------------------|--------|---------|
| Absolute peak surge displacement of the base of hub | 1.6m   | 5m      |
| Absolute peak pitch angle of the rotation of the base of the hub | $1.42^\circ$ | $3^\circ$ |
| Absolute peak top displacement of the hub        | 6m     | 10m     |
| Absolute peak top acceleration of the hub        | $2.8m/s^2$ | $8m/s^2$ |
| Peak top elastic displacement of the hub (fluctuating part) | 0.08m  | 0.12m   |
| Maximum base bending moment of the hub (fluctuating part) | 12000kNm | 19000kNm |

The maximum values of the responses of the interest are shown in table 3. It is seen from table 3 that both absolute maximum values of surge and pitch are significant due to the combined effect of fluctuating wind and wave. However, as it is expected, the effect of the fluctuating wind on the surge and pitch motions of the spar is not large; they are predominantly governed by the wave actions. This is apparent from the elastic displacement of the tip of the hub (produced by wind), which is much less than the absolute peak displacement of the hub. Further, the maximum bending moment at the base of the hub due to the fluctuating component of the wind is not large. Figure 9 shows the PSDFs of absolute acceleration of the tip of the hub. The dominant peak of the PSDFs occurs at the predominant wave frequencies. Since the energy contents of the wind are less compared to the wave energy at these frequencies, the peaks are predominantly governed by the wave actions. Further, the resonant component of the response of the hub due to the wind is negligible in comparison to the wave actions. Table 3 also shows the maximum values of the response quantities. It can be interpreted from the table that the absolute maximum top displacement and acceleration of the hub are considerably large at
extreme wave conditions (12m-18s) considered here. Such a large displacement of the tip of the hub will have an adverse effect on the rotational stability of the blades of the turbine. Further, the absolute tip acceleration of the hub is large enough to deteriorate the smooth operation of the rotor. The large angle pitch rotation and the large surge displacement shown in table 3 may cause significant tension fluctuation in the wires. Moreover, the anchor forces may also be very high.

![Figure 4](image_url). P-M spectra (a) 5m-10s, (b) 12m-18s.

![Figure 5](image_url). Kaimal’s wind spectra (a) 5m-10s, (b) 12m-18s.

![Figure 6](image_url). Pseudo TRF (a) surge, (b) pitch.
Figure 7. PSDF of surge (a) 5m-10s, (b) 12m-18s.

Figure 8. PSDF of pitch (a) 5m-10s, (b) 12m-18s.

Figure 9. PSDF of top tip acceleration of hub (a) 5m-10s, (b) 12m-18s.
4. Conclusions
Frequency-domain spectral analysis is carried out to obtain the response quantities of the interest of the spar offshore wind turbine. The responses include the maximum values of the surge and pitch of the spar and the absolute displacement and acceleration of the tip of the hub. The effect of the wind-wave interaction on the response is considered by an interactive technique. Both fluctuating components of wind and wave are modeled as a random process, which is represented by Kaimal’s wind spectrum and wind-driven P-M spectrum. A 90m wind turbine supported by a spar platform in a water depth of 80m is taken as an illustrative problem. Two sea states designated as the operating and extreme conditions are considered in the analysis. The results of the numerical studies lead to the following conclusions:

a) Top displacement and acceleration of the hub are predominantly governed by wave action in this spar.

b) Under the extreme sea state, significant surge and pitch motion of the spar may take place, providing large tip displacement of the hub, even under the operating sea state, the peak tip displacement of the hub may be of concern.

c) The maximum absolute acceleration produced at the top of the hub for both sea states may affect the smooth operation of the rotor.

d) The large fluctuation in tension in the mooring lines produced due to the large surge and pitch motions of the spar could take place, which may lead to the fatigue failure of the mooring lines.

e) Elastic displacement and resulting base moments of the hub due to fluctuating components of the wind are not significant.

f) From the results of the present example problem, it appears that a good estimate of the hub displacement and acceleration may be obtained by considering the analysis of the spar alone under the wave action.

5. References
[1] Roddier D, Cernelli C, Aubault A and Weinstein A 2010 WindFloat: a floating foundation for offshore wind turbines J. Renew. Sustain. Energy 2 324-59
[2] Karmouche S 2016 Tower Design and Analysis for a Small Wind Turbine Report (Morocco: School of Science and Engineering Al-Akhwayn) p 43
[3] Sun C and Jahangiri V 2019 Fatigue damage mitigation of offshore wind turbines under real wind and wave conditions Eng. Struct. 178 472-83
[4] Karimirad M and Moan T 2012 Wave and wind-induced dynamic response of a spar-type offshore wind turbine J. Waterway, Port, Coastal and Ocean Eng. 138 9–20
[5] Xu N and Ishihara T 2014 Prediction of tower loading of floating offshore wind turbine systems in the extreme wind and wave conditions Wind Eng. 38 463-76
[6] Waris M B and Ishihara T 2012 Dynamic response analysis of floating offshore wind turbine with different types of heave plates and mooring systems by using a fully nonlinear model Coupled Syst. Mech. 3 247-68
[7] Taylor J W 2017 Probabilistic forecasting of wind power ramp events using autoregressive logit models Eur. J. Oper. Res 259 703-12
[8] Taylor J and Jeon J 2018 Probabilistic forecasting of wave height for offshore wind turbine maintenance Eur. J. Oper. Res. 267 877–90
[9] Dinwoodie I A, Catterson V M and McMillan D 2013 Wave height forecasting to improve offshore access and maintenance scheduling IEEE PES p 34
[10] Tong K C 1998 Technical and economic aspects of a floating offshore wind farm J. Wind Eng. Ind. Aerodyn. 74–76 399–410
[11] Tran T, Kim D and Song J 2014 Computational fluid dynamic analysis of a floating offshore wind turbine experiencing platform pitching motion Energies 7 5011–26
[12] Liu Y, Li S, Yi Q and Chen D 2017 Wind profiles and wave spectra for potential wind farms in South China Sea. Part II: Wave spectrum model Energies 10 9-24
[13] Yang Z, Lu Q, Yan J, Chen J and Yue Q 2018 Multidisciplinary optimization design for the section layout of umbilicals based on intelligent algorithm J. Offshore Mech. Arct. Eng. 140 101-27
[14] Karimirad M and Moan T 2011 Extreme dynamic structural response analysis of catenary moored spar wind turbine in harsh environmental conditions J. Offshore Mech. Arct. Eng. 133 251–14
[15] Antonutti R, Peyrard C, Johanning L, Incecik A, and Ingram D 2016 The effects of wind-induced inclination on the dynamics of semi-submersible floating wind turbines in the time domain Renew. Energy 88 83–94.
[16] King R 1977 A review of vortex shedding research and its application Ocean Eng. 4 141–71
[17] Liu X, Lu C, Li G, Godbole A and Chen Y 2017 Effects of aerodynamic damping on the tower load of offshore horizontal axis wind turbines Appl. Energy 204 1101–14
[18] Silva M T 2015 Ocean surface wave spectrum p 7
[19] Davenport A G 1961 The spectrum of horizontal gustiness near the ground in high winds Q. J. R. Meteorol. Soc. 87 194–211
[20] Kaimal J C, Wyngaard J C, Izumi Y and Cote O R 1972 Spectral characteristics of surface-layer turbulence Quart. J. R. Met. Soc. 98 563-89