RESEARCH OF STRESS CONCENTRATION AT CLOSELY PLACED HOLES IN WING BEARING AREA IN ANISOTROPIC PLATES

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Abstract

In this article effective approach of the study of high-stress concentration at closely placed holes in wing bearing area (in anisotropic plates) is proposed. It is based on the boundary integral equation method with the additional use of the asymptotic method. The simplicity, precision of the approach and the stability of the solution are illustrated in the calculation of stresses in the plate with a circular hole, an elliptical hole, elongated holes, a plate with two closely spaced elliptical holes

Keywords: stresses; composite materials; stress concentration, boundary integral equations method; plates with holes; wing bearing area

List of Symbols/Acronyms

BIEM – boundary integral equation method;
SSS – stress-strain state;
SCF – stress concentration factors;
LSM – least squares method;
SIF – Stress Intensification Factors

1. INTRODUCTION

High strength properties, low specific weight, resistance to the action of aggressive environments led to the wide use of composite materials in aviation technology. In heavy aviation, auxiliary structures are mostly made from composite materials. In light aviation, especially unmanned 1st and 2nd class - this is a glider design. In mechanical engineering and other industries, composite structural elements weakened by hole systems are also widely used. The assessment of the strength and durability of composite structural elements is based on the analysis of their stress-strain state (SSS) with full consideration of the anisotropy of mechanical and strength characteristics [5]. To study the SSS of isotropic and anisotropic plates with holes, the method of boundary integral equations (BIEM) is widely used [3, 14, 19].

Using this method, stresses near holes of different shapes in composite plates were studied [1, 2, 4, 15, 20, 22]. In composite elements of aviation equipment, a system of holes is often created. In particular, the rigid skin of aircraft has closely spaced holes in the places of its attachment to the power frame of the aircraft. Cladding is made from separate sheets or panels of different types of materials. BIEM also proved to be effective for calculating stresses in such plates taking into account the interaction of holes [6, 7, 21].

When considering closely spaced holes, stress concentration coefficients (SCF) increase and can become infinite [9, 16, 18]. Therefore, direct numerical methods of studying stresses for such openings, which were used in works [2-4], are ineffective. For two circular holes in isotropic plates, asymptotic formulas for the SCF at close-to-zero distances between them have been established [18]. Based on these formulas, [18] proposed an approach in which the structure of the formula for determining the SCF at small distances containing unknown steels is pre-established. These steels were determined based on the calculated SCFs using BIEM for selected distances followed by the method of least-squares. It was shown in [21] that this approach is also effective when considering isotropic plates with holes of a different shape (in particular, elliptical). In this work, a similar approach is proposed for the study of stresses near closely spaced holes in composite plates. To do this, we first proposed an asymptotic formula for stresses near holes in anisotropic plates. The implementation of the approach was carried out with the combined
application of the method of integral equations [10-
13, 18] and the method of least squares. At the
same time, simple relations for determining the
SCF were obtained, which turned out to be
practically accurate for a wide range of distances
between the holes - both infinitely small and
commensurate with the radius of the hole.

2. FORMULATION OF THE PROBLEM

It is considered an orthotropic plate (Fig. 1) with
two identical elliptic holes with a semiaxes, b,
whose centers are at the points (-a,d,0), (a + d,0)
under stretching of the plate in the direction of Oy
axis with tractions p. At close distances between
the holes, the concentration of stresses around the
points (-d, 0) increases quickly, and therefore, for
calculations with controlled accuracy, it is
necessary to increase the number of nodal points.

![Fig. 1. Scheme of loading the plate with a holes](image)

To illustrate the difficulties encountered in the
direct application of BIEM, the studying of the
influence of the number of nodal points N in the
quadrature method on the accuracy of calculations
of maximum stresses on the contour at short distances
between the holes was performed. The stress state
was calculated on the basis of the integral representation
[11-13], which takes into account the symmetry of the
problem about the Oy axis and the integral equations
are written only at the boundary of the right hole.

A detailed study was performed for two composite
materials CF1 (carbon fiber reinforced plastic) and EF
(reinforced epoxy phenolic plastic) [23,24].

Modules of elasticity, Poisson's ratios, shear
modulus for these material given in table1 [23].

| Material | $E_1$ (GPa) | $E_2$ (GPa) | $G_{12}$ (GPa) | $v_{12}$ | $v_{13}$ |
|----------|-------------|-------------|----------------|---------|---------|
| CF1      | 8.62        | 400         | 2.8            | 0.35    | 0.007   |
| EF       | 21          | 32.8        | 8.62           | 0.35    | 0.007   |

The first of these materials belongs to the class of
highly anisotropic, and the second to slightly
anisotropic.

Table 2 shows the values of the maximum hoop
stresses $\sigma_\theta$ divided by $p$ (it are indicated by the
symbol $\sigma_{pmax} = \sigma_{max} / p$), $\sigma_{pmax} = max(\sigma_\theta)$
are written only at the boundary of the right
hole.

| $D$ | $N$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ |
|-----|-----|----------------|----------------|----------------|----------------|----------------|
| 0.001 | 100 | 143.043 | 100.39 | 20.733 | 20.463 | 6.106 |
| 0.01  | 200 | 131.519 | 76.378 | 19.658 | 20.363 | 6.106 |
| 0.1   | 300 | 114.989 | 69.825 | 20.131 | 20.363 | – |
| 0.5   | 400 | 103.956 | 68.249 | 20.312 | 20.363 | – |
| 1.0   | 500 | 96.082 | 67.808 | 20.353 | 20.363 | – |

The performed calculations showed that high
stresses occur only in the vicinity of the semi axes of
the ellipse placed on the Ox axis. In such cases,
the parametric giving of the contour is modified for
improve efficiency BIEM. Let's write the boundary
contour equation in parametric form as $x = \phi(\theta), y =
\psi(\theta)$, where $0 < \theta \leq 2\pi$. In particular, at considering
elliptical holes, giving $\phi = a \cos \theta, \psi = b \sin \theta$ was
used. Modified givings are written as $x = \tilde{\phi}(\tau), y =
\tilde{\psi}(\tau)$ where $\theta = g(\tau), 0 < \tau \leq 2\pi$. The function $g(\tau)$ is
chosen in such a way that it is monotonically increasing
and that the derivative of it was highly anisotropic, and the second to slightly
anisotropic.

| Table 3. The accuracy of calculations of stresses in a
| plate made of material CF1
| $D$ | $N$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ |
|-----|-----|----------------|----------------|----------------|----------------|----------------|
| 0.001 | 100 | 262.941 | 226.578 | 55.201 | 55.352 | 20.996 |
| 0.01  | 200 | 232.226 | 225.046 | 55.189 | 55.313 | 20.996 |
| 0.1   | 300 | 226.869 | 224.424 | 55.256 | 55.312 | – |
| 0.5   | 400 | 225.484 | 224.192 | 55.291 | 55.312 | – |
| 1.0   | 500 | 224.778 | 224.104 | 55.305 | 55.312 | – |

Table 4. The accuracy of calculations of stresses in a
plate made of material CF1

| $D$ | $N$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ | $\sigma_{max}$ |
|-----|-----|----------------|----------------|----------------|----------------|----------------|
| 0.001 | 100 | 132.853 | 14.401 | 19.203 | 5.928 | 6.004 |
| 0.01  | 200 | 106.956 | 14.601 | 19.365 | 6.001 | 6.017 |
| 0.1   | 300 | 84.388 | 13.822 | 19.479 | 6.015 | 6.018 |
| 0.5   | 400 | 71.953 | 13.686 | 19.353 | 6.018 | 6.018 |
| 1.0   | 500 | 64.463 | 13.743 | 18.736 | 19.563 | 6.018 | 6.018 |
The data in the Tables shows that a large number of nodal points must be selected to ensure the controlled accuracy of calculations using the mechanical quadrature method. In particular, at relative distances d/R~0.01, using the direct BIEM method, it is necessary to select up to 500 nodes, and at d/R~0.001 it is necessary to solve systems ~1000-4000 equations.

Tables 1-3 show that nonlinear transformations of the parameter allow reducing the number of nodal points when calculating the maximum stresses with controlled accuracy. It should be noted that a similar method is used in the finite element method, when the element sizes are reduced in regions with increased stress concentration.

Nonlinear parameter transformations significantly increase the accuracy of the calculations.

3. ASYMPTOTIC METHOD OF INVESTIGATING OF STRESSES NEAR CLOSELY SPACED HOLES

The complexity of the study of stress concentration is that with zero distance between the holes, the stresses are infinite. In [16], it is established that the maximum stresses in isotropic plate at d → 0 are of the order \( \sigma_{\text{max}} \sim C/\sqrt{D} \), where C is constant. Assume that in anisotropic plates \( \sigma_{\text{max}} \sim C/D^m \), m is a constant that must be determined.

Let us further introduce the relative hoop stress \( \sigma = \sigma_{\text{max}} / \sigma_0 \), where \( \sigma_0 \) is the maximum stress on the single hole for the chosen load. With small distances between the holes the relative stresses are determined in the form \( \sigma = f(\delta) \) [16], where

\[
f(\delta) = \frac{a_2}{\delta^m} + a_2 + a_3 \delta^m,
\]

where \( \delta = \frac{d}{1+D} \) are constants, \( j = 1,2,3 \).

The constants are determined by least squares method (LSM) with conditions is the minimum.

\[
I = \sum_{j=1}^{M} \left[ f(\delta_j) - \sigma_j \right]^2,
\]

Herein \( \delta_j \) are the sequence the parameter's value \( \delta \), that were selected at interval \( \delta_{\text{min}} < \delta < \delta_{\text{max}} \), \( \sigma_j \) are the maximum relative stress values at a relative distance \( \delta, m = 0.5 \) [18] is obtained for isotropic plates. For anisotropic plates, this constant was obtained by a successive approximations in the least-squares method. The performed calculations showed that based on the criteria (2) are enough precise determined of the constants \( m, a_1 \). Other constants are more precisely obtained by minimization of value,

\[
I_1 = \sum_{j=1}^{M} \left[ f(\delta_j) - \sigma_j \right]^2,
\]

where the limited values are under the sign of the sum.

Let's give the value of stresses at the points of the boundary \( A(a,0), B(0,b) \) of a single elliptical hole under the stretching of the orthotropic plate to infinity, by which the value is determined \( \sigma_1 \) [8]:

\[
\sigma_0(A)/p = 1 + \left( \frac{1}{\beta_1} + \frac{1}{\beta_2} \right) a, \quad \sigma_0(B)/p = \beta_1 \beta_2,
\]

where the plate is stretched in the direction of the \( Ox \) axis with tractions \( p \);

\[
\sigma_0(B)/q = 1 + \left( \beta_1 + \beta_2 \right) a/b, \quad \sigma_0(A)/p = \frac{1}{\beta_1 \beta_2}, \quad (5)
\]

where the plate is stretched in the direction of the \( Ox \) axis with tractions \( q \). Here \( \beta_{1,2} = \text{Im}(s_{1,2}) \).

4. RESULTS OF THE CALCULATIONS

At calculations we accepted \( \delta_{\text{min}} = 0.0004 \), \( \delta_{\text{max}} = 0.5 \). At minimum distances in the calculations, we chose up to 2000 nodal points. The calculated value \( F = \delta^m \sigma_{\text{max}} / \sigma_0 \) for the isotropic plate at the ratio of the semi axis \( b/a=1 \) (circular hole) and \( b/a=1/2 \) depending on \( m, \delta \) is shown in Fig. 2.

![Fig. 2. The relative maximum stress in isotropic plate, 1 - circular holes, 1/2 - elliptical holes](image)

The relative maximum stress which are calculated by BIEM are depicted by circles, those ones which are obtained by the formula (1) are depicted by the curves—after using the least squares method for obtaining the coefficients \( a_1, a_2, a_3 \). The ratio of the semi-axes of the elliptical hole \( b/a \) is indicated near the curves. On the interspace \( \delta_{\text{min}} < \delta < \delta_{\text{max}} \), formula (1) belongs to the class of interpolation, since it is obtained on the basis of almost exact values of stresses on this interspace (depicted in Fig. 2 by circles). In Figure 2 and below, the stresses obtained on this interspace according to formula (1) are depicted by solid lines, and on the interspace \( 0 < \delta < \delta_{\text{min}} \) by dashed lines.

The coefficients \( a_1, a_2, a_3 \) determined by least-squares method are shown in Table 5. The values of these coefficients at \( b/a = 1 \) was obtained in [16] are equal to \( a_1 = 0.7330, a_2 = -0.6497, a_3 = 0.8127 \) (here the difference in the factors in [16] near the coefficients is taken into account).

| Table 5. Table of coefficients of formula (1) for isotropic material |
|-----------------|---|---|---|---|
| \( b/a \)       | \( m \) | \( a_1 \) | \( a_2 \) | \( a_3 \) |
| 1               | 0.5 | 0.734 | -0.717 | 0.732 |
| 1/2             | 0.5 | 0.783 | -2.826 | 0.774 |

We can see that the first coefficient almost coincides with the obtained value by us 0.7338. At \( b/a = 0.5 \), the first coefficient obtained in [16] is equal to \( a_1 = 0.786 \) which also coincides with the obtained value by us 0.7831. The shown in Fig. 2 curves are
close to the obtained stress distribution in [16] by another method. Similar results are obtained for anisotropic materials. In plate made of material CF1 the obtained power factor is equal to $m = 0.6338$. The results of the relative stress calculations are shown in Fig. 3.

The calculated relative stresses and coefficients of the extrapolation formula for the material EF are shown in Figs. 5-7 and in Tables 8-10 for cases where the direction with bigger stiffness of the material EF is parallel to the axis $Ox$, $Ox$ and the parallel line inclined at an angle of $45^\circ$ respectively.

We considered case where the direction with the bigger stiffness of CF1 material is parallel to the $Ox$ axis. The results of the calculations are shown in Fig. 4 and Table 7.

In addition, the stress at small distances on the interspace $0 < \delta < \delta_{\text{m}}$ was calculated by BIEM. In Fig. 3 calculated relative stresses are represented by circles (red), which are almost exactly located on the curves indicated by dashed lines. That is, formula (1), which is an extrapolation for this interspace, allows you to obtained the stress with close to zero distances between the holes with high accuracy.

![Fig. 3. The relative maximum stress in a plate made of material CF1, 1 - circular holes, 1/2 - elliptical holes](image)

![Fig. 5. The relative maximum stress in a plate made of material EF, 1 - circular holes, 1/2 - elliptical holes](image)

![Fig. 6. The relative maximum stress in a plate made of material EF$_{90}$, 1 - circular holes, 1/2 - elliptical holes](image)

![Fig. 7. The relative maximum stress in a plate made of material EF$_{45}$, 1 - circular holes, 1/2 - elliptical holes](image)
The results of calculations for the isotropic plate and for plates made of materials CF1 and EF for elongated ellipses at \( b/a = 2 \) are shown in Fig. 8.

Table 10. Table of coefficients of formula (1) for material EF

| \( b/a \) | \( m \) | \( a_1 \) | \( a_2 \) | \( a_3 \) |
|-----------|-------|--------|--------|
| 1         | 0.495 | 0.730  | -0.315 | 0.729  |
| 1/2       | 0.495 | 0.795  | -1.907 | 0.789  |

Fig. 8. The relative maximum stress in a plate with elongated ellipse holes at \( b/a=2 \)

5. STRESS AT DISTANT POINTS AT THE BOUNDARY OF THE HOLES

Let us give the values of the divided by \( p \) stresses at distant points on the holes – points \((\pm (2a + d), 0)\) that are obtained at relative distances \( \delta = 0.0004 \) and correspond to the case of practical contact of the holes. The calculated stresses for different materials with the ratio of the semi axes \( b/a \) equal to 1, 0.5 and 2 are shown in Table 11 above the line.

Table 11. Relative stresses at distant points in the hole

| \( b/a \) | I | CF1 | CF190 | EF | EF90 | EF90 |
|-----------|---|-----|-------|----|------|------|
| 1         | 3.860 | 18.170 | 4.029 | 4.181 | 4.931 | 3.265 |
| 3.829 | 18.651 | 4.002 | 4.167 | 4.958 | 3.451 |
| 0.5       | 6.619 | 35.044 | 6.187 | 7.337 | 7.810 | 5.450 |
| 6.657 | 36.303 | 6.182 | 7.334 | 8.917 | 5.993 |
| 2         | 2.407 | 9.782 | 3.090 |      |      |      |
| 2.414 | 9.826 | 2.295 |      |      |      |

The exact value of the relative stresses in the isotropic plate with two circular holes that touch is equal to 3.861 and it agrees well with the value given in the table.

Assume that a plate has two closely placed elliptical holes with semi axes \( a, b \). The minimum radius of curvature of these ellipses is equal to \( p = b^2/a \). Let’s consider an equivalent ellipse with the semi axes \( a_1, b_1 \) such that \( a_1 = 2a \). We choose the half axes \( b_1 \) so that its minimum radius of curvature is also equal to \( p \). From here we obtained \( b_1 = \sqrt{2b} \). The semi axes of the equivalent ellipse will be \( a_1/b_1 = \sqrt{2}a/b \).

For the holes considered above, for which \( b/a=1,0.5,2 \), the minimum radii of curvature of the equivalent ellipse will be: \( \rho = a, 0.25a, 4a \).

The obtained relative stresses in the equivalent ellipse are shown in Table 11 under line. It can be seen that the stresses for both isotropic and anisotropic materials at distant points at tangent holes can be calculated based on consideration of an equivalent ellipse. A slightly larger discrepancy is in the case of asymmetrical locations of the holes relative to the orthotrop axes (see last column). Similar data to the data in Table 11 is obtained for isotropic plates in [16].

6. HALF-PLANE WITH A CIRCULAR HOLE

Consider a half-plane \( x > 0 \) with a circular hole of radius \( R \), the center of which is located at a point \((0, R + d)\) at stretched in the direction of the axis Ox by the forces of \( p \). The algorithm for determining the stresses for such a plate is given in [13,17]. The formula for describing the maximum stresses near the hole at small ion distances to the half-plane boundary was also described by formula (1). The relative stresses for the isotropic (izo) plate and the plate made of EF material, depending on the value, are calculated \( \delta = d/R \) shown n Fig. 9 and Table 12.

Table 12. Table of coefficients of formula (1) for half-plane with a hole

| \( m \) | \( a_1 \) | \( a_2 \) | \( a_3 \) |
|-------|-------|-------|-------|
| Izo   | 0.5   | 2.830 | -0.015 | 2.830 |
| EF    | 0.508 | 3.339 | -1.368 | 3.335 |
| EF90  | 0.504 | 2.860 | 0.039  | 2.860 |

Note that for anisotropic plate at short distances, the extrapolation formula proved to be valid

\[
\sigma_{\text{extr}} = \sigma + 2.83(\sqrt{0.01} + 1/\sqrt{0.01}) = 0.0154.
\]

A detailed half-plane with a circular hole at \( d/R = 0.01 \) is considered. Calculated by the integral equations method, the stress concentration factor at the hole boundary for isotropic material is equal to SCF = 28.5676 (28.3739). In parentheses is given value SCF, which was obtained by formula (1) and table 12.

Similar results for plates made of EF when the modulus of elasticity is maximum \( E \), we have SCF = 33.5712 (33.7551) and SCF = 29.4118 (29.2846) at the maximum \( E \).
At holes close to the half-plane boundary, large stresses arise along its boundary. Figure 10 shows the values of the stresses divided to tractions $p$.

Maximum stresses values are reached at $x = \pm 0.15d/R$.

7. STRESS CONCENTRATION AT REMOTE HOLES IN ANISOTROPIC PLATES

Relationships (1) describe quite accurately the stress distribution at $0 < \beta < 0.01$. At higher values of the parameter to determine the relative stresses $\sigma = \sigma_{\max}/\sigma_1$ approximate formula used $\sigma = S(\delta)$, where

$$S(\delta) = (C_1 + C_2\delta + C_3\delta^2 + C_4\delta^3)/\delta^n.$$ (6)

The coefficients obtained by the method of least squares in this formula for the above cases are shown in Table 13 for composite (EF,CF1) and isotropic (Izo) materials.

![Fig. 10. Stress at the boundary of the half-plane (EF material end isotropic material)](image_url)

Fig. 10. Stress at the boundary of the half-plane (EF material end isotropic material)

FOR THE PURPOSE OF TESTING, CALCULATIONS OF STRESSES NEAR TWO HOLES IN ANISOTROPIC PLATES BEGUN USING THE FINITE ELEMENT METHOD BY M. Batista [13].

For the purpose of testing, calculations of stresses near two holes in an isotropic plate were performed using the finite element method by the Ansys system. The shape and dimensions of the plate are shown in Fig. 10. The calculated maximum stresses at $d/R = 0.1$ when the plate is stretched by tractions $p = l$ are $\max(\sigma_{\theta}) = 6.15$, which is close to the value we obtained of 6.102 (Table 1). To achieve this result, it was necessary to reduce the size of the elements near the holes (8320 elements were chosen). For a smaller value of $d/R = 0.01$, we were unable to achieve stability of the results of the solution with the version of the Ansys program available to us. At that time, when the stresses were investigated in calculations using the integral equations we selected, the stresses were obtained with controlled accuracy at an arbitrary distance between the holes.

8. CONCLUSION

In the article, based on the boundary integral equation method with the additional use of the asymptotic method, a high concentration of stresses near closely spaced holes in the wing bearing area is investigated ground. Known modified nonlinear parametric contour givens were used to increase the convergence of the solution at small distances between the holes. The results of calculations for isotropic plates are close in nature and magnitude to the values obtained by V.V. Panasyuk and M.P. Savruk [16]. Thus, simple formulas were obtained to calculate the SIF, which proved to be practically accurate for a wide range of distances between holes, both infinitesimally small and commensurate with their sizes.

**Author contributions:** research concept and design, O.M., M.M., A.L.; Collection and/or assembly of data, O.M., M.M., A.L.; Data analysis and interpretation, O.M., O.M.D., M.M., A.L.; Writing the article, O.M., O.M.D.; Critical revision of the article, O.M., O.M.D.; Final approval of the article, O.M., O.M.D.

**Declaration of competing interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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