Leader–follower close formation control for underactuated surface vessel via terminal hierarchical sliding mode

Huizi Chen¹, Yan Peng¹, Yueying Wang¹, Shaorong Xie¹ and Huaicheng Yan²

Abstract
This article is concerned with the close formation problem of multiple underactuated surface vessels in the presence of model uncertainties, roll motion, and environmental disturbances. To effectively address these issues, a novel control scheme considering roll stabilization is designed by combing terminal hierarchical sliding mode control with Lyapunov direct method, which can quickly ensure a small formation error in a finite-time for vessels. Meanwhile, a new switching gain adaptation mechanism is utilized to reduce chattering and acquire faster adaptive rate without the excessive temporary tracking errors. Radial basis function neural network and finite-time observer are employed to deal with model uncertainties and disturbances, respectively. Furthermore, dynamic surface control technology is introduced to reduce the complexity of control law. Various simulations and comparison results are conducted to verify the effectiveness of theoretical results.

Keywords
Close formation control, underactuated surface vessel, roll stabilization, terminal hierarchical sliding mode control, switching gain adaptation mechanism, dynamics surface control

Introduction
Close formation control is receiving a considerable attention due to its important applications, such as cooperative exploration of ocean resources, underwater pipe-laying, and marine replenishment.¹ Since the relative distance among vessels is small in close formation, the vessels are easily subjected to this kind of nonlinear complex disturbances from adjacent vessels and ocean simultaneously, resulting in serious roll motion, which will affect the navigation of vessels and the operation of instruments. Meanwhile, the drift caused by rolling will bring formation errors and even cause collisions. For close formation control, not only the motion in the horizontal plane but also the roll motion should be considered. Besides the above challenge, there are more degrees of freedom (DoFs) to be controlled than the number of independent control inputs for underactuated surface vessels (USVs).²,³ Therefore, it is a great significance to implement and maintain a predefined formation mode so that multiple USVs can cooperate to complete a given task.

Currently, there are several popular formation control strategies, such as behavior-based control,⁴ event-triggered

¹School of Mechanical Engineering and Automation, Shanghai University, Shanghai, People’s Republic of China
²School of Information Science and Engineering, East China University of Science and Technology, Shanghai, People’s Republic of China

Corresponding author:
Shaorong Xie, School of Mechanical Engineering and Automation, Shanghai University, Shanghai 200444, People’s Republic of China.
Email: srxie@shu.edu.cn.
real-time scheduling control, virtual structures, and leader–follower architecture. Although the control methods are different in the existing literature, the research has always focused on the estimation of leader’s information, obstacle avoidance, and compensation lumped uncertainties including actuator failure, marine disturbances, and system uncertainties. Among them, most of the scholars have estimated the leader information by state estimator and lumped uncertainties by neural network approximation or takagi-sugeno fuzzy-approximation. Coping with information limitation of leader, a robust adaptive formation control was applied by combining the minimal learning parameter (MLP) algorithm, neural networks, and disturbance observer. To solve unavailable velocity measurements, the fuzzy sliding mode robust controllers with distributed and self-organized capability based on the center-of-swarm guidance scheme were designed to estimate the unmeasured velocities and complete the predefined formation. With less communications among USVs, a velocity-free formation controller was constructed by the finite-time observer (FTO) and time-varying in-type barrier Lyapunov function method. To deal with model uncertainties and environmental disturbances, a sliding mode control scheme was developed by estimating unknown parameter and upper bound. Under a static communication network, a distributed continuous controller was presented to perform time-varying formation tracking control in dynamic, complex, and unpredictable marine environments. Using directed graph theories, backstepping and the MLP algorithm, a distributed robust formation controller with the MLP-based and auxiliary adaptive laws was acquired to complete the cooperative formation under uncertain dynamics and external disturbances. Without the prior knowledge of the environment and reference trajectory, a finite control set model predictive control was devised to deal with the problem of formation collision avoidance. To ensure connectivity preservation and collision avoidance among networked uncertain USVs with different communication ranges, an adaptive output-feedback controller was obtained to accomplish formation tracking problem. And then, the swarm control strategy based on swarm center position guidance was developed for each vehicle to follow the desired path and avoid collisions autonomously. A distributed formation control strategy was proposed by combining the graph theory and the robust compensation theory to achieve the desired formation trajectory and time-varying formation pattern and to adjust the attitude of vessel. To avoid the singularity problem, a linear partitioned sliding mode controller was presented to achieve the desired formation within a finite time. For solving the problem of formation-containment tracking with multiple leader vessels, a practical formation controller was adopted based on distributed extended state observer. In Fu and Wang, a bioinspired model-based hybrid strategy was proposed to avoid the chattering problem in sliding mode and speed jump that occurs in the backstepping method. Besides this, some other control strategies can also be applied for formation control.

Although researchers have done a lot of scientific research on the formation control of USVs, we found that, only few research results have studied the close formation control of USVs and the effect of rolling for formation. However, the formation safety will be threatened in case of the serious roll disturbances and small formation distance. Based on the above considerations, a novel control scheme with roll stabilization is designed by combing terminal hierarchical sliding mode control (THSMC) with Lyapunov direct method. THSMC includes two parts: hierarchical sliding mode control and terminal sliding mode control. The former is coped with the underactuation of vessel, and the latter ensure the finite time convergence of the formation errors. Compared with the references, the proposed THSMC has faster convergence rate and smaller formation error to ensure superior formation performance and navigation safety.

Main contributions can be summarized as follows:

1. As the roll motion in close formation cannot be neglected, a novel roll stabilization control strategy is adopted to compensate the complex disturbances caused by adjacent vessels and marine environment to ensure the smaller formation errors.
2. For the small distance among USVs and underactuated characteristics of vessels, the THSMC is presented to make the USVs quickly adjust its position and attitude in a finite-time to avoid collisions. Meanwhile, the sliding surface is designed in terms of the velocity errors and switching gain. Moreover, a novel switching gain adaptation mechanism helps to reduce chattering and acquires faster adaptive rate without the excessive temporary tracking errors.
3. On the basis of the surge, sway, and yaw motion, considering the roll motion, the number of the control law items increases greatly, resulting in complex control structure and huge calculation. In this article, dynamic surface control (DSC) technology is introduced to reduce the complexity of control law.

The rest of this article is organized as follows: The next section provides some preliminaries and problem formulations. Then, the control design and finite-time stability analysis of formation control are described. In the following, the results of conducted numerical simulations are illustrated. Finally, some conclusions and future works are summarized.

Problems formulation

Preliminaries

Lemma 1. For any real numbers $a > 0, b > 0$ and $0 < l < 1$, the Lyapunov condition of finite-time stability can be given in the form $\dot{V}(x) \leq -aV^l(x) + b$, and the
The corresponding nonlinear system of \( V(x) \) is semi-global practical finite-time stable.\(^{34}\)

**Lemma 2.** For any real variables \( \omega, \varsigma \), there are positive constants \( \mu, \theta, t \), the inequality holds as follows\(^{34}\)

\[
|\omega|^\mu|\varsigma|^\theta \leq \frac{\mu}{\mu + \theta} |\omega|^\mu + \theta + \frac{\theta}{\mu + \theta} |\varsigma|^\theta
\]

**Lemma 3.** For \( \omega_m \in R, m = 1, 2, \ldots, n, 0 < l < 1 \), the inequality relation sets up as follows\(^{34}\)

\[
\left( \sum_{m=1}^{n} |\omega_m| \right)^l \leq \sum_{m=1}^{n} |\omega_m|^l
\]

**Lemma 4.** The radial basis function neural network (RBFNN) is used to approximate continuous and unknown smooth function. It can be described as follows\(^{34}\)

\[
f_n(Z) = W^TH(Z) + \varepsilon
\]

where \( Z \in R^n \) is the input vector; \( \varepsilon \) is the optimal approximation error satisfying \( \varepsilon \leq \hat{\varepsilon} \), where \( \hat{\varepsilon} \) is an unknown small positive constant. \( W^* = [w_{11}^*, w_{12}^*, \ldots, w_{n}^*] \in R^n \) denotes the optimal weight vector, \( n \) is the weight number; the unknown optimal weight value \( W^* \) can be described as

\[
W^* = \arg \min_W \left\{ \sup_{Z \in \Omega} |f(Z) - \hat{W}^TH(Z)| \right\}
\]

where \( \hat{W} \) represents the estimation of \( W^* \).

\[H(Z) = [h_1(z), h_2(z), \ldots, h_n(z)]^T \in R^n \] is the basis function vector, where \( h_q(z) \) is chosen as Gaussian function

\[h_q(z) = \exp \left( - \frac{|z - \sigma_q|^2}{\delta_q^2} \right), (q = 1, 2, \ldots, n)\]

where \( \sigma_q = [\sigma_{q1}, \sigma_{q2}, \ldots, \sigma_{qn}]^T \) is the center of the receptive field and \( \delta_q \) is the width of the Gaussian function.

**Lemma 5.** The following systems\(^{35}\)

\[
\dot{x} = u \cos \psi - v \cos \phi \sin \psi
\]
\[
\dot{y} = u \sin \psi + v \cos \phi \cos \psi
\]
\[
\dot{\psi} = r \cos \phi
\]
\[
\dot{\phi} = p
\]
\[
\dot{u} = \frac{m_{22} v r - d_{11} u^2 + \tau_1 + d_u}{m_{11}}
\]
\[
\dot{v} = -\frac{m_{11} u r - d_{22} v + d_v}{m_{22}}
\]
\[
\dot{r} = \frac{m_{11} - m_{22} uv - d_{33} r + \tau_2 + d_r}{m_{33}}
\]
\[
\dot{p} = -\frac{d_{44} p - l_p \phi + \tau_3 + d_p}{m_{44}}
\]

where \( x, y, \psi, \phi \) are longitudinal displacement, lateral displacement, roll angle, and yaw angle, respectively. \( u, v, r, p \) are surge, sway, yaw, and roll velocity, respectively. \( \tau_1, \tau_2, \tau_3 \) denote the surge force, the yaw moment, and roll moment, respectively. \( d_u, d_v, d_r, d_p \) represent the unknown and time-variant external environmental disturbances due to wind, waves, ocean currents, and adjacent vessels. \( m_{11}, m_{22}, m_{33}, m_{44} \) denote the USV’s inertia coefficients including added mass effects. \( d_{11}, d_{22}, d_{33}, d_{44} \) represent the hydrodynamic damping coefficients. \( l_p = \rho g \nabla GM/m_{44} \), where \( \rho, g, \nabla \) are the water density, gravitational acceleration, and displacement volume of the vessel, respectively, and GM is the transverse metacentric height.

**Remark 1.** In the study of ship roll stability, most of the literature decouple the roll motion and other DoFs to simplify the control objectives. The dynamic model studied in this article is the coupling of the roll motion with the surge, sway, and yaw motion, which is close to the reality.

Define \( f_u = \frac{m_{22} v \cos \psi - d_{11} u^2}{m_{11}}, f_v = -\frac{m_{11} u \cos \psi - d_{22} v}{m_{22}} \), \( f_r = \frac{m_{11} - m_{22} uv - d_{33} r}{m_{33}} \), and \( f_p = -\frac{d_{44} u \cos \psi - \tau_3}{m_{44}} \).

The system (1) can be rewritten as follows

\[
\dot{x} = u \cos \psi - v \cos \phi \sin \psi
\]
\[
\dot{y} = u \sin \psi + v \cos \phi \cos \psi
\]
\[
\dot{\psi} = r \cos \phi
\]
\[
\dot{\phi} = p
\]
\[
\dot{u} = \frac{m_{22} v r - d_{11} u^2 + \tau_1 + d_u}{m_{11}}
\]
\[
\dot{v} = -\frac{m_{11} u r - d_{22} v + d_v}{m_{22}}
\]
\[
\dot{r} = \frac{m_{11} - m_{22} uv - d_{33} r + \tau_2 + d_r}{m_{33}}
\]
\[
\dot{p} = -\frac{d_{44} p - l_p \phi + \tau_3 + d_p}{m_{44}}
\]

**Dynamic model of USV**

A 4-DoF USV motion including surge, sway, roll, and yaw is considered. It can be described as\(^{32}\)
Problems formulation

Figure 1 presents the basic geometric structure about two USVs moving in a leader–follower formation. This position and angle relationship can be extended to multiple USVs.

The leader position is defined as

\[ r = \sqrt{(x_L - x)^2 + (y_L - y)^2} \]

\[ \lambda = \arctan(y_L - y, x_L - x) \]

where \((x_L, y_L)\) is the position of leader and \((x, y)\) is the position of follower.

Suppose there is a virtual vessel that can maintain the desired formation with the leader in real time. If the follower can track the virtual vessel, the desired formation can be completed. The virtual vessel trajectory is defined as

\[
\begin{pmatrix}
    x_d \\
    y_d \\
    \psi_d
\end{pmatrix} = \begin{pmatrix}
    x_L \\
    y_L \\
    \psi_L
\end{pmatrix} + \begin{pmatrix}
    \cos\psi_L - \sin\psi_L & 0 & \rho_d \cos\lambda_d \\
    \sin\psi_L & \cos\psi_L & 0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    \rho_d \\
    \sin\lambda_d
\end{pmatrix}
\]

where \((x_d, y_d)\) is the position of virtual vessel and denotes position that followers need to track to complete the desired formation. \(\beta\) is the heading angle between the virtual vessel trajectory and the follower.

Remark 2. The leader position \((x_L, y_L)\) and the desired formation \((\rho_d, \lambda_d)\) are known \((x_L, y_L)\) is obtained by equation (1) and \((\rho_d, \lambda_d)\) is the preset constant. Firstly, according to definition in virtual vessel trajectory, the virtual vessel position \((x_d, y_d)\) is obtained by \((x_L, y_L, \psi_L), (\rho_d, \lambda_d), \) and \(\beta\). Secondly, followers track the virtual vessel in the presence of model uncertainties and environmental disturbances. Once \(x = x_d, y = y_d, \psi = \psi_d\), which means \(\rho = \rho_d, \lambda = \lambda_d\), the desired formation control is achieved. Therefore, the formation problem of leader and followers can be transformed into the path following of followers.

Assumption 1. The environmental disturbances \(d_u, d_v, d_r, d_p\) are bounded.

Assumption 2. The states of the desired trajectory \(x_d, \dot{x}_d, \dot{y}_d, \dot{y}_d, \dot{\psi}_d, \dot{\psi}_d\) are all bounded.

Following the coordinate transformation,\(^{32}\) we have

\[
\begin{align*}
    z_1 &= x \cos\psi + y \sin\psi \\
    z_2 &= -x \sin\psi + y \cos\psi \\
    z_3 &= \psi \\
    z_4 &= \phi
\end{align*}
\]

Define the desired trajectory as

\[
\begin{align*}
    z_{1d} &= x_d \cos\psi_d + y_d \sin\psi_d \\
    z_{2d} &= -x_d \sin\psi_d + y_d \cos\psi_d \\
    z_{3d} &= \psi_d = \dot{\lambda}_d = \arctan\left(\frac{x_d}{y_d}\right) \\
    z_{4d} &= \phi_d = 0
\end{align*}
\]

Consequently, the path following problem is transformed into the stabilization of error system (3).

Remark 3. Apparently, when meeting \(z_{1e} = z_{2e} = z_{3e} = z_{4e} = 0, z_1 = z_{1d}, z_2 = z_{2d}, \) and \(z_3 = z_{3d}, z_4 = 0\). It can be concluded that \(x = x_d, y = y_d, \psi = \psi_d\), which means desired formation can be completed.

Control design and stability analysis

This section has two tasks to accomplish: One is the path following of followers in the horizontal plane, and the other one is the roll stabilization of USVs. We stabilize the surge and roll error subsystem with a common sliding mode, and stabilize the sway and yaw error subsystems simultaneously with hierarchical sliding mode due to underactuation of the USVs.

Subsystem control design

Step 1. Stabilizing position error \(z_{1e}\).

Choose the following Lyapunov candidate

\[
V_1 = \frac{1}{2} z_{1e}^2
\]

Differentiating \(V_1\), we have
\[ \dot{V}_1 = z_1 [u - \chi r \sin \psi \cos \phi + \chi r \cos \psi \cos \phi - \chi_d \cos \phi + x_d \psi_d \sin \psi_d - \chi_d \sin \psi_d - y_d \psi_d \cos \psi_d] \]

Designing virtual control law as
\[ u_d = x r \sin \psi \cos \phi - \chi r \cos \psi \cos \phi + \chi_d \cos \phi - x_d \psi_d \sin \psi_d + \chi_d \sin \psi_d + y_d \psi_d \cos \psi_d - k_1 z_1 \]

where \( k_1 \) is a positive constant.

Then, the derivatives of Lyapunov function \( V_1 \) is negative
\[ \dot{V}_1 = -k_1 z_1^2 \leq 0 \]

Surge velocity error is defined as \( u_e = u - u_d \).
To stabilize error \( u_e \), a sliding surface is chosen as
\[ S_1 = c_1 u_e^{\beta_1} + c_2 \int_0^t u_e d\tau + (\varepsilon f_u - \hat{\eta}_1) \]

where \( c_1, c_2, \beta_1, \varepsilon f_u \) are positive constants, and \( \varepsilon f_u \) denotes the boundary of approximation error, \( 1 < \beta_1 < 2 \), \( |f_u - \hat{f}_u| \leq \varepsilon f_u \), and \( \hat{f}_u \) is the estimation of \( f_u \).

Remark 4. The PI sliding mode surface can reduce the chattering of the system and also relax some assumptions in Sun et al. such as the existences of first-order derivatives of environmental disturbance and control input.

Differentiating \( u_e \), we have
\[ \dot{u}_e = \dot{u} - \dot{u}_d \quad \text{(4)} \]

To avoid computational expansion of \( \dot{u}_d \), this virtual control laws pass through the first-order filter
\[ \varepsilon u \dot{\alpha}_u + \alpha_u = u_d, \alpha_u(0) = u_d(0) \]

where \( \varepsilon u \) is a positive constant.

Remark 5. Due to \( u_d \) includes roll angle, the amount of calculation will be further increased greatly on the original basis when \( u_d \) is the derivative. DSC is induced to obtain the derivative of \( u_d \) without increasing the amount of computation.

Equation (4) can be rewritten as
\[ \dot{u}_e = \dot{u} - \dot{\alpha}_u \quad \text{(5)} \]

According to equations (2) and (5), it can obtain the derivative of \( S_1 \) as
\[ \dot{S}_1 = c_1 \beta_1 u_e^{\beta_1 - 1} \dot{u}_e + c_2 u_e - \dot{\eta}_1 \]

Employing RBFNN to approximate \( f_u \), we have
\[ \hat{f}_u = \hat{W}_1^T H_1(Z) \]

where \( \hat{W}_1 \) is the estimation of \( W_1^* \).

The equivalent control law of the surge error subsystem is designed as
\[ \tau_{eq} = m_1 \left( -\dot{f}_u + \dot{\alpha}_u - \frac{c_2 u_e^2 - \beta_1}{c_1 \beta_1} \right) - \dot{d}_u \]

where \( \ddot{d}_u \) is the estimation of \( d_u \).

Moreover, the saturation function \( f_{sat}(\cdot) \) is used to replace the \( sign(\cdot) \) to avoid the chattering effectively, and described as
\[ f_{sat}(S_1) = \begin{cases} \text{sign}(S_1) & |S_1| > \varepsilon_{sat} \\ \text{sign}(S_1)|S_1|^\varepsilon_{sat} & |S_1| \leq \varepsilon_{sat} \end{cases} \]

where \( \text{sign}(S_1) = \text{sign}(S_1)|S_1|^{\varepsilon_{sat}}, \forall \varepsilon_{sat} \) are positive parameters.

The switch control law of this subsystem is chosen as
\[ \tau_{1sw} = m_1 (-\ddot{\eta}_1 f_{sat}(S_1) - k_{1sw} S_1) \]

where \( k_{1sw} \) is a positive parameter.

The switching gain employs a new adaptive law as follows
\[ \dot{\eta}_1 = K_{2sw} \text{sign}(S_1) ||S_1||_{\infty} - \varepsilon_1 \text{sign}(||S_1||_{\infty} - \varepsilon_1) e(||S_1||_{\infty} - \varepsilon_1) \]

where \( K_{2sw}, \varepsilon_1 \) are positive parameters and \( \varepsilon_1 \) represents sliding mode boundary threshold.

Remark 6. \( \text{sign}(S_1) \) is to make the switching gain to increase or decrease according to the sliding mode variable \( S_1 ; e(||S_1||_{\infty} - \varepsilon_1) \) guarantees a reasonable convergence trend according to the error, \( ||S_1||_{\infty} - \varepsilon_1 \text{sign}(||S_1||_{\infty} - \varepsilon_1) \) adjusts the amplitude of \( e(||S_1||_{\infty} - \varepsilon_1) \) to avoid too fast or slow convergence rate of sliding mode variables. The adaptation rate of switching gain \( \ddot{\eta}_1 \) is highly affected by \( \varepsilon_1 \).

Remark 7. As seen from the proposed adaptive law, the upper and lower bound information of system uncertainty is not required. When \( ||S_1||_{\infty} \geq \varepsilon_1, \ddot{\eta}_1 = k_{2sw} \text{sign}(S_1) \left(||S_1||_{\infty} - \varepsilon_1\right) e(||S_1||_{\infty} - \varepsilon_1) \). The adaptive law \( \ddot{\eta}_1 \) has a larger growth and attenuation rate to make the error \( ||S_1||_{\infty} - \varepsilon_1 \) faster convergence until \( S_1 \) reaches the vicinity \( \varepsilon_1 \) of the sliding surface, and hence provides higher gain to ensure the good tracking performance in the transient response. Once the sliding variable \( ||S_1||_{\infty} < \varepsilon_1, \ddot{\eta}_1 = k_{2sw} \text{sign}(S_1) \left(||S_1||_{\infty} - \varepsilon_1\right) e(||S_1||_{\infty} - \varepsilon_1) \). The adaptive law \( \ddot{\eta}_1 \) has a smaller growth and attenuation rate to reduce chattering and avoid excessive transient gain, while the sliding variable stays in the vicinity of the sliding manifold. This adaptation mechanism contributes to chattering reduction and the fast adaptation of switching gains without temporary large tracking errors.
The total control law of surge subsystem can be obtained as

\[ \tau_1 = \tau_{1eq} + \tau_{1sw} \]  

(8)

Define Lyapunov candidate function \( V_2 \) as follows

\[
\dot{V}_2 = S_1 \dot{S}_1 - \frac{1}{\gamma_1} \hat{W}_1^T \dot{\hat{W}}_1 \\
= S_1 \left[ c_1 \beta_1 u_{e1}^{\beta_1-1} \left( f_u + \tau_1 + d_u - \hat{\alpha}_u \right) + c_2 u_c - \hat{\eta}_1 \right] - \frac{1}{\gamma_1} \hat{W}_1^T \dot{\hat{W}}_1 \\
= S_1 \left[ c_1 \beta_1 u_{e1}^{\beta_1-1} \left( \hat{W}_1 \dot{H}_1(Z) + \frac{\hat{d}_u}{m_1} - \frac{c_2 u_c}{c_1 \beta_1 u_{e1}^{\beta_1-1}} - \hat{\eta}_1 \text{sat}(S_1) - k_{1sw} S_1 \right) + c_2 u_c - \hat{\eta}_1 \right] - \frac{1}{\gamma_1} \hat{W}_1^T \dot{\hat{W}}_1 \\
= -S_1 \left[ c_1 \beta_1 u_{e1}^{\beta_1-1} \left( \hat{\eta}_1 \text{sat}(S_1) + k_{1sw} S_1 + \hat{\eta}_1 \right) + \hat{W}_1^T \left( c_1 \beta_1 u_{e1}^{\beta_1-1} S_1 \dot{H}_1(Z) - \frac{1}{\gamma_1} \hat{W}_1 \right) \right] \\
+ \hat{W}_1^T \left( c_1 \beta_1 u_{e1}^{\beta_1-1} S_1 \dot{H}_1(Z) - \frac{1}{\gamma_1} \hat{W}_1 \right) \\
\]

where \( \hat{d}_u = d_u - \hat{d}_u \). Based on Lemma 5, \( \hat{d}_u \) is stable in a finite time and \( \hat{d}_u = 0 \).

Designing the adaptive law \( \dot{\hat{W}}_1 \) as follows

\[
\dot{\hat{W}}_1 = \gamma_1 \left( c_1 \beta_1 u_{e1}^{\beta_1-1} S_1 \dot{H}_1(Z) - \hat{W}_1 \right) \\
As \ k_{2sw} |S_1|||S_1||_{\infty} - \varepsilon_1 \| \text{sign}(||S_1||_{\infty} - \varepsilon_1) e(||S_1||_{\infty} - \varepsilon_1) \| \geq 0 \]

and \( \hat{\eta}_1 S_1 \text{sat}(S_1) > 0 \), \( V_2 \) can be rewritten as

\[
\dot{V}_2 = -c_1 \beta_1 u_{e1}^{\beta_1-1} \left( \hat{\eta}_1 S_1 \text{sat}(S_1) + k_{1sw} S_1^2 \right) + k_{1sw} \hat{W}_1 \dot{W}_1 \\
\leq -c_1 \beta_1 k_{1sw} u_{e1}^{\beta_1-1} S_1^2 + k_{1sw} \hat{W}_1 \dot{W}_1 \\
\]

Using Young’s inequality, we obtain

\[
k_{1sw} \hat{W}_1 \dot{W}_1 \leq \frac{k_{1sw} \hat{W}_1^{-1} \hat{W}_1^{-1}}{2} + \frac{k_{1sw} \dot{W}_1 \dot{W}_1}{2} \\
\]

Utilizing Lemma 2, let \( \omega = 1 \), \( \varepsilon = \frac{k_{1sw} \hat{W}_1^{-1} \hat{W}_1}{2} \), \( \mu = 1 - l \), and \( \theta = l \), \( \theta = l^2 \), and \( 0 < l < 1 \), the following inequality can be obtained

\[
\left( \frac{k_{1sw} \hat{W}_1^{-1} \hat{W}_1}{2} \right)^l \leq (1 - l)^l + \frac{k_{1sw} \dot{W}_1 \dot{W}_1}{2} \\
\]

Remark 8. According to Lemma 1, inequality (9) satisfies the theory of finite time stability, that is, the corresponding approximation error variable \( \hat{W}_1 \) is semi-global practical finite-time stable. Furthermore, the finite time stability of formation errors can be guaranteed by extending the inequality to the whole error system.

Since the upper and lower bounds of \( u_e \) are determined by \( u_e \) and \( u \) and \( u_d \) are bounded, so \( u_e \) is also bounded function. Define \( \rho_1 = c_1 \beta_1 k_{1sw} u_{e1}^{\beta_1-1} > 0 \).

Similarly, we can obtain

\[
\left( \rho_1 S_1^2 \right)^l \leq (1 - l) l + \rho_1 S_1^2 \\
\]

(10)

Combining \( \dot{V}_2 \) with equations (9) and (10), it holds that

\[
\dot{V}_2 \leq -\left( \rho_1 S_1^2 \right)^l - \left( \frac{k_{1sw} \hat{W}_1 \dot{W}_1}{2} \right) + 2(1 - l) l \\
+ \frac{k_{1sw} \dot{W}_1 \dot{W}_1}{2} \\
\]

Step 2. Stabilizing position and orientation error \( z_{2r}, z_{3r} \).

Since subsystem \( z_{2r} \) has no control input, we utilize the hierarchical sliding mode to design control input to stabilize both subsystem \( z_{2r} \) and \( z_{3r} \). Firstly, we design two first-order sliding surfaces. Then, the second-order sliding surface is formed by two first-order sliding surfaces on certain ratio.

Choose the following Lyapunov candidate

\[
V_3 = \frac{1}{2} z_{2r}^2 + \frac{1}{2} z_{3r}^2 \\
\]

Differentiating \( V_3 \), we have
\[ \dot{V}_3 = z_{2e}\ddot{z}_{2e} + z_{3e}\ddot{z}_{3e} \]
\[ = z_{2e}(v \cos \phi - z_{1r} \cos \phi + \dot{x}_d \sin \psi_d + x_d \dot{\psi}_d \cos \psi_d) \]
\[ - y_d \dot{\psi}_d \sin \phi - z_{2r} \dot{z}_{2e} \]
\[ = z_{2r}r + v_{1d} \]
\[ r_d = \frac{1}{\cos \phi}(\dot{\psi}_d - k_3z_{3e}) \]

where \( k_2 \) and \( k_3 \) are positive constants.

Then, the derivatives of Lyapunov function \( V_3 \) is negative.

\[ \dot{V}_3 = -k_2z_{2e}^2 - k_3z_{3e}^2 \leq 0 \]

Sway and yaw velocity errors are defined as

\[ v_c = v - v_{1d}, r_c = r - r_d \]

To stabilize error \( v_c \), the first first-order sliding surface is chosen as

\[ \sigma_1 = c_3v_c^3 + c_4 \int_0^\tau v_c d\tau \]

where \( c_3, c_4, \beta_2 \) are positive constants and \( 1 < \beta_2 < 2 \).

Differentiating \( v_c \), we have

\[ v_c = \dot{v} - \ddot{v}_{1d} - z_{1r}r - z_{1r}r \] (11)

To avoid computational expansion of \( \dot{v}_{1d} \), this virtual control laws pass through the first-order filter

\[ \varepsilon_{v_{1d}} \dot{v}_{1d} + \alpha_{v_{1d}} = v_{1d} \]

where \( \varepsilon_{v_{1d}} \) is a positive constant.

Equation (11) can be rewritten as

\[ \dot{v}_c = \dot{v} - \dot{v}_{1d} - z_{1r}r - z_{1r}r \] (12)

Differentiating \( \sigma_1 \) along with equations (2) and (12), we have

\[ \dot{\sigma}_1 = c_3\beta_2v_c^{3-1}\dot{v}_c + c_4v_c \]
\[ = c_3\beta_2v_c^{3-1}(f_v + \frac{d_r}{m_{32}} - \dot{\alpha}_{v_{1d}} - z_{1r}r - z_{1r}r) + c_4v_c \]
\[ = c_3\beta_2v_c^{3-1}(f_v + \frac{d_r}{m_{32}} - \dot{\alpha}_{v_{1d}} - (u + z_{2r} \cos \phi)r) \]
\[ - z_1 \left( f_r + \frac{\tau_2 + d_r}{m_{33}} \right) + c_4v_c \]

Employing RBFNN to approximate \( f_v \) and \( f_r \), we obtain

\[ \hat{f}_v = \hat{W}_v^T H_2(Z) \]
\[ \hat{f}_r = \hat{W}_r^T H_3(Z) \]

where \( \hat{f}_v \) and \( \hat{f}_r \) are the estimations of \( f_v \) and \( f_r \), \( \hat{W}_2 \) and \( \hat{W}_3 \) are the estimations of \( W_2^T \) and \( W_3^T \).

The equivalent control law of the sway error subsystem is designed as

\[ \tau_{21eq} = \frac{m_{33}}{c_3} \left[ c_4v_c^{3-1} + f_v + \frac{d_r}{m_{22}} - \dot{\alpha}_{v_{1d}} - (u + z_{2r} \cos \phi)r - z_{1r}\hat{f}_r \right] - \dot{\hat{d}}_r \]

where \( \dot{\hat{d}}_r, \hat{d}_r \) are the estimations of \( d_r, d_r \).

To stabilize error \( r_c \), the second first-order sliding surface is chosen as

\[ \sigma_2 = c_5r_c^3 + c_6 \int_0^\tau_r d\tau \]

where \( c_5, c_6, \beta_3 \) are positive constants and \( 1 < \beta_3 < 2 \).

Differentiating \( r_c \), we have

\[ \dot{r}_c = \dot{r} - \dot{\hat{r}}_c \] (13)

To avoid computational expansion of \( \dot{r}_c \), this virtual control laws pass through the first-order filter

\[ \varepsilon_{r_{1d}} \dot{r}_{1d} + \alpha_{r_{1d}} = r_{1d} \]

where \( \varepsilon_{r_{1d}} \) is a positive constant.

Equation (13) can be rewritten as

\[ \dot{r}_c = \dot{r} - \dot{\hat{r}}_c \] (14)

Differentiating \( \sigma_2 \) along with equations (2) and (14), we have

\[ \dot{\sigma}_2 = c_5\beta_3r_c^{3-1}\dot{r}_c + c_6r_c \]
\[ = c_5\beta_3r_c^{3-1} \left( f_r + \frac{\tau_2 + d_r}{m_{33}} - \dot{\alpha}_r \right) + c_6r_c \]

The equivalent control law of the yaw error subsystem is designed as

\[ \tau_{22eq} = \frac{m_{33}}{c_3} \left( \frac{c_4v_c^{3-1} + f_v + \frac{d_r}{m_{22}} - \dot{\alpha}_{v_{1d}} - (u + z_{2r} \cos \phi)r}{\varepsilon_{r_{1d}} + z_{1r}\hat{f}_r} \right) - \dot{\hat{d}}_r \]

Defining the second-order sliding mode surface as

\[ S_2 = c_7\sigma_1 + c_8\sigma_2 + (\varepsilon_{r_{1d}} - \hat{\eta}_2) \]

where \( c_7, c_8, \varepsilon_{r_{1d}} \) are positive constants and \( \varepsilon_{r_{1d}} \) denotes the boundary of approximation error, \( \hat{\eta}_2 \) is the positive switching gain estimation, \( \max |f_v - \dot{\hat{f}}_v| \), \( |f_r - \dot{\hat{f}}_r| \leq \varepsilon_{r_{1d}} \).

The switch control law is designed as
\[
\tau_{2sw} = \frac{-m_{33}(\dot{\eta}^2_{2}\text{sat}(S_2) + k_{3sw}S_2) - c_3e_7\beta_2\nu_e^{2\beta_e-1}z_1\tau_{2eq} - c_5e_8\beta_3\nu_e^{2\beta_e-1}\tau_{21eq}}{c_3e_7\beta_2\nu_e^{2\beta_e-1}z_1 + c_5e_8\beta_3\nu_e^{2\beta_e-1}}
\]

where \(k_{3sw}\) is a positive parameter.

The switching gain employs a new adaptive law as follows
\[
\dot{\eta}_2 = k_{4sw}\text{sign}(S_2)||S_2||_\infty - \varepsilon_2|\text{sign}(||S_2||_\infty - \varepsilon_2)|e(||S_2||_\infty - \varepsilon_2)
\]

(15)

where \(k_{4sw}, \varepsilon_2\) are positive parameters and \(\varepsilon_2\) represents sliding mode boundary threshold.

The total control law of the sway and yaw subsystem can be obtained as
\[
\tau_2 = \tau_{21eq} + \tau_{22eq} + \tau_{2sw}
\]

(16)

**Remark 9.** For underactuated systems, there is not an independent sway actuator to perform sway motion. Therefore, USV accomplishes the sway motion indirectly through the yaw control. Different from most nonlinear control, this hierarchical controller has the double layer structure. In the first layer, the terminal sliding surfaces are hierarchically designed for the sway subsystem and yaw subsystem, respectively, and in the second layer, the whole sliding surface is designed as the linear combination of terminal sliding surfaces. Therefore, the control input \(\tau_2\) includes sway force \(\tau_{21eq}\) and yaw moment \(\tau_{22eq}\), which can ensure the error stability of the sway and yaw subsystem.

Differentiating \(S_2\) along with \(\dot{\sigma}_1, \dot{\sigma}_2\), we have
\[
\dot{S}_2 = c_3e_7\beta_2\nu_e^{2\beta_e-1}\left[f_r + \frac{d_r}{m_{32}} - \dot{\alpha}_{i1d} - (u + z_2\tau \cos \phi)\right] - \dot{z}_1\left(f_r + \frac{\tau_{2eq} + d_r}{m_{32}}\right) + c_4e_7\nu_e + c_5e_8\beta_3\nu_e^{2\beta_e-1}\left(f_r + \frac{\tau_{2eq} + d_r}{m_{33}} - \dot{\alpha}_r\right) + c_6e_8\nu_e - \dot{\eta}_2
\]

(17)

Define the following Lyapunov candidate function
\[
V_4 = \frac{1}{2}S_2^2 + \frac{1}{2}\gamma_2\tilde{W}_2^T\tilde{W}_2 + \frac{1}{2}\gamma_3\tilde{W}_3^T\tilde{W}_3
\]

where \(\tilde{W}_2 = W_2^T - \dot{W}_2, \tilde{W}_3 = W_3^T - \dot{W}_3, \gamma_2, \gamma_3\) are positive parameters.

Based on equations (15) to (17), the derivative of \(V_4\) can be written as

\[
\dot{V}_4 = S_2\left\{c_3e_7\beta_2\nu_e^{2\beta_e-1}\left[f_r + \frac{d_r}{m_{32}} - \dot{\alpha}_{i1d} - (u + z_2\tau \cos \phi)\right] - \dot{z}_1\left(f_r + \frac{\tau_{2eq} + d_r}{m_{32}}\right) + c_4e_7\nu_e + c_5e_8\beta_3\nu_e^{2\beta_e-1}\left(f_r + \frac{\tau_{2eq} + d_r}{m_{33}} - \dot{\alpha}_r\right) + c_6e_8\nu_e - \dot{\eta}_2\right\}
\]

\[
= -\dot{\eta}_2S_2\text{sat}(S_2) - k_{3sw}S_2^2 - k_{4sw}|S_2||S_2|_\infty - \varepsilon_2|\text{sign}(||S_2||_\infty - \varepsilon_2)|e(||S_2||_\infty - \varepsilon_2)
\]

\[
+ c_3e_7\beta_2\nu_e^{2\beta_e-1}S_2\left(\tilde{W}_2^T\tilde{H}_2(Z) + z_1\tilde{W}_3^T\tilde{H}_3(Z) + \frac{d_r}{m_{32}} + \frac{\tau_{2eq}}{m_{32}}\right)
\]

\[
+ c_5e_8\beta_3\nu_e^{2\beta_e-1}S_2\left(\tilde{W}_3^T\tilde{H}_3(Z) + \frac{d_r}{m_{33}}\right) - \frac{1}{\gamma_2}\tilde{W}_2^T\tilde{W}_2 - \frac{1}{\gamma_3}\tilde{W}_3^T\tilde{W}_3
\]

\[
= -\dot{\eta}_2S_2\text{sat}(S_2) - k_{3sw}S_2^2 - k_{4sw}|S_2||S_2|_\infty - \varepsilon_2|\text{sign}(||S_2||_\infty - \varepsilon_2)|e(||S_2||_\infty - \varepsilon_2)
\]

\[
+ \tilde{W}_2^T\left(c_3e_7\beta_2\nu_e^{2\beta_e-1}S_2\tilde{H}_2(Z) - \frac{1}{\gamma_2}\tilde{W}_2\right)
\]

\[
+ \tilde{W}_3^T\left(c_3e_7\beta_2\nu_e^{2\beta_e-1}z_1 + c_5e_8\beta_3\nu_e^{2\beta_e-1}\right)S_2\tilde{H}_3(Z) - \frac{1}{\gamma_3}\tilde{W}_3\right]\]
where \( \hat{d}_v, \hat{d}_r \) are stable in a finite time on Lemma 5 and \( \hat{d}_v = d_r = 0 \).

Designing the adaptive law \( \hat{W}_2, \hat{W}_3 \) as follows

\[
\dot{\hat{W}}_2 = \gamma_2 \left( e_3 \eta_4 \beta_2 \hat{e}_r^{2-1} S_2 H_2(Z) - k_{\hat{W}}_2 \hat{W}_2 \right)
\]

\[
\dot{\hat{W}}_3 = \gamma_3 \left[ (e_3 \eta_4 \beta_2 \hat{e}_r^{2-1} \hat{e}_z + e_4 \eta_4 \beta_4 \hat{e}_r^{2-1}) S_2 H_3(Z) - k_{\hat{W}}_3 \hat{W}_3 \right]
\]

As \( \hat{\eta}_2 S_2 f_{sat}(S_2) > 0 \), \( \dot{V}_4 \) can be rewritten as

\[
\dot{V}_4 = -\hat{\eta}_2 S_2 f_{sat}(S_2) - k_{3_{sw}} S_2^2 + k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2
\]

\[
+ k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3 \leq -k_{3_{sw}} S_2^2 + k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2 + k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3
\]

Using Young’s inequality and Lemma 2, we obtain

\[
k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2 \leq -\left( \frac{k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2}{2} \right)^T + \frac{1}{2}
\]

\[
\leq -\left( \frac{k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2}{2} \right)^T + (1 - l) + \frac{k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2}{2}
\]

\[
(18)
\]

\[
k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3 \leq -\left( \frac{k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3}{2} \right)^T + \frac{1}{2}
\]

\[
\leq -\left( \frac{k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3}{2} \right)^T + (1 - l) + \frac{k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3}{2}
\]

\[
(19)
\]

Similarly, we can obtain

\[-k_{3_{sw}} S_2^2 \leq -(k_{3_{sw}} S_2^2)^T + (1 - l) \]

Combining \( \dot{V}_4 \) with equations (18) and (19), it holds that

\[
\dot{V}_4 \leq -(k_{3_{sw}} S_2^2)^T - \left( \frac{k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2}{2} \right)^T - \left( \frac{k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3}{2} \right)^T
\]

\[
+ 3(1 - l) + \frac{k_{\hat{W}}_2 \hat{W}_2^T \hat{W}_2}{2} + \frac{k_{\hat{W}}_3 \hat{W}_3^T \hat{W}_3}{2}
\]

Step 3. Stabilizing rolling error \( \hat{z}_{ac} \).

Choose the following Lyapunov candidare

\[
V_5 = \frac{1}{2} \hat{z}_{ac}^2
\]

Differentiating \( V_5 \), we have

\[
\dot{V}_5 = \hat{z}_{ac} \hat{p}_d
\]

Designing virtual control law as

\[
\hat{p}_d = -k_4 \hat{z}_{ac}
\]

where \( k_4 \) is a positive constant.

Then, the derivatives of Lyapunov function \( V_5 \) is negative.

\[
\dot{V}_5 = -k_4 \hat{z}_{ac} \leq 0
\]

Roll velocity error is defined as \( p_v = p - p_d \).

To stabilize error \( p_v \), a sliding surface is chosen as

\[
S_3 = c_9 p_v + c_{10} \int_0^t p_v dt + \left( \epsilon_{f_p} - \hat{\eta}_3 \right)
\]

where \( c_9, c_{10}, \eta_4, \epsilon_{f_p} \) are positive constants, and \( \epsilon_{f_p} \) denotes the boundary of approximation error, \( 1 < \beta_4 < 2 \).

\( \left[ \tilde{f}_p - \hat{f}_p \right] \leq \epsilon_{f_p}, \hat{f}_p \) is the estimation of \( f_p, \hat{\eta}_3 \) is the positive switching gain estimation.

Differentiating \( S_3 \), we have

\[
\dot{S}_3 = c_9 \beta_4 p_v^{\beta_4 - 1} \hat{p}_e + c_{10} p_v - \hat{\eta}_3 \]

\[
= c_9 \beta_4 p_v^{\beta_4 - 1} \left( f_p^\tau + \frac{\tau_3 + d_p}{m_{44}} \hat{p}_d + c_{10} p_v - \hat{\eta}_3 \right)
\]

Employing RBFNN to approximate \( f_p \), we have

\[
\hat{f}_p = \hat{W}_4^T H_4(Z)
\]

where \( \hat{f}_p \) is the estimation of \( f_p \), \( \hat{W}_4 \) is the estimation of \( W_4 \).

The equivalent control law of the roll error subsystem is designed as

\[
\tau_{3_{eq}} = m_{44} \left( -\frac{c_{10} a_v^{2-\beta_4}}{c_9 \beta_4} \hat{f}_p + \hat{\eta}_3 - \hat{p}_d \right) - \hat{d}_p
\]

The switch control law of this subsystem is chosen as

\[
\tau_{3_{sw}} = m_{44} (\hat{\eta}_s f_{sat}(S_3) - k_{3_{sw}} S_3)
\]

where \( k_{3_{sw}} \) is a positive parameter.

The total control law of roll subsystem can be obtained as

\[
\tau_3 = \tau_{3_{eq}} + \tau_{3_{sw}}
\]

The switching gain employs a new adaptive law as follows

\[
\hat{\eta}_3 = k_{6_{sw}} \text{sign}(S_3) ||S_3||_{\infty} - \epsilon_3 \text{sign}(||S_3||_{\infty} - \epsilon_3) \epsilon_3( ||S_3||_{\infty} - \epsilon_3 )
\]

where \( k_{6_{sw}}, \epsilon_3 \) are positive parameters and \( \epsilon_3 \) represents sliding mode boundary threshold.

Define the Lyapunov candidate function as follows

\[
V_6 = \frac{1}{2} S_3^2 + \frac{1}{2} \gamma_4 \hat{W}_4^T \hat{W}_4
\]

Differentiating \( V_6 \) along with equations (20) to (22), we have
\[ \dot{V}_6 = S_3 \left[ c_9 \beta_4 p_{c}^{\beta_3 - 1} \left( f_p + \frac{\tau_4 + d_p}{m_{44}} - \dot{p}_d \right) + c_{10} p_{c} - \dot{n}_3 \right] - \frac{1}{\gamma_4} \hat{W}_{4}^T \dot{\hat{W}}_4 \]

\[ = S_3 \left[ c_9 \beta_4 p_{c}^{\beta_3 - 1} \left( \hat{W}_{4} H_4(Z) + \frac{\hat{d}_p}{m_{44}} - \frac{c_{10}}{c_9 \beta_4} p_{c}^{2 - \beta_4} - \dot{n}_3 f_{sat}(S_3) - k_{5m} S_3 \right) + c_{10} p_{c} - \dot{n}_3 \right] - \frac{1}{\gamma_4} \hat{W}_{4}^T \dot{\hat{W}}_4 \]

\[ = -S_3 \left[ c_9 \beta_4 p_{c}^{\beta_3 - 1} \left( \dot{n}_3 f_{sat}(S_3) + k_{5m} S_3 \right) + k_{\beta_2} \text{sign}(S_3) \right] ||S_3||_{\infty} - \varepsilon_3 \text{sign}(||S_3||_{\infty} - \varepsilon_3) \left( ||S_3||_{\infty} - \varepsilon_3 \right) \]

\[ + \hat{W}_{4}^T \left( c_9 \beta_4 p_{c}^{\beta_3 - 1} S_3 H_4(Z) - \frac{1}{\gamma_4} \dot{\hat{W}}_4 \right) \]

where \( \hat{d}_p \) is stable in a finite time and \( \hat{d}_p = 0 \).

Designing the adaptive law \( \hat{W}_4 \) as follows

\[ \dot{\hat{W}}_4 = \gamma_4 \left( c_9 \beta_4 p_{c}^{\beta_3 - 1} S_3 H_4(Z) - k_{\hat{W}_4} \hat{W}_4 \right) \]

As \( \dot{n}_3 f_{sat}(S_3) > 0 \), \( \dot{V}_6 \) can be rewritten as

\[ \dot{V}_6 = -c_9 \beta_4 k_{5m}^{\beta_3 - 1} \left( \dot{n}_3 f_{sat}(S_3) - k_{5m} S_3^2 \right) + k_{\hat{W}_4} \hat{W}_4^T \dot{\hat{W}}_4 \]

Using Young’s inequality and Lemma 2, we obtain

\[ k_{\hat{W}_4} \hat{W}_4^T \dot{\hat{W}}_4 \leq -k_{\hat{W}_4} \hat{W}_4^T \hat{W}_4 \frac{1}{2} + k_{\hat{W}_4} \hat{W}_4^T \dot{\hat{W}}_4 \]

\[ \leq -\left( \frac{k_{\hat{W}_4} \hat{W}_4^T \hat{W}_4}{2} \right) + \left( 1 - \ell \right) t + \frac{k_{\hat{W}_4} \hat{W}_4^T \dot{\hat{W}}_4}{2} \]

(23)

Since the upper and lower bounds of \( p_c \) are determined by \( p \) and \( p_d, p, p_d \) are bounded, so \( p_c \) is also bounded function. Define \( \rho_2 = c_9 \beta_4 k_{5m}^{\beta_3 - 1} > 0 \).

Similarly, we can obtain

\[ (\rho_2 S_3^2)^{\ell} \leq \left( 1 - \ell \right) t + \rho_2 S_3^2 \]

(24)

Combining \( \dot{V}_6 \) with equations (23) and (24), it holds that

\[ \dot{V}_6 \leq -(\rho_2 S_3^2)^{\ell} - \left( \frac{k_{\hat{W}_4} \hat{W}_4^T \hat{W}_4}{2} \right) + 2(1 - \ell) t + \frac{k_{\hat{W}_4} \hat{W}_4^T \dot{\hat{W}}_4}{2} \]

Filtering error analysis

Defining filtering error as \( z_u = \dot{u}_d, z_{v_1} = \dot{v}_1 d, z_{r} = \dot{r}_d \).

Calculating the derivatives of filtering errors separately

\[ \dot{z}_u = \dot{\dot{u}}_d = -\frac{z_u}{\varepsilon_u} - B_u \left( x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, \psi, \dot{\psi}, \ddot{\psi}, x_d, \dot{x}_d, \ddot{x}_d, y_d, \dot{y}_d, \ddot{y}_d, \psi_d, \dot{\psi}_d, \ddot{\psi}_d, \alpha, \dot{\alpha}, \ddot{\alpha}, z_{1e}, \dot{z}_{1e} \right) \]

\[ \dot{z}_{v_1} = \dot{\dot{v}}_1 d = -\frac{z_{v_1}}{\varepsilon_{v_1}} - B_{v_1} \left( x_d, \dot{x}_d, \ddot{x}_d, y_d, \dot{y}_d, \ddot{y}_d, \psi_d, \dot{\psi}_d, \ddot{\psi}_d, \psi_d, \dot{\psi}_d, \ddot{\psi}_d, z_{2e}, \dot{z}_{2e} \right) \]

\[ \dot{z}_r = \dot{\dot{r}}_d = -\frac{z_r}{\varepsilon_r} - B_r \left( \dot{\psi}_d, \ddot{\psi}_d, \psi_d, \dot{\psi}_d, \ddot{\psi}_d, z_{1e}, \dot{z}_{1e} \right) \]

where \( B_u(\cdot) = \ddot{u}_d, B_{v_1}(\cdot) = \ddot{v}_1 d, B_r(\cdot) = \ddot{r}_d \) are continuous functions.

Based on Young inequality, it holds that

\[ z_j \dot{z}_j = -\frac{z_j^2}{\varepsilon_j} - z_j B_j \leq -\frac{z_j^2}{\varepsilon_j} + \frac{B_j^2 z_j^2}{2 \varepsilon_j} + \frac{\varepsilon_F}{2} \]

where \( j = u, v_1, r \), and \( \varepsilon_F \) is a positive constant.

Since the desired states of \( B_j \) is bounded according to Assumption 2, there exists a positive constant \( M_j \) such that \( |B_j| \leq M_j \). Then, we further have

\[ z_j \dot{z}_j \leq -\frac{z_j^2}{\varepsilon_j} + \frac{M_j^2 z_j^2}{2 \varepsilon_F} + \frac{\varepsilon_F}{2} \]

(25)

Let \( \rho_j = \frac{1}{\varepsilon_j} \frac{M_j^2}{\varepsilon_F} \) and satisfying \( \varepsilon_j \leq \frac{2\mu_j}{M_j^2} \).

Equation (25) can be rewritten as

\[ z_j \dot{z}_j \leq -\rho_j z_j^2 + \frac{\varepsilon_F}{2} \]

Based on Lemma 2, we obtain

\[ z_j \dot{z}_j \leq -\rho_j z_j^2 + \frac{\varepsilon_F}{2} \]
Defining the Lyapunov candidate function as follows
\[
V_7 = \frac{1}{2} \dot{z}_u^2 + \frac{1}{2} \dot{z}_v^2 + \frac{1}{2} \dot{z}_r^2
\]
The derivative of \(V_7\) can be rewritten as
\[
\dot{V}_7 = z_u \dot{z}_u + z_v \dot{z}_v + z_r \dot{z}_r \leq - (\rho_u z_u^2) + (\rho_v z_v^2) + (\rho_r z_r^2) + 3(1 - l) \epsilon + \frac{3 \epsilon_F}{2}
\]

**Finite-time stability analysis of formation control**

Defining the total Lyapunov function as follows
\[
V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7
\]
Differentiating \(V\), we have
\[
\dot{V} \leq - \sum_{i=1}^{7} \rho_i \dot{V}_i + \sum_{i=1}^{7} \Delta_i
\]
where
\[
\rho_1 = (2k_1), \quad \rho_2 = \min \left(2\rho_1, \left(k_{\text{in}}, \gamma_1 \right) \right), \quad \rho_3 = \min \left(2k_2, (2k_3) \right),
\]
\[
\rho_4 = \min \left(2k_{3uw}, \left(k_{\text{in}}, \gamma_2 \right) \right), \quad \rho_5 = \min \left(2k_4, \left(k_{\text{in}}, \gamma_3 \right) \right),
\]
\[
\rho_6 = \min \left(2\rho_u, (2\rho_v), (2\rho_r) \right).
\]
\[
\Delta_i = 0 (i = 1, 3, 5), \quad \Delta_2 = 2(1 - l) \epsilon + \frac{k_{\text{in}} W_1^T W_1}{2}, \quad \Delta_3 = 3(1 - l) \epsilon + \frac{k_{\text{in}} W_2^T W_2}{2} + \frac{k_{\text{in}} W_3^T W_3}{2},
\]
\[
\Delta_6 = 2(1 - l) \epsilon + \frac{k_{\text{in}} W_4^T W_4}{2}, \quad \Delta_7 = 3(1 - l) \epsilon + \frac{3 \epsilon_F}{2}
\]

Based on Lemma 3, we can obtain
\[
\dot{V} \leq - \rho_V \dot{V} + \Delta \tag{27}
\]
where \(\rho_V = \min(\rho_i)\), \(\Delta = \sum_{i=1}^{7} \Delta_i\).

Defining \(\Omega_z = \{ z | V(z) \leq [\Delta/(1 - \varpi) \rho_V] \}, \Omega_z = \{ z | V(z) \leq [\Delta/(1 - \varpi) \rho_V] \}, \text{ and } 0 < \varpi < 1 \).

Case 1: If \(z(t) \in \Omega_z\), equation (27) can be rewritten as
\[
\dot{V} \leq - \rho_V \varpi \dot{V} - (1 - \varpi) \rho_V \dot{V} + \Delta \leq - \rho_V \varpi \dot{V} \tag{28}
\]
From equation (28), one concludes that
\[
\int_0^T \frac{\dot{V}(z)}{V(z)} \, dt \leq - \int_0^T \varpi \rho_V \, dt
\]
Furthermore, the following inequality holds
\[
\frac{1}{1 - l} V^{1-l}(z(T^*)) - \frac{1}{1 - l} V^{1-l}(z(0)) \leq - \varpi \rho_V T^*
\]
The settling time can be estimated as
\[
T^* \leq \frac{1}{(1 - l) \varpi \rho_V} \left[ V^{1-l}(0) - \left( \frac{\Delta}{(1 - \varpi) \rho_V} \right)^{\frac{1}{1-l}} \right]
\]
where \(V(0)\) denotes the starting value of \(V(t)\).
Table 1. Model parameters of the container ship.

| Parameter | Value |
|-----------|-------|
| $m_{11}$  | 0.00103 |
| $m_{22}$  | 0.0150 |
| $m_{33}$  | 0.0050 |
| $m_{44}$  | 0.000021 |
| $d_{11}$  | 0.0004226 |
| $d_{22}$  | 0.0116 |
| $d_{33}$  | 0.0022 |
| $d_{44}$  | 0.0000075 |
| $I_p$     | 3.8537 |

Table 2. Parameters of control.

| Parameter | Value  |
|-----------|--------|
| $k_1$     | 20     |
| $k_2$     | 0.1    |
| $k_3$     | 0.8    |
| $k_4$     | 0.1    |
| $k_{sw1}$ | 0.1    |
| $k_{sw2}$ | 0.1    |
| $k_{sw3}$ | 0.001  |
| $k_{sw4}$ | 0.5    |
| $c_1$     | 100    |
| $c_2$     | 0.1    |
| $c_3$     | 100    |
| $c_4$     | 100    |
| $c_5$     | 100    |
| $c_6$     | 100    |
| $c_7$     | 0.001  |
| $c_8$     | 10     |
| $c_9$     | 1    |
| $c_{10}$  | 1    |
| $\beta_1$ | 1.5 |
| $\beta_2$ | 1.5 |
| $\beta_3$ | 1.5 |
| $\beta_4$ | 1.5 |

Case 2: If $z(t) \in \Omega_2$, $V'(z) \leq [\Delta/(1 - \omega r)]p_r$, so the trajectory of $z(t)$ does not exceed the set $\Omega_2$.

In conclusion, the whole nonlinear error system is semi-global practical finite-time stable.

Simulation

In this section, the experiments of the control system are executed in MATLAB R2014a. Numerical simulations are included to efficiently demonstrate the feasibility and effectiveness of the proposed scheme in robustness and transient performance for the USV. Detailed model information of the container ship is given in Table 1 and relevant control parameters are listed in Table 2.

Environmental disturbances

The external disturbances acting on USVs are mainly caused by the ocean and the induced wave generated by the adjacent vessels. The P-M wave spectrums are adapted to produce these disturbances, which have been defined as an International Towing Tank Conference standard. One harmonic wave component is now represented by five parameters: wave spectrum $S(\omega_a, \psi_r)$, direction $\psi_r$, frequency $\omega_a$, amplitude $\sqrt{2S(\omega_a, \psi_r)\Delta \omega \Delta \psi}$, and phase angle $\phi_{ab}$. The total surface elevation $h(x, y, t)$ of all wave components at the point $(x, y)$ at time $t$ for $N$ frequencies and $M$ directions will be $h(x, y, t) = \sum_{a=1}^N \sum_{b=1}^M \sqrt{2S(\omega_a, \psi_r)\Delta \omega \Delta \psi} \sin(\omega_a t + \phi_{ab} - k_a(x \cos \psi_r + y \sin \psi_r))$

where $k_a$ is the wave number, which equals $2\pi/\lambda_a$, $\lambda_a = 0.2583$ is the wave length.

Figure 2 shows waves with a significant wave height $H_s = 3$ m (sea state code is 5), which is caused by ocean. The waves shown in Figure 3 are composed of the waves in Figure 2 and the waves $H_s = 5$ generated by the adjacent USVs.

Sørensen has detailed the description and construction of the surge, sway force, yaw, and roll moment and they are omitted in this section.

Formation performance

A group of three USVs in a leader–follower formation is considered. The desired formation configuration is set as: $\rho_{d1} = \rho_{d2} = 15$ m, $\lambda_{d1} = -\pi/2$ rad, and $\lambda_{d2} = \pi/2$ rad. Initial states of the leader are given as $[x_L(0), y_L(0), \psi_L(0), \phi_L(0)] = [1m, 0m, 0^\circ, 0^\circ]^T$, $[u_L(0), v_L(0), r_L(0), p_L(0)] = [2m/s, 0m/s, 0^\circ/s, 0^\circ/s]^T$. And the trajectory of the leader is generated by the following kinematics equations

\[
\begin{align*}
\dot{x}_L &= u_L \cos \psi_L - v_L \cos \phi_L \sin \psi_L \\
\dot{y}_L &= u_L \sin \psi_L + v_L \cos \phi_L \cos \psi_L \\
\dot{\psi}_L &= r_L \cos \phi_L \\
\dot{p}_L &= p_L \\
\dot{r}_L &= -m_{21} u_L r_L - \frac{d_{22}}{m_{22}} v_L
\end{align*}
\]

where the yaw angular velocity $r_L$ of the leader satisfies that $r_L(t) = 2^\circ/s$ when $20s \leq t \leq 100$ and $r_L(t) = 0^\circ/s$ otherwise.
Initial states of follower 1 and follower 2 are given as \(x_1, y_1, \psi_1, \phi_1\) = \([-6m, 18m, -4^\circ, 5^\circ]^T\), \(x_2, y_2, \psi_2, \phi_2\) = \([-6m, -18m, 5^\circ, 0^\circ]^T\), and \([u_1, v_1, r_1, p_1]\) = \([u_2, v_2, r_2, p_2]\) = \([0m/s, 0m/s, 0^\circ/s, 0^\circ/s]^T\). For follower 1 and follower 2, control parameters are listed in Table 2. RBFNNs for \(f_u, f_v, f_r\), and \(f_p\) include 17 nodes, with width 2 and centers spaced in \([-5m/s, 5m/s]\) for the surge velocity, \([-2m/s, 2m/s]\) for the sway velocity, \([-5^\circ/s, 5^\circ/s]\) for the yaw velocity, \([-1^\circ/s, 1^\circ/s]\) for the roll velocity.

Formation trajectories of three USVs are shown in Figure 4. It can be observed that the prescribed formation can be guaranteed and followers are well tracked the leader under the uncertain dynamics and environmental disturbances.

Formation tracking errors of the followers are shown in Figure 5, where \(e_1, e_2, e_3, e_4\) represent the tracking error in position \(x, y\) and angle \(\psi, \phi\), respectively. From Figure 5, it can see that the tracking error and attitude error \(e_1, e_2, e_3, e_4\) are converge to a small neighborhood of the origin, which indicates the followers can track the desired trajectory with bounded errors by the proposed controller.

Figure 6 shows the learning behavior of RBFNNs. It can be observed that the model uncertainties in the surge, sway, yaw, and roll motion are efficiently compensated by the outputs of RBFNNs.

To verify the robustness of the proposed controller under systematical parametric uncertainties and environmental disturbances, the dynamic sliding surface is also displayed in Figure 7.

**Comparison results**

As can be seen from Figure 8, the error of range \(r_e\) between the leader and the follower 1 can rapidly converge to a
small neighborhood of the origin under the adjustment of these controllers, despite the disturbances and uncertainties, which indicated that these different controllers have stronger anti-disturbance ability. The sliding mode control (SMC) with parameter estimation is proposed to solve the problem of the unknown plant parameters and environmental disturbances in formation control, which enhances the robustness of the closed-loop system, and reduces the chattering of the control system by building a continuous function. However, roll motion is not considered in this method, and roll compensation cannot be carried out in close formation, while the horizontal plane error caused by rolling can only be indirectly compensated through surge and yaw control, so it can be seen in Figure 8 that the formation error is always large. The hierarchical sliding mode control (HSMC) with fin roll reduction is employed to deal with the underactuation of the system in the presence of uncertainties and nonlinearities. Although the roll motion is effectively controlled, the PD sliding surface still causes the chattering of the control system. In addition, this method cannot guarantee the formation error convergence in a finite time, which increases the possibility of collision among adjacent USVs. The terminal sliding mode control (TSMC) with a finite-time observer is addressed for trajectory tracking of USVs with complex unknowns disturbances. This TSMC is modified by adding the roll motion of freedom to be more suitable for performance comparison with the other methods. From Figure 8, since this method includes the finite-time disturbance observer and roll motion control, the ocean disturbances are compensated in a finite time and the roll parameter oscillation is greatly reduced, so the formation error is less than the SMC and HSMC. Meanwhile, from the enlarged image in Figure 8, this method has faster convergence speed than the SMC and HSMC. The proposed THSMC can quickly ensure a small formation error in a finite-time for vessels by applying roll stabilization technology, finite time stability theory, and robustness sliding mode control. Meanwhile, a new switching gain adaptation mechanism is utilized to reduce chattering and acquire faster adaptive rate without the excessive temporary tracking errors. Therefore, the proposed THSMC has faster convergence rate and smaller formation error than the above methods.

Figure 7. Time evolution of the sliding surface with wave disturbances.

Figure 8. Formation tracking errors of the leader and follower 1 under different control schemes.

Figure 9. Formation orientation errors of the leader and follower 1 under different control schemes.

Table 3. RMS values of formation range errors and orientation errors.

| Control strategies | Formation range errors | Orientation errors |
|-------------------|------------------------|-------------------|
| SMC\textsuperscript{11} | 0.5824 | 0.2505 |
| HSMC\textsuperscript{32} | 0.3145 | 0.2276 |
| TSMC\textsuperscript{33} | 0.1925 | 0.1659 |
| Proposed THSMC | 0.1593 | 0.1208 |

RMS: root mean square; SMC: sliding mode control; HSMC: hierarchical sliding mode control; TSMC: terminal hierarchical sliding mode control.

Conclusion

In this article, a novel control scheme is presented to accomplish the leader–follower close formation control for USVs under the model uncertainties, roll motion, and environmental disturbances. To ensure the formation of USVs within a finite time, THSMC technology is designed and a new switching gain adaptation mechanism is applied to
reduce chattering and acquires faster adaptive rate. Furthermore, RBFNN is used to approximate the model parameters which vary with their own state. In the meantime, FTO is employed to quickly and accurately estimate the disturbances from the winds, waves, currents, and the adjacent vessels with zero error. Besides this, the derivative of the virtual control law is more complex than the original one after considering the roll motion. To simplify the control law, the DSC is introduced. Finally, simulations and comparison results show that the proposed method has better formation control effect.

In the future, our work will consider more constraints, such as input saturation, dead zone, actuator faults, and so on. Avoidance among adjacent vessels and obstacles may also be considered in close formation. Besides, since the plant in this article is 4-DoF, it is necessary for rollover prevention to avoid excessive roll motion.

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ORCID iD
Huizi Chen https://orcid.org/0000-0002-0418-5730

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