Deformation regularities of carbon fiber reinforced plastic under time variable loading

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Abstract. The effect of strain rate on the stress-strain deformation process of unidirectional composites was investigated. As taking into account the influence of structural and technological factors, engineering methods of identification for mechanical characteristics were created. Describing the mechanical properties of a laminate for a given structure was usually carried out based on the regularities of its constituent layers. The analysis of the viscoelastic properties of composite materials showed that along with the manifestation of rheological properties, a physical nonlinearity was observed, which may be a consequence of shear stresses. This paper is devoted to the development of a phenomenological model of the hereditary type, which makes it possible to evaluate the anisotropy of the rheological properties of a unidirectional composite by the results of tests over a wide range of strain rates. To obtain the constitutive equations, we used the relations of the anisotropic theory of elasticity, the Volterra correspondence principle and the algebra of resolvent operators, which simplify the transformation of the equations. To derive the constitutive equations for the layer, it was assumed that the dependence of the properties on time in the reinforcement direction can be neglected. In the direction perpendicular to the reinforcement, hereditary-elastic behaviour was assumed. The comparison of calculated data and experimental results was presented.

1. Introduction

Carbon fiber reinforced plastics have become widespread as structural materials for elements of various structures. It should be noted that most of the research devoted to the deformation and failure of composite materials relating to the study of mechanical properties under quasi-static loading. It should be particularly emphasized that physical nonlinearity of the layer and multilayered carbon fiber reinforced plastics (CFRP) is concerned with nonlinear properties of the layer under intralaminar shear. Recently a significant number of papers have been devoted to the study of the time dependent properties of polymer composites. In this paper, an attempt to create a structurally phenomenological model for constructing the constitutive equations, which makes it possible to describe the anisotropy of the rheological properties of unidirectional carbon-epoxy plastics.

The CFRP structure elements in service are usually under time variable loading. In connection with this, the study of the regularities of the mechanical behavior of CFRP is really important. Most of papers have been devoted to the experimental and theoretical investigation of the time dependent properties of CFRP [1-4]. In study [2] stress-strain curves of unidirectional carbon epoxy plastics...
AS4/3501-6 with the different strain-rate on compression under different angles between the reinforcement and loading direction have been obtained and its experimental data was used for analysis. The digitized results of tests are presented in Figure 1. It can be seen notable sensitivity to strain-rate for almost all the angles. The most obvious difference in stress-strain curves is observed at angles equal to 30° and 45°, and physical nonlinearity is manifested for all angles with the various directions of the loads. When loading in the reinforcement direction the main stress is in brittle stiff fibers and in consequence of which time-dependent properties can be neglected.

![Figure 1. Stress-strain curves carbon epoxy AS4/3501-6 under compression at angles 15° (180 s⁻¹; 0.75 s⁻¹; 10⁻⁷ s⁻¹), 30° (330 s⁻¹; 0.9 s⁻¹; 10⁻⁴ s⁻¹), 45° (370 s⁻¹; 0.8 s⁻¹; 10⁻⁴ s⁻¹), 60° (380 s⁻¹; 1 s⁻¹; 10⁻⁴ s⁻¹), 75° (400 s⁻¹; 0.85 s⁻¹; 10⁻⁴ s⁻¹) and 90° (380 s⁻¹; 0.8 s⁻¹; 10⁻⁴ s⁻¹)](image)

To describe the influence of the strain-rate on the stress-strain curves, we use the Volterra principle [6], which consists in replacing the elastic moduli of the layer with the corresponding operator expressions. The integral operators with Abel kernel [6], Rabotnov fraction-exponential kernel [6], Prony series [7] can be used to derive the corresponding expression. The algebra of resolvent operators makes it possible to obtain the relations allowing one to describe the behavior of composite materials under time variable loading.

In this study, the operator expression of elasticity modulus was obtained and regularities of CFRP (AS4/3501-6) samples with angles 45° and 90° were investigated.

2. Model Description

The known formula of orthotropic modulus dependence of angle can be written as

\[
E_\theta = \frac{E_1}{m^4 + \frac{E_1}{G_{12}} - 2\nu_{12}} m^2 n^2 + \frac{E_2}{E_1} n^4,
\]

(1)

where \(E_1, E_2, G_{12}, \nu_{12}\) - moduli of elasticity of the layer, \(m = \cos \theta, n = \sin \theta\) - trigonometric functions of the angle \(\theta\) to the direction of main axis of orthotropy.
The property of resolvent operators in theory of hereditary elasticity has been used to take into account the influence of time-dependent effects. Using the Volterra principle, we assume that the operator expressions for the moduli can be represented as \( \frac{1}{E^*} = \frac{1}{E_0^*} \left(1 + \frac{1}{\lambda}K^*\right) \), where \( K^* \) is hereditary operator, \( E^* \) is instantaneous modulus of elasticity. In the absence of temporal effects, the value of \( E^* \) can be determined from stress-strain curves under quasistatic loading.

We assume that the hereditary elastic properties are conditioned by shear and loading in the direction perpendicular to the reinforcement. In the direction of the reinforcement, the influence of the temporal effects can be considered negligibly small. Operator form of hereditary modulus corresponding (1) can be written as follows

\[
E^*_o = \frac{E_1}{m^4 + \left(\frac{E_1}{G_0} - 2\nu_{12}\right)\nu_{12}^2 + \frac{E_1}{E_2} \left(1 + K^*\right)^n}
\]

where the operator with Abel kernel is chosen as the hereditary operator which determines the time dependent properties of the material: \( K^*f = I^*_o f = \frac{1}{\Gamma(1+\alpha)} \int_{0}^{t} (t - \tau)^\alpha f(\tau) d\tau \), the singularity parameter must satisfy the condition \(-1 < \alpha < 0\).

The operator expression of the elasticity modulus can be written as

\[
E^*_o = \frac{E_1}{A_\theta + B_\theta I^*_o}
\]

where \( A_\theta = m^4 + \left(\frac{E_1}{G_0} - 2\nu_{12}\right)\nu_{12}^2 + \frac{E_1}{E_2} n^4 \), \( B_\theta = \frac{E_1}{G_0} m^2 n^2 k_{12} + \frac{E_1}{E_2} n^4 k_{s2} \).

Those operators belong to the class of resolvent ones and can be represented as [6]

\[
(1 + k_\theta I^*_o)^{-1} = 1 - k_\theta \mathcal{C}_\alpha \left(-k_\theta\right),
\]

where \( \mathcal{C}_\alpha \left(-k_\theta\right) \) is Rabotnov fraction-exponential function and resolvent with respect to Abel operator \( I^*_o \) [6].

From (4), an explicit operator expression can be obtained for the modulus of hereditary elasticity:

\[
E^*_o = E_0^* \left(1 - k_\theta \mathcal{C}_\alpha \left(-k_\theta\right)\right)
\]

where \( E_0^* = \frac{E_1}{A_\theta} \) is instant modulus, \( k_\theta = \frac{B_\theta}{A_\theta} \) is parameter and

\[
\mathcal{C}_\alpha \left(-\beta, t\right) = t^\alpha \sum_{n=0}^{\infty} \frac{(-\beta t^{\alpha})^n}{\Gamma\left[1 + \alpha (1 + n)\right]}, \quad \beta > 0, \quad \Gamma\left(\cdot\right) - \text{gamma-function}. \quad \text{The last relation expresses the action of fraction-exponential function on Heaviside unit step function. The kernel of fraction-exponential operator is:} \quad \mathcal{C}_\alpha \left(-\beta, t\right) = t^\alpha \sum_{n=0}^{\infty} \frac{(-\beta t^{\alpha})^n}{\Gamma\left[1 + \alpha (1 + n)\right]}
\]

The determination of the parameters in (2) can be carried out sequentially on the basis of the results of testing the strain-stress curves of samples being cut with 90° and 45°. At angle 90° from relation (2) can be obtained the following relationship
\[ \varepsilon_2 = \frac{1}{E_2^0} (1 + k_2 \varepsilon_2^*) \sigma_2 \]  

(6)

Inversing equation (6) with the aid of (4) the following relation can be obtained

\[ \sigma_2 = E_2^0 \left( 1 - k_2 \mathcal{J}_a^* (-k_2) \right) \varepsilon_2. \]  

(7)

Constitutive equation describing stress-strain curves under strain rate loading is derived

\[ \sigma_2 = E_2^0 \left[ 1 - k_2 \left( \frac{\varepsilon_2}{\dot{\varepsilon}} \right)^{1+\alpha} \sum_{n=0}^{\infty} \frac{\varepsilon_2}{\dot{\varepsilon}^{1+\alpha}} \frac{1}{\Gamma \left[ 2 + (1 + \alpha)(1+n) \right]} \right] \varepsilon_2, \]  

(8)

In (8) it was accounted for strain rate: \( t = \frac{\varepsilon}{\dot{\varepsilon}} \). Expression in square brackets of (8) defines the degree of modulus decreasing depending on the strain rate. From the previous experience it can be accepted that within the experiment errors for CFRP singularity parameter may be taken -0.9. It is also known [6] that at small values of time, a fractional-exponential kernel is close to the Abel kernel, i.e. \( \mathcal{J}_a (-\beta, t) \approx I_a (t) \). Then, the constitutive equation takes the simplified form

\[ \sigma_2 = E_2^0 \left[ 1 - \frac{k_2}{\Gamma (3+\alpha)} \left( \frac{\varepsilon_2}{\dot{\varepsilon}} \right)^{1+\alpha} \right] \varepsilon_2, \]  

(9)

Using (9) other parameters can be easily evaluated: \( E_2^0 = 15.5 \) GPa, \( k_2 = 0.2018 \ s^{-(1+\alpha)} \).

A comparison of the calculated and experimental data is shown in figure 2. In figure 2 it is noted that the strain values are about 1.3% relatively weak deviation from the linear law. For small strain rate, it is necessary to use Rabotnov fraction-exponential function as a kernel in the constitutive equations.

For determining the other parameters of the model we will use some of the elastic characteristics of the unidirectional AS4/3501-6 taken from [8]: \( E_1 = 126 \) GPa, \( E_2 = 11 \) GPa, \( G_{12} = 6.6 \) GPa, \( \nu_{12} = 0.28 \). It has been adopted that in the direction of reinforcement there is no dependence properties on time instant modulus being equal to \( E_1 \), also the value of Poisson ratio remains steady.

**Figure 2.** Strain-stress curves of CFRP samples with the angle of 90° (calculation and experiment)
For determination the remained parameters of the model \( G_{12}^0 \) and \( k_{12} \), characterizing rheological characteristics of unidirectional carbon reinforced plastic AS4/3501-6 the constitutive equation at \( \theta = 45^\circ \) can be written as

\[
\sigma_{45} = E_{45}^0 \left[ 1 - k_{45} \left( \frac{\varepsilon_{45}}{\dot{\varepsilon}} \right)^{1+\alpha} \sum_{n=0}^{\infty} \frac{\left( -k_{45} \frac{\varepsilon_{45}}{\dot{\varepsilon}} \right)^n}{2 + (1 + \alpha)(1 + n)} \right] \varepsilon_{45} \quad (10)
\]

The procedure has been similar to data processing for samples \( \theta = 90^\circ \) gives: \( E_{45}^0 = 25.15 \) GPa, \( k_{45} = 0.2564 \) s\(^{(1+\alpha)}\). A comparison of the calculated data within linear area and experimental results is shown in figure 3. It should be noted that for model completing necessary to broaden the model in area of nonlinear strain.

In figure 3 it is seen that approximately to strain value of 1% the calculated data and experimental results have a satisfactory agreement.

With the values being obtained, the parameters of the constitutive equations are derived, which makes it possible to describe the in-plane shear of the lamina. The parameters of \( A_{45} \) and \( B_{45} \) have been used respectively \( A_{45} = \frac{E_1}{E_{45}^0} \) and \( B_{45} = k_{45} A_{45} \). Then, from the following expressions we can obtain 

\[
A_{45} = 1 + \frac{1}{4} \left( \frac{E_1}{E_{12}^0} - 2\nu_{12} \right) + \frac{1}{4} \frac{E_1}{E_2} \quad \text{and} \quad B_{45} = \frac{1}{4} \frac{E_1}{E_{12}^0} k_{12} + \frac{1}{4} \frac{E_1}{E_2} k_2 ,
\]

the instantaneous shear modulus \( G_{12}^0 \) and the parameter \( k_{12} \) are also obtained. So

\[
G_{12}^0 = \left[ \frac{4}{E_{45}^0} \frac{1 - 2\nu_{12}}{E_1} - \frac{1}{E_2} \right]^{-1}
\]

\[
k_{12} = \frac{G_{12}^0}{E_1} \left[ 4B_{45} \frac{E_1}{E_2} k_2 \right] \quad \text{can be derived. The required values turned out to be} \quad G_{12}^0 = 10.1 \text{ GPa},
\]

\[
k_{12} = 0.301 \text{ s}^{(1+\alpha)}.
\]

Figure 3. Calculation of the deformation curves in the linear region and the experimental results of the deformation curves under loading at an angle of 45°.
3. Summary and conclusions
Linear model based on constitutive equations of hereditary mechanics allows one to predict anisotropy of mechanical properties of unidirectional CFRP under time-variable loading has been suggested. To derive constitutive equations the relations of anisotropic theory of elasticity, Volterra correspondence principle and relationships of resolvent operators’ algebra have been used. The offered approach has been applied to experimental readings of unidirectional AS4/3501-6 CFRP. In linear area a satisfactory agreement between the experimental results and calculated data has been obtained.

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