Analytical Modeling of Modulated Rotating-Blade Noise: the Skipping Rope and the Darrieus Wind Turbine

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Abstract. The present study is about the modulations of the aerodynamic broadband noise heard from slowly rotating rotors with few blades. It is aimed at producing a fast-running prediction tool that could be used to assess the nuisance of Vertical-Axis Wind Turbines (VAWT) in the context of urban installation. The simpler case of a skipping rope that only involves part of the modulation effect is addressed as a first step. The rope is split into segments for which an instantaneous sound-radiation model exists to yield an overall spectrogram. Finally, its time signature is reconstructed by additive synthesis. After inspection of existing databases from oscillatory-airfoil experiments, needs for data representative of the dynamic stall of VAWT blades are seen as the missing block to reconstruct a synthetic spectrogram.

1. Introduction

Sound radiation from rotating-blade technologies with small numbers of blades and rotational frequencies well below the audibility range involves modulation effects which make the annoyance much larger than for stationary sounds of same averaged physical level. Estimating these effects is as crucial as modeling the basic aerodynamic sound sources in such cases. The modulations occur at the rotational frequency and are a matter of concern as the latter is typically of the order of magnitude of 1 Hz. They are in fact twofold. The first modulation results from the relative motion of a blade with respect to the observer, combined with the directivity of the radiated sound; the second one takes place if the flow features responsible for the sound emission vary over the revolution of a blade. Both are referred to as the acoustic and aerodynamic modulations for convenience in this work, respectively. They are well known for the large, conventional three-bladed wind turbines the dominant noise contribution of which is the broadband trailing-edge noise caused by the convection of boundary-layer turbulence past the trailing edges of the blades. Darrieus-type wind turbines, referred to as VAWT (Vertical-Axis Wind Turbines) involve a more pronounced aerodynamic modulation because the large cyclic variations of the relative angle of attack on a blade often lead to dynamic stall. Furthermore each blade interacts with the wakes
and shed vortices of other blades, on the one hand, and with the wake of the mast, on the other hand. This complexity makes the generated noise much higher than basic trailing-edge noise, as confirmed by predictions based on numerical flow simulations [11, 12] and by dedicated measurements [13]. It also makes the development of analytical models quite challenging when such models would be a real need for the assessment of VAWT-noise impact on the environment. Indeed the VAWT is a possible technology that could be used in urban areas to recover wind energy, provided that the associated aerodynamic noise is shown to remain below acceptable levels. Analytical models are good candidates to answer the question in the context of urban planning. Furthermore they provide physical insight from which basic low-noise design rules could be defined.

The swishing sound from a skipping rope involves the same acoustic modulation as an isolated VAWT blade but no aerodynamic modulation. Indeed the vortex shedding produced by the motion of the rope through the air and responsible for the sound is a stationary aerodynamic feature. Yet both examples are very similar, also because the arc of the rope and the best shape for a full VAWT blade are similar [4]. This motivated the authors in considering the skipping rope as a test case for a general methodology aimed at including modulations in basic analytical models of aeroacoustic sources. Aeroacoustics of the skipping rope is first addressed in section 2 based on a vortex-shedding sound model for a rigid cylinder of circular cross-section. It is verified that a realistic time signal can be reconstructed by additive synthesis. The specific additional features of the VAWT are addressed in the preliminary discussion of section 3, where dynamic stall is considered as a basic aerodynamic modulation, from reported experiments. Key requirements for future analytical modeling of the sources are identified.

2. Skipping Rope Model

2.1. Vortex-Shedding Sound Model

The swishing sound of a skipping rope is the acoustic signature of the von Kármán vortex shedding that takes place in its wake. For each elementary segment of the rope the sound is in fact a random, narrow-band emission, the dominant frequency of which \( f_0 \) is defined by the Strouhal number \( St = f_0 D/U_0 \approx 0.2 \), where \( U_0 \) is the local relative flow speed normal to the segment and \( D \) the diameter of its cross-section. The associated far-field acoustic intensity is expressed in a reference frame attached to the segment (see Fig. 1-left) by Goldstein’s formula [1]

\[
I(x) = \frac{CL}{r^2} \sin^2 \theta \cos^2 \phi, \quad C = \frac{\sqrt{2\pi} St^2 (\chi^2/\ell/D) \rho_0 D U_0^6}{32\pi^3} \quad (1)
\]

where \( r \) is the distance from the mid-point of the segment to the observer at point \( \mathbf{x} \), \((\theta, \phi)\) the spherical angles shown in the figure, \( L \) the length of the segment, \( \rho_0 \) the density of air, \( \ell \) the correlation length of the random vortex shedding and \( \chi \) an unsteady lift coefficient which expresses the strength of the equivalent dipole. According to reported experiments the factor \((\chi^2/\ell/D)\) is about 1. The formula holds for a cylinder of large aspect ratio \( L/D \) in the limit of low Mach number \( M_0 \ll 1 \), with \( M_0 = U_0/c_0 \), for an observer in the acoustic and geometrical far field.

2.2. Instantaneous Spectra of Rope Segments

At any time a change of reference frame is required to go from the stationary frame (defined in Fig. 1-right) to the moving frame attached to a segment (defined in Fig. 1-left) in which Eq. (1) is expressed. Geometric considerations lead to the passage formulæ

\[
r^2 = h^2 + z^2 + R^2 - 2zR \cos \Theta - 2hR \sin \Theta \cos(\Phi - \Omega t),
\]

\[
\sin^2 \theta = \left( \frac{z - R \cos \Theta}{r} \right)^2 + \left[ \frac{h}{r} - \frac{R \sin \Theta}{r} \cos(\Phi - \Omega t) \right]^2,
\]

\[2\]
Figure 1. Left: reference frame attached to a moving cylinder, with associated source and observer coordinates. Right: instantaneous rope configuration and main coordinates associated with a segment of mid-span point coordinate \( z \).

\[
\cos^2 \phi = \frac{(\cos \zeta (h - R \sin \Theta \cos (\Phi - \Omega t)) + \sin \zeta (z - R \cos \Theta))^2}{(z - R \cos \Theta)^2 + [h - R \sin \Theta \cos (\Phi - \Omega t)]^2},
\]  

(4)

in which \((R, \Theta, \Phi)\) stand for the spherical coordinates of the observation point \( x \) in the axes \( e_X, e_Y, e_Z \), the rope being assumed in a rotating meridian plane in Fig. 1-right. \( h(z) \) is the radius of the circular path of a segment and \( h_m \) its maximum value at \( z = 0 \). Though the actual shape of the rope arc is a Jacobi’s elliptic function \([4]\) it can be assimilated to a sine arc of equations

\[
\zeta = \frac{\pi}{L_{AB}} \cos \left[ \frac{\pi}{L_{AB}} (z + \frac{L_{AB}}{2}) \right],
\]

where \( L_{AB} \) denotes the straight distance between points A and B.

The last needed information to generate a complete time-frequency analysis of the sound is the spectrum to be associated with Eq.(1). Measurements of the vortex-shedding sound from cylinders tested in aeroacoustic facilities make evidence of a narrow-band spectral peak that can be approximated by a double-exponential shape when plotted in log-frequency scale \([5]\). This signature has been reproduced by numerical simulations \([7]\). Sound spectra in the sub-critical regime obtained from either measurements or simulations are plotted as functions of the Strouhal number in Fig. 2-a. An acceptable collapse is obtained as the power spectral density is scaled from Goldstein’s formula, which also confirms the relevance of the latter. A generic expression can be proposed to fit the results as

\[
10 \log_{10} \left[ \frac{r^2 S_{pp}}{L D U_0^6} \right] \approx 50 e^{-7 |\log_{10}(St) - \log_{10}(0.2)|} - 35
\]

from which a prediction of the quantity \( S_{pp}(\omega) \) is enabled. The fit is plotted as the dashed black line in the figure. Typical time variations of the sectional lift and drags forces deduced from numerical simulations are also shown in Fig. 2-b to illustrate the vortex-shedding modulation. The model spectral shape applied to any segment of the skipping rope allows synthesizing the total acoustic signature. Because of the turbulent character of the von Kármán vortex street the associated acoustic energy is spread around the Strouhal frequency \( 0.2 U_0/D \). Therefore the mean-squared pressure should be arbitrarily integrated between, say \( St = 0.1 \) and \( St = 0.4 \) in the present model for the integrated value to be equivalent to the quantity \( \rho c_0 I(x) \) of Eq. (1).

At any time step and for a given rope segment at the azimuth \( \Psi = \Omega t \), the contribution to the sound-pressure Power Spectral Density (PSD) reads

\[
S_{pp}(x,\omega') = \frac{L D U_0^6}{r^2} F(\omega') \sin^2 \theta \cos^2 \phi,
\]
Figure 2. (a): measured or computed far-field sound pressure PSD for rigid cylinders of large aspect ratios \(L/D\) (ECL data and [5]). (b): computed unsteady sectional lift and drag coefficients [6].

where \(F(\omega')\) stands for the expression of the spectral shape in the physical variables deduced from the aforementioned exponential fit. Once expressed in the stationary reference frame this contribution is an explicit function of time, keeping in mind that \(\omega'\) denotes the emitted angular frequency in the moving reference frame, different from the received frequency because of the Doppler effect and related to it by the equation

\[
\omega = \frac{\omega'}{1 - M_r}, \quad M_r = \frac{\Omega h}{c_0} \sin \frac{R}{r} \sin(\phi - (\Omega t)),
\]

\(M_r\) being the relative Mach number of the segment in the direction of the observer. The transposition to the stationary reference frame completed by an azimuthal averaging has been defined by Schlinker & Amiet [2] and revised by Sinayoko et al. [3]. It leads to the averaged PSD

\[
S_{pp}(x, \omega) = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{\omega'}{\omega}\right)^2 S_{pp}(x, \omega') \, d\Psi
\]

2.3. Time-Frequency Prediction of Rope Sound

The vortex-shedding sound model of previous sections has been implemented by just discretizing the rope arc into \(N\) segments of same axial length \(L/N\). The test case shown in Fig. 3-a corresponds to an observer distance of 5 m, a rope end-to-end length \(L_{AB} = 2\) m, a maximum arc height at rope center \(h_m = 0.8\) m and a whirling frequency \(f_W = 2\) Hz. The diameter of the rope is 4 mm and the observation angle 85°. Three whirling periods are shown, each of which features two humps separated by sharp extinctions. Indeed the observer goes close to the extinction planes of the individual dipoles associated with the segments twice per revolution. The humps correspond to the passages of the directivity lobes of the lift dipoles, occurring for the shortest and longest source-to-observer distances. It must be noted that the averaged drag cannot contribute at very low Mach number and that the fluctuating drag has a negligible amplitude as emphasized by numerical simulations reproduced in Fig. 2-b [6]. Therefore the sound from the drag is not included in the present model. Setting \(N = 20\) generates an artificial sawtoothed first edge of the broad spectral peak because of the jumps
Figure 3. (a): typical spectrogram of the vortex-shedding sound of a skipping rope as generated by the analytical model. $N = 20$ segments (blue-gray mesh) and $N = 120$ segments (red mesh). Main parameters indicated on the plot. (b): time signal synthesized from the spectrogram ($N = 120$ segments, two random-phase runs as red and black plots).

of the relative flow speed between adjacent segments. The discontinuities are smoothed with a finer discretization $N = 120$ without modification of the overall envelope, which indicates that the model is robust whatever the value of $L/D$ could be despite $L/D$ is assumed large in Goldstein’s model. The reason for this is that the correlation length is already accounted for in Eq. (1). A similar approach can therefore be applied to any configuration for which random sound sources are spanwise distributed on a body with some characteristic correlation length explicitly included in the model: the acoustic intensity is found proportional to the product of the span and correlation length. In particular, airfoil trailing-edge noise and stall noise mechanisms have this property [8, 9].

A typical time signal as produced by additive synthesis and found convincing when heard is plotted in Fig. 3-b. All frequency components have been considered with time-varying amplitudes according to the spectrogram and summed after being given individually a random phase, uniformly distributed between $0$ and $2\pi$. The same period of the signal has been repeated twice and two runs are shown (red and black plots) to feature variations. As emphasized by the zoom the signal has a clear resulting frequency with strong amplitude modulations; the same qualitative feature was observed for the sectional lift in the simulation of the source term reported in Fig. 2-b.

3. Aeroacoustics of the Darrieus Wind Turbine

Unlike the skipping rope, the Darrieus wind turbine also involves modulated aerodynamic sound sources because of the periodically varying angle of attack. This effect combines with the modulation induced by the motion relative to the observer.

3.1. Unsteady Aerodynamics and Dynamic Stall

The aerodynamic features of a Darrieus wind turbine with rectangular vertical blades as illustrated in Fig. 4-a depend on the velocity ratio $\xi = \Omega R_0/U_0$ where $\Omega$ is the rotational speed, $R_0$ the radius and $U_0$ the wind speed. For the sake of mathematical modeling the latter is
assumed along the axis $e_Y$ and the azimuth of a reference blade centre-point is $\Omega t$, assuming that the point is on the axis $e_X$ at the origin of time. The instantaneous velocity vector relative to a blade therefore reads

$$W = \Omega R_0 \sin(\Omega t) e_X + U_0 \left(1 - \xi \cos(\Omega t)\right) e_Y$$

and the associated instantaneous angle of attack $\alpha(t)$ is such that

$$\cos(\alpha(t)) = \frac{U_0}{W} \left(\xi - \cos(\Omega t)\right), \quad W = |W| = U_0 \left[1 + \xi^2 - 2 \xi \cos(\Omega t)\right]^{1/2},$$

$$\tan(\alpha(t)) = \frac{\sin(\Omega t)}{\xi - \cos(\Omega t)}. \quad (6)$$

Cyclic variations of the angle of attack on a blade according to Eq. (6) are plotted in Fig. 4-b for moderate and high values of the velocity ratio $\xi$. For $\xi = 2$ dynamic stall probably occurs. This regime is identified as a priority for urban applications because of the expected louder acoustic signature. Following the procedure illustrated in section 2 for the skipping rope, model sound synthesis dedicated to acoustic perception estimates requires a theoretical spectrogram as a first step. Simple model expressions have been proposed only for low frequencies and static-airfoil stall noise, for instance by Moreau et al. [9]; a similar model for a wind-turbine blade entering dynamic stall is still to be developed. This is why a data set from the research program PIBE is used in section 3.2 for a preliminary investigation.

### 3.2. Oscillatory-Airfoil Results

The French program PIBE dedicated to large, conventional horizontal-axis wind turbines includes an oscillatory-airfoil experiment. The measurements have been performed at 50 m/s in the open-jet anechoic facility of ECL on a NACA-0012 airfoil of 12 cm chord and 30 cm span in a nozzle flow of width 40 cm. The airfoil is tripped in order to avoid tonal noise associated with unstable laminar boundary layers. The chord-based Reynolds number is $4 \times 10^5$, the oscillation frequency 1.33 Hz and the oscillation amplitude is $7^\circ$ around the geometrical angle of attack $15^\circ$. Similar Reynolds number and conditions have been investigated by Mayer et al. [10] in a Kevlar-walled wind-tunnel facility. The time-frequency analysis is reported in Fig. 5. Four periods of
Figure 5. Far-field sound time-frequency analysis for an oscillating NACA-0012 airfoil. Geometrical angle of attack $15^\circ + 7^\circ \sin(1.33 \times 2\pi t)$. (a): spectrogram. (b): instantaneous spectra (raw in black and filtered in red) every 1/20 of the period of oscillation. Arbitrary dB scale. Courtesy of the research program PIBE.
reference [9] tuned on the results in Fig. 5-b and a classical trailing-edge noise model [8]. In contrast this would not be relevant for higher oscillating frequencies. This confirms that only oscillatory-airfoil experiments are able to produce reliable results for the definition of a model.

4. Concluding Remarks

A methodological background for the prediction of the time-frequency signature of VAWT blades has been established, starting from the simpler case of a skipping rope. The short-term objective is to propose a fast-running tool that could be run by non-expert users for noise exposure assessment in urban areas provided that main parameters such as chord length and rotational speed are specified. The first step of the approach is to split each blade into segments for which a locally uniform flow is reasonably assumed. If an instantaneous sound-radiation model is available for each segment a global model spectrogram can then be reconstructed. The second step is to generate a time signal by standard additive synthesis that could be played back to a panel of listeners. For the dynamic stall of VAWT blades a complete instantaneous model is still missing. It could be empirically deduced from measured data in oscillatory-airfoil experiments.

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