Phenomenology of Axial-Vector Mesons from an Extended Linear Sigma Model *

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We discuss the phenomenology of the axial-vector mesons within a three-flavour Linear Sigma Model containing scalar, pseudoscalar, vector and axial-vector degrees of freedom.

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1. Introduction

A correct description of axial-vector mesons is important in Quantum Chromodynamics (QCD) for several reasons. The lightest measured axial-vector meson – the $a_1(1260)$ resonance – is experimentally ambiguous already in vacuum: according to listings of the Particle Data Group (PDG\cite{pdg}), the decay width of $a_1(1260)$ possesses values between 250 MeV and 600 MeV. There is only one class of mesons distinct from axial-vectors with similar values of decay widths: the scalar mesons, the theoretical and experimental description of which is famously ambiguous (see, e.g., Refs.\cite{2,3,4,5}). The large decay width renders experimental as well as theoretical determination of the properties of $a_1(1260)$ rather problematic. Additionally, it is well-known that the QCD Lagrangian with $N_f$ massless quark flavours possesses an exact $SU(N_f)_A$ axial symmetry. Consequently, a conserved axial-vector current of the form $\bar{q}_f\gamma^\mu\gamma_5t^iq_f$ arises, where $q_f$

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denotes a quark flavour, \( \gamma^\mu \) and \( \gamma^5 \) are Dirac matrices and \( t^i \) represent generators of the chiral \( SU(N_f)_L \times SU(N_f)_R \) chiral group with \( N_f \) flavours, \( i = 1, \ldots, N_f^2 - 1 \). An axial rotation of this current leads to a conserved vector current of the form \( \bar{q}_f \gamma^\mu t^i q_f \). Identifying (putatively) the latter two currents with the \( a_1(1260) \) and \( \rho(770) \) mesons, respectively, leads to the assertion that the latter two resonances should be degenerate in vacuum. The opposite is observed due to the Spontaneous Breaking of the Chiral Symmetry \([6]\); however, the mentioned degeneration is expected to be restored at finite temperatures and densities (the so-called chiral transition). A viable theoretical description of this postulated high-temperature phenomenon requires a satisfactory description of axial-vectors already in vacuum – and such a description is an objective of this article. (Note that claims have been made \([7]\) that the vector current is actually a mixture of two rotated chiral-partner fields, an axial-vector and a pseudovector one. As a first approximation, we will neglect this possibility.)

In this article, we present a Linear Sigma Model containing scalar, pseudoscalar, vector and axial-vector mesons both in the non-strange and strange sectors (extended Linear Sigma Model or eLSM\([3, 5, 8, 9, 10]\)). The model contains only \( \bar{q}q \) states \([3, 4, 5]\) rendering it appropriate to study not only general features (masses/decays) of mesons but also their structure – if a physical resonance can be accommodated within our model, then it possesses the \( \bar{q}q \) structure. This criterion is important for scalars \([4, 8]\) but also for axial-vectors considering the claims that, e.g., \( a_1(1260) \) represents a meson-meson molecule rather than a quarkonium \([11]\). Consequently, this paper will consider the issue whether the \( a_1(1260) \) meson can be accommodated within eLSM, i.e., if \( a_1 \) can be described as (predominantly) a \( \bar{q}q \) state.

The outline of the paper is as follows. In Sec. 2 we present the three-flavour Lagrangian with vector and axial-vector mesons. Consequences of a global fit of observables for all states present in the model except scalar isosinglets and \( K_1 \) are presented in Sec. 3. We provide our conclusions in Sec. 4.

2. The Model

The Lagrangian of the Extended Linear Sigma Model with the chiral \( U(3)_L \times U(3)_R \) symmetry (eLSM) reads \([3, 5, 8, 9, 10]\):

\[
\mathcal{L} = \text{Tr}[\left(D^\mu \Phi \right)\left(D^\nu \Phi \right)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 [\text{Tr}(\Phi^\dagger \Phi)]^2 \\
- \frac{1}{4} \text{Tr}(L^{\mu\nu})^2 + (R^{\mu\nu})^2 + \text{Tr} \left[ \left( \frac{m_1^2 + \Delta}{2} \right) (L^\mu)^2 + (R^\mu)^2 \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
+ c_1 (\text{det} \Phi - \text{det} \Phi^\dagger)^2 + i \frac{g}{2} \left[ \text{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) + \text{Tr}(R_{\mu\nu}[R^\mu, R^\nu]) \right] \\
+ \frac{\sqrt{2}}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}((L^\mu)^2 + (R^\mu)^2) + h_2 \text{Tr}[\Phi R^\mu \Phi^\dagger L^\mu + 2h_3 \text{Tr}(\Phi R^\mu \Phi^\dagger L^\mu]] (1)
\]
assignment \( \vec{a} \) GeV in the physical spectrum \( [1] \); results from our model prefer the latter K observe that mixing terms containing axial-vectors and pseudoscalars and \( \sigma \) to \( \vec{a} \) states; these states, together with the non-strange isovector state \( K \). The isoscalar fields \( \sigma \) assumed). Explicit symmetry breaking in the (pseudo)scalar sector is described by the matrix containing the scalar and pseudoscalar degrees of freedom, \( L^\mu = V^\mu + A^\mu \) and \( R^\mu = V^\mu - A^\mu \) are, respectively, the left-handed and the right-handed matrices containing vector and axial-vector degrees of freedom with

\[
V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \omega_\mu \left[ K^0 + iK^+ \right] \\ K^+ + iK^- \end{pmatrix}, \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} f_1^{\eta} + \eta \nu \left[ K^0 + iK^+ \right] \\ f_1^{\eta} - \eta \nu \left[ K^0 + iK^+ \right] \end{pmatrix}
\]

and \( \Delta = \text{diag}(\delta_N, \delta_N, \delta_S) \) describes the explicit breaking of the chiral symmetry in the (axial-)vector channel (in terms of masses of \( u, d \) and \( s \) quarks, \( \delta_N \sim m^2_{u,d} \) and \( \delta_S \sim m^2_s \); isospin symmetry for non-strange quarks has been assumed). Explicit symmetry breaking in the (pseudo)scalar sector is described by

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \left( \pi^0 + \eta \right) + i(n_n + n_p) \\ \pi^0 + i\eta \end{pmatrix} \begin{pmatrix} a_0^- + i\pi^- \\ K_0^- + iK^0 \end{pmatrix} \left( \phi - \phi \sigma \right) \left( \phi + \phi \sigma \right)
\]

is a matrix containing the scalar and pseudoscalar degrees of freedom, \( L^\mu = V^\mu + A^\mu \) and \( R^\mu = V^\mu - A^\mu \) are, respectively, the left-handed and the right-handed matrices containing vector and axial-vector degrees of freedom with

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Subsequently, renormalisation coefficients need to be introduced for the pseudoscalar fields and $K_0^*$ (more details in Refs. 3, 5, 9).

Lagrangian 1 contains 14 parameters: $\lambda_1$, $\lambda_2$, $c_1$, $h_{0N}$, $h_{0S}$, $h_1$, $h_2$, $h_3$, $m_0^2$, $g_1$, $g_2$, $m_1$, $\delta_N$, $\delta_S$. Parameters $h_{0N}$ and $h_{0S}$ are determined from the extremum condition for the potential obtained from Eq. 1. Parameter $\delta_N$ is set to zero throughout this paper because the explicit breaking of the chiral symmetry is small in the non-strange quark sector. The other 11 parameters are calculated from a global fit including 21 observables: $f_\pi$, $f_K$, $m_\pi$, $m_K$, $m_\eta$, $m_{\eta'}$, $m_{\rho}$, $m_{K^*}$, $m_{\omega_S}$, $m_{\phi}(1020)$, $m_{f_1S}$, $m_{f_1(1420)}$, $m_{a_1}$, $m_{a_0}$, $m_{a_0(1450)}$, $m_{K_0^0(1430)}$, $\Gamma_{\rho\to\pi\pi}$, $\Gamma_{K^*\to\pi\pi}$, $\Gamma_{\phi\to KK}$, $\Gamma_{a_1\to\rho\pi}$, $\Gamma_{a_1\to\pi\gamma}$, $\Gamma_{f_1(1420)\to K^*K}$, $\Gamma_{a_0(1450)}$, $\Gamma_{K_0^0(1430)\to\pi\pi}$ (data from Ref. 1). Note that the observables entering the fit allow us to determine only linear combinations $m_0^2 + \lambda_1 (\phi_N^2 + \phi_S^2)$ and $m_1^2 + h_1 (\phi_N^2 + \phi_S^2)/2$ rather than parameters $m_0$, $m_1$, $\lambda_1$ and $h_1$ by themselves. However, it is nonetheless possible to calculate axial-vector masses and decay widths (explicit formulas in Refs. 3, 5) – see Sec. 3.

3. Fit Results

Results for the observables from our best fit are presented in Table 1. The fit yields $\chi^2 = 12.33$, i.e. $\chi^2/(21$ observables $- 11$ parameters) $= 1.23$. Table 1 demonstrates a remarkable correspondence of our results with experimental data in all meson channels. In particular, our fit yields $m_{a_1} = (1186 \pm 6)$ MeV, $\Gamma_{a_1\to\rho\pi} = (549 \pm 43)$ MeV and $\Gamma_{a_1\to\pi\gamma} = (0.66 \pm 0.01)$ MeV. Note that $\Gamma_{a_1(1260)\to\pi\gamma} = (0.64 \pm 0.25)$ MeV is the only experimental information regarding the $a_1(1260)$ meson which is not an estimate; our result for $\Gamma_{a_1\to\pi\gamma}$ is within the experimental interval and, being constrained by abundant experimental input, it even produces an error for $\Gamma_{a_1(1260)\to\pi\gamma}$ that is smaller than the one quoted by PDG. Thus results in the non-strange axial-vector channel justify the interpretation of $a_1(1260)$ as (predominantly) a $q\bar{q}$ state.

Our fit allows us to calculate mass and decay width of the strange axial-vector state $K_1$ as a prediction. We obtain $m_{K_1} = 1282$ MeV, $\Gamma_{K_1\to K^*\pi} = 205$ MeV, $\Gamma_{K_1\to\rho K} = 44$ MeV and $\Gamma_{K_1\to\omega K} = 15$ MeV. The mass is very close to the PDG value $m_{K_1} = (1272 \pm 7)$ MeV. However, the full decay width is 264 MeV, while the PDG data read $\Gamma_{K_1(1270)} = (90 \pm 20)$ MeV and $\Gamma_{K_1(1400)} = (174 \pm 13)$ MeV. Our result is therefore approximately three times too large when compared to the data for $K_1(1270)$ and approximately 50% too large when compared to the data for $K_1(1400)$, even with errors omitted from the calculation. These results demonstrate the necessity to include a pseudovector $I(J^{PC}) = 1(1^{-+})$ nonet into our model and implement its mixing with the already present axial-vector nonet.
| Observable | Fit [MeV] | Experiment [MeV] |
|------------|-----------|-----------------|
| $f_\pi$    | 96.3 ± 0.7 | 92.2 ± 4.6      |
| $f_K$      | 106.9 ± 0.6 | 110.4 ± 5.5    |
| $m_\pi$    | 141.0 ± 5.8 | 137.3 ± 6.9    |
| $m_K$      | 485.6 ± 3.0 | 495.6 ± 24.8   |
| $m_{\eta}$ | 509.4 ± 3.0 | 547.9 ± 27.4   |
| $m_{\eta'}$| 962.5 ± 5.6 | 957.8 ± 47.9   |
| $m_\rho$   | 783.1 ± 7.0 | 775.5 ± 38.8   |
| $m_{K^*}$  | 885.1 ± 6.3 | 893.8 ± 44.7   |
| $m_{a_1}$  | 1186 ± 6   | 1230 ± 62      |
| $m_{f_1(1420)}$ | 1372.5 ± 5.3 | 1426.4 ± 71.3 |
| $m_{a_0}$  | 1363 ± 1   | 1474 ± 74      |
| $m_{K^*_0}$| 1450 ± 1   | 1425 ± 71      |
| $\Gamma_{\rho \to \pi\pi}$ | 160.9 ± 4.4 | 149.1 ± 7.4   |
| $\Gamma_{K^* \to K\pi}$  | 44.6 ± 1.9   | 46.2 ± 2.3    |
| $\Gamma_{\phi \to KK}$   | 3.34 ± 0.14  | 3.54 ± 0.18   |
| $\Gamma_{a_1 \to \rho\pi}$ | 549 ± 43    | 425 ± 175     |
| $\Gamma_{a_1 \to \pi\gamma}$ | 0.66 ± 0.01 | 0.64 ± 0.25   |
| $\Gamma_{f_1(1420) \to K^*K}$ | 44.6 ± 39.9 | 43.9 ± 2.2    |
| $\Gamma_{a_0}$ | 266 ± 12   | 265 ± 13      |
| $\Gamma_{K^*_0 \to K\pi}$ | 285 ± 12   | 270 ± 80      |

Table 1. Best-fit results for masses and decay widths compared with experiment. Central values of observables are from Ref. [1]. Errors of observables are considered according to the criterion max(5%, experimental error). The reason is that already isospin-breaking effects in the physical hadron mass spectrum are of the order of 5% [for instance the difference between the charged and neutral pion masses or the masses of $a_1(1260)$ and $f_1(1285)$] but are completely neglected in our model. We have therefore artificially increased the experimental errors to 5% if the actual error is smaller or used the experimental values if the error is larger than 5%.

4. Conclusions

We have presented an extended Linear Sigma Model containing (axial-)vector mesons (eLSM). We have performed a global fit of masses and decay widths from which we have drawn two conclusions: (i) the non-strange axial-vector meson $a_1(1260)$ can be accommodated as a $qq$ state within our model and (ii) a correct description of the strange axial-vector states $K_1(1270)$ and $K_1(1400)$ requires the implementation of mixing between an axial-vector and a pseudovector nonet. The latter also represents an outlook for further investigation of axial-vector mesons; the model can, however, also be applied...
to further study other mesons in vacuum and at finite temperatures and densities (see Refs. [3, 5] for details).

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