Quantum dynamics of resonant molecule formation in waveguides

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Abstract. We explore the quantum dynamics of heteronuclear atomic collisions in waveguides and demonstrate the existence of a novel mechanism for the resonant formation of polar molecules. The molecular formation probabilities can be tuned by changing the trap frequencies that characterize the transverse modes of the atomic species. The origin of this effect is the confinement-induced mixing of the relative and center of mass motions in the atomic collision process leading to a coupling of the diatomic continuum to the center of mass excited molecular states in closed transverse channels.

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1. Introduction

Dimensionality plays a crucial role in interacting ultracold quantum gases [1]–[4]. For a strong two-dimensional (2D) confinement, i.e. a tight waveguide, and a close to resonant scattering process, the atoms can feel the trapping potential within an individual atom–atom collision. A striking manifestation thereof is the confinement-induced resonance (CIR) effect [5]. The narrow transversal confinement leads to a singularity of the coupling constant for the corresponding effective longitudinal motion, and as a result we encounter a 1D strongly interacting gas of impenetrable bosons that can be mapped at resonance on a gas of free fermions, the so-called Tonks–Girardeau gas [6]. The CIR has been interpreted as a Feshbach resonance involving the atom–atom continuum and a bound state of a transversally excited channel of the harmonic waveguide [7]–[9]. CIRs have subsequently been discovered for three-body [10, 11] and four-body [12] scattering under confinement and for a pure p-wave scattering of fermions [13]. The CIR behavior has been found experimentally for s-wave scattering bosons [14, 15] and for p-wave interacting fermions [16]. Recently a dual-CIR, which leads to a confinement-induced transparency effect [8, 17], has been demonstrated. It is characterized by a complete suppression of strong s- and p-wave heteronuclear atomic scattering in 3D because of the presence of the waveguide.

It is to be expected that waveguides with an anharmonic and/or anisotropic transversal confinement will lead to novel ultracold atomic collision properties and consequently to an intriguing many-body dynamics of the quantum gas. Very first results in this direction include the observation of a modified CIR due to a coupling of the center of mass (CM) and the relative motion of the atoms [8, 18]. This nonseparability can be achieved either by employing anharmonic waveguides or, in the case of heteronuclear collisions, by the fact that the two species experience different harmonic frequencies.

In the present work, we show that the quantum dynamics of the coupled CM and relative motion of two atomic species having different confinement potentials, i.e. waveguides, exhibits a resonant formation process of ultracold molecules. The latter is analyzed in detail and its existence is found to be independent of the specifics of the atomic interaction. As a consequence, it is anticipated that heteronuclear atomic-molecular 1D quantum gases in an anisotropic waveguide should explore a rich many-body quantum dynamics.

2. Wave-packet propagation method

We employ a wave-packet dynamical approach to study the atomic scattering in waveguides, which has been developed recently [8, 17]. The collisional dynamics of two (distinguishable) atoms with coordinates \(r_1, r_2\) and masses \(m_1, m_2\) moving in the harmonic waveguide with the transverse potential \(\frac{1}{2} \sum_i m_i \omega_i^2 \rho_i^2\) \((\rho_i = r_i \sin \theta_i)\) is described by the 4D time-dependent Schrödinger equation (\(\hbar = 1\))

\[
i \frac{\partial}{\partial t} \psi(\rho_R, r, t) = H(\rho_R, r) \psi(\rho_R, r, t)
\]

with the transformed Hamiltonian

\[
H(\rho_R, r) = H_{\perp}(\rho_R) + H_{\mu}(r) + W(\rho_R, r) + V(r).
\]
Here
\[ H_{\perp} = -\frac{1}{2M} \left( \frac{\partial^2}{\partial \rho_R^2} + \frac{1}{4\rho_R^2} \right) - \frac{1}{2M\rho_R^2} \left( \frac{\partial}{\partial \rho_R} - \frac{\partial}{\partial \phi} \right)^2 + \frac{1}{2} (m_1\omega_1^2 + m_2\omega_2^2) \rho_R^2 \]  
(2)
and
\[ H_\mu = -\frac{1}{2\mu} \frac{\partial^2}{\partial r^2} + \frac{L^2(\theta, \phi)}{2r^2} + \frac{\mu^2}{2} \left( \frac{\omega_1^2}{m_1} + \frac{\omega_2^2}{m_2} \right) \rho^2 \]  
(3)
describe the CM and relative atomic motions, \( V(r) \) describes the atom–atom interaction, \( \rho_R \) and \( r = r_1 - r_2 \mapsto (\rho, \phi, z) \) are the polar radial CM and the relative coordinates and \( M = m_1 + m_2, \mu = m_1m_2/M. \) The transformed Hamiltonian (1) is given in a rotated frame, which exploits the conservation of the component \((L_1 + L_2)\) of the total angular momentum and allows one to eliminate the angular dependence of \( \phi_R \) of the CM degrees of freedom [8, 19].

Our investigation will focus on the case \( M_{\phi_R} = 0, \) the latter being the quantum number belonging to \( \frac{1}{\sqrt{2\pi}} \).

The term
\[ W(\rho_R, r) = \mu \left( \omega_1^2 - \omega_2^2 \right) r \rho_R \sin \theta \cos \phi \]  
(4)
in the Hamiltonian (1) leads for two distinguishable atoms that feel different confining frequencies \( \omega_1 \neq \omega_2 \) to a nonseparability of the CM and relative atomic motion. We integrate the Schrödinger equation from time \( t = 0 \) to the asymptotic region \( t \to +\infty \) with the initial wave-packet
\[ \psi(\rho_R, r, t = 0) = Nr\sqrt{\rho_R} \exp \left\{ -\frac{\rho_1^2}{2a_1^2} - \frac{\rho_2^2}{2a_2^2} - \frac{(z - z_0)^2}{2a_z^2} + ik_0z \right\} \]  
(5)
representing two different noninteracting atoms in the transversal ground state of the waveguide with \( a_i = (1/m_i\omega_i)^{1/2} \) and the overall normalization constant \( N \) defined by \( \langle \psi(0)|\psi(0) \rangle = 1. \)

We choose \( z_0 \to -\infty \) to be far from the origin \( z = 0 \) and \( a_z \to \infty \) to obtain a narrow width in momentum and energy space for the initial wave-packet. Our wave-packet moves with a positive interatomic velocity \( v_0 = k_0/\mu = \sqrt{2\epsilon_{\|}/\mu} \), thereby approaching the scattering region located at \( z = 0. \) In the course of the collision, the wave-packet splits into two parts moving in opposite directions \( z \to \pm \infty. \) We model the interatomic interaction \( V(r) \) via the Lennard-Jones 6–12 potential \( V(r) = C_{12}/r^{12} - C_6/r^6. \) Note that the effects discussed in the following exist for even qualitatively very different shapes of the interatomic potential (like 6–12 and screened Coulomb [8, 9]). In this sense, our results will be universal, although the molecular formation probabilities, to some limited extent, depend on the appearance of \( V(r). \) To be specific, we consider the pair collisions of \(^{40}\text{K} \) and \(^{87}\text{Rb} \) atoms with mass ratio \( m_1/m_2 = 40/87 \) and the trapping frequency \( \omega_2 = 2\pi \times 200 \) kHz for Rb. Hereafter, we use the units \( \mu = \hbar = \omega_0 = 1 \) with \( \omega_0 = 2\pi \times 10 \) MHz. For the case of a decoupled CM motion, \( \omega_1 = \omega_2 = 0.02 \) in these units, whereas for the coupled case, \( \omega_1 \) varies in the limits \( \omega_2 < \omega_1 \leq 2.2\omega_2. \) Our focus is the analysis of the population dynamics of molecular resonance states (for reasons of brevity we will simply refer to them as molecular bound states with respect to \( V(r) \) for ultracold atom–atom collision in the waveguide.

3. Two-body bound states in a harmonic waveguide

First, we analyze how the molecular spectrum changes under the action of the confining waveguide (see figure 1).

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We allow $C_{12}$ to vary for fixed $C_6 = 1.847$ [20] and focus on the regime of the appearance of a bound state of $V(r)$. In the absence of the confining potential, the first bound molecular state $\varepsilon_0 < 0$ appears at $C_{12} \simeq 0.13$ and becomes increasingly bound with decreasing $C_{12}$. To explore the dependence of the binding energy of this state on $C_{12}$ in the presence of the waveguide, we solve the corresponding eigenvalue problem of the 4D Hamiltonian $H(\rho_r, r)$ (1) employing the spectral method elaborated in [21]. It is based on the computation of the autocorrelation function $\langle \psi(\rho_R, r, t = 0) | \psi(\rho_R, r, t) \rangle$, where the initial state $\psi(\rho_R, r, t = 0)$ can be considered as a test function of the spectrum. The solution $\psi(\rho_R, r, t)$ of the time-dependent Schrödinger equation is obtained via our wave-packet propagation method [8]. For the case $\omega_1 = \omega_2 = \omega$, i.e. for a decoupled CM motion, every unconfined bound state with energy $\varepsilon_0(C_{12})$ transforms by the action of the waveguide into the spectrum $\varepsilon_{n_1n_2}(C_{12})$, which represents at $C_{12} \geq 0.15$

$$\varepsilon_{n_1n_2}(C_{12}) \rightarrow (\omega_1 + \omega_2) + 2n_1\omega_1 + 2n_2\omega_2 = 2\omega(1 + n_1 + n_2),$$

the spectrum of two independent identical 2D oscillators with radial quantum numbers $n_1 = 0, 1, \ldots$ and $n_2 = 0, 1, \ldots$ characterizing the transverse excitations of two noninteracting atoms in the waveguide. We use $n_1, n_2$ as labels of the molecular bound states $\varepsilon_{n_1n_2}(C_{12})$ at fixed $C_{12}$ correlating to the corresponding dissociation limit (6) where they become exact quantum numbers. It is evident from (6) that for $\omega_1 = \omega_2$ the excited states $(n_1, n_2)$ become degenerate $\varepsilon_{n_1n_2} = \varepsilon_{n_2n_1}$. Since the coupling between the CM and the relative motion is absent, the quantum numbers $n$ and $N$ of the harmonic spectrum of the relative and the CM transverse eigenstates could be equally used to classify the spectrum.

For $\omega_1 \neq \omega_2$, the presence of the coupling between the CM and the relative motion leads to a lifting of the above-mentioned degeneracy. In figure 1, we show the corresponding splitting $\varepsilon_{01} \neq \varepsilon_{10}$ of the first excited state and the shift of the ground state.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Molecular energy levels $\varepsilon_0(C_{12})$ and $\varepsilon_{n_1n_2}(C_{12})$ as a function of the parameter $C_{12}$ of the atomic interaction in free space (open crossed circles) and in the waveguide. $\omega_1/2.2 = \omega_2 = 0.02$ for different transversal frequencies and $\omega_1 = \omega_2 = 0.02$ for the same frequencies.}
\end{figure}

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[20] J. A. C. Balthazar et al., J. Phys. B 38, 2845 (2005).

[21] K. Krüger et al., J. Phys. B 38, 2845 (2005).
4. Molecular resonance states in waveguides

The above-discussed spectral structure of the diatomic molecule in the waveguide permits us to analyze the dynamics of atomic pair collisions with respect to the formation of molecular final states. We choose herein $C_{12}$ to be 0.109 and the energy of the colliding atoms to be $\varepsilon = \omega_1 + \omega_2 + \varepsilon_0$ between the thresholds of the lowest $\omega_1 + \omega_2$ and the first excited $\omega_1 + 3\omega_2$ transverse channels ($0 \leq \varepsilon_1 \leq 2\omega_2$). Figure 1 indicates that for the chosen potential and collision energies one can observe one bound state $\varepsilon_{01}$ of the closed channel $n_1 = 0, n_2 = 1$, which becomes degenerate $\varepsilon_{01} = \varepsilon_{10}$ for $\omega_1 = \omega_2$. Figure 2 illustrates the time evolution of the wave-packet in the course of the atomic collision for the case of CM nonseparability $\omega_1 \neq \omega_2$ (figure 2(a)) and CM separation $\omega_1 = \omega_2$ (figure 2(b)).

For $\omega_1 \neq \omega_2$ we observe that a considerable part of the scattered wave-packet (see the probability density distribution $W(\rho_R, r, t)$ at $t = 8t_0$) is located near $r = 0$ after the collision and corresponds to a molecular bound state $\varepsilon_{01}$, which is $(n, N)$-representation is a mixture of the two states $n = 0, N = 1$ and $n = 1, N = 0$ with dominating $n = 0, N = 1$ contribution. Note that during collision $t \sim 3t_0$, the main part of the wave-packet is temporarily in the ground state $\varepsilon_{00}(n = N = 0)$, decays thereafter rapidly into the continuum but part of it goes into the excited molecular state $\varepsilon_{01}$. In contrast to this, the case $\omega_1 = \omega_2$ in figure 2(b) (CM separation) shows an almost complete decay into the continuum: the remaining minor part near $r = 0$ is much smaller compared to the case $\omega_1 \neq \omega_2$.

To quantitatively investigate the molecular formation process accompanied by CM excitation, we calculate the population probability $P_N(t)$ of the molecular bound states for $N = 0$ and 1:

$$P_N(t) = \int_0^r dr \int d\Omega \left| \int_0^\infty d\rho_R \psi(\rho_R, r, t) \Phi_N(\rho_R) \right|^2,$$

where $\Phi_N$ are the 2D oscillator states of the potential $(1/2)(m_1 \omega_1^2 + m_2 \omega_2^2)\rho_R^2$ and $d\Omega = \sin \theta d\theta d\phi$. For the chosen interval of collision energies $0 \leq \varepsilon_1 \leq 2\omega_2$ and $C_{12} = 0.109$, the molecular states $N = 0$ and 1 are the ones that dominate during the collision process (see figure 1). Note that the integration in (7) over the interatomic distance $r$ is limited to the region $r \leq r_m = 10$ of the action of the 6–12 potential. For sufficiently long times, i.e. after the collision process is concluded, all the unbound part of the scattered wave-packet resides outside this region. The complementary part is given by $P_N(t)$ and can be interpreted as a molecular bound state. For $r_m \to \infty$, we obtain $P_0(t) + P_1(t) \approx 1$ for any time and any energy within the above-provided interval. Figure 3 shows $(\omega_1 \neq \omega_2)$ that during the time interval of close collision the wave-packet temporarily occupies the molecular ground state $N = 0$, i.e. $P_0$ becomes large. However, with further increasing time $P_0$ decays to zero and $P_1$ increases rapidly, which corresponds to the population of the excited state $N = 1$. $P_1$ decays very slowly for long enough times after the collision. We interpret this state as the first excited state $n_1 = 0, n_2 = 1$ of the molecular spectrum (see figure 1). For $\omega_1 = \omega_2$ the coupling between the $n_1 = n_2 = 0$ ($N = 0$) and $n_1 = 0, n_2 = 1$ ($N = 1$) molecular states is absent and the temporary population in the course of the collision of the molecular ground state decays back into the continuum after the collision; see the behavior of $P_0(\omega_1 = \omega_2)$. In particular, $P_1(\omega_1 = \omega_2)$ is always negligible.

In figure 4, we illustrate the mechanism of molecule formation for the ground state $n_1 = n_2 = 0$ ($N = 0$) with transversal $\omega_1 + \omega_2$ and relative longitudinal $\varepsilon_\parallel = \mu v_0^2/2$ energies.
Figure 2. Time evolution of the probability density distribution averaged over the angular variables $W(\rho_R, r, t) = \int |\psi(\rho_R, r, \theta, \phi, t)|^2 (r^2 \rho_R)^{-1} \sin \theta \, d\theta \, d\phi$.

(a) For the waveguide with $\omega_1/2.2 = \omega_2 = 0.02$ (b) for $\omega_1 = \omega_2 = 0.02$. For $t = 8t_0$ corresponding to a time after the collision, the insets show a more detailed view of $W$ on the scales $0 \leq W \leq 8 \times 10^{-7}$ corresponding to the continuum part of the wave-packet. $\varepsilon_\parallel = 0.004$; time is given in units of $t_0 = \pi/\omega_2$.

Under the action of the attractive tail $-C_6/r^6$ of interatomic interaction, the atoms accelerate up to the energy $\mu v^2(t)/2 = \varepsilon_\parallel + \omega_1 + \omega_2 - \varepsilon_{00}$ at the instant of collision. The occurrence of the molecular state with energy $\varepsilon_{00}$ (bound molecular state with respect to the ground transversal channel), as an intermediate in the course of the collision process, can be seen by inspecting the intermediate probability density shown in figures 2(a) and 3. The released energy subsequently
Figure 3. $P_N(t)$ of the states $N = 0$ and $1$ as a function of collision time $t$. Full circles and squares correspond to $\omega_1/2.2 = \omega_2 = 0.02$. Open circles and squares belong to $\omega_1 = \omega_2 = 0.02$. $\varepsilon_\parallel = 0.004$; $t_0 = \pi/\omega_2$.

Figure 4. Schematic illustration of the resonant formation of a diatomic molecule. Transfer of the released energy $\varepsilon_{\parallel} + (\omega_1 + \omega_2) - \varepsilon_{00}$ to the excitation $\varepsilon_{01} - \varepsilon_{00}$ of the CM in the course of molecular formation. Solid lines indicate the positions of the molecular energy levels $\varepsilon_{n_1,n_2}$ for the case $\omega_1/2.2 = \omega_2 = 0.02$. The shifts of the levels due to the elimination of the coupling term at $\omega_1 = \omega_2 = 0.02$ are shown by dashed lines.

Transfers to the closed channel $n_1 = 0, n_2 = 1 (N = 1)$ via the formation of a molecule in a CM excited state since this closed channel is coupled to the $N = 0$ by the term $W(\rho_R, r) = \mu(\omega_1^2 - \omega_2^2) r \rho_R \sin \theta \cos \phi$ (4) of the total Hamiltonian (1). This mechanism also explains the
delay in the onset of the collisional interaction observed in figure 3 in the absence of the coupling term $\omega_1 = \omega_2$ as compared to the case $\omega_1 \neq \omega_2$. For $\omega_1 = \omega_2$ the collisional interaction happens later than for $\omega_1 \neq \omega_2$ because the binding energy in the channel $\epsilon_{00}(\omega_1 \neq \omega_2)$ is considerably lower than the binding energy in the channel $\epsilon_{00}(\omega_1 = \omega_2)$, leading to a stronger atomic attraction and acceleration of the colliding atoms for $\omega_1 \neq \omega_2$. A simple semiclassical estimate of the time difference between the onsets of collisional interactions for the two cases is in good agreement with the numerical result of figure 3.

From the above scheme it also follows that if the energy release $\epsilon_\parallel + \omega_1 + \omega_2 - \epsilon_{00}$ is equal to the excitation energy $\epsilon_{01} - \epsilon_{00}$, we have to expect a resonant enhancement of the molecular formation process, i.e. we encounter the resonance condition

$$\epsilon_\parallel + \omega_1 + \omega_2 = \epsilon_{01}.$$  

(8)

To analyze the energy dependence of the atom–molecule reaction, we inspect the population probabilities $P_N(\epsilon_\parallel)$ for the molecular states $N = 0$ and 1 as a function of the longitudinal energy $\epsilon_\parallel$ before the collision. Full circles and squares correspond to $\omega_1/2.2 = \omega_2 = 0.02$. Open circles and squares correspond to $\omega_1 = \omega_2 = 0.02$.

From the above scheme it also follows that if the energy release $\epsilon_\parallel + \omega_1 + \omega_2 - \epsilon_{00}$ is equal to the excitation energy $\epsilon_{01} - \epsilon_{00}$, we have to expect a resonant enhancement of the molecular formation process, i.e. we encounter the resonance condition

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To analyze the energy dependence of the atom–molecule reaction, we inspect the population probabilities $P_N(\epsilon_\parallel)$ for the molecular states $N = 0$ and 1 as a function of the longitudinal energies $\epsilon_\parallel$ (see figure 5). The values $P_N(\epsilon_\parallel) = P_N(\epsilon_\parallel, t \rightarrow \infty)$ were calculated after the collision at $t = 8t_0$, in the asymptotic region where $P_1(\epsilon_\parallel, t)$ is very slowly decaying if $\omega_1 \neq \omega_2$. $P_1(\epsilon_\parallel)$ exhibits a pronounced resonant behavior whereas $P_0(\epsilon_\parallel)$ shows a very weak energy dependence, which is in agreement with the above-discussed reaction mechanism. The calculated position $\epsilon_\parallel = \epsilon_r \sim 0.01$ of the maximum of the population $P_1(\epsilon_\parallel)$ is shifted considerably as compared to $\epsilon_r(\omega_1 \neq \omega_2) = \epsilon_{01}(\omega_1 \neq \omega_2) - \omega_1 - \omega_2 = 0.026$ following from the resonance condition (8). We attribute this difference to the fact that we are using, for the calculation of $P_1(\epsilon_\parallel)$ in equation (7), the oscillator wave function $\Phi_N(\rho_R)$ instead of the exact transversal CM part of the molecular wave function of the state $n_1 = 0, n_2 = 1 (N = 1)$. The resulting deviation from the exact resonance conditions (8) is particularly due to the missing effect of CM coupling which provides the shift $\epsilon_{01}(\omega_1 = \omega_2) - \epsilon_{01}(\omega_1 \neq \omega_2) = 0.014$, resulting in a significantly better agreement of the calculated position of the resonance with the resonant energy from equation (8).
The molecular formation probability $P_1$ also depends on $\Delta \omega = \omega_1 - \omega_2$ and reaches a maximum at a value corresponding to resonance condition (8). This probability can therefore be controlled by changing the transversal confinement frequencies for the different atomic species. By nature the process we observed is a molecular resonance in the atomic scattering and back decay of the molecule to the atom–atom continuum will occur. However, this resonance possesses a long lifetime due to its two-step character, which is clearly demonstrated in figure 3. Finally, we note that $P_1$ amounting to several percent should not obscure the fact that substantial molecular formation rates $\lambda = n_A \nu_0 P_1$ could be achieved by integrating over many collisions even for low linear atomic densities $n_A$. Thus, for $n_A = 10^4 \text{ cm}^{-1}$ and $T = 100 \text{ nK}$ we obtain the estimate $\lambda \sim 10^3 \text{ s}^{-1}$.

5. Conclusions

Perspectives concerning applications of the molecular formation mechanism are based on the fact that it represents a key ingredient in the preparation and subsequent investigation of the dynamics of mixed atomic and dipolar molecular quantum gases in waveguides. The latter are expected to possess a novel collective excitation dynamics particularly in the ultracold regime discussed here, for which only a few transversal channels are accessible. If wanted, one could stabilize the molecular gas and prevent back decay into atoms by a corresponding switch of the trapping fields such that the CM decouples from the interatomic motion finally. Open questions include the impact of inelastic atom–molecule and molecule–molecule collisions, which, however, goes beyond the scope of the present investigation.

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References

[1] Pethick C J and Smith H 2008 *Bose–Einstein Condensation in Dilute Gases* (Cambridge: Cambridge University Press)
[2] Pitaevskii L and Stringari S 2003 *Bose–Einstein Condensation* (Oxford: Oxford University Press)
[3] Grimm R, Weidemuller M and Ovchinnikov Yu V 2000 *Adv. At. Mol. Opt. Phys.* 42 95
[4] Folman R, Krüger P, Schmiedmayer J, Denschlag J and Henkel C 2002 *Adv. At. Mol. Opt. Phys.* 48 263
[5] Fortagh J and Zimmermann C 2007 *Rev. Mod. Phys.* 79 235–89
[6] Olshanii M 1998 *Phys. Rev. Lett.* 81 938–41
[7] Girardeau M 1960 *J. Math. Phys.* 1 516–23
[8] Melezhik V S, Kim J I and Schmelcher P 2007 *Phys. Rev. A* 76 053611
[9] Saeidian S, Melezhik V S and Schmelcher P 2008 *Phys. Rev. A* 77 042721
[10] Mora C, Egger R, Gogolin A O and Komnik A 2004 *Phys. Rev. Lett.* 93 170403
[11] Mora C, Egger R and Gogolin A O 2005 *Phys. Rev. A* 71 052705
[12] Mora C, Komnik A, Egger R and Gogolin A O 2005 *Phys. Rev. Lett.* 95 080403
[13] Granger B E and Blume D 2004 *Phys. Rev. Lett.* 92 133202
[14] Kinoshita T, Wenger T and Weiss D S 2004 Science 305 1125–8
[15] Paredes B, Widera A, Murg V, Mandel O, Fölling S, Cirac I, Shlypinikov G, Hänsch T and Bloch I 2004 Nature 429 277–81
[16] Günter K, Stöferle T, Moritz H, Köhl M and Esslinger T 2005 Phys. Rev. Lett. 95 230401
[17] Kim J I, Melezhik V S and Schmelcher P 2006 Phys. Rev. Lett. 97 193203
Kim J I, Melezhik V S and Schmelcher P 2007 Rep. Progr. Theor. Phys. Suppl. 166 159–68
[18] Peano V, Thorwart M, Mora C and Egger R 2005 New J. Phys. 7 192
[19] Bock H, Lesanovsky I and Schmelcher P 2005 J. Phys. B: At. Mol. Opt. Phys. 38 893–906
[20] Derevyanko A, Babb J F and Dalgarno A 2001 Phys. Rev. A 63 052704
[21] Feit M D, Fleck J A Jr and Steiger A 1982 J. Comput. Phys. 47 412–33