Creation of nonlocal spin-entangled electrons via Andreev tunneling, Coulomb blockade and resonant transport

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We discuss several scenarios for the creation of nonlocal spin-entangled electrons which provide a source of electronic Einstein-Podolsky-Rosen (EPR) pairs. Such EPR pairs can be used to test nonlocality of electrons in solid state systems, and they form the basic resources for quantum information processing. The central idea is to exploit the spin correlations naturally present in superconductors in form of Cooper pairs possessing spin-singlet wavefunctions. We show that nonlocal spin-entanglement in form of an effective Heisenberg spin interaction is induced between electron spins residing on two quantum dots with no direct coupling between them but each of them being tunnel-coupled to the same superconductor. We then discuss a nonequilibrium setup with an applied bias where mobile and nonlocal spin-entanglement can be created by coherent injection of two electrons in a pair (Andreev) tunneling process into two spatially separated quantum dots and subsequently into two Fermi-liquid leads. The current for injecting two spin-entangled electrons into different leads shows a resonance and allows the injection of electrons at the same orbital energy, which is a crucial requirement for the detection of spin-entanglement via the current noise. On the other hand, tunneling via the same dot into the same lead is suppressed by the Coulomb blockade effect of the quantum dots. We discuss Aharonov-Bohm oscillations in the current and show that they contain $h/e$ and $h/2e$ periods, which provides an experimental means to test the nonlocality of the entangled pair. Finally we discuss a structure consisting of a superconductor weakly coupled to two separate one-dimensional leads with Luttinger liquid properties. We show that strong correlations again suppress the coherent subsequent tunneling of two electrons into the same lead, thus generating again nonlocal spin-entangled electrons in the Luttinger liquid leads.

1. INTRODUCTION

The electron has charge as well as spin. While the control of charge is well established in electronic systems and devices, the spin combined with transport only recently attracted interest, both from a fundamental point of view and also for applications in electronics. The idea of using the spin and its coherence for electronic devices has received strong experimental support, showing unusually long spin dephasing times in semiconductors (approaching microseconds), the injection of spin-polarized currents from a magnetic to a nonmagnetic semiconductor, as well as phase-coherent spin transport over distances of up to $100 \mu m$. Besides the improvement of conventional devices due to spin, e.g. in magnetic read out heads, non-volatile memories etc., electron spins in quantum confined nanostructures like semiconductor quantum dots, atoms, or molecules can be used as a quantum bit (qubit) for quantum computing and quantum communication, where a radically new design of the necessary hardware is required. In this article we focus on the creation of spin-entangled electrons-electronic Einstein-Podolsky-Rosen (EPR) pairs exploiting the spin-correlations naturally present in s-wave superconductors, where the electrons are correlated as Cooper pairs with spin-singlet wavefunctions. We note that such EPR pairs represent the basic resources for quantum communication schemes like dense coding, teleportation, or, more fundamentally, for testing nonlocality via Bell inequalities, which would be particularly interesting to implement for massive particles such as electrons in a condensed matter system.

We describe in the following a few proposals for creating nonlocal spin-entanglement between two electrons. We consider first two quantum dots which are coupled to the same superconductors (but not among themselves). In equilibrium, this coupling then induces an effective tunable Heisenberg interaction among the two electrons on the dots which could be used to implement two-qubit gates. We then consider a nonequilibrium situation where now Cooper pairs can be transported by means of an Andreev (pair tunneling) process from the superconductor via two quantum dots into different normal Fermi liquid leads by applying a bias between the superconductor and the leads. The quantum dots in the Coulomb blockade regime are needed to mediate the necessary interaction between the two tunneling electrons so that they preferably tunnel to different dots and then subsequently to different outgoing leads, thereby maintaining their spin-singlet state. Such a setup then works as an entangler for electron spins, satisfying all necessary requirements, within a parameter regime of experimental accessibility, to detect the spin-entanglement in the leads via the current noise. We finally discuss a further realization for an entangler where the necessary interaction to separate the two electrons is provided by strong
correlations in one-dimensional leads (such as nanotubes) with Luttinger liquid properties.

We refer to related work\textsuperscript{23} which makes also use of Andreev tunneling but with a transparent superconductor/normal interface. The electrons move from the superconductor into a normal fork-shaped wire without Coulomb blockade behavior. The electrons are separated via energy filters so that the electrons enter their corresponding leads at different orbital energies. However, due to the transparent interface the partners of different Cooper pairs are not separated in time and space which is needed to identify the spin-entangled partners. Subsequent work in similar direction uses multiterminal hybrid structures.\textsuperscript{18,19,20}

\section*{II. Creation and Detection of Nonlocal Spin-Entanglement in a Double Dot}

We consider a parallel double quantum dot structure. The quantum dots contain one electron spin each and we assume that there is no direct coupling between the dots but each dot is tunnel-coupled to the same s-wave superconductors with a tunnel amplitude $\Gamma$ (see Fig. 1) on both sides of the dots. The two superconductors are held at the same chemical potential. The s-wave superconductor favors an entangled singlet-state on the dots (like in a Cooper pair) and further provides a mechanism for detecting the spin state via the Josephson current. It turns out that in leading order $\propto \Gamma^4$ the spin coupling is described by a Heisenberg Hamiltonian\textsuperscript{15,16} with an antiferromagnetic exchange coupling $J$

$$H_{\text{eff}} \approx J \left(1 + \cos \varphi \right) \left(S_a \cdot S_b - \frac{1}{4} \right), \quad (2.1)$$

where $J \approx 2\Gamma^2/\epsilon$, and the energy of the dot is $\epsilon$ below the chemical potentials of the superconductors. Here, $\varphi$ is the average phase difference across the superconductor–double-dot–superconductor (S-DD-S) junction. We can modify the exchange coupling between the spins by tuning the external control parameters $\Gamma$ and $\varphi$ so the device can act as a two-qubit quantum gate needed for quantum computing where the electron spin is the fundamental computational unit.\textsuperscript{21} Furthermore, the entangled spin state on the dot can be probed if the superconducting leads are joined with one additional (ordinary) Josephson junction with coupling $J'$ and phase difference $\theta$ into a SQUID ring. The supercurrent $I_S$ through this ring is given by\textsuperscript{21}

$$I_S/I_J = \left\{ \begin{array}{l} \sin(\theta - 2\pi f) + (J'/J) \sin \theta, \quad \text{singlet,} \\ (J'/J) \sin \theta, \quad \text{triplets,} \end{array} \right. \quad (2.2)$$

where $I_J = 2eJ/h$ and $f = \Phi/\Phi_0$ with $\Phi$ being the magnetic flux threading the SQUID-ring and $\Phi_0 = hc/2e$ is the flux quantum. Measurement of the spin- and flux-dependent critical current $I_c = \max_f \{|I_S|\}$ probes the spin state of the double dot. This is realized by biasing the system with a dc current $I$ until a finite voltage $V$ appears for $|I| > I_c$\textsuperscript{21}.

\section*{III. Andreev Entangler}

We now consider a nonequilibrium situation where the superconductor is tunnel-coupled to two quantum dots in the Coulomb blockade regime which are further coupled to two normal Fermi liquid leads\textsuperscript{22}, see Fig. 2. Applying a bias between the superconductor and the leads than results in a stationary current of spin-entangled electrons via Andreev tunneling and resonant transport from the superconductor to the leads. The quantum dots are used to mediate the interaction necessary to separate the two spin-entangled electrons originating from a Cooper pair. The amount of spin-entanglement in the outgoing current could be tested via noise measurements.\textsuperscript{22} We have shown that current-current correlations (noise) are enhanced if the injected electrons are singlets due to bunching behavior whereas the noise is suppressed in the case of spin triplets due to antibunching behavior. For such noise measurements, which are based on two-particle interference effects, it is absolutely crucial that both electrons, coming from different leads, possess the same orbital energy.
According to the energy-time uncertainty relation, a Cooper pair into a quasiparticle in the superconductor. The gap \( \Delta \) is the minimum energy to break up and then defines the time delay between the two coherent transport of two electrons of opposite spins due to Andreev tunneling.

In general, the tunnel-coupling of two electron states with singlet spin-wavefunctions \( |S\rangle \) and \( |L\rangle \), which are weakly coupled to noninteracting normal leads \( L_1 \) and \( L_2 \), with tunneling amplitudes \( T_{DL} \). The superconductor and leads are kept at chemical potentials \( \mu_S \) and \( \mu_l \), respectively.

IV. QUALITATIVE DESCRIPTION OF THE ANDREEV ENTANGLER

We first provide a qualitative description of the entangler and its principal mechanism based on Andreev processes and Coulomb blockade effects and also specify the necessary parameter regime for successful transport of the initial spin-entanglement of the Cooper pairs to the outgoing leads. In subsequent sections we then introduce the Hamiltonian and calculate the stationary current for the electrons will preferably tunnel into separate dots and subsequently into separate leads (this will be quantified in the following).

The chemical potentials \( \epsilon_1 \) and \( \epsilon_2 \) of the quantum dots can be tuned by external gate voltages such that the coherent tunneling of two electrons into different leads is at resonance if \( \epsilon_1 + \epsilon_2 = 2\mu_S \), see Fig. 3. This current resonance condition reflects energy conservation in a tunnel process of a Cooper pair with energy \( 2\mu_S \) from the superconductor to the dots 1,2 (one electron on each dot) with chemical potentials \( \epsilon_1, \epsilon_2 \) and requires that the resonant dot levels have to be adjusted such that one is above \( \mu_S \) and the other (by the same amount) below \( \mu_S \). This is very similar to the more familiar picture of Andreev reflection at a superconductor/normal interface. There an electron on the normal side of the junction, and with energy \( \epsilon \) above \( \mu_S \), is back reflected as a hole with energy \( \epsilon \) below \( \mu_S \) by the simultaneous creation of a Cooper pair in the superconductor. In that sense the empty dot level below \( \mu_S \) can be considered as the hole and the empty dot level above \( \mu_S \) as the empty electron state. In contrast, we will see that the current for the coherent tunneling of two electrons via the same dot into the same lead is suppressed by the on-site Coulomb \( U \) repulsion of a quantum dot and/or by the superconducting gap \( \Delta \).

Next, we introduce the relevant parameters describing the proposed device and specify their regime of interest. First we note that to avoid unwanted correlations with electrons already on the quantum dots one could work in the cotunneling regime where the number of electrons on the dots are fixed and the resonant levels \( \epsilon_l \), \( l = 1,2 \) cannot be occupied. However, we prefer to work at the particular resonance \( \epsilon_1 \approx \mu_S \), since then the total current and the desired suppression of tunneling into the same lead is maximized. Also, the desired injection
of the two electrons into different leads but at the same orbital energy is then achieved. In the resonant regime, we can avoid unwanted correlations between tunneling of subsequent Cooper pairs if we require that the dot-lead coupling is much stronger than the superconductor-dot coupling, i.e. $|T_{SD}| < |T_{DL}|$, so that electrons which enter the dots from the superconductor will leave the quantum dots to the leads much faster than new electrons can be provided from the superconductor. In addition, a stationary occupation due to the coupling to the leads is exponentially small if $\Delta \mu > k_B T$, $T$ being the temperature and $k_B$ the Boltzmann constant. Thus in this asymmetric barrier case, the resonant dot levels $\epsilon_l$ can be occupied only during a virtual process (see also Section 6).

Next, the quantum dots in the ground state are allowed to contain an arbitrary but even number of electrons, $N_D = \text{even}$, with total spin zero (i.e. antiferromagnetic filling of the dots). An odd number $N_D$ must be excluded since a simple spin-flip on the quantum dot would be possible in the transport process and as a result the desired entanglement would be lost. Further, we have to make sure that also spin flip processes of the following kind are excluded. Consider an electron that tunnels from the superconductor into a given dot. Now, it is possible in principle (e.g. in a sequential tunneling process) that another electron with the opposite spin leaves the dot and tunnels into the lead, and, again, the desired entanglement would be lost. However, such spin flip processes will be excluded if the energy level spacing of the quantum dots, $\delta\epsilon$, (assumed to be similar for both dots) exceeds both, temperature $k_B T$ and bias voltage $\Delta\mu$. A serious source of entanglement-loss is given by electron hole-pair excitations out of the Fermi sea of the leads during the resonant tunneling events. Since then a simple spin flip on the dot would be possible due to the coupling to the leads. However, we showed that such many-particle contributions can be suppressed if the resonance width $\gamma_l = 2\pi v_l |T_{DL}|^2$ is smaller than $\Delta\mu$ (for $\epsilon_l \simeq \mu_S$), where $v_l$ is the density of states (DOS) per spin of the leads at the chemical potential $\mu_l$.

To summarize, the regime of interest where the coherence of an initially entangled Cooper pair (spin singlet) is preserved during the transport to the leads is given by

$$\Delta, U, \delta\epsilon > \Delta\mu > \gamma_l, k_B T, \quad \text{and} \quad \gamma_l > \gamma_S. \quad (4.1)$$

As regards possible experimental implementations of the proposed setup and its parameter regime, we would like to mention that, typically, quantum dots are made out of semiconducting heterostructures, which satisfy above inequalities. Furthermore, in recent experiments, it has been shown that the fabrication of hybrid structures with semiconductor and superconductor being tunnel-coupled is possible. Other candidate materials are e.g. carbon nanotubes which also show Coulomb blockade behavior with $U$ and $\delta\epsilon$ being in the regime of interest here. The present work might provide further motivation to implement the structures proposed here.

Our goal in the following is to calculate the stationary charge current of pairwise spin-entangled electrons for two competing transport channels, first for the desired transport of two entangled electrons into different leads ($I_1$) and second for the unwanted transport of both electrons into the same lead ($I_2$). We compare then the two competing processes and show how their ratio, $I_1/I_2$, depends on the various system parameters and how it can be made large. An important finding is that when tunneling of two electrons into different leads occurs, the current is suppressed due to the fact that tunneling into the dots will typically take place from different points $r_1$ and $r_2$ on the superconductor (see Fig. 1) due to the spatial separation of the dots $D_1$ and $D_2$. We show that the distance of separation $\delta r = |r_1 - r_2|$ leads to an exponential suppression of the current via different dots if $\delta r > \xi$ (see (5.3)), where $\xi = v_F/\pi \Delta$ is the coherence length of a Cooper pair. In the relevant regime, $\delta r < \xi$, however, the suppression is only polynomial in the parameter $k_F \delta r$, with $k_F$ being the Fermi wavevector in the superconductor, and depends sensitively on the dimension of the superconductor. We find (see Section 7) that the suppression is less dramatic in lower dimensional superconductors where we find asymptotically smoother power law suppressions in $k_F \delta r$. On the other hand, tunneling via the same dot implies $\delta r = 0$, but suffers a suppression due to $U$ and/or $\Delta$. The suppression of this current is given by the small parameter $(\gamma_1/U)^2$ in the case $U < \Delta$, or by $(\gamma_1/\Delta)^2$, if $U > \Delta$ as will be derived in the following. Thus, to maximize the efficiency of the entangler, we also require $k_F \delta r < \Delta/\gamma_1, U/\gamma_1$.

Finally, we will discuss the effect of a magnetic flux on the entangled current in an Aharonov-Bohm loop, and we will see that this current contains both, single- and two-particle Aharonov-Bohm periods whose amplitudes have different parameter dependences. This allows us to distinguish processes where two electrons travel through the same arm of the loop from the desired processes where two electrons travel through different arms. The relative weight of the amplitudes of the two Aharonov-Bohm periods are directly accessible by flux-dependent current measurements which are then a direct probe of the desired nonlocality of the entangled electrons.

V. HAMILTONIAN OF THE ANDREEV ENTANGLER

We use a tunneling Hamiltonian description of the system, $H = H_0 + H_T$, where

$$H_0 = H_S + \sum_l H_{Dl} + \sum_l H_{Il}, \quad l = 1, 2. \quad (5.1)$$

Here, the superconductor is described by the BCS-Hamiltonian $H_S = \sum_{k, \sigma} E_k \gamma_k^\dagger \gamma_k \sigma$, where $\sigma = \uparrow, \downarrow$, and the quasiparticle operators $\gamma_{k\sigma}$ describe excitations out of the BCS-groundstate $|0\rangle_S$ defined by $\gamma_{k\sigma} |0\rangle_S = 0$. 
They are related to the electron annihilation and creation operators $c_{k|\sigma}$ and $c_{k|\sigma}^\dagger$ through the Bogoliubov transformation
\[ c_{k|\sigma} = u_k \nu_{k|\sigma} + v_k \nu_{-k|\sigma}^\dagger, \]
\[ c_{-k|\sigma} = u_k \nu_{-k|\sigma} + v_k \nu_{k|\sigma}^\dagger, \]
where $u_k = (1/\sqrt{2})(1 + \xi_k/E_k)^{1/2}$ and $v_k = (1/\sqrt{2})(1 - \xi_k/E_k)^{1/2}$ are the usual BCS coherence factors and $\xi_k = \epsilon_k - \mu_S$ is the normal state single-electron energy counted from the Fermi level $\mu_S$, and $E_k = \sqrt{\xi_k^2 + \Delta^2}$ is the quasiparticle energy. We choose energies such that $\mu_S = 0$. Both dots are represented as one localized (spin-degenerate) level with energy $\epsilon_i$ and is modeled by an Anderson-type Hamiltonian $H_{DL} = \mu \sum_{\sigma} d_{i|\sigma}^\dagger d_{i|\sigma} + U n_{i\uparrow} n_{i\downarrow}$, $l = 1, 2$. The resonant dot level $\epsilon_i$ can be tuned by the gate voltage. Other levels of the dots do not participate in transport if $\delta \epsilon > \Delta \mu > k_B T$, where $\Delta \mu = -\mu_l$, and $\mu_l$ is the chemical potential of lead $l = 1, 2$, and $\delta \epsilon$ is the single particle energy level spacing of the leads. The leads $l = 1, 2$ are assumed to be noninteracting (normal) Fermi liquids with Hamiltonian $H_{DL} = \sum_{k\sigma \sigma'} \epsilon_{k\sigma'} c_{k|\sigma}^\dagger c_{k|\sigma'}$. Tunneling from the dot $l$ to the lead $l$ or to the point $r_l$ in the superconductor is described by the tunnel Hamiltonian $H_T = H_{SD} + H_{DL}$ with
\[ H_{SD} = \sum_{l \sigma} T_{SD} d_{l|\sigma}^\dagger \psi_{\sigma}(r_l) + H.c., \]
\[ H_{DL} = \sum_{l \sigma \sigma'} T_{DL} d_{l|\sigma}^\dagger c_{l|\sigma'} + H.c. \]
Here, $\psi_{\sigma}(r_l)$ annihilates an electron with spin $\sigma$ at site $r_l$, and $d_{l|\sigma}$ creates it again (with the same spin) at dot $l$ with amplitude $T_{SD}$. $\psi_{\sigma}(r_l)$ is related to $c_{k|\sigma}$ by the Fourier transform $\psi_{\sigma}(r_l) = \sum_k e^{ikr_l} c_{k|\sigma}$. Tunneling from the dot to the state $k$ in the lead is described by the tunneling amplitude $T_{DL}$. We assume that the $k$-dependence of $T_{DL}$ can be safely neglected.

VI. STATIONARY CURRENT AND T-MATRIX

The stationary current of two electrons passing from the superconductor via virtual dot states to the leads is given by
\[ I = 2e \sum_{j,i} W_{ji} \nu_i, \]
where $W_{ji}$ is the transition rate from the superconductor to the leads. We calculate this transition rate in a T-matrix approach.
\[ W_{ji} = 2\pi \sqrt{|1/T(\epsilon_i)|^2 \delta(\epsilon_l - \epsilon_i)}. \]
Here, $T(\epsilon_i) = H_T [-\epsilon_i + i\nu - H_0]^{-1}(\epsilon_l - H_0)$, is the on-shell transmission or T-matrix, with $\eta$ being a small positive real number which we take to zero at the end of the calculation. Finally, $\nu_i$ is the stationary occupation probability for the entire system to be in the state $|i\rangle$. The T-matrix $T(\epsilon_i)$ can be expanded in powers of the tunnel Hamiltonian $H_T$,
\[ T(\epsilon_i) = H_T + H_T \sum_{n=1}^{\infty} \frac{1}{\epsilon_i + i\nu_n - H_T} H_T^n, \]
where the initial energy is $\epsilon_i = 2\mu_S \equiv 0$. We work in the regime defined in Eq. (1.1), i.e., $\gamma_l > \gamma_S$, and $\Delta U, \delta \epsilon \gg \Delta \mu > \gamma_l, k_B T$, and around the particular resonance $\epsilon_i \approx \mu_S$. Further, $\gamma_S = 2\pi \nu_0 |T_{SD}|^2$ and $\gamma_l = 2\pi \nu_l |T_{DL}|^2$ define the tunneling rates between superconductor and dots, and between dots and leads, respectively, with $\nu_S$ and $\nu_l$ being the DOS per spin at the chemical potentials $\mu_S$ and $\mu_l$, respectively. We will show that the total effective tunneling rate from the superconductor to the leads is given by $\gamma_S^2/\gamma_l$ due to the Andreev process. To specify the initial state $|i\rangle$ of the system we point out that since $\Delta > \Delta \mu > k_B T$, the superconductor contains no quasiparticle initially. Also, due to the asymmetric barrier case, $|T_{DL}| > |T_{SD}|$ (or $\gamma_l > \gamma_S$), an electron on the dot level $\epsilon_i$ leaves the dot to the leads much faster than new electrons can be provided by the superconductor. In addition, a stationary occupation of the resonant levels $\epsilon_l$ is given by the grand canonical distribution function $\propto \exp(-\Delta \mu/k_B T)$ which is exponentially small if $\Delta \mu > k_B T$. This implies $\nu_l \approx 0$ for initial states with occupied levels $\epsilon_l$. Therefore we consider the initial state $|i\rangle = |0\rangle_S |0\rangle_D |0\rangle_D$, where $|0\rangle_S$ is the quasiparticle vacuum for the superconductor, $|0\rangle_D$ means that both dot levels $\epsilon_l$ are unoccupied, and $|0\rangle_D$ defines the occupation of the leads which are filled with electrons up to the chemical potential $\mu_l$. We remark that in our regime of interest no Kondo effects appear which could destroy the spin entanglement, since our dots contain each an even number of electrons in the stationary limit.

VII. CURRENT DUE TO TUNNELING TO DIFFERENT LEADS

We now calculate the current for the simultaneous tunneling of two electrons into different leads. Since we assume that the spin is a good quantum number we can specify the final state for two electrons, one of them being in lead 1 the other in lead 2, according to their total spin $S$. This spin can be either a singlet (in standard notation) $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ with $S = 0$, or a triplet with $S = 1$. In the regime of interest (1.1), and since the total spin is conserved, $|S^2, H\rangle = 0$, the singlet state of the initial Cooper pair will be conserved in the transport process and the final state must also be a singlet. That this is indeed true we can see explicitly if we allow for the possibility that the final state could
also be the $S_z = 0$ triplet $|t_0\rangle = (|↑↓⟩ + |↓↑⟩)/\sqrt{2}$. The not-entangled triplets $|t_+\rangle = |↑↑⟩$ and $|t_-\rangle = |↓↓⟩$ can be excluded right away due to the tunnel Hamiltonian $H_T$ conserves the spin-component $\sigma$ and an Andreev process involves tunneling of two electrons with different spin $\sigma$. Therefore we consider two-particle final states of the form

$$ |f\rangle = (1/\sqrt{2}) \left( a_{1p}^{\dagger} a_{2q}^{\dagger} \pm a_{1p}^{\dagger} a_{2q}^{\dagger} \right) |i\rangle. \quad (7.1) $$

The $-\text{ and } +$ signs belong to the singlet $\langle S \rangle$ and triplet $|t_0\rangle$, resp. Note that this singlet/triplet state is formed out of two electrons, one being in the $p$ state in lead 1 and with energy $\epsilon_p$, while the other is in the $q$ state in lead 2 with energy $\epsilon_q$. Thus, the two electrons are entangled in spin space while separated in orbital space, thereby providing a non-local EPR pair. The tunnel process to different leads appears in the following order. A Cooper pair breaks up due to tunneling of an electron with spin $\sigma$ to one of the dots (with empty level $\epsilon_l$) from the point of the superconductor nearest to this dot. This is a virtual state with energy deficit $E_k > \Delta$. Since $\Delta > \gamma_l$, the second electron of the Cooper pair with spin $\sigma$ tunnels to the other empty dot-level before the electron with spin $\sigma$ escapes to the lead. Therefore, both electrons tunnel almost simultaneously to the dots (within the uncertainty time $\hbar/\Delta$). Since $|T| < |T_0|$, we can therefore write the transition amplitude between initial and final state as

$$ \langle f | T_0 | i \rangle = \frac{1}{\sqrt{2}} \langle i | a_{2q}^{\dagger} a_{1p}^{\dagger} T' d_{1\uparrow}^{\dagger} d_{2\downarrow}^{\dagger} | i \rangle \times \langle i | (d_{2\uparrow} d_{1\uparrow} \pm d_{2\downarrow} d_{1\downarrow}) T'' | i \rangle, \quad (7.2) $$

where $T_0 = T(\epsilon_l = 0)$, and the partial T-matrices $T'$ and $T''$ are given by

$$ T' = \frac{1}{\eta - H_0} H_{SD} \frac{1}{\eta - H_0} H_{SD}, \quad (7.3) $$

and

$$ T'' = H_{DL} \sum_{n=0}^{\infty} \left( \frac{1}{\eta - H_0} H_{DL} \right)^{2n+1}. \quad (7.4) $$

In (7.2) we used that the matrix element containing $T'$ is invariant under spin exchange $\uparrow \leftrightarrow \downarrow$. The part containing $T''$ describes the Andreev process of tunneling of two electrons with opposite spins from the SC to different dots 1,2, while the part containing $T'$ is the resonant dot $\leftrightarrow$ lead tunneling.

We first consider the Andreev process. We insert a complete set of single-quasiparticle (virtual) states between the two $H_{SD}$ in (7.3) and find for the singlet final state

$$ \langle i | (d_{2\uparrow} d_{1\uparrow} - d_{2\downarrow} d_{1\downarrow}) T'' | i \rangle = \frac{4 T_{SD}^2}{\epsilon_1 + \epsilon_2 - i \eta} \sum_k \frac{u_k v_k}{E_k} \cos(\mathbf{k} \cdot \mathbf{\delta r}), \quad (7.5) $$

where $\mathbf{\delta r} = \mathbf{r}_1 - \mathbf{r}_2$ denotes the distance vector between the points on the superconductor from which electron 1 and 2 tunnel into the dots. Note that the triplet contribution vanishes since $u_k v_k = -u_{-k} v_{-k}$ for s-wave superconductors. The sum over $k$ in (7.5) can be calculated by linearizing the spectrum around the Fermi energy and we obtain

$$ \sum_k \frac{u_k v_k}{E_k} \cos(\mathbf{k} \cdot \mathbf{\delta r}) = \frac{\pi}{2} \frac{n_s(\mathbf{k}_F \delta r)}{k_F \delta r} e^{-(\mathbf{\delta r}/\xi)}, \quad (7.6) $$

where $k_F$ is the Fermi wavevector in the superconductor.

VIII. DOMINANT RESONANT TUNNELING EVENTS

We turn to the calculation of the matrix element in (7.2) containing $T'$ where tunneling is treated to all orders in $H_{DL}$. We introduce the ket notation $|12\rangle$, and, for simplicity, suppress the spin index $\sigma$. Here 1 stands for quantum numbers of the electron on dot 1/lead 1 and similar for 2. For example, $|pq\rangle$ stands for $a_{1p}^{\dagger} a_{2q}^{\dagger} |i\rangle$, where $p$ is from lead 1 and $q$ from lead 2, or, correspondingly, $|pD\rangle$ stands for $a_{1p}^{\dagger} d_{1\downarrow} |i\rangle$, etc. We restrict ourselves to the resummation of the following dot $\leftrightarrow$ lead transitions $|DD\rangle \rightarrow |LD\rangle \rightarrow |DD\rangle$ or $|DD\rangle \rightarrow |DL\rangle \rightarrow |DD\rangle$. In this sequence, $|DD\rangle$ is the state with one electron on dot 1 and the other one on dot 2, and $|LD\rangle$ denotes a state where one electron is in lead 1 and the other one on dot 2. We thereby exclude tunneling sequences of the kind $|DD\rangle \rightarrow |LD\rangle \rightarrow |LL\rangle \rightarrow |LD\rangle \rightarrow |DD\rangle$ or $|DD\rangle \rightarrow |DL\rangle \rightarrow |LL\rangle \rightarrow |DL\rangle \rightarrow |DD\rangle$, where both electrons are virtually simultaneously in the leads as well as the creation of electron-hole pair excitations out of the Fermi sea. We showed in Ref.[5] that such contributions are suppressed in the regime (1.1) considered here by the small parameter $\gamma_l/\Delta \mu$. It is quite clear that electron-hole pair excitations, which in principal could spoil the entanglement between the original partner electrons, are suppressed in our regime of interest. The following qualitative argument is in order (for a detailed calculation, see[3]). Suppose an electron initially on, say, dot 1, tunnels to the lead 1. Instead of hopping back to the dot, thereby completing the sequence $|DD\rangle \rightarrow |LD\rangle \rightarrow |DD\rangle$, it disappears in a final state out in the lead (with energy $\epsilon_p \sim \epsilon_l$). Now another electron from the Fermi sea, with energy $\epsilon < -\Delta \mu$, hops on the empty dot. The creation of such an electron-hole pair involves an energy deficit of at least $\Delta \mu$ (for $\epsilon_l = 0$). Since this is a virtual state it can only exist during the uncertainty time $\sim 1/\Delta \mu$. The relaxation of this energy deficit requires the tunneling of this “Fermi sea” electron from the dot back to the
lead. Since the tunneling from the dot to the lead takes a time on the scale of $1/\gamma_1$ this process is suppressed since $\Delta \mu > \gamma_1$. The dominant contributions are then resummed in the following sequence

$$\langle pq | T' | DD \rangle$$

$$= \left\{ \langle pq | H_{D,D} | Dq \rangle \langle Dq | \sum_{n=0}^{\infty} \left( \frac{1}{\eta - H_0} H_{D,D} \right)^{2n} | Dq \rangle \times \langle Dq | \frac{1}{\eta - H_0} H_{D,D} | DD \rangle \right\} + \langle pq | H_{D,D} | pD \rangle \langle pD | \sum_{n=0}^{\infty} \left( \frac{1}{\eta - H_0} H_{D,D} \right)^{2n} | pD \rangle \times \langle pD | \frac{1}{\eta - H_0} H_{D,D} | DD \rangle \right\} \times \langle DD | \sum_{m=0}^{\infty} \left( \frac{1}{\eta - H_0} H_{D,D} \right)^{2m} | DD \rangle \right\}. \quad (8.1)$$

Since the sums for the transition $|DD \rangle \rightarrow |DD \rangle$ via the sequences $|DD \rangle \rightarrow |LD \rangle \rightarrow |DD \rangle$ and $|DD \rangle \rightarrow |DL \rangle \rightarrow |DD \rangle$ are independent, we can write all summations in $\varnothing$ as geometric series which allow for explicit resumptions and we obtain

$$\langle pq | T' | DD \rangle = \frac{-T_{DL}^2 (\epsilon_1 + \epsilon_2 - i\eta)}{(\epsilon_1 + \epsilon_2 - i\gamma_1/2)(\epsilon_2 + \epsilon_p - i\gamma_2/2)}. \quad (8.2)$$

Thus, we see that the resumptions in $\varnothing$ cancel all divergences like the $(\epsilon_1 + \epsilon_2 - i\eta)$ denominator appearing in $\varnothing$ and that, as expected, the resummation of divergent terms leads effectively to the replacement $\eta \rightarrow i\gamma_1/2$ which makes the limit $\epsilon_1 \rightarrow 0$ well-behaved.

In $\varnothing$ we neglected a small logarithmic correction to the (bare) resonant levels $\epsilon_1$ which is given as the real part of the self energy $\Sigma = |T_{DL}|^2 \sum_k (i\eta - \epsilon_1 - \epsilon_k)^{-1}$ with $|\text{Re} \Sigma| \sim \gamma \ln(\epsilon_1/\Delta \mu) < \Delta \mu$ and is therefore not important. The energy $\epsilon_1$ is the conduction band cut-off. Making use of Eqs. $\varnothing$ and $\varnothing$ and of Eqs. $\varnothing$ and $\varnothing$ we finally obtain for the current (denoted by $I_1$) where each of the two entangled electrons tunnels into a different lead

$$I_1 = \frac{e \gamma_1^2 \gamma}{(\epsilon_1 + \epsilon_2)^2 + \gamma^2/4} \left[ \frac{\sin(k_F \delta \epsilon)}{k_F \delta \epsilon} \right]^2 \exp \left( -\frac{2\delta \epsilon}{\pi \xi} \right), \quad (8.3)$$

where, $\gamma = \gamma_1 + \gamma_2$. We note that Eq. $\varnothing$ also holds for the case with $\gamma_1 \neq \gamma_2$. The current becomes exponentially suppressed with increasing distance $\delta \epsilon$ between the tunneling points on the superconductor, the scale given by the Cooper pair coherence length $\xi$. This does not pose severe restrictions for conventional s-wave materials with $\xi$ typically being on the order of micrometers. More severe is the restriction that $k_F \delta \epsilon$ should not be too large compared to unity, especially if $k_F^{-1}$ of the superconductor assumes a typical value on the order of a few Angstroms. Still, since the suppression in $k_F \delta \epsilon$ is only power-law like there is a sufficiently large regime on the nanometer scale for $\delta \epsilon$ where the current $I_1$ can assume a finite measurable value. The power law suppression of the current in $1/k_F \delta \epsilon$ is very sensitive to the dimension of the SC and we suspect that the suppression will be softened by going over to lower dimensional superconductors. We will address this issue in Section $\varnothing$. The current $\varnothing$ has a two-particle Breit-Wigner resonance form which assumes it maximum value when $\epsilon_1 = -\epsilon_2$ (see also Fig. 3, and note that $\mu_\Sigma \equiv 0$),

$$I_1 = \frac{4e \gamma_1^2}{\gamma} \left[ \frac{\sin(k_F \delta \epsilon)}{k_F \delta \epsilon} \right]^2 \exp \left( -\frac{2\delta \epsilon}{\pi \xi} \right). \quad (8.4)$$

This resonance at $\epsilon_1 = -\epsilon_2$ clearly shows that the current is a correlated two-particle effect (even apart from any spin correlation) as we should expect from the Andreev process involving the coherent tunneling of two electrons. Together with the single-particle resonances in Eq. $\varnothing$ and by using energy conservation $\epsilon_1 = \epsilon_f = 0$, which implies $\epsilon_p = -\epsilon_q$, we thus see that the current is carried by correlated pairs of electrons whose lead energies satisfy $|\epsilon_p - \epsilon_1| \lesssim \gamma_1$ and $|\epsilon_q - \epsilon_2| \lesssim \gamma_2$.

A particularly interesting case occurs when the energies of the dots, $\epsilon_1$ and $\epsilon_2$, are both tuned to zero, i.e. $\epsilon_1 = \epsilon_2 = \mu_\Sigma = 0$. We stress that in this case the electron in lead 1 and its spin-entangled partner in lead 2 possess exactly the same orbital energy. We have shown previously that this degeneracy of orbital energies is a crucial requirement for noise measurements in which the singlets can be detected by an enhanced noise in the current (bunching) due to a symmetrical orbital wavefunction of the singlet state, whereas uncorrelated electrons, or, more generally, electrons in a triplet state, lead to a suppression of noise (antibunching). Note that not all triplets are entangled states. Only the triplet with $S_z = 0$ is entangled. Measurement of noise enhancement is therefore a unique signature of entanglement.

We remark again that the current $I_1$ is carried by electrons which are entangled in spin space and spatially separated in orbital space. In other words, the stationary current $I_1$ is a current of non-local spin-based EPR pairs. Finally, we note that due to the singlet character of the EPR pair we do not know whether the electron in, say, lead 1 carries an up or a down spin, this can be revealed only by a spin-measurement. Of course, any measurement of the spin of one (or both) electrons will immediately destroy the singlet state and thus the entanglement. Such a spin measurement (spin read-out) can be performed e.g. by making use of the spin filtering effect of quantum dots. The singlet state will also be destroyed by spin-dependent scattering (but not by Coulomb exchange interaction in the Fermi sea). However, it is known experimentally that electron spins in a semiconductor environment show unusually long dephasing times approaching microseconds and can be transported phase coherently over distances exceeding $100 \mu m$. This distance is sufficiently long for experiments performed typically on the length scale of quantum confined nanostructures.
IX. CURRENT DUE TO TUNNELING VIA THE SAME DOT

In this section we calculate the current for tunneling of two electrons via the same dot and subsequently into the same lead. We show that such processes are suppressed by a factor $(\gamma_l/U)^2$ and/or $(\gamma_l/\Delta)^2$ compared to the process discussed in the preceding section. But, in contrast to the previous case, we do not get a suppression resulting from the spatial separation of the Cooper pair on the superconductor, since here the two electrons tunnel from the same point either from $r_1$ or $r_2$ (see Fig. 2). As before, a tunnel process starts by breaking up a Cooper pair followed by an Andreev process with two possible tunneling sequences, see Fig. 4, panel a). In a first step, one electron tunnels from the superconductor to, say, dot 1, and in a second step the second electron also tunnels to dot 1. Now two electrons are simultaneously on the same lead which costs additional Coulomb repulsion energy $U$. Finally, the two electrons leave dot 1 and tunnel into lead 1. There is an alternative competing process, see Fig. 4, panel b), which avoids the double occupancy. Here, one electron tunnels to, say, dot 1, and then the same electron tunnels further into lead 1, leaving an excitation on the superconductor which costs additional gap energy $\Delta$ (instead of $U$), before finally the second electron tunnels from the superconductor via dot 1 into lead 1.

We first concentrate on the tunneling process b), and note that the leading contribution comes from the processes where both electrons have left the superconductor so that the system has no energy deficit anymore. We still have to resum the tunnel processes from the dot to the lead to all orders in the tunnel Hamiltonian $H_{DL}$. In what follows we suppress the label $l = 1, 2$ since the setup is assumed to be symmetric and tunneling into either lead 1 or lead 2 gives the same result. The transition amplitude $\langle f | T_0 | i \rangle$ including only leading terms is

$$
\langle f | T_0 | i \rangle = \sum_{p' \sigma} \langle f | H_{DL} \sum_{n=0}^{\infty} \frac{1}{i \eta - H_0} H_{DL} 2^n | Dp'' \sigma \rangle 
\times \langle Dp'' \sigma | \frac{1}{i \eta - H_0} H_{SD} \frac{1}{i \eta - H_0} H_{DL} \frac{1}{i \eta - H_0} H_{SD} | i \rangle,
$$

where again $| f \rangle = (1/\sqrt{2}) (a_{p1}^\dagger a_{p2}^\dagger \pm a_{p2}^\dagger a_{p1}^\dagger) | i \rangle$, with $\pm$ denoting the triplet (+) and singlet (−), resp., and the intermediate state $| Dp'' \sigma \rangle = d_p^\dagger a_{p''p\sigma}^\dagger | i \rangle$. There are some remarks in order regarding Eq. (9.1). The electron which tunnels to the state $| p'' \sigma \rangle$ has not to be resummed further since this would lead either to a double occupancy of the dot which is suppressed by $1/U$, or to the state with two electrons simultaneously in the lead with a virtual summation over the state $p''$. But we already mentioned above that the latter process is suppressed by $\gamma_l/\Delta \mu$. The first factor in (9.1) describes therefore the multiple dot ↔ lead tunneling of the electron with spin $-\sigma$ which resides on the dot in the intermediate state $| Dp'' \sigma \rangle = d_p^\dagger a_{p''p\sigma}^\dagger | i \rangle$ and which eventually tunnels in its final state in the lead. Again, this amplitude can be resummed explicitly with the result for $\sigma = \uparrow$,

$$
\langle f | H_{DL} \sum_{n=0}^{\infty} \frac{1}{i \eta - H_0} H_{DL} 2^n | Dp'' \uparrow \rangle = - \frac{T_{DL}}{\sqrt{2}} \epsilon_l + \epsilon_{p''} - i \eta \left( \delta_{p''p} \mp \delta_{p''p'} \right),
$$

and for $\sigma = \downarrow$ we get

$$
\langle f | H_{DL} \sum_{n=0}^{\infty} \frac{1}{i \eta - H_0} H_{DL} 2^n | Dp'' \downarrow \rangle = - \frac{T_{DL}}{\sqrt{2}} \epsilon_l + \epsilon_{p''} - i \eta \left( \delta_{p''p} \mp \delta_{p''p'} \right).
$$

In (9.2) and (9.3) the upper sign belongs again to the triplet and the lower sign to the singlet. For the amplitude containing the superconductor-dot transitions in (9.1) we obtain

![Figure 4](image-url)
\[ \langle Dp''|\sigma|\frac{1}{i\eta-H_S}H_{SD}\frac{1}{i\eta-H_D}H_{DL}\frac{1}{i\eta-H_S}|i\rangle \]
\[ = \frac{s(TDL^2)^\nu_S}{\Delta(E_i+\epsilon_{p''}-i\eta)}, \quad (9.4) \]

where \( s = +1 \) (-1) for \( \sigma = \uparrow (\downarrow) \). Combining the results \[ (9.2), (9.4) \] we obtain for the amplitude \( (9.5) \)
\[ \langle f|T_0|i\rangle = -\frac{2^{3/2}\nu_S(TSDTDL)^2(E_i-i\gamma/2)}{\Delta(E_i+\epsilon_p-i\gamma/2)(E_i+\epsilon_{p''}-i\gamma/2)} \quad (9.5) \]
for the singlet final state, whereas we get again zero for the triplet.

Next we consider the process where the tunneling involves a double occupancy of the dot, see panel a) in Fig. 4. In this case the transition amplitude is
\[ \langle f|T_0|i\rangle = \sum_{p''\sigma} \langle f|H_{DL}\sum_{n=0}^{\infty} \frac{1}{i\eta-H_0}H_{DL}\rangle^{2n} \langle Dp''|\sigma| \]
\[ \times \langle Dp''|\sigma|\frac{1}{i\eta-H_0}H_{DL}\frac{1}{i\eta-H_0}H_{SD}\frac{1}{i\eta-H_0}H_{SD}|i\rangle. \quad (9.6) \]

Repeating a similar calculation as before we find that the amplitude is given by \( (9.5) \) but with \( \Delta \) being replaced by \( U/\pi \), and again, \( \langle f|T_0|i\rangle \) is only nonzero for the singlet final state. We note that the two amplitudes \( (9.5) \) and \( (9.6) \) have the same initial and same final states and therefore have to be added coherently to obtain the total current due to processes a) and b). Then, using Eq. \( (6.1) \) we find for the total current \( I_2 \) in case of tunneling of two electrons into the same lead \[ I_2 = \frac{e\gamma_2^2\gamma}{\xi^2}, \quad \frac{1}{\xi} = \frac{1}{\pi\Delta} + \frac{1}{U}. \quad (9.7) \]

We see that the effect of the quantum dots shows up in the suppression factor \( (\gamma/\xi)^2 \) for tunneling into the same lead. We remark that in contrast to the previous case (tunneling into different leads) the current does not have a resonant behavior since the virtual dot states are no longer at resonance due the energy costs \( U \) or \( \Delta \) in the tunneling process. We now compare \( I_1 \) given in \( (8.4) \) with \( I_2 \) by forming the ratio of the two currents
\[ \frac{I_1}{I_2} = \frac{4\xi^2}{\gamma^2} \left[ \sin(\kappa F_0\delta r) \right]^2 \exp \left( \frac{-2\delta r}{\pi\xi} \right). \quad (9.8) \]
From this ratio we see that the desired regime for \( I_1 \) dominating \( I_2 \) is obtained when \( \xi/\gamma > k_F\delta r \), and \( \delta r < \xi \). We would like to emphasize that the relative suppression of \( I_2 \) (as well as the absolute value of the current \( I_1 \)) is maximized by working around the resonances \( \epsilon_1 \approx \mu_S = (\nu_2) \) and, in addition, the desired injection of the two electrons at the same orbital energy is then achieved.

\[ \text{X. EFFICIENCY AND DISCUSSION} \]

The current \( I_1 \) and therefore the ratio \( (9.8) \) suffers an exponential suppression on the scale of \( \xi \) if the tunneling of the two (coherent) electrons takes place from different points \( r_1 \) and \( r_2 \) of the superconductor. For conventional s-wave superconductors the coherence length \( \xi \) is typically on the order of micrometers and therefore poses not severe restrictions. So in the interesting regime the suppression of the Andreev amplitude is only polynomial \( \propto 1/k_F\delta r \). It was shown \[ (9.7) \] that a superconductor on top of a two-dimensional electron gas (2DEG) can induce superconductivity (by the proximity effect) in the 2DEG with a finite order parameter. The 2DEG then becomes a two-dimensional (2D) superconductor. One could then desire to implement the two quantum dots in the 2DEG directly. More recently, it was suggested that superconductivity should also be present in ropes of single-walled carbon nanotubes \[ (9.9) \] which are strictly one-dimensional (1D) systems. It is therefore interesting to calculate \( (9.10) \) also in 2D and 1D. In the case of a 2D superconductor we evaluate \( (7.3) \) in leading order in \( \delta r/\pi\xi \) and find
\[ \sum_{k(2D)} \frac{u_{k\nu}v_k}{E_k} \cos(k \cdot \delta r) \]
\[ = \frac{\pi}{\nu_S} \left( J_0(k_F\delta r) + 2\sum_{\nu=1}^\infty \frac{J_{2\nu}(k_F\delta r)}{\pi\nu} \right). \quad (10.1) \]

The right-hand side of \( (10.1) \) can be approximated by \( (\pi/2)\nu_S J_0(k_F\delta r)/(1-(2/\pi)\ln 2) \) in the limit of large \( k_F\delta r \). For large \( k_F\delta r \), the behavior of the zeroth-order Bessel-function is \( J_0(k_F\delta r) \approx \sqrt{2/\pi k_F\delta r} \cos(k_F\delta r - \pi/4) \). So the amplitude decays asymptotically only \( \propto 1/\sqrt{k_F\delta r} \) and the current \( I_1 \) by a factor \( \propto 1/k_F\delta r \), respectively. In the case of 1D we obtain
\[ \sum_{k(1D)} \frac{u_{k\nu}v_k}{E_k} \cos(k \cdot \delta r) = \frac{\pi}{2} \nu_S \cos(k_F\delta r) e^{-(\delta r/\pi\xi)}, \quad (10.2) \]
where there are only oscillations and no decay of the Andreev amplitude (for \( \delta r/\pi\xi < 1 \)). We see that the suppression due to the finite separation of the tunneling points on the superconductor can be reduced considerably (or even excluded completely) by going over to lower-dimensional superconductors. By taking into account the dependence on the dimension of the superconductor we can relax the condition for the entangler to be efficient to
\[ (\xi/\gamma)^2 > (k_F\delta r)^{d-1}, \quad (10.3) \]
where \( d \) is the dimension of the superconductor.

We note that the coherent injection of the two spin-entangled electrons by an Andreev process via the dots into the leads allows for a time resolved detection of individual Cooper pairs in the leads since the delay time between the two partner electrons of a Cooper pair is given
by $1/\Delta$ whereas the separation in time of subsequent Cooper pairs is given approximately by $2e/I_1 \sim \gamma_l/\gamma_L^2$. Since $\Delta > \gamma_S$ and, in addition, $\gamma_l > \gamma_S$ the time delay between the two partners of the original Cooper pair is much shorter than the time difference between different Cooper pairs.

**XI. AHARONOV-BOHM OSCILLATIONS**

In this section we show that the different tunneling paths of the two electrons from the superconductor to the leads can be detected via the flux-dependent Aharonov-Bohm oscillations in the current flowing through a closed loop (see Fig. 5). We show that due to the possibility that two electrons can tunnel either via different dots into different leads (non-local process) or via the same dot into the same lead (local process), the current as a function of magnetic flux $\phi$ penetrating the loop contains $h/e$ and $h/2e$ oscillation periods. To be concrete, we consider a setup where the two leads 1 and 2 are connected such that they form an Aharonov-Bohm loop, (see Fig. 5), where the electrons are injected from the left via the superconductor, traversing the upper (lead 1) and lower (lead 2) arm of the loop before they rejoin to interfere and then exit into the same lead, where the current is then measured as a function of varying magnetic flux $\phi$. In the presence of a magnetic flux, each tunneling amplitude obtains a phase factor, $T_{D_1 L_1} \rightarrow T_{D_1 L_1} e^{i\phi/\phi_0}$, and $T_{D_2 L_2} \rightarrow T_{D_2 L_2} e^{-i\phi/\phi_0}$, where $\phi_0 = h/e$ is the single-electron flux quantum. For simplicity of the discussion we assume that the entire phase is acquired when the electron hops from the dot into the leads, so that the process dot-lead-dot gives basically the full Aharonov-Bohm phase factor $e^{\pm i\phi/\phi_0}$ of the loop (and only a negligible amount of phase is picked up along the path from the superconductor to the dots). We stress that there is no loss of generality in this assumption. The transition amplitude from the initial state to the final state has now the following structure:

$$\langle f | T_0 | i \rangle \sim T_{D_1 L_1} T_{D_2 L_2} + T_{D_1 L_1}^{-} T_{D_2 L_2}^{-} e^{i\phi/\phi_0} + T_{D_2 L_2}^{-} T_{D_1 L_1}^{-} e^{-i\phi/\phi_0}. $$

Here, the first term comes from the process via different leads (see (8.2)), where no Aharonov-Bohm phase is picked up. The Aharonov-Bohm phase appears in the remaining two terms, which come from processes via the same leads, either via lead 1 or lead 2 (see (8.3) and (8.6)). The total current $I$ is now obtained from $|\langle f | T_0 | i \rangle|^2$ together with a summation over the final states, giving $I = I_1 + I_2 + I_{AB}$, and the flux-dependent Aharonov-Bohm current $I_{AB}$ is given by:

$$I_{AB} = \sqrt{8I_1I_2} F(\epsilon_i) \cos (\phi/\phi_0) + I_2 \cos (2\phi/\phi_0) , \quad (11.1)$$

where, for simplicity, we have assumed that $\epsilon_1 = \epsilon_2 = \epsilon_i$, and $\gamma_1 = \gamma_2 = \gamma_L$. Here, the first term (different leads) is periodic in $\phi_0$ like for single-electron Aharonov-Bohm interference effects, while the second one (same leads) is periodic in half the flux quantum $\phi_0/2$, describing thus the interference of two coherent electrons travelling the upper or the lower arm of the loop (similar single- and two-particle Aharonov-Bohm effects occur in the Josephson current through an Aharonov-Bohm loop). It is clear from (11.1) that the $h/e$ oscillation comes from the interference between a contribution where the two electrons travel through different arms with contributions where the two electrons travel through the same arm. Both Aharonov-Bohm oscillations with period $h/e$, and $h/2e$, vanish with decreasing $I_2$, i.e. with increasing on-site repulsion $U$ and/or gap $\Delta$. However, their relative weight is given by $\sqrt{I_1/I_2}$, implying that the $h/2e$ oscillations vanish faster than the $h/e$ ones. This behavior is quite remarkable since it opens up the possibility to tune down the unwanted leakage process $\sim I_2 \cos (2\phi/\phi_0)$ where two electrons proceed via the same dot/lead by increasing $U$ with a gate voltage applied to the dots. The dominant current contribution with period $h/e$ comes then from the desired entangled electrons proceeding via different leads. On the other hand, if $\sqrt{I_1/I_2} < 1$, which could become the case e.g. for $k_F \delta r > E/\gamma$, we are left with $h/2e$ oscillations only. Besides the fact that the Aharonov-Bohm oscillations are interesting in its own right, the Aharonov-Bohm oscillations further provide an experimental probe of the non-locality of the two spin-entangled electrons. Note that dephasing processes which affect the orbital part suppress $I_{AB}$. Still, the flux-independent current $I_1 + I_2$ can remain finite and contain electrons which are entangled in spin-space, provided that there is only negligible spin-orbit coupling so that the spin is still a good quantum number.

We would like to mention another important feature of the Aharonov-Bohm effect under discussion, namely the relative phase shift between the amplitudes of tunneling to the same lead and to different leads, resulting in the additional prefactor $F(\epsilon_i)$ in the first term of the right-hand side of Eq. (11.1). This phase shift is due to the fact that there is a two-particle resonance in the ampli-
tude [3,2] while there is only a single-particle resonance in the amplitudes [6,3] and [6,1] (we recall that the second resonance is suppressed by the Coulomb blockade effect). Thus, when the chemical potential $\mu_S$ of the superconductor crosses the resonance, $|\epsilon_l| \lesssim \gamma_L$, the amplitude [2,2] acquires an extra phase factor $e^{i\delta r}$, where $\delta r = \arg\left[1/(\epsilon_l - i\gamma_L/2)\right]$. Then the interference of the two amplitudes leads to the prefactor $F(\epsilon_l) = \cos \phi_r$ in the first term on the right-hand side of (11.1). In particular, exactly at the middle of the resonance, $\epsilon_l = 0$, the phase shift is $\phi_r = \pi/2$, and thus the $h/e$ oscillations vanish, since $F(0) = \cos(\pi/2) = 0$. Note however, that although $F = \pm 1$ away from the resonance ($|\epsilon_l| > \gamma_L$) the $h/e$ oscillations vanish again, now because the current $I_1 \sim e\gamma_L^2/\gamma_L^2$ vanishes. Thus the optimal regime for the observation of the Aharonov-Bohm effect is $|\epsilon_l| \sim \gamma_L$.

Finally, the preceding discussion shows that even if the spins of two electrons are entangled their associated charge current does not reveal this spin-correlation in a simple Aharonov-Bohm interference experiment [4]. Only if we consider the current-current correlations (noise) in a beam splitter setup, can we detect also this spin-correlation in the transport current via its charge properties.

### XII. ANDREEV ENTANGLER WITH LUTTINGER LIQUID LEADS

We discuss a further implementation of an entangler for electron spins. We replace the quantum dots and the noninteracting leads by interacting one-dimensional wires described as Luttinger liquids (LL) (see Fig. 6). The low energy excitations of these LL are collective charge and spin density oscillations rather than quasiparticles which resemble free electrons. As a consequence tunneling into a LL is strongly suppressed due to interaction in the LL-leads. We show that the interaction can be used to separate two electrons, originating from an Andreev process, so that they preferably tunnel into different leads rather than into the same. Again we take into account a finite tunneling distance on the superconductor when the electrons tunnel to different leads.

![FIG. 6. The entangler setup with Luttinger liquid leads: An s-wave superconductor, SC, with chemical potential $\mu_S$ is weakly coupled to two strongly interacting bulk Luttinger liquids 1,2 held at the same chemical potential $\mu$. The two spin-entangled electrons of a Cooper pair can coherently tunnel with amplitude $t_0$ by means of an Andreev process from the superconductor to the leads. Two competing transport channels are considered. Two electrons can tunnel to different leads from points $r_1$ and $r_2$ (with distance $\delta r$) of the SC or from the same point $r_1$ or $r_2$ into the same lead. The interaction between the two leads is assumed to be negligible.](image)

![FIG. 6. The entangler setup with Luttinger liquid leads: An s-wave superconductor, SC, with chemical potential $\mu_S$ is weakly coupled to two strongly interacting bulk Luttinger liquids 1,2 held at the same chemical potential $\mu$. The two spin-entangled electrons of a Cooper pair can coherently tunnel with amplitude $t_0$ by means of an Andreev process from the superconductor to the leads. Two competing transport channels are considered. Two electrons can tunnel to different leads from points $r_1$ and $r_2$ (with distance $\delta r$) of the SC or from the same point $r_1$ or $r_2$ into the same lead. The interaction between the two leads is assumed to be negligible.](image)

The two leads 1, 2 are assumed to be infinitely extended and interacting one-dimensional systems described by conventional LL-theory. The LL-Hamiltonian for the low energy excitations of lead $n = 1, 2$ is written in a bosonized form as (neglecting backscattering) [37]

$$H_{Ln} - \mu_L N_n =$$

$$\sum_{\nu=\rho,\sigma} \int \frac{d\xi}{L/2} \left( \frac{u_\nu K_\nu}{2} \Pi_{\nu\nu}^2 + \frac{u_\nu}{2\pi K_\nu} (\partial_x \phi_{\nu\nu})^2 \right), \quad (13.1)$$

where $N_n$ is the number operator for lead $n = 1, 2$, and the fields $\Pi_{\nu}(x)$ and $\phi_{\nu\nu}(x)$ satisfy bosonic commutation relations $[\phi_{\nu\nu}(x), \Pi_{\mu\mu}(x')] = i\delta_{\nu\mu} \delta_{\nu\mu} \delta(x - x')$. The Hamiltonian $[3,3]$ describes long-wavelength charge ($\nu = \rho$) and spin ($\nu = \sigma$) density oscillations in the LL propagating with velocities $u_\nu$ and $u_\sigma$, resp. The velocities $u_\nu$ and the stiffness parameters $K_\nu$ depend on the interaction between the electrons in the LL. In the limit of vanishing backscattering, $u_\sigma = v_F$ and $K_\sigma = 1$ and the LL is described by only two parameters $K_\rho < 1$, and $u_\rho = v_F/K_\rho$, with $v_F$ being the Fermi velocity. We decompose the field operator describing electrons with spin $s$ into a right and left moving part, $\psi_{ns}(x) = e^{ik_F x} \psi_{ns+}(x) + e^{-ik_F x} \psi_{ns-}(x)$. The right (left) moving field operator $\psi_{ns+}(x)$ ($\psi_{ns-}(x)$) is then expressed as an exponential of bosonic fields as [36,40]

$$\psi_{ns\pm}(x) = \lim_{\alpha \to 0} \frac{\eta_{\pm,ns}}{\sqrt{2\pi \alpha}} \exp \left\{ \pm \frac{i}{\sqrt{2\alpha}} \left( \phi_{\nu\nu}(x) + s\phi_{\nu\nu}(x) + \theta_{\nu\nu}(x) + s\theta_{\nu\nu}(x) \right) \right\}, \quad (13.2)$$

where $\partial_x \theta_{\nu\nu}(x) = \pi \Pi_{\nu\nu}(x)$. The operators $\eta_{\pm,ns}$ are needed to ensure the correct fermionic anticommutation
relations. In (13.2) and hereafter we adopt the conven- 
tion that \( s = +1 \) for \( s = \uparrow \), and \( s = -1 \) for \( s = \downarrow \), if \( s \) has not the meaning of an index.

Transfer of electrons from the SC to the LL-leads is described by the tunnel Hamiltonian 
\[
H_T = t_0 \sum_n \psi^\dagger_{n s} \Psi_s(x_n) + \text{H.c..}
\]

The operator \( \Psi_s(x_n) \) annihilates an electron with spin \( s \) at the point \( x_n \) on the superconductor nearest to the LL-lead \( n \), and \( \psi^\dagger_{n s} \) creates it again with amplitude \( t_0 \) at point \( x_n \) in LL \( n \) (see Fig. 6). We assume that the tunneling amplitude \( t_0 \) does not depend on spin and also is the same for both leads.

**XIV. CALCULATION OF THE CURRENT: TWO COMPETING CHANNELS**

We now calculate the current for tunneling of two spin-
entangled electrons into different and into the same leads. Again, we use a T-matrix approach introduced in Section [VI] and calculate the current in lowest order in the tunneling Hamiltonian \( H_T \), which describes an Andreev process. For the current \( I_1 \) for tunneling of two electrons into different LL-leads we obtain in leading order in \( \mu/\Delta \) and at zero temperature

\[
I_1 = \frac{I_0^1}{\Gamma(2\gamma_\rho + 2)\; \rho_\rho} \left[ \frac{2\Lambda \mu^2}{\rho_\rho} \right]^{2\gamma_\rho},
\]

where \( \Gamma \) is the Gamma function and \( \Lambda \) is a short distance cut-off on the order of the lattice spacing in the LL. The interaction suppresses the current considerably and the bias dependence has its characteristic non-linear form \( I_1 \propto \mu^{2\gamma_\rho + 1} \) with an interaction dependent exponent \( \gamma_\rho = (K_\rho + K_\rho^{-1})/4 - 1/2 > 0 \). The parameter \( \gamma_\rho \) is the exponent for tunneling into the bulk of a single LL, i.e. \( \rho(\varepsilon) \sim |\varepsilon|^{\gamma_\rho} \), where \( \rho(\varepsilon) \) is the single particle DOS. The noninteracting limit \( I_0^1 \) is given as

\[
I_0^1 = \frac{\pi e \gamma^2}{2} \mu \left( \frac{\sin(k_F \delta r)}{k_F \delta r} \right)^2 \exp \left( -\frac{2\delta r}{\pi \xi} \right),
\]

with \( \gamma = 4\pi \nu_S \nu_L |t_0|^2 \) being the probability per spin to tunnel from the SC to the LL-leads and \( \nu_S \) and \( \nu_L \) are the energy DOS per spin for the superconductor and the LL-leads at the chemical potentials \( \mu_S \) and \( \mu_L \) resp.. We again observe the same dependence on the tunneling distance \( \delta r \) as before in the Andreev Entangler (see [3]).

The results for the lower dimensional superconductors are as before and given in Section [V].

Now we compare this result with the process when the two electrons tunnel into the same lead (with \( \delta r = 0 \)). Note that in this case the correlation in time between sub-
sequent tunneling events of the two electrons of a Cooper pair becomes extremely important due to the interaction in the LL-lead. We find after some calculation that the current \( I_2 \) for tunneling into the same lead (1 or 2) is suppressed if \( \mu < \Delta \) with the result, in leading order in \( \mu/\Delta \),

\[
I_2 = I_1 \sum_{\beta = \pm 1} A_\beta \left( \frac{2\mu}{\Delta} \right)^{2\gamma_\rho \beta}.
\]

The constant \( A_\beta \) in (14.3) is of order one for not too strong interactions, but is decreasing with increasing inter-
actions in the LL-leads, and is given by

\[
A_\beta = \frac{\pi}{2} \frac{\Gamma(2\gamma_\rho + 2)}{\Gamma(2\gamma_\rho + 2\gamma_\rho + 1)} \left\{ \frac{\Gamma(\gamma_\rho + 1)}{\Gamma(2\gamma_\rho + 1)(1 - 2\gamma_\rho)} \right\}^2 + \sin^2 \left( \frac{\gamma_\rho \pi}{2} \right) \Gamma^2(\gamma_\rho + 1) \left\{ \frac{\Gamma(\gamma_\rho + 1)}{\pi \Gamma(2\gamma_\rho + 1)} \right\}^2.
\]

We remark that in (14.3) the current \( I_1 \) is to be taken at \( \delta r = 0 \). The noninteracting limit \( I_2 = I_1 = I_1^0 \) is rediscovered by putting \( \gamma_\rho = \gamma_\rho = 0 \), and \( u_\rho = v_\rho \). The result for \( I_2 \) shows that the unwanted injection of two electrons into the same lead is suppressed compared to \( I_1 \) by a factor of \( (2\mu/\Delta)^{2\gamma_\rho_\rho} \), where \( \gamma_\rho_\rho = \gamma_\rho \), if both electrons are injected into the same branch (left or right movers), or by \( (2\mu/\Delta)^{2\gamma_\rho_\rho} \) if the two electrons travel in different directions. Since it holds that \( \gamma_\rho_- = \gamma_\rho_+ + (1 - K_\rho)/2 > \gamma_\rho_+ \), it is more favorable that the two electrons travel in the same direction than that in opposite directions. The suppression of the current \( I_2 \) by \( 1/\Delta \) shows very nicely the two-particle cor-
relation effect in the LL, when the electrons tunnel into the same lead. The larger \( \Delta \), the shorter is the delay time between the arrivals of the two partner electrons of a Cooper pair, and, in turn, the more the second electron tunneling into the same lead will feel the existence of the first one which is already present in the LL. By increasing the bias \( \mu \) the electrons can tunnel faster through the barrier since there are more channels available into which the electron can tunnel, and therefore the effect of \( \Delta \) is less pronounced. Also note that this correlation effect disappears when interactions are absent (i.e. when \( \gamma_\rho = \gamma_\rho = 0 \) in the LL. Actual experimental systems which show LL-behavior are e.g. metallic carbon nan-
tubes with similar exponents as derived here [3]. The exponents for tunneling into the bulk of a LL and for tunneling into the end of a LL are, in general, different, and it is predicted by theory [2] and consistent with experiment [4,5] that \( \gamma_{\rho \beta < \gamma_{\rho \beta} \rho} \). We therefore expect that if the electrons tunnel into the end of the LL, the suppression is even more pronounced, but to find the correct exponents would require a careful recalculation in that geometry.

Finally, the entangler setup is efficient if approximately

\[
\left( \frac{\Delta}{2\mu} \right)^{2\gamma_\rho_\rho} \propto (k_F \delta r)^{d-1},
\]

(14.5)
where $d$ is the dimension of the superconductor, $k_F$ is the Fermi wavevector of the superconductor, and it is assumed that the coherence length $\xi$ of the SC is large compared to $\delta r$ (see Section [X]).

XV. DECAY OF THE ELECTRON-SINGLET DUE TO LL-INTERACTIONS

Here we consider the decay of a nonlocal singlet where one electron is in lead 1 and the other in lead 2 due to the interactions in the LL-leads. A quantity which accounts for this consideration is the correlation function

$$P(r, t) = \langle \langle S(r, t)|S(0, 0) \rangle \rangle^2.$$ (15.1)

Here, $P(r, t)$ is the probability that a singlet state injected at $t = 0$ and at point $r \equiv (x_1, x_2) = 0$ is found at some later time $t$ at point $r$, and it is therefore a measure of how much of the initial singlet state remains after switching on the interaction during the time interval $t$. The singlet state created on top of the LL-groundstates is

$$|S(r, t)\rangle = \sqrt{\sigma(\psi^\dagger_1(x_1, t)\psi^\dagger_2(x_2, t)$$

$$- \psi^\dagger_1(x_1, t)\psi^\dagger_2(x_2, t))|0\rangle,$$ (15.2)

where the extra normalization factor $\sqrt{2\pi\alpha}$ for the singlet is introduced to guarantee $\int dr P(r, t) = 1$ in the noninteracting limit. The singlet-singlet correlation function factorizes into a product of two single-particle correlation functions due to negligible interactions between the two leads 1 and 2 and we obtain for $P(r, t)$, retaining only nonoscillatory terms,

$$P(r, t) = \prod_n \frac{1}{2} \sum_{r = \pm} F(t) \delta(x_n - ru_F t),$$ (15.3)

with a time decaying weight factor of the $\delta$-function

$$F(t) = \prod_{\nu = \rho, \sigma} \frac{\Lambda^2}{\Lambda^2 + (v_F - u_\nu)^2 t^2} \times$$

$$\left( \frac{\Lambda^4}{(\Lambda^2 + (v_F t^2 - (u_\nu t)^2)^2 + (2\Lambda u_\nu t)^2} \right)^{\gamma_\nu/2}.$$ (15.4)

Without interaction we have $F(t) = 1$ which means that there is no decay of the singlet state. As interactions are turned on we see that for times $t > \Lambda/u_\rho$, the singlet state starts to decay in time. For long times $t$ and for $u_\sigma = v_F$,

$$K_\sigma = 1$$

and $u_\rho = v_F/K_\rho$, the asymptotic behavior of the decay is $F(t) \sim \frac{\Lambda}{u_\rho(1-K_\rho)} [\frac{\Lambda^2}{u_\rho^2(1-K_\rho)^2}]^{\gamma_\rho/2} t^{2\gamma_\rho+1}$, which for very strong interactions in the LL leads, i.e. $K_\rho$ much smaller than one, becomes $F(t) \sim [\Lambda/u_\rho]^2 t^{2\gamma_\rho+1}$. This result together with (15.3) shows that charge and spin of an electron propagate with velocity $v_F$, whereas charge (spin) excitations of the LL propagate with $u_\rho$ ($u_\sigma$) as we confirm explicitly in the next Section.

XVI. PROPAGATION OF CHARGE AND SPIN

Although the singlet state gets destroyed due to interaction the spin information can still be transported through the wires by the spin density excitations as we will show now. The normal ordered charge density $\rho_n(x)$ of lead $n$ is $\rho_n(x) = \sum_{\sigma} \psi^\dagger_n(x)\psi_n(x)$, where we kept only the slow spatial variations of the density operator, and $r = \pm$ denotes the branch of left- and right-movers. Likewise, for the normal ordered spin density we have $\sigma_n^z(x) = \sum_{\sigma} \psi^\dagger_n(x)\psi_n(x)$, where we inject an electron with spin $s$ into branch $r$ of lead $n$, and at time $t = 0$, on top of the LL groundstate. We calculate the time dependent charge and spin density fluctuations for this state and find for the charge density fluctuations

$$(2\pi \alpha) \langle 0| \psi^\dagger_{n\sigma}(x_n) \rho_n(x_n', t) \psi_{n\sigma}(x_n) |0\rangle$$

$$= \frac{1}{2} (1 + r K_\rho) \delta(x_n' - x_n - u_\rho t)$$

$$+ \frac{1}{2} (1 - r K_\rho) \delta(x_n' - x_n + u_\rho t),$$(16.1)

and for the spin density fluctuations

$$(2\pi \alpha) \langle 0| \psi^\dagger_{n\sigma}(x_n) \sigma_n^z(x_n', t) \psi_{n\sigma}(x_n) |0\rangle$$

$$= \frac{s}{2} (1 + r K_\sigma) \delta(x_n' - x_n - u_\sigma t)$$

$$+ \frac{s}{2} (1 - r K_\sigma) \delta(x_n' - x_n + u_\sigma t).$$(16.2)

We see that in contrast to the singlet, the charge and spin density fluctuations in the LL created by the injected electron do not decay and show a pulsed shape with no dispersion in time. This is due to the linear energy dispersion relation of the LL-model. In carbon nanotubes such a highly linear dispersion relation is indeed realized and therefore carbon nanotubes should be well suited for spin transport. Another interesting effect that shows up in (16.1) and (16.2) is the different velocities of spin and charge, which is known as spin-charge separation. It would be interesting to test Bell inequalities [4] via spin-spin correlation measurements between the two LL-leads and see if the initial entanglement of the spin singlet is still observable in the spin density-fluctuations. Although detection of single spins with magnitudes on the order of electron spins has still not been achieved, magnetic resonance force microscopy (MFRM) seems to be very promising in doing so [5]. Another scenario is to use the LL just as an intermediate medium which is needed to first separate the two electrons of a Cooper pair and then to take them (in general other electrons) out again into two (spatially separated) Fermi liquid leads where the (possibly reduced) spin entanglement could be measured via the current noise in a beamsplitter experiment [5]. In this context we note that the decay of the singlet state given by (15.3) sets in almost immediately after the injection into the LLs (the time scale is approximately the...
inverse of the Fermi energy) but at least at zero temperature, the suppression is only polynomial in time which suggests that some fraction of the singlet state can still be recovered.

**XVII. CONCLUSION**

We discussed setups for creating spin-entanglement in quantum confined nanostructures like semiconductor quantum dots and mesoscopic wires based on superconductors which provide a natural source of spin-entanglement in form of Cooper pairs. We showed that a superconductor can induce tunable nonlocal spin-entanglement between two electron spins which reside on different quantum dots without having a direct tunnel coupling between the dots. We then described an entangler device that creates mobile spin-entangled electrons via an Andreev process into different dots which are tunnel-coupled to leads. The unwanted process of both electrons tunneling into the same leads can be suppressed by increasing the Coulomb repulsion on the quantum dots. We have shown that there exists a regime of experimental relevance where the current of entangled electrons shows a resonance and assumes a finite value with both partners of the singlet being in different leads but having the same orbital energy. This entangler thus satisfies the requirements needed to detect spin entanglement via transport and noise measurements. Further, we discussed the flux-dependent oscillations of the current in an Aharonov-Bohm loop which could serve as an experimental means to detect the nonlocality of the two correlated electrons. Finally, we discussed a novel setup consisting of two interacting Luttinger liquid leads (such as nanotubes) weakly coupled to a superconductor to separate the two spin-entangled electrons. We found that the coherent subsequent tunneling of two electrons into the same LL is suppressed if the applied bias is smaller than the superconducting gap in a characteristic power-law manner.

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