TEMPERATURE, CHEMICAL POTENTIAL
AND THE $\rho$–MESON

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1. Introduction. Models of QCD must confront nonperturbative phenomena such as confinement, dynamical chiral symmetry breaking (DCSB) and the formation of bound states. In addition, a unified approach should describe the deconfinement and chiral symmetry restoring phase transition exhibited by strongly-interacting matter under extreme conditions of temperature and density. Nonperturbative Dyson-Schwinger equation (DSE) models [1, 2] provide insight into a wide range of zero temperature hadronic phenomena; e.g., nonhadronic electroweak interactions of light- and heavy-mesons [3], and diverse meson-meson [4] and meson-nucleon [5] form factors. This is the foundation for their application at nonzero-($T, \mu$) [2],[6]–[8]. Herein we describe the calculation of the deconfinement and chiral symmetry restoring phase boundary, and the medium dependence of $\rho$-meson properties. We also introduce an extension to describe the time-evolution in the plasma of the quark’s scalar and vector self energies based on a Vlasov equation.

2. Dyson-Schwinger Equation at Nonzero-($T, \mu$). The dressed-quark DSE at nonzero-($T, \mu$) is

\begin{align*}
S^{-1}(\tilde{p}_k) &= i\gamma \cdot \tilde{p} A(\tilde{p}_k) + i\gamma_4 \omega_{k+} C(\tilde{p}_k) + B(\tilde{p}_k) = i\gamma \cdot \tilde{p} + i\gamma_4 \omega_{k+} + \Sigma(\tilde{p}_k), \quad (1)
\end{align*}

where \(\tilde{p}_k = (\tilde{p}, \omega_{k+})\), \(\omega_{k+} = \omega_k + i\mu\), and \(\omega_k = (2k+1)\pi T\) is the quark’s Matsubara frequency. The complex-valued scalar functions: \(A(\tilde{p}, \omega_{k+})\), \(B(\tilde{p}, \omega_{k+})\) and \(C(\tilde{p}, \omega_{k+})\), depend only on \((|\tilde{p}|^2, \omega_{k+}^2)\). With a given dressed-gluon propagator the solutions are determined by

\begin{align*}
B(\tilde{p}_k) - m_0 &= \frac{8}{3} \int \frac{d^4\tilde{q}}{(2\pi)^4} \frac{B(\tilde{q}_k)}{\tilde{q}_k^2 C(\tilde{q}_k) + B^2(\tilde{q}_k)}, \quad (2)
\end{align*}

\begin{align*}
(C(\tilde{p}_k) - 1)\tilde{p}_k^2 &= \frac{4}{3} \int \frac{d^4\tilde{q}}{(2\pi)^4} \frac{\tilde{p}_k \cdot \tilde{q}_k C(\tilde{q}_k)}{\tilde{q}_k^2 C(\tilde{q}_k) + B^2(\tilde{q}_k)}, \quad (3)
\end{align*}

where herein we only consider models where \(A(p) = C(p)\). It is the interplay between the functions \(B\) and \(C\) that leads to confinement, realised via the absence of a Lehmann representation for the dressed-quark 2-point function [1, 4],[8]. \(B \neq 0\) in the chiral limit signals DCSB.
To provide an illustrative solution of the quark DSE we employ an Ansatz for the scalar function characterising the dressed-gluon propagator [10]:

$$D(p) = 3\pi^2 \frac{\eta^2}{T} \delta_{k_0} \delta^3(p).$$

(4)

The infrared enhancement in this choice ensures quark confinement and DCSB. As an infrared dominant model, Eq. (4) does not represent well the interaction away from $(\tilde{p}_k - \tilde{q}_k)^2 \simeq 0$ and that introduces some model-dependent artefacts. However, they are easily identified and the model yields qualitatively reliable results, preserving features of more sophisticated studies.

Using Eq. (4) in Eqs. (2-3) we obtain a system with two phases. The
Nambu-Goldstone (NG) phase is characterised by dynamically broken chiral symmetry and confined dressed-quarks. The alternative Wigner-Weyl (WW) solution describes a phase of the model with restored chiral symmetry and deconfinement. In studying the phase transition one must consider the relative stability of the confined and deconfined phases, which is measured by the \((T, \mu)-\)dependent pressure difference between the two distinct phases: \(B(T, \mu) = P[S_{\text{NG}}] - P[S_{\text{WW}}] \). \(B(T, \mu) > 0\) indicates the stability of the confined (Nambu-Goldstone) phase and hence the phase boundary is specified by that curve in the \((T, \mu)\)-plane for which \(B(T, \mu) = 0\). The critical line is depicted in Fig. 1. The phase transition is first order for any non-zero \(\mu\) and second order for \(\mu = 0\). The model has mean field critical exponents, which is a feature of the rainbow-ladder truncation [11]. The study of thermodynamic properties shows that it is essential to keep scalar and vector self-energies as well as their momentum dependence [8, 10].

Mesons are quark-antiquark bound states and their masses are obtained by solving the Bethe-Salpeter equation [12]. Here we focus on the vector channel and employing Eq. (4) the eigenvalue equation for the bound state mass is [13]:

\[
\frac{\eta^2}{2} \text{Re}\left\{\sigma_B(\omega^2_{0+} - \frac{1}{4}M^2_{\rho\pm})^2 - \left[\pm\omega^2_{0+} - \frac{1}{4}M^2_{\rho\pm}\right] \sigma_C(\omega^2_{0+} - \frac{1}{4}M^2_{\rho\pm})^2\right\} = 1, \tag{5}\]

where \(\sigma_{B,C}(p^2_k) = \{B(p^2_k), C(p^2_k)\}/[p^2_k C^2(p^2_k) + B^2(p^2_k)]\). The equation for the \(\rho\)-meson’s transverse component is obtained with \([-\omega^2_{0+} - \frac{1}{4}M^2_{\rho-}]\) in Eq. (5) and in the chiral-limit yields \(M^2_{\rho-} = \frac{1}{2} \eta^2\), independent of \(T\) and \(\mu\). This is the \(T = 0 = \mu\) result of Ref. [14]. Even for nonzero current-quark mass, \(M_{\rho-}\) changes by less than 1% as \(T\) and \(\mu\) are increased from zero toward their critical values. Its insensitivity is consistent with the absence of a constant mass-shift in the transverse polarization tensor for a gauge-boson. For the longitudinal component one obtains in the chiral limit:

\[
M^2_{\rho+} = \frac{1}{2} \eta^2 - 4(\mu^2 - \pi^2 T^2). \tag{6}\]

The results for the medium-dependence of the \(\rho\) meson are summarised in Fig. 2. As in the case of the dressed-quark mass function, the response to increasing \(T\) and \(\mu\) is anti-correlated: the \(\rho\)-mass decreases with increasing chemical potential and increases with temperature. This anti-correlation leads to an edge along which the \(T\) and \(\mu\) effects compensate and the mass remains unchanged up to the transition point.

3. Nonequilibrium Application. The time evolution of the self energies
can be studied using Vlasov’s equation

\[ \partial_t f(p, x) + \partial_p E(p, x) \partial_x f(p, x) - \partial_x E(p, x) \partial_p f(p, x) = 0 . \]  

(7)

Solving this equation is complicated for two reasons. (i) The energy is a functional of the scalar and vector self energies, which in general are nonzero and momentum-dependent. While the scalar self energy is small in the plasma phase due to chiral symmetry restoration, the vector self energy remains significant \[10\]. (ii) The absence of a Lehmann representation for the dressed-quark propagator in the confined phase precludes the existence of a single particle distribution function, \( f \), in this phase. Therefore a conventional kinetic theory is only reasonable in the deconfined phase. This situation is adequately represented in DSE models; e.g., Refs. \[9\] describe a quark’s \((T, \mu)\)-evolution from a confined to a propagating mode, and Ref. \[10\] makes use of this evolved quasiparticle behaviour in calculating the plasma’s thermodynamic properties.
Therefore, approaching the phase boundary from the plasma domain we anticipate a discontinuous disappearance of the quark distribution function, \( f \).

As an illustration we employ an instantaneous interaction of the form

\[
D(p) = 3\pi^2 \eta \delta^3(p)
\]

(8)
to represent dynamics in the deconfined phase. In this case the Matsubara sum in Eq. (2) can be performed analytically and we obtain:

\[
\Sigma^B(p, x) = \eta \frac{\Sigma^B(p, x) + m_0}{(1 + \Sigma^C(p, x))E^*(p, x)}[1 - 2f(p, x)],
\]

(9)

\[
\Sigma^C(p, x) = \frac{1}{(1 + \Sigma^C(p, x))E^*(p, x)}[1 - 2f(p, x)],
\]

(10)

with the quasi particle energy: \( E^*(p, x) = \sqrt{(\vec{p}^*)^2 + M^*(p, x)^2} \), the renormalized momentum: \( \vec{p}^* = \vec{p}(1 + \Sigma^C(p, x)) \), and mass: \( M^*(p, x) = m_0 + \Sigma^B(p, x) \).

As a test whether this simplification still yields necessary and qualitatively important features, such as \( C \neq 1, B \neq m_0 \), in Fig. 3 we compare the momentum dependence obtained in the models specified by Eqs. (4,8) in the vicinity of \( T_c \).

Both functions are well reproduced and hence Eq. (8) can be used to model the persistence of non-perturbative effects in the deconfined domain. The solution of Eqs. (7,9-10) provide the time-evolution of the quark self-energy and distribution function.

As in the case of thermal equilibrium, the vector self energy plays an important role. Neglecting \( \Sigma^C \) and the momentum dependence of \( \Sigma^B \) a simpler equation is obtained

\[
\partial_t f(p, x) + \frac{\vec{p}}{E(p, x)}\partial_x f(p, x) - m(x)\partial_x m(x)\partial_p f(p, x) = 0,
\]

(11)

with \( m(x) \) the quark mass obtained as a solution of the gap equation in models without confinement. This equation has been widely studied; e.g. Refs. [15]. However, we anticipate that the numerical solution of Eq. (7) will yield significantly different results because of the presence and persistence of the vector self energy in the deconfined domain.

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Figure 3: Momentum dependence of the quark scalar and vector self energies obtained in the model of Eq. (4) compared with those in the model of Eq. (8).

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