Reinforcement Learning for Signal Temporal Logic using Funnel-Based Approach

Naman Saxena†, Gorantla Sandeep †, and Pushpak Jagtap

Abstract—Signal Temporal Logic (STL) is a powerful framework for describing the complex temporal and logical behavior of the dynamical system. Several works propose a method to find a controller for the satisfaction of STL specification using reinforcement learning but fail to address either the issue of robust satisfaction in continuous state space or ensure the tractability of the approach. In this paper, leveraging the concept of funnel functions, we propose a tractable reinforcement learning algorithm to learn a time-dependent policy for robust satisfaction of STL specification in continuous state space. We demonstrate the utility of our approach on several tasks using a pendulum and mobile robot examples.

I. INTRODUCTION

Temporal logic is an effective method of formally defining the complex behavior of a particular system [1]. The expressiveness of predicate logic combined with temporal dimension led to its widespread use in designing the specification of dynamical systems. Signal Temporal Logic (STL), presented by [2], extends the applicability of temporal logic by allowing us to specify a particular behavior for a fixed time interval. Let us consider the task of navigating on roads where we need to reach particular locations in specific time intervals. Requirements of this nature can be successfully modeled using the framework of STL.

Several works in literature [3], [4] use the dynamics model to design controllers to enforce STL specifications. But recently, controller synthesis for STL specification without a mathematical model of systems using Reinforcement Learning (RL) has started gaining attention. Not only is reinforcement learning beneficial in the context of STL specifications, but at the same time, STL facilitates the designing of rewards for RL in a structured manner, avoiding the loopholes in developing rewards in a heuristic manner [5]. [6] used Q-learning [7] to achieve tasks defined using STL by maximizing the robustness of STL satisfaction. [6] defined the \( \tau - MDP \) framework to allow each state to store the history of states. Storing the history of states is required to check for the satisfaction of STL formulas and, at the same time, raises doubts about the tractability of the proposed method. Further, [8] tries to solve the tractability issue of Q-learning by proposing the use of flag variables. Flag variables are used to avoid storing the history of states and check the satisfaction of STL formulas. One issue with this work is that it does not consider robustness values. [9] proposed a solution for multi-agent system using Deep Q-learning algorithm [10]. The authors used Deep Q-Network (DQN) to overcome the state-space explosion in the multi-agent setting. Their approach considers robustness value for the STL formula but again suffers from the drawback of storing the history of states.

On the other hand, [11] proposed using a funnel-based control approach to enforce STL specification fragments for known dynamical control-affine systems. Motivated by the above results, we propose using a funnel-based approach in reinforcement learning algorithm to enforce STL specification in a tractable manner. The key contributions of the paper are listed below:

• The proposed approach leverage concept of funnel-based control to resolve the issue of tractability raised due to storing state history. It allows us to make the approach independent of state history by using a policy dependent on the current time and to provide an efficient algorithm to deal with continuous state-space.

• The result by [11] that uses funnels for STL specifications is limited to control-affine and fully-actuated known systems. Moreover, the STL specifications handled by [11] are very restricted. For example, the approach can not handle the logical operator ‘OR’ and only works with concave predicates. In contrast, the approach proposed in this paper can be used for any unknown nonlinear systems along with a wide range of STL properties, including the ‘OR’ operator and convex predicates (which are useful in specifications like obstacle avoidance). Please refer to experimental results in Section IV-B and Section IV-C for demonstration.

In summary, the proposed approach resolves the existing limitations (i) on the use of RL for STL (such as robustness, tractability, and continuous state-space) and (ii) on the use of the funnel-based control approach for STL (such as the requirement of fully actuated and control affine dynamics with unbounded inputs, restrictions on the use of convex predicate and ‘OR’ operators in STL specifications, robustness to disturbances).
II. PRELIMINARIES

A. Deep Q-learning

Reinforcement learning is a learning paradigm based on the framework of Markov Decision Processes (MDP) [13]. An MDP is defined by a tuple $\mathcal{M} = (\mathcal{S}, \mathcal{A}, r, \mathcal{P}, \pi, \gamma)$. In our paper, $\mathcal{S} \subseteq \mathbb{R}^m$ refers to the continuous state space, $\mathcal{A}$ refers to the discrete action space and $r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is the reward function. Further, $\mathcal{P}(\cdot|s,a)$ is the transition probability function defined as $\mathcal{P} : \mathcal{S} \times \mathcal{A} \rightarrow [0,1]$ is a probability measure and $\mathcal{B}(\mathcal{S})$ is the Borel $\sigma$–algebra on state space $\mathcal{S}$. Here, $\pi : \mathcal{S} \rightarrow \Delta(\mathcal{A})$ is a stochastic policy (controller) and $\Delta(\mathcal{A})$ is the probability simplex over action space $\mathcal{A}$. $\gamma \in (0,1)$ is the discount factor. The policy $\pi$ is obtained by optimizing the long-term discounted reward objective function $\eta(\pi)$ as defined below:

$$\eta(\pi) = E\left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right]. \quad (1)$$

where $s_t, a_t$ denotes the state and action taken at time $t$. To solve the above optimization problem, Q-learning [7] is one of the most widely used reinforcement learning algorithms. It uses $\varepsilon$–greedy policy (3) based on the Q-value function (2) to explore and optimize the objective function in (1).

$$Q^\pi(s, a) = E\left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) | s_t, a_t \right]$$

$$\pi(a|s) = \begin{cases} 
1 - \varepsilon + \frac{\varepsilon}{|\mathcal{A}|} & a = \text{arg max}_{a'} Q^\pi(s, a') \\
\varepsilon & \text{otherwise}
\end{cases} \quad (2)$$

Deep Q-learning [10] is a function approximation-based Q-learning algorithm that uses a neural network to learn optimal policy online using a replay buffer. The algorithm uses approximate Q-value function $Q_\theta(s, a)$ to satisfy the Bellman equation in (4), where $\theta$ stands for the parameters of the neural network.

$$Q^\pi(s, a) = E[r(s, a) + \gamma \max_{a' \in \mathcal{A}} Q^\ast(s', a')] \quad (4)$$

B. Signal Temporal Logic

Signal temporal logic (STL) [2] provides a formal framework to capture high-level specifications that can handle spatial, temporal, and logical constraints. It consists of a set of predicates $\varphi$ that are evaluated based on their corresponding predicate function $h : \mathcal{S} \rightarrow \mathbb{R}$ as $\varphi := \{ \text{True, if } h(s) \geq 0 \}$, $\{ \text{False, if } h(s) < 0 \}$. The syntax for an STL formula $\phi$ is given by:

$$\phi ::= \text{True} \mid \varphi \mid \neg \phi \mid \varphi_1 \land \varphi_2 \mid \varphi_1 \lor \varphi_2 \mid F_{[a,b]} \phi \mid G_{[a,b]} \phi \quad (5)$$

where $a, b \in \mathbb{R}_0^+$ with $a \leq b$, $\varphi_1$ and $\varphi_2$ are STL formulas, $\neg, \land$ and $\lor$ are logical negation, conjunction and disjunction operator, respectively; and $F$ and $G$ are temporal eventually and always operators, respectively. The relation $s_t =^* \phi$ indicates that the signal $s : \mathcal{R}_t \rightarrow \mathcal{S}$ satisfies the STL formula $\phi$ at time $t$. The STL semantics for a signal $s$ is recursively defined as follows:

$$s_t =^* \varphi \quad \iff \quad \varphi \text{ is True}$$

$$s_t =^* \neg \phi \quad \iff \quad \neg (s_t =^* \phi)$$

$$s_t =^* \varphi_1 \land \varphi_2 \quad \iff \quad s_t =^* \varphi_1 \land s_t =^* \varphi_2$$

$$s_t =^* \varphi_1 \lor \varphi_2 \quad \iff \quad s_t =^* \varphi_1 \lor s_t =^* \varphi_2$$

$$s_t =^* F_{[a,b]} \phi \quad \iff \quad \exists i \in [t+a,t+b] \text{ s.t. } s_i =^* \phi$$

$$s_t =^* G_{[a,b]} \phi \quad \iff \quad \forall i \in [t+a,t+b] \text{ s.t. } s_i =^* \phi \quad (6)$$

Next, we recall the robust semantics for STL formulas introduced by [14], which will later be used to construct rewards.

$$\rho_\varphi(s_t) = h(s_t)$$

$$\rho_{\neg \varphi}(s_t) = -\rho_\varphi(s_t)$$

$$\rho_{\varphi_1 \land \varphi_2}(s_t) = \min (\rho_{\varphi_1}(s_t), \rho_{\varphi_2}(s_t))$$

$$\rho_{\varphi_1 \lor \varphi_2}(s_t) = \max (\rho_{\varphi_1}(s_t), \rho_{\varphi_2}(s_t))$$

$$\rho_{F_{[a,b]} \varphi}(s_t) = \max_{t' \in [t+a,t+b]} \rho_\varphi(s_{t'})$$

$$\rho_{G_{[a,b]} \varphi}(s_t) = \min_{t' \in [t+a,t+b]} \rho_\varphi(s_{t'}) \quad (7)$$

In this paper, we consider the following fragment of specifications:

$$\psi ::= \varphi \mid \neg \psi \mid \psi_1 \land \psi_2 \mid \psi_1 \lor \psi_2$$

$$\phi_{[a,b]} ::= F_{[a,b]} \psi \mid G_{[a,b]} \psi \mid F_{[a,c]} G_{[c,b]} \psi$$

$$\Phi ::= \bigwedge_{i=1}^k \phi_{[a_i,b_i]} \quad (8)$$

where $0 \leq a \leq c_1, c_2 \leq b, b_1 < a_{i+1}, \forall i \in \{1, \ldots, k-1\}$, $\psi$ and $\phi$ denote non-temporal and temporal formulas, respectively. In the next section, we discuss a funnel-based construction of rewards for deep Q-learning to learn a control policy enforcing the fragment of STL specifications given in (8).

III. PROPOSED APPROACH

In this section, we will first describe how we use concepts from funnel-based control to construct time-varying rewards that capture the robust satisfaction of STL specifications. Subsequently, we will discuss including time as part of state space for the deep Q-learning algorithm.
A. Construction of Rewards using Funnel Functions

Funnel based approach was first used by [11] to develop controllers that satisfy a fragment of STL specifications for known control systems. Several works in the literature now build upon this direction [12], [15], [11] proposed to find a controller that satisfies the following relation:

\[ \forall t \geq 0, \quad -\gamma(t) + \rho_{\text{max}} < \rho_{\psi}(s_t) < \rho_{\text{max}}, \tag{9} \]

where \( \gamma(t) \) is a non-increasing and continuously differentiable positive function referred to as funnel and defined as

\[ \gamma(t) = (\gamma_0 - \gamma_{\infty}) e^{-t} + \gamma_{\infty}, \]

where \( \gamma_0, \gamma_{\infty}, \) and \( l \) are positive constants with \( \gamma_0 \geq \gamma_{\infty}, \) and \( \rho_{\text{max}} \) is the maximum robustness defined for the system for corresponding non-temporal specification \( \psi \) and obtained as \( \rho_{\text{max}} = \max_{s \in \mathcal{S}} \rho_{\psi}(s). \)

Let us consider the STL fragment defined in (8). The STL formula \( \Phi \) can consist of a single eventually (\( F \)) or always (\( G \)) operator, or it could contain these operators combined using a conjunction operator. The parameter \( l \) of funnel function \( \gamma(t) \) for \( F[\alpha,b] \psi, G[\alpha,b] \psi, \) and \( F[\alpha,c_1]G[\alpha,c_2] \psi \) is chosen as given in Table I, while \( \gamma_0 = \rho_{\text{max}} - \min_{s \in \mathcal{S}} \rho_{\psi}(s) \) and \( \gamma_{\infty} \) are defined for the system for corresponding non-temporal specification \( \psi \).

| Operator            | \( t^* \)     | \( l \)       |
|---------------------|--------------|--------------|
| \( F[\alpha,b] \psi \) | \( t^* = a \) | \( l \) ln \( \frac{\rho_{\text{max}} - \gamma_{\infty}}{\gamma_0 - \gamma_{\infty}} \) |
| \( G[\alpha,b] \psi \) | \( t^* \in [\alpha,b] \) | \( l \) ln \( \frac{\rho_{\text{max}} - \gamma_{\infty}}{\gamma_0 - \gamma_{\infty}} \) |
| \( F[\alpha,c_1]G[\alpha,c_2] \psi \) | \( t^* \in [a + c_2, c_1 + c_2] \) | \( l \) ln \( \frac{\rho_{\text{max}} - \gamma_{\infty}}{\gamma_0 - \gamma_{\infty}} \) |

### TABLE I

Selection of funnel function parameter \( l \).

The illustration of the funnel for eventually and always operators is shown in Figure 1. The value for \( l \) is chosen according to the interval for temporal operators. For \( F[\alpha,b] \) operator \( l \) is \( \frac{\ln((\gamma_0 - \gamma_{\infty})/(\rho_{\text{max}} - \gamma_{\infty}))}{t^* - \alpha} \), where \( t^* \in [\alpha,b] \), so that \( \gamma(t^*) = 0 \). Note that \( t^* \) lies in \( [\alpha,b] \) because for eventually operator, we want the robustness to be positive at least once in the interval \( [\alpha,b] \). For \( G[\alpha,b] \) operator \( l \) is \( \frac{\ln((\gamma_0 - \gamma_{\infty})/(\rho_{\text{max}} - \gamma_{\infty}))}{\gamma_0 - \gamma_{\infty}} \), where \( t^* = a \), so that \( \gamma(a) = 0 \) and the robustness is positive throughout the interval \( [\alpha,b] \). Similar reasoning follows for \( F[\alpha,c_1]G[\alpha,c_2] \).

Now we will discuss cases where \( F \) and \( G \) operators appear in conjunction. Let us take the STL formula \( \Phi = F[\alpha_1,b_1] \psi_1 \land G[\alpha_2,b_2] \psi_2 \) with \( b_1 < \alpha_2 \), where \( \psi_1 \) and \( \psi_2 \) are as defined in (8). The funnel function for \( \Phi \) is given as

\[ \gamma(t) = \begin{cases} 
(\gamma_0 - \gamma_{\infty}) e^{-\ln(\rho_{\text{max}} - \gamma_{\infty})/t^*} + \gamma_{\infty}, & \text{for } 0 \leq t \leq b_1, \\
(\gamma_0 - \gamma_{\infty}) e^{-\ln(\rho_{\text{max}} - \gamma_{\infty})/t^*} + \gamma_{\infty} + \gamma_0, & \text{for } t > b_1.
\end{cases} \tag{10} \]

and the plot is shown in Figure 2. In (10), \( t^* = c \) lies in \( [\alpha_1,b_1] \). For \( t \leq b_1 \), the funnel function \( \gamma(t) \) is defined according to \( F[\alpha_1,b_1] \) operator and for \( t > b_1 \), the funnel function is defined according to \( G[\alpha_2,b_2] \) operator. In this case, the \( F \) operator is considered with \( G \), but the funnel function can also be designed similarly to handle two \( F \) operators and/or two \( G \) operators. Further, this method of designing funnel function is not limited to two operators but can be extended to several operators in conjunction.

Given the construction of funnel function \( \gamma(t) \) for the temporal part and robustness measure \( \rho_{\psi}(s_t) \) for the non-temporal part of the STL formula, we can now define the reward function for the deep Q-learning algorithm based on funnel function \( \gamma(t) \) as

\[ r'(s_t, a_t, \rho_{\psi}(s_t)) = \gamma(t) - \rho_{\text{max}}. \tag{11} \]

The reward function in (11) is positive at time \( t \) if the current state of the system (agent) follows the bounds given in (9); otherwise, it is negative. Further, the reward is more positive if \( \rho_{\psi}(s_t) \) is close to \( \rho_{\text{max}} \) and more negative as it is farther away from the lower bound in (9) in the negative direction.

B. Time-aware Deep Q-learning

Our time-aware Deep Q-learning algorithm uses the funnel-based time-dependent reward function in (11), which is not only a function of state and action but also a function of time \( t \). The modified MDP is defined as \( \mathcal{M}' = \)
Algorithm 1 Time-aware Deep Q-learning

Initialize Q-value function parameter $\theta$.
Initialize target Q-value function parameter $\tilde{\theta} \leftarrow \theta$
$\alpha$ is the step size for parameter update.

1: $k = 0$, $s_0 = \text{env.reset}(), t = 0$
2: while $k \leq$ total steps do
3: $a_t \sim \pi (\cdot | s_t, i) \{ \pi \text{ is } \epsilon\text{-greedy policy} \}$
4: $s_{t+1} \sim \mathcal{P}(\cdot | s_t, a_t)$ and $r_t = r(s_t, a_t, t)$
5: Store $\{s_t, a_t, r_t, s_{t+1}, t\}$ in Replay Buffer $\{s_{t+1} = s'_t\}$
6: if $k \% \text{eval_freq} == 0$ then
7: Evaluate(agent)
8: end if
9: Sample $\mathbb{B}_k = \{s_i, t_i, s'_i, t'_i\}_{i=0}^{M-1}$ from the Replay Buffer
10: Update $\theta \leftarrow \theta + \alpha \nabla \theta \left( \frac{1}{M} \sum_{i=0}^{M-1} (r(s_i, a_i, t_i) + \gamma \max_a Q_{\pi}(s'_i, a, t'_i) - Q_{\theta}(s_i, a_i, t_i))^2 \right)$
11: if $k \% \text{target_update_freq} == 0$ then
12: Update $\tilde{\theta} \leftarrow \theta$
13: end if
14: $k = k + 1$
15: if $s_{t+1}$ is terminal then
16: $s_t = \text{env.reset}(), t = 0$
17: else
18: $s_t = s_{t+1}, t = t + 1$
19: end if
20: end while

$\mathcal{S}, \mathcal{A}, r', \mathcal{R}, \pi, \gamma$, where $r' : \mathcal{S} \times \mathcal{A} \times \mathbb{N} \cup \{0\} \mapsto \mathbb{R}$ is the new reward function. The stochastic policy is now defined as $\pi : \mathcal{R} \times \mathbb{N} \cup \{0\} \mapsto \Delta(\mathcal{A})$. The Markov property of the transition probability function is intact because the transition to $s_{t+1}$ still depends on $(s_t, a_t)$ and $a_t$ depends on $s_t$ and the current time $t$. Now, the reward depends on time and consequently Q-value function also depends on time and is defined as follows:

$$Q^\pi(s_t, a_t, t) = E \left[ \sum_{k=0}^{\infty} \gamma^k r(s_k, a_k, k) | s_t, a_t, t \right]$$

$$= E \left[ r(s_t, a_t, t) + \gamma Q^\pi(s_{t+1}, a_{t+1}, t+1) | s_t, a_t, t \right].$$

Further, the policy will depend on time because we use the $\epsilon$-greedy policy, which depends on the Q-value function (3). The proposed method is summarized in Algorithm 1. The next section discusses the results obtained using our time-aware Deep Q-learning algorithm.

IV. EXPERIMENTAL RESULTS

We performed experiments on three case studies with various systems and STL properties to demonstrate the merits of the proposed approach. We trained the RL agent using a time-aware Q-learning algorithm (Algorithm 1).

A. Pendulum System

For the first case study, we used an inverted pendulum model defined as:

$$\theta_{t+1} = \theta_t + \tau \omega_t,$$

$$\omega_{t+1} = \omega_t + \tau (\frac{g}{l} \sin \theta_t - \frac{\mu}{ml^2} \omega_t + \frac{1}{ml^2} a_t),$$

where $\theta$, $\omega$, and $a \in \{-3, -2.9, \ldots, 2.9, 3\}$ are the angle of the pendulum, angular velocity, and actions representing torque applied, respectively. $\tau = 0.01$ is the sampling time. The constants $g = 9.8m/s^2$, $m = 0.15l$, $l = 0.5m$, and $\mu$ are 0.05 represent acceleration due to gravity, mass, length of pendulum and friction coefficient, respectively. We consider the following STL specification:

$$\Phi = \bigwedge_{t=500}^{700} (|\theta| \leq 0.05 \land |\omega| \leq 0.05)$$

$$\land \bigwedge_{t=1000}^{1200} (|1.57 - \theta| \leq 0.05 \land |\omega| \leq 0.05)$$

$$\land \bigwedge_{t=1700}^{2000} (|-1.57 - \theta| \leq 0.05 \land |\omega| \leq 0.05) .$$

In simple words, the specification in (12) says that “the pendulum should maintain $\theta = 0$ and $\omega = 0$ in the interval of 400 to 700 timesteps (which is equivalent to 4 to 7 seconds as $\tau$ is 0.01 seconds) with a tolerance value of 0.05. Subsequently, in the interval of 1000 to 1200 time steps, the pendulum should be balanced at $\theta = 1.57$ and $\omega = 0$ followed by $\theta = -1.57$ and $\omega = 0$ in the interval of 1700 to 2000 time steps with a tolerance value of 0.05”. The funnel-based reward function for (12) with $s_t = [\theta_t, \omega_t]$ is computed as discussed in Section III-A and given as follows:

$$r'(s_t, a_t, t) = r(s_t, a_t, t)$$

$$= \begin{cases} 
\rho_{q}(s_t) + (\gamma_{1} - \gamma_{0}) e^{-t/t_0} + \gamma_0 - \rho_{max,1} & t \in [0, 700) \\
\rho_{q}(s_t) + (\gamma_{2} - \gamma_{1}) e^{-t/700} + \gamma_1 - \rho_{max,2} & t \in [700, 1200) \\
\rho_{q}(s_t) + (\gamma_{3} - \gamma_{2}) e^{-t/1200} + \gamma_2 - \rho_{max,3} & t \in [1700, 2000] 
\end{cases},$$

(12)

where $\rho_{q}(s_t) = 0.05 - \min(|\theta_t|, |\omega_t|)$, $\rho_{q}(s_t) = 0.05 - \min(1.57 - |\theta_t|, |\omega_t|)$, $\rho_{q}(s_t) = 0.05 - \min(-1.57 - |\theta_t|, |\omega_t|)$, $l_1 = 0.0103$ $l_2 = 0.0138$ $l_3 = 0.0083$. $\gamma_{i,j} = \pi$, $\gamma_{i,j} = 0.01$, $\rho_{max,i} = 0.05$, for all $i \in \{1, 2, 3\}$. Then, we trained the RL agent using Algorithm 1 to obtain policy enforcing desired STL specifications. Figure 3 shows the evolution of robustness values, angle and angular velocity of the pendulum over time. One can readily observe that the robustness values are always inside the constructed funnels, and the property is satisfied. Notice that the dynamics is under-actuated and hence one can not use results in [11].

B. Mobile Robot Navigation

For the second case study, we consider the differential drive mobile robot described by: $x_{t+1} = x_t + v_t \sin \theta_t \cos \theta_t$, $y_{t+1} = y_t + v_t \cos \theta_t \sin \theta_t$. The location of robot in $x-y$ plane, $\theta$ represents orientation, and $\tau = 0.01$ is the sampling time. The actions $\nu_t \in \{-5, -4.5, \ldots, 5\}$ and $\omega_t \in \{-3, -2.5, \ldots, 2.5, 3\}$

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
Fig. 3. The evolution of robustness values (left), angle (middle), and angular velocity (right) of the pendulum.

Fig. 4. The evolution of robustness values (left), the trajectory in the 2-d plane with controller learned using reward function with (middle) and without (right) funnel for a mobile robot.

The STL specification considered is given by:

$$
\Phi = G_{[900,1300]} \left( (x-25)^2 + (y-25)^2 \leq 2 \right) \\
\land G_{[1600,2000]} \left( (x-30)^2 + (y-30)^2 \leq 2 \right).
$$

To satisfy the above STL specification, the agent has to reach inside a circle of radius two centered at (25,25) in the interval from 900 to 1300 timesteps. Further, it should move inside the circle of radius two centered at (30,30). By constructing funnel-based reward, we learn the time-dependent policy using Algorithm 1 to enforce the STL specification. Figure 4 shows the plot of robustness values following constructed funnel (left plot) and trajectory followed by the trained RL agent in the 2-d plane (middle plot).

To show the importance of a funnel-based reward structure, we modified the reward function by eliminating the funnel part as described below:

$$
r'(s_t,a_t,t) = \begin{cases} 
\rho \psi_1 & \text{for } 0 \leq t \leq 1300 \\
\rho \psi_2 & \text{for } 1300 \leq t \leq 2000.
\end{cases}
$$

Our ablation study reveals that using robustness values without funnel function \( \gamma(t) \) does not help the agent learn the task. The trajectory in the 2-d plane obtained after using the reward function given in (13) is shown in Figure 4 (right).

**Specification with convex predicates:** To showcase the applicability of the results for the convex predicate (which is one of the limitations in [11]), we next consider the following STL specification:

$$
\Phi = G_{[300,2000]} \left( \sqrt{(x-5)^2 + (y-5)^2} \geq 2 \right) \\
\land \sqrt{(x-5)^2 + (y-5)^2} \leq 5 \\
\land (\psi_1 : \text{convex predicate}) \\
\land (\psi_2 : \text{concave predicate}).
$$

According to specification (14), the robot has to stay inside an annular region centered at (5,5) with the inner radius 2 and the outer radius 5 for the interval of 300 to 2000 time steps. The agent is always reset at a randomly chosen point in a \([0,15] \times [0,15]\) grid. Figure 6 shows the plot of robustness values and the trajectory followed by the robot starting from some random point, under the policy learned using our proposed funnel based STL satisfaction approach.
in the 2-d plane. We can clearly observe that the trajectory achieves the best possible robustness by staying in the centre of the annular region and satisfying the convex predicate \(\varphi_1\). Hence our method is capable of handling convex predicate.

C. Discrete-time Integrator for specification with Disjunction inside Temporal Operator

In this section, we show the applicability of our approach to learning policy for STL specification with the disjunction between predicates inside the temporal operator (which is one of the limitations in [11]). Consider the system (discrete-time integrator) with dynamics \(x_{t+1} = x_t + \tau v_t\), where \(x_t\) is the location at time \(t\) and \(v_t \in \{-3, -2.5, \ldots, 2.5, 3\}\) is the velocity given as input to the system. We train the RL agent for the STL specification \(G(0.2000)(\varphi_1 \lor \varphi_2)\). Here, \(\varphi_1 := |x - 5| \leq 5\) and \(\varphi_2 := |x - 45| \leq 5\). We plotted trajectories of the system in Figure 5 for different initial locations and found that the agent has learnt to reach either \(x = 5\) or \(x = 45\).

D. Comparative Study

We compare our proposed method for STL satisfaction with the flag-based method proposed in [8] by training the RL agent for the differential drive mobile robot (Section IV-B) considering the STL specification given in (15). Please note we always initialize the agent within the circular region of radius 1 centered at (2,2), keeping in mind the temporal constraint. We calculated robustness value by taking a summation of robustness for all time steps and obtained a robustness value of 0.938 for our method and a robustness value of 0.102 for the method suggested in [8]. Also, from Figure 7, it is clear that the trajectory generated by using our approach appears more robust.

\[
\Phi = G(0.2000) \left( \sqrt{(x-2)^2 + (y-2)^2} \leq 1 \right) \quad (15)
\]

V. CONCLUSION

In this paper, we proposed a tractable method to learn a controller for robust satisfaction of STL specification using time-aware deep Q-learning. We described how funnel functions could be used to design reward functions for reinforcement learning algorithms that allow learning controllers for STL specifications. We showed the performance of our method on various environments, such as the pendulum and mobile robot, in accomplishing time-constraint sequential goals. One of the significant advantages of the proposed approach is the satisfaction of the STL formulas with convex predicates. Further, using a simple environment, we demonstrated that we can learn the controller for STL formula with disjunction operator inside temporal operator. As part of future work, we would like to extend the funnel-based method to incorporate the STL formula, including nested temporal operators and temporal operators with overlapping time intervals. Further, we could explore the utility of our proposed funnel-based method for actor-critic algorithms.

REFERENCES

[1] C. Baier and J.-P. Katoen, Principles of model checking. MIT press, 2008.
[2] O. Maler and D. Nickovic, “Monitoring temporal properties of continuous signals,” in Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems. Springer, 2004, pp. 152–166.
[3] M. Kamgarpour, J. Ding, S. Summers, A. Abate, J. Lygeros, and C. Tomlin, “Discrete time stochastic hybrid dynamical games: Verification & controller synthesis,” in 2011 50th IEEE Conference on Decision and Control and European Control Conference, 2011, pp. 6122–6127.
[4] M. Lahijanian, S. B. Andersson, and C. Bela, “Formal verification and synthesis for discrete-time stochastic systems,” IEEE Transactions on Automatic Control, vol. 60, no. 8, pp. 2031–2045, 2015.
[5] X. Li, C.-I. Vasiile, and C. Bela, “Reinforcement learning with temporal logic rewards,” in 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2017, pp. 3834–3839.
[6] D. Aksaray, A. Jones, Z. Kong, M. Schwager, and C. Bela, “Q-learning for robust satisfaction of signal temporal logic specifications,” in 2016 IEEE 55th Conference on Decision and Control (CDC). IEEE, 2016, pp. 6565–6570.
[7] C. J. Watkins and P. Dayan, “Q-learning,” Machine learning, vol. 8, no. 3, pp. 279–292, 1992.
[8] H. Venkataraman, D. Aksaray, and P. Seiler, “Tractable reinforcement learning of signal temporal logic objectives,” in Learning for Dynamics and Control. PMLR, 2020, pp. 308–317.
[9] D. Muniraj, K. G. Vamvoudakis, and M. Farhood, “Enforcing signal temporal logic specifications in multi-agent adversarial environments: A deep q-learning approach,” in 2018 IEEE Conference on Decision and Control (CDC). IEEE, 2018, pp. 4141–4146.
[10] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski, et al., “Human-level control through deep reinforcement learning,” nature, vol. 518, no. 7540, pp. 529–533, 2015.
[11] L. Lindemann, C. K. Verginis, and D. V. Dimarogonas, “Prescribed performance control for signal temporal logic specifications,” in 2017 IEEE 56th Annual Conference on Decision and Control (CDC). IEEE, 2017, pp. 2997–3002.
[12] L. Lindemann and D. V. Dimarogonas, “Feedback control strategies for multi-agent systems under a fragment of signal temporal logic tasks,” Automatica, vol. 106, pp. 284–293, 2019.
[13] M. L. Puterman, “Markov decision processes,” Handbooks in operations research and management science, vol. 2, pp. 331–434, 1990.
[14] A. Donzé and O. Maler, “Robust satisfaction of temporal logic over real-valued signals,” in International Conference on Formal Modeling and Analysis of Timed Systems. Springer, 2010, pp. 92–106.
[15] P. Varnai and D. V. Dimarogonas, “Prescribed performance control guided policy improvement for satisfying signal temporal logic tasks,” in 2019 American Control Conference (ACC). IEEE, 2019, pp. 286–291.