Repulsive Core of NN S-Wave Scattering
in a Quark Model with a Condensed Vacuum

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Abstract

We work in a chiral invariant quark model, with a condensed vacuum, characterized by only one parameter. Bound state equations for the nucleon and Δ are solved in order to obtain an updated value of their radii and masses. Nucleon-nucleon S-Wave scattering is studied in the RGM framework both for isospin T=1 and T=0. The phase shifts are calculated and an equivalent local potential, which is consistent with K-N scattering, is derived. The result is a reasonable microscopic short range repulsion in the nucleon-nucleon interaction.

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I. INTRODUCTION

The NN central repulsion is a prominent experimental feature of NN scattering and through the years it has been the subject of many theoretical papers. The explanation to why this NN repulsion happens falls into two main classes: the skyrmion picture \([1]\) and the resonating group method approach (RGM) \([2–4]\). Both approaches yield a sizeable NN central repulsion and the question arises whether or not these models can also explain other aspects of the NN force and whether and if they can be brought to describe other hadronic phenomena. This extensive program is not completely met yet up to the present day.

An effective model \([5,6]\) has been developed in a series of papers, with just one scale besides the quark mass parameter, which amounts to a BCS theory of "Cooper-like" quark antiquark pairs with the quantum numbers of the vacuum: \(3P_0\). This model embodies in a natural way the physics of spontaneous chiral symmetry breaking \(S\chi SB\) and therefore has a "correct" chiral limit so that it is expected to fare well in describing low energy phenomenology. In our model the mechanism of \(S\chi SB\) is triggered by the effective quark potential whose strength besides governing the extent of the quark condensation also furnishes a natural scale (the typical hadronic size) which turns out to be crucial to understand exotic scattering cross sections \([7]\). In this way we were able to describe a considerable number of experimental results ranging from hadronic spectrum and exotic scattering cross sections to the coupling of pions to nucleons and deltas \([8]\). As for the main weak point of this model we must refer that it yields a too small \(f_\pi\). This is a consequence of lack of covariance \([10]\) together with the absence of coupled channels in the BCS treatment of this many-body problem. For recent developments on spontaneous chiral symmetry breaking in quark models, see \([9]\).

So it is interesting to see whether we can obtain a reasonable NN repulsion with the same potential strength used in the above cited papers. The key point is that the physics of the NN central repulsion must be the "same", with the obvious difference of spin-flavour overlaps, as the physics of the \(KN\) exotic scattering so that we must be able to derive a
common origin for these two experimental facts. We were also able to derive reasonable \( \pi N \) and \( \pi \Delta \) couplings for the very same strength of the quark potential which gave good \( KN \) phase shifts, and all this evidence putted together allow us to hope to obtain, at least at a qualitative level, a unified picture for low energy hadronic reactions including a microscopically derived \( NN \) potential in terms of one single parameter, if the quark masses are neglected.

In section II we briefly describe the model we will be using. The masses of the deltas and nucleons, together with the respective Salpeter amplitudes will be obtained in section III. In section IV we discuss the set up of the RGM equations appropriated for the \( NN \) scattering. Section V is devoted to the evaluation of particular overlaps needed to solve the \( NN \) RGM equation and finally in section VI we present and discuss the results. We conclude with some final remarks.

II. QUARK MODEL WITH CHIRAL SYMMETRY BREAKING

A. Generator of the Condensed Vacuum

In this model we assume a new vacuum \( |\tilde{0}\rangle \) generated from the trivial vacuum \( |0\rangle \), through the condensation of \( q\bar{q} \) pairs

\[ |\tilde{0}\rangle = e^{Q_0^\dagger - Q_0} |0\rangle \]

where the generator of the condensed vacuum can be written as

\[ Q_0^\dagger(\Phi) = \int d^3p \Phi(p) M_{s's'}(\theta, \phi) b_{f'cs'}^\dagger(p)d_{fcs}^\dagger(-p). \]

\( b^\dagger \) and \( d^\dagger \) are the creation and annihilation operators of \( q \) and \( \bar{q} \), respectively. \( \Phi(p) \) is the radial term of the \( q\bar{q} \) wave function and

\[ M_{s's}(\theta, \phi) = -\sqrt{8\pi} \sum_{m_L,m_s} (1m_L 1m_s|00)(\frac{1}{2}s_1 \frac{1}{2}s_2|1m_s) Y_{1mL}(\theta, \phi). \]

stands for the angular term coupled with the spin. The vacuum can be understood in terms of a coherent superposition of pairs \( q\bar{q} \) with the vacuum \( J^{PC} \) quantum numbers \( O^{++} \) and
thus can only have $^3L_J$ equal to $^3P_0$. This coupling is contained in (3). The condensed vacuum $|\tilde{0}\rangle$ obtained is orthogonal to $|0\rangle$ if $\Phi(p)$ differs from zero.

The Fock space quark and antiquark annihilators $b$ and $d$, carry indices for flavour, spin and color. In the new vacuum $|\tilde{0}\rangle$, they are related to the old ones by a Valatin-Bogoliubov transformation conserving the usual commutation relations,

$$\{b_{f'sc}(p), b_{f's'c'}(p')\} = \{d_{f'sc}(p), d_{f's'c'}(p')\} = \delta_{ff'}\delta_{ss'}\delta_{cc'}(p - p'). \quad (4)$$

### B. Rotation of Spinors

The field operator $\psi_{fc}(x)$ is defined as

$$\psi_{fc}(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \left[u_s(p)b_{f'sc}(p) + v_s(p)d_{f'sc}(-p)\right] e^{i p \cdot x} \quad (5)$$

Summation over repeated indices is assumed. In order to satisfy the formal invariance of $\psi$ under the Valatin-Bogoliubov transformation, the rotation of operators in the Fock-space must be compensated with an inverse rotation of spinors $u$ and $v$ and we obtain

$$u_s(p) = \frac{1}{\sqrt{2}} \left[\sqrt{1 + \sin \varphi(p)} + \sqrt{1 - \sin \varphi(p)} \hat{p} \cdot \hat{\alpha}\right] u^0_s,$$

$$v_s(p) = \frac{1}{\sqrt{2}} \left[\sqrt{1 + \sin \varphi(p)} - \sqrt{1 - \sin \varphi(p)} \hat{p} \cdot \hat{\alpha}\right] v^0_s \quad (6)$$

where we define the chiral angle

$$\varphi(p) = \arctan \left(\frac{m}{p}\right) + 2\Phi(p) \quad (7)$$

With this expression, where $m$ is the current mass of the quark, it is easy to verify that for massless quarks the condensation of vacuum generates, through the Valatin-Bogoliubov transformation, an effective mass.

### C. Normal Ordered Hamiltonian

Once the vacuum has been rotated, it is convenient to normal order the second quantization Hamiltonian of the model, which is
\[
\hat{H} = \int d^3x [\hat{H}_0(x) + \hat{H}_I(x)],
\]  
(8)

where \(\hat{H}_0\) is the Hamiltonian density of the Dirac field,

\[
\hat{H}_0(x) = \psi^\dagger(x) \left( m_q \beta - i \vec{\alpha} \cdot \vec{\nabla} \right) \psi(x),
\]  
(9)

and \(\hat{H}_I\) an effective instantaneous interaction term

\[
\hat{H}_I(x) = \frac{1}{2} \int d^3y \bar{\psi}(x) \gamma^0 \lambda^a \frac{\psi(x)}{2} V(x - y) \bar{\psi}(y) \gamma^0 \lambda^a \frac{\psi(y)}{2}.
\]  
(10)

The \(\lambda^a\)'s are the Gell-Mann color matrices and \(m_q\) is the quark mass. \(\psi\) carries indices for flavour and color. Flavour is conserved in both \(\hat{H}_0\) and \(\hat{H}_I\).

The normal ordering relatively to the new vacuum gives us the Hamiltonian in the form

\[
\hat{H} = H_0 + \hat{H}_2 + \hat{H}_2^A + \hat{H}_4
\]  
(11)

where the subscript is the number of operators in each term. \(H_0\) represents the energy of the new vacuum.

\[
H_0 = N_f W \int \frac{d^3p}{(2\pi)^3} [2C(p) - 3 (E(p) + m_q \sin \varphi(p) + p \cos \varphi(p))]
\]  
(12)

where \(N_f\) is the number of flavours,

\[
W = \int \frac{d^3x}{(2\pi)^3}, \quad V(p) = \int \frac{d^3x}{(2\pi)^3} V(x) e^{ip \cdot x};
\]

\[
C(p) = \int d^3p' V(p - p'),
\]

\[
E(p) = \sin \varphi(p) A(p) + \cos \varphi(p) B(p),
\]

\[
A(p) = m_q + \frac{2}{3} \int d^3k V(p - k) \sin \varphi(p),
\]

\[
B(p) = p + \frac{2}{3} \int d^3k V(p - k) \cos \varphi(p) \hat{p} \cdot \hat{k}
\]  
(13)

It is possible to show that this energy turns out to be smaller than the energy of the trivial "empty" vacuum (\(|0\rangle\); where \(\varphi = 0\)). The operator \(\hat{H}_2\) is given by

\[
\hat{H}_2 = \int d^3p E(p) \left[ b_{fsc}^\dagger(p) b_{fsc}(p) + d_{fsc}^\dagger(-p) d_{fsc}(-p) \right]
\]  
(14)
where $E(p)$ is the energy of a free quark.

The anomalous Bogoliubov term has the form

$$\hat{H}_2^A = \int d^3p \left[ A(p) \sin \varphi(p) - B(p) \cos \varphi(p) \right]$$

$$\cdot \left[ M_{ss'} b_{fsc}^\dagger(p) d_{f's'c}^\dagger(-p) + M_{s's}^* d_{fsc}(-p) d_{f's'c}(p) \right].$$

(15)

It has the form of the vacuum generator $Q_0$ and can create $q\bar{q}$ pairs on the vacuum. It is now important to remember that the quiral angle, which defines the physical vacuum, has not yet been chosen. To achieve this we can say that $\varphi(p)$ minimize the energy of the new vacuum. With this condition we get

$$A(p) \cos \varphi(p) - B(p) \sin \varphi(p) = 0$$

(16)

and simultaneously we get the anomalous Bogoliubov to vanish. Expression (14) is called mass-gap equation and defines the chiral angle $\varphi(p)$ with an obvious functional dependence on the chosen potential. An alternative approach [11], at least at the BCS level, is to use the Ward identity for the $\hat{H}_4$ kernel in the derivation of the mass-gap equation.

The term $\hat{H}_4$

$$\hat{H}_4 = \frac{1}{2} \int d^3pd^3kd^3q \ V(q) \ \left( \frac{\lambda^a_{12} \lambda^a_{12}}{4} \right) \ \sum_{j,l=1}^4 \ : \Theta_{c_1c_2}^j(p,p+q) \Theta_{c_3c_4}^l(k,k-q) :$$

(17)

gives 16 terms grouped in ten different four-quark amplitudes. Such amplitudes are combinations of the following four vertices:

$$\Theta_{c_1c_2}^1(p,p') \equiv u_{s'}^\dagger(p')u_s(p) \ b_{f's'c}^\dagger(p')b_{fsc}(p),$$

$$\Theta_{c_1c_2}^2(p,p') \equiv -v_{s'}^\dagger(p')v_s(p) \ d_{fsc}^\dagger(-p)d_{f's'c}(-p'),$$

$$\Theta_{c_1c_2}^3(p,p') \equiv u_{s'}^\dagger(p')v_s(p) \ d_{f's'c}^\dagger(p')d_{fsc}(-p),$$

$$\Theta_{c_1c_2}^4(p,p') \equiv v_{s'}^\dagger(p')u_s(p) \ d_{f's'c}^\dagger(-p')b_{fsc}(p).$$

(18)
D. Spontaneous Chiral Symmetry Breaking

It was shown [7] that chiral symmetry is spontaneously broken in this model. We briefly explain the idea. For massless quarks, the Hamiltonian (8) is invariant under the global chiral transformation

$$\psi \rightarrow e^{-i\alpha^a T^a \gamma_5} \psi$$

(19)

where $T^a$ are the $SU(N_f)$ generators and the corresponding axial charges are,

$$Q_a^5 = \int d^3 x \bar{\psi} \gamma_0 \gamma_5 T^a \psi.$$  

(20)

The Bogoliubov transformation on the Fock space of the creation and annihilation operators can be cast as a rotation through an angle $\Phi = \varphi/2$ and this fact, together with the invariance of the field operator $\psi$ under these transformations produces a corresponding counter-rotation on the spinorial basis. This is enough for the $Q_5^a$ to acquire, for non vanishing $\varphi(p)$, an anomalous term which is the composite operator for the pion creation and thus furnishing the only nonzero contribution for $Q_5^a |\tilde{\psi}\rangle$. At the BCS level, $Q_5^a |\tilde{\psi}\rangle \neq 0$ implies that the pion mass must be zero and the solution of the corresponding Salpeter equation for the pion confirms this fact. Furthermore we obtain, as expected, that the pion relative wave function is given by $\sin \varphi(p)$.

E. Harmonic Potential

In what concerns the phenomenological potential $V$, both the harmonic and linear confining potentials have been studied [11], together with different combinations of Dirac vertices. In the present work we will use the Coulomb harmonic confining potential,

$$V(x - y) = -\frac{3}{4} K_0^3 (x - y)^2, \quad V(q) = \frac{3}{4} K_0^3 \Delta_q \delta(q)$$

(21)

which furnished the best results in previous calculations.
Solving Eqs. (13) and (16) both with this potential, we can obtain the explicit expression for the energy of a free quark,

\[ E_k = m_q \sin \varphi + k \cos \varphi - \frac{(\varphi')^2}{2} - \frac{\cos^2 \varphi}{k^2} \tag{22} \]

as well as the mass-gap equation,

\[ p^2 \varphi'' + 2p \varphi' + 2m_q p^2 \cos \varphi - 2p^3 \sin \varphi + \sin 2\varphi = 0, \tag{23} \]

where units of \( K_0 = 1 \) are used from now on.

The mass-gap equation (23) was solved in Ref. [5,6], with the boundary conditions

\[ \varphi(0) = \frac{\pi}{2}, \quad \varphi(p \to \infty) = 0. \tag{24} \]

The chiral angle has been studied in detail but we point out that in the limit of zero current quark mass and zero potential, which forces the chiral angle to be zero, expression (3) yields the massless Dirac spinors. The other trivial limit arises for very massive fermions with \( \varphi = \frac{\pi}{2} \), for all small momenta. In this case and for this region of momentum, Eq. (3) becomes an identity.

III. NUCLEON AND \( \Delta \) BOUND-STATES

Bound-state equation for mesons and baryons were already studied in Ref. [5,6,12]. Here we report on more precise results for the hadronic radii and masses, both for the nucleon and the \( \Delta \).

A. The Nucleon Wave Function

The nucleon internal wave function \( \phi_N(p_1, p_2, p_3) \), where \( p_i \) is the momentum of quark \( i \), includes the internal degrees of freedom of the three quarks as color, spin and isospin. In the moment space, it is usual to use the ground state harmonic oscillator wave function

\[ \phi_{000}(p_i) = \left( \frac{1}{\pi \alpha^2} \right)^{3/4} e^{-\frac{p_i^2}{2\alpha^2}} \tag{25} \]
as a variational trial wave function for the baryon Salpeter equation, with a variational parameter \( \alpha \). It was already shown \([7]\) that it provides an excellent approximation to the solution of Eq. (31) both for the nucleon and the \( \Delta \).

Both internal wave functions are orbitally symmetric, \( SU(4) \) spin-isospin symmetric and \( SU(3) \) color singlets. For instance, the wave function of a nucleon is

\[
\phi_N = \phi_{000}(P_1) \phi_{000}(P_2) \phi_{000}(P_3) \frac{1}{\sqrt{2}} (D_s D_f + F_s F_f) \epsilon 
\]

where \( D \) (\( F \)) is a three quark state which is symmetric (antisymmetric) in the labels of the first and second quarks:

\[
D^\uparrow_s = \frac{1}{\sqrt{6}} (\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2 \uparrow\uparrow\downarrow)
\]
\[
F^\uparrow_s = \frac{1}{\sqrt{2}} (\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)
\]  

and \( D_f \) and \( F_f \) are equivalent expressions provided we make the substitutions \( \uparrow \rightarrow u, \downarrow \rightarrow d \) in the case of protons and \( \uparrow \rightarrow u, \downarrow \rightarrow -d \) in the case of the neutrons. The usual normalized color component is totally antisymmetric,

\[
\epsilon = \frac{1}{\sqrt{6}} \epsilon_{ijk} |ijk\rangle.
\]

B. Baryons Bound State Equation

For the present study the only quantity needed is the radius of the nucleon in order to solve NN scattering. Therefore we solve the bound state equation neglecting the negative-energy component, since it corresponds to channels of much higher energy.

Since our Hamiltonian is instantaneous and there are no negative-energy channels, the bound-state equation can be written as

\[
H |\psi\rangle = M |\psi\rangle,
\]

where \( |\psi\rangle \) is an eigenstate of the Hamiltonian with mass \( M \).
As in Ref. \cite{7,8,12} we will restrict our calculations to bare baryons i.e. hadrons without coupled hadronic channels, in the simplest S-wave configuration, with quarks of the same current mass, leading to only one chiral angle $\varphi$, the same dispersion relation $E(p)$ for the three quarks and only one set of spinors $u$ and $v$ in the vertices $\Theta$.

We begin with the creation operator for baryons which has the form

$$\Psi_B^\dagger = \int d^3p \, \delta(p_1 + p_2 + p_3) \, \psi(p_1, p_2, p_3) \, \epsilon_{c_1c_2c_3} \chi_{f_1f_2f_3s_1s_2s_3} b_{f_1s_1c_1}^\dagger(p_1) b_{f_2s_2c_2}^\dagger(p_2) b_{f_3s_3c_3}^\dagger(p_3) \quad (30)$$

where $\psi$, $\chi$ and $\epsilon$ represent, respectively the momentum, the spin-flavour and the color components of the baryon wave function. Calculating $(\hat{H}_2 + \hat{H}_4)\Psi_B^\dagger |0\rangle$ we have for the baryon bound state equation:

$$[M - 3E(p_1)] \chi_{s_1s_2s_3} \psi(p_1, p_2, p_3) = -2 \int d^3q \, V(q)$$

$$[u_{s_1}^\dagger(p_1) u_{s_3}(p_1 - q) u_{s_2}^\dagger(p_2) u_{s_4}(p_2 + q)] \chi_{s_4s_5s_6} \psi(p_1 - q, p_2 + q, p_3) \quad (31)$$

with $p_1 + p_2 + p_3 = 0$, for any flavour indices in the function $\chi$. The factor $-2$ includes a contribution of $-2/3$ from the contraction of the color matrices times the number of quark pairs. Thus, the bound-state equation \cite{31} has the form of a Schrödinger equation with an effective interaction

$$V = -\frac{2}{3} \int d^3q \, V(q) \, [u_{s_1}^\dagger(p_1) u_{s_3}(p_1 - q) u_{s_2}^\dagger(p_2) u_{s_4}(p_2 + q)] \chi_{s_4s_5s_6} \psi(p_1 - q, p_2 + q, p_3). \quad (32)$$

Calculations made with the harmonic potential \cite{21} give

$$V_{q_1q_2} = V_{q_1q_2}^{SI} + V_{q_1q_2}^{SS} + V_{q_1q_2}^{T} + V_{q_1q_2}^{SO} \quad (33)$$

including the spin independent, hyperfine, tensor and spin-orbit interactions \cite{12}.
C. Nucleon and $\Delta$ masses

In order to check the consistence of our model, we calculate the $N$ and $\Delta$ masses. In the approximation of $N$ and $\Delta$ with only one S-wave channel, the spin-orbit and tensorial terms vanish and the remainder bound-state equation is

$$\left\{ 3E(p_1) - M - \frac{3}{2} \nabla^2_{p_{12}} + \frac{3}{4} (\varphi'(p_1))^2 + \frac{3}{p_1^2} \left[ 1 - \sin \varphi(p_1) \right] \right\} \psi(p_1, p_2) = 0,$$

where $S$ is the total spin and $\nabla^2_{p_{12}}$ stands for the Laplacian with respect to the relative momentum $(p_1 - p_2)/2$. The mass of the baryon is then given by

$$M = \langle 3E(p_1) - M - \frac{3}{2} \nabla^2_{p_{12}} + \frac{3}{4} (\varphi'(p_1))^2 + \frac{3}{p_1^2} \left[ 1 - \sin \varphi(p_1) \right] \rangle_{p_1 + p_2 + p_3 = 0}$$

$$+ \left[ \frac{3}{4} - \frac{1}{3} S(S + 1) \right] \langle [1 - \sin \varphi(p_1)] [1 - \sin \varphi(p_2)] \frac{\hat{p}_1 \cdot \hat{p}_2}{p_1 p_2} \rangle_{p_1 + p_2 + p_3 = 0}$$

(35)

where the following notation has been introduced:

$$\langle f(p_1, p_2, p_3) \rangle_{p_1 + p_2 + p_3 = 0} \equiv \int d^3p \delta(p_1 + p_2 + p_3) f(p_1, p_2, p_3) |\psi(p_1, p_2, p_3)|^2$$

(36)

Since we do not have an analytic expression for the chiral angle $\varphi$, the calculations were done with the good approximation

$$\sin \varphi(p) = e^{-p^2/(2\beta^2)}$$

(37)

where $\beta = 0.582$.

Figure VII shows the results for the $N\Delta$ masses, in units of $K_0$, as a function of the variational parameter $\alpha$, for the case of massless current quarks. The results are presented in table VII considering the harmonic potential with a strength constant 247 MeV.

The bare nucleon mass obtained gives a mean radius of 0.66 fm, which is reduced when the strength of potential is increased. As an example, $K_0 = 290$ MeV gives 0.57 fm. The introduction of finite current quark masses almost does not change the results we have obtained.
The difference between the bare nucleon mass and its physical counterpart is a consequence of the coupling to $N\pi$ channels which is expected to lower the nucleon bare mass somewhere between $(-300)$ and $(-400)$ MeV \[13\]. The effect of coupled channels should also contribute to the physical $N-\Delta$ difference ($100-200$ MeV).

**IV. RGM EQUATION FOR NN SCATTERING**

It is well known from potential scattering theory that a free spherical wave converging towards the origin is almost unaffected by a potentials which range is of the order of $\sqrt{L(L+1)/k}$. The repulsive short range potential between two nucleons has almost no effect on waves for which $\sqrt{L(L+1)/k} > 0.8$. This fact motivates the choice of the relative S wave to test the repulsive core of NN scattering also revealed by experimental low-energy scattering phase shifts in $^1S_0$ and $^3S_1$ partial waves.

To study the central microscopic NN repulsion it suffices to consider the hyperfine quark-quark potential $V_{q\bar{q}}^{SS}$,

$$V_{q\bar{q}}^{SS} = \frac{\lambda_1^a}{2} \frac{\lambda_1^b}{2} [1 - \sin \varphi(p_i)] [1 - \sin \varphi(p_j)] \frac{\hat{p}_i \hat{p}_j S_i S_j}{p_ip_j}. \quad (38)$$

As we are neglecting the pionic cloud responsible for the long and intermediate range forces, it is obvious that we will not be able to reproduce the NN phase shifts. However, the results will allow us to show how much the quark structure of the nucleons influences the NN S-wave scattering.

To investigate the Nucleon-Nucleon scattering we make use of the Resonating Group Method (RGM) introduced by Wheeler \[14\], and profusely applied by several groups \[2,3,15,16\]. In this method the two clusters relative motion wave function is obtained by requiring it to minimize energy of the system. Then, we must to minimize the matrix element

$$I = \langle N_1 N_2 | (\hat{H} - E) | N_1 N_2 \rangle \quad (39)$$

where $\hat{H} = \hat{H}_2 + \hat{H}_4$ is the Hamiltonian in the second quantization formalism ones, $E$ represents the total energy of the system and $|N_1 N_2\rangle$ describes the six quarks system clustered.
in nucleons \( N_1 \) and \( N_2 \).

The Pauli principle is taken account by demanding the antisymmetry of the total six quarks ensemble which arises naturally through the Wick ordering of quark fields. Since the wave functions of nucleons are already antisymmetrized, we obtain,

\[
\langle N_1 N_2 | (\hat{H} - E) | N_1 N_2 \rangle = \langle N_1 N_2 | (H - E) 36(1 - 9P_{14})(1 - \mathcal{P}) | N_1 N_2 \rangle
\]

\[
\equiv \langle N_1 N_2 | (H - E) \mathcal{A} | N_1 N_2 \rangle \tag{40}
\]

where \( P_{ij} \) stands for the permutation of the quarks \( i \in N_1 \) and \( j \in N_2 \) and \( \mathcal{P} = P_{14}P_{25}P_{36} \) gives the permutation of the nucleons 1 and 2. \( \mathcal{A} \) is the usual designation of the antisymmetrizer of six quarks in two nucleons. The operator \( H \) is defined in first quantization.

At this stage it is interesting to see that the factor \( (1 - \mathcal{P}) \) implies, in a natural fashion, the usual antisymmetry condition at the baryon level. In fact, \( (1 - \mathcal{P}) \) acts on the system of two nucleons and makes the matrix element vanish unless the condition \( L + S + T = odd \) holds.

The Jacobi coordinates adapted to this problem are

\[
\begin{align*}
\mathbf{p}_\rho &= \frac{1}{\sqrt{2}}(\mathbf{p}_1 - \mathbf{p}_2), \quad \mathbf{p}_\lambda = \frac{1}{\sqrt{6}}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3) \\
\mathbf{p}_\xi &= \frac{1}{\sqrt{2}}(\mathbf{p}_4 - \mathbf{p}_5), \quad \mathbf{p}_\gamma = \frac{1}{\sqrt{6}}(\mathbf{p}_4 + \mathbf{p}_5 - 2\mathbf{p}_6) \\
\mathbf{p}_r &= \frac{1}{\sqrt{6}}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) - \frac{1}{\sqrt{6}}(\mathbf{p}_1 + \mathbf{p}_5 + \mathbf{p}_6) \\
\mathbf{p}_{cm} &= \frac{1}{\sqrt{6}}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6). \tag{41}
\end{align*}
\]

As a result of this choice, each nucleon is described by one of the two sets \( \{(\rho, \lambda)\}, \{(\xi, \gamma)\} \) of coordinates. The relative motion is described through \( \mathbf{p}_r \).

The Hamiltonian in first quantization, in the center of mass frame, becomes

\[
H = T_\rho + T_\lambda + T_\xi + T_\gamma + T_r + \sum_{i<j} V_{ij} \tag{42}
\]

The subscripts \( \rho, \lambda, \xi, \gamma, r \) and \( cm \) correspond to the Jacobi coordinates of equation (41) and \( V_{ij} \) is the potential of interaction between the quarks \( i \) and \( j \). Using a more explicit notation,
\[ I = 36\langle \chi(p_r)|\langle \phi^1(\xi_1)\phi^2(\xi_2)|(H - E)(1 - 9P_{14})(1 - P)|\phi^1(\xi_1)\phi^2(\xi_2)|\chi(p_r)\rangle_{p_{cm}=0} \]  

(43)

where the arguments \( \xi_1 \) and \( \xi_2 \) denote the sets of internal degrees of freedom in the wave function \( \phi^{1,2} \) of each nucleon, including \( p_\rho, p_\lambda, p_\xi, p_\gamma \) in the momenta space, and \( \chi(p_r) \) is the wave function for the relative motion of the two nucleons.

Expanding \( \chi(p_r) \) in spherical harmonics,

\[ \chi(p_r) = \sum_m \sum_l \chi_{lm}(p_r), \]  

(44)

where \( \chi_{lm}(p_r) \) includes both radial and orbital components, we obtain that for the particular case of S relative wave function \( \chi_S(p_r) = \chi_S(-p_r) \), and we can drop the permutation \( P \) between the two nucleons, providing the product \( \phi^1(\xi_1) \phi^2(\xi_2) \) of the two wave functions is antisymmetrized. The RGM equation is obtained minimizing the matrix element \( I \) with respect to \( \chi \),

\[ \frac{\partial I}{\partial \langle \chi_S \rangle} = 0 \Leftrightarrow \langle \phi^1(\xi_1)\phi^2(\xi_2)|(H - E)(1 - 9P_{14})|\phi^1(\xi_1)\phi^2(\xi_2)|\chi_S \rangle = 0. \]  

(45)

It is easy to verify that the specific dependence of color in \( V_{ij} \) yields a null matrix element when the permutation is not present. The nucleon mass \( M_N \) is defined by

\[ H_N|N_1⟩ = (T_\rho + T_\lambda + V_{12} + V_{13} + V_{23})|N⟩ = M_N|N_2⟩, \]  

(46)

with an analogous expression for the second nucleon.

Now we introduce the eigenfunctions \( \phi_{nlm}^\alpha \equiv \phi_a \) of the harmonic oscillator with parameter \( \alpha \) as a complete set for the expansion of the relative wave function \( |\chi_S⟩ \)

\[ |\chi_S⟩ = \sum_{ab} |\phi_a⟩⟨\phi_b| \phi_a|\chi_S⟩. \]  

(47)

Using the fact that \( E_r = E - 2M_N \), we get

\[ [(T_r - E_r)(1 + \sigma_{ab}|\phi_a⟩⟨\phi_b|) + v_{ab}|\phi_a⟩⟨\phi_b|] |\chi_S⟩ = 0, \]  

(48)

where
\[
\sigma_{ab} = -9\langle \phi_a(p_r) | \phi^1(\xi_1) \phi^2(\xi_2) | P_{14} | [\phi^1(\xi_1), \phi^2(\xi_2)] | \phi_b(p_r) \rangle \equiv -9\langle P_{14} \rangle,
\]
\[
v_{ab} = -9\langle \phi_a(p_r) | \phi^1(\xi_1) \phi^2(\xi_2) | \sum_{i \in N_1, j \in N_2} V_{ij} P_{14} | [\phi^1(\xi_1), \phi^2(\xi_2)] | \phi_b(p_r) \rangle. \tag{49}
\]

We have to note that Eq. (47), associated to expansion (44), implies that \( \phi_b = \phi_{n,0,0} \). We will retain \( a = b = (0,0,0) \) as an approximation that is reasonable due to the fact that the overlap kernel being compact, happens to filter out higher wave excitations quite effectively.

Now, if we use

\[
|\psi\rangle = (1 + \sigma_{00} |\phi_{000}\rangle \langle \phi_{000} |) |\chi\rangle \tag{50}
\]

where \( \phi_{000} \) is the wave function of the tridimensional harmonic oscillator with the relative Jacobi momenta \( p_r \) it is possible to rewrite Eq. (48) as a non-local Schrödinger type equation:

\[
\left( T_r - E_r + \frac{v_{00}}{1 + \sigma_{00}} |\phi_{000}\rangle \langle \phi_{000} | \right) |\psi\rangle = 0 \tag{51}
\]

Of course, wave function (50) gives the same phase shifts than \( |\chi\rangle \).

V. EVALUATION OF \( \sigma \) AND \( V \)

The nine diagrams we have to consider are given in Fig. VII. The values of \( \sigma_{00} \) for the color and spin-flavour spaces were directly calculated numerically, performing all the permutations involved. To obtain the value of \( v_{00} \) in the same spaces, we must consider the nine graphs of Fig. VII. In order to simplify calculations, the interactions of color and spin can be commuted in permutations given by relations:

\[
\frac{\lambda_i^a \lambda_j^b}{2} = \frac{1}{2}(P_{ij}^{\text{color}} + 2\delta_{ij} - \frac{1}{3}) \]

\[
\vec S_i \cdot \vec S_j = \frac{1}{2}(P_{ij}^{\text{spin}} - \frac{1}{2}) \tag{52}
\]

The evaluations in momenta space could be made with the appropriate graphical rules \[17\]. But in our order of approximation, we do not need to use them. For instance, to obtain \( \langle P_{14} \rangle \), we just have to note that \( P_{14} \) does not change the momenta part of the wave function.
which describes six quarks grouped in two nucleons and with relative wave function $\phi_{000}$.

This result is instantaneously obtained by inspection of expression

$$(\phi_{N1} \phi_{N2})_{\text{mom}} = \phi_{000}(p_\rho) \phi_{000}(p_\lambda) \phi_{000}(p_\xi) \phi_{000}(p_\gamma) \phi_{000}(p_r)$$

(53)

Therefore $\sigma = 1$.

For each one of the nine terms of $v_{00}$, the procedure is the same we describe. We start observing that for the reason given above, permutation $P_{14}$ can be removed. In momenta space, each term of $v_{00}$ reveals to be proportional to

$$V_{00} \equiv \langle \phi_{000}(p_r) \rangle \langle \phi_{000}(p_\rho) \rangle \langle \phi_{000}(p_\lambda) \rangle \langle \phi_{000}(p_\xi) \rangle \langle \phi_{000}(p_\gamma) \rangle \langle \phi_{000}(p_r) \rangle_{p_{cm}=0},$$

(54)

where $(V_{ij})_{\text{mom}}$ is the momentum dependent part of the hyperfine quark potential.

A numerical evaluation gives $V_{00} = -0.0671 K_0$. The calculations were done for the cases $(T,S) = (1,0)$ and $(T,S) = (0,1)$ and the results are in table VII, where the evaluations in the spin-flavour space must be multiplied by a factor $(1/18)^2$.

For instance, we have, for $(T,S)=(1,0)$:

$$\sigma_{00} = \langle P_{14}\rangle_c \langle P_{14} \rangle_{s x f} \langle P_{14} \rangle_{\text{mom}} = \frac{1}{9}$$

$$v_{00} = -9 \sum_{i \in N_1, j \in N_2} (\langle V_{ij} P_{14} \rangle_c \langle V_{ij} P_{14} \rangle_{s x f} \langle V_{ij} P_{14} \rangle_{\text{mom}}) = -\frac{5}{3} V_{00}$$

(55)

These results are summarized in table VII.

**VI. RESULTS AND DISCUSSION**

The value of $K_0$, the unique parameter of this model, was fixed to $K_0 = 247$ MeV in order to obtain good KN phase shifts. This value also gave good $f_{N\pi N}$ and $f_{N\pi\Delta}$ and was used successfully by the Orsay quark group [5]. We obtain $\alpha = 296.4$ MeV and $-V_{00} = 16.6$ MeV. In the most conventional quark models, the oscillator parameter has values of 250 MeV [13] to 410 – 420 MeV [16,20].

Since the potential is separable, the phase shifts were obtained analytically
\[
\delta(z) = -\text{atan} \frac{4 z \sqrt{\pi} \exp(-z^2)}{\alpha^4 + 4 - 8z \exp(-z^2) \int_0^z e^{t^2} dt} \tag{56}
\]

with

\[
\lambda = \frac{v_{00}}{1 + \sigma_{00}}, \quad z = \frac{p_r}{\alpha}, \quad E_r = \frac{\left(\frac{\sqrt{6}}{2} p_r\right)^2}{2m} \tag{57}
\]

where \(m\) is the physical reduced mass of the system of two nucleons and \(E_r\) stands for the energy of the relative motion in the center of mass frame. The factor \(\sqrt{6}/2\) is needed because \(p_r\) is a Jacobi coordinate. The phase shifts obtained are presented in Figs. VII for the \((T,S) = (1,0); (0,1)\) channels.

In order to derive an equivalent local potential \(U_{ELP}\) defined, according to Ref. [3], as

\[
\left[\frac{h^2}{2\mu} \nabla^2 + E_r - U_{ELP}(r)\right] \psi(r) = 0, \tag{58}
\]

the nonlocal Schrödinger equation (51) was solved. This local potential happens to be energy-dependent. The numerical accuracy was tested by reproducing numerically the previous phase shifts.

It is expected in a microscopic calculation of the \(NN\) interaction, to obtain for the short range part a non-local description, besides the usual meson-exchange picture which is also natural in this framework. There are a few realistic potentials that fit in this description, with a different height for the core, describing very well the two body data. The latter does not seem to be very sensitive to the height of the core. The Nijmegen potential was chosen for comparison because it has a version, the NijmI [18], with a very soft core, comparable to ours, and has an optimal \(\chi^2\) per datum for \(np\) data. In Fig VII, both our equivalent local potential \(U_{ELP}\) and the NijmI potential are depicted. It is well known that they should differ in the medium range, due to the meson exchange contributions which are present in any realistic potential, and that we have not yet included. Nevertheless it is remarkable that, contrary to past experiences, such a soft repulsive core can be accommodated to reproduce experimental phase shifts.

Almost all the \(NN\) potentials obtained in the framework of quark models have comparable range values, but they differ largely in the magnitudes. For example, Faessler et al and
Oka [3,16] find potentials with peak values between 600 and 1300 MeV. Ribeiro [2] built a square potential of magnitude 500 MeV that was approximately phase shift equivalent to the ones obtained solving the RGM equation. The Liberman potential [21] has a peak of 450 MeV. Of course, a direct comparison of the potentials is not absolutely correct because the methods used to derive each potential are quite different. In order to avoid this problem, we can compare the equivalent local potentials obtained by Suzuki and Hecht [22], who used a common method for the models we have referred above. However we can accept this large range of results if we take into account the small relative weight of the potential near the origin.

Although it is usually expected that quark exchange processes give the most relevant contribution to the $NN$ repulsion, $\omega$ and $\rho$ can still give contributions to the short range potential. These mesons, and the other possible ones required to describe the intermediate and long range attraction of the nuclear force, can be introduced in a consistent way within this model.

VII. CONCLUSION

In this work we have derived, from a chiral invariant quark model, the short range part of the $NN$ interaction. There is only one free parameter in this model, which was already fixed in previous calculations. A non-local potential was then derived, with a soft core, consistent with the recent NijmI realistic interaction, that fully describe the experimental $NN$ data. This is part of a wider program to describe the full nucleon-nucleon interaction within a quark model.

Finally we find remarkable that we are able to understand, with the same parameter, the $NN$ repulsion and, at the same time, to reproduce the meson and baryon spectra, the vector meson decays, the $f_{N\pi N}$ coupling and to obtain a reasonable $KN$ repulsion.
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FIGURES

FIG. 1. Masses of $N$ and $\Delta$ as a function of the variational parameter $\alpha$. Both are in units of $K_0 = 1$.

FIG. 2. The nine diagrams that contribute to the potential overlap.

FIG. 3. Phase shifts of $NN$ scattering for $(T, S) = (1, 0)$ and $(T, S) = (0, 1)$ channels.

FIG. 4. Our equivalent local potential (solid line) and the realistic NijmI potential (dotted line). The deviation for large distances is due, as usual, to attractive meson exchange.
TABLES

|         | N                  | Δ                  |
|---------|--------------------|--------------------|
| Mass ($K_0$) | 4.88 (1207 MeV)    | 5.08 (1255 MeV)    |
| $\alpha$ ($K_0$) | 1.20               | 1.21               |

TABLE I. $N, \Delta$ masses and inverse radius $\alpha$ in units of $K_0 = 1$. Values in MeV were obtained with $K_0 = 247$ MeV.

|                  | (T,S)=(1,0) | (T,S)=(0,1) |
|------------------|-------------|-------------|
|                  | color       | spin-flavour| mom. | color       | spin-flavour| mom. |
| $\langle P_{14} \rangle$ | $\frac{1}{3}$ | -12         | 1    | $\frac{1}{3}$ | -12         | 1    |
| $\langle V_{14} P_{14} \rangle$ | $\frac{4}{7}$ | 93          | $V_{00}$ | $\frac{4}{7}$ | 57          | $V_{00}$ |
| $\langle V_{15} P_{14} \rangle$ | $-\frac{2}{7}$ | -21         | $V_{00}$ | $-\frac{2}{7}$ | -21         | $V_{00}$ |
| $\langle V_{16} P_{14} \rangle$ | $-\frac{2}{7}$ | -21         | $V_{00}$ | $-\frac{2}{7}$ | -21         | $V_{00}$ |
| $\langle V_{24} P_{14} \rangle$ | $-\frac{2}{9}$ | -21         | $V_{00}$ | $-\frac{2}{9}$ | -21         | $V_{00}$ |
| $\langle V_{25} P_{14} \rangle$ | $\frac{1}{9}$ | 0           | $V_{00}$ | $\frac{1}{9}$ | 5           | $V_{00}$ |
| $\langle V_{26} P_{14} \rangle$ | $\frac{1}{9}$ | 0           | $V_{00}$ | $\frac{1}{9}$ | 5           | $V_{00}$ |
| $\langle V_{34} P_{14} \rangle$ | $-\frac{2}{9}$ | -21         | $V_{00}$ | $-\frac{2}{9}$ | -21         | $V_{00}$ |
| $\langle V_{35} P_{14} \rangle$ | $\frac{1}{9}$ | 0           | $V_{00}$ | $\frac{1}{9}$ | 5           | $V_{00}$ |
| $\langle V_{36} P_{14} \rangle$ | $\frac{1}{9}$ | 0           | $V_{00}$ | $\frac{1}{9}$ | 5           | $V_{00}$ |

TABLE II. Contribution of each diagram in color, spin-flavour and momenta spaces.
\[(T, S) = (1, 0)\quad (T, S) = (0, 1)\]

|       | \(\sigma_{00}\) | \(\frac{1}{5}\)  | \(\frac{1}{9}\)  |
|-------|-----------------|-----------------|-----------------|
| \(v_{00}\) | \(-\frac{5}{3}v_{00}\) | \(-\frac{104}{81}v_{00}\) | \(-\frac{52}{45}v_{00}\) |
| \(\lambda \equiv \frac{v_{00}}{1+\sigma_{00}}\) | \(-\frac{3}{2}v_{00}\) | \(-\frac{52}{45}v_{00}\) |  |

TABLE III. Final values for \(\sigma_{00}\) and \(v_{00}\). The relevant quantity to obtain phase shifts is \(\lambda\).
$E_{\text{lab}} = 100 \text{ MeV}$