NGC 1300 Dynamics: II. The response models

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ABSTRACT
We study the stellar response in a spectrum of potentials describing the barred spiral galaxy NGC 1300. These potentials have been presented in a previous paper and correspond to three different assumptions as regards the geometry of the galaxy. For each potential we consider a wide range of \( \Omega_p \) pattern speed values. Our goal is to discover the geometries and the \( \Omega_p \) supporting specific morphological features of NGC 1300. For this purpose we use the method of response models. In order to compare the images of NGC 1300 with the density maps of our models, we define a new index which is a generalization of the Hausdorff distance. This index helps us to find out quantitatively which cases reproduce specific features of NGC 1300 in an objective way. Furthermore, we construct alternative models following a Schwarzschild type technique. By this method we vary the weights of the various energy levels, and thus the orbital contribution of each energy, in order to minimize the differences between the response density and that deduced from the surface density of the galaxy, under certain assumptions. We find that the models corresponding to \( \Omega_p \approx 16 \text{ km s}^{-1} \text{ kpc}^{-1} \) and \( \Omega_p \approx 22 \text{ km s}^{-1} \text{ kpc}^{-1} \) are able to reproduce efficiently certain morphological features of NGC 1300, with each one having its advantages and drawbacks.

Key words: Galaxies: kinematics and dynamics – Galaxies: spiral – Galaxies: structure – ISM:kinematics and dynamics

1 INTRODUCTION
In [Kalapotharakos et al. (2010) (hereafter Paper I), we have presented three different general models representing the potential of NGC 1300. The morphology we investigate can be observed at a deprojected near-infrared image of the galaxy (Fig. 1). Each of these model-potentials corresponds to different assumptions regarding the distribution of the luminous matter of the galaxy in the third dimension, perpendicular to the equatorial plane. They represent limiting cases for the geometry of the system, and vary from the pure 2D (Model A) to 3D cases. The two 3D cases correspond either to a pure cylindrical geometry of a 3D disc with a constant scale height (Model B) or to a combination of a spheroidal component representing the central part of the bar with the cylindrical geometry of a 3D disc for the rest of the luminous mass (Model C). All of them have the option of the inclusion of two additional terms in the potential representing the central mass concentration and the dark halo component respectively. These terms are constrained by the kinematical data derived by Lindblad et al. (1997).

We note that in our calculation we do not take into account the outer spiral arms that exist beyond the edges of the frame of Fig. 1. These spiral extensions are very weak in the near-infrared, while they are conspicuous in the optical. This indicates that they consist mainly of young objects, thus their contribution to the mass distribution of the galaxy is small (Grosbøl et al. in preparation).

In the present paper we investigate the detailed dynamics in the three cases using response models (hereafter RM). As we have already mentioned in Paper I the global dynamical behaviour in rotating galaxies, is crucially determined by the assumed value of the pattern speed \( \Omega_p \). Thus, in this paper we use the method of response models (Patsis 2006) in order to find which \( \Omega_p \) values, in each Model (A, B and C), are able to reproduce the various morphological features of NGC 1300. We construct our models under the assumption of a single pattern speed and we will assess the results at the end. For this reason, we introduce an index that is a generalization of the Hausdorff distance (see e.g. Deza & Deza 2009) so that we can quantify the (dis)similarity between the density maps of the response models and the K-band deprojected NGC 1300 image. Comparisons that involve the surface density of the galactic disc are done under the assumption of a constant \( M/L \) ratio (see
Paper I). We remind also, that from the image we used (Fig. 1) we have removed point-like sources, that are likely to be young stellar clusters (Paper I).

This study allows also the comparison between various response models and consequently indicates, which kind of geometry applies better to the NGC 1300 case. Nevertheless, our ultimate goal is the detection of the dynamical mechanisms behind the observed structures (bar and spiral arms). This needs a detailed analysis of the orbital stellar dynamics for each case and is presented in Patsis et al. (2010) (hereafter Paper III).

Our paper is structured as follows: in Section 2 we present the method of response models and the corresponding results. In Section 3 we describe the generalized Hausdorff distance that helps us quantify the similarity between the various response models and the galaxy. In Section 4 we check the ability of the response models to describe better the morphological features of NGC 1300 by varying the relative contribution of the energy levels. Finally, we discuss our conclusions in Section 5.

2 RESPONSE MODELS

These models show the response of an initially axisymmetric stellar disc, with particles moving at the beginning of the simulation in circular orbits determined in the axisymmetric part of the potential.

The steps we follow in our numerical experiments are:

(i) we choose a potential (among Models A, B, C) and the pattern speed value \( \Omega_p \);

(ii) we populate uniformly the disc up to \( R_{\text{max}} = 15 \) kpc (the \((x, y)\) of 10\(^6\) particle positions on the plane are taken at random positions),

(iii) we set each particle in circular motion (in the rotating frame) with velocity \( v_{\text{circ}} = \left( \sqrt{\frac{d\Phi_0}{dR} \frac{1}{R} - \Omega_p} \right) R \), where \( \Phi_0 \) is the axisymmetric part of the potential,

(iv) we grow the non-axisymmetric terms of the potential linearly from 0 towards their full amplitude within two pattern rotations,

(v) we integrate the particles’ orbits for 25 pattern rotations (we consider that a particle has escaped and we stop the integration of its orbit when it reaches a radius \( R > 22 \) kpc),

(vi) Finally we construct density maps, by converting our data files to images. For this we consider a grid and we take into account the numerical density of the test particles on the disc.

In Paper I we have studied the distribution of the Lagrangian points in all general models (A, B, C) and for an extended range of \( \Omega_p \) values (see fig. 10 of Paper I). In Fig. 2 we plot the ratio \( R_{\text{CR}}/R_{\text{bar}} \) of the corotation radius \( R_{\text{CR}} \) over the semi-major axis of the bar \( R_{\text{bar}} \) as a function of the corresponding \( \Omega_p \) value. We have omitted the points corresponding to Model C since they are very close to those of Model B. We have considered the bar extending up to the outer white isocontour of Fig. 1 (third white contour starting counting from inside). This contour is not symmetric to the origin, since the right semi-major axis of the bar is longer than the left one. Thus, we set \( R_{\text{bar}} = 8.9 \) kpc for the right semi-major axis of the bar and \( R_{\text{bar}} = 8.0 \) kpc for the left one. From the above it is obvious that the ratio \( R_{\text{CR}}/R_{\text{bar}} \) depends on the side of the bar. The considered \( R_{\text{CR}} \) value is the radius of the Lagrangian points \((L_1, L_2)\) lying in the corresponding side. In Fig. 2 the filled circles and squares correspond to Model A, while the empty ones to Model B. The squares (colored blue in the online version) and the circles (colored red in the online version) correspond to the long and the short semi-major axis of the bar, respectively. Note that there are ranges of \( \Omega_p \) values that give multiple Lagrangian points.

![Figure 1](image1.png)

**Figure 1.** The near-infrared surface brightness of NGC 1300, in gray scale (color scale in online version). This image has been obtained after applying all the necessary amendments (see Paper I) and after deprojecting the image with the considered position and inclination angles \((PA, IA)=(87°,35°)\). The solid curves are six iso-density contours describing the basic contour shapes of NGC 1300. The adopted distance is \( D=19.6 \) kpc.

![Figure 2](image2.png)

**Figure 2.** The \( R_{\text{CR}}/R_{\text{bar}} \) ratio as a function of the pattern speed for our models A (filled symbols) and B (empty symbols). Squares (blue colored in the online version) and circles (red colored in the online version) correspond to the long and the short semi-major axis of the bar, respectively. Note that there are ranges of \( \Omega_p \) values that give multiple Lagrangian points.
Figure 3. The surface density in gray scale (color scale in the online version) of the RMs corresponding to the potential of the pure 2D case (Model A). At the top of each panel is indicated the $\Omega_p$ value. In each frame we plot over the density maps of the RMs six representative contours (solid lines) of the K-band image of NGC 1300 (the same with those plotted in Fig. 1). The density scale is the same as in Fig. 1.
Figure 4. As Fig. 3 but for the potential corresponding to the cylindrical geometry (Model B).
Figure 5. As Fig. 3 but for the potential corresponding to the (spheroidal + cylindrical) geometry (Model C).
This argument is sustained by dynamical studies according to which the orbital content beyond corotation is not able to support the bar shape (Contopoulos 1980). In our models, as we have mentioned already in Paper I, we see that there are ranges of $\Omega_p$ values with multiple Lagrangian points (especially in Model A). Particularly, in some cases the ratios $R_{CR}/R_{bar}$ corresponding to multiple Lagrangian points are quite different when we consider as corotation distance the distance of the one or the other Lagrangian point. E.g. in Model A at $\Omega \approx 25 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ the ratio $R_{CR}/R_{bar}$ varies from $\approx 0.5$ (unphysical value since it is significantly lower than 1) up to $\approx 1.1$ (reasonable value). These cases are quite complicated and lie beyond the standard paradigm. Thus, there is no a priori expectation distance the distance of the one or the other Lagrangian point.

E.g. in Model A at $\Omega \approx 16 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ we realize that beyond the limits of this range the response models were particularly problematic in the sense that there was obviously no good agreement with the image of the galaxy. The models with $\Omega_p \gtrsim 17 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ correspond to “fast bars” as are usually called these with $R_{CR}/R_{bar} < 1.4$ (Rautiainen et al. 2008; Debattista & Sellwood 2000).

In this study we present the results corresponding to the $\Omega_p$ values from 12 to 26 $\, \text{km s}^{-1} \, \text{kpc}^{-1}$. Empirically it has been also realized that beyond the limits of this range the response models were particularly problematic in the sense that there was obviously no good agreement with the image of the galaxy. The models with $\Omega_p \gtrsim 17 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ correspond to “fast bars” as are usually called these with $R_{CR}/R_{bar} < 1.4$ (Rautiainen et al. 2008; Debattista & Sellwood 2000).

In Fig. 3 we plot, in gray scale (color scale in online version), the density maps we obtain from the RMs corresponding to the potential of Model A (2D case) for the $12 \leq \Omega_p \leq 26 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ values. Here, we plot the RMs corresponding to the 8 indicated $\Omega_p$ values. However, we have data for models with totally 15 $\Omega_p$ values within the above range of $\Omega_p$. On each panel of this figure we have also overplotted the same (as in Fig. 1) six iso-density contours (solid lines) of the K-band image of NGC 1300, outlining its major morphological features.

In this figure we observe that the low $\Omega_p$ values (first row) fail to reproduce both the bar and the spiral structure. The response bar is conspicuously broader than the galactic bar, while the spiral arms are absent or flocculent. For higher values of $\Omega_p$ we get spiral arms in good agreement with the real ones (see panel for $\Omega_p = 16 \, \text{km s}^{-1} \, \text{kpc}^{-1}$) although the bar is still broad. For even higher $\Omega_p$ values (third row) the formed spiral arms of the models do not match with the spirals of the galaxy. Nevertheless the bar clearly gets an ansae character in agreement with the NGC 1300 bar morphology (see panel for $\Omega_p = 22 \, \text{km s}^{-1} \, \text{kpc}^{-1}$ as best example). Finally, the highest $\Omega_p$ values (last row) produce very open spiral arms and/or rings and a disintegrated bar.

Figure 3 presents the RMs for Model B (thick disc, cylindrical geometry) at the same $\Omega_p$ values as for Model A (Fig. 3). In this case the response bars have always high density regions at their ends, underlying a clear ansae-type character. A common feature is also a density enhancement along the minor axis. The bar morphology has a good relation with the galactic bar for low $\Omega_p$ values (two first rows). This indicates that a proper selection of the bar-supporting orbits might lead to a best fitting of the NGC 1300 bar. As $\Omega_p$ increases (panels in the third and forth rows) the response bar becomes shorter than that of the galaxy. The response spiral arms are not always well described but there is a range of $\Omega_p = 16 - 22 \, \text{km s}^{-1} \, \text{kpc}^{-1}$, where occasionally we find high densities in the models at the regions corresponding to the galactic spiral arms.

Figure 3 is similar to Figs. 2 and 4 but now for Model C (spherical and cylindrical geometry). The morphology of the response bar resembles more the morphology we get in Model A especially for the lower $\Omega_p$ values (first two rows). However, the inner structure of the NGC 1300 bar ($R \lesssim 2 \, \text{kpc}$) is fitted clearly better in this case. The general spiral response is closer to that of Model B while it is in general fainter.

Up to this point, the similarity between the various RMs and the morphological features of the galaxy have been made by eye. Although the eye is usually a good selecting tool it lacks of objectivity and capability of quantification. For this reason, in the next section, we introduce an index in order to quantify the comparison between the NGC 1300 morphology and the response models.

### 3 Quantitative Comparison

In Fig. 1 we have plotted six iso-density contours of NGC 1300 reproducing its basic morphological features. Our goal is to “measure” in an objective way the similarity of an RM density map with the galactic morphology. Thus, we want to establish a quantitative criterion for this resemblance.

The methodology of this study is related with image comparison and pattern recognition techniques. A simple and well known index used in such studies is the “Hausdorff Distance” $d_H$, that is defined as follows: let $A$, $B$ be two non-empty subsets of a metric space $(M, d)$. In this case the Hausdorff distance $d_H$ between $A$ and $B$ reads

$$d_H = \max \left( \sup_{a \in A} \inf_{b \in B} d(a, b), \sup_{b \in B} \inf_{a \in A} d(a, b) \right).$$

Considering two sets of points corresponding to two contour lines and $d$ being the Euclidean distance, $d_H$ measures the maximum of the distances between the points of each curve from the other one (as a whole). It is obvious from the above definition that $d_H = 0$ corresponds to identical lines. The major issue about $d_H$ is its sensitivity to outliers. Let us suppose that a contour line consists of two parts, the major one being identical to a contour line of NGC 1300, while the minor one being a tiny part far away. In such a case although we have a good matching of the response model with the galaxy morphology (at least for the specific contour), $d_H$ reads a false high value. There have been proposed many modifications of $d_H$ that aim to reduce this drawback (Zhao et al. 2005, and references therein). Here we introduce an index which is a generalization of $d_H$ and it is appropriate for our study.

We call this index “Generalized Hausdorff Distance”, $d_GH$, and it is defined as follows: let $C_1$, $C_2$ be two curves on the plane of non-zero length $L_{C_1}$ and $L_{C_2}$, respectively. The $d_GH$ between $C_1$ and $C_2$ is

$$d_{GH} = \frac{1}{L_{C_1}} \int_{b \in C_1} \inf_{a \in C_2} \frac{d(a, b)}{||b||} \, dl + \frac{1}{L_{C_2}} \int_{b \in C_2} \inf_{a \in C_1} \frac{d(a, b)}{||b||} \, dl,$$

where the integrations are over the curves $C_1$ and $C_2$ and $|| \cdot ||$ denotes the Euclidean norm. The integration adds the contributions from all points, while the division with the curve lengths and the norm $||b||$ aims to the normalization of the final $d_{GH}$ value so that it can be comparable for two totally different contour lines. In practice, for the computation of $d_{GH}$ we sample both curves with a considerable number of points $N_{C_1}$, $N_{C_2}$ and we calculate the quantity

$$d_{GH} = \frac{1}{N_{C_1}} \sum_{i=1}^{N_{C_1}} \frac{1}{\sqrt{||b||}} \min_{a \in C_2} d(a, b) + \frac{1}{N_{C_2}} \sum_{i=1}^{N_{C_2}} \frac{1}{\sqrt{||b||}} \min_{a \in C_1} d(a, b).$$

What we do is to find which contour line of each RM corresponds
Figure 6. In the left-hand column we plot for the RM, the minimum generalized Hausdorff distance $d_{GH}$ value relative to the NGC 1300 contour line shown in the right-hand column (same row). In the middle column we plot the minimum $d_{GH}$ values corresponding to SMs (see Sect. 4). Note that filled circles, empty circles and empty squares (red, blue and green colored lines in the online version) correspond to pure 2D geometry (Model A), to cylindrical geometry (Model B) and to spheroidal-cylindrical geometry (Model C), respectively. The star symbol (light blue line in the online version) in the left-hand column corresponds to a scenario with time-dependent $\Omega_p$ (see Sect. 5).
to the minimum \( d_{GH} \) value relative to each one of the contour lines of NGC 1300 we have plotted in Figs. [135].

In Fig. [5] (left-hand column) we have plotted the minimum \( d_{GH} \) values corresponding to the NGC 1300 contour lines shown in the right-hand column of the same figure. Filled circles, empty circles and empty squares (red, blue and green colored lines in the online version) correspond to Models A, B and C, respectively. Note that in this figure we have also included the models corresponding to \( \Omega_p \) values not plotted in Figs. [35]. Based on this criterion, the basic trends that are concluded by observing the behavior of \( d_{GH} \) for all the Models (A, B, C) and for the studied range of \( \Omega_p \) values are:

- the lower \( \Omega_p \) values in Model A are more efficient (than the higher ones) in portraying the outer contours corresponding mainly to the spiral structure of NGC 1300 (see the two first rows of Fig. [5]).
- the higher \( \Omega_p \) values in Model A are more efficient (than the lower ones) in portraying the inner contours corresponding to the barred structure of NGC 1300 (see third to fifth rows of Fig. [5]).
- Model B is better than Model A in portraying the spiral structure in high \( \Omega_p \) values, while for low \( \Omega_p \)'s, Model A is in general better than B, especially at the outermost isocontour (first row of Fig. [5]).
- Model C is in general worst than Models A and B in portraying the spiral and bar structure, while it is definitely superior in portraying the very inner area especially for the middle and high \( \Omega_p \) values (see last row of Fig. [5]).

Nevertheless, we should have in mind, that the quantification of the similarity of the contours of the model with those of the galactic image by means of \( d_{GH} \), cannot in general account for small morphological differences, essentially in the phases, of the two curves. Despite this drawback \( d_{GH} \) is a reliable index in most cases encountered in the present study.

All the above results are related with the response models and depend to a large extent on their initial set up. These models provide useful information about the underlying dynamics and the morphological features they can support. Despite the fact that the setting up of the models is done in an unbiased way, there is no physical reason to exclude weighting some energies more than others in an effort to improve the similarity between RMs and galaxy morphology. In order to explore this we modify our models by introducing weights attached to each energy level. The energies of the particles are determined by giving them circular velocities in the axisymmetric potential. This determines where they will be end up in the well.
Figure 8. Similar to Fig. 7 but for the potential corresponding to the cylindrical geometry (Model B).

of the full potential. However, one can consider a totally different energy distribution of the particles of the model. The procedure and the corresponding results are described in the following section.

4 WEIGHTING ENERGY LEVELS

Schwarzschild (1979) in a pioneer study, presented a method for constructing time independent self-consistent solutions of specific models. His method has been widely used since then (Richstone 1980, 1982; Schwarzschild 1982; Richstone 1984; Richstone & Tremaine 1984; Schwarzschild 1993; Merritt & Fridman 1994; Cretton et al. 1999; Antonini et al. 2009) mostly for models representing elliptical galaxies. The basic idea of this method is very simple: One considers a density distribution function and then finds the associated potential via the Poisson equation. The next step is to calculate a library of orbits corresponding to this potential for the desired energy range. By assigning a weight to each orbit one tries with the superposition of the weighted orbits to reproduce the initially imposed density. The density distribution for each energy level is considered being the one we get from the corresponding RM. This assumption is true only for phase space domains where Chaos dominates and for time scales for which the corresponding initial conditions have been evolved and are close to a dynamical equilibrium due to chaotic mixing. For phase space domains (of specific energy levels) with significant fractions of regular orbits the above assumption is just a simplification since the distribution of the particles on the existing tori can be supposed to be different than that corresponding to the RMs.

The determination of the weights is made by the following procedure. Let \( N_e \) be the number of Jacobi constant (energy) intervals, which we give to the particles of a response model and \( N_c \), the number of grid cells we divide the plane \((x, y)\). In this situation we derive the weights \( w_i, (i = 1, \ldots, N_e, w(i) > 0) \), that minimize the self-consistency in 12 galaxies, while in barred-spiral systems has been applied by Kaufmann & Contopoulos (1996).

In practice what one is looking for is a way to minimize the difference between the imposed density and that corresponding to the superposed orbits. This detailed task is very demanding and is beyond the scope of this paper. Here we will construct our “best” models acting in a more rough way. We follow the rationale of Schwarzschild’s method but we assign weights to the various energy levels. The density distribution for each energy level is considered being the one we get from the corresponding RM. This assumption is true only for phase space domains where Chaos dominates and for time scales for which the corresponding initial conditions have been evolved and are close to a dynamical equilibrium due to chaotic mixing. For phase space domains (of specific energy levels) with significant fractions of regular orbits the above assumption is just a simplification since the distribution of the particles on the existing tori can be supposed to be different than that corresponding to the RMs.

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quantity

\[
N_c \sum_{j=1}^{N} \left( \sum_{i=1}^{N_e} w_i \frac{m_{ei}}{m_j} - 1 \right)^2
\]  

(4)

where \(m_{ei}\) is the mass contribution of the \(i\)-th energy interval on the \(j\)-th grid cell and \(m_j\) the mass contribution we consider corresponding to the same region at the NGC 1300 image on the \(j\)-th grid cell. We consider the ratio \(m_{ei}/m_j\) in order to treat equally high and low density regions. The final considered surface density distribution is given by

\[
d_j = \frac{\sum_{i=1}^{N_e} w_i m_{ei}}{S_j}, \quad j = 1, \ldots, N_c
\]  

(5)

where \(d_j, S_j\) are the density and the surface area of the \(j\)-th grid cell, respectively. In the present study, we used \(N_e = 30\) and \(N_c = 24 \times 24 = 576\) for the central 576 kpc² area.

Thereupon, we repeat the procedure of the determination of the contour lines of each of these “Schwarzschild type” models (hereafter SM) corresponding to the minimum \(d_{GH}\) value relative to each contour line of NGC 1300 plotted in Figs. 1 and 3-5.

Going back to Fig. 6, in the middle column, we plot the minimum \(d_{GH}\) values, similar to the panels in the left-hand column (RMs), but for the density distribution corresponding to Eq. (5) (SMs). The comparison between the RMs and SMs behavior of \(d_{GH}\), shows the following:

- In 2D geometry (Model A, filled circles or red color line in the online version) the SMs with the high \(\Omega_p\) values are more efficient (than the corresponding RMs) in portraying the spiral structure of NGC 1300 (panels in the two first rows in Fig. 6).
- In cylindrical geometry (Model B, empty circles or blue color line in the online version) the SMs with the low \(\Omega_p\) values are more efficient (than the corresponding RMs) in portraying the bar of NGC 1300 (third to sixth rows of Fig. 6).
- In spheroidal-cylindrical geometry (Model C, empty squares or green color line in the online version) the SMs with the lowest \(\Omega_p\) values (\(\Omega_p < 16 \text{ km s}^{-1}\text{kpc}^{-1}\)) are more efficient (than the corresponding RMs) in portraying the NGC 1300 bar (third to sixth rows of Fig. 6).

In Figs. 7-9 we plot the isocontours (dashed lines) corresponding to the minimum \(d_{GH}\) with respect to the isocontours of NGC 1300 (solid lines), for a sample of the SMs of the Models A, B and C, respectively. These samples include characteristic cases such as those with \(\Omega_p = 16 \text{ km s}^{-1}\text{kpc}^{-1}\) and \(\Omega_p = 22 \text{ km s}^{-1}\text{kpc}^{-1}\). Note that the corresponding isocontour curves (of model and

Figure 9. Similar to Fig. 4 but for the potential corresponding to the (spheroidal + cylindrical) geometry (Model C).
galaxy) are plotted in the same color. From these figures, it becomes evident the nice behavior of the models. The plotting styles are as in Fig. 6.

In almost all cases low (or high) \( \Omega_p \) values around \( \Omega_p \approx 22 \, \text{km s}^{-1} \, \text{kpc}^{-1} \) resemble between the corresponding curves.

Since the \( d_{GH} \) values for the various isocontours are by definition comparable, we can add all the values of each model in order to get a unique value describing globally the model. Such an index puts of course equally weights to all isocontours and it could be low valued in models with no matching curves. Nevertheless, it is reliable in cases with many model curves showing moderate resemblance with those of the galactic image. In Fig. 10 we plot, for all RMs and SMs, this “global” \( d_{GH} \) index for the isocontours of Fig. 6.

The general conclusions we draw by observing Fig. 10 can be summarized as follows:

- For \( \Omega_p \leq 20 \, \text{km s}^{-1} \, \text{kpc}^{-1} \) values, the RM (left-hand panel of Fig. 10) with the lowest global \( d_{GH} \) index are those corresponding to Model A.
- The RMs corresponding to Model B and C improve their behavior from low \( \Omega_p \) values towards high \( \Omega_p \) values (for Model C only up to \( \Omega_p = 24 \, \text{km s}^{-1} \, \text{kpc}^{-1} \).)
- The overall behavior of SMs corresponding to Model A are improved (relative to those of the corresponding RMs) only for \( \Omega_p \leq 22 \, \text{km s}^{-1} \, \text{kpc}^{-1} \).
- The overall behavior of SMs corresponding to Model B are improved (relative to those of the corresponding RMs) for \( \Omega_p \leq 23 \, \text{km s}^{-1} \, \text{kpc}^{-1} \).
- The overall behavior of SMs corresponding to Model C are improved (relative to those of the corresponding RMs) only for \( \Omega_p \leq 14 \, \text{km s}^{-1} \, \text{kpc}^{-1} \).

5 DISCUSSION AND CONCLUSIONS

We studied the stellar response, under the assumption of a single pattern speed, for the potential models of the barred-spiral galaxy NGC 1300 estimated in Paper I and for a wide range of \( \Omega_p \) values. The three potential models of Paper I, correspond to three different geometries as regards the mass distribution in the third dimension (perpendicularly to the equatorial plane). Our goal was to find out, which geometries and which values of \( \Omega_p \) were able to reproduce the fundamental morphological features of NGC 1300, i.e. the bar and the spirals. We used the method of response models (RM), as in Patsis (2006). So, we start with a uniformly populated disk of stars moving initially in circular orbits, which are determined in the axisymmetric part of the potential. We integrate the orbits of the test particles for many pattern rotations under the full potential and we get the density maps of the response models as described in Sect. 2.

The “by eye” comparison between the RM morphologies and the image of the galaxy gives useful qualitative information but it lacks of objectivity and quantification. For this reason we introduced a new index which is a generalized modification of the Hausdorff distance. This index quantifies the resemblance of two isocontour lines and enables the determination of the best matching contour lines (of a model) to each one of the six preselected isocontours of the deprojected K-band image of NGC 1300 (Fig. 7). This new index testifies the general remarks made by eye and furthermore it describes the quality of the fitting for the individual components of the galaxy. Moreover, we investigated possible improvements of the response models as regards their resemblance with the NGC 1300 morphology, by assigning weights to the contributions of the various energy levels. This method follows the general concept of Schwarzschild’s methods, but it is simpler in the sense that it requires weights for the energy levels and not for individual orbits. Some of these modified Schwarzschild models (SM) improve the similarity of specific features of NGC 1300 in comparison to the original response models.

Below we enumerate our conclusions:

(1) The spiral structure of NGC 1300 is reproduced:

- Best by the RM corresponding to the potential of the pure 2D geometry and for \( \Omega_p \) values around 16 \, km s\(^{-1}\) \, kpc\(^{-1}\). The big advantage of this model is that the spirals of RM and galaxy have similar pitch angles. The density enhancement of the the RM at the spiral region is along the same direction as the corresponding arms of NGC 1300.
- Acceptable by the SM corresponding to the potential of the pure 2D geometry and for \( \Omega_p \) values around 22 \, km s\(^{-1}\) \, kpc\(^{-1}\). However, the drawback of this case is that the spirals, especially the one appearing at the lower part of the figures, have different
pitch angles than that of the galaxy. The spiral arms of the model share common regions with the NGC 1300 arms but essentially cross each other. They also add additional features beyond the region within which the NGC 1300 barred-spiral structure extends.

- **Moderately** by the RM and SM corresponding to the potential of the 3D cylindrical geometry and for \( \Omega_p \) values around 22 km s\(^{-1}\) kpc\(^{-1}\). However, the remarks about the pitch angles of the spirals we mention above for the 2D case, yield for the thick disc 3D case as well. On top of that the upper arm is not nicely reproduced by this model.

(2) The ansae-type bar of NGC 1300 is reproduced:

- **Best** by the RM and SM corresponding to the potential of the pure 2D geometry, for \( \Omega_p \) values around 22 km s\(^{-1}\) kpc\(^{-1}\) (very good is also the \( d_{\text{GH}} \) index of the RM corresponding to the potential of the 3D cylindrical geometry again for \( \Omega_p \) around 22 km s\(^{-1}\) kpc\(^{-1}\). However, the ansae are reproduced in the correct regions only in the 2D case)

- **Very well** by the SM corresponding to the potential of the 3D cylindrical geometry and for \( \Omega_p \) values around 17 km s\(^{-1}\) kpc\(^{-1}\).

- **Moderately** by the SM corresponding to the potential of the 3D cylindrical geometry and for \( \Omega_p \) values around 23 km s\(^{-1}\) kpc\(^{-1}\).

(3) The inner oval-like shape structure of NGC 1300 is reproduced:

- **Best** by the RM and SM corresponding to the potential of the 3D spheroidal-cylindrical geometry and for almost all \( \Omega_p \) values

- **Well** by the SM corresponding to the potential of the 3D cylindrical geometry and for \( \Omega_p \) values around 22 km s\(^{-1}\) kpc\(^{-1}\).

- **Moderately** by the SM corresponding to the potential of the pure 2D geometry and for \( \Omega_p \) values around 23 km s\(^{-1}\) kpc\(^{-1}\).

All the above indicate the tendencies of the models to improve their similarity with the K-band image of the galaxy for a given geometry or for a certain \( \Omega_p \) range. However, the relation between “good” models and the NGC 1300 morphology is not unambiguous. It is obvious that some geometries support better certain morphological features. This may suggest the appropriate geometry for the description of specific regions of the galaxy (e.g. the central area should be considered as a spheroidal). Another issue is that a morphological feature may be reproduced equally well by models that differ in their pattern speeds. In addition we remind our assumption of the time-independence of the potential and a constant time-independent \( \Omega_p \). The study of the stellar dynamics of our system under these assumptions not only helps us see some basic trends in the dynamical behaviour that favours the formation of the one or the other morphological feature, but after all it will indicate the necessity of different assumptions. The evolution of the models under a time-dependent potential and \( \Omega_p \) is very dubious in the response model approach, since it requires specific assumptions for the evolution laws for both potential and \( \Omega_p \). The assumptions are free and therefore the number of the possible combinations is large. Such a study is beyond the scope of this paper and should be attacked by means of N-body simulations. However, in order to get a crude impression about how different can be the resulting morphologies if a basic parameter varies with time, we test a scenario with time dependent \( \Omega_p \) in a fixed potential. We have chosen for our calculations a pure 2D geometry. The choice is justified by the fact that the RM corresponding to the pure 2D geometry portrays on the one hand the spiral structure for \( \Omega_p \approx 16 \text{ km s}^{-1} \text{ kpc}^{-1} \) and on the other hand the ansae-type bar for \( \Omega_p \approx 22 \text{ km s}^{-1} \text{ kpc}^{-1} \).

We adopted a rather arbitrary evolution law for the increase of \( \Omega_p \), which reads

\[
\Omega_p(t) = 19 + 3 \tanh \left( \frac{1}{5}(t-15) \right) \text{ km s}^{-1} \text{ kpc}^{-1},
\]

where the time \( t \) is measured in periods corresponding to a pattern rotating with \( \Omega_p = 23 \text{ km s}^{-1} \text{ kpc}^{-1} \). The integration time of the orbits was up to \( t = 30 \). In this numerical experiment the initial value of \( \Omega_p = 16 \text{ km s}^{-1} \text{ kpc}^{-1} \), while the final one is \( \approx 22 \text{ km s}^{-1} \text{ kpc}^{-1} \). In the left-hand columns of Figs. 6 and 10 we have plotted (with star symbol or light blue lines in the online version) the \( d_{\text{GH}} \) values corresponding to each snapshot’s \( \Omega_p \) value. In these figures we observe that for the time corresponding to \( \Omega_p \approx 17 \text{ km s}^{-1} \text{ kpc}^{-1} \) and \( \Omega_p \approx 22 \text{ km s}^{-1} \text{ kpc}^{-1} \) we get a good resemblance with many isocontours of NGC 1300. This could be an indication that the present NGC 1300 structure is just a snapshot in an evolutionary scenario, according to which the observed morphology undergoes successive transient phases. However, whatever good results we find, we find for increasing \( \Omega_p \) from one value to the other. The opposite procedure, i.e. decreasing \( \Omega_p \), according to the law in Eq. 6 does not lead to equally nice results. Decrease of \( \Omega_p \) during an N-body simulation is frequently observed due to dynamical friction (Chandrasekhar 1943; Donner & Sundelius 1993; Debattista & Sellwood 1998), but there is no obvious physical reason for the opposite. In our model the particles that participate in the spiral structure stay for longer times at the outer regions of the disc, i.e. in regions that evolve dynamically slower than particles at the central region that contribute to the bar structure. So, by increasing \( \Omega_p \), the particles that stay longer at the outer disc do not have enough time to feel the potential. On the other hand particles staying at the central region of the system respond fast during the period our simulation reaches the fast \( \Omega_p \) domain.

Concluding, we remark that taking into account all the models we constructed as well as the techniques we used to assess them, there are clearly two \( \Omega_p \) intervals, where we find morphological features of the models resembling the structures of NGC 1300. These are \( \Omega_p \) values close to 16 and 22 km s\(^{-1}\) kpc\(^{-1}\).

Comparing this result with pattern speeds proposed by other authors for NGC 1300 (England 1989; Lindblad & Kristen 1998), there is a relative agreement between the fast model of Lindblad & Kristen (1998) and the group of models around \( \Omega_p = 22 \text{ km s}^{-1} \text{ kpc}^{-1} \), which we find matching the NGC 1300 bar (Lindblad & Kristen 1996) propose \( \Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1} \) for a distance of the galaxy D=20 Mpc. The alternative solution proposed by the same authors (\( \Omega_p = 12 \text{ km s}^{-1} \text{ kpc}^{-1} \) is in a range of \( \Omega_p \) values, where the bars of our RMs are much thicker than the one of NGC 1300 and the stellar spirals are absent. As regards the work of England (1989), the proposed pattern speed gives equilibrium points in a narrow zone compatible again with the \( \Omega_p = 22 \text{ km s}^{-1} \text{ kpc}^{-1} \) values, as it will become evident in Paper III.

In any case, the understanding of the observed NGC 1300 morphology goes through the understanding of the underlying dynamical mechanisms and this can be done only by studying the orbital behavior of the “successful” models. This will allow us also the comparison among them, and is done in Paper III in this series.
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REFERENCES

Antonini F., Capuzzo-Dolcetta R., Merritt D., 2009, MNRAS, 399, 671
Chandrasekhar S., 1943, ApJ, 97, 255
Contopoulos G., 1980, A&A, 81, 198
Cretton N., de Zeeuw P. T., van der Marel R. P., Rix H., 1999, ApJSS, 124, 383
Debattista V., Sellwood J., 1998, ApJ, 493, L5
Debattista V. P., Sellwood J. A., 2000, ApJ, 543, 704
Deza M., Deza E., 2009, Encyclopedia of Distances. Springer, Dordrecht, Heidelberg, London, New York
Donner K., Sundelius B., 1993, MNRAS, 265, 88
England M. N., 1989, ApJ, 344, 669
Kalapotharakos C., Patsis P. A., Grosbøl P., 2010, MNRAS, 403, 83
Kaufmann D. E., Contopoulos G., 1996, A&A, 309, 381
Lindblad P., Kristen H., 1996, A&A, 313, 733
Lindblad P. A. B., Kristen H., Joersaeter S., Hoegbom J., 1997, A&A, 317, 36
Merritt D., Fridman T., 1996, ApJ, 460, 136
Patsis P. A., 2006, MNRAS, 369L, 56
Patsis P. A., Contopoulos G., Grosbøl P., 1991, A&A, 243, 373
Patsis P. A., Kalapotharakos C., Grosbøl P., 2010, MNRAS, submitted - Paper III
Rautiainen P., Salo H., Laurikainen E., 2008, MNRAS, 388, 1803
Richstone D., Tremaine S., 1984, ApJ, 286, 27
Richstone D. O., 1980, ApJ, 238, 103
Richstone D. O., 1982, ApJ, 252, 496
Richstone D. O., 1984, ApJ, 281, 100
Schwarzschild M., 1979, ApJ, 232, 236
Schwarzschild M., 1982, ApJ, 263, 599
Schwarzschild M., 1993, ApJ, 409, 563
Zhao C., Shi W., Deng Y. e. a., 2005, Pattern Recognition Letters, 26, 581