SPIN AND TOTAL ANGULAR MOMENTUM OF THE GLUE

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We briefly review the present theoretical and experimental knowledge on $\Delta G$ and $J_g$.

1 Introduction

Polarized DIS probes the singlet axial charge of the proton $a_0$, i.e. the matrix element

$$
(P, S|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|P, S) = 2M a_0 S_\mu
$$

The surprising small value of $a_0$ ($\sim 0.2$) is explained as a consequence of the axial anomaly and of a relatively large gluon contribution to the proton spin.

Due to the axial anomaly $a_0$ does not coincide with the quark spin $\Delta \Sigma$

$$
\Delta \Sigma = \int_0^1 dx (\Delta u + \Delta d + \Delta s)
$$

but mixes quark and gluon contributions in a scheme dependent way. In particular, in the Adler-Bardeen (AB) scheme $a_0$ is decomposed as

$$
a_0(Q^2) = \Delta \Sigma - N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2),
$$

where $\Delta G$ is the gluon spin

$$
\Delta G = \int_0^1 dx \Delta g .
$$

In this scheme $\Delta \Sigma$ is conserved and it is natural to identify it with the total quark spin as provided by the constituent quark model.

Naively one has $\Delta \Sigma = 1$. Due to relativistic effects and extra-degrees of freedom (e.g., mesons), $\Delta \Sigma$ is reduced to 0.6–0.7, still a moderately large value.
According to (3) the small $a_0$ measured by the experiments is compatible with
the quark model estimate for $\Delta \Sigma$ if one allows for a sizeable $\Delta G$.

Two important questions are:

- Is it possible to infer the value of $\Delta G$ from polarized DIS data?
- Do quark models, or other nonperturbative approaches, provide a $\Delta G$
compatible with the DIS findings?

The answer to the first question is affirmative (although the result is af-
fected by a large error) and comes from the global QCD analyses of the data.
In Ref. 3 it has been shown that the DIS data suggest

$$\Delta G = 1.4 \pm 0.9, \quad \text{at } Q^2 = 1 \text{ GeV}^2,$$

(5)

where the error takes into account various sources of experimental and theo-
retical uncertainties (including different choices for the functional form of the
helicity distributions).

It is clear that a more direct measurement of $\Delta G$, based on the investiga-
tion of semininclusive processes, is needed. A first result in this direction has
recently come from the HERMES study of photoproduction of high-$p_T$ hadron
pairs. The value of $\Delta g(x)/g(x)$ at one average $x$ point has been extracted
from the longitudinal spin asymmetry by means of a Monte Carlo simulation.
The result, referred to a scale $\mu^2 = 2.1 \text{ GeV}^2$, and obtained in LO QCD, is

$$\frac{\Delta g(x)}{g(x)} = 0.41 \pm 0.18 \text{ (stat.)} \pm 0.03 \text{ (exp. syst.)} \quad \text{at } < x > = 0.17$$

(6)

Thus $\Delta g$ is found to be positive in the intermediate $x$ region. The available
LO parametrizations are in agreement with the value (3) (the exception being fit C of Gehrmann–Stirling
where $\Delta g$ becomes negative above $x \simeq 0.1$ – a behavior which seems to be ruled out by the HERMES finding). Although
it is not possible, on the basis of the HERMES datum alone, to discriminate
among different fits and to draw definite conclusions about the integral of $\Delta g$,
we can say that, if the distribution has no bizarre behavior, its first moment
$\Delta G$ should also be positive.

As for the second question, the answer is affirmative as well. No dis-
crepancies emerge between quark model expectations and results of DIS data
analyses.

Another interesting (and still open) problem is the value of the total an-
gular momentum of the glue $J_g$, and of its orbital component $L_g = J_g - \Delta G$.

All these issues will be briefly discussed in the following. To start with,
let us first of all define the quantities at hand.
2 Angular momentum sum rule

The angular momentum sum rule for the nucleon reads

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q(\mu^2) + J_g(\mu^2) \]  

where \( \Delta \Sigma \) is the quark helicity, eq. (2), \( L_q \) is the quark canonical orbital momentum and \( J_g \) the total angular momentum of the glue. All quantities on the r.h.s. of eq. (1) are gauge invariant and depend in general on a scale \( \mu^2 \) (remember however that in the AB scheme \( \Delta \Sigma \) is a conserved quantity). The gluonic term in eq. (7) is given by

\[ J_g(\mu^2) = \langle P \uparrow | \int d^3r \{ \vec{E}(\vec{r}) \times \vec{B}(\vec{r}) \}^3 | P \uparrow \rangle. \]  

(8)

\( J_g \) admits a gauge invariant decomposition into an orbital (\( L_g \)) and a spin (\( \Delta G \)) part, but these two components cannot be written in terms of local operators. Adopting the \( A^+ = 0 \) gauge, however, it is possible to obtain a local expression for the gluon spin

\[ \Delta G(\mu^2) = \langle P \uparrow | \int d^3r \{ \vec{E}(\vec{r}) \times \vec{A}(\vec{r}) \}^3 + \vec{A}_\perp(\vec{r}) \cdot \vec{B}_\perp(\vec{r}) \} | P \uparrow \rangle. \]  

(9)

Both \( J_g \) and \( \Delta G \) have a scale dependence due to the renormalization of the operators appearing in eqs. (8, 9).

Obviously, \( \Delta G \) and \( J_g \) cannot be computed in perturbative QCD. However it is possible to evaluate them by means of nonperturbative tools. Calculations have been performed so far in the QCD sum rule framework or using quark models. In the first case \( J_g \) and \( \Delta G \) are related to the quark and gluon condensates at a given scale. In the second case one uses model wavefunctions to compute the matrix elements in eqs. (8, 9). The results are then assumed to be valid at some fixed scale, which is usually very low (\( \lesssim 0.5 \text{ GeV}^2 \)), and then evolved by the Altarelli-Parisi equations to a higher \( Q^2 \).

3 Sign and magnitude of \( \Delta G \)

The sign of \( \Delta G \) is an interesting matter. Most of the existing fits yield a positive \( \Delta G \) at \( Q^2 \geq 1 \text{ GeV}^2 \). The nonperturbative results accumulated so far also give positive values for \( \Delta G \).

The only indication for a negative \( \Delta G \) came few years ago from a model calculation by Jaffe, who found \( \Delta G \sim -0.4 \) in the MIT bag model at a
scale $\mu_0^2 \simeq 0.25$ GeV$^2$ and an even more negative value ($\sim -0.7$) in the non-relativistic quark model. It was later on explained that Jaffe’s negative result is a consequence of neglecting self-interaction effects in the computation of $\langle \bar{\beta} \rangle$.

To clarify this issue, let us see how $\langle \bar{\beta} \rangle$ reads in a quark model. Denoting by $\langle \cdot \rangle_c$, $\langle \cdot \rangle_{osf}$ the color and orbital–spin–flavor expectation values, respectively, from $\langle \bar{\beta} \rangle$ one gets, after some manipulations

$$\Delta G(\mu^2) = \frac{1}{4} \sum_{i,j=1}^{3} \sum_{a=1}^{8} \langle \lambda^a_i \lambda^a_j \rangle_c \times \int d^3r \langle P \uparrow | \left\{ 2 \left[ \vec{E}_i(\vec{r}) \times \vec{A}_j(\vec{r}) \right] \right\}^3 + \vec{A}_{\perp i}(\vec{r}) \cdot \vec{B}_{\perp j}(\vec{r}) \rangle \rangle_{osf}.$$

(10)

Now if one replaces the color fields by their expectation values in the nucleon eigenstate and includes the self-interaction terms with $i = j$, $\Delta G$ turns out to be exactly zero. The reason is simple. Since the ground state of the nucleon is symmetric with respect to the exchange of any pair of quarks, the matrix elements in the integral of eq. (10) do not depend on the particle indices $i$ and $j$. Thus the integral factorizes out and $\Delta G$ vanishes exactly because $\sum_{i,j} \sum_{a} \langle \lambda^a_i \lambda^a_j \rangle = 0$. On the other hand, if the self–interaction terms are neglected, a negative $\Delta G$ is obtained, as a direct consequence of $\sum_{a} \lambda^a_i \lambda^a_j = -8/3$ for $i \neq j$.

As shown in Ref. 13, the correct way to proceed is to insert in the matrix element of (10) a complete set of intermediate states, which include the orbital excitations of the three-quark system, and to take into account the self-interaction contributions.

The integral in eq. (10) no longer factorizes out and $\Delta G$ is not forced to vanish, even including the self-field terms. We omit the details of this procedure (the reader can find them in Ref. 13). We just mention that, since we work in an effective model, which is supposed to be valid up to excitation energies of the order of 1 GeV, the convergence of the series obtained from the mode expansion of (10) raises no problems: a cutoff is introduced, which excludes excited states with energies of more than $\sim 1$ GeV above the ground state.

In Ref. 13 we evaluated $\Delta G$ using the Isgur-Karl (IK) model. We found

$$\Delta G = 0.26 \alpha_s.$$

Here $\Delta G$ is expressed in terms of the strong coupling constant $\alpha_s$, which is fixed in the IK model by reproducing the $\Delta - N$ mass splitting. This gives
\[ \alpha_s = 0.9 \] which corresponds to a scale \( \mu_0^2 \simeq 0.25 \text{ GeV}^2 \). The dynamical quantities computed in the model must be taken at this scale. Thus our final estimate is
\[ \Delta G (\mu_0^2) \simeq 0.24, \quad \mu_0^2 = 0.25 \text{ GeV}^2. \tag{11} \]

The evaluation of the total angular momentum \( J_g \) proceeds along a similar line and leads to
\[ J_g (\mu_0^2) \simeq \Delta G (\mu_0^2) \simeq 0.24, \quad \mu_0^2 = 0.25 \text{ GeV}^2. \tag{12} \]

We conclude that at the model scale very little room (if any) is left for the orbital angular momentum of gluons:
\[ L_g (\mu_0^2) = J_g (\mu_0^2) - \Delta G (\mu_0^2) \simeq 0. \tag{12} \]

In order to evolve \( \Delta G \) at NLO we need to know \( \Delta \Sigma \). We cannot extract this quantity from the angular momentum sum rule unless we know \( L_q \), the canonical orbital momentum of quarks. We assume \( L_q (\mu_0^2) = \pm 0.10 \) (the available estimates \(^8, ^{15} \) fall in this range). With (12), eq. (7) then implies \( \Delta \Sigma = 0 \pm 0.20 \).

The NLO evolution of \( \Delta G \) is shown in Fig. 1 where the shaded area corresponds to the allowed variation of \( \Delta \Sigma \). We use \( \Lambda = 250 \text{ MeV} \) for 3 flavors. Starting from \( \Delta G = 0.24 \) at \( \mu_0^2 \) we obtain, at 1 GeV\(^2 \), \( \Delta G = 0.59 \pm 0.07 \) which is compatible with the result \(^3 \) of the global analysis of Ref. 3 (the black dot with the error bar in figure).

One may wonder whether a positive value of \( \Delta G \) at some scale \( Q^2 > 1 \text{ GeV}^2 \) is compatible with a negative value at a smaller scale \( \mu^2 \lesssim 0.5 \text{ GeV}^2 \) (a typical quark model scale). In other terms, is it possible to obtain by QCD evolution a positive \( \Delta G \) starting from a negative input at low energy? In principle, the answer is yes. In practice, however, a \( \Delta G \) such as that demanded by the DIS data implies a positive gluon polarization even at very small scales, as we shall now see.

Let us take a look at the NLO evolution for \( \Delta G \) in the AB scheme:
\[ \Delta G (Q^2) = \left[ 1 - \frac{2 N_f}{\pi \beta_0} (\alpha_s(\mu^2) - \alpha_s(Q^2)) \right] \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \Delta G (\mu^2) - \frac{4}{\beta_0} \left( 1 - \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right) \Delta \Sigma, \tag{13} \]

with \( \beta_0 = 11 - 2 N_f/3 \). It is clear that the second term of (13) has no definite sign, so nothing prevents a negative \( \Delta G \) at low \( Q^2 \) from producing a positive one at high \( Q^2 \). But if we evolve back the value \(^5 \) we find that \( \Delta G \) remains positive down to very small \( Q^2 \) values \( \sim 0.15 - 0.20 \text{ GeV}^2 \), below which the perturbative evolution equations are simply inapplicable. This is shown in Fig. 2 where the backward evolution of \( \Delta G \) is plotted. The shaded area corresponds to the error in \(^5 \). For \( \Delta \Sigma \) we used the estimate of Ref. 3: \( \Delta \Sigma =
0.44. The uncertainty quoted on this value (±0.09) does not produce any appreciable variation on the final result.

4 The total angular momentum of the glue

An open issue is the relative weight of $\Delta G$ and $J_g$, the gluon total angular momentum, or – equivalently – the sign and the size of the gluon orbital angular momentum $L_g$.

We have already mentioned that the calculation of eq. (8) in the IK model gives $J_g(\mu_0^2) \simeq \Delta G(\mu_0^2) \simeq 0.24$ and hence $L_g(\mu_0^2) \simeq 0$.

To see what happens at a larger scale we perform a leading–order QCD evolution of $J_g$ (a semiquantitative result is enough for our purposes). The result (with $\Lambda = 200$ MeV for 3 flavors) is

$$J_g(1 \text{ GeV}^2) = 0.29.$$ (14)

A QCD sum rule calculation gives a similar value

$$J_g(1 \text{ GeV}^2) = 0.35 \pm 0.13.$$ (15)

Note that $\Delta G$ evolves with $Q^2$ differently from $J_g$. At LO, for instance, whereas $\Delta G$ increases as $\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)$, $J_g$ has a limiting asymptotic value $8/(16 + 3N_f) = 0.32$.

Hence, with growing $Q^2$, $\Delta G$ gets larger than $J_g$ and the gluon orbital angular momentum $L_g$ becomes increasingly negative. This scenario is similar
to the one suggested by the QCD sum rule approach of Ref. 11 and by the quark model calculation of Ref. 15.

What remains to be done, in the framework of quark models, is a check of the angular momentum sum rule, eq. (5), with a self-consistent evaluation of all terms appearing in it. This work is in progress.

5 Summary

In conclusion, let us summarize the present information on $\Delta G$ and $J_g$.

- Inclusive DIS data require

$$\Delta G (1 \text{ GeV}^2) = 1.4 \pm 0.9 ,$$  \hfill (16)

where the error takes into account various uncertainties.

Evolving this value back in $Q^2$ one still gets a positive $\Delta G$ down to $Q^2 \simeq 0.15 - 0.20 \text{ GeV}^2$, which is the limit of applicability of the evolution equations.

- A recent semiinclusive measurement gives at a scale $\mu^2 = 2.1 \text{ GeV}^2$

$$\frac{\Delta g}{g} = 0.41 \pm 0.18 \text{(stat.)} \pm 0.03 \text{(exp. syst.)} , \quad \text{at } < x > = 0.17 . \quad (17)$$

This hints to a positive $\Delta G$. More precise and extensive determinations are needed in order to draw any definite conclusion.

- Contrary to some claims, quark models do not prescribe a negative gluon helicity: this is only an artifact of neglecting self-interaction contributions. If these are included, which is the correct thing to do, a positive $\Delta G$ results at the model scale. In particular, in the IK model, we found

$$\Delta G = 0.24$$

Setting $\Delta \Sigma = 0.52 \pm 0.20$, $\Delta G$ becomes at 1 GeV$^2$

$$\Delta G = 0.59 \pm 0.07 ,$$

to be compared with (16). The two values are compatible within the errors.

- Quark model and QCD sum rule calculations indicate a small gluon orbital contribution at low $Q^2$. In the IK model, for instance, we obtained

$$\Delta G \simeq J_g , \quad \text{i.e. } L_g \simeq 0 \quad \text{at } \mu_0^2 = 0.25 \text{ GeV}^2$$

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which implies at 1 GeV$^2$

$$J_g = 0.29.$$  

A close result was found in the QCD sum rule calculation of Ref. 11. Since $J_g$ evolves slower than $\Delta G$ we expect the gluon orbital momentum to be increasingly negative as $Q^2$ gets larger.

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