Degenerate Sub-keV Fermion Dark Matter from a Solution to the Hubble Tension

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We present a dark sector model addressing both the Hubble tension and the core-cusp problem. The model is based on a hidden Abelian gauge symmetry group with some chiral fermions required by the anomaly cancellation conditions, producing a candidate for the decaying fermion dark matter as a solution to the Hubble tension. Moreover, the sub-keV mass regime and the thermal history of the dark sector help the dark matter candidate resolve the core-cusp problem occurring in the standard ΛCDM cosmology.

I. INTRODUCTION

The presence of dark matter (DM) becomes unquestionable fact thanks to various observational evidences. Nevertheless, its nature still remains unclear. So far, at least three physical features of DM are known, i.e., its stability, non-zero mass and very weak interaction with the ordinary matter. Unfortunately, however, as for the three aspects of DM, none was examined clearly thus far. It still remains unanswered whether its stability is permanent or effective only for the time scale of the age of Universe today. Also, the uncertainty in a possible DM mass has not been narrowed down yet, ranging from $O(10^{-22})$eV for the ultralight bosonic DM [1, 2] to $O(10) M_{\odot}$ for primordial black holes 1 or a macroscopic compact halo object (MACHO). Above all, lacking knowledge about non-gravitational interaction which DM does, we are still even unaware of if DM is, either of directly or indirectly, coupled to or totally decoupled from the SM sector.

In an effort to address aforementioned questions, we give a special attention to two arguments based on cosmological and astrophysical phenomena. The recently raised Hubble tension is one of them. This regards discrepancy reaching $\sim 4\sigma$ level [4] between a local measurement of the Hubble expansion rate ($H_0$) [5–8] and that inferred from the cosmic microwave background (CMB) observation [9]. While unknown systematics may affect the discrepancy [10–12], it could be a clue for a nature of DM. In particular, a decaying DM (DDM) solution [13] 2 to the tension amongst several resolutions proposes a possibility that the decay of DM with a lifetime $\sim 35$Gyrs can relieve the tension. 3 The advanced starting of the dark energy dominated era due to DM decay enables the faster expansion of the Universe at late times ($z \simeq O(0.1)$) and eventually the Hubble expansion rate at recombination era determined by CMB data can evolve to a value close to SHOES measurement of $H_0 \simeq 73 - 74$Km/sec/Mpc [6]. Inspired by this, we take a DM scenario with a finite lifetime greater than the current age of the Universe as far as the question about stability of DM is concerned.

On the other hand, whereas the correlation of structures on the large scale is in excellent accord with predictions from ΛCDM paradigm (cosmological constant + cold dark matter), discrepancies between results from simulations based on ΛCDM (e.g. cuspy halos [21], too many subhalos [22, 23], dense cores [24]) and the experimental observations on galactic scales may be hinting for a nature of DM distinct from CDM (for review, see [25]). Particularly, from disagreement between the cuspy DM density profile for galaxies predicted by ΛCDM simulations [26–28] and observations of rotation curves for low mass galaxies [29–32], there arises the so-called cusp-core problem. As one of resolutions to the problem, presence of degenerate fermion DM with sub-keV mass has been suggested in [33–38]. The quantum pressure applied by a degenerate fermion gas can counterbalance the gravity to prevent gravitational collapse, yielding cored DM profiles in dwarf spheroidal (dSphs) galaxies. Modelling DM halo as a degenerate fermion gas with the use of the stellar velocity dispersion data of dSphs and the inferred core size of Fornax dwarf galaxy favor the fermion DM mass in sub-keV range.

In this paper, we present a sub-keV fermion DDM scenario which addresses both the Hubble tension and the core-cusp problem. The model is based on a hidden Abelian gauge symmetry group and the fermion DDM is one of the chiral fermions required to cancel the chiral anomaly. The dark matter is non-thermally produced and sufficiently cooled down before virialization to follow the degenerate configuration. The Hubble tension is reconciled by the dark matter decaying into a massive dark photon and a light chiral fermion. We also comment ample, dark radiation [5, 16], dark energy at early time [17–20], etc.

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1 See e.g. [3] for the possibility of primordial black holes as all dark matter.
2 For an exemplary particle physics model for DDM solution to the Hubble tension, see [14, 15].
3 Other scenarios proposed to solve the tension include, for ex-
about the constraint from the Lyman-α forest [39, 40] and showed that our model is not inconsistent with its mass bound.

II. A MODEL FOR THE DARK SECTOR

Let us begin with the basic framework of our model for the dark sector. As a minimal extension of the gauge sector in a full theory, we consider a hidden Abelian gauge symmetry $U(1)_X$ under which matters in the dark sector are charged while SM particles are neutral.

As will be discussed below, the picture for DDM solution to the Hubble tension in [13] assumes at least three particle candidates for (1) a decaying dark matter, (2) a radiating resulting from the decay and (3) a daughter dark matter resulting from the decay. Thus in the case where none of three is a SM particle, introduction of other fields in the dark sector than the $U(1)_X$ gauge field is necessary. Anomaly-free condition for $U(1)_X$ is an useful guide line for how the introduction is made. As a matter of fact, a simple set-up dubbed “Number Theory Dark Matter” for the dark sector along this line of logic was already discussed in [41, 42].

As for the total number of Weyl fermions ($N$) and their $U(1)_X$ charges ($Q_i$), anomaly-free conditions demand

$$
\sum_{i=1}^{N} Q_i^3 = 0, \quad \sum_{i=1}^{N} Q_i = 0, \quad (1)
$$

for cancellation of $U(1)_X^3$ anomaly and gravitational $U(1)_X \times \text{[gravity]}^2$ anomaly. If the theory allows the vector-like fermions in the dark sector, values of their mass parameters are naturally close to a cut-off scale.\(^4\) This point makes the vector-like fermions become irrelevant to the low-energy physics once integrating out heavy degrees of freedom is done. Thus, we take an option to exclude presence of the vector-like fermions in the dark sector. DM being massive, this option automatically requires the breaking of $U(1)_X$.

In order to have particle contents required for the DDM scenario, we realize that the minimum number of Weyl fermions in the dark sector satisfying Eq. (1) amounts to five.\(^5\) Apart from the gauge field and the five Weyl fields, we introduce two complex scalar fields for the spontaneous breaking of $U(1)_X$ and to make fermions massive. Among several possibilities for a set of $U(1)_X$ charges discussed in [41, 42], we consider the following charge assignment

$$
\psi_{-9}, \psi_{-5}, \psi_{-1}, \psi_7, \psi_8, \Phi_1, \Phi_6, \quad (2)
$$

where the subscripts denote $U(1)_X$ charges of each field. The first five are Weyl fields and the remaining two are complex scalars. One can see that the anomaly free conditions in Eq. (1) are satisfied indeed by charges of Weyl fields. Writing the complex scalars as $\Phi_1 \equiv \phi_1 e^{iA_1/V_1}/\sqrt{2}$ and $\Phi_6 \equiv \phi_6 e^{iA_6/V_6}/\sqrt{2}$, we assume $V_1 \equiv \langle \phi_1 \rangle$ is greater than $V_6 \equiv \langle \phi_6 \rangle$ so that the breaking of $U(1)_X$ is induced dominantly by the condensation of $\Phi_1$ and hence the gauge boson mass is given by $m'_A \simeq g V_1$, where $g$ is the gauge coupling constant.\(^6\)

The Yukawa coupling of the dark sector reads

$$
\mathcal{L}_{Yuk} = y_1 \Phi_1 \psi_{-9} \psi_8 + y_2 \Phi_6 \psi_{-5} \psi_{-1} \\
\quad + y_3 \Phi_6^\dagger \psi_{-1} \psi_7 + \text{h.c.},
$$

(3)

where $y_i$ ($i = 1 - 3$) are taken to be real parameters without loss of generality. We diagonalize the mass matrix for $\psi_{-5}, \psi_{-1}$ and $\psi_7$ to obtain mass eigenstates. Together with $\psi_{-1}$, the following linear combination $\chi$ of $\psi_{-5}$ and $\psi_7$

$$
\chi \equiv \left( \frac{y_2}{\sqrt{y_2^2 + y_3^2}} \right) \psi_{-5} + \left( \frac{y_3}{\sqrt{y_2^2 + y_3^2}} \right) \psi_7,
$$

(4)

forms a Dirac fermion $\Psi_{DM} = (\psi_{-1}, \chi^*)^T$ of the mass $m_{DM} \equiv \sqrt{y_2^2 + y_3^2} V_6$. And also the other orthogonal combination to $\chi$ forms a massless Weyl field $\xi$

$$
\xi \equiv \left( \frac{y_3}{\sqrt{y_2^2 + y_3^2}} \right) \psi_{-5} - \left( \frac{y_2}{\sqrt{y_2^2 + y_3^2}} \right) \psi_7.
$$

(5)

From here on, we assume $y_2$ and $y_3$ are of a similar order and $y_2 \simeq y_3 \equiv y_s$ for simplicity.

III. A CONCRETE SCENARIO

In this section, we discuss how each resolution to phenomenological problems can fit into our model for the dark sector presented above. To have a simple thermal history of the dark sector, we first assume that the mass ($\simeq y_1 V_1$) of $\psi_{-9}$ and $\psi_8$ is greater than the inflaton mass
\[ m_1 \] so that we can neglect them in cosmological history of the Universe.\(^7\)

After the inflation ends, both the SM and the dark sector are reheated by the inflaton decay. As a \( U(1)_X \) singlet, the inflaton \( \Phi_I \) couples to both \( \Phi_1 \) and \( \Phi_6 \) through the following renormalizable operators\(^8\)

\[
L_\Phi = b_1 \Phi_I |\Phi_1|^2 + b_6 \Phi_I |\Phi_6|^2 ,
\]

where \( b_i (i=1,6) \) is a dimensionful coupling coefficient. Assuming quartic couplings \( \lambda_1 \simeq \lambda_6 \simeq \mathcal{O}(1) \) for each of complex scalars \( \Phi_1 \) and \( \Phi_6 \), we realize that decay of the inflaton to two \( \phi_1$s is kinematically suppressed because of \( m_I < m_1 \simeq \sqrt{\lambda_1} V_1 \). Hence, for the dark sector, reheating is accomplished via inflaton decay to two \( \phi_6$s. We define the decay rate to SM particles and the dark sector species \( \phi_6 \) to be \( \Gamma_{SM} \equiv \Gamma(\Phi \to SM) \) and \( \Gamma_{DS} \equiv \Gamma(\Phi \to \phi_6 + \phi_6) \) respectively, and make an approximation \( \Gamma_{tot} = \Gamma_{SM} + \Gamma_{DS} \simeq \Gamma_{SM} \). Unless a reheating temperature is so large as to be close to the Planck scale, \( \phi_6$s form a dark thermal bath via scattering with one another among them. Thus, below we consider evolution of the dark thermal bath comprising purely \( \phi_6$s. Notice that the purity of the thermal bath is guaranteed due to smallness of both \( y_s \) and \( U(1)_X \) gauge coupling \( g \) (which will be shown later).

The concrete thermal history which we will show below as a viable physical scenario is the following. After the reheating era, the dark sector begins with the temperature

\[
T_{DS}(a_{RH}) \simeq 0.43 \times \left( \frac{m_{DM}}{1 \text{keV}} \right)^{-1/3} \times T_{RH},
\]

where \( T_{DS}(a) \) (\( T_{SM}(a) \)) denotes the temperature of the dark sector (the SM sector) at the time of the scale factor \( a \). \( a_{RH} \) is the scale factor at the reheating time, and \( T_{RH} \equiv T_{SM}(a_{RH}) \) is assumed.\(^9\) From here on, whenever we encounter \( T_{RH} \), we can convert it using \((a_{RH}T_{RH}) = (a_{EW}T_{EW}) \simeq 10^{-13}\text{GeV} \) with \( a_{EW} \simeq 10^{-15} \) and \( T_{SM}(a_{EW}) \simeq 100 \text{GeV} \). Afterwards, the temperature of the dark thermal bath continues to decrease as

\[
T_{DS}(a) \simeq 0.43 \times \left( \frac{m_{DM}}{1 \text{keV}} \right)^{-1/3} \times \frac{10^{-13}}{a} \text{ GeV} ,
\]

and becomes lower than the mass of \( \phi_6 \). The comoving number density of \( \phi_6 \) is preserved until the time when the rate of \( \phi_6 \) decay to a DM pair \( (\psi^- \chi) \) becomes comparable to the Hubble expansion rate, i.e., \( \Gamma(\phi_6 \to DM + DM) \simeq H \). Then, non-relativistic \( \phi_6$s start to decay to produce free-streaming DMs. The free-streaming of DM is ensured owing to the small \( m_6 \). From \( \Gamma(\phi_6 \to DM + DM) \simeq H \), we infer the temperature of the SM thermal bath at this time

\[
T_{SM}(a_{FS}) \simeq 537 \times g_{SM}(a_{FS})^{-1/4} \times \left( \frac{V_6}{1 \text{GeV}} \right)^{-1} \left( \frac{m_{DM}}{1 \text{keV}} \right)^{-1/3} \left( \frac{m_6}{1 \text{GeV}} \right)^{1/2},
\]

where \( a_{FS} \) is the scale factor for the onset of the free-streaming of DM, \( g_{SM}(a) \) is the number of relativistic degrees of freedom in the SM sector evaluated at the scale factor \( a \) and \( m_6 \) is the mass of \( \phi_6 \). Here we used \( m_{DM} \simeq y_s V_6 \). Note that decay of \( \phi_6 \) and the onset of DM’s free-streaming take place at the same time since \( \phi_6 \) is non-relativistic already at this time. For consistency, \( T_{DS}(a_{FS}) \) should be less than \( m_6 \), which constrains \( V_6 (\simeq m_6 / \sqrt{\lambda_6} \simeq m_6) \) for each \( m_{DM} \). In Fig 2, the region above the green dashed line satisfies \( T_{DS}(a_{FS}) < m_6 \). Due to the entropy conservation, on the other hand, we can write down the temperature of the SM thermal bath as

\[
T_{SM}(a) = \left( \frac{106.75}{g_{SM}(a)} \right)^{1/3} \frac{10^{-13}}{a} \text{ GeV} .
\]

Equating Eq. (11) with Eq. (12) gives

\[
a_{FS} = 8.8 \times 10^{-16} \times g_{SM}(a_{FS})^{-1/12} \times \left( \frac{V_6}{1 \text{GeV}} \right)^{-1} \left( \frac{m_{DM}}{1 \text{keV}} \right)^{-1/3} \left( \frac{m_6}{1 \text{GeV}} \right)^{1/2} ,
\]

which will be used later for computing the free-streaming length for DM. After recombination, DM (\( \Psi_{DM} \)) gradually decays to a massless radiation \( \xi \) and a \( U(1)_X \) gauge boson \( A'_\mu \) serving as a daughter DM. On the other hand, due to a large redshift, DM gets into the motionless stage near recombination era, completely behaving as a matter. Furthermore, its “would-be” temperature today without taking into account virialization due to gravity in the galaxy formation is sufficiently low to enable DM to be in degenerate configuration.

In the following two subsections, we probe the parameter space of the model by applying the constraints on the lifetime of DDM and mass obtained in [13, 33, 34, 36]. Eventually we show there exists a parameter space where the desired physics can be realized in the model.
A. Constraints from the Hubble Tension

DDM solution to the Hubble tension is described by four parameters: lifetime of DDM, a fraction of a rest mass of DDM transferred to energy of the resulting radiation (ε), DM abundance today ΩDM,0 ≡ ρDM,0/ρcr,0 and a reduced Hubble parameter h ≡ H0/Km/sec/Mpc [13]. Below we refer to constraints on these parameters reported in [13] which were found by carrying out a Monte Carlo Markov Chain (MCMC) against the late universe measurements of the Hubble expansion rate for 0 ≤ ε ≤ 2.4. The underlying idea of the solution is to introduce a DDM whose decay leads to production of a massless and a massive particle with four momenta pμ = (em, p) and p′ μ = ((1 − ε)mDM, − p), respectively. Decrease in ρDMa3 after recombination due to decay of DDM is compensated by increase in ρA, which induces the earlier transition from the matter-dominated era to the dark energy dominated era. In the end, H(a) evolves to a larger H0 value than that resulting from CMB based on ΛCDM. As phenomenological consequences, enhancement of the late integrated Sachs-Wolfe (LISW) effect for low ℓ regime (ℓ ∼ 10) can be induced regarding CMB temperature anisotropy power spectrum. [43, 44]. Also, reduction of fσ8 in comparison with that from ΛCDM and suppression of the kinetic Sunyaev-Zel’dovich (kSZ) power spectrum were further discussed in [44] as impacts that DDM can make. (Here f is the matter growth rate, defined to be f ≡ d ln δm/d ln a).

In our model, we identify the Dirac fermion ΨDM = (ψ−1, χ−1)T generated from the decay of φ6 with DDM, the gauge boson of U(1)X with the resulting massive particle, and the massless Weyl fermion ξ with the resulting massless radiation. The dispersion relation of the resulting massive particle gives

$$\frac{m_{A'}}{m_{DM}} = \sqrt{1 - 2\epsilon}. \quad (14)$$

From the coupling of the Dirac fermion DM to the gauge boson, the decay rate of DDM can be read as

$$\Gamma_{DM\rightarrow A'\xi} = \frac{9}{4\pi} g^2 m_{A'} \left[ \left( \frac{m_{DM}}{m_{A'}} \right)^3 - 2 \frac{m_{A'}}{m_{DM}} + \frac{m_{DM}}{m_{A'}} \right]$$

$$\quad = \frac{9g^2 m_{A'}}{2\pi} \times \frac{\epsilon^2 (3 - 4\epsilon)}{(1 - 2\epsilon)^{3/2}}. \quad (15)$$

Applying 1σ level constraints $1.376 \times 10^{-43}$GeV ≲ $\Gamma_{DM\rightarrow A'\xi} \lesssim 1.04 \times 10^{-42}$GeV [10] and 0.0013 ≤ ε ≤ 0.229 [13] to Eq. (15), we obtain the constraints on (g, V1) which is shown as the yellow shaded region in Fig 1. The right panel is the magnification of the left panel for the relatively smaller coupling regime. The yellow shaded region between the yellow solid and dashed line is mapping onto (g, V1) space of constraints on the life of DDM and ε parameter in DDM solution to the Hubble tension [13] at 1σ level. (On the right panel, we showed the same mapping at 2σ level with the green solid line and green shaded region.) The blue shaded region corresponds to (g, V1) values that produce sub-keV decaying fermion DM as a solution to not only the Hubble tension, but the core-cusp problem, satisfying 1σ level constraints.

B. Constraints from the Core-Cusp Problem

Because of DM’s fermionic nature, provided that a thermal history of DM can allow for its degenerate configuration near the time of the structure formation, then

10 The constraint on DDM lifetime discussed in Refs. [45, 46] is not applied here because DDM only decaying into dark radiations is assumed there.

11 The lifetime of A’ can be longer than that of ΨDM depending on y2 and y3.
our DM candidate may form cored density profiles in Milky way’s dSphs satellites by balancing the attractive gravitational force with the repulsive Fermi quantum pressure. To see whether our DM can go through sufficient redshift for achieving the degenerate configuration, we need to estimate DM’s “would-be” temperature today $T_{\text{DM},0}$, in the absence of structure formation and compare it to a degeneracy temperature, $T_{\text{DEG}}$, for dSphs to check if $T_{\text{DM},0}$ is less than $T_{\text{DEG}}$ of dSphs [34]. Starting with $T_{\text{DM}}(a_{\text{FS}}) \approx m_6/2$, we obtain $T_{\text{DM}}(a_{\text{NR}}) \approx (m_6 a_{\text{FS}})/(2 a_{\text{NR}}) \approx m_D$ where $a_{\text{NR}}$ is the scale factor when DM becomes non-relativistic. Combined with Eq. (13), this yields

$$a_{\text{NR}} = 4.4 \times 10^{-10} \times g_{\text{SM}}(a_{\text{FS}})^{-1/12} \times \left( \frac{V_6}{1 \text{GeV}} \right) \left( \frac{m_{\text{DM}}}{1 \text{KeV}} \right)^{-2} \left( \frac{m_6}{1 \text{GeV}} \right)^{1/2}.$$ (16)

Since $a \approx a_{\text{NR}}$, $T_{\text{DM}}$ scales as $a^{-2}$, implying that

$$T_{\text{DM},0} \approx \frac{m_6 a_{\text{FS}}}{2 a_{\text{NR}}} \left( \frac{a_{\text{NR}}}{a_0} \right)^2,$$ (17)

where $a_0 = 1$ is the scale factor today. Below we will see values of $V_1$ obtained from the free-streaming length constraint lie in $V_6 \approx O(10) - O(100) \text{GeV}$. Using Eq. (16) and Eq. (17) with $V_6 \approx O(10) - O(100) \text{GeV}$, we estimate “would-be” temperature today of sub-keV DM to obtain $O(10^{-14}) - O(10^{-11}) \text{eV}$. This corresponds to $O(10^{-10}) \text{K} - O(10^{-6}) \text{K}$ which is lower than the degeneracy temperature for dSphs $T_{\text{DEG}} \approx O(10^{-4}) \text{K} - O(10^{-3}) \text{K}$ [34]. Hence, it is confirmed that our DM is capable of achieving the degenerate configuration today when the heat-up due to the virialization is neglected.

As such, the mass of our DM is subject to Tremaine-Gunn bound [47] which arises due to presence of maximum phase space density of the degenerate Fermi gas in a galaxy. When applied to dSphs, for a generic fermion DM, the logic underlying the Tremaine-Gunn bound gives $m_{\text{DM,min}} \approx 70 - 300 \text{ eV}$ as a lower bound on mass [36, 48]. Also, recently, a fitting analysis for the stellar kinematics of the relatively smaller dSphs (Leo II, Willman I, Segue I) set a conservative mass lower bound $\sim 100 \text{ eV}$ [49]. In the light of these results, we focus on the fermion DM mass greater than $\sim 100 \text{ eV}$ in our model. Combined with constraint on $\epsilon$ parameter in DDM solution to the Hubble tension, Eq. (14) tells us that $m_{A'}$ is at least 70% of $m_{\text{DM}}$. Thus, we may approximate $m_{\text{DM}} \approx m_{A'} = g V_1$ and apply $m_{\text{DM}} > 100 \text{ eV}$ to $(g, V_1)$ plane to improve constraints obtained in Sec III A. In Fig 1, $m_{\text{DM}} = 100 \text{ eV}$ is represented as the blue solid line. Thereby for the right panel the allowed parameter space at $1\sigma$ (2$\sigma$) level reduces to the sub-area of the yellow (green) colored region which intersects with the region above the blue solid line.

Next, in order for the degenerate fermion DM to serve as a solution to the core-cusp problem, it cannot have too large a mass because the core-size of the fermion DM density profile tends to decrease when $m_{\text{DM}}$ increases. In this context, there have been efforts to constrain fermion DM mass by using the kinematic of dSphs. In [34], the use of fermion DM density profile obtained by solving Lane-Emden equation achieves a good fit to the stellar velocity dispersion of eight classical dwarf galaxies especially for the DM mass regime $100-200 \text{ eV}$ [13]. In addition, in [36], the use of the core-size of Fornax dSphs gives $70 \text{ eV} \leq m_{\text{DM}} \leq 400 \text{ eV}$ under the assumption of quasi-degenerate Fermi gas as DM. In contrast, relatively large fermion DM mass near $1-2 \text{ keV}$ was pointed out in [33] by requiring the mass of dSph Willman I to be greater than the minimum halo mass made up of the degenerate fermion DM. Given these arguments for $m_{\text{DM}}$, in this work we concentrate on sub-keV fermion DM mass regime, i.e. $m_{\text{DM}} < 1 \text{ keV}$. Similarly to the previous section, we apply $m_{\text{DM}} \lesssim 1 \text{ keV}$ to $(g, V_1)$ plane to get the blue dashed line and area below it in Fig 1. As the final intersection of several constraints at $1\sigma$ level discussed so far, the blue shaded region in Fig 1 is obtained, which allows our model to produce a degenerate decaying fermion DM resolving the Hubble tension and the core-cusp problem. [14]

Intriguingly, we observe that the blue shaded region (1$\sigma$) in Fig 1 can cover $V_1$ value up to $10^{12} \text{ GeV}$ for the gauge coupling as small as $g \approx 10^{-18}$. Besides, when 2$\sigma$ level constraints on the $(g, V_1)$ plane is considered, even $V_1$ value as large as $10^{13} \text{ GeV}$ is allowed. As we mentioned above, $V_1$ is assumed to be larger than the inflaton mass $m_I$ in the model. This implies that the model can be consistent with even the high scale inflation models such as chaotic inflation [50] or topological inflation [51] where the inflaton mass is $m_I \approx 10^{13} \text{ GeV}$.

### C. Constraints from the Free-Streaming Length

Here we discuss constraints on Yukawa coupling $y$, as well as $V_6(\approx m_6)$. As a decay product of a non-relativistic $\phi_6$, which was never in a thermal equilibrium with other species, our Dirac fermion DM ($\Psi_{\text{DM}}$) is expected to

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12 These small size galaxies are particularly important for studying possibility of a degenerate fermion DM and its mass since increase in DM temperature due to virialization is minimized. For large size galaxies, fermion DM would be present there in non-degenerate configurations, which makes inferring fermion DM mass from the stellar kinematics of the large size galaxies more difficult.

13 The Lane-Emden equation results from combining the continuity equation and the hydrostatic equilibrium equation with the Fermi pressure.

14 In the right panel, the intersection coming from constraints at 2$\sigma$ level is the subspace between the blue solid and dashed lines which overlaps the green shaded region above the green solid line.
follow delta-function like distribution in its momentum space, centered on $|\vec{p}|=m_6/2$ at $a=a_{\text{FS}}$. From then on, DM becomes non-relativistic around the time $a=a_{\text{NR}}$ given in Eq. (16). Bearing in mind $a_{\text{BBN}} \approx 10^{-10}$--$10^{-9}$, we realize that for sub-keV DM, $V_6(\approx m_6)$ around GeV scale or greater than that makes DM still relativistic at BBN era. DM may contribute to energy budget of the universe at BBN era as a radiation. Therefore, we apply $\Delta N_{\text{eff,BBN}} \lesssim 1$ [52] to DM energy density at BBN era to constrain $V_6$. Using Eq. (8) and Eq. (13) with

$$\rho_{\text{DM}}(a_{\text{BBN}}) = \sqrt{m_{6}^{2} + \left(\frac{m_{6}a_{\text{FS}}}{2a_{\text{BBN}}}\right)^{2}} Y_{\text{DM}} s_{\text{SM}}(a_{\text{BBN}}),$$

yields the parameter space in $(m_{\text{DM}},V_6)$ plane satisfying the condition $\Delta N_{\text{eff,BBN}} \lesssim 1$. In Fig 2, this corresponds to the region below the black dashed line.

Given constraint on a value of $V_6$ around $\mathcal{O}(10)$ -- $\mathcal{O}(100)$ GeV (the region between the black and green dashed lines in Fig 2), now it becomes clear that sub-keV DM is produced from $\phi_6$-decay as a relativistic particle. So the next question that should be further asked is whether DM in the model can be consistent with Lyman-$\alpha$ forest data or not. Should the free-streaming length ($\lambda_{\text{FS}}$) of DM is too large, the structure formation at the scale below $\lambda_{\text{FS}}$ would have been erased, likely to cause a disagreement with non-vanishing matter power spectrum at the large scales. On the contrary, if $\lambda_{\text{FS}}$ is too short (i.e. $\lambda_{\text{FS}}<0.1\text{Mpc}$), DM model tends to produce too many satellite galaxies of Milky and Andromeda as opposed to the observed number. Therefore, we require $0.3\text{Mpc}<\lambda_{\text{FS}}<0.5\text{Mpc}$ to further constrain $V_6$ [34, 53–55].

The free-streaming length is given by

$$\lambda_{\text{FS}}=\int_{t_{\text{FS}}}^{t_{0}}\frac{v(t)}{a(t)}dt$$

where $v(t)$ is the DM velocity, and $\Omega_{\text{rad,0}}$, $\Omega_{m,0}$ denote the radiation and matter density parameter, respectively. The dependence of $V_6$ appears through $a_{\text{FS}}$ in Eq. (13). We show the result of mapping $0.3\text{Mpc}<\lambda_{\text{FS}}<0.5\text{Mpc}$ to $(m_{\text{DM}},V_6)$ plane in Fig 2 as the space between the two red solid and dotted red lines. The two red lines correspond to each of $\lambda_{\text{FS}}=0.5\text{Mpc}$ and $0.3\text{Mpc}$. The eventual intersection in $(m_{\text{DM}},V_6)$ plane reflecting four constraints discussed so far is shown as the red shaded region in Fig 2. We found that for the mass regime as low as $100$ -- $300$ eV, the non-thermally produced DM in our model travels the larger $\lambda_{\text{FS}}$ than $0.5\text{Mpc}$ while there exists a consistent parameter space for $300$--$1000\text{eV}$. The strength of the Yukawa $y_{\phi}$ associated with this $V_6$ regime reads $10^{-8}$--$10^{-7}$. This may appear to be in tension with [34, 36]. However, note that to resolve the core-cusp problem with $100\text{eV} \lesssim m_{\text{DM}} \lesssim 400\text{eV}$, either of a fully or quasi degenerate fermion DM configuration was assumed in [34, 36] in the absence of considering a baryonic effect in classical dSphs. As was pointed out in [36], when additional effects such as baryonic feedback or non-trivial DM momentum distribution skewed to lower energies are taken into account together with Fermi repulsion, the upper bound of $m_{\text{DM}}$ can be alleviated. Additionally, in our work, we expect actual momentum distribution of our DM on production to have a broader width due to non-zero velocity dispersion of DM when produced and a center smaller than $m_6/2$ although we approximated it as a delta-function like peak with the assumption of instantaneous decay of $\phi_6$. This consideration may help some part of $100\text{eV} \lesssim m_{\text{DM}} \lesssim 300\text{eV}$ in Fig 2 produce the smaller $\lambda_{\text{FS}}$ than we obtained in the present analysis.

IV. DISCUSSION

In this letter, we present a particle physics model which describes a completely isolated dark sector lacking any way to communicate with the SM sector. The dark sector enjoys an Abelian gauge symmetry $U(1)_{X}$ which meets the anomaly free conditions solely due to fields in the dark sector. We assume $U(1)_{X}$ without the kinetic mixing with $U(1)_{\text{EM}}$ in the SM. This set-up trivially does not affect any known experimental result in the SM. With the purpose to keep the model minimal and consistent with the decaying DM solution to the Hubble tension, we introduced two darkly charged complex scalars and five darkly charged Weyl fermions along with the $U(1)_{X}$ gauge boson. Starting from an inflation model with inflaton mass smaller than $U(1)_{X}$ breaking scale, we showed that one linear combination of two of five Weyl fermions with sub-keV mass can play a role of DM which starts to decay after recombination with the lifetime $\sim 35\text{Gyr}$. Within the parameter space of the model we found, the
redshift that DM candidate experiences since its production from decay of a complex scalar makes “would-be” temperature of DM today without virialization low enough to form a cored density profile for Milky way’s dwarf spheroidal satellites.

As such, DM candidate in our model is expected to serve as a solution to both the Hubble tension and the core-cusp problem. Originated from the dark sector lacking coupling to the SM sector, our DM candidate interacts with the SM particles only via gravitational interaction. We found that for resolving the Hubble tension, DM needs to decay via the gauge interaction with the coupling as tiny as $10^{-15}$. For sub-keV DM mass regime required to solve the core-cusp problem, this coupling strength is still larger than the strength of gravitational interaction that DM does, i.e., $\sim m_{DM}/M_P \approx 10^{-24}$. As for the sub-keV DM mass regime, we found that DM mass allowed in the model is in a slight tension with what is required for a degenerate fermion DM solution to the core-cusp problem, which we believe can be alleviated by a more detailed analysis about the phase-space distribution of DM and the use of unknown baryonic effects on dSphs galaxy formation.

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