Energy and Centrality Dependent Study of Deconfinement Phase Transition in a Color String Percolation Approach at RHIC Energies

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We take the experimental data for transverse momentum spectra of identified charged hadrons in different centrality classes for nucleus-nucleus (A+A) collisions at various Relativistic Heavy-Ion Collider (RHIC) energies measured by the STAR collaboration. We analyse these data in the framework of color string percolation model (CSPM) in order to extract various percolation parameters at different centralities at RHIC energies to study the effect of collision geometry and collision energy. We use these parameters to study the centrality dependent behaviour of initial temperature of the percolation cluster, energy density, average transverse momentum, shear viscosity to entropy density ratio (\(\eta/s\)) and trace anomaly for different energies at RHIC from \(\sqrt{s_{NN}} = 19.6\) to 200 GeV. These observables are found to strongly depend on centrality at various collision energies. The critical percolation density, which is related to the deconfinement phase transition is achieved in the most central nucleus-nucleus collisions, while it fails in the peripheral collisions. A universal scaling is observed in color suppression factor and initial temperatures when studied as a function of charged particle pseudorapidity distribution scaled by nuclear overlap area at RHIC energies. The minimum of \(\eta/s\) is observed in the most central collisions at \(\sqrt{s_{NN}} = 130\) and 200 GeV.

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I. INTRODUCTION

The color string percolation model (CSPM) describes the initial collision of two heavy ions in terms of color strings stretched between the projectile and target. Color strings may be viewed as small discs in the transverse space filled with the color field created by colliding partons [1, 2]. Particles are produced by the Schwinger mechanism, emitting \(q\bar{q}\) pairs in this field [3]. With the growing energy and size of the colliding nuclei the number of strings grow and start to overlap to form clusters in the transverse plane. This procedure is very much like discs in the 2-dimensional percolation theory, a non-thermal second order phase transition [4]. At a certain critical string density a macroscopic cluster appears, which defines the percolation phase transition [5]. In CSPM the Schwinger barrier penetration mechanism for particle production and the fluctuations in the associated string tension due to the strong string interactions make it possible to define temperature. The critical density of percolation is related to the effective critical temperature and thus percolation may provide the information of the deconfinement in the heavy-ion collisions [6]. The CSPM has been successfully used to describe the initial stages in the soft region of the high energy heavy-ion collisions [5].

The aim of this paper is to study the centrality dependence of percolation parameters at various RHIC energies ranging from \(\sqrt{s_{NN}} = 19.6\) to 200 GeV using CSPM. We use these parameters to calculate the centrality dependence of thermodynamical observables such as initial temperature of the percolation cluster, energy density, average transverse momentum, shear viscosity to entropy density ratio, and trace anomaly at RHIC energies. The paper runs as follows. First, we present the formulation and methodology in section II. In section III, we present the results and discussions. Finally, the findings of this work are summarized in section IV.

II. FORMULATION AND METHODOLOGY

In CSPM, the hadron multiplicity \((\mu_n)\) reduces as the interactions of strings increase and the mean transverse momentum squared, \(\langle p_T^2 \rangle_n\) of these hadrons increases, to conserve the total transverse momentum. The hadron multiplicity and \(\langle p_T^2 \rangle_n\) are directly related to the field strength of the color sources and thus to the generating color. For a cluster of \(n\) individual strings, we have [4]

\[
n = \frac{\mu_n \langle p_T^2 \rangle_n}{\mu_0 \langle p_T^2 \rangle_1},
\]

where \(\mu_0\) and \(\langle p_T^2 \rangle_1\) are the multiplicity of hadrons and the mean transverse momentum squared of a single string, respectively.

The multiplicity, \(\mu_n\) and the mean transverse momentum squared \(\langle p_T^2 \rangle_n\) of the particles produced by a cluster of \(n\) strings is defined as [5]:

\[
\mu_n = \sqrt{n S_n / S_1} \mu_0,
\]

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\[ \langle p_T^2 \rangle_n = \sqrt{\frac{n S_1}{S_n}} \langle p_T^2 \rangle_1, \]  
\[ \xi = \frac{N_S S_1}{S_n}, \]  
where \( S_1 \) is the transverse area of a single string, \( S_n \) is the transverse area covered by \( n \) strings. The percolation density parameter \( (\xi) \) can be written as:

\[ \xi = \frac{N_S S_1}{S_n}, \]  
where \( N_S \) is the number of strings formed in the nuclear collisions.

\( \xi \) is evaluated by using the parameterisation of pp collisions at \( \sqrt{s} = 200 \text{ GeV} \) as it was performed in our earlier work [7]. The \( p_T \) spectrum of charged particles can be described by a power law:

\[ \frac{d^2 N_{ch}}{dp_T^2} = \frac{a}{(p_0 + p_T)^\alpha}, \]  
where \( a \) is the normalisation factor and \( p_0, \alpha \) are fitting parameters given as, \( p_0 = 1.982 \) and \( \alpha = 12.877 \) [8]. Due to the low string overlap probability, the fit parameters obtained from pp collisions are then used for \( \text{Au+Au} \) collisions in order to evaluate the interactions of the strings.

In order to include the interactions of strings in nucleus-nucleus collisions, the parameter \( p_0 \) is modified as [5],

\[ p_0 \rightarrow p_0 \left( \frac{\langle n S_1/S_n \rangle_{\text{Au+Au}}}{\langle n S_1/S_n \rangle_{\text{pp}}} \right)^{1/4}. \]

Using thermodynamic limit, i.e. \( n \) and \( S_n \rightarrow \infty \) and keeping \( \xi \) fixed, we get

\[ \langle n S_1/S_n \rangle = \frac{1}{F^2(\xi)}. \]

where \( F(\xi) \) is the color suppression factor which reduces the hadron multiplicity from \( n \mu_0 \) to the interacting string value, \( \mu \) as,

\[ \mu = F(\xi)n\mu_0, \]

where,

\[ F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}. \]

Using eqs. 5 and 7, we calculate for A+A collisions as,

\[ \frac{d^2 N_{ch}}{dp_T^2} = \frac{a}{(p_0 \sqrt{F(\xi)_{pp}/F(\xi)_{\text{Au+Au}} + p_T)^\alpha}}. \]

Here \( F(\xi)_{\text{Au+Au}} \) is the centrality dependent color suppression factor and \( F(\xi)_{pp} \sim 1 \) at low energies due to the low overlap probability.

The initial temperature of the percolation cluster, \( T(\xi) \) can be represented in terms of \( F(\xi) \) as [5]:

\[ T(\xi) = \sqrt{\langle p_T^2 \rangle_1/2F(\xi)}. \]

The mean transverse momentum squared of a single string, \( \langle p_T^2 \rangle_1 \) is calculated using eq. 11 at critical temperature, \( T_c = 167.7 \pm 2.76 \text{ MeV} \) [9] and \( \xi = 1.2 \). We get \( \sqrt{\langle p_T^2 \rangle_1} = 207.2 \pm 3.3 \text{ MeV} \) [5], which is \( \approx 200 \text{ MeV} \), obtained in the previous calculation using percolation model [6]. The initial temperature of the percolation cluster is in reasonable agreement with the initial temperature estimated from the direct photon measurement by the PHENIX collaboration at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) [10].

### III. RESULTS AND DISCUSSIONS

In the present work, we have extracted \( \xi \) and \( F(\xi) \) at mid-rapidity \( (|y| < 0.5) \) for different centrality classes using the transverse momentum spectra of charged particles produced in \( \text{Au+Au} \) collisions at RHIC energies from \( \sqrt{s_{NN}} = 19.6 \) to 200 GeV [11–14].

Figure 1 shows the extracted percolation density parameter, \( \xi \) as a function of number of participants \( (N_{\text{part}}) \) in \( \text{Au+Au} \) collisions for \( \sqrt{s_{NN}} = 19.6 \) to 200 GeV. The proton to pion ratio is affected by baryon stopping at lower center-of-mass energies. We do not consider the results of percolation density parameter for \( \sqrt{s_{NN}} = 7.7 \) and 11.5 GeV, as for these energies the proton to pion ratios are much larger compared to higher collision energies.

The \( \xi \) values are obtained by fitting the experimentally measured transverse momentum spectra of charged particles. \( N_{\text{part}} \) values for different centrality classes are obtained by using Glauber model calculation [15]. The uncertainties in \( \xi \) are estimated by varying the fitting conditions. The associated systematic uncertainties in percolation density parameters are \( \sim 3\% \). We have propagated the uncertainties for other observables accordingly.

It is observed that \( \xi \) values are higher for higher center-of-mass energies and increase with \( N_{\text{part}} \) for a particular energy. This indicates that the string overlap is greater for central collisions. The horizontal line corresponds to the critical percolation density parameter, \( \xi_c \sim 1.2 \). The critical density for the onset of continuum percolation has been determined in numerical studies for a variety of different systems. In 2-dimensions the threshold for per-
It is observed that $\xi_s > 1.2$ for $N_{\text{part}} > 50$ at $\sqrt{s_{NN}} = 130$ and 200 GeV. Whereas, for the lower collision energies starting from $\sqrt{s_{NN}} = 19.6$ - 62.4 GeV, $\xi_s > 1.2$ for central collisions. This suggests that the percolation phase transition is expected for $N_{\text{part}} > 50$ at $\sqrt{s_{NN}} = 130$ and 200 GeV and for central collisions at lower energies. This is in conformity with the higher effective energy available for particle production in central collisions [19–22].

In figure 2, we show the extracted values of $F(\xi)$ as a function of pseudorapidity density of the charged particles ($dN_{ch}$/d$\eta$) for Au + Au collisions at RHIC energies from 19.6 to 200 GeV. $dN_{ch}$/d$\eta$ values for different centralities are taken from experimental data [11, 12]. $F(\xi)$ decreases with increasing collision energies as well as centralities. The decreasing trend of $F(\xi)$ is due to the production of high string density, which reveals more suppression of color charges in comparison to lower collision energies and centralities. $F(\xi)$ in heavy-ion collisions is presented as a function of $dN_{ch}$/d$\eta$ scaled with the transverse overlap area $S_N$, which are estimated by using the Glauber model [11] along with the results in high-multiplicity pp collisions at $\sqrt{s} = 1.8$ TeV from FNAL (Fermi National Accelerator Laboratory) E735 experiment [23] in figure 3. It is observed that $F(\xi)$ falls onto a universal scaling curve for hadron-hadron and nucleus-nucleus collisions. Particularly, in the most central Au+Au collisions and high multiplicity pp collisions, $F(\xi)$ values fall in a line. This suggests that the percolation string densities are independent of collision energies and collision systems.

The initial temperature, $T(\xi)$ for different centrality classes is obtained from $F(\xi)$ by using eq. 11 for all the energies. Figure 4 shows $T(\xi)$ as a function of $N_{\text{part}}$. It is found that $T(\xi)$ increases with $N_{\text{part}}$ as well as with the centre-of-mass energy. Figure 5 represents the initial temperatures versus $dN_{ch}$/d$\eta$/S$_N$ for various collision energies and systems and we notice that it follows a universal curve for hadron-hadron and nucleus-nucleus collisions. The horizontal line at $T \sim 165$ MeV is obtained by comparing the hadron yields measured in different collision systems such as pp, A+A, and $e^+e^-$ with the statistical hadron gas model [9]. This temperature corresponds to the critical percolation density, $\xi_c$, where percolation phase transition takes place. The initial temperatures obtained in most and mid-central collisions at RHIC energies and in high multiplicity hadron-hadron collisions at $\sqrt{s} = 1.8$ TeV are above the hadronization temperature, which evokes the creation of deconfined matter in these collision energies and systems.

The mean transverse momentum $\langle p_T \rangle$ of pions as a function of initial temperature $T(\xi)$ is shown in figure 6. The results are presented for different centralities at RHIC energies. The available $\langle p_T \rangle$ values are taken from the experimental data [11, 12] and $T(\xi)$ are calculated using the CSPM as mentioned above. $\langle p_T \rangle$ values show a scaling behaviour as a function of the initial temperature for all the colliding energies. We notice that, $\langle p_T \rangle$ in the most central collisions of lower energies overlaps with that obtained in the most peripheral collisions of higher energies, which may hint the formation of similar systems in the peripheral collisions at higher energies as formed in the most central collisions at lower energies. Again, this supports the hypothesis of effective energy for particle production [19–22].

The initial energy density can also be calculated using CSPM for different centrality classes at RHIC ener-
FIG. 3: (Color online) Color Suppression factor $F(\xi)$, as a function of pseudorapidity density divided by the nuclear overlap function, $(dN_{ch}/d\eta)/S_N$. Different symbols represent different centre-of-mass energies along with high-multiplicity non-jet $p\bar{p}$ collisions at $\sqrt{s}=1.8$ TeV at Tevatron [23].

FIG. 4: (Color online) The initial temperatures at RHIC energies estimated in CSPM as a function of number of participants ($N_{part}$) for Au+Au collisions.

FIG. 5: (Color online) Temperature as a function of $(dN_{ch}/d\eta)/S_N$ from pp and Au + Au collisions. The horizontal line corresponds to $T \sim 165$ MeV, which is the universal hadronisation temperature [9].

FIG. 6: (Color online) Average momentum $\langle p_T \rangle$ of pions as a function of initial temperature, $T$. $\langle p_T \rangle$ are obtained from experimental data [11] and initial temperatures are estimated by using CSPM. Different symbols are for different centre-of-mass energies.

Assuming the boost invariant Bjorken hydrodynamics [24] with CSPM, the initial energy density is expressed as follows,

$$\varepsilon = \frac{3}{2} \frac{dN_{ch}}{dy} \langle m_T^2 \rangle \frac{2}{S_N \tau_{pro}},$$

(12)

where $S_N$ is the nuclear overlap area and $\tau_{pro}$, the production time for a boson (gluon), is described by [3, 7]

$$\tau_{pro} = \frac{2.405h}{\langle m_T \rangle}.$$  

(13)

Here, $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass. For evaluating $\varepsilon$, we use the charged pion multiplicity $dN_{ch}/dy$ at mid-rapidity and $S_N$ values for each centrality bin for Au+Au collisions at $\sqrt{s_{NN}} = 19.6 - 200$ GeV [11, 12].


Figure 7 shows the variation of the energy density, estimated by using eq. 12 with $\xi$ for different centrality classes at RHIC energies. It is found that $\varepsilon$ is proportional to $\xi$ for all the centrality bins. In our previous work [7], we observed that $\varepsilon$ is proportional to $\xi$. Similarly, here we fit the results with a linear function, $\varepsilon = A\times \xi$ (GeV$/f m^3$) for different centrality classes and extract the parameters. The parameter, $A = 0.786, 0.693, 0.654, 0.649$ (GeV$/f m^3$) for 0-10%, 10-20%, 20-30%, 30-40% centrality classes, respectively. It shows a decreasing trend from central to peripheral collisions as expected.

Figure 8 shows the initial temperature estimated by using CSPM at RHIC energies and the chemical freeze-out temperatures as a function of centre-of-mass energy. The chemical freeze-out temperatures are obtained by fitting the experimental yield of hadrons [12] for different centralities in Au+Au collisions at $\sqrt{s_{NN}} = 19.6$ - 200 GeV. We observe that the initial temperature increases with the collision energies for all the centralities while the chemical freeze-out temperatures remain constant within the uncertainties. Also, the initial temperatures are found close to the freeze-out temperatures at lower energies and the difference between them increases as we move towards the higher collision energies. A strong centrality dependence is found at higher energies in particular, at $\sqrt{s_{NN}} = 130$ and 200 GeV compared to the lower energies.

We first investigate the centrality dependence of shear viscosity to entropy density ratio ($\eta/s$) for various RHIC energies, which act as a measure of the fluidity of the matter formed in heavy-ion collisions. In the framework of the color string percolation model, $\eta/s$ is given by [7],

$$\eta/s \simeq \frac{T L}{5(1-e^{-\xi})},$$

where $T$ is the initial temperature and $L$ is the longitudinal extension of the string $\sim 1$ fm. We use eq. 14 to study the centrality dependence of $\eta/s$ as a function of $T$ as shown in the figure 9 for various RHIC energies, where higher $T$ corresponds to the most central collisions and decreases as we go towards peripheral collisions for all energies. The horizontal line shown in this figure is the conjectured lower bound for $\eta/s$ by Ads/CFT calculations [25]. It is perceived that, $\eta/s$ decreases as we go from peripheral to central collisions at all the collision energies and found to be minimum at most central collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV.

Figure 10 shows the centrality dependence of trace anomaly ($\Delta = (\varepsilon - 3P)/T^4$), which is the reciprocal of $\eta/s$ versus temperature for different RHIC energies. We also compare the CSPM calculations with the LQCD data [26, 27] and find that CSPM results are closer to the HOT QCD collaboration results. $\Delta$ increases with centrality for all the RHIC energies and has its maximum value for the most central collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV.

IV. SUMMARY AND OUTLOOK

We have presented the calculation of percolation parameters and various thermodynamical observables in nucleus - nucleus collisions at RHIC energies starting from $\sqrt{s_{NN}} = 19.6$ to 200 GeV as a function of collid-
gas to Quark-Gluon Plasma is described by the critical percolation density in CSPM. We have studied the percolation density parameter as a function of the number of participants at various RHIC energies. We find that, the critical percolation density ($\xi_c \sim 1.2$) is reached for the most central collisions at all the analysed collision energies. However, as we go towards peripheral collisions, the critical percolation density is difficult to achieve at lower collision energies.

- We have studied the color suppression factor ($F(\xi)$) in CSPM as a function of pseudorapidity density normalised by nuclear overlap area ($S_N$) at RHIC energies. A universal scaling behaviour is observed for all the collision energies. The results from hadron-hadron collisions at $\sqrt{s} = 1.8$ TeV strengthen the observation showing a similar behaviour as obtained in Au+Au collisions. This indicates that the percolation string densities are independent of collision systems and collision energies.

- A centrality dependence of initial temperatures is presented for RHIC energies and high multiplicity pp collisions at $\sqrt{s} = 1.8$ TeV. Initial temperatures increase with the number of participants for all the collision energies. We have also shown the initial temperature as a function of $dN_{ch}/d\eta$ scaled by $S_N$ and find a universal scaling behaviour both in nucleus-nucleus and hadron-hadron collisions. It is also observed that the initial temperatures obtained in central collisions at RHIC energies and in high multiplicity hadron-hadron collisions at $\sqrt{s} = 1.8$ TeV are above the hadronisation temperature, which advocates the possibility of the creation of deconfined matter in these collision energies and systems. This also justifies a deep look into the LHC pp high multiplicity events.

- We have presented the average transverse momentum ($\langle p_T \rangle$) of pions as a function of temperature for various centralities at RHIC energies. It is found that $\langle p_T \rangle$ in the most central collisions of lower energies overlap with that obtained in the most peripheral collisions of higher energies. This suggests that, the system formed in the peripheral collisions at higher energies is similar to that formed in the most central collisions at lower energies, justifying the hypothesis of effective energy.

- $\eta/s$ as a function of temperature for different centralities are studied in CSPM. A strong centrality dependence of $\eta/s$ is found at RHIC energies. The minimum $\eta/s$ is observed for the most central collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV, which envisages the formation of perfect fluid at these energies.
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