Analyses of different methodologies employed for correcting systematic effect

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Abstract. Correcting indications, or compensating for systematic errors, is an indispensable factor in ensuring the reliability of measurement results in routine calibration laboratories. Different methodologies for the application of this correction can be adopted, since the guide for the expression of uncertainty in measurement ISO GUM is not sufficiently clear about whether: a) the correction procedure should be executed on each measure indication individually, or b) the correction procedure should be applied to the average of the measure indications. The main objective of the present work is to discuss and analyze the different methodologies that can be applied to the correction of indications through simulations and the various ways for defining the systematic effect itself. The expected result is to show that Type A uncertainty estimation depends directly on the method utilized for correcting indications. The main consequence is that measurement results always depend on the correction method chosen.

1. Introduction
Metrology is a multidisciplinary science that covers several areas of knowledge since it is not possible to monitor, control or even investigate [1] the effect of a given measurement without taking measurements. The International Vocabulary of Metrology [2] states that measurement is the process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity. Thus, because it is an experimental process, there are a number of factors that can be considered when performing a measurement.

ISO GUM [3] states that although systematic error, like random error, cannot be totally eliminated, it can often be reduced. If a systematic error arises from a recognized effect of an influence’s quantity on the result of a measurand, hereafter termed a systematic effect, the effect can be quantified and, if it is significant in size relative to the required accuracy of the measurement, a correction or correction factor can be applied to compensate for this effect. It is assumed that, after correction, the expectation or expected value of the error arising from a systematic effect is zero.

1.1. Measurement uncertainty
An uncertainty is a “non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used” [2]. It is necessary to evaluate the uncertainties related to frequency distribution and probability-related experience or to other
information to carry out an evaluation correctly. Measurement uncertainty evaluation consists of Type A and Type B uncertainty.

The components related to the evaluation of Type A uncertainty are generally those that most impact a measurement result, since they evaluate the variability of the measurements, both of the measurand, and the standard. There is a method for possible evaluation of the effect of the contribution of the random error, characterized by the standard deviation, according to equation 1 [3].

\[ \mu = s_A/\sqrt{n} \]  

Where:
- \( \mu \) corresponds to Type A uncertainty;
- \( s_A \) corresponds to the sample standard deviation;
- \( n \) is the number of observations.

The standard deviation is the positive square root of the variance, as shown in equation 2.

\[ S_A = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{X})^2} \]  

Where:
- \( s_A \) corresponds to the sample standard deviation;
- \( x_i \) corresponds to the indication;
- \( \bar{X} \) corresponds to the mean of the measurand or standard;
- \( n \) is the number of observations.

It is necessary to define measurement error and measurement uncertainty as different concepts. However, in the present work, the analysis concerns how much the method of correction of the systematic effect, known as a trend in the cases of finite measurements, can change the value of the estimated Type A uncertainty.

2. Methodology

At the present work was applied the methodology for defining systematic effect (bias) and the methodology for systematic effect correction.

2.1. Methodology for defining systematic effect (bias)

Bias can be defined by three different methods: through instrumental bias; through linear regression; and through interpolation with data of a calibration certificate [4].

Table 1 shows an example of data presented in the calibration certificate of a temperature sensor.

| Standard (°C) | Measurand (°C) | Bias (°C) | Measurement uncertainty (°C) | k |
|--------------|----------------|-----------|-----------------------------|---|
| 20.000       | 20.01          | 0.010     | 0.05                        | 2.10 |
| 40.000       | 40.07          | 0.070     | 0.05                        | 2.10 |
| 100.000      | 100.20         | 0.200     | 0.05                        | 2.10 |

In Method 1, the instrumental bias method, the value used is the one closest to the read value identified in the calibration certificate of the measuring instrument. For example, if the value indicated by the instrument was 20.08 °C, according to Table 1, the value set for the systematic effect would be +0.010 °C.

In Method 2, the linear regression method, it is necessary to determine the equation that correlates the value read with its respective trend, considering that the behavior of the instrument is linear. Figure 1 shows the linear correlation of the worked example.
In this method, to define the bias value, it is necessary to replace the value of $x$ in equation 3 with the value read by the instrument.

$$y = 0.0023x - 0.0307$$

Where:
- $y$ corresponds to the estimated value of bias;
- $x$ corresponds to the indication of the measurand.

For example, if the value indicated by the instrument was 20.08 °C, according to equation 3, the value set for the systematic effect would be approximately 0.015 °C. This method produces an uncertainty because the value calculated by the equation is different from the actual value entered for generating it, known as the uncertainty of the regression adjustment.

In Method 3 - interpolation method, the instrumental bias is determined from the consideration that it varies linearly between the calibration points neighboring the reading, according to equation 4:

$$T = \frac{L_2 - l_2}{L_1 - l_1} (T_2 - T_1) + T_1$$

Where:
- $T$ corresponds to the estimated value of instrumental bias;
- $L$ is the indication of the measurand;
- $T_1$ corresponds to the bias value in the calibration certificate of the point below the read value;
- $T_2$ corresponds to the trend value in the calibration certificate of the point above the read value;
- $L_1$ corresponds to the value of the measurand in the calibration certificate of the point below the read value;
- $L_2$ corresponds to the value of the measurand in the calibration certificate of the point above the read value.

For example, if the value indicated by the instrument was 20.08 °C, according to equation 4, the value set for the systematic effect would be $+ 0.010209$ °C, approximately $0.010$ °C.

### 2.2. Methodology for systematic effect correction

Given that the correction is compensation for the systematic effect, that is, the bias with the signal exchanged, the following methods have been defined for carrying out the present work: individually for each reading presented by the measuring instrument and applied only to the averages of the measurements.
In Method A - systematic effect correction is performed for each reading, before determining the sample standard deviation, according to equation 5:

$$L_c = (L_i - T_i)$$  \hspace{1cm} (5)

Where:
- $L_c$ corresponds to the corrected indication of the individual measurand;
- $L_i$ corresponds to each indication of the measurand;
- $T_i$ corresponds to the estimated value of the instrumental bias obtained using the methodology to define the systematic effect individually.

In Method B - a correction of the systematic effect is made in the mean value of the measurements, after determining the sample standard deviation, utilizing equation 6:

$$ar{X}_c = (\bar{X} - T)$$  \hspace{1cm} (6)

Where:
- $\bar{X}_c$ corresponds to the corrected mean of the indications of the measurand;
- $\bar{X}$ corresponds to the average of the indications of the measurand;
- $T$ corresponds to the estimated value of the instrumental bias obtained through the systematic effect definition methodology.

3. Results

Figure 2 presents a hypothetical chart of "bias $T_i$ versus readings $X_i$" obtained through the calibration of a specific measurement instrument. In the same figure the point adjustment line (black line) is also presented that is calculated through the use of the least squares method.

![Figure 2 - Linear regression of the bias.](image)

In order to find the value of the bias $T_w$ corresponding to the reading $X_w$, such that $X_2 < X_w < X_3$, there are three different methodologies for reaching this objective, as previously described in the present work.

In Method 1, the bias value ($T_2$ or $T_3$) corresponding to the closest reading to $X_w$ ($X_2$ or $X_3$) would be adopted from the calibration certificate of the instrument. Adopting this methodology, the quality of the $T_{w1}$ bias value will depend on how close the $X_w$ reading is to $X_2$ or $X_3$. Considering that the $X_w$ value is not chosen, but obtained through measurement, this means that this method presents a considerable level of unpredictability in the quality of the result obtained.

Through Method 2, the bias value adopted would be the value $T_{w2}$ (shown in the figure) obtained through the equation of the adjustment line.

In Method 3, the bias value is obtained by linear interpolation from the points ($X_2$, $T_2$) and ($X_3$, $T_3$), i.e. the point ($X_w$, $T_w$) would be situated somewhere on the line (red line) connecting the points ($X_2$, $T_2$) and ($X_3$, $T_3$).
, (X₃, T₃). It is possible to infer that in the single case in which the adjustment line used in Method 2 coincides with the line connecting the points (X₂, T₂) and (X₃, T₃), the result of bias Tₚ obtained will be the same through the two methods. In any other case, where these lines do not coincide, linear interpolation always provides a Tₚ result more coherent with the initial hypothesis that can be assumed as linear the behavior of the function between (X₂, T₂) and (X₃, T₃). Thus, for instruments of predominantly linear behavior, Method 3 is recommended among the 3 presented, since it does not give the level of unpredictability present in Method 1 and, in addition, provides the most coherent trend value, considering the initial hypothesis of function linearization and the points presented in the instrument calibration certificate.

Simulations were carried out to obtain uncertainty (Type A) values by utilizing data from the calibration certificate presented in Table 1 as well as data from Figure 1. Methods A and B (see section 2.2), for systematic effect correction, were carried out in investigating the three methodologies (methods 1, 2 and 3) for calculating x systematic effect (bias). The results are presented in Tables 2 and 3.

**Table 2.** Simulation results obtained with Method A for the correction of systematic effect. Methods 1, 2 and 3, for calculating systematic effect (bias) were tested.

| Method A | Method 1 | Method 2 | Method 3 |
|----------|----------|----------|----------|
| Nominal Value (°C) | Measure (°C) | Uncertainty Type A (°C) | Measure (°C) | Uncertainty Type A (°C) | Measure (°C) | Uncertainty Type A (°C) |
| 40 | 42.2633333 | 0.3333333 | 42.2666667 | **0.3325667** | 42.2565636 | 0.3323363 |
| 60 | 67.2633333 | 0.3333333 | 67.2091667 | **0.3325667** | 67.2043905 | 0.3326127 |
| 80 | 86.4666667 | 0.8819171 | 86.4980333 | **0.8798887** | 86.4959255 | 0.8800104 |

**Table 3.** Simulation results obtained with Method B for the correction of systematic effect. Methods 1, 2 and 3, for calculating systematic effect (bias) were tested.

| Method B | Method 1 | Method 2 | Method 3 |
|----------|----------|----------|----------|
| Nominal Value (°C) | Measure (°C) | Uncertainty Type A (°C) | Measure (°C) | Uncertainty Type A (°C) | Measure (°C) | Uncertainty Type A (°C) |
| 40 | 42.2633333 | 0.3333333 | 42.2666667 | 0.3333333 | 42.2565636 | 0.3333333 |
| 60 | 67.2633333 | 0.3333333 | 67.2091667 | 0.3333333 | 67.2043905 | 0.3333333 |
| 80 | 86.4666667 | 0.8819171 | 86.4980333 | 0.8819171 | 86.4959255 | 0.8819171 |

Based on Tables 2 and 3, it can be concluded that Method 1, is indifferent for correcting mean value or values individually, since this does not generate change in the uncertainty of Type A. In Methods 2 and 3, correcting the values individually generates a contribution to Type A uncertainty smaller than correcting it in the mean value. Considering that the bias behavior of the simulated instrument is linear in most of the simulated results, it can be observed that Method 2 associated with Method A (in bold on the table) generated smaller contributions to Type A uncertainty when compared to the other method combinations. It should be noted that Method 2 also adds an uncertainty due to the approximation of the correction by a regression, which is not being considered in this paper, nor is it being taken into account some additional uncertainty contribution of Method 3.
4. Conclusion

The choice of a reading correction method should be based on prioritizing the practicality, and ease of operation, that it provides the user. However, the results presented may influence this choice, since reducing uncertainty is often the main objective. From this perspective, it can be concluded that using reading correction Method A (correcting the values individually before the statistical treatment) gives slightly lower uncertainty values in methods 2 and 3.

Method 1, on the other hand, has the advantage of being quite simple and direct, and it does not influence a decision to make corrections by Method A or B, since the uncertainty of this method is the same. However, it is not indicated when the measured value is too far from the points where performance is known. The results show that, regardless of the correction method employed (1, 2 or 3), the uncertainty values of Type A do not change, and only the change in the mean value is corrected. This value, in many cases, is the central value of the measurement result, which leads us to a more detailed analysis on which of the methods presents results that we can consider "more correct".

Furthermore, Method 2 (of linear regression) is probably the proper choice when a large volume of data is required and, combined with individual reading corrections (correction Method A), has generated results with the lowest Type A uncertainties. Future work is recommended to analyze the contribution of regression uncertainty, which is aggregated by this method, in expanded uncertainty.

On the other hand, through the use of individual reading correction (Method A) by interpolation (Method 3) we also obtained uncertainty values due to the dispersion of values significantly better than in Method 1, which means that the option for this correction strategy should be considered, for example in cases where the volume of data processed is lower, but especially in the case of instruments where the number of calibrated points is low, which leads to generally bad regression equations.

During the course of this work we observed several nuances that should be considered, such as the effect of instrument linearity in the choice of a correction strategy, the impact of the corrections on expanded uncertainty (not only in Type A), as well as methodology standardization for estimating the uncertainty arising from interpolation, among others. However, as these points distance us from the main objective of this paper, which is to compare the methods of correction A and B, we leave these observations as a suggestion for future work.

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