THE DISK-MAGNETOSPHERE INTERACTION IN THE ACCRETION-POWERED MILLISECOND PULSAR SAX J1808.4–3658

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ABSTRACT

The recent discovery of the first known accretion-powered millisecond pulsar with the Rossi X-Ray Timing Explorer provides the first direct probe of the interaction of an accretion disk with the magnetic field of a weakly magnetic \((B \lesssim 10^{10} \, \text{G})\) neutron star. We demonstrate that the presence of coherent pulsations from a weakly magnetic neutron star over a wide range of accretion rates places strong constraints on models of the disk-magnetosphere interaction. We argue that the simple \(M^{3/7}\) scaling law for the Keplerian frequency at the magnetic interaction radius, widely used to model disk accretion onto magnetic stars, is not consistent with observations of SAX J1808.4–3658 for most proposed equations of state for stable neutron stars. We show that the usually neglected effects of multipole magnetic moments, radiation drag forces, and general relativity must be considered when modeling such weakly magnetic systems. Using only very general assumptions, we obtain a robust estimate of \(\mu \approx (1-10) \times 10^{26} \, \text{G cm}^3\) for the dipole magnetic moment of SAX J1808.4–3658, implying a surface dipole field of \(\sim 10^9–10^{10} \, \text{G}\) at the stellar equator. We therefore infer that after the end of its accretion phase, this source will become a normal millisecond radio pulsar. Finally, we compare the physical properties of this pulsar with those of the nonpulsing, weakly magnetic neutron stars in low-mass X-ray binaries and argue that the absence of coherent pulsations from the latter does not necessarily imply that these neutron stars have significantly different magnetic field strengths from SAX J1808.4–3658.

Subject headings: accretion, accretion disks — pulsars: individual (SAX J1808.4–3658) — stars: neutron — X-rays: stars

1. INTRODUCTION

The basic framework for understanding accretion-powered pulsars emerged soon after their discovery (Giacconi et al. 1971). These systems are rotating neutron stars accreting matter from a binary companion, with magnetic fields strong enough to disrupt the accretion flow above the stellar surface (Pringle & Rees 1972; Davidson & Ostriker 1973; Lamb, Pethick, & Pines 1973). When threaded by the stellar magnetic field, the accreting gas is brought into corotation with the star and is channeled along field lines to the polar caps, releasing its potential and kinetic energy mostly in X-rays. The rotation of these hot spots through our line of sight produces X-ray pulses at the spin frequency of the neutron star. Most of the \(\approx 50\) accretion-powered pulsars have spin frequencies and inferred dipole magnetic field strengths in the range \(\sim 1-10^3 \, \text{s}\) and \(\sim 10^{11}–10^{13} \, \text{G}\), respectively (White, Nagase, & Parmar 1995).

In strongly magnetic \((B \gtrsim 10^{11} \, \text{G})\) accreting neutron stars, models for the disk-magnetosphere interaction based on this framework (see Ghosh & Lamb 1991 for a review) are generally consistent with the accretion torque behavior of the Be/X-ray pulsar transients, as observed by the Compton Gamma-Ray Observatory/BATSE all-sky monitor (see Bildsten et al. 1997 and references therein). The Be/X-ray pulsar transients allow for a direct test of the predicted scaling of torque with accretion rate, since they span a wide range of accretion rates during their outbursts. However, the same models can only account for the bimodal torque reversals observed in several persistent accreting pulsars if additional assumptions are introduced (Chakrabarty et al. 1997a, 1997b; Bildsten et al. 1997). Examples of suitable assumptions are a bimodal distribution of the mass transfer rate onto the pulsar, or a bimodal dependence on accretion rate of the orientation of the disk (Nelson et al. 1998; van Kerkwijk et al. 1998), of the orbital angular velocity of the accreting gas (Yi & Wheeler 1998), or even of the strength and orientation of any magnetic field produced in the disk (Torkelsson 1998).

In weakly magnetic \((B \lesssim 10^{10} \, \text{G})\) accreting neutron stars, models of the disk-magnetosphere interaction have only been tested indirectly so far. Most neutron stars in low-mass X-ray binaries (LMXBs) show no periodic oscillations in their persistent emission. They have dipole magnetic fields \(\lesssim 10^{10} \, \text{G}\), as inferred from their bursting (Lewin, van Paradijs, & Taam 1995) and rapid variability behavior (see van der Klis 1998 for a review) as well as from their X-ray spectra (Psaltis & Lamb 1998). Their power density spectra show various types of quasi-periodic oscillations (QPOs), and in particular the most luminous of the neutron star LMXBs show \(\sim 15–60 \, \text{Hz}\) QPOs. These are called horizontal-branch oscillations (HBOs) and have frequencies that increase with mass accretion rate (van der Klis et al. 1985; van der Klis 1989). They have been interpreted as occurring at the beat frequency between the Keplerian frequency at the magnetic interaction radius (where the stellar magnetic field strongly interacts with the accretion disk) and the neutron star spin frequency (Alpar & Shaham 1985; Lamb et al. 1985). Models of the inner accretion disk and
the disk-magnetosphere interaction can account for the observed scaling of HBO frequency with accretion rate if the neutron stars in these systems are near their magnetic spin equilibrium and if the inner accretion disks are radiation pressure dominated (Psaltis et al. 1999b).

The discovery of $\sim 200$–$1200$ Hz QPOs (often occurring in pairs, hereafter kHz QPOs; see van der Klis 1998 and references therein) in the X-ray flux of many neutron star LMXBs and the identification of the higher frequency QPO with a Keplerian orbital frequency in the accretion disk (van der Klis et al. 1996; Strohmayer et al. 1996; Miller, Lamb, & Psaltis 1998) has introduced additional complications in studying the disk-magnetosphere interaction using HBO observations (see also Psaltis, Belloni, & van der Klis 1999a). The magnetic interaction radius inferred from the HBO frequencies is larger than the disk radius responsible for the higher frequency kHz QPO. Therefore, if the higher frequency kHz QPO is a Keplerian orbital frequency in the disk, then the magnetospheric beat-frequency model for the HBO can be valid only if a nonnegligible fraction of the disk plasma is not threaded by the stellar magnetic field but remains in the disk plane inside the interaction radius (see Miller et al. 1998; Psaltis et al. 1999b for a discussion). This requirement has not yet been addressed in any theoretical model for the disk-magnetosphere interaction and therefore cannot be tested directly.

The recent discovery of the first weakly magnetic accretion-powered pulsar with the Rossi X-Ray Timing Explorer (RXTE) allows the first direct test of disk-magnetosphere interaction models in the weak-field limit. SAX J1808.4–3658 is a transient X-ray source that shows type I X-ray bursts (in ’t Zand et al. 1998) and coherent 40 Hz X-ray pulsations (Wijnands & van der Klis 1998a) and is a member of a low-mass binary in a 2 hour orbit (Chakrabarty & Morgan 1998). Its distance is $\approx 4$ kpc, as inferred from flat-topped type I X-ray bursts that are thought to be Eddington limited (in ’t Zand et al. 1998), and its luminosity varied by $\geq 2$ orders of magnitude during the 1998 April/May outburst (Cui, Morgan, & Titarchuk 1998).

In this paper we study the disk-magnetosphere interaction in SAX J1808.4–3658. In § 2 we compare the predictions of theoretical models for the inner accretion disk and the disk-magnetosphere interaction with observations of SAX J1808.4–3658, and in § 3 we infer the magnetic field strength of the pulsar. In § 4 we discuss our results and their implications for disk-magnetosphere interaction models and compare SAX J1808.4–3658 with the millisecond radio pulsars and the nonpulsing neutron stars in LMXBs.

### 2. Disk-Magnetosphere Interaction

#### 2.1. Assumptions and Formalism

Throughout this paper we assume that most of the accreting gas around SAX J1808.4–3658 is confined in a geometrically thin accretion disk before interacting with the pulsar magnetic field. This assumption is supported by the transient nature of the source, which can be understood in terms of dwarf nova–like accretion disk instabilities (see Chakrabarty & Morgan 1998). We also neglect the effect of wind mass loss from the inner accretion disk and of radiation drag forces, as well as all general relativistic effects.

The radius $r_0$ at which magnetic stresses remove the angular momentum of the disk flow and disrupt it can be estimated by balancing the magnetic and material stresses (Ghosh & Lamb 1978, 1979a),

$$\frac{B_p B_o}{4 \pi} 4 \pi r_0^2 \Delta r_0 = M \Omega r_0^2,$$

where $B_p$ and $B_o$ are the poloidal and toroidal components of the magnetic field, $\Delta r_0$ is the radial width of the interaction region, $M$ is the mass of the neutron star, and $\Omega(r)$ is the angular velocity of the gas at radius $r$. Assuming that the poloidal magnetic field is dipolar with magnetic moment $\mu$ and that the accretion flow is Keplerian, we obtain

$$r_0 = \gamma_B^2 \left(\frac{\mu^4}{G M M^2}\right)^{1/7},$$

where $\gamma_B \equiv (B_p/B_o)(\Delta r_0/r_0)$, $M$ is the mass of the neutron star, and $G$ is the gravitational constant.

The boundary layer parameter $\gamma_B$ in equation (2) depends in general on all the other physical quantities. Hence, different models for the inner accretion disk’s environment and the disk-magnetosphere interaction generally predict different coefficients and scalings (Ghosh & Lamb 1992; see also Ghosh & Lamb 1978, 1979a; Wang 1996; Ghosh 1996). However, if the disk-magnetosphere interaction takes place in a region of the accretion disk where all physical quantities have a power-law dependence on radius (as is the case for most accretion disk models away from the stellar surface) and equation (2) describes angular momentum balance in the interaction region, then the Keplerian orbital frequency at $r_0$ can be written as

$$v_0 \simeq v_{K,0} \left(\frac{M}{M_\odot}\right) \left(\frac{M_\odot}{10^{27} G \text{ cm}^3}\right)^{\beta} \left(\frac{M}{M_\odot}\right)^\alpha,$$

where $M_\odot$ is the solar mass; $M_\odot = 2 \times 10^{-8} M_\odot \text{ yr}^{-1}$ is the Eddington critical accretion rate (at which the outward radiation forces in a spherically symmetric hydrogen flow balance gravity); and $\alpha, \beta, \gamma,$ and $v_{K,0}$ are parameters which depend on the nature of the inner accretion disk. For our study, we used the parameters from four different inner disk models (see Table 1). We chose these models both because they span a wide range of physical conditions in the inner disk (one- and two-temperature plasmas and gas and radiation pressure–dominated flows) and a range of radiation processes, and because detailed calculations of the interaction with a stellar magnetic field have been performed for

| Disk Modelb | $v_{K,0}$ (Hz) | $\alpha$ | $\beta$ | $\gamma$ |
|-------------|----------------|----------|---------|---------|
| 1G          | 430            | 0.38     | -0.87   | 0.82    |
| 1R          | 210            | 0.23     | -0.77   | 0.70    |
| 2B          | 80             | 0.72     | -0.86   | 0.43    |
| 2S          | 50             | 2.55     | -1.20   | -0.60   |

*After Ghosh & Lamb 1992.
b1G: Optically thick, gas pressure–dominated (GPD) disk; 1R: Optically thick, radiation pressure–dominated disk; 2B: Two-temperature, optically thin GPD disk with Comptonized bremsstrahlung; 2S: Two-temperature, optically thin GPD disk with Comptonized soft photons (for references see Ghosh & Lamb 1992).
these models (see Table 1 and Ghosh & Lamb 1992 for a discussion of the physical processes involved in these models).

2.2. Limits from the Detection of Coherent Pulsations

We can use the presence of coherent X-ray pulsations in SAX J1808.4 – 3658 over a wide range of mass accretion rates to test the scaling equation (3) in the weak magnetic field regime. For a rotating neutron star to appear as an accretion-powered pulsar, the stellar magnetic field must be strong enough to disrupt the Keplerian disk flow above the stellar surface. Note that the orbital frequency \( v_o \) at the interaction radius \( r_0 \) increases with mass accretion rate \( (\alpha > 0) \). Therefore, the stellar magnetic field must be strong enough to disrupt the disk flow at radii larger than the neutron star radius \( R \) at the maximum mass accretion rate \( M_{\text{max}} \) for which coherent pulsations were detected. This leads to a lower limit on the magnetic dipole moment,

\[
\mu \geq 10^{27} \left( \frac{4\pi^2 v_o^2 R^3}{GM_\odot} \right)^{-1/2\beta} \left( \frac{M}{M_\odot} \right)^{(1-2\beta)/2\beta} \times \left( \frac{M_{\text{max}}}{M_\odot} \right)^{-\alpha/\beta} \text{ G cm}^3. \tag{4}
\]

At the same time, the stellar magnetic field must be weak enough that accretion is not centrifugally inhibited at the minimum mass accretion rate \( M_{\text{min}} \) for which coherent pulsations were detected. Therefore, the orbital frequency \( v_o \) at this accretion rate must be larger than the spin frequency of the neutron star \( v_s \). This leads to an upper limit on the magnetic dipole moment,

\[
\mu \leq 10^{27} \left( \frac{v_s}{v_{k,0}} \right)^{1/\beta} \left( \frac{M}{M_\odot} \right)^{-\gamma/\beta} \left( \frac{M_{\text{min}}}{M_\odot} \right)^{-\alpha/\beta} \text{ G cm}^3. \tag{5}
\]

Of course, X-ray brightness modulations at the pulsar spin frequency may be possible even if one of the above two requirements is not met. For example, azimuthal variations in the efficiency of processes dependent on magnetic field strength (such as cyclotron emission, absorption, and resonant scattering) may be strong enough to produce X-ray pulsations, even if the magnetic field of the neutron star is dynamically unimportant and the accretion disk extends down to the stellar surface. However, if at any point during the outburst either the disk had reached the stellar surface or accretion had been centrifugally inhibited, then either the X-ray spectrum or the pulse amplitude should have changed abruptly. Instead, the X-ray spectrum of SAX J1808.4 – 3658 was found to be remarkably stable (Gilfanov et al. 1998; Heindl & Smith 1998) and the pulse amplitude to increase only slightly from 4% to 7% as the inferred luminosity declined by more than 2 orders of magnitude during the outburst (Cui et al. 1998).

Combining the constraints of equations (4) and (5) we obtain

\[
\frac{M}{M_\odot} > 0.047 \left( \frac{v_s}{401 \text{ Hz}} \right)^2 \left( \frac{M_{\text{max}}}{M_{\text{min}}} \right)^{2\alpha} \left( \frac{R}{10 \text{ km}} \right)^3. \tag{6}
\]

Given the observed spin frequency and range of accretion rates at which coherent pulsations are detected, equation (6) defines a limiting curve in the \( M-R \) parameter space for neutron stars. The scaling of this curve is cubic in \( R \), independent of the details of the disk–magnetosphere interaction model, with a coefficient that depends on a single model parameter. As mentioned above, the X-ray spectrum of SAX J1808.4 – 3658 was found to be remarkably stable during the outburst, so we can assume here that the ratio \( M_{\text{max}}/M_{\text{min}} \) scales as the ratio of the corresponding count rates in a given X-ray bandpass, independent of (unknown) bolometric corrections. For the 1998 April/May outburst of SAX J1808.4 – 3658, this ratio was \( \approx 130 \) (Cui et al. 1998).

Figure 1 compares the \( M-R \) relations for several proposed equations of state for stable neutron stars (see Cook, Shapiro, & Teukolsky 1994) with the limit of equation (6) for the four previously mentioned inner disk models applied to SAX J1808.4 – 3658. Only the limiting curve for the radiation pressure–dominated 1R model, for which \( \alpha \approx 0.2 \), is consistent with many proposed equations of state. In a thin accretion disk, radiation pressure dominates over gas pressure at radii (see Treves, Maraschi, & Abramowicz 1988)

\[
\frac{r}{R} \leq 0.5 \alpha^{0.1} \left( \frac{\eta}{500} \right) \left( \frac{M}{M_\odot} \right)^{0.42} \left( \frac{R}{10 \text{ km}} \right)^{-0.24} \times \left( \frac{M}{10^{-4} M_\odot} \right)^{0.76}, \tag{7}
\]

Note that this is a necessary but not a sufficient condition: the X-ray brightness at infinity may not be modulated with any detectable amplitude if, e.g., the spin of the neutron star is perfectly aligned to its magnetic moment or the neutron star is surrounded by a scattering medium that attenuates the pulsations (Brainerd & Lamb 1986; Kylafis &phinney 1989).

While this work was in progress, we learned of a paper by Burderi & King (1998) in which a similar argument is used to constrain \( R \) by assuming a particular disk–magnetosphere interaction model (essentially the 1G model) and hence a specific value for \( \alpha \). However, the applicability of the 1G model to SAX J1808.4 – 3658 is dubious, as we demonstrate here. In contrast, we use eq. (6) to constrain disk–magnetosphere interaction models and hence \( \alpha \).
where $\eta$ is a parameter in the range $10^2$–$10^3$ and $x \leq 1$ is the Shakura-Sunyaev viscosity parameter. During the decline phase of the outburst, the mass accretion rate was so small that $r/R \ll 1$ and the inner accretion disk in SAX J1808.4–3658 was certainly not radiation pressure dominated. Whether or not the disk was radiation pressure dominated at the peak of the outburst is not clear given the uncertainty in the model parameter $\eta$. However, it is very unlikely that there was a transition between a radiation pressure– and a gas pressure–dominated inner accretion disk during the outburst, given the stability of the X-ray spectrum of the source and of the pulsed fraction of the emission (see discussion below). Radiation pressure–dominated models for the inner accretion disk, like the one shown in Figure 1, are therefore not applicable to SAX J1808.4–3658 since its inferred mass accretion rate is significantly sub-Eddington (in ’t Zand et al. 1998).

From the three gas pressure–dominated models, the two-temperature ones (2B and 2S) are inconsistent with any of the proposed equations of state and are thus ruled out for SAX J1808.4–3658. The one-temperature gas pressure–dominated model (1G) is barely consistent with a very restricted range of $M$ and $R$ allowed by only a few equations of state. However, the maximum and minimum accretion rates at which RXTE detected coherent pulsations from SAX J1808.4–3658 were determined by the peak luminosity of the outburst and the instrumental sensitivity, respectively, which are obviously unrelated to the limiting equalities in equations (4) and (5). This makes it quite unlikely that the actual mass and radius of SAX J1808.4–3658 lies very close to the limiting curve 1G in Figure 1. Therefore, either a remarkable coincidence has occurred or (more probably) the 1G model is also ruled out for SAX J1808.4–3658. This is particularly interesting because the dependence of $v_0$ on $M$ predicted by the 1G model is very similar to the simple $v_0 \propto M^{1/7}$ scaling law derived from equation (2) for constant $\gamma_B$. This simple scaling law is widely used to describe the disk-magnetosphere interaction in accreting magnetic stars (and is in fact generally considered the standard model). Figure 1 implies that the scaling of $v_0$ with $M$ for SAX J1808.4–3658 should be weaker than predicted by this simple scaling law. In particular, we find that $x$ must be less than 0.4 and hence $\gamma_B$ must be a function of $M$.

2.3. Predicted Limits on the Accretion Torque in SAX J1808.4–3658

An independent test of the scaling equation (3) in the weak magnetic field regime is possible if the spin frequency derivative $\dot{v}_s$ in SAX J1808.4–3658 due to accretion torques can be measured. For a Keplerian disk flow terminated at a radius $r_o$, the accretion torque on the neutron star is given by

$$2\pi I \dot{v}_s = \eta \dot{M} (GMr_o)^{1/2},$$  \hspace{1cm} (8)

where $I$ is the neutron star moment of inertia and $\eta$ is a dimensionless quantity in which all the physics of the disk-magnetosphere interaction is parameterized (Ghosh & Lamb 1979b). For a prograde accretion disk, $\eta$ is positive and a strong function of both the magnetic field strength and the mass accretion rate (Ghosh & Lamb 1979b). For a retrograde accretion disk $\eta$ is negative and has an absolute value of order unity (Daumerie 1996).

Applying the constraints of equations (4) and (5) to equation (8), we obtain an upper and a lower limit on the spin frequency derivative for SAX J1808.4–3658 predicted by accretion torque theory,

$$\dot{v}_s \leq 1.8 \times 10^{-13} \eta \left( \frac{10^4}{M/M_\odot} \right)^{-1} \left( \frac{v_s}{401 \text{ Hz}} \right)^{-1/3} \left( \frac{R_s}{10 \text{ km}} \right) \left( \frac{M}{M_\odot} \right)^{-1/3} \left( \frac{M_{\text{max}}}{0.06 M_\odot} \right)^{2/3}, \hspace{1cm} (9)$$

$$\dot{v}_s \geq 1.0 \times 10^{-13} \eta \left( \frac{10^4}{M/M_\odot} \right)^{-1} \left( \frac{R_s}{10 \text{ km}} \right)^{3/2} \left( \frac{M}{M_\odot} \right)^{-1/2} \left( \frac{M_{\text{max}}}{0.06 M_\odot} \right). \hspace{1cm} (10)$$

The maximum observed 2–30 keV flux of SAX J1808.4–3658 at the peak of the outburst was $\approx 3 \times 10^9$ ergs cm$^{-2}$ s$^{-1}$ (Cui et al. 1998), which corresponds to a 2–30 keV luminosity of $\approx 0.03(\eta _\odot /4\pi r_L^2)$, where $\eta _\odot$ is the solid angle of emission and $r_L = 2 \times 10^{14}$ ergs s$^{-1}$ is the Eddington critical luminosity that corresponds to $M_{\odot}$. However, the photon spectrum of SAX J1808.4–3658 over most of the RXTE 2–100 keV bandpass is well described by a power law of the form $dN/dE \propto E^{-2}$, where $E$ is the photon energy. The energy spectrum is thus very flat, $\nu L_\nu \propto E^0$. As a result, the bolometric luminosity of the source must be larger than estimated above and is probably about twice the 2–30 keV luminosity, given the expected upper and lower energy cutoffs of the spectrum (see also Heindl & Smith 1998 for spectral fits over the entire RXTE bandpass that show this effect). We therefore adopt $M_{\text{max}} = 0.06 M_\odot$.

Given a measurement of the spin frequency derivative in SAX J1808.4–3658, we could use equations (9) and (10) to place additional constraints on $x$ for inner disk models or $M$ and $R$ for neutron stars, similar to the discussion above for the presence of coherent pulsations (Fig. 1). However, variations of the spin frequency were not detected during the peak of the 1998 April/May outburst of SAX J1808.4–3658, leading to an upper limit on the spin frequency derivative of $|\dot{v}_s| < 7 \times 10^{-13}$ Hz s$^{-1}$ (Chakrabarty & Morgan 1998). This measured upper limit is consistent with our predicted bounds on the spin frequency derivative equations (9) and (10) if $\eta \lesssim 7$.

3. THE MAGNETIC FIELD STRENGTH OF SAX J1808.4–3658

In their discovery paper for SAX J1808.4–3658, Wijnands & van der Klis (1998a) applied “standard” magnetosphere-disk accretion theory (essentially the 1G model discussed in § 2) and used the absence of centrifugal inhibition of accretion during the decline of the 1998 outburst to infer that $B \lesssim (2–6) \times 10^8$ G. As the outburst declined, pulsations continued to be detected, leading to revised upper limits on the magnetic field strength [$B \lesssim (0.4–1.3) \times 10^8$ G: Cui et al. 1998; $B \lesssim \text{ few} \times 10^7$ G: Gilfanov et al. 1998]. However, as we showed in § 2.2, simple scaling arguments or models of the form of equation (3) for the asymptotic regions of gas pressure–dominated accretion disks cannot easily account for the coherent pulsations observed from SAX J1808.4–3658 throughout its 1998 outburst. Therefore, the previous estimates of the magnetic field strength of SAX J1808.4–3658, which are based on similar scalings, are not valid. A more careful calculation is necessary to
infer the magnetic field strength in this system. Because no spin frequency derivative has been detected yet from this source (Chakrabarty & Morgan 1998), we cannot uniquely determine the stellar magnetic field strength but can only constrain it.

We begin with equation (2) for the interaction radius, which is derived directly from the angular momentum equation (1), and impose the same constraints that led to equations (4) and (5). We set the maximum and minimum mass accretion rates at which coherent pulsations were detected to $M_{\text{max}} \approx 0.06 M_\odot$ and $M_{\text{min}} \leq M_{\text{max}}/130$ as inferred from observations (see Cui et al. 1998 and § 2.3). We assume a neutron star radius in the 10–15 km range, as predicted by current equations of state (see Fig. 1). We also assume a neutron star mass in the 1.4–2.3 $M_\odot$ range, consistent with the inferred masses of recycled millisecond pulsars (Thorsett & Chakrabarty 1999), the upper limits on neutron star masses in LMXBs inferred from kHz QPO observations (Miller et al. 1998; see also Zhang, Strohmayer, & Swank 1997), and the possible neutron star mass measurement in 4U 1820−30 (Zhang et al. 1998a).

The limits on the magnetic field strength of SAX J1808.4−3658 also depend on the allowed range for the parameter $\gamma_b$, which in turn depends on the fractional width $\Delta r_0/r_0$ of the boundary layer and the magnetic pitch $B_0/B_\ast$ within this layer (see Ghosh & Lamb 1991 for a detailed discussion). The fractional width should be significantly smaller than unity in order for the boundary layer of equation (2) to be valid. Assuming a dipolar poloidal field and neglecting mass loss from the disk as well as toroidal screening currents in both the disk and the magnetosphere, one finds $\Delta r_0/r_0 \approx 0.3$ (Ghosh & Lamb 1991; Daumerie 1996). However, relaxing any of these assumptions leads to a significantly narrower boundary layer. We therefore conservatively adopt $0.01 \leq \Delta r_0/r_0 < 1$.

The toroidal component $B_0$ of the magnetic field is produced by the differential rotation of gas in the accretion disk with respect to the stellar spin. The magnetic energy stored in the magnetosphere increases with the twisting of the field lines (see Zylstra 1988 and references therein). There is an upper bound on this energy (and hence on the twisting of the field lines and the magnetic pitch) for a semi-infinite space in which the magnetic field strength goes to zero at infinity (Aly 1984, 1991; Zylstra 1988). It has been argued that, above this upper bound, the only existing configurations are those with field lines that close at infinity (i.e., open field lines; Aly 1984, 1991). Simple estimates of the maximum twisting of the field lines (see Ghosh & Lamb 1991 and reference therein) as well as detailed numerical calculations of the magnetospheric structure (Zylstra 1988) lead to maximum values of $B_0/B_\ast \sim 1$. Combining this with our constraints on $\Delta r_0/r_0$, we therefore adopt $0.01 \leq \gamma_b(M) \leq 1$.

Applying all these constraints to SAX J1808.4−3658, we obtain

$$\mu \geq 0.3 \times 10^{26} [\gamma_b(M_{\text{max}})]^{-1/2} \left( \frac{M}{1.4 M_\odot} \right)^{1/4} \times \left( \frac{R_a}{10 \text{ km}} \right)^{9/4} \left( \frac{M_{\text{max}}}{0.06 M_\odot} \right)^{1/2} \text{ G cm}^3,$$

$$\mu \leq 10 \times 10^{26} \left[ \gamma_b(M_{\text{min}}) \right]^{-1/2} \left( \frac{M}{2.3 M_\odot} \right)^{5/6} \times \left( \frac{R_a}{15 \text{ km}} \right)^{1/2} \left( \frac{v_s}{401 \text{ Hz}} \right)^{-7/6} \times \left( \frac{M_{\text{min}}}{4.6 \times 10^{-2} M_\odot} \right)^{1/2} \text{ G cm}^3,$$

which corresponds to a dipolar magnetic field of a few times $10^8$ G at the stellar pole. Figure 2 shows the limits on the magnetic dipole moment as a function of the (unknown) parameter $\gamma_b(M)$.

Finally, we can estimate the mass accretion rate $M_{\text{eq}}$ for which the observed spin frequency of the neutron star is the equilibrium frequency (at which the accretion torque is zero). The result is

$$M_{\text{eq}} = 2 \times 10^{-11} \omega_c^{-7/3} \left( \frac{\gamma_b}{0.1} \right) \left( \frac{\mu}{10^{26} \text{ G cm}^3} \right)^2 \times \left( \frac{M}{1.4 M_\odot} \right)^{-5/3} \left( \frac{v_s}{401 \text{ Hz}} \right)^{7/3} M_\odot \text{ yr}^{-1},$$

where $\omega_c$ is the critical fastness parameter (Ghosh & Lamb 1979b). The value of $M_{\text{eq}}$ depends sensitively on the (unknown) magnetic field strength and on $\gamma_b(M)$. However, for values of these parameters consistent with the constraints imposed above, $M_{\text{eq}}$ agrees with the long-term average accretion rate of SAX J1808.4−3658 inferred by Chakrabarty & Morgan (1998) from the fluence of the X-ray outburst and the outburst recurrence time. Moreover, in order for an LMXB with a 2 hr binary period to be below the separatrix between transient and persistent systems in the diagram of donor mass versus orbital period, it must have $M \lesssim 3 \times 10^{-11} M_\odot \text{ yr}^{-1}$ (van Paradijs 1996;
King, Kolb, & Szuszkiewicz 1997), also consistent with the value estimated above (Chakrabarty & Morgan 1998).

4. DISCUSSION

In this paper we have studied the disk-magnetosphere interaction in the first known accretion-powered, millisecond pulsar, SAX J1808.4—3658. We have demonstrated that various simple models of inner accretion disks are not consistent with observations. We have also argued that the magnetic field strength of SAX J1808.4—3658 is a few times $10^8$ G at the stellar pole, using very general constraints on the properties of the neutron star and on the physics of the inner accretion disk flow. In this section, we discuss the implications of our results for models of the disk-magnetosphere interaction around weakly magnetic neutron stars. We also compare SAX J1808.4—3658 with the recycled millisecond radio pulsars and with the nonpulsing neutron star LMXBs.

4.1. The Disk-Magnetosphere Interaction in Weakly Magnetic Neutron Stars

BATSE observations of strongly magnetic ($B \sim 10^{12}$ G) accretion-powered pulsars in Be/X-ray transients have shown that the scaling of the radius $r_0$ of interaction between the stellar magnetic field and a gas pressure-dominated disk flow can account for the observations (Ghosh 1996; Finger, Wilson, & Harmon 1996; Bildsten et al. 1997). However, in §2 we showed that this same scaling is inconsistent with the detection of coherent pulsations throughout the 1998 April/May outburst of SAX J1808.4—3658, for most equations of state of neutron star matter. This is not surprising, given the very different physical conditions in the interaction regions around weakly and strongly magnetic neutron stars.

In writing the stress balance equation (1) and the scaling equation (2), we made a number of implicit assumptions regarding the properties of the neutron star and the inner accretion disk flow: first, that the stellar magnetic field is dipolar; second, that the pulsar magnetic moment is parallel to its spin axis; third, that the dominant mechanism for removing angular momentum from the accreting gas in the disk is magnetic stress; and finally, that the gravitational field is Newtonian everywhere and that stable, circular Keplerian orbits exist at all radii. Although these assumptions are valid (or at least are reasonable approximations) in the case of a strongly magnetic neutron star, they break down when $r_0$ is comparable to the stellar radius (see Lai 1998 for solutions of the structure of the inner accretion disk where some of these effects have been taken into account in a simple way).

Let us examine the validity of each of these assumptions. First, magnetic stresses would remove angular momentum faster than predicted by equation (2) if higher order multipoles were present in the pulsar magnetic field. For example, if the stellar magnetic field could be described entirely by a multipole of order $l$ (i.e., the magnetic field strength in the equator was $B = \mu_l/\mu^{l+1}$), then the Keplerian frequency at $r_0$ would be

$$v_0 = (2\pi)^{-1} \gamma^3 \mu^{3/(4l-1)} (GM)^{2l+1/(4l-1)} \mu^{-6/(4l-1)} M^{3/(4l-1)}.$$  

Equation (14) shows that if $\gamma$ depends only weakly on $M$, the dependence of $v_0$ on $M$ weakens as the order $l$ of the multipole increases: for a dipolar ($l = 2$) field $v_0 \propto M^{0.43}$, for a quadrupolar ($l = 4$) field $v_0 \propto M^{0.2}$, etc. As a result, the existence of intrinsic or induced multipole moments in the magnetosphere can significantly weaken the $M$-dependence of $v_0$, hence making the scaling of equation (14) consistent with observations of SAX J1808.4—3658. As an illustration of this, Figure 3 shows the constraints imposed by the 1G model on the mass and radius of the neutron star in SAX J1808.4—3658 for various contributions of an aligned quadrupole moment to the pulsar magnetic field. Clearly, even a modest quadrupole moment (which would be completely negligible at distances larger than a few stellar radii from the surface) is enough to make the 1G model consistent with observations.

The intrinsic multipole moments in neutron star LMXBs are thought to be weaker than their dipole moments, based on the 1998 spin frequencies and spin frequency derivatives of the millisecond radio pulsars thought to be their descendants (Arons 1993). However, electrical currents in the accretion disk and the magnetosphere can significantly enhance the strength of multipole moments by altering the magnetic field topology in the interaction region (see, e.g., Ghosh & Lamb 1979a). Moreover, electrical currents on the neutron star induced by the presence of an accretion disk can produce multipoles as strong as the intrinsic dipole, provided that the disk extends very close to the stellar surface (Psaltis, Lamb, & Zylstra 1996). The corotation radius $r_0 = (GM/4\pi^2v_0^2)^{1/3}$ (which is an upper bound on $r_0$ when coherent pulsations are detected; see, e.g., Ghosh & Lamb 1979a) is only about 2.8 times the neutron star radius in SAX J1808.4—3658, and therefore none of the above effects are negligible.

Second, if any of the magnetic moments of SAX J1808.4—3658 is not aligned to the stellar rotation axis...
Figure 4.—Inferred pulsar dipole magnetic field strength at the stellar equator, as a function of spin period. The small dots are normal radio pulsars (Taylor et al. 1993). The open circles are millisecond radio pulsars which are not members of globular clusters and for which the inferred field strengths have been corrected for the apparent period derivative caused by their transverse velocities (Camilo et al. 1994). The error bar shows the allowed range of field strengths for SAX J1808.4—3658. The spin-up line (solid) is shown here for illustrative reasons only and represents the minimum period of recycled pulsars for a given field strength, assuming a specific model for the inner accretion disk and the disk-magnetosphere interaction. The death line (dashed) is an empirical estimate of the maximum period at which pulsars have detectable radio emission for a given field strength.

(which is actually a necessary condition for the system to appear as a pulsar) then the scaling of the inner Keplerian disk radius with accretion rate may be different than what is predicted by the models considered above. This case has never been treated in the literature because it corresponds to a time-dependent non-steady state problem. It is beyond the scope of the current paper to generalize models of the interaction between the pulsar magnetic field and the accretion disk to time-dependent situations. We will therefore not consider this case any further.

Third, when the accretion disk penetrates very close to the neutron star, the effect of magnetic stresses is amplified by radiation drag forces that can efficiently remove angular momentum from the accreting gas (Miller & Lamb 1996; Miller et al. 1998). Finally, the specific angular momentum of gas at radii comparable to the innermost stable orbit around a compact object has a weaker dependence on radius than estimated in a Newtonian gravitational field.

All the effects discussed above result in a weaker $M$-dependence of $v_0$, thus making the scaling of equation (2) consistent with observations of SAX J1808.4—3658.

4.2. Comparison with Millisecond Radio Pulsars

When mass transfer onto the neutron star ends, SAX J1808.4—3658 may appear as a millisecond radio pulsar (Wijnands & van der Klis 1998a; see also Bhattacharya & van den Heuvel 1991). Figure 4 compares the current spin frequency and inferred magnetic field strength of SAX J1808.4—3658 at the stellar equator with those of the known normal and millisecond radio pulsars (Taylor, Manchester, & Lyne 1993). We only include millisecond radio pulsars in the Galactic disk, since the inferred spin frequency derivatives (and hence the magnetic field strengths) of pulsars in globular clusters are probably contaminated by gravitational acceleration in the potential of the cluster. Moreover, we use magnetic field strengths that have been corrected for the effect of the apparent spin frequency derivative caused by the nonnegligible transverse velocities of the pulsars (Camilo, Thorsett, & Kulkarni 1994; Toscano et al. 1999). Note, however, that the inferred magnetic dipole moments assume orthogonal dipole rotators, and model-dependent systematic effects have not been taken into account.

The mass of the companion of SAX J1808.4—3658 is $\lesssim 0.3$ $M_\odot$, and hence the binary must be close to the termination of its X-ray phase (Chakrabarty & Morgan 1998). The current magnetic field strength and spin frequency of the neutron star should thus be very similar to that of the descendant millisecond radio pulsar. Figure 4 shows that the descendant of SAX J1808.4—3658 should appear as a normal millisecond radio pulsar in the Galactic disk. Given the small number statistics and the uncertainty in our estimate of the magnetic field strength in SAX J1808.4—3658 and in the millisecond radio pulsars, it is not possible to determine whether the field strength of SAX J1808.4—3658 is lower or higher than those in the known millisecond radio pulsars. Such a determination would be of interest in comparing SAX J1808.4—3658 with the other nonpulsing LMXBs.

4.3. Comparison with Nonpulsing LMXBs

SAX J1808.4—3658 is the only known weakly magnetic accreting neutron star with persistent coherent pulsations at the neutron star spin frequency. Upper limits on the amplitudes of periodic oscillations in other neutron star LMXBs range from $\lesssim 1\%$ for the most luminous Z sources to a few percent for the less luminous atoll sources (see Vaughan et al. 1994 and references therein). Coherent millisecond oscillations, however, possibly at the spin frequencies of the neutron stars, have been observed in several neutron star LMXBs during type I X-ray bursts (Strohmayer, Swank, & Zhang 1998).

The absence of persistent coherent pulsations in most LMXBs might be due to neutron star magnetic field strengths that are significantly different than that in SAX J1808.4—3658. Specifically, the field strengths in the nonpulsing LMXBs may be so strong that accretion is centrifugally inhibited or so weak that the accretion disk extends all the way to the neutron star surface. Alternatively, the neutron stars in the nonpulsing LMXBs may be surrounded by optically thick scattering media that spread the pulsar beams and attenuate the oscillations below present detection thresholds (Brainerd & Lamb 1987; Kylafis & Phinney 1989). We consider here each possibility separately.

It is rather unlikely that the field strengths of all nonpulsing LMXBs are so strong that accretion is centrifugally inhibited, since at least some are accreting at near-Eddington rates. Moreover, if the field strengths were all so strong, then the observational fact that all known millisecond radio pulsars lie below the so-called spin-up line in the $B$-$P$ diagram (Fig. 4) would have to be a selection effect or a coincidence. This is because the spin-up line defines the minimum spin period of a recycled pulsar (which is the equilibrium period at the Eddington accretion rate) for a
A number of additional effects may be responsible for the detectability of coherent pulsations in SAX J1808.4—3658. For example, its very small measured mass function is suggestive of a nearly face-on viewing angle of the binary (Chakrabarty & Morgan 1998). In this case, the pulsed emission may propagate through a region of lower optical depth close to the pulsar rotation poles, whereas the majority of the (nonpulsed) X-ray photons may travel through optically thick regions nearby. However, it is important to note that the presence of a shallow partial eclipse in SAX J1808.4—3658 has been suggested (Chakrabarty & Morgan 1998; Heindl & Smith 1998). Confirmation of this feature would rule out a face-on orientation of the binary, thus challenging the above explanation. Alternatively, different stellar field topologies in SAX J1808.4—3658 and the nonpulsing neutron star LMXBs, possibly related to different prior evolution or even to different magnetic inclinations, may be responsible.

A number of other LMXBs share many characteristics of SAX J1808.4—3658 and are thus good candidates for detecting periodic pulsations in their persistent emission. For example, as we discussed above, Aql X-1 and 4U 1608—52 are transients with X-ray spectra very similar to SAX J1808.4—3658, suggesting similar field strengths and inner disk flows. The nearly sinusoidal optical orbital phase light curves of three other LMXBs (4U 1636—53, GX 9+9, and 4U 1735—44) suggest that these are also viewed nearly face-on (see van Paradijs & McClintock 1995). Finally, GS 1826—26 (Homer, Charles, & O’Donoghue 1998) and MS 1603+2600 (Hakala et al. 1998) are other LMXBs with low-mass companions in ~2 hr orbits, in which mass transfer is probably driven by angular momentum losses due to gravitational radiation. These binaries are thus in an evolutionary stage very similar to that of SAX J1808.4—3658. In particular, GS 1826—26 is a transient X-ray burster with a low-amplitude modulation of its optical brightness, again possibly suggesting a face-on orientation (Homer et al. 1998). Detecting or imposing stringent upper limits on coherent pulsations in the persistent emission of any of these sources will be crucial for understanding the characteristics of SAX J1808.4—3658 and the relationship between LMXBs and millisecond radio pulsars.

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