Portrait of Theobalda as a young asteroid family

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Accepted 2010 May 12. Received 2010 May 12; in original form 2010 March 29

1 INTRODUCTION

Asteroid families are believed to originate in the catastrophic disruption of large asteroids. To identify an asteroid family, we look for clusters of asteroids in the space of proper orbital elements (Milani & Knežević 1990, 1994): the proper semimajor axis \( a_p \), proper eccentricity \( e_p \) and proper inclination \( I_p \). The orbital elements describe the size, shape and tilt of orbits. Proper orbital elements, which are more constant over time than instantaneous osculating orbital elements, provide a dynamical criterion of whether or not a group of bodies has a common ancestor. Up to now, ejecta from a few tens of major collisions have been discovered in the main belt (e.g. Zappalà et al. 1994; Mothé-Diniz, Roig & Carvano 2005).

The size and velocity distributions of the family members provide important constraints for testing our understanding of the breakup process, but erosion and dynamical evolution of the orbits over time can alter the original signature of the collision. Nowadays, it is well known that the kinematical structures of asteroid families evolved over time, with respect to the original post-impact situations, as a result of chaotic diffusion and gravitational and non-gravitational perturbations (Milani & Farinella 1994; Bottke et al. 2001; Carruba et al. 2003; Dell’Oro et al. 2004). These mechanisms changed the original shapes of the families produced in collisions, and consequently complicated the physical studies of high-velocity collisions.

Unfortunately, most of the observed asteroid families are old enough (older than 100 Myr; Nesvorný et al. 2006a) to be substantially eroded and dispersed. However, young asteroid families (younger than 10 Myr) such as Karin, Veritas and Iannini (Nesvorný et al. 2002, 2003) or even very young families (younger than 1 Myr) such as Datura, Emilkovalski, 1992YC2 and Lucascavin (Nesvorný & Vokrouhlický 2006), suffer little erosion during the period of time after a breakup event. Thus, they provide a unique opportunity to study a collisional outcome almost unaffected by orbit evolution.

In this paper, we study the Theobalda asteroid family. We present its basic properties, including the identification of its membership and the study of the cumulative absolute magnitude distribution of the family members. Moreover, the diameter of the parent body has been estimated. We have studied in detail the dynamical characteristics in the region occupied by the Theobalda asteroid family and we have analysed the role of the dynamics in shaping the family.

As was noted by Novaković, Tsiganis & Knežević (2010), this family is a very good candidate for estimating its age using the method of chaotic chronology (MCC). In order to apply the MCC, the family has to be located in the region of the main asteroid belt where diffusion takes place. Also, it is necessary that diffusion is fast enough to cause measurable effects, but slow enough so that most of the family members are still forming a robust family structure. As we show, it turns out that the Theobalda family is an excellent case in this respect. Given that, as our main result, we have estimated the age of the family. Using two different methods, the MCC (Tsiganis, Knežević & Varvoglis 2007; Novaković, Tsiganis & Knežević 2010a) and backward integrations (Nesvorný et al. 2002), we estimate the age of the family to be 6.9 ± 2.3 Myr. Thus, we establish it as another young asteroid family.

The paper is organized as follows. In Section 2, we present the basic properties of the Theobalda family. We use the hierarchical clustering method (HCM), proposed by Zappalà et al. (1990), to
identify family members. Next, we discuss the cumulative absolute magnitude distribution of identified family members, and we estimate the size of the parent body. We present and discuss the dynamical characteristics of the region occupied by the Theobalda family, and we identify the main mean motion and secular resonances, in that region. We analyse the dynamical stability of the family members, in particular the stability of the largest member of the family (778 Theobalda). In Section 3, we estimate the age of the family. This is performed first by using the backward integration method, and then by using the MCC. The good agreement between these two results indicates a reliable age determination. Finally, in Section 4, we summarize our results, discuss some possibly interesting relations to other works, and draw our conclusions.

2 THEOBALDA FAMILY: THE BASIC FACTS

This asteroid family has attracted little attention so far, mainly because the number of asteroids associated with it was relatively small. However, at present the situation is different, and, as we show later, the Theobalda family now has over 100 known members. This number is large enough that the family characteristics can be reliably determined.

2.1 Identification of the family members

The identification of family members is the first step in our study of the Theobalda family. This is done by applying the HCM to the catalogue of synthetic proper elements of numbered asteroids (Knežević & Milani 2000, 2003) from AstDys\(^1\) (data base as of 2009 October). The HCM requires that distances among the family members, in the proper element space, are less than the so-called cut-off distance \((d_{\text{cut-off}})\), which has the dimension of velocity. As the cut-off distance is a free parameter of the HCM, we tested different values ranging from 20 to 135 ms\(^{-1}\). Also, we apply the HCM using two different central objects: (i) (778) Theobalda, which has a chaotic orbit and (ii) (84892) 2003QD\(_{79}\), which is on a relatively stable orbit. The results are shown in the top panel of Fig. 1. The HCM identified the family around (778) Theobalda for \(d_{\text{cut-off}} \geq 60\) ms\(^{-1}\), while around (84892) 2003QD\(_{79}\) the family exists even for the lowest tested value of \(d_{\text{cut-off}} = 20\) ms\(^{-1}\). For \(d_{\text{cut-off}} \geq 60\) ms\(^{-1}\), the resulting family is the same. This suggests that (778) Theobalda has probably been displaced from its original position as a result of chaotic diffusion.\(^2\)

The bottom panel of Fig. 1 shows the best-fitting power-law index \(\gamma\) of the form \(N(<H) \propto 10^{\gamma H}\) of the cumulative absolute magnitude \((H)\) distribution in the range \(H \in [13–15]\), as a function of cut-off distance \((d_{\text{cut-off}})\).\(^3\) For \(d_{\text{cut-off}} \in [75, 115]\) ms\(^{-1}\), the number of asteroids as well as index \(\gamma\) are nearly constant, and probably each value from this interval can be safely used to identify family members using the HCM.\(^4\) We have adopted the value of \(d_{\text{cut-off}} = 85\) ms\(^{-1}\) to identify the nominal family. For this value of \(d_{\text{cut-off}}\), the HCM linked 128 asteroids to the Theobalda family.

\(^1\) http://hamilton.dm.unipi.it/astdys/
\(^2\) Note that this is very similar to the Veritas family, and the situation with the largest member of this family, (490) Veritas (Tsiganas et al. 2007).
\(^3\) Instead of the index \(\gamma\), the exponent of the cumulative distribution can be obtained in terms of diameters rather than absolute magnitudes (Dell’Oro & Cellino 2007). However, as we do not know the albedos for most of the asteroids, which are necessary to convert from absolute magnitudes to diameters, we have chosen to work with \(\gamma\).
\(^4\) Usually we adopt the value of \(d_{\text{cut-off}}\) that corresponds to the centre of the interval over which the index \(\gamma\) is constant (Vokrouhlický et al. 2006).

There are two main reasons for our choice. The first is that this value of velocity cut-off corresponds to the centre of the plateau, which can be seen in Fig. 1. The second reason is the very good agreement between the ages of the family estimated by applying the MCC to two different groups of family members. We explain this in more detail in Section 3.

Note that the values of \(\gamma\) for family members are always larger than the value of the same index calculated for background asteroids. This is the first indication that the family is relatively young. However, Parker et al. (2008) estimated \(\gamma = 0.44\) for \(H \in [13.0–15.5]\). This value is much lower than ours, as we found \(\gamma = 0.60 \pm 0.02\) for the nominal family, very close to the value that we found for the background population in the region of the Theobalda family. Probably, Parker et al. (2008) underestimated this value because of observational incompleteness, as they worked in the range \(H \in [13–15.5]\) and used a smaller data set for which Sloan Digital Sky Survey (SDSS) colours were available. Although they linked...
100 asteroids with the family, a significant number of these asteroids are probably interlopers.

### 2.2 Size of the parent body

To estimate the diameter of the parent body ($D_{PB}$) of the Theobalda asteroid family, it is necessary to account for small and still undiscovered family members. In general, the data set on asteroids below $H = 15$ mag is basically complete (Gladman et al. 2009). However, as we are dealing with a family at the edge of outer belt, the family members are C-type asteroids, which are several times darker than S-type asteroids. Also, as we are using a catalogue of synthetic proper elements that does not include all known asteroids, the completeness limit for our sample has to be analysed. An indication of the completeness limit can be obtained simply by looking at Fig. 2 for the value of $H$ where the slopes of the two distribution curves change. This is approximately at about $H = 14.5$ mag. A better estimation of the completeness limit can be inferred using the catalogue of asteroids that are not included in the catalogue of synthetic proper elements we deal with (i.e. the catalogue of multi-opposition objects maintained at the AstDys web site). As about 99 per cent of multi-opposition objects, with osculating semimajor axes in the range [3.15, 3.20] au, have $H \geq 14.2$ mag (see Fig. 3), we assume that the catalogue of synthetic proper elements of asteroids in this region is complete up to $H = 14.2$ mag.

In order to overcome the problem of observational incompleteness, using the size–frequency distribution (SFD) of the main-belt asteroids estimated by Gladman et al. (2009) and the fact that the SFD of asteroid families is considered to be shallower than that of the background (Morbidelli et al. 2003), we added some fictitious bodies with $H \in [14.2, 17.0]$ to the family. This was performed in such a way to make the SFD of the ‘extended’ family (real asteroids + fictitious objects) shallower than the obtained SFD for background asteroids. More precisely, in order to be able to estimate the uncertainty of our approach, we generated 100 different sets of fictitious objects and then estimated the size of the parent body from each of the 100 sets.

$D_{PB}$ corresponds to a spherical body with volume equal to the estimated total volume of all the family members, including these added members with $H \in [14.2, 17.0]$. Next, we assume all family members have the same geometric albedo ($p_v$) as (778) Theobalda (i.e. $p_v = 0.0589$ according to Tedesco et al. 2002). Having the values of $H$ and $p_v$, the radius $R$ of a body can be estimated, using the relation (e.g. Bowell et al. 1989)

$$R (\text{km}) = \frac{1329 \times 10^{(\frac{H}{5})}}{2 \sqrt{p_v}}.$$  

This allows us to infer that the diameter$^6$ of the parent body is $D_{PB} \approx 78 \pm 9$ km. The estimated uncertainty accounts for uncertainties in albedos, absolute magnitudes and the SFD. Also, it takes into account the dependence on the HCM cut-off. However, the real uncertainty is larger (e.g. because of the possible interlopers). According to our estimate, the largest remnant, (778) Theobalda, contains 87 per cent of the mass of the parent body. Although this should be considered as an upper bound, because it does not account for small family members with $H > 17$, our result suggests that the Theobalda family was produced by a cratering impact. The typical density ($\rho$) for C-type asteroids is $\rho = 1500$ kg m$^{-3}$ (e.g. Brož et al. 2005). Given this, the escape velocity$^7$ from the Theobalda family parent body is $V_{esc} \approx 32$ m s$^{-1}$.

### Figure 2.

Distribution (top) and cumulative distribution (bottom) of the members of the Theobalda asteroid family as a function of the absolute magnitude.

### Figure 3.

Distribution of multi-opposition asteroids, in the $(a, H)$ plane, from the AstDys web site (data base as of 2009 October). The catalogue consists mostly of the objects discovered more recently than objects included in the catalogue of synthetic proper elements. Thus, it provides a good opportunity to estimate the completeness limit of the catalogue of synthetic proper elements, which is marked by a red line. The green dots represent the members of the Theobalda family.

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5 The asteroid size distribution at diameters $D < 10$ km is still poorly known. Various models and extrapolations yield very different estimates of the number of km-sized and smaller main-belt asteroids. However, other possible estimates (e.g. Ivěříč et al. 2001; Tedesco, Cellino & Zappalà 2005; Wiegert et al. 2007; Yoshida & Nakamura 2007) would not affect our results significantly.

6 Probably the better way to estimate the size of the parent body is that proposed by Durda et al. (2007). However, we were unable to find an appropriate match with the SFDs published in that paper using a simple visual comparison of plots.

7 Compensating for collective effects in the cloud of dispersing fragments, $V_{esc} = 1.64 \times GM/R$, where $GM$ is the product of the gravitational constant and the parent body mass, $R$ is the radius of the parent body, while 1.64 is an empirical factor (Petit & Farinella 1993).
2.3 Dynamical characteristics

The dynamics in the region of the phase space occupied by Theobalda family members is much like in the case of the Veritas family because the two families stretch over a similar range of proper semimajor axes. The dynamics in the region of the Veritas family is very well studied (see, for example, Milani & Farinella 1994; Knežević & Pavlović 2002; Nesvorný et al. 2003; Tsiganis et al. 2007). Therefore, here we focus only on some differences between the dynamics in the regions occupied by the two families. The differences arise from the fact that the proper eccentricities of the Theobalda family members ($e_p \approx 0.25$) are significantly higher than those of the Veritas family ($e_p \approx 0.06$). Also, the proper inclinations are about $5^\circ$ higher. These make the Theobalda family members even more strongly chaotic than the Veritas family members.

Fig. 4 shows the Lyapunov characteristic exponents (LCEs), as a function of proper semimajor axis and eccentricity, in the region occupied by the Theobalda family members, as well as in the surrounding area. Most of the orbits in this region are unstable, even for comparatively low values of proper eccentricity, while for eccentricity above 0.3 almost all chaotic zones are connected, forming a wide chaotic sea with a fast diffusion therein. The chaos also dominates in the region where the Theobalda family is located (inside or close to the equivelocity ellipse). This can be better appreciated from Fig. 5 where the LCEs of the real family members are shown. The vertical strip of the largest, at $a_p \approx 3.174$ au values of LCEs, is associated with the $(5, -2, -2)$ three-body mean motion resonance (MMR), but it seems that this chaotic zone also includes the $(3, 3, -2)$ and $(7, -7, -2)$ three-body MMRs. Most of the bodies have LCE $\geq 1 \times 10^{-4}$ yr$^{-1}$, which corresponds to the Lyapunov times $T_{\text{lyap}} \leq 10000$ yr. These bodies are probably in the so-called Chirikov regime (Guzzo, Knežević & Milani 2002; Morbidelli 2002).

Using the proper frequencies $g$ (the average rate of the perihelion longitude $\sigma$) and $s$ (the average rate of the node longitude $\Omega$), we found that the Theobalda family region is also crossed by two secular resonances $g + s - g_5 - s_5$ and $g + s - g_6 - s_6$ (Fig. 6). Both are of the order 4 (i.e. they arise from the perturbing terms of degree of at least 4 in eccentricity and inclination; Milani & Knežević 1990; Knežević et al. 1991). However, we did not find evidence

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Figure 4. The dynamical structure of the region occupied by the Theobalda family along with the surrounding area. The colour scale codes LCEs (in units of $10^{-6}$ yr$^{-1}$) for 10000 test particles. The ellipse represents the assumed positions of the Theobalda family members immediately after breakup.

Figure 5. The same as in Fig. 4, but for LCEs of the real family members. The linear interpolation is used in order to cover the complete region shown in the figure.

Figure 6. Time evolution of the critical angles $\sigma_1 = \sigma + \Omega - \sigma_5 - \Omega_6$ (top) and $\sigma_2 = \sigma + \Omega - \sigma_6 - \Omega_6$ (bottom) of the secular resonances for the period from 1.2 to 1.6 Myr in the past. The short episodes of trapping inside the resonance seem possible, but several events of resonance crossing are clearly visible. The reversal of the direction of circulation is related to periods when the orbit interacts with these secular resonances.
The secular resonance \( \sigma = \sigma_1 + \Omega - \sigma_5 - \Omega_6 \) has much more influence on the dynamics of family members in the case of the Padua family (see Carruba 2009).

(see Section 3.2.1) that these resonances increase the diffusion speed. Probably this is because these resonances are effective only in narrow bands within the Theobalda family. Consequently, some of the family members, during their secular cycles, might be temporarily trapped in one or both secular resonances, but most of the time these asteroids are outside the secular resonances. Fig. 6 shows the time evolution of the critical angles \( \sigma_1 = \sigma + \Omega - \sigma_5 - \Omega_6 \) and \( \sigma_2 = \sigma + \Omega - \sigma_6 - \Omega \) for asteroid (778) Theobalda. This asteroid might be temporally trapped in both secular resonances. Although short episodes of ‘libration’ are visible, these events may be related to resonance crossing rather than to resonance trapping. Most of the time, both critical arguments circulate.

Fig. 7 shows the distributions of family members, as identified by the HCM, along with the positions of the main mean motion and secular resonances. Obviously, the structure of the family is a result of the dynamical mechanisms at work, which are mainly controlled by the MMRs. The largest spread of family members, in both \( (a_p, e_p) \) and \( (d_p, I_p) \) planes, is associated with the \((5, -2, -2)\) resonance. A smaller spread is observable in the \((3, 3, -2)\) resonance, while the \((7, -7, -2)\) resonance caused only a small diffusion of asteroids. This agrees very well with the obtained values of LCEs (see Fig. 5).

It is interesting to note that there are gaps (without family members) between the \((3, 3, -2)\) and \((5, -2, -2)\) resonances, as well as between the \((5, -2, -2)\) and \((7, -7, -2)\) resonances. We suggest that this is another confirmation that all these three resonances are connected and make one wide chaotic zone. Because of this, all asteroids from \( a_p \approx 3.167 \) au to \( a_p \approx 3.181 \) au reside in one of these three resonances. The asteroids can switch from one resonance to another. However, on a time-scale of a few Myr, this is a rare event, so that each asteroid spends most of the time in one of the resonances. As a result, because of some uncertainty in the procedure of computation of synthetic proper elements for resonant asteroids (i.e. averaging does not work well), all bodies appear to be located in (or close to) the centre of one of the resonances. This can be verified by using the analytical proper elements\(^{11}\) of Milani & Knežević (1990). These elements are calculated by means of analytical theory based on the series development of the perturbing Hamiltonian, and which does not include averaging. The distribution of Theobalda family members in the space of analytical proper elements (in the \( a_p, e_p \) plane) is shown in Fig. 8. The shown distribution is roughly random and without gaps in terms of proper semimajor axis, which confirms our claim that the gaps appear as a result of the averaging procedure. Moreover, this means that switching from one resonance to another must be a rare event. However, the fact that not all of the asteroids are located in the centre of one of the resonances is further evidence that resonance switching is possible (i.e. these three resonances are connected).

The position of the largest remnant, asteroid (778) Theobalda, is not close to the centre of the family. This is also evident in the

\[^{11}\] We did not use analytical proper elements, in our other analysis throughout the paper, because they are not accurate enough in this high eccentricity region. This can be appreciated by comparing the distributions of Theobalda family members with \( a_p \geq 3.183 \) au, shown for two different types of proper elements. The obvious grouping of the regular members, which is clearly visible in the space of synthetic proper elements (Fig. 7), disappears in the space of analytical proper elements (Fig. 8).
its real motion quantitatively, but qualitatively. Still, the behaviour of its semimajor axis suggests that chaos may be responsible for the displacement of (778) Theobalda from the centre of the family in terms of $e_p$ and $I_p$. However, if this were the case, this asteroid would probably spend some time residing in the $(5, -2, -2)$ resonance, which is strong enough to increase its eccentricity from $e_p \approx 0.253$ to $e_p \approx 0.259$, on the time-scale of several Myr.

Studying the distribution of family members shown in Fig. 7, it can be noted that there are no family members located inside the equivelocity ellipses, at $a_p \approx 3.165$ au. Contrary to the gaps between the resonances, the absence of asteroids that belong to the family in this region cannot be explained by dynamical instability or by ‘weakness’ of the procedure of proper-element calculation. Although a detailed study of this problem is beyond the scope of our work, we believe that this may be related to the impact characteristics (cratering event), which ‘forced’ fragments to be symmetrically distributed around the semimajor axis of the largest fragment (778) Theobalda.

3 AGE OF THEOBALDA FAMILY

3.1 Backward integration

The backward integration of orbits is a very accurate method for estimating the age of a family, and it works well with young families. It is based on the fact that because of planetary perturbations the orientation of orbits in the space changes over time. Consequently, two angles that determine the orientation of orbits in space, the longitude of the ascending node ($\Omega$) and the longitude of perihelion ($\sigma$), evolve with different but nearly constant speeds for individual orbits. After some time, this effect spreads out $\Omega$ and $\sigma$ uniformly around 360°. However, immediately after the disruption of the parent body, the orientations of the orbits of the fragments must have been nearly the same. Given this, the age of an asteroid family can be determined by integrating the orbits of the family members backwards, until the orbital orientation angles cluster around some value. This method was used by Nesvorny et al. (2002, 2003) to determine the ages of the Karin cluster (5.8 ± 0.2 Myr) and Veritas family (8.3 ± 0.5 Myr).

Here, we applied the method of Nesvorny et al. to try to estimate the age of the Theobalda family. By integrating the orbits of the Theobalda asteroid family back in time, hopefully we can find a conjunction of orbital elements ($\Omega$ and $\sigma$), which occurred only immediately after the disruption of the parent body. This method is, however, limited to groups of objects moving on regular orbits, which, even in this case, can be accurately tracked up to about 20 Myr in the past. Similarly, as in the case of the Veritas family (Nesvorny et al. 2003; Tsiganis et al. 2007), only a fraction of the Theobalda family members satisfy this condition and can be accurately integrated back in time. Also, as pointed out by Nesvorny et al. (2003) (see also Nesvorny et al. 2008) the region around $a_p = 3.175$ au is close enough to the 2/1 MMR with Jupiter to undergo fast differential evolution of the arguments of perihelion. This induces variability in the evolution histories and complicates any attempt to determine the age of the Theobalda family using arguments of perihelion. Thus, we selected 13 Theobalda family members, which have Lyanupov times $T_{lyap} \geq 10^7$ yr, and propagated their orbits 20 Myr backwards. All these members are located at $a_p \geq 3.183$ au.

Fig. 10 shows the average value of $\Delta \Omega$ for these 13 asteroids. The conjunction of nodal longitudes at $\approx 6.2$ Myr suggests that the Theobalda family, or at least a part of the family located at $a_p \geq 3.183$ au, was formed by a catastrophic collision at that time.

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12 All integrations presented in this paper are performed using the public domain \texttt{gravity9} integrator embedded in the multipurpose \texttt{gravity} package (http://hamilton.dm.unipi.it/astdys/), and the dynamical model which includes the four major planets (from Jupiter to Neptune) as perturbing bodies. The indirect effect of the inner planets is accounted for by applying a barycentric correction to the initial conditions.
The average $\Delta \Omega$, at $\approx 6.2$ Myr, is $\approx 58^\circ$, much smaller than at any other time. This suggests a statistical significance of the $\approx 6.2$ Myr event. In this case, however, the $\langle \Delta \Omega \rangle$ values are substantially more spread at $\approx 6.2$ Myr than in the case of the Karin cluster ($\langle \Delta \Omega \rangle$ is $\approx 10^\circ$) or the Veritas family ($\langle \Delta \Omega \rangle$ $\approx 40^\circ$). This is primarily for two reasons. (i) At least a few MMRs exist in the semimajor axis range from 3.18 to 3.19 au. Thus, despite the present long Lyapunov times of the selected orbits, these orbits might have experienced periods of chaotic motion in the past. (ii) All regular bodies, whose orbits can be accurately tracked back in time, are small bodies ($\lesssim$5 km) and consequently subject to Yarkovsky thermal force, which, even on this relatively short time-scale, can produce large enough changes in the semimajor axes, and consequently affect the secular frequencies in a way that is difficult to reconstruct.

In order to estimate how sensitive this result is on the semimajor axis drift as a result of the Yarkovsky effect, an additional investigation should be carried out. As the Yarkovsky induced drift depends on several parameters, we had to decide the values of the parameters characterizing it. These are asteroid spin axis orientation ($\gamma$), rotational period ($P$), surface and bulk densities ($\rho$), surface thermal conductivity ($K$) and specific heat capacity ($C$). As the Theobalda family members are most likely C-type asteroids, we have adopted the following values of parameters: $K = 0.01–0.5$ W (m K)$^{-1}$, $C = 680–1500$ J (K kg)$^{-1}$ and the same value for surface and bulk density $\rho = 1300–1500$ kg m$^{-3}$. The rotational periods are chosen according to a Gaussian distribution peaked at $P = 8$ h, while the distribution of spin-axis orientation is assumed to be uniform. These values are consistent with C-type asteroids (Brož 2006; Brož & Vokrouhlický 2008).

Next, we made 20 ‘yarko’ clones for each of the 13 regular members, by assigning random values of the parameters, from adopted intervals, to each clone. Then, we integrated$^{13}$ the orbits of all clones (260 orbits in total), but accounting not only for gravitational perturbations but also for the Yarkovsky effect. The initial orbital elements of the asteroids and planets were the same as in the previous experiment. Finally, we checked how the values of average $\Delta \Omega$ change with different combinations of clones. We found that the result shown in Fig. 10 is very sensitive to the Yarkovsky induced drift, as expected. In a few cases, any significant clustering even disappeared, but in most of the cases we obtained a deeper minimum. The deepest minimum that we found is related to the clustering within $\approx 31^\circ$ about 6.4 Myr ago (Fig. 10), which is still within the error bars obtained from the integrations without Yarkovsky force.

Although the clustering at about 6.2 Myr within $\approx 58^\circ$ is the most significant on the time-scale of 20 Myr, there is another clustering at about 15.5 Myr within $\approx 65^\circ$ (see Fig. 10). As this clustering appears in the more distant past where we would expect less tight clustering, it is not possible to rule out its significance. Also, to use the argument of perihelions is impossible, because the changes in the semimajor axes, caused by the Yarkovsky effect, coupled with the large gradient$^{14}$ of secular frequency ($dg/da \approx 0.3$ yr$^{-1}$ au, where g is the longitude of perihelion frequency), erase the evolution histories of these angles. Given this, we believe that, in the case of the Theobalda family, the backward integration method is not enough to draw a firm conclusion about the age of the family.

### 3.2 Chaotic chronology

In this section, we present the results obtained by using the MCC in order to estimate the age of the Theobalda family. This model was successfully applied by Novaković et al. (2010a) to estimate the ages of the Veritas and Lixiaohua asteroid families (see also Novaković, Tsiganis & Knežević 2010b). In order to apply the MCC, the family has to be located in the region of the main belt where diffusion takes place. Also, it is necessary that diffusion is fast enough to cause measurable effects, but slow enough so that most of the family members are still forming a robust family structure. As our results about diffusion speed suggest, the Theobalda family is an excellent example in this respect (see Section 3.2.1).

The basic steps and the model that we used in our Markov chain Monte Carlo (MCMC) simulations are explained in Novaković et al. (2010a), and thus we describe these here only briefly. Our model simulates the evolution of the family in three-dimensional (3D) space (i.e. the proper semimajor axis $a_p$ and two actions $J_1, J_2$; see Section 3.2.1 for a definition of these actions). At the beginning of a simulation, the random walkers are distributed in the region that was presumably occupied by the family members immediately after the impact event. Then, at each time-step $dt$, the random walkers can change their positions in every direction, in the 3D space. The length of the jump in $a_p$ is controlled by the Yarkovsky thermal force (Farinella & Vokrouhlický 1999), while the lengths of the jumps in $J_1$ and $J_2$ depend on diffusion speed (i.e. on the diffusion

$^{13}$ These integrations were performed using the oumr9 integrator in the Grid environment (Novaković et al. 2009).

$^{14}$ Caused by the proximity of the Theobalda family to the 2/1 resonance with Jupiter.
coefficients). At the time-step when 0.3 per cent of random walkers leave an ellipse in the \((J_1, J_2)\) plane, which corresponds to a 3σ confidence interval of a two-dimensional Gaussian distribution, the simulation stops. The number of time-steps multiplied by the time-step \(dt\) gives the age of the family.

3.2.1 Diffusion coefficients

Some of the most important information, needed as input for the MCMC simulations, are the values of diffusion coefficients in the region of interest. As shown by Novaković et al. (2010a), to obtain a good estimate of the family age using the MCC, it is enough to determine the diffusion coefficients as a function of the proper semimajor axis \(a_p\). This is our next step.

As well as the MCC, the procedure for determining the diffusion coefficients, as functions of the proper semimajor axis, is described in Novaković et al. (2010a). Let us mention here only its main features and numbers related to this work. The orbits of \(~5000\) fictitious bodies, distributed randomly in the same ranges of osculating orbital elements as the real family members at present, are propagated for 10 Myr. Then, the time series of mean proper elements (Milani & Knežević 1998) for all of these are calculated. The mean proper elements are transformed to actions according to the relations:

\[
J_1 \approx \frac{1}{2} \sqrt{\frac{m}{a_1}} e_m^2
\]

and

\[
J_2 \approx \frac{1}{2} \sqrt{\frac{m}{a_1}} \sin^2 I_m.
\]

Here, \(a_1\) denotes Jupiter’s semimajor axis, \(e_m\) is the mean eccentricity and \(I_m\) is the mean inclination of the asteroids. Next, the family is split in the small cells, in terms of \(a_p\), using a kind of moving-average technique with a cell size of \(\Delta a_p = 5 \times 10^{-4}\) au and a step size of \(\Delta a_p = 2 \times 10^{-4}\) au. Finally, the mean squared displacements \(\langle \Delta J^2 \rangle\), for both actions, are calculated, and the diffusion coefficients \(D(J_1), D(J_2)\) are determined for each cell, as the least-squares fit slope of the \(\langle \Delta J^2 \rangle(t)\) curve.

The obtained values of diffusion coefficients \(D(J_1)\) and \(D(J_2)\) in the Theobalda family region are shown in Fig. 11. The fastest diffusion is associated with the \((5, -2, -2)\) three-body MMR, but the diffusion is also very fast in the \((3, 3, -2)\) three-body MMR, and these two chaotic zones seem to be connected. The third chaotic zone, associated with the \((7, -7, -2)\) resonance, is connected to the first two in terms of diffusion in \(J_1\), but not in terms of diffusion in \(J_2\). This is in relatively good agreement with the results presented in Section 2.3. The diffusion is faster in \(J_1\) than in \(J_2\). While the local minimum near the centre of the \((5, -2, -2)\) resonance exists for \(D(J_1)\), as in the case of the Veritas family (see Novaković et al. 2010a), there is no such feature for \(D(J_2)\). In the region for \(a_p \geq 3.185\) au the values of \(D(J_2)\) are practically zero, but there is some diffusion in \(J_1\). It should be noted here that this might affect age estimation using the backward integration method, but not significantly. This is because most of the 13 asteroids that we have used to apply the backward integration method are located close to \(a_p = 3.185\) au, where both values, \(D(J_1)\) and \(D(J_2)\), are close to zero.

\footnote{The ellipse is determined by the present size of the family or, as in this case, by the present size of a particular part of the family. It should not be confused with equivelocity ellipses shown, for example, in Fig. 7.}

An important general conclusion can be drawn by comparing the values of the diffusion coefficients obtained for the region occupied by the Veritas family (Novaković et al. 2010a) with the values obtained here. The two families stretch over a similar range of the semimajor axis, but members of the Theobalda family have higher inclinations and significantly higher eccentricities. The estimated diffusion is about one order of magnitude faster in the region occupied by the Theobalda family than in the region occupied by the Veritas family. This confirms the fact that chaos is dominant at higher eccentricities.

3.2.2 Monte Carlo simulations

Having obtained the values of the diffusion coefficients, we are ready to apply the MCC to estimate the age of the family. There are two separate parts of the Theobalda family suitable for application of the MCC. These are bodies inside the \((5, -2, -2)\) and \((3, 3, -2)\) three-body MMRs. Following Tsiganis et al. (2007), who deal with the Veritas family, we have called these bodies group A (5, -2, -2) and group B (3, 3, -2). As the results about the diffusion coefficients have confirmed, there exists significant diffusion in both groups. This gives a unique opportunity to apply the MCC to both groups and to obtain two independent age estimates. A good agreement between these two estimates, as well as with the age derived using the backward integration method, would suggest a reliable result.

As the present size of the chaotic zone is a critical parameter in our model, we start with the family identified by applying the HCM for the velocity cut-off of \(d_{cut-off} = 65\) ms\(^{-1}\). This is probably the lowest acceptable value of \(d_{cut-off}\) in the case of the Theobalda family. With this cut-off velocity, we identified 30 bodies from group A and 16 bodies from group B. The corresponding sizes of these groups in \(J_1\) and \(J_2\) are: group A \((10.67 \pm 1.03) \times 10^{-4}\) and \((3.82 \pm 0.47) \times 10^{-4}\); group B \((10.32 \pm 1.05) \times 10^{-4}\) and \((4.00 \pm 1.06) \times 10^{-4}\).

Using these sizes of the two chaotic groups, for each group, we performed 16 sets of MCMC simulations (each set consisting of 100 runs), using a different number of random walkers \(n\) (2000 or 5000), time-step \(dt\) (from 100 to 2000 yr) and for two initial sizes of the family, which correspond to velocities of \(v = 35\) ms\(^{-1}\) and \(v = 40\) ms\(^{-1}\) (see Fig. 7). From these simulations, we derived
The age of the Theobalda family to identify the nominal family. It should be noted here that \(\mu_0\) and \(\sigma^2\) are the mean and standard deviation of the ith subsample, respectively. The obvious discrepancy between the ages derived from the two different groups is the reason why we reject the age of 2.1 Myr, derived using group A bodies. We have found that the family now consists of 128 members. By analysing the SF of the identified family members, we have been able to infer the diameter of the parent body to be \(D_\text{P} \approx 78 \pm 9\) km. However, this estimate is based on the assumption that all family members have the same albedo as the largest family member, asteroid (778) Theobalda. In order to obtain a better estimate, the albedos of as many family members as possible are desirable. Ongoing projects, such as the Wide-Field Infrared Survey Explorer (WISE), should improve the situation significantly in this respect.

Most, but not all, of the Theobalda family members move on chaotic orbits, thus giving rise to the significant chaotic diffusion that has been changing the kinematical structure of the family over time. The study of the dynamical characteristics, in the region occupied by the family, has shown that three-body MMRs are the most efficient in shaping the family. These are the (3, 3, 7), (5, –2, –2) and (7, –7, –2) resonances. These are connected to the high eccentricity region, allowing bodies to switch from one resonance to another.

The fact that some of the family members have stable orbits is the reason why we were able to apply the backward integration method to estimate the age of the family. However, the presence of chaos in the region occupied by the family allows us to use the MCC in order to estimate the age. Using both methods, and combining the results, we found the age of the Theobalda family to be 6.9 ± 2.3 Myr. Given the very good agreement between the results obtained with different methods, as well as when applied to different groups, we believe this estimate is very robust. Thus, this is another family younger than 10 Myr. This result has several important implications, some of which we mention below.

Young asteroid families are also known to be a source of Solar system dust bands (see, for example, Grün et al. 1985; Nesvorný et al. 2003). The origin of three main dust bands is known, and these correspond to the Karin, Veritas and Beagle families (Nesvorný et al. 2009). However, the origin of some less prominent bands, such as the so-called M/N dust band, is still not quite clear. The young age of the Theobalda family and its proper inclination of \(I_\text{P} \approx 14.3\) suggest that it might be a possible source of the M/N dust band (\(I_\text{P} \approx 15^\circ\); Sykes 1990). However, this dust band has been linked to the (170) Maria asteroid family (Reach, Franz & Weiland 1997), and more recently to the (1521) Seinajoki cluster (Nesvorný et al. 2003). In any case, the dust band produced by such a young family
as Theobalda should be observable. The size of its parent body also suggests that it should produce a prominent dust band. If not the M/N dust band, then there must be another dust band that can be linked to this family. Alternatively, it should be explained why and how this dust band has disappeared.

The Theobalda asteroid family is located very close to the region where three (out of four) so-called main-belt comets (MBCs) \(^{18}\) have been discovered (see, for example, Jewitt, Yang & Haghhighipour 2009). As suggested by Hsieh (2009), it is possible that this type of body can be found among the members of other young families, probably many of which are waiting to be discovered. Being young and dominated by C-type asteroids, we believe the Theobalda family is very good place to start.

ACKNOWLEDGMENTS

I am grateful to Zoran Knežević and Rade Pavlović for their useful suggestions on the manuscript. I also would like to thank David Nesvorný, the referee, for his useful comments and suggestions, which helped me to improve this paper. This work has been supported by the Ministry of Science and Technological Development of the Republic of Serbia (Project No 146004 ‘Dynamics of Celestial Bodies, Systems and Populations’).

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\(^{18}\) MBCs are bodies with asteroid-like dynamical properties but comet-like physical properties (Hsieh, Jewitt & Fernández 2004). These are dynamically ordinary main-belt asteroids on which, probably, subsurface ice has recently been exposed (e.g. because of a collision).

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