Multiple Ultralight Axionic Wave Dark Matter and Astronomical Structures

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An ultralight scalar boson with mass \( m_1 \simeq 10^{-22} \) eV is gaining credence as a Dark Matter (DM) candidate that explains the dark cores of dwarf galaxies as soliton waves. Such a boson is naturally interpreted as an axion generic in String Theory, with multiple light axions predicted in this context. We examine the possibility of soliton structures over a wide range of scales, accounting for galaxy core masses and the common presence of nuclear star clusters. We present a diagnostic soliton core mass-radius plot that provides a global view, indicating the existence of an additional axion with mass \( m_2 \simeq 4 \times 10^{-20} \) eV, with the possibility of a third axion with mass \( m_3 \gtrsim 6 \times 10^{-18} \) eV.

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INTRODUCTION AND SUMMARY

Dark Matter (DM) is consistently established from a wide range of astronomical evidence, including dynamical, lensing and Cosmic Microwave Background (CMB) data. However, the nature of dark matter is far from clear, requiring new physics beyond the standard particle physics model that describes only the \( \simeq 17\% \) baryonic contribution to the total cosmological mass density [1, 2]. It is understood that the majority of the dark matter must be non-relativistic, to the earliest limits of observation, otherwise CMB and the large-scale distribution of galaxies would be featureless on small scales. However, no evidence for the weakly interacting massive particle (WIMP) for the non-relativistic Cold Dark Matter (CDM) scenario has been found, despite increasingly stringent laboratory searches. Alternatively, dark matter as a Bose-Einstein condensate also satisfies the non-relativistic requirement for dark matter, as very light bosons in the ground state behave as Bose-Einstein condensate because of the high occupation number of such light bosons. The uncertainty principle means bosons cannot be confined within the de Broglie scale, naturally avoiding some of the problems encountered in the weakly interacting massive particle model in the CDM scenario, as raised in the fuzzy dark matter scenario [3] with an ultralight boson.

Ultralight bosonic dark matter has recently been successfully simulated for the first time, revealing an unforeseen rich wavelike substructure that may be termed "wave dark matter" (ψDM) model [4, 5]. These ψDM simulations evolve the coupled Schrödinger-Poisson equations [8], under the simplest assumption of negligible interaction other than gravity, producing halos with a central core that is a stationary, minimum-energy solution, or soliton, surrounded by an envelope resembling a CDM halo when averaged azimuthally [4, 5], comprising turbulent, interference pattern of de Broglie scale density fluctuations. The prominent solitonic wave at the base of every virialised potential represents the ground state of the condensate, where self gravity is matched by effective pressure from the uncertainty principle. The solitons found in the simulations have flat cored density profiles that accurately match the known time independent solution of the Schrödinger-Poisson equation [4, 5, 9, 10], for which the soliton mass scales inversely with its radius [11].

Here we take seriously the generic String Theory prediction of multiple axion fields, and search for astronomical evidence of multiple axions. An axion mass has been derived in this context to be \( m_3 \simeq 10^{-22} \) eV [8], corresponding to a solitonic core of 1 kpc of mass \( 10^8 \) \( M_{\odot} \), for the well studied Fornax galaxy [4], representing the most common class of dwarf spheroidal galaxy, for which dark matter dominates over stellar mass. This axion is predicted to form a denser solitonic core in more massive galaxies, like the Milky Way, of about \( \simeq 10^9 \) \( M_{\odot} \) and radius \( \sim 100 \) pc [4, 12]. We also advocate a solitonic origin for the puzzling dynamically distinct, nuclear star cluster of \( 10^7 \) \( M_{\odot} \) at the center of the Milky Way that surrounds the central black hole on a scale of \( \sim 1 \) pc. We find evidence that the inner density profile of this nuclear star cluster (NSC) is fitted by a dense soliton of dark matter corresponding to a heavier axion with mass \( \simeq 4 \times 10^{-20} \) eV. This inner soliton amounts to a small dark matter contribution in addition to the dominant \( 10^{-22} \) eV axion responsible for galaxy formation in this context, forming two concentric soliton structures within the Milky Way...
on scales of 100 pc and 1 pc for axions of $10^{-22}$ eV and $10^{-20}$ eV respectively. This “nested” soliton structure is also consistent with our interpretation of the puzzling presence of a central star cluster in the ultra-faint dwarf galaxy Eridanus II [13].

We form a global view of the $\psi_{DM}$ scenario by displaying all the claimed relevant data into a plane of soliton core mass versus core radius, in Fig. [4] which is the main result of this paper. This preliminary plot clearly suggests the existence of at least 2 distinct ultralight axions, with masses $m_1 \simeq 10^{-22}$ eV and $m_2 \simeq 4 \times 10^{-20}$ eV, and also the possibility of a third axion with mass $m_3 \gtrsim 6 \times 10^{-18}$ eV. Based on the very general form of this plot, we can accommodate recently claimed compact masses at the centers of globular clusters (GCs) as solitons (rather than black holes). We also predict that the core within the massive lensing galaxy cluster (MGC) to be very compact, with $r_c \approx 10$ pc. Further focused testing of this unifying conclusion will clarify the extent to which these widely different astronomical structures can be understood as a manifestation of multi-ultralight axions.

![Image](image1.png)

**FIG. 1:** The Soliton Core Mass-Radius Plot illustrates the map of soliton core mass $M_c$ versus the core radius $r_c$ from cosmological structures at hierarchically different scales of the observational data points plotted together with theoretical prediction of solitons. The Milky Way (MW) and dSph galaxies data suggest an axion mass $m_1 \approx 10^{-22}$ eV. The nuclear star cluster (NSC) and Eri-II suggest an axion with $m_2 \approx 4 \times 10^{-20}$ eV. The Globular Clusters M15 and 47 Tuc data suggest an axion with mass $m_3 \gtrsim 6 \times 10^{-18}$ eV. The triangles indicate data given for the two ultra-faint dwarf galaxies computed originally in Ref. [14] while the corresponding points with errors are from our re-analysis. The core radius of a compact deflector near the center of a massive galaxy cluster (MGC) is undetermined, but bounded by 1 kpc. We predict that it must be extremely compact ($r_c \approx 10$ pc) to fit in this picture with axion $m_1$.

In this paper, we first consider the solitonic properties of the multiple axion scenario. We find that, to a reasonable approximation, solitons from different axions have negligible effect on each other. We illustrate this point with the solution of two “nested” solitons corresponding to two axions with different masses. Next, we constrain the mass of the soliton dark matter contribution to the motion of stars within the central nuclear star cluster in our galaxy. We combine this with the published results on galaxy scales to present the soliton core mass - core radius plot in Fig. [4] supporting the multi-axion picture. This plot also suggests a re-analysis of the dwarf galaxy data, which indicates a possible convergence in the picture. We end with the discussion and conclusion.

## Multiple Axions and Nested Solitons

It is easy to extend the $\psi_{DM}$ formalism to that for the multiple axion case. At the first order perturbative theory and in non-relativistic limit, we have the Schrödinger-Poisson equations for $N$ axion fields evolving on a Newtonian expanding background as follows

$$i\hbar \frac{\partial \psi_1}{\partial t} = \left( -\frac{\hbar^2}{2m_1 a^2} \nabla^2 + m_1 \Phi \right) \psi_1,$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \left( -\frac{\hbar^2}{2m_2 a^2} \nabla^2 + m_2 \Phi \right) \psi_2,$$

$$\vdots$$

$$i\hbar \frac{\partial \psi_N}{\partial t} = \left( -\frac{\hbar^2}{2m_N a^2} \nabla^2 + m_N \Phi \right) \psi_N,$$

$$\frac{\nabla^2 \Phi}{4\pi G a^2} = |\psi_1|^2 + |\psi_2|^2 - \frac{3H^2}{8\pi G}.$$  

In this non-relativistic regime, the multiple-axion features can be captured in the two-axion case, so let us focus on this simplified case. For simplicity, we take into account the fact that the characteristic time for evolution of the system is short compared to the age of the universe, so $a$ becomes unity and $H$ vanishes. In addition, we also consider the system in symmetrically spherical coordinate and find the stationary solution expressed by $\psi_i(x,t) = \psi_i(x) e^{-iE_i t/\hbar}$. Furthermore, to simplify physical constants, we rescale these quantities into the dimensionless variables [15]

$$r = \frac{\hbar^2}{2m^2 G M} \tilde{r}, \quad \psi_i = \left( \frac{2G^3 m^6 M^4}{\pi \hbar^6 M_1} \right)^{1/2} \psi_i,$$

$$\Phi = \frac{2G^2 M^2}{\hbar^2} \tilde{\Phi}, \quad E_i = \frac{2G^2 M^2 m^2}{\hbar^2} m_i \tilde{E}_i,$$

where $M_1, M_2$ are the total masses of the gravitational structure formed by each axion and $m, M$ are the scale parameters which could be determined in a specific system. $\Phi, E_1, E_2$ are one-particle potential and one-particle energy respectively, $\psi_1, \psi_2$ are one-particle wavefunctions which are normalized individually, $\int |\psi_i|^2 d^3 x = 1$. This normalization of wavefunctions also implies $M_{tot} =$
\[ \int \rho(x) d^3x = M_1 + M_2, \quad \text{where} \quad \rho(x) = M_1 |\psi_1|^2 + M_2 |\psi_2|^2. \]

Finally, we obtain a system of scale-invariant equations

\[ \begin{align*}
\frac{\partial^2 \tilde{\psi}_1}{\partial \tilde{r}^2} &= \frac{2}{\tilde{r}} \frac{\partial \tilde{\psi}_1}{\partial \tilde{r}} + \left( \frac{m_1}{m} \right)^2 (\tilde{\Phi} - \tilde{E}_1) \tilde{\psi}_1, \\
\frac{\partial^2 \tilde{\psi}_2}{\partial \tilde{r}^2} &= \frac{2}{\tilde{r}} \frac{\partial \tilde{\psi}_2}{\partial \tilde{r}} + \left( \frac{m_2}{m} \right)^2 (\tilde{\Phi} - \tilde{E}_2) \tilde{\psi}_2, \\
\frac{\partial^2 \tilde{\Phi}}{\partial \tilde{r}^2} &= -2 \frac{\partial \tilde{\Phi}}{\partial \tilde{r}} + \left| \tilde{\psi}_1 \right|^2 + \left| \tilde{\psi}_2 \right|^2.
\end{align*} \]

These equations can be solved numerically under some necessary constraints

\[ \begin{align*}
&\tilde{\psi}_1(0) = \tilde{\psi}_2(0) = \tilde{\Phi}(0) = 0, \\
&\tilde{\psi}_1(\infty) = \tilde{\psi}_2(\infty) = 0, \\
&\int_0^\infty |\tilde{\psi}_1|^2 \tilde{r}^2 d\tilde{r} = \frac{M_i}{M},
\end{align*} \]

given at least the ratio \( m_1/m_2 \) and \( M_1/M_2 \), once we have solution in term of \( \tilde{\psi}_i(\tilde{r}) \) we can find the corresponding density profile at any scale by choosing appropriate value for \( m \) and \( M \), the origin of gravitational potential does not change the final solution because of the shift symmetry in the equations. As an illustration, we approach the problem by initially setting the central values of wavefunctions \( \tilde{\psi}_1(\tilde{0})/\tilde{\psi}_2(\tilde{0}) \) and identify the normalization factors after the corresponding solution has found, as a result, if \( \int_0^\infty |\tilde{\psi}_1|^2 \tilde{r}^2 d\tilde{r} = \alpha_i \), the following density profile

\[ \rho(r) = \frac{2M_i^4 m^6 G^3}{\pi \hbar^6} \left( \left| \tilde{\psi}_1 \right|^2 + \left| \tilde{\psi}_2 \right|^2 \right), \]

where \( \tilde{\psi}_1, \tilde{\psi}_2 \) are solution of [\( \Box \)], describes a soliton with the total mass \( M_{\text{tot}} = (\alpha_1 + \alpha_2) M \). Assume \( m_1/m_2 = 3 \) and \( m^2 = m_1 m_2 \), a few solutions are listed in Table I.

Fig. 2 compares these solutions with the universal single-axion profile suggested in [4]. Using the same setup with \( m_2 = m_1/3 = 8 \times 10^{-23} \text{ eV} \) and \( \tilde{\psi}_1(\tilde{0}) = 3 \tilde{\psi}_2(\tilde{0}) \), we compare the gravitationally-coupled solution to the sum of the two solitons, as shown in Fig. 3. We see that the coupling between the two solitons is negligibly small for radii not much bigger than the core radii.

We conclude that soliton mass density profile from different axions hardly influence each other’s presence; so we may accurately describe overlapping solitons from different axions with the simple sum of individual mass density profiles, each of which follows the individual core mass - core radius relation for the solitons using the general soliton profile [4].

\[ \begin{align*}
M_c &\simeq \frac{5.5 \times 10^9}{(m/10^{-23} \text{ eV})^2(r_c/\text{kpc})} M_\odot, \\
\rho_s(r) &\simeq \frac{1.9(m/10^{-23} \text{ eV})^{-2}(r_c/\text{kpc})^{-4}}{[1 + 9.1 \times 10^{-2} (r,r_c)^2]^8} M_\odot / \text{pc}^3,
\end{align*} \]

\[ \begin{array}{|c|c|c|c|c|}
\hline
\psi_1(0) & \psi_2(0) & \tilde{E}_1 & \tilde{E}_2 & M_1/M_2 \\
\hline
0.5 & 1.0 & 0.69765202 & 1.68251566 & 0.202 & 7.392 \\
1.0 & 1.0 & 0.83895681 & 1.91670679 & 0.638 & 6.621 \\
2.0 & 1.0 & 1.24841522 & 2.59599377 & 1.488 & 5.000 \\
3.0 & 1.0 & 1.72370456 & 3.38136093 & 2.116 & 3.938 \\
\hline
\end{array} \]

\( \text{TABLE I: Stationary solutions in terms of} \tilde{E}_1 \text{ and} \tilde{E}_2 \text{ with different central values of rescaled wavefunctions, using Newton-Raphson method. Notice that} \tilde{E}_1 \text{ and} \tilde{E}_2 \text{ are extremely sensitive to the accuracy of solution in this two-axion problem and can vary in their two-dimensional parameter space depending on the bins size of} \tilde{r}. \)

\[ \text{FIG. 2: Soliton mass density profiles in various scenarios where} M_{\text{tot}} = M_1 + M_2 = 10^5 M_\odot, \quad m_2 = m_1/3 = 8 \times 10^{-23} \text{ eV}, \quad \text{and} \quad M_i \text{ is the total masses of individual soliton. We see that as the fraction of light axion increases, the two-axion profile approaches the single-axion one and vice versa.} \]

\[ \text{SOLITONIC CORE MASS - CORE RADIUS} \]

Now we can discuss the observational data for the \( \psi \text{DM model in Fig. 4 and Table I. This soliton core mass-core radius plot provides evidence that more than one axion exists; it also allows us to make predictions (e.g., the size of the massive galaxy clusters). Note that the masses involved spans 7 orders of magnitude while the distance scale spans 5 orders of magnitude. The only theoretical parameters are the axion masses.} \]

- **Dwarf Spheroidal Fornax, Carina, Sculptor:** By matching the phase space distribution of stars within the well-studied Fornax galaxy and 7 other classical dSph galaxies, Ref. 11[10] give strong evidence for an axion of order \( \simeq 10^{-22} \text{ eV} \), which is most
commonly realized in ψDM framework. We list in Table II and show in Fig. 1 just three of them. The others, including Sextans, Draco Leo I, Leo II, Ursa Minor, yield similar values for the axion-soliton parameters.

- **Ultrafaint Dwarf Draco II, Triangulum II:** The stellar dynamics of Draco II (Dra-II) and Triangulum II (Tri-II) imply their half-light masses of $\log_{10}[M_{1/2}] = 5.5^{+0.4}_{-0.2}$ and $\log_{10}[M_{1/2}] = 5.9^{+0.4}_{-0.2}$ together with their half-light radius $r_h = 19^{+8}_{-6}$ pc and $r_h = 34^{+9}_{-8}$ pc respectively [17, 18]. By adopting an assumption that the maximum halo mass of $\sim 2 \times 10^{10} M_\odot$ Ref. [14] estimates values of appropriate axion masses for each galaxy, which are given in Table II. It is not clear that the measured stellar velocity dispersion is sampled to beyond the soliton radius of these two galaxies, in which case the total soliton mass may be underestimated and hence the axion mass is overestimated. So if we instead adopt the axion mass $m_1$ derived for Fornax [4] located in these dwarf galaxies, we find that the result is shifted and prefers a lighter axion mass as shown in Fig. 1. Substantially increased spectroscopy will help extend this mass profile measurement to larger radius for a reliable soliton mass estimate.

- **Milky Way:** Using a scaling relation derived from the simulations between the mass of soliton and its host virial (halo) mass, $M_{\text{soliton}} \propto M_{\text{virial}}^{1/3}$ [12], Ref. [12] matches the missing mass recently found within the central $\sim 100$ pc of Milky Way [19, 20] with a soliton of $\sim 10^9 M_\odot$ and radius $\sim 100$ pc through Jeans analysis, corresponds to an axion with mass $\sim 8 \times 10^{-23}$ eV. This massive concentrated soliton explains well the projected radial enhancement of bulge star velocity dispersion peaking at $\sim 130$ km/s, that is 50 km/s above the general bulge level of $\sim 80$ km/s.

- **Massive Galaxy Cluster:** Ref. [21] reports a compact dark mass of $\sim 2 \times 10^{10} M_\odot$ near the center of a massive galaxy cluster (MGC), from the radius of curvature of a small lensed structure in a well resolved background galaxy lensed through the center of a massive lensing cluster MACS1149 in recent deep Hubble Frontier Fields images. However, the size is undetermined, with a radius bounded by about 4 kpc (or $r_c < 1$ kpc). This mass may be an offset black hole ejected orbiting the nearby massive galaxy cluster, or a compact soliton with mass of $10^9 M_\odot$ that is expected at the bottom of the potential of a massive cluster of $10^{15} M_\odot$. Assuming that this is due to the $m_1$ axion, we predict an extremely compact soliton with core radius $\sim 10$ pc, corresponding to the smaller de-Broglie wavelength of a massive cluster.

- **Ultrafaint Dwarf Eridanus II:** The dwarf galaxy Eridanus II (Eri-II) is similar to the other dwarf galaxies described above, but has a curious central star cluster with mass $\sim 2 \times 10^9 M_\odot$, with half-light radius $r_h \sim 13$ pc [22]. From the stability of the star cluster centrally residing inside, Ref. [13] infers an axion mass $m \lesssim 10^{-19}$ eV. The half-light mass of Eri-II,

![FIG. 3: Soliton profiles calculated with three different methods, here we consider the same central density in all three cases, $\rho_1(0) = 9\rho_2(0) \approx 6.3 M_\odot/pc^3$, and $m_1 = 3m_2 = 8 \times 10^{-23}$ eV. The red-solid curve shows the numerical solution of the two-axion system with a full treatment of non-linear gravitational interaction between two axions while the black-dotted one is obtained when we simply add two single-axion soliton profile in which the coupling between them is neglected, the corresponding core radius of each soliton is also shown. Therefore, the dashed-blue curve is served as a numerical solution solved from where the heavier axion is just influenced by gravitational potential generated by a flat potential of the lighter one. This approximation approaches the exact solution when $m_1 \ll m_2$ and $\rho_1(0) \ll \rho_2(0)$ and it is realized even in a non-extreme context shown in the graph above. For radii not much bigger than the core radii, the potential of a massive cluster of $10^{15} M_\odot$ and core radius $r_c \sim 10^9 M_\odot$, Ref. [12] matches the missing mass recently found within the central $\sim 100$ pc of Milky Way [19, 20] with a soliton of $\sim 10^9 M_\odot$ and radius $\sim 100$ pc through Jeans analysis, corresponds to an axion with mass $\sim 8 \times 10^{-23}$ eV. This massive concentrated soliton explains well the projected radial enhancement of bulge star velocity dispersion peaking at $\sim 130$ km/s, that is 50 km/s above the general bulge level of $\sim 80$ km/s.

| Structure | $m$ (eV) | $M_c (M_\odot)$ | $r_c$ (pc) |
|-----------|----------|----------------|----------|
| Fornax    | $8.1^{+0.5}_{-0.7} \times 10^{-23}$ | $9.1^{+0.7}_{-0.9} \times 10^7$ | $920^{+200}_{-100}$ |
| Carina    | $1.2^{+0.6}_{-0.7} \times 10^{-22}$ | $5.1^{+0.6}_{-0.7} \times 10^7$ | $741^{+171}_{-106}$ |
| Sculptor  | $1.3^{+0.5}_{-0.3} \times 10^{-22}$ | $5.5^{+0.6}_{-0.3} \times 10^7$ | $589^{+119}_{-146}$ |
| Tri-II    | $2.1^{+0.8}_{-0.9} \times 10^{-22}$ | $4.5^{+3.5}_{-2.6} \times 10^7$ | $289^{+225}_{-164}$ |
| (13)      | $\sim 3.8 \times 10^{-22}$ | $\sim 2.3 \times 10^7$ | $\sim 100$ |
| Dra-II    | $3.1^{+0.2}_{-0.1} \times 10^{-22}$ | $3.0^{+0.1}_{-0.1} \times 10^7$ | $185^{+144}_{-121}$ |
| (13)      | $\sim 5.6 \times 10^{-22}$ | $\sim 1.5 \times 10^7$ | $\sim 105$ |
| MW        | $1.0^{+0.4}_{-0.3} \times 10^{-22}$ | $3.3^{+1.5}_{-0.6} \times 10^8$ | $163^{+103}_{-46}$ |
| MGC       | $\lesssim 10^{-22}$ | $\sim 5 \times 10^9$ | $< 1000$ |
| Eri-II    | $\sim 1.5 - 9.2 \times 10^{-20}$ | $\sim 0.3 - 5.0 \times 10^4$ | $\sim 20 - 50$ |
| NSC       | $\sim 2.2 - 10 \times 10^{-20}$ | $\sim 3.5 - 23 \times 10^5$ | $\sim 0.15 - 0.45$ |
| M15       | $\gtrsim 5.8 \times 10^{-18}$ | $591^{+196}_{-189}$ | $\lesssim 0.022$ |
| 47 Tuc    | $\gtrsim 6.4 \times 10^{-18}$ | $544^{+354}_{-201}$ | $\lesssim 0.014$ |
\[ M_h = M_{1/2} = 1.2^{+0.4}_{-0.3} \times 10^7 \, M_\odot \] with radius \( \sim 280 \) pc \[ ^{22, 23} \] may be due to a soliton similar to that of the other dwarf galaxies. We propose the presence of another soliton inside to stabilize the star cluster. Taking the nested soliton picture, we just consider of another soliton inside to stabilize the star cluster. \[ p_c \] [22, 23] may be due to a soliton similar to that \[ M \] pc. \[ m \] profile on a scale of \( \simeq \) remains unclear, in particular the presence of a core ically distinct nuclear star cluster in the Milky Way \( \simeq \) to an axion of mass \[ h \] dark excesses of both 47 Tuc and M15 [30] corresponding and stellar mass black holes combined on a scale of 0.1 pc, \[ m \] Preferring a small but extended excess of binary stars [29] which tends to exclude such a central point mass, in a new high resolution stellar proper motion study mass has been shown to be lacking in the case of 47 Tuc intermediate mass black holes. However, support for a point mass has been shown to be lacking in the case of 47 Tuc in a new high resolution stellar proper motion study [29] which tends to exclude such a central point mass, preferring a small but extended excess of binary stars and stellar mass black holes combined on a scale of 0.1 or. A soliton explanation has been advocated for the dark excesses of both 47 Tuc and M15 [30] corresponding to an axion of mass \( \simeq 6 \times 10^{-18} \) eV, to account for the central dark core mass with an upper bound for the soliton core radius \( \sim 0.03 \) pc. For a smaller \( r_c \), the axion becomes heavier. Three other GCs, namely M79, M62, M54, yield similar results. It is entirely possible that, instead of a soliton, a black hole is the source of the core masses [31, 92]. More data is needed to definitively test this tentative axion-soliton explanation.

**Nuclear Star Cluster:** The origin of the dynamically distinct nuclear star cluster in the Milky Way remains unclear, in particular the presence of a core profile on a scale of \( \simeq 1 \) pc [24] in the old star population, for which a cusp is firmly predicted but not seen [25]. Also the possible presence of excess unseen matter of \( \simeq 10^6 \, M_\odot \) is implied on a scale of \( \simeq 0.4 \) pc, revealed by the high velocity orbit of the maser star IRS9, and the excessive proper motions of other stars at this radius [20], that imply an extended mass that is additional to the central black hole on a parsec scale. Whether this mass can be accounted for by stars of the surrounding NSC is unclear, requiring a better understanding of the stellar mass function in this region. An upper limit to the mass of the DM is set by the dynamics of the stellar motion in the NSC which implies a total mass of \( 3 \times 10^7 \, M_\odot \) on a scale of 3-5 pc radius of the NSC. Most of this dynamical based mass is thought to be stellar, though the uncertain choice of initial stellar mass function (IMF) means that as much as half this mass may not be stellar in the case of a Chabrier IMF rather than the Salpeter form [26, 27]. Under the conditions provided above, we derive a smaller soliton by matching the enclosed mass at the location of star IRS9 while also satisfying an upper bound closer to the center from the orbit of star S2 [28]. This ”inner” soliton implies a heavier axion, \( m_2 \simeq 4 \times 10^{-20} \) eV, than the lighter axion \( m_1 \) responsible for the ”outer” soliton associated with bulge star dynamics of the Milky Way described above and shown in Fig. 4.

**Globular Clusters M15 and 47 Tuc:** Compact dark masses are reported in two of the best studied Globular Cluster within the Milky Way of \( \simeq 2000 \, M_\odot \) and naturally interpreted as long anticipated intermediate mass black holes. However, support for a point mass has been shown to be lacking in the case of 47 Tuc in a new high resolution stellar proper motion study [29] which tends to exclude such a central point mass, preferring a small but extended excess of binary stars and stellar mass black holes combined on a scale of 0.1 pc. A soliton explanation has been advocated for the dark excesses of both 47 Tuc and M15 [30] corresponding to an axion of mass \( \simeq 6 \times 10^{-18} \) eV, to account for the central dark core mass with an upper bound for the soliton core radius \( \sim 0.03 \) pc. For a smaller \( r_c \), the axion becomes heavier. Three other GCs, namely M79, M62, M54, yield similar results. It is entirely possible that, instead of a soliton, a black hole is the source of the core masses [31, 92]. More data is needed to definitively test this tentative axion-soliton explanation.

![Fig. 4: Fitting the soliton profile with the enclosed mass of Nuclear Star Cluster from the motion of the star IRS9 at \( r \sim 0.365 \) pc from the central black hole [33]. Here the enclosed mass \( M(r) \) inside radius \( r \) is computed by integrating the soliton profile (7), after subtracting a black hole mass of \( \sim 4 \times 10^6 \, M_\odot \) and stars at the center of NSC. As a reference for comparison, we also highlight the mass range (the blue band) for the mass profile of solitons corresponding to the lighter axion \( m_1 \) which have a much wider core radius and relatively low central density [12]. Note, the accurate orbit of the closest orbiting star S2 provides a useful upper bound on any extended matter additional to the black hole [28].

**DISCUSSIONS AND CONCLUSION**

We have focused on the soliton feature as a central prediction of the wave dark matter picture and generalized to the case of multiple axions, motivated by String Theory, deriving a stable time-independent, joint ground state solution. We conclude that when the mass scales of two axions are spaced by more than a factor of a few in mass, the combined solution asymptotes to a nested pair of concentric solitons that are dynamically independent and this allows us to make clear comparisons with the observed structure at the center of our galaxy and other well resolved nearby galaxies where a dense nuclear star cluster is commonly present.

We have shown that the lightest axion of \( m_1 \simeq 8 \times 10^{-23} \) eV implied by both the Fornax Galaxy and the central Milky Way, can also help interpret the massive compact lens of \( 10^{10} \, M_\odot \) uncovered recently near the center of a massive galaxy cluster [21]. We can also the excess of missing mass measured within dense Nuclear
Star Cluster at the center of the Milky Way, which is well fitted by an axion of \( m_\chi \geq 4 \times 10^{-20} \text{ eV} \). This mass scale may also be supported by the compact star cluster in the local dwarf spheroidal galaxy Eri-II. In addition, the compact inner masses of \( \gtrsim 2000 \text{ M}_\odot \) found in the globular clusters M15 data and 47 Tuc data may imply a further axion mass of \( m_3 \gtrsim 6 \times 10^{-18} \text{ eV} \). We have combined this information in a single core mass-radius plot (Fig. 1), providing a powerful "birds-eye" view of the axion role in structure formation, concluding that these distinctive astronomical structures require more than one axion in the wave dark matter context.

A definitive test for the presence of such a nested soliton structure, may be sought using pulsar timing residuals imprinted on millisecond pulsars detected a the Galactic Center [31, 35]. Many thousands are expected [36], and can account for the GeV gamma-ray excess [37, 38] and are being searched with some success [39] and expected to be detected with the SKA, including in the NSC region a product of the intense long history of star formation in this cluster. Depending on the mass ratio of the inner and outer solitons a distinctive multifrequency spectrum is expected on the respective Compton time scales of these independently oscillating scalar fields, from a few hours to a few months, providing a unique soliton signature of the generic multi-axion prediction of String Theory and a definitive solution to the long standing Dark Matter puzzle.

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