Evolution of the gauge couplings and Weinberg angle in 5-dimensions for an SU(5) and flipped SU(5) gauge group

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Abstract. We explicitly test, in a simplified 5-dimensional model with SU(5) and SU(5) × U(1)' gauge symmetries, the evolution of the gauge couplings. We assume that all the matter fields are propagating in the bulk, and consider orbifolds based on Abelian discrete groups which lead to 5-dimensional gauge theories compactified on an S^1/Z_2. The gauge couplings evolution is derived at one-loop level and used to test the impact on lower energy observables, in particular the Weinberg angle.

1. Introduction

The greatest achievement of the Large Hadron Collider (LHC) has been the discovery of the missing building block of the Standard Model (SM)\cite{1, 2}, the Higgs particle (or, at least, a particle which most likely is the SM Higgs particle). This discovery gave good support to the idea of the unification of electromagnetic and weak forces, as described by the electroweak gauge group SU(2)_L × U(1)_Y, which is spontaneously broken to U(1)_EM via the vacuum expectation value (VEV) of the Higgs fields\cite{3}.

The idea of the Grand Unification Theory (GUT) is to embed the SM gauge group (G_{SM} ≡ SU(3)_C × SU(2)_L × U(1)_Y) into a large group G. The SM group is rank 4, which means that the gauge group G must be at least rank 4. In this work, we shall study the non-supersymmetric extensions of the SM based on the gauge group SU(5) and flipped SU(5). In particular, we will study higher-dimensional non-supersymmetric orbifold models\cite{4}.

We consider orbifolds based on Abelian discrete groups which lead to 5-dimensional gauge theory compactified on an S^1/Z_2, where we assume that all matter fields are propagating in the bulk. The extra dimension is compactified on a circle of radius R with Z_2 orbifold. The 5-dimensional Kaluza-Klein (KK) modes of the weak doublet (Q) and singlet (q), as well as
Higgs and gauge fields \((A)\), are given by \([5, 6]\)

\[
A(x, y) = \frac{1}{\sqrt{2\pi R}} A_0(x) + \frac{1}{2\sqrt{\pi R}} \sum_{n=1}^{\infty} A_n(x) \cos \left( \frac{ny}{R} \right),
\]

\[
Q(x, y) = \frac{1}{\sqrt{2\pi R}} Q_L^0(x) + \frac{1}{2\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ Q_L^0(x) \cos \left( \frac{ny}{R} \right) + Q_R^0(x) \sin \left( \frac{ny}{R} \right) \right],
\]

\[
q(x, y) = \frac{1}{\sqrt{2\pi R}} q_L^0(x) + \frac{1}{2\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ q_R^0(x) \cos \left( \frac{ny}{R} \right) + q_L^0(x) \sin \left( \frac{ny}{R} \right) \right],
\]

where the zero modes are the 4-dimensional SM fields and there is a left- and a right-handed KK mode for each SM chiral fermion, whilst the Higgs and the gauge fields are \(Z_2\) even fields \([7]\).

The paper is structured as follows: In section (2) we discuss the evolution of the gauge couplings and Weinberg angle in 5-dimensions for an \(SU(5)\) gauge group, whilst in section (3) we discuss the evolution of the gauge couplings and Weinberg angle in 5-dimensions for an \(SU(5) \times U(1)'\) gauge group. In section (4) we conclude.

2. The gauge coupling evolution equations for an \(SU(5)\) gauge group

In this section shall explore the evolution of the gauge couplings and Weinberg angle in five dimension for an \(SU(5)\) gauge group, in order to have a unified theory above some energy scale \(M_X\), with \(n_g\) generation of fermions, where we need at least 12 new gauge bosons, an \(SU(2)_L\) doublet, colour triplet and their antiparticles, also a colour triplet; \(SU(2)_L\) singlet of the Higgs scalars \(h_{\alpha}\) \([8]\). We put the electro-weak doublet and the colour triplet in the 5-dimensional fundamental representation

\[
5_H = \begin{pmatrix}
    h^r_L \\ h_b \\ \Phi^+ \\ \Phi^0
\end{pmatrix},
\]

where \(SU(3)_C\) acts on the first 3 components, and \(SU(2)_L\) acts on the last two. The \(SU(5)\) gauge group breaks into the SM gauge group, \(SU(3)_c \times SU(2)_L \times U(1)_Y\), when a scalar field \(24_H\) such as the Higgs field acquires VEV, and this VEV is proportional to hypercharge generator \([8]\).

The Higgs sector is made up of an adjoint \(24_H\), which acquires a VEV from the spontaneous breaking \(SU(5) \rightarrow SU(3)_C \times SU(2)_W \times U(1)_Y\):

\[
< 24_H >= \frac{\nu}{\sqrt{30}} \begin{pmatrix}
    2 & 0 & 0 & 0 & 0 \\
    0 & 2 & 0 & 0 & 0 \\
    0 & 0 & 2 & 0 & 0 \\
    0 & 0 & 0 & -3 & 0 \\
    0 & 0 & 0 & 0 & -3
\end{pmatrix}.
\]

We can write the full \(SU(3)_C \times SU(2)_L \times U(1)_Y\) right handed representation of the creation operators as follows \([9]\):

\[
u^\dagger \oplus d^\dagger \oplus e^\dagger \oplus Q^\dagger \oplus \bar{L}^\dagger = (3, 1, 2/3) \oplus (3, 1, -1/3) \oplus (1, 1, -1) \oplus (3, 2, -1/6) \oplus (1, 2, 1/2).
\]
The new gauge bosons are called \( X \) and \( Y \) and they violate baryon and lepton number and carry flavour and colour. The gauge bosons are given by the adjoint representation of the \( SU(5) \) gauge group:

\[
24 \rightarrow (8, 1, 0) \oplus (1, 3, 0) \oplus (1, 1, 0) \oplus (3, 2, -5/6) \oplus (3, 2, 5/6),
\]

where \((8,1,0)\) is identified as the \( SU(3)_C \) gauge bosons \( G_\beta^a \), \((1,3,0)\) is identified as the \( W^\pm \) and \( W^0 \) gauge bosons, \((1,1,0)\) is identified as the \( B \) gauge boson, \((3,2,-5/6)\) is identified as the \( A_\alpha^a = (X_\alpha, Y_\alpha) \) gauge boson and \((3,2,5/6)\) identified as \( A_\beta^a = (X_\alpha, Y_\alpha)^T \) gauge boson. The covariant derivative for a fundamental representation is given by

\[
D_\mu = \partial_\mu - ig \sum_{a=1}^{24} \frac{\lambda_a}{2} A_\mu^a \equiv \partial_\mu - ig A_\mu,
\]

and the matrix of the gauge bosons becomes

\[
A_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
G^c \quad G^r \\
G^d \quad G^g \\
G^b \quad G^y \\
X^g \quad X^b \\
Y^y \quad Y^b \\
\end{pmatrix} + \sqrt{\frac{3}{5}} \begin{pmatrix}
-\frac{B_3}{3} & 0 & 0 & 0 & 0 \\
0 & -\frac{B_3}{3} & 0 & 0 & 0 \\
0 & 0 & \frac{B_3}{3} & 0 & 0 \\
0 & 0 & 0 & \frac{B_3}{3} & 0 \\
0 & 0 & 0 & 0 & \frac{B_3}{3}
\end{pmatrix}.
\]

The new gauge bosons \( X \) and \( Y \) are clearly carrying colour and they have electric charge \( 4/3 \) and \( 1/3 \) respectively.

The one-loop beta functions for the gauge couplings in 4-dimensions for \( SU(5) \) are given by:

\[
16\pi^2 g_1^{-3} \beta_{g_1} = \frac{81}{20},
\]

\[
16\pi^2 g_2^{-3} \beta_{g_2} = \frac{19}{6},
\]

\[
16\pi^2 g_3^{-3} \beta_{g_3} = -\frac{41}{6}.
\]

The one-loop beta functions for the gauge couplings in 5-dimension for \( SU(5) \) are given by:

\[
16\pi^2 g_1^{-3} \beta_{g_1} = (S(t) - 1) \left( \frac{81}{10} \right),
\]

\[
16\pi^2 g_2^{-3} \beta_{g_2} = (S(t) - 1) \left( \frac{7}{6} \right),
\]

\[
16\pi^2 g_3^{-3} \beta_{g_3} = (S(t) - 1) \left( -\frac{5}{2} \right),
\]

where \( t = \ln(S(t)/M_Z R) \), \( S(t) = \mu R \) for \( M_Z < \mu < \ln(1/M_Z R) \). For our numerical calculation we choose the initial values for the gauge couplings based on the renormalisation point \( M_Z \) scale as: \( \alpha_1(M_Z) = 0.01696, \alpha_2(M_Z) = 0.03377 \) and \( \alpha_3(M_Z) = 0.1184 \) [10]. In Figure 1, left panel, we present the evolution of the inverse fine structure constants in five dimensions for the one-loop beta-function, by assuming that all the matter fields are in the bulk. We see that \( \alpha_1^{-1} \) and \( \alpha_2^{-1} \) approximately meet at \( \log(E/\text{GeV}) \sim 3.67 \), \( \alpha_1^{-1} \) and \( \alpha_3^{-1} \) approximately meet at \( \log(E/\text{GeV}) \sim 3.72 \) and \( \alpha_2^{-1} \) and \( \alpha_3^{-1} \) approximately meet at \( \log_2(E/\text{GeV}) \sim 3.78 \). In Figure 1, right panel, we present the evolution of the Weinberg angle \( \sin^2 \theta_W \) for the one-loop
In this section we shall explore the evolution of the gauge couplings and Weinberg angle in 3. The gauge coupling evolution equations for an SU(5) gauge group. The flipped SU(5) gauge group is different from SU(5) gauge group in the electric charge generators, it does not lie completely in SU(5) gauge group. The flipped SU(5) is just embedding of SU(5) × U(1) into SO(10) gauge group, flipped SU(5) model is very special GUT [11]. The flipped SU(5) gauge group containing three generations of the quark and lepton, (10, 10) Higgs boson, they have the following presentation [12]:

\[ F_{(10)} = [Q, d^c, \nu^c]; \quad \tilde{f}_{(5)} = [L, u^c]; \quad l_1 = e^c, \]  

and

\[ H_{(10)} = [Q_H, d^c_H, \Phi_H] \quad \tilde{H}_{(10)} = [Q_R, d^c_R, \Phi_R], \]

where the components \( \Phi_H, \Phi_R \) breaking the flipped SU(5) gauge group to the SM gauge group, once \( \Phi_H, \Phi_R \) acquires a VEV:

\[ SU(5) \times U(1)' \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)', \]  

and then the gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \), is spontaneous breaking to \( SU(3)_C \times U(1)_{em} \), once \( \Phi_5 \) acquires a VEV:

\[ SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \rightarrow SU(3)_C \times U(1)_{em}. \]
Fields & SU(3)$_C$ & SU(2)$_L$ & $Y_5/2$
\hline
Q & 3 & 2 & $\frac{1}{5}$
L & 1 & 2 & $-\frac{1}{2}$
u\& & 3 & 1 & $-\frac{2}{3}$
d\& & 3 & 1 & $\frac{1}{3}$
e\& & 1 & 1 & 1
$\Phi_5$ & 1 & 2 & $\frac{1}{2}$
$\bar{H}_{10}$ & 3 & 1 & $-\frac{1}{5}$
\hline

Table 2. Summary the quark, lepton and Higgs field content in the flipped SU(5) model and their quantum numbers.

The flipped SU(5) gauge group includes SU(5) gauge bosons $W^\pm$, $W_3$, $B$, $X$, $Y$ and $\tilde{B}$, which is a $U(1)'$ gauge boson. The electric charge generator $Q$ in flipped SU(5) model is given as:

$$Q = T_3 - \frac{1}{5} Y' + \frac{2}{5} \tilde{Y},$$

where $Y'$ is the $U(1)$ inside SU(5) and $\tilde{Y}$ is the one outside SU(5).

The hypercharges of the known quark, lepton and Higgs field in the flipped SU(5) gauge group are given as shown in Table 2 [11, 12]:

The one-loop beta functions for the gauge couplings in 4-dimensions for SU(5) × U(1)′ are given by:

$$16\pi^2 g_1^{-3} \beta_{g_1} = \frac{53}{6},$$

$$16\pi^2 g_2^{-3} \beta_{g_2} = -\frac{19}{6},$$

$$16\pi^2 g_3^{-3} \beta_{g_3} = -\frac{41}{6}.$$

The one-loop beta functions for the gauge couplings in 5-dimensions for SU(5) × U(1)′ are given by:

$$16\pi^2 g_1^{-3} \beta_{g_1} = (S(t) - 1) \left(\frac{105}{24}\right),$$

$$16\pi^2 g_2^{-3} \beta_{g_2} = (S(t) - 1) \left(-\frac{7}{24}\right),$$

$$16\pi^2 g_3^{-3} \beta_{g_3} = (S(t) - 1) \left(-\frac{5}{2}\right).$$

In Figure 2, left panel, we showing the evolution of the $\alpha_i^{-1}$ in 5-dimensions for the one-loop beta-function and in this case, one can see that $\alpha_1^{-1}$, $\alpha_2^{-1}$ and $\alpha_3^{-1}$ approximately unified at $\log(E/\text{GeV}) \sim 4.0$ for compactification scale $R^{-1} = 5$ TeV. In the right panel, we present the evolution of Weinberg angle for the one-loop beta-function, for different values of compactification scales $R^{-1} = 1$ TeV and 5 TeV, for the flipped SU(5) model, as an example, for $R^{-1} = 5$ TeV, $\sin^2 \theta_W \sim 0.37$ at $t \sim 5.08$. 
Figure 2. Left panel: The evolution of the inverse fine structure constants $\alpha^{-1}_i(\mu)$ in five dimension as a function of $\log(E/\text{GeV})$ for compactification scale $R^{-1} = 5 \text{ TeV}$. Right panel: Evolution of the Weinberg angle $\sin \theta_W$ for all matter fields in the bulk, for two different compactification scales as a function of $t$, for an $SU(5) \times U(1)'$ gauge group.

| Scenario | $t(R_1)$ | $t(R_2)$ |
|----------|----------|----------|
| 5D flipped $SU(5)$ | 4.55 | 5.08 |

Table 3. The cut-offs in 5 dimensions for the flipped $SU(5)$ gauge group for two different compactification radii $R^{-1} = 1$ and $10 \text{ TeV}$, where $t = \ln(\mu/M_Z)$.

4. Conclusion

In this paper we derived the one-loop renormalisation group equations in five-dimensions for an $SU(5)$ and $SU(5) \times U(1)'$ gauge group compactified on an $S_1/Z_2$. We observed that when the fifth dimension KK-modes became kinematically accessible the evolution of the Weinberg angle rapidly increased by approximately 20% for $SU(5)$ and 7% for $SU(5) \times U(1)'$.

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