On Self-Dual Warped AdS$_3$ Black Holes

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Abstract

We study a new class of solutions of three-dimensional topological massive gravity. These solutions can be taken as non-extremal black holes, with their extremal counterparts being discrete quotients of spacelike warped AdS$_3$ along the $U(1)_L$ isometry. We study the thermodynamics of these black holes and show that the first law is satisfied. We also show that for consistent boundary conditions, the asymptotic symmetry generators form only one copy of the Virasoro algebra with central charge $c_L = \frac{4\ell}{G(\nu^2 + 3)}$, with which the Cardy formula reproduces the black hole entropy. We compute the real-time correlators of scalar perturbations and find a perfect match with the dual CFT predictions. Our study provides a novel example of warped AdS/CFT correspondence: the self-dual warped AdS$_3$ black hole is dual to a CFT with non-vanishing left central charge. Moreover our investigation suggests that the quantum topological massive gravity asymptotic to the same spacelike warped AdS$_3$ in different consistent ways may be dual to different 2D CFTs.
1 Introduction

Three-dimensional topological massive gravity (TMG)\cite{1, 2} is described by the following action
\begin{equation}
I_{TMG} = \frac{1}{16\pi G} \left[ \int d^3 x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right) + \frac{1}{\mu} I_{CS} \right],
\end{equation}
where $I_{CS}$ is the gravitational Chern-Simons action and we take both $G$ and $\mu$ positive. For every value of the coupling $\mu$, TMG admits an AdS$_3$ vacuum solution of radius $\ell$, which is known to be perturbatively unstable except at the chiral point $\mu \ell = 1$ \cite{3}. However, for generic values of the coupling $\mu$, it has been suggested that the theory could possess other stable backgrounds, namely the spacelike, timelike or null warped AdS$_3$ spaces \cite{4}. The spacelike warped AdS$_3$ admits the left-broken isometry group $U(1)_L \times SL(2, \mathbb{R})_R$, and there exist black hole solutions which can be obtained by performing discrete identifications in this background. For $\mu \ell > 3$, the spacelike warped AdS background is said to be stretched and the black holes are regular. It has been conjectured in \cite{4} that TMG defined with suitable “warped AdS” boundary conditions would be dual to a two-dimensional CFT with central charges
\begin{equation}
c_L = \frac{4\nu\ell}{G(\nu^2 + 3)}, \quad c_R = \frac{(5\nu^2 + 3)\ell}{G\nu(\nu^2 + 3)}
\end{equation}
where $\nu = \mu \ell/3$. It was shown that these central extensions can be derived from the asymptotic symmetry algebra associated to the spacelike warped AdS geometries \cite{5, 6, 7}, giving support to the warped AdS/CFT correspondence. Further support to this conjecture comes from the study of the quasi-normal modes and real-time correlators of the spacelike warped AdS$_3$ black hole in \cite{8, 9, 10}. One subtle point in this correspondence is that the asymptotic geometry of the warped black hole is related to the one of the global warped AdS$_3$ by a local coordinate transformation, which then induces the identification of quantum numbers to set up the dictionary of the warped AdS/CFT correspondence. Similar thing happens in the null warped case.

In this paper we provide another significant support to the conjectured warped AdS/CFT correspondence by studying a new kind of self-dual warped AdS$_3$ black hole. Their extremal counterparts have been exhibited in different coordinates in \cite{4}, where an heuristic expression of their entropy is given in the form of Cardy’s formula, suggesting the existence of a dual CFT. Our self-dual warped black hole metric takes a similar form to the near-NHEK geometry \cite{11} and is asymptotic to the spacelike warped AdS$_3$ without any coordinate transformation. We show that for appropriate boundary conditions, the asymptotic symmetry generators form one copy of the Virasoro algebra with central charge $c_L$, with which the application of the Cardy formula \cite{12} precisely reproduces the Bekenstein-Hawking entropy. This result is in favor of the warped AdS/CFT conjecture, in a chiral
The motivation of proposing the self-dual warped black hole solution comes from the Kerr/CFT correspondence \[13\], which could be taken as a generalization of the warped AdS/CFT correspondence. Geometrically, a slice of NHEK geometry at fixed polar angle is locally a self-dual warped $\text{AdS}_3$ spacetime. It was suggested that the quantum gravity in NHEK is dual to a CFT with only left-moving temperature. Moreover, further study suggests that the near-horizon geometry of the near-extremal Kerr black hole (Near-NHEK) is dual to the same CFT with nonvanishing right temperature, as the right-moving sector gets excited \[11\]. The near-NHEK geometry differs slightly from the NHEK geometry in the $\text{AdS}_2$ sector. We are inspired to ask whether there are near-NHEK like solutions in TMG. It turns out that such metrics indeed solve the equation of motion of TMG, and their extremal counterparts are precisely the self-dual solutions found in \[4\]. These solutions are locally equivalent to the spacelike warped $\text{AdS}_3$ by a singular coordinate transformation. This is in accordance with the fact that all the solutions of TMG are locally $\text{AdS}$ or warped $\text{AdS}$ if they are asymptotically related \[1\].

Strictly speaking, our solutions do not belong to the category of regular black holes, which require the existence of a geometric or causal singularity shielded by an event horizon. In fact, the solutions are free of curvature singularity and regular everywhere, even though there exist Killing horizons corresponding to the Killing vector of time translations. The situation is reminiscent of $\text{AdS}_2$ black hole. On the other hand, we show that similar to the black holes, these spacetimes have the regular thermodynamic behavior, satisfying the first law of thermodynamics. We find that the conserved charges and the entropy is independent of the right-moving sector, suggesting the dual CFT is chiral. However we also notice that in the dual CFT both left-moving and right-moving temperatures are non-vanishing. The right-moving one can be explained in a similar way as the temperature of the $\text{AdS}_2$ black hole \[14\], while the left-moving one originates from the identification of the angular coordinate.

Similar to the extreme Kerr case, the asymptotic symmetry group is an enhancement of the $U(1)_L$ isometry. However, there exists significant difference from the Kerr case on consistent asymptotic boundary conditions. Actually since the context here is TMG rather than Einstein gravity, we need to propose a set of slightly different boundary conditions to account for the correction from the Chern-Simons term. More interestingly, we find that the asymptotic symmetry generators form only one copy of the Virasoro algebra with nonvanishing central charge. This is quite different from the case of the spacelike warped

\[1\] all Einstein solutions in three dimensions are locally AdS. We are unaware of a theorem generalizing this to locally warped AdS for suitable asymptotic behaviour.
AdS$_3$ black hole, studied in [5, 6, 7], where both left and right central charges are nonzero, even though the left central charge in both cases are the same. Obviously our study provides a different playground to explore the warped AdS/CFT correspondence.

In the next section we introduce the self-dual warped AdS$_3$ black holes in TMG. In section 3 we study their black hole thermodynamics and verify that the first law holds. In section 4 we yield the left and right temperatures of the dual CFT and explain their different origin. We specify the boundary conditions in section 5, show that the centrally extended asymptotic symmetry algebra gives a left central charge $c_L = \frac{4\nu\ell}{G(\nu^2 + 3)}$, with which the application of the Cardy formula reproduces the Bekenstein-Hawking entropy. We further test the warped AdS/CFT correspondence in section 6 by calculating the scalar real-time correlator from gravity, finding perfect agreement with the CFT prediction. We end with discussions in section 7.

2 Self-dual warped black hole

2.1 Topologically massive gravity

The action of three-dimensional topologically massive gravity with a negative cosmological constant is

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left( R + 2\ell^2 \right) + \frac{\ell}{96\pi G\nu} \int d^3x \sqrt{-g} e^{\lambda\mu\nu} \Gamma_{\lambda\nu} \left( \partial_\mu \Gamma_{\sigma\nu} + \frac{2}{3} \Gamma_{\mu\tau} \Gamma_{\nu\tau} \right),$$

where $e^{\tau\mu\nu} = +1/\sqrt{-g}$ is the Levi-Civita tensor and $G$ has positive sign. The dimensionless coupling $\nu$ in the coefficient of the Chern-Simons action is related to the graviton mass $\mu$ by

$$\nu = \frac{\mu\ell}{3}.$$  

Without loss of generality we take $\nu$ to be positive. Varying the above action with respect to the metric gives the equation of motion

$$G_{\mu\nu} - \frac{1}{\ell^2} g_{\mu\nu} + \frac{\ell}{3\nu} C_{\mu\nu} = 0,$$

where $G_{\mu\nu}$ is the Einstein tensor and $C_{\mu\nu}$ is the Cotton tensor:

$$C_{\mu\nu} = e^{\alpha\beta\gamma} \nabla_\alpha \left( R_{\beta\gamma} - \frac{1}{4} g_{\beta\gamma} R \right).$$

Any Einstein solution with $G_{\mu\nu} = g_{\mu\nu}/\ell^2$ is also a solution of TMG; there are also non-Einstein solutions that satisfy the equation of motion.
2.2 Self-dual warped black hole solution

An interesting class of non-Einstein black hole solutions of TMG, which are asymptotic to spacelike warped AdS$_3$ spacetime, is the following

\[
\begin{align*}
ds^2 &= \frac{\ell^2}{v^2 + 3} \left( -(x - x_+)(x - x_-) d\tau^2 + \frac{1}{(x - x_+)(x - x_-)} dx^2 \\
&\quad + \frac{4v^2}{v^2 + 3} (\alpha d\phi + (x - \frac{x_+ + x_-}{2}) d\tau)^2 \right),
\end{align*}
\]

where the coordinates range as $\tau \in [-\infty, \infty], x \in [-\infty, \infty]$ and $\phi \sim \phi + 2\pi$. In the metric, $\alpha$ is a free parameter, which will be seen to be related to the entropy and the left-temperature of the black hole. The Killing vectors are manifestly the spacelike $\partial_\phi$ and $\partial_\tau$. A coordinate shift in $x$ and reciprocal rescaling of $x$ and $\tau$ brings the metric to a canonical form $x_+ = x_- = 1$, the difference $x_+ - x_-$ is thus seen to correspond to a choice of time $\tau$. We will refer to these as self-dual warped black holes, in analogy to the self-dual solutions studied in [15]. The horizons are located at $x_+$ and $x_-$ where $1/g_{xx}$ vanishes. The vacuum solution for the black holes is given by $x_+ = x_- = 0$ and $\alpha = 1$, which is spacelike warped AdS$_3$ in Poincaré coordinates and under a periodic identification of $u = u + 2\pi$. For $v^2 > 1$, these solutions are free of naked CTCs.

The self-dual warped black hole (7) is closely related to the self-dual solution mentioned in [4]. The self-dual solution in [4] is the discrete quotient of the spacelike warped AdS$_3$ by identifying along the $J_2$ isometry. In Poincaré-like coordinates, the metric is of the form

\[
ds^2 = \frac{\ell^2}{v^2 + 3} \left( -\tilde{x}^2 d\tilde{\tau}^2 + \frac{d\tilde{x}^2}{\tilde{x}^2} + \frac{4v^2}{v^2 + 3} \left( \alpha d\tilde{\phi} + \tilde{x} d\tilde{\tau} \right)^2 \right),
\]

with $\tilde{\phi} \sim \tilde{\phi} + 2\pi$. The parameter $\alpha$ just characterizes a family of different quotients. The self-dual warped black hole (7) is related to this metric through coordinate transformation

\[
\tilde{x} = \frac{1}{\tilde{x}} = \tanh \left( \frac{1}{4} \left( (x_+ - x_-) \tau \pm \ln \frac{x - x_+}{x - x_-} \right) \right),
\]

\[
\tilde{\phi} = \phi + \frac{1}{2} \ln \left[ \frac{1 - (\tilde{\tau}^+)^2}{1 - (\tilde{\tau}^-)^2} \right].
\]

Globally, the maximal analytic extension of the self-dual warped black hole is diffeomorphic to (8). However, the above coordinate transformations are singular at the boundary $x \to \infty$, which indicates that the physics behind the two solutions (8) and (7) are different. The situation here is very similar to the relation between near horizon geometry of extreme Kerr (NHEK) and near horizon geometry of near-extreme Kerr (near-NHEK) [11], or the relation between AdS$_2$ and AdS$_2$ black hole [14]. In a sense, the self-dual warped AdS black hole may be taken as a $U(1)$-fibred AdS$_2$ black hole. As in near-NHEK or AdS$_2$
black holes, we will see that an observer in (7) at fixed $x$ measure a Hawking temperature proportional to $x_+ - x_-$. Moreover, as in near-Kerr case, the entropy does not depend on $x_+ - x_-$, but the scattering amplitudes do depend on $x_+ - x_-$. 

### 2.3 Diffeomorphism

The above self-dual warped black hole metric is locally equivalent to spacelike warped AdS$_3$

$$ds^2 = \frac{\ell^2}{\nu^2 + 3} \left( -\cosh^2 \sigma dv^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2 + 3} (du + \sinh \sigma dv)^2 \right) \quad (10)$$

through the coordinate transformation

$$
\begin{align*}
v &= \tan^{-1} \left[ \frac{2 \sqrt{(x - x_+)(x - x_-)}}{2x - x_+ - x_-} \sinh \left( \frac{x_+ - x_-}{2} \tau \right) \right], \\
\sigma &= \sinh^{-1} \left[ \frac{2 \sqrt{(x - x_+)(x - x_-)}}{x_+ - x_-} \cosh \left( \frac{x_+ - x_-}{2} \tau \right) \right], \\
u &= \alpha \phi + \tanh^{-1} \left[ \frac{2x - x_+ - x_-}{x_+ - x_-} \coth \left( \frac{x_+ - x_-}{2} \tau \right) \right].
\end{align*} \quad (11)
$$

In the coordinates of (10), the Killing vectors of spacelike warped AdS$_3$ are

$$
\begin{align*}
J_2 &= 2 \partial_u, \\
\tilde{J}_1 &= 2 \sin \nu \tanh \sigma \partial_\nu - 2 \cos \nu \partial_\sigma + \frac{2 \sin \nu}{\cosh \sigma} \partial_u, \\
\tilde{J}_2 &= -2 \cos \nu \tanh \sigma \partial_\nu - 2 \sin \nu \partial_\sigma - \frac{2 \cos \nu}{\cosh \sigma} \partial_u, \\
\tilde{J}_0 &= 2 \partial_\nu,
\end{align*}
$$

and satisfy the algebra $[\tilde{J}_1, \tilde{J}_2] = 2 \tilde{J}_0$, $[\tilde{J}_0, \tilde{J}_1] = -2 \tilde{J}_2$, $[\tilde{J}_0, \tilde{J}_2] = 2 \tilde{J}_1$ and $[J_2, \tilde{J}_{1,2,3}] = 0$.

### 3 Thermodynamics

In this section we look into the thermodynamics of self-dual warped black holes. We will see that after accounting for the effects of the Chern-Simons term, the various thermodynamic quantities obey the first law.

#### 3.1 Conserved charges

The ADT mass $\mathcal{M}^{ADT}$ and angular momentum $\mathcal{J}^{ADT}$ of self-dual warped black hole can be calculated following the procedure developed in [16], where they computed the conserved
charges associated to the Killing vectors $\partial_\tau$ and $\partial_\phi$ for a TMG solution linearized around an arbitrary background using the surface integral expressions derived in [17, 18, 19, 20]. The final result is

$$M^{ADT} = 0, \quad J^{ADT} = \frac{(\alpha^2 - 1)\nu\ell}{6G(\nu^2 + 3)}.$$  \hspace{1cm} (12)

That is, the ADT mass and angular momentum do not depend on the parameters $x_+$ and $x_-$. The situation here is similar to the case of near-NHEK geometry[11] of which the ADM mass defined in the asymptotically flat region is given by its extremal value in that limit and does not depend on $T_R$. We will see that the entropy $S$ does not depend on $x_+$ and $x_-$ either. However, the observers at fixed $x$ in self-dual warped black hole measure a Hawking temperature proportional to $(x_+ - x_-)$.

### 3.2 Entropy

The entropy of the self-dual warped black hole is composed of two terms, one comes from the Einstein action and the other is due to the Chern-Simons contribution[21, 22]. The total entropy is given by

$$S = S_E + S_{CS} = \frac{2\pi\alpha\nu\ell}{3G(\nu^2 + 3)}.$$ \hspace{1cm} (13)

### 3.3 First law

It is a nontrivial check of black hole thermodynamics that the mass and angular momentum as well as the entropy are related by the first law. From the self-dual warped black hole metric (7) which is already in ADM form, we read the Hawking temperature $T_H$ and the angular velocity of the horizon $\Omega_h$,

$$T_H = \frac{x_+ - x_-}{4\pi\ell}, \quad \Omega_h = -\frac{x_+ - x_-}{2\alpha\ell}.$$ \hspace{1cm} (14)

Then we check explicitly that the first law

$$dM^{ADT} = T_H dS + \Omega_h dJ^{ADT},$$ \hspace{1cm} (15)

is satisfied for a variation of the black hole parameter $\alpha$.

### 3.4 On the vacuum solution

In the above discussion, we have taken the vacuum solution to be the one with $\alpha_0 = 1$. However, we have no good reason to prefer this value. In fact, the computation of the entropy is independent of the choice of $\alpha_0$, while the computations on the ADT mass and
angular momentum depends on the choice of the vacuum. Nevertheless, even with a different choice of $\alpha_0$, the first law of thermodynamics still holds.

From the entropy and the left temperature $T_L$ of the dual CFT, it seems that the natural choice of the vacuum solution should be $\alpha_0 = 0$. Unfortunately, in this case, the metric becomes degenerate. It would be nice to have a criterion to decide the vacuum solution.

4 Temperatures

In this section we will derive the left- and right-moving temperatures of the self-dual warped black holes and show that they have different origins.

Since the metric (7) looks much similar to the near-NHEK geometry[11], it is sensible to define a quantum vacuum in analogy to the Frolov-Thorne vacuum[23, 24, 25]. The construction begins by expanding the quantum fields in eigenmodes of the asymptotic energy $\omega$ and angular momentum $k$. Consider a scalar field $\Phi$, we may write

$$\Phi = \sum_{\omega, k, l} \phi_{\omega k l} e^{-i\omega \tau + ik \phi} R_l(x).$$

(16)

After tracing over the region inside the horizon, the vacuum is a diagonal density matrix in the energy-angular momentum eigenbasis with a Boltzmann weighting factor

$$\exp \left(-\frac{\hbar \omega - k \Omega_H}{T_H} \right).$$

(17)

This reduces to the Hartle-Hawking vacuum in the non-rotating $\Omega_H = 0$ case. The left and right charges $n_L$, $n_R$ associated to $\partial_\phi$ and $\partial_\tau$ are

$$n_L \equiv k, \quad n_R \equiv \omega.$$  

(18)

Identifying

$$\exp \left(-\frac{\hbar \omega - k \Omega_H}{T_H} \right) = \exp \left(-\frac{n_L}{T_L} - \frac{n_R}{T_R} \right)$$

(19)

defines the left and right temperatures

$$T_L = \frac{\alpha}{2\pi \ell}, \quad T_R = \frac{x_+ - x_-}{4\pi \ell}.$$  

(20)

In the extremal limit $x_+ = x_-$, these reduce to

$$T_L = \frac{\alpha}{2\pi \ell}, \quad T_R = 0.$$  

(21)

The right temperature denotes the deviation from extremality. As discussed in detail in [26, 14] in the AdS$_2$ context, the observers moving along worldlines of fixed $x$ will
detect a thermal bath of particles at temperature $T_R$ in the global Hartle-Hawking vacuum, essentially similar to the case that Rindler observers detect radiations in the Minkowski vacuum.

The left temperature, on the other hand, arises from the periodical identification of points in warped AdS$_3$. From (11) we see that in order for the coordinate transformation to reproduce the black hole we need to identify points along the $\partial\phi$ direction such that $\phi \sim \phi + 2\pi$. Expressing the $\partial\phi$ Killing vector in terms of the original warped AdS$_3$ coordinates, we discretely quotient along the isometry

$$\partial\phi = \frac{q}{2} J_2 = \pi \ell J_2,$$

where $J_2 \in U(1)_L$ is the Killing vector. In analogy with the BTZ case\[29, 30\], the coefficient of the shift is the temperature of the dual 2D CFT. Note that this periodical identification makes no contribution to the right temperature.

5 Asymptotic behavior

Since the self-dual warped black hole solution is locally isometric to the spacelike warped AdS$_3$, it is natural to expect the existence of a dual 2-dimensional CFT. In this section we will show that with suitable boundary conditions, the asymptotic symmetry generators form one copy of a Virasoro algebra with central charge $c_L = \frac{4\nu\ell}{G(\nu^2+3)}$, which is precisely what Anninos et al\[4\] conjectured for $\nu > 1$ quantum TMG. An application of the Cardy formula to the CFT density of states reproduces the black hole entropy.

5.1 Boundary conditions

We impose the following boundary conditions

$$
\begin{align*}
  h_{\tau\tau} &= O(1) & h_{\tau x} &= O(1/x^3) & h_{\tau\phi} &= O(x) \\
  h_{x\tau} &= h_{\tau x} & h_{xx} &= O(1/x^3) & h_{x\phi} &= O(1/x) \\
  h_{\phi\tau} &= h_{\tau\phi} & h_{\phi x} &= h_{x\phi} & h_{\phi\phi} &= O(1)
\end{align*}
$$

where $h_{\mu\nu}$ is the deviation of the full metric from the vacuum. Here the allowed deviations $h_{\tau\phi}$ and $h_{\phi\phi}$ are of the same order as the leading terms in (7). In this regard, the boundary conditions differ from both the spacelike warped AdS$_3$ boundary conditions \[5, 6, 7\], where all deviations are subleading, and from the Kerr boundary conditions \[13\], where $h_{tt}$ rather than $h_{\tau\phi}$ is of leading order. The most general infinitesimal diffeomorphism which preserves the boundary conditions (23) is of the form

$$\zeta = \left[C + O(1/x^3)\right] \partial_\tau + O(1) \partial_x + \left[\epsilon (\phi) + O(1/x^3)\right] \partial_\phi,$$

where $C$ is a constant and $\epsilon (\phi)$ is a function of $\phi$. The condition that $\zeta$ preserves the boundary conditions (23) requires

$$C = 0$$

and

$$\epsilon (\phi) = 0.$$
where $\epsilon(\phi)$ is an arbitrary smooth function of the coordinate $\phi$, and $C$ is an arbitrary constant. The subleading terms indicated above can be seen to correspond to trivial diffeomorphisms by computing the generators. Therefore the asymptotic symmetry group contains one copy of the conformal group of the circle generated by

$$\xi_\epsilon = \epsilon(\phi) \partial_\phi.$$  \hfill (25)

This Virasoro algebra contains a $U(1)$ isometry subgroup. Since $\phi \sim \phi + 2\pi$, it is convenient to define $\epsilon_n(\phi) = e^{in\phi}$ and $\xi_n = \xi(\epsilon_n)$. These generators admit the following commutators

$$i [\xi_m, \xi_n] = (m - n) \xi_{m+n},$$  \hfill (26)

and $\xi_0$ generates the $U(1)$ rotational isometry.

The allowed symmetry transformations (24) also include $\tau$ translations generated by $\partial_\tau$. The conjugate conserved charge $E_R$, with the expression given in the next subsection, could be well-defined by imposing the supplementary boundary condition

$$2 \alpha h_{[0]}^{\tau\phi} - h_{[0]}^{\phi\phi} = 0,$$  \hfill (27)

where $h_{[0]}^{\mu\nu}$ is the coefficient of the leading order deviation $h_{[0]}^{\mu\nu}$ in the neighborhood of our metrics.

The boundary conditions (23)(27) are very different from the usual spacelike warped AdS$_3$ boundary conditions [5, 6, 7]. In the latter case, consistent boundary conditions were imposed in which the $SL(2, \mathbb{R})$ isometry is enhanced to a Virasoro algebra and the $U(1)$ isometry is enhanced to a current algebra. In the present situation, the $SL(2, \mathbb{R})$ isometry becomes trivial and the $U(1)$ isometry is enhanced to a Virasoro, similarly to the case of Kerr [13].

5.2 Asymptotically conserved charges

The conserved charge associated with an asymptotic Killing vector $\xi$ is constructed in the covariant formalism by Barnich, Brandt and Compère [31, 32, 33]. Here we will adopt the formulae in [5] where the covariant theory of asymptotic charges is applied specifically for TMG. For the original application in AdS$_3$ using the Hamiltonian formalism see [34], while an alternative background independent definition of charges can be found in [35].

The charge differences $Q_\xi[g, \bar{g}]$ between the reference solution $\bar{g}$ and the solution of interest $g$ are by

$$Q_\xi [g; \bar{g}] = \int_\bar{g}^g \int_S k^{\mu[\xi}_\nu[\delta g; g]e_{\nu\rho} dX^\rho$$  \hfill (28)
where the first integration is performed in the phase space of solutions, \( S \) is a circle at asymptotic infinity, the two-form \( k^{\mu \nu}[\delta g; g] \) is a one-form defined in the linearized theory by

\[
(16\pi G) k^{\mu \nu}_\xi[\delta g; g] = (16\pi G) k^{\mu \nu}_{Ein,ξ} [\delta g; g] - \frac{1}{\mu} \xi_\lambda \left( 2\epsilon^{\nu \rho \lambda} \delta (G^\lambda_\rho) - \epsilon^{\mu \nu \lambda} \delta G \right) \\
- \frac{1}{\mu} \epsilon^{\nu \rho \lambda} \left( \xi_\rho h^{\lambda \sigma} G_{\sigma \lambda} + \frac{1}{2} h(\xi_\sigma G^\sigma_\rho + \frac{1}{2} \xi_\rho R) \right) \\
+ (16\pi G) E^{\mu \nu}[\delta g; L_{\xi} g],
\]

(29)

\[
\xi_{tot}^\mu = \xi^\mu + \frac{1}{2\mu} \epsilon^{\nu \rho \sigma} D_\rho \xi_\sigma,
\]

the two-form \( k^{\mu \nu}_{Ein,ξ} [\delta g; g] \) is the Iyer-Wald expression \[36\] for general relativity

\[
(16\pi G) \sqrt{-g} k^{\mu \nu}_{Ein,ξ} [\delta g; g] = \sqrt{-g} \epsilon^{\mu \nu}(D_\lambda h^{\lambda \nu} - D^\lambda h) - \delta(\sqrt{-g} D^\nu \xi^\mu) - (\mu \leftrightarrow \nu)
\]

(30)

where the variation acts only on \( g \), and \( E^{\mu \nu}[\delta g; L_\xi g] \) is an additional contribution linear in the Killing equation and its derivatives

\[
(16\pi G) E^{\mu \nu}[\delta g; L_\xi g] = \frac{1}{2} \left( h^{\nu \lambda} L_{\xi} g_\lambda^{\lambda \mu} - h^{\mu \lambda} L_{\xi} g_\lambda^{\lambda \nu} \right) + \frac{1}{4\mu} \epsilon^{\nu \rho \lambda \sigma} \left( D_\lambda L_{\xi} g_{\rho}^{\lambda \sigma} - D_\rho L_{\xi} g_{\lambda}^{\rho \sigma} \right) h.
\]

(31)

The charges do not depend on the path chosen in the integration if the integrability condition \( \delta \oint_S k^{\mu \nu}_\xi[\delta g; g] \epsilon_{\mu \nu \rho} dx^\rho = 0 \) holds. The crucial property of these charges is that they represent the Lie algebra of asymptotic symmetries via a covariant Poisson bracket up to central charges

\[
\{ Q_\xi[g; \bar{g}], Q_{\xi'}[g; \bar{g}] \} = Q_{[\xi, \xi']}[g; \bar{g}] + K_{\xi, \xi'}[\bar{g}].
\]

(32)

Here \( [\xi, \xi'] \) is the Lie bracket and \( K_{\xi, \xi'}[\bar{g}] \equiv \oint_S k^{\mu \nu}_\xi[\mathcal{L}_{\xi'} \bar{g}; \bar{g}] \). More precisely this result holds modulo a technical assumption (see (4.3) of \[33\]) which will be checked for the case at hand.

### 5.3 Central charge

Let us denote the charge differences between self-dual warped black hole metric \( g \) and the vacuum \( \bar{g} \) (\( x_+ = x_- = 0, \alpha = 1 \)) by \( Q_n \equiv Q_{[\xi, \xi']}[g; \bar{g}] \) and \( E_R \equiv Q_{[\xi, \xi']}[g; \bar{g}] \). One can check that \( Q_n \) is finite under the boundary condition \( \mathcal{S} \), while the finiteness of \( E_R \) requires an additional boundary condition \( \mathcal{S} \). In order that the asymptotic symmetries form an algebra, additional conditions on \( E^{\mu \nu} \) were suggested in \[33\] and a simpler sufficient condition was proposed in \[37\], requiring that \( \oint_S E^{\mu \nu}[\delta g; L_\xi g] = 0 \). For the generators \( Q_n \), using the definition of \( E^{\mu \nu} \) \[31\] one can check that this condition is satisfied in the phase space \( \mathcal{S} \). The
term $E^{\mu\nu}$ simply does not contribute to any charge. Therefore, the charges form a representation of the asymptotic symmetry algebra. The central charge $K_{\xi\xi'}[\bar{g}] \equiv \int_{\mathcal{S}} k_{\xi}^{\mu\nu}[\mathcal{L}_{\xi'}\bar{g}; \bar{g}]$ could be calculated by implementing the formula (29) in a Mathematica code. We find the following centrally extended Virasoro algebra

$$i\{Q_m, Q_n\} = (m - n)Q_{m+n} + \frac{c_L}{12}m(m^2 - 2)\delta_{m+n,0},$$  \hspace{1cm} (33)

where

$$c_L = \frac{4\nu \ell}{G(\nu^2 + 3)}$$  \hspace{1cm} (34)

is the Virasoro central charge. This is exactly the value of the left central charge conjectured in [4] for spacelike warped AdS$_3$. Here we have derived it without a Sugawara-type procedure from a current algebra as in [7]. Since the other Virasoro sector vanishes we simply set $c_R = 0$.

We verify that the entropy of the self-dual warped black hole can be reproduced from the Cardy formula describing the density of states of the dual CFT

$$S = \frac{2\pi \alpha \nu \ell}{3G(\nu^2 + 3)} = \frac{\pi^2 \ell}{3} \frac{4\nu \ell}{G(\nu^2 + 3)} \frac{\alpha}{2\pi \ell} \equiv \frac{\pi^2 \ell}{3} c_L T_L,$$  \hspace{1cm} (35)

which provides strong evidence in favour of the warped AdS/CFT correspondence.

The boundary conditions (23) can be relaxed to $h_{\tau x} = O(1/x)$, in which case we find the asymptotic symmetries are augmented with the vectors

$$\tilde{\xi} = f(\tau) \partial_\tau - f'(\tau) x \partial_x.$$  \hspace{1cm} (36)

The charges associated to the $\tilde{\xi}$ are finite provided the condition (27) holds, while the Lie derivative of $\tilde{\xi}$ on the class of self-dual black holes (7) preserves that condition. With

$$f(\tau) = \left(\tanh \tau + \frac{i}{\cosh \tau}\right)^n \cosh \tau, \quad n \in \mathbb{Z},$$  \hspace{1cm} (37)

we obtain a second Virasoro that commutes with the first, gives zero charge for the self-dual black holes, and a zero central extension $c_R = 0$. Whether or not we include these, the results of this section remain unchanged.

6 Scalar perturbation

In order to check the warped AdS/CFT correspondence further, we calculate the real-time correlator of scalar perturbations in this section. We will show that the results from gravity agree precisely with predictions from the dual CFT description.
Consider a scalar field $\Phi$ with mass $m$ in the background (7), expanding in modes as

$$\Phi = e^{-i\omega x + ik\phi(R(x))},$$

the radial wave function $R(x)$ satisfies the equation

$$\frac{d}{dx} \left( (x - x_+)(x - x_-) \frac{d}{dx} R(x) \right) = \left( \frac{v^2 + 3k^2}{4v^2} \frac{\ell^2}{\alpha^2} + \frac{\ell^2}{y^2 + 3} m^2 - \frac{(\omega + \frac{k}{\alpha}(x - \frac{x_+ + x_-}{2}))^2}{(x - x_+)(x - x_-)} \right) R(x) = 0. \quad (38)$$

We would like to calculate the retarded Green's function, for which purpose the ingoing boundary condition at the horizon is chosen. Then the solution is

$$R(x) = N \left( \frac{x - x_+}{x - x_-} \right)^{-\frac{1}{2} \left( \frac{1}{2} - \beta \right)} \cdot \text{F} \left( \left[ \frac{1}{2} - \beta - i \frac{k}{\alpha}, 1 - \beta - i \frac{2\omega}{x_+ - x_-}, 1 - i \left( \frac{k}{\alpha} + \frac{2\omega}{x_+ - x_-} \right); \frac{x - x_+}{x - x_-} \right) \right), \quad (39)$$

where $N$ is an arbitrary constant and

$$\beta^2 = \frac{1}{4} - \frac{3(v^2 - 1)}{4v^2} \frac{k^2}{\alpha^2} + \frac{\ell^2}{y^2 + 3} m^2. \quad (40)$$

At asymptotical infinity, the radial eigenfunction has the behavior

$$R(x) \sim Ax^{-\frac{1}{2} - \beta} + Bx^{-\frac{1}{2} + \beta} \quad (41)$$

where

$$A = N (x_+ - x_-)^{\frac{1}{2} + \beta} \frac{\Gamma(-2\beta) \Gamma \left( 1 - i \left( \frac{k}{\alpha} + \frac{2\omega}{x_+ - x_-} \right) \right)}{\Gamma \left( \frac{1}{2} - \beta - i \frac{k}{\alpha} \right) \Gamma \left( \frac{1}{2} - \beta - i \frac{2\omega}{x_+ - x_-} \right)}; \quad (42)$$

$$B = A (\beta \to -\beta). \quad (43)$$

The real-time retarded Green's function could be computed in terms of the boundary values of the bulk fields using the prescription proposed in [38] and later recast in [39, 40]. This prescription works well not just for asymptotically AdS metrics, but also for the warped AdS/CFT correspondence [10] as well as for the Kerr/CFT correspondence [41, 42]. However, for the latter cases, a careful study of the holographic renormalization is needed as in the usual AdS/CFT correspondence [43]. For a scalar field with the asymptotic behavior (42), consider a real $\beta > 0$ without loss of generality, the retarded correlator is given by

$$G_R \sim \frac{A}{B} = (x_+ - x_-)^{2\beta} \frac{\Gamma(-2\beta) \Gamma \left( \frac{1}{2} + \beta - i \frac{k}{\alpha} \right) \Gamma \left( \frac{1}{2} + \beta - i \frac{2\omega}{x_+ - x_-} \right)}{\Gamma(2\beta) \Gamma \left( \frac{1}{2} - \beta - i \frac{k}{\alpha} \right) \Gamma \left( \frac{1}{2} - \beta - i \frac{2\omega}{x_+ - x_-} \right)}. \quad (44)$$
On the other hand, throwing a scalar $\Phi$ at the black hole is dual to exciting the CFT by acting with an operator $O_\Phi$. For an operator of dimensions $(h_L, h_R)$ at temperature $(T_L, T_R)$, the momentum-space Euclidean Green’s function is determined by conformal invariance and takes the form

$$G_E(\omega_{L,E}, \omega_{R,E}) \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{i\frac{\omega_{L,E}}{T_L}} e^{i\frac{\omega_{R,E}}{T_R}} \Gamma(h_L + \frac{\omega_{L,E}}{2\pi T_L}) \Gamma(h_L - \frac{\omega_{L,E}}{2\pi T_L}) \cdot \Gamma(h_R + \frac{\omega_{R,E}}{2\pi T_R}) \Gamma(h_R - \frac{\omega_{R,E}}{2\pi T_R}),$$

where $\omega_{L,E}, \omega_{R,E}$ are the Euclidean frequencies, which are related to the Minkowskian ones by $\omega_{L,E} = i\omega_L, \omega_{R,E} = i\omega_R$. At finite temperature, $\omega_{L,E}$ and $\omega_{R,E}$ take discrete values of the Matsubara frequencies $\omega_{L,E} = 2\pi m_L T_L, \omega_{R,E} = 2\pi m_R T_R$, where $m_L, m_R$ are integers for bosonic modes. The Euclidean correlator $G_E(\omega_{L,E}, \omega_{R,E})$ is related to the value of the retarded correlator $G_R(\omega_L, \omega_R)$ by

$$G_E(\omega_{L,E}, \omega_{R,E}) = G_R(i\omega_{L,E}, i\omega_{R,E}), \quad \omega_{L,E}, \omega_{R,E} > 0.$$  

Comparing the arguments of the Gamma functions among (45) and (47), one finds precise agreement under the following identification

$$h_L = h_R = \frac{1}{2} + \beta, \quad \omega_L = k/\ell, \quad \omega_R = \omega/\ell, \quad T_L = \frac{a}{2\pi \ell}, \quad T_R = \frac{x_+ - x_-}{4\pi \ell},$$

up to an normalization factor. Note that the conformal dimension $h_{L,R}$ above is the same as the ones discussed in [9]. As the asymptotic geometry of self-dual warped solutions is the same as the one of global warped AdS$_3$, we do not need to make any extra identification of quantum numbers to find the agreements.

This argument still holds for imaginary $\beta$, for which the complex conformal weight indicates an instability of the AdS spacetime due to pair production, similar to the situation of the Kerr/CFT correspondence [11].

From the real-time correlator, we can read the greybody factor from its imaginary part, which is in perfect agreement with the CFT prediction as expected. The quasi-normal modes could be read from the poles in the retarded Green’s function. These poles are characterized by

$$k = -i(2\pi T_L l)(n_1 + h_L), \quad \omega = -i(2\pi T_R l)(n_2 + h_R),$$

where $n_{1,2}$ are non-negative integers. Strictly speaking, only the ones involving the right temperature could be called quasi-normal modes, whose frequencies are vanishing in the extremal limit.
7 Conclusions and discussions

In this paper we studied the self-dual warped black hole solutions in topological massive gravity. These are the TMG analogs of the near-NHEK metric and are asymptotic to the spacelike warped AdS$_3$. We showed that there exist consistent boundary conditions, for which the asymptotic symmetry generators form one sector of Virasoro algebra with central charge $c_L = \frac{4\nu}{\nu^2 + 3}$. This implies that the black hole quantum states can be identified with those of a chiral(left) half of a two-dimensional conformal field theory. Our investigation suggests that the quantum topological massive gravity asymptotic to the same spacelike warped AdS$_3$ in different consistent ways may be dual to different 2D CFTs.

The extremal self-dual warped black holes, which have been exhibited in [4] in different coordinates, are discrete quotients of spacelike warped AdS$_3$ along the $U(1)_L$ isometry, the dual CFT of which has left-moving temperature $T_L = \frac{\nu}{2\nu \ell}$ and vanishing right-moving temperature. For the non-extremal self-dual warped black holes, the right-moving modes are excited and the dual CFT obtains a non-vanishing right-moving temperature $T_R = \frac{\nu - k}{4\nu \ell}$. For both extremal and non-extremal self-dual warped black holes, the entropy is the same and can be reproduced by Cardy’s formula albeit using only a left-moving sector. This gives strong support to the warped AdS/CFT correspondence, and is further corroborated by the perfect agreement of the retarded Green’s function from both the gravity computation and the CFT prediction.

Another interesting feature of the self-dual warped black holes is that they have the same asymptotic geometry as the one of the global warped AdS$_3$. While for the spacelike warped AdS$_3$ black hole, the asymptotic geometry could only be related to the one of global warped spacetime by a local coordinate transformation, but is different globally [45]. In other words, the global warped AdS$_3$ is not the ground state of the warped AdS$_3$ black holes. In our case, we can take the global warped AdS$_3$ as the ground state after periodically identifying $u$.

The warped AdS/CFT correspondence proposed here is a chiral one, very different from the one suggested in [4]. The boundary conditions we proposed are different from those suggested in [5, 6]. It is remarkable that we obtain the left central charge conjectured in [4] naturally through a centrally extended Virasoro algebra which is an enhancement of the $U(1)_L$ isometry, instead of a Sugawara-type procedure from a current algebra as in [7]. Since for non-extremal self-dual warped black holes, the right-moving temperature doesn’t vanish, it is natural to expect from the Cardy formula that the dual CFT has a right central charge $c_R = 0$. For the boundary conditions (23) (27), the right-moving Virasoro sector vanishes. We leave open the question of whether another set of boundary conditions, such
as the one leading to [36], admits a consistent Virasoro sector with central charge \( c_R = 0 \) [6].

In the Kerr/CFT correspondence, the left central charge could be obtained from the asymptotic symmetry of NHEK. The right central charge could be read from the study of AdS\(_2\) gravity after reduction [45]. It would be interesting to study the relation between TMG and AdS\(_2\) gravity and understand the absence of right central charge [47].

We computed the scalar real-time correlators in the self-dual warped black hole and found perfect agreement with the CFT prediction. It would also be interesting to check the warped AdS/CFT correspondence further by computing the real-time correlators for vector, spinor as well as gravitational perturbations.

**Acknowledgments**

We are very grateful to G. Moutsopoulos for his contributions and comments on this project. The work was partially supported by NSFC Grant No.10775002, 10975005 and NKBPRPC (No. 2006CB805905).

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