Y(4220) and Y(4390)

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The cross sections of $e^+e^-\rightarrow\omega_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, $\pi^+\pi^-\psi(3686)$ and $\pi^+D^0D^{*-}+c.c.$ have been measured in experiment. A combined fit is performed to these cross sections by using three resonances Y(4220), Y(4390) and Y(4660). The resonant parameters for the three resonances are obtained. We emphasize that two resonances Y(4220) and Y(4390) are sufficient to explain these cross sections below 4.6 GeV. The lower limits of Y(4220) and Y(4390)’s leptonic decay widths are also determined to be $(36.4\pm 2.0\pm 4.2)$ and $(123.8\pm 6.5\pm 9.0)$ eV.

Y(4260), the first observed charmonium-like state in the process $e^+e^-\rightarrow\pi^+\pi^-J/\psi$ by the BABAR experiment using an initial-state-radiation (ISR) technique. This observation was immediately confirmed by the CLEO [2] and Belle experiments [3] in the same process. Being produced in $e^+e^-$ annihilation, the Y state has quantum numbers $J^{PC}=1^{-+}$. Y(4360), the second observed Y state in the $e^+e^->\gamma_{ISR}Y(4360)\rightarrow\gamma_{ISR}\pi^+\pi^-\psi(3686)$ by BABAR [4] and subsequently confirmed by Belle experiment [5]. Belle also observed another structure, Y(4660), in the $\pi^+\pi^-\psi(3686)$ [3]. The observation of these Y states has stimulated substantial theoretical discussions on their nature [6].

Recently, with higher statistic data, the $e^+e^-\rightarrow\pi^+\pi^-J/\psi$ cross section was measured by BESIII experiment more precisely [7]. The fine structure was observed for Y(4260) in $e^+e^-\rightarrow\pi^+\pi^-J/\psi$. The Y(4260) structure is a combination of two resonances, the lower one is Y(4220) and the higher is Y(4390). Using the results for $e^+e^-\rightarrow\pi^+\pi^-\psi(3686)$ from Belle [8], BABAR [9] and BESIII experiments [10], the authors of Ref. [11] also observed the fine structure for Y(4360) in $e^+e^-\rightarrow\pi^+\pi^-\psi(3686)$, inferring that the Y(4360) structure is also a combination of two resonances, the lower one is Y(4220) and the higher is Y(4360). The Y(4220) state also is observed in the processes $e^+e^-\rightarrow\omega_{c0}$ [12, 13], $\pi^+\pi^-h_c$ [14] and $\pi^+D^0D^{*-}+c.c.$ [15] by BESIII experiment. In the $e^+e^-\rightarrow\pi^+\pi^-h_c$ and $\pi^+D^0D^{*-}+c.c.$, besides the Y(4220), another Y state Y(4390) is observed [14, 15]. The parameters for Y(4220), Y(4320), Y(4360) and Y(4390) states in different processes are listed in Table I.

These states challenge the understanding of charmonium spectroscopy as well as QCD calculations [16]. According to potential models, there are five vector charmonium states between the 1D state $\psi(3770)$ and 4.7 GeV/$c^2$, namely, the 3S, 2D, 4S, 3D and 5S states [16]. Besides the three well-established structures observed in the inclusive hadronic cross section [17], i.e., $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$, five Y states, i.e., Y(4220), Y(4320), Y(4360), Y(4390) and Y(4660) have been observed. These newly-observed Y states exceed the number of vector charmonium states predicted by potential models in this energy region. They are thus good candidates for exotic states, such as hybrid states, tetraquark states and molecule states [18].

Figure 1 shows the cross sections of $e^+e^-\rightarrow\omega_{c0}$ [12, 13], $\pi^+\pi^-h_c$ [14, 15], $\pi^+\pi^-J/\psi$ [7, 20, 21], $\pi^+\pi^-\psi(3686)$ [8–10] and $\pi^+D^0D^{*-}+c.c.$ [15] measured by Belle, BABAR, CLEO and BESIII experiments. For data from BESIII, “XYZ” data sample refers to the energy points with integrated luminosity larger than 40 pb$^{-1}$ and “scan” data sample refers to the energy points with integrated luminosity smaller than 20 pb$^{-1}$. In this paper, we perform a combined fit to these cross sections.

![FIG. 1. Cross sections of $e^+e^-\rightarrow\omega_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, $\pi^+\pi^-\psi(3686)$ and $\pi^+D^0D^{*-}+c.c.$ measured by Belle, BABAR, CLEO and BESIII experiments.](image_url)

We parameterize the cross section with the coherent sum of a few amplitudes, either resonance represented by a Breit-Wigner (BW) function or non-resonant pro-


TABLE I. The parameters for \( Y(4220) \) (\( \omega_{\pi\pi}, \pi^+\pi^- h_c, \pi^+\pi^- J/\psi, \pi^+\pi^- \psi(3686) \) and \( \pi^+ D^0 D^{*-} + c.c. \)), \( Y(4320) \) (\( \pi^+\pi^- J/\psi, Y(4300) \) (\( \pi^+\pi^- \psi(3686) \)) and \( Y(4390) \) (\( \pi^+\pi^- h_c \) and \( \pi^+ D^0 D^{*+} + c.c. \)) states in different processes. The first uncertainties are statistical, and the second systematic.

| \( Y(4220) \) | \( Y(4320)/Y(4360)/Y(4390) \) |
|----------------|---------------------|
| \( \omega_{\pi\pi} \) | 4226 ± 8 ± 6 | 39 ± 12 ± 2 |
| \( \pi^+\pi^- h_c \) | 4218.4±5.8±0.9 | 66.0±12.3±0.4 |
| \( \pi^+\pi^- J/\psi \) | 4222.0±3.1±1.4 | 44.1±4.3±2.0 |
| \( \pi^+\pi^- \psi(3686) \) | 4209.1±6.8±7.0 | 76.6±14.2±2.4 |
| \( \pi^+ D^0 D^{*+} + c.c. \) | 4242.8±5.6±4.0 | 72.3±9.1±0.9 |

\[
\sigma_{\pi^+ D^0 D^{*-} + c.c.}(\sqrt{s}) = |c_2| \sqrt{PS(\sqrt{s})} + BW_1(\sqrt{s})e^{i\phi_6} + BW_2(\sqrt{s})e^{i\phi_7}|^2,
\]

where \( BW_1, BW_2 \) and \( BW_3 \) denote the resonances \( Y(4220), Y(4320) \) and \( Y(4660) \), respectively; \( PS(\sqrt{s}) \) is the phase space factor; \( EXP(\sqrt{s}) = e^{-p_0(3M_{ih} - M_{ih})}/PS(\sqrt{s}) \), is an exponential function, where \( p_0 \) is the free parameter, \( M_{ih} = 2m_{\pi} + m_{J/\psi} \) is the mass threshold of the \( \pi^+\pi^- J/\psi \) system; \( \phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6 \) and \( \phi_7 \) are relative phases; \( c_1 \) and \( c_2 \) are amplitudes of exponential function term and phase space term.

We fit to the cross sections of \( e^+e^- \rightarrow \omega_{\pi\pi}, \pi^+\pi^- h_c, \pi^+\pi^- J/\psi, \pi^+\pi^- \psi(3686) \) and \( \pi^+ D^0 D^{*-} + c.c. \) simultaneously. The fits for \( e^+e^- \rightarrow \omega_{\pi\pi}, \pi^+\pi^- h_c, \pi^+\pi^- J/\psi, \pi^+\pi^- \psi(3686) \) and \( \pi^+ D^0 D^{*-} + c.c. \) are found to have one solution, two solutions, four solutions, four solutions and four solutions with the same minimum values of \( \chi^2 \), respectively. The masses and widths of the resonances are identical, but the \( \Gamma_{e^+e^-}B_f \) are different in different solutions for each process.

Figure 2 shows the fit results with a goodness of the fit is \( \chi^2/ndf = 460/474 = 0.97 \), corresponding to a confidence level of 67%. The good fit indicates that the assumption that the two resonances \( Y(4220) \) and \( Y(4390) \) are same two states in these processes is reasonable. From fit results, we can get \( M_{Y(4220)} = 4216.5±1.4 \) MeV/c\(^2\), \( \Gamma_{Y(4220)} = 61.1±2.3 \) MeV; \( M_{Y(4390)} = 4383.5±1.9 \) MeV/c\(^2\), \( \Gamma_{Y(4390)} = 141.5±5.4 \) MeV; \( M_{Y(4660)} = 4623.4±2.5 \) MeV/c\(^2\), \( \Gamma_{Y(4660)} = 106.1±16.2 \) MeV. The all obtained resonant parameters from fit are listed in Table II.

The systematic uncertainties on the resonant parameters in the combined fit to the cross sections of \( e^+e^- \rightarrow \omega_{\pi\pi}, \pi^+\pi^- h_c, \pi^+\pi^- J/\psi, \pi^+\pi^- \psi(3686) \) and \( \pi^+ D^0 D^{*-} + c.c. \) are mainly from the uncertainty of the center-of-mass energy determination, parametrization of the BW function, background shape and the cross section measurements.

Since the uncertainty of the beam energy is about 0.8 MeV at BESIII, so the uncertainty of the resonant parameters caused by the beam energy is estimated by varying \( \sqrt{s} \) within 0.8 MeV for BESIII data. Instead of using a constant total width, we assume a mass dependent
width to estimate the uncertainty due to parametrization of BW function. To model the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ cross section near 4 GeV, a BW function is used to replace the exponential function, and the difference of the fit results in the two methods are taken as the uncertainty from background shape. The uncertainty of the cross section measurements will affect the resonant parameters in fit, we vary the cross sections within the systematic uncertainty, and the difference in the final results are taken as the uncertainty. By assuming all these sources of systematic uncertainties are independent, we add them in quadrature.

The leptonic decay width for a vector state is an important quantity for discriminating various theoretical models. By considering the isospin symmetric modes of the measured channels, we can estimate the lower limits on the leptonic partial width of the $Y(4220)$ and $Y(4390)$ decays. For an isospin-zero charmonium-like state, we expect

$$B(Y \rightarrow \pi \pi h_c) = \frac{3}{2} \times B(Y \rightarrow \pi^+\pi^- h_c),$$

$$B(Y \rightarrow \pi \pi J/\psi) = \frac{3}{2} \times B(Y \rightarrow \pi^+\pi^- J/\psi),$$

$$B(Y \rightarrow \pi \pi \psi(3686)) = \frac{3}{2} \times B(Y \rightarrow \pi^+\pi^- \psi(3686)),$$

$$B(Y \rightarrow \pi DD^*) = 3 \times B(Y \rightarrow\pi^+ D^0 D^{*-} + c.c.),$$

so we have

$$\Gamma_{e^+e^-}^Y = \sum_f B(Y(4220) \rightarrow f) \times \Gamma_{e^+e^-}^{Y(4220)}$$

$$= B(Y(4220) \rightarrow \omega \chi_{c0}) \times \Gamma_{e^+e^-}^{Y(4220)} + B(Y(4220) \rightarrow \pi \pi h_c) \times \Gamma_{e^+e^-}^{Y(4220)} + B(Y(4220) \rightarrow \pi \pi J/\psi) \times \Gamma_{e^+e^-}^{Y(4220)} + B(Y(4220) \rightarrow \pi \pi \psi(3686)) \times \Gamma_{e^+e^-}^{Y(4220)}.$$
On the other hand, if we take the results with the uncertainties from Table II, considering the solutions with the smallest $\Gamma(Y(4220) \to f) \times \Gamma_{ee}^{Y(4220)}$ and $B(Y(4490) \to f) \times \Gamma_{ee}^{Y(4490)}$, we obtain

\[
\Gamma_{ee}^{Y(4220)} = \left(3.5 \pm 0.4 \pm 0.5\right) + \left(\frac{3}{2} \times 3.1 \pm 0.2 \pm 0.8\right) + \frac{3}{2} \times 3.1 \pm 0.3 \pm 0.6 + \frac{1}{2} \times 1.5 \pm 0.3 \pm 0.3 + 3 \times 7.1 \pm 0.6 \pm 1.3 + \cdots \text{ eV}
\]

\[
= (36.4 \pm 2.0 \pm 4.2) + \cdots \text{ eV}
\]

\[
> (36.4 \pm 2.0 \pm 4.2) \text{ eV},
\]

and

\[
\Gamma_{ee}^{Y(4390)} = \left(\frac{3}{2} \times 7.5 \pm 0.6 \pm 1.8\right) + \frac{3}{2} \times (0.3 \pm 0.1 \pm 0.1) + \frac{3}{2} \times (9.9 \pm 1.0 \pm 1.2) + \frac{3}{2} \times (32.4 \pm 2.1 \pm 2.8) + \cdots \text{ eV}
\]

\[
= (123.8 \pm 6.5 \pm 9.0) + \cdots \text{ eV}
\]

\[
> (123.8 \pm 6.5 \pm 9.0) \text{ eV},
\]

where the first uncertainties are statistical, and the second systematic.

On the other hand, if we take the results with the largest $B(Y(4220) \to f) \times \Gamma_{ee}^{Y(4220)}$ and $B(Y(4390) \to f) \times \Gamma_{ee}^{Y(4390)}$ in Table II, we obtain $\Gamma_{ee}^{Y(4220)} = (206.6 \pm 9.1 \pm 18.7) + \cdots$ and $\Gamma_{ee}^{Y(4390)} = (1001.7 \pm 41.8 \pm 79.5) + \cdots$ eV. This means that the leptonic partial widths of $Y(4220)$ and $Y(4390)$ can be as large as 200 and 1000 eV or even higher based on current information, because maybe there are some other decay channels for $Y(4220)$ and $Y(4390)$ that we have not observed.

In summary, a combined fit is performed to the cross sections of $e^+e^- \to \omega\chi_{c0}$, $\pi^+\pi^-h_c$, $\pi^+\pi^-J/\psi$, $\pi^+\pi^-\psi(3686)$ and $\pi^+D^0D^{*-} + c.c.$ by using three resonances $Y(4220)$, $Y(4390)$ and $Y(4660)$. The parameters are determined to be $M_{Y(4220)} = (4216.5 \pm 1.4 \pm 3.2)$ MeV/$c^2$, $\Gamma_{Y(4220)} = (61.1 \pm 2.3 \pm 3.1)$ MeV; $M_{Y(4390)} = (4838.5 \pm 1.9 \pm 6.0)$ MeV/$c^2$, $\Gamma_{Y(4390)} = (114.5 \pm 5.4 \pm 9.9)$ MeV; $M_{Y(4660)} = (4623.4 \pm 10.5 \pm 16.1)$ MeV/$c^2$, $\Gamma_{Y(4660)} = (106.1 \pm 16.2 \pm 17.5)$ MeV, where the first uncertainties are statistical and the second systematic.

We emphasize that two resonances $Y(4220)$ and $Y(4390)$ are sufficient to explain these cross sections below 4.6 GeV. The resonances $Y(4360)$, $Y(4390)$ and $Y(4660)$ should be one state. The lower limits of $Y(4220)$ and $Y(4390)$’s leptonic decay widths are also determined to be $(36.4 \pm 2.0 \pm 4.2)$ and $(123.8 \pm 6.5 \pm 9.0)$ eV. These results will be useful in understanding the nature of charmonium-(like) states in this energy region. Higher precision measurements around this energy region are desired, this can be achieved in BESIII and BelleII experiments in the further.

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