Fundamentalization of mathematical training: looking for promising solutions

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Abstract. Mathematics has a special place in ensuring the interdisciplinary integration of various Sciences. Its language is universal. It acts as a tool in various scientific fields of knowledge, as well as in many technological and industrial spheres of human activity. This means that the importance of functional mathematical training of the younger generation increases, which leads to the search for promising solutions for training future teachers of physical and mathematical, natural science, and technological profiles. The article discusses one of the solutions proposed by the authors. The authors’ proposals are aimed at solving the problem of training teachers who will be able to make the school mathematics course accessible to many students without losing quality, providing fundamental and functional mathematical knowledge. And the first necessary step in the proposed proposals is the implementation of the process of forming the ability to apply mathematical knowledge in the construction of models of real objects, processes, and phenomena. The authors propose a scheme of such a process, which allows the studied mathematical models to be embedded in a holistic context, and to study them from the point of view of practical applicability in solving various problems. The experimental work carried out confirmed the effectiveness of the implementation of the process under consideration. It showed that the formation of the ability to apply mathematical knowledge in the construction of models of real objects, processes, phenomena in future teachers of physics and mathematics, natural science, technology ensures fundamentalization of their mathematical background. The next necessary step to solve problem proposed by authors consists in changing the pattern of methodical training of future teachers.

1. Introduction

The universality of mathematical knowledge is generally recognized [1]. Patterns described in the language of mathematics, mathematical methods for solving problems, methods for constructing statements and conducting proofs, etc. are necessary attributes of various scientific fields of knowledge. And, consequently, there is a need to create fundamental opportunities to make the course of school mathematics accessible to the majority of students. At the same time, it is important to ensure the fundamental nature of mathematical training for schoolchildren, which involves strengthening the relationship between theoretical and practical aspects of mathematical knowledge. Fundamentality appears as a principle focused “on bringing fundamental knowledge to priority positions and giving this knowledge the value of the basis or core for accumulating a lot of knowledge and forming skills based on it” [2].
The fundamental nature of students’ training depends directly on the fundamental nature of the mathematical training of future teachers of physical and mathematical, natural science, and technological profiles. For teachers of selected profiles, the logical potential of mathematics is necessary not only for the implementation of internal tasks in the development of the subject, but also for solving professional problems. After all, the fundamental nature of training is the "basis for the formation of professional competencies of a specialist: ability to analyze, design, research (theoretical, empirical) and the ability to apply knowledge in practice in the context of professional activity" [3]. At the same time, "... the fundamentalization of mathematical training of future teachers plays a leading role in the formation and improvement of their logical and professional innovative thinking through the comprehension of multifunctional cognitive structures and schemes (as tools and methods of knowledge), which are analogous with mathematical schemes and structures" [4].

2. Problem statement

It should be recognized that mathematics, as an abstract science, is very difficult to perceive, and becomes inaccessible to many students. The proposed abstractions are easily mastered by units. And it makes sense to teach such gifted students in special mathematical classes based on selection, including the results of mathematical Olympiads. That is why it is so important to find a way to make the school mathematics course accessible to many students. This implies its presentation in a language that will allow us to study and describe various social and natural phenomena, objects of the technosphere, as well as various processes of functioning and interaction of objects and environments. And it is this language, understandable and accessible, that future teachers of physics and mathematics, natural science, and technology profiles should master in the course of training at the University. Learning of language, which will be used to teach mathematics in school, should be combined with fundamentality of mathematical conception of teachers, their experiences to introduce schoolchildren into the world of mathematical culture as a basic foundation of many scientific fields of knowledge.

3. Research questions

To solve this problem, it is important to realize that the main result of teaching basic theoretical concepts in the subject field of school mathematics is not just the development of mathematical concepts, but also the ability to apply them in solving practice-oriented problems. Knowledge base formed under the study of such branches of mathematics as algebra basics, basics of mathematical analysis, geometry, foundations of the theory of probability, etc., ability to operate with mathematical symbols, to apply mathematical methods in solving different applied tasks are extremely necessary for specialists in various fields of science, industry, government, education, etc.

Therefore, it is necessary that teachers who carry out mathematical training at school or actively use the mathematical apparatus in subject training have mathematical knowledge at a fundamental level, allowing them to present it in the relationship of theoretical and practical aspects, immersing students in the world of mathematical methods for solving various applied problems. "In this regard, mathematics in high school should be studied as a unified tool for studying other scientific disciplines, i.e. it is necessary to teach to understand abstract mathematical symbols and translate them into practical terms, to build mathematical models. In this case, we mean that the model is a scheme of a phenomenon that is simpler than the original, but reflects its main properties, and the mathematical model is a description of this scheme in mathematical language" [5]. In the process of studying them, the discovery of knowledge will occur – the discovery of "ideal theoretical constructs" for student (facts, notions, laws). Thus, as follows from didactic idea of A. V. Khutorskoy [6], if students recognize created knowledge and used methods of cognition as “personal educational product”, they can use this to perceive real world. And at the same time it is important that obtained personal educational product should be comparable with cultural-historic analogs – products of human activities in this field of knowledge.

Of particular interest is the search for real objects and processes for cognition aimed at obtaining a personal educational product in the form of mathematical models. It seems that such real objects and processes of cognition can be the simplest mechanisms, processes related to changes in body position,
processes related to the distribution of financial resources. Therefore, it is quite justified to rely on mechanics when mastering mathematical knowledge, because it operates with mathematical models of movement, financial mathematics, which requires the construction of models for conducting financial transactions and calculations.

To implement this approach, it is necessary to organize the interaction of future teachers of physics and mathematics, natural science, and technology profiles with real objects, processes, and phenomena. It is important to direct the process of their preparation to the formation of skills to apply mathematical knowledge in the construction of models of real objects, processes, and phenomena. The specificity of the process of formation of skills to apply mathematical knowledge to develop models of real objects, processes, phenomena manifested in the fact that the studied mathematical model embedded in a holistic context, are examined from the point of view of their practical applicability in solving various tasks.

The process of forming the ability to apply mathematical knowledge in the construction of models of real objects, processes, and phenomena is presented in the form of a diagram in figure 1.

![Diagram of building models of real objects, processes, and phenomena](image)

**Figure 1.** Diagram of building models of real objects, processes, and phenomena [7].

Turning to classical Newton's mechanics and financial mathematics allows us to consider the object and process as a whole, putting the principles of an integral organization at the head of the study, rather than individual elements.

To implement the described process, the following conditions must be implemented:

- immersion of future teachers in real situations;
- presentation to future teachers the simulation as a strategy for solving the problem;
- organization of design and research activities of future teachers to find solutions in a real situation;
- lead to the understanding of the essence of the studied objects and processes based on the identification of fundamental relationships, patterns, principles that are not set by algorithms and formulas.

4. **Purpose of the study**

The purpose of the experimental work is to evaluate the effectiveness of the developed scheme of the process of forming the ability to apply mathematical knowledge in the construction of models of real phenomena.

Experimental work was carried out on the basis of the Krasnoyarsk state pedagogical University named after V. P. Astafiev in the preparation of future technology teachers in the first and second years. It was attended by more than 80 people.

5. **Research methods**

The main method of research was a formative experiment aimed at mastering the skills of future teachers to apply mathematical knowledge in the construction of models of real phenomena. During the formative
experiment with teachers, the above scheme was used to master mathematical knowledge about the equations of motion of material points, solids and mechanical systems; to study the laws of classical mechanics for the movement of a material point and a mechanical system; to identify the elementary structure of the simplest mechanisms (movable block, crank, slider, connecting rod, transfer mechanisms, etc.) and the principles of their operation; to discuss the principles of financial transactions of various complexity. Thus, the conditions were created for building systematic ideas about classical mechanics of I. Newton, as well as understanding the basic foundations of financial mathematics. The breadth and consistency of ideas and understanding of the basic foundations are necessary for the future teacher to subsequently find various approaches to the study of objects and processes together with students in a holistic context, to understand the dynamics of processes in the studied systems.

The evaluation of the effectiveness of formative actions was carried out on the basis of testing and analysis of the products of future teachers’ activities. Testing was aimed at recognizing an object, process, or phenomenon that was described by the proposed mathematical model; recognizing a pattern described using a certain sequence of mathematical symbols. The analysis of products of activity involved testing the formation of the ability to choose and design a real object that can explain individual elements of mathematical knowledge, specific mathematical models, and ways to use mathematical symbols to describe patterns.

6. Findings
The main indicator of mastering the ability to apply mathematical knowledge in the construction of models of real objects, processes, and phenomena was the level of development of activities, rather than the amount of knowledge acquired [8]. The assessment of knowledge and skills of future teachers was limited to level II-III. The output to the II level of assimilation (reproductive activity without prompting) was determined by recognizing an object, process, or phenomenon based on the presentation of standard mathematical models; selecting and constructing a real object based on the proposed model. Achievement level III (productive heuristic activity) was established, if recognition of an object, process, phenomenon was conducted to complicated mathematical models; selecting, designing real object proposed independent non-typical search for solutions, the use of different apparatus.

The average coefficient of the quality of knowledge and skills acquisition was evaluated based on the results of six control sections. The dynamics of the coefficient of assimilation of knowledge and the formation of skills to apply mathematical knowledge in the construction of models of real objects, processes, phenomena indicates the development of reproductive (Ka2 = 0,68; 0,69; 0,71; 0,74; 0,79; 0,83) and productive (KA3 = 0,61; 0,62; 0,65; 0,67; 0,68; 0,71) level of knowledge and skills in the above context.

The analysis of the obtained data (Ka≥0,7) allows us to conclude that this degree of mastering the necessary knowledges, formed competences in the use of mathematical knowledges to the construction of models of real objects will permit to improve their knowledges and competences in the future in the course of the self-education. The development of productive (heuristic) experience of future teachers confirms a sufficient level of quality of the fundamentalization of mathematical training, which is expressed in the ability to apply the obtained knowledges in new conditions.

7. Conclusion
Experimental work has shown that the formation of the ability to apply mathematical knowledges in the construction of models of real objects, processes, phenomena in future teachers of physical and mathematical, natural science, and technological profiles provides the fundamentalization of their mathematical training. The proposed method of mastering mathematical knowledge allows us to significantly eliminate the contradictions between the need to integrate knowledge of several scientific and subject areas, on the one hand, and the existing significant disciplinary splitting of the studied educational material, on the other hand.

The proposed solution to the fundamentalization of mathematical training of future teachers in the development and implementation of basic educational programs for physical and mathematical, natural
science, and technological profiles of pedagogical direction is relevant and promising. Its implementation in practice will allow to prepare teachers who are able to ensure the fundamental and functional mathematical training of students. At the same time, it should be understood that it is also necessary to change the nature of methodological training of such teachers. It is aimed at forming the readiness of teachers to use the proposed scheme for building models of real objects, processes, and phenomena in their own teaching activities.

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