Seismological constraints on the high-gravity DOV stars PG2131+066 and PG 1707+427

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Abstract. A seismological study of the pulsating PG1159 stars PG2131+066 and PG 1707+427 is presented. We perform extensive adiabatic computations of g-mode pulsation periods of PG1159 evolutionary models with stellar masses ranging from 0.530 to 0.741 M☉. We constrain the stellar mass of PG2131+066 and PG 1707+427 by comparing the observed period spacing of each star with the theoretical asymptotic period spacings and with the average of the computed period spacings. We also employ the individual observed periods to find representative seismological models for both stars.

1. Introduction
Pulsating PG1159 stars (also called GW Vir or DOV stars) are very hot hydrogen-deficient post-Asymptotic Giant Branch stars with surface layers rich in He, C and O (Werner & Herwig 2006) exhibiting multimode pulsations (non-radial g-modes) with periods ranging from 5 to 50 minutes (Winget & Kepler 2008). It is generally accepted that these stars have their origin in a born-again episode induced by a post-AGB He thermal pulse (Althaus et al. 2005). Recently, considerable observational effort has been invested to study pulsating PG1159 stars. Particularly noteworthy are the observational efforts devoted to RXJ 2117.1+3412 (Vauclair et al. 2002), PG 0122+200 (Fu et al. 2007) and PG 1159–035 (Costa et al. 2008). On the theoretical front, recent important progress in the numerical modeling of PG1159 stars (Althaus et al. 2005; Miller Bertolami & Althaus 2006) has paved the way for unprecedented seismological inferences for the mentioned stars (Córsico et al. 2007a; Córsico et al. 2007b; Córsico et al. 2008). Here, we present a seismological study of the high-gravity pulsating PG1159 stars PG 2131+066 and PG 1707+427, two stars that have been intensively scrutinized with the Whole Earth Telescope.

Our pulsation analysis is based on a new generation of stellar models that take into account the complete evolution of PG1159 progenitor stars with initial masses on the ZAMS ranging from 1 to 3.75 M☉ (Althaus et al. 2005; Miller Bertolami & Althaus 2006). All the post-AGB evolutionary sequences were computed using the LPCODE evolutionary code and were followed through the very late thermal pulse and the resulting born-again episode that gives rise to the H-deficient, He-, C- and O-rich composition characteristic of PG1159 stars. The masses of
the resulting remnants are 0.530, 0.542, 0.556, 0.565, 0.589, 0.609, 0.664, and 0.741 $M_{\odot}$. We computed $\ell = 1$ $g$-mode adiabatic pulsation periods for these equilibrium models with the same numerical code and methods we employed in our previous works (Córneo & Althaus 2006).

2. Mass determination from the observed period spacing
Here, we constrain the stellar mass of PG 2131+066 and PG 1707+427 by comparing the asymptotic period spacing and the average of the computed period spacings with the observed period spacing. These approaches are based on the fact that the period spacing of PG1159 pulsators depends primarily on the stellar mass, and the dependence on the luminosity and the He-rich envelope mass fraction is negligible (Kawaler & Bradley 1994; Córneo & Althaus 2006). Fig. 1 displays the asymptotic period spacing for $\ell = 1$ modes as a function of the effective temperature for different stellar masses. Also shown in this diagram is the location of PG 2131+066, with $T_{\text{eff}} = 95 \pm 5$ kK (Dreizler & Henber 1998) and $\Delta\Pi_{\ell=1}^{O} = 21.6 \pm 0.4$ s (Reed et al. 2000), and PG 1707+427, with $T_{\text{eff}} = 85 \pm 5$ kK (Dreizler & Heber 1998) and $\Delta\Pi_{\ell=1}^{O} = 23.0 \pm 0.3$ s (Kawaler et al. 2004). The asymptotic period spacing is computed as in Tassoul et al. (1990). From the comparison between the observed $\Delta\Pi_{\ell=1}^{O}$ and $\Delta\Pi_{\ell}$, we found a stellar mass of 0.627 $M_{\odot}$ for PG 2131+066 and 0.597 $M_{\odot}$ for PG 1707+427. In Fig. 2 we show the run of average of the computed ($\ell = 2$) period spacings ($\ell = 1$) for PG 2131+066 (left panel), and PG 1707+427 (right panel) in terms of the effective temperature for all of our PG1159 evolutionary sequences. From the comparison between the observed $\Delta\Pi_{\ell=1}^{O}$ and $\Delta\Pi_{\ell}$, we found a stellar mass of 0.578 $M_{\odot}$ and 0.566 $M_{\odot}$ for PG 2131+066 and PG 1707+427, respectively. These values are 8.5% (for PG 2131+066) and 5.5% (for PG 1707+427) smaller than those derived through the asymptotic period spacing, showing once again that the asymptotic approach overestimates the stellar mass of PG1159 stars that, like PG 2131+066 and PG 1707+427, exhibit short and intermediate pulsation periods.

3. Constraints from the individual observed periods
In this approach we seek pulsation models that best matches the individual pulsation periods of PG 2131+066 and PG 1707+427. For both stars, we assume that all of the observed periods correspond to $\ell = 1$ modes (Kawaler et al. 1995; Kawaler et al. 2004). The goodness of the match between the theoretical pulsation periods ($\Pi_{\ell}^{T}$) and the observed individual periods ($\Pi_{\ell}^{O}$) is measured by means of a merit function, $\chi^{2}(M_{*}, T_{\text{eff}})$, which takes into account the differences between the observed and theoretical periods squared. For each star, the PG 1159 model that shows the lowest value of $\chi^{2}$ will be adopted as the “best-fit model”. For PG 2131+066, we find a clear seismological solution corresponding to a model with $M_{*} = 0.589 M_{\odot}$ and $T_{\text{eff}} \approx 102$ kK. This model provides an excellent agreement between the theoretical and observed periods.
Table 1. The main characteristics of PG 2131+066. Column (1) corresponds to spectroscopic results from Dreizler & Heber (1998), column (2) presents results from previous pulsational studies (Kawaler et al. 1995; Reed et al. 2000) and column (3) provides the results from the seismological model of this work, respectively.

|         | (1)       | (2)       | (3)       |
|---------|-----------|-----------|-----------|
| $T_{\text{eff}}$ [kK] | 95 ± 5    | —         | 102.18±3.0 |
| $M_\star$ [$M_\odot$] | 0.55 ± 0.1 | 0.608 ± 0.01 | 0.589±0.020 |
| log $g$ [cm/s$^2$] | 7.5 ± 0.5 | —         | 7.63±0.14 |
| log($L_\star$/$L_\odot$) | 1.6 ± 0.3 | 1.41 ± 0.5 | 1.57±0.07 |
| log($R_\star$/$R_\odot$) | —        | −1.73     | −1.71±0.05 |
| $d$ [pc] | 681±170   | 668±78    | 830±224   |
| $\pi$ [mas] | 1.47±0.4  | 1.50 ± 0.2 | 1.2±0.3   |

This is quantitatively reflected by the average of the absolute period differences, $\overline{\delta \Pi}$ = 1.57 and the root-mean-square residual $\sigma_{\delta \Pi} = 2.32$ s. For PG 1707+427, we find a unambiguous seismological solution for a model with $M_\star = 0.542 M_\odot$ and $T_{\text{eff}} \approx 89.5$ kK, an effective temperature compatible with the spectroscopic determination. We found $\overline{\delta \Pi}$ = 1.75 s and $\sigma_{\delta \Pi} = 1.99$ s.

The main features of our best-fit model for PG 2131+066 are summarized in Table 1. The total mass of the best-fit model ($M_\star = 0.589 M_\odot$) is in agreement with the value derived from the average of the computed period spacings ($M_\star \sim 0.578 M_\odot$), but at odds (~6% smaller) with that inferred from the asymptotic period spacing ($M_\star = 0.627 M_\odot$). Also, the $M_\star$ value of our best-fit model is substantially larger than the spectroscopic mass of 0.53 ± 0.1 $M_\odot$ (Dreizler & Heber 1998; Miller Bertolami & Althaus 2006) for PG 2131+066. A discrepancy between the seismological and the spectroscopic values of $M_\star$ is generally encountered among PG1159 pulsators (Córso et al. 2007a; Córso et al. 2007b). Until now, the seismological mass of PG 2131+066 has been about 11% larger ($\Delta M_\star \approx 0.06 M_\odot$) than the spectroscopic mass if we consider the early estimation for the seismological mass quoted in Reed et al. (2000). In light of the best-fit model derived in this paper, this discrepancy is slightly reduced to about 7% ($\Delta M_\star \approx 0.04 M_\odot$).

The main properties of our best-fit model for PG 1707+427 are shown in Table 2. The stellar mass of our best-fit model is $M_\star = 0.542 M_\odot$, in agreement with the value derived from
the average of the computed period spacings ($M_* \sim 0.566 M_\odot$), but at odds ($\sim 9\%$ lower) with that inferred from the asymptotic period spacing ($M_* = 0.597 M_\odot$). On the other hand, we note that $M_*$ for the best-fit model is in excellent agreement with the spectroscopic derivation ($0.542 M_\odot$ versus $0.53 M_\odot$). Until now, the seismological mass of PG 1707$+$427 has been more than $7\%$ larger ($\Delta M_* \approx 0.04 M_\odot$) than the spectroscopic mass if we adopt for the seismological mass the value found previously (Kawaler et al. 2004). In light of our best-fit model, this discrepancy is strongly reduced to about $2\%$ ($\Delta M_* \approx 0.012 M_\odot$).

4. Summary and conclusions
In this work we have estimated the stellar mass of PG 2131$+$066 and PG 1707$+$427 on the basis of the period-spacing information alone, and we have also been successful in finding seismological models for both stars from period-to-period fits. The present study closes our short series of seismological studies of PG1159 stars based on a grid of full evolutionary models characterized by thick He-rich envelopes, as dictated by canonical stellar evolution.

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