Modal Response Characteristics of a Multiple-Degree-Of-Freedom Structure Incorporated with Tuned Viscous Mass Dampers

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Abstract
A new seismic control device using a ball screw mechanism as an apparent mass amplifier has been developed, which is referred to as the tuned viscous mass damper (TVMD). This device enables effective seismic control using a tuned mass. For a multiple-degree-of-freedom (MDOF) seismic control system incorporated with the TVMD, a design method based on numerical optimization has been previously presented by the authors. However, simpler design methods that are suitable for a practical design have not yet been presented. At the preliminary design stage, it is essential for structural designers to understand the seismic response characteristics of the structure in terms of modal responses. However, a complex-valued eigenvalue analysis, which most structural designers are unfamiliar with, is required for accurate seismic response estimation. This is because the seismic control system incorporated with TVMDs is nonproportionally damped. In this paper, the authors propose a seismic response estimation method that does not require a complex valued analysis. An analysis example illustrates that the square root of the sum of the square (SRSS) of the maximum modal responses derived from the undamped real eigenvalue analysis gives a good approximation in practical terms.

Keywords: seismic control; tuned mass damper; fixed point method; optimum design; complex-valued eigenvalue analysis

1. Introduction
Saito et al. (2008) developed a new seismic control system using a rotational viscous mass damper (Fig.1.) connected with a soft spring. This device is referred to as the tuned viscous mass damper (TVMD). The basic concept of the TVMD is the same as that of a tuned mass damper (TMD) or a dynamic vibration absorber. Furthermore, its optimum design is obtained using fixed points on its resonance curve (Den Hartog 1956). TMD for buildings is effective against wind-induced vibrations (McNamara 1979); however, a statistical study on TMD systems with a secondary mass ratio of less than 0.02 showed that it is not necessarily effective against earthquake-induced vibrations (Kaynia et al. 1981).

Hence, a large secondary mass (larger than the effective mass of the primary structure by several percent) is required to achieve reduction in seismic vibrations. However, installing such a large mass in a building might cause many problems. A large apparent mass (larger than the effective mass of the primary structure by several percent) can be easily obtained by a mass amplifying mechanism using a ball screw and a cylindrical flywheel with a small actual mass in the TVMD (Ikago et al. 2012).

For a multiple-degree-of-freedom (MDOF) TVMD seismic control system, design methods based on numerical optimization have been previously presented by the authors (Ikago et al., 2011a, b). However, simpler design methods that are suitable for practical designs have not yet been presented.
At the preliminary design stage, it is essential for practicing structural designers to understand the seismic response characteristics of the structure in terms of modal responses. Although a complex-valued eigenvalue analysis is required because the seismic control system with TVMDs is nonproportionally damped, complex modes are not commonly used in practice; instead, undamped real modes are used.

In this paper, an analysis example elucidates the modal response characteristics of the MDOF system incorporated with TVMDs. Moreover, using the square root of the sum of the squares (SRSS) of the maximum modal responses derived from the undamped real eigenvalue analysis, the example illustrates the effectiveness of a simple seismic response estimation method that is useful for the seismic control design of a structure incorporated with TVMDs.

2. Analysis Model

In this study, the authors use a 10-story benchmark structure provided by the Japan Society of Seismic Isolation (JSSI, 2007) as a seismic control analysis example. As shown in Fig.2., a TVMD is configured with a rotational mass and a dashpot arranged in parallel, which are connected by a soft spring. The schematic and characteristics of the analytical model are shown in Fig.3. and Tables 1. and 2., respectively.

The benchmark structure is controlled by a TVMD system, in which the mass ratio of the secondary structure to the primary structure is 0.1.

3. Equations of Motion
3.1 Equation of motion for the uncontrolled primary system

As shown in Fig.3. and Table 1., \( m_i, c_i, \) and \( k_i \) represent the mass, damping coefficient, and shear stiffness of the \( i \)th story, respectively. \( m_{r,i}, c_{d,i}, \) and \( k_{b,i} \) represent the mass, damping coefficient, and spring stiffness of the TVMD installed in the \( i \)th story, respectively. The equation of motion for the uncontrolled primary structure is as follows:

\[
M \ddot{\mathbf{x}}_p + C \dot{\mathbf{x}}_p + K \mathbf{x}_p = -M \mathbf{1} \ddot{\mathbf{w}}_p,
\]  

where \( \mathbf{x}_p = \{x_1, x_2, \ldots, x_n\}^T \) is the displacement vector of the primary system relative to the ground, \( \mathbf{1} = \{1, 1, 1, \ldots, 1\}^T \) is the influence coefficient vector, \( C_p \) is the inherent damping matrix for the primary structure, \( x_0 \) is the ground displacement, and superscript \( T \) denotes a matrix transposition.

\[
M = \begin{bmatrix}
    m_1 & 0 & \cdots & 0 \\
    0 & m_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & \cdots & 0 & m_n
\end{bmatrix}, 
\]  

\[
K = \begin{bmatrix}
    k_1 & -k_2 & \cdots & 0 \\
    -k_2 & k_2 + k_3 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    -k_{n-1} & k_{n-1} + k_n & -k_n
\end{bmatrix},
\]  

If we assume that \( C_p \) is proportional to the stiffness matrix and the inherent damping ratio for the 1st mode of the primary structure \( \xi \) equals 0.02, \( C_p \) is given by

\[
C_p = \frac{2\xi}{i\omega_p} K_p = \frac{0.04}{i\omega_p} K_p,
\]  

where \( i\omega_p \) is the lowest fundamental angular frequency of the undamped primary structure.
3.2 Design of the TVMD parameters

Because the TVMD system is activated by the interstory displacements of the primary structure, the effective modal mass of the additional vibration system tuned to the first mode is expressed as follows:

\[ M_d = m_{d,1} \phi_1^2 + \sum_{k=2}^{n} m\phi_k^2 + \sum_{j=1}^{n} k_{ij}(\phi_j - \phi_{j-1})^2, \]  

(5)

where, \( \phi_k \) is the \( k \) th component of the \( j \) th mode vector of the undamped primary system.

Thus, the effective mass ratio is given by \( \mu = \frac{M_d}{\phi^T K_p \phi} \). In this study, an additional mass ratio of 0.1 is specified, and the distribution of the additional masses is set such that it is proportional to that of the story stiffness.

\[ m_{d,i} = \alpha k_i \]  

(6)

Substituting Eq. (6) into Eq. (5) yields

\[ M_d = \alpha \left[ k_1 \phi_1^2 + \sum_{j=2}^{n} k_j (\phi_j - \phi_{j-1})^2 \right] \]

\[ = \alpha \phi^TK_p \phi \]  

(7)

Thus,

\[ \alpha = \frac{M_d}{\phi^TK_p \phi} = \frac{\mu}{\phi^TK_p \phi} = \frac{\mu}{\Omega_p^2}. \]

(8)

For the equivalent single-degree-of-freedom (SDOF) system incorporated with the TVMD, the optimal angular frequency \( \omega_{\text{opt}} \) and damping ratio \( \zeta_{\text{opt}} \) for the specified mass ratio \( \mu \) is obtained using the fixed point method (Saito et al. 2008).

\[ k_{d,i} = \left( \omega_{\text{opt}}^2 \right)^{2} m_i \]  

(9)

\[ c_{d,i} = 2c_{d,i} \omega_{\text{opt}} m_i \]  

(10)

The calculated values are shown in Table 3.

Although each secondary mass is larger than each primary mass in the corresponding story, as shown in Table 3., the actual masses required are reduced to several thousandths by the mass amplifying mechanism.

3.3 Equation of motion for the MDOF seismic control system incorporated with TVMDs

The equation of motion of a damped \( n \) -degree-of-freedom structure incorporated with one TVMD in each story, i.e., \( n \) TVMDs in the entire structure, is described as follows:

\[ M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_d, \]  

(11)

where \( x \) is a \( 2n \) dimensional vector and consists of an \( n \) dimensional displacement vector of the primary system relative to the ground and an \( n \) dimension vector of each damper in each story; in this study, \( n = 10 \).

\[ x = \{x_p^T, x_{d,1}, x_{d,2}, \ldots, x_{d,10} \}^T \]

(12)

The lower half of the influence coefficient vector \( r \) is \( n \) dimensional zero vector, whereas its upper half is \( n \) dimensional unit vector. This is because the TVMDs are not activated by the ground motion but by the relative displacement in each story.

\[ r = \{1, 1, \ldots, 1, 0, 0, \ldots, 0 \}^T \]  

(13)

\[ M, C, \text{ and } K \] denote the following 2\( n \) dimension mass, damping, and stiffness matrices, respectively.

\[ M = \begin{bmatrix} M_p & 0 \\ 0 & M_d \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0 \\ 0 & C_d \end{bmatrix}, \quad K = \begin{bmatrix} K_p & K_{d,1} \\ K_{d,1} & K_{d,2} \end{bmatrix}. \]

(14)

(15)

\[ \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & m_n \end{bmatrix}, \quad \begin{bmatrix} c_{d,1} & 0 & \cdots & 0 \\ 0 & c_{d,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & c_{d,n} \end{bmatrix}, \quad \begin{bmatrix} k_{b,1} + k_{b,2} & -k_{b,2} & \cdots & 0 \\ -k_{b,2} & k_{b,2} + k_{b,3} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -k_{l,n-1} & k_{l,n-1} + k_{l,n} \end{bmatrix}. \]

(16)

(17)

(18)

Table 3. Specifications for the TVMD System

| Story | \( m_{d,i} \) [ton] | \( k_{d,i} \) [kN/m] | \( c_{d,i} \) [kNs/m] |
|-------|-----------------|-----------------|-----------------|
| 10    | 1626            | 20138           | 2353            |
| 9     | 1847            | 22877           | 2673            |
| 8     | 2259            | 27975           | 3269            |
| 7     | 2511            | 31092           | 3633            |
| 6     | 2994            | 37075           | 4332            |
| 5     | 3140            | 38887           | 4544            |
| 4     | 3367            | 41694           | 4872            |
| 3     | 3929            | 48650           | 5684            |
| 2     | 3934            | 48717           | 5692            |
| 1     | 2872            | 35560           | 4155            |
4. Eigenvalue Analyses
4.1 Eigenvalue analysis of the primary structure

The characteristic equation derived from the eigenvalue problem for the uncontrolled primary structure is described as follows:

\[ [\mathbf{K} - \Omega^2 \mathbf{M}] \mathbf{u} = 0. \quad (22) \]

Let \( \Omega \) and \( \phi \) denote the \( j \)th eigenvalue and eigenvector derived from Eq. (22), respectively, and the \( j \)th fundamental angular frequency \( \omega_j \) for the undamped primary system equals the square root of the \( j \)th eigenvalue.

\[ \omega_j = \sqrt{\Omega_j}. \quad (23) \]

Because the authors assume that the damping matrix \( \mathbf{C} \) is proportional to the stiffness matrix, premultiplying Eq. (1) by \( \mathbf{\phi}_j^T \) yields \( n \) modal equations by using the transformation \( \mathbf{x} = \Phi \mathbf{u} \), where \( \Phi = [\phi_1, \phi_2, \ldots, \phi_n] \) and \( \mathbf{u} = [u_j] \) is the modal coordinate.

\[ \ddot{u} + 2 \xi \omega_j \dot{u} + \omega_j^2 u = -\Gamma_j \dot{\mathbf{u}}(t). \quad (24) \]

Here, \( \Gamma_j \) is the modal participation factor, \( \xi = (\mathbf{\phi}_j^T \mathbf{C}_p \mathbf{\phi}_j)/2\omega_j M \), \( \omega_j = i\xi/\omega \) is the damping ratio for mode \( j \) and \( M \) is the generalized \( j \)th modal mass.

The solution of the second-order equation, Eq. (23), is given by

\[ u(t) = -\Gamma_j \int_{t_0}^{t} \dot{\mathbf{u}}(\tau)e^{-\xi \omega_j(t-\tau)} \sin \omega_j (t-\tau) d\tau. \quad (25) \]

where \( \dot{\mathbf{u}}(t; \omega_j, \xi) \) is referred to as the Duhamel integral.

\[ \dot{\mathbf{u}}(t; \omega_j, \xi) = \frac{1}{2\omega_j} \int_{t_0}^{t} \dot{\mathbf{u}}(\tau)e^{-\xi \omega_j(\tau-t)} \sin \omega_j (t-\tau) d\tau. \quad (26) \]

and \( \omega_j = \sqrt{1 - \frac{\xi^2}{\omega^2}} \omega \).

Thus, the \( j \)th modal displacement response \( \mathbf{x}_j(t) \) is obtained as follows:

\[ \mathbf{x}_j(t) = \phi_j u(t) = -\Gamma_j \phi_j h(t, \omega_j, \xi). \quad (27) \]

4.2 Eigenvalue analysis of the controlled structure

Eq. (11) is converted into the following \( 4n \) first order matrix equation to obtain complex modes:

\[ \mathbf{A} \dot{\mathbf{y}} + \mathbf{B} \dot{\mathbf{y}} = -\mathbf{A} \mathbf{w}_0(t), \quad (28) \]

where

\[ \mathbf{A} = \begin{bmatrix} 0 & \mathbf{M} & 0 & \mathbf{K} \\ \mathbf{M} & 0 & \mathbf{C} & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -\mathbf{M} \mathbf{G} \\ 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} \mathbf{r} \\ \mathbf{x} \end{bmatrix}. \quad (29) \]

The eigenvalue problem of Eq. (28) can be expressed as follows:

\[ \mathbf{B} \dot{\phi} = -\lambda \mathbf{A} \phi. \quad (30) \]

If we assume that this seismic control system is underdamped, the eigenvalues and eigenvectors given by Eq. (30) are \( 2n \) complex conjugate pairs.

Here, the authors express the \( j \)th pair of eigenvalues and eigenvectors as \( \lambda_j, \phi_j \) and \( \lambda_j, \phi_j \), respectively.

From the definition of \( \mathbf{y} \), it is obvious that \( \dot{\phi} \) is of the form

\[ \dot{\phi} = \begin{bmatrix} \lambda_j \phi_j \\ \phi_j \end{bmatrix}. \quad (31) \]

where, \( \phi_j \) is the lower half of \( \phi_j \).

The \( j \)th fundamental angular frequency \( \omega_j \) and the corresponding damping ratio \( \xi_j \) can be obtained as follows:

\[ \omega_j = |\lambda_j|, \xi_j = \frac{\text{Re}[\lambda_j]}{|\lambda_j|}, \quad (32) \]

Eq. (28) can be reduced to \( 4n \) decoupled modal equations by using the modal coordinates \( \{z\} = \mathbf{z} \) obtained from the transformation \( \mathbf{y} = \Phi \mathbf{z} \), where \( \Phi = [\phi_1, \phi_2, \ldots, \phi_n] \).

The \( j \)th equation of motion is

\[ \mathbf{A} \dot{\mathbf{z}} + \mathbf{B} \mathbf{z} = -\dot{\phi}_j^T \mathbf{A} \mathbf{w}_0(t), \quad (34) \]

or alternatively

\[ \dot{\mathbf{z}} - \lambda \mathbf{z} = i\mathbf{r}_0(t), \quad (35) \]

where

\[ \dot{\mathbf{A}} = \dot{\phi}_j^T \mathbf{A} \dot{\phi}_j \], \quad (36)

\[ \dot{\mathbf{B}} = \dot{\phi}_j^T \mathbf{B} \dot{\phi}_j = -\lambda \mathbf{A}, \quad (37) \]

\[ \dot{\mathbf{r}} = -\dot{\phi}_j^T \mathbf{A} \mathbf{r} / \mathbf{A} = -\dot{\phi}_j^T \mathbf{M} \mathbf{r} / \mathbf{A}. \quad (38) \]

Fig. 4 compares the participation mode vectors of the uncontrolled primary system \( \Gamma_{\phi_j} \) and those
of the controlled system $r, \lambda, \phi$. By adding the secondary vibration system to the primary system, the uncontrolled $1^{st}$ mode is split into the $1^{st}$ to $11^{th}$ complex conjugate pairs in the controlled system. As shown in Fig.4.(a), the combination of the $1^{st}$ and $11^{th}$ pairs of the split modes is almost identical to the $1^{st}$ mode of the uncontrolled primary system. The $12^{th}$ and $13^{th}$ conjugate pairs of modes of the TVMD system correspond to the $2^{nd}$ and $3^{rd}$ modes of the uncontrolled primary system, respectively. In Fig.4.(b), only the $1^{st}$ and $11^{th}$ conjugate pairs of participation vectors are significant, and the others are insignificant because of their small components.

Table 4. compares the fundamental angular frequencies and the corresponding damping ratios of the uncontrolled primary system and the TVMD seismic control system. The values of the fundamental angular frequencies of the $1^{st}$ to $11^{th}$ modes of the TVMD system are close to each other. They are also close to the $1^{st}$ fundamental angular frequency value of the uncontrolled primary system. The damping ratios of the $2^{nd}$ and $3^{rd}$ modes of the uncontrolled primary system are almost unchanged by the addition of the TVMDs, whereas those of the $1^{st}$ to $11^{th}$ modes of the TVMD system are substantially increased. This means that the TVMD seismic control system can increase the damping ratio of the specified mode, and it almost never changes those of the others. This is an advantageous feature of the TVMD seismic control system.
Fig. 5 shows the relationship between the mass and damping ratios obtained by the complex valued eigenvalue analysis of the equivalent SDOF structure incorporated with a TVMD having a specified mass ratio of 0.1 (Fig. 6). The estimated damping ratio for the MDOF TVMD system can be readily obtained from Fig. 5. The damping ratios of the 1st and 11th conjugate pair modes are well approximated by those of the 1st and 2nd conjugate pair modes of the equivalent SDOF structure, respectively.

\[
\begin{align*}
\phi_{21} = \frac{3(1 - \sqrt{1 - 4\mu})}{4} = 0.206. \\
\end{align*}
\]

This is because almost only the secondary masses are activated in these modes and are independent of the primary responses.

The solution of the first-order equation, Eq. (35), is the following integral:

\[
\dot{x}_i(t) = r \int_0^t \ddot{x}_i(\tau)e^{(\mu_i - \omega_i \tau)\xi} d\tau. 
\]

Combining the i th conjugate pairs of responses gives the i th modal displacement response \( \dot{x}_i(t) \).

\[
\begin{align*}
\dot{x}(t) &= 2\left\{ -\text{Im}(2\pi \phi_{21}) \omega_{b1} \int_0^t \ddot{x}_y(\tau)e^{-(\xi_i + \omega_{b1}^2)(t - \tau))} d\tau \\
&+ \phi_{21} \omega_{b1} \int_0^t \ddot{x}_y(\tau)e^{-i(\xi_i + \omega_{b1}^2)(t - \tau))} \sin \omega_{b1}(t - \tau)d\tau \\
&+ \text{Re}(2\pi \phi_{21}) \omega_{b1} \int_0^t \ddot{x}_y(\tau)e^{-(\xi_i + \omega_{b1}^2)(t - \tau))} \cos \omega_{b1}(t - \tau)d\tau \right\}
\end{align*}
\]

where \( \omega_{b1} = \sqrt{-\xi_i \omega_i} \).

The first integral in Eq. (40) can be readily expressed in terms of the Duhamel integral \( \hat{h}(t;\omega_i, \xi_i) \), whereas the second integral with a cosine term can be obtained in terms of the derivative of the Duhamel integral.

\[
\hat{h}(t;\omega_i, \xi_i) = \int_0^t \ddot{x}_y(\tau)e^{-(\xi_i + \omega_{b1}^2)(t - \tau))} \cos \omega_{b1}(t - \tau)d\tau
\]

By substituting Eq. (41) into Eq. (40), we obtain

\[
\begin{align*}
\dot{x}(t) &= 2\left\{ -\text{Im}(2\pi \phi_{21}) \omega_{b1} \int_0^t \ddot{x}_y(\tau)e^{-(\xi_i + \omega_{b1}^2)(t - \tau))} d\tau \\
&+ \text{Re}(2\pi \phi_{21}) \omega_{b1} \int_0^t \ddot{x}_y(\tau)e^{-(\xi_i + \omega_{b1}^2)(t - \tau))} \sin \omega_{b1}(t - \tau)d\tau \right\}
\end{align*}
\]

From Eqs. (32) and (33), \( \lambda_{21} \) and \( \lambda_{22} \) can be rewritten as follows:

\[
\begin{align*}
\lambda_{21} &= -\xi_i \omega_i + i \omega_{b1} \xi_i \\
\lambda_{22} &= -\xi_i \omega_i - i \omega_{b1} \xi_i.
\end{align*}
\]

Using the above relationship, Eq. (42) reduces to

\[
\dot{x}(t) = a \hat{h}(t;\omega_i, \xi_i) + b \hat{h}(t;\omega_i, \xi_i),
\]

where

\[
\begin{align*}
a &= -2 \text{Re}(2\pi \phi_{21}) \omega_{b1} \\
b &= -2 \text{Re}(2\pi \phi_{21}) \omega_{b1}.
\end{align*}
\]

\( \hat{h}(t;\omega_i, \xi_i) \) and \( \hat{h}(t;\omega_i, \xi_i) \) are readily evaluated using numerical methods such as Newmark’s β method.

In this study, as the input, the authors used the ground motion recorded at Tohoku University during the 2011 Tohoku earthquake (M = 9.0, PGA = 3.33 m/ s/s), which occurred on March 11, 2011 and was the most powerful known earthquake to have hit Japan. The accelerogram and displacement spectra of the ground motion are shown at the top of Fig. 7 and Fig. 8, respectively.

Because the 1st mode of the uncontrolled primary system is split into 11 conjugate pairs modes in the TVMD system, a combination of the 1st and 11th modal responses can be considered to correspond to the 1st primary modal response, as shown by Eq. (46).

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{11} \left\{ a_i \hat{h}(t;\omega_i, \xi_i) + b_i \hat{h}(t;\omega_i, \xi_i) \right\}
\end{align*}
\]

The 1st modal interstory responses at the 10th, 5th, and 1st stories obtained from Eq. (46) are shown in Fig. 7. They are compared with the approximated responses obtained from Eq. (27) by substituting \( \xi_i \) with 0.114, which is the lower damping ratio obtained from the 2nd modal damping ratio of the equivalent SDOF structure incorporated with the TVMD (Fig. 5.).

As can be observed in Fig. 7., the 1st modal responses obtained by the 1st participation mode vector of the uncontrolled primary system and the approximated damping ratio for the 1st mode obtained from the equivalent SDOF structure with the TVMD (Fig. 5) give a good approximation to the complex valued analysis.

For the higher modes, the time histories are readily approximated by those of the uncontrolled primary system, because the mode shapes and modal damping ratios were almost unchanged by the addition of the TVMD system.
As the equivalent mass and damping ratios for the tuned mode is readily obtained from the eigenvalue analysis of the uncontrolled primary system, the complex valued analysis for the MDOF system is no longer required to approximate the modal responses. This suggests that the maximum response can be well estimated by the SRSS method.

Fig.9.(a) compares the maximum responses obtained from the time history analysis based on Newmark’s $\beta$ method, the complex complete quadratic combination method (CCQC) (Yang and Sarkani, 1990), and the SRSS method. As shown in Fig.9.(a), the SRSS estimation gives a good approximation in practical terms. Fig.9.(b) compares the time history responses of systems in which secondary masses are distributed uniformly and proportionally to the stiffness. It shows that the variation of the mass distribution has little effect on the accuracy of the approximation method.
Although further investigation should be conducted with respect to various secondary mass ratios and distribution patterns, this study illustrated the basic modal response characteristics of an MDOF structure incorporated with TVMDs.

5. Conclusions

In this paper, the formulation of the equation of motion of the seismic control system incorporated with the recently developed devices called the tuned viscous mass dampers (TVMD) is presented. An advantageous feature of the TVMD is that it can specify the modal damping ratio of the tuning mode. The modal damping added to the tuning mode can be readily obtained from the complex-valued eigenvalue analysis of the equivalent SDOF structure incorporated with the TVMD.

The modal responses derived from the complex valued modal analyses of the TVMD seismic control system and those derived from the real modal analysis using an uncontrolled primary system were compared. It was found that the estimation using the latter method gives a good approximation of the actual maximum modal responses in practical terms.

Thus, a simple and easy seismic response estimation method that is suitable for the preliminary design stage using the SRSS method is proposed.

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