The Tragedy of the AI Commons

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Abstract

Policy and guideline proposals for ethical artificial-intelligence research have proliferated in recent years. These are supposed to guide the socially-responsible development of AI for the common good. However, there typically exist incentives for non-cooperation (i.e., non-adherence to such policies and guidelines); and, these proposals often lack effective mechanisms to enforce their own normative claims. The situation just described constitutes a social dilemma—namely, a situation where no one has an individual incentive to cooperate, though mutual cooperation would lead to the best outcome for all involved. In this paper, we use stochastic evolutionary game dynamics to model this social dilemma in the context of the ethical development of artificial intelligence. This formalism allows us to isolate variables that may be intervened upon, thus providing actionable suggestions for increased cooperation amongst numerous stakeholders in AI. Our results show how stochastic effects can help make cooperation viable in such a scenario. They suggest that coordination for a common good should be attempted in smaller groups in which the cost for cooperation is low, and the perceived risk of failure is high. This provides insight into the conditions under which we should expect such ethics proposals to be successful with regard to their scope, scale, and content.

1 Introduction

Artificial intelligence promises to fundamentally change nearly every facet of our lives, for better or worse [66,70,103]. In response to this reality, there has been a proliferation of policy and guideline proposals for ethical artificial-intelligence and machine-learning (AI/ML) research. Jobin et al. [83] survey several global initiatives for AI/ML and find no fewer than 84 documents containing ethics principles for AI research, with 88% of these having been released since 2016. These documents are meant to specify ‘best practices’ to which engineers, developers, researchers, etc. ought to adhere.

For example, the Montréal Declaration for the Responsible Development of AI identifies a set of abstract normative principles and values intended to promote the fundamental interests of stakeholders; signatories are invited to commit to ‘the development of AI at the service of the individual and the common good’ [171]. Proposals of this sort typically highlight issues concerning transparency, justice and fairness, non-maleficence, responsibility, and privacy, among others [83]. These initiatives generally take one of two approaches to foster the ethical practice of AI research: proposing principles to guide the socially-responsible development of AI or examining the societal impacts of AI [101].

However, policies-and-procedures documents, like the Montréal Declaration, are examples of ‘non-legislative policy instruments’ or ‘soft law’ [152], which are instruments for cooperation that are not legally binding. This stands in contrast to ‘hard law’, which consists in legally-binding regulations

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passed by legislatures. By definition, then, these reports are not intended to produce enforceable rules but are meant merely as guides for ethical practice. Therefore, the proliferation of such guiding principles raises pressing questions about their efficacy. It has been suggested that such declarations have little to no practical effect [60]; at least one study finds that the effectiveness of ethical codes for influencing the behaviour of professionals in the technology community is virtually nonexistent [106].

Part of the problem is that, in the AI research community, as in many other social contexts, there are typically costs associated with cooperation—e.g., the additional time and financial resources that are required to ensure research and business practices adhere to the proposed normative standards—in addition to incentives for non-cooperation or non-adherence to ethics agreements—e.g., a lucrative defence contract to develop autonomous weapons. Furthermore, non-legislative policy instruments, like guiding principles, lack effective mechanisms to enforce (or reinforce) their own normative claims, because they depend on voluntary, non-binding cooperation between the relevant parties—e.g., individual researchers, labs, academic bodies, corporations, governments, international bodies, etc.

This failure should come as no surprise to game theorists and economists: in the situation just described, no one has an individual incentive to cooperate, although mutual cooperation would lead to the best outcome for all those involved. This constitutes a social dilemma [2; 31; 71; 76; 136; 144].

In this paper, we use stochastic evolutionary game dynamics to model this social dilemma in the context of the ethical development of AI. This model allows us to isolate variables that may be intervened upon; this, in turn, helps to formulate actionable suggestions for increased cooperation amongst numerous stakeholders in AI. Our results show how stochastic effects can help make cooperation viable in such a scenario, and they suggest that coordination for a common good should be attempted in smaller groups in which the cost for cooperation is low, and the perceived risk of failure is high. This analysis provides insight into the conditions under which we may expect such policy and guideline proposals for safe and ethical AI to be successful in terms of their scope, scale, and content. This, in turn, yields plausible solutions to the social dilemmas that the AI/ML community face for acting in a socially responsible and ethical way that is aligned with a common good.

In Section 2, we discuss related work. In Section 3, we present an evolutionary game which models cooperation (with respect to proposed ethical norms and principles in AI) using a social dynamics framework. Select results are presented in Section 4. Section 5 concludes.

2 Background and Related Work

In this section, we discuss related work and highlight how our model-based analysis of adherence to AI ethics guidelines, when understood as a social dilemma, provides substantive and actionable suggestions for fostering cooperation.

Artificial Intelligence and Normative Guides. As mentioned, non-legislative policy instruments, in the form of ethics guidelines, have proliferated in recent years. These guidelines, codes, and principles for the responsible creation and use of new technologies come from a wide array of sources, including academia, professional associations, and non-profit organisations [47; 53; 61; 81; 128; 137; 154; 160; 161; 162; 163; 170; 171; 172; governments [37; 53; 55; 74]; and industry, including for-profit corporations [33; 52; 78; 79; 108; 123; 139; 143; 151]. Several researchers have noted that the very fact that a diverse set of stakeholders would exert such an effort to issue AI principles and policies is strongly indicative that these stakeholders have a vested interest in shaping policies on AI ethics to fit their own priorities [15; 57; 83; 173].

Perhaps unsurprisingly, it has also been widely noted that ethics policies for AI research are typically or relatively ineffective: they do not have an actual impact on human decision-making in the field of AI/ML [60]; they are often too broad or high-level to guide ethics in practice [181]; and they are, by their very nature, voluntary and nonbinding [24; 180].

The content of ethics guidelines can be varied to promote adherence. For example, substantial changes in abstraction can help the application, impact, and influence of ethics principles for AI by way of targeted suggestions that can be implemented easily and concretely in practice [48; 112]. However, while several authors have noted the inefficacy of guiding principles, and have offered proposals

3 Of course, it is nontrivial to determine precisely what a ‘common good’ is; see discussion in Green [56].
to surmount this inefficacy, such proposals are, themselves, often couched in inherently normative language. Therefore, they too fail to instantiate any mechanisms for their own reinforcement.

For example, ethics policies and guidelines can be understood deontically, insofar as they provide static, universal principles to be adhered to \[4; 110\]. Hagendorff \[60\] takes the inefficacy of such policies to be a failure of deontic ethics, proposing that AI ethics should instead be couched in a virtue-ethics framework.\[\] But these types of solutions are themselves ineffective insofar as they are merely meta-level normative principles which recommend adherence to object-level principles. Some reflection shows that we can then ask the same question of the meta-level principles as we did of the object-level ones: Should we expect these to be effective? By dint of what? Since there is no mechanism by which these meta-level principles can be enforced, they provide no solution to the inefficacy created by the proliferation of principles and guides for ethical AI. Instead, the problem is merely reproduced a level up.

In asking how the application and fulfilment of AI ethics guidelines can be improved, we shift the focus from the content of the principles and related meta-ethical considerations to the cooperative, social aspects of the dilemma. Namely, we examine the circumstances under which we should expect cooperative success to be possible or likely in a socio-dynamical context. This analysis has significant practical implications for regulatory principles in AI/ML, which are made all the more pressing as these technologies are increasingly integrated into society.

3 A Socio-dynamic Model of Cooperation for Ethical AI Research

In this section, we describe the model that we employ using standard techniques from evolutionary game theory. (See Appendix A for some additional background details.) This formal framework has been applied to social dilemmas like climate change \[42; 126; 174\] nuclear proliferation \[21; 22; 88; 185\], conflicts over water resources \[102\], etc. However, it has yet to be applied to the cooperative challenges that are faced in the context of AI safety and ethics. We begin by discussing the mathematical structure of the model, including the payoff structure and dynamics. In Section 4 we investigate the effect of several variables on the possibility or likelihood of cooperation in the context of non-legislative policy instruments for AI research.

Model and Parameters. A population of size \(Z\) is organised (divided) into groups of size \(N\). Each individual in the population may be interpreted as an AI researcher who can realise or ignore some proposed norm(s) of research and practice. The population can be thought of as the global AI community at large, and each group of size \(N\) in the population can be thought of as an organisational unit—e.g., a research laboratory, a university department, a collaborative research group, etc.—which can sustain shared norms. Individuals are classified according to the strategy that they choose with respect to the proposed norms for ethical behaviour within the community: they can choose either to cooperate \((C)\), by adhering to the proposed norm, or to defect \((D)\), by flouting the proposed norm.\[\]

Each individual has an initial endowment, \(b \in \mathbb{R}^+\). Contributing to a collective, cooperative effort imposes a cost, \(c \in [0, 1]\), to the cooperator, consisting in a fraction of their endowment; defecting requires no such contribution. The cost may be construed as an additional effort relative to the non-cooperative baseline—e.g., taking extra precautions to ensure research and business practices adhere to the proposed normative standards—or in terms of opportunity costs—e.g., by refusing lucrative projects that would violate the proposed normative standards.

A normative agreement is successful if the fraction of contributors exceeds some threshold, \(p^* \in (0, 1)\).\[\] If this cooperative threshold is not achieved, then with some probability, \(r\), everyone in the group loses a fraction, \(m\), of their endowment. \(r\) can be interpreted as the perceived risk of negative consequences in the event of a failure to cooperate, and \(m\) can be understood as the perceived magnitude of those consequences. That is, when \(m = 1\) (the failure to cooperate is perceived to be
maximally impactful), individuals risk losing all of their endowment; \( m = 0 \) represents the situation in which failure to reach an agreement has no perceived negative consequence.\(^7\)

This provides a five-dimensional space to analyse the possibility of cooperation in our social dilemma: the size of the groups in which cooperation is attempted, \( N \); the perceived risk of negative consequences when cooperation fails, \( r \); the perceived magnitude of said consequences, \( m \); the cost of cooperation, \( c \); and the critical threshold of cooperators required to avoid negative consequences, \( p^* \).

**Payoffs.** The payoffs for each strategy determine the game. An individual’s payoff depends upon what action the individual chooses and what everyone else in the community is doing. The payoffs to cooperators in a group of size \( N \) when \( n_C \) individuals are cooperating is given in Equation \( 1 \)

\[
\pi_C(n_C) = b \cdot \Theta(k) + b \cdot (1 - rm) \cdot \Theta(k - cb), \quad k = n_C - n^*.
\]

\( n^* = \lceil p^* \cdot N \rceil \) is the critical number of cooperators in a group, and \( \Theta \) is the Heaviside step function.\(^8\)

Equation \( 1 \) has two key terms. The left-hand summand \( b \cdot \Theta(k) \) provides payoff \( b \) to the cooperator just in case \( k = n_C - n^* \geq 0 \); this is the payoff when cooperation succeeds. When cooperation fails \( (n_C - n^* \leq 0) \), the right-hand summand \( b \cdot (1 - rm) \cdot \Theta(k - cb) \) comes into play. This payoff is weighted by the cost of failure as a function of the risk and magnitude of some negative consequence, \((1 - rm)\). So, when \( r \) and \( m \) are maximal \((rm = 1)\), the entire endowment is lost if cooperation fails; when the risk or magnitude are nonexistent \((rm = 0)\), none of the endowment is lost. Finally, the cost of cooperation, \( cb \), is subtracted from the payoff for successful cooperation or failure to cooperate. The payoff to defectors is defined in terms of the payoff to cooperators, as in Equation \( 2 \)

\[
\pi_D(n_C) = \pi_C(n_C) + cb.
\]

We calculate the average payoff to each type, \( C, D \), as a function of the group size, \( N \), and the fraction of each type in the broader population, \( x_C \) and \( x_D = 1 - x_C \), respectively. First, we find the fraction of cooperators in the population, \( x_C = n_C^Z / Z \). We then calculate the vector of (binomially-distributed) probabilities for each possible group, composed of \( k \) cooperators and \( N - k \) defectors. We then compute the vector of conditional probabilities \((k/N)\) of being a cooperator in each combination. We compute the average payoff to the cooperators by weighing the payoff for cooperation by the probability of being a cooperator, as described. And, \( \textit{mutatis mutandis} \) for the average payoff to defectors. The mean payoffs capture the expectation if the entire population, \( Z \), were randomly paired into groups of size \( N \). See Appendix \( 3 \) for formal details.

**Dynamics.** We consider the dynamics of small, finite populations. The dynamics determine how the population changes based on assumptions about how the payoffs affect the fitness of strategies, given what others are doing. Our model uses a dynamics called the \( \text{Fermi process} \)\(^{164}\). This involves a pairwise comparison where two individuals—a focal individual and a role model—are sampled from the population at random. The focal individual copies the strategy of the role model with probability \( p \), depending on a comparison of the payoffs of those strategies. If both individuals have the same payoff, the focal individual randomises between the two strategies. The probability is a nonlinear function of the payoff difference for \( p \), called the \( \text{Fermi function} \)\(^{35}41\),

\[
p = \left[1 + e^{\lambda(p_f - p_r)}\right]^{-1}, \quad \lambda, \mu \in \mathbb{R}^+ \]

where \( \lambda \) is the intensity of selection, which specifies the importance of neutral drift compared to the selection dynamics. \( \pi_f, \pi_r \) are the payoffs of the focal individual and the role model, respectively. In addition to the parameters described above, we further assume some small rate of mutation, \( \mu \), which can be interpreted as mutation, experimentation, error, or noise. For all the results we present in Section \( 4 \) we set \( \lambda = 5, \mu = 0.10 \).

In our model, we specify the transition probabilities of the population changing from a state containing \( n_C^Z \) cooperators to one with \((n_C^Z + 1)\) or \((n_C^Z - 1)\) cooperators (i.e., the transition from the dynamics yielding one more or one fewer cooperator in the population at large). A transition matrix (for all possible states) is populated by recursive application of the transition probabilities. The transition

\(^7\)Note that perceived risk of collective failure has proved important for successful collective action in dilemmas of this sort\(^{109}128142\).\(^8\)That is, \( \Theta(k) = 1 \) when \( k \geq 0 \), and \( \Theta(k) = 0 \) otherwise.
matrix is used to compute the gradient of selection for our dynamics. This is defined by the difference between the probability that the number of cooperators increases and the probability that the number of cooperators decreases. We compute the stationary distribution of the process using standard techniques [141]. Again, see Appendix B for formal details.

4 Results

In this section, we examine the results of our model under simulation. This is useful since human behaviour involves so many degrees of freedom that meaningful analytic results are often unlikely to be obtained. As is common practice in game-theoretic models of social behaviour, we are primarily interested in high-level qualitative results—such as whether cooperation will tend to succeed—rather than exact quantities.

Simulation Parameter Values. In our simulations, we fix the population, \( Z = 100 \), and the endowment, \( b = 1 \). All other parameters are variable: this includes the size of the groups, \( N \in \{1, 2, \ldots, 20\} \); the perceived risk of negative consequences occurring if cooperation fails, \( r \in [0, 1] \); the cost of those consequences, understood in terms of the proportion of endowments lost, if cooperation fails, \( m \in [0, 1] \); the proportion of the endowment that is contributed by cooperators, \( c \in [0, 1] \); and the critical threshold of cooperators required in order for cooperation to succeed, \( p^* \in [0, 1] \), which determines the number of cooperators required, \( n^* = \lceil p^* \cdot N \rceil \).

Qualitative Dynamics. We compute the mean payoff to each strategy, the gradient of selection, and the stationary distribution for the dynamics as described in Section 3. The gradient of selection gives the probable direction of evolution in the short run, whereas the stationary distribution captures the long-run proportion of time that the process spends at any given state. This is the appropriate method of analysis for such finite-population dynamics [111; 164]; see Figure 1 for examples.

![Figure 1](image-url)

Figure 1: Examples of each qualitative dynamic for the gradient of selection (top) and stationary distribution (bottom) of the mean-field dynamics, resulting in (a) a prisoner's dilemma favouring ALL DEFECT; (b) a bi-stable dynamics favouring the ALL DEFECT equilibrium; (c) a bi-stable dynamics favouring the POLYMORPHIC equilibrium; and (d) a prisoner’s delight favouring ALL COOPERATE.

The gradient may change from selection for an equilibrium where ALL DEFECT is the unique stable state (top of 1a), displaying the qualitative dynamics of a prisoner’s dilemma [144]; to a bi-stable dynamics with both an ALL DEFECT equilibrium and a POLYMORPHIC equilibrium, where both strategies can coexist in the population (top of 1b, 1c—the latter corresponds to an anti-coordination game [134]); to (in rare circumstances) a unique equilibrium of ALL COOPERATE (top of 1d, where the qualitative dynamics yield a prisoner’s delight [13]).

When there is a unique stable state, the stationary distribution reflects that the process spends the majority of its time near this state (bottom of 1a, 1d). However, when there are multiple stable states (1b, 1c), the dynamics may or may not guarantee population-level success relative to the demands of the cooperative challenge—the stationary distribution shows us the proportion of time spent at each state. Hence, although a mixed-population of cooperators and defectors is stable in Figure 1b, the stationary distribution indicates that the stable configuration of ALL DEFECT is most probable.

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9Our code outputs visual graphics of the selection gradient, average payoffs to each strategy, and the stationary distribution. A GUI with variable parameters is available online at [https://amohseni.shinyapps.io/tragedies-of-the-commons/](https://amohseni.shinyapps.io/tragedies-of-the-commons/)
In sum, there are four possible qualitative outcomes for our population under the dynamics described: (1) ALL DEFECT is the unique stable state, near which the process spends all its time; (2) both ALL DEFECT and a POLYMORPHIC state in which both cooperators and defectors co-exists are stable, yet the process spends a majority of its time near the ALL DEFECT equilibrium; (3) both ALL DEFECT and a POLYMORPHIC state are stable, but the process spends the majority of its time near the polymorphic mixture; and (4) ALL COOPERATE is uniquely stable and the process tends toward this state.

Simulation Results. We want to know how the parameters of interest interact with the achievement of cooperative success (the situations shown in 1c or 1d), and which of those will have the greatest effect. Our dynamics encode a process by which strategies that are more successful proliferate via imitation; so, for cooperation to succeed it must be that cooperators do better on average than defectors. Each of the parameters will either work for or against cooperative success. We will examine their respective effects by first considering their extremal values.

It should be obvious that when the cost of cooperation is maximal \((c = 1)\), defect \((D)\) will dominate since cooperators have nothing to gain from cooperation in this scenario. Therefore, the lower the cost of cooperation, the more likely individuals will be to cooperate. However, we will see that even a low cost of cooperation does not ensure cooperative success.

At the other extreme, when the cost of cooperation is minimal \((c = 0)\), cooperate \((C)\) is the strictly dominant strategy in almost all cases—the exception is when there is no benefit to cooperation \((r \cdot m = 0)\), and so the payoffs for \(C\) and \(D\) are equivalent. On the other hand, if the cost of cooperation is nonzero \((c > 0)\), and there is no payoff for cooperative success \((r \cdot m = 0)\), then \(D\) is strictly dominant. That is, improbable but highly consequential failures to cooperate and highly probable but inconsequential failures to cooperate will evoke the same cooperative (or non-cooperative) response. Transitions between qualitative dynamics as a function of the risk, \(r\), and magnitude, \(m\), of the consequences of cooperative failure are given in Figure 2.

Further, Figure 3 makes clear that cooperative success requires the ratio of the benefit of avoiding the failure to cooperate \((r \cdot m)\) and the cost of cooperation \((c)\) to be sufficiently favourable.

Figures 2 and 3 also illustrate the critical role that group size plays in the prospects of cooperative success. In particular, all else equal, smaller group sizes are more favourable for cooperation. This is
perhaps counter-intuitive; however, the result can be explained as follows. Recall that cooperators must obtain higher average payoffs for cooperation to be favoured. Moreover, agents contribute to the prevalence of cooperation in their respective groups by choosing to cooperate (or not).

When group size is minimal ($N = 1$), an agent who chooses to cooperate ensures cooperative success by dint of her action alone. Thus, all groups composed of cooperators succeed, and all groups composed of defectors fail. (This holds for non-extremal values; i.e., $c > 0$, $m \cdot r > 0$, and $p^* > 0$.) This is sufficient to ensure selection for cooperation.

At the other extreme, when the cooperative group grows arbitrarily large ($N \to \infty$), an agent’s action makes a vanishing contribution to the prospect of cooperative success. Furthermore, by the law of large numbers [16; 131], the proportion of cooperators and defectors in large groups will approach the precise proportion of cooperators and defectors in the whole population. Hence, any group will tend to be equally likely to succeed or fail. Yet, in such groups, defectors will obtain higher payoffs by dint of not having paid the cost to cooperate. This is sufficient to ensure selection for defection.

In between these extremes, smaller group sizes will tend to favour cooperation and disfavour defection. Thus, we find a special instance of the general pattern that correlation between strategic actions conduces to pro-social behaviour [63; 64; 147], where here the relevant form of correlation is realised by the contribution of the individual’s action to the composition of the group’s actions.

Finally, we can consider the effect of the critical threshold for cooperation, $p^*$, on the prospects for cooperative success; see Figure 4.

![Figure 4: Qualitative dynamics as a function of the critical fraction of cooperators required for success, $p^*$, and the cost of cooperation, $c$. In each case, $r \cdot m = 0.50$.](image)

Logically, at the extreme where the critical threshold is minimal ($p^* = 0$), cooperation ‘succeeds’ without need of cooperators, and so defection is dominant. However, when the critical threshold is nonzero ($p^* > 0$), the general pattern is that the smaller the fraction of cooperators required for cooperation, the more likely cooperation is to succeed. Indeed, sufficiently demanding cooperative endeavours mean that while a POLYMORPHIC equilibrium in which cooperation co-exists with defection is stable under the gradient of selection, the stationary distribution demonstrates that arriving and remaining at such a cooperative state may be vanishingly unlikely.

5 Conclusion

A priori, one might have thought that inhibitive costs provide the biggest obstacle for cooperative success. Our model provides evidence for this intuition. Thus, our first moral is the obvious one.

**Moral 1:** Lowering the cost of cooperation increases the likelihood of cooperative success.

10 The reader may identify that this strategic structure is analogous to that of the Paradox of Voting [32].

11 Correlation can be realised variously in a social dilemma—e.g., assortative mating [36; 65], kin selection [62; 105], homophily [107], and network effects [23], among others. All of these support cooperation insofar as they make cooperators more likely to interact with one another, and less likely to interact with defectors. Although we lack the space to discuss these here, they constitute an important further dimension of our analysis.

12 This moral pertains to the likelihood of signing on to an agreement in the first place, but there is also a question of whether individuals who say they will cooperate in fact do so cooperate [93]. When signals are cheap, they can be uninformative or dishonest [28; 38; 39; 178]. It is well-understood that costly signals can promote honesty [64; 90; 132; 186; 187].
However, our model highlights that the dynamics of cooperation here are more subtle than this. We show that even for small costs, the dynamics may not guarantee population-level success relative to the demands of the cooperative challenge for a range of parameter values. Perhaps most surprising is the magnitude of the effect of group size on these outcomes. Thus, we propose the following.

**Moral 2:** Small, decentralised groups may benefit sustained cooperation for responsible AI research.

In the context of globalisation, one might have thought that the best approach for fostering AI ethics and safety was to have a universal set of principles to which everyone agrees. However, our model identifies that cooperation spreads as individuals observe other groups succeeding in their cooperative endeavours; this is fostered by the existence of many smaller groups where cooperation can more easily get off the ground. Hence, targeting policy agreements at smaller, independent groups, such as individual research labs or professional organisations, may constitute a more effective path to the aims of the universal adoption of ethical guidelines.

We mention how small group sizes help cooperation *in virtue of* the relationship between positive correlation between strategies and selection for pro-social behaviour. Yet another way to realise such correlation is via free partner choice. When individuals who are inclined to cooperate in adherence to norms are able to freely join one another in their efforts, this can increase their relative likelihood of success and so spread their example throughout the population. Thus, we get the following corollary.

**Moral 3:** Voluntary participation in AI policy agreements may catalyse the spread of cooperation.

Individuals choosing to adhere to a set of principles (or not) creates positive correlation. So, affording individuals the option of choosing their cooperative group may be more beneficial than stipulating that one must be a part of the group. However, even in small groups cooperation can fail. Hence,

**Moral 4:** It is important to accurately figure both the risks and the consequences of non-cooperation.

The action-relevant parameters in our model are perceived risk and magnitude. Therefore, it is essential to accurately figure the actual risks and potential consequences of non-cooperation. Namely, if the perceived risk is significantly lower than the actual risk, then cooperation may fail to obtain despite its (understated) real-world importance. Failing to cooperate, when cooperation matters, incurs a greater loss than cooperating when it was not entirely necessary to do so; hence, there may be less harm in overestimating the negative consequences of failing to cooperate. One real-world suggestion that we might glean from this is that it may be beneficial to further incorporate education on AI ethics and safety across a breadth of educational curricula and public outreach efforts.

The insights thus far centre primarily on group dynamics; this says nothing whatsoever about the actual content of ethics guidelines for AI. Of course, the content of these guides can significantly affect the likelihood that community-wide norms are taken up. For example, if the proposed norms are impossible to be instantiated, then they will fail to be adopted. Our final moral addresses this.

**Moral 5:** Combining many proposals may undermine their prospects for success.

The likelihood of being willing to adhere to and able to fulfil a set of policies is bounded by the likelihood of adhering to and fulfilling the conjunction of the policies it contains. We show that policies can vary in the magnitude of the cooperative challenge they present. The combination of these facts should make us worry of how demanding we make our policy proposals, if initial and sustained adherence is our aim. Instead, a piecemeal approach in which agents can sign on and gain from participation in single policies may provide a better first step on the path to the ultimate fixation of a robust set of norms for safe and ethical AI.

In closing, we note that even disciplines and fields whose norms of self-regulation are relatively well-entrenched, as in medical practice and biomedical ethics, guiding principles can be ineffective, or at least inefficient, in guiding actual practice when there are competing incentives. This

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13 In our model, the cost for cooperation is nonnegative. So, we do not account for *incentives* to cooperate—i.e., rewards. Conversely, we could lower the payoff for defectors by introducing punishment for non-cooperation. This is already something that has been done by, e.g., the ACM [8]. Although, empirical data suggests rewards are better than punishments for promoting cooperative behaviour in similar social dilemmas [34, 86].

14 This is a typical line of argument in much of the existential risk literature; see, e.g., Russell [138].

15 Although, this is not to say that the solution is to simply impose *hard* laws—this will likely also be ineffective; see discussion in LaCroix and Bengio [92].
is true even when there is consensus amongst researchers and practitioners about the guidelines themselves—and, it should be apparent that no such consensus exists in AI/ML.

In contrast to other work that has highlighted the inefficacy of non-legislative policy agreements in AI/ML, our analysis focused on the socio-dynamics of cooperation. In doing so, we were able to discuss the circumstances under which we should expect cooperation to arise in the first place. Further, instead of providing normative suggestions for increasing the effectiveness of extant guidelines by varying their content, the normative suggestions that we forward here pertain to the social aspects of cooperation; as such, we do not fall prey to the meta-normative inefficacy described in Section 2.

Broader Impact

Epistemic humility is a virtue in modelling work. The results derived from mathematical models of socio-dynamic phenomena are best understood as alerting us to the lower-bound on the complexity of those phenomena, and as providing tentative hypotheses that we must combine with empirical evidence and experimentation to adequately inform our judgements. It would be an error to observe the results of such a model and to become convinced that one fully apprehends the cooperative challenge it depicts.

It would also be a mistake to interpret our results and arguments as a dismissal of current proposals or guidelines for the safe and ethical development of AI. To the contrary, we believe that such efforts are laudable and necessary; but to give ourselves reasonable odds for success, we must appraise ourselves of an understanding of the basic dynamics of such coordination problems. These mistaken interpretations of our results—with their concomitant negative impacts—must be avoided.

From nuclear proliferation and climate change to ethical AI and AI safety, social dilemmas characterise the gulf between us and the futures for which we hope. The potential positive social impacts of our work are plain. Artificial intelligence promises to fundamentally change nearly every facet of our lives, for better or worse. Hence, effective adherence to ethical norms throughout the process of making numerous inevitable advances in AI will make a difference to the tally of instances in which the process promotes or deranges the prospects of human flourishing.

Our contribution to this count is simple. We deploy theoretical tools from evolutionary game theory to analyse the nature of the social dilemma at play in promoting participation in, and adherence to, the proposed policies and norms. We provide insights that, on the one hand, have not obviously informed extant guidelines and policies, and that, on the other hand, correspond to tractable changes in those proposals which may yield significant impact. If we succeed, stakeholders—research laboratories, university departments, collaborative research groups, and so on—aiming to formulate and coordinate around policy guidelines may have a richer awareness of the challenges involved that may inform the scope, scale, and content of such proposals.

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References

[1] Alexander, J. McKenzie (2007). *The Structural Evolution of Morality*. Cambridge University Press, Cambridge.

[2] Allison, Scott T. and Norbert L. Kerr (1994). Group Correspondence Biases and the Provision of Public Goods. *Journal of Personality and Social Psychology*, 66(4): 688–698.

[3] Altrock, Philipp M. and Arne Traulsen (2009). Fixation Times in Evolutionary Games under Weak Selection. *New Journal of Physics*, 11: 013012.

[4] Ananny, Mike (2016). Toward an Ethics of Algorithms: Convening, Observation, Probability, and Timeliness. *Science, Technology, & Human Values*, 41(1): 93–117.

[5] Anscombe, G. E. M. (1958). Modern Moral Philosophy. *Philosophy*, 33(124): 1–19.

[6] Aristotle (1995). Nichomachean ethics. In Barnes, Jonathan, editor, *The Complete Works of Aristotle, The Revised Oxford Translation*, volume 2, pages 1729–1867. Princeton University Press, Princeton.

[7] Ashcroft, Peter, Philipp M. Altrock, and Tobias Galla (2014). Fixation in Finite Populations Evolving in Fluctuating Environments. *Journal of the Royal Society Interface*, 11: 20140663.

[8] Association for Computing Machinery, ACM (2020). ACM code of ethics enforcement procedures. [https://www.acm.org/code-of-ethics/enforcement-procedures](https://www.acm.org/code-of-ethics/enforcement-procedures).

[9] Aumann, Robert and Sergiu Hart (1992). *Handbook of Game Theory with Economic Applications*, volume 1. Elsevier, North-Holland.

[10] Aumann, Robert and Sergiu Hart (1994). *Handbook of Game Theory with Economic Applications*, volume 2. Elsevier, North-Holland.

[11] Aumann, Robert and Sergiu Hart (2002). *Handbook of Game Theory with Economic Applications*, volume 3. Elsevier, North-Holland.

[12] Axelrod, Robert (1981). An Evolutionary Approach to Norms. *American Political Science Review*, 80(4): 1095–1111.

[13] Axelrod, Robert and William D. Hamilton (1981). The Evolution of Cooperation. *Science*, 211(4489): 1390–1396.

[14] Barrett, Jeffrey (2007). Dynamic Partitioning and the Conventionality of Kinds. *Philosophy of Science*, 74: 527–546.

[15] Benkler, Yochai (2019). Don’t Let Industry Write the Rules for AI. *Nature*, 569: 161.

[16] Bernoulli, Jacob (1713/2005). *Ars Conjectandi: Usum & Applicationem Praecedentis Doctrinae in Civilibus, Moralibus & Oeconomicis [The Art of Conjecture]*. John Hopkins University Press, Baltimore.

[17] Bicchieri, Cristina (2006). *The Grammar of Society*. Cambridge University Press, Cambridge.

[18] Binmore, Ken G. (2004). Reciprocity and the Social Contract. *Politics, Philosophy & Economics*, 3: 5–35.

[19] Binmore, Ken G. and Larry Samuelson (1994). An Economist’s Perspective on the Evolution of Norms. *Journal of Institutional and Theoretical Economics*, 150(1): 45–63.

[20] Boehm, C. (1982). The Evolutionary Development of Morality as an Effect of Dominance Behavior and Conflict Interference. *Journal of Social and Biological Structures*, 5: 413–421.

[21] Brams, Steven J. and D. Marc Kilgour (1987a). Threat Escalation and Crisis Stability: A Game-theoretic Analysis. *American Political Science Review*, 81(3): 833–850.

[22] Brams, Steven J. and D. Marc Kilgour (1987b). Winding Down if Preemption or Escalation Occurs: A Game-Theoretic Analysis. *Journal of Conflict Resolution*, 31(4): 547–572.
[23] Broere, Joris, Vincent Buskens, Jeroen Weesie, and Henk Stoof (2017). Network Effects on Coordination in Asymmetric Games. *Scientific Reports*, 7: 17016.

[24] Campolo, Alex, Madelyn Sanfilippo, Meredith Whittaker, and Kate Crawford (2017). *AI Now 2017 Report*. AI Now Institute at New York University.

[25] Chalub, Fabio A. C. C., Francisco C. Santos, and Jorge M. Pacheco (2006). The evolution of norms. *Journal of Theoretical Biology*, 241: 233–240.

[26] Chen, Xiaojie, Attila Szolnoki, and Matjaž Perc (2012). Risk-driven Migration and the Collective-risk Social Dilemma. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 86: 036101.

[27] Claussen, Jens Christian and Arne Traulsen (2005). Non-Gaussian Fluctuations Arising from Finite Populations: Exact Results for the Evolutionary Moran process. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 71(2 pt. 2): 025010.

[28] Crawford, Vincent P. and Joel Sobel (1982). Strategic Information Transmission. *Econometrica*, 50(6): 1431–1451.

[29] Crisp, Roger and Michael Slote (1997). *Virtue Ethics*. Oxford University Press, Oxford.

[30] Darwin, Charles (1981/1871). *The Descent of Man, and Selection in Relation to Sex*. Princeton University Press, Princeton.

[31] Dawes, Robyn (1980). Social Dilemmas. *Annual Review of Psychology*, 31: 169–193.

[32] de Caritat Condorcet, Marie Jean Antoine Nicolas (1793). *Essai sur l’application de l’analyse à la probabilité des décisions rendues à la pluralité des voix [Essay on the Application of Analysis to the Probability of Majority Decisions]*. L’imprimerie Royale, Paris.

[33] DeepMind (2017). *DeepMind Ethics & Society Principles*. https://deepmind.com/applied/deepmind-ethics-society/principles/.

[34] DeSombre, Elizabeth R. (2000). The Experience of the Montréal Protocol: Particularly Remarkable, and Remarkably Particular. *UCLA Journal of Environmental Law and Policy*, 19: 49–82.

[35] Dirac, Paul A. M. (1926). On the Theory of Quantum Mechanics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 112(762): 661–677.

[36] Eshel, Ilan and L. L. Cavalli-Sforza (1982). Assortment of Encounters and the Evolution of Cooperativeness. *Proceedings of the National Academy of Sciences of the United States of America*, 79: 1331–1335.

[37] European Group on Ethics in Science and New Technologies (2018). Statement on artificial intelligence, robotics and ‘autonomous’ systems. http://ec.europa.eu/research/ege/pdf/ege_ai_statement_2018.pdf.

[38] Farrell, Joseph (1987). Cheap Talk, Coordination, and Entry. *The RAND Journal of Economics*, 18(1): 34–39.

[39] Farrell, Joseph and Matthew Rabin (1996). Cheap Talk. *Journal of Economic Perspectives*, 10(3): 103–118.

[40] Fehl, Katrin, Daniel J. van der Post, and Dirk Semmann (2011). Co-evolution of Behaviour and Social Network Structure Promotes Human Cooperation. *Ecology Letters*, 14(6): 546–551.

[41] Fermi, Enrico (1926). Sulla quantizzazione del gas perfetto monoatomico [On the Quantization of the Monoatomic Ideal Gas]. *Rendiconti Lincei. Scienze Fisiche e Naturali*, 3: 181–185.

[42] Finus, Michael (2008). Game Theoretic Research on the Design of International Environmental Agreements: Insights, Critical Remarks, and Future Challenges. *International Review of Environmental and Resource Economics*, 2(1): 29–67.
[43] Fishman, Michael A. (2006). Involuntary Defection and the Evolutionary Origins of Empathy. *Journal of Theoretical Biology*, 242: 873–879.

[44] Fletcher, Jeffrey A. and Martin Zwick (2007). The Evolution of Altruism: Game Theory in Multilevel Selection and Inclusive Fitness. *Journal of Theoretical Biology*, 245: 26–36.

[45] Foot, Philippa (1978). *Virtues and Vices and Other Essays in Moral Philosophy*. Oxford University Press, Oxford.

[46] Fudenberg, Drew and Jean Tirole (1991). *Game theory*. The MIT Press, Cambridge, MA.

[47] Future of Life Institute (2017). Asilomar AI Principles. [https://futureoflife.org/ai-principles/](https://futureoflife.org/ai-principles/)

[48] Gebru, Timnit, Jamie Morgenstern, Briana Vecchione, Jennifer Wortman Vaughan, Hanna Wallach, Hal Daumeé III, and Kate Crawford (2020). Datasheets for Datasets. *arXiv pre-print*, abs/1803.09010. [https://arxiv.org/abs/1803.09010](https://arxiv.org/abs/1803.09010)

[49] Gintis, Herbert (2000). *Game Theory Evolving: A Problem-Centered Introduction to Modeling Strategic Behavior*. Princeton University Press, Princeton.

[50] Gintis, Herbert, Samuel Bowles, Robert Boyd, and Ernst Fehr (2003). Explaining Altruistic Behavior in Humans. *Evolution and Human Behavior*, 24: 153–172.

[51] Gokhale, Chaitanya S. and Arne Traulsen (2010). Evolutionary Games in the Multiverse. *Proceedings of the National Academy of Sciences of the United States of America*, 107(12): 5500.

[52] Google (2018). AI at Google: Our Principles. [https://ai.google/principles](https://ai.google/principles)

[53] Gotterbarn, Don, Amy Bruckman, Catherine Flick, Keith Miller, and Marty J. Wolf (2018). ACM Code of Ethics: A Guide for Positive Action. *Communications of the ACM*, 61(1): 121–128.

[54] Government of Japan, Ministry of Internal Affairs & Communications (MIC) (2017). AI R&D principles. [http://www.soumu.go.jp/main_content/000507517.pdf](http://www.soumu.go.jp/main_content/000507517.pdf)

[55] Government of Japan, Ministry of Internal Affairs & Communications (MIC) (2018). Draft AI utilization principles. [http://www.soumu.go.jp/main_content/000581310.pdf](http://www.soumu.go.jp/main_content/000581310.pdf)

[56] Green, Ben (2019). ‘Good’ Isn’t Good Enough. *Proceedings of the AI for Social Good workshop at NeurIPS*, pages 1–7.

[57] Greene, Daniel, Anna Lauren Hoffmann, and Luke Stark (2019). Better, Nicer, Clearer, Fairer: A Critical Assessment of the Movement for Ethical Artificial Intelligence and Machine Learning. In *22nd Hawaii International Conference on System Sciences*, pages 2122–2131.

[58] Grujić, Jelena, Carlos Gracia-Lazaro, Manfred Milinski, Dirk Semmann, Arne Traulsen, José A. Cuesta, Yamir Moreno, and Angel Sánchez (2015). A comparative analysis of spatial Prisoner’s Dilemma experiments: Conditional cooperation and payoff irrelevance. *Sci. Rep.*, 4(4615): 4615.

[59] Grujić, Jelena, Torsten Rohl, Dirk Semmann, Manfred Milinski, and Arne Traulsen (2012). Consistent strategy updating in spatial and non-spatial behavioral experiments does not promote cooperation in social networks. *PLoS One*, 7(11): e47718.

[60] Hagendorff, Thilo (2019). The Ethics of AI Ethics: An Evaluation of Guidelines. *arXiv pre-print*, abs/1903.03425. [http://arxiv.org/abs/1903.03425](http://arxiv.org/abs/1903.03425)

[61] HAIP Initiative (2018). Harmonious Artificial Intelligence Principles (HAIP). [http://biaia.ac.cn/haip/index.php](http://biaia.ac.cn/haip/index.php)

[62] Hamilton, William D. (1963). The Evolution of Altruistic Behavior. *The American Naturalist*, 9: 354–356.

[63] Hamilton, William D. (1964a). The Genetical Evolution of Social Behaviour. I. *Journal of Theoretical Biology*, 7: 1–16.
[64] Hamilton, William D. (1964b). The Genetical Evolution of Social Behaviour. II. *Journal of Theoretical Biology*, 7: 17–52.

[65] Hamilton, William D. (1971). Selection of Selfish and Altruistic Behavior in Some Extreme Models. In Eisenberg, J. F. and W. S. Dillon, editors, *Man and Beast*, pages 59–91. Smithsonian Institution Press, Washington.

[66] Harari, Yuval Noah (2017). Reboot for the AI revolution. *Nature*, 550: 324–327.

[67] Harms, William and Brian Skyrms (2008). Evolution of Moral Norms. In Ruse, Michael, editor, *The Oxford Handbook of Philosophy of Biology*, pages 434–450. Oxford University Press, Oxford.

[68] Hauert, Christoph, Miranda Holmes, and Michael Doebeli (2006). Evolutionary Games and Population Dynamics: Maintenance of Cooperation in Public Goods Games. *Proceeding of the Royal Society B: Biological Science*, 273(1600): 2565–2570.

[69] Hausken, Kjell and Jack Hirshleifer (2008). Truthful Signalling, the Heritability Paradox, and the Malthusian Equi-Marginal Principle. *Theoretical Population Biology*, 73: 11–23.

[70] Helbing, Dirk (2019). *Towards Digital Enlightenment: Essays on the Dark and Light Sides of the Digital Revolution*. Springer, Cham.

[71] Hofbauer, Josef and Karl Sigmund (1994/1651). *Leviathan, with Selected Variants from the Latin Edition of 1668*. Hackett Publishing Company, Inc., Indianapolis/Cambridge.

[72] Hofbauer, Josef and Karl Sigmund (1998). *Evolutionary Game and Population Dynamics*. Cambridge University Press, Cambridge.

[73] Hofbauer, Josef and Karl Sigmund (2003). Evolutionary Game Dynamics. *Bulletin of the American Mathematical Society*, 40: 479–519.

[74] House of Lords, UK (2018). AI in the UK: Ready, willing and able? [https://publications.parliament.uk/pa/id201719/idselect/idai/100/100.pdf].

[75] Huang, Weini and Arne Traulsen (2010). Fixation Probabilities of Random Mutants under Frequency Dependent Selection. *Journal of Theoretical Biology*, 263(2): 262—268.

[76] Hume, David (1739). *A Treatise of Human Nature*. John Noon, London.

[77] Hurd, Peter L. (1995). Communication in Discrete Action-Response Games. *Journal of Theoretical Biology*, 174: 217–222.

[78] IBM (2017). Principles for the Cognitive Era. [https://www.ibm.com/blogs/think/2017/01/ibm-cognitive-principles/]

[79] IBM (2018). Principles for Trust and Transparency. [https://www.ibm.com/blogs/policy/trust-principles/]

[80] Imhof, Lorens A. and Martin A. Nowak (2006). Evolutionary Game Dynamics in a Wright-Fisher Process. *Journal of Mathematical Biology*, 52(5): 667–681.

[81] Information Technology Industry Council (2017). AI Policy Principles. [https://www.itic.org/public-policy/ITIAIPolicyPrinciplesFINAL.pdf]

[82] Jäger, Gerhard (2008). Evolutionary Stability Conditions for Signaling Games with Costly Signals. *Journal of Theoretical Biology*, 253: 131–141.

[83] Jobin, Anna, Marcello Ienca, and Effy Vayena (2019). The Global Landscape of AI Ethics Guidelines. *Nature: Machine Intelligence*, 1: 389–399.

[84] Johnstone, Rufus A. (1995). Sexual Selection, Honest Advertisement and the Handicap Principle: Reviewing the Evidence. *Biological Reviews*, 7: 1–65.

[85] Kameda, Tatsuya and Daisuke Nakanishi (2003). Does Social/Cultural Learning Increase Human Adaptability? Rogers’s question revisited. *Evolution and Human Behavior*, 24: 242–260.
[86] Kaniaru, Donald, Rajendra Shende, Scott Stone, and Durwood Zaelke (2007). Strengthening the Montréal Protocol: Insurance against Abrupt Climate Change. **Sustainable Development Law & Policy**, 7(2): 3–9, 74–76.

[87] Kendal, Jeremy, Marcus W. Feldman, and Kenichi Aoki (2006). Cultural Coevolution of Norm Adoption and Enforcement when Punishers are Rewarded or Non-punishers Are Punished. **Theoretical Population Biology**, 70: 10–25.

[88] Kraig, Michael R. (1999). Nuclear Deterrence in the Developing World: A Game-Theoretic Treatment. **Journal of Peace Research**, 36(2): 141–167.

[89] Kurokawa, Shun and Yasuo Ihara (2009). Emergence of cooperation in public goods games. **Proceedings of the Royal Society B: Biological Science**, 276(1660): 1379–1384.

[90] Lachmann, Michael, Szabolcs Szamado, and Carl T. Bergstrom (2001). Cost and Conflict in Animal Signals and Human Language. **Proceedings of the National Academy of Sciences**, 98(23): 13189–13194.

[91] LaCroix, Travis (2019). Using logic to evolve more logic: Composing logical operators via self-assembly. **British Journal for the Philosophy of Science**. [https://doi.org/10.1093/BJPS/axz049](https://doi.org/10.1093/BJPS/axz049).

[92] LaCroix, Travis and Yoshua Bengio (2019). Learning from Learning Machines: Optimisation, Rules, and Social Norms. **arXiv pre-print**, abs/2001.00006. [https://arxiv.org/abs/2001.00006](https://arxiv.org/abs/2001.00006).

[93] LaCroix, Travis and Aydin Mohseni (2020). Cheap-Talk Declarations and the Tragedy of the AI Commons. **Unpublished Manuscript**, June 2020.

[94] LaCroix, Travis and Cailin O’Connor (2020). Power by Association. **PhilSci Archive pre-print**, 14318. [http://philsci-archive.pitt.edu/14318/](http://philsci-archive.pitt.edu/14318/).

[95] Littman, Michael L. (1994). Markov Games as a Framework for Multi-agent Reinforcement Learning. **ICML’94: Proceedings of the Eleventh International Conference on International Conference on Machine Learning**, pages 157–163.

[96] Liu, Xuesong, Mingfeng He, Yibin Kang, and Qiuhui Pan (2017). Fixation of Strategies with the Moran and Fermi Processes in Evolutionary Games. **Physica A**, 484: 336–344.

[97] Liu, Xuesong, Qiuhiu Pan, Yibin Kang, and Mingfeng He (2015). Fixation Probabilities in Evolutionary Games with the Moran and Fermi Processes. **Journal of Theoretical Biology**, 364: 242–248.

[98] Liu, Yongkui, Xiaojie Chen, Long Wang, Bin Li, Wenge Zhang, and Huifeng Wang (2011). Aspiration-based Learning Promotes Cooperation in Spatial Prisoner’s Dilemma Games. **EPL (Europhysics Letters)**, 94(6): 60002.

[99] Lomas, Jonathan (1991). Words without Action? The Production, Dissemination, and Impact of Consensus Recommendations. **Annual Review of Public Health**, 12(1): 41–65.

[100] Lomas, Jonathan, Geoffrey M. Anderson, Karin Domnick-Pierre, Eugene Vayda, Murray W. Enkin, and Walter J. Hannan (1989). Do Practice Guidelines Guide Practice? **New England Journal of Medicine**, 321(19): 1306–1311.

[101] Luccioni, Alexandra and Yoshua Bengio (2019). On the Morality of Artificial Intelligence. **arXiv pre-print**, abs/1912.11945. [http://arxiv.org/abs/1912.11945](http://arxiv.org/abs/1912.11945).

[102] Madani, Kaveh (2010). Game Theory and Water Resources. **Journal of Hydrology**, 381(3–4): 225–238.

[103] Makridakis, Spyros (2017). The Forthcoming Artificial Intelligence (AI) Revolution: Its Impact on Society and Firms. **Futures**, 90: 46–60.

[104] Maynard Smith, John (1982). **Evolution and the Theory of Games**. Cambridge University Press, Cambridge.
[105] Maynard Smith, John and George R. Price (1964). Group Selection and Kin Selection. Nature, 201: 1145–1147.

[106] McNamara, Andrew, Justin Smith, and Emerson Murphy-Hill (2018). Does ACM’s Code of Ethics Change Ethical Decision Making in Software Development? In Leavens, Gary T., Alessandro Garcia, and Corina S. Păsăreanu, editors, Proceedings of the 2018 26th ACM Joint Meeting on European Software Engineering Conference and Symposium on the Foundations of Software Engineering - ESEC/FSE 2018, pages 1–7. ACM Press, New York.

[107] McPherson, Miller, Lynn Smith-Lovin, and James M. Cook (2001). Birds of a Feather: Homophily in Social Networks. Annual Review of Sociology, 27: 415–444.

[108] Microsoft (2018). Microsoft AI Principles. https://www.microsoft.com/en-us/ai/our-approach-to-ai.

[109] Milinski, M., R. D. Sommerfeld, H. J. Krambeck, F. A. Reed, and J. Marotzke (2008). The Collective-risk Social Dilemma and the Prevention of Simulated Dangerous Climate Change. Proceedings of the National Academy of Sciences of the United States of America, 105(7): 2291–2294.

[110] Mittelstadt, Brent (2019). Principles alone cannot guarantee ethical AI. Nature Machine Intelligence, 1(11): 501–507.

[111] Mohseni, Aydin (2019). Stochastic Stability & Disagreement in Evolutionary Dynamics. Philosophy of Science, 86(3): 497–521.

[112] Morley, Jessica, Luciano Floridi, Libby Kinsey, and Anat Elhalal (2019). From What to How: An Initial Review of Publicly Available AI Ethics Tools, Methods and Research to Translate Principles into Practices. arXiv pre-print, abs/1905.06876. https://arxiv.org/abs/1905.06876.

[113] Moyano, Luis G. and Angel Sánchez (2009). Evolving Learning Rules and Emergence of Cooperation in Spatial Prisoner’s Dilemma. Journal of Theoretical Biology, 259(1): 84–95.

[114] Nakahashi, Wataru (2007). The Evolution of Conformist Transmission in Social Learning when the Environment Changes Periodically. Theoretical Population Biology, 72: 52–66.

[115] Neumann, John Von and Oskar Morgenstern (2007/1944). Theory of Games and Economic Behavior. Princeton university press.

[116] Nowak, Martin A. (2012). Evolving Cooperation. Journal of Theoretical Biology, 299: 1–8.

[117] Nowak, Martin A., Joshua B. Plotkin, and David C. Krakauer (1999). The Evolutionary Language Game. Journal of Theoretical Biology, 200: 147–162.

[118] Nowak, Martin A., Akira Sasaki, Christine Taylor, and Drew Fudenberg (2004). Emergence of Cooperation and Evolutionary Stability In Finite Populations. Nature, 428: 646–650.

[119] Nowak, Martin A. and Karl Sigmund (2004). Evolutionary Dynamics of Biological Games. Science, 303: 793–799.

[120] Ohtsuki, Hisashi, Pedro Bordalob, and Martin A. Nowak (2007). The One-third Law of Evolutionary Dynamics. Journal of Theoretical Biology, 249(2): 289–295.

[121] Ohtsuki, Hisashi and Martin A. Nowak (2006). Evolutionary Games on Cycles. Proceedings of the Royal Society B: Biological Science, 273(1598): 2249–2256.

[122] Ohtsuki, Hisashi and Martin A. Nowak (2008). Evolutionary Stability on Graphs. Journal of Theoretical Biology, 251: 698–707.

[123] OpenAI (2018). OpenAI Charter. https://blog.openai.com/openai-charter/.

[124] Ostrom, Elinor (2000). Collective Action and the Evolution of Social Norms. Journal of Economic Perspectives, 14(3): 137–158.
[125] Pacheco, Jorge M, Francisco C. Santos, Max O. Souza, and Brian Skyrms (2009). Evolutionary Dynamics of Collective Action in n-Person Stag Hunt Dilemmas. *Proceedings of the Royal Society B: Biological Sciences*, 276(1655): 315.

[126] Pacheco, Jorge M., Vítor V. Vasconcelos, and Francisco C. Santos (2014). Climate Change Governance, Cooperation and Self-organization. *Physics of Life Reviews*, 11(4): 573–586.

[127] Page, Karen M. and Martin A. Nowak (2002). Empathy leads to fairness. *Bulletin of Mathematical Biology*, 64: 1101–1116.

[128] Partnership on AI (2016). Tenets. [https://wwwpartnershiponaiorg/tenets](https://www.partnershiponai.org/tenets/)

[129] Pawlowitsch, Christina (2007). Finite Populations Choose an Optimal Language. *Journal of Theoretical Biology*, 249: 606–616.

[130] Pawlowitsch, Christina (2008). Why Evolution Does Not Always Lead to an Optimal Signaling System. *Games and Economic Behavior*, 63(1): 203–226.

[131] Poisson, Siméon Denis (1837). *Recherches sur la probabilité des jugements en matière criminelle et en matière civile, précédées des règles générales du calcul des probabilités*. Bachelier, Paris.

[132] Pomiankowski, Andrew (1987). Sexual Selection: The Handicap Principle Does Work – Sometimes. *Proceedings of the Royal Society B: Biological Science*, 231: 123–145.

[133] Rand, David G. and Martin A. Nowak (2013). Human Cooperation. *Trends in Cognitive Science*, 17(8): 413–425.

[134] Rapoport, Anatol and Albert M. Chammah (1966). The Game of Chicken. *American Behavioral Scientist*, 10(3): 10–28.

[135] Rogers, Alan R. (1988). Does Biology Constrain Culture? *American Anthropologist*, 90: 819–831.

[136] Ross, Don (2019). Game Theory. In Zalta, Edward N., editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2019 edition.

[137] Royal Statistical Society and the Institute and Faculty of Actuaries (2019). A Guide for Ethical Data Science: A Collaboration between the Royal Statistical Society (RSS) and the Institute and Faculty of Actuaries (IFoA). [https://wwwactuariesorguk/system/files/field/document/an%20ethical%20charter%20for%20data%20science%20WEB%20FINALPDF](https://www.actuaries.org.uk/system/files/field/document/an%20ethical%20charter%20for%20data%20science%20WEB%20FINAL.PDF)

[138] Russell, Stuart (2019). *Human Compatible: Artificial Intelligence and the Control Problem*. Viking, New York.

[139] Sage (2017). The Ethics of Code: Developing AI for Business with Five Core Principles. [https://www.sage.com/ca/our-news/press-releases/2017/06/designing-AI-for-business](https://www.sage.com/ca/our-news/press-releases/2017/06/designing-AI-for-business)

[140] Sánchez, Angel and José A. Cuesta (2005). Altruism May Arise from Individual Selection. *Journal of Theoretical Biology*, 235: 233–240.

[141] Sandholm, William H. (2007). Simple Formulas for Stationary Distributions and Stochastically Stable States. *Games and Economic Behavior*, 59(1): 154–162.

[142] Santos, Francisco C. and Jorge M. Pacheco (2011). Risk of Collective Failure Provides an Escape from the Tragedy of the Commons. *Proceedings of the National Academy of Sciences of the United States of America*, 108(26): 10421–10425.

[143] SAP (2018). Sap’s guiding principles for artificial intelligence. [https://news.sap.com/201809sap-guiding-principles-for-artificial-intelligence](https://news.sap.com/201809sap-guiding-principles-for-artificial-intelligence)

[144] Serrano, Roberto and Allan M. Feldman (2013). *A Short Course in Intermediate Microeconomics with Calculus*. Cambridge University Press, Cambridge.
[145] Shapley, Lloyd S. (1953). Stochastic Games. *Proceedings of the National Academy of Sciences of the United States of America*, 39: 1095–1100.

[146] Sigmund, Karl (2010). *The Calculus of Selfishness*. Cambridge University Press, Cambridge.

[147] Skyrms, Brian (1994). Darwin Meets the Logic of Decision: Correlation in Evolutionary Game Theory. *Philosophy of Science*, 61: 503–528.

[148] Skyrms, Brian (2004). *The Stag Hunt and the Evolution of Social Structure*. Cambridge University Press, Cambridge.

[149] Skyrms, Brian (2010). *Signals: Evolution, Learning, & Information*. Oxford University Press, Oxford.

[150] Skyrms, Brian (2014/1996). *Evolution of the Social Contract*. Cambridge University Press, Cambridge.

[151] Sony (2018). Sony group AI ethics guidelines. https://www.sony.net/SonyInfo/CSR_report/humanrights/hkrfmg0000007rtj-att/AI_Engagement_within_Sony_Group.pdf.

[152] Sossin, Lorne and Charles W. Smith (2003). Hard Choices and Soft Law: Ethical Codes, Policy Guidelines and the Role of the Courts in Regulating Government. *Alberta Law Review*, 40: 867–893.

[153] Souza, Max O., Jorge M. Pacheco, and Francisco C. Santos (2009). Evolution of Cooperation under N-person Snowdrift Games. *Journal of Theoretical Biology*, 260(4): 581–588.

[154] Stanford University (2018). The Stanford Human-Centered AI Initiative (HAI). http://hai.stanford.edu/news/introducing_stanfords_human_centered_ai_initiative/

[155] Szabo, Gyorgy, Attila Szolnoki, and Jeromos Vukov (2009). Selection of Dynamical Rules in Spatial Prisoner’s Dilemma Games. *EPL (Europhysics Letters)*, 87(1): 18007.

[156] Szolnoki, Attila, Jeromos Vukov, and Gyorgy Szabo (2009). Selection of Noise Level in Strategy Adoption for Spatial Social Dilemmas. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 80(5 pt. 2): 056112.

[157] Taylor, Christine, Drew Fudenberg, Akira Sasaki, and Martin A. Nowak (2004). Evolutionary Game Dynamics in Finite Populations. *Bulletin of Mathematical Biology*, 66(6): 1621–1644.

[158] Taylor, Christine, Yoh Iwasa, and Martin A. Nowak (2006). A Symmetry of Fixation Times in Evolutionary Dynamics. *Journal of Theoretical Biology*, 243(2): 245—245.

[159] Taylor, Peter D. and Leo B. Jonker (1978). Evolutionarily Stable Strategies and Game Dynamics. *Mathematical Biosciences*, 40: 145–156.

[160] The Future Society (2017). Principles for the Governance of AI. http://www.thefuturesociety.org/science-law-society-sls-initiative/#1516790384127-3ea0ef44-2aae

[161] The IEEE Global Initiative on Ethics of Autonomous and Intelligent Systems (2017). Ethically Aligned Design, Version 2. http://standards.ieee.org/develop/indconn/ec/autonomous_systems.html

[162] The Japanese Society for Artificial Intelligence (2017). The Japanese Society for Artificial Intelligence Ethical Guidelines. http://ai-elsi.org/wp-content/uploads/2017/05/JSAI-Ethical-Guidelines-1.pdf

[163] The Public Voice (2018). Universal Guidelines for Artificial Intelligence. https://thepublicvoice.org/ai-universal-guidelines/

[164] Traulsen, Arne and Christoph Hauert (2009). Stochastic Evolutionary Game Dynamics. In Schuster, H. G., editor, *Reviews of Nonlinear Dynamics and Complexity*, volume 2, pages 25–62. Wiley-VCH, Weinheim.
[165] Traulsen, Arne, Martin A. Nowak, and Jorge M. Pacheco (2006a). Stochastic Dynamics of Invasion and Fixation. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 74(1 pt. 1): 011909.

[166] Traulsen, Arne, Jorge M. Pacheco, and Lorens Imhof (2006b). Stochasticity and Evolutionary Stability. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 74(2 pt. 1): 021905.

[167] Traulsen, Arne, Jorge M. Pacheco, and Martin A. Nowak (2007). Pairwise Comparison and Selection Temperature in Evolutionary Game Dynamics. *Journal of Theoretical Biology*, 246(3): 522–529.

[168] Traulsen, Arne, Dirk Semmann, Ralf D. Sommerfeld, Hans-Jürgen Krambeck, and Manfred Milinski (2009). Human Strategy Updating in Evolutionary Games. *Proceedings of the National Academy of Sciences of the United States of America*, 107(7): 2962–2966.

[169] Trivers, Robert L. (1971). The Evolution of Reciprocal Altruism. *The Quarterly Review of Biology*, 46(3): 35–57.

[170] UNI Global Union (2017). Top 10 Principles For Ethical Artificial Intelligence. [http://www.thefutureworldofwork.org/media/35420/uni_ethical_ai.pdf](http://www.thefutureworldofwork.org/media/35420/uni_ethical_ai.pdf).

[171] Université de Montréal (2017). The Montreal Declaration for a Responsible Development of Artificial Intelligence. [https://www.montrealdeclaration-responsibleai.com/the-declaration](https://www.montrealdeclaration-responsibleai.com/the-declaration).

[172] US Public Policy Council, Association for Computing Machinery (2017). Principles for Algorithmic Transparency and Accountability. [https://www.acm.org/binaries/content/assets/public-policy/2017_usacm_statement_algorithms.pdf](https://www.acm.org/binaries/content/assets/public-policy/2017_usacm_statement_algorithms.pdf).

[173] Wagner, Ben (2018). Ethics as an escape from regulation: From ‘ethics-washing’ to ethics-shopping? In Bayamlioglu, E., I. Baraliuc, L. A. W. Janssens, and M. Hildebrandt, editors, *Being Profiled: Cogitas Ergo Sum: 10 Years of Profiling the European Citizen*, pages 84–89. Amsterdam University Press, Amsterdam.

[174] Wagner, Ulrich J. (2001). The Design of Stable International Environmental Agreements: Economic Theory and Political Economy. *Journal of Economic Surveys*, 15(3): 377–411.

[175] Wakano, Joe Yuichiro and Kenichi Aoki (2006). A Mixed Strategy Model for the Emergence and Intensification of Social Learning in a Periodically Changing Natural Environment. *Theoretical Population Biology*, 70: 486–497.

[176] Wakano, Joe Yuichiro, Kenichi Aoki, and Marcus W. Feldman (2004). Evolution of Social Learning: A Mathematical Analysis. *Theoretical Population Biology*, 66: 249–258.

[177] Wang, Jing, Feng Fu, Te Wu, and Long Wang (2009). Emergence of Social Cooperation in Threshold Public Goods Games with Collective Risk. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 80: 016101.

[178] Wärneryd, Karl (1993). Cheap Talk, Coordination and Evolutionary Stability. *Games and Economic Behavior*, 5(4): 532–546.

[179] Weibull, Jörgen M. (1997). *Evolutionary Game Theory*. The MIT Press, Cambridge, MA.

[180] Whittaker, Meredith, Kate Crawford, Roel Dobbe, Genevieve Fried, Elizabeth Kaziunas, Varoon Mathur, Sarah Mysers West, Rashida Richardson, Jason Schultz, and Oscar Schwartz (2018). *AI now report 2018*. AI Now Institute at New York University. [https://ainowinstitute.org/AI_Now_2018_Report.pdf](https://ainowinstitute.org/AI_Now_2018_Report.pdf).

[181] Whittlestone, Jess, Rune Nyrup, Anna Alexandrova, and Stephen Cave (2019). The Role and Limits of Principles in AI Ethics: Towards a Focus on Tensions. In *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*, AIES ’19, page 195–200, New York. Association for Computing Machinery (ACM).
[182] Wu, Bin, Philipp M. Altrock, Long Wang, and Arne Traulsen (2010). Universality of Weak Selection. *Physical Review E: Statistical, Nonlinear, Biological, and Soft Matter Physics*, 82: 046106.

[183] Wu, Bin, Benedikt Bauer, Tobias Galla, and Arne Traulsen (2015). Fitness-based Models and Pairwise Comparison Models of Evolutionary Games are Typically Different—Even in Unstructured Populations. *New Journal of Physics*, 17: 023043.

[184] Young, H. Peyton and Shmuel Zamir (2014). *Handbook of Game Theory*, volume 4. Elsevier, North-Holland.

[185] Zagare, Frank C. (1987). *The Dynamics of Deterrence*. University of Chicago Press, Chicago.

[186] Zahavi, Amotz (1975). Mate Selection: A Selection for a Handicap. *Journal of Theoretical Biology*, 53(1): 205–214.

[187] Zahavi, Amotz and Avishag Zahavi (1997). *The Handicap Principle*. Oxford University Press, New York.

[188] Zhang, Kaiqing, Zhuoran Yang, and Tamer Basar (2019). Multi-Agent Reinforcement Learning: A Selective Overview of Theories and Algorithms. *arXiv pre-print*, abs/1911.10635. [https://arxiv.org/abs/1911.10635](https://arxiv.org/abs/1911.10635)

[189] Zollman, Kevin J. S. (2005). Talking to Neighbors: The Evolution of Regional Meaning. *Philosophy of Science*, 72(1): 69–85.
A Game Theory

In this brief appendix, we provide some further game-theoretic background than we had space to discuss in Section 3. For more comprehensive introductions to game theory, see, e.g., Aumann and Hart [9, 10, 11]; Maynard Smith [104]; Neumann and Morgenstern [115]; Weibull [179]; Young and Zamir [184].

Game-Theoretic Analysis of Cooperation and Conflict. Cooperative behaviour persists in human and non-human animal populations alike, but it provides something of an evolutionary puzzle [13; 30; 68; 113; 116; 118; 157]: How can cooperation be maintained despite incentives for non-cooperative behaviour (i.e., defection)? Evolutionary game theory provides useful tools for analysing the evolution of cooperative behaviour quantitatively in both human and non-human animals [7; 40; 49; 58; 59; 72; 73; 80; 89; 104; 119; 121; 122; 133; 168; 179].

Game theory can be used to study the ways in which independent choices between actors interact to produce outcomes. In game theory, a game is determined by the payoffs. For example, the payoff matrix for a generic, $2 \times 2$, symmetric, normal form game is displayed in Figure 5.

Each actor (Player 1 and Player 2) in this example can choose one of two strategies, $C$ or $D$. The payoffs to each of the players are given by the respective entries in each cell—i.e., the first number in the top-right cell ($b$) is the payoff afforded to Player 1 when she plays $C$ and her partner plays $D$; the second number ($c$) is the payoff afforded to Player 2 in the same situation (i.e., when Player 2 plays $D$ and Player 1 plays $C$).

As discussed in the paper, social dilemmas are games where (i) the payoff to each individual for non-cooperative behaviour is higher than the payoff for cooperative behaviour, and (ii) every individual receives a lower payoff when everyone defects than they would have, had everyone cooperated [31].

When $c > a > d > b$, in Figure 5, we have a Prisoner’s Dilemma. Note that when both actors cooperate (i.e., both play $C$), their payoff is higher than if they both defect ($a > d$), thus satisfying criterion (ii) mentioned above. However, each actor has an individual incentive to defect (i.e., play $D$) regardless of what the other actor does; Player 1 would prefer to defect when Player 2 cooperates ($c > a$), and she would prefer to defect when Player 2 defects ($d > b$)—and mutatis mutandis for Player 2. This satisfies criterion (i) above.

In this case, we say that $\text{defect}$ is a strictly dominant strategy for each player, which leads to the unique Nash equilibrium: $\langle D, D \rangle$—that is, a combination of strategies where no actor can increase her payoff by unilateral deviation from her strategy. The ‘dilemma’ is that mutual cooperation yields a better outcome for all parties than mutual defection, but, from an individual perspective, it is never rational to cooperate.

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16See Ross [136] for a philosophical overview.
17Named and formalised by Canadian mathematician Albert W. Tucker in 1952, based on Merrill M. Flood and Melvin Dresher’s 1950 model; see Serrano and Feldman [144].
Evolutionary Game Dynamics. In an evolutionary context, the payoffs are identified with reproductive fitness, so that more-successful strategies are more likely to propagate, reproduce, be replicated, be imitated, etc. This provides a natural way to incorporate dynamics to the underlying game.

There are two natural interpretations of evolutionary game dynamics. The first is biological, where strategies are encoded in the genome of individuals, and those who are successful pass on their genes at higher rates; the second is cultural, where successful behaviours are reproduced through learning and imitation. We are primarily concerned with processes of cultural evolution. This process should be familiar to those in AI/ML who work on multi-agent reinforcement learning (MARL) [95, 145, 188].

In addition to the game, an evolutionary model requires a specification of the dynamics—namely, a set of rules for determining how the strategies of actors in a population will update (under a cultural interpretation), or how the proportions of strategies being played in the population will shift as they proliferate or are driven to extinction (under a biological interpretation). Evolutionary game dynamics are often studied in infinite populations using deterministic differential equations. For example, the replicator dynamic [159] captures how strategies with higher-than-average fitness tend to increase, and strategies with lower-than-average fitness tend to decrease. A population state is evolutionarily stable only if it is an asymptotically stable rest point of the dynamics [104].

Stochastic Game Dynamics. In finite populations, stochastic game dynamics are used to study the selection of traits with frequency-dependent fitness [98, 120, 146, 155, 156, 165].

A standard stochastic game dynamics that is used extensively is the Moran process. This is a simple birth-death process where an individual is chosen proportional to their fitness and replaces a randomly chosen individual with an offspring of its own type [3, 27, 75, 96, 97, 158, 167, 182, 183].

In the standard Fermi process, which we discuss in Section 3, an individual is chosen randomly from a finite population, and its reproductive success is evaluated by comparing its payoff to a second, randomly-selected individual from the population [96, 166, 167].

As mentioned in Section 3, the pairwise comparison of payoffs of the focal individual and the role model informs the probability, \( p \), that the focal individual copies the strategy of the role model; the probability function, called the Fermi function, was presented in Equation 3, and repeated here for convenience:

\[
p = \left[ 1 + e^{\lambda(\pi_f - \pi_r)} \right]^{-1}.
\]

Again, if both individuals have the same payoff, the focal individual randomises between the two strategies. Note, then, that the focal individual does not always switch to a better strategy; the individual may switch to one that is strictly worse.

When the intensity of selection \( \lambda = 0 \), selection is random; when selection is weak (\( \lambda \ll 1 \)), \( p \) reduces to a linear function of the payoff difference; when \( \lambda = 1 \), our model gives us back the replicator dynamic; and, when \( \lambda \rightarrow \infty \), we get the best-response dynamic [46].

Evolutionary game dynamics have been used to shed light upon many aspects of human behaviour, including altruism [44, 50, 140, 169], moral behaviour [1, 20, 67, 148, 150], empathy [43, 127], social learning [85, 114, 135, 175, 176], social norms [12, 17, 19, 25, 87, 94, 124], and the evolution of communication, proto-language, and compositional syntax [14, 69, 77, 82, 91, 117, 129, 130, 149, 189], among many others. See Ross [136] for further details.
B Technical Details

In this brief appendix, we provide some further formal details for our model than we had space to discuss in Section 3.

Mean Payoffs. Recall that the payoffs to each cooperators, $C$, and defectors, $D$, in a group of size $N$ are given as a function of the number of cooperators in that group, $n_C$, as follows:

$$\pi_C(n_C) = b \cdot \Theta(n_C - n^*) + b \cdot (1 - rm) \cdot \Theta(n_C - n^*) - cb,$$
$$\pi_D(n_C) = \pi_C(n_C) + cb,$$

where $\Theta$ is the Heaviside step function. The mean payoffs to each type in a population of size $Z$, where groups are determined by random mixing, is then given as a function of the total fraction of cooperators in the population, $x_C = n_C^Z / Z$, as follows:

$$\Pi_C(x_C) = \sum_{n_c=0}^{N} \frac{n_c}{N} \binom{N}{n_c} x_C^{n_c} (1 - x_C)^{N - n_c} \pi_C(n_c),$$
$$\Pi_D(x_C) = \sum_{n_c=0}^{N} \frac{N - n_c}{N} \binom{N}{n_c} x_C^{n_c} (1 - x_C)^{N - n_c} \pi_D(n_c).$$

Fermi Dynamics. The Fermi dynamics uses the average payoffs to each type to determine the probability that a randomly-chosen individual from the population will imitate the strategy of a second randomly-chosen individual from the population. Such a change will produce one of three outcomes: the number of cooperators in the population, $k$, will increase, decrease, or remain the same. This is captured by the following transition probabilities, which yield a tri-diagonal transition matrix, $T$, for our birth-death process:

$$T^+(k) = (1 - \mu) \frac{k}{Z} \frac{Z - k}{Z - 1} \left(1 + e^{-\lambda(\Pi_C - \Pi_D)}\right)^{-1} + \frac{\mu}{2},$$
$$T^-(k) = (1 - \mu) \frac{Z - k}{Z} \frac{k}{Z - 1} \left(1 + e^{-\lambda(\Pi_D - \Pi_C)}\right)^{-1} + \frac{\mu}{2},$$
$$T^0(k) = 1 - T^+(k) - T^-(k)$$

where $\lambda$ is the inverse temperature associated with the influence of selection versus drift, and $\mu$ is the rate of mutation. This produces an ergodic Markov process.

Gradient of Selection. The gradient of selection of the process captures the expected direction of selection as a function of the number of cooperators in the population, $k$, in a way that is analogous to the mean-field dynamics for the infinite-population case. This is given by

$$G(k) = T^+(k) - T^-(k) = \frac{k}{Z} \frac{Z - k}{Z - 1} \left(\tanh \frac{\lambda}{2} (\Pi_C(k) - \Pi_D(k))\right),$$

where $G(k) > 0$ implies that selection will favour cooperation, and $G(k) < 0$ implies that defection is favoured.

Stationary Distribution. The stationary distribution of the process captures the long run distribution of time the process spends at each state. For an ergodic process, the stationary distribution is known to be unique and independent of initial conditions of that process. We compute it as follows:

$$\sigma_k = \frac{\prod_{i=1}^{k} \frac{T^+(i-1)}{T^-(i-1)} \prod_{j=1}^{Z} \frac{T^+(j-1)}{T^-(j-1)}}{\sum_{i=1}^{Z} \prod_{j=1}^{i} \frac{T^+(j-1)}{T^-(j-1)}} \text{ for } k \in \{1, \ldots, Z\}.$$