One of the most interesting discoveries at HERA has been the observation of large rapidity-gap events, which give us the possibility of investigating Diffractive DIS at small $x$, raising the hope of relating Regge theory with the calculations in perturbative QCD and, in particular, of understanding the nature of the dominant Regge trajectory at high energies, the "pomeron".

A fundamental tool for these studies is the colour–dipole scheme, introduced by Nikolaev and Zakharov and by Mueller, which offers a unified approach of small $x$ inclusive DIS and Diffractive DIS. This scheme derives from the observation that at small $x$ the scattering process can be conceived as the virtual photon fluctuating into a quark–antiquark pair, which interacts with the proton by the exchange of gluons. The $q\bar{q}$ fluctuation lives a time $\Delta t \approx \frac{1}{x m_p}$ ($m_p$ is the proton mass) much longer at small $x$ than the interaction time. Therefore the transverse dimension of the pair can be considered frozen during the scattering. For what concerns the inclusive DIS, the interaction, at the lowest order, is given by the exchange of a gluon and the calculated cross section can be written as:

$$\sigma_{T,L}(\gamma^* N) = \int_0^1 dz \int d\vec{\rho} |\Psi_{T,L}^\gamma(z,\rho)|^2 \sigma(\rho, x)$$

(1)

in terms of a photon (with Transverse or Longitudinal polarization) wave function $\Psi_{T,L}^\gamma(z, \rho)$ (see for its explicit form), whose modulus squared gives the probability of producing a pair with transverse size $\rho$ and with a fraction $z$ of the photon light–cone momentum, and a cross section $\sigma(\rho, x)$, which can be directly related to the unintegrated glue distribution of the proton:

$$\sigma(x, \rho) = \frac{4 \pi \alpha_s}{3} \int \frac{d\vec{k}}{k^4} (1 - e^{i\vec{k} \cdot \vec{\rho}}) \frac{d[xg(x, k^2)]}{d\ln k^2}$$

(2)

An important observation is that the dependences on the flavour of the pair and on the photon polarization are contained in $\Psi_{T,L}^\gamma$, while $\sigma(\rho, x)$ is universal. A similar analysis can be carried on for the DDIS, where one needs, to the lowest
order, the exchange of two gluons. The result can be written as:

$$\frac{d\sigma_{T,L}}{dt} \bigg|_{t=0} = \int \frac{dz}{16\pi} \int d\tilde{\rho} |\Psi_{T,L}(z,\rho)|^2 \sigma(\rho, x)^2$$

(3)

Incidentally, it must be noticed that in the literature Eq.s (1,3) are used both in the momentum or in the dipole-size representations.

Once $\sigma(\rho, x)$ is known, many processes can be calculated. This allows to move from processes where one selects rather small sizes (large $k^2$), and thus in the perturbative regime, up to cases where rather large sizes are relevant and, therefore, there is a penetration into the non-perturbative region. The evaluation of the contribution from the soft region requires some modelling, as stating that the diagrams included in the calculation continue to be dominant and a parameterization of the unintegrated glue at small scales (see Eq. 3). However, one can fix the relatively small number of parameters in some process and then use them for producing a large amount of predictions for other processes: the agreement with experimental data permits to check this ansatz.

The fact of selecting different scales varying the process leads to the prediction of different effective pomeron intercepts: this is a fundamental testable prediction of the colour–dipole scheme. For example, the vector meson production amplitude can be written as:

$$<V|M|\gamma^*> = \int dz \int d\tilde{\rho} \Psi_V(z,\rho) M(\rho) \Psi_{T,L}^\gamma(z,\rho)$$

(4)

where $M(\rho)$ is the amplitude for the scattering of a coloured dipole of size $\rho$ and $\Psi_V$ the vector meson wave function. The analysis of the integrand shows that one is selecting a scale $q^2 \approx (0.1 - 0.2) \cdot (Q^2 + M_V^2)$, which for real photoproduction is rather small, leading to a small effective intercept, while increasing $Q^2$, it goes deeper and deeper in the realm of perturbative QCD giving a rising effective intercept. The variation of the effective intercept with the scale can be nicely observed looking at the experimental data on $\rho, \phi$ and $J/\Psi$ production.

For what concerns the low-$x$ inclusive DIS, the resummation of leading $log(1/x)$ in colour–dipole model recovers the BFKL equation. A plausible scenario for running BFKL equation is that we have a sequence of BFKL poles plus a soft term

$$F_2(x, Q^2) \approx \sum_i F_i(Q^2) \cdot \left(\frac{1}{x}\right)^{\Delta_i} + F_{soft}(Q^2) \cdot \left(\frac{1}{x}\right)^{\Delta_{soft}}$$

(5)

which, in a limited region of $x$, can be parameterized with an effective intercept $F_2 \propto (\frac{1}{x})^{\Delta_{eff}}$. However, at small $x$ one should also include unitarization.
effects, as the one coming from triple pomeron, which, in the colour–dipole scheme, derives from the $q\bar{q}g$ Fock states of the photon. Some first attempt of evaluating unitarization in colour–dipole model have already appeared, but a deeper analysis has yet to be performed. Anyway, experimentally the scale dependence of the effective intercept has been clearly observed.

The most detailed analysis in the colour–dipole scheme has been carried out for DDIS, where various results have been derived in a good agreement with the experimental data.

The study of the transverse diffractive structure function at intermediate values of $\beta$, where excitation of the $q\bar{q}$ Fock component of the photon dominates, has been accomplished giving

$$F_T^{D(3)}(x_{IP}, \beta, Q^2) \approx \frac{\beta(1-\beta)^2(3+4\beta+8\beta^2)}{6m_f^2B_d(\beta)} \cdot \frac{e_f^2}{12} \cdot [\alpha_S(q^2)G(x_{IP}, q^2)]^2$$

For light quarks, a substantial penetration into the low scales is found, for the process is dominated by the scale $q^2 \approx k^2 + m_f^2$. However, for heavy quarks, the large quark mass gives a perturbative scale, leading to a larger effective intercept than for the light quarks component (an interesting breaking of Regge factorization, which we cannot discuss here, is found for charged currents DDIS, see [1]).

On the other hand, for a longitudinal polarization of the $\gamma^*$ the colour–dipole calculation shows that one selects a hard scale $q_{L}^2 \approx \frac{Q^2}{4m_f^2}$, independently on the flavour,

$$F_L^{D(3)}(x_{IP}, \beta, Q^2) \approx \frac{4\beta(1-2\beta)^2}{Q^2B_d(x_{IP})} \cdot \frac{e_f^2}{12} \left[\alpha_S(\frac{1}{4}Q^2)G(x_{IP}, q_{L}^2)\right]^2$$

One finds the same scale for the twist-4 component of the transverse structure function (which appears with a negative sign), studied in Ref. [1]. The relevant point is that for these two last components the $\frac{1}{Q^2}$ factor, due to the higher twist behaviour, is partially compensated by the growth of the glue with $q_{L}^2$. $F_2^{D(3)}$ at large $\beta$ comes from the sum of all these terms, and at $\beta \gtrsim 0.8$ is dominated by $F_L^{D(3)}$ even at relatively large $Q^2$ (see figure 1). Thus at large $\beta$ one cannot apply to diffractive structure function the usual QCD evolution. The different $x_{IP}$ dependences of these different components and their comparison with experimental data are presented in figure 1.

For what concerns the small $\beta$ region, it is dominated by excitation of the $q\bar{q}g$ Fock states of the photon, calculated in Ref. [4]. In this case an approximate factorization and the QCD evolution are recovered.
still a penetration in the small momenta region, albeit less deep than for the light–quark transverse $q\bar{q}$ component, thus the effective intercept is larger. Altogether the prediction of this scheme is that a fit of the form $F^D(3) \propto x^{-\delta}$ is expected to give a large exponent at large $\beta$ ($\delta \approx 0.3$), where $F_L^D$ dominates, which decreases in the region dominated by the transverse $ud \, q\bar{q}$ component reaching a minimum for $\beta \approx 0.6$, where $\delta \approx 0.15$, and increases again in the low $\beta$ region. The available DDIS experimental data do not yet have a sufficient statistics for fitting these exponents with different $\beta$ bins (and excluding the regions were large contributions from other Regge trajectories are expected. For a first attempt see 17). However, altogether, predicting the variation of the effective pomeron intercept with the scale is one of the most relevant results obtained in the colour–dipole scheme. In figure 2 the measured effective intercepts $\delta$ in different processes are shown in function of the effective scales (see 18 for definitions). The scale dependence is clearly observed, although the available fits did not explore the predicted $\beta$-dependence of the effective intercept.

In conclusion, the colour–dipole model has been applied to a large number of different processes, giving a unified treatment of low $x$ inclusive and diffractive DIS [4]. For different processes one selects different scales, with the possi-

\footnote{The model has also been applied successfully to the study of nuclear structure functions, see Ref. 4}
Figure 2: Experimental effective pomeron intercepts in function of the scale $q^2$, for rho (filled circle), $\phi$ (empty quadrangle) and $J/\Psi$ (cross) production. ZEUS (filled rhombus) and H1 (empty circle) DDIS and H1 inclusive DIS (filled quadrangle)

bility of studying the transition between perturbative and non-perturbative QCD. Up to now a good agreement with available experimental data has been obtained; more refined tests of this scheme will soon be possible.

1. N.N.Nikolaev and B.G.Zakharov, Z. Phys. C 49, 607 (91), Z. Phys. C 53, 331 (92), Z. Phys. C 64, 631 (94).
2. A.Mueller and B. Patel, Nucl. Phys. B 415, 373 (94); A.Mueller, Nucl. Phys. B 425, 471 (94).
3. M. Bertini, M. Genovese, N. N. Nikolaev, A.Pronyaev and B.G. Zakharov, Phys. Lett. B 422, 238 (98).
4. B. Kopeliovich, J. Nemchik, N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B309 179, 93 (;) Phys. Lett. B 324, 469 (94); J. Nemchik, N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B 341, 228 (94); J. Nemchik, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys. Lett. B 374, 199
(96), Z. Phys. C 75, 71 (95), hep-ph/9712469.

5. ZEUS coll., Phys. Lett. B 356, 601 (95), Phys. Lett. B 377, 259 (96), Z. Phys. C 76, 599 (97); H1 coll., Nucl. Phys. B 472, 3 (96), Z. Phys. C 75, 607 (97). S. Kananov, these proceedings.

6. N. N. Nikolaev and B. G. Zakharov, Z. Phys. C 64, 631 (94); M. Genovese, N.N. Nikolaev and B. G. Zakharov, JETP 81, 633 (95); J. Bartels and M. Wüstoff, Z. Phys. C 66, 157 (95); J. Bartels, H. Lotter and M. Wüstoff, Z. Phys. C 68, 121 (95); A. Bialas, H. Navelet and R. Peschanski, hep-ph 9711230, 9711442; G.P. Korchemsky hep-ph 9711277; M.Braun and G.P. Vacca, hep-ph 9711480.

7. V.Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys. Lett. B 326, 161 (94); A.L.Ayala, M.B. Gay Ducati and E.M. Levin, Nucl. Phys. B 511, 355 (98).

8. H1 coll., Nucl. Phys. B 470, 3 (96); ZEUS coll., Z. Phys. C 72, 399 (96).

9. N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B 332, 177 (94); M. Genovese, N.N. Nikolaev and B. G. Zakharov, JETP 81, 625 (95); J.Bartels, H.Lotter and M.Wüsthoff, Phys. Lett. B 379, 239 (96); E.Gotsman, E.M. Levin and U.Maor, Nucl. Phys. B 493, 239 (96); H. Lotter, Phys. Lett. B 406, 71 (96); E.M. Levin et al. Z. Phys. C 74, 671 (97).

10. M. Genovese, N.N. Nikolaev and B. G. Zakharov, Phys. Lett. B 378, 347 (96); Phys. Lett. B 380, 213 (96).

11. M. Genovese, M. Bertini, N.N. Nikolaev and B. G. Zakharov, to appear in proceedings of Lafex 98, Rio de Janeiro, hep-ph 9803423.

12. N.N. Nikolaev, proc. of DIS and QCD. 5th Int. Workshop. Chicago, USA, April 14-18, 1997. AIP conf. proc. no.407, Woodbury, New York, editors J.Repond and D.Krakauer; J. Bartels, same proceeding.

13. M. Genovese, N.N. Nikolaev and B.G. Zakharov, JETP 81, 525 (95).

14. H1 coll., Z. Phys. C 76, 613 (97).

15. ZEUS coll., Z. Phys. C 68, 569 (95).

16. N.N. Nikolaev and B.G. Zakharov, Z. Phys. C 49, 631 (94); V.Barone, et al Z. Phys. C 66, 157 (95).

17. R. Fiore et al, hep-ph 9801302.

18. N. N. Nikolaev, overview prepared for WGII at DIS98, http://web.iihe.ac.be/dis98/workinggroup2.html