Research Article
A Case Study Analysis of EEG Signals under Conditions of Cognition

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Abstract: Since Electro Encephalo Graphic (EEG) signal is considered chaotic, Nonlinear Dynamics and Deterministic Chaos theory may supply effective descriptors of the dynamics and underlying chaos in the brain. The EEG signals are highly subjective and the information about the various states may appear at random in the time scale. Therefore, EEG signal parameters, extracted and analyzed using computers, are highly useful. This study was undertaken to evaluate the linear and nonlinear parameters such as Approximate Entropy (ApEn), Correlation Dimension (D2), Pearson Autocorrelation, Bi-correlation, Hurst exponent and phase space plots from the EEG signals under different cognitive states.

Keywords: Cognition, Electroencephalograph (EEG), linear features (properties), non-linear features (properties)

INTRODUCTION

Many interesting phenomena in nature is due to the presence of nonlinearity. The theory of nonlinear dynamical systems, also called 'chaos theory', has now progressed to a stage, where it is possible to study self-organization and pattern formation in the complex neuronal networks of the brain. According to Stephen (1994), chaos theory is "the qualitative study of unstable aperiodic behavior in deterministic dynamical systems".

The brain is a highly complex and vital organ of a human body whose neurons interact with the local as well as the remote ones in a very complicated way. These interactions evolve as the spatio-temporal electro-magnetic field of the brain and are recorded as Electroencphalo Gram (EEG). It has been established that EEG recordings exhibit chaotic behavior from experiments such as the EEG models proposed by Skarda and Freeman (1987) and Wright and Liley (1996) for chaotic dynamics to meet requirements in neurobiology. EEG data are very important for many branches of the neurosciences and, many sophisticated experiments in cognitive science have shown that EEG and evoked potentials are strongly correlated with specific cognitive tasks. Many pathologic states have been examined as well, ranging from toxic states, seizures (Pijn et al., 1997) and psychiatric disorders to Alzheimer's (Jaeseung et al., 2001), Parkinson's and Creutzfeldt-Jakob's disease. Computer-assisted EEG signal analysis increased the desire for effective quantitative interpretation of EEG data and of describing properties of the EEG which often cannot be perceived by human eye. The aim of this study is to study the relation of the nonlinear characteristics of EEG signals with the intelligence activity of the human brain under various mental conditions (Molle et al., 1999; Wang et al., 2010).

Today's cognitive psychology (McCarthy and Warrington, 1990) differs from classical approaches in the methods they use as well as in the interdisciplinary connections to other sciences. Apart from rejecting introspection as a valid method to analyze mental processes, cognitive psychology introduces computer-based techniques that had not been in the range of methods used by classical psychology so far. The realization that there are important links between brain activity and cognitive functions is the key assumption for present and future research. Complete psychological accounts of cognitive functioning require considerations of the computational level, algorithmic level and the brain levels, about how the representation and the algorithm be realized physically. EEG has many advantages in measuring brain activity including the convenience and low cost. However, it is very difficult to estimate cognitive or mental state from EEG signals for a number of reasons. Nonlinear dynamics theory opens new window for understanding behavior of EEG (Korn and Faure, 2003; Ulbikas and Cenys, 1994). The literature on the study of the application of the nonlinear dynamics theory to analyze physiological signals shows that nonlinear approaches were used for analysis of renal blood flow, arterial pressure, EEG and respiratory signals, heart rate and nerve activity (Hoyer et al., 1997; Stein et al., 1992). Lindenberg (1996) points out that the nonlinear characteristic of the
physiological EEG signals greatly differs from that of the pathology; when clear-headed, the brain has higher chaotic degree, processes information more quickly and can make more responses.

Ferri et al. (2002) in this study the author introduced Phenomenological model for consciousness and selfhood which relates time, awareness, and emotion within one framework. The consciousness state space (CSS) model is a theoretical one. Pincus (2001) Cognitive-processing bias in chronic pain, a review and integration, it helps for patients with chronic pain selectively process pain- and illness-related stimuli. The evidence with regard to attention, interpretation, and recall biases is critically reviewed. Theiler (1987) the author dealt about two different signal properties and its nonlinear dynamics and self-organization of brain and behavior of EEG and MEG signals. Bruce J. West dealt about fractal physiology and chaos in medicine, in that mathematical modeling for physiological systems (West, 2013).

**MATERIALS AND METHODS**

In this study five cognitive tasks such as relaxation or baseline task, mental arithmetic multiplication task, 3d figure rotation task, letter composition task, visual counting task were administered to 4 healthy subjects with no history of neurological disorders from an age group of 22-45. Data were collected from 254 scalp, neck, face and eye locations using the Bio Semi Active Two system. The 10-20 system for EEG electrode placement in Fig. 1.

An experiment paradigm was designed for the study and the protocol was explained to all the participants before conducting the experiment. The subject was asked to comfortably lie down with eyes closed and the electrode cap was placed. The subject was instructed to remain as relaxed as possible without thinking anything for 2 min. After assuring the normal relaxed state by checking the alpha waves, the EEG was recorded for 50 sec, collecting five sessions of 10 sec epochs each for the relaxed state. This was used as the baseline reference for further analysis of other tasks. Then the subject was asked to perform a mental task on presentation of an audio cue. Five sessions of 10 sec epochs for each mental task were recorded, each with a time gap of 5 min. The whole experiment lasted for about one hour including electrode placement. The same procedure was repeated for all the subjects for several days. The EEG signal waveforms of a subject performing the cognitive tasks are shown in Fig. 2.
FEATURE EXTRACTION FOR LINEAR AND NONLINEAR ANALYSIS

Approximate entropy: Approximate entropy (ApEn) is defined as the “logarithmic likelihood that runs of patterns of data that are close to each other will remain close on next incremental comparisons”. ApEn is a statistical metric that quantifies the complexity or irregularity of signals both deterministic and stochastic (Flores Vega et al., 2013). It reflects the rate of new pattern generation and is thus related to the concept of entropy. This method was first proposed by Pincus (1991). One of the main advantages of ApEn is that it is very useful for short datasets that may be polluted by noise and interference because it is not sensitive to them. ApEn has two user-specified parameters: a run length m and tolerance window r small values of ApEn imply a greater likelihood that certain patterns of measurements will be followed by similar measurements. If the time-series is highly irregular, the occurrence of similar patterns in the future is less likely.

The algorithm is as follows:

1) Form a m vector \( X(1), ..., X(N - m + 1) \) define by \( X(i) = [x(i), x(i + 1), ..., x(i + m - 1)] \) where the original data are \{x(1), x(2), ..., x(N)\} and N is the total number of data points.

2) Set the distance between \( x(i) \) and \( x(j) \) by \( d[x(i), x(j)] \), define as the maximum absolute difference between their respective scalar elements, \( d[X(i), X(j)] = \max [u(i + k) - u(j + k)] \), for \( k = 0 \sim m - 1 \)

3) For \( x(i) \), find the number of \( j(j = 1, ..., N - m + 1) \) so that \( \Delta x = x(i) - x(j) \leq r \), denoted as \( Nm \). Then, for \( i = 1, ..., N - m + 1, C^m(i) = \frac{N^m(i)}{N - m + 1} \)

4) For each \( C^m(i) \) compute the natural algorithm and average it over \( i \):

\[
\theta^m(r) = \frac{1}{N - m + 1} \sum_{i=1}^{N-m+1} \ln C^m_i(i)
\]

5) Increase the dimension to \( m + 1 \), repeat steps 1 to 4 and find \( C^{m+1}(i), \theta^{m+1}(r) \)

6) The number of data point N is finite and the approximate entropy is defined when the data length is N and it is denoted as ApEn(m, r, N) = \( \theta^m(r) - \theta^{m+1}(r) \).

Although \( m \) and \( r \) are critical in determining the outcome of ApEn, no guidelines exist for optimizing their values. In principle, the accuracy and confidence of the entropy estimate improve as the number of matches of length \( m \) and \( m + 1 \) increases. The number of matches can be increased by choosing small \( m \) (short templates) and large \( r \) (wide tolerance). However, there are consequences for criteria that are too relaxed (Pincus, 1991). For smaller \( r \) values, one usually achieves poor conditional probability estimates, while for larger \( r \) values, too much detailed system information is lost. To avoid a significant contribution of noise in an ApEn calculation, one must choose \( r \) larger than most of the noise (Pincus, 1991). It is possible to automatically select the appropriate tolerance threshold value \( r \), which corresponds to the maximum ApEn value, without resorting to the calculation of ApEn for each of the threshold values selected in the range of zero and one times the standard deviation (Chon et al., 2009; Lu et al., 2008). Furthermore, as \( m \) decreases underlying physical processes that are not optimally apparent at smaller values of \( m \) may be obscured (Lake et al., 2002).

For this study, \( m \) is set to 2 and \( r \) is set to 15% of the standard deviation of each time series. These values are selected on the basis of previous studies indicating good statistical validity for ApEn within these variable ranges (Pincus and Goldberger, 1994).

**Correlation dimension**: Correlation dimension (D2) describes the dimensionality of the underlying process in relation to its geometrical reconstruction in phase space. It is used for detecting chaotic behavior in dynamical systems (Sprott and Rowlands, 2001). Since in principle D2 converges to finite values for deterministic systems and does not converge in the case of a random signal, D2 is a good parameter for evaluating the deterministic or noisy inherent nature of a system.

Grassberger and Procaccia (1983) algorithm (GPA) computes correlation dimension based on the following approximation: The probability that two points of the set are in the same cell of size \( r \) is approximately equal to the probability that two points of the set are separated by a distance \( \rho \) less than or equal to \( r \). Thus \( C(r) \) is approximately given by:

\[
C(r) \approx \frac{\sum_{i=1}^{N} H(r - \rho(x_i, x_j))}{\frac{1}{2}N(N-1)}
\]

where, \( N \) is the number of data points and the Heaviside function \( H \) is defined as:

\[
H(s) = \begin{cases} 
1 & \text{for } s > 0, \\
0 & \text{for } s \leq 0. 
\end{cases}
\]

The most common metric employed to measure the distance \( \rho \) is the Euclidean metric:

\[
\rho(x_i, x_j) = \sqrt{\sum_{k=1}^{m} (x_i(k) - x_j(k))^2}
\]

The choice of metric should not affect the scaling of the Correlation sum with \( r \).
Theiler (1986) made a correction to this method in order to avoid spurious temporal correlations. He proposed that the vectors to be compared when calculating the correlation integral, should be distanced at least W data points (\(|i-j|>W\)), where W is a measure of first minimum of mutual information i.e., delay T.

For small \(r\), \(C(r)\) behaves as according to a power law:

\[
C(r) \sim r^{-D_2}
\]

If number of data and embedding dimensions are sufficiently large we obtain:

\[
D_2 = \frac{d \log C(r)}{d \log r}
\]

Then Correlation Dimension (\(D_2\)) is given by the slope of the log-log plot of \(C(r)\) versus \(r\). Its numerical value describes the coherence of the underlying dynamics-the more coherent the system, the smaller the value of \(D_2\). The graph of \(C(r)\) versus \(\log r\) has a linear region called the scaling region. The GPA assumes that most of the information about the dimension is contained in the scaling region (Babloyantz et al., 1985).

The selection of an appropriate time lag and embedding dimension for phase space reconstruction is important (Liebert and Schuster, 1989). In this study, the delay time T is determined by Mutual Information (MI) method (Fraser and Swinney, 1986) (by finding the place where MI first attains a minimum) and the embedding dimension is estimated by the False Nearest Neighbors method proposed by Kennel et al. (1992). In this study, Correlation dimension of EEG is calculated while \(r\) is fixed to 4.

**Pearson autocorrelation:** Pearson Autocorrelation is a basic tool of linear analysis and statistical description (Box et al., 1994) that gives information on the correlations in time present in the signal. It is the standard autocorrelation for the given range of delays. The delay \(t\) can be an array of positive integers or a single integer.

In addition the Cumulative Pearson Autocorrelation is computed for the same range of delays. Also, if the autocorrelation for delays up to the maximum given delay crosses the \(1/e\) or zero level the delay of de-correlation or zero-autocorrelation, respectively, is assigned a value. The cumulative Pearson autocorrelation and the delay of de-correlation and zero autocorrelation can then be simply assigned to the respective measures if these are selected with the same set of delay values.

**Bicorrelation:** The bi-correlation, of three point autocorrelation (Kugiumtzis, 2001; Schreiber and Schmitz, 1997), or higher order correlation, is the joint moment of three variables formed from the time series and two delays \(t\) and \(s\). A simplified scenario for the delays is implemented, \(s = 2t\), so the bi-correlation is \(E[x(i), x(i+t), x(i+2t)]\), where the mean value is estimated by the sample average. In this way, the Bi-correlation is a function of a single delay \(t\). Bi-correlation is the extension of the standard Pearson autocorrelation to three variables and it is computed for the given range of the delay \(t\). The Bi-correlation is not a widely discussed measure, but the cumulative bi-correlation has been used as a statistic for the test of linearity (or non-linearity as it is best known in the dynamical systems approach of time series analysis), the so-called Hinich (1996).

Cumulative Bi-correlation is the cumulative function of the bi-correlation for the given range of delays. The Cumulative Bi-correlation for each delay \(t\) is the sum of the absolute values of the Bi-correlation up to the delay \(t\).

**Hurst exponent:** The Hurst exponent is a measure that has been widely used to evaluate the self-similarity and correlation properties of fractional Brownian noise, the time-series produced by a fractional (fractal) Gaussian process. It is used to evaluate the presence or absence of long range dependence and its degree in a time series. The Hurst exponent is the measure of the smoothness of a fractal time series based on the asymptotic behavior of the rescaled range of the process. However, non-stationaries are often present in physiological data and may compromise the ability of some methods to measure self-similarity. Hurst Exponent is used by Dangel et al. (1999) to characterize the non-stationary behavior of the sleep EEG episodes. The Hurst exponent \(H\) is defined as:

\[
H = \frac{\log \left( \frac{R}{S} \right)}{\log (T)}
\]

where, \(T\) is the duration of the sample of data and \(R/S\) the corresponding value of rescaled range. The above expression is obtained from the Hurst’s generalized equation of time series that is also valid for Brownian motion. A Hurst exponent, \(H\), between 0 to 0.5 is said to correspond to a mean reverting process (anti-persistent), \(H = 0.5\) corresponds to Geometric Brownian Motion (Random Walk), while \(H > 0.5\) corresponds to a process which is trending (persistent). \(H\) is related to the fractal dimension \(D\) given by: \(H = E+1-D\) (3) where \(E\) is the Euclidean dimension.

**Phase space plot:** Any chaotic system is described by strange attractors in the phase space. In this approach, a phase space plot is obtained with the \(Y\)-axis representing the EEG signal \(x(t)\) and the \(X\)-axis representing the EEG signal after a delay \(x(t+T)\). The choice of an appropriate delay \(T\) is calculated during the
minimal mutual information technique (Fraser and Swinney, 1986).

If \( \tau \) is undersize, the track of the phase space will approach to a straight line; on the contrary if \( \tau \) is oversize, the data point will centralize in a small range of the phase space and we can’t get the attractors’ local structures from the reconstructed phase graph. It has been observed that the patterns are unique to the various mental states.

This kind of analysis gives first time evidence that though the EEG time series look similar in all the channels, different dynamics may be occurring in different areas of the brain.

RESULTS AND DISCUSSION

In this study, the Nonlinear and linear measures such as Approximate Entropy, Correlation Dimension, Pearson Autocorrelation, Bi-correlation, Hurst exponent and phase space plots are applied to the EEG signals of 4 subjects performing cognitive tasks like Resting or Relaxation (T1), Figure Rotation (T2), Letter Composition (T3), Arithmetic (T4) and Visual Counting (T5), respectively. The tabular and graphical representations of the results are given as follows:

\( \text{ApEn} \) is the measure of dynamic changes of the EEG signal in time domain. A decrease in entropy indicates high predictability and a reduced stochastic behavior. A time series containing many repetitive patterns has a relatively small \( \text{ApEn} \), a less predictable process has a higher \( \text{ApEn} \). Theoretically, a perfectly repeatable time series has an \( \text{ApEn} \sim 0 \) and a perfectly random time series has an \( \text{ApEn} \sim 2 \) (Yentes et al., 2012).

From Table 1 and Fig. 3, we can see that performing activities such as mental arithmetic (T4) and visualizing numbers (T5) which require less consciousness have a weaker ability to form a new pattern and hence the time series is less predictable and more complex.

This also corresponds to practice. Since arithmetic computation is according to a fixed rule, its ability to form a new pattern in the future is naturally less.

Correlation dimension is a sensitive parameter in the analysis of electrical brain activity (Lamberts et al., 2000). The technique can be used to distinguish between (deterministic) chaotic and truly random behavior. It quantifies the variability in a time series.

As can be inferred from Table 2 and Fig. 4, there are slight differences in the correlation dimension of the subjects between the tasks. Nevertheless the table indicates that the letter composition activity (T3) is consistently higher despite the relatively small changes.

![Fig. 3: Graphical representation of approximate entropy for various cognitive tasks performed by subjects](image)

![Fig. 4: Graphical representation of correlation dimension for various cognitive tasks performed by subjects](image)

| Table 1: Result of approximate entropy for various cognitive tasks performed by subjects |
|-----------------------------------|------------------|------------------|------------------|------------------|------------------|
| T1  | 0.215 | 0.201 | 0.201 | 0.201 | 0.319 |
| T2  | 0.482 | 0.451 | 0.451 | 0.451 | 0.451 |
| T3  | 0.083 | 0.077 | 0.231 | 0.077 | 0.077 |
| T4  | 0.25  | 0.231 | 0.231 | 0.231 | 0.231 |
| T5  | 0.73  | 0.758 | 0.781 | 0.758 | 0.718 |

| Table 2: Result of correlation dimension for various cognitive tasks performed by subjects |
|-----------------------------------|------------------|------------------|------------------|------------------|------------------|
| T1  | 0.73  | 0.758 | 0.781 | 0.758 | 0.718 |
| T2  | 0.736 | 0.719 | 0.777 | 0.752 | 0.725 |
| T3  | 0.789 | 0.769 | 0.982 | 0.769 | 0.774 |
| T4  | 0.785 | 0.717 | 1.100 | 0.718 | 0.936 |
| T5  | 0.73  | 0.758 | 0.781 | 0.758 | 0.718 |

| Table 3: Result of bi-correlation for various cognitive tasks performed by subjects at t=1 |
|-----------------------------------|------------------|------------------|------------------|------------------|------------------|
| T1  | -0.0195 | 0.0025 | 0.034 | 0.0541 | 0.178 |
| T2  | -0.0700 | -0.327 | -0.134 | -0.145 | 0.087 |
| T3  | -0.0030 | -0.089 | -0.018 | -0.079 | 0.019 |
| T4  | -0.1120 | -0.199 | -0.073 | -0.135 | 0.047 |
The correlation dimension of most mental activities appeared to be higher compared to the relaxation condition. Hence it is concluded that cognitive and mental activity is associated with a higher correlation dimension in the EEG.

Bi correlation is a nonlinear third-order correlation measure. It is used as a statistic for the test of linearity or nonlinearity called the Hinich test in the time domain.

There is no predefined range of expected values for bi-correlation and cumulative bi-correlation, other than [-1, 1]. If the EEG time series shows a higher value of bi-correlation it is said to be linear. A decrease in the value of bi-correlation corresponds to the EEG time series being nonlinear in nature. From Table 3 and Fig. 5, the activity of imagining an object rotating (T2) has negative correlation and visual counting activity (T5) has positive correlation thus showing that T2 is more complex and random and T5 is more predictable.

Pearson autocorrelation describes the correlation between values of the time series at different times, as a function of the two times or of the time lag. It gives a value between [-1, 1] with 1 indicating perfect correlation, 0 is no correlation and -1 indicating negative correlation.

From Table 4 and Fig. 6, it is seen that visual counting activity (T5) has the most positive correlation and the task of figure rotation (T2) has the least positive correlation.

If H = 0.5, the behavior of the time-series is similar to a random walk. If H <0.5, the time-series cover less “distance” than a random walk (i.e., if the time series increases, it is more probable that then it will decrease). But if H >0.5, the time-series covers more “distance” than a random walk (if the time-series increases, it is more probable that it will continue to increase). The Hurst exponent is limited to a value between 0 and 1, as it corresponds to a fractal dimension.

The self-similarity parameter, H has a value higher than 0.5 for all the states compared to the resting condition. This means that randomness increases if the subject is performing the tasks. This value is maximum in object rotation task (T2) from which it gradually decreases in the other states indicating higher self-similarity. Higher the Hurst exponent, the process is said to be smoother and vice versa. You could calculate an H>1, but it would not have any meaning using the accepted definition and fractal dimension boundaries (fractions between integer dimensions must always be less than one) (Table 5 and Fig. 7).

We construct the EEG attractors of all five kinds of mental activities of 4 subjects and find that EEG attractors of various patterns have similar characteristics. As can be seen from Fig. 8, the attractors' track often rotates in an extremely complex way but there is still internal structure when the attractors is magnified. The attractors of resting, composition of a letter and visualizing a 3-dimensional object being rotating about an axis often distribute in a small ellipse region, while the point in the attractors of mental arithmetic and visualizing numbers being written centralize nearby the 45 degree line and there is a large distributing range along the 45 degree line. This is because during rational computation such as mathematics or imagination, the value of the adjacent sampling points of EEG signals are close and the amplitude values of the whole EEG signals are great.
DISCUSSION

In this study, we use the approximate entropy, the correlation dimension, bi-correlation, Pearson autocorrelation Hurst exponent and phase space methods to study the EEG signal of 5 kinds of cognitive activities of 4 subjects. Although in every method there are merits and flaws, the results show the nonlinear dynamic characteristics of the human brain.

The presence of repetitive patterns of fluctuation in a time series renders it more predictable than a time series in which such patterns are absent. The analysis of the approximate entropy presents the degree of various mental activities on generating new pattern (Zhou et al., 2005). The authors think that larger the approximate entropy of the subject at the same state, the more innovational he has.

The correlation dimension shows the variability of different consciousness states as well (Alexey, 2008), which can better indicate the activity degree of the human mind, combining with the approximate entropy. It is well known that the dimension of EEG time series is closely related to the cognitive activity of the brain. The correlation dimension increases with the degree of the cognitive activity (Natarajan et al., 2004). Our results show that dimension is higher during the letter...
writing activity indicating that the brain is involved in thinking rigorously, meaning that the brain is in an active cognitive state. A decreased value indicates that the randomness of the brain activity is reduced.

Both bi-correlation and Pearson autocorrelation are measures of testing linear or nonlinear correlation in the time series. From the results the authors think that a higher value corresponds to linear nature and a decrease in value to nonlinear nature of the EEG time series of various mental states. The self-similarity parameter is higher when the subject is performing tasks which require greater consciousness than the rest owing to a smoother time series without noise. The phase space plots show unique patterns for each consciousness state useful in computing correlation dimension.

The above analyses indicate that different cognitive activities have profound nonlinear dynamic characteristics. Some differences are difficult to perceive and the linear and nonlinear quantitative parameters of different individuals have great differences. Hence it is a critical problem to find a widely applicable criterion, which needs to be explored for a long time.

CONCLUSION AND FUTURE SCOPE

The various task data are analyzed effectively by considering the linear and nonlinear parameters. For certain analysis linear hypothesis is replaced by nonlinear behavior. Higher order statistics are more useful for chaotic as well as different cognitive state measurement which was not possible by linear methods such as spectral methods and other conventional methods like parametric methods. By taking suitable parameters the analysis of human mind can be applied to ADHD and dementia and other cognitive disorders in the brain.

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