Asymptotic optimal location of facilities in a competition between population and industries

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Abstract. We consider the problem of optimally locating a given number $k$ of points in $\mathbb{R}^n$ for an integral cost function which takes into account two measures $\varphi^+$ and $\varphi^-$. The points represent for example new industrial facilities that have to be located, the measure $\varphi^+$ representing in this case already existing industries that want to be close to the new ones, and $\varphi^-$ representing private citizens who want to stay far away. The asymptotic analysis as $k \to \infty$ is performed, providing the asymptotic density of optimal locations.

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1. Introduction

A typical problem in facility location can be mathematically described through the choice of a given number of points in a domain so as to minimize an “average distance” criterion, the average being computed with respect to a measure $\varphi$. More precisely, for every subset $\Sigma \subset \mathbb{R}^n$ define

$$F(\Sigma) := \int_{\mathbb{R}^n} \text{dist}(x, \Sigma) \, d\varphi(x),$$

where $\text{dist}(x, \Sigma) := \inf_{y \in \Sigma} d(x, y)$ is the distance between $x$ and $\Sigma$. In this paper we study the following problem.

Problem 1.1. Find a $\Sigma = \Sigma_{opt} \subset \mathbb{R}^n$ minimizing the functional $F$ among all sets $\Sigma \subset \mathbb{R}^n$ satisfying $\# \Sigma \leq k$. In other words, denoting by $\mathcal{A}_k$ the set of admissible $\Sigma$, i.e.

$$\mathcal{A}_k := \{ \Sigma \subset \mathbb{R}^n : \# \Sigma \leq k \},$$

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