Is the $\nu_\mu \rightarrow \nu_s$ oscillation solution to the atmospheric neutrino anomaly excluded by the SuperKamiokande data?

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Abstract
Recently the SuperKamiokande collaboration have claimed that their data exclude the $\nu_\mu \rightarrow \nu_s$ solution to the atmospheric neutrino anomaly at more than 99% C.L. We critically examine this claim.
Something mysterious with neutrinos is a foot. It is clear that about half of the upward going atmospheric $\nu_\mu$'s are missing [1,2]. Furthermore, about half of the solar $\nu_e$'s have also disappeared [3]. There is also strong evidence that $\nu_e \leftrightarrow \nu_\mu$ oscillations take place with small mixing angles from the LSND experiment [4]. An elegant explanation of these facts is that each neutrino oscillates maximally with an approximately sterile partner, with small angles between generations [5]. For the status of the maximal $\nu_e \rightarrow \nu_s$ solution to the solar neutrino problem, see Ref. [6]. The status of the maximal $\nu_\mu \rightarrow \nu_s$ solution to the atmospheric neutrino problem is the subject of this paper.

As was pointed out sometime ago [7], both $\nu_\mu \rightarrow \nu_s$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations are able to explain the sub-GeV and multi-GeV SuperKamiokande single ring events (while 2 flavour $\nu_\mu \rightarrow \nu_e$ oscillations cannot because there is no observed anomaly with the electron events [8]). Recently, however, the SuperKamiokande Collaboration have argued that the $\nu_\mu \rightarrow \nu_s$ oscillation explanation of the observed deficit of atmospheric neutrinos is disfavoured at more than 99% C.L. [9], while the interpretation in terms of $\nu_\mu \rightarrow \nu_\tau$ oscillations fits all of their data extremely well. This conclusion relies on an analysis of the upward through going muon data (UTM), the partially contained events with $E_{\text{visible}} > 5 \text{ GeV}$ (PC) and the neutral current enriched multi-ring events (NC). These data sets lead to slightly different expectations for the $\nu_\mu \rightarrow \nu_\tau$ vs $\nu_\mu \rightarrow \nu_s$ oscillations because of earth matter effects for $\nu_\mu \rightarrow \nu_s$ oscillations which are important for UTM [10] and PC events [11,7] while neutral current interactions in the detector are utilized for the NC events [12]. These three data sets, obtained from Ref. [9] (for 1100 live days), are shown in Figures 1a,b,c. Also shown is the theoretically expected result for maximal $\nu_\mu \rightarrow \nu_s$ oscillations with $\delta m^2 = 3 \times 10^{-3} \text{ eV}^2$ also obtained from Ref. [9]. SuperKamiokande analyse the data by taking particular ratios and have not as yet provided detailed justification of the systematic uncertainties in the theoretically expected rates.

Let us discuss each of the three data sets in turn:

a) Upward through going muons: The overall normalization of the through going muon fluxes have an estimated 20% uncertainty, however the uncertainty in the expected shape of the zenith angle distribution is significantly lower (for some discussion of these uncertainties, see Ref. [13,14]). A recent estimate [13] of the uncertainty in the vertical/horizontal ratio due to the uncertainties in the atmospheric fluxes is of order 4%. This systematic uncertainty is dominated by the uncertainty in the ratio $K/\pi$ produced in the atmosphere from the interactions of cosmic rays [13]. In addition there will be other uncertainties in the shape of the zenith angle distribution due to the uncertainty in the energy dependence of the neutrino - nucleon cross section and from cosmic ray muons masquerading as neutrino induced muons. The latter uncertainty, while mainly affecting the most horizontal bin ($-0.1 < \cos \Theta < 0$) may be very important, as we will show.

b) Partially contained events (with $E_{\text{visible}} > 5 \text{ GeV}$). The systematic uncertainty in the

† Uncertainties in the energy dependence of the cross section leads to uncertainties in the expected shape of the zenith angle distribution of UTM events, because the zenith angle dependence of the atmospheric neutrino flux is energy dependent.
expected normalization of these events is quite large, again of order 20% [13,14]. The systematic uncertainty on the expected shape of the zenith angle distribution of these events should be relatively small (\( \lesssim \) few\% in the up/down ratio).

c) Neutral current enriched multi-ring events. The systematic uncertainty in the expected normalization is again quite large, of order 20-40\% due to the highly uncertain cross sections (as well as the uncertain atmospheric fluxes). The uncertainty in the expected shape of the zenith angle distribution will of course be much smaller, but may be significant (i.e. of order 5\% in the up/down ratio). The uncertainty is due in part to the uncertainty in the relative contributions due to \( \nu_e \) interactions (which are expected to be approximately up/down symmetric) and \( \nu_\mu \) interactions (which are up/down asymmetric due to the oscillations affecting the upward going \( \nu_\mu \)'s). The relative contributions due to the neutral current weak interactions and the charged current weak interactions will also be uncertain. In addition to the cross section uncertainties there are also the uncertainties in the scattering angle distribution between the angles of the multi-ring events and the incident neutrino.

Note that the normalization uncertainties between the three data sets will be largely uncorrelated because of the different energy ranges for the atmospheric neutrino fluxes and also because of the different cross sections involved. Nevertheless, some weak correlation between UTM and PC events may be expected. While analysing the data using ratio’s does eliminate the normalization uncertainty, the remaining uncertainties will be important for the data sets a) and c). Furthermore, a conclusion based on particular ratios could only be robust if it agreed with a \( \chi^2 \) fit of the binned data points. SuperKamiokande are in the best position to do this for their data, and we hope that they will do this at some point.

In the meantime we will do this using the superKamiokande theoretical Monte-Carlo results for their given test point of maximal mixing with \( \delta m^2 = 3 \times 10^{-3} \text{ eV}^2 \) (which is not expected to be the best fit for \( \nu_\mu \rightarrow \nu_s \) oscillations). We define the \( \chi^2 \) by:

\[
\chi^2_{\text{total}} = \chi^2_{\text{UTM}} + \chi^2_{\text{PC}} + \chi^2_{\text{NC}},
\]

(1)

with

\[
\chi^2_y = \sum_{i=1}^{10} \left( \frac{\text{data}_y(i) - f_y \times \text{theory}_y(i)}{\delta \text{data}_y(i)} \right)^2 + \left( \frac{f_y - 1}{\delta f} \right)^2,
\]

(2)

where \( y = \text{UTM}, \text{PC}, \text{NC} \) and the sum runs over the 10 zenith angle bins, and \( \delta \text{data}_y \) is the statistical uncertainty in the data, \( \text{data}_y(i) \). The normalization factor, \( f_y \) parameterizes the overall normalization uncertainty in the theoretical expected value, \( \text{theory}_y(i) \), and \( \delta f_y \) is the expected normalization uncertainty, and we take \( \delta f_y = 0.2 \) for \( y = \text{UTM}, \text{PC}, \text{NC} \). It is understood that \( \chi^2_y \) is minimized with respect to \( f_y \).

Doing this exercise (using the superKamiokande experimental data, \( \text{data}_y(i) \), \( \delta \text{data}_y(i) \) and also the superKamiokande theoretically expected results \( \text{theory}_y(i) \) for maximal mixing with \( \delta m^2 = 3 \times 10^{-3} \text{ eV}^2 \)), we find that \( \chi^2_y \) is minimized when \( f_{\text{UTM}} \simeq 0.90 \), \( f_{\text{PC}} \simeq 0.87 \) and \( f_{\text{NC}} \simeq 1.07 \). In Figures 2a,b,c we compare the data with \( f_y \text{theory}_y(i) \), which is the theoretical prediction for \( \delta m^2 = 3 \times 10^{-3} \text{ eV}^2 \) (neglecting systematic uncertainties in the shape). We obtain the following \( \chi^2_y \) values:

\[\chi^2_{\text{UTM}} = 17.0 \text{ for 10 degrees of freedom},\]
\[ \chi_{PC}^2 = 13.4 \text{ for 10 degrees of freedom}, \]
\[ \chi_{NC}^2 = 16.0 \text{ for 10 degrees of freedom}. \]  

(3)

Thus we obtain \( \chi_{\text{total}}^2 \simeq 46 \) for 30 degrees of freedom which corresponds to an allowed C. L. of about 3%. While this allowed C.L. is low, it is only a lower limit because we haven’t varied \( \delta m^2 \) or incorporated the systematic uncertainties in the shape of \( \text{theory}_y(i) \), which we now discuss.

Varying \( \delta m^2 \) within the allowed region identified from a fit to the contained events should improve \( \chi_{PC}^2 \) somewhat as well as slightly improving \( \chi_{\text{UTM,NC}}^2 \). For example, for \( \delta m^2 = 5 \times 10^{-3} \text{ eV}^2 \) using our code developed in Ref. [7] we find that \( \chi_{PC}^2 \simeq 12 \) (c.f. 13.4 for \( \delta m^2 = 3 \times 10^{-3} \text{ eV}^2 \)).

With regard to the UTM and NC events the effect of systematic uncertainties on \( \chi^2 \) can be very dramatic. We illustrate this by introducing a slope factor \( s(i) \) defined by

\[ s(i) = 0.95 + 0.01 \times i, \]

(4)

where \( i = 1, \ldots, 10 \) (with \( i = 1 \) the vertical upward going bin). In Eq.(2) we replace \( f_{\text{UTM}}(i) \to s(i) \times f_{\text{UTM}}(i) \), which is roughly within the estimated 1-sigma systematic uncertainty for the UTM events. In fact, this would be roughly equivalent to reducing the atmospheric \( K/\pi \) ratio by about 30 – 40% to be compared with the estimated 25% uncertainty for the \( K/\pi \) ratio [13]. While the uncertainty in the \( K/\pi \) ratio may be the largest single contribution to the uncertainty in the shape of the zenith angle distribution of UTM events, the total systematic uncertainty in the shape of the zenith angle distribution gets many contributions [1] which is why the slope factor in Eq.(4) might be expected to be roughly within the 1-sigma systematic uncertainty. With the above slope factor, we find \( \chi_{\text{UTM}}^2 \simeq 13 \), which represents a significantly improved fit. From our earlier discussion, the systematic uncertainties in the shape of the zenith angle distribution of the events for UTM and NC events are expected to be completely uncorrelated. This means that the best fit for the NC events can have a slope factor with a slope of a different sign, and this is needed to improve the fit. To illustrate the effect then, for NC we replace \( f_{\text{NC}}(i) \to f_{\text{NC}}(i)/s(i) \) and find \( \chi_{\text{NC}}^2 \simeq 12 \). This demonstrates that a \( \chi^2 \) fit to the three data sets incorporating the systematic uncertainties and varying \( \delta m^2 \) would be expected to reduce \( \chi_{\text{total}}^2 \) by at least 9 leading to a \( \chi_{\text{total}}^2 \) of about 37 or less. This corresponds to an allowed C.L. of 15% or more. Of course a global fit of all the superKamiokande data gives a much larger allowed C.L. because of the excellent fit of the \( \nu_\mu \to \nu_\tau \) oscillations to the lower energy contained events (both sub-GeV and multi-GeV) [13]. The results obtained for UTM and NC events using the slope factor \( s(i) \) are given by the dotted lines in Figure 2a,c.

We would also like to emphasise that the poor \( \chi^2 \) fit for UTM events is due largely to the most horizontal bin \((-0.1 < \cos \Theta < 0)\). Excluding this bin we find that

\[ \chi_{\text{UTM}}^2 = 12.5 \text{ for 9 bins} \]

(5)

\[ ^\ddagger \text{Due to e.g uncertainty in the energy dependence of the neutrino nucleon cross section, uncertainty in the interaction length of the cosmic rays in the upper atmosphere, modelling of the atmosphere, uncertainty in the primary cosmic ray energy spectrum and composition of cosmic rays etc.} \]
excluding any systematic uncertainty in the shape of the zenith angle distribution (i.e. with $s(i) = 1$). The reason for questioning the horizontal bin is clear: It is expected that the systematic uncertainty for the most horizontal bin should be relatively large. This is because atmospheric muons can contribute. (In fact the Kamiokande collaboration \cite{16} made the cut $\cos \Theta < -0.04$ and incorporated large systematic errors for this bin). SuperKamiokande, in their published analysis of 537 days \cite{17} included the whole horizontal bin, and made an estimate of the contamination of atmospheric muons in this bin (of order 4%) and subtracted it off. This is based on an extrapolation from $\cos \Theta > 0$ where the background falls off exponentially. This exponential assumption is not discussed in any detail, and needs to be justified if it can be. In fact, from their Figure 1 \cite{17}, which compares the distribution of through-going muons near the horizon observed by SuperKamiokande for regions with thick and thin rock overburden, it seems possible that the atmospheric muon background could be higher by a factor of two or three or even more. This is rather important. For example, if a background of 10% is assumed (which means that we must lower the superKamiokande data value by 6% for this bin), then we obtain a $\chi^2_{UTM} \simeq 14$ for 10 degrees of freedom (excluding the effects of the systematic uncertainties in the shape of the zenith angle distribution, i.e. $s(i) = 1$) or $\chi^2_{UTM} \simeq 11$ including the modest slope factor in Eq.\textup{(4)}. Unless the level of contamination of atmospheric muons in the horizontal bin can be rigorously justified, it is probably safest to exclude the horizontal bin altogether because the systematic uncertainties may be so large as to make it too uncertain to be useful\textsuperscript{5}.

Thus, we have shown that a $\chi^2$ analysis of the recent upward through going muon binned data, partially contained events with $E_{\text{visible}} > 5 \text{ GeV}$ and neutral current enriched multi-ring events does not exclude maximal $\nu_\mu \rightarrow \nu_s$ oscillation solution to the atmospheric neutrino problem with any significant confidence level. This is not in conflict with the SuperKamiokande results since they fit three particular ratio’s rather than the binned data. However it does show that the conclusion that the $\nu_\mu \rightarrow \nu_s$ oscillations are disfavoured does depend on how one analyses the data. Furthermore, the overall fit (i.e. including also the lower energy single ring events) of the $\nu_\mu \rightarrow \nu_s$ oscillations to the SuperKamiokande data is good. Fortunately future data will eventually decide the issue. In the meantime, important work needs to be done on carefully estimating and checking the possible systematic uncertainties.

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Figure Captions

Figure 1: SuperKamiokande data for the upward through going muons (Fig.1a), partially

\textsuperscript{5} In terms of analysis with ratio’s we suggest that the vertical be defined as $-1 < \cos \Theta < -0.5$ and the horizontal as $-0.5 < \cos \Theta < -0.9$. 

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contained events with $E_{\text{visible}} > 5 \text{ GeV}$ (Fig.1b) and neutral current enriched multi-ring events (Fig.1c), all obtained from Ref. [9]. Also shown are the superKamiokande expected results for maximal $\nu_\mu \rightarrow \nu_s$ oscillations with $\delta m^2 = 3 \times 10^{-3} \text{ eV}^2$, also obtained from Ref. [9].

Figure 2: Same as Figure 1 except that the theoretical expectation for maximal $\nu_\mu \rightarrow \nu_s$ oscillations with $\delta m^2 = 3 \times 10^{-3} \text{ eV}^2$, are renormalized by an overall scale factor (as discussed in the text). In Figures 2a and 2c, the dotted line includes the effect of a modest correction to the expected shape of the zenith angle distribution given by Eq.(4), as discussed in the text.
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