Optimization of the connecting rod of a two-stroke engine using finite element analysis

L B Valero-Páez¹, J L Chacón-Velasco¹, and O A González-Estrada²
¹ Grupo de Investigación en Energía y Medio Ambiente, Universidad Industrial de Santander, Bucaramanga, Colombia
² Grupo de Investigaciones en Corrosión, Universidad Industrial de Santander, Bucaramanga, Colombia

E-mail: agonzale@uis.edu.co

Abstract. In this work, the mechanical behavior of a connecting rod of a two-stroke single-cylinder engine is studied by means of finite element analysis to reduce its weight. We investigated the static and dynamic stresses on the parts of the connecting rod, considering the boundary conditions of the Jovaj theory for the design of automotive conrods. Then, the maximum stresses of the models were compared to perform the optimization of the weight of the connecting rod, evaluating the sensitivity of the prototype, and geometry restrictions. For the study using finite element analysis, a connecting rod prototype designed at Universidad Industrial de Santander was used, within the project of reengineering the connecting rod of a two-stroke engine of small displacement for agricultural applications. The mechanical response of the designed conrod was evaluated, taking into account the mechanical stresses to which it is subjected during the operation of the engine. Finally, the design was revised to propose preliminary improvements in the geometry using topology optimization.

1. Introduction
The internal combustion engine (ICE) is the prime mover for industrial applications and research to improve its performance is widely available [1–4]. In recent years, the study of the automotive cranks has been of great importance for the automotive industry, because they are responsible for approximately 24.1% of the failures in the engine system [5]. In addition, this mechanical element allows improving the performance of the engine with the reduction of its weight. In the mechanical design of connecting rods, analytical calculations and numerical methods are used to evaluate the stresses, displacements, and contact pressures, followed by geometry optimization to improve engine performance, considering the power/weight ratio [6].

Nowadays, the paradigm of design has changed thanks to the use of numerical methods such as, for example, the finite element method, which allow complex materials, geometries and load conditions to be considered [7–10]. Webster et al. [11] performed the finite element analysis (FEA) in three dimensions of a high-speed diesel engine connecting rod. For this analysis, they used the maximum compression force that was measured experimentally, and the maximum tensile load that is essentially the inertial load of the piston mass joint. They determined the regions of the rod with high values of tension and deformation at different load conditions. According to Balasubramanian [12], the 2D finite element (FE) models can be used to obtain fast trend values and the 3D FE models for a more accurate investigation of the mechanical behavior. In [12], the different individual loads that act on the connecting
rod were used for the modeling, and the real stress distribution was obtained by superposition. The loads included inertial load, compression load, pressure adjustment of the bearing bush and bolt forces.

Shamim [13] studied the connecting rod used in four-stroke single-cylinder gasoline engines using finite elements. The static structural analysis was carried out by fixing the end of the piston and applying a load to the crankshaft end of the connecting rod. The output parameters in the static stress analysis were von Mises stresses, shear stress, total strain, and equivalent elastic deformation.

Pathade et al. [14] performed the rod stress analysis using the finite element method, obtaining as a result that the maximum stresses developed in the inner section of the two ends of the connecting rod. The comparison and verification of the results obtained by FEA were carried out experimentally using photoelasticity. Yongqi et al. [15] analyzed the static stress distribution, the safety factor and the fatigue of the connecting rod using 3D FEA. The results showed that the most critical location is the transition area between the foot and the body in the condition of maximum compression. In the traction condition, the most critical region is at the junction of the head and the I cross section. Kumar et al. [16] used a connecting rod with a cross-section in I to perform a parametric model and FEA analysis. The objective to know the maximum stresses generated on the steel rod and optimize its weight, with a reduction of 11.23%.

Pranav et al. [17] carried out the FEA and connecting rod optimization using ANSYS Workbench. The study consisted of two types of analysis: static and fatigue. The objective was to explore the reduction of weight for the production of forged steel connecting rods. The weight reduction was achieved by static analysis, obtaining a 9.24% reduction in weight. Recently, Bittencourt et al. [18] applied an alternative methodology based on topology optimization to design a connecting rod reducing its mass. The topology optimization generated a conrod 3% lighter and with better lubrication performance compared to the design obtained from the conventional design methodology.

In this work, the analysis of the mechanical behavior of a prototype connecting rod of a two-stroke engine is carried out by means of the finite element method, considering boundary conditions obtained from the operation of the engine and conventional design theories. First, there is a general description of the connecting rod and the two-stroke engine. Theories for load distribution on the connecting rod are presented for static and dynamic analysis. In addition, the elastic problem formulation and topological optimization are presented by the finite element method. Then, the connecting rod of the case study is presented, and the finite element model to be used is defined, the boundary conditions are specified, and the analysis of the stress distributions for the static and dynamic results is performed. Subsequently, the process of topological optimization and the new model of the connecting rod is detailed. Finally, the conclusions obtained from the study of the different analyzed models are summarized.

2. Materials and methods
The School of Mechanical Engineering at Universidad Industrial de Santander is reengineering the parts of a small displacement two-stroke engine, taking the Shindaiwa B450 engine as a reference. In the power system, we find the crank-connecting-sliding mechanism, of these three mechanical elements, the connecting rod has the function of transferring the rotating and alternative forces between the piston and the crankshaft. The connecting rod has a complex movement of rotation in the head and reciprocating in the foot. Because of this, the connecting rod is subjected to a compressive load due to the gas forces exerted inside the cylinder and to a tensile load due to inertial forces, caused by the accelerated and decelerated movement of the connecting rod masses and the piston. Additionally, the effects of the pressing fit of the bushing are present, which can be considered by superposition. The lateral deflection in the plane of oscillation of the connecting rod generates centrifugal forces that lead to flexion, however, these can be neglected [19].

To obtain the stresses on the foot and the head, experimental tests were carried out in [2] and compared with calculated stresses, considering a connecting rod with I-section. The formulation of the beam of small curvature was taken into account for the calculations. We can find the stresses of the
external, \( \sigma_e \), and internal surfaces, \( \sigma_i \), either for traction or compression, in the top area of the ends, i.e., in the region that comprises between \( \alpha' \) and \( \alpha_a \), as shown in Equation (1):

\[
\sigma_e = 2M \left( \frac{6r_m + h}{h(2r_m + h)} \right) + kN \left( \frac{1}{\text{th}} \right), \quad \sigma_i = -2M \left( \frac{6r_m - h}{h(2r_m - h)} \right) kN \left( \frac{1}{\text{th}} \right),
\]

with

\[
h = \frac{D_p - d}{2}, \quad k = \frac{E_b A_s}{E_b A_s + E_c A_c},
\]

where \( M \) and \( N \) are the moment and normal force from \( \alpha'=0 \) to \( \alpha_a \), \( k \) is the stiffness ratio between the foot/head and the bushing, \( h \) is the wall thickness of the, \( E_b \) and \( E_c \) are the elastic moduli of the conrod and the bushing, \( A_s \) is the area of the foot/head, and \( A_c \) is the area of the bushing.

To calculate the forces on the ends of the connecting rod, the masses that produce the inertia must be calculated, caused by the complex movement of the connecting rod that has rotary movement in the crankshaft journal and reciprocating in the piston pin. For this purpose, the method of equivalent masses is used, where the rod is modeled as two concentrated point masses, one on the pin of the crank that is assumed to be in pure rotation and another on the piston pin that is assumed in pure translation [20].

Figure 1 shows the values of the forces acting on the foot of the connecting rod, considered in the dynamic analysis: the axial force \( F_{ST} \), the force resulting from the combustion and inertial forces \( F_K \), the total force on the head of the connecting rod \( F_{HZ} \). Figure 2 shows the forces on the head, \( F_R \) is the radial force imposed on the crankshaft, \( F_T \) is the force that produces the moment of the crankshaft, and \( F_{HZ} \) is the total force on the head of the connecting rod depending on the axial force of the connecting rod and rotary inertial forces.

3. Results

3.1. Connecting rod model

To study the forces acting on the connecting rod, it is necessary to know the values of the pressure of the gases and of the masses. The thermodynamic analysis of the engine and the theory of equivalent masses are carried out considering the pressure and temperature values of the processes of intake, compression, and combustion of the thermodynamic cycle of the engine used, as well as the value of the masses that cause the inertia of the connecting rod. The maximum pressure of the gases is 6.11 MPa,
equivalent to a force of 7678 N, the alternative masses cause a force of inertia of 1910 N on the foot, and the rotating masses on the head cause a force of 2754 N.

To avoid wear of the internal diameters at the ends of the connecting rod, bearings such as needles rollers or bronze bushes are used, which function as the connection between the stump and the head, and the piston pin and the foot. The connecting rods caps are normally embedded with thermal processes which produce a residual pressure on the connecting rod, which corresponds in the case of study to approximately 12 MPa for the two ends. To model the press fit adjustment, a non-linear contact condition with friction was defined, with a coefficient of friction of 0.18 between the materials, and a radial interference of 3E-6 m.

For the mechanical analysis of the rod, a comparison of the stress behavior of the numerical model with a conventional I-section connecting rod, and with the analytical results of Jovaj’s theory was made. We verify that the results of the numerical model with boundary conditions of the indicated theory, comply with the mechanical behavior on the outer and inner surfaces of the ends of the rod, see Figure 3 and Figure 4. The difference of the values of stress is due to the correction factor $k$ of the equations, which takes into account the area and the stiffness that exists between the bushing and the ends.

Figure 3. Internal stresses in the foot under tensile load for the connecting rod with I section.  
Figure 4. External stresses in the foot under tensile load for the connecting rod with I section.

Figure 5 shows the prototype of the connecting rod used for the study. The rod was manufactured by water jet cutting, with a 10 mm thick HARDOX-450 ($S_y = 1200$ MPa) wear-resistant steel sheet, in addition to a heat treatment of cementation, finishing with electrodeposition and, finally, a rectification of the internal diameters of the foot and the head. The properties of the connecting rod and the characteristics of the two-stroke engine used for static and dynamic calculations are: mass 0.0316 Kg, density 7850 Kgm$^{-3}$, and volume 4.03E-06 m$^3$.

3.2. Static analysis

For compression, a cosine load is applied to the connecting rod foot and at the other end the movement is restricted in all directions. The maximum cosine radial pressure according to Jovaj’s theory, applied at $\alpha' = 180^\circ$, is approximately 52 MPa, and at $\alpha' = 90^\circ$ is a minimum of 0 MPa. The tensile pressure is
considered uniformly distributed from $\alpha' = 0^\circ$ to $\alpha' = 90^\circ$ and is equivalent to approximately 13 MPa. Figure 6 and Figure 7 show the compression and traction boundary conditions, $\alpha' = 0^\circ$ is located at the upper end of the foot, on the axial axis of the connecting rod, and increases counterclockwise.

**Figure 6.** Boundary conditions for the FEM model under tensile loads. Variable pressure (Pa).

**Figure 7.** Boundary conditions for the FEM model under compressive loads. Uniform pressure (Pa).

Figure 8 shows the FEM results for the von Mises stress considering radial pressure applied on the head and foot of the connecting rod. The behavior of the stresses on the ends and on the body of the connecting rod can be observed. The maximum stress is obtained for the tensile load applied on the head and is equivalent to 335 MPa.

**Figure 8.** Von Mises stresses (Pa) for: (a) compression in the head, (b) traction in the head, (c) compression in the foot, (d) traction in the foot.

### 3.3. Dynamic analysis

The static analysis was performed at maximum engine speed, however, at this speed, the engine works without load and for short times. For the dynamic analysis, the calculated forces are used at the
maximum power rotation regime, equivalent to 7500 rpm. Performing the dynamic analysis with finite elements at all angles of rotation of the crankshaft would not be necessary or efficient, therefore, representative values are taken between the top dead center (TDC) and the bottom dead center (BDC), which correspond to 0°, 20°, 60°, 180°, 290° and 350° of crankshaft rotation. Table 1 shows the values of forces $F_{ST}$, $F_K$, $F_{HZ}$, and radial pressure in compression with their respective direction, taking into account the angle of rotation of the crankshaft.

The results indicate that for the study case at 20° the stresses are greater on the whole rod, with a maximum value of 356 MPa, and the radial pressure on the ends is greater than for the other dynamic cases. In contrast, for the analysis at 290°, the configuration of the radial pressures between the foot and the head causes the lowest stresses of the engine cycle, with a maximum value of 22.4 MPa, as a consequence of the traction forces counteracting the forces of gases.

| $\phi$ [°] | $F_{ST}$ [N] | Forces on the foot | $F_K$ [N] | $\Theta$ [°] | Radial pressure [Pa] | Forces on the head | $F_{HZ}$ [N] | $\Theta$ [°] | Radial pressure [Pa] |
|-----------|-------------|-------------------|----------|-----------|---------------------|-------------------|----------|-----------|---------------------|
| 0         | 4679        | 4679              | 0        | 4.0E+07   | 4499                | 0.0                | 2.60E+07 |
| 20        | 6812        | 6782              | 2.6      | 5.9E+07   | 6650                | 0.4                | 3.80E+07 |
| 60        | 2884        | 2810              | 13       | 2.4E+07   | 2837                | 3.5                | 1.60E+07 |
| 180       | 1370        | 1370              | 0        | 1.2E+07   | 1549                | 0.0                | 8.96E+06 |
| 290       | 451         | 437               | 345      | 3.8E+06   | 469                 | 337                | 3.32E+06 |
| 350       | 3180        | 3176              | 357      | 2.7E+07   | 3005                | 359                | 1.70E+07 |

4. Conclusions

The distribution of tensile and compressive loads on the connecting rod is of great importance for the definition of the boundary conditions of the numerical model. The Jovaj’s theory was chosen for static and quasi-static analyzes considering that the connecting rod is subjected most of the thermodynamic cycle under compression. First, the numerical model for a connecting rod with I cross section was validated using the reference solution from Jovaj’s theory.

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