Analytical study of temperature field during microwave drying of the material

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Abstract. Mathematical models of heating wet materials in a microwave field are analyzed. A thermal conductivity model is presented that considers two internal sources of heat, positive and negative. A negative heat source considers moisture output during drying of the material. A formula is obtained for calculating the temperature of a semi-bounded body under the action of internal heat sources for boundary conditions of the first kind. The results of calculations of the temperature of water and a dense layer of wheat grain are presented, depending on the duration of the microwave field and the specific power for the heating period. It is shown that in order to obtain reliable results, it is necessary to correctly set the efficiency of the microwave chamber. The results of calculations of moisture content and temperature of a layer of wheat grain for a period of constant drying speed are presented.

1. Introduction

The objective of this study is to compile a mathematical model and obtain a formula for calculating the temperature of a semi-limited body during microwave heating using accurate analytical methods. The mathematical model of thermal conductivity is written in differential form and is based on the law of conservation of energy. Depending on the complexity of the task, exact analytical methods, approximate and numerical, are used to solve mathematical models in a differential form. The use of accurate analytical methods allows us to obtain formulas for calculations in which there is no approximation error.

An analysis of existing theoretical ideas about the processes of converting the energy of the microwave field into the internal energy of the body [1-4] indicates the particular difficulty of modeling heat and moisture transfer during microwave heating. At the same time, quite successful attempts have been made to create mathematical models [5-7], which allow one to obtain solutions to some particular problems of heat conduction under the action of internal heat sources, for example, during microwave heating. The equation for the temperature distribution in a single solid sphere with steady state conditions taking into account the dependence of the thermal conductivity on temperature subjected to a uniform heat generation is presented in [7]. Of particular interest are models of A.V. Lykov [8], which are based on the parabolic partial differential heat equations. Solutions given in [8] describe the temperature field under the action of internal sources of heat under conditions under which the heat flux is directed from the environment to the surface of the material, which is not performed during microwave heating. There is a need for solutions in the direction of the heat flux from the surface of the material into the environment. Thus, the formula for calculating the temperature during microwave drying for boundary conditions of the third kind is given in [9].
However, the complexity of the formula for calculating the temperature leads to the appearance of areas with results that do not reflect the physical nature of the phenomena of heat transfer. In many heat transfer processes, there are heat sources inside a body. They may be positive (for example, heating a body by microwave field) and negative (moisture evaporation in a moist body when being heated)\cite{8}. The feasibility of obtaining analytical solutions is associated with a practical interest in microwave drying. Information on the temperature distribution in the material is important for various technological processes, for example, drying of seed grain.

2. Theoretical Method

2.1. Problem configuration and assumptions

A semi-infinite body is considered, the temperature of which at all points is the same and equal to \( t_0 \). At time \( \tau_0 \), the semi-infinite body is placed in a microwave (MW) field, heat sources inside the semi-infinite body begin to act. It is assumed that the surface temperature \( t_w \) remains constant: boundary conditions of the first kind are satisfied. The action of the microwave field leads to an increase in temperature in the semi-infinite body. A one-component model has been adopted, according to which the semi-infinite body is considered as a quasi-homogeneous medium with effective characteristics. It is assumed that the physical characteristics of the material are temperature independent. The temperature changes along the \( x \) coordinate and in time \( \tau \).

The volumetric nature of heating a material in a microwave field allows us to consider the material as a medium in which positive internal heat sources act. The exponential nature of the decrease in the intensity of a positive source over the thickness of the semi-infinite body according to the Beer’s and Lambert’s law is accepted:

\[
q_{v1} = q_{v10} \cdot e^{-\alpha x},
\]

where \( q_{v10} \) is the maximum specific power of a positive heat source, W/m\(^3\); \( \alpha \) is the attenuation factor of electromagnetic energy in the material, 1/m; \( x \) is the longitudinal coordinate, m.

The moisture output during drying was represented by the action of a negative heat source:

\[
q_{v2} = q_{v20} \cdot e^{-\beta x},
\]

where \( q_{v20} \) is the maximum specific power of a negative heat source, W/m\(^3\); \( r \) is the specific heat of vaporization, J/kg, \( \rho \) is the density of the material, kg/m\(^3\); \( N \) is the drying rate, 1/s.

An exponential law of variation in the intensity of a negative source over the semi-infinite body thickness is adopted:

\[
q_{v2} = q_{v20} \cdot e^{-\beta x},
\]

where \( \beta \) is the attenuation factor of the negative internal source, 1/m.

2.2. Governing equations

The problem stated may be written mathematically as follows:

\[
\frac{\partial t(x, \tau)}{\partial \tau} = a \frac{\partial^2 t(x, \tau)}{\partial x^2} + \frac{q_{v10}}{c \cdot \rho} \cdot e^{-\alpha x} + \frac{q_{v20}}{c \cdot \rho} \cdot e^{-\beta x},
\]

\[
t(x, 0) = t_0; \quad \frac{\partial t(x, \tau)}{\partial x} = 0, \quad t(0, \tau) = t_w = \text{const},
\]

where \( a \) is the thermal diffusivity, m\(^2\)/s; \( c \) is the heat capacity of the material, J/(kgK).

To solve equation (4), the Laplace transform method was applied.
3. Results and Discussion

3.1. The result of solving a mathematical model of thermal conductivity

The solution of the differential heat equation (4) with initial and boundary conditions (5) made it possible to obtain an expression for calculating the temperature of a semi-bounded body, which is applicable for conditions when the ambient temperature is lower than the temperature of the material. This condition reflects the actual physical process of microwave heating.

\[
 t(x, \tau) = \left(1 - \text{erfc} \left(\frac{x}{2\sqrt{\alpha \tau}}\right)\right) t_0 + \text{erfc} \left(\frac{x}{2\sqrt{\alpha \tau}}\right) \cdot t_w -
\]

\[
 - \frac{q_{v,0}}{c\rho a^2} \left[ e^{-\alpha x} \frac{1}{2} e^{\alpha^2 \alpha^2 - \alpha x} \text{erfc}\left(\frac{x}{2\sqrt{\alpha \tau}}\right) - \text{erfc}\left(\frac{x}{2\sqrt{\alpha \tau}}\right) \right]
\]

\[
 - \frac{q_{v,0}}{c\rho a^2} \left[ e^{-\beta x} \frac{1}{2} e^{\beta^2 \alpha^2 - \beta x} \text{erfc}\left(\frac{x}{2\sqrt{\beta \tau}}\right) - \text{erfc}\left(\frac{x}{2\sqrt{\beta \tau}}\right) \right]
\]

where \( \alpha \) is the thermal diffusivity, \( m^2/c \); the subscript 0 at specific power indicates that the value refers to the maximum value.

3.2. Testing the calculation formula for the temperature of a semi-infinite body without taking into account the moisture output

Functional check of the calculation formula was carried out on wheat grain and water. Figure 1 shows the results of calculating the temperature of grain and water depending on the duration of exposure to the MW field at \( q_{v,0} = 0 \). The initial data for the calculation are as follows: the initial temperatures of the material and the medium are equal: \( t_w = t_0 = 20 \) °C, attenuation factor for water \( \alpha_w = 125 \) m\(^{-1}\), attenuation factor for grain \( \alpha_g = 30 \) m\(^{-1}\). The calculation of the temperature of water and grain at the same efficiency of converting microwave energy into the internal energy of the material (in this case, the efficiency of the chamber is \( \eta = 1 \)) showed that the temperature curves for water (lines 3, 4) are lower than the curves for grains (lines 1, 2). This is because for the correct temperature comparison, the value of the dielectric loss factor \( \varepsilon'' \) of the material should be taken into account (for example, for water at 20° C, \( \varepsilon'' = 10.3 \) at frequency 2450 MHz [10], for wheat grain by moisture content 18% \( \varepsilon'' = 0.45 \) [11]). For this purpose, it is necessary to use the efficiency data of the microwave chamber \( \eta \). The efficiency of absorption of microwave energy by water is higher than that of grain. The efficiency of the MW chamber during water heating was close to 1.

The experimental dependence for determining the efficiency of the camera when it is loaded with grain is given in [10]. In accordance with this dependence, for wheat grain weighing 100 g we obtain efficiency \( \eta = 0.47 \). In this case, the calculation led to the distribution of temperature curves, correctly representing the physical process of microwave heating. Grain temperature (lines 5, 6) are lower than water temperature, and their values satisfactorily correlate with experimental ones [12].
3.3. Testing the calculation formula of a semi-infinity body, considering the moisture output

In a period of constant drying speed, it is assumed that the moisture content of each layer in time varies according to a linear law (7). The change in moisture content in the layer deep into the material is determined by the dependence (8).

\[ u = u_0 - N_i \cdot \tau, \]  \hspace{1cm} (7)
\[ N_i = N_0 e^{-\beta x}, \]  \hspace{1cm} (8)

where \( N_i \) is the drying speed of the \( i \)-th layer, 1/s; \( N_0 \) is the drying rate of the upper layer, 1/s.

According to the above dependences (6), (7), the distribution of moisture content and temperature of the material during drying by microwave heat supply have been obtained. The calculated values of the maximum specific powers of the sources \( q_{v10} \) and \( q_{v20} \), as well as the attenuation factor \( \alpha \) and \( \beta \), are given in Table 1.

|   | \( q_{v10} \), W/m³ | \( q_{v20} \), 10⁵, 1/s | \( \alpha \), 1/m | \( \beta \), 1/m |
|---|-----------------|-----------------|--------------|--------------|
| 1 | 110             | -4.26           | 30.2         | -6.6         |
| 2 | 165             | -8.9            | 30.2         | -6.6         |
| 3 | 275             | -13.9           | 30.2         | -6.6         |

In Figure 2 shows the curves of changes in moisture content and temperature. The temperature profile over the thickness of the layer, when changing from the period of heating to the period of constant drying speed, changes (see Figure 3). If during the warm-up period the temperature decreases exponentially, then in the period of constant drying speed this dependence changes, becomes close to the linear law (see curve 3 of Figure 3) and proceeds to the convex form of the curve. Warm-up period: 1- \( \tau = 60 \) s; 2- \( \tau = 120 \) s; the period of constant drying speed: 3- \( \tau = 360 \) s; 4- \( \tau = 480 \) s; 5- \( \tau = 600 \) s. A power of the MW source \( P = 160 \) W.
To determine the average grain drying rate, an empirical dependence (9) can be proposed, which is valid at $200 \leq q_m \leq 1285$ W/kg [12].

$$N = 1,58 \cdot 10^{-7} (q_m)^{1.17}$$

(9)

where $q_m = Q / M$ is the specific heat flux, W/kg.

To calculate the useful heat flux $Q$, formula (10) is proposed, which considers the energy consumed from the network $P$, the efficiency of the microwave chamber $\eta$ and the efficiency of the magnetron $\eta_m$:

$$Q = P \cdot \eta \cdot \eta_m$$

(10)

It was found that the analytical dependences for calculating the temperature of the material during drying, obtained from the solution of the heat equation taking into account two heat sources, work satisfactorily in the range of operating parameters corresponding to the drying regimes of grain: drying speed $N=10^5$…$10^4$ s$^{-1}$, initial moisture content $u=0.2$ kg/kg [11]. Calculation of temperatures according to the developed mathematical models, considering two internal sources of heat, allows obtaining satisfactory data and predicting the thermal state of the material at an arbitrary point in time.

4. Conclusions

A mathematical model of the thermal conductivity of a semi-limited array, taking into account the action of two sources of heat, positive and negative, satisfactorily describes the process of heating a layer of material in a microwave field, taking into account the evaporation of moisture during drying. The obtained analytical dependence for calculating the local temperatures of a semi-bounded array under microwave heating conditions may apply in the analysis of the influence of heating duration, input power, and initial temperatures on the temperature distribution over the layer thickness.

5. References

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