The Quantum Self-eraser

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Abstract. A scheme for an atomic beam quantum self-eraser is presented. The proposal is based on time reversal invariance on a quantum optical Ramsey fringes experiment, where a realization of complementarity for atomic coherence can be achieved. It consists of two high finesse resonators that are pumped and probed by the same atom. This property relates quantum erasing with time reversal symmetry, allowing for a full quantum erasing of the which-way information stored in the cavity fields. The outlined scheme also prepares and observes a non-local state in the fields of the resonators: a coherent superposition between correlated states of macroscopically separated quantum systems. The proposed scheme emphasizes the role of entanglement swapping in delayed-choice experiments. Finally, we show that the quantum self-eraser violates temporal Bell inequalities and analyze the relation between this violation and the erasability of which-way information.

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1. Introduction

The complementarity principle has traditionally been addressed in order to illustrate the role of observer in Quantum Mechanics. One of the most intriguing aspect of the issue is the possibility of delayed-choice experiments [1]. According to it, the experimenter may decide for instance to display wavelike aspects of an atomic system, long after it has been forced into a particle-like behavior [2]. If the atomic state is entangled with a memory system available to the observer, i.e., a which-way detector (WWD), the experimenter may decide to postpone his decision even after the atom has been detected. Interference fringes can be recovered, provided the which-way information (WWI) is erased from the WWD [3]. An experimental demonstration of a quantum eraser have been given using entangled photon pairs [4, 5, 6], using single photon entanglement between spatial mode and polarization state [7], and using atom interferometry [8].
The great progress in cavity quantum electrodynamics offers the possibility of designing a quantum detector to retain which-way information for rather long time after the system has been detected [9]. On the other hand, microwave resonators can couple strongly to Rydberg atoms. As a matter of fact, high-finesse microwave resonators have been used to generate Einstein-Podolsky-Rosen pairs of atoms [10] and in the construction of quantum logic-gates [11]. Thus, quantum optical Ramsey interferometers are good candidates for complementarity experiments. Several models of cavity-QED methods for demonstrating complementarity have been proposed. Some of them supplement the Young double-slit experiment with WWD based on dispersive atom-field interaction [12, 13, 14]. Other proposal uses two atomic beams as WWD of each other using a cavity field in order to entangle them [15]. Dispersive cavity-QED experiments were reported in [10] and more recently in [17], where the quantum-classical limit of complementarity have been explored using a cavity field continuously tuned from microscopic to macroscopic regimes. A simplified model for quantum erasing have been found in [18].

We present in this paper a novel quantum-erasing scheme: the quantum self-eraser, based on a different strategy. The innovative idea behind is to use time reversibility to "undo" the interactions that deposited the WWI into the WWD. Thus, we launch back the same atom (or a velocity reversal replica of it) to act as the eraser atom (Erason). Controlling time reversibility, we can let the Erason absorb or erase its own WWI that was deposited previously on the WWDs. This scheme will emphasize the role of entanglement swapping in quantum erasing operation. Moreover, the phase difference $\phi$ between the interfering alternatives is mapped into the non-local phase of a superposition between the two macroscopically separated cavity-field states. The self-eraser probes this phase, allowing for a joint demonstration of complementarity and the generation of a non-local coherent macroscopic superposition. Finally, we analyze the relation between the violation of temporal Bell inequalities as expression of the establishment of macroscopic coherence, and the erasability of the WWI in the quantum self-eraser.

The paper is organized as follows. In section 2, complementarity in a quantum optical Ramsey interferometers (QORI) is discussed in order to set up the notation. In section 3, we prepare the stage for the discussion showing how the QORI scheme can be used to prepare in an unitary fashion a non-local field state. Section 4 is devoted to probe of the phase of the macroscopic superposition via quantum self-erasing. Section 5 studies the violation of temporal Bell inequalities. Finally, we end up with a discussion and a summary of our results.

2. The quantum optical Ramsey interferometer

In this section, we describe a quantum optical Ramsey interferometer (QORI). We can summarize the system as follows. It consists in an atomic interferometer, as depicted in Figure 1a. The interfering ways are realized on the internal states of a Rydberg two-level atom, that we denote by $\{|a\rangle, |b\rangle\}$. Before entering into the interferometer,
the atoms are prepared in the upper state $|a\rangle$. A microwave resonator provides the quantized beam splitter (BS) as well as the WWD. Following the strategy of [19], the phase shifter (PS) is provided by an electrostatic external field applied in the region between the cavities. The differential Stark shift between upper and lower levels induces a relative phase $\phi$ that enables an external control of the off-diagonal elements of the density matrix. Subsequently, a classical microwave -$\frac{\pi}{2}$ pulse provides the beam merger (BM). The combined interferometer action performs a transformation from the initial state

$$\rho^o = \frac{1}{2}(1 + \sigma_z) \otimes \rho_D^o,$$  

(1)

to the final state

$$\rho^f = \frac{1}{2} \left( \frac{1 - \sigma_z}{2} \right) C \rho_D^o C + \frac{1}{2} \left( \frac{1 + \sigma_z}{2} \right) a^\dagger S \rho_D^o Sa \right) + \left( \frac{\sigma_z + i \sigma_y}{2} \right) C \rho_D^o Sa e^{-i\phi} + \left( \frac{\sigma_z - i \sigma_y}{2} \right) a^\dagger S \rho_D^o C e^{i\phi},$$  

(2)

where the operators $[19]

$$S = \frac{\sin(\Omega \tau \sqrt{aa^\dagger})}{\sqrt{aa^\dagger}} = S^\dagger, \quad C = \cos(\Omega \tau \sqrt{aa^\dagger}) = C^\dagger,$$  

(3)

have been introduced, $\{\sigma_x, \sigma_y, \sigma_z\}$ are the Pauli spin matrices of the two-level atom, and $\rho_D^o$ is the initial state of the WWD, i.e., the cavity-field inside the resonator. Here $a$ and $a^\dagger$ are annihilation and creation operators of the cavity field mode, $\Omega$ is the vacuum Rabi frequency of the atom-field interaction, and $\tau$ the interaction time, i.e., the time of flight of the atom through the cavity. In the derivation of (2) resonant interaction has been assumed, and cavity damping and spontaneous atomic decay have been neglected.

The final state of the WWD is obtained after tracing out the atomic degree of freedom. Thus

$$\rho_D^f = \text{tr}_{\text{atom}} \{\rho^f\} = w_+ \rho_D^+ + w_- \rho_D^-,$$  

(4)

where $\rho_D^+, \rho_D^-$ are the detector’s two final states corresponding to each way. The WWD can be regarded as a memory system, with two internal pointer states correlated to the interfering ways, given by $\rho_D^+, \rho_D^-$ in (4). The quantities

$$w_\pm \equiv \text{tr} \left\{ \frac{1 \pm \sigma_x}{2} \rho^f \right\},$$  

(5)

are the probabilities for taking each way after the BS. Thus, the asymmetry of the BS is measured by the predictability

$$\mathcal{P} = |w_+ - w_-|.$$  

(6)

The final state of the exit atom is measured by means of state selective field ionization techniques in $D_A$. Quantum optical Ramsey fringes are exhibited in the probability of detecting the outgoing atoms in a definite internal state, in case
Figure 1. Quantum optical Ramsey interferometer. The interfering ways are realized on the internal states \( \{ |a\rangle, |b\rangle \} \) of the Quanton: a two-level Rydberg atom. A quantized BS provides also the WWD in cavity \( M_1 \). After the phase shifter PS a classical microwave field recombines the ways (BM). The state of the atom is measured finally at \( D_A \).

Complementarity allows for them. For instance, \( P_{aa}(P_{ab}) \), the probability of detecting the Quanton in the upper (lower) state can be calculated to yield

\[
P_{aa} = \frac{1}{2} \left( \langle C^2 \rangle_o + \langle S^2a^a \rangle_o + 2 \text{Re} \{ C e^{-i\phi} \} \right),
\]

\[
P_{ab} = 1 - P_{aa},
\]

where the contrast factor \( C \) is given by the expression

\[
C = 2 \langle SaC \rangle_o.
\]

The averages in this equation are taken over the initial field prepared inside the cavity, and can be calculated after an appropriated expansion of the state in the photon number bases.

The stage is now ready for the analysis of complementarity in the interferometer. Complementarity demands that distinguishability of the ways must be followed by lost of coherence in the visibility \( V \) of the interference pattern in \( \text{[17]} \). A measure of the distinguishability \( D \) of the ways in a two-ways interferometer have been given in \( \text{[20-21]} \), quantifying the maximum potential WWI that can be available to the experimenter. Two sources of WWI contribute to \( D \). One is the a-priori WWI the experimenter has about the ways, stemming from the asymmetric preparation of the beam splitter, and thus, quantified by the predictability \( P \) in \( \text{[6]} \). The other source of WWI depends on the quantum "Quality" \( Q \) of the WWD, i.e., its ability to trace down the ways of the two-level system (Quanton) via quantum correlations. Recently, a formalism has been developed \( \text{[22]} \) which allows one to separate both contributions, even in the case where beam splitter (BS) and WWD are provided by the same physical interaction \( \text{[23]} \). Assuming pure state preparation in \( \text{[1]} \), we have

\[
D = \sqrt{(1 - P^2) Q^2 + P^2}.
\]
As can be seen from the above equation, a high-quality $Q = 1$ WWD implies full distinguishability $D = 1$, no matter the value of $P$ and vice versa. In particular, a perfect-quality WWD will prevent any fringes from being displayed at the output port of the interferometer, for any value of the predictability $P$. In fact, for pure state preparation the following equality holds $^{23}$

$$
(1 - P^2) Q^2 + P^2 + V^2 = 1.
$$

The visibility can be rapidly calculated from (7) to be $V = |C|^{20}$, where the contrast factor $C$ ranging from 0 to 1 is given in (8). Fringes are degraded by the potential availability of WWI stemming from the introduction of a quantum WWD into the interferometer. Thus, the observation of an interference pattern in the detection probability of the atom depends on the state preparation of the cavities. If they are unable to acquire which-way information ($Q = 0$), quantum interference is observable due to the indistinguishability of the path leading to the same final state. On the other hand, if we prepare the cavities to act as which-way detectors, for instance in a Fock state ($Q = 1$), then no interference effects are observable ($V = 0$), as ensued by complementarity $^{23}$. As a matter of fact, $V$ has been measured in a recent experiment $^{17}$ for different preparation of the initial cavity field state, showing the transition between both complementary situations.

Consider now the initial cavity field prepared in the vacuum state $\rho_0^D = \langle 0|0 \rangle$. The relevant quantities of this section can be rapidly calculated to yield

$$
P_{ab} = 1/2,
\quad P = |\cos(2\Omega \tau)|,
\quad Q = 1 \Rightarrow D = 1, V = 0.
$$

No interference pattern can be measured at the output port of the interferometer, since the cavity-field stores full WWI about the alternatives taken by the atom ($Q = 1$). The upper (lower) way is perfectly correlated to the no-photon (one photon) state of the cavity field. In order to restore the interference pattern, the WWI must be erased so it is no longer available to the experimenter. This can be done by launching a second Rydberg atom through the cavity, in order to absorb the "which-way" photon stored in the cavity field. An interference pattern can be built in the correlated detection probability of both Quanton and Erason atoms $^{3}$.

3. Unitary preparation of a non-local field state

We follow a different strategy in order to achieve full quantum erasing which is based in time-reversal invariance. In order to preserve the time symmetry property of closed systems, the second microwave pulse providing the beam merger (BM) must be quantized too. We can use a QORI scheme described in $^{24}$. It consists in two high-Q microwave resonator successively transversed by a beam of monoenergetic two-level atoms, at such a low rate that only one atom is present in both cavities at a time (see
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Figure 2. (a) Asymmetric quantum optical Ramsey interferometer. The Quanton leaves the $M_1$ cavity at $t_1$, in a superposition of two alternatives. The BM is now also quantized, performing a conditional $\pi$-pulse, leaving the atom with certainty in state $|b\rangle_A$ after crossing cavity $M_2$. (b) The atom is launched back to self-erase its own WWI. At $t_2$ ($t_E$) the Erason enters (leaves) the $M_1$ resonator and is finally measured in $D_E$.

Figure 2a). The symbol "A" will be used in the sequel to label the Quanton. Thus, we denote the upper and lower levels by $\{|a\rangle_A, |b\rangle_A\}$. Before entering the first cavity the atoms are prepared in the upper level $|a\rangle_A$. We choose now an asymmetrical design for the cavities, as depicted in Figure 2(a). It consists of two asymmetrical cavities (different lengths), such that

\begin{align}
  s_1 &\equiv \sin(\Omega \tau_1) \neq 1, \\
  s_2 &\equiv \sin(\Omega \tau_2) = 1, \\
\end{align}

where $\tau_i$ ($i = 1,2$) is the interaction time of the atom in the cavity $M_i$. The cavities are prepared initially to be empty, i.e., the state vector of the combined cavities-atom system is given by

\[ |\Psi_0\rangle = |0,0\rangle |a\rangle_A \]

Assuming resonant interaction, the probability from the upper-to-lower level transition in each cavity is given by the square of the sine in Eq. (12). Under this preparation, the atoms is left with certainty in its lower state after passage of the second cavity. In this energy transferring situation, there are alternative routes along which
The final lower level state can be reached. Different routes are realized by changing the photon number in different cavities. These paths, sketched in Figure 2(a), will be called the "upper-" and "lower-paths". It should be noticed that the conditional dynamics on the alternative path transitions in $M_2$ is due to the mixing of two different state manifolds of the Jaynes-Cummings interaction. In the case the atom enters $M_2$ in the upper level, it will undergo a $\pi$-pulse leaving a photon in the cavity for the upper path. On the other hand, if the atom enter in the lower level no transitions occur since the atomic-cavity system coincides with the ground state of the interaction Hamiltonian.

After atom $A$ has exited the cavity $M_2$ at time $t_A$, the state of the system reads

$$|\Psi(t_A)\rangle = \{-is_1 |1,0\rangle + c_1 e^{-i\phi} |0,1\rangle\} \otimes |b\rangle_A,$$

where $s_1$ is given in (12) and $c_1 = \sqrt{1-s_1^2}$. After the passage of a single atom the cavities have been prepared in a non-local field state, i.e., in a coherent superposition of correlated states of macroscopically separated quantum systems. It should be noticed that the stated preparation give by Eq. (14) is completely unitary. After the passage of the pumping atom, the cavity is left, with unit probability, in a macroscopic superposition with an externally controllable non-local coherence. Due to the asymmetric design of the resonators, the phase difference $\phi$ of the interfering alternatives is mapped, in an unitary fashion, into the non-local phase of the superposition state of the two WWDs.

Note that we have two WWDs, $M_1$ and $M_2$, both with $Q_1 = Q_2 = 1$. Also, the Predictability in $M_2$ is $P_2 = 1$, so the probability of detecting atom $A$ in the lower level will exhibit a featureless dependence on $\phi$. However, as will be shown in the next section, fringes of unit visibility can be extracted from a suitable measurement of the WWD, provided that full WWI is erased from both cavity fields.

4. The quantum self-eraser and non-local phase observation.

We launch backwards the Quanton to erase its own WWI. For instance, gravity could play the role of an atomic mirror for the Quanton [25]. Alternatively, we can take advantage of the factorization property in (14) in order to simplify the experimental setup. As a matter of fact, after $t_A$, a subsequent measure of the Quanton’s internal state can be performed without introducing projection noise. We can use as the Erason a replica of the Rydberg two-level atom, with upper and lower states that we denote by $\{|e\rangle_E, |g\rangle_E\}$, from a second beam $E$ (see Figure 2(b)). Thus, the detection of the system-atom in $D_A$ can eventually be used to trigger the excitation laser preparing the Erason in the lower level $|g\rangle_E$. The Erason is sent through both cavities in reversed order with the same velocity magnitude as that selected for the Quanton, and thus, it can be regarded as a velocity reversal version of the Quanton. After calculating the coherent transients developed by the Erason, the state of the entangled system is

$$|\Psi(t_E)\rangle = \{-i[s_1^2 + c_1^2 e^{-2i\phi}] |0,0\rangle |e\rangle_E + [c_1 s_1 - s_1 c_1 e^{-2i\phi}] |1,0\rangle |g\rangle_E\} \otimes |b\rangle_A,$$

(15)
where $t_E$ is the final time at which the Erason leaves the cavity $M_1$. The contributions to the amplitude coefficients in (15) can be regarded as labels for the four possible paths of the global system. Before reaching $M_1$, the paths of the returning Erason perfectly correlate with those of the Quanton. Thus, a relative phase of $2\phi$ is accumulated between the upper and lower atomic paths. At this stage, the role of the Erason is to probe the which-way information stored in the cavity $M_2$, using time reversal symmetry to transfer it to its own internal state. However, when the atom crosses cavity $M_1$, a rotation of the Bloch vector is performed such that each of the two possible outcomes in the subsequent measurement of state of the Erason is correlated with both upper and lower paths. Due to the asymmetric design of the interferometer, WWI or quantum erasure can be obtained by the experimenters by means of letting the Erason cross one or both cavities, respectively. After detection of the Quanton, the experimenter may decide at will, in a delayed-choice fashion, to display either particle or wavelike aspects of the quantum system, deciding how many cavities the Erason will be allowed to fly through.

The correlations between the Quanton and the final state of the Erason after crossing both cavities are given by the final detection probabilities

$$P_{ge} = \text{tr} \{ |e\rangle_{EE} \langle e | \rho(t_E) \} = c_1^4 + s_1^4 + 2c_1^2s_1^2 \cos(2\phi),$$

$$P_{gg} = \text{tr} \{ |g\rangle_{EE} \langle g | \rho(t_E) \} = 2c_1^2s_1^2(1 - \cos(2\phi)).$$

After many repetitions of the experiment, the detection probabilities will exhibit fringes and antifringes upon variation of the phase $\phi$. Within repetitions, the cavities should be reinitialized to the vacuum state. It should also be noticed that the erasure could be performed at any time after $t_A$ within the lifetime of the cavity, and in particular after the atom $A$ has been detected \[26\]. As in the usual quantum eraser, fringes and antifringes add destructively, and the featureless pattern $P_{ba} = P_{ge} + P_{gg} = 1$ is recovered when both Erason detection alternatives are summed up. On the other hand, in spite the factorization property of the final state of the Quanton, we would like to remark that the self-eraser do perform quantum erasing operation. The non-local phase is constructed on the interfering alternatives of the Quanton, i.e., its wave-like properties, transferred to the cavity fields via entanglement swapping. Thus, the full recovery of the interference fringes in Eqs. \[16, 17\] demands the complete erasure of WWI about the particle-like properties of the quantum stored in the WWDS, as imposed by complementarity.

As discussed in the previous section, the system-atom prepares the cavity fields in a non-local state. In order to demonstrate the generation of the macroscopic superposition, interference effects sensitive to the non-local coherence must be measured, in order to distinguish this state from an incoherent mixture. The Ramsey scheme converts the non-local phase into a population difference. Thus, according to \[16\], a simple population measurement can be used to probe non-local phase of the cavity fields in \[14\]. Observation of fringes in the detection probability of the Erason demonstrates the coherent character of the macroscopic superposition \[14\].
The visibility of the interference pattern implicit in \( P_{ge} \) of Eqs. (16) is computed to be

\[
\nu_{ge} = \frac{2c_1^2s_1^2}{c_1^2 + s_1^2}.
\]  

(18)

In the case \( M_1 \) is prepared to act also as a \( \pi \)-pulse, \( c_1s_1 = 0 \) and no fringes are obtained as consequence of the single path situation. Perfect visibility fringes are obtained for \( |c_1| = |s_1| \). This situation correspond to a symmetrical mixing (\( P = 0 \)) of the correlation between eraser and system paths in \( M_1 \). \( P_{ge} \) and \( P_{gg} \), given in Eqs. (16) are plotted in Figure 3 as a function of the induced phase \( \phi \) and of the single-cavity transition probability \( s_1^2 \), showing both visibility limits. Perfect visibility is obtained for atomic velocities matching the condition

\[
v = \frac{4\Omega L_1}{(2k + 1)\pi} \quad k = 0, 1, ...
\]  

(19)

where \( L_1 \) is the length of cavity \( M_1 \). Note that Eq. (19) is compatible with the \( \pi \)-pulse preparation of \( M_2 \), provided this cavity is built twice as long as \( M_1 \).

Further insight into the system can be gained by means of calling upon time reversal symmetry. The Erason is designed to act as the velocity reversal version of the Quanton. In fact, time reversal invariance is used in \( M_2 \) in order to swap the WWI from the cavity field to the internal state of the atom. However, the passage coursed by the Erason cannot be regarded as a complete time reversal of the Quanton passage: the open phase shifter breaks the symmetry, allowing for different alternatives in the final detection.

Figure 3. Conditional detection probability \( P_{ge} \) and \( P_{gg} \) plotted versus the externally induce phase \( \phi \) and the single cavity transition probability \( s_1^2 \).
probability. Indeed, the relative phase induced by the PS, adds up to $2\phi$ after the Erason passage, instead of canceling each other. Only in the latter case, the whole interferometer would exhibit complete time reversal invariance and the Erason would end up in the initial state of the Quanton. This can be seen from Eqs. (16) where, consistently with time reversal invariance, $P_{ge} = 1$ for $\phi = 0$.

5. Violation of temporal Bell inequalities

A signature of quantum coherence can be found in the violation of temporal Bell inequalities of the type of [27]. We show in this section that the quantum self-eraser violates a temporal Bell inequalities for certain ranges of the Rabi phase. According to realistic descriptions, dynamical variables possesses definite values at definite times. Following [28], a stochastic process for a two-state system can be defined as the dichotomic random variable $\chi(t)$ assuming the values $1 (-1)$ when the system is on upper (lower) state at time $t$. Let us consider now the quantity

$$\Delta_{\pm} = K_{13} \pm K_{12} \pm K_{23},$$

where $K_{ij}$ are the two-time autocorrelation function

$$K_{ij} \equiv K(t_i, t_j) = \langle \chi(t_i) \chi(t_j) \rangle,$$

and we take $t_1, t_2$ as the entering and exit time of the Quanton through $M_1$ ($t_2 = t_1 + \tau_1$), and $t_3$ as the exit time of the returning Erason after crossing $M_1$, i.e., $t_3 = t_E$ (see Figure 2). Any two-state stochastic process must satisfy the inequality [28]

$$-1 \leq \Delta_{\pm}.$$

On the other hand, from the quantum mechanical point of view, the autocorrelation function given by $K(t_i, t_j) = \langle \sigma_z(t_i) \sigma_z(t_j) \rangle$ can be calculated for the quantum self-eraser to yield

$$K_{12} = c_1^2 - s_1^2,$$

$$K_{23} = c_1^4 - s_1^4,$$

$$K_{13} = 1 - 8c_1^2s_1^2.$$

Inserting (23) into (20), we obtain

$$\Delta_{\pm} = \cos(4\Omega\tau_1) \pm 2\cos(2\Omega\tau_1),$$

which coincides with the value derived for a single cavity preparation in [28]. This coincidence can be understood taking into account that for any two times $t, t'$ between $t_2$ and $t_3 - \tau_1$ we have $K(t, t') = 1$, due to the perfect correlations provided by the $\pi$-pulse in $M_2$. The quantity (24) is plotted in Figure 4 showing violation of the inequality (22) for certain ranges of the vacuum Rabi phase $\Omega\tau_1$. As can be seen in this plot, there is
no violation at $\Omega \tau_1 = \pi/2$ (for which $\Delta_+ = -1, \Delta_- = 3$), which is precisely the point of full recovery of the interference fringes in the detection probability of the Erason.

It is also important to remark that Bell experiments involving Rydberg atoms are especially of interest, since they can close the communication and the detection loopholes [29]. A proposal to measure Bell’s inequality violation with a Rydberg atom sequentially interacting with two classically driven cavities has been given in [30]. On the other hand, Rydberg atoms are mesoscopic systems, which are of interest to study deviations from macroscopic realism [31]. Thus, the proposed self-eraser interferometer posses a wide play-ground to test the role of measurement in quantum mechanics at the mesoscopic level.

6. Discussion

A scheme for a atomic beam quantum self-eraser has been presented. In contrast to previous models, the present proposal is based on time reversal invariance. This property demands an experimental setup in which the Quanton- (Erason-) atoms pump (probe) the cavities in a symmetrical fashion. The present proposal uses an asymmetric design of the microwave resonators: the second resonator allowing one to factorize the state of the exit atom from the state of the cavities. Thus, we can send another atom from a reversed beam, to play the role of the self-eraser atom, provided it is send as a velocity reversal of the pump atom. On the other hand, the factorizing conditions extend our view about quantum erasing operation. Interference fringes are recovered directly: we do not need to correlate the measurements on the detectors $D_A$ and $D_E$. Although in an unconventional fashion [32], the Erason performs quantum erasing operation. In fact, due to the
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experimental configuration, the actual full recovery of the interference pattern demands the quantum erasure of WWI from the WWDs, as required by complementarity. At the point of maximum erasure, we have $Q_1 = Q_2 = 0$, allowing for a full recovery of the interference pattern ($V = 1$). The self-eraser highlights the role of complementarity in the recovery of interference fringes via entanglement swapping.

Also resulting from the asymmetric design of the resonators, the scheme allows one to prepare the correlated cavity fields in a macroscopic superposition state in an unitary fashion.\[24\]. When the Quanton leaves the second cavity, the phase information accumulated by the atom in the interference region is kept in the cavity fields as the non-local coherence between the two components of its superposition state. This coherence is transferred back to the Erason as probed in the fringe structure of its final state. Observation of an interference pattern in the final detection probability of the Erason can be used for a joint demonstration of quantum erasing operation and the generation of non-local superposition in the correlated field state of the macroscopically separated cavity systems.

Violation of temporal Bell inequalities provides a criterion for a signature of the establishment of macroscopic quantum coherence. On the other hand, the quantum self-eraser is built on entanglement swapping of macroscopic coherence. Thus, it is interesting to analyze how both manifestations of quantum coherence are mutually related. We have shown that the quantum self-eraser violates temporal Bell inequalities. However, this violation is not directly connected to quantum erasing capability. In fact, the point of maximum erasability is outside of the parameter range leading to violation.

Thus, the quantum self-eraser appears as an useful playground to study the relation between non-locality, macroscopic realism and complementarity.

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