SPIN VARIATIONS OF BLACK HOLE BINARIES IN AGN DISKS
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ABSTRACT
The spin-orbit misalignments of stellar-mass black hole binaries (BHB) provide important constraints on the formation channels of merging BHBs. Here, we study the spin evolution of a black-hole component in a BHB around a supermassive BH (SMBH) in an AGN disk. We consider the BH’s spin-precession due to the J2 moment introduced by a circum-BH disk within the warping/breaking radius of the disk. We find that the BH’s spin-orbit misalignment (obliquity) can be excited via spin-orbit resonance between the BHB’s orbital nodal precession and the BH spin-precession driven by the circum-BH disk. Assuming a 10^7 M☉ SMBH, this typically occurs at a distance of 10^2−4 AU to the SMBH or 10^5−7 GM_{SMBH}/c^2. In many cases, spin-orbit resonance leads to a high BH obliquity, and a broad distribution of the binary components’ obliquities and effective spin parameters.

1. INTRODUCTION
The detection of gravitational waves from black hole (BH) mergers provides an unprecedented opportunity to probe the properties of BH binaries (BHBs). For instance, the measurement of the spin-orbit misalignment and the spin precession reveal key information to better characterize the BHBs (Cutler & Flanagan 1994; Chatziioannou et al. 2015), and to determine the origin of the BHBs (Rodriguez et al. 2016; Farr et al. 2017; Gerosa & Berti 2017). Hundreds of mergers of stellar mass compact binaries will be detected within the next decade, with the improved sensitivity of LIGO-Virgo-KAGRA and the expected commissioning of LIGO India. This prospect will enable detailed explorations of the origin and evolution of these compact binaries based on the statistical properties of the BHBs.

The dynamics of BHB spin-orbit coupling is rich. For isolated binaries, spinning BHBs can “flip-flop” when the binary components are close to each other during the merger process within a separation of ~10^3 M☉ (Lousto & Healy 2015; Gerosa et al. 2019), where M is the total mass in units G = c = 1. Such occurrence leads to large BH’s spin-orbit misalignment up to 180° between the BH’s spin and the orbital normal direction. Numerical-relativity simulations of BHBs revealed that spin-orbit misalignment can lead to large BH recoils, with important astrophysical implications (Campanelli et al. 2007; Brügmann et al. 2008; Kesden et al. 2010; Lousto & Zlochower 2011; Gerosa et al. 2018).

For stellar mass black hole binaries around a supermassive black hole (SMBH), large BH spin-orbit misalignment can be produced when the binaries are highly misaligned with respect to their orbits around the SMBH prior to the merger process. Specifically, the Von Zeipel-Lidov-Kozai mechanism due to the Newtonian perturbation of the SMBH can lead to inclination oscillation and eccentricity excitation of the BHB’s orbit around each other and induce BH merger (see review by Naoz 2016). Meanwhile, the BH spin undergoes chaotic evolution (Liu & Lai 2017; Fragione & Kocsis 2020), leading to a wide range (0°−180°) of final spin-orbit misalignment from an initially aligned configuration. However, when the binary orbital inclination relative to its orbit around the SMBH is low (≲40°), the binary eccentricity is not excited and only modest (≲20°) spin-orbit misalignment can be produced.

Here, we show the spin-orbit resonance can increase the BHs’ obliquities when forming a BHB in the AGN disk. AGN disks are dense with stars and compact objects (Bartos et al. 2017; Tagawa et al. 2020b), and such environments are conducive to forming and merging of compact-object binaries. How do the spins of the BHs evolve? McKernan et al. (2020) adopted Monte Carlo simulations and found the distribution of the effective spin parameter χ_eff is naturally centred around χ_eff ≈ 0.0 with a narrow width of the χ_eff distribution for low natal spins. In addition, Tagawa et al. (2020a) found that the strong binary-single encounters can randomize the orbital inclination and lead to non-zero but equal obliquity for the binary components due to Bardeen-Petterson effect and binary-single hard encounters. This can be tested with observations. However, detailed analysis has not been conducted on the spin-orbit coupling of the BHs in the AGN disk.

In this letter, we propose a mechanism to change the spin orientation of the stellar mass BHs. Specifically, inner part of the circum-BH disk within its Laplace radius strongly couples to the spin of the BH and is driven...
The binary is composed of stellar mass BH a SMBH AGN disk. The configuration is shown in Figure 1. This can broaden the distribution of planetary disk (Ward & Hamilton 2004; Rogoszinski & Pringle 1983), and while the outer parts of the disk align with the BH orbital axis (e.g., Martin et al. 2009; Tremaine & Davis 2014). This process is analogous to the orbital precession of a satellite around its host planet and perturbed by the star (Goldreich 1966). We use $a_w$ to denote the warping radius, which marks the limit where the precession direction of the disk changes significantly. Tears of the disks can occur when the warp is significant (e.g., $\gtrsim 40^\circ$ depending on the disk properties), and this hinders the subsequent realignment of the BH spin with the outer disk (e.g., Nixon et al. 2012; Gerosa et al. 2020; Nealon et al. 2022).

The warping of the disk typically occurs near the Laplace radius of the disk ($a_w \lesssim a_1$). The Laplace radius of the disk locates where the direction of the disk particle’s nodal precession changes and the Lense-Thirring precession rate is equal to the nodal precession rate. For simplicity, we assume the eccentricities of the disk particles and that of the BHB to be zero. Omitting factors of order unity, we obtain:

$$a_L = a_2^{2/3} R_d^{1/3} \left( \frac{m_1}{m_2} \right)^{2/9}$$

where $R_d = \frac{G m_1}{c^2}$ is the gravitational radius of $m_1$. This expression is within an order of unity of the warping radius obtained in Martin et al. (2009); Tremaine & Davis (2014).

We plot in Figure 2 the Laplace radius of disk particles around a 10 M$_\odot$ and 50 M$_\odot$ central BH with separations of $a$ = 1 AU. Different types of lines represent different BH spin coefficients. The $y$-axis label on the right shows the Laplace radius in terms of the gravitational radius of the central BH. The blue lines and labels correspond to that around a 10 M$_\odot$ BH, and purple corresponds to a 50 M$_\odot$ BH.

When the spin of the BH is misaligned with the binary orbit, the disk becomes warped (as shown in the zoom-in view in Figure 1). Specifically, the inner region of the circum-BH disk aligns with the BH spin due to Bardeen-Petterson effect (Bardeen & Petterson 1975; Papaloizou & Pringle 1983), and while the outer parts of the disk align with the BHB orbital axis (e.g., Martin et al. 2009; Tremaine & Davis 2014). This process is analogous to the orbital precession of a satellite around its host planet and perturbed by the star (Goldreich 1966). We use $a_w$ to denote the warping radius, which marks the limit where the precession direction of the disk changes significantly. Tears of the disks can occur when the warp is significant (e.g., $\gtrsim 40^\circ$ depending on the disk properties), and this hinders the subsequent realignment of the BH spin with the outer disk (e.g., Nixon et al. 2012; Gerosa et al. 2020; Nealon et al. 2022).

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Laplace radius is smaller for smaller BHB separation.

Recently, it has been shown in 3-dimensional hydrodynamical simulations that significant warping as well as breaking can indeed occur for BH spins misaligned with the binary orbit (Nealon et al. 2022). The viscosity of the disk determines whether the breaking occurs (Pringle 1992, Ogilvie 1999), and the breaking radius is close to \(a_L\). The exact location of the breaking radius depends on the obliquity of the BH spin, where retrograde spins allow for larger breaking radius. Overall, the breaking radius is about five times smaller than the Laplace radius. Thus, for simplicity, we set the warping/breaking radius \((a_w)\) in terms of the Laplace radius \((a_L)\) in the following discussions.

2.3. **Spin precession with circum-BH disks**

Within the Laplace radius, the circum-BH disk strongly couples with the BH spin and precesses with it, similar to the precession of Saturn with its satellites and circum-planet disk within the Laplace radius (Ward & Hamilton 2004, Rogoszinski & Hamilton 2020). Here, we consider the spin precession of the central BH, taking into account both the de Sitter precession of the BH itself, due to the curved spacetime produced by its BH companion, as well as the precession of the disk due to the Newtonian tidal force of the companion.

The disk around the central BH produces a quadrupole moment \(J_2\), and the torque due to the BH companion on the \(J_2\) moment can be expressed in the secular limit as follows (Goldreich 1966):

\[
\frac{d\hat{s}}{dt}_{\text{disk}} = -\frac{3Gm_2J_2m_1R^2}{a_3^3L_{\text{spin}}}(\hat{1} \cdot \hat{s})(\hat{1} \times \hat{s})
\]  

(2)

where \(\hat{s} = L_{\text{spin}}/L_{\text{spin}}\) is the unit vector pointing in the direction of the spin, and \(\hat{1}\) is the unit vector along the orbit normal. \(L_{\text{spin}}\) is the total angular momentum of the BH and the close-in disk strongly coupled to the BH spin. For simplicity, we assume that the disk mass is much lower than that of the central BH and the angular momentum of the BH dominates:

\[
L_{\text{spin}} = \frac{GM_2^2\chi}{c}
\]  

(3)

and \(J_2\) is the quadrupole moment introduced by the disk, and it can be expressed as follows, similar to planets in a circumplanetary disk (e.g., Rogoszinski & Hamilton 2020):

\[
J_2 = f a m^2 \Sigma(a) a_3^3 da/m_1 R^2
\]  

For simplicity, we express the integral \(\int f a m^2 \Sigma(a) a_3^3 da/m_1 R^2\) as \(f_J m_1 a_3^3\). Then, \(ds/dt\) can be simplified as follows:

\[
\left(\frac{d\hat{s}}{dt}\right)_{\text{disk}} = -\frac{3f_J c m_2 a_3^2}{a_2^3 m_1 \chi}(\hat{1} \cdot \hat{s})(\hat{1} \times \hat{s})
\]  

(4)

Meanwhile, the BH spin precesses due to the de Sitter precession caused by the BH companion’s spacetime curvature. The precession rate can be expressed as follows assuming the eccentricity is zero (Barker & O’Connell 1975):

\[
\left(\frac{d\hat{s}}{dt}\right)_{\text{CMB}} = \frac{3Gn(m_2 + \mu/3)}{2c^2 a_2^3}(\hat{1} \times \hat{s})
\]  

(5)

where \(\mu = m_1 m_2/(m_1 + m_2)\) is the reduced mass of the BHB, and \(n = \sqrt{G(m_1 + m_2)/a^3}\) is the mean motion of the binary. Note that the de Sitter precession is opposite in direction to the Newtonian disk precession.

To compare the two precession rates, we show in Figure 3 their magnitudes for \(m_1 = 10M_\odot\) and \(M_\odot\), respectively, with \(f_J = 10^{-4}\). We set \(e_2\) to be zero for simplicity, and we assume that the spin is nearly aligned with the orbit (\(\hat{1} \cdot \hat{s} \sim 1\)). For illustration, we set \(\chi = 0.9\). The precession due to the circum-BH disk dominates over de Sitter precession. Note that the precession rate due to the disk is larger for BHs with lower spin coefficients.

3. **ORBITAL PRECESSION AND SPIN-ORBIT RESONANCE**

Spin-orbit resonance occurs when the spin precession frequency coincides with that of the orbit. The BHB orbit precesses due to the \(J_2\) potential of the disk and the Lense-Thirring precession (Barker & O’Connell 1975).

Assuming eccentricities to be zero for simplicity, we get:

\[
\left(\frac{d\hat{l}}{dt}\right)_{LT} = \frac{GS_1(4 + 3m_2/m_1)}{2c^2 a_2^3}(\hat{s}_1 - 3(\hat{1} \cdot \hat{s}_1)\hat{1} + GS_2(4 + 3m_1/m_2)\frac{2c^2 a_2^3}{a_2^3}(\hat{s}_2 - 3(\hat{1} \cdot \hat{s}_2)\hat{1}) \times \hat{1}
\]  

(6)

\[
\left(\frac{d\hat{l}}{dt}\right)_{J_2} = -\frac{3}{2}n_2 f_J \frac{(a_1 a_2)}{a_2^3}(\hat{a}_1 \cdot \hat{l})(\hat{s}_1 \times \hat{l}) - \frac{3}{2}n_2 f_J \frac{(a_1 a_2)}{a_2^3}(\hat{a}_2 \cdot \hat{l})(\hat{s}_2 \times \hat{l})
\]  

(7)

Here, we consider the spin of both of the BHB components, and subscript 1 and 2 denote that properties of \(m_1\) and \(m_2\) separately. Note that we neglected the BHB spin-spin coupling, assuming the orbital angular momentum to be much larger than that of the spin.

In addition, when orbiting around the SMBH \((m_3\) in Figure 4), the binary precesses due to the Newtonian perturbation of \(m_3\) (Naoz 2016). Assuming the inclination of the binary relative to the orbit around \(m_3\) to be low, the nodal precession of the orbit is simple and can be...
expressed as follows:

$$\frac{\dot{a}_3}{a_3} = \frac{-3n_2 m_3}{4(m_1 + m_2)} \left( \frac{a_2}{a_3 \sqrt{1 - e_3^2}} \right)^3 (l_3 \times \hat{l})$$  \hspace{1cm} (8)$$

Note that we neglect the precession due to the circum-SMBH disk and the circum-BHB disk. The disks produce retrograde BHB nodal precession, so the effects are qualitatively similar. Nevertheless for simplicity, we assume that the precession due to the SMBH dominates as the SMBH mass is much more massive than the disk.

Spin-orbit resonance occurs when the spin precession rate matches that of the orbit. We note that the orbital Lense-Thirring precession rate and the $J_2$ precession rate are much lower than the spin precession rate due to de Sitter precession and $J_2$ separately. Thus, we only consider the orbital precession due to the SMBH here.

Equating the spin precession due to the circum-BH disk (eq. 4) and the orbital nodal precession due to the SMBH (eq. 8), we determine the location of the BHB where the spin-orbit resonance occurs in the AGN disk:

$$a_{3,\text{crit}} = \left( \frac{1}{4} \frac{a_2^3}{c} \frac{n_2}{m_1 m_3} \frac{m_1 m_3}{(m_1 + m_2) m_2} \frac{\chi}{f_j} \right)^{1/3}$$  \hspace{1cm} (9)$$

substituting the expression for the Laplace radius (eq. 1), we obtain:

$$a_{3,\text{crit}} = \left[ \frac{1}{4} \frac{a_2^3}{c} \frac{n_2}{R_g} \left( \frac{a_2}{R_g} \right)^{2/3} \frac{m_3}{m_1 + m_2} \left( \frac{\chi m_1}{m_2} \right)^{5/9} \right]^{1/3},$$  \hspace{1cm} (10)$$

where $R_g$ is the gravitational radius of the central BH.

In the upper panel of Figure 4 we show the critical orbital radius around the SMBH where the BHB component spins may be excited. The solid line represents the analytical results using eq. (10). We set $m_1 = 20M_\odot$, $m_2 = 50 M_\odot$, $m_3 = 10^7 M_\odot$, and we set the spin coefficient of $m_3$ to be $\chi = 0.7$. The $f_j$ coefficient is set to be $10^{-4}$. The figure shows that spin-orbit resonance occurs at a distance of $\sim 10^{-2}$ AU from the SMBH.

To compare our analytical results with numerical simulation, we overplot the maximum obliquity of $m_1$ obtained by integrating equation 4 and 8 with a constant disk $J_2$ moment ($f_j = 10^{-4}$) (shown in the upper panel of Figure 4). We adopt the same parameters as in the analytical approach, and we set the initial obliquity to be 0° and the initial inclination to be 5°. The analytical and numerical results agree well as the obliquity gets excited near the solid line. Below the critical semi-major axis ($a_{3,\text{crit}}$), the obliquity increases from zero to $\sim 10^4$° due to orbital precession driven by the companion BH ($m_2$). Above the critical semi-major axis, the obliquity remains zero due to the strong spin-orbit coupling. The dashed green line corresponds to the tidal disruption radius of the BHB.

Next, in the lower panel of Figure 4 we include a specific example, in order to illustrate the evolution of the spin-orbit misalignment over time. As the disk mass decays, the spin-precession rate of the BH decays and possibly crossing the spin-orbit resonance. Figure 4 illustrates the obliquity excitation due to the spin-orbit resonance. We integrate the spin and orbital evolution using eq. 3, 5, and 8 using a fourth order Runge Kutta method. We set the BH masses to be 50 $M_\odot$ and 100 $M_\odot$, and the SMBH mass to be $10^7 M_\odot$. We assume that $J_2$ moment of the disk decays exponentially with different timescales as shown. The initial $J_2$ coefficient of the disk is $f_{j,0} = 10^{-4}$. The blue line corresponds to a decay timescale of $10^6$ yr, and the red one corresponds to that of $10^9$ yr.

The lower panel of Figure 4 shows that the obliquity can indeed be excited during the spin-orbit resonance. The blue line corresponds to a shorter disk decay timescale $10^6$ yrs and the red line corresponds to a 10$^9$yr disk decay timescale. The final obliquity is insensitive to the decay timescale as long as the decay timescale is much longer than the oscillation timescale of the spin-orbit misalignment.

The timescale of the obliquity excitation depends on the disk decay timescale (or the resonance sweeping timescale). This can be shorter than the realignment timescale of the disk ($\sim 10$Myr depending on the detailed disk properties as discussed in King et al. 2008; Shan & Li 2018).
This allows the BH spin not to be realigned with the outer disk immediately, even when the inner disk is not torn apart from the outer disk. Furthermore, it is shorter than the Salpeter timescale, and this allows the spin-orbit resonance to dominate over accretion effects.

4. MONTE CARLO SIMULATIONS

How does spin-orbit resonance affect the spin orientations of the BHBs in general? In this section, we run a suite of Monte Carlo simulations to investigate the distribution of spin-orbit misalignment ($\epsilon$), the effective and precession spin parameters ($\chi_{\text{eff}}$, $\chi_p$).

For illustration, we sample the distances from the SMBH uniformly between $a_3 = 1000 - 5000$ AU, and we set $a_2 = 1$ AU and $\chi_{\text{eff}} = 0.7$. In addition, we set $f_J(t = 0) = 10^{-4}$ and the exponential decay timescale of $f_J$ to be $10^5$ yr to ensure that most of the runs encounter the spin-orbit resonance. The total simulation time is set to be $0.23$ Myr, so that $f_J$ decays to $10^{-5}$ at the end of the simulation. We sample the mass $m_1$ from an initial mass function:

$$
\frac{dN}{dm} \propto m^{-\beta}
$$

in the mass range $20-150$ $\text{M}_\odot$. We set $\beta = 2.3$, reflecting the progenitor of the BH (Kroupa 2001). In addition, we adopt a flat mass ratio distribution for the inner binary $m_2/m_1$.

Figure 5 shows the results of the Monte Carlo simulations. We ran a total of 1000 simulations. The initial obliquities are all zero, and the initial effective spin parameters are all 0.7.

At the end of the simulations, the obliquity of the BHs can reach $\sim 60-70^\circ$ due to the spin-orbit resonance, and the spin-spin misalignment can reach $\sim 100^\circ$. As a result, the distribution of the effective spin parameter $\chi_{\text{eff}}$ becomes broadened and reaches $\sim 0.3$. This can affect the final distribution of the effective spin parameters of the BHs through their evolution in the disks. Note that we set the inclination of the binary to be 5°, and higher orbital inclination can lead to larger obliquities and a wider spread of spin parameters.

The upper panel of Figure 5 shows the scatter plot of the binary components’ obliquities. The color shows the mass ratio of the binary ($q = m_2/m_1$). The lower the mass ratio, the larger the scatter in $\epsilon_1$ versus $\epsilon_2$. This is different from Monte Carlo simulations that neglected spin-orbit resonance (Tagawa et al. 2020a), where the obliquities of the BHB components typically get excited to the same values during hard binary-single encounters. This is because the spin-orbit resonance depends on the mass of the BHs, and the larger the difference in the BH masses, the more different are the obliquities of the binary components.

We find that there are two distinct clusters in the $\epsilon_1$ versus $\epsilon_2$ plot respectively peaked at $10 - 20^\circ$ and at $40 - 50^\circ$. This is because the obliquities are typically excited above $\sim 40^\circ$, which is 90° in the past, and this leads to a dearth of systems with obliquities near $\sim 20-30^\circ$.

The lower panel of Figure 5 plots the scatter plot of $\chi_{\text{eff}}$ versus $\chi_p$. The final $\chi_{\text{eff}}$ and $\chi_p$ are well correlated, as the tilt of the BH spins leads to lower $\chi_{\text{eff}}$ and larger $\chi_p$ values. There are also two clusters in the plane of $\chi_{\text{eff}}$ versus $\chi_p$, also due to the dearth of systems with obliquities near $\sim 20-30^\circ$ via the spin-orbit resonance.

5. CONCLUSION AND DISCUSSIONS

In this letter, we studied changes in the spin orientation of BHB components embedded in the AGN-disk due to spin-orbit resonance. Considering the torque acting on the inner disk of a BHB component, we find that spin-orbit resonance can drive large BH spin obliquities ($\sim 60^\circ$). This is analogous to the tilt of Saturn’s axis considering the effects of the satellites and the close-in circumplanetary disk (Ward & Hamilton 2004; Rogoszinski & Hamilton 2020).

We find that spin-orbit resonance can take place for BHBs orbiting a SMBH ($10^7$ $\text{M}_\odot$) at a distance of $\sim 10^2-4$ AU with a binary separation of $\sim 0.1 - 10$ AU. Using a set of Monte Carlo simulations, we find that the spin-orbit resonance can broaden the distributions of obliquities of the BHB components and effective spin parameters. In addition, the final obliquities show a bimodal distribution centers around $\lesssim 20^\circ$ and $\gtrsim 40^\circ$ respectively, and this can serve as a smoking gun for the spin-orbit resonance. Moreover, GW echos can identify sources situated in the vicinity of a SMBH and further constrain whether the obliquity increases could be due to spin-orbit resonance (Gondán & Kocsis 2021).

We note that we only focus on the effects due to the
spin-orbit resonance in this letter. Subsequent evolution of the BH spin-orbit misalignment due to interactions with stellar flyby/BHs can greatly enhance the inclination of the orbits with respect to the disk and increase the spin-orbit misalignment of the BHs. In addition, gas accretion onto the BHs in a turbulent disk can also change the spin-orientation of the BHs. These effects can allow $\chi_{\text{eff}}$ to center around zero and agree with observational results, as discussed in McKernan et al. (2020); Tagawa et al. (2020a).

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APPENDIX

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