Integer Linear Programming Model and Algorithm to Integrate Heuristics Scheduling EDD, Inventory Control and Distribution Problems in a Modular Production System

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Abstract. In this paper, a two-stage algorithm developed for the cutting sequencing problem in a modular manufacturing system. This system has a level of flexibility depends on the cutting stage of raw material. This paper focuses primarily on the work station. In the first stage of the algorithm, an integer linear programming model is used to determine the number of hardboards that will be cut. The model was tested with two different objective function is to minimize waste and to minimize the number of hardboards used. In the second stage, a heuristic developed a scheduling pattern of cutting, where the due date and the number of requests demand a consideration. This algorithm is further implemented in a furniture manufacturer that operates using the make-to-order basis.

Keywords: Manufacturing system, sequential production, algorithm, linear programming, EDD scheduling

1. Introduction
Currently, rapid industrial development leads to high competition. To be able to survive in the competition and the development of this industry, the response of companies to meet consumer demand for products that often change at any time become one of the important goals for each manufacturing company who wants to achieve success. One of them is the timber industry must always deal with consumer demand for wood products size variations produced.

Wood processing industry usually considers the stage of cutting wood in minimizing the residual waste or congestion in the work station. Much research has been done to solve the problems. This can be seen from the formulation of the first model proposed by [1-6].

[1] discussed several basic formulation issues and procedures to solve a cutting problem. Settlement procedures are summarized in a linear programming model. Further sequencing pattern to cut or allocation patterns, analyzed in a study conducted by [2] in order to minimize the maximum queue in the process of cutting. [3-6] proposed a heuristic approach in allocating sawmill.

In this study, a two-stage algorithm was developed for the cutting sequencing problem of a furniture manufacturer XYZ. In the first stage, linear integer programming model is used to determine the number of sheets of wood to be cut by each type of pattern. This cutting pattern produced by a special software program. This cutting pattern is one of the input models. The model was tested by using two different objective functions, which is about minimizing waste, and others minimize the amount of wood to be
cut. The second stage consists in a heuristic determined sequence cutting patterns obtained in Phase 1 by considering the maturity date of the order, and taking into consideration the demands of the item.

This paper is organized in the following order: the first part is introduction, the second part presents two-stage algorithm, the third part of the implementation of the algorithm on a furniture company XYZ and the final section presents conclusions and recommendations.

2. Two-phase algorithm

Phase 1: Integer Linear Programming Models

At this first phase, a linear programming model is developed. This model was used to determine the number of hardboards that will be cut by each cutting pattern. The number of hardboards that will cut the \( i \)-th cutting pattern given the symbol \( x_i \). There are three types of constraints used in this model, namely:

a) Constraints that ensure that all products demanded by customers will be produced.

\[
\sum_{i=1}^{p} \alpha_{ik} x_i \geq d_k, \quad \forall k
\]

where:

\( \alpha_{ik} \) = the number of product type \( k \) produced using the cutting pattern \( i \)
\( d_k \) = total demand of product type \( k \)
\( P \) = the number of cutting pattern

b) Constraints which considers the capacity of the hardboards used.

\[
\sum_{i=1}^{p} x_i \leq Q
\]

where the value of \( Q \) is the available capacity of the hardboards used.

c) Constraints that ensure nonnegativity integer number for all variables.

\[
x_i \geq 0 \quad \text{for all integer } i
\]

The objective to be achieved in this model is to determine the number of sheets of cut blocks in each cutting pattern. This model was tested by applying two different objective functions, but uses the same constraints for both formulations as a comparison.

The first objective function aims to minimize the total waste (called by Model 1), that is

\[
Z = \sum_{i=1}^{p} \left( LWH - \sum_{k=1}^{N} l_k w_k h_k \alpha_{ik} \right) x_i
\]

where:

\( l \times w \times h \) = product size \((\text{length} \times \text{width} \times \text{height})\)
\( L \times W \times H \) = raw material size \((\text{length} \times \text{width} \times \text{height})\)

The second objective function aims to minimize the number of hardboards that will be cut or used (called by Model 2), that is

\[
Z = \sum_{i=1}^{p} x_i
\]

Data used in both models are the size of the hardboards, hardboards capacities, demand for products and various combinations of cutting patterns. If the both above mathematical model is rewritten back, then we gained two models as follows.
Model 1: Mathematical model with the aim of minimizing waste

Minimize waste:

\[ Z = \sum_{i=1}^{p} \left( LWH - \sum_{k=1}^{N} l_k w_k h_k \alpha_{ik} \right) x_i \]

Subject to:

\[ \sum_{i=1}^{p} \alpha_{ik} x_i \geq d_k \quad \text{for } \forall k \]
\[ \sum_{i=1}^{p} x_i \leq Q \]
\[ x_i \geq 0, \text{ integer for } \forall i. \]

Model 2: Mathematical model with the aim of minimizing the number of hardboards used.

Minimize the number of hardboards:

\[ Z = \sum_{i=1}^{p} x_i \]

Subject to:

\[ \sum_{i=1}^{p} \alpha_{ik} x_i \geq d_k \quad \text{for } \forall k \]
\[ \sum_{i=1}^{p} x_i \leq Q \]
\[ x_i \geq 0, \text{ integer for } \forall i. \]

Phase 2: Heuristics Scheduling

In the second phase contains about heuristic calculation to determine the scheduling of cutting patterns obtained from the Phase 1. This phase also considers the number of demand for each product on each request. Calculation of the heuristic scheduling is to follow the EDD (Earliest Due Date) rule. This is done in achieving one goal to minimize delays in production scheduling and delivery in order to meet consumer demand and companies are not penalized due to delay of the consumer.

Initialization:

Input the following values:

\[ x_i = \] the number of hardboards that will cut by the i-th cutting pattern
\[ \alpha_{ik} = \] the number of product k produced by the i-th cutting pattern
\[ d_{jk} = \] total demand of product k in order-j

Step 1

Sequence the order according to the EDD rule.

Step 2

Write the vector of demand-j using formula:

\[ O_j = (d_{j1}, d_{j2}, \ldots, d_{jk}) \]

Step 3

Compute the weight of the element of demand-j by normalizing the value using formula:

\[ w_{jk} = \frac{d_{jk}}{\max_k \{d_{jk}\}} \quad \forall k \]

Step 4

Compute the matching score between demand-j and pattern-i using formula:
\[ MS_i = \sum_{k=1}^{N} \alpha_{ik} w_{jk} \quad \forall i \]  

where \( N \) is the number of product type.

**Step 5**

Select the best pattern \( \theta \) having the highest machine value \( MS_i \) using formula:

\[ MS_{\theta} = \max_i \{ MS_i \} \]  

**Step 6**

Determine the required number of hardboards cut by pattern \( \theta \) for all product \( k \) using formula:

\[
NC_{\theta k} = \begin{cases} 
  d_{jk} / \alpha_{\theta k} , & \text{if } \alpha_{\theta k} > 0 \text{ and } d_{jk} > 0 \\
  0 , & \text{elsewhere}
\end{cases}
\]  

**Step 7**

Determine the required maximum number of hardboards cut by pattern \( \theta \) from among \( NC_{\theta k} \) determined in Step 6 using formula:

\[ NC_{\theta k^*} = \max_k \{ NC_{\theta k} \} \]  

**Step 8**

Determine the actual number of hardboards cut by pattern \( \theta \) using formula:

\[
NC_{\theta k^*} = \begin{cases} 
  x_{\theta} , & \text{if } NC_{\theta k^*} > x_{\theta} , \\
  \text{set } x_{\theta} \leftarrow 0 \\
  nc_{\theta k^*} , & \text{if } x_{\theta} \geq NC_{\theta k^*} , \\
  \text{set } x_{\theta} \leftarrow x_{\theta} - nc_{\theta k^*}
\end{cases}
\]  

**Step 9**

Update the vector of demand-\( j \) using formula:

\[ ns_k \leftarrow \alpha_{\theta k} nc_{\theta k^*} + ns_k \]  

\[ d_{jk} \leftarrow d_{jk} - ns_k \quad \forall k \]  

where \( ns_k \) is the number of product \( k \) in stock. Update the vector of pattern \( \theta \) by:

\[ \alpha_{\theta k} \leftarrow \begin{cases} 
  0 , & \text{if } x_{\theta} = 0 \\
  \alpha_{\theta k} , & \text{elsewhere}
\end{cases} \]  

If \( d_{jk} \leq 0 \), for all product \( k \), go to Step 2, and read the next demand-\( j \). If not, go to Step 4, and select another pattern, while renormalize the element of vector demand-\( j \) (Step 3).

**Step 10**

Repeat Step 2 – 9 until all order vectors are exhausted.

3. Algorithm Implementation

The proposed two-stage algorithm is implemented in a medium-size furniture manufacturer located in Jakarta Indonesia, named XYZ Company. XYZ Co. is a manufacturing company engaged in timber production. Products produced by the XYZ Co. are wooden sticks that have the length, width and thickness in accordance with consumer demand. Usually the consumer will change the wood products into other products are ready for use. Therefore, XYZ Co. maybe said as a company that produces semi-finished goods for consumers, which will be reprocessed into finished goods.

Raw material used by this company is the raw material sheet of wood beams. In this study, the size of the raw materials used in the hardboards has a length of 2000 mm, width 200 mm and 90 mm thick. Whereas the capacity availability of raw material of hardboards sent by the supplier is about 350 m\(^3\) which is equivalent to 9722 units of the hardboards. This study used 5 products derived from the reservation that are product A, B, C, D and E.
Here is a consumer demand for each product that is written in Table 1.

Table 1. Consumers’ demand for each product (in mm$^3$)

| Product Type | Demand 1 | Demand 2 | Demand 3 | Total Demand |
|--------------|----------|----------|----------|--------------|
| A            | 1062     | 892      | 937      | 2891         |
| B            | 904      | 867      | 880      | 2651         |
| C            | 672      | 604      | 0        | 1276         |
| D            | 28       | 3        | 33       | 64           |
| E            | 43       | 64       | 7        | 114          |

Where the due date demand 1 < due date demand 2 < due date demand 3.

Variety of combination is obtained by using the help of Visual Basic 6.0 software to produce 13 combinations as shown in Table 2.

Table 2. Variety of combination of cutting

| No. | Combination Type | A  | B  | C  | D  | E  | Remaining Volum (mm$^3$) | Code |
|-----|------------------|----|----|----|----|----|---------------------------|------|
| 1   | BA               | 6  | 4  | 0  | 0  | 0  | 5350040                   | LL   |
| 2   | AE               | 6  | 0  | 0  | 0  | 2  | 5393336                   | LL   |
| 3   | BB               | 0  | 8  | 0  | 0  | 0  | 9445120                   | LL   |
| 4   | EB               | 0  | 4  | 0  | 0  | 2  | 9488416                   | LL   |
| 5   | E                | 0  | 0  | 0  | 0  | 4  | 9531712                   | PP   |
| 6   | CA               | 6  | 0  | 2  | 0  | 0  | 9787880                   | LL   |
| 7   | AC               | 2  | 0  | 4  | 0  | 0  | 12529960                  | PP   |
| 8   | CB               | 0  | 4  | 2  | 0  | 0  | 13882960                  | LL   |
| 9   | EC               | 0  | 0  | 2  | 0  | 2  | 13926256                  | LL   |
| 10  | BA               | 2  | 4  | 0  | 0  | 0  | 16931720                  | PP   |
| 11  | D                | 0  | 0  | 0  | 2  | 0  | 18092880                  | LL   |
| 12  | CC               | 0  | 0  | 4  | 0  | 0  | 18320800                  | LL   |
| 13  | AAA              | 6  | 0  | 0  | 0  | 0  | 18627480                  | PP   |

Phase 1 Integer Linear Programming Model

Model 1

Using the data shown on Table 1 and Table 2 and considering the available capacity, then Model 1 becomes:

Objective function:

Minimize $Z = 5350040 x_1 + 5393336 x_2 + 9445120 x_3 + 9488416 x_4 + 9531712 x_5 + 9787880 x_6 + 12529960 x_7 + 13882960 x_8 + 13926256 x_9 + 16931720 x_{10} + 18092880 x_{11} + 18320800 x_{12} + 18627480 x_{13}$

Subject to:

$6x_1 + 6x_2 + 6x_6 + 2x_7 + 2x_{10} + 2x_{13} \geq 2891$

$4x_1 + 8x_3 + 4x_4 + 4x_8 + 4x_{10} + 4x_{12} \geq 2651$

$2x_6 + 4x_7 + 2x_8 + 2x_9 + 4x_{12} \geq 1276$

$2x_{11} \geq 64$

$2x_2 + 2x_4 + 4x_5 + 2x_9 \geq 114$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} \leq 9722$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13} \geq 0$ dan integer

where:

$Z = $ waste

$X_i = $ the number of hardboards that will be cut by cutting pattern $i (i = 1, 2, \ldots, 13)$
By Using software WinQSB, we get the results as follows:

\[ x_1 = 375 \quad x_8 = 0 \quad \Sigma x = 899 \text{ units} \]
\[ x_2 = 1 \quad x_9 = 0 \]
\[ x_3 = 142 \quad x_{10} = 0 \]
\[ x_4 = 4 \quad x_{11} = 32 \]
\[ x_5 = 26 \quad x_{12} = 0 \]
\[ x_6 = 0 \quad x_{13} = 0 \]
\[ x_7 = 319 \quad Z = 8214672000 \text{ mm}^3 \]

**Model 2**

Using the data shown on Table 1 and Table 2 and considering the available capacity, then Model 2 become:

**Objective function:**

\[
\text{Minimize } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}
\]

**Subject to:**

\[
\begin{align*}
6x_1 + 6x_2 + 6x_6 + 2x_7 + 2x_{10} + 2x_{13} & \geq 2891 \\
4x_1 + 8x_3 + 4x_4 + 4x_8 + 4x_{10} & \geq 2651 \\
4x_6 + 4x_7 + 2x_8 + 2x_9 + 4x_{12} & \geq 1276 \\
2x_2 + 2x_4 + 4x_5 + 2x_9 & \geq 114 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} & \leq 9722
\end{align*}
\]

where:

- \( Z \) = waste
- \( x_i \) = the number of hardboards that will be cut by cutting pattern \( i \) (\( i = 1, 2, \ldots, 13 \))

By Using software WinQSB, we get the results as follows:

\[ x_1 = 376 \quad x_8 = 0 \]
\[ x_2 = 0 \quad x_9 = 0 \]
\[ x_3 = 143 \quad x_{10} = 0 \]
\[ x_4 = 1 \quad x_{11} = 32 \]
\[ x_5 = 28 \quad x_{12} = 0 \]
\[ x_6 = 0 \quad x_{13} = 0 \]
\[ x_7 = 319 \quad Z = 899 \text{ units} \]

**Phase 2: Heuristics Scheduling**

**Heuristics Scheduling of Model 1**

- \( i \) = cutting pattern \( i \)-th (1, 2, 3, ..., 13)
- \( j \) = demand \( j \)-th (1, 2, 3)
- \( k \) = product type (A, B, C, D, E)
- \( x_i \) = the number of hardboards that will be cut by cutting pattern \( i \)-th
- \( \alpha_{ik} \) = the number of product type \( k \) produced by cutting pattern \( i \)-th
- \( d_{jk} \) = the total demand of product type \( k \) in demand \( j \)-th

Cutting scheduling to minimize waste shown on the following table.
Table 3 Cutting scheduling to minimize waste

| Scheduling No. | Cutting pattern (i) | The number of hardboards that will be cut by cutting pattern i-th \(x_i\) | Total demand |
|----------------|--------------------|-------------------------------------------------|--------------|
| 1              | 1                  | 226                                             | 1            |
| 2              | 7                  | 168                                             | 1            |
| 3              | 5                  | 11                                              | 1            |
| 4              | 11                 | 14                                              | 1            |
| 5              | 3                  | 109                                             | 2            |
| 6              | 7                  | 151                                             | 2            |
| 7              | 5                  | 15                                              | 2            |
| 8              | 11                 | 2                                               | 2            |
| 9              | 4                  | 2                                               | 2            |
| 10             | 1                  | 149                                             | 3            |
| 11             | 3                  | 33                                              | 3            |
| 12             | 11                 | 16                                              | 3            |
| 13             | 4                  | 2                                               | 3            |
| 14             | 2                  | 1                                               | 3            |

The number of hardboards used = 375 + 1 + 142 + 4 + 26 + 0 + 319 + 0 + 0 + 32 + 0 + 0 = 899 units
Total waste = 8214672952 mm\(^3\)
The percentage of total waste of all the sheets beam used is
\[
\frac{\text{Total Waste}}{\sum \text{total sheet beam used} \times \text{Volum of the sheet beam}} \times 100\% = 25.38\%
\]

Heuristics Scheduling of Model 2
Cutting scheduling to minimize the number of hardboards shown on the following table.

Table 4 Scheduling cutting patterns for minimizing beam sheets

| Scheduling No. | Cutting pattern (i) | The number of hardboards that will be cut by cutting pattern i-th \(x_i\) | Total demand |
|----------------|--------------------|-------------------------------------------------|--------------|
| 1              | 1                  | 226                                             | 1            |
| 2              | 7                  | 168                                             | 1            |
| 3              | 5                  | 11                                              | 1            |
| 4              | 11                 | 14                                              | 1            |
| 5              | 3                  | 109                                             | 2            |
| 6              | 7                  | 151                                             | 2            |
| 7              | 5                  | 16                                              | 2            |
| 8              | 11                 | 2                                               | 2            |
| 9              | 1                  | 150                                             | 3            |
| 10             | 3                  | 34                                              | 3            |
| 11             | 11                 | 16                                              | 3            |
| 12             | 4                  | 1                                               | 3            |
| 13             | 5                  | 1                                               | 3            |

The number of hardboards used = 376 + 0 + 143 + 1 + 28 + 0 + 319 + 0 + 0 + 32 + 0 + 0 = 899 unit beam sheet
Total waste = 8214672952 mm\(^3\)
The percentage of total waste of all the sheets beam used is
\[
\frac{\text{Total Waste}}{\sum \text{total sheet beam used} \times \text{Volum of the sheet beam}} \times 100\% = 25.38\%
\]
From these two model formulations obtained that the total waste of raw materials are the same that is 8214672952 mm³, and the percentage of total waste of all the sheets beam used is is equal to 25.38%.

Table 5 Comparison results of Model 1 and Model 2

| Model      | Total number of hardboards used (units) |
|------------|----------------------------------------|
| Model 1    | 899                                    |
| Model 2    | 899                                    |

The difference results of both calculations minimizing waste and minimization the hardboards occurs on the order of scheduling based on the EDD, which model of minimize waste obtained 14 cutting pattern scheduling sequence and the model of minimize the total number of hardboards used obtained 13 cutting pattern scheduling sequence. Because the results of both formulations are the same, then the decision to apply one of the two model formulations and cutting scheduling handed to the company.

4. Conclusions and Recommendations

In this study, a two-stage algorithm is developed for the cutting sequencing problem of a furniture manufacturer located in Jakarta. In the first stage, an integer linear programming model is used to determine the number of hardboards that will be cut by each pattern type. The model is tested with two different objective functions for comparison. The cutting patterns, generated by a special software program, are among the inputs of this model. The second stage consists of a heuristic that decides on the sequencing of the cutting patterns that are obtained in stage 1 by considering the due date of the orders, and taking into account the demands of the items. The implementation results were discussed with the planners working in the furniture manufacturer, and they indicated that the proposed algorithm could be employed as a useful tool for the cutting sequencing problem of the company.

5. References

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