Quantum coherence and speed limit in the mean-field Dicke model of superradiance

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Dicke superradiance is a cooperative phenomenon which arises from the collective coupling of an ensemble of atoms to the electromagnetic radiation. Here we discuss the quantifying of quantum coherence for the Dicke model of superradiance in the mean-field approximation. We found the single-atom \( l_1 \)-norm of coherence is proportional to the square root of the average intensity of radiation emitted by the superradiant system, thus showing that quantum coherence stands as a crucial figure of merit towards to the understanding of superradiance phenomenon in the mean-field approach. Furthermore, given the nonlinear unitary dynamics of the time-dependent single-atom state that effectively describes the system of \( N \) atoms, we analyze the quantum speed limit time and its interplay with the \( l_1 \)-norm of coherence. We verify the quantum coherence speeds up the evolution of the superradiant system, i.e., the more coherence stored on the single-atom state, the faster the evolution. These findings unveil the role played by quantum coherence in superradiant systems, which in turn could be of interest in condensed matter physics and quantum optics platforms.

I. INTRODUCTION

Light-matter interaction plays a striking role on the understanding of several physical phenomena [1], thus being a subject of significant interest to research on laser cooling and atomic trapping [2–5], cavity quantum electrodynamics (CQED) [6, 7], and more recently quantum computing [8]. Noteworthy, Dicke model, which describes the coupling of a single mode of the radiation field with an ensemble of two-level systems, stands as a paradigmatic toy model from quantum optics [9–12]. In turn, the collective and coherent interaction can promote the well-known Dicke superradiance, in which the system spontaneously emits radiation at high intensity in a short time window [13, 14]. Experimental realization of superradiance has been proposed in optically pumped gas [15], quantum dots [16], and superfluid gas in optical cavity [17].

Dicke superradiance has been recently addressed under the viewpoint of local quantum uncertainty quantifiers [18], and also quantum correlations [19, 20]. This could motivate a considerable interest in the investigation of the superradiance employing another resource as quantum coherence, i.e., a remarkable fingerprint of non-classical systems linked to the quantum superposition principle which plays an essential role on quantum optics [21], quantum thermodynamics [22], condensed matter physics [23], and biological systems [24]. Furthermore, one can ask if Dicke superradiance somehow accelerates the evolution of the quantum system, the latter remaining as a challenge for the design of faster quantum information-processing devices [25, 26]. In particular, this quantum signature could be captured by the so-called quantum speed limit (QSL), i.e., the minimum time of evolution required for a quantum system evolve between two given states [27–29]. Nowadays, QSL find applications in quantum computation and quantum communication [30], quantum metrology [31], and quantum thermodynamics [32].

In this paper, we discuss the quantifying of quantum coherence, and also the quantum speed limit time, for the Dicke model of superradiance in the mean-field approximation. Coherence is a ubiquitous, basis-dependent quantity in quantum mechanics, which has received widespread attention since its rigorous axiomatic formulation a half decade ago [33]. We find quantum coherence is suddenly suppressed for a large number of atoms, while exhibits a maximum value at the time delay of superradiance (time of maximum intensity), at which in turn is robust to the number of atom increasing. Moreover, we explore the relation of quantum coherence and QSL time, and thus analyzing the former as resource capable to speed up the unitary dynamics of each single two-level atom. Noteworthy, QSL bound saturates as one increases the number of atoms \( N \) in the system.

The manuscript is organized as follows. In Sec. II we briefly review the basic features of the Dicke model of superradiance in view of mean-field approximation. In Sec. III we discuss the role of quantum coherence in the referred model. In Sec. IV we study the quantum speed limit time with regards to the effective nonlinear unitary evolution of two non-orthogonal single-atom pure states. In addition, we investigate the interplay between the QSL bound and quantum coherence. Finally, we summarize our main results and present the conclusions.

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Let us consider the Dicke model of superradiance, a system of \(N\) identical two-level atoms with transition frequency \(\omega\), which interacts collectively with their surrounding electromagnetic field in the vacuum state (zero temperature) \([11, 34]\). For transitions between Dicke states \([35]\), considering \(N \gg 1\) and the system weakly coupled to its environment, the dynamics of this system can be described by the Lindblad-type mean-field master equation \((\hbar = 1)\) \([14, 36]\):

\[
\frac{d\rho_N}{dt} = -i\omega [J^z, \rho_N] + \gamma_0 (J^- \rho_N J^+ - \{J^+ J^-, \rho_N\}),
\]

where \(J^z = (1/2) \sum_{j=1}^N \sigma_j^z\) and \(J^\pm = \sum_{j=1}^N \sigma_j^\pm\) are collective operators, with \(\sigma_j^x\) and \(\sigma_j^z\) denoting the Pauli matrices associated with the \(j\)-th atom, \([X, Y] = XY - YX\) is the commutator, and \([X, Y] = XY + YX\) is the anticommutator. In particular, this mean-field master equation can be mapped onto a nonlinear Schrödinger-type equation by embedding the dynamics of the \(N\)-atom mean-field state \(\rho_N \approx (|\psi_t\rangle \langle \psi_t|)^{\otimes N}\) into an effective unitary dynamics of each two-level atom, described by \(|\psi_t\rangle\), which in turn evolves according to the nonlinear Hamiltonian \(H_t = (\omega/2) \sigma_z + i (N\gamma_0/2) \langle \psi_t | \sigma_x | \psi_t \rangle \sigma_+ - \langle \psi_t | \sigma_- \rangle \langle \psi_t | \sigma_+ \rangle \sigma_-\), where \(\sigma_+ (\sigma_-)\) is the raising (lowering) operator, and \(\gamma_0\) is the spontaneous emission rate of each single two-level atom \([36]\).

The solution for the nonlinear Schrödinger equation \((d/dt)|\psi_t\rangle = -iH_t|\psi_t\rangle\) is described by the time-dependent single-atom state \([36]\):

\[
|\psi_t\rangle = \sqrt{1 - p_t e^{i \frac{\omega}{2} t}} |g\rangle + \sqrt{p_t} e^{-i \frac{\omega}{2} t} |e\rangle,
\]

where \(|g\rangle (|e\rangle)\) stands for the ground (excited) state of the two-level atom. Here

\[
p_t = \left[ e^{\gamma_0 N(t-t_D)} + 1 \right]^{-1}
\]

denotes the probability of finding a single atom in the excited state at time \(t\), with \(0 \leq p_t \leq 1\), while \(t_D = (\gamma_0 N)^{-1} \ln(N)\) stands for the time delay of the superradiance (time of maximum intensity) \([14]\). Finally, with \(|\psi_t\rangle\) given by Eq. (2), the nonlinear Hamiltonian can be written as

\[
H_t = \frac{\omega}{2} \sigma_z - i \frac{N\gamma_0}{2} \sqrt{p_t(1-p_t)} (\sigma_+ e^{-i\omega t} - \sigma_- e^{i\omega t}).
\]

Noteworthy, given the probability distribution \(p_t\) in Eq. (3), the average intensity of radiation emitted by the system of \(N\) atoms can be written as \(I(t) = -N\omega (dp_t/dt) = (N^2\omega\gamma_0/4) \text{sech}^2[(N\gamma_0/2)(t - t_D)]\), which is proportional to \(N^2\), characterizing the superradiance \([36]\). For \(t = t_D\), the system emits radiation with the maximum intensity value \(I_{\text{max}} = I(t = t_D) = N^2\omega\gamma_0/4\).

In this section we briefly introduce the main concepts of quantum coherence and discuss its role on the superradiance phenomenon addressed in Sec. II. Over half decade ago, the seminal work by Baumgratz, Cramer and Plenio \([33]\) have introduced the minimal theoretical framework for the quantification of quantum coherence. This approach opened an avenue for the characterization of quantum coherence under the scope of resource theories, which currently still as a matter of intense debate \([37–41]\).

Here we will briefly review the key aspects of quantifying quantum coherence discussed in Ref. \([33]\). We shall consider a physical system defined on a \(d\)-dimensional Hilbert space \(\mathcal{H}\) endowed with some reference basis \(|\{j\}\rangle\). In turn, the state of the system is described by a density matrix \(\rho \in \mathcal{D}(\mathcal{H})\), where \(\mathcal{D}(\mathcal{H}) = \{\rho^t = \rho, \rho \geq 0, \text{Tr}(\rho) = 1\}\) stands for the convex space of positive semi-definite density operators. Particularly, the subset \(\mathcal{I} \subset \mathcal{D}(\mathcal{H})\) of incoherent states encompasses the family of density matrices as \(\delta = \sum_j q_j |j\rangle \langle j|\) that are diagonal in the reference basis, with \(0 \leq q_j \leq 1\) and \(\sum_j q_j = 1\). In summary, a \textit{bona fide} quantum coherence quantifier \(C(\rho)\) must satisfy the properties \([33, 42]\) (i) non-negativity, i.e., \(C(\rho) \geq 0\) for all state \(\rho\), with \(C(\rho) = 0 \iff \rho \in \mathcal{I}\); (ii) convexity under mixing, i.e., \(\sum_j q_n \rho_n \leq \sum_n q_n C(\rho_n)\), with \(\rho_n \in \mathcal{D}(\mathcal{H})\), \(0 \leq q_n \leq 1\), and \(\sum_n q_n = 1\); (iii) monotonicity under incoherent completely positive and trace-preserving (ICPTP) maps, i.e., \(C(\mathcal{E}[\rho]) \leq C(\rho)\), for all ICPTP map \(\mathcal{E}[\bullet]\); (iv) strong monotonicity, i.e., \(C(\rho) \geq \sum_n q_n C(\rho_n)\), where \(q_n = q_n^{-1} K_n \rho K_n^\dagger\) sets the post-measured states for arbitrary Kraus operators \(\{ K_n\}\) satisfying \(\sum_n K_n^\dagger K_n = \mathbb{1}\) and \(K_n^\dagger K_n \subset \mathcal{I}\), with \(q_n = \text{Tr}(K_n^\dagger K_n)\).

Some widely-known quantum coherence measures include relative entropy of coherence \([33]\), geometric coherence \([43]\), and robustness of coherence \([44]\). In addition, the \(l_1\)-norm of coherence, which is defined in terms of \(l_1\)-distance between \(\rho\) and its closest incoherent state, is of fundamental interest since its exact calculation is readily given by \(C(\rho) = \sum_{j,l,|j\neq l|} |\rho_{jl}|\), where \(|\rho_{jl}|\) sets the off-diagonal elements of \(\rho\) evaluated with respect to the reference basis \(|\{j\}\rangle\). Importantly, for a pure single-qubit state (i.e., \(d = 2\) and \(\rho = |\phi\rangle \langle \phi|\)), the referred family of quantum coherence measures are monotonically related to each other \([42]\). In this context, without any loss of generality, from now on we will address \(l_1\)-norm of coherence to characterize the role of quantum coherence in the superradiant system previously discussed.

From Sec. II, fixing the reference basis \(|\{e\}, |g\}\rangle\) Eq. (2) implies the single-atom density operator

\[
\rho_t = (1 - p_t) |g\rangle \langle g| + p_t |e\rangle \langle e| + \sqrt{p_t(1-p_t)} (e^{i\omega t} |g\rangle \langle e| + e^{-i\omega t} |e\rangle \langle g|),
\]

with \(p_t\) given in Eq. (3). In this case, \(l_1\)-norm of cohe-
ence is given by
\[ C(\rho_t) = \text{sech}\left(\frac{N\gamma_0}{2}(t-t_D)\right). \tag{6} \]

Note that \( C(\rho_t) \) is a bell-shaped symmetric function over time \( t \), centered at \( t = t_D \), with a full width at half maximum scaling as \( (\gamma_0 N)^{-1} \). For \( t = 0 \), Eq. (6) reduces to \( C(\rho_0) = \text{sech}(N\gamma_0 t_D/2) \), which in turn approaches to zero in the limit \( N \to \infty \) for \( t_D \neq 0 \).

Figure 1(a) shows the density plot of \( C(\rho_t) \) as a function of dimensionless parameters \( \omega(t-t_D) \) and \( N\gamma_0/(2\omega) \). For \( 0 < t \leq t_D \), \( C(\rho_t) \) starts increasing monotonically and reaches its maximum value \( C(\rho_D) = 1 \) at the time delay \( t = t_D \). In fact, \( C(\rho_D) \) is always maximum regardless the value of \( N\gamma_0/(2\omega) \). For \( t > t_D \), \( C(\rho_t) \) decreases monotonically and approaches to zero for \( N\gamma_0/(2\omega) \ll 1 \), the quantum coherence remains approximately constant around its maximum value \( C(\rho_t) \approx 1 \), while for the very overdamped regime, \( N\gamma_0/(2\omega) \gg 1 \), it follows that \( C(\rho_t) \approx 0 \) for all \( t \neq t_D \). In other words, the more atoms the system has, the less quantum coherence stored on each single-atom state and therefore on the \( N \)-atom state (see Eq. (8)), except at time \( t = t_D \).

Quite interestingly, Eq. (6) can be written in terms of the average intensity of radiation, \( I(t) = (N^2 \omega_\gamma^2/4) \text{sech}^2[(N\gamma_0/2)(t-t_D)] \). Indeed, one readily concludes
\[ C(\rho_t) = \frac{\sqrt{I(t)}}{I_{\text{max}}}, \tag{7} \]
with \( I_{\text{max}} = N^2 \omega_\gamma^2/4 \) the maximum intensity value. From Eq. (7), note the single-atom coherence depends on the square root of the normalized average intensity cooperatively emitted by the whole system. This means that, when experimentally measuring the intensity \( I(t) \), one should directly inferring the quantum coherence of a single two-level atom of the system. We shall stress the intensity \( I(t) \) immediately vanishes for the case in which quantum coherence \( C(t) \) is identically zero. Therefore, quantum coherence, instead of quantum entanglement, stands as a crucial figure of merit towards to the understanding of superradiance phenomenon in the mean-field approach. In addition, one may prove the quantum coherence for the uncorrelated \( N \)-particle state \( \rho_N = \rho_t^{\otimes N} \), i.e., the coherence of the entire system, is given by
\[ C(\rho_t^{\otimes N}) = [1 + C(\rho_t)]^N - 1. \tag{8} \]
Particularly, for the case in which the single-particle quantum coherence is much smaller than one, i.e., \( C(\rho_t) \ll 1 \), the \( N \)-particle quantum coherence in Eq. (8) approximately becomes \( C(\rho_t^{\otimes N}) \approx NC(\rho_t) \).

\[ \text{IV. QUANTUM SPEED LIMIT} \]

In this section we briefly introduce the quantum speed limit (QSL) time and relate it to the superradiance phenomenon. Quantum mechanics imposes a threshold on the minimum evolution time required for a system evolve between two given quantum states, which in turn is certified by the QSL [27–29]. Quite remarkable, given a unitary evolution of pure states \( |\psi_0\rangle \) and \( |\psi_\tau\rangle \) generated by a time-independent Hamiltonian \( H \), Mandelstam and Tamm (MT) [27] have proved the QSL bound \( \tau \geq h \text{arccos}(|\langle \psi_0|\psi_\tau\rangle|)/\Delta E \), in which \( (\Delta E)^2 = \langle \psi_0|H^2|\psi_0\rangle - \langle \psi_0|H|\psi_0\rangle^2 \) stands for the variance of \( H \). Later on, Margolous and Levitin (ML) [28] have derived a novel QSL bound for closed quantum systems evolving between two orthogonal states, with time-independent Hamiltonian \( H \), which reads \( \tau \geq \hbar \pi/(2E) \), in which \( E = \langle \psi_0|H|\psi_0\rangle - E_0 \) is the mean energy, and \( E_0 \) the ground state energy of the system. Over a decade after this result, Levitin and Toffoli [29] have shown that, by focusing on the case of orthogonal pure states evolving unitarily, the tightest QSL sets \( \tau_{QSL} = \max\{\pi/(2\Delta E),h\pi/(2E)\} \).

Giovannetti et al. [45] have addressed the case of QSLs for mixed states undergoing unitary evolutions, also concluding that entanglement is able to speed up the evolution of composite systems. For more details on QSLs for closed quantum systems, see Refs. [46–60].

QSL has been also largely investigated for the dynamics of open quantum systems. Indeed, Taddei et al. [61] and del Campo et al. [62] have derived the MT bound for arbitrary physical processes, which can be either unitary or nonunitary. Furthermore, Deffner and Lutz [63] derived another class of MT and ML bounds, also showing that non-Markovian signatures can speed up the nonunitary dynamics. Nevertheless, it has been proved the link between speeding up the evolution and non-Markovianity exists only for a certain class of dynamical maps and initial states [64]. For completeness, we refer to Refs. [65–76] for other derivations and applications of QSLs for open quantum systems.

In Sec. II we have shown that each single two-level atom of the system undergoes an effective unitary evolution governed by the time-dependent nonlinear Hamiltonian \( H_t \). The effective two-level system is initialized in the pure state \( |\psi_0\rangle \), and thus the evolved state \( |\psi_\tau\rangle \) will also be pure during the unitary dynamics for any \( t \in [0,\tau] \). Therefore, here we will dealing with a quantum system undergoing a nonlinear physical process, but still unitary. In this case, the lower bound on \( \tau \), which holds for initial and final pure states undergoing a unitary physical process, is obtained from the inequality \( \tau \geq \tau_{QSL} \), with the QSL time given by [54, 55]
\[ \tau_{QSL} = \frac{\mathcal{L}(\psi_0,|\psi_\tau\rangle)}{\Delta E_{\tau}}, \tag{9} \]
where \( \mathcal{L}(\psi_0,|\psi_\tau\rangle) = \text{arccos}(|\langle \psi_0|\psi_\tau\rangle|) \) is the Bures angle, i.e., a distance measure between quantum states, while \( \Delta E_{\tau} = \tau^{-1} \int_0^\tau dt \Delta E_t \) is the time-average of the variance \( \Delta E_t = \langle \psi_t|H_t^2|\psi_t\rangle - \langle \psi_t|H_t|\psi_t\rangle^2 \) of the time-dependent Hamiltonian \( H_t \). In the particular case where \( |\psi_0\rangle \) is orthogonal to \( |\psi_\tau\rangle \), Eq. (9) reduces to \( \tau_{QSL} = \pi/(2\Delta E_{\tau}) \).
Physically, $\tau_{\text{QSL}}$ sets the minimal time the system requires to evolve between states $|\psi_0\rangle$ and $|\psi_\tau\rangle$, also presenting a geometric interpretation discussed as follows. On the one hand, the unitary evolution of $|\psi_\tau\rangle$ describes an arbitrary path in the manifold of pure states for $t \in [0, \tau]$, thus connecting states $|\psi_0\rangle$ and $|\psi_\tau\rangle$. The length of this path, which generally is not the shortest one with respect to the set of paths draw by $|\psi_\tau\rangle$, is written as $\int_0^\tau dt \Delta E_t$ and depends on the variance of the Hamiltonian $H_t$, which in turn is nothing but the quantum Fisher information metric for the case of pure states. On the other hand, Bures angle describes the length of the geodesic path connecting states $|\psi_0\rangle$ and $|\psi_\tau\rangle$, and is a function of the overlap of both states. Quite remarkable, Bures angle plays the role of a distinguishability measure of quantum states, and stands as the geodesic distance regarding to the quantum Fisher information metric. For a detailed discussion on geometric QSLs, by exploiting the family of Riemannian information metrics defined on the space of quantum states, which in turn encompasses open and closed quantum systems, pure and mixed states, see Ref. [70].

Now we will discuss the role played by the superradiance phenomenon and the collective excitations of Dicke states into the QSL time. In order to see this, we will first proceed with the analytical calculation of QSL ratio $\tau_{\text{QSL}}/\tau$ in Eq. (9). Given that $\langle \psi_t | H_t^2 | \psi_t \rangle = (\omega/2)^2 + (N\gamma_0/2)^2 p_t (1 - p_t)$, and $\langle \psi_t | H_t | \psi_t \rangle = (\omega/2)(2p_t - 1)$, one obtains the time-average of variance as

$$\overline{\Delta E_t} = \frac{1}{2\tau} \sqrt{1 + \alpha^{-2}} \arccos \left[ (1 - 2p_\tau)(1 - 2p_0) + 4\sqrt{p_0 p_\tau (1 - p_0)(1 - p_\tau)} \right], \quad (10)$$

with $\alpha := N\gamma_0/(2\omega)$, while the Bures angle becomes

$$\mathcal{L}(|\psi_0\rangle, |\psi_\tau\rangle) = \frac{1}{2} \arccos \left[ (1 - 2p_\tau)(1 - 2p_0) + 4\sqrt{p_0 p_\tau (1 - p_0)(1 - p_\tau) \cos(\omega \tau)} \right]. \quad (11)$$

Next, we discuss some remarkable features involving the QSL of the evolution due to superradiant transitions between Dicke states. The state $|\psi_\tau\rangle$ becomes maximally distinguishable, i.e., orthogonal, to the initial state $|\psi_0\rangle$, in the limit $\alpha \rightarrow \infty$ for $\tau > t_D$, with $t_D \neq 0$. Moreover, the bound $\tau \geq \tau_{\text{QSL}}$ saturates in the limit $\alpha \rightarrow \infty$, as long as $t_D \neq 0$ ($p_0 \rightarrow 1$). In other words, the increasing of the number of atoms $N$ in the system saturates the QSL, and thus the system evolves along the geodesic path connecting $|\psi_0\rangle$ and $|\psi_\tau\rangle$ in the manifold of pure quantum states.

Let us now discuss the role played by quantum coherence into the QSL time. Without loss of generality, here we will set the number of atoms as $N = 10^6$. It has been proved that entanglement [45] can promote a speed up in the time evolution of a quantum system, while the quantum coherence also plays a non-trivial role on the time evolution [57, 58, 70]. Figure 1(b) shows the density plot for the ratio $\tau_{\text{QSL}}/\tau$ as a function of dimensionless parameters $\omega(\tau - t_D)$ and $N\gamma_0/(2\omega)$. On the one hand, the inequality $\tau \geq \tau_{\text{QSL}}$ suddenly saturates in the overdamped regime $[N\gamma_0/(2\omega) \gtrsim 1]$, region in which $C(\rho_t) \rightarrow 0$ when $t \neq t_D$, as depicted in Fig. 1(a). On the other hand, in the very underdamped regime $[N\gamma_0/(2\omega) \ll 1]$ where $C(\rho_t) \sim 1$, the ratio $\tau_{\text{QSL}}/\tau$ approaches zero, which in turn implies a speed up into the evolution of the two-level system. Therefore, for a fixed difference $\tau - t_D$, the ratio $\tau_{\text{QSL}}/\tau$ decreases as the coherence increases [see Fig. 1(c)], i.e., quantum coherence speeds up the dynamics. In other words, the more coherence, the faster
the evolution. Note the QSL time saturates whenever $C(\rho_t) \approx 0$, or close to the peak of the superradiance intensity, which is marked by the delaying time $\tau \approx t_D$.

It is worth to mention that the aforementioned relation

$$\frac{\tau_{\text{QSL}}}{\tau} = \frac{1}{2} \frac{\arccos \left( C(\rho_0)C(\rho_t) \cos(\omega \tau) - \text{sgn}(\tau - t_D) \sqrt{(1 - C(\rho_0)^2)(1 - C(\rho_t)^2)} \right)}{\sqrt{1 + \alpha^2} \arccos \left( C(\rho_0)C(\rho_t) - \text{sgn}(\tau - t_D) \sqrt{(1 - C(\rho_0)^2)(1 - C(\rho_t)^2)} \right)}.$$ 

(12)

V. CONCLUSIONS

Dicke superradiance, a phenomenon triggered by the collective coupling of atomic levels with the electromagnetic field, is a subject of wide interest in quantum optics, condensed matter, and solid state physics. In this work we discussed the quantifying of quantum coherence in the Dicke model of superradiance, under the mean-field approximation description, particularly focusing on the $l_1$-norm of coherence. We found that, for all $t \neq t_D$, the more particles in the atomic ensemble, the less quantum coherence is stored in the state of the system. Noteworthy, quantum coherence exhibits its maximum value at time delay $t_D$, for any number $N$ of atoms. Furthermore, we show that quantum coherence stands as a crucial figure of merit towards to the understanding of superradiance phenomenon in the mean-field approach.

In addition, given the time-dependent single-atom state describing the effective dynamics of each two-level atom [see Eq. (2)], we address the quantum speed limit time (QSL) $\tau_{\text{QSL}}$, regarding the unitary evolution between two non-orthogonal pure states. We observed the increasing of the number of atoms $N$ in the system implies the saturation of the QSL time. Furthermore, since between QSL and quantum coherence can be directly observed by rewriting Eqs. (10) and (11) in terms of $C(\rho_t)$, such that

QSL time can be recast in terms of $l_1$-norm of coherence [see Eq. (12)], we have seen the QSL ratio $\tau_{\text{QSL}}/\tau$ decreases as the quantum coherence increases, thus concluding the quantum coherence is a resource that speeds up the overall dynamics. Finally, Eq. (7) suggests that by measuring superradiance intensity, one could make possible access experimentally the single-atom quantum coherence in superradiant systems.

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