NEW DEVELOPMENTS IN SPIRAL STRUCTURE THEORY

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ABSTRACT

After a short review of the principal theories of spiral structure in galaxies, I describe two new developments. First, it now seems clear that linear theory cannot yield a full description for the development of spiral patterns because $N$-body simulations suggest that non-linear effects manifest themselves at a relative overdensity of $\sim 2\%$, which is well below the believed spiral amplitudes in galaxies. Second, I summarize the evidence that some stars in the solar neighborhood have been scattered at an inner Lindblad resonance. This evidence strongly supports a picture of spirals as a recurring cycle of transient instabilities, each caused by resonant scattering by a previous wave.

Subject headings: galaxies: evolution – galaxies: haloes – galaxies: kinematics and dynamics – galaxies: spiral

1. INTRODUCTION

Despite many decades of effort, we do not yet have a widely accepted theory to account for the graceful spiral patterns in disk galaxies. Most workers in this field agree that spiral patterns are gravitationally driven variations in surface density in the old stellar disk, as supported by photometric data (e.g. Zibetti et al. 2009) and streaming motions in high spatial resolution velocity maps (e.g. Shetty et al. 2007). There is also general agreement that gas seems to be essential.

There seems little doubt that some spiral patterns are tidally driven, while others could be the driven responses to bars. Although these two ideas may account for a large fraction of the cases, especially if orbiting dark matter clumps can excite patterns (Dubinski et al. 2008), spirals can still develop in the absence of either trigger, as revealed in simulations.

Here, I focus on the idea that spirals are self-excited oscillations of the stellar disk, which represents the greatest theoretical challenge. Two camps have long advanced quite distinct theories. One idea (e.g. Bertin & Lin 1996), is that spiral features are manifestations of quasi-steady global modes of the underlying disk. The other is that they are short-lived, recurrent transient patterns that originate either from shearing bits and pieces (e.g. Toomre 1990), or something more subtle (e.g. Sellwood 2000).

A serious barrier to progress in this area has been the absence of observational discriminants that would favor one of these radically differing viewpoints over the other. The predictions for density variations or gas responses at a single instant are essentially independent of the generating mechanism and do not depend strongly on the lifetime of the pattern.

In his presentation, but not in the written version, Toomre (1981) likened theoretical work on spiral structure to the (apocryphal) blind men examining an elephant. Several important features of the beast have been described in papers by many different authors over the past 50 years. Here I first review many of these components before attempting to weave them into a distinct, yet still incomplete, picture.

The key idea described in this review resulted from a collaborative visit to Doug Lin in Santa Cruz in the summer of 1987. I developed it further for a few years, but worked on other problems whilst awaiting observational support for the somewhat complicated picture we proposed. The desired evidence appeared with the publication of the Geneva-Copenhagen Survey (Nordström et al. 2004, hereafter GCS), as I will explain below, and results from the Gaia satellite should improve matters still further.

2. GLOBAL MODES OF ROTATIONALLY-SUPPORTED DISKS

Simple models of disk galaxies support many linear instabilities (e.g. Korchagin et al. 2003, Jalali 2007). The bar-forming mode is generally the fastest growing, but it has almost no spirality. These studies are therefore important to understand stability, but do not appear promising for spiral generation.

The “density wave” theory for spiral modes, described in detail by Bertin & Lin (1996), invokes a more specific galaxy model with a cool outer disk and hot inner disk. The local stability parameter, $Q = \sigma_R/\sigma_{R,cr}$ (Toomre 1964), is postulated to be $Q \gtrsim 1$ in the outer disk and to rise steeply to $Q > 2$ near the center. Under these specific conditions, Bertin & Lin find slowly evolving spiral modes that grow by their WASER mechanism. They invoke shocks in the gas to limit the amplitude of the slowly growing mode, leading to a quasi-steady global spiral pattern (Law et al. 1994) present a model of this kind to account for the spiral structure of M81. The main objection to their picture is that it is likely that such a lively outer disk will support other, more vigorous, collective responses that will quickly alter the background state by heating the outer disk, as described below.

3. RECURRENT TRANSIENTS

$N$-body simulations of cool, shearing disks always exhibit recurrent transient spiral activity, and this basic result has not changed for several decades as numerical quality has improved. Sellwood & Carlberg (1984) reported that patterns fade in simple simulations that do not include the effects of gas dissipation; the reason is the disk becomes less responsive as random motion rises due to particle scattering by the spiral activity (Carlberg & Sellwood 1985; Binney & Lacey 1988), which is therefore self-limiting.
Selwood & Carlberg (1984) also show that mimicking the effects of dissipative infall of gas, such as by adding fresh particles on circular orbits, allowed patterns to recur “indefinitely.” Later work (e.g. Carberg & Freedman 1985; Toomre 1990; Chakrabarti 2008) has shown that almost any method of dynamical cooling can maintain spiral activity, as also happens in galaxy formation simulations (e.g. Governato et al. 2007).

4. INDIRECT EVIDENCE FOR TRANSIENT SPIRALS

Thus the transient spiral picture offers a natural explanation for the absence of spiral patterns in S0 disk galaxies that have little or no gas; maintenance of spiral activity requires a constant supply of new stars on near-circular orbits. There are several other indirect pieces of evidence that also favor the transient spiral picture.

It has been clear for sometime that the velocity dispersion of disk stars in the solar neighborhood rises with age (e.g. GCS) and also, for main sequence stars, with color (Ammer & Binney 2009), which is a surrogate for mean age. The highest velocities cannot be produced by cloud scattering (Lacey 1991; Hänninen & Flynn 2002) and some other accelerating agent, such as transient spirals, seems to be required (Binney & Lacey 1988).

Studies of the metallicities and ages of nearby stars (Edvardsson et al. 1993; Nordström et al. 2004; Reid et al. 2007; Holmberg et al. 2007) find that older stars tend to have lower metallicities on average. As it is difficult to estimate the ages of individual stars, the precise form of the relation is still disputed. However, there seems to be general agreement that there is a spread of metallicities at each age, which is also supported by other studies (Chen et al. 2003; Haywood 2008). A metallicity spread amongst coeval stars is inconsistent with a simple chemical evolution model in which the metallicity of the disk rises monotonically in each annular bin, without mixing in the radial direction (Schönrich & Binney 2009). Again Sellwood & Binney (2002) showed that the needed radial mixing arises naturally when the disk supports recurrent transient spirals. (See also Roškar et al. 2008ab).

5. WAVE-PARTICLE SCATTERING

Figure 1 shows, for an axisymmetric disk having a flat rotation curve, how the classical integrals of specific energy, \( E \), and angular momentum, \( L_z \), are changed for stars that are scattered by a steadily rotating mild potential perturbation. The solid curve shows the locus of circular orbits, which has slope \( \Omega \), while stars with more energy for their angular momentum pursue non-circular orbits. The principal resonances for an \( m \)-fold rotationally symmetric disturbance that rotates at angular rate \( \Omega \) are marked. The familiar corotation and Lindblad resonances for near circular orbits can be generalized for non-circular orbits by using the angular frequencies, \( \Omega_\phi \) and \( \Omega_R \), of their doubly periodic motion defined in (Binney & Tremaine 2008, hereafter BT08, pp 146-147). For nearly circular orbits, \( \Omega_\phi \to \Omega_\phi \) and \( \Omega_R \to \kappa \), which are the angular frequencies of circular and epicycle motion respectively. The dotted curves show the loci of the principal resonances where \( m(\Omega_\phi - \Omega_\rho) = l\Omega_R \); here \( l = 0 \) for corotation and \( l = \pm 1 \) of the inner and outer Lindblad resonances respectively.

Lynden-Bell & Kahač (1972) show that stars are scattered by spiral waves only at resonances. Since Jacobi’s constant (BT08, eq. 3.112), is conserved in axes that rotate with the perturbation, the slope of all scattering vectors in this plot is \( \Delta E/\Delta L_z = \Omega_\rho \). Since \( \Omega_\rho = \Omega_\rho \) at corotation, scattering vectors are parallel to the circular orbit curve at this resonance, where angular momentum changes do not alter the energy of random motion. Outward transfer of angular momentum involving exchanges at the Lindblad resonances, on the other hand, does extract energy from the potential well that is converted to increased random motion, as is well known.

Selwood & Binney (2002) show that scattering at corotation causes very effective mixing. In a few Gyr, multiple transient spirals cause stars to diffuse in radius over time. These changes at corotation of the spirals, which occur with no associated heating, are able to account for the apparent metallicity spread with age.

Notice also from Fig. 1 that the direction of the scattering vector closely follows the resonant locus (dotted curve) at the ILR only. Thus, when stars are scattered at this resonance, they stay on resonance as they gain random energy, allowing very strong scatterings to occur. The opposite case arises at the outer Lindblad resonance (OLR), where the star quickly moves off resonance when it gains a small amount of angular momentum.

6. SWING AMPLIFICATION

The classic figure of dust to ashes presented by Toomre (1981) and reproduced in BT08 (Fig. 6.19) shows the dramatic transient trailing spiral that results from a small input leading disturbance. This linear calculation shows that the disturbance does not persist, but decays after its transient flourish. In the late stages, the “wave action” propagates at the group velocity (Toomre 1966, and BT08 pp 499-500) away from corotation, where it is absorbed at the Lindblad resonances.

Toomre (1990) suggests that a large part of the spiral activity observed in disk galaxies is the collective response of the disk to shot noise in the density distribu-
Fig. 2.— The time evolution of the peak overdensity in a series of simulations of the half-mass Mestel disk with different numbers of particles. The model is predicted by Toomre (1981) to be globally stable. The ordinate is the maximum values of $\delta$ on grid rings. Linear theory predicts the amplitude should remain proportional to $N^{-1/2}$.

7. GROOVE MODES

Sellwood & Kahn (1991) showed that disks are destabilized by a deficiency of stars over a narrow range of angular momentum, which creates a “groove” in a disk without random motion. The groove itself is unstable, and the instability becomes a global mode through the vigorous supporting response of the surrounding stellar disk. The resulting linear instability, which also develops in a disk with random motion, produces a large-scale spiral pattern. Sellwood & Binney (2002) showed that the amplitude of the mode was limited by the onset of horseshoe orbits at corotation.

Sellwood & Kahn (1991) showed that almost any narrow feature in the angular momentum density is destabilizing. Thus the common starting assumption of spiral structure studies, that the underlying disk is featureless and smooth, may throw the spiral baby out with the bathwater.

8. A LINEARLY STABLE DISK

The so-called “Mestel” disk has the scale-free surface density profile $\Sigma = V_c^2/(2\pi GR)$, with $V_c$ being the circular orbital speed that is independent of $R$. Zang (1976) and Toomre (1981) carried through a global, linear stability analysis of this disk model with random motions described by a smooth distribution function. They introduced a central cutout and an outer taper in the active mass density, and replaced the removed mass by rigid components in order that the central attraction remained unchanged at all radii. The dominant linear instabilities they derived were confirmed in $N$-body simulations by Sellwood & Evans (2001).

Fig. 3.— A power spectrum of $m = 2$ disturbances during the period of rapid rise in amplitude in the $N = 5 \times 10^7$ experiment from Fig. 2. The solid curve shows $2\delta(R)$ and the dashed curves $2\delta(R) \pm \kappa(R)$.
est amplitude. Note that the largest number of particles, $N = 500M$, is within a factor of 100 of the number of stars in a real galaxy disk, where in reality the mass distribution is far less smooth, owing to the existence of star clusters and giant gas clouds.

9. MORE DETAILED ANALYSIS

The runaway growth once $\delta \gtrsim 2\%$ is of particular interest. A power spectrum analysis of non-axisymmetric density variations during this period of rapid growth reveals multiple waves of many different frequencies, each extending from the inner Lindblad resonance (ILR) to corotation, as shown in Fig. 3. We see that disturbances are present at several frequencies at any one time, and that each appears and fades over a period of ~100 dynamical times, with those of lower frequency generally developing later.

Figure 4 shows the transient wave that was prominent over a short time interval during which the wave grew and decayed while the pattern speed remained approximately constant. The measured $\Omega_p = 0.182V_0/R_c$, placing corotation and the Lindblad resonances for circular orbits at the radii shown by the circles in this Figure. The peak amplitude of the pattern is near corotation and the wave extends to the Lindblad resonances on either side; the outer half has lower amplitude because the wave is spread over a larger area. As the wave decays, the corotation peak disperses (Sellwood & Binney 2002) and the “wave action” is carried radially at the group velocity where it is absorbed at the Lindblad resonances.

It is convenient to separate the specific energy of a star in an axisymmetric potential, $\Phi(R, z)$, into three distinct terms $E = E_{\text{circ}}(L_z) + E_{\text{ran}} + E_z$, where

$$E_{\text{circ}} = \Phi(R_c, 0) + L_z^2/(2R_c^2)$$
$$E_{\text{ran}} = \Phi(R, 0) - \Phi(R_c, 0) + \frac{1}{2} \left[ v_R^2 + L_z^2 \left( \frac{1}{R^2} - \frac{1}{R_c^2} \right) \right]$$
$$E_z \approx \frac{1}{2} (z^2 v_R^2 + v_z^2).$$

Here $R_c$ is the radius of a circular orbit having the same $L_z$, and $v_R$ is the radial velocity. The approximate form for $E_z$ (BT08, eq. 3.86) holds for stars that do not climb to great heights above the mid-plane, with $\nu$ being the vertical oscillation frequency of such stars.

The change in random energy caused by an angular momentum change $\Delta L_z$ is $\Delta E_{\text{ran}} = \Omega_p \Delta L_z - E_{\text{circ}}(L_z + \Delta L_z) + E_{\text{circ}}(L_z)$. It is easy to show that scattering from a circular orbit according to this formula takes place along a line in $(L_z, E_{\text{ran}})$-space that is has a steep negative slope for the ILR and positive slope for the OLR.

Figure 5 shows the distribution of particles in the space of angular momentum (here labeled $J$) and energy of random motion (here labeled $E - E_c$) in this simulation at $t = 600$. It differs from the initial distribution by the pronounced tongues of particles at low angular momentum that reach up to high random energies on an upward curving, negatively sloped trajectory that is almost completely obscured by the loci of the Lindblad resonances (solid lines), and the scattering trajectories (dashed lines). These lines are for the measured pattern speed of the wave shown in Fig. 4 and therefore have no free parameters. The fact that the dashed line at the ILR overlays the tongue of particles so perfectly that they are barely visible indicates that these particles were scattered to more eccentric orbits by the ILR of the identified pattern. Recall from Fig. 4 that particles stay on resonance as they gain angular momentum, which is the reason some reach very high energies.

10. RECURRENT CYCLE?

Each coherent wave leaves behind an altered distribution function and apparently thereby creates the conditions for a new instability. Despite having shown this already in Sellwood & Lin (1989), I am still unclear how this happens in detail. I have not put much effort into pursuing this question since then, because I was concerned that the behavior I was studying might turn out to be a horrible artifact of the simulations and thought it prudent to await evidence that something similar occurs in nature. I hoped
The distribution of GCS stars in the space of the two integrals $E_{\text{van}}$ and $L_{\alpha}$. Aside from the general decrease in density towards large $E_{\text{van}}$ and a skew to lower $L_{\alpha}$ caused in part by the asymmetric drift, the DF in this projection also shows significant substructure.

Sellwood (1994) to see evidence of resonance scattering in the HIPPARCOS measurements of the local stellar kinematics.

11. GENEVA-COPENHAGEN SURVEY

Nordström et al. (2004) followed up the accurate distances and proper motions from HIPPARCOS to obtain the missing radial velocities from ground-based spectra. Their survey has yielded known distances and full space motions of over 14,000 local F & G dwarfs – a sample that is believed to be almost complete within 40 pc and largely free from selection biases to 100 pc. I use the latest revision of their survey (Holmberg et al. 2009), discarding only stars having distances $>200$ pc and all known Hyades cluster stars, which leaves a sample of 13,045 stars.

The addition of radial velocities confirmed the extraordinary substructure in the $U-V$ plane already discovered by Dehnen (1998) in the absence of radial velocities. The local distribution function is far from the smooth double Gaussian postulated by Schwarzschild (BT08, p. 323); there is little evidence for an underlying smooth component, rather the distribution has the appearance of several distinct streams (Bovy et al. 2009), which cannot be dissolved clusters (Bensby et al. 2007; Famaey et al. 2007; Bovy & Hogg 2009). Various groups have attempted to account for this substructure by resonant scattering by the Milky Way bar (Dehnen 2000), spiral arms (De Simone et al. 2004), or both (Quillen & Minchev 2003; Chakrabarty 2007), which may indeed account for parts of the observed structure. But none of these studies was self-consistent, and all attempted to explain the structure in velocity space.

It is more instructive to project the data into integral space, as shown in Figure 6. The distribution in the upper panel assumes a locally flat rotation curve for the Milky Way, $R_0 = 8$ kpc and $V_0 = 220$ km/s, but the features in the plot are insensitive to these assumptions. The almost parabolic lower boundary is a selection effect, because stars whose angular momenta differ increasingly from that of the LSR do not visit the Solar neighborhood unless they have greater energies of random motion. The left-right bias results from the fact that the density of stars in the Galaxy declines outwards, and therefore larger numbers visit the solar neighborhood from the inner Galaxy than from larger radii (i.e. the asymmetric drift).

The distribution in Figure 6 manifests one strong tongue with a steep negative slope rising from an abscissa $L_{\alpha} \approx 0.96R_0V_0$. This could be the scattering feature at an ILR that I suggested (Sellwood 1994) might be present, but in fact trapping at any of the principal resonances would cause an excess of stars along a line of similar slope. Nevertheless the feature is a highly significant density excess, with $\geq 99\%$ confidence from a bootstrap analysis (Sellwood 2010). While there are hints of other features, none but the obvious one is statistically significant.

In order to determine which resonance could have caused this feature, we must examine the phases of the stars, which should exhibit different correlations depending upon the resonance responsible. I therefore computed action-angle coordinates for each star (Sellwood 2010), and show the results in Figure 7. The radial coordinate in the upper panel shows the approximate epicycle size $a = \sqrt{2J_R/\kappa}$ while the azimuth is $2w_\phi - w_R$. The concentration of stars at one phase, marked by the arrow, is highly significant. Furthermore, the stars colored red in the skinny triangle in the upper panel are exactly the stars, also colored red in the lower panel, that defined the scattering tongue. This Figure therefore confirms that the tongue of stars in integral space has been caused by scattering at an ILR.

Unfortunately, it is not possible to tell from this analysis the angular periodicity of the disturbance that created this feature: it could be an $m = 2$ spiral, but higher angular periodicities would give rise to very similar features in these plots. However, an $m = 2$ wave seems the most likely, since the alignment of the scattering trajectory and resonance locus in Fig. 6 is closest for this angular periodicity, making it easier for strong scattering to produce a clear feature.

The identification of an ILR provides strong support for the picture of spiral generation that I have been developing from the simulations. It suggests that spirals in the Milky Way are transient, with the decay of one pattern seeded in part by scattering at an ILR. An ILR also excludes the Bertin & Lin (1996) quasi-steady wave hypothesis as the cause of the pattern that gave rise to this particular feature, although this one case clearly does not preclude the possibility that it may still apply elsewhere.

12. CONCLUSIONS

Globally stable N-body simulations have manifested recurrent transient spiral patterns for many years, and the phenomenon has not changed as numerical quality has risen. The transient nature of spiral patterns receives indirect support both from the importance of gas and secular heating of disks. The more recently discovered further consequence of radial mixing in disks (Sellwood & Binney 2002) is important for structural evolution, metallicity gran-

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1 A similar excluded region is visible in the quantities plotted by Arifyanom & Fuchs (2009); the omission of the small correction for the different Galacto-centric distances of the guiding centers in their energy-like term is unimportant.
shows the distribution in (\(\sim\) action, while the azimuthal coordinate is 2\(w_w\) angle space. The radial coordinate is the square root of the radial gradients, dynamo theory, etc.

Simulations of the only known stable disk with non-uniform rotation have revealed that linear perturbation theory breaks down at relative overdensities of just \(\sim 2\%\). Real galaxy disks, which contain both star clusters and molecular clouds, are not that smooth and it would seem therefore that their behavior cannot be fully described by linear theory. I show how non-linear effects could lead to a recurrent cycle of spirals, although the exact mechanism remains obscure.

The Geneva-Copenhagen survey has full phase space coordinates of an unbiased sample of local F & G dwarf stars. Examining the distribution as a function of integrals has yielded clear evidence for the existence of an inner Lindblad resonance in the solar neighborhood. This discovery provides strong support for the mechanism for recurrent spiral activity that I have been developing.

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FIG. 7.— Top panel shows the distribution of EGCS stars in action-angle space. The radial coordinate is the square root of the radial coordinate. Note the excess in \(L_z \big/ (F_v^2\phi)\) space. The red points are the same stars in each plot.
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