Study of Joint Automatic Gain Control and MMSE Receiver Design Techniques for Quantized Multiuser Multiple-Antenna Systems

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Abstract—This paper presents the development of a joint optimization of an automatic gain control (AGC) algorithm and a linear minimum mean square error (MMSE) receiver for multi-user multiple input multiple output (MU-MIMO) systems with coarsely quantized signals. The optimization of the AGC is based on the minimization of the mean square error (MSE) and the proposed receive filter takes into account the presence of the AGC and the effects due to quantization. Moreover, we provide a lower bound on the capacity of the MU-MIMO system by deriving an expression for the achievable rate. The performance of the proposed Low-Resolution Aware MMSE (LRA-MMSE) receiver and AGC algorithm is evaluated by simulations, and compared with the conventional MMSE receive filter and Zero-Forcing (ZF) receiver using quantization resolution of 2, 3, 4 and 5 bits.

Index Terms—Coarse Quantization, AGC, MU-MIMO detection, MMSE receiver

I. INTRODUCTION

In 5G cellular systems, high data rates, reliable links, low cost and power consumption are key requirements. Multiple-input multiple-output (MIMO) systems in wireless communications provide significant improvements in wireless link reliability and achievable rates. However, as the number of antennas scales up, the energy consumption and circuit complexity increases accordingly [1], [2], [3], [4], [5]. For example, the energy consumption of an analog to digital converter (ADC) grows exponentially as a function of the number of transmit antennas each is. Each entry of the vector $\mathbf{x}_i$ is a standard Zero-Forcing filter at the receiver were examined in [6]. However, the authors have not optimized the AGC algorithm nor used a detector that considers the quantization effects.

This work presents a framework for jointly designing the AGC and a linear receive filter according to the MMSE criterion for a large-scale MU-MIMO system operating with coarsely quantized signals. The procedure consists of computing the modified MMSE receiver presented in [1] and, after that, computing the derivative of the cost function that takes into account the presence of the AGC in order to obtain the optimal AGC coefficients. Then, a Low-Resolution Aware MMSE (LRA-MMSE) receiver that considers both quantization effects and the AGC is derived. A lower bound on the capacity of this system is investigated and an expression to compute the achievable rates is developed.

Notation: Vectors and matrices are denoted by lower and upper case italic bold letters. The operators $(\cdot)^T$, $(\cdot)^H$ and $tr(\cdot)$ stand for transpose, Hermitian transpose and trace of a matrix, respectively. $\mathbf{1}$ denotes a column vector of ones and $\mathbf{I}$ denotes an identity matrix. The operator $E[\cdot]$ stands for expectation with respect to the random variables and the operator $\odot$ corresponds to the Hadamard product. Finally, $\text{diag}(\mathbf{A})$ denotes a diagonal matrix containing only the diagonal elements of $\mathbf{A}$ and $\text{non} \text{diag}(\mathbf{A}) = \mathbf{A} - \text{diag}(\mathbf{A})$.

II. SYSTEM DESCRIPTION

A large-scale uplink MU-MIMO system [12], [14] consisting of a base station (BS) with $N_R$ receive antennas, and $K$ users equipped with $N_T$ transmit antennas each is considered. At each time instant $i$, each user transmits $N_T$ symbols which are organized into a $N_T \times 1$ vector $\mathbf{x}_k[i] = [x_1[i], x_2[i], ..., x_{N_T}[i]]^T$. Each entry of the vector $x_k[i]$ is a coarse quantization of the received signal. The optimization of the AGC is based on the minimization of the mean square error (MSE) and the proposed receive filter takes into account the presence of the AGC and the effects due to quantization. Moreover, we provide a lower bound on the capacity of the MU-MIMO system by deriving an expression for the achievable rate. The performance of the proposed Low-Resolution Aware MMSE (LRA-MMSE) receiver and AGC algorithm is evaluated by simulations, and compared with the conventional MMSE receive filter and Zero-Forcing (ZF) receiver using quantization resolution of 2, 3, 4 and 5 bits.

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symbol vector is then transmitted through flat fading channels and corrupted by additive white Gaussian noise (AWGN). The received signal collected by the receive antennas at the BS is given by the following equation:

$$y[i] = \sum_{k=1}^{K} H_k x_k[i] + n[i] = H x[i] + n[i],$$

where $y[i] \in \mathbb{C}^{N_R \times 1}$, and $H_k \in \mathbb{C}^{N_R \times N_T}$ is a matrix that contains the complex channel gains from the $N_T$ transmit antennas of user $k$ to the $N_R$ receive antennas of the BS. The $N_R \times 1$ vector $n[i]$ is a zero-mean complex circular symmetric Gaussian noise vector with covariance matrix $E[n[i]n^H[i]] = \sigma_n^2 I$. $H$ is a $N_R \times K N_T$ matrix that contains the coefficients of the flat fading channels between the transmit antennas of the $K$ users and the receive antennas of the BS. The symbol vector $x[i] = [x_1[i], ..., x_k[i], ..., x_K[i]]^T$ contains all symbols that are transmitted by the users at time instant $i$. The channel state information (CSI) is assumed to be unknown to the users at the transmit side. Therefore, we assume the same symbol energy per user and transmit antenna, i.e. $E[xx^H] = \sigma_x^2 I$.

![Fig. 1. An uplink quantized MU-MIMO system](image)

As depicted in Fig. 1, before to be quantized, $y$ is jointly pre-multiplied by the clipping level factor $\alpha$ and the AGC matrix $G$ to minimize the granular and the overload distortions. After that, the product $\alpha G y$ is quantized, and the estimation of the transmitted symbol vector $x$ is performed by the LRA-MMSE receiver, represented by $L$. The estimated symbol vector is represented by $\hat{x}$. The real, $y_i, r, i.$ and imaginary, $i, i.i.$, parts of the complex received signal at each antenna are quantized separately by uniform $b$-bit resolution ADCs. Therefore, the resulting quantized signals read as

$$r_{i,l} = Q(y_{i,l}) = y_{i,l} + q_{i,l}, \quad l \in \{R, I\}, \quad 1 \leq i \leq N_R,$$

where $Q(\cdot)$ denotes the quantization operation and $q_{i,l}$ is the resulting quantization error.

The distortion factor indicates the relative amount of quantization noise generated by the quantizer, and is given by $\rho_q = \sigma_{q_{i,l}}^2 / \sigma_{y_{i,l}}^2$, where $\sigma_{y_{i,l}}^2$ is the variance of the input $y_{i,l}$ and $\sigma_{q_{i,l}}^2$ is the variance of the output $q_{i,l}$. This factor depends on the number of quantization bits $b$, the quantizer type, and the probability density function of $y_{i,l}$. In this work the scalar uniform quantizer processes the real and imaginary parts of the input signal $y_{i,l}$ in a range $\pm \frac{\sigma_{y_{i,l}}}{2^b}$. With a high number of antennas the input signals of the quantizer are approximately Gaussian distributed and they undergo nearly the same distortion factor $\rho_q$. It was shown in [3] for the uniform quantizer case, that optimal quantization step $\Delta$ for a Gaussian source decreases as $\sqrt{\rho_q^{-1}}$ and that $\rho_q$ is asymptotically well approximated by $\frac{2^b}{2^{b-1}}$.

**III. PROPOSED JOINT AGC AND LINEAR MMSE RECEIVER DESIGN**

The procedure for joint optimization of the AGC algorithm and the LRA-MMSE receiver carries out alternating computations between the AGC and the LRA-MMSE receiver. The first step consists of computing an LRA-MMSE receive filter that considers the quantization effects. Other approaches to computing linear MMSE or related filters [18], [19], [20], [21] can also be considered. After that, we compute the derivative of the cost function to obtain the optimal AGC coefficients. Then, an updated LRA-MMSE receiver is computed.

**A. Linear LRA-MMSE Receive Filter Design**

In this first step we do not consider the presence of the AGC in the system. Thus, the received signal after the quantizer is expressed, with the Bussgang decomposition [9], as a linear model $r = y + q$. To develop the linear receive filter $W$ that minimizes the MSE we use the Wiener-Hopf equations:

$$W = R_{rr}^{-1} R_{yr},$$

where the auto-correlation matrix $R_{rr}$ is given by

$$R_{rr} = E[rr^H] = R_{yy} + R_{yy}^H + R_{qq},$$

and the cross-correlation matrix $R_{yr}$ can be expressed as

$$R_{yr} = E[xr^H] = R_{xy} + R_{qx}$$

We get the auto-correlation matrix $R_{yy}$ and the cross-correlation matrix $R_{xy}$ directly from the MIMO model as

$$R_{yy} = E[yy^H] = H R_{xx} H^H + R_{nn},$$

and,

$$R_{xy} = E[xy^H] = R_{xx} H^H$$

To compute (4) and (5) we need to obtain the covariance matrices $R_{yy}$, $R_{qq}$ and $R_{xy}$ as a function of the channel parameters and the distortion factor $\rho_q$. The procedure of how to obtain these matrices was developed in [1] and we will use some of these results in this work. The cross-correlation between the received signal vector and the quantization error is approximated by

$$R_{qq} \approx -\rho_q R_{yy}$$

The covariance matrix of the quantization error is deduced from

$$R_{qq} \approx \rho_q \text{diag}(R_{yy}) + \rho_q^2 \text{nondiag}(R_{yy}) = \rho_q R_{yy} - (1 - \rho_q) \rho_q \text{nondiag}(R_{yy}),$$

and the cross-correlation matrix between the desired signal vector and the quantization error can be obtained by

$$R_{qx} = -\rho_q R_{xy}$$
Substituting (10) in (5) we get

$$R_{xx} = (1 - \rho_q)R_{xy}$$

and substituting (6), (8) and (9) in (4) we get

$$R_{rr} \approx (1 - \rho_q)(R_{yy} - \rho_q \text{ nondiag}(R_{yy}))$$

Finally, by substituting (11) and (12) in (3) we get the optimum

equation.

With the presence of the AGC the expression of the received
term can be computed by $z = G y + q$.

III. AGC Design

In [6], the authors proposed a standard AGC algorithm by
using a diagonal matrix $G$ with real coefficients. This matrix
is used to compensate the gain differences of the propagation
channel and involves a search over a transmitted symbol
alphabet. This approach is very computationally demanding
in an environment with a high number of antennas. Writing $G$
as $\text{diag}(g)$, where $g$ is a column vector with the diagonal
elements of $G$, the proposed AGC algorithm is based on the
minimization of the cost function:

$$\varepsilon = E[||x - \hat{x}||^2] = E[||x - W(\alpha \text{ diag}(g)y + q)||^2]$$

and since $G$ is a diagonal matrix with real coefficients we have
$\text{diag}(g)^H = \text{diag}(g)$. Then,

$$\varepsilon = \text{tr}(R_{xx} - \alpha R_{xy} \text{ diag}(g)W^H - R_{xq} W^H - \alpha \text{ diag}(g)R_{xy}^H + \alpha^2 \text{ diag}(g) \text{ diag}(g)W^H + \alpha W \text{ diag}(g)R_{yy}^H - W R_{xq}^H - \alpha \text{ diag}(g) \text{ diag}(g)W^H + \alpha W R_{yy}^H \text{ diag}(g)W^H + \alpha W R_{yy}^H \text{ diag}(g)W^H)$$

To obtain the optimum $G$ matrix we compute the derivative
of the MSE cost function with respect to $\text{diag}(g)$, equate the
derivative terms to zero and solve for $g$:

$$\frac{\partial \varepsilon}{\partial g} = -\alpha \frac{\partial}{\partial g} \text{ tr}(R_{xy} \text{ diag}(g)W^H) - \alpha \frac{\partial}{\partial g} \text{ tr}(W \text{ diag}(g)R_{xy}^H)
+ \alpha^2 \frac{\partial}{\partial g} \text{ tr}(W \text{ diag}(g)R_{xy} \text{ diag}(g)W^H)
+ \alpha^2 \frac{\partial}{\partial g} \text{ tr}(W \text{ diag}(g)R_{yy} \text{ diag}(g)W^H)
+ \alpha \frac{\partial}{\partial g} \text{ tr}(W R_{yy} \text{ diag}(g)W^H)
+ \alpha \frac{\partial}{\partial g} \text{ tr}(W R_{yy} \text{ diag}(g)W^H)$$

We have to take the derivative of each term of Eq. (15). Consider
the conversion between matrix notation and index
notation and the tricky case of a $\text{diag}()$ operator

$$[AB]_{ik} = \sum_j A_{ij}B_{jk}$$

From Eq. (15), we get

$$f = \text{ tr}[A \text{ diag}(g)B] = \sum_i \sum_j A_{ij}g_jB_{ji}$$

Taking the derivative with respect to the coefficients $g_j$ of the
diagonal operator we have

$$\frac{\partial f}{\partial g_j} = \sum_i A_{ij}B_{ji} = [(A^T \circ B)1]_j$$

Therefore, we can write

$$\frac{\partial \text{ tr}[A \text{ diag}(g)B]}{\partial g} = (A^T \circ B)1$$

With these considerations we can take the derivative of terms $I$, $II$, $III$, $IV$ and $V$ from Eq. (15). The derivatives
of the terms $I$ and $II$ can be computed by

$$I = \frac{\partial \text{ tr}[R_{xy} \text{ diag}(g)W^H]}{\partial g} = [(R_{xy}^H \circ W^H)1]$$

$$II = \frac{\partial \text{ tr}[W \text{ diag}(g)R_{xy}^H]}{\partial g} = [(R_{xy}^H \circ W^T)1]$$

To compute the derivative of term $III$ we apply the chain
rule

$$III = \frac{\partial \text{ tr}[W \text{ diag}(g)A] + \partial \text{ tr}[B \text{ diag}(g)W^H]}{\partial g}_{III.1} + \frac{\partial \text{ tr}[B \text{ diag}(g)W^H]}{\partial g}_{III.2}$$

where $A = R_{yy} \text{ diag}(g)W^H$ and $B = W \text{ diag}(g)R_{yy}$. The term $III.1$ can be computed by

$$III.1 = [(W^T \circ (R_{yy} \text{ diag}(g)W^H))1]$$
and the term $III.2$ as

$$III.2 = [((R_{yy}^T \text{diag}(g)W^T) \odot W^H)1]$$

Substituting (23) and (24) in (25) we have

$$III = [((W^T \odot (R_{yy} \text{diag}(g)W^H))1]$$

The derivative of the term $IV$ is given by

$$IV = \frac{\partial tr[W \text{diag}(g)C]}{\partial g} = [((R_{yy}^T \odot W^H)1]$$

where $C = R_{yy}W^H$. Finally, the derivative of the term $V$ can be computed by

$$V = \frac{\partial tr[D \text{diag}(g)W^H]}{\partial g} = [((R_{yy}^* W^T) \odot W^H)1]$$

where $D = WBR_{yy}$. Substituting (20), (21), (25), (26) and (27) in (15) and equating the derivatives to zero we have

$$W^T \odot (R_{yy} \text{diag}(g)W^H) + (R_{yy}^T \text{diag}(g)W^T) \odot W^H]1 = \frac{1}{\alpha}[(R_{yy}^T \odot W^H)1] + (R_{yy}^T \odot (W^H)1] +$$

$$- [(W^T \odot (R_{yy}W^H))1] - (R_{yy}^T \odot (W^H)1)]$$

To achieve the desired $g$ we have to do some manipulations with the first term of (28). To do this we will write the first and second terms of $g$ with the index notation and after that we will return to the matrix notation. We can write the first term as

$$[(W^T \odot (R_{yy} \text{diag}(g)W^H)1] = \sum_{j=1}^{KN_T} \sum_{l=1}^{N_R} W_{ji}R_{yy,il}g_ilW_{ij}^H$$

and the second term as

$$[(W^H \odot (R_{yy}^T \text{diag}(g)W^T)1] = \sum_{j=1}^{KN_T} \sum_{l=1}^{N_R} W_{ij}^H R_{yy,il}g_ilW_{jl}$$

With some manipulations we can isolate the vector $g$

$$W^T \odot (R_{yy} \text{diag}(g)W^H) + W^H \odot (R_{yy}^T \text{diag}(g)W^T)]1 =$$

$$\sum_{j=1}^{KN_T} \sum_{l=1}^{N_R} W_{ji}R_{yy,il}g_ilW_{ij}^H + \sum_{j=1}^{KN_T} \sum_{l=1}^{N_R} W_{ij}^H R_{yy,il}g_ilW_{jl}$$

$$= \sum_{l=1}^{N_R} ([(W^T \text{diag}(g)W^H) \odot R_{yy} + (W^H \text{diag}(g)W) \odot R_{yy}^T]1)g_l$$

$$= [(W^T \text{diag}(g)W^H) \odot R_{yy} + (W^H \text{diag}(g)W) \odot R_{yy}^T]$$

Substituting (31) in (29) and solving with respect to $g$ we have

$$g = [(W^T \text{diag}(g)W^H) \odot R_{yy} + (W^H \text{diag}(g)W) \odot R_{yy}^T]^{-1}$$

$$\cdot \left(\frac{2}{\alpha}(\text{Re}[(W^T \odot (R_{yy}W^H)1)] - \text{Re}[(W^T \odot (R_{yy}W^H)1)])ight)$$

IV. CLIP-LEVEL ADJUSTMENT

In the following we outline the computation of the clipping factor $\alpha$ based on the signal power. This factor conforms the received signal power between the quantizer range to minimize the overload distortion. The received signal power can be computed by

$$P = \text{tr}(E[(y + q)(y + q)^H])$$

$$= \text{tr}(R_y + R_q + R_y^H + R_q)$$

and received symbol energy by

$$E_{rx} = \sqrt{\frac{\text{tr}(R_y + R_q + R_y^H + R_q)}{N_R}}$$

Thus, the clipping factor $\alpha$ can be obtained from

$$\alpha = \beta \cdot \sqrt{\frac{\text{tr}(R_y + R_q + R_y^H + R_q)}{N_R}}$$

where $\beta$ is a calibration factor. In our simulations the value of $\beta$ was set to $\sqrt{b}$ which corresponds to the quantizer output range, to ensure an optimized performance.

V. CAPACITY LOWER BOUND

In [1] a lower bound on the mutual information between the input sequence $x$ and the quantized output sequence $r$ of a quantized MIMO system was developed, based on the MSE approach. We will use a similar procedure to consider a capacity lower bound of our quantized MU-MIMO system with the optimal AGC and to derive an expression for computing the achievable rates for the proposed AGC and LRA-MMSE receiver. We remark that similar analyses can be considered for the downlink with the use of precoding techniques [15], [16], [17]. As described in [11] the mutual information of this channel can be expressed as

$$I(x, r) = h(x) - h(x|r)$$

Given $R_{xx}$ under a power constraint $tr(R_{xx}) \leq P_{tr}$, we choose $x$ to be Gaussian, which is not necessarily the capacity achieving distribution for our quantized system. Then, we can obtain a lower bound for $I(x, r)$ (in bit/transmission) as

$$I(x, r) = \log_2 \text{det}(R_{xx}) - h(x|r)$$

$$= \log_2 \text{det}(R_{xx}) - h(x - \hat{x}|r)$$

$$\geq \log_2 \text{det}(R_{xx}) - h(x - \hat{x})$$

Substituting (37) and solving with respect to $\epsilon$ we have

$$\epsilon \geq \log_2 \frac{\text{det}(R_{xx})}{\text{det}(R_{xx})}$$

The second term in (37) is upper bounded by the entropy of a Gaussian random variable whose covariance is equal to the error covariance matrix $R_{xx}$ of the LRA-MMSE estimate of $x$. Thus, we have to compute the expressions of $R_{xx}$ and $R_{xx}$ for our system. Considering unknown CSI at the transmitter, the autocorrelation matrix $R_{xx}$ is given by

$$R_{xx} = \sigma_x^2 k_{KN_T}$$
and the error covariance matrix can be computed by

\[
R_{ee} = E[(x - \hat{x})(x - \hat{x})^H]
\]

\[
= R_{xx} - R_{xq}GW^H - R_{xq}W^H - WGR_{qy}^H + WGR_{qy}GW^H + WGR_{qy}W^H - WR_{xq}^H + WR_{xq}GW^H + WR_{xq}W^H
\]

(40)

Substituting (39) and (40) in (38) we obtain an expression to compute the achievable rates for the MU-MIMO system with coarsely quantized signals.

VI. RESULTS

To evaluate the results obtained in previous sections we consider a MU-MIMO system with \( K = 16 \) users who are each equipped with \( N_T = 2 \) transmit antennas and one BS with \( N_R = 64 \) receive antennas. At each time instant the users transmit data packets with 100 symbols using BPSK modulation. The channels are obtained with independent and identically distributed complex Gaussian random variables with zero mean and unit variance. For each simulation 10000 packets are transmitted, by each transmit antenna, over a flat-fading channel. The received signal is quantized with 2, 3, 4 and 5 bits.

In Fig. 2 we illustrate the BER performance of the proposed joint AGC and LRA-MMSE receiver design. As expected, the standard MMSE detector achieved, even in a quantized environment, a better performance than the ZF detector. This occurs because the MMSE filter incorporates the variance of the receive antenna noise which improves the accuracy of the MMSE detector at low SNR values. Moreover, we can see that among all receivers the LRA-MMSE with the proposed AGC obtained the best performance. The design of this receiver aggregates the gains by incorporating the AGC and the effects due to the coarse quantization. The curves also show that, with the presented approximations, the joint AGC and LRA-MMSE receiver design achieves a performance very close to the performance of the Full Resolution standard MMSE receiver (FR standard MMSE).

In Fig. 3 we illustrate the achievable sum rates of the MU-MIMO system with the joint AGC and LRA-MMSE receiver design for different numbers of quantization bits. This result shows that, as the number of quantization bits increases, the sum-rate also increases approaching those values obtained by the FR standard MMSE receiver in an unquantized environment.

VII. CONCLUSIONS

In this work we have discussed the joint design of an AGC and a LRA-MMSE receive filter for coarsely quantized MU-MIMO systems. Simulations results have shown that the joint AGC and the LRA-MMSE receiver obtained a performance close to the full resolution MMSE receiver in a quantized MU-MIMO system with 4 and 5 bits of resolution. Furthermore, we have derived an expression for computing the achievable rates for the system. The results have shown that with 4 and 5 quantization bits we achieve a rate very close to the capacity of an unquantized large-scale MU-MIMO system.

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