Calculation method of index weight for facility support capability evaluation in Airfield

Huang Xuelin\textsuperscript{1, a}, Wang Guanhu\textsuperscript{1, b}, Wang Wei\textsuperscript{1, c}, Tang Wei\textsuperscript{1, d}, Yuan Yingjie\textsuperscript{1, e}, Xia Wei\textsuperscript{1, f}

\textsuperscript{1}Air Force Engineering University, Xi'an, China

\textsuperscript{a}1939666916@qq.com, \textsuperscript{b}kgdw0103@163.com, \textsuperscript{c}596564345@qq.com, \textsuperscript{d}1026096161@qq.com, \textsuperscript{e}972380305@qq.com, \textsuperscript{f}432287422@qq.com

Abstract. In order to effectively evaluate the facilities in each airport airfield, this paper proposes four methods: analytic hierarchy process (AHP), subjective weighting method based on Improved Analytic Hierarchy Process (AHP), objective weighting method based on entropy weight method and combination weighting method based on game theory. Combined with the actual situation of the airport, the corresponding calculation method is adopted to introduce the mathematical model into the construction of the airport. After verification, the proposed method can be applied to 90\% of airports, and the applicability of the index is improved by 50\%. The weight of obstacle capability evaluation index provides an important basis.

1. INTRODUCTION
Airfield facilities are an important part of the airport, and the evaluation of its support capability is conducive to the generation and development of combat effectiveness. The evaluation of facility support capability in airfield can provide decision-making basis for superior departments and provide important theoretical guidance for investment direction of airport engineering construction. Each facility includes many underlying indicators, such as size, strength, quality status. At the same time, there will be cross between these indicators, so the indicators involved will be very miscellaneous. Therefore, it is necessary to classify and sort out all the indicators before evaluation, and to summarize the indicators through certain principles and methods to build the index system of capacity evaluation. In addition, the determination of index weight is also an important work in the capability evaluation of facilities in flight area. Therefore, this paper will systematically study the index system and weight determination method of facility support capability in airfield. The determination of index weight is the key and difficult point in the capability evaluation of facilities in the flight area, because whether the index weight is reasonable or not will directly affect the reliability and persuasiveness of the evaluation results. At the same time, there is no fixed standard for how to determine the weight value of each index, which brings great difficulties to the evaluation work.

In the existing research, the weight determination method is mainly divided into two categories: fixed weight method and variable weight method. Among them, the weight determination method can be divided into subjective weighting method, objective weighting method and combination weighting method \cite{1}, and each weight determination method has many corresponding types; variable weight method can also be divided into punishment type, incentive type and combination type \cite{2}.

2. ANALYTIC HIERARCHY PROCESS
Using AHP to determine the index weight, considering the correlation and feedback effect between the
indicators, can more accurately describe the relationship between the influencing factors, construct pairwise judgment matrix, and compare the superiority degree, so as to make the transition from qualitative evaluation to quantitative calculation, so as to determine the index weight. The nine scale method is used to determine the weight of indicators[3].

2.1. Determining judgment matrix

Suppose there are m experts to determine the weight of the index system. There are n indicators \( C_1, C_2, \ldots, C_n \) in a certain layer of the index system. Any expert compares all the indicators according to the scale of 1-9, and obtains the judgment matrix \( A \) of the N indicators:

\[
A = \begin{bmatrix}
1 & \ldots & a_{1n} \\
\vdots & \ddots & \vdots \\
a_{m1} & \ldots & 1
\end{bmatrix} = a_{ij(n\times n)}
\]

(1)

\(a_j\) is the comparative quantitative value of factor \( i \) and factor \( j \).

Then the judgment matrix obtained by m experts is as follows:

\[
A_m = \begin{bmatrix}
1 & \ldots & a_{1n}' \\
\vdots & \ddots & \vdots \\
a_{m1}' & \ldots & 1
\end{bmatrix} = a_{ij(n\times n)}'
\]

(2)

\[
a_{ij(n\times n)}' = \frac{1}{m} \sum_{p=1}^{m} a_{ij(p)} \quad (p = 1, 2, \ldots, m; i, j = 1, 2, \ldots, n)
\]

(3)

2.2. Calculate eigenvalues and eigenvectors

For matrix \( A \), the maximum eigenvalue \( \lambda_{\text{max}} \) and eigenvector \( W_i \) are obtained:

\[
\lambda_{\text{max}} = \sum_{i=1}^{n} \frac{(AW)_i}{nW_i} \quad (i = 1, 2, \ldots, n)
\]

(4)

\( N \) is the order of matrix \( A \). After normalization, the eigenvector is as follows:

\[
W_i = \frac{\prod_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n} \prod_{j=1}^{n} a_{ij}} \quad (i, j = 1, 2, \ldots, n)
\]

(5)

The normalized eigenvector \( W_i \) is the primary weight vector of each index in this layer.

2.3. Consistency test

The coordination between the importance of each index can be completed by consistency test, while the analytic hierarchy process (AHP) can judge the consistency by consistency ratio \( C.R. \) which makes the results more rigorous.

\[
C.R. = \frac{C.I.}{R.I.}
\]

(6)

\[
C.I. = \frac{\lambda_{\text{max}} - n}{n-1}
\]

(7)
3. SUBJECT WEIGHTING METHOD BASED on IMPROVED ANALYTIC HIERARCHY of CONSISTENCY

In order to solve the problem that it is difficult for experts to make pairwise comparison of each index to give a more accurate judgment of importance and the workload caused by the construction of judgment matrix can not pass the consistency test, a new scaling method, three scaling method\[4\], is adopted. The specific steps are as follows:

3.1. Construct judgment matrix

According to the professional knowledge and level of experts and engineers in related fields, the pairwise comparison judgment matrix $A$ of index layer relative to sub target layer and sub target layer to total target layer is constructed according to three scale method.

$$
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix} = (a_{ij})_{n \times n}
$$

(8)

3.2. The optimal transfer matrix is obtained

According to matrix $A$, the optimal transfer matrix $R$ is obtained, $R = (r_{ij})_{n \times n}$,

$$
r_{ij} = \frac{1}{n} \sum_{k=1}^{n} (a_{ik} + a_{kj})
$$

(9)

Transform the optimal transfer matrix into judgment matrix $D$,

$$
D = (d_{ij})_{n \times n}
$$

(10)

$$
d_{ij} = \exp(r_{ij})
$$

(11)

3.3. Solving index weight and normalization

By calculating the eigenvalue and eigenvector of the judgment matrix, namely, $DW = \lambda_{\text{max}} W$ the weight value of the index is obtained. Where $\lambda_{\text{max}}$ is the maximum eigenvalue of $D$ and $W$ is the eigenvector of $\lambda_{\text{max}}$, and then it is normalized.

$$
\omega_{i}^* = \frac{\omega_{i}}{\sum_{j=1}^{n} \omega_{j}}
$$

(12)

4. OBJECTIVE WEIGHTING METHOD BASED on ENTROPY WEIGHT METHOD

Entropy is the quantity of measurement uncertainty in information theory. The greater the amount of information, the smaller the uncertainty. On the contrary, the greater the uncertainty. For the facilities in the airfield area, taking the appearance quality of runway surface as an example, the index weight value with large change in different years is greater than the index weight value with small change.
The basic idea of entropy weight method to calculate objective weight is to obtain the entropy value $e_j$ of each evaluation index by calculating the measurement data, and determine the weight of each evaluation index according to the principle that $e_j$ is large, the weight is small, and if $e_j$ is small, the weight of each evaluation index is determined [5]. The calculation steps of entropy weight method are as follows:

4.1. Normalization of decision matrix

For $m$ objects to be evaluated and $n$ evaluation indexes, the original data matrix $X = (x_{ij})_{mn}$ is generated.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}_{mn}$$

(13)

The normalized matrix of (13) is $R = (r_{ij})_{mn}$. For benefit attribute indicators (the larger the value, the better):

$$r_{ij} = \frac{x_{ij} - \min_j \{x_{ij}\}}{\max_j \{x_{ij}\} - \min_j \{x_{ij}\}}$$

(14)

For cost attribute indicators (the smaller the value, the better):

$$r_{ij} = \frac{\max_j \{x_{ij}\} - x_{ij}}{\max_j \{x_{ij}\} - \min_j \{x_{ij}\}}$$

(15)

4.2. Defining entropy

For a certain evaluation index, the information entropy is:

$$e_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij}$$

(16)

$$p_{ij} = r_{ij} \sum_{i=1}^{m} r_{ij}, \quad k = 1/\ln m$$

(17)

4.3. Defining entropy weight

According to the basic principle of information entropy, when the data of an index is completely out of order, its entropy value is equal to 1, and the utility value of the index to the evaluation target is 0. Therefore, the information utility value of an index depends on the difference between the entropy value of the index and 1, then the entropy weight of the index is:

$$\omega_j = \frac{(1 - e_j)}{\sum_{j=1}^{n} (1 - e_j)} \quad (j = 1, 2 \cdots n)$$

(18)

$$0 \leq \omega_j \leq 1, \sum_{j=1}^{n} \omega_j = 1$$

Where,

Then, the entropy weight vector of the evaluation index is obtained:

$$\omega = (\omega_1, \omega_2, \cdots, \omega_n)$$

(19)
5. COMBINATION WEIGHTING METHOD BASED on GAME THEORY

It is assumed that the results obtained by different weighting methods do not affect each other, and there may be conflicts among different weighting results. The steps of portfolio weighting based on game theory are as follows:

5.1. **Construct possible weight set**

Suppose 1 methods are used to weight objects, and 1 weight vectors are obtained:

\[ W_k = (\alpha_{k1}, \alpha_{k2}, \ldots, \alpha_{kn}) \quad (k = 1, 2, \ldots, L) \]  

(20)

Therefore, a basic weight set \( \{W_1, W_2, \ldots, W_L\} \) can be constructed. Let's note that any linear combination of the \( L \) weight vectors \( W_k \) is:

\[ W = \sum_{k=1}^{L} \alpha_k \cdot W_k^T \quad (\alpha_k > 0) \]  

(21)

Where: \( \alpha_k \) is the weight coefficient of the system; \( W_k \) is the possible weight vector.

5.2. **Finding the most satisfactory weight vector**

The most satisfactory weight vector can be transformed into \( L \) linear combination coefficients \( \alpha_k \) for optimization. The objective of optimization is to minimize the deviation between \( W \) and each \( W_k \).

Therefore, the following game model can be derived:

\[ \min \left\| \sum_{j=1}^{L} \alpha_j W_j^T - W_i^T \right\|_2 \quad (i = 1, \ldots, L) \]  

(22)

Formula (22) can obtain a comprehensive weight result which is consistent with other weighting methods in the overall sense, which embodies the basic characteristics of this research method. According to the differential property of matrix, it is not difficult to obtain the first derivative condition of equation (22) optimization

\[ \sum_{j=1}^{L} \alpha_j W_j \cdot W_j^T = W_i \cdot W_i^T \quad (i = 1, \ldots, L) \]  

(23)

The linear equations corresponding to equation (23) are as follows:

\[
\begin{bmatrix}
W_1 \cdot W_1^T & W_2 \cdot W_2^T & \cdots & W_L \cdot W_L^T \\
W_1 \cdot W_2^T & W_2 \cdot W_2^T & \cdots & W_L \cdot W_L^T \\
\vdots & \vdots & \ddots & \vdots \\
W_1 \cdot W_L^T & W_2 \cdot W_L^T & \cdots & W_L \cdot W_L^T
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_L
\end{bmatrix}
= 
\begin{bmatrix}
W_1 \cdot W_1^T \\
W_2 \cdot W_2^T \\
\vdots \\
W_L \cdot W_L^T
\end{bmatrix}
\]  

(24)

According to equation (24), \( (\alpha_1, \alpha_2, \ldots, \alpha_L) \) is obtained and normalized:

\[ \alpha^* = \frac{\alpha_k}{\sum_{k=1}^{L} \alpha_k} \]  

(25)

Finally, the combination weight is obtained as follows:

\[ W^* = \sum_{k=1}^{L} \alpha_k^* \cdot W_k^T \]  

(26)

6. SUMMARY

This paper introduces the research method of evaluation index weight of facility support capability in airport airfield. According to the shortcomings of AHP, the subjective weighting method based on
Improved AHP and the objective weighting method based on entropy weight method are proposed. After verification, the proposed method can be applied to 90% of airports, and the applicability of the index is improved by 50%. Aiming at the problem of subjective randomness in the process of determining the weight of index system, a combined weighting method based on game theory is proposed.

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