Nontrivial Pure Zero-Scattering Regime Delivered by a Hybrid Anapole State

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Abstract. The ability to manipulate electric and magnetic components of light at the nanoscale delivered by dielectric and semiconductor components is paving the way to a plethora of new optical effects, leading towards novel types of sources and nanoantennae with exceptional electromagnetic signatures, flexible and tunable metasurface architectures, enhanced light harvesting structures, etc. Recently, the “anapole” states arising from the destructive interference of basic multipoles and their toroidal counterparts have been widely exploited to cancel radiation from an individual scattering channel of isolated nanoresonators, while displaying nontrivial near fields. As such, anapole states have been claimed to correspond to non-radiating sources. Nevertheless, these states are commonly found together with high order multipole moments featuring non-zero overall far-field. In this paper, we theoretically and experimentally demonstrate a fully non-scattering state governed by a novel 4-fold hybrid anapole with all the dominant multipoles suppressed by their corresponding toroidal (retarded) terms, i.e. a dark analogue of the superscattering effect. This invisibility state, however, allows for non-trivial near-field maps enabled by the unique interplay of the resonant Mie-like and Fabry-Perot modes as demonstrated by the quasi-normal modal expansion. Moreover, the hybrid anapole state is shown to be “protected”: the spectral position of the non-scattering point remains unperturbed in the presence of a substrate with significantly high refractive index. We experimentally verify our novel effect by means of dark field measurements of the scattering response of individual nanocylinders. The obtained results are of high demand for efficient sensing and Raman scattering setups with enhanced signal-to-noise ratio, highly transmissive metasurfaces for phase manipulation, holograms, and a large span of linear and non-linear applications in dielectric nanophotonics.

Introduction

Over the past few years, all-dielectric nanophotonics has become one of the cornerstones of the modern research in nano-optics.¹ Unlike plasmonic structures, dielectric ones allow to overcome the fundamental limitation of Ohmic losses. Utilizing electric and magnetic Mie-like resonances of nanoparticles consisting of low-loss high-index semiconductor or dielectric materials, such as Si, TiO₂, Ge, GaAs,²,³ enables manipulating both the electric and magnetic components of light at the nanoscale. This emerging field has already led to a wide range of exciting applications, such as low-loss discrete dielectric waveguides⁴-⁵ different types of dielectric nanoantennas (Yagi-Uda⁶, super-directive antennas⁷, etc.), directional sources⁸, light harvesting and antireflective coatings⁹-¹², all-dielectric metasurfaces with artificially tailored optical response¹³-¹⁶, to mention just few.

Therefore, the ability to properly describe and predict electromagnetic scattering is of a prime importance in order to manipulate the behavior of light at the nanoscale. For this purpose, different types of electromagnetic multipole expansions were introduced¹⁷-²¹. Among them the charge-current Cartesian decomposition is one of the most common and widely used for describing optical signatures of nano-objects of arbitrary shape¹⁷-¹⁹. One of the most intriguing
possibilities delivered by the Cartesian multipole expansion is the ability to define the so-called toroidal moment family allowing for an additional flexibility in light governing. The electric toroidal dipole moment is the lowest order member of the toroidal family. Its formal meaning is associated with the poloidal currents flowing along the meridians of a torus and generating a magnetic field loop. Higher order toroidal moments, also known as mean square radii, feature more complex current distributions recently investigated in Refs.

After the first experimental demonstration of artificial metamaterials presenting significant electric toroidal dipole contributions, toroidal moments are paid ever-increasing attention and are now widely utilized in nanophotonics and metamaterials, active photonics, ultrasensitive biosensing, and applications requiring strong near field localization. The fields radiated by toroidal moments share the same angular momentum and far-field properties as their Cartesian electric or magnetic multipolar counterparts, allowing for the realization of two exciting effects: enhanced multipolar response enabled by the constructive interference of the fields, and mutual cancellation of the far-field contributions via the destructive one, so-called “anapole” states. The electric dipole anapole was recently observed in a dielectric nanodisk.

In this scenario the far-field radiation from a given electric or magnetic multipole is suppressed. Consequently, anapole states, in principle, could drive low-scattering regimes. This promising feature is of high demand for, e.g., cloaking and invisibility research and a variety of nonlinear applications enabled by strong near-fields. However, the great majority of the investigations on anapole-states are limited to the electric dipole term only. Recently, the authors of Ref. pointed out the exciting possibility to simultaneously cancel several multipoles in spherical dielectric particles, which can be well understood in terms of Mie theory. However, in this case, “hybrid” anapole states are always hidden by the contributions of other multipole moments, driving non-negligible scattering, which is a strict limitation of the spherical geometry.

Fortunately, state of the art developments in the theory of multipole expansions have opened the possibility to qualitatively and quantitatively investigate higher order electric and magnetic anapole states in scatterers with arbitrary shape, and allow their physical interpretation in terms of the interfering higher order electric and magnetic multipoles and their corresponding toroidal counterparts.

Here we demonstrate for the first time the existence of such exotic dark states, schematically illustrated in Figure 1. We theoretically predict a novel type of non-trivial invisibility (non-scattering regime), accompanied by an effective internal field concentration, governed by the spectral overlap of four electric and magnetic anapole states of different orders. This ‘true’ Hybrid Anapole state originates from the far-field destructive interference of all the leading electric and magnetic Cartesian multipoles of a cylindrical scatterer with their associated toroidal moments. Complementing the new multipolar approach, the near-field maps are interpreted in terms of the fundamental modes of an open cavity (quasi-normal modes), thus providing a complete physical picture of the effect. Moreover, we have observed experimentally these hybrid anapole states in the visible range (with very good agreement with the theoretical predictions), in high-index silicon nanoparticles having sizes comparable to the wavelength of the incident light.

The manuscript is organized as follows:

We start by giving a short overview of the approach utilized for the calculation of the multipoles providing a quantitative description of high order anapole states in terms of the interference between basic and toroidal moments. Then we proceed to the particular situation of a four-fold anapole state realized in a high-index cylindrical nanoparticle. The physical origin and the dynamics of the effect are then discussed in terms of both the irreducible multipole and
quasinormal mode expansions. Next, the influence of different dielectric substrates on the near- and far-fields is investigated. Finally, we provide a short description of the experimental setup, details of the measurements and a direct comparison between the theoretical and experimental results.

**Figure 1.** Artistic representation of the novel effect. A normally incident plane wave excites nontrivial modal contributions in a Si nanocylinder whose interference with the background field leads to a four-fold hybrid anapole state (see Figure 2a), yielding the nanoaentenna virtually invisible in the far and near fields. The same eigenmodes are responsible for enhanced electromagnetic energy storing inside the resonator. Inset depicts the current distributions of the two resonant eigenmodes arising due to standing waves between the top and bottom walls (red) and lateral walls (blue) in the plane of incidence.

**The Cartesian multipole expansion and high-order anapole conditions**

The analysis of the optical response of a nanoparticle is usually carried out via the decomposition of the scattering cross section as a sum of multipoles, which represent independent scattering channels of the object. Here we utilize the irreducible Cartesian multipole expansion derived in Ref.27 (for completeness, also given in the Supplementing Information), which explicitly takes into account higher order toroidal moments. The latter provides the conceptual framework necessary for the physical understanding of higher order anapole states.

Within this approach, in the long wavelength limit the condition for an electric or magnetic anapole of order $n$ is:

$$P^{(e,m)}_{n} + i \frac{k_d}{v_d} T^{(e,m)}_{n} = 0$$  \hspace{1cm} (1)

Here we have denoted $n$th order electric or magnetic moments with $P^{(e)}$ and $P^{(m)}$ and corresponding electric and magnetic toroidal moments with $T^{(e)}$ and $T^{(m)}$, respectively. The
number of subscripts indicates the order of each Cartesian tensor, i.e., one subscript corresponds to dipole, two correspond to quadrupole, etc. $k_d, \varepsilon_d$ are the wavenumber and the dielectric permittivity of the host medium, and $v_d$ is the speed of light in the medium. A hybrid anapole state occurs when more than one multipole moment fulfills Eq. (1) at a given wavelength, resulting in a simultaneous suppression of scattering of two or more channels. However, as mentioned above, light, in general, can be radiated out through other non-zero multipole moments, destroying the overall effect. Thus, only the cancellation of all the leading multipoles can enable true low-scattering or “invisible” regimes.

For the sake of clarity, in the rest of the manuscript we will rely on the widespread notation for low-order multipoles, i.e. $p, m$ for electric and magnetic dipoles, and $Q^{(e)}, Q^{(m)}$ for electric and magnetic quadrupoles.

Near-zero scattering hybrid anapole states

Under conventional plane wave illumination, hybrid anapole states in homogeneous spherical particles are unfortunately hidden by the contributions of high order multipoles.\(^{47,50}\) Note that this restriction naturally vanishes for nanoobjects with additional geometrical degrees of freedom, like cylinders or parallelepipeds.\(^{51}\) Throughout this Letter, and particularly in the next section, we will unveil the fundamental reason behind this remarkable behavior.

Let us consider a cylindrical constituted of amorphous silicon nanoparticle embedded in air (the dielectric function is measured directly from the bulk material, see Supplementing Information). Hereafter we will use this material in both theoretical and experimental studies. The illumination scheme is presented in the left inset of Figure 2a (normally incident $x$-polarized plane wave propagating along $−z$ direction).

![Figure 2a](image)

**Figure 2.** (a) Multipole reconstruction of the numerically obtained scattering cross section for the cylindrical amorphous silicon nanoparticle (see Supplementing Information for the measured refractive index dispersion). In the legend caption, “total” implies that both basic and toroidal contributions of a given multipole are plotted. Right inset of (a) corresponds to the total electromagnetic field near the nanoparticle, left inset schematically depicts the cylindrical particle and the illumination setup. The geometrical parameters of the cylinder are: height $H=367$ nm, diameter $D=252$ nm. Point A ($\lambda = 780$ nm ) corresponds
to the hybrid anapole state. (b) Dimensionless scattering efficiency \( Q_{\text{sc}} = 4\sigma_{\text{sc}} / \pi D^2 \) map as a function of the diameter. Each panel in (c): Amplitudes and phase differences between the multipoles and their toroidal counterparts; from left to right, respectively, the electric and electric toroidal dipoles, the magnetic and magnetic toroidal dipoles, and the electric and electric toroidal quadrupoles. Amplitudes correspond to the left ordinate-axis, and phase differences are read from the right ordinate-axis. Vertical black dashed lines correspond to the point A from (a).

The design methodology is based on the following: firstly, the excitation wave setup restricts the excited electric multipole components to the plane of incidence (k-E plane), and the magnetic multipoles to the perpendicular k-H plane, (see Supplementing Information). The incident field induces, respectively, the \( p_x \) and \( Q_{1x}^{(e)} = Q_{1x}^{(m)} \) components of the electric dipole and quadrupole moments, and the \( m_y \) and \( Q_{1y}^{(m)} = Q_{1y}^{(m)} \) components of the magnetic dipole and magnetic quadrupole respectively. Toroidal multipoles follow the same rule as their basic counterparts, and a similar scenario takes place for higher order multipoles. Consequently, according to the multipole expansion (Eq. (1) in the Supplementing Information), only x and z components of the polarization currents are non-zero, as proven numerically and shown in Figure 2c.

Secondly, we note that the spectral positions of the full (basic and toroidal parts) electric dipole and magnetic quadrupole anapoles are mainly dependent on the cylinder’s radius, while the wavelengths of the full magnetic dipole and electric quadrupole anapoles (see 27 for details) change both as functions of the cylinder height and radius. Figure S3 in the Supplementing Information illustrates the dynamics of the multipoles in detail. Thus, carefully tailoring these two geometrical degrees of freedom makes possible to place the anapoles of all the leading terms ultimately close to each other (Figure 2b), providing a strong scattering minimum (Figure 2a, point A).

The total scattering cross section and its multipole decomposition after the numerical optimization are shown in Figure 2a. Perfect agreement between the sum of the multipole contributions given by Eq.(1) in the Supplementing Information and the result of the full-wave simulations in Comsol Multiphysics is demonstrated, proving that only the first four multipoles are sufficient in the visible range. Therefore, the low-scattering regime delivered by this state provides almost perfect particle invisibility in terms of far-fields (see the right inset on Figure 2a).

The different panels in Figure 2c show the amplitudes and phase differences of the three most relevant multipoles with their toroidal moments. The results further confirm that the generalized anapole condition in Eq.(1) is well fulfilled for each pair (the amplitudes are equal, and the phase shift is about \( \pi / 2 \) rad) at the hybrid anapole wavelength \( \lambda = 780 \text{ nm} \).

Noteworthy, the near-zero values of the full scattering coefficients do not imply the induced polarization currents in the particle to be also close to zero. This is in agreement with the usual anapole behavior 52, and, due to the suppression of several multipoles simultaneously, the hybrid anapole also displays well confined internal fields. Figure 3b demonstrates the average electromagnetic energy density inside the cylinder at the hybrid anapole wavelength to exceed 12 times the value of free space almost without leakage of the field outside the particle volume. This, in its turn, further enhances invisibility (see Figure S1a in the Supplementing Information) and reduces the interaction with the surrounding (see the following sections).

### Quasi-normal mode analysis of the hybrid anapole state

While Cartesian multipoles are suitable for the description of far-fields, in this section we introduce the natural quasi-normal mode (QNM) expansion 49 of near-fields and internal currents,
which in the following will allow us to further unveil the physics behind the hybrid anapole state. QNMs provide a suitable basis for the induced polarization currents:

\[
\mathbf{J}(\omega, \mathbf{r}) = \sum_s \alpha_s(\omega) \mathbf{J}_s(\omega, \mathbf{r}) - i\omega \varepsilon_0 (\varepsilon_p - \varepsilon_m) \mathbf{E}_{\text{inc}}(\omega, \mathbf{r}) .
\]

Here \( \mathbf{J}_s(\mathbf{r}) = -i\omega \varepsilon_0 (\varepsilon_p - \varepsilon_m) \hat{\mathbf{E}}_s(\mathbf{r}) \) and \( \alpha_s(\omega) \) are, respectively, the induced modal current distribution as a function of the internal mode field, and the excitation coefficient of the \( sth \) mode describing its individual contribution to the total current at a given frequency.

We use a modified version of the freeware QNMEig developed by the authors of Ref. 48. More details on the approach can be found in the Supplementing Information. For simplicity, we consider a dispersionless, lossless nanocylinder with a constant refractive index \( n \approx 3.87 \) (corresponding to amorphous Si at 780 nm), so that the excitation coefficients depend solely on the fields of an individual QNM. Losses and refractive index dispersion of the original design are negligible in the considered spectral range, and therefore this approximation does not remarkably change the scattering cross section and average electromagnetic energy density.

The results of the QNM expansion are displayed in the different panels of Figure 3. The correctness of our calculations in the studied spectral range, particularly near the scattering dip, is well validated in Figures 3a-b by comparing the sum of the individual QNM contributions with the numerically obtained total scattering cross section (a) and average electromagnetic energy density inside the cylinder (b). For hereon we shall label the QNMs with the standardized notation for the modes of isolated cylindrical cavities, i.e. \( (TE,TM)_{uv} \), where the sub-indices denote the number of standing wave maxima in the azimuthal (\( u \)), radial (\( v \)) and axial (\( \ell \)) directions. \( TE \) and \( TM \) indicate the predominant nature of the internal field distribution. Specifically, \( TM \) (transverse magnetic) modes have \( H_z \approx 0 \), while \( TE \) (transverse electric) have \( E_z \approx 0 \).
**Figure 3.** (a) Alternative scattering cross section decomposition by means of the QNM expansion method. The full-wave simulation is nearly the same (without losses) as in **Figure 2**, but in linear scale. Colored lines are the individual contributions of the physical QNMs. The contributions of modes having their resonances outside the considered spectral range are added up in the green dashed line. Resonant modes in the considered spectral range are associated with the $TE_{111}, TE_{110}$ and $TM_{113}$ modes of the isolated cylinder. Blue line corresponds to the reconstructed scattering cross section, confirming that all the resonant spectral features can be successfully recovered via this method, and demonstrating good agreement near the hybrid anapole, point A. (b) Spectra of the volume averaged electromagnetic field energy inside the cylinder, and individual contributions of the excited modes. Colors and legends are the same as in (a). The electromagnetic energy density has been normalized with respect to the incident electromagnetic energy density in vacuum $\omega_{\text{EM}} = \varepsilon_0 c^2$. Very good agreement is obtained with the full-wave simulations. (c) Normalized internal electric field distributions of the two most relevant modal contributions near point A, from left to right, associated with Fabry-Perot ($TM_{113}$) and Mie-like ($TE_{110}$) standing wave patterns, respectively. Their addition via Eq. (2), together with the background modes (Bckg.), leads to the reconstruction of the internal fields of the hybrid anapole, also displayed on the right-hand side of (c).

The spectral behavior of the resonant QNMs can be well described by means of the well-known Fano formula with two critical points, representing maxima and minima in scattering $^{54}$, as demonstrated in section 5 of the Supplementing Information. The other nearby QNMs constitute the background scattering contribution of the particle.

In **Figure 3a**, we note that a total of three QNMs resonate in the visible range. The correct reconstruction of the scattering cross section requires taking into account background modes, despite their resonances being outside the considered spectral range (green dashed line). Nevertheless, at point A, only the $TM_{113}$ and $TE_{110}$ modes present a resonant ‘Fano-like’ response.

Now the invisibility effect can be easily grasped as a consequence of modal interference: a clear signature that this is indeed the case are the resonant negative contributions to scattering presented by both the $TE_{110}$ and $TM_{113}$ modes. This implies that, when the incident field impinges in the resonator, energy exchange takes place between the two and the background QNM fields$^{55}$. This is a consequence of the QNMs not obeying the usual conjugate inner product relation of orthogonal modes in Hermitian systems$^{56}$. Here it is important to emphasize the unusual feature of the hybrid anapole: the two resonant QNMs dominating the spectra are simultaneously negatively suppressed by interference with the background.

A completely different picture arises within the resonator. **Figure 3b** presents the modal decomposition of the internal energy stored in the cylinder in the vicinity of point A. This is one of the main results of the section, since, contrarily to the multipole expansion, the QNM decomposition allows us to clearly distinguish the contributions of the different modes to the internal fields. Firstly, we note that, similarly to an electric dipole anapole particle$^{35}$, the electromagnetic energy is significantly enhanced (around 9 times) with respect to the incident plane wave. Secondly, it is clearly seen that the stored energy at the hybrid anapole is mainly driven by the $TM_{113}$ mode due to its higher quality factor and the proximity of its resonant wavelength, and in a minor measure by the $TE_{110}$ and the sum of the background contributions. Energy exchange between the internal fields of the QNMs is strongly minimized, as reflected in the fact that no negative contributions to the internal energy can be appreciated. Overall, the QNM analysis given in **Figure 3** demonstrates that both the invisibility effect (outside the cylinder) and the internal energy enhancement at the hybrid anapole state are mediated by the simultaneous resonant response of the $TM_{113}$ and the $TE_{110}$ modes. The background modes, in the other hand, while they do not apparently define significantly the spectral features of the figures of merit, play
also an important role since their interference with the resonant ones gives rise to the invisibility effect.

The electric field distributions of the $TE_{110}$ and $TM_{113}$ modes are shown in Figure 3c. Following Refs. 57,58 we can classify the first as a ‘Mie’ type mode, similar to the ones supported by an infinite cylinder, while the second is of the ‘Fabry-Perot’ (FP) type 57, arising due to the formation of a standing wave pattern between the top and bottom walls of the resonator, i.e. having non-zero axial wavenumber ($\ell \geq 1$). Their distinct origin unveils the reason why it is possible to obtain a hybrid anapole state in this particular geometry, contrarily to spherical scatterers. As shown in the Supplementing Information, in a first approximation, the real parts of the eigenwavelengths of the modes in the cylinder can be approximated as

$$\lambda_{uv} \approx \frac{\pi D}{n_p \sqrt{\left(\frac{\ell \pi D}{2H}\right)^2 + \zeta^2_{uv}}} ,$$

where $\zeta_{uv}$ is the $u$th root of the $v$th Bessel function for $TE$ modes, or its first derivative for $TM$ modes. For FP modes, $\ell \neq 0$ and the denominator in Eq. (3) displays a strong dependence on the aspect ratio $D/H$ of the cylinder. In contrast, since $\ell = 0$ for Mie modes, their eigenwavelengths only change with $D$. Thus, the eigenwavelengths and the multipolar content of these two mode types are independently tunable from each other, resulting in a flexible control over the optical response of the resonator and enabling the simultaneous scattering suppression observed in Figure 2a, and Figure 3a - the hybrid anapole state.

A straightforward comparison between the QNM and multipolar methods allows to determine the multipoles radiated by a given QNM (see section 6, Figure S3, in the Supplementing Information). Specifically, Figure S3 leads us to the conclusion that the $TE_{110}$ mode radiates primarily as $p$, with a minor $Q^{(m)}$, while the $TM_{113}$ mode scatters as a combination of $m$ and $Q^{(e)}$ (in both cases, referring to both their basic and toroidal counterparts). When scattering from a mode is resonantly suppressed, radiation from the multipoles associated to it is also strongly minimized, and results in the different multipole anapole states observed in the decomposed spectra of Figure 2a. Thus, the close proximity of the destructive interference points of the $TE_{110}$ and $TM_{113}$ modes at point A leads to an overlap of the anapole states of the four dominant multipoles. In this fashion, using the QNM expansion approach, we have shown an alternative and general physical explanation of dark scattering states and qualitatively illustrated its link to the multipolar response of the particle.

Summarizing the results of this section, we have investigated the fundamental origin behind the hybrid anapole state and found that this novel effect arises from the modal interference of scattering by resonant FP-type and Mie-like modes excited in the resonator with the background off-resonant modes. Contrarily to a vast majority of the previous works on the conventional electric dipole anapole state, the modal decomposition has allowed us to study the separate contributions to the internal field, highlighting the connection between the irreducible Cartesian multipoles and the QNM expansion. We utilized the unique advantages of both approaches, delivering a unified physical picture of the effect in the near- and far-field regions. Importantly, the considered analysis and explanations developed here are general, and can be utilized to understand the mechanisms behind anapole states of arbitrary order, once a sufficient set of the QNMs is known.

**Evolution of the hybrid anapole state in the presence of a substrate**
Realistic experimental conditions and possible applications require nanoparticles to be deposited on a substrate. Therefore, we should address and explain its influence on the hybrid anapole state.

In a general situation, the incident and scattered fields (including near and far field contributions) are partially reflected back from the substrate inducing ‘effective’ multipoles, different from the case of a homogeneous environment \(^59\). The latter implies that any ‘anomalous’ scattering feature displayed by the nanoparticle, such as, e.g., Kerker-like points \(^60\), relying on a delicate balance between the phases and amplitudes of the leading multipoles, will be perturbed. More formally, the substrate breaks reflection symmetry of the surrounding environment and introduces different possible multipole coupling channels in the effective polarizability tensor describing the particle response to an external electromagnetic field \(^59\).

In comparison with an arbitrary spectral point, the hybrid anapole state is remarkably robust in the presence of a substrate. This particular behavior is illustrated in Figure 4a, where we have plotted the calculated scattering cross sections of the cylinder in free space and deposited over glass \((n_s = 1.5)\), hypothetical substrates with \(n_s = 2, 3\) and over amorphous silicon \((n_s = 3.87)\).

It can be appreciated that the amplitude and spectral position of the scattering dip is not displaced from the free-space values while the refractive index contrast at the bottom of the particle remains non-zero. However, we observe drastic changes in the shape of the Fano profile and anapole state efficiency, once the contrast is absent (silicon particle over silicon substrate). This effect suggests an underlying mechanism by which the hybrid anapole is “protected” against modifications of the substrate refractive index, that will be unveiled in the following.

Intuitively, the substrate invariance can be understood by considering the distinct nature of the two resonant modes involved in the formation of the hybrid anapole (Figure 4c). On the one hand, increasing the refractive index of the substrate leads to a decrease in the lower wall reflectivity, which is crucial to contain the standing waves in the Fabry-Perot mode \(TM_{113}\) inside the resonator. The simulations show that an increasing \(n_s\) results in a higher energy leak towards the substrate and a substantial decrease of the mode quality factor (see Figure 4c). Consequently, its resonance flattens, and disappears when the lower boundary becomes transparent, as shown in Figure 4a.

The standing wave pattern of the \(TM_{113}\) bears a close resemblance with the modes sustained by a one-dimensional Fabry-Perot resonator with the same height \(H\) as the nanocylinder, deposited over a dielectric substrate. The QNMs of this simplified model have the advantage of being analytically solvable\(^59\), thus providing valuable physical insight easy to extrapolate to the problem at hand. As we derive in the Supplementing Information, the QNMs are formed by two interfering plane waves traveling in opposite directions inside the cavity, when the driving wavelength satisfies the condition

\[
\frac{r_{21}r_{23}}{w^2} = 1
\]

where \(w = \exp(iknH)\) and \(r_{21}, r_{23}\) are the Fresnel reflection coefficients from the cavity-air and the cavity-substrate interfaces, respectively. The quality factor of a QNM with axial index \(\ell\) is calculated as

\[
Q_\ell = \frac{\ell \pi}{\ln(1/r_{23}r_{21})}.
\]

Equation (5) serves very well to illustrate the influence of the substrate on the \(TM_{113}\) mode. To do so, it is also important to notice that the numerator is proportional to the real part of the eigenwavelength, while the denominator is linked to its imaginary part. When the two reflection coefficients are unitary, the energy is completely stored inside the resonator and the quality factor
is infinite. Similarly, a decrease in the reflection coefficient from the substrate leads to radiative losses and a decrease in the quality factor, effectively becoming zero when the substrate refractive index matches that of the particle (i.e., no refractive index contrast). Indeed, a lower quality factor leads to less appreciable spectral features, as observed in the simulations (Figure 4a). Another important conclusion that one can draw from the numerator in Eq. (5) is that the real part of the resonant wavelength of the QNM is independent of the refractive index at the walls. Thus, modifying the substrate refractive index does not shift the spectral position of the resonance (i.e., does not shift the hybrid anapole wavelength), but simply changes the amplitude and width of the Fano profile.

With proper normalization, and employing the notation of the inset in Figure 4a, the amplitudes of the incoming and outgoing plane waves inside the resonator are $|A^+_2| = \sqrt{\alpha r_{23} W}$ and $|A^-_2| = \sqrt{\alpha r_{23} W}$, with $\alpha = 1/(4HE)$ (see Supplementing Information for details). As could be expected, plane waves reflected from the substrate are decreased in amplitude when the refractive index contrast becomes lower. Consequently, the field maxima of the $TM_{113}$ mode closer to the substrate are reduced in amplitude. While this prediction is observed in the simulations for relatively low contrasts, the behavior at very small contrasts is very different (see the case with $n_z = 3$ in Figure 4c). In the latter situation, the standing waves along $z$ become negligible in comparison with the initially weaker standing waves in the $x$-$y$ plane, and the QNM can only be well described numerically.

Contrarily, it is noteworthy that even comparably small contrast (3 – 3.87) leads to enough contribution of the $TE_{110}$ mode to still preserve the scattering dip (see Figure 4a). The standing wave pattern of this second mode develops in the lateral walls of the cylinder, and therefore depends much less on variations of the reflectivity of the lower wall, keeping an almost constant quality factor (see Supplementing information). Most of the QNM energy is then stored in the resonator even in the case of zero effective contrast with the substrate, as demonstrated in Figure 4c. This results in a larger contribution of the $TE_{110}$ mode to extinction at small contrasts in comparison with the $TM_{113}$ mode. As a consequence, the electric quadrupole and magnetic dipole contributions are no longer dominant, and the hybrid anapole is mainly driven by electric dipole radiation stemming from the $TE_{110}$ mode (see Supplementing Information).

To conserve the four-fold hybrid anapole, it suffices that the particle-substrate interface reflectivity is high enough to retain the standing waves in the $z$ direction of the $TM_{113}$ mode. The panels in Figure 4b show the total (left) and scattered fields (right) corresponding to the hybrid anapole state with and without a glass substrate. Both designs demonstrate nearly perfect scattering cancellation - the scattered amplitude does not exceed 2% - at the close vicinity of the nanoparticle and is even much less in the far-field zone. Since the detectable signature of an object is the wave-front distortion (in terms of phase and/or amplitude), our cylinder clearly displays the features characterizing an invisibility state while still providing strong internal fields due to the excited modes.

The stability of the hybrid anapole state against changes in the substrate refractive index is an important result, since it renders the effect as an ideal building block for the practical implementation and study of non-linear phenomena, as well as for the development of novel functional fully transparent metasurfaces, enhanced Raman scattering setups, and a variety of applications requiring high signal/noise ratios.
Figure 4. Evolution of the hybrid anapole state with different substrates (a) Comparison between the numerically obtained scattering cross sections for the nanocylinder with the size from Figure 2, deposited on substrates with increasing refractive index. Left inset of (a): Schematic view of the particle placed on a substrate, under normally incident plane wave illumination. Right inset: One-dimensional Fabry Perot model of the $TM_{113}$ mode. (b) Electric field distribution for the indicated plane wave illumination: (1) the particle in free space, left panel: total field, right panel: scattered field $E_{\text{sc}}=E-E_{\text{inc}}$ (2) Deposited on the glass substrate, left panel: total field, right panel: scattered field. The incident field amplitude is $E_{\text{inc}}=1\, \text{V/m}$. Colorbar is set equal for both total and scattered fields for better visualization. (c) Field distributions of the QNMs $TE_{110}$ and $TM_{113}$ when the cylinder is deposited over substrates with different refractive index. As predicted by our qualitative theory, losses from the $TM_{113}$ mode increase when decreasing the refractive index contrast. Contrarily, the $TE_{110}$ mode remains confined in the scatterer.

Thus, we conclude that the changes in the Fano profile even in the limiting case of zero contrast can be well understood by investigating the dissimilar behavior of the two resonant QNMs involved. In this fashion, we have provided a simple and intuitive physical picture which

**Experimental Observations**

To confirm our theoretical predictions, nanocylinders supporting the novel state were fabricated over a glass substrate using standard deposition techniques (see SEM micrography of one of the samples in Figure 5b). The scattering response of the individual nanoresonators was then measured utilizing a high aperture dark field microscopy setup. The general idea is schematically depicted in Figure 5c, more details on the methods can be found in the Supplementing Information.

The shift in the scattering dip with $D$ is in excellent agreement with the calculations (dashed lines, Figure 5a). As expected, an increase of the lateral dimension of the samples leads to a redshift of
the hybrid anapole state. Moreover, since the Fano resonances governing the hybrid state have different dynamics (see Supplementing Information), the amplitude and the overall shape of the scattering dip are also slightly changed with variations in the cylinder diameter. The most pronounced scattering dip is confirmed at $D = 252 \text{ nm}$, corresponding to an almost complete invisibility due to the fine overlap of the four multipole anapole states. Deviations from the theoretical values appear at long wavelengths and are a consequence of the decreased sensitivity of the photodetector.

The observations undoubtedly prove the existence of hybrid anapole states in our configuration, which can be potentially exploited for a large span of applications in linear and nonlinear optics.

**Figure 5.** (a) Solid lines: Dark field measurements of a single isolated nanocylinder. Dashed lines: fitted numerical calculations with different cylinder diameters D. (b) SEM micrograph of one of the samples. (c) Schematic view of the experimental measurement setup.

**Realization of non-Huygens transparent metasurfaces**

Our novel effect can be harnessed to design fully transmissive, all-dielectric metasurfaces without relying on the well-known Huygens condition$^{61,62}$. Contrarily to the latter, the light traverses the structure without significant phase variation, thus rendering the metasurface truly invisible (see Figure 6b). This is a direct consequence of Eq.(1). It can be easily seen by writing the transmission coefficient as a sum of the relevant multipole contributions of the meta-atoms$^{63}$:

$$t = 1 + t_p + t_m + t_{Q^{(e)}} + t_{Q^{(o)}}$$

Each term in the previous sum is proportional to the corresponding total multipole moment (basic and toroidal contributions). For example, the contribution of the total electric quadrupole radiation to transmission when the system is illuminated by x-polarized light is simply:
\[ t(q^0) = A \left( Q_y^0 + \frac{ik_d}{v_d} \tau_y^0 \right) \] (7)

**Figure 6.** Design of a highly compact hybrid anapole-based metasurface. The spacing between the nanocylinders is set to 300 nm, i.e. at a value much smaller than the incident wavelength. (a) Transmitted intensity and phase of the transmitted wave with respect to the incident one. As predicted by Eq. (1), a zero-phase difference at the hybrid anapole wavelength (\( \lambda = 780 \text{ nm} \)), is observed. (b) x component of the electric field in the metasurface at the hybrid anapole state, when the structure is illuminated from above by an incident x-polarized plane wave.

where \( A \) is a constant depending on the lattice period, incident wavelength and refractive index of the host environment\(^{63}\). When Eq. (1) is fulfilled, the term in brackets in Eq. (7) is exactly zero, and the same takes place for the other multipoles. Consequently, we are left with \( t = 1 \) in Eq. (6), and the incident wave leaves the system unperturbed (see Figure 6a-b). Importantly, due to the multipolar suppression, the near field of each individual nanocylinder is exceptionally well confined (see e.g. Figure 4b). As a result, near field lattice coupling is strongly minimized, and the hybrid anapole state can be observed in very dense metasurfaces, such as the one presented in Figure 6, where the cylinder spacing is only 2/5ths of the incident hybrid anapole wavelength. Finally, we note that the results presented above are in remarkable contrast with the optical behavior of a solid aSi thin film at the hybrid anapole wavelength (reflection around 70%).

**Conclusion**

In summary, we have obtained a non-trivial invisibility state in a non-spherical all-dielectric particle with dimensions comparable to the incident wavelength. For the first time to our knowledge, we have been able to describe the present configuration as a combination of anapole states originating from different terms in the Cartesian charge-current multipole expansion of the particle, and thus demonstrated simultaneous cancellation of all leading contributions to scattering. Furthermore, the effect has been shown to be a consequence of the resonant interaction of a Fabry Perot and a Mie-like modes excited inside the scatterer, which are responsible for strong internal electric and magnetic hotspots and, consequently – dynamic energy storing. The hybrid anapole state is then shown both numerically and analytically to be “protected” against significant changes in the substrate refractive index. In this regard, we observe and explain a transition from a four-fold to a two-fold hybrid anapole as a consequence of the nature of the modes involved.

We have confirmed our results by realizing the first experimental measurement in the visible range of a dark hybrid anapole nanoparticle. Finally, we have demonstrated that the effect can be
exploited for the realization of truly invisible (both in amplitude and phase) and highly dense metasurfaces in the visible range. The latter results are not only of fundamental interest, but also suggest a route towards enhanced nonlinear effects in the absence of an elastic scattering background. We anticipate that these states can also be obtained in the visible range for non-centrosymmetric high-index materials such as AlGaAs, thus providing an ideal platform for both third and second harmonic sources. More importantly, in this context the hybrid electric and magnetic nature of the modes at play could lead to new interesting interference effects between the high order nonlinear currents.

Our theoretical and experimental findings have broadened our knowledge of anapole electrodynamics beyond the electric dipole approximation, revealing new perspectives in the field and giving for the first time a solid physical picture in terms of the fundamental QNMs driving the resonator response. The results of this work will open new pathways for the design of the efficient scattering suppression and sensing devices in the field of all-dielectric nanophotonics.

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