PARTIAL SAFETY FACTORS FOR PRESTRESSED CONCRETE GIRDERS STRENGTHENED WITH CFRP LAMINATES

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Abstract

This paper provides a framework for the calibration of partial safety factors in prestressed concrete (PC) girders strengthened in flexure with carbon fibre-reinforced polymer (CFRP) laminates. A hybrid approach was proposed to take advantage of comprehensive non-linear numerical models in reliability analysis using a first order reliability method (FORM) in conjunction to the response surface method (RSM). The PC girders selected for analyses were taken from real structures designed and built since the 1980s, based on old standards, now requiring strengthening and upgrade due to partial corrosion of prestressing strands. Using the proposed approach, a sensitivity analysis was performed to identify the most relevant variables and assess the area of CFRP laminates needed to restore the capacity to new design standards. Following this study, a partial safety factor was proposed for strengthening PC girders using CFRP laminates. A sensitivity analysis also showed the traffic loads and model uncertainties to be the most important variables for calibration.

Keywords: CFRP laminates; concrete girder; reliability; numerical models; partial safety factors; target reliability index.
Introduction

Many reinforced concrete (RC) bridges built in the last decades using precast prestressed concrete (PC) girders are currently in need of retrofit or upgrade due to degradation and increasing traffic. As interventions are progressively undertaken, the use of externally bonded fibre-reinforced polymer (EB FRP) laminates is a competitive technique when compared with other options (e.g., concrete jacketing or epoxy-bonded steel plates). This is due to the low weight and thickness of FRP laminates, easy application, high stiffness and strength, corrosion protection and reasonable costs (CEB-FIB 2001).

There are currently several guidelines applicable to EB FRP laminates, such as the CEB-FIB (2001), ACI 440.2R-08 (2008), CNR (2001), TR-55 (2000), and the AS 5100.8 (2017). To design strengthening solutions using FRP laminates, the guidelines adopt a limit state approach, where safety or reduction factors, respectively, \( g \) and \( f \), are either applied to the overall resistance or to each material property, depending on the standard. A summary of these factors for carbon FRP (CFRP) laminates is shown in Table 1.

Despite the standards available, there are still limitations in terms of their scope of application. Specifically, the partial safety factors were mostly developed for new construction and may not directly apply to rehabilitation/strengthening of existing structures. In this case, the assessment of the partial safety factor to be adopted certainly depends on the type of structure and loading conditions such as flexural, on the confinement (Baji 2017), on the standard adopted in the original design, current state of damage, as well as, on the new standard in place when rehabilitation is sought. In the European context, for example, this issue is particularly critical given that many structures were designed using former national guidelines, which are often less demanding than the new guidelines now in use by all partner countries.

Several researchers (Coelho et al. 2018; El-Tawil and Okeil 2002; Monti and Santini 2002; Okeil et al. 2002; Plevris et al. 1995) addressed the uncertainties in the quantification of safety factors applicable to structures strengthened in flexure with FRP laminates. The general approach is based on the creation of a database with a wide range of parameters and Monte Carlo simulations for each designed member. The resulting randomly generated data sets are then used to develop a resistance model for strength. The probability of failure and the reliability index are normally assessed using a first order reliability method (FORM) with the subsequent calibration of flexural resistance factors.

One of the first studies in this scope was carried out by Plevris et al. (1995) focusing on reinforced concrete beams strengthened with CFRP laminates. The authors classified the strength and the
ultimate strain of the concrete and area of the CFRP laminates as most relevant properties, and calibrated safety factors for a target reliability index of 3.0. It should be mentioned that this study did not include structural rehabilitation. This was addressed later in the studies from Okeil et al. (2002), and El-Tawil and Okeil (2002), where CFRP laminates were used to restore the capacity of degraded bridge girders. The strength reduction factors were calibrated for target reliability index factors of around 3.75. Atadero and Karbhari (2008) performed a reliability study on RC T-beams strengthened with CFRP laminates using real design situations. They developed a methodology to calibrate the strength factors for flexural strengthening based on three reliability indices (2.5, 3.0 and 3.5). They used a simplified analytical model for the debonding of CFRP laminates and showed the reliability of beams to strongly depend on the amount of reinforcement that remains uncorroded in the damaged structure. Intermediate crack and end debonding failure modes on FRP-retrofitted RC T-beams were considered by Pham and Al-Mahaidi (2008). Their study found that the type of debonding significantly decreases reduction factors in the reliability analysis.

It is quite common to perform reliability studies using simplified analytical models with a common assumption of perfect bond between FRP and substrate. In fact, limited research has considered intermediate crack debonding – even if this effect is critical in the analysis of safety factors (Pham and Al-Mahaidi 2008). It is not yet known to which extent the underlying simplifications are safe for design. For example, the interaction of cracks and the debonding, or the failure of the FRP laminates, all are highly related phenomena, and their consideration in numerical models could potentially lead to more demanding safety/reduction factors based on reliability analyses. The study presented in this paper contributes towards these research questions by focusing on the reliability analysis of PC bridge girders strengthened in flexure with CFRP laminates.

The girders selected for analysis are taken from existing PC bridges requiring strengthening based on a set of idealised damage due to corrosion of prestressing steel. The girders were originally designed and built since the 1980s using old standards, in which case any strengthening solution sought here – attachment of CFRP laminates– needs to comply with new standards, in this case European Standards EN1991-2 (2002) and EN1992-2 (2005). The paper quantifies the partial safety factors that could be used for designing the strengthening solution with CFRP laminates and presents a new hybrid procedure to take advantage of non-linear FEM models to accurately simulate the material and structural behaviour thus obtaining a more refined solution in reliability analysis.

The proposed hybrid method combines an analytical simplified model to obtain a first estimate on the reliability index, after which an advanced FEM model searches for a more refined solution for designing the strengthened structure. The study also focusses on the requirements created by the replacement of old standards by the Eurocodes, since these are often significantly more demanding
in terms of safety and loads. Up to the authors knowledge, the study presented in this paper is the first that directly proposes a hybrid method for reliability analysis and quantifies partial safety factors for damaged prestressed concrete girders strengthened with bonded CFRP laminates in the scope of the Eurocodes. The new framework is quite general and can be easily adapted to other codes.

105 Design cases

The bridges studied in this paper are based on a simply-supported structural scheme widely used in main roads connecting mid-sized towns in Portugal. The span is relatively short when compared with most recent practice in construction and ranges between 13 and 19 m. The bridge was designed for one traffic lane in each direction, with a side-walk on both sides. The structure was composed of three prestressed concrete ‘I’-shaped girders – see cross-section defined in Figure 1 and dimensions in Table 2. The mean concrete compressive strength, $f_{cm}$, was 43 MPa, the mean tensile strength, $f_{ctm}$, was 3.2 MPa and the mean Young’s modulus, $E_{cm}$, was 34 GPa. Please note that the notation adopted is in accordance with Eurocode 2 EN1992-2 (2005).

The bridges complied with the provisions from REBAP (1983) with design loads given in RSA (1983). It is worth mentioning that both ultimate and service limit states were considered in the original design. The exterior girder is typically the most critical and is herein considered for further analyses. The three representative spans adopted are, 13 m, 16 m and 19 m, in which case the corresponding main dimensions of the exterior girder are summarised in Table 2.

It should be mentioned that the design loads required by the new European Standards EN1991-2 (2002) and EN1992-2 (2005) can be significantly higher than those obtained with the old standard. For example, the ratio between live and dead bending moments for the shortest span reaches a factor of 3 in the new standard, whereas the same factor drops to 2 in the old one. This means that upgrading the girder also requires strengthening to meet the new standard. The unstrengthened (or undamaged) situation is the reference (D0) in the study that follows. In addition, six damaged scenarios are chosen for the same girder caused by corrosion of prestressing strands. Such scenarios are defined by assuming the loss of area for the prestressing strands ranging from 10 to 30% affecting one (Dx) or the two (2Dx) levels of reinforcement. A summary of all scenarios and the remaining (i.e. uncorroded) area of the prestressing strands, $A_p$, is given in Table 3.

The strengthening of the PC girders is to be achieved using CFRP laminates with anchorage at both ends, as to obtain the maximum benefit from strengthening with CFRP laminates (Garden and Hollaway 1998; Quantrill and Hollaway 1998). The area of the CFRP laminates should restore the structural capacity of the girders.
Proposed hybrid model for reliability analysis

This section proposes a hybrid model for reliability analysis using the design cases defined in the previous section, with the purpose of determining the design area of CFRP laminates to comply with the reliability index as defined by EN 1990 (2002).

General background

Failure is herein defined as the random structural resistance, \( R \), being lower than the current random load demand, \( S \), in which case (Bucher 2009; FERUM 2010; Melchers 2017):

\[
P_f = P(R - S < 0),
\]

and structural reliability is defined by \( 1 - P_f \), which identifies the probability of the structure performing its intended function. The relationship \( R - S \) is designated by limit state function and is a boundary separating acceptable and unacceptable structural performance depending on the random variables defined. Graphically, the probability of failure corresponds to the grey volume represented in Figure 2.a if only two variables are considered.

The reliability index, \( b \), and the probability of failure, can both be shown to be equivalent.

Geometrically, the former parameter directly measures the minimum distance from the origin to the failure domain. This point is the so-called design point – see representation in Figure 2.b \((r^*, s^*)\) – and its cosines direction measure the importance of each parameter on the probability of failure, where a positive value means that an increase of the mean value also increases safety (see Figure 2.c).

The limit state function is typically defined using several variables that may not be normally distributed. In this case, the random variables are transformed from the original space to a standard normal space, which simplifies calculations since the transformed variables will follow an approximated normalised distribution. This normalisation can be applied using the Nataf transformation described by Melchers (2017). It should be mentioned that there is not usually a closed-form equation available for the limit state function. Therefore, the derivation of the reliability index requires an iterative approach to identify the design point. FORM uses a Taylor expansion in the neighbourhood of the design point that is progressively refined. For highly non-linear problems, however, a combination of FORM with the response surface method (RSM) can be more effective. The RSM is used to approximate the non-linear limit state function by a regression function of lower-order polynomials (Bucher, 2009) using selected support points for each random variable. The
reliability index is then determined within two iterative cycles, the first uses RSM to compute an
approximated limit state function, and the second applies FORM to determine the reliability index for the approximated limit state function. Both are applied sequentially until converging into a design point within an acceptable threshold.

In the following section, a new methodology is proposed for the efficient use of RSM and FORM with advanced non-linear numerical models for prestressed concrete girders strengthened with CFRP laminates. The methodology combines both analytical and numerical models to limit the use of time-consuming calculations in the search for the design point.

**Methodology implemented**

The limit state function, \( G \), was herein defined by the difference between the resistance and standardised traffic loads, as follows:

\[
G = g_{mtl} - g_t, \quad 2
\]

where \( g_{mtl} \) is the maximum traffic load scale factor supported by the girder – obtained using the analytical and FEM models as described ahead – and \( g_t \) is the traffic load scale factor. The model uncertainties are considered as:

\[
g^q q q q g \quad 6
\]

\[
G = g_{mtl} \left( E \right)^q R - g_t, \quad 3
\]

where \( E \) is the load model uncertainty. The resistance uncertainty is directly multiplied by the scale factor, whereas the load uncertainty is assigned to the structural model to affect both traffic and remaining loads.

The maximum traffic scale factor is obtained from the ultimate load, and is a function of all remaining random variables, including dead loads. Therefore, the limit state function can be written as:

\[
G = g_{mtl} \left( q_E : n_1 : n_2 : n_3 : \ldots : n_n \right)^q q_R - g_t, \quad 4
\]
where $i$ stands for the random variables.

A hybrid process is herein proposed using RSM and FORM to efficiently take advantage of comprehensive non-linear numerical models. For this purpose, analytical and numerical models are progressively used in the analysis to calculate the area of laminates needed for strengthening the structure and reach the necessary target index. This parameter is taken from EN 1990 (2002) for a level of high economic, social and environmental consequences for structural failure, in which case $b_i$ is 4.3. Please note that more details about each model are provided ahead.
The analytical model is used to calculate the area of the strengthening laminate, \( A_f \), corresponding to the target reliability index defined in the standard. The area of the laminate is searched incrementally, so that each step starts by guessing the area and then obtaining the reliability index using RSM and FORM. If this index is within 1% error of the target, the non-linear numerical model is engaged in a second stage of analysis leading to a more accurate search. This procedure is very efficient, since the use of a computationally demanding non-linear model is minimal and only applied to fine-tune the final reliability index.

Within each cycle of analysis, the iterative procedure first calculates the reliability index, as represented in Figure 3. This is carried out by initialising the design points, \( dp_0 \), and reliability index, \( b \), with mean values used for the random variables. RSM is then applied to define the response surface in the neighbourhood of the design and support points. Analytical and numerical models are used to run structural analyses and define the response surfaces. Finally, a new estimate for the design points, \( dp_{n+1} \), is obtained from FORM by calculating the maximum load scale factor at the design point, \( dp_n \), including an updated reliability index, \( b_{n+1} \). If the change in the reliability index is less than 1% the procedure stops and convergence is found. Otherwise, a new surface approximation is calculated with RSM based on the most recent approximation for the design points and the whole cycle starts.

**Probabilistic models**

Only the most significant variables are herein considered random following the recommendations found in (Gomes et al. 2014). These are the steel strand strength, \( f_p \), CFRP laminates strength, \( f_s \), and resistance model uncertainties, \( q_r \), on the side of the resistance models, and traffic loads scale factor, \( g_t \), dead loads, \( g_d \), self-weight of concrete, \( g_e \), and load model uncertainties, \( q_l \), on the side of the load model. Table 4 summarises the statistical descriptions for the adopted variables.

In the definition of the distribution types and coefficients of variation (COV) for each variable, available bibliography was considered. The steel strand strength model was selected based on the study from Jacinto et al. (2012). The CFRP laminates strength model was defined after a probabilistic study by Gomes et al. (2018), where the Weibull distribution was shown to be accurate for probabilistic analyses. The dead loads corresponding to the weight of sidewalks, guard rails and asphalt are considered uniformly distributed over the girder, following a normal distribution with a
COV of 0.10 (von Scholten and Vejdirektoratet 2004).
The statistic values of the traffic loads, $Q$, can be assumed to have a normal distribution according to von Scholten and Vejdirektoratet (2004). Considering that the nominal values defined in the standard correspond to the 95th percentile and that the bridge lifetime horizon is 50 years for the strengthened situation, the distribution for the maximum load asymptotically approaches a Gumbel distribution with mean and standard deviation values (Ang and Tang 2007) provided by the following:

$$m = u_n + \frac{g}{a_n}$$

$$s = \frac{p}{\sqrt{6a_n}}$$

where $g$ is the Euler–Mascheroni constant (0.5772), $n$ is the time in years, $u_n$ is the shape parameter and $a_n$ is the scale parameter.

The characteristic value of traffic loads from Gumbel distribution is found using the following equation:

$$Q_{k} = \frac{m}{1+1.866V_d}$$

where $\mu$ and $V_d$ are respectively the mean and the coefficient of variation (COV) of the traffic loads, $Q$. The 95th percentile loads scale factor is herein taken as 1 and the COV 0.15 (Atadero and Karbhari 2008; El-Tawil and Okeil 2002; Wisniewski 2007).

The model uncertainties were defined following the range of recommendations in (El-Tawil and Okeil 2002; JCSS 2001), with a mean value of 1.05 and COV of 0.105.

It should be mentioned that effects of the ageing of the epoxy and fatigue loads were not considered in the analyses.

**Structural analysis**

This section briefly describes the two approaches used for performing the structural analysis of the girders.

- Analytical model

The analytical model is based on a cross-sectional analysis using the stress-strain diagram shown in
Figure 4 for a linear strain distribution over the girder depth. Plane sections were assumed to remain plane after bending, in which case the flexural moment is computed as follows:
\[ M = F_c z_c + F_p z_p + F_f z_f \]

where \( F_c \) is the compressive force in concrete, \( z_c \) is the distance from the neutral axis to the upper fibre, \( x \), to the concrete force, \( F_p \) is the force due to prestressing strands, \( z_p \) is the distance between the prestressing strands and the neutral axis, \( F_f \) is the force due to CFRP laminates, and \( z_f \) is the distance from the CFRP laminates to the neutral axis.

The constitutive model for concrete under compression is modelled using the stress-strain relation given in EN 1992-1-1 (2004):

\[
\varepsilon_c = \frac{k h^2 - h}{f_c} \left( \frac{1}{1 + (k - 2) h} \right) 
\]

with

\[ h = \frac{\varepsilon_c}{\varepsilon_{c1}} \quad \text{and} \quad k = 1.05 \frac{E_c}{f_c} \left| \frac{\varepsilon_{c1}}{f_c} \right| \]

where \( \varepsilon_{c1} \) is the strain at peak stress according to EN 1992-1-1 (2004), \( E_c \) is the secant Young’s modulus of concrete, and \( f_c \) is the concrete cylinder compressive strength. The tensile strength of concrete is disregarded in the analysis.

The ultimate strength of the cross-section is calculated in two main stages of analysis. In the first stage, the stress state at the cross-section is calculated before strengthening, so that the stress/strains installed just before applying the CFRP laminates are known. This first step is critical to assess the initial strain at the soffit of the girder, where CFRP laminates are going to be applied, and that will no longer be transferred to the laminates once the strengthening system is fully operational. In the second stage of analysis, the ultimate moment of the girder is finally obtained by identifying which failure mode occurs first, i.e. the mode with the lowest bending moment. All possible situations are accounted for, e.g. debonding and failure at CFRP laminates, crushing of concrete, prestressing strands reaching the 0.1% proof stress before (or simultaneously) with failure at CFRP laminates.

During each stage of analysis all calculations are performed following a standard iterative procedure that searches for the location of the neutral axis and assures the balance of forces inside the cross-section.

\[
\frac{\varepsilon_c}{\varepsilon_{c1}} \quad \text{and} \quad k = 1.05 \frac{E_c}{f_c} \left| \frac{\varepsilon_{c1}}{f_c} \right| 
\]
A finite element model based on the discrete strong discontinuity approach (DSDA) is used to perform the advanced analysis of the structural behaviour of the concrete girders strengthened with
CFRP laminates (Dias-da-Costa et al. 2018b). The model is based on finite elements enhanced by additional degrees of freedom that are progressively placed along the crack paths to measure their widths. The effect of the crack opening is then transmitted to the edges of the enhanced element as a rigid body movement that increases the overall deformability of the structure due to damage propagation and development (see Figure 5a). During the structural analysis, new cracks are activated inside each element whenever the strength of concrete is reached, therefore preventing the maximum tensile stress to rise above it. Each crack undergoes a traction-separation law that softens the tensile stress, simultaneously reducing the stiffness of the element while increasing the crack width. The development of the model from a conceptual and mathematical points of view can be found in (Dias-da-Costa et. al 2009).

The embedded cracks can naturally interact with steel and strengthening material, thus capturing the local debonding and increased deformation due to damage of the materials. This capability is critical to accurately predict the ultimate strength of the member (Dias-da-Costa et al. 2018b). Figure 5b compares a discrete crack model with smeared models in the neighbourhood of highly-localised stress fields caused by the opening of a crack and represents the local stretching and failure of the CFRP laminates. Such model was shown to provide reliable results in terms of crack propagation, crack patterns and crack openings for both service and ultimate loads in concrete members under flexural loads (Dias-da-Costa et al. 2010; 2017 and 2018aa). A detailed presentation about the implementation aspects can be found in (Dias-da-Costa et al. 2009; 2013 and 2013).

The numerical model is validated using experimental data from flexural tests performed on PC girders, one with and two without CFRP laminates (Fernandes 2007; Fernandes et al. 2013) – see Figure 6a. The ‘I’-shaped girders were tested under flexural loading. The active reinforcement was composed by twelve 3/8” prestressing bonded strands at bottom and two 3/8” unbonded post-tensioning strands at the top of the cross-section – see Figure 6b. The stirrups in the web were 5 mm bars with 500 MPa yield stress in a two-legged arrangement with 150 mm spacing along the span.

The pre-tensioning strands were initially stretched to 1,430 MPa before pouring concrete and kept attached to the precast table, i.e., not engaged with the girder until day 5. At that age, six pre-tensioning strands were released and the post-tensioning strands were stretched to 1,160 MPa. Next, the six remaining pre-tensioning strands were cut from the table and the girder fully demoulded. The post-tensioning strands were only installed to avoid premature failure due to the high level of stress applied by the pre-tensioning strands at such an early age. It should be highlighted that once the girders are finally taken to the construction site to erect the bridge, the post-tensioning strands are meant to be deactivated after enough vertical load is applied to the structure. These girders were part of a linkage project with precast industry to study the economical and practical possibility of
extremely short turnaround times, in which case high-strength concrete was used to allow enough
compressive strength when releasing the pre-tensioning strands at an early age. The girders were
designed and experimentally tested by one of the co-authors and were selected for validation of the
numerical model given that very detailed information was available. The properties for the high-
strength concrete were experimentally characterised and are listed in Table 5. The strengthening of
the girder was also addressed by the original experimental programme. The CFRP laminates used in
the strengthened girder consisted of two CFK 150/2000 with rectangular cross-section of
100 ′ 1.4 mm² anchored at the extremities. Figure 7 shows the failure after the tests.

The numerical simulations are based on 2D analysis using the finite element mesh shown in Figure
8. Plane stress bilinear (i.e. 4-node) elements are adopted for simulating concrete, whereas linear truss
(i.e. 2-node) elements are used for simulating steel reinforcements and CFRP laminates. Since the
CFRP laminates are very thin, their bending is negligible compared to the axial component. This
makes it particularly suitable for simulation using 2-node elements showing only axial stiffness, i.e.,
standard truss elements – these are also used for modelling the strands and stirrups. The truss elements
are connected to the concrete elements using zero-thickness interface elements, which directly follow
the bond-slip law of the CFRP laminates. Further details can be found in (Dias-da-Costa et al. 2018b),
where focus was given to the modelling of concrete slabs strengthened with CFRP laminates and its
interaction with fracture. Interface elements are also adopted to model the bond behaviour of the
prestressed strands. The stirrups are modelled using the 2-node truss elements directly connected to
the concrete elements with 150 mm along the beam with the area of the two-legged arrangement
described earlier. Given that the stress state in the girder is mostly bidimensional for the structural
and loading schemes adopted, this assures that the confinement provided by the stirrups and resistance
against shear are adequately approximated.

The concrete is assumed elastoplastic under compression as defined in EN 1992-1-1 (2004). For
tension, embedded cracks are used to capture the non-linear effect using a bilinear softening law with
the fracture energy defined by CEB-FIP Model Code 1990 (1991). The prestressing strands and
stirrups are modelled considering the bilinear law defined in EN 1992-1-1 (2004) with the parameters
shown in Table 5, whereas the CFRP laminates are modelled with a linear elastic behaviour. Perfect
bond conditions are assumed between concrete and reinforcements, whereas the bond between CFRP
laminates and concrete is modelled using the simplified model proposed by Lu et al. (2005) – see
Appendix A for more details. The automatic method proposed by Graça-e-Costa et al. (2013) is used
to overcome convergence issues during the stages of concrete cracking and crushing, yielding of steel
reinforcements, and local debonding at CFRP laminates-concrete interfaces (Graça-e-Costa et al
2012; 2013).
It should be mentioned that two different procedures can be usually followed to simulate the forces due to the pre-tensioning strands. The first option consists in applying a negative uniform temperature variation to the pre-tensioning strands at day 5 corresponding to the opposite stretch that was applied to the strands before casting, i.e. corresponding to 1,430 MPa. The second option available – which was the one followed in this paper – directly applies the compressive force caused by the pre-tensioning strand to the girder. These stresses are the same that appear when the pre-tensioning strand is finally cut from the precast table. The initial tensile strain/stress in the pre-tensioning strands is stored and considered when computing the total tensile stress at the strand. Naturally, the tensile stress of the strand obtained just after releasing is lower than 1,430 MPa, both numerically and experimentally, due to the bending and axial shortening of the girder caused by compressive forces. Figure 8 represents all forces applied to the girder – external load F and internal forces \( P_1, P_2 \) and \( P_3 \) due to the strands. All material parameters adopted in the validation simulations are summarised in Table 5.

Figure 9 shows a comparison between experimental and numerical results for both strengthened and non-strengthened girders. The main stages related with the onset of cracking, the yielding of prestressed reinforcement and concrete crushing are also represented. In summary, a good agreement is observed between numerical and experimental data. In summary, a good agreement is observed between numerical and experimental data. It should be mentioned that even though the numerical model can simulate the global debonding of CFRP laminates accurately – see (Dias-da-Costa et al. 2018b) for a detailed validation of the model– this failure mode could not develop due to the material properties of the cross-section and anchorage of the CFRP laminates. Therefore, the strengthened girder fails with crushing of concrete after the yielding of prestressed tendons, thus confirming the experimental findings. The crack pattern at failure is shown in Figure 10 for both strengthened and non-strengthened models.

### Results and discussion

#### Reliability index and prestressing area before strengthening

Figure 11 shows the variation of the reliability index, \( \gamma \), with the area of pre-tensioning steel, \( A_p \), based on RSA (1983) and EN1991-2 (2002) for the girders with strengthening laminates. In both cases, the reliability index increases with the amount of uncorroded prestressing area in the girder. For similar areas, the reliability values based on the former standard are significantly higher than the
ones obtained with the latter code, and in some cases this difference can reach more than 200% – e.g.
when the area of pre-tensioning steel is 1,652 mm². This difference is mainly related to the more
demanding traffic load requirements in the current standard. Reliability is strongly influenced by the
amount of prestressing area of the girder (Atadero and Karbhari 2008) and results show that
strengthening is required to reach the target reliability index defined in the new standard in all design
cases.

A sensitivity plot for both traffic load models is shown in Figure 12. The most influential variable is
the traffic load, $g_d$, followed by the load and resistance uncertainties, $q_E$ and $q_R$. The dead loads,$g_{dl}$, and the concrete self-weight, $g_c$, have a reduced influence in general, whereas the steel strand
strength, $f_p$, has a sensitivity factor close to 0.18. Despite the differences in traffic load models and
safety requirements, the cosines direction at design point are nearly the same in both standards.

**Sensitivity analysis and design point for the strengthened girders**

The area of the CFRP laminates calculated according to the procedure described previously is
summarised in Table 6. During the analyses, the possibility of the CFRP laminates debonding and
fracturing before concrete crushing and/or the yielding of the steel strands is properly accounted for.
All design cases require CFRP laminates to reach the target reliability index, with the flexural
capacity of the girders increasing up to 74% for the most degraded cases, B13-2D3, to restore the full
capacity according to the EN1991-2 (2002). The reference design case, i.e. the undamaged girder,
only requires an upgrade of a maximum of 25%, which directly reflects the increment due to the
provisions of the current standard.

The relative importance of the seven random variables considered in the reliability study is presented
in Figures 13a-c based on the cosines direction at design points in the normalised space. For each
random variable, the several scenarios of corroded pre-tensioning strands are considered. The
leftmost bar corresponds to the highest area of pre-tensioning steel and the rightmost bar to the lowest.
From these charts, it can be observed that traffic loads, $g_d$, play a fundamental role in the analysis,
being always the most significant variable, in some cases reaching an importance of almost 0.80.

The load uncertainties also have an important weight in the analysis, ranging from 0.40 to 0.60. In
respect to the other loads, namely concrete self-weight, $g_c$, and dead loads, $g_{dl}$, both of them present
lower sensitivity factors, usually smaller than 0.15. On the other hand, the resistance parameter
showing the highest importance is the resistance uncertainty, $q_R$, presenting values around -0.40 for
all analyses. The steel strand strength, $f_p$, shows values of nearly -0.10 for bridges B13 and B19 and
can reach -0.20 for bridge B16. The CFRP laminates strength, $f_f$, exhibits values up to -0.30, assuming
greater importance than the steel strand strength in most analyses. This can be related to the loss of pre-tensioning steel.

Table 7 shows the reliability index and design points used for the calibration of CFRP laminates partial safety factors, i.e., the cases for which the area of CFRP laminates leads to values closest to the target reliability index. Reliability indices are slightly higher than the target of 4.3. This occurs because the numerical model is more accurate than the analytical model for simulating the structural behaviour. However, it should be mentioned that the differences are in the order of 6%, meaning that design using simplified models is safe for the calibration of partial safety factors for CFRP.

As expected, the design values in Table 7 show that the resistance variables, \( f_p, f_f \) and \( q_R \) are generally lower than the corresponding mean values. The opposite trend is observed in the load variables, \( q_r, q_x, q_L \) and \( q_r \). Traffic loads exhibit the higher deviation from the mean value, which shows the importance they have for the calibration process.

Calculation of partial safety factors
Table 8 presents the partial safety factors calculated for each model based on the characteristic value for the distribution. The partial safety factors obtained for the CFRP laminates have an average of 1.16. This is consistent with the recommendations found in design guides. For instance, for the design of concrete structures using CFRP end anchored laminates, CEB-FIB (2001) recommends the use of a safety factor of 1.20, a value slightly higher than the one found in this paper. CNR (2001) recommends a factor of 1.10 and TR-55 (2000) is more conservative, suggesting a factor of 1.54 for the same type of strengthening.

428 Conclusions

This paper proposed a new hybrid procedure to perform reliability analyses efficiently combining analytical and advanced non-linear FEM models to overcome the simplifications normally assumed in reliability studies. The framework developed uses a discrete crack model to capture the interaction between concrete cracks and local debonding of CFRP laminates and was applied to calibrate the partial safety factor required for designing the strengthened PC girders.

The PC girders were taken from existing bridges built to connect small cities since the 1980s, with spans ranging from 13 to 19 m, and originally designed with previous standards. Several corrosion damage scenarios were considered when determining the area of CFRP laminates needed to restore the structural capacity to current standards. The following conclusions are highlighted:

- the partial safety factor for designing strengthening of PC girders with CFRP laminates is in the range of 1.16, and was observed not to change significantly with the span;

- the use of advanced non-linear models entails higher accuracy in the simulation of both material and structural behaviour, particularly concerning the interaction of concrete cracking with the local debonding of CFRP laminates. However, given that the differences relatively to simplified analytical models were found to be in the range of 6%, the use of simplified models for future studies targeting code calibration of partial safety factors for CFRP laminates could be sufficient;

- the sensitivity analysis carried out shown the traffic loads and model uncertainties to be the most significant parameters for the calibration process, assuming high values compared with dead load, concrete self-weight and steel strand strength. Thus, it is important to assess the
model uncertainties for further reliability analysis, particularly in the case of more advanced models now widely available.

It is worthwhile mentioning that the methodology presented in this paper is fully general and can easily be adapted to different standards and geometries where the stress-state is predominantly two-dimensional. The generalisation to three-dimensional structures, however, will require the development of discrete crack models with more robust algorithms to be able to reliably track the geometry of crack propagation, and therefore remain more precise than the uncertainty of the input data.

Acknowledgements

S. Gomes acknowledges the financial support of the Portuguese Science and Technology Foundation (FCT) through the Ph.D. grant number SFRH/BD/76345/2011. D. Dias-da-Costa would like to acknowledge the support from the Australian Research Council through its Discovery Early Career Researcher Award (DE150101703) and Linkage grant (LP140100591). This work was also supported by FCT, within ISISE, project UID/ECI/04029/2013.

Data Availability Statement

Some or all data, models, or code generated or used during the study are available from the corresponding author by request.
Appendix A - Adopted bond-slip law

Lu et al. (2005) model, which is adopted as FRP-to-concrete bond-slip law in this paper, is defined by the following equations:

\[ t = t_{\text{max}} \sqrt{\frac{s}{s_0}} \quad \text{if } s \leq s_0, \]  
\[ t = t_{\text{max}} e^{\frac{s - s_0}{G_f}} \quad \text{if } s > s_0, \]

with

\[ s_0 = 0.0195 b_c f_c, \]

\[ t_{\text{max}} = \alpha_1 b_c f_c, \]

\[ \alpha = \frac{1}{\frac{G_f}{t_{\text{max}} s_0} - \frac{2}{3}}, \]

\[ b_c = \frac{2.25 - b_f / b_c}{1.25 + b_f / b_c}, \]

\[ G_f = 0.309 b_c^2 \sqrt{f_t}, \]

where \( t_{\text{max}} \) is the maximum local bond stress, \( s \) is the local slip, \( s_0 \) is the local slip at \( t_{\text{max}} \), \( f_t \) is the concrete tensile strength, \( b_c \) and \( b_f \) are, respectively, the widths of concrete prism and FRP plate, and \( G_f \) is the interfacial fracture energy.
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Figure 1. Cross-section and details of the exterior girder (dimensions are in meters if not stated otherwise).

Figure 2. (a) Joint density function $f_{R,S}(r,s)$ of two random variables with marginal density functions $f_r$ and $f_s$; (b) Reliability index and design point in the standard space, assuming a linear limit state function and two random variables $u_1$ and $u_2$; and (c) cosines direction $\mathbf{a}$ at design point $u^*$. Figure adapted from Schneider (1997).

Figure 3. Flowchart showing both stages of analysis and iterative cycles.

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Figure 11. Reliability index as a function of the area of the pre-tensioning steel in non-strengthened girders according to: (a) RSA (1983); and (b) EN1991-2 (2002)

Figure 12. Cosines direction at design point.
Figure 13. Cosines direction at design point as a function of the pre-tensioning steel (a) B13, (b) B16, and (c) B19.
Table 1. Summary of safety and reduction factors, respectively $g$ and $f_r$.

| Design guideline       | Safety/Reduction factor |
|------------------------|-------------------------|
| CEB-FIB (2001)         | $g = 1.20$ to $1.35$    |
| CNR (2001)             | $g = 1.10$ to $1.50$    |
| TR-55 (2000)           | $g = 1.10$ to $3.50$    |
| JSCE (2001)            | $g = 1.20$ to $1.30$    |
| ACI 440.2R-08 (2008)   | $f_r = 0.85$ to $0.95$  |
| AASHTO (2012)          | $f_r = 0.85$            |
| ISIS (2001)            | $f_r = 0.75$            |
| AS 5100.8 (2017)       | $f_r = 0.65$ to $0.80$  |
Table 2. Geometry of the bridge girders considered in the study.

| Bridge | $h$  | $b$  | $b_\text{w}$ | Span (m) |
|--------|------|------|-------------|---------|
| B13    | 0.6  | 0.4  | 0.15        | 13      |
| B16    | 0.9  | 0.6  | 0.2         | 16      |
| B19    | 1.2  | 0.6  | 0.2         | 19      |
Table 3. Cases of structural deterioration.

| Case | % of loss | $A_p$ (mm$^2$) |
|------|-----------|----------------|
| D0   | 0         | 2,240          |
| D1   | 10        | 2,142          |
| D2   | 20        | 2,044          |
| D3   | 30        | 1,946          |
| 2D1  | 20        | 2,044          |
| 2D2  | 20        | 1,848          |
| 2D3  | 30        | 1,652          |
Table 4. Statistical properties for the random parameters.

| Variable                                      | Mean  | Standard deviation | COV  | Distribution type |
|-----------------------------------------------|-------|--------------------|------|-------------------|
| Steel strand strength, \( f_p \) (MPa)        | 1674  | 50                 | 0.03 | Normal            |
| CFRP strength \( f_t \) (MPa)                 | 2686  | 215                | 0.08 | Weibull           |
| Resistance model uncertainties, \( q_R \)    | 1.0   | 0.13               | 0.13 | Log-normal        |
| Traffic loads, \( g_{tl} \)                  | 0.78  | 0.12               | 0.15 | Gumbel            |
| Dead loads, \( g_{dl} \) (kN/m)              | 10.83 | 1.08               | 0.10 | Normal            |
| Concrete self-weight, \( g_c \) (kN/m^3)     | 25.0  | 1.0                | 0.04 | Normal            |
| Load model uncertainties, \( q_E \)          | 1.05  | 0.11               | 0.10 | Log-normal        |
Table 5. Main material parameters for the validation models.

| Parameter                | Value          |
|--------------------------|----------------|
| Concrete                 |                |
| Compressive strength     | 120 MPa        |
| Tensile strength         | 5.5 MPa        |
| Young’s modulus          | 59 GPa         |
| Fracture energy          | 0.2 N/mm       |
| Prestressing steel       |                |
| 0.1% proof-stress        | 1,640 MPa      |
| Young’s modulus          | 200 GPa        |
| Steel Reinforcement      |                |
| Tensile strength         | 500 MPa        |
| Young’s modulus          | 200 GPa        |
| CFRP                     |                |
| Ultimate strength        | 2,300 MPa      |
| Young’s modulus          | 165 GPa        |
Table 6. Summary of bridges used for calibration.

| Bridge   | % steel loss | $A_p$ (mm$^2$) | $A_f$ (mm$^2$) | Flexural resistance (kNm) |
|----------|--------------|----------------|----------------|---------------------------|
|          |              | Initial        | Strengthened   |                           |
| B13      | 0            | 2,240          | 477            | 2,352                     |
| B13-D1   | 10           | 2,142          | 531            | 2,239                     |
| B13-D2   | 20           | 2,044          | 586            | 2,125                     |
| B13-D3   | 30           | 1,946          | 641            | 2,011                     |
| B13-2D1  | 2°10         | 2,044          | 586            | 2,135                     |
| B13-2D2  | 2°20         | 1,848          | 688            | 1,916                     |
| B13-2D3  | 2°30         | 1,652          | 781            | 1,696                     |
| B16      | 0            | 2,240          | 453            | 3,336                     |
| B16-D1   | 10           | 2,142          | 508            | 3,174                     |
| B16-D2   | 20           | 2,044          | 570            | 3,011                     |
| B16-D3   | 30           | 1,946          | 625            | 2,849                     |
| B16-2D1  | 2°10         | 2,044          | 563            | 3,021                     |
| B16-2D2  | 2°20         | 1,848          | 672            | 2,704                     |
| B16-2D3  | 2°30         | 1,652          | 781            | 2,385                     |
| B19      | 0            | 2,240          | 445            | 4,354                     |
| B19-D1   | 10           | 2,142          | 508            | 4,140                     |
| B19-D2   | 20           | 2,044          | 563            | 3,926                     |
| B19-D3   | 30           | 1,946          | 625            | 3,712                     |
| B19-2D1  | 2°10         | 2,044          | 563            | 3,936                     |
| B19-2D2  | 2°20         | 1,848          | 680            | 3,518                     |
| B19-2D3  | 2°30         | 1,652          | 789            | 3,097                     |
Table 7. Reliability index and design points used for calibration.

| Bridge | $b$   | $f_p^*$ | $f_f^*$ | $g_c^*$ | $g_{ul}^*$ | $g_{ul}^*$ | $q_{q_1}^*$ | $q_{q_2}^*$ |
|--------|-------|---------|---------|---------|------------|------------|-------------|-------------|
| B13    | 4.36  | 1,652   | 2,640   | 25.4    | 11.2       | 1.41       | 0.78        | 1.29        |
| B13-D1 | 4.38  | 1,666   | 2,676   | 25.0    | 11.2       | 1.48       | 0.77        | 1.25        |
| B13-D2 | 4.35  | 1,648   | 2,539   | 25.0    | 10.6       | 1.42       | 0.78        | 1.27        |
| B13-D3 | 4.41  | 1,649   | 2,512   | 25.3    | 10.6       | 1.42       | 0.78        | 1.28        |
| B13-2D1| 4.32  | 1,653   | 2,471   | 24.9    | 11.0       | 1.42       | 0.78        | 1.24        |
| B13-2D2| 4.33  | 1,644   | 2,398   | 25.1    | 11.1       | 1.32       | 0.80        | 1.29        |
| B13-2D3| 4.34  | 1,651   | 2,405   | 25.1    | 10.3       | 1.28       | 0.80        | 1.34        |
| B16    | 4.43  | 1,649   | 2,679   | 25.1    | 11.3       | 1.51       | 0.77        | 1.23        |
| B16-D1 | 4.47  | 1,627   | 2,585   | 25.5    | 11.4       | 1.30       | 0.80        | 1.36        |
| B16-D2 | 4.57  | 1,656   | 2,445   | 25.3    | 11.6       | 1.30       | 0.80        | 1.38        |
| B16-D3 | 4.65  | 1,615   | 2,421   | 24.6    | 10.6       | 1.32       | 0.79        | 1.35        |
| B16-2D1| 4.44  | 1,628   | 2,484   | 25.2    | 11.0       | 1.33       | 0.79        | 1.33        |
| B16-2D2| 4.62  | 1,672   | 2,512   | 25.1    | 10.8       | 1.49       | 0.77        | 1.27        |
| B16-2D3| 4.42  | 1,641   | 2,524   | 25.4    | 10.8       | 1.41       | 0.78        | 1.29        |
| B19    | 4.58  | 1,649   | 2,752   | 24.9    | 11.1       | 1.48       | 0.77        | 1.30        |
| B19-D1 | 4.45  | 1,658   | 2,561   | 26.0    | 11.9       | 1.31       | 0.80        | 1.34        |
| B19-D2 | 4.45  | 1,657   | 2,493   | 25.3    | 11.7       | 1.48       | 0.77        | 1.21        |
| B19-D3 | 4.57  | 1,649   | 2,492   | 25.2    | 10.9       | 1.38       | 0.79        | 1.35        |
| B19-2D1| 4.53  | 1,656   | 2,568   | 25.3    | 11.0       | 1.42       | 0.78        | 1.33        |
| B19-2D2| 4.53  | 1,660   | 2,513   | 25.3    | 11.3       | 1.42       | 0.78        | 1.31        |
| B19-2D3| 4.48  | 1,650   | 2,521   | 25.7    | 10.6       | 1.47       | 0.77        | 1.24        |
Table 8. Partial safety factors for CFRP.

| Bridge    | $g_f$ | Bridge    | $g_f$ | Bridge    | $g_f$ |
|-----------|-------|-----------|-------|-----------|-------|
| B13       | 1.12  | B16       | 1.12  | B19       | 1.09  |
| B13-D1    | 1.12  | B16-D1    | 1.12  | B19-D1    | 1.13  |
| B13-D2    | 1.16  | B16-D2    | 1.18  | B19-D2    | 1.20  |
| B13-D3    | 1.17  | B16-D3    | 1.20  | B19-D3    | 1.18  |
| B13-2D1   | 1.19  | B16-2D1   | 1.18  | B19-2D1   | 1.15  |
| B13-2D2   | 1.21  | B16-2D2   | 1.18  | B19-2D2   | 1.18  |
| B13-2D3   | 1.19  | B16-2D3   | 1.17  | B19-2D3   | 1.18  |
Figure 1. Cross-section and details of the exterior girder (dimensions are in meters if not stated otherwise).
Figure 2. (a) Joint density function \( f_{R,S}(r,s) \) of two random variables with marginal density functions \( f_r \) and \( f_s \); (b) Reliability index and design point in the standard space, assuming a linear limit state function and two random variables \( u_1 \) and \( u_2 \); and (c) cosines direction at design point \( u^* \). Figure adapted from Schneider (1997).
Figure 3. Flowchart showing both stages of analysis and iterative cycles.
Figure 4. Stress-strain diagram for cross-sectional analysis of PC girders.
Figure 5. (a) Finite elements with embedded cracks. (b) Interaction between FRP and cracks in the scope of discrete and smeared models.
Figure 6. Loading scheme and cross-section of the girder: (a) side view (dimensions in m); and (b) cross-section (dimensions in mm unless stated otherwise).
Figure 7. Tested PC girder: (a) control, and (b) CFRP-strengthened girders.
Figure 8. Mesh used in finite element analysis including loading and boundary conditions.
Figure 9. Load-displacement curves for: (a) non-strengthened; and (b) CFRP-strengthened girders.
Figure 10. Detail of the deformed shape and crack pattern at failure: (a) identification of the detailed region; (b) non-strengthened; and (c) CFRP-strengthened girders. Note: for illustration purpose crack widths are magnified by a factor of 20 and only widths above 0.25 mm are shown.
Figure 11. Reliability index as a function of the area of the pre-tensioning steel in non-strengthened girders according to: (a) RSA (1983); and (b) EN1991-2 (2002)
Figure 12. Cosines direction at design point.
Figure 13. Cosines direction at design point as a function of the pre-tensioning steel (a) B13, (b) B16, and (c) B19.