Collective Sideband Cooling in an Optical Ring Cavity

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We propose a cavity based laser cooling and trapping scheme, providing tight confinement and cooling to very low temperatures, without degradation at high particle densities. A bidirectionally pumped ring cavity builds up a resonantly enhanced optical standing wave which acts to confine polarizable particles in deep potential wells. The particle localization yields a coupling of the degenerate travelling wave modes via coherent photon redistribution. This induces a splitting of the cavity resonances with a high frequency component, that is tuned to the anti-Stokes Raman sideband of the particles oscillating in the potential wells, yielding cooling due to excess anti-Stokes scattering. Tight confinement in the optical lattice together with the prediction, that more than 50% of the trapped particles can be cooled into the motional ground state, promise high phase space densities.

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Laser cooling is a powerful technique to obtain the lowest temperatures accessible in laboratories to date, which has paved the path for the formation of Bose-Einstein condensates of atomic gases. Unfortunately, laser cooling has so far been constrained to a limited number of atomic species prepared at relatively low densities below $10^{13}$ cm$^{-3}$. The reason is that all laser cooling schemes have used spontaneous emission in one or the other way to optically dissipate motional energy. To enable repeated excitation and spontaneous emission cycles, the particle to be laser cooled after spontaneous decay needs to return to a state that can be re-excited by laser radiation. This is only possible for a small class of atoms (e.g. alkali metals) with a sufficiently simple structure of the ground state. Molecules, for example, with their typically complex rotational and vibrational structure, would afford an unfeasibly large number of excitation frequencies. At high particle densities spontaneous emission poses the additional problem of uncontrolled re-absorption of fluorescence photons, which do not provide the highly ordered characteristics of the exciting laser radiation and thus yield undesired heating. These deficiencies of conventional laser cooling have inspired a quest for new optical cooling schemes which rely on coherent scattering, resonantly enhanced by optical resonators.

The interaction of atoms with a light mode of a high finesse cavity has been a subject of extensive research in the past. For long, most work has focused on very small cavities with high values of the electric field per photon and a few photons coupled to a few atoms. Recent studies on such cavities have also incorporated motional degrees of freedom. In the past few years, high finesse cavities with larger mode volumes have attracted interest as a possible means to cool large atomic samples without the need for spontaneous photons. In a typical scenario referred to as cavity Doppler cooling, polarizable particles placed inside a linear resonator are irradiated from the side by an off-resonant light pulse, yielding elastic Rayleigh scattering with a scattering rate into resonant cavity modes, which is significantly enhanced over the free space value. By tuning the cavity resonance slightly above the frequency of the incident light pulse, energy is extracted from the motional degrees of freedom.

Unfortunately, such cavity based cooling techniques impose significant technical challenges. Because the temperature limit scales with the cavity linewidth, for accessing low temperatures extremely sharp resonance conditions have to be maintained, although the number of photons inside the resonator, that carry the information, required for frequency control techniques, is tiny. Particles to be cooled have to be well confined inside the mode volume, which for technical reasons is usually limited. As proposed in ref. an additional confinement potential may be applied, posing the problem of a good matching with the mode volume.

In this article we propose a simple and robust cavity cooling scheme, which may provide a solution to the aforementioned obstacles, combining cooling to very low temperatures with tight confinement in a single light field. A bidirectionally pumped ring cavity forms a resonantly enhanced intense optical standing which acts to confine polarizable particles. The particle localization yields a coupling of the degenerate counter-propagating travelling wave modes via coherent photon redistribution. This leads to a splitting of the cavity resonance with a high frequency component, that can be tuned to the anti-Stokes Raman sideband of the particles oscillating in the potential wells. As a consequence scattering on the anti-Stokes Raman sideband is resonantly enhanced, exceeding Stokes scattering into resonator or free space modes, and thus vibrational energy is dissipated into the light field. Tight trap potentials in the optical lattice together with the prediction, that more than 50% of the trapped particles can be cooled into the motional ground state, promise high phase space densities.

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The cooling scenario is sketched in Fig.1(a). For simplicity, we refer to the particles to be cooled as atoms, bearing in mind, that nothing essential changes in the following discussion, if molecules were addressed. Laser light is resonantly coupled into both counter–propagating traveling wave modes of a ring cavity, which exhibits very high finesse (typically above $10^5$) and a large mode volume (typically a few mm$^3$). Inside the resonator an intense standing light wave is formed. Its frequency is tuned far away from resonant excitation frequencies of the atoms into a region of normal dispersion. The standing light field forms a one–dimensional optical lattice, i.e., steep light shift potentials can tightly trap the atoms well inside the Lamb-Dicke regime (given by $\omega_V \gg \omega_R$, where $\omega_V$ is the fundamental vibrational energy and $\omega_R$ is the single photon recoil energy) and the regime of resolved sidebands (given by $\omega_V \gg \gamma_c$, where $\gamma_c$ is the intra–cavity field decay rate). If the sample of $N$ atoms is homogeneously distributed, only forward scattering arises, i.e., the degenerate travelling wave modes are not coupled. Both modes remain eigenmodes, however shifted in frequency by an amount $N\Delta_0$, where $\Delta_0$ is the lightshift per photon (see Fig.1(b)). This shift, which is negative for the relevant case of normal dispersion, is readily calculated by considering the refractive index of $N$ atoms distributed over the mode volume.

The optical lattice acts to localize the atoms in the antinodes. The degree of localization may be described by a parameter $g \equiv \frac{1}{\sqrt{N}} \sum_{\nu=1}^{N} e^{-i2kz_\nu}$ which takes values inside the complex unity sphere, where $z_\nu$ denote the deviations of the atomic positions from the adjacent potential minimum. For perfect localization, i.e., all atoms are positioned exactly in an anti–node ($z_\nu = 0$), $g = 1$, while for a homogeneous atomic distribution $g = 0$. Generally, the complex phase of $g$ scales with the center of mass and the modulus of $g$ describes the spread of the atomic sample. In fact, for small deviations $kz_\nu \ll 1$, we may write $g = 1 - i2kz_{cm}$, where $z_{cm}$ is the center of mass coordinate. If the phases $kz_\nu$ are assumed to follow a Gaussian distribution, we may write $g = exp(-2k^2\Delta_{rms}^2)exp(-2ikz_{cm})$, where $\Delta_{rms}$ denotes the root mean square spread of the atomic positions.

For localized atoms additional back–scattering arises which couples the counter–propagating travelling wave modes and lifts their degeneracy. In fact, two modified eigenmodes with orthogonal standing wave geometries arise. External pumping populates only one of the modes which provides the lattice. This mode suffers an increased negative shift $N\Delta_0 (1 + |g|)$ for the relevant case of normal dispersion, is readily calculated by considering the refractive index of $N$ atoms distributed over the mode volume, i.e., setting $\omega_V = 2N|\Delta_0| |g|$ in Fig.1(c). This is readily achieved by adjusting $\omega_V$ via the intra-cavity light intensity.

The rate for resonant scattering into a resonator mode is $\eta c \Gamma$, where $\eta c = 12F/\pi (w_0)^2$ ($F$ = Finesse, $w_0 = e^{-2}$ radius of cavity mode) denotes the ratio of the scattering rate into a resonator mode to the free space scattering rate $\Gamma$. For particles confined in the Lamb-Dicke regime scattering acquires a discrete frequency spectrum with an elastic component, which preserves the atomic motion, and modulation sidebands shifted by trap oscillation frequencies. The sideband intensities are suppressed by Lamb-Dicke factors which account for the spatial mismatch resulting from the change of the motional quantum state $\Delta$. In the harmonic approximation, the suppression of scattering into the cavity modes is $\eta LD = \omega_R/\omega_V$, 

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{(a) Sketch of cooling scenario. (b) Sketch of mode splitting mechanism. The travelling wave modes, indicated by the unfilled box in the center are assumed to be resonant at $\omega_c$. When localized polarizable particles are present, the degeneracy is lifted and a resonance doublet arises (grey boxes on the right). (c) Energy budget of self-induced cavity sideband cooling. The lower frequency component of the resonance doublet in (b) provides the optical lattice at frequency $\omega_L$, operating in the Lamb-Dicke regime. The higher frequency component $\omega_L + \Omega$ is tuned the anti-Stokes Raman band, i.e., $\Omega = \omega_V$, where $\omega_V$ is the vibrational frequency in the potential wells.}
\end{figure}
while scattering into all other modes is suppressed by \( \xi \eta_{LD} \). The extra geometry factor 2/5 accounts for the angular distribution of the emitted k-vectors for scattering into free space modes. The rate for a resonant inelastic Raman scattering event \(|n\rangle \rightarrow |n-1\rangle\), reducing the number of vibrational quanta by one, is thus given by \( \Gamma_n = n \eta_{LD} (\xi + \frac{2}{5}) \Gamma \). Raman scattering on the red Stokes sideband \(|n-1\rangle \rightarrow |n\rangle\) is not resonant with the cavity, i.e., for scattering into the cavity mode an extra suppression factor \( \xi = (1 + (2 \omega_{V}/\gamma c)^2)^{-1} \) results from the cavity resonance profile. In the sideband regime \( \xi \) is much smaller than one. The corresponding rate for \(|n-1\rangle \rightarrow |n\rangle\) transitions is \( \gamma_n = n \eta_{LD} (\xi \eta + \frac{2}{5} \Gamma) \). Cooling is expected, if \( \gamma_n \) is significantly smaller than \( \Gamma_n \), indicating that values of \( \eta \) larger than the geometry factor 2/5 are required.

Solving the rate equations for the populations \( \Pi_n \) of the vibrational levels \(|n\rangle\), connected by the rates \( \Gamma_n \) and \( \gamma_n \), yields a steady state with \( \Pi_0/\Pi_{n-1} = (\xi \eta + \frac{2}{5}) / (\eta + \frac{2}{5}) \), a corresponding population of the vibrational ground state of \( \Pi_0 = (1 - \xi) \eta_c / (\eta + \frac{2}{5}) \), and a mean vibrational quantum number \( \langle n \rangle = (\eta_c \xi + \frac{2}{5}) / (1 - \xi) \eta_c \). For high finesse cavities we can realize values of \( \eta_c \) on the order of one and \( \xi \) on the order of \( 10^{-2} \), i.e., ground state populations above 50% and mean vibrational quantum numbers below 0.5 should be readily accessible. Calculating the total change of kinetic energy

\[
d\frac{d}{dt} W = -\sum_{n=0}^{\infty} (\Gamma_n - \gamma_{n+1}) \Pi_n
\]

in the classical limit \( \hbar \omega_V \ll k_B T \) yields exponential cooling according to

\[
dT/dt = -2 (1 - \xi) \eta_{LD} \eta_c \Gamma T + \left( \eta_c \xi + \frac{2}{5} \right) \Gamma T_R,
\]

with a cooling rate \( 2 (1 - \xi) \eta_{LD} \eta_c \Gamma \). If \( \xi \ll 1 \), the main contribution to the temperature limiting second term on the right hand side involving the recoil temperature \( T_R \) results from scattering into free space.

To substantiate the results of the simple physical picture presented so far, a theoretical treatment starting from the dynamical equations of the system is required. In particular, a quantitative account of the mode coupling introduced by the atoms and the corresponding resonance condition should be produced. Furthermore, the potential role of cavity mediated collective interactions needs to be clarified. In the following we take a semiclassical viewpoint, where the cavity field and the atomic motion is treated classically, while the atomic polarizability is taken from quantum mechanics. We start from the dynamic equations for the complex field amplitudes of the two degenerate modes of the empty cavity \( \alpha_{\pm}(t, z, t) = \alpha_{\pm}(t) \exp(i kz) \) \( k \) is wave number. According to ref. [12] these equations write in the limit of low saturation and large detunings as

\[
\frac{d}{dt} \left( \begin{array}{c} \alpha_+ \\ \alpha_- \\ \eta_+ \\ \eta_- \end{array} \right) = \mathbf{M} \left( \begin{array}{c} \alpha_+ \\ \alpha_- \\ \eta_+ \\ \eta_- \end{array} \right) + \gamma_0 \left( \begin{array}{c} \eta_+ \\ \eta_- \end{array} \right)
\]

\[
\mathbf{M} = \left( \begin{array}{cccc} i (\delta_c - N \Delta_0) - \gamma_c & -i N \Delta_0 g & -i N \Delta_0 g^* & i (\delta_c - N \Delta_0) - \gamma_c \\ -i N \Delta_0 g & \gamma_c & -i N \Delta_0 g^* & -i N \Delta_0 g \\ -i N \Delta_0 g^* & -i N \Delta_0 g & \gamma_c & -i N \Delta_0 g^* \\ i (\delta_c - N \Delta_0) - \gamma_c & -i N \Delta_0 g & -i N \Delta_0 g^* & \gamma_c \end{array} \right)
\]

where \( \delta_c \) is the detuning of the incoupled frequency from the resonance frequency of the empty cavity, \( \gamma_0 = c/L \) \( c = \) speed of light, \( L = \) cavity roundtrip length is the free spectral range and \( \eta_+ \), \( \eta_- \) are the complex field amplitudes of the incoupled light beams (all field amplitudes are scaled to the field per photon).

In order to describe cooling dynamics, we are looking for solutions of eq.3, which account for the presence of modulation sidebands. As already mentioned, such sidebands are expected to arise in the Lamb-Dicke regime, because the atoms oscillate in their potential wells with some frequency \( \Omega \). We introduce the ansatz \( \alpha_{\pm}(t) = \alpha_{\pm} + \beta_{\Omega \pm} \exp(-i \Omega t) + \gamma_{\Omega \pm} \exp(i \Omega t) \) and \( g = |g|(1 + i c \cos(\Omega t)) \) into eq.3, assuming symmetric pumping \( \eta_+ = \eta_- = \eta \) and find

\[
\alpha_{\pm} = \frac{\gamma_0 \eta}{\gamma_c - i (\delta_c - N \Delta_0 (1 + |g|))} \alpha_{\pm} + \frac{N \Delta_0 \epsilon}{\gamma_c - i (\delta_c + \Omega - N \Delta_0 (1 - |g|))} \beta_{\Omega \pm}
\]

\[
\beta_{\Omega \pm} = \frac{\gamma_0 \eta}{\gamma_c - i (\delta_c - N \Delta_0 (1 + |g|))} \beta_{\Omega \pm} + \frac{N \Delta_0 \epsilon}{\gamma_c - i (\delta_c + \Omega - N \Delta_0 (1 - |g|))} \alpha_{\pm}
\]

The small phase parameter \( \epsilon = 2k_z z_{cm,0} \) measures the maximum amplitude \( z_{cm,0} \) of the center of mass oscillation of the sample. For the carrier fields \( \alpha_{\pm} \) forming the lattice, the resonance condition, obtained from eq.6 is \( \delta_c = N \Delta_0 (1 + |g|) \), i.e., the detuning \( \delta_c \) acquires a negative value \( \Delta_0 < 0 \), for normal dispersion). We set \( \alpha_{\pm} \) to its resonant values \( \gamma_0 \eta / \gamma_c \) in the following. The resonance condition for \( \beta_{\Omega \pm} \) reads \( \Omega = -2 N \Delta_0 |g| \) (eq.7), and the corresponding frequency detuning of the empty cavity is \( \delta_c + \Omega = N \Delta_0 (1 - |g|) \). Obviously, \( \delta_c + \Omega > \delta_c \), showing that at resonance the field amplitudes \( \beta_{\Omega \pm} \) correspond to a blue detuned modulation sideband. Similarly, the resonance condition for \( \gamma_{\Omega \pm} \) is \( \Omega = 2 N \Delta_0 |g| \) (eq.7), and the corresponding frequency detuning of the empty cavity is \( \delta_c - \Omega = N \Delta_0 (1 - |g|) > \delta_c \). In this case the field amplitudes \( \gamma_{\Omega \pm} \) correspond to a blue detuned resonant modulation sideband. Obviously, either choice yields a resonant sideband at blue detuning, in accordance with our simple model (Fig.1(b)), i.e., emission on this sideband extracts kinetic energy from the system.

We are now in the position to calculate the change of kinetic energy per particle \( W \) by means of emission of photons on the high frequency sideband. This energy change is given by

\[
dW/dt = -\frac{2}{N} \frac{\Omega}{\omega_{\perp}} T_{\text{loss}} (P_\beta - P_\gamma),
\]
where $P_B$ and $P_L$ are the powers in the traveling waves $\beta_{\Omega \pm}$ and $\gamma_{\Omega \pm}$ connected with the blue detuned resonant modulation sideband and the red detuned off-resonant counterpart respectively, and $T_{\text{loss}}$ is the total loss from the resonator given by $2\gamma_0/\gamma_0$. Eq.8 is evaluated as follows. We account for the fact that $\Omega$ is determined by the lattice potential wells by setting $\Omega = \omega_V$ in the following. We use eq.7 for the case of resonance for $\alpha_{\pm}$ to express $P_B$ and $P_L$ by the power $P_s$ in each of the traveling wave modes $\alpha_{\pm}$, which can be further expressed by the free space scattering rate $\tilde{\Gamma}$. Upon the assumption of a Gaussian ensemble, the square of $\epsilon$ is expressed as $\epsilon^2 = 4k^2\Delta z_{\text{rms}}^2/N$. The relations $2W = k_B T = m\Delta z_{\text{rms}}^2 \omega_V^2$ ($m =$ particle mass) are used to finally obtain

$$\frac{d}{dt}T = -2 \eta_c \eta_{LD} \tilde{\Gamma} (L(\omega_V) - L(-\omega_V)) \quad (9)$$

$$L(\omega_V) = \frac{\sqrt{\eta_c}}{\sqrt{\eta_c^2 + (2N\Delta_0|g| + \omega_V)^2}}, \quad (10)$$

which reproduces the exponential cooling dynamics with the same cooling rate as found in eq.2. Note that at resonance $\omega_V = -2N\Delta_0|g|$ the term in the brackets on the right hand side of eq.9 becomes $1 - \xi$. The second (temperature limiting) term of eq.2 is naturally not reproduced, since free space scattering is not accounted for in eq.9. Our dynamical derivation of the cooling rate reveals the important role of the center of mass motion which is damped with a rate proportional to the number of particles $N$. The relative motional modes of freedom are only damped via their small statistical coupling to the center of mass motion scaling with the inverse of $N$. Our calculation confirms, that the resonance splitting, which is essential for the cooling mechanism, results from the common action of all atoms.

Finally, we briefly sketch a realistic scenario, to experimentally test the new cooling scheme with rubidium atoms. We assume a triangular ring cavity similar to that reported in ref.[14] with a finesse of $1.8 \times 10^5$, a bandwidth $\gamma_c = \pi \times 17$ kHz and a mode radius $w_0 = 130 \mu$m, i.e. $\eta_c \approx 0.6$. At 0.1 nm red detuning of the lattice frequency with respect to the D2 line at 780.24 nm and with 50 mW power circulating in each propagation direction the free space scattering rate becomes $\tilde{\Gamma} \approx 8 \times 10^3$ s$^{-1}$ and the Lamb-Dicke parameter is $\eta_{LD} \approx 10^{-2}$. The corresponding cooling rate is $2\eta_{LD} \eta_c \tilde{\Gamma} \approx 100$ s$^{-1}$, i.e., if the resonance condition $\omega_V = -2N\Delta_0|g|$ is maintained, the temperature decreases by 1/e in 10 ms. The sample is cooled to a mean vibrational quantum number of $\langle n \rangle \approx 0.66$ which corresponds to a kinetic temperature of approximately 12 $\mu$K. Since the light shift per photon is $\Delta_0 \approx 1$ s$^{-1}$ and the vibrational frequency is $\omega_V \approx 2\pi \times 380$ kHz (corresponding to a trap depth of 460 $\mu$K), the resonance condition $\omega_V = -2N\Delta_0|g|$ is satisfied with $N \approx 1.3 \times 10^6$ atoms and a value for the localization of $|g| = 0.9$. According to ref. [14] this can be realized with no difficulties by loading the lattice with a standard magneto-optic trap yielding initial phase space densities of several $10^{-6}$ at a temperature of about 120 $\mu$K.

In summary, we have proposed a novel laser cooling scheme, which can provide tight confinement and nearly zero vibrational temperature in an optical lattice operating in the Lamb-Dicke regime. Because this scheme relies on coherent Raman scattering, it is applicable to any polarizable particle and does not suffer degradation at high particle densities. In this article we have discussed a one-dimensional scenario, however, an extension to two and three dimensions should not present any difficulties. A particularly simple way to add cooling in the transverse lattice planes is to combine our scheme with cavity Doppler cooling.

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[1] C. Adams and E. Riis, Prog. Quant. Electr. 21, 1-79 (1997).
[2] M. H. Anderson et al., Science 269, 198 (1995).
[3] P. Pinkske et al., Nature 404, 365-368 (2000).
[4] C. Hood et al., Science 287, 1457 (2000).
[5] A. Doherty et al., Phys. Rev. A. 56, 833 (1997).
[6] P. Horak et al., Phys. Rev. Lett. 79, 4974 (1997).
[7] G Hechenblaikner et al., Phys. Rev. A. 58, 4030 (1998).
[8] V. Vuletic and S. Chu, Phys. Rev. Lett. 84, 3787 (2000).
[9] H. Chan et al., Phys. Rev. Lett. 90, 063003 (2003).
[10] V. Vuletic et al., Phys. Rev. A. 64, 033405 (2000).
[11] A. Hemmerich, Phys. Rev. A. 60, 943 (1999).
[12] D. Wineland and W. Itano, Phys. Rev. A. 20, 1521 (1979).
[13] M. Gangl and H. Ritsch, Phys. Rev. A. 61, 043405 (2000).
[14] B. Nagorny et al., Phys. Rev. A. xx, Rxyz (2003).