Rock Slope Stability Analysis Based on Comprehensive Failure Mechanism and Engineering Application

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Abstract. The sliding modes of rock slopes are closely related to the strength yield criterion of the rock mass adopted in numerical analysis. In order to make theoretical calculation results meet engineering practice, particularly for complex rock slopes with multiple empty surfaces, multiple control sliding surfaces and potential sliding modes, numerical analysis method used for stability evaluation should take comprehensive failure of the rock mass into consideration. This paper proposes a stability evaluation method based on a comprehensive failure mechanism of the rock mass based on discrete element theory and a modified M-C strength yield criterion to overcome the shortcomings of traditional strength yield criteria, which are mainly derived from shear strength failure. However, the method is used to analyze deformation and failure modes of a typical rock slope in Shuang Jiang Kou Hydropower Station. The results of the findings demonstrated that the method mentioned above can describe different failure modes of rock blocks and joints, deformation trend and stable state of the slope. The experimental results are approved by geological exploring. To adapt to the yield modes of the rock block and joint, a space support system with variable angles, lengths, and design forces is chosen for this typical rock slope. This support system has been proven to be practical and can improve the stability coefficient of the slope. The results presented in this study are useful in engineering practice.

1. Introduction

Instability modes of rock slopes are complex because, in addition to the physical and mechanical properties of the rock mass, spatial distribution and contact condition of the structural plane, many other factors affect the failure modes(e.g., Zhou, J.W.2021). Although the theory of the rigid limit equilibrium method is simple, the calculation results may be inaccurate and it may be difficult to satisfy the actual analysis requirements due to the limitations of assumptions. In recent years, the finite element analysis method has been applied extensively in slope engineering, but there are limitations to this method because the stress-deformation characteristics of the interactions between rock blocks do not conform to the continuum assumption, especially for rock slopes with multiple empty surfaces, multiple control sliding surfaces, and potential sliding modes. Discrete element theory has been proposed and successfully applied to analyze the stability analysis of rock slopes in order to take into consideration the continuity of the blocks and the discontinuity of the surfaces(e.g., Scholtes, L.2012, L, and Y.2017).

The number of engineering examples indicates instability of the rock slopes is usually accompanied by comprehensive failure modes of the rock mass, including tension strength failure, shear strength...
failure, and tension-shear strength failure. As the failure modes of the rock mass are closely related to the strength yield criteria adopted, to overcome the shortcomings of traditional strength yield criteria mainly derived from shear failure, a stability evaluation method should be based on the comprehensive failure mechanism of the rock mass. As we all know, it is very difficult to propose a completely new yield criterion, and therefore, it is feasible and effective to modify the existing criteria. A lot of studies have demonstrated that the Mohr-Coulomb criterion is one of the most widely used criteria for stability analysis of rock slopes. Using the discrete element theory of deformable body and the Mohr-Coulomb strength yield criteria, this study proposes an anti-sliding stability analysis technique that integrates the activities of shear failure and tension failure. The application values are then verified using a typical engineering case.

2. Discrete element theory of deformable block

For rock slopes, the first step in analyzing stability is to solve the challenge of how to calculate stress and deformation of the rock mass. Both continuous deformation of the rock blocks and discontinuous deformation with elative displacement and rotation included among rock blocks should be calculated in the stability evaluation of the rock slope as they are affected by the spatial distribution and mechanical properties of the structural plane. If the rock mass is regarded as a collection of discrete deformable blocks in mechanical solution, the system should satisfy the equilibrium equation shown in Equation (1) and the physical equation shown in Equation (2). Under specific stress and deformation boundary conditions,

$$d\varepsilon_{ij} = \frac{1}{2} (\partial u_{ij} + \partial u_{ji}) dt$$  \hspace{1cm} (1)

In Equation (1), $d\varepsilon$ is the strain increment, $u_i$ is the strain rate, and $dt$ is the infinitesimal time in iterative computation. In addition, indices $i,j$ take the values of 1,2,3.

$$d\sigma_{ij} = C_{ij} (d\varepsilon_{ij} - d\varepsilon_{ij}^p)$$  \hspace{1cm} (2)

In Equation (2), $d\sigma_{ij}$ is the stress increment, $d\varepsilon_{ij}^p$ is the plastic strain increment, and $C_{ij}$ is the elastic stiffness matrix.

For the interactions among rock blocks, a suitable load-deformation mechanics criterion should be used to describe the sliding and detachment behaviours of the rock blocks at common boundaries. Generally, the types of contacts among rock blocks are varied, with corner-corner contact, corner-edge contact, corner-plane contact, edge-edge contact, edge-plane contact, and plane-plane contact included. According to the mechanics of the contact, tangential stress and normal stress on contact pairs can be calculated as Equation (3) ~ Equation (5).

$$F_n = F_n - k_n \Delta u_n$$  \hspace{1cm} (3)

$$F_s = F_s - k_s \Delta u_s$$  \hspace{1cm} (4)

$$F_s = \min\{\mu F_n, F_s\} \text{sgn}(F_s)$$  \hspace{1cm} (5)

From Equation (3) ~ Equation (5), for the specific contact, $F_n$ is the normal stress; $F_s$ is the tangential stress; $k_n$ is the normal stiffness; $k_s$ is the tangential stiffness; $\Delta u_n$ is the normal displacement; $\Delta u_s$ is the tangential displacement, and $\mu$ is the friction coefficient.

If normal stress exceeds the limited tensile strength, the contact pair is detached. The normal and tangential stress on contact pairs should be modified as shown in equation (6).

$$F_n = 0, \quad F_s = 0$$  \hspace{1cm} (6)

Based on the aforementioned theories, the motion equation is used to determine the velocities and displacements of the grid nodes from the stress and external forces, and then the strain rates of the elements can be calculated from the spatial derivative. The constitutive relationship of the rock mass can be used to update the new element stress
3. Strength criterion based on composite failure

With reference to relevant mechanical analysis (e.g., Dai, Z.H. 2006), sliding surface of the slope may be composed of a tensile strength failure section and a shear strength failure section, and the conclusion has been verified by field survey and engineering practice. Presently, the strength yield criteria of rock mass widely used are derived from shear strength failure. It is necessary to adjust the strength yield criteria according to the physical and mechanical properties of rock mass. Taking the Mohr-Coulomb strength criterion as an example, the comprehensive failure modes are considered as follows:

The Mohr-Coulomb strength criterion establishes the relationship between normal stress and shear strength at a point in force space. If the three principal stresses of the space point satisfy the agreement $\sigma_1 \geq \sigma_2 \geq \sigma_3$ (tension is positive, and pressure is negative), the stress state outside the plastic yield envelope may be considered as a failure state, as shown in Figure 1. To differentiate various possible failure modes, considering the tensile strength of the rock mass, the maximum tensile strength of the rock mass should be converted into the linear Mohr-Coulomb strength criterion.

![Figure 1. Relationship between spatial stress and shear strength in M-C criterion based on comprehensive failure modes](image)

4. Strength reduction method with composite failure consideration

When the strength reduction method is used to calculate the stability of the specific slope, the strength reduction coefficient $F_{red}$ is selected to reduce the shear strength parameters of the rock mass as shown in Equation (7) and Equation (8) until the slope becomes unstable (e.g., Chi, S.C. 2004, Zhen, Y.R. 2005). In the Equation (7) and (8), $c$ is cohesion of the rock mass, and $\varphi$ is the friction angle of the rock mass.

$$c_{red} = c(F_{red})^{-1}$$ (7)

$$\varphi_{red} = \arctan(\tan(\varphi)(F_{red})^{-1})$$ (8)

A tensile-shear composite strength yield criteria is required to assess the potential tensile failure of the rock mass, as shown in Figure 1. For rock slopes, the tensile strength of rock blocks and structural planes is usually significant and definitely affects the stability calculation results. Therefore, the tensile strength of the rock mass and structural planes should be reduced in an appropriate way in the process of stability calculation. As the uniaxial tensile strength and shear strength of the rock mass conform to the M-C strength criterion, the relationship between these two indicators is satisfied by Equation (9).

Therefore, tensile strength is reduced by subsisting strength parameters as shown in Equation (10), where $\sigma_{t}$ is the tensile strength of the rock mass (e.g., Zhou, Z.J. 2014).

$$\sigma_{t} = -2c(\tan \varphi + (1 + \tan^2 \varphi)^{1/2})^{-1}$$ (9)
\[ \sigma_{red} = -2c(\tan \varphi + (F_{rel}^2 + \tan^2 \varphi)^{1/2})^{-1} \]  \hspace{1cm} (10)

5. Engineering application

5.1 Project introduction

There are three main faults in the rock slope, which is located at the entrance of the spillway at Shuang Jiang Kou Hydropower Station, and the faults are described as follows:

1. Large fault F3 with occurrence of N50° ~ 60°W/NE θ 65° ~ 85° is composed of several small faults with poor properties, and width of the main fault is about 1~3 m.

2. The high dip small fault f2 with the occurrence of N55° ~ 70°W/SW θ 70° ~ 80°, and the impact zone of this fault is about 10 ~ 20 cm;

3. Moderately inclined small fault f2 with occurrence of N25° ~ 30°E/NW θ 39° ~ 45°, fault width of 5~20 cm.

Relative positions are shown in Figure 2, and the three groups of structural planes are interlaced and combined.

In order to evaluate stability of the slope more intuitively, entire excavation process of the slope was simulated with comprehensive failure modes of the rock mass consideration. The stability of the slope was analyzed in terms of stress, deformation distribution, and potential failure modes.

5.2 Model and parameters of the rock mass

Based on regional surface contours and preliminary survey data, a model including rock blocks and main faults is established, as shown in Figure 3. And the model is 395m long in X-direction and 219m wide in the Y-direction. The bottom altitude of the model is about 2340m.

The natural state is taken as the specific working condition for exhibiting in this study. With reference to the test results and engineering analogy, the parameters of rock mass used in calculation are listed in Table 1 and Table 2 respectively. In conditions of water saturation, the strength parameters of the rock mass take 90% of the natural state, and the strength parameters of the faults take 80% of the natural state.

Table 1. Physical and mechanical parameters of the rock block in natural state

| Category     | \( \rho \) | \( E \) | \( \nu \) | \( \psi \) | \( \sigma^t \) | \( c \) | \( \phi \) |
|--------------|------------|--------|--------|--------|-------------|------|--------|
| Rock in rank IV | 2.35       | 5      | 0.35   | 8      | 0.27        | 0.4  | 35.0   |
Table 2. Physical and mechanical parameters of the faults in natural state

| Category | kn (N·m⁻¹) | ks (N·m⁻¹) | ψ (°) | σt (MPa) | c (MPa) | φ (°) |
|----------|------------|------------|-------|----------|---------|-------|
| f1       | 1.6×10¹⁰  | 1.6×10¹⁰  | 10    | 0.08     | 0.075   | 21.8  |
| f2 (the part above 2840m) | 1.6×10¹⁰  | 1.6×10¹⁰  | 10    | 0.08     | 0.075   | 21.8  |
| f2 (the part below 2840m) | 1.6×10¹⁰  | 1.6×10¹⁰  | 10    | 0.08     | 0.075   | 29.9  |
| F3       | 1.6×10¹⁰  | 1.6×10¹⁰  | 10    | 0.08     | 0.075   | 21.8  |

5.3 Results analysis

The deformation distribution map after excavation, as shown in Figure 4, shows that the potential wedge is the zone with significant deformation in condition without support. The overall sliding displacement of the wedge in its natural state is about 1.5~6 cm, with the maximum deformation of the slope occurring at the top of the arch and the side wall on the left side of the entry spillway. After excavation, the wedge-shaped body takes fault F3 as the trailing edge surface and fault f2 as the bottom surface, and may slide along fault f1 and fault f2 to the front edge of the slope. The potential slipping trend is shown in Figure 5.

![Figure 4. Deformation distribution map of the slope after excavation without support(unit: m)](image)

![Figure 5. Deformation vector of the slope after excavation without support](image)

The distribution of the plastic zone shown in Figure 6 indicates that there is no block divided completely by the plastic zone after excavation, but the distribution area of the plastic zone is relatively concentrated in the scope of about 45 m along the axial direction of the tunnel. Although the slope is still stable, the concentration phenomenon of the plastic zone deserves more attention in the course of support design. The strength reduction technique based on comprehensive failure modes is used to evaluate the stability of the rock slope, and the overall stability coefficient is listed in the following Table 3. Although the stability coefficient is greater than 1, the calculated results and related requirements show that it does not conform to the design standards, implying that support design is definitely required.

Based on the calculation results of the slope excavation without support, with spatial distribution of the stress deformation and plastic yield distribution, and structural properties of the faults combined, a
large number of calculations on optimization of support design are implemented gradually. A spatial support system with variable angles, lengths, and tonnages is recommended to implement precise support for this rock slope. The final support plan is as follows:

(1) On the side of the entry, 16 rows of pre-stressed anchor cables are used to support the slope at a height of 2513m ~ 2594m. The design lengths of the anchor cables are from 35m ~ 65m, and their depression angles are around 10°~17° following the real topography.

(2) The interval distance between the anchor cables in the same excavation layer is 4 m, but 10 m for different excavation layers.

(3) In this spatial supporting system illustrated in Figure 7, 70 pieces of pressurized anchor cables with a bearing capacity of 2000 kN, and 111 pieces of pressurized anchor cables with a bearing capacity of 1500 kN are used.

Under natural conditions, the maximum deformation of the slope with final support in Figure 8 is definitely reduced by 25~30% compared with results without support. It indicates that the reinforcement effect of the anchor cables in the suggested design scheme is remarkable for slope stability. From the yield states of the anchor cables after excavation shown in Figure 9, the overall efficiency of the anchor cable on the left side of the slope (in the axial direction of the spillway) is higher than the others, and even some of them get close to their design tension strength. The results above verify the accuracy and efficiency of the spatial support system suggested.
Evaluated by the strength reduction method based on comprehensive failure modes, the overall safety factors of the rock slope at Shuang Jiang Kou Hydropower Station under each condition are calculated and listed in Table 3. The findings show that the support design scheme can successfully improve the overall stability of the slope and satisfy the design requirements.

| Case                | Natural condition | Saturated condition | Accidental conditions (earthquake) |
|---------------------|-------------------|---------------------|-----------------------------------|
| Unsporting condition| 1.54              | 1.13                | 1.11                              |
| Supporting condition| 1.63              | 1.25                | 1.22                              |
| allowable value      | 1.25–1.30         | 1.15–1.20           | 1.05–1.10                         |

6. Conclusion

The technique of stability calculation for rock slopes is adopted with comprehensive failure modes consideration based on discrete element theory and the M-C strength yield criterion. The application values are then verified using a typical engineering case. Relevant conclusions are as follows:

1. A stability evaluation method based on the comprehensive failure mechanism of the rock mass, which includes stress and deformation calculation method, judgment criteria for potential yield failure modes, and an adaptive strength reduction procedure, is adopted.

2. The engineering application shows that the method can better reflect different plastic yield modes of the rock blocks and joints, deformation trend and stable state of the slope. And the analysis results are approved by geological exploration.

3. A spatial support design with variable angles, lengths, and tonnages is suggested to implement precise support based on the calculation results of the slope excavation without support, with spatial distribution of the stress deformation and plastic yield distribution, and structural properties of the faults combined. Furthermore, the results of the calculation show that the overall efficiency of the support system is advanced.

As the slope of the Shuang Jiang Kou spillway is still under excavation, there is no systematic monitoring data of stress and deformation. Further and deeper studies are needed to test the practical value of the method mentioned in this paper.

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