Large Coulomb corrections to the $e^+e^-$ pair production at relativistic heavy ion colliders

D. Yu. Ivanov
Institute of Mathematics, Novosibirsk, 630090, Russia

A. Schiller
Institut für Theoretische Physik and NTZ, Universität Leipzig, D-04109 Leipzig, Germany

V. G. Serbo
Novosibirsk State University, Novosibirsk, 630090, Russia

We consider the Coulomb correction (CC) to the $e^+e^-$ pair production related to multiphoton exchange of the produced $e^\pm$ with nuclei. The contribution of CC to the energy distribution of $e^+$ and $e^-$ as well as to the total pair production cross section are calculated with an accuracy of the order of 1%. The found correction to the total Born cross section is negative and equal to $-25\%$ at the RHIC for Au–Au and $-14\%$ at the LHC for Pb–Pb collisions.

Introduction. Two new large colliders with relativistic heavy nuclei, the RHIC and the LHC, are scheduled to be in operation in the nearest future. The charge numbers $Z_1 = Z_2 = Z$ of the nuclei with masses $M_1 = M_2 = M$ and their Lorentz factors $\gamma_1 = \gamma_2 = \gamma = E/M$ are the following

- $Z = 79, \gamma = 108$ for RHIC (Au–Au collisions)
- $Z = 82, \gamma = 3000$ for LHC (Pb–Pb collisions).

Here $E$ is the heavy ion energy in the c.m.s. One of the important processes at these colliders is

$$Z_1 Z_2 \rightarrow Z_1 Z_2 e^+ e^-.$$  \hspace{1cm} (2)

Its cross section is huge. In the Born approximation (see Fig. 1 with $n = n'$ = 1) the total cross section according to the Racah formula \cite{Racah} is equal to $\sigma_{\text{Born}} = 36$ kbarn

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig1.jpg}
\caption{The amplitude $M_{nn'}$ of the process (2) with $n (n')$ virtual photon emitted by the first (second) nucleus.}
\end{figure}

for the RHIC and 227 kbarn for the LHC. Therefore it will contribute as a serious background to a number of experiments, besides, this process is the leading beam loss mechanism (for details see review \cite{Collins}.

The cross sections of the process (2) in the Born approximation are known with accuracy $\sim 1/\gamma^2$ (see, for example, Refs. \cite{Collins} and more recent calculations reviewed in Refs. \cite{Collins}). However, besides of the Born amplitude $M_{\text{Born}} = M_{11}$, also other amplitudes $M_{nn'}$ (see Fig. 1) have to be taken into account for heavy nuclei since in this case the parameter of the perturbation series $Z\alpha$ is of the order of unity. Therefore, the whole series in $Z\alpha$ has to be summed to obtain the cross section with sufficient accuracy. Following Ref. \cite{Collins}, we call the Coulomb correction (CC) the difference $d\sigma_{\text{Coul}}$ between the whole sum $d\sigma$ and the Born approximation

$$d\sigma = d\sigma_{\text{Born}} + d\sigma_{\text{Coul}}.$$  \hspace{1cm} (3)

Such kind of CC is well known in the photoproduction of $e^+e^-$ pairs on atoms (see Ref. \cite{Collins} and §98 of \cite{Collins}). The Coulomb correction to the total cross section of that process decreases the Born contribution by about $10\%$ for a Pb target. For the pair production of reaction (2) with $Z\alpha \ll 1$ and $Z\alpha \sim 1$ CC has been obtained in Refs. \cite{Collins}. Recently this correction has been calculated for the pair production in the collisions of muons with heavy nuclei \cite{Collins}. The results of Refs. \cite{Collins} agree with each other in the corresponding kinematic regions and noticeably change the Born cross sections. Formulae for CC for two heavy ions were suggested ad hoc in Sect. 7.3 of \cite{Collins}. However, our calculations presented here do show that this suggestion is incorrect.

In the present paper we calculate the Coulomb correction for process (2) omitting terms of the order of $1\%$ compared with the main term given by the Born cross section. We find that these corrections are negative and quite important:

$$\sigma_{\text{Coul}}/\sigma_{\text{Born}} = -25\%$$  \hspace{1cm} for RHIC,
\[ \frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}} = -14\% \text{ for LHC}. \] (4)

This means that at the RHIC the background process with the largest cross section will have a production rate 25\% smaller than expected.

Our main notations are given in Eq. (1) and Fig. 1, besides, \((P_1 + P_2)^2 = 4E^2 = 4\gamma^2 M^2\), \(q_i = (\omega_i, q_i) = P_i - P_i'\), \(\varepsilon = \varepsilon_+ + \varepsilon_-\) and

\[ \sigma_0 = \frac{\alpha^2 Z_1^2 Z_2^2}{\pi m^2}, \quad L = \ln \frac{P_1 P_2}{2M_1 M_2} = \ln \gamma^2 \] (5)

where \(m\) is the electron mass. The quantities \(q_{i\perp}\) and \(p_{\pm\perp}\) denote the transverse part of the corresponding three–momenta. Throughout the paper we use the well known function [3]

\[ f(Z) = Z^2 \alpha^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + Z^2 \alpha^2)}, \] (6)

its particular values for the colliders under discussion are \(f(79) = 0.313\) and \(f(82) = 0.332\).

Selection of the leading diagrams and the structure of the amplitude. Let \(M\) be the sum of the amplitudes \(M_{nn'}\) of Fig. 1. It can be presented in the form

\[ M = \sum_{nn' \geq 1} M_{nn'}, \quad M_1 = \sum_{n' \geq 2} M_{1n'}, \quad M_2 = \sum_{nn' \geq 2} M_{nn'}. \] (7)

The Born amplitude \(M_{\text{Born}}\) contains the one–photon exchange both with the first and the second nucleus, whereas the amplitude \(M_1 (M_2)\) contains the one–photon exchange only with the upper (lower) nucleus. In the last amplitude \(M_2\) we have no one–photon exchange. According to this classification we write the total cross section as

\[ \sigma = \sigma_{\text{Born}} + \sigma_1 + \bar{\sigma}_1 + \sigma_2 \] (8)

where

\[
\begin{align*}
\frac{d\sigma_{\text{Born}}}{\sigma} &\propto |M_{\text{Born}}|^2, \\
\frac{d\sigma_1}{\sigma} &\propto 2\text{Re}(M_{\text{Born}} M_1^*) + |M_1|^2, \\
\frac{d\bar{\sigma}_1}{\sigma} &\propto 2\text{Re}(M_{\text{Born}} M_2^*) + |M_2|^2, \\
\frac{d\sigma_2}{\sigma} &\propto 2\text{Re} \left( M_{\text{Born}} M_2^* + M_1 M_2^* + M_1 \tilde{M}_2 + M_2 \tilde{M}_1 \right) + |M_2|^2.
\end{align*}
\]

It is not difficult to show that the ratio \(\sigma_1/\sigma_{\text{Born}}\) is a function of \((Z\alpha)^2\) only but not of \(Z\alpha\) itself. Additionally we estimate the leading logarithms appearing in the cross sections \(\sigma_i\). The integration over the transferred momentum squared \(q_1^2\) and \(q_2^2\) results in two large Weizsäcker–Williams (WW) logarithms \(\sim L^2\) for the \(\sigma_{\text{Born}}\), in one large WW logarithm \(\sim L\) for \(\sigma_1\) and \(\bar{\sigma}_1\). The cross section \(\sigma_2\) contains no large WW logarithm. Therefore, the relative contribution of the cross sections \(\sigma_i\) is \(\sigma_1/\sigma_{\text{Born}} = \bar{\sigma}_1/\sigma_{\text{Born}} \sim (Z\alpha)^2/L\) and \(\sigma_2/\sigma_{\text{Born}} \sim (Z\alpha)^2/L^2 < 0.4\%\) for the colliders [1]. As a result, with an accuracy of the order of 1\% we can neglect \(\sigma_2\) in the total cross section and use the equation

\[ \sigma = \sigma_{\text{Born}} + \sigma_1 + \bar{\sigma}_1. \] (9)

With that accuracy it is sufficient to calculate \(\sigma_1\) and \(\bar{\sigma}_1\) in the leading logarithmic approximation (LLA) only since the next to leading log terms are of the order of \((Z\alpha/L)^2\). This fact greatly simplifies the calculations.

The calculation in the LLA can be performed using the equivalent photon or WW approximation. The main contribution to \(\sigma_1\) and \(\bar{\sigma}_1\) is given by the region \((\omega_1/\gamma)^2 \ll -q_1^2 \ll m^2\) and \((\omega_2/\gamma)^2 \ll -q_2^2 \ll m^2\), respectively. In the first region the main contribution arises from the amplitudes \(M_{\text{Born}} + M_1\) (in the second region \(M_{\text{Born}} + M_1\)). The virtual photon with four–momentum \(q_1\) is almost real and the amplitude can be expressed via the amplitude \(M_\gamma\) for the real photoproduction \(\gamma Z_2 \rightarrow Z_2 e^+ e^-\) (see, for example, §9B of Ref. [1])

\[ M_{\text{Born}} + M_1 \approx \frac{\sqrt{4\pi\alpha Z_1}}{(-q_1^2)} \frac{2E_\gamma}{\omega_1} M_\gamma. \] (10)

The amplitude \(M_\gamma\) has been calculated in Ref. [1]. We use the convenient form of that amplitude derived in the works [1] and [11]:

\[ M_\gamma = (f_1 M_{\text{Born}} + i f_2 \Delta M_\gamma) e^{i\Phi}. \] (11)

where \(M_{\text{Born}}\) is the Born amplitude for the \(\gamma Z_2 \rightarrow Z_2 e^+ e^-\) process. This Born amplitude depends on the transverse momenta \(p_{\pm\perp}\) only via the two combinations \(A = \xi_+ - \xi_-\) and \(B = \xi_+ p_{\perp} + \xi_- p_{-\perp}\) where \(\xi_{\pm} = m^2/(m^2 + p_{\pm\perp}^2)\). The quantity \(\Delta M_\gamma\) is obtained from \(M_{\text{Born}}^\gamma\) replacing \(A \rightarrow \xi_+ + \xi_- - 1\) and \(B \rightarrow \xi_+ p_{\perp} - \xi_- p_{-\perp}\).

All the nontrivial dependence on the parameter \(Z_2 \alpha \equiv \nu\) are accumulated in the Bethe-Maximon phase

\[ \Phi = \nu \ln \left( \frac{p_+ P_2}{p_- P_2} \xi_+ \right), \] (12)

and in the two functions (with \(z = 1 - (-q_2^2/m^2) \xi_+ \xi_-\))

\[ f_1 = \frac{F(i \nu, -i \nu; 1; z)}{F(i \nu, -i \nu; 1; 1)}, \quad f_2 = \frac{1 - z}{\nu} f_1'(z). \] (13)

The function \(f_1(z)\) and its derivative \(f_1'(z)\) are given with the help of the Gauss hypergeometric function \(F(a, b; c; z)\).

It can be clearly seen that in the region \(p_{\pm\perp}^2 \sim m^2\) the amplitude \(M_\gamma\) differs considerably from the \(M_{\text{Born}}^\gamma\) amplitude and, therefore, the whole amplitude \(M\) differs
from its Born limit $M_{\text{Born}}$. Let us stress that just this transverse momentum region $p_{\perp}^2 \sim m^2$ gives the main contribution into the total Born cross section $\sigma_{\text{Born}}$ and into $\sigma_1$.

Outside this region the CC vanishes. Indeed, for $p_{\perp}^2 \ll m^2$ or $p_{\perp}^2 \gg m^2$ the variable $z \approx 1$, therefore, $f_1 \approx 1$, $f_2 \approx 0$ and

$$M_{\text{Born}} + M_1 = M_{\text{Born}} \Phi.$$  \hspace{1cm} (14)

Note that the region $p_{\perp}^2 \gg m^2$ gives a negligible contribution to the total cross section $\sigma$, however, this region might be of interest for some experiments.

The results of Ref. [5], which are used here in the form of Eqs. (10)-(14) are the basis for our consideration. These results were confirmed in a number of papers (see, for example, Refs. [13-16]) using various approaches.

Recently in Refs. [11] the Coulomb effects were studied within the framework of a light–cone or an eikonal approach. However, the approximations used in Refs. [11] fail to reproduce the classical results of Bethe and Maximon [6]. To show this explicitly, we consider the simplest case where in the collider system ($\gamma_1 = E_1/M_1 \sim \gamma_2 = E_2/M_2$) both $e^+$ and $e^-$ are ultrarelativistic ($\epsilon_{\pm} \gg m$). We assume that the $z$-axis is directed along the initial three-momentum of the first nucleus $P_1$.

To obtain the energy distribution of $e^+$ and $e^-$ in the LLA we have to take into account two regions $p_{\perp \pm} \gg m$ and ($-p_{\perp \pm}$) $\gg m$ where the lepton pair is produced either in forward or backward direction. In the first region we have $x_{\pm} = \epsilon_{\mp}/\epsilon$, $y = \epsilon/E_1$, and from Eq. (13-14) we obtain in the LLA

$$d\sigma_1^{(1)} = -4\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{m\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+ d\epsilon_-}{\epsilon^2}, \hspace{1cm} (18)$$

In the second region we have $x_{\pm} \approx \epsilon_{\mp}/\epsilon$, $y \approx m^2 \epsilon/(4E_1 \epsilon_{\mp})$ and

$$d\sigma_1^{(2)} = -4\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+ d\epsilon_-}{m^2 \epsilon^2}, \hspace{1cm} (19)$$

Summing up these two contributions, we find

$$d\sigma_1 = -8\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+ d\epsilon_-}{\epsilon^2}, \hspace{1cm} (20)$$

To obtain $\sigma_1$ we have to integrate the expressions (18) and (19) over $\epsilon_{\mp}$ (with logarithmic accuracy)

$$d\sigma_1^{(1)} = -\frac{28}{9}\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+}{\epsilon_{\mp}}, \hspace{1cm} (21)$$

$$d\sigma_1^{(2)} = -\frac{28}{9}\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+}{m^2 \epsilon_{\mp}}, \hspace{1cm} (22)$$

from which it follows that

$$d\sigma_1 = -\frac{28}{9}\sigma_0 f(Z_2) \left(1 - \frac{4\epsilon_+ \epsilon_-}{3\epsilon^2} \right) \ln \left(\frac{\gamma_1^2}{\epsilon_{\mp}} \right) \frac{d\epsilon_+}{\epsilon_{\mp}}. \hspace{1cm} (23)$$

The further integration of Eqs. (21), (22) over $\epsilon_{\mp}$ results in

$$\sigma_1 = -\frac{28}{9}\sigma_0 f(Z_2) \left[\ln \frac{P_1 P_2}{2M_1 M_2} \right]^2. \hspace{1cm} (24)$$
This expression is in agreement with the similar result for the $\mu Z$ scattering (see Eq. (31) from [8] for $Z_1 = 1, Z_2 = Z$).

The corresponding formulae for $\tilde{\sigma}_1$ can be obtained from Eqs. (20), (23) and (24) by replacing $\gamma_1 \leftrightarrow \gamma_2, Z_1 \leftrightarrow Z_2$. The whole CC contribution $d\sigma_{\text{Coul}} = d(\sigma_1 + \tilde{\sigma}_1)$ for the symmetric case $Z_1 = Z_2 = Z$ and $\gamma_1 = \gamma_2 = \gamma$ takes the following form

$$d\sigma_{\text{Coul}} = -16 \sigma_0 f(Z) \left(1 - \frac{4\varepsilon_+ \varepsilon_-}{3\varepsilon^2}\right) L \frac{d\varepsilon_+ d\varepsilon_-}{\varepsilon^2}$$

(25)

at $m \ll \varepsilon_\pm \ll m\gamma$,

$$d\sigma_{\text{Coul}} = -\frac{112}{9} \sigma_0 f(Z) L \frac{d\varepsilon_+}{\varepsilon_+}$$

(26)

at $m \ll \varepsilon_+ \ll m\gamma$, and

$$\sigma_{\text{Coul}} = -\frac{56}{9} \sigma_0 f(Z) L^2.$$  

(27)

The size of this correction for the two colliders was given before in Eq. (4). The total cross section with and without Coulomb correction as function of the Lorentz factor $\gamma$ is illustrated in Fig. 2 for Pb nuclei.

![Graph showing total cross section with and without Coulomb correction](image)

FIG. 2. The total cross section of the process $ZZ \rightarrow ZZe^+e^-$ with (solid line) and without (dashed line) Coulomb correction as function of the Lorentz factor $\gamma$ of Pb nuclei ($Z = 82$).

Conclusion. We have calculated the Coulomb corrections to $e^+e^-$ pair production in relativistic heavy ion collisions for the case of colliding beams. Our main results are given in Eqs. (25)-(27). We have restricted ourselves to the Coulomb corrections for the energy distribution of electrons and positrons and for the total cross section. In our analysis we neglected contributions which are of the relative order of $\sim (Z\alpha)^2/L^2$. The CC to the angular distribution of $e^+e^-$ can be easily obtained in a similar way, however only with an accuracy $Z\alpha/L^2$.

Since our basic formulae (25), (27) are given in the invariant form, a similar calculation can be easily repeated for fixed-target experiments. This interesting question will be considered in a future work.

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Email address: d-ivanov@math.nsc.ru
Email address: schiller@tph204.physik.uni-leipzig.de
Email address: serbo@math.nsc.ru

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