Low-frequency fluorescence spectrum of a laser driven polar quantum emitter damped by squeezed vacuum with finite bandwidth

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Abstract. A two-level quantum emitter with broken inversion symmetry driven by external semiclassical monochromatic high-frequency electromagnetic (e.g., laser) field and damped by squeezed vacuum reservoir with finite bandwidth is presented. The squeezed vacuum source is assumed to be either degenerate parametric oscillator (DPO) or a non-degenerate parametric oscillator (NDPO). It is shown that the shape of low-frequency fluorescence spectrum of the emitter can be effectively alternated by controlling the degree of the squeezed vacuum source degeneration and phase of the squeezing.

Keywords: polar emitter; fluorescence spectrum; squeezed vacuum; two-level atom; broken inversion symmetry; asymmetric quantum dot; polar molecule

1. Introduction
As a rule, theoretical and experimental studies have been carried out under the assumption of symmetric quantum emitters where inversion symmetry is assumed. Nevertheless, violation of this symmetry is common in such natural systems as polar molecules as well as in artificially manufactured systems, like quantum dots. The cause for this violation is different for different systems. For example, in polar molecules its origin is due to the parity mixing of the molecular states [1], while in quantum dots it is induced by the asymmetry of the confining potential of the dot. However, in all cases this violation results in nonzero permanent dipole matrix elements of the ground and excited states. To our knowledge, the idea that a simple two-level quantum system driven by high-frequency classical electromagnetic (EM) field can emit EM field of much lower frequency if its dipole operator possesses permanent non-equal diagonal matrix elements, was introduced in [2]. This phenomenon was further studied in [3, 4, 5], where the properties of this low-frequency radiation were thoroughly investigated for the case of a two-level system driven by external EM field and damped by a dissipative reservoir. It was also found that the same system can also be used for the amplification of the weak low-frequency EM field [4]. This property of a relatively simple quantum system may help develop useful techniques for manipulation of the low-frequency EM radiation, especially in the terahertz range of frequencies. Present research aims to expand the investigation of the low-frequency EM fluorescence radiation phenomenon to the case of an externally driven two-level system with broken inversion symmetry interacting with a finite-band squeezed vacuum dissipative reservoir, which properties can be
tuned appropriately in order to control the shape of the fluorescence radiation spectrum. The case of interaction with a broadband squeezed vacuum dissipative reservoir was studied earlier for weak driving EM field in [6, 7].

2. Model Hamiltonian

In this study we consider a two-level atom with ground state \( |g⟩ \), excited state \( |e⟩ \), transition frequency \( ω_0 \) and the electric dipole moment \( d \), driven by external classical monochromatic field \( E(t) = E \cos(ω_f t) \) with an amplitude \( E \) and frequency \( ω_f \). The atom is also coupled to a reservoir \( B \) made of a plurality of modes of quantized electromagnetic field being in the squeezed vacuum state. It is assumed that the frequency Lamb shift due to interaction with the reservoir is already incorporated into the atomic transition frequency \( ω_0 \). Thus, the model Hamiltonian reads

\[
H = H_S(t) + \hbar \sum_k ω_k b_k^+(ω_k)b(ω_k) + \sum_k (g(ω_k)S_k^+b(ω_k) + g^*(ω_k)b^+(ω_k)S_k^-). \tag{1}
\]

Here \( S_k^+ = |e⟩⟨g| \) and \( S_k^- = |g⟩⟨e| \) are the usual raising and lowering atomic operators and \( S^2 = 1/2(|e⟩⟨e| - |g⟩⟨g|) \) is the atomic population inversion operator.

The operators \( b(ω_k) \) and \( b^+(ω_k) \) are the annihilation and creation operators for the vacuum modes satisfying the commutation relations

\[
[b(ω), b^+(ω')] = δ(ω - ω'), \quad [b(ω), b(ω')] = 0, \quad [b^+(ω), b^+(ω')] = 0, \tag{2}
\]

and the term

\[
H_S(t) = \hbar ω_0 S^2 + \hbar/2 Ω_R (S^- e^{iω_f t} + S^+ e^{-iω_f t}) + \hbar/2 (e^{iω_f t} + e^{-iω_f t}) \left[ δ_a S^2 - \frac{δ_s}{2}(|e⟩⟨e| + |g⟩⟨g|) \right]. \tag{3}
\]

contains an interaction between the driving field and the atom in the rotating wave approximation (RWA). Here \( Ω_R = -Ed_{eg}/\hbar \) is the Rabi frequency being made real and positive by the appropriate choice of the phase factors of the states \( |e⟩ \) and \( |g⟩ \), and \( d_{eg} = e⟨e|g⟩ \), \( d_{ge} = e⟨g|e⟩ \), \( d_{ee} = e⟨e|e⟩ \), \( d_{gg} = e⟨g|g⟩ \) are the atomic dipole moment operator matrix elements. As a rule, it is assumed that \( d_{ee} = d_{gg} = 0 \), because typical physical systems, like atoms and molecules, possess the inversion symmetry, and each of the states \( |g⟩ \) and \( |e⟩ \) is either symmetric or antisymmetric. Contrary to this view, we assume that the inversion symmetry of the system in question is violated, \( d_{ee} \neq d_{gg} \), so that \( δ_a = E(d_{gg} - d_{ee})/\hbar \) and \( δ_s = E(d_{gg} + d_{ee})/\hbar \). The term proportional to \( δ_a \) does not influence the dynamics of the system and can be omitted, while the term proportional to the symmetry violation parameter \( δ_s \) is retained. The squeezed vacuum reservoir source is assumed to be either a degenerate parametric oscillator (DPO) or a non-degenerate parametric oscillator (NDPO). The output fields from this oscillators are characterized by the following correlation functions [8]

\[
\langle b^+(ω_k)b(ω_{k'})⟩_{svac} = N(ω_k)δ(ω_k - ω_{k'}); \quad ⟨b(ω_k)b(ω_{k'})⟩_{svac} = -M(ω_k, θ)δ(ω_k + ω_{k'} - 2ω_s), \tag{4}
\]

\[
\langle b(ω_k)b^+(ω_{k'})⟩_{svac} = (N(ω_k)+1)δ(ω_k - ω_{k'}); \quad ⟨b^+(ω_k)b^+(ω_{k'})⟩_{svac} = -M^*(ω_k, θ)δ(ω_{k'} + ω_k - 2ω_s), \tag{5}
\]

where \( ω_s \) is the carrier frequency of the squeezed field, \( θ \) is the phase of squeezing, \( N(ω) \) is related to the mean number of photons at frequency \( ω \) and \( M(ω, θ) \) is characteristic of the squeezed vacuum field and describes the correlation between the two photons created in the down-conversion process. They satisfy the inequality \( |M(ω, θ)| ≤ \sqrt{N(ω)(N(ω) + 1)} \), in which in the case of ideal squeezed state produced by an optical parametric oscillator transforms into the equality. The frequency dependencies of \( N(ω) \) and \( M(ω, θ) \) for an optical parametric oscillator below threshold for the ideal DPO are given by [9]

\[
N(ω) = \frac{λ^2 - μ^2}{4} \left[ \frac{1}{(ω - ω_s)^2 + μ^2} - \frac{1}{(ω - ω_s)^2 + λ^2} \right], \tag{6}
\]
$$M(\omega, \theta) = e^{i\theta} \frac{\lambda^2 - \mu^2}{4} \left[ \frac{1}{(\omega - \omega_s)^2 + \mu^2} + \frac{1}{(\omega - \omega_s + \lambda^2)^2 + \lambda^2} \right],$$

(7)

while for the ideal NDPO they assume more general form [10]

$$N(\omega) = \frac{\lambda^2 - \mu^2}{8} \left[ \frac{1}{(\omega - \omega_s - \alpha)^2 + \mu^2} + \frac{1}{(\omega - \omega_s + \lambda^2) + \mu^2} \right] - \frac{1}{(\omega - \omega_s - \alpha)^2 + \lambda^2} - \frac{1}{(\omega - \omega_s + \alpha)^2 + \lambda^2},$$

(8)

$$M(\omega, \theta) = e^{i\theta} \frac{\lambda^2 - \mu^2}{8} \left[ \frac{1}{(\omega - \omega_s - \alpha)^2 + \mu^2} + \frac{1}{(\omega - \omega_s + \lambda^2) + \mu^2} \right] + \frac{1}{(\omega - \omega_s - \alpha)^2 + \lambda^2} + \frac{1}{(\omega - \omega_s + \alpha)^2 + \lambda^2}. $$

(9)

Here the parameters \( \lambda \) and \( \mu \) are expressed in terms of the parametric oscillator cavity damping rate \( \gamma_c \) and the effective pump amplitude \( \epsilon \) of the coherent field driving the parametric oscillator

\[
\lambda = \gamma_c + \epsilon, \quad \mu = \gamma_c - \epsilon, \quad \epsilon = E_s/E_c,
\]

(10)

where \( E_s \) is the amplitude of the pump coherent field and \( E_c \) is its threshold value for parametric oscillator. In optical parametric oscillator (OPO) the amplitude \( E_s \) is related to the power of pumping \( P \) [11, 12], so the effective pump amplitude is related to the ratio of input pump power to the critical power, \( r = P/P_c \), and we have \( \epsilon = \sqrt{\gamma_c/2} \). The noise spectrum and the squeezing level of the output light from OPO is related to \( \epsilon/(\gamma_c/2) \). When \( \epsilon \) goes to 1 and therefore \( r \to 1 \), the threshold happens in OPO. The parameter \( \alpha = (\omega_1 - \omega_2)/2 \) is the displacement from the central frequency of the squeezing at which the two-mode squeezed vacuum is maximally squeezed. It is worth noticing that Eqs.(6,7,8,9) are only valid sufficiently below of the threshold, i.e. when \( 0 < \epsilon < \gamma_c/2 \), both parameters \( \lambda \) and \( \mu \) are positive and \( \lambda > \mu \), and the squeezing values are not too large. When the parameters \( \lambda \) and \( \mu \) are much greater than all other relaxation rates in the problem, the frequency dependence of \( N(\omega) \) and \( M(\omega, \theta) \) can be neglected. This case is referred to as broadband squeezed vacuum in which there is no difference between the output fields from DPO and NDPO.

3. Equations of Motion for Atomic Variables

In what follows, it is assumed that \( \delta_s \ll \Omega_s \), so that the interaction of the driving field with the permanent dipole moment is much weaker than its interaction with the transitional dipole moment while, at the same time, the driving field is strong enough to be viewed as a dressing field for the two-level system. To derive the master equation, two successive unitary transformations are performed. The first one

$$U_1(t) = \exp \left[ -i \left( \omega_f S^z + \sum_k \omega_k b_k^+ (\omega_k) b(\omega_k) \right) t \right]$$

(11)

transforms the Hamiltonian (1) to the frame rotating with the driving field frequency \( \omega_f \) and to the interaction picture with the vacuum as the driving field modes, thus yielding the resulting Hamiltonian

$$H_1(t) = U_1^+(t) H U_1(t) - i h U_1^+(t) \frac{\partial U_1(t)}{\partial t} = -\frac{\hbar}{2} \Delta S^z + \frac{\hbar}{2} \Omega_R (S^+ + S^-) + \frac{\hbar}{2} \delta_a (e^{i \omega_f t} + e^{-i \omega_f t}) S^z + \sum_k \left( g(\omega_k) S^+ b(\omega_k) e^{i(\omega_f - \omega_k)t} + g^*(\omega_k) b^+ (\omega_k) S^- e^{-i(\omega_f - \omega_k)t} \right), $$

(12)

where \( \Delta = \omega_f - \omega_0 \) is the detuning of the driving field frequency \( \omega_f \) from the atomic frequency \( \omega_0 \). The second transformation is the unitary transformation
\[ U_2(t) = \frac{\tau}{\hbar} \exp \left[ -\frac{i}{\hbar} \int_0^t dt \left( -\frac{\hbar}{2} \Delta S_z + \frac{\hbar}{2} \Omega_R (S^+ + S^-) + \frac{\hbar}{2} \delta_a (e^{i\omega_f t} + e^{-i\omega_f t}) S^z \right) \right] = \exp \left[ -\frac{i}{2} \left( \Omega_R (S^+ + S^-) - \Delta S_z \right) t \right] \tau \exp \left[ -\frac{i}{2} \int_0^t d\tau \frac{\delta_a (e^{i\omega_f t} + e^{-i\omega_f t}) \hat{S}^z (t)}{\hbar} \right], \tag{13} \]

where
\[ \hat{S}^z (t) = \exp \left[ \frac{i}{2} \left( \Omega_R (S^+ + S^-) - \Delta S_z \right) t \right] S^z \exp \left[ -\frac{i}{2} \left( \Omega_R (S^+ + S^-) - \Delta S_z \right) t \right]. \tag{14} \]

As a result of this transformation, the model Hamiltonian takes the form
\[ H_2(t) = U_2^+(t) H_1(t) U_2(t) - i\hbar U_2^+(t) \frac{\partial U_2(t)}{\partial t} = \sum_k \left( g(\omega_k) \hat{S}(t)^+ \rho(\omega_k) e^{i(\omega_f - \omega)t} + g^*(\omega_k) \rho^\dagger(\omega_k) \hat{S}^\dagger(t) e^{-i(\omega_f - \omega)t} \right), \tag{15} \]

where
\[ \hat{S}^\dagger(t) = U_2^+(t) S^\dagger U_2(t) \approx e^{\frac{i}{2}(\Omega_R (S^+ + S^-) - \Delta S_z) t} S^\dagger e^{-\frac{i}{2}(\Omega_R (S^+ + S^-) - \Delta S_z) t}, \tag{16} \]

if the condition of weak inversion symmetry violation \( \delta_a \ll \Omega_R \) is accounted for. Therefore,
\[ \hat{S}(t) \approx \frac{1}{2} \left[ S_a \pm (1 \mp \tilde{\Delta}) S_b e^{-i\Omega t} \pm (1 \pm \tilde{\Delta}) S_c e^{i\Omega t} \right], \tag{17} \]

where
\[ S_a = \tilde{\Omega} \left[ \tilde{\Omega} (S^+ + S^-) - 2\tilde{\Delta} S^z \right], S_b = \frac{1}{2} \left[ (1 - \tilde{\Delta}) S^+ - (1 + \tilde{\Delta}) S^- - 2\tilde{\Omega} S^z \right], \tag{18} \]
\[ S_c = \frac{1}{2} \left[ (1 + \tilde{\Delta}) S^+ - (1 - \tilde{\Delta}) S^- + 2\tilde{\Omega} S^z \right], \tag{19} \]

and \( \tilde{\Omega} = \Omega_R / \Omega', \tilde{\Delta} = \Delta / \Omega', \Omega' = \sqrt{\Omega_R^2 + \tilde{\Delta}^2} \). As a consequence of the performed transformations, the approximate master equation for the transformed atomic reduced density operator \( \rho_S^\text{trans}(t) \) can be derived using standard methods [13], and in the Born approximation is given by
\[ \frac{\partial \rho_S^\text{trans}(t)}{\partial t} = -\frac{1}{\hbar^2} \int_0^t d\tau S_{PR} \left( [H_2(t), [H_2(t - \tau), \rho_R(0) \rho_S^\text{trans}(t - \tau)]] \right), \tag{20} \]

where \( \rho_S^\text{trans}(t) = S_{PR} \left( U_2^+(t) U_1(t)^\dagger \rho(t) U_1(t) U_2(t)^\dagger \right) \), \( \rho(t) \) is the density operator for the total system (1), \( \rho_R(0) \) is the density operator for the vacuum field reservoir and \( S_{PR}(...) \) is the trace over the reservoir states. As usual, in the Markoff approximation the upper limit of the integration over \( \tau \) can be extended to infinity, and the master equation in the frame rotating with the driving field frequency \( \omega_f \) can be written, under the assumption that the carrier frequency \( \omega_c \) of the squeezed field coincides with the frequency \( \omega_f \), as
\[ \frac{\partial \rho_S^f(t)}{\partial t} = i\Gamma S_z \rho_S^f(t) - \frac{\gamma}{2} \delta_a (e^{i\omega_f t} + e^{-i\omega_f t}) [S^z, \rho_S^f(t)] + \frac{i}{2} \Gamma N (2S + \rho_S^f(t)) S^- - S^- S^+ \rho_S^f(t) - \rho_S^f(t) S^+ S^- + \frac{i}{2} \Gamma (N + 1)(2S^- \rho_S^f(t) S^+ - S^+ S^- \rho_S^f(t) - \rho_S^f(t) S^- S^+) - \Gamma M S^+ \rho_S^f(t) S^- - \Gamma M^* S^- \rho_S^f(t) S^+ - \frac{1}{2} i\Omega_R [S^+ + S^-, \rho_S^f(t)] + \frac{1}{2} i(\beta [S^+, [S^z, \rho_S^f(t)]] - \beta^* [S^-, [S^z, \rho_S^f(t)]]), \tag{21} \]

where
\[ \rho_S^f(t) = U_2(t) \rho_S^{trans}(t) U_2^+(t) = e^{i\omega_f S^z t} \rho_S(t) e^{-i\omega_f S^z t}, \quad \tilde{N} = N(\omega_f + \Omega') + \frac{1}{2}(1 - \tilde{\Delta}^2)\gamma_n, \quad (22) \]

\[ \tilde{M} = (|M(\omega_f + \Omega', \theta)| + i\tilde{\Delta}\delta\mu) \exp(i\theta) - \frac{1}{2}(1 - \tilde{\Delta}^2)(\gamma_n - i\delta_n), \quad (23) \]

\[ \delta = \Delta/\Gamma + \tilde{\Delta}\delta\mu + \frac{1}{2}(1 - \tilde{\Delta}^2)\delta_n, \quad \beta = \Gamma \Omega_{2r} |\delta_n + \delta\mu \exp(i\theta) - i\tilde{\Delta}(\gamma_n - i\delta_n)|, \quad (24) \]

\[ \gamma_n = N(\omega_f') - N(\omega_f + \Omega') - (|M(\omega_f, \theta)| - |M(\omega_f + \Omega', \theta)|) \cos(\theta), \quad (25) \]

\[ \delta_n = (|M(\omega_f, \theta)| - |M(\omega_f + \Omega', \theta)|) \sin(\theta), \quad (26) \]

\[ \delta_N = \frac{1}{2} \pi \int_{-\infty}^{\infty} d\omega \frac{\langle N(\omega) \rangle}{\omega - \omega_s + \Omega'}, \quad \delta_M = \frac{1}{2} \pi \int_{-\infty}^{\infty} d\omega \frac{|M(\omega, \theta)|}{\omega - \omega_s + \Omega'}, \quad (27) \]

Here \( \Gamma \) is the radiative damping constant. The principal values of the integrals (27) can be evaluated by means of the contour integration [14] as

\[ \delta_N = \delta_{\mu} - \delta_{\lambda}, \quad \delta_M = \delta_{\mu} + \delta_{\lambda}, \quad (28) \]

where for the degenerate DPO case

\[ \delta_{\mu}^{dpo} = \Omega' \frac{\lambda^2 - \mu^2}{4\mu(\Omega'^2 + \mu^2)}, \quad \delta_{\lambda}^{dpo} = \Omega' \frac{\lambda^2 - \mu^2}{4\lambda(\Omega'^2 + \lambda^2)}, \quad (29) \]

and for the non-degenerate NDPO case

\[ \delta_{\mu}^{ndpo} = \frac{\lambda^2 - \mu^2}{8\mu} \left[ \frac{(\Omega' + \alpha)}{(\Omega' + \alpha)^2 + \mu^2} + \frac{(\Omega' - \alpha)}{(\Omega' - \alpha)^2 + \mu^2} \right], \quad (30) \]

\[ \delta_{\lambda}^{ndpo} = \frac{\lambda^2 - \mu^2}{8\lambda} \left[ \frac{(\Omega' + \alpha)}{(\Omega' + \alpha)^2 + \lambda^2} + \frac{(\Omega' - \alpha)}{(\Omega' - \alpha)^2 + \lambda^2} \right]. \quad (31) \]

This equation is similar to the master equation obtained earlier in [15] for a two-level non-polar emitter without broken inversion symmetry and is derived assuming that the system-reservoir and the system-field interactions are weak and the reservoir correlation time is small compared with the time \( t \) of observation. So, a closed set of equations follows from Eq.(21):

\[ \frac{d}{dt} \langle \tilde{S}^- (t) \rangle = -\Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + i\frac{1}{2} \delta_n (e^{i\omega_f t} + e^{-i\omega_f t}) \right) \langle \tilde{S}^- (t) \rangle + \Gamma M \langle \tilde{S}^+ (t) \rangle + \Omega_R \langle S^z (t) \rangle, \quad (32) \]

\[ \frac{d}{dt} \langle \tilde{S}^+ (t) \rangle = -\Gamma \left( \frac{1}{2} + \tilde{N} + i\delta - i\frac{1}{2} \delta_n (e^{i\omega_f t} + e^{-i\omega_f t}) \right) \langle \tilde{S}^+ (t) \rangle + \Gamma M^* \langle \tilde{S}^- (t) \rangle + \Omega_R \langle S^z (t) \rangle, \quad (33) \]

\[ \frac{d}{dt} \langle S^z (t) \rangle = -\frac{1}{2} (\Omega_R + \beta^*) \langle S^z (t) \rangle - \frac{1}{2} (\Omega_R + \beta^*) \langle S^+ (t) \rangle - \Gamma (2\tilde{N} + 1) \langle S^z (t) \rangle - \Gamma/2, \quad (34) \]

where \( \langle \tilde{S}^\pm (t) \rangle = \pm i \langle S^\pm (t) e^{i\omega_f t} \rangle \) are slowly varying parts of the atomic operators. The system of equations (32-34) can be solved numerically by means of the technique employed earlier in [19], where the components of the vector \( \vec{X}(t) = (\langle \tilde{S}^- (t) \rangle, \langle \tilde{S}^+ (t) \rangle, \langle S^z (t) \rangle) \) are decomposed as

\[ X_i(t) = \sum_{l=-\infty}^{+\infty} X_i^{(l)}(t) e^{i\omega_f t}, \quad i = 1, 2, 3, \] and the slowly varying amplitudes \( X_i^{(l)}(t) \) obey the system of equations

\[ \frac{d}{dt} X_1^{(l)}(t) = -\Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + i\frac{\omega_f}{\Gamma} \right) X_1^{(l)}(t) - \frac{i}{2} \delta_n (X_1^{(l-1)}(t) + X_1^{(l+1)}(t)) + \Gamma M X_2^{(l)}(t) + \Omega_R X_3^{(l)}(t), \quad (35) \]

\[ \frac{d}{dt} X_2^{(l)}(t) = -\Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + i\frac{\omega_f}{\Gamma} \right) X_2^{(l)}(t) + i \frac{\delta_n}{2} (X_2^{(l-1)}(t) + X_2^{(l+1)}(t)) + \Gamma M^* X_1^{(l)}(t) + \Omega_R X_3^{(l)}(t), \quad (36) \]

\[ \frac{d}{dt} X_3^{(l)}(t) = -\Gamma \frac{\delta_n}{2} - (\Gamma (2\tilde{N} + 1) + i\omega_f) X_3^{(l)}(t) - \frac{\Omega_R + \beta^*}{2} X_1^{(l)}(t) - \frac{\Omega_R + \beta}{2} X_2^{(l)}(t). \quad (37) \]
Fluorescence spectrum

The incoherent part of the steady-state fluorescence spectrum is given by [16, 17]

\[ F_{\text{inc}}(\omega) = \frac{\Gamma}{\pi} \Re \lim_{t \to \infty} \int_0^\infty d\tau \lim_{\tau \to -\infty} [(\tilde{S}^+(t)\tilde{S}^-(-t + \tau)) - (\tilde{S}^+(t)\tilde{S}^-(-t + \tau))] e^{i(\omega - \omega_f)\tau}, \]  

(38)

where the coherent contribution from the incident driving field scattered by the atom is subtracted, as usual. In accordance with the so-called quantum regression hypothesis [13, 18], the fluctuation correlation functions \( Y_1(t, t + \tau) = \langle \tilde{S}^+(t)\tilde{S}^-(-t + \tau) \rangle - \langle \tilde{S}^+(t)\tilde{S}^-(-t + \tau) \rangle \), \( Y_2(t, t + \tau) = \langle \tilde{S}^+(t)\tilde{S}^-(-t + \tau) \rangle - \langle \tilde{S}^+(t)\tilde{S}^-(-t + \tau) \rangle \), satisfy virtually the same set of equations of motion (32-34) for the correspondent averages \( \langle \tilde{S}^-(-\tau) \rangle \), \( \langle \tilde{S}^+(\tau) \rangle \) and \( \langle \tilde{S}^+(\tau) \rangle \) with the only difference that the inhomogeneity \(-\Gamma/2\) disappears due to the subtraction of the mean. These correlation functions can be decomposed as

\[
Y_i(t, t + \tau) = \sum_{l=-\infty}^{+\infty} Y_i^{(l)}(t, t + \tau) e^{i\omega_f(t + \tau)} \quad , \quad i = 1, 2, 3, \]  

so that

\[
\frac{d}{dt} Y_1^{(l)}(t, t + \tau) = -\Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + il\omega_f \right) Y_1^{(l)}(t, t + \tau) - \frac{i\delta_a}{2} \left( Y_1^{(-1)}(t, t + \tau) + Y_1^{(l+1)}(t, t + \tau) \right) + \Gamma \tilde{Y}_1^{(l)}(t, t + \tau) + \Omega R Y_3^{(l)}(t, t + \tau), \]  

(39)

\[
\frac{d}{dt} Y_2^{(l)}(t, t + \tau) = -\Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + il\omega_f \right) Y_2^{(l)}(t, t + \tau) + \frac{i\delta_a}{2} \left( Y_2^{(-1)}(t, t + \tau) + Y_2^{(l+1)}(t, t + \tau) \right) + \Gamma \tilde{Y}_2^{(l)}(t, t + \tau) + \Omega R Y_3^{(l)}(t, t + \tau), \]  

(40)

and the Laplace transforms \( Y_i^{(l)}(t, z) = \int_0^\infty e^{-zt} Y_i^{(l)}(t, \tau) d\tau \) of the components \( Y_i^{(l)}(t, \tau) \) will satisfy the following set of equations:

\[
\frac{d}{dz} Y_1^{(l)}(t, z) + \Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + il\omega_f \right) Y_1^{(l)}(t, z) + \frac{i\delta_a}{2} \left( Y_1^{(-1)}(t, z) + Y_1^{(l+1)}(t, z) \right) = \frac{1}{2} \delta_{l,0} + X_3^{(-1)}(z), \]  

\[
\frac{d}{dz} Y_2^{(l)}(t, z) + \Gamma \left( \frac{1}{2} + \tilde{N} - i\delta + il\omega_f \right) Y_2^{(l)}(t, z) - \frac{i\delta_a}{2} \left( Y_2^{(-1)}(t, z) + Y_2^{(l+1)}(t, z) \right) = \frac{1}{2} \delta_{l,0} + X_3^{(-1)}(z), \]  

\[
\frac{d}{dz} Y_3^{(l)}(t, z) + \frac{\omega R + \beta^*}{2} Y_1^{(l)}(t, z) + \frac{\omega R + \beta^*}{2} Y_2^{(l)}(t, z) = \frac{1}{2} \delta_{l,0} + X_3^{(-1)}(z), \]  

(42)

(43)

(44)

In the steady state limit \( (t \to \infty) \) only the zero-order component \( Y_1^{(0)}(t, z) \) contributes to \( F_{\text{inc}}(\omega) \), and the incoherent part of the spectrum (38) reads as

\[
F_{\text{inc}}(\omega) = \lim_{\tau \to \infty} Y_1^{(0)}(t, z) \bigg|_{z = -i(\omega - \omega_f)}. \]  

(45)
4. Numerical Results

Equations (35)-(37) and (42)-(44) were solved numerically, as usual [19], in the steady state limit \((t \to \infty)\) by truncation of the number of the harmonic amplitudes \(X_i^{(l)}(t)\) and \(Y_i^{(l)}(t, z)\) involved, and the case of the driving laser field frequency \(\omega_f\) and the carrier frequency of the squeezed field \(\omega_s\) being simultaneously in resonance with the atomic transition frequency \(\omega_0\) was studied. It was already found [3] in the case of a two-level system with broken symmetry interacting with non-squeezed vacuum reservoir that for \(\delta_a \neq 0\) a low-frequency radiation peak centered nearly exactly at the frequency \(\omega = \Omega_R\) appears in the fluorescence spectrum. The amplitude of the peak increases steadily with the increase of the symmetry violation parameter \(\delta_a\) and decreases with the increase of the driving field frequency \(\omega_f\). The central frequency of the peak is defined for the most part by the Rabi frequency \(\Omega_R\) and depends weakly on the symmetry violation parameter \(\delta_a\), so that it drifts very slowly toward \(\omega = 0\) along with the increase of this parameter. Here it was found that the interaction with the finite bandwidth squeezed vacuum reservoir does not change these aspects of the spectral peak behavior. At the same time, the amplitude of this peak strongly depends on the NDPO non-degeneracy parameter \(\alpha\). It is maximal in the DPO case \(\alpha = 0\) and decreases with the increase of the non-degeneracy while the width of the peak increases, see Fig.1. At the same time, the position of the peak is not significantly affected and, consequently, the NDPO degeneracy can be employed to control the fluorescence intensity output of the polar emitter at the fixed frequency \(\Omega_R\). The same goal of the fluorescence intensity output control can be achieved for fixed non-degeneracy parameter \(\alpha\) by varying the phase of squeezing \(\theta\) instead, see Fig.2. It was also found that the fluorescence peak is most pronounced for the case of the OPO at the threshold when \(2\epsilon/\gamma_c = 1\). Below the threshold the amplitude of the peak decreases while the position of the peak is not shifted, see Fig.3 and Fig.4. This property provides additional means for the control of the fluorescence intensity output.

\[\text{Figure 1.} \quad \text{Fluorescence spectrum at } \omega = \Omega_R \text{ for various values of } \alpha \text{ at the OPO threshold.} \]
\[
\Gamma = 1, \alpha = 0 : 10 : 50, \gamma_c = 10, \epsilon = 5, \theta = 0, \omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10.
\]

\[\text{Figure 2.} \quad \text{Fluorescence spectrum at } \omega = \Omega_R \text{ for various values of } \theta \text{ at the OPO threshold.} \]
\[
\Gamma = 1, \alpha = 0, \gamma_c = 10, \epsilon = 5, \theta = 0 : \pi/6 : \pi, \omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10.
\]
Figure 3. Fluorescence spectrum at \( \omega = \Omega_R \) for various values of \( \alpha \) below the OPO threshold. \( \Gamma = 1, \alpha = 0 : 10 : 50, \gamma_c = 10, \epsilon = 2, \theta = 0, \omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10. \)

Figure 4. Fluorescence spectrum at \( \omega = \Omega_R \) for various values of \( \theta \) below the OPO threshold. \( \Gamma = 1, \alpha = 0, \gamma_c = 10, \epsilon = 2, \theta = 0 : \pi / 6 : \pi, \omega_f = \omega_s = \omega_0 = 5000, \Omega_R = 100, \delta_a = 10. \)

5. Conclusion
In conclusion, we investigated the effect of the vacuum dissipative reservoir finite bandwidth squeezing on the phenomenon of the low-frequency fluorescence by a damped quantum two-level polar system with broken inversion symmetry driven by external high-frequency classical EM (laser) field. The source of the squeezed vacuum was represented by the OPO being in either the NDPO or the DPO mode of operation. It was shown that the parameters of the OPO, such as the non-degeneracy degree and the phase of squeezing, provide efficient means for the control of the intensity and spectral width of the fluorescence radiation output.

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