Steady-state squeezing transfer in hybrid optomechanics

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A hybrid scheme is presented that allows the transfer of squeezed states (TSS) from the mechanical part to an optical cavity in the steady-state. In a standard optomechanical scheme, a three-level atom acts as an intermediate element for TSS. Two different procedures are developed that allow the visualization of the TSS effect: In the first one, we apply a coherent pump of squeezed phonons in our hybrid system, and the second method is achieved by placing the system in contact with a phonon squeezed bath. Our model and procedures show that in optomechanical systems TSS can be achieved with a high fidelity.

With the remarkable state-of-the-art of the hybrid systems composed by mechanical oscillators (MO), cavities, spins, etc., it is becoming more and more feasible to control such systems in their quantum regime in search for non-classical features of their elements. Besides MO have the ability to easily interact with a wide range of physical systems, such as ultracold atomic BECs, superconducting qubits, spin states in quantum dots and color/NV centers, cavity fields in the opto-mechanical systems, etc.

The hybrid setups are of paramount importance regarding the development of the quantum technologies. Therefore, to move forward the limits of the fields such as quantum computing, quantum networks, quantum metrology and sensing, a very demanded tool should be the high-fidelity transfer of the quantum states between the components of a hybrid system. For instance, in the realm of quantum networks, the mechanical object can serve as a light-matter transducer or to map/encode information from a qubit. Particularly, spin-mechanical systems have attained major attention, mainly because spin systems exhibit long-coherence times and they can be easily manipulated and read-out. A rapidly emerging field is the quantum metrology and sensing by using the quantum states and protocols with the aim to overcome the precisions unachievable by the classical sensing. There are several experiments proving the sensing near and beyond the standard quantum limit, e.g. some theoretical work over the last two decades. To advance in this direction, in the present work we propose an optomechanical protocol of high-fidelity squeezing transfer from the mechanical mode to the cavity field. Our results show that such a transfer could be realized by a coherent mechanical squeezed pump, as well as under the coupling with a squeezed phononic bath.

As illustrated in Fig. 1(a), we consider a hybrid system where a confined photon mode is coupled both to two upper levels of the three-level atom and to a mechanical oscillator as \( \hbar = 1 \)

\[
\mathcal{H}_0 = \sum_{i=0}^{2} \omega_i \sigma_{ii} + \omega_c a^\dagger a + \omega_m b^\dagger b + i g_{ac} (a a_{21}^\dagger - a^\dagger a_{21}) - i g_{cm} a^\dagger a \left( b^\dagger - b \right), \tag{1}
\]

where \( \sigma_{ij}^\pm \) are the ladder operators for the atom, \( a(b) \) is the annihilation operator of the cavity (mechanical) mode of frequency \( \omega_c(\omega_m) \) and \( \omega_i \) are the energy levels...
Here we used the Hermitian operator \( F \) coupling takes very small values. In experiments in optomechanics \[1\], it is expected that magnitude of the coherent squeezing pump. Considering the frequency \( \Delta = \omega \) the rotating wave approximation under the blue-detuned regime \( \Delta = \omega_m \), and keeping only the time independent terms, the above Hamiltonian becomes

\[
\hat{H}_2 = iJ \sigma_0^+ ab^\dagger + iqb^\dagger b - 2iK a^\dagger a b^\dagger + H.c.,
\]

where \( J = g_{ac} \cdot g_{cm}/w_m \) is the tripartite atom-photon-phonon interaction strength.

In order to produce squeezed light in the optical cavity, two lasers \( E_1 \) and \( E_2 \) (proportional to the field strengths) are introduced in the system, which are resonant with the transitions of the levels \( |2\rangle \leftrightarrow |0\rangle \) and \( |1\rangle \leftrightarrow |0\rangle \), respectively. These coherent drives are described by the Hamiltonian in the interaction picture \( \hat{H}_E = iE_1 (\sigma_{20}^+ - \sigma_{02}^-) + iE_2 (\sigma_{10}^- - \sigma_{10}^+) \). If we now include the dissipation caused by the system-environment coupling, the dissipative dynamics of the hybrid quantum system is described by the Markovian master equation

\[
\frac{d\rho}{dt} = -i[\hat{H}_2 + \hat{H}_E, \rho] + \frac{\gamma_{21}}{2} \mathcal{L}[\sigma_{21}]\rho + \frac{\gamma_{10}}{2} \mathcal{L}[\sigma_{10}]\rho + \frac{\kappa_a}{2} \mathcal{L}[a]\rho + \frac{\kappa_b}{2} \mathcal{L}[b]\rho,
\]

where \( \rho \) is the density matrix of the hybrid system and the common Lindblad dissipative terms are defined by: \( \mathcal{L}[O] = 2\gamma O \mathcal{O}^\dagger - \mathcal{O} \mathcal{O}^\dagger - \rho \mathcal{O} \mathcal{O}^\dagger - \mathcal{O}^\dagger \mathcal{O} \rho \), with all the baths at \( n_{th} = 0 \). Here \( \gamma_{21} (\gamma_{10}) \) correspond to spontaneous emission rates from level 2 to 1 (level 1 to 0), respectively and \( \kappa_a (\kappa_b) \) are the decay rates of the optical (mechanical) mode. The approach of \( n_{th} = 0 \) is realistic in recent experiments, for example for a hybrid system with atoms, cavity and mechanics in the regime of microwave frequencies, as in recent experiments \[8 12\] the mechanical mode has \( \omega_m \approx 30 \text{ GHz} \), and by cooling the system to the temperatures of \( \sim 30 \text{ mK} \), one gets \( n_{th} \equiv [\exp(h\omega_m/k_B T)]^{-1} \approx 10^{-4} \).

After building the theoretical model, we discuss the degree of squeezing present in the states of the cavity and the mechanical part. For this, we rely on numerical methods according to \[10\] to solve Eq. \[4\] in the steady-state, i.e. \( \dot{\rho} = 0 \), and so calculating the quadrature fluctuations. Here, we study the parameter space in order to minimize fluctuations and achieve an optimal result for TSS in the stable region.

For a better understanding the TSS effect and stability of the hybrid system, we get a set of first order differential equations from Eq. \[4\]

\[
\frac{d\langle a^\dagger a \rangle}{dt} = -2J\langle a^\dagger b \rangle - \kappa_a \langle a^\dagger a \rangle,
\]

\[
\frac{d\langle b^\dagger b \rangle}{dt} = 2J\langle a^\dagger b \rangle + 4q\langle b^2 \rangle - \kappa_b \langle b^\dagger b \rangle,
\]

\[
\frac{d\langle a^2 \rangle}{dt} = -2J\langle a b \rangle - \kappa_a \langle a^2 \rangle,
\]

\[
\frac{d\langle b^2 \rangle}{dt} = 2J\langle a b \rangle + 4q\langle b^2 \rangle - \kappa_b \langle b^2 \rangle + 2q.
\]
The stability of the system is obtained by solving the system of equations of the form \( \frac{d\hat{\mathbf{x}}}{dt} = M\hat{\mathbf{x}} + \mathbf{c} \), where we have defined a vector \( \hat{\mathbf{x}} \) with the moments shown in the left-side of Eqs. \( \text{(5)}\). \( M \) is a matrix built with the elements on the right-side of Eqs. \( \text{(5)}\) and \( \mathbf{c} \) is a constant vector arising from the constant term \( 2q \) in the equation for \( \langle b^2 \rangle \). The steady-state is only achieved if the real part of the eigenvalues of the matrix are all negative.

\[
\begin{align*}
\frac{d\langle a\dagger b \rangle}{dt} &= -J \left( \langle b\dagger b \rangle - \langle a\dagger a \rangle \right) + 2q \langle ab \rangle \\
&\quad - \frac{\kappa_a}{2} \langle a\dagger a \rangle - \frac{\kappa_b}{2} \langle b\dagger b \rangle, \quad (9) \\
\frac{d\langle ab \rangle}{dt} &= -J \left( \langle b^2 \rangle \langle \sigma_{21}^- \rangle - \langle a^2 \rangle \langle \sigma_{21}^+ \rangle \right) + 2q \langle a\dagger b \rangle \\
&\quad - \frac{\kappa_a}{2} \langle ab \rangle - \frac{\kappa_b}{2} \langle ab \rangle, \quad (10)
\end{align*}
\]

where we have considered the factorization of the form \( \langle a\dagger b \sigma_{21}^- \rangle = \langle a\dagger b \rangle \langle \sigma_{21}^- \rangle \) (adiabatic approximation) and since \( \omega_m \gg \{g_{cm},q\} \) we have considered \( K = 0 \).

By considering \( \{E_1, E_2, \gamma_{10}\} \gg \{J, q, \kappa_a, \kappa_b\} \) and \( \gamma_{21} = 0 \), we get the following semiclassical results for the quadrature fluctuations in steady-state:

\[
\begin{align*}
\langle \Delta y_a \rangle^2 &= \frac{m + n + s + 1}{4(m+1)(n+1)}, \\
\langle \Delta y_b \rangle^2 &= \frac{n + s}{4(m+1)(n+1)},
\end{align*}
\]

where

\[
\begin{align*}
m &= \frac{4q}{\kappa_a + \kappa_b}, \\
n &= \frac{\kappa_a \kappa_b + 4J^2 \langle \sigma_{21}^+ \rangle^2}{4q \kappa_a}, \\
s &= \frac{\kappa_b}{\kappa_a + \kappa_b}, \\
\langle \sigma_{21}^+ \rangle &= \frac{2 \sqrt{E_1 E_2}}{\gamma_{10} + 4 (E_1^2 + E_2^2)}.
\end{align*}
\]

The stability of the system is obtained by solving the system of equations of the form \( \frac{d\hat{\mathbf{x}}}{dt} = M\hat{\mathbf{x}} + \mathbf{c} \), where we have defined a vector \( \hat{\mathbf{x}} \) with the moments shown in the left-side of Eqs. \( \text{(5)}\). \( M \) is a matrix built with the elements on the right-side of Eqs. \( \text{(5)}\) and \( \mathbf{c} \) is a constant vector arising from the constant term \( 2q \) in the equation for \( \langle b^2 \rangle \). The steady-state is only achieved if the real part of the eigenvalues of the matrix are all negative.
losses (for $n_{th}^{(at,cav)} = 0$), as well as a phonon squeezed vacuum reservoir:
\[
\frac{dp}{dt} = -i[H_3 + \hat{H}_E, \hat{\rho}] + \frac{\gamma_{21}}{2} \mathcal{L}[\sigma_{21}]\rho + \frac{\gamma_{10}}{2} \mathcal{L}[\sigma_{10}]\rho + \frac{\kappa_a}{2} \mathcal{L}[\sigma_a] \rho + \frac{\kappa_b}{2} \mathcal{L}[\sigma_b] \rho,
\]
with the Lindbladian corresponding to the incoherent pump of squeezed phonons as $\mathcal{L}_{sq}[b] = (N_{sq} + 1)(2b\rho b - b\rho b - b b\rho b) + N_{sq}(2b\rho b - bb\rho b - b b\rho b) + M_s (2b\rho b - b\rho b - b b\rho b) + M_s^* (2b\rho b - bb\rho b - b b\rho b)$; here $N_{sq} = \sinh^2 r$ and $M_s = -\exp(\imath \theta) \sinh r \cos r$, obey the relation $\sqrt{N_{sq}(N_{sq} + 1)} = |M_s|$.

Using the above scheme, we explore the effects of the squeezing parameters ($r, \theta$) on the quantum fluctuations and fidelity, in order to achieve the best TSS. In the semiclassical approximation we get following results for the quadrature fluctuations in steady-state:
\[
(\Delta x_a)^2 = \frac{1}{4} p + l,
\]
\[
(\Delta x_b)^2 = \frac{1}{4} p \left[2(N_{sq} - M_{sq}) + 1\right] + l,
\]
where
\[
p = \frac{\kappa_a \kappa_b}{4 J^2 \langle \sigma_{21}^2 \rangle^2 + \kappa_a \kappa_b},
\]
\[
l = \frac{J^2 \langle \sigma_{21}^2 \rangle^2 \left[2(N_{sq} - M_{sq}) + \gamma_{21}\right] + \kappa_a}{\kappa_a + \kappa_b \left[2J^2 \langle \sigma_{21}^2 \rangle^2 + \kappa_a \kappa_b\right]}.
\]

Now, we study the parameters in our hybrid system for which both quadratures reach their maximum squeezing and that allow the optimal TSS via optomechanical coupling. In panel (a) of the Fig. 4 they are presented as functions of the parameter $r$. The results show that for $g_{cm} = 0.01$, the TSS performed in an optimal way, since the curves are closer, that is, both fluctuations are similar. In panel (b) of the Fig. 4 we show the time evolution of the quadrature fluctuations. Without loss of generality, we have fixed the parameters $r = 0.3$ and $g_{cm} = 0.01$. Analogously to the aforementioned process of the coherent pump, we can see how the injection of a squeezed bath leads to cavity and mechanical squeezing in steady state.

In [33] the authors propose a setup that allows the transmission from an atomic squeezed state to a membrane with a high fidelity in presence of relevant decoherence rates. Motivated for this, we simulate the influence of the Jaynes-Cummings coupling on the fidelity of the TSS in our system. In panel (a) of Fig. 5 we show the fidelity as a function of $g_{cm}$ for some couplings $g_{ac}$. Notice that the fidelity is close to unity as the optomechanical and Jaynes-Cummings couplings increase, even in the presence of dissipation in the system. In panel (b) of Fig. 5 we show the fidelity as a function of $g_{ac}$ for some values of $g_{cm}$. This result allows us to conclude that in the strong coupling regime the open system allows the TSS with a reliability close to 100%.

In conclusion, we have proposed a hybrid system consisting of a three-level atom, an optical cavity, and mechanical resonator that allows the steady-state squeezing conversion. We demonstrated that by pumping coherently and incoherently phonon squeezing in our hybrid system and also adding two coherent atomic drives, we can transfer steady-state squeezing to the cavity photons with an extremely high fidelity, close the unity (see Figs. 3 and 4). Furthermore, in the parameter range used in this work, as we increase the optomechanical coupling, we get better results in producing closer phonon and photon squeezed states, i.e. improving the protocol of TSS. The parameters used in this work are compatible with recent experiments for the optomechanical hybrid setups, e.g. [1, 3, 12].

![Fig. 4](image-url) (a) Quadrature fluctuations as functions of the parameter $r$. Here $(\Delta x_a)^2$ (red and blue curves) and $(\Delta x_b)^2$ (green and magenta curves). (b) Time evolution of the fluctuations evidencing the steady-state. The parameters are: $g_{ac} = 10^2$, $\kappa_a = \kappa_b = 0.2$, $\gamma_{10} = 20$, $\gamma_{21} = 0$, $E_1 = E_2 = 25$, $\theta = \pi$.

![Fig. 5](image-url) (a) Fidelity between the cavity field and mechanical state as a function of the couplings $g_{cm}$ and $g_{ac}$. (b) Fidelity as a function of $g_{ac}$ for some couplings $g_{cm}$. As is observed, the fidelity is close to unity as the optomechanical and Jaynes-Cummings couplings increase. The parameters here are the same as in Fig. 4 with $r = 0.3$. 
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