Likelihood analysis of the Local Group acceleration revisited

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ABSTRACT

We re-examine likelihood analyses of the Local Group (LG) acceleration, paying particular attention to non-linear effects. Under the approximation that the joint distribution of the LG acceleration and velocity is Gaussian, two quantities describing non-linear effects enter these analyses. The first one is the coherence function, i.e. the cross-correlation coefficient of the Fourier modes of gravity and velocity fields. The second one is the ratio of velocity power spectrum to gravity power spectrum. To date, in all analyses of the LG acceleration, the second quantity has not been accounted for. Extending our previous work, we study both the coherence function and the ratio of the power spectra. With the aid of numerical simulations we obtain expressions for the two as functions of wavevector and $\sigma_b$. Adopting WMAP's best determination of $\sigma_b$, we estimate the most likely value of the parameter $\beta$ and its errors. As the observed values of the LG velocity and gravity, we adopt respectively an estimate of the LG velocity based on the cosmic microwave background, and Schmoldt et al.’s estimate of the LG acceleration from the PSCz catalogue. We obtain $\beta = 0.66^{-0.21}_{+0.07}$, thus our error bars are significantly smaller than those of Schmoldt et al. This is not surprising, because the coherence function they used greatly overestimates actual decoherence between non-linear gravity and velocity.

Key words: methods: analytical – methods: numerical – cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Analyses of large-scale structure of the Universe provide estimates of cosmological parameters that are complementary to those from the cosmic microwave background (CMB) measurements. In particular, comparing the large-scale distribution of galaxies to their peculiar velocities enables one to constrain the quantity $\beta \equiv \Omega_m^{0.6}/b$. Here, $\Omega_m$ is the cosmological matter density parameter and $b$ is the linear bias of galaxies that are used to trace the underlying mass distribution. This is so because the peculiar velocity field, $v$, is induced gravitationally and therefore is tightly coupled to the matter distribution. In the linear regime, this relationship takes the form

$$v = \Omega_m^{0.6} \frac{d^3r}{4\pi} \delta(r) r^3,$$

where $\delta$ denotes the mass density contrast and distances have been expressed in km s\textsuperscript{-1}. Under the assumption of linear bias, $\delta = b^{-1}\delta_g$, where $\delta_g$ denotes the density contrast of galaxies, and the amplitude of peculiar velocities depends linearly on $\beta$.

These comparisons are done by extracting the density field from full-sky redshift surveys (such as the PSCz; Saunders et al. 2000), and comparing it to the observed velocity field from peculiar velocity surveys. The methods for doing this fall into two broad categories. One can use equation (1) to calculate the predicted velocity field from a redshift survey, and compare the result with the measured peculiar velocity field; this is referred to as a velocity–velocity comparison. Alternatively, one can use the differential form of this equation, and calculate the divergence of the observed velocity field to compare directly with the density field from a redshift survey; this is called a density–density comparison.

Peculiar velocities of galaxies and groups of galaxies are generally determined by measuring their distances independently of redshifts. However, the motion of the Local Group (LG) of galaxies can be deduced in another way, namely from the observed dipole anisotropy of the CMB temperature. This dipole reflects, via the Doppler shift, the motion of the Earth with respect to the CMB rest frame. The components of this motion of local, non-cosmological origin (the Earth’s motion around the Sun, the Sun’s motion in the Milky Way, and the motion of the Milky Way in the LG) are known and can be subtracted (e.g. Courteau & van den Bergh 1999). When transformed to the barycentre of the LG, the motion is towards $(l, b) = (276^\circ \pm 3^\circ, 30^\circ \pm 2^\circ)$, and of amplitude $v_{\text{LG}} = 627 \pm 22$ km s\textsuperscript{-1}, as inferred from the four-year COBE data (Lineweaver et al. 1996). This can be compared to that predicted from a redshift survey, and historically this was the first velocity–velocity comparison.

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[with angular data only (e.g. Meiksin & Davis 1986; Yahil, Walker & Rowan-Robinson 1986); with redshift information (Strauss & Davis 1988; Lynden-Bell, Lahav & Burstein 1989)]. This velocity estimate is much more accurate than estimates of peculiar velocities based on redshift-independent distances. Therefore, the analysis of the LG motion remains an interesting alternative to current velocity–velocity comparisons, performed simultaneously for many galaxies, but with less accurately measured velocities.

Let us define the scaled gravity, $g$, by the equation:

$$ g = \int \frac{d^3r}{4\pi} \delta(r) \frac{r}{r^3}, $$

where again distances have been expressed in km s$^{-1}$. The scaled gravity is proportional to the gravitational acceleration, and can be measured from a redshift survey. Equation (1) yields

$$ v = \Omega_0 g, $$

so a velocity–velocity comparison is in fact a velocity–gravity one. Hereafter, we will refer to ‘scaled gravity’ simply by ‘gravity’. It is also often called ‘the clustering dipole’.

A number of effects (non-linear effects, observational windows through which the velocity and gravity of the LG are observed, shot noise) spoil the linear relationship (3). Therefore, to estimate $\beta$ one cannot simply equate the two dipoles – rather, a more sophisticated approach is needed. Commonly adopted is a maximum-likelihood approach, which provides a way to account for these effects and to compute errors of estimated cosmological parameters. Here we re-examine a likelihood analysis of Schmoldt et al. (1999), paying particular attention to proper modelling of non-linear effects (NE), which affect the estimated values of parameters and their errors.

In a previous work (Chodorowski & Ciecielag 2002, hereafter C02) we concentrated on one quantity describing NE in the LG velocity–gravity comparison, the coherence function (CF). This is the cross-correlation coefficient of the Fourier modes of the gravity and velocity fields. We showed that the form of the function adopted by Schmoldt et al. (1999) drastically overestimates actual decoherence between non-linear gravity and velocity. This implies that the true random error of $\beta$ is significantly smaller. In the present study we give a complete description of NE present in the analysis. In particular, we show that not only the CF is relevant, but also the ratio of the power spectrum of velocity to the power spectrum of gravity. With the aid of numerical simulations we model the two quantities as functions of the wavevector and of cosmological parameters. We then combine these results with observational estimates of $\nu_{LG}$ and $g_{LG}$, and obtain the ‘best’ value of $\beta$ and its errors.

The paper is organized as follows. In Section 2 we outline an analytical description of the likelihood for cosmological parameters, based on the measured values of the LG velocity and acceleration. In Section 3 we describe our numerical simulations, and we model numerically the coherence function and the ratio of the power spectra. An estimate of the most likely value of $\beta$ and its errors is presented in Section 4. Summary and conclusions are in Section 5.

## 2 Analytical Description of the Likelihood

Let $f(g, v)$ denote the joint distribution function for the LG gravity and velocity. It is commonly approximated by a multivariate Gaussian (Strauss et al. 1992, hereafter S92; Schmoldt et al. 1999, hereafter S99). This approximation has support from numerical simulations (Kofman et al. 1994; Ciecielag et al. 2003), where the measured non-Gaussianity of $g$ and $v$ is small. This is rather natural to expect since, for example, gravity is an integral of density over effectively a large volume, so the central limit theorem can at least partly be applicable (but see Catelan & Moscardini 1994).

In a Bayesian approach, one ascribes a priori equal probabilities to values of unknown parameters, which allows one to express their likelihood function via $f$:

$$ L(\text{param.}) = f(v, g | \text{param.}) $$

As the parameters to be estimated we adopt $\beta$ and $b$. Using statistical isotropy of $g$ and $v$, the distribution function can be cast into the form (Juszkiewicz, Vittorio & Wyse 1990; Lahav, Kaiser & Hoffman 1990):

$$ f(g, v) = \frac{1}{(2\pi)^{3/2} \sigma_v \sigma_g} \exp \left[ -\frac{x^2 + y^2 - 2x\mu_{xy}}{2(1 - r^2)} \right], $$

where $\sigma_v$ and $\sigma_g$ are the rms values of a single Cartesian component of gravity and velocity, respectively. From isotropy, $\sigma_g^2 = \langle g \cdot g \rangle / 3$ and $\sigma_v^2 = \langle v \cdot v \rangle / 3$, where $\langle \cdot \rangle$ denotes ensemble averaging. Next, $(x, y) = (g \cdot \sigma_g, v / \sigma_v)$ and $\mu = \cos \psi$ being the misalignment angle between $g$ and $v$. Finally, $r$ is the cross-correlation coefficient of $g_{0n}$ with $v_{0n}$, where $g_{0n}$ ($v_{0n}$) denotes an arbitrary Cartesian component of $g$ ($v$). From isotropy,

$$ r = \frac{\langle g \cdot v \rangle}{(g^2)^{1/2} (v^2)^{1/2}}. $$

Also from isotropy,

$$ \langle x_n x_m \rangle = \delta_{nm}, $$

where $\delta_{nm}$ denotes the Kronecker delta. In other words, there are no cross-correlations between different spatial components.

We have to take into account that the LG gravity is measured through the window $W_g$:

$$ g = \int \frac{d^3r}{4\pi} \delta(r) W_g(r) \frac{r}{r^3}. $$

If $v$ is irrotational like $g$, then similarly

$$ v = \int \frac{d^3r}{4\pi} \theta(r) W_v(r) \frac{r}{r^3}, $$

where $\theta$ is the (minus) velocity divergence, $\theta = - \nabla \cdot v$, and $W_v$ is the velocity window. As a consequence of Kelvin’s circulation theorem, the cosmic velocity field is vorticity-free as long as there is no shell crossing. $N$-body simulations (Bertschinger & Dekel 1989; Mancini et al. 1994; Pichon & Bernardeau 1999) show that the vorticity of velocity is small in comparison to its divergence even in the fully non-linear regime.

The best current estimate of the LG gravity is inferred from the PSCz catalogue of IRAS galaxies (Saunders et al. 2000). S99 follow S92 and measure the LG gravity through the standard IRAS window.

$$ W_g = \begin{cases} (r/r_s)^3, & r < r_s, \\ 1, & r_s < r < R_{\text{max}}, \\ 0, & R_{\text{max}} < r. \end{cases} $$

The window is characterized by a small-scale smoothing and a sharp large-scale cut-off. Following S99, we adopt the values $r_s = 500$ km s$^{-1}$ and $R_{\text{max}} = 15000$ km s$^{-1}$, appropriate for the PSCz catalogue. The window function relevant to the LG velocity is

$$ W_v = \begin{cases} 0, & r < r_{\text{min}}, \\ 1, & \text{otherwise}, \end{cases} $$

which has a small-scale cut-off, $r_{\text{min}} = 100$ km s$^{-1}$, to reflect the finite size of the LG (S92). Using numerical simulations, we have
checked that the velocity field remains approximately Gaussian even for such a small smoothing scale.

In Fourier space, relations (8) and (9) read:

$$g_k = \frac{ik}{k^2} \delta_k \hat{W}_k(k),$$  \hspace{1cm} (12)

$$v_k = \frac{ik}{k^2} \delta_k \hat{W}_k(k),$$  \hspace{1cm} (13)

where the subscript $k$ denotes the Fourier transform. The quantity $\hat{W}$ is not a Fourier transform of $W$, but is related to it in the following way (S92):

$$\hat{W}(k) = k \int_{0}^{\infty} W(r) j_1(kr) dr.$$  \hspace{1cm} (14)

Here and below $j_i$ represents the spherical Bessel function of order $i$. In particular,

$$\hat{W}_k(k) = \frac{3 j_1(kr_c)}{kr_c} - j_0(k R_{\text{max}})$$  \hspace{1cm} (15)

and

$$\hat{W}_e(k) = j_0(k r_{\text{max}}).$$  \hspace{1cm} (16)

From equations (12) and (13) we have

$$\langle g \cdot g \rangle = \frac{1}{2\pi^2} \int_{0}^{\infty} \hat{W}^2(k) P(k) dk$$  \hspace{1cm} (17)

and

$$\langle v \cdot v \rangle = \frac{1}{2\pi^2} \int_{0}^{\infty} \hat{W}^2(k) \mathcal{R}(k) P(k) dk.$$  \hspace{1cm} (18)

Here, $P(k)$ and $P_\theta(k)$ are respectively the power spectrum of the density and the power spectrum of the velocity divergence. Thus,

$$\langle v \cdot v \rangle = \frac{1}{2\pi^2} \int_{0}^{\infty} \hat{W}^2(k) \mathcal{R}(k) P(k) dk,$$  \hspace{1cm} (19)

where

$$\mathcal{R}(k) \equiv \frac{P_\theta(k)}{P_v(k)}.$$  \hspace{1cm} (20)

and $P_v$ and $P_\theta$ are the power spectra respectively of velocity and gravity. Furthermore,

$$\langle g \cdot v \rangle = \frac{1}{2\pi^2} \int_{0}^{\infty} \hat{W}_e(k) \hat{W}_e(k) C(k) P_\theta^{1/2}(k) P_v^{1/2}(k) dk,$$  \hspace{1cm} (21)

where $C(k)$ is the so-called coherence function\(^1\) (S92), or the correlation coefficient of the Fourier components of the gravity and velocity fields:

$$C(k) \equiv \frac{\langle g_k \cdot v_k \rangle}{\langle |g_k|^2 \rangle^{1/2} \langle |v_k|^2 \rangle^{1/2}} = \frac{\langle |\delta_k|^2 \rangle^{1/2} \langle |\delta_k|^2 \rangle^{1/2}}{\langle |\delta_k|^2 \rangle^{1/2} \langle |\delta_k|^2 \rangle^{1/2}}.$$  \hspace{1cm} (22)

Hence, we obtain

$$r = \left[ \int_{0}^{\infty} \hat{W}_e^2(k) P_\theta(k) dk \right]^{1/2} \left[ \int_{0}^{\infty} \hat{W}_e^2(k) \mathcal{R}(k) P_v(k) dk \right]^{1/2}.$$  \hspace{1cm} (23)

Equations (17), (19) and (23) specify all the parameters (the variances and the correlation coefficient) that determine distribution (5) in the absence of observational errors. The deviation of the correlation coefficient from unity is then due to the different windows through which the gravity and the velocity of the LG are measured, and due to non-linear effects. The latter are described by two functions: the coherence function, and the ratio of the power spectra. In the linear regime, $C(k) = 1$ and $\mathcal{R} = \Omega_m^{1/2}$. This yields

$$r = \left[ \int_{0}^{\infty} \hat{W}_e^2(k) P_\theta(k) dk \right]^{1/2} \left[ \int_{0}^{\infty} \hat{W}_e^2(k) \mathcal{R}(k) P_v(k) dk \right]^{1/2},$$  \hspace{1cm} (24)

so for linear fields the correlation coefficient is determined solely by the windows, as expected.

3 MODELLING NON-LINEAR EFFECTS

3.1 Numerical simulations

We follow the evolution of the dark matter distribution using the pressureless hydrodynamic code CPAP (Cosmological Pressureless Parabolic Advection; see Kudlicki, Plewa & Różycka 1996; Kudlicki et al. 2000, for details). It employs an Eulerian scheme with third-order accuracy in space and second-order in time, which assures low numerical diffusion and an accurate treatment of high-density contrasts. Standard applications of hydrodynamic codes involve a collisional fluid; however, we implemented a simple flux interchange procedure to mimic collisionless fluid behaviour. Thanks to this approach we avoid a few problems of $N$-body codes. The main advantage of a hydrodynamic code over an $N$-body code is accurate treatment of low-density regions. Moreover it directly produces a volume-weighted velocity field. This is important because, in the definition of the CF equation (22), the velocity field is volume-weighted, not mass-weighted. Furthermore, the fast Fourier transform can be directly applied to the data on a uniform grid.

The computational domain forms a $(400 h^{-1} \text{ Mpc})^3$ cube with 256\(^3\) grid cells and periodic boundary conditions. This setup allows us to cover a broad range of wavenumbers, $k \in [0.016, 2.0] h \text{ Mpc}^{-1}$, which is important for accurate calculation of the integrals in equation (23). The initial distribution of the density fluctuations is Gaussian and their power spectrum is given by a cold dark matter (CDM) model (as in equation 7 of Elstathou, Bond & White 1992) with the shape parameter $\Gamma = 0.19$, as inferred from the IRAS PSCz survey (Sutherland et al. 1999) and in agreement with recent determinations from WMAP (Spergel et al. 2003). The initial value of the square root of the variance of the density field is in spheres of radius $8 h^{-1} \text{ Mpc}$, $\delta_\Omega(t_i)$, is 0.019. We studied a range of outputs from the simulations, corresponding to different values of $\delta_\Omega$. Specifically, full output data were dumped in constant intervals of the scale factor, $\Delta \alpha = 0.025$.

All our simulations assume $\Omega_m = 0.3$, $\Omega_\Lambda = 0$. In the mildly non-linear regime, the quantity $\delta_\Omega \equiv \Omega_m^{1/3} \delta_\Omega$ is insensitive to the cosmological density parameter and cosmological constant, as demonstrated both analytically (Bouchet et al. 1995; Nusser & Colberg 1998; see also appendix B3 of Scoccimarro et al. 1998) and by means of $N$-body simulations (Bernardeau et al. 1999). Therefore, our results should be valid for any cosmology. Specifically, we have

$$\mathcal{R} = \Omega_m^{1/2} \bar{\mathcal{R}},$$  \hspace{1cm} (25)

and

$$C = \bar{C},$$  \hspace{1cm} (26)

where

$$\bar{\mathcal{R}}(k) = \frac{P_\theta(k)}{P_v(k)}.$$  \hspace{1cm} (27)

\(^1\) S92 call it the decoherence function. We prefer the name ‘coherence’, because higher values of the function imply higher, not lower, correlation between gravity and velocity.
and
\[ C(k) = \frac{\langle \mathbf{k} \cdot \mathbf{v} \rangle}{\langle |\mathbf{k}|^2 \rangle^{1/2} \langle |\mathbf{v}|^2 \rangle^{1/2}}. \] (28)

In a previous paper (C02), we tested numerically the dependence of the coherence function on \( \Omega_m \) and found it to be extremely weak.

In C02, we also investigated numerical effects. In short, we found that the effects of resolution affect numerical determination of the CF at scales smaller than four grid cells, i.e. twice the Nyquist wavelength. Therefore, all the results we present here are for \( k < 1 \, h \, \text{Mpc}^{-1} \). For these wavenumbers, both simulated \( C \) and \( \mathcal{R} \) practically do not depend on resolution. Also, here we use a larger box size than in C02 in order to model longer modes, while the short-wavelength limit (\( k < 1 \, h \, \text{Mpc}^{-1} \)) still allows us to calculate accurately the integrals in equation (23), as will be shown in Subsection 3.4.

### 3.2 Coherence function

We studied the CF in C02. We found there that the characteristic decoherence scale is an order of magnitude smaller than previously used (S92). The weak point of the formula fitted in C02 was a poor description of the CF for long-wavelength modes. These modes are important however when calculating the integrals in equation (23). [From equations (12) and (13) it follows that the gravity and velocity fields are more sensitive to long-wavelength modes than the density field.] Therefore, here we use another fitting function, which is more accurate for low values of \( k \):
\[ C(k) = \left[ 1 + \left( a_0 k - a_2 k^{1.5} + a_3 k^2 \right)^{2.5} \right]^{-0.2}. \] (29)

Parameters \( a_i \) were obtained for 35 different values of \( \sigma_8 \) in the range [0.1, 1], and we found the following, power-law, scaling relations:
\[
\begin{align*}
  a_0 &= 4.908 \sigma_8^{0.75}, \\
  a_1 &= 2.663 \sigma_8, \\
  a_2 &= 5.889 \sigma_8^{1.74}.
\end{align*}
\] (30)

The fit was calculated for \( k \in [0.1, 1] \, h \, \text{Mpc}^{-1} \), with the imposed constraint \( C(k = 0) = 1 \). Unlike the previous fit, the new one results in a value of the correlation coefficient of gravity and velocity that agrees with the value measured directly in our simulations. In Fig. 1 we show, for various values of \( \sigma_8 \), the CF from simulations, its fit (29) and results of perturbative calculations described in C02. We see that the perturbative approximation breaks down for \( \sigma_8 > 0.5 \).

### 3.3 Ratio of the power spectra

The ratio of the velocity to the gravity power spectrum, \( \mathcal{R} \), is related to its scaled counterpart, \( \tilde{\mathcal{R}} \), by equation (25). The quantity \( \tilde{\mathcal{R}} \), defined in equation (27), practically does not depend on the background cosmological model. It departs from unity in the non-linear regime because the velocity grows more slowly than would be expected from the linear approximation.

We have found that \( \tilde{\mathcal{R}} \) obtained from simulation can be fitted with the following formula:
\[ \tilde{\mathcal{R}}(k) = [1 + (7.071 k)^4]^{-\alpha}, \] (31)
with
\[ \alpha = -0.06574 + 0.29195 \sigma_8 \] for \( 0.3 < \sigma_8 < 1 \).

We stress that the above relation between \( \alpha \) and \( \sigma_8 \) is valid for \( \sigma_8 \in [0.3, 1] \), which is still a sufficiently wide range of values. Fig. 2 shows the ratio of the power spectra from simulation and our fit.

### 3.4 Convergence of \( r \)

We calculate the correlation coefficient, \( r \), by inserting fits (29) and (31) into formula (23). However, the integrals in this formula extend over the whole \( k \) space, while the fits have been obtained for a limited range of wavenumbers between 0.016 and \( 1 \, h \, \text{Mpc}^{-1} \). Therefore, the question has to be answered whether this extrapolation is justified.

Fig. 3 shows the integrands of the integrals in formula (23) for \( \sigma_8 = 0.84 \). It is evident that contributions from wavenumbers greater than unity are negligible. This is so because the observational windows of the LG gravity and velocity filter out smaller scales.\(^2\) This is not quite the case for wavenumbers smaller than 0.016 \( h \, \text{Mpc}^{-1} \), but these scales are well within the linear regime, for which the limiting values of the CF and the ratio of the power spectra are known to converge to unity.

\(^2\) Strictly speaking, the velocity window passes contributions from wavenumbers up to about 2 \( h \, \text{Mpc}^{-1} \), but the ratio of the power spectra damps them additionally for \( k > 1 \, h \, \text{Mpc}^{-1} \).
4 PARAMETER ESTIMATION

We now apply our formalism to the PSCz survey. As the value of $\sigma_6$, we adopt the WMAP estimate, $\sigma_6 = 0.84$ (± 0.04; Spergel et al. 2003). This specifies the coherence function and the ratio of the power spectrum of velocity to the power spectrum of gravity. Also, this provides a normalization for the power spectrum of density.

4.1 Parameter dependence of the model

As stated before, the likelihood of specific values of $\beta$ and $b$ is determined by the distribution (5). In this distribution, the observables are $g$ and $v$, or $g$, $v$ and the misalignment angle $\psi$. Following S99, we adopt for them the following values: $g = 933$ km s$^{-1}$ (from the distribution of the PSCz galaxies up to 150 h$^{-1}$ Mpc), $v = 627$ km s$^{-1}$ (inferred from the four-year COBE data by Lineweaver et al. 1996), and $\psi = 15^\circ$.

The theoretical quantities are $\sigma_g$, $\sigma_v$ and $r$. The variance of a single spatial component of measured gravity, $\sigma_g^2$, is a sum of the cosmological component, $\sigma_{g,c}^2$, and errors, $\epsilon^2$. Since gravity here is inferred from a galaxy, rather than mass, density field, we have $\sigma_g^2 = b^2 \sigma_{g,c}^2$, where $\sigma_{g,c}^2$ is the variance of a single component of the true (i.e. mass-induced) gravity. From equation (17),

$$s_g^2 = \frac{1}{6\pi^2} \int_0^\infty \tilde{W}_g^2(k) P(k) \, dk.$$  \hspace{1cm} (33)

The gravity errors are two-fold: due to finite sampling of the galaxy density field, and due to the reconstruction of the galaxy density field in real space. Therefore, $\epsilon^2 = (\sigma_{SN}^2 + \sigma_{rec}^2)/3$, where $\sigma_{SN}^2$ and $\sigma_{rec}^2$ are respectively the shot noise (or sampling) variance and the reconstruction variance. (Both $\sigma_{SN}^2$ and $\sigma_{rec}^2$ are full, i.e. 3D, variances.) Estimated by S99 using mock catalogues, the cumulative shot noise at 150 h$^{-1}$ Mpc amounts to $\sigma_{SN} = 160$ km s$^{-1}$. An average reconstruction error in the differential contribution to the cumulative gravity, produced by a shell of matter 10 h$^{-1}$ Mpc wide, is 15 km s$^{-1}$. Since up to 150 h$^{-1}$ Mpc there are 15 such shells, we have $\sigma_{rec} = \sqrt{15} \times 15$ km s$^{-1} = 58$ km s$^{-1}$. To sum up,

$$\sigma_g^2 = b^2 \sigma_{g,c}^2 + \sigma_{SN}^2 + \sigma_{rec}^2/3,$$  \hspace{1cm} (34)

where

$$\sigma_{SN} = 160 \text{ km s}^{-1} \quad \text{and} \quad \sigma_{rec} = 58 \text{ km s}^{-1}.$$  \hspace{1cm} (35)

Errors in the measured velocity of the LG are negligible compared to those in the gravity. Equations (19) and (25) yield

$$\sigma_v = \Omega_m^{0.6} s_v,$$  \hspace{1cm} (36)

where

$$s_v^2 = \frac{1}{6\pi^2} \int_0^\infty \tilde{W}_v^2(k) \tilde{R}(k) P(k) \, dk.$$  \hspace{1cm} (37)

Finally, errors in the estimate of the LG gravity do not affect the cross-correlation between the LG gravity and velocity, but increase the gravity variance. This has the effect of lowering the value of the cross-correlation coefficient. Specifically, from equations (25), (26) and (34) we have

$$r = \rho \left( 1 + \frac{\sigma_{SN}^2 + \sigma_{rec}^2}{3b^2 \sigma_g^2} \right)^{-1/2}$$  \hspace{1cm} (38)

where

$$\rho = \left[ \int_0^\infty \tilde{W}_g^2(k) P(k) \, dk \right]^{1/2} \left[ \int_0^\infty \tilde{W}_v^2(k) \tilde{R}(k) P(k) \, dk \right]^{1/2}. $$  \hspace{1cm} (39)

Thus, the likelihood depends explicitly on the parameters $b$ and $\Omega_m$. However, for reasons that will become evident later on, as the parameters to be estimated we choose $b$ and $\beta \equiv \Omega_m^{0.6}/b$. Since $\Omega_m^{0.6} = b \beta$, then $\sigma_g$, $\sigma_v$ and $r$ all depend on $b$. On the other hand, only $\sigma_g$ depends on $\beta$. This makes the derivation of the most likely value of $\beta$, given $b$, simple.

4.2 $\beta$ for known $b$

Using equations (5) and (36), and the equality $\Omega_m^{0.6} = b \beta$, the logarithmic likelihood for $\beta$ takes on the form:

$$\ln \mathcal{L}(\beta) = -3 \ln (2\pi r) - 3 \ln \left[ \frac{\sigma_v}{b \sigma_g} (1 - r^2)^{1/2} \right] - 3 \ln \beta - \frac{1}{2(1-r^2)} \left( \frac{g^2}{\sigma_g^2} + \frac{v^2}{\sigma_v^2} - \frac{2r \mu g v}{\sigma_g \sigma_v} \right).$$  \hspace{1cm} (40)

To find its maximum, we calculate its partial derivative with respect to $\beta$, $\partial \ln \mathcal{L}/\partial \beta$, and equate it to zero. This yields the following equation:

$$3(1 - r^2) \beta^2 + \frac{r \mu g v}{\sigma_g \sigma_v} \beta - \frac{v^2}{b^2 \sigma_v^2} = 0.$$  \hspace{1cm} (41)

The LG gravity, inferred from the PSCz survey, is tightly coupled to its velocity, $1 - r \ll 1$ and $1 - \mu \ll 1$. (Specifically, $\mu = 0.97$ and, for $\sigma_g$ around 0.8, $r \simeq 0.93$.) At first approximation we can therefore assume $r = \mu = 1$, hence

$$\beta_1 = \frac{\sigma_g}{b \sigma_v} v.$$  \hspace{1cm} (42)

Using equation (34) we obtain finally

$$\beta_1 = \frac{s_g}{s_v} \left( 1 + \frac{\sigma_{SN}^2 + \sigma_{rec}^2}{3b^2 \sigma_g^2} \right)^{1/2} v.$$  \hspace{1cm} (43)

Thus, the best estimate of $\beta$ is not just the ratio of the LG velocity to its gravity: it is modified by non-linear effects (which affect $s_v$,...
4.3 Joint likelihood for $\beta$ and $b$

Fig. 4 shows isocontours of the joint likelihood for $\beta$ and $b$, corresponding to the confidence levels of 68, 90 and 95 per cent. The maximum of the likelihood function is denoted by the dot.

Through the function $R_\beta$, different observational windows (which affect $s_k$ and $s_v$ differently) and observational errors.

At next approximation, in equation (41) one could approximate $\beta^2$ by $\beta^2_\beta$. However, we have considered the case of known $b$ for illustrative purposes only and from now on we relax this assumption. In the next subsection we will analyse the joint likelihood for $\beta$ and $b$.

Figure 5. The marginal distribution for $\beta$. The result of marginalizing over all possible values of $b$ (from zero to infinity) is shown as the dashed line. The result of marginalizing over the values of $b$ in the range [0.7, 1.1] is shown as the solid line.

$\beta$ is shown in Fig. 5 as the solid line. This distribution is also skewed and more peaked than the previous one. We obtain $\beta = 0.66^{+0.21}_{-0.07}$ (68 per cent confidence limits).

5 SUMMARY AND CONCLUSIONS

We have performed a likelihood analysis of the LG acceleration, paying particular attention to non-linear effects. We have adopted a widely accepted assumption that the joint distribution of the LG acceleration and velocity is Gaussian. Then, two quantities describing non-linear effects are relevant. The first one is the coherence function, or the cross-correlation coefficient of the Fourier components of the gravity and velocity fields. The second one is the ratio of the power spectrum of the velocity to the power spectrum of the gravity. Extending our previous work, we have studied both the coherence function and the ratio of the power spectra. Using numerical simulations we have performed fits to the two as functions of the wavevector and $\sigma_g$. Then, we have estimated the best values of the parameter $\beta$ and its errors. We have obtained $\beta = 0.66^{+0.21}_{-0.07}$ at 68 per cent confidence level.

The analysis of the LG acceleration performed by S92 and S99 was in a sense more sophisticated than ours. Both teams analysed a differential growth of the gravity dipole in subsequent shells around the LG. Instead, here we used just one measurement of the total (integrated) gravity within a radius of 150 $h^{-1}$ Mpc. Nevertheless, the errors on $\beta$ that we have obtained are significantly smaller than those of S92 and S99. In particular, S99 obtained $\beta = 0.70^{+0.35}_{-0.20}$ at $1\sigma$ confidence level. Comparing these errors to ours should be done with caution, because S99 considered the joint likelihood for $\beta$ and the index of the power spectrum, $\Gamma$.4 (To obtain a constraint on $\beta$, they marginalized the distribution over the values of $\Gamma$ allowed by the constraints on the Hubble constant.) Still, it is striking that, while our best value of $\beta$ is close to theirs, our errors are significantly smaller. The reason is our careful modelling of non-linear effects. In a previous paper (C02) we showed that the coherence function used by S99 greatly overestimates actual decoherence between non-linear gravity and velocity. Tighter correlation between the LG gravity and velocity should result in a smaller random error of $\beta$; in the present work we have shown this to be indeed the case.

The second factor in the LG acceleration analysis, describing non-linear effects, is the ratio of the power spectra of velocity to gravity.

3 Our best value of the bias, 0.94, is, given the errors, surprisingly close to this estimate.

4 The constraints on $\Gamma$ obtained by S99 were extremely weak, so we decided to fix the value of $\Gamma$ and instead to study the dependence on $b$. 

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Unlike the coherence function, this quantity has not been accounted for previously. It affects the value of the correlation coefficient, hence the random error, to a lesser extent than the CF. However, it does affect the most likely value of $\beta$, as can easily be seen from illustrative equation (42) (via $s_v$). Neglecting the different non-linear growth rates of the gravity and the velocity is equivalent to setting $\tilde{R} = 1$; we have checked that then the best value of $\beta$ is 0.62. Therefore, a small discrepancy between our and S99’s most likely value of $\beta$ is not due to non-linear effects.

Unlike ours, the analysis of S99 included also other constraints on the velocity field around the LG: its (small) shear, and the value of the bulk flow within $30 h^{-1}$ Mpc. This may have had an effect on the best value of $\beta$. Moreover, an approach with multiple windows should allow further tightening of the errors. It is therefore interesting to repeat the analysis of S99 exactly, but with proper treatment of non-linear effects. We plan to do this in the future.

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5 If we write $\tilde{R}(k) = 1 - \epsilon(k)$, from equations (38) and (39) it is straightforward to show that $r = r|_{\tilde{R}=1} + O(\epsilon^2)$.

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