From Tomonaga-Luttinger to Fermi liquid in transport through a tunneling barrier.

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(November 16, 2017)

Abstract

Finite length of a one channel wire results in crossover from a Tomonaga-Luttinger to Fermi liquid behavior with lowering energy scale. In condition that voltage drop ($V$) mostly occurs across a tunnel barrier inside the wire we found coefficients of temperature/voltage expansion of low energy conductance as a function of constant of interaction, right and left traversal times. At higher voltage the finite length contribution exhibits oscillations related to both traversal times and becomes a slowly decaying correction to the scale-invariant $V^{1/g-1}$ dependence of the conductance.

72.10.Bg, 72.15.-v, 73.20.Dx
Quantum transport in Tomonaga-Luttinger liquids (TLL) has attracted a great deal of interest as it was suggested to be realized in a 1D constriction [1,2] and the edge state of the fractional quantum Hall (FQH) liquid [3]. Both suggestions have been supported in recent experiments on FQH [4] and on 1D [2] transport. Even more experimentalists [3] made claim on observation of the interaction effects in the 1D transport, however, without comprehensible connection with a theoretical model. Unless there is a resonant tunneling the repulsive interaction typically suppresses conductance at low energy. The experiment by Tarucha’s group [5] on 1D transport through a long wire with a weak impurity random potential inside demonstrated a crossover from TL liquid to Fermi liquid behavior at low temperature. This crossover is a finite length effect [7] and may be described in an inhomogeneous TLL model (ITLL) [8–10]. The ITLL model predicts both the conductance behavior in the Fermi liquid region up to a renormalization constant (i.e., relations between the coefficients at temperature/voltage in different integer degrees) and the interaction dependent non-analytical behavior in the TLL region.

The aim of this work is to examine this crossover in the low voltage conductance of the one channel wire where its suppression is mostly determined by a high point barrier located inside the wire. Position and height of the barrier are assumed to be due to an external gate. Therefore both distances $L_{R(L)}$ from the barrier to the right/left reservoir and the ratio $\zeta = L_R/(L_R + L_L)$ are assumed to be known. Then it will be shown below that just two more parameters, which are the total traversal time $t_0$ equal to sum of the right and left ones: $t_0 = t_R + t_L$ and the constant of the forward scattering $g$, are necessary for description of the crossover, and that the ITTL model gives quite a few ways to determine these parameters from either high voltage ($V > 1/t_0$) or low voltage ($V < 1/t_0$) conductance measurements. Temperature dependence of the conductance in this model was considered in [11]. Similar model turned out to be useful for experimental study of the FQH transport [4]. It was noticed recently [12] that there is a special way of connection between a $\nu = 1/3$ FQH liquid and leads when the whole setup corresponds to the $g = 1/3$ ITLL model. If so, the results obtained below for spinless electrons could be directly addressed to that FQH
device when the FQH liquid embeds a point scatterer.

Under condition that the link between two parts of the wire is weak enough it suffices to apply the tunneling Hamiltonian approach in the lowest order to describe the transport \[2,13,14\]. Current flowing through the weak link located, say, at \(x = 0\) inside the wire is given by the operator \(J(t) = -i[A\psi^+_R(0, t)\psi_L(0, t) - h.c.]\), \((e, \hbar = 1)\), where \(A\) is the tunneling amplitude and \(\psi_{R,L}(x, t)\) are the electron annihilation operators in the right \(0 \leq x < L_R\) and in the left \(-L_L < x \leq 0\) part of the wire, respectively. The average current under voltage \(V\) applied to the left lead can be written as:

\[
\langle J \rangle = \frac{2 \pi |A|^2}{\hbar} \int d\epsilon [f(\epsilon - V) - f(\epsilon)] \rho_R(\epsilon) \rho_L(\epsilon - V),
\]

where \(f\) is Fermi distribution. The problem reduces to finding of the tunneling density of states of the right (left) end of the junction \(\rho_{R(L)}(\epsilon)\) which are the sum of the particle \(\rho_p(\epsilon) = (1 - f(\epsilon))\rho(\epsilon)\) and hole \(\rho_h(\epsilon) = f(\epsilon)\rho(\epsilon)\) densities. The latters relate to the particle correlator as \(\rho_p(\epsilon) = 1/(2\pi) \int dt e^{i\epsilon t} <\psi(0, t)\psi^+(0, 0)>\) and to the hole one as \(\rho_h(\epsilon) = 1/(2\pi) \int dt e^{i\epsilon t} <\psi^+(0, 0)\psi(0, t)>\).

Tunneling density of states - To calculate the tunneling density of states \(\rho_R\) on the right side of the weak link let us first consider spinless fermions and apply bosonization to the \(\psi\) field under condition of an elastic reflection from the boundary located at \(x = 0\) \[15,16\]. (Carrying out this calculations we will omit index "R" below.) Bosonic repersentation of the \(\psi\) field reads \(\psi(x, t) = \sum_{a=r,l} \psi_a(x, t) = (2\pi \alpha)^{-1} \sum_{\pm} exp\{i(\theta(x, t) \pm \phi(x, t))/2\}\), where \(\psi_{r(l)}\) is the right (left) going chiral component of \(\psi\) and the \(\theta\) and \(\phi\) fields are bosonic and mutually conjugated \([\theta(x, t), \phi(y, t)] = 2\pi i \text{sgn}(x - y)\). The elastic reflection means that \(\psi_l(0, t) = e^{i\delta} \psi_r(0, t)\) with an appropiate phase shift \(\delta\). This results in both:

\[
\phi(0, t) = \delta, \quad \frac{1}{2\pi} \partial_x \theta(x, t)|_{x=0} = \psi^+_r(0, t)\psi_r(0, t) - \psi^+_l(0, t)\psi_l(0, t) = 0.
\]

Then the density of particle states could be found as

\[
\rho_p(\epsilon) = \rho_O \frac{E_F}{2\pi} \int_{-\infty}^{+\infty} dt e^{i\epsilon t + \frac{1}{4}[\langle\theta(0, t)\theta(0, 0)\rangle - \langle\theta^2(0, 0)\rangle]} \]

where the value of the free electron tuneling density was introduced as: \(\rho_O = (1+\cos \delta)/(\pi v)\).

The problem reduces to finding the \(\theta\) field correlator. It can be done for the finite length piece of the wire adiabatically connected to the lead making use of the ITTL model \[8,10\]. In
this model the Tomonaga-Luttinger interaction \((\sum_{\mathbf{r},j} \rho_a)^2\) is switched on in the wire \(x < L_R\) and switched off outside. Then the Hamiltonian takes a bosonized form

\[
\mathcal{H} = \int_0^\infty dx \frac{v}{2} \left\{ u^2(x) \left( \frac{\partial_x \phi(x)}{\sqrt{4\pi}} \right)^2 + \left( \frac{\partial_x \theta(x)}{\sqrt{4\pi}} \right)^2 \right\}
\]

(3)

where function \(u(x)\) ensuing from the interaction can be approximated in the low energy limit by a step-function: \(u(x) = 1\) if \(x > L_R\) and \(u(x) = u = 1/g < 1\), otherwise. The correlator of the \(\theta\) field ordered in imaginary time \(T(x, y, \tau) \equiv T_\tau \theta(x, \tau) \theta(y, 0)\) can be shown to satisfy the following equation

\[
\left\{ \frac{\omega^2}{v^2 u^2(x)} - \partial_x^2 \right\} f_\omega(x) = \left\{ \frac{\omega^2}{v^2 u^2(x)} - \partial_x^2 \right\} h_\omega(x) = 0
\]

(5)

under the boundary conditions \(\partial_x T(x, y, \tau)|_{x=0} = 0\) following from (1). Fourier transform of this correlator \(T(x, y, \omega)\) is symmetrical under \(\omega \rightarrow -\omega\). It can be constructed from the solutions of the homogeneous equation corresponding to Eq.(4)

\[
T(x, y, \omega) = \frac{4\pi}{v W(\omega)} \left[ \theta(x - y) f_\omega(x) h_\omega(y) + \theta(y - x) f_\omega(y) h_\omega(x) \right]
\]

if these solutions meet boundary conditions: \(h'_\omega(0) = 0, f_\omega(x) = \exp(-\omega x/v)\) at \(x \rightarrow \infty\) and positive \(\omega\). The Wronskian \(W(\omega)\) is equal to \(-f'_\omega(0) h_\omega(0)\) and, hence, \(T(0, 0, \omega) = -4\pi/[v (\ln f_\omega(0))^\prime]\). The only solution we need can be written as right going plus reflected left going waves at \(x < L_R\). The reflection amplitude \(r \equiv r_\theta = -e^{-2\eta}, \ (\tanh(\eta) = 1/u)\) for the \(\theta\) field is negative for the repulsive interaction. It is related to the one \(r_\phi\) for the \(\phi\) field \([3, 11]\) as \(r_\theta = -r_\phi\) by the duality symmetry. Substituting this solution one can find \(T(0, 0, \omega) = \frac{4\pi u}{\omega} \tanh(\omega t_R + \eta)\) with \(t_R\) equal to the time of travelling from the junction to the right lead. Analytical continuation of this function \([-T(0, 0, -i\omega + 0)]\) is the retarded Green function for the \(\theta\) field. Imaginary part of the latter multiplied by the Bose distribution function for holes \(1 + f_B(\omega)\) and by a factor \((-2)\) coincides with the Fourier transform of the correlator at \(\omega\). Then the particle density of states \([2]\) in dimensionless units is obtained as

\[
\rho_\nu(\epsilon) = \frac{\rho_0}{2\pi\gamma} \int_{-\infty}^{+\infty} d\epsilon \exp\left\{ i\epsilon p + \int_{-\infty}^{+\infty} d\tilde{\omega} e^{-\gamma |\tilde{\omega}|} (1 + f_B(\tilde{\omega})) \frac{e^{-i\tilde{\omega}p}}{\tilde{\omega}} - iM \tan(\tilde{\omega} + i\eta) \right\}
\]

(6)
where the inverse temperature $\beta$ and energy $\epsilon, \omega$ were scaled as $\beta = 1/(T t_R)$, $\epsilon = t_R \epsilon$, $\tilde{\omega} = t_R \omega$ ($T$ is the temperature in energy units). The dimensionless cut-off parameter $\gamma$ is $(E_F t_R)^{-1}$, and $p$ is dimensionless time $p = t/t_R$. The hole density of states $\rho_h(\epsilon)$ can be found as $\rho_h(\epsilon) = \rho_p(-\epsilon)$.

The correlator used in (6) could be represented as a product after expansion of the imaginary part of the tangent in a sum of exponents:

$$<\psi_r(0, p)\psi_r^+(0, 0) >= \frac{E_F}{2\pi v} \left( \frac{-i\gamma \pi/\beta}{\sinh((p - i\gamma)\pi/\beta)} \right)^u \prod_{n \geq 1} \left( \frac{\sinh((2n + i\gamma)\pi/\beta)}{\prod_{\pm} \sinh((2n \pm (p - i\gamma))\pi/\beta)} \right)^{wr^n}$$

(7)

This expression can be easily understood as the product of the contributions of the $2n$ length paths connecting $(0, p)$ and $(0, 0)$ points and undergoing $n$ reflections from a $x = L_R$ non-elastic boundary with the negative reflection amplitude $r = -e^{-2\eta}$ and $n$ reflections from the $x = 0$ elastic boundary with unit reflection amplitude. The product has a good convergence due to exponential decrease of $r^n$ with $n$. Substituting it into (6) one can find low ($\epsilon \ll 1$) and high ($\epsilon \gg 1$) energy behavior of the tunneling density of states. Similar calculations in the spinful case require change of $Im[\tan(\tilde{\omega} + i\eta)]/\tanh(\eta)$ in (6) into $(1 + Im[\tan(\tilde{\omega} + i\eta)]/\tanh(\eta))/2$. The product representation of the correlator can be obtained from (7) if the exponent of the second multiplier $u$ is changed into $(u+1)/2$ and the exponents of all the rest of the product $wr^n$ into $wr^n/2$.

The calculation of the low energy expansion is tiresome but straightforward. Up to the fourth order in energy it gives in the spinless and spinful cases, respectively:

$$\rho(\epsilon) = c_1 \rho_o \gamma^{u-1} \left[ 1 + (1 - g^2) \left( \frac{\epsilon^2}{2} + \frac{1}{6} \left( \frac{\pi}{\beta} \right)^2 \right) + (1 - g^2)(4.5 - 6.5g^2) \right]$$

$$\times \left[ 0.104\epsilon^2 \left( \frac{\pi}{\beta} \right)^2 + 0.01\epsilon^4 \right] + (1 - g^2)[1.3 - 1.9g^2] \left( \frac{\pi}{\beta} \right)^4$$

(8)

$$\rho(\epsilon) = \sqrt{c_1 \rho_o} \gamma^{u-1} \left[ 1 + (1 - g^2) \left( \frac{\epsilon^2}{4} + \frac{1}{12} \left( \frac{\pi}{\beta} \right)^2 \right) + (1 - g^2)(2.22 - 3.22g^2) \right]$$

$$\times \left[ 0.104\epsilon^2 \left( \frac{\pi}{\beta} \right)^2 + 0.01\epsilon^4 \right] + (1 - g^2)[0.644 - 0.926g^2] \left( \frac{\pi}{\beta} \right)^4$$

(9)
where the constant $c_1 = \prod_{n>0} (2n)^{2ur^n}$ modifies a renormalization parameter $\gamma^{u-1}$ and is model dependent. Except for this parameter the functions $\rho(\varepsilon)$ and, hence, all coefficients of (8,9) in the curly brackets are universal functions of $g$ and $t_R$. The coefficients in the spinful case are approximately two times less than the ones in the spinless. In the second order this correspondence is exact.

The high energy behavior of the tunneling density at zero temperature and positive $\varepsilon$ can be obtained making transform of the counter of integration in (3) into a sum of the contours going around the cuts of the correlator (7) in the complex time plane from $i\infty + 2n$ to $2n$ and back. The result takes the form:

$\rho(\varepsilon)/\rho_0 = \frac{\gamma^{u-1}}{\pi}(\sin(\pi u)\Gamma(1-u)\varepsilon^{u-1} + 2r(\varepsilon))$ in the spinless case. The spinful expression just needs replacement of $u$ by $(u + 1)/2$ in the spinless one. It reveals that finite length corrections $r(\varepsilon)$ to the scale-invariant power law dependence display an interference structure. Here the constant $a_n$ in the spinless density of states (10) is given by $a_n^{-1} = 2^un^n(1-r^n)\prod_{m \neq n>0} \left|1 - \left(\frac{n}{m}\right)^2\right|^{ur^n}$ and the constant $b_n$ in the spinful density (11) is $b_n = \sqrt{a_n/(2n)}$. Both of them are on the order of 1 at small $n$ and decrease with increase of the number. However, a quick convergence in (10, 11) is mostly due to the $ur^n$ factor. The first two terms of the sums bring the leading contribution. They describe weakly decaying oscillations with the periods equal to $2t_R$ ($n = 1$) and $4t_R$ ($n = 2$), respectively. The second term always dominates at large energies as $r < 0$. However, the crossover energy is exponentially large at small $u$. Comparing coefficients of the first two oscillating terms of the sum in (11) one can gather that the finite length oscillations of the $2t_R$ period dominate the ones of the $4t_R$ period above $\varepsilon \approx 1$ unless $u > 1.8$ in the spinless case or $u > 2.1$ in the spinful one. It is in accordance with the numerical calculations [17]. This structure in the density of states may be understood as a quantization of the plasmonic modes inside the wire [18].

**Conductance** - It is convenient below to redefine the dimensionless energy in units of the
inverse total traversal time as $T \equiv t_0 T, V \equiv t_0 V$. Low energy conductance is an analytical function of $T, V < \zeta^{-1}, (1 - \zeta)^{-1}$. Integrating the low energy tunneling density of states (8,9), it can be found in the spinless and spinful cases, respectively:

$$
\sigma = \frac{c_i^2 \gamma^{2(u-1)}}{R_O \zeta_1^{u-1}} \left( 1 + (1 - g^2) \zeta_2 \left[ \frac{V^2}{2} + \frac{(\pi T)^2}{3} \right] + \left[ \frac{(1 - g^2)^2}{6} \zeta_1^2 + (1 - g^2) \frac{9 - 13g^2}{16} \zeta_3 \right] (\pi TV)^2 \right) + \Delta_T \sigma
$$

(12)

$$
\sigma = \frac{c_i \gamma^{u-1}}{R_O \zeta_1^{(u-1)/2}} \left( 1 + \frac{1 - g^2}{2} \zeta_2 \left[ \frac{V^2}{2} + \frac{(\pi T)^2}{3} \right] + \frac{(1 - g^2)^2}{4} \left( \frac{\zeta_1^2}{6} - \frac{\zeta_3}{64} \right) + (1 - g^2) \frac{9 - 13g^2}{32} \zeta_3 \right) (\pi TV)^2 + \Delta_T \sigma
$$

(13)

where $\gamma^{-1} = t_0 E_F$ and $\zeta_1 = \zeta(1 - \zeta), \zeta_2 = 1 - 2\zeta_1, \zeta_3 = \zeta^4 + (1 - \zeta)^4$ are geometrical coefficients determined by the only parameter $\zeta$ of the barrier position and $R_O^{-1}$ is a free electron conductance of the junction. $\Delta_T \sigma$ is produced by the finite difference between the temperatures of the right and left reservoirs $\Delta T = T_R - T_L$ and $T = (T_R + T_L)/2$. In the lowest linear order it is non-zero unless $\zeta = 1/2$ and reads for the spinless and spinful conductance, respectively:

$$
\Delta_T \sigma = \frac{c_i^2 \gamma^{2(u-1)}}{R_O \zeta_1^{u-1}} \frac{(1 - g^2)(9 - 13g^2)}{72} \zeta_4 V^2 \pi^2 T \Delta T
$$

(14)

$$
\Delta_T \sigma = \frac{c_i \gamma^{u-1}}{R_O \zeta_1^{(u-1)/2}} \frac{1 - g^2}{12} (0.74 - 1.073g^2) \zeta_4 V^2 \pi^2 T \Delta T
$$

(15)

where $\zeta_4 = \zeta^4 - (1 - \zeta)^4$. For $\zeta = 1/2$ $\Delta_T \sigma$ in (14), (15) vanishes and the conductance is always increasing due to $(\Delta T)^2$ term as the temperature difference is introduced. Relation of the $T^2$ term coefficient to the $V^2$ one is universal. Following Weiss’s consideration [20] one can prove it is equal to $(2\pi^2)/3$ in all orders of the perturbation expansion in the tunneling amplitude [19]. This is similar to Wilson relation in the Kondo problem. One can see from (12), (13) that both the interaction constant $g$ and the traversal time $t_0$ may be determined from the low energy expansion by comparing coefficients for the terms of the second and fourth order in energy.

Next we consider the high energy behavior of the conductance. To describe the latter for $V \gg 1/(1 - \zeta), 1/\zeta$ it is convenient to start from a representation of the current
in the form: 

\[ <J> = |A|^2 \int dt (e^{iVt} - e^{-iVt}) <\psi_R(0,t)\psi_R^+(0,0) > <\psi_L^+(0,t)\psi_L(0,0) > \]

The correlators may be expressed in the form \([\square]\). Making asymptotic integration we come to the conductance which in the spinless case could be written as 

\[
\partial J/\partial V = (\pi R_{O})^{-1}(t_0 E_F)^{2(1-u)}[\sin \pi (2u - 1)\Gamma(2 - 2u)V^{2u-2} + \Delta \sigma(V)].
\]

Expression for the spinful conductance is obtained by replacing \(u\) by \((u + 1)/2\). At zero temperature the finite length corrections \(\Delta \sigma(V)\) of spinless and spinful conductance may be found for irrational values of \(\zeta\), respectively, as

\[
\Delta \sigma(V) = 2V^{2u-2} \sum_{n>0} \left[ \frac{\cos(2nV\zeta + 2\varphi_{n,R})}{(V\zeta)^{2u(1-r^n/2)-1}g_n'(1-\zeta)} + \frac{\cos(2nV(1-\zeta) + 2\varphi_{n,L})}{(V(1-\zeta))^{2u(1-r^n/2)-1}g_n'(1-\zeta)} \right] \times a_n \sin(\pi ur^n) \Gamma(1 - ur^n) \quad (16)
\]

\[
\Delta \sigma(V) = 2V^{u-1} \sum_{n>0} \left[ \frac{\sin(2nV\zeta + \varphi_{n,R})}{(V\zeta)^{n(1-r^n/2)}g_n'(1-\zeta)} + \frac{\sin(2nV(1-\zeta) + \varphi_{n,L})}{(V(1-\zeta))^{n(1-r^n/2)}g_n'(1-\zeta)} \right] \times b_n \sin(\pi ur^n/2) \Gamma(1 - ur^n/2) \quad (17)
\]

where \(g_n'(x) = (2n)^{u-1} \prod_{m>0} \left| 1 - (xn/m)^2 \right|^{ur^n} \). The phase shifts are \((r^n - (u - 1)r^{[nx]})\pi/4\) \(([nx] \) denotes the integer part of \(nx\)\) with \(x = \zeta/(1-\zeta)\) for \(\varphi_{n,R}\) and \(x = (1-\zeta)/\zeta\) for \(\varphi_{n,L}\). Again one can see that only the \(n = 2\) term is important at high \(V\). However, if \(-r \ll 1\) the \(n = 1\) oscillations dominate over a large range of energy above 1. With decrease of \(\zeta < 1/2\) the oscillations of the \(\pi/\zeta t_0\) periodicity acquire much larger amplitude than those of \(\pi/(1 - \zeta)\).

If \(\zeta\) is rational: \(\zeta/(1-\zeta) = n_2/n_1\) the resonant enhancement of the oscillations with the frequency \(2n_1\zeta V = 2n_2(1 - \zeta) V\) occurs. Since only lowest \(n\)’s contribute to the sum \((\square)\) this resonance is important when both \(n_i\)’s are small: \(\zeta = 1/2, 1/3, 2/3\). For \(\zeta = 1/2\) the finite length correction of the spinless and spinful conductances reads

\[
\Delta \sigma(V) = V^{2u-2} \sum_{n>0} 4na_n^2 \cos(2nV\zeta + \pi(r^n - 1)) \sin(2\pi ur^n) \Gamma(1 - 2ur^n)(V\zeta)^{2u(r^n-1)-1} \quad (18)
\]

\[
\Delta \sigma(V) = V^{u-1} \sum_{n>0} 2a_n \sin(2nV\zeta + \pi r^n/2) \sin(\pi ur^n) \Gamma(1 - ur^n)(V\zeta)^{u(r^n-1)} \quad (19)
\]

The resonant enhancement strengthens oscillations of the \(4\zeta\) frequency and weakens the ones of the \(2\zeta\). These interference structures are shown in Fig.1 for spinful electrons.
With decrease of $V$ the high voltage behavior of the conductance ([16], [17]) will meet the low voltage one ([12], [13]). However, if $\zeta \ll 1/2$ there is a region $1/(1 - \zeta) \ll V \ll 1/\zeta$ where conductance relates to the low and high energy tunneling densities of states of the right and left parts of the wire, appropriately. In the leading order it is described by

$$\sigma = \rho_R(0)\rho((1 - \zeta)V)/(RO\rho_O^2).$$

In summary, we considered suppression of the conductance through a tunneling barrier in a 1D channel constriction in the ITLL model accounting for the finite length of the wire. Starting from the zero energy the conductance increases analytically as $T^2, V^2$ and higher even integer degrees. The relations between all coefficients are determined by the constant of interaction and two traversal times. At voltages higher than inverse of the sum of the traversal times conductance approaches the infinite length scale-invariant dependence. The deviation from the latter produced by the finite length contribution decays as a negative degree of the voltage oscillating with the periodicities related to double traversal times for a weak repulsion and to quadrupole ones if the interaction become stronger.

We acknowledge S. Tarucha for useful discussions. It is a special pleasure for one of us (V.P.) to thank T. Iitaka for his help in conducting calculations. This work was supported by the Center of Excellence and partially by the fund of the JSPS for development of collaboration between the former Soviet Union and Japan.
FIGURES

Dependences of the finite length correction $(E_F t_0)^{(1-u)} \Delta \sigma(V)/R_O$ to the differential conductance on voltage $V$ measured in $\pi$ over traversal time $t_0$ unit for spinful electrons at $u = 1.396$: the solid line relates to the symmetrical position of the weak link; the dotted line to the case when the ratio between the lengths of the right and left shoulders is $1/4$. 
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Fig. 1