Hybrid Beamforming for Terahertz MIMO-OFDM Systems Over Frequency Selective Fading

Hang Yuan, Nan Yang, Kai Yang, Chong Han, and Jianping An

Abstract

We propose novel hybrid beamforming schemes for the terahertz (THz) wireless system where a multi-antenna base station (BS) communicates with a multi-antenna user over frequency selective channels. Here, we assume that the BS employs sub-connected hybrid beamforming, due to its low complexity, and orthogonal frequency-division multiplexing (OFDM) to deliver ultra high data rate. First, we build a three-dimensional wideband THz channel model by incorporating the joint effect of molecular absorption, high reflection loss, and multipath fading, and consider the carrier frequency offset in OFDM. With this model, we propose a two-stage hybrid beamforming scheme which includes a normalized beamsteering codebook searching algorithm for analog beamforming and a regularized channel inversion method for digital beamforming. We then propose a novel wideband hybrid beamforming scheme with two digital beamformers. In this scheme, an additional digital beamformer is developed to compensate for the performance loss caused by the difference among subcarriers and hardware constraints. Furthermore, we consider imperfect channel state information (CSI) and propose a probabilistic robust hybrid beamforming scheme to combat channel estimation errors. Numerical results demonstrate the benefits of our proposed schemes for the sake of practical implementation, especially considering its high spectral efficiency, low complexity, and robustness against imperfect CSI.

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Index Terms
Terahertz communications, hybrid beamforming, massive multiple-input and multiple-output, frequency selective fading, imperfect channel knowledge.

I. INTRODUCTION
Terahertz (THz) communications is an avant-garde wireless technology in the future fifth generation (5G)-beyond and sixth generation (6G) wireless networks [2], due to its huge potential to support ultra-high-data-rate transmission [3], the Internet of nano-things [4], and massive connectivity [5]–[9] in the THz band (0.1–10 THz). Particularly, the drastically increasing traffic demand, which is expected to reach terabit per second (Tbps) within the next 5–10 years [3], can hardly be satisfied with the current wireless technologies that are designed in the microwave band below 6 gigahertz (GHz). The scarcity of available spectrum resources in the microwave band extremely restricts the achievable data rate, which has triggered tremendous research activities in higher-frequency bands [10]–[12], such as the millimeter wave (mmWave) band ranging from 30 to 300 GHz. However, the data rate provided in the mmWave band is in the order of 10 gigabits per second (Gbps) [13], which is still below the expected traffic demand. Against this background, THz communications has become an indispensable enabler for Tbps links in future wireless data applications since it uses the ultra-large usable bandwidth which ranges from tens of GHz up to several THz [3]. Importantly, the rapidly advancing THz device technologies, e.g., new graphene-based THz transceivers [14] and ultra-broadband antennas that operate at THz frequencies [15], make THz communications a reality.

One critical factor that affects the propagation of THz waves is severe path loss, which may restrict THz transmission distance into a few meters. Thus, THz communications systems are very promising to be applied in an indoor environment [2]. The extremely short wavelength, however, has a superiority, i.e., a large number of antennas can be tightly packed into a small area at a transceiver. Thus, the high beamsteering gain, multiplexing gain, and spatial diversity gain [12] enabled by massive multiple-input and multiple-output (MIMO) techniques can be exploited to combat severe path loss.

One challenge in realizing THz communications with massive MIMO is to deal with hardware constraints. Current THz devices have large power consumption and high complexity [16], imposing strict constraints on massive MIMO systems. To tackle this challenge, a low-complexity indoor THz wireless system with hybrid beamforming was investigated in [17], where
a criterion for designing the number and size of antenna arrays was provided by analyzing the performance degradation caused by the uncertainty in THz phase shifters. In [18], two hybrid beamforming architectures, namely, the fully-connected and sub-connected structures, were examined. Specifically, [18] showed that both the spectral efficiency and energy efficiency of the sub-connected structure are higher than those of the fully-connected structure under the consideration of insertion loss. Despite that [17], [18] stand on their own merits, none of them has touched frequency selective fading in wideband THz systems. Since THz communications is anticipated to operate over broadband channels [19], the design of frequency selective hybrid beamforming schemes is of great significance for wideband THz systems.

It is worthwhile noting that the research related to the design of frequency selective hybrid beamforming in THz systems is still in the infant stage. Some recent research efforts have been devoted to studying hybrid beamforming for mmWave systems over frequency selective channels. In [20], an optimal hybrid beamforming scheme that maximizes the achievable mutual information under total power and unitary power constraints was proposed for the orthogonal frequency-division multiplexing (OFDM)-based mmWave system with limited feedback channels. A useful criterion for hybrid codebook construction was also developed in [20], where both the baseband and radio frequency (RF) precoders are taken from quantized codebooks. Following [20], [21] considered the dynamic subarrays architecture for wideband hybrid beamforming, where a criterion for constructing the optimal subarrays that maximize the spectral efficiency was proposed. Furthermore, [22] showed that hybrid beamforming with a small number of RF chains can asymptotically approach the performance of fully digital beamforming for a sufficiently large number of transceiver antennas. The important findings in [20]–[22] provide valuable insights and guidelines to the design of frequency selective hybrid beamforming, while having several limitations, as follows:

1) The molecular absorption effect which may hamper the signal propagation significantly in the mmWave and THz bands was not considered.

2) They did not exploit the high channel correlation between THz subcarriers, thus incurring a high computational complexity.

3) They assumed fully digital receiver and perfect carrier frequency offset (CFO) synchronization, which may be not practical. The CFO is a pervasive problem in OFDM systems and very likely to degrade the system performance.

4) The robust design against imperfect channel state information (CSI) was not addressed.
For massive MIMO systems, it is very difficult to obtain perfect CSI at the transmitter, especially for frequency selective channels. Due to the aforementioned limitations, the schemes developed in [20]–[22] cannot be directly used in wideband THz wireless systems.

Against this background, we propose novel hybrid beamforming schemes to enable the transmission over frequency selective channels in a THz wireless system. In this system, a multi-antenna base station (BS) which employs the sub-connected architecture transmits to a multi-antenna user equipment (UE), during which OFDM is used to combat frequency selectivity. The rationale behind using OFDM is to harness its tremendous benefits such as high spectral efficiency and strong capabilities of parallel data processing, despite its requirements on frequency synchronization and digital processors at transceivers [23].

The contributions of this paper, especially relative to our previous work [1], are summarized as follows.

- We first establish a three-dimensional (3D) propagation model to characterize the special properties of frequency selective THz channels. This model accommodates the joint effect of molecular absorption, high reflection loss, and multipath fading in THz channels. With this model, we theoretically characterize the inter-band interference (IBI) to analyze the effect of CFO in the considered system.
- We propose a wideband codebook-based hybrid beamforming scheme with constraints on THz phase shifters. We develop a novel beamsteering codebook searching algorithm which aims to jointly maximize the sum of the normalized equivalent channel modulus over all subcarriers. The normalized factor which equals to the LOS channel gain of each subcarrier can be estimated at the BS due to the unique properties of THz channels. We also develop a regularized channel inversion (RCI) method at the baseband with special power constraints. The rationale behind this scheme is to maximize the long-term average signal power for analog beamforming and minimize the IBI for digital beamforming.
- We propose a wideband hybrid beamforming scheme with two digital beamformers where spatial channel covariance matrices are exploited to reduce the computational complexity. An additional digital beamformer is used to compensate for the performance loss caused by the difference among subcarriers and hardware constraints. Along with the compensation digital beamformer, the statistical eigen analog beamformer can approximate the unconstrained optimal beamformer. This scheme achieves a comparable performance to fully digital beam-
forming while requires much lower hardware complexity and power consumption.

- We consider channel estimation errors and propose a robust hybrid beamforming scheme. Instead of maximizing the SINR directly, our robust scheme maximizes the average signal power and introduces a probabilistic constraint to control the interference power, which ensures a very low probability that the interference power is higher than an acceptable level. The probabilistic constraint is transformed into a deterministic one, based on which a greedy amplitude-angle separate optimization method is developed to solve the problem.

Aided by numerical results, we show that our proposed schemes achieve higher spectral efficiencies than the existing hybrid beamforming scheme proposed in [18]. We also show that the spectral efficiencies achieved by our proposed schemes are close to that achieved by the high-complexity fully digital beamforming scheme. We further show that the proposed robust hybrid beamforming scheme provides a clear performance advantage over the non-robust one in the presence of imperfect CSI. Overall, our numerical results demonstrate the benefits of our proposed schemes for practical implementation, especially considering its high spectral efficiency, low complexity, and robustness.

**Notation:** Scalar variables are denoted by italic symbols. Vectors and matrices are denoted by lower-case and upper-case boldface symbols, respectively. Given a vector \( z \), \( (z)_i \) denotes the \( i \)th entry in \( z \) and \( \angle(z) \) denotes the angle of \( z \). Given a matrix \( Z \), \( \|Z\|_F \) denotes the Frobenius norm of \( Z \), \( Z^{-1} \) denotes the inverse of \( Z \), \( Z^T \) denotes the transpose of \( Z \), \( Z^H \) denotes the Hermitian transpose of \( Z \), \( \text{tr}\{Z\} \) denotes the trace of \( Z \), \( \text{eig}_1(Z) \) denotes the first eigenvector of \( Z \) corresponding to the largest eigenvalue, and \( [Z]_{r,:} \) and \( [Z]_{:,c} \) denote the \( r \)th row and \( c \)th column of \( Z \), respectively. Furthermore, \( I_m \) denotes the \( m \times m \) identity matrix, \( \text{diag}\{\cdot\} \) denotes a diagonal matrix with indicated vector along the diagonal, and \( \mathcal{CN}(\mu, \nu) \) denotes the complex Gaussian distribution with mean \( \mu \) and covariance \( \nu \), and \( \mathbb{E}[\cdot] \) denotes the expectation.

**II. SYSTEM AND CHANNEL MODELS**

**A. System Model**

We consider an indoor THz wireless communication system, where an \( N_{BS} \)-antenna BS transmits to an \( N_U \)-antenna UE using the ultra-large THz band. The BS employs the sub-connected architecture to deploy a large number of antennas via \( N_{RF} \) RF chains. With this architecture, each RF chain drives one disjoint subarray only. We assume that each subarray at the BS is an UPA with \( M_t \times N_t \) tightly-packed directional antennas, each of which is attached
to a THz phase shifter [17]. Hence, the total number of antennas at the BS is
\[ N_{BS} = N_{RF}M_tN_t. \]
At the UE, only one subarray driven by a single RF chain is equipped due to UE’s constraints on hardware and signal processing complexity [18]. The subarray at the UE is an UPA with \( M_r \times N_r \) tightly-packed antennas. The total number of antennas at the UE is \( N_U = M_rN_r \).

We assume that the antennas of each subarray are equally spaced, where the space between two adjacent antennas, denoted by \( a \), is smaller than the wavelength. This assumption guarantees a high degree of correlation between antennas within a subarray. We also assume that the space between two adjacent subarrays, denoted by \( b \), is larger than the wavelength, which indicates that the channels among different subarrays are approximately independent. With the sub-connected architecture, all signal processing is conducted at the subarray level. Thus, the basic component of the system becomes a subarray. The large channel matrix between the BS and UE can be decomposed into \( N_{RF} \) independent sub-matrices.

The BS adopts OFDM to transmit signals. We assume that the data transmitted to the UE is modulated and spread across \( K \) subcarriers. Therefore, the BS transmits a length-\( K \) data symbol to the UE during one block. At the BS, \( s = [s_1, s_2, \ldots, s_K]^T \) is a \( K \times 1 \) vector, where \( s_k, k \in \{1, 2, \ldots, K\} \), denotes the data transmitted to the UE at the \( k \)th subcarrier. Also, \( s \) is constrained by the transmit power given by \( E[ss^H] = P_sI_K \), where \( P_s \) is the total transmit power over all subcarriers.

At the baseband, digital beamforming is employed to control and route data streams through different RF chains. For example, \( s_k \) is precoded by an \( N_{RF} \times 1 \) digital baseband beamformer \( f_k \) in the frequency domain. Then, the precoded signal is transformed into the time domain using the \( K \)-point inverse fast Fourier transform (IFFT). After this, a length-\( Q \) cyclic prefix is added to the signal on each RF chain to eliminate the inter-symbol interference, before applying the \( N_{BS} \times N_{RF} \) analog RF beamformer \( W \) in the time domain. Since the RF beamforming is implemented after the IFFT, the analog RF beamformer is consistent for all subcarriers. Hence, the transmitted symbol at the \( k \)th subcarrier is expressed as \( x_k = Wf_k s_k \). For the sub-connected architecture, the analog RF beamformer \( W \) is a block diagonal matrix which can be expressed as

\[
W = \begin{bmatrix}
  w_1 & 0 & \cdots & 0 \\
  0 & w_2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & w_{N_{RF}}
\end{bmatrix},
\]
where \( w_n, n \in \{1, 2, \ldots, N_{RF}\} \), is an \( M_tN_t \times 1 \) vector. Each entry in \( w_n \) is limited by a constant modulus constraint such that \( |(w_n)_i| = 1/\sqrt{M_tN_t}, \ i \in \{1, 2, \ldots, M_tN_t\} \). Furthermore, we consider the power constraint for each subcarrier which is imposed by normalizing \( f_k \) such that \( \|Wf_k\|_F^2 = 1 \).

At the UE, the received signal at the \( k \)th subcarrier is given by
\[
y_k = v^H H_k x_k + n_k,
\]
where \( v \) is the \( N_U \times 1 \) receive analog beamforming vector, \( H_k \) is the \( N_U \times N_{BS} \) THz channel matrix between the BS and the UE, and \( n_k \) denotes the additive Gaussian white noise (AWGN) at the UE with the power of \( \sigma_n^2 \). We note that \( v \) is also limited by the constant modulus constraint such that \( |(v)_j| = 1/\sqrt{N_U}, \ j \in \{1, 2, \ldots, N_U\} \).

### B. Frequency Selective THz Channel Model

In wideband THz channels, each transmission window experiences frequency selective fading. This is because that the width of the window, which is approximately 0.2 THz [12], is much larger than the coherence bandwidth, which is approximately 5 GHz in the indoor environment [19]. Due to the high frequency dependence in THz channels, we build up the channel model as the combination of many subcarriers. In each subcarrier, the arrival paths generally consist of a LOS ray and several non-LOS (NLOS) rays that are caused by reflection. The scattering and diffraction can be neglected since they impose a negligible effect on propagation in the THz band. Thus, the delay-\( q \) channel response matrix for the OFDM system is given by [24]–[26]

\[
H_q(f_k, d) = \alpha^L p_r(qT_s - \tau) G_t G_r a_r(\theta^r, \phi^r) a_t^H(\theta^t, \phi^t) \\
+ \sum_{i=1}^{N_{clus}} \sum_{l=1}^{L_{ray}^i} \alpha_{i,l}^{NL} p_r(qT_s - \tau_{i,l}) G_t G_r a_r(\theta_{i,l}^r, \phi_{i,l}^r) a_t^H(\theta_{i,l}^t, \phi_{i,l}^t),
\]

where \( f_k \) is the center frequency of the \( k \)th subcarrier, \( N_{clus} \) is the number of clusters, \( L_{ray}^i \) is the number of rays in the \( i \)th cluster, \( \alpha^L \) and \( \alpha_{i,l}^{NL} \) are the complex gains of the LOS ray component and the NLOS ray component, respectively, \( G_t \) and \( G_r \) are the associated transmit and receive antenna gains, respectively, and \( p_r(qT_s - \tau) \) is the pulse-shaping function for \( T_s \) space signaling at the time delay of \( \tau \) [20]. Moreover, we denote \( a_t(\cdot, \cdot) \) and \( a_r(\cdot, \cdot) \) as the antenna array response vectors at the transmitter and receiver, respectively, where \( \theta^t, \phi^t, \theta^r, \phi^r \) denote the azimuth AoD (A-AoD), elevation AoD (E-AoD), azimuth AoA (A-AoA), and elevation AoA (E-AoA) for the LOS ray, respectively, and \( \theta_{i,l}^t, \phi_{i,l}^t, \theta_{i,l}^r, \phi_{i,l}^r \) denote the A-AoD, E-AoD, A-AoA, and E-AoA for the \( l \)th NLOS ray in the \( i \)th cluster, respectively.
Next, we introduce some important characteristics of THz channels that are different from those of mmWave channels. Such characteristics play a fundamental role in our design and analysis.

1) Path Gain: At very high frequencies, the molecular absorption loss drastically affects the wireless channel. Thus, similar to mmWave channels, the path loss in THz channels consists of spreading loss and molecular absorption loss. The path gain of the LOS ray, \(|\alpha^L|^2\), is expressed as a function of \(f\) and \(d\), given by 
\[ |\alpha^L|^2 = L_{spr}(f, d) L_{abs}(f, d), \]
where \(L_{spr}(f, d)\) and \(L_{abs}(f, d)\) are expressed as 
\[ L_{spr}(f, d) = \left( \frac{c}{4\pi fd} \right)^2 \]
and 
\[ L_{abs}(f, d) = e^{-k_{abs}(f)d}, \]
with \(c\) being the speed of light and \(k_{abs}(f)\) being the frequency-dependent medium absorption coefficient that is determined by the composition of the transmission medium at the molecular level \[24\]. Compared with mmWave channels, both spreading loss and molecular absorption loss become much more severe in THz channels. Also, the major cause of the molecular absorption in THz channels comes from water vapor, which indicates that the molecular absorption loss is humidity-dependent.

To incorporate the losses of NLOS rays, the Fresnel reflection coefficient \(\mathcal{F}_{i,l}\) needs to be considered \[17\]. This coefficient is a function of \(f\). Moreover, we denote \(d_1\) as the distance between the transmitter and the reflector and \(d_2\) as the distance between the reflector and the receiver. The path gain of the \(l\)th NLOS ray in the \(i\)th cluster with one reflection, \(|\alpha_{NL}^{i,l}|^2\), is expressed as a function of \(f\), \(d_1\), and \(d_2\), given by 
\[ |\alpha_{NL}^{i,l}|^2 = \mathcal{F}_{i,l}(f)L_{spr}(f, d_1 + d_2) L_{abs}(f, d_1 + d_2). \]
Due to the high reflection loss in THz channels, only up to the second order reflections are considered, where the path gain of NLOS rays can be calculated with two Fresnel reflection coefficients \[25\], \[27\]. We note that THz channels only have a few NLOS paths and are much sparser than mmWave channels. The gap between the LOS and NLOS path gains in THz channels (e.g., more than 15 dB on average) is more significant than that in mmWave channels. Hence, THz channels are apparently LOS-dominant and NLOS-assisted, and more sensitive to blockages than mmWave channels. This property also motivates us to build a 3D THz channel model since THz BSs are very likely to be deployed at a high place to prevent blockages.

2) Angles of Departure and Arrival: The angular spread in the THz channel is much smaller than that in mmWave or microwave channels, due to the significant increase in reflection and scattering loss. We clarify that \(\theta_{i,l}^t\) and \(\phi_{i,l}^t\) in the same cluster consist of the mean AoD and the AoD shift for the \(l\)th NLOS ray, while \(\theta_{i,l}^r\) and \(\phi_{i,l}^r\) consist of the mean AoA and the AoA shift for the \(l\)th NLOS ray. Thus, \(\theta_{i,l}^t, \phi_{i,l}^t, \theta_{i,l}^r, \phi_{i,l}^r\) are expressed as 
\[ \theta_{i,l}^t = \Theta_i^t + \vartheta_{i,l}^t, \phi_{i,l}^t = \Phi_i^t + \varphi_{i,l}^t, \]
\[ \theta_{i,l}^r = \Theta_i^r + \vartheta_{i,l}^r, \phi_{i,l}^r = \Phi_i^r + \varphi_{i,l}^r, \]
respectively, where \(\Theta_i^t/\Phi_i^t\) and \(\Theta_i^r/\Phi_i^r\) denote mean AoD...
and mean AoA of the $i$th cluster, and $\varphi_{t,i,l}^t$, $\varphi_{r,i,l}^r$, $\vartheta_{t,i,l}$, and $\vartheta_{r,i,l}$ denote angle shifts. We note that $\Theta_{t,i,l}^t/\Phi_{t,i,l}^t$ and $\Theta_{r,i,l}^r/\Phi_{r,i,l}^r$ follow uniform distributions on $[-\pi, \pi]$ and $[-\frac{\pi}{2}, \frac{\pi}{2}]$, respectively. We also note that $\varphi_{t,i,l}^t$, $\varphi_{r,i,l}^r$, $\vartheta_{t,i,l}$, and $\vartheta_{r,i,l}$ can be characterized by a zero-mean second order Gaussian mixture model (GMM) [28].

Given the delay-$q$ THz channel model in (2), the channel at the $k$th subcarrier in the OFDM system is expressed as

$$H_k = \sum_{q=1}^{Q} H_q(f_k, d)e^{-j\frac{2\pi}{K}q}$$

$$= \alpha_L^k G_t G_r a_r(\theta^r,\phi^r) a_t^H(\theta^t,\phi^t) P_r(k,\tau)$$

$$+ \sum_{i=1}^{N_{\text{ delayed}}} \sum_{l=1}^{L_{\text{ ray}}} \alpha_{NL}^{k,i,l} G_t G_r a_r(\theta_{r,i,l}^r,\phi_{r,i,l}^r) a_t^H(\theta_{t,i,l}^t,\phi_{t,i,l}^t) P_r(k,\tau_{i,l}),$$

where $P_r(k,\tau)$ is defined as $P_r(k,\tau) = \sum_{q=1}^{Q} p_r(qT_s - \tau)e^{-j\frac{2\pi}{K}q}$. Based on (3), we clarify that all subcarriers are low-rank channels. Indeed, $H_k$ contains a very limited number of NLOS rays. We also clarify that the channel coefficients across subcarriers are highly correlated since $a_r(\theta^r,\phi^r)$ and $a_t(\theta^t,\phi^t)$ are the same for all subcarriers.

So far, we have established the system and channel models. Such models enable us to formulate the hybrid beamforming design problem in the next subsection.

C. Problem Formulation of Hybrid Beamforming Design

In OFDM systems, the accurate frequency synchronization operating in the THz band is extremely challenging since it requires very high sampling rates (e.g., over multi-giga- or tera-samples per second) [27]. Hence, we consider the CFO of OFDM systems in this paper for the sake of practicality. In practice, the CFO may be caused by misalignment in carrier frequencies or Doppler shift. The combined signal at the $k$th subcarrier at the UE can be written as [29]

$$\tilde{y}_k = S_0 v^H H_k W_f s_k + \Delta + n_k,$$

where $\Delta = \sum_{\lambda=1,\lambda\neq k}^{K} S_{\lambda-k} v^H H_\lambda W_f s_k$ denotes the IBI caused by other subcarriers. The sequence $S_i, i \in \{1-K,\ldots,0,\ldots,K-1\}$, denotes the IBI coefficient which depends on the CFO and is given by [30]

$$S_i = \frac{\sin \pi(i+\varepsilon)}{K \sin \frac{\pi}{K}(i+\varepsilon)} e^{j\pi(1-\frac{1}{K})(i+\varepsilon)},$$
where $\varepsilon$ is the ratio between the CFO and the subcarrier spacing. For zero CFO, $S_i$ reduces to the unit impulse sequence. We clarify that IBI mainly captures the power leakage from neighboring subcarriers. Thus, the IBI decreases when the bandwidth of subcarriers increases. With the employment of directional antennas, the path loss and delay spread of channels are reduced and the IBI also increases [30].

Given the received signal in (4), the average achievable data rate is derived as

$$R = \frac{1}{K} \sum_{k=1}^{K} \log_2 (1 + \gamma_k)$$

$$= \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{|S_0|^2 |v^H H_k W f_k|^2}{\sum_{\lambda=1,\lambda\neq k}^{K} |S_\lambda-k|^2 |v^H H_\lambda W f_k|^2 + \psi} \right), \quad (6)$$

where $\gamma_k$ is the SINR at the $k$th subcarrier and $\psi = K \sigma_n^2 / P_s$. Adopting $R$ as the system performance metric, the hybrid beamforming design problem is to find $v$, $\{w_n\}_{n=1}^{N_{RF}}$, and $\{f_k\}_{k=1}^{K}$ such that $R$ is maximized. Due to the constraints on RF hardware, the analog beamforming vectors generally take certain values. Hence, in the next section we will investigate the hybrid beamforming problem when analog beamformers are taken from quantized codebooks while no quantization constraints are imposed on digital beamformers.

III. WIDEBAND HYBRID BEAMFORMING SCHEME WITH BEAMSTEERING CODEBOOKS

In this section, we propose a two-stage wideband codebook-based hybrid beamforming scheme for the considered system. In the first stage, we develop a novel beamsteering codebook searching algorithm for analog beamforming. Here, we propose to jointly maximize the sum of normalized equivalent channel modulus over all subcarriers to ensure the fairness among subcarriers. In the second stage, we design a revised RCI method with IBI elimination for digital beamforming. In our considered system, analog beamforming is implemented by THz phase shifters. Since THz phase shifters are mostly digitally controlled, only quantized angles are available. We now relax the optimization problem in Section II-C by adopting the assumption of quantized analog beamformers. Here, we consider the beamsteering codebooks where the codewords have the same form as the antenna array response vectors [20]. With the transmit beamsteering codebook at the BS, denoted by $\mathcal{W}$, and the receive beamsteering codebook at the UE, denoted by $\mathcal{V}$, we next design the hybrid beamforming, including analog beamforming in the RF domain and digital beamforming at the baseband.
A. Analog Beamforming Design

At the BS, each antenna subarray has one beamsteering direction within the sub-connected architecture. Since the channels among different subarrays are considered as being independent, we can decompose the channel matrix approximately as $H_k = [H_{k,1}, H_{k,2}, \ldots, H_{k,N_{RF}}]$, where $H_{k,n}$ is an $M_rN_r \times M_tN_t$ channel matrix from the $n$th antenna subarray to the UE. Given the target A-AoA, $\theta_r^0$, and the target E-AoA, $\phi_r^0$, the corresponding beamsteering vector, denoted by $a_r(\theta_r^0, \phi_r^0)$, is adopted as the ideal analog beamformer at the receiver. Similarly, given the target A-AoD, $\theta_0^t$, and the target E-AoD, $\phi_0^t$, the corresponding beamsteering vector, denoted by $\hat{a}_t(\theta_0^t, \phi_0^t)$, is adopted as the ideal analog beamformer at the transmitter. Hence, the equivalent channel of the subarray at the baseband is expressed as

$$
\hat{h}_{k,n} = a_r^H(\theta_0^t, \phi_0^t)H_{k,n}\hat{a}_t(\theta_0^t, \phi_0^t)
$$

$$
= \alpha_k^tG_tG_rA_r^c(\theta^r, \phi^r)A_t^c(\theta^t, \phi^t)P_r(k, \tau)
$$

$$
+ \sum_{i=1}^{N_{\text{lay}}} \sum_{l=1}^{L_{\text{lay}}} \alpha_{k,i,l}^N G_tG_rA_r^c(\theta_{i,l}^r, \phi_{i,l}^r)A_t^c(\theta_{i,l}^t, \phi_{i,l}^t)P_r(k, \tau_{i,l}), \quad (7)
$$

where

$$
A_r^c(\theta, \phi) = \frac{1}{\sqrt{M_rN_r}} \sum_{m_r=0}^{M_r-1} \sum_{n_r=0}^{N_r-1} e^{j\frac{2\pi}{N_r}(\Omega_{m_r,n_r}(\theta,\phi) - \Omega_{m_r,n_r}(\theta_0^r,\phi_0^r))}
$$

$$
\approx \frac{1 - e^{j\pi M_r(\cos \theta \sin \phi - \cos \theta_0^r \sin \phi_0^r)}}{\sqrt{M_rN_r}} \frac{1 - e^{j\pi N_r(\sin \theta \sin \phi - \sin \theta_0^r \sin \phi_0^r)}}{1 - e^{j\pi(\sin \theta \sin \phi - \sin \theta_0^r \sin \phi_0^r)}} \times \frac{\sin [\pi M_r(\cos \theta \sin \phi - \cos \theta_0^r \sin \phi_0^r)]}{\sin [\pi N_r(\sin \theta \sin \phi - \sin \theta_0^r \sin \phi_0^r)]}
$$

and

$$
A_t^c(\theta, \phi) = \frac{1}{\sqrt{M_tN_t}} \sum_{m_t=0}^{M_t-1} \sum_{n_t=0}^{N_t-1} e^{j\frac{2\pi}{N_t}(\Omega_{m_t,n_t}(\theta,\phi) - \Omega_{m_t,n_t}(\theta_0^t,\phi_0^t))}
$$

$$
\approx \frac{\sin [\pi M_t(\cos \theta \sin \phi - \cos \theta_0^t \sin \phi_0^t)]}{\sin [\pi N_t(\sin \theta \sin \phi - \sin \theta_0^t \sin \phi_0^t)]} \times \frac{\sin [\pi N_t(\sin \theta \sin \phi - \sin \theta_0^t \sin \phi_0^t)]}{\sin [\pi (\sin \theta \sin \phi - \sin \theta_0^t \sin \phi_0^t)]}. \quad (9)
$$

We find that when $\theta^r$ and $\phi^r$ approach the ideal beamforming angles at the receiver, the modulus of $A_r^c(\theta^r, \phi^r)$ approaches its maximum. When $\theta^t$ and $\phi^t$ approach the ideal beamforming angles at the transmitter, the modulus of $A_t^c(\theta^t, \phi^t)$ approaches its maximum. Hence, the optimal beamforming angles can be selected from codebooks, which aims to jointly maximize the sum of normalized equivalent channel modulus over all subcarriers, i.e., $\sum_{k=1}^{K} \frac{|a_r^H(\theta^r, \phi^r)H_{k,n}\hat{a}_t(\theta^t, \phi^t)|^2}{F_{(f_k,d)}}$. 
Algorithm 1 Normalized Codebook Searching Algorithm for Analog Beamforming Design

1: **Input:** The receive beamsteering codebook at the UE, $\mathcal{V}$, and the transmit beamsteering codebook at the BS, $\mathcal{W}$.

2: Estimate the normalized factor for each subcarrier, $F(f_k, d) = \left(\frac{c}{4\pi f_k d}\right)^2 e^{-k_{\text{abs}}(f_k)d}$.

3: Search $\mathcal{V}$ to find the optimal analog beamforming angles at the UE, $\hat{\theta}^r$ and $\hat{\phi}^r$, such that
   
   $$\left\{a_r(\hat{\theta}^r, \hat{\phi}^r)\right\} = \arg\max \sum_{n=1}^{N_{\text{RF}}} \sum_{k=1}^{K} \left|a_r^H(\theta^r, \phi^r)H_{k,n}\right|^2 F(f_k, d).$$

4: for $n = 1 : N_{\text{RF}}$ do

5: Search $\mathcal{W}$ to find the optimal analog beamforming angles at each RF chain, $\hat{\theta}^t_n$ and $\hat{\phi}^t_n$, such that
   
   $$\left\{\hat{a}_t(\hat{\theta}^t_n, \hat{\phi}^t_n)\right\} = \arg\max \sum_{k=1}^{K} \left|\frac{a_t^H(\theta^t, \phi^t)H_{k,n}}{F(f_k, d)}\right|^2.$$

6: end for

7: **Output:** $v = a_r(\hat{\theta}^r, \hat{\phi}^r)$ and $w_n = \hat{a}_t(\hat{\theta}^t_n, \hat{\phi}^t_n), \; n \in \{1, 2, \ldots, N_{\text{RF}}\}$.

Here, $F(f_k, d)$ is the normalized factor which is equal to the LOS path gain of the $k$th subcarrier channel $H_{k,n}$. Also, the effective channel gains are normalized based on center frequencies to ensure the fairness among subcarriers. It is noted that the use of normalized equivalent channel modulus is unique in THz wireless systems and cannot be extended from mmWave or microwave systems. Actually, THz channel gain is approximately deterministic since THz channels are absolutely LOS-dominant and small-scale fading can be ignored. Thus, the BS can estimate the channel gain only based on the transmission frequency and distance. Based on this, we propose the normalized codebook searching algorithm for analog beamforming and present it in Algorithm 1. For each subarray, we search the given beamsteering codebook to find the optimal transmit beamforming vectors serving all subcarriers. Since the channels of all subcarriers are low-rank and highly correlated, the obtained consistent analog beamformer is expected to work well across all subcarriers.

With the obtained analog beamformer, the effective channel at the baseband, $\hat{h}_k$, can be viewed as a MISO channel which is expressed as

$$\hat{h}_k = v^H H_k W$$

$$= a_r^H(\hat{\theta}^r, \hat{\phi}^r)H_k\begin{bmatrix} \hat{a}_t(\hat{\theta}_1^t, \hat{\phi}_1^t) & 0 & \cdots & 0 \\ 0 & \hat{a}_t(\hat{\theta}_2^t, \hat{\phi}_2^t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{a}_t(\hat{\theta}_{N_{\text{RF}}}^t, \hat{\phi}_{N_{\text{RF}}}^t) \end{bmatrix}$$
Accordingly, the SINR is rewritten as
\[ \gamma_k = \frac{|S_0|^2 |\hat{h}_k f_k|^2}{\sum_{\lambda=1, \lambda \neq k}^K |S_{\lambda-k}|^2 |\hat{h}_\lambda f_k|^2 + \psi}. \] (11)

Hence, the channel information that needs to be fed back is \( \hat{h}_k \). This indicates that the exact CSI, \( H_k \), is no longer needed for digital beamforming design, which significantly reduces the feedback overhead.

**B. Digital Beamforming Design**

We now design digital beamforming for IBI elimination. For this design, the effective channels at the baseband, \( \{ \hat{h}_k \}_{k=1}^K \), are assumed to be known at the BS through feedback. We adopt the RCI method to eliminate the IBI, which is implemented as [37]

\[
\mathbf{f}_k = \left( (\mathbf{H}_{\text{comb},k}^H \mathbf{H}_{\text{comb},k} + \beta \mathbf{I}_{N_{\text{RF}}})^{-1} \mathbf{H}_{\text{comb},k}^H \right)_{:,k},
\] (12)

where \( \beta \) is a regularization parameter to be optimized. In (12), \( \mathbf{H}_{\text{comb},k} \) is the combined effective channel given by

\[
\mathbf{H}_{\text{comb},k} = \left[ S_{1-k} \hat{h}_1^T, \ldots, S_0 \hat{h}_k^T, \ldots, S_{K-k} \hat{h}_K^T \right]^T.
\]

The rationale behind using the RCI method is to maximize the SINR optimally. In the following theorem, we show the optimal value of \( \beta \) which maximizes the SINR in (11).

**Theorem 1:** \( \gamma_k \) is maximized when \( \beta \) reaches the optimal value, \( \beta^* = \frac{\psi}{\| \mathbf{f}_k \|_F^2} \).

**Proof:** We first rewrite the SINR as

\[
\gamma_k = \frac{|S_0|^2 f_k^H \hat{h}_k^H \hat{h}_k f_k}{\sum_{\lambda=1, \lambda \neq k}^K |S_{\lambda-k}|^2 f_k^H \hat{h}_\lambda^H \hat{h}_\lambda f_k + \psi} = f_k^H \left( \mathbf{H}_{\text{comb},k}^H \mathbf{H}_{\text{comb},k} - |S_0|^2 \hat{h}_k^H \hat{h}_k \right) \mathbf{f}_k + \psi = f_k^H \left( \mathbf{H}_{\text{comb},k}^H \mathbf{H}_{\text{comb},k} + \frac{\psi}{\| \mathbf{f}_k \|_F^2} \mathbf{I}_{N_{\text{RF}}}) \mathbf{f}_k - |S_0|^2 f_k^H \hat{h}_k^H \hat{h}_k f_k = \frac{\omega_k}{1 - \omega_k},
\] (13)

where we define \( \omega_k \) as

\[
\omega_k = \frac{f_k^H |S_0|^2 \hat{h}_k^H \hat{h}_k f_k}{f_k^H \left( \mathbf{H}_{\text{comb},k}^H \mathbf{H}_{\text{comb},k} + \frac{\psi}{\| \mathbf{f}_k \|_F^2} \mathbf{I}_{N_{\text{RF}}}) \mathbf{f}_k}.
\] (14)
Based on (14), we find that $0 \leq \omega_k < 1$ and $\gamma_k$ is a monotonically increasing function of $\omega_k$. Moreover, $\omega_k$ has the form of generalized Rayleigh quotient and the optimal $f_k$ that maximizes $\omega_k$ has the same direction as the generalized eigenvector of $\left( H_{comb,k}^H H_{comb,k} + \frac{\psi}{\|f_k\|_F^2} I_{N_{RF}} \right)^{-1} \hat{h}_k^H \hat{h}_k$. Since $H_{comb,k}^H H_{comb,k} + \frac{\psi}{\|f_k\|_F^2} I_{N_{RF}}$ is invertible, the optimal $f_k$ that maximizes $\gamma_k$ becomes the dominant eigenvector of $\left( H_{comb,k}^H H_{comb,k} + \frac{\psi}{\|f_k\|_F^2} I_{N_{RF}} \right)^{-1} \hat{h}_k^H \hat{h}_k$. Given $f_k$ in (12), the optimal regularization parameter that maximizes SINR is $\beta^* = \frac{\|h_k\|_F^2}{\|f_k\|_F^2}$, which completes the proof.

We note that compared with the conventional RCI method, the optimal regularization parameter value in our scheme is normalized by $\|f_k\|_F^2$. In our proposed scheme, the analog beamformer is integrated into the combined effective channel, but hybrid beamforming vectors are constrained by the joint power constraint, i.e., $\|Wf_k\|_F^2 = 1$. Hence, the norm of $f_k$ is not deterministic and needs to be normalized here. In addition, the possible drawback of the aforementioned digital beamforming design may be that the dimension of $H_{comb,k}$ is too large and thus the computational complexity may not be tolerable. Fortunately, the interference caused by non-adjacent subcarriers can be ignored, since $|S_m|^2$ is close to zero when $m \leq -2$ or $m \geq 2$. Hence, we simplify the combined effective channels as

$$
\hat{H}_{comb,k} \approx \left[ S_{-1} \hat{h}_{k-1}^T, S_0 \hat{h}_k^T, S_1 \hat{h}_{k+1}^T \right]^T.
$$

(15)

Based on (15), the original $N_{RF} \times K$ channel matrix is simplified as an $N_{RF} \times 3$ matrix. When $f_k = \left( \hat{H}_{comb,k}^H \hat{H}_{comb,k} + \frac{\psi}{\|f_k\|_F^2} I_{N_{RF}} \right)^{-1} \hat{h}_k^H$, the SINR achieves its maximum value, $\gamma_k^{\text{max}}$, which is derived as

$$
\gamma_k^{\text{max}} = \frac{|S_0|^2 \hat{h}_k^H \left( \hat{H}_{comb,k}^H \hat{H}_{comb,k} + \psi I_{N_{RF}} \right)^{-1} \hat{h}_k}{1 - |S_0|^2 \hat{h}_k^H \left( \hat{H}_{comb,k}^H \hat{H}_{comb,k} + \psi I_{N_{RF}} \right)^{-1} \hat{h}_k} = |S_0|^2 \hat{h}_k^H \left( \sum_{\lambda=1, \lambda \neq k}^K |S_{\lambda-k}|^2 \hat{h}_{\lambda}^H \hat{h}_{\lambda} + \psi I_{N_{RF}} \right)^{-1} \hat{h}_k^H = \text{tr} \left\{ |S_0|^2 \hat{R}_k \left( \sum_{\lambda=1, \lambda \neq k}^K |S_{\lambda-k}|^2 \hat{R}_\lambda + \psi I_{N_{RF}} \right)^{-1} \right\},
$$

(16)

where $\hat{R}_k$ is defined as $\hat{R}_k = \hat{h}_k^H \hat{h}_k$. Based on the aforementioned simplification, our proposed hybrid beamforming scheme avoids the processing of large channel matrices and ensures low complexity. This makes our scheme particularly suitable for practical implementation.
IV. WIDEBAND HYBRID BEAMFORMING SCHEME WITH TWO DIGITAL BEAMFORMERS

In this section, we relax the assumption made in Section III that analog beamformers are selected from beamsteering codebooks. We note that the hybrid beamforming scheme with beamsteering codebooks, proposed in Section III, is very compatible under the constraints on digital controlled phase shifters. However, an exhaustive search over the codebook is required to find the optimal analog beamforming vectors. This search may be complicated, especially for large codebooks and massive MIMO systems. Hence, in this section we develop a novel wideband hybrid beamforming scheme without using the exhaustive search while achieving high performance for the considered system. Here, we propose to decompose the digital beamforming matrix into two matrices, namely, the compensation matrix and the RCI matrix. The conventional statistical eigen analog beamformer using spatial channel covariance matrices across all subcarriers cannot work well alone in wideband systems due to hardware constraints and the subcarrier channel difference. Thus, the compensation matrix is proposed to assist the analog beamformer and approximate the unconstrained optimal beamformer, while the RCI matrix is used to eliminate the IBI at the baseband. The design of the compensation matrix is challenging due to the low dimensional constraint of digital beamforming matrices in hybrid beamforming systems.

The wideband hybrid beamforming design problem in Section II-C is a fractional optimization problem, which is in general intractable. The major difficulties in solving the problem are:

1) The joint optimization of the transmitter and receiver;
2) The coupling between analog and digital beamforming matrices, caused by the power constraint given by $\|Wf_k\|_F^2 = 1$;
3) The non-convex constraints imposed on analog beamformers [31];
4) The consistent of analog beamformers, $v$ and $W$, for all subcarriers.

To conquer the first three difficulties, prior studies in narrow-band hybrid beamforming design [32]–[36] approximated the original problem as a convex one and obtained a near-optimal hybrid beamformer with low complexity. Although these heuristic methods used in such studies were shown to exhibit good performance, they cannot be directly used to design wideband hybrid beamforming in frequency selective THz wireless systems, due to their lack of consideration of the fourth difficulty.

Against this background, in this section we tackle all the four difficulties and propose a
wideband hybrid beamforming scheme with two digital beamformers, aiming to achieve a comparable performance to the fully digital beamforming design while eliminating the IBI caused by CFO. To this end, we develop a novel design as follows:

- First, W and v are jointly designed to maximize the desired signal power at the UE using the statistical eigenvalue decomposition (EVD) of spatial channel covariance matrices across all subcarriers, while neglecting the IBI.
- Second, we develop FC,k to compensate for the performance loss of the obtained analog beamformer caused by the constant modulus constraint and the difference among subcarriers.
- Third, we develop another digital beamformer, FI,k, to eliminate the IBI, using the obtained hybrid beamformers at the transmitter and the receiver.

We note that compared with the hybrid beamforming scheme in [21], we consider a single-RF-chain receiver and imperfect carrier frequency synchronization, which is more practical. Also, a joint analog beamforming design at the transmitter and receiver needs to be considered in our proposed scheme. Most importantly, we develop a hybrid beamforming design strategy with two low dimensional digital beamformers to approximate the optimal beamformer and eliminate interference.

A. Joint Analog Beamforming Design

Following the aforementioned design strategy, the first step is to jointly optimize W and v, such that the desired signal power at the UE is maximized. Here, we neglect the IBI. In narrow-band hybrid beamforming systems, the optimal analog beamformer at the transmitter is composed of eigenvectors of HKHk and the optimal analog combiner at the receiver is composed of eigenvectors of HKHk [37], [38]. In frequency selective THz wireless systems, subcarriers have different channels, which results in different analog beamformers. However, the analog beamformer is frequency flat in the considered system, which means that the single analog beamformer needs to serve all subcarriers. This is an important property of the hybrid beamforming system which distinguishes it from the conventional fully-digital beamforming system [20]. Hence, the conventional eigen beamforming method does not work in the considered system since it cannot perform the joint optimization of all subcarriers.

Fortunately, the channel coefficients across subcarriers are highly correlated. We note from [3] that antenna array response vectors, ar(θr, φr) and at(θt, φt), are the same for all subcarriers. Moreover, according to [22], the eigenvectors of HKHk are approximately the transmit antenna
array response vectors when the number of antennas is sufficiently large. Thus, the dominant eigenvectors of channel covariance matrices for different subcarriers are approximately the same. The analog beamforming can be jointly designed with all subcarriers. We propose to incorporate all channel covariance matrices over different subcarriers and design the analog beamforming using a statistical eigen hybrid beamforming scheme, where analog beamforming vectors at the transmitter are designed as the dominant eigenvectors of \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \) and analog combining vectors at the receiver are designed as the dominant eigenvectors of \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k \mathbf{H}_k^H \). Next, we will analyze the asymptotic performance of the proposed scheme compared with the fully digital beamforming scheme and present Theorem 2. For analytical tractability, we focus on the hybrid beamforming at the transmitter and consider the unconstrained analog beamforming.

**Theorem 2:** For low-rank wideband correlated THz channels, i.e., the rank of \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \) is lower than or equal to the number of RF chains, if the constant modulus constraint is not considered, the proposed statistical eigen hybrid beamforming scheme at the transmitter has approximately the same SINR as that of the fully digital beamforming scheme.

**Proof:** We assume that the rank of \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \), \( N_H \), is lower than or equal to the number of RF chains, i.e., \( N_H \leq N_{RF} \), and the eigenvectors of \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \) associated with its nonzero eigenvalues are \( \mathbf{\tilde{U}} \). Thus, \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \) can be represented as \( \mathbf{\tilde{U}} \mathbf{\tilde{\Lambda}} \mathbf{\tilde{U}}^H \). We note that \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \) is not a full-rank matrix and \( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k = \mathbf{\tilde{U}} \mathbf{\tilde{\Lambda}} \mathbf{\tilde{U}}^H \) is not a strictly defined EVD. Thus, \( \mathbf{\tilde{\Lambda}} \in \mathbb{C}^{N_H \times N_H} \) is generally not a diagonal matrix.

In the hybrid beamforming design, the analog beamformer without the constant modulus constraint and the sub-connected constraint is given by \( \mathbf{W} = [\mathbf{\tilde{U}} \ 0] \), which indicates that only \( N_H \) RF chains are effective. Hence, the effective channel covariance matrix is expressed as

\[
\mathbf{\hat{R}}_{\text{HB}} = \frac{1}{K} \sum_{k=1}^{K} (\mathbf{H}_k \mathbf{W})^H \mathbf{H}_k \mathbf{W}
\]

\[
= \mathbf{W}^H \left( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{H}_k \right) \mathbf{W}
\]

\[
= \begin{bmatrix}
\mathbf{\tilde{U}}^H \\
0
\end{bmatrix} \mathbf{\tilde{\Lambda}} \begin{bmatrix}
\mathbf{\tilde{U}}^H \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\mathbf{\tilde{\Lambda}} \\
0
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} \in \mathbb{C}^{N_{RF} \times N_{RF}}.
\]
From (16), the average SINR for all subcarriers with the RCI method is expressed as
\[
\gamma^{HB} = \text{tr} \left\{ |S_0|^2 \hat{R}^{HB} \left( \sum_{\lambda=0, \lambda \neq k}^{K-1} |S_{\lambda-k}|^2 \hat{R}^{HB} + \psi I_{N_{RF}} \right)^{-1} \right\}
\]
\[
= \text{tr} \left\{ |S_0|^2 \tilde{\Lambda} \left( \sum_{\lambda=0, \lambda \neq k}^{K-1} |S_{\lambda-k}|^2 \tilde{\Lambda} + \psi I_{N_t} \right)^{-1} \right\}.
\] (18)

In the fully digital beamforming design, we assume that the eigenvectors of \( H_k^H H_k \) associated with its nonzero eigenvalues are \( \tilde{U}_k \). Let \( W_k = [\tilde{U}_k \ \ V_k] \) be a unitary matrix, where \( V_k \) is the null space of \( \tilde{U}_k \) such that \( \tilde{U}_k^H V_k = 0 \) and \( V_k^H V_k = I_{N_{BS} - N_{t}} \). The effective channel covariance matrix is expressed as
\[
\hat{R}^{FD} = \frac{1}{K} \sum_{k=1}^{K} (H_k W_k)^H H_k W_k
\]
\[
= \frac{1}{K} \sum_{k=1}^{K} W_k^H (H_k^H H_k) W_k
\]
\[
= \frac{1}{K} \sum_{k=1}^{K} \begin{bmatrix} \tilde{U}_k^H \\ V_k \end{bmatrix} \tilde{U}_k \tilde{\Lambda}_k \tilde{U}_k^H \begin{bmatrix} \tilde{U}_k \\ V_k \end{bmatrix}
\]
\[
= \frac{1}{K} \sum_{k=1}^{K} \begin{bmatrix} \tilde{\Lambda}_k & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{C}^{N_{BS} \times N_{BS}}.
\] (19)

For frequency selective THz channels, we find from (3) that all subcarriers are highly correlated, i.e., the antenna array response vectors are approximately the same for all subcarriers. Hence, we have \( \frac{1}{K} \sum_{k=1}^{K} \tilde{\Lambda}_k \approx \tilde{\Lambda} \). The average SINR for the fully digital beamforming design is then expressed as
\[
\gamma^{FD} = \text{tr} \left\{ |S_0|^2 \hat{R}^{FD} \left( \sum_{\lambda=0, \lambda \neq k}^{K-1} |S_{\lambda-k}|^2 \hat{R}^{FD} + \psi I_{N_{BS}} \right)^{-1} \right\}
\]
\[
\approx \text{tr} \left\{ |S_0|^2 \tilde{\Lambda} \left( \sum_{\lambda=0, \lambda \neq k}^{K-1} |S_{\lambda-k}|^2 \tilde{\Lambda} + \psi I_{N_t} \right)^{-1} \right\}.
\] (20)

Since (18) is identical to (20), the proof is completed.

We emphasize that the assumption that the rank of \( \frac{1}{K} \sum_{k=1}^{K} H_k^H H_k \) is lower than or equal to the number of RF chains is reasonable, since THz channels are low-rank channels and contain a very limited number of NLOS rays. For the sub-connected architecture, however, RF chains are connected to disjoint subsets of antennas, each of which has one unique beamsteering direction.
Hence, each RF chain has a unique optimal beamforming direction that maximizes the SINR. It follows that we need to design them separately. For the \( n \)th RF chain, the analog beamformer can be designed as

\[
\mathbf{w}_{n}^{opt} = \text{eig} \left( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_{k,n} \mathbf{H}_{k,n}^H \right),
\]

where \( i \in \{1, 2, \ldots, M_t N_t\} \). This approach is similar to [38], [39] where each RF chain is separated and becomes the basic operational unit. We also note that the practical RF beamformer is not the optimal one, since its amplitude needs to keep constant, constrained by the structure of phase shifters. Hence, we propose that the RF beamformer is designed to have the same angle with the optimal one. The practical RF beamforming vector in the \( n \)th chain is designed as

\[
(w_n)_i = \frac{1}{\sqrt{M_t N_t}} \exp \left( j \angle ((w_n^{opt})_i) \right),
\]

where \( i \in \{1, 2, \ldots, M_t N_t\} \). Similarly, the analog combiner at the receiver is also designed by the statistical eigen hybrid beamforming scheme, which is expressed as

\[
(v)_j = \frac{1}{\sqrt{M_r N_r}} \exp \left( j \angle ((v^{opt})_j) \right),
\]

where \( j \in \{1, 2, \ldots, M_r N_r\} \) and \( v^{opt} = \text{eig} \left( \frac{1}{K} \sum_{k=1}^{K} \mathbf{H}_{k} \mathbf{H}_{k}^H \right) \). Obviously, the performance of the obtained analog beamformer at the transmitter cannot approach that of the unconstrained fully digital beamformer, due to the concession in (21) and (22). This motivates us to design an additional compensation matrix in digital beamforming.

**B. Two Digital Beamformers Design**

In this subsection, we present the digital beamforming design at the transmitter. With the aforementioned design strategy, we propose to decompose the digital beamformer into two low dimensional matrices. The first compensation matrix is used to compensate for the performance loss caused by the difference among subcarriers and hardware constraints. The second RCI matrix is used to eliminate the IBI. Here, we develop the compensation matrix using Corollary 2 in [20]. This baseband beamformer is designed jointly with the analog beamformer at the transmitter to approach the performance of unconstrained fully digital beamforming, which is given by

\[
\mathbf{F}_{c,k} = (\mathbf{W}^H \mathbf{W})^{-\frac{1}{2}} \mathbf{V}_{k,f},
\]

where \( \mathbf{V}_{k,f} \) is the output of the analog beamforming at the transmitter.
Algorithm 2 Wideband Hybrid Beamforming Scheme with Two Digital Beamformers

1: **Input:** The exact CSI for all subcarriers, $H_k$, $k \in \{1, 2, \ldots, K\}$.

2: **for** $n = 1 : N_{RF}$ **do**

3: Construct the average covariance matrix across all subcarriers as $\overline{R}_n = \frac{1}{K} \sum_{k=1}^{K} H_{k,n}^H H_{k,n}$, $k \in \{1, 2, \ldots, K\}$.

4: Perform EVD of $\overline{R}_n$ as $U_n \Lambda_n U_n^H$.

5: Construct the RF beamformer on each RF chain as $w_n = [U_n]_{:,1}$ and then normalize $w_n$.

6: **end for**

7: Construct the complete analog beamforming matrix at the transmitter according to (1).

8: Construct the analog combining matrix at the receiver according to (23).

9: Construct the compensation baseband beamformer according to (24).

10: Construct the equivalent channels as $\hat{h}_k = \sqrt{V} H_k W F_{c,k}$.

11: Combine the equivalent channels as $\hat{H}_{\text{comb},k} = \begin{bmatrix} S_{-1} \hat{h}_{k-1}^T & S_0 \hat{h}_k^T & S_1 \hat{h}_{k+1}^T \end{bmatrix}^T$.

12: Construct the baseband beamformer for IBI elimination as $f_{i,k} = \left( \hat{H}_{\text{comb},k}^H \hat{H}_{\text{comb},k} + \frac{\psi}{\|f_{i,k}\|_2^2} I_{N_{RF}} \right)^{-1} \hat{h}_k^H$.

13: Construct the complete baseband beamformer as $f_{BB,k} = F_{c,k} f_{i,k}$.

14: **Output:** $v, w_n, n \in \{1, 2, \ldots, N_{RF}\}$, and $f_{BB,k}, k \in \{1, 2, \ldots, K\}$.

where $\sqrt{V}_k$ is resulting from the singular value decomposition (SVD) of $H_k W (W^H W)^{-\frac{1}{2}} = \hat{U}_k \hat{\Sigma}_k \sqrt{V}_k$. We note that $F_{c,k}$ is a low dimensional matrix with the dimension of $N_{RF} \times N_{RF}$, but the combination of $WF_{c,k}$ is sufficiently close to the optimal digital beamformer, which benefits from the unique sparsity of THz channels.

The RCI baseband beamformer, $F_{i,k}$, can be obtained using (12). The complete digital baseband beamforming vectors are given by $f_{BB,k} = F_{c,k} f_{i,k}, \quad k \in \{1, 2, \ldots, K\}$. The overall process for the proposed wideband hybrid beamforming scheme with two digital beamforming matrices is summarized in Algorithm 2.

V. ROBUST HYBRID BEAMFORMING DESIGN IN THE PRESENCE OF IMPERFECT CSI

In Sections III and IV, we have developed hybrid beamforming schemes under the assumption of perfect CSI. That is, so far we have not considered any channel estimation error in our design. In most practical scenarios, only imperfect CSI can be accessed at the BS due to channel uncertainty. Thus, any non-robust design that does not address the imperfect CSI may degrade the
system performance significantly. Against this background, we analyze the impact of imperfect CSI and then design a robust hybrid beamforming scheme using a probabilistic approach to combat estimation errors in this section. The conventional statistical robust approach requires certain statistical assumptions on estimation errors and pays no attention to some extreme cases. The robustness of the adopted probabilistic approach lies in ensuring high quality of service (QoS) performance with high probability even in extreme cases [40]. In our proposed robust hybrid beamforming scheme, we add a probabilistic constraint to the robust hybrid beamforming design. Instead of maximizing the SINR directly, our scheme aims to maximize average signal power and restrain interference power by ensuring a very low probability that the interference power is higher than or equal to the given threshold.

We first model the estimated THz channel matrix under the assumption of imperfect CSI as $H_k = H_{kp} + E_k$, where $H_{kp}$ is the presumed channel matrix, given by (3), and $E_k \in \mathbb{C}^{N_U \times N_{BS}}$ is the channel estimation error matrix consisting of independent and identically distributed (i.i.d.) complex zero-mean Gaussian entries with the variance of $\sigma_e^2$ [40], [41]. For simplicity, we assume that the error variances for all subcarriers are equal. We also assume that $H_{kp}$ and $E_k$ are statistically independent, i.e., $E[H_k^H E_k] = E[E_k^H H_{kp}] = 0$.

We now focus on the robust hybrid beamforming design at the BS. According to the THz channel model with estimation errors, the SINR at the $k$th subcarrier becomes a function of presumed channels, $H_{kp}$, and estimation errors, $E_k$. It is expressed as

$$\gamma_{ke} = \frac{|S_0|^2 |(H_{kp} + E_k)Wf_k|^2}{\sum_{\lambda=1, \lambda \neq k}^K |S_{\lambda-k}|^2 |(H_{\lambda p} + E_{\lambda})Wf_k|^2 + \psi},$$

where $\tilde{H}_{kp}$ denotes an extended presumed channel matrix that excludes $H_{kp}$, i.e., $\tilde{H}_{kp} = \begin{bmatrix} S_{1-k}H_{1p}^T, \ldots, S_{1-k}H_{k-1p}^T, S_1H_{k+1p}^T, \ldots, S_KH_{Kp}^T \end{bmatrix}^T$, and $\tilde{E}_k$ denotes an extended error matrix that excludes $E_k$, i.e., $\tilde{E}_k = \begin{bmatrix} S_{1-k}E_{1p}^T, \ldots, S_{1-k}E_{k-1p}^T, S_1E_{k+1p}^T, \ldots, S_{K-k}E_{Kp}^T \end{bmatrix}^T$.

With estimation errors on both the numerator and denominator in (25), it may be difficult to maximize the SINR directly. Hence, our robust scheme takes the probabilistic approach which maximizes average signal power of each subcarrier and ensuring the probability of high interference power very low. Mathematically, the robust hybrid beamforming design problem
with the probabilistic constraint is formulated as

\[
\left\{ \{W\}, \{f_k\}_{k=1}^K \right\} = \arg \max \mathbb{E} \left[ |S_0 (H_{kp} + E_k) W f_k|^2 \right]
\]

\[
\text{s.t. } Pr \left\{ |(H_{kp} + \hat{E}_k) W f_k|^2 \geq T_k \right\} \leq p_k,
\]

(26)

where \(Pr\{A\}\) denotes the probability of the event \(A\), \(T_k\) denotes a pre-specified interference power threshold, and \(p_k\) is a given probability.

The objective function is obtained by taking the expectation of signal power allocated on the \(k\)th subcarrier with respect to estimation errors, which is presented as

\[
\mathbb{E} \left[ |S_0 (H_{kp} + E_k) W f_k|^2 \right] = |S_0|^2 \mathbb{E} \left[ (W f_k)^H (H_{kp} + E_k)^H (H_{kp} + E_k) W f_k \right]
\]

\[
= |S_0|^2 \mathbb{E} \left[ \text{tr} \left\{ (H_k^H H_k + E_k^H E_k) M_k \right\} \right]
\]

\[
= |S_0|^2 \text{tr} \left\{ (H_k^H H_k + \sigma_E^2 I_{N_{\text{BS}}}) M_k \right\},
\]

(27)

where \(M_k \triangleq (W f_k)(W f_k)^H\) is a symmetric positive semi-definite matrix.

We introduce a constraint in (26) to guarantee that the probability for the interference power of the \(k\)th subcarrier to exceed a pre-specified threshold \(T_k\) is less than \(p_k\). In order to design the hybrid beamforming, the major challenge now is to convert the probabilistic constraint into a deterministic form. We define a non-negative random variable \(Z_k = (W f_k)^H (\hat{H}_{kp} + \hat{E}_k)^H (\hat{H}_{kp} + \hat{E}_k)(W f_k)\). Using Markov’s inequality, the left hand side of the constraint satisfies

\[
Pr \{ Z_k \geq T_k \} \leq \frac{\mathbb{E} [Z_k]}{T_k}.
\]

(28)

Taking expectation with respect to the error matrix, we obtain \(\mathbb{E} [Z_k]\) as

\[
\mathbb{E} [Z_k] = \mathbb{E} \left[ \text{tr} \left\{ (\hat{H}_k^H \hat{H}_k + \hat{E}_k^H \hat{E}_k) M_k \right\} \right]
\]

\[
= \text{tr} \left\{ (\hat{H}_{kp}^H \hat{H}_{kp} + (K - 1)\sigma_E^2 I_{N_{\text{BS}}}) M_k \right\}.
\]

(29)

With (28) and (29), it can be easily verified that the probabilistic constraint is satisfied if

\[
\text{tr} \left\{ (\hat{H}_{kp}^H \hat{H}_{kp} + (K - 1)\sigma_E^2 I_{N_{\text{BS}}}) M_k \right\} \leq p_k T_k.
\]

Hence, the probabilistic problem in (26) can be transformed into the following form by reformulating the objective function and the probabilistic constraint, which is presented by

\[
\{M_k\} = \arg \max \text{tr} \left\{ (H_{kp}^H H_{kp} + \sigma_E^2 I_{N_{\text{BS}}}) M_k \right\}
\]

\[
\text{s.t. } \text{tr} \left\{ (\hat{H}_{kp}^H \hat{H}_{kp} + (K - 1)\sigma_E^2 I_{N_{\text{BS}}}) M_k \right\} \leq p_k T_k,
\]

(30)
where $M_k$ is a matrix to be optimized. We emphasize that only the expectation of channel covariance matrices, i.e., $\mathbb{E}\left[(H_{kp} + E_k)^H(H_{kp} + E_k)\right]$, is required at the BS. This indicates that the exact estimation error variances for all subcarriers are no longer needed for our robust scheme, which is more practical than the conventional statistical scheme. The optimal solution can be obtained by a standard mathematical programming CVX in [42]. We note that the objective function is a convex expression since $\text{tr}\left\{\left(H_{kp}^H H_{kp} + \sigma^2_k \mathbf{I}_{\text{BS}}\right) M_k\right\}$ is a complex value. However, maximizing a convex expression in the objective function cannot be solved in the CVX. Here, we propose a greedy separate optimization method where the amplitude and the angle of complex entries in $M_k$ are optimized separately. First, we propose to solve the problem in (30) only considering the amplitude information and the obtained optimal solution with CVX, $M_k^{\text{opt}}$, is a real matrix. Then, the angle information of entries in $M_k^{\text{opt}}$ is optimized by applying randomization technique. The idea of the randomization technique is to generate a set of candidate complex vectors using $M_k^{\text{opt}}$ and then choose the best solution from these candidate complex vectors. More specifically, we calculate the EVD of $M_k^{\text{opt}} = \hat{U}_k \hat{\Lambda}_k \hat{U}_k^H$ and obtain the candidate complex vectors by $f_{k,i}^{\text{cond}} = \hat{U}_k \hat{\Lambda}_k^{1/2} v_i$, where $v_i$ is a zero-mean complex Gaussian vector whose covariance matrix is $\mathbf{I}$. It is necessary to check whether the constraint are violated by the candidate vector. The optimal hybrid beamforming vector for each subcarrier, $f_{\text{hy},k}^{\text{opt}}$, is selected among those candidate complex vectors, which has the smallest norm. With $f_{\text{hy},k}^{\text{opt}}$, we can recover the optimal analog beamforming matrix and digital beamforming vectors using the method in Section IV.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we evaluate the spectral efficiencies of our proposed hybrid beamforming schemes. Here, the spectral efficiency is defined as the average achievable data rate per subcarrier. To demonstrate the benefits of our proposed schemes, we consider two benchmark schemes: i) The unconstrained fully digital beamforming scheme with no IBI and ii) The existing hybrid beamforming scheme proposed in [18]. Throughout this section, the operating frequency is set as 0.35 THz to 0.45 THz and the bandwidth of each subcarrier is $B = 5$ GHz. For the considered massive MIMO-OFDM system, the number of subcarriers is $K = 20$. The angles of phase shifters are quantized with 3 bits. The channels consist of one LOS path and four NLOS paths. The azimuth AoAs and AoDs of the LOS path are assumed to be uniformly distributed in $[0, 2\pi]$.
and the elevation AoAs and AoDs of the LOS path are uniformly distributed in \([-\pi/2, \pi/2]\). The CFO ratio is set as \(\epsilon = 0.3\). The AWGN power is \(\sigma_n^2 = -75 \text{ dBm}\).

Fig. 1 plots the spectral efficiencies of proposed hybrid beamforming schemes with IBI versus \(P_s\). For the fairness of comparison, the receivers in all schemes are set as the same analog combiner, considering that only one RF chain is equipped at the UE. Our proposed hybrid beamforming schemes employ the RCI method to eliminate the IBI, while the existing hybrid beamforming scheme proposed in [18] employs the ZF method. We also plot the spectral efficiency of the hybrid beamforming scheme where the IBI is treated as noise, i.e., no interference elimination, for comparison. We first observe that the performance of our proposed hybrid beamforming schemes, especially the wideband hybrid beamforming scheme with two digital beamforming matrices, is close to that of the unconstrained fully digital beamforming scheme, which demonstrates the accuracy of Theorem 2. Considering that our proposed schemes have much lower hardware complexity than the unconstrained fully digital beamforming scheme, this observation demonstrates the significant benefit of our proposed hybrid schemes, especially for the sake of practical implementation. Second, we observe that the performance of our proposed hybrid beamforming scheme with two digital beamforming matrices is better than that of our
proposed codebook-based hybrid beamforming scheme. This performance gap mainly comes from the compensation beamforming at the baseband and the quantification to phase shifters. Although the proposed scheme using codebooks is not optimal for rate maximization, it is also an attractive alternative since phase shifters are mostly digitally controlled in the THz band. Third, our proposed hybrid beamforming schemes significantly outperform the existing hybrid beamforming scheme proposed in [18]. This observation reveals the advantage of our proposed hybrid beamforming schemes, especially over frequency selective fading. We also observe that the spectral efficiency of the proposed hybrid scheme with the revised RCI method is much higher than that of the scheme without IBI elimination, especially when the transmit power is high. This indicates that the IBI has a significant impact on the system performance and the revised RCI method is effective to eliminate the IBI.

Fig. 2 plots the spectral efficiencies of our proposed hybrid beamforming schemes versus the number of antennas at each subarray. In this figure, the number of subarrays keep the same. The number of RF chains is fixed as 8 for hybrid beamforming schemes while the number of RF chains is equal to the number of antennas for the fully digital beamforming scheme. First, we observe that the performance of all four beamforming schemes improves when the number of
Fig. 3. Spectral efficiencies of hybrid beamforming schemes versus $d$ with $P_s = 20$ dBm, $N_{BS} = 128$, and $M_r \times N_r = 4 \times 4$.

antennas at each subarray increases. This indicates that the spectral efficiency can be improved by increasing $N_{BS}$, instead of increasing $N_{RF}$, since the use of RF chains leads to high hardware complexity and power consumption. Second, we observe that the performance of all beamforming schemes improves slowly when $N_{BS}$ is large. For the fully digital beamforming scheme, the slowdown in growth mainly comes from the correlation effect among massive antennas. For hybrid beamforming schemes, the slowdown in growth is more obvious than that of the fully digital scheme, which is because the performance of hybrid beamforming schemes is mainly limited by the number of RF chains when $N_{BS}$ is large.

Fig. 3 plots the spectral efficiencies of our proposed hybrid beamforming schemes versus $d$. Fig. 3 shows that the spectral efficiencies decrease rapidly as the BS-UE distance increases, which is resulting from the severe path loss in THz channels. Although a massive number of antennas are equipped at the BS, the maximum effective transmission distance is generally limited to 10 m. We also plot the spectral efficiencies of our proposed hybrid beamforming schemes with different $N_{RF}$. While $N_{BS}$ is fixed, the number of RF chains is set as $N_{RF} = 4$ or $N_{RF} = 8$. We observe that the spectral efficiencies of our proposed hybrid beamforming schemes with 8 RF chains are higher than those with 4 RF chains. Specifically, the performance of our proposed wideband hybrid beamforming scheme with two digital beamforming matrices is comparable
to that of the unconstrained fully digital beamforming scheme when $N_{RF} = 8$. However, when $N_{RF} = 4$, the performance of our proposed hybrid beamforming schemes is much lower than that of the unconstrained fully digital beamforming scheme. Considering that the number of NLOS channel paths is 4, this observation demonstrates that $N_{RF}$ need to be higher than the rank of THz channels.

Fig. 4 plots the spectral efficiencies of the proposed wideband hybrid beamforming scheme with two digital beamforming matrices versus $P_s$. In this figure, we compare the performance in three cases. In the first case, the perfect CSI is assumed to be known at the BS. In the second and third cases, only imperfect CSI is known at the BS where the variance of channel estimation error is $1 \times 10^{-8}$. The proposed robust hybrid beamforming scheme is adopted to mitigate the channel estimation error in the second case, where the interference power threshold in the probabilistic constraint is set as $T_k = 1 \times 10^{-8}$ and the probability threshold is set as $p_k = 5\%$. The non-robust hybrid beamforming scheme is used in the third case. We observe that the proposed robust hybrid beamforming scheme provides a clear performance advantage over the non-robust one. When $P_s \leq 12$ dBm, the performance gap between the robust scheme and the non-robust scheme increases with $P_s$. This is because that the variance of channel estimation
error is assumed to be constant and independent of the transmit power. When $P_s$ is low, the interference can be effectively controlled by our proposed probabilistic approach and the Gaussian noise plays a major role for the performance loss. However, when $P_s$ becomes very high, the performance gap between the robust scheme and the non-robust scheme almost keeps constant. This is because when $P_s$ is very high, the detrimental impact of the IBI becomes extremely severe. The probabilistic constraint might be too strict and the probabilistic approach cannot eliminate the interference, which is limited by the performance of the mathematical programming. Thus, the IBI plays an important role for the performance loss in the extreme high transmit power regime.

VII. CONCLUSION

In this work, we proposed new wideband hybrid beamforming schemes for the THz massive MIMO-OFDM system over frequency selective fading. Importantly, we considered the CFO and unique THz channel characteristics in the system modeling and design. We first built a 3D THz channel model to characterize the frequency selective propagation in an indoor environment. Based on this model, we proposed a two-stage wideband hybrid beamforming scheme using a normalized beamsteering codebook searching algorithm for analog beamforming and a revised RCI method for digital beamforming. To avoid the exhaustive search over codebooks, we proposed a novel wideband hybrid beamforming scheme with two digital beamformers using the EVD of channel covariance matrices. Furthermore, we analyzed the impact of channel estimation errors and then designed a robust hybrid beamforming scheme to combat the imperfect CSI. Using numerical results, we showed that our proposed schemes exhibit a comparable spectral efficiency to the fully digital beamforming. Also, our proposed schemes achieve a significantly higher spectral efficiency than the existing hybrid beamforming scheme. Thus, our design reveals the profound practicality to employ frequency selective hybrid beamforming in THz wireless communications.

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