CARMA: Fair and efficient bottleneck congestion management with karma

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Abstract

In this paper, we propose a traffic demand management scheme, named CARMA, to address the morning commute congestion with heterogeneous travelers. We introduce karma as a non-tradable mobility credit used by the commuters to bid for access to a fast lane that is kept in free-flow. Commuters who did not bid enough to enter the fast lane instead use a slow lane in which they might experience congestion. At the end of each day, the karma spent by those entering the fast lane is evenly redistributed to all commuters, and this process repeats indefinitely. A core novelty of our approach is that we generalize the notion of Value of Time (VOT) by allowing it to vary dynamically each day, e.g., as a function of the trip purpose, rather than being a static characteristic of each individual. This allows us to assess the efficiency and fairness of the system over the long-term instead of based on a single day. We formalize the recurrent interactions among self-interested commuters as a dynamic population game, and prove the existence of a Stationary Nash Equilibrium (SNE). We then investigate the performance of CARMA through three numerical case studies with different assumptions on the heterogeneity in VOT. Our results demonstrate that CARMA closely approaches the efficiency of classical optimal congestion pricing, while preventing individuals with a higher (monetary) VOT from consistently taking advantage of the fast lane, thereby providing a fair and efficient solution to the bottleneck congestion.

Keywords: karma mechanism, bottleneck model, dynamic population game, traffic demand management
1. Introduction

For decades, traffic congestion has been causing tremendous social cost to major cities around the world. According to the INRIX 2019 Global Traffic Scorecard report, drivers in the U.S. lost an average of 99 hours a year due to congestion, costing nearly $88 billion in total. The situation was even worse in the U.K., where drivers on average spent 115 hours in traffic \[1\]. To manage rush hour traffic, a wide variety of tools have been proposed in the literature as well as implemented in practice. Among them, congestion pricing is the most widely known due to its theoretical efficiency. It suggests to internalize the cost of the negative externalities of individual travelers on the social cost \[2, 3, 4\]. However, the classical congestion pricing is often arguably politically and socially infeasible \[5\] as it tends to favor wealthier travelers \[6, 7, 8\]. Moreover, it is difficult to determine and implement the theoretically optimal tolls in real time \[9, 10, 4\]. As a result, most existing congestion pricing schemes are restricted to zone-, cordon- and facility-based schemes with flat rates during certain time windows \[4, 11\].

Due to the limitations of congestion pricing, a growing attention has been drawn to alternative quantity- and credit-based approaches. The former directly limit the number of vehicles on the road, e.g., through license-plate rationing \[12, 13\] or highway reservation \[14, 15\], while the latter assign a limited number of travel credits/permits to road users which can be traded in a monetary market \[16, 17, 18\]. These approaches are preferable to classical congestion pricing as they avoid a net financial flow from road users to authorities, as well as mitigate some of the aforementioned practical difficulties \[19, 5\]. Nevertheless, these approaches fail to address the fairness issue. On the one hand, the license-plate rationing implemented in China has induced wealthy travelers to purchase additional vehicles \[20\]. On the other hand, wealthy travelers can also take advantage of the credit-based schemes since they have a larger capacity to buy credits than others \[21\]. As a result, more transport capacities would be allocated to travelers with higher income levels. Instead, a fair and efficient mechanism should allocate these capacities based on the true urgency of the travelers that could vary on a daily basis. Therefore, rather than quantifying Value of Time (VOT) on a universal scale, we consider it to follow a dynamic process over days. In our setting, a higher VOT means that the individual has a more urgent need for congestion-free travel today in comparison to other days, e.g., because it is late for an important appointment or must catch an important flight.
In this study, we propose CARMA, a novel congestion management scheme that uses non-monetary mobility credits called karma. Similar to other credit schemes, CARMA gives rise to a highly efficient traffic assignment without requiring any private information about the travelers. Differently, CARMA also manages to address the fairness issue discussed above because karma is not exchanged in a monetary market.

1.1. Related work

As a major contributor to urban traffic congestion, the morning commute problem has been studied extensively in the literature using the stylized bottleneck model \cite{3, 22, 23, 24}, where commuters between a single origin-destination pair arrive at a bottleneck dynamically and form a queue due to its limited capacity. It is shown that, without intervention, the selfish choice of departure time by the commuters leads to costly traffic congestion. In contrast, a time-varying monetary toll, known as Vickrey’s toll, is theoretically capable of achieving the socially optimal traffic assignment where the bottleneck congestion is completely eliminated.

The original bottleneck model has been extended in various directions. One of them regards the design of a special lane that can only be used by a certain group of vehicles. The discussion started with the High Occupancy Vehicle (HOV) lanes that are exclusive for carpoolers \cite{25}, then extended to the High Occupancy Toll (HOT) lanes to increase the utilization rate of HOV lanes meanwhile generating toll revenue \cite{26}. It is found that both HOV and HOT promote carpooling and reduce social cost, and converting an HOV lane into an HOT lane may further increase the carpool ratio \cite{27}. Recent studies also discuss the capacity allocation between regular and HOV lanes \cite{28}. Moreover, it has been recently proposed to dedicate lanes for connected and autonomous vehicles, driven by their potential in increasing road capacity \cite{29}.

Another line of research examines the performance of tradable credits in managing traffic during the rush hours. In \cite{21}, Vickrey’s toll is substituted by a time-varying credit charging scheme coupled with a daily initial allocation of credits. When late arrival is not allowed, the authors have shown that an optimal scheme could achieve the system optimum and eliminate congestion. Yet, similar to Vickrey’s toll, a time-varying charging scheme could be challenging to compute in practice. Hence, a step-wise charging scheme is proposed in \cite{30}. Specifically, travelers who pass the bottleneck during the peak time either pay a fixed amount of credits or a higher toll, while
those traveling during the off-peak are rewarded with credits. The analysis is extended in [19] to consider elastic demand with alternative mode choice. Although the tradable credit scheme solves several shortcomings of congestion pricing, the authority still needs to design the credit charging scheme. The design is hardly optimal since the private information of travelers, e.g., their VOT, is often not available. Moreover, the monetary trading of credits leads to favoring high income travelers, as discussed above.

In essence, traffic demand management regards the efficient allocation of the scarce transport resources to the transport users. Accordingly, a few studies introduced auctions as a mechanism design approach to perform this resource allocation under private user information. In [17], a one-shot auction is designed to determine the times and duration each traveler may visit the downtown area. The optimal allocation is solved via a combinatorial optimization problem, albeit the bidding strategy of each traveler is not explicitly modeled (the bids are considered exogenous). A similar combinatorial auction is proposed in [31] to allocate traffic in a more general network. To induce truthful reporting of private values, the Vickrey-Clarke-Groves (VCG) mechanism is used to compute the payment of each traveler for its assigned path. The same mechanism has been introduced to the optimal pricing and matching in ride-sharing [32, 33, 34] and car-sharing [35, 36]. A few auction-based mechanisms are also proposed to manage the bottleneck traffic. In [37], the peak period is divided into small intervals and travelers may bid for each interval over multiple days. To promote ride-sharing, [38] designs a mechanism that jointly determines the permit allocation and the shared trip assignment. Since the auctions in all of these approaches are monetary, they are also subject to the fairness issue discussed above.

The sensitivity towards the use of money is not exclusive to traffic demand management. In the economics literature, a significant body of works has regarded the design of mechanisms without money [39], such as organ donations [40, 41, 42] and college admissions [43, 44]. Despite of some successes, non-monetary mechanisms are in general difficult to design in a truth-revealing manner since they cannot rely on a general incentive instrument like money [45, 46]. However, when the resource is repeatedly contested, a few studies have shown that repetition can be leveraged to incentivize truthfulness [47, 48]. The core principle is to let the resource users trade-off between immediate and future access to the resource. In recent work, this principle is formalized in a practical mechanism, called karma mechanism, in which users are allocated karma points to bid for the resource, and karma is transferred
directly from those who manage to acquire the resource to those who yield it [49]. This provides the departure point for the present study, which applies karma mechanisms to bottleneck congestion management.

1.2. Paper contribution

In this study, a bottleneck model is constructed to represent the dynamic traffic during rush hour. The model consists of a fast lane operated at or below capacity and a slow lane subject to congestion. Our proposed scheme, called CARMA, regulates access to the fast lane as follows. All commuters are given an equal initial endowment of karma points which they use to bid on accessing the fast lane on a daily basis, based on their current VOT. At the end of each day, karma is transferred directly from commuters who entered the fast lane to those who entered the slow lane. In contrast to previous works, there exists no monetary market to trade karma. This could encourage dedicating more capacity to the fast lane and assist in further decongestion. Moreover, since karma cannot be purchased by money, it stands in sharp contrast to existing tradeable credit and monetary auction schemes. We demonstrate that this non-monetary instrument fosters a fair and efficient use of the transport capacity.

The main contributions of the paper can be summarized as follows:

• we formalize CARMA as a dynamic population game [50] which rigorously incorporates the commuters’ strategic behavior. We prove that a Stationary Nash Equilibrium (SNE) exists in this game;

• through numerical analysis, we demonstrate the efficiency of CARMA in reducing the bottleneck congestion, which is close to the theoretical efficiency of an optimal monetary tolling scheme;

• we further analyze the fairness of CARMA from the perspective of individual commuters with heterogeneous VOT processes and show that it outperforms the classical monetary policy.

1.3. Notation

Let $a, d \in D \subseteq \mathbb{N}$ and let $c \in C \subseteq \mathbb{R}^n$, then for a function $f : D \times C \to \mathbb{R}$, we distinguish discrete and continuous arguments through the notation $f[d](c)$. Alternatively, we write $f : C \to \mathbb{R}^{[D]}$ as the vector-valued function $f(c)$, with $f[d](c)$ denoting its $d^{th}$ element. Similarly, $g[a \mid d](c)$ denotes the conditional probability of $a$ given $d$ and $c$. Specifically, $g[d^+] \mid d](c)$
denotes one-step transition probabilities for \( d \). We denote by \( p \in \Delta(D) := \{ \sigma \in \mathbb{R}^D_+ \mid \sum_{d \in D} \sigma[d] = 1 \} \) a probability distribution over the elements of \( D \), with \( p[d] \) denoting the probability of element \( d \). Finally, when considering heterogeneous commuter types, we denote by \( x_\tau \) a quantity associated to type \( \tau \).

2. Preliminaries

2.1. Bottleneck model with lane segmentation

To formalize CARMA, we first recast a classical analysis framework for the morning commute problem \([15]\) that is suitable to the karma mechanism. Consider a bottleneck with \( N \) commuters sharing the same desired arrival time \( t^* \) and experiencing a single source of congestion identified by the bottleneck capacity \( s \). The bottleneck is split into two lanes: the fast lane with capacity \( s_{\text{fast}} \) is tolled and free of congestion, while the regular (or slow) lane with capacity \( s_{\text{slow}} \) is subject to congestion. Let \( \alpha, \beta, \gamma \) be the normalized queuing, early arrival, and late arrival penalty, respectively. As in [51] and [21], we assume that commuters are heterogeneous in their Value of Time (VOT) but share the same ratios of queuing penalty over early/late arrival penalty, i.e., \( \frac{\alpha}{\beta} \) and \( \frac{\alpha}{\gamma} \). This setup can be modeled by classifying the commuters into \( M \) groups. Each group \( j \in \{1, \ldots, M\} \) is associated with a multiplier \( u_j \) describing their VOT, and without loss of generality we consider that \( u_1 < \cdots < u_M \). Then, the travel cost for commuters who enter the fast lane at \( t \) reads

\[
c_{\text{fast}}[u](t) = \begin{cases} u \beta (t^* - t), & t \leq t^*, \\ u \gamma (t - t^*), & t > t^*, \end{cases}
\]

and their total cost is given by \( \hat{c}_{\text{fast}}[u](t) = c_{\text{fast}}[u](t) + p(t) \), where \( p(t) \) is the toll price at time \( t \). If they commute via the slow lane, then their cost is

\[
c_{\text{slow}}[u](t) = \begin{cases} u \left[ \alpha \frac{q(t)}{s_{\text{slow}}} + \beta (t^* - t) - \frac{q(t)}{s_{\text{slow}}} \right], & t + \frac{q(t)}{s_{\text{slow}}} \leq t^*, \\ u \left[ \alpha \frac{q(t)}{s_{\text{slow}}} + \gamma (t + \frac{q(t)}{s_{\text{slow}}} - t^*) \right], & t + \frac{q(t)}{s_{\text{slow}}} > t^*, \end{cases}
\]

where \( q(t) \) denotes the queue length on the slow lane at time \( t \). Given the departure rate \( \lambda_{\text{slow}}(t) \), the queue is computed as \( q(t) = \int_{t_{\text{start}}}^{t} (\lambda_{\text{slow}}(x) - s_{\text{slow}})dx \), where \( t_{\text{start}} \) is the time when the queue emerges (given below).
Since the VOT ratios are the same among all commuters, the departure rate on the slow lane follows the same pattern as in the homogeneous case. According to \[52\] Eq. 7-10, the starting and ending times of the peak hours (\(t_{\text{start}}\) and \(t_{\text{end}}\)), as well as the time with maximum queuing delay (\(t_{\text{peak}}\)), read respectively

\[
\begin{align*}
  t_{\text{start}} &= t^* - c^*/\beta, \\
  t_{\text{end}} &= t^* + c^*/\gamma, \\
  t_{\text{peak}} &= t^* - c^*/\alpha,
\end{align*}
\] (3)

where \(c^* = \frac{\beta\gamma}{\beta + \gamma} \frac{N}{s}\). The equilibrium cost of each group on the slow lane is given by \(c_j^* = u_j c^*\).

Figure 1: Illustration of equilibrium in base scenario with optimal pricing of the fast lane.

Figure 1 illustrates the cumulative departures and arrivals on the slow lane, as well as the optimal toll price on the fast lane at equilibrium. The departure and arrival curves on the fast lane are omitted as they coincide with each other with constant rate \(s_{\text{fast}}\). We follow a reasoning similar to \[53\] to compute the optimal toll price \(p^*(t)\) on the fast lane that leads to the social optimum. Consequently, the tolling policy is piece-wise linear and commuters with higher VOT multipliers would depart closer to \(t^*\) (see Figure 1b). Letting \([t_j, t_{j+1}]\) and \([t'_{j+1}, t'_j]\) be the time intervals during which group \(j\) enter the fast lane, the slope of the optimal toll price \(p^*(t)\) satisfies

\[
\dot{p}^*(t) = \begin{cases} 
  u_j \beta, & t \in [t_j, t_{j+1}], \\
  -u_j \gamma, & t \in [t'_{j+1}, t'_j].
\end{cases}
\] (4)
2.2. Karma mechanism: theoretical framework

In this section, we revisit the karma mechanism firstly introduced in [49]. In brief, it repeatedly allocates a resource to a group of competing agents over an infinite time horizon. At each time step, an agent endowed with an integer quantity \( k \in \mathbb{N} \), called karma, can submit an integer bid \( b \in \mathcal{B}[k] = \{0, \ldots, k\} \) that cannot exceed the karma, hereafter referred to as a karma bid. The resource is then allocated through an auction-like mechanism, and karma is transferred from the agents who acquire the resource to those who yield it in accordance to a karma payment and redistribution rule to be designed. Apart from the karma, each agent also features an urgency state \( u \in \mathcal{U} = \{u_1, \ldots, u_M\}, u > 0 \). It describes the urgency to acquire the resource (e.g., the VOT in a morning commute scenario). Given a finite number of agent types \( \tau \in \Gamma \subset \mathbb{N} \), the urgency of an agent of type \( \tau \) evolves according to an exogenous Markov chain, denoted by \( \phi_\tau[u^+ | u] \). The principle of a karma mechanism is to facilitate a fair and efficient allocation of the resource. Specifically, it incentivizes agents to save karma for periods of high urgency and thus enables the resource to be assigned to those who need it the most.

Formally, the karma mechanism is modeled as a dynamic population game [50]. Namely, the number of agents \( N \) is assumed large, thus they can be approximated by a continuum of mass. The distribution of agent types is compactly denoted by \( g \in \Delta(\Gamma) \), where \( g_\tau \in [0,1] \) is the mass of agents in type \( \tau \in \Gamma \). Accordingly, the time-varying joint type-state distribution is given by

\[
d \in \mathcal{D} = \left\{ d \in \mathbb{R}^{|\Gamma| \times |\mathcal{X}|} \left| \sum_{u,k} d_\tau[u,k] = g_\tau, \quad \forall \tau \in \Gamma \right. \right\},
\]

where \( d_\tau[u,k] \) denotes the mass of agents in the static type \( \tau \) and dynamic state \((u,k) \in \mathcal{X}\), and \( \mathcal{X} = \mathcal{U} \times \mathbb{N} \).

At each time step, each agent can choose an action \( a \) from a finite state-dependent discrete set \( \mathcal{A}[u,k] \). The action includes the agent’s bid \( b \) as well as other decisions, e.g., the departure time in a morning commute scenario. Agents of the same type \( \tau \) follow the homogeneous randomized policy \( \pi_\tau : \mathcal{X} \to \Delta(\mathcal{A}[u,k]) \), where \( \pi_\tau[a | u,k] \) denotes the probabilistic weight that these agents place on action \( a \) when in state \((u,k)\). The concatenation of the policies of all types \( \pi = (\pi_\tau)_{\tau \in \Gamma} \) is simply referred to as the policy, and the space of policies is denoted by \( \Pi \).
In the dynamic population game, the tuple of type-state distribution and policy \((d, \pi)\) is referred to as the social state because it gives a macroscopic description of the distribution of agents in the population as well as their behaviors. Hence, all the outcomes at each time step are functions of \((d, \pi)\). Let \(\kappa[k^+ | k, a](d, \pi)\) be the karma transition function that describes how the agent’s karma changes between two consecutive time steps given its current karma \(k\) and action \(a\). Then, together with the urgency transition function \(\phi(u^+ | u)\), the joint state transition function is given by

\[
\rho_{\tau}[u^+, k^+ | u, k, a](d, \pi) = \phi(u^+ | u) \kappa[k^+ | k, a](d, \pi).
\] (6)

Figure 2 illustrates a one-step state transition of an agent of type \(\tau\). Moreover, we define \(\zeta[u, a](d, \pi)\) as the immediate reward function of each agent in urgency \(u\) taking action \(a\). Both the immediate reward and the karma transition will be further specified in Section 3. Yet, two conditions are required to ensure that the karma mechanism is well defined.

**Assumption 1** (Continuity). The immediate reward function \(\zeta[u, a](d, \pi)\) and the karma transition function \(\kappa[k^+ | k, a](d, \pi)\) are continuous in the social state \((d, \pi)\).

**Assumption 2** (Karma preservation in expectation). Karma is preserved in expectation for all \((d, \pi)\), i.e., \(E[k^+] = E[k]\), which expands to

\[
\sum_{\tau, u, k} d_{\tau}[u, k] \sum_{a} \pi_{\tau}[a | u, k] \sum_{k^+} \kappa[k^+ | k, a](d, \pi) k^+ = \sum_{\tau, u, k} d_{\tau}[u, k] k.
\] (7)

In brief, Assumption 2 requires that the total amount of karma in the system remains the same, i.e., the amount paid by agents equals the amount
received by agents. The readers are referred to [49] for an in depth discussion on the assumptions and functions introduced above.

Given the social state, each agent faces a Markov decision process. Specifically, the expected reward of the agents of type $\tau$ is given by

$$ R_\tau[u,k](d,\pi) = \sum_a \pi_\tau[a | u,k] \zeta[u,a](d,\pi), \quad (8) $$

and the state transition follows

$$ P_\tau[u^+,k^+ | u,k](d,\pi) = \sum_a \pi_\tau[a | u,k] \rho_\tau[u^+,k^+ | u,k,a](d,\pi). \quad (9) $$

Accordingly, the expected return in the infinite horizon, also known as the value function, is derived as

$$ V_\tau[u,k](d,\pi) = R_\tau[u,k](d,\pi) + \delta \sum_{u^+,k^+} P_\tau[u^+,k^+ | u,k](d,\pi) V_\tau[u^+,k^+](d,\pi), \quad (10) $$

where $\delta \in (0,1]$ is the discount factor. A smaller $\delta$ models a more myopic agent behavior. To describe the rational decision of each agent, we also need to define the state-action value function, also called $Q$-function, as

$$ Q_\tau[u,k,a](d,\pi) = \zeta[u,a](d,\pi) + \delta \sum_{u^+,k^+} \rho_\tau[u^+,k^+ | u,k,a](d,\pi) V_\tau[u^+,k^+](d,\pi). \quad (11) $$

Then, to maximize the long-term return, each agent chooses a policy based on the best response correspondence, given by

$$ B_\tau[u,k](d,\pi) = \left\{ \sigma \in \Delta(\mathcal{A}(u,k)) \mid \forall \sigma' \in \Delta(\mathcal{A}(u,k)), \right. \left. \sum_a (\sigma[a] - \sigma'[a]) Q_\tau[u,k,a](d,\pi) \geq 0 \right\}. \quad (12) $$

By definition, $B_\tau[u,k](d,\pi)$ gives the set of randomized individual actions that maximize the $Q$-function in state $(u,k)$ for agents of type $\tau$.

We are finally ready to define the equilibrium state under a karma mechanism.
**Definition 1** (Stationary Nash Equilibrium (SNE)). A **Stationary Nash Equilibrium** is a social state \((d^*, \pi^*) \in D \times \Pi\) such that, for all \((\tau, u, k) \in \Gamma \times U \times N\),
\[
d^*_\tau[u,k] = \sum_{u^-,k^-} d^*_\tau[u^-,k^-] P_{\tau}[u,k \mid u^-,k^-](d^*, \pi^*),
\]
\[(13)\]
\[
\pi^*_\tau[\cdot \mid u,k] \in B_{\tau}[u,k](d^*, \pi^*).
\]
\[(14)\]

The **SNE** is similar to the classical notion of Nash equilibrium in that it denotes a state of the game where all agents have no incentive to unilaterally deviate from the equilibrium policy \(\pi^*\) given by Eq. (14), but it additionally requires that the type-state distribution \(d^*\) is **stationary** as per Eq. (13).

We conclude this section by stating the conditions for a karma mechanism to guarantee the existence of a **SNE** (see [49] for the proof).

**Theorem 1** (Existence of Stationary Nash Equilibrium, [49] Theorem 1). If Assumptions 1 and 2 hold, then, given \(\bar{k} \in \mathbb{N}\), there always exists a **SNE** \((d^*, \pi^*)\) such that \(\sum_{\tau,u,k} d^*_\tau[u,k] k = \bar{k}\), and \(\bar{k}\) is the average amount of karma per agent in the system.

### 3. CARMA: bottleneck congestion management with karma

We now specialize the general karma mechanism above to model the morning commute problem. We consider \(N\) commuters that travel daily through a bottleneck. The commuters are heterogeneous in their **VOT process**, that is, the process by which their daily VOT multiplier \(u\) changes. We discretize the feasible departure times into \(T\) intervals. Therefore, on every morning, commuters with state \((u,k)\) make a decision on their departure time \(t \in T = \{1, \ldots, T\}\) and bid to enter the fast lane \(b \in \mathcal{B}[k]\). Consequently, the highest \(s_{\text{fast}}\) bidders departing at \(t\) are allowed to enter the fast lane, while all others have to use the slow lane. The mechanism is illustrated in Figure 3.

In what follows, we first specify the key elements for **CARMA**, i.e., the immediate reward and karma transition function, along with their properties to ensure the existence of equilibrium.

#### 3.1. Immediate reward function \(\zeta[u,t,b](d, \pi)\)

Following the classical bottleneck model, we define the immediate reward in two parts: queuing delay \(t^q\) and early or late schedule delay \((t^e\) or \(t^l\), respectively). Given the departure time and bid, how much delay each commuter
Figure 3: Illustration of Karma mechanism in bottleneck.

endsures depends on the outcome of the karma auction. Let $\psi[o \mid t, b](d, \pi)$ denote the probability of an ego commuter finally entering lane $o \in \{\text{fast, slow}\}$, given its choice of $t$, bid $b$ and the other commuters’ actions (function of the social state $(d, \pi)$). Then, the immediate reward can be written as

$$
\zeta[u, t, b](d, \pi) = -u \sum_o \psi[o \mid t, b](d, \pi) \left( \alpha t^q[t, o](d, \pi) + \beta t^e[t, o](d, \pi) + \gamma t^l[t, o](d, \pi) \right),
$$

(15)

where $t^q[t, o](d, \pi)$, $t^e[t, o](d, \pi)$ and $t^l[t, o](d, \pi)$ are given by

$$
t^q[t, o](d, \pi) = \begin{cases} 0, & o = \text{fast}, \\
\frac{q[t](d, \pi)}{q_{\text{slow}}}, & o = \text{slow}, \end{cases}
$$

(16)

$$
t^e[t, o](d, \pi) = \max\{0, t^* - t - t^q[t, o](d, \pi)\},
$$

(17)

$$
t^l[t, o](d, \pi) = \max\{0, t + t^q[t, o](d, \pi) - t^*\}.
$$

(18)

In Eq. 16 $q[t](d, \pi)$ gives the queue length on the slow lane at time $t$, i.e., it is the discrete-time equivalent of $q(t)$ defined in Section 2.1. Accordingly, if the commuter enters the fast lane, Eq. 15 reduces to Eq. 1 (negated for rewards instead of costs). Otherwise, it reduces to Eq. 2.

To complete the definition of Eq. 15, we now derive $\psi[o \mid t, b](d, \pi)$. We define a threshold bid $b^*$ such that

- if $b > b^*$, the commuter enters the fast lane for sure, i.e., $o = \text{fast}$;
- if $b < b^*$, the commuter enters the slow lane for sure, i.e., $o = \text{slow}$;
- if $b = b^*$, the commuter ties with others and enters the fast lane via a random draw on the remaining capacity.
Let $\nu[t, b](d, \pi)$ be the mass of commuters departing at $t$ and bidding $b$, i.e.,

$$
\nu[t, b](d, \pi) = \sum_{\tau, u, k} d_{\tau}[u, k] \pi_{\tau}[t, b | u, k].
$$

(19)

Then, $b^*$ is given by

$$
b^* = \max \left\{ b \in \mathbb{N} \left| \sum_{b' \geq b} \nu[t, b'](d, \pi) \geq \frac{s_{\text{fast}}}{N} \right. \right\}.
$$

(20)

Accordingly, the probability of entering the fast lane is derived as

$$
\psi(o = \text{fast} | t, b](d, \pi) =
\begin{cases}
1, & b > b^*, \\
0, & b < b^*, \\
\frac{s_{\text{fast}}/N - \sum_{b' > b^*} \nu[t, b'](d, \pi)}{\nu[t, b](d, \pi)} + \epsilon, & b = b^*.
\end{cases}
$$

(21)

Note that $\psi(o = \text{fast} | t, b](d, \pi)$ is continuous in $(d, \pi)$ except where $\nu[t, b](d, \pi) = 0$. To ensure that Assumption 1 is satisfied, we approximate Eq. 21 with a function that is continuous everywhere, given by

$$
\psi^\epsilon(o = \text{fast} | t, b](d, \pi) =
\begin{cases}
1, & \sum_{b' > b} \nu[t, b'](d, \pi) \leq \frac{s_{\text{fast}}}{N} - \nu[t, b](d, \pi) - \epsilon, \\
0, & \sum_{b' > b} \nu[t, b'](d, \pi) \geq \frac{s_{\text{fast}}}{N}, \\
\frac{s_{\text{fast}}/N - \nu[t, b](d, \pi) - \epsilon < \sum_{b' > b} \nu[t, b'](d, \pi) < s_{\text{fast}}/N}{\nu[t, b](d, \pi) + \epsilon},
\end{cases}
$$

(22)

where $\epsilon > 0$ is an arbitrarily small approximation parameter. A graphic illustration is shown in Figure 4, demonstrating the continuity of $\psi^\epsilon$ and that $\psi^\epsilon \to \psi$ as $\epsilon \to 0$.

3.2. Karma transition function $\kappa[k^+ | k, t, b](d, \pi)$

The karma transition function encodes the rules of how karma is transferred between the commuters. There are significant degrees of freedom in the design of these rules, though in this work we opt for a simple scheme. The experimentation of different schemes is an interesting extension that is left for future works. Suppose that all commuters entering the fast lane pay their bids, and at the end of each day, the total payments are uniformly
Probability of entering fast lane

$$\frac{\nu[\sigma t, b']}{N_{s\text{fast}}}$$

Mass of commuters bidding \(b' > b\)

Figure 4: Illustration of the exact and approximate outcome probability.

redistributed to all commuters in the system (including those who entered the fast lane). Formally, similarly to Eq. 1, let \(p[b, o]\) be the karma payment made by a commuter who bids \(b\), then we have

$$p[b, o] = \begin{cases} b, & o = \text{fast,} \\ 0, & o = \text{slow.} \end{cases}$$

Accordingly, the \textit{average payment} is computed by aggregating Eq. 23 over all commuters, i.e.,

$$\bar{p}(d, \pi) = \sum_{t, b} \nu[t, b|(d, \pi)} \sum_{o} \psi[o\mid t, b|(d, \pi)} p[b, o]$$

$$= \sum_{t, b} \nu[t, b|(d, \pi)} \psi[o = \text{fast} \mid t, b|(d, \pi)} b.$$  (24)

To preserve the integer value of karma, \([\bar{p}(d, \pi)]\) is randomly distributed to some commuters and \([-\bar{p}(d, \pi)]\) to the others. Specifically, the mass of commuters receiving \([\bar{p}(d, \pi)]\) units of karma is given by \(f(d, \pi) = \bar{p}(d, \pi) - [\bar{p}(d, \pi)]\). This yields the following karma transition probabilities, conditional
on the outcome $o$:

$$
P[k^+ | k, t, b, o](d, \pi) = \begin{cases} 
  f(d, \pi), & o = \text{fast}, \text{ and } k^+ = k - b + \lceil \bar{p}(d, \pi) \rceil, \\
  1 - f(d, \pi), & o = \text{fast}, \text{ and } k^+ = k - b + \lfloor \bar{p}(d, \pi) \rfloor, \\
  f(d, \pi), & o = \text{slow}, \text{ and } k^+ = k + \lceil \bar{p}(d, \pi) \rceil, \\
  1 - f(d, \pi), & o = \text{slow}, \text{ and } k^+ = k + \lfloor \bar{p}(d, \pi) \rfloor, \\
  0, & \text{otherwise.} 
\end{cases} \quad (25)$$

Note that Eq. 25 does not explicitly depend on the departure time $t$, but we write a dependency here for generality.\(^1\) It is straightforward to verify that Eq. 25 is continuous in $(d, \pi)$. Finally, we can construct the karma transition function as

$$
k[k^+ | k, t, b](d, \pi) = \sum_o \psi[o | t, b](d, \pi) P[k^+ | k, t, b, o](d, \pi). \quad (26)$$

Clearly, Eq. 26 is also continuous in $(d, \pi)$. The following result ensures that Assumption 2, necessary to invoke Theorem 1, also holds.

**Proposition 1.** The karma transition function Eq. 26 satisfies Assumption 2, i.e., karma is preserved in expectation.

**Proof.** We verify that condition Eq. 7 is satisfied as follows (the dependency

\(^1\)This allows further extensions on the redistribution rule, e.g., a non-uniform redistribution based on the departure time.
on \((d, \pi)\) is omitted in the notation):
\[
\sum_{\tau,u,k} d_{\tau}[u,k] \sum_{t,b} \pi[t,b | u,k] \sum_{k^+} \kappa[k^+ | k, t, b] k^+
\]
\[
= \sum_{\tau,u,k} d_{\tau}[u,k] \sum_{t,b} \pi[t,b | u,k] \sum_{o} \psi[o | t,b] \sum_{k^+} \mathbb{P}[k^+ | k, t, b, o] k^+
\]
\[
= \sum_{\tau,u,k} d_{\tau}[u,k] \sum_{t,b} \pi[t,b | u,k]
\]
\[
(\psi[o = \text{fast} | t,b] (k - b + \bar{p}) + \psi[o = \text{slow} | t,b] (k + \bar{p}))
\]
\[
= \sum_{\tau,u,k} d_{\tau}[u,k] k + \bar{p} - \sum_{\tau,u,k} d_{\tau}[u,k] \sum_{t,b} \pi[t,b | u,k] \psi[o = \text{fast} | b] b
\]
\[
= \sum_{\tau,u,k} d_{\tau}[u,k] k.
\]

The equality from the second to the third line is due to \((1 - f) \lfloor \bar{p} \rfloor + f \lceil \bar{p} \rceil = \bar{p} \).

Given Proposition 1 and the continuity of the immediate reward Eq. 15 and karma transition Eq. 26, the karma mechanism for the bottleneck model is well-posed, i.e., a SNE is guaranteed to exist. The equilibrium can be computed using an evolutionary dynamics \cite{54} inspired algorithm, as described in detail in \cite{49} Algorithm 1.

4. Numerical experiments

In this section, we compare the classical congestion pricing scheme with CARMA through numerical experiments. We first define the performance measures and discuss how they are computed in both models, as well as a non-controlled benchmark. Then, three different case studies that highlight the novel features of CARMA are presented.

4.1. Performance measures and benchmark

The performance measures can be divided into two categories: system level and user type level. Specifically, we investigate the queuing delay and travel cost at the equilibrium traffic assignment. As a benchmark, we consider a classical bottleneck, where commuters also have varying VOT multipliers but there is no lane segmentation or tolling.
In Section 2.1 we introduced $M$ commuter groups with different VOT multipliers $u_j$, while in Section 3, commuters are classified into types with different dynamic VOT processes. Accordingly, $u_j$ in Eqs. 1 and 2 is to be regarded as the one-day realization of the VOT process in Section 3. In the classical model, the commuters act myopically according to their current VOT realization, since there is no dynamic element linking their immediate actions to the future repetitions. Let $P_{\tau}[u]$ denote the probability of having VOT $u$ for commuters in type $\tau$. Then, the type average VOT is

$$\bar{u}_{\tau} = \sum_u P_{\tau}[u] u,$$  \hspace{1cm} (27)

and the system average VOT is

$$\bar{u} = \sum_u P[u] u = \sum_u \left( \sum_{\tau} g_{\tau} P_{\tau}[u] \right) u.$$ \hspace{1cm} (28)

Table 1 summarizes the performance measures, where those associated with the benchmark (“NOM”) are from [24]. Notice that here the average queuing delay of each type is equal to the system average [21], and the system average queuing delay in TOLL is related to NOM by the factor $s_{\text{slow}}/s \leq 1$.

At the equilibrium of TOLL, there exists a VOT index $m$ such that only the commuters with VOT higher than or equal to $u_m$ can enter the fast lane. We further define $r_m$ as the fraction of commuters in group $m$ entering the

| Name                      | Benchmark (“NOM”) | Optimal tolling (“TOLL”) | Karma mechanism (“CARMA”) |
|---------------------------|-------------------|--------------------------|---------------------------|
| System average queuing delay $t_{q}$ | $\frac{t}{2\sigma}$ | $\frac{s_{\text{slow}}}{2\sigma}$ | $\sum_{t,b} \psi[t, b] \psi[\bar{\tau}[t, b] t_{q}[t, b]$ |
| System average travel cost $c_{\bar{\tau}}$ | $u_{\tau} c^*$ | $\sum_{\tau} g_{\tau} u_{\bar{\tau}}$ | $- \sum_{\tau,u,k} d_{\tau}[u, k] R_{\tau}[u, k]$ |
| Type average queuing delay $t_{q}^{\tau}$ | $\frac{t}{2\sigma}$ | $\sum_{u} P_{\tau}[u] t_{q}[u]$ | $\frac{1}{d_{\tau}} \sum_{t,b} \psi[t, b] \psi[\bar{\tau}[t, b] t_{q}[t, b]$ |
| Type average travel cost $c_{\bar{\tau}}$ | $u_{\tau} c^*$ | $\sum_{u} P_{\tau}[u] \bar{c}[u]$ | $- \frac{1}{g_{\tau}} \sum_{u,k} d_{\tau}[u, k] R_{\tau}[u, k]$ |

*All measures are computed at the SNE.
slow lane, given by
\[ r_m = \frac{1}{\mathbb{P}[u_m]} \left( \frac{s_{\text{slow}}}{s} - \sum_{j < m} \mathbb{P}[u_j] \right). \]  
(29)

The average queuing delay and travel cost at \( \text{VOT} \) \( u_j < u_m \) (\( \bar{\pi}[u_j] \) and \( \bar{c}[u_j] \), respectively) is the same as the type average measures in \( \text{NOM} \). Differently, commuters on the fast lane incur no queuing delay and a lower travel cost. The normalized travel cost \( \bar{c}[u_j]/u_j \) varies among commuter groups because of their departure order. As per the equilibrium condition and Eq. 4, we can solve the departure interval of each group and derive the average travel cost as
\[ \bar{c}[u_j] = u_j \left( c^* - \frac{1}{2} \frac{\beta \gamma}{\beta + \gamma} \frac{N_j}{s_{\text{fast}}} \right) = u_j c^* \left( 1 - \frac{1}{2} \frac{s}{s_{\text{fast}}} \frac{N_j}{N} \right), \]
(30)
where \( N_j \) is the number of commuters in group \( j \) on the fast lane. Except for group \( m \), it is equal to the group size \( \mathbb{P}[u_j] N \). Putting everything together, the average queuing delay and travel cost for each \( \text{VOT} \) level in \( \text{TOLL} \) are
\[ \bar{\pi}[u_j] = \begin{cases} \frac{c^*}{2}, & j < m, \\ 0, & j > m, \\ r_m \frac{c^*}{2}, & j = m, \end{cases} \]
(31)
\[ \bar{c}[u_j] = \begin{cases} u_j c^*, & j < m, \\ u_j c^* \left( 1 - \frac{1}{2} \frac{s}{s_{\text{fast}}} \mathbb{P}[u_j] \right), & j > m, \\ u_j c^* \left( 1 - (1 - r_m) \frac{1}{2} \frac{s}{s_{\text{fast}}} \mathbb{P}[u_j] \right), & j = m. \end{cases} \]
(32)

The default values of the model parameters are reported in Table 2.

4.2. Case 1: Homogeneous commuters
4.2.1. Setting

We first consider a scenario in which all commuters have the same in-dependent and identically distributed (i.i.d.) \( \text{VOT} \) process, namely, they have low \( \text{VOT} (u_l = 1) \) 80% of the time and high \( \text{VOT} (u_h = 6) \) 20% of the time. This is represented by a Markov chain with stochastic matrix \( \phi = \begin{pmatrix} 0.8 & 0.2 \\ 0.8 & 0.2 \end{pmatrix} \).

The purpose of this case study is to a) provide insights on the strategic behaviors of commuters under \( \text{CARMA} \), and b) compare the performance of \( \text{CARMA}, \text{TOLL}, \) and \( \text{NOM} \).
Table 2: Default values of model parameters.

| Name                             | Notation | Unit        | Value |
|----------------------------------|----------|-------------|-------|
| Number of commuters              | $N$      |             | 9000  |
| Bottleneck capacity              | $s$      | veh/min     | 60    |
| - fast lane                      | $s_{fast}$|             | 12    |
| - low lane                       | $s_{slow}$|             | 48    |
| Length of discrete time step     | $\Delta t$| min        | 15    |
| Normalized VOT                   |          | cost/hour*  |       |
| - queuing delay                  | $\alpha$ |             | 6.4   |
| - early arrival                  | $\beta$  |             | 4     |
| - late arrival                   | $\gamma$ |             | 16    |
| Desired arrival time             | $t^*$    | min         | 120   |
| Discount factor                  | $\delta$ |             | 0.99  |
| Parameter for model continuity   | $\epsilon$ |             | $10^{-4}$ |
| Average karma per commuter       | $\bar{k}$|             | 10    |

*In TOLL the unit is $$/hour.

4.2.2. Results and discussion (Figure 5)

The SNE under CARMA is shown on Figures 5a (equilibrium policy $\pi^*$) and 5b (stationary karma distribution $d^*$). Figure 5a reports the bidding policy for the low VOT state $u_l = 1$ (left column) and the high VOT state $u_h = 6$ (right column). Each row indicates one discrete departure time. In each subplot, the probability that a commuter with $k$ karma ($x$-axis) bids $b$ ($y$-axis) is depicted as the intensity of red color (the higher probability the darker color), while the infeasible bids are displayed in grey. The threshold bid $b^*$ at each departure time is also plotted for reference.
Figure 5: Results for Case 1. (a) Stationary Nash Equilibrium (SNE) policy. (b) SNE karma distribution. (c) Threshold bid $b^*$ in CARMA versus optimal toll price $p^*$ in TOLL. (d) Left: departure rate in CARMA versus TOLL. Right: queuing delay over time. (e) Performance comparison of CARMA versus TOLL and NOM.
As expected, **low-VOT commuters** bid below $b^*$ most of the time and their bids are rather random because they would enter the slow lane without a payment anyway. This result also indicates their intention to spare karma for a future high VOT event. In contrast, **high-VOT commuters** always bid exactly $b^*$ so that they can enter the fast lane without overpaying for it. The only exception arises at $t = 15$. The high-VOT commuters with $k = 1$ would bid $b = 1$ to ensure they could enter the fast lane, given that all low-VOT commuters would bid $b = 0$. With more karma in hand, high-VOT commuters tend to depart closer to $t^*$ and bid more for the fast lane. Consequently, $b^*$ is the highest for the most contested time $t^*$, and decreases linearly away from it, as shown in Figure 5c. Despite that karma and money are incomparable, the trend of $b^*$ over time resembles the optimal toll ($p^*$ in TOLL) remarkably well.

The similarity between CARMA and TOLL is also demonstrated in the aggregate outcomes. As shown in Figure 5d, they lead to almost the same departure and congestion patterns. Specifically, the departure rate on the slow lane is constant and higher than its capacity before the peak time $t_{\text{peak}} = 45$ and then drops to a constant below the capacity till the end of the rush hours. Moreover, the fast lane is largely occupied by high-VOT commuters under CARMA, also in line with the prediction of TOLL. In other words, CARMA enables the most urgent commuters to use the fast lane and thus allocates it efficiently.

Compared to the noncontrolled benchmark (NOM), both CARMA and TOLL help reduce the average queuing delay and travel cost. As shown in Figure 5e, their improvements are also quite close to each other (20% in queuing delay and 30% in travel cost). Recalling that TOLL leads to the system optimum (when only the fast lane can be regulated), we can conclude that CARMA closely achieves the system optimum with homogeneous commuters. Furthermore, it does so without any prior knowledge of the commuters' VOT, which is, however, required to determine the optimal toll price in TOLL.

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2In CARMA, a linear interpolation of the queuing delays was performed in order to counter-act the effect of the course time discretization (see, e.g., [55]).
4.3. Case 2: Heterogeneous commuters with constant VOT

4.3.1. Setting

We next consider two types of commuters: $\tau_1$ has $VOT = 1$ all the time, and $\tau_2$ has $VOT = 6$ all the time. This is the counterpart of the static heterogeneous VOT in the classical bottleneck model. Empirical evidence has shown that the individual (monetary) VOT is highly correlated with the income level [56]. Hence, we may consider commuters in type $\tau_2$ as wealthier than those in type $\tau_1$. We further assume type $\tau_1$ occupies 80% of the population and type $\tau_2$ the remaining 20%. The purpose of this case study is to demonstrate the fairness of CARMA with respect to heterogeneous income classes.

4.3.2. Results and discussion (Figure 6)

As can be seen from Figure 6a, CARMA and TOLL achieve the same aggregate departure and congestion patterns. The total departure rates as well as the queuing delays are identical to Case 1. However, the two models behave quite differently when we compare the results by commuter type. Under TOLL, the fast lane is fully occupied by high-income commuters (type $\tau_2$) whereas, under CARMA, the fast lane capacity is split between the two types in the same ratio as their proportions in the population. In the former case, the optimal toll price essentially discriminates against the low-income commuters from using the fast lane. As a result, TOLL leads to a serious fairness issue in terms of both queuing delay and travel cost (normalized by the VOT level$^3$), as demonstrated in Figure 6b. Specifically, the high-income commuters enjoy no queuing delay at all while the low-income commuters on average get stuck in the queue for 37min. They also incur double the travel cost compared to the high-income commuters.

In contrast, under CARMA, the two types of commuters share the same queuing delay and normalized cost. Figure 6c sheds light on the cause. At the SNE, both types of commuters have the same distribution of karma. Accordingly, they have equal opportunity to bid for the fast lane. In fact, their departure and bidding strategies are exactly the same (see the top two

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$^3$The normalization brings the costs of both types on the same scale to facilitate the interpersonal comparability [57]. If commuters in type $\tau_2$ are truly persistently more urgent than those in type $\tau_1$ and deserve preferential treatment from the social point of view, the karma mechanism can be designed to explicitly subsidize them, e.g., through non-uniform karma redistribution.
subplots in Figure 6c for an example at $t = t^*$ because they essentially face the same optimization problem (up to a constant difference in the scaling of costs).

Another important observation from Figure 6b is that only high-income commuters benefit from TOLL, while the low-income commuters remain the same as in NOM. Differently, all commuters enjoy a shorter queuing delay and a lower travel cost thanks to CARMA. Furthermore, we note that the fairness of CARMA does not come at the cost of efficiency, as we still have the optimal reduction in congestion with respect to NOM (same average queuing delay as in Figure 5e).

Figure 6: Results for Case 2. (a) Left: departure rate in CARMA versus TOLL. Right: queuing delay over time. (b) Fairness of CARMA versus TOLL and NOM. (c) SNE with equilibrium policy at $t^* = 120$ min (top) and stationary karma distribution (bottom) per type $\tau$. 
4.4. Case 3: Heterogeneous commuters with different temporal VOT spread

4.4.1. Setting

Our final case considers four types of commuters each constituting a fourth of the population. All types have the same average VOT $\bar{u} = 2$, but differ in the magnitude and frequency of the high VOT. The type-specific VOT processes are summarized in Table 3.

Table 3: VOT processes of heterogeneous types.

| Type | Low VOT ($u_l$) | High VOT ($u_h$) | Probability of $u_h$ | VOT spread |
|------|----------------|------------------|----------------------|------------|
| $\tau_1$ | 1 | 11 | 10% | 10 |
| $\tau_2$ | 1 | 6 | 20% | 5 |
| $\tau_3$ | 1 | 3 | 50% | 2 |
| $\tau_4$ | 2 | 2 | 100% | 0 |

Note that from type $\tau_1$ to $\tau_4$, the commuters have a decreasing spread in their VOT levels. Type $\tau_1$ has the widest spread, incurring the very high urgency $u_h = 11$ on relatively rare occasions. On the other hand, type $\tau_4$ has the constant urgency $u = 2$ all the time. The purpose of this case study is to showcase a new kind of heterogeneity in the VOT that has not been considered in the literature, namely, the heterogeneity in how the VOT dynamically varies from one day to the next.

4.4.2. Results and discussion (Figure 7)

Similar to the previous two case studies, we first compare the departure patterns in CARMA and TOLL. The left column in Figure 7a illustrates the departure rates under CARMA, where the commuter type and VOT are plotted in different colors (each hue represents one type and the high VOT levels are displayed with higher saturation). The departure rates under TOLL are plotted in the right column and they are solely determined by the immediate VOT. In line with classical results [58], there is a perfect separation in the departure intervals: commuters with higher VOT ($u = 3, 6, 11$) enter the fast lane and depart closer to $t^*$, while those with lower VOT ($u = 1, 2$) only use the slow lane.

As expected, the aggregate departure pattern under CARMA is exactly the same as that under TOLL. However, they are drastically different when analyzed by VOT type. Under CARMA, commuters on the fast lane are largely heterogeneous, and the departure intervals of different types and VOT are partially overlapping. Nevertheless, we can observe that the departures on
the fast lane at $t^*$ are dominated by commuters with the highest urgency ($u = 11$), while those with the lowest urgency level ($u = 1$) tend to use the slow lane. Even if they enter the fast lane, they would choose a departure time far from $t^*$ to save their karma.

![Figure 7](image)

Figure 7: Results for Case 3. (a) Departure rate in CARMA (left) versus TOLL (right) (in veh/15min). (b) Fairness of CARMA versus TOLL and NOM. (c) SNE stationary karma distribution per type $\tau$.

Figure 7 plots the average queuing delay and travel cost by commuter type in the three schemes. As the average VOT is identical among the commuters, they share the same average queuing delay and travel cost in NOM. Similar to the observation in Case 2, TOLL results in a serious fairness
issue. Specifically, commuters in type $\tau_4$ (with constant urgency $u = 2$) do not benefit from the control scheme at all because they never use the fast lane. On the other hand, commuters in type $\tau_3$ has the largest reduction in queuing delay while those in type $\tau_1$ enjoy the most travel cost saving.

Compared to TOLL, CARMA benefits all commuters meanwhile achieving a considerable equalizing effect. However, unlike in Case 2 (Figure 6b), strict equality among types is not obtained in this case. This is expected because, despite of sharing the same average VOT, the heterogeneous VOT processes, and in particular the different VOT spread, lead to different value functions (Eq. 11) and, in turn, different optimal policies. Another interesting observation is that CARMA encourages yielding: those suffering from a longer queue enjoy a lower cost. This result also implies that, if commuters are free to choose their VOT processes to minimize their average travel cost, they would tend to be cautious and yielding most of the time and be urgent only sparingly. In other words, CARMA fosters a more collaborative environment where commuters carefully assess when they truly need access to the fast lane.

To better illustrate the commuters’ bidding policies under CARMA, we plot the distribution of karma at the SNE in Figure 7c. It can be found that the most yielding commuters (type $\tau_1$) hold more karma on average than others and the average karma per commuter decreases with the VOT spread. This result is also expected because commuters who anticipate high VOT events in the future are more likely to yield on low VOT days to reserve karma. If the variation in the VOT processes is small, commuters tend to avoid congestion whenever they have enough karma.

5. Conclusions

This study proposes CARMA as a solution to the bottleneck congestion in the morning commute. Under CARMA, each commuter is endowed with non-monetary credits called karma used to bid for access to a noncongested fast lane. Different from existing credit-based schemes, karma is not tradable in a monetary market. We model the commuter behaviors under CARMA as a dynamic population game and prove the existence of a Stationary Nash Equilibrium (SNE). This dynamic model enables us to introduce a novel kind of heterogeneity in the Value of Time (VOT), namely, heterogeneity in how the VOT varies dynamically each day.
To demonstrate the performance of CARMA, we conducted three case studies with different settings. When commuters are homogeneous, CARMA leads to similar departure and congestion patterns as an optimal tolling scheme and closely approaches the system optimum (when only the fast lane can be regulated). When commuters are heterogeneous, CARMA is demonstrably fairer than monetary tolling while preserving near-optimal decongestion. In particular, CARMA does not discriminate against low-income commuters who have equal chances of entering the fast lane as high-income commuters. Moreover, in CARMA all commuters strictly benefit with respect to no policy interventions, whereas under monetary tolling only commuters with high VOT benefit.

There are several directions to extend the current work. First, it is interesting to explore non-uniform karma redistribution schemes, e.g., where more karma is distributed to commuters departing earlier/later. Such a scheme could improve congestion also on the unregulated slow lane. Second, it is still unclear how heterogeneity affects CARMA in general. Our numerical results show that CARMA allows all commuters to strictly benefit with respect to no policy intervention, while a thorough analysis is needed to better understand this property. Finally, we believe that karma mechanisms could have wide-ranging applicability in transportation systems. We hope that this study could inspire a series of work on transport management with karma.

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