The invulnerability assessment of two-layer railway network under the uncertain environmental hazard risk

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Abstract. The failure of the railway station or line will cause the reduction of the railway network efficiency and bring travel inconvenience to the passengers, especially for the key stations and tracks. Reasonable invulnerability assessment is beneficial to maintain the system operation ability and prevent passengers from abandoning railway when the uncertain environmental hazard risk is appeared. At present, larger numbers of research about the railway network invulnerability assessment are purely based on complex network theory, while ignored the nature of railways. In this paper, the Railway Network Invulnerability Model (RNIM) is introduced from the perspective of infrastructure network and service network, and the efficiency of the proposed RNIM is verified according to compare the structural connectively and functional accessibility using different attack strategies. Furthermore, the RNIM framework is applied on the China high-speed railway network, and the results show the validity of the proposed model for the invulnerability of railway network assessment in accurately and efficiently. Finally, some protection suggestions are proposed to reduce the negative effects after the inevitable disruptions emerged.

1. Introduction

Due to the occurrence of disruptions in train operation, the safety of the railway network is under various huge threats. For example, the disruptions caused by the harsh natural environment along the railway causes the railway transportation to be interrupted more than 100 times a year on average, and the highest has reached 211 times a year. Disaster-incurred sections account for more than 20% of the total operating mileage of the entire rail. A sound invulnerability assessment can make targeted recommendations for dealing with disruptions, thereby reducing the hazards of disruptions and the loss of life and property. Because, the invulnerability of the railway network refers to the ability of the network to maintain the railway function under the condition of deliberate attacks or natural damage to the railway site or railway line [1]. Hence, the research on the invulnerability of railway networks is crucial in order to ensure the safe operation of the railway.

At present, there are several ways to study invulnerability. The first and common is based on graph-topological measures, originating in theory of complex networks [2-4]. Then, Chen et al.[5] optimized the invulnerability of China railway network by adding edges to the network, and evaluated it based on the natural connectivity; Wang et al.[6] established an invulnerability entropy model based on the degree of network invulnerability and from the perspective of network recovery after being attacked, Sun et
al.[7] proposed a new recovery strategy and demonstrated the role of network invulnerability. Otherwise, Cao xiongb[8] proposed cascading invulnerability of mathematical analysis model. Khaled et.al.[9] researched invulnerability by proposing a new network total cost in the freight railway network and Santos et.al. [10] calculated the average increase of travel cost in a road network to assess the network invulnerability. However, none of these studies of railway network invulnerability has been assessed in terms of the nature of the railway, i.e. they have not linked the structural and functional characteristics of the railway network to its invulnerability.

In this paper, we proposed a new Railway Network Invulnerability Model (RNIM) to assess the invulnerability of the system based on the properties of the railway network by dividing the network into a two-layer network, a railway infrastructure network and a railway service network. The invulnerability of the railway network is divided into structural invulnerability and functional invulnerability for research. The structural invulnerability is measured based on the connectivity of the infrastructure network. And functional invulnerability is measured based on the accessibility of the service network. Then, we demonstrated the performance of the RNIM framework on the real-world instance of a part of the China railway network and the importance and effectiveness of railway invulnerability. Last, we proved that this new Railway Network Invulnerability Model could simulate the railway network and estimate an accurate assessment of the network and make suggestions on railway operation to ensure the safe operation of the railway and reduce the losses and hazards caused by disruptions.

The remainder of the paper is structured as follows. Section 2 gives the framework of the invulnerability model and explains the indicators in it. And the framework is applied on a case study in Section 3, where we demonstrate the framework’s features, its general performance and discuss our findings. Section 4 gives the final conclusions.

2. Railway network invulnerability assessment methods and metrics

In this section, the railway network is divided into two layers, including the railway infrastructure network and the railway service network. As for the infrastructure network is concerned, the station, track and structure of railway network are selected to describe the characteristics of the network. As for the railway service network, the origin and destination of passenger travel and the demand of passenger travel are considered as the characteristics to describe the network.

Based on the stratification of the railway network, invulnerability is divided into structural and functional invulnerability, which correspond to the network performance of the infrastructure network and the service network, respectively after disruption. From the two perspectives, the ability to keep the railway line as connected as possible and the ability to ensure the passengers or goods could reach their destination, which are the railway’s core function, would be assessed.

2.1. Connectivity-centered invulnerability

Structural invulnerability as a connectivity-centered invulnerability (CcI) is the ability of the railway network to remain physically connected to each other after the disruption. The connectivity-centered invulnerability is evaluated based on the trend of connectivity changes in infrastructure networks. And the connectivity of railway infrastructure network is composed of the following three connectivity degrees, the average station connectivity, the average railway connectivity and connectivity of railway network structure.

In the railway infrastructure network, stations and track are the most basic components of the infrastructure. Whether a station or track in an infrastructure network is attacked, it can directly affect the connectivity of the network. Hence, the influence of station and railway track changes should be considered at the same time after the disruption. Meanwhile, because the infrastructure network is a network with BA network properties, the analysis of network structure is also important to assessing network connectivity. When a key station or track is destroyed, the infrastructure network may split into multiple networks rather than simply losing a node or side. This may greatly reduce the connectivity of the network, so it is necessary to select an indicator which evaluate the overall structure of the network.
to reflect the current connectivity of the network. Therefore, by comparing the connectivity before and after the disruption, the structural invulnerability of the infrastructure network can be comprehensively evaluated. And the connectivity model of infrastructure network is as follows:

\[
\text{Con} = \alpha_1 \times \text{ASC}^i / \text{ASC} + \alpha_2 \times \text{ARC}^i / \text{ARC} + \alpha_3 \times \text{RNSC}^i / \text{RNSC} \cdot \alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{1}
\]

Where, \( \text{Con} \) represents the connectivity of the network in the case of disruption \( i \); \( \text{ASC}^i, \text{ARC}^i, \) and \( \text{RNSC}^i \) is the connectivity index of the following three items when the disruption occurs; \( \text{ASC}, \text{ARC}, \) and \( \text{RNSC} \) are three connectivity indexes of railway network under normal operation; \( \alpha_1, \alpha_2, \alpha_3 \) are the weight coefficients assigned to each kind of connectivity, based on the AHP analytic hierarchy process, the weight allocation of the three types of connectivity degree is determined according to the emphasis angle of the railway network evaluated. The factors used in connectivity are introduced respectively in the following parts.

### 2.1.1. Average station connectivity (ASC).

Station is one of the two main parts of the railway infrastructure network. The station connectivity reflects the ability of a station connect to other stations. Therefore, the concept of average station connectivity is introduced. Average station connectivity is a measure indicator of the ability of each station in railway infrastructure network to connect to other stations. Its model is as follows:

\[
\text{ASC} = 2/N(N-1) \times \sum_{i<j} d_{ij} \tag{2}
\]

Where, \( N \) is the number of total stations in the basic railway network, and \( d_{ij} \) represents the total number of stations that need to pass from station \( i \) to station \( j \); \( 2/N(N-1) \) is the total number of possible stations connected in the network.

### 2.1.2. Average railway connectivity (ARC).

Another major component of the railway infrastructure network is the railway tracks that connect stations to each other. Average railway connectivity is a measure indicator reflecting the influence of railway lines on network connectivity in railway network. Because, the actual distance between stations can reflect the difficulty of connecting the two sites. In addition, we take the reciprocal of the distance, it proves that the shorter the average distance between stations in the network is, the better the connectivity of the network is.

Therefore, the average railway connectivity is selected as an indicator to evaluate the railway infrastructure network. Its model is as follows:

\[
\text{ARC} = 2/N(N-1) \times \sum_{i<j} 1 / D_{ij} \tag{3}
\]

Where, \( N \) is the total number of all stations in the network; \( D_{ij} \) represents the actual shortest railway line distance between stations. If two stations cannot be connected, there is no such line. \( 2/N(N-1) \) represents the maximum number of possible lines in the network.

### 2.1.3. Railway network structure connectivity (RNSC).

Connectivity of railway network structure is a measure indicator of the connectivity of existing network structure. For the railway network, the size of the largest sub-network indicates how many stations can be connected and how much traffic demand can be met in the network. In order to evaluate the change of the railway network connectivity, it is necessary to analyze the relationship between the railway network and its largest sub-network. Therefore, RNSC is selected as an indicator to evaluate railway infrastructure network. Its model is as follows:

\[
\text{RNSC} = n^i / n \tag{4}
\]

Where, \( i \) is the number of stations that are still connected in the largest sub-network of the network under the state \( i \); \( n \) is the number of terminus points of the railway network under normal operation.
2.2. Accessibility-centered invulnerability

Functional invulnerability as accessibility-centered invulnerability (AcI) is the ability to ensure that passengers or cargo can reach the destination after a disruption. AcI is assessed by trends in accessibility changes on the railway service network. Accessibility is quantified as the ratio of the revenue of the current railway service network to the total cost of passengers on the transportation network after the disruption. Through the analysis of changes in railway revenues and share of total transportation costs, it reflects whether passengers can reach the original destination through the service network and the willingness of passengers to choose the railway travel mode. This shows the service capability and competitiveness of the network, and comprehensively evaluates the AcI. The accessibility model is as follows:

\[ \text{Acc}^{\lambda} = \frac{P^\lambda}{P_{\text{all}}^{\lambda}} \]  

Where, \( \lambda \) represents in the case of the disruption \( \lambda \); \( \text{Acc}^\lambda \) represents the network accessibility; \( P^\lambda \) represents the revenue of the current railway service network; \( P_{\text{all}}^{\lambda} \) represents total cost of passengers on the transportation network.

In order to simulate the actual travel behavior of passengers, the airplane travel mode is selected to perform a complete passenger travel simulation. For passengers, choosing a certain travel mode may have different strategy but is mainly based on the two angles, time cost and money cost. Hence, the two cost are used as the constraints in accessibility and introduced respectively in the following parts.

For a travel OD pair, time cost is divided into three parts, the entry and exit time which considered as a constant based on travel mode, the transit waiting time which is a linear equation based on the number of transit waiting stations and the time required for vehicle operation, which is the ratio between the actual distance required for train operation and the maximum train speed. In terms of money cost, it is composed of the product of the price coefficient per kilometer of the selected travel mode and the actual travel distance.

Therefore, cost model is as follows:

\[
\begin{aligned}
\text{time}_{i_0}^\alpha &= \text{M}^\alpha + \text{time}_{i_0 \to j}^\alpha + \text{time}_{\text{ope} \to j}^\alpha \\
\text{time}_{i_0 \to j}^\alpha &= k^\alpha \times d_{ij} \\
\text{time}_{\text{ope} \to j}^\alpha &= D_{ij}^\alpha / v^\alpha \\
& \text{Where, } \alpha \text{ represents travel mode; time}_{i_0}^\alpha \text{ represents the time spent from station } i \text{ to } j; \text{time}_{i_0 \to j}^\alpha \text{ represents the cost of entering and leaving the station; time}_{\text{ope} \to j}^\alpha \text{ represents the cost of overstay time; time}_{\text{ope} \to j}^\alpha \text{ represents the running time cost; P}_{ij}^\alpha \text{ represents the money cost form station } i \text{ to } j; M^\alpha \text{ is a constant representing the cost of the in and out of the station; c is the price coefficient per kilometer of a travel mode. k is the transit waiting coefficient representing the time spent at a stop; d_{ij} \text{ is the number of stop-over from station } i \text{ to station } j; D_{ij}^\alpha \text{ is the actual transportation distance between station } i \text{ and station } j; v \text{ is the vehicle speed.}
\end{aligned}
\]

The model framework analyzes the changing trend of the connectivity based on the railway infrastructure network and the accessibility based on the railway service network so that it could evaluate the structural and functional invulnerability reflecting the invulnerability of the railway network.

3. Results and discussion

This section presents a case study on Railway Network Invulnerability Model to verify that the model can better evaluate the invulnerability of railway network. The China high-speed railway network, one of the largest railway networks in the world, has been chosen for performing our experiments. This network includes more than 900 infrastructure arcs and 865 stations, with the operated timetable from a
working day which includes a large part of China high-speed railway network. Since a big amount of information in the network has to be analyzed, the new model described above has been converted into computer code.

3.1. Strategies and parameters
Disruptions in the rail network are caused by malicious attacks or contingency attacks. To conduct a comprehensive assessment of the invulnerability of the railway network, we have selected targeted attack strategy and random attack strategy. Because, targeted attacks simulate malicious attacks and random attacks simulate contingency attacks. Among the targeted attack strategy, targeted attacks are divided into two methods. One is to give priority to attacking the stations in the railway network with the most connections to other stations which is called the node degree-based attack strategy; the other is to give priority to attacking the stations in the railway network that need to be passed the most times on which is called node betweenness-based attack strategy. In addition, in order to evaluate the performance of the railway network under different intensities of disruptions, the intensity of the disruptions was divided into 100 levels, and the intensity of one level represents that 1% stations (9 stations) in the railway network are attacked and failed.

The strategy of transportation travel mode selection is that passengers will give priority to high-speed rail travel when the railway is in normal operation and interruption. But in the following two situations, the airplane travel mode will be selected. The first situation is that either the origin or destination of the passenger is disconnected from the service network; the other situation is that the monetary cost of the airplane travel mode is lower than that of high-speed rail travel or the time spent on high-speed rail travel exceeds 1.5 times the time cost of normal railway operation.

For evaluating the invulnerability of the railway network, 200 OD pairs were randomly generated to simulate the travel behavior of passengers at a certain time. Other accessibility-related parameters were assumed in order to perform the invulnerability analysis of the rail service network based on publicly available data. They are shown in Table 1.

| Table 1. Parameters of the invulnerability model |
|-----------------------------------------------|
| Parameters | Description | Value |
| $M^a$ | Entry and exit time in a certain mode | Train: 2 h, Air: 4 h |
| $v^a$ | Average speed of vehicle | Train: 250 km/h, Air: 900 km/h |
| $c^a$ | Average unit price of vehicle per km | Train: 0.45 yuan/km, Air: 1.55 yuan/km |
| $k^a$ | Transit waiting time in a certain mode | Train: 1/12 h, Air: 1.5 h |

3.2. Railway network assessment
To analyze and discuss the efficiency and adaptability of the proposed invulnerability model, two experiments are designed to evaluate connectivity-centered and accessibility-centered invulnerability.

3.2.1. Experiment 1: connectivity-centered invulnerability.
CcI is evaluated by the variation tendency of connectivity under three different attack strategies. The connectivity variation under the three attack strategies are shown in figure 1.
Figure 1. Simulation of connectivity-centered invulnerability

The attack strategy based on node degree caused the fastest rate of network connectivity decline, so that the CcI is less than 10% reached 8.5% of the normal operation network structure after 72 stations with higher node degree failure. Among it, the average station connectivity is most affected by the attack strategy based on node degree and dropped to 3.8% after only 36 stations failed. And the CcI decreased to 9.8% after 99 stations failed based on the node betweenness-based attack strategy, and to 9.97% after 153 stations failed based on the random attack strategy.

This demonstrates that malicious attacks aimed at the interconnection of stations are more harmful to the infrastructure network. Therefore, the maintenance of high-node stations, such as: Nanjing south railway station with 6 degree, Zhengzhou east station with 5 degree and Wuhan station with 5 degree, etc., should be emphasized. These stations are the key stations for infrastructure network.

3.2.2. Experiment 2: accessibility-centered invulnerability.

AcI is assessed by the variation tendency of accessibility under these three attack strategies and the change is shown in figure 2.
Based on the fact that railway travel was abandoned by all passengers before the 30-level interruption, only the change trend of the first 30-level was selected for display in Figure 2. The attack strategy based on node betweenness caused the fastest decline in accessibility, the percentage of AcI dropped low than 5% to 1.19% after 36 stations with higher betweenness failed. And the AcI based on the node degree attack strategy and the random attack strategy is reduced to 2.36% and 2.95% with 45 and 126 stations failed respectively.

This result shows that attacking stations that have been passed more times in railway transportation will have a greater impact on the railway service network, causing the network to lose a large number of passengers. When the top 27 stations fail, 79% of passengers will abandon the railway to travel. Under the random attack strategy, when nearly 100 stations in the network fail, nearly 80% of passengers will be lost. Therefore, a station with a larger betweenness, such as such as Changsha South, Xuzhou East and Suzhou Dong, etc., is more important to the railway service network as the core station of the network.

3.3. Operation Suggestions
According to the results of railway network invulnerability model assessment of railway network under the environmental hazard risk, some operation suggestions are presented as follows:

- The node degree-based attack strategy which attacks the stations with the largest number of direct connections is more damage to infrastructure network. Hence, stations with high nodes degree should be paid more attention in the environmental disaster prevention and control by the railway maintenance department to ensure the performance of the railway infrastructure network.

- The node betweenness-based attack strategy which is to attack the station with the highest number of passes makes the service network cause greater loss of passengers. Therefore, the railway operation department should adjust the train operation strategy when the environmental disaster is appeared to reduce the number of stations with high betweenness for preventing the loss of passengers.

4. Conclusions
This paper presents a new Railway Network Invulnerability Model according to dividing the network into two layers. Considering the invulnerability from the railway infrastructure network and service network, the two measures of connectivity and accessibility based on the inherent attributes of the railway are analyzed. And the The reasonableness and validity of the proposed model is demonstrated by using a real case on a high-speed railway network. Through this case, it reflects the influence trend of different levels of disruptions on the invulnerability change of the railway network, and finds out the key stations in the railway network, which provides effective information for railway workers to operate and maintain the railway network. Through the proposed recommendations, railway operators can reduce the loss of passengers' lives and property in the environmental hazard risk prevention and control, improve the attractiveness of the railway for passengers, thus improving the status of the railway network in the transportation network and increasing the revenue of the railway.

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