Statistical modelling of tropical cyclone tracks: a comparison of models for the variance of trajectories

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Abstract

We describe results from the second stage of a project to build a statistical model for hurricane tracks. In the first stage we modelled the unconditional mean track. We now attempt to model the unconditional variance of fluctuations around the mean. The variance models we describe use a semi-parametric nearest neighbours approach in which the optimal averaging length-scale is estimated using a jack-knife out-of-sample fitting procedure. We test three different models. These models consider the variance structure of the deviations from the unconditional mean track to be isotropic, anisotropic but uncorrelated, and anisotropic and correlated, respectively. The results show that, of these models, the anisotropic correlated model gives the best predictions of the distribution of future positions of hurricanes.

1 Introduction

We are interested in developing accurate methods to estimate the probability of extreme hurricanes making landfall at different locations. Estimating the probability of extreme events is difficult because, by the definition of what makes an extreme, the data is sparse. If we try and estimate hurricane landfall probabilities using local data then this problem is acute: severe hurricanes only strike individual geographical locations very rarely. For some locations there are no strikes at all in the historical record, even though such strikes may be possible. However, the problem of lack of data can be reduced, and the risk estimated more accurately, by appropriate use of data from surrounding regions. Various methods have been described that use this principle to estimate hurricane risks, such as those of Clark (1986), Darling (1991) and Chu and Wang (1998). One of the most interesting approaches is to build a statistical model for hurricane tracks and intensities across the whole of the Atlantic basin. This is the approach taken in Drayton (2000), Vickery et al. (2000) and Emanuel et al. (2005), and is the approach we are taking ourselves.

In general, when using surrounding data to estimate local parameters, one must decide on the size and shape of the region from which data is to be taken and how the data within that region is to be weighted. There is a balance between two effects: using more data gives more precise estimates, but using less relevant data can introduce biases. Finding where the optimal balance lies is crucial to any attempt to build an accurate model.

In Hall and Jewson (2005) we have started building a new model for hurricane tracks in the Atlantic basin, and we address the question of how best to use surrounding data by calculating the optimum size of the data region using cross-validation. The track model that we describe is based on a two dimensional linear stochastic process, with the two dimensions representing the longitude and latitude of the location of a hurricane. In Hall and Jewson (2005), and also in this article, we assume that the innovations driving this stochastic process are Gaussian. Because of this simplifying assumption the model can be completely specified by the appropriate means, variances and covariances. As a start, in Hall and Jewson (2005) we describe a model for the unconditional mean. That is, a model for the expected motion of a hurricane, given no information at all except the current location of the hurricane. The model uses a nearest neighbours approach with an isotropic Gaussian weighting function incorporating a free parameter length-scale. The cross-validation fitting procedure gives an optimal length-scale of 300km.

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In this article we describe what we consider to be the most appropriate next stage in building our track model, which is the modelling of the unconditional variance of fluctuations around the unconditional mean tracks. What this adds to the model for the unconditional mean tracks can be described as follows. The model for the unconditional mean can be used to make one step (6 hour) predictions for the future location of a hurricane. A model for the unconditional mean and variance goes a step further and gives one step predictions for the whole distribution of future locations of the hurricane. Again, the meaning of ‘unconditional’ is that this prediction assumes no information at all except for the current location. We are ignoring autocorrelations in time as well as other influences on the track such as the effects of intensity, time of year and ENSO state. All of these will be included in the model in due course.

We estimate the unconditional variance using a nearest neighbours approach, very similar to the method used to estimate the unconditional mean. Broadly speaking, this approach works as follows. Using the model for the unconditional mean we separate the observed hurricane tracks into an unconditional mean component and a deviation from the unconditional mean. We then model the variance of these deviations at each point in the basin as being equal to an empirical variance estimated from the observed deviations near to that point. As before, a length-scale defines what we mean by ‘near’. We calculate a new length-scale specifically for the variance to allow for the possibility that the length-scale could be different from that derived for the mean.

Within this overall framework we will compare three different models for the variance, ranging from simple to complex. We take this systematic simple-to-complex approach to try to ensure that we avoid over-fitting (i.e. we want to avoid using a complex model which performs less well than a simpler model could). The models we are test are (a) one in which the variance structure of the deviations from the mean is isotropic, (b) one in which the variance structure of the deviations is anisotropic but uncorrelated relative to orthogonal axes defined along and across the local unconditional mean trajectory, and (c) one in which the variance structure of the deviations is anisotropic and correlated relative to these axes.

We now describe the data we will use for this study (section 2), the three models (section 3), and the results from a comparison between the results of the three models (section 4). We then summarise the results and describe our future plans (section 5). Finally we include two appendices which contain some additional information and discussion.

2 The data

The basic data set for this study is a subset of the Hurdat data, as described in Hall and Jewson (2005). In that study we used a nearest neighbours method to predict the unconditional mean of our stochastic process. We use that model to make one-step (6 hour) predictions of the observed hurricane tracks. By comparing these predictions with the actual tracks we can generate forecast errors. We take these forecast errors as the starting point for the current study. We will refer to these forecast errors as the deviations from the unconditional mean track.

3 The models

3.1 The isotropic model

Our initial model for the deviations from the unconditional mean track is isotropic in that we assume that the variance of the deviations is the same in all directions. In other words, the contours of constant probability density in the predicted distribution of the deviations are circular. In the continuous time limit we can write this model as:

\[
\begin{align*}
\frac{dX}{dt} &= \mu_x(\theta, \phi)dt + \sigma(\theta, \phi)dW_x \\
\frac{dY}{dt} &= \mu_y(\theta, \phi)dt + \sigma(\theta, \phi)dW_y
\end{align*}
\]

where \(X\) and \(Y\) are the longitude and latitude of the hurricane, \(\theta\) and \(\phi\) are longitude and latitude, \(\mu_x\) and \(\mu_y\) are the unconditional mean velocities (determined by the model for the unconditional mean), \(\sigma\) is the standard deviation of the deviations from the unconditional mean track and \(dW_x\) and \(dW_y\) are independent Brownian motions. The standard deviation \(\sigma(\theta, \phi)\) is determined by the lengthscale \(\lambda\) and the historical data, so we could write:

\[
\sigma(\theta, \phi) = \sigma(\text{historical data, } \lambda)
\]

In order to fit \(\sigma\) at the point \((\theta, \phi)\) we calculate the weighted variance of the observed deviations from the unconditional mean track model, where the deviations are weighted using a Gaussian weighting function
so that nearest errors are much more important than distant errors. We vary the lengthscale in the weighting function to find which length-scale gives the optimal results.

How should we define ‘optimal’ for a probabilistic prediction model of this type? When fitting the model for the unconditional mean track, we used RMSE as the cost function. However, RMSE cannot be used to fit the variance, since changing the predicted variance doesn’t affect it. The most obvious generalisation of RMSE seems to be (minus one times) the log-likelihood of classical statistics, and this is what we use as our cost function. We calculate the log-likelihood out of sample: an in-sample maximisation of the log-likelihood would lead to an optimal lengthscale of zero. It would also not take into account parameter uncertainty when comparing the results from different models, and would thus not penalise over-fitted models.

We now derive an expression for the log-likelihood for the isotropic model.

### 3.1.1 Likelihood for the isotropic model

The multivariate normal distribution with dimension $p$ has the density:

$$f = \frac{1}{(2\pi)^{p/2} D^{1/2}} \exp \left( -\frac{1}{2} (z - \mu)^T \Sigma^{-1} (z - \mu) \right)$$

(3)

where $\Sigma$ is the covariance matrix (size $p$ by $p$), $D$ is the determinant of the covariance matrix (a single number), $z$ is a vector length $p$ and $\mu$ is a vector length $p$.

In our two dimensional ($p = 2$) isotropic case we let $z = (x, y)$ be the deviations from the unconditional mean track, we define $\sigma^2$ to be the variance of both $x$ and $y$, and we set $\mu = 0$ because we are looking at deviations. The assumption of isotropy means that $x$ and $y$ are independent, and so the covariance matrix $\Sigma$ is just $\sigma^2$ times the unit matrix:

$$\Sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}. \quad (4)$$

It follows that the inverse of the covariance matrix is given by:

$$\Sigma^{-1} = \begin{pmatrix} \sigma^{-2} & 0 \\ 0 & \sigma^{-2} \end{pmatrix} \quad (5)$$

and

$$D = \sigma^4. \quad (6)$$

This gives:

$$(z - \mu)^T \Sigma^{-1} (z - \mu) = \frac{1}{\sigma^2} (x^2 + y^2) \quad (7)$$

For $N$ data points the likelihood is thus:

$$f = \prod_{i=1}^{N} \frac{1}{2\pi \sigma_i^2} \exp \left( -\frac{x_i^2 + y_i^2}{2\sigma_i^2} \right) \quad (8)$$

and the log of this is

$$\ln f = \sum_{i=1}^{N} -\ln(2\pi \sigma_i^2) + \sum -\frac{x_i^2 + y_i^2}{2\sigma_i^2} \quad (9)$$

### 3.2 The anisotropic uncorrelated model

Our second model for the deviations from the mean track is anisotropic in that we allow for the deviations to have different variances in the different directions defined by the coordinate system. Rather than use the ($\theta, \phi$) coordinate system of lines of longitude and latitude, it seems to make sense to use a coordinate system defined to suit the problem. The obvious choice is to use an orthogonal curvilinear coordinate system based on the local unconditional mean tracks, as defined in [Hall and Jewson (2005)](see figure 3 in that paper). We write the deviations from the unconditional mean track within this coordinate system as $(u, v)$. $u$ represents displacements along the mean track while $v$ represents displacements across the mean track. Within this coordinate system we assume that the forecast errors are independent. The contours of constant probability density in this model are thus ellipses with their principal axes lying
along and across the directions given by the unconditional mean tracks. By comparing the fitted values of the variance in the $u$ and $v$ directions we will be able to see which of the principle axes is the longer of the two.

We can write this model as:

$$
\begin{align*}
    dU &= \mu_u(\theta, \phi) dt + \sigma_u(\theta, \phi) dW_u \\
    dV &= \mu_v(\theta, \phi) dt + \sigma_v(\theta, \phi) dW_v
\end{align*}
$$

where $U$ and $V$ are the projections of the hurricane motion in the directions parallel and perpendicular, respectively, to the local mean track, and $dW_u$ and $dW_v$ are uncorrelated.

### 3.2.1 Likelihood for the anisotropic uncorrelated model

We define $z = (u, v)$ (deviations from the mean track in the along-mean-track and across-mean-track directions), and the variances of $u$ and $v$ to be $\sigma_u^2$ and $\sigma_v^2$. $\mu = 0$ because we are looking at deviations from the mean tracks, and because we assume that $u$ and $v$ are independent the covariance matrix $\Sigma$ is diagonal and given by:

$$
\Sigma = \begin{pmatrix}
    \sigma_u^2 & 0 \\
    0 & \sigma_v^2
\end{pmatrix}.
$$

The inverse of the covariance matrix is:

$$
\sigma^{-1} = \frac{1}{\sigma_u^2 \sigma_v^2} \begin{pmatrix}
    \sigma_v^2 & 0 \\
    0 & \sigma_u^2
\end{pmatrix}
$$

and

$$
D = \sigma_u^2 \sigma_v^2.
$$

This gives:

$$
(z - \mu)^T \Sigma^{-1}(z - \mu) = \frac{1}{\sigma_u^2 \sigma_v^2} (u^2 \sigma_v^2 + v^2 \sigma_u^2)
$$

and so the log-likelihood for $N$ data points is:

$$
\ln f = \sum_{i=1}^{N} -\ln(2\pi \sigma_u \sigma_v) + \sum -\frac{1}{2\sigma_u^2 \sigma_v^2} (u_i^2 \sigma_v^2 + v_i^2 \sigma_u^2)
$$

where we have suppressed the $i$ subscripts on $\sigma_u$ and $\sigma_v$ for clarity.

### 3.3 The anisotropic correlated model

Our third model for the deviations from the unconditional mean tracks is isotropic but correlated. As with the previous model we use the $(u, v)$ coordinate system based on the local unconditional mean tracks, but now we also calculate the correlation between along-track and across-track errors in this coordinate system. The contours of constant probability in this model are now ellipses but with arbitrary alignment relative to the mean tracks. This alignment is determined by the correlation between the deviations in the $u$ and $v$ directions.

We can write this model as:

$$
\begin{align*}
    dU &= \mu_u(\theta, \phi) dt + \sigma_u(\theta, \phi) dW_u \\
    dV &= \mu_v(\theta, \phi) dt + \sigma_v(\theta, \phi) dW_v
\end{align*}
$$

where $dW_u$ and $dW_v$ are correlated with linear correlation coefficient $r$. 

3.3.1 Likelihood for the anisotropic correlated model

The covariance matrix $\Sigma$ is now given by

$$
\Sigma = \begin{pmatrix}
\sigma_u^2 & r\sigma_u \sigma_v \\
\sigma_u \sigma_v & \sigma_v^2
\end{pmatrix}
$$

(17)

The inverse of the covariance matrix is

$$
\Sigma^{-1} = \frac{1}{(1-r^2)\sigma_u^2 \sigma_v^2} \begin{pmatrix}
\sigma_v^2 & -r\sigma_u \sigma_v \\
-r\sigma_u \sigma_v & \sigma_u^2
\end{pmatrix}
$$

(18)

and

$$
D = (1-r^2)\sigma_u^2 \sigma_v^2.
$$

(19)

This gives:

$$
(z - \mu)^T \Sigma^{-1} (z - \mu) = \frac{1}{(1-r^2)\sigma_u^2 \sigma_v^2} (u^2 \sigma_u^2 + v^2 \sigma_v^2 - 2ruv \sigma_u \sigma_v)
$$

(20)

and so the log-likelihood for $N$ data points is:

$$
\ln f = \sum_{i=1}^{N} -\ln[2\pi(1-r^2)\sigma_u \sigma_v] + \sum -\frac{1}{2(1-r^2)\sigma_u^2 \sigma_v^2} (u_i^2 \sigma_u^2 + v_i^2 \sigma_v^2 - 2ruv \sigma_u \sigma_v)
$$

(21)

Again we’ve suppressed the $i$ subscripts on $\sigma_u$, $\sigma_v$, and $r$ for clarity.

4 Results

4.1 Log-likelihood scoring results

Figure 1 shows the log-likelihood score versus averaging length-scale for the three models. For the isotropic model the optimal averaging length-scale is 380km (panel A). For the anisotropic uncorrelated model the optimal averaging length-scale is 300km (panel B), and for the anisotropic correlated model the optimal averaging length-scale is also 300km (panel C). These optimal length-scales were calculated to within 20km.

In figure 1, panel D, we compare the log-likelihood scores for the three models. We see that the isotropic model gives the worst results, the anisotropic uncorrelated model gives somewhat better results and the anisotropic correlated model gives the best of the three. One could, on the basis of this comparison, conclude that the anisotropic correlated model is the best model to use. However, it is possible that these differences are not really significant i.e. that in fact the differences arise just because of the particular sample of data we are working with, and that other samples would give a different ordering for the models. If this were the case then it might be better to use the isotropic model on the basis that it is simpler to understand and implement. It is therefore important to assess the significance of the differences in log-likelihood scores. One way to assess whether the differences are significant is to look at them for each individual year, and this is shown in figure 2. The black curve shows the differences in the log-likelihood scores between the isotropic and anisotropic uncorrelated models, and the dashed curve shows the differences between the anisotropic uncorrelated model and the anisotropic correlated model. Comparing the isotropic and anisotropic uncorrelated models we see that the anisotropic uncorrelated model wins in 50 out of the 54 years of data on which we have tested. Comparing the anisotropic uncorrelated model with the anisotropic correlated model we see that the anisotropic correlated model wins in 42 of the 54 years we have tested.

How might we actually quantify the significance of these differences? It would be wrong to test the differences in the means between these models using a t-test or bootstrap, since the different years are sampled from different distributions. This is because the likelihood values in each year depend on the number of hurricanes in that year, as well as the length of each track. The likelihood score for a poor model in a year with many long hurricane tracks could easily be greater than that for a good model in a year in which there are only a few short hurricane tracks.

One way to avoid this difficulty might be to normalise the likelihood scores in each year using the number of tracks and the number of points on the tracks, but that is hard to do in a fair way because of correlations between errors along the tracks. As an alternative, we simply test whether 50 wins in 54 contests (and
42 wins in 54 contests) is significant, against the null hypothesis that winning is equally likely for either model. This is then equivalent to testing whether 50 or more heads in 54 coin tosses would be significant (and hence evidence for a biased coin). Using the CDF of the binomial distribution for 54 contests this gives probabilities of over 0.99999999999 and over 0.99999 for our two tests. We see that the differences between the three models are indeed highly significant. Since our log-likelihood scores are all calculated out-of-sample we can conclude that the anisotropic correlated model is the best of the three models at predicting the distribution of the future positions of the hurricane track. Thus, the contours of constant probability in the distribution of future positions of a hurricane are significantly elliptical, and the principal axes of these ellipses are significantly rotated relative to the mean track.

4.2 Variance and correlation maps

Having determined the optimum averaging length-scale within each model we can then calculate the implied variance field. These variance fields can be considered as an optimally smoothed estimate of the real variance field within the context of that particular model. Figure 8 shows these variance fields for two of the models. In the top left panel we see the variance field for the isotropic model. The main feature of this field is a steep gradient in variance from south to north, with the standard deviation increasing from around 50km in the trade-wind region to over 100km in the westerlies. In the top right and bottom left panels we see the variance fields for the anisotropic correlated model, for along-mean-track and across-mean-track directions respectively. The along-mean-track variances are slightly greater than the across-mean-track variances, but both components show the strong north-south gradient seen in the isotropic model variance field. Finally in the lower right panel we show the correlation between the along and across track deviations as determined by the optimal lengthscale in the anisotropic correlated model. The correlations are everywhere rather weak, with values between -0.2 and 0.2. This suggests that the ellipses describing the probability distribution for the deviations are only slightly rotated relative to the unconditional mean tracks.

In figure 9 we show the ratio between the \( u \) and \( v \) variances from the anisotropic uncorrelated model. This ratio is mostly positive, implying that errors in the along-mean-track direction are typically greater than errors in the across-mean-track direction (as can already be seen by comparing the second and third panels in figure 8). There is also significant spatial structure in this ratio field, which is perhaps related to the typical forward speeds of hurricanes in each region, as shown in figure 5.

4.3 Simulated tracks

Although we are still ignoring a number of features that are possibly rather important, we now have the ingredients for a minimal stochastic model for hurricane tracks. Figure 10 shows some tracks generated from this model. For comparison, one example of a real hurricane track is shown by the blue line, and other examples are give in figure 1 in [Hall and Jewson (2005)]. The simulated tracks certainly show some of the features of observed hurricane tracks: for instance, they move westward in the subtropical Atlantic and eastward in mid-latitudes. However, it seems clear that these simulated tracks are less smooth than the observations. Presumably this is because we are ignoring the autocorrelations between successive deviations along each track. The incorporation of autocorrelations is the next stage in the development of the model.

5 Summary

In a previous paper we have described a simple semi-parametric statistical model for the unconditional mean motion of Atlantic hurricanes. In this paper we have investigated how to extend that model to include the unconditional variance of the motion of the hurricanes too. The three models for the variance that we describe all rely on a simple nearest-neighbour fitting technique, with the length-scale that defines ‘near’ being optimised using cross-validation. Of these models, the simplest, which models the deviations from the unconditional mean track to be variance-isotropic, performs the least well. A model in which the deviations are modelled to be uncorrelated in directions along and across the unconditional mean track, but with different variances in these two directions, performs much better. Finally a model which represents the deviations in these two directions as correlated performs better still, and is thus the best of the models that we have tested.
The cost function we use to fit and compare our models is the out-of-sample likelihood. We propose this cost function as a sensible and objective way to compare any two hurricane track models. For instance, the models of Vickery et al. (2000) and Emanuel et al. (2005) could be evaluated and compared with our model using this cost function. As long as the models are fitted on the same underlying data, and the cross-validation is performed correctly, this would be a fair way to compare the models.

Using the models described in this paper we can normalise the observed deviations from the unconditional mean tracks so that they become stationary in variance (as well as mean zero). This will hopefully make them much easier to model using standard statistical methods (which typically assume stationarity and constant variance). The next stage of our modelling strategy is therefore to attempt to model these standardised deviations using well-known time-series techniques such as AR and ARMA, and thus incorporate memory into the model.

A Likelihood for the anisotropic correlated model

For reference, we now give the likelihood for the anisotropic correlated model in terms of the covariance $c$ rather than the correlation $r$.

The covariance matrix $\Sigma$ is given by

$$\Sigma = \begin{pmatrix} \sigma_u^2 & c \\ c & \sigma_v^2 \end{pmatrix}. \tag{22}$$

The inverse of the covariance matrix is

$$\Sigma^{-1} = \frac{1}{\sigma_u^2 \sigma_v^2 - c^2} \begin{pmatrix} \sigma_v^2 & -c \\ -c & \sigma_u^2 \end{pmatrix} \tag{23}$$

and

$$D = \sigma_u^2 \sigma_v^2 - c^2. \tag{24}$$

This gives:

$$(z - \mu)^T \Sigma^{-1} (z - \mu) = \frac{1}{\sigma_u^2 \sigma_v^2 - c^2} (u^2 \sigma_v^2 + v^2 \sigma_u^2 - 2uvc) \tag{25}$$

The log-likelihood for $N$ data points is then:

$$\ln f = N \ln \left[2\pi (\sigma_u^2 \sigma_v^2 - c^2)^{\frac{3}{2}} \right] + \sum_{i=1}^{N} \frac{1}{2(\sigma_u^2 \sigma_v^2 - c^2)} (u_i^2 \sigma_v^2 + v_i^2 \sigma_u^2 - 2uvc) \tag{26}$$

Again we’ve suppressed the $i$ subscripts on $\sigma_u$, $\sigma_v$ and $c$ for simplicity of notation.

B Derivation of an advection-diffusion differential equation

Purely for interest sake we now show that the models described above can be reformulated as an advection-diffusion equation.

Given the stochastic differential equation in equation 1 one can apply some of the standard machinery of mathematical physics. In particular, one can derive a partial differential equation (usually known as the Fokker-Planck equation or the Kolmogorov equation) that governs the evolution of a density field in space and time (Gardiner, 1985). In our case the density refers to the probability density of hurricanes. If we write this density as $f$, and ignore curvature effects, this gives:

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mu f) + \frac{\partial^2 f}{\partial \theta^2} = \frac{1}{2} \frac{\partial^2 f}{\partial \phi^2}$$

or, more succinctly, as

$$\frac{\partial f}{\partial t} + \nabla \cdot (\mu f) = \nabla^2 f \tag{28}$$

Slightly more esoterically, there is also a partial differential equation for the evolution of the probability density backwards in time (the backward Fokker-Planck or backwards Kolmogorov equation). This can be written as:

$$\frac{\partial f}{\partial t} = \mu \cdot \nabla f + \sigma \nabla^2 f \tag{29}$$
We present these equations mainly for curiosity value. They don’t seem to be particularly useful for solving the practical problem of modelling hurricane risk since they don’t generalise easily to the case where there is memory along the trajectory. Also, we are ultimately interested in modelling the intensity along the track and the damage caused by individual hurricanes, and these also don’t fit into this framework. However, we find the analogy suggested by equation 28 reasonably interesting: the distribution of possible hurricanes is advected by a mean flow field, including the effects of compression and expansion, and is diffused by a diffusive field. The equation governing this behaviour is exactly the same as the equation that governs the advection and diffusion of the density of a compressible fluid.

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P Vickery, P Skerlj, and L Twisdale. Simulation of hurricane risk in the US using an empirical track model. *Journal of Structural Engineering*, 126:1222–1237, 2000.
Figure 1: Panels A, B and C show the log-likelihood score for the isotropic, anisotropic and uncorrelated, and anisotropic and correlated models discussed in the text, as a function of averaging length scale. We see that the models have optimum averaging length-scales of 380km, 300km and 300km respectively. Panels D shows the curves from panels A, B and C together. The top curve (and hence the best model) is the anisotropic correlated model. The middle curve is the anisotropic uncorrelated model and the lower curve is the isotropic model.
Figure 2: Differences between the log-likelihood scores for the three variance models on a year by year basis. The solid curve shows differences between the two simplest models: the anisotropic uncorrelated model and the isotropic model. Since these differences are almost all positive we conclude that the anisotropic uncorrelated model beats the isotropic model in almost all years. The dashed line shows the differences between the anisotropic correlated model and the anisotropic uncorrelated model. Again the differences are mostly positive, and we can conclude that the correlated model beats the uncorrelated model in most years.
Figure 3: Panel A shows the variance field for the isotropic model. Panels B and C show the variance field for the anisotropic uncorrelated model, in the along-mean-track and across-mean-track directions respectively, and panel D shows the correlations from the anisotropic correlated model.
Figure 4: The ratio of the along-mean-track variances to the across-mean-track variances in the anisotropic uncorrelated model. This ratio indicates the extent to which the contours of constant probability density deviate from circles and become elliptical. A value near to 1 indicates the contours are nearly circular, while a value above 1 indicates that the contours are elliptical, with the longest axis along the direction of the unconditional mean tracks.
Figure 5: The forward speed of the unconditional mean tracks.
Figure 6: The thin black curves show simulated hurricane tracks from the anisotropic correlated model, all originating from the same point. The thick red curve shows the unconditional mean track from the same point, and the blue curve shows an observed hurricane track.