Enhancement of the critical slowing down influenced by extended defects

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We study an influence of the quenched extended defects on the critical dynamics of the \(d=3\)-dimensional systems with \(m\)-component non-conserved order parameter (model A dynamics). Considering defects to be correlated in \(\varepsilon_d\) dimensions and randomly distributed in \(d-\varepsilon_d\) dimensions we obtain reliable numerical values for the critical exponents governing divergence of the relaxation time as function of \(m\) and \(\varepsilon_d\).

Keywords: quenched disorder, extended defects, critical dynamics, renormalization

Critical slowing down constitutes one of the most prominent effects accompanying critical phenomena. Approaching the critical point \(T_c\) the relaxation time \(\tau\) increases and becomes infinite at \(T_c\). Its divergence is related to the divergence of the correlation length \(\xi\) by

\[
\tau \sim \xi^z,
\]

where \(z\) is the dynamical critical exponent \(z\) \cite{1}.

Study of the influence of structural disorder on the critical behaviour is an important and non-trivial task both for the theory and the experiment. For fluids, an experimental situation may be realized by a criticality of fluids in porous medium \cite{2}. Traditionally, such objects are modelled by \(m\)-vector systems with quenched uncorrelated point-like defects \cite{3}. A more realistic description is given by models, where defects are extended in space \cite{4,5}. Here, we present analysis of the critical dynamics for \(d=3\)-dimensional system with \(m\)-component non-conserved order parameter (model A dynamics) influenced by presence of \(\varepsilon_d\)-dimensional parallel
defects, randomly distributed in $d$-ε dimensions [3,4].

Reliable estimates for the exponents describing static critical behaviour of such a model were obtained only recently [6] exploiting results of Refs. [5,7]. Estimates of similar accuracy for the dynamic exponents were obtained in Ref. [8]. Here, we continue studies initiated in Ref. [8].

Presence of the extended defects of parallel orientation introduces anisotropy and two correlation lengths naturally arise in system description: one is perpendicular to the extended impurities direction, $ξ_⊥$, and the second one is parallel, $ξ_∥$ [3]. This modifies dynamical scaling as well. Indeed, the relaxation time in direction perpendicular to the impurities extension, $τ_⊥$, differs from that in direction along the impurities extension, $τ_∥$ and their scaling approaching the critical point is governed by:

$$
τ_⊥ \sim ξ^z_⊥, \quad τ_∥ \sim ξ^z_∥
$$

with two independent dynamical exponents $z_⊥$ and $z_∥$ [5].

To analyse asymptotic scaling behaviour (2) we make use of the standard tools of the field-theoretical renormalization group (RG) in minimal subtraction scheme [9]. We work within Bausch-Janssen-Wagner formalism [10], which reduces analysis of the dynamical equations of motion for the order parameter to the study of corresponding Lagrangian. Renormalising its parameters, such as bare couplings $u_0$, $v_0$, order parameter relaxation coefficient $λ_0$ and anisotropy constant $a_0$ we obtain RG functions $β_u(u,v), β_v(u,v), ζ(u,v), ζ_α(u,v)$ needed to calculate dynamical critical exponents. The $β$-functions are characterized by presence of the stable and an accessible fixed point (FP) $u^*, v^*$ of RG transformation, which corresponds to the critical point. And in this FP critical exponents (2) are calculated from the relations:
\[ z_\perp = 2 + \zeta(u^*, v^*) \quad \text{(3)} \]

\[ z_\parallel = \frac{z_\perp}{1 - \zeta(u^*, v^*) / 2} \quad \text{(4)} \]

Here we use the two-loop RG functions of Refs. [5,7]. They are obtained as expansions in powers of the renormalized couplings \( u, v \). The RG perturbative expansions are known to be divergent, thus the application of resummation is required [11] to get reliable data on their basis. We apply the Chisholm-Borel resummation [9,11] to obtain dynamical critical exponents \( z_\perp \) from (3). And as far as the series for \( z_\parallel \) (4) contains only two terms we use for it only direct substitution of the FP coordinates.

In the Fig. 1 we present the obtained results for the critical exponents \( z_\perp \) and \( z_\parallel \) as functions of impurity dimension \( \varepsilon_d \) for different \( m \). As it is seen from the figure, with an increase of \( \varepsilon_d \) for \( m=2, 3, 4 \) the critical exponents remain constant and equal to the corresponding exponents of pure model, until \( \varepsilon_d \) is smaller than \( \varepsilon'_d(m) \). For \( \varepsilon_d > \varepsilon'_d(m) \) they begin to increase. For the Ising system \( (m=1) \) the values of exponents start to increase as soon as \( \varepsilon_d > 0 \). This behaviour is explained by generalisation of usual Harris criterion [12] for extended defects [4]. It states that the extended defects become relevant for \( \varepsilon_d > d - 2/\nu_p \), where \( \nu_p \) is the correlation length critical exponent of a system without defects. The above relation defines for each \( \varepsilon_d \) the critical value \( m_c \), below which the extended defects influences universal critical properties. It turns out, that the disorder with extended defects is relevant for \( d=3 \) over a wider range of \( m \) than the point defect disorder.

The typical numerical values for the dynamical critical exponents of Fig. 1 are larger then those of the pure \( d=3 \) Ising model \((z=2.18 [13])\) as well as for the \( d=3 \) Ising model with point-like defects \((z=2.017 [14])\). It means that the behaviour of the relaxation time in the vicinity of a critical point is characterized by a stronger...
singularity. Therefore presence of extended defects leads to further enhancement of the critical slowing down in comparison with a pure system.

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Fig.1 The values of dynamical critical exponents $z_{\perp}$ and $z_{\parallel}$ of three-dimensional $m$-component magnets at different fixed values of extended defect dimension $\varepsilon_d$.

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