Abstract

In this paper we calculate the contributions to the branching ratio of $B \to X_s \gamma$ from the charged Pseudo-Goldstone bosons appeared in one generation Technicolor model. The current CLEO experimental results can eliminate large part of the parameter space in the $m(P^+) - m(P_S^8)$ plane, and specifically, one can put a strong lower bound on the masses of color octet charged PGBs $P_S^8$: $m(P_S^8) > 400 \text{GeV}$ at 90\%C.L for free $m(P^+)$. 

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1 Introduction

Recently the CLEO collaboration has observed\cite{1} the exclusive radiative decay $B \rightarrow K^*\gamma$ with a branching fraction of $BR(B \rightarrow K^*\gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$. The inclusive $b \rightarrow s\gamma$ branching ratio measured by CLEO\cite{2} is:

$$BR(B \rightarrow X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \quad (1)$$

The newest upper and lower limits of this decay branching ratio are

$$1.0 \times 10^{-4} < BR(B \rightarrow X_s\gamma) < 4.2 \times 10^{-4}, \quad \text{at \ 95\% \ C.L..} \quad (2)$$

As a loop-induced flavor changing neutral current (FCNC) process the inclusive decay (at quark level) $b \rightarrow s\gamma$ is in particular sensitive to contributions from those new physics beyond the Standard Model (SM)\cite{3}. There is a vast interest in this decay.

The decay $b \rightarrow s\gamma$ and its large leading log QCD corrections have been evaluated in the SM by several groups \cite{4}. The reliability of the calculations of this decay is improving as partial calculations of the next-to-leading logarithmic QCD corrections to the effective Hamiltonian\cite{5, 6}.

On the other hand the discovery of the top quark and the measurement of its mass (in this paper we use the weighted average $m_t = 180 \pm 12 \ GeV$ from the announced results of $m_t$ by CDF and D0\cite{7} wherever possible) at FERMILAB basically eliminated a source of uncertainties for the calculation of the decay $b \rightarrow s\gamma$ in the SM and in beyond theories. The great progress in theoretical studies and in experiments achieved recently encourage us to do more investigations about this decay in Technicolor theories.

In this paper, we estimate the possible contributions to the decay $b \rightarrow s\gamma$ from the exchange of the charged Pseudo-Goldstone bosons which will appear in no-minimal Technicolor models, such as the Farhi-Susskind one-generation Technicolor model (OGTM)\cite{8}. We know that the experimental data seems disfavor the OGTM which generally tend to predict $S$ parameter large and positive \cite{9}. Why we here still choose it to do the calculations? The reasons are the following:

(1) At first, presence of the Pseudo-Goldstone bosons in the particle spectrum is a common feature of those non-minimal TC models with ordinary or novel ETC sectors, no matter the specific differences of structures between those models. The gauge couplings of the PGBs are determined by their quantum numbers, while the Yukawa couplings of PGBs to ordinary fermions are generally proportional to fermion masses for many TC/ETC models. Among the non-minimal TC models, the OGTM\cite{8} is the simplest and most frequently studied model. Many relevant works \cite{10} have been done since the late 1970’s. One can use those existed results directly in further investigations.
On the other hand, the constraints on the $S$ parameter could be relaxed considerably by introducing three additional parameters $(V, W, X)$ [11]. A global fit to the data in which all six oblique parameters $S$ through $X$ are allowed to vary simultaneously gives the one standard deviation bound on $S$: $S \sim -0.93 \pm 1.7$ [12]. This fact means that the constraint on the OGTM from the parameter $S$ could be considerably weakened if we consider the effects from light technifermions and light PGBs [13].

In this paper, we estimate the possible contributions to the rare decay $b \to s\gamma$ from the charged PGBs in the framework of the OGTM. At least, one can regard our results as an estimation for the “correct” output of the future “realistic” TC models.

This paper is organized as the following: In Section 2, we present the basic ingredients of the OGTM and then calculate the PGB contributions to $b \to s\gamma$ decay, together with the full leading log QCD corrections. In Section 3, we obtain the branching ratios of this decay, and derive out the constraints on masses of charged PGBs by phenomenological analysis. The conclusions are also included in this section.

2 Charged PGBs and QCD Corrections to $b \to s\gamma$

In the OGTM [8], when the technifermion condensate $\langle \overline{T} T \rangle \neq 0$ was formed, the global flavor symmetry will break as follows: $SU(8)_L \times SU(8)_R \to SU(8)_{L+R}$. Consequently, 63 (Pseudo)-Goldstone bosons will be produced from this breaking. When all other interactions but the Technicolor are turned off, these 63 Goldstone bosons are exactly massless. Three of them are eaten by the $W^\pm$ and $Z^0$ gauge bosons. The others acquire masses when one turned on the gauge interactions, and therefore they are Pseudo-Goldstone Bosons(PGBs).

According to previous studies, the phenomenology of those color-singlet charged PGBs in the OGTM is very similar with that of the elementary charged Higgs bosons $H^\pm$ of Type-I Two-Higgs-Doublet Model(2HDM) [14]. And consequently, the contributions to the decay $b \to s\gamma$ from the color-singlet charged PGBs in the OGTM will be very similar with that from charged Higgs bosons in the 2HDM. As for the color-octet charged PGBs, the situation is more complicated because of the involvement of the color interactions. Other neutral PGB’s don’t contribute to the rare decay $b \to s\gamma$.

The gauge couplings of the PGBs are determined by their quantum numbers. The Yukawa couplings of PGBs to ordinary fermions are induced by ETC interactions and hence are model dependent. However, these Yukawa couplings are generally proportional to fermion masses with small differences in the magnitude of the coefficients for different TC/ETC models. The relevant couplings needed in our calculation are directly quoted.
from refs.\[15, 16, 17\] and summarized in Table 1, where the $V_{ud}$ is the corresponding element of Kobayashi-Maskawa matrix. For the OGTM, the Goldstone boson decay constant $F_\pi$ in Table 1 should be $F_\pi = v/2 = 123 \text{ GeV}$, in order to ensure the correct masses for the gauge bosons $Z^0$ and $W^\pm$. \[10\]

Table 1: The relevant gauge couplings and Effective Yukawa couplings for the OGTM.

| Operator | Coupling |
|----------|----------|
| $P^+ P^- \gamma_\mu$ | $-ie(p_+ - p_-)_\mu$ |
| $P_{8a}^+ P_{8b}^- \gamma_\mu$ | $-ie(p_+ - p_-)_\mu \delta_{ab}$ |
| $P^+ u d$ | $i\frac{V_{ud}}{2F_\pi} \sqrt{\frac{2}{3}} [M_u(1 - \gamma_5) - M_d(1 + \gamma_5)]$ |
| $P_{8a}^+ u d$ | $i\frac{V_{ud}}{2F_\pi} 2\lambda_a [M_u(1 - \gamma_5) - M_d(1 + \gamma_5)]$ |
| $P_{8a}^+ P_{8b}^- g_\mu$ | $-gf_{abc}(p_a - p_b)_\mu$ |

In ref.\[18\], Randall and Sundrum have estimated the contributions to $b \rightarrow s \gamma$ from the exchange of ETC gauge bosons in various ETC scenarios. In the case of “traditional" ETC (just the case which will be studied here), the dominant contribution to $b \rightarrow s \gamma$ occurs when the ETC gauge boson is exchanged between purely left-handed doublets and when the photon is emitted from the technifermion line. But the resulted ETC contribution is strongly suppressed with respect to the SM by a factor of $m_t/(4\pi v) < 0.09$ for $m_t < 200 \text{ GeV}$\[18\]. In short, the ETC contribution to the decay $b \rightarrow s \gamma$ is small and will be masked by still large experimental and theoretical uncertainties. We therefore can neglect the ETC Contributions to $b \rightarrow s \gamma$ at present phenomenological analysis.

In Fig.1, we draw the relevant Feynman diagrams which contribute to the decay $b \rightarrow s \gamma$, where the half-circle lines represent the W gauge boson of SM as well as the charged PGBs $P^\pm$ and $P_8^\pm$ of OGTM. In the evaluation we at first integrate out the top quark and the weak W bosons at $\mu = M_W$ scale, generating an effective five-quark theory. By using the renormalization group equation, we run the effective field theory down to $b$-quark scale to give the leading log QCD corrections, then at this scale, we calculate the rate of radiative $b$ decay.

After applying the full QCD equations of motion\[19\], a complete set of dimension-6 operators relevant for $b \rightarrow s \gamma$ decay can be chosen to be:

\[
O_1 = \langle \bar{c}_{L\beta} \gamma^\mu b_{L\alpha} \rangle \langle \bar{s}_{L\alpha} \gamma_\mu c_{L\beta} \rangle ,
\]

\[
O_2 = \langle \bar{c}_{L\alpha} \gamma^\mu b_{L\alpha} \rangle \langle \bar{s}_{L\beta} \gamma_\mu c_{L\beta} \rangle ,
\]

\[
O_3 = \langle \bar{s}_{L\alpha} \gamma^\mu b_{L\alpha} \rangle \sum_{q=u,d,s,c,b} \langle \bar{q}_{L\beta} \gamma_\mu q_{L\beta} \rangle ,
\]
\[
O_4 = (\bar{s}_L \gamma^\mu b_L \delta) \sum_{q=u,d,s,c,b} (\bar{q}_{L\delta} \gamma_\mu q_{L\alpha}),
\]
(6)
\[
O_5 = (\bar{s}_L \gamma^\mu b_L \delta) \sum_{q=u,d,s,c,b} (\bar{q}_{R\delta} \gamma_\mu q_{R\alpha}),
\]
(7)
\[
O_6 = (\bar{s}_L \gamma^\mu b_L \delta) \sum_{q=u,d,s,c,b} (\bar{q}_{R\delta} \gamma_\mu q_{R\alpha}),
\]
(8)
\[
O_7 = (e/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},
\]
(9)
\[
O_8 = (g/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a.
\]
(10)

The effective Hamiltonian appears just below the W-scale is given as
\[
H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(M_W) O_i(M_W).
\]
(11)

The coefficients of 8 operators are calculated from diagrams of Fig.1:
\[
C_i(M_W) = 0, \quad i = 1, 3, 4, 5, 6, \quad C_2(M_W) = -1,
\]
(12)
\[
C_7(M_W) = A(\delta) + \frac{B(x)}{3\sqrt{2}G_F F_\pi^2} + \frac{8B(y)}{3\sqrt{2}G_F F_\pi^2},
\]
(13)
\[
C_8(M_W) = C(\delta) + \frac{D(x)}{3\sqrt{2}G_F F_\pi^2} + \frac{8D(y) + E(y)}{3\sqrt{2}G_F F_\pi^2},
\]
(14)

with \( \delta = M_W^2/m_t^2, \ x = (m(P^+) + m_t) \) and \( y = (m(P^+) + m_t) \). The functions \( A \) and \( C \) arise from graphs with W boson exchange are already known contributions from SM; while the functions \( B, D, \) and \( E \) arise from diagrams with color-singlet and color-octet charged PGBs of OGTM. They are given by,
\[
A(\delta) = \frac{1}{3} + \frac{5}{27} \delta - \frac{7}{27} \delta^2 + \frac{3}{4} \delta - \frac{1}{4} \delta^2 \log[\delta],
\]
(15)
\[
B(y) = \frac{-11}{36} + \frac{53}{27} y - \frac{25}{72} y^2 + \frac{-1}{3} y + \frac{2}{3} y^2 - \frac{1}{3} y^3 \log[y],
\]
(16)
\[
C(\delta) = \frac{1}{3} - \frac{5}{8} \delta - \frac{1}{4} \delta^2 - \frac{3}{4} \delta^2 \log[\delta],
\]
(17)
\[
D(y) = \frac{-5}{24} + \frac{10}{27} y - \frac{5}{6} y^2 + \frac{1}{2} y^2 - \frac{1}{2} y^3 \log[y],
\]
(18)
\[
E(y) = \frac{3}{2} - \frac{15}{8} y - \frac{9}{8} y^2 + \frac{9}{3} y - \frac{9}{2} y^2 \log[y].
\]
(19)

It is shown from these expressions that, for \( \delta < 1, \ x, y >> 1 \), the OGTM contribution \( B, D, \) and \( E \) have always a relative minus sign with the SM contribution \( A \) and \( C \). As a result, the OGTM contribution always destructively interferes with the SM contribution. This can also be seen from the numerical results and discussion in the next section.
The running of the coefficients of operators from $\mu = M_W$ to $\mu = m_b$ was well described in refs.\[4\]. After renormalization group running we have the QCD corrected coefficients of operators at $\mu = m_b$ scale.

\[
C_{7}^{\text{eff}}(m_b) = \eta^{16/23}C_7(M_W) + \frac{8}{3}(\eta^{14/23} - \eta^{16/23})C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \eta^{a_i}. \tag{20}
\]

With $\eta = \alpha_s(M_W)/\alpha_s(m_b)$,

\[
h_i = \left( \frac{62612}{272277}, -\frac{56281}{51730}, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057 \right),
\]

\[
a_i = \left( \frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456 \right).
\]

### 3 The $B \rightarrow X_s \gamma$ decay rate and phenomenology

Following refs.\[1\], applying a spectator model,

\[
BR(B \rightarrow X_s \gamma)/BR(B \rightarrow X_c e\nu) \simeq \Gamma(b \rightarrow s\gamma)/\Gamma(b \rightarrow c\bar{c}\nu). \tag{21}
\]

Then when have

\[
\frac{BR(B \rightarrow X_s \gamma)}{BR(B \rightarrow X_c e\nu)} \simeq \left| \frac{V_{tb}V_{ts}^*}{V_{cb}} \right|^2 \frac{6\alpha_{\text{QED}}}{\pi g(m_c/m_b)} |C_{7}^{\text{eff}}(m_b)|^2 \left( 1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b) \right)^{-1}, \tag{22}
\]

where the phase space factor $g(z)$ is given by:

\[
g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \log z, \tag{23}
\]

and the factor $f(m_c/m_b)$ of one-loop QCD correction to the semileptonic decay is,

\[
f(m_c/m_b) = (\pi^2 - 31/4)(1 - m_c^2/m_b^2) + 3/2. \tag{24}
\]

Afterwards one obtains the $B \rightarrow X_s \gamma$ decay rate normalized to the quite well established semileptonic decay rate $Br(B \rightarrow X_c e\nu)$. If we take experimental result $BR(B \rightarrow X_c e\nu) = 10.8\%$\[20\], the branching ratios of $B \rightarrow X_s \gamma$ is found to be:

\[
BR(B \rightarrow X_s \gamma) \simeq 10.8\% \times \left| \frac{V_{tb}V_{ts}^*}{V_{cb}} \right|^2 \frac{6\alpha_{\text{QED}} |C_{7}^{\text{eff}}(m_b)|^2}{\pi g(m_c/m_b)} \left( 1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b) \right)^{-1}. \tag{25}
\]

In numerical calculations we always use $M_W = 80.22$ GeV, $\alpha_s(m_Z) = 0.117$, $m_c = 1.5$ GeV, $m_b = 4.8$ GeV and $|V_{tb}V_{ts}^*|^2/|V_{cb}|^2 = 0.95$\[20\] as input parameters.

Generally speaking, the contribution to the decay $b \rightarrow s\gamma$ from color singlet $P^\pm$ is small when compared with the contribution from the color octet $P_8^\pm$, since there is a
color enhancement factor 8 appeared in the third terms in eqs. (3) for the functions $B(y)$ and $D(y)$. Fig.2 is the plot of the branching ratio $Br(B \to X_s \gamma)$ as a function of the top quark mass. The upper dashed curve in Fig.2 represents the branching ratio in the standard model, while the solid curve shows the same ratio with the inclusion of the contributions from $P^\pm$ and $P_8^\pm$ assuming $m(P^\pm) = 300 \text{ GeV}$ and $m(P_8^\pm) = 600 \text{ GeV}$. The band between two dash-dotted lines corresponds to the newest CLEO limits: $1.0 \times 10^{-4} < BR(B \to X_s \gamma) < 4.2 \times 10^{-4}$ at 95% C.L. [2]. The branching ratio of $b \to s \gamma$ with large contribution from OGTM, is much more sensitive with the top quark mass, compared with the case of pure SM.

It is known from the decoupling theorem that for heavy enough nonstandard boson, we should recover the SM result. So for sufficiently large values of $m(P^\pm)$, $m(P_8^\pm)$ (e.g. $m(P^\pm) > 600 \text{ GeV}$, $m(P_8^\pm) > 2000 \text{ GeV}$), the contributions from OGTM shall be negligible. This can also be seen from the fact that the functions $B, D$ and $E$ go to zero, as $x, y \to \infty$. For not so large $m(P^\pm)$, $m(P_8^\pm)$, the OGTM contribution cancels much of the SM contribution because of the relative minus sign between their contribution. As a result, the branching of $b \to s \gamma$ reached the lower limit of the CLEO experiment. So a large region of $m(P^\pm)$, $m(P_8^\pm)$ (i.e. $1000 \text{ GeV} < m(P_8^\pm) < 2000 \text{ GeV}$, for all $m(P^\pm)$) is ruled out. When $m(P^\pm)$, $m(P_8^\pm)$ go on smaller, their contribution is about two times as large as contribution of SM (recall there is a relative minus sign), the branching ratio of $b \to s \gamma$ resumes to experiment allowed region. But if the $m(P^\pm)$, $m(P_8^\pm)$ are smaller enough, the contribution of OGTM is more larger, the region is also excluded by the upper limit of CLEO experiment. The whole result is illustrated at Fig.3, large part of the parameter space in the $m(P^\pm) - m(P_8^\pm)$ plane can be excluded according to the current CLEO 95% C.L. limits on the ratio $BR(B \to X_s \gamma)$ [2]. It is easy to see that no direct limits on $m(P^\pm)$ can be obtained at present for free $m(P_8^\pm)$, but at the same time, one can simply read out the lower bound on the mass of color octet PGBs: $m(P_8^\pm) > 440 \text{ GeV}$ for free $m(P^\pm)$ (assuming $m_t = 180 \text{ GeV}$), if we simply interpret the CLEO 95% C.L. limits on the ratio $BR(B \to X_s \gamma)$ as the bounds on the masses of charged PGBs.

Of course, we have not considered the effects of other possible uncertainties, such as that of $\alpha_s(m_Z)$, next-to-leading-log QCD contribution [5], QCD correction from $m_{\text{top}}$ to $M_W$ [6] etc. The inclusion of those additional uncertainties will broaden the border lines between the allowed regions and excluded regions in Fig.3. The limitations drawn from the calculations will be surely weaken, i.e., the lower limit will become $m(P_8^\pm) > 400 \text{ GeV}$ at 90% C.L. if we include an additional 20% theoretical uncertainties.

As a conclusion, the size of contribution to the rare decay of $b \to s \gamma$ from the PGBs strongly depends on the values of the masses of the top quark and the charged PGBs. This is quite different from the SM case. By the comparison of the theoretical prediction.
with the current data one can derives out the constraints on the masses of the color octet charged PGBs: \( m(P_8^\pm) > 400 \text{ GeV} \) at 90\%C.L. for free \( m(P^\pm) \), assuming \( m_t = 180 \text{ GeV} \).

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**Figure Captions**

**Fig.1:** The Feynman diagrams which contribute to the rare radiative decay $b \to s\gamma$. The half-circle-lines in the loop represent the W gauge boson and charged PGBs propagators.

**Fig.2:** The plot of the branching ratio of $b \to s\gamma$ versus the top quark mass $m_t$ assuming $m(P^\pm) = 300$ GeV and $m(P_8^\pm) = 600$ GeV. For more details see the text.

**Fig.3:** Allowed range in the $m(P^\pm) - m(P_8^\pm)$ plan for $m_t = 180$ GeV, the band is corresponding to the current CLEO 95% C.L. limits on the ratio $BR(B \to X_s\gamma)$ as given in eq.(2).
