STRING FORMATION IN THE MODEL OF THE STOCHASTIC VACUUM AND CONSISTENCY WITH LOW ENERGY THEOREMS

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Abstract

We re-derive and discuss two low energy theorems which relate the potential of a static quark-antiquark pair with the total energy and action stored in the flux tube between the sources. In lattice QCD these relations are known as Michael’s sum-rules, but we give an essential correction to one of them. Then we relate the low energy theorems to the virial theorem for a heavy quark-antiquark bound state. Finally we compare the results for the flux tube formation, which have been calculated in the model of the stochastic vacuum, with the low energy theorems and obtain consistency. From this we conclude that the model describes the non perturbative gluon dynamics of QCD at a renormalization scale, where the strong coupling constant is given by $\alpha_s = 0.57$.

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1 Introduction

As it is well known there is no analytical tool comparable to perturbation theory for treating QCD processes involving large distances. The most direct approach is a numerical evaluation of the lattice regularized version of QCD. Nevertheless it seems useful also to try analytical approaches, based on model assumptions, which may reveal some of the underlying structure of the theory. The success of the QCD sum-rules makes it plausible that the properties of the nontrivial QCD-vacuum play an essential role for the infra-red behavior of the theory.

The model of the stochastic vacuum (MSV) makes the crucial assumption that the complicated infra-red behavior of non-Abelian gauge theories can be adequately approximated by a cluster expansion of the stochastic process describing the quantized theory [1][2][3]. It has been shown that such a model leads to linear confinement. In order to make more specific and quantitative predictions one has to make more radical assumptions, the most radical one being that all higher cumulants can be neglected as compared to the two point function. Then the stochastic process is a Gaussian one, i.e. all higher correlators can be reduced to products of two point functions by factorization. We refer to [3][4] for the description of the model and its applications. In this note we present some consistency checks by comparing results of the model for heavy quark-antiquark systems with low energy theorems.

Our paper is organized as follows: In section 2 we re-derive some low energy theorems, in section 3 we show how these theorems relate to the virial theorem for a heavy quark-antiquark bound state. In section 4 we present the results for the string formation calculated in the MSV and in section 5 we compare these findings with the low energy theorems and discuss them.

Except for section 3 we work with an Euclidean field theory (EFT), where the square of the electric field has the opposite sign of the same quantity in a Minkowski field theory (MFT), whereas the square of the magnetic field has the same sign in both theories:

\[ \vec{E}_{EFT}^2 = -\vec{E}_{MFT}^2, \quad \vec{B}_{EFT}^2 = \vec{B}_{MFT}^2. \]

Here and in the following a sum over the \( N_c^2 - 1 \) color components is understood.

2 The Low Energy Theorems

In this section we consider pure gluon QCD and static quark-antiquark sources. We follow the procedure of Novikov, Shifman, Vainshtein and Zakharov (NSVZ) [5] and redefine the unrenormalized matrix valued gluon field tensor by multiplying it with the bare coupling constant \( g_0 \):

\[ \vec{F}_{\mu\nu} = g_0 \vec{F}_{\mu\nu}^{(0)}. \] (1)
The QCD-action $S_QCD$ then depends on $\alpha_{s0} = \frac{g_0^2}{4\pi}$ only in the form

$$S_{QCD} \equiv \frac{1}{8\pi\alpha_{s0}} \int d^4x \Tr [\bar{F}_{\mu\nu}(x)F_{\mu\nu}(x)] .$$  \hfill (2)$$

Let us consider the vacuum expectation value of a Wegner-Wilson loop $[6][7]$:

$$\langle W[L] \rangle \equiv \langle Tr \exp[-i \int_L \bar{A}_\mu dx_\mu] \rangle .$$  \hfill (3)$$

Here $L$ denotes a closed loop and $\mathcal{P}$ path ordering. In a somewhat symbolic notation we write this as a functional integral:

$$\langle W[L] \rangle = \frac{1}{N} \int DA \exp[-S_{QCD}] \Tr \exp[-i \int L \bar{A}_\mu dx_\mu] ,$$  \hfill (4)$$

$$N \equiv \int DA \exp[-S_{QCD}] .$$

Differentiating $\log \langle W[L] \rangle$ with respect to $-\frac{1}{8\pi\alpha_{s0}}$ we obtain:

$$8\pi\alpha_{s0}^2 \frac{\partial \log \langle W[L] \rangle}{\partial \alpha_{s0}} =$$

$$\frac{1}{\langle W[L] \rangle} \langle \int d^4x \Tr [\bar{F}_{\mu\nu}(x)F_{\mu\nu}(x)] \Tr \exp[-i \int L \bar{A}_\mu dx_\mu] \rangle$$

$$- \langle \int d^4x \Tr [\bar{F}_{\mu\nu}(x)F_{\mu\nu}(x)] \rangle .$$  \hfill (5)$$

If the loop $L$ is a rectangle with “spatial” extension $-R/2 < x_3 < R/2$ and “temporal” extension $-T/2 < x_4 < T/2$ the right-hand side (rhs) of eq.(5) can be simplified further in the limit of large $T$. Let the length $a$ be of the order of the correlation length of the gluon field strengths. Then for $|x_4| > T/2 + a$ the rhs of eq.(5) is zero and for $|x_4| < T/2 - a$ independent of $x_4$. This gives for eq.(5) in the limit of large $T$:

$$8\pi\alpha_{s0}^2 \frac{\partial \log \langle W[L] \rangle}{\partial \alpha_{s0}} = T \langle \int d^3x \Tr [\bar{F}_{\mu\nu}(x)F_{\mu\nu}(x)] \rangle_R .$$  \hfill (6)$$

Here and in the following $x_4$ is zero and the expectation value $\langle . \rangle_R$ shall denote the EFT expectation value in the presence of a static quark-antiquark pair at distance $R$, i.e. static sources in the fundamental representation, where the expectation value in absence of the sources has been subtracted.

We define the potential $V(R)$ by:

$$V(R) \equiv \lim_{T \to \infty} -\frac{\log \langle W[L] \rangle}{T}$$  \hfill (7)$$
and thus obtain the low energy theorem:

\[-4\pi\alpha_s^2 \frac{\partial V(R)}{\partial \alpha_s} = \frac{1}{2} < \int d^3x \, \text{Tr} \left[ \mathbf{F}_{\mu\nu}(x) \mathbf{F}_{\mu\nu}(x) \right] >_R . \quad (8)\]

In eq. (7) and eq. (8) self energies are understood to be subtracted.

Up to now we have used the bare coupling $g_0$ and bare fields. If we use the background gauge fixing \cite{8} we have

\[g = Z_g^{-1} g_0 , \quad A_\mu = Z_{g}^{-1/2} A^{(0)}_\mu , \quad Z_g Z^{1/2} = 1 , \quad \tilde{A}_\mu = g_0 A^{(0)}_\mu = g A_\mu , \quad (9)\]

where $g$ and $A_\mu$ are renormalized quantities. The renormalized squared field strength tensor is given by:

\[\text{Tr} \left[ \mathbf{F}_{\mu\nu}(x) \mathbf{F}_{\mu\nu}(x) \right] = Z_{F^2}^{-1} \text{Tr} \left[ \mathbf{F}^{(0)}_{\mu\nu}(x) \mathbf{F}^{(0)}_{\mu\nu}(x) \right] , \quad (10)\]

where $Z_{F^2}$ was calculated \cite{9} in background gauge with Landau gauge fixing to be

\[Z_{F^2} = 1 + \frac{1}{2} g_0 \frac{\partial}{\partial g_0} \log Z\]

and thus we obtain

\[\frac{\partial \alpha_s}{\partial \alpha_{s0}} = Z Z_{F^2} . \quad (11)\]

We use eq. (9)-eq. (11) to express eq. (8) in terms of renormalized quantities:

\[-\alpha_s \frac{\partial V(R)}{\partial \alpha_s} = \frac{1}{2} < \int d^3x \, \text{Tr} \left[ \mathbf{F}_{\mu\nu}(x) \mathbf{F}_{\mu\nu}(x) \right] >_R . \quad (12)\]

In order to evaluate the left-hand side (lhs) of eq. (12) we use standard renormalization group arguments: The only scales are the renormalization scale $\mu$ and $R$, thus the potential $V = V(R, \mu, \alpha_s)$ satisfies on dimensional grounds:

\[\left( R \frac{\partial}{\partial R} - \mu \frac{\partial}{\partial \mu} \right) V(R, \mu, \alpha_s) = -V(R, \mu, \alpha_s) . \quad (13)\]

But for $V$ as a physical quantity the total derivative with respect to $\mu$ must vanish:

\[\mu \frac{d}{d\mu} V(R, \mu, \alpha_s) = \left( \mu \frac{\partial}{\partial \mu} + \tilde{\beta}(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) V(R, \mu, \alpha_s) = 0 , \quad (14)\]
where $\tilde{\beta}(\alpha_s) \equiv \mu \frac{d}{d\mu} \alpha_s(\mu)$. We get from eq.(13) and eq.(14):

$$ \frac{\partial V(R)}{\partial \alpha_s} = -\frac{1}{\beta} \left\{ V(R) + R \frac{\partial V(R)}{\partial R} \right\} \tag{15} $$

and we finally obtain with eq.(12) the low energy theorem

$$ \left\{ V(R) + R \frac{\partial V(R)}{\partial R} \right\} = \frac{1}{2} \alpha_s < \int d^3x \text{ Tr} \left[ \mathbf{F}_{\mu\nu}(x) \mathbf{F}_{\mu\nu}(x) \right] > R \tag{16} $$

or equivalently

$$ \left\{ V(R) + R \frac{\partial V(R)}{\partial R} \right\} = \frac{1}{2} \alpha_s < \int d^3x \left( \vec{E}(x)^2 + \vec{B}(x)^2 \right) > R . \tag{17} $$

The eq.(17) is for the case of a linear potential just one of the many low energy theorems derived by NSVZ \cite{5}. But there renormalization was only discussed to leading order.

We have also the usual relation between the potential $V(R)$ and the energy density $\Theta_{00}(x)$:

$$ V(R) = \int d^3x < \Theta_{00}(x) > R = \frac{1}{2} < \int d^3x \left( -\vec{E}(x)^2 + \vec{B}(x)^2 \right) > R . \tag{18} $$

Note that we are in a Euclidean field theory, hence the minus sign of the electric field, and that on the rhs of eq.(17) and eq.(18) we have renormalized composite operators.

In lattice QCD eq.(17) and eq.(18) are known as Michael’s sum-rules \cite{10}. But in the derivation of the action sum-rule (eq.(17)) scaling of $V(R)$ with $R$ was not taken properly into account and hence the second term on the lhs of eq.(17) is missing in \cite{10}.

### 3 The Low Energy Theorems For A Quark-Antiquark Bound State

In this section we work in Minkowski field theory of gluons interacting with a dynamical heavy quark field $q$. The symmetrized energy momentum tensor is given by:

$$ \Theta_{k\lambda}^{\text{sym.}} = \Theta_{k\lambda}^{G} + \Theta_{k\lambda}^{Q} , \quad \Theta_{k\lambda}^{G} = 2 \text{ Tr} \left[ \mathbf{F}_{\kappa\rho} \mathbf{F}_{\lambda}^{\rho} \right] + \frac{1}{2} g_{k\lambda} \text{ Tr} \left[ \mathbf{F}_{\mu\nu} \mathbf{F}_{\mu\nu} \right] , \quad \Theta_{k\lambda}^{Q} = \frac{i}{4} \bar{q} \left( \gamma_\kappa \mathbf{D}_\lambda + \gamma_\lambda \mathbf{D}_\kappa \right) q . \tag{19} $$
Here $D_\lambda$ is the covariant derivative.

Consider a quark-antiquark bound state, which is an eigenstate of energy and momentum:

$$|M(p)\rangle = |qq(p)\rangle,$$

$$<M(p')|M(p)\rangle = 2p^0 (2\pi)^3 \delta^3 (p' - p). \quad (20)$$

We have from Lorentz invariance

$$<M(p)|\Theta_{\mu\nu}^{\text{sym}}(0)|M(p)\rangle = 2p_{\mu}p_{\nu}. \quad (21)$$

In the rest frame of the bound state the total energy is given by $p_R^0 = 2m_q + E_B$, where the binding energy $E_B$ is the sum of the mean values of the potential and kinetic energies:

$$E_B = <V> + <T>. \quad (22)$$

With eq.(21) and $\Theta_{\mu\nu}^{\text{sym}} \equiv g^{\mu\nu} \Theta_{\mu\nu}^{\text{sym}}$ we obtain for $|E_B| \ll m_q$:

$$<M(p_R)|\Theta_{00}^{\text{sym}}(0)|M(p_R)\rangle = <M(p_R)|\Theta_{00}^{\text{sym}}(0)|M(p_R)\rangle = 8m_q^2 + 8m_qE_B. \quad (23)$$

The kinetic part of the energy momentum tensor of the quark-antiquark bound state is $\Theta_{\mu\nu}^{Q}$. With the kinetic momenta of the quarks, $(p_q)^2 = (p_{\bar{q}})^2 = m_q^2$, we have

$$\frac{1}{p_R^0} <M(p_R)|\Theta_{\mu\nu}^{Q}(0)|M(p_R)\rangle = <\frac{2p_q\mu p_q\nu}{p_q^0} + \frac{2p_{\bar{q}}\mu p_{\bar{q}}\nu}{p_{\bar{q}}^0}>. \quad (24)$$

This gives for $\Theta^{Q} \equiv g^{\mu\nu} \Theta_{\mu\nu}^{Q}$ and $\Theta_{00}^{Q}$:

$$\frac{1}{2p_R^0} <M(p_R)|\Theta^{Q}(0)|M(p_R)\rangle = 2m_q - <T>,$$

$$\frac{1}{2p_R^0} <M(p_R)|\Theta_{00}^{Q}(0)|M(p_R)\rangle = 2m_q + <T>. \quad (25)$$

More details on the derivation of eq.(24) and eq.(25) will be given in a future publication. Now we evaluate the gluonic part of the energy momentum tensor by subtracting the kinetic part (eq.(25)) from the total tensor (eq.(23)). We obtain

$$\frac{1}{2p_R^0} <M(p_R)|\Theta^{G}(0)|M(p_R)\rangle = <V> + 2 <T>,$$

$$\frac{1}{2p_R^0} <M(p_R)|\Theta_{00}^{G}(0)|M(p_R)\rangle = <V>. \quad (26)$$
with $\Theta^G \equiv g^{\mu\nu} \Theta^G_{\mu\nu}$. In order to relate the kinetic energy to the potential one we use the virial theorem:

$$< T > = \frac{1}{2} < R \frac{\partial V}{\partial R} > .$$

Eq. (26) then gives

$$\frac{1}{2p^0_R} < M(p_R)|\Theta^G(0)|M(p_R) > = < V > + < R \frac{\partial V}{\partial R} > ,$$

$$\frac{1}{2p^0_R} < M(p_R)|\Theta^G_{00}(0)|M(p_R) > = < V > .$$  \hspace{1cm} (27)$$

Using the trace anomaly of the energy momentum tensor \[11\]

$$\Theta^G = \frac{\tilde{\beta}}{2\alpha_s} \text{Tr} [F_{\mu\nu}F^{\mu\nu}]$$

we finally obtain:

$$< V > + < R \frac{\partial V}{\partial R} > = \frac{1}{2p^0_R} < M(p_R)|\frac{\tilde{\beta}}{2\alpha_s} \text{Tr} [F_{\mu\nu}(0)F^{\mu\nu}(0)]|M(p_R) > ,$$

$$< V > = \frac{1}{2p^0_R} < M(p_R)|\Theta^G_{00}(0)|M(p_R) > .$$  \hspace{1cm} (28)$$

It is straightforward to show that here we have again the relations (16) and (18), but in eq. (28) and eq. (29) we have mean values as obtained by integration with the absolute square of the wave function of the quark-antiquark bound state. The integration over the space points in eq. (16) and eq. (18) corresponds to the translational invariance of the quark-antiquark bound state in eq. (28) and eq. (29).

4 The Flux Tube In The MSV

In this section we present the main results from a calculation of the flux tube between a static quark-antiquark pair in the MSV. For all technical details we refer to our previous publication \[12\].

The expectation values of the squared elements of the gluon field strength tensor in presence of the quark-antiquark pair minus the vacuum values are given by:

$$\Delta F^2_{\alpha\beta}(x) \equiv \frac{4}{R^4_p} < W[L] P_{\alpha\beta}(x) > - < W[L] >= < P_{\alpha\beta}(x) > .$$  \hspace{1cm} (30)$$
Here $P_{\alpha\beta}(x)$ is a Wegner-Wilson loop over a plaquette in $\alpha, \beta$-direction centered at $x$ with side length $R_P$. For $R_P \to 0$ we have:

$$\lim_{R_P \to 0} \Delta F_{\alpha\beta}^2(x) = -2 < g^2 \text{Tr} [F_{\alpha\beta}(x)F_{\alpha\beta}(x)] > R \tag{31}$$

where there is no summation over $\alpha$ and $\beta$.

It was shown in [12] that by symmetry arguments $< g^2 \vec{B}^2 > R = 0$. The square of the electric field perpendicular to the loop $L (< g^2 E_\perp^2 >)$ is also practically not influenced but only the squared electric field parallel to $L (< g^2 E_\parallel^2 >)$ is affected by the static sources. In fig. (1) we display $- < g^2 E_\parallel^2 > R$ as a function of the perpendicular distance $x_\perp$ from the loop $L$ and of the position along the quark-antiquark axis $x_3$. The lengths are given in units of the correlation length $a$ of the gluon field strengths, which is fixed to $a = 0.35$ fm in the MSV [12].

$R = 4a, a = 0.35$fm $R = 9a, a = 0.35$fm

Figure 1: Difference of the squared electric field parallel to the quark-antiquark ($x_3$) axis ($- < g^2 E_\parallel^2(x_3, x_\perp) > R$) in GeV fm$^3$ for different quark separations $R$. $x_\perp$ is the distance transverse to the $x_3$-axis and the dots denote the quark positions.

As can be seen directly from eq.(17) and eq.(18) the squared electric and magnetic field strengths are separately not renormalization group invariant, thus statements like $< g^2 \vec{B}^2 > R = 0$ or $< g^2 E_\perp^2 > R \approx 0$ are scale dependent. In [12] we have used eq.(18) in order to determine at which $\alpha_s$ the MSV is supposed to work. We have calculated the total energy stored in the flux tube and compared it to the potential $V(R)$, which can be determined directly in the MSV [1,2]. In fig.(2) we present this comparison using our fitted value $\alpha_s = 0.57$ [12].

Figure 2: The total energy stored in the field (dots) calculated with the energy sum-rule (18) as compared with the potential of a $q\bar{q}$-pair obtained by evaluation of the Wegner-Wilson loop in the MSV [1,2] (solid line)

5 Discussion

We first consider the region where the potential is linear. Then eq.(17) reads:

$$\frac{1}{2 \alpha_s} < \tilde{\beta} > \int d^3x \left( \vec{E}(x)^2 + \vec{B}(x)^2 \right) > R = 2 V(R) \tag{32}$$

Since in the model of the stochastic vacuum the static sources do not modify the color magnetic field, i.e. $< \vec{B}^2 > R = 0$, we have consistency between eq.(18)
and eq. (32) only if

\[ \tilde{\beta} / \alpha_s = -2. \]  

(33)

From the energy sum-rule (eq. (18)) we have already obtained \( \alpha_s = 0.57 \), thus consistency of the MSV requires that eq. (33) should be satisfied for this value of \( \alpha_s \).

The \( \tilde{\beta} \)-function can be expanded in a power series in \( \alpha_s \):

\[ \tilde{\beta}(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 + \ldots. \]  

(34)

The n-loop expansion gives the series up to \( \alpha_s^{n+1} \). The first two terms of the power series are scheme independent, the numerical value of the term proportional to \( \alpha_s^4 \) has been evaluated \([13]\) in \( \overline{MS} \). For pure gauge SU(3) theory we have \([14]\):

\[ \beta_0 = 11, \quad \beta_1 = 51, \quad \beta_2'_{\overline{MS}} = 2857. \]  

(35)

Numerical investigations of the running coupling constant in lattice gauge Monte Carlo simulations \([13]\) indicate that the power series \([14]\) for \( \tilde{\beta}(\alpha_s) \) truncated at \( \alpha_s^4 \) can be used up to rather large values of \( \alpha_s \) in their scheme if the coefficient \( \beta_2 \) is fitted numerically with the result \( \beta_2 \approx 6270 \).

From fig. (3) we see that our condition (33) yields \( \alpha_s = 1.14 \) using \( \tilde{\beta} \) on the one loop level, \( \alpha_s = 0.74 \) on the two loop level and \( \alpha_s = 0.64 \) on the three loop level in \( \overline{MS} \). This value is already very close to the value of \( \alpha_s = 0.57 \) determined from the flux tube calculation using eq. (18). If we use for \( \tilde{\beta} \) the series \([14]\) truncated at order \( \alpha_s^4 \) and impose condition (33) for \( \alpha_s = 0.57 \) and determine the coefficient \( \beta_2 \) accordingly we find \( \beta_2 = 6250 \) (see fig. (3)), i.e. about twice the \( \overline{MS} \) value. Curiously this agrees almost exactly with the numerically determined value of \([15]\) quoted above.

Figure 3: \( \tilde{\beta} / \alpha_s \) as a function of \( \alpha_s \) for different \( \tilde{\beta} \)-functions

We see from fig. (2) that for distances \( R/a \leq 1 \) \( (a = 0.35 \text{ fm}) \) the MSV does no longer yield a linearly rising potential and hence we have

\[ V(R) + R \frac{\partial V(R)}{\partial R} \neq 2V(R), \quad R/a \leq 1. \]

Therefore the conditions \([18]\) and \([32]\) cannot be fulfilled simultaneously for \( < \tilde{B}^2 >_R = 0 \). This does by no means speak against the model, since in the derivation of the results in \([12]\) it was stressed that the spatial extension of the loop had to be large as compared to the correlation length of the field strengths in order to justify the evaluation of \( \tilde{E}^2 \) and \( \tilde{B}^2 \).
6 Conclusions

We have re-derived and discussed low energy theorems and sum-rules relating densities of squared field strengths to the potential of a static quark-antiquark pair. We found that for a dynamic quark-antiquark bound state these sum-rules are in a sense equivalent to the dilatation trace anomaly and the virial theorem. We have applied the sum-rules to quantities obtained in the model of the stochastic vacuum. This model was introduced as a phenomenological approximation to the complicated measure for functional integration over the slowly varying gluon field strengths. No use of the equations of motion has been made explicitly. The factorization hypothesis (Gaussian measure) enters crucially in the evaluation of the squared color fields. In view of the drastic approximations made it is gratifying that relation (33), which is a highly non-trivial consequence of the dynamics of the system, is so well fulfilled.

The scale \( \mu \) at which the MSV works was found to be the one where \( \alpha_s(\mu) = 0.57 \) in [12] by using the energy sum-rule. With the help of the action sum-rule we now checked the consistency of the MSV at this scale and obtained an effective \( \tilde{\beta} \)-function.

As can be seen from eq.(17) and eq.(18) the squared field strengths \( \vec{B}^2 \) and \( \vec{E}^2 \) depend on the renormalization scale \( \mu \) and hence on \( \alpha_s(\mu) \). We may use relations (18) and (33) in order to predict the ratio of

\[
Q = \frac{\int d^3x \langle \vec{B}(x)^2 \rangle_R}{\int d^3x \langle \vec{E}(x)^2 \rangle_R} = \frac{2 + \tilde{\beta}/\alpha_s}{2 - \tilde{\beta}/\alpha_s}
\]

as a function of the strong coupling \( \alpha_s \). The result is plotted in fig.(4) (we have used there our numerically determined value of \( \beta_2 \)). This ratio can be checked in lattice gauge calculations.

Figure 4: The ratio \( Q \) as a function of \( \alpha_s \)

In this paper we have worked in pure gauge theory and in a theory with gluons and one heavy quark field. We leave the investigation of similar sum-rules in a theory including light quarks for future work.

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Figure 1: Difference of the squared electric field parallel to the quark-antiquark \((x_3)\) axis \((- g^2 E^2_{||}(x_3, x_{\perp}) > R)\) in \(\text{GeV/fm}^3\) for different quark separations \(R\). \(x_{\perp}\) is the distance transverse to the \(x_3\)-axis and the dots denote the quark positions.

Figure 2: The total energy stored in the field (dots) calculated with the energy sum-rule (18) as compared with the potential of a q-\(\bar{q}\)-pair obtained by evaluation of the Wegner-Wilson loop in the MSV [1][2] (solid line)
Figure 3: $\frac{\beta}{\alpha_s}$ as a function of $\alpha_s$ for different $\beta$-functions.

Figure 4: The ratio $Q$ as a function of $\alpha_s$. 

\[ Q \]