Effect of delamination on vibration behaviour of woven Glass/Epoxy composite plate-An experimental study

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Abstract. We have analysed the free vibration responses of the laminated composite plate with delamination numerically and validated with subsequent experiment. In order to compute the numerical frequencies, the delaminated composite plate is modelled with two sub-laminates approaches in the commercial finite element package (ANSYS) using ANSYS parametric design language code in ANSYS environment. For the experimental analysis, the woven Glass/Epoxy composite plate is fabricated using hand layup method with the desired delamination. The natural frequencies of the delaminated plate are also computed experimentally with the help of the vibration analyser (NI-CDAQ) and validated by comparing with the simulation result. Further, the simulation model is extended for various design parameter and discussed in detail.

Keywords: Delaminated composites plate; Vibration; Experimental; ANSYS

1. Introduction
Composite materials have their own advantages of high strength and stiffness to weight ratio. These materials are not only satisfy the high strength criterion but also suitable for weight sensitive application. Delamination is one of the reason of failure of these material which occurs due to different loading, imperfect fabrication and edge effect. Delamination plays the significant role in the reduction in stiffness which seriously affect the vibration characteristics. So it is very important to predict the effect of delamination on vibration characteristics of the laminated composites. Few important work on the delaminated composites are discussed in the following line to make the article self-standing.

Borst and Remmers [1] proposed a method for the modelling of delaminated composite assuming the degraded properties using the damage law. A two sub-laminates model of delaminated composite plate is developed by Rajendran and Song [2] in ANSYS environment to perform the buckling analysis. Finite element model (two dimensional) of delaminated beam is presented by Çallioğlu et al. [3] using finite element software package ANSYS to analyse the vibration characteristics. Tenek et al. [4] studied the vibration behaviour of the delaminated composites plate using finite element based on the three-dimensional theory of linear elasticity. Zheng and Sun [5] model the delamination by using triple plate model at two different thickness location to analyse the delamination interaction. Ju et al. [6] developed a finite element model to analyse the effect of delamination size, delamination location and number of delamination on the vibration characteristic of the delaminated composites plate using first ordered shear deformation theory (FSDT). Numerical investigation on vibration characteristic of the delaminated composites plates have been presented by Nanda and Sahu [7] using finite element method based on FSDT. Parhi et al. [8] presented a finite element model based on first-order shear deformation theory (FSDT) to analyse the vibration behaviour of a plate with arbitrary located multiple delaminations.
Experimental as well as finite element modelling study of the vibration of laminated composites plate with circular delamination is conducted by Hou and Jeronimidis [9]. Panda et al. [10] performed experimental and numerical study on vibration analysis of woven fiber Glass/Epoxy delaminated composite plates. For numerical study finite element formulation is prepared in the MATLAB environment. Mohanty et al. [11] presented the experimental, as well as numerical investigation on the buckling behaviour of multiple, delaminate composites plates.

From the literature review, it is clear that limited number of work on the finite element modelling and the subsequent experimental validation are reported. The aim of the present work is to compute the fundamental frequencies of the delaminated composite plate numerically using the commercial FE package (ANSYS) through the ANSYS parametric design language (APDL) code and compared with previously published literature. In addition to that, the laminated composite plate of the woven glass fiber is fabricated using the hand lay-up method with desired delamination and the experimental frequencies are computed through NI-CDAQ. Finally, the effect of different design parameters (thickness ratio and support conditions) on the frequency responses are computed and discussed in detailed.

2. Theoretical Formulations

2.1. Mathematical model for without delamination

In this analysis, the laminated composite plate composed of the finite number of orthotropic layers of uniform thickness is considered as in Fig. 1. A simulation model has been developed in ANSYS using APDL code. For the discretization purpose, Shell 281 element has been chosen from ANSYS element library. The shell element has eight nodes, more than other shell elements and hence it is suitable for analysing thin to moderately thick shell structure. The plate displacement field kinematics is modelled using the FSDT:

\[
\begin{align*}
    u(x, y, z, t) &= u_0(x, y) + z\theta_x(x, y) \\
    v(x, y, z, t) &= v_0(x, y) + z\theta_y(x, y) \\
    w(x, y, z, t) &= w_0(x, y) + z\theta_z(x, y)
\end{align*}
\] (1)

where, t is the time and \( u, v \) and \( w \) are the displacements of any point along the \( x, y \) and \( z \) coordinate axes respectively. \( u_0, v_0 \) and \( w_0 \) are corresponding displacements of a point on the mid-plane and \( \theta_x \) and \( \theta_y \) are the rotations of normal to the mid-surface, i.e., \( z=0 \) about the \( y \) and \( x \)-axes, respectively.
The stress-strain relations for any $k^{th}$ lamina oriented at an arbitrary angle $\phi$ about any arbitrary axes are given by:

$$\{\sigma\} = \{Q_{r}\}\{\epsilon\}$$

(2)

where, $\{\sigma\}$, $\{Q_{r}\}$ and $\{\epsilon\}$ are the stress tensor, reduced stiffness matrix, and strain tensor, respectively.

The stress tensor further modified in force form as:

$$\{F\} = [D]\{\epsilon\}$$

(3)

where, $[D]$ is the property matrix.

2.2. Finite Element Formulation

The displacement fields for the assumed kinematic models are expressed in terms of desired field variables, and the models are discretized using suitable FEM steps. The displacement vector $d$ at any point within the mid-plane is given by:

$$d = \sum_{i=1}^{n} N_i(x, y) d_i$$

(4)

where, $\{d_i\} = \{u_0, v_0, w_0, \theta_x, \theta_y, \theta_z\}^T$ is the nodal displacement vector and $N_i$ is the corresponding interpolating function for the $i^{th}$ node.

The strain vector can be expressed in the matrix form after introducing the FEM steps and condensed as:

$$\{\epsilon\} = [T]\{\tilde{\epsilon}\}$$

(5)
where, \( [T] \) is the thickness coordinate matrix and \( \{ \varepsilon \} \) is the mid plane strain vector which can be further reduced as:

\[
\{ \varepsilon \} = [B_e]\{ d \}
\]  

(6)

where, \([B_e]\) is a general strain displacement relation matrix.

The total strain energy of the laminate can be expressed as:

\[
U = \frac{1}{2} \int_{-h/2}^{+h/2} \int_{-h/2}^{+h/2} \{ \varepsilon \}^T \{ \sigma \} dz \, dxy
\]

(7)

Equation (7) can be rewritten by substituting the strain and stress tensors as:

\[
U = \frac{1}{2} \int \int (\{ \varepsilon \}^T [D] \{ \varepsilon \}) \, dxy
\]

(8)

where, \([D] = \int_{-h/2}^{+h/2} [T]^T [Q_r] [T] \, dz\)

The kinetic energy of the laminate can be expressed as:

\[
T = \frac{1}{2} \int_\Omega \rho \{ \delta \}^T \{ \delta \} \, dV
\]

(9)

where, \( \rho \) and \( \{ \delta \} \) are the mass density and the global velocity vector, respectively.

The stiffness matrix \([K]\) and the mass matrix \([M]\) can be further expressed as:

\[
[K] = \int_\Omega \sum_{k=1}^{n} \sum_{l=1}^{m} [B_e^T]\{D\}[B_e] \, dz \, dA
\]

\[
[M] = \int_\Omega \sum_{k=1}^{n} \sum_{l=1}^{m} [N]^T[N] \rho dz \, dA
\]

(10)

2.3. Mathematical model for the delaminated panel

Now the earlier laminated model is extended further for the delaminated cases and for that a typical laminate with \(p\) number of delamination as shown in Fig. 2. The local coordinate system for the element is \(x', y'\) and \(z'\) which is similar to that used segment without delamination. The displacement field of the delaminated element is assumed to be of the following form relative to its own local coordinate system.

\[
\begin{align*}
\mathbf{u}(x', y', z', t) & = \mathbf{u}_0(x', y') + z \, \theta_{x'}(x', y') \\
\mathbf{v}(x', y', z', t) & = \mathbf{v}_0(x', y') + z \, \theta_{y'}(x', y') \\
\mathbf{w}(x', y', z', t) & = \mathbf{w}_0(x', y') + z \, \theta_z(x', y')
\end{align*}
\]  

(11)

where, \(t\) is the time and \(u', v'\) and \(w'\) are the displacements of any point along the \(x', y'\) and \(z'\) coordinate axes, respectively and it is similar to that of the without delamination. \(u_0, v_0\) and \(w_0\) are corresponding displacements of a point within the mid-plane and \(\theta_{x'}\) and \(\theta_{y'}\) are the rotations of normal to the mid-surface, i.e., \(z' = 0\) about the \(x'\) and \(y'\) axes, respectively.
The corresponding element stiffness and mass matrices can be obtained as:

\[
[K'] = \int \left( \sum_{k=d+1}^{d+p} \int [B_i^k] [D][B_i^k] d\hat{z} \right) dA
\]

\[
[M'] = \int \left( \sum_{k=d+1}^{d+p} \int [N] [N] \rho d\hat{z} \right) dA
\]

where, \( p \) is the number of layers in the delaminated element, and two delaminations which form the bottom surface and the top surface of the element occur between layer \( d \) and layer \( d + 1 \), and layer \( d + p \) and layer \( d + p + 1 \), respectively. The other symbols have the same meanings as before. The nodal displacement vector is \( \{d\} = \{u_0, v_0, w_0, \theta_{x0}, \theta_{y0}, \theta_{z0}\} \).

### 2.3.1 Displacement continuity conditions

In the previous sections, the elements in the segment without delaminations and delaminated segment are derived separately, and all the variables in the displacement field are independent. However, at the boundary connecting these two segments, the continuity conditions for the displacement field must be satisfied.

Fig. 3. shows the elements for the case where the delaminated section is on the right. Consider one of the delaminated elements, element \( m \), in the delaminated segment, where element 1 is an element in the laminated segment on the left of the element \( m \). The common nodes 1, 4 and 8 are at the connecting boundary. Let the element stiffness and element mass matrices and element nodal displacement vectors be denoted by \([k], [m] \) and \( \{d\} \) for element 1, and \([k'], [m'] \) and \( \{d'\} \) for element \( m \).
For common nodes 1, 4 and 8, let

\[ \{d_i\} = \{u_i, v_i, w_i, \theta_{x_i}, \theta_{y_i}, \theta_{z_i}\}^T \quad i=1, 4, 8 \]  

for element 1

\[ \{d_i\} = \{u_i, v_i, w_i, \theta_{x_i}, \theta_{y_i}, \theta_{z_i}\}^T \quad i=1, 4, 8 \]  

for element m

In order to satisfy the displacement continuity conditions at the connecting boundary, the following relations must hold:

\[
\begin{align*}
\dot{u}_i &= u_i + e\dot{\theta}_i \\
\dot{v}_i &= v_i + e\dot{\theta}_i \\
\dot{w}_i &= w_i + e\dot{\theta}_i \\
\theta_{x_i} &= \theta_{y_i} \\
\theta_{y_i} &= \theta_{z_i} \\
\theta_{z_i} &= \theta_{x_i}
\end{align*}
\]

for \( i=1,4 \) and 8

(14)

where \( e \) is the distance between the mid-plane of element 1 and the mid-plane of element m, and is positive in the sense indicated in Fig. 3.

Therefore,

\[ \{d_i\} = [\lambda]\{d_i\} \quad \text{for } i=1, 4 \text{ and } 8 \]  

(15)

where, \( [\lambda] = \)

\[
\begin{bmatrix}
1 & 0 & 0 & e & 0 & 0 \\
0 & 1 & 0 & 0 & e & 0 \\
0 & 0 & 1 & 0 & 0 & e \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Using equation 11, the following transformation of element m can be written as

\[ \{d\} = [T]\{\tilde{d}\} \]  

(16.a)

where,

\[ \{\tilde{d}\} = [d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8]^T \]  

(16.b)
\[
[T] = \text{Diag}[\lambda I I I I I I I \lambda]
\]  
(16.c)

where, the matrix \([I]\) is a \(8 \times 8\) unit matrix.

Now, the transformations for the element stiffness and element mass matrices of element \(m\) can be expressed as

\[
[k] = [T]^T[k][T]
\]
\[
[m] = [T]^T[m][T]
\]

(17)

The displacement continuity conditions have been incorporated into \([\vec{k}]\) and \([\vec{m}]\), and the nodal displacement \(\{d_i\}\) is replaced by \(\{\tilde{d}_i\}\) for \(i = 1, 4, 8\). The matrices \([\vec{k}]\) and \([\vec{m}]\), are subsequently used to assemble the global stiffness and global mass matrices. Obviously, a similar treatment can be carried out for cases when the connecting boundaries of the two segments are at the other edges.

2.4. Governing equations

The final form of governing equation of free vibrated composite plate is obtained by using Hamilton’s principle and expressed as:

\[
\delta \left\{ (T - U)dt \right\} = 0
\]

(18)

Substituting Equations (4), (8) and (9) into equation (18), the final form of the equation will be conceded as:

\[
[M]\{\ddot{d}_i\} + [K]\{d_i\} = 0
\]

(19)

where, \(\ddot{d}_i\) is the acceleration, \(d_i\) is the displacement

Neglecting the required matrices, the eigenvalue form of the governing equation to obtain the natural frequency of the system is conceded as:

\[
([K] - \omega^2[M])\Delta = 0
\]

(20)

where, \(\omega\) and \(\Delta\) are the natural frequency and the corresponding eigenvector, respectively.

The Equation 20 is solved by using the following sets of support conditions to avoid any rigid body motion as well as reduce the number of unknowns.

Simply supported (SSSS):

\[
v_0 = w_0 = \theta_y = \theta_z = 0 \quad \text{at} \ x=0 \ \text{and} \ a;
\]

\[
u_0 = w_0 = \theta_x = \theta_z = 0 \quad \text{at} \ y=0 \ \text{and} \ b;
\]

(21)
Clamped (CCCC):

\[ u_0 = v_0 = w_0 = \theta_x = \theta_y = \theta_z = 0 \quad \text{at} \quad x=0 \quad \text{and} \quad y=0 \quad \text{and} \quad z; \quad (22) \]

3. Result and discussion

The free vibration responses of the composite plate with delamination is computed using developed FE code in ANSYS environment. The validation of the proposed model is examined by comparing the responses with those available published results. Consequently, the woven Glass/Epoxy plate is fabricated with the help of hand layup method and the desired delamination also introduced by adding Teflon tape in between the laminates. The natural frequencies of the fabricated delaminated plate are obtained experimentally and validated with the simulation model. The effect of various parameters such as thickness ratios and different support conditions are computed using the present simulation model.

3.1. Validation and comparison study

The finite element model developed in the present study is validated by comparing the results with those available previously published literature. The free vibration responses of a twenty layered cantilever square composite plates are computed using M1 material and geometrical properties as in Table 1. same as to the reference considering three cases namely Case-I, Case-II and Case-III. The Case-I represents natural frequencies of the laminated plate and the Case-II and the Case-III are the square delamination of size \( a/2 \) and \( 3a/4 \), respectively. The delamination is located at the middle of the plate or at the mid-plane of the structure. The natural frequencies of the delaminated plate are computed for different lamination schemes and compared with Parhi et al. [12] and presented in Table 2. It can be seen that the results are showing very good agreement with that of the references for each case as presented in the table.

| Properties                      | MATERIAL-1 (M1) | MATERIAL-2 (M2) |
|---------------------------------|-----------------|-----------------|
| Young’s modulus x direction (\( E_x \)) | 172.5GPa        | 7.366GPa        |
| Young’s modulus y direction (\( E_y \)) | 6.9GPa          | 5.568GPa        |
| Young’s modulus z direction (\( E_z \)) | 6.9GPa          | 5.568GPa        |
| Shear modulus (\( G_{xy} \))    | 3.45GPa         | 2.79GPa         |
| Shear modulus (\( G_{yz} \))    | 1.38GPa         | 1.395GPa        |
| Shear modulus (\( G_{xz} \))    | 3.45GPa         | 2.79GPa         |
| Poisson’s ratio (\( \nu_{xy} \)) | 0.25            | 0.17            |
| Poisson’s ratio (\( \nu_{yz} \)) | 0.25            | 0.17            |
| Poisson’s ratio (\( \nu_{zx} \)) | 0.25            | 0.17            |
| Density (\( \rho \))           | 1600 Kgm\(^{-3}\) | 1100 Kgm\(^{-3}\) |
Table 2. Natural frequency (Hz) of cantilever composite plate with centrally placed delamination.

| Lamination scheme | Case no. | Natural Frequency (Hz) |
|-------------------|----------|------------------------|
|                   |          | Present | Parhi et al. [12]     |
| (60°/-60°)_{10}   | Case I   | 9.528   | 9.5                   |
|                   | Case II  | 8.3184  | 8.43                  |
|                   | Case III | 6.4981  | 6.84                  |
| (90°/-90°)_{10}   | Case I   | 6.7217  | 6.71                  |
|                   | Case II  | 6.3129  | 6.12                  |
|                   | Case III | 5.02    | 5.0168                |

3.2. Preparation of the woven Glass/Epoxy delaminated composite plate

Glass/Epoxy composite plate specimens were fabricated using hand layup method. Very firstly, four layers of woven Glass fibers sheets were cut out in required size. On a rigid flat plywood platform, a plastic sheet was placed as a mold releasing sheet and a releasing agent i.e. a thin film of polyvinyl alcohol was applied to it. Two layers of Glass fibers sheets were glued by applying a gel coat (a mixture of Epoxy and Hardener) with the help of a brush, and a smooth steel roller was rolled over the laminate to minimize the void content in the laminates. Delamination was introduced by placing Teflon film of an area, 6.25% of the total laminate area in the centre of the prepared laminate. Complete care was taken in the casting of the plates with the presence of delamination. On top of it, other two layers of the Glass fiber sheets were glued with gel coat. The process of hand layup was continued for all layers of reinforcement before the gel coat fully hardened as shown in Fig. 4 (a). After completion of all the layers, again a plastic sheet sprayed with polyvinyl alcohol was covered at the top of the last ply. Now the hot-press (Fig. 4 (b).) is employed for pre-curing of the laminate where the temperature is maintain 60°C for half an hour and then for two days it is left in atmosphere condition for post curing. After proper curing of laminates, the release sheets were detached.

Fig. 4 (a):- Woven Glass/Epoxy delaminated plate after hand layup.  
Fig. 4 (b):- Hot press
3.3. Material property evaluation

The mechanical properties say, Young’s modulus, shear modulus and Poisson’s ratio in principal material directions of the delaminated composite plate has been obtained for the computation of the desired natural frequency of the plate structure. For that the unidirectional tensile test was conducted via Universal Testing Machine (UTM) INSTRON 1195 at National Institute of Technology, Rourkela on the specimens in the longitudinal direction, transverse direction and at an angle of 45° inclined to the longitudinal direction of the plate. The ASTM standard: D 3039/D 3039M is used for the evaluation of Young’s modulus. The tensile test was conducted with a recommended rate of loading, i.e., 1 mm/minute as shown in Fig. 5 (a). Tensile test specimen is shown in Fig. 5 (b), including grip length. The material properties, obtained and presented in Table 1, as M2 material properties. Poisson’s ratio is taken as 0.17 as in Crawley et al. [13]. Similarly, the shear modulus was determined using the following formula as in Jones [14]:

$$G_b = \frac{1}{4E_{45}} \left( \frac{1}{E_i} - \frac{1}{E_j} \right) \frac{2\nu_{12}}{E_j}$$

Fig. 5 (a):- UTM INSTRON 1195.

Fig. 5 (b):- Glass/Epoxy laminated composite specimen after tensile test.

3.4. Modal test

Now, the free vibration responses of the woven Glass/Epoxy delaminated composite plate is analyzed experimentally using the CDAQ-9178 (National Instruments) for the plate of dimension $a = b = 15cm$ and $h = 0.12cm$ with delamination of a size of $a/4$ at the midpoint of the laminate. The various components in the experimental set-up are shown in Fig. 6 (a) and (b). The vibration responses of the plate are recorded under CFFF support condition, with the help of a fixture via an accelerometer mounted on the plate. The plate is excited with the help of an impact hammer on any random points, and the desired data are received through the CDAQ-9178, which is a compact data acquisition system of USB chassis comprising of the eight channels. These signals are again processed with the help of LABVIEW software. In LABVIEW, a virtual instrument (VI) program circuit is developed which primarily comprises of three components: a back panel, a front panel and a connector panel. These three component helps in supply of the input and show the output on the computer screen Fig. 6(c), depicts...
the block diagram of the LABVIEW software for the data acquisition of the desired signal and the subsequent analysis. The acceleration signals are passed through the power spectrum module of the LABVIEW for fast Fourier transformation to obtain the frequency domain responses. The peaks of the frequency response spectrum give the natural frequencies of vibration for different modes. Finally, the experimental results are compared with those the simulation model and presented in Table 3. The results of the simulation model are in decent agreement with the experimental results.

Fig. 6 (a):- NI CDAQ 9178.  
Fig. 6 (b):- Experimental set-up.  
Fig. 6 (c):- Block diagram of the LABVIEW for experimental data recording.
Table 3. Natural frequency (Hz) of cantilever woven Glass/Epoxy composite plate with centrally placed delamination.

| Mode no. | Natural Frequency (Hz) | Present | Experimental |
|----------|------------------------|---------|--------------|
| 1        | 21.053                 |         | 19.5         |
| 2        | 55.079                 |         | 44.5         |
| 3        | 133.13                 |         | 126.5        |
| 4        | 167.96                 |         | 163          |
| 5        | 199.5                  |         | 196.5        |

3.5. Numerical illustrations

Based on the validation and the comparison study, it is evident that the presently developed simulation model is capable to compute the free vibration responses of the delaminated composite quite accurately. Now, to highlight the importance of the simulation model for the analysis of the delaminated structure responses are computed for different design parameters like the thickness ratios and the support conditions. Based on the responses, the quantitative understanding of the free vibration behaviour of the delaminated composite plates are discussed in the following lines. The responses are computed for twenty layered cross-ply composite plates with Case-II type of delamination placed centrally in the mid laminate having M1 material properties as in Table 1. unless otherwise stated.

3.5.1. Effect of support condition on vibration response of delaminated composited plate

The natural frequencies of the delaminated plate has been computed for different support conditions and the aspect ratios \((ab=1, 2, 3, 4 \text{ and } 5)\) with \(b/h=100\). In general, the support conditions constrained the degrees of freedom of the structure, and it is well-known fact that the increase in the number of constraints makes the structure stiffer. The effect of four types of support condition (CCCC, CFCF, SSSS and CFFF) on the delaminated plate is computed and presented in Fig. 7. It is interesting to note that the responses are increasing with increase in number of constrained DOF.

![Fig. 7: - Natural frequency of twenty layered cross-ply composite plate with central delamination of case-II for varying support conditions.](image)

3.5.2. Effect of thickness ratio on free vibration response of delaminated composited plate

It is well known that the thickness ratio of any structure or the structural component increases then the structure becomes thinner, and the structural stiffness of the laminated panel is inversely proportional to that. The free vibration response has been examined using simulation model for the clamped delaminated plate by varying its thickness ratios \((b/h=100, 125, 150, 175 \text{ and } 200)\). The first natural
frequency of the plates is computed for different aspect ratio \( (a/b=1, 2, 3, 4 \text{ and } 5) \) and presented in Fig. 8. It is clearly observed that the natural frequency decreases as the thickness ratio increases and the responses are within the expected line.

![Fig. 8: Natural frequency of twenty layered cross-ply composite plate with central delamination of case-II for varying thickness ratio.](image)

4. Conclusion
In this present article, the free vibration responses of the delaminated composite plate have been numerically using the commercial FE package (ANSYS) and validated subsequently with that of the available numerical results. Further, the mechanical properties of the fabricated delaminated composite plate are computed experimentally, and the desired frequencies are also obtained experimentally with the help of in-house vibration analyser via CDAQ-9178. The frequencies obtained using the simulation, and the experimental results are compared for the validation purpose as well. Finally, few examples are solved numerically with the help of the present simulation model to highlight the effect of design parameter on the frequency responses. The parametric study indicated that the natural frequencies of the delaminated composite plate increases with the decrease in thickness ratio and increase in constrained.

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