A study of $cc\bar{c}\bar{c}$ tetraquark decays in 4 muons and in $D^{(*)}\bar{D}^{(*)}$ at LHC

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Abstract

We perform a quantitative analysis of the decays of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{++}, 2^{++}$ into 4 muons and into hidden- and open-charm mesons and estimate, for the first time, the fully charmed tetraquark decay width. The calculated cross section upper limit is $\sim 40$ fb for the 4 muons channel, and $\sim 28$ nb for the $D^{(*)}\bar{D}^{(*)} \to e\mu$ channel. On the basis of our results, with the present sensitivity LHCb should detect both signals, for $0^{++}$ and $2^{++}$ fully-charmed tetraquarks.

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I. INTRODUCTION

In this note we consider production and decay at proton colliders of the fully charmed tetraquarks, $T = ccc\bar{c}$. In particular, we consider the $4\mu$ and meson-meson decays, the latter revealed through the $e\mu$ signature of their weak decays. We focus on the ground states with $J^{PC} = 0^{++}, 2^{++}$. We shall use the method applied recently to production and decay of fully bottom tetraquarks, $b\bar{b}b\bar{b}$ in [1], briefly described in the following.

Preliminary evidence for a $4\mu$ resonance has been presented in a recent seminar of the LHCb Collaboration [2], which is in line with our estimates and indicates that $4\mu$ and meson-meson channels may be the key to the study of these truly exotic hadrons.

The hypothetical existence of hadronic states with more than minimal quark content ($q\bar{q}$ or $qqq$) was proposed by Gell-Mann in 1964 [3] and Zweig [4], followed by a quantitative model by Jaffe [5] for the lightest scalar mesons, described as diquark anti diquark pairs. Recent years have seen considerable growth in the observations of four valence quark states that cannot be included in the well-known systematics of $q\bar{q}$ mesons, like $Z(4430)$ [6, 7] and $Z(4248)$ [8]. Similar particles have also been found in the bottom sector, $Z_b(10610)$ and $Z_b(10650)$, observed by the Belle collaboration [9] (see [10, 11] for recent reviews).

The first predictions of a fully-charmed $ccc\bar{c}$ tetraquark below the $2J/\Psi$ threshold were made in Refs. [12, 13], and were supported by more recent contributions in [14–20].

Refs. [21] have estimated the $J^{PC} = 0^{++}$, fully-bottom tetraquark decay width.

Theoretically, $J^{PC} = 0^{++}$ is expected for the $ccc\bar{c}$ ground-state. Following Ref. [1] we present a calculation of decay widths and branching ratios of the main, hidden- and open-charm channels of $ccc\bar{c}$ tetraquarks. Our estimates apply to tetraquarks close (below or above) to the $2J/\Psi$ threshold.

Our results are as follows.

Decay rates are proportional to the ratio of overlap probabilities of the annihilating $c\bar{c}$ pairs in $T$ and $J/\Psi$:

$$\xi = \frac{|\Psi_T(0)|^2}{|\Psi_{J/\Psi}(0)|^2}$$

Branching ratios are uniquely determined and reported in Table I, which is the basis of our results. In particular, we find

$$B(T \to 4\mu) = 2.7 \cdot 10^{-6} \ (J^{PC} = 0^{++});$$
\[ B(T \rightarrow 4\mu) = 16 \cdot 10^{-6} \ (J^{PC} = 2^{++}). \]  

(2)

The total width is expressed as:

\[ \Gamma(T(J = 0^{++}) = 21 \cdot \xi \ \text{MeV} \]  

(3)

With two alternative definitions of the coordinates, as explained later, the model in [19] gives the two estimates

\[ \xi = (1.8 \text{ or } 5.1) \]  

(4)

The same model gives

\[ |\Psi_{J/\Psi}(0)|^2 = 0.14 \ \text{GeV}^3 \]  

(5)

in excellent agreement with the value obtained from charmonium decay into a muon pair, \[ |\Psi_{J/\Psi}(0)|^2 = 0.13 \ \text{GeV}^3 \], see also [23].

To estimate the \( T \) width, we take for \( \xi \) the geometric average of the two estimates and obtain

\[ \xi = 3.0^{+1.2}_{-2.1} \]  

(6)

Our best estimate is then

\[ \Gamma(T(J = 0^{++}) = 62^{+25}_{-43} \ \text{MeV}. \]  

(7)

We extend the calculation to the \( J^{PC} = 2^{++} \), fully-charmed tetraquark. \( J = 2 \) tetraquarks are produced in \( p + p \) collisions with a statistical factor of 5 with respect to the spin 0 state. The decay \( T \rightarrow \eta_c + \text{light hadrons} \) is suppressed but annihilations into meson pairs take place at a greater rate.

| [\(ccc\bar{c}\)] | \(\eta_c+\) any | \(D_q\bar{D}_q\) \((m_q < m_c)\) | \(D_q^*\bar{D}_q^*\) | \(J/\Psi+\) any | \(J/\Psi + \mu^+\mu^-\) | \(4\mu\) |
|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|
| \(J^{PC} = 0^{++}\) | 0.77 | 0.019 | 0.057 | 7.5 \cdot 10^{-4} | 4.5 \cdot 10^{-5} | 2.7 \cdot 10^{-6} |
| \(J^{PC} = 2^{++}\) | 0 | 0 | 0.333 | 4.4 \cdot 10^{-3} | 2.6 \cdot 10^{-4} | 1.6 \cdot 10^{-5} |

TABLE I: Branching fractions of fully-charmed tetraquarks, assuming \( S \)-wave decay.

With [6], we find:

\[ \Gamma(T(J = 2^{++}) = 42^{+18}_{-30} \ \text{MeV} \]  

(8)

Branching fractions and upper limits to the cross sections of final states in \( pp \) collisions are summarised in Tabs. I and II.
The results of Tab. I combined with the recent determination by LHCb of the cross section for $2J/\Psi$ production at 13 TeV [22], give encouraging upper bounds to the production of $T \rightarrow 4\mu$ at LHC

$$\sigma(p + p \rightarrow T(0^{++}) + \cdots \rightarrow 4\mu + \cdots) < 40 \text{ fb}$$
$$\sigma(p + p \rightarrow T(2^{++}) + \cdots \rightarrow 4\mu + \cdots) < 238 \text{ fb}$$ (9)

II. DETAILS OF THE CALCULATION

We give here a brief description of our method, the reader may consult Ref. [1] for more details. The starting point is the Fierz transformation, which brings $c\bar{c}$ together [10]:

$$T(J = 0^{++}) = |(cc)_{\frac{1}{3}} (c\bar{c})_{\frac{1}{3}}{^0}_1 = -\frac{1}{2} \left( \sqrt{\frac{1}{3}} |(c\bar{c})_{1}^{1} (c\bar{c})_{1}^{1}{^0}_1 - \sqrt{\frac{2}{3}} |(c\bar{c})_{8}^{1} (c\bar{c})_{8}^{1}{^0}_1 \right) +$$
$$+\frac{\sqrt{3}}{2} \left( \sqrt{\frac{1}{3}} |(c\bar{c})_{1}^{0} (c\bar{c})_{1}^{0}{^0}_1 - \sqrt{\frac{2}{3}} |(c\bar{c})_{8}^{0} (c\bar{c})_{8}^{0}{^0}_1 \right).$$

(10)

quark bilinears are normalised to unity, subscripts denote the dimension of colour representations, and superscripts the total spin. Similarly, for the $J = 2$ tetraquark, one finds:

$$T(J = 2^{++}) = |(cc)_{\frac{1}{3}} (c\bar{c})_{\frac{1}{3}}{^2}_1 = \left( \sqrt{\frac{1}{3}} |(c\bar{c})_{1}^{1} (c\bar{c})_{1}^{1}{^2}_1 - \sqrt{\frac{2}{3}} |(c\bar{c})_{8}^{1} (c\bar{c})_{8}^{1}{^2}_1 \right).$$

(11)

We describe $T$ decay as due to individual decays of one of the $c\bar{c}$ pairs in (10), described as follows (see [1] for details).

1. The colour singlet, spin 0 pair decays into 2 gluons, which are converted into confined, light hadrons with a rate of order $\alpha_S^2$; taking the spectator $c\bar{c}$ pair into account, this decay leads to: $T \rightarrow \eta_c + \text{light hadrons}.$

2. The colour singlet, spin 1 pair decays into 3 gluons, which are converted into confined light hadrons leading to: $T \rightarrow J/\Psi + \text{light hadrons};$ final states $J/\Psi + \mu^+\mu^- + 4\mu$ are also produced. The rate is of order $\alpha_S^3.$

3. The colour octet, spin 1 pairs annihilate into one gluon, which materialises into a pair of light quark flavours, $q = u,d,s;$ the latter recombine with the spectator pair to produce a pair of open-charm mesons $D_q\bar{D}_q$ and $D^*_q\bar{D}^*_q$, with a rate of order $\alpha_S^2.$
4. The colour octet, spin 0 pairs annihilate into a pair of light quarks (necessary to neutralize the colour of the spectator $c\bar{c}$ pair) with amplitude of order $\alpha^2_S$ and rate of the order of $\alpha^4_S$, which we neglect.

Total $\mathcal{T}$ decay rate is the sum of individual decay rates, obtained from the simple formula \[24\]

$$\Gamma((c\bar{c})_c^0) = |\Psi(0)_T|^2 v \sigma((c\bar{c})_c^0 \rightarrow f)$$

(12)

$|\Psi(0)_T|^2$ is the overlap probability of the annihilating pair, $v$ the relative velocity, $\sigma$ the spin-averaged annihilation cross section in the final state $f$ and suffixes $s$ and $c$ denote spin and color \(^1\). For tetraquarks near the $2J/\Psi$ threshold, the spectator $c\bar{c}$ pair appears as $\eta_c$ or $J/\Psi$ on the mass shell, or combines with the outgoing $q\bar{q}$ pair into an open-charm meson pair.

We normalise the overlap probabilities to $|\Psi_{J/\Psi}(0)|^2$, derived from the $J/\Psi$ decay rate into lepton pairs. Eq. (12) applied to this case gives:

$$\Gamma(J/\Psi \rightarrow \mu^+\mu^-) = Q_c^2 \frac{4\pi\alpha^2}{3} \frac{4}{m_{J/\Psi}^2} |\Psi_{J/\Psi}(0)|^2.$$ 

(13)

In terms of the Vector Meson Dominance parameter \[25\] defined by

$$J^\mu(x) = \bar{c}(x)\gamma^\mu c(x) = \frac{m_{J/\Psi}^2}{f} \psi^\mu(x)$$

(14)

with $f$ a pure number, one obtains \[26\]:

$$|\Psi_{J/\Psi}(0)|^2 = \frac{m_{J/\Psi}^3}{4f^2}; \quad f = 7.4; \quad |\Psi_{J/\Psi}(0)|^2 \sim 0.13 \text{ GeV}^3.$$ 

(15)

**Numerical results.** The contribution to the $\mathcal{T}$ decay rate of the colour singlet, spin 0 decay is

$$\Gamma_0 = \Gamma(\mathcal{T} \rightarrow \eta_c + \text{light hadrons}) = 2 \cdot \frac{1}{4} \cdot |\Psi(0)_T|^2 v \sigma((c\bar{c})_1^0 \rightarrow 2 \text{ gluons})$$

$$= \frac{1}{2} \Gamma(\eta_c) \cdot \xi = 16 \text{ MeV} \cdot \xi$$

(16)

\(^1\) Our method of calculation is borrowed from the theory of $K$ electron capture, where an atomic electron reacts with a proton in the nucleus to give a final nucleus and a neutrino, see \[\Pi\].
We have used the spectroscopic coefficient in [10] and have set

\[ |\Psi(0)_{J/\Psi}|^2 \nu \sigma((c\bar{c})_1^0 \to 2 \text{ gluons}) \sim \Gamma(\eta_c) = 32 \text{ MeV}. \] (17)

Similarly

\[
\begin{align*}
\Gamma_1 &= \Gamma(\mathcal{T} \to J/\Psi + \text{light hadrons}) = 2 \cdot \frac{1}{12} \cdot |\Psi(0)_{T}|^2 \nu \sigma((c\bar{c})_1^0 \to 3 \text{ gluons}) = \\
&= \frac{1}{6} \Gamma(J/\Psi) \cdot \xi = 16 \cdot \xi \text{ keV} \\
\Gamma_2 &= \Gamma(\mathcal{T} \to J/\Psi + \mu^+ \mu^-) = B_{\mu\mu} \Gamma_1 = 0.92 \cdot \xi \text{ keV} \\
\Gamma_4 &= \Gamma(\mathcal{T} \to 4\mu) = B_{\mu\mu}^2 \Gamma_1 = 5.6 \cdot 10^{-2} \cdot \xi \text{ keV}
\end{align*}
\] (18)

![Diagram of colour flow in $b\bar{b}$ annihilation](image)

FIG. 1: Colour flow in $b\bar{b}$ annihilation. Open circles represent the insertion of quark bilinears, and black dots the QCD vertices. Colour matrices and normalizations are indicated.

Finally we consider the annihilation of $(c\bar{c})_k^1$ into light quark pairs, Fig. 1. The numerical factor associated to the traces of the colour matrices along fermion closed paths, C (the Chan-Paton factor [27]) gives the effective coupling constant of the process, $\alpha_{eff} = C\alpha_S$, which is what replaces $Q_c\alpha$ in Eq. (13). From Fig. 1 we read $C = \sqrt{2}/3$ and find $^2$:

\[
\begin{align*}
\Gamma_5 &= \Gamma(\mathcal{T} \to M(c\bar{q}) + M(q\bar{c})) = 2 \cdot \frac{1}{6} \cdot \frac{2}{9} \cdot \left( \frac{4\pi\alpha_S^2}{3} \frac{4}{m_{J/\Psi}^2} \right) |\Psi(0)_{J/\Psi}|^2 \cdot \xi \\
&= \frac{8\pi}{81} \left( \frac{\alpha_S}{f} \right)^2 m_{J/\Psi} \cdot \xi = 1.56 \text{ MeV} \cdot \xi
\end{align*}
\] (19)

Using Eq. (15), $\alpha_S = 0.3$ and massless $q$, we obtain

\[
\Gamma_5 = \frac{8\pi}{81} \left( \frac{\alpha_S}{f} \right)^2 m_{J/\Psi} \cdot \xi = 1.56 \text{ MeV} \cdot \xi
\] (20)

$^2$ The factor 2 arises from the two choices of the annihilating bilinear: given the symmetry of the tetraquark, we may call $c_1$ the annihilating $c$ quark and pair it to either $\bar{c}_1$ or $\bar{c}_2$; the spectroscopic factor is from [10]; in parenthesis $\nu \sigma(c\bar{c} \to q\bar{q})$
\[
\Gamma(T) = \Gamma_0 + \Gamma_1 + 3\Gamma_5 = 21 \cdot \xi \text{ MeV} \tag{21}
\]

Eq. (20) gives the total decay rate into pseudoscalar and vector meson pairs. It is easy to see that the rate is shared between pseudoscalar and vector mesons in the ratio 1 : 3 \[1\].

III. THE VALUE OF \(\xi\)

To minimize systematic errors, it is desirable to estimate \(\xi = |\Psi_T(0)|^2/|\Psi_{J/\Psi}(0)|^2\) with the same method for numerator and denominator. In the so-called gaussian approximation, constituents wave functions in charmonia and tetraquarks are taken as simple products of gaussians, with the shape parameters obtained by minimising the expectation of the QCD Hamiltonian given in \[19, 28\], see \[1\] for more details and references.

In the gaussian approximation one finds

\[
|\Psi_{J/\Psi}(0)|^2 = 0.14 \text{ GeV}^3 \text{ (gaussian approximation)} \tag{22}
\]

Reassuringly, the value is in excellent agreement with the one derived from the \(J/\Psi\) leptonic width, \[15\].

Constituent coordinates in the tetraquark are defined as \[19\]

\[x, y : \text{ antiquarks};\]
\[z, 0 : \text{ quarks}\]

and one introduces the Jacobi coordinates

\[
\xi_1 = x - y; \quad \xi_2 = z; \quad \xi_3 = x + y - (z + 0) \tag{23}
\]

A probability function describing \(c\bar{c}\) separation can be obtained in two ways: (i) integrating \(|\Psi|^2\) over \(y\) and \(z\) gives the probability distribution of the distance of one \(c\) from the \(\bar{c}\) sitting in the origin; (ii) integrating over \(\xi_1\) and \(\xi_2\) gives the probability distribution of the distance of the diquark center of mass to the di-antiquark one, which we indicate by \(\ell = (\xi_3/2)\). We obtain

\[
\xi_x = \frac{|\Psi_T(x = 0)|^2}{|\Psi_{J/\Psi}(0)|^2} = 1.8; \\
\xi_\ell = \frac{|\Psi_T(\ell = 0)|^2}{|\Psi_{J/\Psi}(0)|^2} = 5.1 \tag{24}
\]
As a compromise, to estimate the tetraquark total width, (7) and (8), we have used the geometrical mean, with the previous results used as an error estimate:

\[ \xi_{\text{th}} = \sqrt{\xi_\ell \xi_\ell} = 3.0^{+1.2}_{-2.1} \text{ (best guess)} \]  

(25)

Branching ratios do not depend on \( \xi \).

**IV. TETRAQUARK CROSS SECTIONS**

Combining Eqs. (18) and (21) we obtain:

\[ B_{4\mu} = B(T \rightarrow 4\mu) = 2.6 \times 10^{-6} \]  

(26)

corresponding to the cross section upper bound

\[ \sigma_{\text{theo.}}(T \rightarrow 4\mu) \leq \sigma(pp \rightarrow 2 \Upsilon)B_{4\mu} = 40 \text{ fb} \]  

(27)

where \( \sigma(pp \rightarrow 2J/\Psi) \simeq 15.2 \text{ nb} \) is the two-\( J/\Psi \) production cross section measured by LHCb at 13 TeV [22].

| \( [cc][\bar{c}\bar{c}] \) | Decay Channel | Cross section upper limit (fb) |
|-----------------|-----------------|-------------------------------|
| \( T \rightarrow D^{(*)+}D^{(*)-} \rightarrow e + \mu + \ldots \) | 4.3 \( 10^{-3} \) | \( 6.5 \times 10^{4} \) |
| \( T \rightarrow D^{(*)0}\bar{D}^{(*)0} \rightarrow e + \mu + \ldots \) | 0.67 \( 10^{-3} \) | \( 1.0 \times 10^{4} \) |
| \( T \rightarrow 4\mu \) | 2.7 \( 10^{-6} \) | 40 |
| \( T \rightarrow D^{(*)+}\bar{D}^{(*)-} \rightarrow e + \mu + \ldots \) | 6.3 \( 10^{-3} \) | \( 9.6 \times 10^{4} \) |
| \( T \rightarrow D^{(*)0}\bar{D}^{(*)0} \rightarrow e + \mu + \ldots \) | 0.98 \( 10^{-3} \) | \( 1.5 \times 10^{4} \) |
| \( T \rightarrow 4\mu \) | 1.6 \( 10^{-5} \) | 238 |

**TABLE II**: Upper limits of two- and four-lepton cross sections via \( T \) production, estimated from the production cross sections of 2 \( \psi(1S) \) (LHC, 13 TeV) [22].

We focus on the \( e\mu \) inclusive channel and give in Tab. II the upper limits to \( \sigma_{\text{theo.}}(T \rightarrow 2D^{(*)}_q \rightarrow e\mu + \ldots) \), calculated as

\[ \sigma_{\text{theo.}}(T \rightarrow 2D^{(*)}_q \rightarrow e\mu + \ldots) = \sigma(pp \rightarrow T + \ldots)BF(T \rightarrow 2D^{(*)}_q \rightarrow e\mu + \ldots) \leq \sigma(pp \rightarrow 2J/\Psi + \ldots)BF(T \rightarrow 2D^{(*)}_q \rightarrow e\mu + \ldots) \]  

(28)
The largest part of the signal (the total signal for $J^{PC} = 2^{++}$) arises from the decay of $\mathcal{T}$ into a pair of vector mesons. Vector particles decay promptly into a pseudoscalar plus a soft pion or photon(s) and contribute to the signal on the same basis as the pseudoscalars.

In conclusion, production in the $4\mu$ channel and decay rates that we estimate for the $cc\bar{c}\bar{c}$ tetraquarks are tantalizingly similar to the preliminary results presented by the LHCb Collaboration [2]. The meson-meson channel with the $e\mu$ signature may provide an additional, complementary tool to identify and study the spectacular, exotic $cc\bar{c}\bar{c}$ tetraquarks.

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