Markov matrixes for random sequences imitation in fatigue testing and design in the problem of safety of steel structures

I Gadolina1, R Voronkov2, A Bautin2, I Serebrjakova3, A Erpalov4

1 IMASH RAS, Structural materials science Department, Bardina 4, Moscow, Russia
2 Central Aerohydrodynamic Institute, Structural Integrity Department, Moscow Region, Zhukovsky 1, Russia
3 Bauman Moscow state technical university, Department Computer Software and Information Technology, Moscow, Russia
4 South Ural State University, Chelyabinsk, Russia

E-mail: gadolina@mail.ru

Abstract. To guarantee the safety of steel structures it is important to deal properly with the problem of representation of exploitation loading. For metal fatigue testing and design, the proper choice of random (irregular) loading type is very important. The principles of random loading are discussed and some alternative approaches with their pros and cons are shortly reviewed in the paper. As a sound decision for random, but taking into account some specific features of the exploitation loading process, the target Markov method is proposed. According to this method, the important information of the real random processes in the form of the turning point is used for filling the square Markov matrix (analysis phase) and later on, with employing the random number generator, serves as a source for creating of the so-called replicas. The replicas are the random trial for numerical estimation of longevity scatter. Due to the fact, that all these manipulations are performed with the aim of metal fatigue investigation, some important processes’ characteristics for fatigue, like irregularity factor, fullness factor and machine part longevities were compared. Some important suggestion for the future development of this method, that is taking into consideration the sequence of the events, is discussed.

Key words. Random processes, metal fatigue, Markov’s matrixes

1. Introduction
The process of metal fatigue failure is complex by its nature and possesses huge scatter especially considering the working duration of machines in exploitation under random loading. Even for nominal conditions, the duration before failure might be varied by several times and more. There are two main sources for scatter - the instability of physical material properties and variation of the loading conditions [1]. Scatter of mechanical properties are being investigated during static strength tests (example in [2]) and by regular fatigue testing while building the fatigue curve. To investigate the
scatter of the results due to the variability of loading condition, it is necessary to perform testing under random loading.

For investigation of the random loads on massive off-shore structures, caused mostly by random waves, the package WAFO (Wave Analysis for Fatigue and Oceanography) was developed. This package includes analysis of the sequence of turning points [3].

Fatigue testing under random loading plays a big role nowadays mostly for two reasons: 1) the test results provide useful data about the fatigue damage accumulation, taking into account the peculiarities of irregular loading; 2) there are excellent technical and control decisions nowadays allowing to realize many options and methods of random loading during the test. The good example of the testing under random loading is given in [4]. Some questions of peculiarities of distributions of random values are discussed in [5]. Extreme loads prediction plays an important role while evaluating the risk in varied fields of industry, considering the different scenarios of the events [6]. In [7] the recordings of random loads in wave energy converter of micro power plants are presented. This fact also indicates the need for developing the methods for simulating the random loading sequence.

In metal fatigue, usually, the researchers consider two fatigue stages with different mechanics: crack initiation and crack growth stages. Effect of sequence in loading history exists at both stages. The degree of degradation during the initial stage depends on the sequence of applied load cycles. On the other hand, from the studies of technological operation influencing on fatigue properties, it is known, that fatigue strength could be more than doubled by a single high overload [8]. Overloads are the cause of residual compressive stresses in the concentration areas and reduce the damage from the rest of the loading cycles, therefore the duration also increases. Even at the greater degree the sequence effect exists at the crack propagation stage.

2. Testing method review

2.1. Sinus-like fatigue testing
This type of testing had been performed since the first fatigue testing machine appeared (since ~1850). The result of this testing is a fatigue curve, or additionally, and sometimes alternatively, the estimation of the so-called fatigue limit. Nowadays many scientific sources imply that probably there is no such a thing like a fatigue limit. Further application of these results requires applying the Miner-summation rule [9] to get the reasonable estimation of machine-part longevity

2.2. Block testing
Historically the first mode of irregular loading was testing with the blocks. E. Gassner proposed the ideology of block testing for the aviation structures [10]. Block steps are presented in the form of simple sinusoidal loading with the amplitudes vary stepwise. Usually, it is taken 8 steps for each block. The block testing should be planned in such way, that blocks could be repeated several times before failure. The amplitudes and their repetition numbers are estimated on the basis of real loading by applying the cycle counting methods for the transformation of irregular loading process into the set of simple cycles. The example of block is shown in figure 1 (the values of loadings and time are shown non-dimensional).
2.3. One-to one random testing
The idea of testing one-to-one by simply repeating the exploitation loading has many shortcomings. First of all, it is very time-consuming. Machine parts serve years and conclusion of their reliability should be done during few weeks. Even if the researcher excludes time-out and down-states of the machines, the testing time would be extremely long. The second problem is representability of rare events, like overloading. It might come out, that in the particular single realization the rare event just would not be fixed. In this way, the testing estimation would be overoptimistic. It is also important to have in mind that in reality the process might be different comparing to the single registered realization. For this testing option, the servo-hydraulic machine is required.

2.4. Imitation of random processes using spectral density
One of the modes of early days testing was controlling the power spectral density function of the random exploitation process and later on use this information not only in simulating the random process during laboratory testing [4], but also for fatigue durability estimation [11]. This approach was especially popular in 1970-1980, when in researchers’ disposition mostly only the vibration testing machines existed. Those machines were controlled by frequency. Similar methods are still popular nowadays when the parts of the apparatus with minor dimensions are being tested (the solder joints, for example).

It should be noted here, that in situation where the researchers are dealing with the machine parts, where the loading process could be easily registered through the tensor gages, the preferable solution is to perform the cycle counting procedure, namely, rain-flow [12]. While doing this, the information about physical frequencies is being lost. The rain-flow method saves only the information about stress amplitudes and their repetition [13].

2.5. Applying the Markov’s matrixes
A good decision for simulating the random sequence of the turning points is the utilization of the Markov’s model. Markov’s matrixes were proposed as a method for modeling random sequences for
academicals research for imitation of random processes with different irregularity factor IF [12, 14]. Irregularity factor features the degree of the complexity of the random process and equals IF=No/N_E. Here No is the number of the mean level crossing and N_E is the turning points number. Both values should be taken on the representative enough realization part. In [15] the representative distributions were analyzed for three options: IF = 0.3 (process with complicated structure), IF = 0.7 (medium process) and IF = 0.99 (almost regular process, but the amplitudes vary significantly), figure 2.

In [16] the Markov’s modeling is compared to the other random process simulation technique, namely autoregressive moving average modeling. In [16] the loading process is considered to be unceasing, not just the turning points, like in Markov’s modeling.

The short parts of standard sequences the examples are shown in figure 2.

![Figure 2. Parts of standard Markov’s sequences [14]](image)

3. Method: target Markov’s matrixes simulation

The method was developed with the purpose of investigating the scatter produced by the randomness of loading. In this paper target Markov’s modelling had been proposed for the better representation of individual peculiarities of the investigated machines loading. Some unique loading features, including randomness and overloads, deserve to be saved and later on to be employed while simulation of so-called replicas. To create a Markov’s matrix the real loading process firstly some treatment should be done.

3.1. Preliminary treating

Before creating the matrixes some steps for process preparation should be done [12]:

- Process discretization. After performing it, instead of the continuous stress process σ(t) the researcher gets the set of digital stress reading σ₁, σ₂, ...... σ_z, where z – is the number of readings in the realization;
- Division by classes – that means the finite number of digits, say, N, classes, by which the values σᵢ are represented. Value N depends on required representation accuracy and usually is taken as N=2k, where k=5 … 7. While doing this substitution, the researchers implement the preliminary process filtration. The data later on are presented in integer form that speed up the future treatment;
- Turning points definition. After this procedure the initial process is represented by the sequence of turning points (extremums-“the saw”):

  \[ e₁, e₂, ...... e_{z₁} \]  

  (1)

  Note, that z₁<z, because of the transformation described above.

3.2. Analysis stage
The analysis implies the creation of Markov’s matrix of frequencies on the base of a real process. During the analysis phase, the square matrix [32x32] or [64x64] is filled up on the basis of sequence (1) following the simple rules described below.

For the explanation, a short sequence of events is shown in figure 3, left. On the main diagonal of the matrix figure 3, right, are zeros. The triangle to the right of the main matrix diagonal contains the numbers of ascending ranges (ei, ei+1), (i∈ 2k - even indexes) from the minimum to the maximum. In the triangle to the left of the main diagonal are the numbers of descending from the maximum to the minimum ranges (i∈ 2k+1 - uneven indexes. In this manner, the analysis – "convolution" of the initial sequence to Markov’s matrix is carried out.

Figure 3. Analysis stage in target Markov’s matrixes modeling

It is worth mentioning, that this filled matrix by the form and by its content is very similar to the matrix of ranges, produced by range cycle counting method [12].

3.3. Synthesis stage
After the matrix, like one shown in figure 3, right, has been filled, the next step would be the creation of the series of replicas on the base of the initial sequence. This step might be called “synthesis” because on the base of the target matrix the random sequences are being produced. To investigate the statistical uncertainty, more than one replica had been produced (namely nine, in this example).

The algorithm of this modeling is as follow. The filled matrix and a random number generator are used. The initial sequence minimum (class α) is chosen arbitrarily (figure 4). Next, the ascending range starting from this minimum α is modeled. In the string α contains some frequencies, and their sum equals Tα. The random value RAND * Tα is calculated (RAND is a random number, RAND=0...1). When the accumulated sum of the parts in the line α from the main diagonal to the right exceeds RAND * Tα, it gives a signal that the desired maximum is found and its value is β. This will be the first ascending range: α→β. The next step is to model the next minimum after β. The transition to the main diagonal is performed and the accumulated sum of the parts to the left of the main diagonal Tβ is calculated. The newly drawn random number RAND (it will be, of course, the different value), and the RAND * Tβ is computed. Similarly, when the sum of frequencies in the line of β starting from the main diagonal to the left will exceeds RAND * Tβ, it will determine the following the minimum and its value will be γ. After this step the simulation continues up to the required length of realization z₁, i.e. turning points number.
4. Results and discussion

In accordance to the scope of the paper, that is to create a method for simulation the similar, but varied in some detail’s replicas of the initial random process for metal fatigue testing and design, some replicas were modelled. Some important characteristics of realizations were compared and their scatter was analysed. The analysis was performed on the base of stress realization of the part of the transport machine.

The Markov’s matrix was filled up accordingly 3.2. Nine replica realizations (1, 2 …. 9) were modelled using the method described in 3.3. The pictures of some replicas processes are shown in figure 5.

\[
V_{cor} = \frac{0.25}{h_{sum}} \sum h_i \left( \frac{\sigma_{nx}}{\sigma_a} \right)^m
\]  

(2)
In (2) \( m \) - is the fatigue exponent, \( h_{SUM} \) – is the total number of cycles in the block; \( h_i \) - is the number of cycles at the \( i \)-th step; \( \sigma_{ai} \) – is the current value of the stress amplitude at the \( i \)-th step; \( \sigma_{\alpha} \) - is the maximum amplitude in the block.

With known data of fatigue curve of the machine part, known \( \sigma_{\alpha} \) and \( V \), it is possible to estimate medium longevity \( L \) [cycles], with applying the corrected linear hypothesis of fatigue damage summation, it is possible to estimate the number of blocks until failure \( \lambda_b \) and later on to estimate the longevity \( L \):

\[
L = \lambda_b \cdot h_{SUM}
\]

The parameters of a convenient fatigue curve were taken arbitrary for the sake of comparison of the results.

Because the longevity of the machine parts is known to be distributed logarithmically normal, the distribution of the random value of decimal logarithm of \( L \), namely, \( \lg L \) was investigated on the base of the set of replicas. The mean value \( \lg L = 0.92 \); standard deviation \( \lg L = 0.12 \) and variation factor \( \lg L = 0.119 \). The distribution of \( \lg L \) is shown in figure 6. This graph shows theoretical and sample quantiles. It can be seen, that the distribution is close to a normal one.

![Image](Longevity_distribution.png)

**Figure 6.** Variability analysis of the longevities estimated by replicas

In Table 1 the summary of initial (“init”) process and nine target replicas with the most important characteristics for fatigue is presented.

| Realization and replicas | \( \sigma_{\alpha} \) | \( V \) | \( IF \) | \( L \) | \( \lg L \) |
|--------------------------|------------------|-----|-----|-----|-----|
| init                     | 13.5             | 0.546 | 0.433 | 12.44 | 1.091 |
| 1                        | 14.5             | 0.527 | 0.497 | 10.04 | 1.002 |
| 2                        | 15.5             | 0.537 | 0.503 | 6.01  | 0.779 |
As can be seen from the Table 1 and Fig.6, although the replica processes demonstrate some scatter in their characteristics, in general, the data are close to the initial ones. Because of the proper choice of physical model, one can expect that this scatter might characterize the scatter of the longevity in exploitation.

5. Further directions for the research

As it was mentioned in Introduction, many investigations have shown, that the consequence of the events might have influenced on the longevity at both stages of fatigue (crack initiation and crack propagation).

It is known that the durability can be increased by residual compressive stresses, which are induced in various ways. Sometimes they are intentionally being produced by special technologies, like shot peening. It worth mentioning, that the impact of the load sequence also needs consideration, especially when the number of cycles to failure is not so big – that is called low cycle fatigue. Some methods, like described in [19] consider the load sequence, because it influences for the estimation of mean stress effect.

The same situation even in the greater degree is true at crack propagation stage. For the crack propagation estimation, the special methods were developed [20-23]. The idea of the models is to calculate effective values of the stress intensity factors (SIF) for the cycles that follow after overloads (i.e. for large-scale cycles, occurring at some segment of the construction). Physically these models based on the formation of a large area of plastic deformation in the top of the crack because of the overload, which slows down the crack growth. In this case, the following after overload cycles form the next zones of plasticity, which are smaller in size. And before crack does not "come out" of the zone of plasticity formed by overload, the growth rate decreases significantly because of the residual compressive stress. These effects are taken into account by the so-called effective values of SIF further are applied in the kinetic diagram of fatigue crack growth.

As it is seen from that short explanation, for both fatigue stages the consequence of the events plays important role. The experiments have shown that under the same distribution of rain-flow cycles the situation with high cycles appearing first will be preferable for slowing down the metal fatigue process. There is a need to create some criteria, which indicate the distribution of the biggest maxima among the realization. In this way, it can be summarized, that there is a need for creating some process parameter, which will show the distribution of overloading during the testing time.

The other fact, which should be considered while modeling by Markov matrix, is the interaction (aftereffect) of the random events of the turning points. It follows from the fact, that the energy is finite, and it is impossible to produce the great leaps from one point to another in no time.
All calculations, as well as simulation programming, were performed in freely distributed programming language and free software environment for statistical computing and graphics program R [24].

References
[1] Kogaev V P 1993 Strength design under non-stationary stresses (Moscow, Mashinostroenie) p 363
[2] Gubenko S I. Ivanov I A and Kononov D P 2018 Zavad. Lab. Diagn. Mater 3 The impact of steel quality on the fatigue strength of wrought wheels pp 52-60
[3] Per Andreas Brodtkorb 2000 WAFO- Paper no.ISOPE -GFC0-02 A Matlab toolbox for an analysis of random waves and loads www.researchgate.net/publication/282371975
[4] Erpalov AV and Shefer L A 2016 Procedia Engineering 2nd International Conference on Industrial Engineering (ICIE-2016) Fatigue-based Classification of Loading Processes 150 pp 144-149
[5] Syzrantseva K, Syzrantsev V 2017 Key Engineering Materials Determination of parameters of endurance limit distribution law of material by the methods of nonparametric statistics and kinetic theory of high-cycle fatigue 736 52 DOI: 10.4028/www.scientific.net/KEM
[6] Makhtutov N A, Reznikov D O 2019. Assessment of large-scale catastrophes in complex engineering systems IOP Conference Series: Materials Science and Engineering, Volume 481 012002 IOP Publishing. doi:10.1088/1757-899X/481/1/012002 pp 1-8
[7] Hossain I, Velkin V I, Shcheklein S E and Eliseev A V. 2019 Safety 2018. IOP Conf. Series: Materials Science and Engineering 481 Structural Design Development of a Float Type Wave MicroPower Plant. IOP Publishing doi:10.1088/1757-899X/481/1/012007 pp 1-6
[8] Fuchs H O, Stephens R I 2002 Metal fatigue in engineering (John Wiley&Sons. New York) p 318
[9] Miner M.A. 1945 Journal of Applied Mechanics, Trans. ASME Cumulative Damage in Fatigue 12
[10] Gassner E. 1939 Dt. Luftwacht, Ausg. Luftwiss 6 pp 61-64
[11] Benasciutti D, Tovo R. 2018 Mat.-Werkstofftech Frequency-based analysis of random fatigue loads: Models, hypotheses, reality 49 pp 345-361
[12] GOST 25.101-83 2005 Strength calculation and testing. Representation of random loading of machine elements and structures and statistical evaluation of results (Moscow: Standartinform) p 25
[13] Gadolina I and Zaynetdinov R 2019 IOP Conference Series: Materials Science and Engineering Advantages of the rain-flow method at the post-processing stage in comparison with the spectral approach 481 012005 IOP Publishing. doi:10.1088/1757-899X/481/1/012002 1 pp 1-9
[14] Fisher R, Haibach E. 1983 in book: Behavior of steel under cyclic loads ed Dahl V Modeling loading functions in experiments on the evaluation of materials pp.368-405
[15] Gadolina I V, Monahova A A, Petrova I M. and Serjabjjakova I L 2016 Proc. of the Conference “Survivability and Structural Materials Science. SSMS LABORATORY AND SIMULATION EXPERIMENT UNDER RANDOM LOADING AS THE ALTERNATIVE TO TESTING UNDER REGULAR LOADING pp 4-9
[16] Ling Y et al 2011 Int J Fatigue Stochastic prediction of fatigue loading using real-time monitoring data doi:10.1016/j.ijfatigue
[17] Dubin D A, Nakaznoi O A, Smirnov I A and Shleev A. N. 2016 Trudy NAMI Experimental determination of kinematic and power parameters of loading of system elements suspension of high-speed tracked vehicle 266 pp 45-53
[18] Savkin A N, Sedov A A, Boyko G V and Badikov K A 2017 AIP Conference Proceedings 1909, 020187 Estimation of contribution of different parties of random loading spectra to damage accumulation https://doi.org/10.1063/1.5013868
[19] Martin J E, Topper T H, Sinclair I G M 1971 *Material Research and Standards, MTRS* Computer Based Simulation of Cyclic Stress-Strain Behavior with Applications to Fatigue 11 pp 23-28

[20] Schijve J, Jacobs P A, Tromp P J 1976 *NLR TR 76065* The effect of load sequence on fatigue crack propagation under random loading and program loading

[21] Svirsky Yu A., Basov V N 1994. *1st International Conference FUNDAMENTAL RESEARCH IN AEROSPACE SCIENCE* Accounting for non - linearities of damage accumulation in aircraft structures under irregular loading Russia. (Zhukovsky TsAGI) pp 93-96

[22] Willenborg J, Engle R H, Wood H A 1971 *AFFDL-TM-71-1 FBR, WPAFB* A Crack Growth Retardation Based on an Effective Stress Concept

[23] Romanov A N, Nesterenko G I and Filimonova N I 2018 *Journal of Machinery Manufacture and Reliability* Damage Accumulation under Variable Loading of Cyclically Hardening Material at the Stages of Formation and Development of Cracks 47 pp 414-419. DOI: 10.3103/S1052618818050102

[24] R Development Core Team 2018 R: A language and environment for statistical computing. R Foundation for Statistical Computing Vienna Austria. ISBN 3-900051-07-0, URL http://www.R-project.org