Voltage stability analysis: Case study in Madura 150 kV transmission system

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Abstract. Voltage stability problem during the operation of the power system cause serious failures in the power system, so that making the development of stability analysis an important necessity. In this study, the data taken were Gilitimur, Bangkalan, Sampang, Pamekasan and Sumenep bus obtained from PT. PLN (Persero) UP2B East Java Madura 150 kV system area. In March 2019 there was a large voltage drop on the Sumenep bus so that the voltage was 133 kV. In power systems, the system voltage must be maintained within the limits (+ 5%) and (-10%) of the nominal voltage. Based on these regulations, the voltage condition on the Sumenep load bus experiences a voltage drop of 17 kV or >10% so that the voltage drop has exceeded the allowable voltage standard, whereas on other buses in Madura the region has also experienced a voltage drop that is almost close to the standard limit as in the Sampang bus has a voltage drop of 137 kV. To predict the voltage stability of the system in Madura, it can be done with a voltage stability analysis study using the Q-V Sensivity Analysis method. This analysis can be useful to determine which buses are prioritized for maintenance. In the Q-V Sensivity Analysis it is assumed that the bus voltage and load is absolute, when the value of Jacobian reduction (JR) is negative, indicating that the bus condition is unstable. In the analysis of the Sumenep bus the negative value is -0.015 while the other buses are positive namely the Gilitimur bus = 1.225, Bangkalan bus = 0.502, Sampang bus = 0.448 and Pamekasan bus = 0.287, so it can be concluded that the 150 kV Madura transmission system has unstable buses is the Sumenep bus.

1. Introduction
PT. PLN (Persero) East Java Region 4 Load Regulatory Service Unit (UP2B) is one of the working areas of the PLN Bali Bali Distribution and Control Center (P3B JB) which is responsible for operating 150kV and 70kV system in the region. data obtained from observations at PT. PLN (Persero) East Java Region 4 Load Management Service Unit (UP2B) on the voltage metering data using SCADA on Bus 70 kV, 150 kV, and 500 kV obtained a drop voltage value, Sunday 10 March 2019 at 10.00 WIB on the bus Sumenep drop voltage occurs up to 133kV. In general, the power system voltage must be maintained within the limits (+ 5%) and (-10%) in accordance with the Regulation of the Minister of Energy and Mineral Resources No. 03/2007 concerning the Rules for the Java-Madura-Bali Power System [1]. By looking at the voltage conditions based on the above data compared to the standard allowable voltage limit, the Sumenep load bus experiences a drop voltage of 17 kV or >10% so that the drop voltage that
occurs exceeds the allowable voltage standard limit of more than 10% or 135 kV while on other buses in Madura the area also experienced a voltage drop which was almost close to the standard limit as in the Sampang bus the voltage drop was 137 kV.

Voltage Stability is defined as the ability of a power system to stabilize the voltage to normal again after a disturbance in the system [2-4]. Voltage problems have become a major concern during the planning and operation of the power system due to the number of serious failures caused by this phenomenon which makes the development of voltage stability analysis becomes necessary [5-9]. Therefore, this paper will discuss voltage stability in the 150 kV system in the Madura area using the Q-V sensitivity analysis method [10-14]. This analysis can be useful to determine which buses are prioritized for maintenance. The basic of this method is to calculate the value of $J_R$ (Jacobian reduction) [15] associated with the form of changes in voltage and reactive power. The voltage stability condition is stable if the $J_R$ value is positive. The smaller the positive value of $J_R$, the more stable the condition of voltage stability in the system, if the value of $J_R$ is negative, the voltage stability condition is unstable. The smaller the negative value of $J_R$, the more unstable the voltage stability condition in the system. In this paper, voltage stability analysis of Madura 150 kV transmission system is presented. Based on the model and calculation derived in this paper, negative and low value of $J_R$ indicate bus Sumenep is unstable, while four others buses are considered stable with positive value of $J_R$. The authors leave the analysis in this paper as a recommendation for PT PLN to take further steps to avoid future failures the network.

2. Main Theory

2.1. Power Flow Analysis

In this voltage stability analysis, power flow analysis using Newton Raphson's method is used to obtain the jacobian matrix element, jacobian also provides very useful information about voltage stability [2][4][15]. Before using the Newton Raphson method, admittance must be calculated on each bus. Admittance is the ratio of the effective current to the effective voltage, admittance can be determined from the resistance and reactance on the transmission line according to the following equation:

$$Y = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2}$$

(1)

Where:

$Y =$ line admittance

$R =$ line resistance

$X =$ line reactance

While the Ybus (admittance bus) matrix can be made from calculation of admittance for each bus, the rules when forming Ybus elements as follows:

1. Element $Y$ on the inter-bus will be negative in the admittance matrix
2. Element $Y$ on the bus itself will be equal to the total value of $Y$ connected to the bus.
3. Element $Y$ that is not/ not connected to the inter-bus will be zero in the admittance matrix.

In the bus admittance matrix, the values in rectangular form are obtained, for conversion from rectangular to polar form with the aim of finding the angle of each admittance in the matrix as follows: rectangular form:

$$g + je$$

conversion to polar form:

$$\sqrt{g^2 + e^2} \angle \tan^{-1} \frac{e}{g}$$

(3)

After the bus Y admittance matrix formed in a rectangular form is changed to polar. Then determine the active power and reactive power on bus k are:

$$P_k = V_k \sum_{m=1}^{n} (G_{km} V_m \cos \Theta_{km} + B_{km} V_m \sin \Theta_{km})$$

(4)
\[ Q_k = V_k \sum_{m=1}^{n} (G_{km}V_m \sin \theta_{km} - B_{km}V_m \sin \theta_{km}) \]  

Where: \( \theta_{km} = \theta_k - \theta_m \) and \( \theta_{11} = \theta_{22} = \cdots = \theta_{xx} = 0 \)

The equation above forms a nonlinear algebraic equation with its own variable. The magnitude of each variable is expressed in units per unit. Next, Jacobian matrix was formed as follows:

\[
J = \begin{bmatrix}
\frac{\partial P_1}{\partial \theta_1} & \cdots & \frac{\partial P_1}{\partial V_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial P_n}{\partial \theta_1} & \cdots & \frac{\partial P_n}{\partial V_n}
\end{bmatrix}
\]

**Figure 1. Jacobian Matrix**

Based on Figure 1, this Jacobian matrix consists of 4 submatrices, namely submatrix H, N, M and L or with other expressions J1, J2, J3 and J4.

For submatrix j1 or H can be calculated with the following formula

\[
\frac{\partial P}{\partial \theta}
\]

For submatrix j2 or N can be calculated with the following formula

\[
\frac{\partial P}{\partial V}
\]

For submatrix j3 or M can be calculated with the following formula

\[
\frac{\partial Q}{\partial \theta}
\]

For submatrix j4 or L can be calculated with the following formula

\[
\frac{\partial Q}{\partial V}
\]

### 2.2. Case Study Transmission Line Data

Transmission lines in the Madura area are generally radial in shape, the 150 kV transmission system line in Madura uses ASCR-HAWK type conductor material, that is aluminum reinforced steel conductors with 150 kV voltage. This line has a current capacity of up to 600 A with a maximum temperature of 75°C [4].

### 2.3. Q-V Sensitivity Analysis

Q-V Sensitivity Analysis of Voltage Stability, this method was first introduced by Morrison and Kundur who have discovered a technique to find out the voltage stability in the system. The basic of this method is to calculate the value of the Jacobian Reduction matrix system. Jacobian reduction is a representation of changes in reactive power to changes in voltage. The Q-V sensitivity analysis is based on the Jacobian Reduction matrix obtained from power flow studies using Newton Raphson's method.

Power flow equation using Newton Raphson method as follows:

Linear equation power flow:
\[ \Delta P_2 = J_{P\theta} \Delta \Theta + J_{PV} \Delta V_2 \] 
\[ \Delta Q_2 = J_{Q\theta} \Delta \Theta + J_{QV} \Delta V_2 \] 

With assumption \( \Delta P_2 = 0 \), and get:

\[ \Delta Q_2 = (J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}) \Delta V \] 
\[ \Delta Q_2 = J_R \Delta V \] 

With notes:

\[ J_R = [J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV}] \] 

The JR matrix represents a linear relationship between changes in voltage and changes in reactive power injection on a bus according to Eq. 13. The voltage stability condition is stable if the JR value is positive (\( JR > 0 \)). The smaller the positive value of JR, the more stable the voltage stability condition in the system, otherwise if the JR value is negative (\( JR < 0 \)), the voltage stability condition is in an unstable condition. The smaller the negative value of JR, the more unstable the voltage stability condition in the system. [2].

3. Results and Discussion

Before looking for the admittance value (Y) bus, it is necessary to model the equivalent circuit of a single line diagram with the target of finding the admittance matrix formula in the system shown in Figure 2.

3.1. Modelling the Equivalent Circuit

This is modelling the equivalent circuit of the 150 kV system in the Madura region shown in Figure 3:

\[ Figure 2. \] Single line diagram 150 kV Madura
Figure 3. Modelling Equivalent Circuit 150 kV Madura

Through the admittance matrix rule, an admittance matrix formula is obtained from the equivalent circuit:

Table 1. Admittance matrix formula

|     | 1       | 2       | 3       | 4       | 5       |
|-----|---------|---------|---------|---------|---------|
| 1   | \(Y_a + Y_c\) | \(-Y_c\) | 0       | 0       | 0       |
| 2   | \(-Y_c\)  | \((Y_b + Y_c)\) | \(-Y_d\) | 0       | 0       |
| 3   | 0       | \(-Y_d\)  | \((Y_a + Y_c + Y_g)\) | \(-Y_c\) | \(-Y_g\) |
| 4   | 0       | 0       | \(-Y_e\)  | \((Y_e + Y_f)\) | \(-Y_f\) |
| 5   | 0       | 0       | \(-Y_g\)  | \(-Y_f\)  | \((Y_g + Y_f)\) |

The line admittance model of Madura transmission line is derived and formulated in Table 1. Subsequently, to calculate the admittance value (Y), the resistance value (R) and the reactance value (X) on the transmission line are needed. The per unit line admittance for each line is calculated based on the data in Table 2, and summarized in Table 3. Then, the value of each Y is entered into the admittance matrix formula according to Table 1 so that the admittance matrix is obtained as depicted in Table 4.

Table 2. Resistance and reactance data between bus

| Resistance data | Reactance data | Label                          |
|-----------------|----------------|-------------------------------|
| \(R_a\) = 0.79  | \(X_a\) = 2.15 | Gilitimur bus and Infinite bus |
| \(R_b\) = 1.97  | \(X_b\) = 5.36 | Bangkalan bus and Infinite bus |
| \(R_c\) = 1.34  | \(X_c\) = 3.19 | Gilitimur bus and Bangkalan bus |
| \(R_d\) = 2.21  | \(X_d\) = 5.25 | Bangkalan bus and Sampang bus  |
| \(R_e\) = 2.21  | \(X_e\) = 5.24 | Sampang buses and Pamekasan buses |
| \(R_f\) = 3.88  | \(X_f\) = 9.21 | Pamekasan bus and Sumenep bus  |
| \(R_g\) = 6.14  | \(X_g\) = 14.56| Sampang bus and Sumenep bus    |
Table 3. Line Admittance

| Expression | Admittance Value (pu) |
|------------|-----------------------|
| $Y_a$      | 0.1505737049 − j0.4097891968 |
| $Y_b$      | 0.06040999065 − j0.1643642385 |
| $Y_c$      | 0.1119306364 − j0.2666667389 |
| $Y_d$      | 0.06811191311 − j0.1618043185 |
| $Y_e$      | 0.06833283746 − j0.1620199309 |
| $Y_f$      | 0.03888471993 − j0.092212037 |
| $Y_g$      | 0.02459017707 − j0.0583115595 |

Table 4. Rectangular form admittance matrix

|   | 1          | 2          | 3          | 4          | 5          |
|---|------------|------------|------------|------------|------------|
| 1 | 0.2625 − j0.6762 | −0.1119    | 0          | 0          | 0          |
| 2 | + j0.2664  | 0.1723     | −0.0681    | + j0.1618  | 0          |
| 3 | 0          | + j0.1618  | − j0.3821  | + j0.1620  | + j0.5083  |
| 4 | 0          | 0          | −0.0681    | 0.1071     | −0.388     |
| 5 | 0          | 0          | + j0.1620  | − j0.2542  | + j0.0922  |
|   |            |            |            |            |            |

To simplify the mathematical analysis, the form of admittance equations from Table 4 is converted to polar form by utilizing Equation 3. The modified admittance equations are included in the matrix in Table 5.

Table 5. Admittance matrix in polar form

|   | 1          | 2          | 3          | 4          | 5          |
|---|------------|------------|------------|------------|------------|
| 1 | 0.72 ∠ −68.7° | 0.18 ∠ −67.2° | 0          | 0          | 0          |
| 2 | 0.18 ∠ −67.2° | 0.35 ∠ −68.2° | 0.09 ∠ −67.1° | 0          | 0          |
| 3 | 0          | 0.09 ∠ −67.1° | 0.3 ∠ −67.1° | 0.09 ∠ −67.1° | 0.02 ∠ −67.2° |
| 4 | 0          | 0          | 0.09 ∠ −67.1° | 0.17 ∠ −67.1° | 0.04 ∠ −67.1° |
| 5 | 0          | 0          | 0.02 ∠ −67.2° | 0.04 ∠ −67.1° | 0.08 ∠ −67.1° |

3.2. Active and Reactive Power Expressions

The expressions of active and reactive power for each bus are derived using eq. 4 and 5, and are shown in Table 6. Table 7-11 shows the components of Jacobian matrix, defined as submatrices, using eq. 6 – 9 to obtain the partial derivative, denoted as $J_{\phi}$, $J_{PV}$, $J_{Q\phi}$, $J_{QV}$ for each Gilitimur, Bangkalan, Sampang, Pamekasan, and Sumenep.
Table 6. Active and Reactive Power

| Bus         | Active Power (pu)                                                                 | Reactive Power (pu)                                                                                        |
|-------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Gilitimur   | $0.2625 V_1^2 - 0.1119 V_1 V_2 \cos \Theta_{12}$                              | $0.6762 V_1^2 - 0.1119 V_2 V_1 \sin \Theta_{12}$                                                        |
|             | $+ 0.2664 V_1 V_2 \sin \Theta_{12}$                                            | $- 0.2664 V_2 V_1 \cos \Theta_{12}$                                                                       |
| Bangkalan   | $-0.1119 V_1 V_2 \cos \Theta_{21}$                                             | $-0.1119 V_1 V_2 \sin \Theta_{21} - 0.2664 V_1 V_2 \cos \Theta_{21}$                                     |
|             | $+ 0.2664 V_1 V_2 \sin \Theta_{21}$                                            | $+ 0.4308 V_2 \cos \Theta_{21}$                                                                            |
|             | $+ 0.1723 V_2^2$                                                               | $- 0.0681 V_2 V_2 \sin \Theta_{23}$                                                                       |
|             | $- 0.0681 V_2 V_2 \cos \Theta_{23}$                                            | $- 0.1618 V_2 V_2 \cos \Theta_{23}$                                                                       |
| Sampang     | $-0.0681 V_2 V_3 \cos \Theta_{32}$                                             | $-0.0681 V_2 V_3 \sin \Theta_{32}$                                                                         |
|             | $+ 0.1618 V_2 V_3 \sin \Theta_{32}$                                            | $0.1618 V_2 V_3 \cos \Theta_{32} + 0.3821 V_3^2 - 0.2664 V_3 \sin \Theta_{33}$                         |
|             | $+ 0.1610 V_3^2$                                                              | $0.0683 V_4 V_4 \sin \Theta_{34}$                                                                         |
|             | $- 0.0683 V_4 V_4 \cos \Theta_{34}$                                            | $0.1620 V_4 V_4 \cos \Theta_{34}$                                                                         |
|             | $+ 0.1620 V_4 V_4 \sin \Theta_{34}$                                            | $0.0245 V_5 V_5 \sin \Theta_{35} - 0.0583 V_5 V_5 \cos \Theta_{35}$                                     |
|             | $- 0.0245 V_5 V_5 \cos \Theta_{35}$                                            | $+ 0.0583 V_5 V_5 \sin \Theta_{35}$                                                                       |
| Pamekasan   | $-0.0683 V_3 V_4 \cos \Theta_{43}$                                             | $-0.0683 V_3 V_4 \sin \Theta_{43} - 0.1620 V_3 V_4 \cos \Theta_{43}$                                     |
|             | $+ 0.1620 V_3 V_4 \sin \Theta_{43}$                                            | $+ 0.2542 V_4^2 - 0.0388 V_5 V_5 \sin \Theta_{45}$                                                      |
|             | $+ 0.1071 V_4^2$                                                              | $- 0.0922 V_4 V_4 \cos \Theta_{45}$                                                                       |
|             | $- 0.0388 V_5 V_5 \cos \Theta_{45}$                                            | $0.0922 V_4 V_4 \sin \Theta_{45}$                                                                          |
|             | $+ 0.0922 V_5 V_5 \sin \Theta_{45}$                                            | $+ 0.0634 V_4^2 - 0.0388 V_5 V_5 \cos \Theta_{45}$                                                      |
| Sumenep     | $-0.0245 V_3 V_5 \cos \Theta_{53}$                                             | $-0.0245 V_3 V_5 \sin \Theta_{53} + 0.0583 V_3 V_5 \sin \Theta_{53}$                                     |
|             | $+ 0.0583 V_3 V_5 \sin \Theta_{53}$                                            | $+ 0.0603 V_5^2 - 0.0388 V_4 V_4 \cos \Theta_{54}$                                                      |
|             | $+ 0.0634 V_5^2$                                                              | $- 0.0922 V_4 V_4 \sin \Theta_{54}$                                                                       |
|             | $- 0.0388 V_4 V_4 \cos \Theta_{54}$                                            | $+ 0.0922 V_4 V_4 \sin \Theta_{54}$                                                                          |
|             | $+ 0.0922 V_4 V_4 \sin \Theta_{54}$                                            | $0.0922 V_4 V_4 \sin \Theta_{54}$                                                                          |

Table 7. Submatrix of Bus Gilitimur

| Operator | Expression                                             |
|----------|--------------------------------------------------------|
| $J_{P\Theta}$ | $\frac{\partial P}{\partial \Theta}$ = 0.1119 $V_1 V_2 \sin \Theta_{12}$ + 0.2664 $V_1 V_2 \cos \Theta_{12}$ |
| $J_{PV}$ | $\frac{\partial P}{\partial V}$ = 0.5250 $V_1 - 0.1119 V_2 \cos \Theta_{12}$ + 0.2664 $V_2 \sin \Theta_{12}$ |
| $J_{Q\Theta}$ | $\frac{\partial Q}{\partial \Theta}$ = $-0.1119 V_1 V_2 \cos \Theta_{12}$ + 0.2664 $V_1 \sin \Theta_{12}$ |
| $J_{QV}$ | $\frac{\partial Q}{\partial V}$ = 1.3524 $V_1 - 0.119 V_2 \sin \Theta_{12}$ - 0.2664 $V_2 \cos \Theta_{12}$ |

Table 8. Submatrix of Bus Bangkalan

| Operator | Expression                                             |
|----------|--------------------------------------------------------|
| $J_{P\Theta}$ | $\frac{\partial P}{\partial \Theta}$ = 0.1119 $V_1 V_2 \sin \Theta_{21}$ + 0.2664 $V_1 V_2 \cos \Theta_{21}$ + 0.0681 $V_3 V_2 \sin \Theta_{23}$ |
| $J_{PV}$ | $\frac{\partial P}{\partial V}$ = $0.3446 V_2 - 0.1119 V_1 \cos \Theta_{21}$ + 0.2664 $V_1 \sin \Theta_{21}$ - 0.0681 $V_3 \cos \Theta_{23}$ + 0.1618 $V_3 \sin \Theta_{23}$ |
| $J_{Q\Theta}$ | $\frac{\partial Q}{\partial \Theta}$ = $-0.1119 V_1 V_2 \cos \Theta_{21}$ + 0.2664 $V_1 V_2 \sin \Theta_{21}$ - 0.0681 $V_3 V_2 \cos \Theta_{23}$ + 0.1618 $V_3 V_2 \sin \Theta_{23}$ |
| $J_{QV}$ | $\frac{\partial Q}{\partial V}$ = $0.8616 V_2 - 0.1119 V_1 \sin \Theta_{21}$ - 0.2664 $V_1 \cos \Theta_{21}$ - 0.0681 $V_3 \sin \Theta_{23}$ - 0.1618 $V_3 \cos \Theta_{23}$ |
Table 9. Submatrix of Bus Sampang

| Active Power (pu) |
|-------------------|
| \( J_{P\theta} = \frac{\partial P}{\partial \theta} \) 0.0681 \( V_3 V_2 \sin \theta_{32} \) + 0.1618 \( V_3 V_2 \cos \theta_{32} \) + 0.0683 \( V_4 \) \( V_3 \sin \theta_{34} \) + 0.1620 \( V_4 \) \( V_3 \cos \theta_{34} \) + 0.0245 \( V_5 \) \( V_3 \sin \theta_{35} \) + 0.0583 \( V_5 \) \( V_3 \cos \theta_{35} \) |
| \( J_{P\nu} = \frac{\partial P}{\partial \nu} \) 0.322 \( V_3 \) - 0.0681 \( V_2 \) \( \cos \theta_{32} \) + 0.1618 \( V_4 \) \( V_3 \sin \theta_{34} \) - 0.0683 \( V_4 \) \( \cos \theta_{34} \) + 0.162 \( V_4 \) \( V_3 \sin \theta_{34} \) - 0.0245 \( V_5 \) \( V_3 \cos \theta_{35} \) + 0.0583 \( V_5 \) \( V_3 \sin \theta_{35} \) |
| \( J_{Q\theta} = \frac{\partial Q}{\partial \theta} \) 0.7642 \( V_3 \) - 0.0681 \( V_2 \) \( \sin \theta_{32} \) - 0.1618 \( V_2 \) \( \cos \theta_{32} \) - 0.0683 \( V_4 \) \( \sin \theta_{34} \) + 0.162 \( V_4 \) \( V_3 \sin \theta_{34} \) - 0.0245 \( V_5 \) \( V_3 \cos \theta_{35} \) - 0.0583 \( V_5 \) \( V_3 \sin \theta_{35} \) |
| \( J_{Q\nu} = \frac{\partial Q}{\partial \nu} \) 0.0683 \( V_4 \) \( V_3 \sin \theta_{43} \) + 0.1620 \( V_4 \) \( V_3 \cos \theta_{43} \) + 0.0388 \( V_5 \) \( V_4 \sin \theta_{45} \) |

Table 10. Submatrix of Bus Pamekasen

| Active Power (pu) |
|-------------------|
| \( J_{P\theta} = \frac{\partial P}{\partial \theta} \) 0.0683 \( V_4 \) \( V_3 \sin \theta_{43} \) + 0.1620 \( V_4 \) \( V_3 \cos \theta_{43} \) + 0.0388 \( V_5 \) \( V_4 \sin \theta_{45} \) + 0.0922 \( V_5 \) \( V_4 \cos \theta_{45} \) |
| \( J_{P\nu} = \frac{\partial P}{\partial \nu} \) 0.2142 \( V_4 \) - 0.0683 \( V_3 \) \( \cos \theta_{43} \) + 0.1620 \( V_3 \) \( \sin \theta_{43} \) - 0.0388 \( V_5 \) \( \cos \theta_{45} \) |
| \( J_{Q\theta} = \frac{\partial Q}{\partial \theta} \) -0.0683 \( V_4 \) \( V_3 \cos \theta_{43} \) + 0.162 \( V_4 \) \( V_3 \sin \theta_{43} \) - 0.0388 \( V_5 \) \( V_4 \cos \theta_{45} \) + 0.0922 \( V_5 \) \( V_4 \sin \theta_{45} \) |
| \( J_{Q\nu} = \frac{\partial Q}{\partial \nu} \) 0.5084 \( V_4 \) - 0.0683 \( V_3 \) \( \sin \theta_{43} \) - 0.162 \( V_3 \) \( \cos \theta_{43} \) - 0.0388 \( V_5 \) \( \sin \theta_{45} \) - 0.0922 \( V_5 \) \( \cos \theta_{45} \) |

Table 11. Submatrix of Bus Sumenep

| Active Power (pu) |
|-------------------|
| \( J_{P\theta} = \frac{\partial P}{\partial \theta} \) 0.0245 \( V_5 \) \( V_3 \sin \theta_{53} \) + 0.0583 \( V_5 \) \( V_3 \cos \theta_{53} \) + 0.0388 \( V_5 \) \( V_4 \sin \theta_{54} \) + 0.0922 \( V_5 \) \( V_4 \cos \theta_{54} \) |
| \( J_{P\nu} = \frac{\partial P}{\partial \nu} \) 0.1268 \( V_5 \) - 0.0245 \( V_4 \) \( \cos \theta_{53} \) + 0.0583 \( V_5 \) \( \sin \theta_{53} \) - 0.0388 \( V_4 \) \( \cos \theta_{54} \) + 0.0922 \( V_5 \) \( V_4 \sin \theta_{54} \) |
| \( J_{Q\theta} = \frac{\partial Q}{\partial \theta} \) -0.245 \( V_5 \) \( V_3 \cos \theta_{53} \) + 0.0583 \( V_5 \) \( V_3 \sin \theta_{53} \) - 0.0388 \( V_5 \) \( V_4 \cos \theta_{54} \) + 0.0922 \( V_5 \) \( V_4 \sin \theta_{54} \) |
| \( J_{Q\nu} = \frac{\partial Q}{\partial \nu} \) 0.301 \( V_5 \) - 0.245 \( V_3 \) \( \sin \theta_{53} \) - 0.0583 \( V_3 \) \( \cos \theta_{53} \) - 0.0388 \( V_4 \) \( \sin \theta_{54} \) - 0.0922 \( V_4 \) \( \cos \theta_{54} \) |

3.3. Jacobian Values

After obtaining all the variables to determine the Jacobian value which represents a linear relationship, then the Jacobian value can be calculated on each bus with the formula refers to Eq.14, as summarized in Table 12.

Table 12. Jacobian value each bus

| Bus Name | Jacobian Reduction \( (J_R) \) | Explanation |
|----------|-------------------------------|-------------|
| Giritimur | 1.225                         | Stable      |
| Bangkalan | 0.502                         | Stable      |
| Sampang  | 0.448                         | Stable      |
| Pamekasen | 0.287                         | Stable      |
| Sumenep  | -0.015                        | Unstable    |
Table 12 shows the results of Jacobian value of each bus derived from the preceding steps. It can be noted in the table, the value of \( J_r \) of Gilitimur is highest among other buses. This shows that Gilitimur is considerably more sensitive to the change of voltage since the jacobian value indicates the slope between the voltage to reactive power injection. The smaller positive \( J_r \) value indicate the more stable operation, thus Pamekasan is considered the most stable because its value is closest to zero, and therefore less sensitive to the variation of Q. On the contrary, Sumenep suffers from the negative value of \( J_r \), meaning that the system is unstable, according to the Q-V voltage stability criterion [2].

Based on the Q-V analysis, it confirms the unstable operation of voltage on Sumenep bus, according the event of voltage drop (\( \geq -10\% \)) mention in the report as discussed previously in the introduction. As far as the evidence is concerned, it is possible that the sudden drop in Sumenep bus is attributed to the bus inability to meet the reactive power demand. The authors suggest this analysis to state electric company (PT PLN) to further explore the account of the event and avoid ones in the future operation.

4. Conclusion
In this paper, Q-V Sensitivity analysis methods are implemented on Madura 150 kV transmission system. According to observation that on Madura transmission system get the drop voltage, drop voltage occurs on the Sumenep bus. On the analysis of voltage stability by using Q-V Sensitivity analysis, it can be known that the Sumenep bus is getting negative value of jacobian reduction (-0.015) which is regarded as unstable operation of voltage. The other four buses get positive results value that is Gilitimur bus = 1.225, Bangkalan bus = 0.502, Sampang bus = 0.448 and Pamekasan bus = 0.287, thus are stable.

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