Probing new physics scenarios in accelerator and reactor neutrino experiments

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We perform a detailed combined fit to the $\nu_e \to \nu_e$ disappearance data of the Daya Bay experiment and the appearance $\nu_\mu \to \nu_e$ and disappearance $\nu_\mu \to \nu_\mu$ data of the T2K one in the presence of two models of new physics affecting neutrino oscillations, namely a model where sterile neutrinos can propagate in a large compactified extra dimension and a model where non-standard interactions (NSI) affect the neutrino production and detection. We find that the Daya Bay $\oplus$ T2K data combination constrains the largest radius of the compactified extra dimensions to be $R \lesssim 0.17 \mu m$ at 2$\sigma$ C.L. (for the inverted ordering of the neutrino mass spectrum) and the relevant NSI parameters in the range $O(10^{-3}) - O(10^{-2})$, for particular choices of the CP violating phases.

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INTRODUCTION

After the recent discovery of the reactor angle $\theta_{13}$ and its good measurement in the Daya Bay [1] and RENO [2] reactor experiments, the experimental efforts in the neutrino sector is now devoted to establish the presence of CP violation in the lepton sector, the neutrino mass ordering and the absolute neutrino mass scale. In fact, the relatively large value of $\theta_{13} \sim 9^o$ opens the possibility to search for possible non-vanishing CP violating phase $\delta$ in the Long Baseline (LBL) neutrino experiments T2K [3] and NO$\nu$A [4] and in the future LBL experiments like Hyper-Kamiokande [5] and LBNE [6]. The recent observation of 28 electron neutrino events in T2K [3] confirmed the $\nu_\mu \to \nu_e$ transition at more than 7$\sigma$ and provided a first (although weak) indication for the value of $\delta$. Indeed a preliminary combined joint analysis [7] of the appearance and disappearance channels in T2K, which also includes the reactor constraints on $\theta_{13}$, disfavors $\delta/\pi \in [0.1,0.8]$ at more than 90% C.L., with a best fit point around $\delta = 3\pi/2$. This shows the large increase of sensitivity in the determination of $\delta$ when performing a combined analysis of reactor and super-beam data [8-10]. The strength of such a procedure can also be used to test the presence of physics beyond the Standard Model (SM) in the neutrino sector (affecting neutrino oscillation probabilities) and, to some extent, to analyze its impact on the determination of the standard oscillation parameters. In this paper we consider two possible of such scenarios: the so called non-standard neutrino interactions (NSI), where the neutrino interactions with ordinary matter are parametrized at low energy in terms of effective flavour-dependent couplings $\varepsilon_{\alpha\beta}$ [11], and the large extra dimensions (LED) model, where sterile neutrinos can propagate in a larger than three dimensional space whereas the SM left-handed neutrinos are confined to a four dimensional (4-D) space-time brane [12-14].

The aim of this paper is to take full advantage of whole T2K ($\nu_\mu \to \nu_e$ appearance [3] and $\nu_\mu \to \nu_\mu$ disappearance [15]) and $\bar\nu_e \to \bar\nu_e$ Daya Bay [16] data in order to:

• study in details how the correlation between the standard oscillation parameters are modified by the presence of new physics [17,18];
• constrain the parameter space of the LED and NSI models.

In what follows, we will first recall the main features of the NSI and LED models. Then, after a brief description of the statistical technique employed in analyzing the experimental data, we will discuss how
the presence of new physics modifies the correlation among the standard oscillation parameters. Eventually, we will give the constraints on the model parameters obtained from the joint analysis of the Daya Bay and T2K data.

**BASIC FORMALISM**

The NSI model

The NSI approach describes a large class of new physics models where the neutrino interactions with ordinary matter are parametrized at low energy in terms of effective flavour-dependent couplings $\varepsilon_{\alpha\beta}$, of $\mathcal{O}(M_{EW}/M_{NP})$, where $M_{EW}$ is the electroweak scale and $M_{NP}$ is the scale where new physics sets in. The presence of these new couplings can affect the neutrino production and detection [11, 19]. Many authors have studied the impact of NSI on conventional neutrino beams [20–25], and on taking data (or ended) reactor experiments [26–28], finding that the sensitivity reach of such experiments on NSI parameters can be of the order of $\mathcal{O}(10^{-2})$.

In what follows, we consider the analytic treatment of the NSI as described in [20]. The starting point is to assume that the neutrino states at the source (s) and at the detector (d) are a superposition of the orthonormal flavor eigenstates $|\nu_e\rangle$, $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$ [29–31]:

$$|\nu_s^\alpha\rangle = |\nu_\alpha\rangle + \sum_{\beta=e,\mu,\tau} \varepsilon_{\alpha\beta}^s |\nu_\beta\rangle \quad (1)$$

$$|\nu_d^\beta\rangle = |\nu_\beta\rangle + \sum_{\alpha=e,\mu,\tau} \varepsilon_{\alpha\beta}^d |\nu_\alpha\rangle \quad (2)$$

The oscillation probability can be obtained by squaring the amplitude $\langle \nu_d^\beta|e^{-iH_L}|\nu_s^\alpha\rangle$:

$$P_{\nu_s^\alpha \rightarrow \nu_d^\beta} = |\langle \nu_d^\beta|e^{-iH_L}|\nu_s^\alpha\rangle|^2 = |(1 + \varepsilon^d)_{\gamma\beta} (e^{-iH_L})_{\gamma\delta}(1 + \varepsilon^s)_{\alpha\delta}|^2 \quad (3)$$

Since the parameters $\varepsilon_{e\alpha}^s$ and $\varepsilon_{e\alpha}^d$ receive contributions from the same higher dimensional operators [20], one can constrain them by the relation:

$$\varepsilon_{e\alpha}^s = \varepsilon_{e\alpha}^d \equiv \varepsilon_{e\alpha} e^{i\phi_{e\alpha}} \quad (4)$$

whereas for the related CP violating phases $\phi_{e\alpha}$ no constraints have been obtained so far. These bounds can be improved by future reactor neutrino experiments [29] and, in particular, at neutrino factories [33], where the non-diagonal couplings are expected to be constrained at the level of $\mathcal{O}(10^{-3})$.

The Hamiltonian of the system is given by:

$$H_{\alpha\beta} = \frac{1}{2E_\nu} \left[ U_{\alpha j} \begin{pmatrix} 0 \\ \Delta m_{21}^2 \\ \Delta m_{31}^2 \\ \end{pmatrix} (U^\dagger)_{k\beta} \right] \quad (6)$$

where $U$ is the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) matrix [34], for which we assume the standard parameterization [35]. For the analysis of the Daya Bay data, the $P(\nu_e \rightarrow \nu_e)$ can be obtained
from Eq. (3) and Eq. (4), expanding for small $\varepsilon$ and neglecting terms of order $O(\Delta m^2_{31} L/(4E_\nu))$ and $O(\varepsilon^2)$:

\[ P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left[ \frac{\Delta m^2_{31} L}{4E_\nu} \right] + 4\varepsilon_{ee} \cos \phi_{ee} \left[ 1 - \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \right] \sin^2 2\theta_{13} \]
\[ - 4\varepsilon_{e\mu} \sin 2\theta_{13} \sin \theta_{23} \cos \theta_{23} \cos(\delta - \phi_{e\mu}) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 4\varepsilon_{e\tau} \sin 2\theta_{13} \sin \theta_{23} \cos \theta_{23} \cos(\delta - \phi_{e\tau}) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ + O(\varepsilon^2) + O(\Delta m^2_{31} L/(4E_\nu)) . \]

On the other hand, $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_\mu \rightarrow \nu_\mu)$ (relevant for the analysis of the T2K data), cannot be evaluated neglecting terms of $O(\Delta m^2_{31} L/(4E_\nu))$, otherwise the correct dependence on the standard CP phase $\delta$ would be lost. Thus, $P(\nu_\mu \rightarrow \nu_e)$ can be written as:

\[ P(\nu_\mu \rightarrow \nu_e) \sim \sin^2 2\theta_{23} \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 2\sin 2\theta_{12} \sin 2\theta_{23} \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \sin \theta_{13} \cos^2 \theta_{13} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \sin \delta + P_0 + P_1 , \]

where $P_0$ and $P_1$ are the zero and the first order contributions of the NSI expanded for small $\Delta m^2_{31} L/(4E_\nu)$, respectively. Using the constraints in Eq. (4) and defining $\varepsilon^{s,d}_{\alpha\beta} = |\varepsilon^{s,d}_{\alpha\beta}| \exp(i\phi^{s,d}_{\alpha\beta})$, one finds:

\[ P_0 = - 4|\varepsilon^{s}_{\mu e}| \sin \theta_{13} \sin \theta_{23} \cos(\delta + \phi^{s}_{\mu e}) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 4|\varepsilon^{s}_{e\mu}| \sin \theta_{13} \sin \theta_{23} \sin(\delta + \phi^{s}_{e\mu}) \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \cos \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 4|\varepsilon^{s}_{e\mu}| \sin \theta_{13} \sin \theta_{23} \cos(\delta - \phi^{s}_{e\mu}) \cos \theta_{23} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 4|\varepsilon^{s}_{e\mu}| \sin \theta_{13} \sin \theta_{23} \sin(\delta - \phi^{s}_{e\mu}) \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \cos \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ + 8|\varepsilon^{s}_{e\tau}| \sin \theta_{13} \sin^2 \theta_{23} \cos \theta_{23} \cos(\delta - \phi^{s}_{e\tau}) \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) + O(\varepsilon^{s}_{ee} \sin^2 \theta_{13}) \]
\[ + O(\varepsilon^{s}_{e\mu} \sin^2 \theta_{13}) + O(\varepsilon^{s}_{e\mu} \sin^2 \theta_{13}) + O(\varepsilon^{s}_{e\mu} \sin^3 \theta_{13}) + O(\varepsilon^{s}_{e\mu} \cos^2 \theta_{13}) + O(\varepsilon^2) , \]

and

\[ P_1 = - |\varepsilon^{s}_{\mu e}| \sin 2\theta_{12} \sin \theta_{23} \sin \phi^{s}_{\mu e} \frac{\Delta m^2_{31} L}{2E_\nu} \]
\[ + 2|\varepsilon^{s}_{e\mu}| \sin 2\theta_{12} \sin^2 \theta_{23} \cos \theta_{23} \cos \phi^{s}_{e\mu} \sin \left( \frac{\Delta m^2_{31} L}{2E_\nu} \right) \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ + |\varepsilon^{s}_{e\tau}| \sin 2\theta_{12} \cos \theta_{23} \sin \phi^{s}_{e\tau} \frac{\Delta m^2_{31} L}{2E_\nu} \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ + 2|\varepsilon^{s}_{e\tau}| \sin 2\theta_{12} \sin \theta_{23} \cos^2 \theta_{23} \cos \phi^{s}_{e\tau} \sin \left( \frac{\Delta m^2_{31} L}{2E_\nu} \right) \sin \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ - 2|\varepsilon^{s}_{e\tau}| \sin 2\theta_{12} \sin \theta_{23} \cos^2 \theta_{23} \sin \phi^{s}_{e\tau} \frac{\Delta m^2_{31} L}{2E_\nu} \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) \]
\[ + O \left( \varepsilon \sin \theta_{13} \frac{\Delta m^2_{31} L}{4E_\nu} \right) + O(\varepsilon^2) . \]

Note that Eqs. (7)-(10) coincide with those derived in [17] for $\delta = 0$. Within the same approximations $P(\nu_\mu \rightarrow \nu_\mu)$ reads:
at the same time, to solve the hierarchy problem \[37\]. In order to avoid the strong constraints on scenarios been introduced to explain the weakness of gravity, since it propagates in the higher dimensional space, and, whereas the SM left-handed neutrinos are confined to a 4-D space-time brane \[12–14\]. This framework has In LED models, sterile neutrinos can propagate, as well as gravity, in a larger than three dimensional space

\[\text{After the EWSB the neutrino mass matrix can be extracted from the following Lagrangian} \[14\]:\]

\[
P(\nu_\mu \rightarrow \nu_\mu) = P^{SM}(\nu_\mu \rightarrow \nu_\mu) + 2|\varepsilon|_{\mu\mu} \cos \phi_{\mu\mu} + 2|\varepsilon|_{\mu\mu} \cos \phi_{\mu\mu}^d \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \\
- 2 \left|\varepsilon_{\mu\tau}\right| \cos \phi_{\mu\tau}^s + 2 \left|\varepsilon_{\tau\mu}\right| \cos \phi_{\tau\mu}^d \cos 2\theta_{23} \sin 2\theta_{23} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \\
+ \left(\gamma_{\mu\tau}/\sin \phi_{\mu\tau}^s + \gamma_{\tau\mu}/\sin \phi_{\tau\mu}^d\right) \sin 2\theta_{23} \sin \left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) + O\left(\frac{\Delta m_{23}^2}{\Delta m_{31}^2}\right) + O(\sin \theta_{13}\varepsilon) + O(\varepsilon^2),
\]

where the approximate formula for \(P^{SM}(\nu_\mu \rightarrow \nu_\mu)\) can be found in \[36\]:

\[
P^{SM}(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{23} - \sin^2 \theta_{23} \sin^2 2\theta_{13} \cos 2\theta_{23} \sin^2 \left(\frac{\Delta m_{23}^2 L}{4E_\nu}\right) \\
- \left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right)^2 \sin^2 \theta_{12} \sin^2 2\theta_{23} + J \sin^2 \theta_{23} \cos \delta \sin \left(\frac{\Delta m_{23}^2 L}{2E_\nu}\right) \\
- \left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right)^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} + \sin^2 \theta_{12} \sin^2 2\theta_{23} \sin \left(\frac{\Delta m_{23}^2 L}{4E_\nu}\right),
\]

with \(J = \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}\). We use Eq. \[11\] to compute the theoretical predictions for the number of events at both near and far detectors in the disappearance channel at the T2K experiment.

**The LED model**

In LED models, sterile neutrinos can propagate, as well as gravity, in a larger than three dimensional space whereas the SM left-handed neutrinos are confined to a 4-D space-time brane \[12\]. This framework has been introduced to explain the weakness of gravity, since it propagates in the higher dimensional space, and, at the same time, to solve the hierarchy problem \[37\]. In order to avoid the strong constraints on scenarios with one extra dimension only \[35\], we assume to work in a \(\delta + 4\) dimensional space in which one of the \(\delta\) extra dimensions is much larger than the others, so we can use an effective five dimensional formalism to compute the neutrino oscillation probabilities. The effects of, for instance, a sixth dimension much smaller than the fifth one can be safely neglected in our study: in fact, the mass separation among the KK states is proportional to \(1/R\) so, in the case of \(R_{\delta=2} \ll R_{\delta=1}\), the "new" KK excitations would be at a much higher scale and then energetically less accessible by the oscillation phenomenon. The strongest bounds on the radius \(R\) of the largest extra dimension at 95% C.L. \[38\] is:

\[R < 44 \text{ mm},\]

reached in experiments based on the torsion pendulum instrument testing deviations from the Newtonian theory of gravity.

In this paper we are interested in scenarios in which 3 bulk sterile neutrinos give a Dirac mass term for the 3 active ones. This model is often indicated as the (3, 3) LED model. The action of 5-dimensional massless bulk neutrinos \(\Psi_i(x, y)\), interacting with the standard left-handed neutrinos \(\nu_\alpha\), is as follows:

\[
S = \int d^4x \, dy \, i \, \bar{\Psi} \Gamma_A \dot{\Psi}^A + \int d^4x \, \left( i \, \bar{\nu}^\alpha_L \gamma_0 \dot{\nu}^\alpha_L + \lambda_{\alpha\beta} H \, \bar{\nu}^\alpha_L \nu^\beta_R(x, 0) + \text{h.c.} \right),
\]

where \(\Gamma_A\) are the Dirac matrices in five dimensions, \(\lambda_{\alpha\beta}\) the Yukawa couplings and \(H\) the Higgs doublet. After the EWSB the neutrino mass matrix can be extracted from the following Lagrangian \[14\]:

\[
\mathcal{L}_\text{eff} = \sum_{\alpha, \beta} n_{\alpha\beta}^D \left[ \bar{\nu}^{(0)}_R \nu^\beta_L + \sqrt{2} \sum_{n=1}^\infty \bar{\nu}^{(n)}_R \nu^\beta_L \right] + \sum_{\alpha} \sum_{n=1}^\infty \frac{n}{R} \bar{\nu}^{(n)}_R \nu^\alpha_L + \text{h.c.},
\]
where $\alpha, \beta = e, \mu, \tau$, $m^D_{\alpha\beta}$ is a Dirac mass matrix and $\nu_R^{(0)}$, $\nu_R^{(n)}$, and $\nu_L^{(n)}$ are linear combinations of the bulk fermions. The diagonalization of the mass matrix allows to find the neutrino mass eigenstates and then to compute the oscillation probability in vacuum \[14\]:

$$ P(\nu_\alpha \rightarrow \nu_\beta) = |A_{\alpha\beta}(L)|^2, $$

$$ A_{\alpha\beta}(L) = \sum_{i=1}^{3} \sum_{n=0}^{\infty} U^{\alpha i}_{i1} U^{\beta i}_{i1} |U^{(n)}_{i1}|^2 \exp \left( \frac{i \lambda_{i1}^2 L}{2 E_{\nu} R^2} \right), \tag{16} $$

where $U^{\alpha i}$ is the matrix element of the PMNS matrix $U$ and $U^{(n)}_{i1}$ the matrix element that connects the zero mode, i.e. the usual standard neutrinos, with the $n$-th KK mode [31]. The value of $U^{(n)}_{i1}$ can be computed by solving the eigenvalues problem for the mass matrix defined in Eq. (15). The dimensionless parameters $\lambda_{i1}^2$ are defined as $\lambda_{i1}^2 \equiv m_{i1}^2 R$ and can be calculated in a perturbative scheme, as briefly reported in Appendix A. In the case of reactor experiments, the above-mentioned procedure allows to calculate the LED contribution to the amplitude $A_{\nu e}^{(LED)}$. Introducing the expansion parameter $\xi_i \equiv \sqrt{2}m_i R$, this reads [39]:

$$ A_{\nu e}^{(LED)} \sim \xi_i^2 |U_{e1}|^2 + \xi_j^2 |U_{e2}|^2 + \xi_k^2 |U_{e3}|^2 \sim \xi_i^2 \cos^2 \theta_{12} \sin^2 \theta_{13} + \xi_j^2 \cos^2 \theta_{12} \sin^2 \theta_{13} + \xi_k^2 \sin^2 \theta_{12} + \xi_l^2 \sin^2 \theta_{13}. \tag{17} $$

In the normal ordering (NO) case ($m_1 > m_2 > m_3 = m_0$), $A_{\nu e}^{(LED)}$ is dominated by the last term and thus suppressed by the small reactor angle. For the inverted ordering (IO) case ($m_2 > m_1 > m_3 = m_0$) the first two terms dominate the amplitude and no suppressing factor is at work. We then expect the IO scenario to give better constraints on $R$ and $m_0$ than the NO case.

The situation is quite different for the $\nu_\mu \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_e$ probabilities. Indeed for the disappearance channel we have:

$$ A_{\nu_\mu e}^{(LED)} \sim \xi_i^2 |U_{e1}|^2 + \xi_j^2 |U_{e2}|^2 + \xi_k^2 |U_{e3}|^2 \sim \xi_i^2 \cos^2 \theta_{23} \sin^2 \theta_{12} + \xi_j^2 \cos^2 \theta_{12} \sin^2 \theta_{23} + \xi_k^2 \cos^2 \theta_{13} \sin^2 \theta_{23} + 2(\xi_i^2 - \xi_j^2) \cos \theta_{12} \cos \theta_{23} \sin \theta_{13} \sin \theta_{23} \cos \delta + O(\sin^2 \theta_{13}), \tag{18} $$

and, due to the absence of the $\sin \theta_{13}$ suppression in the $\xi_1^2$ term, we do not expect significant difference in sensitivity between NO and IO. This channel is also expected to give better constraints than the $\nu_\mu \rightarrow \nu_e$ appearance one. Indeed in the latter case the amplitude reads:

$$ A_{\nu_\mu e}^{(LED)} \sim \xi_i^2 U_{e1} U_{\mu 1} + \xi_j^2 U_{e2} U_{\mu 2} + \xi_k^2 U_{e3} U_{\mu 3} \sim (\xi_i^2 - \xi_j^2) \cos \theta_{12} \cos \theta_{23} \cos \theta_{13} \sin \theta_{23} + \xi_k^2 \sin \theta_{12} \cos \theta_{13} \sin \theta_{23} e^{-i\delta} + \xi_{13}^2 \sin \theta_{12} \cos \theta_{13} \sin \theta_{23} e^{-i\delta} \tag{19} $$

and every term is suppressed by either $\Delta m_{21}^2$ or $\sin \theta_{13}$.

**NEUTRINO FACILITIES AND DETAILS OF THE STATISTICAL ANALYSIS**

The Daya Bay experimental setup that we take into account consists of six reactors \[1\], emitting antineutrinos $\bar{\nu}_e$ whose spectra have been recently estimated in Refs. [48] [41]. The total flux of arriving $\bar{\nu}_e$ at the six antineutrino detectors has been estimated using the convenient parametrization discussed in Ref. [40] and taking into account all the distances between the detectors and the reactors (summarised in Tab. 2 of Ref. [1]). For this analysis we use the data set accumulated during 217 days extracted from Fig. 2 of Ref. [10]. The antineutrino energy $E$ is reconstructed by the prompt energy deposited by the positron $E_{\text{prompt}}$ using the approximated relation \[1\] $E \simeq E_{\text{prompt}} + 0.8$ MeV. The energy resolution function is a Gaussian function, parametrized according to:

$$ \sigma(E)[\text{MeV}] = \begin{cases} \gamma \sqrt{E/\text{MeV} - 0.8}, & \text{for } E > 1.8 \text{ MeV}, \\ \gamma, & \text{for } E \leq 1.8 \text{ MeV}, \end{cases} \tag{20} $$
with \( \gamma = 0.08 \text{ MeV} \). The antineutrino cross section for the inverse beta decay (IBD) process has been taken from [12]. The statistical analysis of the data has been performed using a modified version of the GLoBES software [13] with the \( \chi^2 \) function defined as follows [11]:

\[
\chi^2_{DBJ}(\theta, \Delta m^2, \vec{S}, \alpha_r, \epsilon, \varepsilon_d, \eta_d) = \sum_{d=1}^{36} \sum_{i=1}^{36} \frac{[M_i^d - T_i^d \cdot (1 + \epsilon + \sum_r \omega_r^d \sigma_r + \varepsilon_d) + \eta_d]^2}{M_i^d + B_i^d} + \frac{\varepsilon^2}{\sigma^2} + \sum_r \frac{\alpha_r^2}{\sigma_r^2} + \sum_{d=1}^{6} \frac{\varepsilon_d^2}{\sigma_d^2 + \sigma_{B_d}^2} \quad \text{Priors},
\]

where \( \vec{S} \) is a vector containing the new physics parameters, \( M_i^d \) are the measured inverse beta decay (IBD) events of the \( d \)-th detector ADs in the \( i \)-th bin, \( B_i^d \) the corresponding background and \( T_i^d = T_i(\theta, \Delta m^2, \vec{S}) \) are the theoretical predictions for the rates. The parameter \( \omega_r^d \) is the fraction of IBD contribution of the \( r \)-th reactor to the \( d \)-th detector AD, determined by the approximate relation \( \omega_r^d \sim L_{rd}^{-2}/(\sum_r 1/L_{rd}^2) \), where \( L_{rd} \) is the distance between the \( d \)-th detector and the \( r \)-th reactor. The parameter \( \sigma_\epsilon \) is the reactor flux uncertainty (\( \sigma_\epsilon \approx 3\% \)), \( \sigma_\varepsilon \) is the uncorrelated detection uncertainty (\( \sigma_\varepsilon = 0.2\% \)) and \( \sigma_{B_d} \) is the background uncertainty of the \( d \)-th detector obtained using the information given in [10]: \( \sigma_{B_1} = \sigma_{B_2} = 8.21 \), \( \sigma_{B_3} = 5.95 \), \( \sigma_{B_4} = \sigma_{B_5} = \sigma_{B_6} = 1.15 \). Eventually, \( \sigma_\epsilon \approx 0.8\% \) are the uncorrelated reactor uncertainties. The corresponding pull parameters are \( \epsilon, \varepsilon_d, \eta_d \) and \( \alpha_r \). With this choice of nuisance parameters we are able to reproduce the \( 1\sigma, 2\sigma \) and \( 3\sigma \) confidence level results presented in Fig. 3 of Ref. [10] with high accuracy. The differences are at the level of few percent (see Tab. I and Tab. II of Ref. [13]).

The T2K experiment [3] consists of two separate detectors, both of which are 2.5 degrees off axis of the neutrino beam. The far detector is located at \( L_F = 295 \text{ km} \) from the source, the ND280 near detector is \( L_N = 280 \text{ meters} \) from the target. In our analysis we used the public data in [3, 15], which reported 28 events in the appearance channel and 120 events in the disappearance channel (constrained with the 17369 CC\( \pi0 \) events at the ND280 near detector). The neutrino flux has been estimated from [44]. We fixed the fiducial mass of the near and the far detector as \( F_M_{ND280} = 1529 \text{ Kg} \) and \( F_M_{SK} = 22.5 \text{ Kton} \) [45], respectively; a bin to bin normalization has been fixed in order to reproduce the T2K best fit events. For the energy resolution function we adopt a Gaussian function as in Eq. (20), with \( \gamma = 0.085 \text{ GeV} \).

The \( \chi^2_{T2K} \) is defined as:

\[
\chi^2_{T2K}(\theta, \Delta m^2, \vec{S}, \rho_d, \Omega_d, \alpha_d, \alpha_N) = \sum_{d=1}^{2} \sum_{i=1}^{n_{bins}} \left[ M_i^d - T_i^d \cdot (1 + \rho_d + \Omega_d) + M_i^d \log \frac{M_i^d}{T_i^d \cdot (1 + \rho_d + \Omega_d)} \right] + \sum_{i=1}^{n_{bins}} \left[ M_i^{N} - T_i^{N} \cdot (1 + \rho_d + \Omega_d) + M_i^{N} \log \frac{M_i^{N}}{T_i^{N} \cdot (1 + \rho_d + \Omega_d)} \right] + \sum_{d=1}^{2} \left( \frac{\rho_d^2}{\sigma_{\rho_d}^2} + \frac{\Omega_d^2}{\sigma_{\Omega_d}^2} + \frac{\sigma_{\rho_d}^2}{\sigma_{\Omega_d}^2} \right) + \text{Priors}.
\]

In the previous formula, \( \vec{S} \) is a vector containing the new physics parameters, \( M_i^d \) are the measured events, including the backgrounds, of the \( d \)-th channel of the far detector in the \( i \)-th bin, \( T_i^d = T_i^d(\theta, \Delta m^2, \vec{S}, \alpha_d) \) are the theoretical predictions for the rates, \( \theta \) and \( \Delta m^2 \) are respectively the mixing angles and the squared mass differences contained in the oscillation probability, \( n_{bins} \) is the number of bins for the \( d \)-th channel of the far detector. With obvious notation, \( M_i^{N} \) and \( T_i^{N} = T_i^{N}(\theta, \Delta m^2, \vec{S}, \alpha_N) \) are the measured and theoretical event rates at the near detector, respectively. The parameter \( \sigma_{\rho_d} \) contains the systematics uncertainties in the \( d \)-th channel: \( (\sigma_{\rho_1}, \sigma_{\rho_2}) = (8.8\%, 8.1\%) \) which are extracted from Table II of [3] and Table I of [13]; \( \sigma_{\Omega_d} \) are the fiducial mass uncertainties for the \( d \)-th detector (\( \sigma_{\Omega_d} \) have been estimated at the order of 1\% for the far and the near detectors similarly to [46]), \( \alpha_d \) and \( \alpha_N \) are free parameters which represent the energy scale for predicted signal events with uncertainty \( \sigma_{\alpha_d} \) and \( \sigma_{\alpha_N} \) (\( \sigma_{\alpha_d} = 1\% \)). The corresponding pull parameters are \( (\rho_d, \Omega_d, \Omega_N, \alpha_d, \alpha_N) \). The experimental event rates at the near detector
have been estimated rescaling the non oscillated event rates at the far detector (extracted from [15]) using the scale factor $L_F^2/L_N^2 \times FM_{ND280}/FM_{SK}$.

The whole Daya Bay and T2K data sample is analyzed using:

$$\chi^2_{tot} = \chi^2_{DB} + \chi^2_{2K}.$$  \hfill (23)

**NUMERICAL RESULTS**

In the following plots, unless explicitly stated, all the not shown parameters have been marginalized over. In particular, the SM quantities $\theta_{13}$, $\theta_{23}$, $\delta$ and $\Delta m^2_{31}$ are unconstrained, since they have to be reconstructed from the data themselves, whereas for the solar angle and the solar mass difference we used external best fit points [8] and assumed an error to follow a gaussian distribution: $\sin^2 \theta_{12} = 0.308 \pm 20\%$ and $\Delta m^2_{21} = (7.54 \pm 5\%) \times 10^{-5} \text{eV}^2$. The new physics parameters, in both NSI and LED scenarios, are considered as free parameters constrained only by the current upper limits, Eqs. [5] and [13], and by the additional restriction to be in the perturbative regime, e.g., $\xi_i < 0.2$.

**Standard Model**

We first consider the fit to the data in the standard three-neutrino framework, with the intent to make easier the comparison of the standard results with the ones obtained with the contribution of new physics. In Fig. I we show our results in the $[\sin^2 \theta_{13}, \delta]$, $[\sin^2 \theta_{23}, \delta]$, $[\sin^2 \theta_{23}, \sin^2 \theta_{13}]$, $[\sin^2 \theta_{23, \Delta m^2_{31}}]$ and $[\sin^2 2\theta_{13, \Delta m^2_{31}}]$ planes. The curves represent the $1\sigma$, $2\sigma$, $3\sigma$ confidence level regions for 1 degree of freedom (dof). The case of normal ordering of the neutrino mass spectrum is represented with dotted, dashed and solid lines whereas the inverted spectrum in red (dark-gray), orange (gray) and yellow (light-gray). The obtained best fit points are indicated with a circle for NO and with a cross for IO. The figures have been obtained using the standard oscillation probabilities relevant for the $\nu_e \to \nu_e$, $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\mu$ transitions. In Tab. I we summarize the best fit points and the $1\sigma$, $2\sigma$ and $3\sigma$ confidence level regions. Our results are in agreement with Ref. [8] and confirm the recent hint for maximal CP violation in the neutrino sector, $\delta \simeq 3\pi/2$.

| Parameter | Best-fit ($\pm 1\sigma$) | $1\sigma$ range | $2\sigma$ range | $3\sigma$ range |
|-----------|--------------------------|-----------------|-----------------|-----------------|
| $\Delta m^2_{31}$ (NO) $[10^{-3} \text{eV}^2]$ | 2.51$^{+0.06}_{-0.06}$ | 2.45 - 2.57 | 2.40 - 2.63 | 2.34 - 2.69 |
| $\Delta m^2_{31}$ (IO) $[10^{-3} \text{eV}^2]$ | 2.54$^{+0.06}_{-0.06}$ | 2.48 - 2.60 | 2.42 - 2.67 | 2.37 - 2.73 |
| $\sin^2 \theta_{23}/10^{-1}$ | 5.3$^{+0.4}_{-0.6}$ | 4.7 - 5.7 | 4.3 - 6.0 | 4.0 - 6.3 |
| $\sin^2 \theta_{13}/10^{-2}$ | 2.3$^{+0.3}_{-0.2}$ | 2.1 - 2.6 | 1.9 - 2.8 | 1.6 - 3.0 |
| $\delta/\pi$ | 1.50$^{+0.33}_{-0.37}$ | 1.16 - 1.86 | 0.00 - 0.24 $\oplus$ 0.75 - 2.00 | — |
| | 1.48$^{+0.36}_{-0.35}$ | 1.13 - 1.84 | 0.00 - 0.26 $\oplus$ 0.73 - 2.00 | — |

**TABLE I**: Best fit and $1\sigma$, $2\sigma$ and $3\sigma$ errors of the standard oscillation parameters obtained in the combined fit of the Daya Bay and T2K data using the standard oscillation probabilities. If two values are given, the upper one corresponds to NO and the lower one to IO.

**Effects of including NSI and LED**

The modification of the relevant transition probabilities due to the presence of NSI and LED parameters can result in a distortion of the allowed regions of the standard neutrino mixing parameters. In order to quantify
such effects, we repeat the previous fit on the T2K and Daya Bay data, using the modified expressions of the transition probabilities in the $\nu_e \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_e$ channels, illustrated in Eqs. (7)-(12) and Eq. (16). For the sake of a more clear presentation, we limit ourselves to $2\sigma$ and $3\sigma$ confidence level. Our results are presented in Fig. 2 for LED and in Fig. 3 for NSI models. We use the same conventions as in Fig. 1 and give the obtained best fit points and confidence level regions in Tab. III for NSI and in Tab. II for LED. For completeness, we also show in Appendix B the one dimensional projections of $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ as a function of the standard oscillation parameters $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$, $\Delta m_{31}^2$ and $\delta$.

The presence of LED parameters in the oscillation formulae does not affect too much the shape of the contours (Fig. 2); in this respect, the importance of including the T2K data in our analysis is mainly visible in the determination of $\Delta m_{31}^2$: in fact, in the analysis of the Daya Bay data only performed in [18], the $3\sigma$ confidence region for $\Delta m_{31}^2$ was roughly 5% larger with respect to the SM determination, whereas in the present analysis this difference is reduced to roughly 1%.

In the NSI scenario the presence of the new couplings $\varepsilon_{\alpha\beta}$ enlarges sensibly the confidence regions of the standard oscillation parameters and, in particular, reduces the hints for a maximal CP violation since the whole $[0, 2\pi]$ range for $\delta$ is allowed at $2\sigma$ confidence level. This effect is caused by the new sources of the CP violation, encoded in the unconstrained phases $\phi_{\alpha\beta}$ of Eq. (7) and Eqs. (9)-(10). A large effect is also found in the determination of the reactor angle $\sin^2 \theta_{13}$. Indeed the $3\sigma$ confidence region in the NSI case is roughly twice larger than in the SM case. The main reason for such a behavior is the strong correlation
among \( \theta_{13} \) and the NSI parameters: for large enough \( \varepsilon_{\mu e} \) and/or \( \varepsilon_{\mu \tau} \) (and an appropriate choice of the related CP phases), huge cancellations can occur with the standard part of the probability, thus causing an increase of the allowed \( \theta_{13} \); the opposite can also happen: positive interferences can decrease the expected value of the reactor angle \([18]\).

![Graphical representation](image-url)

**FIG. 2:** 2\(\sigma\) and 3\(\sigma\) confidence level regions on the standard neutrino oscillation parameters for 1 dof from the combined fit to the Daya Bay and T2K data, obtained using the LED oscillation probabilities. Same conventions as in Fig. 1.

| Parameter | Best-fit (±1\(\sigma\)) | 1\(\sigma\) range | 2\(\sigma\) range | 3\(\sigma\) range |
|-----------|----------------|-----------------|-----------------|-----------------|
| \(\Delta m_{31}^2\) (NO) [10\(^{-3}\) eV\(^2\)] | 2.53\(^{+0.07}_{-0.05}\) | 2.48 – 2.60 | 2.42 – 2.66 | 2.37 – 2.73 |
| \(\Delta m_{31}^2\) (IO) [10\(^{-3}\) eV\(^2\)] | 2.54\(^{+0.07}_{-0.07}\) | 2.47 – 2.61 | 2.41 – 2.65 | 2.35 – 2.71 |
| \(\sin^2\theta_{23}/10^{-1}\) | 5.3\(^{+0.4}_{-0.5}\) | 4.8 – 5.7 | 4.3 – 6.0 | 4.1 – 6.3 |
| \(\sin^2\theta_{13}/10^{-2}\) | 2.3\(^{+0.2}_{-0.4}\) | 1.9 – 2.5 | 1.7 – 2.7 | 1.4 – 3.0 |
| \(\delta/\pi\) | 1.47\(^{+0.35}_{-0.31}\) | 1.16 – 1.82 | 0.00 – 0.21 \(\oplus\) 0.72 – 2.00 | — |
| 1.59\(^{+0.30}_{-0.36}\) | 1.23 – 1.89 | 0.00 – 0.28 \(\oplus\) 0.78 – 2.00 | — |

**TABLE II:** Best fit and 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) errors of the standard oscillation parameters obtained in the combined fit of the Daya Bay and T2K data using the LED probabilities. If two values are given, the upper one corresponds to neutrino mass spectrum with normal ordering (NO) and the lower one to spectrum with inverted ordering (IO) (see text for further details).
**FIG. 3:** Same as in Fig. 2 but using the NSI oscillation probabilities.

| Parameter                      | Best-fit (±1σ) | 1σ range      | 2σ range      | 3σ range      |
|--------------------------------|----------------|---------------|---------------|---------------|
| $\Delta m^2_{31}$ (NO) [10^{-3} eV^2] | $2.56^{+0.06}_{-0.09}$ | $2.47 - 2.62$ | $2.38 - 2.68$ | $2.31 - 2.74$ |
| $|\Delta m^2_{31}|$ (IO) [10^{-3} eV^2] | $2.56^{+0.09}_{-0.08}$ | $2.47 - 2.64$ | $2.39 - 2.71$ | $2.32 - 2.77$ |
| $\sin^2 \theta_{23}/10^{-1}$ | $5.2^{+0.6}_{-0.3}$ | $4.6 - 5.8$   | $4.1 - 6.2$   | $3.8 - 6.5$   |
| $\sin^2 \theta_{13}/10^{-2}$ | $3.9^{+0.4}_{-0.2}$ | $1.3 - 4.3$   | $0.9 - 4.7$   | $0.7 - 5.1$   |
| $\delta/\pi$                  | $1.65^{+0.55}_{-0.78}$ | $0.00 - 0.20 \oplus 0.87 - 2.00$ | — — | — — |
|                                | $1.35^{+0.75}_{-0.49}$ | $0.00 - 0.10 \oplus 0.15 - 0.20$ | — — | — — |

**TABLE III:** Best fit and 1σ, 2σ and 3σ errors of the standard oscillation parameters obtained in the combined fit of the Daya Bay and T2K data using the NSI probabilities. If two values are given, the upper one corresponds to neutrino mass spectrum with normal ordering (NO) and the lower one to spectrum with inverted ordering (IO) (see text for further details).
Bounds on LED parameters

We now consider the bounds on \( m_0 \) and on the size of the largest extra dimension \( R \). We perform a fit on the T2K data only (left panel of Fig. 4) and also show the results for a combined analysis of the T2K and the Daya Bay data (right panel of Fig. 4). In both the figures, the horizontal dashed line represents the expected sensitivity on the lightest neutrino mass from KATRIN [49], whereas the \( 2\sigma \) and \( 3\sigma \) exclusion limits are represented with the dashed and solid lines for NO and with the orange (gray) and yellow (light gray) regions for IO neutrino mass spectrum. In addition, the circles and the stars indicate the \( 2\sigma \) bounds (for 1 dof) obtained using the IceCube IC-40 and IC-79 data set [50], respectively, from which we have the following constraints: \( R < 0.54 \) (0.34) \( \mu m \) using the IC-40 (IC-79) data set.

Using the T2K data only, we obtain the same upper bound, \( R \leq 0.93 \) (1.0) \( \mu m \), for both normal and inverted orderings at \( 2\sigma \) (3\( \sigma \)) C.L., in agreement with our expectations, Eq. (18). However, it is not possible to give better constraints on \( m_0 \), a part from a small region for large enough \( R \) where \( m_0 \lesssim 0.1 \) eV. The combined T2K and Daya Bay analysis gives better limits on \( R \) only: \( R \leq 0.60 \) \( \mu m \) for NO (a bit worst than the IceCube limits) and \( R \leq 0.17 \) \( \mu m \) for IO (roughly a factor of 2 better than the IceCube bounds).

Since the combined analysis is clearly dominated by the Daya Bay experiment (due to the higher statistics of the \( \bar{\nu}_e \) disappearance channel), the bounds on \( R \) are similar to the ones given in [18].

Bounds on NSI parameters

Finally, we analyze the bounds on the new couplings \( \epsilon_{\alpha\beta} \) but \( \epsilon_{ee} \), which has been already discussed in details in [18] and for which the T2K data do not add any additional constraints. Given the large numbers of new moduli and phases affecting the transition probabilities, sensitive limits can only be put under the supplementary hypotheses of fixed parameters. We have checked that no relevant bounds can be obtained on the various \( |\epsilon_{\alpha\beta}| \) if we marginalize the \( \chi^2 \) function over all the other parameters. In Fig. 5 we show the \( 2\sigma \) and \( 3\sigma \) confidence regions for the \( \epsilon_{\mu\tau}^s \) and \( \epsilon_{\mu\mu}^s \) parameters, which enter the \( \nu_\mu \to \nu_\mu \) probability, obtained

FIG. 4: Exclusion regions at 2\( \sigma \) and 3\( \sigma \) confidence levels for 1 dof in the \([\log_{10}(R), \log_{10}(m_0)]\)-plane (LED model). Dashed and solid lines refer to NO, whereas the orange (gray) and yellow (light gray) regions are for IO neutrino mass spectrum. Left panel: results obtained from the fit to the T2K data only. Right panel: results obtained from the fit to the T2K \oplus Data Bay data. The circles and the stars represent the 2\( \sigma \) bounds obtained using the IceCube IC-40 and IC-79 data set [50], respectively.
setting to zero the standard CP phase and all the NSI parameters not shown in the plots (results obtained for other fixed values of $\delta$ are similar to the case of $\delta = 0$, due to its $\theta_{13}$-suppressed dependence shown in Eq. (11)). As we can see the obtained bounds are weaker then those of Eq. (5) and so not particularly interesting.

![Graph](image)

**FIG. 5:** Left panel: allowed regions in the $[\varepsilon^s_{\mu\tau}, \phi^s_{\mu\tau}]$-plane at 2$\sigma$ and 3$\sigma$ confidence level for 1 dof (the standard CP violation phase $\delta$ and the not shown NSI parameters are set to zero). The dashed vertical line corresponds to $\varepsilon^s_{\mu\tau} = 0.013$. Right panel: the same as in the left panel but in the $[\varepsilon^s_{\mu\mu}, \phi^s_{\mu\mu}]$-plane. The dashed vertical line is at $\varepsilon^s_{\mu\mu} = 0.078$.

The bounds on $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$ and $\varepsilon^s_{e\mu}$ are shown in Fig. 6 for $\delta = 0$ (upper panel), $\delta = \pi$ (middle panel) and $\delta = 3\pi/2$ (lower panel). As it can be seen, the obtained bounds for the absolute values of $\varepsilon_{\alpha\beta}$ are modulated by the relative phases $\phi_{\alpha\beta}$: for example, for $\delta = 0$ and $\phi_{e\mu}/\pi \sim 0$ we get $\varepsilon_{e\mu} < O(10^{-3})$ at 2$\sigma$ C.L., a bit stronger than the model independent limit derived in [32], whereas for $\phi_{e\mu}/\pi \sim 1/2$ we have no bound whatsoever. In Table IV we summarize the bounds on the NSI parameters obtained for particular choices of the phases $\phi_{\alpha\beta}$ and $\delta$. Since the parameter $\varepsilon_{e\tau}$ cannot be better constrained by the current data, we do not present the obtained upper bounds in the table.

| $\phi_{\alpha\beta}/\pi$ | $\delta$ | Upper bound (2$\sigma$ C.L.) | $\phi_{\alpha\beta}/\pi$ | $\delta$ | Upper bound (2$\sigma$ C.L.) |
|-----------------------|---------|-----------------------------|-----------------------|---------|-----------------------------|
| $\phi_{e\mu}/\pi \sim 0$ | 0       | $4.85 \times 10^{-3}$       | $\phi^s_{\mu\mu}/\pi \sim 0$ | 0       | $6.28 \times 10^{-3}$       |
| $\phi_{e\mu}/\pi \sim 1.0$ | $\pi$   | $9.94 \times 10^{-3}$       | $\phi^s_{\mu\mu}/\pi \sim 1.0$ | $\pi$   | $9.96 \times 10^{-3}$       |
| $\phi_{e\mu}/\pi \sim -0.5$ | $3\pi/2$ | $3.50 \times 10^{-2}$       | $\phi^s_{\mu\mu}/\pi \sim 0.5$ | $3\pi/2$ | $3.12 \times 10^{-2}$       |

**TABLE IV:** Upper bounds on the parameters $\varepsilon_{e\mu}$ and $\varepsilon^s_{\mu\mu}$ at 2$\sigma$ C.L. for particular choices of the phases $\phi_{e\mu}$, $\phi^s_{\mu\mu}$ and $\delta$, obtained from Fig. 6.

**SUMMARY AND CONCLUSIONS**

In this paper we have analyzed the recent appearance [3] and disappearance [15] data of T2K experiment and the disappearance data [16] of the Daya Bay reactor experiment to constrain the parameter space of two models of physics beyond the SM, namely the non standard neutrino interactions and large extra dimensions models, and to quantify the impact of this kind of new physics on the determination of the standard oscillation parameters.

While the impact of LED on the best fit values and 1$\sigma$ errors of the standard oscillation parameters is almost negligible (the largest difference is found for $\sin^2 \theta_{13}$ where the 3$\sigma$ LED confidence region is almost 10%
larger than the standard model), this is not the case for the NSI scenario, where particularly the allowed values of $\theta_{13}$ and $\delta$ are different from the standard determination. Indeed the $1\sigma$ confidence region for $\theta_{13}$ is roughly six times larger than the standard model analysis. The situation is similar for the phase $\delta$, where the presence of new phases from the NSI complex couplings $\varepsilon_{\alpha\beta}$ reduces the sensitivity with respect to the
standard physics. In fact, although the best fit is still around the standard solution $\delta \sim 3\pi/2$ (as found in [8, 9]), the presence of NSI effects makes this value statistically less significant.

As for the bounds on the parameters of the LED and NSI models, we have found the following interesting results:

- the strongest 2$\sigma$ C.L. bounds are for $\varepsilon_{\mu e} < 4.85 \times 10^{-3}$ (for $\delta \sim 0$) and $\varepsilon_{\mu e}^* < 6.28 \times 10^{-3}$ (for $\delta \sim 0$);
- for the radius of the largest extra dimension $R$ we obtain $R \leq 0.60$ $\mu$m (NO) and $R \leq 0.17$ $\mu$m (IO) at 2$\sigma$ confidence level, similarly to [18].

Following the discussion of the previous section, the current bounds on the NSI parameters are expected to be improved after a better determination of the standard CP phase $\delta$. For the LED parameters, an effort must be done in order to constrain the absolute mass $m_0$ and, consequently, the value of $R$.

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**APPENDIX A: APPROXIMATE FORMULAE LED**

For the sake of completeness, we give here the approximate formulae of the eigenvalues and the rotation matrices $U_{0n}$ for the LED model. Following the results of [14], we can write the eigenvalues equation for $\lambda_i = m_{i}^{(n)} R$ as:

$$\lambda_i - \frac{\pi}{2} \xi_i^2 \cot(\pi \lambda_i) = 0,$$

with $\xi_i \equiv \sqrt{2}m_i^D R$. In the case of small $\xi_i$ ($R^{-1} \gg m_j^D$), we get:

$$\lambda_i^{(0)} = \frac{1}{\sqrt{2}} \xi_i - \frac{1}{12\sqrt{2}} \pi^2 \xi_i^3 + \frac{1}{90\sqrt{2}} \frac{11}{16} \pi^4 \xi_i^5 \text{ + } O(\xi_i^7),$$

$$\lambda_i^{(k)} = k + \frac{1}{2k} \xi_i^2 - \frac{1}{4k^3} \xi_i^4 \text{ + } O(\xi_i^6) \text{ for } k > 0.$$  

Given the rotation matrix $U_{0n}$ [14]:

$$(U_{0n}^{(n)})^2 = \frac{2}{1 + \pi^2 \xi_i^2 / 2 + 2\lambda_i^{(n)2} / \xi_i^2},$$

we can easily derive their expressions in terms of $\xi_i$:

$$(U_{00}^{(0)})^2 = 1 - \frac{\pi^2}{6} \xi_i^2 + \frac{\pi^4}{60} \xi_i^4 \text{ + } O(\xi_i^6),$$

$$(U_{0k}^{(k)})^2 = \frac{\xi_i^2}{k^2} - \frac{3 \xi_i^4}{2k^4} + \frac{(10 - k^2 \pi^2)}{4k^6} \xi_i^6 \text{ + } O(\xi_i^8) \text{ for } k > 0.$$  

Notice that the expansion parameter $\xi_i$ depends on the mass ordering and thus affects both the neutrino eigenvalues and the rotation matrices.
APPENDIX B: ONE DIMENSIONAL PROJECTIONS

In this section we give the one dimensional projections of $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ as a function of $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$, $\Delta m^2_{31}$ and the CP phase $\delta$. The case of NO is shown in Fig. 7, where the values are given for the SM (solid blue line), the LED case (small dashed orange line) and the NSI case (large dashed green line). Notice that, as expected, the $\chi^2$ for $\sin^2 \theta_{13}$ in the NSI case is almost flat in the $1\sigma$ allowed region. The case for the IO

![Graphs showing one dimensional projections of $\Delta \chi^2$ as a function of various parameters for NO and IO neutrino mass spectra.](http://www.ba.infn.it/~now/now2014/web-content/index.html)

FIG. 7: Upper panels. $\Delta \chi^2$ as a function of $\sin^2 \theta_{13}$ (left panel) and $\sin^2 \theta_{23}$ (right panel) using the SM (solid blue line), LED (small dashed orange line) and NSI (large dashed green line) oscillation probabilities for NO neutrino mass spectrum. Lower panels. Same as in the upper but for $\Delta \chi^2$ as a function of $\Delta m^2_{31}$ (left panel) and $\delta$ (right panel).

is given in Fig. 8.

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FIG. 8: Same as Fig. 7 but for the IO case.

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[51] In Appendix A we give the expressions of the unitary transformations $U^\nu_{0n}$ in terms of the radius $R$ and the neutrino masses.