Bulk viscosity for pion and nucleon thermal fluctuation in the hadron resonance gas model

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We have calculated microscopically bulk viscosity of hadronic matter, where equilibrium thermodynamics for all hadrons in medium are described by Hadron Resonance Gas (HRG) model. Considering pions and nucleons as abundant medium constituents, we have calculated their thermal widths, which inversely control the strength of bulk viscosities for respective components and represent their in-medium scattering probabilities with other mesonic and baryonic resonances, present in the medium. Our calculations show that bulk viscosity increases with both temperature and baryon chemical potential, whereas viscosity to entropy density ratio decreases with temperature and with baryon chemical potential, the ratio increases first and then decreases. The decreasing nature of the ratio with temperature is observed in most of the earlier investigations with few exceptions. We find that the temperature dependence of bulk viscosity crucially depends on the structure of the relaxation time. Along the chemical freeze-out line in nucleus-nucleus collisions with increasing collision energy, bulk viscosity as well as the bulk viscosity to entropy density ratio decreases, which also agrees with earlier references. Our results indicate the picture of a strongly coupled hadronic medium.

PACS numbers: 11.10.Wx, 12.39.Ki

I. INTRODUCTION

The extraction of the transport properties of the strongly interacting medium created in heavy ion collision (HIC) experiments is currently a very active topic of research in the HIC community. The methods of relativistic hydrodynamics with minimal viscous correction have been quite successful in describing the time evolution of the hot and dense fireball created in the HIC experiments. These kind of investigations have also concluded that the shear viscosity ($\eta$) to entropy density ($s$) ratio, $\eta/s$, of the medium created in HIC experiments is very close to its quantum lower bound $1/4\pi$. Similar to $\eta$, another transport coefficient is the bulk viscosity, $\zeta$, which is defined as the proportionality constant between the non-zero trace of the viscous stress tensor to the divergence of the fluid velocity, and usually it appears associated with processes accompanied by a change in fluid volume or density. The viscous coefficient $\zeta$ has received much less attention than the $\eta$ in hydrodynamical simulations because its numerical value is assumed to be very small, as it is directly proportional to the trace of the energy-momentum tensor, which generally vanishes for conformally symmetric matter. However, according to Lattice Quantum Chromo Dynamics (LQCD) calculations, the trace of the energy momentum tensor of hot QCD medium might be large near the QCD phase transition, which indicates the possibility of a non-zero and large value of $\zeta$ as well as of $\zeta/s$ near the transition temperature. This indication is confirmed by the Refs. 4-5, related with LQCD estimation, where Ref. 7 exposes the possibility of divergence of $\zeta$ near the transition temperature. In recent times, different phenomenological investigations 4-18 demonstrated that bulk viscosity can have a non-negligible effect on heavy ion observables, where the values of $\zeta/s$ in Ref. 18 is assumed to be quite large.

On the basis of phenomenological importance, microscopic calculations of $\zeta$ for quark gluon plasma (QGP) and hadronic matter is a matter of contemporary interest in the community of HIC. A list of references are 2-19-37, where Ref. 19 addressed high temperature perturbative QCD calculations of $\zeta$, Refs. 20-25 have gone through Nambu-Jona-Lasinio (NJL) model calculations of $\zeta$ and Refs. 26-28 provided the discussions on Linear Sigma Model (LSM) estimation of $\zeta$. These effective QCD model calculations 20-28 cover both QGP and hadronic phases while hadronic-model calculations of Refs. 26-37 are restricted within hadronic phase only. The present work is also addressing the estimation of $\zeta$ in the hadronic phase only. At vanishing baryonic chemical potential, most of the microscopic calculations predict that $\zeta(T)$ increases but $\zeta/s(T)$ decreases in the hadronic temperature domain. However, few exceptions are there depending on different scenario. For example, Ref. 28 showed that the decreasing function of $\zeta/s(T)$ is transformed to an increasing function in the hadronic temperature domain, when its medium constituents sigma meson becomes heavier. Similar kind of fact is also observed in Ref. 27 depending on the different nature of phase transition as well as methodological differences of LSM calculations. In the hadronic temperature domain, a decreasing nature of $\zeta(T)$ is observed in Ref. 21 while...
Ref. 33 estimated increasing $\zeta/s(T)$, these knowledge from the earlier investigations suggest that the nature of $\zeta(T)$ and $\zeta/s(T)$ are still not very settled issues. Again, the numerical strength of $\zeta$ and $\zeta/s$ from different model calculations exhibit a large band - $\zeta \sim 10^{-5}$ GeV$^3$ 32 to $10^{-2}$ GeV$^3$ 20, or, $\zeta/s \sim 10^{-3}$ 32 to 10$^0$ 20. These uncertainty in nature as well as numerical values of $\zeta(T)$ from the earlier investigations demand for further research on these kind of microscopic calculations. Owning to that motivation, we have gone through a microscopic calculations of $\zeta$ and $\zeta/s$, where equilibrium situations of hadronic matter are controlled by the standard HRG model and non-equilibrium picture of medium constituents is introduced via quantum fluctuation of pion and nucleon in medium. With respect to the earlier HRG calculations of $\zeta$ 33 36, the main distinguishable contribution is in the non-equilibrium properties of medium constituents, quantified by their thermal width. Assuming pions and nucleons as most abundant constituents of medium, we have calculated their thermal width, which are coming from their in-medium scattering with different possible mesonic and baryonic resonances. The main formalism for this thermal width calculations of pion and nucleon are explicitly described in the Section III, which is started with a brief description HRG model, handling the equilibrium part. Next, the numerical results are discussed in Section III and lastly, our investigations have been summarized and concluded in Section IV.

II. FORMALISM

The HRG system is an ideal gas of hadrons and their resonances are taken from the Particle Data Book 38. Here we consider all resonances up to 2 GeV masses. The recent LQCD data at zero baryon chemical potential ($\mu_B$) show that for temperatures up to the crossover region (150 - 160 MeV), HRG provides a reasonably good description of the LQCD thermodynamics 29 31. All thermodynamic quantities of the HRG can be computed from the logarithm of total partition function

$$\ln Z_{HRG} (T, \mu_B, \mu_Q, \mu_S) = \sum_i \ln Z_i^s (T, \mu_B, \mu_Q, \mu_S) ,$$

where

$$\ln Z_i^s = \frac{g_i}{2\pi^2} V T^3 \sum_{n=1}^{\infty} \frac{(\pm)^{(n+1)}}{n^4} \left( \frac{m_i}{T} \right)^2 K_2 \left( \frac{m_i}{T} \right) e^{n\beta \mu_i}$$

is the single particle partition function of the i-th hadron. In Eq. 2, $g_i$ is the degeneracy factor of i-th particle with mass $m_i$, $V$ is volume of the medium, and $K_2(\cdot)$ is the modified Bessel function. Under the condition of complete chemical equilibrium, all the hadron chemical potentials can be expressed in terms of only three chemical potentials corresponding to the QCD conserved charges

$$\mu_i = B_i \mu_B + Q_i \mu_Q + S_i \mu_S \,$$

where $B_i$, $Q_i$, and $S_i$ are the baryon number, electric charge and strangeness of the i-th hadron. It is straightforward to compute other thermodynamic quantities from $Z_{HRG}$, such as pressure ($P$), energy density ($\epsilon$), entropy density ($s$):

$$P = -\frac{T}{V} \ln Z_{HRG} ,$$

$$\epsilon = \frac{1}{V} \left\{ T^2 \frac{\partial \ln Z_{HRG}}{\partial T} + \sum_i \mu_i T \frac{\partial \ln Z_{HRG}}{\partial \mu_i} \right\} ,$$

$$s = \frac{1}{T} \left\{ \epsilon + P - \frac{1}{V} \sum_i \mu_i T \frac{\partial \ln Z_{HRG}}{\partial \mu_i} \right\} .$$

Square of the speed of sound is defined as

$$c_s^2 = \left( \frac{\partial P}{\partial \epsilon} \right)_{\rho_B} ,$$

where $\rho_B$ is net baryon density.

From the Relaxation Time Approximation (RTA) of kinetic theory approach 28 31 or from the one-loop expression of diagrammatic approach based on Kubo formula 37, we can get standard expressions of bulk viscosity coefficient for pion and nucleon components 28 31 34 37 :

$$\zeta_\pi = \left( \frac{g_\pi}{T} \right) \int \frac{d^3k}{(2\pi)^3} \frac{n_\pi [1 + n_\pi]}{\omega_\pi^3 \Gamma_\pi} \left\{ \left( \frac{1}{3} - c_s^2 \right) k^2 - c_s^2 m_\pi^2 \right\}^2$$

and

$$\zeta_N = \left( \frac{g_N}{T} \right) \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_N^3 \Gamma_N} \left\{ \left( \frac{1}{3} - c_s^2 \right) k^2 - c_s^2 m_N^2 \right\}$$

$$- \omega_N \left( \frac{\partial P}{\partial \rho_B} \right)_{\epsilon} n_N^{-1} \left( 1 - n_N^+ \right) + \left( \frac{1}{3} - c_s^2 \right) k^2$$

$$- c_s^2 m_N^2 + \omega_N \left( \frac{\partial P}{\partial \rho_B} \right)_{\epsilon} n_N^{-1} \left( 1 - n_N^+ \right) ,$$

where $n_\pi = 1/\{e^{\omega_\pi/T} - 1\}$ is the Bose-Einstein (BE) distribution function of pion with energy $\omega_\pi = \{k^2 + m_\pi^2\}^{1/2}$, $n_N^+ = 1/\{e^{(\omega_N + \mu_B)/T} + 1\}$ are the Fermi-Dirac (FD) distribution functions of nucleon and anti-nucleon respectively with energy $\omega_N = \{k^2 + m_N^2\}^{1/2}$ at finite temperature $T$ and baryon chemical potential $\mu_B$. The degeneracy factors of pion and nucleon components are $g_\pi = 3$ and $g_N = 2 \times 2$ respectively. Next, let us come to the important quantities $\Gamma_\pi$ and $\Gamma_N$ of Eq. 8 and 9, which are called thermal widths of pion and nucleon respectively. During propagation in the medium, pion and nucleon may go through different on-shell scattering with other mesonic ($M$) and baryonic ($B$) resonances, which can be quantified by their different possible self-energy diagrams. From the imaginary part of their self-energy functions, their respective thermal widths $\Gamma_\pi$ and $\Gamma_N$ can be found. Fig. 1(a) represents pion self-energy with internal lines of pion ($\pi$) and other mesonic resonances
FIG. 1: Pion self-energy diagram with mesonic loops (a) and baryonic loops [(b) and (c) are direct and cross diagrams] and nucleon self-energy diagram (d).

(M), which we can shortly call $\pi M$ loop. We will take $M = \sigma$ and $\rho$, as they are dominant resonances of $\pi\pi$ decay channel (within the invariant mass range of 1 GeV).

Now, from the retarded self-energy of pion for $\pi M$ loop $\Pi^R_{\pi(\pi M)}(k)$, the corresponding thermal width $\Gamma_{\pi(\pi M)}$ can be obtained as

$$\Gamma_{\pi(\pi M)} = -\text{Im} \Pi^R_{\pi(\pi M)}(k_0 = \omega, \vec{k})/m_\pi ,$$  \hspace{1cm} (10)

where subscript notation stands for external (outside the bracket) and internal (inside the bracket) particles for the diagram (a). Following Similar notation, we can define

$$\Gamma_{\pi(NB)} = -\text{Im} \Pi^R_{\pi(NB)}(k_0 = \omega, \vec{k})/m_\pi ,$$  \hspace{1cm} (11)

where intermediate states of pion self-energy are nucleon $N$ and other baryonic resonance $B$ as shown in Fig. (b) along with its cross diagram (c). As a dominant 4-star baryons with spin $J_B = 1/2$ and 3/2, we have taken $B = \Delta(1232), N^*(1440), N^*(1520), N^*(1535), \Delta^*(1600), \Delta^*(1620), N^*(1650), \Delta^*(1700), N^*(1700), N^*(1710)$ and $N^*(1720)$. Adding all these mesonic ($\pi M$) and baryonic (NB) loops, the total thermal width of pion $\Gamma_{\pi M}$ can be obtained as

$$\Gamma_{\pi} = \Gamma^M_{\pi} + \Gamma^B_{\pi} = \sum_M \Gamma_{\pi(M)} + \sum_B \Gamma_{\pi(NB)} .$$  \hspace{1cm} (12)

Similarly, one-loop self-energy of nucleon with pion ($\pi$) and baryon ($B$) intermediate states, which is denoted as $\Sigma^R_{N(\pi B)}$ (retarded part), will be our matter of interest to estimate corresponding nucleon thermal width $\Gamma_{N(\pi B)}$. The diagramatic anatomy of $\Sigma^R_{N(\pi B)}$ is shown in Fig. (d). Here we have taken all the 4-star spin 1/2 and 3/2 baryons, mentioned above. Hence, summing all the $\pi B$ loops, we get our total nucleon thermal width:

$$\Gamma_N = \sum_B \Gamma_{N(\pi B)} = -\sum_B \text{Im} \Sigma^R_{N(\pi B)}(k_0 = \omega_N, \vec{k}) .$$  \hspace{1cm} (13)

The imaginary part of self-energies, given in Eqs. (10), (11) and (13), have been derived with help of standard thermal field theoretical techniques. At first, the expression for $\text{Im} \Pi^R_{\pi(\pi M)}$ is

$$\text{Im} \Pi^R_{\pi(\pi M)}(k_0 = \omega, \vec{k}) = \int \frac{d^3l}{32\pi^2\omega\omega_u} L_{\pi\pi M}(k, l) \delta(\omega - \omega_u - \omega_l) ,$$  \hspace{1cm} (14)

where $n_l, n_u$ are BE distribution functions of $\pi, M$ mesons respectively at energies $\omega_l = \{l^2 + m^2_\pi\}^{1/2}$ and $\omega_u = \{(k - l)^2 + m^2_\pi\}^{1/2}$. The vertex factors $L_{\pi(\pi M)}(k, l)$ have been calculated by using the effective Lagrangian density,

$$L_{\pi N M} = g_\rho \bar{\rho}_\mu \frac{\pi}{\pi/2} \partial^\mu \frac{\pi}{\pi/2} + \frac{g_\sigma}{2} m_\pi \bar{\pi} \pi \sigma ,$$  \hspace{1cm} (15)

where $g_\rho$ and $g_\sigma$ are respectively effective coupling constants of $\rho$ meson field ($\bar{\rho}_\mu$) and $\sigma$ meson field ($\sigma$), which are coupled with the pion field ($\pi$).

Next, the direct and cross diagrams of pion self-energy for $NB$ loop are combinedly expressed as

$$\text{Im} \Pi^R_{\pi(NB)}(k_0 = \omega, \vec{k}) = \int \frac{d^3l}{32\pi^2\omega\omega_u} L_{\pi NB}(k, l) \delta(\omega - \omega_u - \omega_l) + \delta(\omega - \omega_l + \omega_u) \{ -n_l^+ + n_u^- \} \delta(\omega - \omega_l + \omega_u) ,$$  \hspace{1cm} (16)

where $n_l^+, n_u^-$ are FD distribution functions of $N$ and $B$ (± for particle and anti-particle) respectively at energies $\omega_l = \{l^2 + m^2_N\}^{1/2}$ and $\omega_u = \{(k - l)^2 + m^2_B\}^{1/2}$ (± for diagrams (b) and (c) respectively). With the help of the effective Lagrangian densities for $\pi NB$ interactions [43],

$$\text{L}_{\pi NB} = \frac{\psi_N}\{J_B = 1/2\} \gamma^\mu \gamma^5 \partial^\mu \psi_N + \text{h.c. for } J_B = \frac{3}{2} ,$$

$$(P \text{ stands for parity of } B)$$  \hspace{1cm} (17)

one can deduced the vertex factors $L_{\pi NB}(k, l)$.

At last, the expression for $\text{Im} \Pi^R_{\pi(NB)}$ is

$$\text{Im} \Pi^R_{\pi(NB)}(k_0 = \omega, \vec{k}) = \int \frac{d^3l}{32\pi^2\omega\omega_u} L_{\pi NB}(k, l) \delta(\omega - \omega_u - \omega_l) + \delta(\omega - \omega_l + \omega_u) \{ n_l^+ + n_u^- \} \delta(\omega - \omega_l + \omega_u) ,$$  \hspace{1cm} (18)

where $n_l$ is BE distribution functions of $\pi$ at energy $\omega_l = \{(l^2 + m^2_\pi)\}^{1/2}$ and $n_u^+$ is FD distribution of $B$ at energy
\[ \omega_u = \left\{ \left( \vec{k} - \vec{l} \right)^2 + m_B^2 \right\}^{1/2}. \]

With the help of the interaction Lagrangian densities from Eq. (17), the vertex factors \( L_{N\pi B}(k,l) \) have been obtained.

### III. RESULTS AND DISCUSSION

Let us start our numerical discussion with the Fig. (2), where momentum distribution of thermal widths of pion and nucleon have been displayed. With the help of Eqs. (10), (11), (12), (14) and (16), \( \Gamma_{\pi} \), \( \Gamma_B \) and nucleon thermal width (dashed line) at three different medium parameters: (a) \( (T, \mu_B) = (0.130 \text{ GeV}, 0) \), (b) \( (0.170 \text{ GeV}, 0) \) and (c) \( (0.130 \text{ GeV}, 0.300 \text{ GeV}) \).

The momentum distribution of thermal widths of pion can be deduced by integrating out when we will estimate \( \zeta \) from Eqs. (8) and (9) respectively.

Let us come to the different loop contributions of pion and nucleon thermal width in bulk viscosity coefficient of hadronic matter. Fig. (3) shows individual contributions of \( \pi\sigma \) (dotted line) and \( \pi\rho \) (dash line) loops in \( \zeta \), which reveals that they are respectively important in low \( (T < 0.080 \text{ GeV}) \) and high \( (T > 0.080 \text{ GeV}) \) temperature domain for getting a non-divergent values of \( \zeta \). These are respectively obtained by putting \( \Gamma_{\pi(\pi\sigma)} \) and \( \Gamma_{\pi(\pi\rho)} \) in place of \( \Gamma_{\pi} \) of Eq. (5). Putting \( \Gamma_{\pi} = \Gamma_{\pi(\pi\sigma)} + \Gamma_{\pi(\pi\rho)} \) in place of \( \Gamma_{\pi} \) of Eq. (5), we get the solid line, representing total bulk viscosity of pionic component due to meson loops. After a mild decrement in low \( T \) \((< 0.080 \text{ GeV})\), it receives an increment nature in high \( T \) \((> 0.080 \text{ GeV})\). Along with Fig. (3) a), where an explicit temperature dependent \( c_2^\pi \) is taken from HRG model, the results for \( c_2^\pi = 0 \) and \( c_2^\pi = 1/3 \) are also displayed in Fig. (3) a) and (b), which are little different in nature. Just to show the phase space sensitivity of bulk viscosity via \( c_2^\pi \), these two results are displaying two extreme limits of \( c_2^\pi \). Therefore, we can understand Fig. (3) c) as some sort of superposition of (a) and (b).

According to Eq. (12) different baryon loops contribution \( (\Gamma_B^2) \) should have to add with meson loops con-
distribution \((\Gamma_\pi^M)\) to get total pion thermal width \(\Gamma_\pi\). In Fig. 4(a), changing the nature of dash-dotted line to dotted line indicates that inclusion of baryon loops with meson loops becomes the reason for reducing the rate of increment of \(\zeta_\pi(T)\) at high temperature region, \(T > 0.100\) GeV. Putting our calculated nucleon thermal width \(\Gamma_N\) in Eq. 3, we get \(\zeta_N\) as shown by dash line in Fig. 4(a). Now adding \(\zeta_N\) with \(\zeta_\pi\) we have total bulk viscosity \(\zeta_T = \zeta_\pi + \zeta_N\), \(19\) as shown by solid line in Fig. 4(a). In Fig. 4(b), this \(\zeta_T\) (solid line) has been compared with the results generated for two constant values of \(c_s^2\) (\(c_s^2 = 0.25\): dash line and \(c_s^2 = 0.15\): dotted line), within which \(c_s^2(T, \mu_B = 0)\) from HRG model more or less varies.

At two different values of \(\mu_B\), \(\zeta(T)\) due to nucleon thermal width \((\Gamma_N)\), pion thermal width for meson loops \((\Gamma_\pi^M)\) and meson + baryon loops \((\Gamma_\pi)\) are shown in Fig. 5(a), (b) and (c) respectively. Similarly, Fig. 6(a), (b) and (c) are displaying different loop contributions in \(\zeta(\mu_B)\) at \(T = 0.050\) GeV (dotted line), \(0.100\) GeV (dashed line) and \(0.150\) GeV (solid line). From Fig. 5(a) and 6(a), we see that \(\zeta_N\) increases with \(T\) as well as \(\mu_B\). From Fig. 6(b), we see the \(\zeta_\pi\) due to \(\Gamma_\pi^M\) at finite \(\mu_B\) first decreases at low \(T\) then increases at high \(T\). The nature

\[ \zeta_T = \zeta_\pi + \zeta_N, \]
of these curves are quite similar to the curve of \(\zeta_\pi(T)\) at vanishing \(\mu_B\) but their minima are only shifted towards lower \(T\) as \(\mu_B\) increases. Following the same story of vanishing \(\mu_B\), inclusion of baryon loops in pion self-energy is again influencing on \(\zeta_\pi(T)\) in high temperature domain. The variation with \(\mu_B\) of \(\zeta_N(\mu_B)\) in Fig. 8(a) and \(\zeta_\pi(\mu_B)\) in Fig. 8(b) and (c) are grossly same as their temperature dependence. For small \(T\) and \(\mu_B\), \(\zeta_N\) and \(\zeta_\pi\) are of similar order. However, with increasing \(T\) and \(\mu_B\), \(\zeta_N\) dominates over \(\zeta_\pi\). \(\zeta_N\) receives additional contribution from \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\). One should keep in mind that the term \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\) goes to zero for \(\mu_B = 0\). The \(T\) and \(\mu_B\) dependence of \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\) are shown in Fig. 7(b) and (d) respectively while Fig. 7(a) and (c) are displaying the \(T\) and \(\mu_B\) dependence of \(\zeta_\pi\). From Fig. 7(a), we see that our \(\zeta_\pi(T,\mu_B = 0)\) curve (solid line) is in good agreement with LQCD results (circles) within the hadronic temperature domain \((T < 0.160\) GeV). Total bulk viscosity \(\zeta_T\) (a), entropy density \(s\) (b) and their ratio \(\zeta/s\) (c) are plotted against \(T\) in Fig. 8 and \(\mu_B\) in Fig. 9 at three different values \(\mu_B\) and \(T\) respectively. Since increment of \(s(T)\) is larger than the increment of \(\zeta(T)\), therefore, \(\zeta/s\) is appeared as a decreasing function of \(T\). On the other hand, both \(\zeta(\mu_B)\) and \(s(\mu_B)\) monotonically increase with \(\mu_B\) but the ratio \(\zeta/s(\mu_B)\) increases first and then decreases at high \(\mu_B\) domain. Next, Fig. 10(a), (b), and (c) reveal respectively the variation of total bulk viscosity \(\zeta\), entropy density \(s\) and their ratio with the variation of center of mass energy \(\sqrt{s}\) (Reader are requested to be careful on the same symbol used for entropy density and square of beam energy). The beam energy dependence of \(T\) and \(\mu_B\) used in computation are those obtained from fits to hadron yields. We have used the parameterization from Ref. [45]. We notice in Fig. 11 that \(\zeta\) (a) and \(\zeta/s\) (c) are decreasing with \(\sqrt{s}\), which is qualitatively agreeing with the results of earlier studies [32] [34]. The decreasing trend of \(\zeta\) and \(\zeta/s\) with \(\sqrt{s}\) can be understood from the fact that \(\mu_B\) decreases with \(\sqrt{s}\) while \(T\) remains fairly constant in the range of \(\sqrt{s}\) analyzed here and according to Fig. 9(a) and (c), the \(\zeta\) and \(\zeta/s\) decreases with decreasing of \(\mu_B\).

Fig. 11 is dedicated for comparative understanding of our results with respect to the earlier investigations. As most of the works have been done at \(\mu_B = 0\), so we have plotted \(\zeta\) (a) and \(\zeta/s\) (b) against \(T\) for \(\mu_B = 0\), where our results for \(\pi\) - component (red lines) and \((\pi + N)\)- components (black lines), using our calculated \(\tau(\vec{k}, T, \mu_B = 0)\) along with LQCD results (green circles), Hostler et al. (Blue solid circles), Hostler et al. (Open squares [36]), Chakraborty et al. (Pink solid triangle), Marty et al. (Green triangles down [20]), Deb et al. (Brown stars [28]), Kadam et al. (Violet plus [31]), Fraile et al. (black solid squares), Sasaki et al. (Red pluses [33]), Chakraborty et al. (Brown open squares, [32]), Mirza et al. (Green open circles) and our results (red dash line). The variation with \(\mu_B\) of \(\zeta\) from HRG (b) and entropy density from HRG (a) and their ratios are quite similar to the curve of \(\zeta_\pi(T)\) at vanishing \(\mu_B\) and their minima are only shifted towards lower \(T\) as \(\mu_B\) increases. Following the same story of vanishing \(\mu_B\), inclusion of baryon loops in pion self-energy is again influencing on \(\zeta_\pi(T)\) in high temperature domain. The variation with \(\mu_B\) of \(\zeta_N(\mu_B)\) in Fig. 8(a) and \(\zeta_\pi(\mu_B)\) in Fig. 8(b) and (c) are grossly same as their temperature dependence. For small \(T\) and \(\mu_B\), \(\zeta_N\) and \(\zeta_\pi\) are of similar order. However, with increasing \(T\) and \(\mu_B\), \(\zeta_N\) dominates over \(\zeta_\pi\). \(\zeta_N\) receives additional contribution from \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\). One should keep in mind that the term \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\) goes to zero for \(\mu_B = 0\). The \(T\) and \(\mu_B\) dependence of \(\left(\frac{\partial P}{\partial \rho_B}\right)_T\) are shown in Fig. 7(b) and (d) respectively while Fig. 7(a) and (c) are displaying the \(T\) and \(\mu_B\) dependence of \(\zeta_\pi\). From Fig. 7(a), we see that our \(\zeta_\pi(T,\mu_B = 0)\) curve (solid line) is in good agreement with LQCD results (circles) within the hadronic temperature domain \((T < 0.160\) GeV). Total bulk viscosity \(\zeta_T\) (a), entropy density \(s\) (b) and their ratio \(\zeta/s\) (c) are plotted against \(T\) in Fig. 8 and \(\mu_B\) in Fig. 9 at three different values \(\mu_B\) and \(T\) respectively. Since increment of \(s(T)\) is larger than the increment of \(\zeta(T)\), therefore, \(\zeta/s\) is appeared as a decreasing function of \(T\). On the other hand, both \(\zeta(\mu_B)\) and \(s(\mu_B)\) monotonically increase with \(\mu_B\) but the ratio \(\zeta/s(\mu_B)\) increases first and then decreases at high \(\mu_B\) domain. Next, Fig. 10(a), (b), and (c) reveal respectively the variation of total bulk viscosity \(\zeta\), entropy density \(s\) and their ratio with the variation of center of mass energy \(\sqrt{s}\) (Reader are requested to be careful on the same symbol used for entropy density and square of beam energy). The beam energy dependence of \(T\) and \(\mu_B\) used in computation are those obtained from fits to hadron yields. We have used the parameterization from Ref. [45]. We notice in Fig. 11 that \(\zeta\) (a) as well as \(\zeta/s\) (c) are decreasing with \(\sqrt{s}\), which is qualitatively agreeing with the results of earlier studies [32] [34]. The decreasing trend of \(\zeta\) and \(\zeta/s\) with \(\sqrt{s}\) can be understood from the fact that \(\mu_B\) decreases with \(\sqrt{s}\) while \(T\) remains fairly constant in the range of \(\sqrt{s}\) analyzed here and according to Fig. 9(a) and (c), the \(\zeta\) and \(\zeta/s\) decreases with decreasing of \(\mu_B\).
(dashed lines) and constant $\tau$ (solid lines), are compared with the results, obtained by Sasaki et al. (Green triangles down [20]), Deb et al. (Pink solid squares [23]), Chakraborty et al. (Brown stars [28]), Marty et al. (open circles [21]), Kadam et al. (Violet pluses [33]), Fraile et al. (Blue solid circles), Hostler et al. (Open squares [32]). We see a large numerical band for $\zeta (10^{-5}-10^{-2} \text{ GeV}^3)$ or $\zeta/s (10^{-5}-10^{0})$, within which earlier estimations are located. The results of the present work and Fraile et al. [37] both show similar kind of temperature dependence of $\zeta$ - it decreases at low $T$ domain ($<0.100 \text{ GeV}$) and then increases at high $T$ domain ($>0.100 \text{ GeV}$). Monotonically increasing nature of $\zeta(T)$ for constant value of $\tau$ (solid lines) discloses the fact that the origin of non-monotonic behavior of dashed lines are because of explicit structure of $\tau(\vec{k}, T, \mu_B = 0)$. The $\zeta(T)$ of Ref. [34] decreases up to $T \sim 0.150 \text{ GeV}$ after which a mild increment is observed. Most of the earlier works [20, 21, 23, 28, 33, 37] based on effective QCD model calculations [21, 22, 27, 28, 33] as well as effective hadronic model calculations [22, 23, 27, 28] predicted a decreasing function of $\zeta/s(T)$ in the hadronic temperature domain, which is qualitatively similar with our results (dashed lines). These are not supporting the fact that $\zeta/s$ diverges or becomes large near the transition temperature as indicated by Refs. [2, 4, 5], within the temperature domain of quark phase. Some of the effective QCD model calculations [22, 23, 27, 28] which can predict estimations of $\zeta/s$ in both temperature domain, exposed a peak structure near the transition temperature. While some of the HRG model calculations [33, 36] have supported this behavior by displaying an increasing tendency of $\zeta/s(T)$ as one goes towards the transition temperature from the hadronic temperature domain. This kind of increasing $\zeta/s(T)$ is also observed in our work when we consider the constant value of $\tau$ (solid lines). Regarding this two opposite nature of $\zeta/s(T, \mu_B = 0)$ within hadronic temperature domain, Ref. [24, 39] have exposed the possibility of both nature. Ref. [39] shows that inclusion Hagedorn states (HS) in HRG model can convert $\zeta/s(T)$ from decreasing to increasing function. In this context, our results for explicit $T, \mu_B$ dependent $\tau$ and constant value of $\tau$ are also displaying both type of nature. Taking shear viscosity $\eta(T, \mu_B = 0)$ from Ref. [13], based on same pion and nucleon thermal fluctuations, we get $\zeta/\{(1/3 - c_\pi^2)^2/2\} \eta \approx 5 - 4$ and $\zeta/\{(1/3 - c_\pi^2)^2/2\} \eta \approx 0.8 - 0.7$. This is supporting the estimation of gravity dual theory [45] instead of the relation $\zeta/\{(1/3 - c_\pi^2)^2/2\} \eta \approx 15$, followed by photon fields [51], scalar fields [31] or QCD theory [19]. So our estimation within the hadronic temperature domain is representing the strongly coupled picture instead of weakly coupled scenario [14]. Again, at high temperature domain, our numerical values of $\zeta/s$ are matching (after extrapolation) with high temperature values of Refs. [12, 52] - $\zeta/s(T \approx 0.200 - 0.400) \approx 0.002 - 0.001$, obtained from the perturbative QCD calculations. In this regard, our estimation is indicating a smooth transformation from the strongly coupled picture of the hadronic temperature domain to a weakly coupled medium of quarks, instead of divergence or peak structure of $\zeta/s$ near transition temperature.

IV. SUMMARY

We have gone through a detailed microscopic calculation of bulk viscosity coefficient for hadronic matter, where thermodynamical equilibrium conditions of all hadrons in medium have been treated by standard HRG model, which is very successful to generate LQCD thermodynamics up to the transition temperature. The thermal widths of medium constituents in the bulk viscosity expression inversely determine their numerical strength. Assuming pions and nucleons as most abundant medium constituents, we have concentrated on the bulk viscosity contributions from pion and nucleon components, where their corresponding thermal widths are derived from their in-medium scattering probabilities with different mesonic and baryonic resonances in the hadronic matter. Owing to the field theory version of optical theorem, the imaginary part of pion and nucleon self-energy (on-shell) at finite temperature give the estimation of their corresponding thermal widths. In the one-loop diagrams of pion self-energy, we have taken different mesonic and baryonic loops, while pion-baryon intermediate states are considered in the one-loop diagrams of nucleon self-energy. Their thermal widths are basically on-shell values of their corresponding Landau cut contributions, which disappear in the absence of medium and therefore, these are inversely interpreted as their respective relaxation time, which proportionally control the numerical strength of transport coefficients like $\zeta$. Our result show that $\zeta(T)$ at $\mu_B = 0$ increases in the high temperature domain $(0.080 < T(\text{GeV}) < 0.175)$ but a decreasing nature of $\zeta(T)$ has also been observed at low $T (< 0.08 \text{ GeV})$. The $\pi\pi$ and $\pi\rho$ loops of pion self-energy are respectively responsible for the decreasing and increasing nature of $\zeta(T)$ at low and high $T$ domain. Addition of baryon loops in pion self-energy mainly make $\zeta(T)$ reduce at high $T$ domain. Bulk viscosity for nucleon component monotonically increases with $T$. At finite $\mu_B$, the nucleon component of bulk viscosity is highly dominating over the pion component. Adding nucleon and pion components, the total $\zeta$ increases with both $T$ and $\mu_B$. However, after dividing by total entropy density, $\zeta/s$ appear as a decreasing function of $T$ and with the variation of $\mu_B$, it increases first at low $\mu_B$ region and then decreases at high $\mu_B$ region. Along the beam energy axis, the $\zeta$ and $\zeta/s$ both decreases, as noticed in some earlier works [33, 35].

During comparison with earlier results of $\zeta/s(T)$ at $\mu_B = 0$, one can notice that the qualitative as well as quantitative nature is not a very settled issue. Some of them [4, 4, 8] indicated divergence tendency of $\zeta/s$ near transition temperature, some of effective QCD model cal-
citations [22, 24, 27] revealed peak structure near transition temperature, whereas most of the effective QCD model calculations [20, 21, 24, 27, 28] as well as effective hadronic model calculations [32, 34–37], including our present work, predict a decreasing function of $\zeta/s(T)$ in the hadronic temperature domain, with few exceptional HRG calculations [33, 36]. Our decreasing $\zeta/s(T, \mu_B = 0)$ is representing a strongly coupled picture in the hadronic temperature domain, whose smooth extrapolation to high temperature domain agrees with a weakly coupled picture [13].

Acknowledgment: During first and major part of this work, SG is financially supported by the DST project with no NISER/R&D-34/DST/PH1002, (with title “Study of QCD phase Structure through high-energy heavy ion collisions” and principal investigator Prof. B. Mohanty). During the last part of the work, SG is supported from UGC Dr. D. S. Kothari Post Doctoral Fellowship under grant No. F.4-2/2006 (BSR)/PH/15-16/0660. SC acknowledges XIIth plan project no. 12-R&D-NIS-5.11-0300 and CNT project PIC XII-R&D-VECC-5.02.0500 for support. SG thanks to high energy group of NISER (Prof. B. Mohanty, Dr. A. Das, Dr. C. Jena, Dr. R. Singh, R. Haque, V. Bairathi, K. Nayak, V. Lyer, S. Kundu and others) and group of Calcutta University (Prof. A. Bhattacharyya, Prof. G. Gangopadhyay) for getting various academic and non-academic support at NISER and CU during this work and also to Dr. V. Roy and Prof. H. Mishra for some discussion regarding this work.

[1] P. Kovtun, D. T. Son, and O. A. Starinets, Phys. Rev. Lett. 94, 111601 (2005).
[2] D. Kharzeev and K. Tuchin, J. High Energy Phys. 09 (2008) 093.
[3] A. Bazavov et al. (HotQCD Collaboration), Phys. Rev. D 90, 094503 (2014).
[4] H. B. Meyer, Phys. Rev. Lett. 100, 162001 (2008).
[5] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B 663, 217 (2008).
[6] G. Torrieri and I. Mishustin, Phys. Rev. C 78, 021901 (2008).
[7] G. Sarwar, S. Chatterjee, Jane Alam arXiv:1312.0649[v1] [nucl-th].
[8] S. Mitra and S. Sarkar, Phys. Rev. C 90, no. 3, 034907 (2014).
[9] J. Noronha-Hostler, J. Noronha and F. Grassi, Phys. Rev. C 88, 044916 (2013).
[10] J. Noronha-Hostler, G. S. Denicol, J. Noronha, R. P. G. Andrade and F. Grassi, Phys. Rev. C 88, 044916 (2013).
[11] J. Noronha-Hostler, J. Noronha and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[12] J. Noronha-Hostler, G. S. Denicol, J. Noronha, R. P. G. Andrade and F. Grassi, Phys. Rev. C 88, 044916 (2013).
[13] J. Noronha-Hostler, J. Noronha, and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[14] J. Noronha-Hostler and K. Tuchin, J. High Energy Phys. 06 (2010) 085021 (2009).
[15] J. Noronha-Hostler, J. Noronha, and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[16] J. Noronha-Hostler, J. Noronha, and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[17] J. Noronha-Hostler, J. Noronha, and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[18] J. Noronha-Hostler, J. Noronha, and F. Grassi, Phys. Rev. C 90, no. 3, 034907 (2014).
[19] A. Bazavov, T. Bhattacharyya, M. Cheng, et al., Phys. Rev. D 80, 014504 (2009).
[20] S. Borsanyi et al., J. High. Ener. Phys. 0109, 73 (2010).
[21] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, J. High. Ener. Phys. 1201, 138 (2012).
[22] S. Ghosh, G. Krein, S. Sarkar, Phys. Rev. C 88, 045205 (2014).
[47] S. Ghosh, Braz. J. Phys. 44, 789 (2014).
[48] F. Karsch and K. Redlich, Phys. Lett. B 695, 136-142 (2011).
[49] P. Benincasa, A. Buchel, and A. O. Starinets, Nucl. Phys. B 733, 160 (2006); A. Buchel, Phys. Rev. D 72, 106002 (2005).
[50] S. Weinberg, Astrophys. J. 168, 175 (1971).
[51] R. Horsley and W. Schoenmaker, Nucl. Phys. B 280, 716 (1987).
[52] J. I. Kapusta, Relativistic Nuclear Collisions, Landolt-Bornstein New Series, Vol. I/23, ed. R. Stock (Springer-Verlag, Berlin Heidelberg 2010); L. P. Csernai, J. I. Kapusta, and L. D. McLerran, Phys. Rev. Lett. 97, 152303 (2006).