Article

Techno-Economic Power Dispatching of Combined Heat and Power Plant Considering Prohibited Operating Zones and Valve Point Loading

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Abstract: Co-generation units (i.e., combined heat and power plants—CHPs) are playing an important role in fulfilling the heat and power demand in the energy system. Due to the depletion of fossil fuels and rising carbon footprints in the environment, it is necessary to develop some alternatives as well as energy efficient operating strategies. By utilising the waste heat from thermal plants, cogeneration units help to decrease energy generation costs as well as reduce emitted pollutants. The combined heat and power economic dispatch (CHPED) operation strategy is becoming an important optimisation task in the energy efficient operation of a power system. The optimisation of CHPED is quite complex due to the dual dependence of heat and power in the cogeneration units. The valve point loading effect and prohibited operating zones of a thermal power unit make the objective function non-linear and non-convex. In this work, to address these issues more effectively, the viable operational area of the CHP and power system network losses are considered for the optimal allocation of power output and heat generation. A metaheuristic optimisation algorithm is proposed to solve the CHPED to minimize the fuel supply, thus satisfying the constraints. To handle equality and inequality constraints, an external penalty factor-based constraint handling technique is used.

The success of the proposed CHPED algorithm is tested on three considered cases. In all the cases, the results show the effectiveness in terms of solution accuracy and better convergence.

Keywords: cogeneration unit; combined heat and power (CHP) plant; constraint handling; techno-economic power dispatch

1. Introduction

Smart energy networks need intelligent operation for optimally managing electrical and thermal energy requirements. The electrical and thermal energy demands can be managed through combined heat and power (CHP) plants. CHP plants are effectively contributing to the efficient operation of electrical and thermal energy networks as well as to the reduction in carbon emissions, facilitating an increasing penetration of intermittent renewable energy sources. CHPs are designed and operated to provide stable thermal and electrical energy supplies, and they are capable to overcome the load dynamics. Over the years, the most pressing challenges in electric power system operation have been the economic load dispatches from conventional power plants [1,2]. The energy efficiency of combined cycle plants is around 50–60%, since a majority of the energy is lost during the energy conversion process. The cogeneration system needs to be implemented in thermal power plants for utilizing the waste heat in a useful way to reduce the overall energy cost and increase the environmental benefits. In the utility market, cogeneration units are used as district power plants for supplying the electrical and thermal energy demands as well as to facilitate intermittent renewable energy sources. The use of a CHP system can increase
fuel proficiency around 90%, reduce the operating charges by 10–40%, and decrease carbon emissions around 13–18% [3,4].

As a result, CHP economic dispatch (CHPED) can play an important role in increasing an overall system’s energy efficiency. The CHPED needs to be optimized considering different operational strategies (e.g., thermal or electrical prioritized operations, steam turbine operational limits, etc.). The CHEPD objective function becomes non-linear and non-convex due to the valve point loading (VPL) effect and prohibited operating zones (POZs) of a traditional thermal power system. Due to mutual dependency of heat and power in a cogeneration unit, the CHPED optimisation problem faces some technical challenges [5,6]. To better understanding, the feasible operating region of the cogeneration unit and transmission losses should be taken into consideration [7].

Several optimisation strategies are proposed to resolve the CHPED problem, and they are classified in two types: mathematical and meta-heuristic. Mathematical methods consist of dual and quadratic programming [8], Lagrangian relaxation [9], two-layer Lagrangian relaxation algorithm [10], branch and bound algorithms [11], etc. These methods are not sufficiently suitable to address the non-convex, discontinues, and non-differentiable fuel cost functions. Therefore, there is a significant challenge to resolve CHPED problems. As a result, new metaheuristic strategies for CHPED problem formulation can be used to solve these shortcomings. These metaheuristic algorithms demonstrate the ability to locate optimal solutions through a global search. As a result, researchers applied several swarm intelligence optimisation techniques to resolve CHPED complications and find better solutions. Initially, researchers used a metaheuristic algorithm to solve the CHPED problem on a small scale. For example, in [12], genetic algorithm with multiplier updating technique has been proposed with a penalty value to solve the CHPED issue. A stochastic model is introduced with improved particle swarm optimisation to solve CHPED in a real situation [13]. In [14], a self-adaptive real-coded GA (SARGA) based on mutation and crossover phenomena has been proposed for CHPED, and, to address the equality and inequality constraints, a penalty mechanism has been implemented. In [15], the authors have proposed a harmony search (HS) algorithm for minimising the fuel cost function by adjusting the pitch of basic search operators. In [16], author(s) have employed the firefly algorithm (FA), influenced by the behaviour of fireflies, as they attract each other at different rates depending on the intensity of the light. Therefore, this intensity of the light is a good analogy to optimise any objective function value. In [17], the authors have suggested a genetic algorithm based on an enhanced penalty function technique; to realize an equilibrium between exploration and probability, a distance-based constraint handling approach has been included to solve the CHPED problem. In [18], the authors have proposed cuckoo search optimisation (CSO) algorithm with penalty factor to minimise the overall fuel cost function accurately. However, in [19], the bender decomposition algorithm the CHPED problem has divided into master and sub-problem. The subproblem generates the bender cuts, which has been used to update the master problem. In [20], a new variant of PSO has been proposed, where only the largest particle is chosen whose fitness is greater than the average fitness value. Whereas in [21], the hybridization of genetic algorithm and harmony search has been introduced to solve the CHPED problem. In [22], non-convexity has been replaced by convex formulations by using the two binary variables. However, in [23], the authors have proposed simulated annealing with basic bio-geography-based optimisation, which increases the convergence capability and avoids finding local minima. The main disadvantage of mentioned algorithms is that they do not consider the valve point loading effects including losses. However, differential evolution (DE) and bee colony optimisation (BCO) overcome these drawbacks [24,25]. Similarly, in [26], the authors have proposed novel self-adaptive learning method to solve the CHPED problem considering VPL, ramp rate limit, reserve constraints, and network losses.

In [27], the authors have been proposed cuckoo search algorithm (CSA) to resolve the CHPED issue. CSA is relatively easy to implement and provides better alternatives for CHPED problems due to its fewer control parameters and computational timings. In [28],
the authors have used the gravity search algorithm (GSA) to explain the CHPED issue. The gravitation law of moving particles inspired GSA, which includes various physical quantities such as the gravitational constant, mass, Euclidean distance, gravitation, acceleration, and velocity. GSA gives a better solution for the CHPED problem, having minimal fuel prices by dynamic adjustment of control parameters. In [29], cuckoo optimisation has been applied as a powerful and robust solution technique for large scale units. An improved differential evolution algorithm has been proposed, which enhances the performance of basic DE with a modified repair process to handle equality constraints [30]. In [31], the authors have suggested a new optimisation method for analysing heat and power demand uncertainties using information gap decision theory and interval analysis to explain the CHPED optimisation process. In [32], a Lagrangian-relaxation-based alternative approach has used to divide the non-convex area of the CHP unit into several convex regions using the Big M method to solve the CHPED issue efficiently.

To improve system performance and accuracy, several other studies have been projected to explain the CHPED optimisation task. For example, in [33], the acceleration constants have dynamically altered to increase the optimum search and avoid premature convergence during CHPED optimisation. However, in [34], the authors have suggested an oppositional teaching learning-based (OTLBO) algorithm to improve CHP plant solution accuracy and speed of convergence. In [35], to solve CHPED effectively, the authors have suggested a crisscross optimisation algorithm in which two exploration operator crossings exist, flat and perpendicular. Whereas in [36], a real coded genetic algorithm (RCGA) through progressive transformation evolution has been applied on six benchmark systems of the CHP plant, considering VPL and network losses.

Thermal power plant’s prohibited operating zones (POZs) make the system more challenging due to the discontinuous nature of fuel supply. It increases the complexity to solve the CHPED system. To solve CHPED with POZs, only a few optimisation algorithms have been implemented. In [37,38], the author proposed group search optimisation to reduce the operating cost as well as enhance the solution accuracy of the CHP economic dispatch problem. To handle the non-convex CHPED problem, an oppositional-based group search optimisation based on guess and opposite guess has been developed [39]. The authors of [40] have suggested a heat transfer search technique that included three stages: conduction, convection, and radiation. In contrast, the authors of [41] have reported enhanced GA, employing unique crossover and mutation techniques. To recover mutation offspring and assist them in reaching a feasible range, an effective constraint handling technique has been used to resolve the CHPED issue. To address the CHPED problem with different constraints, a bio-geography-based learning particle swarm optimisation is developed, which uses migration operator to attain the best position [42]. In [43], the authors have proposed a modified PSO, where Gaussian random variables are taken to improve the global search of particles. However, in [44], a self-regulating PSO has given the higher speed of convergence, when considering VPL and POZs. Differential evolution with a dual population has been planned to solve the CHPED problem for multifuel selections of generating units [45]. In [46], differential evolution with migrating variables has been used to increase the direction of search and replace those variables that were targeted earlier. A mesh adaptive direct search algorithm has been used in CHP dispatch in [47], but it has not sufficiently covered impacts of POZs. Similarly, presented algorithms in [48,49] for CHP dispatching, POZs and VPLs have not been adequately considered for finding operations’ of CHPs.

It is required to investigate a more effective metaheuristic algorithm, which can solve a large and complex non-linear and non-convex CHPED problem. Metaheuristic algorithm can only provide the global optimum solutions. They cannot give accurate solutions. Most of the techniques explained by the different authors are complicated due to consideration of a large number of parameters. This paper focuses on the use of a simple algorithm to solve the CHPED problem, and it is based on [50]. In the considered algorithm, a random communication occurs between all the candidates throughout the iteration process. The
simplicity of the considered algorithm is that there are only two design parameters such as population size and number of iterations. To address the CHPED problem more accurately, the feasible operating region of the cogeneration unit and network losses are taken into account with the objective of minimizing fuel cost. To handle equality and inequality constraints, a suitable penalty factor-based constraint handling strategy has introduced. The VPLs and POZs are used for considering the feasible operating region of a CHP plant.

The CHEPD formulations have been presented in [51–55] and they have used in their presented soft computing techniques for CHP dispatch. The VPLs description have been provided in [56], but not covered sufficiently role of POZs. Also, some computing techniques have been described in [57–60] with CHP operations’ constraints for maintaining the heat and power balance operation within the considered energy network.

In this work, to validate the proposed algorithm, three typical test case systems, including 4-units, 7-units, and 24-units, are considered. The results of these cases are compared with the well-established algorithm to show the superiority among them. The following sections of the article are structured as: (i) Section 2 presents the mathematical modelling of the CHP economic dispatch optimisation problem; (ii) Section 3 represents the proposed algorithm; (iii) Section 4 provides the analysis of considered three cases; And (iv) finally, the conclusion and future scope of the article are given in Section 5.

2. Mathematical Modelling of CHPED
2.1. Objective Function

The main objective of the CHPED is to identify the most cost-effective way to schedule power and heat generation, to fulfil both the heat and power demand while satisfying all constraints. The mathematical equation, which is based on [18,33,56,59,60] for total operating cost function, is expressed in Equation (1).

\[
\text{Min } C = \sum_{i=1}^{N_{TH}} C_i(P_{i,TH}) + \sum_{j=1}^{N_{CHP}} C_j(P_{j,CHP}, h_{j,CHP}) + \sum_{k=1}^{N_{H}} C_k(P_{k,H}) \text{ ($/h$)}
\]  

(1)

where, \(C\) is entire operating price, \(C_i(P_{i,TH})\) is the fuel cost function of the ith thermal power unit, \(C_j(P_{j,CHP}, h_{j,CHP})\) is cost function of the jth CHP unit, \(C_k(P_{k,H})\) is the cost function of the kth heat-only unit, \(N_{TH}, N_{CHP}, N_{H}\) are the number of thermal power, cogeneration, and heat units. \(P_{i,TH}\) is the power output of the ith conventional thermal power unit, \(P_{j,CHP}\) and \(h_{j,CHP}\) are the power and heat outputs of the jth CHP unit, and \(P_{k,H}\) is the heat output of the kth heat unit.

The quadratic cost function (based on [18,33,56,59,60],) of the thermal power unit is expressed in Equation (2).

\[
\sum_{i=1}^{N_{TH}} C_i(P_{i,TH}) = \sum_{i=1}^{N_{TH}} [a_i + b_i P_{i,TH} + C_i P_{i,TH}^2] \text{ ($/h$)}
\]  

(2)

where, \(a_i, b_i,\) and \(C_i\) are its cost coefficients of the thermal power unit.

Due to VPL (i.e., valve-point loading) effects, objective function becomes nonconvex, nonlinear, and discontinuous, creating several local minima. A rectified sinusoidal function is needed to include in the cost function for the accurate modelling of VPL effects [53,54,56]. The mathematical equation of cost function, including VPL, is expressed in Equation (3).

\[
\sum_{i=1}^{N_{TH}} C_i(P_{i,TH}) = \sum_{i=1}^{N_{TH}} [a_i + b_i P_{i,TH} + C_i P_{i,TH}^2 + \epsilon_i \sin \left\{ f_i (P_{i,TH}^{min} - P_{i,TH}) \right\}] \| \text{ ($/h$)}
\]  

(3)

where, \(\epsilon_i\) and \(f_i\) are the cost coefficient of the ith unit for valve point loading in a conventional thermal power unit.
The mathematical equation of the cost function for the cogeneration unit and heat-only unit is expressed in Equations (4) and (5), and it is based on [18,33,56].

\[
\sum_{j=1}^{N_{\text{CHP}}} C_j \left( P_{\text{CHP}}^j, h_{\text{CHP}}^j \right) = \sum_{j=1}^{N_{\text{CHP}}} \left[ a_j + b_j P_{\text{CHP}}^j + c_j P_{\text{CHP}}^j h_{\text{CHP}}^j + d_j h_{\text{CHP}}^j + e_j h_{\text{CHP}}^{j2} + f_j h_{\text{CHP}}^j \right] \text{($)}/h) \tag{4}
\]

\[
\sum_{k=1}^{N_H} C_k \left( h_{\text{H}}^k \right) = \sum_{k=1}^{N_H} \left[ a_k + b_k h_{\text{H}}^k + c_k h_{\text{H}}^{k2} \right] \text{($)}/h) \tag{5}
\]

where, \( C_j \left( P_{\text{CHP}}^j, h_{\text{CHP}}^j \right) \) is the cost function of the \( j \)-th cogeneration unit; \( a_j, b_j, c_j, d_j, e_j, f_j \) are the cost coefficient of the \( j \)-th cogeneration unit; \( C_k \left( h_{\text{H}}^k \right) \) is the cost of the \( k \)-th heat-only units, and \( a_k, b_k, \) and \( c_k \) are the cost coefficient of the \( k \)-th heat-only unit.

### 2.2. Constraints

#### 2.2.1. Power Output Balance

The entire power generation of the thermal power and cogeneration units should be equal to the total power demand. If any network losses are existing, they would be added to the power demand shown in Equation (6), based on [18,33,56].

\[
\sum_{i=1}^{N_{\text{TH}}} P_{\text{TH}}^i + \sum_{j=1}^{N_{\text{CHP}}} P_{\text{CHP}}^j = P_d + P_{\text{loss}} \tag{6}
\]

where \( P_d \) is generated power demand, \( P_{\text{loss}} \) is transmission line losses, determined by Kron’s loss formulation [55], and it is expressed by Equation (7), based on [18,33,56].

\[
P_{\text{loss}} = \sum_{i=1}^{N_{\text{TH}}} \sum_{j=1}^{N_{\text{TH}}} P_{\text{TH}}^i B_{ij} P_{\text{TH}}^j + \sum_{i=1}^{N_{\text{TH}}} \sum_{j=1}^{N_{\text{CHP}}} P_{\text{TH}}^i B_{ij} P_{\text{CHP}}^j + \sum_{i=1}^{N_{\text{CHP}}} \sum_{j=1}^{N_{\text{CHP}}} P_{\text{CHP}}^i B_{ij} P_{\text{CHP}}^j + \sum_{i=1}^{N_{\text{CHP}}} P_{\text{CHP}}^i B_{oi} + \sum_{j=1}^{N_{\text{CHP}}} P_{\text{CHP}}^j B_{oj} + B_{oo} \tag{7}
\]

where \( B_{ij}, B_{oi}, B_{oo} \) are the B-matrix coefficient of the transmission line.

#### 2.2.2. Heat Production Balance

The entire heat produced through the cogeneration and heat unit should be equivalent to total heat demand, expressed in Equation (8), based on [18,33,56].

\[
\sum_{j=1}^{N_{\text{CHP}}} h_{\text{CHP}}^j + \sum_{k=1}^{N_H} h_{\text{H}}^k = h_d \tag{8}
\]

where \( h_{\text{CHP}}^i \) is the heat output of the \( i \)-th CHP unit, \( h_{\text{H}}^k \) is the heat output of the \( k \)-th heat unit, and \( h_d \) is heat demand.

#### 2.2.3. Capacity Limits of Thermal Power Unit

The capacity limits of thermal power units are required and expressed in Equation (9), based on [18,33,56].

\[
P_{\text{TH}}^{\text{min}} \leq P_{\text{TH}}^i \leq P_{\text{TH}}^{\text{max}}; \text{where } i = 1, \ldots, N_{\text{TH}} \tag{9}
\]

where \( P_{\text{TH}}^{\text{min}} \) and \( P_{\text{TH}}^{\text{max}} \) show the lower and higher limits of the \( i \)-th thermal power unit in MW.
2.2.4. Capacity Limits of Cogeneration Unit

The capacity limits of CHP units are required and expressed in Equations (10) and (11), and they are based on [18,33,56].

\[
P_{\text{CHP, min}}^j(h_{\text{CHP}}^j) \leq P_{\text{CHP}}^j \leq P_{\text{CHP, max}}^j(h_{\text{CHP}}^j) \quad \text{where } j = 1, \ldots, N_{\text{CHP}} \tag{10}
\]

\[
h_{\text{CHP, min}}^j(P_{\text{CHP}}^j) \leq h_{\text{CHP}}^j \leq h_{\text{CHP, max}}^j(P_{\text{CHP}}^j) \quad \text{where } j = 1, \ldots, N_{\text{CHP}} \tag{11}
\]

where \( P_{\text{CHP, min}}^j \) and \( P_{\text{CHP, max}}^j \) are the lower and higher limits of the power generation of the jth cogeneration unit in MW, and the function of heat generation given by \( h_{\text{CHP}}^j \), \( h_{\text{CHP, min}}^j \), and \( h_{\text{CHP, max}}^j \) are the min and max heat produced by the jth CHP unit in MWth and the function of generated power given by \( P_{\text{CHP}}^j \).

2.2.5. Capacity Limits of Heat Unit

The capacity limits of only heat units are required and expressed in Equation (12), based on [18,33,56].

\[
h_{k, \text{min}} \leq h_k \leq h_{k, \text{max}} \quad \text{where } k = 1, \ldots, h_k \tag{12}
\]

where, \( h_{k, \text{min}} \) and \( h_{k, \text{max}} \) show the minimum and maximum limit of the heat unit k, expressed in MWth.

2.2.6. Constraint of Prohibited Operating Zones (POZs)

Due to some problems in the machinery or its components, for example pumps or boilers, practical generating units include the forbidden operating zone. The input-output characteristic of a device having prohibited operation zones (POZs) is discontinuous [53,54]. The prohibited zones of thermal power units are expressed by Equation (13), based on [18,33,53–56].

\[
\begin{aligned}
P_{\text{TH, min}}^{i,m-1} \leq P_i & \leq P_{\text{TH, max}}^{i,m-1}, \text{ where } m = 2, 3, \ldots, Z_i \\
P_i & \leq P_{\text{TH, min}}^{i,zi} \leq P_i \leq P_{\text{TH, max}}^{i}\end{aligned}
\]

where, \( P_{\text{TH, min}}^{i,zi} \) and \( P_{\text{TH, max}}^{i} \) are the minimum and maximum limit of the mth POZ of the ith conventional thermal power unit. \( Z_i \) represents the quantity of POZs for the ith generation unit.

2.3. Constraint Handling Technique

For single-objective functions, handling equality and inequality restrictions are accomplished via external penalty parameters, which punish infeasible solutions throughout the iterative process of transforming the constrained CHPED issue into an unconstrained problem. After multiple trials, the right values of the static penalty factors must be carefully adjusted in order to achieve better optimal solutions, while adhering to the limitations [18]. This method is also known as the sequence of unconstrained minimization techniques. Since different constraints have different orders of magnitude [18,33,56], it is more appropriate to normalise each constraint; for example, the ith equality constraint \( G_i(Y) - \theta_i \leq 0 \) and the jth inequality constraint \( H_j(Y) - \eta_j \leq 0 \) can be normalised to \( G_i(Y) = G_i(Y)/\theta_i - 1 = 0 \) and \( H_j(Y) = H_j(Y)/\eta_j - 1 \leq 0 \). The optimisation issue can be written in compact form after normalising the constraints, where \( \theta_i \) and \( \eta_j \) are constants, and \( Y = (Y_1, Y_2, \ldots, Y_N) \) represent n number of state variables. The optimisation problem can be written in compact form after normalising the constraints in the following manner:

\[
\text{Min } C (Y_1, Y_2, \ldots, Y_N)
\]
Subject to constraints

\[ G_i(Y_1, Y_2, \ldots, Y_N) = 0; \text{ where } i = 1, 2, \ldots, Ne \]

\[ H_j(Y_1, Y_2, \ldots, Y_N) \leq 0; \text{ where } j = 1, 2, \ldots, Nie \]

\[ y_k^{\text{min}} \leq y_k \leq y_k^{\text{max}}; \text{ where } k = 1, \ldots, N \]

where \( N, Ne, Nie \) are the number of unknown state variables, equality constraints, and inequality constraints, respectively [18,33,53-56]. The modified objective function is expressed in Equation (14).

\[
F = \text{Min } C(Y_1, Y_2, \ldots, Y_N) + R \left( \sum_{i=1}^{Ne} |G_i(Y_1, Y_2, \ldots, Y_N)|^2 \right) + \sum_{j=1}^{Nie} |\text{Max}(0, H_j(Y_1, Y_2, \ldots, Y_N))|^2 \quad (14)
\]

where \( R \) is a suitable penalty value, and its suitable value is estimated after several trials to get the best solution.

3. Optimisation Algorithm

The issues (discussed in Section 2) need to be addressed more effectively. For the viable operational area of the CHP, power system network losses or an optimal allocation of power output and heat generation is required. In this work, a metaheuristic optimisation algorithm is proposed to solve the CHPED to minimise the fuel supply, thus satisfying the constraints. To handle equality and inequality constraints, an external penalty factor-based constraint handling technique is used. This paper focuses on use of a simple algorithm to solve the CHPED problem and it is based on [50]. The programming technique is outlined in the steps, given below. The algorithm flow chart is shown in the Figure 1.

![Flow chart of CHPED optimization.](image)

Figure 1. Flow chart of CHPED optimization.

Step 1. Design the fitness function: Formulate the mathematical equation as a fitness function \( f(x) \) to be minimised.
Step 2. Initialization of system input parameters: Input the CHPED designing variable for the power and heat units, electrical power demand and heat demand, population size, and termination criteria. Consider the number of design variables, and population size and candidate solution are set to be m and n, respectively.

Step 3. Select the required solutions: Find the minimum value of fuel cost $F(X)$ of CHP economic dispatch for the best particle as $F(X)_{\text{best}}$ and the maximum value of $F(X)$ as $F(X)_{\text{worst}}$ throughout the algorithm step. Identify the corresponding power and heat generation $X_{j,k,i}$ of the CHP plant, according to the minimum and maximum values of fuel cost. Where, $X_{j,k,i}$ is the value of the jth design variable during the ith iteration for the kth particle.

Step 4. Modify the solutions: Modify the solution based on the minimum and maximum fuel cost of the CHP system and the random interaction between the generated values of power and heat. Modifying the power and heat generation values, marked by the following Equation (15), which is based on [50]:

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i} \left( X_{j,\text{best},i} - |X_{j,\text{worst},i}| \right) + r_{2,j,i} \left( |X_{j,k,i} \text{ or } X_{j,l,i}| - |X_{j,l,i} \text{ or } X_{j,k,i}| \right)$$  (15)

where $X_{j,\text{best},i}$ is the best particle value for variable j during the ith iteration, $X_{j,\text{worst},i}$ is the worst particle value for variable j during the ith iteration, $X'_{j,k,i}$ is the updated value of $X_{j,k,i}$, and $r_{1,j,i}$ and $r_{2,j,i}$ are two random numbers for the jth variable during the ith iteration between (0,1).

$X_{j,k,i}$ or $X_{j,l,i}$ shows any particle result k is related with any randomly chosen particle solution i, and information is replaced based on their fitness value. If the objective function value of the kth particle is better than the objective function value of the ith solution, then $X_{j,k,i}$ or $X_{j,l,i}$ is replaced with $X_{j,k,i}$. However, if the objective function value of the ith solution is better than the objective function value of the kth solution, then $X_{j,l,i}$ or $X_{j,k,i}$ is replaced with $X_{j,l,i}$. Similarly, if the objective function value of the ith solution is better than the objective function value of the kth solution, then $X_{j,l,i}$ or $X_{j,k,i}$ becomes $X_{j,l,i}$. On the other hand, if the objective function value of the kth solution is better than the objective function value of the ith solution, then “$X_{j,l,i}$ or $X_{j,k,i}$” becomes $X_{j,k,i}$.

Step 5. Report the required solutions: As a check, if the new value of fuel cost $F(X)_{\text{new}}$ is corresponding to the modified value of power and heat generation $X'_{j,k,i}$, then take the modified value in place of the previous value, otherwise, keep the previous value. Repeat the same process until the termination criteria has been satisfied and the optimum solution has been found out.

4. Results and Discussion

CHP units have their own feasible operating zones (FORs), and they are required for finding the operational points of them, considering the combination of power and thermal supplies. In this work, the four types of FOR of CHP units are considered, based on [18], and they are depicted in Figure 2. Figure 2a shows the FORs of a type-1 CHP unit. It includes the second unit of test system 1, the fifth unit of test system 2, and the fourteenth and sixteenth units of test system 3. Figure 2b shows the FORs of the type-2 CHP unit, which includes the third unit of test system 1, the sixth unit of test system 2, and the fifteenth and seventeenth units of test system 3. Figure 2c shows the FORs of the type-3 CHP unit, and it considers the eighteenth unit of test system 3. Figure 2d shows the FORs of type-4 CHP, and it includes the nineteenth unit of test system 3. Due to the random results of the stochastic algorithm, the effective solutions are receiving over 50 independent trials. The statistical simulation results of minimum cost, maximum cost, mean cost, and computational time are analysed. To prove the utility of the proposed optimisation technique, based on [50], for the CHPED issue, three case systems have been considered, and the results have been compared with other techniques available in the literature.
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Figure 2. CHP units’ feasible operating regions (FOR) (a) Type-1; (b) Type-2; (c) Type-3; (d) Type-4 [18].

4.1. Test System 1

Test system 1 includes four units having one thermal power unit, two cogeneration units, and a heat unit. Test structure data has taken from [33]. The power demand $P_d$ and heat demand $h_d$ are 200 MW and 115 MWth, respectively. The two cases are used for analysing the proposed CHPED technique.

4.1.1. Case 1

The VPL effects have been considered in Case 1. The population size and maximum number of iterations are set to 50 and 150. Table 1 shows the best allocation of power and heat generation from the proposed CHPED technique. Table 2 summarises the statistical study in terms of minimum, mean, maximum, and computational time, including those of GA-PF [17], FA [16], MADS-LHS [47], MADS-PSO [47], MADS-DACE [47], CSA [27], TVAC-PSO [33], MCSA [27], GWO [37], and IGA-NCM [41]. It is observed from Table 2, after applying the proposed CHPED algorithm, that the minimum cost obtained is 9256.0075 ($/h). Table 2 shows that the cost is the lowest of all the other approaches. The minimum fuel cost convergence curve found using the suggested CHPED algorithm is shown in Figure 3a.

Table 1. Best allocation of power output and heat generation for case 1 of test system 1.

| Algorithm | $P_1$ | 0 | $P_3$ | 40 | $H_3$ | 75.2427 | $P_2$ | 159.9358 | $H_2$ | 39.8401 | $H_4$ | 0 |

Table 2. Statistical analysis for case 1 of test system 1.

| Algorithm | GA-PF [17] | FA [16] | MADS-LHS [47] | MADS-PSO [47] | MADS-DACE [47] | CSA [27] | TVAC-PSO [33] | MCSA [27] | GWO [37] | IGA-NCM [41] | Proposed CHPED Algorithm |
|-----------|------------|--------|----------------|----------------|----------------|--------|----------------|--------|--------|----------|------------------------|
| Maximum cost ($/h) | - | - | 10,005.3805 | 9997.6576 | 9260.4317 | 9303.183 | - | - | - | 9257.9014 | 9352.1458 |
| Mean cost ($/h) | - | - | 9547.9151 | 9531.277 | 9257.5148 | 9258.168 | - | - | - | 9257.1553 | 9295.0657 |
| Minimum cost ($/h) | 9267.28 | 9257.10 | 9301.357 | 9257.07 | 9257.07 | 9257.07 | 9257.07 | 9257.07 | 9257.07 | 9257.07 | 9256.0075 |
| Computational time (s) | - | - | 7.0422 | 7.5994 | 2.3078 | 0.59 | 1.33 | 1.35 | 1.082 | 2.66 | 1.846 |
4.1.2. Case 2

Both the VPL effects and POZs of conventional thermal power units are considered in Case 2. The test case 2 data for prohibited operating zones have been taken from [41]. Using proposed CHEPD algorithm, the results are analysed. The population size and maximum number of iterations are set to 50 and 150, respectively. The power output and heat generation towards minimum fuel cost is summarised in Table 3. The minimum cost, mean cost, maximum cost, and computational time are shown in Table 4, including those of GSO [38], OGSO [39], DEGM [51], and DE [51]. It is clear from Table 4, after applying the proposed CHEPD algorithm, that the obtained minimum cost is 9262.8770 ($/h), which is less when compared to other reported results. Figure 3b shows the fuel cost convergence characteristic.

**Table 3.** Best allocation of power output and heat generation for case 2 of test system 1.

|        | P1 | P2  | P3   | H2  | H3   | H4  | Power Demand (MW) |
|--------|----|-----|------|-----|------|-----|-------------------|
|        | 0  | 159.7143 | 40.2763 | 40.6534 | 74.3513 | 0   | 600               |

**Table 4.** Statistical analysis for case 2 of test system 1.

| Algorithm | GSO [38] | OGSO [39] | DEGM [51] | DE [51] | Proposed CHEPD Algorithm |
|-----------|----------|-----------|-----------|---------|--------------------------|
| Maximum cost ($/h) | 9292.6631 | 9290.7735 | 9290.5810 | 9292.4286 | 9477.2720 |
| Mean cost ($/h)   | 9291.7326 | 9290.6026 | 9290.5331 | 9291.6290 | 9324.8554 |
| Minimum cost ($/h) | 9291.2717 | 9290.5459 | 9290.4804 | 9291.1375 | 9262.8770 |
| Computational time (s) | 1.846 | 1.33 | 1.082 | 1.35 | 1.988 |

4.2. Test System 2

Test system 2 includes four thermal power unit, two cogeneration units, and a heat unit. Moreover, transmission network losses are considered in test system 2. Test system 2 data have been taken from [33]. The power demand and heat demand are 600 MW and 150 MWth. Two cases are selected for test system 2, and the considered transmission line loss matrix coefficients are based on [56].

4.2.1. Case 1

The VPL effect of thermal power unit is considered in case 1. The population size and maximum number of iterations are set to 50 and 500. Table 5 shows the best values of power and heat generation obtained from the proposed CHPED algorithm. Table 6 summarises the statistical summary in terms of minimum cost, mean cost, maximum cost, and computational time, including those of BCO [25], TVAC-PSO [33], TLBO [34],
OTLBO [34], AIS [48], EP [48], PSO [48], CSA [27], OGSO [39], and IGA-NCM [41]. It is observed from Table 6, after applying the proposed CHPED algorithm, the minimum cost obtained is 10,079.2151 ($/h). Table 6, shows that the cost is the lowest of all the other approaches. The minimum fuel cost convergence curve is depicted in Figure 4a.

Table 5. Best allocation of power output and heat generation for case 1 of test system 2.

| Algorithm | BCO [28] | TVAC-PSO [33] | TLBO [34] | OTLBO [34] | AIS [48] | EP [48] | PSO [48] | CSA [27] | OGSO [39] | IGA-NCM [41] | Proposed CHPED Algorithm |
|-----------|-----------|----------------|-----------|------------|----------|--------|---------|---------|-----------|--------------|------------------------|
| Maximum cost ($/h) | - | - | 10,133.6130 | 10,106.8314 | - | - | - | 12,602.920 | 10,097.3801 | 10,108.6241 | 10,107.6677 |
| Mean cost ($/h) | - | - | 10,114.1539 | 10,099.4057 | - | - | - | 10,878.882 | 10,095.8455 | 10,096.8481 | 10,090.9149 |
| Minimum cost ($/h) | 10,317 | 10,100.3164 | 10,094.3529 | 10,355 | 10,390 | 10,613 | 10,177.6 | 10,094.2407 | 10,097.3801 | 10,096.8481 | 10,079.2151 |
| Computational time (s) | 5.1563 | 3.25 | 2.86 | 3.06 | 5.2956 | 5.3274 | - | 1.2 | 2.5681 | 7.69 | 8.023 |

Figure 4. Minimum cost convergence curve of test system 2 for two cases, listed as (a) case 1 and (b) case 2.

4.2.2. Case 2

Both the VPL effect and POZs of the thermal power unit are considered in Case 2. The test case data of POZs have been taken from [41]. The population size and maximum number of iterations are set to 50 and 500. The power output and heat generation towards minimum fuel cost is summarised in Table 7. Table 8 compares the statistical results of minimum cost, mean cost, maximum cost, and computational time of the proposed CHPED algorithm and other algorithms, including those of GSO [38], OGSO [39], IGA-NCM [41], and BLPSO [42]. It is clear from Table 8, after applying the proposed CHPED algorithm, the minimum cost obtained is 10,075.3202 ($/h), which is less when compared to other reported results. The minimum fuel cost convergence curve is shown in Figure 4b.

Table 6. Statistical analysis for case 1 of test system 2.

| Algorithm       | BCO [28] | TVAC-PSO [33] | TLBO [34] | OTLBO [34] | AIS [48] | EP [48] | PSO [48] | CSA [27] | OGSO [39] | IGA-NCM [41] | Proposed CHPED Algorithm |
|-----------------|----------|----------------|-----------|------------|----------|--------|---------|---------|-----------|--------------|------------------------|
| Maximum cost    | -        | -              | 10,133.6130 | 10,106.8314 | -        | -      | -       | 12,602.920 | 10,097.3801 | 10,108.6241 | 10,107.6677 |
| Mean cost       | -        | -              | 10,114.1539 | 10,099.4057 | -        | -      | -       | 10,878.882 | 10,095.8455 | 10,096.8481 | 10,090.9149 |
| Minimum cost    | 10,317   | 10,100.3164    | 10,094.3529 | 10,355     | 10,390   | 10,613 | 10,177.6 | 10,094.2407 | 10,097.3801 | 10,096.8481 | 10,079.2151 |
| Computational time (s) | 5.1563 | 3.25 | 2.86 | 3.06 | 5.2956 | 5.3274 | - | 1.2 | 2.5681 | 7.69 | 8.023 |

Table 7. Best allocation of power output and heat generation for case 2 of test system 2.

| P1    | 47.0388 | P3    | 112.5798 | P5    | 92.0439 | H5    | 28.7711 | H7    | 46.0278 |
|-------|---------|-------|----------|-------|---------|-------|---------|-------|---------|
| P2    | 98.5481 | P4    | 209.8352 | P6    | 40.0004 | H6    | 75.1990 |       |         |
Table 8. Statistical analysis for case 2 of test system 2.

| Algorithm           | GSO [38] | OGSO [39] | IGA-NCM [41] | BLPSO [42] | Proposed CHEPD Algorithm |
|---------------------|----------|-----------|--------------|------------|--------------------------|
| Maximum cost ($/h)  | 10,103.7203 | 10,102.3585 | 10,116.3024 | 10,102.1864 | 10,080.2645              |
| Mean cost ($/h)     | 10,102.2168 | 10,101.8566 | 10,115.0872 | 10,101.5626 | 10,077.4625              |
| Minimum cost ($/h)  | 10,101.3483 | 10,101.3160 | 10,114.8410 | 10,101.3079 | 10,075.3202              |
| Computational time (s) | 2.5903 | 2.8073 | 8.17 | 6.18 | 9.536 |

4.3. Test System 3

Test system 3 is a combination of twenty-four thermal power units, six cogeneration units, and five heat units. The test data have been taken from [33]. The power demand $P_d$ and heat demand $h_d$ are 2350 MW and 1250 MWh. The following two cases are considered.

4.3.1. Case 1

The VPL effects of thermal power units are considered. Here, the population size and the maximum number of iterations are 50 and 1000. Table 9 shows the best allocation of power and heat generation. Table 10 gives the statistical analysis in terms of the minimum cost, mean cost, maximum cost, and computational time obtained from the proposed CHEPD algorithm, including those of CPSO [33], TVAC-PSO [33], TLBO [34], GSA [28], and GSO [49]. Figure 5a depicts the minimum fuel cost convergence curve obtained from the proposed CHEPD algorithm hm. Table 10 shows that the minimum fuel cost 57,861.1978 ($/h), obtained from the proposed CHEPD algorithm, is minimal when compared to other algorithms.

Table 9. Best allocation of power output and heat generation for case 1 of test system 3.

| P1  | 628.3130 | P11 | 40.0243 | h15 | 81.0654 |
|-----|----------|-----|---------|-----|---------|
| P2  | 359.9973 | P12 | 55.0398 | h16 | 105.4930 |
| P3  | 0.0045   | P13 | 55.0067 | h17 | 77.3073 |
| P4  | 159.7227 | P14 | 81.0460 | h18 | 40.5253 |
| P5  | 109.8682 | P15 | 40.0038 | h19 | 20.7161 |
| P6  | 60.0000  | P16 | 81.0099 | h20 | 459.2565 |
| P7  | 159.7445 | P17 | 40.0119 | h21 | 59.9983 |
| P8  | 159.7982 | P18 | 10.0000 | h22 | 59.9780 |
| P9  | 159.6977 | P19 | 35.0186 | h23 | 120.0000 |
| P10 | 115.7025 | h14 | 105.6650| h24 | 119.9832 |

Table 10. Statistical analysis for case 1 of test system 3.

| Algorithm          | CPSO [33] | TVAC-PSO [33] | TLBO [34] | GSA [28] | GSO [49] | Proposed CHEPD Algorithm |
|--------------------|-----------|---------------|-----------|----------|----------|--------------------------|
| Maximum cost ($/h) | 60,076.6903 | 58,359.5520 | 58,038.5273 | -         | 58,453.9227 | 57,864.0758              |
| Mean cost ($/h)    | 59,853.478 | 58,198.3106 | 58,014.3685 | -         | 58,217.0254 | 57,862.6973              |
| Minimum cost ($/h) | 59,736.2635 | 58,122.7460 | 58,036.9992 | 58,121.8640 | 58,122.7088 | 57,861.1978              |
| Computational time (s) | 53.36 | 52.25 | 5.67 | 26.483 | 54.31 | 10.469 |

Figure 5. Minimum cost convergence curve of test system 3 for two cases, listed as (a) case 1 and (b) case 2.
4.3.2. Case 2

The VPL and POZs of the thermal power unit are considered in case 2. The test case data of the prohibited operating zones have been taken from [41]. The population size and maximum number of iterations are set to 50 and 1000. The power output and heat generation towards minimum fuel cost is summarised in Table 11. The minimum cost, mean cost, maximum cost, and computational time is shown in Table 12, including those of GSO [38], OGSO [39], CCO-PPS [52], and HTS [40]. It is clear from Table 12, after applying the proposed CHPED algorithm, that the minimum cost found is $57,895.6050/h, which is less when compared to other reported results. The minimum fuel cost convergence curve obtained from the proposed CHPED algorithm is shown in Figure 5b.

Table 11. Best allocation of power output and heat generation for case 2 of test system 3.

| P1  | P2  | P3  | P4  | P5  | P6  | P7  | P8  | P9  | P10 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 628.9019 | 359.7078 | 0.0210 | 109.8662 | 159.7324 | 60.0000 | 109.8561 | 159.7088 | 114.5164 |
| 77.3854 | 55.0000 | 118.2993 | 81.0225 | 40.0000 | 81.0000 | 10.0000 | 35.0000 | 109.9887 |
| 79.2938 | 105.8367 | 78.4357 | 40.8931 | 21.0661 | 454.0547 | 59.9983 | 120.0000 | 120.0000 |

Table 12. Statistical analysis for case 2 of test system 3.

| Algorithm | GSO [38] | OGSO [39] | CCO-PPS [52] | HTS [40] | Proposed CHPED Algorithm |
|-----------|-----------|------------|--------------|----------|-------------------------|
| Maximum cost ($/h) | 58,119.1635 | 57,953.3522 | 57,945 | 57,960.73 | 57,897.8129 |
| Mean cost ($/h) | 58,114.6060 | 57,946.0934 | 57,940 | 57,959.92 | 57,896.7356 |
| Minimum cost ($/h) | 58,110.0900 | 57,942.5577 | 57,935 | 57,959.41 | 57,895.6050 |
| Computational time (s) | 5.8017 | 5.9869 | 4.43 | 6.6877 | 11.271 |

The presented methodology can be used in a power system network integrated with thermal power plants, CHP plants, and heat supply units, for providing heat and power demand within the considered network. The CHPED algorithm can be implemented in coordinating the economic dispatch of power and heat from the power plants, considering the technical constraints as well as transmission network’s line loss coefficients.

5. Conclusions and Future Scope

In this work, the combined heat and power economic dispatches of thermal and power supplies, from the combinations of power units, have used the proposed CHPED optimisation algorithm. The proposed CHPED algorithm is used to find thermal and power dispatches, considering the valve point loading effects as well as prohibited operating zones of conventional thermal power units, feasible operating region of cogeneration units, and transmission network losses. The considered three case systems have been analysed for the CHPED, and the obtained results have demonstrated the effectiveness of the proposed algorithm. The obtained results of three case systems are compared with the well-established algorithms, and it is observed that the proposed CHPED algorithm provides the best results for the economic dispatch of thermal and power supplies from the combinations of different power units. To achieve the feasible optimum results in the CHPED problem, all equality and inequality constraints are handled by the exterior static penalty method considered in the proposed CHEPD algorithm. Through considered for different test systems and case studies, it is observed that the presented method for CHEPD is suitable for small- and large-scale power units. The research work presented in this article is limited to combined heat and power dispatching that consider the VPL and POZs of a thermal power unit.
Therefore, the presented work can be further enhanced for integrating the combined heat and power units with intermittent renewable energy resources, for the optimum dispatching of power to maintain secure and reliable energy supplies. Moreover, the other constraints of thermal power units, such as ramp rate limit and reserve constraints, can be included for making operation more reliable and stable.

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**Abbreviations**

| Abbreviation | Description |
|--------------|-------------|
| AIS          | artificial immune system |
| BCA          | bee colony algorithm |
| BCO          | bee colony optimisation |
| BLPSO        | biogeography-based learning particle swarm optimisation |
| CHPED        | combined heat and power economic dispatch |
| CPSO         | classic particle swarm optimisation |
| CSA          | cuckoo search algorithm |
| CSO-PPS      | civilized swarm optimisation and Powell’s pattern search |
| DE           | differential evolution |
| DEGM         | differential evolution with Gaussian mutation |
| EP           | Evolutionary programming |
| FA           | firefly algorithm |
| GA           | genetic algorithm |
| GA-PF        | genetic algorithm-based penalty function |
| GWO          | grey wolf optimisation |
| GSA          | gravitational search algorithm |
| GSO          | group search optimisation |
| HS           | harmony search |
| HTS          | heat Transfer Search |
| IDE          | improved differential evolution |
| GA-NCM       | improved genetic algorithm with novel crossover and mutation |
| MADS-DACE    | mesh adaptive direct search-design and analysis of computer experiment |
| MADS-LHS     | mesh adaptive direct search-Latin hypercube |
| POZ          | prohibited operating zone |
| SARGA        | self-adaptive real-coded genetic algorithm |
| TVAC-PSO     | time-varying acceleration coefficients particle swarm optimisation |
| TLBO         | teaching learning-based optimisation |
| VPL          | valve point loading |
| \( h_{CHP}^i \) | heat output of ith CHP unit |
| \( h_k \) | heat output of kth heat unit |
| \( h_d \) | heat demand |
| \( h_{H,min}^k \) | minimum limit for the heat output of kth heat unit |
| \( h_{H,max}^k \) | maximum limit for the heat output of kth heat unit |
| \( h_{CHP, min}^j \) | minimum limit of heat produced of jth CHP unit associated with generated power \( p_{CHP}^j \) |
\[ h_{\text{CHP}, \text{max}} (p_{\text{CHP}}) \]

maximum limit of heat produced of jth CHP unit associated with generated power \( p_{\text{CHP}, i} \)

\( m \)

number of design variables

\( n \)

number of candidate solutions

\( N_{\text{TH}} \)

number of conventional thermal power units

\( N_{\text{H}} \)

number of heat units

\( N_{\text{CHP}} \)

number of CHP units

\( p_d \)

generated power demand

\( p_{\text{loss}} \)

transmission line losses

\( p_{\text{TH}} \)

power output of i\textsuperscript{th} conventional thermal power unit

\( p_{\text{TH}, \text{min}} \)

minimum limit for power output of i\textsuperscript{th} conventional thermal power unit

\( p_{\text{TH}, \text{max}} \)

maximum limit for power output of i\textsuperscript{th} conventional thermal power unit

\( p_{\text{CHP}} \)

power output of j\textsuperscript{th} CHP unit

\( p_{\text{CHP}, \text{min}} (h_{\text{CHP}}) \)

minimum limit of power generation of j\textsuperscript{th} cogeneration unit associated with generated heat

\( p_{\text{CHP}, \text{max}} (h_{\text{CHP}}) \)

maximum limit of power generation of j\textsuperscript{th} cogeneration unit associated with generated heat

\( p_{i,m} \)

minimum limit of m\textsuperscript{th} POZ of i\textsuperscript{th} conventional thermal power unit

\( p_{i,m}^\text{max} \)

maximum limit of m\textsuperscript{th} POZ of i\textsuperscript{th} conventional thermal power unit

decision variables

\( x_1, x_2, \ldots, x_n \)

value of the j\textsuperscript{th} variable during the i\textsuperscript{th} iteration for the k\textsuperscript{th} candidate

\( X_{j,k,i} \)

value of best candidate for variable j during i\textsuperscript{th} iteration

\( X_{j,\text{worst},i} \)

value of worst candidate for variable j during i\textsuperscript{th} iteration

\( X_{j,k,i}^{*} \)

updated value of \( X_{j,k,i} \)

\( r_{1,j,i}, r_{2,j,i} \)

random numbers for j\textsuperscript{th} variable during i\textsuperscript{th} iteration

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