THE STUDY OF THE STIFFNESS COEFFICIENT OF THE SEAM DEPENDING ON THE QUANTITY OF SYMMETRICALLY LOCATED SHIFT CONNECTIONS IN AN OVAL TWO-LAYER PLATE

Andrey Viktorovich Turkov1* Ekaterina Karpova2
1Southwest State University, Faculty of Building and Architecture, Kursk, Russia
2Orel State University, Orel, Russia

The paper considers a two-layer isotropic plate on pliable connections, for which the theory of calculation of composite plates by A.R. Rzhanitsin was adopted. With the help of bending moments Mx(My), the numerical stiffness coefficient of the seam is calculated. The numerical basic studies were carried out on an oval two-layer plate in the case of a rigid and hinged support of the plate along the contour. It was proved that the stiffness coefficient of the seam \( \xi \) depends on the main coefficient after the hard oscillation frequency \( \omega \). The authors constructed the graph of the change of the natural oscillation frequency \( \omega \) and the graph of the change of the stiffness coefficient of the seam \( \xi \) on the number of shift connections \( nss./ne \).

Key words: Composite plate, Layer of the composite plate, Stiffness coefficient of the seam, Basic frequency of oscillation, Bending moment, Shift connections

INTRODUCTION

A large number of works are devoted to the calculation of solid and composite plates [01, 02, 03, 04]. In [05-08], composite plates of square and round shapes were studied depending on the number of symmetrically and unevenly located shift connections, and the stiffness coefficients of the seams were determined depending on the frequency of the natural oscillations of composite plates. The authors also studied the stiffness coefficients of the seam for triangular composite plates [09, 10]. The present paper researched the stiffness coefficients of seams depending on the frequency of natural oscillations of oval composite plates with a different number of evenly and symmetrically located shift connections.

Let us consider a composite plate consisting of a number of layers joined together not only by the shift connections, but also by transverse connections, which prevent the removal or rapprochement of the layers. This approach is based on A.R. Rzhanitsin's theory of composite plates. For each layer, a hypothesis of direct normals is valid. The number of seams or gaps between plates is \( n \), and the total number of layers is \( n + 1 \) (Figure 1).

There is a linear dependence between the differences of longitudinal displacements and tangential stresses in the connections of the \( i \)-th seam shift:

\[ \tau_x^i = \xi_i \Delta u_i, \quad \tau_y^i = \xi_i \Delta v_i \]  

where,

\[ \Delta u_i = \frac{dw}{dx} c_i + u_{i+1} - u_i \]

\[ \Delta v_i = \frac{dw}{dy} c_i + v_{i+1} - v_i \]

\( c_i \) is the distance between the median planes of the layers lying on both sides of the \( i \)-th seam; \( \xi_i \) is the stiffness coefficient of the connections of the \( i \)-th seam shift.

Figure 1: Accepted designations for a composite plate
For bending and twisting moments in the $i$-th seam: $M_x^i$, $M_y^i$, and $M_{xy}^i$, one can draw equations:

\[
\begin{align*}
\frac{dM_x^i}{dx} + \frac{dM_{xy}^i}{dy} &= Q_x^i + m_x^i + m_{xy}^{i-1} \\
\frac{dM_y^i}{dy} + \frac{dM_{xy}^i}{dx} &= Q_y^i + m_y^i + m_{xy}^{i-1}
\end{align*}
\]

By transforming the right-hand side of the system and expressing lateral forces through the sum of the momentary loads in adjacent layers, we have:

\[
\begin{align*}
\frac{d^2 M_x^i}{dx^2} + 2 \frac{d^2 M_{xy}^i}{dx dy} + \frac{d^2 M_y^i}{dy^2} &= -q_i \frac{d(m_x^i + m_{xy}^{i-1})}{dx} + \frac{d(m_y^i + m_{xy}^{i-1})}{dy}
\end{align*}
\]

As a result of the transformations, we get:

\[
\begin{align*}
\frac{d^2 M_x^i}{dx^2} + 2 \frac{d^2 M_{xy}^i}{dx dy} + \frac{d^2 M_y^i}{dy^2} &= -q - \sum_{i=1}^{n} c_i \left( \frac{d^2 \tau_x^i}{dx^2} + \frac{d \tau_y^i}{dy} \right)
\end{align*}
\]

We express the left side of equation through the biharmonic operator from the deflection:

\[
D \nabla^2 \nabla^2 w = q + \sum_{i=1}^{n} c_i \left( \frac{d^2 \tau_x^i}{dx^2} + \frac{d \tau_y^i}{dy} \right)
\]

where $D_0$ is the cylindrical rigidity of a composite plate devoid of shift connections, which is determined by the formula:

\[
D_0 = \frac{E}{12(1-\mu^2)} \sum_{i=1}^{n} h_i^3
\]

$\mu$ is Poisson’s ratio; $h_i$ is the thickness of the $i$-th layer.

Accepting

\[
A_i = \frac{\partial \tau_x^i}{\partial x} + \frac{\partial \tau_y^i}{\partial y}
\]

we substitute $A_i$ into equation (8):

\[
D \nabla^2 \nabla^2 w = q + \sum_{i=1}^{n} c_i A_i
\]

and lowering the order, we get:

\[
D \nabla^2 \nabla^2 W = -M + \sum_{i=1}^{n} c_i T_j
\]

Equation (10) is analogous to the equation of the theory of elastic compound rods with absolutely rigid cross connections [02]. Carrying out similar transformations in accordance with [02], we get:

\[
\begin{align*}
\frac{\nabla^2 T_i}{\tau} &= \sum_{j=1}^{n} \frac{1+\mu}{E_j h_j} N_j - \frac{1-\mu}{E_i h_i} N_i - \frac{c M}{D_0}
\end{align*}
\]

In the composite plate, $T_i$ is the total shifting force in the $i$-th plate, which is equal to $\int \tau dx$, and $N_i$ is the longitudinal forces in the $i$-th layer.  

The bending moments $M_x$ and $M_y$ will be considered equal to the total bending moments in the composite plate devoid of shift connections:

\[
\begin{align*}
\sum_{i=1}^{n+1} M_x^i &= M_x + \sum_{i=1}^{n+1} M_y^i = M_y; \\
\sum_{i=1}^{n+1} M_{xy}^i &= M_{xy}; \sum_{i=1}^{n+1} q_i = q \\
M &= M_x + M_y
\end{align*}
\]

where $\mu_{sc}$ is Poisson’s ratio of a conventional solid plate.

Let us consider a specific case of a composite plate of two layers. For this, we set $n = 1$ for equations (12) and (13). We have the system of equations:

\[
\begin{align*}
\frac{\nabla^2 T}{\tau} &= \delta T + \frac{N_1}{E_1 h_1} - \frac{N_2}{E_2 h_2} - \frac{c M}{D_0} \\
D_0 \nabla^2 W &= -M + c T
\end{align*}
\]

where $D_0$ is actual cylindrical stiffness equal to

\[
D_0 = \sum_{i=1}^{n} D_i; D_i = \frac{E_i h_i^3}{12(1-\mu_i^2)} (i = 1,2)
\]

\[
\delta = \frac{c^2}{D} + \frac{1}{E_1 h_1} + \frac{1}{E_2 h_2}
\]

\[
E_j^* = \frac{E_j}{1-\mu_j^2} (j = 1,2)
\]

where $E_1^*$ is the modulus of elasticity of the layers in the composition of the composite plate, while the indices of the seams are omitted, since the seam is one.
Eliminating \( T \) from the system of equations (14), we get:

\[
\begin{align*}
T &= \frac{D_0}{c} \nabla^2 w + \frac{M}{c}; \\
\frac{D_0}{c^2} \nabla^2 \nabla^2 w + \frac{1}{c^2} \nabla^2 w &= \frac{\delta D_0}{c} \nabla^2 w + \\
\frac{\delta}{M} \left( \frac{N_1}{E_1 h_1} - \frac{N_2}{E_2 h_2} - \frac{cM}{D_0} \right)
\end{align*}
\]

or

\[
\nabla^2 \nabla^2 w - \xi \nabla^2 w = -\frac{\nabla^2 M}{D_0} + \frac{\xi \delta}{D_0} M + \\
\frac{c \xi}{D_0} \left( \frac{N_1}{E_1 h_1} - \frac{N_2}{E_2 h_2} \right) \frac{\xi c^2 M}{D^2}
\]

In the absence of axial loads \( N_1 \) and \( N_2 \), equation (17) takes the form:

\[
\nabla^2 \nabla^2 W - \xi \nabla^2 W = -\frac{\nabla^2 M}{D_0} + \xi M \frac{\delta D_0 - c^2}{\delta D_0} M
\]

Knowing that

\[
\nabla^2 M = -q, \quad \nabla^2 \nabla^2 W - \frac{\nabla^2 M}{D_{ycn}} = \frac{q}{D_{ycn}}
\]

\[
\nabla^2 w = -\frac{M}{D_{ycn}}
\]

where \( D_{ycn} \) is cylindrical rigidity of a conventional solid plate.

\[
\frac{q}{D_{ycn}} + \xi \cdot \frac{M}{D_{ycn}} = \frac{q}{D_0} + \xi \cdot \frac{M}{D_0} - \frac{\delta \cdot D_0 - c^2}{\delta D_0}
\]

For the plate:

\[
D_M = \frac{\delta \cdot D_0^2}{(\delta \cdot D_0 - c^2)}
\]

where \( D_m \) is cylindrical rigidity of a monolithic plate with a longitudinal modulus of elasticity in the seam zone and it is equal to zero. Then:

\[
\frac{q}{D_{ycn}} + \xi \cdot \frac{M}{D_{ycn}} = \frac{q}{D_0} + \xi \cdot \frac{M}{D_M}
\]

We express the stiffness coefficient of the seam from this equation:

\[
\xi = \frac{q \left( \frac{1}{D_0} - \frac{1}{D_{ycn}} \right)}{\xi \cdot \frac{M}{D_{ycn}} - \frac{M}{D_M}}
\]

\textit{THE STUDY OF THE SEAM STIFFNESS COEFFICIENT}

In the framework of the present study, the research task of studying the stiffness coefficient of the seam \( \xi \) depending on the the boundary conditions of the layers and the ratio of the number of elements with shift connections \( n_s \) to the number of finite elements of one layer \( n_e = 288 \) symmetrically located along the area of the plate was being solved. In turn, we introduce symmetrically shift connections into the net of finite elements, according to schemes a-e in Figure 3, while the rigidity of the shift connections in all cases is constant and equal to \( EA_{sh} = 10 \text{ kN} \).

The calculation was performed in the SCAD software package by the finite element method [11]. As a result of the calculation, the fundamental frequency of the transverse oscillations and the value of the distributed moments were determined.

The following schemes of composite plates with symmetrically located shift connections were considered (Figure 2).

The studies of oval composite two-layer plates were carried out using the method of finite elements; for this purpose, both layers were conventionally divided into sectors (Figure 2), and this division of the dependence allows approximating the original plate rather precisely.

Two conditions were considered for the plate support: hinged and rigid pinching. The supports along the contour of the plate were located at the nodes of the finite elements of the layers, while their boundary conditions were the same.

![Figure 2: The split of the composite oval plate into the final elements](image)
The composite material on a wood base (a chipboard plate) with a thickness $\delta = 10$ mm is used as layers. All the characteristics were taken from the product passport: the thickness is $\delta = 10$ mm, the average density is $\rho = 740$ kg / m$^3$, the modulus of elasticity at bending is $E = 260,000$ MPa. To determine the dynamic calculation of the mass in the nodes, the results were collected in accordance with the construction with a volumetric weight and the loading node area. In the case of a static calculation, a uniformly distributed load with intensity of 1kN / m$^2$ was applied to the top layer. The distance between the layers was assumed to be equal to the distance between the gravity centers of layers. The calculation was performed in the SCAD software package by the finite element method [11].

The results of numerical studies of composite plates, which are pivotally supported and rigidly clamped along the contour, are shown in Table 1. By the results of the calculation, the bending moments in the layers of the composite plate and the natural frequencies of the structure oscillations were determined, depending on the ratio $n_{ss}/n_{e}$.

According to the data in Table 1, graphs of the oscillation frequency change depending on the stiffness coefficient of the seam (Figure 4), as well as changes in oscillation frequencies and stiffness coefficient of the seam, depending on the ratio of the number of shift connections to the number of final elements in the layer $n_{ss}/n_{e}$ (Figures 5 and 6) were constructed.
Table 1: Numerical studies of the composite plate when changing the quantity of symmetrically arranged shift connections in accordance with schemes a-f in Figure 3

| №  | Quantity of finite elements with shift connections, \(n_{ss}/n_e\) | The circular frequency of the basic tone, \(\omega\) (c\(^{-1}\)) | Distributed moment, \(M_x, M_y\) (N×m/m) | Maximum moment, \(M\) (N×m/m) | Coefficient of stiffness of the seam, \(\xi \times 10^6\) (N/m\(^3\)) |
|----|---------------------------------------------------------------|---------------------------------------------------------------|------------------------------------------|-----------------------------------|--------------------------------------------------|
| 1  | 0 (0.000)                                                     | 161.2229                                                     | 12.26674                                  | 17.7459                           | 97.44331                                          |
| 2  | 8 (0.028)                                                     | 161.291                                                     | 12.25693                                  | 17.73081                           | 97.52243                                          |
| 3  | 18 (0.063)                                                    | 162.5401                                                   | 11.99214                                  | 17.32344                           | 99.71077                                          |
| 4  | 24 (0.083)                                                    | 163.9788                                                   | 11.85484                                  | 17.11222                           | 100.887                                           |
| 5  | 32 (0.110)                                                    | 167.771                                                    | 11.24681                                  | 16.17678                           | 106.4702                                          |
| 6  | 48 (0.167)                                                    | 172.6747                                                   | 10.52109                                  | 15.06029                           | 114.0551                                          |
| 7  | 72 (0.250)                                                    | 184.8238                                                   | 9.001003                                  | 13.7217                            | 134.3309                                          |
| 8  | 288 (1.000)                                                   | 224.3665                                                   | 5.960833                                  | 8.044512                           | 211.2151                                          |

The plate rigidly clamped along the contour

| №  | Quantity of finite elements with shift connections, \(n_{ss}/n_e\) | The circular frequency of the basic tone, \(\omega\) (c\(^{-1}\)) | Distributed moment, \(M_x, M_y\) (N×m/m) | Maximum moment, \(M\) (N×m/m) | Coefficient of stiffness of the seam, \(\xi \times 10^6\) (N/m\(^3\)) |
|----|---------------------------------------------------------------|---------------------------------------------------------------|------------------------------------------|-----------------------------------|--------------------------------------------------|
| 1  | 0 (0.000)                                                     | 101.4194                                                    | 33.21119                                  | 43.97822                           | 51.95415                                          |
| 2  | 8 (0.028)                                                     | 103.5157                                                   | 32.39724                                  | 42.72599                           | 53.50067                                          |
| 3  | 18 (0.063)                                                    | 105.3268                                                   | 31.34793                                  | 41.11166                           | 55.6631                                           |
| 4  | 24 (0.083)                                                    | 107.5365                                                   | 30.48494                                  | 39.78399                           | 57.60233                                          |
| 5  | 32 (0.110)                                                    | 112.9102                                                   | 28.17058                                  | 36.22342                           | 63.68014                                          |
| 6  | 48 (0.167)                                                    | 119.1424                                                   | 25.87582                                  | 32.69303                           | 71.38537                                          |
| 7  | 72 (0.250)                                                    | 132.6553                                                   | 22.11987                                  | 26.91465                           | 89.96421                                          |
| 8  | 288 (1.000)                                                   | 169.6148                                                   | 16.79486                                  | 18.72232                           | 148.3558                                          |

The plate pivotally supported along the contour

Figure 4: Frequency oscillation graphs depending on the stiffness coefficient of the seam (a – at a rigid pinching of the plate along the contour, b – at a hinged support along the contour)

Figure 5: Change in frequencies of natural oscillations (\(\omega\)) and stiffness of the seam (\(\xi\)) from the number of shift connections (\(n_{ss}/n_e\)) at a rigid contour seal
Andrey Viktorovich Turkov - The study of the stiffness coefficient of the seam depending on the quantity of symmetrically located shift connections in an oval two-layer plate

CONCLUSION

The analysis of the data shows that with the growth of the quantity of symmetrically arranged shift connections in the two-layer plate increases both the frequency of natural oscillations and the stiffness coefficient of the seam, which does not contradict the physical meaning of the problem. In practical terms, the stiffness of the seam can be estimated by determining the frequency of the basic tone of the oscillations of the composite plate.

REFERENCES

1. Rzhanitsyn A. (1976) [Calculation of composite plates with absolutely tight cross couplings]. Researches on the theory of constructions, issue XXII, M.: Stroyizdat, pp. 120-133. (in Russian)
2. Rzhanitsyn A. (1986) [Composite rods and plates]. M.: Stroyizdat, 316 p. (in Russian)
3. Kalmanok A. (1950) [Construction mechanics of plates]. M.: Mashstroyizdat, 303 p. (in Russian)
4. Shaposhnikov N. (1968) [Calculation of plates on a bend according to the finite-element method]. Works of MITE, issue 250, pp. 134 – 144. (in Russian)
5. Turkov A.V., Marfin K.V. (2012) [Interrelation of maximum deflections and natural oscillation frequencies of composite square isotropic plates depending on the number of symmetrically located shift connections]. Stroitel'stroykonstrukcija [Construction and Reconstruction], no 2, pp. 44-48. (in Russian)
6. Turkov A.V., Marfin K.V. (2012) [Determination of the coefficient of the seam stiffness of a square compound isotropic plate based on its fundamental oscillation frequency]. [Construction mechanics and calculation of structures], no 5, pp. 38-44. (in Russian)
7. Turkov A.V., Marfin K.V. (2013) [Determination of the coefficient of the seam stiffness of a circular composite isotropic plate at its fundamental frequency of oscillations]. [Construction mechanics and calculation of structures], no 4, pp. 58-62. (in Russian)
8. Turkov A.V., Marfin K.V. (2013) Determination of the coefficient of the seam stiffness of a circular composite isotropic plate at its fundamental frequency of oscillations. XVI International scientific and practical conference of students, Master students, postgraduate students and young scientists “Construction – the formation of the environment of life”. Moscow, 24-26 April, pp. 187-190.
9. Turkov A., Karpova E. (2015) [The study of a seam stiffness coefficient for a triangular composite isotropic plate depending on its basic frequency of oscillations in case of different rigidity of shift bindings]. Construction mechanics and calculation of constructions, no. 2, pp. 66-69. (in Russian)
10. Karpova E.V. (2015) The study of the coefficient of the seam stiffness of composite triangular isotropic plates, depending on the number of symmetrically located shift connections. Collected Works of the Eighteenth International Interuniversity Scientific and Practical Conference of Students, Master Students and Young Scientists. Moscow, April 22-24, pp. 238-242.
11. Semenov A., Gabitov A. (2005) [The SCAD project computer system in educational process. Part I. Static calculation]. M.: ASV publishing house, 152 p. (in Russian)

Paper submitted: 24.10.2017.
Paper accepted: 28.11.2017.
This is an open access article distributed under the CC BY-NC-ND 4.0 terms and conditions.