Micromechanics and statistics of slipping events in a granular seismic fault model

L. de Arcangelis
Department of Information Engineering and CNISM, Second University of Naples, Aversa (CE), Italy

M. Pica Ciamarra
CNR-SPIN, Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Italy

E. Lippiello
Department of Environmental Sciences and CNISM, Second University of Naples, Caserta, Italy

C. Godano
Department of Environmental Sciences and CNISM, Second University of Naples, Caserta, Italy
E-mail: dearcangelis@na.infn.it

Abstract. The stick-slip is investigated in a seismic fault model made of a confined granular system under shear stress via three dimensional Molecular Dynamics simulations. We study the statistics of slipping events and, in particular, the dependence of the distribution on model parameters. The distribution consistently exhibits two regimes: an initial power law and a bump at large slips. The initial power law decay is in agreement with the the Gutenberg-Richter law characterizing real seismic occurrence. The exponent of the initial regime is quite independent of model parameters and its value is in agreement with experimental results. Conversely, the position of the bump is solely controlled by the ratio of the drive elastic constant and the system size. Large slips also become less probable in absence of fault gouge and tend to disappear for stiff drives. A two-time force-force correlation function, and a susceptibility related to the system response to pressure changes, characterize the micromechanics of slipping events. The correlation function unveils the micromechanical changes occurring both during microslips and slips. The mechanical susceptibility encodes the magnitude of the incoming microslip. Numerical results for the cellular-automaton version of the spring-block model confirm the parameter dependence observed for size distribution in the granular model.

1. Introduction
The understanding of the earthquake phenomenology represents a challenge also due to the impossibility of simulating seismic events in laboratory. It is therefore extremely important to develop models for earthquakes which can be investigated, at the same time, theoretically, experimentally and numerically. An earthquake is a mechanical process characterized by two
Figure 1. The two considered models of a seismic fault. Left panel: granular particles (gray grains) are enclosed between two rigid plates (red grains) under constant pressure, and represent the fault gouge. The bottom plate is kept fixed. The top plate is driven via a spring: one extreme of the spring is attached to the plate, while the free one moves with constant velocity $V$. Right panel: the usual spring block model where the top plate is driven at constant velocity.

phases: one in which the fault is locked and elastic energy is stored and one in which the elastic energy is suddenly released as soon as the system becomes unable to sustain the applied stress. In this mechanical transition, the fault gouge appears to play an important role. Gouge is the granular material present between the planes of a fault, being the product of past wearing, and it controls the macroscopic sliding friction and the frictional stability of the fault. This observation has triggered the experimental study of the stick–slip dynamics of granular models of faults [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], as the one shown in Fig. 1 (left panel), in which the shear stress is applied via a spring mechanism, at constant pressure $P$. A particular interest has received the study of the role of the granular medium on the mechanical properties, such as strain hardening, crystallization, velocity dependent friction and shear localization [1, 2, 3, 4, 8]. Experimental results have also shown evidence [9, 10] of novel features of the non linear elastic relaxation in a granular medium perturbed by an acoustic seismic wave. This has provided a novel interpretation for some highly non linear behaviour of seismic occurrence, as for instance triggering of earthquakes at very large distances [12]. In fact, the non linear behaviour could be explained considering that small perturbations applied to a granular system modify the contact force network, setting up a memory mechanisms which could be responsible of the spatio–temporal correlations between earthquakes.

Numerical simulations of granular models for seismic faults allow to investigate the mechanical properties of the system at a level of spatial and temporal resolution not accessible experimentally [13, 19, 14, 15, 16, 17, 18, 20]. Some studies investigate the effect of fragmentation of the gouge [19, 14, 15], others focus on the mechanisms at the basis of the unjamming transition [16, 17, 18, 20]. An important open question concerns the ability of granular fault models to reproduce fundamental phenomenological laws for seismic occurrence, as for instance the Gutenberg-Richter (GR) law [21]. This states that earthquake magnitudes are exponentially distributed. The logarithmic relation between earthquake magnitude and fault slip implies that slips follow a power law distribution, which is found to be characterized by an exponent $a \approx 1.7$ independent of the geographic region and quite stable in time [22, 23]. Such a power law distribution is reproduced by the cellular-automaton version of the spring–block model [24, 25, 26, 27] (Fig. 1, right panel) which is a model of an elastic fault, where a slip may involve the displacement of a few as well as of many blocks. Conversely, in the granular model of fault shown in Fig. 1 (left panel), slips of different size typically involve the entire top plate. It could be therefore surprising that this and similar granular models of faults have been shown
to reproduce the GR distribution experimentally [5, 6, 7] and numerically [20, 13]. However, it is not clear what is the role of the granular media in the overall dynamics, and how the slip size probability distribution is affected by the model parameters.

In this paper, we review results from three dimensional Molecular Dynamics simulations of a seismic fault model, where granular media play the role of fault gouge. Moreover we compare the statistics for the size of slipping events with numerical simulations of the cellular-automaton version of the spring-block model. The paper is organized as follows. We first present the granular seismic fault model in Sec. 2 and its dynamical behaviour in Sec. 3, then discuss the slip size distribution in Sec. 4. We analyze the micromechanical properties of slipping events in Section 5. We present the spring-block results in Sec. 6, where we compare the size distribution with the granular model results. Conclusions are drawn in the last section.

2. The granular model

We perform Molecular Dynamics simulations of a fault model, where the fault gouge is represented by spherical granular particles, as illustrated in Fig. 1. The fault gouge is enclosed between two rigid rough plates of dimension $L_x \times L_y$. Each plate is made of $L_x \times L_y/d^2$ spheres of diameter $d$ placed in random position in the $xy$ plane. Spheres are shifted by a random $\Delta z \in [0, d/2]$ in the perpendicular direction and then glued to each other. In order to make the plates rigid, the particles keep their relative positions. While the bottom plate is kept fixed, the top one is subject to a constant pressure $P$ and attached to a spring of elastic constant $k_m$ (see Fig. 1). The free end of the spring moves with constant velocity $V$ in the $x$ direction. Accordingly, in the jammed phase, when the top plate does not move, the shear stress $\sigma = k_m \Delta l/L_x L_y$ increases linearly in time, $\Delta l$ being the deformation of the spring.

We consider a system of $N$ monodisperse spheres of diameter $d$ and mass $m$, enclosed between the rough plates. We keep the transverse length $L_y$ fixed and explore both the effect of changing the system length $L_x$, as well as the number of particles $N$. Since each layer contains roughly $L_x L_y/d^2$ particles, in the following we will focus on the effect of changing the number of layers, defined as $h = Nd^2/L_x L_y$ (note that $N$ does not include the particles forming the confining plates). We use periodic boundary conditions along $x$ and $y$. We employ a contact force model that captures the major features of granular interactions, known as spring-dashpot model, also taking into account the presence of static friction [28].

Static friction is implemented by recording the shear displacement of contacting grains. Let’s consider two grains $\{i, j\}$ at positions $\{r_i, r_j\}$, with velocities $\{v_i, v_j\}$ and angular velocities $\{\omega_i, \omega_j\}$. To describe the force acting on particle $i$ we introduce the normal compression $\delta_{ij}$, the relative normal velocity $v_{n_{ij}}$, and the relative tangential velocity $v_{t_{g_{ij}}}$, given by

\[ \delta_{ij} = d - r_{ij}, \]
\[ v_{n_{ij}} = (v_{ij}, n_{ij})n_{ij}, \]
\[ v_{t_{g_{ij}}} = v_{ij} - v_{n_{ij}} - \frac{1}{2}(\omega_i + \omega_j) \times r_{ij}, \]

where $r_{ij} = r_i - r_j$, $n_{ij} = r_{ij} / r_{ij}$, with $r_{ij} = |r_{ij}|$, and $v_{ij} = v_i - v_j$. The elastic tangential displacements $u_{t_{g_{ij}}}$, set to zero at the initiation of a contact, evolve according to

\[ \frac{du_{t_{g_{ij}}}}{dt} = v_{t_{g_{ij}}} - \frac{(u_{t_{g_{ij}}}, v_{ij})r_{ij}}{r_{ij}^2}, \]

where the second term in Eq.4 arises from the rigid body rotation around the contact point and ensures that $u_{t_{g_{ij}}}$ always lies in the local tangent plane of contact. Normal and tangential forces acting on particle $i$ are given by

\[ F_{n_{ij}} = k_n \delta_{ij} n_{ij} - \gamma_n m_{eff} v_{n_{ij}}, \]

\[ F_{t_{g_{ij}}} = \frac{k}{2} \frac{d}{dt} \left( \frac{1}{2} v_{t_{g_{ij}}}^2 \right) - \gamma_n m_{eff} v_{t_{g_{ij}}}, \]

\[ F_{t_{d_{ij}}} = \frac{k}{2} \frac{d}{dt} \left( \frac{1}{2} v_{t_{d_{ij}}}^2 \right) - \gamma_n m_{eff} v_{t_{d_{ij}}}, \]

\[ F_{t_{ij}} = \frac{k}{2} \frac{d}{dt} \left( \frac{1}{2} v_{t_{ij}}^2 \right) - \gamma_n m_{eff} v_{t_{ij}}. \]
where $k_{n,tg}$ and $\gamma_{n,tg}$ are elastic [29] and viscoelastic constants, respectively, and $m_{eff} = m_i m_j / (m_i + m_j)$. Here the subscripts $n$ and $tg$ indicate the normal and tangential components. We use $\gamma_{tg} = 0$ and $\gamma_n$, chosen in such a way that the restitution coefficient is $e = |v_{in}/v_{out}| = 0.8$, where $v_{in}$ ($v_{out}$) is the relative normal velocity of two particles before (after) their collision. Coulomb friction is taken into account via the introduction of a friction coefficient $\mu$, chosen to be $\mu = 0.1$, so that $|\mu F_{nij}| \geq |F_{tgij}|$.

In this manuscript, we measure the length in units of $d$ and time in units of $\sqrt{m/k_n}$. The fixed bottom plate is treated as an object with infinite mass, while the mass of the top plate is $L_x L_y / d^2$. When not stated otherwise, we fix $L_y = 5$, $m = 1$, $k_n = 2 \times 10^5$ and $k_t = 2k_n / 7$, the spring constant of the drive mechanisms to $k_m = k_0 = 10^2$, and the driving velocity to $V = 5 \times 10^{-2}$.

3. Phase diagram

The rate at which the shear stress increases in absence of plate motion, $kV/L_x L_y$, and the confining pressure, $P$, are the two parameters which most greatly affect the dynamics of the system, which either flows continuously, or exhibits stick–slip motion. The crossover between these two regimes is shown Fig. 2, where we plot the time evolution of the top plate position $x(t)$ for $V = 1$, and three different values of the confining pressure, $P = 10^1, 10^2, 10^3$. At high pressure, the system is characterized by a stick–slip dynamics: when the system is jammed the applied stress increases linearly in time, and the flow begins when the system becomes unable to sustain the applied stress. In the flow regime the stress decreases, and the system eventually jams again. Conversely, at small values of the pressure the system flows continuously, as it is never able to sustain the applied shear stress. The role of the two control parameters $P$ and $V$ is summarized in Fig. 3. In agreement with experimental results of a similar system [11], as well as with numerical results found in two dimensions [18], we find either stick–slip motion at high pressure and small loading velocity, or continuous flow at low pressure and high loading velocity.

In this paper, we choose the parameter values in order to observe stick–slip behaviour for the system, more precisely we set $V = 5 \times 10^{-2}$ and $P = 100$. In the stick phase, the system evolves as
Figure 3. Different dynamical regimes are observed depending on the value of the control parameters, the imposed pressure $P$, and the velocity of the free extreme of the spring $V$. At low pressure and high velocity, the system continuously flows (diamonds), while at high pressure and small velocity stick-slip motion is observed (circles). In the transition region, an irregular motion is observed (stars).

It adapts to the increasing stress. Its evolution consists of large slips and small slips, in which the top plate moves by a fraction of a particle diameter $d$. These microslips are difficult to identify from the top plate displacement. A more accurate identification is based on the evaluation of the top plate velocity, which exhibits peaks in correspondence of each slips, as illustrated in Fig. 4 (lower panel). We define the starting time $t_{s}^{(i)}$ of the $i$-th slip as the time when $v$ becomes larger than a given threshold $v_t = 10^{-5}$ (other threshold values give similar results, if small enough). Similarly, the ending time $t_{e}^{(i)}$ is the time when $v$ goes back to values smaller than $v_t$. Having identified the starting and the ending time of each slip, we define, as customary, the slip size as $S_{i} = x(t_{e}^{(i)}) - x(t_{s}^{(i)})$.

4. Slip size distribution

The slip size distribution, $p(S)$, in the case of real earthquakes can be derived from the Gutenberg–Richter law for the magnitude distribution, $P(M) \propto 10^{-bM}$, where $b \simeq 1$. In fact, the slip size $S$ is proportional to the seismic moment, $M = \mu AS$, where $A$ is the slipping area and $\mu$ the shear modulus. The seismic moment is related to the magnitude $M$ by a logarithmic relation $M = 2/3 \log(M) + \text{const}$ [30]. Accordingly, since in our case the slipping area is constant, one finds $p(S) \propto S^{-a}$, with $a = 1 + (2/3)b \simeq 1.7$, for $b \simeq 1$. Here we examine the dependence of this probability distribution on the number of layers, on the system length and on the spring constant of the driving mechanism.

4.1. Effect of the number of layers

Fig. 5 shows the slip size distribution for a number of layers ranging from $h = 0$ to $h = 8$. In all cases we identify two different regimes in the distribution: a power law behaviour at small slip sizes, named microlips, with $S < 1$, and a bump at a characteristic size $S \sim \ell_x$. These events are named slips. The power law behaviour, $p(S) \propto S^{-a}$, holds over about four decades with an exponent $a$ in good agreement with the one predicted by the Gutenberg–Richter law, $a \simeq 1.7$, except for the cases $h = 1$ and 2. Note that the exponent $a \simeq 1.7$ is also observed for $h = 0$, which corresponds to the case of solid–on–solid friction. In the case of one or two granular layers,
Figure 4. Evolution of the top plate position (upper panel) and velocity (lower panel) as a function of time. The insets illustrate the typical stick slip motion. The main panels correspond to a zoom on a small time interval across a single large slip. The velocity evolution has a peaked structure, which signals the presence of small slips, so small that they are difficult to identify from the behavior of $x(t)$ plotted in the upper panel. The dashed horizontal line in the lower panel marks the threshold value of the velocity used to identify the slips. Here $L_x = 20$, and $h = 1$.

Figure 5. Slip size distribution for $L_x = 20$, $L_y = 5$ and different numbers of granular layers $h$. 
the distribution shows a larger probability for intermediate slip sizes, and therefore a smaller scaling exponent. For \( h > 2 \) the scaling behaviour is independent of the number of layers. This result can be understood measuring the average displacement, \( \Delta x \), of particles at height \( z \) from the bottom plate during a slip event. The displacement configuration indicates that particles in the two uppermost granular layers respond more intensively to the external shear and therefore the system reorganization during a slip event involves mostly few uppermost layers, not the entire system. Accordingly, only when \( h < 2 \) the finite thickness of the system interferes with the dynamical process, changing the scaling behaviour of the probability distribution \( p(S) \). This is also consistent with the fact that, for \( h > 2 \), the bump is centered at about the same slip size, regardless of \( h \).

This analysis clarifies that the thickness \( h \) of the fault gouge does not affect much the slip size probability distribution, as long as \( h > 2 \). This is consistent with the finding of a universal exponent \( a \) for the slip size of real earthquakes, which occur on faults certainly having different gouge thickness.

4.2. Role of different \( L \) and \( k_m \)

In Fig. 6 and 7 we show the effects of the different system size \( L_x \) and the elastic constant of the driving spring, \( k_m \), on the slip size distribution for \( h = 8 \) and \( L_y = 5 \). We first discuss the behaviour of the distribution for microslips. We observe that, keeping \( k_m \) fixed, the initial power law regime is independent of \( L_x \) (Fig. 6, upper panel). Moreover, Fig. 7 indicates that at small \( S \) the distribution is neither affected by the \( k_m \) values. On the other hand, \( k_m \) introduces a large size cut-off. The maximum slip, indeed, decreases by increasing \( k_m \), being roughly inversely proportional to \( k_m \). Therefore, for very stiff external drive, large slips are not observed. In general for large \( k_m \), separation of length scales between microslips and slips is no longer observed, and for instance for \( k_m = 10k_0 \) the largest events merge into the tail of the power law behaviour. The above results suggest that the self-similar organization of microslips is independent of parameters with a stable exponent \( a \approx 1.7 \), in agreement with experimental values for earthquakes. Note that \( k_m \) could be related to the shear modulus of the earth crust close to the fault, which is different on different faults as it depends on the particular rock composition. The initial power-law regime being independent of \( k_m \) is therefore in agreement with the experimental finding of a fault–independent \( a \) exponent [22, 23].

Conversely, the position of the bump depends on both \( L_x \) and \( k_m \). Since in a large slip most of the accumulated stress is released, in order to understand the \( L_x \) and \( k_m \) dependence we consider, as a simple example, a toy model made of a single block of dimension \( L_x \times L_y \), pulled on a rough surface by a spring of elastic constant \( k_m \). Assuming that a slip, occurring when the stress reaches a critical threshold \( \sigma_c \), releases the whole accumulated stress, the corresponding slip size is \( S = \sigma_c L_x L_y / k_m \), and scales as \( L_x / k_m \). We will show in the following that the large slips of our granular model present this scaling behaviour.

First, we investigate the dependence on the system size \( L_x \) at fixed values of \( k_m \). Fig. 6 (upper panel), shows that \( L_x \) affects the position of the bump which moves towards larger slip sizes as \( L_x \) increases. The position of the bump, in agreement with the simple model, linearly scales with \( L_x \), as illustrated in Fig. 6 (lower panel), where the bump collapse is obtained by plotting \( p(S) \) versus \( S / L_x \). Second, in order to verify the scaling of the bump position with \( k_m \), as suggested by the single block model, we perform simulations changing at the same time \( L_x \) and \( k_m \), keeping their ratio \( k_m / L_x \) fixed. Results, plotted in Fig. 8 clearly show that the dependence on \( k_m \) and \( L_x \) enters via the variable \( k_m / L_x \). Indeed, data corresponding to different \( L_x \) collapse onto the same curve, as predicted by the single block calculation. Notice that not only the bumps but the whole distributions collapse onto a unique curve if the ratio \( k_m / L_x \) is kept fixed.
5. Micromechanics of an earthquake

5.1. Onset of a slip

To understand the mechanisms acting at the onset of a slip, we need to identify its precise starting time \( t_u \). Here we describe the analysis performed on the particular slip occurring at time \( t \approx 1600 \). Similar results are obtained for other slips. We consider a replica of the system at time \( t \), and follow its time evolution at zero driving velocity \( V = 0 \). If the replica made at time \( t \) resists to the applied stress, then \( t < t_u \). Conversely, if a slip is observed, \( t > t_u \). We define \( t_u \) as the largest time where no slip occurs, and we identify it (one for each slip) with an accuracy equal to the time step of integration of the equation of motion \( \delta t \). This procedure is equivalent to a quasi-static simulation [31] around the un-jamming time \( t_u \) and gives the value of the shear stress above which the system starts to flow. The identification of \( t_u \) is possible for both slips and microslips. We here concentrate our analysis on slips, where major rearrangements of the system occur.

We have performed a number of checks which suggest that no structural changes occur at \( t_u \). For instance, the comparison of the state of the system at time \( t_u \) with the one at shortly earlier

Figure 6. Slip size distribution \( p(S) \) versus \( S \) for \( L_y = 5, k_m = 100, h = 8 \) and different system size \( L_x \) (upper panel). Same distributions for slip size rescaled by system size \( S/L_x \) (lower panel).
and later times, shows that no contact breaks at $t_u$. We have also considered the distribution of the parameter $\lambda = |f_t|/\mu f_n$ where $f_t$ and $f_n$ are the tangential and the normal forces. When $\lambda > 1$ a contact breaks as the Coulomb condition is violated. The maximum of the probability distribution $P(\lambda)$ gradually moves toward 1 as $t_u$ is approached, indicating the weakening of the solid [18]. However, neither at $t_u$ the number of contacts with $\lambda \simeq 1$ overcomes a given fraction, nor they appear to be spatially organized. The absence of structural changes at $t_u$ supports a scenario in which the system is located in an energy minimum which slowly becomes an inflection point at time $t_u$, and therefore the smallest eigenvalue of the dynamical matrix

**Figure 7.** Slip size distribution $p(S)$ versus $S$ for $L_x = 20$, $L_y = 5$, $h = 8$ and different elastic spring constants $k_m$, for $k_0 = 100$.

**Figure 8.** Slip size distribution $p(S)$ versus $S$ for $L_y = 5$, $h = 8$ and different $L_x$ and $k_m$, with fixed ratio $k_m/L_x = 5$. 
continuously decreases to zero \[32, 33\].

5.2. Evolution in the force space

When the system sticks and the shear stress increases, no macroscopic motion is observed. However, the system microscopically changes since it sustains an increasing shear stress. The time evolution of the top plate position \(X(t)\) (Fig. 9a), consists in an elastic deformation, where \(X(t)\) increases very slowly in time, interrupted by sudden microslips. To characterize the evolution of the system in the force space, we introduce a two time force-force correlation function for the normal forces, defined as

\[
C^n(t_0, t) = \frac{\sum_{ij} |f^n_{ij}(t_0)||f^n_{ij}(t)|}{\sum_{ij} |f^n_{ij}(t_0)|^2},
\]

where the sum running over all couples of particles \((i, j)\) corresponds to a spatial average. An equivalent definition holds for the correlation of tangential forces, \(C^t\). This function could be used to monitor the temporal evolution of the force network approaching any characteristic time \(t_0\). Being interested in the unjamming transition of the slip event found previously, here we fix \(t_0 = t_u\), and consider the evolution of the correlation function \(C^n(t_u, t)\) for earlier times, \(t < t_u\) (Fig. 9b). Similar results are found for other slips. The force correlation function \(C^n\) (and \(C^t\), not shown) exhibits small jumps in correspondence of microslips (Fig. 9a), revealing the unusual occurrence of bursts in the reorganization of the force network. During these bursts, the energy due to the tangential interaction decreases, whereas the one due to the normal interaction increases. A possible interpretation is in terms of a two force network scenario, in which the applied stress \(\sigma\) is supported by a stress \(\sigma_n\) due to the normal force network, and by a stress \(\sigma_t\) due to the tangential forces, \(\sigma = \sigma_n + \sigma_t\). In a burst, few contacts break, leading to a decrease of \(\sigma_t\), \(\sigma_t \to \sigma_t - \delta\sigma\). A microscopic slips is observed since the normal forces quickly adapt and succeed in sustaining the applied stress, \(\sigma_n \to \sigma_n + \delta\sigma\). This physical interpretation is supported by the inset of Fig. 9b, which shows both \(C^n\) and \(C^t\) across a microslip, with \(t_0\) its starting time. \(C^n\) slightly increases and overcomes 1, while \(C^t\) exhibits a sharp drop due to the breaking of several contacts.

The force correlation function (Eq. 7) gives also insights into the system evolution during a slip. In Fig. 10 we plot \(C^{n,t}(t_u, t)\) for \(t > t_u\), and for comparison the scaled top plate position \((X(t) - X(t_u))/(X(t_\infty) - X(t_u))\), where \(t_\infty\) is a time following the slip event, whose precise value does not influence our results. We first notice that the forces evolve on a timescale much shorter than the plate motion. For instance the correlation functions reach the value 0.1, denoting an almost complete relaxation, when the top plate moved only by 10% of its total displacement. The presence of different time scales in the relaxation process is evidenced by the self-scattering correlation function \(F(q, t) = \frac{1}{N} \sum_j \exp[iq \cdot (r_j(t) - r_j(t_u))]^2\), where \(r_j\) represents the position of the \(j\)th particle. Since the system is sheared along \(x\) and confined along \(z\), we have considered wave vectors along \(y\), \(q = (0, q, 0)\). For large \(q\), \(F(q, t)\) probes small scale relaxation, and coincides with \(C^n(t_u, t)\), as shown in Fig. 10. Conversely, at small \(q\), \(F(q, t)\) relaxes on a time scale comparable to that of the upper plate motion. The relaxation time \(\tau_q\), \(F(q, \tau_q) = 1/e\) indeed increases as \(q\) decreases. This scenario also holds for the approach to jamming. Moreover, tangential forces decorrelate before normal ones. This can be explained considering the unjamming transition as a buckling-like instability of the chains of large normal forces, which are sustained by weaker tangential contacts. When the weaker sustaining contacts break, either the normal forces adapt to sustain the extra load, leading to a microslips, or a buckling-like instability occurs, giving rise to a slip. The same quantities can be used to investigate the subsequent jamming transition. To this end, the force network at time \(t\) is compared with the force network after the slip event studying \(C^{n,t}(t, t_\infty)\). Normal forces
Figure 9. Time evolution of the top plate position (a) and of the correlation function of the normal forces $C^n(t,t_u)$ (b). Dashed lines identify the unjamming time $t_u$, i.e. the time slip begins. The inset show $C^n$ and $C^{tg}$ across a microslip at $t = 1579.5$.

Figure 10. Correlation functions at the unjamming (upper panels) and at the subsequent jamming transitions (lower panels). The left panels show the normal and tangential force correlation functions $C^{n,tg}(t_0,t)$ and $C^{n,tg}(t,t_\infty)$ (symbols) during a slip event. The dashed blue lines indicate the time evolution of the plate position scaled between 0 and 1. The plain lines show the self-scattering function $F(q,t)$ for $q = 3, 5, 9, 17$. The corresponding $\tau_q$ are shown in the right panels. $t_0$ is fixed equal to $t_u$ for the unjamming transition, whereas $t_\infty = 1614$ for the subsequent jamming.

correlate before tangential ones, confirming the stabilizing role of tangential forces in the force network.
5.3. Response to perturbations: slips versus microslips
The correlation length $\xi$ in equilibrium systems is measured from the response to an external perturbation. The susceptibility, for instance, scales like $\xi^{2-\eta}$ near critical points, where $\eta$ is the correlation function critical exponent. Here we measure the response of the system to a pressure change at the onset of both slips and microslips. More precisely, at each time $t$ we stop the external drive, setting $V = 0$, and introduce a perturbation in the external pressure $P$, fixed to $P' = P(1 - \alpha)$ for a time interval $\delta t_{\text{pert}} = 0.1$. The response $\chi_\alpha$ at time $t$ is defined as

$$
\chi_\alpha(t) = \frac{1}{\alpha P} \lim_{t \to \infty} \left[ \frac{1}{N} \sum_i (\mathbf{r}^\alpha_i(t+\tau) - \mathbf{r}^0_i(t+\tau))^2 \right]^{1/2}
$$

where $\{\mathbf{r}^\alpha_i\}$ and $\{\mathbf{r}^0_i\}$ are the asymptotic states of the perturbed and of the unperturbed systems. In the unjammed phase, the susceptibility is divergent. In the jammed phase, it measures the linear size of the region of correlated particles that respond to the external perturbation. It therefore provides an estimate of the correlation length. It is a static quantity since the time $t$ only indicates the instant at which the perturbation is applied. $\chi_\alpha$ can be also defined without setting $V = 0$, provided that the characteristic response time of the system is much smaller than the timescale over which the applied stress varies. For a wide range of $\alpha$, the response of the system at times far from $t_u$ is linear in the perturbation, since $\chi_\alpha$ does not depend on $\alpha$ (Fig. 11). In particular, $\chi_\alpha$ gradually increases in time and drops in correspondence to microslips. The inset shows that the microslip size $\Delta X$ depends on $\chi_\alpha$ evaluated just before the slip, as $\Delta X \simeq \chi_\alpha^b$. This indicates that the size of a microslip is already encoded in the system state. In fact, considering that $\chi_\alpha$ increases in time on approaching a microslip, the measured value of $\chi_\alpha$ provides a lower bound for the magnitude of the incoming event.

As the unjamming time is approached, the response is no longer linear. $\chi_\alpha$ remains roughly constant until it abruptly increases at a time which depends on $\alpha$, the sooner the greater $\alpha$. This increase is consistent with a power-law divergence. Accordingly at each time, namely at each value of the applied shear stress, there is a minimum value of the perturbation intensity for slip.

Figure 11. Time evolution of the susceptibility $\chi_\alpha$ for different $\alpha$. (Inset) Size of a microslip $\Delta X$ versus the corresponding value $\chi_\alpha$. The straight line is $\chi_\alpha^b$ with $b \simeq 1.2$. 
triggering. This behavior is in line with the existence of a minimum threshold amplitude in the deformation associated with seismic waves for earthquake remote triggering [34]. A difference between slips and microslips is in how different particles contribute to $\chi_\alpha$ in the sum of Eq. 8. Indeed, these contributions are very similar for microslips, and highly heterogeneous for slips. The presence of an heterogeneous response is consistent with previous numerical results found at $\sigma = 0$ [35, 36, 37, 38].

6. Spring-block model

Here we compare the results of the granular model with those of the cellular-automaton version of the spring-block model, originally introduced by Burridge & Knopoff [24]. In particular, we investigate the scaling behaviour of the slip distribution of a spring-block model and its robustness to parameter changes. We focus on the one-dimensional version of the model introduced by Nakanishi [27, 39]. This comparison is instructive since in the granular model the fault is rigid, the particles forming the shearing plate having fixed relative positions, while the Nakanishi model is the simplest representation of an elastic fault under tectonic drive forces.

In the one-dimensional version of this model (Fig. 1, lower panel) $L$ blocks are interconnected by springs with elastic constant $k$ and to a moving plate driven by a spring of constant $k_m$. Indicating by $V$ the velocity of the moving plate, the force $f_i$ acting on the i-th block is given by

$$f_i = -k_m(x_i - Vt) + k(x_{i+1} + x_{i-1} - 2x_i)$$  \hspace{1cm} (9)$$

where $x_i$ is the position of the i-th block and $t$ is the evolution time. If the force is smaller than the local friction value $f_{th}$, the block is locked. As soon as $f_i$ overcomes $f_{th}$, the i-th block is no longer able to sustain the external stress and slips to a new position $x'_i$. The displacement of the i-th block produces a change of the forces acting on nearest neighbor blocks. If this change leads to $f_{i+1} > f_{th}$ or $f_{i-1} > f_{th}$ the neighbor block becomes unstable, it slips to a new equilibrium position and the event propagates. The evolution of the system is governed by the evolution equation of the forces acting on each block. More precisely, when all $f_i < f_{th}$ the system is locked and forces grow due to the external drive

$$df_i(t)/dt = k_m V.$$  \hspace{1cm} (10)$$

When at the generic site $j$, $f_j$ becomes larger than $f_{th}$, the external drive is stopped and the forces $f_j$ and $f_{j\pm1}$ are updated to the new values $f'_j$ and $f'_{j\pm1}$ according to the following rules

$$f'_j = \phi(f_j - f_{th})$$

$$f'_{j\pm1} = f_{j\pm1} + \frac{1}{2}\Delta(f_j - f'_j).$$  \hspace{1cm} (11)$$

Here

$$\Delta = \frac{2k}{2k + k_m}$$  \hspace{1cm} (12)$$

measures the fraction of elastic energy dissipated. Energy conservation is recovered in the limit $k \gg k_m$. If $f_{j\pm1} > f_{th}$, the update rules Eq.s 11 are iterated and the process goes on until all sites have $f_i < f_{th}$. The total slip $S$ is then given by

$$S = \sum_i f'_i - f_i.$$  \hspace{1cm} (13)$$

where $f'_i$ is the final value of the force acting on the $i$th block. Stopping the external drive during the propagation process reflects the large separation of time scales between the fracture
propagation and the drive load. The function $\phi(x)$ controlling the new value of the force in unstable sites must satisfy the constraint that the new force $f_j' < f_{th}$. In the following we will use the functional form

$$\phi(x) = \frac{(2 - \delta f)^2/\alpha}{x + (2 - \delta f)/\alpha} - 1$$

proposed by Nakanishi in Ref. [27] to reproduce velocity weakening, with $\alpha$ and $\delta f$ two model parameters. The above equations allow a simple interpretation of the role of $k$, the spring constant parameters. The role of $k$ slightly influences the exponent $a$ of the initial power law regime of the slip distribution $p(S)$, and in the following we set $\alpha = 1.75$, which provides $a \simeq 1.65$ in agreement with the experimental GR distribution and results of the MD simulations. We also fix $k = 10^3$ and investigate different values of $k_m$ and $L$.

In Fig. 12 (upper panel) the size distributions $p(S)$ for $k_m = 10$ and three values of $L = 20, 30, 40$ are plotted. The distribution is very similar to the one observed in Fig. 5: an initial power law decay with an exponent $a \simeq 1.65$, followed by a bump at large sizes $S$. This bump indicates the existence of a typical slip size $S_c$. We wish to stress that, as in the granular model, this size cannot be straightforwardly related to the system size $L$. Indeed, the value of $S_c$ is inversely proportional to $k_m$. Therefore, by changing $k_m$ and $k$ keeping $\Delta$ fixed, $S_c$ can take all values from zero to infinity. However, for $\Delta$ and $k_m$ given, the position of the bump depends on $L$ and moves towards larger $S$ as $L$ increases. Differently from the MD model, $S_c$ does not scale with $L$ but rather as $L^{1.5}$ (Fig. 12 lower panel). The quality of the bump collapse is significantly worse than the one observed in Fig. 6 (lower panel).

Next we turn to consider the case of fixed $L$ and different $k_m$. Since $k$ is fixed, changes in $k_m$ produce two effects: the scaling of $S$ with $k_m$ and the $\Delta$ change. In particular for larger $k_m$, $\Delta$ becomes smaller and dissipation increases. Both effects make the probability of large slip size smaller for larger $k_m$. This is confirmed by Fig. 13 where the largest observed slip decreases by increasing $k_m$. In Fig. 13 (lower panel) we plot $p(S)$ versus $S k_m^{1.5}$ and observe that the largest slip size again decreases by increasing $k_m$, as the consequence of a smaller $\Delta$: as $\Delta$ decreases, energy dissipation increases, making less probable the observation of events involving all blocks.

We finally consider the case of different $k_m$ and $L$ but with the fixed ratio $k_m/L = 0.5$. According to the above observations, since $k_m$ varies in a small range, $k_m \in [10, 20]$, $\Delta$ is roughly constant and we can expect a scaling with $k_m/L$. Fig. 14 shows that curves for different $L$ exhibit a similar behavior with the bump located at about the same position. In order to compare the results of the Nakanishi and the granular model, we observe that the scaling with $k_m/L$ in the granular model suggests that the elastic coupling among the grains is very stiff, which corresponds for the Nakanishi model to $k \gg k_m$ and $\Delta \simeq 1$, so that small changes in $k_m$ do not modify $\Delta$. This is a reasonable condition since in the granular simulations the typical ratio between the spring constant of the grain-grain interaction, $k_m$, and the spring constant of the drive, $k_m$, is $k_m/k_m = 2 \times 10^3$. We finally notice that similar slip size distributions have been also observed in the original spring block model with real continuum-time dynamics [40].

### 7. Conclusions

We have investigated the statistics of slips sizes in a model of fault in which granular materials play the role of fault gouge, as function of model parameters.
Figure 12. Slip size distribution \( p(S) \) versus \( S \) for different system size \( L \) (upper panel). Same distributions for slip size rescaled by system size as \( S/L^{1.5} \) (lower panel).

Our results indicate that the slip distribution is characterized by two regimes, the first one well described by a power law, the second one by a bump at a characteristic slip size. The microslips involve mostly the shearing of the two uppermost layers of the granular medium, which explains why the exponent of the slip size distribution depends on the number of layers \( h \) only when \( h = 1, 2 \). For \( h > 2 \) the resulting exponent is in very good agreement with the measured value for earthquake catalogs, \( a \approx 1.7 \).

Beside being almost independent of the number of layers, the exponent \( a \) results to be also independent of other model parameters, and precisely the system size and the elastic constant of the driving spring. This is in agreement with the existence of a universal exponent of the slip size distribution, which is independent of the fault size and the mechanical properties of the rocks the fault walls are made of. Conversely, the bump at large slip sizes depends on \( L_x \) and \( k_m \), the relevant variable controlling its location being \( L_x/k_m \). The separation between the power law regime and the bump disappears as \( L_x/k_m \) decreases, namely for very stiff faults. This result is particularly interesting since it suggests that large slip behaviour for very large systems can be investigated by tuning the elastic constant of the drive in smaller systems.

The absence of precise structural changes at the unjamming time, and the bursts observed
Figure 13. Slip size distribution $p(S)$ versus $S$ for $L = 20$ and different spring elastic constant $k_m$ (upper panel). Same distributions for slip size rescaled by the elastic constant as $S k_m^{1.5}$ (lower panel).

Figure 14. Slip size distribution $p(S)$ versus $S$ for different $L$ and $k_m$ with fixed ratio $k_m/L = 0.5$. 
in the prior dynamics, suggest that the increasing external stress progressively modifies the underlying energy landscape. At each time the system is in an equilibrium position which can be seen as local energy minimum of an effective energy landscape which depends on the applied shear stress. Microslips occur when the local energy minimum flattens down as the applied shear stress increases, letting the system fall in a neighbor minimum. If there are no close minima, a slip occurs, and the system jumps to a far away configuration. The deforming energy landscape picture also suggests that soft-modes could be found not only when unjamming is approached at $\sigma = 0$ by decreasing the volume fraction $\phi$ [35, 38], but also [41] when unjamming is approached increasing $\sigma$.

We have also compared the granular model and the Nakanishi model, obtaining very similar results for the probability distribution of slip sizes. This is a surprising finding considering that the two models are very different. Indeed, the first one (granular) models a rigid fault, because of the rigidity of the confining plate, in the sense that the relative positions of the particles of each plate is fixed. Conversely, the Nakanishi model is paradigmatic of an elastic fault. The scaling results suggest that both models behave as a system at the critical point and belong to the same universality class. This behaviour is robust for the granular model, whereas it weakly depends on the choice of the parameter $\alpha$ for the Nakanishi model.

The ability of the granular fault model to reproduce the statistics of slipping events suggests the existence of a deep relation between earthquakes and the jamming and un-jamming transitions of athermal particulate systems (see Ref.s [42, 43] for recent reviews). This opens the possibility of investigating whereas theoretical tools and concepts developed in the study of the jamming transition, such as the presence of soft-modes, of related diverging length scales [44] and hysteretic behavior [45], may provide new insights in the earthquakes phenomenology.

[1] Marone C, Raleigh C B and Scholz C H 1990 J. Geophys. Res. 95 7007
[2] Beeler N M, Tullis T E, Blanpied M L and Weeks J D 1996 J. Geophys. Res. 101 8697
[3] Nasuno S, Kudrolli A and Gollub J P 1997 Phys. Rev. Lett. 79 949
[4] Tsai J C, Voth G and Gollub J P 2003 Phys. Rev. Lett. 91 064301
[5] Dalton F and Corcoran D 2001 Phys. Rev. E 63 061312; 2002 Phys. Rev. E 65 031310
[6] Bretz M, Zaretzki R, Field S B, Mitarai N and Nori F 2006 Europhys. Lett. 74 1116
[7] Daniels K and Hayman N W 2008 J. Geophys. Res. 113 B11411
[8] Rathbun A P and Marone C 2010 J. Geophys. Res. 115 B01204
[9] Johnson P A and Jia X 2005 Nature 437 871
[10] Johnson P A, Savage H, Knuth M, Gomberg J and Marone C 2008 Nature 451 57
[11] Nasuno S, Kudrolli A, Bak A and Gollub J P 1998 Phys. Rev. E 58 2161
[12] Hill D P et al 1993 Science 260 1617
[13] Peña A A, McNamara S, Lind P G and Herrmann H J 2009 Granular Matter 11 243
[14] Mair K and Abe S 2008 Earth Planet. Sci. Lett. 274 72
[15] Abe S and Mair K 2009 Geophys. Res. Lett. 36 L23302
[16] Mora P and Place D 1998 J. Geophys. Res. 103 21067
[17] Aharonov E and Sparks D 1999 Phys. Rev. E 60 6890
[18] Aharonov E and Sparks D 2004 J. Geophys. Res. 109 B09306
[19] Abe S and Mair K 2005 Geophys. Res. Lett. 32 L05305
[20] Pica Ciamarra M, Lippiello E, Godano C and de Arcangelis L 2010 Phys. Rev. Lett. 104 238001
[21] Gutenberg B and Richter C F 1944 Bull. Seismol. Soc. Am. 34 185
[22] Kagan Y Y 1991 Geophys. J. Int. 106 123
[23] Godano C and Pingue F 2000 Geophys. J. Int. 142 193
[24] Burridge R and Knopoff L 1967 Bull. Seismol. Soc. Am. 57 341
[25] Carlson J M and Langer J S 1989 Phys. Rev. Lett. 62 2632; 1989 Phys. Rev. A 40 6470
[26] Omami Z, Feder H J S and Christensen K 1992 Phys. Rev. Lett. 68 1244
[27] Nakanishi H 1990 Phys. Rev. A 41 7086
[28] Silbert L E, Ertas D, Halsey T C, Levine D and Plimpton S J 2001 Phys. Rev. E 64 051302
[29] Cundall P A and Strack O D L 1979 Geotechnique 29 47
[30] Kanamori H and Anderson D 1975 Bull. Seismol. Soc. Am. 65 1073
[31] Heussinger C and Barrat J L 2009 Phys. Rev. Lett. 102 218303
[32] Maloney C E and Lemaitre A 2006 Phys. Rev. E 74 016118
[33] Welker P R and McNamara S C 2009 Phys. Rev. E 79 061305
[34] Gomberg J and Johnson P A 2005 Nature 437 830
[35] Silbert L E, Liu A J and Nagel S R 2005 Phys. Rev. Lett. 95 098301
[36] Somfai E, van Hecke W, Ellenbroek W G, Shundyak K and van Saarloos W 2007 Phys. Rev. E 75 020301
[37] Ellenbroek W G, Somfai E, van Hecke M and van Saarloos W 2006 Phys. Rev. Lett. 97 258001
[38] Wyart M, Liang H, Kabla A, Mahadevan L 2008 Phys. Rev. Lett. 101 215501
[39] Nakanishi H 1991 Phys. Rev. A 43 6613
[40] Mori T and Kawamura H (2005) Phys. Rev. Lett. 94 058501; (2006) J. Geophys. Res. 111 B07302
[41] Pica Ciamarra M and Coniglio A 2009 Phys. Rev. Lett. 103 235701
[42] Liu A J and Nagel S R 2010 Annu. Rev. of Condens Matter Phys. 1 347
[43] van Hecke M 2010 J. Phys.: Condens. Matter 22 033101
[44] Wyart M, Silbert L E, Nagel S R and Witten T A 2005 Phys. Rev. E 72 051306; Silbert L E, Liu A J and Nagel S R 2005 Phys. Rev. Lett. 95 098301; Somfai E, van Hecke M, Ellenbroek W G, Shundyak K and van Saarloos W 2007 Phys. Rev. E 75 020301(R)
[45] Vanel L, Howell D, Clark D, Behringer R P and Clement E 1999 Phys. Rev. E 60 R5040; Ciamarra M P, Pastore R, Nicodemi M and Coniglio A e-print arXiv:0912.3140; Grebenkov D S, Ciamarra M P, Nicodemi M and Coniglio A 2008 Phys. Rev. Lett. 100 07801; Forterre Y and Pouliquen O 2008 Ann. Rev. of Fluid Mech. 40 1