Electron Green’s function in the planar \( t - J \) model

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Dedicated to Prof. Wolfgang Götze on the occasion of his 60th birthday.

Theoretical investigations have been to large extent motivated by the observation of anomalous electronic properties of superconducting cuprates. We shall discuss here only the state of the ‘strange’ metal, as manifested in cuprates at \( T > T_c \). Our study is devoted to the single-particle response, which is in cuprates investigated directly by the angle resolved photoemission (ARPES) probing the spectral function \( A(k, \omega) \). ARPES reveals in compounds with intermediate doping a quasiparticle (QP) dispersion consistent with a large electronic Fermi surface. On the other hand the conclusions on the QP character, as deduced from the low-energy \( \omega \sim 0 \) spectral properties, are less clear. Still it seems that spectral shapes are never underdamped, as expected for the QP excitations near the Fermi surface within the usual Fermi liquid (FL) theory. In order to explain the anomalous response, a phenomenological marginal Fermi liquid (MFL) theory has been proposed \[1\], which assumes at low \( T \) a frequency-dependent QP damping of the form \( \Sigma''(\omega) \propto \omega \), rather than the FL behaviour \( \Sigma''(\omega) \propto \omega^2 \). While for ARPES the interpretation of results is still controversial \[2\], there is more agreement on the evidence for the anomalous spin dynamics, as manifested within the NMR relaxation experiments \[3\] and the neutron scattering \[4\], and for the anomalous charge dynamics, as deduced from the resistivity \( \rho(T) \propto T \) and from the non-Drude form of the optical conductivity \[5\]. Mentioned anomalies seem to be well accounted by the MFL scenario \[6\].

Theoretical investigations of electron Green’s functions and related spectral properties, starting on the level of prototype microscopic models for correlated electrons, as the Hubbard model or the \( t - J \) model, have reached quite a level of consensus for a single mobile carrier (hole) introduced into the reference insulator \[7\], representing the spin polaron in the case of the antiferromagnetic (AFM) spin background \[8\]. Even here there are open questions in the relation of model results with relevant ARPES experiments \[1\]. While single-hole results could be possibly extended to the low-doping regime, the spectral properties of the intermediate-doping regime are much more difficult to approach theoretically. So far there has been a number of numerical studies, where \( A(k, \omega) \) has been calculated via the exact diagonalization and the quantum Monte Carlo methods in small systems \[9\]–\[12\]. Whereas these investigations obtained valuable information on the QP dispersion and on the location of the Fermi surface (FS), due to several restrictions it was not possible to resolve spectral widths and shapes, so results were quite restricted in determining the low-energy QP properties, being at the core of different scenarios for ‘strange’ metal. Recently a novel numerical method \[13\], combining Lanczos diagonalization and random sampling for finite systems at \( T > 0 \), has been applied to the study of \( A(k, \omega) \) within the \( t - J \) model \[14\]. It was possible to extract self energies \( \Sigma(k, \omega) \), which revealed the anomalous behavior following the MFL scenario, i.e. for \( k \sim k_F \) it was found \( \Sigma''(k, \omega) \propto |\omega| + \xi T \). This finding is consistent with the MFL-type charge and spin dynamics, as found previously in numerical studies within the \( t - J \) model \[15\], as well as in experiments on cuprates \[16\].

There have been few attempts to treat electron Green’s functions analytically at intermediate doping. Starting from the one-band Hubbard model, self energies have been related to the AFM spin fluctuations within the random phase approximation and weak coupling \[17\], and within a selfconsistent (FLEX) theory \[18\]. Both approaches rely on the assumption of modest correlations, i.e. not too large \( U/t \). At least within the weak coupling the analysis yields a low-energy behaviour according to normal FL. Spectral functions in the strong-correlation regime, where the \( t - J \) model should be more appropriate starting point, have proven to be even harder due to projections involved in fermionic operators and due to composite character of electrons within the slave-boson theories \[19\].
In this paper we introduce a simple theory for the electron Green's function \( G(\mathbf{k}, \omega) \) in the strong-correlation regime. The central observation is that within the \( t - J \) model one could work directly with projected fermionic operators. Since these prevent the application of usual diagrammatic techniques, we apply the method of equations of motion to express the self energy \( \Sigma(\mathbf{k}, \omega) \). Working at \( T = 0 \) we divide \( \Sigma \) into a contribution coherent at \( \omega \to 0 \), and an incoherent part. We approximate the former by performing a decoupling into the single-particle propagation and spin fluctuations. We do not intend to evaluate within the same framework the spin dynamics, so we assume it of the MFL-type form, as found in previous studies [24]. In spite of its simplicity such an analysis yields promising features.

II. PROJECTED ELECTRON PROPAGATOR

In the following we study the \( t - J \) model [19] as a prototype model for strongly correlated electrons, and for electronic properties of cuprates in particular,

\[
H = -t \sum_{(ij)s} \langle \hat{c}_{j \alpha}^\dagger \hat{c}_{i \alpha} \rangle + J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \tag{1}
\]

where we have omitted usual but less important density-density coupling in the \( J \) term, and

\[
\hat{c}_{i \alpha}^\dagger = (1 - n_{i \alpha}) \hat{c}_{i \alpha}^\dagger \tag{2}
\]

are local fermionic operators, which project out the states with the double occupancy. \( \mathbf{S}_i = \frac{1}{2} \sum_{s s'} \hat{c}_{i s}^\dagger \sigma_{ss'} \hat{c}_{i s'} \) are local spin operators.

Our aim is to calculate the electron propagator

\[
G(\mathbf{k}, z) = \left\langle \left\langle \hat{c}_{\mathbf{k} \alpha} \right\rangle \right\rangle_z = -i \int_0^\infty e^{i(\omega + \mu)t} \left\langle \{ \hat{c}_{\mathbf{k} \alpha}(t), \hat{c}_{\mathbf{k} \alpha}^\dagger \} \right\rangle dt, \tag{3}
\]

where \( z = \omega + i \delta \), \( \delta > 0 \), \( \mu \) is the chemical potential, and

\[
\hat{c}_{\mathbf{k} \alpha}^\dagger = \frac{1}{\sqrt{N}} \sum_i e^{i \mathbf{k} \cdot \mathbf{r}_i} \hat{c}_{i \alpha}. \tag{4}
\]

Model Eq. (3) is defined within the basis set, which does not contain doubly occupied sites. Within this subspace the Green’s function, Eq. (3), expressed with projected operators, is the same as the usual electronic propagator. Since commutation relations for projected operators Eq. (3) are not simple, the usual diagrammatic derivation of the fermionic self energy is not applicable. Hence we employ the method of equations of motion, introduced to treat dynamics of more general operators [20].

Note that general (anticommutator) correlation functions obey the equations of motion [21]

\[
z \left\langle \left\langle A; B \right\rangle \right\rangle_z = \left\langle \left\langle A \right\rangle \right\rangle_z + \left\langle \left\langle B \right\rangle \right\rangle_z = \left\langle \left\langle A; \{B \} \right\rangle \right\rangle_z. \tag{5}
\]

Let us apply Eqs. (3) to the autocorrelation function \( G(z) = \left\langle \left\langle A; A^+ \right\rangle \right\rangle_z \). If we define the operator \( C \) as

\[
[\mathbf{A}, \mathbf{H}] = \zeta \mathbf{A} - i \mathbf{C}, \quad \{ \mathbf{C}, \mathbf{A}^+ \} = 0, \tag{6}
\]

we can express \( G \) via Eq. (3),

\[
G(z) = G_0(z) + \frac{1}{\alpha^2} G_0(z)^2 \left\langle \left\langle C; C^+ \right\rangle \right\rangle_z, \quad G_0(z) = \frac{\alpha}{z - \zeta}, \quad \alpha = \left\langle \left\langle \{A, A^+ \} \right\rangle \right\rangle. \tag{7}
\]

It is an evident intention to reexpress Eq. (7) in terms of self energy \( \Sigma(z) \)

\[
G(z) = \frac{\alpha}{z - \zeta - \Sigma(z)}, \quad \Sigma(z) \sim \frac{1}{\alpha^2} \left\langle \left\langle C; C^+ \right\rangle \right\rangle_z. \tag{8}
\]

Such an expression for \( \Sigma(z) \) would within the diagrammatic techniques correspond to the contribution of ‘irreducible’ diagrams. It is valid within the perturbation theory, as applied e.g. to the analysis of the dynamical conductivity [21]. More generally it could be considered as a memory function in analogy with the Mori’s projection method [22], where again only the ‘irreducible’ part of \( \left\langle \left\langle C; C^+ \right\rangle \right\rangle_z \) would contribute to \( \Sigma(z) \). Recently a treatment of Green’s functions, analogous to Eqs. (3-7), has been applied also to some problems of correlated electrons [23, 24].

The ‘irreducible’ contribution is in the present analysis approximated by an appropriate decoupling on this level, in the sense of mode-coupling theories. It is evident that in our application the decoupling is justified in terms of underlying physics, since the theory does not have a small parameter.

The application of Eqs. (3-7) to the electron \( G(\mathbf{k}, \omega) \) is straightforward. We define in analogy to Eq. (8),

\[
G(\mathbf{k}, z) = \frac{\alpha_k}{z + \mu - \zeta_k - \Sigma(\mathbf{k}, z)}, \tag{9}
\]

and the corresponding spectral function \( A(\mathbf{k}, \omega) = -(1/\pi) \text{Im} G(\mathbf{k}, \omega) \). First we note that

\[
\alpha_k = \left\langle \left\langle \{ \hat{c}_{\mathbf{k} \alpha}, \hat{c}_{\mathbf{k} \alpha}^\dagger \} \right\rangle \right\rangle = \frac{1}{N} \sum_i \left\langle \left\langle \hat{c}_{i \alpha}, \hat{c}_{i \alpha}^\dagger \right\rangle \right\rangle = 1 - \frac{c_e}{2} = \frac{1}{2} (1 + c_h), \tag{10}
\]

where \( c_e, c_h = 1 - c_e \) are the electron and the hole concentration, respectively. Eqs. (8, 10) imply that within the \( t - J \) model the spectral function is not normalized to unity [10-13], as the consequence of the projected fermionic basis. Still the normalization constant \( \alpha \) is \( \mathbf{k} \)-independent. It should be also noted that due
to $\alpha < 1$ the self energy $\Sigma$ in Eq. (10) does not coincide with the standard definition $\Sigma$ which requires $\alpha = 1$ in the numerator of Eq. (10), but can be related to it, i.e. $\Sigma = [\Sigma + (\alpha - 1)z]/\alpha$. Note however that $\Sigma$ would be less convenient since $\Sigma(\omega \to \pm \infty) \neq 0$ [14].

Next we consider the equations of motion

$$[\hat{c}_{ks}, H] = -t \sum_{j n n i} [(1 - n_{i-s}) \hat{c}_{js} + S^T_{s} \hat{c}_{j-s}] +$$

$$+ \frac{1}{2} J \sum_{j n n i} (sS^z_{s} \hat{c}_{is} + S^z_{s} \hat{c}_{j-s}), \quad s = \pm 1.$$  (11)

It is convenient to express $n_{i-s} = n_{i}/2 - S^z_{i}$ in terms of local density and spin operators. We are interested in the normal metallic phase without any spin or charge long range order, i.e. $(\mathbf{S}_i) = 0$ and $(\mathbf{n}_i) = c_{e_i}$. Introducing $n_{t} = c_{e} + \bar{n}_{t}$, we can rewrite Eq. (11) in the $k$-representation,

$$[\hat{c}_{ks}, H] = -(1 - \frac{c_{e}}{2}) t_\gamma \hat{c}_{ks} + \frac{t}{2} \sum_{k'} \gamma_{k-k'} \tilde{n}_{k-k'} \hat{c}_{k's} +$$

$$+ \frac{1}{2} sS^z_{s} \hat{c}_{i} + S^z_{s} \hat{c}_{j-s}] +$$  (12)

where $\gamma_{k} = \sum_{n i} \exp(i k \cdot r_{i}).$

Using Eqs. (6,9,12) we can express the ‘free’ propagation term $\zeta_{k}$ as

$$\zeta_{k} = \frac{1}{\alpha} \langle \{[\hat{c}_{ks}, H], \hat{c}^\dagger_{ks}\}_+ \rangle = \zeta + \epsilon_{k},$$

$$\epsilon_{k} = -\frac{t}{\alpha} \sum_{n n i} \langle (1 - n_{i-s})(1 - n_{j-s}) \rangle \exp(i k \cdot \mathbf{r}_{i} - \mathbf{r}_{j}),$$  (13)

where $\eta < \alpha$, due to AFM correlations.

Note that the spectral function with $\alpha, \zeta_{k}$, following from Eqs. (8,10,13), has correct lowest $l = 0, 1$ frequency moments

$$m_{l} = \int \omega^{l} A(k, \omega) d\omega,$$  (14)

while moments $m_{l>1}$ will not be exact due to approximations involved in the calculation of $\Sigma(k, \omega)$, described in the next section.

III. SELF ENERGY

Eq. (12) is determining the operator $C_{k}$ corresponding to Eq. (6) and is the starting point for approximations to $\Sigma(k, z)$, Eq. (8). The first term in Eq. (12) is the ‘free’ fermion propagation, which is absorbed into $\zeta_{k}$, Eq. (13). The second and the third term represent coupling of the fermion to density and spin fluctuations, respectively.

We are interested mainly in the processes which at low $T \ll J, t$ dominate the low-energy QP relaxation (damping). We still expect that at $T = 0$, or at least at low $T$ (assuming the metallic phase extending to low $T$) there exists a well defined Fermi energy and the corresponding FS determined by $\Sigma''(k, \omega = 0) = 0$. The relaxation rate $|\Sigma''(k, \omega = 0)| > 0$ is then dominated by the coupling to low-energy spin and charge fluctuation modes.

It is well documented that the spin dynamics in the normal phase of cuprates at $T > T_{c}$ is anomalous [34], with an enhanced response at low $T$. This is best seen in the NMR relaxation on Cu sites with $T_{1}^{-1} \sim \text{const}$, rather than Korringa law $T_{1}^{-1} \propto T$. So it seems evident that down to low $T \sim T_{c}$ spin fluctuations behave as quite independent degrees of freedom, although not as a coherent mode. This naturally leads to the idea of decoupling of spin dynamics from the fermion propagation. The density fluctuations $\tilde{n}_{k}$ on the other hand, seem to be less important. One reason is that at low doping their scaling is with $c_{h}$. Also there is so far no indication for pronounced low-$\omega$ density fluctuations, and the dynamics is restricted predominantly to $\omega > t$ [22].

The assumption that spin degrees behave as independent modes, leads to the decoupling of correlations, e.g.,

$$(\tilde{S}^z_{k-k'}(t) \tilde{c}_{k's}(t) \tilde{c}^\dagger_{k's}(t) \tilde{S}^z_{k-k'}(t')) \sim$$

$$\sim \delta_{k-k'}(\delta_{k'^{z}_{k-k'}}(t) \tilde{S}^{z}_{k-k'}(t'') \tilde{c}^\dagger_{k'^{z}_{k-k'}}(t''')),$$  (15)

and to the expression for spin fluctuation contribution $\Sigma_{sf}(k, \omega) = \langle C_{k'} C_{k''} \rangle^{\text{irr}} / \alpha,$

$$\Sigma_{sf}(k, \omega) = \sum_{k'} M_{kk'} \int \frac{d\omega_{1} d\omega_{2}}{\pi^{2}} g(\omega_{1}, \omega_{2}) \times$$

$$\times A(k', \omega_{1}) \chi''(k - k', \omega_{2}) \frac{1}{z \omega_{1} - \omega_{2}},$$  (16)

where $\chi$ is the dynamical spin susceptibility

$$\chi(q, \omega) = -i \int_{0}^{\infty} e^{i \omega t} \langle [S^z_{q}(t), S^z_{-q}] \rangle,$$  (17)

and

$$M_{kk'} = \frac{3}{\alpha} \frac{t(\gamma_{k'} - \frac{J}{2} \gamma_{k-k'})^{2}}{\epsilon_{k-k'}},$$  (18)

$$g(\omega_{1}, \omega_{2}) = 1 - f(\omega_{1}) + \bar{n}(\omega_{2}),$$  (19)

where $f$ and $\bar{n}$ are Fermi and Bose distribution functions, at $T = 0$ leading to

$$g(\omega_{1}, \omega_{2}) = 1, \quad \omega_{1}, \omega_{2} > 0,$$

$$= 1, \quad \omega_{1}, \omega_{2} < 0,$$

$$= 0 \quad \text{otherwise.}$$  (20)

In the derivation of Eq. (13) we have assumed that in the spin system there is no long range order or spin anisotropy, i.e. $\chi^{\alpha\beta}(k, z) = \delta_{\alpha\beta} \chi(k, z)$, and $G(k, z)$ is spin invariant as well.
Let us briefly comment Eq. (16). Similar expressions, coupling QP dynamics to spin fluctuations, have been obtained for the Hubbard model within the perturbation theory, or beyond that using a diagrammatic approach \[14,17\], as well as within the strongly correlated \(t - J\) model for the hole dynamics in the low-doping regime \[24\]. Using a phenomenological starting point, analogous expressions for \(\Sigma(k, \omega)\) have been considered also within the MFL theory \[8\] and in the nearly AFM scenario \[23\]. It is important to note that in our theory the coupling \(\zeta\) involves besides \(I\) explicitly also \(t\), coming directly from Eq. (11), i.e. from the restriction of no double occupancy.

Eq. (16) is intended primarily to describe the QP damping close to the Fermi energy, i.e. \(\Sigma''(k, \omega \sim 0)\). On the other hand, it is easy to establish, e.g. by calculating some higher frequency moments \(m_{t>1}\), Eq. (14), that \(\Sigma_{sf}\) needs corrections, in particular for \(\omega \ll 0\). These arise due to neglected density fluctuations in Eq. (12), and also due to the oversimplified decoupling, Eq. (10). We remedy this deficiency by adding a \(k\)-independent contribution,

\[
\Sigma(k, z) = \Sigma_{sf}(k, z) + \Sigma_{inc}(z).
\]  

(21)

The incoherent \(\Sigma_{inc}\) has been studied in detail in connection with a single hole introduced into the \(J = 0\) magnetic insulator \[7\]. It persists also at \(J > 0\) \[8\] and at finite doping \[12,14\]. We will lateron choose \(\Sigma_{inc}(\omega)\) qualitatively consistent with previous studies, introducing however two essential requirements on its form. First, \(\Sigma_{inc}\) should not influence the QP damping at \(\omega \sim 0\) and not spoil the existence of the FS, i.e. \(\Sigma_{inc}'(\omega \to 0) \propto \omega^\prime, \nu \gtrsim 2\). On the other hand, we require also the conservation of the FS volume (Luttinger theorem) within the correlated system \[23\]. The latter seems to be confirmed with ARPES experiments on cuprates \[1\] and also with numerical studies of small model systems \[10,12,14\]. Note that the Fermi surface is determined by conditions \[27\]

\[
\zeta_F + \Sigma'(k_F, 0) = \mu, \quad c_e = \frac{V_{FS}}{V_0}.
\]  

(22)

whereby the FS volume \(V_{FS}\) should correspond to the fermion density, where \(V_0\) is the volume of the first Brillouin zone. In our case \(c_e\) is given by

\[
c_e = \frac{1}{N} \sum_k \int_{-\infty}^{0} A(k, \omega) d\omega.
\]  

(23)

Working with fixed \(c_e\) we can satisfy Eq. (23) by choosing appropriate \(\mu\). Since our decoupling approximation is not conserving automatically the FS volume, Eq. (22), we can achieve this only via certain restrictions on \(\Sigma_{inc}\). We are mainly interested in the regime \(c_e \lesssim 1\), which requires generally quite large \(\Sigma_{inc}(\omega < 0)\).

In the present work we do not intend to calculate spin susceptibilities \(\chi(q, \omega)\), which enter Eq. (14). Rather we consider them as input. We assume \(\chi''(q, \omega)\) according to recent numerical analysis of the \(t - J\) model \[15\] and consistent with anomalous spin dynamics found in NMR \[5\] and neutron scattering experiments on cuprates \[3\]. Within the intermediate doping regime results seem to imply the MFL form \[4\], at least for the local susceptibility at \(T > 0\), \[13\]

\[
\chi''_L(\omega) = \frac{1}{N} \sum_q \chi''(q, \omega) = \tanh\left(\frac{\omega}{2T}\right) \tilde{S}_L(\omega),
\]  

(24)

where \(\tilde{S}_L(\omega)\) is essentially \(T\)-independent, and \(\tilde{S}_L(\omega \to 0) = S^0_L > 0\). Since we consider in this paper only \(T = 0\), this implies a nonanalytical \(\chi''_L(\omega \sim 0) = \text{sign}(\omega) S^0_L\).

At the same time we assume that the \(k\)-dependence of \(\chi(k, \omega)\) is less critical on approaching \(T = 0\). The latter assumption distinguishes the MFL theory \[4\] from some other scenarios \[23\].

After choosing particular forms for \(\chi(q, \omega)\) and \(\Sigma_{inc}(\omega)\), and at fixed model parameters and \(c_e\), we are faced with a closed set of selfconsistent (SC) equations \[10,16\], which we discuss in the following section.

**IV. RESULTS**

We apply our analysis to the case of the \(t - J\) model on 2D square lattice, as relevant for strongly correlated electrons in doped AFM, and particularly in cuprates \[9\]. This implies the regime \(J < t\) and \(c_e \lesssim 1\). Due to specific approximations our approach is applicable in a restricted window of \(c_e\). We expect that this regime corresponds to the intermediate doping. For very low doping \(c_e \ll 1\) electrons are closer to a description in terms of independent AFM spin polarons \[1\], where it is hard to establish the existence of the FS \[14\], and the spin background is nearly an ordered AFM state. Such facts are not properly incorporated within our decoupling approximation.

On the other side, within the strongly ‘overdoped’ regime the spin fluctuations lose their identity as independent degrees of freedom. Within the \(t - J\) model the ‘optimum’ doping is plausibly determined by the competition between the kinetic energy density \(\propto c_e t\) and the AFM exchange energy density \(\propto J\), so that the ‘intermediate’ regime is for us a broad range around \(c_e \sim J/t\). We furtheron consider only \(T = 0\).

Since we are interested only in a qualitative description of QP, we assume the simplest form for dynamical spin susceptibility \(\chi(q, \omega)\), consistent with the MFL form for the \(\omega\)-dependence, Eq. (24), while the \(q\)-dependence is determined by the AFM inverse correlation length \(\kappa\),

\[
\chi''(q, \omega) = \frac{W}{(|q - Q|^2 + \kappa^2)(\omega^2 + \omega_0^2)} \text{sign}(\omega),
\]  

(25)

where \(Q = (\pi/2, \pi/2)\) is the AFM wavevector, and the characteristic exchange frequency \(\omega_0 \propto J\). \(W\) is estimated via the sum rule.
\[
\frac{1}{N} \sum_{q} \int_{0}^{\infty} \chi''(q, \omega) d\omega = \langle S_i^z \rangle^2 = \frac{1}{4}(1 - c_h),
\] (26)

For \( \kappa < 1 \) this yields \( W \sim \omega_0 \kappa^2 / 2 \). A similar form to Eq.(25) has been indeed found in the numerical analysis of finite systems [14].

Let us consider some qualitative consequences of the theory. First we discuss the hole part \( \omega < 0 \), corresponding to the photoemission (PES) spectra [4]. The crucial contribution to \( \Sigma'_{\alpha}(k, \omega \sim 0) \) in Eq.(18) comes from spin fluctuations with \( q \sim Q \) and \( k, k' \sim k_F \). Hence we can estimate

\[
M_{kk'} \sim M_0 = \frac{3}{2}(t \gamma_{k_F} + 2J)^2, \quad \gamma_{k_F} \sim \frac{4c_h}{\lambda},
\] (27)

and \( \lambda \gg 1 \). Since on a square lattice at \( c_h \ll 1 \) the FS is nearly nested with a wavevector \( \tilde{Q} \), \( \tilde{Q} \lesssim Q \), we get from Eqs.(16,25) for \( |\omega| < \omega_0 \)

\[
\Sigma''(k_F, \omega) \sim -M_0 \chi''(\tilde{Q}, \omega) \int_{0}^{\infty} \frac{d\omega_1}{\pi} A(k_F - \tilde{Q}, \omega_1)
\]

\[
\propto \frac{-M_0 N_F}{\omega_0} |\omega|,
\] (28)

provided that \( \tilde{Q} - Q < \kappa \), and \( N_F \) is the density of states at the FS. It is not surprising that we recover the MFL form \( \Sigma''(\omega) \propto |\omega| \), since Eq.(16) has a close analogy to the phenomenological derivation [3], where the \( \omega \)-dependence of the boson spectrum was assumed the same as in Eq.(25). The essential difference is that within the phenomenological theory \( k \)-dependences are neglected. In our approach the \( q \)-dependence of spin fluctuations is important, and we expect the MFL form only if the approximate nesting condition is fulfilled, taking into account the width of the maximum \( \sim \kappa \) in Eq.(22).

When we estimate \( r = \Sigma''(k_F, \omega) / \omega_0 \) from Eq.(28) for the 'optimum' doping \( c_h \sim J / t \), it is characteristic that \( |r| > 1 \). This means that the hole-like QP peaks are generally overdamped (with damping larger than the characteristic QP frequency), as indeed observed in recent numerical studies [4], and consistent with ARPES experiments on cuprates [4].

Another consequence of Eq.(16) is the asymmetry between the hole part, \( \omega < 0 \), and the electron part \( \omega > 0 \) of the spectra, also evident in numerical studies [4]. Within the present theory this phenomenon originates partially from \( M_{kk'} \), Eq.(18), which reaches minimum effectively for \( k > k_F \). Unlike in \( M_0 \), Eq.(27), where both terms inside the bracket have the same sign and are of the same order of magnitude, terms in Eq.(18) are cancelling each other for \( k > k_F \). Another difference comes from \( \Sigma'_{inc}(\omega) \) which should be added predominantly at \( \omega < 0 \), in order to satisfy the conservation of \( V_{FS} \), Eq.(22). This enhances strongly only the QP damping within the hole part.

Let us finally turn to the numerical analysis of SC equations. Here we present only some general features of QP spectra \( A(k, \omega) \). To be close to the regime of cuprates we choose \( J = 0.3 \ t \) [9] and \( \omega_0 \sim 2J \) [3], while the lattice is a 2D square one. Although it is necessary to acknowledge the importance of short range AFM correlations even in the regime of intermediate doping, we assume for simplicity in Eq.(13) \( \eta = \alpha \). Namely, in Eq.(13) AFM correlations enter only to modify the width of the effective band \( c_k \), and thus have a less crucial quantitative effect. We also put \( \bar{\epsilon} = 0 \). We are interested in the intermediate doping regime with short range AFM correlations \( \xi = \kappa^{-1} \sim 1 \). For fixed \( \mu \) (which has some technical advantage over fixing \( c_h \)) the numerical task is now to calculate \( c_h \) from Eq.(23) and to use such \( \Sigma_{inc} \) to simultaneously satisfy Eq.(22). The form of \( \Sigma_{inc}(\omega) \) in our theory is quite arbitrary provided that \( \Sigma_{inc}(0) \) is zero, since it enters Eq.(22) only via \( \Sigma_{inc}(0) \). In the calculation we choose a simple parabolic form for \( \Sigma_{inc}(\omega) \) within the interval \( [\omega_1, \omega_2] \), \( \omega_1 < \omega_2 < 0 \) with the maximum value \( \max[\Sigma_{inc}(\omega)] = \Sigma_0 \) as a parameter. To eliminate discontinuities we smoothly connect the latter parabola with another one in the interval \( [\omega_2, 0] \) following the analytic FL form \( \Sigma_{inc}(\omega) = -c_\omega^2 \).

In the following we present the specific case with \( c_h = 0.3 \), where we fix \( \xi = 1.0 \). Choosing \( \omega_1/t = -6 \), \( \omega_2/t = -1 \) we still have to use quite large \( \Sigma_0 = 3.5 \ t \) to satisfy Eq.(23). In Fig.1 we present \( A(k, \omega) \) for selected direction \( k = (x, x) \), \( 0 < x < 1 \), and in Figs.2,3 the corresponding \( \Sigma''(k, \omega) \) and \( \Sigma'(k, \omega) = \Sigma'(k, \omega) + \zeta_k - \omega - \mu \). We can comment on several characteristic features. While the damping \( \Sigma''(k, \omega \ll 0) \) is dominated by the incoherent contribution \( \Sigma_{inc} \), the behaviour at \( \omega \sim 0 \) is governed by the spin-fluctuation part \( \Sigma_{ff} \). It is evident that for \( k \sim k_F \) (here the FS is at \( x \sim 0.42 \)) the form is MFL-like \( \Sigma'(k, \omega) \propto \omega^2 \) for \( \omega \ll k_F \) as well as for \( k \gg k_F \). Both these facts and the observation that \( \zeta_k > 1 \) are consistent with recent numerical results within the \( t-J \) model [14]. Note that also the magnitude and the overall shape of \( \Sigma''(k, \omega) \) is in agreement with numerical results [4], supporting in addition our choice of \( \Sigma_{inc} \). Evident from Fig.1 is also the pronounced asymmetry between the hole part \( \omega < 0 \) and the electron part \( \omega > 0 \). QP excitations are generally overdamped for \( k < k_F \), and moreover immersed in an incoherent background for \( k \ll k_F \). On the other hand, the QP peaks for \( k > k_F \) are better pronounced and less damped [13]. It should be however noted that for \( \omega > 0 \) the maximum in the damping \( \Sigma''(k > k_F, \omega) \sim t \), seen in Fig.2, is probably overestimated within our decoupling. Hence in Fig.1 the QP peaks at \( k \gg k_F \) seem to be more damped than found in the numerical study [13].

In Fig.4 we show also the corresponding density of states \( N(\omega) = (2/N) \sum_k A(k, \omega) \), which should be in
general less sensitive to strong correlations, and its bandwidth close to the free fermion case. \( N(\omega \ll 0) \) is clearly dominated by the incoherent part \( \Sigma_{\text{inc}}'' \). A peculiar maximum (which is not a singularity) appears at \( \omega \gtrsim 0 \) and is related to the minimal damping just above FS.

The qualitative features remain similar for a whole range of intermediate dopings 0.15 < \( c_h < 0.4 \). It is however evident that on approaching \( c_h < 0.2 \) it becomes harder to satisfy the FS requirement, Eq. (23), and the results depends stronger on the form chosen for \( \Sigma_{\text{inc}}'' \). This is plausible since \( G(k, \omega) \) becomes dominated by the incoherent contributions, studied extensively in connection with a single mobile hole [7]. In our approximation the treatment of such an incoherent motion is too crude. Nevertheless we can still study low \( \omega \sim 0 \) dynamics, if we relax somewhat the condition Eq. (22).

V. CONCLUSIONS

In this paper we have presented a simple theory for the electron Green’s function in the doped AFM. Note that our treatment differs essentially from the analysis of a single hole in an AFM, or slave-boson approaches for finite doping, where the spinless hole (holon) propagators are studied in the first place. We study directly the equations of motion for electron operators. This prevents the use of usual diagrammatic techniques. Still we argue that in the sense of mode-coupling theories our approximations, decoupling the self energy into the single-electron propagation and spin fluctuations, are reasonable at low \( \omega \), as far as the spin fluctuations behave as independent degrees of freedom. It is well possible that such assumption breaks down at certain energy or temperature scale \( \omega^* \sim T^* \). Nevertheless such \( \omega^* \ll t, J \) is clearly quite low, so far not seen in small-system studies [13], and also hard to resolve from e.g. ARPES experiments [14], but it is possibly related to the onset of superconductivity in cuprates.

The main message of our theory, feasible for the intermediate-doping regime, is that MFL-type spin dynamics implies also the MFL-type QP damping and related spectral shapes. The coupling to spin fluctuations is necessarily strong, since it is related to both \( J \) and \( t \), Eq. (27), and consequently QP features in the hole part are generally overdamped. Our analysis relies on the existence of the well defined FS and on the conservation of its volume (Luttinger theorem). It should be however noted, that the obtained (large) FS within the \( t-J \) model is close to a circular one, unlike the experimental ones in cuprates [1]. It is probably easy to change the shape of FS by including additional terms in the model Eq. (1), e.g. by introducing the n.n.n. hopping.

It is clear that the present theory is not fully selfconsistent, since it does not describe the origin of the anomalous spin dynamics, which seems to be the essential ingredient and also challenge within the intermediate-doping regime, and could arise again from the frustration induced by mobile holes.

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FIG. 1. Spectral functions $A(k, \omega)$ for $c_h = 0.3$ and $J/t = 0.3$, for various $k = x(\pi, \pi)$. $x$ are presented in steps of 0.1.

FIG. 2. Imaginary part of the self energy $\Sigma''(k, \omega)$, corresponding to results on Fig. 1.

FIG. 3. Real part of the self energy $\tilde{\Sigma}'(k, \omega) = \Sigma'(k, \omega) + \zeta_k - \omega - \mu$, corresponding to Fig. 1.

FIG. 4. Density of states $N(\omega)$, for data as above.
$A(k,\omega)$

$x=0.0$

$x=1.0$
\[ \text{Re} \Sigma(k, \omega)/t \sim x = 0.0, 0.5, 1.0 \]
