Evolution of pressure waves acting on a bubble liquid through adjacent boundaries

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Abstract. The paper presents the results on the dynamics of two-dimensional compression waves in a bubble liquid, when the effect on the system occurs through adjacent boundaries. The cases of a bell-shaped and a step-shaped impulses are considered. A comparison of the dependence of the wave evolution on the shape of the initial impulse is carried out.

1. Introduction

The main studies on the dynamics of one-dimensional waves in two-phase vapor-gas-droplet media are described in detail in [1–6]. In the first papers on the modeling of two-dimensional waves in bubble liquid [7–10] several statements of problems on the effect of a pulsed signal on a bubble liquid and a bubble layer in a pure liquid were considered. Based on numerical studies, criteria for strengthening and damping of the signal by means of a bubble region of finite dimension were established.

In contrast to the first papers, works where a wide-range equation of state of water and vapor was used [11–14] appeared later. The authors considered the effect of pressure, vapor-gas content on the speed of sound in the gas-liquid mixture. On the basis of the Rankine-Hugoniot relations, the parameters of incident and reflected shock waves in the gas-liquid medium are obtained for the cases of isothermal, adiabatic and shock compression of the gas component.

2. Mathematical model

Let there be some region filled with a bubble liquid, which is a parallelepiped with a square section whose generatrix is parallel to the \(z\)-axis (the zone longitudinal size is much longer than the transverse one). We consider two-dimensional wave impulses obtaining, for example, when occurs a boundary pressure independent of the \(z\)-coordinate \((p = p^0(t, y)\) for \(x = x_0\) and \(p = p^0(t, x)\) for \(y = y_0\) (Fig. 1)).

We assume that in each unit volume all the bubbles are spherical and of the same radius, the viscosity and thermal conductivity are significant only in the interfacial interaction and, in particular, in the pulsation of bubbles, there is no crushing and fusion of the bubbles. The system of equations of mass conservation, the number of bubbles, pulses and pressure in bubbles with such assumptions for single-velocity motion has the form [7]:
\[
d\rho_i + \rho_i \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad (i = l, g), \quad d n + n \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0,
\]
\[
\rho \frac{du}{dt} + \frac{\partial u}{\partial x} \rho_i = 0, \quad \rho \frac{dv}{dt} + \frac{\partial v}{\partial y} \rho_i = 0, \quad \rho = \rho_l + \rho_g,
\]
\[
\frac{dp_g}{dt} = -\frac{3\gamma p_g}{a} w - \frac{3(\gamma - 1) q}{a_0}, \quad w = \frac{da}{dt}, \quad \left( \frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right),
\]
\[
\alpha_i + \alpha_g = 1, \quad \rho_i = \rho_i^0 \alpha_i, \quad \alpha_g = \frac{4 \pi n a^3}{\gamma}.
\]

Here \(\alpha\) is the bubble radius, \(p_i\) is the phase pressure, \(q\) is the heat-transfer coefficient, \(n\) is the number of bubbles per unit volume, \(w\) is the radial bubble velocity, \(\rho_i^0\) are phase true densities, \(\gamma\) is the heat capacity ratio for a gas and \(\alpha_i\) are volume phase contents. The velocities \(u\) and \(v\) correspond to the motion along the \(x\) and \(y\) coordinates. The parameters of the liquid and gaseous phases marked by the subscripts \(i = l, g\) respectively.

Fig. 1. Scheme of the calculation area, where D1, D2, D3 and D4 are imaginary sensors, \(x_L\) and \(y_L\) are the dimensions of the one.

In describing the radial motion, in accordance with the correction proposed in [15], we assume that the velocity of this motion consists of two components \(w = w_R + w_A\), where \(w_R\) is described by the Rayleigh–Lamb equation
\[
a \frac{dv_R}{dt} + \frac{3}{2} w_R^2 + 4v_l w_R \frac{\rho_g}{\rho_l^0} = \frac{(p_g - p_l)}{\rho_l^0},
\]
where \(v_l\) is the liquid viscosity.

The additional velocity \(w_A\) is determined from the solution (in the acoustical approximation) to the problem of spherical rarefaction on a sphere with radius \(a\) in a carrier liquid:
\[
w_A = \frac{p_g - p_l}{\rho_l^0 C_l \alpha_g^{1/3}},
\]
where \(C_l\) is the sound velocity in the liquid.

We take the liquid acoustically compressible, and the gas is calorically perfect
\[
p_l = p_0 + C_l^2 (\rho_l^0 - \rho_0^0), \quad p_g = \rho_g^0 RT_g,
\]
where \(R\) is the gas constant. Here and below, the indices 0 at the bottom are furnished with parameters relating to the initial unperturbed state.
The accepted system of equations allows to describe the dynamics of waves, when the compression of bubbles is determined not only by the effects of radial inertia of the fluid, but also by acoustic discharge on the bubbles, and consequently by the compressibility of the liquid.

3. Numerical results

Below we present the results of a numerical simulation of the evolution of two wave impulses bell and step-shaped in a homogeneous bubble mixture with an initial volume content of the dispersed phase $\alpha_0 = 10^{-3}$ acting through adjacent boundaries as in Fig. 1. A bell-shaped impulse was set in the form

$$p^0(t, y) = p^0(t, x) = p_0 + \Delta p_0 \exp\left[-\left(\frac{t-t_0/2}{t_0/6}\right)^2\right].$$

The impulse of the step shape was given in the following form

$$p^0(t, y) = p^0(t, x) = p_0 + \Delta p_0.$$

The following data were used as initial values of the system: $\alpha_0 = 10^{-3}$ m, $p_0 = 0.1$ MPa, $T_0 = 300$ K, $\lambda_g = 2.6 \cdot 10^{-2}$ J/(K·sec·m), $\rho_0^0 = 1000$ kg/m$^3$, $c_g = 1006$ J/(K·kg), $\rho_{g0}^0 = 1.29$ kg/m$^3$, $\nu_f = 2 \cdot 10^{-6}$ m$^2$/sec, $\gamma = 1.4$. The amplitude of the pulse is $\Delta p_0 = 0.3$ MPa.

Fig. 2 shows the calculated oscillograms corresponding to the readings of the sensors D1, D2, D3 and D4 located diagonally in the calculated region, respectively, at a distance of 0.1, 0.3, 0.5 and 0.7 m from the origin (Fig. 1). Blue lines correspond to the case when the Lagrangian boundaries $x_0 = 0$ and $y_0 = 0$ are acted upon by wave pulses of a bell shape with a time length $t_s = 10^{-4}$ sec. Red line corresponds to the bell-shaped pulse with a time length of $t_s = 10^{-3}$ sec. The black line corresponds to a step-shaped pulse.

Because of the considering area geometry when the pulses interact, the maximum of their interaction will propagate along the diagonal of the calculated region. It can be seen from the oscillograms that the bell-shaped pulses become weaker with time, and the short-wave pulses do it rapidly. This decrease in the wave amplitude is explained by the proximity of its duration to the period of natural oscillations of the bubbles ($t_s \approx t_M$, $t_M = 2\pi a_0 \left(\rho_{g0}^0 / 3\gamma p_0\right)^{1/2}$). In this case, there are
anomalously strong dispersion and dissipative mechanisms [2]. Therefore, when the short-wave pulses interact, the pressure does not exceed 0.2 MPa. In the case of interaction of longer wavelength signals ($t_s = 10^{-3}$ sec), as they pass into the bubble medium, the total wave pulse, due to the radial inertia of the bubble liquid, amplifies in amplitude and the pressure recorded by the sensors can exceed 0.9 MPa.

![Fig. 3](image-url)

Fig. 3. The evolution of two wave impulses of bell-shaped form (a and b) and a "step" (c) type, acting through adjacent boundaries

In the case of a step-shaped wave the total amplitude from two pulses that propagates along the diagonal of the calculated region increases as the wave passes. The amplitude value for the calculations performed could reach 1.2 MPa. This increase can be explained as follows: in contrast to the case of a bell-shaped impulse, when the media relaxes after the compression and the bubbles expansion dissipate a part of the wave energy, when the pulse has the step shape, the bubbles remain compressed constantly due to the increase in the background pressure behind the wave. Consequently, the dissipative mechanisms work weaker. Some increase in the amplitude of the wave is associated with secondary waves formed behind the front of the main wave due to small oscillations of the bubbles and catch up the main pulse propagating through a medium with a higher initial pressure.

For visual representation of the described results, contour maps of the calculating area are shown (Fig. 3). In Fig. 3a and 3b show the effect of a bell-shaped pulse, and in Fig. 3c – an impulse of a step kind. According to the presented maps it is seen that in the case of a step wave the value of the peak pressure in the system exceeded previous two cases.

4. Conclusion
As a result of numerical studies it was found that in contrast to the case of a bell-shaped impulse signal, when a step-shaped pulse passes, the bubbles remain compressed constantly due to the increase
in the background pressure behind the wave and perform minor fluctuations. In this connection, the
dissipation of the amplitude of the original signal decreases with oscillations of the bubbles. Some
increase in the amplitude of the wave signal is associated with secondary waves, which are formed
behind the front of the main wave due to small oscillations of the bubbles and catch up the main pulse
propagating through a medium with a higher initial pressure.

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