LHAASO J2226+6057 as a pulsar wind nebula

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Received 30 August 2022 / Accepted 22 September 2022

ABSTRACT

Context. The Large High Altitude Air Shower Observatory has reported the detection of cosmic-ray sources in the Milky Way that can accelerate particles up to PeV ($=10^{15}$ eV) energies. These sources, so-called “PeVatrons", are mostly unidentified. Several classes of sources, such as supernova remnants, pulsar wind nebula, or young stellar clusters can potentially be the counterparts of these PeVatrons.

Aims. The aim of this work is to study a pulsar wind nebula interpretation of one of these PeVatrons, LHAASO J2226+6057, which has a relatively well covered multifrequency spectrum.

Methods. We have performed a leptonic, time-dependent modeling of the pulsar wind nebula (PWN) associated with PSR J2229+6114 considering a time-energy-dependent diffusion-loss equation. Injection, energy losses, as well as the escape of particles were considered to balance the time-dependent lepton population. We have also included the dynamics of the PWN and the associated supernova remnant and their interaction via the reverse shock to study the reverberation phase of the system.

Results. We have considered different values of the braking index ($n$) and true age ($t_{\text{age}}$) for the fitting of the MultiWaveLength (MWL) spectral energy distribution (SED) of LHAASO J2226+6057. The best-fit PWN model parameters and their 1σ confidence intervals have been evaluated. We have also demonstrated the impact of reverberation on the MWL SED with increasing time. Additionally, we have discussed the resultant large radius and low magnetic field associated with the PWN in question, as caveats for the possible physical connection of the pulsar as the origin of this high energy source.

Key words. radiation mechanisms: non-thermal – pulsars: general – gamma rays: general

1. Introduction

Recent observations by state-of-the-art observatories, such as the Large High Altitude Air Shower Observatory (LHAASO), Tibet ASy, the High Altitude Water Cherenkov (HAWC), among others, have paved the way for the detection of multiple Galactic ultra-high energy (UHE; $E_\gamma \geq 100$ TeV) gamma-ray sources (Abe+20; Amenomori+19, 2021; Cao+20). Upcoming observatories such as the Cherenkov Telescope Array (CTA; Cherenkov Telescope Array Consortium 2019) and the Southern Wide-field Gamma-ray Observatory (SWGO; Albert+19) will be of importance to identify and characterize these PeVatrons; Galactic CR sources that accelerate particles up to PeV energies.

LHAASO is a state-of-the-art dual-task facility designed for CRs and gamma-ray studies at a few hundred GeV to a few PeV, located at 4410 m above sea level in China (Cao+10). The recent data reported by the LHAASO observatory show the existence of 12 significantly detected sources (>7σ) that emit gamma rays with energies above several hundred TeVs (Cao+21). Most of the sources reported by LHAASO have significantly extended gamma-ray emission regions up to ~1°. The very high energy (VHE; 100 GeV ≤ $E_\gamma$ ≤ 100 TeV) counterparts of these sources residing in the Galactic plane have been associated with pulsar wind nebulae (PWNe), based on the spatial proximity with highly energetic pulsars and typically extended morphological features (H. E. S. S. Collaboration 2018). It has already been posited that UHE gamma-ray emission spatially coincident or in very close proximity of energetic pulsars with a high spin-down luminosity ($\dot{E} > 10^{38}$ erg s$^{-1}$) may be a universal feature; for more details, readers can refer to Albert+21. Moreover, the Crab nebula, associated with pulsar PSR B0531+21, was confirmed to be a PeVatron by recent LHAASO observations (Cao+21). Bearing all of this in mind, it is natural to consider PWNe as possible Galactic PeVatrons, from which UHE gamma rays are detected.

PWNe, considered to be one of the most efficient lepton accelerators in the Galaxy, are powered by highly energetic pulsars. Pulsars dissipate most of their rotational energy via the injection of ultrarelativistic electron-positron pairs, which form a cold, ultrarelativistic wind of particles. Since the bulk velocity of this ultrarelativistic wind is supersonic with respect to the ambient medium, this wind creates a termination shock. Injected
particles can be accelerated to very high energies at this termination shock. The accelerated leptons can then interact with the ambient matter, photon fields, and the magnetic field through Bremsstrahlung, inverse-Compton (IC), and synchrotron processes. The cooling of the accelerated leptons results in a MWL spectrum ranging from radio to gamma-ray energies. Here, we consider LHAASO J2226+6057 as our source of interest, as this is the only UHE gamma-ray source for which data have been observed across radio, X-ray, and gamma-ray energy ranges. It has been observed that LHAASO J2226+6057 is situated in a complex morphological region. Due to the close spatial proximity of Boomerang PWN, as well as supernova remnant (SNR) G106.3+2.7 and associated molecular clouds (MCs), it is hard to confirm the exact source responsible for the UHE gamma-ray emission observed. From the observations of Imaging Air Cherenkov Telescopes (IACTs), such as MAGIC and VERITAS, it was found that the emission region was divided into two morphological regions: the head and the tail. The head region contains Boomerang PWN and PSR J2229+6114, and the tail region contains VER J2227+608, which is likely to be associated with SNR G106.3+2.7 and the MCs in the region. Faint and diffuse radio and X-ray emission were also observed from the tail region. It is possible that the emission from Boomerang PWN illuminates the head region, whereas hadronic interaction occurring between the SNR and the MCs are responsible for the gamma-ray emission in the tail. Although it is difficult to confirm which source is actually responsible for the UHE gamma-ray emission, comprehensive studies are needed to explore both scenarios. New data from upcoming observations with high angular resolution by MAGIC and VERITAS will be crucial to shed new light on this source. We discuss the features of LHAASO J2226+6057, as well as the previous works done for this source in Sect. 2. Then we comment on the PWN model used in this work in Sect. 3, and present the results in Sect. 4. We finally conclude in Sect. 5.

2. LHAASO J2226+6057 features

LHAASO J2226+6057 was detected at RA = 336.75° and Dec = 60.95° with a significance of 13.6σ above 100 TeV. Its gamma-ray spectrum reaches up to a maximum energy of 0.57 ± 0.19 PeV (Cao et al. 2021). This source is spatially associated with SNR G106.3+2.7, as well as the pulsar J2229+6114 and its wind nebula, known as the “Boomerang” nebula (Kothes et al. 2001). PSR J2229+6114 is a bright gamma-ray pulsar with a spin period of 51.6 ms, a characteristic age of 10.460 yr, and a spin-down luminosity of 2.2 × 10^{37} erg s^{-1} (Halpern et al. 2001a). Pulsed GeV gamma-ray emission from this pulsar was detected by Fermi-LAT (Abdo et al. 2009a). In the VHE range, SNR G106.3+2.7 was observed by VERITAS as VHE J2229+608, located 0.7° away from PSR J2229+6114 (Acciari et al. 2009). Gamma rays with GeV energies (Xin et al. 2019), diffuse nonthermal X-rays (Fujita et al. 2021), as well as radio data (Pineault & Joncas 2000) have also been observed from the source region. The distance of the source suffers from great uncertainty. It was estimated to be 7.5 kpc (based on the pulsar dispersion measure; Abdo et al. 2009a), 3 kpc (based on X-ray absorption measurements; Halpern et al. 2001a), and 0.8 kpc (based on measurements of radial velocities of atomic hydrogen and molecular material; Kothes et al. 2001). In this work we consider the distance to be 3 kpc, similar to Joshi et al. (2022) and Yu et al. (2022).

The possible connection between LHAASO J2226+6057 and PSR J2229+6114 has previously been studied by Breuhaus et al. (2022), Joshi et al. (2022), and Yu et al. (2022). Breuhaus et al. (2022) performed a time-independent one-zone treatment, which is a steady-state leptonic scenario from the PWN, to explain only the highest energy gamma-ray data. The evolution of the source was neglected, and also MWL data were not used. Joshi et al. (2022) used a time-dependent one-zone leptonic scenario from the PWN. However, the authors did not consider the effect of escape for LHAASO J2226+6057, which they only accounted for LHAASO J1908+0621, nor did they explore the effect of age and the braking index on the evolution of the injected leptonic population. The impact of the SNR reverse shock and its effects on the PWN radius evolution were not taken into account, assuming that such effects are important only if the age of the PWN is greater than 10 kyr, which is not necessarily the case; for more details, readers can refer to Martín et al. (2016), for example. Yu et al. (2022) also performed a similar study. The authors argue that a distorted nebula, which was created due to the impact of a SNR reverse shock, is responsible for the GeV gamma-ray emission observed from the source region by Xin et al. (2019). Their results are similar to those of Joshi et al. (2022). Our PWN model here intends to test their conclusions after relaxing assumptions or adding additional physical details.

A recent paper by Tibet ASgamma Collaboration (2021) has proposed a hadronic origin of the LHAASO source based on the spatial proximity of a molecular cloud (MC) with the gamma-ray centroid of the source. However, as pointed out by Breuhaus et al. (2022), the associated SNR is quite old to produce the observed hard gamma-ray spectrum at the highest energies. Consequently, a hadronic scenario from a SNR+MC association would need a peculiar modeling to explain the UHE gamma-ray spectrum observed from the source. A novel approach was explored in De Sarkar & Gupta (2022) to explain the hadronic origin of LHAASO J1908+0621 and, perhaps, a similar approach may participate here as well.

3. Brief description of the model

For this work, we have used the code TIDE, for which earlier applications can be found in Torres et al. (2014), Martín et al. (2016), and Torres & Lin (2018), for example. The numerical code solves the evolution of leptonic pair distribution in the PWN, as a function of Lorentz factor γ at time t described by the following equation:

\[
\frac{\partial N(\gamma,t)}{\partial t} = Q(\gamma,t) - \frac{\partial}{\partial \gamma} [\dot{\gamma}(\gamma,t)N(\gamma,t)] - \frac{N(\gamma,t)}{\tau(\gamma,t)}. \tag{1}
\]

The left-hand side of Eq. (1) describes the variation of lepton distribution in time. The first term on the right-hand side is the lepton injection function Q(γ, t), which is usually assumed as a broken power law:

\[
Q(\gamma,t) = Q_0(t) \begin{cases} \left(\frac{\gamma}{\gamma_b}\right)^{-\alpha_1} & \gamma \leq \gamma_b, \\ \left(\frac{\gamma}{\gamma_b}\right)^{-\alpha_2} & \gamma > \gamma_b. \end{cases}
\]

The second and third terms on the right-hand side take radiative losses into account – such as synchrotron, IC, Bremsstrahlung, adiabatic losses, or heating – as well as the escape of the particles (we assume Bohm diffusion), respectively (see Martín et al. 2012 for the incorporated formulae). The normalization factor Q_0(t) is calculated using the spin-down luminosity L(t) of the pulsar through the equation, \((1 - \eta)L(t) = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \gamma n_{e}\gamma c^2 Q(\gamma,t)d\gamma\), where \(\eta\) is the fraction of spin down power that goes on to...
power the magnetic field of the PWN. The magnetic field varies in time as a result of the balance between the magnetic field power and adiabatic gains or losses of the field due to the contraction or the expansion of the PWN (Torres et al. 2012). The reverberation phase of the PWN included in the model has been considered to be the same as that in Torres (2017). The variation of the PWN radius is calculated by taking into account the age, SNR explosion energy, ambient medium density, expansion velocity of the nebula, and pressure profiles of the SNR at the position of the PWN. The model takes into account the change in pressure profiles depending on whether the PWN shell is surrounded by unshocked ejecta (i.e., $R < R_{RS}$), or the shocked ejecta (i.e., $R_{RS} < R < R_{SNR}$), where $R_{RS}$ and $R_{SNR}$ are the radii of the SNR reverse shock and the SNR, respectively. After reverberation, an assumed Sedov expansion follows when the PWN reaches the pressure of the SNR. For more details, readers can refer to Martin et al. (2016), Torres (2017), and Torres & Lin (2018), for example. For a more thorough description of the reverberation period, we need to account for the fact that the ejecta pressure is not constant (Bandiera et al. 2020, 2022). This is beyond the scope of this work since, at the moment, a prescription to deal with this fact in the context of the radiative PWN is unavailable. Given the likely young age of the source, a PWN would be before the time of largest compression (see below), and thus we expect the current approach to be acceptable.

4. Results

4.1. Braking index and true age exploration

In the works of Joshi et al. (2022) and Yu et al. (2022), the true age and the braking index were chosen to be 7000 yr and 3, respectively. Since the choice of the true age is essential for a time evolutionary model of PWNe, we explore other values here as well. On the other hand, if the pulsar is assumed to be a dipole in vacuum, and considered to be spinning down by emitting only magnetic dipole radiation (MDR), then the corresponding braking index associated with the spin down is calculated to be 3 (Manchester & Taylor 1977). Alternatively, the spin down of a pulsar driven entirely by a particle wind would result in a braking index of 1 (Michel 1969; Manchester et al. 1985). A combination of magnetic dipole radiation and wind braking would result in a braking index with a value in between 1 and 3 (Archibald et al. 2016). Most of the observed pulsar braking indices fall within this range (Espinoza et al. 2011b, Pons et al. 2012, and references therein). Although, there are exceptions: for example, PSR J1640–4631, which has a braking index of $n = 3.15 \pm 0.03$ (Archibald et al. 2016), and PSR J1734–3333, which has a braking index of $n = 0.9 \pm 0.2$ (Espinoza et al. 2011a). For this work, we only consider the braking index range $1 < n < 3$. Since the choice of the braking index affects the characteristics of the pulsar spin down, it is also important to explore whether the variation of $n$ affects the SED.

To explore the effect of $n$ and $t_{age}$ on the MWL SED, we have chosen $t_{age} = 1000, 4000,$ and 7000 yr, and $n = 1.5, 2.0, 2.5,$ and 3.0. Given the characteristic age $\tau_c \approx 10.5$ kyr and the present-day spin-down luminosity $L(t_{age}) = 2.25 \times 10^{37}$ erg s$^{-1}$, the initial spin-down age ($t_0$) and initial spin-down luminosity ($L_0$) were calculated using the relations,

$$L(t) = L_0 \left(1 + \frac{t}{t_0}\right)^{-\frac{3}{n-1}},$$

and,

$$t_0 = \frac{2\tau_c}{n-1} - r,$$

for a specific choice of $n$ and $t_{age}$. We also assume the values of the following parameters are the same throughout the work: minimum Lorentz factor $\gamma_{\text{min}} = 1$; SN explosion energy $E_{SN} = 10^{51}$ erg; interstellar medium (ISM) ambient density $\rho_{\text{ISM}} = 0.1$ cm$^{-3}$; SNR core density index $n = 0$; SNR envelope density index $n = 9$ (we assume a type II SN as the progenitor, as in Chevalier & Fransson 1992; Blondin et al. 2001; Gelfand et al. 2009); PWN adiabatic index $\gamma = 1.333$ and SNR adiabatic index $\gamma = 1.667$; containment factor $\epsilon$ (ratio of the Larmor radius of particles to the radius of the termination shock) = 0.5; and magnetic compression ratio $\kappa = 3$ (strong shock condition). The ejected mass of the progenitor SNR was also fixed at $M_{age} = 8 M_\odot$ for the $n-t_{age}$ exploration. For the target radiation fields for the IC interaction, we considered the cosmic microwave background, far-infrared (FIR) and near-infrared (NIR) radiation fields at the source position with temperatures of $T_{FIR} = 25 K$ and $T_{NIR} = 5000 K$, respectively. The energy densities associated with the FIR and NIR field were taken from Porter et al. (2006) and fixed at those values for $n-t_{age}$ exploration. It has almost always been observed (see Torres et al. 2014) that the energy densities of the FIR and NIR fields required to explain the IC emission from PWNe, differ from the interstellar radiation fields reported by Porter et al. (2006). Breuhaus et al. (2021) also argue that an enhancement of the radiation field is needed to explain the hard gamma-ray spectrum observed at highest energies. However, since the aim of this subsection is to compare the effect of $n$ and $t_{age}$ on the computed SED, we have fixed the energy densities of FIR and NIR fields at the values given by Porter et al. (2006), that is $\omega_{FIR} = 0.29$ eV cm$^{-3}$ and $\omega_{NIR} = 0.45$ eV cm$^{-3}$, so there are fewer free parameters left to vary. The maximum Lorentz factor of the lepton distribution was fixed by the most restrictive condition between the synchrotron limit (de Jager et al. 1996) or gyroradius limit (de Jager & Drijnamata-Atai 2009) during the evolution. For a specific choice of $n$ and $t_{age}$, we varied the injection function parameters, that is to say low energy index $\alpha_1$, high energy index $\alpha_2$, energy break $\gamma_b$, as well as magnetic fraction $\eta$, to describe the MWL SED of the source.

First, we fixed the braking index at the value of $n = 2.5$, and considered three different cases of $t_{age}$. The MWL SED along with the computed model flux, injected lepton spectrum, and magnetic field at the considered $t_{age}$ are shown in Fig. 1. From the figure, it can be seen that the fit corresponding to $t_{age} = 7000$ yr is the best out of the three ages considered. For $t_{age} = 1000$ yr and $t_{age} = 4000$ yr, the X-ray and radio data can be adequately explained. However, for $t_{age} = 1000$ yr, the IC emission is not significant enough to explain the high-energy data. Also, for $t_{age} = 4000$ yr, the IC emission only partially explains it. From this exploration, it is apparent that the age of the PWN lies within 4000 and 7000 yr. In this particular work, we consider $t_{age} = 7000$ yr to compare our results with those obtained by Joshi et al. (2022) and Yu et al. (2022), albeit these authors did not explore other options. We also discuss the results of considering the true age as a free parameter in a later subsection.

After limiting the age of the system, we subsequently explored the effect of the braking index on the evolution of the source. Assuming a fixed $t_{age} = 7000$ yr, we changed the braking index and tried to describe the MWL SED. We calculated $L_0$ and $t_0$ using Eqs. (3) and (4). We again changed $\alpha_1$, $\alpha_2$, $\gamma_b$, A23, page 3 of 9
and $\eta$, similarly to the previous case. The calculated spectra for $n = 1.5$, $2.0$, $2.5$, and $3.0$ are given in Fig. 2. We note that $n$ does not significantly affect the computed SEDs and we have selected $n = 2.5$ for further study.

4.2. $\chi^2$ fitting of the MWL SED

We have considered typical PWN parameters as initial input, and, using the TIDEFIT code Martin & Torres (2022), subsequently varied them to solve Eq. (1) and computed the best $\chi^2$-fitted model spectrum. To find the best-fit spectra, we used $t_{\text{br}} = 7000$ yr and $n = 2.5$. Similar to the previous discussion, $L_0$ and $t_0$ have been fixed at the value calculated by Eqs. (3) and (4). The PWN parameters that had already been fixed in the above discussion have also been fixed in this calculation as well, except for the cases of the ejected mass and FIR and NIR energy densities (between 0.01 eV cm$^{-3}$ and 5 eV cm$^{-3}$). Since the ejected mass of the progenitor SNR directly affects the size, as well as the magnetic field of the PWN, it was left free within the typical ejected mass range of $7 M_{\odot}$ to $15 M_{\odot}$. Apart from these changes and similarly to the previous discussion, $\alpha_1$, $\alpha_2$, $\gamma_b$, and $\eta$ were left free to vary so as to find the best-fit MWL spectrum. The parameters used in the model are reported in Table 1, in which we have divided the parameters among measured or assumed, derived, and fitted values. The resulting plots are given in Fig. 3. The time evolution of the calculated MWL spectrum and injection spectrum are given in Fig. 4.

From the top row of Fig. 3, it can be seen that the computed MWL spectrum using the model along with the 1$\sigma$ confidence interval, matches with the observed MWL data well. The goodness of fit can also be seen from the bottom residual plot. The systematic uncertainty associated with the model is 0.32, and the $\chi^2$/d.o.f. for the given fit is 35.65/30. Also from the figure, as well as from Table 1, it can be seen that FIR and NIR radiation fields do not contribute to the IC emission needed to explain the VHE-UHE data, rather it was found that the CMB is most likely solely responsible as the target photon field required. A recent work by de Oña Wilhelmi et al. (2022) has also considered CMB photons as the most relevant target for IC scattering. However, such a result is somewhat different from what was assumed by Joshi et al. (2022) who considered radiation fields 1.5–3.0 times that of CMB to fit the data.
From the bottom row of Fig. 3 and Table 1, it can be seen that the PWN would be very extended at the present age according to the model, and concurrently the associated magnetic field would be very low and close to the Galactic average value. The large radius of the PWN may agree with the large extension measured by LHAASO, but it contradicts the radio and X-ray sizes observed for the Boomerang PWN (Halpern et al. 2001a,b), as these sizes are much smaller compared to the calculated PWN radius. The required magnetic field to fit the data is also uncomfortably low (we discuss this below), and begs the question of how the particles are confined in such a diluted PWN.

### 4.3. Possible impact of reverberation

From Fig. 3, and from the resulting values given in Table 1, it can be seen that the radius of the SNR reverse shock is smaller than the PWN radius, which means that the reverse shock has just reached the position of the PWN shell at the onset of the reverberation phase of the PWN. If the age of the system is increased further from the considered age, the effect of reverberation will be apparent on the MWL spectrum. Since the PWN has just started to contract at the considered age of 7000 yr, it is unlikely that it is heavily distorted.

### 4.4. $t_{\text{age}}$ as a free parameter

From the above discussion, it is apparent that the true age of the PWN cannot be much greater than 7000 yr due to reverberation beginning afterwards. As discussed in Sect. 4.1, the true age cannot also be lower than 4000 yr. Thus, we have used TIDEFIT, leaving the true age parameter free. Apart from $t_{\text{age}}$, the free parameters considered in this case are the same as that discussed in Sect. 4.2. The best-fit $t_{\text{age}}$, from the fitting of the MWL SED, is 4880.3 (3871.4, 6060.4) yr. This value is consistent with the range of the said parameter discussed above. The best fitted model spectrum is similar to that shown in Fig. 3. Although, the best-fit values of the free parameters obtained from the $\chi^2$ fitting of the MWL SED, in this case, are not exactly the same as those obtained assuming $t_{\text{age}} = 7000$ yr (see Sect. 4.2 and Table 1), the 1σ confidence intervals of these free parameters for the two cases overlap with each other. Both solutions are essentially the same within uncertainties. So we decided not to report the best-fit results obtained in this case. We have also obtained a comparatively large PWN radius ($R_{\text{PWN}} = 8.26$ pc) and low magnetic field ($B_{\text{PWN}} = 1.87$ µG) associated with the PWN at the best fitted $t_{\text{age}}$, similar to the case discussed in Sect. 4.2 for $t_{\text{age}} = 7000$ yr, which we discuss next.

### 5. Concluding remarks: Large radius and low magnetic field issue

In this paper, we have provided a detailed, time-dependent, one-zone model to explain the UHE gamma-ray emission observed from the direction of LHAASO J2226+6057 using the emission from PWN associated with PSR J2229+6114. We summarize the main points obtained in this work, and compare them with previous studies.

1. The effects due to the variation of the true age and braking index were not explored in previous studies performed for this source.

2. From our study, we found that a true age between 4000 and 7000 yr is most suitable for the fitting of the SED of the source. Considering $t_{\text{age}}$ as a free parameter during the fitting of the MWL SED also revealed a compatible result. Our study found that no significant effects can be seen on the MWL spectrum if we consider different braking indices.

3. We used the TIDEFIT code to fit the observed MWL SED by computing the best $\chi^2$ fit model spectrum. From the fit, we found that CMB is the target photon field responsible for the IC cooling of the injected leptons from

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**Fig. 2.** MWL SEDs of LHAASO J2226+6057 for $n = 1.5, 2.0, 2.5,$ and 3.0. The color scheme of the data points is same as given in Fig. 1.
Table 1. Physical parameters used by and resulting from the fit.

| Definition | Parameter | Value |
|------------|-----------|-------|
| Age        | $t_{\text{age}}$ [kyr] | 7     |
| Characteristic age | $\tau_c$ [kyr] | 10.5  |
| Braking index | $n$ | 2.5   |
| Present day spin-down luminosity | $L(t_{\text{age}})$ [erg s$^{-1}$] | 2.25 $\times$ 10$^{37}$ |
| Distance | $D$ [kpc] | 3     |
| Minimum energy at injection | $\gamma_{\text{min}}$ | 1     |
| SN explosion energy | $E_{\text{SN}}$ [erg] | 10$^{51}$ |
| ISM density | $\rho_{\text{ISM}}$ [cm$^{-3}$] | 0.1   |
| SNR core density index | $w_{\text{core}}$ | 0     |
| SNR envelope density index | $w_{\text{env}}$ | 9     |
| PWN adiabatic index | $\gamma_{\text{PWN}}$ | 1.333 |
| SNR adiabatic index | $\gamma_{\text{SNR}}$ | 1.667 |
| Containment factor | $\epsilon$ | 0.5   |
| Magnetic compression ratio | $\kappa$ | 3     |
| CMB temperature | $T_{\text{CMB}}$ [K] | 2.73  |
| CMB energy density | $\omega_{\text{CMB}}$ [eV cm$^{-3}$] | 0.25  |
| FIR temperature | $T_{\text{FIR}}$ [K] | 25    |
| NIR temperature | $T_{\text{NIR}}$ [K] | 5000  |

| Derived parameters |
|--------------------|
| Initial spin-down luminosity | $L_0$ [erg s$^{-1}$] | 1.13 $\times$ 10$^{38}$ |
| Initial spin-down age | $\tau_0$ [kyr] | 7     |

| Fitted parameters |
|--------------------|
| Energy break at injection | $\gamma_b$ | 3338.00 (2082.91, 10597.30) |
| Low energy index at injection | $\alpha_1$ | 1.4522 (1.0000, 1.6432) |
| High energy index at injection | $\alpha_2$ | 2.3727 (2.3316, 2.3890) |
| Ejected mass | $M_{\text{ej}}$ [M$_\odot$] | 8.8927 (8.1735, 9.3202) |
| Magnetic fraction | $\eta$ | 0.0033 (0.0026, 0.0060) |
| FIR energy density | $\omega_{\text{FIR}}$ [eV cm$^{-3}$] | 0.0100 (0.0100, 0.4611) |
| NIR energy density | $\omega_{\text{NIR}}$ [eV cm$^{-3}$] | 0.0100 (0.0100, 5.0000) |

| Resulting features |
|--------------------|
| PWN radius | $R_{\text{PWN}}$ ($t_{\text{age}}$) [pc] | 9.33  |
| SNR forward shock radius | $R_{\text{FS}}$ ($t_{\text{age}}$) [pc] | 16.23 |
| SNR reverse shock radius | $R_{\text{RS}}$ ($t_{\text{age}}$) [pc] | 8.98  |
| PWN magnetic field | $B_{\text{PWN}}$ ($t_{\text{age}}$) [uG] | 1.91  |

**Notes.** The bracketed terms in the fitted parameters section signify the lower and upper bounds of a 1σ confidence interval.

3. We have taken the effect of reverberation into account in our modeling. It was found that the PWN is at the onset of compression due to the impact of a reverse shock hitting the shell of the PWN. The effect of reverberation on the MWL spectrum was also explored by gradually increasing the true age of the PWN to the characteristic age. The MWL description worsened with increasing age. The true age cannot be much larger than the considered age of 7000 yr if a PWN is responsible for the gamma-ray emission detected. Since reverberation does not provide a better fit, its inclusion does not solve the large radius and low magnetic field issue.

4. We have also found that a very low magnetic field (~2 uG) is needed to explain the MWL SED of the source, which is comparable with the Galactic average magnetic field value. A similarly low magnetic field (a few uG) was found in previous studies as the one needed to describe this source (Joshi et al. 2022; Yu et al. 2022). Moreover, a low magnetic field was found for other LHAASO detected PeVatron candidates as well (De Sarkar & Gupta 2022; Joshi et al. 2022; Crestan et al. 2021; Li et al. 2021; Burgess et al. 2022), which is an a priori obvious outcome of requesting a leptonically generated high energy emission. Further complications of the model, as we have discussed, do not significantly alleviate this.

As discussed earlier, an uncertainty regarding the distance of the source remains. We have explored this uncertainty by considering two very different values $D = 800$ pc and $D = 7.5$ kpc. For $D = 7.5$ kpc, we have found the fit is comparatively worse compared to that discussed in Sect. 4.2, with $\chi^2$/d.o.f. = 36.78/30 and a systematic uncertainty of 0.41. On the other hand, the fit in the case of $D = 800$ pc is comparable to that discussed in Sect. 4.2, with $\chi^2$/d.o.f. = 35.88/30 and a systematic uncertainty of 0.5.
Fig. 3. Calculated best-fit MWL spectrum is shown at the top row, along with the MWL data points. The color scheme of the data points are the same as that in Fig. 1. In the bottom panel of the figure, the residuals are also plotted. The color scheme of the residuals are the same as the data points. The middle row shows the timescales of radiative losses, adiabatic losses, and the escape of particles considered in the model (left) and the injected lepton spectrum at the present age (right). Also the time evolution of the magnetic field (left), as well as the SNR forward shock, SNR reverse shock, and PWN radius (right) are given in the bottom row.

of 0.32. There is no clear evidence to overrule one distance over the other, so we only report the case of $D = 3$ kpc in detail to directly compare our results with those of Joshi et al. (2022) and Yu et al. (2022). It is to be noted that the power law lepton injection spectrum is more favored in the case of $D = 800$ pc, rather than a broken power law spectrum. However, the issue
of a large radius and low magnetic field remains in this case as well.

The large estimated PWN size appears to be a caveat of the PWN interpretation of the LHAASO source. It is likely that both the head and the tail region contribute to the total observed emission from the source. In light of such a complicated source morphology, a one-zone treatment of the source region considered in the model proves to be a simplistic take on the same. The large PWN radius obtained from the calculation could be a consequence of such a simplistic assumption. Nevertheless, our model tends to be the most definitive PWN approach considered thus far to explain the MWL emission from the LHAASO source, complete with a reverberation consideration and PWN true age estimation. It is to be noted that the PWN radius obtained by Joshi et al. (2022) (~3.1 pc) is different from that obtained in this work. Although one zone model has been considered in both cases, the differences in the formalism adopted to compute the radius evolution in the presence of the background SNR may be the reason behind this. In any case, further investigation will be important to properly address the complexity of the source morphology.

The Galactic magnetic field (GMF) plays an important role in the cosmic ray propagation. The intensity and orientation of GMF are constrained by several methods such as Zeeman-splitting observations (Crutcher 1999); infrared, synchrotron, and starlight polarization studies (Nishiyama et al. 2010; Jaffe et al. 2010; Heiles 1996); and Faraday rotation measurements (Han et al. 2006; Pshirkov et al. 2011). The GMF model typically has three components, namely the disc \( B_{\text{Disc}} \), halo \( B_{\text{Halo}} \), and turbulent \( B_{\text{Turb}} \) contributions. Typical values for \( B_{\text{Disc}} \) and \( B_{\text{Halo}} \) lie within the range of 2–11 \( \mu G \) (De Sarkar et al. 2021; Di Bernardo et al. 2013); although, the value of the turbulent component depends on the halo height of the Galaxy (Di Bernardo et al. 2013), for example, for a typical halo height of 8 kpc, the value of \( B_{\text{Turb}} \) comes out to be \( \approx 6 \mu G \) (De Sarkar et al. 2021). Although the exact structure of the small-scale GMF is not known yet, from the models given by Pshirkov et al. (2011) and Jansson & Farrar (2012), and also from the observed secondary-to-primary ratio (De Sarkar et al. 2021, and references therein), the large-scale, average GMF can be estimated to be in the range 2–6 \( \mu G \). The PWN magnetic field...
(\(-2\mu G\)) would be marginally close to, or even lower than, any of the estimates of the average GMF value.

We see two possible ways out of this. On the one side, the local environment of the PWN could have been evacuated by the explosion of the supernova, and/or by earlier explosions, so that the local field in the vicinity is actually much lower than a few \(\mu G\), allowing for a magnetic field contrast to appear between the PWN and its environment. On the other hand, there is still a chance that the approximate representation of reverberation that we have adopted here is still misleading. We know that assuming the ejecta pressure as constant (as all models of PWNe have so far done) is in fact an oversimplification (Bandiera et al. 2020, 2022). A better treatment of the ejecta pressure could plausibly change the reverberation results, and we shall explore this in the future. Of course, it is also possible that the PWN explanation of the source is not realized at all, and an additional hadronic component provides the dominant contribution. For the moment, a conflicting low value for the PWN magnetic field (not solved by age, braking indices, or the currently assumed behavior of the reverberation process) leaves the PWN origin of LHAASO J2226+6057, and other similar sources, in search for further observational tests.

Acknowledgements. This work has been supported by grant PID2021-124581OB-I00. D.F.T. acknowledges as well USTC and the Chinese Academy of Sciences Presidential International Fellowship Initiative 2021VMA0001. This work was also supported by the Spanish program Unidad de Excelencia María de Maeztu CEX2020-001058-M. A.D.S. acknowledges hospitality provided by the Institute of Space Sciences, where majority of this work was carried out. A.D.S. also acknowledges RRI HPC2020 cluster for providing facilities to perform numerical computations reported in this work. A.D.S. thanks Nayanatara Gupta for helpful comments.

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