Temperature dependence of sodium and ionized calcium resonance lines perturbed by helium

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Abstract.
Traces of heavy metals in cool DZ white dwarf stars may be attributed to the accretion of circumstellar dust thought to originate from tidal disruption of rocky parent bodies. Spectra of such stars therefore provide a unique opportunity to study the composition of extrasolar planetary systems. The determination of metal abundances from stellar spectra depends on stellar atmospheric parameters and an accurate prior knowledge of the collision broadening of the line profiles by the most common constituents of the stellar atmosphere. For this purpose, we present theoretical absorption spectra of Na and Ca\textsuperscript{+} broadened by He for the conditions prevailing in cool white dwarfs.

1. Introduction

Ultracool stellar atmospheres show absorption by alkali resonance lines significantly broadened by collisions with neutral perturbers. In the coolest and densest atmospheres, such as those of T dwarfs, Na I and K I broadened by H\textsubscript{2} and He can come to dominate the entire optical spectrum [1]. Na I line profiles have also been observed in extremely cool, metal-rich white dwarfs [2, 3]. The studies of cool DZ white dwarfs observed in the Sloan Digital Sky Survey (SDSS) by [4] showed clearly the importance of extended wings of calcium doublets. They pointed out the need for more accurate line profile calculations than Lorentzian profiles based on approximate impact broadening assumptions. In actuality, the lines are significantly non-Lorentzian, and are broadened due to collisions with helium with detectable wings extended as much as 1000 Å either side of line core. Through their blanketing effect these lines have a dominant influence...
on the model structure and thus on the determination of atmospheric parameters and element abundances.

The theory of spectral line shapes, especially the unified approach we have developed, makes possible accurate models of stellar spectra that account both for the centers of spectral lines and their extreme wings in one consistent treatment. Complete details and the derivation of the theory are given by Allard et al (1999) \[5\].

In this continuation of previous work \[6, 7\] we illustrate the evolution of the absorption spectra of Na-He and Ca \(^+\)-He for the temperatures prevailing in atmosphere of cool white dwarf stars.

2. NaHe line absorption in ultra-cool white dwarf stars

Observations of the oldest and colder white dwarfs following the study of \[2, 3\] have revealed two stars showing very unusual wide and deep absorption at 5000-6000 Å.

Homeier \textit{et al.} (2007) \[8\] investigated effects on the Na doublet of high perturber densities occurring for metal-rich white dwarfs with a helium-dominated atmosphere. They found that the density of neutral atomic helium in these two very cool white dwarfs showing very strong Na absorption could reach several \(10^{21}\) to \(10^{22}\) cm\(^{-3}\) (Fig. 1). Cool white dwarfs require a specific treatment for line broadening due to the high perturber density which is involved.

![Figure 1](image_url)

\textbf{Figure 1.} Number density of helium (left scale) and temperature profile (right scale) for 3000 K and 5000 K WD atmosphere models with varying [He/H] ratio (extracted from Homeier \textit{et al.} 2007 \[8\]).

In dense plasmas, the possibility of several atoms interacting strongly is high, and the effects play a role in the wavelength of the line center, e.g. the shift of the line, as well as the continuum generated far from the line center. The tools the physicist uses to determine line profiles in stellar spectra are very refined, and the days when simple analytic expressions for line widths and shifts would suffice for these applications are long past.

For such high helium density the collisional effects should be treated by using the autocorrelation formalism in order to take into account simultaneous collisions with more than one perturbing atom. The line shape theory requires accurate atomic potentials and transition
moments which are regrettably not yet available for many systems of astrophysical interest. For \textit{ab initio} calculations, it is the selection of basis states and the optimization techniques that may determine the final accuracy. The current state-of-the-art in \textit{ab initio} theories of molecular potentials (atom-atom potentials and dipole transition moments as a function of separation) yield data of spectroscopic accuracy. Our calculations reported here are based on the very recent \textit{ab initio} Na-He potentials of [9] and the earlier radiative dipole moments of Pascale (1983) [10].

![Graph showing \( \Delta V(R) \) and the temperature dependence of modulated dipole \( D(R) \) corresponding to the 3s \( X \rightarrow 3p \, B \, P_{3/2} \) transition of the Na \( D2 \) line.]

**Figure 2.** \( \Delta V(R) \) and the temperature dependence of modulated dipole \( D(R) \) corresponding to the 3s \( X \rightarrow 3p \, B \, P_{3/2} \) transition of the Na \( D2 \) line.

3. Temperature dependence of absorption spectra of Na and Ca\(^{+} \) in dense helium

Detailed line profiles for the Na I D resonance doublets at 0.59\( \mu \)m calculated by Allard et al. (2003) [11] are now updated from the Na-He molecular potentials of [10] to the newer ones of [9].

The unified theory predicts that there will be line satellites centered periodically at frequencies corresponding to integer multiples of the extrema of the difference potential \( \Delta V(R) \):

\[
\Delta V(R) \equiv V_{e'e'}[R(t)] = V_{e'}[R(t)] - V_e[R(t)] = V_e'[R(t)] - V_{e'}[R(t)],
\]

which represents the difference between the electronic energies of a quasi-molecular \( e-e' \) transition [12]. As part of a more general or unified concept of spectral line formation, satellites are the binary-collision manifestation of a ubiquitous phenomena, the many-body nature of spectral lines from dense gases.

While the position of the line satellites critically depends on the interaction potential, their strength depends on both the interaction potential and the radiative dipole moments, \( D(R_{\text{ext}}) \), in the internuclear region where the line satellite is formed. In Allard \textit{et al}. 1999 [5] we define \( d_{ee'}(R(t)) \) as a modulated dipole

\[
D(R) \equiv \tilde{d}_{ee'}[R(t)] = d_{ee'}[R(t)]e^{-\frac{V_e[R(t)]}{kT}},
\]
$V_e$ is the lower state potential as we consider absorption profiles. $\Delta V$ and variation of $D(R)$ with temperatures of Na-He modulated dipole are displayed in Fig. 2.

The presence of line satellite features is very sensitive to the temperature due to the fast variation of the modulated dipole moment with temperature in the internuclear region where the line satellite is formed (Fig 2). NaHe satellite is apparent for $T \geq 1000$ K but disappears for decreasing $T$ when the transition moment $\tilde{d} = [R(t)]$ becomes very small (Fig. 1 of Allard et al. (2003) [11]). In Fig. 3 we show the absorption cross section for the resonance line of Na for a He density of $10^{21}$ cm$^{-3}$ and temperatures from 10000 to 3000 K. The NaHe satellite is at 0.53 $\mu$m which might be the large feature observed in the spectrum of WD2356-209 (Fig. 2B of Oppenheimer et al. (2001) [2]). There is a total blend of the line satellite in the core of the line when the density reaches $10^{22}$ cm$^{-3}$ (Fig. 4).

![Figure 3](image1.png)

**Figure 3.** Variation of the absorption cross section of the Na $D_2$ line (left) and the $D_1$ line (right). ($n_{He}=10^{21}$ cm$^{-3}$, from top to the bottom $T=10000$, 5000 and 3000 K).

![Figure 4](image2.png)

**Figure 4.** Variation of the absorption cross section of the Na $D_2$ (blue lines) and $D_1$ (red lines) with helium density ($T=3000$ K, from top to the bottom $n_{He}=5 \times 10^{21}$ and $10^{22}$ cm$^{-3}$).

In Fig. 5 we show the variation of the absorption cross section of the $K$ and $H$ lines, of Ca$^+$-He at various temperatures from 10000 to 4000 K. The strength of the satellites at about 0.45 $\mu$m increases with temperature. From this dependence it is apparent that the sensitivity of the spectrum to temperature and pressure is a tool for determining basic parameters of white dwarf atmospheres, provided that the physics underlying its formation is well described.
Figure 5. Variation of the absorption cross section of the Ca II K line (left) and the H line (right). ($n_{\text{He}} = 5 \times 10^{21}$ cm$^{-3}$, from top to the bottom $T = 10000$, 6000 and 4000 K).

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