Electron random walk in ideal phonon gas. 
Spectra of density matrix evolution 
and electron mobility 1/f noise

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Abstract. The previously derived exact evolution equations for density matrix of electron (quantum particle) in phonon field (boson thermostat) are qualitatively analysed. Their statistical interpretation is explained in detail, and their main symmetry and spectral properties are expounded. In application to the electron’s random walk, it is shown that these properties certainly forbid conventionally assumed Gaussian long-range asymptotic of the walk statistics. Instead, the exact equations imply super-linear dependence of fourth-order cumulant of total electron’s path on observation time, which signifies existence of 1/f-type low-frequency fluctuations in electro’s diffusivity and mobility. Physical meaning of this result is discussed, along with general origin of 1/f-noise in classical and quantum Hamiltonian many-particle systems.

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1. Introduction

This paper is direct continuation (second part) of the work [1], where exact equation for a quantum particle (“electron”) in interaction with harmonic boson thermostat (“ideal phonon gas”) were reduced to a visual shortened form lightening their consideration, and, besides, their relation to 1/f noise (1/f- type fluctuations of electron’s mobility) was discussed.

Here, we start systematic investigation of these equations. First, we will consider more carefully their statistical meaning, as it looks from the viewpoint of probability theory. Then we will formulate their main principal symmetry properties and spectral properties and show that they are incompatible with such “white-noise” statistics of the electron’s random walk what follows from standard kinetic approximations. The
alternative predicted by the exact equations is such statistics which combines white noise in velocity of electron and 1/f noise (or “flicker” noise) in its diffusivity and mobility.

We will finish the paper by discussion of this our concrete result in the framework of one more attempt to make our understanding of universal nature of 1/f noise being clear also for scientific community.

2. The model and the problem

First, let us recollect main results of [1]. We have considered the simplest Hamiltonian of quantum particle (“electron”) in quantum boson (“phonon”) field,

\[ H = H_e + H_{ph} + H_{int}, \]

\[ H_e = \frac{p^2}{2m}, \quad H_{ph} = \sum_k \hbar \omega_k a_k^\dagger a_k, \]

\[ H_{int} = \frac{1}{\sqrt{\Omega}} \sum_k \left[ c_k^* e^{ikr} a_k + c_k e^{-ikr} a_k^\dagger \right], \]

and under the thermodynamical limit, when the system’s volume \( \Omega \to \infty \), derived the following exact hierarchy of evolution equations:

\[ \dot{\Delta}_n = -\hat{V} \nabla_X \Delta_n - i \sum_{k \in K_n} \sigma_k (\omega_k - k\hat{V}) \Delta_n - \]

\[ - (i\hbar \sqrt{\Omega_0})^{-1} \sum_{k \in K_n} \sigma_k c_k^* \left[ e^{ikY/2} - e^{-ikY/2} (1 + N_k^{-1}) \right] \Delta_{n-1} + \]

\[ + \frac{\sqrt{\Omega_0}}{i\hbar} \sum_{\sigma \in \{+, -\}} \sigma \int c_{q-\sigma}^\dagger \left( e^{iqY/2} - e^{-iqY/2} \right) N_q \Delta_{n+1} \frac{d^3q}{(2\pi)^3} \]

Here \( \Delta_0 = \Delta_0(t, X, Y) \) is density matrix of the electron, in the coordinate representation, and \( \Delta_n = \Delta_n(t, X, Y, K_n, \Sigma_n) \) are definite functions characterizing statistical correlations of electron with quasi-classic (coherent-state) amplitudes \( A_k^\pm \) (\( A_k^- \equiv A_k^\dagger \equiv a_k \exp(ikX) \), \( A_k^+ \equiv A_k^* \equiv a_k^\dagger \exp(-ikX) \)) of \( n \) phonon modes marked by \( K_n = \{k_1 \ldots k_n\} \) and and \( \Sigma_n = \{\sigma_1 \ldots \sigma_n\} \), with \( \sigma_j = \pm \) (or \( \pm 1 \));

\( K_{n-1} = K_n \ominus k \) and \( \Sigma_{n-1} = \Sigma_n \ominus \sigma_k \) in the third term on the right, and \( K_{n+1} = K_n \oplus q \) and \( \Sigma_{n+1} = \Sigma_n \oplus \sigma \) in the last term (“collision integral”);

\( X \) is electron’s coordinate, and \( Y \) is spatial variable conjugated with electron’s momentum \( p \), so that \( \hat{V} = -i(h/\hbar)\nabla_Y \) is electron’s velocity operator, and the Fourier transform

\[ \int \exp(-ipY/\hbar) \Delta_n d^3Y/(2\pi\hbar)^3 \]

leads to the Wigner representation;

\( N_k = [\exp(\hbar \omega_k/T) - 1]^{-1} \) are equilibrium occupancies of phonon modes;

\( c_k^- = c_k^* \), \( c_k^+ = c_k \),

and \( \Omega_0 \) is some fixed (formally arbitrary) volume.
Initial conditions to Eqs. 2 can be taken in the form

$$\Delta_n(t = 0) = \delta_{n,0} \Delta_0^0 \quad (3)$$

(one particular reasonable choice of $\Delta_0^0 = \Delta_0^0(X, Y)$ was mentioned in [1]), or

$$\Delta_n(t = 0) = W_0(X) \Delta_{eq}^n(Y, K_n, \Sigma_n) \quad (4)$$

with $\Delta_{eq}^n$ being stationary (equilibrium and spatially homogeneous) solution of Eqs. 2 (normalized to $\Delta_{eq}^0(Y = 0) = 1$).

For more details, please, see [1].

Notice that derivation of Eqs. 2 there does not touch the purely electron’s part of full quantum Liouville (von Neumann) operator, therefore the resulting equations can be trivially generalized to electron under action of an external force $f$ (switched on at $t = 0$) by mere adding terms $(-fY/\hbar) \Delta_n$ to right-hand sides of (2).

Our main problem is qualitatively complete and quantitatively correct analysis of large-time evolution of electron’s position (and total path) probability distribution,

$$W(t, X) = \Delta_0(t, X, Y = 0) \ ,$$

if initially the electron was located in some finite space region, e.g. in vicinity of $X = 0$.

3. Statistical contents of the evolution equations

To perceive more carefully statistical meaning of the functions $\Delta_n \ (n > 0)$, let us return to their basic definition, - i.e. formula (21) from [1], - and rewrite it in equivalent form

$$\Delta_n = \lim_{\Omega \to \infty} \int F(A) \left[ \prod_{k \in K_n} \frac{\sqrt{\Omega}}{\sqrt{\Omega_0}} \cdot \frac{\partial}{\partial A_k^{\sigma_k}} \right] F^{-1}(A) P(A) dA \quad , \quad (5)$$

where $P(A) = P(t, X, Y, A)$ is the quasi-probability density distribution of all the phonon amplitudes $A \equiv \{A_k^{-}, A_k^{+}\}$, $\int \ldots dA$ is integral over all them, and

$$F(A) \equiv \exp \left( - \sum_k \frac{A_k^{-}A_k^{+}}{N_k} \right)$$

The equivalence directly follows from the expansion (20) in [1] (along with relations between functions $P, Q, P_n$ and $Q_n$ pointed out there). The integration by parts turns (5) into

$$\Delta_n = \lim_{\Omega \to \infty} \int P(A) \left\{ F^{-1}(A) \prod_{k \in K_n} \left[ -\frac{\sqrt{\Omega}}{\sqrt{\Omega_0}} \cdot \frac{\partial}{\partial A_k^{\sigma_k}} \right] F(A) \right\} dA \quad (6)$$

If all wave vectors of the set $K_n$ are different one from another then this expression reduces to formula (38) from [1]. We now will consider general case allowing coincidence of some of the wave vectors even in the thermodynamic limit, that is after transition from discrete to continuous phonon spectrum.
Symbolically, this transition reduces to mere re-scaling of the phonon amplitudes, so that
\[
\sqrt{\Omega} A_k^\sigma \Rightarrow A_k^\sigma , \quad \sqrt{\Omega} \frac{\partial}{\partial A_k^\sigma} \Rightarrow (2\pi)^3 \frac{\delta}{\delta A_k^\sigma} ,
\]
\[
\frac{1}{\Omega} \sum_k \ldots \Rightarrow \int \ldots \frac{d^3k}{(2\pi)^3} ,
\]
\[
F(A) \Rightarrow \exp \left( - \int \frac{A_k^- A_k^+}{N_k} \frac{d^3k}{(2\pi)^3} \right) ,
\]
and similar redefinition of the parent creation and annihilation operators, so that their commutator turns to
\[
[a_k, a_k^\dagger] \Rightarrow (2\pi)^3 \delta(k - q)
\]
(see e.g. [1]) while phonon-related part the Hamiltonian (1) to
\[
H_{ph} = \int \hbar \omega_k a_k^\dagger a_k dk , \quad H_{int} = \int [ c_k^* e^{ikr} a_k + c_k e^{-ikr} a_k^\dagger ] dk ,
\]
with \(dk \equiv d^3k/(2\pi)^3\). Consequently, instead of (6) we can write
\[
\Delta_n = \int P(A) \left\{ F^{-1}(A) \prod_{k \in K_n} \left[ - \frac{(2\pi)^3}{\sqrt{\Omega_0}} \cdot \frac{\delta}{\delta A_k^\sigma} \right] F(A) \right\} dA ,
\]
where now \(\int P(A) \ldots dA\) means functional integration.

The Eqs.6 and 9, together with the mentioned formula (38) from [1], prompt to redefine functions \(\Delta_n\) as follows:
\[
\Delta_n(t, X, Y, K_n, \Sigma_n) \Rightarrow \Delta_n(t, X, Y, K_n, -\Sigma_n) \prod_{k \in K_n} \frac{1}{N_k \sqrt{\Omega_0}}
\]
Then our basic Eqs.2 take form
\[
\dot{\Delta}_n = - \hat{V} \nabla_X \Delta_n + i \sum_{k \in K_n} \sigma_k (\omega_k - k\hat{V}) \Delta_n + \nonumber
\]
\[
+ (i\hbar)^{-1} \sum_{k \in K_n} \sigma_k c_k^{-\sigma_k} [ N_k e^{ikY/2} - (N_k + 1) e^{-ikY/2} ] \Delta_{n-1} - \nonumber
\]
\[
- (i\hbar)^{-1} \sum_{\sigma \in \{+, -\}} \sigma \int c_0^\sigma (e^{iqY/2} - e^{-iqY/2}) \Delta_{n+1} \frac{d^3q}{(2\pi)^3} ,
\]
where, as above, \(\Delta_n = \Delta_n(t, X, Y, K_n, \Sigma_n)\), \(\Delta_{n-1} = \Delta_{n-1}(t, X, Y, K_n \ominus k, \Sigma_n \ominus \sigma_k)\), \(\Delta_{n+1} = \Delta_{n+1}(t, X, Y, K_n \oplus q, \Sigma_n \oplus \sigma)\). We may write them in compact form
\[
\dot{\Delta} = - \hat{V} \nabla_X \Delta + \hat{\Lambda} \Delta ,
\]
with \(\Delta = \{\Delta_0, \Delta_1, \ldots\}\) and operator \(\hat{\Lambda} = \hat{\Lambda}(Y, \nabla_Y)\) unifying all the phonon-related operators from right-hand sides of (10). Then Eq.9 yields
\[
\Delta_n = \int P(A) \left\{ F^{-1}(A) \prod_{k \in K_n} \left[ - \frac{(2\pi)^3}{N_k} \frac{\delta}{\delta A_k^\sigma} \right] F(A) \right\} dA
\]
In the case of non-coinciding wave vectors this reduces to

\[ \Delta_n = \int P(A) \left\{ \prod_{k \in K_n} A_k^{\sigma_k} \right\} dA, \quad (13) \]

or, equivalently,

\[ \Delta_n = \left\langle \delta(X(t) - X) \exp \left( iYp(t)/\hbar \right) \prod_{k \in K_n} A_k^{\sigma_k}(t) \right\rangle \quad (14) \]

in the coordinate representation, or

\[ \Delta_n = \left\langle \delta(X(t) - X) \delta(p(t) - p) \prod_{k \in K_n} A_k^{\sigma_k}(t) \right\rangle \quad (15) \]

in the Wigner representation, where \( X(t), p(t) \) and \( A_k^{\sigma_k}(t) \) mean electron’s coordinate and momentum and phonon amplitudes considered as random processes.

If some of wave vectors from \( K_n \) are equal and, besides, their counterparts \( \sigma_k = \pm \) have opposite signs, then - in contrast to (13), - the braces in (6), (9) and (12) represent definite Hermite polynomials of phonon amplitudes. Statistical meaning of the resulting expressions becomes clear if one unifies all the redefined functions \( \Delta_n \)'s, - with arbitrary \( K_n \)'s, - into generating functional

\[ \Delta \equiv \Delta_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{\sigma_1 \ldots \sigma_n} \int \Delta_n \prod_{j=1}^{n} z_{\sigma_j}(k_j) \, d^3k_j \quad (16) \]

Application of Eq.12 gives

\[ \Delta = \exp \left[ - \int (2\pi)^3 N_k \, z_+(k)z_-(k) \, d^3k \right] \times \]

\[ \times \int \exp \left[ \sum_{\sigma} \int z_{\sigma}(k)A_k^{\sigma} \, d^3k \right] P(A) \, dA \quad (17) \]

First exponential on the right here is nothing but inverse of equilibrium characteristic functional of free phonon field (i.e. in absence of the electron:

\[ \exp \left[ \int (2\pi)^3 N_k \, z_+(k)z_-(k) \, d^3k \right] = \]

\[ = \int \exp \left[ \sum_{\sigma} \int z_{\sigma}(k)A_k^{\sigma} \, d^3k \right] P_{\text{free}}(A) \, dA \equiv \]

\[ \equiv \left\langle \exp \left[ \sum_{\sigma} \int z_{\sigma}(k)A_k^{\sigma} \, d^3k \right] \right\rangle_{\text{free}} \equiv F_{\text{free}}\{z\} , \quad (18) \]

where

\[ P_{\text{free}}(A) = F(A) \left[ \int F(A) \, dA \right]^{-1} \]

is continuous limit of the discrete measure

\[ \prod_k f_k = \prod_k (2\pi N_k)^{-1} \exp (-|A_k|^2/N_k) \]
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(see [1]). Hence, in the frames of coordinate representation Eq.17 can be written as

\[ \Delta = \mathcal{F}\{t, X, Y, z\} \]

(19)

\[ \mathcal{F}\{t, X, Y, z\} = \frac{\delta(X(t) - X) \exp \left( \frac{i Y p(t)}{\hbar} \right) \exp \left( \sum_{\sigma} \int z_\sigma(k) A^\sigma_k(t) d^3k \right)}{\mathcal{F}_{\text{free}}\{z\}} \]

(20)

And in the Wigner representation

\[ \Delta = \Delta\{t, X, p, z\} = \mathcal{F}\{t, X, p, z\} \]

(21)

\[ \mathcal{F}\{t, X, p, z\} = \frac{\delta(X(t) - X) \delta(p(t) - p) \exp \left( \sum_{\sigma} \int z_\sigma(k) A^\sigma_k(t) d^3k \right)}{\mathcal{F}_{\text{free}}\{z\}} \]

(22)

Formulae (17)-(21) give complete and transparent statistical interpretation of generating functional of \( \Delta_n \)'s and thus \( \Delta_n \)'s themselves. It remains to notice that in (symbolic) terms of the full density matrix of the system, \( \rho \) (see formula (4) in [1]),

\[ \mathcal{F}\{t, X, Y, z\} = \operatorname{Tr}_{\text{ph}} \exp \left( \int \rho \right) \exp \left( \sum_{\sigma} \int z_\sigma(k) A^\sigma_k(t) d^3k \right) \times \]

\[ \times \langle r | \rho | r' \rangle \exp \left( \int \rho \right) \exp \left( \sum_{\sigma} \int z_\sigma(k) A^\sigma_k(t) d^3k \right) \]

(23)

where \( r = X + Y/2 \), \( r' = X - Y/2 \), \( |r\rangle \) and \( |r'\rangle \) are eigenstates of the electron's coordinate operator, and \( \operatorname{Tr}_{\text{ph}} \) is trace over all phonon states.

4. Generating evolution equation

The Eqs.10 are useful equivalent of Eqs.37 in [1] for functions \( D_n \) representing the same correlations as \( \Delta_n \) in Eqs.14 and 15 but in dimensionless relative units (obviously,

\[ \Delta_n(K_n, \Sigma_n) = D_n(K_n, -\Sigma_n) \prod_{k \in K_n} \left( \frac{-c^\sigma_k}{\hbar \omega_k} \right) , \]

after our above redefinition of \( \Delta_n \).) Corresponding equivalent of the functional evolution equation (40) from [1] is

\[ \dot{\Delta} = -\hat{V} \nabla_X \Delta + \sum_{\sigma} \int d^3k \ z_\sigma(k) \hat{L}_{k\sigma} \frac{\delta}{\delta z_\sigma(k)} \Delta + \]

\[ + \sum_{\sigma} \int d^3k \ z_\sigma(k) \hat{B}_{k\sigma} \Delta + \sum_{\sigma} \int d^3k \ \hat{A}_{k\sigma} \frac{\delta}{\delta z_\sigma(k)} \Delta \equiv \]

\[ \equiv -\hat{V} \nabla_X \Delta + \hat{L} \left\{ z, \frac{\delta}{\delta z} \right\} \Delta , \]

(24)

where now

\[ \hat{L}_{k\sigma} = i \sigma (\omega_k - k \hat{V}) , \]
\[
\hat{B}_{k\sigma} \equiv \frac{\sigma c_k^{-\sigma}}{i\hbar} \left[ e^{i k Y / 2} N_k - e^{-i k Y / 2} (N_k + 1) \right],
\]
(25)
\[
\hat{A}_{k\sigma} \equiv -\frac{c_k^{\sigma}}{i\hbar (2\pi)^3} \left( e^{i k Y / 2} - e^{-i k Y / 2} \right).
\]

Correspondingly, the functional \( \mathcal{F} \) undergoes equation
\[
\dot{\mathcal{F}} = \hat{\mathcal{V}} \nabla_X \mathcal{F} + \hat{\mathcal{L}}_0 \left\{ z, \frac{\delta}{\delta z} \right\} \mathcal{F},
\]
(26)
where
\[
\hat{\mathcal{L}}_0 \equiv \mathcal{F}_{\text{free}} \hat{\mathcal{L}} \mathcal{F}_{\text{free}}^{-1} =
\sum_{\sigma} \int d^3 k \ z_{\sigma}(k) \hat{L}_{k\sigma} \frac{\delta}{\delta z_{\sigma}(k)} +
\sum_{\sigma} \int d^3 k \ z_{\sigma}(k) \hat{B}^0_{k\sigma} + \sum_{\sigma} \int d^3 k \ \hat{A}_{k\sigma} \frac{\delta}{\delta z_{\sigma}(k)}.
\]
(27)

with
\[
\hat{B}^0_{k\sigma} \equiv -\frac{\sigma c_k^{-\sigma}}{i\hbar} e^{-i k Y / 2}
\]
(28)
Notice that the evolution operator \( \hat{\mathcal{L}}_0 \) does not include the phonon mode occupancies (which instead must appear in initial conditions for \( \mathcal{F} \)).

5. Symmetry properties of correlation functions and their evolutions

According to previous section, both the hierarchy \( \Delta = \{ \Delta_0, \Delta_1, \ldots \} \) as the whole and the functional (16) are equivalent to full density matrix of our system. The hermicity of this density matrix means that
\[
\Delta_n(-Y, -\Sigma_n) = \Delta^*_n(Y, \Sigma_n),
\]
(29)
in the coordinate representation, or
\[
\hat{\mathcal{H}} \Delta_n(Y, \Sigma_n) \equiv \Delta^*_n(-Y, -\Sigma_n) = \Delta_n(Y, \Sigma_n)
\]
Here and below we write out only arguments what are under current attention. In terms of the generating functional,
\[
\hat{\mathcal{H}} \Delta\{Y, z_{\sigma}\} \equiv \Delta^*\{-Y, z_{-\sigma}\} = \Delta\{Y, z_{\sigma}\},
\]
(30)
where \( * \) means complex conjugation of coefficients of the functional (but not its argument \( z_{\sigma}(k) \)). Indeed, one can verify that evolution operator in Eqs.10 and 24 satisfies
\[
\hat{\mathcal{H}} \left[ -\hat{\mathcal{V}} \nabla_X + \hat{\Lambda} \right] \hat{\mathcal{H}}^{-1} = \left[ -\hat{\mathcal{V}} \nabla_X + \hat{\Lambda} \right],
\]
\[
\hat{\mathcal{H}} \left[ -\hat{\mathcal{V}} \nabla_X + \hat{\mathcal{L}} \right] \hat{\mathcal{H}}^{-1} = \left[ -\hat{\mathcal{V}} \nabla_X + \hat{\mathcal{L}} \right]
\]
(31)
Hence, if the hermicity property is satisfied at any one time moment (e.g. when initial conditions (3) take place) then it keeps at all other time moments. In the Wigner representation the same property looks as
\[
\hat{\mathcal{H}} \Delta_n(p, \Sigma_n) \equiv \Delta^*_n(p, -\Sigma_n) = \Delta_n(p, \Sigma_n),
\]
(32)
with similar changes in (30) and (31) and with \( \hat{V} \Rightarrow p/m \).

Further, if the system’s Hamiltonian expressed by (1) and (8) is invariant in respect to time inversion, - which is the case when
\[
\omega_{-k} = \omega_k, \quad c_{-k} = c_k^*.
\]
- then
\[
\Theta [ -\hat{V} \nabla_X + \hat{\Lambda} ] \Theta^{-1} = - [ -\hat{V} \nabla_X + \hat{\Lambda} ],
\]
\[
\Theta [ -\hat{V} \nabla_X + \hat{\mathcal{L}} ] \Theta^{-1} = - [ -\hat{V} \nabla_X + \hat{\mathcal{L}} ],
\]
where \( \Theta \) is time reversal operator defined by
\[
\Theta \Delta_n(Y, K_n, \Sigma_n) = \Delta_n(-Y, -K_n, -\Sigma_n)
\]
\[
\Theta \Delta\{Y, z_\sigma(k)\} = \Delta\{-Y, z_{-\sigma}(-k)\}
\]
in the coordinate representation and by
\[
\Theta \Delta_n(p, K_n, \Sigma_n) = \Delta_n(-p, -K_n, -\Sigma_n),
\]
\[
\Theta \Delta\{p, z_\sigma(k)\} = \Delta\{-p, z_{-\sigma}(-k)\}
\]
in the Wigner’s one (in any time-reversed process, all momenta of particles and quanta are inverted while their creations are replaced by annihilations and vice versa).

Hence, if \( \Delta_n(t, X, Y, K_n, \Sigma_n) \) and \( \Delta\{t, X, Y, z_\sigma(k)\} \) are some solutions of Eqs.10 and 24, respectively, then \( \Theta \Delta_n(t_0 - t, X, Y, K_n, \Sigma_n) = \Delta_n(t_0 - t, X, -Y, -K_n, -\Sigma_n) \) and \( \Theta \Delta\{t_0 - t, X, Y, z_{\sigma}(k)\} = \Delta\{t_0 - t, X, -Y, z_{-\sigma}(-k)\} \) also are their solutions. As the consequence, the equilibrium stationary solution of evolution equations satisfies
\[
\Theta \Delta_n^{eq} = \Delta_n^{eq}
\]

The properties (31) and (34) are characteristic principal properties of (quantum) Liouville operators. This is quite natural: relations (19), (21) and (23) do show that the evolution operators \( -\hat{V} \nabla_X + \hat{\Lambda} \) and \( -\hat{V} \nabla_X + \hat{\mathcal{L}} \) are nothing but particular representations of the full system’s Liouville operator, i.e. \([H, \ldots]/i\hbar\).

This means that operators \( -\hat{V} \nabla_X + \hat{\Lambda} \) and \( -\hat{V} \nabla_X + \hat{\mathcal{L}} \) have the same spectrum as \([H, \ldots]/i\hbar\) do. That is all their eigenvalues are purely imaginary, since spectrum of \([H, \ldots]/i\hbar\) consists of \((E_S - E_{S'})/i\hbar\), with \(E_S\) being eigenvalues of the full Hamiltonian: \(H|S\rangle = E_S|S\rangle\). According to relation (23), the corresponding eigenvectors of \( -\hat{V} \nabla_X + \hat{\Lambda} \) and thus \( -\hat{V} \nabla_X + \hat{\mathcal{L}} \) are determined by expression
\[
\Delta_{S'S'}\{X, Y, z\} = F^{-1}_{free}\{z\} \operatorname{Tr}_{ph} \exp \left[ \int z_-(k) a_k e^{ikr} d^3k \right] \times \langle r| S\rangle \langle S'| r'\rangle \exp \left[ \int z_+(k) a_k^* e^{-ikr'} d^3k \right]
\]

Clearly, the functionals introduced by this expression form definite complete set of eigenfunctions of our evolution operators. Then the imaginary character of corresponding spectrum means that these operators can not have real eigenvalues or complex ones with nonzero real parts. Next, we will consider most principal and important consequence of this circumstance, while Appendix A contains a its simple visual illustration.
6. Joint characteristic function of total electron’s path and instant system’s state, and their cumulants

Let initially the electron is located in some finite spatial region around the coordinate origin, so that, for example,

$$\Delta_0^0(X, Y = 0) = W_0(X) = (2\pi x_0^2)^{-3/2} \exp \left( -X^2/2x_0^2 \right)$$

in (3) and (4), respectively, with some reasonable uncertainty \(x_0\) (e.g. \(x_0^2 = \hbar^2/4mT\) [1]). In the first case, the electron is suddenly thrown in the phonon gas just at initial time moment \(t = 0\) and therefore at \(t = 0\) has no correlation with phonons. In the second case, at \(t = 0\) it already possesses all equilibrium correlations with phonons correspondingly to momentum and energy balance with them.

This difference, however, must become unimportant after sufficiently large time, \(t \gg \tau_0\), where \(t \gg \tau_0\) is characteristic time of relaxation (thermalization) of electron’s momentum probability distribution. Therefore at \(t \gg \tau_0\), - or, formally, at \(t \to \infty\), - we can concentrate on statistics of electron’s random walk, i.e. on probability distribution, \(W(t, X)\), of its coordinate \(X = X(t)\). At that, since the latter unboundedly spreads in space, - so that \(\langle X^2(t) \rangle/x_0^2 \to \infty\), - we can treat \(X(t)\) as total electron’s path accumulated during time interval \((0, t)\).

Then, as usually, it is convenient to investigate firstly characteristic function of the path distribution, by considering Fourier transforms

$$\Delta_n(t, i\kappa, Y, K_n, \Sigma_n) = \int \exp (i\kappa X) \Delta_n(t, X, Y, K_n, \Sigma_n) \, d^3X\ ,$$

$$\Delta\{t, i\kappa, Y, z\} = \int \exp (i\kappa X) \Delta\{t, X, Y, z\} \, d^3X\ ,$$

so that the evolution equations take form

$$\dot{\Delta}_n = i\kappa \hat{V} \Delta_n + \sum_{k \in K_n} \hat{L}_{k\sigma_k} \Delta_n +$$

$$\sum_{k \in K_n} \hat{B}_{k\sigma_k} \Delta_{n-1} + \sum_{\sigma \in \{+,-\}} \int \hat{A}_{q\sigma} \Delta_{n+1} \, d^3q$$

(39)

(\text{where, of course, } K_{n-1} = K_n \ominus k\ , \Sigma_{n-1} = \Sigma_n \ominus \sigma_k , K_{n+1} = K_n \oplus q \text{ and, } \Sigma_{n+1} = \Sigma_n \oplus \sigma\) , - or shortly

$$\dot{\Delta} = [i\kappa \hat{V} + \hat{\Lambda}] \Delta\ ,$$

(40)

- and

$$\dot{\Delta} = i\kappa \hat{V} \Delta + \hat{\Lambda} \Delta\ ,$$

(41)

with operators defined in (24)-(25).

According to Eq.19,

$$\Delta\{t, i\kappa, Y, z\} = \frac{F\{t, i\kappa, Y, z\}}{F_{\text{free}}\{z\}}\ ,$$

(42)

where

$$F\{t, i\kappa, Y, z\} = \int \exp (i\kappa X) F\{t, X, Y, z\} \, d^3X =$$

(43)
\[
\left\langle \exp \left[ i\kappa X(t) + i\xi V(t) + \sum_\sigma \int z_\sigma(k) A_\sigma^\prime(t) \, d^3k \right] \right\rangle,
\]

with \( V(t) = p(t)/m \) being electron's velocity, and

\[ \xi \equiv mY/\hbar \]

Thus, \( F\{t, i\kappa, Y, z\} \) in fact is full characteristic function of our system (in the sense of the probability theory), while \( \Delta\{t, i\kappa, Y, z\} \) differs from it only by exclusion of undisturbed part of self-correlations of phonon modes. At the same time, since

\[ X(t) = X(0) + \int_0^t V(\tau) \, d\tau \]

and \( |X(0)| \lesssim x_0 \), function \( \Delta\{t, i\kappa, Y, z\} \) serves as characteristic functional of electron's velocity on the interval \((0, t)\).

As usually, it is reasonable to consider this function in terms of various cumulants and their generating functional:

\[
C\{t, i\kappa, \xi, z\} \equiv \ln \Delta\{t, i\kappa, Y, z\} = \sum_{j+l+n>0} \frac{(i\kappa)^j(i\xi)^l}{j!l!n!} \sum_{\sigma_1...\sigma_n} \int_{k_1} z_{\sigma_1}(k_1) \cdots \int_{k_n} z_{\sigma_n}(k_n) \, C_{jln}(t, K_n, \Sigma_n),
\]

\[
C_{jln}(t, K_n, \Sigma_n) = \left\langle \left\langle X^j(t) V^l(t) \prod_{s=1}^n A_{\sigma_s}^\sigma(t) \right\rangle \right\rangle,
\]

where \( \int_k \ldots = \int \ldots \, d^3k \), \( K_n = \{k_1 \ldots k_n\} \), \( \Sigma_n = \{\sigma_1 \ldots \sigma_n\} \), the double angle brackets denote \((j+l+n)\)-order cumulant, i.e. purely irreducible part of \((j+l+n)\)-order mutual correlation, of the enveloped multipliers, and equity of some of indices \( j, l, n \) to zero means absence of corresponding multipliers (only in special case, when \( j = l = 0, n = 2 \), \( C_{002} \) is difference between full cumulant and its “seed” value determined by \( \ln F_{\text{free}} \)).

In particular,

\[
C\{t, i\kappa, \xi, 0\} = \ln \Delta_0\{t, i\kappa, Y\} = \sum_{j+l>0} \frac{(i\kappa)^j(i\xi)^l}{j!l!} \left\langle \left\langle X^j(t) V^l(t) \right\rangle \right\rangle
\]

describes most interesting for us full joint statistics of electron's velocity and total path in themselves.

Notice that in terms of cumulants Eq.41 becomes nonlinear:

\[
\dot{C} = \left[ i\kappa - \sum_\sigma \int d^3k \, z_\sigma(k) i\sigma k \frac{\delta C}{\delta z_\sigma(k)} \right] \, \hat{V} \, C + \sum_\sigma \int d^3k \, z_\sigma(k) \, \hat{L}_k \frac{\delta C}{\delta z_\sigma(k)} + \sum_\sigma \int d^3k \, z_\sigma(k) \, \hat{B}_k \sigma + \sum_\sigma \int d^3k \, \hat{A}_k \sigma \frac{\delta C}{\delta z_\sigma(k)},
\]

with quadratic nonlinearity.
7. Long-time asymptotic of the cumulants and electron’s diffusivity/mobility low-frequency fluctuations

Now, let us consider long-range time behavior of the electron’s path and related cumulants at \( t/\tau_0 \to \infty \).

Our sole assumption will be that electron’s interaction with the phonon thermostat ensures fast enough, - i.e. integrable, - relaxation of the electron’s velocity correlation function \( \langle \langle V(t)V(t') \rangle \rangle \) and thus diffusive character of the random walk, in the sense that

\[
C_{200}(t) = \langle \langle X^2(t) \rangle \rangle \to 2Dt + \text{const} \quad (47)
\]

Undoubtedly, this is the case under suitable choice of electron-phonon couplings \( |c_k|^2 \) and phonon frequencies \( \omega_k \). In particular, without essential loss of generality we may take

\[
c_k = c(|k|), \quad \omega_k = \omega(|k|), \quad (48)
\]

with \( c(|k|) = c^*(|k|) \geq 0 \) (phases of \( c_k \)'s anyway are of no importance). This guarantees spherical symmetry of the random walk.

A suitable choice can be recognized from the simplest approximation of exact equations, i.e. the “kinetic equation”, - see Eq.58 in Sec.5.4 in [1]. In the representation under consideration it can be written as

\[
\Delta_0(t, i\kappa, Y) = [i\kappa \hat{V} + \hat{K}] \Delta_0(t, i\kappa, Y), \quad (49)
\]

with \( \epsilon = +0 \) ‡.

Provided the mean square of electron’s path obeys the diffusive law (47), let us discuss possible long-term asymptotic of higher-order path cumulants and other cumulants.

If we were basing on habitual intuition, in turn based on standard kinetic theory, we would predict that all the cumulants from (44) and (45) tend to finite limits, except self-cumulants of the electron’s path:

\[
C_{sln}(t) \to \text{const} \quad \text{if} \quad l + n > 0, \quad (50)
\]

\[
C_{s00}(t) = \langle \langle X^s(t) \rangle \rangle \to sD_s t + \text{const}, \quad (51)
\]

where

\[
D_s = \frac{1}{s} \frac{d}{dt} \langle \langle X^s(t) \rangle \rangle = \langle \langle V(t)X^{s-1}(t) \rangle \rangle \to \text{const}
\]

‡ Notice that \( \hat{L}_{k\sigma} \) is imaginary-valued operator. Addition of infinitely small real positive \( \epsilon = +0 \) to it is necessary for artificial breaking of the time reversal symmetry. Otherwise, at \( \epsilon = 0 \), as one can easy verify, the integral in (49), i.e. \( \hat{K} \), would turn to zero (see also Appendix A below).
and \( D_2 = D \) (of course, \( D_s = 0 \) at odd \( s \) because of the spherical symmetry). Such behavior means that, at any \( n > 0 \) and any \( \eta_j(t) \) being either \( V(t) \) or some of \( A_k^s(t) \),
\[
\left\langle \eta_1(t) \ldots \eta_s(t) V(t_1) \ldots V(t_n) \right\rangle \to 0
\]
when some of \( t - t_j \to \infty \),
where the zero limit is achieved in so fast way, that these cumulants are integrable over all \( t_j \) simultaneouly. Orally, all irreducible correlations between present state of the system, - characterized by electron’s velocity and phonon amplitudes, - and past values of velocity disappear with time. Then we can write
\[
\Delta\{t, i\kappa, Y, z\} = e^{\lambda(i\kappa)t} \Delta'\{t, i\kappa, Y, z\} \to e^{\lambda(i\kappa)t} \Delta'\{\infty, i\kappa, Y, z\}
\]
with
\[
\lambda(i\kappa) = \lim_{t \to \infty} \frac{C\{t, i\kappa, 0, 0\}}{t} = \sum_{s=1}^{\infty} \frac{D_{2s}}{(2s-1)!} (-\kappa^2)^s < 0
\]
Correspondingly, the path probability distribution is asymptotically Gaussian:
\[
W(t, X) \to (4\pi D t)^{-3/2} \exp \left( -\frac{X^2}{4Dt} \right)
\]
Just such type of asymptotical behavior follows from the kinetic equation. At that, \( \lambda(i\kappa) \) is real negative quantity determined by the eigenvalue problem
\[
\lambda(i\kappa) f\{X, Y, z\} = \left[ -\hat{V} \nabla_X + \hat{K} \right] f\{i\kappa, Y\}
\]
But, unfortunately, from the viewpoint of exact equations, (39) or (40) or (41), so nice behavior is impossible!

Indeed, if the assumed asymptotic (50),(51),(53) was true, then, evidently, the Eq.41 would result in equality
\[
\left[ i\kappa \hat{V} + \hat{L} \right] \Delta'\{\infty, i\kappa, Y, z\} = \lambda(i\kappa) \Delta'\{\infty, i\kappa, Y, z\}
\]
thus stating that operator \( i\kappa \hat{V} + \hat{L} \) has a real eigenvalue. Then this would imply, - as it is easy to see, - that operator \(-\hat{V} \nabla_X + \hat{\Lambda}\) (and, equivalently, \(-\hat{V} \nabla_X + \hat{\Lambda}\)) has the same real eigenvalue,
\[
\lambda(i\kappa) f\{X, Y, z\} = \left[ -\hat{V} \nabla_X + \hat{\Lambda} \right] f\{X, Y, z\}
\]
with corresponding eigenvector expressed by
\[
f\{X, Y, z\} = e^{-i\kappa X} \Delta'\{\infty, i\kappa, Y, z\}
\]
However, this is impossible, since, - as we have shown above, - the whole spectrum of the exact evolution operator is purely imaginary. It is useful to add that this reasoning stays valid even if the constants in (47) and (51) are replaced by arbitrary sub-linear time functions.

Hence, the assumptions (50) and (51) (at \( s > 2 \)), and thus (53) and the hypothetical asymptotic (54), in spite of their seeming plausibility, all are qualitatively wrong.
One more proof of incompatibility of these assumptions with exact evolution
equations comes from the property (34). The latter implies, firstly, that if (57) is
eigenfunction of the operator $-\hat{V} \nabla X + \hat{L}$, with eigenvalue $\lambda(i\kappa)$, then this operator
has also eigenvalue $-\lambda(i\kappa)$, corresponding to eigenfunction $e^{-i\kappa X} \hat{\Theta} \Delta'\{\infty, i\kappa, Y, z\}$. Therefore, secondly, starting at $t = 0$ from initial conditions (e.g. (3) or (4)) what
satisfy time-reversal symmetry, like (37), we must come to asymptotic which, in contrast
to (53), with equal rights includes two exponentials, $\exp[\lambda(i\kappa)t]$ and $\exp[-\lambda(i\kappa)t]$, and thus is certainly physically senseless!

Consequently, we have to conclude that in reality some of the cumulants in (52)
are not fast enough decaying, i.e. integral of some of them over all $t_j$ from 0 to $t$ are
unboundedly growing with time $t$. The natural first candidate for such role is $\hat{\Theta}$
that is
\[
\langle \langle V(t) V(t_1) V(t_2) V(t_3) \rangle \rangle \rightarrow \infty \tag{58}
\]
This implies that
\[
\frac{\langle \langle X^4(t) \rangle \rangle}{t} = 4 \int_0^t \frac{\langle \langle V(t) X^3(t) \rangle \rangle}{t} dt \sim \langle \langle V(t) X^3(t) \rangle \rangle \rightarrow \infty \tag{59}
\]
Thus, the fourth-order cumulant of electron’s path grows with time obeying
a super-linear law. This conclusion is main final result of the present paper.

We see that long-range behavior of (even equilibrium) electron’s random walk can
not be adequately described with the help of single parameter $D$, which now determines
mean square value of path only, but not its higher-order statistical moments.

Further consequences of this result, concerning the 1/f-noise, do follow already
known general relations [3, 4, 6, 7, 8]. Namely, the asymptotic (59) shows that the
electron’s path $X(t)$ behaves as if electron’s diffusivity was a random variable, $\tilde{D}(t)$, 
with mean value $D$ and effective correlation function
\[
\langle \langle \tilde{D}(t) \tilde{D}(0) \rangle \rangle = \frac{1}{24} \frac{d^2}{dt^2} \langle \langle X^4(t) \rangle \rangle = \frac{1}{2} \langle \langle V(t) X^2(t) V(0) \rangle \rangle \tag{60}
\]
From (58)-(59) one finds that this “diffusivity correlation function” is slowly decaying
(definitely non-integrable) time function $\parallel$.

$\parallel$
The cumulant $\langle \langle V(t) V(t_1) \rangle \rangle$, can not play such role as far as the mean square diffusive law is
fulfilled, either in the form (47) or $\langle \langle X^2(t) \rangle \rangle / t \rightarrow 2D = \text{const}$. 

$\parallel$
Formulae like (60) appeared in [3, 4, 6]. They presume that observation of (equilibrium) random
path starts from nearly equilibrium initial condition like our (4).

One of possible forms of the diffusivity correlation function was predicted and considered in [3, 4, 6, 8].
In principle, generally speaking, this function, being introduced by (60), can be even not decaying but
instead growing with time. Such possibility was demonstrated and explained from physical point of
view in [7, 8, 9, 10, 16].
Analogous consideration of our equations in presence of external force (as was mentioned above) or, alternatively, application to our results of the generalized fluctuation-dissipation relations shows that the diffusivity fluctuations always convert themselves into similar fluctuations of (low-field) electron’s mobility, with the same correlation function (60) \[ \eta \].

Obviously, the corresponding spectrum of diffusivity/mobility fluctuations is of 1/f type \(^+\) - or, more generally, of 1/f\(^\gamma\) type, - showing unbounded growth at \( f \tau_0 \to 0 \).

A concrete form of this spectrum produced by our exact equations will be investigated separately.

8. Discussion

As it was realized already in [3, 4, 5, 6, 7, 14], low-frequency 1/f-type fluctuations in diffusivity and mobility of charge carriers, - as well as other molecular-size (quasi-) particles, - originates from indifference of many-particle systems to a number and relative frequency of collisions (elementary acts of interactions, scattering, etc.) of any concrete particle. In other words, indifference of many-particle systems to rates of participation of a concrete particle in various irreversible and random processes, first of all, in the particle’s thermalization (e.g. momentum and energy relaxation), diffusion and drift under external forces.

Let, for example, during last 1 second some air particle or charge carrier has undergone two times greater (or lesser) number of collisions than it must have “on average”. Such incident will make no influence upon next particle’s motion, since any detail of its motion becomes forgotten by the system after characteristic time \( \tau_0 \) much shorter than 1 second. In other words, even strong fluctuation of number of collisions, - and hence of rate of particle’s diffusion, - does not cause a “back reaction” of the system (which would enforce this particle to make “opposite fluctuation” in number of collisions during next 1 second and “compensate” the old incident).

This reasoning shows principal difference of fluctuations in relative frequency of collisions (and connected physical quantities) from, say, particle’s energy fluctuations (which cause back reaction suppressing energy deviations from its mean value during time \( \sim \tau_0 \)).

The 1 second in above reasoning can be replaced by arbitrary time interval greater than \( \tau_0 \), and instead of 1 second we can take 10 seconds, 100 seconds, and so on *.

\[ \] See [5, 6, 7, 8, 11, 12] and, for the fluctuation-dissipation relations, references mentioned in [6, 7, 8, 13] and in preprints [Yu.E.Kuzovlev, arXiv: 1106.0589-,, 1108.1740].

\(^+\) For details and examples see e.g. [4, 6, 7, 8, 9, 10, 11].

* Strictly speaking, this interval can be bounded above by some characteristic “non-ergodicity time”, which, however, is very large for real, even closed, many-particle systems [10]) and infinitely large for open or infinitely many-particle systems.
At that, because the system constantly forgets history of the particle’s collisions, it, figuratively speaking, can not distinguish what part of collisions is “average” and what is “deviation from average”. Hence, it allows the latter to be as fast growing with time as the former. This characteristic law, \( N(t) - \langle N(t) \rangle \propto \langle N(t) \rangle \propto t \) (with \( N(t) \) standing number of collisions) just means that spectrum of fluctuations of instant relative frequency of collisions (and thus of rate of diffusion, etc.) at low frequencies \( f \ll 1/\tau_0 \) has nearly 1/f form (differing from it by some logarithmic factors).

Additional explanations of so simple origin of 1/f-noise can be found in [1, 8, 9, 10, 11, 12, 13, 15, 16].

Quite obvious conclusion from the aforesaid is that theoreticians should honestly investigate actual Hamiltonian dynamics of many-particle systems and their dynamical chaos, - basing on exact equations of statistical mechanics, - instead of thoughtless demonstration of Bernoulli’s “art of conjectures”.

In essence, the same was claimed by N. Krylov [17] as far ago as in 1950. One of important results of [17] is statement that even in “good” Hamiltonian systems (possessing excellent mixing properties) relative frequencies of a given sort of events on different phase trajectories, - i.e. in different experiments, - generally differ one from another and thus from ensemble average, regardless of duration of experiments. Therefore, - as N. Krylov underlined, - it is impossible to describe actual randomness of physical systems in terms of any a priori introduced “probabilities of events” (e.g. collisions). In other words, “probabilities of events” have no certain values.

Hence, all probability-theoretical models of noise inevitably lose fundamental (in the above explained sense) 1/f-noise. That is why we have to investigate dynamical models without any a priori truncation of their exact equations (for any truncation, - like one leading to the “kinetic equation” (49), - acts as artificial introduction of some a priori probabilities).

One more important Krylov’s result is that in statistical mechanics statistical correlations between particles or events do not necessarily say about some real “physical” (cause-and-consequence) correlations or connections between them. Our above treatment of collisions’ number fluctuations well illustrates this fact. Thus it would be wrong to interpret the 1/f-noise under discussion as manifestation of some physical long-living correlations (e.g. between charge carriers) or some “slow relaxation”. There are only purely statistical correlations, and they manifest only so rich randomness which can not be adequately imitated by traditional probability-theoretical (kinetic and stochastic) models.

The situation with 1/f-noise in semiconductors [18, 19] serves as remarkable example (see [6, 8, 12]). On one hand, many experiments give strong evidences that main source of this noise is (independent) 1/f-type fluctuations of mobilities of charge carriers resulting from their scattering by lattice vibrations, i.e. phonon gas [18, 19]. On the other hand, the traditional way of thinking enforces to switch attention from the electron-phonon interaction and scattering itself to the phonon subsystem in itself and
suggest 1/f fluctuations of occupancies of phonon modes (see e.g. [19] and references therein). Even in spite of clear contradiction between such idea and kinetic theory which establishes quite definite (finite) relaxation times of phonon modes! In fact, however, low-frequency 1/f-type fluctuations observed in phonon systems themselves (e.g. fluctuations of internal friction in quartz crystals or intensity of light scattering by them) come from fluctuations in relaxation rates of phonon modes there (see [13] and references therein), - in accordance with the above reasonings, - but in no way from fluctuations of phonon numbers.

In our Hamiltonian model, considered in this paper, perturbation of any particular phonon mode by the electron vanishes under the thermodynamic limit. In other words, any finite sequence of $\sim t/\tau_0$ electron’s scattering events during finite time $t$ involves only vanishingly small portion of total number of phonon modes. Therefore, all their occupancies do not fluctuate at all, staying mere random constants. At that, since they are independent one on another, all effects of their randomness are suppressed by the infiniteness of their density in $k$-space (any finite volume there includes infinitely many modes). Hence, equations of our model describe just what is most interesting for us and for applications, i.e. 1/f-noise resulting from uncertainty of rate of electron’s scattering by phonons.

Of course, real samples under measurements are finite and rather small, but one should remember about very many degrees of freedom in their surroundings which inevitably somehow interact with any sample.

9. Conclusion

To resume, we have shown that if an interaction of electron (quantum particle) with ideal phonon gas (harmonic boson thermostat) enforces the electron to move like Brownian particle, - so that mean square of its path obeys the diffusion law, $\langle X^2(t)\rangle \propto t$, - then such interaction certainly supplies this motion with 1/f (or “flicker”) low-frequency fluctuations in electron’s diffusivity and thus mobility.

In essence, this our result was deduced from most general properties of exact statistical-mechanical equations of the electron-phonon system only. This fact prompts that our result can be extended to a wide class of different systems, in the form of some general theorem (which surely will include the molecular random walk in fluids considered in [7, 10, 11, 16]). This seems very attractive task. But not less intriguing

Thus authors of such idea are even more “revolutionary” than I am! I do not suggest that results of kinetic theory are invalid, stating only that they are incomplete, and that, to supplement them properly, one should return from habitual probability-theoretical scheme

\[
\text{Probabilities} \Rightarrow \text{Stochastic model} \Rightarrow \text{Noise}
\]

to canonical statistical-mechanical scheme

\[
\text{Hamiltonian model} \Rightarrow \text{Noise} \Rightarrow \text{Probabilities}
\]
Appendix A. Imaginary spectrum of the evolution operator and falsity of the kinetic equation

Notice that properties (31) and (34) remain formally valid under truncations of the evolution equations. Therefore character of spectrum of the evolution operator remains unchanged. And, in order to feel it, we can for simplicity make truncation already at \( n = 2 \), by formal putting \( \Delta_2 = 0 \), as was done in [1] when deriving the ”kinetic equation”, Eq.58 in [1]. Besides, for more simplicity, we will confine ourselves by spatially homogeneous eigenfunctions.

Thus, let us consider the eigenvalue problem

\[
\lambda \Delta_0(Y) = -\frac{1}{i\hbar} \sum_{\sigma \in \{+,-\}} \sigma \int c_k^\sigma (e^{ikY/2} - e^{-ikY/2}) \Delta_1(Y, k, \sigma) \frac{d^3k}{(2\pi)^3},
\]

\[
\lambda \Delta_1(Y, k, \sigma) = \frac{1}{i\hbar} \sigma c_k^{-\sigma} \left[ N_k e^{ikY/2} - (N_k + 1) e^{-ikY/2} \right] \Delta_0(Y) +
\]

\[
+ i\sigma (\omega_k - k\tilde{V}) \Delta_1(Y, k, \sigma),
\]  

(A.1)

and assume that it has solution with a real non-zero \( \lambda \). From viewpoint of the kinetic equation this seems undoubted, because kinetic equation arose in the same approximation, and spectrum of its \( X \)-independent eigenfunctions (spectrum of momentum and energy relaxation) certainly is real.

Excluding \( \Delta_1 \) from this pair of equations and going to the Wigner representation, we come to

\[
\lambda \Delta_0(p) = \frac{2\pi}{\hbar^2} \int |c_k|^2 \left\{ \delta_\lambda(\omega_k - kV + \frac{\hbar k^2}{2m}) \times
\right.
\]

\[
\times \left[ N_k \Delta_0(p - \hbar k) - (N_k + 1) \Delta_0(p) \right] +
\]

\[
+ \delta_\lambda(\omega_k - kV - \frac{\hbar k^2}{2m}) \times
\]

\[
\times \left[ (N_k + 1) \Delta_0(p + \hbar k) - N_k \Delta_0(p) \right] \} \frac{d^3k}{(2\pi)^3} =
\]

\[
= \int \left[ V_\lambda(p|p') \Delta_0(p') - V_\lambda(p'|p) \Delta_0(p) \right] d^3p',
\]

(A.2)

where

\[
\delta_\lambda(x) \equiv \frac{1}{\pi} \frac{\lambda}{\lambda^2 + x^2},
\]

(A.3)

\[
V_\lambda(p|p') \equiv \frac{2\pi}{\hbar^2} \int |c_k|^2 \delta(p - p' - \hbar k) \left[ N_k \delta_\lambda(\omega_k + \frac{\hbar k^2}{2m} - kV) +
\right.
\]

\[
+ (N_k + 1) \delta_\lambda(\omega_k - \frac{\hbar k^2}{2m} + kV) \right] \frac{d^3k}{(2\pi)^3}
\]

(A.4)
At $\lambda \to +0$ operator on the right in Eq.A.2 turns to the kinetic operator from the Eq.58 in [1], that is operator $\hat{K}$ from the same Eq.49) above, whose spectrum is real non-positive. At finite $\lambda > 0$, evidently, the operator in Eq.A.2 has the same structure as general kinetic operators, therefore it also is non-positively defined. Hence, Eq.A.2 can not have non-trivial solution with $\lambda > 0$. If $\lambda < 0$ then signs of both sides of $\lambda < 0$ do change simultaneously and thus remain opposite. Therefore $\lambda < 0$ also can not give the solution. Clearly, addition to $\lambda$ of imaginary component does not change the situation. Consequently, all non-zero eigenvalues in the Eqs.A.1 are purely imaginary.

This well illustrates that the kinetic equation, in spite of its usefulness, is in principal contrast with its parent evolution equations.

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