Lower Excitation Spectrum of the Nucleon and Delta in a Relativistic Chiral Quark Model

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The lower excitation spectrum of the nucleon and Δ is calculated in a relativistic chiral quark model. Contributions of the second order self-energy and exchange diagrams due to pion fields to the mass spectrum of the SU(2) baryons are estimated. A splitting between N(939) and positive parity nucleon resonance (Roper resonance) N* (1440) is reproduced with a reasonable accuracy. The obtained structure of one-meson exchange interaction confirms a prediction of the large Nc limit approach stating that the mass splitting between various baryon states receive contributions from operators which simultaneously couple spin, isospin and orbital momentum. It is shown that one-meson exchange interaction generates a splitting between negative parity N*(1/2−) and N*(3/2−) states, and also between Δ*(3/2−) and Δ*(1/2−) states in contrast to the non-relativistic Goldstone-Boson Exchange based quark models. This splitting is due to a relativistic operator which couples the lower and upper orbital momentum of two interacting valence quarks.

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I. INTRODUCTION

There is a growing interest to the baryon spectroscopy in the light of recent discovery of pentaquark baryon Θ+ with a positive strangeness and mass value M = 1540 ± 10 MeV and width Γ < 25 MeV by the LEPS group at the SPRING-8 facility in Harima, Japan [11] and another pentaquark baryon Ξ− with M = 1862 MeV and Γ < 18 MeV by the NA49 group at CERN [2]. The minimal possible quark content of the Θ+ and Ξ− is u2d2s and s2d2¯u respectively and therefore they are called pentaquarks. The discovery of pentaquarks in particular inspired a new overview on the light hadron spectroscopy, the lowest excitation spectrum of the Nucleon and Δ. One new point here is that the long standing puzzle of the Roper resonance N*(1440)(1/2+) is suggested to be resolved with an idea that this baryon state has a pentaquark content u2d2¯d [3]. In addition to the long discussed exotic structure of the Roper resonance [4], the new point requires a detailed study of the lowest excitation spectrum of light baryons.

The most successful study of the spectroscopy of the lowest excited states of the N and Δ was done in the traditional One-Gluon Exchange (OGE) Constituent Quark Model [4, 5, 6]. Another prominent, Goldstone Boson Exchange (GBE) Constituent Quark Model [7] is based on the idea that the spontaneous breaking of the chiral symmetry of QCD plays a crucial role for the hadron structure. Continuing debate of the two above models over last decade [10, 11] was focused mainly on the so-called “three-body spin orbit problem” associated with the OGE mechanism. The authors of the GBE based model claims that the Goldstone boson exchange, rather than the gluon exchange, is the source of the hyperfine splitting, and that there is no any spin-orbit splitting between for-example, negative parity N*(1/2−) and N*(3/2−) resonances, and also between Δ*(3/2−) and Δ*(1/2−) resonances. Unfortunately, the current status of the experimental data still does not allow to overcome these problems [12]. Although, the new analysis of the experimental data [13] confirms the splitting in above nucleon and Δ bands.

On the other hand, neither of these models of the baryon structure is relativistic and neither is QCD and it is impossible to draw any conclusion concerning the nature of the hyperfine interaction in baryons. It is clear now that all results of the constituent quark models have to be reexamined in some relativistic quark model. Up to now, the QCD based models like the MIT bag [14], cloudy bag [15] and chiral bag [16] have not been extended to the excited baryon sector. Most detailed study of the masses of light hadrons in cloudy and chiral bag models was done by Saito [17], however this work also did not include lower excitation spectrum of the Nucleon and Delta. The inclusion of the Instanton induced interaction mechanism in the frame of the MIT bag model [18] also was restricted to the octet baryons.

The octet and decuplet baryons also were studied successfully in a fully relativistic Poincare covariant Diquark-Quark Model based on Bethe-Salpeter equation [19]. Another model which respects the Poincare covariance and which preserves all results of the original cloudy bag model [17] is the Light Front Cloudy Bag Model [20]. The model recently was extended for the extrapolation of lattice QCD results to the physical values of m∗ [21]. This work have demonstrated the importance of the development of cavity models for hadron structure. On the other hand, the
lattice studies still did not reproduce the main properties of the excited \( N \) and \( \Delta \) sector, although the ground states \( N(939) \) and \( \Delta(1232) \) are described well (see recent review \cite{22}).

The aim of the present work is to develop a relativistic chiral quark model based on field-theoretical description of the interquark interaction for the excited Nucleon and \( \Delta \) spectroscopy. The model was firstly suggested in Ref. \cite{23} and used for the calculations of the Nucleon ground state properties in \cite{24,25}. A modification of the model, the so-called Perturbative Chiral Quark Model was recently extended to the systematic study of the nucleon properties \cite{26}, the mass-spectrum of the octet-baryons \cite{27} and the pentaquark systems \cite{28}. In these approaches baryons are considered as bound states of valence quarks surrounded by a cloud of Goldstone bosons (\( \pi-, K-, \eta- \)mesons) as required by the chiral symmetry.

As was found in the model independent analyses of baryons in the large \( N_c \) limit \cite{29}, the mass splitting between various baryon states receive important contributions from operators which simultaneously couple spin, isospin and orbital momentum. On the other hand (see \cite{24,25}), the relativistic structure of the one-meson exchange forces for the ground states of the \( N \) and \( \Delta \) is the same as in the Goldstone Boson Exchange Model of Glozman et al., i.e. it is

\[
V(r_{ij}) \hat{\tau}_i \hat{\tau}_j \hat{\sigma}_i \hat{\sigma}_j,
\]

where \( \hat{\tau}_i \) and \( \hat{\sigma}_j \) are spin and isospin operators respectively. It is important to note that the above structure of GBE interaction yields the correct ordering of the radially and orbitally excited nucleon resonances, namely the Roper resonance \( N(1440)(1/2^+) \) and negative parity \( N(1520)(3/2^-), N(1535)(1/2^-) \) resonances \cite{24}. We found also that this structure holds for all \( S^- \)wave baryons (ground and radially excited baryon states). However the important question, does this structure hold for orbitally excited baryon resonances, is still open. Below we show that in the latter case the one-meson exchange interaction has more complicated structure. More precisely, it contains additionally an operator which couples the upper orbital momentum of valence quark emitting a single meson with the lower orbital momentum of another valence quark absorbing this meson. In the \( S^- \)wave limit this operator is proportional to the spin operator, in other words it supports the above well-known structure. However, for \( P-, D- \) wave baryons the exchange operator yields different matrix elements and the above structure of the one-meson exchange does not hold. It is clear that this relativistic effect can not be obtained in non-relativistic meson exchange models. As a result we have a splitting due-to one-meson exchange in the excited baryon sector. This finding concerns the long-standing ”three-body spin-orbit puzzle” in baryons. It is important to note that the obtained structure of the one-meson exchange operator confirms the prediction of the large \( N_c \) approach. It couples upper and lower orbital momentum, spin, isospin and full momentum of the two interacting valence quarks.

The relativistic quark model is based on an effective chiral Lagrangian describing quarks as relativistic fermions moving in a confining static potential. The potential is described by a Lorentz scalar and the time component of a vector potential, where the latter term is responsible for short-range fluctuations of the gluon field configurations \cite{30}. The model potential defines unperturbed wave functions of the quarks which are subsequently used in the calculations of baryon properties. Interaction of quarks with Goldstone bosons is introduced on the basis of the nonlinear \( \sigma \)-model \cite{31}. All calculations are performed at one loop or at order of accuracy \( o(1/f_\pi^2) \). Due to negligible contribution of the \( K^- \) and \( \eta^- \) meson loop diagrams in our model to the Nucleon and Delta sectors we restrict to the \( \pi^- \) meson loop diagrams.

In the following we proceed as follows: we first describe the basic formalism of our approach. Then we indicate the main derivations relevant to the problem and finally present the numerical results.

II. MODEL

The effective Lagrangian of our model \( \mathcal{L}(x) \) contains the quark core part \( \mathcal{L}_Q(x) \) the quark-pion \( \mathcal{L}_I^{(\pi\pi)}(x) \) interaction part, and the kinetic part for the pion field \( \mathcal{L}_\pi(x) \):

\[
\mathcal{L}(x) = \mathcal{L}_Q(x) + \mathcal{L}_I^{(\pi\pi)}(x) + \mathcal{L}_\pi(x)
\]

\[
= \bar{\psi}(x)[i \not\!\!\!D(r) - \gamma^0 V(r)]\psi(x) - 1/f_\pi \bar{\psi}[S(r)i\gamma^5\tau^i\phi_i]\psi + \frac{1}{2}(\partial_\mu\phi_i)^2 - \frac{1}{2}m_i^2\phi_i^2. \tag{2}
\]

Here, \( \psi(x) \) and \( \phi_i, i = 1, 2, 3 \) are the quark and pion field operators, respectively. The matrices \( \tau^i (i = 1, 2, 3) \) are the isospin matrices. The pion decay constant is \( f_\pi = 93 \text{ MeV} \). The scalar part of the static confinement potential

\[
S(r) = cr + m \tag{3}
\]
where $c$ and $m$ are constants. The constant part of the scalar potential can be interpreted as the current quark mass term.

At short distances, transverse fluctuations of the string are dominating, with some indication that they transform like the time component of the Lorentz vector. They are given by a Coulomb type vector potential as

$$V(r) = -\alpha/r$$

(4)

where $\alpha$ is approximated by a constant. The quark fields are obtained from solving the Dirac equation with the corresponding scalar plus vector potentials

$$[i\gamma^\mu \partial_\mu - S(r) - \gamma^0 V(r)]\psi(x) = 0$$

(5)

The respective positive and negative energy eigenstates as solutions to the Dirac equation with a spherically symmetric mean field, are given in a general form as

$$u_\alpha(x) = \left(\begin{array}{c} g_{N\kappa}(r) \\
-i f_{N\kappa}(r) \bar{\sigma} \chi \end{array}\right) \mathcal{Y}^{m_j}(\hat{x}) \chi_m \chi_m \exp(-iE_\alpha t)$$

(6)

$$v_\beta(x) = \left(\begin{array}{c} g_{N\kappa}(r) \\
-i f_{N\kappa}(r) \bar{\sigma} \chi \end{array}\right) \mathcal{Y}^{m_j}(\hat{x}) \chi_m \chi_m \exp(+iE_\beta t)$$

(7)

The quark and antiquark eigenstates $u$ and $v$ are labeled by the radial, angular, azimuthal, isospin and color quantum numbers $N, \kappa, m_j$, $m\bar{m}$ and $m_c$, which are collectively denoted by $\alpha$ and $\beta$, respectively. The spin-angular part of the quark field operators

$$\mathcal{Y}^{m_j}(\hat{x}) = [Y_i(\hat{x}) \otimes \chi_{1/2}] m_j \quad j = |\kappa| - 1/2.$$ 

(8)

For a given total angular momentum $j$ and projection $m_j$, the upper and lower components of Eq.(6) and Eq.(7) are expanded in a harmonic oscillator basis. The quark fields $\psi$ are expanded over the basis of positive and negative energy eigenstates as

$$\psi(x) = \sum_\alpha u_\alpha(x)b_\alpha + \sum_\beta v_\beta(x)d_\beta.$$ 

(9)

The expansion coefficients $b_\alpha$ and $d_\beta$ are operators, which annihilate a quark and create an antiquark in the orbits $\alpha$ and $\beta$, respectively.

The free pion field operator is expanded over plane wave solutions as

$$\phi_j(x) = (2\pi)^{-3/2} \int \frac{d^3k}{(2\omega_k)^{1/2}} [a_{jk} \exp(-ikx) + a_{jk}^\dagger \exp(ikx)]$$

(10)

with the usual destruction and creation operators $a_{jk}$ and $a_{jk}^\dagger$ respectively. The pion energy is defined as $\omega_k = \sqrt{k^2 + m_\pi^2}$.

In denoting the three-quark vacuum state by $|0>$, the corresponding noninteracting many-body quark Green’s function (propagator) is given by the customary vacuum Feynman propagator for a binding potential $^{32}$:

$$iG(x, x') = iG^F(x, x') = <0|T\{\psi(x)\bar{\psi}(x')\}|0> = \sum_\alpha u_\alpha(x)\bar{u}_\alpha(x')\theta(t - t') + \sum_\beta v_\beta(x)\bar{v}_\beta(x')\theta(t' - t)$$

(11)

Since the three-quark vacuum state $|0>$ does not contain any pions, the pion Green’s functions are given by the usual free Feynman propagator for a boson field:

$$i\Delta_{ij}(x - x') = <0|T\{\phi_i(x)\bar{\phi}_j(x')\}|0> = -\delta_{ij} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2 + i\epsilon} \exp[-ik(x - x')]$$

(12)

Using the effective Lagrangian we calculate the lowest excitation spectrum of the nucleon and delta. In the model the quark core result ($E_Q$) is obtained by solving the Dirac equation for the single quark system numerically. Since we work in the independent particle model, the bare three-quark state of the $SU(2)$-flavor baryons has the structure
coordinates and setting in intrinsic wave function (center of mass system). The second method uses the Fourier transformation of the c.m. corrections. For the excited states we calculate the splitting from ground state both due to quark core and one-pion perturbative the center of mass motion in the product wave function, similar to the non-relativistic shell model. In present work, we do CM correction using above three methods for the quark core results of the nucleon and delta ground states. For the excited states we calculate the splitting from ground states both due to quark core and one-pion perturbative corrections.

The second order perturbative corrections to the energy spectrum of the SU(2) baryons due to pions (ΔE(π)) are calculated on the basis of the Gell-Mann and Low theorem:

$$\Delta E = \langle \phi_0 | \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int i\delta(t_1) d^4x_1 \ldots d^4x_n T[H_I(x_1) \ldots H_I(x_n)] | \phi_0 \rangle_c$$

with \(n = 2\), where the relevant quark-pion interaction Hamiltonian density is

$$H^{(\pi)}_I(x) = \frac{i}{f_\pi} \bar{\psi}(x) \gamma^5 \tau \phi(x) S(r) \psi(x),$$

(13)

The index (c) in Eq.(13) denotes the contributions only from connected graphs. The stationary bare three-quark state (3-quark core) |φ₀> is constructed from the vacuum state using the usual creation operators:

$$| \phi_0 >_{\alpha\beta\gamma} = b^+_\alpha b^+_\beta b^+_\gamma |0>, \tag{15}$$

where α, β and γ represent the quantum numbers of the single quark states, which are coupled to the respective baryon configuration. The energy shift of Eq.(13) is evaluated up to second order in the quark-pion interaction, and generates self-energy and exchange contributions.

### A. Self-energy contributions

The self-energy terms contain contributions both from intermediate quark (\(E > 0\)) and antiquark (\(E < 0\)) states. The self energy term due to pion fields (see Fig.1) is evaluated as

$$\Delta E^{(\pi)}_{\alpha c} = -\frac{1}{2f_\pi^2} \sum_{\alpha=1}^{3} \sum_{\alpha' \leq \alpha} \int \frac{d^3\vec{p}}{(2\pi)^3 p_0} \left\{ \sum_{\alpha} V^{\alpha+}_{\alpha\alpha'}(\vec{p}) V^{\alpha'}_{\alpha\alpha}(\vec{p}) - \sum_{\beta} V^{\beta+}_{\beta\alpha'}(\vec{p}) V^{\alpha}_{\beta\beta'}(\vec{p}) \right\} + \frac{m^2}{E_\alpha - E_{\alpha'} + p_0}, \tag{16}$$

with \(p_0^2 = \vec{p}^2 + m^2\). The transition form factors are defined by:

$$V^{\alpha}_{\alpha\alpha'}(\vec{p}) = \int d^3x \bar{\alpha}(\vec{x}) \Gamma^{\alpha}(\vec{x}) u_{\alpha'}(\vec{x}) e^{-i\vec{p} \cdot \vec{x}}, \tag{17}$$

$$V^{\beta}_{\beta\alpha'}(\vec{p}) = \int d^3x \bar{\alpha}(\vec{x}) \Gamma^{\alpha}(\vec{x}) u_{\alpha'}(\vec{x}) e^{-i\vec{p} \cdot \vec{x}}, \tag{18}$$

The vertex function of the \(\pi - q - q\) and \(\pi - q - \bar{q}\) transition is

$$\Gamma^{\alpha} = S(r) \gamma^5 \gamma^a I_c, \tag{19}$$

where \(I_c\) is the color unity matrix. The sum in Eq.(16) is performed over \(\alpha'\) up to and including the Fermi level with quantum number \(\alpha\) and over all quantum numbers \(\alpha\) and \(\beta\) of the intermediate quark state with it’s positive and negative energy solutions. After estimation of the transition form factors (see Appendix) and putting into equation

$$E_Q = 2E(1S_{1/2}) + E(nlj)$$

The result for \(E_Q\) still contains the contribution of the center of mass motion. To remove this additional term we resort to three approximate methods, which correct for the center of mass motion: the \(R = 0\) and \(P = 0\) and LHO methods. All these methods were examined in [36] for the center of mass correction of the ground state nucleon and delta and they give similar results. The first method is based on the extraction of the center of mass using the expression of the baryon wave function in terms of the Jacobi coordinates and putting

$$P = 0 \text{ under Fourier integral. The last method is based on the keeping the lowest s-state for the center of mass motion in the product wave function, similar to the non-relativistic shell model. In present work, we do CM correction using above three methods for the quark core results of the nucleon and delta ground states. For the excited states we calculate the splitting from ground states both due to quark core and one-pion perturbative corrections.}$$
(16) and integration over angular part in the momentum space, we obtain next expression for the energy shift of the SU(2) baryon state due to the second order self-energy diagrams:

\[
\Delta E_{s.e.} = -\frac{1}{16\pi^3 f_\pi^2} \int \frac{dp^2}{p_0} \sum_{N',l',j'} \sum_{l_n} \left\{ \sum_\alpha \left[ \int \frac{dr^2 G_{\alpha\alpha'}(r) S(r) j_{l_n}(pr)}{E_\alpha - E_{\alpha'} + p_0} \right]^2 Q_{s.e.}(l,l',l_n,j,j') - \sum_\beta \left[ \int \frac{dr^2 G_{\beta\alpha'}(r) S(r) j_{l_n}(pr)}{E_\beta + E_{\alpha'} + p_0} \right]^2 Q_{s.e.}(l,l',l_n,j,j') \right\}
\]

where \(j_{l_n}\) is the Bessel function. The radial overlap of the single quark state \(\alpha = (N,l,j,m_{jt},m_{tc})\) and \(\alpha' = (N',l',j',m_{j't},m_{tc}')\) is defined as

\[
G_{\alpha\alpha'}(r) = f_\alpha(r) g_{\alpha'}(r) + f_{\alpha'}(r) g_\alpha(r).
\]

The angular momentum coefficients \(Q\) are evaluated for all SU(2) baryons as

\[
Q_{s.e.}(l,l',l_n,j,j') = 12\pi \left[ t^l l_n j \right] \left[ c^l l_n j \right] \sum_{m_j} \sum_{m_{j'}} \left[ C_{lm_{j'}m_j} W(t^l l_n) \right] \sum \left[ C_{l_{n}m_{j}} \right] \left[ C^l_{lm_{j'}} \right] \left[ C^l_{lm_{j'}} \right] \left[ C^l_{lm_{j'}} \right],
\]

where \(C\) and \(W\) are the Clebsch-Gordan and Wigner coefficients, respectively. The sum in Eq.(22) does not depend on the orientation of the full momentum of the valence quark \(m_{j'} = -j', -j' + 1, ..., j' - 1, j'\). In the case of the ground state \(N\) and \(\Delta\) the sum over parameter-set \((N',l',j')\) is replaced by the factor 3, since a valence quark in this case can be only in \(1S_{1/2}\) state. The first term in the sum of Eq.(20) represents the contribution from intermediate quark states, and the second term corresponds to the contribution of intermediate antiquark states to the energy shift of the SU(2) baryon state.

We note also that the role of meson cloud (self-energy) corrections can be also pinned down by sigma-terms and when considering the chiral limit [27].

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**FIG. 1:** Second order self energy diagrams due-to \(\pi\)-meson fields

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**B. Exchange diagrams contribution**

The exchange term due-to pion fields (see Fig.2) is evaluated as:

\[
\Delta E_{ex.}^{(\pi)} = -\frac{1}{2f_\pi^2} \sum_{a=1}^{3} \sum_{\alpha \leq \alpha_F} \sum_{\alpha' \leq \alpha_F} \int \frac{d^3 \vec{p}}{(2\pi)^3 p_0^2} \left\{ V_{\alpha\alpha'}^a(\vec{p}) V_{\alpha\alpha'}^a(\vec{p}) - V_{\alpha\alpha'}^{a+}(\vec{p}) V_{\alpha\alpha'}^{a+}(\vec{p}) \right\}.
\]
By using the explicit form of the transition form factor (see Appendix) and Wick’s theorem we can write more convenient expression for the energy shift due to the second order pion exchange diagrams:

\[
\Delta E^{(\pi)}_{\text{ex.}} = -\frac{1}{16\pi^3 f_\pi^2} \int \frac{dp}{p^2} \sum_{l_n} \vec{\Pi}_{l_n}(p)
\]

(24)

where

\[
\vec{\Pi}_{l_n}(p) = <\phi_B | \sum_{i \neq j} \hat{\tau}(i) \hat{\tau}(j) \hat{T}_{l_n}(i) \hat{T}_{l_n}(j) \hat{K}_{l_n}(i) \hat{K}^+_{l_n}(j) | \phi_B >
\]

(25)

and the operators \( \hat{\tau}, \hat{T}_{l_n} \) and \( \hat{K}_{l_n} \) are summed over single quark levels \( i \neq j \) of the SU(2) baryon. In the quark model, the baryon wave function \( |\phi_B > \) is presented as a bound state of three quarks, and it can be written down commonly as

\[
|\phi_B > = |\alpha \beta \gamma > = \sum_{J_0 T_0} |\alpha \beta ; \gamma >^{T M_T(T_0)}_{J M(J_0)}
\]

\[
= \sum_{J_0 T_0} \hat{S} \left[ |\psi_\alpha(r_1) \psi_\beta(r_2) \psi_\gamma(r_3) \rangle \chi^{J M}_{J_0} (x_1, x_2, x_3) > |\alpha \beta ; \gamma >^{T M_T(T_0)}_{J M(J_0)} \right],
\]

(26)

where \( J_0 \) and \( T_0 \) are intermediate spin and isospin couplings respectively. The states \( \psi \) are the single particle states, labeled by a set of quantum numbers \( \alpha, \beta \) and \( \gamma \), excluding the color degree of freedom.

The operator \( \hat{T}_{l_n} \) in Eq.(25) is the radial integration operator with the factor \( j_{l_n}(pr) S(r) \):

\[
< \alpha | \hat{T}_{l_n} | \beta > = \int dr \left[ r^2 S(r) j_{l_n}(pr) G_{\alpha \beta}(r) \right].
\]

(27)

The matrix elements of the operator \( \hat{K}_{l_n} \) are given by

\[
< \alpha | \hat{K}_{l_n} | \beta > = \left( 4\pi [l^+(\alpha)] [l_n] [j(\alpha)] \right)^{1/2} C_{l^+(\alpha)0}^{l_n(\alpha)} W(j(\alpha) l_n; l(\beta); j^+(\beta)) C^{l(\beta)m(\beta)}_{j(\alpha)m(\alpha) l_n(m(\beta)-m(\alpha))},
\]

(28)

where \( j(\alpha), l(\alpha), l^+(\alpha), m(\alpha) \) and \( j(\beta), l(\beta), l^+(\beta), m(\beta) \) are the quantum numbers of the single quark states \( < \alpha | \) and \( < \beta | \) : the full momentum, upper orbital momentum, lower orbital momentum, and projection of the full momentum respectively. The Hermitian conjugation operator is defined as

\[
< \alpha | \hat{K}^+_{l_n} | \beta > = < \beta | \hat{K}_{l_n} | \alpha >.
\]

(29)

It is important to note that operator \( \hat{K}_{l_n} \) couples the upper component of the single quark state \( \alpha \) (which corresponds to the valence quark emitting a single pion) with the lower component of the single quark state \( \beta \) (which corresponds to the valence quark absorbing this pion). And operator \( \hat{K}^+_{l_n} \) acts vice versa. These operators correspond to the coefficient \( F \) in Appendix. It is clear that they define the relativistic structure of the one-meson exchange. For the S-wave baryons, i.e., when \( \alpha \) and \( \beta \) single quark states are S-quarks, the operators \( \hat{K}_{l_n}(i) \) and \( \hat{K}^+_{l_n}(j) \) are proportional to the spin operators \( \hat{\sigma}_i \) and \( \hat{\sigma}_j \) respectively. It means that for the S-wave baryons the relativistic and non-relativistic structure of one-meson exchange are the same. However for the orbitally excited baryon states (P-, D-wave baryons) they are different due to operator \( \hat{K}_{l_n} \) which couples the lower and upper components of the interacting quarks.

### III. NUMERICAL RESULTS

In order to account for the finite size effect of the pion we introduce a one-pion vertex regularization function in the momentum space, parameterized in the dipole form as

\[
F_p(p^2) = \frac{A^2 - m^2}{A^2 + p^2}.
\]
According to the study of long-wavelength transverse fluctuations of the flux-tube [30] we use the value \( \alpha = 0.26 \approx \pi/12 \). We choose two sets of parameters: model A and model B. The parameters of the confining potential (\( c \) and \( m \)) are fitted to reproduce correct axial charge of the proton which remains unaltered when including the effects of the pion cloud, the empirical value of the pion-nucleon coupling constant

\[
G_{\pi NN}^2/4\pi \approx 14
\]

and a normal value for the quark core RMS radius 0.5 fm of the proton. Thus, we use only few parameters to reproduce the SU(2) baryons spectrum contrary to the traditional OGE and GBE based quark models.

In Table 1 we indicate the model parameters together with the corresponding single-quark energies. We note here that the first orbitally excited single valence quark level is \( 1S_{1/2} \). Therefore we associate the first orbitally excited nucleon and delta states with the structure \((1S)^21P_{3/2}\).

Table 2 contains the quark core results [24] for the static properties of the proton. The results include the center of mass correction. After CM correction, as was shown in [24], the magnetic moment of the proton increases by about 20\%, and the axial charge decreases by about 5\%. The CM correction reduces the RMS radius of the proton approximately by 10\% of the uncorrected value. From the table one can see, that a larger value of the strength parameter \( c \) of the confining potential yields a smaller value for the RMS radius of the proton. An estimation of the pion-nucleon coupling constant is close to the empirical value for both Model A and Model B.

Perturbative corrections to the lower SU(2) baryon states energy shift values due-to self-energy diagrams are given in Table 3. Contribution from intermediate quark and antiquark states are taken with the full momentum up to \( j = 25/2 \). As was shown in [24] for the ground state nucleon, the pion self-energy contribution is positive and convergent also for excited Nucleon and Delta states in contrast to the bag models. Contributions from self-energy diagrams for the radially excited Nucleon and Delta states with the last valence quark in \( 2S_{1/2} \) state are larger than for excited Nucleon and Delta states with the last valence quark in orbitally excited \( 1P_{3/2} \) and \( 1P_{1/2} \) states.

Corresponding energy shift values due-to exchange diagrams are given in Table 4. As we noted above, the relativistic structure of one-meson exchange operator results a splitting between negative parity nucleon resonances and also between delta resonances. The nucleon states with the structure \((1S)^21P_{3/2}\) split according to the energy shift values -132.8 and -150 MeV. In contrast to the GBE based quark models, lowest radial excitation of the Nucleon \( N^*(1440)(1/2^+) \) which has one valence quark in \( 2S_{1/2} \) state, in our model is less bound with 120.35 MeV than the lowest orbital excitations of the Nucleon \( N^*(1520)(3/2^-) \) and \( N^*(1535)(1/2^-) \), with 132.8 and 150 MeV respectively, which have the last valence quark in \( 1P_{3/2} \) state.

In Table 5 we give the mass values for the g.s. N(939) with and without CM correction for both parameter sets A and B. In the case of the model A, a reasonable value (1280-1320 MeV) for the mass of the \( \Delta \) is obtained. However, for the nucleon ground state the estimation is still large. Model B yields too large value for the mass of the nucleon. This means that larger values of the strength parameter \( c \) of the confining potential is not likely. From the Table one can see that the three methods for the correction of center of mass motion agree within 50 MeV which seems...
too large. However, these three methods always give corrections with systematic differences. For example, the LHO method yields correction larger than the \( P = 0 \) method, but smaller than the \( R = 0 \) method. Thus, we can fix one of these methods and analyse the excited sector.

And finally in Table 6 we compare the theoretical energy splitting values with experimental data from PDG \cite{12} and recent analysis \cite{13}. The new analysis of Arndt et al. does not contain the resonance \( \Delta(1600)(3^+/2^+) \). We note that the results presented in the last table do not include any correction due-to CM motion. The quark core and self-energy contributions to the Nucleon and Delta states with the same structure (for example \( (1S)^2(1P_{3/2}) \)) are equal. Therefore the contributions from quark core and self-energy terms to the \( N(939) - \Delta(1232) \), \( N(1520) - N(1535) \) and \( \Delta(1620) - \Delta(1700) \) splitting are exactly zero. The only correction due-to CM motion in the excited Nucleon sector in Table 6 is needed for \( N(939) - N(1440) \) and \( N(1440) - N(1535) \) splitting. However, we can estimate this correction by using an approximate factor from 1/2 to 2/3. First of all we note a good agreement of the theoretical value for the splitting of the first radial excitation of the Nucleon, Roper resonance \( N^*(1440)(1/2^+) \). The quark core result 415 MeV in the Model A should restrict to about 200-250 MeV after CM correction. That gives a good agreement with the experimental data when adding a perturbative correction (284 MeV for the model A). However the ordering and splitting of the Roper resonance and orbital Nucleon excitations \( N^*(1520)(3/2^-) \) and \( N^*(1535)(1/2^-) \) is not reproduced in our model contrary to the non-relativistic Constituent Quark Model of Glozman et al. \cite{9}. We suggest that the difference comes mainly from the relativistic structure of the one-meson exchange.

We also note that the splitting between Delta states is well reproduced though the last analysis of Arndt et al. shows some different picture.

**IV. SUMMARY AND CONCLUSIONS**

We demonstrated that a relativistic chiral quark model can yield a reasonable description of the energy spectrum of the ground state \( N \) and \( \Delta \) and their lowest radial and orbital excitations. The model yields a good splitting of the first radial excitation of the nucleon, the Roper resonance \( N^*(1440) \) from ground state \( N(939) \) and splitting between negative parity nucleon \( N^*(1520)(3/2^-) \) and \( N^*(1535)(1/2^-) \) resonances. The splitting in the \( \Delta \) sector also was reproduced. It is important to note that these results correspond to the parameters of the model, which yield a normal quark core radius of the nucleon, being about 0.5 fm and empirical value of the pion-nucleon coupling constant \( G_{\pi NN}^2/4\pi \approx 14 \).

We found that the relativistic structure of one-meson exchange interaction confirms a prediction of the large \( N_c \) approach \cite{29}, which stated that the mass splitting between various baryons receive important contributions from operators which simultaneously couple spin, isospin and orbital momentum. In particular, the one-pion exchange interaction includes an operator which couples the lower and upper orbital momentum of two interacting valence quarks. In the S-wave limit (ground and radially excited Nucleon and Delta states) this operator is proportional to the simple spin operator. However, for the orbitally excited baryon resonances it yields different matrix elements and splitting between for example negative parity nucleon resonances and delta resonances. As a result we conclude that relativistic one-meson exchange interaction does not yield the correct ordering of lowest positive and negative parity nucleon resonances and a new mechanism is needed for the explanation of this effect.

Since the developed model gives only a part of the \( N(939) - \Delta(1232) \) mass splitting and that the Delta states are reproduced quite well, we see two possible development of the model. The first is to include Instanton induced exchange mechanism, which should give another part of the \( N(939) - \Delta(1232) \) splitting. It seems that these forces are responsible for the correct ordering of the radially and orbitally excited Nucleon resonances. On the other hand, they don’t change the spectrum of the Delta states and thus keep a good description of this sector. The second development would include the one-gluon exchange forces which should give some part of the above splitting, and possible inclusion of the Instanton induced interaction mechanisms. The obtained splitting between negative parity nucleon resonances due-to relativistic one-meson exchange forces in our model would cancel a large value of the spin-orbit interaction due-to one-gluon exchange. This would help to understand a long standing "three body spin-orbit puzzle" in baryons.

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Appendix: Transition form factors

Putting explicit expression of the vertex matrix $\Gamma^a(\bar{p})$ from Eq.(19) into Eq.(17) we receive next equation:

$$V^a_{\alpha\alpha'}(\bar{p}) = -i \int d\bar{r} r^2 \left[ g_{\alpha}(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_{\alpha}(r) \right] S(r)$$

$$\int d\bar{r} \left[ Y_{j\ell}^{m_j}(\bar{r})(\bar{e}\bar{r})Y_{j'\ell'}^{m_{j'}}(\bar{r})e^{-i\bar{p}\bar{r}} \right] < m_\alpha | r^\alpha | m'_\alpha > < m_c | I_c | m'_c >$$  \hspace{1cm} (30)

Now using

$$Y_{j\ell}^{m_j}(\bar{r})(\bar{e}\bar{r}) = -Y_{j\ell}^{-m_j}(\bar{r})$$

which couples the lower orbital momentum to the spin, and expanding the exponential function over spherical Bessel functions and integrating over angular part of the variable $\bar{r}$, we get next equation for the integral

$$\int d\bar{r} \left[ Y_{j\ell}^{m_j}(\bar{r})(\bar{e}\bar{r})Y_{j'\ell'}^{m_{j'}}(\bar{r})e^{-i\bar{p}\bar{r}} \right] = \sum_{l_n} (-i)^l_n j_{l_n}(pr)Y_{l_n}^{m_j-m_{j'}}(\bar{p})F(l^\pm, l', l, j, j', m_j, m_{j'})$$

where coefficients $F$ are defined as

$$F(l^\pm, l', l, j, j', m_j, m_{j'}) = - \left( 4\pi |l^\pm||l_n||j | \right)^{1/2} C_{l_n, l_m} W(j, l, l', j', j) C_{l_m, l_n} m_{j'}.$$ 

For the transition form-factor now it is easy to write the next expression:

$$V^a_{\alpha\alpha'}(\bar{p}) = \sum_{l_n} (-i)^l_n + 1 \int d\bar{r} r^2 \left[ g_{\alpha}(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_{\alpha}(r) \right] S(r) j_{l_n}(pr)$$

$$Y_{l_n}^{m_j-m_{j'}}(\bar{p})F(l^\pm, l', l, j, j', m_j, m_{j'}) < m_\ell | r^\ell | m'_\ell > < m_c | I_c | m'_c > .$$  \hspace{1cm} (31)

The Hermitian conjunction of the transition form factor

$$V^{a^+}_{\alpha\alpha'}(\bar{p}) = \sum_{l_n} (i)^l_n + 1 \int d\bar{r} r^2 \left[ g_{\alpha}(r)f_{\alpha'}(r) + g_{\alpha'}(r)f_{\alpha}(r) \right] S(r) j_{l_n}(pr)$$

$$Y_{l_n}^{m_j-m_{j'}}(\bar{p})F(l^\pm, l', l, j, j', m_j, m_{j'}) < m_\ell | r^\ell | m'_\ell > < m'_c | I_c | m_c > .$$  \hspace{1cm} (32)

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| Model | c, GeV | m, GeV | Λπ, GeV | α | E(1S) | E(2S) | E(1P_{3/2}) | E(1P_{1/2}) |
|-------|--------|--------|----------|---|--------|--------|--------------|--------------|
| A     | 0.16   | 0.06   | 1.0      | 0.26 | 571.7  | 986.7  | 822.8        | 860.7        |
| B     | 0.20   | 0.07   | 1.2      | 0.26 | 641.4  | 1105.7 | 922.0        | 964.4        |

**TABLE II: Quark core contributions to the static properties of the proton** [24]

| Model | g_A | μ_p, N.M. | RMS radius, fm | G^2_{NN}/(4π) |
|-------|-----|-----------|----------------|----------------|
| A     | 1.26 | 1.58      | 0.52           | 13.919         |
| B     | 1.26 | 1.41      | 0.47           | 13.984         |

**TABLE III: Second order perturbative corrections due to one-pion self energy diagrams for the energy shift of the single valence quarks in MeV**

| (J,T) | (1S)^A | (1S)^2S | (1S)^S | (1S)^2P_{3/2} | (1S)^2P_{1/2} |
|-------|--------|---------|--------|---------------|---------------|
|       | -179.5 | -120.35 | 3.7    | -132.8        | -132.8        |

**TABLE IV: Second order perturbative corrections due to one-pion exchange diagrams for the mass spectrum of lowest N and Δ states in MeV for the Model A**

| (J,T) | A 3 | A 2S | A 1P_{3/2} | A 1P_{1/2} | B 3 | B 2S | B 1P_{3/2} | B 1P_{1/2} |
|-------|-----|------|------------|------------|-----|------|------------|------------|
|       | 1715| 940  | 985        | 966        | 1924| 1057 | 1110       | 1088       |

**TABLE V: The mass value of the g.s. nucleon in MeV with and without center of mass (CM) correction**

| Model | No CM | R=0, [33] | P=0, [34] | LHO, [35] |
|-------|-------|-----------|-----------|-----------|
| A     | E_Q  | 1715      | 940       | 985       | 966      |
| B     | E_Q  | 1924      | 1057      | 1110      | 1088     |
|       | E_Q + ΔE | 2225      | 1358      | 1411      | 1389     |

**TABLE VI: Energy splitting values between lowest N and Δ states in MeV**

| N(939)(1/2^+) - Δ(1232)(3/2^+) | 0 | 144 | 0 | 223 | 293 | 293 |
| N(939)(1/2^+) - N(1440)(1/2^+) | 415 | 284 | 464 | 439 | 490 ÷ 530 | 528 ± 4.5 |
| N(1440)(1/2^+) - N(1535)(1/2^+) | -164 | -101 | -184 | -161 | 50 ÷ 125 | 78 ± 6.5 |
| N(1520)(3/2^+) - N(1535)(1/2^+) | 0 | 17 | 0 | 26 | 0 ± 30 | 30.4 ± 3 |
| Δ(1232)(3/2^+) - Δ(1600)(3/2^+) | 415 | 237 | 464 | 366 | 320 ± 470 | 580 ± 2.4 |
| Δ(1232)(3/2^+) - Δ(1620)(1/2^-) | 251 | 164 | 280 | 250 | 380 ± 445 | 381 ± 2 |
| Δ(1620)(1/2^-) - Δ(1700)(3/2^-) | 0 | -10 | 0 | -15 | (-5) ÷ 185 | 74 ± 3.5 |