Non-thermal leptogenesis in supersymmetric 3-3-1 model with inflationary scenario

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Abstract.

We study a leptogenesis scenario in which the heavy Majorana neutrinos are produced non-thermally in inflaton decays in the supersymmetric economical SU(3)\textsubscript{C} \otimes SU(3)\textsubscript{L} \otimes U(1)\textsubscript{X} model with inflationary scenario, and for this purpose neutrino masses play the key role. Due to the inflaton with mass in the GUT scale, the model under consideration provides successful neutrino masses, which is different from ones without inflationary scenario. The lepton-number-violating interactions among the inflaton and right-handed neutrinos appear at the one-loop level, and this is a reason for non-thermal leptogenesis scenario. The bound followed from the gravitino abundance and the cosmological constraint on neutrino mass/the neutrino oscillation data is $m_{\nu 3} \simeq \frac{0.05}{\delta_{\text{eff}}} \text{eV}$. By taking the reheating temperature as low as $T_R = 10^6$ GeV, we get a limit on the ratio of masses of the light heavies neutrino to those of the inflaton to be $\frac{M_{\nu 3}}{M_\phi} = 0.87$.

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1. Introduction

The recent experimental results confirm that neutrinos have tiny masses and oscillate [1], which implies that the standard model (SM) must be extended. Among the beyond-SM extensions, the models based on the SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_X$ (3-3-1) gauge group [2, 3] have some intriguing features. First, they can give partial explanation of the generation number problem. Second, the third quark generation has to be different from the first two, so this leads to the possible explanation of why top quark is uncharacteristically heavy. An additional motivation to study this kind of the models is that they can also predict the electric charge quantization [4].

Depending on the electric charge of particle at the bottom of the lepton triplet, the 3-3-1 models are classified into two main versions: the minimal model [2] with the lepton triplet $(\nu, l, l^c)_L$ and the version with right-handed (RH) neutrinos [3], where the RH neutrinos place at the bottom of the triplet: $(\nu, l, \nu^c)_L$. In the 3-3-1 model with right-handed neutrinos, the scalar sector requires three Higgs triplets. It is interesting to note that two Higgs triplets of this model have the same U(1)$_X$ charges with two neutral components at their top and bottom. In the model under consideration, the new charge $X$ is connected with the electric charge operator through a relation

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X.$$

Assigning these neutral components vacuum expectation values (VEVs) we can reduce the number of Higgs triplets to two. Therefore we have a resulting 3-3-1 model with two Higgs triplets [5]. As a result, the dynamical symmetry breaking also affects the lepton number. Hence it follows that the lepton number is also broken spontaneously at a high scale of energy. Note that the mentioned model contains a very important advantage, namely, there is no new parameter, but it contains very simple Higgs sector; therefore, the significant number of free parameters is reduced. To mark the minimal content of the Higgs sector, this version that includes right-handed neutrinos is going to be called the economical 3-3-1 model.

By this time, the cosmology becomes one of the most important sciences giving deep knowledge on the origin of our Universe. The critical moment for the development of modern cosmology was discovery of the 2.7 $K$ microwave background radiation arriving from the farthest reaches of the universe. The existence of the microwave background had been predicted by the hot-universe theory, which gained immediate and widespread acceptance after the discovery. Despite successes, there are a lot of difficulties (see, for example, [6]) in modern cosmology such as flatness, horizon, primordial monopole problems, etc. It is all the more surprising, then, that many of these problems, together with a number of others that predate the hot universe theory, have been resolved in the context of one fairly simple scenario for the development of the universe - the so-called inflationary universe scenario [7]. Inflation assumes that there was a period in the very early universe when the potential and vacuum energy density dominated the energy of the universe, so that the cosmic scale factor grew exponentially. The important
ingredient of the inflationary scenario is a scalar field $\phi$ having effective potential $V(\phi)$ with some properties (satisfying many constrains that are rather unnatural). This scalar field is called inflaton.

On the other hand, to explain the well-known matter-antimatter asymmetry, the baryogenesis plays an important role. In addition, primordial lepton asymmetry is converted to baryon asymmetry in the early universe through the “sphaleron” effects of electroweak gauge theories \cite{8} if it is produced before the electroweak phase transition. Thus, the leptogenesis scenario \cite{9} seems to be the most plausible mechanism for creating the cosmological baryon asymmetry.

The aim of this work is to consider leptogenesis in the supersymmetric economical 3-3-1 model with inflationary scenario. This paper is organized as follows. In Section 2 we present the particle content in the supersymmetric economical 3-3-1 model. Section 3 is devoted to neutrino mass in the supersymmetric economical 3-3-1 model without the inflationary scenario. We will show that in this case, the neutrino mass matrix is unrealistic. In Section 4 we present the seesaw mechanism in the model with inflationary scenario. At the one-loop level, the neutrino mass matrix gives the necessary hierarchy. The non-thermal leptogenesis scenario is presented in section 5. We summary our results and make conclusions in section 6.

2. A brief review of the model

To proceed further, the necessary features of the supersymmetric economical 3-3-1 model \cite{10,11} will be presented. The superfield content in this model is defined in a standard way as follows:

$$\hat{F} = (\bar{F}, F), \quad \hat{S} = (S, \bar{S}), \quad \hat{V} = (\lambda, V),$$

where the components $F$, $S$ and $V$ stand for the fermion, scalar and vector fields of the economical 3-3-1 model while their superpartners are denoted as $\bar{F}$, $\bar{S}$ and $\lambda$, respectively \cite{12,13}.

The superfields for the leptons under the 3-3-1 gauge group transform as

$$\hat{L}_a^L = (\tilde{\nu}_a, \tilde{\ell}_a, \tilde{\nu}_c^a)^T \sim (1, 3, -1/3), \quad \tilde{\nu}_a^c \sim (1, 1, 1),$$

where $\tilde{\nu}_a^c = (\tilde{\nu}_R)^c$ and $a = 1, 2, 3$ is a generation index.

It is worth mentioning that in the economical version the first generation of quarks should be different from others \cite{5}. The superfields for the left-handed quarks of the first generation are in triplets

$$\hat{Q}_{1L} = \left(\tilde{u}_1, \tilde{d}_1, \tilde{u}'_1\right)^T \sim (3, 3, 1/3),$$

where the right-handed singlet counterparts are given by

$$\tilde{u}_1^c \sim (3^*, 1, -2/3), \quad \tilde{d}_1^c \sim (3^*, 1, 1/3).$$

Conversely, the superfields for the last two generations transform as antitriplets

$$\hat{Q}_{\alpha L} = \left(\tilde{d}_{\alpha}, -\tilde{u}_{\alpha}, \tilde{d}_{\alpha}'\right)^T \sim (3, 3^*, 0), \quad \alpha = 2, 3,$$
where the right-handed counterparts are in singlets
\[
\tilde{d}_{cL} \sim (3^*, 1, -2/3), \quad \tilde{d}_{cL}^c \sim (3^*, 1, 1/3).
\] (7)

The primes superscript on usual quark types \((u', w')\) with the electric charge \(q_{u'} = 2/3\) and \(d'\) with \(q_{d'} = -1/3\) indicate that those quarks are exotic ones. The mentioned fermion content, which belongs to that of the 3-3-1 model with right-handed neutrinos [3, 5] is, of course, free from anomaly.

The two superfields \(\hat{\chi}\) and \(\hat{\rho}\) are introduced to span the scalar sector of the economical 3-3-1 model [5]:
\[
\hat{\chi} = \left(\chi^0_1, \chi^-_2, \chi^0_2\right)^T \sim (1, 3, -1/3),
\] (8)
\[
\hat{\rho} = \left(\rho^+_1, \rho^0_1, \rho^-_2\right)^T \sim (1, 3, 2/3).
\] (9)

To cancel the chiral anomalies of Higgsino sector, the two extra superfields \(\hat{\chi}'\) and \(\hat{\rho}'\) must be added as follows:
\[
\hat{\chi}' = \left(\chi'^0_1, \chi'^+_2, \chi'^0_2\right)^T \sim (1, 3^*, 1/3),
\] (10)
\[
\hat{\rho}' = \left(\rho'^+_1, \rho'^0_1, \rho'^-_2\right)^T \sim (1, 3^*, -2/3).
\] (11)

In this model, the \(SU(3)_L \otimes U(1)_X\) gauge group is broken via two steps:
\[
SU(3)_L \otimes U(1)_X \xrightarrow{w, w'} SU(2)_L \otimes U(1)_Y \xrightarrow{v, v', u, u'} U(1)_Q,
\] (12)
where the VEVs are defined by
\[
\sqrt{2}\langle \chi \rangle^T = (u, 0, w), \quad \sqrt{2}\langle \chi' \rangle^T = (u', 0, w'),
\] (13)
\[
\sqrt{2}\langle \rho \rangle^T = (0, v, 0), \quad \sqrt{2}\langle \rho' \rangle^T = (0, v', 0).
\] (14)

The VEVs \(w\) and \(w'\) are responsible for the first step of the symmetry breaking while \(u, u'\) and \(v, v'\) are for the second one. The VEVs \(w, w'\) give mass for the exotic quarks and new gauge bosons while the VEVs \(u, u', v, v'\) give mass for SM particles. Therefore they have to satisfy the constraints
\[
u, v', u, u' \ll w, w'.
\] (15)

On the other hand, we can drive constraint \(v, v' \simeq v_{\text{electroweak}} = 246\) GeV from the bound of \(W\) boson mass and \(u, u' < 2.46\) GeV (for details, see [5]). Note that \(u\) and \(u'\) carry lepton number 2 [14], so they are the kinds of lepton-number-violating parameter.

Hence, it leads to the limit
\[
u, v' \ll v, v'.
\] (16)

The vector superfields \(\hat{V}_c, \hat{V}\) and \(\hat{V}'\) containing the usual gauge bosons are, respectively, associated with the \(SU(3)_C, SU(3)_L\) and \(U(1)_X\) group factors. The colour and flavour vector superfields have expansions in the Gell-Mann matrix bases \(T^d = \lambda^d/2\) \((d = 1, 2, ..., 8)\) as follows:
\[
\hat{V}_c = \frac{1}{2}\lambda^d \hat{V}_{cd}, \quad \hat{V}_c^c = -\frac{1}{2}\lambda^{dc} \hat{V}_{cd}^c, \quad \hat{V} = \frac{1}{2}\lambda^d \hat{V}_d, \quad \hat{V}' = -\frac{1}{2}\lambda^{dc} \hat{V}_{d}^c.
\] (17)
3. Neutrino mass in supersymmetric 3-3-1 model without the inflationary scenario

Let us mention that recent data from neutrino oscillations produced the following results:

\[
0.36 \leq \sin^2 \theta_{23} \leq 0.67, \quad 0.27 \leq \sin^2 \theta_{12} \leq 0.38 \quad \text{sin}^2 \theta_{13} < 0.053,
\]

and

\[
2.07 \times 10^{-3} \text{ eV}^2 \leq \Delta m^2_{\text{atm}} \leq 2.75 \times 10^{-3} \text{ eV}^2,
\]

\[
7.03 \times 10^{-5} \text{ eV}^2 \leq \Delta m^2_{\text{sol}} \leq 8.27 \times 10^{-5} \text{ eV}^2,
\]

at 99.73% CL [16].

This gives the constraints on neutrino masses and mixing. Let us consider the above problem in the model the without inflationary scenario.
3.1. Tree-level Dirac mass

At the tree-level, the neutrinos get masses from the term
\[- \lambda'_a b L_a L_b \rho + H.c., \tag{24}\]
which gives us
\[- \lambda'_a b (\nu_a L \nu_b L - \nu_a L \nu_b L - \nu_a L \nu_b L) \rho^0. \tag{25}\]

This mass term can now be rewritten in terms of a 6 × 6 matrix \( X_\nu \) by defining the following column vector:
\[(\psi_0^\nu)^T = (\nu_1 L \nu_2 L \nu_3 L \nu_1 R \nu_2 R \nu_3 R)^T. \tag{26}\]

Now we can rewrite our mass term as
\[- \mathcal{L} = \frac{1}{2} \left( (\psi_\nu^0)^T X_\nu \psi_\nu^0 + H.c. \right), \tag{27}\]
with
\[X_\nu = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 & G_{21} & G_{31} \\ 0 & 0 & 0 & G_{12} & 0 & G_{32} \\ 0 & 0 & 0 & G_{13} & G_{23} & 0 \\ 0 & G_{12} & G_{13} & G_{32} & 0 & 0 \\ G_{21} & 0 & G_{23} & 0 & 0 & 0 \\ G_{31} & G_{32} & 0 & 0 & 0 & 0 \end{pmatrix} \equiv \begin{pmatrix} 0 & M_D^T \\ M_D & 0 \end{pmatrix} \tag{28}\]

where
\[G_{ab} = (\lambda'_{ab} - \lambda'_b a). \tag{29}\]

Due to the fact that \( G_{ab} = -G_{ba} \), the mass pattern of this sector is 0, 0, \( m_\nu \), \( m_\nu \), \( m_\nu \), \( m_\nu \), where \( \sqrt{2} m_\nu = v \sqrt{G_{31}^2 + G_{32}^2 + G_{21}^2} \). Noting that this mass spectrum is the same as that of the non-supersymmetric version and the mass spectrum is not realistic [17]. The most general neutrino mass spectrum is in the following form:
\[M_\nu = \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{30}\]
where \( M_{L,R} \) (vanish at the tree-level) and \( M_D \) get possible corrections.

3.2. The one-loop corrections to the Dirac and Majorana masses

The Yukawa couplings of the leptons and the relevant Higgs self-couplings are explicitly rewritten as follows:
\[L_{\text{Y}}^\text{lep} = \lambda'_{a b} \nu_a L L_b \rho^2 + \lambda'_{a b} \nu_a R L_b \rho^2 + \gamma_{a b} \nu_a L L_b \rho^2 + \gamma_{a b} \nu_a R L_b \rho^2 + H.c., \]
\[L_{\text{H}}^\text{rel} = \frac{g^2}{8} (\chi_i L e^b \chi_j - \chi_i L e^b \chi_j + \rho^a L e^b \rho_j - \rho^a L e^b \rho_j)^2 \]
\[+ \frac{g'^2}{12} \left( -\frac{1}{3} \chi_i L e^b \chi_j + \frac{1}{3} \chi_i L e^b \chi_j + \frac{2}{3} \rho^a L e^b \rho_j - \frac{2}{3} \rho^a L e^b \rho_j \right)^2, \tag{31}\]

In the limit \( v', u', w', w' \ll w, w' \), the masses of the charged Higgs bosons get approximate values such as [11]: \( m_{\rho^+} \simeq m_W, m_{\rho^0} \simeq 0, m_{\chi_2} \simeq m_{\chi_3} = 0, m_{\rho^-} \simeq m_{\chi_3} = 0. \)
With the couplings given in (31), the right- and left-handed neutrino mass matrices are given by a sum of two one-loop diagrams, shown in Figs. 1 and 2 respectively:

\[
i(M_L)_{ab} P_L = \int \frac{d^4p}{(2\pi)^4} \left( i2\lambda'_{ac} P_L \right) \frac{i(\bar{\rho} + m_c)}{p^2 - m_c^2} \left( i\gamma'_{cd} \frac{v}{\sqrt{2}} P_R \right) \frac{i(\bar{\rho} + m_d)}{p^2 - m_d^2} \\
\times \left( i\gamma'_{bd} P_L \right) \frac{-1}{(p^2 - m_b^2)(p^2 - m_d^2)} \left( ig^2\nu' \right) \\
+ \int \frac{d^4p}{(2\pi)^4} \left( i\gamma_{ac}^* P_L \right) \frac{i(-\rho + m_c)}{p^2 - m_c^2} \left( i\gamma_{cd} \frac{v}{\sqrt{2}} P_R \right) \frac{i(-\rho + m_d)}{p^2 - m_d^2}
\]
as the same as in the non-supersymmetric economical 3-3-1 model [17]. This gives six
different values: two lights and four heavies. Let us consider the one-loop contribution
the dominant matrix is
\begin{equation}
\text{with } a, b \text{ are not summed.}
\end{equation}
Similarly, we have
\begin{equation}
(M_R)_{ab} = - (M_L)_{ab}.
\end{equation}
Because of \(m_{\rho_1^+} = m_W, m_{\rho_3^-} = 0\), we obtain
\begin{equation}
I(m^2, m_{\rho_3}^2, m_{\rho_3}^2) \simeq - \frac{i}{16\pi^2 m^2},
\end{equation}
\begin{equation}
I(m_a^2, m_{\rho_1^+}, m_{\rho_3}^2) \simeq - \frac{i}{16\pi^2 m_a^2 - m_{\rho_1^+}^2} \left( 1 - \frac{m_{\rho_1^+}^2}{m_a^2 - m_{\rho_1^-}^2} \ln \frac{m_a^2}{m_{\rho_1^-}^2} \right),
\end{equation}
With the functions given in Eq.(34), the one loop correction to the mass matrix \(M_L\) can be written as
\begin{equation}
(M_L)_{ab} \propto - (M_R)_{ab}
\end{equation}
\begin{equation}
\simeq (M_D^{\text{tree}})_{ab} \propto v
\end{equation}
Thus, the one-loop correction leads to the relationship \(M_L = - M_R\), which is similar
to the case of non-supersymmetric economical 3-3-1 model [17]. These mass matrices
are proportional to the value \(v\) but they are suppressed by an extra factor \(\frac{g^2}{16\pi^2}\). Hence, the
dominant matrix is \(M_D\), and it can be diagonalized by biunitary transformation
as the same as in the non-supersymmetric economical 3-3-1 model [17]. This gives six
different values: two lights and four heavies. Let us consider the one-loop contribution
to the Dirac neutrino masses. Applying the Feynman rules to the Fig. we obtain
correction to the mass matrix \(M_D\) of the form
\begin{equation}
- i(M_D^{\text{tree}})_{ab} P_L = \int \frac{d^4 p}{(2\pi)^4} (-i2\lambda'_{ac} P_L) \frac{i(p + m_e)}{p^2 - m_e^2} \left( i\gamma_{cd} \frac{v}{\sqrt{2}} P_R \right) \frac{i(p + m_d)}{p^2 - m_d^2} \frac{-1}{(p^2 - m_{\rho_1}^2)(p^2 - m_{\rho_2}^2)} \left( g^2 v v' \right)
\end{equation}
\begin{equation}
+ \int \frac{d^4 p}{(2\pi)^4} (i\lambda'_{bc} P_L) \frac{i(-p + m_e)}{p^2 - m_e^2} \left( i\gamma_{cd} \frac{v}{\sqrt{2}} P_R \right) \frac{i(-p + m_d)}{p^2 - m_d^2} \frac{-1}{(p^2 - m_{\rho_1}^2)(p^2 - m_{\rho_2}^2)} \left( g^2 v v' \right).
\end{equation}
We rewrite the above result as
\[
(M_{D}^{\text{rad}})_{ab} = \frac{g^2}{16\pi^2} \lambda_{ab}^\prime v \left[ 1 - \frac{m^2_a}{m^2_\rho} \left( 1 - \ln \frac{m^2_a}{m^2_\rho} \right) \right]
\]
\[
\propto v.
\] (37)

It is very interesting that the scale for one-loop correction to the Dirac masses is proportional to the expectation values v, the same as that of the tree level. However, unlike the case of the tree level, the mass matrix given in (37) is non-antisymmetric in \(a\) and \(b\). Hence, after including the one-loop correction to the Dirac neutrino mass, all three eigenvalues of the Dirac mass matrix are non-zero. On the other hand, the left and right handed neutrino mass matrices are gained at the one-loop correction. However, there is no larger hierarchy between \(M_L, M_R\) and \(M_D\). It is difficult to obtain the seesaw mechanism in this scenario. To solve this puzzle, as in the non-supersymmetric economical 3-3-1 model, it is necessary to introduce a new mass of the GUT scale \(\mathcal{M} \approx 10^{16}\) GeV [17].

Below we shall show that, in the model with an inflationary scenario, the type I seesaw mechanism can appear naturally.

4. The seesaw mechanism in supersymmetric economical 3-3-1 model with an inflationary scenario

We have constructed a hybrid inflationary scheme based on a realistic supersymmetric 3-3-1 model by adding a singlet superfield \(\hat{\Phi}\) which plays the role of the inflation, namely the inflaton superfield [18]. Let us recall that the inflationary potential is given by
\[
W_{\text{inf}}(\hat{\Phi}, \hat{\chi}, \hat{\chi}^\prime) = \alpha \hat{\Phi} \hat{\chi} \hat{\chi}^\prime - \mu^2 \hat{\Phi}.
\] (38)

The superpotential related to the neutrino masses is
\[
W_{\text{neut}} = \mu_{ab}^\prime \tilde{L}_a \tilde{\chi}^\prime \tilde{\phi}
\] (39)

Integrating out the superspace gives the relevant interaction Lagrangian for the one-loop correction to neutrino mass
\[
L_{\text{int}} = \mu_{ab}^\prime \nu_{aL} \tilde{\phi} \chi_1^0 + \mu_{ab}^\prime \nu_{aR} \tilde{\phi} \chi_3^0 + H.c.,
\] (40)

\[
V_{\text{Higgs}}^{\text{rel.}} = \alpha^2 (\chi \chi')^2
\] (41)

Besides the relevant Higgs self-coupling given in Eq. (41), there is another Higgs potential contributing to the neutrino mass at the one-loop correction, namely
\[
V_D = \frac{g^2}{12} \left( -\frac{1}{3} \chi^\dagger \chi + \frac{2}{3} \delta^\dagger \delta - \frac{2}{3} \rho^\dagger \rho \right)^2
\]
\[
+ \frac{g^2}{8} \left( \chi_i^{\dagger} \lambda_{ij} \lambda_{ji}^\dagger - \chi_i^{\dagger} \lambda_{ij}^* \lambda_{ji}^\dagger + \rho_i^{\dagger} \lambda_{ij} \rho_j - \rho_i^{\dagger} \lambda_{ij}^* \rho_j^\dagger \right)^2
\] (42)

with \(g', g\) are the gauge couplings of \(U(1), SU(3)_L\) groups, respectively. Because of this, the \(g'\) coupling constant is the co-variant function of energy and the \(g\) coupling constant is the contravariant function of energy. At the inflationary and preheating times, the \(g'\)
coupling constant is dominated and we will ignore the self-Higgs coupling in the second line of Eq. (12). On the other hand, requiring the nonadiabatic string contribution to the quadrupole to be less than 10%, the coupling $\alpha$ belongs to $10^{-4} \div 10^{-8}$ \[18\]. If we compare this value with that of $g'$ coupling constant at the early time of the universe, the values of $\alpha$ coupling is tiny enough to ignore the Higgs self-coupling given in Eq. (41). In short, at the inflationary and preheating times, the Lagrangian related to the one-loop correction to neutrino mass is given by

$$L_{int} = \mu'_0 \mu_x \tilde{\phi} \chi^0 + \mu'_a v \phi_x \tilde{\phi} \chi^0 + H.c.,$$

$$V_D^{(1)} = \frac{g^2}{12} \left( -\frac{1}{3} \chi^+ \chi + \frac{1}{3} \chi^\dagger \chi' + \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho^\dagger \rho \right)^2$$

At the one-loop order, there is no correction to the mass matrix $M_D$ but there is correction to the mass matrices $M_L$ and $M_R$ given in Figs. 4 and 5.

We assume that the vacuum expectation values $w, u, v$ are the same as $w', u', v'$, respectively. With this assumption, the contributions from diagrams 5 (c) and (d) are canceled by each other and similarly for diagrams 5(e) and (f). Hence, the total contribution to the neutrino mass matrix $M_L$ is obtained from diagrams 5(a) and (b) as follows: [see (A.4)]

$$-i M^{inf}_{Lab} P_L = \int \frac{d^4 p}{(2\pi)^4} (\nu'_{oa} P_L) \frac{i(\nu + m_{\tilde{\phi}})}{p^2} \left( -im_{\tilde{\phi}} \right) \frac{i(\nu + m_{\tilde{\phi}})}{p^2}$$

$$\times (\nu'_{oa} P_L) \frac{1}{(p^2 - m_{\chi^0}^2)^2} \left( \frac{u^2 g^2}{54} \right)$$

$$= 2m_{\tilde{\phi}} \frac{g^2}{54} \mu'_{oa} \mu'_{ob} u^2 P_L \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{(p^2 - m_{\chi^0}^2)^2(p^2 - m_{\tilde{\phi}}^2)^2}$$

$$+ 2m_{\tilde{\phi}} \frac{g^2}{54} \mu'_{oa} \mu'_{ob} u^2 P_L \int \frac{d^4 p}{(2\pi)^4} \frac{m_{\tilde{\phi}}^2}{(p^2 - m_{\chi^0}^2)^2(p^2 - m_{\tilde{\phi}}^2)^2}$$

$$= m_{\tilde{\phi}} \frac{g^2}{27} \mu'_{oa} \mu'_{ob} u^2 P_L \left[ I(m_{\tilde{\phi}}, m_{\chi^0}) + m_{\tilde{\phi}} I_1(m_{\tilde{\phi}}, m_{\chi^0}) \right]$$

(45)

Note that $\tilde{\phi}$ is a super partner of inflaton; hence their mass must be larger than those of inflaton. It means that $m_{\tilde{\phi}} \gg m_{\chi^0}$. If we take that the ratio of $m_{\chi^0}$ to $m_{\tilde{\phi}}$ is of the order $10^{-x}$, we obtain

$$I(m_{\tilde{\phi}}, m_{\chi^0}) \approx -\frac{i}{16\pi^2 m_{\tilde{\phi}}^2},$$

$$I_1(m_{\tilde{\phi}}, m_{\chi^0}) \approx -\frac{i}{16\pi^2 m_{\tilde{\phi}}^2} (2 - x \ln 10)$$

(46)

Substitution of Eq. (46) into Eq. (45) gives

$$M^{inf}_{Lab} \approx -\frac{i g^2}{16\pi^2} \frac{27}{27} \mu'_{oa} \mu'_{ob} u^2$$

(47)
Making similar steps to the mass matrix $M_R$, we obtain the result

$$M_{Rab}^{inf} \simeq -\frac{i}{16\pi^2}\frac{g^2}{2\hat{\mu}_0^w}\frac{w^2}{m_{\tilde{\phi}}}$$  \hspace{1cm} (48)

The neutrino masses are the eigenvalues of the matrix

$$\begin{pmatrix} M_{Lab}^{inf} & M_D^T \\ M_D & M_{Rab}^{inf} \end{pmatrix}$$  \hspace{1cm} (49)

Because of the condition $w', w \gg u', u, v, v'$ and $u', u \ll v', v$ [see Eq.(16)] and ($M_R \propto w^2, M_D \propto v^2, M_L \propto u^2$), we obtain a hierarchy in values of the elements of the neutrino mass:

$$M_{Rab}^{inf} \gg M_D \gg M_{Lab}^{inf}$$  \hspace{1cm} (50)
The heavy and light eigenvectors are found to be diagonalize the matrices:

\[ m_R = M_{Rab}^{inf}, \quad m_\nu = M_D M_{Rab}^{inf} M_D^T. \] (51)

Let us mention again that in the framework of the non-supersymmetric economical models as well as the supersymmetric version without inflationary scenario, to get successful neutrino masses, it is necessary to introduce a new mass of the GUT scale \( \mathcal{M} \approx 10^{16} \) GeV [17]. While in the supersymmetric model with an inflationary scenario, with the help of the interactions among the inflaton and right handed neutrinos [11], the above puzzle is solved. Thus the inflaton with mass around \( 10^{17} \) GeV plays the role of new physics in the economical models with the inflationary scenario.
5. Non-thermal leptogenesis via inflaton decay

Let us consider the non-thermal leptogenesis scenario in our model. In the non-thermal leptogenesis scenario, the right handed neutrinos are produced through the direct non-thermal decay of the inflaton. In our scenario, there is no interaction term which describes that decay process at the tree level. However, the necessary interaction arises at the one-loop level. The relevant self-Higgs and inflaton couplings is given by

\[ L_{\text{thermal}} = \left| \frac{\partial W_{\text{inf}}}{\partial \chi} \right|^2 + \left| \frac{\partial W_{\text{inf}}}{\partial \chi'} \right|^2 = \alpha^2 (|\chi|^2 + |\chi'|^2) \phi \]  

(52)

From the Lagrangian given in (43) and (52), the effective interaction relevant for the right handed neutrinos and inflaton at the one-loop correction is given in Fig. 6.

![Feynman diagram for the process $\phi \rightarrow \nu_R \nu_R$](image)

Figure 6. Feynman diagram for the process $\phi \rightarrow \nu_R \nu_R$

The effective Lagrangian for the process $\phi \rightarrow \nu_R \nu_R$ is given by

\[ L_{\nu_R \nu_R} = A_{eff} \phi \nu_R \nu_R + H.c \]  

(53)

where $A_{eff}$ stands for effective coupling, which is obtained as

\[ A_{eff} \propto 2m_\phi^2 \alpha^2 \mu_{0b}^2 \mu_{0a}^2 [I(m_{\phi}^2, m_{\chi}^2) + m_{\phi}^2 I_1(m_{\phi}^2, m_{\chi}^2)] \propto 54 \frac{M_R}{g^2 w^2} \alpha^2 \]  

(54)

The inflaton decay rate is given by

\[ \Gamma(\phi \rightarrow \nu_R \nu_R) \simeq \frac{|A_{eff}|^2}{4\pi} m_\phi \]  

(55)

with $m_\phi$ the inflaton mass. The produced reheating temperature is obtained by

\[ T_R = \left( \frac{45}{2\pi^2 g^*} \right)^{\frac{1}{4}} (\Gamma M_P)^{\frac{1}{2}} \]  

(56)

where $g^*$ is the effective degree of the freedom in the universe at $T \sim M_R$. In our model, the effective degree of the freedom is taken approximately 140 (for more details, see [19]).

We assume that the inflaton $\phi$ decays dominantly into a pair of the lightest heavy Majorana neutrino, $\phi \rightarrow \nu_{R1}, \nu_{R1}$, and other decay modes including these into pair
\(\nu_{R2}, \nu_{R3}\) are forbidden. The inflaton decays to lightest heavy neutrino and that neutrino decay to charged leptons and Higgs reheats the Universe, producing not only the lepton-number asymmetry but also entropy for thermal bath. The interference between the tree-level decay amplitude and the absorptive part of the one-loop diagram can lead to a lepton asymmetry of the right order of magnitude to explain the observed baryon asymmetry. The \(N_1\) decays immediately after being produced by the inflaton decays and hence we obtain lepton-to-entropy ratio [20]

\[
\frac{n_L}{s} \simeq \frac{3}{2} \epsilon \times B_r \times \frac{T_R}{m_\phi} 
\]

(57)

where \(B_r\) is the branching ratio of the inflaton decay into the \(N_1\) channel. The lepton asymmetry (in Eq.57) is converted to the baryon asymmetry through the “sphaleron” effects which is given by

\[
\frac{n_B}{s} = a \frac{n_L}{s} 
\]

(58)

with \(a = -\frac{8}{23}\) in the MSSM. The ratio of the lepton number to entropy density after preheating is estimated to be [20]

\[
\frac{n_B}{s} = -0.35 \times \frac{3}{2} B_r, (\phi \to \nu_{R1}, \nu_{R1}) \frac{T_R}{M_\phi} \times \epsilon. 
\]

(59)

The lepton asymmetry parameter \(\epsilon\) is produced by the interference between the tree level and one-loop level of the \(\nu_R \to l_L \rho\) or \(\nu_R \to l_L \rho'\) decay process. The thermal leptogenesis scenario, in detail, in the economical 3-3-1 model will be presented elsewhere [21]. The CP violating parameter [22] is given by

\[
\epsilon = \frac{1}{(8\pi \lambda' \lambda''_{11})_{11}} \sum_{j=2,3} Im \left[ (\lambda' \lambda''_{1j})_{1j} \left[ f(M_{Rj}^2/M_{R1}^2) + 2g(M_{Rj}^2/M_{R1}^2) \right] \right] 
\]

(60)

with \(f(x)\) and \(g(x)\) the vertex and the wave functions, respectively. In the limit \(x \gg 1\), the CP violating parameter \(\epsilon\) can be written as

\[
\epsilon = -\frac{3}{16\pi (\lambda' \lambda''_{11})_{11}} \sum_{j=2,3} Im \left[ (\lambda' \lambda''_{1j})_{1j} \frac{M_{R1}}{M_{Rj}} \right] 
\]

(61)

As mentioned in the last section, we have type I seesaw mechanism \(m_\nu = M_D M_R^{-1} M_D^T = \lambda' \lambda'' \rho^2\), hence the CP violating parameter can be written as

\[
\epsilon = -\frac{3}{16\pi} \frac{M_{R1}}{\langle \rho \rangle^2} \frac{Im[\lambda' \lambda''_{1j}]}{(\lambda' \lambda''_{11})_{11}} 
\]

\[
= -\frac{3}{16\pi} \frac{m_{\nu3} M_{R1} \delta_{eff}}{\langle \rho \rangle^2} 
\]

(62)

where the effective CP-violating phase \(\delta_{eff}\) is given by

\[
\delta_{eff} = \frac{Im \left[ \lambda_{13}^2 + \frac{m_{\nu3}}{m_{\nu3}} \lambda_{12}^2 + \frac{m_{\nu3}}{m_{\nu3}} \lambda_{11}^2 \right]}{\left| \lambda_{13} \right|^2 + \left| \lambda_{12} \right|^2 + \left| \lambda_{11} \right|^2} 
\]

(63)

Numerically, taking \(\langle \rho \rangle = v \simeq v_{\text{electroweak}} = 246\) GeV, we obtain

\[
\epsilon \simeq -2 \times 10^{-6} \left( \frac{M_{R1}}{10^{10} \text{ GeV}} \right) \left( \frac{m_{\nu3}}{0.05 \text{ eV}} \right) \delta_{eff} 
\]

(64)
As considered in section 4, there is no loop correction to the Dirac mass matrix $M_D$; the effective coupling $\lambda'_{11} = 0$ is the same as the coupling at the tree level. Assuming the coupling $\lambda'_{12} = |\lambda|e^{i\delta_{12}}$, $\lambda'_{13} = |\lambda|e^{i\delta_{13}}$, we get the effective CP-violating phase

$$\delta_{\text{eff}} = \frac{\sin \delta_{13} + \frac{m_{\nu_3}}{m_{\nu_3}} \sin \delta_{12}}{2}. \quad (65)$$

As far as we know, the neutrino oscillation data is given in [23] as follows:

$$\Delta_{12}^2 = 7.59 \times 10^{-5} \text{eV}^2, \Delta_{13}^2 = 2.43 \times 10^{-3} \text{eV}^2 \quad (66)$$

Assuming that the neutrino mass spectrum has a normal hierarchy,

$$M_{\nu} = \text{Diag} \left( m_0, \sqrt{m_0^2 + \Delta_{12}^2}, \sqrt{m_0^2 + \Delta_{13}^2} \right), \quad (67)$$

leads to the product of the maximal CP asymmetry and the heaviest light neutrino mass, which is presented in Fig. 7.

![Figure 7](image-url)

**Figure 7.** The product of $m_{\nu_3}[\text{eV}]$ and $\delta_{\text{eff}}$ is a function of $m_0$ by taking the maximal CP violating phases ($\sin \delta_{13} = \sin \delta_{12} = 1$).

On the other hand, the ratio of the lepton number to entropy density after preheating can be written as

$$\frac{n_B}{s} \simeq 10^{-10} B_r(\phi \to \nu_{R1}\nu_{R1}) \left( \frac{T_R}{10^6 \text{ GeV}} \right) \left( \frac{M_{R1}}{M_{\phi}} \right) \left( \frac{\delta_{\text{eff}} m_{\nu_3}}{0.05 \text{ eV}} \right). \quad (68)$$

The cosmological constraint on the gravitino abundance gives a bound on the reheating temperature [24]: $T_R < 10^7 \text{ GeV}$. Assuming that the reheating temperature is $T_R = 10^6 \text{ GeV}$ and combining with the observed baryon number to entropy ratio, we get a constraint on the heaviest light neutrino as

$$m_{\nu_3} > 0.01 \text{ eV}. \quad (69)$$

Taking the maximal CP violating phases from Fig. 7 we can roughly estimate the value $\delta_{\text{eff}} m_{\nu_3} = 0.05$. Hence, in order to satisfy the observed value of the baryon asymmetry [24]

$$Y_B = \frac{n_B}{s} = 0.87 \times 10^{-10}, \quad (70)$$
the ratio $\frac{M_{R1}}{M_φ}$ must satisfy
\[ \frac{M_{R1}}{M_φ} = 0.87. \] (71)

If we combine the cosmological constraint on the gravitino abundance ($T_R < 10^7$ GeV) with Eqs. (55) and (54), we obtain the constraint on the effective coupling
\[ A_{eff} < \frac{10^{-\frac{7}{2}}}{M_φ^2}. \] (72)

From Eq.(54) and Eq.(72), the constraint on the inflaton mass is given by
\[ M_φ^3 \leq 2 \times \frac{10^{-\frac{7}{2}}}{\alpha^2} \times g^2 w^2. \] (73)

Taking into account $g' w \propto 10^3$ GeV, we get the constraint on the inflaton mass:
\[ M_φ^3 \leq 2 \times \frac{10^2}{\alpha^2}. \] (74)

Note that the constraint on the coupling $\alpha$ has been given in [18], namely the value of coupling $\alpha$ should be smaller than $10^{-4}$. The inflaton mass is roughly estimated in Table 1.

**Table 1.** Coupling constant $\alpha$ and inflaton mass

| $\alpha$   | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ | $10^{-7}$ | $10^{-8}$ | $10^{-9}$ | $10^{-10}$ |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|------------|
| $M_φ^{max}$ [GeV] | $2 \times 10^7$ | $4 \times 10^8$ | $9.2 \times 10^9$ | $2 \times 10^{11}$ | $4 \times 10^{12}$ | $9.2 \times 10^{13}$ | $2 \times 10^{15}$ |

Table 1 shows that the constraint on the coupling $\alpha$ as ($\alpha \in [10^{-4}, 10^{-10}]$) leads to the inflaton mass around $M_φ \in [10^7, 10^{15}]$ GeV. These values not only produce the observed value of the baryon asymmetry but also are suitable to the hybrid inflationary scenario given in [18].

In short, non-thermal leptogenesis scenario via inflaton decay to the pair of right handed neutrinos is forbidden at the tree level. However, this process is available at the one-loop level. By taking the reheating temperature $T_R = 10^6$ GeV, we can solve the gravitino problem. Due to $δ_{eff} < 1$, the heaviest light neutrinos mass satisfies both the cosmological constraints and the oscillation data $m_{νβ} \simeq \frac{0.05}{δ_{eff}}$ eV. We have obtained the constraint on the lightest heavy right-handed neutrino: its mass is smaller than those of inflaton, namely $\frac{M_{R1}}{M_φ} = 0.87$. It is worth noting that the cosmological constraint on the gravitino abundance gives a bound on the Higgs-self couplings and inflation mass, which naturally fit to our inflation scenario.
6. Summary and conclusions

In this paper, non-thermal leptogenesis in which the heavy Majorana neutrinos are produced through inflaton decays in the supersymmetric economical 3-3-1 model with the inflationary scenario has been considered.

We have shown that the problem in the supersymmetric economical 3-3-1 model (without the inflationary scenario) is the same as in the non-supersymmetric version: neutrino masses are unrealistic: there is no larger hierarchy between $M_L, M_R$ and $M_D$. It is difficult to obtain the seesaw mechanism in this scenario.

Fortunately, in the model with inflationary scenario, the lepton-number-violating interactions among the inflaton and right-handed neutrinos appear at the one-loop level. Thus, it not only gives a solution for the above puzzle but also gives a chance for studying non-thermal leptogenesis scenario.

Our analysis has shown that the leptogenesis works without overproduction of gravitinos if reheating temperature $T_R = 10^6$ GeV and the lightest heavy right-handed neutrino mass satisfies $M_{R1} = \frac{M_0}{0.87}$. This result satisfies also the cosmological constraint $m_{\nu3} \simeq \frac{0.05}{\delta_{eff}}$ eV with $\delta_{eff} < 1$.

One of the criteria for the inflationary scenario, beside providing the predictions in good agreement with observations of the microwave background and large scale structure formation, is an explanation of the origin of the observed baryon asymmetry. For this aim, we note that the model under consideration contains the lepton-number-violating interactions among the inflation and the right-handed neutrinos at one-loop level, and this is a reason for the successful leptogenesis scenario considered in this work.

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Appendix A. Feynman integration

In this Appendix, we present evaluation of the integral.

\[ I_1(a,b,c) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)}, \]  \hspace{1cm} (A.1)

\[ I(a,b,c) \equiv \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{(p^2 - a)(p^2 - b)(p^2 - c)}, \]  \hspace{1cm} (A.2)
where \(a, b, c > 0\) and \(I(a, b, c) = I(a, c, b)\) should be noted in use. 

\[
I_1(a, b, c) = \frac{-i}{16\pi^2} \left\{ \frac{a \ln a}{(a - b)(a - c)} + \frac{b \ln b}{(b - a)(b - c)} + \frac{c \ln c}{(c - b)(c - a)} \right\} \quad \text{(A.3)}
\]

\[
I(a, b, c) = \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)} + \frac{a}{(p^2 - a)^2(p^2 - b)(p^2 - c)} \right]
= \frac{-i}{16\pi^2} \left\{ \frac{a(2\ln a + 1)}{(a - b)(a - c)} - \frac{a^2(2a - b - c)}{(a - b)^2(a - c)^2} \right. \\
+ \left. \left\{ \frac{b^2 \ln b}{(b - a)^2(b - c)} + \frac{c^2 \ln c}{(c - a)^2(c - b)} \right\} \right\}.
\]

If \(a, b \gg c\) or \(c \simeq 0\), we have an approximation as follows

\[
I(a, b, c) \simeq -\frac{i}{16\pi^2} \frac{1}{a - b} \left[ 1 - \frac{b}{a - b} \ln \frac{a}{b} \right]. \quad \text{(A.4)}
\]

In the other case with \(b = c\) and \(b \neq a\), we have also

\[
I(a, b) \equiv I(a, b, b) = -\frac{i}{16\pi^2} \left[ \frac{a + b}{(a - b)^2} - \frac{2ab}{(a - b)^3} \ln \frac{a}{b} \right], \quad \text{(A.5)}
\]

where, also, \(I(a, b) = I(b, a)\) should be noted in use.

If \(b \gg a\) or \(a \simeq 0\), we have the following approximation

\[
I(a, b) \simeq -\frac{i}{16\pi^2} b. \quad \text{(A.6)}
\]

Let us note that the above approximations \(aI(a, b, c)\) (or \(bI(a, b, c)\)) and \(bI(a, b)\) are kept in the orders up to \(\mathcal{O}(c/a, c/b)\) and \(\mathcal{O}(a/b)\), respectively.

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