THERMODYNAMICS OF DECAYING VACUUM COSMOLOGIES

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Abstract

The thermodynamic behavior of vacuum decaying cosmologies is investigated within a manifestly covariant formulation. Such a process corresponds to a continuous irreversible energy flow from the vacuum component to the created matter constituents. It is shown that if the specific entropy per particle remains constant during the process, the equilibrium relations are preserved. In particular, if the vacuum decays into photons, the energy density $\rho$ and average number density of photons $n$ scale with the temperature as $\rho \sim T^4$ and $n \sim T^3$. The temperature law is determined and a generalized Planckian type form of the spectrum, which is preserved in the course of the evolution, is also proposed. Some consequences of these results for decaying vacuum FRW type cosmologies as well as for models with “adiabatic” photon creation are discussed.

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1 Introduction

Cosmological applications of vacuum decay have been rather investigated in the literature, mainly in connection with inflationary universe scenarios[1,4]. More recently, motivated by the so-called “cosmological constant problem” as well as by the “age problem” of the standard Friedmann-Robertson-Walker(FRW) model(for reviews of such problems see Refs.[5-7]), many authors have also proposed phenomenological models with a slowly decaying vacuum energy density[8-20]. Roughly speaking, the basic difference between these two kind of models comes from the fact that in the former, the vacuum decays completely in a very short period in the very early universe(phase transition), whereas in the latter, it decays continuously(slowly) in the course of the cosmic evolution. In the second class of models, the attempts to invent a mechanisms rendering the cosmological constant almost exactly or exactly vanishing are replaced by the opposite and somewhat more natural idea that the vacuum energy density is a dynamic variable. It is assumed that the effective Λ-term behaves like a fluid interacting with the other matter fields of the universe (as in a multifluid model). As a consequence, the vacuum energy density is not constant since the energy momentum tensor of the mixture must be conserved in the course of the expansion. In such Λ-variable models, the slow decay of the vacuum energy density may also provide the source term for matter and radiation, thereby suggesting a natural solution for the aforementioned puzzles. Firstly, the explanation accounting for the present smallness of the effective cosmological constant may be deceptively simple: the cosmological constant is very small today because the universe is too old[9]. Secondly, although small in comparison with the usual micro-physics scales, the “remnant” cosmological constant may provide a good fit to the age of the universe[15, 18](see also Ref.[16] for other kinematical tests).

In what follows, although the results presented here may be interesting for the first class of models, we are more interested in the macroscopic approach for continuous vacuum decay(variable Λ) models. To the best of our
knowledge, the temperature evolution for the created matter at the expense of the vacuum component, has not been computed from first principles. In particular, for the case of radiation, the lack of a well defined temperature law as well as the related spectrum implies that the constraints coming from the measurements of the cosmic microwave background radiation (CMBR) cannot be studied without additional hypotheses. As we know, the isotropy of the CMBR and the Planckian form of its spectrum may be a crucial test for this kind of cosmologies. For instance, when the distortions of the Planck spectrum are discussed in the model proposed by Freese et al. [11], it is explicitly assumed that the vacuum does not decay into photons fully equilibrated to a Planck spectrum since in this case there are no distortions at all. On the other hand, in their nucleosynthesis analysis, the created photons are supposed to be quickly (and continuously) thermalized, with the total radiation energy density always satisfying the equilibrium relation $\rho_r \sim T^4$ during the radiation phase. Indeed, they had to make this assumption, in order to be able to determine how the radiation number density, the temperature and other physical quantities change with time [22]. This kind of assumption was further extensively adopted (see, for instance, Refs. [13, 15, 21]).

In this article, we focus our attention on the thermodynamic aspects of decaying vacuum models. As we shall see, if the vacuum is regarded as a second fluid component transferring energy continuously to the material component, the second law of thermodynamics constrains the whole process in such a way that the temperature law may be easily determined. In particular, we will establish under which conditions the equilibrium relations are preserved. These constraints leads us to introduce the idea of an “adiabatic” vacuum decay which, due to its simplicity, seems to be the most relevant process from a physical point of view. The related spectral distribution is derived and some consequences of this approach to FRW decaying vacuum models are also discussed.
2 Thermodynamics and Vacuum Decay

Let us consider a self-gravitating fluid satisfying the Einstein field equations (EFE) with a variable $\Lambda$-term:

\[ G^\alpha_\beta = \chi T^\alpha_\beta + \Lambda \gamma^\alpha_\beta , \]  

(1)

where $\chi = 8\pi G$ and the $\Lambda$-term is the vacuum energy momentum tensor (EMT), corresponding to an energy density $\rho_v = \frac{\Lambda}{8\pi G}$ and pressure $p_v = -\rho_v$ (in our units $c=1$). $T^\alpha_\beta$ is the EMT of the material component which is defined by

\[ T^\alpha_\beta = (\rho + p)u^\alpha u^\beta - pg^\alpha_\beta , \]  

(2)

where $\rho$ is the fluid energy density and $p$ is the pressure.

Since we are assuming a continuous energy transfer from the vacuum to the material component, the effective cosmological constant is a time-dependent parameter. In this way, the energy conservation law ($u_\alpha T^\alpha_\beta ;_\beta = 0$), which is contained in the EFE, assumes the following form:

\[ \dot{\rho} + (\rho + p)\theta = -\frac{\dot{\Lambda}}{8\pi G} , \]  

(3)

where the overdot denotes covariant derivative along the world lines (for instance, $\dot{\rho} := u^\alpha \rho_\alpha$) and $\theta = u^\alpha_\alpha$ is the scalar of expansion. For a FRW geometry, for instance, $\dot{\rho}$ is the time comoving derivative and $\theta = 3H$, where $H$ is the Hubble parameter.

As we know, in order to have a complete fluid description, besides its EMT, it is necessary to define the particle current $N^\alpha$ and the entropy current $S^\alpha$ in terms of the fluid variables. The current $N^\alpha$ is given by

\[ N^\alpha = nu^\alpha , \]  

(4)

where $n$ is the particle number density of the fluid component. Since material constituents are continuously generated by the decaying vacuum, the above
four-vector satisfies a balance equation $N^\alpha_{\dot{\alpha}} = \psi$ or equivalently

$$\dot{n} + n\theta = \psi \quad ,$$

where $\psi$ is the particle source ($\psi > 0$) or sink ($\psi < 0$) term. For vacuum decaying models $\psi$ is positive, and must be related in a very definite way with the variation rate of $\Lambda$. Since the EMT of both components are isotropic, without loss of generality, we may define the entropy current in the form below

$$S^\alpha = n\sigma u^\alpha \quad ,$$

where $\sigma$ is the specific entropy (per particle). If the $\Lambda$-term is constant, the above entropy current is conserved. This is a consequence of the fact that in our approach we are neglecting the usual dissipative processes arising in relativistic simple fluids as well as a possible irreversible matter creation process at the expense of the gravitational field (see Refs. [23-26]). Accordingly, the existence of a nonequilibrium decay process means that

$$S^\alpha_{\dot{\alpha}} \geq 0 \quad ,$$

as required by the second law of thermodynamics. Some words are necessary to clarify the meaning of the above condition. In principle, one might argue that the second law should be applied for the system as a whole, that is, including the vacuum component. However, assuming that the chemical potential of the vacuum is identically zero, it follows from the vacuum equation of state that $\sigma_v = 0$ and so also its entropy current is zero (see Eq. (10) below). In other words, the vacuum plays the role of a condensate carrying no entropy, as happens in the two-fluid description usually employed in superfluid dynamics [27].

Before discussing the temperature evolution law, we need to obtain an expression relating $\dot{\Lambda}$ and $\psi$. As usual for nonequilibrium processes, such an expression must be defined in such a way that the entropy source is
nonnegative. To do that, we first remark that the equilibrium variables are related by Gibbs’ law:

\[ nT d\sigma = \frac{\rho + p}{n} dn \]

where \( T \) is the temperature. Hence, taking the time comoving derivative of the above expression and using (3) and (5), it is readily obtained that

\[ S^\alpha_{,\alpha} := n\dot{\sigma} + \sigma \psi = -\frac{\dot{\Lambda}}{8\pi GT} - \frac{\mu \psi}{T} \]

where \( \mu \) denotes the chemical potential of the created matter, which is defined by the usual Euler’s relation:

\[ \mu = \frac{\rho + p}{n} - T\sigma \]

It should be noted that when \( \psi = 0 \), we expect a vanishing time variation of \( \Lambda \) and so also of the entropy production. We recall that the vacuum decaying is the unique source of irreversibility (particle creation) considered in the present treatment. Such a condition can be expressed by the following phenomenological ansatz

\[ \frac{\dot{\Lambda}}{8\pi G} = -\beta \psi \]

where \( \beta \) is a positive definite parameter, in order to guarantee that for \( \psi > 0 \) we shall have \( \dot{\Lambda} < 0 \). With this choice, (9) can be rewritten as

\[ S^\alpha_{,\alpha} := n\dot{\sigma} + \sigma \psi = \frac{\psi}{T}(\beta - \mu) \]

Hence, the entropy production rate will be nonnegative in accordance with (7) only when the phenomenological coefficient \( \beta \) satisfies either \( \beta \geq \mu \) if \( \psi > 0 \), or \( \beta \leq \mu \) if \( \psi < 0 \). As first remarked by Salim and Waga\[18\], in the case of photons(\( \mu = 0 \)), we see from (7) and (9) that only a cosmological constant decreasing with time is thermodynamically allowed. As a self-consistency
check, we notice that the same result is derived from our phenomenological ansatz (11), together with (12) and the second law of thermodynamics. Keeping these considerations in mind, we discuss next the temperature law for a continuously decaying vacuum.

### 3 Temperature Evolution Law

The time dependence of the temperature may be easily established from Eqs.(3), (5) and (8), by adopting $T$ and $n$ as basic thermodynamic variables. Inserting $\rho(T, n)$ into (3) and using (5), it follows that

$$
\left(\frac{\partial \rho}{\partial T}\right)_n \frac{\dot{T}}{T} = \left[ n \left(\frac{\partial \rho}{\partial n}\right)_T - \rho - p \right] \theta - \left(\frac{\partial \rho}{\partial n}\right)_T \psi - \frac{\dot{\Lambda}}{8\pi G} .
$$

(13)

Now, since $d\sigma$ is an exact differential, Gibbs’ law (8) yields the well known thermodynamic identity

$$
T \left(\frac{\partial p}{\partial T}\right)_n = \rho + p - n \left(\frac{\partial \rho}{\partial n}\right)_T ,
$$

(14)

and inserting (14) into (13) we obtain

$$
\frac{\dot{T}}{T} = - \left(\frac{\partial p}{\partial \rho}\right)_n \theta - \left(\frac{\partial \rho}{\partial n}\right)_T \psi - \frac{\dot{\Lambda}}{8\pi GT} \left(\frac{\partial \rho}{\partial T}\right)_n .
$$

(15)

The first term on the right-hand side (RHS) of (15) is the usual equilibrium contribution. In this case, we see that for an expanding fluid, $\theta > 0$ leads to $\dot{T} < 0$ as it should be. The remaining terms display the out of equilibrium contributions due to the particle creation rate $\psi$ and its source $\dot{\Lambda}$, which are related by equation (11). Note that Eq.(15) is a pure consequence of the relativistic nonequilibrium first order thermodynamics e.g., the EFE does not played any special role in its derivation. Besides, since many different processes may take place simultaneously, other contributions like, bulk viscosity, gravitational matter creation, heat flow, diffusion and all possible
cross effects should be taken into account. In any case, the method applied here may be easily extended by including the corresponding terms into the basic thermodynamic quantities. In what follows, I will discuss in detail an interesting particular case, which due to its simplicity and possible physical applications (see section 5) deserves a special attention.

4 “Adiabatic” Case

Let us first discuss under which conditions the equilibrium relations for the particle number and energy density are preserved in the presence of a decaying vacuum. Inserting the value of \( \theta \) obtained from (5) and the value of \( \Lambda \) as given by the phenomenological law (11) into (15), it follows that

\[
\frac{\dot{T}}{T} = (\frac{\partial p}{\partial \rho})_n \frac{\dot{n}}{n} - \frac{1}{nT(\frac{\partial \rho}{\partial T})_n} \left[ T(\frac{\partial p}{\partial T})_n + n(\frac{\partial \rho}{\partial n})_T - n\beta \right] \psi .
\]

(16)

The first term on the RHS of the above equation still resembles a typical equilibrium term, however, there are also out of equilibrium contributions encoded in it. To be more precise, suppose that the second term on RHS of (16) is absent and that the fluid satisfies the usual “\( \gamma \)-law” equation of state

\[
p = (\gamma - 1)\rho,
\]

(17)

where the “adiabatic index” \( \gamma \) lies in the interval \([0, 2]\). In this case, a straightforward integration of (16) furnishes \( n^{(1-\gamma)T} = \text{const.} \), and for \( \gamma \neq 1 \)

\[
n = \text{const} \times T^{\frac{1}{\gamma-1}}.
\]

(18)

which has the same form for \( n(T) \) as perfect adiabatic simple fluid. However, the number density of particles no longer satisfies the usual conservation law (see Eq.(5)). It thus follows, that the equilibrium relations will be preserved only if the second term on the RHS of (16) is identically zero. In this case, us-
ing (14) we see that the phenomenological parameter $\beta$ assumes a remarkably simple form

$$\beta = \frac{\rho + p}{n}.$$  \hspace{1cm} (19)

The next step is to show that the value of $\beta$ deduced above also guarantees the equilibrium relation for the energy density. In fact, by combining (3), (11), (17) and (19) we readily obtain

$$\dot{\rho} + \gamma \rho \theta = \gamma \rho \frac{\psi}{n}.$$  \hspace{1cm} (20)

Therefore, comparing (20) with (5) it follows that

$$\frac{\dot{\rho}}{\rho} = \gamma \frac{\dot{n}}{n},$$  \hspace{1cm} (21)

the solution of which is $\rho = \text{const} \times n^\gamma$, or using (18)

$$\rho = \eta T^{\frac{\gamma}{\gamma - 1}},$$  \hspace{1cm} (22)

where $\eta$ is a $\gamma$-dependent integration constant. The above expression is the equilibrium energy density-temperature relation for a $\gamma$-fluid. In particular, for a photon fluid ($\gamma = 4/3$), one obtains from (18) and (22), respectively, $n \sim T^3$ and $\rho \sim T^4$, just the well known relations valid for blackbody radiation. Therefore, the condition expressed by (19) preserve the usual equilibrium relations and so it should have a rather simple physical interpretation.

In order to clarify the physical meaning of relation (19) we return to the entropy production expression given by (12). Inserting the value of $\beta$ given above into (12), it is easy to see that the variation rate of the specific entropy may be written as

$$\dot{\sigma} = \frac{\psi}{nT} \left( \frac{\rho + p}{n} - \mu - T\sigma \right),$$  \hspace{1cm} (23)

and from Euler’s relation (10) we see that $\dot{\sigma} = 0$. Therefore, the equilibrium relations are preserved only if the specific entropy per particle of the created
particles is constant. In other words, when the specific entropy remains constant, or equivalently, $\beta$ is given by (19), no finite thermalization time is required since the particles originated from the decaying vacuum are created in equilibrium with the already existing ones. Naturally, the process as a whole is out of equilibrium. In fact, since $\sigma = \frac{s}{n}$, the condition $\dot{\sigma} = 0$ leads, with the help of (5), to a balance equation for the entropy density (see also the discussion below Eq.(29))

$$\dot{s} + s\theta = \frac{s\psi}{n},$$

(24)

which for $\psi = 0$ reduces to the usual “continuity” equation for an adiabatic flow.

5 "Adiabatic" Decaying Vacuum and FRW Type Cosmologies

Let us now consider the FRW line element:

$$ds^2 = dt^2 - R^2(t)(\frac{dr^2}{1 - kr^2} + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2) ,$$

(25)

where $R$ is the scale factor and $k = 0, \pm 1$ is the curvature parameter.

In such a background the Einstein field equations (EFE) for the nonvacuum component plus a cosmological $\Lambda$-term are:

$$8\pi G\rho + \Lambda = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} ,$$

(26)

$$8\pi Gp - \Lambda = -2\frac{\dot{R}}{R} - \frac{\ddot{R}^2}{R^2} - \frac{k}{R^2} ,$$

(27)

where $\rho$ and $p$, as usual, are assumed to obey the $\gamma$-law equation of state (17).

As we have seen, in the “adiabatic” case, the temperature $T$ satisfies (18) and the energy density is given by (22) regardless of the microscopic details.
of the vacuum decay. Besides, for a FRW geometry, the comoving volume scales as $V \sim R^3$ and, up to a constant factor, $N = nR^3$. Thus, using (18) we may write the following temperature evolution law:

$$N^{1-\gamma}TR^{3(\gamma-1)} = \text{constant} .$$  \hspace{1cm} (28)

As expected, if $N$ is conserved (no vacuum decay), the usual equilibrium law is recovered. It should be noticed that the above temperature law has a rather general character. It can be applied regardless of the specific creation mechanism operating in the FRW geometry. As a matter of fact, it depends only on the validity of the “adiabaticity” condition (19), thereby implying that the equilibrium relation (18) is preserved. For instance, the above law is the same as the one deduced in Ref.[25] for an “adiabatic” particle creation at the expenses of the gravitational field. In particular for photon creation($\gamma = 4/3$), equation (28) reduces to

$$N^{-\frac{4}{3}}TR = \text{const} . ,$$  \hspace{1cm} (29)

instead of the usual $TR = \text{const}$ of the standard FRW model.

In the homogeneous case, $\sigma = S/N$, where $S$ and $N$ are the total entropy and number of particles, and since we are considering the “adiabatic” case, it follows that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} .$$  \hspace{1cm} (30)

Hence, the burst of entropy is closely related with the created matter due to the decaying vacuum.

It is worth mentioning that, in comparison with the standard model, the macroscopic formulation discussed here has only one additional free parameter, namely the variation rate of the $\Lambda$-term, or equivalently from (11), the particle creation rate $\psi$. In fact, in the “adiabatic” case, the phenomenological $\beta$ parameter is completely determined by condition (19). In principle,
either \( \dot{\Lambda} \) or \( \psi \) must be computed from a more fundamental model for the decaying vacuum. Of course, at the level of a definite equation of motion, this is equivalent to assuming a priori a functional expression for \( \Lambda(t) \) itself, as has been usually done in the literature (see Refs. [10-20]). Note also that any “adiabatic” decaying vacuum FRW type cosmology has its temperature law determined by (29). However, how \( T \) scales with \( R \) depends, naturally, on the specific decay rate of \( \Lambda \), since it will determine from (11) and (5) the specific \( N(R) \) function. Such a function will also define through (30) the amount of entropy produced. In this way, both the cooling rate and entropy generation in decaying vacuum cosmologies are highly model dependent functions, as should be expected.

Another interesting question is closely related to the spectrum of the CMBR. As we know, in the standard model, both the equilibrium relations and the Planckian form of the spectrum are preserved in the course of the expansion. The latter result follows naturally from the fact that \( \nu \sim R^{-1} \) (kinematical condition for FRW geometry) and \( T \sim R^{-1} \). Our results show that when “adiabatic” photon creation takes place the equilibrium relations are preserved, while \( T \) necessarily follows a more general temperature law given by (29). In this case, one may be tempted to conclude that all models with photon creation fail the crucial test provided by the present isotropy and spectral distribution of the CMBR (see, for instance, Steigman [28] and Refs. quoted therein). However, such a conclusion is not so neat as it appears at first sight. For instance, suppose that in the “adiabatic” case, the spectrum assumes the following form (a derivation is outlined in the appendix)

\[
\rho_T(\nu) = \left( \frac{N(t)}{N_o} \right)^\frac{4}{3} \frac{8\pi\hbar}{c^3} \frac{\nu^3}{\exp\left[ \frac{(N(t)}{N_o} \right] \frac{\hbar\nu}{kT} - 1},
\]

where \( N(t) \) is the comoving time dependent number of photons and \( N_o \) is the constant value of \( N \) evaluated at some fixed epoch, say, the present time. When there is no creation, \( N(t) = N_o \), and the usual Planckian form is
recovered. Since the frequency scales as $\nu \sim R^{-1}$, as a consequence of the temperature law (29), the exponential factor in (31) is clearly preserved in the course of the expansion. In addition, it is readily seen that the equilibrium relations are recovered using such a spectrum. In fact, from (21) follows that $n \sim \rho^{\frac{3}{4}}$, and by introducing a new variable $x = (\frac{N}{N_o})^{\frac{1}{3}} \frac{b\nu}{kT}$, it is easy to see that

$$\rho(T) = \int_0^\infty \rho_T(\nu) d\nu = aT^4,$$

(32)

where $a$ is the usual radiation density constant. In this way, the spectrum given by (31) seems to be the most natural generalization of Planck's radiation formula in the presence of “adiabatic” photon production. Since it cannot, on experimental grounds, be distinguished from the usual blackbody spectrum with no matter creation, models with “adiabatic” photon creation may be compatible with the present isotropy and spectral distribution of the microwave background. This conclusion is extremely general and may be applied even for Dirac type cosmologies (the case for G-variable cosmologies will be discussed in detail elsewhere). It is worth mentioning that (31) is quite different from the form originally proposed by Canuto and Narlikar[30] to circumvent the criticism of Steigman[29] (see also the paper of Narlikar and Rana[31]). The main difference comes from the fact that the temperature evolution law (29) has now been incorporated in the exponential factor of the above spectrum. Although the usual Planckian spectrum cannot be distinguished at present from (31), this does not mean that the same happens for high redshifts. For instance, one may check that the wavelength $\lambda_m$ for which $\rho_T(\lambda)$ assumes its maximum value now satisfies the following displacement law

$$\lambda_m T = 0.289(\frac{N(t)}{N_o})^{\frac{1}{3}} cm. K,$$

(33)

which reduces to the usual Wien’s law for $N = N_o$. Hence, since in the past, $N(t) < N_o$, for a given redshift $z$, the typical energy of photons whose
spectral distribution is given by (31), will be smaller than that described by
the usual Planckian spectrum. More precisely, since the scale factor as a
function of the redshift is given by $R = R_o(1 + z)^{-1}$, we see from (29) that

$$T = T_o(1 + z)\left(\frac{N(t)}{N_o}\right)^\frac{4}{3}, \quad (34)$$

where $T_o$ is the present day value of $T$. This relation has some interesting
physical consequences. Firstly, we observe that universes with “adiabatic”
photon creation are, for any value of $z > 0$, cooler than the standard model.
Such a prediction may be experimentally verified, for instance, observing
atomic or molecular transitions in absorbing clouds at high redshifts. In this
way, it provides a crucial test for models endowed with “adiabatic” photon
production, which is accessible with the present day technology. In this
connection, Songaila et al. [32], reported recently the detection of the first
fine-structure of neutral carbon atoms in the $z = 1.776$ absorption-line sys-
tem. Assuming that no other significant sources of excitation are present,
the relative population of the level yielded a temperature of $T = 7.4 \pm 0.8K$
while using the standard relation it should be $7.58K$. Although in accordance
with this prediction, it is too early to interpret such measurements as a new
successful test of the standard model. As remarked by Mayer [33], it is very
difficult to pin down the amount of local excitation in the observed clouds
using independent observations. In this way, this result must strictly be con-
sidered as an upper bound for the temperature of the universe in the above
mentioned redshift. In principle, improved observational techniques as well as
some reasonable estimates of the possible sources of excitations, may lead to
a smaller value of the temperature, in conflict with the standard prediction.
In this case, as happens with the cosmological constant and age problems,
a decaying vacuum cosmology may become an interesting possibility to fit
the data. In the future, temperature-redshift measurements of sufficient ac-
ccuracy may constrain the free parameters of any specific decaying $\Lambda$ model,
or more generally, any kind of cosmology endowed with “adiabatic” creation
of photons. Note also that (34) give us a simple qualitative explanation of
why models with decaying vacuum may solve the cosmological age problem which plagues the class of FRW models. In fact, since for a given redshift \( z \) the universe is cooler than the standard model, more time is required to attain a fixed temperature scale in the early universe. Some quantitative examples are given in Refs.[13-17].

As is well known, there is no derivation specifying either how fast the vacuum decays nor how it couples with matter and/or radiation. As discussed earlier, this is equivalent to determining the function \( N(t) \). In the present case, regardless of the form of such a function, it seems that the radiation must be produced through an induced decay mechanism, because the energy is always injected “adiabatically”, that is, fully equilibrated with the generalized Planck spectrum. Such a possibility was noted but not discussed by Freese et al.[11]. On the other hand, if one assumes that the vacuum couples only with the radiation, this means that baryons (and antibaryons) are not produced in the course of the evolution. Therefore, like in the standard FRW model, the number density of the nonrelativistic component is conserved, that is, \( n_b \) scales with \( R^{-3} \), and from (34) we may write

\[
\sigma_{rb} = \sigma_{ro} \frac{N(t)}{N_0},
\]

where, \( \sigma_{rb} = \frac{4aT_r^3}{3n_b} \) is the radiation specific entropy (per baryon) and \( \sigma_{ro} \) its present value. The above expression means that the photon-baryon ratio increases as the universe expands, however, in a rate which is strongly model dependent. Since the function \( N(t) \) has not been determined from first principles, the constraint from nucleosynthesis code cannot be seen as a definitive answer, at least while we do not know how to select the best phenomenological description for the decaying vacuum. In this way, it is not surprising that models based on different decay rates and/or initial conditions, lead to somewhat opposite conclusions about such constraints (compare, for instance, Refs.[11][14]).
6 Conclusion

The thermodynamic behavior of variable $\Lambda$-models has been investigated in the framework of a first-order relativistic theory for irreversible processes. The main results derived here may be summarized in the following statements:

1) Vacuum decaying models with photon creation may be compatible with the constraints of the cosmic background radiation only when the creation occurs under “adiabatic” conditions e.g., when the equilibrium relations are preserved. This is equivalent to say that the entropy per photon remains constant during the creation process (see section 4).

2) If the vacuum decays “adiabatically” in particles obeying the equation $p = (\gamma - 1)\rho$, the temperature law is given by $N^{1-\gamma}T^3V^{\gamma-1} = \text{const.}$, where $N(t)$ is the instantaneous number of particles and $V$ the comoving volume. In the case of radiation ($\gamma = 4/3$), it reduces to $N^{-1}T^3V = \text{const.}$, and for a FRW geometry, $V \sim R^3$, we have $N^{-\frac{4}{3}}TR = \text{const.}$ (see Eq.(29)).

3) The usual Planckian spectrum has been generalized to include “adiabatic” photon creation (see Eq.(31) and appendix). Such a form is uniquely determined by the radiation temperature law. In this way, when the “adiabatic” condition has been implemented, it can be applied regardless of the specific creation mechanism operating in the spacetime. For instance, it holds for models endowed with “adiabatic” creation at the expense of the gravitational field as discussed in Refs.[23-26]. The new spectrum is preserved in the course of the evolution and clearly consistent with the present isotropy of the CMBR. Instead of the matter creation process, the slight observed anisotropy must (like in the standard model) be associated with the usual physical processes taking place during the matter dominated phase.

4. The measurement of the universe temperature at high redshifts is a crucial test for models endowed with “adiabatic” creation, in particular, for vacuum decaying cosmologies. For a given redshift $z$, the temperature is smaller than that one predicted by the standard model (see Eq.(34)).
Finally, we remark that the results presented here may be generalized by allowing additional contributions for the EMT (different creation mechanisms). However, if the “adiabatic” condition is imposed both the temperature law and the form of the spectrum will not be modified. Specific models will be studied in a forthcoming communication.
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A “Adiabatic” Blackbody Spectrum

In this appendix, a formula for blackbody radiation when the photon creation process takes place “adiabatically”, is derived. As we have seen, in this case the temperature law is given by (see our Eq.(29))

$$N^{-\frac{1}{3}}TR = \text{const.} \quad .$$

Since the wavelength $\lambda$ scales with $R$, the above equation means that if one compress or expand a hollow cavity containing blackbody radiation, in such a way that photons are “adiabatically” introduced in it, we may write for each wave component

$$N^{-\frac{1}{3}}\lambda T = \text{const.} \quad .$$

The above quantity plays the role of a generalized “adiabatic” invariant in the sense of Ehrenfest[34]. When $N$ is constant the usual adiabatic invariant for expanding blackbody radiation is recovered.

Let $T_1$ be the temperature in the instant $t = t_1$, and focus our attention on the band $\Delta \lambda_1$ centered on the wavelength $\lambda_1$ whose energy density is $\rho_{T_1}(\lambda_1)\Delta \lambda_1$. In a subsequent time $t = t_2$, when the temperature $T_1$ changed
to \(T_2\), due to an “adiabatic” expansion, the energy of the band changed to 
\(\rho_{T_2}(\lambda_2)\Delta\lambda_2\) and according to (36) \(\Delta\lambda_1\) and \(\Delta\lambda_2\) are related by

\[
\frac{\Delta\lambda_2}{\Delta\lambda_1} = \left(\frac{N(t_2)}{N(t_1)}\right)^{\frac{1}{3}} \frac{T_1}{T_2}.
\]

(38)

As shown earlier in section 4, the thermodynamic equilibrium relations are preserved and since distinct bands do not interact, it follows that

\[
\frac{\rho_{T_2}(\lambda_2)\Delta\lambda_2}{\rho_{T_1}(\lambda_1)\Delta\lambda_1} = \left(\frac{T_2}{T_1}\right)^4.
\]

(39)

By combining the above result with (36) and using again (35), we obtain for an arbitrary component

\[
\rho_T(\lambda)^5 = \text{const} N^{\frac{a}{3}}.
\]

(40)

In the Planckian case, \(N = N_o = \text{const.}\), the above expression reduces to \(\rho_T(\lambda)^5 = \text{const}\), as it should be. Without loss of generality, taking into account (36), the above result may be rewritten as (we have normalized \(N\) by its value \(N_o\) without photon creation)

\[
\rho_T(\lambda) = \left(\frac{N}{N_o}\right)^{\frac{a}{3}} \lambda^{-5} \phi^*\left(\left(\frac{N}{N_o}\right)^{-\frac{1}{3}} \lambda T\right).
\]

(41)

where \(\phi^*\) is an arbitrary function of its argument. In terms of frequency, since \(\rho_T(\nu)\,d\nu = \rho_T(\lambda) \left|\frac{d\nu}{d\lambda}\right| \,d\lambda\), it follows that

\[
\rho_T(\nu) = \left(\frac{N}{N_o}\right)^{\frac{a}{3}} \nu^3 \phi^*\left(\left(\frac{N}{N_o}\right)^{-\frac{1}{3}} \frac{T}{\nu}\right).
\]

(42)

where \(\phi\) is proportional to \(\phi^*\). The above equation is a generalized form of the well known Wien’s law. In order to recover the usual Planckian distribution the arbitrary function must be \(\phi = \frac{8\pi h}{c^3} \exp\left[\frac{1}{\nu}\frac{\nu^3}{kT}\right]^{-1}\), with (41) taking the form assumed in (31): namely,

\[
\rho_T(\nu) = \left(\frac{N(t)}{N_o}\right)^{\frac{a}{3}} \frac{8\pi h}{c^3} \exp\left[\frac{1}{\nu}\frac{\nu^3}{kT}\right]^{-1} - 1.
\]

(43)
Note that no reference has been made to the specific source of photons. The above sketched derivation depends only on the temperature law as given by (36), or equivalently, that creation occurs preserving the equilibrium relations (“adiabatic” creation).

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