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Distribution of plastic anisotropy in thickness direction for plate

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Abstract

A small-cube compression test is proposed to measure local plastic anisotropy for bulk material. The small-cubic specimen with a side length of 1 mm was cut out of a thick plate of pure aluminum. It was compressed in specific directions under a well-lubricated condition. The initial square cross-section was transformed into a rectangular when the plastic-anisotropic metal was tested. For the sheet material, the Lankford-value of small-cube compression test is nearly equal to that of a conventional tensile test. The anisotropic coefficients, namely $F$, $G$, $H$, and $N$, of Hill's quadratic yield criterion (1948) are calculated from Lankford-values which are measure by small-cube compression test. Also anisotropic coefficients $L$ and $M$ are calculated to measure planar anisotropy on the planes which normal to rolling- and width direction. It is found that plastic anisotropy is distributed in thickness direction for thick plate of pure aluminum. And the distribution is influence on the deformation of thick plate upsetting. Then, this deformation is verified by FE analysis considering anisotropy.

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1. Introduction

Recently, plates have been increasingly used for bulk forming [1]. The plate has uniform thickness due to rolling process. And this uniformity of the thickness is advantageous to precision forging. However, the thick plate also

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has plastic anisotropy like in sheet forming. Probably, plastic anisotropy in the thickness direction will be important for plate forging. So, consideration of plastic anisotropy like sheet forming is required for more precise FE-analysis of plate forging. For sheet forming, plastic anisotropy is measured for Lankford-values [2] by using the conventional tension test. However, it is difficult to measure plastic anisotropy in the thickness direction and local plastic anisotropy because of limitation of the specimens size. In this report, a small-cube compression test [3] is carried out for aluminum plate with 1 mm and 15 mm in thickness. Local plastic anisotropy is measured for this 15-mm-thick plate. Upsetting tests are carried out and are analyzed by FEM considering local plastic anisotropy.

| Nomenclature               |
|---------------------------|
| x, y, z                   | rolling-, transverse-, thickness-direction |
| X, Y, Z                   | normal plane to the x-, y-, and z-direction |
| t                         | position from plate surface |
| r₀                          | Lankford-value, ε₀/ε₀ |
| X₀, Y₀, Z₀                | Lankford-value on X-, Y-, Z-plane |
| γ, β, α                   | rotated angle from y-, z-, and x-direction |
| F, G, H, L, M, N          | anisotropic coefficient of Hill's quadratic yield criterion |
| C, C₀                     | yield stress, yield stress in z-direction |

2. Small-cube compression test

2.1. Suitability of small-cube compression test

The Lankford-value, r₀, is useful in expressing plastic anisotropy in a rolled sheet. The value is naturally measured by the tensile test. We have proposed a small-cube compression test to measure a strain ratio which is close to the Lankford-value, for a bar and a tube material [4]. Fig. 1 shows the loading directions and orientation of specimens for the tensile test and the small-cube compression test. A cubic specimen has 1.0 mm on a side. The cubes are cut out by using a wire electric discharge machining. The maximum surface roughness is about 10 μmRz. For the compression test, the die-set with flat dies is used. The dies move in parallel by rigid guide bars. The material of dies is a cold working tool steel. The surface is finished 0.3 μmRz. Beef-tallow is used for the lubricant. Compression speed is set at 0.1 mm/s.

Fig. 2 shows the values of r₀ for the conventional tensile test and for the small-cube compression test. The material is rolled sheet of Al-Mg-Si alloy. The thickness is 1.0 mm. The r₀ of the small-cube compression test correspond to those of the conventional tensile test when the reduction in height is 15% and 30%. Thus the r₀-values can be obtained by the small-cube compression test like a conventional tensile test. However the r₀-value of the small-cube compression at 50% reduction in height is a little larger than that of the tensile test. It maybe influence of friction between the dies and specimen [5]. So, it is note that the small-cube compression test need to be carried out under a well-lubricated condition and light compressive reduction.

2.2. Hill’s quadratic yield criterion and Lankford-value on x-, y-, and z-plane

In a rolled sheet, the principal axes of plastic anisotropy coincide with x-, y-, and z-directions, which mean the rolling-, transverse-, and thickness-directions, respectively. Hill’s quadratic yield criterion was proposed in 1948 [6]. The criterion is simple to generalize the von-Mises criterion of the plastic anisotropy materials. It has the form:

\[
2f(\sigma_{ij}) = F(\sigma_y - \sigma_x)^2 + G(\sigma_x - \sigma_y)^2 + H(\sigma_x - \sigma_z)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 2C^2, \tag{2}
\]

where \( F, G, H, L, M, N \) are coefficients that characterize plastic anisotropy. And \( C \) is a yield stress in a reference direction. When \( C \) is selected a yield stress in z-direction, \( C_z \),

\[
F + G = 2. \tag{3}
\]
In sheet metal forming, the coefficients $F$, $G$, $H$, and $N$ are generally determined by Lankford-values. Referring to Lankford-values, we define the $X_r$-, $Y_r$-, and $Z_a$-values which are Lankford-values on the $X$-, $Y$-, and $Z$-plane. Here the $X$-, $Y$-, and $Z$-planes are normal plane to the $x$-, $y$-, and $z$-axis, respectively.

\[
X_r = \left[ F + (2L - G - H - 4F) \sin^2 \gamma \cos^2 \gamma \right] / \left( G \sin^2 \gamma + H \cos^2 \gamma \right), \quad (4a)
\]

\[
Y_r = \left[ G + (2M - H - F - 4G) \sin^2 \beta \cos^2 \beta \right] / \left( H \sin^2 \beta + F \cos^2 \beta \right), \quad (4b)
\]

\[
Z_a = \left[ H + (2N - F - G - 4H) \sin^2 \alpha \cos^2 \alpha \right] / \left( F \sin^2 \alpha + G \cos^2 \alpha \right) = r_a, \quad (4c)
\]

where $\gamma$, $\beta$, and $\alpha$ are rotated angle from $y$-, $z$-, and $x$-directions, respectively. By using these values Hill’s coefficients are determined as follows:

\[
F = X_0H = H/Z_0, \quad (5a)
\]

\[
G = Y_0H = H/Z_0, \quad (5b)
\]

\[
L = (0.5 + X_{45})(1/X_0 + 1/X_{90})F = (0.5 + X_{45})(1 + 1/Z_0)H, \quad (5c)
\]

\[
M = (0.5 + Y_{45})(1/Y_0 + 1/Y_{90})F = (0.5 + Y_{45})(1 + 1/Z_0)H, \quad (5d)
\]

\[
N = (0.5 + Z_{45})(1/Z_0 + 1/Z_{90})H. \quad (5e)
\]

Then, the coefficient $H$ is calculated by equations (3) and (5a, b).

\[
H = 2Z_0/Z_0. \quad (6)
\]

Therefore, Hill’s coefficients are determined by a yield stress of $C_z$ and five values of $Z_0$, $Z_{45}$, $Z_{90}$, $X_{45}$, and $Y_{45}$.

![Fig. 1. Loading directions for tensile test and small-cube compression test.](image)

![Fig. 2. Comparison between conventional tensile test and small-cube compression test.](image)

### 2.3. Local anisotropy for plate

Distribution of plastic anisotropy in the thickness-, $z$-, direction is measured by the small-cube compression test. The material is pure aluminum as rolled plate. The thickness is 15 mm. Fig. 3 shows the cutting positions on the $Y$-plane. The cubes cut out by three positions in the thickness direction which are $t = 0.5$ mm (Surface), 3.75 mm (Middle layer), and 7.5 mm (Center layer) from the surface. The texture of the specimen is rolled texture. The grain is elongated in the rolling-direction. The average grain size is measured by using EBSD and OIM-Analysis 7. The grain size of high-angle boundary, which misorientation is more than 15 degrees, is $6 \mu$m at the surface, $5 \mu$m at the middle layer, and $8 \mu$m at the center layer.

Fig. 4 shows the typical examples of the compression surface to be compressed in the $y$-direction at the position of (a) surface, (b) middle layer, and (c) center layer for A1050 pure aluminum. The two-dot chain line means the initial outline of the cube. At the position (a), (b) and (c), the elongation in the $z$-direction is larger than that in the $x$-direction. But at the position (c), the elongation in the $z$-direction is not so larger comparison with positions (a) and (b). The strain ratio $Z_{90} = \varepsilon_z/\varepsilon_x = 1/X_0$ shows 0.17 at (a), 0.19 at (b), and 0.80 at (c).
Fig. 5 shows (001) pole figure at the position of (a) surface, (b) middle layer, and (c) center layer. And Fig. 6 shows IPF (Inverse Pole Figure) Map on Y-plane. At the positions (a) and (b), the orientation of <001> is mainly orientated to the z-direction and the orientation of <011> is mainly orientated to the x- and y-direction. But at the position (c), the orientation of <111> is mainly orientated to the y-direction and the orientation of <011> is mainly orientated to the z-direction. So the result of small-cube compression test corresponds to the crystal orientation. A further discussion would require a CPFEM (Crystal Plasticity Finite Element Method) analysis.

Fig. 7 shows planar anisotropy on Z-plane. Planer anisotropy at the center layer is stronger than that at the surface and the middle layer. Also, planar anisotropy on X- and Y-planes are also measured. Normal anisotropy on X- and Y-plane are stronger than that on Z-plane.

Table 1 shows the Hill’s coefficients for A1050 thick plate. These coefficients are calculated by X- , Y- , and Z-values using equations (3), (5a-e), and (6). At the surface and the middle layer, F and G indicate the almost same value. And H is smaller than F and G. At the center layer, F is smaller than the other positions. Plastic anisotropy is distributed in the thickness direction in the thick plate.
Table 1. Hill’s coefficients for A1050 plate.

|                | $F$ | $G$ | $H$ | $L$ | $M$ | $N$ | Flow stress |
|----------------|-----|-----|-----|-----|-----|-----|-------------|
| Surface ($t = 0.5$ mm) | 1.6 | 1.7 | 0.27 | 2.7 | 3.6 | 4.6 | $C_z = 135e^{0.15}$ |
| Middle layer ($t = 3.75$ mm) | 1.9 | 1.6 | 0.37 | 2.7 | 3.0 | 4.8 | $C_z = 134e^{0.16}$ |
| Center layer ($t = 7.5$ mm) | 0.66 | 1.5 | 0.53 | 3.5 | 2.7 | 5.5 | $C_z = 140e^{0.20}$ |
| Isotropic metal | 1.0 | 1.0 | 1.0 | 3.0 | 3.0 | 3.0 |             |

3. Influence of plastic anisotropy on sheet bulk forming

3.1. Cylinder and cube compression test

A cylinder having 10 mm in diameter and 15 mm in height is cut out of the thick pure aluminum plate, A1050, which thickness is 15 mm. Fig. 8 (a) shows the result of the cylinder compression test. The shape of compression surface changes into ellipse which has a longer axis in the $x$-direction. The strain ratio indicates $\varepsilon_x / \varepsilon_y = 0.39 / 0.30 = 1.3$ at the surface. So, this metal has plastic anisotropy. And the side shape is not uniform. At the center layer, the strain ratio $\varepsilon_x / \varepsilon_y = 0.39 / 0.23 = 1.7$. The distribution of plastic anisotropy in the thickness direction may influence the side shape.

A cube with 15-mm-long edges also cut out of same A1050 plate. And the cubes are compressed in the $z$- and $y$-directions. Fig. 8 (b) and (c) are the examples of cube compression test. The strain ratio indicates $\varepsilon_x / \varepsilon_y = 1.3$. It is the same strain ratio as the one in the cylinder compression test.

3.2. FE-analysis considering plastic anisotropy

The FE-analysis considering local plastic anisotropy is carried out. DEFORM-3D was used as a FE-code. This code can consider Hill’s quadratic yield criterion. Three different layers, which are applied different Hill’s anisotropic coefficients and strain hardening rules as shown Table 1, are used for the simulation. The frictional coefficient between the die and the specimen is set to 0.05. Fig. 9 shows the results of the FE-analysis. The shape after compression is well-predicted by considering the distribution of plastic anisotropy in the thickness direction.

(a) Cylinder compressed in $z$-direction (b) Cube compressed in $z$-direction (c) Cube compressed in $y$-direction

Fig. 8. Examples of cylinder compression test and cube compression test.
4. Conclusions

The distribution of the plastic anisotropy in the thickness direction for the pure aluminum thick plate was measured by using small-cube compression test. We would like to sum up the results as follows:

1. The small-cube compression test was useful to measure plastic anisotropy like the conventional tension test. But friction between the die and the specimen should be reduced.
2. The small-cube compression test is also useful to measure local anisotropy. The experimented thick plate ($t = 15$ mm) showed strong plastic anisotropy. And plastic anisotropy was distributed in the thickness direction.
3. The compressed cylinder and cube indicated non-uniform and unique deformation in the thickness direction. This non-uniform deformation was precisely simulated by using FE-analysis considering local plastic anisotropy.

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