Constraints on rainbow gravity functions from black-hole thermodynamics

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Abstract – In this paper, we investigate the thermodynamic properties of black holes in the framework of rainbow gravity. By considering rainbow functions in the metric of Schwarzschild and Reissner-Nordström black holes, remnant and critical masses are found to exist. Demanding the universality of logarithmic corrections to the semi-classical area law for the entropy leads to constraining the form of the rainbow functions. The mass output and the radiation rate for these constrained form of rainbow functions have been computed for different values of the rainbow parameter \(\eta\) and have striking similarity to those derived from the generalized uncertainty principle.

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Introduction. – There is a strong indication of an observer independent minimum length scale in all theories of quantum gravity, e.g., in string theory \(^{[1]}\), noncommutative geometry \(^{[2]}\), loop quantum gravity \(^{[3,4]}\) and Lorentzian dynamical triangulations \(^{[5-7]}\) to name a few. This minimum measurable length scale is assumed to be the Planck scale. The mathematical apparatus of general theory of relativity is based on a smooth manifold. This picture breaks down when spacetime is probed at energies of the order of Planck energy \(^{[8,9]}\). One therefore expects a radically new picture of spacetime. This includes departing from the standard relativistic dispersion relation implying the breaking of Lorentz invariance. Lorentz symmetry which is well known to be one of the most remarkable symmetries in Nature fixes the standard form of energy-momentum dispersion relation to be \(E^2 - p^2 = m^2\). It has been suggested in most theories of quantum gravity \(^{[10-14]}\) that in the ultraviolet limit, the standard energy-momentum dispersion relation gets modified. For example, in Horava-Lifshitz gravity, the energy-momentum dispersion relation gets modified in the ultraviolet region \(^{[15,16]}\). It should be noted, however, that while this is true for gravitons and scalar field, for the rest of the fields such a modification would require non-natural fine-tuning to avoid strong Lorentz violation. Hence one is essentially forced to couple matter to gravity in the usual way \(^{[17]}\). One way out of this, suggested in \(^{[18]}\), leads to the possibility of nontrivial coupling. Due to the presence of a maximum energy scale \(E_p\) leading eventually to the breaking of Lorentz invariance, theories have been constructed with two universal invariants, namely, the velocity of light \(c\) and the Planck energy \(E_p\). This leads to modified energy-momentum dispersion relations (MDR). This generalization of special relativity is called double special relativity (DSR) \(^{[19-21]}\).

In curved spacetime it is possible to make generalization of DSR leading to double general relativity \(^{[22]}\). In these theories the geometry of spacetime depends on the energy of the particle used to probe it. Therefore, the geometry here is represented by a one parameter family of energy-dependent metrics forming a rainbow of metrics. It is for this reason that the name rainbow gravity originates. The effects of rainbow gravity have led to considerable changes in the thermodynamic properties of black
holes [23–25]. However, the entropy expression does not contain the logarithmic corrections (which are well known to be universal) to the semi-classical area law for all values of the parameter \( n \) appearing in the rainbow functions.

In this paper we start by studying the modification of the thermodynamic properties, namely, the Hawking temperature, heat capacity and entropy for the rainbow-gravity–inspired Schwarzschild and Reissner-Nordström (RN) black-holes. The existence of black-hole remnants has been considered seriously in recent times [26–32] despite some of the challenges it faces in the context of information loss paradox [29]. For example, noncommutative inspired black holes have led to a departure from the conventional scenario of complete evaporation of the black holes [23–25]. However, the entropy expression does not motivate any specific value for \( n \).

In the subsequent discussion, we shall use natural units \( c = 1 = h \) and \( k_B = 1 \).

Rainbow-gravity–inspired Schwarzschild black holes. – In this section we want to study the thermodynamic properties of a Schwarzschild black hole taking into account the effect of the rainbow functions (3). The metric of the Schwarzschild black hole inspired by rainbow gravity is given by [21]

\[
\frac{\text{d}s^2}{\text{d}t^2} = -\frac{1}{f^2(E/E_p)} \left( 1 - \frac{2MG}{r} \right) \text{d}t^2 + \frac{1}{g^2(E/E_p)} \left( 1 - \frac{2MG}{r} \right)^{-1} \text{d}r^2 + \frac{r^2}{g^2(E/E_p)} \text{d}\Omega^2.
\]

The surface gravity is related with the Hawking temperature as \( T = \frac{\kappa}{2\pi} \) [38], where the surface gravity is defined by the relation \( \kappa = \lim_{r \to r_s} \sqrt{-g^{rr}g^{tt}(g_{tt}r^2)^2} \). Hence, the surface gravity for the rainbow-gravity–inspired Schwarzschild black hole reads \( \kappa = \frac{g(E/E_p)}{f(E/E_p) 4MG} \). Therefore, the Hawking temperature is given by

\[
T = \frac{1}{8\pi G} \sqrt{\frac{1}{M^2 - \frac{\eta}{(2GE_p)^n} M^{n+2}}}. \tag{5}
\]

In obtaining the above expression we have set \( E = 1 \). This equation gives a relation between the temperature and mass of the rainbow-gravity–inspired Schwarzschild black hole. Since the temperature has to be a real quantity, we obtain the following condition:

\[
\frac{1}{M^2} - \frac{\eta}{(2GE_p)^n} M^{n+2} \geq 0. \tag{6}
\]

The above condition readily leads to the existence of a critical mass below which the temperature becomes a complex quantity,

\[
M_{cr} = \frac{\eta^{\frac{1}{n}}}{2GE_p} = \eta^{\frac{1}{n}} M_p. \tag{7}
\]
The heat capacity of the rainbow-gravity-inspired Schwarzschild black hole reads

\[ C = \frac{dM}{dT} = 16\pi G \frac{(\eta + n)(2GE)^n}{(2GE)^n M^{n+2} + 2 M^2} \quad (8) \]

The remnant mass (where the black hole stops evaporating) can be obtained by setting \( C = 0 \) and is found to be identical to the critical mass (7). Thus, we have demonstrated that the remnant mass of the black hole is equal to its critical mass.

The entropy can be calculated using the heat capacity of this black hole as follows:

\[ S = \int C \frac{dT}{T} = -\int \frac{dM}{T} \quad (9) \]

Substituting eq. (5) in eq. (9) and assuming that \( n \geq 3 \) leads to

\[ S = 8\pi G \int \frac{dM}{\sqrt{\frac{1}{M^2} - \frac{(\eta + n)(2GE)^n}{M^{n+2}}}} + 4\pi GM^2 F_1 \left( \frac{1}{2}, \frac{2}{n}, \frac{1}{2} \right) \frac{\eta}{(2GE)^n M^n} \quad (10) \]

where \( F_1 \) is the Gauss hypergeometric function. It is quite remarkable to see that there exists an analytic solution for the entropy. Carrying out an expansion of this result for a small argument \( \eta \) yields

\[ S = 8\pi G \left[ \frac{M^2}{2} + \frac{\eta}{2(2-n)(2GE)^n} M^{n-2} + \frac{3\eta^2}{8(2-2n)(2GE)^{2n} M^{2n-2}} + \frac{5\eta^3}{16(2-3n)(2GE)^{3n} M^{3n-2}} \right] S_{BH} + \frac{\pi^2 \eta}{(2-n)} S_{BH}^{(2-n)} + \frac{3\pi^2 \eta^2}{8(1-n)} S_{BH}^{(n-1)} + \frac{5\pi^2 \eta^3}{8(2-3n)} S_{BH}^{(2-n)} \quad (11) \]

where \( S_{BH} = \frac{\pi M^2}{M_p^2} \) is the semi-classical Bekenstein-Hawking entropy for the Schwarzschild black hole. The reason for assuming \( n \geq 3 \) is that the result of the integration is not valid for \( n = 1, 2 \) as can be easily seen from the integrand. In terms of the area of the horizon \( A = 4\pi M^2 = 4\pi^2 \eta S_{BH} \), the above expression for the entropy can be recast in the form

\[ S = \frac{A}{4} + \frac{\pi^2 \eta}{(2-n)} \left( \frac{A}{4} \right)^{(2-n)} + \frac{3\pi^2 \eta^2}{8(1-n)} \left( \frac{A}{4} \right)^{(n-1)} + \frac{5\pi^2 \eta^3}{8(2-3n)} \left( \frac{A}{4} \right)^{(2-n)} \quad (12) \]

where we have set \( l_p = 1 \). It is evident from this expression for the entropy of the black hole that there are no logarithmic corrections to the semi-classical result for the values of \( n \geq 3 \). However, if one takes into account the universality of the logarithmic corrections then the values of \( n \) get restricted to \( n = 1, 2 \).

For \( n = 1 \), the entropy takes the exact analytic form

\[ S = \frac{\pi}{2} \left[ \frac{8 \frac{M^2}{M_p^2} + 2\eta \frac{M}{M_p} - 3\eta^2}{\sqrt{1 - \frac{M}{M_p} \frac{M}{2M_p}}} + 3\eta^2 \ln \left( \frac{2\sqrt{M}}{M_p} \right) \right. \]

\[ + \left. 3\eta^2 \ln \left( 1 + \sqrt{1 - \frac{M}{2M_p}} \right) \right] \quad (13) \]

Expanding the above expression keeping terms up to \( O(\eta^2) \) yields

\[ S = S_{BH} + \eta \sqrt{\pi} \sqrt{S_{BH}} + \frac{3\pi \eta^2}{8} \ln (S_{BH}) + \frac{3\pi \eta^2}{8} \ln 4\pi \]

\[ = \frac{A}{4} + \eta \sqrt{\pi} \sqrt{A} + \frac{3\pi \eta^2}{8} \ln \left( \frac{A}{4} \right) + \frac{3\pi \eta^2}{8} \ln (4\pi) \quad (14) \]

The above expression involves the logarithmic corrections which are known to be universal. Remarkably, the expression for the entropy has a striking similarity with the expression for entropy obtained using the generalized uncertainty principle with a linear term in the momentum uncertainty [28],

\[ S_{GUP} = \frac{A}{4l_p^2} + \frac{\alpha}{2\sqrt{\pi}} \sqrt{\frac{A}{4l_p^2}} + \frac{\alpha^2 - 2\beta^2}{16\pi} - \frac{\beta^2}{16\pi} \ln \left( \frac{A}{4l_p^2} \right) \]

\[ - \frac{\beta^2}{16\pi} \ln (16\pi) + \frac{\alpha \beta}{32\sqrt{\pi} A} \quad (15) \]

where the parameters \( \alpha \) and \( \beta \) are those appearing in the GUP with a linear term [28]. The equivalence is evident from the presence of the \( \sqrt{A} \) term in the entropy expression. For \( n = 2 \), the entropy expression becomes

\[ S = \pi \left[ \frac{4M^2}{M_p^2} \sqrt{1 - \frac{M^2}{4M_p^2}} + \eta \ln \left( \frac{4M^2}{M_p^2} \right) \right. \]

\[ + \left. \eta \ln \left( 1 + \sqrt{1 - \frac{M^2}{4M_p^2}} \right) \right] \quad (16) \]

Expanding the above expression keeping terms up to \( O(\eta^2) \) yields

\[ S = \frac{A}{4} + \frac{\eta \pi}{2} \ln \left( \frac{A}{4} \right) + \frac{\eta \pi}{2} \ln (4\pi) - \frac{3\pi^2 \eta^2}{8} \left( \frac{1}{4} \right) \quad (17) \]

This expression is similar in structure with the one obtained from the simplest generalized uncertainty principle (without any linear term in momentum uncertainty) which is the \( \alpha = 0 \) limit of eq. (15) [26,27].
Energy output as a function of time. If the temperature of the black hole is greater than the ambient temperature, then it must radiate energy in terms of photons and other ordinary particles. As a consequence of this, the mass of the black hole reduces further while its temperature keeps on increasing. If one assumes that the energy loss is dominated by photons, then the Stefan-Boltzmann law can be employed to estimate the radiated energy as a function of time. For the standard case (for $\eta = 0$) [26], we have

$$\frac{d}{dt} \left( \frac{M}{M_0} \right) = -\frac{\sigma}{256\pi^4G^2M_0^2} \left( \frac{M_0}{M} \right)^2,$$  \quad (18)

where $\sigma$ is the Stefan-Boltzmann constant. Setting $x = \frac{M}{M_0}$ and defining the characteristic time $t_{ch} = \frac{1}{16\pi^3\sigma G^2M_0^2}$, the above equation takes the form $\frac{dx}{dt} = -\frac{3t}{t_x}$.

Equation (18) implies that the black hole evaporates completely in time $t_x = \frac{1}{3} \left( \frac{M}{M_0} \right)^3$ and the rate at which energy is radiated blows up at the end of the process.

We now proceed to carry out the above analysis with the rainbow-gravity–inspired Schwarzschild black hole. In the subsequent discussion, we shall restrict the values of $n$ in the rainbow functions to be 1, 2.

For $n = 1$, the differential equation expressing the rate at which energy is radiated takes the form

$$\frac{dM}{dt} = -\sigma A T^4 = -\frac{\sigma}{256\pi^4G^2M_0^2} \left( \frac{M_0}{M} \right)^2 \left[ 1 - \eta \left( \frac{M_0}{M} \right) \right]^2,$$  \quad (21)

where we have used eq. (5), $E = \frac{1}{2GM}$ and $A = \frac{16\pi G^2M^2}{ct}$ to write down the second equality of the above equation. This leads to

$$\frac{d}{dt} \left( \frac{M}{M_0} \right) = -\frac{\sigma}{256\pi^4G^2M_0^2} \left( \frac{M_0}{M} \right)^2 \left[ 1 - \eta \left( \frac{M_0}{M} \right) \right]^2.$$  \quad (22)

In terms of $x = \frac{M}{M_0}$, the above equation can be recast as

$$\frac{dx}{dt} = -\frac{1}{t_{ch}x^2} \left( 1 - \frac{\eta}{x} \right)^2.$$  \quad (23)

Solving this, we obtain the mass-time relation up to $O(\eta)$ to be

$$x = \left[ \frac{3t}{t_{ch}} + x_i^3 + 3\eta x_i^2 - 3\eta \left( x_i^3 - \frac{3t}{t_{ch}} \right)^{\frac{2}{3}} \right]^{\frac{3}{2}}.$$  \quad (24)

Hence the rate at which energy is radiated as a function of time is given by

$$\frac{dx}{dt} = -\frac{1}{t_{ch} \left[ \frac{3t}{t_{ch}} + x_i^3 + 3\eta x_i^2 - 3\eta \left( x_i^3 - \frac{3t}{t_{ch}} \right)^{\frac{2}{3}} \right]^\frac{3}{2}} \times \left[ 1 - \frac{\eta}{x} \right]^{\frac{2}{3}}.$$  \quad (25)

The time in which the black hole evaporates completely is given by (up to $O(\eta)$)

$$\frac{t}{t_{ch}} = \frac{x_i^3}{3} + \eta x_i^2.$$  \quad (26)

For $n = 2$, the differential equation expressing the rate at which energy is radiated takes the form (in terms of $x$)

$$\frac{dx}{dt} = -\frac{1}{t_{ch}x^2} \left( 1 - \frac{\eta}{x^2} \right)^2.$$  \quad (27)

Solving this, we obtain the mass-time relation up to $O(\eta)$ to be

$$x = \left[ \frac{3t}{t_{ch}} + x_i^3 + 6\eta x_i - 6\eta \left( x_i^3 - \frac{3t}{t_{ch}} \right)^{\frac{1}{3}} \right]^{\frac{1}{2}}.$$  \quad (28)

Hence the rate at which energy is radiated as a function of time is given by

$$\frac{dx}{dt} = -\frac{1}{t_{ch} \left[ \frac{3t}{t_{ch}} + x_i^3 + 6\eta x_i - 6\eta \left( x_i^3 - \frac{3t}{t_{ch}} \right)^{\frac{1}{3}} \right]^\frac{1}{2}} \times \left[ 1 - \frac{2\eta}{x^2} \right]^{\frac{1}{3}}.$$  \quad (29)

Now by solving eq. (27) the time in which the black hole evaporates completely is given by (up to $O(\eta)$)

$$\frac{t}{t_{ch}} = \frac{x_i^3}{3} + 2\eta x_i.$$  \quad (30)

The mass as a function of time given by eqs. (24), (28) and the rate at which energy is radiated as a function of time given by eqs. (25), (29) are shown in figs. 1, 2, 3, 4 for different values of $\eta$.

From the plots we observe that they are similar in appearance to the plots which take into account the effect of the generalized uncertainty principle in black-hole thermodynamics [26].

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Constraints on rainbow gravity functions from black-hole thermodynamics

Rainbow-gravity–inspired Reissner-Nordström black holes. – The rainbow-gravity–inspired Reissner-Nordström black-hole metric reads

\[ ds^2 = -\frac{1}{f(E/E_p)^2} \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \frac{1}{g(E/E_p)^2} \left( 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + \frac{r^2}{g(E/E_p)^2} d\Omega^2. \] (31)

The surface gravity in this case is given by

\[ \kappa = \frac{g(E/E_p)}{f(E/E_p)} \left( \frac{M}{r_0^3} - \frac{Q^2}{r_0^3} \right), \] (32)

where \( r_0 = M + \sqrt{M^2 - Q^2} \) is the radius of the event horizon of the rainbow-gravity–inspired RN black hole. The Hawking temperature therefore reads

\[ T = \frac{1}{2\pi} \sqrt{1 - \frac{\eta}{(E_p)^n} \left( 1 - \frac{M}{r_0^3} - \frac{Q^2}{r_0^3} \right)}. \] (33)

The condition for the reality of the temperature yields

\[ 1 - \frac{\eta}{(E_p)^n} \left( 1 - \frac{M}{r_0^3} - \frac{Q^2}{r_0^3} \right) \geq 0 \] (34)

and this, in turn, implies that the critical mass of the rainbow-gravity–inspired RN black hole is given by

\[ M_{cr} = \sqrt{\frac{Q^2}{2} + \frac{Q^4}{4} + \frac{(\eta/E_p)^{2/n}}{4}}. \] (35)

The heat capacity of this black hole reads

\[ C = 4\pi \left( -\frac{2M}{r_0^3} + \frac{3Q^2}{r_0^5} \right) \left( 1 - \frac{\eta}{(E_p)^n} \right)^{-1} + \frac{\eta}{(E_p)^n} \left( \frac{M}{r_0^3} - \frac{Q^2}{r_0^5} \right). \] (36)

As before, the remnant mass is obtained by setting \( C = 0 \) and is found to be identical to the critical mass.

The entropy of this black hole is now computed and gives

\[ S = 2\pi \int \frac{dM}{\sqrt{1 - \frac{\eta}{(E_p)^n} \left( 1 - \frac{M}{r_0^3} - \frac{Q^2}{r_0^3} \right)}} = \pi r_0^2 F_1 \left( \frac{1}{2} - \frac{2}{n}, 1 - \frac{2}{n} ; \frac{\eta}{(E_p r_0)^n} \right). \] (37)

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Interestingly, here also we find that there exists an analytic solution for the entropy. Carrying out an expansion of this result for a small argument $\eta$ yields

$$S = S_{BH} + \frac{\pi^2 \eta}{(2 - n) S_{BH}^{(n - 1)}} + \frac{3\pi^2 \eta^2}{8(1 - n) S_{BH}^{(n - 1)}}
+ \frac{5\pi^3 \eta^3}{8(2 - 3n) S_{BH}^{(n - 1)}},$$

(38)

where $S_{BH} = \pi r_0^2$ is the semi-classical Bekenstein-Hawking entropy for the rainbow-gravity–inspired RN black hole. Note that the above integration is valid only for $n \geq 3$. Once again no logarithmic corrections are present in the expression for the entropy. However, carrying out the integration for $n = 1, 2$ and making use of the identity

$$\left(\frac{r_0}{M_{(Q_0 - Q)^2}}\right) = \frac{1}{r_0 - M},$$

one gets for $n = 1$

$$S = \frac{\pi r_0}{2E_p} (2E_p r_0 + 3\eta) \sqrt{1 - \frac{\eta}{E_p r_0}}$$

$$+ \frac{3\pi^2 \eta}{4E_p} \ln \left[ \eta - 2E_p r_0 \left( 1 + \sqrt{1 - \frac{\eta}{E_p r_0}} \right) \right].$$

(39)

Expanding this up to $O(\eta^2)$ yields

$$S = \frac{A}{4} + \eta \sqrt{\frac{A}{4}} + \frac{3\pi^2 \eta^2}{8} \ln \left( \frac{A}{4} \right).$$

(40)

For $n = 2$, entropy expression reads

$$S = \pi r_0^2 \sqrt{1 - \frac{\eta}{(E_p r_0)^2}} + \frac{\eta}{2E_p} \ln$$

$$\left[ \eta - 2(E_p r_0)^2 \left( 1 + \sqrt{1 - \frac{\eta}{(E_p r_0)^2}} \right) \right].$$

(41)

Expanding the above expression up to $O(\eta^2)$ gives

$$S = \frac{A}{4} + \frac{\eta \sqrt{\frac{A}{4}}}{2} + \frac{3\pi^2 \eta^2}{8} \frac{1}{\left( \frac{A}{4} \right)^2}.$$

(42)

Once again the expressions for entropy in both cases are similar in structure to that obtained using the GUP [26–28].

Conclusions. – In this paper we have investigated the modifications of the thermodynamic properties of Schwarzschild and Reissner-Nordström black holes taking into account the effects of rainbow gravity functions. We found that rainbow gravity modifies the mass-temperature relationship and leads to the existence of a black-hole remnant. We further observed that as in the case of the generalized uncertainty principle, here also the remnant and critical masses are identical. The computation of the entropy does not contain the universal logarithmic corrections for all values of the parameter $n$ appearing in the rainbow functions. It is only for the values of $n = 1, 2$ appearing in the rainbow functions that the logarithmic corrections to the semi-classical area law for the entropy are found to exist. It is to be noted that loop quantum gravity does not motivate any specific value for $n$. The mass output and the radiation rate are finally computed for these constrained forms of the rainbow functions for different values of the rainbow parameter $\eta$ and have striking similarity to those derived from the generalized uncertainty principle.

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