New conceptions of transitivity and minimal mappings

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Abstract: The concepts of topological δ-transitive maps, α-type transitive maps, δ-minimal and α-minimal mappings were introduced by M. Nokhas Murad Kaki. In this paper, the relationship between two different notions of transitive maps, namely topological δ-type transitive maps and topological α-type transitive maps has been studied and some of their properties in two topological spaces (X, τδ) and (X, τα), τδ denotes the δ-topology (resp. τα denotes the α-topology) of a given topological space (X, τ) has been investigated. Also, we have proved that there exists a dense orbit in X, where X is locally compact Hausdorff space and τ has a countable basis. The main results are the following propositions: Every topologically α-type transitive map is a topologically transitive map which implies topologically δ-transitive map, but the converse not necessarily true., and every α-minimal map is a minimal map which implies δ-minimal map in topological spaces, but the converse not necessarily true. Finally, we have proved that a map which is γr-conjugated to γ-transitive (resp. γ-minimal, γ-mixing) map is γ-transitive (resp. γ-minimal, γ-mixing).

Keywords: Topologically δ-Transitive, δ-Irresolute, δ-Type Transitive, δ-Dense, γ-Dense, γ Transitive

1. Introduction

Let A be a subset of a topological space (X, τ). The closure and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open [1] (resp. preopen [2]) if A = Int(Cl(A)) (resp. A ⊂ Int(Cl(A))). A set A ⊂ X is said to be δ-open [3] if it is the union of regular open sets of a space X. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [3] of A and is denoted by Clδ(A). .. Recall that a set S is called regular closed if S = Cl(Int(S)). A point x ε X is called a δ-cluster point [3] of S if S ∩ U ≠ ∅ for each regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by Clδ(S). A subset S is called δ-closed if Clδ(S) = S. The complement of a δ-closed set is called δ-open. The family of all δ-open sets of a space X is denoted by δ(X, τ). The δ-interior of S is denoted by Intδ(S) and it is defined as follows Intδ(S) = {x ∈ X : x ∈ U ⊆ Int(Cl(U)) ⊆ S} for some open set U of X.

The area of Dynamical Systems where one investigates dynamical properties that can be described in topological terms is called Topological Dynamics. Let X be a compact topological space and let f : X → X be continuous. The pair (X, f) is so called topological system. The topological system (X, f) is called topologically δ-type transitive (or just δ-type transitive [4]) if for every pair of nonempty δ-open sets U and V in X there is a nonnegative integer n such that f^n(U) ∩ V ≠ ∅. If the space X has no isolated points, this is equivalent to the existence of a point x ∈ X whose orbit O_f(x) = {x, f(x), f^2(x), ..., f^n(x), ...,} is δ-dense in X. Consequently, a topologically δ-type transitive topological system cannot be decomposed into two disjoint sets with nonempty δ-interiors. For more information on topological δ-type transitivity see, e.g. [4] and references there.

In this paper, we will study some new class of topological transitive maps called topological δ-type transitive [4], also, we will study the relationship between two types of minimal mappings, namely, δ-minimal mapping and α-minimal mapping, and we will prove that the properties of δ-type transitive, δ-mixing and δ-minimal maps are preserved under δr-conjugacy and study some of its properties.
2. Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let $X$ be a space and $A \subseteq X$. The intersection (resp. closure) of $A$ is denoted by $\text{Int}(A)$ (resp. $\text{Cl}(A)$).

Definition 2.1 Let $(X, \tau)$ be a space. A subset $A$ of $X$ is called dense in $X$ if $\text{Cl}(A) = X$.

Definition 2.2 (i) A space $X$ is said to be 2nd countable if it has a countable basis.

(ii) $X$ is said to be of First Category if it is a countable union of nowhere dense subsets of $X$. It is of second Category if it is not of First Category.

Theorem 2.3 Let $X$ be a non-empty locally compact Hausdorff space. Then the intersection of a countable collection of open dense subsets of $X$ is dense in $X$. Moreover, $X$ is of second Category.

Definition 2.4 Let $(X, \tau)$ be a topological space. $X$ is second countable if and only if the topology of $X$ has a countable basis.

Theorem 2.5 Recall that a space $X$ is said to be separable if $X$ contains a countable dense subset.

Corollary 2.6 A subset $A$ of a space $(X, \tau)$ is dense if and only if $A \cap U \neq \phi$ for all $U \in \tau$ other than $U = \phi$.

Definition 2.7 Let $(X, \tau)$ be a topological space, $f: X \to X$ be a continuous map then $f$ is said to be topologically transitive if every pair of non-empty open sets $U$ and $V$ in $X$ there is a positive integer $n$ such that $f^n(U) \cap V \neq \phi$.

The purpose of the following theorem is to prove that topological transitivity implies dense orbits in a space $X$ where $X$ is a non-empty locally compact Hausdorff topological space.

Theorem 2.8 Let $(X, f)$ be a topological system where $X$ is a non-empty locally compact Hausdorff topological space and $f: X \to X$ is a continuous map and that $X$ is separable. Suppose that $f$ is topologically transitive. Then there is $x \in X$ such that the orbit $O_f(x) = \{x, f(x), f^2(x), \ldots, f^n(x), \ldots\}$ is dense in $X$.

Proof: Let $B = \{A_i\}$, $i = 1, 2, 3, \ldots$ be a countable basis for the topology of $X$. For each $i$, let $O_i = \{x \in X : f^n(x) \in U_i$ for some $n \geq 0\}$

Then, clearly $O_i$ is open and dense. It is open since $f$ is continuous, so, $O_i = \bigcup_{n=0}^{\infty} f^{-n}(U_i)$ is open and dense since $f$ is topological transitive map. Further, for every open set $V$, there is a positive integer $n$ such that $f^n(V) \cap U_j \neq \phi$.

Now, apply theorem 2.3 to the countable dense sets $\{O_i\}$ to say that $\cap_{i=0}^{\infty} O_i$ is dense and so non-empty. Let $y \in \cap_{i=0}^{\infty} O_i$.

This means that, for each $i$, there is a positive integer $n$ such that $f^n(y) \in U_i$ for every $i$. By corollary 2.6 this implies that $O_f(x)$ is dense in $X$.

Definition 2.9 If for $x \in X$ the set $\{f^n(x) : n \in \mathbb{N}\}$ is dense in $X$ then $x$ is said to have a dense orbit. If there exists such an $x \in X$, then $f$ is said to have a dense orbit.

Definition 2.10 A function $f: X \to X$ is called $\gamma$-homeomorphism if $f$ is $\gamma$-irresolute bijective and $f^{-1}: X \to X$ is $\gamma$-irresolute.

Definition 2.11 [19] Two topological systems $f: X \to X$ and $g: Y \to Y$ are said to be topologically $\gamma\phi$-conjugate if there is a $\gamma\phi$-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$ (i.e. $h(f(x)) = g(h(x))$). We will call $h$ a topological $\gamma\phi$-conjugacy.

Remark 2.12 [19] If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{y_0 = h(x_0), y_1 = h(x_1), y_2 = h(x_2), \ldots\}$ yields an orbit of $g$ since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$. In particular, $h$ maps periodic orbits of $f$ onto periodic orbits of $g$.

In [19], we introduced and defined the new type of transitivity called $\gamma\phi$-transitive in such a way that it is preserved under topologically $\gamma\phi$-conjugation. It means; we have proved that a map which is $\gamma\phi$- conjugated to $\gamma\phi$- transitive (resp. $\gamma\phi$-minimal, $\gamma\phi$-mixing) map is $\gamma\phi$-transitive (resp.$\gamma\phi$-minimal, $\gamma\phi$-mixing).

We proceed to prove the following important proposition:

Proposition 2.13 [19] Let $(X, f)$ and $(Y, g)$ be two topological systems, if $f: X \to X$ and $g: Y \to Y$ are topologically $\gamma\phi$-conjugate. Then

(1) $f$ is topologically $\gamma\phi$-transitive if and only if $g$ is topologically $\gamma\phi$-transitive;

(2) $f$ is $\gamma\phi$-minimal if and only if $g$ is $\gamma\phi$-minimal;

(3) $f$ is topologically $\gamma\phi$-mixing if and only if $g$ is topologically $\gamma\phi$-mixing.

Proof (1) Assume that $f: X \to X$ and $g: Y \to Y$ are topologically $\gamma\phi$-conjugated by $h: X \to Y$. Suppose $f$ is $\gamma\phi$-type transitive. Let $A, B$ be $\gamma\phi$-open subsets of $Y$ (to show $g^n(A) \cap B \neq \phi$ for some $n > 0$).

$U = h^{-1}(A)$ and $V = h^{-1}(B)$ are $\gamma\phi$-open subsets of $X$ since $h$ is an $\gamma\phi$- irresolute

Then there exists some $n > 0$ such that $f^n(U) \cap V \neq \phi$ since $f$ is $\gamma\phi$-type transitive. Thus (as $f \circ h^{-1} = h^{-1} \circ g$ implies $f^n \circ h^{-1} = h^{-1} \circ g^n$),

$\phi \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B)$

Therefore, $h^{-1}(g^n(A) \cap B) \neq \phi$ implies $g^n(A) \cap B \neq \phi$ since $h^{-1}$ is invertible.
Proof (2)
Assume that \( f : X \rightarrow X \) and \( g : Y \rightarrow Y \) are topological systems, which are topologically \( \gamma \)-conjugated by \( h : Y \rightarrow X \). Thus, \( h \) is \( \gamma \)-homeomorphism (that is, \( h \) is bijective and thus invertible and both \( h \) and \( h^{-1} \) are \( \gamma \)-irresolute) and \( h \circ g = f \circ h \), that is, the following diagram commutes:

\[
\begin{array}{ccc}
Y & \xrightarrow{g} & Y \\
\downarrow{h} & & \downarrow{h} \\
X & \xrightarrow{f} & X
\end{array}
\]

We show that if \( g \) is \( \gamma \)-minimal, then \( f \) is \( \gamma \)-minimal. We want to show that for any \( x \in X \), \( O_f(x) \) is \( \gamma \)-dense. Since \( h \) is surjective, there exists \( x \in X \) such that \( y = h^{-1}(x) \). Since \( g \) is \( \gamma \)-minimal, \( O_g(y) \) is \( \gamma \)-dense. For any non-empty \( \gamma \)-open subset \( U \) of \( X \), \( h^{-1}(U) \) is an \( \gamma \)-open subset of \( Y \) since \( h^{-1} \) is \( \gamma \)-irresolute because the map \( h \) is \( \gamma \)-homeomorphism and it is non-empty since \( h \) is invertible map. By \( \gamma \)-density of \( O_g(y) \) there exist \( k \) in \( N \) such that \( g^k(y) \in h^{-1}(U) \iff h(g^k(y)) \in U \)

Since \( h \) is \( \gamma \)-conjugacy; as \( f \circ h = h \circ g \) implies \( f^k \circ h = h \circ g^k \) so \( f^k(h(y)) = h(g^k(y)) \in U \) thus \( O_f(h(y)) \) intersects \( U \). This holds for any non-empty \( \gamma \)-open set \( U \) and thus shows that \( O_f(x) = O_f(h(y)) \) is \( \gamma \)-dense.

Proof (3)
We only prove that if \( g \) is topologically \( \gamma \)-mixing then \( f \) is also topologically \( \gamma \)-mixing. Let \( U, V \) be two \( \gamma \)-open subsets of \( X \). We have to show that there is \( N > 0 \) such that for any \( n > N \), \( f^n(U) \cap V \neq \emptyset \).

\( h^{-1}(U) \) and \( h^{-1}(V) \) are two \( \gamma \)-open sets since the map \( h \) is \( \gamma \)-irresolute. If \( g \) is topologically \( \gamma \)-conjugacy then there is \( N > 0 \) such that for any \( n > M \), \( g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \emptyset \). Therefore there exists \( x \in g^n(h^{-1}(U)) \cap h^{-1}(V) \). That is, \( x \in g^n(h^{-1}(U)) \) and \( x \in h^{-1}(V) \) if and only if \( x = g^n(y) \) for \( y \in h^{-1}(U) \) and \( h(x) \in V \).

Thus, since \( h \circ g^n = f^n \circ h \), so that, \( h(x) = h(g^n(y) = f^n(h(y)) \in f^n(U) \) and we have \( h(x) \in V \) that is \( f^n(U) \cap V \neq \emptyset \).

So, \( f \) is \( \gamma \)-mixing.

3. Transitive and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system \( (X, f) \) [15] we mean a topological space \( X \) together with a continuous map \( f : X \rightarrow X \). The space \( X \) is sometimes called the phase space of the system. A set \( A \subseteq X \) is called \( \gamma \)-invariant if \( f(A) \subseteq A \).

A topological system \( (X, f) \) is called minimal if \( X \) does not contain any non-empty, proper, closed \( \gamma \)-invariant subset. In such a case we also say that the map \( f \) itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them. Given a point \( x \) in \( X \), \( O_f(x) = \{x, f(x), f^2(x), ...\} \) denotes its orbit (by an orbit we mean a forward orbit even if \( f \) is a homeomorphism) and \( o_f(x) \) denotes its \( \omega \)-limit set, i.e. the set of limit points of the sequence \( x, f(x), f^2(x), ... \). The following conditions are equivalent:

- \( (X, f) \) is \( \alpha \)-minimal (resp. \( \theta \)-minimal),
- every orbit is \( \alpha \)-dense (resp. \( \theta \)-dense) in \( X \),
- \( \omega_f(x) = X \) for every \( x \in X \).

A minimal map \( f \) is necessarily surjective if \( X \) is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system \( (X, f) \), a set \( A \subseteq X \) is called a minimal set if it is non-empty, closed and invariant and if no proper subset of \( A \) has these three properties. So, \( A \subseteq X \) is a minimal set if and only if \( (A, f|_A) \) is a minimal system. A system \( (X, f) \) is minimal if and only if \( X \) is a minimal set in \( (X, f) \).

Let \( (X, f) \) be a topological system, and \( f : X \rightarrow X \) \( \alpha \)-homeomorphism of \( X \) onto itself. For \( A \) and \( B \) subsets of \( X \), we let \( N(A, B) = \{n \in \mathbb{Z} : f^n(A) \cap B \neq \emptyset \} \) and \( N(x, B) = \{n \in \mathbb{Z} : f^n(x) \in B \} \).

We write \( N(A, B) = N(x, B) = \{n \in \mathbb{Z} : f^n(x) \in B \} \). For a point \( x \in X \) we write \( O_f(x) = \{f^n(x) : n \in \mathbb{Z}\} \) for the orbit of \( x \) and \( Cl_\alpha(O_f(x)) \) for the \( \alpha \)-closure of \( O_f(x) \).

We say that the topological system \( (X, f) \) is \( \alpha \)-type point transitive if there is a point \( x \in X \) with \( O_f(x) \) \( \alpha \)-dense. Such a point is called \( \alpha \)-type transitive. We say that the topological systems \( (X, f) \) is topologically \( \alpha \)-transitive (or just \( \alpha \)-type transitive) if the set \( N(U, V) \) is nonempty for every pair \( U \) and \( V \) of nonempty \( \alpha \)-open subsets of \( X \).

3.1. Topologically \( \alpha \)-Transitive Maps

In [11], we introduced and defined a new class of transitive maps that are called topologically \( \alpha \)-transitive maps on a topological space \( (X, \tau) \), and we studied some of their properties and proved some results associated with...
these new definitions. We also defined and introduced a new class of $\alpha$-minimal maps. In this paper we discuss the relationship between topologically $\alpha$-transitive maps and $\theta$-transitive maps. On the other hand, we discuss the relationship between $\alpha$-minimal and $\theta$-minimal in topological systems.

**Definition 3.1.1** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is called $\alpha$-dense in $X$ if $\text{Cl}_\alpha(A) = X$.

Note that, in general topology, for any subset $A$ of the space $X$, $A \subset \text{Cl}_\alpha(A) \subset \text{Cl}(A)$, therefore if $A$ is $\alpha$-dense, in $X$, then $A$ is dense in $X$.

**Remark 3.1.2** Any $\alpha$-dense subset in $X$ intersects any $\alpha$-open set in $X$.

**Proof:** Let $A$ be an $\alpha$-dense subset in $X$, then by definition, $\text{Cl}_\alpha(A) = X$, and let $U$ be a non-empty $\alpha$-open set in $X$. Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is $\alpha$-closed and $A \subset U^c = B$. So $\text{Cl}_\alpha(A) \subset \text{Cl}_\alpha(B)$, i.e., $\text{Cl}_\alpha(A) \subset B$, but $\text{Cl}_\alpha(A) = X$, so $X \subset B$, this contradicts that $U = \emptyset$.

**Definition 3.1.3** [12] A map $f : X \to Y$ is called $\alpha$-irresolute if for every $\alpha$-open set $H$ of $X$, $f^{-1}(H)$ is $\alpha$-open in $X$.

**Example 3.1.4** [11] Let $(X, \tau)$ be a topological space such that $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, b\}, \{b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, c, d\}\}$ and the set of all $\alpha$-closed sets is $\alpha C(X, \tau) = \{\phi, X, \{c, d\}, \{a, c, d\}, \{a, d\}\}$. Then define the map $f : X \to X$ as follows $f(a) = a$, $f(b) = b$, $f(c) = d$, $f(d) = c$, we have $f$ is $\alpha$-irresolute because $\{b\}$ is $\alpha$-open and $f^{-1}(\{b\}) = \{a, b\}$ is $\alpha$-open; $\{a, b\}$ is $\alpha$-open and $f^{-1}(\{a, b\}) = \{a, b\}$ is $\alpha$-open; $\{b, c\}$ is $\alpha$-open and $f^{-1}(\{b, c\}) = \{b, d\}$. Since $\{a, b, c\}$ is $\alpha$-open and $f^{-1}(\{a, b, c\}) = \{a, b, d\}$ is $\alpha$-open; $\{a, b, d\}$ is $\alpha$-open and $f^{-1}(\{a, b, d\}) = \{a, c, d\}$ is $\alpha$-open.

**Definition 3.1.5** A subset $A$ of a topological space $(X, \tau)$ is said to be nowhere $\theta$-dense, if its $\theta$-closure has an empty $\theta$-interior, that is, $\text{int}_\theta(\text{Cl}_\theta(A)) = \emptyset$.

**Definition 3.1.6** [11] Let $(X, \tau)$ be a topological space, $f : X \to X$ be a $\alpha$-irresolute map then $f$ is said to be topological $\alpha$-transitive if every pair of non-empty $\alpha$-open sets $U$ and $V$ in $X$ there is a positive integer $n$ such that $f^n(U) \cap V \neq \emptyset$. In the foregoing example 3.1.4: we have $f$ is $\alpha$-transitive because $b$ belongs to any non-empty $\alpha$-open set $V$ and also belongs to $f(U)$ for any $\alpha$-open set it means that $f(U) \cap V \neq \emptyset$ for $f$ is a $\alpha$-transitive.

**Example 3.1.7**[11] Let $(X, \tau)$ be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{X\}\}$. Then the set of all $\alpha$-open sets is $\alpha \tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$. Define $f : X \to X$ as follows $f(a) = b$, $f(b) = b$, $f(c) = c$. Clearly $f$ is continuous because $\{a\}$ is open and $f(\{a\}) = \emptyset$ is open. Note that $f$ is transitive because $f(\{a\}) = \{b\}$ implies that $f(\{a\}) \cap \{b\} \neq \emptyset$. But $f$ is not $\alpha$-transitive because for each $n \in N$, $f^n(\{a\}) \cap \{a, c\} = \emptyset$; since $f^0(\{a\}) = \{b\}$ for every $n \in N$, and $\{b\} \cap \{a, c\} = \emptyset$. So we have $f$ is not $\alpha$-transitive, so we show that transitivity not implies $\alpha$-transitivity.

**Definition 3.1.8** Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is called $\theta$-dense in $X$ if $\text{Cl}_\theta(A) = X$.

**Remark 3.1.9** Any $\theta$-dense subset in $X$ intersects any $\theta$-open set in $X$.

**Proof:** Let $A$ be a $\theta$-dense subset in $X$, then by definition, $\text{Cl}_\theta(A) = X$, and let $U$ be a non-empty $\theta$-open set in $X$. Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is $\theta$-closed because $B$ is the complement of $\theta$-open and $A \subset U^c = B$. So $\text{Cl}_\theta(A) \subset \text{Cl}_\theta(B)$, i.e., $\text{Cl}_\theta(A) \subset B$, but $\text{Cl}_\theta(A) = X$, so $X \subset B$, this contradicts that $U = \emptyset$.

**Definition 3.1.10**[14] A function $f : X \to X$ is called $\theta$-irresolute if the inverse image of each $\theta$-open set is a $\theta$-open set in $X$.

**Definition 3.1.11** A subset $A$ of a topological space $(X, \tau)$ is said to be nowhere $\theta$-dense, if its $\theta$-closure has an empty $\theta$-interior, that is, $\text{int}_\theta(\text{Cl}_\theta(A)) = \emptyset$.

**Definition 3.1.12** [15] Let $(X, \tau)$ be a topological space, and $f : X \to X$ be a $\alpha$-irresolute map, then $f$ is said to be topologically $\theta$-type transitive map if for every pair of $\theta$-open sets $U$ and $V$ in $X$ there is a positive integer $n$ such that $f^n(U) \cap V \neq \emptyset$.

Associated with this new definition we can prove the following new theorem.

**Theorem 3.1.13** [11]: Let $(X, \tau)$ be a topological space and $f : X \to X$ be a $\alpha$-irresolute map. Then the following statements are equivalent:

1. $f$ is topological $\alpha$-transitive map
2. For every nonempty $\alpha$-open set $U$ in $X$, $\bigcap_{\alpha} f^n(U)$ is $\alpha$-dense in $X$
3. For every nonempty $\alpha$-open set $U$ in $X$, $\bigcap_{n=0} f^n(U)$ is $\alpha$-dense in $X$
4. If $B \subset X$ is $\alpha$-closed and $B$ is $f$-invariant i.e. $f(B) \subset B$, then $B = X$ or $B$ is nowhere $\alpha$-dense.
5. If $U$ is $\alpha$-open and $f^{-1}(U) \subset U$ then $U$ is either empty set or $\alpha$-dense in $X$.

**Theorem 3.1.14**: [4] Let $(X, \tau)$ be a topological space and $f : X \to X$ be $\theta$-irresolute map. Then the following statements are equivalent:

1. $f$ is $\theta$-type transitive map
2. $\bigcap_{\alpha} f^n(D)$ is $\theta$-dense in $X$, with $D$ is $\theta$-open set in $X$.
3. $\bigcup_{n=0} f^n(D)$ is $\theta$-dense in $X$ with $D$ is $\theta$-open set in $X$
4. If $B \subset X$ is $\theta$-closed and $f(B) \subset B$, then $B = X$ or $B$ is nowhere $\theta$-dense.
5. If $f^{-1}(D) \subset D$ and $D$ is $\theta$-open in $X$ then $D = \emptyset$ or $D$ is $\theta$-dense in $X$.  

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4. Minimal Functions

We introduced a new definition on α-minimal[11] (resp. δ-minimal[4]) maps and studied some new theorems associated with these definitions.

Given a topological space X, we ask whether there exists α-irresolute (resp. δ-irresolute) map on X such that the set \( \{ f^n(x) : n \geq 0 \} \) called the orbit of x and denoted by \( O_f(x) \), is α-dense(resp. δ-dense) in X for each x є X. A partial answer will be given in this section. Let us begin with a new definition.

Definition 4.1 (α-minimal) Let X be a topological space and f be α-irresolute map on X with α-regular operator associated with the topology on X. Then the dynamical system (X, f) is called α-minimal system (or f is called α-minimal map on X) if one of the three equivalent conditions hold[11]:

1) The orbit of each point of X is α-dense in X.
2) \( Cl_\alpha(O_f(x)) = X \) for each x є X
3) Given x є X and a nonempty α-open U in X, there exists n є N such that \( f^n(x) \in U \)

A system \( (X, f) \) is called δ-minimal if X does not contain any non-empty, proper, δ - closed f-invariant subset. In such a case we also say that the map f itself is δ-minimal. Another definition of minimal function is that if the orbit of every point in X is dense in X then the map f said to be minimal.

Theorem 4.2[4] For \( (X, f) \) the following statements are equivalent:

1) f is an δ-minimal map.
2) If E is an δ-closed subset of X with \( f(E) \subseteq E \), we say E is invariant. Then E = ∅ or E = X.
3) If U is a nonempty δ-open subset of X, then \( \bigcup_{n=0}^{\infty} f^n(U) = X \).

5. Topological Systems and Conjugacy

Definition 5.1[4] A map \( h:Y \rightarrow X \) is said to be δr-homeomorphism if h is bijective and thus invertible and both h and \( h^{-1} \) are δr-irresolute.

Definition 5.2 Let \( (X, f) \) and \( (Y, g) \) be topological systems, then \( f: X \rightarrow X \) and \( g: Y \rightarrow Y \) are said to be topologically δr-conjugate if there is δr-homeomorphism \( h: X \rightarrow Y \) such that \( h \circ f = g \circ h \). We will call h a topological δr-conjugacy. Thus, the two topological systems with their respective function acting on them share the same dynamics.

Associated with these definitions we have the following theorem:

Theorem 5.3[4] Let \( (X, f) \) and \( (Y, g) \) be two systems, if \( f: X \rightarrow X \) and \( g: Y \rightarrow Y \) are topologically δr-conjugate. Then

1) f is topologically δ-transitive if and only if g is topologically δ-transitive;
2) f is δ-minimal if and only if g is δ-minimal;
3) f is topologically δ-mixing if and only if g is topologically δ-mixing.

6. Conclusion

The main results are the following:

Proposition 6.1 Every topologically α-type transitive map is a topologically transitive map which implies topologically δ- transitive map, but the converse not necessarily true.

Proposition 6.2 Every α-minimal map is a minimal map which implies δ- minimal map in topological spaces, but the converse not necessarily true.

Theorem 6.3 Let \( (X, f) \) be a topological system where X is a non-empty locally compact Hausdorff topological space and X is separable. Suppose that f is topologically transitive. Then there is \( x \in X \) such that the orbit \( O_f(x) = \{ x, f(x), f^2(x), ... f^n(x), ... \} \) is dense in X.

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