DENSITY SCALING AND ANISOTROPY IN SUPERSONIC MAGNETOHYDRODYNAMIC TURBULENCE

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ABSTRACT

We study the statistics of density for supersonic turbulence in a medium with magnetic pressure larger than the gaseous pressure. Our simulations exhibit clumpy density structures, with the contrast increasing with the Mach number. At Mach 10, the densities of some clumps are 3 orders of magnitude higher than the mean density. These clumps give rise to a flat and approximately isotropic density spectrum corresponding to the random distribution of clumps in space. We claim that the clumps originate from our random, isotropic turbulence driving. When the contribution from those clumps is suppressed by studying the logarithm of density, the density statistics exhibit scale-dependent anisotropy consistent with the models in which density structures arise from shearing by Alfvén waves. It is noteworthy that originally such models were advocated for the case of low-Mach, nearly incompressible turbulence.

Subject headings: ISM: clouds — ISM: kinematics and dynamics — ISM: structure — MHD — turbulence

1. INTRODUCTION

The paradigm of the interstellar medium has undergone substantial changes recently. Instead of a quiescent medium with hanging and slowly evolving clouds, a turbulent picture emerged (see review by Vázquez-Semadeni et al. 2000). With the magnetic field being dynamically important and dominating the gas pressure in molecular clouds, studies of compressible magnetohydrodynamic (MHD) turbulence are called for.

Recent years have also been marked by a substantial progress in our understanding of the MHD turbulence statistics (see review by Cho & Lazarian 2005 and references therein). These statistics allow one to find regularities in turbulence; e.g., the power spectrum allows one to learn how much energy is at a particular range of scales.

A very important insight into the incompressible MHD turbulence by Goldreich & Sridhar (1995, hereafter GS95) has been followed by our progress in our understanding of nearly incompressible \( (M \ll 1) \) and compressible \( (M \sim 1, M > 1) \), MHD turbulence (Lithwick & Goldreich 2001; Cho & Lazarian 2003, hereafter CL03; Cho et al. 2003; Vestuto et al. 2003). In particular, a simulation in CL03 showed that the Alfvénic cascade evolves on its own\(^1\) and that it exhibits Kolmogorov-type scaling (Kolmogorov 1941; i.e., \( E \sim k^{5/3} \)) and scale-dependent anisotropy of the Goldreich-Sridhar type (i.e., \( k_\perp \sim k^{5/3} \) even for high Mach number turbulence. While slow modes exhibit similar scalings and anisotropy, fast modes show isotropy. The density scaling obtained in CL03 was somewhat puzzling. At low Mach numbers, it was similar to slow modes, while it got isotropic for high Mach numbers. The uncertainties associated with the earlier study motivate our present one.

Density statistics are important for understanding the structure of molecular clouds and the associated processes of star formation. The density spectrum and the probability density function (PDF) can reflect the initial mass function for star-forming clouds. The propagation and transfer of radiation are also affected by the density spectrum. Are density perturbations elongated along magnetic field lines? How does this depend on the Mach number of turbulence? These are the questions that we would like to answer.

There are a number of observational implications of the density spectra. A shallow power spectrum can result in a lot of small-scale absorption (Deshpande 2000) that can account for the mysterious tiny-scale atomic structures (Heiles 1997). A power spectrum that is shallower than the Kolmogorov one was reported in a number of observations (Deshpande 2000; Padoan et al. 2003). Boldyrev et al. (2002) proposed a model for density scaling for the super-Alfvénic case \( (M_a > 1) \). Density anisotropies have been observed in scintillation studies at small scales, but it is unclear whether or not we should expect them at all scales.

2. THEORETICAL CONSIDERATIONS

Subsonic compressible MHD is a rather well-studied topic today. It has been suggested that there may be an analogy between the subsonic \( (M < 1) \) MHD turbulence and its incompressible counterpart, namely, the GS95 model. Therefore, the correspondence between the two that was revealed in CL03 is not unexpected.

It could be easily seen that in the low-\( \beta \) case \( (P_{\text{mag}} > P_{\text{gas}}) \), the density is perturbed by the slow mode (CL03). Slow modes are sheared by Alfvén turbulence; therefore, they exhibit Kolmogorov scaling and GS95 anisotropy for low Mach numbers. However, for high Mach numbers, we expect shocks to develop. The density will be perturbed mainly by those shocks. However, the relative perturbation of density is likely to be proportional to the density itself.

One can also approach the problem from the point of view of the underlying hydrodynamic equations. It is well known that...
there is a multiplicative symmetry with respect to density in the ideal flow equations for an isothermal fluid (see, e.g., Passot & Vázquez-Semadeni 1998, hereafter PV98). This assume that if there is some stochastic process disturbing the density, it should be a multiplicative process with respect to density, rather than additive, and the distribution for density values should be lognormal, rather than normal. One-dimensional numerical simulations of high-Mach hydrodynamics confirmed that the distribution is approximately lognormal for an isothermal fluid (PV98).

In MHD, however, the above-described symmetry is broken by the magnetic tension. This could be qualitatively described as the higher density regions having lower Alfvén speed, if we assume there is no significant correlation between the density and the magnetic field. The latter is usually the case with strongly magnetized, low-β fluid. It is interesting to test whether or not this causes a substantial deviation of the distribution from the lognormal law.

In a low-Mach turbulence, the processes leading to the perturbation of density are governed by the sound speed. The self-evolution of density perturbations will be slow in comparison with the shearing by Alfvén waves.

With a high sonic Mach number, we expect a considerable amount of shocks to arise. In the sub-Alfvénic case, however, we expect oblique shocks to be disrupted by Alfvénic shearing, and, as most of the shocks are generated randomly by driving, almost all of them will be sheared to smaller shocks. The evolution of the weak shocks will again be governed by the sonic speed, and structures from shearing, as in a low Mach number case, should arise.

We also note that shearing will not affect the PDF of the density but will affect its spectra and structure function (SF) scaling. In other words, we deal with two distinct physical processes, one of which, i.e., the random multiplication or division of density in the presence of shocks, will affect the PDF, while the other, the Alfvénic shearing, will affect the anisotropy and scaling of the SF of the density. In order to test this, we performed direct numerical simulations.

3. THE CODE

We used data cubes from our direct three-dimensional numerical simulations (see Cho & Lazarian 2004). As we are interested in high-Mach turbulence, we performed simulations on a periodic $512^3$ Cartesian grid with the average sonic Mach numbers of $\sim10$ and $\sim3$. The effects of numerical diffusion are expected to be important at the scales of less than 10 grid points. We observed that, parallel to the magnetic field, velocities stay supersonic down to 8 grid units for Mach 10, and 20 grid units for Mach 3. We used the isothermal equation of state and randomly drove the turbulence on a scale that was about 2.5 times smaller than the box size. The Alfvén velocity of the mean magnetic field was roughly the same as the rms velocity, which corresponds to an Alfvénic Mach number of around unity.

4. RESULTS

In Figure 1 we show the distribution of the log density for various values of Mach numbers for three-dimensional numerical simulations of MHD. In all cases, except subsonic, driving was chosen so that the Alfvénic Mach number $M_A$ is slightly less than unity. This was motivated by the idea that in a strongly super-Alfvénic fluid, given enough time, the magnetic field will grow and approach equipartition. As soon as we observe scales smaller than the driving scale, we will see mildly super-Alfvénic or sub-Alfvénic turbulence.

We see that the distribution shows a significant deviation from the lognormal law. The rms deviation of density for the subsonic case is consistent with the prediction of $M$ for the low-β case (CL03), and the rms deviation of the log density for the supersonic case is around unity regardless of the Mach number. The distributions are notably broader for higher Mach numbers, though. Similar PDFs have been observed in many supersonic simulations (see, e.g., Passot & Vázquez-Semadeni 2003).

The dimension of the high-density structures was between 1 and 2, being viewed as flattened filaments or elongated pancakes. There was no evident preferred orientation for these structures along or perpendicular to the mean magnetic field. The maximum density value in a Mach 10 data cube was around $3 \times 10^7 \rho_0$.

It is obvious that density clumps with values of 3–4 orders of magnitude of mean density can severely distort the power spectrum. It is expected that these clumps can hide density structures created by motions at small scales. We see in Figure 2 that the spectrum of density for a high Mach number is quite shallow and seems to be not as good a power law as a velocity spectrum.

![Fig. 1.—Probability density function for the density in a direct numerical simulation with an Alfvénic Mach number around unity and various sonic Mach numbers.](image1)

![Fig. 2.—$M = 10$, the power spectra of the density (solid line), the velocity (dashed line), and the logarithm of density (dotted line).](image2)
Randomly distributed high-density clumps will also suppress any anisotropy originating from motions at small scales.

We attempted to overcome this effect and reveal an underlying density scaling by using a log density instead of density\(^2\) for spectra and structure functions. We found this way of suppressing the influence of the high peaks on the spectrum or SF to be superior to other filtering procedures. Indeed, if we cut off the peaks at some level, it would give similar results, but the SF would look worse, as the procedure of cutting off, or restricting density to some level, introduces artificial structures in the real space.

The results and the comparison with the scalings of the magnetic field are presented in Figure 3. The magnetic field, being perturbed mostly by the Alfvén mode, shows anisotropy. Anisotropy of the density SF is not noticeable; however, the SF for log density exhibits pronounced anisotropy. Because the left and right panels of Figure 3 are very similar, in order to look specifically for scale-dependent anisotropy of the GS95 type \((r_s \sim k^{2/3})\), we plotted values of \(r_s\) with the same structure function on Figure 4. We see that the magnetic field SF is described with pretty good accuracy by GS95-type anisotropy between scales of 8 grid units and the driving scale. This is consistent with the results of Cho & Lazarian (2004) and with the concept that in the sub-Alfvénic case \((M_s < 1)\), strong Alfvénic turbulence is relatively unaffected by compressible motions, despite a high sonic Mach number. As to the log density, its anisotropy is of the GS95 type around the outer scale and becomes more scale-independent at around 10 grid units. This could be due to the fact that the simplistic model described in § 2 (in which shocks and rarefaction waves provide near white-noise random and isotropic perturbations of density, while Alfvénic waves shear these and provide the anisotropy and spectrum) might not work very well for Alfvénic Mach numbers close to unity (we expect it to work better for \(M_s \ll 1\), but still \(M_s \gg 1\)). This is also manifested by the fact that the power spectrum of the log density is not as close to Kolmogorov as the spectrum of the velocity (Fig. 2).

We also checked for a correlation between density and magnetic field magnitude, which was expected from models of external compression of an ideal MHD fluid. We have not found any significant correlation of this type.

5. DISCUSSION OF RESULTS

It is a well-known concept that supersonic turbulence consists mostly of shocks and other discontinuities. Our driving is incompressible, but the modes are not decoupled at the injection scale when the Alfvénic Mach number is of the order of unity (CL03). Therefore, we expect that the driving excites an appreciable amount of compressible motion. Indeed, our testing of data showed that the rms velocity associated with the slow mode was of the same order as the velocity of the Alfvén mode.

\[r_s \sim k^{2/3}\]
We assume that the resulting flat power spectrum of density is associated with the very large perturbation of density from compressible motions that naturally arise at the driving scale as a result of the coupling of compressible and incompressible motions quantified in CL03. Shocks in isothermal fluid can have very large density contrasts, up to sonic Mach squared, and can act as shocks in the snowplow phase of a supernova; namely, they collect matter, thus keeping the total momentum of the shock constant (see, e.g., Spitzer 1978). However, we do not see strong shocks near density clumps. In the magnetically dominated medium that we deal with here, it is reasonable to assume that the corresponding shocks move material along magnetic field lines in the same way that the slow modes do in the subsonic case. The shocks are randomly oriented, and therefore the clumpy structure that we observe does not reveal any noticeable anisotropy. Density perturbations associated with such shocks should not be correlated with the magnetic field strength enhancement, which is similar to the case of densities induced by slow modes (see CL03). Our analysis of the data confirms this. A correlation between the density and the magnitude of the magnetic field was observed in many super-Alfvénic simulations (see, e.g., Padoan & Nordlund 1999 and compare with the two-dimensional sub- and super-Alfvénic simulations by Passot & Vázquez-Semadeni 2003).

If we associate the clumps in simulations with interstellar clouds, in the interstellar medium with random driving we would expect the clouds not to be particularly oriented in relation to magnetic fields, at least until self-gravity becomes important. We observed substantial variation of the gas pressure, of 3 orders of magnitude, which is consistent with the findings in Jenkins (2002). The flat spectrum observed is roughly consistent with some observational data. Needless to say, a more systematic analysis of the data is required now when we have theoretical expectations to test, e.g., the change of the density spectrum with the Mach number. Testing the anisotropy of density is another interesting project, even though one cannot directly observe the log density, and the effects associated with the projection along the line of sight must be considered carefully (see discussion of this in Esquivel et al. 2003).

Surely, for real clouds, self-gravity can be important. This effect should make the observed spectrum even flatter, as the density peaks will become higher and more delta-function–like. In addition, cooling may make interstellar gas more pliable to compression than the isothermal gas that we used in the simulations. This could result in more density contrast when the original gas is warm. However, usually interstellar warm gas has a Mach number of the order of unity. For molecular clouds with substantial Mach numbers, our isothermal calculations seem to be adequate.

There were attempts to explain the isotropy of the density by the notion that the high-Mach turbulence has to be shock-dominated, and as it has no locality in k-space, driving could affect scalings down to small scales. This explanation is not self-consistent, however, as the power spectrum is rather flat, not the $\sim 2$ slope that it has to be in shock-dominated fields. Moreover, in our simulations we cannot say that the flow is shock-dominated. Also, it is natural to assume that the strong Alfvénic cascade will take over on scales in which the Alfvénic eddy turnover time is much smaller than the average time between the passage of shocks.

We understood the unusual behavior of density scalings as being the result of the density in high-Mach simulations perturbing significantly nonlinearly, therefore making the interpretation of the power spectra more involved. We used filtering to mitigate this effect and succeeded in showing that density scaling is anisotropic. The range of scales for which incompressible turbulent theory is applicable is shortened in a numerical simulation with supersonic driving. Between sub- and supersonic scales, there is a region where compressible motions cascade in a way that is yet to be understood.

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