Relational Diagrams: a pattern-preserving diagrammatic representation of non-disjunctive Relational Queries

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ABSTRACT
Analyzing relational languages by their logical expressiveness is well understood. Something not well understood or even formalized is the vague concept of relational query patterns. What are query patterns? And how can we reason about query patterns across different relational languages, irrespective of their syntax and their procedural or declarative nature? In this paper, we formalize the concept of query patterns with a variant of pattern-preserving mappings between the relational tables of queries. This formalism allows us to analyze the relative pattern expressiveness of relational query languages and to create a hierarchy of languages that have equal logical expressiveness yet different pattern expressiveness. We show that relational calculus can express a greater class of patterns than the basic operators of relational algebra. And we propose a complete sound diagrammatic representation of safe relational calculus that is not only relationally complete, but can also express all query patterns for the large and useful fragment of non-disjunctive relational calculus. Among all diagrammatic representations for relational queries that we are aware of, our Relational Diagrams are the only one that is relationally complete and that can represent all query patterns in the non-disjunctive fragment.

1 INTRODUCTION
When designing and comparing query languages, we are usually concerned about logical expressiveness: can a language express a particular query we want? For relational languages, questions of expressiveness have been studied for decades, and formalisms for comparing expressiveness are well developed and understood. Logical expressiveness basically boils down to the notion of logical equivalence: does a particular query in one language have a logically-equivalent representation in another?

We do not have the same sophisticated machinery to reason about the up-to-now vague notion of relational query patterns: can a given language express a particular query pattern from another language? We posit that identifying patterns in queries may have several advantages, akin to how formalizing best practices in software design patterns has aided software engineers [20]. For one, general and reusable query patterns could assist in teaching students how to write complicated queries. Queries written using common patterns could then potentially be easier to interpret quickly. But how would we actually define a relational query pattern? And what would it mean for a given target language to be able to express a particular pattern? Importantly, a formalisation of relational patterns should be applicable across identical fragments of the four important languages Datalog, Relational Algebra (RA), Relational Calculus (RC), and SQL, and thus be orthogonal to questions of syntax and language design. Furthermore, it should allow us to answer whether such patterns can be expressed to the same extent in all relational query languages, or whether there is a hierarchy of pattern expressiveness among popular languages.

Our 1st contribution: query patterns. We develop a language-independent notion of relational query patterns that allows us to compare the abilities of relational query languages to express query patterns present in other languages, and thus to reason about their relative pattern expressiveness. This approach allows us to contribute a novel hierarchy of pattern-expressiveness among the non-disjunctive fragment of above mentioned four languages. The intuition of our language-independent formalism is to reason in terms of mapping between the extensional relations used in two queries. However, it is not trivial to turn this intuition into a corresponding algorithm that can be applied to any relational query, no matter the language used for expressing it (we include examples to show that more intuitive mappings would fail on queries).

Example 1 (RA vs. Datalog). Consider the Datalog* query in Fig. 1g, which returns all tuples in R(A, B) whose attribute B does not appear in the unary table S(B). The query uses each of the input tables R and S exactly once. As we will later prove, there is no way to express this query in basic Relational Algebra (RA) by...
using each of the tables $R$ and $S$ only once. Figures 1b and 1e show two logically-equivalent queries in RA, each of which uses the table $R(A, B)$ twice (intuitively, Fig. 1b adds a column before applying a negation, whereas Fig. 1e adds the column after the negation). We also added equivalent Datalog$^{\sim}$ queries, which for those two RA expression use the exact same “logical pattern” (a concept we will formalize later). Intuitively (and we prove this later more formally), Datalog$^{\sim}$ can express strictly more query patterns than RA; it has a higher “pattern-expressiveness” despite having the same logical expressiveness. We believe that any diagrammatic language for illustrating and reasoning about query patterns used in queries should be able to express the full range of possible patterns across existing relational query languages (such as the one in Fig. 1e). It follows that any diagrammatic representation of relational queries that relies on a one-to-one mapping with the operators of RA cannot represent the full spectrum of query patterns of relational queries.

Our 2nd contribution: Relational Diagrams. Motivated by prior user studies [31, 37, 38] showing that diagrammatic representations of queries can help users understand them faster, we first discuss the basic limits of diagrammatic languages in contrast to textual languages to represent such patterns. We then design an arguably simple and intuitive diagrammatic representation of relational queries called Relational Diagrams that (i) is relationally complete and (ii) preserves the structure of query patterns in the fragment of non-disjunctive relational calculus.

Example 2. In the rightmost column of Fig. 1, we show Relational Diagrams (that we will formalize later) that use the same query patterns as the associated queries in Datalog$^{\sim}$ and RA to their left.

Outline of the paper. Section 2 defines the non-disjunctive fragment of relational query languages for Datalog, Relational Algebra (RA), Tuple Relational Calculus (TRC), and SQL, and proves that they have equivalent logical expressiveness.

Section 3 develops our formal approach for comparing the relative pattern expressiveness among relational query languages. We apply the approach and contribute a novel hierarchy of pattern expressiveness among the above four languages.

Section 4 shows that the non-disjunctive fragment allows for a very intuitive diagrammatic representation system we term Relational Diagrams. We give the formal translation from the non-disjunctive fragment of TRC to Relational Diagrams and back, and define their formal validity. We also prove that, for this fragment, Relational Diagrams have the same pattern expressiveness as TRC.

Section 5 adds a single visual element (a union at the root) to make Relational Diagrams relationally complete.\(^1\)

Section 6 makes a minor modification to the definition of Relational Diagrams that allows them to also represent logical sentences (or, equivalently, Boolean queries or logical constraints). This extension allows us to compare our diagrammatic formalism against a long history of diagrams for representing logical sentences.

Section 7 contrasts our formalism with selected related work. In particular, we discuss the connection to Peirce’s existential graphs [35, 40, 42] and show that our formalism is more general and solves interpretational problems of Peirce’s graphs, which have been the focus of intense research for over a century.

Due to space constraints, we had to move all proofs and several intuitive illustrating examples to the appendix.

2 THE NON-DISJUNCTIVE FRAGMENT OF RELATIONAL QUERY LANGUAGES

In this section, we define the non-disjunctive fragment of relational query languages. We focus here on non-Boolean queries and discuss Boolean queries (logical sentences) later in Section 6.

We assume the reader to be familiar with Datalog$^{\sim}$ (Non-recursive Datalog with negation), RA (Relational Algebra), TRC (safe Tuple Relational Calculus), SQL (Structured Query Language), and the necessary safety conditions for TRC and Datalog$^{\sim}$ to be equivalent in logical expressiveness to RA. We also assume familiarity with concepts such as relations, predicates, atoms, and the named and unnamed perspective of relational algebra. Currently, the most comprehensive exposition of these topics we know of is Ullman’s 1988 textbook [45], together with resources for translating between SQL and relational calculus [6, 17]. These connections are also discussed in most database textbooks [18, 21, 36, 43], though in less detail. To save space, we build upon those well-established results and focus on the novel concepts introduced in this paper. We do not cover safe Domain Relational Calculus (DRC) as we will show later in Section 7.1 that TRC has a more natural translation into diagrams.

2.1 Why disjunctions are harder to represent

Assume Alice calls Bob and tells him “I see a blue car that has a flat tire.” What is the mental image that Bob has from this information? It is a car with two conditions: it is blue, and it has a flat tire as in Fig. 2a. Next, instead assume that Alice instead tells Bob “I see a car that is either blue or that has a flat tire.” What is the mental image that Bob has from that information? There is no single mental image that could capture that situation. Bob needs to imagine two different images. If Bob sees one image with two different cars (one blue, the other with a flat tire), then he actually sees two separate cars. Bob needs to add some additional visual symbol representing the logic that those are two different overlapping possible worlds. In other words, any situation (think of a concrete arrangement of items) can only display conjunctive information [42]. This diagrammatic representation problem is not as apparent in text: “Car.color = ‘blue’ OR Car.tire = ‘flat’.”

Next, we define non-disjunctive fragments of four query languages. We will show in Section 4 that these languages lend themselves to a rather natural diagrammatic representation that we term Relational Diagrams, which have nice structure-preserving properties (Section 3). Further on in Section 5, we achieve relational completeness by adding an additional visual construct. In the following subsections, we use the SQL query in Fig. 8d over a schema

\(^1\)Although disjunctions can be composed from conjunction and negation using De Morgan’s law ($A \lor B = \neg (\neg A \land \neg B)$), this additional visual symbol is necessary: for safe relational queries, DeMorgan is not enough, as there is no way to write a safe Tuple Relational Calculus (TRC) expression “Return all entries that appear in either $R$ or $S$” that avoids a union operator. This argument is part of the textbook argument for the union operator being an essential, non-redundant operator for relational algebra.

\(^2\)To provide some additional intuition, recall that conjunctions of selections can be simply modeled as concatenation of simple selections, e.g. $\sigma_{C_1 \land C_2} (R)$ is the same as $\sigma_{C_1} (\sigma_{C_2} (R))$. Thus conjunctions are an inherently more natural logical connective than disjunctions; disjunction cannot be represented without additional visual symbols.
We focus on the fragment of basic Algebra (canonically as $\neg(\exists \sigma \in R [r.A = 0])$ because the table variable $r$ is defined outside the scope of the most inner negation around the predicate $r.A = 0$. However, we allow the logically-equivalent $\neg((\exists r \in R[r.A \neq 0])$ where the table variable $r$ is existentially quantified within the same scope as the attribute $r.A \neq 0$.

**Definition 3 (Anchored predicate).** A predicate is anchored if it contains at least one attribute of a table that is existentially quantified inside the same negation scope as that predicate.

Intuitively, this anchoring condition guarantees that the predicates can be applied in the same logical scope where a table is defined. This requirement also avoids a hidden disjunction. To be illustrated, consider the following TRC query:

$$\{ q(A) | \exists r \in R[q.A = r.A \land (\exists s \in S[s.B = r.B]) \} \}$$

This query contains no apparent disjunction, however the predicate "$r.A = 0$" could be pulled outside the negation, and after applying De Morgan’s law on the expression we get a disjunction:

$$\{ q(A) | \exists r \in R[q.A = r.A \
and (\exists s \in S[s.B = r.B]) \} \}$$

To avoid disjunctions and “hidden disjunctions” the non-disjunctive fragment avoids disjunctions entirely and also requires that predicates are pulled up in the nesting hierarchy as much as possible.

**Definition 4 (TRC⁺).** The non-disjunctive fragment of safe TRC restricts predicates to conjunctions of anchored predicates.

In order to express Fig. 8d, we need the disjunction operator. Two possible translations are:

$$\{ q(A) | \exists r \in R, s \in S, t \in T[q.A = r.A \land (r.A = s.A \lor r.A = t.A)] \}$$

and:

$$\{ q(A) | \exists r \in R, s \in S, t \in T[q.A = r.A \land r.A = s.A] \lor 
\exists r \in R, s \in S, t \in T[q.A = r.A \land r.A = s.A] \}$$

**2.5 SQL under set semantics**

Structured Query Language (SQL) uses bag instead of set semantics and uses a ternary logic with NULL values. In order to treat SQL as a logical query language, we assume binary logic and no NULL values in the input database. It has been pointed out that “SQL’s logic of nulls confuses people” and even programmers tend to think in terms of the familiar two-valued logic [13]. Our focus here is devising a general formalism to capture logical query patterns across relational languages, not on devising a visual representation for SQL’s idiosyncrasies. To emphasize the set semantic interpretation, we write the DISTINCT operator in all our SQL statements.

We define the non-disjunctive fragment of SQL as the syntactic shown in Fig. 3, interpreted under set semantics (no duplicates by using DISTINCT) and under binary logic (no null values allowed in the input tables). Notice we also have the same syntactic restriction as for TRC⁺: every predicate needs to be anchored (Definition 3), i.e. reference at least one table within the scope of the last NOT.

**2.2 Non-recursive Datalog with negation**

We start with Datalog since the definition is most straightforward. Datalog expresses disjunction (or union) by repeating an Intentional Database predicate (IDB) in the head of multiple rules. For example, Fig. 8d expressed in Datalog results from disallowing the union operators or the union operator $\cup$ as in:

$$\{ q(A) | \exists r \in R[q.A = r.A \land (\exists s \in S[s.B = r.B]) \} \}$$

Two possible translations are:

$$\{ q(A) | \exists r \in R, s \in S, t \in T[q.A = r.A \land (r.A = s.A \lor r.A = t.A)] \}$$

and:

$$\{ q(A) | \exists r \in R, s \in S, t \in T[q.A = r.A \land r.A = s.A] \lor 
\exists r \in R, s \in S, t \in T[q.A = r.A \land r.A = s.A] \}$$

**2.3 Relational Algebra (RA)**

We focus on the fragment of basic RA that contains no union operator $\cup$ and in which all selection conditions are simple (i.e., they do not use the disjunction operator $\lor$). A simple condition is $C = (XθY)$ where $X$ is an attribute, $Y$ is either an attribute or a constant, and $θ$ is a comparison operator from $\{=, \neq, <, \leq, >, \geq\}$. Notice that conjunctions of selections can be modeled as concatenation of selections, e.g., $σ_{C_1 \land C_2}(R)$ is the same as $σ_{C_1}(σ_{C_2}(R))$. Fig. 8d cannot be expressed in that fragment and requires either the disjunction operator $\lor$ or as in:

$$\pi_A(σ_{A \lor B}(C(T)) \land σ_{A \lor B}(C(T)))$$

or the union operator $\cup$ as in:

$$\pi_A(R \times S \times σ_{A \lor B}(C(T))) \cup \pi_A(R \times T \times σ_{A \lor B}(S))$$

**Definition 2 (RA⁺).** The non-disjunctive fragment of Relational Algebra (RA⁺) results from disallowing the union operators $\cup$ and by restricting selections to conjunctions of simple predicates.

**2.4 Tuple Relational Calculus (TRC)**

Recall that safe TRC only allows existential quantification (and not universal quantification) [45]. Predicates are either join predicate "$r.A \theta s.B$" or selection predicates "$r.A \theta a$", with $r, s$ being table variables and $a$ a domain value. WLOG, every existential quantifier can be pulled out as early as to either be at the start of the query, or directly following a negation operator. For example, instead of $\neg(\exists r \in R[r.A = 0 \land \exists s \in S[s.B = r.B])$, we rather write this sentence canonically as $\neg((\exists r \in R, s \in S[r.A = 0 \land s.B = r.B])$. This canonical representation implies that a set of existential quantifiers is always predated by the negation operator, except for the table variables outside any scope of negation operators.

We will define an additional requirement that each predicate contains a local (or what we refer to as anchored) attribute whose table is quantified within the scope of the last negation. For example, we do not allow $\neg((\exists r \in R[r.A = 0])$ because the table variable $r$ is defined outside the scope of the most inner negation around the predicate $r.A = 0$. However, we allow the logically-equivalent $\neg((\exists r \in R[r.A \neq 0])$ where the table variable $r$ is existentially quantified within the same scope as the attribute $r.A \neq 0$.

**I see a car that is blue and that has a flat tire (“blue” AND “flat tire”)**

**(a)**

**I see a car that is blue or that has a flat tire (“blue” OR “flat tire”)**

**(b)**

**Figure 2: Showing a car that has a blue color or a flat tire.**

$R(A), S(A), T(A)$ as running example for a query that cannot be expressed in the non-disjunctive fragment.
We show that the 4 previously defined non-disjunctive fragments addition, all predicates (joins and selections) need to be anchored.

**Definition 5.** SQL*: Non-disjunctive SQL under set semantics (SQL*) is the syntactic restriction of SQL under binary logic (no NULL values in the input tables) to the grammar defined in Fig. 3, and additionally requiring that every predicate is anchored.

Each such query can be brought into a canonical form that shows a straightforward one-to-one correspondence with TRC⁺, which will simply our later discussion. The idea is to replace membership and quantified subqueries with existential subqueries (Fig. 3) and then unnest any existential quantifiers, i.e., to only use "NOT EXISTS". This pulling up quantification as early as possible is logically identical to the way we defined the canonical form of TRC⁺.

### 2.6 Logical expressiveness of the fragment

We show that the 4 previously defined non-disjunctive fragments are equivalent in their logical expressiveness. The proof is available to the appendix and is an adaptation of the standard proofs of equal expressiveness as found, for example, in [45]. However, the translations also need to pay attention to the restricted fragment (e.g., we cannot use union to define an active domain). The translations between the languages attempt to keep the numbers of existential quantifiers, i.e., language-specific appearances of relational input table in a query. These two definitions allows us to refer to the individual appearance of extensional tables in a query, irrespective of the language.

**Definition 7 (Extensional table).** We call an extensional table any existentially-quantified and language-specific appearance of a relational input table in a query.

**Definition 8 (Query representation).** Given a query \( q(T) \) that maps a set of database tables \( T \) to an output table using a signature \( \overline{F} \) of extensional tables, we call the query representation \( \overline{q}(\overline{F}) \) of query \( q \) the language-specific representation of the query.

The intuition is that every relational query language has language-specific ways to reference tables, which are then used to define the actual query. These two definitions allows us to refer to the existentially-quantified input relations while abstracting away details of the language. Recall that a query can quantify the same input relation more than once. For example, the Datalog query

\[
Q(x) := R(x, y), R(\ldots, y)
\]

has two extensional tables, both of which refer to the same input table \( R \). It thus represents a relational function \( q(R) \) that maps a any valid instance of table \( R \) to an output table. Its query representation however is \( \overline{q}(\overline{R}, R) \) since it uses two occurrences of \( R \), i.e. two extensional tables.

We can now analyze the query defined by a query representation, treating each extensional table as different. Thus, from \( \overline{q}(\overline{R}, R) \), we consider what we call its shattered query \( Q'(R_1, R_2) \) defined as \( Q'(x) := R_1(x, y), R_2(\ldots, y) \). Notice that if we replace the signature \( (R_1, R_2) \) of the shattered

\[\text{THEOREM 6.} \text{ [Logical expressiveness] Datalog}, RA^*, \text{TRC}^*, \text{and SQL}^* \text{ have the same logical expressiveness.}\]

### 3 PATTERN-PRESERVING MAPPINGS BETWEEN QUERIES ACROSS LANGUAGES

Our goal is to establish a formalism that allows us to reason about the so-far vague notion of a relational query pattern. We wish to use this formalism to compare relational query languages by their relative abilities to express the query patterns. Thus, our definitions need to be applicable to all relational query languages, irrespective of their syntax and language-dependent peculiarities.

#### 3.1 Mapping patterns across queries

**Intuition.** Our idea is to formalize patterns based on the only common symbols in queries across languages: the input relations from the database. Intuitively, we will define two queries to be pattern-isomorph if there is a 1-to-1 mapping between the relational input tables across the queries such that appropriately synchronized changes to the input tables (e.g., inserting another tuple) will keep the two queries logically equivalent. We call any such 1-to-1 correspondence between queries a pattern-preserving mapping.

For a definition of pattern-preserving mapping to correctly identify patterns instead of logical equivalence, we need to be able to treat repeated instances of the same input table (also called self-joins) in a query as independent. As example, consider the relational query \( R \leftarrow (\pi_A R \times S) \) from Fig. 1b. From a logical point of view, the query is a function \( q(R, S) \) that maps input relations \( R \) and \( S \) to a binary output table. In that perspective of queries as functions mapping relations to an output relation, we call the signature of this query its relational input \( (R, S) \). We then consider what we call the query representation \( \overline{q}(R, R, S) \) which represents the function mapping three explicitly-referenced tables to an output. We then call the query \( q'((R_1, R_2, S) = R_1 - (\pi_A R_2 \times S) \) that treats all explicit tables as distinct as shattered query of \( q \). And we define the relational query pattern to be that shattered query.

**Formalisation.** To make these intuitions precise across the varying syntax of relational query languages, we need a way to refer to the individual appearance of extensional tables in a query, irrespective of the language.

**Definition 7 (Extensional table).** We call an extensional table any existentially-quantified and language-specific appearance of a relational input table in a query.

**Definition 8 (Query representation).** Given a query \( q(T) \) that maps a set of database tables \( T \) to an output table using a signature \( \overline{F} \) of extensional tables, we call the query representation \( \overline{q}(\overline{F}) \) of query \( q \) the language-specific representation of the query.

The intuition is that every relational query language has language-specific ways to reference tables, which are then used to define the actual query. These two definitions allows us to refer to the existentially-quantified input relations while abstracting away details of the language. Recall that a query can quantify the same input relation more than once. For example, the Datalog query \( Q(x) := R(x, y), R(\ldots, y) \) has two extensional tables, both of which refer to the same input table \( R \). It thus represents a relational function \( q(R) \) that maps a any valid instance of table \( R \) to an output table.

Its query representation however is \( \overline{q}(\overline{R}, R) \) since it uses two occurrences of \( R \), i.e. two extensional tables. We can now analyze the query defined by a query representation, treating each extensional table as different. Thus, from \( \overline{q}(\overline{R}, R) \), we consider what we call its shattered query \( Q'(R_1, R_2) \) defined as \( Q'(x) := R_1(x, y), R_2(\ldots, y) \). Notice that if we replace the signature \( (R_1, R_2) \) of the shattered

\[\text{Recall that an isomorphism is a structure-preserving mapping between two structures that can be reversed. For it to be reversible, it needs to be surjective (each element in the target is mapped to) and injective (different elements in the source need to map to different elements in the target)} \]
query \( Q' \) with the signature \((R, R)\) of the query representation \( \overline{Q} \), we get a logically equivalent formulation of the original query \( Q \).

**Definition 9 (Shattered query).** Given a query \( q(T) \) with representation \( \overline{q}(T) \), we call \( q'(\overline{T}) \) its shattered query if \( q'(\overline{T}) \equiv q(T) \).

In the above example, \( Q' (R_1, R_2) \) with signature \( \overline{T}' = (R_1, R_2) \) is the shattered query of \( Q(R) \) because \( Q'(R, R) \equiv Q(R) \). Both are \( Q(x) : = R(x, y), R(\_ , y) \). The intuition behind our formalism is that the shattered query defines a function that maps a set of extensional tables (not just a set of tables) to an output table. Thus the shattered query is a semantic definition of a relational query pattern across different relational query languages. Two queries the use the same query pattern if their shattered queries are logically equivalent, up to renaming and reordering of the input tables. We first give the formal definition and then illustrate with more examples.

**Definition 10 (Query pattern).** The semantics of a query pattern for a relational query \( q(T) \) is defined by its shattered query \( q'(\overline{T}') \).

**Definition 11 (Pattern isomorphism).** Given two queries \( q_1 \) and \( q_2 \) with shattered queries \( q_1' (\overline{T}') \) and \( q_2' (\overline{T}'') \). The queries are pattern-isomorphic iff there is a bijective homomorphism \( h : \overline{T}' \rightarrow \overline{T}'' \) such that \( q_1'(\overline{T}') \equiv q_2'(h(\overline{T}')) \).

**Example 3 (Different patterns).** We give an example of logically-equivalent queries that use arguably different query patterns. Consider table \( R(A, B) \) and the two queries \( Q_1 (R) \) and \( Q_2 (R) \) with

\[
Q_1 (x) : = R(x, \_), R(x, \_).
\]

\[
Q_2 (x) : = R(\_ , y), R(_, y).
\]

Both queries are logically equivalent to \( Q(x) : = R(x, \_), \) and thus also logically equivalent to each other. However, \( Q_1 \) and \( Q_2 \) represent arguably different patterns: \( Q_1 \) never uses the second attribute of \( R \) whereas \( Q_2 \) uses it to join both occurrences of \( R \). This difference becomes even more apparent when writing the two queries with the same join pattern in SQL: Fig. 4a would even work if \( R \) was unary, whereas Fig. 4c requires \( R \) to be at least binary.

We next show that shatteredness allows us to formally distinguish the two patterns in the queries, i.e., they are not pattern-isomorphic.

The shattered queries are \( Q'_1 (R_1, R_2) \) and \( Q'_2 (R_3, R_4) \) with

\[
Q'_1 (x) : = R_1(x, \_), R_2(x, \_).
\]

\[
Q'_2 (x) : = R_3(x, y), R_4(_, y).
\]

Neither of the two possible mappings between the shattered queries' extensional tables, \( h_1 = \{(R_1, R_3), (R_2, R_4)\} \) nor \( h_2 = \{(R_1, R_4), (R_2, R_3)\} \), preserves logical equivalence for the shattered queries.

However, \( Q_1 \) is pattern-isomorphic to the TRC query \( q_3 (R) \) with

\[
\{q_3(A) \mid \exists r_1 \in R, r_2 \in [q.A = r_1.A \land r_1.A = r_2.A]\}
\]

To see that, notice that its shattered query \( q'_3(R_5, R_6) \) with

\[
\{q'_3(A) \mid \exists r_1 \in R_5, r_2 \in R_6 [q.A = r_1.A \land r_1.A = r_2.A]\}
\]

allows the reversible mapping \( h_1 = \{(R_1, R_5), (R_2, R_6)\} \) from \( Q'_1 \) to \( q'_3 \) that preserves logical equivalence.

By the same arguments, \( Q_1 \) is pattern-isomorphic to the query in Fig. 4a, and \( Q_2 \) is pattern-isomorphic to the query in Fig. 4c.

Our formalism is similar in spirit to edge-preserving graph homomorphisms that map two nodes in graph \( G_1 \) linked by an edge to two nodes in graph \( G_2 \) that are also linked by an edge. In our pattern-preserving isomorphisms between queries, the role of nodes is played by the extensional tables in the queries and the queries themselves play the role of the edges. Notice the difference to well-known homomorphisms between conjunctive queries for determining query containment [9]: in that formalism, the role of nodes is played by variables (and constants) and the relational atoms play the role of edges. Also notice from Example 3 that a simpler mapping between the relational symbols (instead of the repeated extensional tables) between two queries alone would not work.

Notice that—by design—our definition does not classify different join orders as different query patterns. Also—by design—our definition does not include any notion of views or intermediate tables. This is achieved by excluding Intensional Database Predicates (IDBs) (as in Datalog) from the definition of extensional tables. We again illustrate the intuition for that design with examples.

**Example 4 (Join orders and views do not affect patterns).** Assume that the edges of a directed graph are stored in a binary relation \( E(A, B) \). Consider a query returning nodes which form the starting point of a length 3 directed path. We write the query in two different ways in unnamed RA where indices replace attribute names [1]. The first query applies projections as late as possible, whereas the second query applies the projections as early as possible:

\[
q_1(E) = \pi_1 \sigma_{z \leq 3}(E \times E \times E)
\]

\[
q_2(E) = \pi_1 \sigma_{z \leq 3}(E \times E \times \pi_1(E))
\]

We also write these two queries in the more named perspective of RA. These queries encode the same algebraic operations but are more verbose since the named perspective of RA requires a rename operator to unambiguously express the identical queries:

\[
q_1(E) = \pi_{E.A,B,E.F,B,E.G,A}(E \times \rho_{E \rightarrow F}E \times \rho_{E \rightarrow G}E)
\]

\[
q_2(E) = \pi_{E.A,B,E.F,B,E.G,A}(E \times \pi_E.A,F,B,G,A(E \times F.E \times G.E \times \pi_E.G.A(E))
\]

Both RA queries use the same relational pattern according to our definition, and we think the distinction between query patterns and join orders is important: if join orders determined relational query pattern, then relational query patterns would be inherently tied to relational algebra; concepts of join order and early projections are not meaningful in the context of declarative logical query languages. To see that consider the logically-equivalent query in Datalog:

\[
Q_3(x) : = E(x, y), E(y, z), E(z, w).
\]

Query \( Q_3 \) logically specifies on what attributes the three tables need to be joined, but it does not specify any order or joins nor when projections happen. Furthermore notice that RA query \( q_1 \) does not even specify a join order between the three extensional tables.

For a similar reason, temporary tables such as Intensional Database Predicates (IDBs) in Datalog do not count as extensional tables. Thus, the following Datalog query uses the same logical pattern (find three edges that join and keep the starting node), even though it defines the intermediate intensional database predicate 1:

\[
I(y) : = E(y, z), E(z, w).
\]

\[
Q_4(x) : = E(x, y), I(y).
\]
Notice that it follows immediately that two pattern-isomorph queries need to have the same number of extensional tables. We believe that such a pattern-preserving mapping between queries is important if we want to help readers understand the exact logical pattern behind a relational query, irrespective of the language it is written in. In particular, if we want to help users understand the logic of an existing relational query (recall that we focus on set semantics and binary logic), the diagrammatic representation needs to preserve this 1-to-1 correspondence with the query.

3.2 Comparing relational languages in terms of “pattern-expressiveness”

We next add the final definition needed to formally compare relational query languages based on their relative abilities to represent query patterns.

**Definition 12 (Representation equivalence).** We say that a query language \( L_2 \) can pattern-represent a query language \( L_1 \) (written as \( L_1 \sqsubseteq_{\text{rep}} L_2 \)) iff for every legal query \( q_1 \in L_1 \) there is a pattern-isomorphic query \( q_2 \in L_2 \). We call a query languages \( L_2 \) pattern-dominating another language \( L_1 \) (written as \( L_1 \sqsubseteq_{\text{rep}} L_2 \)) iff \( L_1 \sqsubseteq_{\text{rep}} L_2 \) but \( L_1 \not\sqsubseteq_{\text{rep}} L_2 \). We call \( L_1, L_2 \) representation equivalent (written as \( L_1 \equiv_{\text{rep}} L_2 \)) iff \( L_1 \sqsubseteq_{\text{rep}} L_2 \) and \( L_1 \sqsupseteq_{\text{rep}} L_2 \), i.e. if both language can represent the same set of patterns.

We are now ready to state our result on the hierarchy of pattern expressiveness of these four relational query languages. Recall that we are only considering the non-disjunctive fragment of the relational calculus has relational patterns that can represent all relational query patterns from TRC.

**Theorem 13.** \( RA^* \sqsubseteq_{\text{rep}} Datalog^* \sqsubseteq_{\text{rep}} TRC^* \equiv_{\text{rep}} SQL^* \).

Thus, we prove that relational calculus has relational patterns that cannot be expressed in relational algebra. In more detail, our proofs available in the appendix show that there exists a query for which \( RA^* \) needs 50% more extensional tables than Datalog*, and there exists a query for which Datalog* needs 33.3% more extensional tables than TRC* or SQL*. The important consequence is that \( RA^* \), Datalog* or any diagrammatic language modeled after them would not be a suitable target language for helping users understand all existing relational query patterns (including those used by SQL*).

We will later see that most existing visual query representations are modeled after relational algebra in that they model data flowing between relational operators, which implies they cannot truthfully represent all relational query patterns from TRC or SQL*.

4 RELATIONAL DIAGRAMS

This section introduces the basic visual elements of Relational Diagrams (Section 4.1). It gives the formal translation from TRC* (Section 4.2) and back (Section 4.3), showing that there is a one-to-one correspondence between TRC* expressions and Relational Diagrams and proving their validity (Section 4.4).

4.1 Visual elements

In designing our diagrammatic representation, we started from existing widely-used visual metaphors and then added the minimum necessary visual elements to obtain expressiveness for full TRC*. In the following 5 points, we discuss both (i) necessary specifications for Relational Diagrams, and (ii) concrete design choices that are not formally required but justified based on HCI and visualization guidelines and best practices. We use the term canvas to refer to the plane in which the Relational Diagram is displayed. Figure 6d displays several of the examples discussed next.

(1) **Tables and attributes:** We use the set-of-mappings definition of relations [45] in which a tuple is a mapping from attributes’ names to values—in contrast to the set-of-lists representation in which order of presentation matters and which more closely matches the typical vector representation. Thus a table is represented by any visual grouping of its attributes. We use the typical UML convention to values—in contrast to the set-of-lists representation in which order of presentation matters and which more closely matches the typical vector representation. Thus a table is represented by any visual grouping of its attributes. We use the typical UML convention of representing tables as rectangular boxes with table name on top and attribute names below in separate rows. For maximum legibility, we ensure that text has maximal luminance contrast with its background [34]. Table names are shown with white text on a black background and, to differentiate them, attributes use black text on a white background. For example, table \( R \) with attribute \( A \). To simplify query interpretation, we do not use table aliases, similar to Datalog and RA (and different from SQL and TRC). We also reduce visual complexity by only showing attributes that are used in the particular query, similar to SQL and TRC (and different from Datalog). Database users are commonly familiar with relational schema diagrams. Thus, we argue that a simple conjunctive query should be visualized similarly to a typical database schema representation, as used prominently in standard introductory database textbooks [18, 43]. We illustrate with Fig. 6d.

(2) **Selection predicates:** Selection predicates are filters and are shown “in place.” For example, an attribute “\( r_2.C > 1 \)” is shown as \( C>1 \) in the corresponding instance of table \( R \). An attribute participating in multiple selection predicates is repeated at least as many times as there are selections (e.g., to display “\( r_2.C > 1 \land r_2.C < 3 \)”, we would repeat R.C twice as \( C>1 \) and \( C<3 \)). An attribute participating in \( k \) selection predicates, then we repeat it \( k \) times.

(3) **Join predicates:** Equi-join predicates (e.g., “\( r_2.A = t_2.A \)”), which arguably are the most common type of join in practice, are
represented by lines connecting the joined attributes. For the other less-frequent theta join operators \( \{ \neq, <, \leq, >, \geq \} \), we additionally place the operator as a label on the line and use an arrowhead to indicate the reading order and correct application of the operator in the direction of the arrow. For example, for a predicate \( r_1.A > r_2.B \), the label is \( > \) and the arrow points from attribute A of the first R occurrence to B of the second: A \( \rightarrow \) B. Notice that the direction of arrows can be flipped, along with flipping the operator, while maintaining the identical meaning: A \( \leftarrow \) B. To avoid ambiguity with the standard left-to-right reading convention for operators, we normalize arrows to never point from right to left. An attribute participating in multiple join predicates needs to be shown only once and has several lines connecting it to other attributes. An attribute participating in one or more join predicates and \( k \) selection predicates, is shown \( k + 1 \) times.

(4) Negation boxes: In TRC\(^+\), negations are either avoided (e.g., \( \neg(R.A = S.B) \) is identical to \( R.A \neq S.B \)) or placed before the existential quantifiers. We represent a negation with a closed line that partitions the canvas into a subcanvas that is negated (inside the bounding box) and everything else that is not (outside of the bounding box). As convention, we use dashed rounded rectangles.\(^6\) Recursive partitioning of the canvas allows us to represent a tree-based nesting order that corresponds to the nested scopes of quantified tuple variables in TRC (and also the nesting order of subqueries in SQL). We call the main canvas the root of that nesting hierarchy and each node a partition of the canvas.

(5) Output table: We display an output table to emphasize the compositional nature of relational queries: a relational query uses several tables as input, and returns one new table as output. We use the same symbol for that output table as the TRC expression, for which we most commonly use Q. We use a gray background \( \square \) to make this table visually distinct from input tables. As the title is always Q, the reduced luminance contrast of white on gray is acceptable as it little impacts query readability.

### 4.2 From TRC to Relational Diagrams

We next describe the 5-step translation from any valid TRC\(^+\) expression to a Relational Diagram. We also illustrate by translating the TRC\(^+\) expression from Fig. 6a into the Relational Diagram from Fig. 6d. Notice that the translation critically leverages 3 conditions fulfilled by the input: (1) Safe TRC (and thus also TRC\(^+\)) only allows existential and not universal quantification [45]. (2) TRC\(^+\) only allows conjunction between predicates, and (3) all predicates in TRC\(^+\) are anchored (recall Definition 3).

(1) Creating canvas partitions: The scopes of the negations in a TRC are nested by definition. We translate this hierarchy of the scopes for each negation (the negation hierarchy) into a nested partition of the canvas. Fig. 6c illustrates the nested partitions as derived from the negation hierarchy Fig. 6b of the original TRC\(^+\).

\[ \{ q(A, D) \mid \exists r_1 \in R, \exists r_2 \in R, \exists s_1 \in S \mid q.A = r_1.A \land q.D = r_2.C \land r_2.C > 1 \land r_2.C < 3 \land r_1.A > r_2.B \land \neg(\exists t_1 \in T[t_1.A = r_1.A]) \land \neg(\exists s_2 \in S, \exists t_2 \in T, \exists u \in U[s_2.A = t_2.A \land s_2.B > s_1.A \land \neg(\exists r_3 \in R[r_3.A \neq 1]) \land \neg(\exists r_4 \in R[r_4.B \neq s_2.B]) \} \]  

(a) TRC\(^+\)

(b) Negation hierarchy

(c) Canvas partitions

(d) Relational Diagram

Figure 6: Example TRC\(^+\) expression (a), its corresponding Relational Diagram (d), and derivation of the nesting hierarchy (b, c). Colored partitions \( q_1 \) (purple) and table variables \( r_1 \) (blue) are not part of Relational Diagrams and displayed to discuss the correspondence.

expression. Notice that the double negation “\( \neg(\neg(\ldots)) \)” results in the scope \( q_1 \) of the negation hierarchy to be empty.

(2) Placing tables: Each table variable defines a table that gets placed into the canvas partition that corresponds to the respective negation scope. For example, the tables corresponding to the table variables \( r_1, r_2, \) and \( s_1 \) are all outside any negation scope and are thus placed in the root partition \( q_0 \). Notice that similar to Datalog and RA (and in contrast to TRC and SQL), Relational Diagrams do not need table aliases.

(3) Placing selection predicates: The predicates within each scope are combined via conjunction and are thus added one after the other. Since all selection predicates are anchored, the selection predicates can be placed in the same partition as their respective table, which allows correct interpretation (see Section 4.3). For example, for \( \neg(\exists r_3 \in R[r_3.A \neq 1]) \), the predicate “\( A \neq 1 \)” is placed directly below \( R \) in \( q_4 \). An example of a predicate that is not anchored would be \( \exists r_3 \in R[\neg(\exists r_3.A = 1)] \): the scope of the negation contains a predicate of a table that is not existentially quantified in that scope.

(4) Placing join predicates: For each each join predicate, we add the two attributes (if not already present) and connect them via an edge with any comparison operator drawn at the middle. An attribute participating in multiple join predicates needs to be shown only once. Equi-joins are the standard and no operator is shown.
Asymmetric joins include an arrowhead at one end of the edge (see Section 4.1). Since for anchored join predicates at least one of the two attributes is in the partition of a local table, the negation can be correctly interpreted. An example of a predicate that is not anchored would be $\neg(r_4.B = s_2.B)$. What is possible is the logically-equivalent $r_4.B \neq s_2.B$ (as long as one of the two attributes is in the local scope of the last negation. In our example, this is the case in $\neg(\exists r_4 \in R [r_4.B \neq s_2.B])$).

5. **Place and connect output table** The safety conditions for TRC [45] imply that the output predicates can only be chosen from tables outside of all negations, thus in the root scope or partition $q_0$. We place an additional table with a new name (we commonly use $Q$ for query) and use a unique gray background to imply the difference from extensional tables.

**Completeness.** Notice that this 5-step translation guarantees uniqueness of the following aspects: (1) nesting hierarchy (corresponding to the negation hierarchy), (2) where tables are placed (canvas partitions corresponding to the negation scope), (3) which attributes have selection predicates, and (4) which attributes participate in joins and how. The following aspects are not uniquely defined (without impact on the later interpretation): (1) the order of attributes below each table; (2) the direction of arrows can be flipped with simultaneous label flip e.g., $A \leftarrow s_2.B$ and $s_1.A \leftarrow s_2.B$ are identical (by convention we avoid arrows from right-to-left, but allow them up-to-down and down-to-up); (3) the size of visual elements and their relative arrangement; and (4) any optional changes in style (e.g., other than dashed negation boxes, distinct visual appearance between tables and attributes).

### 4.3 From a Relational Diagram to TRC

We next describe the reverse 5-step translation from any valid Relational Diagram to a valid and unique TRC expression. At the end, we summarize the conditions of a Relational Diagram to be valid, which are the set of requirements listed for each of the 5 steps. We again illustrate with the examples from Fig. 6.

1. **Determine the nested scopes of negation:** From the nested canvas partitions (Fig. 6c), create the nested scopes of the negation operators of the later TRC expression (Fig. 6b).

2. **Quantification of table variables:** Each table in a partition corresponds to an existentially-quantified table variable. WLOG, we use a small letter indexed by number of occurrence for repeated tables. We add those quantified table variables in the respective scope of the negation hierarchy (Fig. 6c). For example, table $T$ in $q_2$ becomes $\exists r_1 \in T [\ldots]$ and replaces $q_2$ in Fig. 7. Notice that partition $q_1$ is empty and the resulting negation scope does not contain any expression other than another negation scope. We require that the leaves of the partition are not empty and contain at least one table. Otherwise, expressions $\land \neg()$ and $\land \neg\neg()$ would both have to be true, leaving the meaning of an empty leaf partition ambiguous. This also implies that an empty canvas (there is only one partition, in which root and leaf are empty) is not a valid Relational Diagram.

3. **Selection predicates:** Selection attributes are placed into the scope in which its table is defined. For example, the predicate $R.A \neq 1$ in partition $q_3$ leads to $\neg(\exists r_3 \in R [r_3.A \neq 1])$.

4. **Join predicates:** For join predicates (lines connecting attributes in Relational Diagrams with optional direction and operator), we have a validity condition that they can only connect attributes of tables that are in the same partition or different partitions that are in a direct-descendant relationship. In our example, $T.A$ in $q_2$ connects to $R.A$ in $q_0$ (here $q_0$ is the root and grandparent of $q_2$.) However, we could not connect any attribute in $q_3$ with any attribute in $q_4$ (which are siblings in the nesting hierarchy). This requirement is the topological equivalent of scopes for quantified variables in TRC and guarantees that only already-defined table variables are referenced. Each such predicate is placed in the scope of the lower of the two partitions in the hierarchy, which guarantees the predicate to be anchored. For example, the inequality join connecting $S.B$ in $q_3$ and $R.B$ in $q_5$ is placed in the scope of $q_5$.

5. **Output table:** The validity condition for the output table is that each of its one or more attributes is connected to exactly one attribute from a table in the root partition $q_0$. This corresponds to the standard safety condition of safe TRC. This steps adds the set parentheses, the output tables, and its attribute and output predicates shown in green in Fig. 6a.

**Soundness.** Notice that this 5-step translation guarantees that the resulting TRC* is uniquely determined up to (1) renaming of the tuple variables; (2) reordering the predicates in conjunctions, and (3) flipping the left/right positions of attributes in each predicate. It follows that Relational Diagram are sound, and their logical interpretation unambiguous.

### 4.4 Valid Relational Diagrams

In order for a Relational Diagram to be valid we require that each of the conditions for the 5-step translation process are fulfilled.

**Definition 14 (Validity).** A Relational Diagram is valid iff

1. The nested hierarchy of optional negation boxes partitions the canvas (any two dashed boxes are either disjoint or one is completely contained within the other).
2. Each table, its attributes, and its selection predicates are discernible and reside in exactly one canvas partition.
3. Each leaf in the canvas partition contains at least one table.
4. Joins only happen between attributes of tables in partitions that are descendants (not siblings or their descendants). Join predicates with asymmetric operators such as $<$ and $>$ require a line with directionality (e.g., an arrow head).
5. The output table has at least one attribute, and each attribute connects to exactly one attribute in the root partition $q_0$ (safety condition of TRC).

**Theorem 15 (Unambiguous Relational Diagrams).** Every valid Relational Diagram has an unambiguous interpretation in TRC*. The constructive translation from Section 4.3 forms the proof.
This is arguably a loss of 7 to replace disjunctions in the root by unions of queries. These two extensions together make Relational Diagrams relationally complete: every query expressible in full Datalog, safe TRC, or our prior SQL* fragment extended by disjunctions of predicates\(^7\) can then be represented as Relational Diagram.

We illustrate with two examples. The first shows how to avoid disjunctions if they are not at the root level. The second shows how to replace disjunctions in the root by unions of queries.

**Example 5 (Replacing disjunctions).** Consider the SQL query from Fig. 8a which contains a disjunction and is not in SQL*. Using De Morgan’s Law \(\neg(A \lor B) = \neg A \land \neg B\), we can first reformulate the conditions including disjunction as DNF, and then distribute the quantifier over the conjuncts. This leads to a query without disjunctions, yet comes at the cost of having to repeat relation R:

\[
\{q(A) \mid \exists r \in R[q.A=r.A \land \neg(3s \in S \land \neg(\exists r_2 \in R[r_2.B=s.B \lor r_2.C=s.C]) \land r_2.A=r.A)]\}
\]

\[
\{q(A) \mid \exists r \in R[q.A=r.A \land \neg(3s \in S \land \neg(\exists r_2 \in R[r_2.B=s.B \land r_2.A=r.A]) \land \neg(\exists r_3 \in R[r_3.C=s.C \land r_3.A=r.A])\} \}
\]

Figure 8b shows this query as representation-equivalent SQL* query, and Fig. 8c as Relational Diagram.

\[
\text{Theorem 16 (Representation-equivalence). TRC* and Relational Diagrams are representation-equivalent.}
\]

Hence both languages can represent the same set of query patterns. The translations from Sections 4.2 and 4.3 keep a 1-to-1 correspondence between extensional tables and thus form the proof.

**5 RELATIONAL COMPLETENESS**

Recall from our discussion in Section 2 that any single situation can display only conjunctive information. Shin goes further and claims that “Any diagrammatic system that seeks to represent disjunctive information needs to bring in an artificial syntactic device with its own convention.” [41]. The syntactic device we use is inspired by the representation of disjunction in Datalog: we allow placing several Relational Diagrams on the same canvas, each in a separate union cell. Each cell of the canvas then contains one Relational Diagram displaying only conjunctive information, yet the relation among the different cells is disjunctive. We also allow logical transformations that are not pattern-preserving (thus we focus on logical equivalence only, and not our more strict criterion of representation equivalence). This is arguably a loss of representation power, but in line with the current understanding of the limits of diagrams to express disjunctive information [41, 42].

Two extensions to make Relational Diagrams relationally complete: every query expressible in full RA, safe TRC, Datalog*, or our prior SQL* fragment extended by disjunctions of predicates\(^7\) can then be represented as Relational Diagram.

We illustrate with two examples. The first shows how to avoid disjunctions if they are not at the root level. The second shows how to replace disjunctions in the root by unions of queries.

**Example 6 (Union of queries).** Consider three unary tables \(R(A), S(A),\) and \(T(A)\) and the query from Fig. 8d that we had already used in Section 2. We can replace disjunction by pulling it to the root and replacing the query with a union of disjunction-free SQL* queries:

\[
\{q(A) \mid \exists r \in R, 3s \in S, \exists t \in T[q.A=r.A \land (r.A=s.A \lor r.A=t.A)]\}
\]

\[
\{q(A) \mid \exists r \in R, 3s \in S, \exists t \in T[q.A=r.A \land (r.A=s.A) ] \lor (q(A) \mid \exists r \in R, 3s \in S, \exists t \in T[q.A=r.A \land r.A=t.A])\}
\]

**Theorem 18 (Completeness).** Relational Diagrams extended with union cells are relationally complete.

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\(^7\)Extend the grammar from Fig. 3 with one additional rule: \(P ::= (P \lor P)\).
6 FROM QUERIES TO SENTENCES

This section discusses a simple generalization of Relational Diagrams from relational queries to relational sentences (or Boolean queries). This generalization allows us to express constraints (sentences that need to be true), an extension that is not available in relational algebra. It also allows us to compare our formalism against a long history of formalisms for logical statements.

The idea is simple: we leave away the output table. An additional freedom with sentences is that the safety conditions of relational calculus fall away. Thus, we can express statements that do not have any existentially-quantified relations in the main canvas.

For the Boolean fragment of TRC, we thus allow the first negation before the first existentially-quantified table. In the translation back and forth from Relational Diagrams (Sections 4.2 and 4.3), we leave away point 5. For SQL, we need to make sure that it always returns either true or false (and not only true or the empty set). This can be achieved by adding to the rule for \( Q \) in Fig. 3 two alternative derivations \( Q := \text{SELECT NOT} (P) \) and \( Q := \text{SELECT [NOT] EXISTS} (P) \). We illustrate with an intuitive example here. More illustrative examples are included in our online appendix [23].

**Example 7.** Consider the statement: “All sailors reserve a red boat.”

\[
\neg (\exists s \in \text{Sailor} \neg (\exists b \in \text{Boat}, r \in \text{Reserves} \{b.\text{color} = \text{'red'} \land r.\text{bid} = b.\text{bid} \land r.\text{sid} = s.\text{sid}\}))
\]

The first 4 steps of the translation in Section 4.2 still work: the root canvas \( q_0 \) does not contain any relation (Fig. 9b). Similarly, the equivalent canonical SQL statement contains no FROM clause before the first NOT. Notice that Definition 11 of query pattern isomorphism still works as it is defined based on the relational tables.

7 RELATED WORK

7.1 Peirce’s beta existential graphs

Relational Diagrams represent nested quantifiers in a similar way as the influential and widely-studied Existential Graphs by Charles Sanders Peirce [35, 40, 42] for expressing logical statements (or, equivalently, Boolean queries). Peirce’s graphs come in two variants called alpha and beta. Alpha graphs correspond to propositional logic, whereas beta graphs correspond to first-order logic (FOL). Both variants use so-called cuts to express negation (similar to our nesting boxes), and beta graphs use a syntactical element called the Line of Identity (LI) to denote both the existence of objects and the identity between objects.

**Example 8 (Nested negation).** Figure 10 shows 2 beta graphs:

**Fig. 10a:** There exists a sailor who reserved a red boat.

**Fig. 10b:** All red boats were reserved by some sailor.

Beta graphs cannot represent constants and thus need to replace a selection of boats that are red with a dedicated new predicate “is a red boat.” Their respective translations into DRC are:

**Fig. 10a:** \( \exists x, y \{\text{Sailor}(x) \land \text{RedBoat}(y) \land \text{Reserves}(x, y)\} \)

**Fig. 10b:** \( \neg (\exists x \{\text{RedBoat}(y) \land \neg (\exists x \{\text{Sailor}(x) \land \text{Reserves}(x, y)\})\}) \)

Contrast the beta graphs with their respective Relational Diagrams and TRC:

**Fig. 10c:** \( \exists x \in \text{Sailor}, b \in \text{Boat}, r \in \text{Reserves} \{r.\text{bid} = b.\text{bid} \land r.\text{sid} = s.\text{sid} \land b.\text{color} = \text{'red'}\} \)

**Fig. 10d:** \( \neg (\exists b \in \text{Boat} \{\text{B.color} = \text{'red'} \land \neg (\exists s \in \text{Sailor}, r \in \text{Reserves} \{r.\text{bid} = b.\text{bid} \land r.\text{sid} = s.\text{sid}\})\}) \)

Differences. The 4 key differences of beta graphs vs. Relational Diagrams are: (1) beta graphs can only represent sentences and not queries; (2) beta graphs cannot represent constants, so selections cannot be modeled and instead require dedicated predicates; (3) beta graphs can only represent identity predicates (and no comparisons); and (4) Lines of Identity (LIs) in beta graphs have multiple meanings (existential quantification and identity) and are a primary symbol.\(^8\)

This can make reading the graphs ambiguous. We, in contrast, have predicates inspired from TRC. Lines only connect two attributes and have no loose ends. Interpreting a graph as a TRC formula is straightforward and can be summarized in a simple set of rules (recall Section 4). We now discuss this last point in more detail.

Problems from abusing lines in beta graphs. While over 100 years old, Peirce’s beta system has led to multiple misinterpretations and ongoing discussions about how to interpret a valid beta graph correctly. The literature contains many attempts to provide formal “interpretations” and provide consistent readings of these graphs. How can it be that something that should be unambiguous still gives so much margin of error? In our opinion, beta graphs have one important design problem leading to those misunderstandings: it is the overloading of the meaning of the Lines of Identity (LI), and thus the abuse of lines as symbol. As mentioned before, LIs are used to denote two different concepts: (1) the existence of objects (intuitively an existential quantification of a variable in DRC such as \( \exists x \)), and

---

\(^8\)Every beta graph has lines and graphs with lines but no predicates have meanings. See, e.g., the recursive definition in [42, p. 41].
Example 9 (A red boat). Consider the sentence “There is a red boat” shown in Fig. 11a as a beta graph. As beta graphs cannot represent constants, the graph requires a special unary predicate “red boat.” The LI represents both “there exists something” and “that something is equal to a red boat.” Thus a line (which arguably suggests two items being connected or joined) is meant as a quantifiable variable, and the beta graph can be interpreted in DRC as:

$$\exists x [RedBoat(x)]$$

Figure 11b shows the same sentence as a Relational Diagram. Notice that “there exists something” is represented by just placing this something (a predicate) on the canvas. There is no need for an existential line. Also notice how the modern UML diagram allows predicates (relational atoms) with several attributes, and one of those attributes can be set equal to a constant (here \(\text{color} = \text{'red'}\)). It can be read rather naturally like a TRC statement:

$$\exists b \in \text{Boat} [b.\text{color} = \text{'red'}]$$

The interpretation of beta graphs where one LI represents one existentially-quantified variable can at times be intuitive and simply correspond to a modern DRC interpretation (recall Example 8). However, such a simple interpretation is not always possible.

Example 10 (Exactly one red boat). Consider the sentence “There exists exactly one red boat,” shown in Fig. 11a as a beta graph. Figures 12a and 12b show two beta graphs with different cut nestings that can both be read as

$$\exists x [RedBoat(x)] \land \neg (\exists y [RedBoat(y) \land x \neq y])$$

Now a single LI needs to represent two existentially-quantified variables, and two different nestings of the cuts can represent the same statement. Contrast this with Fig. 12c, read in TRC as:

$$\exists b \in \text{Boat} [b.\text{color} = \text{'red'}] \land \neg (\exists b_2 \in \text{Boat} [b_2.\text{color} = \text{'red'} \land b.\text{bid} \neq b_2.\text{bid}])$$

Notice here that the inequality is simply represented by a label of a join between two predicates. Two tuple variables are represented by two different atoms and the interpretation is unambiguous: There exists a boat whose color is red, and there does not exist another boat whose color is red and whose bid is different.

The fact that one LI can branch into multiple endings (also called ligatures), and may have loose endings, and may represent multiple existentially-quantified variables, together with cuts being applied to such LIs’s can quickly lead to hard-to-interpret diagrams (see e.g., the increasingly-unreadable figures in [42, pp. 42-49]). This led to several attempts in the literature to provide “reading algorithms” of those graphs (e.g., [39, 42, 46]) and rather complicated proofs of the expressiveness of beta graphs [46], assuming a correct reading. As example, the paper by Dau [16] points out an error in Shin’s reading algorithm [42]. However, Dau’s correction to Shin [16] itself also has errors (e.g., the interpretation of the right-most diagram in [16, Fig 2] misses one equality, see [23] for details).

Why Relational Diagrams avoid the problem. Relational Diagrams use the line only for connecting two attributes. The type of connection is unambiguously represented by a label. Quantification is represented by predicates themselves. Thus, on a more philosophical level, we think that our visual formalism solves those problems based on a more modern interpretation of first order logic: TRC was created by Edgar Codd in the 1960s and 1970s in order to provide a declarative database-query language for data manipulation in the relational data model [12]. In contrast, beta graphs were proposed even before first-order logic (FOL), which was only clearly articulated some years after Peirce’s death in the 1928 first edition of David Hilbert and Wilhelm Ackermann’s “Grundzüge der theoretischen Logik” [24]. Zeman, in his 1964 PhD thesis [46], was the first to note that beta graphs are isomorphic to first-order logic with equality. However, the secondary literature, especially Roberts [39] and Shin [42], does not agree on just how this is so [14]. We did not start from Peirce’s beta graphs and attempted to fix the issues that have been occupying a whole community for years. Rather, we started from the modern UML reading of relational schemas and an understanding of TRC, and tried to achieve a minimal visual extension to provide relational completeness and pattern-isomorphism to TRC, which happens to provide a natural solution of interpretation problems of beta graphs. We believe that Relational Diagrams provide a clean, unambiguous, and, in hindsight, simple abstraction of query patterns.

7.2 QueryVis

Some of our design decisions are similar to our earlier query representation called QueryVis [15, 22, 31]. In QueryVis diagrams, grouping boxes are used to group all tables within a local scope, i.e., for each individual query block. Those boxes thus cannot show their respective nesting, and an additional symbol of directed arrows is needed to “encode” the nesting. The high-level consequence of those design decisions is that (1) QueryVis does not guarantee to unambiguously visualize nested queries with nesting depth \(\geq 4\) (please see our online appendix [23] for a minimum example), (2) each grouping box needs to contain at least one relation (thus QueryVis cannot represent the query in Fig. 6), and (3) QueryVis cannot represent general Boolean sentences (e.g., the sentence “All sailors have reserved some red boat”). Thus QueryVis is not sound and not relationally complete, even for the disjunctive fragment.

Problems from abusing lines in QueryVis. Similar to beta graphs, we think, in hindsight, that the design of QueryVis abuses
The development of QBE [47] was strongly influenced by \( \text{query pattern of relational algebra and not relational calculus.} \)
quoting

\[ \text{QBE: (i.e. with one single occurrence of the Sailor relation).} \]

\[ \text{TRC requiring two occurrences of the Sailor table.} \]

\[ \text{pattern from } \]Fig. 13a, \[ \text{and then finds all the other Sailors (Fig. 13b). The pattern of this query in QBE thus matches exactly the one of } \]

\[ \text{DRC,} \]

\[ \text{of implementing relational division} \]

\[ \text{disjunctions, such as } \]

\[ \text{disjunction and even more general features of SQL (such as group-} \]

\[ \text{relational algebra. They thus mirror dataflow-type languages where visual symbols (directed hyperedges) represent operators like set difference connecting two relational symbols, leading to a new third symbol as output. We have proved that there are simple queries in relational calculus (recall Example 1) that cannot be represented in relational algebra with the same number of relational symbols. Thus any visual formalism based on relational algebra cannot represent the full range of relational query patterns.} \]

\[ \text{the line symbol by using it for two purposes: for (i) joining atoms and (ii) for representing the negation hierarchy. In contrast, Rela-} \]

\[ \text{ational Diagrams use the line only for connecting two attributes and represent the negation hierarchy explicitly by nesting negation boxes. Relational Diagrams fix those the completeness and soundness issues, and, in addition, can show logical sentences and queries or sentences lacking tables in one or more of the negation scopes of nested queries.} \]

\[ \text{7.3 Query-By-Example (QBE)} \]

\[ \text{The development of QBE [47] was strongly influenced by DRC. However, QBE can express relational division only by using COUNT or by breaking the query into two logical steps and using a temporary relation} \]

\[ \text{[36, Ch. 6.9]. But in doing so, QBE uses the query pattern from RA and Datalog} “ \]

\[ \text{of implementing relational division (or universal quantification) in a dataflow-type, sequential manner, requiring two occurrences of the Sailor table.} \]

\[ \text{Example 11 (Sailors reserving all red boats in QBE). Consider the query} “ \]

\[ \text{Find sailors who have reserved all red boats” QBE needs to create a temporary relation BadSids in order to express relational division. It does follow the query pattern of relational algebra and not relational calculus.} \]

\[ \text{the line symbol by using it for two purposes: for (i) joining atoms and (ii) for representing the negation hierarchy. In contrast, Relational Diagrams use the line only for connecting two attributes and represent the negation hierarchy explicitly by nesting negation boxes. Relational Diagrams fix those the completeness and soundness issues, and, in addition, can show logical sentences and queries or sentences lacking tables in one or more of the negation scopes of nested queries.} \]

\[ \text{the line symbol by using it for two purposes: for (i) joining atoms and (ii) for representing the negation hierarchy. In contrast, Relational Diagrams use the line only for connecting two attributes and represent the negation hierarchy explicitly by nesting negation boxes. Relational Diagrams fix those the completeness and soundness issues, and, in addition, can show logical sentences and queries or sentences lacking tables in one or more of the negation scopes of nested queries.} \]

\[ \text{7.4 Other relationally-complete formalisms} \]

\[ \text{On a high-level, all visual formalisms that we are aware of and that are proven to be relationally complete (including those listed in [4]) are at their core visualizations of relational algebra operators. This applies even to the more abstract graph data structures (GDS) from [3] and the later graph model (GM) from [5], which are related to our concept of query representation. The key difference is that GDS and GM are formulated inductively based on mappings onto operators of relational algebra. They thus mirror dataflow-type languages where visual symbols (directed hyperedges) represent operators like set difference connecting two relational symbols, leading to a new third symbol as output. We have proved that there are simple queries in relational calculus (recall Example 1) that cannot be represented in relational algebra with the same number of relational symbols. Thus any visual formalism based on relational algebra cannot represent the full range of relational query patterns.} \]

\[ \text{8 CONCLUSIONS AND FUTURE WORK} \]

\[ \text{We motivated a criterion called pattern-preserving that preserves logical query patterns across languages and gave evidence for its importance in designing diagrammatic representations. To the best of our knowledge, our work is the the first to discuss and formalize the concepts of relational query patterns with a semantic definition that is applicable to any relational query language. We also propose the ability to represent query patterns already used in existing query languages as criteria for comparing and evaluating visual query representations. As example, Jarke and Vassiliou’s survey [28] established various criteria for query language selection. Those include usability and functional criteria. No prior work we are aware of discusses diagrams under the view point of truthfully representing query patterns (i.e. at least with the same number of extensional tables) used in relational query languages.} \]

\[ \text{We formulated the non-disjunctive fragments of Datalog} “ \]

\[ \text{RA, safe TRC, and corresponding SQL (interpreted under set semantics) that naturally generalize conjunctive queries to nested queries with negation. We prove that this important fragment allows a rather intuitive and, in hindsight, natural diagrammatic representation that can preserve the query pattern used across all 4 languages. We call this representation Relational Diagrams and further prove that this formalism, extended with a representation of union, is complete for full safe relational calculus (though not pattern-preserving). No prior diagrammatic representation we know is both relationally complete and representation-equivalent to relational calculus, at least for the non-disjunctive fragment.} \]

\[ \text{Finding a pattern-preserving diagrammatic representation for disjunction and even more general features of SQL (such as grouping and aggregates) is an open problem. For example, it is not clear how to achieve an intuitive representation for arbitrary nestings of disjunctions, such as } \]

\[ \text{“RA < S.E ∧ (R.B < S.F ∨ R.C < S.G)” or “(RA > 0 ∧ RA < 10) ∨ (RA > 20 ∧ RA < 30)” with minimal additional notations. Grounded in a long history of diagrammatic representations of logic, we gave intuitive arguments for why visualizing disjunctions is inherently more difficult than conjunctions. However, this is no proof it cannot be done with a novel approach.} \]
A NOMENCLATURE

| Symbol | Definition |
|--------|------------|
| $q(T)$ | Query mapping a signature $T$ of tables to the query output. Example $q(R); Q(x) := R(x, y)$, $R(\_y)$ |
| $q(\bar{T})$ | Query representation with signature $\bar{T}$ of extensional tables. Example $q(R, R'): Q(x) := R(x, y)$, $R(\_y)$ |
| $q'(\bar{T}'')$ | Shattered query mapping signature $\bar{T}''$ of tables to the query output. Example $q'(R_1, R_2): Q(x) := R_1(x, y)$, $R_2(\_y)$ |

$\equiv$ Logical equivalence between a query and its shattered queries with appropriate signature. Example $q'(R, R) = q(R)$.

$L_1 \subseteq_{\text{rep}} L_2$ Language $L_2$ can pattern-represent language $L_1$, i.e. $L_2$ can represent all relational query patterns of language $L_1$.

$L_1 \not\subseteq_{\text{rep}} L_2$ Language $L_2$ pattern-dominates language $L_1$, i.e. $L_2$ can represent all relational query patterns of language $L_1$ and $L_2$ can represent relational query patterns that $L_1$ cannot.

$L_1 \equiv_{\text{rep}} L_2$ Languages $L_1$ and $L_2$ are representation equivalent, i.e. they can express the identical set of relational query patterns.

B PROOFS Section 2

Proof Theorem 6. We prove each of the directions in turn.

• RA* $\rightarrow$ Datalog*: The proof for this direction is an easy induction on the size of the algebraic expression. It is a minor adaptation of the translation from RA to Datalog* proposed in [45], yet it also pays attention to the restricted fragment, and keeps the numbers of atoms constant during the translation, if possible. Formally, we show that if an RA* expression has $i$ occurrences of operators, then there is a Datalog* program that produces, as the relation for one of its IDB predicates, the value of the expression.

The basis is $i = 0$, that is, a single operand. If this operand is a given relation $R$, then $R$ is an EDB relation and thus "available" without the need for any rules. For the induction, consider an expression whose outermost operator is one of 6 operators: Cartesian product ($\times$), selection ($\sigma$), theta join ($\triangleright_v$), projection $\pi$, and difference ($-$). Case 1: $Q = E_1 \times E_2$: Let RA* expressions $E_1$ and $E_2$ have Datalog* predicates $e_1$ and $e_2$ whose rules define their relations, and assume their relations are of arities $m$ and $n$, respectively. Then define $q$, the predicate for $Q$, by:

$q(x_1, \ldots, x_{m+n}) := e_1(x_1, \ldots, x_m), e_2(x_{m+1}, \ldots, x_{m+n})$.

Case 2: $Q = \sigma_c E$: By restricting our language from RA to RA*, we only allow selections $\sigma_c(\varphi)$ where the condition $c$ is a conjunction of simple selections $c = c_1 \land c_2 \land \cdots$, i.e. each selection $c_i$ is of the form $\sigma_{A_{i1}\theta_{i2}}, \sigma_{A_{i2}}$ (selection predicate). Let $e$ be a Datalog* predicate whose relation is the same as the relation for $E$, and suppose $e$ has arity $d$. Then the rule for $Q$ is:

$q(x_1, \ldots, x_d) := e_1(x_1, \ldots, x_d), c_0$.

where $c_0$ is a conjunction of join predicates $x_i \theta x_j$ and selection predicates $x_i \theta b$ where $A_i$ and $A_j$ correspond to the attributes indexed by $x_i$ and $x_j$.

Case 3: $Q = E_1 \triangleright_v E_2$: While the join operator is not a basic operator of relational algebra, built-in predicates are, in practice, commonly expressed directly through join conditions. This case follows immediately from cases 1 and 2, and definition of joins as $Q = E_1 \triangleright_v E_2 = \sigma_c(E_1 \times E_2)$.

From the safety conditions of this rule we know that all variables in negated atoms also need to appear in positive atoms: $\bigcup y_i \subseteq \bigcup x_i$. Let $z$ be the set of complementing attributes, i.e. the attributes that only appear in positive atoms: $z = \bigcup x_i - \bigcup y_i$.

Case 4: $Q = \pi_{x_1,\ldots,x_d} E$: Let $E$’s relation have arity $d$, and let $e$ be a predicate of arity $m$ whose rules produce the relation for $E$. Then the rule for $q$, the predicate corresponding to expression $Q$, is:

$q(x_1, \ldots, x_d) := e(x_1, \ldots, x_m)$.

Case 5: $Q = E_1 - E_2$: We know by definition of the set difference that $E_1$ and $E_2$ must have the same arities. Assume those to be $d$, and that there are predicates $e_1$ and $e_2$ whose rules define their relations to be the same as the relations for $E_1$ and $E_2$, respectively. Then we use rule:

$q(x_1, \ldots, x_n) := e_1(x_1, \ldots, x_d), \neg e_2(x_1, \ldots, x_d)$.

Figure 14: Directions used in proof for Theorem 6 on logical expressiveness.
This expression translates one single rule into a valid relational algebra expression without union or disjunction. Since every IDB predicate \( q \) appears in only one rule, we do not need union or disjunctions even if multiple rules are translated. It then follows by induction on the order in which the IDB predicates are considered that each has a relation defined by some expression in RA*.

\( \text{TRC} \rightarrow \text{Datalog}^* \). In this translation, we start from the canonical representation of \( \text{TRC}^* \) (Section 2.4) where a set of existential quantifiers is always preceded by the negation operator (except for the table variables at the root of the query). This implies that we can decompose any query in \( \text{TRC}^* \) and write it as nested query components, each delimited by the scope of one negation operator. Each query is then of the form:

\[
\{ q(x) \mid p_1 \in P_1, \ldots, p_k \in P_k \} \in \text{(Relational Diagram)}
\]

where \( x \) are attributes chosen from the positive relations \( P_i \) as specified in \( \text{cout} \), and \( \epsilon_\theta \) is conjunction of comparison predicates between the positive relations \( P_i \).

For the induction step, assume that each nested \( q_i(A_j) \) is safe and translated into a rule \( q \). Then safe query \( q \)

\[
\{ q(A) \mid p_1 \in P_1, \ldots, p_k \in P_k \} \in \text{R}
\]

is translated into a rule

\[
q(x) := P_1(x_1), \ldots, P_k(x_k), \neg N_1(y_1), \ldots, \neg N_m(y_m), \epsilon_\theta.
\]

where \( x \) are attributes chosen from the positive relations \( P_i \) as specified in \( \text{cout} \), and \( \epsilon_\theta \) is conjunction of comparison predicates between the positive relations \( P_i \) and \( y_j \) are chosen from the variables used in the positive relations \( P_k(x_j) \).

(2) Next assume that a nested query is valid, yet not safe. This can happen because of two reasons: (i) Either some \( q_j(A_j) \) uses an attribute from the output \( q(A) \) directly; or (ii) some predicate in \( \text{cout} \) connects an output predicate to a predicate from \( P_j(x_j) \) with an inequality predicate. In both cases, we can make this query safe by adding one or more additional tables \( P_{k+1} \) and adding appropriate predicates: For case (i), we add an equality predicate to \( \text{cout} \) and replace the attribute specified in \( q_j(A_j) \). For case (ii), we add an equality predicate specified in \( \text{cout} \). However, nested queries do not need to be safe, which creates the one complication we need to take care of during the translation. We proceed in two steps:

(1) First assume that each query is safe. Then each subquery can be immediately translated into a separate rule by induction on the nesting hierarchy from the inside out. Basis of the induction for the leaf queries which are of the form:

\[
\{ q(A) \mid p_1 \in P_1, \ldots, p_k \in P_k \} \in \text{cout} \land \epsilon_\theta.
\]

A leaf query is translated into

\[
q(x) := P_1(x_1), \ldots, P_k(x_k), \epsilon_\theta.
\]

where \( x \) are attributes chosen from the relations \( P_i \) as specified in \( \text{cout} \), and \( \epsilon_\theta \) is conjunction of comparison predicates between the positive relations \( P_i \).
equality predicate to \( c_p \). We illustrate both cases of the translation with one example each.

**Example 12 (All quantification in Datalog\(^*\)).** We illustrate the translation with the help of the relational division example from Example 15 and Fig. 18:

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(\exists s \in S[\neg(\exists r_2 \in R[r_2.B = s.B \wedge r.2.A = r.A)])])
\]

Based on our extended safety condition for TRC\(^*\) Definitions 3 and 4, all predicates are anchored, i.e. they contain at least one attribute of a table that is existentially quantified inside the same negation scope as that predicate. Those are \( r.A \) in \( r.A = q.A \), \( r_2.B \) in \( r_2.B = s.B \), and \( r_2.A \) in \( r_2.A = r.A \) (shown in red below):

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(\exists s \in S[\neg(\exists r_2 \in R[r_2.B = s.B \wedge r_2.A = r.A)])])
\]

Rewriting the query based on its recursive nested negation hierarchy allows us to identify 3 query components:

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(q_1(r.A))])
\]

\[
(q_1(A) | \exists s \in S[\neg q_2(q_1.A,.B)])
\]

\[
(q_2(A,B) | \exists r_2 \in R[r_2.B = q_2.B \wedge r_2.A = q_2.A])
\]

Now notice that the predicate \( r_2.A = r.A \) (or \( r_2.A = q_2.A \) in the recursive hierarchy) is not limited: it references attribute \( r.A \) that is outside the negation scope of the direct parent of the scope in which it appears. This can also be seen from the recursive call \( q_2(q_1.A,.B) \) that “passes through” a predicate through the hierarchy. It can also be seen from the fact that \( q_1 \) is not safe.

We can limit the predicate (or equivalently make \( q_1 \) safe) by adding another table \( r_3 \in R \) in \( q_1 \) that accepts and hands over that attribute in the call hierarchy:

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(q_1(r.A))])
\]

\[
(q_1(A) | \exists s \in S[\exists r_3 \in R[r_3.A = r.A \wedge \neg q_2(r_3.A,.B)])
\]

\[
(q_2(A,B) | \exists r_2 \in R[r_2.B = q_2.B \wedge r_2.A = q_2.A])
\]

This rewritten query now allows a direct translation into Datalog\(^*\) from the inside out:

\[
Q_2(x,y) := R(x,y).
\]

\[
Q_1(x) := R(x), S(y), \neg Q_2(x,y).
\]

\[
Q(x) := R(x), \neg Q_1(x).
\]

**Example 13 (Built-in predicates in Datalog\(^*\)).** We next illustrate the translation for a built-in predicate with \( Q_3 \) from Example 22 asking for values from \( R \) for which no smaller value appears in \( S \):

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(\exists s \in S[s.A < r.A)])])
\]

Rewriting the query based on its recursive nested negation hierarchy allows us to identify 2 query components:

\[
(q(A) | \exists r \in R[r.A = q.A \wedge \neg(q_1(r.A))])
\]

\[
(q_1(A) | \exists s \in S[s.A < q_1.A])
\]

Now notice that the predicate \( s.A < q_1.A \) is not limited: it references attribute \( r.A \) that is outside the negation scope with an inequality instead of equality predicate. This can also be seen from the fact that \( q_1 \) is not safe.

We can limit the predicate (or equivalently make \( q_1 \) safe) by adding another table \( r_2 \in R \) in \( q_1 \):

\[
q(A) | \exists r \in R[r.A = q.A \wedge \neg(q_1(r.A))])
\]

\[
(q_1(A) | \exists s \in S[r_2.A < r_2.A \wedge r_2.A = q_1.A])
\]

This rewritten query now allows a direct translation into Datalog\(^*\) from the inside out:

\[
Q_2(x,y) := R(x,y) \cup S(y), x > y.
\]

\[
Q_1(x) := R(x), \neg Q_1(x).
\]

It follows that every query in TRC\(^*\) can be translated into a logically equivalent query in Datalog\(^*\):

\[
\text{Datalog}^* \rightarrow \text{TRC}^*;\text{ We consider a general Datalog rule:}
\]

\[
q(x) := p_1(x_1), \ldots, p_k(x_k), \neg n_1(y_1), \ldots, \neg n_m(y_m), c_p.
\]

Here \( c_p \) is a conjunction of built-in predicates that adhere to the standard safety conditions [7]. We know the rule is safe and thus \( \bigcup y_i \subseteq \bigcup x_i \). Let \( z \) again be the set of complementing attributes, i.e. the attributes that only appear in positive atoms: \( z = \bigcup x_i \cup \bigcup y_i \). The rule then translates into a TRC fragment

\[
\{ q(A) | p_1 \in P_1, \ldots, p_k \in P_k, c_p \wedge \neg(\exists n_1 \in N_1, \ldots, n_m \in N_m, c_p) \}
\]

Here \( A \) is a set of attributes that correspond to the variables returned by the Datalog rule (from safety conditions, only attributes from the positive relations can be returned), \( c_p \) is a conjunction of equality joins linking attributes from the output table \( q \) to attributes from the input tables \( P_1 \), \( c_p \) is a conjunction of comparison predicates between the positive relations or constants, and \( c_p \) is a conjunction of equality predicates between exactly one negative relation and either a positive relation or a constant.

\[
\text{TRC}^* \rightarrow \text{SQL}^*;\text{ We prove equivalence in three steps: We first reduce the syntactic variety of SQL}^*, \text{ then define a canonical form, and finally prove a one-to-one mapping between that canonical SQL}^* \text{ and canonical TRC}^*.
\]

1. Starting from Fig. 3, we first transform “membership sub-queries” and “quantified subqueries” into “existential subqueries.” We use the same grammar to describe this transformation. Concretely, replace “membership subqueries” of the form Fig. 16a with “existential subqueries” of the form Fig. 16a, and “quantified subqueries” Figs. 16c and 16e with Figs. 16d and 16f respectively. Here 
   O’ is the complement operator of \( O \) (for example “<” or “>”) and 
   C1 and C2 represent different columns.

2. Similar to TRC\(^*\), we pull existential quantifier of tables (table variables defined FROM clauses) as early as to either be in the root query, or directly following a not exists. We show this recursive pulling out in the transition from Fig. 16g to Fig. 16h.

3. The resulting canonical SQL\(^*\) in now in a direct 1-to-1 correspondence to TRC\(^*\), and the translation between SQL\(^*\) and TRC\(^*\) is then matter of translating the different syntactic expressions between the two languages: The SELECT DISTINCT \( C \{, C \}\) is equivalent to the output definition in TRC\(^*\), each FROM \( R \{, R \}\) defines the existentially quantified tuple variables \( \exists r \in R[], r \).
Theorem 6. We need to only point out which directions are guaranteed to preserve the number of tables (and are thus representation-preserving). For those directions that do not preserve the structure in general, we give a minimum counter example.

- RA* \subseteq^{SF} Datalog*: This direction follows immediately from the proof of Theorem 6 by observing each of the mappings in the 5 cases to be structure-preserving.
- RA* \nsubseteq^{SF} Datalog*: We show that the set difference (or minus \(\sim\)) from RA* cannot isomorphically represent negation from Datalog* if the complementing set of attributes is non-empty (see (6)). We show that with our Example 1 from the introduction:
  \[
  Q(x, y) := R(x, y), \neg S(y)
  \]
The binary minus operator from RA requires the same arity of the two input relations. Thus one cannot apply the minus operator directly to combine \(R\) and \(S\) as in Datalog*. Any possible sequence that includes a minus thus either uses the minus on 1 attribute, or 2 attributes (or 3 or more attributes, but those require even more joins and thus more table instances).

Case 1: Minus on 2 (or more) attributes: Having 2 (or more) attributes for the minus requires us to increase the arity of the right side and thus \(S\). This in turns requires a cross product with the domain from \(RA\) before the minus as in the following translation:
  \[
  R \times \pi_A \times S
  \]
This in turn increases the number of input table instance used from 2 to at least 3, which prevents a structure-preserving representation.

Case 2: Minus on 1 attribute: Having 1 attribute on the minus requires us to increase the arity after applying the minus (because our output has arity 2). This in turn unavoidably increase the number of input table instances to at least 3, which again prevents a structure-preserving representation. An example translation first uses a projection on \(R\) before the minus (to reduce the left input to arity 1) and then a subsequent join again with \(R\) after the minus:
  \[
  R \times \pi_A \times S
  \]
It follows that in whatever way the Datalog* expression (7) is expressed in RA, the expression will have at least 3 references to input tables. Thus RA cannot preserve the representation from (7).
- Datalog* \nsubseteq^{SF} TRC*: This direction follows immediately from the proof of Theorem 6 by observing the mappings of each Datalog rule to be structure-preserving.
- Datalog* \nsubseteq^{SF} TRC*: We show with our Example 15 that RA cannot isomorphically represent relational division from TRC*.

Consider a schema \(R(A), B, S(B)\) and the query asking for attribute values from \(R.A\) that co-occur in \(R\) with all attribute values from \(S.B\). Relational division can be written in TRC as:
  \[
  \{q(A) | \exists r \in R[q.A=r.A \land \neg(\exists s \in S[r.2=r.B, r=s.B \land r.2.A=r.A])]\}
  \]
Notice that (8) uses 2 occurrences of \(R\) and 1 occurrence of \(S\). Further notice that the predicate \(r.2.A=r.A\) joins two \(R\) tables across two different negation scopes. Figure 18b shows that query pattern as Relational Diagram with a join between the two corresponding \(R\) tables across two negation boxes.

We now show that there is no way to represent relational division in Datalog* with this pattern (and thus with only 2 occurrences of the \(R\) symbol). The key ingredient for this proof is the fact that the safety condition of Datalog* requires that each variable occurring in a negated atom also needs to be occur in at least one non-negated atom of the same rule [7]. As such, it can model negation only one rule at a time (each rule only allows application of one negation).
We next illustrate with the help of the more complicated example D MORE ILLUSTRATIONS Section 3 of relational division that there is a structure-preserving mapping proof of Theorem 6 by observing the 1-to-1 correspondences of the \( R \)ationed safety condition of \( S.B \) against the domain from \( S.ÁR \)3 occurrences of \( Datalog \)usions from (8). Instead, it needs to use another occurrence of two \( logically equivalent \) Figure 18: Example 15: Relational division in SQL (a)(c)(e) and as Relational Diagrams (b)(d). All 5 representations are logically equivalent, but only the partitions [(a), (b)] and [(c), (d), (e)] are also pattern-isomorph (which is what we expect).

As a consequence, it cannot model a query pattern with a join predicate across two negations that is needed for the TRC \( ^* \) expressions from (8). Instead, it needs to use another occurrence of \( R \) as "guard" for each negation. Thus Datalog \( ^* \) cannot preserve the pattern from (8).

This is achieved by the standard translation into Datalog \( ^* \) with 3 occurrences of \( R \) and two rules: The first rule finds all the \( R.A \) that do not co-occur with all \( S.B \) values. The second rule then finds the complement against the domain from \( R.A \):

\[
\begin{align*}
  I(x) & \equiv R(x,\_).S(y) \land \neg R(x, y) \\
  Q(x) & \equiv \neg R(x,\_).I(x)
\end{align*}
\]

(9)

Notice that the first "extra" atom \( R(x,\_) \) is needed for the aforementioned safety condition of Datalog \( ^* \). Figure 19b shows that logical pattern with an extra repeated table \( R \).

- SQL \( ^* \Rightarrow \text{TRC} ^* \): This also follows immediately from the proof of Theorem 6 by observing the 1-to-1 correspondences of the mappings in both directions. □

D MORE ILLUSTRATIONS Section 3

We next illustrate with the help of the more complicated example of relational division that there is a structure-preserving mapping from RA to TRC, but not in the other direction.

**Example 15 (TRC and RA are not representation-equivalent).** Assume a schema \( R(A,B),S(B) \). Consider the relational division asking for attribute values from \( R.A \) that co-occur in \( R \) with all attribute values from \( S.B \). The translation into TRC is

\[
\begin{align*}
  q(A) & \equiv \exists r \in R \left[ q.A = r.A \land \neg \exists s \in S \left[ \neg \left( \exists r_2 \in R [r_2.B = s.B \land r_2.A = r.A] \right) \right] \right]
\end{align*}
\]

(10)

The corresponding canonical SQL statement is shown in Fig. 18a. Relational division expressed in primitive RA is

\[
\pi_{\overline{A}}(\overline{A} \in S) = R
\]

(11)

The translation into Datalog \( ^* \) uses two rules:

\[
\begin{align*}
  I(x) & \equiv R(x,\_).S(y) \land \neg R(x, y) \\
  Q(x) & \equiv \neg R(x,\_).I(x)
\end{align*}
\]

(12)

The atoms \( R(x,\_) \) are needed for the safety condition of Datalog \( ^* \). This translation is part of a standard proof for equivalence of expressiveness between RA and safe TRC in textbooks such as [1, 33, 45].

Now notice an arguably important difference between the three expressions: TRC (10) uses the atom \( R \) two times, whereas RA (11) and Datalog \( ^* \) (12) use \( R \) three times. It turns out that there is no way to represent relational division in primitive RA or Datalog \( ^* \) with only two occurrences of the \( R \) symbol (see Theorem 13). There is, however, an alternative representation in TRC that preserves the RA structure with three occurrences of \( R \):

\[
\begin{align*}
  q(A) & \equiv \exists r \in R \left[ q.A = r.A \land \neg \exists s \in S \exists r_3 \in R [r_3.A = r.A \land \neg \exists r_2 \in R [r_2.B = s.B \land r_2.A = r.A] \left] \right] \right]
\end{align*}
\]

(13)

Notice that now there a natural 1-to-1 correspondence between the atoms in TRC (13) and the atoms in RA (11). This correspondence is even more intuitive by mapping the correspondence between two logically-equivalent SQL statements (Figs. 18c and 18e) and RA (11): for example, lines 4–8 in Fig. 18c translate into the RA fragment \( \pi_A((\pi_A \times S) - R) ^* \), which corresponds to the IDB predicate Temp(x) in Datalog \( ^* \) (12).

In other words, while all of these 7 queries are logically equivalent, they partition into two disjoint sets that are "pattern-isomorphic":

Set 1 = \{RA (11), TRC (13), SQL Fig. 18c, SQL Fig. 18e, Datalog \( ^* \) (12)\}

Set 2 = \{TRC (10), SQL Fig. 18a\}

This suggests that TRC and RA are not representation equivalent.

We next prove the pattern-isomorphism between RA query (11) and TRC query (13) with our formalism. First, write their shattered queries \( q_{RA}(R_1, S, R_3) \) and \( q_{TRC}(R_1, S, R_2, R_3) \) with

\[
\begin{align*}
  q_{RA} & \equiv \pi_A R_1 - \pi_A ((\pi_A R_X S) - R_3) \\
  q_{TRC} & \equiv \pi_A R_1 - \pi_A ((\pi_A R_X S) - R_3)
\end{align*}
\]

(14)

The shattered queries are logically equivalent after composition with \( h \):

\[
q_{RA}^h \equiv q_{TRC}^h \equiv h
\]

In other words:

\[
q_{RA}^h (R_1, R_2, S, R_3) \equiv q_{TRC}^h (h(R_1, R_2, S, R_3))
\]

Figure 19 illustrates the pattern isomorphism within two sets of queries with the color-highlighted \( R \) atoms. Notice in Fig. 19a the correspondences between the blue and orange highlighted tables \( R \) across the SQL and the TRC statements and the Relational Diagram. Notice that the constraints between their \( "A" \) attributes \( (\overline{R.A.A.R.A}) \) make use of the nesting hierarchy: two levels of the "not exists" nesting hierarchy in SQL and two levels in the negation hierarchy in TRC. Similarly, in Fig. 19b the two SQL variants use different syntactic constructs to represent the single negation.
Figure 19: Example 15 and Fig. 18 continued: Two logically-equivalent sets (a) and (b) of relational division in 5 query languages (Relational Diagrams, SQL*, RA*, Datalog*, TRC*). The queries in (a) use 2 occurrences of $R$, whereas the ones in (b) use 3 occurrences of $R$. We highlight the two or three occurrences of $R$ across the different languages that can mapped to each other according to our pattern isomorphism defined in Definition 11. We prove in Appendix C that for (a), there is no pattern-isomorphism representation in Datalog* (thus neither in RA).

Figure 20: A few ways to visualize a table and its set of attributes as a group of nodes. We use (a) inspired by UML conventions.

Figure 21: Example 16: A query with simple disjunction "Find sailors who reserve a red or a blue boat" can be represented with double negation in the non-disjunctive fragment of SQL (a) and whereas Relational Diagram.

**E MORE ILLUSTRATIONS FOR Section 4**

A table can be represented by any visual grouping of its attributes (see Fig. 20 for examples). Our choice in Relational Diagrams is to use the typical UML convention of representing tables as rectangular boxes with the table name on top and attribute names below in separate rows (Fig. 20a). This choice may affect the readability and usability of Relational Diagrams, yet does not affect their semantics and pattern expressiveness.

**F MORE ILLUSTRATIONS FOR Section 5**

Example 16 (Red or blue). Consider the following TRC* query asking for sailors who have reserved a red or a blue boat:

$$
\{q(sname) \mid \exists s \in Sailor, r \in Reserves \{ q.sname = s.sname \land \\
    s.sid = r.sid \land \exists b \in Boat \\
    \{ \text{not exists}(SELECT * FROM Boat WHERE \text{color} = 'red' \land \text{b.bid} = b.bid) \land \\
    \text{not exists}(SELECT * FROM Boat WHERE \text{color} = 'blue' \land \text{b.bid} = b.bid) \land \\
    b.bid = r.bid \land (b\text{.color} = 'red' \lor b2\text{.color} = 'blue') \} \}
$$

Using De Morgan’s Law $(A \lor B) = \neg (\neg A \land \neg B)$ applied to quantifiers, we can transform the disjunction into double-negation with conjunction. This transformation comes at the cost of repeated uses of extensional tables and is thus not pattern-preserving:

$$
\{q(sname) \mid \exists s \in Sailor, r \in Reserves \{ q.sname = s.sname \land \\
    s.sid = r.sid \land \neg ( \\
    \neg (\exists b1 \in Boat \{b1.bid = r.bid \land b1.color = 'red'\}) \land \\
    \neg (\exists b2 \in Boat \{b2.bid = r.bid \land b2.color = 'blue'\}) \} \}
$$

Figure 21a shows (14) translated into canonical SQL* and Fig. 21b its translation into a Relational Diagram. Notice how the non-disjunctive fragment repeats the boats table twice.
SELECT DISTINCT S.sname FROM Sailor S WHERE not exists (SELECT * FROM Boat B WHERE B.color = 'red' AND not exists (SELECT * FROM Reserves R WHERE R.bid = B.bid AND R.sid = S.sid)))

SELECT exists (SELECT * FROM Sailor S WHERE not exists (SELECT * FROM Boat B WHERE B.color = 'red' AND not exists (SELECT * FROM Reserves R WHERE R.bid = B.bid AND R.sid = S.sid))))

SELECT not (not exists (SELECT * FROM R WHERE R.A = 1)) AND not exists (SELECT * FROM R2 WHERE R2.A = 2))

Figure 23: Example 18: \( \exists r \in R[R.A = 1 \lor R.A = 2] \).

Figure 22: Example 17: Sailors reserving all red boats.

### G PROOFS Section 5

**Proof Theorem 18.** Given a safe TRC expression. We pull any existential quantifier as early as to either be at the start of the query, or directly following a negation operator.

First, consider a nested query with disjunctions in the WHERE conditions, possibly nested with conjunctions. Rewrite the conditions as DNF, i.e. as

\[ \neg(\exists r \in R[f_1(r') \lor f_2(r') \cdots \lor f_k(r')]) \]

Here, \( r \) is a set of table variables, \( R \) a set of tables, and each \( f_i \) is a conjunction of predicates in free and/or quantified variables \( r' \supset r \). Next rewrite it as:

\[ \neg(\exists r_1 \in R[f_1(r')]) \land \neg(\exists r_2 \in R[f_2(r')]) \land \cdots \land \neg(\exists r_k \in R[f_k(r')]) \]

This fragment is in TRC* and can be visualized by *Relational Diagrams*.

Second, for remaining disjunctions in the top query \( q_0 \), rewrite the query as union over queries without disjunction:

\[
\{ q(A) \mid \exists r \in R[f_0 \land f_1(\cdot)] \lor f_2(\cdot) \cdots \lor f_k(\cdot) \} = \{ q(A) \mid \exists r \in R[f_0 \land f_1(\cdot)] \} \cup \{ q(A) \mid \exists r \in R[f_0 \land f_2(\cdot)] \} \cup \cdots \cup \{ q(A) \mid \exists r \in R[f_0 \land f_k(\cdot)] \}
\]

Here, \( f_0 \) is a conjunction of attribute assignments to the output table \( q \) and nested subqueries.

### H MORE ILLUSTRATIONS FOR Section 6

**Example 17 (Sailors reserving all red boats).** Consider the sailor database [36] that models sailors reserving boats: Sailor(sid, sname, rating, age), Reserves(sid, bid, day), Boat(bid,bname,color), and the query “Find sailors who reserved all red boats.”

\[
\{ q(sname) \mid \exists s \in Sailor(s.name = s.sname \land \\
\neg(\exists b \in Boat(b.color = 'red' \land \\
\neg(\exists r \in Reserves(r.bid = b.bid \land r.sid = s.sid)))) \}
\]

Contrast it with the logical statement “There is a sailor who reserved all red boats.” In TRC*, the difference is achieved by leaving away curly brackets and any mentions of the output table (highlighted for a different example in green color in Fig. 6a):

\[
\exists s \in Sailor\{ q(sname) \mid s.sname \land \\
\neg(\exists b \in Boat(b.color = 'red' \land \\
\neg(\exists r \in Reserves(r.bid = b.bid \land r.sid = s.sid)))) \}
\]

Similarly, *Relational Diagrams* loose the output table (contrast Fig. 22c with Fig. 22d and their respective SQL statements).

We give an example that shows that in order to express sentences (instead of queries), and to be relationally complete (in that we would like to be able to express all logical sentences), we actually would not have to introduce the visual union. This is in stark contrast to the union at the root being necessary for queries.

**Example 18 (Disjunctions).** Consider the simplest disjunction

\[
\exists r \in R[R.A = 1 \lor R.A = 2]
\]

We can remove the disjunction with a double negation:

\[
\neg(\neg(\exists r \in R[R.A = 1] \lor \exists r \in R[R.A = 2]))
\]

The first 4 steps of the translation in Section 4.2 still work and leads to Fig. 23b. For SQL, the query uses the second new rule to express double negation before the first FROM clause.

### I MORE DETAILED RELATED WORK Section 7

#### I.1 Peirce’s existential graphs (Section 7.1)

We mentioned in Section 7.1 the complications arising from LI’s (Lines of Identities) being overloaded. Any LI can branch into multiple endings (also called *ligatures*), and may have *loose endings*, and may represent multiple existentially-quantified variables, together with cuts being applied to such LI’s can quickly lead to hard-to-interpret diagrams (see e.g., the increasingly-unreadable figures in [42, pp. 42-49]). This led to several attempts in the literature to provide “reading algorithms” of those graphs (e.g., [39, 42, 46]) and rather complicated proofs of the expressiveness of beta graphs [46], assuming a correct reading. As example, the paper by Dau [16] points out an error in Shin’s reading algorithm [42]. However, Dau’s correction to Shin [16] itself also has errors. For example, the interpretation of the right-most diagram in [16, Fig 2] (reproduced as Fig. 25) is wrong and misses one equality. The given interpretation
the middle, the ligatures of the graph are split due to Zeman’s algorithm, on the
Below, a sample graph for Zeman’s and Shin’s improved reading is provided. In
formulas which, as Shin writes, ‘

\[ \exists x. \exists y. \exists z. [S(x) \land P(y) \land T(z) \land \neg (x = y \land y = z)] \]

whereas it should be

\[ \exists x. \exists y. \exists z. [S(x) \land P(y) \land T(z) \land \neg (x = y \land y = z \land x = z)] \]

This is just an intuitive example how difficult beta graphs are in practice to interpret, even by the experts, and even by experts pointing out errors from other experts.

### I.2 QueryVis (Section 7.2)
We use the “unique beer taste” query [31] to show the difference in design decisions.

**Example 19 (Unique-set-query).** Consider the SQL query from
Fig. 26a asking to find “drinkers who like a unique set of beers,” i.e. no other drinker likes the exact same set of beers. The looping brackets to the left of the query in Fig. 26a show the content of boxes
used by QueryVis, which include all tables from each individual query block. Without the additional visual symbol of arrows, this diagram becomes ambiguous to interpret. To mitigate this problem, the
design of QueryVis [15, 31] uses directed arrows with an implied reading order (Fig. 26b).

The looping brackets to the right in Fig. 26a show the nesting of the variables scopes in queries, which are also reflected in the

dashed bounding boxes in Relational Diagrams (Fig. 26c).

The design decision by [31] is justified in terms of usability (for “most” queries the diagrams are not ambiguous and the reduction in nesting simplifies their interpretation), yet requires overloading
of the meaning of arrows. Two conceptual problem with these diagrams are: 1. QueryVis requires each partition of the canvas to contain a relation from the relational schema. Our earlier examples from Figs. 21 and 23 show examples that can thus not be handled. 2. QueryVis does not guarantee to unambiguously visualize nested queries with nesting depth \( \geq 4 \). This was alluded to already in [31], and we next give an example to illustrate:

**Example 20 (Ambigious QueryVis).** We next give a minimum example for when QueryVis becomes ambiguous. Consider the two

different SQL queries Figs. 27a and 27b. Following the algorithm
given in [31], both lead to the same visual representation Fig. 27c. In other words, it is not possible to uniquely interpret the diagram in Fig. 27c.

### I.3 DFQL
DFQL (Dataflow Query Language) is an example visual representation that is relationally complete [4, 11] by mapping its visual
symbols to the operators of relational algebra. Aside from providing basic set of operators derived from the requirements for being as expressive as first-order predicate calculus, DFQL also
provides diagrammatic representation of grouping operators in both comparison functions and aggregations. Following the same
procedurality as RA, DFQL expresses the dataflow in a top-down tree-like structure. However, since DFQL focuses on the 1-to-1
correspondence between the number of lines in a graph and the number of the variables in

\[ \exists x. \exists y. \exists z. [S(x) \land P(y) \land T(z) \land \neg (x = y \land y = z)] \]

The join between Sailor S and Sailor S2 is necessary to project column

\[ \pi_{\text{name}}(\text{Sailor} \times (\pi_{\text{bid}}(\pi_{\text{color}}(\text{Boat}) \land \pi_{\text{bid}}(\pi_{\text{color}}(\text{Reserves})))) \]

The join between Sailor S and Sailor S2 is necessary to project column

name from the table. This later query can be visualized by DFQL
in a pattern-preserving way as Fig. 28. One can easily find a 1-to-1
mapping between DFQL operators and this RA expression.

Notice that for the same arguments, there is also no pattern-

isomorph expression of the query shown in Fig. 1g and DFQL needs
two extensional tables for input table R to represent that query.

### I.4 Tools for Query Visualisations
The four projects that we know of that focus on the problem of visualizing existing relational queries are QueryVis [15, 22, 31]
(which we showed is not relationally complete, yet which inspired a lot of our work), GraphSQL [8], Visual SQL [26], (both of which maintain the 1-to-1 correspondence to SQL, and syntactic variants of the same query like Fig. 15 lead to different representations), and Snowflake join [44] (which is a pure query visualization approach that focuses on join queries with optional grouping, but does not support any nested queries with negation). Compared to all these
visual representations, ours is the only one that is relationally complete and that can preserve and represent all logical patterns in the non-disjunctive fragment of relational query languages.

I.5 Applications for Query interpretation

Query Interpretation is the problem of reading and understanding an existing query. It is often as hard as query composition, i.e., creating a new query [37]. In the past, several projects have focused on building Query Management Systems that help users issue queries by leveraging an existing log of queries. Known systems to date include CQMS [29, 30], SQL QuerIE [2, 10], DBease [32], and SQLShare [25]. All of those are motivated by making SQL composition easier and thus databases more usable [27], especially for non-sophisticated database users. An essential ingredient of such systems is a query browse facility, i.e., a way that allows the user to browse and quickly choose between several queries proposed by the system. This, in turn, requires a user to quickly understand existing queries.

Whereas visual systems for specifying queries have been studied extensively (a 1997 survey by Catarci et al. [4] cites over 150 references), the explicit reverse problem of visualizing and thereby helping interpret a relational query that has already been written has not drawn much attention, despite very early [37, 38] and very recent work [31] repeatedly showing that visualizations of relational queries can help users understand them faster than SQL text.
We have shown earlier that Datalog\textsuperscript{*} cannot represent all Query patterns from TRC\textsuperscript{*}. We next use another example to illustrates that this limit of Datalog does not only appear with deeply nested queries; it already appears for simply nested queries and is an immediate consequence of Datalog’s safety conditions for built-in predicates.

### I.6 Limits of Datalog for representing patterns

We have shown earlier that Datalog\textsuperscript{*} cannot represent all Query patterns from TRC\textsuperscript{*}. We next use another example to illustrate that this limit of Datalog does not only appear with deeply nested queries; it already appears for simply nested queries and is an immediate consequence of Datalog’s safety conditions for built-in predicates.

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**Example 22 (Limits of Datalog).** Consider two unary tables \( R(A) \) and \( S(A) \) and three questions:

- **\( Q_1 \)**: Find values from \( R \) that also appear in \( S \).
- **\( Q_2 \)**: Find values from \( R \) that do not appear in \( S \).
- **\( Q_3 \)**: Find values from \( R \) for which no smaller value appears in \( S \).

The first three lines of Fig. 29 show these queries expressed in SQL\textsuperscript{*}, TRC\textsuperscript{*}, RA\textsuperscript{*}, Datalog\textsuperscript{*}, and Relational Diagrams.

Notice that the SQL\textsuperscript{*}, TRC\textsuperscript{*}, and Relational Diagram queries from the third row have no pattern-isomorph query in RA\textsuperscript{*} or Datalog\textsuperscript{*}. The safety condition of Datalog requires each variable to appear in a non-negated atom. This criterion requires a cross-join with the domain of \( R.A \) in a separate rule before the negation can be applied on an equality predicate. For the same reason, RA\textsuperscript{*} cannot apply the set difference directly and also requires an additional cross-join with \( R.A \). The forth row shows the resulting resulting RA\textsuperscript{*} and Datalog\textsuperscript{*} queries together with their pattern-isomorph queries in SQL\textsuperscript{*}, TRC\textsuperscript{*}, and Relational Diagrams.