Reaction $e^+e^- \rightarrow \bar{D}D$ and $\psi'$ mesons

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We study the reaction $e^+e^- \rightarrow \bar{D}D$ near threshold in the $^3P_0$ non-relativistic quark model, including as intermediate states the $J/\psi$, $\psi(2S)$, $\psi(3770)$ and $\psi(4040)$ mesons. The work reveals that experimental data strongly favor one of the two $\psi(2S) - \psi(3770)$ mixing angles derived by fitting to the $e^-e^+$ partial decay widths of the $\psi(2S)$ and $\psi(3770)$ mesons. The meson $X(3940)$ as well as the resonance around 3.9 GeV observed by Belle and BaBar Collaborations in the reaction $e^+e^- \rightarrow \bar{D}D$ is unlikely to be a $c\bar{c}I^{G(JPC)} = 0^-(1^{--})$ state.

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I. INTRODUCTION

The recent studies of the exclusive initial state radiation production $e^+e^- \rightarrow \bar{D}D$ near threshold from Belle [1] and BaBar [2, 3] collaborations have consistently reported an enhancement around 3.9 GeV in the $\bar{D}D$ mass spectrum. The $D\bar{D}$ production in $e^+e^-$ annihilations near threshold is investigated in an effective Lagrangian approach [4], where the $X(3900)$ is included as a $J^{PC} = 1^{--}$ meson. It is concluded in [4] that the inclusion of the $X(3900)$ is essential to reproduce the experimental data [1, 8]. As a vector charmonium with $J^{PC} = 1^{--}$ at 3.9 GeV will cause great concern about the non-relativistic $c\bar{c}$ phenomenology, the alternative explanation of the bump structure by the $\bar{D}+D+c.c.$ open charm effects via intermediate meson loops is investigated [4].

By employing a partial reconstruction technique to increase the detection efficiency and suppress background, Belle first observed a peak around 3.94 GeV in spectrum of mass recoiling against the $J/\psi$ in the inclusive process $e^+e^- \rightarrow J/\psi X$ with $X$ decaying to $DD^*$ [5]. Later, the processes $e^+e^- \rightarrow J/\psi D^{(*)}D^{(*)}$ was studied and the observation of a charmonium-like state with mass about 3.94 GeV was confirmed [6]. The reaction $e^+e^- \rightarrow J/\psi X(3940)$ is studied in the framework of light cone formalism [7], supposing that the $X(3940)$ is a $3^1S_0$ state or one of the $2^3P$ states. It is suggested in [7] that the $X(3940)$ is a $3^1S_0$ charmonium. The most likely interpretation of the $X(3940)$ is that it is the $3^1S_0(\bar{c}\bar{c})\eta_c(3S)$ state (see Ref. [8] for a recent review).

In this work, we study in the $^3P_0$ non-relativistic quark model the lineshape of the cross section reaction $e^+e^- \rightarrow \bar{D}D$ near threshold, including $X(3900)$ or $X(3940)$ along with the $J/\psi$, $\psi(2S)$, $\psi(3770)$ and $\psi(4040)$ as intermediate mesons. We will show that the meson $X(3940)$ as well as the resonance around 3.9 GeV observed by Belle and BaBar Collaborations is unlikely to be a charmonium state with $I^{G(JPC)} = 0^-(1^{--})$.

II. $e^+e^- \rightarrow \bar{D}D$ IN $^3P_0$ QUARK MODEL

The reaction $e^+e^- \rightarrow \bar{D}D$ may stem from two possible processes, namely, the one-step process where the $e^+e^-$ pair annihilates into a virtual time-like photon, then the virtual photon decays into a $c\bar{c}$ pair, and finally the $c\bar{c}$ pair is dressed directly by an additional quark-antiquark pair pumped out of the vacuum to form the $\bar{D}D$ final state, and the two-step process where the created $c\bar{c}$ pair...
first form a vector meson and then the vector meson decays into $\bar{D}D$. Theoretical works in the $^3P_0$ quark model reveal that the reactions $e^+e^- \rightarrow \pi\pi, \pi\omega, NN$ are dominated by the two-step process at low energies. We expect the reaction $e^+e^- \rightarrow \bar{D}D$ near threshold is mainly a two-step process, in line with our previous works and the vector meson dominance model which is successfully applied to study the reaction $e^+e^- \rightarrow \bar{D}D$ in an effective Lagrangian approach.

The transition amplitude of the reaction $e^+e^- \rightarrow \bar{D}D$ in the two step process takes the form

$$T = \sum_{\psi_i} \langle \bar{D}D|V_{\bar{q}q}|\psi_i\rangle \langle \psi_i|G|\psi_i\rangle \langle \psi_i|\bar{q}q\rangle \langle \bar{q}q|T|e^+e^-\rangle \quad (1)$$

where $\psi_i$ stand for all $\bar{c}c$ $I^G(J^{PC}) = 0^-(1^-)$ mesons such as the $J/\psi$, $\psi(2S)$, $\psi(3770)$ and $\psi(4040)$, and $\langle \psi_i|\bar{q}q\rangle$ are simply the wave functions of the intermediate mesons. $\langle \psi_i|G|\psi_i\rangle$, the Green function describes the propagation of the intermediate mesons, and $\langle \bar{D}D|V_{\bar{q}q}|\psi_i\rangle$ is the transition amplitude of the intermediate meson $\psi_i$ decaying to the $\bar{D}D$ state in the $^3P_0$ non-relativistic quark model.

The transition amplitude of the process $e^+e^- \rightarrow \psi_i$ in eq. (1) can be easily derived in the standard method of quantum field theory, taking the form

$$T_{e^+e^-\rightarrow \psi_i} = \sum_{M_L,M_s} \frac{2}{\sqrt{3}} \frac{C(1,2,S,s_q s_M)C(SLJ,M_s M_L M)}{(2\pi)^3/2E_q} \Psi_{\psi_i L,M}^L M_L (\bar{p}) T_{e^+e^-\rightarrow \bar{c}c} \quad (2)$$

where $S$, $L$ and $J$ are respectively the total spin, orbital angular momentum (either 0 or 2) and total angular momentum (actually equal to 1) of the $\bar{c}c$ pair of the $\psi_i$ meson, $\Psi_{\psi_i L}^L M_L (\bar{p})$ is the spatial wave function of the $\psi_i$ meson in momentum space with $\bar{p}$ being the relative momentum between the quark and antiquark inside, and $T_{e^+e^-\rightarrow \bar{c}c} \equiv \langle \bar{q}q|T|e^+e^-\rangle$ is the transition amplitude of the reaction of $e^+e^- \rightarrow \bar{q}q$, taking the form

$$\langle e^+e^-|T|\bar{q} q \rangle = -\frac{e_q^e e_s}{s} \bar{u}_e(p_{e-}, m_{e-})\gamma^\mu v_e(p_{e+}, m_{e+}) \bar{v}_q(p_{\bar{q}}, m_{\bar{q}})$$

where $s = (p_q + p_{\bar{q}})^2$; $e_q$ is the quark charge, and the Dirac spinors are normalized according to $\bar{u}u = \bar{v}v = 2m_q$.

The Green function in Eq. (1) describing the propagation of the intermediate meson takes the form

$$\langle \psi_i|G|\psi_i\rangle = \frac{e^{i\phi_i}}{E_{cm} - (M_{\psi_i} - i\Gamma_{\psi_i}/2)} \quad (4)$$

where $E_{cm}$ is the center-of-mass energy of the system, $M_{\psi_i}$ and $\Gamma_{\psi_i}$ are the mass and width of the intermediate meson $\psi_i$, and a phase factor $e^{i\phi_i}$ is added to the amplitude of all charmonium resonances except for the $J/\psi$.

The transition amplitudes for the processes $\psi_i \rightarrow \bar{D}D$ are derived in the $^3P_0$ quark model. It was shown that the $^3P_0$ approach is successful in the description of hadronic $\bar{D}D$ decays into $\pi\pi$, $\pi\omega$, $NN$. The $^3P_0$ decay model defines the quantum states of a quark-antiquark pair destroyed into or created from vacuum to be $J = 0$, $L = 1$, $S = 1$ and $T = 0$. The effective vertex in the $^3P_0$ model takes the form as in Refs. [9, 10]

$$V_{ij} = \lambda \sigma^\mu \cdot (\bar{p}_i - \bar{p}_j) F_{ij} \hat{C}_{ij} \delta(\bar{p}_i + \bar{p}_j)$$

$$= \lambda \sum_{\mu} \sqrt{\frac{4\pi}{3}} (-1)^\mu \sigma^\mu_{ij} Y_{1\mu} (\bar{p}_i - \bar{p}_j) \hat{F}_{ij} \hat{C}_{ij} \delta(\bar{p}_i + \bar{p}_j)$$

(5)

where $\sigma^\mu_{ij}$, $\hat{F}_{ij}$, $\hat{C}_{ij}$, and $\lambda$ are respectively the spin, flavor and color operators, and the effective coupling constant. The operations of flavor, color, and spin operators onto a $\bar{q}q$ pair are

$$\langle 0,0|\bar{F}_{ij}|\bar{t}_i \otimes t_j|_{T,\bar{T}}\rangle = \sqrt{2}\delta_{T,0}\delta_{T,\bar{T},0},$$

$$\langle 0,0|\bar{C}_{ij}|\bar{q}^\alpha \bar{q}^\beta\rangle = \delta_{\alpha\beta},$$

$$\langle 0,0|\sigma^\mu_{ij}|\bar{t}_i \otimes t_j|_{J,M}\rangle = (-1)^M \sqrt{2}\delta_{J,1}\delta_{M,-\mu}$$

(6)

where $\chi_i(\bar{\chi}_i)$ and $t_i(\bar{t}_i)$ are the spin and flavor states of quark (antiquark), and $\alpha$ and $\beta$ are the color indices.

In the work we approximate the wave function of all mesons with the Gaussian form,

$$\Psi_{nlm}(\bar{p}) = N_n e^{-a^2 \bar{p}^2/2} L_n^{1+1/2}(ap) Y_{lm}(\theta, \phi)$$

(7)
where \( L_n^{l+1/2}(x) \) are the generalized Laguerre polynomial, \( \vec{p} \) is the relative momentum between the quark and antiquark in a meson, and \( a \) is the length parameter of the Guassian-type wave function. As the final state mesons are spinless, there exists only the P-wave transition amplitude for the processes \( \psi_i \to \bar{D}D \), that is

\[
T_{\psi_i \to \bar{D}D} = \sum_{m=-1}^{1} F_{n,l=1}(k) Y_{l=1,m}(\vec{k})
\]

with \( F_{n,l=1}(k) \) taking the general form

\[
F_{n,l} = A_1 k (1 + A_2 k^2 + A_4 k^2) e^{-\frac{(k^2 a^2)^2}{4(k^2 a^2 + 2b^2)}}
\]

where \( \vec{k} \) is the relative momentum between the two final mesons, and \( b \) and \( B \) are respectively the length parameters of the intermediate \( \psi' \) meson and the final \( D(\bar{D}) \) meson. For the purpose of good documentation, we list obviously the non-zero coefficients in eq. \( (8) \) for the processes \( \psi_i(nS) \to \bar{D}D \) and \( \psi_i(nD) \to \bar{D}D \). We have

\[
\psi(1S) : \quad \frac{8\sqrt{3}b^{3/2}B^3(b^2 + B^2)}{9\sqrt{3}(b^2 + 2B^2)^{7/2}}
\]

\[
\psi(2S) : \quad -\frac{8\sqrt{3}b^{3/2}B^3(b^2 - 3B^2)(3b^4 + 2B^2)}{3\sqrt{3}(b^2 + 2B^2)^{7/2}}
\]

\[
\psi(3S) : \quad \frac{4\sqrt{3}b^{3/2}B^3(b^2 - 2B^2)(3b^4 - 11b^2B^2 - 6B^4)}{3\sqrt{3}(b^2 + 2B^2)^{7/2}}
\]

\[
\psi(1D) : \quad \frac{32\sqrt{3}b^{7/2}B^5}{3\sqrt{3}(b^2 + 2B^2)^{7/2}}
\]

\[
\psi(2D) : \quad -\frac{16\sqrt{3}b^{7/2}B^5(b^2 - 2B^2)}{3\sqrt{3}(b^2 + 2B^2)^{7/2}}
\]

for the coefficient \( A_1 \),

\[
\psi(2S) : \quad \frac{2b^2B^3(b^2 + B^2)}{(b^2 - 3B^2)(b^2 + 2B^2)(3b^2 + 2B^2)}
\]

\[
\psi(3S) : \quad \frac{4b^2B^3(5b^4 - 9b^2B^2 - 10B^4)}{5(b^2 - 2B^2)(b^2 + 2B^2)(3b^4 - 11b^2B^2 - 6B^4)}
\]

\[
\psi(1D) : \quad -\frac{B^2(b^2 + B^2)}{5(b^2 + 2B^2)}
\]

\[
\psi(2D) : \quad -\frac{B^2(b^2 - 3b^2B^2 - 2B^4)}{5(b^2 - 2B^2)(b^2 + 2B^2)}
\]

for the coefficient \( A_2 \), and

\[
\psi(3S) : \quad \frac{4b^4B^8(b^2 + B^2)}{5(b^2 - 2B^2)(b^2 + 2B^2)^4(3b^4 - 11b^2B^2 - 6B^4)}
\]

\[
\psi(2D) : \quad -\frac{2b^3B^6(b^2 + B^2)}{35(b^2 - 2B^2)(b^2 + 2B^2)^2}
\]

for the coefficient \( A_4 \).

In our first calculation we have four intermediate mesons \( J/\psi, \psi(2S), \psi(3770) \) and \( \psi(4040) \) included with their masses and widths taken from the particle data group [12]. While the \( J/\psi \) is kept always as a 1S meson, the study of the reactions \( \psi(2S) \to e^-e^+ \) and \( \psi(3770) \to e^-e^+ \) reveals that the \( \psi(2S) \) possess a small D-wave component [13, 14]. Let

\[
\psi(2S) = \cos \theta_1 |2S\rangle - \sin \theta_1 |1D\rangle
\]

\[
\psi(3770) = \sin \theta_1 |2S\rangle + \cos \theta_1 |1D\rangle
\]

where \( \theta_1 \) is the mixing angle between the 2S and 1D states. In analogy, the \( \psi(4040) \) is also allowed to be the mixture of the 3S and 2D waves, that is

\[
\psi(4040) = \sin \theta_2 |3S\rangle + \cos \theta_2 |2D\rangle
\]

We fit our theoretical result to the experimental data from Belle and BaBar, letting all the relative phase factors \( \phi_i \) as well as the mixing angles \( \theta_1 \) and \( \theta_2 \) and the effective coupling constant \( \lambda \) be free parameters and letting the length parameters \( B \) and \( b \) run in a large region from 1.0 to 5.0 GeV\(^{-1}\). We found that it is impossible to reproduce the lineshape of the \( \psi(3770) \) meson as well as the bump structure observed around 3.9 GeV in the \( e^-e^+ \to \bar{D}D \) cross section.

It is necessary to include as the intermediate state a resonance at 3.9 GeV. The candidate could be the \( X(3940) \) as it is not ruled out that the \( X(3940) \) may have the quantum number \( J^{PC} = 0^-(-++) \). The study of the reaction \( e^-e^+ \to \bar{D}D \) in an effective Lagrangian approach [14] suggests a \( 0^-(-++) \) resonance with a mass about 3.9 GeV and width about 90 MeV (denoted as \( X(3900) \)). We include either the \( X(3900) \) or the \( X(3940) \) by pairing it with the \( \psi(4040) \),

\[
X(3900)(X(3940)) = \cos \theta_2 |3S\rangle - \sin \theta_2 |2D\rangle
\]

\[
\psi(4040) = \sin \theta_2 |3S\rangle + \cos \theta_2 |2D\rangle
\]
We study the reaction $e^+e^- \rightarrow \bar{D}D$ in the $^3P_0$ quark model with as many model parameters as possible predetermined. The size parameter $B$ is determined with the process $D^+ \rightarrow \mu^+\nu_\mu$. The partial decay width of the reaction $D^+ \rightarrow \mu^+\nu_\mu$ is evaluated with
\[
\Gamma = \frac{p_f}{32 M_D \pi^2} \int |T_{D^+\rightarrow \mu^+\nu_\mu}|^2 d\Omega \tag{16}
\]
with
\[
T_{D^+\rightarrow \mu^+\nu_\mu} = \int \frac{d\bar{p}}{2\pi} \psi(\bar{p}) \frac{\sqrt{2M_D}}{\sqrt{2E_1}\sqrt{2E_2}} T_{cd\rightarrow \mu^+\nu_\mu} \tag{17}
\]
where $T_{cd\rightarrow \mu^+\nu_\mu}$ is the transition amplitude of the process $u\bar{d} \rightarrow \mu^+\nu_\mu$ and $\psi(\bar{p})$ is the $D$ meson wave function in momentum space. Used as inputs the weak coupling constant $G = 1.166 \times 10^{-5}$ GeV$^{-2}$, the CKM element $|V_{cd}| = 0.230$, the $D^+$ meson mass $M_D = 1.870$ GeV, the $c$ quark mass $m_c = 1.27$ GeV, the $d$ quark mass as the constituent mass $m_d = 0.35$ GeV, and the experimental value of $\Gamma_{D^+\rightarrow \mu^+\nu_\mu} = 2.42 \times 10^{-7}$ eV, we derive the size parameter $B$ of the $D$ meson to be 2.28 GeV$^{-1}$. Note that it is impossible to estimate an error range for the size parameter $B$ as the CKM element $|V_{cd}|$ alone would lead to a sizable error bar for the $D$ meson decay width.

The size parameter $b$ of the $\psi(2S)$ and $\psi(3770)$ mesons and the mixing angle $\theta_1$ in eq. [13] are determined by the reactions $\psi(2S) \rightarrow e^-e^+$ and $\psi(3770) \rightarrow e^-e^+$ in the present model. The decay width of these two reactions can be worked out the same way as for the process $D^+ \rightarrow \mu^+\nu_\mu$. Fitting the experimental values of $\Gamma_{\psi(2S)\rightarrow e^-e^+} = 2.35 \pm 0.04$ keV and $\Gamma_{\psi(3770)\rightarrow e^-e^+} = 0.262 \pm 0.018$ keV leads to $b = 1.95 \pm 0.01$ GeV$^{-1}$ and the mixing angle $\theta_1$ being 10.69$^{+0.63}_{-0.27}$ or $27.6 \pm 0.69^{+0.69}_{-0.69}$ in the present model.

With $\theta_1$ being $10.69^{+0.63}_{-0.27}$ or $27.6 \pm 0.69^{+0.69}_{-0.69}$, the fit of the theoretical result of the partial decay width of the process $\psi(3770) \rightarrow \bar{D}D$ in the $^3P_0$ model to the experimental data, $\Gamma_{\psi(3770)\rightarrow D^+D^-} = 11.15 \pm 1.09$ MeV and $\Gamma_{\psi(3770)\rightarrow D^0\bar{D}^0} = 14.14 \pm 1.36$ MeV [12] leads to the effective coupling strength $\lambda = 0.68 \pm 0.04$ or $\lambda = 4.15 \pm 0.20$.

We fit the lineshape of the $\psi(3770)$ meson in the $e^+e^- \rightarrow \bar{D}D$ cross section with two sets of model parameters, that is, $\{B = 2.28$ GeV$^{-1}$, $b = 1.95$ GeV$^{-1}$, $\theta_1 = 10.69^0$, $\lambda = 0.68\}$ and $\{B = 2.28$ GeV$^{-1}$, $b = 1.95$ GeV$^{-1}$, $\theta_1 = -27.6^0$, $\lambda = 4.15\}$. Here only the $J/\psi$, $\psi(2S)$ and $\psi(3770)$ are included as the intermediate mesons. It is found that the experimental data strongly favor the first set of parameters as the second set of parameters leads to a $\psi(3770)$ peak over 10 times higher than the data. It is also noted that the error of the size parameter $b$, 0.01 GeV$^{-1}$, has very little effect on the cross section.

With $B = 2.28$ GeV$^{-1}$ for the final $D(\bar{D})$ meson, $b = 1.95$ GeV$^{-1}$ for all intermediate $\psi_i$, 10.69$^0$ for the $\psi(2S)$–$\psi(3770)$ mixing angle and $\lambda = 0.68 \pm 0.04$ for the effective coupling strength of the $^3P_0$ vertex, we fit the Belle and BaBar data of the processes $e^+e^- \rightarrow \bar{D}D D^0$ and $D^+D^-$ at energies from the $\bar{D}D$ threshold to 4.2 GeV, where the errors of the experimental data have been included in the fitting process. The $J/\psi$, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, and $X(3900)$ or $X(3940)$ are included as the intermediate states, and the decay width of the $X(3900)$ or $X(3940)$ and the mixing angle $\theta_2$ as well as all the relative phase factors $e^{-i\phi_\chi}$, are free parameters in the calculations. The fitted parameters and $\chi^2$/DOF with regard to the central values of the parameters are listed in Table I where the second column (Fit I) and the third (Fit II) are from the calculations with the $X(3900)$ and $X(3940)$ included as the intermediate meson, respectively. The theoretical results with the central values of the parameters in Table I are compared with experimental data in Fig. 1 for the cross section of the reaction $e^+e^- \rightarrow \bar{D}D D^0$. The theoretical results for the $e^+e^- \rightarrow D^+D^-$ cross section are similar to the ones for the reaction $e^+e^- \rightarrow D^0\bar{D}^0$. The curves in the first and second panels of Fig. 1 are the results with the $X(3900)$ and $X(3940)$ included as the intermediate state, respectively. The decay widths of the $X(3900)$ and $X(3940)$ are fitted to be $210 \pm 19$ MeV and $268 \pm 17$ MeV, respectively. A decay width of 200 MeV is much larger than the one predicted in Ref. [4]...
for the $X(3900)$ and some three times the experimental upper limit of the $X(3940)$ decay width [12].

TABLE I: Model parameters: Fit I (Fit II) from the calculation with $X(3900)$ ($X(3940)$) included as intermediate meson. The $^3P_0$ coupling strength $\lambda = 0.68 \pm 0.04$ is input for all intermediate mesons.

| Parameters | Fit I       | Fit II      |
|------------|-------------|-------------|
| $\lambda_{D\bar{D}X}$ | $0.68 \pm 0.04$ | $0.68 \pm 0.04$ |
| $\Gamma_X$ (MeV) | $210 \pm 19$ | $268 \pm 17$ |
| $\theta_2$ | $28.6 \pm 1.9$ | $13.4 \pm 1.8$ |
| $\phi_{\psi(2S)}$ | $164 \pm 10$ | $175 \pm 6$ |
| $\phi_{\psi(3770)}$ | $55 \pm 12$ | $65 \pm 9$ |
| $\phi_{\psi(3940)}$ | $250 \pm 11$ | $257 \pm 7$ |
| $\phi_X$ | $170 \pm 13$ | $170 \pm 15$ |
| $\chi^2$/DOF | 0.08 | 0.09 |

In the first calculation mentioned above, we have input the same $^3P_0$ vertex strength $\lambda = 0.68 \pm 0.04$ for all the intermediate mesons and fitted the decay widths of the $X(3900)$ and $X(3940)$ mesons. Instead of doing so in the second calculations, we let the $^3P_0$ vertex strength $\lambda$ free for both the $X(3900)$ and $X(3940)$ mesons and take the decay width from Ref. [1] for the $X(3900)$ and the one from Ref. [12] for the $X(3940)$ as input parameters. The fitted model parameters are listed in Table II, where the second column (Fit III) and the third (Fit IV) are from the calculations with the $X(3900)$ and $X(3940)$ included as the intermediate meson, respectively. To see the errors

![FIG. 1: Theoretical results for the cross section of the reaction $e^+e^- \rightarrow \bar{D}^0D^0$. First panel: $X(3900)$ included and its decay width a free parameter; Second panel: $X(3940)$ included and its decay width a free parameter; Third panel: $X(3900)$ included and the coupling strength $\lambda_{D\bar{D}X(3900)}$ a free parameter; Fourth panel: $X(3940)$ included the coupling strength $\lambda_{D\bar{D}X(3940)}$ a free parameter. The experimental data are taken from the Belle [1] and the BaBar [2].](image-url)
of the fitted parameters, we have considered the errors of the experimental data in the fitting process. The theoretical results with the central values of the parameters in Table I are plotted in the third and fourth panels in Fig. 1 for the cross section of the reaction $e^+e^- \to \bar{D}D$ compared with the Belle and BaBar data. It turns out that the effective coupling strengths of the $^3P_0$ vertex for the reactions $X(3900) \to \bar{D}D$ and $X(3940) \to \bar{D}D$ are much smaller than the one, $\lambda = 0.68$, for the decay processes $\psi(2S) \to \bar{D}D$, $\psi(3770) \to \bar{D}D$ and $\psi(4040) \to \bar{D}D$.

III. DISCUSSION AND CONCLUSIONS

The near threshold $e^+e^- \to \bar{D}D$ reaction is investigated in the $^3P_0$ quark model with a number of model parameters predetermined by other processes. The model study reveals that it is necessary to include as the intermediate states the resonance $X(3900)$ or $X(3940)$ as well as $J/\psi$, $\psi(2S)$, $\psi(3770)$ and $\psi(4040)$ to reproduce the experimental data for the $e^+e^- \to \bar{D}D$ cross section.

It is found that experimental data rule out one of the two $\psi(2S) - \psi(3770)$ mixing angles derived by fitting to the $e^-e^+$ partial decay widths of the $\psi(2S)$ and $\psi(3770)$ mesons.

We have assumed that the $X(3900)$ or $X(3940)$ is a $c\bar{c}$ $J^G(J^{PC}) = 0^-(1^{--})$ state and hence applied the same coupling strength of the $^3P_0$ vertex for the $X(3900) \to \bar{D}D$ and $X(3940) \to \bar{D}D$ decays as for the processes $\psi(2S) \to \bar{D}D$, $\psi(3770) \to \bar{D}D$ and $\psi(4040) \to \bar{D}D$. By fitting to the experimental data, however, the assumption leads to a decay width for either the $X(3900)$ or $X(3940)$, which is much larger than the experimental data [12] or the prediction of other work [11].

Instead of using as inputs the same coupling strength for all the intermediate mesons, we have input the experimental decay width for the $X(3940)$ and the width from Ref. [12] for the $X(3900)$. It turns out that the experimental data of the $e^+e^- \to \bar{D}D$ cross section dictate a much smaller coupling strength of the $^3P_0$ vertex for either $X(3900)$ or $X(3940)$ than the one for the $c\bar{c}$ $I^G(J^{PC}) = 0^-(1^{--})$ states $\psi(2S)$, $\psi(3770)$ and $\psi(4040)$.

The study reveals that, without including the $X(3900)$ or $X(3940)$ as the intermediate state, it is impossible to reproduce the lineshape of the $\psi(3770)$ meson as well as the bump structure observed around 3.9 GeV in the $e^+e^- \to \bar{D}D$ cross section in the present model. We have assumed the $X(3900)$ or $X(3940)$ to be a normal $c\bar{c}$ $I^G(J^{PC}) = 0^-(1^{--})$ meson to fit the experimental data, but derived a much weaker coupling strength of the $^3P_0$ vertex for the reactions $X(3900) \to \bar{D}D$ and

| Parameters | Fit III | Fit IV |
|------------|---------|--------|
| $\Gamma_X$ (MeV) | 90 ± 12 | 70 ± 11 |
| $\lambda_{D\bar{D}X}$ | 0.24 ± 0.03 | 0.15 ± 0.02 |
| $\theta_2$ | 26.9 ± 0.8 | 19.4 ± 1.2 |
| $\phi_{\psi(2S)}$ | 164 ± 2 | 195 ± 2 |
| $\phi_{\psi(3770)}$ | 42 ± 6 | 71 ± 4 |
| $\phi_{\psi(4040)}$ | 207 ± 14 | 209 ± 15 |
| $\phi_X$ | 143 ± 12 | 132 ± 12 |
| $\chi^2$/DOF | 0.10 | 0.11 |
than for the processes $\psi(2S) \to \bar{D}D$, $\psi(3770) \to \bar{D}D$ and $\psi(4040) \to \bar{D}D$. Therefore, one may concludes that the $X(3940)$ and $X(3900)$ are unlikely to be normal $c\bar{c}$ $I^G(J^P_C) = 0^-(1^-)$ states.

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