Dual Channel Supply Chain Model with Delivery Lead Time on The Imperfect Production Process by Notice Into Carbon Emission Capacity Regulation

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Abstract. The development of the internet, adds flexibility of supply chain with an online channel for the product distribution. In this research, the dual channel supply chain model is used on the imperfect production process by notice into carbon capacity regulation and also delivery lead time. We construct the model to maximize the system profit of one manufacturer and one retailer by considering a carbon emissions capacity constrain from the government. Furthermore, we determine the optimal solution with Karush Kuhn Tucker condition. Based on sensitivity analysis maximal profit is obtained when probability of defective product, delivery lead time sensitivity, and carbon emissions are minimal.

1. Introduction
Supply chain is the system that includes processes from production to product distribution to consumer [5]. Production processes and products distribution on supply chain management can cause carbon emissions which contributes to global warming. To solve the problem, the government issued carbon emission regulations in industrial sector. We can construct the supply chain management mathematically through supply chain inventory model. The development of information and communication technology, resulting in the development on supply chain inventory model with products distribution through online and offline channel. In the process of products distribution through online channel, we need to notice the delivery lead time caused by shipping process. Supply chain with distribution process through online and offline channel called dual channel supply chain.

Dual channel supply chain’s related research conducted by reference [15] with consider to carbon emission capacity regulation, consists of one supplier and one retailer. Supplier buys products to manufacturer, so there is no production process. Reference [3] did a research about production process with defective product that are sold at a discounted price. The next research about rework process for defective products [11]. The defective and non defective products are classify through the inspection process. In fact, errors occur in the inspection process. There are two types of inspection errors. Type I of inspection error occur when classify perfect products into defects and type II of inspection error occur when classify defect products to perfect products [13].

In this research, we construct dual channel supply chain model with delivery lead time on the imperfect production process by notice into carbon emission capacity regulations. The defective product which produced from the imperfect production process, selected through the inspection process that notice to inspection error, then repaired through the rework process. In addition, we also pay attention
to delivery lead time because of online channel. The purpose of this research is to maximize the system profit that consist of one manufacturer and one retailer. We determine the optimal selling price through online and offline channels, so the system total profit is maximum.

2. Literature Review
In the dual channel supply chain research, the manufacturer makes green products for the environmental conscious. The research discussed pricing and greening strategies in centralized and decentralized cases, then compare the results [6]. Next research is studied about single channel and dual channel strategy to examine the effect of different return policy. That paper of supply chain system comprised of production, refurbishing, collection, and waste disposal processes [2]. Other research studied optimal product distribution strategy for symmetric manufacturers that use dual channel supply chains. Including an asymmetric distribution policy, where one manufacturer distributes products only through the direct channel, while the other manufacturer distributes through both the direct and retail channel [7]. Pricing and channel priority strategies of dual channel supply chain in the presence of supply shortage caused by random yields [14]. Pricing and service decisions for complementary products in a dual channel supply chain are determined by two manufacturers and one retailer. One of two manufacturers distributes products through the online and traditional retail channel [12]. Then, the research about dual channel supply chain for standard and customize products in centralized system. The manufacturer offers customized products through a direct online channel and standard product through the traditional retail channel [1]. Reference [16] developed the decision making models of the centralized and decentralized dual channel supply chain, which consist of one manufacturer and one retailer from game theoretical perspective. The research is design an improved revenue sharing contract to effectively coordinate the manufacturer and retailer and suggest government make cap and trade regulation to reduce carbon emission efficiently. The research from [15] considered the coordination of a dual-channel supply chain under carbon emission capacity regulation in centralized and decentralized system which consist of one supplier and retailer. Supplier sell products through online channel to brand loyal and offline channel to retailer. Retailer only sell products through offline channel to brand loyal and retailer loyal. Dual channel supply chain in [9] studied about optimal pricing policies for a centralized and decentralized dual channel retailer with same and cross channel returns. Retailer are adopting a dual channel retailing strategy in which products are offered through two channels, there are physical stores and online stores. The demand disruption management in a dual channel supply chain producing and selling green products, composed of one manufacturer and one retailer in centralized and decentralized cases are taken from [10]. The research examines pricing and greening issues for a dual channel green supply chain when the market demand is disrupted. The study from [8] examined about a dual channel supply chain under price and delivery time dependent stochastic customer demand. The research analyze centralized and decentralized systems for unknown distribution function of the random variables through a distribution free approach.

Based on the related research above, the supplier in the dual channel supply chain model from reference [15] replaced by manufacturer. Manufacturer did imperfection production process, and the defective product fix through rework process. The inspection process in manufacturer, also are noticed into inspection error type II. The aim of this research is to maximize the system profit under carbon emission capacity regulation.

3. Model Construction

3.1. Demand Function
Customers are divided into two types, there are brand loyal and retailer loyal, which have initial demand $d_m$ and $d_r$. The retailer loyal customers purchase products only from the retailer. The brand loyal customers can purchase the products from both the manufacturer and retailer. Demand on online channel just done by brand loyal with initial proportion $\theta$. Demand on offline channel is done by brand loyal
with initial proportion \((1 - \theta)\) and retailer loyal. All initial demand is reduced when the price rises. Changes in demand caused by the increase of price describe with elasticity selling price parameter \(\beta\). When the selling prices of the two channels differ, customers will change their priority to the channel that has lower price. The priority of consumer purchases is described with \(\eta\). The delivery lead time on online channel \((l_m)\) affects to the demand of both channel. The longer delivery lead time in online channel makes consumers change their priority to offline channel. Priority of consumer purchases due to delivery lead time is described with \(\alpha\). Then, the demand functions of the online, offline, and both channels are

\[
D_m(p_m, p_r) = d_m(\theta(1 - \beta p_m) - \eta(p_m - p_r) - \alpha l_m),
\]

\[
D_r(p_m, p_r) = d_r(1 - \beta p_r) + d_m((1 - \theta)(1 - \beta p_r) + \eta(p_m - p_r) - \alpha l_m),
\]

\[
D_c(p_m, p_r) = D_m(p_m, p_r) + D_r(p_m, p_r).
\]

### 3.2. Manufacturer Profit Function

The profit function is obtained from total revenue minus total cost. Manufacturer issued manufacturing, inspection, post sale failure, and rework cost, with cost per unit product are \(c_{mp}, c_{imp}, c_{pfp},\) and \(c_{rw}p\). Manufacturer follows the lot for lot policy, so manufacturer produces products according to consumer demand on equation (3). Manufacturer does rework process to fix the defective products with probability \(y\) and inspections error type II with probability \(f_2\). That error makes retailer return the product, then manufacturer does the inspection and rework process. Retailer receives defective product with proportion \(\delta\) and retailer doesn’t make inspection errors, so retailer return the defective products to manufacturer. Manufacturer cost is

\[
TC_M(p_m, p_r) = (c_{mp} + c_{imp}(1 + \gamma f_2 \delta) + c_{pfp} \gamma f_2 \delta + c_{rw}p \gamma(1 - f_2(1 - \delta)))D_c.
\]

Manufacturer sells the products through online channel to brand loyal with price \(p_m\) and through offline channel to retailer with wholesale price \(w\). Manufacturer revenue on online and offline channel are \(p_m D_m\) and \(w D_r\). Then, manufacturer revenue is

\[
TR_M(p_m, p_r) = p_m D_m + w D_r.
\]

Manufacturer sells products through online and offline channel, so the profit function of manufacturer is the sum of online and offline profit. The profit function of manufacturer is

\[
\Pi_M(p_m, p_r) = (p_m - C_M)D_m + (w - C_M)D_r,
\]

with \(C_M = c_{mp} + c_{imp}(1 + \gamma f_2 \delta) + c_{pfp} \gamma f_2 \delta + c_{rw}p \gamma(1 - f_2(1 - \delta))\).

### 3.3. Retailer Profit Function

Retailer issued product purchases, inspection, and operational cost, with cost per unit product are \(w, c_{irp},\) and \(c_{orp}\). Retailer total cost is cost per unit multiplied by offline channel demand on equation (2). So, retailer total cost is

\[
TC_R(p_m, p_r) = (w + c_{irp} + c_{orp})D_R.
\]

Retailer just sells the products through offline channel to brand loyal and retailer loyal with price \(p_r\). Then, retailer revenue is

\[
TR_R(p_m, p_r) = p_r D_r.
\]

The profit function of retailer is total revenue on equation (8) minus total cost on equation (7). Retailer profit function is

\[
\Pi_R(p_m, p_r) = (p_r - C_R)D_R,
\]

with \(C_R = w + c_{irp} + c_{orp}\).
3.4. Dual Channel Supply Chain Model Manufacturer-Retailer
In centralized system, manufacturer and retailer determine the decisions about price together. The system profit is sum of manufacturer and retailer profit on equation (6) and (9). So, the system profit is
\[
\Pi_C(p_m, p_r) = (p_m - C_M) D_m + (p_r - (C_M + c_{irp} + c_{orp})) D_r.
\]
(10)
The government determines carbon emission capacity \(K_c\). The maximal profit should not exceed the \(K_c\) limit. The main carbon emissions are generated from production process \(e\), and product distribution in online and offline channel \((e_m\ and\ e_r\). The total carbon emissions is
\[
E_c(p_m, p_r) = D_m e_1 + D_r e_2
\]
with \(e_1 = e + e_m\) and \(e_2 = e + e_r\). So, we can formulate the mathematical model as
\[
\begin{align*}
\max, & \quad \Pi_C(p_m, p_r) = (p_m - C_M) D_m + (p_r - (C_M + c_{irp} + c_{orp})) D_r, \\
\text{s.t.,} & \quad E_c(p_m, p_r) \leq K_c.
\end{align*}
\]
(12)
4. Optimal Solution
The total profit of the dual channel supply chain system of \(\Pi(p_m, p_r)\) is strictly concave function. So, there is only one value of \((p_m, p_r)\) which is resulted in the maximal value of \(\Pi_C(p_m, p_r)\). The proof of concavity is given in Appendix A. The mathematical model on equation (12) is optimization problem with constraints. The optimal solution of the model is determined by completing Karush Kuhn Tucker conditions. The proof of optimal solutions is given in Appendix B. The optimal value of \(p_m\), \(p_r\), and \(\lambda\) are
\[
p^*_m = \left(\frac{(d_r + m(\eta + (1 - \theta)\beta))(A + c_{irp} + c_{orp}) + \eta(d_m(1 - \theta + l_m\alpha) + d_r)}{(d_r e_\beta + e_\beta(1 - \theta) + \eta))}\lambda^* + \beta(d_m(1 - \theta) + d_r)C_m)\right) / 2B,
\]
(13)
\[
p^*_r = \frac{1}{2\eta}(A - 1/B(\eta + \beta\theta))((d_r + m(\eta + (1 - \theta)\beta))(A + \eta(d_m(1 - \theta + l_m\alpha)
\]
\[
+ d_r + (d_r e_\beta + d_m(\eta + (1 - \theta) + \eta))) + (d_r + m(1 - \theta)\beta + \eta))
\]
\[
(c_{irp} + c_{orp}) + \beta(d_m(1 - \theta) + d_r)C_m).
\]
(14)
\[
\lambda^* = -(e_2 d_c + d_m(e_1 - e_2) (l_m\alpha - \theta) + \beta(e_2 d_c + d_m(e_2(1 - \theta) + e_1\theta)) (1 + f_2\delta\gamma) + c_{irp} + (d_r e_\beta + d_m(-e_1\eta + e_2((1 - \theta)\beta + \eta))) c_{irp}
\]
\[
+ (e_2 d_c + d_m\beta(e_1 - e_2))
\]
\[
(C_M - c_{irp}(1 + f_2\delta\gamma) + e_2 c_{orp}(d_m(1 - \theta) + d_r) - d_m c_{orp}(e_1 - e_2) + 2K_c)
\]
\[
/e_2^* d_c + d_m e_1(2e_2\eta + e_1(\eta + \beta\theta))
\]
(15)
with \(A = -l_m\alpha + \theta - \eta\lambda^*(e_1 - e_2) + \beta(e_1\lambda^* + C_M) - \eta(c_{irp} + c_{orp})\) and \(B = \beta(d_r(\eta + \beta\theta) + d_m(\eta - (\theta - 1)\theta))\).
5. Numerical Example
The parameters values are taken from the research of [4, 11, 15]. There are \(d_c = 6500\) unit, \(\theta = 0.5, \beta = 0.01/\$, \(\eta = 0.05/\$, \(l_m = 4\) days, \(\alpha = 0.025/day, c_{irp} = 25\$, \(c_{irp} = 1\$, \(c_{orp} = 3\$, \(c_{orp} = 25\$, \(\gamma = 0.02, f_2 = 0.05, \delta = 0.5, c_{irp} = 1\$/unit, c_{orp} = 3$/unit, K_c = 13960, e = 3, e_m = 4, \) and \(e_r = 5\).
There are three possible demands from the brand loyal and retailer loyal, \(d_m < d_r\), \(d_m = d_r\), and \(d_m > d_r\). These optimal solutions of three possible demands are in Table 1.

### Table 1. Optimal solution of DCSC model

| \(d_m\) (unit) | \(d_r\) (unit) | \(\lambda^*\) | \(p_m^*\) ($) | \(p_r^*\) ($) | \(\Pi_c^*\) ($) |
|----------------|----------------|--------------|--------------|--------------|---------------|
| 2500           | 4000           | 1.81         | 68.67        | 72.7         | 77026.4       |
| 3250           | 3250           | 1.77         | 68.75        | 72.56        | 77790.2       |
| 4000           | 2500           | 1.72         | 68.63        | 72.42        | 78549.6       |

From Table 1, for every possible demand we obtain the value of \(p_m^* < p_r^*\) and \(\lambda^* > 0\), and the profit system is maximal when the value of \(d_m = 4000\) unit and \(d_r = 2500\) unit. Manufacturer sells products in online channel at 68.63$ and retailer sell products in offline channel at 72.42$. The optimal price obtained when the value of Lagrange multiplier is 1.72. With that optimal solution, we obtain the maximal profit is 78549.6$. Furthermore, we did sensitivity analysis to know the optimal solution when we changed parameter values.

### 6. Sensitivity Analysis
We did sensitivity analysis for \(\beta, \eta, \theta, \alpha, e_1, e_2, y, \delta,\) and \(f_2\). The result of the sensitivity analysis of parameters is in Table 2.

### Table 2. Result of sensitivity analysis

| Parameter | \(\beta\) | \(\eta\) | \(\theta\) | \(\alpha\) | \(e_1\) | \(\Pi_c^*\) ($) |
|-----------|-----------|----------|-----------|-----------|-------|---------------|
|           | 0.01      | 0.01     | 1         | 0.01      | 3.26  | 78549.6       |

| Parameter | \(e_2\) | \(\gamma\) | \(\delta\) | \(f_2\) | \(f_2(\delta = 0.87)\) | \(\Pi_c^*\) ($) |
|-----------|---------|-----------|-----------|--------|------------------------|---------------|
|           | 5.44    | 0.01      | 0         | 1      | 0.01                   | 81562.8       |

From Table 2, the maximal profit is 81562.7$, achieved when brand loyal only choose online channel. The minimal profit is 77741.6$, acquired when the price sensitivity is 0.01. When probability of defective product approach to zero and carbon emissions in online and offline channel are minimal, the profit are maximal. Change in value of type II inspection error in the range \(0 \leq \delta < 0.87\) resulted the maximal profit when the inspection for all defective products are wrong. When \(0.87 \leq \delta \leq 1\), the maximal profit achieved when the probability of type II inspection error approach to zero. The result of the probability of type II inspection error in this paper similar to the result of research investigate by [13].

### 7. Conclusion
This paper explains how the manufacturer and retailer determine the optimal selling prices to maximize their system profit by notice into carbon emission capacity regulation. The maximal profit in centralized system can be obtained when probability of defective product, delivery lead time sensitivity, and carbon emissions are minimal. This paper only focus on manufacturing, inspection, and rework process. Future research could explore a dual channel supply chain model with additional to other production processes.

### Appendix A
To determine if the total profit of the dual channel supply chain strategy is a strictly concave function, we must examine the principal minor determinant from Hessian matrix. The Hessian matrix of \(\Pi_c(p_m, p_r)\) is
\[ \mathbf{H} = \begin{pmatrix} -2d_m(\eta + \beta \theta) & 2d_m \eta \\ 2d_m \eta & -2(d_r \beta + d_m((1- \theta) \beta + \eta)) \end{pmatrix}. \] (A.1)

From (A.1) we get
\[ \mathbf{H}_{11} = -2d_m(\eta + \beta \theta) < 0, \] (A.2)
\[ \mathbf{H}_{22} = 4d_m \beta(\eta + \beta \theta) + d_m(\eta - \beta(\theta - 1) \theta) > 0. \] (A.3)

Sylvester Criterion says if the principal minor determinant from Hessian matrix a function is \((-1)^i\), with \(i = 1, 2, 3, \ldots, n\) then the function is strictly concave function. From (A.2) and (A.3) we can conclude that \( \Pi_c(p_m, p_r) \) is a strictly concave function.

**Appendix B**

Necessary conditions of KKT for maximization problem is
\[ \mathbf{V} f(\mathbf{X}) - \hat{\mathbf{A}} \mathbf{V} g(\mathbf{X}) = 0, \]
\[ \lambda f(\mathbf{X}) = 0, \]
\[ g(\mathbf{X}) \leq 0. \] (A.4)

i. Let \( \frac{\partial (p_m, p_r \lambda)}{\partial p_m} = 0 \) and \( \frac{\partial (p_m, p_r \lambda)}{\partial p_r} = 0 \), we can get the optimal selling prices \( p^*_m \) and \( p^*_r \) in the centralized system as
\[ p^*_m = \left( (d_r \beta + d_m(\eta + (1- \theta) \beta)) \left( A + c_{orp} + c_{orp \lambda} \right) + \eta(d_m(1- \theta + l_m \alpha) + d_r) + \left( d_r e_2 \beta + d_m(-e_1 \eta + e_2 \beta(1- \theta + \eta)) \right) \lambda^* + \beta(d_m(1- \theta + d_e)C_M) / 2B, \right. \]
\[ = \frac{1}{2\eta} \left( (A - 1/B(\eta + \beta \theta)) \left( d_r \beta + d_m(\eta + (1- \theta) \beta) \right) A + \eta(d_m(1- \theta + l_m \alpha) + d_r(1- \theta + l_m \alpha) + d_m(-e_1 \eta + e_2 \beta(1- \theta + \eta)) \right) \lambda^* + \left( d_r \beta + \beta(d_m(1- \theta + d_e)C_M) \right) \). \]

ii. And from the other necessary condition we get the optimal \( \lambda \) is
\[ \lambda^* = -(e_2 d_c + d_m(e_1 - e_2)(l_m \alpha - \theta) + \beta(d_r e_2 + d_m(e_2(1- \theta) + e_1 \theta))(1 + f_2 \delta y) \]
\[ c_{imp} + \left( d_r e_2 \beta + d_m(-e_1 \eta + e_2 \beta(1- \theta + \eta)) \right) c_{orp} + \left( e_2 \beta d_c + d_m \beta(1- \theta - e_2) \right) \]
\[ \left( C_M - c_{imp} (1 + l_f \delta) \right) + e_2 \beta c_{orp} (d_m(1- \theta + d_r) - d_m \eta c_{orp}(e_1 - e_2) + 2K_c \right) \]
\[ / (e_2 \beta d_c + d_m e_1(1 - e_2 \eta + e_1(1 + \beta \theta))). \]

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