Analysis of ordinary and radiative muon capture in liquid hydrogen

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A simultaneous analysis is made of the measured rates of ordinary muon capture (OMC) and radiative muon capture (RMC) in liquid hydrogen, using theoretical estimates for the relevant atomic capture rates that have been obtained in chiral perturbation theory with the use of the most recent values of the coupling constants. We reexamine the basic formulas for relating the atomic OMC and RMC rates to the liquid-hydrogen OMC and RMC rates, respectively. Although the analysis is significantly influenced by ambiguity in the molecular state population, we can demonstrate that, while the OMC data can be reproduced, the RMC data can be explained only with unrealistic values of the coupling constants; the degree of difficulty becomes even more severe when we try to explain the OMC and RMC data simultaneously.

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1. Introduction

Ordinary and radiative muon capture (OMC and RMC) on a proton

\[ \mu^- + p \rightarrow n + \nu_\mu, \quad \text{(OMC)} \]  
\[ \mu^- + p \rightarrow n + \nu_\mu + \gamma, \quad \text{(RMC)} \]  

are fundamental weak-interaction processes in nuclear physics and a primary source of information on \( g_P \), the induced pseudoscalar coupling constant of the weak nucleon current, see e.g. [1, 2]. The most accurate existing measurements of the OMC and RMC rates have been carried out using a liquid hydrogen target, which unfortunately makes the analysis of the data sensitive to the molecular transition rates in liquid hydrogen. We denote by \( \Lambda_{\text{liq}} \) the OMC rate in liquid hydrogen. The experimental value obtained by Bardin et al. [3] is

\[ \Lambda_{\text{exp}}^{\text{liq}} = 460 \pm 20 \text{ [s}^{-1}] \quad \text{(OMC)}. \]  

As for RMC, Jonkmans et al. [4] measured the absolute photon spectrum for \( E_\gamma \geq 60 \text{ MeV} \) and deduced therefrom the partial RMC branching ratio, \( R_\gamma \), which is the number of RMC events (per stopped muon) producing a photon with \( E_\gamma \geq 60 \text{ MeV} \). The measured value of \( R_\gamma \) is [4, 5]

\[ R_\gamma^{\text{exp}} = (2.10 \pm 0.22) \times 10^{-8} \quad \text{(RMC)}. \]  

Surprisingly, the value of \( g_P \) deduced in [4, 5] from the RMC data is \( \sim 1.5 \) times larger than the PCAC prediction [6]. By contrast, the value of \( g_P \) deduced in [7] from the OMC data is in good agreement with the PCAC prediction.

On the theoretical side, the early estimation of \( g_P \) was made using PCAC. Heavy-baryon chiral perturbation theory (HB\( \chi \)PT), a low-energy effective theory of QCD, allows us to go beyond the PCAC approach, but the results of detailed HB\( \chi \)PT calculations [8] up to next-to-next-to-leading order (NNLO) essentially agree with those obtained in the PCAC approach. Thus the theoretical framework for estimating \( g_P \) is robust. The key quantities in analyzing OMC and RMC are the atomic rates, \( \Lambda_s \) and \( \Lambda_t \), where \( \Lambda_s \) (\( \Lambda_t \)) is the capture rate for the hyperfine singlet (triplet) state of the \( \mu^-p \) atom. \footnote{The rates \( \Lambda_s \) and \( \Lambda_t \) are generic symbols for OMC and RMC. When we need to distinguish OMC and RMC, we use the symbols \( \Lambda_{s,t}^{\text{OMC}} \) and \( \Lambda_{s,t}^{\text{RMC}} \).}

The atomic rates for OMC and RMC have also been estimated in the framework of HB\( \chi \)PT [9, 10, 11, 12, 13, 14, 15]. The expressions obtained in HB\( \chi \)PT have been found to be essentially in agreement with those of the earlier work [16, 17, 18, 19, 20]. It has also been confirmed that the chiral expansion converges rapidly, rendering estimates of the OMC and RMC rates obtained in \( \chi \)PT extremely robust. As for the numerical results, however, the earlier estimates of the atomic OMC rates, e.g. [13, 17], need to be revised because some values of the input parameters (\( g_A \), \( g_\pi N \), etc.) used in those estimates are now obsolete. In Ref. [10], we provided updated estimates of \( \Lambda_{s,t}^{\text{OMC}} \) and \( \Lambda_{s,t}^{\text{RMC}} \) based on HB\( \chi \)PT (up to
NNLO). A notable finding in [10] is that the use of the recent larger value of the Gamow-Teller coupling constant, $g_A$, gives a value of $\Lambda^{OMC}_s$ that is significantly larger than the older value commonly quoted in the literature, see Refs. [10, 15].

To make comparison between theory and experiment, one needs to relate the theoretically calculated atomic OMC and RMC rates to $\Lambda_{liq}$ and $R_\gamma$, respectively. For convenience, we refer to this relation as the A-L (atom-liquid) formula. Bakalov et al. [21] made a detailed study of the A-L formula, and they gave an explicit expression for $\Lambda_{liq}$ (see Eq. (56c) in Ref. [21]). In our previous work [10] we analyzed $\Lambda_{liq}$ using the A-L formula of Bakalov et al. and found that the best available estimates of the atomic capture rates based on HB$\chi$PT would lead to a value of $\Lambda_{liq}$ that was significantly larger than $\Lambda^{exp}_{liq}$. We also reported that, by introducing a molecular state mixing parameter, $\xi$, considered by Weinberg [22], it was possible to reproduce $\Lambda^{exp}_{liq}$ and $R^{exp}_\gamma$ simultaneously. However, the A-L formula of Bakalov et al. does not correspond to the experimental condition of OMC; to compare with $\Lambda^{exp}_{liq}$, the time sequence of the experimental measurement should be considered [3].

In this work we reexamine $\Lambda_{liq}$ and $R_\gamma$ by incorporating into our analysis the experimental conditions as well as the updated estimates of the atomic capture rates. In particular, we investigate the influence of ambiguity in the transition rate between the molecular states (to be discussed below); we also examine the dependence of the results on the values of $g_P$ and the molecular parameter $\xi$ (see below).

2. Muonic states in liquid hydrogen

To evaluate $\Lambda_{liq}$ and $R_\gamma$ from the calculated atomic OMC and RMC rates, we need to know the temporal behavior of the various $\mu$-capture components (capture from the atomic states and capture from $p-\mu-p$ molecular states). Fig. 1 schematically depicts various competing atomic and molecular processes occurring in liquid hydrogen. A muon stopped in liquid hydrogen quickly forms a muonic atom ($\mu-p$) in the lowest Bohr state.

Figure 1: Atomic and molecular states relevant to muon capture in liquid hydrogen; $\lambda_{pp\mu}$ is the transition rate from the atomic singlet state to the ortho $p-\mu-p$ molecular state, and $\lambda_{op}$ is that from the ortho to para molecular state.
The atomic hyperfine-triplet state (S=1) decays extremely rapidly to the singlet state (S=0), with a transition rate $\lambda_0 \simeq 1.7 \times 10^{10} \text{ s}^{-1}$. In the liquid hydrogen target a muonic atom and a hydrogen molecule collide with each other and form a $p-\mu-p$ molecule with the molecule predominantly in its ortho state. We denote by $\lambda_{ppp}$ the transition rate from the atomic singlet state to the ortho $p-\mu-p$ molecular state. The ortho $p-\mu-p$ state further decays to the para $p-\mu-p$ molecular state. This rate is denoted by $\lambda_{op}$. Let $N_s(t)$, $N_{om}(t)$, and $N_{pm}(t)$ represent the numbers of muons at time $t$ in the atomic singlet, ortho-molecular, and para-molecular states, respectively. They satisfy coupled kinetic equations, see Eq. (54a) in Ref. [21]. To integrate these coupled differential equations, we need to know the initial conditions.

For illustration purposes, let us consider a case in which there is one muon in the singlet state at $t = 0$; i.e., $N_s(0) = 1$ and $N_{om}(0) = N_{pm}(0) = 0$. We then have

$$
N_s(t) = e^{-\lambda_0 t}, \quad N_{om}(t) = \frac{\lambda_{ppp}}{\lambda_2 - \lambda_3} (e^{-\lambda_3 t} - e^{-\lambda_2 t}),
$$

$$
N_{pm}(t) = \frac{\lambda_{op} \lambda_{ppp}}{(\lambda_3 - \lambda_4)(\lambda_2 - \lambda_4)} e^{-\lambda_4 t} - \frac{\lambda_{op} \lambda_{ppp}}{(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_4)} e^{-\lambda_3 t} + \frac{\lambda_{op} \lambda_{ppp}}{(\lambda_2 - \lambda_3)(\lambda_2 - \lambda_4)} e^{-\lambda_2 t},
$$

(5)

where $\lambda_2 = \lambda_0 + \lambda_{ppp} + \lambda_{OMC}^O + \lambda_{RMC}^O$, $\lambda_3 = \lambda_0 + \lambda_{op} + \lambda_{OMC}^O + \lambda_{om}^O$, $\lambda_4 = \lambda_0 + \lambda_{om}^O + \lambda_{pm}^O + \lambda_{pm}^{RMC}$. Here $\lambda_0$ is the muon natural decay rate. $\lambda_{OMC}^O$ and $\lambda_{om}^O$ are the OMC rates in the ortho molecular and para molecular states, respectively; similarly for $\lambda_{RMC}^O$, and $\lambda_{pm}^{RMC}$.

These rates are given by

$$
\Lambda_{om}^F = 2\gamma_0 \left( \frac{3}{4} \Lambda_s^F + \frac{1}{4} \Lambda_t^F \right), \quad \Lambda_{pm}^F = 2\gamma_P \left( \frac{1}{4} \Lambda_s^F + \frac{3}{4} \Lambda_t^F \right),
$$

(6)

where $F$ stands for “OMC” or “RMC”, and $2\gamma_0 = 1.009$, $2\gamma_P = 1.143$ [21].

At this point we discuss the numerical values of $\lambda_{ppp}$ and $\lambda_{op}$. The former shows a wide scatter in the literature, ranging from $\lambda_{ppp} = (1.89 \pm 0.20) \times 10^6 \text{ s}^{-1}$ to $(2.75 \pm 0.25) \times 10^6 \text{ s}^{-1}$ [23]. In this work, for the sake of definiteness, we employ the averaged value $\lambda_{ppp} = 2.5 \times 10^6 \text{ s}^{-1}$ (the main point of our argument is not affected by this choice). This value is comparable to the muon decay rate $\lambda_0 = 0.455 \times 10^6 \text{ s}^{-1}$. As regards $\lambda_{op}$, there is a significant difference between the experimental and theoretical values; $\lambda_{op}^{ex} = (4.1 \pm 1.4) \times 10^4 \text{ s}^{-1}$ [7] as compared with $\lambda_{op}^{th} = (7.1 \pm 1.2) \times 10^4 \text{ s}^{-1}$ [21].

The dominant state for the OMC and RMC measurements is the ortho molecular state as is evident from Eq. (5). In both measurements, data taking starts at $t = t_i \neq 0$, and it is essential to incorporate this aspect into the A-L formula (see below). Furthermore, in the OMC experiment the time dependence of the population of each state plays an important role.

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2Since $\lambda_{OMC}^O$, $\lambda_{om}^O$, $\lambda_{pm}^{RMC}$ are very small, they can be ignored in the calculation of $N_{s,om,pm}(t)$. In evaluating the RMC rate itself, however, we need these capture rates; see Eq. (10) below.
3. Atom-Liquid (A-L) formula for OMC and RMC

The discussion so far is common for both OMC and RMC, but we now turn to the individual discussion of each case. In the OMC experiment (see Fig. 4 in Ref. [3]), \( \mu^- \) beams arrive at the target on the average in a 3 \( \mu \)s-long burst with repetition rate 3000 Hz. The data collection typically starts 1 \( \mu \)s after the end of the 3 \( \mu \)s-long beam burst, and the measurement lasts until 306 \( \mu \)s after the end of the beam burst. As mentioned, the cascade processes leading to the \( \mu^-p \) ground state and the transition between the atomic hyperfine states are extremely fast. One therefore can safely ignore a time lag between the muon arrival time and the time at which the \( \mu^-p \) atomic hyperfine-singlet state is formed. To proceed with the consideration of OMC, we assume that the average time intervals of Ref. [3] cited above are actual time intervals. Then, provided all the muons arrive at the same time, we can choose with no ambiguity that arrival time as the origin of time (\( t = 0 \)) and let \( t = t_i \), the starting time for data collection, refer to that origin. However, the finite duration (\( t_b = 3 \mu s \)) of the beam burst causes uncertainty in the value of \( t = t_i \) to be used in Eq. (5); \( t_i \) can be anywhere between 1.0 \( \mu \)s and 4.0 \( \mu \)s.

To account for this muon pulse duration time, \( t_b \), we assume for simplicity that the beam pulse has a rectangular shape. Then, at time \( t \) the average number of residual muons are:

\[
\bar{N}_\mu(t) \equiv \frac{1}{t_b} \int_0^{t_b} dt'N_\mu(t-t').
\]

(7)

where \( N_\mu(t) = N_s(t) + N_{om}(t) + N_{pm}(t) \). The OMC experiment [3] counts the number of electrons produced by \( \mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu \), and \( \Lambda_{liq} \) is deduced from the difference between the muon decay rate in liquid hydrogen and that in vacuum; the latter is determined from the number of positrons produced in \( \mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu \). We use the expression of Ref. [3] (and \( t_i = 4 \mu s \))

\[
\Lambda_{liq} = \left( \frac{\int_{t_i}^{\infty} dt \frac{dN_e}{dt} \bar{N}_\mu}{\int_{t_i}^{\infty} dt(t-t_i)\frac{dN_e}{dt}} \right) - \lambda_0,
\]

(8)

where \( \bar{N}_e(t) \) is the averaged number of electrons produced at time \( t \) and \( \frac{dN_e(t)}{dt} = \lambda_0 \bar{N}_\mu(t) \). Here we have used the fact the duration of the measuring time (306 \( \mu \)s) is long enough to be treated as \( \infty \).

On the other hand, for the RMC experiment [4, 5], the muons essentially arrive one by one and the data taking begins at \( t_i = 365 \) ns. We therefore can neglect the beam burst duration time in the RMC case, and we obtain

\[
R_{\gamma} = \frac{N_\gamma(\infty) - N_\gamma(t_i)}{N_\mu(t_i)}.
\]

(9)

Here \( N_{\gamma}(t) \) is the number of photons obtained by integrating the photon spectrum over the interval, \( 60 \leq E_\gamma \leq 99 \) MeV, and the production of photons in RMC is determined by

\[
\frac{dN_\gamma(t)}{dt} = \Lambda_s^{RMC} N_s(t) + \Lambda_{om}^{RMC} N_{om}(t) + \Lambda_{pm}^{RMC} N_{pm}(t),
\]

(10)
Table 1: Coupling constants and the atomic capture rates \([s^{-1}]\) used in the present analysis

|       | \(g_A\) | \(g_{\pi N}\) | \(\Lambda_s^{OMC}\) | \(\Lambda_t^{OMC}\) | \(\Lambda_s^{RMC}\) | \(\Lambda_t^{RMC}\) |
|-------|---------|------------|-----------------|----------------|----------------|----------------|
|       | 1.267   | 13.40      | 695             | 11.9           | \(0.891 \times 10^{-3}\) | \(20.1 \times 10^{-3}\) |

where \(N, (0) = 0\).

4. Numerical results and discussion

We first give the numerical values of inputs to be used in what follows. Table 1 presents the values of the coupling constants and the atomic capture rates. The OMC and RMC rates for the hyperfine-singlet and -triplet states have been calculated in HB\(\chi\)PT up to NNLO \([10, 14]\) and with the use of the most recent values of the coupling constants discussed in \([10]\).

We estimate \(\Lambda_{liq}\) by using the atomic OMC rates in Table 1. Besides the A-L formula in Eq. (8), we consider two others for the sake of comparison; these two A-L formulas are that of Bardin et al. \([7]\) and that of Bakalov et al. \([21]\). For the ortho-para transition rate we employ either \(\lambda_{op}^{exp}\) or \(\lambda_{op}^{th}\). The use of \(\lambda_{op}^{exp}\) leads to \(\Lambda_{liq} = 460 \text{ s}^{-1}\) with Eq. (8) and \(\Lambda_{liq} = 459 \text{ s}^{-1}\) with Bardin et al.’s A-L formula. Those values agree with \(\Lambda_{liq}^{exp} = 460 \pm 20 \text{ s}^{-1}\). Meanwhile, if we employ \(\lambda_{op}^{th}\), we obtain \(\Lambda_{liq} \simeq 421 \text{ s}^{-1}\) with Eq. (8), and \(\Lambda_{liq} \simeq 419 \text{ s}^{-1}\) with Bardin et al.’s formula. Thus, \(\Lambda_{liq}\) is highly sensitive to \(\lambda_{op}\). On the other hand, the use of Bakalov et al.’s A-L formula \([21]\) gives too large a value for \(\Lambda_{liq}\) regardless of whether we use \(\lambda_{op}^{exp}\) or \(\lambda_{op}^{th}\); \(\Lambda_{liq} = 532 \text{ s}^{-1}\) for \(\lambda_{op}^{exp}\), and \(\Lambda_{liq} = 518 \text{ s}^{-1}\) for \(\lambda_{op}^{th}\). As mentioned before, the A-L formula of Bakalov et al., which was adopted in our previous work \([10]\), corresponds to the choice of \(t_i = 0\), and this choice does not simulate the experimental condition.

An estimate of \(R_\gamma\) is obtained from Eq. (9) and the atomic RMC rates in Table 1. With the use of \(\lambda_{op}^{exp}\), the calculated value of \(R_\gamma\) is significantly smaller than \(R_\gamma^{exp}\) in Eq. (4); \(R_\gamma^{exp}/R_\gamma^{th} \approx 1.5\). If in Eq. (4) we use \(\lambda_{op}^{th}\) instead of \(\lambda_{op}^{exp}\), then \(R_\gamma\) is enhanced by about 9% but the increase is not large enough to reconcile \(R_\gamma^{th}\) with \(R_\gamma^{exp}\). Thus it is not possible to reproduce \(R_\gamma^{exp}\) in the existing theoretical framework with the use of the standard set of input parameters. In addition, we remark that our results indicate that the sensitivity of \(R_\gamma\) to \(\lambda_{op}\) is comparable to that of \(\Lambda_{liq}\).

As mentioned, the atomic capture rates calculated using a phenomenological relativistic tree-level model \([18]\) are consistent with those of HB\(\chi\)PT \([13, 14, 15, 24]\) (provided the former uses the updated value of \(g_A\) and the PCAC value of \(g_P\)). Therefore, the above conclusions are not necessarily unique to HB\(\chi\)PT. However, since HB\(\chi\)PT gives \(\Lambda_s\) and \(\Lambda_t\) with high precision (primarily because the value of \(g_P\) is strictly restricted by chiral symmetry), it allows us to draw much sharper conclusions than the phenomenological approach.

Next, we discuss the sensitivity of \(\Lambda_{liq}\) and \(R_\gamma\) to possible changes in the values of \(g_P\) and the molecular mixing parameter \(\xi\). In this discussion we use the phenomenological model of Fearing \([18]\), a model which admits the variation of \(g_P\) within a certain range.
As discussed by Weinberg [22], the possible mixing of the ortho molecular $p$-$\mu$-$p$ spin 3/2 state and spin 1/2 state, parameterized by $\xi$, may change the molecular capture rates to

$$\Lambda_{om}^F = \xi \Lambda_{om}^F (1/2) + (1 - \xi) \Lambda_{om}^F (3/2),$$

where $F$ stands for “OMC” or “RMC”; $\Lambda_{om}^F (1/2) = \Lambda_{om}^F$ [see Eq. (6)] and $\Lambda_{om}^F (3/2) = 2\gamma_{O}\Lambda_{om}^F$. Although the existing theoretical estimate favors $\xi \simeq 1$ [21, 25], we treat it here, as we did in Ref. [10], as a parameter to fit the data. In this phenomenological model, with the use of $\lambda_{exp}$, we can reproduce $R_{\gamma_{exp}}$ by adopting either $g_P = 1.4g_{P\,CAC}$ or $\xi = 0.80$. However, with the same value of $\lambda_{op}$, the OMC data requires $g_P \leq 1.2g_{P\,CAC}$ or $\xi \geq 0.95$. Therefore, it is impossible to simultaneously fit the OMC and the RMC data even by adjusting $g_P$ and $\xi$. If $\lambda_{op}$ is taken to be smaller than $\lambda_{exp}^{\gamma} = 4.1 \times 10^6$ s$^{-1}$, then it is not impossible to explain $\Lambda_{eq}^{\exp}$ and $R_{\gamma}^{\exp}$ within the phenomenological model with a value of $g_P$ larger than that of $g_{P\,CAC}$ and $\xi \leq 0.95$. However, we cannot attach too much significance to this possibility, since HB$\chi$PT constrains the value of $g_P$ with high accuracy, i.e., there is not much room left for adjusting the value of $g_P$. The result of a more precise measurement of $\lambda_{op}$ at TRIUMF [26] will shed much light on this issue.

Our findings are largely in the nature of reconfirming the conclusions stated in one way or another in the literature, but a coherent treatment of OMC and RMC in liquid hydrogen as described here is hoped to be useful. Our treatment is characterized by the use of the best available atomic capture rates obtained in HB$\chi$PT, and by an improved A-L formula. Although we have presented examples of simulation of the experimental conditions, they are only meant to serve illustrative purposes. Definitive analyses can be done only by the people who carried out the relevant experiments. Finally, we remark that a precise measurement of the OMC rate in hydrogen gas is planned at PSI [23]. This experiment would eliminate the ambiguity of the molecular transition rate discussed in this paper and directly test the HB$\chi$PT prediction [10] [13].

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