Strangeness S=-3 and -4 baryon-baryon interactions in chiral EFT

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Abstract. I report on recent progress in the description of baryon-baryon systems within chiral effective field theory. In particular, I discuss results for the strangeness $S = -3$ to $-4$ baryon-baryon systems, obtained to leading order.

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INTRODUCTION

Chiral effective field theory (EFT) as proposed in the pioneering works of Weinberg [1] is a powerful tool for the derivation of nuclear forces. In this scheme there is an underlying power counting which allows to improve calculations systematically by going to higher orders in a perturbative expansion. In addition, it is possible to derive two- and corresponding three-nucleon forces as well as external current operators in a consistent way. For reviews we refer the reader to Refs. [2, 3, 4].

Over the last decade or so it has been demonstrated that the nucleon-nucleon ($NN$) interaction can be described to a high precision within the chiral EFT approach [5, 6]. Following the original suggestion of Steven Weinberg, in these works the power counting is applied to the $NN$ potential rather than to the reaction amplitude. The latter is then obtained from solving a regularized Lippmann-Schwinger equation for the derived interaction potential. The $NN$ potential contains pion-exchanges and a series of contact interactions with an increasing number of derivatives to parameterize the shorter ranged part of the $NN$ force.

Recently, also hadronic systems involving the strange baryons $\Lambda$ and $\Sigma$, and the $S = -2$ baryon $\Xi$ were investigated within EFT by the group in Jülich [7, 8, 9, 10, 11]. Specifically, the interactions in the $\Lambda N$ and $\Sigma N$ channels [7] as well as those in the $S = -2$ sector ($\Lambda\Lambda$, $\Sigma\Sigma$, $\Lambda\Sigma$, $\Xi N$) [8] were considered. In these works the same scheme as applied in Ref. [6] to the $NN$ interaction is adopted. In the present contribution I focus on a recent extension of that study to systems with $S = -3$ and $-4$ [10].

FORMALISM

To leading order (LO) in the power counting, as considered in the aforementioned investigations [7, 8, 10], the baryon-baryon potentials involving strange baryons consist of four-baryon contact terms without derivatives and of one-pseudoscalar-meson exchanges, analogous to the $NN$ potential of [6]. The potentials are derived using constraints from SU(3)$_f$ flavor symmetry. Details on the derivation of the chiral potentials for the $S = -1$ to $S = -4$ sectors at LO using the Weinberg power counting can be found in Ref. [7]. The contributions of one-pseudoscalar-meson exchanges are identical to those already discussed extensively in the literature, see, e.g., [7]. The LO SU(3)$_f$ invariant contact terms for the octet baryon-baryon interactions that are Hermitian and invariant under Lorentz transformations follow from the Lagrangians

$$\mathcal{L}^1 = C_1^{\Gamma} \langle \vec{B}_a \vec{B}_b (\Gamma_i B)^a \rangle \langle (\Gamma_i B)^b \rangle,$$

$$\mathcal{L}^2 = C_2^{\Gamma} \langle \vec{B}_a (\Gamma_i B)^a \vec{B}_b (\Gamma_i B)^b \rangle,$$

$$\mathcal{L}^3 = C_3^\Gamma \langle \vec{B}_a (\Gamma_i B)^a \rangle \langle \vec{B}_b (\Gamma_i B)^b \rangle. \quad (1)$$

As described in [7], in LO the Lagrangians give rise to six independent low-energy coefficients (LECs) $- C_1^\Gamma, C_2^\Gamma, C_3^\Gamma, C_1^\epsilon, C_2^\epsilon$ and $C_3^\epsilon$ – that need to be determined by a fit to experimental data. Here $S$ and $T$ refer to the central and spin-spin interactions.
FIGURE 1. Total cross sections for various reactions in the strangeness $S = -3$ sector as a function of $p_{lab}$. The shaded band shows the chiral EFT results for variations of the cut-off in the range $\Lambda = 550 \ldots 700$ MeV.

RESULTS AND DISCUSSION

The LO chiral EFT interaction for the $S = -3$ and $-4$ baryon-baryon sector depends only on those five contact terms that enter also in the $YN$ interaction, cf. Table 1 in [10]. Thus, based on the values which were fixed in our study of the $YN$ sector [7] we can make genuine predictions for the interaction in the $S = -3$ and $-4$ channels that follow from the imposed SU(3)$_f$ symmetry.

Corresponding results for the $\Xi^0 \Lambda \rightarrow \Xi^0 \Lambda$, $\Xi^0 \Sigma^- \rightarrow \Xi^- \Lambda$, $\Xi^0 \Sigma^- \rightarrow \Xi^- \Sigma^0$, $\Xi^0 \Sigma^- \rightarrow \Xi^0 \Sigma^-$, and $\Xi^0 \Sigma^+ \rightarrow \Xi^0 \Sigma^+$ scattering cross sections are presented in Fig. 1. Partial waves with total angular momentum up-to-and-including $J = 2$ are taken into account. The shaded bands show the cut-off dependence. From that figure one observes that the $\Xi^0 \Lambda \rightarrow \Xi^0 \Lambda$ and $\Xi^0 \Sigma^+ \rightarrow \Xi^0 \Sigma^+$ cross sections are rather large near threshold. Though the cross section for

parts of the potential, respectively. The scattering amplitude is obtained by solving a Lippmann-Schwinger equation for the LO potential. Thereby, the possible coupling between different baryon-baryon channels, $\Lambda N - \Sigma N$ or $\Xi \Lambda - \Xi \Sigma$, say, is taken into account. The potentials in the LS equation are cut off with a regulator function, $\exp \left[ - \left( p'^4 + p^4 \right) / \Lambda^4 \right]$, in order to remove high-energy components of the baryon and pseudoscalar meson fields [6].
we do not observe any sizeable cusp effects in the large scattering length in the corresponding

Predicted cross sections for the \( \Sigma^- \Xi^- \rightarrow \Xi^- \Xi^- \) and \( \Xi^0 \Xi^0 \rightarrow \Xi^0 \Xi^0 \) as a function of \( p_{\text{lab}} \). The shaded band shows the chiral EFT results for variations of the cut-off in the range \( \Lambda = 550 \ldots 700 \) MeV.

### TABLE 1. Selected \( \Sigma \Xi \) and \( \Xi \Xi \) singlet and triplet scattering lengths \( a \) and effective ranges \( r \) (in fm) for various cut-off values \( \Lambda \). The last columns show results for the Nijmegen potential (NSC97a, NSC97f) [15] and the model by Fujiwara et al. (fss2) [16].

| \( \Lambda \) (MeV) | EFT     | NSC97a | NSC97f | fss2   |
|-------------------|---------|--------|--------|--------|
| 550               | 600     | 650    | 700    |        |
| \( a_{\Xi \Lambda} \) | –33.5  | 35.4   | 12.7   | 9.07   | –0.80  | –2.11 | –1.08 |
| \( r_{\Xi \Lambda} \) | 1.00   | 0.93   | 0.87   | 0.84   | 4.71   | 3.21  | 3.55  |
| \( a_{\Xi \Xi} \)   | 0.33   | 0.33   | 0.32   | 0.31   | 0.54   | 0.33  | 0.26  |
| \( r_{\Xi \Xi} \)   | –0.36  | –0.30  | –0.29  | –0.27  | –0.47  | 2.79  | 2.15  |

\( \Xi^0 \Sigma^- \rightarrow \Xi^- \Lambda \) rises too, in this case it is only due to the phase space factor \( p_{\Xi^- \Lambda} / p_{\Xi^0 \Sigma^-} \). There is a clear cusp effect visible in the \( \Xi^0 \Sigma^- \) cross section at \( p_{\text{lab}} \approx 106 \) MeV/c, i.e. at the opening of the \( \Xi^- \Sigma^0 \) channel. On the other hand, we do not observe any sizeable cusp effects in the \( \Xi^0 \Lambda \) cross section around \( p_{\text{lab}} = 690 \) MeV/c, i.e. at the opening of the \( \Sigma \Sigma \) channels. The latter is in line with the results reported by the Nijmegen group for their interactions [15], where a cusp effect in that channel is absent too. In this context I would like to remind the reader that the cusp seen in the corresponding strangeness \( S = –1 \) case, namely in the \( \Lambda \Lambda \) cross section at the \( \Sigma \Sigma \) threshold, is rather pronounced in our chiral EFT interaction [7] but also in conventional meson-exchange potential models [12, 13, 14].

Predicted cross sections for the \( \Xi^0 \Xi^0 \) and \( \Xi^0 \Xi^- \) channels are shown in Fig. 2, again as a function of \( p_{\text{lab}} \) and with shaded bands that indicate the cut-off dependence.

Results for the \( \Xi^0 \Lambda \), \( \Xi^0 \Sigma^+ \), and \( \Xi \Xi \) scattering lengths and effective ranges are listed in Table 1. Here we also include predictions by other models [15, 16] for channels where pertinent results are available in the literature. This Table reveals the reason for the sizeable \( \Xi^0 \Lambda \) cross section predicted by the chiral EFT interactions, namely a rather large scattering length in the corresponding \( ^1S_0 \) partial wave. It is obvious that its value is strongly sensitive to cut-off
that in such a case the predictive power of our LO calculation is rather limited. One has to wait for at least an NLO calculation, where we expect that the cut-off dependence will become much weaker so that more reliable conclusions on the possible existence of a virtual or a real bound state should be possible. The $^1S_0$ scattering lengths of the other potentials suggest also an overall attractive interaction in this partial wave though only a very moderate one.

The results for the $^3S_1$ state of the $\Xi^0\Lambda$ channel are fairly similar for all considered interactions. Moreover, with regard to the chiral EFT interaction there is very little cut-off dependence. The $S$-waves in the $\Xi\Sigma I = 3/2$ channel belong to the same (10$^\ast$ and 27, respectively) irreducible representations where in the $NN$ case real ($^3S_1 - ^3D_1$) or virtual ($^1S_0$) bound states exist, cf. Table 1 in Ref. [10]. Therefore, one expects that such states can also occur for $\Xi\Sigma$. Indeed, bound states are present for both partial waves in the Nijmegen model, cf. the discussion in Sect. III.B in Ref. [15]. Their presence is reflected in the positive and fairly large singlet and triplet scattering lengths for $\Xi^0\Sigma^+$, cf. Table 1. The chiral EFT interaction has positive scattering lengths of comparable magnitude for $^1S_0$, for all cut-off values, and therefore bound states, too. These binding energies lie in the range of $-2.23$ MeV ($\Lambda = 550$ MeV) to $-6.15$ MeV (700 MeV). In the $^3S_1 - ^3D_1$ partial wave the attraction is obviously not strong enough to form a bound state. The same is the case (but for both $S$ waves) for the quark model fss2 of Fujiwara et al. [16].

The $^1S_0$ state of the $\Xi\Xi$ channel belongs also to the $2\Sigma$-plet irreducible representation and also here the Nijmegen as well as the chiral EFT interactions produce bound states. In our case the binding energies lie in the range of $-2.56$ MeV ($\Lambda = 550$ MeV) to $-7.28$ MeV (700 MeV). The predictions of both approaches for the $^3S_1$ scattering length are comparable. The quark model of Fujiwara et al. exhibits a different behavior for the $\Xi\Xi$ channel, see the last column in Table 1. The small and negative $^1S_0$ scattering length signals an interaction that is only moderately attractive. The large and positive scattering length in the $^3S_1 - ^3D_1$ partial wave, produced by that potential model, is usual a sign for the presence of a bound state, though according to the authors this is not the case for this specific interaction. Further results, and specifically $\Xi\Lambda$, $\Xi\Sigma$, and $\Xi\Xi$ phase shifts, can be found in [17].

SUMMARY

Our investigations show that the chiral EFT scheme, successfully applied in Ref. [6] to the $NN$ interaction, also works well for the $\Lambda\Lambda$, $\Sigma\Sigma$ [7, 11] and $\Lambda\Sigma$ [8] interactions. Moreover, as reported here, it can be used to make predictions for the $S = -3$ and $-4$ baryon-baryon interactions invoking constraints from SU(3) flavor symmetry. It will be interesting to see whether the new facilities J-PARC (Tokai, Japan) and FAIR (Darmstadt, Germany) allow access to empirical information about the interaction in the $S = -3$ and $-4$ sectors. Such information could come from formation experiments of corresponding hypernuclei or from proton-proton and antiproton-proton collisions at such high energies that pairs of baryons with strangeness $S = -3$ or $S = -4$ can be produced.

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