Sterile Neutrinos and Global Symmetries

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We use an effective-field-theory approach to construct models with naturally light sterile neutrinos, due to either exact or accidental global symmetries. The most attractive models we find are based on gauge symmetries, either discrete or continuous. We give examples of simple models based on $\mathbb{Z}_N$, $U(1)'$, and $SU(2)'$.

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I. INTRODUCTION

A wide variety of neutrino-oscillation experiments indicate that neutrinos have non-vanishing masses in the sub-eV range.\(^1\) All experiments but one are consistent with three species of neutrinos: an atmospheric pair with $|\Delta m_{23}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$, $\sin 2\theta_{23} \approx 1.0$, and a solar pair with $|\Delta m_{12}^2| \approx 8.0 \times 10^{-5} \text{ eV}^2$, $\tan^2 \theta_{12} \approx 0.45$. The exception is the Liquid Scintillator Neutrino Detector (LSND) experiment\(^2\), which finds $|\Delta m^2| \approx (0.2 - 7) \text{ eV}^2$, $\sin 2\theta \approx 0.003 - 0.04$. This result requires one or more additional neutrinos. Since the decay $Z \to \nu\bar{\nu}$ shows that there are only three species of neutrinos with ordinary weak interactions, these additional neutrinos must be sterile with respect to the standard-model gauge interactions.

In the standard model (SM), there are three species of active neutrinos, and they are massless due to an accidental $U(1)_L$ global symmetry.\(^2\) If one postulates, however, that there is new physics at a scale $M$ much greater than the electroweak scale $v \approx 246 \text{ GeV}$, it is natural for neutrinos to have small masses, independent of the details of the new physics. As long as the physics at $M$ does not respect the accidental $U(1)_L$ symmetry of the SM, it may give rise to a dimension-five interaction in the effective theory at the weak scale proportional to $\frac{1}{M} (L\phi) (L\phi)$, where $L$ and $\phi$ are the lepton and Higgs doublets, respectively.\(^2\) When the Higgs field acquires a vacuum expectation value (vev) $v$, this interaction yields neutrino masses of order $\frac{v^2}{M}$. For $M \approx 10^{14} - 10^{16} \text{ GeV}$, the resulting neutrino masses are in the desired range.

The presence of the scale $M$ raises the question of why the other fermions of the standard model do not acquire masses of order $M$. The reason is that they are forbidden by the SM gauge interactions from acquiring masses until the electroweak symmetry is broken at the scale $v$. Thus the other fermions of the SM have masses of order $v$, regardless of the presence of new physics at the scale $M$.

If sterile neutrinos exist in nature, they are not protected by SM gauge interactions from acquiring masses of order $M$. Thus, within this framework, light ($m \sim \text{ eV}$) sterile neutrinos are unnatural. For this reason there is considerable skepticism regarding their existence. In addition, big bang nucleosynthesis favors just three species of light neutrinos, although more than three is not ruled out ($2.67 \leq N_\nu \leq 3.85$ at 68% CL)\(^4\). A global fit to the world’s neutrino data favors at least two sterile neutrinos\(^5\). The MiniBooNE experiment at Fermilab seeks to confirm the LSND result.

In this paper, we explore extensions of the SM in which sterile neutrinos are light due to a global symmetry. We use an effective-field-theory approach, so our results are independent of the details of the new physics that resides at the scale $M$. We first consider an exact global symmetry, and show that it is straightforward to produce a model with light sterile neutrinos. There is doubt, however, that exact global symmetries exist in nature. An exception is a discrete symmetry that is the remnant of a broken gauge symmetry. We then turn to approximate (accidental) symmetries, and show that light sterile neutrinos require new gauge interactions and a set of new fermions that are SM singlets. Thus an entire sterile sector is required to explain the occurrence of light sterile neutrinos.

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\(^1\) For a recent analysis, see Ref.\(^6\).

\(^2\) Throughout this paper, it is understood that all global symmetries are classical, and may have quantum anomalies. These anomalies figure into the discussion in various places. In the context of the SM, both $U(1)_L$ and $U(1)_B$ are accidental global symmetries at the classical level, but only the combination $U(1)_{B-L}$ is free of global anomalies. Thus the neutrinos are exactly massless in the SM due to the accidental $U(1)_{B-L}$ symmetry.
field $U(1)_X$ charge

| $Q$   | $X_Q$ |
| $u^c$ | $-X_Q + X_L$ |
| $d^c$ | $-X_Q - X_L$ |
| $L$   | $X_L$ |
| $e^c$ | $-2X_L$ |
| $\nu_s$ | 1 |
| $\phi$ | $-X_L$ |
| $S$   | $-1$ |

TABLE I: A simple model for light sterile neutrinos based on an exact global $U(1)_X$ symmetry.

II. EXACT GLOBAL SYMMETRY

A. Continuous Symmetry

There is considerable doubt that exact global symmetries can exist in nature. It is likely that such symmetries are violated by quantum-gravitational processes. Furthermore, there are none known in nature. The only candidates are $U(1)_L$ and $U(1)_B$, but these are accidental global symmetries and are likely violated by higher-dimensional operators. Indeed, this is the most compelling explanation of why neutrinos are so much lighter than the other known fermions, as discussed in the Introduction.

Despite these arguments, it remains a logical possibility that there are exact global symmetries in nature that we have not yet discovered. If a sterile neutrino $\nu_s$ is charged under this symmetry, it could forbid a bare Majorana mass $m_{\nu_s}\nu_s$. Here we consider the possibility of an exact global $U(1)_X$ symmetry, along with the SM gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$, and assign a charge of unity to the sterile neutrino, $X_{\nu_s} = 1$.

In order to explain why the sterile neutrino is much lighter than the weak scale, we need the $U(1)_X$ symmetry to forbid the operator $(L\phi)\nu_s$. Such an operator would generate a Dirac neutrino mass at the weak scale. The symmetry $U(1)_L$ allows this operator (with $L_{\nu_s} = -1$), so it cannot play the role of $U(1)_X$. We want the sterile neutrino to acquire mass via a dimension-five operator, analogous to the operator $(L\phi)\nu_s$ that generates a small mass for the active neutrinos. The only candidate operators are $\nu_s\nu_s(\phi\phi)$ and $\nu_s\nu_s(\phi\bar{\phi})$, where $\phi \equiv \epsilon\phi^*$; the former is forbidden by $U(1)_Y$, the latter by $U(1)_X$ regardless of the charge carried by $\phi$.

Thus, in order to proceed on this tack, we must extend the Higgs sector. Adding a second Higgs doublet $\phi_2$ with hypercharge $-\frac{1}{2}$ would allow the operator $\nu_s\nu_s(\phi_1\phi_2)$, which would generate a small Majorana mass for $\nu_s$ if both $\phi_1$ and $\phi_2$ acquire weak-scale vevs. However, we also need order-unity mixing between active and sterile neutrinos, which requires a dimension-five operator of the form $L\nu_s \times$ (two Higgs fields). Such an operator is forbidden by the $SU(2)_L$ gauge symmetry if the only Higgs fields available are $SU(2)_L$ doublets.

Rather than adding a second Higgs doublet, let’s add a Higgs singlet $S$ – a sterile Higgs. The effective Lagrangian is

$$\mathcal{L} = \frac{c_1}{M}(L\phi)(L\bar{\phi}) + \frac{c_2}{M}\nu_s\nu_sSS + \frac{c_3}{M}(L\phi)\nu_sS + \text{h.c.},$$

where $c_j$ are dimensionless coefficients. The second term requires that the Higgs field carries $U(1)_X$ charge $X_S = -1$. The other terms require that $L$ and $\phi$ carry equal and opposite $U(1)_X$ charge. Also taking into account the Yukawa couplings of the quarks and leptons, we list in Table I the $U(1)_X$ charges of the fields of the SM as well as of the sterile neutrino and sterile Higgs.

There are two free parameters, namely the charges of the quark and lepton doublets relative to the charge of the sterile neutrino (which we have normalized to unity).

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3 All operators considered in this paper are constructed from a pair of left-handed Weyl fermions combined to form a Lorentz scalar, plus additional scalar fields. The $SU(2)_L$ fields contained in parentheses form an $SU(2)_L$ singlet.

4 This Lagrangian may be generalized by adding additional powers of $S/M$ to each of the operators. The resulting neutrino masses would then be much less than the eV scale.
When the sterile Higgs acquires a vev, it breaks the $U(1)_X$ symmetry and generates a Majorana mass for the sterile neutrino via the second term in Eq. (1). Since it is a sterile Higgs field, it does not break the electroweak symmetry. Thus its vev is not required to lie at the weak scale. We only choose it to do so in order to generate sterile neutrino masses in the desired range.

The breaking of the global $U(1)_X$ symmetry produces a Goldstone boson, corresponding to the phase of the sterile Higgs field $S$. This Goldstone boson couples to both active and sterile neutrinos with strength $\frac{1}{\sqrt{2}}$ via the last two interaction terms of Eq. (1). Such a weak coupling means that the Goldstone bosons are not in thermal equilibrium \footnote{Goldstone bosons with stronger couplings can have observable consequences for cosmology \cite{KH, CH, GH, KL}.} during nucleosynthesis. We therefore cannot calculate their contribution to the energy density of the universe, and hence to the expansion rate, during nucleosynthesis. This is in contrast to the sterile neutrinos themselves, which are in thermal equilibrium via their mixing with active neutrinos, and contribute to the density of the universe, and hence to the expansion rate, during nucleosynthesis. This is in contrast to the sterile neutrinos themselves, which are in thermal equilibrium via their mixing with active neutrinos, and contribute to the density of the universe, and hence to the expansion rate, during nucleosynthesis. Thus far we have considered an exact continuous global symmetry to explain why sterile neutrinos are light; however, an exact discrete symmetry could also be employed for this purpose. For example, any $\mathbb{Z}_N$ subgroup ($N > 2$) of $U(1)_X$ would be sufficient to forbid a sterile neutrino bare mass. In this scenario $\nu_s$ carries charge 1 and $S$ carries charge $-1 + \mathbb{N}$, so $\nu_s S = 0 \mod \mathbb{N}$ and the Yukawa interactions of Eq. (1) are allowed. SM particles could also carry nonzero $\mathbb{Z}_N$ charges.

As in the case of a continuous global symmetry, it is widely believed that a discrete symmetry is violated by quantum-gravitational effects. However, unlike the case of a continuous global symmetry, a discrete symmetry can be protected from quantum-gravitational effects if it is a subgroup of a broken gauge symmetry ("discrete gauge symmetry") \cite{EF}. For completeness, we will not restrict our discussion to a discrete gauge symmetry until the end of this section.

The discrete symmetry is broken by the vev of $S$ without producing a Goldstone boson; however, a broken discrete symmetry gives rise to domain walls that are in conflict with cosmological observations \cite{EF}. There are at least two ways to avoid this problem. The first is for the discrete symmetry to be anomalous, thereby lifting the degeneracy among the vacuum states \cite{EF}. However, there are two potential pitfalls with this solution. The first is that if the discrete symmetry is anomalous, then it cannot be a discrete gauge symmetry, and hence it may be violated by...
quantum-gravitational effects. The second is that, as in the case of a continuous global symmetry, it is difficult to interpret an anomalous discrete symmetry as a fundamental symmetry.

The second way to avoid problems with domain walls is for the discrete symmetry to be a discrete gauge symmetry. This solves the problem if the gauge symmetry is broken after the era of inflation. If it is broken before the era of inflation, then there is no mechanism to remove the domain walls [15].

Let us consider a simple anomaly-free discrete symmetry. The simplest candidate is $\mathbb{Z}_3$, with $\nu_5$ carrying charge 1 and $S$ carrying charge 2. However, in addition to the Yukawa interactions of Eq. (1), this symmetry allows the dimension-four operator $\nu_5 \nu_5 S^*$, which would generate a weak-scale mass for the sterile neutrino when $S$ acquires a vev. Thus this model is not viable.

The next simplest candidate is $\mathbb{Z}_4$, where $\nu_5$ carries charge 1 and $S$ carries charge 3. In addition to the Yukawa interactions of Eq. (1), the dimension-five operator $\nu_5 \nu_5 S^* S^*$ is also allowed. This operator also contributes to an eV-scale mass for the sterile neutrino when $S$ acquires a vev. Hence this is the simplest viable model. A single sterile neutrino has a discrete gauge anomaly, so one must add additional sterile neutrinos. The simplest anomaly-free model has two sterile neutrinos with unit $\mathbb{Z}_4$ charge. This satisfies the discrete gravitational anomaly, which requires that the sum of the charges equal 0 mod $N$ (mod $N/2$ for $N$ even) [14, 21]. The other anomaly conditions place no constraint on this model.

The $\mathbb{Z}_4$-symmetric sterile-Higgs potential is given by

$$V = -\mu^2 S^* S + \lambda_1 (S^* S)^2 + \lambda_2 (S^4 + S^{*4})$$

(3)

When the sterile Higgs field acquires a vev, it breaks the $\mathbb{Z}_4$ symmetry and generates sterile neutrino masses and mixing with active neutrinos via Eq. (1) and the additional $\mathbb{Z}_4$-symmetric Yukawa interaction $\nu_5 \nu_5 S^* S^*$ mentioned above.

There are many other possibilities for $N > 4$. Another simple anomaly-free model that yields two eV-scale sterile neutrinos is $\mathbb{Z}_6$ with fermions of charge $(1, 2)$ and a sterile Higgs field of charge 5. Both fermions acquire mass via dimension-five operators, $\psi_1 \bar{\psi}_1 S_5 S_5^*$ and $\psi_2 \bar{\psi}_2 S_1^* S_1^*$. Only $\psi_1$ mixes with the active neutrinos via a dimension-five operator, $(L \phi) \bar{\psi}_1 S_5$. We sketch a simple gauge theory that gives rise to such a $\mathbb{Z}_6$ model in an Appendix.

In this section we have explored the possibilities for explaining the presence of light sterile neutrinos via exact global symmetries, either continuous or discrete. While it is easy to construct such models, they can all be criticized in one way or another. The only model that has no shortcomings, discussed in the paragraphs above, was one based on a discrete gauge symmetry.

III. ACCIDENTAL GLOBAL SYMMETRY

A. $U(1)'$

If $U(1)_X$ were a local rather than a global symmetry, then the absence of a bare Majorana mass term $m\nu_5 \nu_5$ would be ascribed to an accidental flavor symmetry, call it sterile neutrino number, $U(1)_S$, corresponding to $\nu_5 \rightarrow e^{i\theta} \nu_5$. This symmetry is explicitly violated by the last two dimension-five operators of Eq. (1), and is therefore only approximate. This is completely analogous to the case of the active neutrinos, with $U(1)_S$ playing the role of $U(1)_L$. Unfortunately, this rather elegant model has a fatal flaw: the $U(1)_X$ local symmetry is anomalous, as discussed in the previous section. If we choose $X_L = -3X_Q$, then the $U(1)_X$ charges of all SM particles are proportional to their hypercharges, and most anomalies cancel, as discussed above. The only nonvanishing anomalies are the mixed $U(1)_X$-gravitational anomaly and the $[U(1)_X]^3$ anomaly. The contribution of SM particles to these anomalies vanish, so anomaly cancellation must occur in the sterile sector separately.\(^6\)

As in the previous section, simply adding another sterile neutrino with the opposite $U(1)_X$ charge is enough to cancel all anomalies; however, this is not an acceptable solution, as there is no symmetry preventing the two sterile

\(^6\) This conclusion depends on the requirement that neutrino masses and mixings arise from the dimension-five operators of Eq. (1). If one relaxes that constraint, then one may construct $U(1)'$ models in which anomaly cancellation occurs between SM and sterile fermions [21].
neutrinos from pairing up to form a Dirac neutrino of mass $M$. The minimum number of sterile particles needed to both cancel all gauge anomalies and to form a chiral representation of $U(1)_X$ is five \cite{22}. Thus there must exist an entire sterile sector. SM particles may carry $U(1)_X$ charges (proportional to their hypercharges), or they may be $U(1)_X$ singlets (corresponding to $X_L = X_Q = 0$).

The $U(1)_X$ charges of the five sterile particles are not restricted to commensurate values. If we impose such a restriction, the simplest model (in the sense of having the smallest integer charges) is $U(1)_X = (-8, -7, 1, 5, 9)$ \cite{22}. This is much simpler than the solution given in Ref. \cite{22}. We list in Table II the fifty simplest integer solutions; that of Ref. \cite{22} is number 42 in the list. The charges must be commensurate if $U(1)_X$ is a subgroup of a simple gauge group \cite{24}.

In the absence of Yukawa couplings, all five sterile particles are massless, and carry an accidental $[U(1)]^5$ global symmetry. In order to generate masses for the particles, we must explicitly violate this global symmetry with Yukawa couplings, as well as break the $U(1)_X$ gauge symmetry. The pattern of fermion masses depends on the $U(1)_X$ charge of the Higgs field or fields that are chosen.

Let us consider an explicit model as an illustration. We choose the simplest model, with $U(1)_X$ charges \((-8, -7, 1, 5, 9)\). In order to make $\nu_s S$ invariant under $U(1)_X$, we must choose the $U(1)_X$ charge of $S$ to be opposite of one of the five fermions. Let us consider the case where the Higgs field carries charge $-1$. Including terms up to dimension five, the Yukawa interactions allowed by the gauge symmetry are

$$\mathcal{L} = y \psi_9 \psi_{-8} S_{-1} + \frac{c_1}{M} (L \phi) (L \phi) + \frac{c_2}{M} \psi_1 \psi_{-1} S_{-1} + \frac{c_3}{M} (L \phi) \psi_1 S_{-1} + \frac{c_4}{M} \psi_9 \psi_{-7} S_{-1} + \frac{c_5}{M} \psi_5 \psi_{-7} S_{1}^* S_{1}^* + \text{h.c.}, \quad (4)$$

where the subscript on the fields indicate their $U(1)_X$ charge. The field $\psi_1$ plays the role of the sterile neutrino, $\nu_s$, in Eq. (1), and the field $S_{-1}$ plays the role of $S$. When the sterile Higgs field $S_{-1}$ acquires a weak-scale vev, the fields $\psi_9, \psi_{-8}, \psi_5, \psi_{-7}$ form Dirac neutrinos with masses of order $v$ and $\frac{v}{\sqrt{2}}$. Neither of these Dirac neutrinos couples to the active neutrinos. Thus this model yields one light sterile neutrino that mixes with the active neutrinos. The light Dirac neutrino does not contribute in a calculable way to the expansion rate during big bang nucleosynthesis because it is not in thermal equilibrium with the active neutrinos.

Any of the models listed in Table II can be used to generate light sterile neutrinos along the lines above, given a suitable choice of sterile Higgs field(s). However, none of these models is particularly compelling, in that they are not motivated by anything other than the desire to produce naturally light sterile neutrinos.
\[
\begin{array}{ccc}
Q & 2 & 1 \\
Q^c & 1 & 2 \\
L & 2 & 1 \\
L^c & 1 & 2 \\
\end{array}
\]

TABLE III: Particle content and gauge charges of the left-right symmetric model.

### B. SU(2)′

Let us consider replacing the U(1)_X symmetry with SU(2)′. The sterile neutrino must transform nontrivially under this symmetry. In order to avoid having to add yet another particle to the model, let its SU(2)′ partner be the positron field e^c. This is the left-right symmetric model with the gauge group SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \ [25]. The particle content of this anomaly-free model is given in Table III.

In order to generate quark masses, we need a Higgs field Φ in the (2, 2) representation of SU(2)_L \times SU(2)_R, which gives rise to the Yukawa coupling

\[ \mathcal{L} = y_{Q^c} \bar{Q} \Phi Q^c, \]

where Q^c is an SU(2)_R doublet consisting of (u^c, d^c). This coupling generates masses for up-type and down-type quarks when the diagonal components of Φ acquire vevs. Unfortunately, the operator

\[ \mathcal{L} = y_L \bar{L} \Phi L^c \]

is also allowed (where L^c is an SU(2)_R doublet consisting of (ν, e^c)), and generates both charged-lepton and neutrino Dirac masses. Thus there is no symmetry protecting sterile neutrinos from acquiring weak-scale masses in this model.

Let us consider the other possibility, that SU(2)′ is orthogonal to the SM. The gauge group is

\[ SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)' \],

and the SM particles are SU(2)′ singlets. Sterile neutrinos lie in some representation of SU(2)′, all of which are (almost) anomaly-free. Any half-integer representation, χ, of SU(2)′ forbids a bare mass.

Consider a single half-integer SU(2)′ representation. The doublet representation has a global (gauge) anomaly, so it is not a candidate \[26\]. An even number of such representations does not have a global (gauge) anomaly, but allows a mass term \(M^{ij} \chi_i \chi_j\) \((i \neq j)\). Thus the sterile fermions would naturally lie at the high mass scale.

The simplest representation that is free of all anomalies is the spin-3/2 representation, which yields four sterile neutrinos. In addition, one needs a sterile Higgs field, ϕ, also in the spin-3/2 representation of SU(2)′. The effective Lagrangian is

\[ \mathcal{L} = \frac{c_1}{M} (\bar{L} \phi) (\bar{L} \phi) + \frac{c_2}{M} (\bar{L} \phi) (\bar{\chi} \varphi) + \frac{c_3}{M} (\bar{L} \phi) (\bar{\chi} \bar{\varphi}) + \frac{c_4}{M} (\bar{\chi} \varphi) (\bar{\chi} \varphi) + \frac{c_5}{M} (\bar{\chi} \varphi) (\bar{\chi} \varphi) + \frac{c_6}{M} (\bar{\chi} \varphi) (\bar{\chi} \varphi)
\]

\[ + \frac{c_7}{M} (\bar{\chi} \varphi \varphi) + \frac{c_8}{M} (\bar{\chi} \varphi \varphi) + \frac{c_9}{M} (\bar{\chi} \varphi \varphi) + \text{h.c.} \],

where \(\varphi^{ijk} \equiv \epsilon^{ijk} \epsilon^{klm} \varphi_{lmn}\). This model has been analyzed in Ref. \[22\]. It yields two light sterile neutrinos that mix with the active neutrinos, as desired. There are also two other light sterile neutrinos that do not mix with the active neutrinos, due to an unbroken \(\mathbb{Z}_3\) symmetry.

\[ \text{7 We do not impose any additional discrete symmetries.} \]

\[ \text{8 SU(2) representations are free of gauge anomalies but may suffer from a global (gauge) anomaly} \ [26]. \text{We take this into account shortly.} \]

\[ \text{9 More generally, one may consider a model with an odd number of spin-3/2 representations. Although a mass term} \ M^{ij} \chi_i \chi_j \ (i \neq j) \text{is allowed, an antisymmetric mass matrix always has one zero eigenvalue if it is odd dimensional. Thus there would be one light spin-3/2 representation.} \]

\[ \text{10 The SU(2)′ invariants that appear are of the form} \ (\chi \phi) = \chi^{ijk} \phi_{ijk} \text{and} \ (\chi \chi \varphi \varphi) = \chi^{ijk} \chi^{lmn} \varphi_{ijk} \varphi_{lmn}. \]

\[ \text{11 This is an example of a discrete gauge symmetry. However, in this model it is not being used to produce a light sterile neutrino, in contrast to our discussion in Section II B.} \]
From here, the model-building possibilities are endless. Any model of the form $G_{44} \times G'$, where $G'$ is a chiral, anomaly-free gauge group, can potentially yield light sterile neutrinos along the lines of the $G' = U(1)_{X}$ and $G' = SU(2)'$ models discussed above. A variety of models of this type have been analyzed in Ref. [27]. Perhaps the most compelling of these models are the mirror models, $G' = SU(3)'_{C} \times SU(2)'_{Y} \times U(1)'_{Y}$ [28, 29]. While the original versions of these models have been excluded, variations may still be viable [30, 31]. These models yield three light sterile neutrinos. As mentioned in the Introduction, big bang nucleosynthesis constrains the number of light neutrinos. Although it may be possible to circumvent this constraint, there is a preference for models with only a few additional neutrinos beyond the three active ones.\textsuperscript{12}

IV. CONCLUSIONS

We have taken a bottom-up approach to models with naturally light sterile neutrinos. We have argued that there must be a symmetry responsible for the lightness of such particles. We considered two classes of symmetry. One is an exact global symmetry, broken at the weak scale. We found that such models are easy to construct. However, there are serious doubts that exact global symmetries exist in nature, due to quantum-gravitational effects. The only model we found that evades this criticism is based on a discrete $\mathbb{Z}_{N}$ symmetry. If this symmetry is a discrete gauge symmetry, then it is not violated by quantum gravity. The simplest example we found is a $\mathbb{Z}_{4}$ model with two sterile neutrinos of unit charge. The $\mathbb{Z}_{4}$ symmetry is broken at the weak scale by a sterile Higgs field of charge 3. We also discussed a $\mathbb{Z}_{6}$ model with sterile neutrinos of charge $(1, 2)$, and a sterile Higgs field of charge 5.

The other class of models is based on chiral gauge theories that gives rise to accidental global symmetries, analogous to $U(1)_{L}$ in the standard model. All the models we considered are of the form $G_{44} \times G'$. For $G' = U(1)'_{X}$, the simplest anomaly-free model has five sterile fermions, all of which are candidates for light sterile neutrinos, depending on how the $U(1)'_{X}$ gauge symmetry is broken. We gave an example where only one light sterile neutrino is generated that mixes with the active neutrinos. We also discussed the case $G' = SU(2)'_{Y}$, which has been analyzed in Ref. [27].

In summary, the most attractive models of light sterile neutrinos are based on extensions of the standard-model gauge symmetry, either by a discrete or a continuous gauge symmetry. Thus the confirmation of the existence of light sterile neutrinos would be evidence of a sterile sector of particles and forces.

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APPENDIX A: DISCRETE GAUGE SYMMETRY AND $U(1)_{X}$

Although we followed a bottom-up approach throughout this paper, it is an interesting question whether the $\mathbb{Z}_{N}$ models studied in Section II have been obtained from a broken gauge symmetry (discrete gauge symmetry). Rather than addressing this question in general, we ask the more specific question of what $\mathbb{Z}_{N}$ models arise if we break the $U(1)_{X}$ gauge models of Section III A at the high scale via a Higgs field of charge N.

As discussed in Section II the simplest candidate for a discrete gauge symmetry is $\mathbb{Z}_{4}$ with two fermions of charge unity. However, the $U(1)_{X}$ models listed in Table II lead either to four particles with unit $\mathbb{Z}_{4}$ charge, or to a vectorlike representation of $\mathbb{Z}_{4}$. Thus the $\mathbb{Z}_{4}$ model discussed in Section II cannot arise from any of the $U(1)_{X}$ models listed in Table II (that is, models with five commensurate charges). This does not imply, however, that it cannot arise from some other broken gauge theory.

The other simple model we discussed in Section II is $\mathbb{Z}_{6}$ with two fermions of charges $(1, 2)$. This model can result from the $U(1)_{X}$ models listed in Table II. Let’s consider the simplest $U(1)_{X}$ model with charges $(8, 7, -1, -5, -9)$ (the conjugate of the first model listed in Table II). If we break the $U(1)_{X}$ symmetry at the high scale via a Higgs field of

\textsuperscript{12} For a recent review of neutrinos in cosmology, see Ref. [52].
charge 6, then $\psi_{-1}$ and a linear combination of $\psi_7$ and $\psi_{-5}$ pair up to form a massive Dirac fermion and $\psi_{-9}$ acquires a heavy Majorana mass. The orthogonal combination of $\psi_7$ and $\psi_{-5}$ as well as $\psi_8$ remain massless, and these two particles indeed possess $\mathbb{Z}_6$ charges (1, 2), respectively.

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