The SVZ-Expansion and Beyond

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I discuss the standard SVZ-expansion of the QCD two-point correlators in terms of the QCD vacuum condensates and beyond it due to a new unflavoured $1/Q^2$-term (tachyonic gluon mass squared $\lambda^2$) which modelizes the effects of uncalculated higher order perturbative terms. The approach is confronted with low-energy and lattice data. One can notice that, in the different examples studied here, high-dimension condensates are expected to deviate largely from their vacuum saturation (large $N_c$) values, while the new $1/Q^2$-term due to $\lambda^2$ solves the hadronic mass scale hierarchy-puzzle encountered in the early SVZ-sum rule analysis and improves the low-energy phenomenology. From tau-decay data, one can extract the running strange quark mass $m_s(2\text{ GeV}) = (93^{+29}_{-32})$ MeV and a more accurate value of the QCD coupling $\alpha_s(M_Z) = 0.117^{+0.002}_{-0.002}$.

1. Introduction

The SVZ approach [1] (for a review, see e.g. [2]), where the non-perturbative effects are approximated by the contributions of QCD condensates, continues to explain successfully the low-energy hadron phenomenology. Among the different predictions, one has been able to predict the $\rho$-meson mass and coupling by the introduction of the gluon condensate $\langle \alpha_s G^2 \rangle$ [1], while more recently, one has extracted with a high accuracy (after runned until $M_Z$) the QCD coupling $\alpha_s$ using tau-decay data [3]. However, in the beginning, most of QCD perturbative practioners, have not been enthousiastic on these successes due to the relative small mass scale at which, one a priori, (naïvely) expects pQCD to breakdown. The SVZ approach has also been used as a serious alternative and guide to the lattice calculations, both approaches being based on the same uses of the QCD fundamental Lagrangian and related parameters. However, unlike lattice QCD, the accuracy of the SVZ-sum rule is limited and cannot (a priori) be iteratively improved being an approximate scheme. One of the limitation of the approach is the absence of our knowledge of the complete pQCD series. The aim of this talk is, firstly, to review the different values of the QCD condensates extracted from low-energy and lattice data, and, secondly, to present a string-inspired model (tachyonic gluon mass) [4,5] inducing a new $1/Q^2$-term in the OPE which can mimic the unknown higher order short-distance terms of the pQCD series. Consequences of the model on the well-established hadron phenomenology [5–7], on the determination of the strange quark mass from tau-decays [8] will be reviewed. New result on the effect on the accurate determination of $\alpha_s$ from tau decays will be presented.

2. SVZ-expansion and QCD condensates

For a pedagogical introduction, let’s consider the generic two-point hadronic correlator:

$$\Pi(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T J(x) (J(0)) | 0 \rangle$$

(1)

built from the local hadronic current $J(x)$. Following SVZ, the correlator can be approximated by a sum of power corrections:

$$\Pi(Q^2) \approx \sum_{d \geq 2} \frac{C_{2d} \langle O_{2d} \rangle}{(-q^2)^d}$$

(2)

where: $\langle O_{2d} \rangle$ are the QCD non-perturbative condensates of dimension $D = 2d$; $C_{2d}$ is the associated perturbative Wilson coefficient; $q^2 \equiv -(Q^2 > 0)$ is the momentum transfer. Owing to gauge invariance, there is no $D = 2$ term in the chiral limit.
Table 1

| “Standard” Condensates | Dimension D | Values $[\text{GeV}]^D$ |
|-------------------------|-------------|------------------------|
| $(\bar{s}s)/(\bar{u}u)$ (normal ordered) | $3-3=0$ | $0.66 \pm 0.10$ |
| $\langle \alpha_s G^2 \rangle$ | 4 | $(7.1 \pm 0.9) \times 10^{-2}$ |
| $g\langle \bar{\psi} G_a \mu \nu \sigma_{\mu \nu} \psi \rangle \equiv M^2_0 \langle \bar{\psi} \psi \rangle$ | 5 | $M^2_0 = (0.8 \pm 0.1) \text{ GeV}^2$ |
| $\langle g^3 f_{abc}G^a G^b G^c \rangle$ | 6 | $\approx 1.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$ |
| $\langle \bar{\psi} \Gamma_1 \psi \Gamma_2 \psi \rangle$ | 6 | violation of factorization by 2–3 |
| $\langle GGGG \rangle$ | 8 | violation of factorization by $\geq 4$ |

1 Fo the non-normal ordered condensate which possesses a small perturbative piece, the ratio is $0.75 \pm 0.12$.

$m_j = 0$, because the possible $D = 2$ term $\langle A_\mu A^\mu \rangle$ is a priori gauge dependent (see, however [9]). The well-known condensate is the chiral condensate: $\langle \bar{\psi} \psi \rangle$ entering into the GMOR-PCAC relation: 

$$
\langle m_u + m_d \rangle \langle \bar{\psi} \psi \rangle = -f^2 \langle \bar{\psi} \psi \rangle : f = 93 \text{ MeV}.
$$

We show in Table 1 the ratio of the normal ordered condensates $(\bar{s}s)/(\bar{d}d)$, indicating large SU(3) breakings. Less-known condensates are the SVZ-condensates entering into the OPE:

- The Gluon Condensate $\langle \alpha_s G^a \mu \nu G_a^\mu \nu \rangle$, initially introduced by SVZ [1] and estimated phenomenologically by different groups in the literature [1,2,10–17] including lattice calculations [18–20] with a spread of results. However, its value is found to be definitely non-zero and positive. Some of the results including the SVZ value do not satisfy the lower bound derived by Bell and Bertlmann [13] from moment sum rules. Recent estimates in [17] satisfying this bound from light and heavy quark channels are given in Table 1.

- The Mixed Quark-Gluon condensate $g\langle \bar{\psi} G_a \mu \nu \sigma_{\mu \nu} \psi \rangle \simeq M^2_0 \langle \bar{\psi} \psi \rangle$, where $M^2_0$ has been estimated from baryon sum rules [21,22], the $B-B^*$ mass-splittings [23], string model [24] and lattice calculations, with a fair agreement.

- The Four-quark Condensates $\langle \bar{\psi} \Gamma_1 \psi \bar{\psi} \Gamma_2 \psi \rangle$ estimated from different channels [15,2] and lead to a violation of the factorization assumption (large $N_c$ approximation) by at least a factor 2.

- The three gluon condensate $\langle g^3 f_{abc} G^a G^b G^c \rangle$, originally estimated by SVZ using instanton liquid model and confirmed later on by lattice calculations [18].

- The dimension $D = 8$ and higher dimension condensates estimated in the $V + A$ and $P + S$ [7] channels using experimental and lattice data [25].

More recently, condensates of the $V - A$ channel have been estimated from $\tau$-decay data [26] and from QCD at large $N_c$ [27]. Large violations of the vacuum saturation estimates of these condensates have been also observed.

The results from the different estimates of these condensates are summarized in Table 1. Tests of these results from other methods such as lattice with dynamical quarks are mandatory.

3. $1/Q^2$-term beyond the SVZ-expansion

A possible existence of this $1/Q^2$-term as an eventual source of errors in the determination of $\alpha_s$ from tau-decay data has been advocated in [28]. This conjecture has been checked from a fit of the $e^+e^- \rightarrow$ hadrons data [6] which allow the existence of a small but imaginary constituent quark mass:

$$
m^2_q = -(71 \sim 114)^2 \text{ MeV}^2 ,
$$

after the identification of the result with the quark mass correction of $-6m^2/Q^2$ in this channel. Later on, it has been found more convenient to identify this term with the tachyonic gluon mass squared $\lambda^2$ [5].

One possible origin of this term might be the $\langle A_\mu A^\mu \rangle$ condensate which has been found to be non-zero [4,29], but gauge dependent. However, there are some proposals that its minimal value is gauge invariant over the gauge orbit [9]. In this talk, I shall consider that the $1/Q^2$-term is purely of short distance nature and can mainly emerge from the resummation of the infinite terms of the pQCD series (UV renormalon). From the form of the Cornell potential, one can relate the string tension to the squared gluon mass $\lambda^2$ [4]:

$$
V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r \Rightarrow \sigma \approx -\frac{2}{3} \Gamma_0 \lambda^2
$$

(4)
giving $\lambda^2 \approx - (0.5 \sim 1.0) \text{GeV}^2$. A possible indication of the existence of this term comes from the lattice calculation of the plaquette $\langle G^2 \rangle$, which can be written as:

$$
\langle G^2 \rangle \approx \left( \frac{N_c^2 - 1}{a^4} \right) \left[ \sum_n c_n^2 \alpha_s^n + d_n^2 \alpha_s^2 + c_4 \Lambda^4 \right] \tag{5}
$$

where $a$ is the lattice spacing; $\Lambda$ is the QCD scale; $c_n$, $d_n$ are log- or numerical coefficients. In the present model, $d_n^2$ is expected to decrease for increasing $n$. In fact, for $n \leq 11$, the fit of the lattice data requires a quadratic term [19], while for large $n \geq 26$ this term disappears [20]. In the analytic approach where the pQCD series is at best known to $O(\alpha_s^4)$ [30], we then expect that $\lambda^2$ plays an essential role for quantifying the unknown higher order terms. In practice, one can systematically introduce the UV $\lambda^2$-effect by replacing the gluon propagator at short distance [5]:

$$
D^{\mu\nu}(k^2) = \frac{\delta^{\mu\nu}}{k^2} \rightarrow \delta^{\mu\nu} \left( \frac{1}{k^2} + \frac{\lambda^2}{k^4} \right),
$$

where, this operation is gauge invariant to leading order.  

4. Estimate of the tachyonic gluon mass squared $\lambda^2$

- **From $e^+e^- \rightarrow \text{hadrons data}**

The contribution of the tachyonic gluon mass squared $\lambda^2$ to the vector correlator is [5,32]:

$$
\Pi_\rho(M^2) = \Pi^0 \left[ 1 + \left( \frac{\alpha_s}{\pi} \right) \left( 1 - 0.05 \frac{\lambda^2}{M^2} \right) \right] \tag{7}
$$

Using a ratio of exponential moments (less sensitive to $\alpha_s$):

$$
R \equiv -\frac{d}{d\tau} \log \int_0^\infty dt \ e^{-\tau} \frac{1}{\pi} \text{Im} \Pi(t) \tag{8}
$$

$$(\tau \equiv 1/M^2 \text{ sum rule variable}), \text{an earlier fit of the data for a postulate existence of a} \; "1/M^2\text{-term}" \text{ interpreted in term of } \lambda^2 \text{ leads to [6]}:

$$
\lambda^2(1 \text{ GeV}) \approx -(0.2 - 0.5) \text{ GeV}^2 \implies \alpha_s \lambda^2 \approx -(0.03 - 0.09) \text{ GeV}^2, \tag{9}
$$

with: $\alpha_s \equiv (\alpha_s/\pi)(1 \text{ GeV}) \approx 0.17 \pm 0.02$.

- **From the pion channel**

We use again the ratio of moments $R$ in order to eliminate $\alpha_s$ and quark mass terms to leading order. We also use the Pion pole + ChPT parametrization of $3\tau$ contributions+QCD continuum of the spectral function [33]. Then, we deduce the value of $\lambda^2$ in Table 2. One can notice that the introduction of $\lambda^2$ has enlarged the stability/QCD duality region to larger $\tau$ (lower $M^2$) -values which reduces the distance from the pion pole, which is a success of the approach. Taking the average of the previous results from $e^+e^-$ and $\pi$-channel leads to:

$$
(\alpha_s/\pi)\lambda^2 \approx -(0.07 \pm 0.03) \text{ GeV}^2 \implies \lambda^2(\tau \approx 1 \text{ GeV}^{-2}) \approx -(0.41 \pm 0.18) \text{ GeV}^2 \tag{10}
$$

5. Hadronic scale puzzle and $\lambda^2$

In the sum rules with $\lambda^2 = 0$, one has the optimization scale hierarchy:

$$
M_{\rho\rightarrow\text{canal}} \ll M_{\pi\rightarrow\text{canal}} \ll M_{\text{gluonium\rightarrow\text{canal}}}. \tag{11}
$$

There are various ways to get these results [34,5]:

- Detailed sum rules analysis or/and 10% correction criterion leads to:

$$
M_{\rho\rightarrow\text{canal}} \approx (0.6 \sim 0.8) \text{ GeV}^2
$$

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Table 2

$\lambda^2$ from the pion channel

| $\tau$(GeV$^{-2}$) | $\mathcal{L}_{exp}/(2f^2 \pi^2 m^4)$ | $R_{exp}$ | $R^{\lambda=0}_{QCD}$ | $-\alpha_s \lambda^2$(GeV$^2$) |
|-------------------|-------------------------------|----------|-----------------|-----------------|
| 1.4               | 1.20                          | 0.27 ± 0.06 | 0.66 ±0.50     | 0.09 ±0.08     |
| 1.2               | 1.29                          | 0.34 ± 0.07 | 0.78 ±0.42     | 0.10 ±0.09     |
| 1.0               | 1.42                          | 0.46 ± 0.10 | 0.93 ±0.34     | 0.11 ±0.06     |
| 0.8               | 1.61                          | 0.58 ± 0.12 | 1.12 ±0.28     | 0.12 ±0.06     |
| 0.6               | 1.89                          | 0.73 ± 0.15 | 1.24 ±0.17     | 0.12 ±0.06     |
| 0.4               | 2.29                          | 0.89 ± 0.16 | 1.33 ±0.13     | 0.10 ±0.06     |

---

3A generalization of this procedure to the whole range of $k^2$ has been also proposed by 't Hooft [31] in an exploratory analytic approach to confinement.
\[ \approx \left( \frac{10\pi}{3}\alpha_s G^2 \right)^{1/2}, \quad (12) \]

indicating the breaking of Asymptotic Freedom (AF) by infrared phenomena!

- Using the positivity of the pseudoscalar spectral function, one can deduce:

\[ M_{\pi-\text{canal}}^2 \geq \sqrt{\frac{16\pi^2}{3} \frac{f_\pi^2 m_\pi^4}{(m_u + m_d)^2}} \approx 1.8 \text{ GeV}^2, \quad (13) \]

where the factor \( m_\pi/(m_u + m_d) \approx 13 \) brings a large numerical factor! We have seen in the previous analysis that the presence of \( \lambda^2 \) leads to \(^4\):

\[ M_{\pi-\text{canal}}^2 \approx M_{\rho-\text{canal}}^2 \approx 1 \text{ GeV}^2. \quad (14) \]

- The \( I = 1 \) \( a_0 \) scalar meson channel has the same scale as the pseudoscalar one because the alone difference comes from the \( D = 6 \) condensate contribution which is a small correction. Contrary to the case of instanton, the \( \lambda^2 \) contribution is not affected by chirality which is the main difference between the two approaches that can be checked by accurate lattice measurements. One has to remind that the sum rule with \( \lambda^2 = 0 \) reproduces quite well the \( a_0 \) phenomenology \([2,36]\).

- In the scalar glueball channel, the uses of the substracted sum rule plus a low-energy theorem (LET) for \( \Pi_G(0) \) give:

\[ \Pi_G(M^2) \approx \Pi^0_G \left[ 1 + \left( \frac{8\pi}{-\beta_1} \right) \left( \frac{\pi}{\alpha_s} \right)^2 \left( \frac{\alpha_s G^2}{M^4} \right) \right], \quad (15) \]

where \( -\beta_1 = 9/2 \) for 3 flavours, and LET brings a huge factor of about 400 in front of \( 1/M^4 \) \([34]\):

\[ M_{\rho-\text{gluonium}}^2 \approx 20 M_{\pi-\text{canal}}^2 \approx 15 \text{ GeV}^2, \quad (16) \]

which indicates that it is difficult to interpret the breaking of AF in terms of resonances! The inclusion of the \( \lambda^2 \) term leads to:

\[ \Pi_G(M^2) = \Pi^0_G \left( 1 - \frac{3\lambda^2}{M^2} + \ldots \right). \quad (17) \]

Thus, the \( \lambda^2 \) correction is expected to be large due to the absence of an extra power of \( \alpha_s \). A 10% correction like in the \( \rho \)-case gives:

\[ M_{\rho-\text{gluonium}}^2 \approx 13 \text{ GeV}^2, \quad (18) \]

\(^4\)For an alternative explanation using diquarks (see e.g. \([35]\)). In amusing agreement with the independent estimate in Eq. (16). More detailed phenomenology of the (pseudo)scalar gluonia channels lead to a lower value \([37,38]\):

\[ M_{0^+\text{-gluonium}}^2 \approx (3 \sim 5) \text{ GeV}^2 \quad (19) \]

which is still much larger than \( M_{0^+}^2 \), indicating the particularity of this channel. Also contrary to the instanton approach, where the \( 2^{++} \) channel is not affected by instanton \([39]\), the contribution of \( \lambda^2 \) is similar to the (pseudo)scalar channels indicating an universality of the gluonium scales. Large scale is also expected for the hybrid channel as \( \lambda^2 \) does not also have an extra power of \( \alpha_s \) \([40]\).

For an illustration of the previous discussions both for the “standard condensates” and for

![Figure 1. Lattice data of the S+P correlator: the dotted curve is the SVZ value (SET 1); the dot-dashed curve is the prediction for SET 3 where the contribution of the \( x^2 \)-term has been added to SET 2; the bold dashed curve is SET 3 + a fitted value of the \( D = 8 \) condensate contributions (\( \approx (x/0.58)^8 \) compared to \( (x/1.2)^8 \) from factorization); the diamond curve is the prediction from the instanton liquid model of \([39]\).]
Table 3
Different sets of the power corrections values

| Sources | $\langle \alpha_s G^2 \rangle$ | $\alpha_s \langle \psi \psi \rangle^2$ | $(\alpha_s / \pi)\lambda^2$ |
|---------|-----------------|-----------------|---------------------|
| SET 1[1] | 0.04            | 0.255           | 0                   |
| SET 2 [17] | 0.07            | $5.8 \times 10^{-4}$ | 0                   |
| SET 3[5,6] | 0.07            | $5.8 \times 10^{-4}$ | $-0.12$           |

the $\lambda^2$-contribution, we consider explicitly the scalar+pseudoscalar correlators $S + P$ in x-space where the single instanton effect cancels out:

$$R_{P+S} = \frac{1}{2} \left( \frac{\Pi^P}{\Pi^{pert}} + \frac{\Pi^S}{\Pi^{pert}} \right) \rightarrow 1 - \frac{\alpha_s}{2\pi} \lambda^2 x^2$$

$$+ \frac{\pi}{96} (\alpha_s (G_{\mu\nu}^a)^2) x^4 + \frac{4\pi^3}{81} \alpha_s (\bar{q}q)^2 x^6 \ln x^2$$

(20)

Lattice data on $R_{P+S}$ are available [25] and we show the analysis in Fig. 1. It is clear from this figure that the set of parameters (SET 3) given in Table 1 and in Eq. (10) give a much better fit of the data.

7. $\lambda^2$ and the light quark masses

• In the (pseudo)scalar channels, $\lambda^2$ decreases the estimate of the light quark mass by about 5-6% [5], implying the value to order $\alpha_s^3$ [2,41]:

$$\overline{m}_{u+d}(2 \text{ GeV}) = (8.6 \pm 2.1) \text{ MeV} \ .$$

(21)

Using the ChPT ratio: $2m_s/m_d + m_u = 24.4 \pm 1.5$[42], one can deduce:

$$\overline{m}_s(2 \text{ GeV}) = (105 \pm 26) \text{ MeV} \ .$$

(22)

• For the extraction of the strange quark mass from $\tau$-decay, one works with a suitable combination of finite energy sum rules for the $\Delta S = -1$ component of $\tau$-decay [8]:

$$S_{10} = \int_0^{t_c} \frac{1}{t_c} dt \left( 1 + \frac{\bar{t}}{t_c} \right) \frac{1}{\pi} \text{Im} \Pi_{V+A}^{(0+1)} ,$$

(23)

which is directly sensitive to $m_s^2$ and to $\lambda^2$ to leading order and which has a convergent PT series (optimal $t_c \approx M_s^2$). One finds that $m_s$ increases with $|\lambda^2|$. For the value of $\lambda^2$ given in Eq. (10), one obtains:

$$\bar{m}_s = (106_{-37}^{+33}) \text{ MeV} \Rightarrow$$

$$\overline{m}_s(2 \text{ GeV}) = (93_{-32}^{+29}) \text{ MeV} \ .$$

(24)

As one can see in Table 4, the value of $\lambda^2$ = 0 which leads to a too small value of $m_s$ is excluded by the lower bound from (pseudo)scalar channels [43] updated to order $\alpha_s^3$ and including $\lambda^2$ to be $(71.4 \pm 3.7)$ MeV in [2,41], and by the bound of about 80 MeV from the direct extractions of the quark condensate [44].

Table 4
Invariant mass $\bar{m}_s$ from the tau-decays

| $\alpha_s \lambda^2$ in GeV$^2$ | $\bar{m}_s$ in MeV |
|-----------------|-----------------|
| 0.03            | 56 ± 28 ± 8     |
| 0.06            | 81 ± 21 ± 9     |
| 0.07            | 89 ± 21 ± 9     |
| 0.09            | 102 ± 18 ± 11   |
| 0.12            | 118 ± 16 ± 11   |
| 0.15            | 132 ± 15 ± 11   |
| 0.18            | 145 ± 13 ± 12   |

Combining the previous values in Eqs. (22) and (24) with the value:

$$\overline{m}_s(2 \text{ GeV}) = (81 \pm 22) \text{ MeV}$$

(25)

obtained from tau-decays using the difference of the $\Delta S = -1$ with the $\Delta S = 0$ spectral functions [45], one can deduce the average:

$$\langle \overline{m}_s(2 \text{ GeV}) \rangle = (92 \pm 15) \text{ MeV} ,$$

(26)

which can be compared with lattice results [47].

8. $\lambda^2$ and the value of $\alpha_s$ from tau decay

Including $\lambda^2$, the modified expression of the tau hadronic width is [3]:

$$R_{\tau} = 3 \left( |V_{ud}|^2 + |V_{us}|^2 \right) S_{EW}^7 \times \left( 1 + \delta_{EW} + \delta_{P} + \delta_{\tau}^s + \delta_{\tau}^A + \delta_{NP} \right) ,$$

(27)

where: $|V_{ud}| = 0.9751 \pm 0.0006$ and $|V_{us}| = 0.221 \pm 0.003$ are the weak mixing angles; $S_{EW}^7 = 1.0194$

$\lambda^2$ is negligible.

$\lambda^2$ is negligible.

We have not included the result from $e^+e^- \rightarrow$ hadrons data [46], which we are revisiting.
and $\delta_{EW}^f = 0.0010$ are the electroweak corrections; $\delta_{\Gamma_T}^f = a_s + 5.2023a_s^2 + 26.366a_s^3 + ...$ is the pQCD series using Fixed Order Perturbation Theories (FOPT); ... are higher order uncalculated terms which, at present, are model dependent. In [3,49], it has been taken to be $\approx \pm 130a_s^4 \approx \pm 0.020$, which is the main source of theoretical error. In the present approach, this term is replaced by the tachyonic gluon correction [5]:

$$\delta_\lambda^T \simeq -2 \times 1.05 \frac{a_s \lambda^2}{M_Z^2} \simeq 0.0465 \pm 0.0199.$$  (28)

which has about the same size as the $\alpha_s^3$ correction! One can notice that one does not gain much on the precision but the origin of the error is more justified and can be improved; $\delta_\lambda^T$ is the tiny running light quark masses corrections and $\delta_{NP}^T = -(2.8 \pm 0.6) \times 10^{-2}$ is the sum of non-perturbative effects of operators of dimension $D \geq 4$ [17]. Using the average of the hadronic width from the tau-lifetime and leptonic widths:

$$\langle R_\tau^{\text{exp}} \rangle = 3.634 \pm 0.004,$$  (29)

one can deduce: $\delta_{\Gamma_T}^f \simeq 0.1688 \pm 0.0199$. Then:

$$\alpha_s(M_T) = 0.252 \pm 0.025(\text{ContourCoupling})[48] \pm 0.0022 \text{ (FOPT)}[3],$$  (30)

giving the average:

$$\alpha_s(M_T) = 0.313 \pm 0.017,$$  (31)

which, runned until $M_Z$ becomes:

$$\alpha_s(M_Z) = 0.117 \pm 0.002.$$  (32)

Compared to previous estimates [3,48–50], the presence of $\lambda^2$ shifts the central value of $\alpha_s(M_Z)$ from 0.121 to 0.117, while the source of the error is understood and is slightly improved here. This number agrees quite well with the existing world average $0.1187 \pm 0.0020$ given by PDG [49].

9. Conclusions

We have shortly reviewed the different estimates of power corrections entering into the SVZ-expansion, and discussed the phenomenology implied by the existence of a tachyonic gluon mass $\lambda^2$, which induces a new $1/Q^2$ term in the OPE. We found that the size of higher dimension condensates deviates notably from the vacuum saturation assumption valid at large $N_c$, while $\lambda^2$ improves the low-energy phenomenology. We plan to explore its effect in the heavy quark sector [51].

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