NEUTRINOS AND GRAVITATIONAL WAVES FROM COSMOLOGICAL GAMMA-RAY BURSTS

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Abstract

Cosmological gamma ray bursts are very likely powerful sources of high energy neutrinos and gravitational waves. The aim of this paper is to review and update the current predictions about the intensity of emission in this two forms to be expected from GRB’s. In particular a revised calculation of the neutrino emission by photohadronic interaction at the internal shock is obtained by numerical integration, including both the resonant and the hadronization channels. The detectability of gravitational waves from individual bursts could be difficult for presently planned detectors if the GRB’s are beamed, but it is possible, as we have proposed in a paper two years ago, that the incoherent superimposition of small amplitude pulse trains of GW’s impinging on the detector, could be detected as an excess of noise in the full VIRGO detector, integrating over a time of the order of one year.
1 Introduction

The Gamma Ray Bursts (GRB) are intense flashes of soft \(\gamma\)-rays, serendipitously discovered by the Vela test ban treaty (Klebesadel et al. 1973) and immediately confirmed by the Soviet Konus satellites (Mazets et al. 1974). The phenomenology of GRB’s has been discussed in several reviews (see e.g. Djorgovski et al. (2001) and references therein). The most important breakthrough in the GRB’s observations has been the discovery in 1997 of a long lasting X-ray “afterglow” of the burst GRB970228 by the Beppo-Sax Italian-Dutch satellite (Costa et al. 1997), which has allowed the identification of its host galaxy (van Paradijs et al. 1997, Sahu et al. 1997), making the study of GRB’s a “multi-wavelengths adventure” (Klose 2000).

According to a widely accepted (and acceptable) interpretation, the prompt \(\gamma\)-rays burst and the delayed afterglow, are both powered by the dissipation of the kinetic energy of a relativistically expanding “fireball” (or jet), whose primal cause is not known. This mechanism, qualitatively suggested by Cavallo & Rees (1978), has been refined by Paczynski (1986) and put in its final form of a standard model by Rees & Meszaros (1994) (for a recent review see e.g. Mészáros (2002) and references therein). The obvious expectation is that such a powerful highly relativistic explosion should be accompanied by emission of neutrinos and gravitational waves, thus becoming the herald of the new astronomy.

The aim of this paper is to review and update the current predictions about the intensity of emission in this two forms to be expected from GRB’s. In the following I will first review in §2 and §3 the essential quantitative information on the energetics and the large scale distribution of the sources, which has been obtained from prompt and afterglows observations. In §4 I present a new calculation of the photohadronic neutrino emissivity in the fireball/jet system from which predictions about the observability of the diffuse neutrino background can be derived. In §5 I reexamine the possibility that a similar stochastic background could be observed as an additional noise component in the long term integration of the signal from gravitational wave detectors.

2 Energetics

The total energetics involved in the GRB’s is the most important parameter for my subsequent emissivity estimate. A direct estimate of the rest frame energy output can be obtained only for bursts with known redshift. According to a compilation by J. Greiner, available at http://www.mpe.mpg.de/~jcg/grbrsh.html, the number of GRB with reliable redshift measurement of the host galaxy redshift, is about 20.

The intrinsic isotropic bolometric (from 0.1 keV to 10 MeV) energy release, estimated from the fluence of 17 burst in the 30 keV -2 MeV range, spans from \(5.6 \times 10^{51}\) erg (GRB990712) to \(2 \times 10^{54}\) erg (GRB990123), with a median of \((E_\gamma^{iso}) = 1.8 \times 10^{53}\) erg and a dispersion of 0.8 dex(r.m.s.) (Bloom et al. 2001). However, it is to be emphasized that this value applies only to GRBs with measured redshift. Therefore observational biases in burst detection and redshift determination, might obscure the true underlying average energy output of the typical GRB. However, keeping in mind this caveat, one can assume that the
kinetic energy input to the expanding relativistic fireball is of the order of

$$\langle E_{\text{iso}} \rangle \approx 1 \left( \frac{20\%}{\eta_\gamma} \right) M_\odot c^2$$

where \( \eta_\gamma \) is the fraction of the fireball kinetic energy that is converted into \( \gamma \)-rays (Guetta et al. 2001).

The afterglow light curve of many bursts shows an achromatic break, currently interpreted as the evidence for a beamed emission (Sari et al. 1999, Rhoads 1999). In this case the emitted \( \gamma \)-ray energy \( \mathcal{E}_{\gamma}^{\text{jet}} \) is obviously reduced by the factor (Frail et al. 2001)

$$\mathcal{E}_{\gamma}^{\text{jet}} = f_{\text{jet}} \mathcal{E}_{\gamma}^{\text{iso}}$$

where \( f_{\text{jet}} = 1 - \cos\theta_{\text{jet}} \). In a recent paper (Bloom et al. 2003) present a complete sample of 29 gamma-ray bursts (GRBs) for which it has been possible to determine temporal breaks (or limits) from their afterglow light curves. Incorporating realistic estimates of the ambient density and propagating error estimates on the measured quantities, in agreement with the previous analysis of a smaller sample, the derived jet opening angles of those 29 bursts result in a narrow clustering of geometrically corrected gamma-ray energies about \( \langle \mathcal{E}_{\gamma}^{\text{jet}} \rangle = 1.33 \times 10^{51} \) ergs, with a burst-to-burst variance of 0.34 dex.

Three consequences can be drawn, if this is true:

1. The central engines of GRBs release energies of the order of

$$\mathcal{E}_0 \approx \text{few} \times 10^{-2} \left( \frac{20\%}{\eta_\gamma} \right) M_\odot c^2$$

This value is indeed comparable to the kinetic energy output of ordinary SNe, suggesting a connection between the two phenomena, also supported by the observation of bumps in the afterglows light curves (Bloom 2003);

2. The large spread in observed fluence and peak luminosity of GRBs is mostly due to a spread in the opening angle of the beam;

3. Only a small fraction of GRB’s are visible to a given observer and the true GRB’s rate is

$$R_{\text{GRB}}^{\text{jet}} = R_{\text{GRB}}^{\text{iso}} \times \langle f_{\text{jet}}^{-1} \rangle$$

In practice from Table 2 of the paper by Bloom et al. (2003) one can guess that the true GRB’s rate should be about 300 times larger than the observed one.

It is clear that such a radical descoping of the energetics associated with the GRB’s explosion, has a fundamental impact on the physical interpretation, but has little or no impact, at the first order, in the predictions of their high energy neutrino emission. In fact we expect that the aperture of the beam of neutrinos emitted by photohadronic processes will be the same of the beam of \( \gamma \)-rays emitted by electrons.

On the contrary the gravitational wave emission by GRB, if present, will be in any case isotropic. We observe that the beaming could reduce the probability of detecting individual events, because the amount of energy to be radiated is strongly reduced. But this should have no impact on the prediction of a possible cosmological background originated by the GRB, because in case of beaming we should assume that the rate of events is increased by the same factor. Therefore
we have that the total energy released is in the case of beaming will be in the average
\[ E_{\text{jet}}^{GW} \times R_{\text{jet}}^{GW} = E_{\text{iso}}^{GW} \times R_{\text{iso}}^{GRB} \]

As we said already the above given estimate can be applied, only to the bursts with observed afterglow. Since afterglows have been observed only for long lasting bursts with duration \( T_{90\%} \geq 2 \) s, it could be argued that the short bursts could be powered by a totally different central engine, whose energetic could be on a lower scale. Some indications in this direction is given by the fact that the observed fluence of the short bursts is significantly lower then the one of long bursts (Tavani 1998). However one could argue back that the difference in the average fluence could be due to a markedly different large scale distribution of the two classes of bursts. In the absence of a any direct distance estimator, only statistical indicators of the spatial distribution of GRB can be used to test this hypothesis. A recent analysis of the 4B burst catalog (Schmidt 2001) shows that the short bursts have essentially the same characteristic peak luminosity of the long ones, but their local space density is around three times lower. This applies to bursts shorter then 0.25 s, therefore the total energy output of the short bursts should be 1-2 order of magnitude lower then the one of the long burst.

3 Large scale distribution

The common wisdom about the large scale distribution of the bursts is that, being a phenomenon originated by the collapse of short lived massive stars, they should track closely the Star Formation Rate (SFR) cosmological distribution. In a recent paper Porciani & Madau (2000) have used three different SFR derived from other astronomical data to fit the observed log N-Log P distribution for the bursts. Their results imply about 1-2 GRB’s per million SN type II and a characteristic isotropic luminosity of the bursts in the range \( 3 - 20 \times 10^{51} \) ergs s\(^{-1}\). The best fitted models of the rate per unit comoving volume are shown in Fig. 1.

The luminosity-variability relation (Fenimore & Ramirez-Ruiz 2000) is a new entirely empirical connection, which has been proposed in analogy to the well understood luminosity-period relationship of the cepheids. Having calibrated over a sample of 8 GRB’s with known redshift, the authors have applied this fit to derive the redshifts for 220 bursts. Applying non-parametric statistical techniques to the 220 Gamma-Ray Burst (GRB) redshifts and luminosities derived in this way, it has been proposed (Lloyd-Romning et al. 2001) that there exists a significant correlation between the GRB intrinsic luminosity and redshift, which can be parameterized as \( (\Omega_\gamma/4\pi) L_\gamma \propto (1 + z)^{1.4 \pm 0.5} \), where \( z \) is the burst redshift. In addition, this analysis supports a co-moving rate density of GRBs that continues to increase for \((1 + z) > 10\). We have plotted in the Fig. 1 the different possible large scale distribution of the GRB’s, which is compatible with the observed log N-log P distribution.
Figure 1:

4 High energy neutrinos

In the general framework of the relativistic fireball model of GRB’s, the prompt \( \gamma \)-ray emission is originated by synchrotron emission of relativistic electrons accelerated at the internal shocks, while the afterglow is due to the interaction of the external shock with the ambient material (Piran 2000). The radius of the inner shock (or shocks in case of multi-peaked emission) should be of the order of \( R_s \approx 2 \Gamma_b^2 c \Delta t \), where \( \Delta t \) is the smallest timescale observed in the burst emission, usually in the range 1-10 ms.

A lower limit to the bulk Lorentz factor of the expanding fireball can be derived from the \( \gamma \gamma \rightarrow e^+ e^- \) self absorption in the burst at the highest observed \( \gamma \)-ray energy \( \epsilon_{\gamma}^{\text{max}} \approx 100 \) MeV. In fact given the threshold of the reaction \( 2 m_e c^2 \approx 1 \) MeV we must have \( \Gamma_b \geq \epsilon_{\gamma}^{\text{max}} / \sqrt{2 m_e c^2} \approx 100 \). A more detailed calculation (Lithwick & Sari 2001) shows that the optical depth is \( \tau_{\gamma \gamma} \propto \Gamma_b^{-4+2 \alpha} \), where \( \alpha \) is the low energy spectral index of the prompt \( \gamma \)-ray emission. Therefore a bulk Lorentz factor \( \Gamma_b \gtrsim 300 \) is required for the emission of \( \sim 100 \) MeV \( \gamma \)-rays.

It has been shown (Vietri 1995, Waxman 1995) that the high magnetic fields and relativistic velocities of the internal shocks, can accelerate protons up to \( 10^{19} - 10^{20} \) eV, corresponding to the range of the UHE cosmic rays. From first principles one can derive the rate of acceleration from the relativistic shock \( \dot{\epsilon}_p / \epsilon_p \approx \frac{m_p}{\epsilon_p} B \Gamma_b^2 \), thus even in a very short time, smaller than the coherence time of the \( \gamma \)-ray emission, a huge maximum energy

\[
\ln \left( \frac{\epsilon_p^{\text{max}}}{\epsilon_p^{\text{min}}} \right) \approx \frac{e}{m_p} B \Gamma_b \Delta t
\]

can be reached.

However the shock acceleration mechanism is efficient only when the radius of the shock is larger then the Larmor radius \( r_L \approx c \epsilon_p / e B \) of the accelerated particle. Therefore the more stringent limit will be \( r_L \lesssim R_s \), that assuming \( R_s \approx 2 \epsilon_p / \epsilon \approx 2 B \Gamma_b^2 \Delta t \) gives \( \epsilon_p^{\text{max}} \lesssim \epsilon \approx 10^{19} \) eV with field of the order \( \sim 1 \) T. The typical fields required in the burst for electron synchrotron emission peaked in \( \sim 100 \) keV range are \( \gg 1 \) T.
Therefore the maximum energy imparted to the proton will be limited, in practice, by proton synchrotron radiation and inelastic scattering over the radiation field. The latter interaction will copiously produce pions, that subsequently decay into neutrinos and TeV $\gamma$-rays. The pion yield

$$\text{Pion Yield} \approx 4 \times 10^{-5} \text{ mbarn}$$

for the reaction $p\gamma \rightarrow \pi^\pm X$ at the internal shock has been calculated, under simplifying assumptions, by several authors (see e.g. Guetta et al. (2003) and references therein). All this previous calculations have included only the photihadronic production near threshold, where the photon scatters quasi-elastically with the nucleon that is pushed into an excited state corresponding to $N^+$ and $\Delta^+$ baryonic resonances:

$$p\gamma \rightarrow \{ N^+ \Delta^+ \} \rightarrow \{ n \pi^+ \text{p}\pi^0 \}$$

These resonances are produced with high cross sections ($\sim 100 - 500 \mu\text{barn}$) (Hagiwara et al. 2002). In addition to these resonant channels, pions will be also produced at intermediate energies by the hadronization of the photon $\gamma \rightarrow q\bar{q}$. The reaction proceed in practice as

$$\gamma p \rightarrow (q\bar{q})p \rightarrow N, \pi^\pm, \pi^0, \cdots$$

The importance of considering both processes, and not only the resonant channels, in photihadronic pion production calculations has been already stressed in the literature (Mucke et al. 1999). In addition to the arguments given in that
paper, one should consider that the photons which interact via the resonant channel should have energies in the range
\[
\Gamma^2_b \frac{M^2_{N+\Delta^+} - m_p^2}{4\epsilon_p} \leq \epsilon_\gamma \leq \epsilon^{\max}_\gamma
\] (3)

In Fig. 2 we have plotted the cross section versus the energies, in the observer’s frame, of the photon and the \(\gamma\)-ray. If the low-energy spectral index is \(\alpha < 0\), protons with \(\epsilon_p \geq 10^{13} \text{ eV}\) will produce baryonic resonances scattering on photons with \(\epsilon_\gamma \leq 1 \text{ keV}\). Therefore we expect that the resonant photoproduction at high energies will depend strongly on the shape of the \(\gamma\)-ray spectrum, in a range well below the observed one.

The theoretical expectation from the electron synchrotron emission (Sari & Piran 1999) is that the spectrum will level off due to electron cooling at energies \(\epsilon_\gamma \lesssim 100 \text{ eV}\) and be strongly suppressed by self-absorption for \(\epsilon_\gamma \lesssim 10^{-3} \text{ eV}\). Following this indication we have assumed a photon energy distribution given by the form empirically proposed by Band et al. (1993), modified to take into account the cooling:

\[
\phi(\epsilon_\gamma) = A \begin{cases} 
E_\gamma^\alpha e^{(\beta-\alpha)E_\gamma/E_b} & \text{if } \epsilon_\gamma \leq E_c \\
\epsilon_\gamma^\alpha e^{(\beta-\alpha)\epsilon_\gamma/E_b} & \text{if } \epsilon_\gamma < E_b \\
\epsilon_\gamma^\beta E_b^{\alpha-\beta} e^{\beta-\alpha} & \text{if } \epsilon_\gamma \geq E_b
\end{cases}
\] (4)
in the range \(10^{-3} \text{ eV} \leq \epsilon_\gamma \leq 100 \text{ MeV}\) and zero outside this range. The constant \(A\) has been chosen in order to normalize to 1 the function \(\psi(\epsilon_\gamma)\) in the above given range. Under these assumptions the photon density at the source will be

\[
n_\gamma \simeq \frac{L_\gamma \Delta t}{4\pi R_s^2 \Delta R \langle \epsilon_\gamma \rangle} \phi(\epsilon_\gamma)
\] (5)

where \(R_s \simeq 2c \Gamma^2_b \Delta t\).

The full expression for the m.f.p. of a proton with energy \(\epsilon_p\) in a radiation field with photon energies \(\epsilon^{\min}_\gamma \leq \epsilon_\gamma \leq \epsilon^{\max}_\gamma\) in observer’s frame is given by

\[
\lambda^{-1}_{p\gamma} = \frac{m_p^2 + 4\epsilon_p \epsilon^{\max}_\gamma / \Gamma^2_b}{4\epsilon_p} \int_{m_p^2}^{\epsilon^{\max}_\gamma} \sigma_{p\gamma}(s) ds
\] (5)

where, \(\sigma_{p\gamma}\) is given by the sum of Breit-Wigner resonances and hadronization cross sections

\[
\sigma_{p\gamma}(s) \simeq \sum_{k=N^+,\Delta^+} \sigma_{p\gamma}^{\max} \frac{1/4 \Gamma_k^2}{(\sqrt{s} - M_k)^2 + 1/4 \Gamma_k^2} + \sigma_h(s)
\] (7)

, we can calculate the average number of inelastic interactions per proton

\[
\eta_{p}(\epsilon_p) = \frac{\Delta R}{\lambda^{-1}_{p\gamma}(\epsilon_p)} = \frac{L_\gamma \lambda^{-1}_{p\gamma}}{16\pi c^2 \Gamma^4_b \Delta t \langle \epsilon_\gamma \rangle}
\] (8)
The probability of having one or more proton-photon inelastic scattering is
\[ P_\pi = 1 - \exp(\eta_\pi) \] which is \( P_\pi \approx \eta_\pi \) only when \( \eta_\pi \ll 1 \). This is not the case, specially in the hadronization channel, because the inelasticity of the interaction is of the order of \( k_{\text{inel}} \simeq 40\% \). Therefore one has to take into account that the proton actually starts an small hadronic cascade in the considered layer. We

\begin{align}
\eta'_\pi(\xi) &= \sum_{n=1}^{n_{\text{max}}} \xi^{-\gamma n} \bar{n}_\pi(\xi^n \epsilon_\pi) \{ 1 - \exp[\eta_\pi(\xi^n \epsilon_\pi)] \} \\
\end{align}

where is \( \xi = 1/(1-k_{\text{inel}}) \), \( \gamma \) the spectral index of the proton spectrum, and \( \bar{n}_\pi \) is either the average multiplicity of the multiple pion production for the hadronization channel or the branching ratios in the case of the resonant channel. In Fig. 3 we report the fraction of energy lost by protons to pions, assuming a photon energy distribution given by Eq. (4), with low-energy spectral index \( \alpha = -1 \), high-energy index \( \beta = -2.25 \), a break energy \( E_b = 511 \text{ keV} \), and a cooling energy \( E_c = 100 \text{ eV} \). For the fireball’s parameters it is assumed \( L_\gamma = 10^{52} \text{ erg s}^{-1}, \Gamma_b = 300 \) and \( \Delta t = 10 \text{ ms} \).

Figure 3: Fraction of energy lost by protons to pions, assuming a photon energy distribution given by Eq. (4), with low-energy spectral index \( \alpha = -1 \), high-energy index \( \beta = -2.25 \), a break energy \( E_b = 511 \text{ keV} \), and a cooling energy \( E_c = 100 \text{ eV} \). For the fireball’s parameters it is assumed \( L_\gamma = 10^{52} \text{ erg s}^{-1}, \Gamma_b = 300 \) and \( \Delta t = 10 \text{ ms} \).

have integrated numerically Eq. (6) including all the resonant channels and the continuum hadronization cross section, approximated by a polynomial logarithmic fit (Cudell et al. 2000). We have also approximately taken into account the possibility of multiple interactions, simply summing over the histories. In practice we have considered that the pions of a given energy \( \epsilon_\pi \) could have been produced either by a proton of energy \( \epsilon_p \) or by a proton which suffered \( n \) previous scattering, with \( n \leq \log(\epsilon_p/\epsilon_\pi)/\log(1-k_{\text{inel}}) \). Thus we have in practice:

\[ \eta'_\pi(\xi) = \sum_{n=1}^{n_{\text{max}}} \xi^{-\gamma n} \bar{n}_\pi(\xi^n \epsilon_\pi) \{ 1 - \exp[\eta_\pi(\xi^n \epsilon_\pi)] \} \]
Figure 4: Typical neutrino spectrum for a burst with $E_p = 2.2 \times 10^{52}$ erg with spectrum $\propto \epsilon_p^{-2}$ for $10^{10} \text{eV} < \epsilon_p < 10^{20} \text{eV}$, exploding at $z = 1$ in a Universe with $H_0 = 71 \text{km s}^{-1}\text{Mpc}^{-1}$, $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$ and $w = -1$ (see text). The fireball parameters are the same of Fig. 3. This hypothetical burst (assuming $E_e = E_p$ and $f_\gamma = 0.04$) would have a fluence at Earth in the 20-2000 keV band of $3 \times 10^{-7}$ erg cm$^{-2}$.

energy distribution for the neutrino produced in the decay chain

$$\pi \rightarrow \nu_\mu \mu \quad \text{and} \quad \mu \rightarrow \bar{\nu}_\mu \, e \, \nu_e \quad (10)$$

In both cases it is important to take into account the pion (and muon) energy loss, before decaying. This has been done solving analytically the coupled equations for decay and energy loss, which gives as a result when the dominant energy loss is the synchrotron emission:

$$\eta_{\nu_\mu} (\epsilon_{\nu_\mu}) = \frac{m_\pi}{2 \epsilon_{\nu_\mu}} \int_{\epsilon_{\pi}^2}^{\epsilon_{\pi}^\text{max}} \eta_{\pi} (\epsilon_{\pi}) \left( \frac{\epsilon_{\pi}}{\epsilon_{\pi}} \right)^2 e^{-\frac{\epsilon_{\pi}^2 - \epsilon_{\nu_\mu}^2}{2 \epsilon_{\pi}^2 (\epsilon_{\pi}/\epsilon_{\nu_\mu})^2}} \frac{d\epsilon_{\pi}}{\epsilon_{\pi}} \quad (11)$$

where $\epsilon_{\nu_\mu}^* = (m_\pi^2 - m_\mu^2) / 2m_\pi$ is the energy of the neutrino in the muon rest frame, and $\epsilon_{\pi}$ is the energy of the pion for which the rate of energy loss is equal to the decay time. For $\epsilon_{\pi} \gg \epsilon_{\pi c}$ Eq. (11) gives the expected behavior $\propto (\epsilon_{\pi}/\epsilon_{\pi c})^{-2}$, while for $\epsilon_{\pi} \approx \epsilon_{\pi c}$ a pile up peak is generated. We can derive the spectrum of the muons from the same formula with the only difference that the integration limits are in this case $\frac{m_\pi \epsilon_{\pi c}}{2 \epsilon_{\pi c} (1+\beta^*)} \leq \epsilon_{\pi} \leq \frac{m_\mu \epsilon_{\mu c}}{2 \epsilon_{\mu c} (1-\beta^*)}$ where $\epsilon_{\mu c}^* = \sqrt{m_\mu^2 + \epsilon_{\nu_\mu}^2}$ is the energy of the muon in the pion rest frame and $\beta^*$ its velocity. In the muon decay the energy loss before decay will be more severe because the decay constant of the muon is longer (implying $\epsilon_{\mu c} << \epsilon_{\pi c}$), but
Figure 5: Neutrino spectrum for different bulk Lorentz factor keeping all the other parameters constant.

we can use a formula equivalent to Eq. (11). For the sake of simplicity we have calculated the combined neutrinos and anti-neutrinos flux, because in the multiple pion production it can be assumed that positive and negative pions are equally produced. The marked difference between the spectrum of muon neutrinos and the one of the electronic neutrinos is due to the fact that the first type of neutrinos are directly produced in the pion’s decay, while the electronic ones are produced only in the muon’s decay, which have suffered a larger energy loss. In Fig. 4 we show for comparison the neutrino flux calculated by Guetta et al. (2003), which is also in good agreement with this result. In Fig. 5 we report our prediction for the expected neutrino flux from the photopionic channel, for different values of the bulk Lorentz factor. It is to be remarked that I do not find in the final flux estimate the strong dependance $\propto \Gamma_b^{-4}$ which is argued by many authors (see e.g. Guetta et al. (2003)). The reason is that even if the average number of inelastic collision $\eta_{\pi}$ calculated by Eq. (8) is $\propto \Gamma_b^{-4}$, the pion yield is $\eta_{\pi}' \propto \eta_{\pi}$ only when $\eta_{\pi} \ll 1$. As we said above when it becomes $\eta_{\pi} \gtrsim 1$ one should take into account the multiple regenerative interactions, which take place at decreasing energies, as has been taken into account using Eq. (11). In Fig. 6 we report the diffuse cosmological background flux of neutrinos obtained from our emissivity estimate and the different large scale distribution models discussed in §3. The neutrino fluence at Earth of a burst with redshift $z$ has been estimated as

$$\epsilon_{\nu}^2 F_{\nu}(z, \epsilon_{\nu}) = \frac{\eta_{\nu}'((1+z)\epsilon_{\nu})}{4\pi (1+z)d_L^2(z)} \frac{\mathcal{E}_p}{\ln(\epsilon_p^{\max}/\epsilon_p^{\min})}$$

(12)

where $d_L(z)$ is the cosmological luminosity distance, $\mathcal{E}_p$ the total accelerated proton energy with spectral index $-2$, and $\eta_{\nu}'$ is given by Eq. (11). The diffuse
Figure 6: Diffuse neutrino flux for the different models of GRB’s large scale distributions shown in Fig. 4 for a cosmological model with the same parameters of Fig. 4. The normalization of the curves has been obtained assuming an average total energy for accelerated protons in the burst rest frame of $E_p = 10^{52}$ erg. The band shown in this figure corresponds to ±0.8 dex variation of this average.

The flux of neutrinos has been calculated integrating

$$
\Phi_{\nu}(E_{\nu}) = \int_0^\infty \frac{R_{GRB}(z)}{(1+z)} F_{\nu}(z,E_{\nu}) \frac{dV}{dz} dz
$$

where

$$
\frac{dV}{dz} = \frac{c}{(1+z)^2} \frac{dL(z)}{dz}
$$

being for Friedmann-Robertson-Walker metric (Hagiwara et al. 2002)

$$
H(z) = H_0 \sqrt{\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_V(1+z)^{-3(1+w)}}
$$

with $\Omega_K = 1 - \Omega_M - \Omega_V$.

Finally in Fig. 7 we report our prediction for the fluence of the $\gamma$-rays with $E_{\gamma} > 100$ GeV produced by neutral pion decays. This mechanism has been discussed in the literature (for a recent review see Fragile et al. (2002) and references therein) as a possible source for VHE $\gamma$-rays potentially detectable by ground based experiments. It is well known that only one statistically significant detection has been reported until now by Atkins et al. (2003) using the Milagrito detector. This type of signal is expected to be detectable only for very energetic and nearby GRB’s for two reason: 1) the strong self absorption due to $\gamma\gamma \rightarrow e^+e^-$ interactions with the soft $\gamma$-ray radiation field in the source, and 2) the absorption of the VHE photons by the cosmic microwave background photon.
due to the same process. The first process is responsible for the suppression of the emission below $\approx 1 - 10$ TeV, while the second originates the cut-off at 1 PeV.

5 Gravitational waves

Prompt gravitational waves emission has been predicted from GRB since 1993 by Kochanek & Piran (1993) at a level to be detected by upcoming gravitational wave-experiments. As we have discussed in §2, the estimate of the kinetic energy required to power a beamed $\gamma$-ray burst is of the order of $E_0 \gtrsim 0.01 M_\odot c^2$, much larger then the total nuclear binding energy of few-$M_\odot$'s stars. Therefore in any case the energetic of the bursts is compatible only with the collapse of a several solar masses object into a black hole (Woosley 1993). Any reasonable assumption regarding angular momentum then leads to a massive accretion disk that flows, on a viscous time scale, into a black hole that very rapidly approaches the Kerr limit. The dissipation the rotational and gravitational energy within the geometry of a thick torus is very likely to lead to jets, possibly by the purely electromagnetic mechanism proposed by Blandford & Znajek (1977). The energy irradiated in this case as gravitational waves (Stark & Piran 1985) is

$$\mathcal{E}_{GW} = \epsilon^4 \left( \frac{m}{M_{BH}} \right)^2 M_{BH} c^2 \tag{16}$$

where $m$ is the mass of the torus, $M_{BH}$ is the mass of the black hole, and $\epsilon$ is the ellipticity of the torus. It is worth noticing that if the jets are accelerated by Poynting flux (Drenkhahn 2001) the amount of kinetic energy going into the burst $E_0$ and the energy radiated in gravitational waves are strictly related,
because the GW luminosity will be \( L_{GW} \propto \langle Q_{ij} Q^{ij} \rangle \), being \( Q \) the quadrupole moment of the mass distribution of the torus, while the Poynting flux will be \( L_{\text{e.m.}} \propto \langle M_{ij} M^{ij} \rangle \) being in this case \( M \) the quadrupole moment of the magnetic field embedded in the torus. Therefore we assume that the energy radiated as gravitational radiation is \( E_{GW} = \eta_{GW} E_0 \) where \( \eta_{GW} \lesssim 1 \) is an unknown parameter.

The energy flux (viz. energy per unit surface and unit time) of GW produced by a burst of intrinsic luminosity (in gravitational waves) \( L_{GW} \) at a red shift \( z \) is by definition \( \Phi_{GW} = L_{GW} / 4\pi d_L^2(z) \) if \( d_L \) is the luminosity distance. In order to convert this flux into an adimensional amplitude we can use the classical formula (Shapiro & Teukolsky 1983):

\[
\Phi_{GW} = \frac{c^3}{16\pi G} \left\langle \frac{1}{2} \dot{h}_+^2 + \dot{h}_x^2 \right\rangle
\]  

where the average is taken over several wavelengths. The amplitude of the signal produced depends from the direction and the beam pattern of the detector. In the best case we have integrating over time and applying the Parseval’s theorem

\[
\int_\omega \omega^2 \tilde{h}^2(\omega) \, d\omega = \frac{16\pi G (1 + z) E_{GW}}{c^3} \frac{E_0}{4\pi d_L^2(z)}
\]

where \( \tilde{h}^2(\omega) \) will be the Rayleigh power of the signal as a function of the frequency \( \omega \). In order to estimate the order of magnitude of the amplitude of the GW signal we do not need a detailed shape of the spectral power density of the signal, but only the knowledge of the first two moments of the distribution, viz. \( \bar{\omega} \) and \( \Delta \omega \). In fact we can recast Eq. (18) in the form

\[
(\bar{\omega}^2 + \Delta \omega^2) \int_\omega \tilde{h}^2(\omega) \, d\omega = \frac{16\pi G (1 + z) E_{GW}}{c^3} \frac{E_0}{4\pi d_L^2(z)}
\]

It is remarkable that rather natural physical assumption on the first and second moment of the unknown Rayleigh power distribution \( \tilde{h}^2(\omega) \) can be made. In fact we can assume that the first moment will be twice the keplerian angular velocity of the marginally stable orbit around the BH, namely

\[
\bar{\omega} = 2 \sqrt{\frac{2G_N M_{BH}}{\beta^3 r_S^3}} \approx 2 \pi 10^3 \left( \frac{7M_\odot}{M_{BH}} \right) \left( \frac{2.5}{\beta} \right)^{3/2} \text{rad s}^{-1}
\]

where \( r_S \) will be the Schwarzschild radius of the collapsed object, which is function only of the black hole mass, and \( 1 \leq \beta \leq 3 \) depends on the angular momentum of the BH, being minimal for maximally rotating BH, and maximal for non-rotating ones.

The second moment will be the r.m.s. bandwidth of a wave packet that can be estimated \( \Delta \omega \approx 2\pi / \Delta t \) where \( \Delta t \) is the timescale of the emission in the comoving frame. As we have seen in the \( \gamma \)-ray emission has a variability over a timescale \( \Delta t \approx 10 \text{ ms} \), therefore it is rather natural to assume that the timescale of the GW emission by the torus will be of the same order of magnitude. In
this case we have, if $c \Delta t \gg r_S$, for the peak amplitude

$$d_L(z) \tilde{h}_{\text{peak}} \approx \sqrt{\frac{2 \beta^3 G_N}{\pi e^9} M_{BH} \sqrt{E_{GW} \Delta t}} \quad (20)$$

The expected amplitude for typical values of beamed bursts is

$$d_L \tilde{h}_{\text{peak}} \approx 10^{-26} \left( \frac{M_{BH}}{7 M_\odot} \right) \left( \frac{\mathcal{E}_0}{6.5 \times 10^{51} \text{ erg}} \frac{\eta_{GW} \Delta t}{0.01 \text{ ms}} \right)^{\frac{1}{2}} \text{ Gpc}/\sqrt{\text{Hz}}$$

compatible with our previous estimate (Auriemma 2001). The predicted signal by Coward et al. (2002) for a “gentle chirp” spectral model, is well in agreement with the above value, when scaled by a factor $10^5$ to take into account the different assumption on the distance and the total energy, which are in that paper respectively $d_L = 100$ Mpc and $E_{GW} = 3.75 \times 10^{53}$ erg. The current upper limit for the search for emission of GW in correlations with GRB with bar detectors (Tricarico et al. 2001, Astone et al. 2002) has given the upper limit $h_{\text{RMS}} \leq 1.5 \times 10^{-18}$. It is clear from this estimate that the probability of detecting a single burst, even under optimal is very low, unless $\eta_{GW} \gg 1$. However even if the individual burst could not be detected, it is to be remarked that in case of beaming the rate of explosion could be very large ($\approx 1500$ per day), therefore the stochastic accumulation of signal integrated over a long time could emerge from the noise, as we have shown in our previous paper (Auriemma 2001). The same type of calculation, but with more optimistic assumptions on the energetic of the source, as we said before, has been repeated by Coward et al. (2002). The energy flux of GW produced at Earth from a cosmological distribution of sources can be calculated integrating over the cosmological distribution, as we have done for the neutrino in the Eq. (12), namely

$$\Phi_{GW}^{\text{diff}} = \int_0^{z_{\text{max}}} R_{\text{GRB}}(z) \frac{(1 + z) E_{GW}}{4 \pi d_L^2(z)} \frac{dV}{dz} dz \quad (21)$$

where $dV/dz$ is given by Eq. (14). Applying again Eq. (17) we have:

$$\frac{c^3}{16 \pi G} \left \langle h_+^2 + h_\times^2 \right \rangle = \frac{c}{4 \pi} \int_0^{z_{\text{max}}} R_{\text{GRB}}(z) \frac{E_{GW}}{(1 + z)^2 H(z)} dz \quad (22)$$

On the L.H.S. of this equation we have the amplitude of the wave that invests at a certain instant the detector. We have seen in the previous section that each of this burst will not have a detectable intensity, but if we average over an observation time $T$ long compared to the GW burst duration but short compared to Hubble time scale (typically one year) we have a signal

$$\frac{1}{T} \int_{-\infty}^{+\infty} \omega^2 \tilde{h}^2(\omega) d\omega = \frac{8 G}{c^2} \int_0^{z_{\text{max}}} R_{\text{GRB}}(z) E_{GW} \frac{dV}{dz} dz \quad (23)$$

that will be detectable if the power spectral density is greater then the power spectral of the noise, averaged over the same observation time. The uncorrelated superimposition of bursts of gravitational waves will be well approximated, for
the central limit theorem, by the superimposition of redshifted gaussian distributions. Therefore the power spectral density of the signal can be estimated by the integral

$$
\langle \tilde{h}^2(\omega) \rangle_T \approx \frac{8 G}{c^2 \omega^2} \int_0^{z_{\text{max}}} e^{-\frac{(1+z)\omega_0 - \omega)^2}{2 \sigma^2}} \frac{R^{\text{jet}}_{GRB}(z) E_{GW}(1+z)^2 H(z)}{(1+z)^2} \, dz
$$

(24)

where \( \omega_0 \) will the first momentum of the distribution of the frequencies \( \bar{\omega}(M_{BH}) \) given by Eq. (19), namely \( \omega_0 = \int M_i \bar{\omega}(M_{BH}) \psi(M_{BH}) \, dM_{BH} \) where \( \psi(M_{BH}) \) is the normalized mass distribution function for the black holes, and \( \sigma_\omega \) the second momentum. In the lack of any better guess we will assume that \( \psi(M_{BH}) \) is flat for \( 4 M_\odot \leq M_{BH} \leq 14 M_\odot \), thus having \( \omega_0 = 2\pi \times 10^3 \text{ rad s}^{-1} \) and \( \sigma_\omega = 2\pi \times 200 \text{ rad s}^{-1} \).

Finally we have reported in Fig. 8 the Rayleigh power of the stochastic signal from the cosmological background for \( \eta_{GW} = 0.01 \). For comparison we have also reported the noise expected in the VIRGO experiment (Cuoco et al. 1998) averaged over one year of integration time.

6 Discussion

The two central results of this paper are illustrated by Fig. 6 and Fig. 8, where estimates of the neutrino and gravitational waves diffuse background are reported. the diffuse neutrino background has been calculated accurately including many details that were not considered in previous calculations. However the
final estimate given in this paper is in very reasonable agreement with those of previous calculations, indicating that the order of magnitude predictions were rather robust. The comparison of this predictions with the best upper limits obtained so far by AMANDA for the northern sky, indicates that an increase in sensitivity of 2-3 order of magnitude will be required in order to set a stringent constraint on the more plausible models of neutrino emission by cosmological GRB’s. On the other side the detection of extremely powerful bursts of neutrinos in a km$^2$ detector cannot be ruled out, given the extreme variability of the energetic among the various bursts.

In §5 it has been shown that the GW emission from single bursts at cosmological distances, if the $\gamma$-ray prompt emission is beamed with a small angle as suggested by afterglow observations, is expected to be $d_L \dot{h} \lesssim 10^{-27}$ Gpc/$\sqrt{\text{Hz}}$ which is well below the detection threshold of presently planned experiments. However if the emission is beamed only a small fraction (one over 300) of the GRB’s are observed by $\gamma$-ray satellites. This implies that small amplitude pulse trains of GW’s impinge over the detector at a frequency that could be as high as half per minute, which is practically a continuous signal. Even if the individual pulse is small, integrating over a reasonable observation time (order of one year) an excess Rayleigh power should emerge from the instrumental noise. The frequency spectrum of this eventual excess should give a direct information on the characteristic time scale of the collapse and on the cosmological evolution of the GRB rate in the recent past ($z \approx 1 - 5$). The predicted amplitude is conservatively estimated to be in the range of $5 \times 10^{-26} 1/\sqrt{\text{Hz}}$ averaging the Rayleigh power over one year. This estimate is rather robust because does not depends on the beaming factor and depends only slightly from the large scale distribution of the GRB sources. The cosmological origin of the excess noise could be proved by detecting a dipole anisotropy. In addition the auto correlation spectrum of the noise shall carry an imprint of the characteristic duration of the GW pulse trains. The detection of those two signatures, even if perhaps not possible with presently planned experiments, can give the important additional evidence of the cosmological origin of the stochastic signal and informations on the physics of GRB’s.

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